Numerical Simulation and Experimental Study of a Deployable Footbridge

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Abstract. Spatial structures with bars have been used in different fields since the beginning of the 20th century. These were first used in aerospace techniques to obtain lightweight structures [1]. It was as of 1930 when their use in construction became widespread, especially in roofs with big spans. Deployable structures can be considered a special case within the broader class of spatial structures. They can be transformed from a closed compact configuration to a predetermined expanded form. There is a wide range of applications, such as temporary construction and roofing systems, or for movable elements in the aerospace industry. This article describes a deployable structure that has been patented by researchers of San Pablo CEU University and Eduardo Torroja Institute of Madrid, Spain. The presented invention optimizes the material needed to fulfill the safety requirements according to the span it covers. It has a folding and unfolding system that makes transport easier and cheaper. It is versatile, since it adapts perfectly to many different uses with a reduced number of elements and it also reduces the cost by making the best possible use of the materials. Specifically for space deployable structures the main challenge remains to ensure high reliability in deployed geometry, stiffness and function. The goal of this article is to simulate different options to reduce the axial stress and the deflection of a 40-meter span deployable structure. Firstly, the basic elements that define the system will be described; secondly a first analysis on software SAP2000 will be shown; finally, this paper presents the procedure and methodology to improve the shape of a structure through optimization, and to control the deflection at mid span by means of post-tensioning.

1. Introduction
Spatial structures with bars have been used in different technological fields since the beginning of the 20th century. These were first used in aerospace techniques to obtain lightweight structures [1]. It was as of 1930 when their use in construction became widespread, especially in roofs with big spans. Flat roofs and domes made of metal bars are well-known in architecture and engineering applications. Deployable structures have been primarily used for emergency shelters, temporary bridges after natural disasters, retractable roofs for sports facilities and stages. All the improvements made to these structures have to do with joints connecting tension and compression elements. Standardizing prefabricated components increases the time efficiency and safety. It lowers final costs of labour and reduces risks [2, 3]. The deployment of the structure at minimal energy and without external control is key to accomplish
time efficiency and lower costs [4]. Making physical models along with software simulations can help designers develop a control methodology for a deployment of a near full-scale footbridges [5]. A particular kind of deployable structures are characterized by tension elements (strings, tendons or cables) surrounding compression elements (bars or struts) in equilibrium [6]. These structures are suitable for deployment since shape transformations occur by changing lengths of tension elements at low energy costs [7]. The design process of spatial structures has to include identification of geometry, topology, axial stiffness, and self-stressed state [8].

Mathematical investigation of these structures also led to fundamental discoveries in the theory of rigidity and stability of frameworks. A detailed study of mathematical models shows that the shape of a deployable system can be changed substantially with little change in the potential energy of the structure, in contrast to the control of classical structures which require a large amount of energy to change their shape. Tensegrity structures are very special cases of trusses, where members are assigned special functions. Some members are always in tension and others are always in compression. Buckminster Fuller coined the word tensegrity from tension and integrity [9]. There exists a large literature on the geometry, art form, and architectural appeal of tensegrity structures [10] and some authors have studied them for engineering purposes [11-13]. The advantages of tensegrity structures over traditional ones are: i) materials of high strength tend to have very limited displacement capability whereas stowage in a compact volume is a virtue of deployable structures [14]; ii) tensegrity structures are easily tunable and some adjustments can be made after deployment; iii) bending moments are not relevant in this kind of structures for all bars and cables are axially loaded [15-17]. Some authors have studied experimentally the geometrical adaptation and pre-stress properties of a tensegrity module during deployment [18, 19].

Control of active cables has enabled successful connection at mid-span structures [20]. Self-weight significantly influences deployment [21] and simulation models should include joint dimensions for accurate prediction of nodal positions [22, 23].

The system presented in this article consists of a foldable, transportable and deployable structure that allows the construction of temporary walkways, stages and scaffold, with a low-cost system and the possibility of being reused once the purpose for which it has been installed comes to an end. The system reduces the material needed to fulfil the safety requirements according to the span it covers; it has a folding and unfolding system that makes transport easier and cheaper; it is versatile, since it adapts to many different uses with a reduced number of elements and it also reduces the cost by making the best possible use of the materials needed. The fact that it can be used as many times as needed and for different purposes is also remarkable.

