The impact of magnetic properties on atom-wall interaction

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Abstract

The Lifshitz-type formulas for the free energy and Casimir-Polder force acting between an atom possessing a permanent magnetic moment and a wall made of different materials are derived. Simple model allowing analytic results is considered where the atomic magnetic susceptibility is frequency-independent and wall is made of ideal metal. Numerical computations of the Casimir-Polder force are performed for H atom interacting with walls made of ideal metal, nonmagnetic (Au) and ferromagnetic (Fe) metals and of ferromagnetic dielectric. It is shown that for the first three wall materials the inclusion of the magnetic moment of an atom decreases and for the fourth material increases the magnitude of the Casimir-Polder force.

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I. INTRODUCTION

During the last few years atom-wall interaction attracts considerable attention in connection with experiments on quantum reflection [1, 2, 3] and Bose-Einstein condensation [4, 5, 6]. This stimulated investigation of the van der Waals and Casimir-Polder atom-wall interaction potentials including their dependence on atomic and wall properties [7, 8, 9, 10, 11]. Most of previous work was devoted to a pure electrically polarizable atom near a metallic or dielectric wall described by the frequency-dependent dielectric permittivity $\varepsilon$ and equal to unity magnetic permeability $\mu$. However, the role of magnetic properties of an atom and wall material has also been discussed. Magnetic properties received much interest due to highly conjectural possibility of repulsive atom-wall interaction based on the finding [12] that a pure electrically polarizable atom repels a pure magnetically polarizable one. Keeping in mind that both atoms and walls used in cavity quantum electrodynamics may possess magnetic properties, the impact of these properties on atom-wall interaction deserves detailed consideration.

Recent Ref. [13] developed the theory of atom-wall interaction for the case of both a polarizable and a (para)magnetizable atom near a magnetodielectric macrobody. This theory was applied to the case of an atom near a semispace (thick magnetodielectric wall described by the frequency-dependent $\varepsilon$ and $\mu$). It was shown that the resulting potential of atom-wall interaction is very similar to the known respective potential of a polarizable atom interacting with a dielectric wall. It is pertinent to note that Ref. [13] deals with paramagnetic atoms which are magnetizable but have no intrinsic magnetic moment. This is what is referred to as the Van Vleck paramagnetism [14]. It is caused by the deformation of the electron structure of an atom by the external field which creates the induced magnetic moment. Usually such deformation leads to the diamagnetic effect. However, in some specific cases the paramagnetic effect arises [14]. Thus, the Van Vleck paramagnetism is of polarization origin and the respective magnetic susceptibility is temperature-independent.

In this paper we consider the impact of magnetic properties on atom-wall interaction for paramagnetic atoms possessing the intrinsic (permanent) magnetic moment. Such atoms (for instance H or Rb) participate in different physical processes involving atom-surface interaction (see, e.g., Refs. [15, 16]). As wall material, we present computations for a nonmagnetic metal and ferromagnetic metal and dielectric. The case of atoms possessing
a permanent magnetic moment is interesting in two aspects. First, the magnitude of a permanent magnetic moment is much larger than the magnitude of an induced one. Second, the resulting magnetic susceptibility is of orientation origin and it is temperature-dependent. In Sec. II we derive the Lifshitz-type formula for an atom with permanent magnetic moment interacting with a magnetodielectric wall starting from the known formula for two semispaces described by $\varepsilon(\omega)$ and $\mu(\omega)$. Section III is devoted to the case of nonmagnetic wall. We begin with simple model of an ideal metal wall and frequency-independent magnetic susceptibility of an atom and demonstrate that for Rb atom the effect of magnetic moment is negligibly small because the electric polarizability remains much larger than the magnetic susceptibility at all temperatures from 1 to 300 K. For H atom the inclusion of magnetic moment leads to minor decrease of the magnitude of atom-wall force at $T = 1$ K. Then, similar computations are performed for H atom near an Au wall. In Sec. IV the case of H atom near walls made of ferromagnetic materials is considered. It is shown that for ferromagnetic metal (Fe) the inclusion of atomic magnetic moment leads to qualitatively the same result as for nonmagnetic metals. For ferromagnetic dielectrics the inclusion of atomic magnetic moment increases the magnitude of atom-wall force at $T = 1$ K where effect of magnetic properties is most pronounced. Section V is devoted to our conclusions and discussion.

