Decoherence effects on the generation of exciton entangled states in coupled quantum dots

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Abstract

We report on exciton-acoustic-phonon coupling effects on the generation of exciton maximally entangled states in \( N = 2 \) and \( N = 3 \) quantum dot systems. In particular, we address the question of the combined effect of laser pulses, appropriate for generating Bell and Greenberger-Horne-Zeilinger entangled states, together with decoherence mechanisms as provided by a phonon reservoir. By solving numerically the master equation for the optically driven exciton-phonon kinetics, we show that the generation of maximally entangled exciton states is preserved over a reasonable parameter window.

PACS: 71.10.Li, 71.35.-y, 73.20.Dx

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Confined excitons together with ultrafast optical spectroscopy have been shown to be important elements for achieving coherent wavefunction control on the nanometer and femtosecond scales in semiconductors [1]. Maximally entangled states (MES), of Bell-type for excitons in two coupled quantum dots (QDs) and Greenberger-Horne-Zeilinger (GHZ) type for three coupled QDs, have been reported as excellent candidates for achieving quantum entanglement in solid-state based devices [2]. However, the question arises as to how reliable the MES preparation scheme of Ref. [2] will be when decoherence mechanisms are taken into account during the generation step. Exciton decoherence in semiconductor QDs is dominated by acoustic phonon scattering at low temperatures [3].

In this work we present results on the kinetics of the generation of exciton MES in QDs, taking into account an acoustic phonon dephasing mechanism. The Hamiltonian describing a system formed by $N$ QDs in the rotating wave approximation is

$$H(t) = \Delta \omega J_z - V(J^2 - J_z^2) - A(J^+ + J^-) + \sum_{\vec{k}} \omega_{\vec{k}} a_{\vec{k}}^\dagger a_{\vec{k}} + \sum_{\vec{k}} g_{\vec{k}} J_z(a_{\vec{k}}^\dagger + a_{\vec{k}}) \tag{1}$$

where $J_+ = \sum_{n=1}^{N} e_n^\dagger h_n^\dagger$, $J_- = \sum_{n=1}^{N} h_n e_n$ and $J_z = \frac{1}{2} \sum_{n=1}^{N} (e_n^\dagger e_n - h_n h_n^\dagger)$ with $e_n^\dagger (h_n^\dagger)$ describing the electron (hole) creation operator in the $n$’th QD. The collective operators describing the QD excitons, $J$-operators, satisfy the usual angular momentum commutation relationships: $[J_+, J_-] = 2J_z$, $[J_\pm, J_z] = \mp J_\pm$. $\Delta \omega = \epsilon - \omega$ is the resonance detuning, $\epsilon$ denotes the semiconductor energy gap, $\omega$ is the laser central frequency, $V$ the Forster term representing the Coulomb interdot interaction, $A$ the laser pulse amplitude and $a_{\vec{k}}^\dagger (a_{\vec{k}})$ the creation (annihilation) operator of the acoustic phonon with wavevector $\vec{k}$. We put $\hbar = 1$ throughout this paper. We work within corresponding optically active exciton states, i.e. $J = 1$ and $J = 3/2$ for two and three coupled quantum dots, respectively. Mixing with dark exciton states can be induced by exciting selectively a single QD or by a different coupling with the local environment of each QD. These latter effects will not be considered here.
The time evolution from any initial state under the action of $H$ in Eq. (1) is easily performed by means of the pseudo 1/2-spin operator formalism \[2, 4\]. The exact kinetic equations for this system can be obtained by applying the method of operator-equation hierarchy developed for Dicke systems in \[5\]. As a test, we verified that in the limit of zero laser intensity and no Förster term our results coincide with those in \[6\] where two-state systems coupled to a dephasing environment were considered. Until now the experimental identification and quantification of exciton decoherence mechanisms in low dimensional semiconductor heterostructures is rather scarce. As a consequence we adopt here a simplified model. In a standard way, by assuming a very short correlation time for exciton operators, the exact hierarchy of equations transforms into a Markovian master equation. The initial condition is represented by the density matrix $\rho(0) = |0\rangle\langle 0|\rho_{Ph}(T)$, exciton vacuum and the equilibrium phonon reservoir at temperature $T$. At resonance, \(i.e.\) $\Delta\omega = 0$, the dynamical equation for the expectation value of exciton operators is then given by

$$\frac{\partial \langle J_{r-s}^\alpha \rangle}{\partial t} = -iV \langle [J_{r-s}^\alpha, J_z^2] \rangle - iA \langle [J_{r-s}^\alpha, J^+ + J^-] \rangle - \Gamma(2\langle [J_{r-s}^\alpha, J_z] J_z \rangle - \langle [J_{r-s}^\alpha, J_z^2] \rangle)$$

where the decoherence rate is $\Gamma = \int d\omega' \omega'^n e^{-\omega'}/\omega_c(1 + 2N(\omega', T))$ with $n$ depending on the dimensionality of the phonon field, $\omega_c$ is a cut-off frequency (typically the Debye frequency) and $N(\omega', T)$ is the phonon Bose-Einstein occupation factor. We do not attempt here to perform a microscopic calculation of $\Gamma$ but instead we take it as a variable parameter. We consider pure decoherence effects that do not involve energy relaxation of excitons, as indicated by the last term in Eq. (1).

