Extremal Correlator of Three Vertex Operators for Circular Winding Strings in $AdS_5 \times S^5$

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Abstract

We study a three-point correlator of the three heavy vertex operators representing the circular winding string states which are point-like in $AdS_5$ and rotating with two spins and two winding numbers in $S^5$. We restrict ourselves to the case that two of the three vertex operators are located at the same point. We evaluate semiclassically the specific three-point correlator on a stationary splitting string trajectory which is mapped to the complex plane with three punctures. It becomes an extremal and 4d conformal invariant three-point correlator on the boundary. The marginality condition of the vertex operator is discussed.
1 Introduction

The AdS/CFT correspondence [1] has more and more revealed the deep relations between the $\mathcal{N} = 4$ super Yang-Mills (SYM) theory and the string theory in $AdS_5 \times S^5$. A lot of fascinating results have been found in the computation of the planar contribution to the conformal dimensions of non-BPS operators for any value of the coupling constant, using integrability [2].

From the AdS/CFT correspondence the correlation functions in the $\mathcal{N} = 4$ SYM theory can be calculated both at weak and at strong coupling. The three-point correlation functions of BPS operators have been derived at strong coupling in the supergravity approximation [3]. There have been in the past [4, 5, 6, 7, 8, 9, 10] various investigations that correlation functions of operators in $AdS_5 \times S^5$ string theory carrying large charges of order of string tension ($\sim \sqrt{\lambda}$) should be controlled at large $\lambda$ by the semiclassical string solutions.

From the semiclassical procedure the two-point correlators of heavy string vertex operators have been constructed [11, 12, 13] where relevant string surfaces ending at the AdS boundary saturate the correlators. There has been an extension to certain three-point correlator of two heavy string vertex operators with large charges and one light operator representing a BPS state or a non-BPS string light mode [14, 15, 16], where such correlator can be computed by evaluating the light operator on the same classical string solution as saturates the corresponding two-point correlator of the heavy string vertex operators. Further constructions of the three-point correlators of two various heavy string states and a certain light mode have been performed [17, 18, 19, 20, 21]. This approach has been extended to the four-point correlators of two heavy vertex operators and two light operators [22], giant gravitons [23], finite-size effects [24] and various other aspects [25]. The correlators involving Wilson loops and a light local operator have been investigated [26].

In the $\mathcal{N} = 4$ SYM theory side the planar three-point correlators of single trace gauge-invariant operators have been studied at weak coupling [27]. Using the integrability techniques the three-point correlators involving two operators in the SU(3) sector have been computed at weak coupling [28, 29], where a precise match between weak and strong coupling is observed for the correlators of two non-BPS operators in the Frolov-Tseytlin limit and one short BPS operator [29]. This matching has been further demonstrated for the three-point correlators involving two operators in the SL(2,R) closed subsector of $\mathcal{N} = 4$ SYM theory [30] and for the four-point correlators [31].

The three-point correlator of three BMN string states which are point-like in $AdS_5$ and rotating along a great circle of $S^5$ has been computed by making a simple assumption that the three cylinders associated with three external states may be joined together at an intersection point. By taking advantage of the conformal symmetry of $AdS_5$ instead of extremizing with respect to the intersection point the 4d conformal invariant dependence on the three positions of the operator insertion points has been extracted [12]. In ref. (32) using a light-cone gauge for the worldsheet theory the three-point correlator of BMN vertex operators has been computed by explicitly minimizing the action upon varying the intersection point of three Euclidean BMN strings. Further the three-point correlator of the three heavy string vertex operators representing the circular spinning strings with winding numbers in $S^5$ has been constructed to have a 4d conformal invariant expression by solving
the minimization problem for the intersection point.

The classical splitting of strings has been extensively studied \cite{33}, mostly in flat Minkowski space. In the context of the AdS/CFT correspondence, the splitting of the spinning string with large spin in $R_t \times S^5$ has been investigated \cite{34}, and the splitting of the folded spinning string in $AdS_5$ together with spins and winding numbers in $S^5$ has been studied \cite{35}, where a special splitting string solution describing the evolution of the final states after the string splitting is presented. Using the classical integrability of the string $\sigma$-model the general classical splitting string solution in $R_t \times S^5$ has been constructed \cite{36}.

Using the vertex operator prescription we will compute a special type of three-point correlator of the three heavy string vertex operators where two locations of the three vertex operators coincide and each vertex operator represents the circular string state that is point-like in $AdS_5$ and rotating with two spins and two winding numbers in $S^5$. By taking account of the splitting process of a circular winding string into two strings we will construct a stationary splitting string trajectory which controls the three-point correlator. Through the stationary surface sourced by the three vertex operators the semiclassically evaluated correlator will be expressed as an extremal and 4d conformal invariant three-point correlator in the boundary theory. It will be also regarded as a 2d conformal invariant three-point correlator where the three points describe the locations of three punctures that are the vertex operator positions on the worldsheet.

## 2 Three-point correlator and string splitting

Based on the vertex operator prescription \cite{7,11,13} we consider a three-point correlation function of the three heavy string vertex operators which are associated with the circular spinning string solution describing a point-like string in $AdS_5$ and a rotating string with spins and winding numbers $(J_1, m_1)$ and $(J_2, m_2)$ in $S^5$.

