Ultra-violet Finite Noncommutative Theories

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We establish the ultra-violet finiteness of various classes of noncommutative gauge theories.

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There has been a great deal of recent interest in noncommutative (NC) quantum field theories, stimulated by a connection with string theory and $M$-theory; see for example Refs. [1]–[19]. The theories have, moreover, novel properties which make them worthy of attention in their own right; for example NC quantum electrodynamics exhibits both asymptotic freedom and charge quantisation.

The algebra of functions on a noncommutative space is isomorphic to the algebra of functions on a commutative space with coordinates $x^\mu$, with the product $f \ast g(x)$ defined as follows

$$
f \ast g(x) = e^{-i\Theta^{\mu\nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x^\nu}} f(x + \xi)g(x + \eta)|_{\xi,\eta \to 0},$$

where $\Theta$ is a real antisymmetric matrix. Quantum field theories analogous to the corresponding commuting theories are now straightforward to define, with $\ast$-products replacing ordinary products. In the case of gauge theories there are a number of subtleties, however. Consider a field $\phi(x)$ which transforms as follows under a local symmetry transformation:

$$
\phi(x) \rightarrow \phi'(x) = U(x) \ast \phi(x) = e^{i\Lambda(x)} \ast \phi(x),
$$

where

$$
e^{i\Lambda(x)} = 1 + i\Lambda - \frac{1}{2!} \Lambda \ast \Lambda + \cdots
$$

By considering the product $U_1 \ast U_2 = e^{i\Lambda_1} \ast e^{i\Lambda_2}$ it is easy to show that $SU_N$ is not a group under the $\ast$-product, whereas $U_N$ is, so that we will devote our attention to $U_N$ gauge theories. Such gauge theories are constructed using the gauge fields $A_\mu$ and matter fields $\chi, \xi, \phi$ (scalars or fermions) transforming as follows:

$$
A'_\mu = U \ast A_\mu \ast U^{-1} + ig^{-1}U \ast \partial_\mu U^{-1}
$$

$$
\chi' = U \ast \chi
$$

$$
\xi' = \xi \ast U^{-1}
$$

$$
\phi' = U \ast \phi \ast U^{-1}
$$

where $\chi, \xi, \phi$ transform according to the fundamental, the anti-fundamental and the adjoint representations respectively. One may also, of course, have matter singlets; but, as has been noted by previous authors, it is not clear how to construct other representations (such as fractionally charged particles in the $U_1$ case).

In this paper we consider the ultra-violet (UV) divergences of NC theories, and in particular seek theories that are UV finite. Consider the pure (no matter) $U_N$ NC gauge
theory (NCGT). If one computes the one loop corrections and isolates the UV divergence, one finds that this can be described both for \( N = 1 \) and for \( N \geq 2 \) by a single \( \beta \)-function \( \beta_g \), which is moreover identical (for \( N \geq 2 \)) to the corresponding one-loop \( \beta_g \) for the \( SU_N \) commutative theory (CGT). (Contrast this to the \( U_N \) CGT case, where of course, writing \( U_N \equiv SU_N \otimes U_1 \), the \( U_1 \) gauge coupling is unrenormalised).

Although our chief interest here is in supersymmetric theories, an elementary consequence of our methods is that for the pure \( U_N \) gauge theory, the NCGT \( \beta \)-function is identical to all orders to the large \( N \) approximation to the corresponding \( SU_N \) CGT \( \beta \). The NC formalism extends readily to supersymmetric theories. An \( N = 1 \) \( U_N \) gauge theory with a set of adjoint chiral superfields \( \Phi_i \) is described by the Lagrangian

\[
L = \int d^4 \theta \, \text{Tr} \left( e^{-gV} * \Phi_i * e^{gV} * \Phi_i \right) + \left[ \int d^2 \theta \left( W(\Phi_i) + \frac{1}{4} W^\alpha * W_\alpha \right) + \text{c.c.} \right],
\]

where \( V \) is the vector superfield, \( W^\alpha \) the corresponding field strength, and the superpotential \( W(\Phi_i) \) is holomorphic and gauge invariant.

