Influence of the radiation pattern errors for the correlation interferometer

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Abstract. While designing phase-correlation direction finders, as antenna components, the non-directional ones are usually applied. Mounting the non-directional antenna components provides certain new options but makes the bearing computation procedure more problematic. In the present paper, the feasibility of using the directional antenna components is discussed and confirmed in terms of its positive influence on the stability of the bearing results in the case when the deviations of the implementations of the actual diagrams of the antenna components from their reference model occur.

1. Introduction

In the paper [1], the characteristics of the correlation-based interferometric direction finder are studied focusing on the use of the non-directional antenna components, though a number of producers of the direction finding equipment [2], [3] use directional antenna components in their correlation-based interferometric direction finders. In the paper [4], additional options are analyzed arising due to the replacement of the non-directional antenna components by the directional ones, but a more complex bearing computation procedure may lead to the estimation errors as the directional pattern involved in the decision functional of the bearing estimation can be inexact. This inexactness of the directional pattern may result from either a dependence of this characteristic on frequency which is difficult to estimate, or the neglected particularities of the specific direction finding equipment. Some sort of deformation suffered by such equipment during its operation is another possible cause of the directional pattern errors. As it is rather difficult to take into account all these factors at once, the task of the directional pattern error influence estimation is still an urgent one and it will be addressed in this study.

To estimate the influence of the deviation of the real directional pattern from its ideal mathematical model, such indicators are used as the root-mean-square error of the bearing estimation and the percentage of the abnormal errors that depends on the random error caused by the various
inexactnesses in the directional pattern. The object of study is a circular non-directional antenna array with a central non-directional reference component and 9 directional antenna components that are equally arranged in a circle.

2. The model representations of the measurements

Not to violate generality, one starts with stating that the signal under bearing generated by the radio source is the harmonic one. In this case, it can be presented as

\[ y(t) = A_0 \exp(j2\pi v_c t/\lambda). \] (1)

where \( A_0 \) and \( \lambda \) are the amplitude and the wave length of the emitted signal; \( v_c \) is the speed of light; \( t \) is the time; \( j \) is the imaginary unit. At the input of the \( k \)-th antenna component this signal takes the form

\[ z_k(t) = AD_k(n) \exp[j2\pi v_c(t - \Delta t_k)/\lambda] = AD_k(n) \exp[j2\pi v_c(t - L_k/v_c)/\lambda]. \] (2)

In (2), \( A \) is the signal amplitude at the receiving point; \( \Delta t_k \) is the delay corresponding to the distance \( L_k \) between the radio source and the \( k \)-th antenna component, and it can be presented as \( \Delta t_k = L_k/v_c \); \( D_k(n) \) is the directivity of the antenna component (standing for the correction of the amplitude and phase constituents of the received signal depending on the direction of the signal arrival that is expressed by the ort \( n \)). The ort \( n \) used in the formula (2) as the direction to the radio source is in the following relationship with the position angle \( U \) and the azimuth \( \theta \):

\[ n = (\cos U \sin \theta, \cos U \cos \theta, \sin U)^T, \] (3)

where symbol “\(^T\)” stands for the transposition operation.

The bearing estimation carried by the correlation-based interferometric direction finder with the commuted antenna components is implemented by means of the joint measurements at the antenna pairs. The calculated value \( F_{ms}(n, A, \Delta \psi) \) for the pair of the antenna components whose numbers are \( m, s \) is formed using the signals received by the two antennas and it can be presented as

\[ F_{ms}(n, A, \Delta \psi) = z_m(t)z_s(t) = PD_m(n)D_s(n)^H \exp(j\Delta \psi) \exp(j\Delta \Phi_{ms}(n)), \] (4)

where the symbol “\(^H\)” stands for the complex conjugation; \( P = A^2 \); \( \Delta \psi \) is the phase incursion due to the mismatch of the receiver channels; \( \Delta \Phi_{ms}(n) = 2\pi b^T n/\lambda \) is the difference of the phase incursions between the two antenna components; \( b \) is the vector of the coordinate difference between the reference and the switched antenna components in the Cartesian coordinate system. This vector can be written as

\[ b = (x_s - x_m, y_s - y_m, y_s - y_m)^T. \] (5)

As the one of the parts of the registered measurements is the unknown disturbance caused by the error, the actually processed is not the measured signal product (4) but its disturbed values

\[ z_{ms} = F_{ms}(n, A, \Delta \psi) + \xi_{ms}, \] (6)

where \( \xi_{ms} \) is the measurement error. Hereinafter, the antennas with the fixed non-directional reference component are to be considered and thus the index “\( s \)” will be omitted in the calculations.

