Quantum Interference and Phase Mixing in Multistream Plasmas

M. Akbari-Moghanjoughi

Faculty of Sciences, Department of Physics,
Azarbaijan Shahid Madani University, 51745-406 Tabriz, Iran
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Abstract

In this paper the kinetic corrected Schrödinger-Poisson model is used to obtain the pseudoforce system in order to study variety of streaming electron beam-plasmon interaction effects. The noninteracting stream model is used to investigate the quantum electron beam interference and electron fluid Aharonov-Bohm effects. The model is further extended to interacting two-stream quantum fluid model in order to investigate the orbital quasiparticle velocity, acceleration and streaming power. It is shown that quantum phase mixing in the two-stream model is due to quasiparticle conduction band overlap caused by the Doppler shift in streaming electron de Broglie wavenumbers, a phenomenon which is also known to be a cause for two-stream plasma instability. However, in this case the phase mixing leads to some novel phenomena like stream merging and backscattering. To show the effectiveness of model, it is used to investigate the electron beam-phonon and electron beam-lattice interactions in different beam, ion and lattice parametric configurations. Current density of beam is studied in spatially stable and damping quasiparticle orbital for different symmetric and asymmetric momentum-density arrangements. These basic models may be helpful in better understanding of quantum phase mixing and scattering at quantum level and can be elaborated to study electromagnetic electron beam-plasmon interactions in complex quantum plasmas.

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I. INTRODUCTION

Plasmons are high frequency elementary quantized excitations of plasma electron oscillations [1, 2]. They play inevitable role in many fundamental properties of plasmas semiconductors and metallic nanoparticles from electric and heat transport phenomena to optical and dielectric response, etc. [3, 4]. Dynamics of these quantized electromagnetic quasiparticles make an ideal platform for miniaturization of ultrafast terahertz device communications [5], where conventional integrated circuits fail to operate. They also have numerous other interesting applications in nanotechnology [6], plasmonics [7–9], optoelectronics [10], etc. for engineering low-dimensional nano-fabricated semiconductor industry [11–13]. Energy conversion by plasmons is a new way of solar power extraction due to its high efficiency in photovoltaic and catalytic devices. Use of the collective oscillations of electrons instead of single particles makes huge amount of energy extraction in an operation step in plasmonic solar devices [14, 15]. Collective oscillations in local surface plasmon resonance (LSPR) [16] process by surface electrons which are so called hot electrons are collectively excited by electromagnetic radiations in UV-VIS range. The hot electron current are collected in an appropriate contacts of nanoparticle surfaces by an efficient electron collecting material like TiO$_2$ in Schottky configuration [17].

Collective charge screening effect which manifests itself as the characteristic optical edge in metallic surfaces already have may applications in metallic alloys making then optically unique among other solids. Collective electron excitations rule almost every aspect of solid from optical to dielectric response in plasmas [18, 20] and condensed matter. Recent infrared spectroscopic techniques shows that Low dimensional semiconductors [21] such as gapped graphene also demonstrate interesting surface plasmon effects. The collective electron transport property of graphene makes it an ideal element for multilayer composite devices such as compact ultrafast switches, optical modulators, optical lattices, photodetectors, tandem solar cells and biosensors [22–24]. The first theoretical development of the idea of collective electron excitations by Bohm and Pines dates back to mid-nineties, when they coined plasmon name for such excitations due to the long-range electromagnetic nature of interactions [25–29]. The theoretical as well as experimental aspects of collective electron dynamics in quantum level has been the subject of intense investigations over the past few decades [30–35], due to its fundamental importance in many field of physics and chemistry.
Pioneering developments of quantum statistical and kinetic theories [36–39] had a long tradition furnishing a pavement for modern theories of quantum plasmas [40–43]. Many interesting new aspects of collective quantum effects in astrophysical and laboratory plasmas has been recently investigated using quantum plasma theories [44–57]. The quantum kinetic theories like time-dependent density functional theories (TDFT) are, however, less analytic as compared to the quantum hydrodynamic analogues, due mostly to mathematical complexity which require large scale computational programming. Recent investigation reveals [58] that quantum hydrodynamic approaches based on the density functional formalism [43] can reach beyond the previously thought kinetic limitations, such as the collisionless damping if accurately formulated. One of the most effective hydrodynamic formalism for studying the quantum aspects of plasmas is the Schrödinger-Poisson model [59, 60], based on the Madelung quantum fluid theory which originally attempted for the single-electron quantum fluid modeling [61]. It has been recently shown that the analytic investigation of linearized Schrödinger-Poisson system for arbitrary degenerate electron gas provides routes to some novel quantum feature of collective plasmon excitations [62, 63]. In current study we use this model in order investigate the quantum interference and phase mixing in electron gases with arbitrary degree of degeneracy, based on the collective quasiparticle orbital concept based on pseudoforce formulation.

