The effect of an in-plane magnetic field on the interlayer transport of quasiparticles in layered superconductors

L. N. Bulaevskii, M J. Graf, and M. P. Maley
Los Alamos National Laboratory, Los Alamos, NM 87545
(received: January 29, 1999)

We consider the quasiparticle $c$-axis conductivity in highly anisotropic layered compounds in the presence of the magnetic field parallel to the layers. We show that at low temperatures the quasiparticle interlayer conductivity depends strongly on the orientation of the in-plane magnetic field if the excitation gap has nodes on the Fermi surface. Thus measurements of the angle-dependent $c$-axis (out-of-plane) magnetoresistance, as a function of the orientation of the magnetic field in the layers, provide information on the momentum dependence of the superconducting gap (or pseudogap) on the Fermi surface. Clean and highly anisotropic layered superconductors seem to be the best candidates for probing the existence and location of the nodes on the Fermi surface.

PACS: 74.25.Fy, 74.70.Kn, 74.72.-h

The symmetry of the excitation gap in the superconducting and normal state of the cuprates, the quasi-two-dimensional organic salts, and the ruthenate oxides has been the focus of theoretical and experimental studies for the last several years. Now there is consensus that most of the cuprates are not only anomalous metals that develop a pseudogap when underdoped [1], but also that they are unconventional ($d$-wave) superconductors with nodes on the Fermi surface, as demonstrated in phase-sensitive Josephson junction experiments [2,3,4]. The situation is less clear in the organic salts and the ruthenates, in spite of many reports of power laws in the temperature behavior of various transport and thermodynamic properties (see, e.g., Refs. [3,4]). Spin fluctuation models for the organics predict a $d_{xy}$ superconducting state similar to the cuprates [2,3,4]. Since phase-sensitive Josephson junction and angle-resolved photoemission spectroscopy experiments are not available for these materials, other more stringent experiments are required.

In this Letter we propose angle-dependent magnetoresistance oscillation (AMRO) experiments which directly probe the locations of the line nodes of the gap on the Fermi surface, as well as information on the location of these nodes. The conditions that are necessary to apply our method seem to be fulfilled in highly anisotropic and very clean organic salts like the quasi-two-dimensional BEDT-TTF [bis(ethylenedithio)-tetrathiafulvalene] superconductors [4], the Bi- or Tl-based cuprates [5,6,7], and the oxide superconductor Sr$_2$RuO$_4$ [8].

For perfect crystals with translational invariance we use the interlayer Hamiltonian, which describes the conservation of the in-plane momentum when electrons tunnel between the layers. In Josephson coupled superconducting layers the in-plane magnetic field penetrates almost freely into the sample, inducing the vector potential $a_z(\mathbf{r}) = \mathbf{b} \cdot \mathbf{r}$, where $\mathbf{r} = (x, y)$ is the in-plane coordinate. Thus the interlayer Hamiltonian or tunneling Hamiltonian can be written as

$$\mathcal{H}_\perp = t_\perp \sum_{n,\sigma} \int d^2r [\psi_{n+1,\sigma}^+(\mathbf{r})\psi_{n,\sigma}(\mathbf{r})e^{i\chi_n,n+1(\mathbf{r})} + h.c.] ,$$

where $\psi_{n,\sigma}$ is the annihilation operator for electrons in layer $n$ with spin $\sigma$, and $t_\perp$ is the interlayer transfer integral. We assume that $t_\perp$ is isotropic in the layers. The vector potential $a_z(\mathbf{r})$ leads to a change in the in-plane momentum of the tunneling electron, $\mathbf{k} \to \mathbf{k} + \mathbf{b}$, where $\mathbf{k} = (k_x, k_y)$ is the in-plane momentum. In the momentum representation the interlayer tunneling Hamiltonian has the form

$$\mathcal{H}_\perp(\mathbf{b}) = t_\perp \sum_{n,k,\sigma} [\psi_{n,k+\mathbf{b},\sigma}^+\psi_{n+1,k,\sigma} + h.c.] .$$

