Comparing the SFI peculiar velocities with the PSCz gravity field: a VELMOD analysis

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ABSTRACT

We compare the peculiar velocities derived from the I-band Tully–Fisher (TF) relation for 989 field spiral galaxies in the SFI catalogue with the predicted velocity field derived from the IRAS PSCz galaxy redshift survey. We assume linear gravitational instability theory and apply the maximum likelihood technique, VELMOD to SFI galaxies within a redshift \( c_{z_{\text{LG}}} = 6000 \, \text{km} \, \text{s}^{-1} \). The resulting calibration of the TF relation is consistent with a previous, independent calibration for a similar sample of spirals residing in clusters. Our analysis provides an accurate estimate of the quantity \( \beta_{1} = \Omega_{m}^{0.6}/b_{t} \), where \( b_{t} \) is the linear biasing parameter for IRAS galaxies. Using the forward TF relation and smoothing the predicted velocity field with a Gaussian filter of radius 300 km s\(^{-1}\), we obtain \( \beta_{1} = 0.42 \pm 0.04 \) (1σ uncertainty). This value, as well as other parameters in the fit, are robust to varying the smoothing radius to 500 km s\(^{-1}\) and splitting the sample into spherical shells in redshift space. The one exception is the small-scale velocity dispersion, \( \sigma_{v} \), which varies from \( \sim 200 \, \text{km} \, \text{s}^{-1} \) (within \( c_{z_{\text{LG}}} = 4000 \, \text{km} \, \text{s}^{-1} \)) to \( \sim 500 \, \text{km} \, \text{s}^{-1} \) at larger distance. For \( \beta_{1} = 0.42 \), the residuals between the TF data and the PSCz galaxy field are uncorrelated, indicating that the model provides a good fit to the data. More generally, a \( \chi^{2} \) statistic indicates that the PSCz model velocity field provides an acceptable (3σ) fit to the data for \( 0.3 < \beta_{t} < 0.5 \).

Key words: galaxies: clusters: general – galaxies: distances and redshifts – cosmology: observations – cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

Measurements of peculiar motions provide a fundamental tool to probe the mass distribution in the local universe. In the linear regime of gravitational instability, a simple relation between peculiar velocity, \( \mathbf{v} \), and mass density contrast, \( \delta_{m} \), can be easily obtained from mass conservation, either in differential

\[
\nabla \cdot \mathbf{v} = -\Omega_{m}^{0.6} \delta_{m},
\]

or integral,

\[
\mathbf{v}(r) = \frac{\Omega_{m}^{0.6}}{4\pi} \int d^{3}r \frac{\delta_{m}(r'(r' - r))}{|r' - r|^{3}}
\]

\[
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\]

\( \Omega_{m} \) is the cosmological mass density parameter.

Together with the commonly used simplifying assumption of linear biasing, \( \delta_{g} = b_{g} \delta_{m} \), where \( \delta_{g} \) is the galaxy density contrast and \( b_{g} \) the galaxy biasing parameter, these two equations provide a relation between observable quantities: the peculiar velocity of luminous objects, \( \mathbf{v} \), and the density contrast, \( \delta_{g} \), which can be obtained from large all-sky redshift surveys. The IRAS Point Source Catalogue (Beichman et al. 1998) is particularly suited for this purpose because of the good sky coverage and homogeneity of the survey. In this paper, we present a comparison of peculiar velocity data with a redshift survey based on the IRAS catalogue. Throughout the paper, we will use the subscript \( I \) to indicate that an IRAS density field has been used.

Comparing \( \delta_{g} \) with \( \mathbf{v} \) using any of the two equations above allows us to estimate the quantity \( \beta_{g} = \Omega_{m}^{0.6}/b_{g} \) and to test the
validity of the gravitational instability hypothesis. Although the two equations (1) and (2) are mathematically equivalent, they lead to two different strategies for measuring \( \beta_I \). Equation (1) is used to perform the so-called density–density (d–d) comparisons which typically consist of the following steps: a 3D velocity field reconstruction from observed radial velocities; differentiation of \( \mathbf{v}(r) \) and use of equation (1) to compute \( \delta_z \); comparison to the observed galaxy density fields. The first step is the least trivial and requires some additional theoretical assumptions. Bertschinger & Dekel (1989) successfully implemented a d–d technique by assuming that \( \mathbf{v}(r) \) is irrotational, in the potent reconstruction method. The many applications of potent to various data sets have consistently led to large values of \( \beta_I \) consistent with unity (see Sigad et al. 1998 and references therein). Equation (2) is at the core of the so-called velocity–velocity (v–v) comparisons. In this approach, one computes the mass density field obtained from the galaxy distribution in the redshift survey, uses equation (2) to predict a peculiar velocity field and then compares it with the observed galaxy velocities. The v–v methods have been applied to most of the catalogues presently available and have given values of \( \beta_I \) which are typically in the range 0.4–0.6 (see Willick 2001 for an updated summary of the various results).

The v–v methods are commonly regarded as more reliable and robust than the d–d ones because they require less manipulation of the data. The values of \( \beta_I \) obtained by these analyses are significantly smaller than unity, irrespective of the velocity tracers, model gravity field and comparison technique used. However, some of the v–v analyses showed evidence for a poor match between models and data, which would render the estimate of \( \beta_I \) meaningless. Davis, Nusser & Willick (1996) used their ITF technique (Nusser & Davis 1994) to compare the gravity field derived from the IRAS 1.2-Jy survey (Fisher et al. 1995) with the peculiar velocities obtained from the Mark III catalogue (Willick et al. 1995, 1996, 1997a). The coherent dipole residuals that they found were taken as evidence for significant discrepancies between modelled and observed velocity fields. Willick et al. (1997b, hereafter V1) and Willick & Strauss (1998, hereafter V2) also considered the IRAS 1.2-Jy velocity predictions and the Mark III data set but compared them using the VELMOD method. They were able to obtain a good fit to the data only by introducing a physically motivated, external quadrupole contribution to the model velocity field. Da Costa et al. (1998, hereafter D98) found good agreement between the peculiar velocities of galaxies in the SFI catalogue (Haynes et al. 1999a,b) and those derived from the IRAS 1.2-Jy gravity field, by performing an ITF comparison. The same ITF method has been recently applied to compare two different data sets: the IRAS PSCz redshift survey (Saunders et al. 2000) and the peculiar velocities in the recently completed ENEAR catalogue (da Costa et al. 2000). Also in this case the agreement between model and data was satisfactory (Nusser et al. 2000).

In this paper, we use the VELMOD technique to compare the PSCz velocity prediction to the SFI data set. As for any v–v comparison, our main aim is to constrain \( \beta_I \) and to investigate the adequacy of the PSCz model velocity field. However, rather than simply adding one more measurement of \( \beta_I \) to those already in the literature from other v–v comparisons, we hope to address some more specific questions which should help simplify the rather complicated picture that has emerged from the results of the various v–v comparisons. Our goal is to check whether the PSCz–IRAS gravity field still provides a good fit to SFI data when the comparison is performed with the VELMOD method rather than the ITF. We also want to exploit fully the denser and deeper PSCz catalogue and see whether we can improve the agreement between the gravity field and the measured velocities, thus reducing the uncertainties in the estimate of \( \beta_I \).

In Section 2 we review the basics of the VELMOD technique and describe its current implementation. The SFI sample is presented in Section 3 and the IRAS PSCz catalogue and its gravity field are described in Section 4. VELMOD is tested in Section 5 and the results of its application to the SFI catalogue are presented in Section 6. An analysis of errors based on the magnitude and velocity residuals is performed in Section 7. Finally, in Sections 8 and 9 we discuss the results and conclude.

2 THE VELMOD METHOD

VELMOD is a maximum likelihood method introduced by V1 and described in detail in V1 and V2. Here we simply outline the main points of the method, focusing on its implementation in the case of a forward TF relation. In the terminology of Strauss & Willick (1995) VELMOD uses a Method II approach, i.e. takes the TF observables (apparent magnitude and velocity width) and the redshift of an object and quantifies the probability of observing the former given the latter, for a particular model of the velocity field and a TF relation. This probability is then maximized with respect to the free parameters of the velocity model and the TF relation. Unlike previous Method II implementations (e.g. Hudson 1994), VELMOD replaces the redshift–distance relation with a joint probability distribution of distance and redshift. This probabilistic approach allows a statistical treatment of all those effects (small-scale velocity noise, inaccuracy of the velocity model and existence of triple-valued regions) that spoil the uniqueness of the redshift–distance mapping.

