Time-varying volatility in Bitcoin market and information flow at minute-level frequency

Irena Barjašić · Nino Antulov-Fantulin

Received: date / Accepted: date

Abstract In this paper, we analyze the time-series of minute price returns on the Bitcoin market through the statistical models of generalized autoregressive conditional heteroskedasticity (GARCH) family. Several mathematical models have been proposed in finance, to model the dynamics of price returns, each of them introducing a different perspective on the problem, but none without shortcomings. We combine an approach that uses historical values of returns and their volatilities - GARCH family of models, with a so-called "Mixture of Distribution Hypothesis", which states that the dynamics of price returns are governed by the information flow about the market. Using time-series of Bitcoin-related tweets and volume of transactions as external information, we test for improvement in volatility prediction of several GARCH model variants on a minute level Bitcoin price time series. Statistical tests show that the simplest GARCH(1,1) reacts the best to the addition of external signal to model volatility process on out-of-sample data.

Keywords Bitcoin · Volatility · Econometrics

1 Introduction

The first mathematical description of the evolution of price changes in a market dates back to 1900, when, in his seminal work, Bachelier [1] derived an equation (later rediscovered as Brownian motion, or random walk model) to theoretically ground the fact that price increments are uncorrelated and have a stationary mean. These properties emerged from the condition that any information from the previous prices will quickly be noticed by traders, who will then trade in a way that removes it. Nevertheless, as the distribution of the increments was the normal distribution, a very well known stylized fact of the logarithmic price returns, its
leptokurtosis, was unaccounted for. To target that issue, Mandelbrot [2] suggested to model price changes by generalizing the price increments from the Gaussian distribution to a Lévy stable distribution with the stable parameter between 0 and 2. It successfully reproduced the observed fat-tailedness of the price change distribution, although leaving the moments of the distribution (including the variance) infinite. To avoid working with infinite variances, truncated Lévy processes were introduced [3].

An opposing hypothesis (later named “Mixture of Distribution Hypothesis”) was introduced by Clark [4], where the non-normality of price returns distribution is assigned to the varying rate of price series evolution during different time intervals. The process that is driving the rate of price evolution is proposed to be the information flow available to the traders. Due to the governing of the information flow, the number of summed price changes per observed time interval varies substantially, and the central limit theorem cannot be applied to obtain the distribution of price changes. Nevertheless, a generalization of the theorem provides a Gaussian limit distribution conditional on the random variable directing the number of changes. Therefore, if we model the price evolution with a process \(X(t)\) that is normally distributed and consists of prices equidistantly distributed in time, and the speed of price evolution with a process \(T(t)\) that is a positive stochastic process, a new process \(X(T(t))\) represents then the actual price formation. The new process \(X(T(t))\) is said to be directed by \(T(t)\), and subordinated to \(X(t)\). As a result, the direction process \(T(t)\) turns the normal distribution of \(\Delta X(t)\) into a leptokurtic distribution of \(\Delta X(T(t))\).

In a different approach, the autoregressive conditional heteroskedasticity (ARCH) [5] model, originally introduced by Engle, describes the heteroscedastic behavior (time-varying volatility) of logarithmic price returns relying only on the information of previous price movements. In addition to the previous values of price returns, its generalized variant GARCH [6] introduces previous conditional variances as well when calculating the present conditional variance. GARCH is thus able to account for volatility clustering and for the leptokurtic distribution of price returns, both the stylized statistical properties of returns. An alternative view comes from GARCH-Jump model [7], which assumes that the news process can be represented as \(\epsilon_t = \epsilon_{1,t} + \epsilon_{2,t}\), a superposition of a normal component \(\epsilon_{1,t} = \sigma_t z_t\) and a jump-like Poisson component with intensity \(\lambda_t\). The constant intensity was generalized to autoregressive conditional jump intensity \(\lambda_t = f(\lambda_{t-1})\) in [8]. Contrary to other studies about news jump dynamics and impact on daily returns [8,9], we will model the volatility and external signals on a minute-level granularity. On this time-scale, our external signals are not modeled with Poisson-like dynamics, but added directly as an exogenous observable variable \(I_{t-1}\) to form GARCHX model.

