Dynamics of cell wall elasticity pattern shapes the cell during yeast mating morphogenesis.

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Content

Supporting Text S1..............................................................................................................................1
Dynamic Cell Wall Models (DM)........................................................................................................1
Steady State Cell Wall Model (SM).....................................................................................................6
References...........................................................................................................................................9
Figures S1 - S13....................................................................................................................................10
Table S1 Parameters values used in the models..................................................................................24

Supporting Text S1

Dynamic Cell Wall Models (DM)

We developed a computational model to simulate inhomogeneous elasticity and elasto-plastic growth due to internal turgor pressure in space and time. The approach is based on continuum mechanics, where plane stress for thin shells is assumed. The cell wall is discretized into a triangular mesh using the finite element mesh generator Gmsh (Geuzaine and Remacle 2009) and simulations are performed with the general purpose finite element framework DUNE (Bastian et al. 2008). We simulated the model equations in different situations for a wide range of parameters. Furthermore, the analytical results derived for the steady state model (SM) were compared and matched with the dynamic models DM1 and DM2.
First, we introduce the elastic properties of a single triangle in the discretization of the cell wall as suggested by Delingette (Delingette 2008). Let \( T \) be a triangle of the cell wall discretization, where the points of the undeformed triangle are denoted by \( P_1, P_2, P_3 \) and the points of the elastically deformed triangle \( \hat{T} \) are denoted by \( Q_1, Q_2, Q_3 \), as depicted in figure S10. In the remainder, we refer to the elastically undeformed and deformed state of the triangle also as relaxed and stretched triangle, respectively. The area of the relaxed and stretched triangle is denoted as \( A_T \) and \( \hat{A}_T \), respectively. The evolution of the cell wall was described by a deformation function \( \Phi(\tilde{x}, t) \). The right Cauchy-Green deformation tensor was computed from

\[
C = \nabla \Phi^T \nabla \Phi
\]  

and the Green-Lagrange strain tensor from

\[
\varepsilon = \frac{1}{2} (C - I).
\]

These tensors were used to calculate the elastic energy, elastic forces and yield criteria. There are three important quantities that were expressed in terms of angles and length of the sides of the deformed and the relaxed triangle (Delingette 2008):

\[
\text{tr}(C) = \frac{1}{2A_T} \left( l_1^2 \cos \alpha_{T,1} + l_2^2 \cos \alpha_{T,2} + l_3^2 \cos \alpha_{T,3} \right),
\]

\[
\text{tr}(\varepsilon) = \frac{1}{4A_T} \left( (l_1^2 - l_1^2) \cos \alpha_{T,1} + (l_2^2 - l_2^2) \cos \alpha_{T,2} + (l_3^2 - l_3^2) \cos \alpha_{T,3} \right),
\]

\[
\text{det}(C) = \frac{\hat{A}_T}{A_T}.
\]

Here, \( l_{T,i} \) and \( \hat{l}_{T,i} \) are the lengths and \( A_T \) and \( \hat{A}_T \) are areas of the relaxed triangle and stretched triangle, respectively. The angles of the relaxed triangle are denoted by \( \alpha_{T,i} \). A sketch of the relaxed state and stretched triangle can be found in figure S10. The elastic energy of a single triangle \( T \) with thickness \( d \) is given by

\[
W^\text{el}_T = \frac{\lambda}{2} \text{tr}(\varepsilon)^2 + \frac{\mu}{2} \text{tr}(\varepsilon^2)
\]
\[ \sum_{i=1}^{3} k_{T,i} = \sum_{i \neq j}^{3} c_{T,i} \left( l_{T,i}^2 - l_{T,j}^2 \right)^2 \left( l_{T,i}^2 - l_{T,j}^2 \right)^2. \] (6)

The coefficients depend on the Young’s modulus \( E^* \) for plane elasticity, the cell wall thickness \( d \) and the Poisson’s ratio \( \nu \). The Young’s modulus for plane elasticity is related to the 3D Young’s modulus \( E \) by

\[ E^* = \frac{E}{(1 - \nu^2)}. \] (7)