We will focus particularly on the following two theories:

\[
W_1 = h_1 \text{Tr} (\Phi_1 * [\Phi_2, \Phi_3]) = h_1 (W_a - W_b)
\]

\[
W_2 = h_2 \text{Tr} (\Phi_1 * \{\Phi_2, \Phi_3\}) = h_2 (W_a + W_b)
\]

where \( W_a = \text{Tr}(\Phi_1 * \Phi_2 * \Phi_3) \) and \( W_b = \text{Tr}(\Phi_1 * \Phi_3 * \Phi_2) \), and \( \Phi_1...3 \) are adjoint chiral supermultiplets. If we define

\[
\Phi = \frac{1}{\sqrt{2}} \phi^a \lambda^a, \quad a = 0, 1, \cdots N^2 - 1
\]

where \( [\lambda^a, \lambda^b] = 2if^{abc} \lambda^c, \{\lambda^a, \lambda^b\} = 2d^{abc} \lambda^c, \) and \( \text{Tr}(\lambda^a \lambda^b) = 2\delta^{ab} \), then in the commutative versions of the above theories we would have

\[
W_1^C = i\sqrt{2} h_1 f^{abc} \phi_1^a \phi_2^b \phi_3^c
\]

and

\[
W_2^C = \sqrt{2} h_2 d^{abc} \phi_1^a \phi_2^b \phi_3^c
\]

Note in particular that in Ref. [10] the gauge invariance of the one loop effective action for the \( \mathcal{N} = 4 \) theory was demonstrated.
and it is interesting to contrast this with the NC case where we have

\[ W_1 = \frac{h_1}{\sqrt{2}} (d^{abc} \phi_1^a \ast [\phi_2^b, \phi_3^c]_\ast + if^{abc} \phi_1^a \ast \{\phi_2^b, \phi_3^c\}_\ast) \] (11)

and

\[ W_2 = \frac{h_2}{\sqrt{2}} (d^{abc} \phi_1^a \ast \{\phi_2^b, \phi_3^c\}_\ast + if^{abc} \phi_1^a \ast [\phi_2^b, \phi_3^c]_\ast) \] (12)

In both the CGT and the NCGT cases, \( W_1 \) corresponds to \( \mathcal{N} = 4 \) supersymmetry, if we set \( h_1 = g \). It is well-known that the \( \mathcal{N} = 4 \) CGT is all orders finite\(^2\), as we shall see the same is true in the NCGT \( \mathcal{N} = 4 \) case. This is to be expected since in general NC theories have improved UV divergence properties. Somewhat more surprising, however, is the following: in the CGT case, the \( SU_N \) version of \( W_2^C \), for the case

\[ h_2 = gN/\sqrt{N^2 - 4} \] (13)

is the so-called \( \mathcal{N} = 4d \) model discussed in Refs. [20], [21]. It is UV finite through two loops, but has a three (and higher) loop divergence [22], which can, however, be removed [23] by replacing Eq. (13) by

\[ h_2 = gN/\sqrt{N^2 - 4} + a_5g^5 + \cdots \] (14)

where \( a_5, \cdots \) are calculable constants. In the NCGT case the \( U_N \) version of the theory is, as we shall see, all orders UV finite simply given \( h_2 = g \), in other words without recourse to the kind of coupling constant redefinition represented by Eq. (14).

Since the \( \Phi \) are adjoint fields in \( U_N \) we can use the diagrammatic notation originally introduced by ’t Hooft [24], where we represent \( \Phi_a^b \) by a double line as in Fig. 1, the arrow pointing towards the upper index. This is in fact a considerable simplification compared to the generalised \( f^{abc}, d^{abc} \) formulation that has been used in some papers.

**Fig. 1: The propagator for an adjoint \( U(N) \) field**

\(^2\) The \( \mathcal{N} = 4 \) \( U_N \) CGT consists of the direct product of the familiar \( \mathcal{N} = 4 \) \( SU_N \) theory with a \( \mathcal{N} = 4 \) \( U_1 \) free field theory.
The vertices $W_a$, $W_b$, and their complex conjugates $\overline{W}_a$ and $\overline{W}_b$ are then represented as in Fig. 2.

