Abstract: Rail transit network is the backbone of the transport network. In this article, a topology nonlinear model based on $L$-space and $P$-space is proposed to understand the unreliability of urban rail transit network in computer-aided design. The proposed model requires different characteristics of rail transit according to $L$-space and $P$-space. The experiment result shows that the degree value of the network is relatively small in the $L$-space. In the $L$-space, more than 80% of the nodes have degree 2, and the degree distribution is very concentrated. In $P$-space, the overall degree value of the network is higher, and the distribution area is wider. Experiment results have proved that the aggregation coefficient of more than 85% of the nodes in the Shanghai rail transit network is 1, which provides a basis for the comparison of different trains and the research on the changes in future urban trains.

Keywords: rail transit network, topology, reliability, non-linearity, urban rail

1 Introduction

With the development and prosperity of China’s social economy and economy, the scale of cities is also expanding. In mega and super large cities, the existing traffic network cannot meet the growing traffic demand gradually. The phenomenon of traffic congestion is becoming more and more serious. There are more and more traffic accidents, and people’s desire for green transportation is becoming more and more urgent. As a large environment, urban rail transit has become an important part of urban transportation. It has also become an important choice for urban residents due to its advantages of scale, safety, environmental protection, tourism, and energy saving. As of the end of 2017, a total of 34 cities across the country have opened urban rail lines, with a total of 165 lines in operation, of which 32 new lines have been added, an increase of 879.9 km, and a cumulative operating mileage of 5032.7 miles [1]. In recent years, with the continuous expansion of urban railways, many cities have also entered the era of network operation from single line operation, and there are hundreds of rail transit stations in cities. Although urban rail transit has the incomparable efficiency, safety, and punctuality of other transportation modes, which provides great convenience for residents to travel, it is precisely because of the huge network that once an emergency occurs, urban rail transit cannot operate normally, it will have a major impact on traffic in the entire city. Therefore, under the new situation of urban rail transit network, higher requirements are placed on the reliability and flexibility of rail transit to respond to emergencies [2]. How to judge the reliability of urban rail transit network connection, so as to carry out the functional layout and optimization of the network, has become one of the hotspots of modern railway network research to provide theoretical basis for the construction and transformation of urban railways. Aiming at the urban rail transit network in different stages of development, the reliability of its network connectivity is analyzed and researched, which also provides a new perspective for the study of modern urban rail transit network.

2 Literature review

In the past, the research on network traffic mainly focused on the research of roads, mostly on the improvement of traffic organization with the goal of achieving business results, and did not study the form of the entire network facilities. Xu et al. believed that in recent years, complex
network theory has gradually begun to be applied in the research of transportation network and has become more and more extensive. The research on the basic theory of transportation network combined with complex network theory not only has important guiding significance for the planning practice of transportation network, but also promotes the application research of complex network theory [3]. Wu et al. compared the subway networks between Boston and Vienna by choosing three metrics: clustering coefficient, average length, and average vertex degree [4].

Luan et al. established the world’s largest virtual subway network in 2006. Combined with relevant theories, they verified that the degree distribution of network characteristic parameters is an exponential distribution, and proposed that the network can be evaluated according to the change in degree value [5]. In 2008, Hua and Ong selected the Seoul Metro Network in South Korea, analyzed its network characteristic values such as network efficiency, maximum Dalian Tong subgraph, and degree distribution, and also verified that the node degree presents the characteristics of exponential distribution [6]. In 2010, a survey of several urban rail networks found that all types of urban rail are available on a small global, no-scale basis [7].

Later, in 2019, the authors understood the characteristics of Shanghai urban rail transit network through the network topology itself. They put forward the calculation method of network feature analysis, and calculated the eigenvalues of each network based on Space L and Space P topology by using complex network theory. This theory is also able to find the degree of distribution, characteristic path length, and aggregation coefficient in the network. Finally, it is concluded that the integration of Shanghai urban rail transit network is large and the road length is small [8].

In 2012, Yang et al. analyzed the topology of the domestic metropolitan area network at the physical level of the metropolitan area network based on the complex network theory, and came to the conclusion that the average distance of the domestic metropolitan area network is the shortest. The Space L model is larger and the clustering coefficients are located on the soles of the feet. The distribution of degree is similar to Poisson distribution, and the node and medium are linearly correlated. It is considered that under specific attacks, the propagation speed of faults in metro network is faster, and after propagation to a certain extent, the effect of subsequent attacks is less [9].