Here, \( \mu \) and \( \lambda \) are the Lamé parameters, which are given by

\[ \lambda = \frac{E^* \nu}{(1 - \nu^2)}. \] (8)

\[ \mu = \frac{E^*}{(1 + \nu)}. \] (9)

The coefficient \( k_{T,i} \) can be interpreted as tensile stiffness and the coefficient \( c_{T,i} \) as angular stiffness. These coefficients read

\[ k_{T,i} = E^* d \frac{2 \cot^2 \alpha_i + 1 - \nu}{16(1 - \nu^2)A_T}, \] (10)

\[ c_{T,i} = E^* d \frac{2 \cot \alpha_{T,j} \cot \alpha_{T,j} - 1 + \nu}{16(1 - \nu^2)A_T}. \] (11)

From the elastic energy of each triangle the resulting forces at vertex \( i \) were computed as

\[ \vec{p}_{T,i}^{el} = \sum_{i \neq j} k_{T,j} \left( l_{T,k}^2 - l_{T,k}^2 \right) \left( \vec{Q}_{T,j} - \vec{Q}_{T,k} \right) + \sum_{i \neq j} \left( c_{T,j} \left( l_{T,i}^2 - l_{T,i}^2 \right) + c_{T,i} \left( l_{T,j}^2 - l_{T,j}^2 \right) \right) \left( \vec{Q}_{T,j} - \vec{Q}_{T,i} \right). \] (12)
In this representation the coordinates of the stretched triangle are represented by \( \vec{Q}_{T,i} \). The total force at each vertex was computed from the sum of the forces induced by the turgor pressure and the counteracting elastic force:

\[
\vec{F}_{T,i} = \vec{F}_{T,i}^\text{turgor} + \vec{F}_{T,i}^\text{el}.
\]

The forces induced by the turgor pressure were computed from:

\[
\vec{F}_{T,i}^\text{turgor} = \frac{1}{3} A_T P \vec{n}_T,
\]

where \( \vec{n}_T \) is the outward pointing unit normal of triangle \( T \). The equation of motion for every vertex of the mesh reads

\[
\ddot{\vec{x}}_n = \sum_{T} \frac{1}{m_n} \vec{F}_{T,i},
\]

with \( x_n \in T \).

The mass \( m_i \) was computed from the triangle area as \( m_n = \frac{1}{3} \rho A d \), where \( A \) is the area of all triangles that share the vertex \( n \).

**Elasto-plastic growth and yield criteria**

The elasto-plastic growth depends on a yield stress criterion. In case the value calculated from the yield criterion was above a given yield limit, new material was inserted. In case the yield limit was not reached, we assumed only elastic deformations (see figure S10). Computationally this process was modeled by an irreversible deformation of the relaxed triangle. For these plastic deformations, we tested two yield criteria, yield stress \( \sigma_Y \) (model DM1) and yield strain \( \varepsilon_Y \) (model DM2). As yield stress criterion we used the von Mises yield criterion for plane stress (Yu 2006):

\[
\sigma_Y < \sigma_{VM} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2},
\]
where $\sigma_1$ and $\sigma_2$ are the principal stresses. Assuming a spherical shmoo tip with typical radius $0.5 \, \mu m - 1.0 \, \mu m$ and a turgor pressure of $0.2 \, MPa$ the maximum and von Mises stress is approximately $0.5 \, MPa$ to $1.0 \, MPa$. It shall be noted, that in case of a cylinder the Maximum stress differs from the von Mises stress:

$$\sigma_{\text{max}} = \frac{p_r}{d}, \quad \sigma_{\text{VM}} = \frac{3p_r}{4d}.$$ (17)

For the dynamic model the stress criterion was computed from the linear stress tensor:

$$S = \lambda \, \text{tr}(\varepsilon) I + \mu \varepsilon.$$ (18)

