The masses of the mesons and baryons.
Part V. The neutrino branch particles

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We have determined theoretically the rest mass of the muon neutrino at 50 milli-eV and the rest mass of the electron neutrino at 5 meV, as well as, to 1% accuracy, the ratio of the masses of the stable elementary particles which decay by weak decays. We assume that the particles of the neutrino branch consist of a cubic, isotropic nuclear lattice, held together by the weak nuclear force. The eigenfrequencies of the lattice are calculated with Born’s theory of cubic lattices. Only neutrinos are required to explain the so-called stable particles of the neutrino branch.

1 Introduction

We have shown in [1] that it follows from the well-known masses and the well-known decays of the so-called stable elementary particles that the stable particles consist of a γ-branch and a neutrino branch, the γ-branch particles decaying directly or ultimately into photons, whereas the neutrino branch particles decay directly or ultimately into neutrinos and e±. The masses of the γ-branch, the π0, η, Λ, Σ0, Ξ0, Λ0, Σ0c, Ξ0c and Ω−c particles, are integer multiples of the mass of the π0 meson, within at most 3.3%, the average deviation being 0.0073. The masses of the neutrino branch, the π±, K±, n, D± and D±S particles, are integer multiples of the mass of the π± mesons times a factor 0.86 ± 0.02. In [2] we have explained the integer multiple rule of the γ-branch particles with different modes and superpositions of plane, standing electromagnetic waves in a cubic nuclear lattice. For the explanation of the particles of the neutrino branch we follow a similar path. We assume, as appears to be quite natural, that the π± mesons and the other particles of the ν-branch consist of the same particles into which they decay, that means of neutrinos and electrons or positrons. Since the particles of
the $\nu$-branch decay through weak decays, we assume, as appears likewise to be natural, that the weak nuclear force holds the particles of the $\nu$-branch together. Since the range of the weak interaction is only about a thousands of the diameter of the particles, the weak force can hold particles together only if the particles have a lattice structure, just as macroscopic crystals are held together by microscopic forces between atoms. We will, therefore, investigate the energy which is contained in the oscillations of a cubic lattice consisting of electron and muon neutrinos. We will explain the ratios of the masses of the neutrino branch particles, in particular the factor $0.86 \pm 0.02$ which appears in the ratio of the masses of the $\nu$-branch particles divided by the mass of the $\pi^\pm$ mesons.

2 The frequency spectra of the oscillations of diatomic lattices

Since we will explain the particles of the $\nu$-branch with the oscillations of a cubic lattice consisting of muon and electron neutrinos it is necessary to outline the basic aspects of diatomic lattice oscillations. In diatomic lattices the lattice points have alternately the masses $m$ and $M$. The classic example of a diatomic lattice is the salt crystal with the masses of the Na and Cl atoms in the lattice points. The theory of diatomic lattice oscillations was developed by Born and v.Karman [3], referred to as B&K. They first discussed a diatomic chain. The equation of motions in the chain are according to Eq.(22) of B&K

\begin{equation}
\begin{align*}
    m\ddot{u}_{2n} &= \alpha(u_{2n+1} + u_{2n-1} - 2u_{2n}), \\
    M\ddot{u}_{2n+1} &= \alpha(u_{2n+2} + u_{2n} - 2u_{2n+1}),
\end{align*}
\end{equation}

where the $u_n$ are the displacements, $n$ an integer number and $\alpha$ a constant characterizing the force between the particles. As with any spring the restoring forces in (1) increase with increasing distance between the particles. Eq.(1) is solved with

\begin{equation}
\begin{align*}
    u_{2n} &= Ae^{i(\omega t + 2n\phi)}, \\
    u_{2n+1} &= Be^{i(\omega t + (2n+1)\phi)},
\end{align*}
\end{equation}

