SecureBoost+ : A High Performance Gradient Boosting Tree Framework for Large Scale Vertical Federated Learning

Weijing Chen¹, Guoqiang Ma¹, Tao Fan¹, Yan Kang¹, Qian Xu¹, Qiang Yang¹,²
¹AI Department of WeBank, Shenzhen, China
²Hong Kong University of Science and Technology, Hong Kong, China
{weijingchen, zotrseeewma, dylanfan, yangkang, qianxu}@webank.com, qyang@cse.ust.hk

Abstract

Gradient boosting decision tree (GBDT) is a widely used ensemble algorithm in the industry. Its vertical federated learning version, SecureBoost, is one of the most popular algorithms used in cross-silo privacy-preserving modeling. As the area of privacy computation thrives in recent years, demands for large-scale and high-performance federated learning have grown dramatically in real-world applications. In this paper, to fulfill these requirements, we propose SecureBoost+ that is both novel and improved from the prior work SecureBoost. SecureBoost+ integrates several ciphertext calculation optimizations and engineering optimizations. The experimental results demonstrate that SecureBoost+ has significant performance improvements on large and high dimensional data sets compared to SecureBoost. It makes effective and efficient large-scale vertical federated learning possible.

1. Introduction

With the rapid development of information technologies, nowadays a huge amount of data are generated, collected in various domains. Artificial intelligence technologies, especially deep learning, have made striking advances on the basis of accumulated big data. However, data is also a constraint of industrial AI applications. In the actual scene, big data are stored in different institutions, or different departments in the same enterprise, forming a landscape called data isolated islands. As the whole society is now increasingly concerned with privacy information protections and preventing unlawful data usages, most countries are taking action to enact data privacy-related legislation. For example, the European Union has enacted General Data Protection Regulation (GDPR)¹⁷ which is design for enhancing user-data privacy safety. This tendency makes cross-institutions data sharing and machine learning modeling even more challenging.

To break down the barriers of AI applications on scattered large-scale data, The concept of Federated Learning (FL)²⁴ is proposed in recent years, which mainly focuses on building machine learning models based on privacy-computing techniques, such as Homomorphic Encryption (HE)¹³, Multi-Party Computation (MPC)²⁵. Federated learning can be generally divided into vertical (feature-partitioned) federated learning and horizontal (instance-partitioned) federated learning. Recently many efforts are made to develop cross-parties privacy-preserving vertical federated algorithms. For example, researchers have been proposed vertical privacy-preserving linear/logistic regression ²⁴ ²¹ ³ ⁴ ¹⁰, matrix factorization ² ²³ ²² ¹⁴ ¹⁸.

Among these vertical federated algorithms, tree-based algorithms, especially gradient boosting decision trees (GBDT), are one of the most popular kinds. Tree-based methods become very successful for its excellent performance in data-mining competitions⁵ and effectiveness in industrial tasks⁹ ⁷. Since tree models have efficient modeling speed, they are usually used in recommendation⁹ ¹⁵, fraud detection¹¹ and click-through rate prediction¹⁹ tasks with large scale data set. We can see that tree-based vertical federated algorithms have great potential in large-scale cross-institutions modeling. There have been several studies of secure tree algorithms in the vertical federated learning setting ¹² ²⁰ ¹⁶. SecureBoost ⁶ is also one of them, and it is our previous work. However, as far as we know, some of these previous studies concentrate more on the security of algorithms but don’t pay much attention to the gap between research environment and practice. Some of them are only tested on public experimental data sets instead of million-scale or high-dimensional data which are commons in the several above-mentioned tasks, and some of them are tested on relatively larger data sets but can not return results in a reasonable time. And according to our practical experiences, the
performance of Secureboost is still not satisfying. Thus we think it is necessary to invest efforts to tackle the challenge of learning vertical tree models on big data, and in this paper we propose SecureBoost+, which is a vertical gradient boosting tree framework based on HE encryption schema, and it is developed based on our previous work SecureBoost in FATE-1.11.

The main contributions of SecureBoost+ are:

- We propose a cipher-optimization framework which is integrated in SecureBoost+. This framework is specifically designed for HE encryption schema based vertical tree methods.
- We propose three novel training mechanism-level optimizations. They can in addition speed up the training process.
- We implement SecureBoost+ on both large-instances and high-dimension dataset to demonstrate its high efficiency on large-scale vertical federated training.

2. Preliminary and Related Work

2.1 Vertical Federated Learning

The vertical federated learning or feature-partitioned federated learning is in the scenario that several data sets have same instance id are distributed in different parties 24. Each party’s data set has its unique feature space.

In the process of vertical federated learning, each party locally compute model intermediate results and then aggregate results in a privacy-preserving way at certain party to compute the model update information, and model loss. These information will be returned to participated parties for local model update. By this way, every party works collaboratively to build a federated model that makes use of global features.

2.2 Homomorphic Encryption

The Homomorphic Encryption (HE) schema is the key components of many privacy-preserving ML algorithms 3 2 24 23. The core properties of HE are homomorphic addition and multiplication:

\[
[[x_1]] + [[x_2]] = [[x_1 + x_2]] \tag{1}
\]

\[
x_1 \otimes [[x_2]] = [[x_1 \times x_2]] \tag{2}
\]

where \([\ldots]\) denotes the encryption operator. With (1) and (2), SecureBoost+ is able to encrypt and compute on intermediate encrypted results.

2.3 SecureBoost Review

We first review the SecureBoost 6, from which our SecureBoost+ is improved.

2.3.1 Problem setting

All parties conduct a privacy-preserving instance id intersections at the very beginning. After intersections, We get \(\{X^k \in \mathbb{R}^{n \times d_k}\}^{m}_{k=1}\) as the final aligned data matrices distributed on each parties. Each party holds a data matrix \(X^k\) which has \(n\) instances and \(d_k\) features. For the ease of description, we use \(x\) to denote a federated instance whose features are distributed on multiple parties and use \(I = \{1, \ldots, n\}\) to represent the instance space. \(F^k \in \{f_1, f_2, \ldots, f_{d_k}\}\) is the feature set on the k-th party. Each party holds its unique features, for any two feature set \(F^i\) and \(F^j\) from i-th party and j-th party, \(F^i \cap F^j = \emptyset\). Only the guest party holds the label \(y \in \mathbb{R}^n\). The host parties only have features.

