Analysis of the Jun Ishiwara’s "The universal meaning of the quantum of action"

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Here we present an analysis of the paper “Universelle Bedeutung des Wirkungsquantums” (The universal meaning of the quantum of action), published by Jun Ishiwara in German in the “Proceedings of Tokyo Mathematico-Physical Society 8 (1915) 106-116”. In his work, Ishiwara, established in the Sendai University, Japan, proposed - simultaneously with Arnold Sommerfeld, William Wilson and Niels Bohr in Europe - the phase-space-integral quantization, a rule that would be incorporated into the old-quantum-theory formalism. The discussions and analysis render this paper fully accessible to undergraduate students of physics with elementary knowledge of quantum mechanics.

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I. INTRODUCTION

No theory defies our common sense as much as quantum mechanics (QM). Richard Feynman (1918-1988) said explicitly on his lecture "Probability and Uncertainty: The Quantum Mechanical View of Nature" [Feynman 1985] that nobody understands the theory. Mário Schenberg (1914-1990) said that QM is the most important scientific revolution of the history of humanity [Schenberg 1984] and the debates between Albert Einstein (1879-1955) and Niels Bohr (1885-1962) about the interpretation of QM are notorious (see the first chapters of [Wheeler 1983]). The QM reached its standard form with the modern wave theory based on the Schrödinger (presented on his set of papers "Quantization as a problem of proper values I-IV" [Ludwig 1968, Schrödinger 1978]) and Dirac [Dirac 1928] equations, with the probabilistic interpretation for the wave function by Max Born (1882-1970) (presented on his papers "On the quantum mechanics of collisions" [Wheeler 1983] and "Quantum mechanics of collision processes" [Ludwig 1968], and Paul Dirac (1902-1984) [Dirac 1928]. However, the path since the establishment of the concept of energy quantum by Max Planck (1858-1947) (translations from german of his original papers can be found on [Haar 1967, Planck 2000a, Planck 2000b] and analysis about them on [Feldens 2010, Studart 2000]) – in a process described by himself as "an act of desperation [done because] a theoretical explanation [to the black-body-radiation spectrum] had to be supplied at all cost, whatever the price" [Jammer 1966] – until the wave equations lasted about 3 decades [Gamow 1958, Jammer 1966].

During that time, the physics of the atomic phenomena evolved into a theory now known as the old quantum theory (OQT) [Bucher 2008, Jammer 1966, Tomonaga 1968], which was based on a heuristic approach to atomic phenomena according to the following rules [Jammer 1966]:

- i) the use of classical mechanics to determine the possible motions of the system;
- ii) the imposition of certain quantum conditions to select the actual or allowed motions;
- iii) the treatment of radiative processes as transitions between allowed motions, subject to Bohr’s frequency formula.

From the third rule, we can see the important role played by Bohr [Kragh 2013, Parente 2013, Svidzinsky 2014] with his atomic model [Bohr 1913a, Bohr 1913b, Bohr 1913c, Bohr 1963], who spurred one of the greatest achievements in physics during the first decades of the twentieth century: the theoretical derivation of Balmer’s formula [Banet 1966, Banet 1970]. The Bohr model was based on the following postulates (as originally published in [Bohr 1913a]):

1. "That the dynamical equilibrium of the systems in the stationary states can be discussed by help of the ordinary mechanics, while the passing of the systems between different stationary states cannot be treated on that basis;

2. That the latter process is followed by the emission of a homogeneous radiation, for which the relation between the frequency and the amount of energy emitted is the one given by Planck’s theory."

We can see that the third rule of the OQT was based on Bohr’s second postulate. At last, in the calculations, an adequate version of the correspondence principle, formulated in distinct forms by Bohr, Planck and Werner Heisenberg (1901-1976) [Darrigol 1992, Jammer 1966, Liboff 1984, Makowski 2006], which states that when we take the limits of some basic parameters of QM we must find the classical results again [Bhattacharyya 2006, Hassoun 1969, Jammer 1966, Liboff 1984, Makowski 2006]. The pioneer example of the application of the correspondence principle was made by Bohr [Parente 2013] on Sec. 3 ("General considerations continued") of Ref. [Bohr 1913a], where he uses it to obtain the correct dependence between the electronic frequencies of emission/absorption and translation around the nucleus.

