I. CPT TEST WITH NEUTRINO

The CPT theorem, which connects three discrete symmetries: charge conjugation (C), parity (P), and time reversal (T), and has been theoretically proved in different ways [1–5], states that any Lorentz invariant local quantum field theory of point-particle must be CPT invariant. If it is discovered that CPT symmetry is not conserved, one of the three foundational assumptions (Lorentz invariance, Hamiltonian hermiticity, and locality) must be sternly reconsidered. A consequence of the CPT invariance is that the particle and its anti-particle must have the same energy spectra. This important property opens a possibility for direct testing CPT invariance by comparing the mass spectra, or other properties such as lifetime or magnetic moment of a particle and its anti-particle. Ref. [6] provides the latest results on Lorentz and CPT violation searches in the context of Standard Model Extension. A summary of the model-independent CPT testing based on different properties of the different systems of particle and anti-particle can be found in Ref. [7]. In terms of relative precision, the most stringent constraint on the CPT test was achieved on the neutral kaon system [8]

\[
\left| \frac{m(K^0) - m(K^0)}{m_K} \right| < 6 \times 10^{-19} \text{ at 90\% C. L.} \quad (1)
\]

As pointed out in Ref. [9], when expressed in terms of the mass-squared difference, the bound on the \(K^0 - \bar{K}^0\) mass difference does not appear to be formidable. From Eq.(1), one can get

\[
|m^2(K^0) - m^2(\bar{K}^0)| < 0.3 \text{ eV}^2. \quad (2)
\]

Comparing this to the two mass-squared differences of the three neutrino mass eigenstates [8], \(m_{\nu_2}^2 - m_{\nu_1}^2 \approx 7.4 \times 10^{-5} \text{ eV}^2\) and \(m_{\nu_3}^2 - m_{\nu_2}^2 \approx 2.45 \times 10^{-3} \text{ eV}^2\), it becomes clear that neutrino measurements, rather than neutral kaons, provide the best constraint on the CPT test in terms of the mass-squared difference [9, 10]. The aforementioned neutrino mass-squared differences come from measuring the neutrino oscillation, which is a macroscopic quantum phenomenon establishing that neutrinos are massive and thus beyond the Standard Model’s description. It is worth noting that the neutrino mass spectrum cannot be calculated solely from neutrino oscillations, but must be combined with cosmological constraints and beta decay, as recently discussed in Ref. [11]. Neutrinos, unlike neutral B and K mesons,
are neutral elementary particles, and it is intriguing that this particle could be a Majorana particle, where neutrino and anti-neutrino are indistinguishable in the conventional sense of the CPT invariant paradigm. The neutrino nature under the CPT-violating scenario has been explored in Ref. [12]. Here we focus on the phenomenological consequence of the CPT violation in the observable neutrino oscillation.

In context of three-flavor PMNS framework [13, 14], for a given propagation distance \(L\) and matter density \(\rho\), the probabilities \(P_{\nu_\alpha \rightarrow \nu_\beta}\), \(P_{\nu_\alpha \rightarrow \nu_\beta}\) for a neutrino and anti-neutrino at a specific energy \(E_\nu, E_\bar{\nu}\) oscillating from one flavor \((\nu_\alpha, \bar{\nu}_\alpha)\) to another flavor \((\nu_\beta, \bar{\nu}_\beta)\) are completely and commonly described with six oscillation parameters including three leptonic mixing angles \((\theta_{12}, \theta_{13}, \theta_{23})\), one Dirac CPT-violating phase \(\delta_{CP}\), and two mass-squared differences \((\Delta m^2_{21}, \Delta m^2_{31})\). Under CPT symmetry, the neutrino and anti-neutrino oscillation probabilities are well connected as follows:

\[
P_{\nu_\alpha \rightarrow \nu_\beta}^{\text{CPT}} = P_{\nu_\alpha \rightarrow \nu_\beta} = f(\theta_{12}, \theta_{13}, \theta_{23}, \delta_{CP}, \Delta m^2_{21}, \Delta m^2_{31}).
\]

If the CPT is violated in neutrino sector, the underlying sets of oscillation parameters in neutrino and anti-neutrino may differ. Empirically, we assume

\[
P_{\nu_\alpha \rightarrow \nu_\beta} = f(\theta_{12}, \theta_{13}, \theta_{23}, \delta_{CP}, \Delta m^2_{21}, \Delta m^2_{31}),
\]

(3)

for describing the neutrino oscillations, and

\[
P_{\bar{\nu}_\alpha \rightarrow \nu_\beta} = f(\bar{\theta}_{12}, \bar{\theta}_{13}, \bar{\theta}_{23}, \bar{\delta}_{CP}, \bar{\Delta} m^2_{21}, \bar{\Delta} m^2_{31}),
\]

(4)

for anti-neutrino oscillations.