Let \(f\) be a federated decision tree, the prediction on guest party for a federated instance is given by the sum of all \(K\) decision tree:

\[
\hat{y}_i = \sum_{k=1}^{K} f_k(x_i) \tag{3}
\]

Our goal is to train an ensemble gradient boosting tree model on the federated dataset \(\{X^k \in \mathbb{R}^{n \times d_k}\}^{m}_{k=1}\) and label \(y\). Notice that SecureBoost adopt a histogram based split finding strategy similar to Lightgbm. So every party will transform their feature values into bin indices using quantile binning. We here use \(X_{(b)}\) to represent data after binning.

2.3.2 Federated Split Finding

When training an ensemble tree model, we add a new decision tree \(f_t\) at iteration \(t\) to minimize the following second-order approximation loss:

\[
L^{(t)} = \sum_{i \in I} \left[ l(y_i, \hat{y}^{(t-1)}_i) + g_t f_t(x_i) + \frac{1}{2} h_t f_t^2(x_i) \right] + \Omega(f_t) \tag{4}
\]

where \(\Omega(f_t)\) is the regularization term and \(g_t, h_t\) are the first and second derivatives of \(l(y_i, \hat{y}^{(t-1)}_i)\) with respect to \(\hat{y}_i\).

We rewrite equation (4) in a leaf-weight format 5:

\[
L^{(t)} = \sum_{j \in \Gamma} \sum_{i \in l_j} \left[ l(y_i, \hat{y}^{(t-1)}_i) + g_j w_j + \frac{1}{2} h_j w_j^2 \right] + \frac{\lambda}{2} w_j^2 \tag{5}
\]

By setting the second-order approximation function above, we derive the split gain function for splitting a node:

1https://github.com/FederatedAI/FATE
\[ \text{gain} = \frac{1}{2} \left[ \frac{(\sum_{i \in I_L} g_i)^2}{\sum_{i \in I_L} h_i + \lambda} + \frac{(\sum_{i \in I_R} g_i)^2}{\sum_{i \in I_R} h_i + \lambda} - \frac{(\sum_{i \in I} g_i)^2}{\sum_{i \in I} h_i + \lambda} \right] \]

where \( I_L, I_R, I \) are the instances space of left child node, right child node and parent node respectively. And the leaf output of an arbitrary leaf \( j \) put is given by:

\[ w_j = \frac{\sum_{i \in I_L} g_i}{\sum_{i \in I_L} h_i + \lambda} \]

Because it is easy to infer guest labels from \( g_i \) and \( h_i \), in order to compute the global best split gain using features of all parties while protecting guest labels from leaking, guest needs to conduct homomorphic encryption on \( g_i \) and \( h_i \) of all instances and send them to other host parties. With the additive property of homomorphic encrypted \( g_i \) and \( h_i \), hosts are able to compute ciphertext histograms, and then construct split-info. For the consideration of protecting host feature information, Hosts mark split-infos with unique ids, randomly shuffle these split-infos and then send them to guest. Details of the host-side computations are shown in Algorithm 1 below.

**Algorithm 1 Split-info Construction on k-th Host**

**Data:** \( I \); instance space of current node; \( X^k \): \{([g_i]), ([h_i])\} \( i \in I \); encrypted \( g, h \) from guest party;

\( F^k \); feature space

**Result:** Encrypted Split-info

Initialize Histogram \( H \):

for each instance \( x_{(b)} \) in \( X^k_{(b)} \), \( i \in I \) do

for each feature \( f_j \) in \( F^k \) do

\( \text{bid} = x_{(b)}[j] \) (bin index)

\( H[j][\text{bid}][0, 1] = ([g_i]), ([h_i]) \)

end

for each feature \( f_j \) in \( F^k \) do

for \( \text{bid} = 1 \) to the last bin index do

\( H[j][\text{bid}][0, 1] = H[j][\text{bid} - 1][0, 1] \)

\( g_i, h_i = H[j][\text{bid}][0, 1] \)

AddSplitInfo(\( g_i, h_i \))

end

end

ShuffleAndSendToGuest()

The guest side can compute local split-infos at first, then receive encrypted split-info from host parties and decode them. Then guest now are able to know the best split point without leaking labels and knowing information about host parties’ features. The whole process of global split-finding is shown below.

**Algorithm 2 Split Finding**

**Data:** \( \{s^0_{(b)}, ..., s^k_{(b)}\} \); Host Encrypted Split-infos; \( s_g \); Guest Split-infos; \( I \); instance space on current node;

\( \{g_i, h_i\}_{i \in I} \); \( g \) and \( h \)

**Result:** Best split-info

Initialize \( \text{best_gain}, \text{best_split} \);

Add \( s_g \) into \( \{s^0_{(b)}, ..., s^k_{(b)}\} \); \( g, h = \sum_{i \in I} g_i, \sum_{i \in I} h_i \)

for \( s \in \{s^0_{(b)}, ..., s^k_{(b)}, s_g\} \) do

for split info \( s \in s \) do

if \( s \) is from host then

\( \text{decrypt}(s.g, s.h) \)

end

\( g_r, h_r = g - s.g, h - s.h \)

\( \text{gain} = \text{Gain}(s.g, s.h, g_r, h_r, g, h) \)

if \( \text{gain} > \text{best_gain} \) then

\( \text{best_gain} = \text{gain}, \text{best_split} = s \)

end

end

SendSplitInfoIdToBelongingParty()

The party who holds the best split-info will be responsible for splitting and assigning current node instances to child nodes. The result of instances assignment will be synchronized to all parties. Started from the root node, We repeat the split-finding in a layer-wise manner until we reach the max depth or meet the stop conditions.