An improvement of the OQT would be attained through phase-space analysis, something that is not surprising, since Planck’s constant $h$ (see translations of original Planck’s papers "On an improvement of Wien’s equation for the spectrum" and "On the theory of the energy distribution law of the normal spectrum", with historical analysis, on [Haar 1967, Planck 2000a, Planck 2000b]) has the dimension of an action (and angular momentum) [Bucher 2008, Eckert 2014, Feldens 2010, Studart 2000]. New conditions for quantization then emerged, proposed by William Wilson (1875-1965), Jun Ishiwara (1881-1947), Arnold Sommerfeld (1868-1951) and Bohr. Bohr’s (see the first part of his paper "On the quantum theory of line-spectra" on [Warburg 2007]) and Sommerfeld’s [Sommerfeld 1916a, Sommerfeld 1916b, Sommerfeld 2014a, Sommerfeld 2014b] contributions have been subject of detailed scrutiny - for the recent literature see [Aaserud 2013, Duncan 2014, Eckert 2014, Kragh 2012, Seth 2008] - while Wilson’s [Wilson 1913, Wilson 1916] and Ishiwara’s [Ishiwara 1913, Ishiwara 2017] papers still await historical analysis.

The original papers of Wilson, Bohr and Sommerfeld are accessible in English. In the case of Sommerfeld, see [Sommerfeld 2014a, Sommerfeld 2014b]; Sommerfeld’s book Atombau und Spektrallinien - then regarded as the
**II. A BRIEF BIOGRAPHY OF JUN ISHIWARA**

Jun Ishiwara (1881-1947) – his name is sometimes translated as Ishihara – was educated at the Tokyo Imperial University, graduating in theoretical physics in 1906. He was, together with Yoshio Nishina (1890-1951), one of Hantaro Nagaoka’s students (1865-1950) [Frédéric 2002] - Nagaoka was the author of the *Saturnian atomic model* [Nagaoka 1904]. In 1908, Ishiwara became a teacher at the Army Gunnery Engineering School [Mehra 2001] and in 1911 he was appointed assistant professor at the Graduate School of Science and Faculty of Science, Tohoku University, Sendai [Mehra 2001]. From 1912 to 1914, Ishiwara studied in Europe at the University of Munich, the E. T. H. (Eidgenössische Technische Hochschule) in Zurich and at the University of Leyden, studying with Sommerfeld and Einstein [Einstein 1982; Mehra 2001]. After returning to Japan, he was named for the chair of Full Professor of Physics at Tohoku University, in Sandai and, in 1919, he received, for his works on relativity and QM, the Gakushin award by the Imperial Academy of Sciences [Einstein 1982; Frédéric 2002; Sigeko 2000].

Ishiwara dedicated most of his time to Einstein’s theory of relativity, and he was a scholar and debater of the theory, gaining reputation both in Japan and China, where he was known as “the only expert of relativity studies in Japan (…)" [Hu 2007]. The first paper by Ishiwara on the theory was published in 1909 and was the first in Japan on this topic [Sigeko 2000].

When, in late 1922, the publishing company Kaizosha invited Einstein to visit Japan [Low 2001], it was Ishiwara who hosted him [Hu 2007; Low 2001]. Einstein arrived in Japan on November 17 and stayed for six weeks [Low 2001; Pais 2005].

With an invitation by Kitaro Nishida (1870-1945), professor of Philosophy at Kyoto University, Einstein gave a talk entitled “How I created the theory of relativity”, when he clarified several aspects of his creative process and the route to his main works. Einstein made no written notes for the talk, however, and the only record of Einstein’s words are the careful notes taken by Ishiwara, who also provided a running translation from German to Japanese [Einstein 1982; Pais 2005]. In fact, it was in his stay in Japan that Einstein received the news of his Nobel Prize [Einstein 1982; Einstein 2015a]. Kaizosha’s collection of Einstein’s complete works was the first in the world, selling about 4000 copies, and was edited by Ishiwara [Hu 2007; Low 2001].

There are several letters between both physicists in Einstein’s archive [Einstein 2015b]. For example, on January 12th 1923, soon after Einstein’s visit, Ishiwara wrote to Einstein beginning with:

“Esteemed Professor, Your stay in our country was a special pleasure for me that I shall always cherish as such a fine memory. You are probably still happily continuing your journey!”

followed by a discussion on tensorial analysis [Ishiwara 2015].

