Hadron yields in Au+Au/Pb+Pb at RHIC and LHC from thermalized minijets

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Abstract

We calculate the yields of a variety of hadrons for RHIC and LHC energies assuming thermodynamical equilibration of the produced minijets, and using as input results from pQCD for the energy densities at midrapidity. In the calculation of the production of partons and of transverse energy one has to account for nuclear shadowing. By using two parametrizations for the gluon shadowing one derives energy densities differing strongly in magnitude. In this publication we link those perturbatively calculated energy densities of partons via entropy conservation in an ideal fluid to the hadron multiplicities at chemical freeze-out.
1 Introduction

Particle production in high-energy heavy-ion reactions at the BNL-RHIC and CERN-LHC colliders will soon provide interesting insight into nuclear modifications of semi-hard processes \[1\]. This is because pQCD processes involving gluons in the initial state will dominate the inelastic AA cross-section at collider energies. In particular, they might lead to a better understanding of the gluon distribution in large nuclei, which is not accessible in DIS.

In \[2\] the effect of nuclear shadowing of the parton distribution functions on the charged particle multiplicity at midrapidity has been investigated assuming no rescattering between the produced minijets and the hadrons they fragment into. Here, we will take the opposite point of view and assume maximal rescattering, i.e. local thermal and chemical equilibrium of the minijets. We compute final-state hadron multiplicities of various hadron species under the assumption of entropy conservation.

In \[3\] we calculated the initial conditions at RHIC and LHC by means of pQCD above the semihard scale \(p_T = 2\) GeV to derive the number and energy densities of partons at midrapidity. In that calculation we explicitly included the shadowing effect on the parton distribution functions entering the formulas for the production of flavor \(f = g, q, \bar{q}\) in the minijet approach. We employed two different parametrizations for the shadowing effect accounting for weak and strong gluon shadowing, respectively, shown in figure \[4\]. A direct consequence of the shadowing effect is the decrease in the production of partons of given momentum \(p_T\), i.e. a decrease of transverse energy production at midrapidity (for a more detailed description of the calculation, the variables, and the hard partonic subprocesses see \[3, 4, 5, 6\]). We calculate the first \(E_T\) moment with and without shadowed pdf’s and with a cut-off function \(\epsilon(y)\) ensuring that we only count scatterings into the central rapidity region \((|y| \leq 0.5)\):

\[
\sigma_{\text{hard}} \langle E_T \rangle_{\text{hard}} = \int dE_T \frac{d\sigma}{dE_T} \langle E_T \rangle = \int dp_T^2 \ dy_1 \ dy_2 \ \sum_{ij,kl} x_1 f_i(x_1, Q^2) x_2 f_j(x_2, Q^2) \left[ \delta_{fk} \frac{d\hat{\sigma}^{ij \rightarrow kl}}{dt}(\hat{t}, \hat{u}) + \delta_{fl} \frac{d\hat{\sigma}^{ij \rightarrow kl}}{dt}(\hat{u}, \hat{t}) \right] \frac{p_T \epsilon(y)}{1 + \delta_{kl}}. \tag{1}
\]
Figure 1: Shadowed parton distributions as parametrized in [7] and in [3].

|        | no shad. | weak shad. | strong shad. |
|--------|----------|------------|--------------|
| \(\sigma^g \langle E_T \rangle\) | 26.74    | 27.25      | 17.80        |
| \(\sigma^q \langle E_T \rangle\) | 3.20     | 2.87       | 1.00         |
| \(\sigma^{\bar{q}} \langle E_T \rangle\) | 1.89     | 1.73       | 0.59         |

Table 1: First \(E_T\)-Moment in mb GeV at midrapidity (\(|y| \leq 0.5\)) for parton production at RHIC for the different shadowing parametrizations and no shadowing, respectively.

The results for the production of transverse energy by the semihard partons at RHIC for unshadowed, weakly shadowed, and strongly shadowed gluons are shown in table 1 (for the results from the different subprocesses see [3]).

