Velocity-Dependent Forces and an Accelerating Universe

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Abstract

In recent work, it was shown that velocity-dependent forces between parallel fundamental strings moving apart in a $D$-dimensional spacetime implied an expanding universe in $D - 1$-dimensional spacetime. Here we expand on this work to obtain exact solutions for various string/brane cosmological toy models.
1. Introduction

In a recent paper [1], it was shown that the velocity-dependent forces between parallel fundamental strings in \( D \) spacetime dimensions, with certain initial conditions, lead to an expanding universe in \( D - 1 \) dimensions. The findings were consistent with recent observations [2] of an accelerating universe, and predict an asymptotically constant late time expansion rate.

We start with the action

\[
I_D = \frac{1}{2\kappa^2} \int d^Dx \sqrt{-g} e^{-2\phi} \left( R + 4(\partial\phi)^2 - \frac{1}{12}H^2_3 \right)
\] (1.1)

is the \( D \)-dimensional string low-energy effective spacetime action and

\[
S_2 = -\frac{\mu}{2} \int d^2\zeta \left( \sqrt{-\gamma} \gamma^{\mu\nu} \partial_{\mu}X^M \partial_{\nu}X^N g_{MN} + \epsilon^{\mu\nu} \partial_{\mu}X^M \partial_{\nu}X^N B_{MN} \right)
\] (1.2)

is the two-dimensional worldsheet sigma-model source action. \( g_{MN}, B_{MN} \) and \( \phi \) are the spacetime sigma-model metric, antisymmetric tensor and dilaton, respectively, while \( \gamma_{\mu\nu} \) is the worldsheet metric. \( H_3 = dB_2 \) and \( \mu \) is the string tension. The fundamental string solution to the combined action, representing stationary macroscopic strings parallel to the \( x^1 \) direction, is given by [3]

\[
ds^2 = h^{-1} \left( -dt^2 + (dx^i)^2 \right) + \delta_{ij} dx^i dx^j,
\]

\[
e^{-2\phi} = h = 1 + \frac{k_n}{r^n}, \quad B_{01} = -h^{-1}
\] (1.3)

where \( n = D - 4, r^2 = x^i x_i \) and the indices \( i \) and \( j \) run through the \( D - 2 \)-dimensional space transverse to the string. The constant \( k_n = 2\kappa^2 T_1 / n\Omega_{n+1} \), where \( T_1 = \mu \) is the tension of the string, equal to its mass/length, and \( \Omega_{n+1} \) is the volume of \( S^{n+1} \), the \( n + 1 \)-dimensional unit sphere.

This solution can be extended to a multi-static string solution owing to the existence of a zero-force condition. This condition in turn arises from the cancellation between the
attractive gravitational and dilatational forces of exchange with the repulsive antisymmetric field exchange, and is based on the existence of supersymmetry and the saturation of a BPS bound [4].

It was subsequently shown that, in addition to the zero static force, the leading order \((O(v^2))\) velocity-dependent forces cancel for moving strings as well [5] (see also [6]). This result too is associated with the existence of higher supersymmetry [7]. Following [7], it is straightforward to verify that the four-point amplitude corresponding to the scattering of two such fundamental string states approaches zero in the small velocity limit. This is identical to the result found for the \(a = \sqrt{3}\) black holes, which also preserve half of the total spacetime symmetries, the maximum for such black hole, string or \(p\)-brane solutions.

The Lagrangian for a test fundamental string moving in the background of a parallel source string is then given by [1,5]

\[
L = -m h^{-1} \left( \sqrt{1 - h\dot{x}^2} - 1 \right),
\]

where \(m\) is the mass of the string, \(\dot{x}^2 = \dot{x}^i \dot{x}_i\) and the “\(\cdot\)” represents a time derivative. It was shown in [1] that the velocity-dependent force following from this Lagrangian is repulsive whenever the strings are moving away from each other, and this leads to a further separation of the strings. Since this type of interaction occurs for any two strings, if we start with any number of close, parallel strings initially moving apart in the transverse space, they will continue to do so indefinitely and will fuel an expanding universe in the \(D - 2\)-dimensional transverse space and therefore in the \(D - 1\)-dimensional spacetime orthogonal to the strings. For example, five-dimensional fundamental strings lead to an expanding universe in \(D = 4\) spacetime dimensions.