The trace of $S$ is given by

$$\text{tr}(S) = 2\lambda \, \text{tr}(\varepsilon) + \mu \text{tr}(\varepsilon),$$ (19)

and the determinant by

$$\det(S) = (\lambda^2 + \lambda \mu)\text{tr}(\varepsilon)^2 + \frac{\mu^2}{4}(\det(C) - \text{tr}(C) - 1).$$ (20)

Using these quantities, the von Mises stress criterion can be computed as follows

$$\sigma_{\text{VM}} = \sqrt{\text{tr}(S)^2 - 3\det(S)}.$$ (21)

The subsequent expansion rate was calculated from

$$\alpha = \begin{cases} \Phi \cdot (\sigma_{\text{VM}} - \sigma_Y) & \text{if } \sigma_{\text{VM}} > \sigma_Y, \\ 0, & \text{else.} \end{cases}$$ (22)

For the extensibility $\Phi$ a bell-shaped distribution around the tip was assumed

$$\Phi(\vec{x}) = \lambda e^{-\frac{|\vec{x} - \vec{x}_{\text{tip}}|^2}{2R_{\text{growth}}^2}}.$$ (23)
The expansion of the cell wall was modeled as elongation of the edges of the relaxed triangle. Let \( T \) and \( T' \) be two adjacent triangles and \( L_{T,T'} \) the relaxed length of the edge that both triangles share. The expansion rates \( \alpha_T \) and \( \alpha_{T'} \) were calculated as above for the triangles \( T \) and \( T' \), respectively. The elongation of the edge \( L_{T,T'} \) is given by

\[
\frac{1}{L_{T,T'}} \frac{dL_{T,T'}}{dt} = \frac{1}{2} (\alpha_T + \alpha_{T'}). 
\] (24)

For the DM2 we modelled cell wall expansion upon a yield strain. Here, we used the volumetric strain as a measure for yielding:

\[
\varepsilon_V = \text{tr}(\mathbf{E}). 
\] (25)

The extensibility is in this case given by \( \Phi^* = (\lambda^*/\lambda) \Phi \). Which leads to the expansion rate

\[
\alpha^* = \begin{cases} 
\Phi^* \cdot (\varepsilon_V - \varepsilon_Y) & \text{if } \varepsilon_V > \varepsilon_Y \\
0 & \text{else.}
\end{cases} 
\] (26)

The elongation of the edge \( L_{T,T'} \) is then analogously given by

\[
\frac{1}{L_{T,T'}} \frac{dL_{T,T'}}{dt} = \frac{1}{2} (\alpha_T^* + \alpha_{T'}^*). 
\] (27)

**Steady State Cell Wall Model (SM)**

In a mechanistic model for the cell shape, it is crucial to describe the distribution of forces due to turgor pressure, material insertion and the counterbalancing forces, which can be derived from the material properties of the cell wall. The first basic relationship connects the stresses (force per unit area) on the cell wall to the turgor pressure \( P \), the cell wall thickness \( d \) and the local geometry. The latter is characterized by the principal curvatures. We assumed the cell shape to have a rotational symmetry and the shape of the cell was described as a surface of rotation.
The distribution of the corresponding stresses are expressed in terms of curvatures (Flügge 1973):

\[
\sigma_s(s) = \frac{P}{2d\kappa_\theta(s)}, \quad \sigma_\theta(s) = \frac{P}{2d\kappa_\theta(s)} \left(2 - \frac{\kappa_s(s)}{\kappa_\theta(s)}\right).
\] (28)

Here, the principal curvatures are given by the meridional curvature, \(\kappa_s\), and the circumferential curvature, \(\kappa_\theta\). The stresses were connected to the strains with a constitutive relationship for linear elasticity (Flügge 1973)

\[
\begin{pmatrix}
\sigma_s(s) \\
\sigma_\theta(s)
\end{pmatrix} = \begin{pmatrix}
E^*(s) \\
(1-v^2)
\end{pmatrix} \begin{pmatrix}
1 & \nu \\
\nu & 1
\end{pmatrix} \begin{pmatrix}
\varepsilon_s(s) \\
\varepsilon_\theta(s)
\end{pmatrix},
\] (29)

where \(E^*(s)\) is the Young’s modulus for plane elasticity, \(v\) is the Poisson’s ratio, \(\varepsilon_s(s)\) and \(\varepsilon_\theta(s)\) are the meridional and circumferential strain, respectively. The Young’s modulus and strains are functions depending on the arc length.

