1. Introduction

Corruption is a form of dishonesty or criminal activity undertaken by a person, group of people or organization entrusted with a position of authority often to acquire illicit benefits (Okwuagbala 2018). Nigeria, the most populous country in Africa, is highly ranked in corruption by Transparency International and other notable organizations that monitor corrupt practices around the world. High corruption cases had been linked to most Nigerians in foreign countries, so a lot of people have the perception that Nigerians are corrupt (Sirajo 2015). The emergence of a new government that promised to fight corruption to a standstill in the year 2015 increased the hope of many in the country that corruption will be minimized. In year 2015, out of the 168 countries surveyed, Nigeria was at the bottom of the table in the category of number 136. This implies that Nigeria was the 32nd most corrupt country in the world in 2015 according to the Transparency International (Transparency International 2015; Oyinlola 2011). The situation has made so many people feel a lot of pains as the money which would have been used to reduce poverty in the country are being channeled into the pockets of a small group of persons. It is hard to enter any sector in Nigeria without observing one corrupt practice or the other (Yusuf 2016).

Mathematical modeling is the process of using mathematical concepts like equations and graphs to represent real life situations. A model is an abstraction that reduces a problem to its essential characteristics. Models are designed to focus on certain aspects of the object of study; other aspects are abstracted away. Mathematical models are useful because they exemplify the mathematical core of a situation without extraneous information (Akinsola and Oluyo 2019).

In this paper, a mathematical model of the transmission dynamics of corruption among populace is analyzed. The corruption free equilibrium state, characteristic equation and Eigen values of the corruption model were obtained. The basic reproductive number of the corruption model was also determined using the next generation operator technique at the corruption free equilibrium points. The condition for the stability of the corruption free equilibrium state was determined. The local stability analysis of the mathematical model of corruption was done and the results were presented and discussed accordingly. Recommendations were made from the results on measures to reduce the rate of corrupt practices among the populace.
2. The Mathematical Model

2.1 Model Formulation

The total population of the populace (N) is sub-grouped into five compartments namely: Susceptible Class (S), Immune Class (I), Corrupt Class (C), Jailed Class (J) and the Reformed Class (R) according to the following definitions:

(a) Susceptible Class (S): This class consists of individuals who have never been involved in any corrupt practices that will have harmful effects on the country’s national growth and development but vulnerable to being infected with the corrupt practices in the society.

(b) Immune Class (I): This class consists of individuals who can never be involved in corrupt practices irrespective of the circumstances around them.

(c) Corrupt Class (C): This class consists of individuals who are often involved in corrupt practices and are capable of influencing the susceptible and immune individuals to become corrupt.

(d) Jailed Class (J): This is a class of individuals who have been convicted or punished of corrupt practices and imprisoned for a specific period of time during which he/she cannot be involved in any corrupt act and cannot influence others during the imprisonment.

(e) Reformed Class (R): This class consists of the ex-convicts who have been reformed while serving their jail term and can become susceptible to corruption.

The susceptible class is generated from daily recruitment of individuals born into homes with good moral standards and are vulnerable to being infected by the corrupt practices at a rate $\theta \beta$ while the immune class are those who has moral standards from their homes and can never become corrupt at a rate $(1 - \theta)\beta$ (Eguda, Oguntolu and Ashezua 2017) while the corrupt individuals are jailed at a rate $\delta$. Susceptible individuals acquire corruption infection/tendencies from corrupt individuals and become corrupt at a rate $\alpha$ thereby leaving the susceptible class for the corrupt class. The susceptible-immune individual leaves the susceptible class for the immune class and the reformed individual leaves the reformed class into the susceptible class. The corrupted-immune individual leaves the immune class for the corrupt class. A corrupt individual after right orientation through public enlightenment leaves the corrupt class for the reformed class. A prosecuted and imprisoned corrupt individual leaves the corrupt class for the jailed class and also a jailed individual become reformed while still serving his/her jail terms at the rate $\rho$. Corrupt and jailed individuals become reformed in the reformed class while serving their jail terms at rates $\tau$ and $\rho$ respectively. Individuals in the reformed class become susceptible after a while at a rate $\omega$ while susceptible individuals prone to corruption become immune at a rate $\upsilon$ due to moral and religious beliefs as well as public enlightenment campaign. All the classes are subjected to natural death at a rate $\mu$ (Binuyo 2019).

