Heavy quark expansion for heavy-light light-cone operators

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We generalize the celebrated heavy quark expansion to nonlocal QCD operators. By taking nonlocal heavy-light current on the light-cone as an example, we confirm that the collinear singularities are common between QCD operator and the corresponding operator in heavy quark effective theory (HQET), at the leading power of \(1/M\) expansion. Based on a perturbative calculation in operator form at one-loop level, a factorization formula linking QCD and HQET operators is investigated and the matching coefficient is determined. The matching between QCD and HQET light-cone distribution amplitudes (LCDAs) as well as other momentum distributions of hadron can be derived as a consequence.

I. INTRODUCTION

Hadrons are multi-scale strong interaction systems. Heavy hadron—the hydrogen atom of strong interaction, plays an unique role of understanding and examining quantum chromodynamics (QCD). When one of the quarks in a hadron is heavy comparing with strong interaction scale, i.e., \(M \gg \Lambda_{\text{QCD}}\), the hard scale \(M\) is expected to disentangle from the infrared scale. This leads to the heavy quark effective theory (HQET) \cite{1,2,3}, which has proved an effective approach of studying heavy flavor hadrons, especially in \(B\)-meson physics. For a review of HQET, see Refs. \cite{4,5}.

The HQET action can be derived by expanding the QCD action in series of the inverse powers of \(M\), which is known as the heavy quark expansion (HQE). The HQE for local composite operators is also extensively explored. For example, consider the heavy-light axial-vector current \(\bar{q}\gamma^\mu\gamma^5Q\), its HQE gives

\[
\bar{q}\gamma^\mu\gamma^5Q = C(M, \mu)\bar{q}\gamma^\mu\gamma^5h_v + \mathcal{O}(1/M),
\]

where \(\bar{q}\) is light quark, \(Q\) is the heavy quark field in QCD, while \(h_v\) is the heavy quark field in HQET, with velocity index \(v\). A matching coefficient \(C(M, \mu)\) is introduced due to the different ultraviolet (UV) behavior of the full and effective theories. The matching coefficient can be calculated in perturbation theory, while the infrared physics is only enclosed in the operators. This relation holds at operator level, so the matching equation as well as the matching coefficient are independent of hadron states.

Even in local field theories, one can construct not only local composite operators, but also nonlocal operators. In QCD and its effective theories, the nonlocal operators are crucial for understanding inner structure of hadrons. One important type of such operators are the bilocal quark operators \(\bar{q}(z)|z,0\rangle\Gamma Q(0)\), in which the two quark fields are located on the light-cone (i.e., \(z^2 = 0\) but \(z \neq 0\)), with \(\mu\) being the renormalization scale that defines the operator. The parton momentum distributions in a hadron, e.g., parton distribution functions (PDFs) and light-cone distribution amplitudes (LCDAs), are defined through the matrix elements of light-cone operators. These distributions are indispensable ingredients for QCD factorization theorems. For example, for many \(B\)-meson exclusive decay processes, the decay amplitude can be factorized in terms of perturbation functions and \(B\)-meson LCDAs \cite{6–10}, where the \(B\)-meson LCDAs are defined by the matrix elements of heavy-light operators on the light-cone in HQET \cite{11}. The other case is that the two parton fields are separated off the light-cone. The space-like operators attract lots of attentions in the past few years, thanks to the development of large momentum effective theory \cite{12,13} and many other approaches designed for accessing parton physics from lattice calculation, e.g., pseudo-PDFs \cite{14,15} and lattice cross-sections \cite{16,17}.

When the heavy quark mass \(M \gg \Lambda_{\text{QCD}}\), analogous to local operators, the bilocal operators are also expected to be factorized into hard functions and HQET bilocal operators. A related factorization theorem was proposed recently \cite{18}, which connects \(B\)-meson LCDAs defined in QCD and HQET, based on the perturbative calculation on the LCDAs of heavy-light mesons \cite{19}. In this work, we will focus on the operators instead of the momentum distributions, because factorization holds at operator level, taking matrix elements and Fourier transformations are irrelevant for establishing a factorization theorem. The goal of this work is to derive the HQE for nonlocal QCD

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operators, or in other words, the nonlocal generalization of Eq. (1). Without loss of generality, we will study the HQE for the nonlocal heavy-light current in which two quark fields are separated on the light-cone, similar discussions might be easily generalized to other nonlocal operators. Based on the factorization formula in the operator form, the factorization for $B$-meson LCDA$s$ and other structure functions can be naturally derived.