If there are observable differences in the parameters of the two sets, it may indicate a CPT violation in the lepton sector. Since the discovery of neutrino oscillations [15, 16] at the end of the twentieth century, neutrino oscillation experiments [8] using both natural and man-made neutrino sources have transitioned into the precision measurement phase of three mixing angles and two mass-squared differences, and being explored three remained known unknowns including the neutrino mass ordering, whether CP is violated, and whether the mixing angle \(\theta_{23}\) is maximal \((\theta_{23} = 45^\circ)\) or belong to a lower \((\theta_{23} < 45^\circ)\) or higher \((\theta_{23} > 45^\circ)\) octant. Each experiment is typically sensitive to a subset of the oscillation parameters but not the entire set. The experiments with solar neutrinos provide the most constraints on the \((\theta_{12}, \Delta m^2_{21})\) parameters while the reactor-based long-baseline neutrino (R-LBL) experiments can measure precisely the \((\bar{\theta}_{12}, \bar{\Delta} m^2_{31})\) parameters. The reactor-based short-baseline (order of 1 km) neutrino (R-SBL) experiments play a central role in measuring the \((\bar{\theta}_{13}, \bar{\Delta} m^2_{21})\) parameters. The under-developing reactor-based medium-baseline neutrino (R-MBL) experiment JUNO, which will be discussed later, takes advantage of interference of oscillations at different wavelengths, huge statistics, and good energy resolution to achieve sub-percent precision in measuring the \((\bar{\theta}_{12}, \bar{\Delta} m^2_{21}, \bar{\Delta} m^2_{31})\) parameters. Experiments with the atmospheric neutrino and accelerator-based neutrino sources can precisely measure the \((\theta_{23}, \bar{\theta}_{23}, \Delta m^2_{31}, \Delta m^2_{31})\) parameters. Besides, this type of experiment is also sensitive to the \((\theta_{13}, \bar{\theta}_{13})\) parameters, but the precision of these parameters is much lower in comparison to the R-SBL experiment due to the statistical limit and their strong correlation with two known unknowns, CP-violating phase and neutrino mass ordering. Although there is some hint [17] of non-zero CP-violating phase \(\delta_{CP}\), precise measurement on this parameter is not possible until the next generation of the accelerator-based long-baseline (A-LBL) experiments. It is provided in Ref. [18] the most recent update at 3σ confidence level (C. L.) on the bounds of CPT violation on each individual parameter with global neutrino data.

\[
|\delta_{\nu} (\Delta m^2_{31})| < 4.7 \times 10^{-5} \text{ eV}^2,
\]

\[
|\delta_{\nu} (\Delta m^2_{31})| < 2.5 \times 10^{-4} \text{ eV}^2,
\]

\[
|\delta_{\nu} (\sin^2 \theta_{23})| < 0.14,
\]

\[
|\delta_{\nu} (\sin^2 \theta_{13})| < 0.029,
\]

(5)

\[
|\delta_{\nu} (\sin^2 \theta_{23})| < 0.19,
\]

where \(\delta_{\nu}(X) = X - \bar{X}\) for the X neutrino oscillation parameter and the \(\bar{X}\) anti-neutrino oscillation parameter. In this study, we focus on the synergy between two on-going A-LBL experiments (T2K and NOνA) and one under-developing R-MBL experiment (JUNO) to explore the potential sensitivity to the measurement of \(\delta_{CP}(\Delta m^2_{31})\) and \(\delta_{CP}(\sin^2 \theta_{23})\) parameters. The A-LBL experiments utilize the highly intense beam of the almost pure muon neutrinos \(\nu_\mu\) and muon anti-neutrinos \(\bar{\nu}_\mu\) for measuring the four transitions categorized into two channels, \textit{appearance} channels \((\nu_\mu \rightarrow \nu_e, \bar{\nu}_\mu \rightarrow \bar{\nu}_e)\), and \textit{disappearance} channels \((\nu_\mu \rightarrow \nu_\mu, \bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)\). While the \textit{appearance} channels are sensitive to a wider subset of parameters and being explored for searching the CP violation in the lepton sector, measuring \((\nu_\mu \rightarrow \nu_e, \bar{\nu}_\mu \rightarrow \bar{\nu}_e)\) is not sufficient to test CP directly since the corresponding CPT-mirrored processes are missing. The \textit{disappearance} channels, on the other hand, are well-suited for testing CP since they are two CPT-mirrored processes. We characterize the difference in the probabilities of the muon neutrino \textit{disappearance} and muon anti-neutrino \textit{disappearance}, \(A_{\mu}^{\text{CPT}} = P_{\nu_\mu \rightarrow \nu_\mu} - P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu}\) as an observable measure of the CPT-violating effect.

