Quenched disorder forbids discontinuous transitions in non-equilibrium low-dimensional systems

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Quenched disorder affects significantly the behavior of phase transitions. The Imry-Ma-Aizenman-Wehr-Berker argument prohibits first-order/discontinuous transitions and their concomitant phase coexistence in low-dimensional equilibrium systems in the presence of random fields. Instead, discontinuous transitions become rounded or even continuous once disorder is introduced. Here we show that phase coexistence and first-order phase transitions are also precluded in non-equilibrium low-dimensional systems with quenched disorder: discontinuous transitions in two-dimensional systems with absorbing states become continuous in the presence of quenched disorder. We also study the universal features of this disorder-induced criticality and find them to be compatible with the universality class of the directed percolation with quenched disorder. Thus, we conclude that first order transitions do not exist in low-dimensional disordered systems, not even in genuinely non-equilibrium systems with absorbing states.

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I. INTRODUCTION

Quenched disorder has a dramatic effect on both the statics and the dynamics of phase transitions [1-3]. A time-honored argument by Imry and Ma explains in a simple and parsimonious way why symmetries cannot be spontaneously broken in low-dimensional systems in the presence of quenched random fields [4]. In a nutshell, the argument is as follows. Suppose a discrete symmetry (e.g., \( Z_2 \) or up-down) was actually spontaneously broken in a \( d \)-dimensional system and imagine a region of linear size \( L \) with a majority of random fields opposing the broken-symmetry state. As a direct consequence of the central limit theorem, by reversing the state of such a region the bulk-free energy would decrease proportionally to \( L^{d/2} \), but this inversion would also lead to an interfacial energy cost proportional to \( L^{d-1} \). Comparing these two opposing contributions for large region sizes, it follows that for \( d \leq 2 \) the first dominates, making the broken-symmetry state unstable. If the distinct phases are related by a continuous symmetry, soft modes reduce the effect of the boundary conditions to \( L^{d-2} \) and the marginal dimension is \( d = 4 \) [5]. Thus, the energetics of low-dimensional systems is controlled by the random field, which is symmetric, thus preventing symmetries from being spontaneously broken and continuous phase transitions from existing. Instead, in higher dimensional systems, the situation is reversed, symmetries can be spontaneously broken, and phase transitions exist.

The Imry-Ma argument \( i \) holds for equilibrium systems (where the free energy is well defined), \( ii \) is backed by more rigorous renormalization group calculations, which prove that no symmetry breaking occurs even at the marginal case \( d = 2 \) (where rough interfaces could potentially break the argument above [4]), \( iii \) has been verified in countless examples both experimentally and numerically, and \( iv \) has been extended to quantum phase transitions [6, 7].

In contrast with the equilibrium case, recent work by Barghathi and Vojta [8], shows that second-order phase transitions may survive to the introduction of random fields even in one-dimensional cases [9, 10] in genuine non-equilibrium systems with absorbing states for which there is not such a thing as free energy [11–14]. Therefore, the Imry-Ma argument does not apply to these non-equilibrium systems owing to the presence of absorbing states, and, in consequence, states of broken symmetry can exist in the presence of random fields.

Let us now shift the discussion to first-order phase transitions, for which system properties such as the magnetization, energy, density, etc., change abruptly as a control parameter crosses a threshold value at which two distinct phases coexist. As shown by Aizenman and Wehr, first-order phase transitions in low-dimensional equilibrium systems are rounded (made less sharp) by disorder, and, even more remarkably, the rounding may result into a critical point; i.e., first-order/discontinuous phase transitions become second-order/continuous ones.
upon introducing (random-field) disorder. A similar conclusion applies to the case of random interactions; indeed, a random distribution of interactions (e.g., bonds) locally favors one of the two phases, and thus, this has the same effects as random fields. Different Monte Carlo results support this conclusion; furthermore they suggest that the disorder-induced continuous transition exhibits critical exponents which are consistent with those of the corresponding pure model. An argument explaining these findings was put forward by Kardar et al. 

In close analogy with the argument above for the absence of symmetry-breaking, in the case of phase coexistence as well, regions (or “islands”) of arbitrary size of one of the phases appear in a stable way within the other. Therefore, islands exist within islands in any of the two phases in a nested way, leading always to hybrid states. Hence, two distinct phases cannot possibly coexist and first-order transitions are precluded in disordered low-dimensional equilibrium systems.