In this paper, we compare price volatility predictions of GARCH(1,1) with those of GARCHX(1,1) to explore how information is absorbed into the emerging cryptocurrency market of Bitcoin. The Bitcoin [10] is a cryptocurrency system operated through the peer-to-peer network nodes, with a public distributed ledger, called blockchain [11]. Similar to the foreign exchange markets, Bitcoin markets [12,13] allow the exchange to fiat currencies and back. Different studies on Bitcoin quantify the price formation [14,15], bubbles [16,17], volatility [18,19], systems dynamics [20,21,22] and economic value [23,24,25]. Although various studies [26,27,28,29] have used social signals from social media, WWW, search queries, sentiment, comments, and replies on forums, there still exists a gap in understanding Bitcoin volatility process through the autoregressive conditional heteroskedasticity models and exogenous signals. In this work we will focus on the statistical quantification of the “Mixture of Distribution Hypothesis” [30] for the Bitcoin market on a minute level time-scale.
2 Methods

“Mixture of Distribution Hypothesis” [4], models the non-normality of price returns distribution with the varying rate of price series evolution due to the different information flow during different time intervals. Practically, Clark [4] hypothesizes that this can be observed as a linear relationship between the proxy for the information flow $I(t)$ and the price change variance $[\Delta X(T(t))]^2$, and suggests trading volume $V(t)$ as the proxy. Tauchen and Pitts [31] state a bivariate normal mixture model which conditions the price returns and trading volume on the information flow as:

$$r_t = \sigma_1 \sqrt{I_t} z_1 t, \quad z_1 t \in N(0,1).$$

$$v_t = \mu_2 I_t + \sigma_2 \sqrt{I_t} z_2 t, \quad z_2 t \in N(0,1).$$

The relationship between price variance and trading volume immediately follows:

$$\text{Cov}(r_t^2, v_t) = \sigma_1 \mu_2 \text{Var}(I_t),$$

and the stochastic term in (2) shows that the above proposed linear relationship is only an approximation. The aforementioned GARCH(1,1) model [6] conditions the volatility on its previous value and the previous value of price returns:

$$r_t = \mu_t + \varepsilon_t, \quad \varepsilon_t = \sigma_t z_t, \quad z_t \in N(0,1).$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2.$$ 

Large $\alpha$ coefficient indicates that the volatility reacts intensely to market movements, while large $\beta$ shows that the impact of large volatilities slowly dies out. The volatilities defined by the model display volatility clustering and the respective distribution of price returns is leptokurtic, which agrees with the observations in the real data.

The central part of our study will be the volatility and external signals interdependence on a minute-level granularity. For that purpose, we formed a GARCHX model by adding the proxy for the information flow $I_{t-1}$ directly to the GARCH volatility equation.

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma I_{t-1}.$$ 

We will compare price volatility predictions of GARCH(1,1) with those of GARCHX(1,1) to explore how information is absorbed into the emerging cryptocurrency market of Bitcoin.

3 Results

We used minute-level Bitcoin prices from the Bitfinex exchange and calculated logarithmic returns as a natural logarithm of two consecutive prices. The period we observed spans from April 18th, 2019 to May 30th, 2019, with 58,000 observations in total, 50,000 observations as in-sample and 8,000 as out-of-sample. In Table 1 we can see the descriptive statistics of logarithmic returns; the mean of the returns is very close to zero ($8 \cdot 10^{-6}$), with a standard deviation of $9.41 \cdot 10^{-4}$, the distribution is negatively skewed and leptokurtic.

Two different datasets for external signals were available as the external information proxy - a time series of tweets mentioning cryptocurrency-related news on a second level [32], and a time series of Bitcoin trade volumes from Bitfinex also on a second level, shown on Fig. 1a and Fig. 1b. Both time series were aggregated to the minute level.
|          |          |          |          |          |
|----------|----------|----------|----------|----------|
| Mean     | 0.000008 | Median   | 0.0      | Maximum  | 0.026982 |
|          |          |          |          | Minimum  | -0.019309|
| Std.dev. | 0.000941 | Skewness | -0.44813 | Kurtosis | 41.286157|

Table 1: Descriptive statistics of logarithmic returns for Bitcoin market.