In 2014, Lin selected the rail transit networks in Beijing, Shanghai, and Guangzhou for topology abstraction, analyzed and explained the basic structure of the network, by calculating the variability of the measured parameters of various network characteristics. After random and selective attacks, the security of railway transportation is evaluated, and finally the stops and vehicles are isolated to the main stations in the urban rail transit network [10]. Xu and Li studied the changes in urban rail transit network connectivity reliability through three different attack strategies and network attack strategies in 2012 [11].

In 2013, Jiang et al. developed a new model to test the network connectivity reliability of urban rail transit through the maximum connectivity subgraph of Shanghai rail transit network and the change in network efficiency before and after stops were used to measure optimization [12]. In 2014, Li et al. analyzed the connectivity reliability of Beijing Metro network by analyzing the static characteristic parameters of the network, such as degree and intermediate index, and established the network topology model based on point and edge [13].

Similarly, in 2014, Xu et al. took Beijing Metro as an example, by analyzing the changes in network characteristic parameters before and after the attack, they obtained the nodes and lines with high importance in Beijing urban rail transit network [14]. The public transport topology proposed by Zhang has been investigated in 22 cities in Poland, and it is found that the distribution of high-rise buildings conforms to the exponential distribution law [15]. The Boston subway has always been the forerunner of the world’s small network capabilities, and analyzed its effectiveness and fault tolerance through network efficiency [16].

Due to the relatively narrow distribution range of urban rail transit entity network structure (line network), the current research tends to adopt a research method similar to the public transport network to prove that it has the characteristics of complex network. But in fact, in the urban rail transit network, the body line network is the basis of network operation, and the simple study of the abstract operation network cannot fully reflect its network characteristics.

Therefore, using complex network theory and graph theory, this article analyzes the two network topologies of Space L and Space P of subway network. This article also studies the characteristic indexes of subway network under different topologies, and reveals the characteristics of different topologies. The network topology is shown in Figure 1.

The network has many characteristics, and the description of its characteristics is realized by some specific properties. At present, in the existing research, there are the following geometric quantities: degree, diameter, path length, and clustering coefficient. These geometric quantities can
describe the structural characteristics of different aspects of the network, and can comprehensively summarize the inherent characteristics of the network.

3 Research methods

In order to determine the characteristics and legitimacy of urban railways in China, Shanghai, which is currently very efficient in rail transit, is selected. The following points are proposed for research needs.

Definition 1. Degree: The degree of a base in a network represents all edges connected to a node.

Definition 2. Determining the 2-degree distribution: The $P(k)$ class distribution of network nodes can be used to represent the percentage of nodes with degree $k$ to the total number of nodes.

Definition 3. Cumulative range distribution: The rank distribution represents the nonlinear probability distribution function $P(k)$ of all nodes in the network, according to Eq. (1):

$$P(k \geq k') = \sum_{k' \geq k} P(k'),$$

Definition 4. Define a 4-node clustering factor: In the network, node $i$ has $k_i$ edges connecting to other nodes, and at most $k_i(k_i - 1)/2$ edges between $k_i$ neighbors. The edges of $E_i$ are connected, and the clustering coefficient of $i$ is defined by Eq. (2):

$$C_i = \frac{E_i}{k_i(k_i - 1)/2} = \frac{2E_i}{k_i(k_i - 1)}.$$  \hspace{0.1cm} (2)

Definition 5. Network clustering coefficient: As shown in Eq. (3), the clustering coefficient of all networks is the average of the clustering coefficients of all nodes in the network:

$$C = \frac{1}{N} \sum_{i=1}^{N} C_i.$$  \hspace{0.1cm} (3)

Definition 6. L-Location and P-Location: There are various representations of urban rail transit networks. The characteristics of these networks can be explored separately to explore the topological patterns of the same traffic.

L-Space (SpaceL): A station is considered a station, if two stations are adjacent in a lane, and they have connecting edges;

P-Space (SpaceP): A station is considered a station, if two stations have direct rail, they are connected, and otherwise the stations are not connected. It is not difficult to see that the network created by the SpaceL method is a sub-network of the network created by the SpaceP method.

Figure 1: Network topology.
4 Result analysis

Through analysis, some eigenvalues of the network topology are obtained. The highest point of the entire topology is 10, which is line 4, that is, line 10 that can be changed from line 4 to other 11 lines, because line 4 is a line. The minimum value is 1, i.e., only row 5 is connected to row 1, i.e., row 5 can be moved to row 1 after one pass. The total area of the network is 6.33, that is to say, one line connects 6.33 lines on average, which represents the change and access of the entire network [18]. The clustering coefficients of each network node are shown in Table 1. The average network clustering coefficient is 0.7745, and all networks are relatively compact.