II. THE QUANTUM HYDRODYNAMIC MODEL

In order to investigate the dynamics of quantum plasma we consider the following quantum hydrodynamic equations consisting of the continuity, force balance including the quantum force and the Poisson’s equation [42]

\[ \frac{\partial n_s}{\partial t} + \frac{\partial n_s v_s}{\partial x} = 0, \]  
\[ \frac{\partial v_s}{\partial t} + v_s \frac{\partial v_s}{\partial x} = \frac{q_s}{m_s} \frac{\partial \phi}{\partial x} - \frac{1}{m_s} \frac{\partial \mu_s}{\partial x} + \frac{\gamma_S \hbar^2}{2 m_s^2} \frac{\partial}{\partial x} \left( \frac{1}{\sqrt{n_s}} \frac{\partial^2 \sqrt{n_s}}{\partial x^2} \right), \]  
\[ \frac{\partial^2 \phi}{\partial x^2} = -4\pi \sum_s q_s n_s, \]

where the index \( s \) refers to streams, \( q_s \) is the charge state of given stream, \( n_s \) denotes the stream number density, \( v_s \) the fluid velocity, \( \phi \), the electrostatic potential and \( \mu_s \), the chemical potential of given stream in the quantum fluid. Also, \( \hbar \) is the reduced Planck
constant and $m_s$ is the stream species mass. The chemical potential-temperature dependent parameter $\gamma_s$ denotes the kinetic correction to plasmon excitations in long wavelength and low phase speed plasma regime, which is given as

$$\gamma_s = \frac{\text{Li}_{3/2}[-\exp(\beta \mu_0)] \text{Li}_{-1/2}[-\exp(\beta \mu_0)]}{3 \text{Li}_{1/2}[-\exp(\beta \mu_0)]^2},$$

(2)

in which $\beta = 1/k_B T$, $\mu_0$ being the equilibrium chemical potential and the polylog function $\text{Li}$ is defined through the Fermi integrals

$$F_k(\eta) = \int_0^\infty \frac{x^k}{\exp(x - \eta) + 1} dx = -\Gamma(k + 1) \text{Li}_{k+1}[-\exp(\eta)],$$

(3)

where $\Gamma$ is the gamma function. The fundamental thermodynamic quantities depend through the equation of state (EOS) which is used to close the hydrodynamic system. Here we choose the isothermal EOS which is appropriate in the low phase speed plasmon phenomenon

$$n_s(\eta_s, T) = \frac{2^{7/2} \pi m_s^{3/2}}{\hbar^3} F_{1/2}(\eta) = \frac{2^{5/2} (\pi m_s k_B T)^{3/2}}{\hbar^3} \text{Li}_{3/2}[-\exp(\eta_s)],$$

(4a)

$$P_s(\eta_s, T) = \frac{2^{9/2} \pi m_s^{3/2}}{3 \hbar^3} F_{3/2}(\eta) = \frac{2^{5/2} (\pi m_s k_B T)^{3/2}}{k_B T} \text{Li}_{5/2}[-\exp(\eta_s)],$$

(4b)

in which $P_s$ is the statistical pressure of given stream satisfying, $\partial P_s / \partial \mu_s = n_s$, and $\eta_s = \beta \mu_s$.

The hydrodynamic system (1) is used to formulate an effective Schrödinger-Poisson model for the system using the Madelung transformation

$$i \hbar \sqrt{\gamma_s} \frac{\partial \mathcal{N}_s}{\partial t} = -\frac{\gamma_s h^2}{2m_s} \frac{\partial^2 \mathcal{N}_s}{\partial x^2} + q_s \phi \mathcal{N}_s + \mu_s(n_s, T) \mathcal{N}_s,$$

(5a)

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi \sum_s q_s |\mathcal{N}_s|^2,$$

(5b)

where $\mathcal{N}_s = \sqrt{n_s(x, t)} \exp[i S_s(x, t)/\hbar \sqrt{\gamma_s}]$ is the state-function characterizing the collective quasiparticle excitations of each stream in the gas. The equivalence between (1) and (5) may be readily examined by separating the real and imaginary parts for each stream, which follows

$$m_s \frac{\partial n_s}{\partial t} + \frac{\partial n_s}{\partial x} \frac{\partial S_s}{\partial x} + n_s \frac{\partial^2 S_s}{\partial x^2} = 0,$$

(6a)

$$\frac{\partial^2 S_s}{\partial t \partial x} + \frac{1}{m_s} \frac{\partial S_s}{\partial x} \frac{\partial^2 S_s}{\partial x^2} = -q_s \frac{\partial \phi}{\partial x} - \frac{\partial \mu_s}{\partial x} + \frac{\partial B_s}{\partial x},$$

(6b)

$$B_s = \frac{\gamma_s h^2}{8 \pi s n_s^2} \left[ 2 n_s \frac{\partial^2 n_s}{\partial x^2} - \left( \frac{\partial n_s}{\partial x} \right)^2 \right],$$