**Fig. 2: The vertices $W_a$, $W_b$, $\overline{W}_a$, $\overline{W}_b$.**

In momentum space, $W_a$ is associated with a factor $e^{i k_1 \wedge k_2}$ where $k_i$ is the momentum associated with $\Phi_i$ and $p \wedge q = \Theta^{\mu \nu} p_\mu q_\nu$. Suppose we associate momenta $p_i$ with the lines as shown in Fig. 2 (flowing in the direction of the arrows), so that for $W_a$, $k_1 = p_3 - p_2$ etc, and for $W_b$, $k_1 = p_2 - p_3$ etc. Then the exponential factor for $W_a$ can be rewritten using

$$k_1 \wedge k_2 = p_1 \wedge p_2 + p_2 \wedge p_3 + p_3 \wedge p_1$$  \hspace{1cm} (15)

as

$$e^{i \sum_{\text{legs}} p_{\text{out}} \wedge p_{\text{in}}} = \prod_{\text{legs}} e^{ip_{\text{out}} \wedge p_{\text{in}}}$$  \hspace{1cm} (16)

where $p_{\text{out}}$, $p_{\text{in}}$ are the momenta associated with the lines with arrows pointing out from, or into, the vertex respectively for each leg. We thereby associate an exponential factor with each leg of the vertex. It is easy to check that the exponential factor can also be written in the form Eq. (16) for $W_b$ and indeed for $\overline{W}_a = \text{Tr} (\Phi_1 \ast \Phi_3 \ast \Phi_2)$ and $\overline{W}_b = \text{Tr} (\Phi_1 \ast \Phi_2 \ast \Phi_3)$. Moreover the $\Phi_i \Phi_i V^n$ vertex is given by the expression $\text{Tr} \left( \frac{1}{n!} \Phi_i [\cdots [\Phi_i, V], V]_{\ast}, \cdots V]_{\ast}, V]_{\ast} \right)$ with $n$ nested commutators. Again, the exponential factor for one of these vertices can be written in the form given in Eq. (16).
We claim that it is only planar graphs constructed using the vertices above which contribute to the renormalisation-group (RG) functions ($\beta$-functions and anomalous dimensions) for the theories with $W_1$ or $W_2$ in the noncommutative case. Let us start by considering the theory with $W_1$. Consider for example the one loop contribution to the anomalous dimension of $\Phi_1$ given by contracting $W_1$ with $\overline{W}_1$. The contractions of $W_a$ with $\overline{W}_a$, or $W_b$ with $\overline{W}_b$, give planar diagrams, as depicted in Figs 3(a,b), while the contractions of $W_a$ with $\overline{W}_b$, or $W_b$ with $\overline{W}_a$, give non-planar graphs, as depicted in Fig. 3(c,d).

![Fig. 3: The one-loop diagrams.](image)

Now these four diagrams all correspond to the same one-loop momentum integral with a single loop momentum. For the planar graphs in Figs. 3(a,b), the loop momentum may be assigned to the closed loop and momenta may be assigned to the other lines consistently with momentum conservation at the two vertices and a given external momentum. It is then clear from Eq. (16) that the exponential factors on the internal pairs of lines cancel in pairs; because the “out” momentum for one vertex is the “in” momentum for its neighbour. The remaining exponential factors from the external legs cancel by momentum conservation. In particular there is no phase factor containing the loop momentum which, if present, would suppress the ultraviolet divergence\[1\]–\[5\]. On the other hand, in the case of the non-planar graphs in Figs. 3(c,d), there is no closed loop to which the loop momentum can be assigned, the above argument breaks down, and therefore there will be a phase factor involving the loop momentum (as can easily be checked) making the diagram ultra-violet finite.
This argument readily extends to higher loop orders, to graphs containing gauge fields, and to other RG-functions. For any planar graph, the loop momenta from the corresponding Feynman graph may be assigned to closed loops of the planar graph, and the exponential factors cancel in pairs on internal pairs of lines. In the case of the non-planar graphs, there are fewer planar loops (of the kind apparent in Figs. 3(a,b)) than loop momenta and this argument breaks down. There will then be an overall exponential factor involving at least one of the loop momenta, and this graph will not contribute to the RG-function. Of course a non-planar graph (with a phase factor) may have a planar (and hence divergent) sub-graph, but this graph will be finite after subtraction of sub-divergences; this is analogous to the way that in commutative $\phi^4$ theory, the $\phi^6$ 1PI Green’s function is finite, in spite of the fact that it includes 4-point sub-graphs.