Defects in the crystal structure lead to spatial variations in $t_\perp$. The spatial average, $\langle t_\perp \rangle$, determines the coher-
tunneling part. We assume that it dominates the low-temperature c-axis transport and use in (1) and (2) \( t_\perp = |t_{\perp}| \), for details see Ref. 7. Scattering inside the layers leads to a change of the momentum while tunneling, \(|\mathbf{k}| = 1/\ell_\parallel\). Here \( \ell_\parallel \) is the effective mean-free-path for scattering of quasiparticles inside the layers. In conductivity measurements a finite voltage \( V \) is applied between neighboring layers and hence the energy of the tunneling electron changes by \( V \). In the limit \( V \to 0 \) the energy is conserved within the accuracy of the temperature \( T \). The important point is that at low temperatures only quasiparticles near the nodes of the gap on the Fermi surface can contribute to the dissipative quasiparticle c-axis transport. Let us denote the positions of the nodes on the Fermi surface by the momenta \( \mathbf{k}_g \). Then at low \( T \) and \( b \gg 1/\ell_\parallel \) the contribution of the quasiparticles near the nodes to the conductivity is significant when the change of the quasiparticle energy, \( E_{k+b} - E_k \), is small, i.e., when \( h \nu_f(\mathbf{k}_g) \cdot b \ll T \ll \Delta_0 \), see Fig. 1. Here \( \mathbf{k} = k/k \) is a unit vector, \( \mathbf{v}_f \) is the Fermi velocity, and \( \Delta_0 \) is the amplitude of the energy gap. This means that the dominant contribution to \( \sigma_q \) comes from quasiparticles in the vicinity of the nodes and contribution of a given node \( \mathbf{k}_g \) to \( \sigma_q \) is maximal for \( b \perp \mathbf{v}_f(\mathbf{k}_g) \).

In the superconducting state the Josephson current contributes to the c-axis transport. Thus to obtain only the quasiparticle contribution the Cooper pair current must be suppressed when the interlayer current is measured as a function of the orientation of the in-plane magnetic field. The suppression of the Josephson current can be achieved by applying a c-axis current significantly exceeding the Josephson critical current \( I_c \). Such a condition is easily fulfilled, considering that the in-plane magnetic field strongly diminishes the Josephson critical current, without significantly affecting the superconductivity inside the layers. Another method for observing the quasiparticle current is to perform the measurements of the c-axis conductivity in the resistive state of the intrinsic Josephson junctions \( 10 \).

We calculate the quasiparticle conductivity \( \sigma_q \) using the BCS theory for d-wave and p-wave pairing in the presence of elastic, isotropic in-plane scattering, characterized by the normal-state scattering rate \( \Gamma \ll \Delta_0 \). Here \( \Delta_0 \) is the amplitude of the superconducting spin-singlet d-wave gap \( \Delta(\mathbf{k}) = \Delta_0(T)(k^2_\perp - k^2_\parallel) \) or \( \Delta(\mathbf{k}) = 2\Delta_0(T)k_\parallel k_\parallel \) or of the spin-triplet p-wave gap along a given spin direction \( \Delta(\mathbf{k}) = \Delta_0(T)k_\parallel \) or \( \Delta(\mathbf{k}) = \Delta_0(T)k_\parallel \). We consider first a circular Fermi surface inside the layers and a cylindrical, three-dimensional open Fermi surface with \( \ell_\perp \) much smaller than any other relevant energy scale in the system. For simplicity, we neglect the effect of the Zeeman term on the in-plane Green functions \( 11 \). Although it will change the field dependence of \( \sigma_q \), it will not affect its angular dependence. The perturbation theory with respect to \( \mathcal{H}_\perp \) gives the expression for the quasiparticle conductivity in terms of the quasiparticle spectral functions \( 12 \):

\[
\sigma_q(\mathbf{b}) = \frac{e^2}{4\pi^2} \int_{-\infty}^{+\infty} d\omega \cosh^{-1} \frac{\omega}{2T} \times \int d^2 k A(\mathbf{k} + \mathbf{b}, \omega) A(\mathbf{k}, \omega),
\]

