Nearly $\text{AdS}_2$ Sugra and the Super-Schwarzian

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**Abstract**

In nearly $\text{AdS}_2$ gravity the Einstein-Hilbert term is supplemented by the Jackiw-Teitelboim action. Integrating out the bulk metric gives rise to the Schwarzian action for the boundary curve. In the present note, we show how the extension to supergravity leads to the super-Schwarzian action for the superspace boundary.
1 Introduction

The theory of gravity simplifies in lower dimensions and thus lower dimensional gravity could serve as a playground for addressing difficult problems such as e.g. in black hole physics. However, the simplification might go too far away from a semi realistic situation. In two dimensions the Einstein tensor vanishes identically, gravity is not dynamical. To capture some non trivial behaviour often an additional scalar field (dilaton) is coupled to the Einstein-Hilbert term and thought of as arising in compactifications of higher dimensional theories of gravity. (For a review on two dimensional models see [1].) A particularly simple model is provided by the Jackiw-Teitelboim action [2,3]. Variation with respect to the dilaton forces the metric to be of constant curvature (whose value is an input parameter). In the Einstein equation (obtained by considering metric variations) the Einstein tensor is replaced by the dilaton’s energy momentum tensor. Jackiw-Teitelboim gravity on $AdS_2$ spaces was recently analysed in [4] in the context of holography. In [5–8] it was shown how the Schwarzian appears as an effective Lagrangian for a UV regulator brane on the $AdS_2$ side. This is interesting because the Schwarzian also arises as a Lagrangian of a Goldstone boson associated to broken reparameterisation invariance of the SYK model [9–15] and alternatives without disorder [16–18]. Hence, nearly $AdS_2$ gravity (or Jackiw-Teitelboim gravity) is related to these models by holographic duality. Further, black hole physics has been connected via the SYK model or its alternatives to random matrix theory [19–22].

In [23] supersymmetric versions of the SYK model were shown to lead to super-Schwarzian Lagrangians. Again there are alternatives without disorder [24] as well as connections to random matrix models [25]. Path integrals with the Schwarzian as well as the super-Schwarzian action have been recently evaluated in [26].

An obvious expectation is that the super-Schwarzian arises as an effective Lagrangian for a UV regulator brane in nearly $AdS_2$ supergravity. In the present note, we will fill in the details of this expectation for the case of minimal supersymmetry. In the next section, we will review the part of [5–8] which will be modified to include supersymmetry.

2 Recap: Schwarzian from Nearly $AdS_2$ Gravity

Here, we repeat the argumentation of [7] (see also [5,6,8]) which will be supersymmetrised in the next section. Our starting point is the gravity action whose relevant part is (Euclidean signature)

$$S = -\frac{1}{16\pi G} \left[ \int_M d^2x \sqrt{g} \phi (R + 2) + 2 \int_{\partial M} du \sqrt{h} \phi K \right], \quad (1)$$

where $\phi$ is the dilaton and the last term is an adapation of the Gibbons-Hawking-York boundary term ensuring that there are no boundary contributions to $\delta S$ once we impose Dirichlet conditions on variations. The extrinsic curvature, $K$, will be discussed later. Variation w.r.t. $\phi$ yields the constraint $R = -2$ which is solved by the Euclidean $AdS_2$...
metric

\[ ds^2 = \frac{dt^2 + dy^2}{y^2}. \]  

(2)

Variation w.r.t. the metric forces the energy momentum tensor of the dilaton to vanish leading to the general solution

\[ \phi = \frac{\alpha + \gamma t + \delta (t^2 + y^2)}{y}. \]  

(3)

The other ingredient is introducing a UV brane, i.e. a curve which approaches the boundary at \( y = 0 \) once a small parameter, \( \epsilon \), is taken to zero. The authors of [7] impose the crucial condition that the proper length of the boundary curve is constant for finite \( \epsilon \). Parameterising the boundary as \( (t(u), y(u)) \), this leads to

\[ \frac{1}{\epsilon^2} = \frac{t'' + y''}{y^2}. \]  

(4)

For later use, we include the first subleading (as \( \epsilon \to 0 \)) contribution into the solution

\[ t = t(u), \quad y = \epsilon t' \left(1 + \frac{\epsilon^2}{2} \left( \frac{t''}{t'} \right)^2 \right) + \ldots. \]  

(5)