On the assumption that a random error usually produced by the receiver during the measurements has a normal distribution, the optimal procedure for determining the angles \( U \) and \( \theta \), in accordance
with the measurements (6), is the least squares method (LSM) [5]. For (6), the decision functional which is to be minimized can be written in the following way:

$$J(n, A, \Delta \psi) = \sum_m \left| z_m - F_m(n, A, \Delta \psi) \right|^2 .$$

(7)

As the expression $P \exp(j \Delta \psi)$ is linearly attached to the decision functional, according to the classic LSM-based solution, it can be calculated explicitly. It allows reducing the enumeration range to the two variables, on which the vector $n$ is dependent, and thus the decision functional can be transformed into

$$J(n) = \sum_{k=1}^M z_k^H z_k - \sum_{k=1}^M z_k^H D_k(n) \exp[j \Delta \Phi_k(n)] \left[ \sum_{m=1}^M D_m(n) D_m(n)^H \right].$$

(8)

As the first summand in the generated functional does not depend on $n$, in the expression (8), minimization problem can be replaced by the equivalent problem of determining the maximum of the functional

$$J(n) = \sum_{k=1}^M D_k(n)^H \exp[-j \Delta \Phi_k(n)] z_k \left[ \sum_{m=1}^M D_m(n) D_m(n)^H \right].$$

(9)

3. The model of the directional pattern of the antenna components

As the directional antenna components, one can use the different antenna designs. The aim of the present study is not to test any specific design variants but to estimate the influence of the inexact practical implementation of the directional pattern, in our calculations the directional patterns close by shape to the cardioid are to be used. The cardioid is a good approximation of the directional pattern of the electric vibrator with the resistive loads in the gaps between the arms in the circular antenna array and with the reflector mounted in the center of the antenna component [6], [7]. To simulate the cardioid, the following formula is introduced

$$D(n)_{\text{card}} = 1 + \cos \gamma ,$$

where $\gamma$ is the angle between the direction onto the signal source, determined using the expression (3), and the direction of the antenna component axis, whose ort is predefined by the formula

$$n_a = (\cos \beta \sin \alpha, \cos \beta \cos \alpha, \cos \beta)^T .$$

(11)

In (11), $\alpha$ is the angle between the direction of the antenna component axis projection on the horizontal plane and the northern orientation of the direction finder, while $\beta$ is the angle between the direction of the antenna component axis and the horizontal plane. It is obvious that the cosine of $\gamma$ is determined this way

$$\cos \gamma = n_a^T n .$$

(12)

As the modifications of the directional patterns, one uses the circular directional pattern and the half-sum of the cardioid and the circular directional pattern that are defined as follows:

$$D(n)_{\text{circle}} = 1, \quad D(n)_{\text{half\_sum}} = \left[D(n)_{\text{card}} + D(n)_{\text{circle}} \right]/2 .$$

(13)

In figure 1, one can see the cross-sections of the different types of the directional patterns by the plane cutting through the symmetry axis general for them all.
4. The model representations of the measurements

To estimate the influence of the inexact directional pattern on the results of the bearing, one set of the directional patterns is used for the simulation of the measurements and another – for the actual bearing. As the estimates of the bearing exactness, the RMSD (root-mean-square deviation) of the direction finding $\sigma_0$ is used (in the absence of the abnormal errors caused by the jump from the incorrect desicion functional minimum to another local minimum) together with the abnormal error probability $P_{\text{abnorm}}$.