The rest of this paper is organized as follows. In Section II, we will study the HQE at tree level, and propose a matching equation that links nonlocal heavy-light current in QCD and HQET; In Section III and Section IV, by employing coordinate representation technique, we will calculate one-loop corrections to QCD and HQET operators, respectively; Based on previous sections, the matching equation will be examined and matching coefficient will be determined in Section V. The last section will be the summary and outlook.

II. HEAVY QUARK EXPANSION AND MATCHING FORMULA

In QCD, the gauge invariant nonlocal light-cone operators are of great interest. A bilocal operator composed by a light quark field and a heavy quark field can be expressed as $\bar{q}(z)\gamma^\mu q(0)$, where $\bar{q}(z)$ denotes the light-quark with mass $m$, and $M$ is the mass of the heavy quark field $Q(0)$, when ultraviolet (UV) singularities exist, then a matching equation that links nonlocal heavy-light current in QCD and HQET will be a superposition of $O(\bar{q}(z)\gamma^\mu q(0))$ and $O(\bar{q}(z)\gamma^\mu K(0))$, where $K$ is the velocity vector of heavy quark. $h_\mu$ is constrained by $\gamma_\mu h_\mu = h_\mu$ and equation of motion $v \cdot D h_\mu = 0$.

When the heavy quark mass $M$ is large, the heavy quark field and QCD Lagrangian can be expanded in series of $1/M$. The full heavy quark field $Q$ is expressed by the effective heavy quark field as (see, e.g., Refs. [4, 5])

$$Q(x) = e^{-iMv \cdot x} \left( 1 + \frac{iD^\mu}{2M} + \ldots \right) h_\mu(x),$$

(2)

where $D^\mu \equiv D^\mu = v^\mu v \cdot D$, with $D$ denoting the covariant derivative. At tree-level, since there is no interaction, we immediately have

$$O(z, 0)^{(0)} = \tilde{O}(z, 0; v)^{(0)} + O \left( \frac{1}{M} \right),$$

(3)

with the help of Eq. (2). The superscript $^{(0)}$ denotes that the operators are at tree-level.

If the radiative corrections are included, however, HQE will generally modify the UV behavior. Taking $M \to \infty$ in radiative correction of $O(z, 0)$ can not reduced to $O(z, 0; v)$ when ultraviolet (UV) singularities exist, then a matching is needed. Because the matching is related to hard scale $M$, the matching coefficient can be evaluated in perturbation theory. When the interaction is included, the position of quarks will be generally shifted, which means that HQE of $O(z, 0)$ will be a superposition of $O(\bar{q}(z)\gamma^\mu q(z))$.

The HQE formula proposed in this work is

$$O(z, 0, \mu) = \int_0^1 d\alpha \int_0^{\alpha} d\beta C(\alpha, \beta, t, M, \mu, \mu) \tilde{O}(\bar{q}(z)\gamma^\mu q(z) + O \left( \frac{\Lambda_{\text{QCD}}}{M}, z^- \Lambda_{\text{QCD}} \right),$$

(4)

for $1/M \ll z^- \ll 1/\Lambda_{\text{QCD}}$, where $t \equiv v \cdot z - i0$, $\alpha \equiv 1 - \alpha$. $C(\alpha, \beta, t, M, \mu, \mu; \alpha s)$ is the matching coefficient, which can be evaluated in perturbation theory. To confirm the matching formula and evaluate the matching coefficient, one should firstly calculate the radiative corrections of both QCD and HQET operators.