The easiest argument for the Schwarzian, which just needs the leading order solution, is to start with a symmetry based guess

\[ S[t(u)] \sim \int du \phi_r(u) \text{Sch}(t, u), \]  

(6)

where

\[ \text{Sch}(t, u) = \frac{t'''}{t'} - \frac{3}{2} \left( \frac{t''}{t'} \right)^2 \]  

(7)

denotes the Schwarzian. Here, \( \phi_r(u) \) is a scalar which is related to the dilaton as will be discussed shortly. We see that the action \( [8] \) is invariant only under an \( \text{SL}(2, \mathbb{R}) \) subgroup of reparameterisations. Variation w.r.t. \( t(u) \) leads to the equation of motion

\[ \left\{ \frac{1}{t'} \left[ (t'\phi_r)' \right]' \right\}' = 0. \]  

(8)

Taking \( \phi_r \) to be the renormalised (i.e. multiplied by \( \epsilon \)) boundary value of the dilaton (to leading order in \( 1/\epsilon \))

\[ \phi_r(u) = \frac{\alpha + \gamma t(u) + \delta t(u)^2}{t'(u)} \]  

(9)

shows that \( [8] \) is solved for any \( t(u) \) which is consistent with our earlier considerations not leading to further conditions on \( t(u) \).
A more direct way to obtain the Schwarzian is to evaluate the 2d action (1) at the metric (2) while cutting off the integration at the UV brane (5). This yields

$$-\frac{1}{8\pi Ge^2} \int du \phi_r(u)K.$$  

(10)

The result for $K$ is given in [7] as

$$K = 1 + \epsilon^2 \text{Sch}(t,u),$$

(11)

and we see that after losing an additive divergent contribution we get indeed (6) now with the prefactor fixed. Since we are going to supersymmetrise also this calculation let us fill in some details on the computation of $K$. The extrinsic curvature is defined as

$$K = g^{\mu\nu} \nabla_\mu n_\nu,$$

(12)

where $\nabla$ denotes the covariant derivative and $n_\mu$ is the normal vectorfield of the UV brane. Its tangent vector is computed by taking the partial $u$ derivatives of its coordinates

$$T^\mu = \left( t', \epsilon t'' \left[ 1 + \epsilon^2 \left( \frac{t''}{t'} - \frac{1}{2} \left( \frac{t''}{t'} \right)^2 \right) \right] \right),$$

(13)

where we used (5) and stopped writing dots indicating the existence of higher order terms in $\epsilon$. The normal vector is defined by the two conditions

$$T^\mu n_\mu = 0, \quad n^\mu n_\mu = 1.$$  

(14)

Explicitly one finds

$$n_\mu = \left( \frac{t''}{t'^2} \left[ 1 + \epsilon^2 \text{Sch}(t,u) \right], -\frac{1}{\epsilon t'} \left[ 1 - \epsilon^2 \left( \frac{t''}{t'} \right)^2 \right] \right).$$

(15)

Because of the normalisation condition in (14) one can replace the metric in (12) by a projector on directions orthogonal to $n$, i.e. on tangent directions. In formulas this is expressed as

$$K = (g^{\mu\nu} - n^\mu n_\nu) \nabla_\mu n_\nu = \frac{T^\mu T^\nu}{T^2} \nabla_\mu n_\nu = \frac{T^\nu}{T^2} \nabla T n_\nu,$$

(16)

with the directional covariant derivative

$$\nabla_{T} n_\mu = \frac{\partial n_\mu}{\partial u} - \Gamma^\rho_{\mu\nu} n_\rho T^\nu.$$  

(17)

Here, the normalisation of the tangent vector is fixed by $T^\mu \partial_\mu = \partial_u$. It is now an easy exercise to reproduce (11). Note, that subleading contributions matter in the $\Gamma^y_{tt} n_y T^{y2}$ contribution. The form of $K$ in (16) will lead us to a supersymmetric version of $K$ in the next section.
3 Super-Schwarzian from Nearly AdS$_2$ Supergravity