The mathematical model of the corruption is called the SICJR model which is depicted in the compartmental diagram as shown in figure 1 and is expressed as the system of nonlinear initial value problem given in the form:

\[
S'(t) = \theta \beta - \frac{\alpha S(t)C(t)}{N(t)} - (\mu + \upsilon)S(t) + \omega R(t)
\]

\[
I'(t) = (1 - \theta)\beta + \upsilon S(t) - (\mu + \gamma)I(t)
\]

\[
C'(t) = \frac{\alpha S(t)C(t)}{N(t)} + \gamma I(t) - (\mu + \tau + \delta)C(t)
\]

\[
J'(t) = \delta C(t) - (\mu - \rho)J(t)
\]

\[
R'(t) = \tau C(t) + \rho J(t) - (\mu + \omega)R(t)
\]

The fractional system of equations (1) to (5) is given thus:

\[
s'(t) = \theta \beta - \alpha s(t)c(t) - (\mu + \upsilon)s(t) + \omega r(t)
\]

\[
n'(t) = (1 - \theta)\beta + \upsilon s(t) - (\mu + \gamma)n(t)
\]

\[
c'(t) = \alpha s(t)c(t) + \gamma n(t) - (\mu + \tau + \delta)c(t)
\]

\[
j'(t) = \delta c(t) - (\mu - \rho)j(t)
\]

\[
r'(t) = \tau c(t) + \rho j(t) - (\mu + \omega)r(t)
\]

The schematic diagram of the corruption transmission dynamic model is shown in the figure 1 below (Binuyo 2019):

![Figure 1: Schematic diagram of the corruption transmission dynamic model.](image-url)
Table 1: Definitions of Parameters used in the Mathematical Model.

| Parameter | Description |
|-----------|-------------|
| $\theta$  | Proportion of individuals not borne immune |
| $\beta$   | Birth rate of individuals borne into the population |
| $\alpha$  | Effective corruption contact rate |
| $\gamma$  | Rate at which susceptible individuals become immune to corruption |
| $\omega$  | Rate at which reformed individuals become susceptible to corruption |
| $\upsilon$ | Rate at which immune individuals are susceptible to corruption |
| $\delta$  | Rate at which prosecution and imprisonment of corrupt individuals occur |
| $\tau$    | Rate at which corrupt individuals become reformed due to public enlightenment |
| $\rho$    | Rate at which jailed individuals become reformed |
| $\mu$     | Natural death rate |
| $S(t)$    | Susceptible individual class at time $t$. |
| $I(t)$    | Immune individual class at time $t$. |
| $C(t)$    | Corrupt individual class at time $t$ |
| $J(t)$    | Jailed individual class at time $t$ |
| $R(t)$    | Reformed individual class at time $t$ |
| $N(t)$    | Number of Individuals in the population at time $t$ |
| $s(t)$    | Fraction of Susceptible Individual at time $t$ |
| $i(t)$    | Fraction of the Immune Individual at time $t$ |
| $c(t)$    | Fraction of the Corrupt Individual at time $t$ |
| $j(t)$    | Fraction of the Jailed Individual at time $t$ |
| $r(t)$    | Fraction of the Reformed Individual at time $t$ |

3. Analysis of the Mathematical Model

3.1 The Equilibrium Points

At Equilibrium point $\frac{ds}{dt} = \frac{di}{dt} = \frac{dc}{dt} = \frac{dj}{dt} = \frac{dr}{dt} = 0$

From Equation (6) – Equation (10), we obtained the following as the equilibrium points

$$s(t)^* = \frac{\theta \beta + \omega r(t)^*}{\alpha c(t)^* + \mu + \upsilon}$$  \hspace{1cm} (11)

$$i(t)^* = \frac{(1 - \theta) \beta + v s(t)^*}{\mu + \gamma}$$  \hspace{1cm} (12)

3.2 Jacobian Matrix of the System

To investigate the possible equilibria of the system (6 - 10), we consider the Jacobian matrix $\frac{\partial (f_1, f_2, f_3, f_4, f_5)}{\partial (s, i, c, j, r)}$ and
impose the restriction on the equilibrium points. By partially differentiating the system (16 - 20) with respect to the state variables, we obtained the Jacobian matrix as follows:

\[ J(s, i, e, f, r) = \begin{pmatrix}
    \alpha s - (\mu + \nu) & 0 & -\alpha s & 0 & \omega \\
    v & -\mu & 0 & 0 & 0 \\
    \alpha c & \gamma & \alpha s - (\mu + \tau + \delta) & 0 & 0 \\
    0 & 0 & \delta & -(\mu - \rho) & 0 \\
    0 & 0 & \tau & \rho & -(\mu + \omega)
\end{pmatrix} \]