### 3. Optimizing SecureBoost

The most computationally expensive parts of histogram-based gradient boosting algorithms are the histogram building and split-point finding [9]. In the vertical-federation scenario, computational costs in these two parts are particularly high. From the review of SecureBoost, we have following observations:

1. Histograms computation on the host side will be especially time-consuming because it is computed on homomorphic ciphertexts. Homomorphic additions of ciphertext, which usually involve multiplication and modulo operations on large integers, are far more expensive than plaintext additions

2. We need to encrypt all gradient and hessianian, and we have to decrypt batches of encrypted split-infos when we are splitting nodes. The encryption and decryption costs are not negligible when training on large-scale data.

3. We need to take communications into consideration when running a federation algorithm. At every epoch, we first need to synchronized encrypted gradient and hessian to host parties. For every node, we have a batch of encrypted split-info to send. These will bring heavy communications overheads.
4. When training on large multi-classification data set, all costs enlarge by several times(times of unique label number), further makes training more expensive.

Therefore, inspired by the four points listed above, we proceed to optimize SecureBoost from three directions:

- Ciphertext-related operations and transmissions are the main overhead in vertical-federated algorithms, as we analyze above. We firstly seek way to directly reduce the cost of homomorphic ciphertext computations on host sides, especially computations related to histogram computations and split-finding. We propose a cipher-optimization framework. Its main ideas are: pack plaintext to reduce ciphertext numbers in computation and transmission, compress ciphertext on host sides using homomorphic properties to reduce decryption and communication costs. We introduce this framework elaborately in section 4.

- The federation parts of algorithms, like split-finding, always involve cipher computations and communications. From training mechanism levels we design two methods: the mix tree mode and the layered tree mode to directly skip partial of federation parts of the default setting of SecureBoost.

We notice that default GBDT multi-classification training setting might not suitable for large-scale federation training, so we propose a new multi-classification mechanism SecureBoost-MO for faster multi-classification training.

These contents will be introduced in the section 5.

- We also merge common engineering optimization techniques into SecureBoost+. They directly reduce instances and data terms involved in boosting tree training. This is benefical when training on large-scale data. We introduce them in the section 6.

For the ease of description, we list all key notations of this paper in the Table 1.

| Notations | Descriptions |
|-----------|--------------|
| I, F      | Set of instance id, features |
| X         | Instance data matrix |
| g, h, g, h, G, H | Scalar, vector, matrix of gradient, hessian |
| gh; gh; GH | Scalar, vector, matrix of packed g, h |
| g_{off}   | The offset number for g |
| g_{max}, h_{max} | The maximum of g and h |
| g_{max}, h_{max}, r | The maximum of fixed point encoded g and h |
| r         | The fixed point encoding precision |
| H         | Histogram, a multi-dimension tensor |
| s, s, p, p | Split point, vector of split points |
| n, f, b, n | Bit assignment for g, h and packed g, h |
| h         | The bit length of the largest integer allowed by the encryption schema |
| b_{g}, b_{h}, b_{gh} | Number of instance, feature, bin, tree node |
| l         | Tree height |
| ηs, ηc    | Bit assignment for g, h and packed g, h |
| n_{k}     | The number of compressed split points, classes |
| [-]       | Ciphertext needed for packing g, h of k classes |

- The communication cost: encrypted g/h + split-infos batches:

\[
\text{cost}_{\text{comm}} = 2 \times n_i + 2 \times n_b \times n_f \times n_n \quad (10)
\]

4.2 GH Packing

We firstly introduce GH Packing.

From algorithm [1] we notice that gradients and hessians have the same operations in the process of histogram building and split-finding: gradients and hessians of an sample will be encrypted at the same time, encrypted gradients and hessian of an examples go to the same bin but added to different indices(grad is added to 0 and hess is added to 1), and gradients and hessians of a split-info will be decrypted together.

It is very straightforward to come up with an idea that we can bundle gradient and hessians together since they share the same operations. Once we found a way to bundle gradients and hessians into one number, and it satisfies the properties of simultaneous addition and subtraction of g and h in a number, cost of all g/h-related operations will be reduced by half.

\[
\text{cost}_{\text{comp}} = 2 \times n_i + 2 \times n_b \times n_f + n_n \times b \quad (8)
\]

\[
\text{cost}_{\text{ende}} = 2 \times n_i + 2 \times n_b \times n_f \times n_n \quad (9)
\]
The Paillier and the Iterative-Affine are the homomorphic cryptosystems available in the SecureBoost. It usually uses a key length of 1024 or 2048 during the training process. Taking Paillier HE as the example, when using a 1024 length key, the upper bound of the positive integer allowed in encryption is usually a 1023 bit-length number. This upper bound is far larger than the gradient/hessian in a fix-point integer format, they usually have a bit length smaller than 100. Therefore, a large plaintext space is wasted. Inspired by recently proposed gradient quantization and gradient batching techniques [26], we here adopt a gradient/hessian packing and method. We combine g and h using bit operation: We move g to the left by a certain number of bits and then add h to it.

Let us take a binary-classification task as an example. In a binary-classification task, the range of g is $[-1, 1]$, maximum of $g$ $g_{\text{max}} = 1$ and the range of h is $[0, 1]$, $h_{\text{max}} = 1$. The SecureBoost adopts a simple fix-point encoding strategy to transform a float number $n_{\text{float}}$ into a large integer $n_{\text{int}}$:

$$n_{\text{int}} = \lfloor n_{\text{float}} \times 2^r \rfloor$$  \hspace{1cm} (11)

where $r$ is a precision parameter, usually set as 53. Concerning that there will be negative numbers in g, to satisfy the property of adding and subtracting, we will offset all g to make sure they are all positive numbers. Another consideration of offsetting g is for the cipher-compressing in the next section. In the classification case, the offset number $g_{\text{off}}$ for g will be 1. We will remove offsets when we are recovering g, h in the split finding step.

