Radiative Decays of the P-wave Charmed Mesons

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Abstract

The predictions of the large mass limit for the radiative decays of the known p-wave charmed mesons are analyzed. Special attention is devoted to the problem of gauge invariance of the transition matrix elements. The width ratios arising from heavy quark symmetry are given for the different multipole components. Finally, some estimates for the rates of these decays are given, using the constituent quark model of Isgur, Scora, Grinstein and Wise.

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1. Introduction

The strong, weak and electromagnetic interactions of mesons containing one heavy quark can be described in terms of an effective theory which is simultaneously invariant under heavy quark spin–flavour symmetry, the chiral $SU_L(2) \times SU_R(2)$ and the $U_{e.m.}(1)$ gauge group. This effective theory has recently been applied \cite{1,2} to predict the $B^* \to B\gamma$ decay widths from the CLEO data\cite{3} on branching ratios for $D^{*+} \to D\gamma(\pi)$. There is, as yet, no similar data on radiative decays of the known $p$–wave D mesons, although the mode $D_{s1}^{*+}(2536) \to D_{s1}^{*}\gamma$ is reported \cite{4} as having possibly been seen. However, the situation might change soon if enough statistics becomes available. It is therefore of some interest to see what information can be obtained about these decays by using solely the symmetries of the large mass limit. In the present paper we study the radiative decays of the members of the $s_{\pi\ell\ell=3/2}^+ \pi$ multiplet of charmed mesons to the respective charmed $s$-wave states. Previous work on the same problem has been done in \cite{3,5,6}. These authors employed a specific dual model and tensor meson dominance and/or made use of the $SU(4)$ flavour symmetry to extract the necessary couplings. Such an approach is clearly not compatible with the large mass effective theories referred to above. We prefer instead to describe the electromagnetic interactions of these mesons using a method which explicitly displays the new spin and flavour symmetries appearing in the heavy mass limit. We show that the heavy quark contribution to the matrix elements of these transitions is, up to $O(1/m_c)$, completely determined in terms of the same Isgur–Wise function $\xi_{3/2}^s(v \cdot v')$ which describes the semileptonic decays of the $\bar{B}$ mesons to excited $s_{\ell}^{\pi} = 3^+ \pi$ p–wave D–mesons\cite{7}. Ratios of partial widths are given for the different multipole components. Finally, the constituent quark model of Isgur, Scora, Grinstein and Wise \cite{9} is used in order to make some estimates for the widths of these decays. The result is that the decays of the $D_{s}^{+}$ mesons are dominated by the magnetic quadrupole $M2$ mode while the $D^{0}$ mesons decay predominantly through an electric dipole $E1$ mode.

2. The method

We are interested in the matrix elements of the electromagnetic current

$$J^{\mu}_{e.m.} = \frac{2}{3} \bar{u} \gamma^{\mu} u - \frac{1}{3} \bar{d} \gamma^{\mu} d - \frac{1}{3} \bar{s} \gamma^{\mu} s + \frac{2}{3} \bar{c} \gamma^{\mu} c \tag{1}$$

taken between one of the $s$–wave charm meson states and one of the two degenerate states of the $s_{\ell}^{\pi} = 3^+ \pi$ p–wave multiplet:

$$\mathcal{M}^{\mu}_{ij} = \langle M_i(v') | J^{\mu}_{e.m.} | M^{*\mu}_j(v) \rangle \tag{2}$$

1More precisely, this is only true for the $E1$ component of the transition.
where $M_i(v'), M_j^{**}(v)$ are generic notations for the respective meson states, $(M_1, M_2) = (D, D^*)$ and $(M_1^{**}, M_2^{**}) = (D_1, D_2^*)$. Their fields can be combined, as usual $\Sigma'$ into a Dirac matrix

$$H = \frac{1 + \gamma'}{2}[D^{\ast \lambda} \gamma_\lambda - D \gamma_5] \quad (3)$$

$$T^\alpha = \frac{1 + \gamma'}{2}\{D_2^{\ast \alpha \lambda} \gamma_\lambda - \frac{1}{\sqrt{6}}D_1^{\lambda} \gamma_5[3 g_\lambda - \gamma_\lambda(\gamma^\alpha - v^\alpha)]\}. \quad (4)$$