### 3.3 Corruption Free Equilibrium Points

At corruption free equilibrium, this implies:

\[ c^*(t) = j^*(t) = r^*(t) = 0 \]

Equation (11) and (12) becomes:

\[ s_0(t)^* = \frac{\theta \beta}{\mu + v} \]

\[ i_0(t)^* = \frac{(1 - \theta) \beta + v s_0(t)^*}{\mu + \gamma} \]

### 3.4 Basic Reproductive Number \( (R_0) \)

The basic reproduction number usually denoted \( R_0 \) is a threshold parameter defined as the average number of secondary infective produced when a single infected individual is introduced into a population consisting entirely of susceptible (Heesterbeck 2002). If \( R_0 > 1 \), the infected individual is infecting more than one further person, so the number of infective will exponentially increase and an epidemic will occur. However, if \( R_0 < 1 \), the infective is not passing the infection on to enough people to replace itself so the incident dies out and seize to persist in the community. There may be some secondary cases, but these will decrease with time and eventually the infection will become extinct. If \( R_0 \approx 1 \), the infection just barely succeeds in reproducing itself and there will be a similar number of cases at any later time (Binuyo 2015). The larger the magnitude of \( R_0 \), the faster the spread of corruption and the more difficult its control will be. In this paper, the basic reproductive number for the corruption model of equations (6) to (10) is obtained using the next generation matrix operator. This method is proposed by (Heesterbeck 2002) and it is given as:

\[ R_0 = \rho \left( FV^{-1} \right) \]

Where \( R_0 \) denotes the basic reproductive number, \( F \) represents the matrix of partial derivatives of the rates secondary infections are produced and \( V \) represents the matrix of the expected time an individual initially introduced into the disease compartment (Driessche and Watmough 2002). Thus:

\[ F = \begin{pmatrix}
    \alpha s^* & 0 \\
    0 & 0
\end{pmatrix} \]
\[ V = \begin{pmatrix} \mu + \tau + \delta & 0 \\ -\delta & 0 \end{pmatrix} \]  
(29)

\[ |V| = (\mu + \tau + \delta)(\mu + \rho) \]  
(30)

\[ V^{-1} = \begin{pmatrix} \frac{\mu + \rho}{(\mu + \tau + \delta)(\mu + \rho)} & 0 \\ \frac{-\delta}{(\mu + \tau + \delta)(\mu + \rho)} & 1 \end{pmatrix} \]  
(31)

\[ FV^{-1} = \begin{pmatrix} \alpha s^* & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{\delta}{(\mu + \tau + \delta)(\mu + \rho)} & 1 \end{pmatrix} \]  
(32)

The basic reproductive number is given as the spectral radius of \( FV^{-1} \) i.e.

\[ R_0 = \rho(FV^{-1}) = \frac{\alpha s^*}{\mu + \tau + \delta} \]  
(33)

### 3.5 Stability Analysis of Corruption Free Equilibrium Points

Let

\[ k_1 = (\mu + \nu) \]  
(34)

\[ k_2 = \mu + \gamma \]  
(35)

\[ k_3 = (\mu + \tau + \delta) - \alpha s^* \]  
(36)

\[ k_4 = \mu - \rho \]  
(37)

\[ k_5 = \mu + \omega \]  
(38)

Equation (21) becomes:

\[ J(H_0) = \begin{pmatrix} -k_1 & 0 & -\alpha s^* & 0 & \omega \\ \nu & -k_2 & 0 & 0 & 0 \\ 0 & \gamma & -k_3 & 0 & 0 \\ 0 & 0 & \delta & -k_4 & 0 \\ 0 & 0 & \tau & \rho & -k_5 \end{pmatrix} \]  
(39)

Reducing Equation (39) into an echelon form, the following steps were taken:

(i) New row 2 (NR2) is formed where row 1 (R1) of (39) is the pivot row:

\[ NR_2 = R_1 + \frac{\nu}{k_1} R_2 \]

(ii) New row 3 (NR3) is formed where new row 2 (NR2) is the pivot row:

\[ NR_3 = R_2 + \frac{\gamma}{k_2} R_3 \]

(iii) New row 4 (NR4) is formed where new row 3 (NR3) is the pivot row:

\[ NR_4 = R_3 + \frac{\delta M_1}{k_3} R_4 \]

Where

\[ M_1 = \left( k_3 + \frac{\gamma \alpha s^* \nu}{k_1 k_2} \right) \]  
(40)

