De-Sitter Type of Cosmological Model in n-Dimensional Space - Time - Mass Theory of Gravitation

G S Khadekar * and Vrishali Patki
Department of Mathematics, Nagpur University
Mahatma Jyotiba Phule Educational Campus, Amravati Road
Nagpur-440010 (INDIA)

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Abstract

Exact solution are obtained for a homogeneous spacially isotropic cosmological model in a matter free space with or without cosmological constant for a n-dimensional Kaluza-Klein type of metric in the rest mass varying theory of gravity proposed by Wesson[1983]. The behavior of the model is discussed.

Key Word: Kaluza-Klein theory, Wesson theory, Higher dimensional space time.

AMS Subject Classification : 83EXX

1 Introduction

In a past few years there have been many attempts to construct a unified field theory based on the idea of multidimensional space time. In need it is generally be-lived that higher dimensional must play a significant role in the early universe. Wesson [1983] proposed a 5-dimensional Space-Time-Mass (STM) theory of gravity with variable rest mass. In this theory 5 coordinates are $x^0 = ct$, the three space co-ordinates are $x^1 = x$, $x^2 = y$, $x^3 = z$ and $x^4 = \frac{Gm}{C^2}$, this new 5-dimensional theory of variable rest mass as a natural extension of general theory of relativity. The existence of constant $C$ suggest that $x^4 = \frac{Gm}{C^2}$ ($m = mass$) be a coordinate in 5-dimensional theory of space- time- mass.

In our present work we extended the work of Chaterjee[1987] for n-dimensional variable mass theory of gravity. We have obtained an exact solutions for a homogeneous spatially isotropic n-dimensional cosmological model in vacuume both

*Tel.91-0712-23946, email:gkhadekar@yahoo.com
with or without cosmological constant. However, it is pointed out that Chaterjee's [1987] solution is a particular case of solution presented here.

1.1 Field Equations

The line element for a n-dimensional homogeneous and spatially isotropic cosmological model is taken as

$$ds^2 = e^\nu dt^2 - e^\omega dx^2 + e^\mu dm^2$$  \hspace{1cm} (1)

where $dx^2 = \sum_{i=1}^{(n-2)} dx_i^2$ and $\mu, \omega$ and $\nu$ are the functions of time and mass. Here the coordinate $x^0 = t$, $x^{1,2,\cdots,(n-2)}$ (space coordinate) and $x^{(n-1)} = m$. For simplicity we have set the magnitudes of both $C$ and $G$ to unity. By applying this metric to the Einstein field equation $G_{ij} = R_{ij} - \frac{1}{2}g_{ij} R = \Lambda g_{ij}$ with the assumption $e^\nu = 1$, we get

$$G_{00} = -(n-2)(n-3) \frac{\ddot{\omega}}{8} - (n-2) \frac{\dot{\omega} \dot{\mu}}{4} - (n-2) e^{-\mu} \left( \frac{\dddot{\omega}}{2} - \frac{\dot{\omega} \dot{\mu}}{4} + (n-1) \frac{\dot{\omega}^2}{8} \right) = -\Lambda$$  \hspace{1cm} (2)

$$G_{11} = e^\omega \left( \frac{\dddot{\mu}}{8} + (n-3) e^\omega \left( \frac{\dddot{\omega}}{2} + \frac{\dot{\omega} \dot{\mu}}{4} + (n-2) \frac{\dot{\omega}^2}{8} \right) \right)$$

$$+ (n-3) e^{\omega-\mu} \left( \frac{\dddot{\omega}}{2} - \frac{\dot{\omega} \dot{\mu}}{4} + (n-2) \frac{\dot{\omega}^2}{8} \right) = \Lambda$$  \hspace{1cm} (3)

$$G_{11} = G_{22} = G_{33} = \cdots = G_{(n-2)(n-2)}$$

$$G_{0(n-1)} = (n-2) \left( \frac{\dddot{\omega}}{2} + \frac{\dot{\omega} \dot{\mu}}{4} - \frac{\dot{\mu} \dot{\omega}}{4} \right) = 0$$  \hspace{1cm} (4)

$$G_{(n-1)(n-1)} = -(n-2)(n-3) \frac{\ddot{\omega}^2}{8} - (n-2) e^\mu \left( \frac{\dddot{\omega}}{2} + (n-1) \frac{\dot{\omega}^2}{8} \right) = -\Lambda e^\mu$$  \hspace{1cm} (5)

where a dot(,) and star(*) denote, respectively partial derivative with respect to time and mass.