We will choose to evaluate the matrix element (2) of the first three terms in (1) (the light quarks’ electromagnetic current) in a somewhat different way from the last one (the heavy quark electromagnetic current). Namely, the former is given in the effective theory by the matrix elements of the most general operator which includes the fields (3-4), has positive parity and is gauge invariant. To second order in the photon momentum it can be written as

$$n_\mu n_\mu^{\text{light}} = f_1 \text{Tr}[\bar{H} T^\alpha \gamma^\beta F_{\alpha \beta}] + f_2 \text{Tr}[\bar{H} T^\alpha v^\beta F_{\alpha \beta}]$$

$$+ g_1 \Lambda_\chi \text{Tr}[\bar{H} T^\alpha \partial_\lambda \sigma^{\lambda \beta} F_{\alpha \beta}] + g_2 \Lambda_\chi \text{Tr}[\bar{H} T^\alpha \partial_\sigma \partial_\mu \partial_\mu F_{\sigma \mu}] + \frac{i g_3}{\Lambda_\chi} \text{Tr}[\bar{H} T^\alpha (\gamma^\lambda v^\beta + \gamma^\beta v^\lambda) \partial_\lambda F_{\mu \nu}] \quad (5)$$

Here $F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor and $n_\mu$ is the polarization vector of the emitted photon. Note that the velocities of the two heavy meson fields in (5) must be taken to be equal $v = v'$, since otherwise the invariance under the $SU_{\text{spin}}(2)$ heavy quark spin symmetry group is lost. Terms with one derivative acting on the heavy meson fields are forbidden by reparametrization invariance constraints, as shown in $\Sigma'$ for the case of the strong interactions of the same mesons. $f, g, h$ and $j$ are unknown constants which have to be determined from a comparison with experiment. $\Lambda_\chi = 1 \text{ GeV}$ suppresses terms with one derivative on the photon field.

The matrix element (2) of the last term in (1) will be evaluated by using usual HQET methods $\Sigma''$. The vector current $\bar{c} \gamma_\mu c$ is matched in the HQET to $\mathcal{O}(1/m_c)$ onto the operator

$$\bar{j}_\mu^{\text{heavy}} = e^{im_c(v' - v) \cdot x} \{ \bar{h}^{(c)}_v \gamma_\mu h^{(c)}_{v'} - \frac{i}{2m_c} \bar{h}^{(c)}_{v'} [\overrightarrow{D} \gamma_\mu - \gamma_\mu \overleftarrow{D}] h^{(c)}_v \} \quad (6)$$

The two velocities $v, v'$ are taken to be different here and satisfy

$$M_{P_{3/2}} v_\mu = M_S v'_\mu + k_\mu \quad (7)$$

with $k_\mu$ the photon momentum. Defining the binding energies

$$\bar{\Lambda}' = M_S - m_c, \quad \bar{\Lambda} = M_{P_{3/2}} - m_c \quad (8)$$

$^2$A factor of $\sqrt{M_S}/(\sqrt{M_{P_{3/2}}})$ is absorbed into the field $H (T^\alpha)$. 

2
we have

\[ v \cdot k = (\bar{\Lambda} - \bar{\Lambda}')[1 - \frac{\bar{\Lambda} - \bar{\Lambda}'}{2m_c} + \frac{\bar{\Lambda}(\bar{\Lambda} - \bar{\Lambda}')}{2m_c^2} + \mathcal{O}(\frac{m^3}{m_c^3}, \frac{\bar{\Lambda}^3}{m_c^3})] \]

\[ v \cdot v' = 1 + \frac{(\bar{\Lambda} - \bar{\Lambda}')^2}{2m_c^2} + \mathcal{O}(\frac{\bar{\Lambda}^3}{m_c^3}, \frac{\bar{\Lambda}'}{m_c^3}). \]

(9) (10)

It can be seen from (10) that \( v \cdot v' \) differs from 1 only through terms of \( \mathcal{O}(1/m_c^2) \).

This provides the justification for our assumption about the equality of the two velocities \( v \) and \( v' \) built into the equation (5). However, in considering the matrix elements of the heavy quark current (6), we will stick to a two–velocities description, as a means for introducing the photon momentum \( k_\mu \) via (7).