(6c)
which reduce to first two equations in (1) by the definition, \(v_s = (1/m_s)\partial S_s/\partial x\). Note also that, the fluid velocity of plasma species satisfy the relation
\[
v_s(x,t) = j_s(x,t)/n_s(x,t),
\]
where, \(j_s(x,t) = i\hbar \gamma_s/(2m_s) \partial S_s/\partial x\). Note also that, the fluid velocity of plasma species satisfy the relation
\[
v_s(x,t) = j_s(x,t)/n_s(x,t),
\]
where, \(j_s(x,t) = i\hbar \gamma_s/(2m_s) \partial N_s(x,t)/\partial x\), which is identical in the form to the velocity of particles in pilot-wave guiding equation. Let us first consider monokinetic streams with the action
\[
S_s(x,t) = p_s x + f_s(t) \text{ with } p_s = m_s v_s \text{ and } f_s(t) \text{ being an arbitrary function of time.}
\]

The space-time separated multistream Schrödinger-Poisson system, assuming \(N_s(x,t) = \psi_s(x) T_s(t) \exp[i(p_s x - f_s t)/\sqrt{\gamma_s}]\), then reads
\[
\begin{align}
\left\{ \frac{\gamma_s \hbar^2}{2m_s} \frac{d^2}{dx^2} - q_s \phi(x) + \epsilon - \mu_s \right\} \psi_s(x) \exp \left( \frac{ik_s x}{\sqrt{\gamma_s}} \right) &= 0, \\
\frac{d^2 \phi(x)}{dx^2} &= -4\pi \sum_s q_s \psi_s(x)^2, \\
i\hbar \sqrt{\gamma_s} \frac{dT_s(t)}{dt} &= \epsilon T_s(t) e^{-i\omega_s(t)},
\end{align}
\]
where \(\epsilon\) is the energy eigenvalue of the multistream quantum system, \(k_s = p_s/\hbar\) being the s-th stream de Broglie’s wavenumber and \(\omega_s(t) = -f_s(t)/\hbar\sqrt{\gamma_s}\).

### III. ELECTRON BEAM INTERFERENCE EFFECTS

For our illustration purpose we would like to study the two stream electron interference effect with \(\gamma_1 = \gamma_2 = \gamma\). To this end, we linearize the system (7) using the perturbations \(\psi_s = \psi_0 + \psi_1, \phi = 0 + \phi_1\) and \(p = 0 + p_1\) which leads to the following driven coupled pseudoforce system after linearization [67]
\[
\begin{align}
\gamma \frac{d^2 \Xi(x)}{dx^2} + \Phi(x) + E \Xi(x) &= \\
\alpha(k_1^2 - E) \exp \left( \frac{ik_1 x}{\sqrt{\gamma}} \right) + (1 - \alpha)(k_2^2 - E) \exp \left( \frac{ik_2 x}{\sqrt{\gamma}} \right), \tag{8b} \\
\frac{d^2 \Phi(x)}{dx^2} &= \Xi(x), \tag{8c}
\end{align}
\]
where \(\Xi(x) = \alpha \Psi_1(x) + (1 - \alpha) \Psi_2(x)\) is the complete wavefunction with \(\alpha = \omega^2_{p_1}/\omega_{p_2}^2\) being the fractional plasmon frequency of streams. We have used the Thomas-Fermi assumption in which the chemical potential variations are neglected \((\mu_1 = \mu_2 = \mu_0)\) and the temperature is fixed in agreement with the single-electron Fermi-Dirac distribution [68, 69]. Further more
we have used the normalization scheme in which $E = (\epsilon - \mu_0)/E_p$ with $E_p = \hbar \omega_p$ being the plasmon energy and $\omega_p = \sqrt{4\pi e^2n_0/m}$ the plasmon frequency. Also, $\Psi_1(x) = \psi_1(x)/\sqrt{n_0}$, $\Psi_2(x) = \psi_2(x)/\sqrt{n_0}$ and $\Phi(x) = e\phi(x)/E_p$. The space, $x$ and time $t$ variables are, respectively, normalized with the inverse of the plasmon wavenumber, $k_p = \sqrt{2mE_p/\hbar}$ and inverse of plasmon frequency. Note that the quasineutrality condition $n_{10} + n_{20} = n_0$ holds. Note that while the model (8) is appropriate for resonant electron-plasmon interactions to study the beam-interference effect it ignores the interaction between the electron beams. In the proceeding sections we will use a generalized model to study such effects. Assuming the nontransient solution to the system (8) reads

$$\Xi(x) = \alpha B_1 e^{ik_1x/\sqrt{\gamma}} + (1 - \alpha) B_2 e^{ik_2x/\sqrt{\gamma}}, \quad B_j = -\frac{k_j^2(k_j^2 - E)}{\gamma + k_j^2(k_j^2 - E)}; \quad (9a)$$

$$\Phi(x) = \alpha A_1 e^{ik_1x/\sqrt{\gamma}} + (1 - \alpha) A_2 e^{ik_2x/\sqrt{\gamma}}, \quad A_j = \frac{\gamma(k_j^2 - E)}{\gamma + k_j^2(k_j^2 - E)}; \quad (9b)$$