where \( \text{Im} G(\mathbf{k}, \omega) = -\pi A(\mathbf{k}, \omega) \) is the imaginary part of the Green function. Within the BCS theory we obtain

\[
A(\mathbf{k}, \omega) = \frac{1 + \xi_\perp/k_\perp}{2\pi |(\omega' - E_k)^2 + \gamma^2|} + \frac{1 - \xi_\perp/k_\perp}{2\pi |(\omega' + E_k)^2 + \gamma^2|},
\]

where \( E_k = |\xi_\perp^2 + \Delta^2(\mathbf{k})|^{1/2} \), and \( \xi_\perp \) is the quasiparticle energy in the normal state, measured from the Fermi level. The scattering rate of the quasiparticles is \( \gamma(\omega) = -2\text{Im} \Sigma(\omega) \), where \( \Sigma(\omega) \) is the diagonal part proportional to the unit matrix of the impurity self-energy, and \( \omega'(\omega) = \omega - \text{Re} \Sigma(\omega) \). The impurity self-energy has to be calculated self-consistently and is energy (temperature) dependent, even for elastic scattering, in the superconducting state \( 13 \).

In unconventional superconductors even nonmagnetic impurities lead to gapless states in the vicinity of the nodes of the gap function on the Fermi surface. At low temperatures, \( T \ll \gamma(0) \ll \Delta_0(0) \), the dominant contribution to the quasiparticle conductivity comes from these gapless states. In rather clean superconductors and at low energies \( \gamma(\omega) \) becomes \( \gamma(0) \sim |\Gamma \Delta_0(0)|^{1/2} \) in the case of strong scattering, and \( \gamma(0) \sim \Delta_0(0) \exp[-\Delta_0(0)/|\Gamma|] \) for weak scattering (Born limit). Note that the relevant phase space for scattering of quasiparticles is restricted to a fraction \( \sim \gamma(0)/\Delta_0(0) \) of phase space. At low temperatures, \( T \ll \gamma(0) \), we obtain

\[
\sigma_q(\mathbf{b}) \approx \frac{e^2}{4\pi^2} \int d^2 k A(\mathbf{k} + \mathbf{b}, 0) A(\mathbf{k}, 0).
\]

In the following, we take into account that regions near the nodes contribute mainly to the integral over \( \mathbf{k} \). We replace \( E_{k+b}^2 \approx \xi_\perp^2 + \Delta^2(\mathbf{k}) \), where \( \xi_\perp \approx \xi_\parallel + \mathbf{b} \cdot \mathbf{v}_f(\mathbf{k}) \), and parameterize \( \mathbf{k} = (\cos \varphi, \sin \varphi) \) and \( \mathbf{b} = b(\cos \theta, \sin \theta) \). For example, the \( N_g = 4 \) nodal angles for d-wave\(-y^2\) pairing are given by \( \varphi_g = (2g - 1)\pi/4 \). Finally, we find at low temperatures and at low fields a universal conductivity, similar to the in-plane transport \( 14 \),

\[
\sigma_q(b = 0) = \frac{e^2}{4\pi^2} \frac{sN_f}{h} \frac{2}{\pi \Delta_0(0)} = \frac{esJ_0}{\pi \Delta_0(0)},
\]

\[
\sigma_q(\theta) = \frac{1}{N_g} \sum_{g=1}^{N_g} \ln \left[ \frac{(1 + \alpha_g^2)^{1/2}}{\alpha_g(1 + \alpha_g^2)^{1/2}} \right],
\]