Thus, the question arises as to whether shifting to the non-equilibrium realm entails the shattering of a fundamental cornerstone of equilibrium statistical mechanics as it happens for continuous phase transitions (see table 1 for a synthetic summary); do first-order phase transitions, and, hence, phase coexistence, exist in low-dimensional non-equilibrium disordered systems?

Aimed at shedding some light on this issue, we study non-equilibrium models with absorbing states in the presence of disorder. More specifically, we study a variant of the well-known “contact process”, sometimes called the “quadratic contact process”, in which two particles are needed to generate an offspring while isolated particles can spontaneously disappear. As a first step, we verify that the pure version of the model exhibits a first-order transition separating an active phase from an absorbing one. Then we introduce disorder in the form of a site-dependent transition rates and investigate whether the discontinuous character of the transition survives.

II. TWO POSSIBLE SCENARIOS

Two alternative scenarios might be expected a priori for the impure/disordered model:

1. the Imry-Ma argument breaks down in this non-equilibrium case and a first order phase transition is observed, or

2. the Imry-Ma prediction holds even if the system is a non-equilibrium one, and a disorder-induced second-order phase transition emerges.

If the latter were true, we could then ask what universality class such a continuous transition belongs to. A priori, it could share universality class with other already-known critical phase transitions in disordered systems

with absorbing states or, instead, belong to a new universality class defined by this disorder-induced criticality.

If no novel universal behavior emerges, then it is expected for the model to behave as a standard two-dimensional contact process (or directed percolation) with quenched disorder with the following main features:

- there should be a critical point separating the active from the absorbing phase,
- at criticality, a logarithmic or activated type of scaling (rather than algebraic) should be observed. For instance, for quantities related to activity spreading such as the survival probability, averaged number of particles, and radius from a localized initial seed, we expect $P_a(t) \sim \ln(t/t_0)^{-\beta}$, $N(t) \sim \ln(t/t_0)^{\theta}$, and $R(t) \sim \ln(t/t_0)^{1/\delta}$, respectively; $t_0$ is some crossover time, and $\delta, \theta, \beta, \gamma$ should take the values already reported in the literature,
- there should be a sub-region of the absorbing phase, right below the critical point, exhibiting generic algebraic scaling with continuously varying exponents, i.e. a Griffiths phase. Griffiths phases stem from the existence of rare regions where the disorder takes values significantly different from its average.

These features follow from a strong-disorder renormalization group approach for the disordered contact process, which concludes that this anomalous critical behavior can be related to the random transverse-field Ising model for sufficiently strong disorder, and have been confirmed in computational studies which suggested that this behavior is universal regardless of disorder strength.
III. MODEL AND RESULTS

We study the simplest non-equilibrium model with absorbing states exhibiting a first-order/discontinuous transition. Given that, owing to different reasons, one-dimensional systems with absorbing states rarely exhibit first-order phase transitions (even in pure systems) here, we focus on the physically more relevant two-dimensional case. In this two-dimensional reaction-diffusion contact-process-like model [11–14], individual particles disappear at a fixed rate, $\mu$, while a pair of nearest-neighbor particles is required to create an offspring at some rate $\lambda$

$$ A \xrightarrow{\mu} \emptyset, \quad 2A \xrightarrow{\lambda} 3A, \quad (1) $$

with the additional (“hard-core” or “Fermionic”) constraint preventing sites from housing more than one particle. This restriction can be relaxed at the cost of introducing a reaction

$$ 3A \xrightarrow{\lambda} 2A, \quad (2) $$

which keeps the number of particles bounded. In either case, the corresponding rate or mean-field equations are (see Appendix)

$$ \rho(t) = -\mu \rho(t) + \lambda \rho(t)^2(1 - \rho(t)), \quad (3) $$

where $\rho$ represents the density of active sites or particles. This equation has the trivial stationary solution, $\rho = 0$ and an additional one at $\rho^* = \frac{1}{2}(1 + \sqrt{1 - 4\mu/\lambda})$ for $\lambda > 4\mu$, with an associated discontinuous transition at $\lambda = 4\mu$.

