Free convective flow of nanoliquids in a partitioned cavity with linearly heating

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Abstract. The numerical study is executed to inspect the laminar free convective characteristics of different nanoliquids in a square partitioned enclosure. An adiabatic baffle is attached on the bottom of the enclosed space. The left barrier is heated linearly whereas the right barrier temperature is kept as constant. The horizontal barriers of enclosed space are adiabatic. The finite volume method is employed to solve the transport equations for various groupings of relevant parameters in the work. It is established that the energy diffusion rate rises with growing the volume fraction of nanoparticles. The increment in averaged Nusselt number powerfully depends on the nanoparticle picked.

Key words: Natural convection; nanofluid; cavity; linearly heating wall.

1. Introduction
Convective cooling augmentation procedure has been a key issue in scientific applications because of thermal control of devices [1-5]. Since the effectiveness of cooling procedure is limited by poor energy transport properties of employed fluid, it needs an improved energy transport medium for convective cooling technique to overcome this limitation. In recent years, nanofluids are considered in research to analyze the convection heat transfer in enclosures [6-10]. Ho et al. [8] numerically examined the uncertainties in the properties of nanoliquids on convection. Sivasankaran et al. [9] deliberated the convection of nanoliquids in a linearly heated box. Ho et al [10] experimentally examined the free convection nanoliquid in a square enclosure of various sizes. Ho et al. [11] explored the influences of sedimentation, thermophoresis, and Brownian movement on the Rayleigh-Bénard convection of Al2O3/water nanoliquid in an enclosed box. Sivasankaran and Pan [12] numerically analyzed the influence of non-uniform warming of the sidewalls on free convection of nanoliquids in a container.

Convection in partly divided enclosed space has been received attention predominantly due to its usage in the strategy of energy efficient structures and flat plate solar collectors. Enclosures with baffles either on their active wall(s) or on adiabatic wall(s) have been studied in several studies. Jetli et al. [13] numerically deliberated free convection of a partitioned box with various baffle positions. Zimmerman and Acharya [14] inspected free convection in an enclosed space with conducting baffles. They established that the energy transmission rate drops on rising baffle conductivity. Frederick [15] numerically examined the buoyant convection in an enclosed box with a partition. Shi and Khodadadi [16] deliberated the influence of the thin baffle on convection in a square enclosed space. Tasnim and Collins [17] inspected the influence of the thin baffle on warm wall of an enclosed space. Sivasankaran and Kandaswamy [18-19] deliberated the influence of size and position of the partition in a rectangular enclosed space. Sivasankaran et al. [20] inspected the magneto-convection of an electrically conducting liquid in a partitioned enclosed space. Effect of several thermal conditions are actively used now in research [21-27].

The above said studies are concerned about convective flow in enclosed spaces with conventional fluids or nanofluid with different boundary conditions. No work has been explored on
convective flow of nanoliquids in partitioned enclosure for linearly varying thermal conditions. Hence, the current study goals to inspect the influence of convection characteristics in a square partitioned enclosed space with linearly heating utilizing different nanoliquids.

2. Mathematical modelling

A two-dimensional enclosed space filled with water-based nanoliquids containing various nanoparticles of length and height L as shown in the Figure 1 is undertaken. The adiabatic baffle of length L/2 is fixed on the bottom barrier of the enclosed space. The left barrier of enclosed space is heated linearly and the right barrier is cooled at a constant temperature. The horizontal walls are insulated. The nanoliquid is a liquid-solid mixture with constant volume fraction, \( \phi \), size and shape of nano-particles dispersed within base liquid of water. Several nanoparticles like, Al\(_2\)O\(_3\), Cu, Ag and TiO\(_2\), are used to examine the heat transfer performances in the study. The flow is presumed to be incompressible & laminar. The thermo-physical properties are presumed to be constant except density. The Boussinesq approximation is valid in the buoyancy term. It is presumed that both base liquid and nanoparticles are in thermally equilibrium. There is no chemical reaction between the base liquid and nanoparticles. Furthermore, the viscous dissipation is negligible. The mathematical model can be given as follows.

\[ u_x + v_y = 0 \]
\[ u_t + uu_x + vv_y = -\frac{1}{\rho_{nf}} p_x + \left( \frac{\mu_{nf}}{\rho_{nf}} \right) \nabla^2 u \]
\[ v_t + uv_x + vv_y = -\frac{1}{\rho_{nf}} p_y + \left( \frac{\mu_{nf}}{\rho_{nf}} \right) \nabla^2 v + \frac{g\beta_{nf}(\theta - \theta_e)}{\rho_{nf}} \]
\[ \theta_t + uu\theta_x + vv\theta_y = \alpha_{nf} \nabla^2 \theta \]

where \((u, v)\) are velocities, \(g, p, t, \rho, \beta, \mu, \alpha, \theta\) are gravitational acceleration, pressure, time, density, thermal expansion coefficient, dynamic viscosity, thermal diffusivity, respectively. The boundary & initial settings are:

For \(t = 0\): \[ u = v = 0, \theta = 0, \text{ at } 0 \leq (x, y) \leq L, \]
For \(t > 0\): \[ u = v = 0, \theta_y = 0, \text{ at } y = 0 \text{ & } L, \]
\[ u = v = 0, \theta = 0, \text{ at } x = 0, \]
\[ u = v = 0, \theta_x = 0, \text{ at } x = L, \]
\[ u = v = 0, \theta_x = 0 \text{ on partition, } 0 \leq y \leq L/2, x = L/2. \]

The active thermo-physical properties of the nanoliquid are estimated by several formulae existing as follows.

Density: \[ \rho_{nf} = \phi \rho_p + (1 - \phi) \rho_f \] (6)
Specific heat: \[ (\rho c_p)_{nf} = \phi (\rho c_p)_p + (1 - \phi) (\rho c_p)_f \] (7)
Thermal expansion coefficient: \[ (\rho \beta)_{nf} = \phi (\rho \beta)_p + (1 - \phi) (\rho \beta)_f \] (8)
The Brinkman's formula for the dynamic viscosity of nanoliquid is taken as
\[ \mu_{nf} = \mu_f \left(1 - \phi\right)^{2.5} \]  

(9)

The Maxwell formula for thermal conductivity is estimated as

\[ k_{nf} = k_f \left[ \frac{2 + k^*_{nf} + 2\phi (k^*_{nf} - 1)}{2 + k^*_{nf} - \phi (k^*_{nf} - 1)} \right] \text{ with } k^*_{nf} = \frac{k^*}{k_f} \]  

(10)

The thermo-physical values of the base liquid (water) and various nanoparticles are provided in the Table 1. The following dimensionless variables are utilized.

\[ (X, Y) = \left( \frac{u(x, y)}{L}, \frac{(U, V)}{\alpha_f} \right), \quad T = \frac{\theta - \theta_s}{(\theta_h - \theta_s)}, \quad F_0 = \frac{t \alpha_{nf}}{L^2}, \quad \text{and} \quad P = \frac{p L^2}{\rho_{nf} \alpha_f}. \]

Using this non-dimensional variables, the governing model is non-dimensionalized and they are:

\[ U_x + V_y = 0 \]  

(11)

\[ U_{x0} + UU_x + VU_y = -P_x + Pr_f \left( \frac{C_{p,nf} \mu_{nf}^{*}}{k_{nf}} \right) \nabla U \]  

(12)

\[ V_{x0} + UV_x + VV_y = -P_y + Pr_f \left( \frac{C_{p,nf} \mu_{nf}^{*}}{k_{nf}} \right) \nabla V + Ra_f Pr_f \beta_{nf} \left( \frac{C_{p,nf}^{*}}{k_{nf}} \right)^2 T \]  

(13)

\[ T_{x0} + UT_x + VT_y = \left[ \frac{\partial}{\partial X} \left( k_{nf}^* \frac{\partial T}{\partial X} \right) \right] + \frac{\partial}{\partial Y} \left[ k_{nf}^* \frac{\partial T}{\partial Y} \right] \]  

(14)

The initial and boundary settings in the dimensionless form are:

\[ F_0 = 0; \quad T = U = V = 0, \quad \text{at} \quad 0 \leq (X, Y) \leq 1, \]

\[ F_0 > 0; \quad T_Y = 0, \quad U = V = 0, \quad \text{at} \quad Y = 0 \& 1, \]

\[ T = Y, \quad U = V = 0, \quad \text{at} \quad X = 0, \]

\[ T = U = V = 0, \quad \text{at} \quad X = 1, \]

\[ T_X = 0, \quad U = V = 0, \quad \text{on} \quad \text{Partition,} \quad 0 \leq Y \leq 0.5, \quad X = 0.5. \]