4.1 Adjacency matrix in L space and P space of Shanghai rail transit network

The adjacency matrix of complex networks is defined as L and P. Obtain the connection matrix $A_l(i, j)$ and $A_P(i, j)$ of Shanghai rail transit network L position and P position, respectively. Both matrices are $250 \times 250$ square matrices, in which $i$ and $j$ are the serial numbers in Shanghai rail transit sequence number station name line table, representing the station serial numbers. At distance L, if there is a direct railway line between nodes i and j without passing other stations, and does not intersect other stations, $A_l(i, j) = 1$, otherwise $A_l(i, j) = 0$. For example, $A_l(1, 2) = 1$ indicates that there is a direct connection from station 1 Xinzhuang to station 2 outer ring roads through line 1. $A_l(1, 3) = 0$ means there is no direct connection between Xinzhuang Station 1 and Lianhua Road Station 3. Although line 1 can run through these two stations, they are separated by station 2 outer ring road. When $i = j$, i.e., from my place to my own place, they are considered discontinuous, i.e., in general $A_l(i, i) = 0$, $A_P(i, j)$, e.g., Eq. (4):

$$A_l(i, j) = \begin{bmatrix} 0 & 1 & \ldots & 1 \\ 1 & 0 & \ldots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \ldots & \ldots & 0 \end{bmatrix}.$$  

If there is a direct rail transit line between nodes i and j from P (regardless of whether it passes through other stations in the middle), $A_P(i, j) = 1$. For example, $A_P(1, 5) = 1$ indicates that there can be direct access from station 1 Xinzhuang to station 5 Shanghai south stations through line 1, although there are station 2 outer ring road, station 3 Lianhua Road, and station 4 Jinjiang paradise [19]. $A_P(1, 103) = 0$, which means that although the line from station 1 Xinzhuang of line 1 to station 103 Dongchuan Road of line 5 is connected on the track network, there is no track line that can directly connect the two stations, that is, the line from Xinzhuang to Dongchuan Road needs to be changed (line 1 changes to line 5), so $A_P(1, 103) = 0$. They are considered disconnected when $i = j$, i.e., when moving from site i to site i, that is, $A_P(i, i) = 0$. The general form of $A_P(i, j)$ is shown in Eq. (5):

$$A_P(i, j) = \begin{bmatrix} 0 & 1 & 1 & \ldots \\ 1 & 1 & 1 & \vdots \\ 1 & 1 & 1 & \vdots \\ \vdots & \vdots & \vdots & \ddots \\ 1 & 1 & 1 & 0 \end{bmatrix}.$$  

### Table 1: Node clustering coefficient of line network topology

| Line node | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Clustering coefficient $C_i$ | 0.7 | 0.7 | 0.8 | 0.7 | 0   | 1   | 0.8 | 0.8 | 0.9 | 1   | 1   | 1   |

4.2 Analysis of topological structure characteristics of Shanghai rail transit network

The different personal values of the network are obtained by running the adjacency matrix, and these personal values are mainly: degree and degree distribution, shorter method and classification, group coefficient, etc.
4.2.1 Degree and degree distribution

From the map topology analysis,
1. At L position: the maximum number of nodes in the network, namely, Century Avenue station is 7. Historically, this station was the intersection of Lines 2, 4, 6, and 9, and the entrance must have been Line 8. However, since the intermediate station of line 4 and line 6 from Lancun road to Century Avenue is called Pu circuit, it is considered to be the same station, so the entire network has a maximum of 7 network nodes. The minimum number of network nodes is 1, which is the origin and destination of each line. The average rank of the entire network is 2.1968, that is, each node is connected to 2.19 adjacent nodes on average, which is due to the relatively small number of nodes in rail transit lines [20–24].

2. At P position: the highest point 104 of the network, namely, Century Avenue station, that is, starting from Ji Avenue station, you can take the subway to 104 stations directly, which is consistent with the situation that Century Avenue station is the largest hub node of Shanghai rail. The lowest degree is 5, which is the node degree of line 13. This is because line 13 only opens 5 stations from Jinhaijiang Road to Jinyun Road, so the degree of the whole line is low. The entire network has an average of 30.856 stations, that is, starting from 1 station, the average number of stations is 30.8 without any change, which is divided at the stations of most lines of Line 20–35.

Figure 2a and b shows the frequency distribution of node degrees at point L and position P in the network. At the L position, the occurrence rate of node degree 2 is the largest, close to 80%, that is, if a node is arbitrarily selected from the network, the probability of degree 2 is about 80%. This is because in L space, except for transfer stations and start and end stations, other nodes are only connected with the front and rear stations of the line, so the stations with degree 2 are the most. Compared with L space, the degree under P space is large, and it is mainly concentrated between 20 and 50, and the distribution is relatively scattered [25,26].