Here, \(\varepsilon_s(s) = \frac{ds-d\bar{s}}{ds}\) and \(\varepsilon_\theta(s) = \frac{r(s)-R(s)}{R(s)}\) are the meridional and circumferential strain, respectively. While \(d\bar{s}\) denotes a small relaxed and \(ds\) the actual extend of a small surface patch in meridional direction, \(S\) is the meridional distance of the relaxed shape measured from the base end. Note, in all figures the arclength \(s\) is plotted from the tip instead of the base for better comparison to the dynamic model (see figure S11). As for the dynamic model, the von Mises stress and volumetric strain are given by:

\[
\sigma_{VM}(s) = \sqrt{\sigma_s^2(s) + \sigma_\theta^2(s) - \sigma_s(s)\sigma_\theta(s)},
\] (30)

\[
\varepsilon_V(s) = \varepsilon_s(s) + \varepsilon_\theta(s).
\] (31)

From equations (28) and (29) we can derive a relationship between circumferential and meridional strain, which only depends on the geometry and the Poisson’s ratio

\[
\varepsilon_s(s) = \varepsilon_\theta(s) \frac{(1-2v)\kappa_\theta(s) + v\kappa_s(s)}{(2-v)\kappa_\theta(s) - \kappa_s(s)}.
\] (32)

Using this relationship, we get the formula
\[ ds = \left( \frac{r(s)}{R(s)} - 1 \right) \left( 1 - 2\nu \kappa_\theta(s) + \nu \kappa_\phi(s) \right) \left( \frac{2\nu}{\kappa_\theta(s) - \kappa_\phi(s)} - 1 \right) dS, \] (33)

(see also (Bernal, Rojas, and Dumais 2007)), which was used to identify points of the relaxed and natural shape of the cell starting from the end of the base. Therefore, strains and stresses were calculated from the parameters \( P, d \) and \( \nu \) and the geometry of the relaxed and natural shape only. Note, that the strains were not calculated in the growth region, since we assumed plastic growth at the shmoo tip. Inserting this relationship in the constitutive model we get:

\[ \varepsilon_\theta(s) = \frac{r(s) - R(s)}{R(s)} = \frac{1}{E^*(s)} \left( \frac{P}{2d\kappa_\theta(s)} \left( 2 - \frac{\kappa_\phi(s)}{\kappa_\theta(s)} \right) - \nu \frac{P}{2d\kappa_\theta(s)} \right). \] (34)

For a given cellular geometry, the elasticity distribution was computed from (Bernal, Rojas, and Dumais 2007):

\[ E^*(s) = \frac{R(s)}{r(s) - R(s)} \left( \frac{P}{2d\kappa_\theta(s)} \left( 2 - \frac{\kappa_\phi(s)}{\kappa_\theta(s)} \right) - \nu \frac{P}{2d\kappa_\theta(s)} \right). \] (35)

We calculated the stress and elasticity distribution for the shmoo shape shown in figure 1 (see main text) and figure S2. For the numerical computations, cubic spline functions were used to characterize the shape. The NumPy and SciPy packages in Python were used for this purpose. Calculations for different assumptions on relaxed and extended cell shape and strains are shown in figure S2. In the special case of a sphere or a cylinder, the strains, the stresses and the elasticity can be computed analytically (see figure S1). For a sphere, the circumferential and meridional strain are given by

\[ \varepsilon_s = \varepsilon_\theta = \frac{r - R}{R} = \frac{1 - \nu}{E^* \frac{P}{2d^3}} \] (36)

and the principal stresses are given by

\[ \sigma_s = \sigma_\theta = \frac{P r}{2d^3} \] (37)

8
Assuming a spherical geometry at the base as well as tip, the Young’s modulus the tip can be approximated by the following formula:

\[ E^* = (1 - \nu) \frac{PrR}{2d(r - R)}. \]  

(38)

This formula was used to compare the Young’s Modulus obtained from AFM measurements and the Young’s modulus estimated by osmotic shock experiments.