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where $A$ and $B$ are constants and $\phi$ is given by

$$\phi = \frac{2\pi a}{\lambda},$$

(3)

where $a$ is the lattice constant and $\lambda$ the wavelength, $\lambda = na$. The solutions of Eq. (2) are obviously periodic in time and space and describe standing waves in which for all times $t$ the displacements are zero at the nodes $2n\phi = \pi/2$ and $(2n+1)\phi = \pi/2$. Using (2) to solve (1) leads to a secular equation from which according to Eq. (24) of B&K the frequencies of the oscillations of the chain follow from

$$4\pi^2\nu^2 = \frac{\alpha}{Mm} \cdot (M + m \pm \sqrt{(M - m)^2 + 4mMc^2\phi}).$$

(4)

Longitudinal and transverse waves are distinguished by the minus or plus sign in front of the square root in (4). We will encounter later on a similar equation for the plane waves in an isotropic three-dimensional lattice. The equations of motion for the oscillations in a three-dimensional diatomic lattice have been developed by Thirring [4].

3 The neutrino lattice oscillations

The particles of the neutrino branch decay primarily by weak decays, see Table 2 in [1]. The characteristic case are the $\pi^\pm$ mesons which decay via e.g. $\pi^+ \rightarrow \mu^+ + \nu_\mu$ (99.988%) followed by $\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$ ($\approx$100%). Only neutrinos result from the decay of the $\pi^\pm$ mesons, but for $e^\pm$ which conserve charge. If the particles consist of the particles into which they decay, then the $\pi^\pm$ mesons and the other particles of the neutrino branch are made of neutrinos and $e^\pm$. The neutrinos must be held together in some form, otherwise the particles could not exist over a finite lifetime, say $10^{-10}$ sec. Since neutrinos interact through the weak force which has a range of order of $10^{-16}$ cm according to p.25 of [5], and since the size of the nucleon is of order of $10^{-13}$ cm, the only apparent way for the weak force to hold the particles together is through a neutrino lattice. We suggest that the lattice is cubic, because a cubic lattice is held together by central forces, which are the most simple forces to consider. It is not known with certainty that neutrinos actually have a rest mass as was originally suggested by Bethe [6] and Bahcall [7] and what the values of $m(\nu_e)$ and $m(\nu_\mu)$ are. However, the results of the Super-Kamiokande experiments [8] indicate that the neutrinos
have rest masses and that the difference of both rest masses is about 70 milli-
eV. The neutrino lattice must be diatomic, meaning that the lattice points
have alternately larger (m(ν_µ)) and smaller (m(ν_e)) masses. We will retain
the traditional term diatomic. The lattice we consider is shown in Fig. 1.

Fig.1: The neutrino lattice. Bold lines mark the forces between
neutrinos and antineutrinos. Thin lines mark the forces between
either neutrinos only, or antineutrinos only.

Since the neutrinos have spin \( \frac{1}{2} \) this is a four-Fermion lattice as is re-
quired for Fermi’s explanation of the \( \beta \)-decay. The first investigation of
cubic Fermion lattices in context with the elementary particles was made by
Wilson [9]. The entire neutrino lattice is electrically neutral and has no spin,
the spin of each neutrino is canceled by the opposite spin of an adjacent neu-
trino. Since we do not know the structure of the electron we cannot consider
charge.

A neutrino lattice takes care of the continuum of frequencies which must,
according to Fourier analysis, be present after the high energy collision which
created the particle. The continuum of frequencies of the Fourier spectrum
can be absorbed by the continuous frequency spectrum of the lattice oscilla-
tions. We will, for the sake of simplicity, first set the sidelength of the lattice
at \( 10^{-13} \) cm that means approximately equal to the size of the nucleon. The
lattice then contains \( 10^9 \) lattice points, since the lattice constant \( a \) is of order
\( 10^{-16} \) cm. The sidelength of the lattice does not enter Eq.(6) for the frequen-
cies of the lattice oscillations. The calculation of the ratios of the energy
of different lattice oscillations is consequently independent of the size of the
lattice. However, the size of the lattice can be determined theoretically as will be shown later.