The embedding coordinates $Y_M$ ($M = 0, \cdots, 5$) for the Minkowski signature $AdS_5$ are expressed in terms of the global coordinates $(t, \rho, \theta, \phi_1, \phi_2)$ as

$$
Y_5 + iy_0 = \cosh \rho e^{it}, \quad Y_1 + iy_2 = \sinh \rho \cos \theta e^{i\phi_1}, \quad Y_3 + iy_4 = \sinh \rho \sin \theta e^{i\phi_2},
$$

$$
Y^M Y_M = -Y_5^2 + Y^m Y_m + Y_4^4 = -1, \quad Y^m Y_m = -Y_0^2 + Y_5 Y_5
$$

with $m = 0, 1, 2, 3, i = 1, 2, 3$. These coordinates are related with the Poincare coordinates $(z, x^m), ds^2 = z^{-2}(dz^2 + dx^m dx_m)$

$$
Y_m = \frac{x_m}{z}, \quad Y_4 = \frac{1}{2z}(-1 + z^2 + x^m x_m), \quad Y_5 = \frac{1}{2z}(1 + z^2 + x^m x_m)
$$

for $S^5$ the embedding coordinates are also defined by

$$
X_1 \equiv X_4 + iX_5 = \sin \gamma \cos \psi e^{i\varphi_1} = r_1 e^{i\varphi_1}, \quad X_2 \equiv X_3 + iX_4 = \sin \gamma \sin \psi e^{i\varphi_2} = r_2 e^{i\varphi_2},
$$

$$
X_3 \equiv X_5 + iX_6 = \cos \gamma e^{i\varphi_3} = r_3 e^{i\varphi_3}, \quad \sum_{k=1}^3 r_k^2 = 1.
$$

Both the worldsheet time $\tau$ and the global AdS time $t$ are rotated to the Euclidean ones simultaneously

$$
\tau_e = i\tau, \quad t_e = it,
$$

3
which lead to the similar rotations for the time-like coordinates

\[ Y_{0e} = i Y_0, \quad x_{0e} = i x_0. \] (5)

We perform the following conformal transformation to map the Euclidean \((\tau_\epsilon, \sigma)\) world-sheet into the \(\xi\) complex plane with three punctures located at \(\xi = \xi_1 (\tau_\epsilon = -\infty), \xi = \xi_2 (\tau_\epsilon = \infty), \xi = \xi_3 (\tau_\epsilon = \infty)\)

\[ e^{\tau_\epsilon + i \sigma} = \frac{\xi - \xi_1}{(\xi - \xi_2)^{\alpha_2}(\xi - \xi_3)^{\alpha_3}}, \quad \alpha_2 > 0, \quad \alpha_3 > 0. \] (6)

We choose

\[ \alpha_2 + \alpha_3 = 1 \] (7)

so that the infinite point \(\xi = \infty\) is mapped to \((\tau_\epsilon, \sigma) = (0, 0)\). These three arbitrary finite positions \(\xi_1, \xi_2, \xi_3\) will be regarded as the positions where three vertex operators are inserted on the string surface.

Let us consider the vertex operator of dimension \(\Delta\) which describes a point-like string located at the origin in AdS\(_5\) and the circular spinning string with quantum numbers like spins \((J_1, J_2)\) and winding numbers \((m_1, m_2)\) in \(S^5\) [37]

\[ t = \kappa \tau, \quad \rho = 0, \quad \theta = 0, \quad \phi_1 = 0, \quad \varphi_1 = \omega_1 \tau - m_1 \sigma, \quad \varphi_2 = \omega_2 \tau + m_2 \sigma, \quad \gamma = \frac{\pi}{2}. \] (8)

For the integrated vertex operator \(V(x') = \int d^2 \xi V(x(\xi) - x', \cdots)\) specified by the four coordinates \(x'_m = (x'_{0e}, x'_I)\) on the boundary of the AdS\(_5\) space, we define

\[ V(a) = \int d^2 \xi [z + z^{-1}(a \xi - a)]^{-\Delta}(X_1 + i X_2)^{J_1}(X_3 + i X_4)^{J_2}, \] (9)

where the location of the vertex operator in the boundary is chosen by \(x'_m = (a, 0, 0, 0)\) and the winding number dependences are implicitly included through the angular coordinates.

We use this vertex operator to compute a special type of correlator of three vertex operators

\[ < V_{\Delta_1, J_1, m_1, J_2, m_2} (-a) V_{\Delta_3, J_3, m_3} (a) >, \] (10)

where we restrict ourselves to the case that the second and the third vertex operators are located at the same point on the boundary. The Euclidean continuation (11) of (8) is given by

\[ t_e = \kappa \tau_e, \quad \rho = 0, \quad \varphi_1 = -i \omega_1 \tau_e - m_1 \sigma, \quad \varphi_2 = -i \omega_2 \tau_e + m_2 \sigma. \] (11)

