On the PQCD prediction for the pion form factor

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Abstract
We comment on the results of a complete leading-twist next-to-leading order QCD analysis of the spacelike pion electromagnetic form factor at large-momentum transfer $Q$. For the asymptotic distribution amplitude, we have examined the sensitivity of the predictions to the choice of the renormalization scale. The results show that, regarding the size of the radiative corrections, reliable perturbative predictions for the pion electromagnetic form factor can already be made at a momentum transfer $Q$ of the order of 5 to 10 GeV.

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ON THE PQCD PREDICTION FOR THE PION FORM FACTOR

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We comment on the results of a complete leading-twist next-to-leading order QCD analysis of the spacelike pion electromagnetic form factor at large-momentum transfer $Q$. For the asymptotic distribution amplitude, we have examined the sensitivity of the predictions to the choice of the renormalization scale. The results show that, regarding the size of the radiative corrections, reliable perturbative predictions for the pion electromagnetic form factor can already be made at a momentum transfer $Q$ of the order of 5 to 10 GeV.

1 Introduction

The study of exclusive processes at large-momentum transfer represents a challenging area for application of perturbative QCD (PQCD). Although the PQCD approach undoubtedly represents an adequate and efficient tool for analyzing exclusive processes at very large momentum transfer, its applicability to these processes at experimentally accessible momentum transfer has been long debated and attracted much attention. In a moderate energy region (a few GeV) soft contributions could still be substantial. Further on, the self-consistency of the PQCD approach was questioned regarding the non-factorizing end-point contributions. It has been shown, that the incorporation of the Sudakov suppression in the so-called modified hard-scattering approach (mHSA) effectively eliminates soft contributions from the end-point regions and that the PQCD approach to the pion form factor begins to be self-consistent for a momentum transfer of about $Q^2 > 4$ GeV$^2$. However, in the PQCD approach to exclusive processes one still has to check its self-consistency by studying radiative corrections. It is well known that, unlike in QED, the leading-order (LO) predictions in PQCD do not have much predictive power, and that higher-order corrections are important. They have a stabilizing effect reducing the dependence of the predictions on the schemes and scales.

In our recent papers we have clarified some discrepancies between previous calculations and by including the complete closed form for the NLO evolution of the pion distribution amplitude (DA) derived recently we have obtained the complete NLO PQCD prediction for the pion electromagnetic form factor (within the so-called standard hard-scattering approach (sHSA).
and using different candidate DAs). The size of the NLO correction as well as the size of the expansion parameter, i.e. QCD running coupling constant, serve as sensible measures of the self-consistency of PQCD prediction. But, as the truncation of the perturbative expansion for the pion form factor at finite order causes the residual dependence of the prediction on the choice of the renormalization and factorization scales (as well as on the renormalization scheme), the choices for these scales represent the major ambiguity in the interpretation of the results.

In this paper we would like to outline our calculation and to address the scale ambiguity problem.

2 Pion electromagnetic form factor in the sHSA

In leading twist, the pion electromagnetic form factor can be expressed by a convolution formula

$$F_\pi(Q^2) = \int_0^1 dx \int_0^1 dy \ \Phi^*(y, \mu_F^2) \ T_H(x, y, Q^2, \mu_R^2, \mu_F^2) \ \Phi(x, \mu_F^2). \quad (1)$$

Here $Q^2 = -q^2$ is the momentum transfer in the process and is supposed to be large, $\mu_R$ is the renormalization scale, and $\mu_F$ is the factorization scale at which soft and hard physics factorize; $x$ and $y$ ($\overline{x} = 1 - x$ and $\overline{y} = 1 - y$) denote incoming and outgoing quark (antiquark) momentum fractions.

The (process dependent) hard-scattering amplitude $T_H(x, y, Q^2, \mu_R^2, \mu_F^2)$ is calculated in perturbation theory and represented as a series in the QCD running coupling constant $\alpha_S(\mu_R^2)$. We have used the dimensional regularization method and the $\overline{MS}$ renormalization scheme in our calculation.

The (process independent) pion DA $\Phi$ is intrinsically nonperturbative quantity, whose evolution can be calculated perturbatively and represented as a series in $\alpha_S(\mu_F^2)$. There are compelling theoretical results which disfavor the end-point concentrated distributions, and one expects that the pion DA does not differ much from the asymptotic form. In this work we comment only on the results obtained with the asymptotic distribution $\phi_{as}(x, \mu_F^2)$.