This paper analyzes procedures for a deployable structure made of modules. The research team has developed a methodology in order to address the common challenges and demonstrate a whole design process. The global behavior of the structure is investigated, showing the potential benefits for simply supported beam-like structures such as footbridges in terms of static behavior. The global stiffness of this system is then examined and design conditions are raised on the self-stress state. Resistance aspects are also taken into account to propose realistic cross sections. Finally, a large-span example is presented and simulated to demonstrate the feasibility of this kind of structure.

2. Analysis of the basic module.

2.1. Geometrical analysis

The system described in this article consists of a set of tubular bars and cables made of steel, aluminum or reinforced polymers. The deployment process starts with a folded structure and ends with the final shape under operating conditions. Tension cable-length changes are relevant when it comes to analyzing the deployment behavior of the structure. These changes were studied empirically and analytically, as it is shown in figure 1. The length of tension cables changes through deployment and folding processes. A deployment path can be pre-defined for both numerical and physical models by means of algebraic equations. The addition of a guiding element helps folds the cable onto itself during the process.
2.2. Building a prototype.
In order to validate the geometric design, the team has built several prototypes to check the feasibility of the structure. Figure 2 shows the final prototype characterized by its ability to maintain an equilibrium shape with all tensile members in tension and in the absence of external forces or torques. The basic frame is the bases of the system. The linear bearing is used for precisely fitting together with axis in sliding pair, the clearance is minimal and can be neglected. So, the model of basic frame can be simplified into a mechanism only with joint pairs. Deployment accuracy has also been successfully tested.

3. A case study for structural optimization.
The structural efficiency is based firstly on the fact that the tensioned elements of the modules are made of cables. In addition to contributing to the overall stiffness and strength of the structure, these cables
are designed to guide the folding and unfolding process of the structure. In each module the tension cable is extended until it is connected to the midpoint of the compression bar, thus regulating its position and avoiding the possibility of unforeseen folding while the structure is fully deployed. A first model for analysis has been created using an educational version of the SAP2000 software (which allows the accurate reproduction of internal links between bars that this structure includes, in particular the scissor link between the diagonals of each module), the results of which are extracted below. The basic module is a 2x2 meter-grid that spans 40 meters with a depth of 2 meters. This ratio span to depth equal to 20 is based on the possibility of creating a rigid joint between lintel and supports and the corrective effect that post-tensioning will have on the system’s deformation, which will be discussed later.

Figure 3 shows the first analysis of the structure. The shift of the cable element from the top chord of the truss to the bottom one occurs on both sides at the third module starting from both ends. This frame has been subjected to the effect of a permanent area load of 1 kN/m² plus an imposed load of 1 kN/m² applied on a width of 2 meters, which could account for the maintenance load of a typical roof. In any case, at this point the determination of an order of magnitude that can then be easily scaled according to specific situations is more important than the value of the load itself. The safety coefficients of 1.35 for permanent loads (Gk) and 1.5 for variable loads (Qk) are applied.

After running the simulation, two conclusions are drawn: i) firstly, the bending effect of the scissor joint between diagonals is quite irrelevant, which is promising for the optimization of the system; ii) secondly, the shift of the cable element from one side of the truss to the other will probably have to move towards its center. In any case, the prototypes include some transition modules in which both sides are prepared to work indistinctly under tension and compression. The existence of preferential stress transmission routes is also observed, which do not always coincide with the real geometry of the frame, particularly at the bottom of the columns and in the joints between lintel and supports. The axial forces are obviously the most decisive ones for the design, with estimated values of 170 kN at the center of the bottom chord, -160 kN at the center of the top chord, 190 kN at both ends of the top chord, 180 kN on the outer side of the support and -275 kN on its inner face. In the case of diagonals, the highest values of axial force occur in the connection between lintel and supports, and are around -140 kN. In this first model the following sections have been defined: the chords have been built with a 100.60.10 hollow rectangular section, in anticipation of having to address some local bending issues at later stages of the design; the diagonals are 60.60.10 square hollow sections and the cables have a diameter of 20 mm. The material chosen for the rigid bars is steel S275, and we have assumed the cables to be made of high-strength steel with a yield limit of 1500 N/mm². These dimensions give a maximum use coefficient for the chords of 58.8%, corresponding to the out-of-plane buckling check. When it comes to the in-plane buckling check, the use coefficient is reduced to 43.6%. In the case of diagonals, the use coefficient remains around 39% if we take its buckling length as 1.41. But if we double this number in the
understanding that the out-of-plane displacement of the midpoint of each module is not restrained, this coefficient would rise to 114%, which would force to slightly increase its size. As far as cables are concerned, a 20mm thick element of such high-strength steel is more than enough to withstand the resulting stresses. In any case, the dimensions are reasonably adjusted to the requirements of the problem.

