Computational Modeling of Flow Characteristics in Three Products Hydrocyclone Screen

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Abstract: Three products hydrocyclone screen (TPHS) can be considered as the combination of a conventional hydrocyclone and a cylindrical screen. In this device, particles are separated based on size under the centrifugal classification coupling screening effect. The objective of this work is to explore the characteristics of fluid flow in TPHS using the computational fluid dynamics (CFD) simulation. The 2 million grid scheme, volume fraction model, and linear pressure–strain Reynolds stress model were utilized to generate the economical grid-independence solution. The pressure profile reveals that the distribution of static pressure was axisymmetric, and its value was reduced with the increasing axial depth. The maximum and minimum were located near the tangential inflection point of the feed inlet and the outlets, respectively. However, local asymmetry was created by the left tangential inlet and the right screen underflow outlet. Furthermore, at the same axial height, the static pressure gradually decreased along the wall to the center. Near the cylindrical screen, the pressure difference between the inside and the outside cylindrical screen dropped from positive to negative as the axial depth increased from −35 to −185 mm. Besides, TPHS shows similar distributions of turbulence intensity $I$, turbulence kinetic energy $k$, and turbulence dissipation rate $\varepsilon$; i.e., the values fell with the decrease in axial height. Meanwhile, from high to low, the pressure values are distributed in the feed chamber, the cylindrical screen, and conical vessel; the value inside the screen was higher than the outer value.

Keywords: flow characteristics; three products hydrocyclone screen; computational fluid dynamics; pressure distribution; turbulence

1. Introduction

Size separation can be considered as the process of dividing a group of particles into two or more size ranges according to their size, which usually plays a key role in connecting the preceding and the following [1]. The processed material can not only become the final product, but also be used as the feed material to enter the next process. Furthermore, the particle separation effect is important for the whole process. For example, in mineral processing, if fine particles cannot be effectively separated from the grinding circuit in time, it will lead to particle overgrinding, which not only increases energy consumption, but also affects the subsequent operation process [2]; even the commercialization of state-of-the-art enhanced gravity separation technologies may be hindered due to the inefficient particle size separation [3]. In general, the size separation methods include screening and classification. Screening refers to the process in which particles (from 300 mm down to around 40 µm) are divided into different sizes after passing through one or more layers of screen surface. Although some screen types can effectively separate particles below 40 µm,
sizes below 250 µm are also undertaken by classification, which is a method of separating into two or more products based on the velocity of particles through the water medium.

The above-mentioned particle size separation method has been widely used in mineral, chemical, environmental, and other industries [4–6]. However, the mineral process suffers from both some new tendencies (for instance, the decreasing processed particle size [7,8]) and inherent weaknesses (such as unsatisfactory efficiency and sharpness [9] and fish-hook effect [10]). This is can be attributed to not only the requirement to grind at fine sizes for mineral liberation [11], but also the market demand [12]. These changes clearly weaken the performance of separation devices, even causing the failure to meet the industrial needs [13,14].

Therefore, according to the above-mentioned defects of particle separation, we proposed the idea of combining screening and classification for the design of three products hydrocyclone screen (TPHS) [15]. The equipment has been optimized and improved based on the results of conventional hydrocyclone (CH). The structure and operation of TPHS are similar to those of CH, while the difference is that a cylindrical screen is added in the former. Thus, particles are separated under the compound of centrifugal classification and screening; moreover, TPHS generates an additional product called screen underflow besides overflow and underflow. Although TPHS has undergone a simple structural change, the particle distributions have been redistributed in the three product streams to eliminate the fish-hook effect for better device performance [15]. The successful pilot-scale experiments show its good application prospect in mineral processing. Compared with CH, TPHS realizes the organic integration of two different particle separation methods. However, it also increases the complexity of the multiphase fluid flow inside the device. Our earlier work exhibits that TPHS has a higher radial velocity and a lower tangential velocity than CH, which can help fine particles to pass through the screen [14]. The following study revealed that due to the unformed negative pressure zone near outlets, the absent air column in TPHS can be observed to reduce the energy consumption [16]. The prior studies reveal some common comprehensions for TPHS; however, the details of flow characteristics including the pressure distribution and turbulent features remain unknown. In the present study, computational fluid dynamics (CFD) modeling was conducted involving a TPHS with a diameter of 75 mm to comprehensively describe the pressure distribution and systematically explore the turbulence features.

