A non-linear resonance model for the black hole and neutron star QPOs: theory supported by observations

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Abstract.
Kilohertz Quasi-Periodic Oscillations (QPOs) have been detected in many accreting X-ray binaries. It has been suggested that the highest QPO frequencies observed in the modulation of the X-ray flux reflect a non-linear resonance between two modes of accreting disk oscillation. This hypothesis implies certain very general predictions, several of which have been borne out by observations. Some of these follow from properties of non-linear oscillators, while the others are specific to oscillations of fluid in strong gravity. A 3:2 resonant ratio of frequencies can be clearly recognized in the black-hole as well as in the neutron-star QPO data.

Keywords: Relativity and gravitation, X-ray binaries, time variability
PACS: 04.70.-s, 95.30.Sf, 97.80.Jp, 97.10.Gz, 95.75.Wx

1. INTRODUCTION
The resonance model of Kluzniak and Abramowicz explains pairs of high frequency QPOs observed in neutron star and black hole (microquasar) sources in Galactic low mass X-ray binaries as being caused by a 3:2 non-linear resonance between two global modes of oscillations in accretion flow in strong gravity. In this report, we shortly review rather convincing observational arguments that support the resonance model:

1. The rational frequency ratio 3:2. For microquasars, the observed ratio \( \nu_u/\nu_l \) of the upper and lower frequency is 3/2. For neutron star sources the ratio varies in time, but its statistical distribution peaks up, within a few percent, at the 3/2 value.

2. The inverse mass scaling. In compact sources, \( \nu_0 \sim 1/M \).

3. The frequency-frequency correlation. The upper and lower frequencies of a particular neutron star source are linearly correlated along the “Bursa line”, \( \nu_u = A \nu_1 + B \), with \( A \neq 3/2 \). Observational points occupy a finite sector of the Bursa line which typically crosses the reference line \( \nu_u = (3/2) \nu_1 \) at the resonance point.

4. The slope-shift anticorrelation. For a sample of several neutron star sources, the coefficients \( A, B \) of the Bursa lines corresponding to individual sources in the sample are anticorrelated, \( A = 3/2 - B/(600\,\text{Hz} \pm \Delta \nu) \), where \( \Delta \nu \ll 600\,\text{Hz} \) is a small scatter.

5. Behavior of the rms amplitudes across the resonance point. For the few sources examined so far, the difference between rms amplitudes of the lower and upper QPO changes sign at the resonance point.

One should have in mind that except the inverse mass scaling the above properties are typical of a non-linear resonance in any system of two weakly coupled oscillatory modes. They give no direct clues on how to answer several fundamental questions connected to the specific physics of the oscillations of accretion disks:

• Which are the two modes in resonance? An often discussed possibility is that they are the epicyclic modes, but there are some difficulties with this interpretation.

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1 For a recent review of observations see and for a review of the theory see.
2 Only a few examples of sources for which the actually known data do not cross the 3:2 line are known [see, e.g.,].
• **What are the eigenfrequencies of the modes in question?** It was recently found that pressure and other non-geodesic effects have stronger influence on the frequencies of some modes than previously thought [6, 7]. If this question is answered, one may be able to precisely determine characteristics of strong gravity, in particular the black hole spin (see Section 2).

• **How are the modes excited? What is the energy source that feeds the resonance?** It was suggested that the excitation in the neutron star case could be by a direct influence of the neutron star spin (e.g. due to a magnetic coupling) [8, 9], and in the black hole case by influence of the turbulence [10, 11, 12]. Understanding of this point may provide strong constrains on the magnitude and nature of turbulence in accretion disks.

• **How is the X-ray flux modulated by accretion disk oscillations?** It was suggested that in the case of neutron star sources modulation occurs at the boundary layer [13], and in the black hole sources it is due to gravitational lensing and Doppler effect — at least partially [see, e.g., 14, 15]. The mechanism of modulation must be related to connection between QPOs and the “spectral states” [16].

2. BLACK HOLE HIGH FREQUENCY QPOS

2.1. The 3:2 ratio and $1/M$ scaling

The 3:2 ratio is a strong argument in favor of explaining the QPOs in terms of a resonance [see, e.g., 5, for details]. In general relativistic phenomena around a mass $M$, all frequencies scale as $1/M$. This agrees with black holes having lower QPO frequencies than neutron stars and is suggestive of a relativistic origin for kHz QPOs [1]. McClintock and Remillard [25] found that in microquasars the frequencies do indeed scale inversely with mass:

$$\nu_u = 2.793 \left( \frac{M_u}{M_\odot} \right)^{-1} \text{kHz}. \quad (1)$$

The $1/M$ scaling could be used to estimate the black hole mass in AGNs and ULXs [27], if HF QPOs were to be discovered in those sources (see Figures 11, 12).