Generally, one can express the NLO form factor as

$$F_\pi(Q^2, \mu_R^2, \mu_F^2) = F_\pi^{(0)}(Q^2, \mu_R^2, \mu_F^2) + F_\pi^{(1)}(Q^2, \mu_R^2, \mu_F^2). \quad (2)$$

The first term in (2) is the LO contribution, while the second term is the NLO contribution coming from the NLO correction to the hard-scattering amplitude as well as arising from the inclusion of the NLO evolution of the DA. For the results obtained using $\phi_{as}(x, \mu_F^2)$ distribution, the effect of the NLO evolution of the DA is negligible ($\approx 1\%$).
Truncation of the perturbative series of $F_\pi(Q^2)$ at any finite order causes a residual dependence on the scheme as well as on the scales (which is explicitly denoted in Eq. (2)). As we approximate $F_\pi(Q^2)$ only by two terms of the perturbative series, we hope that we can minimize higher-order corrections by a suitable choice of $\mu_R$ and $\mu_F$, so that the LO term $F_\pi^{(0)}(Q^2, \mu_R^2, \mu_F^2)$ gives a good approximation to the complete sum $F_\pi(Q^2)$.

We take that a PQCD prediction for pion form factor can be considered reliable provided the corrections to the LO prediction are reasonably small ($< 30\%$) and the expansion parameter (effective QCD coupling constant) is acceptably small ($\alpha_S(\mu_R^2) < 0.3$ or 0.5). The consistency with the experimental data is not of much use here since reliable experimental data for the pion form factor exist for $Q^2 \leq 4$ GeV$^2$ i.e., outside the region in which the perturbative treatment based on Eq. (1) is justified. It should also be mentioned that there are controversial arguments regarding the reliability of existing experimental data. The new data in this energy region are expected from the CEBAF experiment E-93-021.

3 Examining the scale dependence of the NLO prediction

The simplest and widely used choice for the $\mu_R$ and $\mu_F$ scales is

$$\mu_R^2 = \mu_F^2 = Q^2, \quad (3)$$

the justification for the use of which is mainly pragmatic. The prediction for the pion form factor depends very weakly on the choice of the factorization scale $\mu_F$. Actually, taking $\mu_F^2$ to be an effective constant, i.e., $\mu_F^2 = \langle \mu_F^2 \rangle$, the only $\mu_R^2$ dependence of the results obtained using $\phi_{as}(x, \mu_F^2)$ distribution comes from the NLO evolution of the DA which is negligible.

Physically, a more appropriate choice for $\mu_R^2$ would be that corresponding to the characteristic virtualities of the particles in the parton subprocess (which are considerably lower than the overall momentum transfer $Q^2$ i.e., virtuality of the probing photon)

$$\mu_R^2 = a(x, y) \cdot Q^2. \quad (4)$$

For example, some of the physically motivated choices are

$$a(x, y) \in \{x y, \sqrt{x y y}, e^{-5/3} x y\}. \quad (5)$$

These correspond, respectively, to the (LO) gluon virtuality, geometrical mean of the gluon and quark virtualities (an attempt to take into account that the QCD coupling is renormalized not only by the vector particle propagator, but also by the quark-gluon vertex and the quark-propagator), and to the choice of
the renormalization scale according to the Brodsky-Lepage-Mackenzie (BLM) procedure\textsuperscript{12} the essence of which is that all vacuum-polarization effects from the QCD $\beta$ function should be resummed into the running coupling constant (the NLO coefficient of $T_H$ becomes $n_f$ (i.e. $\beta_0$) independent).

A glance at Eq. (1), where the coupling constant $\alpha_S(\mu_R^2)$ appears under the integral sign, reveals that the choice (2) leads immediately to the problem if the usual one-loop formula for the effective QCD running coupling constant is employed. Namely, regardless of how large $Q^2$ is, the integrations over $x$ and $y$ allow $\alpha_S(\mu_R^2)$ to be evaluated at low momenta i.e., in the region where usual one-loop formula is not a good representation of the effective QCD coupling. There are number of proposals\textsuperscript{13},\textsuperscript{14} for the form of the coupling constant $\alpha_S(\mu_R^2)$ for small $\mu_R^2$, but its implementation in this calculation demands the more refined treatment. Alternatively, one can choose $\mu_R^2$ to be an effective constant

$$\mu_R^2 = \langle \mu_R^2 \rangle = \langle a(x, y) \rangle Q^2 = a Q^2. \quad (6)$$

Hence, the expressions (2) get replaced by their respective averages

$$a \in \{ \langle x \rangle^2, \langle x \rangle^{3/2}, e^{-5/3} \langle x \rangle^2 \}. \quad (7)$$

Now, the key quantity in the above expressions is $\langle x \rangle$, the average value of the momentum fraction. Owing to the fact that $\phi_{a,s}(x, \mu^2)$ is centered around the value $x = 0.5$, the simplest choice is $\langle x \rangle = 0.5$.  Void of the renormalization scale ambiguity.