with \( \alpha_g = (\ell_{\parallel}/2\nu_f) b \cdot \mathbf{v}_f(\mathbf{k}_g) = (\ell_{\parallel}/2) \cos(\varphi_g - \theta) \), where \( \ell_{\parallel} = h \nu_f/\gamma(0) \) is the effective scattering mean-free-path, and \( J_0 = 2e^2N_f/\pi h \) is the Josephson critical current density. Here \( N_f = m/(2\pi^2\hbar^2) \) is the 2D density of states.
per spin of quasiparticles with mass \( m \). This result is readily generalized to an elliptical 2D Fermi surface by rescaling the momenta \( k_i = \sqrt{m_i/m} k'_i \), \((i = x, y)\), where \( m_i \) are the effective masses and \( m^2 = m_x m_y \). Then Eq. (6) is valid with \( \alpha_g = (\ell/b)/2 \left[ \sqrt{m_x/m_y} \sin \theta \cos \varphi_g + \sqrt{m/m_b} \sin \theta \sin \varphi_g \right] \).

The largest contribution of a given node to the conductivity comes when \( b \perp \nu_f(k) \). The conductivity is maximal for \( b \) along the nodes with periodicity \( \theta = \pi/2 \), as shown in Fig. 2. For low fields, \( (\ell/b)^2 \ll 1 \), \( \sigma_q(b) \) shows a quadratic field dependence. In the high field limit, \( (\ell/b)^2 \gg 1 \), the conductivity falls off as \( b^{-2} \) in \( b \).

In the case of \( \gamma(0) \ll T \ll \Delta_0(0) \), the quasiparticle scattering rate is energy (temperature) dependent and \( \gamma(T) \sim \Gamma(T/\Delta_0)^n \) with \( n = 1 \) in the weak scattering regime (Born limit), and \( n = -1 \) for strong scattering \([13]\). Thus for fields \( b \ll \Delta_0(0)/\nu_f \) we obtain after integrating over \( \omega \)

\[
\sigma_q(b) = \frac{\pi^2 T^2 s}{2 \pi^2 T_0} \int d^2 k \frac{\gamma(T) \cos^{-2}(E_k/2T)}{E_k^2 - E_k b - E_k^2 + 4 \gamma^2(T)}. \tag{7}
\]

In the next step, we expand \( E_{k+b} - E_k \approx (\partial E_k/\partial k) \cdot b \approx E_k^2 k \nu_f(k) \cdot b \), and take into account that at low temperatures, \( T \ll \Delta_0(0) \), only small values of \( k \) and angles near the nodes, \( \varphi_g \), contribute to the quasiparticle conductivity. Then we obtain

\[
\frac{\sigma_q(b)}{\sigma_q(0)} \approx \frac{1}{N_g} \sum_{g=1}^{N_g} \frac{1}{(1 + \alpha_g^2)^{1/2}}, \tag{8}
\]

where now \( \ell || \sim \nu_f \Delta_0/T \) in the Born limit and \( \ell || \sim \nu_f/T \Delta_0 \) in the strong scattering limit. In the high field limit, \( (\ell/b)^2 \gg 1 \), the quasiparticle -axis conductivity falls off as \( 1/b \).

These results are quite general and also valid for p-wave superconductors. In this case \( \varphi_g = 0, \pi \) or \( \varphi_g = \pi/2, 3\pi/2 \). Now \( \sigma_q(\theta) \) is minimal for fields along the nodes with periodicity \( \pi \). A p-wave state has larger oscillations than a d-wave state for the same parameter \( \ell/b \).

Angle-dependent oscillations of \( \sigma_q(\theta) \) were observed in the normal state in organic conductors \([13]\), cuprates \([8]\), and ruthenates \([8]\), and were explained in terms of Fermi surface anisotropy. In materials with an elliptical 2D Fermi surface the angular periodicity of \( \sigma_q(\theta) \) is \( \pi \). The amplitude of the oscillations is determined by the parameter \( \delta v_f b/\Gamma \), where \( \delta v_f \) is the variation of the anisotropic Fermi velocity.