2.2. Measuring the black hole spin

According to the epicyclic resonance hypothesis, the two modes in resonance have eigenfrequencies $v_{rad}$ (equal to the radial epicyclic frequency) and $v_r$ (equal to the vertical orbital frequency $v_\theta$ or to the Keplerian frequency $v_K$). Several resonances of this kind are possible, and have been discussed [see, e.g., 5].

Formulae for the Keplerian and epicyclic frequencies $v_{vert}$ and $v_{rad}$ in the gravitational field of a rotating Kerr black hole with the mass $M$ and internal angular momentum $a$ (spin $a$) are well known [22]:

$$v_r^2 = v_K^2 \left( 1 - 6x^{-1} + 8ax^{-3/2} - 3a^2x^{-2} \right),$$

$$v_\theta^2 = v_K^2 \left( 1 - 4ax^{-3/2} + 3a^2x^{-2} \right),$$

$$x = \frac{r}{r_G} = \frac{r}{2GM/c^2},$$

$$v_K = \frac{1}{2\pi} \left( \frac{GM_0}{r_G^3} \right)^{1/2} \left( x^{3/2} + a \right)^{-1} . \quad (2)$$

| Source* | $\nu_u [\text{Hz}]$ | $\Delta \nu_{\text{upp}} [\text{Hz}]$ | $\nu_1 [\text{Hz}]$ | $\Delta \nu_1 [\text{Hz}]$ | $2\nu_u/3\nu_1 - 1$ | Mass$^\dagger [M_\odot]$ |
|---------|-------------------|-----------------|-------------------|-----------------|-------------------|------------------|
| GRO 1655–40 | 450 ± 3 | 300 ± 5 | 0.000 | 6.0 — 6.6 |
| XTE 1550–564 | 276 ± 3 | 184 ± 5 | 0.000 | 8.4 — 10.8 |
| H 1743–322 | 240 ± 3 | 166 ± 8 | -0.036 | not measured |
| GRS 1915+105 | 168 ± 3 | 113 ± 5 | 0.009 | 10.0 — 18.0 |

* Twin peak QPOs first reported by [17, 18, 19, 20, 21].

† See [22, 23, 24, 25] for the microquasars. Note that there is also the different estimate for GRO 1655–40: $M = (5.4 \pm 0.3)M_\odot$ [26].
FIGURE 1. The microquasar kHz QPOs frequencies in a 3:2 ratio \[25\] may be extrapolated to the black hole in the Galactic center \[21, 28\].

FIGURE 2. Left: 1/M scaling in black hole sources. The solid line is the best 1/M fit for microquasars by McClintock and Remillard \[25\] and possibly identified with the 3:2 epicyclic resonance frequency for \(a \sim 0.97\), dotted line is plotted for the same resonance and \(a = 0\). Right: Position of the center of a torus fulfilling the 3:2 resonance condition, as a function of the \(\beta\) parameter characterizing non-slenderness of the torus \[7\]. In this Pseudo-Newtonian result, the resonant frequencies increase with \(\beta\). A similar effect in the Kerr metric would lower the black hole spin parameter estimated for a given QPO frequency.

For a particular resonance \(n:m\), the equation

\[ n\nu_r = m\nu_v; \quad \nu_v = \nu_0 \text{ or } \nu_K \]  

(3)
determines the dimensionless resonance radius \(x_{n:m}\) as a function of spin \(a\). Thus, from the observed frequencies and from the estimated mass one can calculate the relevant spin of a central black hole \[30, 31\].

Results of this procedure were summarized, e.g., in \[32\]. Several resonance models give the values of spin in the range \(a \in (0, 1)\). In particular, the 3:2 epicyclic parametric (or internal) resonance model supposed to be most natural in the Einstein gravity \[13\] implies microquasar black hole spin \(a \sim 0.9\). However, the most recent results of the spin estimate for GRO 1655–40 [e.g., \[33\]] claims the spin to be \(a \sim 0.7\). It could be interesting that recently proposed 3:2 periastron precession resonance \[34\] implies the spin of GRS 1915+105 to be also \(a \sim 0.7\).