For the Euclidean point-like string in AdS\(_5\) we make a dilatation such that \(x_{0e}(\infty) = a, x_{0e}(-\infty) = -a\) at the boundary we have

\[ z = \frac{a}{\cosh \kappa \tau_e}, \quad x_{0e} = a \tanh \kappa \tau_e \] (12)

and the Euclidean spinning string configuration in \(S^5\) is specified by

\[ X_1(\sigma, \tau_e) = r_1 e^{\omega_1 \tau_e - i m_1 \sigma}, \quad X_2(\sigma, \tau_e) = r_2 e^{\omega_2 \tau_e + i m_2 \sigma}. \] (13)
with \( r_1 = \cos \psi, r_2 = \sin \psi, \psi = \text{const.} \)

Here we consider a splitting of the classical circular spinning string with winding numbers in \( S^5 \). Under the Schwarz-Christoffel transformation, \( \rho \equiv \tau_e + i \sigma = \ln(\xi - \xi_1) - \alpha_2 \ln(\xi - \xi_2) - \alpha_3 \ln(\xi - \xi_3), \) \( (14) \)

the ends of string worldsheet at the initial and final times \( \tau_e = -\infty \) and \( \tau_e = \infty \) in the \( \rho \) complex plane are transformed onto the three points \( \xi_1, \xi_2, \xi_3 \) in the \( \xi \) complex plane. The mapping \( (13) \) shows that an initial incoming closed string I splits into an outgoing closed string II with length \( 2\pi\alpha_2 \) and an outgoing closed string III with length \( 2\pi\alpha_3 \). Although we are working in a Euclidean formulation, we refer to each string state as “incoming” and “outgoing”.

The worldsheet in the \( \rho \) complex plane is expressed as the ‘pair of pants’ diagram and consists of three cylinders, which are parameterized by

\[
W_I = \{ (\sigma, \tau_e) | 0 < \sigma \leq 2\pi, \tau_e < \tau_e^0 \}, \\
W_{II} = \{ (\sigma, \tau_e) | 0 < \sigma \leq 2\pi\alpha_2, \tau_e > \tau_e^0 \}, \\
W_{III} = \{ (\sigma, \tau_e) | 2\pi\alpha_2 < \sigma \leq 2\pi, \tau_e > \tau_e^0 \},
\]

\( (15) \)

where \( 0 < \alpha_2 < 1 \) and the \( \sigma \)-interval is periodically identified in each case. At the local interaction point \( \sigma, \tau_e \) specified by the initial circular spinning string solution \( X_k(\sigma, \tau_e) \), \( k = 1, 2 \) as

\[
X_k(\sigma, \tau_e) = \{ X_k(\sigma, \tau_e), \text{ for } \tau_e \leq \tau_e^0, 0 < \sigma \leq 2\pi \}, m_k^I = m_k.
\]

We assume that at the time \( \tau_e = \tau_e^0 \), the two points \( \sigma = 0, \sigma = 2\pi\alpha_2 \) on the initial string coincide in target space and their velocities agree

\[
X_k^I(0, \tau_e^0) = X_k^I(2\pi\alpha_2, \tau_e^0), \quad \partial_\sigma X_k^I(0, \tau_e^0) = \partial_\tau X_k^I(2\pi\alpha_2, \tau_e^0),
\]

\( (17) \)

that is the consistency condition of the self-interaction for the closed string splitting to be possible. The continuity of the string variables at the interaction time \( \tau_e^0 \) is imposed as

\[
\begin{align*}
\begin{cases}
X_k^I(\sigma, \tau_e^0) &= X_k^{II}(\sigma, \tau_e^0), \\
\partial_\sigma X_k^I(\sigma, \tau_e^0) &= \partial_\tau X_k^{II}(\sigma, \tau_e^0)
\end{cases} \quad \text{for } 0 < \sigma \leq 2\pi\alpha_2,
\end{align*}
\]

\( \begin{align*}
\begin{cases}
X_k^I(\sigma, \tau_e^0) &= X_k^{III}(\sigma, \tau_e^0), \\
\partial_\sigma X_k^I(\sigma, \tau_e^0) &= \partial_\tau X_k^{III}(\sigma, \tau_e^0)
\end{cases} \quad \text{for } 2\pi\alpha_2 < \sigma \leq 2\pi.
\end{align*} \)

\( (18) \)

The two outgoing closed string solutions are required to satisfy new periodicity conditions

\[
X_k^{II}(\sigma + 2\pi\alpha_2, \tau_e) = X_k^{II}(\sigma, \tau_e), \quad X_k^{III}(\sigma + 2\pi\alpha_3, \tau_e) = X_k^{III}(\sigma, \tau_e).
\]

\( (19) \)

Now we take a simple separation that the initial circular spinning string solution \( X_k(\sigma, \tau_e) \) remains valid in the outgoing two regions \( W_{II}, W_{III} \)

\[
X_k^{II}(\sigma, \tau_e) = \{ X_k(\sigma, \tau_e), \text{ for } \tau_e^0 \leq \tau_e, 0 < \sigma \leq 2\pi\alpha_2 \},
\]

\[
X_k^{III}(\sigma, \tau_e) = \{ X_k(\sigma, \tau_e), \text{ for } \tau_e^0 \leq \tau_e, 2\pi\alpha_2 < \sigma \leq 2\pi \}.
\]

\( (20) \)
We regard (20) as simple special solutions for the outgoing strings which obey the boundary condition (18) and the relevant equations of motion.

