On the $Z_2$ Monopole of $Spin(10)$ Gauge Theories

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An “expanded” description is introduced to examine the spinor-monopole identification proposed by Strassler for four-dimensional $\mathcal{N} = 1$ supersymmetric $Spin(10)$ gauge theories with matter in $F$ vector and $N$ spinor representations. It is shown that a $Z_2$ monopole in the “expanded” theory is associated with massive spinors of the $Spin(10)$ theory. For $N = 2$, two spinor case, we confirm this identification by matching the transformation properties of the two theories under $SU(2)$ flavor symmetry. However, for $N \geq 3$, the transformation properties are not matched between the spinors and the monopole. This disagreement might be due to the fact that the $SU(N)$ flavor symmetry of the $Spin(10)$ theory is partially realized as an $SU(2)$ symmetry in the “expanded” theory.

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1 Introduction

The recent years have witnessed significant progress in the study of strongly coupled dynamics of four-dimensional $\mathcal{N} = 1$ supersymmetric gauge theories \[1\]. See \[2\] for recent reviews and references therein for earlier work. Most of this progress is attributed to the pioneering work of N. Seiberg \[3\] on the outstanding idea of duality: a SUSY gauge theory that is in a non-Abelian Coulomb phase may have an equivalent description in terms of a different gauge groups and matter content. A strongly coupled SUSY gauge theory thus has a weakly coupled dual that allows the theory to be solved. Thanks to Seiberg’s work, many examples of duality involving complicated gauge group and matter content have been uncovered \[4, 5, 6\].

Among various classes of duality that have been worked out, one particular electric-magnetic pair of duality is especially interesting \[7, 8\]. The electric theory is based on an Spin($N$) gauge group with matter in vector and spinor representations; while the magnetic counterparts have in general an SU($M$) gauge group with matter in symmetric tensor, fundamental, and anti-fundamental representations. No rigorous proof on this Spin – SU duality is known, but several non-trivial consistency checks support its existence. One remarkable feature of the Spin – SU duality appears when the spinor representation of Spin($N$) is real. The electric theory becomes non-chiral. However, the matter content of SU($N$) magnetic theory is obviously in chiral representations, the so called chiral–non-chiral duality.

One of the consistency checks on the Spin – SU duality is to add masses to all fields in spinor representations. The gauge group of the electric theory is broken to SO($N$)\[^{4}\]. On the magnetic theory, the effect of adding spinor masses generally forces the symmetric tensor to acquire expectation values and breaks the gauge group SU($M$) to SO($M$)\[^{3}\]. After massive fields are integrated out, the unbroken SO($M$) theory is found to be the dual of SO($N$) theory under the elementary SO duality of \[4\]. This provides strong evidence for the Spin – SU duality.

Recently, Strassler made an interesting observation about the breaking pattern of this Spin – SU duality \[5\]. The SU($M$) $\rightarrow$ SO($M$) breaking pattern on the magnetic theory renders a non-vanishing second homotopy group $\pi_2[SU(M)/SO(M)] = \pi_1[SO(M)] = \mathbb{Z}_2$, for $M > 2$. This indicates that a topological stable monopole carrying a $\mathbb{Z}_2$ charge exists in the theory \[10\]. He then argued that the $\mathbb{Z}_2$ charge associated with the monopole of the magnetic theory has to be carried by those massive spinors of the electric theory, because the spinors also have a $\mathbb{Z}_2$ charge under the center of Spin($N$), i.e. $\mathbb{Z}_2 = Spin(N)/SO(N)$. Therefore, the $\mathbb{Z}_2$ monopole is the image of massive spinors under duality.

It is important to highlight that the monopole in the magnetic theory only exists when all spinors are massive. If one or more of spinors in the electric theory are massless, the massive spinors in general will be surrounded by massless spinor cloud and form neutral bound states. Under this circumstance, the $\mathbb{Z}_2$ charge will not be visible in the magnetic theory. However, if all

\[^{4}\] In Spin(10) theory, no mass term can be written for spinors. Coupling the spinors to a field in vector representation that acquires an expectation value can then perform the check on the duality.

\[^{3}\] This breaking pattern is characteristic of the Spin($N$) theory with one spinor representation.
spinor fields are massive, the cloud that surrounds massive spinors can only be those light fields in vector or adjoint representations of $SO(10)$. Because these fields are neutral under the group $Z_2$, the $Z_2$ charge of massive spinors will not be screened at long distance. One thus expects that the lightest state with non-zero $Z_2$ charge of the electric theory will be visible in the form of the heavy monopole in the magnetic theory. The strongest evidence of this spinor-monopole identification comes from the transformation properties of the two theories under flavor groups. In [9], several examples have been given to verify that the massive spinors and monopole indeed transform in the same way under flavor symmetries.

In this paper, we continue the work on spinor-monopole identification by studying $Spin(10)$ gauge theories with matter in $F$ vector and $N$ spinor representations. The purpose is to check the transformation properties of the monopole under $SU(N)$ flavor symmetry that rotates the $N$ spinors into each other. Section 2 briefly describes the model for this study. The main results of this identification under the $SU(N)$ flavor symmetry is contained in Section 3. Finally, a conclusion is given in Section 4. Note that $SU(F)$ flavor symmetry which rotates $F$ vectors into each other will be discussed only peripherally in this article because it has been reported in [9].

2 The “expanded” theory of $Spin(10)$ theory

The electric theory is based on an $Spin(10)_{\text{local}} \times [SU(F) \times SU(N) \times U(1)_Y \times U(1)_R]_{\text{global}}$ symmetry group with matter in $F$ vector and $N$ spinor representations. The transformation properties of fields under the symmetry group are

$$P_F \sim (10; □, 1, -2N, \xi)$$
$$Ψ \sim (16; 1, □, F, 0),$$

where $\xi = \frac{F + 2N - 8}{F}$. The superpotential of the electric theory is zero, $W = 0$.