By dividing the transverse energy \(\bar{E}_{T,AA}^{f}(b=0) = T_{AA}(0)\sigma_{hard}^{f} \langle E_T \rangle_{hard}\) of parton species \(f\) by the initial volume

\[
V_i = \pi R_A^2 \Delta y/p_0, \quad R_A = A^{1/3} \times 1.1 \text{ fm}
\]

we found the energy densities for the three different cases as shown in table 2. One sees that the results for the energy density with the weak shadowing are even slightly increased due to the onset of \(R_G(x, Q^2)\). We also included the contributions of the soft processes as discussed in [8]. Motivated by the factorization in QCD one can assume that the production of transverse
energy in AA collisions can be split up into a hard and a soft contribution as
\[ \tilde{E}_T(b) = T_{AA}(b) \left[ \sigma_{\text{hard}}^{pp} \langle E_T \rangle_{\text{hard}}^{pp} + \sigma_{s}^{pp} \langle E_T \rangle_{s}^{pp} \right]. \] (3)

With an energy independent value of \( \sigma_{s}^{pp} = 32 \text{ mb} \) one derives \( \sigma_{s}^{pp} \langle E_T \rangle_{s}^{pp} = 15 \text{ mb GeV} \). With \( T_{AuAu} = 29/\text{mb} \) one can derive the soft contribution (i.e. the one for \( p_T \leq 2 \text{ GeV} \)) to the energy density for RHIC as \( \varepsilon_{\text{soft}} = 33.7 \text{ GeV/fm}^3 \). If one assumes the soft contribution to be independent of the shadowing phenomenon one can derive the change in relative weight of hard to soft processes due to the shadowing of the hard contributions. For the total first \( E_T \) moment of gluons, quarks, and antiquarks one finds that \( (\sigma_{g}\text{hard} + \sigma_{q}\text{hard} + \sigma_{\bar{q}}\text{hard}) \langle E_T \rangle_{\text{hard}} \) equals 31.8 mbGeV for no, 19.4 mbGeV for strong, and 31.9 mbGeV for weak shadowing.

When comparing to the soft contributions one finds with \( \sigma_{\text{hard}} \langle E_T \rangle_{h} = (\sigma_{g} + \sigma_{q} + \sigma_{\bar{q}})\text{hard} \langle E_T \rangle_{\text{hard}} \) that the ratio of soft to hard contribution
\[ R_{sh} = \frac{\sigma_{\text{soft}} \langle E_T \rangle_{s}}{\sigma_{\text{hard}} \langle E_T \rangle_{h}} \] (4)
is \( R_{sh} = 0.47 \) for no, \( R_{sh} = 0.77 \) for strong, and \( R_{sh} = 0.47 \) for weak shadowing, respectively. This implies that at RHIC the soft component could even dominate if it were energy independent and also unaffected by the shadowing effect.

The same analysis was done for LHC and we found for the first \( E_T \) moments in the three cases the results depicted in table 3. For LHC we only calculated the contribution of the gluons that strongly dominate all

|       | no shad. | weak shad. | strong shad. |
|-------|----------|------------|--------------|
| \( \varepsilon_{g} \) | 60.0     | 61.2       | 40.0         |
| \( \varepsilon_{q} \) | 7.2      | 6.4        | 2.3          |
| \( \varepsilon_{\bar{q}} \) | 4.3      | 3.9        | 1.3          |

Table 2: Energy densities for unshadowed, weakly and strongly shadowed gluons in GeV/fm\(^3\) for Au + Au at RHIC.
|             | no shad. | weak shad. | strong shad. |
|-------------|---------|------------|-------------|
| $\sigma^g \langle E_T \rangle$ | 513.01  | 286.87     | 60.39       |

Table 3: First $E_T$-Moment in mb GeV at midrapidity $|y| \leq 0.5$ for gluon production at LHC for the different shadowing parametrizations and no shadowing, respectively.

partonic processes due to the large distribution function at small momentum fractions. The results for the energy densities for no, strong, and weak gluon shadowing, then are $\varepsilon_g = 1229.7$ GeV/fm$^3$, $\varepsilon_g = 144.8$ GeV/fm$^3$, and $\varepsilon_g = 678.6$ GeV/fm$^3$. With $T_{pp}\approx 32$/mb one can derive the energy density from the soft part and has $\varepsilon_{soft} = 35.8$ GeV/fm$^3$ which is slightly larger than at RHIC due to the larger nuclear overlap function, i.e. the larger number of effective scatterings in the Glauber picture leading to the transverse energy production. For the various shadowing parametrizations one finds for the total first $E_T$ moment $(\sigma_{hard}^g + \sigma_{hard}^q + \sigma_{hard}^{\bar{q}}) \langle E_T \rangle_{hard}$ is 513 mbGeV for no, 60 mbGeV for strong, and 287 mbGeV for weak shadowing. The relative weight

$$R_{sh} = \frac{\sigma_{soft} \langle E_T \rangle_s}{\sigma_{hard} \langle E_T \rangle_h}$$

between soft and hard contributions therefore changes and becomes $R_{sh} = 0.029$ for no, $R_{sh} = 0.25$ for strong, and $R_{sh} = 0.052$ for weak shadowing, respectively.