Correlation between squared price returns and volume was calculated for different time lags of the volume time series, as shown in Fig. 2. It has a peak when the volume leads the squared price returns by one minute. The significant correlation, i.e. normalized covariance between squared price returns and trading volume, indicates an approximately linear relationship between the volatility and the proxy for information flow (see Eq. 3).

In Appendix we plot the same correlation calculation for cryptocurrency-related tweets (see Fig. 4a). We do not observe a similar correlation (covariance) pattern as for volume signal. Multiple reasons could be behind this: (i) a large noise in the Twitter signal might be covering the information flow w.r.t. trading volume signal, (ii) linear dependence might not be enough to capture the relationship, or (iii) Twitter signal might not contain a sufficient information flow to influence price volatility. If noise is i.i.d., then “integrated external signal” \( \hat{I}(t) = \int_{t-\delta}^{t} I_{t} dt \) should filter the noise component. We observe that the stronger correlation pattern is present after the Twitter series is integrated with \( \delta = 30 \) minutes (see Appendix Fig. 4b), which indicates that strong noise is present in Twitter series.

Fig. 1: (a) Time-series external signal of cryptocurrency-related tweets with minute mean \( \mu = 6.77 \) and standard deviation \( \sigma = 4.15 \). (b) Time-series of trading volume on Bitfinex market for BTC-USD pair, with mean \( \mu = 7.88 \) and standard deviation \( \sigma = 18.44 \).
3.1 Transfer entropy analysis

To check the non-linear dependence argument between the squared returns and external information flow signals (volume and Twitter), we employ transfer entropy (TE), an information-theoretic measure that is both nonlinear and non-symmetric. The non-symmetry allows us to distinguish the direction of information exchange between time series, $i_n$ and $j_n$. It is defined as a Kullback-Leibler entropy that measures the deviation from the generalized Markov property $p(i_{n+1}^{(k)} | i_n^{(k)}) = p(i_{n+1}^{(l)} | i_n^{(l)}, j_n^{(l)})$, where it is abbreviated $i_n^{(k)} = (i_n, ..., i_{n-k+1})$:

$$T_{J\to I} = \sum p(j_{n+1}, i_n^{(k)}, j_n^{(l)}) \log \frac{p(i_{n+1}^{(k)} | i_n^{(k)})}{p(i_{n+1}^{(l)} | i_n^{(l)}, j_n^{(l)})}. \quad (7)$$

In Table 2, we present the results for transfer entropy from external variables to squared returns time series and conversely. Markov process orders $l$ and $k$ are taken to be 1. Results show that values are significant, with the largest one being the transfer entropy from squared returns to trading volume. The statistical significance (p-value) of transfer entropy was estimated by a bootstrap method of the underlying Markov
To account for the finite sample size, we use the effective transfer entropy (ETE) measure:

$$ETE_{J \rightarrow I} = T_{J \rightarrow I} - \frac{1}{M} \sum_{m=1}^{M} T_{J(m) \rightarrow I},$$

(8)

where $J(m)$ is the $m$-th shuffled series of $J$. We observe stronger information transfer from the volume signal to squared return than from the Twitter signal to squared returns. At this point, we conclude that both external signals show significant dependence towards the proxy for volatility signal i.e. squared returns.

|             | tweet count | trading volume |
|-------------|-------------|----------------|
| $v_t \rightarrow r_t^2$ | TE 0.002242* | 0.01125*       |
|             | ETE 0.002091 | 0.01113        |
| $r_t^2 \rightarrow v_t$ | TE 0.002197* | 0.02601*       |
|             | ETE 0.002058 | 0.02586        |

Table 2: Transfer entropy and effective transfer entropy between external signals and squared returns.
* Represents p-value smaller than 0.001