After offsetting g and transforming g,h into large positive integers, we can now pack them into one large integer. In the case of aggregation result overflow during histogram computation, we need to reserve more bits for g and h. Assuming we have $n_i$ instances, so we are sure that the summing value in an arbitrary bin in histograms will no larger than

$$g_{\text{imax}} = n_i \times (g_{\text{max}} + g_{\text{off}}) \times 2^r$$
$$h_{\text{imax}} = n_i \times h_{\text{max}} \times 2^r$$  \hspace{1cm} (12)

So we assign $b_g$ bits for g and $b_h$ bits for h:

$$b_g = \text{BitLength}(g_{\text{imax}})$$
$$b_h = \text{BitLength}(h_{\text{imax}})$$  \hspace{1cm} (13)

We move g to left by $b_g$ bits and then add h to it to combine them.

The packing algorithm is described below:

**Algorithm 3** Gradient Hessian Packing

**Data:** $g, h$, vectors of $g, h, n$, the instances number; $r$, the fix-point parameter

**Result:** $[\lfloor gh \rfloor]$, Encrypted packed GH; $b_{gh}$, Total bit take

$$g_{\text{off}} = \text{abs}(\min(g))$$
$$g = g + g_{\text{off}}$$
$$g_{\text{max}} = \max(g)$$
$$g, h = \text{round}(g \times 2^r), \text{round}(h \times 2^r)$$

compute $b_g, b_h$

initialize empty vector $[\lfloor gh \rfloor]$

for $g_i, h_i \in g, h$ do

$g_i = g_i << h_{\text{bits}}$
$$gh_i = g_i + h_i$$
$$[\lfloor gh \rfloor].\text{add}(|\text{encrypt}(gh_i)|)$$

end

$b_{gh} = b_g + b_h$

This algorithm returns $[\lfloor gh \rfloor]$ and $b_{gh}$. From $b_{gh}$ we can know the bit-length a packed gh will take, and we are sure that histogram statistical result in a split-info will not exceed $b_{gh}$ bits. Guest will synchronize $[\lfloor gh \rfloor]$ to all hosts party. Now host only need to compute on one ciphertext per sample when building histograms, the whole homomorphic computation cost is reduced by half.

Figure 1 demonstrates the process of GH packing.

![Figure 1: The Process of GH Packing](image)

### 4.3 Ciphertext Histogram Subtraction

Histogram-subtraction is a classic and commonly used optimization technique in histogram-based tree algorithms. When splitting a node, samples are distributed to either the left node or the right node. So for every feature, summing the histogram sets of left node and right node we will get the parent histograms. To make use of this property, in SecureBoost, both guest and host parties cache previous layers’ histograms. When growing from the current layer, guest and host firstly compute the histograms of nodes with fewer samples, and finally get their siblings’ histograms by histogram subtractions.

Since ciphertext histogram computations on the host side are always the bulk of overhead, ciphertext histogram subtraction will considerably speed up the computations because at least half of the homomorphic computations are reduced. In SecureBoost+, host parties conduct histogram...
subtractions on packed g, h histograms. Actually, the homomorphic computation cost is somehow reduced more than half because samples are usually not equally divided.

Figure 2 shows the process of ciphertext histogram subtraction.

Figure 2: Ciphertext Histogram Subtraction

4.4 Cipher Compressing

Though GH packing makes use of large plaintext space, the plaintext space is not fully exploited. Remind that we usually have a 1023 bit-length integer as the plaintext upper bound when using a 1024-bit key length Pailier HE. Assuming instances number \( n = 1,000,000 \), \( r = 53 \), we will assign 74 bits for \( g \) and 73 bits for \( h \), according to equation (13). The \( b_{gh} \) is 147, still smaller than 1023.

To make full use of plaintext space, we can conduct cipher compressing on host parties. This technique utilizes the property of our in-built HE algorithms that an addition operation or a scalar multiplication operation has less cost than a decryption operation. We use addition and multiplication operations to compress several encrypted numbers in split-info into one encrypted number and make sure that the plaintext space is fully exploited. The idea of this method is the same as the GH Packing: we multiply a ciphertext by \( 2^{b_{gh}} \) to move its plaintext to left by \( b_{gh} \) bits. Then we can add another ciphertext to it to compress two ciphers into one. We can repeat this process until the plaintext space has no bits left for compressing. Once a guest received an encrypted number, it only has to decrypt one encrypted number to recover several \( gh \) statistical results and reconstruct split-infos.

In the case above, given a 1023 bit-length plaintext space, and an \( b_{gh} \) of 147, We are able to compress \( \lceil 1023/147 \rceil = 6 \) split points into one. The algorithm is described below.

```
Algorithm 4 Cipher Compressing on the Host Parties
Data: s, split_infos_list
Result: SplitInfo Packages List
p
Host receive compressing parameter \( \eta_s \) and \( b_{gh} \) from Guest
init compressing package list \( p \)
init split info id list \( \text{id} \); split info count list \( \text{sc} \)
init \( \text{count} = 0 \), init encrypted number \( e = \text{None} \)
for \( s \in s \) do
    \( [gh_l] = s.gh_l \)
    if \( \text{count} == \eta_s \) then
        \( \text{count} = 0 \)
        \( \text{p}.add([e, \text{id}, \text{sc}]) \)
        \( e = \text{None} \)
        empty(\( \text{id} \)), empty(\( \text{sc} \))
    end
    if \( e == \text{None} \) then
        \( e = [gh_l] \)
        continue
    end
    \( e = e \times 2^{b_{gh}} \)
    \( e = e + [gh_l] \)
    \( \text{sc}.add(s.\text{sample_count}) \)
    \( \text{id}.add(s.\text{id}) \)
    \( \text{count} += 1 \)
end
```

By using cipher-compressing, on one hand, we save time from less decryption operations, on the other hand, the costs in transferring encrypted split-Infos are reduced by several times.