The observable asymmetry \(A_{\mu}^{\text{CPT}}\) may consist of two parts: intrinsic CPT asymmetry and extrinsic CPT asymmetry caused by differences in interactions between neutrinos and anti-neutrinos with the matter of the propagation medium [19–23]. Fig. 1 illustrates the CPT asymmetries \(A_{\mu}^{\text{CPT}}\) calculated in vacuum and in the matter presence at baselines of the T2K experiment (L = 295 km) and of the NOνA experiment (L = 810 km). Here we take the best-fit values of the mainly involved \((\Delta m^2_{31}, \Delta m^2_{31}, \theta_{23}, \bar{\theta}_{23})\) parameters from the recent T2K results [24] and of the others from the global data analysis [25], which are summarized in Table I. It is wor-
TABLE I: Values of nominal parameters, taken from the recent T2K measurements [24] of muon-neutrino and muon-anti-neutrino 
\textit{disappearances} and from the global analysis of the neutrino oscillation data [25]. Our work utilising the data samples of muon-neutrino and muon-anti-neutrino disappearance is insignificantly affected by uncertainty of ($\theta_{12}$, $\bar{\theta}_{12}$, $\theta_{13}$, $\bar{\theta}_{13}$, $\delta_{CP}$, $\bar{\delta}_{CP}$, $\Delta m^2_{21}$, $\Delta m^2_{31}$) parameters. The primary driving parameters in this study are ($\theta_{23}$, $\bar{\theta}_{23}$, $\Delta m^2_{31}$, $\Delta m^2_{32}$) parameters.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Parameter & Value \\
\hline
$\sin^2\theta_{23}$ & 0.51 \\
$\sin^2\bar{\theta}_{23}$ & 0.43 \\
$\Delta m^2_{31}$ & $2.55 \times 10^{-3}$ eV$^2$ \\
$\Delta m^2_{32}$ & $2.58 \times 10^{-3}$ eV$^2$ \\
\hline
$\sin^2\theta_{12}$, $\sin^2\bar{\theta}_{12}$ & 0.318 \\
$\sin^2\theta_{13}$, $\sin^2\bar{\theta}_{13}$ & 0.022 \\
$\delta_{CP}$, $\bar{\delta}_{CP}$ & $1.08\pi$ rad \\
$\Delta m^2_{21}$, $\Delta m^2_{31}$ & $7.50 \times 10^{-5}$ eV$^2$ \\
\hline
\end{tabular}
\end{table}


In this study, we assume that neutrino and anti-neutrino masses are ordered similarly. Table II summarize the measurements of the ($\Delta m^2_{31}$, $\Delta m^2_{32}$, $\theta_{23}$, $\bar{\theta}_{23}$) parameters with the first generation of the A-LBL experiment MINOS [27, 28], on-going second generation T2K [24], NO$\nu$A [29], and precise constraint of the $\Delta m^2_{31}$ parameter from the R-SBL experiment Daya Bay [30]. It is shown that $\Delta m^2_{31}$ is measured with about 3% precision with the A-LBL experiments, while $\Delta m^2_{31}$ is measured with about 10% precision, which can be complemented with the R-SBL experiment with 2.3% precision. For the mixing angle, the precision is varied among experiments due to the fact that we are unsure whether ($\sin^2\theta_{23}$, $\sin^2\bar{\theta}_{23}$) is maximal or belong to a specific octant. The neutrino and anti-neutrino involved parameters agree within 1$\sigma$ C. L.

In this paper, we will investigate the prospects of testing the possible CPT violation via the applicable sensitive $\delta_{CP}(\Delta m^2_{31})$ and $\delta_{CP}(\sin^2\theta_{23})$ parameters with the synergy of T2K-II, NO$\nu$A extension (for convenience, we will denote it NO$\nu$A-II from now on), and JUNO experiments. In particular, we focus on the use of data samples of the $\nu_\mu$ and $\bar{\nu}_\mu$ \textit{disappearance} channels from the T2K-II and NO$\nu$A-II experiments in combination with the \textit{disappearance} of $\nu_e$ collected by the JUNO experiment before 2028, where we expect the operational start of the next generation A-LBL experiments. The paper is organized as follows. We describe the simulation of T2K-II, NO$\nu$A-II and JUNO experiments in Sec. II. The possibly established bounds of the manifested quantities $\delta_{CP}(\Delta m^2_{31})$ and $\delta_{CP}(\sin^2\theta_{23})$ of the CPT violation are presented in Sec. III. Further investigation into the potential significance of CPT-invariant exclusion and its robustness against the variation of the underlying physical parameters are discussed in Sec. IV. Finally, we conclude our study in Sec. V.

II. EXPERIMENTAL SIMULATION

The General Long Baseline Experiment Simulator (GLoBES) [31, 32] is a sophisticated but flexible framework to simulate, explore the physical potentials of neutrino experiments and fit the experimental data. By de-
fault, GLoBES assumes that the oscillation parameters for neutrinos and anti-neutrinos in Eq. (3) and Eq. (4) are identical or CPT-invariant. We extend the package to describe the neutrino and anti-neutrino oscillations independently.

For the oscillation probability formula, we follow the analytical expressions in Ref. [33]. Neutrino (anti-neutrino) oscillation in matter depends on nine variables, including six oscillation parameters listed in Eq. (3) for neutrino (or Eq. (4) for anti-neutrino), as well as neutrino energy $E_{\nu}$, the propagation distance $L$, and the matter density $\rho$. For the CPT test, the oscillation parameters of neutrinos and anti-neutrinos can be treated independently, thus having twelve oscillation parameters as a complete set. However, for this particular study, since the A-LBL experiments have no sensitivity to the solar parameter, we keep $\theta_{12} = \theta_{13}$; $\Delta m_{21}^2 = \Delta m_{32}^2$; $\theta_{13} = \bar{\theta}_{13}$; $\delta_{CP} = \delta_{CP}$ and fixed practically. Four independent parameters of interests ($\Delta m_{31}^2$, $\Delta m_{31}^2$, $\theta_{23}$, $\bar{\theta}_{23}$) remains.