Figure 1: NLO prediction for $Q^2 F_\pi(Q^2)$ obtained using the $\phi_{a,s}(x, \mu^2)$ distribution amplitude and the choices of $\mu_R^2$ given by Eqs. (3), (6), and (7), while $\mu_F^2 = Q^2$ and $\langle x \rangle = 0.5$.

The shaded area denotes the theoretical uncertainty introduced by the renormalization scale ambiguity.
Figure 2: The ratio $F^{(1)}_{\pi}(Q^2)/F^{(0)}_{\pi}(Q^2)$ obtained using the same DA and the same choices for $\mu^2_R$ and $\mu^2_F$ scales as in Fig. 1. The shaded area denotes the region of predictions which corresponds to $|F^{(1)}_{\pi}(Q^2)/F^{(0)}_{\pi}(Q^2)| < 30\%$, and $\alpha_S(\mu^2_R) < 0.5$ ($\alpha_S(\mu^2_R) < 0.3$).

Numerical results of our complete NLO QCD calculation for $Q^2 F_{\pi}(Q^2)$, obtained using the $\phi_{as}(x, \mu^2_F)$ distribution, with $\mu^2_F = Q^2$ and different choices for the renormalization scale $\mu^2_R$ given by (3) (6), (7), and $\langle x \rangle = 1/2$, are displayed in Fig. 1 (in our calculation we take $\Lambda_{\overline{MS}} = 0.2$ and $f_\pi = 0.131$ GeV). The ratio of the NLO to the LO contribution to $F_{\pi}(Q^2)$, i.e., $F^{(1)}_{\pi}(Q^2)/F^{(0)}_{\pi}(Q^2)$, as a useful measure of the importance of the NLO corrections, is plotted as a function of $Q^2$ in Fig. 2.

One notices that the total NLO perturbative prediction for $Q^2 F_{\pi}(Q^2)$ is somewhat below the trend indicated by the presently available experimental data, but what alarms us and seems to question the self-consistency of the PQCD approach is the fact that the ratio $F^{(1)}_{\pi}/F^{(0)}_{\pi}$ corresponding to the often encountered choice $\mu^2_R = Q^2$ (solid line) is rather high: $F^{(1)}_{\pi}/F^{(0)}_{\pi} \leq 30\%$ is not reached until $Q^2 \approx 500$ GeV$^2$. The answer to this problem lies in the previously stated inappropriateness of the choice $\mu^2_R = Q^2$. Namely, owing to the partitioning of the overall momentum transfer $Q^2$ among the particles in the parton subprocess, the essential virtualities of the particles are smaller than $Q^2$, so that the “physical” renormalization scale, better suited for the process of interest, is inevitably lower than $Q^2$. It follows from Fig. 2 that by choosing the renormalization scale determined by the dynamics of the pion rescattering process, the size of the NLO corrections is significantly reduced and reliable predictions are obtained at considerably lower values of $Q^2$, namely, for $Q^2 < 100$ GeV$^2$. 
The total NLO prediction for $Q^2 F_\pi(Q^2)$ depends weakly on the choice of $\mu_R^2$. This is a reflection of the stabilizing effect that the inclusion of the NLO corrections has on the LO predictions. Let us explore this point more closely.

By taking $\mu_R^2$ and $\mu_F^2$ to be effective constants, the LO and NLO contributions to the pion form factor obtained using $\phi_\alpha(x, \mu_F^2)$ distribution amount to

$$Q^2 F^{(0)}_{\pi}(Q^2, \mu_R^2, \mu_F^2) = 8 \pi f^2_{\pi} \alpha_S(\mu_R^2), \quad (8)$$

$$Q^2 F^{(1)}_{\pi}(Q^2, \mu_R^2, \mu_F^2) = 8 f^2_{\pi} \frac{\alpha_S^2(\mu_R^2)}{4} \left(\beta_0 \left(\ln \frac{\mu_R^2}{Q^2} + \frac{14}{3}\right) - 3.92\right), \quad (9)$$