Using the usual trace formalism, we have

\[ \langle M_i(v')|\tilde{J}_\mu{}^{\text{heavy}}(x)|M_j^{**}(v) \rangle = e^{i(M_vv' - M_{P_{3/2}}v)} \sqrt{M_S M_{P_{3/2}}} \text{Tr}[\hat{M}_i^1 \hat{M}_j^{**\alpha\alpha'} \xi_{3/2}(v \cdot v')] \]

where only the leading term in (6) has been retained. \( \hat{M}_i \) and \( \hat{M}_j^{**\alpha} \) are the usual interpolating fields associated with the respective meson fields:

\[ (\hat{M}_1^1, \hat{M}_2^1) = (\gamma_5, \epsilon^*), \]

\[ (\hat{M}_1^{**\alpha}, \hat{M}_2^{**\alpha}) = \left(-\frac{1}{\sqrt{6}} \epsilon^\alpha \gamma_5 [3g^\sigma - \gamma_\lambda (\gamma^\alpha - v^\alpha)] , \epsilon^\alpha \gamma_\lambda \right). \]

(11) (12) (13)

Unfortunately, the matrix element (11) is not gauge invariant. Contracting equ. (11) with \( k_\mu = M_{P_{3/2}}v_\mu - M_{Sv'\mu} \) gives a result of order \( \mathcal{O}(m_0^0) \), proportional to \( (\bar{\Lambda} - \bar{\Lambda}') \).

It is not difficult to trace the origin of this difficulty: it is the nonconservation of the HQET vector current (6) when only the first term is retained. In order to have vector current conservation, the second term in (6) must be included as well. Only then we have

\[ \partial_\mu \tilde{J}_\mu{}^{\text{heavy}} = \mathcal{O}(1/m_c). \]

(14)

Of course, vector current conservation holds true up to any given order in \( 1/m_c \), provided the necessary additional terms are added to (6).

On the other hand, the inclusion of the second term in (6) requires also consideration of the \( \frac{1}{m_c} \)–terms in the heavy quark effective lagrangian

\[ \mathcal{L}_{HQET} = \bar{h}_v^{(c)}(iv \cdot D)h_v^{(c)} + \frac{1}{2m_c} \bar{h}_v^{(c)}(iD)^2h_v^{(c)} - \frac{g}{4m_c} \bar{h}_v^{(c)}\sigma \cdot G h_v^{(c)}, \]

which could give in principle contributions of comparable magnitude. As is well known since Luke’s paper[13] all these bring along new, unknown form–factors. We
will show, however, that to first order in $\frac{v}{m_c}$, no new form–factors show up besides $\xi_{3/2}^* (v \cdot v')$.

We shall first evaluate the contribution of the second term in equ.(6). To this end consider the matrix elements of the operator $-i\hbar_v^{(c)} \vec{D}_\rho \Gamma h_v^{(c)}$, which are given by

$$
\langle M_i (v') | -i\hbar_v^{(c)} \vec{D}_\rho \Gamma h_v^{(c)} | M_j^{**} (v) \rangle = \sqrt{M_S M_{P_0}} \text{Tr} [\hat{M}_i \frac{1 + y' \Gamma}{2} \frac{1 + y' \hat{M}_j^{** \alpha} (f_1 g_{\alpha \rho} + f_2 v_\alpha v_\rho + f_3 v_\alpha' v_\rho' + f_4 v_\alpha' \gamma_\rho)] (16)
$$

Here $f_{1-4}$ are functions of $v \cdot v'$. One gets a first constraint on these unknown form–factors by contracting with $v^{\rho}$ and using the equation of motion $\bar{h}_v^{(c)} \vec{D} \cdot v' = 0$:

$$f_1 + f_2 v \cdot v' + f_3 - f_4 = 0. \tag{17}
$$

A second constraint is obtained by taking the derivative

$$
\partial_\rho \langle M_i (v') | -i\hbar_v^{(c)} \Gamma h_v^{(c)} | M_j^{**} (v) \rangle = (\bar{\Lambda} v' - \bar{\Lambda} v)_\rho \langle M_i (v') | \hbar_v^{(c)} \Gamma h_v^{(c)} | M_j^{**} (v) \rangle = (M_i (v') | -i\hbar_v^{(c)} \vec{D}_\rho \Gamma h_v^{(c)} - i\hbar_v^{(c)} \Gamma \vec{D}_\rho \ h_v^{(c)} | M_j^{**} (v) \rangle. \tag{18}
$$

Contracting with $v^{\rho}$, employing the equation of motion $v \cdot D h_v^{(c)} = 0$ and comparing with (16), a second constraint is obtained

$$f_2 + f_3 v \cdot v' - f_4 = [\bar{\Lambda} v' - \bar{\Lambda}] \xi_{3/2}^* (v \cdot v'). \tag{19}
$$