The toy model presented in [1] consisted of a large number of fundamental strings initially very close to each other. Each pair of such strings interacts as above, so that an initial outward propagation of the strings tends to further push them apart in the transverse space. In a mean-field approximation, the effective force on each string was
approximated by that of a single, very large source fundamental string whose Noether charge $k$ is equal to the total charge of all of the strings in the $D$-dimensional space. The distance $r$ between the test string and the source string in this model then represents the approximate average position of the strings, and hence the size of the universe. The time dependence of $r$ at both early and late times was determined [1] for this expanding model.

In this paper, we expand on the results of [1] to obtain exact solutions for the radial position as a function of time for the mean-field approximation for $D = 5$ and $D = 6$ for the case of zero angular momentum. We also consider a spherically symmetric toy model and obtain similar results in the very early and late time limits as well, to give further support to the mean-field approximation results.

2. Generalized $p$-Branes

Before outlining these solutions, it is interesting to note that the Lagrangian (1.4) also arises whenever we consider the motion of a maximally supersymmetric $p$-brane moving in the background of a parallel, identical $p$-brane. For example, (1.4) is the same Lagrangian one obtains for a test fivebrane moving in the background of a parallel source fivebrane or for a D0-brane moving in the background of a source D0-brane (see, e.g., [8]). This can be seen immediately either from supersymmetry, or through dimensional reduction [9]. For the case of the fivebrane, for example, one replaces the two form $B_{01}$ with a six-form $A_{012345}$ and proceeds in the same manner to obtain (1.4), where now $m$ represents the mass of the test fivebrane moving in the background of a parallel source fivebrane.

The relevant dimension is the number of transverse dimensions, given by $n = D - p - 3$, since the harmonic function $h = 1 + k_n/r^n$ depends only on $n$. Here

$$k_n = \frac{2\kappa^2_D T_p}{n\Omega_{n+1}}, \quad (2.1)$$

where $T_p$, the tension of the $p$-brane, is equal to its mass/$p$-volume [6]. Compactifying $q \leq p$ dimensions, we can relate the $D$-dimensional Newton’s constant $G_D$ to the $D - q$-
dimensional Newton’s constant $G_{D-q}$ via \[6\]

\[
G_D = \kappa_D^2 = \kappa_{D-q}^2 V_q = G_{D-q} V_q,
\]  

(2.2)

where $V_q$ is the compactified $q$-volume. Since $T_p = m/V_p$, it follows that

\[
k_n = \frac{2\kappa_{D-q}^2 T_{p-q}}{n \Omega_{n+1}},
\]

(2.3)

which is just (2.1) for a $p-q$-brane. In particular, for $q = p - 1$, we recover the string formula. For $q = p$, we obtain the formula for D0-branes. As long as the longitudinal directions of the branes are held parallel, the dynamics are independent of $p$, the dimension of the branes, and depend only on the number of transverse directions.

In what follows, we will consider parallel strings for simplicity, keeping in mind that we could equally well consider, say, D0-branes in one less dimension.

3. Mean-Field Approximation

For the case in which we replace the total repulsive force on a single string by an effective, large string at the origin, it was shown in \[1\] that

\[
\dot{r}^2 = \frac{\rho(h\rho + 2)}{(h\rho + 1)^2},
\]

(3.1)

where $E$ is the constant total energy of the string, and where $\rho = E/m$ is the ratio of the energy to the rest energy of the test string. We have set the angular momentum $l = 0$. Following, Chebyshev’s Theorem \[1\], only the two cases of $D = 5$ ($n = 1$) and $D = 6$ ($n = 2$) can be integrated exactly.