We now turn to the theory with superpotential $W_2$. We shall show that its divergences are the same as those of the theory with superpotential $W_1$ (with $h_1 \to h_2$). The difference between the superpotentials $W_1$ and $W_2$ (apart from $h_1 \to h_2$) lies simply in the sign of $W_b$ (and $\overline{W}_b$). For simplicity we start with diagrams which only contain Yukawa vertices. Note that of course by chirality $W$s and $\overline{W}$s must alternate in such a diagram. Once again the only divergent diagrams are the planar ones. Consider any planar diagram. We may assign it an odd or even “parity” according as its sign is changed or unchanged by $W_b \to -W_b$, $\overline{W}_b \to -\overline{W}_b$. We would like to show that every planar diagram has even parity. For simplicity, suppose we join together the external legs of the diagram and imagine it to be drawn on the surface of a sphere. We see that for planar diagrams every closed loop has the same sense of rotation for the arrow (anti-clockwise with our conventions). Therefore the fields $\Phi_1$, $\Phi_2$ and $\Phi_3$ always appear in clockwise order for $W_a$ and anticlockwise order for $W_b$; and conversely, the fields $\overline{\Phi}_1$, $\overline{\Phi}_2$ and $\overline{\Phi}_3$ always appear in anticlockwise order for $\overline{W}_a$ and clockwise order for $\overline{W}_b$.

![Fig. 4: Reduction of diagrams.](image)

If the diagram contains a pair of linked adjacent vertices $W_a$ and $\overline{W}_a$, or $W_b$ and $\overline{W}_b$, (signalled by a sequence of fields such as $\Phi_2\Phi_1\Phi_2$ in some loop) then we may obtain a graph with two fewer vertices and the same parity by the process depicted in Fig. 4.
We now repeat this process until we can do so no further. The process could terminate in one of two ways: the first possibility is that eventually we obtain a diagram consisting of separate closed loops and no vertices, which clearly has even parity by default; and thus the original diagram must have even parity. The second possibility is that eventually every loop consists of a permutation of the sequence $\Phi_1 \Phi_2 \Phi_3$ repeated an integral number $n$ times (where $n$ would be even by chirality). But it is easy to see that this is impossible for planar diagrams. The diagram would then consist entirely of hexagons, dodecagons and so on. Suppose we have a diagram with $n_6$ hexagons, $n_{12}$ dodecagons etc. Let $n_V$ be the number of vertices, $n_P$ the number of propagators and $n_L$ the number of loops. Then we have

$$3n_V = 6n_6 + 12n_{12} + 18n_{18} + \ldots \Rightarrow n_V = 2n_6 + 4n_{12} + 6n_{18} + \ldots,$$

$$2n_P = 6n_6 + 12n_{12} + 18n_{18} + \ldots \Rightarrow n_V = 3n_6 + 6n_{12} + 9n_{18} + \ldots, \quad (17)$$

$$n_L = n_6 + n_{12} + n_{18} + \ldots,$$

and then

$$n_V - n_P + n_L = -n_{12} - 2n_{18} - \ldots \quad (18)$$

so that Euler’s formula

$$n_V - n_P + n_L = 2 - 2G \quad (19)$$

has no solution for the sphere which has genus $G = 0$. We deduce that the second possibility does not in fact occur, and therefore the original diagram is indeed of even parity. It is easy to extend this argument to graphs with gauge propagators, by noting that we may remove a gauge propagator without changing the parity of the graph. It follows that the divergences, and thus the RG-functions, of the theory with superpotential $W_2$ may be obtained from those for superpotential $W_1$ by replacing $h_1$ with $h_2$.