2. Methodology

2.1. Geometric Models

As mentioned, a 75 mm TPHS was adopted, the diagrams for which are shown in Figure 1. In the schematic shown in Figure 1a, it can be seen that the TPHS is composed of inlet, feed chamber, cylindrical envelope, cylindrical screen, conical vessel, vortex finder, outlets, and so on. The fluid is imported into TPHS along the tangential feed inlet, and then it moves downward under the combined action of gravity and centrifugal force, finally flowing out from the underflow outlet, overflow outlet, and screen underflow outlet. To analyze the fluid flow in TPHS, the ANSYS ICEM 16.0 software was used to model the structural grid of the internal flow path, shown in Figure 1b. It should be noted that the overflow cavity was ignored to simplify the numerical simulation compared with the physical equipment (see the red border in Figure 1a). Thus, the overflow outlet in CFD cases is located at the top surface of the feed chamber. The grid model was decomposed into three blocks, namely hydrocyclone (i.e., inside section), cylindrical envelope (i.e., outside section), and cylindrical screen. The inside, outside, and cylindrical screen are focused on in the description of the flow features in the TPHS in the next section. The interface between the adjacent parts was merged using the strategy of mesh interfaces in the ANSYS Fluent software 16.0. Furthermore, the details of mesh quality testing are as follows: The total number of grids was more than 2 million. The maximum cell skewness was below 0.8. The minimum of quality was ~0.48. The minimum of determination was
~0.6. In terms of our previous work [14], the above grid scheme can provide the economical grid-independence solution.

![Diagram of 75 mm three products hydrocyclone screen (TPHS)](image)
2.2. Numerical Models

The fluid flow in TPHS can be considered as liquid–gas flow, where the main and second phases are water and air, respectively. Thus, the volume of fluid (VOF) model [17] was used to trace the interface between different phases by solving the continuity equation of the phase volume fraction, shown in Equations (1) and (2):

\[
\frac{1}{\rho_q} \left[ \frac{\partial}{\partial t} (\alpha_q \rho_q) + \nabla \cdot (\alpha_q \rho_q \vec{u}_q) \right] = \sum_{p=1}^{n} \left( \dot{m}_{pq} - \dot{m}_{qp} \right)
\]  

(1)

\[
\sum_{q=1}^{n} \alpha_q = 1
\]  

(2)

The meanings of variables in the presented formulas are explained in the Nomenclature section. Furthermore, the Reynolds-averaged Navier–Stokes (RANS) equations were composed of the continuity equation (Equation (3)) and motion equation (Equation (5)) to describe the turbulence flow in TPHS [14], where the instantaneous velocity was decomposed into the time mean component and fluctuation component, described as Equation (3).

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} \left( \rho \overline{u_j} \right) = 0
\]  

(3)

\[
\frac{\partial}{\partial t} \left( \rho \overline{u_i} \right) + \frac{\partial}{\partial x_j} \left( \rho \overline{u_i} \overline{u_j} \right) = - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right) \right] + \frac{\partial}{\partial x_j} \left( -\rho \overline{u_i^' u_j^'} \right)
\]  

(4)

\[
\overline{u_i} = \overline{u_i} + u_i^'
\]  

(5)

The linear pressure–strain Reynolds stress model [18–20], given as Equations (6) and (7), was accepted to solve the above RANS.

