Effects of variable heat source on convective motion in an anisotropic porous layer

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Abstract. The onset of convective motion in a fluid saturated anisotropic porous medium layer is examined analytically in the occurrence of the qualitative effect of the source of variable internal heat. Three different kinds of heat source functions: (i) \( N(z) = z \) (linear) (ii) \( N(z) = z^2 \) (parabolic) and (iii) \( N(z) = z^3 \) (cubic) are considered. The resulting eigenvalue problem was analytically solved using regular perturbation technique. A parametric study is carried out by varying the following parameters: the heat source parameter \( \lambda \), thermal anisotropy parameter \( \eta \), and mechanical anisotropy parameter \( \xi \). Results indicate that the effects of the increasing values of \( \lambda \) and \( \xi \) will enhance the convection of an anisotropic porous layer system while the increasing of \( \eta \) will help to stabilize the system. It is also noted that for case (iii) the system is more stable, while for case (i) the system is more unstable.

Keywords: Heat source, mechanical anisotropy parameter, thermal anisotropy parameter

1. Introduction

Due to a large number of technical applications, such as energy storage, casting solidification, petroleum resources, filtration, crystal growth, chemical reactors, nuclear waste and geophysical and environmental applications, the study of convective motion in a horizontal porous matrix has received considerable attention. Similarly, in some physical problems, such as crystal formation, heat exchangers, geophysics, chemistry, nuclear waste and convective motion in biomechanics, variations in the heat source have their own significance. The various understandings of this subject are given in the books by Nield & Bejan[1] and[2].

Penetrative convection is a phenomenon that emerges from unstable equilibrium due to convective instability. In several natural, astrophysical and geophysical phenomena, convection due to internal heating occurs. For instance, heating and cooling of lakes, triggering the pattern of granulation observed on the surface, and radioactive decay and much more as described [2]. An extensive literature on penetrative convection is available. In a system and in thermo-convective instability, Carr
& Putter[3] investigates penetrative convection due to heat source strength and radiation plays an significant role. Straughan [4] defines the effect of radioactive heating through the lower bed to a non-Boussinesq fluid layer. Shivakumara et.al [5] performed the stability study for two layer system with heat source strength. Suma et.al [6] analyzed the combined effect of heat source strength and throughflow on two-layer system. Onset of Benard–Marangoni Convection in a Composite anisotropic porous material is analyzed by [7].

In recent years, more attention has been paid to the study of thermal convection in a different heat source role, and this is because many practical applications have a variable heat source effect. Rionero and Straughan [8] have been studied used the energy method for instability in presence of heat gravity effects on the onset of convective motion in porous matrix. Suma et al.[9] and Gangadharaiah et al.[10] have studied the combined effect of through flow and variable gravity on the onset of convection in a fluid-saturated porous bed using standard perturbation techniques. Nandal & Mahajan[11] is investigating the most recent effects of a various types of heat source strength on fluid-saturated porous media. The stability due to variable heat source and variable gravity field was investigated by Mahabaleshwar[12] and Nagarathnamma et al.[13] by applying the linear stability principle using the Gelerkin method. In this paper, therefore, we analyze the effect of variable internal heat source on instability in an anisotropic porous matrix.

2. Conceptual Model
Fig. 1 illustrates the physical configuration of the present study. The physical model under consideration is a horizontal anisotropic porous bed bounded between planes at $z = 0$ & $z = d$.

![Figure 1. Physical configuration](image)

3. Mathematical Formulation
The porous layer governing equations are :
\[
\nabla \cdot \mathbf{V} = 0
\]
\[
\frac{\rho_0 u}{\phi} \frac{\partial V}{\partial t} = -\nabla p - \mu K^{-1} \cdot \mathbf{V} + \rho_0 \left[ 1 - \beta \left( T - T_b \right) \right] \mathbf{g} (z)
\]
\[
A \frac{\partial T}{\partial t} + \left( \mathbf{V} \cdot \nabla \right) T = k_i \nabla^2 T + Q (z)
\]

The basic steady state solution is of the form
\[
\left( u, v, w, p, T \right) = \left( 0, 0, 0, p_b (z), T_b (z) \right)
\]