III. ONE-LOOP CORRECTION TO QCD OPERATOR

Since the nonlocal operator is defined in coordinate space, it is natural to perform calculation in coordinate-representation. Furthermore, the coordinate-representation calculation can be done in operator form. So in this paper we adopt the coordinate representation to evaluate the one-loop corrections. We work in $D = 4 - 2\epsilon$ dimensions so that the UV and soft singularities are regularized in dimensional regularization (DR). The light-quark mass $m$ serves as the regulator for the collinear (mass) singularity.

The radiative corrections to operator $O(z, 0)$ involve UV singularity, so the operator should be renormalized first. In modified minimal subtraction (\text{\overline{MS}}) scheme, the renormalized and bare operators are related by

$$O(z, 0, \mu)\text{ren.} = Z_{2, q}^{1/2} Z_{2, Q}^{1/2} \int_0^1 d\alpha \int_0^{\alpha} d\beta Z(\alpha, \beta; \alpha s(\mu)) O(\bar{q}(z)\gamma^\mu q(z))\text{bare},$$

(5)
where 
\[
Z_{2,q} = Z_{2,q} = 1 - \frac{\alpha_s C_F}{4\pi} \frac{1}{\epsilon_{\text{UV}}} + \mathcal{O}(\alpha_s^2)
\]  
(6)
are the renormalization constants of light and heavy quark, respectively, and 
\[
Z(\alpha, \beta; \alpha_s(\mu)) = \delta(\alpha)\delta(\beta) - \frac{\alpha_s C_F}{2\pi} \frac{1}{\epsilon_{\text{UV}}} \left( \delta(\beta) \left[ \frac{\beta}{\alpha} \right]_+ + \delta(\alpha) \left[ \frac{\beta}{\alpha} \right]_+ + 1 \right) + \mathcal{O}(\alpha_s^2).
\]  
(7)

Here the plus distribution is defined by 
\[
\int_0^1 du \left[ \frac{\bar{u}}{u} \right]_+ T(u) = \int_0^1 du \frac{\bar{u}}{u} [T(u) - T(0)],
\]  
(8)
with \(T(u)\) denoting a test function. The renormalization group equation (RGE) for \(O(z, 0, \mu)\) is 
\[
\frac{\mu^2}{d\mu^2} O(z, 0, \mu) = \int_0^1 d\alpha \int_0^1 d\beta \, V(\alpha, \beta) O(\tilde{\alpha} z, \beta z, \mu),
\]  
(9)
and 
\[
V(\alpha, \beta) = \frac{\alpha_s C_F}{2\pi} \left( \delta(\beta) \left[ \frac{\alpha}{\beta} \right]_+ + \delta(\alpha) \left[ \frac{\beta}{\alpha} \right]_+ + 1 - \frac{1}{2} \delta(\alpha)\delta(\beta) \right) + \mathcal{O}(\alpha_s^2)
\]  
(10)
is the Balitsky-Braun evolution kernel. It indicates that under renormalization, the nonlocal operator will get mixed with all the operators of the same type but with smaller separation between two quarks. By taking the forward hadron-to-hadron or meson-to-vacuum matrix elements and performing Fourier transformation, this equation will be reduced to the nonsinglet part of the Dokshizer-Gribov-Lipatov-Altarelli-Parisi equation for PDFs [21–23], or the Efremov-Radyushkin-Brodsky-Lepage equation for LCDAs [24–26], respectively. Recently the evolution of light-cone operators is known up to three-loops [28–31].