The magnetic theory of [11] constructed in [8] is based on an $SU(F + 2N - 7) \times Sp(2N - 2)$ gauge group. One remarkable structure worthy of attention is the partial $SU(2)$ realization of the $SU(N)$ flavor symmetry in the magnetic theory. The full $SU(N)$ flavor symmetry only exist as a quantum accidental symmetry [11]. However, when $N = 2$, the flavor symmetries of the electric theory and the magnetic theory are equivalent. (The magnetic theory is summarized in appendix B).

If one wishes to analyze the problem of spinor-monopole identification on the magnetic theory, he/she will soon discover that the complex and intricate breaking pattern in the theory is too difficult to perform. We thus use a dual description of the magnetic theory for study. This new description is derived from expanding the symmetric tensor of the magnetic theory with the method of deconfinement. According to [12, 13], there are two such approaches available. The first approach treats the symmetric tensor as a bound state of an auxiliary gauge theory, whereas the second approach considers the symmetric tensor and the fundamentals as bound states of another auxiliary gauge theory. Both approaches are able to yield dual descriptions
of the magnetic theory. Yet, the latter method suggested in [13] renders a good theory with controllable breaking pattern when masses of spinors are introduced.

This new theory is based on an \( [SO(10) \times SU(2N + 4) \times Sp(2N - 2)]_\text{local} \times [SU(F) \times SU(2) \times U(1)_Y \times U(1)_R]_\text{global} \) symmetry group and will be called the “expanded” theory of the Spin(10) theory. As in the magnetic theory, the \( SU(N) \) flavor symmetry is also partially realized in this theory. The field content is

\[
\begin{align*}
P_F & \sim (\Box, 1, 1; \Box, 1, -2N, \xi) \\
S & \sim (1, \Box, 1, 1, 1, 2F - \frac{6F}{N + 2}, \frac{1}{N + 2}) \\
Q & \sim (1, \Box, 1, 1, 1, 2N - 1, -F - \frac{3F}{N + 2}, 1 + \frac{1}{2(N + 2)}) \\
\bar{Q}_A & \sim (\Box, \Box^*, 1, 1, 1, -F + \frac{3F}{N + 2}, 1 - \frac{1}{2(N + 2)}) \\
\bar{Q}_B & \sim (1, \Box^*, \Box, 1, 1, (2N - 1)F + \frac{3F}{N + 2}, -2(N + 1) - \frac{1}{2(N + 2)}) \\
R & \sim (1, 1, \Box; 1, 1, 1, 2N - 2, 2F, 0) \\
P & \sim (\Box, 1, 1; 1, 1, 2N - 1, 1, 2F, 0) \\
N & \sim (1, 1, 1, 1, 2N - 1, -2(N - 1)F, 2N + 3) \\
u & \sim (1, 1, 1; 1, 1, -4NF, 4N + 6).
\end{align*}
\]

The “expanded” theory has this superpotential

\[
W = Q_A^2 S + u Q_B^2 S + Q^2 S + P Q_A Q + N Q_B Q + Q R Q.
\]

Note that all parameters, including those needed for dimensional consistency, are set to unity.

At first glance, the “expanded” theory is far more complicated than the magnetic theory because the former is consisted of a product of three gauge groups and many chiral superfields. Yet, it will be shown later that this poses no obstacle to calculation. As can be seen from the matter content, all fields in the “expanded” theory, except for \( P_F \), transform as singlet fields under \( SU(F) \) flavor symmetry. The “expanded” theory is thus like the Spin(10) theory with spinor representations being substituted by vector representations of an \( SO(10) \) theory. This \( SO(10) \) theory is then extended to a theory with three gauge groups for the purposes that all local anomalies will cancel, all global anomalies can match, and all composite operators can map to those of the Spin(10) theory.

The term “expanded” is derived from the fact that a product of two Spin(\( N \)) spinors can always be decomposed into a direct sum of anti-symmetric tensors of \( SO(N) \). For instance, the product of two Spin(10) spinors has this decomposition: \( \mathbf{16} \times \mathbf{16} = [1]_S + [3]_A + [5]_S \). Here,
\([n]\) denotes an anti-symmetric rank-\(n\) tensor representation, “S” and “A” subscripts indicate symmetry and anti-symmetry under spinor exchange, and the tilde over the last term implies that the rank-5 representation is complex self-dual. These anti-symmetric tensors are then further deconfined into the fields in fundamental representations of the \(SO(10)\times SU(2N+4)\times Sp(2N-2)\) gauge theory. That is

\[
\begin{align*}
\Psi_2^{[1]} & \leftrightarrow P + \bar{Q}_A \bar{Q}^{2N+2} \bar{Q}_B \\
\Psi_2^{[3]} & \leftrightarrow \bar{Q}_A^3 \bar{Q}^{2N} \bar{Q}_B \\
\Psi_2^{[5]} & \leftrightarrow \bar{Q}_A^5 \bar{Q}^{2N-2} \bar{Q}_B,
\end{align*}
\]

where the subscript \([n]\) denotes an \(SO(10)\) anti-symmetric tensor of rank-\(n\).

Does the “expanded” theory have a solution of topological stable monopole as the magnetic theory has when masses of all spinors are introduced? The answer is yes. Because all fields in the “expanded” theory are neutral under the \(Z_2\) group, they cannot screen the topological charge. The \(Z_2\) charge has to be carried by a monopole of the “expanded” theory. This statement can be easily checked for one spinor case, \(N = 1\). On one side, the \(Spin(10)\) theory is broken to \(SO(9)\) upon introducing a coupling of spinor to a vector that acquires expectation value. On the other side, the \(SO(10)\times SU(6)\) gauge group of the “expanded” theory is first broken to \(SO(9)\times SU(5)\) and then to \(SO(9)\times SO(5)\). At the latter stage of breaking sequences, a \(Z_2\) monopole will be generated since \(\pi_2[SU(5)/SO(5)] = Z_2\). \((N = 1 \ Spin(10)\) theory and its “expanded” theory are summarized in appendix A\). We will show in next section that the breaking pattern \(SU(5) \rightarrow SO(5)\) also exists for \(N \geq 2\) spinor cases. A remark is noted. The \(SO(9)\times SO(5)\) “expanded” theory will be reduced to an \(SO(9)\) theory since the \(SO(5)\) theory confines. After massive fields are integrated out, this \(SO(9)\) theory is equivalent to the electric \(SO(9)\) theory. The “expanded” theory is thus self-dual, rather than dual, to the \(Spin(10)\) theory under duality.