On QM the first contribution of Ishiwara was published in the inaugural volume of *The science reports of the Tohoku University* [Sigeko 2000]. He was studying the electron theory on metals, a research - on Sigeko’s words [Sigeko 2000] - "important in that it paralleled Bohr’s degree thesis of 1911 on the electron theory of metals and his subsequent well-known quantum theory of atomic structure". As noted by Abiko [Abiko 2015]:

"In 1911, [Ishiwara] submitted (though published in 1912) his own original paper in German 'Contribution to light-quantum theory', in which he deduced Planck’s radiation formula from the viewpoint of radiation as a collection of light-quanta, in a similar way to Bose’s statistics of 1924 (…). Also in that
paper of 1911, similarly to de Broglie did by way of introducing the phase waves in 1923, Ishiwara tried to explain wave-like behaviour of radiation as a collection of light-quanta, by associating minute electric and magnetic vectors to each light quantum."

We see here that Ishiwara anticipated by almost 15 years fundamental ideas of Satyendra Nath Bose (1894-1974) and Louis de Broglie (1892-1987) [Abiko 2015]. In another work in the same year, Ishiwara supported the light-quantum hypothesis about the constitution of x-rays and γ-rays in an pioneering attitude, against the opinion of the Braggs - William Henry (1862-1942) and William Lawrence (1890-1971) - for who these rays were constituted by neutral particles. To emphasizing the pioneeerism of Ishiwara, it is important to say that Einstein’s light quantum theory, at the time, was supported only by a minority of physics [Abiko 2015, Sigeko 2000]. In his 1915 paper analyzed here, he suggested the application of the phase-space-integral quantization to the hydrogen atom, supposing elliptical orbits [Ishiwara 1915, Ishiwara 2017] - that was the first published reference to Bohr’s theory by a non-western physicist [Kragh 2012].

By 1918, 4 years before Einstein’s visit to Japan, Ishiwara unfortunately diminished his research activity [Sigeko 2000, Tabata 2015]. He took leave from the university in 1921 and formally retired in 1923, working then mainly in divulgation and journalism of science [Abiko 2015] - indeed, he was pioneer in those fields in Japan [Tabata 2015] - and dedicated himself to tanka (a type of short Japanese poetry [Frédéric 2002]). He continued to influence many young physicists with his literary and scientific divulgation activities [Sigeko 2000]. At last, two treatises of Ishiwara, "The fundamental problems of physics", were published in his retirement time - the first volume on 1923 and the second on 1926 - including sections about quantum theory and atomic constitution. These books were references among the specialists of physics [Abiko 2015].

III. ANALYSIS

A. Precedents

The analysis of phase-space has been conducted over many years, before the Ishiwara work. Three forms of quantization for the phase-space were proposed at the first Solvay Congress [Jammer 1966, Straumann 2011]. It is important, here, to emphasize that the 'phase-space' expression was coined by Paul Ehrenfest (1880-1933) in a review paper of the work of Ludwig Boltzmann (1844-1906), published on 1911 [Nolte 2010]. Because it is a relatively recent expression at the time, we should not be surprised that Ishiwara did not used it on his paper, writing "state space" instead. However, in our discussions here, we will use the now familiar and consolidated expression, independently of its use (or not) by the followed mentioned authors.

Planck [Eckert 2014, Jammer 1966, Straumann 2011] suggested the rule defined by

$$\int \int dp \, dq = h$$

(1)

where, and this elicited a discussion with his colleagues, including Sommerfeld and Einstein. Following the Planck presentation, Henri Poincaré (1854-1912) [Sigeko 2000] suggested the expansion of Planck’s rule, for a system with n degrees of freedom, to

$$\int \int (dp_1 \, dq_1 + dp_2 \, dq_2 + \ldots + dp_n \, dq_n) = h$$

(2)

By his time, Sommerfeld presented in his works in the field that, for every molecular process, the quantity of action

$$\int_0^\tau L \, dt = \frac{h}{2\pi}$$

(3)

where τ is the duration of the process and L is the Lagrangian. This is the same form presented on the Karlsruhe meeting [Sommerfeld 1911], originating discussions with other participants on both events [Jammer 1966, Straumann 2011]. The precedents for the Sommerfeld formula of 1911 were his works on a theory about γ-rays and β-decay, where both emissions were mutually related - it is convenient to emphasize that, at the time, there was no nuclear theory yet since Rutherford published the results of his pioneering experiments of scattering during 1911-1914 [Hermann 1971, Jammer 1966]. Prior to (3), after an exchange of letters with Planck, Sommerfeld wrote the expression
\[
\int_0^T H \, dt = \frac{\hbar}{2\pi}
\]  

(4)

where \( H \) was the dynamical potential and for "the most part we shall view \( H \) as the mere abbreviation for \( T - V \)". Where \( T \) and \( V \) are the kinetic and potential energy, respectively [Hermann 1971]. Then, on 1913, were published the classic set of papers [Bohr 1913a, Bohr 1913b, Bohr 1913c] where Bohr presents his atomic model.