From (17) and (19) we get

$$f_1 (1) = -(\bar{\Lambda}' - \bar{\Lambda}) \xi_{3/2}^* (1). \tag{20}
$$

The matrix element of the second term in (6) can be expressed with the help of equ. (18) as

$$
-i \langle M_i (v') | \hbar_v^{(c)} (\vec{D} \gamma_\mu - \gamma_\mu \vec{D}) h_v^{(c)} | M_j^{**} (v) \rangle = -2i \langle M_i (v') | \hbar_v^{(c)} \vec{D}_\mu h_v^{(c)} | M_j^{**} (v) \rangle - (\bar{\Lambda} v' - \bar{\Lambda} v)_\rho \langle M_i (v') | \hbar_v^{(c)} \gamma_\rho h_v^{(c)} | M_j^{**} (v) \rangle. \tag{21}
$$

The first term on the r.h.s. is given by a formula like (16) with $\Gamma = 1$, where it can be seen that for $v_\mu = v'_\mu$ only the term with $f_1$ survives because of the orthogonality relation $\hat{M}_j^{** \alpha} v_\alpha = 0$. But from (20) one sees that at $v \cdot v' = 1$, $f_1$ can be expressed only in terms of $\xi_{3/2}^* (1)$. This is essentially the explanation for our simple final result.
Now for the contribution of the non–leading terms in the HQET Lagrangian (15). The \( \bar{h}_v^{(c)}(iD)^2h_v^{(c)} \) term just ”renormalizes” \( \xi_{3/2}^* \) by effectively changing

\[
\xi_{3/2}^*(v \cdot v') \rightarrow \xi_{3/2}^*(v \cdot v') + \frac{1}{m_c} \eta(v \cdot v') \quad (22)
\]

and the chromomagnetic term \( \bar{h}_v^{(c)} \sigma \cdot G h_v^{(c)} \) gives a contribution to the matrix element of interest equal to

\[
\sqrt{M_S M_{P_{3/2}}^2} \frac{1}{m_c} \text{Tr}[\hat{M}_i^1 + \frac{y''}{2}\gamma_\mu 1 + \frac{y'}{2}\sigma^{\rho \lambda} 1 + \frac{y}{2}\bar{M}_{j}^{*\alpha\mu} v_\alpha (\chi_1 \sigma_{\rho \lambda} + i\chi_2 (v'_\rho \gamma_\lambda - v'_\lambda \gamma_\rho))] \\
+ \sqrt{M_S M_{P_{3/2}}^2} \frac{1}{m_c} \text{Tr}[\hat{M}_i^1 + \frac{y''}{2}\gamma_\mu 1 + \frac{y'}{2}\sigma^{\rho \lambda} 1 + \frac{y}{2}\bar{M}_{j}^{*\alpha\mu} v_\alpha (\chi_1 \sigma_{\rho \lambda} - i\chi_2 (v_\rho \gamma_\lambda - v_\lambda \gamma_\rho))] \quad (23)
\]

\( \eta, \chi_1 \) and \( \chi_2 \) are real unknown functions. There are thus five unknown form factors besides the Isgur–Wise function \( \xi_{3/2}^* \) which determine the matrix elements of the (approximatively) conserved vector current \( \bar{c} \gamma_\mu c \) in the heavy quark effective theory up to \( \mathcal{O}(1/m_c) \). In the expressions for these matrix elements we insert \( v'_\mu \) as a function of \( v_\mu \) and \( k_\mu \) from (7), the meson masses \( M_S \) and \( M_{P_{3/2}} \) are expressed in terms of \( \tilde{\Lambda} \) and \( \tilde{\Lambda}' \) and the result is expanded in powers of \( k/m_c \). In this expansion, one power of \( k_\mu \) counts as much as one power of \( \tilde{\Lambda}, \tilde{\Lambda}', f_{1-4}, \eta \) or \( \chi_{1,2} \) as they are of the same order of magnitude. The leading terms of this expansion look as follows:

\[
\langle D^*(v', \epsilon_2) | \bar{J}_\mu^{heavy} | D^*_2(v, \epsilon_1) \rangle = \sqrt{M_S M_{P_{3/2}}^2} \left\{ -2 \frac{\epsilon_1^{\mu \lambda} \epsilon_2^{* \alpha \rho}}{m_c} k^\alpha v_\mu + 2 \frac{\tilde{\Lambda} - \tilde{\Lambda}'}{m_c} \epsilon_1^{\mu \lambda} \epsilon_2^{* \alpha \rho} \right\} \xi_{3/2}^*(1), \quad (24)
\]

\[
\langle D^*(v', \epsilon_2) | \bar{J}_\mu^{heavy} | D_1(v, \epsilon_1) \rangle = \sqrt{M_S M_{P_{3/2}}^2} \frac{2}{\sqrt{6}m_c} i \epsilon_{\alpha \beta \gamma \epsilon_1} \epsilon_2^{* \beta} k^\gamma \gamma^{* \gamma} \xi_{3/2}^*(1), \quad (25)
\]