II. LIFSHITZ-TYPE FORMULA FOR ATOM-WALL INTERACTION WITH ACCOUNT OF MAGNETIC PROPERTIES

We start from the Lifshitz formula for the free energy per unit area in configuration of two parallel magnetodielectric semispaces separated by a distance $a$, at temperature $T$ in thermal equilibrium \cite{17}

$$
\mathcal{F}(a, T) = \frac{k_B T}{2\pi} \sum_{l=0}^{\infty} \int_{0}^{\infty} k_\perp dk_\perp \left\{ \ln \left[ 1 - r_{TM}^{(1)}(i\xi_t, k_\perp)r_{TM}^{(2)}(i\xi_t, k_\perp)e^{-2aq_l} \right] \\
+ \ln \left[ 1 - r_{TE}^{(1)}(i\xi_t, k_\perp)r_{TE}^{(2)}(i\xi_t, k_\perp)e^{-2aq_l} \right] \right\}. \quad (1)
$$
Here the reflection coefficients $r^{(n)}_{TM,TE}$ for two magnetodielectric semispaces ($n = 1, 2$) are given by [18, 19]

$$
\begin{align*}
  r^{(n)}_{TM}(i\xi_l, k_\perp) &= \frac{\varepsilon_l^{(n)}(i\xi_l, k_\perp) - k^{(n)}(i\xi_l, k_\perp)}{\varepsilon_l^{(n)}(i\xi_l, k_\perp) + k^{(n)}(i\xi_l, k_\perp)}, \\
  r^{(n)}_{TE}(i\xi_l, k_\perp) &= \frac{\mu_l^{(n)}(i\xi_l, k_\perp) - k^{(n)}(i\xi_l, k_\perp)}{\mu_l^{(n)}(i\xi_l, k_\perp) + k^{(n)}(i\xi_l, k_\perp)},
\end{align*}
$$

(2)

and the dielectric permittivities and magnetic permeabilities, $\varepsilon_l^{(n)}(i\xi_l) = \varepsilon_l^{(n)}(i\xi_l), \mu_l^{(n)}(i\xi_l) = \mu_l^{(n)}(i\xi_l)$, are calculated at the imaginary Matsubara frequencies, $\xi_l = 2\pi k_B T l / \hbar$ where $l = 0, 1, 2, \ldots$, $k_B$ is the Boltzmann constant. The other notations in Eqs. (1), (2) are as follows:

$$
\begin{align*}
  q_l &= q(i\xi_l, k_\perp) = \sqrt{k^2_\perp + \xi_l^2 / c^2}, \\
  k_l^{(n)} &= k^{(n)}(i\xi_l, k_\perp) = \sqrt{k^2_\perp + \varepsilon_l^{(n)}(i\xi_l, k_\perp) / \mu_l^{(n)}(i\xi_l, k_\perp)}.
\end{align*}
$$

(3)

$k_\perp$ is the wave vector projection onto the boundary planes restricting both semispaces. A prime near the summation sign means that the term for $l = 0$ has to be multiplied by 1/2.

At both zero and nonzero temperature, Eq. (1) with frequency-independent $\varepsilon^{(n)}$ and $\mu^{(n)}$ was used to determine the values of $\varepsilon^{(n)}$ and $\mu^{(n)}$ leading to a positive energy (free energy) and respective repulsive Casimir force between two magnetodielectric semispaces [20]. It was shown [21], however, that in the range of frequencies which give the major contribution to the Casimir force $\mu$ is nearly equal to unity far away from the values needed for the realization of Casimir repulsion.