### 3 The spinor-monopole identification

In this section, the “expanded” theory (3) is chosen for the study of spinor-monopole identification of the \(Spin(10)\) theory with matter in \(F\) vector and \(N\) spinor representations. Because the spinor representation of \(Spin(10)\) is chiral, no mass can be introduced for spinor fields. However, mass terms can be given when \(Spin(10)\) is broken to a smaller \(Spin\) group. In the \(Spin(10)\) theory (1), by coupling spinors to a vector that acquires an expectation value, the spinors become massive in an \(Spin(9)\) theory. This coupling also breaks the “expanded” theory (3) to an \([SO(9)\times SU(2N+4)\times Sp(2N-2)]_{\text{local}} \times [SU(F-1)\times SU(2)]_{\text{global}}\) theory, which differs from (3) with regard to \(SO\) gauge group. In other words, this new theory will have matter content as in (3) except for those fields in vector representations of \(SO(10)\). The \(SO(10)\) vectors \(\bar{Q}_A\) and \(P\) of (3) get split.

‡ In principle, the “expanded” theory of any \(Spin(N)\) theory can be determined from this rule.
according to this: $\tilde{Q}_A \rightarrow (\tilde{Q}_A, \bar{q}_A)$ and $P \rightarrow (P, p)$, where $\tilde{Q}_A$ and $P$ are $SO(9)$ vectors and $\bar{q}_A$ and $p$ are $SO(9)$ scalars. Note that we use the same notation for both $SO(10)$ and $SO(9)$ vectors. Taking the effects of field splitting and mass perturbation into account, we conclude that this theory has the effective superpotential

$$W_{N,j} = \sum_{a,b=1}^{2N+4} \left\{ S^{ab} Q^a_A Q^b_A + u S^{ab} Q^a_B Q^b_B + S^{ab} \bar{q}^a_A \bar{q}^b_A + \sum_{i=1}^{2N-1} \left[ P_i Q^a_A + N_i Q^a_i Q^a_i + p_i \bar{Q}^a_A \right] \right\} + p_j. \tag{5}$$

where the notation $W_{N,j}$ denotes that the superpotential of an $SO(9) \times SU(2N+4) \times Sp(2N-2)$ theory is perturbed by $p_j$ in the $j$-th direction. Here, $a, b$ superscripts denote $SU(2N+4)$ gauge indices, $\alpha, \beta$ superscripts indicate $Sp(2N-2)$ gauge indices, and $i, j$ subscripts are $SU(2)$ flavor indices. $J_{\alpha\beta}$ is an anti-symmetric tensor taken to be $J = 1_{N-1} \otimes i\sigma_2$. $R^0_0 = R^a_{2N-1} = 0$ are introduced to make the expression compact. All Clebsch-Gordan coefficients of $SU(2)$ multiplication are set to unity for simplicity. The mass term introduced for spinors in the electric theory is mapped to the last term $p_j$ in (5). The other candidate $\bar{q}_A \bar{Q}^{2N+2} \bar{Q}_B$ that also maps to the mass term is of no interest to the dynamics, and will not be discussed. It is noted that masses of all spinors are taken to be equal, and $SO(9)$ gauge indices are omitted because they are spectators to the breaking pattern of Higgs mechanism.

In (5), the F-flatness condition $\partial W/\partial p_j$ ensures that $\langle \bar{q}_A^a Q^a_j \rangle$ is nonzero, and the gauge group of the theory will be broken to a smaller group. The detailed breaking pattern is of course dependent on what direction of $p_j$ in the $(2N-1)$ representation of $SU(2)$ is assigned to. Because of the reflection symmetry of $SU(2)$, there are only $N$ possibilities for $p_j$. It will be shown below that the choice of $p_j = p_N$ in (5) corresponds to give masses to all spinors in the electric theory. A monopole state which carries a $Z_2$ topological charge is found to exist in the “expanded” theory at the final stage of Higgs breaking sequences. According to the prediction of (5), this monopole will be accompanied by $2(N-1)$ zero modes that after quantization make the monopole an $(N-1)$-index multispinor under the $SU(2)$ flavor symmetry.\footnote{There are other types of zero modes that make the same monopole an multispinor under another flavor symmetry.}

Next, this prediction will be checked on the “expanded” theory.

### 3.1 The $Spin(10)$ theory with two spinors

For two-spinor case, $N = 2$, the “expanded” theory is based on an $SU(8) \times Sp(2)$ gauge group. (The notation of $SO(9)$ gauge group is suppressed). The scalar field $p_j$ forms a three dimensional representation of $SU(2)$, that is, $p_j = (p_1, p_2, p_3)$.

First, let us investigate the choice of $p_j = p_1$ in (5). The F-flatness condition of $p_1$ implies that $\bar{q}_A^a$ and $Q^a_1$ get expectation values, which by the D-term equation must be in the same direction.
of the color group. Thus, the gauge group is broken to $SU(7) \times Sp(2)$ with the superpotential

$$W^{(1)}_{2,1} = \sum_{a,b=1}^{7} \left\{ S^{ab} \bar{Q}_A^a Q_B^b + u S^{ab} \bar{Q}_B^a Q_B^b + \sum_{i=2}^{3} \left[ P_i Q_i^a \bar{Q}_A^a + N_i Q_i^a \bar{Q}_B^a \right] \right\} + 2 \sum_{a,\beta=1}^{2} J_{a\beta} \left[ Q_2^a Q_2^a, R_2^\beta + Q_3^a Q_2^a, R_2^\beta + S^{ab}(\bar{Q}_1^a, \bar{Q}_2^a, - \bar{Q}_2^a, \bar{Q}_1^a, \bar{Q}_2^b, - \bar{Q}_2^b, \bar{Q}_1^b, \bar{Q}_2^b) + S^{8a}(Q_2^a Q_2^a, - \bar{Q}_2^a, \bar{Q}_1^a, \bar{Q}_2^b, - \bar{Q}_2^b, \bar{Q}_1^b, \bar{Q}_2^b, \bar{Q}_1^b) \right\} + S^{88}, \tag{6}$$

where we pick $\langle \bar{q}_1^a \rangle \langle Q_2^a \rangle \neq 0$ and the superscript $(n)$ denotes that the superpotential $W^{(n)}_{2,1}$ is derived after $n$ steps of symmetry breaking.