3.2 Volatility GARCHX process analysis

Next, we turn our attention to the statistical quantification of the GARCH volatility processes. Apart from expanding GARCH(1,1) to GARCHX(1,1), we add the exogenous variable to models eGARCH(1,1), cGARCH(1,1) and TGARCH(1,1) as well, to check for improvement in volatility predictions. The conditional variance equations corresponding to these models (see Table 3) are extensions of Eq. 5. eGARCH [36] and TGARCH [37] capture the asymmetry between positive and negative shocks, giving greater weight to the later ones, with the difference between them being the multiplicative and the additive contribution of historical values, and cGARCH [38] separates long and short-run volatility components.

|             | eGARCH     | cGARCH     |
|-------------|------------|------------|
| $\ln(\sigma_t^2) = \omega + \alpha \left[ \epsilon_{t-1}^2 / \sigma_{t-1}^2 \right] - E \left[ \epsilon_{t-1}^2 / \sigma_{t-1}^2 \right] + \beta \ln(\sigma_{t-1}^2)$ | $\sigma_t^2 = q_t + \alpha (\epsilon_{t-1}^2 - q_{t-1}) + \beta (\sigma_{t-1}^2 - q_{t-1})$ |
|             | $q_t = \omega + \rho q_{t-1} + \theta (\epsilon_{t-1}^2 - \sigma_{t-1}^2)$ | $\sigma_t = \omega + \alpha \epsilon_{t-1} + \beta q_{t-1} + \phi \epsilon_{t-1}^3 [\epsilon_{t-1} < 0]$ |

Table 3: GARCH family
One way to get the intuition how good the GARCH volatility models are at explaining the volatility is to regress \( a \cdot \sigma_t^2 + b \) on squared returns \( r_t^2 \), where \( \sigma_t^2 \) is the squared GARCH volatility estimate. Then, we measure the coefficient of determination \( R^2 \) i.e. the proportion of the variance in the dependent variable that is predictable from the independent variable. We determine the statistical significance of \( R^2 \) with the F-test. Additionally, we measure the Pearson correlation coefficient (PCC) of estimated \( \sigma_t^2 \) and squared returns \( r_t^2 \), along with its statistical significance, Table 4.

|                 | GARCH | eGARCH | cGARCH | TGARCH |
|----------------|-------|--------|--------|--------|
| \( R^2 \)      | 0.1467| 0.1412 | 0.1529 | 0.1581 |
| F-statistic    | 1375* | 1315*  | 1444*  | 1502*  |
| PCC            | 0.383*| 0.3758*| 0.3911*| 0.3976*|

Table 4: Out-of-sample measures for the GARCH volatility process. In-sample consists of 50,000 points, out-of-sample consists of 8000 points. * Represents p-value smaller than 0.001.

However, for a more precise statistical quantification of the difference between models and their GARCHX variants more advanced statistical tests are needed. For that purpose, we employ predictive negative log-likelihood (NLLH) \ ([40] \). 

\[
\tilde{\mathcal{L}} = -\ln(\mathcal{L}(\mu_1, ..., \mu_n, \sigma_1, ..., \sigma_n)) = -\sum_{i=1}^{n} \left( \frac{1}{2} \ln(\sigma_i) + \frac{1}{2} \ln(2\pi) - \frac{(r_i - \mu_i)^2}{2\sigma_i^2} \right) .
\] (9)

We evaluated predictive negative log-likelihood (NLLH) on the out-of-sample period. Values of \( \{\mu_i\}_{i=1}^{n} \) and \( \{\sigma_i\}_{i=1}^{n} \) are predictions of the model, and \( \{x_i\}_{i=1}^{n} \) are observed price returns. To show whether the improvements can be considered significant, we employed the likelihood ratio test. It takes the natural logarithm of the ratio of two log-likelihoods as the statistic:

\[
LR = -2 \ln \left( \frac{\mathcal{L}(\theta_0)}{\mathcal{L}(\theta)} \right) .
\] (10)

Since its asymptotic distribution is \( \chi^2 \)-distribution, a p-value is obtained using Pearson’s chi-squared test. In Table 5, we see from the p-values that the exogenous variables improve the NLLH significantly for all the models except for eGARCH.