For better understanding, Figure 3 demonstrates the process of cipher compressing.

4.5 The Framework Summary

We put three optimization methods together to rewrite optimized Split-info Construction and Split Finding algorithms:

- At the beginning, guest pre-process \( g, h \) using algorithm 3. Assuming that the bit length of the plaintext space of HE schema is \( i \). Then guest can derive capacity needed for cipher compressing: \( \eta_s = \lfloor i/b_{gh} \rfloor \) and then synchronizes capacity and \( b_{gh} \) to all party.

- Host parties firstly find out nodes with less instances, then host parties build histogram and compute split-info using algorithm 5 which combines GH packing, Histogram subtraction and Cipher Compressing.
Algorithm 5 Optimized Split-info Construction on Host

Data: $I$, instance space node with less instances; $X^k_{b(i)}\{[[gh_i]]\}_{i\in I}$, packed and encrypted $gh$ from guest party $F^k$, feature space $H_p$, cached parent ciphertext histogram

Result: Compressed Split-info Packages

Initialize Histogram $H$ and sibling Histogram $H_{sib}$;

for each instance of $X^k_{b(i)}$, $x_i(b)$, $i \in I$ do

  for $f_j \in F^k$ do
    bid = $x_{i(b)}[j]$ 
    $H[j][bid] += [[[gh_i]]]$ 
  end

end

$H_{sib} = H_p - H$, cache($H$, $H_{sib}$)

for $H_i \in \{H, H_{sib}\}$ do

  Initialize split-info list $s$;

  for $f_j \in F^k$ do
    for bin_idx ← 1 to the last bin index do
      $H_i[j][bid] += H_i[j][bid-1]$ 
      $gh = H_i[j][bid]$ 
      AddSplitInfo($gh_i$, $s$)
    end
  end

  AddtoResult(CipherCompressing($s$))
end

ShuffleAndSendToGuest()

Guest receive split-info packages and decompress, unpack $gh$, remove accumulated $g_{off}$ and recover split-infos. Then with split-infos guest runs the algorithm 2 to get the best split. The whole process is illustrated in algorithm 6.

4.6 Re-estimate Cost after Cipher Optimization

We return to the cost estimation in this subsection. To be more detailed, assuming that we are using a paillier encryption schema of 1024 key-length, we have 1 million instances and 2,000 features, tree depth is 5, bin number is 32, the precision parameter $r$ of equation (11) is 53.

Firstly with GH packing and histogram subtraction, ciphertext histogram computation costs is reduced by 4 times and operations in bin cumsum is reduced by half.

The homomorphic computation cost: ciphertext histogram computations + bin cumsum =
We bring the values into formula (8) and (14), the cipher computational cost is reduced by 75%.

Because we pack g, h into one, so encryption/decryption operations is reduced by half. Under current paillier encryption setting, we use c to denote the compress number, we can compress: \( \eta = \lfloor 1023 / 147 \rfloor = 6 \) ciphertext into one, so we get:

\[
\text{cost}^*_\text{comp} = \frac{1}{2} \times n_i \times h \times n_f + n_n \times n_f \times n_b \]

We bring the values into formula (8) and (14), the cipher computational cost is reduced by 75%.

Because we pack g, h into one, so encryption/decryption operations is reduced by half. Under current paillier encryption setting, we use c to denote the compress number, we can compress: \( \eta = \lfloor 1023 / 147 \rfloor = 6 \) ciphertext into one, so we get:

\[
\text{cost}^*_\text{ende} = n_i + \frac{n_b \times n_f \times n_n}{\eta_s} \]

The communication cost is:

\[
\text{cost}^*_\text{comm} = n_i + \frac{n_b \times n_f \times n_n}{\eta_s} \]

We bring the setting values into equations (15), (16), and equations (9), (10), result shows that the cost is reduced by 78%.

From the estimation we can see that cipher-optimization is theoretically effective.

5. Training Mechanism Optimization

The SecureBoost can already have a significant speed improvement with the cipher-optimization framework. We secondly try to find ways to reduce cipher-related computations costs and communication costs at a training mechanism level. In this section we propose three mechanism-level optimization methods. These methods are integrated in SecureBoost+ framework and can be used to speed up training under certain scenarios.

5.1 Mix Tree Training Mode

In the Mix Tree Training Mode, every participated party will build a certain number of trees using their local features, and this procedure will be repeated until reach the max epoch or meet the stop conditions.

While building a guest tree, the guest party side simply builds this decision tree locally, other host parties will skip this tree and wait. While building a host tree, the related host party will receive encrypted g/h to find the best split points with the assistant of the guest party. The structures of host trees and split points will be preserved on the host side while leaf weights will be saved on the guest side. This mode is suitable for data-balanced-distributed vertical federated learning. By this way, we can speed up the training by skipping the federation process.

5.2 Layered Tree Training Mode

The Layered Tree Training Mode is another mechanism we designed for data-balanced-distributed vertical federated learning. Similar to The Mix Tree Training Mode, the guest and the host are responsible for different layers when building a decision tree. Every decision tree has in total \( h = h_{\text{guest}} + h_{\text{host}} \) layers. The host will be responsible for building the first \( h_{\text{host}} \) layers, with the help of the guest, and the guest will build the next \( h_{\text{guest}} \) layers. All trees in the boosting model will be built in this 'layered' manner. Like the Mix Tree Training mode, parts of communication costs and encryption costs will be saved by skipping the federation parts.

5.3 SecureBoost-MO

In the traditional GBDT setting, the strategy of multi-classification learning is to separate the gradient/hessian of each class and learn a tree for each class independently and each tree is responsible for predicting a single variable. In the local training, we are able to fit a tree for each class parallelly. However, in the vertical federated scenario, using a traditional single-output-tree-based multi-classification strategy has its limitations: All the computation costs and communication overheads are amplified by the times of the number of labels. It will be extremely time-consuming if we learn a multi-classification model tree by tree. But if we train every epoch in a parallel manner, it will bring a heavy computation and communication workload. This will make multi-classification tree learning expensive.