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1 In order to integrate (3.1) we need to evaluate integrals of the form $\int x^m (a + bx^n)^p dx$ (so called binomial differentials), where $m, n, p$ are any rational numbers and $a, b$ any constants. Chebyshev proved that integrals of this form can be expressed through algebraic, logarithmic and inverse circular functions in only three cases:

1) $p$ is an integer (positive, negative or zero)
2) $\frac{m+1}{n}$ is an integer
3) $\frac{m+1}{n} + p$ is an integer.
A straightforward integration for \( n = 1 \) yields

\[
\left( \frac{\rho + 3}{\rho + 1} \right) \ln \left( \sqrt{\frac{r}{a}} + \sqrt{\frac{r}{a} + 1} \right) + \sqrt{\frac{r}{a}} \sqrt{\frac{r}{a} + 1} = \sqrt{\frac{(\rho + 2)^3}{\rho (\rho + 1)^2}} \left( \frac{t - t_0}{k_1} \right),
\]

(3.2)

where \( a = \frac{\rho k_1}{\rho + 2} \). For small \( r \) (or early time \( t \)), \( r \simeq \left( \frac{\rho + 2}{\rho + 1} \right)^2 t^2/k_1 \), while for large \( r \) (or late time), \( r \simeq \sqrt{\rho (\rho + 2)/(\rho + 1)^2} t \), both in agreement with the findings of [1]. In this three-dimensional transverse space, \( r(t) \) represents the mean size of the universe in this toy model. Restoring factors of the speed of light \( c \) in (2.1), it follows that \( k_1 = \frac{2G_5 M}{Lc^2} = \frac{G_4 M}{2\pi c^2} \), where \( L \) is the length of each string and \( M \) is the mass of the source string, representing the effective total mass of the universe in the mean-field approximation.

In [1], it was claimed that \( r << k_1 \) and \( r >> k_1 \) corresponded to early and late times (relative to the current epoch), respectively, in the expansion of the universe as implied in (3.2). Let us verify this assumption using estimates of cosmological parameters obtained in [10]. The current matter density \( \rho \) is very close to the critical density \( \rho_0 = 3H_0^2/8\pi G_4 \), and the age of the universe \( t_0 = H_0^{-1} \), where \( H_0 \) is the Hubble’s constant at present time. Ignoring numerical factors of \( O(1) \), we take the current size of the universe \( r_0 \sim ct_0 \) and its mass as \( M \sim \rho r_0^3 \). It is then straightforward to show that \( k_1 \) is at most an order of magnitude less than \( r_0 \). It follows that \( r << k_1 \) corresponds to much earlier times than present \( t << k_1/c \) and \( r >> k_1 \) corresponds to much later times \( t >> k_1/c \). The model for \( n = 1 \) has the defect that the ratio of relative velocities to relative positions is not immediately a spatial constant, unless the spatial dimensions of the universe are restricted to two (see next section).

For the more interesting case of \( n = 2 \), a straightforward integration yields

\[
\sqrt{r^2 + a} + \sqrt{a} \left( \frac{\rho + 2}{\rho + 1} \right) \ln \left( \frac{r + \sqrt{r^2 + a - \sqrt{a}}}{r + \sqrt{r^2 + a + \sqrt{a}}} \right) = \sqrt{\frac{\rho(\rho + 2)}{(\rho + 1)^2}} (t - t_0),
\]