These are the precedents for the final form of the phase-space quantization rule,

\[
\int q_i \, dp_i = n_i \hbar
\]

(5)

where \( q \) indicates the position and the \( p \) indicates the momentum.

With these pieces, it is not difficult to have a view of the route to the works of Wilson, Ishiwara, Sommerfeld and the subsequent work of Bohr.

The first author to present the definitive rule was Wilson [Mehra 1982, Mehra 2001] in a paper for the *Philosophical Magazine* from 1915 [Wilson 1915]; communicated by John William Nicholson (1881-1955) - the priority of discovery was, indeed, recognized by Sommerfeld [Mehra 1982]. Wilson wanted to establish the "possibility of deducing the results of Planck and Bohr from a single form of quantum theory" [Wilson 1915] and he made his theory with the hypothesis of steady states of a system, that can be analyzed by Hamiltonian dynamics behind (5), and postulating the existence of discontinuous process between these steady states with absorption or emission of energy. With this he obtained, in his first paper [Wilson 1915], Bohr's formula for the kinetic energy of the electron and the Planck distribution law. In an other paper [Wilson 1916], also in *Philosophical Magazine*, Wilson stated more clearly the assumptions of his theory and developed it for the emission of spectral lines, citing both the rule (1) and the work of Ishiwara, published between Wilson’s two works.

By other side, Sommerfeld began to work in Bohr’s model, as described by Eckert [Eckert 2014], motivated by his quest for a theory of the Zeeman and Stark effects [Eckert 2014, Jammer 1966]. Indeed, "Sommerfeld’s response to Bohr’s atomic model was the earliest reaction from outside Rutherford’s circle, where Bohr had spent some time as a ‘postdoc’, and it revealed a vivid interest in the theory of the [at the time] unknown author" [Eckert 2014]. Sommerfeld was impressed with the expression founded by Bohr for the Rydberg constant in terms of fundamental parameters (in Sommerfeld’s own words, ‘calculating this constant is undoubtedly a great feat’ [Eckert 2014]), and was intrigued with the possibilities concerning the Zeeman and Stark effects [Eckert 2014, Jammer 1966].

During the First World War (from which Sommerfeld was dispensed), Sommerfeld corresponded with Karl Schwarzschild (1873-1916) - who died from a skin disease that he brought home from the front (see the bio profile on Voigt 1992) - Friedrich Paschen (1865-1947), Wilhelm Lenz (1888-1957) and Wilhelm Wien (1864-1928) and soon he become convinced of the consistency of Bohr’s model. To explain the decomposition of spectral lines in electric fields published by Stark in 1914, Sommerfeld proposed the existence of elliptic orbits for the electrons, accompanying the circular Bohrian orbits [Eckert 2014, Jammer 1966]. New developments on hydrogen and X-ray spectra culminated on his treatises presented to the Bavarian Academy of Science on December, 1915 [Sommerfeld 1914a] and January, 1916 [Sommerfeld 1914b]. The treatment of two-dimensional motion of the electron in its orbital plane indicated how systems with several degrees of freedom could be approached, at first with the rule (1) of Planck in 1911 [Eckert 2014, Sommerfeld 1914a, Sommerfeld 1914b]; soon after, having recognized that the quantization of the angular *momentum* could be expressed by requiring that

\[
\oint p_\varphi d\varphi = n_\varphi \hbar
\]