Note that the kinetic correction in the normalized form reads

$$\gamma = \frac{\text{Li}_{3/2}[-\exp(\sigma/\theta)] \text{Li}_{-1/2}[-\exp(\sigma/\theta)]}{3\text{Li}_{1/2}[-\exp(\sigma/\theta)]^2}, \quad (10)$$

in which $\theta = T/T_p$ with $T_p = E_p/k_B$ being the characteristic plasmon temperature. The number density distribution corresponding to quasiparticle orbital, $E$, is given by

$$n(E) = \Xi(x)\Xi^*(x) = \alpha^2 B_1^2 + (1 - \alpha^2) B_2^2 + 2\alpha(1 - \alpha) B_1 B_2 \cos \left[ \frac{(k_1 - k_2)x}{\sqrt{\gamma}} \right]. \quad (11)$$

The corresponding current density, on the other hand, reads

$$j(E) = \frac{i\hbar}{2m} \frac{\sqrt{\gamma}}{\Xi(x) - \Xi^*(x)} \left[ \Xi^*(x) \frac{\partial \Xi}{\partial x} - \Xi(x) \frac{\partial \Xi^*}{\partial x} \right]; \quad (12a)$$

$$j(E) = \frac{\hbar}{m} \left\{ \alpha^2 B_1^2k_1 + (1 - \alpha)^2 B_2^2k_2 + \alpha(1 - \alpha)(k_1 + k_2) B_1 B_2 \cos \left[ \frac{(k_1 - k_2)x}{\sqrt{\gamma}} \right] \right\}. \quad (12b)$$

The quasiparticle velocity corresponding to the energy orbital level, $E$, may be written in the following form

$$v(E) = \frac{j(E)}{n(E)} = \frac{i\hbar \sqrt{\gamma}}{2m} \left[ \frac{1}{\Xi^*(x)} \frac{\partial \Xi}{\partial x} - \frac{1}{\Xi(x)} \frac{\partial \Xi^*}{\partial x} \right] = \frac{\hbar \sqrt{\gamma}}{m} \text{Im} \left[ \frac{d\Xi(x)/dx}{\Xi(x)} \right], \quad (13)$$

Note that the quasiparticle orbital velocity is the same as the velocity of guiding equation in pilot wave theory in Bohmian quantum mechanics [70, 71]. For our two stream electron gas model, the quasiparticle orbital velocity reads

$$v(E) = \left( \frac{\hbar}{m} \right) \frac{\alpha^2 B_1^2k_1 + (1 - \alpha)^2 B_2^2k_2 + \alpha(1 - \alpha)(k_1 + k_2) B_1 B_2 \cos \left[ \frac{(k_1 - k_2)x}{\sqrt{\gamma}} \right]}{\alpha^2 B_1^2 + (1 - \alpha^2) B_2^2 + 2\alpha(1 - \alpha) B_1 B_2 \cos \left[ \frac{(k_1 - k_2)x}{\sqrt{\gamma}} \right]}, \quad (14)$$
For the case of single stream with the plane-wave wavefunction solution, \((\alpha = 0, 1)\), we find the quasiparticle orbital velocity to be the same as the average beam velocity. Moreover, for \(k_1 = k_2\), the result also reduces to the single stream case with constant beam velocity.

Equations (11) and (12) clearly indicate that the quantum interference of two electron beams occur as sinusoidal patterns in the electron number density and current density profiles. The average quantities over all possible orbitals from \(E = E_0 = 2\sqrt{\gamma}\) (for ground state quasiparticle level) up to infinity may be obtained as follows

\[
J_t = \int_{E_0}^{\infty} j(E)D(E)f(E)dE, \quad v_t = \int_{E_0}^{\infty} v(E)D(E)f(E)dE, \quad (15)
\]

where \(f(E) = 1/[1 + \exp(E/\theta)]\) is the occupation number for fermion quasiparticles and \(D(E)\) is the 1D density of states (DOS) given by

\[
D(E) = \frac{(E^2 - 4\gamma)(L^4 + \gamma) + E(L^4 - \gamma)\sqrt{E^2 - 4\gamma}}{4\pi L\sqrt{2}(E^2 - 4\gamma)\sqrt{\gamma(L^4 - \gamma)}\sqrt{E^2 - 4\gamma} - 4L^2\gamma^2 + E(L^4 + \gamma)} \quad (16)
\]

in which \(L\) is the normalized size of the quantum plasma. In all our simulations we have chosen \(L = 100\).

Figures 1(a) and 1(b) depicts the current density and velocity of collective excitations at different quasiparticle orbital. It is remarked that, while the orbital current density has a sinusoidal form the velocity has a nonlinear shape. Moreover, Figs. 1(c) and 1(d) show the total average current density and velocity over all posible quasiparticle orbital. It is seen that the main contribution to the average quantity comes from the ground state orbital, \(E = E_0\). However, due to important phase contribution from different orbital they may not be neglected in statistical summation.