Figure 3a and b shows the equation distributions for the L and P position equations, respectively. The results show about classification of exponential division type, which is also similar to the previous network degree distribution.

4.2.2 Shortest path analysis

At position L, the shortest maximum value in the network is 42, that is, 42 stations need to pass from Gangcheng road of line 6 to Anting of line 11, of which the bifurcation of line 11 is calculated as the whole line, that is, line 11 reaches Jiading new city – Bailyin road – Jiading West – Jiading North – Shanghai Race yard – Changji East Road – Shanghai Auto City – Anting. The average shortest path of the network is 14.519, that is, it takes an average of 14–15 stations to reach any other station from any station. As can be seen from Figure 4, the shortest paths of nodes are usually divided between 5 and 30, that is, from any station, the average probability of reaching the destination through 5–30 stations is the highest. In P space, the maximum and minimum distance is 4, that is, it takes three transfers from one node to another. For example, the stations from Century Avenue to Gangcheng road from line 5 to line 6 need three transfers, so their minimum distance is 4.
The shortest path distance is 1, i.e., the shortest path from one of the nodes to all other paths is 1. The average shortest distance of the network is 2.1464, that is, at position P, the average distance from one point in the network to another is only 2.1464.

### 4.3 Analysis results of urban rail transit network in L and P space

By analyzing the coefficients of previous degree and classification, shortest method, and group, it can be seen that the topological properties of various networks in L space and P space are quite different. The two methods have their own characteristics for describing the properties of networks. For example, a level tester in the L position can know the number of lines adjacent to the network, while in the P position it can know the number of stations that the network node can directly reach without changing. Analysis of the shortest path at distance L tells the number of stations between that station and another station, while the shortest path at distance P tells any other station how many passes.

### 4.4 Suggestions

The urban rail transit network of many cities in my country is in the expansion stage. Due to various limitations, the main purpose of studying the reliability of
urban rail transit network connection is not how to rebuild the entire urban rail transit network, but to put forward planning suggestions for new lines and optimization measures for daily operation and passenger flow organization based on the existing urban rail transit network. From the comments above, the following points can be improved:

1) Optimize the network structure from the perspective of planning:

By calculating the benefits of the sample model, it can be seen that with the continuous expansion of the urban train crossing scale, the reliability of the network connection is also improving. With the further development of the city, more lines will be built to join the original cable network in the later stage, and some ordinary stations will be “upgraded” to transfer stations, and the importance of transfer stations is obvious. This traffic volume also affects the importance of its adjacent stations, which means that the importance of most of the stations in the network will gradually increase to some extent, and thus the network reliability of all urban train transportation will increase. Therefore, in the planning stage of the new line of the urban rail transit network, how to choose the new interchange station will have a positive impact on the reliability of all urban rail transit networks.

2) Strengthen the early warning of key stations and lines:

In the sample model, when a higher priority site is attacked, the reliability of the network connection is greatly reduced. Therefore, in order to improve the operation efficiency of urban railways, it is necessary to strengthen the protection of stations, which is of greater importance. First of all, in order to ensure the safety and stability of the facilities and equipment in the parking lot, special personnel should be arranged to inspect and manage the facilities and equipment in the parking lot to avoid affecting the daily operation of the station. The second is to strengthen the employee training, carry out continuous training on employee safety, improve employee quality, and enhance trust through the ideas of good employees. Finally, strengthen the commuting capacity of key stations. Commuters at major stations will increase further during morning and evening rush hours or on weekends. This will lead to low site performance and affect the reliability of all network connections.

3) Complete station emergency plan

Computational results on the sample model show that the connection reliability of the network is very low at the beginning of the attack and gradually becomes flat after the attack phase. Therefore, in the early stage of failure, the first emergency plan is activated to process the connected lines, which can prevent the transmission of the faulty node. The main task of the railway is to use various markings according to the actual situation of the urban railway, formulate emergency plans, and eliminate them in time and during an emergency.

5 Conclusion

Taking the Shanghai subway network as an example, the statistical analysis of the Space L and Space P topology models shows that one line of the Shanghai rail transit network connects 6.33 lines on average, and the average coefficient of the network cluster is 0.7745. The entire network is more compact. In space L, the degree value of the network is small; more than 80% of the nodes have degree 2. The degree distribution is very concentrated, and the length of the shortest path is also large. Overall, the whole network representation actually needs to be passed from one point to another. The average value of the points in the network or the integration of the network is the smallest which is close to 0. This is the basis for a comparative study of various trains in the future urban train transformation. In order to reduce the amount of calculation, this article mainly selects some sites in central Shanghai as one of the endpoints of the connecting line. In future studies, connections between other lines can be considered, and the line endpoints do not need to be fixed.

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