(3.3)

\[2\] The constant energy \( E \) does not include the constant rest energy \( m \). It is straightforward to show that, in the late time limit, \( \gamma = (1 - \beta^2)^{-1/2} = \rho + 1 \), so that the interaction energy is ultimately transformed into kinetic energy \((= (\gamma - 1) m)\).
where again \( a = \frac{\rho k_2}{\rho + 2} \) and \( t_0 \) is a constant. For small \( r \), \( r \approx r_0 \exp \frac{t}{\sqrt{k_2}} \), while for large \( r \) we again find \( r \approx \sqrt{\rho(\rho + 2)/(\rho + 1)^2} t \), both again in agreement with the findings of [1]. In this four-dimensional transverse space, the three-dimensional universe may be regarded as an expanding spherical shell with radius \( r(t) \). A subtle point here arises as to the connection between \( G_5 \) and \( G_4 \). In going from a five-dimensional universe to a four-dimensional one whose constant time slices consist of the expanding three-sphere, it follows that \( G_5 \sim G_4 r(t) \). So \( G_5 \) and \( G_4 \) cannot both be constant. For constant \( G_5 \), \( k_2 = G_5 M/2\pi^2 c^2 \) (from (2.1)) is also constant, but the four-dimensional Newton’s law changes with time as the universe expands. The alternative picture is to demand a constant \( G_4 \), but then allow for changing \( k_2 \), so that the mean-field model in this case should be thought of as an approximation to a cosmological \( p \)-brane solution with \( k_2 \) a function of time. In either case, within the limits of these toy models, a straightforward calculation shows that \( k_2 \sim \bar{r}^2 \), where \( \bar{r} \) is the current size of the universe. It again follows that the early \( (r << \sqrt{k_2}) \) and late \( (r >> \sqrt{k_2}) \) time limits are valid, as in the \( n = 1 \) case. Especially interesting features of the \( n = 2 \) case, other than allowing for a spatially constant Hubble’s constant, are the inflationary expansion at early times and the asymptotically constant expansion rate for late times. This latter feature is generic to these string/brane models, since the velocity-dependent forces vanish at asymptotically large distances.

4. Spherical Shell Model

Now consider the following model of a string-seeded universe for both \( n = 1 \) and \( n = 2 \). \( N \) parallel, identical strings, with \( N >> 1 \), are all located at the same distance \( R \) from the center of the transverse \( D - 2 \) dimensional space and move with the same, purely radial, velocity \( \vec{v} = \dot{R} \hat{R} \) outward from the center. We would again like to determine \( R(t) \) and to compare our findings with the mean-field approximation. Before doing so, we note that such a model is consistent with the cosmological observation of a Hubble’s constant. In the \( n = 1 \) (\( n = 2 \)) case, the spatial universe consists of an expanding 2-
sphere (3-sphere). It is straightforward to show that the relative position is given by
\[ r_{21} = |\vec{r}_{21}| = |\vec{r}_2 - \vec{r}_1| = 2R \sin \theta / 2, \]
where \( \theta \) is the angle between the position vectors \( \vec{r}_1 \) and \( \vec{r}_2 \). Similarly, the relative speed between two strings \( v_{21} = |\vec{v}_{21}| = |\vec{v}_2 - \vec{v}_1| = 2v \sin \theta / 2, \)
where \( \vec{v}_1 \) and \( \vec{v}_2 \) are the velocities of the two strings. It follows that \( v_{21}/r_{21} = v/R \) is a constant over each sphere (or for a given time slice), representing the Hubble’s constant for this model.

The Hamiltonian for the system of test string moving in the source string background can be easily obtained from the Lagrangian (1.4) of this system
\[ H = \frac{m}{\hbar} \left( \frac{1}{\sqrt{1 - h\dot{x}^2}} - 1 \right), \quad (4.1) \]

For \( D = 5 \) (\( D = 6 \)) this model is equivalent to a system of particles on the surface of a 2-sphere (3-sphere) with two-particle energy of interaction. This interaction energy is just the difference between the conserved total energy of the 2-particle system given by (1.4) and the kinetic energy of the test string, since the source string is assumed to be stationary in the mean-field approximation. Note also that in the mean-field approximation (see also [1]), the energy (4.1) was taken to be a constant, which led to a solution for the motion of a single test string in the background of a much larger source string, which approximated the aggregate effect of the velocity-dependent forces of all the other strings. In the shell model, the interaction energies obtained from (4.1) are not individually constant but must be added into a total energy for the system, which is then set equal to a constant total energy. The center of mass of the system remains at the center of the sphere. The interaction energy of one string in the background of another is then given by

\[ \frac{4}{3} \]