Our main results now follow immediately upon setting $h_1 = h_2 = g$. Firstly, the theory with $W_1$ now becomes $\mathcal{N} = 4$ NCGT. So we see that the $\mathcal{N} = 4$ NCGT $\beta$ functions are derived from the planar graphs. We now note that these planar graphs are exactly those which give the leading $N$ contribution to the $\beta$-function for the $\mathcal{N} = 4$ CGT, since at $L$ loops they contain the maximum number ($L$) of closed loops. Since the $\mathcal{N} = 4$ CGT is finite, the leading $N$ contributions must vanish individually at each loop order. Therefore the $\mathcal{N} = 4$ NCGT $\beta$ functions must also vanish, and $\mathcal{N} = 4$ NCGT is finite to all orders. Secondly, since the RG-functions for the theory with $W_2$ are identical to those of the theory with $W_1$, the theory with $W_2$ is also finite to all orders (for $h_2 = g$).
Clearly the fact that both $W_1$ and $W_2$ lead (for $h_1 = h_2 = g$) to finite theories, and the obvious similarity between Eq. (11) and Eq. (12), suggest that $W_2$ also represents a theory with $\mathcal{N} > 1$ supersymmetry; however we have been unable to demonstrate this. The presence of the commutator in Eq. (4) (as opposed to the anti-commutator in Eq. (7)) is crucial for the additional symmetries (as given, for example, in Ref [25]) associated with the $\mathcal{N} = 4$ invariance. It would clearly be interesting to compare the two theories in the infra-red; it has been argued[7] that the $\mathcal{N} = 4$ theory is free of divergences as $\Theta \to 0$, although such divergences are characteristic of NC theories in general.

By similar reasoning we can use the finiteness of commutative $\mathcal{N} = 2$ theories beyond one loop to establish the corresponding result in the NCGT case, that is for superpotentials of the form

$$W = h \sum_{n=1}^{N_f} \xi_n \Phi^* \chi_n \quad (20)$$

where $\xi_n, \phi, \chi_n$ transform according to the superfield generalisation of Eq. (4), and for $\mathcal{N} = 2$ supersymmetry we require $h = \sqrt{2}g$. The contributions to RG-functions are associated once again with cancellation of phase factors in planar graphs; here these contributions are (at $L \geq 2$ loops) precisely given by the terms of order $N^L, N^{L-1}N_f, N^{L-1}N^2_f, \ldots NN_f^{L-1}$ from the corresponding RG-functions for the CGT. (This corresponds to the Veneziano[26] (as opposed to the 't Hooft) limit, i.e. both $N, N_f \to \infty$ with $N/N_f$ fixed.) Since in the CGT case the RG-functions vanish beyond loop, it follows that all these contributions cancel. By choosing $N_f = 2N$ for one loop finiteness we obtain another class of all orders UV finite theories.

In conclusion: we have established the UV finiteness to all orders of the $\mathcal{N} = 4 \, U_N$ NCGT, a closely related $\mathcal{N} = 1$ theory and the class of one-loop finite $\mathcal{N} = 2 \, U_N$ theories. A simple corollary of our methods is that $\beta_g$ for the pure non-supersymmetric $U_N$ NCGT is identical to the large-$\mathcal{N}$ (or planar) approximation to the $\beta_g$ for the corresponding $SU_N$ CGT; and for NCQCD (with $N_f$ flavours) the $L$-loop contribution to the $U_N$ RG-functions are given by the terms from the corresponding commutative $SU_N$ QCD RG-functions of the form $N^a N_f^b$ where $a + b = L$, corresponding once again to the Veneziano[26] limit.
Note Added

After this paper was submitted to the archive we were made aware of some related work:

The UV/IR connection (the existence of infra-red singularities arising from large virtual momenta) was described in Ref. [27]. This paper deals mainly with scalar theories, and in fact describes the cancellation of phase factors involving internal momenta in planar graphs by use of momentum assignments like those shown in our Fig. 2. A rigorous proof of renormalisability for various massive NC scalar theories (in particular $\phi^* \phi \phi^* \phi$ for $d = 4$) was given in Ref. [28]. The relevance of the Veneziano limit for NCQCD described above was remarked in Ref. [29]. A general proof of the renormalisability of a particular supersymmetric noncommutative theory is given for the Wess-Zumino model in Ref. [30]. It was pointed out in Ref. [5] and re-emphasised in Ref. [31] that the divergences of pure $U_N$ noncommutative gauge theory are dictated by the large $N$ limit of the commutative theory. The latter paper also raises the interesting possibility of finite, possibly non-supersymmetric noncommutative theories obtained by orbifold truncation of the $\mathcal{N} = 4$ theory. We also mention the possibility of defining finite noncommutative theories on fuzzy spheres [32]. (See Ref. [33] for the $q$-deformed case.)