Note, that for two models with fixed parameters, the likelihood ratio test is the most powerful test at given significance level \( \alpha \), by Neyman–Pearson lemma.

In order to further test the robustness of the conclusions on different samples, we perform the bootstrapping. We restrict the lengths of in-sample and out-of-sample to \( T = 1000 \) points each and bootstrap \( N = 100 \) such segments from the original time series. Then, for each segment we fit a model on its in-sample data segment and calculate predictive out-of-sample NLLH \( \{\tilde{\mathcal{L}}_i\}_{i=1}^{N} \). Model \( M_i \) represents a model from GARCH family \{GARCH, cGARCH, eGARCH, TGARCH\} and \( M_{i,j} \) denotes its corresponding GARCHX extension, where external signal \( j \in \{\text{Volume, Twitter}\} \). Models \( M_i \) and \( M_{i,j} \) will have empirical distribution functions \( \psi_{M_i}(\tilde{\mathcal{L}}) \) and \( \psi_{M_{i,j}}(\tilde{\mathcal{L}}) \), respectively (see boxplots estimates in Fig. 3). We calculate the Kolmogorov-Smirnov (KS) statistics between corresponding empirical predictive out-of-sample NLLH distributions:

\[
KS_{i,j} = \sup_{\tilde{\mathcal{L}}} |\psi_{M_i}(\tilde{\mathcal{L}}) - \psi_{M_{i,j}}(\tilde{\mathcal{L}})| .
\] (11)
Fig. 3: Bootstrap robustness check over $N = 100$ splitting points with $T = 1000$ training points and $T = 1000$ test size for GARCH and GARCHX models. The non-parametric Kolmogorov–Smirnov test of the equality of the NLLH out-of-sample distributions between the GARCH and GARCHX models is done. a) KS test implies a significant difference for both external signals for the GARCH model. b) KS test implies no significant difference for external signals for eGARCH model. c) KS test implies no significant difference for both external signals for the cGARCH model. d) KS test implies no significant difference for external signals for the TGARCH model.


Table 5: Out-of-sample measures for predictive negative log-likelihood and likelihood ratio test. In-sample consists of 50,000 points, out-of-sample consists of 8,000 points.

*Represents the p-value smaller than 0.001. NA - some algorithms had convergence problems.

and obtain its statistical significance. Only the GARCH model shows significant improvements with both external variables, under the bootstrapping KS-NLLH robustness check. That is not surprising, as the non-parametric KS test is not very powerful [41]. However, significant differences for the GARCH model allow us to confirm that the "Mixture of Distribution Hypothesis" is robust under temporal bootstrapping conditions. Finally, we take the GARCH volatility process as representative and perform additional bootstrapping KS-NLLH robustness checks on two additional segments (March – April 2019 and November – December 2019) and we see similar results (See Fig. 5, Appendix).

4 Conclusion

Although the theoretical foundations of the effects of information on markets have been proposed a long time ago [1][2], they were further developed in 1970, as "weak", "semi-strong", and "strong" forms of efficient market hypothesis [42]. The mathematical models of information effects continued to advance in the 70s as well, by the proposition of the Mixture of Distribution Hypothesis [4], which states that the dynamics of price returns are governed by the information flow available to the traders. Following the growth of computerized systems and the availability of empirical data in the 80s, more elaborate statistical models were proposed, such as generalized autoregressive conditional heteroscedasticity models (GARCH) [6] and news Poisson-jump processes [7] with constant intensity. Furthermore, studies from the 2000s generalized the news Poisson-jump processes by introducing time-varying jump effects, supporting it with the statistical evidence of time variation in the jump size distribution [8][9].