T2K [34] and NOvA [35] are two world-leading A-LBL experiments. For convenience, we denote T2K run up to 2027 by T2K-II and NOvA extension up to 2024 by NOvA-II. The similarity in experimental configuration and operating principle makes it interesting to have a joint fit between the two experiments [36, 37]. Both experiments use intense muon (anti-)neutrino beams created by accelerators to study oscillation phenomena. The off-axis technique adopted by both experiments can produce a narrow-band beam of neutrinos to enhance the sensitivity of neutrino oscillation measurements and mitigate the effect of possible bias in the neutrino energy reconstruction from their interaction products. The ability to focus either positive or negative particles (mainly pions and kaons) offers the A-LBL experiment a unique opportunity to operate in both neutrino-mode and anti-neutrino-mode. This important feature enables the testing of CPT invariance in the A-LBL experiments.

JUNO [38] is a R-MBL experiment which studies electron anti-neutrino disappearance ($\bar{\nu}_e \rightarrow \bar{\nu}_e$). The experiment uses electron anti-neutrino flux produced from nuclear reactors to study neutrino oscillation at a medium baseline (about 50 km) to take advantage of the interference of two oscillation lengths, which are driven by two mass-squared splittings, $\Delta m_{21}^2$ and $\Delta m_{31}^2$. Achieving a neutrino energy resolution of less than 3% is essential for JUNO to resolve these two oscillation patterns and measure oscillation parameters $\sin^2 2\theta_{12}$, $\Delta m_{21}^2$ and $|\Delta m_{31}^2|$ at precision less than 0.5% [39]. The JUNO experiment also can resolve neutrino mass hierarchy at $3\sigma$ C. L. after six years of operation. Combining data samples from JUNO and from the A-LBL experiments, T2K-II and NOvA-II, will definitely resolve the neutrino mass ordering [26].

We follow closely the experimental specifications for T2K-II, NOvA-II, and JUNO in the Ref. [26], except for some updates in T2K-II and JUNO. In original proposal [40], T2K-II is expected to operate until 2027, exposing $20 \times 10^{21}$ protons-on-target (POT). According to the most recent plan [36], statistics may be cut in half. Thus, we use $10 \times 10^{21}$ POT for T2K-II in this work. We also updated the T2K flux, which was released in 2020 [41]. For JUNO, a total thermal of 26.6 GWth [39] is used instead of 36 GWth as in the previous report. Also, the energy resolution is set at 2.9% [39] to reflect closely the JUNO’s prospect.

In terms of the data samples for analysis, for T2K-II and NOvA-II, we used the disappearance channels only, with statistics equally divided into $\nu$-mode and $\bar{\nu}$-mode. As we will show later in Sec. III, the CPT test on the $\delta_{CP}(\Delta m_{31}^2)$ will be limited due to the precision of $\Delta m_{31}^2$ measurement by the A-LBL experiment, and thus the bound established in this parameter can be elevated if we have more neutrino data. However, this scenario is unlikely since running an experiment in anti-neutrino mode is very important for the CP violation measurement. For JUNO, $\bar{\nu}_e$ disappearance data is used. We assume neutrino masses are in normal ordering throughout the study.

![FIG. 1: CPT asymmetries in disappearance channels for T2K baseline L = 295 km (left) and NOvA baseline L = 810 km (right). The differences in solid lines and dashed lines indicate extrinsic CPT effects caused by matter.](image-url)
in Sec. III and Sec. IV. The study in Sec. III is done with the values of nominal parameters listed in Table I, in which we follow the measurements of T2K [24] for atmospheric parameters ($\Delta m^2_{31}$, $\Delta m^2_{31}$, $\sin^2 \theta_{23}$, $\sin^2 \theta_{13}$) and global fit [25] for the rest.

The bounds and the sensitivities to rule out CPT invariant hypothesis with $\delta_3(\nu)(X)$ parameter are explored. The $\chi^2$ of individual experiment is calculated for given true values of $X$ for neutrinos and $X$ for anti-neutrinos, where $X$ can be $\sin^2 \theta_{23}$ or $\Delta m^2_{31}$. We use a log-likelihood $\chi^2$ function for T2K-II and NO$\nu$A-II, while a Gaussian formula is used for JUNO due to its high statistics. The calculation of $\chi^2$ is then minimized over the nuisance parameters and other oscillation parameters except for $X$ and $\bar{X}$. The two-dimensional distributions of $\Delta \chi^2$ which is the sum of all the individual ones of the three experiments, are obtained. The minimum of $\Delta \chi^2$ as a function of $\delta_3(\nu)(X) = X - \bar{X}$ is then found. The statistical significance of excluding CPT conservation is expressed as the squared root of the minimum $\Delta \chi^2$.