In order to obtain the free energy of a magnetic atom near a magnetodielectric semispace we use the same method as was suggested for the electrically polarizable atom near a dielectric semispace [17]. For this purpose we remain the semispace with $n = 1$ unchanged but consider a rarefied magnetodielectric semispace with $n = 2$ as a paramagnetic gas. Expanding the dielectric permittivity and magnetic permeability of the latter in powers of the number of atoms per unit volume $N$ and preserving only the first-order contributions one obtains [17, 22]

$$
\begin{align*}
  \varepsilon_l^{(2)} &= 1 + 4\pi\Gamma(i\xi_l) = 1 + 4\pi N \alpha(i\xi_l) + O \left(N^2\right), \\
  \mu_l^{(2)} &= 1 + 4\pi\chi(i\xi_l) = 1 + 4\pi N \beta(i\xi_l) + O \left(N^2\right).
\end{align*}
$$

(4)

Here, $\Gamma(i\xi_l), \chi(i\xi_l)$ are dynamic electric and magnetic susceptibilities of the rarified material of the semispace with $n = 2$, $\alpha(i\xi_l)$ and $\beta(i\xi_l)$ are the respective quantities, as applied to one
atom. It should be remembered that the quantity $\alpha(i\xi_l)$ is usually temperature-independent, whereas $\beta(i\xi_l)$ for paramagnetic materials with the orientation polarization is the reciprocal to the temperature (see Sec. III).

Substituting Eq. (4) in Eq. (2), using Eq. (3) and expanding up to the first power in $N$, one arrives at

$$r^{(2)}_{\text{TM}}(i\xi_l, k_\perp) = \pi N \left[ 2\alpha(i\xi_l) - \frac{\beta(i\xi_l) + \alpha(i\xi_l) \xi_l^2}{q_l^2 c^2} \right] + O\left(N^2\right),$$

$$r^{(2)}_{\text{TE}}(i\xi_l, k_\perp) = \pi N \left[ 2\beta(i\xi_l) - \frac{\beta(i\xi_l) + \alpha(i\xi_l) \xi_l^2}{q_l^2 c^2} \right] + O\left(N^2\right).$$

(5)

Rewriting Eq. (1) in terms of the result (5) we obtain

$$F(a, T) = -k_B T N \int_0^\infty \int_0^\infty k_\perp \, dk_\perp \, e^{-2aq_l} \left\{ \begin{array}{l} 2\alpha(i\xi_l) - \frac{\beta(i\xi_l) + \alpha(i\xi_l) \xi_l^2}{q_l^2 c^2} \, r^{(1)}_{\text{TM}}(i\xi_l, k_\perp) \\ + \left[ 2\beta(i\xi_l) - \frac{\beta(i\xi_l) + \alpha(i\xi_l) \xi_l^2}{q_l^2 c^2} \right] \, r^{(1)}_{\text{TE}}(i\xi_l, k_\perp) \end{array} \right\} + O\left(N^2\right).$$

(6)

Alternatively, the additivity of the first order term in the expansion of the free energy in powers of $N$ results in

$$F(a, T) = N \int_a^\infty F^A(z, T) \, dz + O\left(N^2\right),$$

(7)

where $F^A(z, T)$ is the free energy of a magnetic atom spaced $z$ apart of a magnetodielectric wall.

Now we equate the right-hand sides of Eqs. (6) and (7) and calculate the derivative with respect to $a$. Then in the limit $N \to 0$ we obtain