It is convenient to use this velocity dependent form of the Hamiltonian. In the same way, one can easily obtain the conventional form \( H = H(r, p) \) by using the expression for the momenta \( p_i = \frac{m \dot{x}_i}{\sqrt{1 - h \dot{x}^2}} \). Then \( H = \frac{m}{\hbar} \left( \sqrt{1 + h \frac{\dot{x}^2}{m^2}} - 1 \right). \)

\[ 4 \]
Strictly speaking, we should use the more cumbersome relativistic form of the kinetic energy. However, since most of the subsequent analysis involves the early time expansion, the
\[ E_{int12} = \frac{m}{h_{12}} \left( \frac{1}{\sqrt{1 - h_{12}r_{12}^2}} - 1 \right) - \frac{m \dot{r}_{12}^2}{2}, \] (4.2)

where \( h_{12} = 1 + \frac{k}{r_{12}} \) and \( r_{12} \) is the relative position of the strings.

For \( D = 5 \) we assume for simplicity that the 1st string is located at the north pole of the 2-sphere with radius \( R \). Then, by symmetry, the energy of interaction between the 1st string and \( dN \) strings which are located inside the belt with azimuthal angles between \( \theta \) and \( \theta + d\theta \) is given by

\[ dE_{int12} = dN \left[ \frac{m}{h_{12}} \left( \frac{1}{\sqrt{1 - h_{12}r_{12}^2}} - 1 \right) - \frac{m \dot{r}_{12}^2}{2} \right], \] (4.3)

where \( dN = \frac{N}{2} \sin \frac{\theta}{2} d\theta \), \( r_{12} = 2R \sin \frac{\theta}{2} \), \( \dot{r}_{12} = 2 \dot{R} \sin \frac{\theta}{2} \) and \( h_{12} = 1 + \frac{k}{r_{12}} \).

In order to obtain the energy of interaction between 1st string and all other strings we need to integrate (4.3) over \( \theta \) from \( \theta = 0 \) to \( \theta = \pi \). This can be done explicitly, but leads to a rather complicated, and not especially illuminating, expression. In order to make a connection with the early-time mean-field approximation, we first make the assumption that \( \frac{k}{r_{12}} >> 1 \). This means that the distance between any two strings \( r_{12} << k_{1} \) and our model can describe the system during the time when this condition holds, i.e. early times.

Thus we replace \( h_{12} \simeq \frac{k_{1}}{r_{12}} \) in (4.3). The integration over \( \theta \) is easy to perform, and we obtain for the energy of interaction between the 1st string and all the other strings

\[ E_{1int} = -\frac{4mNR}{k_{1}} \left\{ \frac{2}{15a^3} \left[ \sqrt{1-a} (3a^2 + 4a + 8) - 8 \right] + \frac{a}{8} + \frac{1}{3} \right\} \] (4.4)

where

\[ a = \frac{2 \dot{R}^2 k_{1}}{R}. \] (4.5)

Note that \( a \) is restricted to be in the domain \( 0 \leq a \leq 1 \) for this model to be valid.

By symmetry, the total energy of interaction of \( N \) strings is non-relativistic approximation used for this model is valid. Furthermore, as we shall see later, the result for late times is not affected by making this simplification either.
\[ E_{int} = NE_{1int} \]  

(4.6)

and the total conserved energy of the system is

\[ NE_1 = E = E_{int} + N \frac{m \dot{R}^2}{2}, \]  

(4.7)

where \( E_1 \) is the total energy of a single string.