(6)

where \( p_\varphi \) is the angular (not the linear) momentum associated to the azimuthal angle \( \varphi \), with the integration extended over the period, Sommerfeld postulated the "final" rule (5). His ideas were detailed in a treatise for the *Annalen der Physik* "On quantum theory of spectral lines" [Sommerfeld 1916a, Sommerfeld 1916b] and, finally, presented in his classical book [Sommerfeld 1923]. Sommerfeld refers in his papers [Sommerfeld 1916a, Sommerfeld 1916b, Sommerfeld 1916c, Sommerfeld 1914a, Sommerfeld 1914b] to the Planck works, but did not refer to the earlier publications of Ishiwara and Wilson. This omission was corrected in his book [Sommerfeld 1923]. His work received major notoriety, because he made the theory more robust and detailed [Jungnickel 1986], generalized the quantum law of phase integrals to apply to systems of any number of degrees of freedom, revealing the close connection of the phase integrals with the Hamilton-Jacobi theory and applying to Bohr’s atom (a deep discussion about his calculations can be found on Castro 2016), considering elliptical orbits that allowed to deduce not just Balmer’s rule but the formula for the fine structure of hydrogen spectra. It was considered the highest point of OQT [Bucher 2008, Jammer 1966], referred to by Planck as "an achievement fully comparable with that of the famous
discovery of the planet Neptune whose existence and orbit was calculated by Leverrier before the human eye had seen it" [Planck 2016].

At last, it is convenient to mention that the meaning of the Planck’s constant has changed to Sommerfeld between 1911 and his final communications. When researching the \(\gamma\) and \(x\)-rays (at the time, named "Röntgen rays" [Sommerfeld 2014a]), he postulated in 1911 - he remebered it in his 1915 presentation [Sommerfeld 2014a] - the relation \(\text{Energy} \times \text{Time} = h\) as general, but on 1915, he restricted the hypothesis "essentially to periodic motions, and envisages an integral multiple of \(h\), furthermore equality is replaced by a proportionality depending on the force law" [Sommerfeld 2014a]. It was 1911 Sommerfeld’s hypothesis that Ishiwara used in the analysis of his paper [Ishiwara 1915, Ishiwara 2017].

The last work considered here is the Bohr one, the communication “On the quantum theory of line-spectra” published in English in 1918 in the Mémoires de l’académie royale des sciences et des lettres de Danemark (the first part can be found on the reference [Waerden 2007]). As the title itself indicates, the main concern of Bohr is the spectral analysis. Emphasizing the limitations of classical electrodynamics to atomic systems and remembering his own model, he approaches systems with one degree of freedom by Hamilton dynamics. For \(s\) degrees of freedom, Bohr forms the expression

\[
I = \int_0^\sigma \sum_{k=1}^s p_k q_k \, dt \tag{7}
\]

where \(\sigma\) is the period of the motion, \(q_k\) the coordinates and \(p_k\) the momentum. Analyzing small variations - "\(I\) will be invariant for any finite transformation of the system which is sufficiently slowly performed, provided the motion at any moment during the process is periodic and the effect of the variation is calculated on ordinary mechanics" - and making use of Planck’s theory for harmonic oscillator, Bohr arrives at

\[
I = \int_0^\sigma p \, dq = n \, h \tag{8}
\]

His concern, then, is to generalize the theory, with an expression equivalent to (5), and to apply it for other periodic systems, like a particle moving under the influence of the attractions from two fixed centers or a particle moving in a plane executing harmonic vibrations in two perpendicular directions with distinct frequencies. Bohr recognizes the works of Sommerfeld and Wilson, but did not mention Ishiwara.

Summarizing the Wilson, Sommerfeld and Bohr approaches, all of them are based on the (5) formula, the problems treated were the hydrogen spectra with both Balmer’s and the fine structure formula, and the Planck distribution. They were all published in the period between 1915 and 1918, but were based on discussions and works from the beginning of the decade. After this little historical review, the work of Ishiwara will be discussed.

B. Ishiwara’s "The universal meaning of the quantum of action"

Ishiwara’s paper [Ishiwara 1915, Ishiwara 2017] is divided in four sections: a short introduction, followed by the proposal of his phase-space quantization rule, the analysis of the Bohr atomic model (perhaps the principal part of the paper), concluding with the photoelectric effect. The latter application was unique with the considered authors.