(iv) New row 5 (NR5) is formed where new row 3 (NR3) is the pivot row:

\[ NR_5 = R_3 + \frac{\tau M_1}{k_5} R_5 \]

The matrix from the above steps gives:

\[ J(H_0) = \begin{pmatrix} -k_1 & 0 & -\alpha s^* & 0 & \omega \\ 0 & -k_2 & -\frac{\alpha s^* \nu}{k_1} & 0 & \frac{\nu \omega}{k_1} \\ 0 & 0 & -M_3 & 0 & \frac{\gamma \nu \omega}{k_1 k_2} \\ 0 & 0 & 0 & -k_4 & \frac{\delta \gamma \nu \omega}{k_1 k_2 M_3} \\ 0 & 0 & 0 & \rho & -k_5 \end{pmatrix} \]  
(41)

The reduced echelon matrix is
The new row 5 of (42) is obtained where row 4 of (42) is the pivot row thus:

\[ NR_k = \frac{\rho \delta \gamma \omega \upsilon}{k \epsilon k_2 k_4} - \frac{\tau \gamma \omega \upsilon}{k \epsilon k_2 M_3} \]  

(43)

### 3.6 Characteristic Equation and Eigenvalues

The eigenvalues of the row-transformed Jacobian matrix (39) are obtained as follows;

\[-k_1 \lambda_1 \left(-k_2 - \lambda_2\right) (-M_3 - \lambda_3) (-k_4 - \lambda_4) (-M_5 - \lambda_5) = 0 \]  

(44)

Hence,  \[ \lambda_1 = -k_1 = - (\mu + \upsilon) < 0 \]  

(45)

\[ \lambda_2 = -k_2 = - (\mu + \gamma) < 0 \]  

(46)

\[ \lambda_3 = -k_4 = - (\mu + \rho) < 0 \]  

(47)

\[ \lambda_4 = -M_3 = - \left( k_3 + \frac{\gamma \alpha s^* \upsilon}{k \epsilon k_2} \right) < 0 \]  

(48)

\[ \lambda_5 = -M_5 = - \left( k_3 + \frac{\gamma \alpha s^* \upsilon}{k \epsilon k_2} \right) < 0 \]  

(49)

From (48) and (49)  

\[ M_3 = k_3 + \frac{\gamma \alpha s^* \upsilon}{k_2} > 0 \]  

(50)

\[ M_5 = k_3 - \frac{\rho \delta \gamma \omega \upsilon}{k_2 k_4 k_5 M} - \frac{\tau \gamma \omega \upsilon}{k_2 M_3} > 0 \]  

(51)

From (49)

\[ \mu + \tau + \delta = k_1 + \alpha s^* \]  

(52)

For Corruption free equilibrium, the Reproduction number  \[ R_0 < 1. \]  

Hence

\[ R_0 = \frac{\alpha s^*}{\mu + \tau + \delta} < 1 \]  

(53)

\[ \therefore R_0 = \frac{\alpha s^*}{k_3 + \alpha s^*} < 1 \]  

(54)

This implies that

\[ \alpha s^* + k_3 > \alpha s^* \]  

(55)

Hence  \[ k_3 > 0 \]  

(56)

Similarly,

\[ k_3 = (\mu + \tau + \delta) - \alpha s^* > 0 \]  

(57)

\[ \mu + \tau + \delta > \alpha s^* \]  

(58)

\[ \frac{\alpha s^*}{\mu + \tau + \delta} < 1 \]  

(59)

\[ R_0 < 1 \]  

(60)

Furthermore,

\[ \lambda_4 = -M_3 = - \left( k_3 + \frac{\gamma \alpha s^* \upsilon}{k \epsilon k_2} \right) < 0 \]  

(61)

\[ \lambda_4 = - \left( \frac{k \epsilon k_4 k_5 + \alpha \gamma \alpha s^* \upsilon}{k \epsilon k_2} \right) < 0 \]  

(62)

\[ -k_1 k_2 k_3 - \alpha \gamma \upsilon < 0 \]  

(63)

\[ k_1 k_2 k_3, \alpha \gamma \upsilon > 0 \]  

(64)

\[ k_1 k_2 (\mu + \tau + \delta) - \gamma \alpha s^* \upsilon < 0 \]  

(65)

\[ k_1 k_2 (\alpha s^* - (\mu + \tau + \delta)) - \gamma \alpha s^* \upsilon < 0 \]  