First assume \( a << 1 \) and expand the total interaction energy of the system (4.7) in powers of \( a \)

\[ E_{int} = \frac{3mR}{10k_1} N^2 \left[ a^2 + O(a^3) \right]. \]  

(4.8)

Thus up to 2nd order in \( a \), Eq. (4.7) takes the form

\[ \rho = \frac{3NR}{10k_1} a^2 + \frac{R}{4k_1} a \]  

(4.9)

where \( \rho = E_1/m = E/mN \) is again the ratio of the total energy (not including the rest energy) of each string to its rest energy.

Solving the quadratic equation in (4.9) for \( a \), and using \( R << k_1 \), it easily follows that the linear term in \( a \) in (4.3) may be dropped. It then follows that up to an \( O(1) \) numerical factor, \( a \simeq \sqrt{\frac{k_1 \rho}{NR}} << 1 \). Since \( \rho \) is at least of \( O(1) \), it follows that from \( a << 1 \) that \( R >> \frac{k_1}{N} \). Dropping the 2nd term in the r.h.s. of (4.9), solving (4.9) with respect to \( \dot{R} \) and integrating we obtain

\[ R \simeq \left( \frac{\rho}{kN} \right)^{1/3} t^{4/3}. \]  

(4.10)

Another domain of interest is when \( a \approx 1 \). This corresponds to an even earlier time, since from (4.10) it follows that \( a \sim t^{-2/3} \), so that an earlier time corresponds to larger \( a \). For \( a \to 1 \), the solution \( R = R(t) \) is easily obtained from the definition of \( a \) (4.5)
\[ R \simeq \frac{t^2}{k_1} \quad (4.11) \]

and is valid for \( R \sim k_1/N \), which can be seen from \((4.7)\) with \( E_{int} \) given by \((4.6)\) and 
\[ E_{1int} \] by \((4.4)\) with \( a \simeq 1 \). Note that this quadratic expansion in time is consistent with
the early-time approximation of the mean-field model.

For large \( R \) \((R >> k_1)\), we can assume that \( h = 1 + k_1/R \simeq 1 \), so that \( \partial H/\partial R = 0 \). From \((4.3)\), \((4.7)\) it then follows that the constant energy \( E_1 \) depends only on \( \dot{R} \) and \( N \) \(^5\) so that \( \dot{R} \) depends only on \( N \) and \( \rho \). So the radial velocity \( \dot{R} \) is constant. Thus \( R \propto t \) for
large \( R \), again in agreement with the mean-field approximation.

As mentioned above, for \( D = 6 \), the transverse motion of parallel strings is equivalent
to the motion of particles with two particle energy of interaction given by \((4.2)\) with

\[ h_{12} = 1 + \frac{k}{r_{12}^2} \quad (4.12) \]

where \( r_{12} = 2R \sin \frac{\chi}{2} \), \( \dot{r}_{12} = 2\dot{R} \sin \frac{\chi}{2} \) and \( \chi \) is the azimuthal angle, where we assume that
the 1\(^{st}\) string is located at the north pole of the 3-sphere.

If we assume (as before) that the \( N >> 1 \) strings are distributed homogeneously
on the surface of the 3-sphere, then the number of strings located inside the belt with
azimuthal angles between \( \chi \) and \( \chi + d\chi \) is

\[ dN = \frac{2N}{\pi} \sin^2 \chi d\chi \quad (4.13) \]

and the energy of interaction between the 1\(^{st}\) string (at the north pole) and \( dN \) strings
inside the belt is given by the same expression \((4.3)\) with \( h_{12} \) given by \((4.12)\) and \( dN \) by
\((4.13)\). Assuming (as for \( D = 5 \)) that \( k_2 r_{12}^2 >> 1 \) (i.e. \( R << \sqrt{k_2} \)) and replacing \( h_{12} \simeq \frac{k_2}{r_{12}^2} \)
we obtain

\(^5\) This last statement is obviously also valid for the correct relativistic expression for the kinetic
energy.
\[ dE_{\text{int}} = \frac{2Nm}{\pi} \sin^2 \chi d\chi \left[ \frac{4R^2 \sin^2 \frac{\chi}{2}}{k_2} \left( \frac{1}{\sqrt{1 - \frac{k_2 R^2}{R^2}}} - 1 \right) - 2\dot{R}^2 \sin^2 \frac{\chi}{2} \right] \]  