The supersymmetric version of Jackiw-Teitelboim gravity has been formulated in [27]. Conveniently one uses two dimensional superspace spanned by the coordinates $z, \bar{z}, \theta, \bar{\theta}$, where $z, \bar{z}, (\theta, \bar{\theta})$ values are complex (Grassmann) numbers related by complex conjugation. Fields are promoted to superfields depending on superspace coordinates. The supersymmetric version of (1) reads [27]

$$
S = -\frac{1}{16\pi G} \left[ i \int d^2z d^2\theta E \Phi \left( R_{+-} - 2 \right) + 2 \int_{\partial M} dud\bar{\theta} \Phi K \right], \tag{18}
$$

where $E$ is the superdeterminant of the vielbein in superspace, $\Phi$ is the dilaton superfield, $R_{+-}$ is a superfield containing the usual scalar curvature in the coefficient at the $\theta \bar{\theta}$ term when Taylor expanded in Grassmann coordinates, and finally the last term is a supersymmetric version of the Gibbons-Hawking-York term where $u$ and $\vartheta$ are coordinates on one dimensional superspace. More details will be discussed later. Since the author of [27] had closed string worldsheets in mind this last term is not discussed there. The boundary curve is now described by two dimensional superspace coordinates being functions of $u$ and $\vartheta$. We replace condition (4) by its natural supersymmetrisation

$$
\frac{du^2 + 2\vartheta d\vartheta du}{4e^2} = dz^\xi E^1_\xi dz^\pi E^\pi_\xi, \tag{19}
$$

where Greek (Einstein) indices run over two dimensional superspace coordinates, whereas 1 and $\bar{1}$ are flat space Euclidean indices. The $E$’s denote vielbein components and on the left hand side the corresponding expression for one dimensional flat superspace has been divided by the constant $4e^2$ (the factor four is there for later convenience). We fix the superconformal gauge

$$
E_+ = e^{-\frac{\Sigma}{2}} D_\theta \equiv e^{-\frac{\Sigma}{2}} \left( \frac{\partial}{\partial \theta} + \theta \frac{\partial}{\partial z} \right), \quad E_1 = \frac{1}{2} \left\{ E_+, E_+ \right\}, \tag{20}
$$

$$
E_- = e^{-\frac{\Sigma}{2}} D_{\bar{\theta}} \equiv e^{-\frac{\Sigma}{2}} \left( \frac{\partial}{\partial \bar{\theta}} + \bar{\theta} \frac{\partial}{\partial \bar{z}} \right), \quad E_{\bar{1}} = \frac{1}{2} \left\{ E_-, E_- \right\}, \tag{21}
$$

where braces denote the anti-commutator. In this gauge the supercurvature is [27],

$$
R_{+-} = -2ie^{-\Sigma} D_\theta D_{\bar{\theta}} \Sigma. \tag{22}
$$

The constraint $R_{+-} = 2$ is solved by the superconformal factor

$$
e^\Sigma = \frac{1}{2Imz} \left( 1 - \frac{i\theta \bar{\theta}}{2Imz} \right). \tag{23}
$$

The supervielbein is a four by four matrix with a diagonal two by two block structure. The holomorphic block is

$$
\begin{pmatrix}
E^\theta_+ & E^\theta_1 \\
E^\theta_{\bar{1}} & E^\pi_1
\end{pmatrix}
= \begin{pmatrix}
e^{-\frac{\Sigma}{2}} & 0 \\
e^{-\frac{\Sigma}{2}} D_\theta e^{-\frac{\Sigma}{2}} & e^{-\Sigma} - \theta e^{-\frac{\Sigma}{2}} D_\theta e^{-\frac{\Sigma}{2}}
\end{pmatrix}. \tag{24}
$$
Its inverse is easily computed to
\[
\begin{pmatrix}
E^+_\theta & E^0_\theta \\
E^+_z & E^0_z
\end{pmatrix} =
\begin{pmatrix}
e^{\frac{\Sigma}{2}} + \theta e^{\Sigma} D_\theta e^{-\frac{\Sigma}{2}} & -\theta e^{\Sigma} \\
-e^{\Sigma} D_\theta e^{-\frac{\Sigma}{2}} & e^{\Sigma}
\end{pmatrix}.
\] (25)

For the anti-holomorphic block corresponding expressions hold. Plugging all this into (19) yields
\[
\frac{du^2 + 2\vartheta d\vartheta du}{4e^2} = e^{2\Sigma} \left\{ \left| \frac{\partial z}{\partial u} + \theta \frac{\partial \theta}{\partial u} \right|^2 du^2 + \left[ \left( \theta \frac{\partial \vartheta}{\partial \vartheta} - \frac{\partial z}{\partial \vartheta} \right) \left( \frac{\partial \bar{z}}{\partial u} + \bar{\theta} \frac{\partial \bar{\theta}}{\partial u} \right) + \left( \frac{\partial z}{\partial u} + \theta \frac{\partial \theta}{\partial u} \right) \right] d\vartheta du \right\}.
\] (26)