The renormalized operators including radiative correction can be generally expressed as 
\[
O(z, 0, \mu)^{\text{ren}} = \int_0^1 d\alpha \int_0^1 d\beta \, K(\alpha, \beta, m, \mu; \alpha_s) O(\tilde{\alpha} z, \beta z)^{(0)} + \text{higher twist operators}.
\]  
(11)
Here the operators that vanished by equation of motion are also eliminated. The kernel \(K(\alpha, \beta, m, \mu; \alpha_s)\) is a series in \(\alpha_s\)
\[
K(\alpha, \beta, m, \mu; \alpha_s) = K^{(0)}(\alpha, \beta) + \frac{\alpha_s C_F}{2\pi} K^{(1)}(\alpha, \beta, m, \mu) + \cdots,
\]  
(12)
with tree-level kernel \(K^{(0)}(\alpha, \beta) = \delta(\alpha)\delta(\beta)\). The one-loop term can be calculated in coordinate representation, the result reads 
\[
K^{(1)}(\alpha, \beta, m, \mu) = \delta Z \, \delta(\alpha)\delta(\beta) + \left[ \frac{\alpha}{\beta} \ln \frac{\mu^2}{\alpha^2 u_0^2 M_H^2} \right] \delta(\beta) + \left[ \frac{\beta}{\alpha} \ln \frac{\mu^2}{\beta^2 u_0^2 M_H^2} \right] \delta(\alpha) \\
+ \frac{2 u_0 \bar{u}_0 + (\alpha u_0 - \beta \bar{u}_0)(u_0 - \bar{u}_0) - (\alpha u_0 - \beta \bar{u}_0)^2}{[(\alpha u_0 - \beta \bar{u}_0)^2]^{1+\epsilon_{\text{IR}}}} \Gamma(1 + \epsilon_{\text{IR}}) \left( \frac{\mu_{\text{IR}}^2 e^{\gamma_E}}{M_H^2} \right)^{\epsilon_{\text{IR}}} + \ln \frac{\mu^2}{M_H^2 (\alpha u_0 - \beta \bar{u}_0)^2},
\]  
(13)
where \(M_H \equiv m + M\), and \(u_0 \equiv m/M_H\), \(\mu\) and \(\mu_{\text{IR}}\) are the renormalization and soft scales, respectively, \(\gamma_E\) is the EulerMascheroni constant, \((\alpha_s C_F/2\pi)\delta Z = \sqrt{Z_{2,q}^{\text{OS}} Z_{2,q}^{\text{OS}} - 1} \), and 
\[
Z_{2,q}^{\text{OS}} = 1 - \frac{\alpha_s C_F}{2\pi} \left( \frac{1}{\epsilon_{\text{IR}}} + \frac{1}{2} \ln \frac{\mu^2}{m^2} + \ln \frac{\mu_{\text{IR}}^2}{m^2} + 2 \right) + \mathcal{O}(\alpha_s^2),
\]  
\[
Z_{2,q}^{\text{OS}} = 1 - \frac{\alpha_s C_F}{2\pi} \left( \frac{1}{\epsilon_{\text{IR}}} + \frac{1}{2} \ln \frac{\mu^2}{M^2} + \ln \frac{\mu_{\text{IR}}^2}{M^2} + 2 \right) + \mathcal{O}(\alpha_s^2)
\]  
(14)
are the \(\overline{\text{MS}}\) subtracted on-shell renormalization constants for \(q\) and \(Q\), respectively. The second term in Eq. [13] is from the interaction between light quark and Wilson line, while the third term is from heavy quark—Wilson line.
interaction. The last two terms are from light quark—heavy quark interaction. Note that there is a scheme dependence on the treatment of $\gamma^5$ in DR: one is the naive DR scheme that $\gamma^5$ anti-commutes with all $\gamma^\mu$ [32]; another choice is the ‘t Hooft-Veltman scheme [33, 34], in which $\gamma^5$ anti-commutes with $\gamma^\mu$ for $\mu = 0, 1, 2, 3$ but commutes with $\gamma^\mu$ for $\mu = 4, \cdots, d - 1$. Without loss of generality, we simply adopt naive scheme in this work. We also note that $\epsilon_{1R}$ is not expanded at this stage, because the existence of soft singularities located at $\alpha = \beta = 0$. Such expansion is only safe when the soft singularities are isolated. For this purpose, we introduce following plus distributions for $T$

where $\tilde{\epsilon} = \delta (x - \tilde{\alpha} \tilde{u}_0 - \beta \tilde{v}_0)$. $\tilde{f}$ is the test function with two variables. The plus-preservation for single variable function has already been defined in Eq. (8). With the help of these plus distributions, Eq. (13) can be expanded in $\epsilon_{1R}$ and reorganized as