The two stream model \([8]\) may be generalized by including the vector magnetic potential in the effective Schrödinger-Poisson model. Using the generalized action \(\nabla S_s = m_s v_s + q_s A_s\), we find the generalized electron beam wavenumber, \(k_s = (p_s - eA_s)/\hbar\) for given stream. Therefore, the Aharonov-Bohm effect may be visualized for single-momentum narrow electron beams splitting around the current carrying solenoid \([72, 73]\). In this case we have \(k_1 = (p - eA)/\hbar\) and \(k_2 = (p + eA)/\hbar\), where \(A\) is the constant magnetic vector potential and \(p\) is the electron beam momentum. The phase shift between the clockwise and counterclockwise electron beams amounts to \(\delta \varphi = (e/\hbar) \oint A \cdot dl\), as in the Aharonov-Bohm configuration.
FIG. 1: (a) Current density of different quasiparticle orbitals for given values of normalized chemical potential, $\sigma$, temperature $\theta$ and stream wavenumbers. (b) Quasiparticle velocity of different quasiparticle orbitals for given values of normalized chemical potential, $\sigma$, temperature $\theta$ and stream wavenumbers. (c) Total current density averaged over all orbitals for parameters given in (a). (d) Total quasiparticle velocity averaged over all orbitals for parameters given in (a).
IV. ELECTRON BEAM-PHONON INTERACTIONS

In order to study the plasmon-phonon interaction effects, let us first consider the energy dispersion of the following system with only one electron stream

\[ \gamma \frac{d^2 \Psi(x)}{dx^2} \pm 2i\sqrt{\gamma}k_d \frac{d\Psi(x)}{dx} + \Phi(x) + (E - k_d^2) \Psi(x) = 0, \]  
\[ \frac{d^2 \Phi(x)}{dx^2} - \Psi(x) = 0, \]  

(17a)  

(17b)

with \( k_d \) being the de Broglie’s wavenumber of the stream. The system (22) which can be obtained by direct expansion of (7) admits the following dispersion relation

\[ E_\pm = \frac{1}{k^2} + (\sqrt{\gamma} k \pm k_d)^2, \]  

(18)

where the plus/minus signs refer to the right/left going streams, respectively. Note that introduction of the electron stream leads to a Doppler shift in the particle-like branch of the energy dispersion and destroys the quasiparticle wavenumber symmetry. Such a Doppler shift is familiar from two-stream instability in plasma environments [74].

Figure 2 shows the effect of Doppler shift on the energy dispersion of collective free electron excitations. The effect of fractional chemical potential \( \sigma = \mu_0/E_p \) on the dispersion curve is shown in Fig. 2(a), in the absence of electron stream. It is remarked that, in fixed temperature \( \theta \), the quasiparticle conduction minimum \( (E_0 = 2\sqrt{\gamma}) \) is lowered by increase in the \( \sigma \). Also, the conduction minimum wavenumber \( k_0 = \pm \gamma^{-1/4} \) shifts slightly to higher wavenumbers. Figure 2(b) shows that for fixed chemical potential the increase in the normalized temperature \( \theta \) has the converse effect by shifting the conduction minimum to higher energy and lower wavenumbers. Effect of the electron stream is shown in Figs. 2(c) and 2(d). It is clearly remarked that the introduction of stream destroys the dispersion symmetry shifting the conduction band energy minimum, making collective excitation orbital more available/unavailable to quasiparticles in the counterstream/stream direction. Such effect may be the origin of well-known two-stream instability. Note that \( \epsilon \) is the exact quasiparticle normalized energy eigenvalue, while, in forthcoming analysis we use the scaled eigenvalue \( E = \epsilon - \sigma \), which depends on the chemical potential of the electron beam, for simplicity.

In the model (22) neutralizing ion background are static (jellium model). This model may
FIG. 2: (a) Variations in energy dispersion zero-stream plasmon excitations due to changes in the normalized chemical potential. (b) Variations in energy dispersion zero-stream plasmon excitations due to changes in the normalized temperature. (c) Variations in energy dispersion noninteracting two-stream plasma excitations due to changes in the positive stream wavenumber. (d) Variations in energy dispersion noninteracting two-stream plasma excitations due to changes in the negative stream wavenumber.
be generalized considering ion dynamics as follows

\[ \gamma \frac{d^2 \Psi_e(x)}{dx^2} \pm 2i \sqrt{\gamma k_d} \frac{d \Psi_e(x)}{dx} + \Phi(x) + (E - k_d^2) \Psi_e(x) = 0, \]  
\[ \zeta \frac{d^2 \Psi_i(x)}{dx^2} - \Phi(x) + (E + \mu) \Psi_i(x) = 0, \]  
\[ \frac{d^2 \Phi(x)}{dx^2} - \Psi_e(x) + \Psi_i(x) = 0, \]

where \( \zeta = m_e/m_i \). The system (19) admits the following eigenvalue system

\[
\begin{pmatrix}
\Psi_e \\
\Psi_i \\
\Phi
\end{pmatrix}
= \begin{pmatrix}
\gamma k^2 - 2 \sqrt{\gamma} k k_1 - (E - k_1^2) & 0 & -1 \\
0 & \zeta k^2 - (E + \mu) & -1 \\
k^2 & \alpha & 1 - \alpha
\end{pmatrix}
\begin{pmatrix}
\Psi_{e1} \\
\Psi_{i1} \\
\Phi_1
\end{pmatrix},
\]

which admits the following energy dispersion relation

\[ E_{\pm} = \frac{2 + \xi k^2 \pm \sqrt{4 + \xi^2 k^4}}{2 k^2}, \quad \xi = (k_d - k \sqrt{\gamma})^2 - \zeta k^2 + \sigma, \]

where the plus/minus sign refers to the upper/lower energy band.