Now work out the F-flatness conditions for this superpotential. The vanishing $\partial W^{(1)}_{2,1}/\partial S^{88}$ condition ensures that $\langle J_{a\beta} \bar{Q}_2^a Q_2^a, - \bar{Q}_2^a, \bar{Q}_1^a, \bar{Q}_2^b, - \bar{Q}_2^b, \bar{Q}_1^b, \bar{Q}_2^b \rangle$ is non-zero. $\langle Q_2^a \rangle (\langle Q_2^a \rangle, \langle Q_2^b \rangle) \neq 0$ is adopted in computation. This breaks the $SU(7)$ gauge symmetry down to $SU(6)$ and completely breaks the $Sp(2)$ gauge group in which a dynamical superpotential is generated by a weak coupling instanton process. After combining this quantum correction with the classical terms (9), we obtain the superpotential of this $SU(6)$ gauge theory,

$$W^{(2)}_{2,1} = \sum_{a,b=1}^{6} \left\{ S^{ab} \bar{Q}_A^a Q_B^b + u S^{ab} \bar{Q}_B^a Q_B^b + P_3 Q_3^a Q_A^a + N_3 Q_3^a Q_B^a + S^{ab} \bar{q}^a \bar{q}^a + R_3 Q_3^a \bar{q}^a + \det S^{ab} \right\}, \tag{7}$$

where $\bar{q}^a \equiv \bar{Q}_1^a, 1 = \bar{Q}_2^a, 2$ and the last term on the equation comes from the instanton effect.

Note that the theory with superpotential (9) is the “expanded” theory of an $Spin(9)$ theory with matter in $(F-1)$ vector and one spinor representations. Readers can confirm this point from appendix A. Hence, the choice of $p_j = p_1$ in the $SU(8) \times Sp(2)$ “expanded” theory is mapped to a mass perturbation for one of the two spinors in the $Spin(9)$ theory with $N = 2$. As mentioned earlier, a similar breaking result, $SU(8) \times Sp(2) \rightarrow SU(7) \times Sp(2) \rightarrow SU(6)$, is obtainable when $p_j = p_3$.

Next, let us consider the same $SU(8) \times Sp(2)$ gauge theory with this choice $p_j = p_2$. The theory is first broken to an $SU(7) \times Sp(2)$ theory by $\langle \bar{q}_2^a Q_2^a \rangle \equiv \langle \bar{q}_1^a \rangle \langle Q_2^a \rangle \neq 0$ with the superpotential

$$W^{(1)}_{2,2} = \sum_{a,b=1}^{7} \left\{ S^{ab} \bar{Q}_3^a Q_A^b + u S^{ab} \bar{Q}_4^a Q_B^b + \sum_{i=1,3} \left[ P_i Q_i^a \bar{Q}_3^a + N_i Q_i^a \bar{Q}_3^a \right] \right\} + 2 \sum_{a,\beta=1}^{2} J_{a\beta} \left[ S^{ab}(Q_1^a, Q_2^b, - Q_2^a, Q_1^b, Q_2^b, - Q_2^b, Q_1^b, Q_2^b) + S^{8a}(Q_3^b Q_2^a, Q_2^a, - Q_1^b, Q_1^a, Q_2^b, - Q_2^b, Q_1^b, Q_2^b, Q_1^b) \right\} + S^{88}. \tag{8}$$
In a similar vein, by working on the F-flatness condition \( \partial W_{2,2}^{(1)} / \partial S^{88} \) of this superpotential, we arrive at a non-vanishing \( \langle J_{\alpha \beta} Q^a_1 Q^a_3 \rangle \). This indicates that the \( SU(7) \times Sp(2) \) gauge group is further broken to \( SU(5) \) in addition to a non-perturbative superpotential that is generated by \( Sp(2) \) instanton process. The result of the choice of expectation values, \( \langle Q^7_1 \rangle \langle Q^7_3 \rangle \langle Q^6_1 \rangle \langle Q^6_2 \rangle \), produces this superpotential

\[
W_{2,2}^{(2)} = \sum_{a,b=1}^{5} \left\{ S^{ab} \bar{Q}^a_A Q^b_B + u S^{ab} \bar{Q}^a_B Q^b_A + S^{ab} \bar{q}^a q^b + S^{6a} S^{7b} [S^4]^{ab} + S^{67} \det S^{ab} \right\} + S^{67},
\]

where \( [S^4]^{ab} \equiv \epsilon^{a_{a_1} \cdots a_4} \epsilon^{b_{b_1} \cdots b_4} S^{a_1 b_1} \cdots S^{a_4 b_4} \) and the last two terms inside the brace signs are from instanton contribution. Another consistency checks of (9) is provided in appendix C.

Yet, the symmetry breaking does not cease at theory (9). The vanishing of the F-flatness condition for field \( S^{67} \) ensures that \( \langle \det S^{ab} \rangle \) is nonzero. This will break the gauge group \( SU(5) \) down to \( SO(5) \) and thus generates a \( Z_2 \) monopole solution. Before massive fields are integrated out, the \( SO(5) \) gauge theory has the superpotential

\[
W_{2,2}^{(3)} = \sum_{a=1}^{5} \left\{ \langle S \rangle Q^a_A \bar{Q}^a_A + \langle S \rangle u \bar{Q}^a_B Q^a_B + \langle S \rangle \bar{q}^a q^a + \langle S^4 \rangle S^{6a} S^{7a} \right\}.
\]

As can be seen from (10), fields \( \bar{Q}^a_A, \bar{q}^a, S^{6a}, \) and \( S^{7a} \) become massive as a result of their coupling to \( S \). In the presence of the monopole, \( \bar{Q}^a_A \) bears an extra \( SO(9) \) gauge index and has nine zero modes, making the monopole an \( Spin(9) \) spinor after quantization. Similarly, \( \bar{q}^a \) has one zero mode and will make the monopole an “\( Spin(1) \)” spinor. Analogously, \( S^{6a} \) and \( S^{7a} \) each has a single zero mode that, upon quantization, constructs the monopole a two dimensional spinor under \( SU(2) \) flavor symmetry. This agrees with the transformation properties of the spinors of the electric theory under the \( Spin(9) \) color group and the \( SU(2) \) flavor group. We thus verify the prediction of spinor-monopole identification in the model of \( Spin(10) \) gauge theory with two spinors.