Thus, it is necessary that we make effort to specifically optimize the multi-classification training. Inspired by [27], which introduces a general framework of multi-output gradient boosting tree, we here propose a novel multi-output-tree based vertical boosting tree techniques for multi-classification tasks. Leaves of multi-output-tree give multi-dimension output, corresponding to every class. Instead of learning trees for every class separately, now we only need to fit one tree at every boosting epoch. Figure 5 tells the differences between traditional multi-classification trees and MO-trees.

5.3.1 Gain computation of SecureBoost-MO

Here we develop a splitting gain function for multi-output decision tree. Given a \( l \) label multi-classification task, we use \( g \) and \( h \) to denote the gradient/hessian vectors which have \( l \) elements, each element corresponds to the \( g/h \) of each label. In a multi-output decision tree, every leaf gives an output \( w \), which also has \( l \) elements. We rewrite equation (10) to get:
Figure 4: The Mix Tree Mode and the Layered Tree Mode

Figure 5: Default Trees and Multi-Output Trees

\[ L(t) = \sum_{j \in T} \left[ l(y_i, \hat{y}_i^{(t-1)}) + g_i^T w_j + \frac{1}{2} w_j^T H w_j \right] \]

\[ + \frac{\lambda}{2} w_j^T w_j \]  

(17)

where \( y_i, \hat{y}_i^{(t-1)} \) are on-hot label vector and predict score vector, \( H \) is the hessian matrix. Because in SBT-MO we use cross-entropy as the multi-classification objective, so \( H \) is a diagonal matrix. By minimizing \( L(t) \) we get

\[ w_j = -\frac{\sum_{i \in I_j} g_i}{\sum_{i \in I_j} h_i + \lambda} \]  

(18)

as the weight function and the splitting gain function:

\[ Score = -\frac{1}{2} \sum_{j=1}^{l} \frac{1}{\sum_{i \in I_j} (h_j)_i} \]  

(19)

\[ gain = Score_{parent} - (Score_{left} + Score_{right}) \]  

(20)

Then we are able to develop SBT-MO based on the leaf weight and split finding functions.

5.3.2 Split Finding of SecureBoost-MO

The splitting finding process of Multi-output boosting tree has the nearly same process as the standard GBDT trees, but there are only two differences:

- In standard GBDT trees bin values are scalar \( g, h \) values. When computing the histogram in MO-trees, aggregated terms in the bin are \( g, h \) vectors.

- When computing the split gain, leaf weights, we use mo-gain, mo-weight equation (20), (18).

As a result, in the vertical federated learning scenario, we need to design a new structure that supports encryption, decryption of \( g, h \) vectors, and support operations in the histogram computation, histogram subtraction, and split-info building.

To fulfill this requirement, we design a Multi-Class packing algorithms based on our cipher-optimization framework. Remind that \( b_{gh} \) represent the plaintext bit length for a pair of \( g, h \). For a HE schema that has a plaintext space of \( t \) bits, and we have \( k \) classes in total, we can pack:

\[ \eta_c = \left\lceil \frac{t}{b_{gh}} \right\rceil \]  

(21)

classes in a ciphertext, and in total we need:

\[ n_k = \left\lceil \frac{k}{\eta_c} \right\rceil \]  

(22)

integers to pack \( gh \) vectors of an instance. With this strategy we are able to pack \( g \) and \( h \) vectors of a sample into a small amount of integers and then we encrypt them to yield a ciphertext vector. The algorithm is shown below:
Algorithm 7 Multi Class GH Packing

Data: G, H. Each row are g, h, corresponds to each instances, n, the instances number; r, the fix-point parameter.

Result: [[GH]], each row of matrix is a packing cipher vector.

g_{off} = \text{abs}(\text{min}(G)), G = G + g_{off}
G, H = \text{round}(G \times 2^r), \text{round}(G \times 2^r)
g_{max} = \text{max}(G)

compute \(\eta_c, b_g, b_h\)
initialize empty matrix \([\text{GH}]\)

for \(g_i, h_j \in G, H\) do
  init empty list \([\text{vec}]\)
  for \(g_k, h_l \in g_i, h_j\) do
    if count == \(\eta_c\) then
      \([\text{vec}], \text{add(encrypt(e))}\)
      count, e = 0, 0
    end
    \(g_j = g_j << b_h, gh_j = g_j + h_j\)
    \(e << b_h, e += gh_j\)
    count += 1
  end
  \([\text{GH}], \text{add([vec])}\)
end

Guest decrypt returned split-info, recover accumulated g, h vectors using Algorithm [8] and find out global best splits.

Algorithm 8 SBT-MO Split Info Recovery

Data: s, a split info from host; l, label numbers; \(\eta_c\), number of class a ciphertext holds.

Result: s, the recovered split info

init count = 0; init empty list g, h

for e in s, [vec]] do
  decrypt(e)

  \(gh = e \& (2^{bh} - 1), e = e \gg b_h\)
  \(h = gh \& (2^{bh} - 1), g = gh \gg b_g\)
  \(g = g - g_{off} \times s, \text{sample_count}\)
  g.add(g), h.add(h)
  count += 1
  if count == \(\eta_c\) then
    break
end

\(s, g, s, h = g, h\)

6. Subsample and Sparse Optimization

We also adopt two simple yet efficient engineering optimization methods that are commonly used in the existing tree-boosting framework.

6.1 GOSS

Gradient-based One-Side Sampling is firstly proposed in Lightgbm [9]. The idea of GOSS sampling is very forthright: instances are sampled according to their gradients, when the number of training instances is large, we put more attention on those hard-to-fit samples, which may have large gradients, and discard most of those well-trained instances whose gradients are small. The sample ratios of large gradient samples and small gradient samples are controlled by parameter \(top\_rate\) (usually is 0.2) and \(other\_rate\) (usually is 0.1). As proved in [9], tree model will still reach a performance close to full-instance training performance. Thus, in a federated scenario, GOSS can play a very important role because we can reduce the number of training instances by \(1 - (top\_rate + other\_rate)\) percent. As a result, overall costs are reduced greatly.