where the NLO contribution coming from the NLO evolution of the DA is neglected. As it is seen from (8), all of the $\mu_R$ dependence of the LO result $Q^2 F^{(0)}_{\pi}$ is contained in the strong coupling constant $\alpha_S(\mu_R^2)$. Thus, as $\mu_R$ decreases the LO result increases, and it increases without bound. In contrast to the LO, the NLO contribution $Q^2 F^{(1)}_{\pi}$, as evident from the explicit expression given in (9), decreases (becomes more negative) with decreasing $\mu_R$. Upon adding up the LO and NLO contributions, we find that the full NLO result, as a function of $\mu_R$, stabilizes and reaches a maximum value for $\mu_R^2 = \mu^2_{extreme} \approx Q^2/18$. If we take that the renormalization scale continuously changes in the interval defined by $a \in [1/18, 1]$ the curves representing the NLO predictions for $Q^2 F_{\pi}(Q^2)$ fill out the shaded region in Fig. 1, and this shaded region essentially determines the scale ambiguity related theoretical uncertainty of the NLO calculation.

4 Towards resolving the renormalization scale ambiguity problem

The optimization of the scale and scheme choice according to some sensible criteria remains an important task for the application of PQCD. Several scale-setting procedures were proposed in the literature: the principle of fastest apparent convergence (FAC) $^1$ ($F^{(1)}_{\pi}(Q^2, \mu^2_{FA}) = 0$), the principle of minimal sensitivity (PMS) $^2$ ($\mu^2_{PMS} = \mu^2_{extreme}$), and the BLM method. The application of those methods can give strikingly different results in some calculations.$^3$

As it is known, the relations between physical observables must be independent of renormalization scale and scheme conventions to any fixed order of perturbation theory. It was argued$^4$ that applying the BLM scale-fixing to perturbative predictions of two observables in, for example, $\overline{MS}$ scheme and then algebraically eliminating $\alpha_{\overline{MS}}$ one can relate any perturbatively calculable observables without scale and scheme ambiguity, where the choice of BLM scale ensures that the resulting “commensurate scale relation” (CSR) is independent of the choice of the intermediate renormalization scheme. Following this approach, in paper by Brodsky et al.$^5$ the exclusive hadronic amplitudes
were calculated in $\alpha_V$ scheme, in which the effective coupling $\alpha_V(\mu^2)$ is defined from the heavy-quark potential $V(\mu^2)$.

It follows from (9) that the $\beta_0$ dependent term vanishes for $\mu_R^2 = \langle \mu_{BLM}^2 \rangle = \mu_{BLM}^2 = e^{-14/3}Q^2$. Using the scale-fixed relation between $\alpha_V(\mu_R^2)$ and $\alpha_{\overline{MS}}(\mu_{\overline{MS}}^2)$, and Eqs. (8-9) one obtains the NLO prediction for the pion form factor in $\alpha_V$ scheme:

$$Q^2 F_\pi(Q^2, \mu_R^2) = 8 \pi f_\pi^2 \alpha_V(\mu_R^2) \left( 1 - 1.92 \frac{\alpha_V(\mu_R^2)}{\pi} \right),$$

where $\mu_R^2 = e^{5/3} \mu_R^2 = e^{-3}Q^2 \approx Q^2/20$. Considering the energy region we are interested in, for the purpose of this calculation we approximate $\alpha_V$ with usual one-loop expression. Thus obtained numerical predictions are given in Table 1. Considering the size of the $F_\pi^{(1)}(Q^2)/F_\pi^{(0)}(Q^2)$ ratio ($\approx -20\%$) and the size of the effective coupling, we find that NLO PQCD predictions for the pion form factor obtained in $\alpha_V$ scheme can be considered reliable for $Q^2$ below 100 GeV$^2$ i.e., already for momentum transfer $Q$ of the order of 5 – 9 GeV.

5 Conclusions

We have shown that, regarding the size of the radiative corrections, the sHSA can be consistently applied to the calculation of the pion electromagnetic form factor already for momentum transfer $Q$ of the order of 5 – 10 GeV. In order to improve this PQCD based prediction it is necessary to obtain and apply the proper form of the QCD coupling in the low-momentum regime as well as to investigate the corrections (present in the few GeV region) introduced by the mHSA. Reliable experimental data and inclusion of the soft contributions should then enable complete confrontation between theory and experiment.

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Table 1: NLO PQCD results for the pion form factor, $Q^2 F_\pi(Q^2)$ in the $\alpha_V$ scheme.

| $Q^2$ [GeV$^2$] | $\alpha_V(\mu_R^2)$ | $F_\pi^{(1)}(Q^2)/F_\pi^{(0)}(Q^2)$ [%] | $Q^2 F_\pi(Q^2)$ [GeV$^2$] |
|----------------|---------------------|----------------------------------|------------------|
| 20             | 0.434               | -26.5                            | 0.138            |
| 100            | 0.289               | -17.7                            | 0.103            |
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