VELMOD does not require smoothing of the TF data which, along with the allowance for triple-valued regions and small-scale velocity noise, allows one to probe the velocity field in high-density regions, thus exploiting the denser sampling of the new PSCz galaxy catalogue. Another convenient feature of VELMOD is that it does not require an a priori calibration of the TF relation, which is a common issue of concern in peculiar velocity studies. Instead, a fit of the parameters of the TF relation is performed simultaneously with a fit of the parameters of the velocity field.

2.1 Implementation of VELMOD

For each galaxy, the angular position \((l, b)\), redshift \((cz)\), apparent magnitude \((m)\), and velocity width parameter \(\eta = \log_{10}(W)\) = 2.5, where \(W\) is twice the rotation velocity of the galaxy) are taken from the SFI catalogue. Hereafter, we will always use the redshift measured in the Local Group (LG) frame unless otherwise specified. In addition, a model for the density field is needed to account for inhomogeneous Malmquist bias. Such a model is obtained from the distribution of IRAS PSCz galaxies, together with a self-consistent model for the peculiar velocity field (as described in Section 4). The velocity model is completely specified by the value of the \(\beta_I\) parameter and a one-dimensional velocity dispersion, \(\sigma_v\), which quantifies both the inaccuracy of the velocity model and the true velocity noise arising from small-scale nonlinear motions. Strauss, Ostriker & Cen (1998) and V2 have shown that the velocity dispersion on small scales is an increasing function of the local density. The SFI galaxy sample considered in this work consists of field spirals which avoid high-density environments and thus we ignore such dependency and assume a constant \(\sigma_v\), independent of the environment.
The probability of observing a magnitude \( m \) for a galaxy with a given \( \eta \) and distance \( r \) is modelled by a linear TF relation,  
\[
m = M(\eta) + 5 \log(r) = A_{TF} + b_{TF} \eta + 5 \log(r) + \sigma_r,
\]  
where \( \sigma_r \) is a Gaussian random distribution with zero mean and dispersion \( \sigma_{TF} \). This TF relation is completely specified by its zero point \( (A_{TF}) \), slope \( (b_{TF}) \) and scatter \( (\sigma_{TF}) \). In this work we do not model a possible dependency of the TF scatter on the luminosity or velocity width. In Section 6.4, we will find that our results are insensitive to this approximation.

The final ingredient needed to compute the probability of observing the measurements of the SFI catalogue are the selection function of the observational quantities and their correlations. Systematic errors can affect peculiar velocities computed with the forward TF relation if selection effects are not properly accounted for. For that purpose, we have used the correlation and selection models of Freudling et al. (1995).

With the above assumptions, one can compute the probability that the \( i \)th object of the sample with recession velocity \( cz \), and velocity width parameter \( \eta \) will have an apparent magnitude \( m \), \( P(m, \eta, cz) \). To evaluate this conditional probability one needs to integrate the joint probability distribution \( P(m, \eta, cz) \) over \( m \). Although analytic approximations for this integral have been introduced by V2, which are valid away from triple-valued regions and at distances much larger than \( \eta \), these have not been adopted by V2, which are valid away from triple-valued regions and at distances much larger than \( \eta \). Therefore, we choose not to allow for any external contribution to reduce such discrepancies, allowing us to drop this extra parameter.

The models for the density and velocity fields are obtained from the PSCz catalogue. The models for the density and velocity fields are obtained from the PSCz catalogue. The models for the density and velocity fields are obtained from the PSCz catalogue. The models for the density and velocity fields are obtained from the PSCz catalogue.

3 The SFI Sample

The SFI sample of galaxies (Giovanelli et al. 1997a; Haynes et al. 1999a,b) is a homogenous all-sky sample of galaxies for which I-band Tully–Fisher parameters are available. The sample uses new observations for declinations \( \delta > -40^\circ \) and re-reduced data from Mathewson, Ford & Buchorn (1992) in the southern polar cap. The sample is, by design, angular diameter limited, with different limiting diameters, \( D_{lim} \) for different redshift shells. The limiting diameters, expressed in unit of 0.1 arcmin, are \( D_{lim} = 25 \), 16 and 13 in the redshift ranges \( cz_{lim} < 3000 \text{ km s}^{-1} \), \( 3000 < cz_{lim} < 5000 \text{ km s}^{-1} \) and \( 5000 < cz_{lim} < 7500 \text{ km s}^{-1} \), respectively. However, for a number of different reasons, some bright galaxies smaller than the stated diameter limit were included in the sample. These additional galaxies, which amount to \( \sim 15 \) per cent of the total, do not constitute a strictly magnitude-limited sample. All of them are brighter than an apparent Zwicky magnitude of \( m_z = 14.5 \).

In a detailed investigation of these extra galaxies, we have not found any dependence of the selection on distance or apparent diameter. This is different from the distance-dependent selection function that Willick et al. (1996) have used to describe the so-called MAT sample. For the present analysis, we have therefore approximated the SFI selection sample as the combination of a strictly diameter-limited sample with the above criteria, and a magnitude-limited sample with limiting Zwicky magnitude of 14.5. For such a case, Willick (1994) has derived the VELMOD formalism. In his terminology, this is the Two-Catalogue Selection case and the selection function, expressed in a form suitable for the VELMOD analysis, is given by equation (63) of Willick (1994). We have found that the results presented in this paper do not change significantly when using \( m_z = 13.5 \) for the putative magnitude limit of the extra bright galaxies.

Freudling et al. (1995) have modelled the correlations between the quantities used to define the selection criteria, \( (m_z, D) \), and the TF observables, \( (m, \eta) \), as

\[
m_z = 1.6 + m + 0.5 \eta
\]  
with dispersion \( \sigma_{m_z} = 0.59 \) and

\[
D = 3.82 + 0.205m - 0.102\eta
\]  
with dispersion \( \sigma_D = 0.121 \), where \( m \) indicates the I-band apparent magnitude. These correlations can be easily translated into the VELMOD formalism.

Below, we will consider the following subsamples drawn from the SFI catalogue. Most of the analysis is carried out with the 989 galaxies with \( cz_{lim} < 6000 \text{ km s}^{-1} \) and \( \eta > -0.25 \). These cuts allow us to perform a homogeneous comparison with the ITF analysis of D98. In addition, we will consider separate subsamples restricted to three independent redshift-space shells \( 2000 \text{ km s}^{-1} \) thick and with external radii of \( 2000, 4000 \) and \( 6000 \text{ km s}^{-1} \). These subsamples mix different nominal selection criteria and serve as an additional check that selection effects are well accounted for. The three subsamples contain 158, 355 and 496 galaxies, respectively.

SFI galaxies avoid rich clusters and high-density environments. In particular none of them belong to the Virgo cluster. Therefore, there is no need to adopt any grouping procedure and the hypothesis of a small-scale velocity dispersion, \( \sigma_v \), independent of the environment is well justified.

4 Model Density and Velocity Fields from the PSCz Survey

The models for the density and velocity fields are obtained from the distribution of IRAS galaxies in the recently completed PSCz all-sky redshift survey (Saunders et al. 2000). This catalogue, which basically extends the old 1.2-Jy one (Fisher et al. 1995), contains \( \sim 15 \) 500 IRAS PSC galaxies with a flux at 60 \( \mu \)m larger than 0.6 Jy. A complete description of the data set, its selection criteria and the procedures adopted to avoid stellar contamination and galactic...
cirus are given in Saunders et al. (2000). For our purposes, the most interesting features of the catalogue are the large area sampled (∼84 per cent of the sky), its depth (the median redshift is 8500 km s\(^{-1}\)) and dense sampling (the mean galaxy separation at 10000 km s\(^{-1}\) is (\(\ell\) ∼ 1000 km s\(^{-1}\)), compared with a value of (\(\ell\) ∼ 1500 km s\(^{-1}\) for the IRAS 1.2-Jy catalogue).