\[
\langle D(v') | \bar{J}_\mu^{heavy} | D_2^*(v, \epsilon_1) \rangle = \sqrt{M_S M_{P_{3/2}}^2} \frac{1}{m_c^2} i \epsilon_{\alpha \beta \gamma \epsilon_1} \epsilon_2^{* \beta} k^\gamma v_\alpha \gamma_\mu \gamma^{* \gamma} \xi_{3/2}^*(1), \quad (26)
\]

\[
\langle D(v') | \bar{J}_\mu^{heavy} | D_1(v, \epsilon_1) \rangle = \sqrt{M_S M_{P_{3/2}}^2} \left( \frac{4}{\sqrt{6}m_c} (k \cdot \epsilon_1) v_\mu - \frac{4}{\sqrt{6}m_c} (\tilde{\Lambda} - \tilde{\Lambda}') \epsilon_{1 \mu} \right) \xi_{3/2}^*(1). \quad (27)
\]

It is apparent that all the matrix elements (24–27) can be expressed only in terms of \( \xi_{3/2}^*(1) \) and that none of the five subleading form factors contributes. An important role is played here by eqn. (10) which prevents the appearance of terms proportional to the derivative of \( \xi_{3/2}^*(x) \) at \( x = 1 \). These matrix elements exhibit approximate gauge invariance (up to \( \mathcal{O}(1/m_c) \)), as shown in (14). In practice this is slightly inconvenient, so that we will henceforth replace \( \tilde{\Lambda} - \tilde{\Lambda}' \) by \( v \cdot k \) in the above equations, which only changes the result by a next-to-leading quantity, as is apparent from (9). After this change our matrix elements are explicitly gauge invariant.

\( ^3 \)Similar results were obtained also in the Ref. [14].
3. Results

The four possible electromagnetic decays of the \( s_{\ell}^{\pi} = \frac{3}{2}^{+} \) charmed mesons to the \( s_{\ell}^{\pi} = \frac{1}{2}^{-} \) charmed mesons have, in the infinite mass limit, the following multipole content: \( D_{2}^{*} \rightarrow D^{*} \), \( D_{2}^{*} \rightarrow D \), \( D_{1}^{*} \rightarrow D^{*} \), \( D_{1}^{*} \rightarrow D \).

The \( D_{2}^{*} \rightarrow D^{*} \) transition can in general proceed also through a \( E3 \) mode, as far as angular momentum and parity are concerned. In the infinite mass limit this is forbidden. However, this prediction is not very likely to be easily tested experimentally.

More interesting are the heavy quark symmetry predictions for the ratios of partial amplitudes for these decays:

\[
A_{E1}(D_{2}^{*} \rightarrow D^{*} \gamma) : A_{E1}(D_{2}^{*} \rightarrow D \gamma) : A_{E1}(D_{1} \rightarrow D^{*} \gamma) : A_{E1}(D_{1} \rightarrow D \gamma) = \sqrt{3} : 0 : 1 : \sqrt{2}, \tag{28}
\]

\[
A_{M2}(D_{2}^{*} \rightarrow D^{*} \gamma) : A_{M2}(D_{2}^{*} \rightarrow D \gamma) : A_{M2}(D_{1} \rightarrow D^{*} \gamma) : A_{M2}(D_{1} \rightarrow D \gamma) = \sqrt{3} : \sqrt{2} : \sqrt{5} : 0, \tag{29}
\]

which can e.g. be obtained along the lines of Ref. [14]. Using (5) and the matrix elements (24-27) we obtain the following one-photon widths, which automatically satisfy the above relations:

\[
\Gamma(D_{2}^{*} \rightarrow D^{*} \gamma) = \frac{4\alpha M_{D^{*}}}{3 M_{D_{2}^{*}}} |\vec{k}|^{3} (e_{q} F + e_{Q} \frac{\xi_{3/2}^{*(1)}}{m_{c}})^{2} + \frac{3\alpha M_{D^{*}}}{5 M_{D_{2}^{*}}} |\vec{k}|^{5} (e_{q} G)^{2}, \tag{30}
\]

\[
\Gamma(D_{2}^{*} \rightarrow D \gamma) = \frac{2\alpha M_{D}}{5 M_{D_{2}^{*}}} |\vec{k}|^{5} (e_{q} G + e_{Q} \frac{\xi_{3/2}^{*(1)}}{2m_{c}^{2}})^{2}, \tag{31}
\]