We require that the lattice is isotropic which is the most simple possible case. Isotropy means that the ratio of the force constant $\alpha$ for the lattice points at distance $a$ to the force constant $\gamma$ for the lattice points at distance $a\sqrt{2}$ is $\gamma/\alpha = 0.5$. That $\gamma/\alpha = 0.5$, in the isotropic case, follows from the definition of $\gamma$ and $\alpha$. According to Eq.(13) of B&K it is

$$\alpha = a(c_{11} - c_{12} - c_{44}),$$
$$4\gamma = a(c_{44} + c_{12}),$$

(5)

where $c_{11}$, $c_{12}$ and $c_{44}$ are the elastic constants of an elastic body in continuum mechanics which applies in the limit $a \to 0$. The neutrino lattice is actually the best approximation to an elastic body of continuum mechanics. If we consider central forces then $c_{12} = c_{44}$ which is the classical Cauchy relation. Isotropy requires that $c_{44} = (c_{11} - c_{12})/2$ or that in this case $3c_{44} = c_{11}$.

Using Thirring’s equations of motion for a diatomic three-dimensional lattice and assuming that the displacements $u_{l,m}$ of the smaller masses and $U_{l+1,m}$ of the larger masses are periodic in space and time, similar to Eq.(2), one arrives at a characteristic equation from which after a very long but straightforward calculation the frequency equation for the two-dimensional, i.e. plane oscillations of a cubic, isotropic, diatomic lattice is obtained. It is

$$\nu_{\pm}^2 = \frac{\alpha}{4\pi^2 M} \left[ \left( \frac{M}{m} + 1 \right) \left( 1 + \frac{2\gamma}{\alpha} (1 - \cos \phi_1 \cos \phi_2 + \sin \phi_1 \sin \phi_2) \right) \right]$$

$$\pm \sqrt{ \left( \frac{M}{m} - 1 \right)^2 \left( 1 + \frac{2\gamma}{\alpha} (1 - \cos \phi_1 \cos \phi_2 + \sin \phi_1 \sin \phi_2) \right)^2 + \frac{4M}{m} \cos^2 \phi_1 }$$

(6)

This is a two-dimensional version of Eq.(4) above which applied to a diatomic chain. In Eq.(6) are, for each pair $\phi_1, \phi_2$, two different frequencies $\nu_-$ or $\nu_+$ caused by the $\pm$ sign in front of the square root. These frequencies correspond to longitudinal (minus sign) and transverse oscillations (plus sign). It is important to note that each value of $\nu_+$ and $\nu_-$ comes with a plus as well as with a minus sign, because Eq.(6) determines the square of $\nu_+$ and $\nu_-$. From Eq.(6) we can calculate the frequency distribution for all values $-\pi \leq \phi_1, \phi_2 \leq \pi$. These values depend on the ratio $M/m$, where $M$ is the
mass of the muon neutrino and m the mass of the electron neutrino. The distribution of $\nu_-/\nu_0$ following from Eq.(6) with $\nu_0 = \sqrt{\alpha/4\pi^2M}$ is shown in Fig. 2 for the range $0 \leq \phi_1, \phi_2 \leq \pi$.

![Fig. 2: The frequency distribution $\nu_-/\nu_0$ of the basic mode according to Eq.(6). Diatomic, isotropic case with $M/m = 9.3$.](image)

The value of $M/m$ originates from a trial run. There are some easily verifiable values of the frequencies $\nu_-$ of the longitudinal waves. At $\phi_1, \phi_2 = 0, 0$ or $\pi, \pi$, or $\pi, -\pi$ it must be that $\nu_- = 0$. At $\phi_1, \phi_2 = \pi/2, \pi/2$ it must be that $\nu_- = \nu_0\sqrt{6}$. At $\phi_1, \phi_2 = \pi/2, -\pi/2$ it must be that $\nu_- = \nu_0\sqrt{2}$.

