Abstract

Using the recently proposed $gg\eta'$ effective vertex, we investigate the production of $\eta'$ from gluon fusion in polarized $pp$ collisions. We show that by measuring $A_{LL}$ in $\eta'$ production, one can extract the polarized gluon distribution $\Delta G(x, Q^2)$ at $Q^2 \sim 1\text{ GeV}^2$ and in a wide range of $x$. 
1 Introduction

The Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory will collide polarized protons at a center of mass energy of $\sqrt{s} = 250$ GeV in very near future (for a review of the spin program at RHIC, see [1]). One of the most important goals of the experiment is to measure the poorly known polarized gluon distribution in a proton. In general, polarized distributions will be measured at RHIC in a more extended kinematic region and with higher accuracy than the previous experiments.

Even though Deep Inelastic Scattering of electrons on protons is the best process in which to measure QCD structure functions and the parton distribution functions, it is also possible to extract the parton distribution functions from Drell-Yan, direct photons, $J/\psi$'s, etc. Recently, we proposed [2] measuring $\eta'$ in (unpolarized) proton-nucleus collisions as a way to extract the gluon distribution function in nuclei. This process allows one to directly (without any deconvolution) extract the nuclear gluon distribution function. Furthermore, the accessible kinematic region in $x$ in this measurement is orders of magnitude smaller than other processes in hadronic collisions. One can also investigate the possibility of the restoration of $U_A(1)$ symmetry at zero temperature due to high gluon density effects in nuclei [3]. Production of $\eta'$ in heavy ion collisions was considered in [4].

In this short note, we extend our previous work on $\eta'$ production in proton-nucleus collisions to polarized proton-proton collisions at RHIC and estimate the double spin asymmetry $A_{LL}$. We show that the predicted asymmetry is large enough to be experimentally measurable in a wide range of rapidity. We predict the asymmetry $A_{LL}$ to be $\sim 0.001$ at mid rapidity using the available parameterization of the polarized gluon distribution functions. Alternatively, by measuring the asymmetry $A_{LL}$ in the mid rapidity region, one can probe the polarized gluon distribution function at $x \sim 0.004$ and $Q^2 = 1$ GeV$^2$.

2 $\eta'$ production cross section

The $\eta'$ meson is much heavier than its fellow pseudo-scalar mesons, $\pi, K$ and $\eta$. This fact has intrigued the physics community for decades. It is now understood that $\eta'$ gets most of its mass through instantons and breaking of $U_A(1)$ symmetry

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1. The possibility of using $\eta'$ at low $P_t$ to probe the polarization of gluons inside the proton was first mentioned in Ref. [1].
due to the triangle anomaly \[\Im\]. Atwood and Soni used this anomaly to propose the following effective vertex for gluon-gluon-\(\eta'\) coupling in \[\Re\]

\[ T_{ab}^{\alpha\beta}(p, q, P) = H(p^2, q^2, P^2) \delta_{ab} \epsilon_{\mu\nu\lambda\gamma} p^\mu q^\nu q^\nu \epsilon^\lambda(p, \alpha) \epsilon^\gamma(q, \beta) \quad (1) \]

\(p\) and \(q\) are the momenta of the gluons while \(P\) is the momentum of the produced \(\eta'\). Here, \(\epsilon(\alpha, p)\) is the polarization vector of a gluon with momentum \(p\) and helicity \(\alpha = \pm 1\). The Kronecker delta \(\delta_{ab}\) indicates that only the color singlet combination of the gluons contribute. In Ref.\[\Re\], the form factor \(H(p^2, q^2, P^2)\) was estimated to be \(1.8\, \text{GeV}^{-1}\) in the limit where the incoming gluons and the produced \(\eta'\) are on-shell. Using this effective vertex, it is easy to calculate the differential cross section for \(gg \to \eta'\) which is given by \[\Re\]

\[
d\hat{\sigma}_{ab,\alpha\beta}^{gg \to \eta'} = \frac{1}{4\sqrt{(p \cdot q)^2}} |T_{ab}^{\alpha\beta}|^2 (2\pi)^4 \delta^4(P - p - q) \frac{d^3P}{(2\pi)^3 2E_P} \quad (2)
\]

with \(p^2 = q^2 = 0\) and \(P^2 = M_{\eta'}^2\).