The flux-limited nature of the catalogue causes the number of objects to decrease with distance. This decrease is quantified by a radial selection function. Various authors have used different estimators to compute the selection function (Springel 1996; Canavez et al. 1998; Sutherland et al. 1999; Branchini et al. 1999; Monaco & Efstathiou 1999; Saunders et al. 2000). Branchini et al. (1999, hereafter B99) found that different selection functions induce variations smaller than 5 per cent in the model density and velocity fields within a distance of 20000 km s\(^{-1}\). In this work we use the selection function specified by equation (1) of B99.

The density and velocity field in real space are obtained from the redshift-space distribution of PSC\(_2\) galaxies by implementing Method 1 of B99, which uses the iterative technique of Yahil et al. (1991) to minimize redshift-space distortions. The procedure relies on gravitational instability theory, assumes linear biasing, and is valid in the limit of small density fluctuations where linear theory applies. At each step of the iteration the gravity field, \(g\), is computed from the distribution of the 11 206 PSC\(_2\) galaxies within 20000 km s\(^{-1}\). The gravity field is subsequently smoothed with a top hat filter of radius ∼ 500 km s\(^{-1}\) which allows us to assume linear theory and thus to obtain the smoothed peculiar velocity of each galaxy from the acceleration, \(v = \beta g\). The new distances of the objects, \(r\), are then assigned assuming a unique redshift–distance relation \(r = cz - u\), where \(u\) is the radial component of the peculiar velocity vector. The procedure is repeated until convergence is reached.

The final products are the real space positions of the galaxies and their velocities for a given value of \(\beta\). We ran 19 different reconstructions with \(\beta_l = 0.1, 0.15\ldots1.0\). The continuous density field is obtained by smoothing the galaxy distribution on 129 points of a cubic grid inside a box of 19200 km s\(^{-1}\) a side with the LG at the centre, using a Gaussian filter of 300 km s\(^{-1}\) (G3, hereafter). The associated smoothed velocity fields are computed for the appropriate value of \(\beta_l\) from equation (1).

The procedure returns 19 models of the density field (one for each value of \(\beta_l\)), along with their associated peculiar velocity fields, both defined at the gridpoint positions. Velocity predictions are made in the LG frame to minimize the uncertainties derived from the lack of information about the mass distributions on scales larger than the size of the PSC\(_2\) sample.

5 TESTING THE VELMOD IMPLEMENTATION

In this work we rely on the error analysis performed by V1, based on an extensive application of VELMOD to realistic mock catalogues, which we do not repeat here. Instead, we present the results of two tests aimed at checking the reliability of our VELMOD implementation.

5.1 Tests with ideal PSC\(_2\) and SFI mock catalogues

As a first test, we applied VELMOD to a suite of SFI and PSC\(_2\)

mock catalogues. Errors in the VELMOD analysis are dominated by inhomogeneous Malmquist bias, with uncertainties in the model velocity fields being an important role only in the innermost part of the sample. To account properly for Malmquist bias, the mass distribution in the mock catalogues needs to mimic the one in our local universe as close as possible. Unlike V1, however, we do not extract mock catalogues from constrained N-body simulations (i.e. similar to those produced by Kolatt et al. 1996). Instead, we construct ‘ideal’ PSC\(_2\) and SFI mock catalogues from the G3-smoothed PSC\(_2\) density and reconstructed linear velocity fields (described in the previous section) for \(\beta_l = 1.0\). Non-linear effects were mimicked by adding a constant Gaussian noise of 150 km s\(^{-1}\) to each of the Cartesian components of the model velocity vectors. Such a small value is meant to mimic the observed ‘coldness’ of the cosmic velocity field (e.g. Strauss et al. 1998).

20 mock PSC\(_2\) catalogues where generated with Monte Carlo techniques using different random seeds. Selection criteria close to the observational ones were applied to mimic the PSC\(_2\) selection function and the presence of unobserved regions. Mock PSC\(_2\) galaxies were generated assuming a probability proportional to the local density, i.e. assuming that they trace the mass distribution. The same reconstruction method used for the real data was then applied to each of these mocks to obtain 20 mock PSC\(_2\) velocity models for each of the 20 values of \(\beta_l = 0.5, 0.55\ldots1.5\).

Similarly, we generate 20 mock SFI catalogues by Monte Carlo resampling the original PSC\(_2\) G3-smoothed density and velocity fields with the added thermal noise. First, catalogues with large numbers of galaxies were created. Next, absolute magnitudes were assigned according to the luminosity function. Subsequently, diameters and velocity widths were assigned according to the correlations described in Section 3. TF parameters close to the ones found in our final analysis were used to assign velocity widths. Finally, the nominal selection criteria of the SFI sample were used to select galaxies for the final catalogues. The additional galaxies mentioned in Section 3 were simulated by randomly including galaxies up to the magnitude limit, until the number of galaxies not satisfying the nominal selection criteria matched that of the SFI sample. The resulting mock samples reproduce the observed redshift distribution of the SFI sample nicely.

The results of applying VELMOD to the 20 SFI mock catalogues are summarized in Table 1. The true values of the free parameters used in the mock catalogues are listed in the first row while those obtained from the VELMOD analysis are shown in the second row. For each parameter we report the average value from the 20 mocks, the error in the mean and, in parenthesis, the typical error in a single realization. These results indicate that VELMOD returns an unbiased estimate of the free parameters.

Given the ideal nature of the velocity field in these mock catalogues the errors displayed in Table 1 are likely to underestimate the real ones. In what follows, however, we will be mainly interested in the errors on \(\beta_l\) which, as V1 demonstrated, can be obtained from the values at which \(\chi^2_{\text{min}}(\beta_l)\) differs by one unit from its minimum value at \(\beta_{\text{min}}\).

5.2 Tests with the Mark III catalogue

In the second test, we repeated part of the VELMOD analysis performed by V1 and V2, but using the PSC\(_2\) model velocity field instead of the IRAS 1.2-Jy one. We applied VELMOD to the Aaronson et al. (1982, hereafter A82) and Mathewson et al. (1992, hereafter MAT) subsamples selected according to the V1 prescriptions. The selection functions for A82 and MAT and their...
coefficients were taken from Willick et al. (1996). Both samples are spatially limited to a redshift \( c_{\text{LGO}} = 3000 \text{ km s}^{-1} \). A third subsample we use, MAT2, also obtained from the Mathewson et al. (1992) catalogue, coincides with the one considered by V2 and extends out to \( c_{\text{LGO}} = 7500 \text{ km s}^{-1} \).

The results are summarized in Table 2 where the values of the free parameters obtained from our VELMOD analysis are compared to those obtained by V1 and V2 (in parenthesis). Only errors in \( \beta_T \) are quoted. Our results generally agree with those of V1 and V2. The analysis of the MAT and MAT2 samples returns values of \( \beta_T \) smaller than those obtained by V1 and V2. The difference is within the 1σ error bar and probably reflects the difference between the \( \text{IRAS} \) 1.2-Jy and \( \text{PSCz} \) model velocity fields.

The largest discrepancy is the \( \text{v}_{\text{p}} \) parameter for which we consistently obtain values larger by 30–70 per cent than those estimated by V1 and V2. Such discrepancy is statistically significant because the expected errors on \( \text{v}_{\text{p}} \) estimated from the mock catalogues and, more qualitatively, from the likelihood curve, are of the order of 40 km s\(^{-1}\). To interpret this discrepancy, one has to keep in mind that the value of \( \sigma_v \) is approximately given by the sum in quadrature of random velocity noise (\( \sigma_{\text{v}} \)) and the uncertainties in the model velocity field (\( \sigma_{\text{m}} \)).