\[
\Gamma(D_{1} \rightarrow D^{*} \gamma) = \frac{4\alpha M_{D^{*}}}{9 M_{D_{1}}} |\vec{k}|^{3} (e_{q} F + e_{Q} \frac{\xi_{3/2}^{*(1)}}{m_{c}})^{2} + \alpha \frac{M_{D^{*}}}{M_{D_{1}}} |\vec{k}|^{5} (e_{q} G)^{2}, \tag{32}
\]

\[
\Gamma(D_{1} \rightarrow D \gamma) = \frac{8\alpha M_{D}}{9 M_{D_{1}}} |\vec{k}|^{3} (e_{q} F + e_{Q} \frac{\xi_{3/2}^{*(1)}}{m_{c}})^{2}. \tag{33}
\]

Here \( e_{q} \) and \( e_{Q} (= \frac{2}{3} \) for \( Q = c \)) are the light and the heavy quark electric charges and \( F \), \( G \) are given by

\[
F = f_{1} - f_{2} + \frac{v}{2\Lambda_{\chi}} (g_{1} + 2g_{2} + 4g_{3}), \tag{34}
\]

\[
G = \frac{1}{\Lambda_{\chi}} (g_{1} + 2g_{2}). \tag{35}
\]

The heavy quark contribution to the \( M2 \) partial width has been written only for the transition \( D_{2}^{*} \rightarrow D \gamma \), since it is only in this case that it can be expressed solely
in terms of $\xi_{3/2}^*(1)$. For all the other decays, it depends also on some of the five unknown subleading form factors.

We will use in the following the ISGW value\(^8\)

$$\xi_{3/2}^*(1) = 0.584.$$  \hfill (36)

This corresponds to a transition $c \rightarrow c$ and is slightly different from the corresponding value for a $b \rightarrow c$ transition, of 0.537\(^9\). A similar value is also obtained from a recent QCD sum rule calculation\(^[15]\). Although this value is the result of a model calculation, $\xi_{3/2}^*(1)$ can be obtained experimentally, from a study of the differential cross-section for the semileptonic decays of $B$ mesons into $p$-wave D mesons. In this sense the relations to be derived below are truly model–independent.

One first set of predictions is nothing else but the mass corrections to the ratios (28-29). In the approximation that $F$ and $G$ are constants independent of the photon energy, these are:

$$\Gamma_{E1}(D^+_s \rightarrow D^+_s \gamma) : \Gamma_{E1}(D^+_s \rightarrow D^+_s \gamma) : \Gamma_{E1}(D^+_s \rightarrow D^+_s \gamma)$$

$$= 3.694 : 0 : 1 : 4.071$$

$$\Gamma_{M2}(D^+_s \rightarrow D^+_s \gamma) : \Gamma_{M2}(D^+_s \rightarrow D^+_s \gamma) : \Gamma_{M2}(D^+_s \rightarrow D^+_s \gamma)$$

$$= 0.965 : 2 : 1.212 : 0.$$  \hfill (37)

and similarly for the $D^0$ mesons:

$$\Gamma_{E1}(D^0_s \rightarrow D^{*0}_s \gamma) : \Gamma_{E1}(D^0_s \rightarrow D^{*0}_s \gamma) : \Gamma_{E1}(D^0_s \rightarrow D^{*0}_s \gamma) : \Gamma_{E1}(D^0_s \rightarrow D^{*0}_s \gamma)$$

$$= 3.539 : 0 : 1 : 4.022$$

$$\Gamma_{M2}(D^0_s \rightarrow D^{*0}_s \gamma) : \Gamma_{M2}(D^0_s \rightarrow D^{*0}_s \gamma) : \Gamma_{M2}(D^0_s \rightarrow D^{*0}_s \gamma) : \Gamma_{M2}(D^0_s \rightarrow D^{*0}_s \gamma)$$

$$= 0.964 : 2 : 1.125 : 0.$$  \hfill (39)

In the corresponding one pion decay case a small mixing between the $D_1$ and the axial vector member of the $s^{1/2}_\pi$ multiplet could significantly alter the predicted amplitude ratio. However, in our case this is no longer true. Both these states can decay to the $s$–wave mesons through a $E1$ mode, and as far as the $M2$ channel is concerned, it is only the decay of the vector member of the $s^{1/2}_\pi$ multiplet which is forbidden by large mass limit selection rules.