A. Pure model

Among the many possible ways in which the above particle system can be implemented [29,31], we employ the model proposed in Ref. [31], which was numerically studied in two-dimensions and verified to exhibit a first-order phase transition separating an active from an absorbing phase [31].

We consider a two-dimensional square lattice and define a binary occupation variable $s = 0, 1$ (empty/occupied) at each site. We consider some initial conditions and perform a sequential updating following the standard procedure [11–14]: $i)$ an active site is randomly selected (from a list including all active ones); $ii)$ with probability $p_d$ (death) the particle is annihilated, otherwise, with complementary probability $1 - p_d$ a nearest neighbor site is chosen; $iii)$ if this latter is empty, the selected particle diffuses to it, and otherwise an offspring particle is created at a randomly chosen neighboring site with probability $p_b$ (birth) provided it was empty; otherwise nothing happens. We keep $p_b = 0.5$ fixed and use $p_d$ as the control parameter.

As customarily done, we perform two types of experiments [11–14], considering as initial condition either a homogeneous state, i.e., a fully occupied lattice of linear size $L$, or a localized seed, consisting in this case of a few, at least a couple, neighboring particles in an otherwise empty lattice.

Homogeneous initial conditions—Figure 1 shows results of computer simulations for the temporal decay of the particle density from $\rho(t = 0) = 1$. The upper panel shows an abrupt change of behavior at a threshold value $p_{d_{th}} \approx 0.0747$; activity survives indefinitely for $p_d < p_{d_{th}}$ (at least up to the considered maximum time) and the particle density converges to relatively large steady state values ($\rho \approx 0.6$), while activity dies off exponentially for any $p_d > p_{d_{th}}$. This behavior is compatible with a first order phase transition, but the location of the threshold value has to be considered as a rough estimate.

To better locate the transition point, we study the mean survival time (MST) as a function of system size. Figure 1b shows a non-standard non-monotonous dependence of the MST as a function of size $N = L^2$. As we see, there are two regimes: $i)$ for $L < L_c$, there is an exponential increase of the MST with system size; $ii)$ for $L_c < L$, and quite counter-intuitively, the MST decreases with increasing system size. This behavior can be rationalized following recent work where a particle system very similar to ours is studied by employing a semiclassical approach [32] (see also Ref. [33]). Following this study, the first regime corresponds to the standard Arrhenius law, i.e., the fact that a quasi-stationary state with a finite particle density experiences a large fluctuation extinguishing the activity in a characteristic time which grows exponentially with system size [33]. On the other hand, there is a “critical system size” above which the most likely route to “extinction” consists on the formation of a critical nucleus that then expands in a ballistic way, destabilizing the quasi-stationary state. Obviously, the larger the system size the most likely that a critical nucleus is spontaneously formed by fluctuations. Finally, for sufficiently large system sizes there is a last “multi-droplet” regime in which many nuclei are formed and the MST ceases to depend on system size, reaching and asymptotic value [32]. This picture fits perfectly well with our numerical findings.

From this analysis, we conclude that, with the present computational resolution, we can just give a rough estimation for the location of the transition point $0.070 < p_{d_{th}} < 0.075$.

To show further evidence of the discontinuous nature of the phase transition, figure 2 illustrates the system bistability around the transition point: depending on the density of the initial configuration, a homogeneous steady state may converge either to a stationary state of large density (active) or to the absorbing state. A separatrix marks the distinction between the two different basins of attraction. Let us remark that systems exhibiting a first-order transition are bistable only at exactly the transition
FIG. 1: (Color online) Decay from a homogeneous initial condition in the pure model. (a) Time evolution of the total averaged particle density for $N = L^2 = 256^2$; a first order phase transition can be observed near $p_{d,thr} \approx 0.0747$. (b) Mean survival time, $t_F$, required to reach an arbitrarily small density value here fixed to 0.01 (results are robust against variations of this choice) as a function of system size. Up to $10^3$ realizations have been used to average these results. From this finite size analysis, the threshold point can be bounded to lie in the interval $[0.070, 0.075]$. Point but for finite system sizes the coexistence region has some non-vanishing thickness. The existence of bistability makes a strong case for the discontinuous character of the transition.

