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Spatial Coherence and Optical Beam Shifts

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A beam of light, reflected at a planar interface, does not follow perfectly the ray optics prediction. Diffractive corrections lead to beam shifts; the reflected beam is displaced (spatial Goos-Hänchen type shifts) and/or travels in a different direction (angular Imbert-Fedorov type shifts), as compared to geometric optics. How does the degree of spatial coherence of light influence these shifts? We investigate this issue first experimentally and find that the degree of spatial coherence influences the angular beam shifts, while the spatial beam shifts are unaffected.

We start by briefly reviewing the theory. We consider a monochromatic partially coherent beam with a Gaussian envelope [24–26], a so-called Gaussian Schell-model beam, where both the intensity distribution \( I(\mathbf{r}) \) and the spatial degree of coherence \( g(\Delta \mathbf{r}) \) are Gaussian [27] (\( \mathbf{r} = r_2 - r_1 \) are the transverse position, and the relative transverse position, respectively). We obtain for the Gaussian Schell-model beam

\[
I(\mathbf{r}) \propto \exp \left( -\frac{\mathbf{r}^2}{2\sigma_s^2} \right), \quad g(\Delta \mathbf{r}) = \exp \left( -\frac{\Delta \mathbf{r}^2}{2\sigma_s^2} \right)
\]  

(1)

In the source plane, \( \sigma_s \) is the coherent (Gaussian) mode waist, and \( \sigma_s \) determines the correlation length. The latter approaches infinity for a fully coherent mode, and is a measure of the speckle size in case of partial spatial coherence. After propagating over a distance \( z \), these quantities evolve into \( \sigma_s(z) \) and \( \sigma_s(z) \); however, it turns out that their ratio \( \sigma_s(z)/\sigma_s \) is independent of propagation [29]: \( \sigma_s(z)/\sigma_s = \sigma_s/\sigma_s \); therefore, we use this ratio to quantify transverse coherence. Figure 1(a) shows three examples of such beams, from fully coherent (i) to the case where the coherence length is below one tenth of the beam size (iii). By calculating the intensity autocorrelation, the two length scales which are involved become visible: The Gaussian envelope leads to a wide background, while the emerging speckles in case (ii) and (iii) add a short-range correlation as is easily visible in the cross-section curves in Fig. 1(b). The number of participating modes can be estimated from the number of speckles in Fig. 1(a), or be approximated by \([1 + (\sigma_s/\sigma_s)^{-1}]^2\), which is \((1, \approx 50, \approx 200)\) for the cases (i,ii,iii) respectively. The three beams shown in Fig. 1 have been used in the experiments reported below.

To be able to discuss such a Gaussian Schell-model beam within the unifying beam shift framework developed by Aiello and Woerdman [30], we consider a paraxial, monochromatic, and homogeneously polarized
FIG. 1 (color online). Study of the statistical properties of the Gaussian Schell-model beams for different degrees of spatial coherence, which are used in the experiments ($\lambda = 675$ nm). The first row (a) shows exemplary intensity profiles out of which the statistical ensemble beam consists; these are obtained by stopping the rotation of the diffuser plate. The second row (b) shows the intensity autocorrelation of the corresponding fields in (a) as false-color plots and cross-sectional curves. This shows clearly the two scales involved, i.e., the Gaussian beam width and the speckle size. We have determined the ratio $\sigma_g/\sigma_S$ by measuring the beam waist together with the far field divergence angle [24].

($\lambda = 1, 2 = p, s$), but otherwise arbitrary, incoming optical field $U'(x, y) = \sum_\lambda U(x, y) a_\lambda x'_\lambda$. It propagates along $x'_\lambda$ (z coordinate), and $(a_1, a_2)$ is its polarization Jones vector. We use dimensionless quantities in units of $1/k$, where $k$ is the wave vector. The coordinate systems and their unit vectors $x'^1_\lambda$, $y'^1_\lambda$ are attached to the incoming ($i$) and reflected ($r$) beam, respectively [see Fig. 2(a)]. After reflection at a dielectric interface, the polarization and spatial degree of freedom are coupled by the Fresnel coefficients $r_{p,s}$ as [7]

$$U'(x, y, z) = \sum_\lambda a_\lambda r_\lambda U(-x + X_\lambda, y - Y_\lambda, z) x_\lambda,$$

(2)

$X_{1,2}$ and $Y_{1,2}$ are the polarization-dependent dimensionless beam shifts:

$$X_1 = -i \partial_\phi[\ln r_1(\theta)], \quad Y_1 = i \frac{a_2}{a_1} \left(1 + \frac{r_2}{r_1}\right) \cot \theta,$$

(3a)

$$X_2 = -i \partial_\phi[\ln r_2(\theta)], \quad Y_2 = -i \frac{a_1}{a_2} \left(1 + \frac{r_1}{r_2}\right) \cot \theta.$$