The consistency condition (17) implies \( m_k^I \alpha_2 \in \mathbb{Z}^+ \) so that we put

\[
m_k^{II} = m_k^I \alpha_2, \quad (k = 1, 2)
\]

by introducing positive integers \( m_k^{II} \). The interaction position is fixed as \( \sigma^0 = 2\pi m_k^{II}/m_k^I \).

The periodicity condition (19) constrains the behaviors of the two fragments to yield

\[
m_k^I \alpha_2 \in \mathbb{Z}^+, \quad m_k^I \alpha_3 = m_k^I (1 - \alpha_2) \in \mathbb{Z}^+,
\]

which hold owing to (21). We introduce positive integers \( m_k^{III} \) again to put

\[
m_k^{III} = m_k^I \alpha_3, \quad (k = 1, 2).
\]

After rescaling of the worldsheet space coordinate \( \sigma \) to \( \sigma^{II}, \sigma^{III} \) in such a way that

\[
0 < \sigma < 2\pi \alpha_2 \rightarrow 0 < \sigma^{II} \leq 2\pi, \quad 2\pi \alpha_2 < \sigma < 2\pi \rightarrow 0 < \sigma^{III} \leq 2\pi
\]

\[
\sigma^{II} = \frac{\sigma}{\alpha_2}, \quad \sigma^{III} = \frac{\sigma - 2\pi \alpha_2}{\alpha_3},
\]

we rewrite the two outgoing string configurations (20) as

\[
X_k^{II}(\sigma^{II}, \tau_e) = r_k e^{i\omega_k \tau_e + (-1)^k i m_k^{II} \sigma^{II}},
\]

\[
X_k^{III}(\sigma^{III}, \tau_e) = r_k e^{i\omega_k \tau_e + (-1)^k i m_k^{III} \sigma^{III}},
\]

which are specified by the common frequencies \( \omega_k \). From these expressions the positive integers \( m_k^{II} \) are interpreted as the winding numbers of the string II on the outgoing cylinder \( W_{II} \), while the positive integers \( m_k^{III} \) as the winding numbers of the string III on the outgoing cylinder \( W_{III} \). We observe that in this simple separation of string the winding numbers are conserved owing to (7)

\[
m_k^I = m_k^{II} + m_k^{III}, \quad (k = 1, 2).
\]

Thus the splitting process is characterized by the separations of the winding numbers of the initial incoming string through the self-interaction of the closed string, where the winding number \( m_1^I \) is separated into \( m_1^{II} \) and \( m_1^{III} \) in the same way as \( m_2^I \) into \( m_2^{II} \) and \( m_2^{III} \). The world surface does not change such that under suitable rescaling of the worldsheet space coordinate \( \sigma \), the two outgoing string solutions (25) described by the corresponding winding numbers become the same type expression as the initial string solution.

The energy of the initial string is given by

\[
E^I = \sqrt{\lambda} \int_0^{2\pi} \frac{d\sigma}{2\pi \kappa} = \kappa \sqrt{\lambda}
\]

and the energies of the two fragments are obtained by

\[
E^{II} = \sqrt{\lambda} \int_0^{2\pi \alpha_2} \frac{d\sigma}{2\pi \kappa} = \alpha_2 \kappa \sqrt{\lambda},
\]

\[
E^{III} = \sqrt{\lambda} \int_{2\pi \alpha_2}^{2\pi} \frac{d\sigma}{2\pi \kappa} = \alpha_3 \kappa \sqrt{\lambda}.
\]
Owing to (7) the string energy is conserved in the splitting process
\[ E^I = E^{II} + E^{III}. \]  
Similarly the two spins \( J^I_1, J^I_2 \) of the initial string are given by
\[ J^I_1 = \sqrt{\lambda} \int_0^{2\pi} \frac{d\sigma}{2\pi} r^2_1 \omega_1 = \sqrt{\lambda} r^2_1 \omega_1, \quad J^I_2 = \sqrt{\lambda} \int_0^{2\pi} \frac{d\sigma}{2\pi} r^2_2 \omega_2 = \sqrt{\lambda} r^2_2 \omega_2, \]  
while the two spins \( J^{II}_1, J^{II}_2 \) of the fragment II and the two spins \( J^{III}_1, J^{III}_2 \) of the fragment III are expressed as
\[ J^{II}_1 = \alpha_2 \sqrt{\lambda} r^2_1 \omega_1, \quad J^{II}_2 = \alpha_2 \sqrt{\lambda} r^2_2 \omega_2, \]
\[ J^{III}_1 = \alpha_3 \sqrt{\lambda} r^2_1 \omega_1, \quad J^{III}_2 = \alpha_3 \sqrt{\lambda} r^2_2 \omega_2. \]  
We have the conservation of each spin
\[ J^I_1 = J^{II}_1 + J^{III}_1, \quad J^I_2 = J^{II}_2 + J^{III}_2. \]  