The physical properties ratios existing in equations are as follows. \( k_{nf}^* = \frac{k_{nf}}{k_f} \), \( \mu_{nf}^* = \frac{\mu_{nf}}{\mu_f} \), \( C_{p,nf}^{*} = \frac{C_{p,nf}}{C_{p,f}} \), \( \rho_{nf}^* = \frac{\rho_{nf}}{\rho_f} \), and \( \beta_{nf}^* = \frac{\beta_{nf}}{\beta_f} \), where the subscripts \( nf \) and \( f \) denote the nanofluid and the base liquid, respectively. The dimensionless parameters are, \( Pr_f = \frac{V_f}{\alpha_f} \), Prandtl number, and \( Ra_f = \frac{g \beta_f (\theta_h - \theta_s) L^4}{\nu_f \alpha_f} \), Rayleigh number. The stream function is estimated by using \( U = \Psi_Y \) and \( V = -\Psi_X \). The heat energy transfer rate at warmer wall of the enclosed space is calculated by the Nusselt number, which is given as follows \( Nu_h = \frac{k_h L}{k_f} = -k_{nf}^* \frac{\partial T}{\partial Y} \bigg|_{x=0} \). The averaged Nusselt number alongside the warmer wall is achieved as follows \( \overline{Nu} = \frac{1}{0} Nu_h dY \). Also, it is essential to estimate the energy transfer efficacy of using the nanoliquid to that of the base liquid. The ratio of the averaged heat transfer coefficient at the warm wall to that of the base liquid, \( \omega_s \), is calculated as
\[ \varepsilon_n = \frac{\nu_c}{\nu_f}. \]

The dimensionless governing partial differential equations are solved by control volume method. The complete solution procedure is provided in Sivasankaran et al. [9].

3. Results and discussion

The calculations are done for the Rayleigh numbers stretching from \(10^3\) to \(10^6\), and the volume fraction of nanoparticles from 0% - 4%. The Prandtl number for base liquid (water) is used to be 6.7. The results obtained are deliberated under various groupings of relevant parameters in the work.

Figure 2 shows stream pattern and energy transfer characteristics of Ag-nanofluid for different Rayleigh numbers. The stream pattern is involves of single eddy occupying the entire enclosed space for low Rayleigh number. The fluid near the right-bottom corner is almost stagnant due to existing of the partition. The central region of the cell is elongated on raising the values of Rayleigh number. The dual cell structure exists at higher values of Ra (Ra=\(10^6\)). The secondary eddy in bottom-right corner exists due to the partition. The isotherms are distributed almost equally in the left side of the cavity and heat transfer is poor in the right side of the enclosed space, particularly in the right-bottom corner of the enclosed space for all Rayleigh number. Here convection diminishes and conduction is ruled mode of energy transfer at Ra=\(10^3\). The isotherms show the vertical temperature stratification inside the cavity for Ra\(\geq\)10^5. The isotherms are bunched alongside the upper part of the thermally active walls and forming the thermal boundary layers. It clearly indicates that convection mode of energy transport is dominated here.

![Figure 2. Streamlines (up) and isotherms (down) of Ag-nanofluid with \(\phi=4\%\) for different Rayleigh numbers.](image)

The influence of local energy transport rate among the different nanoliquid is exhibited in Figure 3 for Ra=\(10^3\) and Ra=\(10^6\). It is observed that the peak value of local Nusselt number is attained at Y=0.75 for Ra=\(10^6\), that is, \(3/4\) from the bottom of the warmer wall and at Y=1 for Ra=\(10^3\), that is, at the top of the warmer wall. To find the outcome of total energy transport rate through the enclosed space among various nanoliquids, the averaged Nusselt number is drawn against the nanoparticle volume fraction for various Ra in Figure 4. The favorable outcome of the particle fraction on the averaged heat transfer trend can be seen clearly on raising the particle fraction. A rise in averaged Nusselt number is found for all Ra. It is established that the maximum energy transport rate is witnessed for using Ag-nanoparticle comparing among other nanoparticles, like Cu, Al_2O_3 and TiO_2. When growing the volume fraction of nanoliquid, the variance in the energy transport rate among the several nanoliquids is raised. The variance among several nanoparticles plays a chief factor on the convective energy transport rate, which is evidently grasped from the Figure 4.
Figure 3. Local Nusselt number for various nanoliquids with $\phi=4\%$.

Figure 4. Average Nusselt number for various volume fractions and nanoliquids.

Figure 5 demonstrated the energy transfer efficacy of the nanoliquids for various volume fraction of the nanoparticle by plotting the energy transport coefficient ratio $\varepsilon$ against the Rayleigh number for various volume fraction of the nanoparticles with various nanoliquids. It is observed that energy transport coefficient ratio is above unity. The heat transfer ratio of oxide-nanoliquids ($\text{Al}_2\text{O}_3 \& \text{TiO}_2$) gets smallest when $\text{Ra}=10^4$. Though, the opposite tendency is witnessed while using metal nanoparticles ($\text{Ag} \& \text{Cu}$), that is, maximum energy transfer ratio is achieved at $\text{Ra}=10^6$. It is concluded from these figures that Cu and Ag-nanofluids are having similar behaviour while $\text{Al}_2\text{O}_3$ and $\text{TiO}_2$-nanofluids are so. The peak value of the energy transport ratio for the nanoliquids with the oxide particle is obtained for $\text{Ra}=10^5$ whereas highest value of the heat energy transport coefficient ratio for $\text{Ag}$- and $\text{Cu}$-nanofluids is obtained for $\text{Ra}=10^6$.