As a side remark, after all massive fields are integrated out, the \( SO(5) \) theory confines without generating a superpotential. The final result is a pure \( SO(9) \) theory with matter in \( (F-1) \) vector representations and a heavy monopole. Under duality, this theory is self-due to the \( Spin(9) \) gauge theory with \( (F-1) \) vectors and two massive spinors.

### 3.2 The \( Spin(10) \) theory with \( N \geq 3 \) spinors

For \( N \)-spinor case, the “expanded” theory (3) has an \( SU(2N+4) \times Sp(2N-2) \) gauge group. The field \( p_j \) forms a \( (2N-1) \) dimensional representation of \( SU(2) \), that is, \( p_j = (p_1, \cdots, p_{2N-1}) \). The reflection symmetry of \( SU(2) \) connotes that the choices of \( p_k \) and \( p_{2N-k} \) \( (1 \leq k \leq N) \) generate the same symmetry breaking pattern. We thus focus on the range of \( p_j \) with \( j = 1, \cdots, N \).

As discussed in Section 3.1, it can be demonstrated that, for the choice of \( p_j \) with \( j \leq N-2 \), the theory is eventually broken to an \( SU(2(N-j)+4) \times Sp(2(N-j)-2) \) gauge symmetry
in which the superpotential takes the form $W_{N-j,0}$ (see (3)). Here the subscript “0” indicates the removal of the field $p_j$ in (3). (The result is summarized in appendix D.) This theory is apparently the “expanded” theory of an $Spin(9)$ gauge theory with matter in $(F-1)$ vector and $(N-j)$ spinor representations. Similarly, for $p_j = p_{N-1}$ situation, the resulting theory is based on an $SU(6)$ gauge symmetry, the “expanded” theory of an $Spin(9)$ theory with $(F-1)$ vectors and one spinor.

Let us continue by choosing $j = N$ for $p_j$. The final theory will be reduced to an $SO(9)$ gauge theory with matter in $(F-1)$ vector representations and a monopole solution. This can be illustrated as follows. The F-flatness condition $\partial W_{N,N}/\partial p_N$ of (3) implies that $\langle q_A^a \bar{Q}_N^a \rangle$ is non-zero. Thus the $SU(2N+4) \times Sp(2N-2)$ gauge group is broken to $SO(2N+3) \times Sp(2N-2)$ with superpotential generically denoted by $W^{(1)}_{N,N}$. Following this superpotential would be the second step of symmetry breaking, it produces a theory based on an $SO(2N+1) \times Sp(2N-4)$ gauge group with superpotential

$$W^{(2)}_{N,N} = \sum_{a,b=1}^{2N-1} \left\{ S^{ab} \bar{Q}_A^a \bar{Q}_A^b + u S^{ab} \bar{Q}_B^a \bar{Q}_B^b + S^{ab} q^a q^b + \sum_{i=1}^{2N-1} \left[ P_i q^a \bar{Q}^a_A + N_i q^a \bar{Q}^a_B + h_i q^a q^a \right] 
+ \sum_{a,\beta=1}^{2N-2} J_{\alpha\beta} \left[ S^{ab} (Q_1^{a,\alpha} Q_2^{b,\beta} - Q_2^{a,\alpha} Q_1^{b,\beta}) + \sum_{i=1}^{2N-1} Q_i^{a,\alpha} R_i^{\beta} + Q_2^{a,\alpha} R_i^{\beta} \right] 
+ S^{2N+3,a} Q_{N-2}^{b,a} Q_{N+2}^{b,a} + S^{2N+3,a} Q_{N-2}^{b,a} Q_{N+2}^{b,a} \bar{Q}_1^{a,\beta} + Q_1^{2N+2} Q_{N+2}^{a,\alpha} Q_1^{a,\beta} \right) + S^{2N+2,2N+3}, \quad (11)$$

where $\sum_{i=1}^{2N-1}$ denotes that the sum is taken from the interval $(1, \cdots, N-2, N+2, \cdots, 2N-1)$. The fields $\bar{q}^a \equiv Q_1^{a,2N-2} = Q_2^{a,2N-3}$, $h_i \equiv (R_i^{2N-2} + R_i^{2N-3})$, and $R_0 = R_0^{a,2N-2} = R_0^{a,2N-1} = 0$ are introduced to pack the expression.

The theory (11) is further broken to a smaller gauge group by the F-flatness condition of $S^{2N+2,2N+3}$. The result of integrating out massive fields is an $SU(2N-1) \times Sp(2N-6)$ gauge group with the superpotential

$$W^{(3)}_{N,N} = \sum_{a,b=1}^{2N-1} \left\{ S^{ab} \bar{Q}_A^a \bar{Q}_A^b + u S^{ab} \bar{Q}_B^a \bar{Q}_B^b + S^{ab} q^a q^b + \sum_{i=1}^{2N-1} \left[ P_i q^a \bar{Q}^a_A + N_i q^a \bar{Q}^a_B + h_i q^a q^a \right] 
+ \sum_{a,\beta=1}^{2N-6} J_{\alpha\beta} \left[ S^{ab} (Q_1^{a,\alpha} Q_2^{b,\beta} - Q_2^{a,\alpha} Q_1^{b,\beta}) + \sum_{i=1}^{2N-1} Q_i^{a,\alpha} R_i^{\beta} + Q_2^{a,\alpha} R_i^{\beta} \right] 
+ S^{2N+1,a} Q_{N-3}^{b,a} Q_{N+3}^{b,a} + S^{2N+1,a} Q_{N-3}^{b,a} Q_{N+3}^{b,a} \bar{Q}_1^{a,\beta} + Q_1^{2N} Q_{N+3}^{a,\alpha} Q_1^{a,\beta} \right) + S^{2N,2N+1}, \quad (12)$$

where $\sum_{i=1}^{2N-1}$ denotes that the sum is taken from $1, \cdots, N-3, N+3, \cdots, 2N-1$ and $R_0 = R_0^{a,2N-3} = R_0^{a,2N+2} = R_0^{a,2N-1} = 0$. 