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Figures S1 - S13

Figure S1. Geometrical considerations and principal plane stresses and strains. (a) shows the used coordinates: circumferential angle $\theta$, meridional distance $s$, radius $r$ and shell thickness $d$. (b) shows principal stresses and strains for a given shell element. (c) Circumferential and meridional stresses ($\sigma_\theta$, $\sigma_s$) and strains ($\varepsilon_\theta$, $\varepsilon_s$) are equal for a sphere, while $\sigma_\theta$ is twice as high as $\sigma_s$ for the lateral surface of a cylinder. Additionally, $\varepsilon_\theta$ exceeds $\varepsilon_s$, given that $\nu \leq 0.5$. 

\[
\sigma_\theta = \sigma_s = \frac{P}{2d}, \\
\varepsilon_\theta = \varepsilon_s = \frac{\sigma_s}{E} (1-\nu), \\
\sigma_0 = 2\sigma_s = P \frac{d}{d} \\
\varepsilon_s = \frac{\sigma_s}{E} (1-2\nu), \\
\varepsilon_\theta = \frac{\sigma_s}{E} (2-\nu)
\]
Figure S2. Stress, strain and elasticity profiles of the SM. (a) Contours for the relaxed (dashed line) and expanded (solid line) cell shape. The tip was the origin of the meridional coordinate, indicated with a dashed arrow. The regions base (I), neck (II), shaft (III), and tip (IV), are separated by dashed lines for orientation. For the cell shape in (a) profiles of: (b) Young’s modulus $E$, von Mises stress $\sigma_{\text{VM}}$ and volumetric strain $\epsilon_V$, C meridional stress $\sigma_s$, circumferential stress $\sigma_\theta$ and $\sigma_{\text{VM}}$, (d) meridional strain $\epsilon_s$, circumferential strain $\epsilon_\theta$ and $\epsilon_V$ are shown. (e), (f) $\epsilon_V$- and $E$-profiles for different relaxed shapes with varying radius $R_{\text{shaft}}$ were calculated. (g), (h) Profiles of $\epsilon_s$ and $\epsilon_\theta$ or $E$ and $\sigma_{\text{VM}}$ assuming a constant volumetric strain $\epsilon_V$ and the expanded cell shape in (a).
Figure S3. Mapping the cell wall elasticity of entrapped haploid *S. cerevisiae* cells. (a) Scheme of the experimental setup: the previously trapped yeast cell is scanned by an AFM in QI™ Mode. For each pixel, a nano-indentation measurement was performed. (b) 3D representation of the shape detected in (a). (c) Hypothetic approach curve for one pixel, along with a Sneddon fit (green) of these data assuming a conical indenter. The resulting spatial information on the elasticity (0 – 20 MPa) of the probed material was displayed in 2D images (d) or used as a texture for 3D representations (b). (b) and (d) show the same *MATa bar1Δ* cell with characteristically stiffer bud scar regions (indicated by arrows); scale bar is 1 µm.
Figure S4. Indentation depth varies between stiffer and softer regions. (a) Exemplary force-distance curves of nano-indentation measurements at the marked region in (b) and (c). (b) shows elasticity map and (c) indentation map of shmooing *MATa bar1Δ* cell shown in figure 2. If both curves reached a similar maximum force, the conical tip of the cantilever indented the cell wall in softer region (shaft, green dots) deeper than the stiffer region (base, orange squares), due to the smaller slope of the curve (b). The indentation depth $\delta$ (mean ± ra, N = 900) of a quadratic region (550 × 550nm) at the top of the cell was at the shaft ($\delta_{\text{shaft}} = 97 ± 26$nm) and at the base ($\delta_{\text{base}} = 38 ± 10$nm) less or equal than 115nm, which is supposed to be the thickness of the cell wall.
Figure S5. Comparison of height and elasticity patterns between α-factor treated and non-treated MATa bar1Δ cells. (a), (b) Height images and (e), (f) elasticity maps of two non-treated cells and (c), (d) height images and (g), (h) elasticity maps of two individual MATa bar1Δ cells treated with 10µM α-factor; black and white bars correspond to 1µm. The white arrows represent position length and direction of the selected cross-sections in (i), (j), (k), (l). Manually selected regions for base (I), shaft (II) and tip (IV) are framed with white rectangles.
Figure S6. The cell wall, at shaft of the mating projection, showed significant lower $E$-values. (a) Scatter Dot Plot of mean $E$-values for selected regions at the base, shaft and tip, respectively; bar represents the mean. (b) Mean $E$-values of selected regions at the shaft, base and tip plotted against the mean $E$-values at the base for each measured cell. Lines correspond to linear regressions forced through $(0,0)$ with a slope of $E_{\text{shaft}}/E_{\text{base}} = 0.28 \pm 0.06$ (Sy.x=0.41, DF 6), $E_{\text{tip}}/E_{\text{base}}=0.71 \pm 0.2$ (Sy.x=1.7, DF 6) and $E_{\text{base}}/E_{\text{base}}=1.00 \pm 0.00$ (Sy.x=0.00, DF 6) Broken lines correspond to respective confidence intervals (95%). (c) Elasticity profiles over tip, shaft and base for all analyzed shmooing cells showed reduction of $E$ from base to shaft and tip to shaft for every cell.