In this paper, we have analyzed the effects of information flow on the cryptocurrency Bitcoin exchange market that appeared with the introduction of blockchain technology in 2008 [11], using the GARCH volatility models on minute-level. Although, in the last 10 years the trading volume in the largest cryptocurrency markets has grown exponentially, still the research on their (in)efficiency quantification is ongoing [43][44]. In this work, we have tried to quantify the "Mixture of Distribution Hypothesis" (MDH), by using trading volume and Twitter signal. We have focused on the Bitcoin, the largest cryptocurrency w.r.t. market capitalization. We have used the reliable data of price returns and traded volume from Bitfinex exchange market [45] on a minute-level granularity. Another reason, why we have concentrated on the Bitcoin, was the availability of Twitter-related data [32]. We have used the trading volume and social media signals from Twitter as a proxy for information flow together with the GARCH family of [46] processes to quantify the price volatility and MDH.
We have made extensive experiments on the following models: GARCH, eGARCH, cGARCH, and TGARCH on the minute level data of price returns, Twitter volume, and exchange volume data. Our testing procedure consisted of multi-stage statistical checks: (i) out-of-sample $R^2$ and Pearson correlation measurements, (ii) out-of-sample predictive likelihood measurements with the likelihood ratio test on 8000 points, (iii) bootstrapped predictive likelihood measurements with the non-parametric Kolmogorov-Smirnov test. Additionally, we have employed recently developed non-parametric information-theoretic transfer entropy measures [33,35,34], to confirm the nonlinear relationship between the exogenous proxy for information (trading volume and cryptocurrency related tweets) and squared price returns (proxy for volatility). From the predictive perspective, of the non-linear parametric GARCH model, we have found that exogenous proxy for information flow significantly improves out-of-sample minute volatility predictions for the GARCH model. It is not surprising that the basic GARCH model is outperforming more advanced models [47,39] such as eGARCH [36], TGARCH [37], and cGARCH [48] on out-of-sample data. Note, that the previous study [18], found that the cGARCH model on the Bitcoin market was performing the best on in-sample daily returns.

Finally, we have taken the GARCH model and applied the bootstrapping on two additional segments (March – April 2019 with 38 000 points and November – December 2019 with 52 000 points) and we observe that our observations still hold (see Appendix, Fig. 5.). These observations give support to the mixture of distribution hypothesis [4,30] on the Bitcoin market and thus a better explanation of the time-varying volatility process.

5 Appendix

![Fig. 4](image) (a) Correlation between squared price returns and Twitter volume. Permutation significance check indicates no statistically significant correlation between time-permutated squared price returns and Twitter time series. (b) Correlation between squared price returns and integrated Twitter volume (over a 30-minute moving window). This test is only used to check whether the integrating operator is filtering noise. Correlation between squared price returns and Twitter time series. All values of correlation are statistically significant (p-value $\leq 0.001$).
Time-varying volatility in Bitcoin market and information flow at minute-level frequency

Fig. 5: Bootstrap robustness check over $N = 100$ splitting points with $T = 1000$ training points and $T = 1000$ points in test size for GARCH and GARCHX models. The non-parametric Kolmogorov–Smirnov test of the equality of the NLLH out-of-sample distributions between GARCH and GARCHX models is done. a) KS test implies a significant difference for all external signals for the GARCH model in the period from November 3rd 2019 to December 9th 2019 with 52 000 observations. b) KS test implies a significant difference for all external signals for the GARCH model in the period from March 18th 2019 to April 9th 2019 with 38 000 observations.