1.2 Solutions

By solving equation (4) we get

$$e^\mu = \frac{\ddot{\omega}}{\alpha(m)}$$  \hspace{1cm} (6)

where $\alpha(m)$ is an arbitrary function of mass only.

Since $\dot{\omega} \neq 0$ we get, using equation (6) in (5)

$$e^\omega \left[ \dddot{\omega} + \frac{(n-1)}{4} \dddot{\omega} - \frac{2\Lambda}{(n-2)} \right] + (n-3) \frac{\dot{\omega}^2}{4} \alpha(m) = 0$$  \hspace{1cm} (7)

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Replacing $e^\omega$ by $y$ we get

$$\ddot{y} - \frac{2\Lambda}{(n-2)}y + \frac{(n-3)}{4} \alpha = 0 \quad (8)$$

After first integration we get

$$\dot{y}^2 = \frac{8\Lambda}{(n-1)(n-2)}y^2 - \alpha y + \frac{\gamma}{y^{n-2}} \quad (9)$$

where $\gamma$ is an arbitrary function of mass only.

Different cases for the equation (9) may arise. We shall consider four of them.

**Case I**: $\Lambda = 0$, $\gamma = 0$

In this case simple solution of equation (9) is

$$e^\omega = \alpha_1 t^2 + \beta_1 t + \gamma \quad (10)$$

where $\alpha_1 = -\frac{\alpha}{4}$, $\beta_1$ and $\gamma_1$ are arbitrary functions of mass only. This result is exactly identical to those obtained by Chaterjee [1987] for 5-dimension.

**Case II**: $\Lambda = 0$, $\gamma \neq 0$

After an extremely tedious but straightforward calculation we get the general solution of equation (9) as

$$\frac{2}{\alpha N_1} \left[ \frac{-1}{(\gamma - x^2)^{\frac{N_2}{2}}} + N_2 \gamma \int \frac{dx}{(\gamma - x^2)^{N_3}} \right] = t + C_1 \quad (11)$$

where $C_1$ is an arbitrary function of mass and $x^2 = (\gamma - \alpha e^{(n-2)\omega})$, $N_1 = \frac{(n-1)}{2(n-3)}$, $N_2 = \frac{(n-5)}{2(n-3)}$, $N_3 = \frac{(3n-1)}{2(n-3)}$.

**Case III**: $\Lambda \neq 0$, $\gamma = 0$

In this case the solution of equation (9) is

$$\frac{4\Lambda}{3} e^\omega - \alpha + 2 \left( \frac{2\Lambda}{3} \right)^{\frac{1}{2}} \left[ \frac{2 \Lambda e^{2\omega}}{3} - \alpha e^\omega \right]^{\frac{1}{2}} = e^{(t+C_2)} \quad (12)$$

where $C_2$ is an arbitrary function of mass only.

This solution is formerly the same as the solution obtained by Chaterjee [1987] for 5-dimension.

**Case IV**: $\Lambda \neq 0$, $\gamma \neq 0$

From equation (9) we get the general solution

$$\frac{e^{K_1 \omega}}{K_1} - \frac{K_2 e^{K_3 \omega}}{2 \gamma K_3} + \frac{\alpha e^{K_4 \omega}}{2 \gamma K_4} = (\gamma)^{\frac{1}{2}} t + C_3 \quad (13)$$

where $K_1 = \frac{(n-1)}{4}$, $K_2 = \frac{8\Lambda}{(n-1)(n-2)}$, $K_3 = \frac{3(n+1)}{4}$, $K_4 = \frac{(3n-7)}{4}$.
1.3 Conclusion

In this paper we have considered the the n-dimensional Kaluza-Klein type metric in rest mass varying theory of gravity. The solution obtained here is more general by Chaterjee[1987] for 5-dimensional case. Chaterjee’s solution is a particular case of the solution presented here. We think that this new exact higher dimensional solution together with cosmological consideration should bring some additional information and as such they need to be further investigated. It is our hope that the higher dimensional solution presented here can be used as the starting point to investigate the behavior of the rest of the particles in more realistic universe model.

References

[1] Wesson P S (1983) Astro.Asrophys. 119, 145

[2] Chaterjee S (1987) Astron.Astrophy.179,122