From the frequency distribution follows the group velocity $d\omega/dk$ at each point $\phi_1, \phi_2$. With $k = 2\pi/\lambda$ and $\phi = 2\pi a/\lambda$ it follows that

$$d\omega/dk = 2\pi a d\nu/d\phi.$$  

(7)

With $\nu = \nu_0 f(\phi_1, \phi_2)$ we have

$$c_g = d\omega/dk = a\sqrt{\alpha/M} \cdot df(\phi_1, \phi_2)/d\phi.$$  

(8)

In order to determine the value of $d\omega/dk$ we have to know the value of $\sqrt{\alpha/M}$. From Eq.(5) for $\alpha$ follows that $\alpha = ac_{44}$ in the isotropic case with
central forces, when $c_{12} = c_{44}$ (Cauchy’s relation) and $c_{11} = 3c_{44}$. The group velocity is therefore

$$c_g = \sqrt{a^3 c_{44} / M} \cdot df / d\phi = \sqrt{c_{11} / 3\rho} \cdot df / d\phi.$$  \hspace{1cm} (9)

We now set $a\sqrt{\alpha / M} = c_s$, where $c_s$ is the velocity of light, which is a necessity because the neutrinos in the lattice soon approach the velocity of light. In a formula

$$c_s = \sqrt{a^3 c_{11} / 3M} = a\sqrt{\alpha / M}.$$  \hspace{1cm} (10)

It follows that

$$c_g = c_s \cdot df / d\phi,$$  \hspace{1cm} (11)

as it was with the $\gamma$-branch in [2]. Since $c_g$ must be $\leq c_s$ Eq.(11) limits the value of $df(\phi_1, \phi_2)/d\phi$ to $\leq 1$, regardless whether we consider $\nu_-$ or $\nu_+$. There is no extra spectrum for transverse oscillations.

From Eq.(10) follows the mass of the muon neutrino, it is

$$M = a^3 c_{11} / 3c_s^2.$$  \hspace{1cm} (12)

According to Eq.(18) of [10] $c_{11}$ can be determined theoretically from an exact copy of the determination of $c_{11}$ in Born’s lattice theory, assuming weak nuclear forces between lattice points at distance $a = 10^{-16}$ cm, and replacing $e^2$ by $g^2$, where $g^2$ is the interaction constant of the weak force. We note that the value of $c_{11}$ in [10] is, within the uncertainty of its determination, compatible with a completely different theoretical determination of the compression modulus of the nucleon [11]. Using the equation for the elasticity constant $c_{11} = 0.538g^2\epsilon / a^4$ from Eq.(17) of [10], and with $\epsilon = 1.5 \cdot 10^{-12}$ from Eq.(10) of [10], we find that

$$M = m(\nu_\mu) = 0.538g^2\epsilon / 3ac_s^2 = 2.69 \cdot 10^{-13}g^2/ac_s^2,$$  \hspace{1cm} (13)

which shows that $m(\nu_\mu)c_s^2$ depends only on the weak interaction constant $g^2$ and the range $a$ of the weak force. Using now the same values for $a = 10^{-16}$ cm and $g^2 = 2.946 \cdot 10^{-17}$ erg cm we used before in [10] we find

$$M = m(\nu_\mu) = 0.9 \cdot 10^{-34} gr = 5 \cdot 10^{-2}eV/c_s^2.$$  \hspace{1cm} (14)

Within the accuracy of the parameters $g^2$ and $a$ the mass of the muon neutrino is 50 milli-electron-Volt. It can be verified easily that the mass of
the muon neutrino we have found makes sense. The energy of the rest mass of the $\pi^\pm$ mesons is 139 MeV, and we have about $10^9$ lattice points. Therefore there can be on the average no more energy per lattice point than about 0.14 eV. According to Eq.(14) about 50% of the available energy per lattice point goes into a muon neutrino mass, a small part, as we will see, goes into an electron neutrino mass, the rest goes into the lattice oscillations.