We will now consider the collision of longitudinally polarized protons at RHIC. The double spin asymmetry \(A_{LL}\) is defined as (for an introduction to spin effects in high energy collisions, see \[\Re\])

\[
A_{LL} \equiv \frac{d\sigma_{++} - d\sigma_{+-}}{d\sigma_{++} + d\sigma_{+-}} \equiv \frac{d\Delta\sigma}{2d\sigma} \quad (3)
\]

where \(d\sigma_{++}\) denotes the cross section where both protons have their spins parallel to their momenta while \(d\sigma_{+-}\) denotes the cross section when one proton has its spin anti-parallel to its momentum. The unpolarized proton-proton differential cross section is denoted by \(d\sigma\).

The differential cross section \(d\Delta\sigma\) is related to the polarized gluon distribution function through

\[
d\Delta\sigma^{pp \to \eta'X} = \int dx_1 dx_2 \Delta G(x_1, Q_f^2) \Delta G(x_2, Q_f^2) d\Delta \hat{\sigma}_{gg \to \eta'} \quad (4)
\]

where \(x_1\) and \(x_2\) are the momentum fractions of the incoming gluons, \(Q_f^2\) is the factorization scale and \(d\Delta \hat{\sigma} \equiv d\hat{\sigma}_{++} - d\hat{\sigma}_{+-}\). Here \(d\hat{\sigma}_{++}\) is the differential cross section for scattering of two positive helicity gluons averaged over the color. Explicitly,

\[
d\hat{\sigma}_{++} = \frac{1}{(N_c^2 - 1)^2} \sum_{a,b} d\hat{\sigma}_{ab,++}^{gg \to \eta'} \quad (5)
\]
and similarly for $d\sigma_{+-}$. However, since $\eta'$ is a pseudo-scalar, it is clear that only positive helicity gluons will contribute. In other words,

$$d\Delta\sigma^{gg\rightarrow \eta'} = d\hat{\sigma}_{++}$$

(6)

Calculation of $|T_{ab}^{++}|^2$ is straightforward and gives

$$|T_{ab}^{++}|^2 = \delta_{ab} \frac{M_{\eta'}^4 H_0^2}{4}$$

(7)

where we defined $H_0 = H(0,0,M_{\eta'}^2)$. The differential cross section for inclusive $\eta'$ production in polarized proton-proton collisions is then

$$\frac{d\Delta\sigma^{pp\rightarrow \eta'X}}{dx_L} = \frac{\pi H_0^2}{32 x_E} x_+ \Delta G(x_+, Q^2_f) x_- \Delta G(x_-, Q^2_f)$$

(9)

where $x_L = 2P_z/\sqrt{s}$ and $x_E = 2E_P/\sqrt{s}$ are the momentum and energy fractions of the produced $\eta'$ respectively and

$$x_\pm \equiv \frac{x_E \pm x_L}{2} = \frac{E_P \pm P_z}{\sqrt{s}}$$

(10)

Using (9) in (3) leads to the following result for the asymmetry $A_{LL}$

$$A_{LL} = \frac{\Delta G(x_+, Q^2_f) \Delta G(x_-, Q^2_f)}{G(x_+, Q^2_f) G(x_-, Q^2_f)}$$

(11)

Equation (11) is our main result. It simply relates the measured double spin asymmetry to the polarized gluon distribution in a proton.

3 Results

To estimate the cross-sections and the theoretical uncertainties, we use following two sets of gluon distribution functions. The first set consists of the MRST99\textsuperscript{12}

\begin{footnote}{The unpolarized cross-section calculated in Ref.\textsuperscript{3} is

$$\frac{d\sigma^{pp\rightarrow \eta'X}}{dx_L} = \frac{\pi H_0^2}{64 x_E} x_+ G(x_+, Q^2_f) x_- G(x_-, Q^2_f)$$

(8)

which is the average of the polarized cross-sections $d\sigma^{pp\rightarrow \eta'X}_{++} / dx_L$ and $d\sigma^{pp\rightarrow \eta'X}_{+-} / dx_L$.}


Figure 1: The gluon distribution function $xG(x, Q_f^2)$ from MRST99 (solid line) and GRV98 (dashed line). Both are calculated at $Q_f^2 = 1.25 \text{ GeV}^2$.

The first set consists of MRST99 unpolarized gluon distribution function and LSS2001\cite{13} polarized gluon distribution function which is based on MRST99. The second set consists of GRV98\cite{14} unpolarized gluon distribution function and GRSV2000\cite{15} polarized gluon distribution function which is based on GRV98. The minimum $Q^2$ allowed by MRST99 is $1.25 \text{ GeV}^2$ and that’s what we set our value of $Q_f^2$ to be.