The periodicity and height of the AMRO, induced by the topology of the Fermi surface in the normal state, changes when a superconducting gap or pseudogap emerges. If an orthorhombic (organic) crystal undergoes a superconducting transition with s-wave pairing, the \( \pi \)-periodicity will preserve, but \( \sigma_q \) will tend to zero as \( T \to 0 \). For a d-wave state an additional \( \pi/2 \)-periodicity emerges on cooling and \( \sigma_q \) saturates as \( T \to 0 \). In tetragonal cuprate crystals the periodicity is \( \pi/2 \), both in the superconducting and normal state \([8]\). The positions of the AMRO maxima in the normal and superconducting states may differ (as in Tl-2201 \([8]\)) or coincide. In the former case, the maxima observed in the normal state should drop and new ones should appear on cooling below \( T_c \) and further increase with increasing field \( b \). In the latter case, the positions of the AMRO maxima do not change below \( T_c \), but \( \sigma_q \) saturates on cooling at \( T \to 0 \) for d-wave superconductivity, while for s-wave pairing \( \sigma_q \) vanishes exponentially. The main parameter which determines the angular behavior of \( \sigma_q(\theta) \) at low temperatures, \( T \ll T_c \), is \( \ell || b \). We note that \( \gamma \gg \Gamma \), while \( \delta v_f \) can be of order \( \nu_f \). Thus, we anticipate that the amplitude of oscillations will be about the same above and below \( T_c \) in tetragonal crystals and weaker in orthorhombic crystals.

Using typical values for the interlayer spacing, \( s \approx 15 \text{ Å} \), in organic and cuprate superconductors, e.g., \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8 \) \((\text{Bi}-2212)\) crystals, we obtain \( b \approx 4.56 B (T \mu m)^{-1} \). Then at low temperatures a mean-free-path of \( \ell \gg 200 \text{ Å} \) is needed for a d-wave state in order to observe significant oscillations, i.e., oscillations that are larger than 1% of the total magnetoresistivity for magnetic fields of order 10 T. It is feasible that this condition is fulfilled in the organic BEDT-TTF superconductors, where \( \ell \approx 1000 \text{ Å} \) was deduced from de Haas-van Alphen and Shubnikov-de Haas measurements \([16]\). In the high-temperature superconductors Bi-2201 and Bi-2212 the mean-free-path, \( \ell || b \), of quasiparticles at low temperatures is sufficiently long \([16]\) to observe this effect, too.

Until now we have discussed the dependence of the quasiparticle conductivity on the orientation of \( b \). More generally, the dependence of \( \sigma_q(b) \) on \( b \), described by Eq. (8), provides information on the spatial correlations in the system. Namely, if \( \sigma_q(b) \) is obtained experimentally then the spatial dependence of the spectral density of the Green function may be extracted by an inverse Fourier transformation with respect to \( b \),

\[
(1/4T) \int_{-\infty}^{\infty} d\omega \cosh^{-2} \frac{\omega}{2T} A(r, \omega) A(-r, \omega) \approx \left( \pi^2 h/e^2 \right) \int d^2 b e^{2b} r \sigma_q(b). \tag{9}
\]

In the low temperature regime the left-hand side is simply \( A(r, 0) A(-r, 0) = A^2(r, 0) \) and the coordinate dependence of the spectral function at zero energy may be calculated directly from measurements of \( \sigma_q(b) \). A similar method was used previously to obtain information on the correlation function of pancake vortices in a vortex liquid from the dependence of the plasma resonance frequency or from the dependence of \( \sigma_q(b) \) on the magnetic field parallel to the layers with a fixed out-of-plane component \([17,18]\). For superconductors, in the
The absence of a perpendicular magnetic field, the coordinate dependence of $A(r,0)$ provides information on the superconducting correlation length. At low temperatures in highly anisotropic layered metals in the normal state the combination of parallel and perpendicular magnetic field components leads to oscillations of the c-axis conductivity as a function of $b$ or as a function of the angle between the magnetic field and the c-axis [19, 20].