\[
\frac{\partial}{\partial t} \left( \rho \overline{u_i u_j} \right) + \frac{\partial}{\partial x_k} \left( \rho u_k \overline{u_i u_j} \right) = - \frac{\partial}{\partial x_k} \left( \rho \overline{u_i u_j u_k} \right) + p \left( \delta_{ij} \frac{\partial \overline{u_i}}{\partial x_k} \right) - \rho \left( \frac{\partial \overline{u_i}}{\partial x_k} \frac{\partial \overline{u_j}}{\partial x_k} \right) - 2\mu \frac{\partial \overline{u_i}}{\partial x_k} \frac{\partial \overline{u_j}}{\partial x_k} \right] \]  

\[
\Phi_{ij} = -1.8 \rho_k \left( \overline{u_i u_j} - \frac{2}{3} \delta_{ij} \overline{u_k^2} \right) - \frac{2}{3} \delta_{ij} \left( \frac{P_{ij} - C_{ij}}{\Phi_{ijk}} \right) - \frac{1}{2} \delta_{ij} \left( \frac{P_{kk} - C_{kk}}{\Phi_{ijk}} \right) - \frac{0.294}{2} \left[ \left( \overline{u_i u_j u_k n_k} + \overline{u_i u_j u_k n_k} \right) - \frac{0.179}{2} \phi_{ijk} n_k \right]
\]  

(7)

In addition, the fluid behavior near the wall region was modeled using the stand wall function, shown as follows [16]:

\[
\frac{U_p 0.09^{1/3} k_p^{1/2}}{\tau_w / \rho} = \frac{1}{0.4187} \ln \left( 9.793 \phi_{0.09^{1/4} k_p^{1/2} y_p / \mu} \right)
\]  

(8)

2.3. Boundary Conditions and Initialization

In this research, the inlet and outlet of TPHS were set as velocity inlet and pressure outlet, respectively. In detail, the inlet velocity was 5, 10, and 15 m/s, and all the outlets were the ambient atmospheric pressure, i.e., 1 atm. The hydraulic diameter was adopted
as the characteristic size of each opening, and the empirical value of 5% was considered as the turbulence intensity. In addition, considering the backflow phenomenon caused by negative pressure at each outlet, the reflux air volume fraction was 1, which means that the backflow was air. The standard initialization computed from all zones was considered as the solution initialization. The reference frame was relative to the cell zone. The initial values of gauge pressure, X velocity, Y velocity, Z velocity, turbulent kinetic energy, turbulent dissipation rate, and air volume fraction were set as 0 pascal, 0 m/s, 0 m/s, 0 m/s, 0.0234 m²/s², 0.6717 m²/s³, and 0. The time step and maximum iteration step were 0.001 s and 20, respectively. To obtain a high-precision solution, the residual of continuity, velocity, turbulent kinetic energy, dissipation rate, stresses, and volume fraction were set as $10^{-6}$. Note that the subrelaxation factor was adjusted according to the iterative convergence during the simulation process to prevent calculation divergence. So as to obtain a time-independent solution, the simulation time was 20 s, while all the numerical results were averaged over 16–20 s.

3. Results and Discussions

In this section, prior to exploring the fluid flow inside TPHS, the validity of the simulation results is verified by comparing with the earlier literature. Following this, the flow characteristics comprising pressure distribution and turbulence feature are described clearly. Subsequently, the optimization of the flow field is analyzed critically.

3.1. Validation

In view of our previous work [14,16], the combination of PIV test and CFD simulation has been used to comprehensively verify the above-mentioned numerical methods consisting of geometric modeling, numerical models, and solution conditions (see Section 2 for details). Thus, as a series of studies, to simplify the verification process, the water split was adopted to verify the simulation results. Note that the water split means that the ratio of water reporting through each outlet, which clearly influences the device classification efficiency [21]; thus, this parameter can be used to verify the CFD simulation [22].