III. POSSIBLY ESTABLISHED BOUNDS OF $\delta_3(\nu)(\Delta m^2_{31})$ AND $\delta_3(\nu)(\sin^2 \theta_{23})$ ON CPT VIOLATION

In this study, assuming that CPT is exactly conserved or extremely small for detection, we estimate the expected bound of the two sensitive parameters, asymmetry in the mass-squared differences $\delta_3(\nu)(\Delta m^2_{31})$ and asymmetry in the leptonic mixing angles $\delta_3(\nu)(\sin^2 \theta_{23})$, on the possible CPT violation. In particular, $\Delta m^2_{31} = \Delta \overline{m}^2_{31} = 2.55 \times 10^{-3}$ eV$^2$ and $\sin^2 \theta_{23} = \sin^2 \theta_{13} = 0.51$, which are the T2K’s best-fit points with recent measurement [24], are assumed to be true. To compute the allowed region of the $\delta_3(\nu)(\Delta m^2_{31})$ and $\delta_3(\nu)(\sin^2 \theta_{23})$ parameters, we build up the $\chi^2$ profiles on a two-dimensional grid points of neutrino and anti-neutrino corresponding parameters ($\Delta m^2_{31}$, $\Delta \overline{m}^2_{31}$) and ($\sin^2 \theta_{23}$, $\sin^2 \theta_{13}$), respectively. The $\chi^2$ profiles take into account the correlations among the oscillation parameters. The $\Delta \chi^2$ profiles are attained by subtracting to the minimum value of the according $\chi^2$, which is essentially located at the true values.

Fig. 2 shows 3$\sigma$ C. L. allowed regions of pairs of parameters ($\Delta m^2_{31}$, $\Delta \overline{m}^2_{31}$) and ($\sin^2 \theta_{23}$, $\sin^2 \theta_{13}$) under the assumption that CPT is conserved. Three different analyses are presented: (i) T2K-II only, (ii) a joint of T2K-II and NO$\nu$A-II, and (iii) a joint of T2K-II, NO$\nu$A-II, and JUNO. It is expected that a joint analysis of T2K-II and NO$\nu$A-II improves significantly the precision of four involved ($\Delta m^2_{31}$, $\Delta \overline{m}^2_{31}$, $\sin^2 \theta_{23}$, $\sin^2 \theta_{13}$) parameters while JUNO mainly contribute to the precision of $\Delta \overline{m}^2_{31}$.

To answer for the question about the allowed parameter magnitudes in the mass-squared difference $\delta_3(\nu)(\Delta m^2_{31})$ and the leptonic mixing angle $\delta_3(\nu)(\sin^2 \theta_{23})$, projections of $\Delta \chi^2$ profiles on these two variables are constructed and depicted in Fig. 3. The upper limits of these two CPT-sensitive variables at 3$\sigma$ C. L. are extracted and summarized in Table III. With total exposure of $10 \times 10^{21}$ POT, T2K-II alone can set more stringent limits on the CPT violation search, if it will be not found, both with atmospheric mass-squared splitting $|\delta_3(\nu)(\Delta m^2_{31})| \leq 2.0 \times 10^{-4}$ eV$^2$ and leptonic mixing angles $\delta_3(\nu)(\sin^2 \theta_{23}) \leq 0.14$, than the combined data of current neutrino experiments. By adding NO$\nu$A-II, the 3$\sigma$ C. L. limit on $|\delta_3(\nu)(\sin^2 \theta_{23})|$ for CPT violation is reduced to 0.10, a 47% improvement over the current limit. Meanwhile, if no evidence of CPT violation is found, the potential bound on $|\delta_3(\nu)(\Delta m^2_{31})|$ at 3$\sigma$ C. L. will be expected to be $5.3 \times 10^{-5}$ eV$^2$ for the combined analysis of the three experiments. This prospective bound on the possible CPT violation search is slightly better than the DUNE sensitivity [42], $|\delta_3(\nu)(\Delta m^2_{31})| < 8.1 \times 10^{-5}$ eV$^2$ at 3$\sigma$ C. L.

IV. SIGNIFICANCE OF CPT EXCLUSION: DEPENDENCE AND PROJECTION

Apparently if the analyses with real data shows the asymmetries of $|\delta_3(\nu)(\Delta m^2_{31})|$ or $|\delta_3(\nu)(\sin^2 \theta_{23})|$ larger than the corresponding upper limits presented in Table III, it would imply the CPT violation in the lepton sector. However, one raised question is whether these anticipated limits are affected by the true values of the underlying parameters, which can fluctuate from the current best-fit values. To investigate this issue, we performed the full joint analysis of T2K-II, NO$\nu$A-II, and JUNO under various assumptions of the involved parameters. In particular, for the potential effect on $\delta_3(\nu)(\Delta m^2_{31})$, we examine the CPT sensitivity at three points ($2.46 \times 10^{-3}$, $2.55 \times 10^{-3}$, $2.63 \times 10^{-3}$ eV$^2$) of $\Delta m^2_{31}$, taken as the T2K best-fit and $\pm \sigma$ shifted values, in combination with a variation of $\Delta \overline{m}^2_{31}$ such that $|\delta_3(\nu)(\Delta m^2_{31})| < 0.15 \times 10^{-8}$ eV$^2$. In this case of study, $\sin^2 \theta_{23} = \sin^2 \theta_{13} = 0.51$ is assumed to be true. In addition, we check the sensitivities of CPT violation on the $\delta_3(\nu)(\Delta m^2_{31})$ parameter at three shared values (0.44, 0.51, 0.57) of ($\sin^2 \theta_{23}$, $\sin^2 \theta_{13}$). For each case, the statistical significance to exclude the corresponding form of the CPT invariance is extracted as function of $\delta_3(\nu)(\Delta m^2_{31})$ and the results are shown in Fig. 4. It is observed that the CPT violation sensitivity manifested on the $\delta_3(\nu)(\Delta m^2_{31})$ parameter depend marginally on the central