(66)
\[ \alpha s^* - (\mu + \tau + \delta) < \frac{\gamma \alpha s^* v}{k_1 k_2} \]  \hspace{1cm} (67)

\[ \alpha s^* - \frac{\gamma \alpha s^* v}{k_1 k_2} < \mu + \tau + \delta \]  \hspace{1cm} (68)

\[ \alpha s^* \left(1 - \frac{\gamma v}{k_1 k_2}\right) < \mu + \tau + \delta \]  \hspace{1cm} (69)

\[ \frac{\alpha s^*}{\mu + \tau + \delta} \left(1 - \frac{\gamma v}{k_1 k_2}\right) < 1 \]  \hspace{1cm} (70)

Such that,

\[ \frac{\alpha s^*}{\mu + \tau + \delta} < \frac{1}{1 - \frac{\gamma v}{k_1 k_2}} < 1 \]  \hspace{1cm} (71)

Hence, the basic reproductive number \( R_0 \) is

\[ R_0 = \frac{\alpha s^*}{\mu + \tau + \delta} < 1 \]  \hspace{1cm} (72)

Then,

\[ R_0 = \frac{\alpha \theta \beta}{(\mu + v)(\mu + \tau + \delta)} = \frac{\alpha \theta \beta}{k_1 (\mu + \tau + \delta)} \]  \hspace{1cm} (73)

as obtained in (33).

Therefore, the corruption free equilibrium is locally asymptotically stable.

4. Discussions/Recommendations

From the eigenvalues obtained from the stability analysis of the corruption free equilibrium having the negative values, then the corruption free equilibrium is locally asymptotically stable for which \( R_0 < 1 \). The basic reproductive number \( (R_0) \) of the corruption model is evaluated from the averaging of the number of secondary cases produced when one case of corrupt practice is introduced into a susceptible corrupt population. It can be deduced that the effective corruption contact rate \( (\alpha) \) must be very small for corruption to be eradicated in Nigeria. Contact rate of corrupt individual with the susceptible population must be put to check. Rate at which corrupt individuals become reformed due to public enlightenment \( (\tau) \), and rate at which prosecution and imprisonment of corrupt individuals occur \( (\delta) \) must be increased tremendously for anti-corruption war to be won.

The following recommendations are made to reduce the rate of contacting or spreading corrupt practices in our society:

1. Strengthening of investigative system to ensure timely discovery and apprehension of corrupt individuals in the society. Training and re-training of security and law enforcement agents like State security service and Police personnel. Agencies like the Economic and Financial Crime Commission (EFCC), Code of Conduct Bureau (CCB) and Independent Corrupt Practices Commission (ICPC) should be fortified and supported in carrying out their mandate without fear and undue favouritism.

2. Speedy justice: judicial reformation and digitalization of court processes or procedures to aid timely delivery of pronouncements and sanctions on corrupt individuals. This will serve as deterrent to others.

3. There must be no selective and partial justice. A single corrupt individual is potent enough to make perpetuate the act. All corrupt individuals must be brought to book irrespective of their affiliations politically, ethnically, economically, religiously or otherwise.

4. Efficient correctional system should be put in place. The changing of the name of Nigerian Prison Service is a welcome development. It is now more of a correctional center by name and this should reflect in her services and operations. Measures to regenerate and re-orientate the inmates into more responsible citizens should be implemented.

5. Public enlightenment on reducing the rate of spreading corrupt practices should be strengthened. Organizations like National Orientation Agency (NOA), News Agency of Nigeria (NAN) and other Mass Media outfits (Print & Voice) should not relent in their responsibilities to discourage the spread of corrupt practices in the society.

6. Constant employment of guidance and counsellors in our primary, secondary and tertiary institutions must be done periodically.

7. All hands must be on deck to nip the corruption menace in the mud, such that the rate of spreading corrupt practices will be reduced drastically in our Nation.

Conclusion

In this study, the stability analysis of the corruption mathematical model had been analyzed using the linearization technique via the Jacobian matrix approach. The corruption free equilibrium point were obtained and analyzed. The basic reproductive number of the model was obtained to be the alternative way for identifying how the spread or the outbreak of the corrupt practices among the populace can be greatly reduced. It was observed that the mathematical model produced an asymptotically stable population such that corrupt practices among the populace die out from the populace as time increases when adequate
measures mentioned in the recommendations are done by the right people and at the right time. Furthermore, in the nearest future the corruption endemic equilibrium points of the model shall be investigated and its stability analyzed critically. The numerical solution of the model shall also be determined.

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