It is clear that without any knowledge of the constants $F$ and $G$ no further advance can be made. Since the value (36) has been obtained by using the constituent

\footnote{The relation to the function $\tilde{\tau}_{3/2}$ used in \(^8\) is $\xi_{3/2}^* = \sqrt{3}\tilde{\tau}_{3/2}$.}

\footnote{Note that what we call $\xi_{3/2}^*$ is not the usual Isgur–Wise function, but rather its renormalization group invariant version, which includes a scaling factor with the respective velocity dependent anomalous dimension and has a logarithmic dependence on the heavy quark mass. This in part explains the two different numerical values quoted above.}
quark model of Isgur, Scora, Grinstein and Wise, consistency requires that we use the same model for determining them. Although the authors of Ref.[9] made use of their model only to calculate matrix elements of heavy quark current bilinears, it can give matrix elements of light quark currents as well, thereby providing a way for evaluating our parameters $F$ and $G$. The model is well tested and is known to give reliable results for both the $s$– and $p$–wave charmed mesons. Furthermore, it provides a simple way for including $SU(3)$ violating effects induced by a constituent strange quark mass $m_s$ different from $m_u$, $m_d$.

To determine $F$ and $G$ for the $D^0$ meson case, consider the following matrix element of the light quark current $\bar{u}\gamma_\mu u$, which according to the effective lagrangian (5) is given by

$$\langle D^0(v)|\bar{u}\gamma_\mu u|D^0_1(v,\epsilon)\rangle = 4\sqrt{M_{D^0}M_{D^0_1}}[-(v\cdot k)\epsilon_\mu + (\epsilon \cdot k)v_\mu]F_{D^0}.$$  \hspace{1cm} (41)

The same matrix element is written as

$$\langle D^0(v)|\bar{u}\gamma_\mu u|D^0_1(v,\epsilon)\rangle = \sqrt{M_{D^0}M_{D^0_1}}[-2.673(v\cdot k)\epsilon_\mu + 2.396(\epsilon \cdot k)v_\mu] \text{ (GeV)}, \hspace{1cm} (42)$$

where we make use of the relations given in the Appendix B of Ref.[9]. Here some care is needed, as the light quark in a $D$ meson is an antiquark. Therefore the direct application of the relations of Ref. [9] gives rather the matrix element between the corresponding $\bar{D}$ mesons. However, these can be related by crossing to the matrix elements of interest. Unfortunately, the transition matrix element (42) is not gauge invariant. This is a typical problem of constituent quark model calculations of radiative transition matrix elements (see e.g. [17]) and we will deal with it by adopting as the value of the coupling $4F$ the average of the two numbers in (42), with an error given by their difference. This should give at the same time some measure of the model dependence of the result. Thus we take

$$F_{D^0} = (0.634 \pm 0.034) \text{ GeV}^{-1} \hspace{1cm} (43)$$

In obtaining this number we have used a value $\kappa = 1$ for the ”relativistic correction factor” $\kappa$ (see Ref.23 in [8]). Adopting $\kappa = 0.7$ lowers $F$ by a factor of about 0.5. Similarly the value of $G$ can be obtained by calculating the matrix element

$$\langle D^0(v)|\bar{u}\gamma_\mu u|D^0_2(v,\epsilon)\rangle = -2i\sqrt{M_{D^0}M_{D^0_2}}G_{D^0}\epsilon_{\mu\nu\lambda\rho}\epsilon^{\rho\alpha}k_\alpha v^\lambda k_\rho \hspace{1cm} (44)$$

with the result

$$G_{D^0} = -2.168 \text{ GeV}^{-2}.$$  \hspace{1cm} (45)

Inserting these values for $F_{D^0}$ and $G_{D^0}$ in the rate formulae (30-33), we obtain the partial widths in Table 1. Here a charmed quark mass $m_c = 1.82$ GeV has been used.
A similar calculation gives the following values for the corresponding couplings for the $D_s^+$ mesons:

\begin{align}
F_{D_s^+} &= (0.493 \pm 0.083) \text{ GeV}^{-1} \\
G_{D_s^+} &= -1.254 \text{ GeV}^{-2}.
\end{align}

In obtaining these numbers the following values for the oscillator strength parameters of the model have been used: $\beta_S = 0.40$ GeV and $\beta_P = 0.35$ GeV. For the flavour content $s\bar{c}$ these parameters are not available in the Ref.[9] and therefore we simply took them slightly larger than for $u\bar{c}$, as seems to be the case for the light mesons. The mass of the tensor state $D_{2s}^{*+}$ has been taken equal to $M_{D_{2s}^{*+}} = 2564$ MeV [18]. The partial rates for this case are also shown in the Table 1. Several remarks are in order about these results:

- The heavy quark contribution amounts to about 34% in absolute value in the $E1$ amplitude for the $D^0$ mesons’ case and 56% for the $D_s^+$ case. In the former it has an enhancement effect while in the latter case it contributes with an opposite sign. On the other hand, in the $M2$ amplitude for the $D_2^* \to D$ transition the heavy quark current contributes negligibly (under 4%). Very likely, the same is true for all the other $M2$ amplitudes.

