DOUBLE PARTON CORRELATIONS IN PERTURBATIVE QCD

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This talk brings attention to what is knowable from perturbative QCD theory on two-parton distribution functions in the light of CDF measurements of the inclusive cross section for double parton scattering.

The Collider Detector at Fermilab (CDF) Collaboration has measured a large number of double parton scattering [1] providing new and complementary information on the structure of the proton and parton-parton correlations. The possibility of observing two separate hard collisions has been proposed since long [2], and from that has also developed in a number of works [3, 4]. A brief review of the current situation and some progress in the modeling account of correlated flavour, colour, longitudinal and transverse momentum distributions can be found in Ref. [5]. Multiple interactions require an ansatz for the structure of the incoming beams, i.e. correlations between the constituent partons. As a simple ansatz, usually, the two-parton distributions are supposed to be the product of two single-parton distributions times a momentum conserving phase space factor. In recent papers [6] it has been shown that this hypothesis is in some contradiction with the leading logarithm approximation of perturbative QCD (in the framework of which a parton model, as a matter of fact, was established in the quantum field theories [7]).

In order to be clear and to introduce the denotations let us recall that, for instance, the differential cross section for the four-jet process (due to the simultaneous interaction of two parton pairs) is given by [3]

\[ \frac{d\sigma_{ij}}{d\sigma_{eff}} D_p(x_1, x_3) D_p(x_2, x_4), \]

where \( d\sigma_{ij} \) stands for the two-jet cross section. The dimensional factor \( \sigma_{eff} \) in the denominator represents the total inelastic cross section which is an estimate of the size of the
hadron, $\sigma_{\text{eff}} \simeq 2\pi r_p^2$ (the factor 2 is introduced due to the identity of the two parton processes). With the effective cross section measured by CDF, $(\sigma_{\text{eff}})_{\text{CDF}} = (14.5 \pm 1.7^{+1.7}_{-2.3})$ mb [1], one can estimate the transverse size $r_p \simeq 0.5$ fm, which is too small in comparison with the proton radius $R_p$ extracted from $ep$ elastic scattering experiments. The relatively small value of $(\sigma_{\text{eff}})_{\text{CDF}}$ with respect to the naive expectation $2\pi R_p^2$ was, in fact, considered [4] as evidence of nontrivial correlation effects in transverse space. But, apart from these correlations, the longitudinal momentum correlations can also exist and they were investigated in Ref. [6]. The factorization ansatz is just applied to the two-parton distributions incoming in Eq. (1):

$$D_p(x_i, x_j) = D_p(x_i, Q^2) D_p(x_j, Q^2) (1 - x_i - x_j), \quad (2)$$

where $D_p(x_i, Q^2)$ are the single quark/gluon momentum distributions at the scale $Q^2$ (determined by a hard process).

However many parton distribution functions satisfy the generalized Gribov-Lipatov-Altarelli-Parisi-Dokshitzer (GLAPD) evolution equations derived for the first time in Refs [8, 9] as well as single parton distributions satisfy more known and cited GLAPD equations [7, 10]. Under certain initial conditions these generalized equations lead to solutions, which are identical with the jet calculus rules proposed originally for multiparton fragmentation functions by Konishi-Ukawa-Veneziano [11] and are in some contradiction with the factorization hypothesis (2). Here one should note that at the parton level this is the strict assertion within the leading logarithm approximation.

After introducing the natural dimensionless variable

$$t = \frac{1}{2\pi b} \ln \left[ 1 + \frac{g^2(\mu^2)}{4\pi b} \ln \left( \frac{Q^2}{\mu^2} \right) \right] = \frac{1}{2\pi b} \ln \left[ \frac{\ln(Q^2_{\Lambda^2})}{\ln(Q^2_{\Lambda^2})} \right], \quad b = \frac{33 - 2n_f}{12\pi} \quad \text{in QCD},$$

where $g(\mu^2)$ is the running coupling constant at the reference scale $\mu^2$, $n_f$ is the number of active flavours, $\Lambda_{\text{QCD}}$ is the dimensional QCD parameter, the GLAPD equations read [7, 10]

$$\frac{dD_i^j(x, t)}{dt} = \sum_{j'} \int \frac{dx'}{x'} D_{i'}^{j'}(x', t) P_{j' \rightarrow j} \left( \frac{x}{x'} \right). \quad (3)$$

They describe the scaling violation of the parton distributions $D_i^j(x, t)$ inside a dressed quark or gluon $(i, j = q/g)$. 
We will not write the kernels $P$ explicitly and derive the generalized equations for two-parton distributions $D_{i}^{j_{1}j_{2}}(x_{1}, x_{2}, t)$, representing the probability that in a dressed constituent $i$ one finds two bare partons of types $j_{1}$ and $j_{2}$ with the given longitudinal momentum fractions $x_{1}$ and $x_{2}$ (referring to [6, 7, 8, 9, 10] for details), we note only that their solutions can be represented as the convolution of single distributions [8, 9]. This convolution coincides with the jet calculus rules [11] as mentioned above and is the generalization of the well-known Gribov-Lipatov relation installed for single functions [7] (the distribution of bare partons inside a dressed constituent is identical to the distribution of dressed constituents in the fragmentation of a bare parton in the leading logarithm approximation). The obtained solution shows also that the double distribution of partons is correlated in the leading logarithm approximation:

$$D_{i}^{j_{1}j_{2}}(x_{1}, x_{2}, t) \neq D_{i}^{j_{1}}(x_{1}, t)D_{i}^{j_{2}}(x_{2}, t).$$