The only possible explanation for the discrepancy in \( \sigma_v \) is that \( \sigma_{\text{m}} \), which is an intrinsic property of the velocity catalogue, is the same in both analyses, but the \( \text{PSCz} \) velocity model has a larger \( \sigma_{\text{m}} \). Indeed, because B99 estimate \( \sigma_{\text{m}} = 130 \text{ km s}^{-1} \) for the \( \text{PSCz} \) model, while V1 find \( \sigma_{\text{m}} \approx 84 \text{ km s}^{-1} \) for the \( \text{IRAS} \) 1.2-Jy model, one may think that the intrinsic velocity noise would then be similar in the \( \text{PSCz} \) (\( \sigma_v \approx 120 \text{ km s}^{-1} \)) and \( \text{IRAS} \) 1.2-Jy (\( \sigma_v \approx 100 \text{ km s}^{-1} \)). However, it is difficult to understand why errors in the \( \text{PSCz} \) model should be larger than in the \( \text{IRAS} \) 1.2-Jy one. The denser sampling and larger depth of the \( \text{PSCz} \) catalogue should decrease the uncertainties in the model velocity field, not increase them. In fact, the errors quoted by V1 and B99 were estimated using different mock catalogues. V1 used mock derived from the reconstruction technique of Kolatt et al. (1996) which are based on a PM N-body code and are intrinsically ‘colder’ than those used by B99 which were taken from the higher-resolution AP\(^3\)M N-body simulations of Cole et al. (1998). As the precision of the velocity reconstruction decreases with increasing small-scale random velocity noise, it is reasonable to suspect that the difference in the values of \( \sigma_v \) estimated by V1 and B99 simply reflects the differences in the mock catalogues rather than differences in the errors of the two model velocity fields, and that an error analysis based on the same set of mock catalogues would show that the value of \( \sigma_v \) for the \( \text{PSCz} \) model is comparable, if not smaller, than for the \( \text{IRAS} \) 1.2-Jy model velocity field.

Nevertheless, the \( \sigma_v \) discrepancy must reflect some difference in the \( \text{IRAS} \) \( \text{PSCz} \) and 1.2-Jy velocity prediction. The only remaining possibility is that such a difference is systematic rather than purely random. Indeed, the two models are different because they are based on two different redshift catalogues and because they are derived using two different reconstruction methods. The \( \text{PSCz} \) catalogue allows a denser sampling and therefore traces small-scale density fluctuations which are washed out by shot noise in the \( \text{IRAS} \) 1.2-Jy catalogue. The \( \text{IRAS} \) 1.2-Jy model used by V1 and V2 was obtained using the technique of Sigad et al. (1998) which differs from the one of B99 in the treatment of triple-valued regions, the use of Wiener filtering and, most importantly, in the allowance for mildly non-linear motions. As a result, the \( \text{PSCz} \) velocity model is intrinsically more linear than the \( \text{IRAS} \) 1.2-Jy model, which also lacks power on small scales. The linearity of the \( \text{PSCz} \) model forces any discrepancy between true and model velocities to contribute to \( \sigma_v \). In the \( \text{IRAS} \) 1.2-Jy case, such discrepancies are, on average, smaller because of the lack of small-scale power and because they are partially absorbed by the mildly non-linear motions. The net result is that VELMOD analyses based on the \( \text{IRAS} \) 1.2-Jy velocity model return values of \( \sigma_v \) that are systematically smaller and a value of \( \beta \) that is slightly larger than those based on the \( \text{PSCz} \) velocity model.

### 6 RESULTS

In this section we present the results of applying the VELMOD analysis to the SFI sample within 6000 km s\(^{-1}\) and we check the robustness of our results using smaller subsamples, smoothing scale and errors in the LG velocity.

#### 6.1 Application to the full SFI catalogue

We ran VELMOD on the full SFI sample of 989 galaxies with redshift \( c_{\text{LGO}} < 6000 \text{ km s}^{-1} \) and \( \eta > -0.25 \). The \( \text{PSCz} \) density and velocity fields used in this calculation were smoothed with a Gaussian filter of effective radius 300 km s\(^{-1}\). The likelihood was minimized at each of 19 values of \( \beta_T = 0.1, 0.15, \ldots, 1.0 \) by varying the free parameters \( A_{\text{TF}}, b_{\text{TF}}, \sigma_{\text{TF}}, \sigma_v \) and \( \sigma_T \). The resulting values of \( L_{\text{min}}(\beta_T) \) are represented by filled dots in Fig. 1. In order to find the actual minimum, we fit a cubic function to \( L_{\text{min}}(\beta_T) \):

\[
L_{\text{min}} = L_0 + a_1(\beta_T - \beta_{\text{min}})^2 + a_2(\beta_T - \beta_{\text{min}})^3
\]

The value of \( \beta_{\text{min}} \) and its errors are indicated in the plot and are also listed in Table 3 together with the values of the other parameters found at the minimum of the likelihood curve.

The values for the TF parameters listed in Table 3 should be compared with the values found by Giovannelli et al. (1997b, hereafter G97) for an associated sample of cluster galaxies, termed SCI, selected in a similar way to the SFI sample, which are commonly used as the assumed TF relation for the SFI sample. They find zero point, \( A_{\text{TF}} \), of \( -6.00 \pm 0.02 \) and slope, \( b_{\text{TF}} \), of \( 7.5 \pm 0.2 \) with a scatter of about 0.36 mag. While the range of values for the slope of this determination from cluster galaxies\(^2\)...

\[^2\text{In this work, absolute magnitudes refer to distances in units of \text{km s}^{-1}, and not of 10 pc, as often adopted in the literature.}\]
overlaps with the current results, we obtain a smaller value for the zero point and a larger scatter. This is in agreement with other analyses of the SFI data set (da Costa et al. 1998, hereafter D98; Freudling et al. 1999). A similar difference in the TF scatter of cluster and field samples has been found in other data sets (e.g. Bothun & Mould 1987; Freudling, Martel & Haynes 1991).

The recovered value of \( \sigma_v = 250 \text{ km s}^{-1} \) is twice as large as the value obtained by V1 and V2. As we have already discussed, part of this discrepancy may simply reflect differences between our model velocity field and the one used by V1 and V2. It may also indicate that the PSCz velocity model does not provide a satisfactory fit to the SFI data set. We will return to this important point in Section 7.

Fig. 2 shows the variation of the four parameters with \( \beta_1 \), where the other parameters are allowed to vary. The velocity dispersion reaches a minimum when \( \beta_1 \sim \beta_{\text{min}} \), a behaviour consistent with the analysis of the mock catalogues performed by V1 (but not with their analysis of the real A82 + MAT sample). The TF parameters are very insensitive to \( \beta_1 \). The largest variation (\( \sim 5 \) per cent) occurs for the TF scatter which has a minimum around \( \beta_1 = 0.55 \), significantly larger than \( \beta_{\text{min}} \). This is not alarming because, as stressed by V1, minimizing the TF scatter does not correspond to maximizing the likelihood. This is probably a result of the covariance between \( \alpha_v \) and \( \sigma_{\text{TF}} \); the increase of \( \alpha_v \) after its minimum is compensated by a further decrease of \( \sigma_{\text{TF}} \).

### 6.2 The effect of smoothing

In their analysis, V1 noticed that smoothing the model velocity field with a G5 filter had little effect on the VELMOD results. Along with their small value of \( \sigma_v \), this was taken as an indication that density fluctuations on scales between 300 and 500 km s\(^{-1}\) contribute little to both the Mark III and the IRAS 1.2-Jy velocity fields. However, the larger value of \( \sigma_v \) we have found in our VELMOD analysis of the A82, MAT and SFI samples seems to indicate that contributions from small scales are non-negligible.

To investigate this issue, we have repeated the same exercise and performed a VELMOD analysis using a G5-smoothed PSCz model velocity field instead of the G3 one. We find that the results are insensitive to the smoothing scale, with \( \beta_{\text{min}} \) increasing only by \( \sim 5 \) per cent, consistent at the 1\( \sigma \) level with the value found in the G3 case. The same is true for the other free parameters with the exception of \( \sigma_v \), which increases by \( \sim 15 \) per cent. Moreover, the value of \( \mathcal{L}_{\text{min}} \) at \( \beta_{\text{min}} \) increases by 10 units with respect to the G3 case, corresponding to a probability decrease by a factor \( e^{-20} \approx 90 \) (as \( \mathcal{L} = -2 \ln P \)). How do we interpret these results? With a G5 filter, the model velocity field does not receive contributions from scales smaller than \( 500 \text{ km s}^{-1} \) and larger values of \( \beta_1 \) and \( \sigma_v \) are needed to match the amplitude of the observed velocities. This means that the linear PSCz model velocity field does receive a non-negligible contribution from small scales. The significant increase in the likelihood indicates that the G3 velocity field provides a better fit to the SFI velocities, i.e. small-scale fluctuations do contribute to the peculiar velocities of galaxies in the real world.