(4.14)

integrating this expression over \( \chi \) from 0 to \( \pi \) we obtain the energy of interaction between the 1\textsuperscript{st} string and all other strings

\[ E_{\text{int}} = \frac{2mNR^2}{k_2} \left( \frac{1}{\sqrt{1 - \frac{k_2 R^2}{R^2}}} - 1 \right) - mN\dot{R}^2 \]  

(4.15)

and conservation of energy condition can be written in the same form (4.7) as before with \( E_{\text{int}} \) given by (4.15) or

\[ \frac{1}{\sqrt{1 - b}} - 1 - \frac{b}{2} = \frac{\rho k_2}{2NR^2} \]  

(4.16)

where

\[ b = \frac{k_2 \dot{R}^2}{R^2}, \]  

(4.17)

where 0 < b < 1. For \( R << \sqrt{k_2/N} \), it follows from (4.16) that \( 1 - b << 1 \). Alternatively, expanding (4.16) in powers of \( 1 - b \) and keeping only the first nonvanishing term we easily obtain that condition \( b \approx 1 \) (or \( 1 - b << 1 \)) leads to \( R << \sqrt{k_2/N} \). From the definition of \( b \) (4.17) we see that \( R \approx R_0 \exp \sqrt{k_2} \) in this case, again corresponding to the exponentially inflationary expansion in the \( D = 6 \) mean-field model.

On the other hand, the condition \( \frac{\rho k_2}{2NR^2} << 1 \) or \( R >> \sqrt{k_2/N} \) (again \( \rho \) is at least of \( O(1) \)) leads to \( b << 1 \), which can also be easily seen from (4.16). As before, expanding (4.16) in powers of \( b \), keeping the lowest nonvanishing term and then integrating (in order to get \( R = R(t) \)) we obtain

\[ R \approx \sqrt{\frac{\rho}{12Nk_2}} t^2 \]  

(4.18)

For large \( R \) (i.e. \( R >> \sqrt{k_2} \)) we can drop the 1 in Eq (4.12). In this case we obviously obtain the same result as for \( D = 5 \) (and generally any \( D \)): expansion with constant
radial speed, once more in agreement with the mean-field limit. We emphasize again that
the nonrelativistic approximation for the kinetic energy in both cases does not affect the
results for either early or late times.

An interesting possibility in this case is that the moving strings in $D = 6$ lead to an
expanding five-dimensional universe, in which an effective four-dimensional brane universe
resides, following [11]. The asymptotic late time expansion rate is also intriguing, and may
represent a testable prediction for this type of model.

One possible advantage to the type of asymptotically flat universe shown in these
models is that, in contrast to a de Sitter universe, S-matrices would be well-defined [12].
At the same time, these models allow for an accelerating universe without assuming the
existence of a cosmological constant. One can regard the changing acceleration as cor-
responding to an effective cosmological “constant” which varies with time. For example,
the $D = 6$ ($n = 2$) model which has exponential growth in the early universe, has a con-
stant effective cosmological constant, $\Lambda \sim k_2^{-1}$ ([13]) which, however, is due entirely to
the velocity-dependent forces between the strings/branes. At very late times, the effective
cosmological constant is zero. It is then a straightforward but tedious exercise to determine
the exact time dependence of the effective cosmological constant in these models.

Further investigations of this type of model are clearly merited. More complicated and
far more realistic models, possibly involving different species of branes could be considered.
Furthermore, it would be interesting to go beyond the analytic, classical results obtained
above, using a possible combination of numerical computations, quantum string effects and
nonequilibrium thermodynamics. Nevertheless, it is likely that the leading order behaviour
of the type of accelerating string/brane universe considered above is well-described in the
classical approximation. Needless to say, these results await further verification and a
better understanding of the underlying many-body interactions.

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