Mechanics of alpine skiing: carve turn with angulation

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Abstract. The article deals with the problem of modeling the movement of an athlete-skier when performing a cut ski turn with angulation. A system of integral equations for dimensionless motion parameters is formulated. Its numerical solution, taking into account individual styles of angulation control, allows us to construct the trajectory of the center of mass of the system, estimate its speed, angles that characterize the position of the skier's body and its skis relative to the slope, and the time of its movement. Examples of calculating the parameters of a carve turn with different style angulations are considered.

1. Introduction
In the works devoted to the consideration of models of movement of a skier in a carve turn without side slip \([1-2]\), it is assumed that the support reaction acting from the slope to the skis is directed along the normal to the sliding surface of the skis. This assumption means that the value of the edging angle of the skis always corresponds to the angle between the supporting reaction and the normal to the slope, which excludes the possibility of controlling the skier's own movement when turning. In reality, the skier controls the movement in a carve turn, using the angulations as a way to create additional angle of the skis when he adopt a specific, "angulations" position \([3]\). In this position, the skier's body forms an obtuse angle in the transverse plane to the direction of movement, with its vertex in the area of the skier's pelvis, or his knees, facing the center of the turn. Due to angulations, the edging angle of changes and, consequently, the radius of the turn, which allows the skier to effectively control the movement in the turn, changing the trajectory of the center of mass. This work is devoted to the creation of a numerical model of the movement of a skier in a carve turn, taking into account the angulations controlled by the skier, which is an urgent task when modeling the passage of various sports tracks and describing the biomechanics of the ski turn as such.

2. The model of the system skier–skis
When describing the movement of the mechanical system "skier-skis" on the slope, we will consider the movement of the center of mass of the mechanical system "skier-skis" (point C) in an inertial system with the x, y, and z axes connected to the slope (Fig. 1), and the relative motion of the system itself in a reference frame with parallel axes passing through the center of mass C. Since the mobile reference frame is not generally inertial, it is necessary to enter the inertia force \(\Phi = -ma\) to describe the dynamics of relative motion. Given that the distances to the center of curvature of the trajectory...
from the center of mass (point C) and the supporting point (point O) differ by no more than 3%, we assume that the speeds of these points are almost equal, i.e. $V_c = V_o = V$ (Fig. 1). As shown in [4], in order to avoid loss of stability during the arcing phase of turning, the skier must be in relative balance. This means that the system of applied forces, including the force of inertia, must be equivalent to zero. Because the force of gravity and the inertial force applied to the center of mass (we will neglect the small sliding friction force along the skis) the line of action of support reaction $R_O$ should pass through the center of mass C (the forces of resistance and inertia are not shown in Fig. 1)

3. The movement of the center of mass

\[ \text{Figure 1. Diagram of the arc AB of the carve turn} \]

C – center of mass of the skier-ski system;
C’ – projection of the center of mass of the system on the slope;
G – gravity;
O – the point of application of the resultant supporting reactions $R_O$ (the middle of the conditional ski);
OC – supporting line;
OC’ – projection of the supporting line to the slope;
OD – direction of the normal n axis;
OH and the y-axis is the direction of the slope lines;
CC’ and the z-axis are perpendicular to the slope;
$V$ – the velocity vector is directed tangentially to the trajectory along the axis $\tau$;
$\alpha$ – the angle of the slope;
$\zeta$ – the angle of precipitation due to the presence of friction force as a component of the supporting reaction [5];
$\delta$ – the slope of the reference line, or the angle between the reference line and the normal to the slope;
$\theta$ – edging angle, or the angle between the slope plane and the sliding surface of the ski;
$\beta$ – the angle of movement, or the angle between the speed vector and the slope line.
In [2], the authors obtained expressions for tangent ACN normal acn components of the acceleration of the center of mass under conditions of relative equilibrium and ignoring changes in the normal component of the slope reaction:

\[
a_{x} = \frac{dV}{dt} = g(\sin \alpha \cos \beta - \cos \alpha \tan \delta \sin \zeta) - \frac{F_{r}}{m},
\]

(1)

\[
a_{y} = \frac{V^2}{\rho} = g(\tan \delta \cos \alpha \cos \zeta - \sin \alpha \sin \beta),
\]

(2)

where \( \rho \) is the radius of curvature of the trajectory of the point C, \( F_{r} \) is the force of resistance to movement, including the force of sliding friction and the force of aerodynamic drag. As shown there, the radius of curvature is determined by the radius of the side cutout of the ski \( R \) and the edging angle \( \theta \):

\[
\rho = R \cos \theta.
\]

(3)