**Figure S7.** Time series replica for estimation of the cell wall dynamics during mating projection formation. (a), (b) Time-lapse sequence of height and elasticity development during formation of a mating projection, obtained with AFM; from left to right, consecutive images of the $E$-distribution (top) and 3D reconstruction with elasticity pattern (bottom), from a continuous measurement. *MATa bar1Δ* cells were induced with 12 µM (a) and 10 µM (b) α-factor for 122 min (a) and 42 min (b), respectively, before the first image was acquired. White arrows indicate a region of stiffer cell wall material at the tip. Note, the first image of sequence (b) shows a barely noticeable reduction in $E$ at the side of the emerging protrusion. Both time-lapse series show typical AFM-artifacts for high objects ((a) doubling of the tip shape and (b) height “shadow”), indicated with black arrows.
Figure S8. Cell stiffness saturated at 1.5 N/m for high loading forces and large indentations. The cell stiffness, obtained with indentation experiments of three untreated bar1Δ cells, was plotted against loading forces and corresponding indentation depths. The plotted stiffness $k$ (mean, rms, N=1024), represents the maximal slope of the indentation curve, using a cantilever with $k = 0.64$ N/m. The fitted maximum, assuming a Hill function, was used to calculate the turgor pressure applying the formula: $P = k / \pi R$. 
Figure S9. Steady state model shows that relaxed shell volume declines at higher turgor pressure. (a) shows the relaxed radius of the spherical shell with resting radius 2.5 µm, (b) shows the relative volume confined by the relaxed shell and (c) shows the relative circumferential strain depending on turgor pressure.
Figure S10. The DM is based on elasto-plastic deformations of triangular surface elements. (a) shows the triangular meshed surface of the simulated shmooing cell. Note, concentric rings at the protrusion result from mesh refinement steps during the simulation. (b) In each simulation step, the triangles deform according to the applied stresses (DM1) or strains (DM2). If the triangle is not in the
defined growth zone, the triangle deforms purely elastically. $L_1, L_2, L_3$ are the relaxed lengths of the unstressed triangle $T$ and and $l_1, l_2, l_3$ are the elastically expanded lengths of the resulting triangle $T'$. Correspondingly, $\alpha_1, \alpha_2, \alpha_3$ represent relaxed angles and $\beta_1, \beta_2, \beta_3$ angles of the deformed triangle. Additionally, triangles in the defined growth zone deform plastically if $\sigma_{VM} > \sigma_Y$ (DM1) or $\varepsilon_V > \varepsilon_Y$ (DM2). Thereby, relaxed lengths expand to new relaxed lengths, $L_1^{new}, L_2^{new}$ and $L_3^{new}$, while angles remain unaltered.
Figure S11. Simulated stress, strain and elasticity profiles along the cell contour. (a) Contours of the resulting cell shape of DM1 (yellow) and DM2 (blue) at time 3250 s with indicated regions: base (I), shaft (II), neck (III) and tip (IV). (b), (c), (d) Contour plots along the arc length $s$ of the resulting von Mises stress $\sigma_{VM}$, the resulting volumetric strain $\varepsilon_V$ and the assumed Young’s modulus $E$. 
Figure S12. Sensitivity analysis of the yield limit for DM1 and DM2. (a) and (b) show cell shapes obtained from simulations with various limits for yield stress $\sigma_Y$ and yield strain $\varepsilon_Y$, respectively at 1000 s, 1500 s and 2000 s. Lower $\sigma_Y$ or lower $\varepsilon_Y$ result in faster growth. When varying parameters in a certain range, similar shapes are obtained at different time points.
Figure S13. Decrease in Young's modulus with increasing indentation velocity was negligible within the scope of this study ($v = 67\mu m/s$). The mean Young's modulus of a selected $400 \times 400$nm region in the center of a trapped cell (left) plotted against the applied indentation velocity (dots with error bars). Linear regression showed significant but minor decrease in $E$ (slope = $-0.0046 \pm 0.0008$ MPa/($\mu m/s$), $R^2=0.85$, $F(1, 6)=33.93$ p=0.001).
Table S1 Parameters values used in the models