Therefore the soft contributions gain much more weight in this naive picture due to the strong effect of shadowing on the small-$x$ gluons (for further details of the calculation, the shadowing parametrizations, etc. see [3]).

## 2 Hadron Multiplicities at Chemical Freeze-Out

Having calculated the energy densities at midrapidity for RHIC and LHC in pQCD we connect $\varepsilon_i$ to the number of hadrons at midrapidity by assuming an ideal fluid that is characterized by entropy conservation from the quark-gluon
plasma to the hadron gas, \(dS_i/dy = dS_f/dy\) [4]. We relate the energy density of the quark-gluon plasma to its entropy density via the bag model equation of state [10]. We account for \(u\), \(d\), \(s\) quarks (with masses \(m_u = m_d = 0\), \(m_s = 150\) MeV), the antiquarks, and gluons.

The total produced entropy \(dS_i/dy\) is obtained from the entropy density at midrapidity as \(dS_i/dy = V_is_i\) with the initial volume of the central region \(V_i = \pi R_A^2\tau_i\), which is numerically \(V_i = 12.9\) fm\(^3\) for \(Au + Au\) and \(V_i = 13.4\) fm\(^3\) for \(Pb + Pb\) at \(b = 0\) with \(\tau = 0.1\) fm/c. Since we assumed an ideal fluid the total entropy is conserved throughout the expansion until freeze-out which is chosen here to happen at a temperature \(T_{FO} = 160\) MeV. For simplicity we furthermore assume vanishing chemical potentials in the central rapidity region, i.e. that all conserved currents are identically zero. If this were not true one would have to multiply by factors \(\exp(\mu_i/T)\) (in Boltzmann approximation).

The entropy density of the hadronic fluid is calculated assuming an ideal gas composed of all hadrons up to a rest-mass of 2 GeV. Their respective occupation numbers are given by Fermi-Dirac or Bose-Einstein distribution functions, respectively. Thus, \(T_{FO}\) and \(dS_f/dy = dS_i/dy\) determine the multiplicity of each hadron species uniquely [11, 12]. Feeding from post freeze-out decays of heavier resonances is also taken into account.

### 3 Results

With the model outlined above and the energy densities derived above we calculated the number of a variety of hadrons at midrapidity. We also include the multiplicities due to the soft contributions and quote the initial temperatures for a QGP of three flavors. For LHC we derived the yields shown in table 4 and for RHIC the ones in table 5. For the latter one, one can clearly see that there is no change in the hadron yield for weak shadowing and the unshadowed case, respectively, due to the almost identical energy density serving as an input for the calculation.
| LHC    | \(\pi^+ + \pi^-\) | K   | \(\phi\) | p  | \(\Lambda\) | \(\Xi\) | \(\Omega\) | \(T_i\) |
|--------|-------------------|-----|----------|----|------------|-------|---------|------|
| no shad. | 2680            | 478 | 32.1     | 91 | 58         | 9.3   | 1.4     | 881 MeV |
| weak shad. | 1720            | 306 | 20.6     | 58.3| 37.2       | 5.9   | 0.9     | 760 MeV |
| strong shad. | 538             | 95.9| 6.5      | 18.3| 11.6       | 1.9   | 0.3     | 516 MeV |
| soft contrib. | 187             | 33  | 2        | 6  | 4          | 0.7   | 0.1     |       |

Table 4: Hadron yields at freeze-out with initial conditions from pQCD for LHC. The initial temperatures are for a three flavor quark-gluon plasma with two massless quarks and \(m_s = 150\) MeV. Since the soft processes do not significantly contribute to the temperature we did not calculate \(T_i\) for the soft partons.