In the beginning, Ishiwara is concerned about the different meanings of \(h\), based, fundamentally, on the fact of the Planck’ s constant being identified with scalar (basic dimension on phase-space or a parameter related to fundamental amount of energy and difference of energy on atomic levels) and vectorial (angular momentum) quantities. The title of the paper itself refers to the quest for an ultimate meaning of \(h\). There are two interpretations considered by Ishiwara as opposites: the Bohr one, where \(h\) is closely related to the quantized value of angular momentum and that of Sommerfeld, where \(h\) is related to the fundamental volume of the phase-space. Moreover, he states three questions at the beginning, answering them along the paper:

1. Are those different interpretations of \(h\) identical to each other? No, because the angular momentum is not quantized a priori - its quantization is a consequence of (11) - and his final value, founded on section B - The Bohr model of the atom,

\[
f = n \frac{\hbar}{\pi} \sqrt{1 - \varepsilon^2} \tag{9}
\]

- equation (12) on Ishiwara’s paper [Ishiwara 1915, Ishiwara 2017] - for general/elliptical orbits, is distinct from the value found by Bohr

\[
f = n \frac{\hbar}{\pi} \tag{10}
\]
2. Which viewpoint of the universal meaning of $h$ is the right one? The position of Sommerfeld on 1911, where $h$ is the fundamental length on phase space, related to the general postulate $Energy \times Time = h$ [Sommerfeld 2014a].

3. Should all phenomena be explained by one and the same basic assumption? Yes, and the basic assumption is (11).

1. The basic assumption

The version proposed by Ishiwara to the phase-space quantization rule is

$$h = \frac{1}{j} \sum_{i=1}^{j} q_i \, dp_i \quad (11)$$

That is the first equation of his paper - equation (1) in Ishiwara’s paper - and we can observe that it resembles to Poincaré’s version, (2), by the presence of a sum. Unfortunately, Ishiwara does not mention the calculations that conducted him to (11) and, though there exists the possibility that he had read the proceedings of the Solvay Congress, there is nothing to confirm it [Sigeko 2000].

Ishiwara cites the works of Sackur and Tetrode. In addition to the works cited in the paper, it is also important to mention that Sackur tried to construct a theory of gases by employing the concept of finite cells in phase space, which size determined by Planck’s constant. He and Tetrode advanced - in parallel with Planck and Sommerfeld - to the first attempts to construct a theory of degenerate gases, applying chemical constant method - a constant which occurred in the expression for the absolute entropy of a gaseous substance, that was used by them to the quantization of translational motion of atomic particles on monoatomic gases and to analyze the deviation of the classical equation at low temperatures - then anticipating the ideas of Bose and Einstein [Mehra 1982, Mehra 2001].

2. The Bohr model of the atom

Supposing an electron submitted to a central nuclear force, with stationary motion, elliptical orbits and with no consideration of other electrons of the atom, Ishiwara calculates, with his rule (11), the frequency for the emitted radiation,

$$\nu_1 = \frac{2\pi^2 m_0 e^4}{\hbar^3} \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad (12)$$

- equation (21) on Ishiwara’s paper - "which is just what is found in Bohr’s theory". Curiously, Ishiwara made the hypothesis "that the central charge of the hydrogen atom consists of two electrical elementary ones". Bohr later pointed out that Ishiwara’s theory was inconsistent with the usual one-electron model of neutral hydrogen [Kragh 2012]. This hypothesis was needed because of the form of Ishiwara’s equation (11) and it was not necessary in the formulation of Wilson, Sommerfeld and Bohr. It is appropriate, however, to question the sources of this odd hypothesis. The most likely answer is the work of Nicholson (who communicated the papers of Wilson) [Kragh 2012].

Nicholson was a student of line spectra of celestial bodies and supposed that the matter in stars could be more elementary structures than the known chemical atoms found on Earth. Following Thomson’s model, he proposed an atomic model based on classical mechanics and electrodynamics, imagining primary atoms consisting of small spheres of negative electricity, rotating around a smaller spherical positive nucleus. Some of these structures were "coronium", the most simple, contained two electrons; the next, "hydrogen", three electrons, followed by more complexes "nebulium" and "protofluorine". With the primary atoms, he constructed the chemical elements, and the hydrogen element consisted of two atoms of primary hydrogen [Kragh 2012, Mehra 1982]. Nicholson made criticisms about Bohr’s non-classical model and theory, in particular about the mechanical instability of Bohr’s orbits - some results were supported by calculations of Ludwig Föppl (1887-1976) and mentioned on Sommerfeld’s book [Sommerfeld 1923]. In Nicholson’s model, there were no quantum jumps and the spectral frequencies were vibration frequencies of the electrons in their positions. Yet this model gained some notoriety until World War I, thereafter it was set aside in favor of the Bohr-Sommerfeld model [Kragh 2012].
3. The photoelectric phenomenon

Here it is important to note, at first, that at that time there were several theories to explain the photoelectric effect \cite{Jammer1966, Stuewer1970, Wheaton2008}. We can cite here the theories of Arthur Erich Haas (1884-1941) and Joseph John Thomson (1856-1940), before our approach to the Sommerfeld one.