\[
K^{(1)} (\alpha, \beta, m, M, \mu) = \left\{ \frac{\tilde{\alpha}}{\alpha} \ln \frac{\mu^2}{\alpha_0^2 M_H^2} \right\} \delta (\beta) + \frac{1}{2} \delta (\alpha) \delta (\beta) \left( \frac{3}{2} (u_0 - \tilde{u}_0) \ln \frac{u_0}{\tilde{u}_0} - 3 \right) + 2 u_0 \tilde{u}_0 \delta^\prime (\alpha) \delta (\beta) \left( \frac{1}{u_0} + \ln \frac{\tilde{u}_0}{u_0} \right) + \left[ \frac{u_0 \tilde{u}_0}{(\alpha u_0 - \beta \tilde{u}_0)^2} \right]_{+} + \frac{1}{2} \left[ \frac{u_0 - \tilde{u}_0}{\alpha_0 u_0 - \beta \tilde{u}_0} + \ln \frac{M_H^2}{\alpha_0 u_0 - \beta \tilde{u}_0} - 1 \right]_{+} (\alpha \leftrightarrow \beta, u_0 \leftrightarrow \tilde{u}_0).
\]

One can easily find that this kernel has no soft divergence. The soft singularity in the light quark—heavy quark interaction is canceled by the one in quark self-energy $\delta Z$, while the interactions between Wilson line and quarks can be written as plus distributions themselves and free of soft singularity.

Our result in Eq. (16) is valid for arbitrary $m$ and $M$. To compare with previous result on LCDA for mesons with non-equal quark masses (e.g., $K$ and $B_c$), one can sandwich the operator $O(z, 0, \mu)$ between vacuum and the lowest Fock state, then Fourier transform to momentum space. By recalling Eq. (11), this is equivalent to a convolution

\[
f_H (x, \mu) = \int_0^1 d\alpha \int_0^\alpha d\beta K (\alpha, \beta, m, M, \mu; \alpha) \delta (x - \tilde{\alpha} u_0 - \beta \tilde{u}_0),
\]

where $f_H$ is the decay constant defined by the matrix element of local operator, and $\phi (x, \mu)$ is the leading twist LCDA for a meson with non-equal quark masses. With the kernel given in Eq. (16), we have

\[
f_H (x, \mu) = f_H^{(0)} (x - u_0) + \frac{\alpha_C F}{\mu^2} f_H^{(0)} \left\{ \left[ x - \left( u_0 + \frac{u_0 - \tilde{u}_0}{u_0} \ln \frac{u_0}{\tilde{u}_0} + \delta (x - u_0) \left( \frac{3}{2} (u_0 - \tilde{u}_0) \ln \frac{u_0}{\tilde{u}_0} - 3 \right) \right] \right\} + O (\alpha_s^2),
\]

where the last term is the same as the one-loop correction of decay constant. By eliminating this term one will arrive at the result for LCDA, which was firstly calculated by Bell and Feldmann [19] and later further explored in NRQCD re-factorization approach [33, 36].

Since the topic of this work is the heavy-light operator, what we are interested in is the $m \to 0$ and $M \to \infty$ limit. In fact the factorization only holds under this limit. To do this, a better starting point might be Eq. (13). After some efforts, we finally arrive at

\[
K^{(1)} (\alpha, \beta, m, M, \mu) \xrightarrow{\frac{m}{M} \to \infty} \left( \frac{3}{4} \ln \frac{M^2}{m^2} - 3 \right) \delta (\alpha) \delta (\beta) + \left[ \frac{\tilde{\alpha}}{\alpha} \ln \frac{\mu^2}{\alpha_0^2 m^2} + \frac{1}{2} \ln \frac{\tilde{\alpha}^2 \mu^2}{\alpha_0^2 m^2} + \frac{2}{\alpha} + \frac{3}{2} \right] \delta (\beta) + \left[ \frac{\tilde{\beta}}{\beta} \ln \frac{\mu^2}{\beta^2 M^2} \right] \delta (\alpha) + \left[ \frac{\tilde{\beta}}{\beta} + \frac{1}{\beta^2} \ln \frac{\mu^2}{\beta^2 M^2} \right] + O \left( \frac{1}{M} \right).
\]

Another special case is $u_0 = 1/2$, i.e., $m = M$, then Eq. (10) describes the one-loop correction to the operator with equal quark masses, which can be used to mesons like $\pi^0$ and $\eta_c$, etc.