(3b)

Their real parts correspond to spatial beam shifts, and their imaginary parts to angular beam shifts. For either variant, beam displacements $X_\lambda$ (along the $x$ coordinate) correspond to longitudinal Goos–Hänchen (GH) type shifts [1], while transverse displacements $Y_\lambda$ along $y$ have Imbert–Fedorov character [3,4]. We observe that the transverse shifts $Y_\lambda$ require simultaneously finite $a_1$ and $a_2$, such as is present in circularly polarized light; this is not necessary for the longitudinal shifts $X_\lambda$. This explains why

the spatial shifts depend in the GH case only on one reflection phase, $\phi_1$ or $\phi_2$ with $\phi_\lambda = \arg(r_\lambda)$, while in the Imbert–Fedorov case, the spatial shift depends on the phase difference (e.g., $\phi_1 - \phi_2$).

In the lab, beam shifts are usually measured via the centroid of the reflected beam

$$\langle R(z) \rangle = \sum_\lambda w_\lambda \frac{\int \rho[U(-x + X_\lambda, y - Y_\lambda, z)]^2 dxdy}{\int[U(-x + X_\lambda, y - Y_\lambda, z)]^2 dxdy},$$

(4)

where $w_\lambda = |r_\lambda a_\lambda|^2/\sum_\lambda |r_\lambda a_\lambda|^2$ is the fraction of the reflected intensity with polarization $\lambda$. Equation (4) can be calculated by Taylor expansion around zero shift ($X_\lambda = Y_\lambda = 0$). With the spatial $\Delta_\lambda = \Re(X_\lambda, Y_\lambda)$ and angular $\Theta_\lambda = \Im(X_\lambda, Y_\lambda)$ shift vectors we obtain for the centroid $\langle R(z) \rangle = \sum_\lambda w_\lambda (\Delta_\lambda + M(z) \Theta_\lambda)$, where $M(z)$ is a polarization-independent $2 \times 2$ matrix which couples longitudinal and transverse beam shifts depending on the transverse mode of the field [7].

For a spatially incoherent beam, the incoming field $U^i$ corresponds to one realization of the ensemble of random fields with equal statistical properties. For our case of a
Gaussian Schell-model beam, \( M(z) \) turns out to be diagonal [21], and we finally obtain
\[
\langle R \rangle(z) = \sum_{\lambda=1}^{2} w_{\lambda} [\Delta_{\lambda} + \Theta_{\lambda} \theta_{S}^{2} z].
\]  

The first term is independent of \( z \), it therefore describes shifts of purely spatial nature. Since spatial coherence enters the discussion only via the parameter \( \theta_{S} \) (which we discuss below), and the first term is independent thereof, we conclude that spatial shifts are expected to be independent of the degree of transverse coherence.

We test this in our first experiment [Fig. 2(a)], where we examine the spatial Goos-Hänchen shift (extension to the spatial Imbert-Fedorov case is straightforward). A single Gaussian mode from a fiber-coupled 675 nm superluminescent diode (FWHM spectral width 20 nm) is focused loosely (**(fL1 = 20 cm, beam waist at focus 50 \( \mu m \)) close to the outer edge of a holographic diffuser (Edmund Optics NT47-988 light shaping diffuser, 25 mm diameter, scattering angle 0.5\(^{\circ}\)) [25]. This imprints a random phase on the beam, which in turn leads to speckle pattern formation by random interference. To average over many different realizations of this field, the diffuser is rotated at 70 Hz, which leads to a modulation in the speckle pattern at \( \sim 30 \text{ kHz} \) (this is related to the microscopic structure of the diffuser). This frequency is much higher than the polarization modulation frequency, see below. We collimate the far field (**(fL2 = 10 cm) from the plate and use an adjustable diaphragm [see Fig. 2(a)] to gain full control over the key parameter \( \sigma_{S} / \sigma_{S} \). We implement polarization modulation (10 Hz) using a polarizer in combination with a liquid-crystal variable retarder to generate an \( s \) or \( p \) polarization beam. This beam is reflected under total internal reflection in a 45\(^{\circ} \) - 90\(^{\circ} \) - 45\(^{\circ} \) prism (BK7, \( n = 1.514 \) at 675 nm), and refraction at the side faces of the prism is taken into account for determination of the angle of incidence \( \theta \). A quadrant detector in combination with a lock-in amplifier (locked to the polarization modulation) is used to measure the relative beam displacement (the quadrant detector is binned so that it effectively acts as a binary split detector).