Figure 4 shows the deflection in the middle of the span is up to 50 cm. In relative terms, this is L/80, well above the usual codes requirements. It is at this point where the post-tensioning of the cable element will show the most noticeable improvements in the system.

On the same SAP2000 model, the effect of introducing a post-tensioning load on the cable of the bottom chord capable of generating strains between 0 and -20 per thousand has been studied. In terms of internal forces, the bending moments are still as irrelevant as in the previous version and a better adjustment of the axial force distribution to the expected behavior of the truss can be seen. There are elements subjected to higher axial loads than in the previous version. Nevertheless, the axial forces are more homogenous and their maximum values, responsible for the design of the different families of bars, become smaller. Figure 5 shows that when the strain imposed on the cable of the bottom chord is equal to -1%, the most compressed parts of both chords are entitled to resist an axial force of -250 kN. The maximum compression load on the first analysis was -275 kN. The diagonals would be designed to resist an axial force of -80 kN as opposed to the previous -140 kN.

In any case, the greatest improvement due to the action of the post-tensioning must be assessed in terms of stiffness. Figure 6 shows that the deflection has been reduced at the midpoint of the span below 20 cm, L/210 in relative terms, much closer to the values accepted by building codes.
Figure 6. Deflection analysis of the frame for the load combination $G_k+Q_k+P_k$, where $P_k$ is an imposed strain of -1%.

The effect of post-tensioning the bottom chord cable on the structure as a whole can be assessed in Figure 7. The X axis represents strains imposed on the different cable elements of the frame, from 0 to -20 per thousand. The red line indicates the deflection at the midpoint of the span related to the imposed strain at the bottom chord of the truss. The red line relates the deflection at this same point with the strain imposed at the exterior side of the columns and the ends of the top chord. The blue line relates the maximum tensile force at the bottom chord with the strain imposed in that same element. The purple line relates the maximum tensile force at the top chord with the strain imposed at the exterior side of the columns and the ends of the top chord. Positive values for deflections indicate downward displacements.

Figure 7. Influence of cable post-tensioning on deflections and internal forces.

When the strain is applied on the bottom chord of the truss, there is a clearly linear reduction of the lintel midpoint’s deflection that might go as far as moving this point upwards. The counter-effect of this operation is the increase of axial forces, particularly at the same point where the strain is introduced. When the strains are introduced in the cables at the columns and the top chord, the effectiveness of this operation is much smaller. In terms of deflections, the reduction is only one third of that obtained applying the post-tensioning load at the bottom chord. And, in terms of internal forces, although for small strains the increase of the maximum tensile forces seems to be very insignificant, it soon gets similar to that caused by post-tensioning at the bottom chord. Therefore, a continuous strain applied to
the cable at the bottom chord of the truss seems to be the most promising way of stiffening the design for a deployable frame.

4. Conclusions
This paper has presented the geometric and structural characteristics of deployable system. Firstly, the geometry of the basic module has been developed in order to achieve minimum size and weight when the structure is folded. The guiding element can help minimize risks on inaccuracy during the deployment process. Secondly, the structural analysis of a 40 meter span footbridge has showed that the maximum deflection at the mid-span is above the code requirements for deflection and stiffness. Cross sectional characteristics as well as the geometric configuration have a direct influence on the self-weight and the structural behavior. In consequence, an optimal realistic solution that verifies Service Limit State (SLS) conditions requires further testing. Finally, the maximum deflection has been reduced by means of introducing a post-tensioning load on the cable of the bottom chord. The positive impact of this modification is shown in the comparative analysis between the two options.

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