On the other hand, above mentioned resonance spin estimates are based on a resonance between epicyclic frequencies equal to those for exactly geodesic motion \[\text{4}\]. Recently \[\text{6, 7}\], it was found that pressure and other non-geodesic effects have stronger influence on the frequencies than previously thought. At the moment, it is difficult to give plausible estimate how this effects change the spin predictions as the theory is not developed enough, but work on this is in progress. A rough estimate, based on Pseudo-Newtonian calculations of \[\text{7}\] and the assumption of the Kerr case being qualitatively same, says that corrected values of spin required from 3:2 parametric resonance model will be rather lower then was supposed (see Fig. 2 right panel).
3. NEUTRON STAR HIGH FREQUENCY QPOS

As was shortly reminded above, for the black holes $\nu_u$ and $\nu_l$ appear to be fairly fixed having the well defined ratio $\nu_u/\nu_l = 3/2$. Contrary to this, $\nu_u$, $\nu_l$ in neutron star sources vary by hundreds of Hertz, along “Bursa lines”

$$\nu_u = A \nu_l + B.$$  \hspace{1cm} (4)

**FIGURE 3.** Bursa plot: The linear fit of the neutron-stars data (dashed line) is obviously inconsistent with the 3:2 relation, but the frequency ratio is peaked to the 3:2 value, i.e. close to the 3:2 resonance \[35\]. Psaltis et al. \[36\] has shown, in a different context, that frequency pairs of all sources cluster along a single line. Bursa (2002, unpublished) also pointed that each source follows a slightly different line. It was demonstrated by \[37, 38, 39\] that variations of $\nu_u$ and $\nu_l$ along the line \(4) can be explained within the non-linear resonance model for QPOs (see Section 3.2).

3.1. Ratio vs. frequency distribution

The distribution of the $\nu_u/\nu_l$ frequency ratio in neutron star sources was shown by Abramowicz et al. \[35\] to cluster near the 3:2 value. This result was later criticised by Belloni et al. \[40\] who thought that the relevant quantity must be the distribution of a single frequency.

We stress that this is a misunderstanding and these two distributions touch different problems. Linear correlation between the frequencies doesn’t imply the ratio clustering because for this the location of data segment is important as well (see Figures 4 and 5). Nevertheless, Belloni et al. \[40\] confirmed the 3:2 clustering in the neutron star sources and shown also the very interesting clustering close to the related resonant frequency.

**FIGURE 4.** A simulated segment of 57 frequency-frequency points (related by $\nu_u = 0.75\nu_l + 450$) with a flat distribution in the lower frequency is placed approximately at the position of Sco X-1 data in $\nu$-$\nu$ diagram. Resulting histogram of ratio is peaked at the 3:2 value.
FIGURE 5. Displacing the data segment from its position consistently with the same fixed relation $v_u = 0.75v_l + 450$ results in completely different histogram even if the displacement is of the order of length of datasegment.

3.2. Anticorrelation between slope and shift

Abramowicz et al. [41] have shown that linear relations (4) for twelve neutron star sources are anticorrelated. Below we shortly remind the connection between this anticorrelation and properties of weakly coupled nonlinear oscillators [see [42, for details].

The frequencies of non-linear oscillations may be written in the form

\[ v_l = v_0^l + \Delta v_l, \quad \Delta v_l = v_0^l (\kappa_l a_l^2 + \kappa_u a_u^2), \]
\[ v_u = v_0^u + \Delta v_u, \quad \Delta v_u = v_0^u (\kappa_l a_l^2 + \kappa_u a_u^2), \]

where $v_0^l$ and $v_0^u$ are the eigenfrequencies of oscillator and $a_l, a_u$ are the amplitudes of oscillations.

If the two amplitudes, $a_l$ and $a_u$, are correlated, i.e., if they are functions of a single parameter $s$, one gets a linear relationship between $v_u$ and $v_l$, obtained from lowest order expansion in the parameter $s$.