The ratio between the two components on the RHS is compatible with the LHS if
\[
Dz = \theta D\theta , \quad D\bar{z} = \bar{\theta} D\bar{\theta},
\] (27)
where $D$ denotes the one dimensional superderivative
\[
D \equiv \frac{\partial}{\partial \vartheta} + \vartheta \frac{\partial}{\partial u}.
\] (28)

After imposing (27) the first iterated solution to (26) reads
\[
\theta = \bar{\theta} = \xi , \quad z = t + i \epsilon (D\xi)^2,
\] (29)
where $t$ and $\xi$ are functions of $u$ and $\vartheta$ satisfying
\[
Dt = \xi D\xi.
\] (30)

Eq. (30) can be solved in terms of a commuting function $f$ and an anti-commuting function $\eta$,
\[
t = f(u + \vartheta \eta) , \quad \xi = \sqrt{f'(u)} \left[ \eta(u) + \vartheta \left( 1 + \frac{\eta(u)\eta'(u)}{2} \right) \right].
\] (31)

A natural guess for an effective action of the functions $f$ and $\eta$ is
\[
S \sim \int dud\vartheta \Phi_r(u, \vartheta) S [t, \xi; u, \vartheta],
\] (32)
where we adopted the notation of [23] for the super-Schwarzian, i.e.
\[
S [t, \xi; u, \vartheta] = \frac{D^4 \xi}{D\xi} - 2 \frac{D^3 \xi D^2 \xi}{(D\xi)^2},
\] (33)
with $\xi$ given by (31). $\Phi_r$ is a renormalised version of the dilaton superfield evaluated at the boundary. Before determining it by the sugra equations let us derive equations of motion from (32). To this end, it is useful to decompose
\[
\Phi_r(u, \vartheta) = \phi_r(u) + \vartheta \lambda_r(u).
\] (34)
Eq. (32) takes the form
\[ S \sim \int du \left( \phi_r S_b(t, \xi; u, \vartheta) + \lambda_r S_f(t, \xi; u, \vartheta) \right), \] (35)
where the super-Schwarzian components are given by
\[ S_b = \frac{1}{2} \text{Sch} \left( f, u \right) (1 - \eta \eta') + \frac{3 \eta' \eta''}{2} + \frac{\eta''' \eta''}{2}, \] (36)
\[ S_f = \text{Sch} \left( f, u \right) \frac{\eta}{2} + \eta'' + \frac{\eta' \eta''}{2}. \] (37)
The equations of motion are obtained by variations of \( f \) and \( \eta \)
\[ 0 = \left\{ \frac{1}{f'} \left[ \left( \frac{f' \left[ \phi_r (1 - \eta \eta') + \lambda_r \eta \right]'}{f'} \right)' \right] \right\}', \] (38)
\[ 0 = \phi_r \text{Sch} \left( f, u \right) \eta' + (\phi_r \text{Sch} \left( f, u \right) \eta)' + 3 \left( \phi_r \eta'' \right)' + 3 \left( \phi_r \eta'' \right)'' - \phi_r \eta''' - (\phi_r \eta)' \]
\[ + \lambda_r \text{Sch} \left( f, u \right) + 2 \lambda_r' + \lambda_r \eta'' + \left( \lambda_r \eta'' \right)' + \left( \lambda_r \eta'' \right)''. \] (39)
It remains to determine the boundary values of the dilaton. Using the details given in \[27\] it is easy to see that (3) is again a solution to the bulk equations up to terms subleading as the boundary is approached (e.g. terms containing \( \theta \bar{\theta} \)). The renormalisation prescription is to multiply with \( \epsilon \) yielding
\[ \Phi_r = \frac{\alpha + \gamma \eta + \delta \eta''}{(D \xi)^2}. \] (40)
Plugging in (31) leads to
\[ \phi_r = \frac{\alpha + \gamma f + \delta f^2}{f'} (1 - \eta \eta'), \quad \lambda_r = \gamma \eta + 2 \delta \eta f - \left( \frac{f'' \eta}{f'^2} + \frac{f'' \eta'}{f'} \right) \left( \alpha + \gamma f + \delta f^2 \right). \] (41)
Consistency can be established by checking that (38) and (39) are satisfied which is indeed the case as can be seen by a straightforward, though tedious, calculation.