| Experiments | $|\delta_3(\nu)(\Delta m^2_{31})|$| 3$\sigma$ C. L. upper limits | $|\delta_3(\nu)(\sin^2 \theta_{23})|$| 3$\sigma$ C. L. upper limits |
|-------------|----------------|------------------|----------------|------------------|
| T2K-II      | $2.0 \times 10^{-4}$ eV$^2$ | 0.14             |                     |
| T2K-II+NO$\nu$A-II | $1.2 \times 10^{-4}$ eV$^2$ | 0.10             |                     |
| T2K-II+NO$\nu$A-II+JUNO | $5.3 \times 10^{-5}$ eV$^2$ | 0.10             |                     |

TABLE III: The bounds on CPT violation with atmospheric mass-squared difference and mixing angle at 3$\sigma$ C. L. for three analyses: T2K-II only, a joint of T2K-II and NO$\nu$A-II, a joint of T2K-II, NO$\nu$A-II, and JUNO.
FIG. 2: The 3σ C. L. regions of $\Delta m_{31}^2$ and $\Delta m_{12}^2$ (left), $\sin^2 \theta_{23}$ and $\sin^2 \theta_{13}$ (right). The black, red, and blue lines are for an analysis with T2K-II only, a joint of T2K-II and NO$\nu$A-II, and a joint of T2K-II, NO$\nu$A-II, and JUNO, respectively.

FIG. 3: The bounds on possible CPT violation manifested in the asymmetries of the mass-squared splittings $|\delta_{\nu\tau}(\Delta m_{31}^2)|$ (left) and of the leptonic mixing angles $|\delta_{\nu\tau}(\sin^2 \theta_{23})|$ (right). The black, red, and blue lines correspond to an analysis with T2K-II only, a joint of T2K-II and NO$\nu$A-II, and a joint of T2K-II, NO$\nu$A-II, and JUNO, respectively.

FIG. 4: Statistical significance to exclude CPT is computed as a function of true $\delta_{\nu\tau}(\Delta m_{31}^2)$ under various scenarios of the involved parameters. The left plot is when $\Delta m_{31}^2$ is examined at three different true values, while $\sin^2 \theta_{23} = \sin^2 \theta_{13} = 0.51$ is assumed to be true. The right plot presents the CPT sensitivity of $\delta_{\nu\tau}(\Delta m_{31}^2)$ at different true values of $\sin^2 \theta_{23}$ and $\sin^2 \theta_{13}$ while $\Delta m_{31}^2 = 2.55 \times 10^{-3}$eV$^2$ is assumed to be true.
value of $\Delta m^2_{31}$ and $\Delta m^2_{31}$ in the current allowed range of this parameter. Also the dependence of the $\delta_{\nu,\sigma}(\Delta m^2_{31})$ sensitivity on the true value of the mixing parameter ($\sin^2 \theta_{23}, \sin^2 \overline{\theta}_{23}$) is relatively small. Apparently, due to the octant degeneracy of ($\sin^2 \theta_{23}, \sin^2 \overline{\theta}_{23}$) presented in the disappearance probabilities of muon (anti-)neutrinos, the significance of the CPT test is slightly worse than the case where ($\sin^2 \theta_{23}, \sin^2 \overline{\theta}_{23}$) is exactly equal or near the maximal mixing. The lower limit of true $\delta_{\nu,\sigma}(\Delta m^2_{31})$ magnitude to exclude the CPT at 3$\sigma$ C. L. or higher significance is presented in Table IV. We find that if the deviation of $\delta_{\nu,\sigma}(\Delta m^2_{31})$ from zero is greater than 6.0 $\times$ 10$^{-5}$eV$^2$ the CPT invariance will be excluded at 3$\sigma$ C. L. for almost the entire currently-allowed range of the involved parameters. The range of possible $\delta_{\nu,\sigma}(\Delta m^2_{31})$ asymmetry to be explored significantly is slightly extended ([5.36, 5.46] $\times$ 10$^{-5}$eV$^2$) if the mixing angle is near the maximal mixing. Due to the aforementioned octant degeneracy of the (anti-)neutrino oscillation probabilities in the disappearance samples, the deviation of $\delta_{\nu,\sigma}(\Delta m^2_{31})$ from zero must be moderately greater ([5.77, 5.99] $\times$ 10$^{-5}$eV$^2$) for attaining a same level of significance to exclude the CPT invariance. To see how impressive the improvement in the CPT test sensitivity from this three-experiment combined analysis is, we project the statistical significance from the current measurements. As summarized in the Table II, the difference in mass-squared splitting at the best-fit values of ($\Delta m^2_{31}, \Delta m^2_{31}$) measured by T2K [24] is $|\delta_{\nu,\sigma}(\Delta m^2_{31})| = 3 \times 10^{-5}$eV$^2$, well consistent within 1$\sigma$ uncertainty of 20 $\times$ 10$^{-5}$eV$^2$. However, if this asymmetry persists as the true, it will correspond to 1.7$\sigma$ C. L. exclusion of CPT conservation by the combined analysis of T2K-II, NO$\nu$A-II, and JUNO. If the level of asymmetrical $\delta_{\nu,\sigma}(\Delta m^2_{31})$ in the neutrino and anti-neutrino best-fit values of NO$\nu$A and MINOS (+), which is 7.0 $\times$ 10$^{-5}$eV$^2$, are assumed to be persisted as the true, the synergy of the three experiments can exclude CPT conservation at 4$\sigma$ C. L.