### IV. ONE-LOOP CORRECTION TO HQET OPERATOR

The one-loop correction to HQET operator can be calculated in the same manner with QCD case. We denote HQET operator as $O (z, 0; \mu) = \bar{q} (z) [z, 0] \gamma_5 b_c (0)$, and add a tilde upon other related variables to distinguish from the QCD
ones. The one-loop correction to $\tilde{O}(z, 0; v)$ is also UV divergent and should be renormalized. The renormalization of HQET operator in $\overline{\text{MS}}$ scheme is given by

$$\tilde{O}(z, 0, \bar{\mu}; v)^{\text{ren.}} = Z_{2, d} Z_{h}^{-1} \int_{0}^{1} \, d\tilde{\alpha} \int_{0}^{\tilde{\alpha}} \, d\beta \, \tilde{Z}(\alpha, \beta, t; \alpha_{s}(\bar{\mu})) \tilde{O}(\bar{\alpha} z, \beta z; v)^{\text{bare}},$$

(20)

here $Z_{h}$ is the field renormalization constant for $h_{c}$, with the value $[4]$ $Z_{h} = 1 + \frac{\alpha_{s} C_{F}}{2\pi} \frac{1}{\epsilon_{UV}},$ (21)

and $[37]$

$$\tilde{Z}(\alpha, \beta, t; \alpha_{s}(\bar{\mu})) = \delta(\alpha)\delta(\beta) + \frac{\alpha_{s} C_{F}}{2\pi} \left\{ \frac{1}{2\epsilon_{UV}} + \frac{1}{\epsilon_{UV}} \ln(-\bar{\mu}^{2} t^{2} e^{2\gamma_{E}}) \right\} \delta(\alpha) - \frac{1}{\epsilon_{UV}} \left[ \frac{\tilde{a}}{\alpha} \right] \delta(\beta) + \mathcal{O}(\alpha_{s}^{2}).$$

(22)

Unlike the QCD case, there is a $1/\epsilon_{UV}^{2}$ UV divergence. In HQET, the heavy quark is described by a Wilson line along the $v$-direction. The interaction between the $v$- and $n$-Wilson lines generates a cusp singularity, therefore light-cone singularity and cusp singularity appear simultaneously and leads to the $1/\epsilon_{UV}^{2}$ pole. The cusp singularity and corresponding cusp anomalous dimension was computed at two-loop order long time ago $[38, 39]$ and has been known up to three-loops $[40, 41]$. The light quark—Wilson line interaction contributes equally to both QCD and HQET operators. The heavy quark—light quark interaction is UV finite. It is also valuable to point out that the location of of $h_{c}$ is fixed to 0 because of the constraint of $\delta(\beta)$. As we will see below, it is also true for finite terms. It indicates that because heavy quark has infinite mass in HQET, the position of heavy quark will not be shifted by interaction. Based on the renormalization relation, the RGE for $\tilde{O}(z, 0, \bar{\mu}; v)$ can be written down as

$$\tilde{\mu}^{2} \frac{d}{d\tilde{\mu}^{2}} \tilde{O}(z, 0, \bar{\mu}; v) = -\frac{\alpha_{s} C_{F}}{2\pi} \left\{ \frac{1}{2} \ln(-\bar{\mu}^{2} t^{2} e^{2\gamma_{E}}) - \frac{1}{4} \right\} \tilde{O}(z, 0, \bar{\mu}; v) + \frac{\alpha_{s} C_{F}}{2\pi} \int_{0}^{1} \, d\tilde{\alpha} \left[ \frac{\tilde{a}}{\alpha} \right] \tilde{O}(\tilde{\alpha} z, \bar{\alpha} \beta z; v).$$