Spreading experiments from a localized seed– We consider a few (at least 2) neighboring particles at the center of an otherwise empty lattice, and monitor how activity spreads from that seed. Each simulation run ends whenever the absorbing state is reached or when activity first touches the boundary of the system. We monitor the averaged squared radius from the origin $R^2(t)$, the averaged number of particles over surviving trials, $N_s(t)$, and the survival probability, $P_s(t)$ [11]. Figure 4 shows log-log plots of these three quantities as a function of time. In all cases, we find a threshold value $p_{d,thr} \approx 0.073$ that marks a change of tendency, signaling the frontier between the absorbing and active phases. In the active phase ($p_d < p_{d,thr}$) and for large values of $t$, both $N_s(t)$ and $R^2$ grow approximately as $t^2$ (as expected for ballistic expansion), while $P_s(t)$ converges to a constant (i.e. some runs do survive indefinitely). On the other hand, in the absorbing phase all three quantities curve downwards indicating exponential extinction.

Thus, the pure model exhibits a discontinuous transition at some value of $p_{d,thr} \approx 0.073$, which separates a phase of high activity from an absorbing one. Observe, that the estimation of the transition point is compatible with the interval obtained above.

B. Disordered model

In the disordered version of the model, each lattice site has a random uncorrelated (death) probability. In particular, we take $p_d(x) = p_d r$ where $p_d$ is a constant and $r$ is a homogeneously-distributed random number $r \in [0, 2]$ (and, thus, the mean value is $p_d$). Spatial disorder is refreshed for each run, to ensure that averages are independent of any specific realization of the disorder.

Homogeneous initial conditions– We have computed time series for i) the mean particle density averaged over all runs and ii) the mean particle density for surviving runs (i.e. those which have not reached the absorbing state). Figure 5 shows time evolution after up to $2 \times 10^4$ realizations. Results are strikingly different from those of the pure-model.

For values below threshold, $p_d < p_{d,c} \approx 0.077$, the par-
FIG. 3: (Color online) Spreading experiments for the pure model. Double-logarithmic plot of the (from bottom to top), (i) the survival probability $P_s(t)$, (ii) the averaged number of particles $N_s(t)$ (averaged over surviving runs), and (iii) the averaged squared radius (averaged over all runs), as a function of time, for $N = 1024^2$ using up to $4 \times 10^{10}$ experiments. Curves for $R^2(t)$ have been shifted upwards for clarity. In spite of the large number of runs used to average, curves are still noisy. This is due to the fact that, being very close to the transition point, a large fluctuation is needed for the system to “jump” to the active phase from the vicinity of the absorbing one and only a few runs reach large times.

Particle density converges to a constant value for asymptotically large times, while for $p_d > 0.077$ curves decay as power laws (a much more precise estimation of the critical point will be computed below). The generic algebraic decay is observed for a wide range of $p_d$; however, the transient before the power-law regime increases with $p_d$, which makes it difficult to determine the exact boundaries of the mentioned range. The presence of generic algebraic scaling in an extended region is the trademark of Griffiths phases.

Plotting the activity over the surviving trials [Fig. 4b], we observe that the evolution is non-monotonous in the absorbing (Griffiths) phase: the curves decrease up to a minimum value and then increase. This stems from the fact that realizations with large rare active regions remain active for longer times than those with smaller ones; as realizations with only relatively small rare-regions progressively die out, those with larger and larger rare-regions are filtered through and, thus, the overall average density grows as a function of time, being limited only by system size.

In addition, we observe that, contrarily to the pure case, there is no bistability around the transition point (figure 3). Indeed, very near to the transition point ($p_d = 0.07650$), all curves regardless of their initial value converge to a unique well-defined stationary density close to zero, as appropriate for a continuous transition to an absorbing state.

Spreading experiments from a localized seed—Figure 6 shows results for three spreading observables as a function of time; for all of them, we clearly observe generic asymptotic power laws with continuously varying exponents. These spreading quantities also allow us to scrutinize the behavior at the critical point. As discussed in the Introduction, in a disordered system as the one under study, we expect logarithmic (activated scaling) at criticality. Indeed, Figure 7 shows results for the usual spreading quantities represented in a double logarithmic plot of the different quantities as a function of $ln(t/t_0)$. The value of $t_0$ is in principle unknown and constitutes a significant error source [23]. We fix it as the value of $t$ such that it gives the best straight lines at the transition point for all three quantities [23]. Right at the critical point ($p_c \approx 0.07652$ to be obtained with more accuracy below) a straight asymptotic behavior in-
FIG. 5: (Color online) Absence of bistability in the disordered model. Double-logarithmic plot of the averaged particle density as a function of time for $p_d = 0.0765$, with $N = 256^2$, and up to $10^6$ realizations. Initial densities are $\rho_0 = 0.00006, 0.01, 0.2, 0.3, 0.4, 0.7, 1$. Regardless of the initial condition, the system stabilizes to a constant small value of the density, as expected for a second-order phase transition.