$$F^A(a, T) = -k_B T \sum_{l=0}^\infty \int_0^\infty k_\perp \, dk_\perp \, q_l e^{-2aq_l} \left\{ \begin{array}{l} 2\alpha(i\xi_l) r^{(1)}_{\text{TM}}(i\xi_l, k_\perp) + \beta(i\xi_l) r^{(1)}_{\text{TE}}(i\xi_l, k_\perp) \\ -\frac{\xi_l^2}{q_l^2 c^2} \left[ \alpha(i\xi_l) \xi_l^2 + \beta(i\xi_l) \right] \left[ r^{(1)}_{\text{TM}}(i\xi_l, k_\perp) + r^{(1)}_{\text{TE}}(i\xi_l, k_\perp) \right] \end{array} \right\}. \quad \text{At zero temperature similar formula for the energy of a magnetizable atom was obtained in Ref. [13] using the Green function method. For a nonmagnetic atom, } \beta(i\xi_l) = 0, \text{ near a dielectric wall, } \mu(i\xi_l) = 1, \text{ Eq. (8) coincides with the results of Refs. [12, 17] (one should take} \frac{1}{2} \text{ instead of } N. \text{ Otherwise, for } \beta(i\xi_l) \neq 0, \text{ we have the above formula.} \right.$$
into account different convention for the phase multiple in the definition of the TE reflection coefficient used in [7]).

From Eq. (8) it is straightforward matter to derive the expression for the force acting on a magnetic atom near a magnetodielectric wall

$$F^A(a, T) = -\frac{\partial F^A(a, T)}{\partial a} = -2k_BT\sum_{l=0}^{\infty} \int_0^\infty k_{\perp} dk_{\perp} q_{l}^2 e^{-2q_{l}a}$$

$$\times \left\{ 2[\alpha(i\xi_l)r_{TM}^{(1)}(i\xi_l, k_{\perp}) + \beta(i\xi_l)r_{TE}^{(1)}(i\xi_l, k_{\perp})] - \frac{\xi_l^2}{q^2} [\alpha(i\xi_l) + \beta(i\xi_l)] [r_{TM}^{(1)}(i\xi_l, k_{\perp}) + r_{TE}^{(1)}(i\xi_l, k_{\perp})] \right\}.$$  (9)

It is interesting to note that both the free energy (8) and the force (9) of atom-wall interaction are represented as the sums of two contributions

$$F^A(a, T) = F_{\alpha}^A(a, T) + F_{\beta}^A(a, T),$$

$$F^A(a, T) = F_{\alpha}^A(a, T) + F_{\beta}^A(a, T),$$  (10)

depending on the dynamic atomic polarizability $\alpha$ and magnetic susceptibility $\beta$, respectively. However, magnetic properties of wall material influence on both contributions to the free energy and force through the magnetic permeability $\mu$ entering the reflection coefficients $r_{TM,TE}^{(1)}$ defined in Eq. (2). In the next section, Eq. (9) is used in numerical computations to determine the impact of magnetic properties on atom-wall interaction.

### III. ATOMS WITH PERMANENT MAGNETIC MOMENT NEAR A NONMAGNETIC WALL

To perform computations of the force acting between an atom and a wall using Eq. (9), one needs sufficiently precise expressions for the atomic dynamic polarizability $\alpha$ and magnetic susceptibility $\beta$. For a rarefied gas of paramagnetic atoms the magnetic susceptibility along the imaginary frequency axis is given by [23, 24]

$$\chi(i\xi_l) = N\beta(i\xi_l) = N \frac{g^2 \mu_B^2 J(J+1)}{3k_BT} \frac{1}{1 + \tau\xi_l},$$  (11)

where $g$ is the Lande factor, $\mu_B = e\hbar/(2m_ec)$ is the Bohr magneton, $m_e$ is the electron mass, $J$ is the total momentum and $\tau$ is the relaxation time. Below we consider the ground state
atoms of H and $^{87}$Rb which have approximately equal magnetic moments \[^{25}\]. For these atoms $g = 1$ and $J = 1/2$ (the magnetic moment of H atoms was determined in Ref. \[^{26}\]; the relativistic and radiative corrections to it are discussed in Ref. \[^{27}\]). For different atoms at $T = 300$ K, $\tau$ varies in the range from $10^{-10}$ to $10^{-4}$ s and increases with the decrease of temperature.