It is a well known fact that very narrow linewidth of the photoluminescence signal of a single QD does exist due to the elimination of inhomogeneous broadening effects. Consequently, the decoherence rate $\Gamma$ in our calculations should be associated with just homogeneous broadening effects. At low temperature the main decoherence mechanism is indeed acoustic phonon scattering pro-
cesses. The decoherence parameter $\Gamma$ is temperature dependent and it amounts for 20-50 $\mu\text{eV}$ for typical III-V semiconductor QDs in a temperature range from 10 K to 30 K \cite{3}. We solve numerically the coupled differential linear equations for the time dependent pseudo-spin expectation values (8 for Bell states and 15 for GHZ states). For $\Gamma$ we take typical values which can represent real situations for QDs at low temperatures. Other common parameters for the results shown below are: resonance condition $\epsilon = \omega = 1$ and Forster term $V = \epsilon/10$. Laser strengths and decoherence rates are to be expressed in units of $V$. As a quantitative measure of the successful generation of exciton MES we present our results in terms of the time dependent overlaps $O_B(t) = Tr\{\rho_{\text{Bell}}\rho(t)\}$ and $O_G(t) = Tr\{\rho_{\text{GHZ}}\rho(t)\}$ where $\rho_{\text{Bell}} = (1 + J_0^{0-1} - J_1^{1-2})/3 - J_0^{0-2}$ and $\rho_{\text{GHZ}} = 1/4 + (J_0^{0-1} - J_2^{2-3})/2 - J_0^{0-3}$ (we use the same notation as in \cite{2}). $|0\rangle$ represents the exciton vacuum, $|1\rangle$ denotes a single-exciton state, $|2\rangle$ represents the biexciton state and $|3\rangle$ is the triexciton state.

In order to appreciate the importance of the non-linear Forster term to generate exciton MES we present in Fig. 1 the evolution of the overlaps $O_B(t)$ and $O_G(t)$ in the limit of very weak light excitation and zero decoherence \cite{2}. It is worth noting that no exciton MES generation is possible if the Forster interaction is turned off. This implies that efficient exciton MES generation should be helped by compact QD systems where the Forster term can take a significant value.

Next, we discuss the $N = 2$ case and Bell-state generation in presence of noise. In Fig. 2a results are shown for a decoherence rate $\Gamma = 0.001$ and different laser intensities ($A = 0.1$ and $A = 0.4$). Bell-state generation time is significantly shortened by applying stronger laser pulses. Therefore, decoherence effects can be minimized by using higher excitation levels. However, a higher laser intensity also implies a sharper evolution which therefore requires a very precise pulse length. In Fig. 3a Bell-state generation is shown for different values of the decoherence parameter ($\Gamma = 0.001, 0.01$ and 0.1). It is evident that at high temperature $\Gamma = 0.1$ no MES generation is possible. However, we estimate that $\Gamma$ values between 0.001 – 0.01 are typical in the
temperature range from 10 K to 50 K. We conclude that a parameter window exists where successful generation of Bell MES can be produced.

Now we address the GHZ MES generation in a $\mathcal{N} = 3$ QD system. As for the Bell case, using higher laser excitation levels it is possible to obtain in shorter times a total overlap with the GHZ density matrix as depicted in Fig. 2b ($\Gamma = 0.001$). Temperature effects through the variation of $\Gamma$ are depicted in Fig. 3b ($\nu = 0.4$). It is evident that similar decoherence rates yield a more dramatic reduction of the MES coherence in the GHZ case than in the Bell case. However, as for Bell generation, a parameter window does exist where the generation of such entangled states can be feasible.

It is worth noting the different scaling behaviour of the generation frequency of these MES at very low temperature, i.e. vanishing $\Gamma$ and very low laser excitation. While selective $\pi/2$ laser pulse length for the Bell case scales like $V/A^2$, selective $\pi/2$ pulse length for the GHZ case scales like $V^2/A^3$. This property of $\pi/2$ pulses to generate exciton MES was demonstrated in an analytical way in [2] and can be verified in our numerical results by looking at Fig. 2a and Fig. 2b.

In summary, we have shown that decoherence effects can be minimized in the generation of maximally entangled states by applying stronger laser pulses and working at low temperatures where acoustic phonon scattering is the main decoherence mechanism.

This work has been partially supported by COLCIENCIAS.
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Figure Captions

Figure 1: Exciton MES generation in the zero decoherence limit. Thick lines represents the Bell-state overlap with $A = 0.1$: solid, Forster term included; dotted, Forster term not included. Thin lines represent the GHZ-state overlap with $A = 0.2$ and similar meaning for solid and dotted lines.

Figure 2: Exciton MES generation in the presence of decoherence (a) $\langle O_B(t) \rangle$ for $A = 0.1$, dotted line and $A = 0.4$, solid line. (b) $\langle O_G(t) \rangle$ for $A = 0.2$, dotted line and $A = 0.4$, solid line. $\Gamma = 0.001$.

Figure 3: Exciton MES generation in the presence of decoherence (a) $\langle O_B(t) \rangle$ and (b) $\langle O_G(t) \rangle$. $A = 0.4$, $\Gamma = 0.001$, dotted line, $\Gamma = 0.01$, solid line and $\Gamma = 0.1$, dashed line for both (a) and (b).
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Fig. 2 a. F. J. Rodriguez et al. "Decoherence effects on the generation of exciton entangled states in coupled quantum dots"
Fig. 2 b. F. J. Rodriguez et al. "Decoherence effects on the generation of exciton entangled states in coupled quantum dots"
Fig. 3a. F. J. Rodriguez et al. “Decoherence effects on the generation of exciton entangled states in coupled quantum dots"
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