dicates that results are compatible with logarithmic (i.e. activated) type of scaling. The best estimates for the (pseudo)-exponents listed in section II are: $\bar{\delta} \approx 1.90$, $\bar{\theta} \approx 2.09$, and $\Psi = 0.43$, which are compatible with the values reported in the literature for the universality class of directed percolation with quenched disorder (i.e. $\bar{\delta} = 1.9(2), \bar{\theta} = 2.05(20), \Psi = 0.51(6)$).

Similarly, following the work of Vojta and collaborators [23], we represent in Fig. 8 one of the spreading quantities as a function of another one, e.g. $N(t)$ as a function of $P_s(t)$ to eliminate the free variable $t_0$ from the plot. This type of plot allows for the identification of power-law dependencies rather than logarithmic ones, i.e. $N(t) \sim P_s(t)^{-\bar{\theta}/\bar{\delta}}$. If the second-order phase transition belongs to the universality class of the directed percolation with quenched disorder (see section II), we should have $N(t) \sim P_s(t)^{-1.08(15)}$, using as a reference the values in the literature [23]. Indeed, as shown in Figure 8 we obtain $N(t) \sim P_s(t)^{-1.10(2)}$, in very good agreement with the expected value [23], and this is the method by which the critical point location, $p_d \approx 0.07652$, is obtained with best accuracy.

IV. CONCLUSIONS AND DISCUSSION

In contrast with the pure model, in the disordered case we have found a Griffiths phase and a second-order phase transition with an activated type of scaling. Therefore, in this non-equilibrium system with one absorbing state the situation remains much as in equilibrium situations: disorder annihilates discontinuous transitions and induces criticality.

Results are rather similar to those reported for the standard contact process with quenched disorder. Indeed, results are fully compatible (up to numerical precision) with the standard strong-disorder fixed point of the universality class of the directed percolation with quenched disorder [20, 24]. We believe that our results are robust upon considering other types of (weaker) disorders [26]. Thus, two different models with signifi-
FIG. 7: (Color online) Spreading experiments in the disordered model. Double logarithmic plot of the three usual spreading quantities as a function of \( \ln(t/t_0) \) for different parameter values (from top to bottom 0.07, 0.073, 0.075, 0.07652, 0.077, 0.0775, 0.078 (curves in the active phase are plotted with dashed lines). Same network sizes as in the homogeneous case and up to \( 10^6 \) realizations. By conveniently choosing \( t_0 = 0.01 \) (see main text) we observe straight lines at the critical point, \( p_d \approx 0.07652 \). From their corresponding slopes we measure the associated (pseudo)-exponents: \( \bar{\theta} \approx 2.09, \bar{\delta} \approx 1.9, \) and \( 2/\Psi \approx 4.6 \) (slopes marked by dashed lines). These values have a large uncertainty, as changes in the value of \( t_0 \) severely affect them. Estimating these exponents with larger precision is computationally very demanding.

FIG. 8: (Color online) Double logarithmic plot of \( N(t) \) as a function of \( 1/P_s(t) \) for spreading experiments at criticality in the disordered model [24]. Our best estimate for the slope of at the critical point (separatrix of the curves, see main panel) is compatible with the value reported in the literature \( N(t) \sim P_s(t)^{-\beta/\delta = -1.08} \) corresponding to the universality class of the directed percolation with quenched disorder. The inset shows a zoom around the critical point. Lattice size \( N = 1024^2 \); averages up to \( 5 \times 10^7 \) realizations, and same parameters as in Figure 7.

Griffiths phases and activated scaling at the transition point.