Now we consider the dynamic atomic polarizability. As is seen from Eq. (11), at high frequencies the magnetic properties cannot have a pronounced effect on the force between an atom and a cavity wall. Because of this, below we consider the impact of magnetic properties on the Casimir-Polder force in the separation range from 1 to $10 \mu$m where the relevant frequencies are relatively low. In this range of separations sufficiently precise results for the Casimir-Polder force are obtained by using the single-oscillator model \[^{7}\]

$$
\alpha(i\xi_l) = \frac{\alpha(0)}{1 + \frac{\xi_l^2}{\omega_a^2}},
$$

where $\alpha(0)$ is the static atomic polarizability and $\omega_a$ is the eigenfrequency. For H atom $\alpha(0) = 6.67 \times 10^{-25}$ cm$^3$ and the characteristic energy is $\hbar \omega_a = 11.65$ eV \[^{28}\]. For $^{87}$Rb atom it holds $\alpha(0) = 4.73 \times 10^{-23}$ cm$^3$ \[^{29}\] and the characteristic energy is $\hbar \omega_a = 1.68$ eV \[^{30}\].

In this section we consider the case of nonmagnetic metal walls. Note that for nonmagnetic dielectric walls the magnetic moment of an atom leaves the Casimir-Polder force unaffected. This is because the magnetic susceptibility \[^{11}\] is dominant at zero frequency. It is well known \[^{31}\], however, that for nonmagnetic dielectrics $r_{TE}^{(1)}(0, k_{\perp}) = 0$. Thus, from Eqs. (8) and (9) it follows that for nonmagnetic dielectrics there is no impact of the atomic magnetic moment on atom-wall interaction. The case of walls made of magnetic materials is considered in the next section.

It is more convenient to perform computations by using the dimensionless variables

$$
\zeta_l = \frac{2a\xi_l}{c} = \frac{\xi_l}{\omega_c}, \quad y = 2aq_l,
$$

where $\omega_c \equiv c/(2a)$ is the characteristic frequency of the Casimir-Polder interaction. In terms of these variables Eq. (9) takes the form

$$
F_A(a, T) = -\frac{k_B T}{8a^4} \sum_{l=0}^{\infty} \int_{\zeta_l}^{\infty} y dy e^{-y} \left\{ 2y^2 \left[ \alpha(i\omega_c\zeta_l) r_{TM}^{(1)}(i\omega_c\zeta_l, y) + \beta(i\omega_c\zeta_l) r_{TE}^{(1)}(i\omega_c\zeta_l, y) \right] - \zeta_l^2 \left[ \alpha(i\omega_c\zeta_l) + \beta(i\omega_c\zeta_l) \right] \right\}.
$$

\[^{11}\]
where the reflection coefficients (12) are

\[
\begin{align*}
    r_{TM}^{(1)}(i\omega, \zeta_l, y) &= \frac{\varepsilon_l^{(1)} y - \sqrt{y^2 + \zeta_l^2 (\varepsilon_l^{(1)} \mu_l^{(1)} - 1)}}{\varepsilon_l^{(1)} y + \sqrt{y^2 + \zeta_l^2 (\varepsilon_l^{(1)} \mu_l^{(1)} - 1)}} \\
    r_{TE}^{(1)}(i\omega, \zeta_l, y) &= \frac{\mu_l^{(1)} y - \sqrt{y^2 + \zeta_l^2 (\varepsilon_l^{(1)} \mu_l^{(1)} - 1)}}{\mu_l^{(1)} y + \sqrt{y^2 + \zeta_l^2 (\varepsilon_l^{(1)} \mu_l^{(1)} - 1)}},
\end{align*}
\]

(15)

with \(\varepsilon_l^{(1)} = \varepsilon^{(1)}(i\omega, \zeta_l), \mu_l^{(1)} = \mu^{(1)}(i\omega, \zeta_l)\)

As a simple model, we first consider atom with frequency-independent electric polarizability \(\alpha(0)\) and magnetic susceptibility \(\beta(0)\) near an ideal metal wall. Then from Eq. (15) one obtains \(r_{TM}^{(1)} = 1, r_{TE}^{(1)} = -1\) and Eq. (14) results in

\[
F^A(a, T) = -\frac{k_B T}{4a^4} [\alpha(0) - \beta(0)] \sum_{l=0}^{\infty} \int_{\zeta_l}^{\infty} y^3 e^{-y} dy. 
\]