Figure 2 exhibits the comparison between the CFD simulation and the literature under the inlet velocity of 5 m/s. In this investigation, the residual values of continuity, velocity, turbulent kinetic energy, dissipation rate, stresses, and volume fraction were below $10^{-4}$ in each simulation case, which means that the present simulations were convergent. From Figure 2, it is apparent that the water split of each outlet in the present CFD simulation displays a value that is almost consistent with that in the literature [14]. This trend means that the numerical methods adopted in our work can accurately describe the fluid flow in TPHS.

![Figure 2](image-url)

**Figure 2.** Comparison between the Computational fluid dynamics (CFD) simulation and the literature [14] under the same inlet velocity of 5 m/s.
3.2. Pressure Distribution

Referring to the previous work [14], the inlet velocity of 10 m/s was analyzed. Figure 3 shows the contour of static pressure (a) front view and (b) back view in vertical planes (y = 0 mm and y = 35 mm) and (c) different horizontal planes (z = 55 to −350 mm). It is noted that the coordinate origin is located at the interface between the feed chamber and cylindrical envelope (see the red coordinate in Figure 1a). Thus, the planes y = 0 mm and y = 35 mm are located across the center of the device and the feed inlet, respectively; the horizontal plane z = 55 to −350 mm is located from the feed chamber to the underflow outlet.

Comparing Figure 3a–c, it can be found that the distribution of static pressure was generally axisymmetric. However, due to the left tangential inlet and the right screen underflow outlet, the static pressure presents local asymmetry, especially near the feed inlet and screen underflow outlet. This phenomenon also can be clearly observed in Figure 4 (see the next paragraphs for details). The static pressure at the inlet is clearly higher than that at other parts. The maximum static pressure appears in the tangential inflection point of the feed inlet (the red area in Figure 3c). This can be attributed to the block of the feed chamber wall which forces the fluid from the linear motion to rotary motion. With the change of the flow state, the fluid dynamic pressure rapidly changes to static pressure. In addition, the static pressure near each outlet was smallest and negative, which means that the air reflux occurred here. However, the reflux was too weak to form the air column [16].

Furthermore, Figure 4 demonstrates the distribution of static pressure along the radial direction at (a) feed chamber, (b) cylindrical screen, and (c) conical vessel in the y = 0 mm plane. It can be seen that the distribution of static pressure was commonly symmetrical, while differences can be observed in some local areas, such as the aforesaid near inlet region (i.e., line z = 20 mm in Figure 4a) in the previous paragraph. Compared to Figure 4a–c, the static pressure decreased gradually along the wall to the center. In the vertical depth direction, the closer to the underflow outlet, the smaller the drop in static pressure.
Figure 3. Contour of static pressure distribution: (a) front view and (b) back view in vertical planes (y = 0 mm and y = 35 mm) and (c) different horizontal planes (z = 55 to -350 mm) in TPHS with the inlet velocity of 10 m/s.

Figure 4. Distribution of static pressure along the radial direction at (a) feed chamber, (b) cylindrical screen, and (c) conical vessel in the y = 0 mm plane.

In Figure 4a, the static pressure curve is clearly divided into two parts. The gap in this curve (see the blue dotted lines) is attributed to the vortex finder wall. From the lines z = 20 mm to z = 55 mm, it can be seen that the static pressure gradually increased with the increasing radius, while its value in the vortex finder was smaller than that in the feed chamber. Meanwhile, the analogous static distribution can be detected in the conical vessel area (Figure 4c). These trends demonstrate that in these areas, the fluid static pressure close to the outside wall was higher than that close to the center. This results in that the fluid tends to flow from the wall to the center. However, Figure 4b reveals the different pressure changes. With the increasing radius, the static pressure rose gradually and reached the maximum value near the screen aperture; then, it went down rapidly (see the red dotted lines in Figure 4b) and finally slowly increased. This trend can be attributed to the rapid exchange between the velocity and static pressure during the fluid passing through the screen.