Haas was the author on 1910 of a model of the hydrogen atom which allowed him to deduce, for the first time (and three years before Bohr’s deduction \cite{Bohr1913a, Bohr1913b, Bohr1913c}), the Rydberg constant in terms of the charge and mass of the electron and the Planck constant, but with a wrong numerical factor \cite{Jammer1966}. However, in his theory for the photoelectric effect, Haas used the more known Thomson’s model (where electrons were immersed on a larger positive sphere) \cite{Jammer1966} and supposed that, the electron inside the atomic positive sphere would oscillate when a wave incided over the atom. For an electron on the boundary of the atomic sphere, its energy would be $h\nu$ and, if the energy of the incident wave was larger than that amount, the electron would be ejected and the $h\nu$ would be simultaneously abstracted from the incident wave \cite{Stuewer1970}. The Haas theory was supported by Hendrik Antoon Lorentz (1853-1928), who presented it on a lecture on Göttingen in October 1910 \cite{Stuewer1970, Wheaton2008} and used his own theory of the electron \cite{Lorentz2004} to analyze how classical waves might deliver quantum units of energy to bound electrons of the surface.

Thomson, by this time, propose two distinct theories, associated with two distinct atomic models, different from his former model too. In both theories, the emission of the electrons would be by resonance between the frequency of the incident radiation and the characteristic frequency (associated to revolutions or oscillations) of the electron in the atom and, in the second one, he reached the relation

$$T = h\nu$$

(13)

where $T$ is the kinetic energy.

Some other names related to the photoelectric effect were Philipp Lenard (1862-1947), Carl Ramsauer (1879–1955), Erich Marx (1879-1955) and Alexandr Stoletov (1839-1896). It is interesting to mention, however, the theories of Peter Franken (1828-1999) \cite{Stuewer1970} and Owen Richardson (1879-1959) \cite{Wheaton2008}, both them containing without further no assumptions about the nature of light. Richardson’s theory was developed on 1914 and had a thermodynamic appeal, treating only with macroscopic quantities, by analogy between the effect and the evaporation of molecules from a liquid \cite{Wheaton2008}. Franken’s theory, curiously, is a contemporary one and uses time-dependent perturbation theory with Schrödinger’s equation. It assumes that the atom is in its ground state and is subjected to a perturbation represented by a classical electromagnetic wave of well defined frequency, with this wave having enough energy to induce over the electron the transition to continuum. Franken obtained the same expression of Einstein’s theory (including the work function), and Planck’s constant $h$ is introduced by Schrödinger’s equation \cite{Stuewer1970}.

Let us return to the Sommerfeld model to photoelectric effect \cite{Delve1913, Sommerfeld1911}. It was innovative because he introduced $h$ from the beginning with the rule of Eq. \cite{Jammer1966, Stuewer1970, Wheaton2008}, contrary to the models of Haas and Thomson, who introduces $h$ at the end of the calculations. The incident radiation made the electron oscillates and, in resonance, after a certain time $\tau$, the electron will be ejected of the atom. This is very similar to the explanation of Ishiwara in the section C of his paper. It is important to note that none of the mentioned theories considers the work function on the law for photoelectric effect. For Sommerfeld, this function was not related to the effect itself, but associated to the path of the photoelectron from the ejection from atom to the surface of the metal under analysis. At last, for the Sommerfeld theory, a plot between photoelectron’s energy and the atomic characteristics frequencies will have, to each atomic frequency, a maximum (Sommerfeld shows some graphics on his paper with Debye \cite{Delve1913}, contrary to Einstein’s theory, where the frequency of the photoelectrons depends of any atomic frequency \cite{Stuewer1970}). A deep analysis of Sommerfeld theory can be found on the chapter 7 of Wheaton’s book \cite{Wheaton2008}.