To obtain the super-Schwarzian in a more direct way we have to specify the last (boundary) term in the action (18). We do so by covariantising with respect to superspace. To this end, we consider expression (16) where we replace \( \partial_u \) by \( D \) (see (28)) such that its transformation under super-reparameterisations cancels the Berezinian of the super-line measure \[23\]. Further, we need to change from Einstein indices to Lorentz indices by means of the two dimensional supervielbein and hence replace the pulled back Christoffel symbols by their spin connection equivalent. This leads to
\[ K = \frac{T^A D_T n_A}{T^A T_A}, \] (42)
with \( A \in \{1, \bar{1}\} \) and
\[ D_T n_A = D n_A + \frac{\partial z}{\partial u} \Omega_{\xi} \right) + \vartheta \frac{\partial z}{\partial u} \Omega_{\xi}; \] (43)
where $\xi$ is a two dimensional (curved) super-space index ($\xi \in \{\theta, z, \bar{\theta}, \bar{z}\}$). Further, we should replace
\[ dud\Phi \rightarrow \frac{1}{2\epsilon^2}dud\Phi_r. \] (44)

For the computation of the surface term it turns out that first order corrections in $\epsilon$ are sufficient. However, in addition to (29) we also need the linear correction to $\theta$,
\[ \theta = \xi + i\epsilon \rho + \ldots. \] (45)

It is important to notice that this leads to a correction of $\text{Im}z$ at linear order in $\epsilon$. Indeed, after imposing (27) condition (26) is solved by
\[ \text{Im}z = \epsilon D\theta D\bar{\theta} \left( 1 - \frac{i\theta}{2\epsilon D\theta D\bar{\theta}} \right). \] (46)

Plugging in (45) yields the complete first order correction
\[ \text{Im}z = \epsilon (D\xi)^2 \left( 1 - \frac{\xi \rho}{(D\xi)^2} \right) + \ldots. \] (47)

Finally,
\[ \rho = D^2 \xi, \] (48)
can be fixed by imposing (27). In summary, the solution to the boundary conditions up to linear order in $\epsilon$ is
\[ \theta = \xi + i\epsilon D^2 \xi, \quad \text{Im}z = \epsilon \left( (D\xi)^2 - \xi D^2 \xi \right). \] (49)

The tangent vector in (42) is computed as
\[ T^1 = E^1 \frac{\partial z^\xi}{\partial u} = e^\Sigma (D\theta)^2 = \frac{1}{2\epsilon} \frac{D\theta}{D\bar{\theta}}, \] (50)
and $T^\bar{1}$ is obtained by complex conjugation. Normalising $|n_1|^2 = n_1 n_\bar{1} = 1/4$ and imposing $n_A T^A = 0$ yields
\[ n_1 = \frac{i D\bar{\theta}}{2 D\theta}, \] (51)

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1. This looks a bit ad hoc but can be motivated as follows. Condition (26) implies an induced one dimensional supervielbein $e_+ = \sqrt{2\epsilon}D$ in superconformal gauge. Its super-determinant is $1/\sqrt{2\epsilon}$. With that measure we should replace $D$ by $e_+$ which cancels the superdeterminant from the measure. Another power of $1/\epsilon$ appears after changing the tangent vector normalisation to one (see (50)). (In the bosonic case the effects of replacing $\partial_u$ by $e_u$ and changing the normalisation of the tangent vector cancel.) Together with $\Phi = \Phi_r/\epsilon$ one obtains the factor introduced in (44). We will not make use of the one dimensional vielbein in the rest of the paper.

2. The non vanishing components of the 2d flat metric and its inverse are $g_{1\bar{1}} = g_{\bar{1}1} = 1/2$ and $g^{1\bar{1}} = g^\bar{1}1 = 2$. 