To describe the static pressure inside the cylindrical screen (see the dotted lines in Figure 2b), the pressure distributions on the line x = 37.1–43.0 mm in plane y = 0 mm are represented in Figure 5. Because the thickness of the screen bar was 5 mm (see Table 1), the investigation area could cover the whole screen aperture. In Figure 5, it can be seen that as the axial depth increased from −35 to −185 mm, the higher static pressure inside the
cylindrical screen (i.e., $x < 37.5$ mm) gradually decreased, while the lower static pressure in other regions (namely $x > 37.5$ mm) shows the opposite trend. As the axial depth increased to about $-85$ mm, the two sides showed highly similar static pressure. Then, the pressure on the outside of the screen became higher than that on the inside of the screen. This means that in the upper part of the cylindrical screen, the fluid has a tendency to move outward along the screen aperture, while in the lower part, the tendency is opposite, that is, from the outside to the inside. This phenomenon is consistent with previous studies [14].

![Distribution of static pressure near the cylindrical screen.](image)

**Figure 5.** Distribution of static pressure near the cylindrical screen.

| Items                      | Size      | Items                               | Size       |
|---------------------------|-----------|-------------------------------------|------------|
| Inner diameter of screen mesh | 75 mm     | Height of cylindrical envelope      | 185 mm     |
| Size of aperture          | 0.65 mm   | Inner diameter of cylindrical column| 75 mm      |
| Thickness of screen bar   | 5 mm      | Inner diameter of vortex finder     | 25 mm      |
| Width of screen bar       | 3.25 mm   | Height of cylindrical envelope      | 185 mm     |
| Height of screen mesh     | 185 mm    | Inner diameter of screen underflow outlet | 30 mm |
| Size of feed inlet (length × width) | 29 mm × 8 mm | Angle of cone section                | 20°        |
| Length of feed cavity     | 70 mm     | Inner diameter of underflow outlet  | 10 mm      |

### 3.3. Turbulence Feature

In this study, the fluid turbulence feature was described by the turbulence intensity $I$, turbulence kinetic energy $k$, and turbulence dissipation rate $\varepsilon$, which are shown in Figures 6–8, respectively, under different feed inlet velocities: (a) 5 m/s, (b) 10 m/s, and (c) 15 m/s. It is clear that although the inlet velocities were different, the $I$, $k$, and $\varepsilon$ of fluid flow in TPHS exhibit similar distributions. Namely, their values decrease from top to bottom, and the maximum values are located at the tangential end of the feed and the screen underflow outlet, while the minimum values are located near the underflow outlet. In addition, with the increase in the inlet velocity, the $I$, $k$, and $\varepsilon$ increased obviously. The following takes the inlet velocity of 10 m/s as an example to elaborate the turbulent flow distribution in TPHS:
(a) 5 m/s: (i) 1–5% (ii) 5–20% (iii) 20–70% 

(b) 10 m/s: (i) 1–15% (ii) 15–60% (iii) 60–100% 

Figure 6. Cont.
Figure 6. Distribution of turbulence intensity I (%) in TPHS.

(a) $v = 5 \text{ m/s}$: (i) $0 \sim 0.003 \text{ m}^2/\text{s}^2$ (ii) $0.003 \sim 0.03 \text{ m}^2/\text{s}^2$ (iii) $0.03 \sim 0.3 \text{ m}^2/\text{s}^2$

(c) $15 \text{ m/s}$: (i) $1 \sim 10\%$ (ii) $10 \sim 40\%$ (iii) $40 \sim 70\%$

Figure 7. Cont.
(b) \( v = 10 \) m/s: (i) \( 0 \sim 0.01 \) m\(^2\)/s\(^2\) (ii) \( 0.01 \sim 0.1 \) m\(^2\)/s\(^2\) (iii) \( 0.1 \sim 1 \) m\(^2\)/s\(^2\)