4 The consequences of the rest masses of the neutrinos

If the neutrinos have a rest mass and a continuous frequency distribution then the masses of the particles of the neutrino branch follow from the formula

$$m(n) = \sum_{i=1}^{N} \left[ \frac{m(\nu_\mu)_0}{\sqrt{1 - \beta^2_{i\mu n}}} + \frac{m(\nu_e)_0}{\sqrt{1 - \beta^2_{ien}}} \right].$$  \hspace{1cm} (15)

The $\beta_i = v_i/c_*$ are the time averages of the velocity of each oscillation, the index $n$ goes from 1 to 5, $n = 1$ marking the $\pi^\pm$ mesons and $n = 5$ the $D^\pm_S$ mesons. The $\beta_{i\mu n}$ of the muon neutrinos are different from the $\beta_{ien}$ of the electron neutrinos, both neutrino types oscillate with the same frequencies which means that they have the same energies $E_i = h\nu_i$ but since $m(\nu_e) < m(\nu_\mu)$ the $\beta_i$ of each electron neutrino is larger than the $\beta_i$ of the corresponding muon neutrino. The different velocities reflect the different displacements of both neutrino types. In order to evaluate the sums in Eq.(15) we have to know the number of muon or electron neutrinos in the lattice. Recent measurements and theoretical analysis put the value of the proton radius at $r_p = (0.88 \pm 0.015) \cdot 10^{-13}$ cm [12,13]. The radius of the $\pi$ mesons from experiments is given as $(0.74 \pm 0.03) \cdot 10^{-13}$ cm [14]. Theoretical analysis raises the pion radius to $r_\pi = 0.83 \cdot 10^{-13}$ cm [15]. In the following we use $r_p = r_\pi = 0.88 \cdot 10^{-13}$ cm. From this follows with $a = 10^{-16}$ cm that the number of the muon as well as of the electron neutrinos in the cubic nuclear lattice is $N = 1.427 \cdot 10^9$ or that the total number of neutrinos in the lattice is $2N = 2.854 \cdot 10^9$.

Since we cannot determine the velocity of each of the $10^9$ particles of the lattice we determine the maximal value of the velocity of the neutrinos in the lattice using the well-known fact that the sum of the potential energy and the kinetic energy of a harmonic oscillator is constant and equal to the
maximal kinetic energy. Averaged over i and time, we have in the case of the muon neutrinos, for the particle with index n

\[ Nm(\nu_\mu)_0 \cdot \left[ \frac{1}{\sqrt{1 - \beta_n^2}} - 1 \right] = 1/2 \cdot (m(n) - N[m(\nu_\mu)_0 + m(\nu_e)_0]), \]  

(16)

with a corresponding equation for the electron neutrinos. The sum of the maximal kinetic energies or the sum of potential and kinetic energy of the muon and electron neutrinos is, according to Eq. (16), equal to the difference between the rest mass of the particle \( m(n) \) and the sum of the rest masses of both neutrino types. The \( \beta \) of the particles of the neutrino branch with \( N = 0.6 \cdot 10^9 \) are listed in Table 1. For \( m(\nu_\mu)_0 \) we have used 50 meV according to Eq.(14), and for \( m(\nu_e)_0 \) we have used 5 meV, which value will be justified soon from the ratios of the masses \( m(n)/m(\pi^\pm) \). Replacing the many \( \sqrt{1 - \beta_n^2} \) in Eq.(15) with the averages \( \sqrt{1 - \beta_n^2} \) from Eq.(16) we obtain correctly the masses of the different particles \( m(n) \).

| particle | \( \pi^\pm \) | \( K^\pm \) | n | \( D^\pm \) | \( D_S^5 \) |
|----------|---------------|---------------|---|-------------|-------------|
| index n  | 1             | 2             | 3 | 4           | 5           |
| mode \((i_1, i_2)\) | (1.1)         | (2.2)         | 2\,(2.2) | 4\,(2.2)   | 2\,(2.2) + (3.3) |
| \( \beta_\mu \) | 0.9329        | 0.9933        | 0.9981 | 0.9995      | 0.99955     |
| \( \beta_e \) | 0.9986        | 0.99992       | 0.99998 | 0.99999     | 0.99999     |
| \( E(n)/E(1) \) | 1             | 0.8713 \cdot 4 | 0.8467 \cdot 8 | 0.8328 \cdot 16 | 0.8321 \cdot 17 |
| \( m(n)/m(\pi^\pm) \) | 1             | 0.8843 \cdot 4 | 0.8415 \cdot 8 | 0.8371 \cdot 16 | 0.8296 \cdot 17 |

Table 1: The average velocities and the calculated and experimental mass ratios of the neutrino branch particles.