(16)

The calculation of the integral in Eq. (16) leads to

\[
F^A(a, T) = -\frac{k_B T}{4a^4} [\alpha(0) - \beta(0)] \left[ 3 + \sum_{l=1}^{\infty} \left( 6 + 6\zeta_l + 3\zeta_l^2 + \zeta_l^3 \right) e^{-\zeta_l} \right]. 
\]

(17)

By performing all summations in Eq. (17) one obtains

\[
F^A(a, T) = -\frac{k_B T}{4a^4} [\alpha(0) - \beta(0)] \left[ 3 + \frac{6}{e^\tau - 1} + \frac{6\tau}{(e^\tau - 1)^2} \right. \\
+ \frac{3\tau^2 e^\tau (1 + e^\tau)}{(e^\tau - 1)^3} + \left. \frac{\tau^3 e^\tau (1 + 4e^\tau + e^{2\tau})}{(e^\tau - 1)^4} \right],
\]

(18)

where the parameter \(\tau\) has the meaning of the normalized temperature \(\tau = 2\pi T/T_{\text{eff}}\), and the effective temperature is defined as \(k_B T_{\text{eff}} = \hbar c/(2a)\).

Equations (18) shows that the impact of atomic magnetic moment on atom-wall interaction is determined by the relationship between the static electric polarizability \(\alpha(0)\) and static atomic susceptibility \(\beta(0)\). From Eq. (11) one arrives at the following values for \(\beta(0)\) of H and \(^{87}\text{Rb}\) atoms at \(T = 300\) K and \(T = 1\) K, respectively: \(\beta(0; T = 300\) K\) = 5.2 \(\times 10^{-28}\) cm\(^3\), \(\beta(0; T = 1\) K\) = 1.56 \(\times 10^{-25}\) cm\(^3\). From the comparison with the values of \(\alpha(0)\) for H and \(^{87}\text{Rb}\) atoms presented below Eq. (12) it follows that the impact of atomic magnetic moment on the interaction of Rb atoms with a cavity wall is negligibly small. We emphasize that this conclusion is obtained in the model where \(\beta\) is frequency-independent and equal to its static value. The more so in situations when the decrease of
\( \beta \) with increasing frequency is taken into consideration. As to \( H \) atoms, the impact of their magnetic moment is also negligibly small at \( T = 300 \text{ K} \) but is comparable with the role of electric polarizability at \( T = 1 \text{ K} \) keeping in mind that \( \beta(0) = 0.23\alpha(0) \). Here we do not consider very low temperatures \( T \ll 1 \text{ K} \), where \( \beta(0) \) might become even larger than \( \alpha(0) \) as is suggested by Eq. (11). The reason is that at very low temperature even weak magnetic interaction between separate atoms in the rarefied paramagnetic gaseous medium influences on its magnetic properties and makes Eq. (11) inapplicable [32]. Thus, the case of very low temperature deserves further investigation.

Now we present the results of numerical computations in more realistic situations. We begin with the case of \( H \) atom characterized by the frequency-dependent \( \alpha(i\xi_l) \) and \( \beta(i\xi_l) \) interacting with an ideal metal wall. In this case Eq. (14) results in

\[
F^A(a, T) = -\frac{k_B T}{4a^4} \sum_{l=0}^\infty \int_{\xi_l}^\infty y^3 e^{-y} dy [\alpha(i\omega_c \xi_l) - \beta(i\omega_c \xi_l)].
\]