The coordinate dependence of the spectral density of the Green function at arbitrary energy may be extracted from c-axis I-V measurements in the presence of an in-plane magnetic field. The c-axis current density is

$$J_q(b, V) = \frac{e t^2 s}{2\pi^3 h} \int_{-\infty}^{+\infty} d\omega \left[ \frac{\tanh(\omega + eV/2T)}{2T} - \frac{\tanh(\omega/2T)}{2T} \right]$$

$$\times \int d^2 k A(k + b, \omega + eV) A(k, \omega).$$

(10)

In the limit of low temperatures, $T \to 0$, the spectral densities are only slowly varying with respect to $\omega$. In this limit we solve the integral equation (10) for $A(r, \omega)$ at known $J_q(b, V)$ by Fourier and Laplace transformation,

$$\int_0^\infty d\omega e^{-\omega t} A(r, \omega) = \left[2\pi^3 h/t^2 s\right]^{1/2}$$

$$\times \left[ \int_0^\infty dV e^{-V t} \int d^2 b e^{i b \cdot r} J_q(b, V) \right]^{1/2}.$$  

(11)

In conclusion, we have shown that, in principle, at low temperatures the spatial dependence of the spectral function $A(r, \omega)$ can be determined from measurements of the c-axis I-V characteristic as a function of the magnetic field $B_0$. The dependence of $\sigma_0$ on the orientation of the in-plane field $B_0$ provides information on the existence of line nodes on the Fermi surface as well as on their locations. Measurements of the angle-dependent in-plane magnetoresistance will be useful to map out the momentum distribution of the superconducting gap or pseudogap in organic salts and cuprates.

We thank V.M. Yakovenko and A.V. Balatsky for many useful discussions. This work was supported by the Los Alamos National Laboratory under the auspices of the U.S. Department of Energy.

[1] M.R. Norman et al., Nature 392, 157 (1998).
[2] C.C. Tsuei and J.R. Kirtley, Physica C 282-287, 4 (1997); C.C. Tsuei et al., Nature 387, 481 (1997).
[3] K. Kanoda, Hyperfine Interact. 104, 235 (1997).
[4] T. Ishiguro, K. Yamaji, and G. Saito, Organic Superconductors, 2nd ed., Springer-Verlag, Berlin (1998).
[5] J. Schmalian, Phys. Rev. Lett. 81, 4232 (1998).
[6] Yu.I. Latyshev et al., e-print cond-mat/9903256.
[7] N.E. Hussey et al., Phys. Rev. Lett. 76, 122 (1996).
[8] E. Ohmichi et al., Phys. Rev. B 59, 7263 (1999).
[9] M. Suzuki, S. karimoto, and K. Namekawa, J. Phys. Soc. Jpn. 67, 732 (1998).
[10] R. Kleiner and P. Müller, Phys. Rev. B 49, 1327 (1994).
[11] K. Yang and S.L. Sondhi, Phys. Rev. B 57, 8566 (1998).
[12] G.D. Mahan, Many-Particle Physics, 2nd ed., chap. 9, Plenum Press, New York (1990).
[13] C.J. Pethick and D. Pines, Phys. Rev. Lett. 57, 118 (1986); P.J. Hirschfeld, D. Vollhardt, and P. Wölfle, Sol. State Comm. 59, 111 (1986).
[14] P.A. Lee, Phys. Rev. Lett. 71, 1887 (1993); M.J. Graf, S.-K. Yip, J.A. Sauls, and D. Rainer, Phys. Rev. B 53, 15147 (1996).
[15] A.G. Lebed and N.N. Bagmet, Phys. Rev. B 55, R8654 (1997).
[16] N. Toyota et al., Solid State. Comm. 72, 859 (1990).
[17] A.E. Koshelev, L.N. Bulaevskii, and M.P. Maley, Phys. Rev. Lett. 81, 902 (1998).
[18] N. Morozov et al., Phys. Rev. Lett. 82, 1008 (1999).
[19] R.H. McKenzie and P. Moses, Phys. Rev. Lett. 81, 4492 (1998); e-print cond-mat/98121113.
[20] A. Dragulescu, V.M. Yakovenko, and D.J. Singh, e-print cond-mat/9811101.