(c) \( v = 15 \) m/s: (i) \( 0 \sim 0.03 \) m\(^2\)/s\(^2\) (ii) \( 0.03 \sim 0.3 \) m\(^2\)/s\(^2\) (iii) \( 0.3 \sim 3 \) m\(^2\)/s\(^2\)

**Figure 7.** Distribution of turbulence kinetic energy \( k \) (m\(^2\)/s\(^2\)) in TPHS.
(a) $v = 5 \text{ m/s}$: (i) $0 \sim 0.02 \text{ m}^2/\text{s}^3$ (ii) $0.02 \sim 2 \text{ m}^2/\text{s}^3$ (iii) $2 \sim 200 \text{ m}^2/\text{s}^3$

(b) $v = 10 \text{ m/s}$: (i) $0 \sim 0.1 \text{ m}^2/\text{s}^3$ (ii) $0.1 \sim 10 \text{ m}^2/\text{s}^3$ (iii) $10 \sim 1000 \text{ m}^2/\text{s}^3$

Figure 8. Cont.
The cylindrical screen gradually decreases along with the axial height to the vicinity of the cylindrical screen–cone interface, the value of $k$ shows a similar trend to turbulence intensity $I$. In Figure 6b, (i) 0~0.1 m, (ii) 0.1~1 m, and (iii) 1~10 m. Generally, the above analysis reveals that the turbulence intensity $I$ in TPHS is distributed between 10% and 70%; that is, the flow field has high-intensity turbulence. The maximum $I$ value (about 40~70%) in the high-intensity turbulence zone shown in Figure 6b(iii) appears near the feed chamber and the screen underflow outlet. Figure 6b(ii) indicates that as the cylindrical screen gradually decreases along with the axial height to the vicinity of the cylindrical screen–cone interface, the value of $I$ drops 10~40%. As the axial height drops to the cone section area, shown in Figure 6b(i), the turbulent flow of the fluid changes from high-intensity turbulence to medium-intensity turbulence ($I = 1$~10%). Moreover, the $I$ inside the cylindrical screen is higher than that outside the screen, except for the screen underflow outlet under the right screen.

Figure 7b depicts the variation of $k$ in TPHS: (i) 0~0.01 m$^2$/s$^2$, (ii) 0.01~0.1 m$^2$/s$^2$, and (iii) 0.1~1 m$^2$/s$^2$. Obviously, $k$ shows a similar trend to turbulence intensity $I$; that is, the value of $k$ decreases as the axial depth increases, and its value inside the cylindrical screen is higher than that outside; moreover, the maximum value is located near the feed chamber and the screen underflow outlet.

In Figure 8b, (i) 0~0.1 m$^2$/s$^3$ (ii) 0.1~1 m$^2$/s$^3$, and (iii) 1~10 m$^2$/s$^3$ show the distribution of $\epsilon$ in the coupled flow field of TPHS. Obviously, $\epsilon$ also shows a distribution law similar to that of the turbulence intensity $I$. Specifically, as the axial height decreases, the $\epsilon$ value decreases; the $\epsilon$ value inside the screen is higher than that outside the screen, and the distribution area of the maximum $\epsilon$ value is consistent with the $I$ value. This shows that the turbulent energy loss rate $\epsilon$ of the flow field is the highest near the feed chamber and the screen underflow outlet, followed by the cylindrical screen, and is the lowest in the cone section.

Generally, the above analysis reveals that the turbulence intensity $I$, turbulent kinetic energy $k$, and turbulence dissipation rate $\epsilon$ show similar distribution trends in TPHS. The pressure values from high to low are distributed in the feed chamber, the cylindrical screen, and the conical vessel; the value inside the screen was higher than the outer value.