Figure 2(b) shows the measured spatial GH shifts for the three beams with different spatial coherence shown in Fig. 1. We present exclusively polarization-differential shifts \( D_{ps} = D_{p} - D_{s} \), where \( D_{ps} \) are the displacements of \( p \) and \( s \) polarized reflected beams from the geometrical-optics position. For \( \sigma_{S} / \sigma_{S} \gg 1 \) we recover the well-known result that the spatial GH shift appears only for \( \theta > \theta_{c} \) [31], where \( \theta_{c} \) is the critical angle of 41.35\(^{\circ} \). However, the essential point of Fig. 2(b) is, that we find that the spatial beam shift is in fact independent from the degree of spatial coherence. This demonstrates that the theoretical result in Refs. [19,21,23] is correct, contrary to competing claims [20,22].

We turn now to the angular shifts, i.e., to the second term in Eq. (5). The parameter \( \theta_{S} \) is simply the effective beam divergence half-angle for a Gaussian Schell-model beam [24]:
\[
\theta_{S}^{2} = \frac{2}{k^{2}} \left[ \left( \frac{1}{2\sigma_{S}} \right)^{2} + \left( \frac{1}{\sigma_{S}} \right)^{2} \right].
\]

We see that reduced spatial coherence, i.e., reduced \( \sigma_{S} \), leads to increased beam divergence, and this in turn leads to increased angular beam shifts.

We test this in our second experiment, where we investigate the case of the in-plane (Goos-Hänchen type) angular beam shift. For this we use, as shown in Fig. 3(a), an additional lens \( L3 \) (\( f_{L3} = 10 \text{ cm} \)) to focus the beam, which is now reflected externally at the hypotenuse plane of the same prism as used before. The angular shift implies that \( s \) and \( p \) polarized beams follow slightly different paths which both originate at the beam waist [6]. For our experimental parameters and for beam propagation of a few centimeters, this angular shift is expected to lead to many tens of \( \Delta \) displacements of the centroid. We can then use simply a CCD camera to determine the difference in centroid position for \( p \) and \( s \) polarization. From two of such measurements at different propagation distance

![FIG. 3 (color online). Setup to measure angular shifts. (a) Compared to the experiment in Fig. 2(a), we introduce lens \( L3 \) to give the beam a sizable angular spread, and we use external reflection from the prism hypotenuse face. Further, we use a CCD camera and centroid determination by a computer to measure the relative beam position for \( s \) and \( p \) polarization. (b) Angular beam shifts for different degrees of coherence. The experimental data (symbols) agree with theory (curves); the vertical line indicates the Brewster angle (56.55\(^{\circ} \)).](image)
(5 cm apart) we determine the angular Goos-Hänchen shift; see Fig. 3(b). The angular shift shows a dispersive shape around the Brewster angle $\theta_B$. This in itself is well known [6]; it is a consequence of the fact that the amplitude reflectivity flips sign at its zero crossing at $\theta_B$. New is that we find a strong influence of the degree of coherence on the shift, in perfect agreement with the theoretical curves shown. We conclude that also in this case the theoretical predictions in Refs. [19,21,23] are correct.

Further, we note that if we replace the coherent-mode opening angle $\theta_0$ in the angular GH shift formulas by the effective beam opening angle $\theta_5$ [see Eq. (6)], partially coherent beams are well described. We therefore expect that, for a constant angle of incidence $\theta$, the angular beam shift is proportional to $\theta_5^2$. We have demonstrated this experimentally for $\theta = 70^\circ$ (see Fig. 4).

In conclusion, we have found experimentally that partial spatial coherence of a beam does not affect spatial beam shifts, while angular beam shifts are enhanced. Basically, reduced spatial coherence increases the effective angular spread of the beam, and therefore, angular shifts are increased. Our data are in good agreement with the theoretical study of Simon and Tamir [19], as well as later work [21,23]. We can conclude that the dispute in literature [19–23] is now definitively resolved.

We note that partially coherent beams have several advantages: they are less vulnerable to speckle formation and also less susceptible to atmospheric turbulences [32]. Although our results have been obtained for a single dielectric interface, this can be easily extended to the case of multilayer dielectric mirrors and metal mirrors. Also, despite that our experimental results are for longitudinal Goos-Hänchen type shifts only, it is clear from theory that the spatial and angular transverse Imbert-Fedorov shifts depend in the same way on the degree of spatial coherence as the spatial and angular GH shifts do. For completeness we mention that in our work we simulate a truly stochastic beam, namely by using a rotating diffusor; this has a much wider range of applications than when one uses a stationary diffusor [33]. Our findings demonstrate that transverse-incoherent sources, such as light-emitting diodes, can be used in applications which use beam shifts as a sensitive meter, such as in biosensing [34] or position detection [35]; as well as that the use of Goos-Hänchen shifts for beam position control [36,37] is applicable to incoherent beams. Beam shifts may also be relevant in photo lithography where partial spatial coherence plays a role [38]. Finally, our findings are relevant for beam shifts of particle beams (such as electron beams [16] or other matter beams [15]). Such beams are extremely difficult to prepare in a single mode (contrary to light beams) due to the smallness of the de Broglie wavelength; however, we know now that this should not diminish their (spatial) beam shifts.

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