From a more general perspective, deciding whether novel universal behavior emerges in disorder-induced criticality is still an open problem in statistical mechanics. For illustration, let us point out that recent work suggests that disorder-induced second-order phase transitions in an Ising-like system with up-down symmetry does not coincide with Ising transition [35]. Similarly, in Ref. [36] a novel type of critical behavior is found for disorder-induced criticality. In the case studied here, the disorder-induced criticality does not seem to lead to novel behavior (up to numerical precision); indeed, all evidences suggest that it behavior coincides with the universality class of the directed percolation with quenched disorder.

After a careful inspection of the literature in search of discontinuous transitions in disordered non-equilibrium low-dimensional systems, we found a very recent work in which the authors study the popular (two-dimensional) Ziff-Gulari-Barshad (ZGB) model for catalytic oxidation of carbon monoxide [37] in the presence of catalytic impurities (a fraction of inert sites) [38]. The pure ZGB model is known to exhibit, among many other relevant features, a discontinuous transition into an absorbing state. However, after introducing quenched disorder, no matter how small its proportion, the discontinuous transition is replaced by a continuous one [38], similarly to our findings here.

In conclusion, we conjecture that first-order phase transitions cannot appear in low-dimensional disordered systems with an absorbing state. In other words, the Imry-Ma-Aizenman-Wehr-Berker argument for equilib-
rium systems can be extended to non-equilibrium situations including absorbing states. The underlying reason for this is that, even if the absorbing phase is fluctuationless and hence is free from the destabilizing effects the Imry-Ma argument relies on, the other phase is active and subject to fluctuation effects. Therefore, intrinsic fluctuations destabilize it as predicted by the Imry-Ma-Aizenman-Wehr-Berker argument, precluding phase coexistence.

Remarkably, in the case studied by Barghathi and Vojta, in which the Imry-Ma argument is violated in favor of a second-order phase transition, the two broken-symmetry states are absorbing ones: once the symmetry is broken in any of the two possible ways, the system becomes completely frozen, i.e., free from fluctuation effects, and, consequently, the Imry-Ma argument breaks down. Thus, the only possibility to have first-order phase transitions in low-dimensional disordered systems would be to have (in its pure version counterpart) a discontinuous phase transition between two different fluctuationless states, and we are not aware of any such transition. Therefore, we conclude that quenched disorder forbids discontinuous phase transitions in low-dimensional non-equilibrium systems with absorbing states.

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Appendix A: Simple mean field approach

In order to obtain a simple mean-field approach to the present problem, expected to hold in the high-dimensional limit, we consider a fully connected lattice in which each node is nearest neighbor to any other one. $n(t)$ is the total number of particles at a given time and $\rho(t) = n(t)/N$ the corresponding density. Allowed changes at any time-step are of the magnitude $\pm 1/N$. We can thus write the overall transition rates as $W^-(\rho \rightarrow \rho - 1/N) = \mu \rho$ and $W^+(\rho \rightarrow \rho + 1/N) = \lambda \rho^3 (1 - \rho)$ for creation and annihilation processes, respectively. Expanding the associated master equation in power series, and keeping only the first two leading terms we obtain the Fokker-Planck equation:

$$\partial_t P(\rho, t) = -\partial_\rho [(-\lambda \rho^3 + \lambda \rho^2 - \mu \rho)P] + \frac{1}{2N} \partial^2_\rho [(-\lambda \rho^3 + \lambda \rho^2 + \mu \rho)P],$$

equivalent to the (Itô) Langevin equation [35]:

$$\partial_t \rho = -\mu \rho + \lambda \rho^2 - \lambda \rho^3 + \sqrt{\frac{\mu \rho - \lambda \rho^3 + \lambda \rho^2}{N}} \xi(t),$$

where $\xi(t)$ is a Gaussian white noise. In the $N \rightarrow \infty$ (mean-field) limit one recovers the rate equation [Eq.(3)] with its associated discontinuous transitions. Using the noise term it is possible to derive (using the theory of mean first passage times [32,34]) the scaling of the escape time as a function of the system size.

In general, one-component reaction diffusion systems with $l$-particle creation and $k$-particle annihilation [39]:

$$kA \rightarrow (k - n)A, \quad lA \rightarrow (l + m)A, \quad (A1)$$

always exhibit a first-order transition if $l > k$ if the particles are fermionic. Instead, if the hard-core constraint is excluded, an additional reaction $iA \rightarrow jA$ with $i > j$ and $i > l$ is needed to stabilize the system.

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