**Figure 8.** Distribution of turbulence dissipation rate $\epsilon$ (m$^2$/s$^3$) in TPHS.

In Figure 6b, the distribution of turbulence intensity $I$ is shown: (i) 1~10%, (ii) 10~40%, and (iii) 40~70%. The figure exhibits that the value of $I$ in TPHS is distributed between 10% and 70%; that is, the flow field has high-intensity turbulence. The maximum $I$ value (about 40~70%) in the high-intensity turbulence zone shown in Figure 6b(iii) appears near the feed chamber and the screen underflow outlet. Figure 6b(ii) indicates that as the cylindrical screen gradually decreases along with the axial height to the vicinity of the cylindrical screen–cone interface, the value of $I$ drops 10~40%. As the axial height drops to the cone section area, shown in Figure 6b(i), the turbulent flow of the fluid changes from high-intensity turbulence to medium-intensity turbulence ($I = 1$~10%). Moreover, the $I$ inside the cylindrical screen is higher than that outside the screen, except for the screen underflow outlet under the right screen.

Figure 7b depicts the variation of $k$ in TPHS: (i) 0~0.01 m$^2$/s$^2$, (ii) 0.01~0.1 m$^2$/s$^2$, and (iii) 0.1~1 m$^2$/s$^2$. Obviously, $k$ shows a similar trend to turbulence intensity $I$; that is, the value of $k$ decreases as the axial depth increases, and its value inside the cylindrical screen is higher than that outside; moreover, the maximum value is located near the feed chamber and the screen underflow outlet.

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Generally, the above analysis reveals that the turbulence intensity $I$, turbulent kinetic energy $k$, and turbulence dissipation rate $\epsilon$ show similar distribution trends in TPHS. The pressure values from high to low are distributed in the feed chamber, the cylindrical screen, and the conical vessel; the value inside the screen was higher than the outer value.
4. Conclusions

In the present research, a TPHS device was developed. The contrast of CFD simulation and literature was utilized to validate the geometric and numerical models. Based on this, a series of simulations were performed to explore the fluid flow in the TPHS. In terms of the aforementioned discussion, the following conclusions were drawn:

(1) Generally, the distribution of static pressure was axisymmetric, and its value decreased with the increase in the axial depth. The maximum was located near the tangential inflection point of the feed inlet, while the minimum appeared in each outlet. However, due to the left tangential inlet and the right screen underflow outlet, local asymmetry can be observed.

(2) In the same horizontal plane, the static pressure decreased gradually along the wall to the center. Near the cylindrical screen, as the axial depth increased, the static pressure inside the cylindrical screen gradually decreased, while that outside the cylindrical screen regularly increased. As the axial depth increased to about 85 mm, the static pressure of the two sides was closely in equilibrium. Then, the pressure on the outside of the screen became higher than that on the inside of the screen.

(3) The turbulence intensity $I$, turbulence kinetic energy $k$, and turbulence dissipation rate $\varepsilon$ of the fluid flow in TPHS exhibit similar distributions; that is, the values decreased with the increase in axial depth. Meanwhile, from high to low, the pressure values are distributed in the feed chamber, the cylindrical screen, and the conical vessel; the value inside the screen was higher than the outer value.

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Nomenclature

- $\vec{u}_q$: Velocity vector (m/s)
- $u^\prime$: Mean velocity component (m/s)
- $u_i$: Fluctuation velocity component (m/s)
- $i, j$: $x, y, or z$ components
- $p, q$: Phase in fluid
- $\mathbf{m}_{pq}$ ($\mathbf{m}_{qp}$): The mass transfer from phase $p$ ($q$) to phase $q$ ($p$)
- $k_p$: Turbulence kinetic energy at the wall-adjacent cell centroid, $P$
- $t$: time (s)
- $U_p$: mean velocity of the fluid at the wall-adjacent cell centroid, $P$
- $x_i, x_j, x_k$: position (m)
- $y_p$: distance from the centroid of the wall-adjacent cell to the wall, $P$
- $\delta$: Kronecker delta function
- $\rho$: density (kg/m$^3$)
- $\tau_w$: wall shear stress
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