Using MRST99 the cross section integrated between $10^{-5} < x_L < 0.1$ is

$$\sigma = \frac{1}{2}(\sigma_{++} + \sigma_{+-}) = 0.21 \text{ mb}$$

while using GRV98 gives

$$\sigma = \frac{1}{2}(\sigma_{++} + \sigma_{+-}) = 1.0 \text{ mb}$$

in the same range. This difference in the cross-section is mainly due to the difference of the two gluon distribution functions at small $x$. For illustration, we plot $xG(x, Q_f^2)$ for both sets in Figure (1).
From the thermal model as well as exclusive reaction studies\[5, 16\] one would expect the $\eta'$ cross-section to be a few % of the pion production cross-section, or $O(0.1\, \text{mb})$. However, this does not necessarily imply that MRST99 is better suited for our purpose. To the authors’ knowledge, there is no measurement of $\eta'$ inclusive production cross-section. By measuring the $\eta'$ inclusive production cross-section, one can better constraint the behavior of the gluon distribution function at small $x$.

The polarized gluon distributions from LSS2001 and GRSV2000 are also very different. Using LSS2001, we get

$$\Delta\sigma = (\sigma_{++} - \sigma_{+-}) = 0.97 \, \mu\text{b}$$

integrated in the $10^{-5} < x_L < 0.1$ range while using GRSV2000 gives

$$\Delta\sigma = (\sigma_{++} - \sigma_{+-}) = 0.17 \, \mu\text{b}$$

in the same range. These results indicates that the asymmetry $A_{LL}$ can range from $1.7 \times 10^{-4}$ to $4.6 \times 10^{-3}$ or can differ by a factor of 30 or more depending on the choice of current gluon distribution functions. Therefore measuring $\eta'$ cross-section and the asymmetry accurately can greatly help constrain different gluon distribution functions currently in use.

Figure (2) shows $A_{LL}$ as a function of $\eta'$ meson momentum fraction $x_L$ at $\sqrt{s} = 250\, \text{GeV}$. Again we use the two sets of gluon distribution functions as described above. The solid curve in Figure (2) uses Set 1 (MRST99+LSS2001) and the dashed curve uses Set 2 (GRV98+GRSV2000). The difference between two sets is quite large. The values of $A_{LL}$ at small $x$ differ by a factor of almost 40.

One notable feature of this figure is that for $x_L < 0.001$, $A_{LL}$ is constant. To understand the plateau, we note that

$$x_{\pm} = \frac{\sqrt{x_L^2 + 4M_{\eta'}^2/s} \pm x_L}{2}$$

If $x_L \ll M_{\eta'}/\sqrt{s}$, then $x_+ \approx x_- \approx M_{\eta'}/\sqrt{s}$ and hence

$$A_{LL} \approx \left( \frac{\Delta G(x = M_{\eta'}/\sqrt{s})}{G(x = M_{\eta'}/\sqrt{s})} \right)^2$$

will remain almost constant. The value of the asymmetry $A_{LL}$ at $y = 0$ of course is a function of the collision energy. The input parametrization used for the unpolarized gluon distribution function for both LSS2001 and GRSV2000 is of the form

$$x\Delta G(x) = C x^a (1-x)^b xG(x)$$

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Figure 2: The asymmetry $A_{LL}$ as a function of $\eta' x_L$ calculated at $\sqrt{s} = 250$ GeV and $Q_f^2 = 1.25$ GeV$^2$. The upper curve is calculated using MRST99 and LSS2001 parton distribution functions. The lower curve is calculated using GRV98 and GRSV2000 sets.

If our $Q_f^2 = 1.25$ GeV$^2$ is not very much different from the input $Q_0^2$ of the distribution functions, the asymmetry should be given by

$$A_{LL}(x_L) \approx C^2 x_+^a (1 - x_+)^b x_-^a (1 - x_-)^b$$

$$= C^2 \left( \frac{M_{\eta'}^2}{s} \right)^a (1 - x_+)^b (1 - x_-)^b$$  \hspace{1cm} (19)

where we used $x_+ x_- = M_{\eta'}^2 / s$. Near $x_L = 0$, both $x_+$ and $x_-$ are small so that

$$A_{LL} \approx C^2 \left( \frac{M_{\eta'}^2}{s} \right)^a$$  \hspace{1cm} (20)

Therefore measuring the asymmetry at $y = 0$ for various energies can tell us what the exponent $a$ should be without knowing the detailed form of $xG(x)$. For LSS2001,
this exponent turned out to be about 0.6 and for GRSV2000, it is about 0.8. The reason that Set 2 curve deviates from a straight line may be explained by the fact that the input $Q^2_0$ of set 2 (0.4 GeV$^2$) is not so close to our $Q^2_f = 1.25$ GeV$^2$.