### 6.3 Uncertainties in the velocity of the local group

As model predictions are given in the LG frame (i.e. the LG velocity is subtracted from all other velocities), we need to quantify the impact of uncertainties in the predicted and measured LG velocity when performing the VELMOD analysis. V1 and V2 tackled the problem by introducing a free parameter to model a random component to the LG velocity vector. Here we take a different approach aimed at minimizing the number of free parameters in the analysis. Instead of modelling the uncertainties in the LG velocities, we quantify the impact that these uncertainties, independently estimated, may have on our VELMOD analysis. Errors in the LG velocity derive from uncertainties in modelling the LG velocity and in transforming redshifts from the heliocentric to the LG frame. The former have been quantified by Schmoldt

\[
cz_{\text{LG}} < 6000 \text{ km/s}
\]

![Figure 1. The VELMOD likelihood, \( \mathcal{L}_{\text{min}}(\beta_1) \), (filled symbols) for the full SFI sample. The dotted curve represents a cubic fit to the points.](image-url)
et al. (1999), while for the latter we consider the recent work of Courteau & van den Bergh (1999). We assume that these error estimates are independent and so we compute the total uncertainty by adding them in quadrature. The resulting total error in the LG velocity is $120 \beta_1 \, \text{km s}^{-1}$ for each Cartesian component.

To evaluate the impact of these errors we ran 10 VELMOD analyses in which the LG velocity was perturbed with a Gaussian noise of the same amplitude. The effect is to increase the random errors without introducing any systematic bias. Averaging over the 10 experiments we obtain a value of $\beta_1 = 0.43$ with a 1σ error around the mean of 0.05, slightly larger than the typical error in a single realization. Similar considerations apply to the other parameters. In all but one experiment, the value of $L_{\text{min}}(\beta_{\text{min}})$ was larger than in the unperturbed case. In the one case with smaller likelihood (by only two units) the perturbation turned out to be very small, in agreement with the V1 and V2 results in which the extra random components added to the LG velocity vector turned out to be trivially small.

### 6.4 Breakdown by rotation velocities

In their calibration of the TF relation for SCI galaxies, G97 found that the TF scatter is a function of the velocity width: rapidly rotating galaxies have a smaller TF scatter than slow rotators. A similar trend has also been detected by Federspiel, Sandage & Tamman (1994) and by Willick et al. (1997a) in some of the Mark III subsamples. In the SFI catalogue itself this effect has been included for the direct TF relation by Freudling et al. (1999), but not in the inverse TF relation (D98). In the current analysis, we have chosen to neglect this effect. To estimate the impact of this approximation we divided the SFI catalogue into two subsamples according to the rotational velocity of the galaxies. In the first catalogue, we included only objects with a velocity width parameter $h_{20} > 1$, while in the second we included slow rotators with $h_{20} < 1$. The results of applying VELMOD on these two samples are shown in Table 3. The TF dispersion is found to be smaller for fast rotators, as could be expected. However, the

| Sample $c_{z,\text{LG}}$ (km s$^{-1}$) | $N_{\text{gal}}$ | $A_{\text{TF}}$ | $b_{\text{TF}}$ | $\sigma_{\text{TF}}$ | $\sigma_{\beta}$ | $\beta_1$ |
|-------------------------------------|-----------------|----------------|-----------------|-------------------|----------------|----------|
| $\leq 6000$                         | 989             | $-5.89$        | 7.19            | 0.439             | 250            | $0.42 \pm 0.04$ |
| $6000 < c_{z,\text{LG}} < 6000$, $\eta > -0.1$ | 640             | $-5.89$        | 6.74            | 0.382             | 230            | $0.43 \pm 0.04$ |
| $6000 < c_{z,\text{LG}} < 6000$, $\eta < -0.1$ | 349             | $-5.87$        | 7.04            | 0.498             | 248            | $0.39 \pm 0.04$ |
| $c_{z,\text{LG}} < 4000$            | 493             | $-5.86$        | 7.32            | 0.415             | 216            | $0.45 \pm 0.05$ |
| $c_{z,\text{LG}} < 2000$            | 158             | $-5.73$        | 6.67            | 0.454             | 206            | $0.42 \pm 0.09$ |
| $2000 \leq c_{z,\text{LG}} < 4000$  | 335             | $-5.88$        | 7.41            | 0.410             | 196            | $0.44 \pm 0.07$ |
| $4000 \leq c_{z,\text{LG}} < 6000$  | 496             | $-5.85$        | 7.54            | 0.420             | 255            | $0.40 \pm 0.09$ |

![Figure 2](https://academic.oup.com/mnras/article-abstract/326/3/1191/956251) SFI versus PSCz 1197

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values of the other free parameters, in particular $\beta_{\text{min}}$, are very similar in the two subsamples, showing that our approximation of a constant $\sigma_{\text{TF}}$ has little impact on our $\beta_1$ estimates.

### 6.5 Breakdown by redshift

As a last robustness test we have divided our sample into three independent redshift shells $2000\,\text{km}\,\text{s}^{-1}$ thick and applied VELMOD to each of them. As the selection criteria are different in the three shells, this test serves as a check of whether selection effects are properly taken into account by our procedure. The resulting $L_{\text{null}}(\beta)$ curves are displayed in Fig. 3 and the free parameters are listed in Table 3. The values of $\beta_{\text{min}}$ in the three shells are consistent with each other. The remaining parameters are also in good agreement, with only two exceptions. One is the TF slope, which in the innermost shell is $\sim 10$ per cent shallower than in the rest of the sample. The analysis of SFI mock catalogues in Section 5.1 reveals that such a discrepancy is significant at the $1.5\sigma$ level. However, as we have already pointed out, the errors estimated using those catalogues are smaller than the real ones. A VELMOD analysis of more realistic SFI mocks would return larger errors and decrease the statistical significance of the slope variation.

The other, more serious, discrepancy is observed for $\sigma_r$, which in the outermost shell increases by a factor of $\sim 2.7$. Such a variation is much larger than what we observe in the mock catalogues and deserves some further investigation. We have therefore sliced the SFI sample into thinner redshift shells, all of them $1000\,\text{km}\,\text{s}^{-1}$ thick, except the innermost one which we extended to $2000\,\text{km}\,\text{s}^{-1}$ so that a comparable number of objects is contained in each shell. We then repeated the VELMOD analysis in each of these shells. The results are visualized in the histograms of Fig. 4. The two dashed lines represent the $1\sigma$ error about the mean. The upper plot shows the radial behaviour of $\sigma_r$. Within $4000\,\text{km}\,\text{s}^{-1}$, $\sigma_r = 200\,\text{km}\,\text{s}^{-1}$, comparable to what we have obtained from the analysis of the A82, MAT and MAT2 samples. This value is larger than those found by V1 and V2 and, as discussed in Section 5.2, this is probably because the PSCz model velocity field is more sensitive to power on small scales. Around $5000\,\text{km}\,\text{s}^{-1}$, $\sigma_v$ increases to $\sim 600\,\text{km}\,\text{s}^{-1}$ and then stabilizes at a value of $\sim 400\,\text{km}\,\text{s}^{-1}$.

In order to test whether any particular part of the sky is responsible for this dependence of $\sigma_r$ on distance, we have cut the sample into several complementary hemispheres (e.g. above and below the Galactic plane, above and below the supergalactic plane and so on) and, for each pair, we have ran VELMOD on the two sets of redshift shells. In all cases we have found that $\sigma_r$ increases beyond $4000\,\text{km}\,\text{s}^{-1}$, suggesting that no particular cosmic structure is responsible for the increase in $\sigma_r$.

The ability of VELMOD to constrain $\sigma_r$ is a rapidly decreasing function of the redshift and we need to assess the statistical significance of the jump at $4000\,\text{km}\,\text{s}^{-1}$, i.e. we need to estimate the errors on $\sigma_r$. We compute them from the distribution of $\sigma_r$ found from the analysis of our 20 mock SFI samples. This test reveals that the increase of $\sigma_r$ is significant at about the $3\sigma$ level, leaving little doubt about the reality of the change at around $4000\,\text{km}\,\text{s}^{-1}$. However, as we have already stressed, such tests are only indicative, because non-linearities are not properly modelled in our mock samples.