Table 1 illustrates that the energy transfer enhancement for various volume fractions ($\phi\%$) and various nanofluids. The averaged Nusselt number of nanoliquid with 1% volume concentration for $\text{Al}_2\text{O}_3$, $\text{Ag}$, $\text{Cu}$, and $\text{TiO}_2$-particles raises 2.7%, 3.2%, 3.1% and 2.3% for $\text{Ra}=10^3$, respectively, with that of base fluid. The energy transfer rate of nanoliquid with 4% volume concentration for $\text{Al}_2\text{O}_3$, $\text{Ag}$, $\text{Cu}$, and $\text{TiO}_2$-particles raises 6.1%, 12.8%, 14.8% and 5.3% for $\text{Ra}=10^3$, respectively, with that of base liquid. The averaged Nusselt number of nanofluid with 1% volume concentration for $\text{Al}_2\text{O}_3$, $\text{Ag}$, $\text{Cu}$, and $\text{TiO}_2$-particles raises 1.8%, 3.7%, 3.3% and 1.6% for $\text{Ra}=10^6$ respectively, with that of base liquid. The heat transfer rate of nanoliquid with 4% volume concentration for $\text{Al}_2\text{O}_3$, $\text{Ag}$, $\text{Cu}$, and $\text{TiO}_2$-particles rises 7.0%, 14.3%, 12.7%, & 6.1% for $\text{Ra}=10^6$, respectively, with that of base liquid. Therefore, it is concluded from the Table 1 that choosing of nanoparticle is very significant in the convection heat energy transfer applications.
4. Summary

The current work numerically inspects the heat energy transfer enhancement of nanoliquids for several nano-particles in a rectangular enclosed space with linearly heating. The simulation outcomes from the work have been offered in detail.

1. The convective heat transfer ability of base liquid increases when immersing the nano-particles in base liquid and the influence is more noticeable as the particle volume fraction raises. However, the enhancement in average heat energy transmission rate is strongly depending on the nanoparticle chosen.

2. The heat energy transport rate of TiO\textsubscript{2}-nanoliquid is minimum while it is high for Ag-nanofluid for Rayleigh numbers.

3. A major variance on the averaged Nusselt number is established for various nanoparticles. The results evidently illustrate that the type of nanoparticle used is significant on the convective energy transport applications.

4. The highest value of the energy transfer ratio for the nanoliquids with the oxide particle is found for Ra=10\textsuperscript{3} whereas highest value of the heat energy transport coefficient ratio for Ag- and Cu-nanoliquids is obtained for Ra=10\textsuperscript{4}.

5. The partition inhibits the convection flow and it results the energy transport rate reduction.

Figure 5. Heat transfer ratio for different nanofluids.
Table 1. Heat transfer enhancement for various values of $\phi$ (%) and nanofluids

| $Ra$ | Phi (%) | Al$_2$O$_3$ | TiO$_2$ | Cu | Ag |
|------|---------|-------------|---------|----|----|
| $10^4$ | 1 | 2.6482 | 2.2763 | 3.0777 | 3.1826 |
| 2 | 5.3460 | 4.5825 | 6.1830 | 6.3933 |
| 4 | 10.8928 | 9.2959 | 12.4994 | 12.8867 |
| $10^5$ | 1 | 1.5545 | 1.3637 | 3.3395 | 3.8903 |
| 2 | 3.0861 | 2.6945 | 6.5728 | 7.6374 |
| 4 | 6.0802 | 5.2628 | 12.7491 | 14.7581 |
| $10^6$ | 1 | 1.6820 | 1.4670 | 3.3068 | 3.8015 |
| 2 | 3.3275 | 2.9035 | 6.5209 | 7.4729 |
| 4 | 6.5204 | 5.6500 | 12.7106 | 14.5068 |
| $10^7$ | 1 | 1.7895 | 1.5602 | 3.2779 | 3.7269 |
| 2 | 3.5562 | 3.0910 | 6.4796 | 7.3465 |
| 4 | 7.0157 | 6.0651 | 12.6831 | 14.3134 |

A-nanofluid; B-pure fluid

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