3 Extremal three-point correlator

In order to compute semiclassically the correlator (10), we express the Euclidean action accompanied with the vertex contributions as an integral over the \( \xi \) complex plane with three punctures describing the splitting string worldsheet
\[ A_e = \frac{\sqrt{\lambda}}{\pi} \int d^2 \xi \left[ \frac{1}{z^2} (\partial z \bar{\partial} z + \partial x_0 e \bar{\partial} x_0 e) + \frac{1}{2} \sum_{k=1}^{2} (\partial X_k \bar{\partial} X_k + \partial \bar{X}_k \partial X_k) \right] \\
- \Delta^I \int d^2 \xi \delta^2(\xi - \xi_1) \ln \frac{z}{z^2 + (x_{0e} + a)^2} - \Delta^{II} \int d^2 \xi \delta^2(\xi - \xi_2) \ln \frac{z}{z^2 + (x_{0e} - a)^2} \\
- \Delta^{III} \int d^2 \xi \delta^2(\xi - \xi_3) \ln \frac{z}{z^2 + (x_{0e} - a)^2} - \sum_{k=1}^{2} J^I_k \int d^2 \xi \delta^2(\xi - \xi_1) \ln r_k e^{i\varphi_k} \\
- \sum_{k=1}^{2} J^{II}_k \int d^2 \xi \delta^2(\xi - \xi_2) \ln r_k e^{-i\varphi_k} - \sum_{k=1}^{2} J^{III}_k \int d^2 \xi \delta^2(\xi - \xi_3) \ln r_k e^{-i\varphi_k}. \]  

We will show that the Euclidean string solution (12), (13) expressed in terms of the complex worldsheet coordinate \( \xi \) becomes the stationary splitting string trajectory in the presence of three vertex operators as source terms. The equation of motion for \( x_{0e} \) is given by
\[ \frac{\partial^2 x_{0e}}{z^2} + \frac{\partial^2 x_{0e}}{\bar{z}^2} = \frac{2\pi}{\sqrt{\lambda}} \left[ \Delta^I \frac{x_{0e} + a}{z^2 + (x_{0e} + a)^2} \delta^2(\xi - \xi_1) + \Delta^{II} \frac{x_{0e} - a}{z^2 + (x_{0e} - a)^2} \delta^2(\xi - \xi_2) \\
+ \Delta^{III} \frac{x_{0e} - a}{z^2 + (x_{0e} - a)^2} \delta^2(\xi - \xi_3) \right]. \]
The inversion of the conformal transformation (6) is expressed as
\[
\begin{align*}
\tau_e &= \frac{1}{2} \ln \frac{\xi - \xi_1}{\xi - \xi_2}\frac{(\xi - \xi_1)(\xi - \bar{\xi}_1)}{(\xi - \xi_2)(\xi - \bar{\xi}_2)(\xi - \xi_3)(\xi - \bar{\xi}_3)^{\alpha_3}}, \\
\sigma &= \frac{1}{2t} \ln \frac{\xi - \xi_1}{\xi - \xi_2}\frac{(\xi - \xi_1)(\xi - \bar{\xi}_1)(\xi - \bar{\xi}_2)^{\alpha_2}(\xi - \xi_3)^{\alpha_3}},
\end{align*}
\]
which produce
\[
(\partial \bar{\partial} + \bar{\partial} \partial)\tau_e = \pi |\delta^2(\xi - \xi_1) - \alpha_2 \delta^2(\xi - \xi_2) - \alpha_3 \delta^2(\xi - \xi_3)|, \quad (\partial \bar{\partial} + \bar{\partial} \partial)\sigma = 0. \tag{36}
\]

The substitution of (12) into (34) leads to
\[
\kappa (\partial \bar{\partial} + \bar{\partial} \partial)\tau_e = \frac{\pi}{\sqrt{\lambda}} [\Delta^I \delta^2(\xi - \xi_1) - \Delta^I \delta^2(\xi - \xi_2) - \Delta^I \delta^2(\xi - \xi_3)], \tag{37}
\]
which is satisfied through (36) if each dimension is given by
\[
\Delta^I = \kappa \sqrt{\lambda}, \quad \Delta^I = \alpha_2 \kappa \sqrt{\lambda}, \quad \Delta^I = \alpha_3 \kappa \sqrt{\lambda}, \tag{38}
\]
which correspond to (27), (28).

We turn to the equation of motion for \(z\)
\[
\frac{\partial^2 \bar{z}}{z^2} + \frac{\partial^2 z}{z^2} + \frac{2}{z^2} (\partial z \bar{\partial} z + \partial x_{0*} \bar{\partial} x_{0e}) = \frac{\pi}{\sqrt{\lambda} z^2} \left[ \Delta^I \frac{z^2 - (x_{0e} + a)^2}{x^2 + (x_{0e} + a)^2} \delta^2(\xi - \xi_1)
+ \Delta^I \frac{z^2 - (x_{0e} - a)^2}{x^2 + (x_{0e} - a)^2} \delta^2(\xi - \xi_2) + \Delta^I \frac{z^2 - (x_{0e} + a)^2}{x^2 + (x_{0e} - a)^2} \delta^2(\xi - \xi_3) \right]. \tag{39}
\]
Since the limits \(\tau_e \to -\infty\) and \(\tau_e \to \infty\) correspond to the limits \(\xi \to \xi_1\) and \(\xi \to \xi_{2,3}\) respectively it becomes
\[
\kappa (\partial \bar{\partial} + \bar{\partial} \partial)\tau_e = -\frac{\pi}{\sqrt{\lambda} \tanh \kappa \tau_e} [\Delta^I \delta^2(\xi - \xi_1) + \Delta^I \delta^2(\xi - \xi_2) + \Delta^I \delta^2(\xi - \xi_3)], \tag{40}
\]
which is reduced to (37).