The value of the edging angle \( \theta \) is equal to the sum of the slope angle of the supporting line \( \delta \) and the angle of additional edging of skis due to the angulation of the skier (angle of angulation \( \phi \)):

\[
\theta = \delta + \phi.
\]

(4)

The length of the arc element \( ds \) of the trajectory traversed during the time period \( dt \), is determined by the radius of its curvature \( \rho \) and the change in the angle of motion \( d\beta \):

\[
ds = V dt = \rho dB.
\]

(5)

We express elementary movements along the \( dx \) trajectory and the \( dy \) and \( dz \) coordinate axes using (3) and (5):

\[
dx = R \cos \beta dB,
\]

\[
dx = ds \sin \beta = R \cos \theta \sin \beta dB,
\]

\[
dy = ds \cos \beta = R \cos \theta \cos \beta dB.
\]

(6)

We find for small sliding friction forces (\( \cos \zeta \approx 1, \sin \zeta \approx 0 \)) from (1) taking into account (5) and (3) the speed increment:

\[
dV = a_{x} dt = \left[ g \sin \alpha \cos \beta - \frac{F_{r}}{m} \right] \frac{R \cos \theta}{V} dB,
\]

where from:

\[
d(V^2) = 2 \left( \sin \alpha \cos \beta - \frac{F_{r}}{mg} \right) R g \cos \theta dB.
\]

(7)

To get the most general solutions to the task, we introduce dimensionless parameters:

\[
i = \frac{V_{0} t}{R}; \quad \dot{V} = \frac{V}{\sqrt{R g}}; \quad \hat{\rho} = \frac{\rho}{R} = \cos \theta; \quad \hat{\kappa} = \frac{1}{\hat{\rho}} = R \kappa; \quad \hat{s} = \frac{s}{R}; \quad \hat{x} = \frac{x}{R}; \quad \hat{y} = \frac{y}{R}.
\]

(8)

Here \( i, \dot{V}, \hat{\rho}, \hat{\kappa}, \hat{s}, \hat{x}, \hat{y} \) – are dimensionless time, speed, radius of curvature and curvature, path and coordinates of the center of mass \( C \), \( V_{0} \) is the initial speed of movement. Then the previous equations (5-7) are converted to the form that allows us to make integral expressions for dimensionless parameters of the center of mass movement:
4. Determining the angulation angle

In all the obtained integral relations (10–14) there is a dimensionless radius of curvature $\tilde{\rho}$ directly related to the edging angle $\theta$ according to the formula (9). Since the edging angle, in turn, is determined by the formula (4), we find an expression that allows us to associate the angles of the reference line's slope $\delta$ and angulation $\phi$ with the rest of the kinematic parameters of motion. To do this, we transform expression (2) with the following (4), (8), (9) and a small value of the sliding friction force to the species:

$$\tilde{V}^2 = 2\left(\sin\alpha \cos\beta - \frac{F_{\text{coupl}}}{mg}\right)\tilde{\rho}d\beta;$$

If you know the relationship between the angle of angulation and the angle of inclination of the supporting line as a function $\phi = \phi(\delta)$, solving this equation together with (15), you can find the angles $\delta$ and $\phi$. The numerical solution for the angle $\delta$ with the relation $\phi(\delta)$ depends on the values of the current parameters $\tilde{V}$, $\beta$ and $\alpha$, which allows us to find the edging angle $\theta$.

5. Style angulation function $\phi = \phi(\delta)$

In [5], the dependencies of the maximum possible angulation angle were evaluated on a specially created anthropometric stand. Experimental dependences of the maximum possible angle of angulation $\psi$ on the angle of inclination of the supporting line $\delta$ due to geometric and anatomical limitations are obtained. These dependencies are of a particular nature, since they were determined only in one subject, but they revealed a general type of change in the function $\psi(\delta)$ due to anatomical limitations. In accordance with it (curve 1, Fig. 2) when increases the angle of inclination of the supporting's line the limit angles of angulation decrease from the maximum value of $\psi_0$ almost according to the linear law up to the value of $\psi_1$ at a certain intermediate angle $\delta_1$. Then the rate of linear reduction of the limit angle of angulation $\psi$ increases sharply, and the angle itself becomes zero at $\delta_2$. The graphs show the values of the coordinate axes in degrees. According to experimental data from [6], the values of $\delta_1$, $\delta_2$, $\psi_0$ and $\psi_1$ are $50^\circ$, $65^\circ$, $28^\circ$ и $15^\circ$, respectively.
The dependence of the angle of angulation on the angle of inclination of the supporting line \( \varphi = \varphi(\delta) \) is not determined by purely mechanical conditions of movement. This function has an individual, stylistic character, it is chosen by the skier based on the specifics of a particular turn, the state of the snow, the skier's qualifications and skills, his physical condition, and so on. We introduce the degree of angulation \( \eta \) as a parameter that characterizes the ratio of the angulation angle to its limit value \( \psi(\delta) \). Then the angulation function takes the form:

\[
\varphi = \eta \cdot \psi(\delta)
\]