We will now determine the mass ratios of the \( \nu \)-branch particles from the frequencies of the neutrino lattice oscillations. The actually possible oscillations are given by Eq.(11), in which \( df/d\phi \) must be < 1, because the neutrinos cannot move with the velocity of light, \( c \), then \( bc \). The possible frequency distributions are like Fig. 3 only that the slope is \( df/d\phi = \beta \), not 1 as in the \( \gamma \)-branch. The frequency distributions of the higher modes are very similar to Fig. 3 because \( \beta \) of the higher modes is soon practically equal to one, as shown on Table 1. The higher modes of the oscillations are obtained by replacing the variables \( \phi_1 \) and \( \phi_2 \) in the ansatz for the solutions of the diatomic lattice oscillations by the variables \( i_1\phi_1 \) and \( i_2\phi_2 \), where \( i_1 \) and \( i_2 \)
Fig. 3: The frequency distribution $\nu/\nu_0$ of the basic (1.1) mode, according to Eq.(11), with slope 1.

Fig. 4: The frequency distribution $\nu/\nu_0$ of the (2.2) mode, according to Eq.(11), with slope 1. The variables $\phi_1$ and $\phi_2$ are one-half of the actual $\phi$ of the second mode.

are integers. In all cases considered $i_1 = i_2$. For $\phi > 0$ the function $f(\phi_1, \phi_2)$ extends then, for example, from 0 to $2\pi$ for the first higher mode ($i_1, i_2 = 2$).

The frequency distribution of the first higher mode with slope 1 is shown in Fig. 4. The area covered by Fig. 4 is four times the area covered by the first mode (Fig. 3), consequently the number of oscillations is four times the number of oscillations of the first mode, but the frequencies are the same as those of the first mode.

The energy contained in all longitudinal lattice oscillations is given by

$$E = \frac{N h \nu_0}{(2\pi)^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{\nu_-/\nu_0(\phi_1, \phi_2)}{d\phi_1 d\phi_2}.$$  (17)

This equation was used before in [2], it originates from Eq.(50) of B&K. The numerical value of the double integral in Eq.(17) of the (1.1) state shown
in Fig. 3 was found by numerical integration to be 66.9 radians\(^2\). If the frequencies of the oscillations in the neutrino lattice increase with \(\beta\) the value of the double integral in Eq.(17) has to be multiplied by \(\beta\) too. The ratio of the energy of the oscillations of a state \((i_n, i_n)\) to the basic state \((1,1)\) is then given by \(i_n^2/\beta_n/\beta_1\) or by the sum of different \(i_n^2\) times \(\beta_n/\beta_1\) if a particle is the superposition of different modes. The factor \(i_n^2\) originates from the increased area of the higher modes, see e.g. Fig. 4. The particles of the \(\nu\)-branch are, however, not made up exclusively by the energy of the neutrino lattice oscillations but also by the energy contained in the neutrino rest masses. From \(E(n) = m(n)c^2 = E_k + N[m(\nu_\mu)_0 + m(\nu_e)_0]c^2\), where \(E_k\) is the kinetic or oscillation energy, follows that

\[
\frac{E(n)}{E(1)} = \frac{E_k(n)}{E_k(1)} \cdot \frac{1 + \Sigma/E_k(n)}{1 + \Sigma/E_k(1)}.
\]

(18)

where we have abbreviated the sum over the neutrino rest masses by \(\Sigma\). In the case of the photons of the \(\gamma\)-branch particles \(\Sigma\) is zero and \(\beta_n = 1\), so \(E_k(n)/E_k(1)\) is simply \(i_n^2\) or equal to the sum of the \(i_n^2\) of different modes, that means the integer multiple rule of the \(\gamma\)-branch applies.