Regarding the sensitivity of $\delta_{\nu,\sigma}(\sin^2 \theta_{23})$ on the CPT test, we examine and find that their dependence on the fluctuation of the ($\Delta m^2_{31}, \Delta m^2_{31}$) parameters is relatively small while the dependence on the true value of ($\sin^2 \theta_{23}, \sin^2 \overline{\theta}_{23}$) is significant, as shown in Fig. 5. When the true value of $\sin^2 \theta_{23}$ belongs to an octant, there exists a degenerated solution in the other octant. For example, when $\sin^2 \theta_{23}=0.44$, the extrinsic CPT-invariant solution of $\sin^2 \overline{\theta}_{23}=0.58$ (along with the genuine solution of $\sin^2 \overline{\theta}_{23}=0.44$). Similar behavior is observed when $\sin^2 \theta_{23}$ values in the higher octant. The behavior is well-understood due to the dependence of muon (anti-) neutrino disappearance probabilities on the $\sin^2 \theta_{23}$ ($\sin^2 \overline{\theta}_{23}$) rather than $\sin^2 \overline{\theta}_{23}$ ($\sin^2 \overline{\theta}_{23}$). As summarized in Table V, to attain the same significance level to exclude the CPT, compared to the maximal case $\sin^2 \theta_{23}=0.51$, the magnitude of true $\delta_{\nu,\sigma}(\sin^2 \theta_{23})$ asymmetry in the non-maximal cases ($\sin^2 \theta_{23}=0.44$ and $\sin^2 \theta_{23}=0.57$) is required to be larger or smaller depending on whether the $\theta_{23}$ and $\overline{\theta}_{23}$ belong to different or the same octants, respectively. In particular, for $\sin^2 \theta_{23}=0.51$ as indicated by both T2K [24] and NO$\nu$A [29], the magnitude of $\delta_{\nu,\sigma}(\sin^2 \theta_{23})$ asymmetry must be between [0.076, 0.084] to be discovered with 3$\sigma$ C. L. T2K (NO$\nu$A) measured $\delta_{\nu,\sigma}(\sin^2 \theta_{23})=0.08$ (0.10) respectively, and if it remains as true the CPT invariance will be excluded at 3$\sigma$ or higher C. L. If $\theta_{23}$ and $\overline{\theta}_{23}$ are in the same octant and relatively far off from the maximal values, the deviation of $\delta_{\nu,\sigma}(\sin^2 \theta_{23})$ from zero must be greater than 0.051 in order to rule out CPT invariance at 3$\sigma$ C. L. If $\theta_{23}$ and $\overline{\theta}_{23}$ are in different octants, $\theta_{23}$ in lower octant and $\overline{\theta}_{23}$ in higher octant or vice versa, the magnitude of $\delta_{\nu,\sigma}(\sin^2 \theta_{23})$ must be significantly higher, varying in the (0.165,0.190) range, to exclude CPT at the same 3$\sigma$ statistical significance. The sensitivity to detect CPT violation via the $\delta_{\nu,\sigma}(\sin^2 \theta_{23})$ asymmetry is not good due to the aforementioned octant degeneracy in the muon (anti-) neutrino disappearance samples. The sensitivity can be improved by adding the electron (anti-) neutrino appearance samples from the A-LBL experiments. Fig. 6 shows the sensitivity of $\delta_{\nu,\sigma}(\sin^2 \theta_{23})$ on the CPT exclusion with a combination of both disappearance and appearance samples. It is observed that by adding the electron (anti-) neutrino appearance samples, the statistical significance to exclude the extrinsic CPT-invariant solution is enhanced notably. Consequently, the sensitivity of $\delta_{\nu,\sigma}(\sin^2 \theta_{23})$ to the CPT violation has improved. However, one must consider carefully when adding the electron (anti-) neutrino appearance samples. The reason is that the probabilities of $\nu_e (\overline{\nu}_e)$ from $\nu_\mu (\overline{\nu}_\mu)$ depend not only on $\theta_{23} (\overline{\theta}_{23})$ but also on two known unknowns, CP-