Then Eq.(3) can be written for basic temperature $T_b$ as:
\[
\frac{d^2 T_b}{dz^2} - \frac{1}{\kappa} Q (z) = 0
\]

Integrating the above equation twice, we get
\[
T_b (z) = -\frac{1}{\kappa} \int_0^z \int_0^\xi Q (\lambda) d\lambda d\xi + Cz + D
\]

Applying the boundary conditions
\( T_b = T_b \) at \( z = 0 \) \& \( T_b = T_u \) at \( z = d \),

\[ T_b(z) = -\frac{1}{\kappa} \int_0^\xi Q(\lambda) d\lambda d\xi - C z + T_1, \]  

where the constant 
\[ C = \frac{1}{d}(T_1 - T_u) - \frac{1}{\kappa d} \int_0^d Q(\lambda) d\lambda d\xi. \]

Basic state is slightly perturbed using the relation given by
\[ \tilde{V} = \tilde{V}', p = p_b(z) + p', T = T_b(z) + \theta \]

**Small disturbance analysis**

We assume the solution are of the form
\[ (w, T) = \left[ W(z), \Theta(z) \right] e^{(ix+my)} \]  

The linearised equations governing the perturbation are
\[ \left( \frac{1}{\xi} D^2 - a^2 \right) W = -R a^2 \Theta \]  
\[ (D^2 - \eta a^2) \Theta = -\left(1 + Ns N(z)\right) W \]

Where \( N(z) = \delta q(z) = F(z)/C \) with \( N(z) = \delta q(z) = F(z)/C \) with \( F(z) = k_1/k_1 \) is an anisotropic permeability, and \( \eta = k_1/k_1 \) is an anisotropic effective thermal diffusivity.

The boundary conditions take the form
\[ W = DW = D\Theta = 0 \quad \text{at}\; z = 0 \]  
\[ W = D\Theta = D^2W = 0 \quad \text{at}\; z = 1. \]

**4 Analytical Procedure**

The analytical solutions of the eigenvalue problem Eq.(11) and Eq.(12) are obtained by using a using regular perturbation procedure. Accordingly, the variables \( W, and \Theta \) are expanded in powers of \( a^2 \) as
\[ (W, \Theta) = \sum_{i=0}^N (a^2)^i (W_i, \Theta_i) \]  
\[ D^2W_0 = 0 \]  
\[ D^2\Theta_0 = -N(z)W_0 \]

With the boundary conditions
\[ W_0 = DW_0 = D\Theta_0 = 0 \quad \text{at}\; z = 0 \]
\[ W_0 = D^2W_0 = D\Theta_0 = 0 \quad \text{at}\; z = 1 \]

The solution to the zero\(^{th}\) order Eq. (16) and (17) by using boundary conditions in Eq. (18) and (19) are as follow:
\[ W_0 = 0 \quad \text{and}\; \Theta_0 = 1 \]

First- order equations are
\[ D^2\Theta_i - \eta = -N(z)W_i. \]
With the boundary conditions
\[ W_0 = D W_1 = D \Theta_1 = 0 \quad \text{at} \quad z = 0 \] \hspace{1cm} (23)
\[ W_0 = D^2 W_1 = D \Theta_1 = 0 \quad \text{at} \quad z = 1 \] \hspace{1cm} (24)

The Eq. (21) to (22) will be solved for three cases of heat source variations for the three cases:
(i) \( N(z) = z \)  
(ii) \( N(z) = z^2 \)  
(iii) \( N(z) = z^3 \)  and the equation of critical Rayleigh number \( R_C \) will be obtained in term of \( \delta, \xi \) and \( \eta \).

5. Results and discussion
This research presents effects of variable heat generation in an anisotropic porous layer was analytically studied through a regular perturbation procedure. The boundaries are regarded rigid-free and insulating with a linear stability assessment. The outcomes collected are described graphically in figures 2-12 to show the effect of different parameters on the critical Rayleigh number, \( R_C \).