Then, even more than one decade after Einstein’s paper about photoelectric effect was published in Annalen der Physik \cite{Arons1965}, his theory was not the only one, and not widely accepted yet (we may disregard the little known theories of Franken and Richardson here). As a matter of fact, this is also due to the lack of precise experimental data \cite{Jammer1966, Stuewer1970, Wheaton2008}, but the problem was solved by the verification of Einstein’s law (a non-resonance phenomenon with linear energy-frequency relation for the emitted electrons) by Millikan, who was working in it at the time of Ishiwara’s paper \cite{Millikan1914, Millikan1916a, Millikan1916b, Millikan1921}. The Nobel Prize for Einstein \cite{Einstein2015a} "for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect" on 1921 - while he was in Japan - and, for Millikan \cite{Millikan2016} "for his work on the elementary charge of electricity and on the photoelectric effect", on 1923. After them, the questions of Ishiwara on the end of his paper were naturally answered. It is curious that Millikan refers to the numerical value of the elementary charge $e$ as "the author’s value" \cite{Millikan1914} or "my value" \cite{Millikan1916b}. On \cite{Millikan1916b}, we
have a complete description of the experimental apparatus of Millikan, a portrait of his ingenuity, and the original graphics of results.

Ishiwara used the Sommerfeld model from 1911 to show the "special power" of its formula (11), considering the case where the frequency of the incident wave is the same as the natural frequency of the electron and neglecting the damping. Until electron’s ejection, its movement is described by the forced-damping oscillator; after its ejection, Ishiwara applies his equation (11) where he uses \( j = 1 \), preventing the influences of his incorrect condition. He obtains the "accumulation time" in accordance with the value calculated from the Sommerfeld theory. At last, Ishiwara obtains the linear relation between the energy and the frequency of the electron, in accordance with Einstein’s law. However, the potential energy is present on the final expression, disagreeing with Sommerfeld’s theory. Ishiwara purposes the question "Where does the potential energy \( U \) remain?"

IV. CONCLUSION

The work analyzed here is one hundred years old and it take part at the apex of attempts to understand and explain the spectroscopy data [Bucher 2008, Castro 2016, Duncan 2014, Eckert 2014, Haar 1967, Jammer 1966, Mehra 1982, Mehra 2001]. Moreover, it reflects pioneering efforts to understand the relations between quantum and classical worlds in a structure where the question of classicality of nature was more evident, because the set of OQT rules itself, included explicitly the use of classical mechanics from the beginning.

The question of the interpretations of quantum theory and the division of quantum and classical worlds is still subject to discussions [Freire 2011, Jammer 1974, Laloë 2001, Nikolic 2007, Omnès 1994, Pessoa 2006a, Pessoa 2006b, Pinto 2010, Wheeler 1983], but the focus changed with the emergence of concepts [Jammer 1966, Mehra 1982], with which any modern physicist is familiar, and were consolidated 10 years after Ishiwara’s work - uncertainty relations (Heisenberg [Wheeler 1983, 1927], wave particle duality (from Bohr culminating with the proposal of Louis de Broglie (1892-1987) on 1924 [Broglie 1929]), wave equations (Schrödinger, 1926 [Schrödinger 1978] and Dirac 1928 [Dirac 1928]) and wave function with its probabilistic interpretations (Born [Ludwig 1968, Wheeler 1983] and Dirac [Dirac 1926], both on 1926). Today, contrary to OQT, the classical physics enters implicitly in the optical-mechanic analogy (based on the Hamilton-Jacobi theory [Fetter 2003]) of Schrödinger in the deduction of his equation [Joas 2009, Köberle 1979, Wessels 1977] and explicitly just in some specific potentials inspired on classical expressions (e.g., the harmonic and Coulombian ones [Eisberg 1961]) needed to solve the equation.

We hope that this work made clear how rich the first 30 years of QM were, raising questions rarely seen on textbooks - thermodynamics with chemical constants, the atomic models of Nagaoka and Nicholson, the way from Planck to Sommerfeld on phase-space analysis and the alternative theories for photoelectric effect - and stimulates new studies about the career of Jun Ishiwara, a physicist who is rarely remembered but, as we saw on section II, had a wide range of skills. Indeed, besides his contribution to disseminate the theory of relativity in the Eastern world, Ishiwara lost the primacy to the proposal of phase-space quantization rule by just two months (his work was published on May 1915, and Wilson’s work in March of the same year). Though his paper (and also Wilson’s paper) exerted no significant influence on the further development of the quantum theory of atoms [Kragh 2012], Ishiwara rivaled in some aspects with names of stature of Poincaré, Planck, Bohr, and Sommerfeld, although of course his work lacked the depth of that of cited ones.

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