(23)

If the anomalous dimension from decay constant is counted, this evolution equation will match the RGE for $B$-meson LCDA in coordinate space $[42]$. The RGE for $B$-meson LCDA in the name of Lange-Neubert equation was first derived in momentum space $[43]$. The two-loop evolution equation was derived very recently $[44]$. After the UV singularities are removed, the renormalized operator is linked to the tree-level one by

$$\tilde{O}(z, 0, \bar{\mu}; v)^{\text{ren.}} = \int_{0}^{1} \, d\tilde{\alpha} \int_{0}^{\tilde{\alpha}} \, d\beta \, \tilde{K}(\alpha, \beta, m, t; \bar{\mu}; \alpha_{s}) \tilde{O}(\bar{\alpha} z, \beta z; v)^{(0)} + \text{higher twist operators},$$

(24)

where $\tilde{K}(\alpha, \beta, m, t; \bar{\mu}; \alpha_{s})$ can also be expanded in series of $\alpha_{s}$

$$\tilde{K}(\alpha, \beta, m, t; \bar{\mu}; \alpha_{s}) = \tilde{K}^{(0)}(\alpha, \beta) + \frac{\alpha_{s} C_{F}}{2\pi} \tilde{K}^{(1)}(\alpha, \beta, m, t, \bar{\mu}) + \cdots,$$ (25)

with tree-level kernel $\tilde{K}_{0}(\alpha, \beta) = \delta(\alpha)\delta(\beta)$. Our result for the one-loop term is

$$\tilde{K}^{(1)}(\alpha, \beta, m, t, \bar{\mu}) = -\left[ \frac{1}{4} \ln^{2}(\bar{\mu}^{2} t^{2} e^{2\gamma_{E}}) + \frac{5\pi^{2}}{24} \right] \delta(\alpha)\delta(\beta) - \left[ \frac{1}{2} \ln(-m^{2} t^{2}) - \frac{1}{4} \ln(\bar{\mu}^{2} t^{2}) + 2 \right] \delta(\alpha)\delta(\beta)$$

$$+ \left[ \frac{\tilde{a}}{\alpha} \ln(\frac{\bar{\mu}^{2} t^{2}}{m^{2}}) - \frac{1}{2} \ln(-\bar{\mu}^{2} t^{2} e^{2\gamma_{E}}) - \frac{2}{\alpha} \right] \delta(\beta).$$

(26)

The first term arises from the interaction between heavy quark and Wilson line. A similar result in which the collinear divergence is regularized in DR was reported in Refs. $[42, 43]$. In contrast to QCD, the HQET nonlocal operator is non-analytic when $z \to 0$ because of the logarithmic and double-logarithmic dependence on $t$, therefore cannot approach to local operator smoothly, and the local OPE does not exist $[37]$.

V. FACTORIZATION AND MATCHING COEFFICIENT

With the one-loop corrections to QCD and HQET operators, we are now able to see how factorization formula

$$C(\alpha, \beta, t, M, \mu, \bar{\mu}; \alpha_{s}) = C^{(0)}(\alpha, \beta) + \frac{\alpha_{s} C_{F}}{2\pi} C^{(1)}(\alpha, \beta, t, M, \mu, \bar{\mu}) + \mathcal{O}(\alpha_{s}^{2}).$$

(27)

works. Since the matching coefficient is calculable in perturbation theory, one can expand it in series of $\alpha_{s}$
At tree-level, the QCD and HQET operators are same, so the factorization formula Eq. (11) holds and the tree-level matching coefficient is simply $C_0(\alpha, \beta) = \delta(\alpha)\delta(\beta)$.