References

1. L. Bachelier, *Louis Bachelier’s theory of speculation: the origins of modern finance*. Princeton University Press, 2011.
2. B. Mandelbrot, “The variation of certain speculative prices,” *The Journal of Business*, vol. 36, no. 4, pp. 394–419, 1963.
3. H. Stanley and R. Mantegna, *An introduction to econophysics*. Cambridge University Press, Cambridge, 2000.
4. P. K. Clark, “A subordinated stochastic process model with finite variance for speculative prices,” *Econometrica*, vol. 41, no. 1, p. 135, Jan. 1973.
5. R. F. Engle, “Autoregressive conditional heteroskedasticity with estimates of the variance of united kingdom inflation,” *Econometrica: Journal of the Econometric Society*, pp. 987–1007, 1982.
6. T. Bollerslev, “Generalized autoregressive conditional heteroskedasticity,” *Journal of econometrics*, vol. 31, no. 3, pp. 307–327, 1986.
7. P. Jorion, “On jump processes in the foreign exchange and stock markets,” *The Review of Financial Studies*, vol. 1, no. 4, pp. 427–445, 1988.
8. W. H. Chan and J. M. Maheu, “Conditional jump dynamics in stock market returns,” *Journal of Business & Economic Statistics*, vol. 20, no. 3, pp. 377–389, 2002.
9. J. M. Maheu and T. H. McCurdy, “News arrival, jump dynamics, and volatility components for individual stock returns,” *The Journal of Finance*, vol. 59, no. 2, pp. 755–793, 2004.
10. D. Chuen, *Handbook of Digital Currency: Bitcoin, Innovation, Financial Instruments, and Big Data*. Academic Press, 2015.
11. S. Nakamoto, “Bitcoin: A peer-to-peer electronic cash system,” 2008.
12. N. Gandal, J. Hamrick, T. Moore, and T. Oberman, “Price manipulation in the bitcoin ecosystem,” 2017.
13. P. Ciaian, M. Rajcaniova, and d. Kancs, “The economics of bitcoin price formation,” *Applied Economics*, vol. 48, pp. 1799–1815, 2016.
14. E.-T. Cheah and J. Fry, “Speculative bubbles in bitcoin markets? an empirical investigation into the fundamental value of bitcoin,” *Economics Letters*, vol. 130, pp. 32–36, 2015.
15. L. a. Kristoufek, “What are the main drivers of the bitcoin price? evidence from wavelet coherence analysis,” *PLoS one*, vol. 10, no. 4, p. e0139293, 2015.
16. J. Donier and J.-P. Bouclier, “Why do markets crash? bitcoin data offers unprecedented insights,” *PLOS ONE*, vol. 10, pp. 1–11, 2015.
17. S. Wheatley, D. Sorinett, T. Huber, M. Reppen, and R. N. Gantner, “Are bitcoin bubbles predictable? combining a generalized metcalfe’s law and the lpps model,” 2018.
18. F. Katsiampa, “Volatility estimation for bitcoin: A comparison of GARCH models,” *Economics Letters*, vol. 158, pp. 3–6, 2017.
19. T. Guo, A. Bifet, and N. Antulov-Fantulin, “Bitcoin volatility forecasting with a glimpse into buy and sell orders,” in *2018 IEEE International Conference on Data Mining (ICDM)*, Nov 2018, pp. 989–994.
20. D. Ron and A. Shamir, “Quantitative analysis of the full bitcoin transaction graph,” in *International Conference on Financial Cryptography and Data Security*. Springer, 2013, pp. 6–24.
21. A. ElBahrawy, L. Alessandretti, A. Kandler, R. Pastor-Satorras, and A. Baronchelli, “Evolutionary dynamics of the cryptocurrency market,” *Royal Society Open Science*, vol. 4, no. 11, p. 170623, 2017.
22. N. Antulov-Fantulin, D. Tolic, M. Piskorec, Z. Ce, and I. Vodenska, “Inferring short-term volatility indicators from the bitcoin blockchain,” in *Complex Networks and Their Applications VII*. Cham: Springer International Publishing, 2019, pp. 508–520.
23. A. Hayes, “Cryptocurrency value formation: An empirical analysis leading to a cost of production model for valuing bitcoin.” *SSRN Electronic Journal*, 2015.
24. W. Bolt, “On the value of virtual currencies.” *SSRN Electronic Journal*, 2016.
25. S. Nadarajah and J. Chu, “On the inefficiency of bitcoin,” *Economics Letters*, vol. 150, pp. 6–9, 2017.
26. L. Kristoufek, “Bitcoin meets google trends and wikipedia: Quantifying the relationship between phenomena of the internet era,” *Scientific reports*, vol. 3, p. 3415, 2013.