| RHIC   | \(\pi^+ + \pi^-\) | K   | \(\phi\) | p  | \(\Lambda\) | \(\Xi\) | \(\Omega\) | \(T_i\) |
|--------|-------------------|-----|----------|----|------------|-------|---------|------|
| no shad. | 316              | 56.3| 3.8      | 10.7| 6.8        | 1.1   | 0.2     | 433 MeV |
| weak shad. | 316              | 56.3| 3.8      | 10.7| 6.8        | 1.1   | 0.2     | 433 MeV |
| strong shad. | 217             | 38.7| 2.6      | 7.4 | 4.7        | 0.8   | 0.1     | 383 MeV |
| soft contrib. | 179             | 32  | 2        | 6  | 4          | 0.6   | 0.1     |       |

Table 5: Hadron yields at freeze-out with initial conditions from pQCD for RHIC.
4 Conclusions and outlook

We computed the rapidity densities of a variety of hadrons based on the assumption of entropy conservation of an ideal fluid. The entropy densities were derived from the energy densities which in turn were calculated by means of pQCD [3]. We find that in the limit that the minijets equilibrate locally the effect of shadowing on the hadron yield is not as large as on the pure partonic degrees of freedom. This can be seen e.g. in the ratio of energy densities between unshadowed and strongly shadowed gluons at LHC, which is about a factor of 9, while the ratio of the hadron yields is only a factor of 5. Since vanishing net baryon and strangeness densities were assumed, the relative depletion of shadowed to unshadowed gluon distribution is independent of the particle species. For RHIC, even without shadowing we only get about 300 pions since the perturbative calculation, entering via the energy densities, was performed with the cut-off $p_T = 2$ GeV. Therefore, the soft contribution constitutes a significant part of the transverse energy and therefore of the particle multiplicities [12]. E.g. in the UrQMD model the total pion yield at midrapidity is $\approx 1100$ [13] for RHIC. Therefore with an energy density of $\varepsilon_s = 33.7$ GeV/fm$^3$, which was extracted from CERN data, and assumed to be energy independent, we get 316 + 179 pions and therefore get a significantly smaller multiplicity compared to calculations in the hadronic cascade model UrQMD. The key to this discrepancies could stem from the assumed energy independence of the soft contribution. For a high energy process one can write the interaction as a current-current interaction as

$$\sigma(p_T) = H^0 \otimes f^2 \otimes f^2 + \left( \frac{1}{p_T} \right) H^1 \otimes f^3 \otimes f^2 + \mathcal{O} \left( \frac{1}{p_T^4} \right)$$

(6)

where the $f^n$ are non-perturbative IR dominated matrixelement of (lowest) twist $n$ and $H$ depicts the hard part. One should interpret the soft processes at least in part to stem from higher twist effects. For an unpolarized process the next-to-leading twist is $\tau-4$. In general one can relate the matrix elements of higher twist to the well known parton distribution functions of $\tau-2$. In [14]
the twist-4 correlation functions in a nucleus were parametrized as

\[ T_{q,g} = \lambda^2 A^{1/3} f_{i/A}(x, Q^2, A) = \lambda^2 A^{1/3} R_{q,g}(x, Q^2, A) f_{i/p}(x, Q^2) \]  

(7)

where \( \lambda^2 \sim 0.05 - 0.1 \text{GeV}^2 \). This immediately implies that the soft processes are not independent of the energy since \( x \sim 1/\sqrt{s} \) and therefore should also increase from CERN-SPS to BNL-RHIC/CERN-LHC. Equation (7) in turn implies that the soft processes also are affected by shadowing; and since the DGLAP evolution between \( Q \sim 1 \text{ GeV} \) and \( Q \sim 2 \text{ GeV} \) is rather slow, one should expect no change in the relative weight between hard and soft processes. As a result one has two competing effects: as the c.m. energy increases the soft production of transverse energy increases due to the decrease in the momentum fraction \( x \) and eq. (7). On the other hand the shadowing ratio \( R_G(x, Q^2) \) suppresses those contributions as \( x \) decreases. This topic requires a more detailed analysis elsewhere.

The interesting feature at RHIC is the dependence of the hadron yields on the onset of gluon shadowing which differs substantially in the two approaches we compared. Since for the weaker gluon shadowing given in [7] the energy density for RHIC is more or less the same as in the unshadowed case we see the same results for the particle numbers in the two cases. However, for the strong gluon shadowing we see a depletion to about 70% of the unshadowed value. The effect of shadowing on the hadron multiplicities at central rapidity is found to be weaker than on the newly produced partons.

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