The inexact directional pattern presentation is not in itself critical for the results of the bearing. When there are no random errors in the direction finder’s measurements, the inexact directional pattern error (caused by the different directional patterns (10)-(14) used for the simulation of the bearing and for the actual bearing) will influence the exactness of the bearing at the very low frequencies only, up to 100 Hz. But there is still risk that, when combined with the other measurement errors, the inexact directional pattern would greatly influence the bearing exactness. To check this presupposition, the dependencies of the bearing RMSD on the different relative measurement errors are tested. In figure 2 and figure 3, the comparisons of the different values of the bearing RMSD for the cases of the different relative errors, under the absence and the presence of the incorrect directional pattern resulting from both the use of the cardioid for the data simulation and the bearing carried out with the help of the circular directional pattern.

From the comparisons of the dependencies depicted in figure 3 with the diagrams presented in figure 2 it is seen that under $\sigma_0/A < 0.25$ the RMSD of the bearing estimation while using the inexact directional pattern does not differ greatly from the values of the RMSD corresponding to the case of the exact directional pattern used for the calculations, though the abnormal errors start arising already. The abnormal errors probabilities are compared in figure 4 and figure 5. With the increase in the RMSD of the relative measurements up to $\sigma_0/A > 0.3$ under the frequencies of 800-1600 MHz, the inexactness of the directional pattern presentation makes the bearing RMSD by an order of the magnitude higher. In this case, the bearing becomes impossible, if some other errors are also present.

Figure 1. The cross-sections of the different types of the directional patterns.
The next step is to estimate the influence of the different errors in the directional pattern on the bearing exactness. To accomplish this, the table is required where, depending on the antenna component directional pattern and the decision functional directional pattern, the bearing RMSD and the number of the abnormal errors under the fixed radio signal frequency and a relative random error are presented. In table 1 and table 2, the results of such an estimate are provided for the radio signal frequency of 800 MHz and the relative random error $\sigma_0/A = 0.25$ and $\sigma_0/A = 0.5$, respectively.

5. Conclusion
As one can see from the results demonstrated in the tables 1, 2, the implementations using the circular antenna components have proven to be more stable compared with the directional ones. Yet, this effect is actual only under the very great directional patterns disturbances (when the circular directional pattern is replaced by the cardioid one, and vice versa). Under the greater but still not abnormal directional pattern disturbances, the stability of the correlation-based interferometric direction finder.
operation remains rather comparable in the cases when using the directional or the non-directional antenna components.

Table 1. Estimation of direction finding exactness at 800 MHz and under the relative random error $\sigma/A = 0.25$.

| Decision functional directional pattern | Antenna component directional pattern $\sigma_\theta$, $P_{abnormal}$ |
|----------------------------------------|-------------------------------------------------------------|
| Cardioid $(D(n)_{card})$               | Cardioid $(D(n)_{card})$, Half-sum $(D(n)_{half\_sum})$, Circular $(D(n)_{circle})$ |
| Cardioid $(D(n)_{card})$               | $0.26^\circ$; $0.3^\circ$; $3.8^\circ$; $2.2\%$ |
| Half-sum $(D(n)_{half\_sum})$         | $0.27^\circ$; $0.28^\circ$; $0.31^\circ$ |
| Circular $(D(n)_{circle})$            | $0.3^\circ$; $0.2\%$; $0.3^\circ$; $0.28^\circ$ |

Table 2. Estimation of direction finding exactness at 800 MHz and under the relative random error $\sigma/A = 0.5$.

| Decision functional directional pattern | Antenna component directional pattern $\sigma_\theta$, $P_{abnormal}$ |
|----------------------------------------|-------------------------------------------------------------|
| Half-sum $(D(n)_{half\_sum})$         | Half-sum $(D(n)_{half\_sum})$, Circular $(D(n)_{circle})$ |
| Half-sum $(D(n)_{half\_sum})$         | $1.3^\circ$; $0.28\%$; $0.56^\circ$; $0.03\%$; $3.1^\circ$; $4.4\%$ |
| Circular $(D(n)_{circle})$            | $7.5^\circ$; $9.1\%$; $1.4^\circ$; $3.1\%$; $0.9^\circ$; $2.2\%$ |

Acknowledgments
This study was financially supported by the Ministry of Science and Higher Education of the Russian Federation (research project No. FSWF-2020-0022).

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