To further assess the reality of this peak we performed a second test. We ran two VELMOD analyses using only objects with redshifts in the range $4000$–$6000\,\text{km}\,\text{s}^{-1}$. In the two calculations all the parameters were fixed to their maximum likelihood values, except $\sigma_r$ which was set equal to $200\,\text{km}\,\text{s}^{-1}$ in one case and $535\,\text{km}\,\text{s}^{-1}$ (i.e. its maximum likelihood value) in the other. The smaller $\sigma_r$ results in an increase of 12 units in the likelihood or a decrease in probability of $\sim 6^\circ = 400$, indicating that the increase in $\sigma_r$ seen in the external shell is indeed significant. It is therefore possible that the dramatic increase in $\sigma_r$ reflects a disagreement between the velocity model and the SFI data on large scales. We will investigate this possibility further in the next section.

The lower plot of Fig. 4 shows how $\beta_{\text{min}}$ varies with redshift. VELMOD returns a very robust estimate of $\beta_{\text{min}}$ which does not change significantly even when $\sigma_r$ increases. Despite this encouraging evidence, it is sensible to adopt a more conservative approach and repeat the VELMOD analysis within $4000\,\text{km}\,\text{s}^{-1}$, i.e. in the volume where there are no obvious indications of a possible mismatch between model and data. The results, listed in Table 3, are in good agreement with those of the full sample. In particular, the minimum of the likelihood curve (shown in Fig. 5), $\beta_{\text{min}} = 0.45 \pm 0.05$, is fully consistent with the results from the other subsamples. The velocity dispersion is $\sim 200\,\text{km}\,\text{s}^{-1}$, in good agreement with those obtained from the analysis of the A82, MAT and MAT2 samples.

### 7 Analysis of the Velocity and Magnitude Residuals

VELMOD is a maximum likelihood technique in which the sources of variance, $\sigma_r$ and $\sigma_{\text{TF}}$, are treated as free parameters. For this reason, the VELMOD analysis can only tell us which are the best values of $\beta_1$, $A_{\text{TF}}$, $b_{\text{TF}}$, $\sigma_{\text{TF}}$ and $\sigma_r$ for a given velocity field model, but it cannot address the question of whether the velocity model is an acceptable fit to the data. In this respect, the increase of...
\( \sigma_v \) at large radii found in the preceding section is only suggestive of a mismatch between observed and modelled velocities, and a proper error analysis is needed to assess its statistical significance. In this section we address this problem by inspecting the magnitude and velocity residuals, following the formalism and notation of V1.

### 7.1 Maps of velocity residuals

We define the normalized magnitude residuals for each object of magnitude \( m \):

\[
\delta_m = \frac{m - E(m|n, cz)}{\Delta_m},
\]

where the expected apparent magnitude, \( E(m|n, cz) \), and the dispersion around it, \( \Delta_m \), can be obtained by integrating over the constrained probability function, \( P(m|n, cz, LG) \), which is computed in the VELMOD analysis. The magnitude residuals where smoothed on a scale \( S = d/5 \), where \( d \) is the PSC\( z \) predicted distance of the generic object, and then converted into smoothed peculiar velocity residuals, \( u_{SFI} - u_{PSCz} \), according to the V1 prescriptions. The resulting maps of the velocity residuals are shown in Figs 6–8 for three different values of \( \beta_I \) and in three different redshift shells. When looking at the maps one should bear in mind that coherence up to scales of \( \sim 35\,\text{arcmin} \) is induced by the adopted smoothing and therefore systematic mismatches between the model and reality can only be revealed by coherence on much larger angular scales. The visual inspection of the maps clearly shows that the velocity residuals for \( \beta_I = 0.4 \) are less coherent and have smaller amplitudes than those of the models with \( \beta_I = 0.1 \) and \( \beta_I = 1.0 \). The residuals in the \( \beta_I = 0.1 \) map show a pattern similar to \( \beta_I = 0.4 \) case, but have a larger amplitude and coherence. When \( \beta_I = 1.0 \), the residuals grow even larger and the map exhibits a clear dipolar structure. The velocity residuals of the model with \( \beta_I = 0.4 \) look qualitatively similar to the ones between the SFI and IRAS 1.2-Jy velocities for \( \beta_I = 0.6 \), as seen in fig. 7 of D98. However, a one-to-one comparison between the two sets of maps is not possible because different smoothing procedures were used, and because our residuals are computed at the PSC\( z \) predicted distances whereas the ones of D98 are computed at the redshift-space positions.

### 7.2 Residual correlation function

A more quantitative assessment of the goodness of fit can be obtained by computing the correlation function of the unsmoothed magnitude residuals:

\[
\psi(\tau) = \frac{1}{N(\tau)} \sum_{i<j} \frac{\delta_{ni} \delta_{nj}}{\Delta_{ni} \Delta_{nj}},
\]

where \( N(\tau) \) is the number of galaxy pairs with predicted separation \( d_i \leq \tau \leq 100\,\text{km\,s}^{-1} \). This correlation function applies to normalized magnitude residuals, i.e. it does not depend on \( \sigma_v \) or \( \sigma_{TF} \) and is only sensitive to genuine correlation among residuals.

In Fig. 9 we show the correlation function of all SFI galaxies with \( cz_{LG} < 4000\,\text{km\,s}^{-1} \) for the same three values of \( \beta_I \) used to produce the velocity residuals shown in Figs 6–8. The error bars represent Poisson errors \( N(\tau) \frac{1}{N(\tau)} \). In the \( \beta_I = 0.4 \) model, the correlation function appears to be consistent with zero almost everywhere apart from some positive correlation at separations

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**Figure 4.** Dependency of \( \sigma_v \) (upper plot) and \( \beta_{min} \) (lower plot) on redshift. The histograms show the results of running VELMOD on six different redshift shells. In both plots the dashed lines represent 1σ errors around the mean.

**Figure 5.** The VELMOD likelihood, \( L_{\text{min}}(\beta_I) \), (filled symbols) for SFI galaxies with \( cz_{LG} < 4000\,\text{km\,s}^{-1} \). The dotted curve represents a cubic fit to the points.
smaller than 500 km s\(^{-1}\), i.e. of the order of the smoothing scale of the model velocity field. A significant excess correlation on small and large scales is detected for \(b_I \hat{\beta}_0\): to an even greater degree, for \(b_I \hat{\beta}_1\).

To quantify the goodness of the fits we compute the quantity

\[
\chi^2_j = \sum_{i=1}^{N_{\text{bins}}} \frac{\xi^2(\tau)}{N(\tau)},
\]

where \(\xi(\tau) = N(\tau)\psi(\tau)\) and \(N_{\text{bins}}\) is the number of independent distance bins in which \(\psi(\tau)\) is computed. V1 have shown that if the residuals are indeed uncorrelated on a scale \(\tau\) then \(\xi(\tau)\) is a Gaussian random variable with zero mean and variance \(N(\tau)\). Under this approximation, the \(\chi^2\) statistic is distributed as \(\chi^2\) with \(N_{\text{bins}}\) degrees of freedom equal to the number of independent distance bins. Any correlation among residuals will result in a larger \(\chi^2\). Extensive tests with mock catalogues performed by V1 revealed that \(\chi^2\) indeed has properties similar to a \(\chi^2\) statistic, with the same variance, but with a mean of \(\sim 0.87\) per degree of freedom, rather than unity. We have computed the quantity \(\chi^2\) for 10 values of \(\hat{\beta}_l\) ranging from 0.1 to 1.0. The results are shown in Fig. 10. The continuous, heavy line shows the expectation value for a \(\chi^2\) statistic with \(N_{\text{bins}} = 60\) degrees of freedom, while the dashed and long-dashed lines show, respectively, the 1\(\sigma\) and 3\(\sigma\) deviations from that value. The dotted line represents the expectation value of \(\chi^2\) according to the V1 correction, i.e. assuming a number of degrees of freedom of \(0.87 \times 60 = 52.2\).

Despite its limited discriminatory power, the \(\chi^2\) statistic clearly indicates that PSCz velocity models with \(0.3 < \beta_l < 0.5\), and therefore also our best model according to the VELMOD analysis, \(\beta_l = 0.42 \pm 0.04\), provide an acceptable fit to the SFI velocities. All other models with smaller or larger values of \(\beta_l\) can be ruled out at a level \(> 3\sigma\).

We are now in a position to address the question of whether the PSCz model velocity field provides an acceptable fit throughout the whole SFI sample, particularly in the external regions where \(s_v\) is large. We do that by computing the residual correlation function in the same three redshift shells previously used. The results are displayed in Fig. 11 which shows the correlation functions, \(\psi(\tau)\), for the three subsamples.