In the case of the neutrino branch, where the neutrinos have rest masses, the ratio \(E(n)/E(1)\) is not an integer number and is not constant, but is determined by \(i_n^2\) times a rest mass factor \(R\)

\[
\frac{E(n)}{E(1)} = i_n^2 R = i_n^2 \frac{\beta_n}{\beta_1} \left(\frac{1 + \Sigma/E_k(n)}{1 + \Sigma/E_k(1)}\right)
\]

(19)

or, if different modes are involved, \(i_n^2\) is replaced by the sum of different \(i_n^2\). In order to determine \(\Sigma/E_k(n)\) we use the empirical values of \(E_k(n)\) which are the energies of the particles \(m(n)\) minus the sum of the energies of the rest masses of the neutrinos. Since \(E_k(1)\) is smaller than \(E_k(n > 1)\) and since \(\Sigma = \text{const}\) it follows that the term in parenthesis in \(R\) is \(< 1\) and decreases with increased \(n\), whereas \(\beta_n/\beta_1\) is \(> 1\) and increases as \(\beta_n\) approaches 1. That means that the ratio \(E(n)/E(1)\) is an integer number times the rest mass factor which depends on \(n\) and decreases as \(n\) increases. This is so because the contribution of the rest masses of the neutrinos to the energy of the particle decreases. In the asymptotic case of large \(n\) the rest mass factor approaches 0.82. The different values of these factors are listed in Table 1 on the next to last line and can be compared to the factors following from the measured masses of the \(\nu\)-branch in the last line of Table 1. A complete agreement
of the calculated and experimental factors cannot be expected because we
do not take into account the consequences of spin, isospin, strangeness and
charm. The existence of the rest mass factors in the ratios of the $\nu$-branch
particle masses is solely a consequence of the rest masses of the neutrinos.

The ratio of the masses of the $\nu$-branch is a weak function of the mass of
the electron neutrino which enters Eq.(19) through $\Sigma$ of which $\Sigma m(\nu_e)$ is the
smaller part. The value of $m(\nu_e) = 5\text{meV}$ used in the calculations leading
to the numbers in Table 1 has been determined by trial and error, it is the
$m(\nu_e)$ which produced the smallest deviation ($\pm 0.7\%$) of the calculated rest
mass factors to the experimentally found factors on the last line of Table 1.
Only integer values of meV have been considered because the uncertainty of
$m(\nu_\mu)$ does not warrant the consideration of fractions of meV.

As in the case of the particles of the $\gamma$-branch we have also found the
antiparticles of the $\nu$-branch. From Eq.(6) follows that for each positive
frequency there exists also a negative frequency with the same absolute value.
That means, as follows from Eq.(17), that the energy contained in all lattice
oscillations is negative if the frequencies are negative. Since the total energy
of a $\nu$-branch particle consists of the energy of the lattice oscillations plus the
energy of the rest masses of the neutrinos, we have to show that the energy
of the neutrino rest masses can be negative, in other words that there are
antineutrinos with the same absolute mass as those of the neutrinos. Only
then will the absolute value of the mass of an antiparticle of the $\nu$-branch be
equal to the mass of the corresponding particle, as must be.

The existence of the muon antineutrinos follows from Eq.(7) for the group
velocity of the waves in the neutrino lattice. Since $c_g = 2\pi a d\nu/d\phi$ and since
the slope of the negative frequencies $d\nu_{an}/d\phi$ in the antiparticle has the
opposite sign of the slope $d\nu/d\phi$ in the particle and since $c_g$ is the same
in both cases it must be that the lattice distance $a_{an}$ of the antiparticles
has the opposite sign of the lattice distance $a$ of the particles, but has the
same absolute value as $a$. That means that the coordinate system of the
antineutrinos is turned by 180° around a central axis perpendicular to the
plane waves in the neutrino lattice. If $a_{an} = -a$ then the mass of a muon
antineutrino is negative according to Eq.(13), but has the same absolute
value as the muon neutrino. Since we do not have a formula for the mass
of the electron neutrino we cannot show that electron antineutrinos exist as
well. But, since the antiparticles of the $\nu$-branch have the same mass as the
$\nu$-branch particles, electron antineutrinos with the same absolute mass as the
electron neutrinos must exist.
The size of the particles does not enter Eq.(6) for the frequencies of the lattice oscillations, so the particle size has to be determined separately. As we have shown in [16] the particle size of the γ-branch particles is determined by the radiation pressure. The particle will break up at the latest if the outward directed radiation pressure is equal to the inward directed force between one lattice layer and the adjacent layer. A similar consideration applies for the ν-branch particles. The pressure in the neutrino lattice $p = \sum_{i=1}^{N} [m(\nu_\mu)\beta_\mu^2c^2 + m(\nu_e)\beta_e^2c^2]$ is proportional to the size of the lattice because $N$ is proportional to the lattice size. If the outward directed pressure is equal to the force between the lattice layers, in technical terms to the Young's modulus of the lattice, the lattice will break up, at the latest. Details cannot be discussed here.