The critical \( R_C \) is plotted against the internal heat source parameter \( N_s \) for all three cases of variable heat function with comparison between isotropic and anisotropic cases are displayed in figures 2 and 4. From figures it can be observed critical \( R_C \) decrease with an increasing \( N_s \). Furthermore, it is noticed that for the cubic heat variation the system is more stable, while for the linear heat variation the system is more unstable. Figures 4-6 presents the vertical velocity \( W \) for various values of heat parameter for all three different cases of heat variance. It shows from all three cases more accelerate \( W \) in the absence of \( N_s \).

Figures 5-7 displays the deviation of \( R_C \) is plotted against the mechanical anisotropic parameter, \( \xi \), for different values of \( N_s \) for all three cases variable heat variance with \( \eta = 0.5 \). It can be observed from figures the \( R_C \) decrease with an increase in the value of the mechanical anisotropic parameter. Finally from figures 5-7, on increasing the value of \( \xi \), we found that the value of \( R_C \) decrease. The value of \( \xi \) is directly proportional to the x-direction permeability, \( K_x \) of the porous matrix and inversely proportional to the z-direction permeability, \( K_z \) of the porous matrix. This is due to the facts that increasing the value of \( K_x \) will cause the size of the cell to become larger while decreasing the value of \( K_z \) will result in larger temperature difference between the lower and upper plate as reported by Degan et al., [15]. Furthermore, it is noticed that for the cubic heat variation the system is more stable, while for the linear heat variation the system is more unstable.

The critical \( R_C \) with thermal anisotropic parameter, \( \eta \) for various values of \( N_s \) for all three type of heat variance are presented in figures 10-12. It is noted that with increasing the value of \( \eta \) the value of critical \( R_C \) also increase. Increasing \( \eta \) is corresponds to decrease in vertical thermal diffusivity \( \kappa_T \), which will slow down the heat flow vertically through it. Furthermore, it is noticed that for the cubic heat variation the system is more stable, while for the linear heat variation the system is more unstable.

6. Conclusions
The stationary thermal penetrative convection via variable heating in a fluid saturated an anisotropic porous medium is investigated analytically using linear stability analysis. The resulting eigenvalue problem obtained from the governing equations are solved analytically by regular perturbation procedure. The study was conducted in three cases of variance of the heat: (i) \( N(z) = z \)  
(ii) \( N(z) = z^2 \)  
(iii) \( N(z) = z^3 \). The \( R_C \) is obtained with the purpose of investigating the
effect of heat source parameter, $N_s$, mechanical anisotropy, $\xi$, and thermal anisotropy parameter, $\eta$.

Results show that the effects of increasing the increasing values of $\xi$ and $N_s$ will enhance the convection of a porous layer system while the increasing of $\eta$ and $Da$ will help to stabilize the system. Furthermore, it is noticed that for the cubic heat variation the system is more stable, while for the linear heat variation the system is more unstable.

**Figure 2.** $R^c$ against $N_s$ for three cases of heat source variation. For isotropic case

**Figure 3.** $R^c$ against $N_s$ for three cases of heat source variation. For anisotropic case

**Figure 4.** Vertical velocity eigen functions $W$ for various values of $N_s$ for linear case
Figure 5. Vertical velocity eigen functions $W$ for various values of $N_s$ for parabolic case

Figure 6. Vertical velocity eigen functions $W$ for various values of $N_s$ for cubic case

Figure 7. $R^c$ against $\xi$ for various values of $N_s$ for linear case variation with $\eta = 0.5$.

Figure 8. $R^c$ against $\xi$ for various values of $N_s$ for parabolic case variation with $\eta = 0.5$. 
Figure 9. $R^c$ against $\xi$ for various values of $Ns$ for cubic case variation with $\eta = 0.5$.

Figure 10. $R^c$ against $\eta$ for various values of $Ns$ for linear case variation with $\xi = 0.5$.

Figure 11. $R^c$ against $\eta$ for different values of $Ns$ for parabolic case with $\xi = 0.5$. 
Figure 12. $R^c$ against $\eta$ for different values of $N_s$ for cubic case variation with $\xi = 0.5$.

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