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Note that superpotential (14) is similar to superpotential (11) except that the former has less numbers of superfields. The two superpotentials differ only in the summation over gauge indices $a, b$ and over the $SU(2)$ flavor index $i$. The similarity of the two superpotentials (11) and (14) suggests that the structure of superpotential is quite unique and can be formally expressed in the same form at each stage of symmetry breaking sequences. Hence, we would expect that the superpotential (14) will be reduced to a superpotential, denoted by $W_{N,N}^{(4)}$, of the same form. In a similar way, $W_{N,N}^{(4)}$ will be reduced to $W_{N,N}^{(5)}$ of the same form, $W_{N,N}^{(5)}$ will be reduced to $W_{N,N}^{(6)}$ of the same form, and this pattern will continue to hold true before the $Sp$ gauge group is completely broken.

This finding is significant, because it simplifies huge amounts of analysis on the breaking pattern. It assists us in constructing the breaking pattern of the “expanded” theory in a systematic way. That is, when an $SU(2N + 4) \times Sp(2N - 2)$ “expanded” theory (5) is perturbed by $p_j$ along the $N$-th direction, the breaking pattern has this sequence: $SU(2N + 4) \times Sp(2N - 2) \xrightarrow{(1)} SU(2N + 3) \times Sp(2N - 2) \xrightarrow{(2)} SU(2N + 1) \times Sp(2N - 4) \xrightarrow{(3)} \cdots \xrightarrow{(N-2)} SU(9) \times Sp(4) \xrightarrow{(N-1)} SU(7) \times Sp(2)$, with the superpotential takes the same form at each breaking stage. Therefore, the superpotential of the above $SU(7) \times Sp(2)$ theory will have this form

$$W_{N,N}^{(N-1)} = \sum_{a,b=1}^{7} \left\{ S^{ab} Q_A^a \bar{Q}_A^b + u S^{ab} Q_B^a \bar{Q}_B^b + S^{ab} \bar{q}^a \bar{q}^b + \sum_{i=1,2N-1} \left[ P_i Q_i^a \bar{Q}_B^b + h_i Q_i^a q^a \right] + \sum_{\alpha,\beta=1}^{2} J_{\alpha\beta} \left[ S^{4\alpha} (\bar{Q}_2^{a,\alpha} \bar{Q}_2^{b,\beta} - \bar{Q}_2^{a,\beta} \bar{Q}_2^{b,\alpha}) + S^{8a} Q_1^{b,a} \bar{Q}_2^{b,a} \bar{Q}_2^{b,a} + S^{9a} Q_1^{b,a} Q_1^{a,b} + S^{89} Q_1^{a,\alpha} Q_2^{b,\alpha} Q_2^{b,\alpha} \right] + S^{89} \right\}. \quad (13)$$

Interested readers may compare and contrast (8) and (13).

Now, the F-flatness condition $\partial W_{N,N}^{(N-1)}/\partial S^{89}$ of this superpotential implies that the $SU(7) \times Sp(2)$ gauge group is broken to $SU(5)$ with a non-perturbative superpotential generated by $Sp(2)$ instanton effects. This $SU(5)$ theory has superpotential

$$W_{N,N}^{(5)} = \sum_{a,b=1}^{5} \left\{ S^{ab} Q_A^a \bar{Q}_A^b + u S^{ab} Q_B^a \bar{Q}_B^b + S^{ab} \bar{q}^a \bar{q}^b + S^{6a} Q_2^{b,a} \bar{Q}_2^{b,a} + S^{67} [S^4]^{ab} + S^{67} \right\} + S^{67}. \quad (14)$$

By comparing (14) and (4), we find that they are equivalent. As a result, the discussion on the spinor-monopole identification of (5) in previous section can be applied directly to (14) without any modification. That is, the $SU(5)$ gauge symmetry will be broken to $SO(5)$ with superpotential $W_{N,N}^{(N+1)} = W_{2,3}^{(5)}$ and generates a $\mathbb{Z}_2$ monopole solution. Under $SU(2)$ flavor group, the monopole is accompanied with two zero modes which make it a two dimensional spinor. Unfortunately, this goes against the prediction of (5) that suggests the existence of $2(N - 1)$ zero modes for $N \geq 3$ theories.
4 Conclusion

The “expanded” description presented in this article for $\text{Spin}(10)$ gauge theories with matter in $F$ vector and $N$ spinor representations is used for the study of spinor-monopole identification proposed by Strassler. Because this theory is derived by deconfining the magnetic theory, the monopole solution is shown to exist in the “expanded” theory as it does in the magnetic theory. This “expanded” description has a systematic breaking pattern of gauge symmetry, upon all spinors become massive in the $\text{Spin}(10)$ theory.

For the $\text{Spin}(10)$ theory with two spinors, we confirm this identification by matching the transformation properties of the two theories under $SU(2)$ flavor symmetry. We find that the existence of two zero modes in the “expanded” theory makes the monopole an $SU(2)$ spinor that corresponds to two massive spinors of $\text{Spin}(9)$ theory. We also note that the monopole transforms as an $\text{Spin}(9)$ spinor as the massive spinors do. However, for theories with $N \geq 3$ spinor fields, the transformation properties are not matched between the massive spinors and the monopole.