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References

[1] T. Filk, Phys. Lett. B376 (1996) 53
[2] C. P. Martin and D. Sanchez-Ruiz, Phys. Rev. Lett. 83 (1999) 476 [hep-th/9903077]
[3] M.M. Sheikh-Jabbari, JHEP 9906 (1999) 015 [hep-th/9903107]
[4] T. Krajewski and R. Wulkenhaar, Int. J. Mod. Phys. A15 (2000) 1011 [hep-th/9903187]
[5] D. Bigatti and L. Susskind, Phys. Rev. D62 (2000) 066004 [hep-th/9908056]
[6] M. Hayakawa, [hep-th/9912167]
[7] A. Matusis, L. Susskind and N. Toumbas, JHEP 0012 (2000) 002 [hep-th/0002075]
[8] D. Zanon, Phys. Lett. B504 (2001) 101 [hep-th/0009106]
[9] A. Santambrogio and D. Zanon, JHEP 0101 (2001) 024 [hep-th/0010275]
[10] M. Pernici, A. Santambrogio and D. Zanon, Phys. Lett. B504 (2001) 131 [hep-th/0011140]
[11] D. Zanon, Phys. Lett. B502 (2001) 265 [hep-th/0012099]
[12] S. Ferrara and M. A. Lledo, JHEP 0005 (2000) 008 [hep-th/0002084]
[13] A. Armoni, Nucl. Phys. B593 (2001) 229 [hep-th/0005208]
[14] A. Armoni, R. Minasian and S. Theisen, [hep-th/0102007]
[15] S. Terashima, Phys. Lett. B482 (2000) 276 [hep-th/0002119]
[16] F. R. Ruiz, Phys. Lett. B502 (2001) 274 [hep-th/0012171]
[17] C. P. Martin and D. Sanchez-Ruiz, Nucl. Phys. B598 (2001) 348 [hep-th/0012024]
[18] L. Bonora and M. Salizzoni, Phys. Lett. B504 (2001) 80 [hep-th/0011088]
[19] L. Bonora, M. Schnabl, M. M. Sheikh-Jabbari and A. Tomasiello, Nucl. Phys. B589 (2000) 461 [hep-th/0006091]
[20] D.R.T. Jones and L. Mezincescu, Phys. Lett. B138 (1984) 293
[21] A.J. Parkes and P.C. West, Nucl. Phys. B256 (1985) 340
[22] D.R.T. Jones and A.J. Parkes, Phys. Lett. B160 (1985) 267
[23] D.R.T. Jones, Nucl. Phys. B277 (1986) 153;
A.V. Ermushev, D.I. Kazakov and O.V. Tarasov, Nucl. Phys. B281 (1987) 72;
D.I. Kazakov, Phys. Lett. B179 (1986) 352
[24] G. ’t Hooft, Nucl. Phys. B72 (1974) 461;
P. Cvitanovic, P.G. Lauwers and P.N. Scharbach, Nucl. Phys. B203 (1982) 385
[25] M. Grisaru, M. Rocek and W. Siegel, Phys. Rev. Lett. 45 (1980) 1063
[26] G. Veneziano, Nucl. Phys. B117 (1976) 519
[27] S. Minwalla, M. Van Raamsdonk and N. Seiberg, JHEP 0002 (2000) 020 [hep-th/9912072]
[28] I. Chepelev and R. Roiban, JHEP 0103 (2001) 001 [hep-th/0008090]
[29] J. Ambjorn, Y. M. Makeenko, J. Nishimura and R. J. Szabo, Phys. Lett. B480 (2000) 399 [hep-th/0002158]
[30] H.O. Girotti, M. Gomes, V.O. Rivelles and A.J. da Silva, Nucl. Phys. B587 (2000) 299 [hep-th/0005272]
[31] A. Armoni, JHEP 0003 (2000) 033 [hep-th/9910031]
[32] H. Grosse, C. Klimcik and P. Presnajder, Comm. Math. Phys. 180 (1996) 429 [hep-th/9602115]; Int. J. Theor. Phys. 35 (1996) 231 [hep-th/9505175]
[33] H. Grosse, J. Madore and H. Steinacker, J. Geom. Phys. 38 (2001) 308 [hep-th/0103164]