| Property                                      | Abbr. | Value                  |
|-----------------------------------------------|-------|------------------------|
| **General**                                   |       |                        |
| measured turgor pressure                      | $P$   | $0.21 \pm 0.05 \text{ MPa}$ |
| cell wall thickness                           | $d$   | $115 \text{ nm}$       |
| Poisson's ratio                               | $\nu$ | $0.5$                  |
| **Steady state model (SM)**                   |       |                        |
| relaxed base radius                           | $R_{\text{base}}$ | $1.9 \mu\text{m}$ |
| relaxed shaft radius                          | $R_{\text{shaft}}$ | $0.5 \mu\text{m}$ |
| expanded base radius                          | $r_{\text{base}}$ | $2.5 \mu\text{m}$ |
| turgor pressure                               | $P$   | $0.2 \text{ Mpa}$      |
| **Dynamic cell wall models (DM1 & DM2)**      |       |                        |
| mass density                                  | $\rho$ | $0.5$                  |
| yield stress                                  | $\sigma_Y$ | $0.4 \text{ MPa}$ |
| yield strain                                  | $\varepsilon_Y$ | $0.1$                 |
| Young's modulus at the base                   | $E_{\text{base}}$ | $2.5 \text{ Mpa}$ |
| Young's modulus at the shaft                  | $E_{\text{shaft}}$ | $0.7 \text{ Mpa}$ |
| Young's modulus at the tip                    | $E_{\text{tip}}$ | $1.8 \text{ Mpa}$ |
| characteristic elasticity time                | $\tau$ | $200 \text{ s}$        |
| relaxed base radius                           | $R_{\text{base}}$ | $1.9 \mu\text{m}$ |
| initial expanded base radius                  | $r_{\text{base}}$ | $2.5 \mu\text{m}$ |
| radius of enhanced Young's modulus at the tip | $R_{\text{tip}}$ | $0.4 \mu\text{m}$ |
| radius of reduced Young's modulus at the shaft| $R_{\text{shaft}}$ | $0.8 \mu\text{m}$ |
| growth radius of the tip                      | $R_{\text{growth}}$ | $0.45 \mu\text{m}$ |
| extensibility (DM1)                           | $\lambda$ | $0.004 \text{ MPa}^{-1} \text{ s}^{-1}$ |
| extensibility (DM2)                           | $\lambda^*$ | $0.004 \text{ s}^{-1}$ |