- Our estimates for the total one-photon widths agree well with the predictions in [9] for the decay $D_{2s}^{*+} \to D_s^{*+}$ and are lower for the same decay of the $D^0$ meson. Compared to the [9], our estimates for $\Gamma(D_2^* \to D\gamma)$ are down by a factor of 1.8 and 5.5 for the $D^0$ and $D_s^+$ cases respectively. Also, we get a value for $\Gamma(D_2^{*0} \to D^{*0}\gamma)$ which is lower by a factor of 5.2 than the one given in [9]. As a general common point we mention the strong suppression of the decay widths of the $D_s^+$ mesons compared to the $D^0$ mesons.

- These estimates yield branching ratios of about (0.9–1.5)% for the radiative decays of the $D^0$ mesons and larger than (0.02–0.4)% for the $D_{s1}^+$, for which an upper limit for the total width of 3.9 MeV exists[18].

The largest source of errors in our approach seems to be the neglect of higher order terms in the effective lagrangian (5), with more derivatives on the photon field. Such terms would be a source of additional photon-energy dependence of the couplings $F$ and $G$. An estimate for this dependence can be obtained by formally setting $g_3 = 0$ in (34). Then the dependence of $F$ with the photon energy can be expressed simply in terms of $G$. This yields a variation of $F$ among the three allowed $E1$ transitions of about 15 %, with the attendant corrections to the amplitude ratios (37) and (39).

The finite mass corrections are only significant in the $E1$ amplitudes, and we have estimated them to contribute up to 10% of the amplitude. Much more likely
are important the corrections due to $SU(3)$ violation. These can be calculated in a systematical way\[1\] using the heavy hadron chiral perturbation theory. We have only evaluated them in an effective way, from a constituent quark model, to be of about 25% for $F$ and 50% for $G$.

Similar methods can be applied to the description of the electromagnetic decays of the members of the $s_{1/2}^π = 1^{+}$ multiplet of charmed mesons and of the similar excited B mesons. Since the former are expected to be quite broad ($\Gamma \sim 200$ MeV) because of the pionic decay mode, their photonic branching ratios are surely very small. As for the latter, in the view of the absence of any experimental data about these states, such applications must be reserved for the future.
| Decay                                      | $\Gamma_{E1} \ (keV)$ | $\Gamma_{M2} \ (keV)$ | $\Gamma_{tot} \ (keV)$ | $|\vec{k}| \ (MeV)$ |
|-------------------------------------------|-----------------------|------------------------|------------------------|--------------------|
| $\Gamma(D^0_1 \to D^0\gamma)$            | $(245 \pm 18)$        | 0                      | $(245 \pm 18)$         | 494.93             |
| $\Gamma(D^0_1 \to D^{*0}\gamma)$        | $(60 \pm 5)$          | 101                    | $(161 \pm 5)$          | 381.05             |
| $\Gamma(D^{*0}_2 \to D^{*0}\gamma)$     | $(222 \pm 16)$        | 87                     | $(309 \pm 16)$         | 410.38             |
| $\Gamma(D^{*0}_2 \to D^0\gamma)$        | 0                     | 181                    | 181                    | 522.63             |
| $\Gamma(D^+_{s1} \to D^+_s\gamma)$      | $(1.6 \pm 2.3)$       | 0                      | $(1.6 \pm 2.3)$        | 503.77             |
| $\Gamma(D^+_{s1} \to D^{*+}_s\gamma)$   | $(0.4 \pm 1.0)$       | 10.0                   | $(10.4 \pm 1.0)$       | 389.97             |
| $\Gamma(D^{*+}_{s2} \to D^{*+}_s\gamma)$| $(1.4 \pm 2.0)$       | 8.0                    | $(9.4 \pm 2.0)$        | 413.56             |
| $\Gamma(D^{*+}_{s2} \to D^+_s\gamma)$   | 0                     | 16.0                   | 16.0                   | 526.12             |

**Table 1.** The calculated partial widths for the radiative decays of the $s^{\pi_{\ell}} = \frac{3}{2}^+$ p–wave charmed mesons to s-wave charmed mesons. In the last column the corresponding photon momentum is given.

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