The kinetic term for \(S^5\) in (33) is rewritten by \(\sqrt{\lambda}/\pi \int d^2 \xi \sum_k (\partial r_k \bar{\partial} r_k + r_k^2 \partial \varphi_k \bar{\partial} \varphi_k)\). Therefore the equations of motion for \(\varphi_k\) \((k = 1, 2)\) read
\[
r_k^2 (\partial \bar{\partial} + \bar{\partial} \partial) \varphi_k = \frac{i \pi}{\sqrt{\lambda}} [J^I_k \delta^2(\xi - \xi_1) - J^I_k \delta^2(\xi - \xi_2) - J^I_k \delta^2(\xi - \xi_3)], \tag{41}
\]
which are satisfied by \(\varphi_k\) in (11) through (30), (31). The equations of motion for \(r_k\) have additional contributions to the singular part. Here we assume that they are still satisfied by constant \(r_k\).

Now the three-point correlator can be calculated semiclassically by evaluating string action with source terms on the stationary splitting string trajectory. It is convenient to go
back to the Euclidean cylindrical worldsheet coordinates \((\tau_e, \sigma)\) through (33) for computing the string action in (33):

\[
A_{str} = \frac{\sqrt{\lambda}}{4\pi} \left[ \int_{\tau_e(\xi_1)}^{\tau_e(\xi_2)} d\tau_e \int_0^{2\pi} d\sigma + \int_{\tau_e(\xi_3)}^{\tau_e(\xi_4)} d\tau_e \int_0^{2\pi\alpha_2} d\sigma + \int_{\tau_e(\xi_5)}^{\tau_e(\xi_6)} d\tau_e \int_2^{2\pi} d\sigma \right] L_{str},
\]

\[
L_{str} = \frac{1}{z^2} ((\partial_{\tau_e} z)^2 + (\partial_{\tau_e} x_0 e)^2) + \frac{2}{2} \sum_{k=1}^{r_k^2 ((\partial_{\tau_e} \varphi_k)^2 + (\partial_{\sigma} \varphi_k)^2)}
\]

(42) with

\[
\tau_e(\xi_k) = \frac{1}{2} (\ln |\xi - \xi_1|^2 - \alpha_2 \ln |\xi - \xi_2|^2 - \alpha_3 \ln |\xi - \xi_3|^2),
\]

(43)

where the integral region is divided into three sectors according to (15).

Taking into account that the two outgoing string solutions are identical to the incoming string solution we substitute the stationary string solution (11) or the equivalent one (12) with (16), (20) into (42) and then perform the integral over \(\tau_e\) and \(\sigma\) to have

\[
A_{str} = \frac{\sqrt{\lambda}}{2} [k^2 + \frac{2}{2} \sum_{k=1}^{r_k^2 \left(-\omega_k^2 + (m_k^2)^2\right)} (\alpha_2 \ln |\xi_1 - \xi_2|^2 - \alpha_2 \alpha_3 \ln |\xi_2 - \xi_3|^2 + \alpha_3 \ln |\xi_3 - \xi_1|^2)],
\]

(44)

where the one-point function divergence is ignored. It is observed that the dependence on \(\tau_e^0\) disappears, although the splitting time \(\tau_e^0\) is specified by \(\alpha_2, \xi_i (i = 1, 2, 3)\) through the turning point condition. The source terms associated with \(AdS_5\) in (33) are evaluated using the delta-function as

\[
A_{sour}(AdS_5) = -\Delta^I \left[ \frac{\lambda}{2} (\alpha_2 \ln |\xi_1 - \xi_2|^2 + \alpha_3 \ln |\xi_3 - \xi_1|^2) - \ln 2a \right]
\]

\[
- \Delta^II \left[ \frac{\lambda}{2} (\alpha_2 \ln |\xi_1 - \xi_2|^2 - \alpha_3 \ln |\xi_2 - \xi_3|^2) - \ln 2a \right]
\]

\[
- \Delta^III \left[ \frac{\lambda}{2} (\alpha_2 \ln |\xi_3 - \xi_1|^2 - \alpha_2 \ln |\xi_2 - \xi_3|^2) - \ln 2a \right].
\]

The source terms associated with \(S^5\) in (33) read

\[
A_{sour}(S^5) = \frac{1}{4} \ln |\xi_1 - \xi_2|^2 [\omega_1 J_1^I + \omega_2 J_2^I + \alpha_2 (\omega_1 J_1^I + \omega_2 J_2^I)]
\]

\[- \frac{1}{4} \ln |\xi_2 - \xi_3|^2 [\alpha_2 (\omega_1 J_1^III + \omega_2 J_2^III) + \alpha_3 (\omega_1 J_1^I + \omega_2 J_2^I)]
\]

\[+ \frac{1}{4} \ln |\xi_3 - \xi_1|^2 [\alpha_3 (\omega_1 J_1^I + \omega_2 J_2^I) + \omega_1 J_1^III + \omega_2 J_2^III]
\]

\[+ \frac{1}{2} (m_1^I J_1^I - m_1^II J_1^II) \ln \frac{(\xi_1 - \xi_2)^{\alpha_2 (\xi_1 - \xi_2)}^{\alpha_2 (\xi_1 - \xi_2)} - \frac{1}{2} (m_2^I J_1^I - m_1^II J_1^II) \ln \frac{(\xi_2 - \xi_3)^{\alpha_2 (\xi_2 - \xi_3)}^{\alpha_2 (\xi_2 - \xi_3)} - \frac{1}{2} (m_1^I J_1^III - m_2^II J_2^III) \ln \frac{(\xi_2 - \xi_1)^{\alpha_2 (\xi_2 - \xi_1)}^{\alpha_2 (\xi_2 - \xi_1)}}{(\xi_2 - \xi_1)^{\alpha_2 (\xi_2 - \xi_1)}^{\alpha_2 (\xi_2 - \xi_1)}}.
\]