This discrepancy for $N \geq 3$ may be due to the partial $SU(2)$ realization of the $SU(N)$ flavor symmetry in the “expanded” theory (and in the magnetic theory). It can be inferred by breaking the $\text{Spin}(10)$ theory down to $\text{Spin}(10 - k)$ with expectation values of $k$ vectors, then the electric theory has an $SU(N) \times Spin(k)$ flavor symmetry, with all spinors massive. On the other side, the “expanded” theory would have an $[SO(10 - k) \times SO(5)]_{\text{local}} \times [SU(2) \times SO(k)]_{\text{global}}$ symmetry group. It can be demonstrated that the transformation properties of the spinors and the monopole are identical under the $Spin(k)$ flavor symmetry, but not so under the $SU(2)$ flavor symmetry. We thus speculate that the partial realization of flavor symmetry is responsible for the inconsistency. However, to resolve this needs further investigation on the quantum accident symmetry [11].

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A appendix

This appendix summarizes the $\text{Spin}(10)$ theory with matter in $F$ vector and one spinor representations. For the purpose of simplicity, we set all coupling constants and dimensional parameters to unity. For details, please refer to [7].

The electric theory is based on an $\text{Spin}(10)$ gauge group. Under the symmetry groups $\text{Spin}(10)_{\text{local}} \times [SU(F) \times U(1)_Y \times U(1)_R]_{\text{global}}$, the fields of the theory transform as follows,

$$P_F \sim (10; \Box, -2, 1 - \frac{6}{F})$$

The $SU(F - k)$ flavor symmetry is suppressed.
\[ \Psi \sim (16; 1, F, 0). \]  

(15)

The superpotential of the electric theory is zero, \( W = 0 \).

The “expanded” theory has an \( SO(10) \times SU(6) \) gauge group. Under the symmetry group, \([SO(10) \times SU(6)]_{\text{local}} \times [SU(F) \times U(1)_Y \times U(1)_R]_{\text{global}}\), the matter content is

\[
\begin{align*}
P_F & \sim (\square, 1; \square, -2, 1 - \frac{6}{F}) \\
S & \sim (1, \square^*; 1, 0, \frac{1}{3}) \\
Q & \sim (1, \square; 1, -2F, \frac{7}{6}) \\
\bar{Q}_A & \sim (\square, \square^*; 1, 0, \frac{5}{6}) \\
\bar{Q}_B & \sim (1, \square^*; 1, 2F, -\frac{25}{6}) \\
P & \sim (\square, 1; 1, 2F, 0) \\
N & \sim (1, 1; 1, 0, 5) \\
u & \sim (1, 1; 1, -4F, 10).
\end{align*}
\]

(16)

The superpotential of the “expanded” theory is

\[ W = \bar{Q}_A^2 S + u\bar{Q}_B^2 S + PQ_A Q + N\bar{Q}_B Q + S^6. \]

(17)

Under duality, the following \( SO(10) \) gauge covariant operators are identified

\[ \Psi_{[1]} \leftrightarrow P \quad \text{and} \quad \Psi_{[5]} \leftrightarrow \bar{Q}_A \bar{Q}_B, \]

(18)

where the subscript \([n]\) denotes a rank-\(n\) anti-symmetric tensor.

B appendix

The magnetic theory of \( Spin(10) \) theory with matter in \( F \) vector and \( N \) spinor representations is summarized. As in the appendix A, the irrelevant parameters are set to one.

The theory is based on an \([SU(\hat{N}) \times Sp(2N - 2)]_{\text{local}} \times [SU(F) \times SU(2) \times U(1)_Y \times U(1)_R]_{\text{global}}\) symmetry group. Here \( \hat{N} = F + 2N - 7 \). Note that the \( SU(N) \) flavor symmetry of the electric theory \([\square]\) is realized as an \( SU(2) \) symmetry in this theory. The matter content of the theory is as follows,

\[
S \sim (\square \square, 1; 1, 1, \frac{4F(N - 1)}{N}, 2 - \frac{2F\xi}{N}).
\]

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The superpotential of the magnetic theory is

\[ W = M\bar{Q}_F^2 S + \bar{Q}_A^2 S + N\bar{Q}_F Q_f + \bar{Q}_A Q_f R. \]  

(20)

C appendix

This appendix provides another consistency checks for the superpotential (9). In section 3.1, it is derived as a result of \( SU(7) \to SU(5) \) gauge breaking as well as the \( Sp(2) \) instanton effect.

Now let us first take a step backward and consider the \( SU(7) \times Sp(2) \) gauge theory with superpotential \( W^{(1)}_{2,2} \). Because the \( Sp(2) \) group has 14 flavors, \( \bar{Q}^{a,\alpha}_1 \) and \( \bar{Q}^{a,\alpha}_2 \) \((a = 1, \ldots, 7)\), it can be dualized by \( Sp \) duality \[4\]. The new theory has an \([SU(7) \times Sp(8)]_\text{local} \times SU(2)_\text{global}\) group and extra dual quarks and mesons transforming under the symmetry group as

\[
\begin{align*}
\bar{S} &\sim (\Box^*, 1; 1) \\
\bar{A} &\sim (\Box^*, 1; 3) \\
\tilde{Q} &\sim (\Box, \Box; 2),
\end{align*}
\]

(21)

where \( SO(9) \) gauge and \( SU(F - 1) \) flavor symmetries are suppressed.