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The first contribution to (42) is

\[ \frac{T^A Dn_A}{|T^A|^2} = 4\epsilon \text{Im} \frac{D^2 \theta}{D \bar{\theta}} = 4\epsilon^2 \left( \frac{D^4 \xi}{D \xi} - \frac{D^2 \xi D^3 \xi}{(D \xi)^3} \right), \]  

where we suppressed higher orders in \( \epsilon \). Next, we need to compute the contribution due to the induced spin connection. To this end, it is useful to notice that

\[ \partial_z^\xi \Omega^\xi + \bar{\theta} \partial_{\bar{u}}^\xi \Omega^\xi = D \theta (\Omega_\theta + \theta \Omega_z) + D \bar{\theta} (\Omega_{\bar{\theta}} + \bar{\theta} \Omega_{\bar{z}}), \]  

The combination of spin connections is exactly what is needed to make \( D \theta \) respectively \( D \bar{\theta} \) covariant under local rotations. In superconformal gauge this can be found in [28],

\[ \Omega_\theta + \theta \Omega_z = -D \theta \Sigma = D \bar{\theta} \Sigma = \Omega_{\bar{\theta}} + \bar{\theta} \Omega_{\bar{z}} = \frac{i}{2\epsilon^2} \left( \bar{\theta} - \theta \right). \]

Plugging in expansion (49), we obtain (since normal vector components are purely imaginary complex conjugation takes care of the opposite charge under rotations)

\[ \left( \partial_z^\xi \Omega^\xi + \bar{\theta} \partial_{\bar{u}}^\xi \Omega^\xi \right) \frac{T^A n_A}{|T^A|^2} + \text{c.c.} = -4\epsilon^2 \frac{D^2 \xi D^3 \xi}{(D \xi)^2}. \]

Thus we obtain for the extrinsic curvature in (42)

\[ K = 4\epsilon^2 S [t, \xi; u, \bar{\theta}], \]

where the super-Schwarzian had been given in (33). This is the expected result. We fixed the coefficient such that for \( \eta = 0 \) integration over \( d\bar{\theta}/2\epsilon^2 \) (see (44)) reproduces the bosonic result. This seems reasonable since its bulk ‘partner’ \( \int d^2 \theta R_{\perp\perp} \) gives just the bulk part of the Gauss-Bonnet density in the bosonic case [27].

4 Conclusions

In the present note we considered Jackiw-Teitelboim supergravity. The supercurvature is constrained to a constant value such that the actual scalar curvature is constant and negative. The geometry is that of \( AdS_2 \) superspace. We introduced a cutoff line near its boundary by demanding that its arc length element differs from flat one dimensional superspace only by a diverging constant factor. This imposes conditions on superspace coordinates which resemble superconformal transformations related to super-reparameterisations of the boundary. The origins of the respective conditions are quite similar due to demanding that the mixed fermion-boson component of the arc length has a fixed ratio with the boson-boson component. In addition, one finds that the superspace boundary is given by a boson-fermion pair of functions \( (f(u) \text{ and } \eta(u)) \). We argued, that integrating over the bulk leads to an effective super-Schwarzian Lagrangian for these functions. Two arguments are
given. First, we just assumed the Lagrangian to be dilaton times super-Schwarzian. This could also be viewed as an action for the boundary value of the dilaton. We checked that the corresponding equations of motion hold when a bulk solution of the dilaton is plugged in. The dilaton’s boundary value is obtained as the pull back to the boundary. This is consistent with imposing Dirichlet conditions on variations and no further boundary conditions. To ensure that Dirichlet conditions on variations are enough to cancel boundary terms in variations, usually a Gibbons-Hawking-York term is added. In our second argument for the super-Schwarzian we supersymmetrised the bosonic Gibbons-Hawking-York term. Indeed, when plugging in our solution for the boundary curve, it gives rise to a super-Schwarzian Lagrangian. A divergence, present in the bosonic case, is absent in the supersymmetric case. We gave reasonable arguments for our normalisation of the supersymmetric boundary term. It would be nice to confirm our term by direct computation as carried out in Wess-Zumino gauge in [29]. This as well as the extension to \(N = 2\) supersymmetry is left for future work.

In summary, our note suggests the supergravity dual of the supersymmetric SYK model to be given by nearly \(AdS_2\) supergravity (containing the supersymmetric version of Jackiw-Teitelboim gravity). In addition there will be massive particles whose spectrum and interactions have been recently studied in [30] for the non supersymmetric SYK model. It would be interesting to extend this to the supersymmetric case and see whether supersymmetry leads to simplifications.

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