Now we can explain the particles of the neutrino branch. The basic state $i_1, i_2 = (1.1)$ of the neutrino lattice oscillations corresponds to the $\pi^\pm$ mesons. The first higher mode $i_1, i_2 = (2.2)$ corresponds to the $K^\pm$ mesons. A superposition of two coupled (2.2) modes produces the neutron which has spin $\frac{1}{2}$, just as the superposition of two coupled η mesons or (2.2) modes produces the Λ particle with spin $\frac{1}{2}$ in the γ-branch. The superposition of a proton and a neutron or of two neutrons with opposite spin creates the $D^{\pm,0}$ meson. The superposition of two (2.2) states on a (3.3) state creates the $D^+_S$ meson. All ν-branch particles are accounted for, their masses are integer multiples of the basic state times the rest mass factor which is of order 0.85. The agreement of the calculated masses with the experimentally determined masses is in the 1% range.

5 Conclusions

We have shown that the masses of the neutrino branch of the so-called stable elementary particles, the $\pi^\pm$, $K^\pm$, $n$, $D^\pm$, and $D^+_S$ particles, can be explained as the sum of the energies of the oscillations of plane, standing waves in a cubic, diatomic, isotropic neutrino lattice and the rest masses of the neutrinos. In particular, we have explained, as a consequence of the rest masses of the neutrinos, the factors of about 0.85 which appear in the ratios of the ν-branch particles to the $\pi^\pm$ mesons. We have also determined the rest masses of the muon neutrino and the electron neutrino. Both neutrino types as well as the particles of the ν-branch have automatically antiparticles.

Let us consider what we have learned from the standing wave model
we propose. We started with the elementary observation that the so-called stable mesons and baryons consist of a $\gamma$-branch and a neutrino branch, and that the masses of the $\gamma$-branch are, in a good approximation, integer multiples of the $\pi^0$ meson. We have also observed that, according to Fourier analysis, a continuum of frequencies must be present in a particle created by a high energy collision. This continuum of frequencies can be absorbed by the oscillations of a nuclear lattice. We have explained the masses of the particles and antiparticles of the $\gamma$-branch with different modes and superpositions of modes of the frequency distributions of a nuclear lattice, using nothing else but photons. The masses so determined agree on the average within 0.7% with the mass ratios of the measured particles masses, an accuracy not matched in the theory of the particles. The remaining stable particles, the particles of the $\nu$-branch, can also be explained by the standing wave model as we have shown above. In this case we consider the oscillations of a lattice consisting of muon and electron neutrinos. We have determined the rest masses of the muon and electron neutrinos, as well as the ratios of the masses of the particles of the $\nu$-branch. In other words we have, except for the bottom particles, determined the masses of the entire spectrum of stable mesons and baryons using only photons or neutrinos.

We note that a nuclear lattice automatically entails the existence of a strong attractive force between the sides of two lattices, resulting from the unsaturated forces of the $10^6$ lattice points at each side. This force can be calculated following Born and Stern [17] as discussed in [10]. We will show in a forthcoming paper that the spin of the particles can also be explained with standing waves in the particles.

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