The meson \( \tilde{S}^{ab} \) becomes massive because of its coupling to field \( S^{ab} \) (see \[8\]). The superpotential of this \( SU(7) \times Sp(8) \) theory is

\[
\tilde{W}^{(1)}_{2,2} = \sum_{a,b=1}^{7} \left\{ S^{8a}(Q_{3}^{b}A_{1}^{ba} - Q_{1}^{b}A_{1}^{ba}) + S^{88}(Q_{3}^{a}Q_{3}^{b} - Q_{3}^{a}Q_{3}^{b})\tilde{A}^{ab}_{2} + \sum_{i=1,3} P_{i}Q_{i}^{a}Q_{i}^{a} + N_{i}Q_{i}^{a}Q_{i}^{a} \\
+ \sum_{\alpha,\beta=1}^{8} J_{\alpha\beta} \left[ (\tilde{Q}^{a,\alpha}_{1}\tilde{Q}^{b}_{1} + u\tilde{Q}^{a}_{1}\tilde{Q}^{b}_{1})(\tilde{Q}^{a,\alpha}_{2}\tilde{Q}^{b}_{2} - \tilde{Q}^{a,\alpha}_{2}\tilde{Q}^{b}_{2}) + \tilde{A}^{ab}_{1}\tilde{Q}^{a,\alpha}_{1}\tilde{Q}^{b}_{1} + \tilde{A}^{ab}_{2}(\tilde{Q}^{a,\alpha}_{2}\tilde{Q}^{b}_{2} + \tilde{Q}^{a,\alpha}_{2}\tilde{Q}^{b}_{2}) + \tilde{A}^{ab}_{3}\tilde{Q}^{a,\alpha}_{2}\tilde{Q}^{b}_{2} \right] \right\} + S^{88}. \]
\]

(22)
Now, the vanishing F-flatness condition $\partial \tilde{W}_{2,2}^{(1)}/\partial S^{88}$ of this superpotential implies a non-vanishing $(Q_1^a Q_2^a A_2^a)$ expectation value. This paper takes $(Q_1^a)(Q_2^a)(A_2^a)$ and concludes that the theory is broken to an $SU(5) \times Sp(8)$ symmetry in which $Sp(8)$ gauge group has 10 flavors, $\tilde{Q}_1^{a,\alpha}$ and $\tilde{Q}_2^{a,\alpha}$ $(a = 1, \cdots, 5)$. Therefore, this $Sp(8)$ theory confines and generates a constraint equation on the quantum moduli space, i.e. $Pf \tilde{Q}^{a,\alpha} = 1$. The result after including this quantum effect is an $SU(5)$ gauge theory with the superpotential

$$\tilde{W}_{2,2}^{(2)} = \frac{5}{a,b=1} \left\{ S^{ab} \tilde{Q}^a_A \tilde{Q}^b_A + u S^{ab} \tilde{Q}^a_B \tilde{Q}^b_B + S^{ab} \tilde{q}^a \tilde{q}^b + S^{ab} \tilde{A}^{6a} \tilde{A}^{7a} + \lambda (1 - \det S^{ab}) \right\},$$

(23)

where $\tilde{q}^a \equiv \tilde{A}^{7a} = \tilde{A}^{6a}$, $\lambda$ is the Lagrangian multiplier, and $\det S^{ab} \equiv Pf \tilde{Q}^{a,\alpha}$. Note that this superpotential agrees with (3).

The result of (2) can also be obtained by first dualizing the $Sp(2)$ gauge group of (3).

**D appendix**

We briefly report the result when the “expanded” theory (3) is perturbed by $p_j$ $(1 \leq j \leq N - 2)$. The breaking pattern of gauge group is as follows, $SU(2N + 4) \times Sp(2N - 2) \xrightarrow{(1)} SU(2N + 3) \times Sp(2N - 2) \xrightarrow{(2)} SU(2N + 1) \times Sp(2N - 4) \xrightarrow{(3)} \cdots \xrightarrow{(j)} SU(2(N - j) + 5) \times Sp(2(N - j)) \xrightarrow{(j+1)} SU(2(N - j) + 4) \times Sp(2(N - j) - 2)$, with superpotential takes the same form at breaking stages (2) to (j). At the $(j + 1)$th-stage, it is an $SU(2(N - j) + 4) \times Sp(2(N - j) - 2)$ gauge theory with superpotential

$$W_{N,j}^{(j+1)} = \sum_{a,b=1}^{2(N-j)+4} \left\{ S^{ab} \tilde{Q}^a_A \tilde{Q}^b_A + u S^{ab} \tilde{Q}^a_B \tilde{Q}^b_B + S^{ab} \tilde{q}^a \tilde{q}^b + \sum_{i=2j+1}^{2N-1} \left[ P_i Q_i^a \tilde{Q}^a_i + N_i Q_i^a \tilde{Q}^b_i + h_i Q^a_i \tilde{q}^a \right] \\
+ \sum_{\alpha,\beta=1}^{2(N-j)-2} J_{\alpha\beta} \left[ S^{ab} (Q_1^{a,\alpha} Q_2^{b,\beta} - Q_2^{a,\alpha} Q_2^{b,\beta}) + \sum_{i=2j+1}^{2N-1} Q_i^a (Q_1^{a,\alpha} R_i^\beta + Q_2^{a,\alpha} R_i^{\alpha \beta}) \right] \\
+ \sum_{i=2j+1}^{2N-1} \left[ R_i^{(2(N-j)-1)} Q_i^a \tilde{q}^a + R_i^{(2(N-j))} Q_i^a \tilde{q}^a_{2(N-j)-1} \right] \right\},$$

(24)

where $\tilde{q}^a \equiv \tilde{Q}_1^{a,2N-2} = \tilde{Q}_2^{a,2N-3}$. By F-flatness conditions, $R_i^{(2(N-j)-1)} = J_{\alpha\beta} Q_{2j+1}^{b,\alpha} \tilde{Q}_2^{b,\alpha} R_i^{\beta}$ and $\tilde{Q}_1^{a,2(N-j)-1} = J_{\alpha\beta} Q_{2j+1}^{b,\alpha} \tilde{Q}_2^{b,\alpha} \tilde{Q}_1^{b,\alpha}$ are expressed as composite states of the $SU(2(N - j) + 4) \times Sp(2(N - j) - 2)$ gauge group. However, the $Sp(2(N - j) - 2)$ gauge group is in IR free phase, these expressions can be fulfilled only if the $Sp(2(N - j) - 2)$ group is broken to a smaller one.

Because no further Higgs mechanism occurs in (24), $R_i^{(2(N-j)-1)}$ and $\tilde{Q}_1^{a,2(N-j)-1}$ vanish. We thus conclude $W_{N,j}^{(j+1)} = W_{N,j,0}$, where “0” subscript denotes the elimination of $p_j$ in (3).
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