Figure 3 shows the plasmon energy dispersion for quasineutral electron beam-ion plasma with various plasma parameters. Figure 3(a) indicated that in the absence of electron stream there are symmetric upper and lower energy bands. However, the presence of electron stream, as shown in Fig. 3(b), disturbs the symmetry of bands making more quasiparticle levels available to counterstream particles. Figure 3(c), on the other hand, depicts the effect of higher normalized electron chemical potential. It is remarked that, Increase in this parameter lowers the values of both upper and lower energy bands. Finally, Fig. 3(d) reveals that increase in the normalized electron temperature does not have a significant effect on the energy band of collective excitations, as compared to Fig. 3(b).

Figure 4 shows the spatial electron drift velocity variations in electron-ion plasma with dynamic ions for different plasma and drift parameters. Figure 4(a) shows the quasiparticle velocity at orbital \( E = 0.2, 0.5, \sigma = \theta = 0.1 \) and drift wavenumber \( k_d = 0.5 \). It is clearly remarked that the electron beam backscatters at orbital \( E = 0.2 \). The later is evidently due to presence of phonon excitations by dynamic ions. Figure 4(b) depicts the quasiparticle velocity at high energy orbital \( E = 1.5, 2 \) for similar other parameters as in Fig. 4(a). It is remarked that for low energy orbital electrons drift with a constant speed after a transient oscillatory motion at interface. However, for high energy orbital \( E = 2 \) the velocity is in a
FIG. 3: Energy dispersion of electron plasmon-phonon interaction with (a) zero electron drift (b) non-zero electron drift, (c) increased electron normalized chemical potential and (d) increased electron normalized temperature.

periodic oscillatory form. Figures 4(c) and 4(d) show the velocity for the same orbital as 4(a) and 4(b), respectively, for lower electron beam drift wavenumber. It is seen that for orbital $E = 0.2$ the quasiparticle velocity becomes oscillatory with no total backscattering. However, the electrons are partially scattered at specific ranges, where the velocity becomes negative. In Fig. 4(d) the decrease in $k_d$ does not radically change the orbital velocity.
FIG. 4: The quasiparticle velocity at different energy orbital for electron-ion plasma with drifting electrons. (a) $E = 0.2, 0.5, \gamma = 0.2, k_d = 0.5$ (b) $E = 1.5, 2, \gamma = 0.2, k_d = 0.5$, (c) $E = 0.2, 0.5, \sigma = \theta = 0.1$ and $k_d = 0.3$, (d) $E = 1.5, 2, \sigma = \theta = 0.1$ and $k_d = 0.3$. pattern.
V. PLASMON-PLASMON INTERACTIONS AND PHASE-MIXING

Next we consider the following pseudoforce system of interacting two-stream model

\[
\gamma \frac{d^2 \Psi_1}{dx^2} + 2i \sqrt{\gamma} k_1 \frac{d \Psi_1}{dx} + \Phi + (E - k_1^2) \Psi_1 = 0 \quad (22a)
\]

\[
\gamma \frac{d^2 \Psi_2}{dx^2} + 2i \sqrt{\gamma} k_2 \frac{d \Psi_2}{dx} + \Phi + (E - k_2^2) \Psi_2 = 0 \quad (22b)
\]

\[
\frac{d^2 \Phi}{dx^2} - \alpha \Psi_1 - (1 - \alpha) \Psi_2 = 0, \quad (22c)
\]

where \(k_1\) and \(k_2\) are the generalized momentums, as normalized to the plasmon momentum \(k_p = \sqrt{2mE_p/\hbar}\). The coupled system \(22\) does not have straightforward analytical solution and must be solved numerically. Valuable information may be extracted from the energy dispersion relation which is obtained by assuming plane wave expansions, \(\Psi_1(x) = \Psi_{11} \exp(ikx)\), \(\Psi_2(x) = \Psi_{21} \exp(ikx)\) and \(\Phi_1(x) = \Phi_1 \exp(ikx)\), leading to the following eigenvalue system

\[
\begin{pmatrix}
\Psi_1 \\
\Psi_2 \\
\Phi
\end{pmatrix} =
\begin{pmatrix}
\gamma k^2 - 2 \sqrt{\gamma} kk_1 - (E - k_1^2) & 0 & -1/2 \\
0 & -\gamma k^2 + 2 \sqrt{\gamma} kk_2 - (E - k_2^2) & -1/2 \\
k^2 & \alpha & 1 - \alpha
\end{pmatrix}
\begin{pmatrix}
\Psi_{11} \\
\Psi_{21} \\
\Phi_1
\end{pmatrix},
\]

from which one obtains the following energy dispersion relations

\[
E_{\pm} = \frac{1}{4k^2} \left[ 1 \pm \sqrt{1 + 4\beta k^2 (k_1 - k_2) + 2k^2 (k_1^2 + k_2^2 - 2\eta k^2 \sqrt{\gamma})} \right], \quad (24a)
\]

\[
\beta = \eta [(k_1 - k_2) k^2 + 2\alpha - 1], \quad \eta = k_1 + k_2 - 2k^2 \sqrt{\gamma}. \quad (24b)
\]

Note that we assumed \(\gamma_1 = \gamma_2 = \gamma\) and \(\sigma_1 = \sigma_2 = \sigma\), hence \(E_1 = E_2 = E\), for simplicity. However, a more general dispersion relation can be obtained for different streaming conditions. A more general model may also include the screening (pseudodamping) effect.