Figure 2 shows 3 variants of style inhalation functions. Curve 1 (brown color) corresponds to the degree of angulation equal to one, at which the angulation angle is equal to its limit value for any angles of inclination of the reference line \( \delta \). Therefore, curve 1 exactly coincides with the curve for the function \( \psi(\delta) \). Curve 2 (blue, "proportional" angulation) corresponds to the value of \( \eta = 0.35 \) for any angle of inclination \( \delta \). Curve 3 (red, let's call it: "progressive" angulation) reflects a more complex relationship implemented by skier, including in order to maintain a normal slope position of the upper part of their body in the frontal plane. Here, for the zero angle of inclination, the degree of angulation \( \eta_0 = 0.35 \), then increases linearly up to the value \( \eta_1 = 0.9 \) for the angle \( \delta_1 \), then the degree of angulation remains unchanged.

6. Examples of calculating the parameters of the cut turn for various angulation functions.

As can be seen from equation (14), for numerical integration of the system of equations (9−14), it is necessary to describe the forces of resistance to movement, the correct definition of which is an independent research task [7], [8]. To illustrate the chosen approach of numerical estimation of a carve turn with angulation, we consider two simple cases that do not require a description of the values of the resistance forces. The first of them is movement at a constant speed, the second is the absence of resistance to movement. To determine the driving conditions for numerical estimation, we will set the following parameters of movement in a carve turn: the slope angle \( \alpha = 10^\circ \), the initial and final angles of movement \( \beta_0 = -60^\circ \) and \( \beta_e = 60^\circ \), the initial value of the dimensionless speed \( \tilde{V}_0 = 0.5 \).

The figures below show graphs of the dependence of the angles \( \delta, \varphi, \theta \) on the angle of motion \( \beta \) and dimensionless time \( \tilde{t} \), as well as graphs of dimensionless parameters \( \tilde{\rho}, \tilde{V}, \tilde{\kappa}, \tilde{s}, \tilde{x}, \tilde{y} \), defined by formulas (9-14) using the numerical integration method. Matching the angulation angle \( \varphi \) nd the slope angle of the supporting line \( \delta \) and counting the edging angle \( \theta \) (4) were performed at each integration.
step by solving a system of equations (15–16) or various style angulation functions discussed in the previous section, including the option of no angulation ($\eta = 0$). The following symbols are used for the type of inhalation functions for the specified graph curve:

1 – no angulation $\eta = 0$, constant dimensionless speed $\tilde{V} = 0.5$, curves color-brown;
2 – angulation proportional to the slope of the reference line, $\eta = 0.35$, $\tilde{V} = 0.5$, color-blue;
3 – progressive angulation, discussed in the previous section, $\eta_0 = 0.35$, $\eta_1 = 0.9$, $\tilde{V} = 0.5$, color-red;
4 – maximum possible, maximum angulation, $\eta = 1$, $\tilde{V} = 0.5$, color-golden;
5 – progressive angulation, variable dimensionless speed in the absence of resistance to movement, the initial value of the dimensionless speed $\tilde{V}_0 = 0.5$, color-black.

![Figure 3](image3.png)

![Figure 4](image4.png)
Figure 5

Figure 6

Figure 7

Figure 8
Figure 20

Figure 21

Figure 22. The trajectory of the center of mass
It should be noted that the characteristic bends and breaks in the curves in Fig. 3-12 are associated with a change in the nature of angulation from progressive to proportional in accordance with the dependence 3 (Fig. 2).

7. Conclusion

We consider the movement of a skier in a cut turn on a ski slope with angulation as a way to increase the angle of the skis. A system of integral equations of motion is formulated that allows determining the main parameters of the movement of the mechanical system "skier-ski", including changes in the speed of the center of mass, its trajectory and radius of curvature of the trajectory, angles of inclination of the reference line, angulation, edging, time of movement in a carve turn. The solution of the problem is illustrated by examples of motion with a constant speed, as well as movement in conditions of neglect of the sliding friction force and aerodynamic drag. The proposed analytical model allows us to perform quantitative estimates of the parameters of the carve turn for various styles of angulation control and can be used in the work of ski training coaches, as well as in the development of models of skis and sports simulators.

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