Figure 3: The asymmetry $A_{LL}$ at the mid-rapidity calculated at $Q^2_f = 1.25$ GeV$^2$. MRST99 and LSS2001 are used for the upper curve and GRV98 and GRSV2000 are used for the lower curve. The solid lines are linear fits. For MRST99+LSS2001, the slope turned out to be $-1.27$ corresponding to $a = 0.633$ in Eq.(18). For GRV94+GRSV, the slope is $-1.65$ corresponding to $a = 0.827$.

Even though the detectors at RHIC can only go up to the maximum $x$ of about 0.01, it is worth noting the behavior of $A_{LL}$ for larger values of $x$ in Figure (2). When $x_L \gg M_{\eta'}/\sqrt{s}$, $x_\pm$ can be approximated to be\footnote{For RHIC at $\sqrt{s} = 250$ GeV, $M_{\eta'}/\sqrt{s} = 0.0038$. For LHC at $\sqrt{s} = 5500$ GeV, $M_{\eta'}/\sqrt{s} = 0.00017$.}

$$x_+ \approx x_L + \frac{M_{\eta'}^2}{s x_L} \quad \text{and} \quad x_- \approx \frac{M_{\eta'}^2}{s x_L}$$

(21)
and the asymmetry becomes

\[ A_{LL}(x_L) \approx C^2 \left( \frac{M_{\eta'}^2}{s} \right)^a \left( 1 - x_L - \frac{M_{\eta'}^2}{s x_L} \right)^b \left( 1 - \frac{M_{\eta'}^2}{s x_L} \right)^b \]  \hspace{1cm} (22)

In LSS2001, the exponent \( b \) is argued to be 0. In other parametrizations, \( b \) is substantially different from 0. For instance in GRSV2000, \( b \) ranges from 4 to 7. Therefore even at moderately large \( x_L \), the behavior of \( A_{LL} \) as a function of \( x_L \) depends strongly on the exponent \( b \). In this way, very forward \( \eta' \)'s can distinguish between different parameterizations of the unpolarized gluon distribution function.

4 Conclusion

In this paper, we showed the dependence of \( A_{LL} \) on \( \eta', x_L \) as well as the \((pp)\) center of mass energy of the collision using \((\Pi)\). Measuring \( A_{LL} \) in this way directly probes the gluon distribution function in the proton. Had we known the unpolarized gluon distribution function well, \( A_{LL} \) could be used to map out the polarized gluon distribution. As it happens, currently the gluon distribution is not very well constrained by the past experiments in the small \( x \) region. Measuring \( \eta' \) at high energy can greatly constrain the gluon distribution in this region.

There are several caveats to our results. First and perhaps the most important one theoretically, is that \( M_{\eta'} \sim 1\text{GeV} \) is not very large and therefore higher order \((\alpha_s)\) corrections can be quite sizable. This is somewhat “cured” by introducing a \( K \) factor which is supposed to take higher order corrections into effect. In our case, since we are calculating the ratio of cross sections, these effects should largely cancel out. However, without a real calculation of higher order effects, there is no way to know for sure. Another complication arises due to effects such as initial state radiation of gluons, etc. which would lead to the produced particle having a transverse momentum. These effects can be resummed for few limited processes but are mostly included phenomenologically by introducing an intrinsic \( k_t \) for the incoming partons \([17]\). We intend to do this in the near future and investigate the dependence of \( A_{LL} \) on the transverse momentum of \( \eta' \). However, if we consider \( p_t \) smaller than the intrinsic \( k_t \) scale, our analysis should still apply.

Another potential problem is that we have neglected possible contribution of annihilation of quarks and anti quarks into \( \eta' \)'s. We believe we can avoid this problem here for two reasons. First, the values of \( x \) considered are quite small where gluons are the dominant partons. Therefore contribution of sea quarks to this process should be of order of a few percent which we can safely neglect. Furthermore, if
the $q\bar{q}'$ vertex is of the derivative type\cite{8}, then in the on-shell limit the quark anti-quark annihilation matrix element is $O(m_q/M_{\eta'})$ smaller than the gluon fusion matrix element. Hence, in the small $x$ range the gluon fusion process should dominate over quark anti-quark annihilation process.

In summary, we have investigated the production of $\eta'$ in polarized $pp$ collisions and estimated the double spin asymmetry variable $A_{LL}$ using the available polarized gluon distribution functions. Alternatively, measuring $A_{LL}$ at polarized proton-proton collisions at RHIC will enable us to directly extract the polarized gluon distribution function at $Q^2 \sim 1$ and different values of $x$.

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