The one-loop matching coefficient can be extracted by comparing the $O(\alpha_s)$ terms on the both sides of Eq. (11), the result is

$$C^{(1)}(\alpha, \beta, t, M, \mu, \tilde{\mu}) = K^{(1)}(\alpha, \beta, m, M, \mu)e^{-i\beta Mt} - \tilde{K}^{(1)}(\alpha, \beta, m, t, \tilde{\mu}).$$

(28)

The reason for the phase factor $e^{-i\beta Mt}$ is following: the radiative correction changes the location of heavy quark in QCD operator from 0 to $\beta z$, then according to Eq. (2), the heavy quark in QCD and HQET is related by a phase factor $e^{-i\beta Mt}$ at leading order of $1/M$ expansion, this phase factor finally enters the matching coefficient.

By recalling Eqs. (10), (20) and (28), one can evaluate the matching coefficient at one-loop level, the value reads

$$C^{(1)}(\alpha, \beta, t, M, \mu, \tilde{\mu}) = \left[\frac{1}{2}\ln(-\bar{\alpha}^2\mu^2 t^2e^{2\gamma_E}) + \frac{\bar{\alpha}}{\alpha}\ln\frac{\mu^2}{\bar{\mu}^2} + \frac{4}{\alpha} + \frac{3}{2}\right] \delta(\beta) + \left[\frac{\bar{\beta}}{\beta}\ln\frac{\mu^2}{\beta^2 M^2}\right] e^{-i\beta Mt} \delta(\alpha)$$

$$+ \left[\frac{1}{2}\ln(\bar{\alpha}^2\bar{\mu}^2 t^2e^{2\gamma_E}) + \frac{1}{2}\ln(-\bar{\mu}^2 t^2e^{2\gamma_E}) + \frac{1}{2}\ln(-t^2 M^2) - \frac{1}{2}\ln\frac{\bar{\mu}^2}{\mu^2} + \frac{5\pi^2}{24} - 1\right] \delta(\alpha)\delta(\beta).$$

(29)

One can see that the collinear divergences in QCD and HQET operators, which are represented by $\ln m^2$, are canceled. The matching coefficient $C^{(1)}(\alpha, \beta, t, M, \mu, \tilde{\mu})$ is free of collinear and soft singularities, indicating that the factorization also holds at one-loop level. By sandwiching the both sides of matching equation between vacuum and meson states, then performing Fourier transformations that demanded by the definitions of LCDAs, one can get the matching formula for $B$-meson LCDAs defined in QCD and HQET, which has been addressed in Ref. [18].

We also note here that only the $\gamma^5$ component of axial-current is considered in this paper. If the analysis is performed for all the components, i.e., $\gamma^\mu\gamma^5$, Lorentz structures like $\gamma^\mu\gamma^5\gamma^5$ and many others will enter the expansion formula. HQE for a general current will be a straightforward generalization of this work.

VI. SUMMARY AND OUTLOOK

In this paper, we generalize the heavy quark expansion to nonlocal heavy-light current on the light-cone. Based on a perturbative calculation in operator form, we confirm up to one-loop accuracy that the QCD nonlocal heavy-light current can be matched onto the corresponding HQET operator by a factorization theorem. All soft singularities are canceled, both for QCD and HQET operators; while the collinear singularities are common and can be canceled between QCD and HQET operators. The matching coefficient is determined at one-loop and leading power of $1/M$ expansion, which does not involve any infrared scale. The matching between leading twist LCDAs defined in QCD and HQET can be derived by taking matrix elements and Fourier transformations. The results presented in this paper might be useful to resum the large logarithms of $Q/M$ and $M/\Lambda_{QCD}$. Furthermore, if the $B$-meson LDA in QCD is calculable by lattice QCD through large momentum effective theory, it would provide another way of accessing $B$-meson LDA in HQET comparing with Ref. [17].

The work reported in this paper can be generalized along many directions.

- In this paper only axial-vector current is considered. It will be straightforward of applying the method described in this paper to study other heavy-light currents on the light-cone.

- It will be also interesting to study the heavy quark expansion for nonlocal heavy-heavy operators, which would be useful to understand the heavy quark PDFs or the shape functions.

- In this paper the nonlocal current is located on light-cone. A study on the heavy quark expansion for equal-time operators would be important for lattice simulations of heavy meson LCDAs, through large momentum effective theory or Ioffe time pseudo-distribution approach.

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