6.2 Sparse Optimization

The sparse dataset, which contains large fractions of zero terms, is common in real-world modeling scenarios. In SecureBoost, histogram computation is designed as
sparse-aware. In quantile binning step, original data are transformed into key-value format sparse vector wherein keys are feature index and values are bin indices. For those instances with zero values in their features, corresponding zero-value feature indices(keys) in the sparse vector are missed on purpose. We can recover the gradient/hessian sum of zero-value terms of every feature by subtracting current feature gradient/hessian sum from node gradient/hessian sum. With sparse-aware histogram computation, the cost of host-side ciphertext histograms computation is especially reduced because histogram computing on sparse terms needs a large number of homomorphic additions but is now reduced to two-time homomorphic additions.

7. Experiments

In this section, we conduct several experiments to empirically validate the effectiveness of our proposed hetero boosting tree framework. Our experiments mainly focus on these points:

- The cipher-optimization, GOSS and sparse optimization are the default setting for large-scale hetero federation training in our proposed framework. We will find out to what extend they can speed up the hetero federated tree boosting algorithms together.
- We propose three training mechanism level optimizations. We want to show the benefits of our proposed mechanisms that they can further speed training on basis of previous optimization, on the condition of reaching the same model performance.
- Throughout these experiments we will pay attention to model performances, to make sure that our optimization methods are lossless in comparison to local modeling.

7.1 Experiment Setting

**Baseline:** We use Secureboost provided in FATE-1.5 as the federation learning baseline and Xgboost as the local modeling baseline.

**Environment:** The framework is developed based on FATE-1.6. Our experiments are conducted on two machines which are guest and host respectively. Each machine has 16 cores and 32GB RAM. They are deployed in intranet with a network speed of 1GBps.

**Hyper-parameters:** For all experiments, key hyperparameters settings are uniform: We set $tree\_depth = 5$, $max\_bin\_num = 32$ (in Xgboost the split method is 'hist') and $learning\_rate = 0.3$. For encryption parameters, we use Paillier and $IterativeAffine$ as the encryption schema, the $key\_length = 1024$. For the efficiency of conducting experiments, our baseline methods will run $tree\_num = 25$ (epoch) trees.

In our framework, we use goss subsample, the $top\_rate = 0.2$ and the $other\_rate = 0.1$. In the Layered Tree Mode the $guest\_depth = 2$ and $host\_depth = 3$, and in the Mix Tree Mode every party is responsible for building $tree\_per\_party = 1$ tree.

**Data set:** We evaluate our framework on open real-world large-scale data sets to test model performance and training speed. We choose both large-instances and high-dimensional data sets.

The first four data sets are for binary classification tasks:

- Give credit A bank credit binary-classification data set. It contains 150,000 instances and 10 features. It is extracted from UCI Credit Card data set and is a standard data set in FATE.
- Higgs and SVHN data sets are to predict physics events. They have million scales of instances and are widely used in machine-learning training speed evaluations. 
- Epsilon is a binary data set from PASCAL Challenge 2008. It not only contains a large number of samples but also contains 2000 features. So It is an ideal data set to test the performance on the large-scale and high-dimensional modeling.
- Three Multi-classification data sets are prepared for verifying the usefulness of SecureBoost-MO on purpose:
  - Sensorless is for sensorless drive diagnosis, Covtype predicts 7 types of forest covers, and SVHN is a street-view digit image classification task. We choose these three multi-classification data sets for they have more classes of labels while at the same time have relatively more instances. And we choose SVHN specifically for it is a high-dimensional data set.

We vertically and equally divide every data set into two to make guest data set and host data set. Details of data sets are shown in table.

**Metrics** We use AUC for binary tasks and accuracy scores for multi-classification tasks. We use train scores as the model performance metrics.

7.2 Assessment of Default SecureBoost+

**Training speed** We firstly conduct experiments to test the training speed and the model performance of default SecureBoost+, which enables GOSS, sparse optimization and cipher optimization. We only take tree-building time into account and ignore non-tree-building time, like time spent on data I/O, feature engineering and evaluation. For both SecureBoost and SecureBoost+ we build 25 trees, and calculate the average time consuming of a single tree as the result.

Figure shows the average tree building time on 4 data sets. We can see that the proposed SecureBoost+ obviously

\footnotesize{3https://www.csie.ntu.edu.tw/~cjlin/libsvmtools/datasets/
3https://github.com/FederatedAI/FATE/tree/master/examples/data}
Table 2: Data Set Details

| Data set       | # instance | # features | # guest feature | # host features | # labels | task type       |
|----------------|------------|------------|-----------------|-----------------|----------|-----------------|
| Give credit    | 150,000    | 10         | 5               | 5               | 2        | binary classification |
| Susy           | 5,000,000  | 18         | 4               | 14              | 2        | binary classification |
| Higgs          | 11,000,000 | 28         | 13              | 15              | 2        | binary classification |
| Epsilon        | 400,000    | 2000       | 1000            | 1000            | 2        | binary classification |
| Sensorless     | 58,509     | 48         | 24              | 24              | 11       | multi-classification |
| Covtype        | 581,012    | 54         | 27              | 27              | 7        | multi-classification |
| SVHN           | 99,289     | 3072       | 1536            | 1536            | 10       | multi-classification |

Figure 7: Tree Building Time

Table 3: AUC Performances

| Data set   | XGB  | SecureBoost | SecureBoost+ |
|------------|------|-------------|--------------|
| Give-credit| 0.872| 0.874       | 0.873        |
| Susy       | 0.864| 0.873       | 0.873        |
| Higgs      | 0.808| 0.806       | 0.8          |
| Epsilon    | 0.897| 0.897       | 0.894        |

7.3 Assessment of SecureBoost+ Mechanism Optimization

In this subsection we evaluate our proposed training mechanism optimizations.