27. T. R. Li, A. S. Chamrajnagar, X. R. Fong, N. R. Riazi, and F. Fu, “Sentiment-based prediction of alternative cryptocurrency price fluctuations using gradient boosting tree model,” *arXiv preprint arXiv:1805.00558*, 2018.
28. Y. B. Kim, J. G. Kim, W. Kim, J. H. Im, T. H. Kim, S. J. Kang, and C. H. Kim, “Predicting fluctuations in cryptocurrency transactions based on user comments and replies,” *PLoS one*, vol. 11, 2016.
29. D. Garcia and F. Schweitzer, “Social signals and algorithmic trading of bitcoin,” *Royal Society Open Science*, vol. 2, no. 9, p. 150288, 2015.
30. T. G. Andersen, “Return volatility and trading volume: An information flow interpretation of stochastic volatility,” *The Journal of Finance*, vol. 51, no. 1, pp. 169–204, 1996.
31. G. E. Tauchen and M. Pitts, “The price variability-volume relationship on speculative markets,” *Econometrica*, vol. 51, no. 2, pp. 485–505, 1983.
32. J. Beck, R. Huang, D. Lindner, T. Guo, Z. Ce, D. Helbing, and N. Antulov-Fantulin, “Sensing social media signals for cryptocurrency news,” in *Companion Proceedings of The 2019 World Wide Web Conference*, 2019, pp. 1051–1054.
33. T. Schreiber, “Measuring information transfer,” *Physical Review Letters*, vol. 85, no. 2, p. 461–464, Jul 2000.
34. T. Dimpfl and F. J. Peter, “Using transfer entropy to measure information flows between financial markets,” *Studies in Nonlinear Dynamics and Econometrics*, vol. 17, no. 1, pp. 85–102, 2013.
35. R. Marschinski and H. Kantz, “Analysing the information flow between financial time series,” *The European Physical Journal B-Condensed Matter and Complex Systems*, vol. 30, no. 2, pp. 275–281, 2002.
36. D. Nelson, “Conditional heteroskedasticity in asset returns: A new approach,” *Econometrica*, vol. 59, no. 2, pp. 347–70, 1991.
37. J.-M. Zakoian, “Threshold heteroskedastic models,” *Journal of Economic Dynamics and control*, vol. 18, no. 5, pp. 931–955, 1994.
38. G. Lee and R. Engle, “A permanent and transitory component model of stock return volatility,” *Cointegration, Causality and Forecasting: A Festschrift in Honor of Clive W.J. Granger*, pp. 475–497, 1999.
39. T. G. Andersen and T. Bollerslev, “Answering the skeptics: Yes, standard volatility models do provide accurate forecasts,” *International economic review*, pp. 885–905, 1998.
40. Y. Wu, J. M. Hernández-Lobato, and Z. Ghahramani, “Gaussian process volatility model,” in *NIPS*, 2014, pp. 1044–1052.
41. M. Marozzi, “Nonparametric simultaneous tests for location and scale testing: A comparison of several methods,” *Communications in Statistics - Simulation and Computation*, vol. 42, no. 6, pp. 1298–1317, Jul 2013.
42. E. F. Fama, “Efficient capital markets: A review of theory and empirical work,” *The Journal of Finance*, vol. 25, no. 2, p. 383, May 1970.
43. V. L. Tran and T. Leirvik, “Efficiency in the markets of cryptocurrency,” *Finance Research Letters*, p. 101382, Nov. 2019.
44. L. Kristoufek and M. Vosvrda, “Cryptocurrencies market efficiency ranking: Not so straightforward,” *Physica A: Statistical Mechanics and its Applications*, vol. 531, p. 120855, Oct. 2019.
45. M. Hougan, H. Kim, M. Lerner, and B. A. Management, “Economic and non-economic trading in bitcoin: Exploring the real spot market for the world’s first digital commodity,” *Bitwise Asset Management*, 2019.
46. R. Engle, “New frontiers for arch models,” *Journal of Applied Econometrics*, vol. 17, no. 5, pp. 425–446, 2002.
47. G. Jafari, A. Bahraminasab, and P. Norouzzadeh, “Why does the standard garch (1, 1) model work well?” *International Journal of Modern Physics C*, vol. 18, no. 07, pp. 1223–1230, 2007.
48. R. F. Engle and M. E. Sokalska, “Forecasting intraday volatility in the us equity market. multiplicative component garch,” *Journal of Financial Econometrics*, vol. 10, no. 1, pp. 54–83, 2012.