(46)

Since the coefficients in the last two terms in (46) are expressed as

\[
m_1^I J_1^II - m_2^II J_2^II = \alpha_2 (m_1^I J_1^I - m_2^II J_2^II), \quad m_1^I J_1^III - m_2^II J_2^III = \alpha_3 (m_1^I J_1^I - m_2^II J_2^II),
\]

(47)

we put

\[
m_1^I J_1^I = m_2^II J_2^II
\]

(48)
to obtain a 2d conformal invariant expression through (30) and (31)

\[
< V_{\Delta^1, j_1^1, m_1^1, j_2^1, m_2^1, j_3^1, m_3^1} (-a) V_{\Delta^II, j_1^II, m_1^II, j_2^II, m_2^II, j_3^II, m_3^II} (a) V_{\Delta^III, j_1^III, m_1^III, j_2^III, m_2^III, j_3^III, m_3^III} (a) > 
\]

\[
\approx \int d^2 \xi_1 d^2 \xi_2 d^2 \xi_3 e^{-A_{str} - A_{sour}(AdS_5) - A_{sour}(S^5)} 
\]

\[
\approx \frac{1}{(2a)^{\Delta^1 + \Delta^II + \Delta^III}} \int d^2 \xi_1 d^2 \xi_2 d^2 \xi_3 \frac{1}{|\xi_1 - \xi_2|^{\Delta^1} |\xi_2 - \xi_3|^{\Delta^II} |\xi_3 - \xi_1|^{\Delta^III}} 
\]

\[
\times \frac{1}{|\xi_2 - \xi_3| - \sqrt{a_2 \alpha_1 (\kappa^2 + \sum_k r_k^2 (\omega_k^2 + (m_k^2)^2) - \frac{\Delta^1}{\sqrt{a}}) (\alpha_3 \Delta^II + \Delta^III)}} 
\]

\[
\times \frac{1}{|\xi_3 - \xi_1| - \sqrt{a_3 \alpha_1 (\kappa^2 + \sum_k r_k^2 (\omega_k^2 + (m_k^2)^2) - \frac{\Delta^1}{\sqrt{a}}) (\alpha_3 \Delta^II + \Delta^III)}} 
\]

(49)

This correlator is regarded as a space-time three-point correlator of the boundary theory at strong coupling where two space-time points coincide, while it is in itself defined as a worldsheet three-point correlator of the string theory where the three points are associated with the three punctures on the worldsheet. The dimensions $\Delta^1$, $\Delta^II$, $\Delta^III$ are identified with the incoming string energy $E^I$ and the outgoing string energies $E^II$, $E^III$ respectively.

Owing to the relations $\Delta^II = \alpha_2 \Delta^1$, $\Delta^III = \alpha_3 \Delta^1$ in (38) the three-point correlator (49) turns out to be

\[
\frac{1}{(2a)^{\Delta^1 + \Delta^II + \Delta^III}} \int d^2 \xi_1 d^2 \xi_2 d^2 \xi_3 \frac{1}{|\xi_1 - \xi_2|^{\alpha_2 \sqrt{\lambda}} |\xi_2 - \xi_3|^{\alpha_2 \alpha_3 \sqrt{\lambda}} |\xi_3 - \xi_1|^{\alpha_3 \sqrt{\lambda}}} 
\]

(50)

with

\[
f = \kappa^2 + \sum_{k=1}^2 r_k^2 (\omega_k^2 + (m_k^2)^2) - \frac{2 \kappa \Delta^1}{\sqrt{\lambda}}. 
\]

(51)

From the marginality condition of vertex operator the worldsheet three-point correlator should take a 2d scaling behavior $|\xi_1 - \xi_2|^{-2} |\xi_2 - \xi_3|^{-2} |\xi_3 - \xi_1|^{-2}$ which in the large spin leads to

\[
\kappa^2 + \sum_{k=1}^2 r_k^2 (\omega_k^2 + (m_k^2)^2) - \frac{2 \kappa \Delta^1}{\sqrt{\lambda}} = 0. 
\]

(52)

The circular $(J_1, J_2)$ string solution with winding numbers $(m_1, m_2)$ was constructed from the off-diagonal Virasoro constraint that agrees with (48) and the diagonal Virasoro constraint

\[
\frac{2 \kappa E}{\sqrt{\lambda}} - \kappa^2 = 2 \sum_{k=1}^2 \sqrt{m_k^2 + \nu^2} J_k - \nu^2 
\]

(53)

with $J_k = r_k^2 \omega_k$, $\omega_k = \sqrt{m_k^2 + \nu^2}$. The identity $\nu^2 = \sum_k r_k^2 \nu^2 = \sum_k r_k^2 (\omega_k^2 - m_k^2)$ shows that (52) coincides with (53) since dimension $\Delta^1$ is identified with the incoming string energy $E^I = E$.