7.3.1 The Mix Tree Mode and The Layered Tree Mode

Similar to previous experiments, we test the Mix Tree Mode and the Layered Tree Mode on our four binary classification data sets. In this experiments we use Secureboost+ as the baseline. Based on the default setting of Secureboost+, we turn on the mix mode and the layered mode.

Training Speed Figure 8 shows the average tree building time of default setting, mix mode, and layered mode. It is no doubt that the mix mode and layered mode will build tree faster than default setting because it skips global split-finding and inter-party communications with certain strategies. In the IterativeAffine schema, the mix mode reduces average tree building time by 33%, 40%, 40.3%, 38.4%, while the layered mode reduces tree building time by 10%, 24.4%, 16.5%, 30.5%, on four testing data set respectively. In the Paillier schema, the mix mode reduces time by 39.4%, 51.1%, 37.3%, 36.6%, while the layered mode reduces time by 13.2%, 11.7%, 9.4%, 22.8%.

Model Performance The precision losses in cipher optimization and gradient-based sampling may bring performance loss, so we compare the modeling performance of Xgboost, which trains locally, SecureBoost, and SecureBoost+. The result is shown in Table 3. From the result we can see that SecureBoost+, in general, achieves the same performance as local Xgboost modeling and the SecureBoost baseline.
training, they have slight performance loss in comparison to local baseline and default setting, but losses are minor. According to our experiments, these two mode sometimes have to take few more epochs to catch up with the performances of default setting. But from the result of Epsilon data set, we believe that the mix mode and the layered mode are still a good choice for accelerating training when the data set contains high dimensional features and features are balanced distributed.

### 7.3.2 SecureBoost-MO

In this section, we will test SecureBoost-MO. We develop SecureBoost-MO specifically for multi-classification-training optimization. MO-decision trees are shown equally outstanding performance in the multi-classification tasks as traditional GBDT [27], so in this section, we mainly focus on the speed of its vertically federated learning version. Multi-Output Decision Tree has completely different structures compared to default multi-classification GBDT setting, we firstly train a 25-epoch model on Xgboost and SecureBoost+ as the baseline. Then, we will train SecureBoost-MO to reach the baseline, and then analyze the time consuming.

**Baseline** The baseline is shown in Table [5]. We can see that SecureBoost+ have a relatively better accuracy scores, so we will train SecureBoost-MO to reach SecureBoost+ performances.

**Training Speed** As shown in Figure [9] in a 25-epoch model, SecureBoost+ needs to build 275, 175, 250 trees when training on sensorless, Covtype, SVHN respectively. To reach the SecureBoost+ baseline, SecureBoost-MO only needs to build 38, 37, 47 trees respectively. The tree numbers of SecureBoost-MO is far less than default SecureBoost+.

In SecureBoost-MO, host side computes on cipher-vectors and cipher-compressing is disabled, building a single MO tree is more expensive than default trees.

Therefore we count the total time SecureBoost-MO spent on building trees. The time statistics are shown in Figure [10]. From the figure we can see the advantages of using MO trees to learn on large scale data sets with large number of labels. In the IterativeAffine schema, SecureBoost-MO reduces tree building time by 81%, 76.7%, 57.5% on Sensorless, Covtype, SVHN respectively. In the Paillier Schema, SecureBoost-MO reduces tree building time by 74%, 73.1%, 36.4% respectively. All these optimization is on the basis of reaching the same performances.

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**Table 4: The Mix Mode and The Layered Mode AUC Performances**

| Data set  | XGB | Default | Mix | Layered |
|-----------|-----|---------|-----|---------|
| Give-credit | 0.872 | 0.874 | 0.87 | 0.871 |
| Susy     | 0.864 | 0.873 | 0.869 | 0.87 |
| Higgs    | 0.808 | 0.8 | 0.795 | 0.796 |
| Epsilon  | 0.897 | 0.894 | 0.894 | 0.894 |

**Table 5: Multi-classification Accuracy Baseline**

| Data set  | XGB | SecureBoost+ |
|-----------|-----|--------------|
| sensorless | 0.999 | 0.992 |
| cov       | 0.78 | 0.806 |
| SVHN      | 0.686 | 0.686 |

---

![Figure 8: Tree Building Time Comparison](image1)

![Figure 9: Tree number comparison](image2)
7.4 Summary

With the experimental results, we can now answer the three points at the beginning of the experiment sections:

- SecureBoost+ can effectively reduce tree building time with cipher-optimization and engineering optimizations. According to the experiment results, tree building time is reduced by 37.5% to 82.4% in IterativeAffine encryption schema. In the Paillier schema, SecureBoost+ gains more benefits from optimization, tree building time is reduced by more than 80% on each dataset.

- The mix mode and the layered mode can further reduce training time at cost of minor performance losses. The layered mode on average reduce 20% tree building time in IterativeAffine schema, and 14.3% in Paillier schema, while the mix mode reduce more: 37.6% and 41.1% respectively.

The SecureBoost-MO shows its outstanding performance in multi-classification task optimization. It reduces multi-classification tree building time on average by 71.7%, 61.2% in terms of IterativeAffine and Paillier.

- From the perspective of model performances, We compare SecureBoost+ with Xgboost, and SecureBoost. To sum up, SecureBoost+ has the same performance as local modeling. The training mechanism optimizations will not bring obvious performance loss.

8. Conclusion

In this paper we proposed SecureBoost+, a high-performance vertical booting tree framework developed based on SecureBoost. With several optimization mechanisms, from several aspects, we tackle the costly parts of vertical tree learning. The empirical results of experiments show that SecureBoost+ is remarkably better than our baseline, proves the power of our optimization methods. With SecureBoost+ we are able to solve real-world large-scale vertical federated modeling in a reasonable time.
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