Figure 5 shows the two-stream energy dispersion which consists of two distinct lower and upper branches. It is clearly evident that appearance of lower/upper branches is due to mixing of the collective excitation modes caused by dual streams. Dispersion of symmetric stream momentum with asymmetric density is shown in Fig. 5(a) indicating a Doppler shift towards positive stream direction which is less dense, i.e., \(\alpha = 0.1\). In Fig. 5(b) the symmetric momentum case with slightly lower wavenumber is depicted. It is remarked that more quasiparticle orbital are available for lower stream momentum. The case with
FIG. 5: Energy dispersion of interacting two-stream plasma excitations with (a) symmetric momentum with asymmetric density configuration (b) symmetric momentum asymmetric density configuration with different stream wavenumber than in (a), (c) symmetric momentum and density configuration and (d) asymmetric momentum with symmetric density configuration.

symmetric stream momentum and density is shown in Fig. 5(b) resulting in a completely symmetric dispersion curves. However, despite the fact that the total current is zero in this case, the mixing of modes is still present. The case of asymmetric momentum stream with symmetric density in Fig. 5(d) indicates that dispersion curve is broken due to stream
FIG. 6: Current density of orbital $E = 1.5$ for (a) symmetric momentum and stream density (b) asymmetric momentum with symmetric density (c) symmetric momentum with asymmetric density, and (d) symmetric momentum with asymmetric density different from that in (c).

density distribution imbalance.

We have numerically evaluated the two stream system (22) for initial condition $\Psi_{10} = \alpha$, $\Psi_{20} = (1 - \alpha)$, $\Phi_0 = 0$ and $\Psi'_{10} = \Psi'_{20} = \Phi'_0 = 0$. Figure 6 depicts the current density if counter streaming electron gas at orbital $E = 1.5$. The case of complete symmetric stream is shown in Fig. 6(a) where the total current density vanishes. However, Fig. 6(b) shows
that for asymmetric momentum configuration phase mixing occurs, where current of low momentum stream reverses at some points of its current. The latter is cause by the fact the more orbital become available to low momentum beam at the other side of stream leading to backscattering of the low momentum stream. While similar in concept, this is however a different phenomenon than stream instability. The phase mixing effect due to the density imbalance of counter streaming electrons is shown in Fig. 6(c), for $\alpha = 0.1$. Note that strong oscillations in current density of dense stream as compared to the other one. Figure 6(d) reveals that the phase mixing is absent for symmetric momentum and $\alpha = 0.9$.

Figure 7 depicts the quasiparticle orbital velocity for $E = 1.3$ and different steam parameters. The quasiparticle excitations corresponding to this orbital are unstable due to low energy which can not excite collective excitations. However, there is a transient short period at the beginning before the orbital velocity becomes uniform. It is remarked that the low density counter streaming beam merges into the denser one, even though its momentum is higher in Figs. 7(a) and 7(b). In Fig. 7(c) the velocity of negative stream grows sharply while the positive stream velocity decays slowly. Figure 7(d), on the other hand, shows that less dense negative streaming beam rapidly merges into the positively streaming beam. However, there is no obvious formula quantifying the final momentum orbital of the merged beams in these plots.

Figure 8(a) and 8(b) shows the merging of unstable orbital $E = 0.3$ and $E = 0.5$ for overtaking streams. For the low energy orbital of Fig. 8(a) the merging (damping) rate seems to be lower. Figures 8(c) and 8(d), on the other hand, show the stable orbital velocities for $E = 1$ and $E = 1.3$ for symmetric counter streaming electron beams. It is found that, the oscillation frequency in velocity pattern is higher and irregular for high energy orbital, shown in Fig. 8(d).

In Fig. 9 we show the variations in quasiparticle acceleration, $a(x) = v(x)dv(x)/dx$, and dissipated power $p(x) = a(x)v(x)$ for unstable orbital $E = 1.2$ and $E = 0.7$. The force acting on each beam particles, in orbital $E = 1.2$, for slightly asymmetric unstable counter-stream is depicted in Fig. 9(a). It is seen that the force acting on the low and momentum negative stream is stronger and rapidly grows over distance. Figure 9(b) show the orbital velocity of quasiparticle corresponding to Fig. 9(a). The grow and damping of oscillations are also present in the velocity pattern. Figures 9(c) and 9(d) show the acceleration and dissipated power of orbital $E = 0.7$ for counter-streaming electrons. The nonlinear profiles show
FIG. 7: Quasiparticle speed for unstable orbital $E = 1.3$ with different two-stream wavenumber and density parameters.

periodic energy exchange between streams as they merge into single beam. The plasmon stopping power of an electron beam may be readily calculated using $\Delta E/\Delta x = \int_0^\delta a(x)dx$ in which $\delta$ is the penetration depth.
FIG. 8: Quasiparticle speed for different stable and unstable orbital with different two-stream wavenumber and density parameters.
FIG. 9: (a) The quasiparticle acceleration and (b) speed for unstable quasiparticle orbital $E = 1.2$. (c) The acceleration and (d) dissipated stream power for unstable quasiparticle orbital $E = 0.7$. 