Because of the energy conservation (29), that is, $\Delta^I = \Delta^II + \Delta^III$ the correlator (50) exhibits the expected scaling behavior of the extremal three-point correlator

\[
\frac{1}{(2a)^{\Delta^I}} 
\]

(54)
where $2a$ is the distance between the vertex operator of the initial string state I and the vertex operators of the final string states II, III in the boundary of the Euclidean $AdS_5$ space.

The energy-spin relation derived from (53) or (52) is expressed in the $\lambda/(J_1)^2$ expansion with $J^I = J_1^I + J_2^I$ as

$$\Delta^I = E^I = J^I + \frac{\lambda}{2(J_1)^2} \sum_{k=1}^2 (m_k^I)^2 J_k^I - \frac{\lambda^2}{8(J_1)^4} \sum_{k=1}^2 (m_k^I)^4 J_k^I + \cdots.$$  \hspace{1cm} (55)

From this expression we use (27), (28) and (30), (31) to obtain

$$\Delta^{II} = J^{II} + \frac{\lambda \alpha^2}{2(J^{II})^2} \sum_{k=1}^2 (m_k^{II})^2 J_k^{II} - \frac{\lambda^2 \alpha^4}{8(J^{II})^4} \sum_{k=1}^2 (m_k^{II})^4 J_k^{II} + \cdots$$  \hspace{1cm} (56)

with $J^{II} = J_1^{II} + J_2^{II}$ and

$$\Delta^{III} = J^{III} + \frac{\lambda \alpha^2}{2(J^{III})^2} \sum_{k=1}^2 (m_k^{III})^2 J_k^{III} - \frac{\lambda^2 \alpha^4}{8(J^{III})^4} \sum_{k=1}^2 (m_k^{III})^4 J_k^{III} + \cdots.$$  \hspace{1cm} (57)

with $J^{III} = J_1^{III} + J_2^{III}$. Further, the relations (21), (23) yield the following expressions of the energies of the two string fragments

$$\Delta^{II} = J^{II} + \frac{\lambda}{2(J^{II})^2} \sum_{k=1}^2 (m_k^{II})^2 J_k^{II} - \frac{\lambda^2}{8(J^{II})^4} \sum_{k=1}^2 (m_k^{II})^4 J_k^{II} + \cdots,$$

$$\Delta^{III} = J^{III} + \frac{\lambda}{2(J^{III})^2} \sum_{k=1}^2 (m_k^{III})^2 J_k^{III} - \frac{\lambda^2}{8(J^{III})^4} \sum_{k=1}^2 (m_k^{III})^4 J_k^{III} + \cdots.$$  \hspace{1cm} (58)

These spin-energy relations for the outgoing strings II, III are accompanied with

$$m_1^{II} J_1^{II} = m_2^{II} J_2^{II}, \quad m_1^{III} J_1^{III} = m_2^{III} J_2^{III},$$  \hspace{1cm} (59)

which are also given from (48).

4 Conclusion

Using the integrated vertex operators we have computed a specific three-point correlator of the heavy string vertex operators representing the circular winding string states which are point-like in $AdS_5$ and rotating with two spins and two winding numbers in $S^5$, where two locations of vertex operators are the same.

We have constructed a Schwarz-Christoffel mapping from the complex plane with three punctures to the splitting string surface consisting of three cylinders. Taking advantage of this special mapping and considering the appropriate saddle-point surface we have semiclassically evaluated the correlator of the string vertex operators located on three punctures. We have shown that the Euclidean continuation of the circular winding string solution mapped on the complex plane with three punctures solves the relevant equations of motion on the
complex plane with the delta-function sources at three insertion worldsheet points of vertex operators.

In the process of a simple separation of string such that the two outgoing string solutions are identical to the incoming string solution, the worldsheet space position of splitting is determined by using the consistency condition for splitting and specified by the ratio of the relevant winding numbers. The string splitting is characterized by the separation of the initial winding number into two winding numbers at the self-interaction of the incoming closed string so that winding numbers, energy and spins are conserved in the same way.

By analyzing the restricted three-point correlator we have observed that the marginality condition of the vertex operator for the worldsheet scaling behavior of the semiclassically evaluated three-point correlator yields the same dispersion relation among energy (dimension), spins and winding numbers as is obtained from the diagonal Virasoro constraint for the circular winding string solution, while the requirement for the worldsheet three-point correlator to have the 2d conformal invariant expression gives the same relation between spins and winding numbers as follows from the off-diagonal Virasoro constraint.

We have demonstrated that owing to the same separation behavior in the conservations of energy, spins and winding numbers the dispersion relations of the two outgoing strings become the same expression as the dispersion relation of the initial incoming string.

We have observed that the resulting three-point correlator shows the space-time scaling behavior of the extremal correlator on the boundary theory, which is associated with the energy conservation in the string splitting. This extremal behavior seems to be related with the choice of the Schwarz-Christoffel mapping which resembles the conformal mapping describing the three-string interaction in the light-cone gauge for the string field theory.

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