The inferred coefficients $A$ and $B$ of linear relation (4) will vary from source to source, but if the eigenfrequencies of oscillations ($v_0^l$, $v_0^u$) are universal for the sources (being fixed by the common space-time metric and mass), then these linear relations themselves will satisfy

\[ A = v_0^l v_0^u B = A_0 - \frac{1}{v_0^l} B. \]

Therefore, the slope $A$ and the shift $B$ are anticorrelated. If the eigenfrequencies of oscillators slightly differ offactor $\sigma$ (e.g., due to the difference of the mass and connected $1/M$ frequency scaling) but the ratio of eigenfrequencies is fixed, this anticorrelation reads

\[ A = A_0 - \frac{1}{\sigma v_0^l} B. \]

Thus, for given type of resonance, the individual pairs $A, B$ should locate on lines inside a triangle in the $A$-$B$ plane with a quite well determined vertex at $[0, A_0]$, and with the size of its base proportional to the scatter in $v_l$. In particular for the 3:2 resonance, the vertex should be $[0, 1.5]$. Figure 6 suggests that this is indeed the case.

In principle, for the 3:2 parametric epicyclic resonance model the eigenfrequencies of oscillations can be directly identified with the epicyclic frequencies of Keplerian motion at the location of this 3:2 resonance. This implies that the generic neutron star mass connected to the straight line in Fig. 6 is about $1M_\odot$ (first noticed by Bursa 2003, unpublished). However, recent results [7] suggest that due to a pressure and other non-geodesic effects, this mass will be higher.

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3 Note that if one includes more terms in the expansion, the frequency-frequency correlation could deviate from straight line.
FIGURE 6. The coefficients of the linear frequency-frequency relation for kHz QPOs in twelve neutron-star sources:
1) 4U 0614, 2) 4U 1728, 3) 4U 1820, 4) 4U 1608, 5) 4U 1636, 6) 4U 1735 [42]; 7) 4U 1915 [43]; 8) XTE J1807 [4]; 9) GX 17+2 [44];
10) GX 34+0 [45]; 11) GX 5-1 [46]; 12) Sco X-1 [40].

The data are consistent with a linear relationship intercepting the \( A \) axis at the value 1.5 (Note good quality of fit by straight line).
The scatter (shaded area) expressed in terms of \( \sigma \) coefficient from eq. (7) is most likely from the interval \( \sigma \in (0.8, 1.2) \). Under the assumption that difference in the eigenfrequencies is caused by different neutron star mass, the mass should differ of factor \( \sim 1.5 \).

3.3. RMS amplitude evolution across the resonance point

Quite recently, Török discovered the strong change in character of the observed QPO amplitudes (Török et al. [47]). The difference between the rms amplitude of the lower QPO and the rms amplitude of the relevant upper QPO, i.e., the quantity

\[
\Delta A_{rms}(\nu) \equiv A_{rms}^h(\nu) - A_{rms}^l(\nu), \quad \text{where} \quad \nu = \nu_l
\]

is rather well correlated with frequency and changes its sign across the 3:2 resonance line which is illustrated in Figures 7 and 8.

It was noticed by Horák that such phenomenon resembles general behavior of mechanical systems crossing an internal resonance (Horák et al. [48]). As some frequencies approach a rational ratio the modes involved in the resonance start to exchange energy. The slow motion through the resonance perturbs the strict balance between both directions of energy exchange and finally causes increase of one amplitude at the expense of the other one.

FIGURE 7. The difference of the rms amplitudes vs. lower QPO frequency for two neutron star atoll sources 4U 1728-34 and 4U 1608-52. Each datapoint corresponds to one continuous segment of observation (error bars without central circle denote insignificant observations). Cyan points correspond to the difference in double peaks. Orange points follow from single QPOs identified through the Q-factor [see 49], their counterparts are here supposed (probably not quite realistically) to have rms equal zero. It seems that up to the 'resonance point' the amplitudes are larger in the case of the upper frequency, equal when 'Bursa line' pass 3:2 ratio and above this, the lower QPO has the stronger amplitude. Based on [47] in preparation.
FIGURE 8. Left: The difference (yellow curve and symbols as in previous figure) and total (grey curve) of the rms amplitudes for atoll source 4U 1636-53. The continuous curves are calculated from polynomial interpolation of the values of rms for the upper and lower frequency obtained through shift and add over all PDS. Right: Double peaks vs. single QPO which are distinguished using Q-factor and placed in accord to linear fit. Up to the intersection of the fit with the 3:2 line, lower frequencies are often suppressed (i.e., difficult to find) as the single ‘upper’ frequency dominate, while above this the situation is opposite. Based on [47] in preparation.

ACKNOWLEDGMENTS

M.A.A., Z.S., and G.T. were supported by the Czech grant MSM 4781305903 and WK by KBN grant 2P03D01424. We thank Didier Barret, Eva Šrámková and Jirka Horák for help and discussions.

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