| $\Delta m^2_{31}$ [eV$^2$] | Shared values of $\sin^2 \theta_{23}$, $\sin^2 \overline{\theta}_{23}$ | 0.44 | 0.51 | 0.57 |
|-----------------|---------------------------|---------|---------|---------|
| 2.46 $\times$ 10$^{-3}$ | 5.96 $\times$ 10$^{-5}$ | 5.63 $\times$ 10$^{-5}$ | 5.80 $\times$ 10$^{-5}$ |
| 2.55 $\times$ 10$^{-3}$ | 5.95 $\times$ 10$^{-5}$ | 5.30 $\times$ 10$^{-5}$ | 5.77 $\times$ 10$^{-5}$ |
| 2.63 $\times$ 10$^{-3}$ | 5.99 $\times$ 10$^{-5}$ | 5.97 $\times$ 10$^{-5}$ | 7.59 $\times$ 10$^{-5}$ |

$\delta_{\nu,\sigma}(\Delta m^2_{31})$ limit to exclude CPT at 3$\sigma$ C. L.

| $\sin^2 \theta_{23}$ | Shared values of $\Delta m^2_{31}$, $\overline{\Delta m^2_{31}}$ [eV$^2$] | 2.46 $\times$ 10$^{-3}$ | 2.55 $\times$ 10$^{-3}$ | 2.63 $\times$ 10$^{-3}$ |
|-----------------|---------------------------|---------|---------|---------|
| 0.44 | -0.051 (+0.190) | -0.049 (+0.187) | -0.048 (+0.186) |
| 0.51 | -0.084 (+0.082) | -0.080 (+0.078) | -0.078 (+0.076) |
| 0.57 | -0.169 (+0.047) | -0.166 (+0.044) | -0.165 (+0.043) |

$\delta_{\nu,\sigma}(\sin^2 \theta_{23})$ limit to exclude CPT at 3$\sigma$ C. L.
The analysis is expected to happen by 2028, before the operational start of the next generation of accelerator-based long-baseline neutrino experiments, DUNE [43] and Hyper-Kamiokande [44]. In particular, we focus on the asymmetries in the mass-square splitting $\Delta \nu \nu (\Delta m^2_{31})$ and in the leptonic mixing angle $\delta_{\nu \nu}(\sin^2 \theta_{23})$. The synergy of these three experiments will plausibly establish an unprecedented bound of $\delta_{\nu \nu}(\Delta m^2_{31})$ to about $5.3 \times 10^{-5} \text{eV}^2$ at $3\sigma$ C. L. In case the CPT symmetry is conserved to be sensitive. This bound extends substantially the current bound of $2.5 \times 10^{-4} \text{eV}^2$ derived from the global neutrino data analysis. It is noteworthy to stress that this bound of $\delta_{\nu \nu} (\Delta m^2_{31})$ on the possible CPT violation is marginally dependent on the true values of the involved parameters, especially the ambiguity of the $\theta_{23} (\overline{\theta}_{23})$ values. The improvement of CPT sensitivity is very encouraging since if the difference between the best-fit values of $\Delta m^2_{31}$ and $\Delta m^2_{31}$ currently measured by NO$\nu$A and MINOS(+) persists as the true, the statistical significance to exclude the CPT is about $4\sigma$ C. L. For the testable asymmetry in the leptonic mixing angle $\delta_{\nu \nu}(\sin^2 \theta_{23})$, the statistical significance of the CPT sensitivity depends strongly on their own genuine values, which rooted from the parameter degeneracy in the muon (anti-)neutrino disappearance probabilities. In the case of CPT conservation, if the neutrino mixing angle $\theta_{23}$ is close to the maximal, as indicated by both T2K and NO$\nu$A current measurements, the combined analysis of the three experiments will potentially establish a limit of $\delta_{\nu \nu}(\sin^2 \theta_{23})=0.10$, compared to the current bound of 0.19 attained from the global neutrino data analysis. Interestingly, if the difference in the best-fit values of $\sin^2 \theta_{23}$ and $\sin^2 \overline{\theta}_{23}$ measured recently by both T2K and NO$\nu$A persist as the true, the combined analysis of the two with their final data samples will indicate CPT violation with $3\sigma$ C. L. or higher.

Finally, it is important to emphasize that one cannot claim a CPT violation simply by observing sizable violations phase and mass ordering, which will complicate the interpretation of the experimental observation.

V. CONCLUSION

In the paper, we presented the potential of timely combined analysis of the two on-going accelerator-based long-baseline experiments T2K-II, NO$\nu$A-II and a reactor-based medium-baseline JUNO experiment in testing CPT symmetry via the measurable asymmetry in the oscillation parameters of neutrinos and anti-neutrinos.
\[ \delta_{\nu\tau}(2m_{11}^2) \text{ or } \delta_{\nu\tau}(\sin^2\theta_{23}) \] asymmetries because some non-standard interactions, such as those discussed in Ref. [45], can mimic the effect. In any case, investigating the potential differences in the parameters governing neutrino and anti-neutrino oscillations is critical to revealing the new physics.

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