The computations using Eqs. (11), (12) and (19) were performed at \( T = 1 \text{ K} \) at separations from 1 to 10 \( \mu \text{m} \). In Eq. (11) the value \( \tau = 10^{-8} \text{ s} \) was used. It was checked that further increase of \( \tau \) does not influence the force values. The computational results for the magnitude of the Casimir-Polder force multiplied by the fifth power of separation are presented in Fig. 1a. The solid line reproduces conventional results for \( F^A_\alpha \) obtained by discarding the magnetic moment of \( H \) atom [i.e., by assuming \( \beta(i\omega_c \xi_l) = 0 \)]. The dotted line represents the computational results for \( F^A \) with account of both dynamic electric polarizability and magnetic susceptibility of \( H \) atom. The relative deviation between the results of two computations, \( |F^A| - |F^A_\alpha|)/|F^A_\alpha| \), is equal to \(-0.018\% \) at the shortest separation \( a = 1 \mu \text{m} \) and achieves \(-0.18\% \) at \( a = 10 \mu \text{m} \).

For \( H \) atom with frequency-dependent \( \alpha(i\xi_l) \) and \( \beta(i\xi_l) \) near an \( Au \) wall computations were performed using Eqs. (11), (12), (14) and Eq. (15) with \( \mu_1^{(1)} = 1 \). For the dielectric permittivity of \( Au \) the plasma model

\[
\varepsilon(i\xi_l) = 1 + \frac{\omega_p^2}{\xi_l^2},
\]

where \( \omega_p = 9.0 \text{ eV} \) is the plasma frequency, has been used. As was shown in Ref. [33], at separations \( a > 400 \text{ nm} \) the description of the dielectric properties of \( Au \) by means of the plasma model is very accurate. The computational results for \( a^5|F^A| \) at \( T = 1 \text{ K} \) in the separation range from 1 to 10 \( \mu \text{m} \) are shown in Fig. 1b (the solid line is for \( \beta = 0 \) and
the dotted line is for both $\alpha$ and $\beta$ not equal to zero). The relative deviation between the two lines varies from $-0.015\%$ to $-0.15\%$ when separation increases from $1$ to $10 \mu m$. As is seen from the comparison of Fig. 1a and Fig. 1b, the correction due to the nonzero skin depth only quantitatively influences the computational results leaving the relative difference between the solid and dotted lines nearly unchanged.

IV. ATOMS WITH PERMANENT MAGNETIC MOMENT NEAR WALLS MADE OF FERROMAGNETIC MATERIALS

Now we consider the cavity wall made of ferromagnetic materials which are characterized by rather large magnetic permeabilities $\mu$. It is common knowledge that in this case $\mu$ is a function of the strength of the magnetic field $H$ and achieves maximum values at rather large $|H|$. As to the frequency-dependence of $\mu$, it is of the same form as in Eq. (11), but with much larger values of $\tau$ than for a paramagnetic gas. Because of this, the influence of magnetic properties of ferromagnetic materials on atom-wall interaction occurs through the contribution of the zero-frequency term of the Lifshitz formula. Notice that the Lifshitz formulas (1), (8) and (9) were derived using the standard expression for the magnetic induction $B = \mu(\omega)H$, where the magnetic susceptibility depends only on frequency. Thus, in the applications of the Lifshitz formula to ferromagnetic materials it is justified to use $\mu(\omega)$ from the initial point ($B = H = 0$) of the normal magnetization curve where these assumptions are satisfied.

Let us consider the interaction of H atom with Fe wall. The computations of the Casimir-Polder force were performed using Eqs. (14) and (15). For wall material the values $\mu(0) = 1000$ and $\hbar\omega_p = 11.1 \text{ eV}$ were used. The computational results for $a^5|F^A(a, T)|$ at $T = 1 \text{ K}$ are presented in Fig. 2a (with the same notations for the solid and dotted lines as in Fig. 1). In the case of Fe wall the relative contribution of atomic magnetic moment at the shortest separation $a = 1 \mu m$ is equal to only $-8 \times 10^{-5}\%$. At $a = 10 \mu m$ it increases till $-0.13\%$. Thus, for metal walls the inclusion of ferromagnetic properties does not increase the role of magnetic moment of an atom in atom-wall interaction.