The value of \(\psi(\tau)\) appears to be consistent with the null hypothesis of no correlation in nearly all the distance bins in each of the three redshift shells, with the exception of an excess correlation at small separation in the outermost shell. The value of the statistic \(\chi^2\) in each shell is indicated in the plots. It is always smaller than the number of bins used in each shells and comparable to the expectation value for \(\chi^2\) computed using the V1 correction. These results show that, despite the large value of \(s_v\) obtained from the VELMOD analysis, the residuals of the fit are not correlated.
i.e. the differences in magnitude between model and data are randomly distributed.

8 DISCUSSION

The results of our VELMOD analysis generally agree with those of independent analyses, with the exception of the velocity noise, \( \sigma_v \). We find a significantly larger value than that found by V1 and V2 who compared the Mark III velocities to the IRAS 1.2-Jy model velocity field. A VELMOD analysis of SFI subsamples limited in redshift revealed the existence of two regions. An inner sphere of radius 4000 km s\(^{-1}\) in which \( \sigma_v = 200 \) km s\(^{-1}\), a value which is believed to reflect the cumulative effect of velocity noise and errors in the model predictions (as corroborated by the results of the analysis of the A82 and MAT samples), and an external region in which \( \sigma_v \sim 500 \) km s\(^{-1}\).

If these large values of \( \sigma_v \) were caused by systematic deviations of the SFI velocity field from the model predictions, then the residuals should correlate on scales larger than the smoothing length. However, this is not the case (see Fig. 10). In fact, the residuals turn out to be quite small and do not show any significant spatial correlation. This is true for all redshift shells, including the outermost one where \( \sigma_v \) is large. Moreover, a quantitative analysis based on a \( \chi^2 \) statistic shows that the PSC\(_{z} \) velocity model with \( 0.3 < \beta_l < 0.5 \) does provide a satisfactory fit to the SFI data without the need to introduce any external fields as in the V1 and V2 VELMOD analyses. This can be interpreted as implying that discrepancies between the model and the data do exist, but not in the form of peculiar motions coherent on scales larger than \( \sim 300 \) km s\(^{-1}\). Rather, they are randomly distributed and thus properly quantified by means of the total variance, to which both \( \sigma_v \) and \( \sigma_{\text{TF}} \) contribute. It is worth noticing that the increase in \( \sigma_v \) occurs at the distance at which large structures like the Perseus-Pisces supercluster and the Great Attractor appear in the sample. Non-linear effects may be strongest in these structures, leading to a mismatch on small scales between measured and predicted velocity fields which could contribute to the increase in \( \sigma_v \). To investigate whether these structures are indeed responsible for the increase in \( \sigma_v \), we ran two VELMOD analyses in which we have excluded either SFI galaxies in the Perseus-Pisces supercluster (i.e. with \( 120^\circ \leq l \leq 170^\circ, -35^\circ \leq b \leq 5^\circ \) and \( 4000 \) km s\(^{-1} \leq c_{\text{LG}} \leq 6000 \) km s\(^{-1} \) or in the Hydra Centaurus region (i.e. with \( 260^\circ \leq l \leq 330^\circ, -10^\circ \leq b \leq 40^\circ \) and \( 2000 \) km s\(^{-1} \leq c_{\text{LG}} \leq 6000 \) km s\(^{-1} \)). A third analysis has been performed after excluding galaxies in both structures. The results turned out to be very similar to those shown in Fig. 4 for the full sample case (the differences are of the order of \( 20 \) km s\(^{-1} \)). The abrupt increase of \( \sigma_v \) at \( c_{\text{LG}} \geq 4000 \) km s\(^{-1} \) is present in all the cases explored, showing that neither the Perseus-Pisces supercluster nor the Great Attractor contribute appreciably to the variation of \( \sigma_v \).

A second possibility is that the increase in \( \sigma_v \) with redshift is caused by a comparable increase in the errors of the velocity

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model. Indeed, B99 quantified such effect (see their equation 18) which, however, is far too small to explain the observed increase in $s_v$.

The final possibility is that the TF scatter decreases significantly with distance, which seems reasonable, because the scatter tends to decrease for high-linewidth galaxies, and these are seen preferentially at large distances. Because of the covariance between $s_v$ and $s_{TF}$ (see V1), this could lead to an overestimation of $s_v$ and an underestimation of $s_{TF}$. In this context, it is interesting to note that a power spectrum analysis of the same SFI data set, like the one performed by Freudling et al. (1999), also returns different results if computed separately for the closer and more distant halves of the sample. However, such an analysis reveals that it is the inner part of the sample which has an apparently enhanced $s_v$. Given these conflicting results, we consider it unlikely that the apparent increase in $s_v$ reflects a true property of the velocity field. In any case, it is worth stressing that the increase of $s_v$ with distance does not affect the estimate of $b_I$ which, as shown in Fig. 4 and Table 3, does not show significant variations.

9 SUMMARY AND CONCLUSIONS

We have implemented, tested and used the VELMOD technique (V1; V2) to compare the model velocity field obtained from the spatial distribution of PSCz galaxies with the velocities of field spiral galaxies within $6000 \text{ km s}^{-1}$ in the SFI catalogue. This

Figure 8. The sky projection of the smoothed VELMOD velocity residuals, $u_{SFI} - u_{PSCz}$, for $\beta_I = 1.0$.

Figure 9. The correlation function of magnitude residuals plotted for $\beta_I = 0.1$ (lower plot), $\beta_I = 0.4$ (middle plot) and $\beta_I = 1.0$ (upper plot). The results refer to the full SFI sample.
obtained by V1 and V2. Although the meaning of such a large

Figure 10. The statistic \( \chi^2 \) plotted for various values of \( \beta_t \). The heavy line shows the expectation value for a \( \chi^2 \) statistic with the same number of degrees of freedom. The dashed and long-dashed lines show 1\( \sigma \) and 3\( \sigma \) deviations from the expectation value. The dotted line shows the expectation value of \( \chi^2 \) corrected according to V1.

correction allowed us to estimate the value of the \( \beta_t \) parameter, the amplitude of the small-scale velocity noise, \( \sigma_v \), and to calibrate the TF relation of SFI galaxies.

VELMOD returns an estimate of \( \beta_t \) which is very robust to various systematic and random errors that may enter the analysis at various stages. For the full sample, we obtain \( \beta_t = 0.42 \pm 0.04 \), while a value of \( \beta_t = 0.45 \pm 0.05 \) is obtained when a more conservative cut at 4000 km s\(^{-1}\) is applied to the SFI sample.

The slope of our TF relation is in good agreement with that obtained from the calibration of the TF relation by G97 using SCI galaxies. However, we find a significantly larger TF scatter (\( \sigma_{TF} = 0.44 \)) than the average scatter found by G97 (\( \sigma_{TF} = 0.36 \)), and a significantly larger zero point (\( A_{TF} = -5.89 \) versus \( A_{TF} = -6.09 \)). We thus conclude that the scatter in the TF relation for field galaxies is larger and the zero point is smaller than the corresponding values for galaxies in clusters, confirming the results of D98 and Freudling et al. (1999). Our analysis also confirms that the TF scatter decreases with increasing galaxy rotation velocity, but this does not affect our estimates of \( \beta_t \). We have also found that the velocity noise, \( \sigma_v \), increases with distance up to \( \sim 500 \) km s\(^{-1}\) and is significantly larger than the values obtained by V1 and V2. Although the meaning of such a large \( \sigma_v \) is not clear, it is reassuring that it does not affect the general result of the VELMOD analysis and, in particular, the value of \( \beta_t \).

In summary, we have found that the PSC\( z \) velocity model provides a satisfactory fit to the velocities of SFI galaxies when a value of \( \beta_t = 0.42 \pm 0.04 \) is used. This value is in good agreement with most of the recent \( v-v \) analyses that use the IRAS gravity field, inferred either from the 1.2-Jy or the PSC\( z \) surveys, irrespective of the type of distance indicator used. Indeed, values of \( \beta_t \) in the range 0.4–0.6 are found using the TF relation for spiral galaxies (Davis et al. 1996; V1; V2; D98), the \( D_n-\sigma \) relation for early type galaxies (Nusser et al. 2000), Type Ia Supernovae (Riess et al. 1997) and the ‘surface brightness fluctuations’ method in nearby galaxies (Blakeslee et al. 2000). Our estimate of \( \beta_t \) is also consistent with the results of \( v-v \) analyses which use a model gravity field derived from optical galaxies in the Optical Redshift Survey (Santiago et al. 1995, 1996), once the stronger clustering of optical galaxies is taken into account.