(4)

Of course, it is interesting to find out the phenomenological issue of this parton level consideration. This can be done within the well-known factorization of soft and hard stages (physics of short and long distances). As a result the equations (3) describe the evolution of parton distributions in a hadron with $t (Q^{2})$, if one replaces the index $i$ by index $h$ only. However, the initial conditions for new equations at $t = 0 (Q^{2} = \mu^{2})$ are unknown a priori and must be introduced phenomenologically or must be extracted from experiments or some models dealing with physics of long distances [at the parton level: $D_{i}^{j}(x, t = 0) = \delta_{ij}\delta(x - 1); \ D_{i}^{j_{1}j_{2}}(x_{1}, x_{2}, t = 0) = 0$]. Nevertheless the solution of the generalized GLAPD evolution equations with the given initial condition may be written as before via the convolution of single distributions [6, 9]. This result shows that if the two-parton distributions are factorized at some scale $\mu^{2}$, then the evolution violates this factorization inevitably at any different scale ($Q^{2} \neq \mu^{2}$), apart from the violation due to the kinematic correlations induced by the momentum conservation.

For a practical employment it is interesting to know the degree of this violation. Partly this problem was investigated theoretically in Refs. [9, 12] and for the two-particle correlations of fragmentation functions in Ref. [13]. That technique is based on the Mellin transformation of distribution functions and the asymptotic behaviour can be estimated. Namely, with the growth of $t (Q^{2})$ the correlation term becomes dominant for finite $x_{1}$ and $x_{2}$ [12] and thus the two-parton distribution functions “forget” the initial conditions unknown a priori and the correlations perturbatively calculated appear.

The asymptotic prediction “teaches” us a tendency only and tells nothing about the values of $x_{1}, x_{2}, t(Q^{2})$ beginning from which the correlations are significant. Naturally
numerical estimations can give an answer to this specific question. We do it using the CTEQ fit [14] for single distributions as an input. The nonperturbative initial conditions $D_h(x,0)$ are specified in a parametrized form at a fixed low-energy scale $Q_0 = \mu = 1.3 \text{ GeV}$. The particular function forms and the value of $Q_0$ are not crucial for the CTEQ global analysis at the flexible enough parametrization. The results of numerical calculations are presented in Fig. 1 for the ratio:

$$R(x,t) = \left(\frac{D_{p(QCD)}^{gg}(x_1,x_2,t)}{D_{p}^{g}(x_1,t)D_{p}^{g}(x_2,t)(1-x_1-x_2)^2}\right)\bigg|_{x_1=x_2=x}. \quad (5)$$

![Figure 1](image_url)

Figure 1: The ratio of perturbative QCD correlations to the factorized component for the double gluon-gluon distribution in the proton as a function of $x = x_1 = x_2$ for three values of $Q = 5$ (solid), 100 (dashed), 250 (dash-dotted) GeV.

Figure 1 shows that at the scale of CDF hard process ($\sim 5 \text{ GeV}$) the ratio (5) is nearly 10% and increases right up to 30% at the LHC scale ($\sim 100 \text{ GeV}$) for the longitudinal momentum fractions $x \leq 0.1$ accessible to these measurements. For the finite longitudinal momentum fractions $x \sim 0.2 \div 0.4$ the correlations are large right up to 90%. They become important in more and more $x$ region with the growth of $t$ in accordance with the predicted QCD asymptotic behaviour [9,12].
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The correlation effect is strengthened insignificantly (up to 2%) for the longitudinal momentum fractions $x \leq 0.1$ when starting from the slightly lower value $Q_0 = 1$ GeV (early used by CTEQ Collaboration). We conclude also that $R(x, t) \to \text{const at } x \to 0$ most likely, calculating this ratio ($\simeq 0.1$) at $x_{\text{min}} = 10^{-4}$.

Seemingly the correction to the double gluon-gluon distributions at the CDF scale can be smoothly absorbed by uncertainties in the $\sigma_{\text{eff}}$ increasing the transverse effective size $r_p$ by a such way. But this augmentation is still not enough to solve a problem of the relatively small value of $r_p$ with respect to the proton radius without nontrivial correlation effects in transverse space [4].

Recently a nonminor role of the QCD evolution of multiparton distribution functions has been also demonstrated [15]. In the case of multiple production of W bosons with equal sign, the terms with correlations may represent a correction of the order of 40% of the cross sections, for $pp$ collisions at 1 TeV c.m. energy, and a correction of the order of 20% at 14 TeV. In the case of $b\bar{b}$ pairs the correction terms are of the order of 10-15% at 1 TeV and of the order of 5% at 14 TeV.

In summary, the numerical estimations show that the leading logarithm perturbative QCD correlations are quite comparable with the factorized distributions. With increasing a number of observable multiple collisions (statistic) the more precise calculations of their cross section (beyond the factorization hypothesis) will be needed also. In order to obtain the more delicate their characteristics (distributions over various kinematic variables) it is desirable to implement the QCD evolution of two-parton distribution functions in some Monte Carlo event generator as this was done for single distributions.

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