Now we turn to the consideration of walls made of ferromagnetic dielectrics. These are composite materials having physical properties typical for dielectrics, but demonstrating ferromagnetic behavior under the influence of external magnetic field. One example is a sub-
stance on the basis of a magnetically soft iron powder and a polymer compound. Recently it was suggested to design such materials on the basis of air-stable iron-cobalt nanoparticles [35]. Ferromagnetic dielectrics are widely used in magnetooptic waveguides (see, e.g., Ref. [36] and references therein). In the case of ferromagnetic dielectrics, the value of the TE reflection coefficient at zero frequency is approximately equal to unity, whereas the TM reflection coefficient remains to be less than unity.

As an example we perform computations of the Casimir-Polder force acting between H atom and a wall made of polyethylene with $\varepsilon(0) = 3$ with a fraction of iron powder. For the magnetic permeability of such compound material $\mu(0) = 100$ was used. Computations were performed using Eq. (14) in the separation region from 1 to 10 $\mu$m (note that at such large separations the frequency dependence of the dielectric permittivity does not contribute essentially to the obtained results). The computational results for $a^5|F^A(a, T)|$ at $T = 1$ K are presented in Fig. 2b by the solid line (with atomic magnetic properties discarded) and by the dotted line (with atomic magnetic properties included). In qualitative difference with the case of nonmagnetic and ferromagnetic metal wall materials (see Fig. 1a,b and Fig. 2a), for a wall made of ferromagnetic dielectric the inclusion of atomic magnetic moment increases the magnitude of the Casimir-Polder force. At the shortest separation $a = 1 \mu$m the realtive deviation between the computational results with included and discarded atomic magnetic moment is equal to 0.04%. At separation distance of $a = 10 \mu$m this deviation achieves 0.4%. Thus, for wall made of ferromagnetic dielectric the correction to the Casimir-Polder interaction due to the atomic magnetic moment is larger than for other wall materials considered above.

V. CONCLUSIONS AND DISCUSSION

In the foregoing we have derived the Lifshitz-type formulas for the Casimir-Polder free energy and force acting between the atom with a permanent magnetic moment and a wall made of different materials. These formulas express the free energy and force in terms of electric polarizability and magnetic susceptibility of an atom, and dielectric permittivity and magnetic permeability of a wall. Using a simple model of the atom with frequency-independent electric polarizability and magnetic susceptibility near an ideal metal wall the analytical expression for the Casimir-Polder force was obtained. Specifically, for Rb atoms
the influence of magnetic properties on the force was shown to be negligibly small, as compared with H atoms. We have also performed numerical computations of the Casimir-Polder force acting between H atoms with frequency-dependent electric polarizability and magnetic susceptibility and walls made of ideal metal, Au, Fe and ferromagnetic dielectric. In the first three cases the inclusion of an atomic magnetic moment was shown to lead to the decrease of the force magnitude and in the fourth case to the increase of it. Although the impact of the permanent magnetic moment of an atom on atom-wall interaction was found to be equal to only a fraction of percent, it is larger than the effect of the induced (para)magnetic moment previously considered in the literature [13].

Note that our analysis is not applicable to atoms under the influence of an external magnetic field. This is because the inclusion of the magnetic field changes the mathematical expression for the Casimir force between the plates used as a starting point in Sec. II [37, 38]. In fact the most interesting configuration considered above is the H atom near a ferromagnetic dielectric wall. The point is that for metal walls there are supplementary magnetic interactions caused by the magnetic noise from Johnson currents [39, 40]. In the case of an atom interacting with a dielectric wall there is no action of such effects which makes this configuration preferable for further investigations.

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FIG. 1: The magnitude of the Casimir-Polder force acting between H atom and (a) ideal metal and (b) nonmagnetic metal (Au) wall multiplied by the fifth power of separation with discarded (the solid line) and included (the dotted line) atomic magnetic moment.
FIG. 2: The magnitude of the Casimir-Polder force acting between H atom and (a) ferromagnetic metal (Fe) and (b) ferromagnetic dielectric wall multiplied by the fifth power of separation with discarded (the solid line) and included (the dotted line) atomic magnetic moment.