$E = 1.2 \quad \gamma = 0.2, \quad k_1 = -0.6, \quad k_2 = 0.62, \quad \alpha = 0.5$

$E = 0.7 \quad \gamma = 0.2, \quad k_1 = -0.2, \quad k_2 = 1, \quad \alpha = 0.5$
VI. ELECTRON BEAM-LATTICE INTERACTION

In order to prove the effectiveness of current model we would like to study the beam lattice interactions. To this end, we consider the following driven coupled pseudoforce system

\[ \gamma \frac{d^2 \Psi}{dx^2} + 2i \sqrt{k_d} \frac{d\Psi}{dx} + \Phi + (E - k_d^2)\Psi = 0 \]  
(25a)
\[ \frac{d^2 \Phi}{dx^2} - \Psi + U \cos(Gx) = 0, \]  
(25b)

in which \( G = 2\pi/a \) is the reciprocal lattice vector with atomic spacing \( a \) and \( U \) is the lattice potential strength. Note that we use a simple toy model in which the atomic potential is sinusoidal, for our illustration purpose. However, current model may be readily generalized to fourier components of a real Columbic potential [76]. The Doppler shift in the energy dispersion is same as in single stream model. However, in this case the electron stream wavenumber can resonantly interact with intrinsic periodic structure of the plasmonic lattice.

In Fig. 10 we have shown the quasiparticle velocity corresponding to the orbital \( E = 1.8 \) in a beam-lattice interaction by solving the driven pseudoforce system (25) with initial conditions \( \Psi_0 = 1, \Phi_0 = 0 \) and \( \Psi'_0 = \Phi'_0 = 0 \). The orbital with \( k_d = 0.5 \) is found to be stable orbital with lattice parameter \( G = 5 \), as shown in Fig. 10(a). The periodic nondamping velocity profiles is an indication of the orbital stability. However, as the drift wavenumber increases to the value \( k_d = 0.7 \) the orbital becomes unstable, as is evident from Fig. 10(b). An interesting effect occurs for unstable orbital with \( k_d = 0.7 \) when the reciprocal lattice decreases to the value \( G = 2 \), as shown in Fig. 10(c). It is remarked that the electron beam backscatters due to resonant beam-lattice interaction. The effect becomes even more pronounced and forms a regular pattern as the lattice potential increases to the value \( U = 2 \), as is evident from Fig. 10(d).

Figure 11(a) shows the current density of stable orbital \( E = 2 \) for given beam and lattice parameters. There are strong fluctuations in the current density due to beam-plasmon interactions. Moreover, Fig. 11(b) reveals that increase of the beam momentum relatively increases the amplitude of average current density, as expected. Decrease of the reciprocal lattice vector to \( G = 3 \) with other parameter same as in Fig. 11(a) leads to negative current for this orbital, as shown in Fig. 11(c). Finally, increase in the lattice potential strength leads to strong backscattering of the electron beam for this stable beam, producing a chaotic current density pattern in Fig. 11(d). Such a backscattering effect may also be related to
FIG. 10: (a) Stable quasiparticle velocity of electron beam-lattice interaction in orbital $E = 1.8$.
(b) Unstable quasiparticle velocity of electron beam-lattice in orbital $E = 1.8$. (c) Backscattering of electron beam from lattice at orbital $E = 1.8$ with reciprocal lattice vector $G = 2$ and lattice potential strength $U = 1$. (d) Backscattering of electron beam from lattice at orbital $E = 1.8$ with reciprocal lattice vector $G = 2$ and lattice potential strength $U = 2$.

the well-known phenomenon, the Umklapp scattering [4] or U-process which is the dominant ineffective electronic heat transport in metallic elements at low temperatures.
FIG. 11: The current density of stable orbital $E = 2$ for different values of electron beam wavenumber, $k_d$, reciprocal lattice vector, $G = 2\pi/a$ and lattice potential strength, $U$.

VII. CONCLUSION

Using the kinetic corrected Schrödinger-Poisson model we developed a pseudoforce theory to study the multistream quantum plasmas. The noninteracting electron two-stream model was used to study the quantum beam interference and Aharonov-Bohm-like effects in the two beam electron gas. On the other hand, quasiparticle velocity, acceleration and power at given
orbital was used to investigate the stream interactions and quantum phase mixing in two-stream plasma model for different stream momentum and density configurations. We further extend the model to include the electron beam-lattice interactions and backscattering effect. Current research may be important in understanding of electron-plasmon and plasmon-plasmon interaction in quantum orbital level and may be further elaborated to study the quantum wave-particle interactions in complex plasmas.

VIII. DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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