These values of \( \beta_t \) from \( v-v \) analyses are significantly smaller than those found using d–d methods (e.g. Sigad et al. 1998), even when both are applied to identical data sets. Further support for high values of \( \beta \) comes from power spectrum analyses of various velocity catalogues (Zaroubi et al. 1997; Freudling et al. 1999; Zaroubi et al. 2001), although in this case it is possible that accounting for non-linear effects may help reduce the discrepancy (Silberman et al. 2001). As \( v-v \) methods generally rely on the deviations of a set of observed velocities from a set of model velocities, differences between the two sets do not contribute to \( \beta \) but to the random errors, whereas this is not the case in power spectrum analyses.

A different explanation for the discrepancy between \( v-v \) and d–d methods is that the \( v-v \) analyses are affected by non-linear effects. All \( v-v \) methods implemented so far assume linear biasing and most of them also use linear gravitational instability theory. Recently new methods have been introduced to measure the nonlinearity in the bias relation in redshift surveys (e.g. Matarrese, Verde & Heavens 1997; Szapudi 1998; Sigad, Branchini & Dekel 2000; Feldman et al. 2001) and some of these have been applied to the PSC\( z \) survey. For example, Branchini et al. (in preparation) have applied the technique of Sigad et al. (2000) to the PSC\( z \) sample. Their preliminary results indicate that deviations from linear biasing are small and manifest themselves mainly as an anti-bias in low density regions. The amplitude of these effects can only lead to a modest increase of \( \beta_t \) of the order of \( \sim 10 \) per cent.

Non-linear motions might also change the value of \( \beta_t \). However,
Shaya, Peebles & Tully (1995) used a fully non-linear model of the velocity field and still recovered a low value of $\beta$. Similarly, $V_1$ performed a non-linear VELMOD analysis in an attempt to break the degeneracy between $\Omega_m$ and $b_1$ and found indirect evidence that non-linear motions are already accounted for in the model velocity field when this is predicted from a smooth density field. These results suggest that a treatment of nonlinear effects and the inclusion of non-linear biasing prescriptions in $v-v$ analyses will not change the inferred value of $\beta$ significantly. Thus, it seems unlikely that the disagreement between the two methods will be eliminated purely by improving the existing $v-v$ techniques.

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REFERENCES

Aaronson M. et al., 1982, ApJS, 50, 241
Beichman C. A., Neugebauer G., Habing H. J., Clegg P. E., Chester T. J., 1988, IRAS Catalogs and Atlases, Version 2, Explanatory Supplement. NASA Ref. Publ., p. 1190
Bertschinger E., Dekel A., 1989, ApJ, 336, L5
Blakeslee J. et al., 2000, in Crotta S., Strauss M., Willick J. eds, ASP. Conf. Ser. Cosmic Flows 1999: Towards an Understanding of Large-Scale Structures. Astron. Soc. Pac., San Francisco, p. 254
Bothun G., Mould J., 1987, ApJ, 313, 629
Brandini E. et al., 1999, MNRAS, 308, 871 (B99)
Canaveses A. et al., 1998, MNRAS, 297, 777
Cole S., Hatton S., Weinberg D., Frenk C., 1998, MNRAS, 300, 945
Courteau S., van den Bergh S., 1999, AJ, 118, 337
da Costa L., Nusser A., Freudling W., Giovanelli R., Haynes M., Salzer J., Wehrse G., 1998, MNRAS, 299, 452, (D98)
da Costa L. et al., 2000, AJ, 120, 95
Davis M., Nusser A., Willick J., 1996, ApJ, 473, 22
Federspiel M., Sandage A., Tamman G., 1994, ApJ, 430, 29
Feldman H., Frieman J., Fry N., Scoccimarro R., 2001, Phys. Rev. Lett., 86, 1434
Fisher K., Huchra J., Strauss M., Davis M., Yahil A., Schlegel D., 1995, ApJS, 100, 69
Freudling W., Martel H., Haynes M., 1991, ApJ, 317, 1
Freudling W., da Costa L., Wegner G., Giovanelli R., Haynes M., Salzer J., 1995, AJ, 110, 1995
Freudling W., Zehavi I., da Costa L. N., Dekel A., Eldar A., Giovanelli R., Haynes M. P., Salzer J. L., Wegner G., Zaroubi S., 1999, ApJ, 523, 1
Giovanelli R., Haynes M., Herter T., Vogt N., Wegner G., Salzer J., da Costa L., Freudling W., 1997a, AJ, 113, 22
Giovanelli R., Haynes M., Herter T., Vogt N., da Costa L., Freudling W., Salzer J., Wegner G., 1997b, AJ, 113, 53, (G97)
Haynes M., Giovanelli R., Salzer J., Wegner G., Freudling W., da Costa L., Herter T., Vogt N., 1999a, AJ, 117, 1668
Haynes M., Giovanelli R., Chamaraux P., da Costa L., Freudling W., Salzer J., Wegner G., 1999b, AJ, 117, 2039
Hudson M., 1994, MNRAS, 266, 475
Kolatt T., Dekel A., Ganon G., Willick J. A., 1996, ApJ, 458, 419
Mathewson D., Ford V., Buchhorn M., 1992, ApJL, 81, 412, (MAT)
Matarrese S., Verde L., Heavens, 1997, MNRAS, 290, 651
Monaco P., Efstathiou G., 1999, MNRAS, 308, 763
Nusser A., Davis M., 1994, ApJ, 421, 1L
Nusser A., da Costa L., Branchini Enzo., Bernardi M., Alonso M., Wegner Gary, Willmer C., Pellegrini P., 2000, MNRAS, 320, L21
Riess A., Davis M., Baker J., Kirschner R., 1997, 488, L1
Santiago B., Strauss M., Lahav O., Davis M., Dressler A., Huchra J., 1995, ApJ, 446, 457
Santiago B., Strauss M., Lahav O., Davis M., Dressler A., Huchra J., 1996, ApJ, 461, 38
Saunders W. et al., 2000, MNRAS, 317, 55
Schmoldt I. et al., 1999, MNRAS, 304, 893
Shaya E., Beechman J., Tully B., 1995, ApJ, 454, 1
Sigd Y., Dekel A., Strauss M., Yahil A., 1998, ApJ, 495, 516
Sigd Y., Branchini E., Dekel A., 2000, ApJ, 540, 62
Silverman L., Dekel A., Eldar A., Zehavi I., 2001, preprint (astro-ph/0101361)
Springel V., 1996, MSc thesis, Eberhard-Karls-Universität, Tübingen
Sutherland W. et al., 1999, MNRAS, 308, 289
Strauss M., Ostriker J., Cen R., 1998, ApJ, 494, 20
Strauss M., Willick J., 1995, Phys. Rep., 261, 271
Szapudi I., 1998a, MNRAS, 300, L35
Willlick J., 1994, ApJS, 92, 1
Willlick J., 2001, Proceedings of the XXXVth Rencontres de Moriond: Energy Densities in the Universe. in press (astro-ph/0003232)
Willick J., Strauss M., 1998, ApJ, 486, 629, (V2)
Willick J., Courteau S., Faber S., Burstein D., Dekel A., 1995, ApJ, 446, 12
Willick J., Courteau S., Faber S., Burstein D., Dekel A., Kolatt T., 1996, ApJ, 457, 333
Willick J., Courteau S., Faber S., Burstein D., Dekel A., Strauss M., 1997a, ApJS, 109, 333
Willick J., Strauss M., Dekel A., Kolatt T., 1997b, ApJ, 486, 629, (V1)
Yahil A., Strauss M. A., Davis M., Huchra J. P., 1991, ApJ, 372, 380
Zaroubi S., Zehavi I., Dekel A., Hoffman Y., Kolatt T., 1997, ApJ, 486, 21
Zaroubi S., Bernardi M., da Costa L. N., Hoffman Y., Alonso V., Wegner G., Willmer C. N. A., Pellegrini P. S., 2001, MNRAS, in press (astro-ph/000558)

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