On the Space Time of a Galaxy

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We present an exact solution of the averaged Einstein’s field equations in the presence of two real scalar fields and a component of dust with spherical symmetry. We suggest that the space-time found provides the characteristics required by a galactic model that could explain the supermassive central object and the dark matter halo at once, since one of the fields constitutes a central oscillaton surrounded by the dust and the other scalar field distributes far from the coordinate center and can be interpreted as a halo. We show the behavior of the rotation curves all along the background. Thus, the solution could be a first approximation of a “long exposition photograph” of a galaxy.

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I. INTRODUCTION

Doubtless, one of the most interesting open questions in physics by now is the one concerning the nature of the dark matter in the Universe. In a series of recent works we have proposed that the dark matter in Cosmos is of scalar field nature [1–4]. Following an analogous procedure as in particle physics, we wrote a Lagrangian with all the terms needed to reproduce the observed Universe. In particular, using the scalar field potential for the scalar field dark matter (SFDM)

\[ V(Φ) = V_0 [\cosh (\sqrt{\kappa_0} Φ) - 1] \] (1)

where \( \kappa_0 = 8\pi/M^2_{Pl} \), we were able to reproduce all the successes of the standard Lambda cold dark matter model (ΛCDM) at cosmological level very well including galactic scales [1][2]. The free parameters of the scalar potential \( V_0 \) and \( λ \) can be fitted by cosmological observations obtaining \( λ ≃ 20.23 \) and \( V_0 ≃ (3 \times 10^{-27} M_{Pl})^4 \), being \( M_{Pl} \) the Planck mass [3]. The mass of the scalar particle is then \( m_Φ ≃ 9.1 \times 10^{-52} M_{Pl} = 1.1 \times 10^{-23} \) eV (compare this value of the scalar mass with that in [5]). Under galactic scales there are some differences between the ΛCDM and the SFDM. The self interaction of the scalar field Φ could explain the observed dearth of dwarf galaxies and the smoothness of the galaxy core halos as well [2]. The question whether this scalar field is from fundamental origin or it is a combination of some other fundamental particles, is open. Nevertheless, in order to reproduce the high resolution N-body numerical simulations of self interacting dark matter [6], we found that the renormalization scale of this scalar field is of the same order of the Planck mass [3]. This would suggest that this scalar field dark matter could have a fundamental origin. We have also studied the scalar field hypothesis at galactic level in [3][4]. The idea follows the standard idea of galaxy formation, namely that scalar field (dark matter) fluctuations are the responsible for the origin of the galaxies. In the case of the scalar field potential (1), we have \( \cosh(λΦ) → \cosh(λ(Φ + δΦ)) \sim \exp(λδΦ) \) for regions where the scalar field fluctuation dominates \( δΦ > Φ \). Of course, as in the standard theory of galaxy formation, the dark matter fluctuations are of different size in different regions of the Universe, for different galaxies. Therefore, at galactic level, we have a scalar field potential which depends on local variables. Thus, the exponential potential approximates the cosh potential in some regimes of the scalar field and we could develop some interesting aspects of a scalar dark matter halo in galaxies.

On the other hand, new observations show that supermassive central objects lying in active galactic nuclei seem to be correlated with the velocity dispersion of the dark matter composing the dark halo, suggesting that the central object was formed at the same time than the halo [7]. This is probably contrary to the standard idea about galactic nuclei that proposes the existence of a central black hole, since it should be formed a time after the disc, i.e. much more after the formation of the halo. Going further, it has been also shown that the supermassive objects which are supposed to be in the center of galactic nuclei, could be boson stars, obtaining the same predictions for the rate of accretion of matter due to the presence of a completely regular space-time background without surface or horizon [8].
The question we are facing now is whether there is a dynamical mechanism that could provide a realistic scenario of galaxy formation using the scalar field dark matter hypothesis. First of all, a complete evolution of galactic and under galactic fluctuations belong to the non-linear regime of perturbations. The right answer would be provided by numerical evolution of Einstein’s equations. Fortunately, a partial answer is given in numerical research on Einstein’s equations developed since 1990. In particular, the collapse of a scalar field has been studied deeply in [10–12], and it was found that there are final equilibrium and stable configurations for collapsed scalar field particles: boson stars (when the scalar field is complex) and oscillatons (when the scalar field is real and time-dependent), both of them being formed through a process called gravitational cooling [11]. Based on these ideas, we present in section II the motivations inviting us to build a galactic model with scalar fields, followed by the solution to the model in section II; physical features of the model are shown in section IV and finally we draw some conclusions.

II. THE GALACTIC MODEL WITH SCALAR FIELD DARK MATTER

We first recall the main results obtained in [10,11]. Through the gravitational cooling process, a cosmological fluctuation of scalar field would collapse to form a compact oscillaton by ejecting part of the scalar field. This ejected part of the cooling process would carry the excess of kinetic energy out and can play the role of the halo of the collapsed object. The final configuration then consists of a central compact object, an oscillaton, surrounded by a diffuse cloud of scalar field, both formed at the same time due to the same collapse process. This would provide a correlation between the central object and the scalar halo.

If the central object is an oscillaton, its formation is due to coherent scalar oscillations around the minimum of the scalar potential [11]. The scalar field $\Phi$ and the metric coefficients (considering the spherically symmetric case) are time dependent and it was proved that this configuration is stable, non-singular and asymptotically flat [11]. But, this could not be the final answer because, from the quantum-mechanical point of view, a Bose-Einstein condensate should form and we must take into account that the scalar field $\Phi$ can also decay or self annihilate. In such a case, we should have to consider the scattering cross section $\sigma_{\Phi \rightarrow \Phi}$ in order to calculate the relaxation time of such condensation. In the models treated in [13–15], the coefficient of $\Phi^4$ is the responsible both for the size of the compact object and for the scattering cross section $\sigma_{\Phi \rightarrow \Phi}$. With potential (1) it is necessary to take into account all the couplings. Nevertheless, it is possible to find the relaxation time smaller than the age of the Universe if

$$g > 10^{-15}(m_{\Phi}/eV)^{7/2}$$

with $g$ being the effective $\Phi^4$-theory coupling. Then, considering the value $m_{\Phi} \simeq 10^{-23}$ eV (see [2]), $g > 10^{-96}$. This condition is well satisfied for our case since $g \sim 10^{-48}$. This ensures that the relaxation time is shorter than the age of the Universe.

In the case studied in [11], a massive scalar field without self-interaction collapses ejecting out 13% of the initial configuration of scalar field. The critical mass of the final configuration for the oscillaton depends on the mass of the boson [11]. For our model (1), the mass of the scalar field is $m = 1.1 \times 10^{-23}$ eV and it is found that

$$M_{\text{crit}} \sim 0.6 \frac{M_{\odot}^2}{m} \sim 10^{12} M_{\odot}$$

which is a surprising result: the critical mass needed for the scalar field to collapse is of the same order of magnitude than the dark matter contents of a standard galaxy. Then, we expect that the fluctuations of this scalar field due to Jeans instabilities will in general collapse to form objects with mass of the same order than the mass of the halo of a typical galaxy. This is the reason by means of which we expect that the SFDM model can also work at galactic level. Summarizing, we consider two working hypothesis up to now. First, we identify the formation of a central compact object and a halo with the gravitational collapse of a scalar field. The compact object could be an oscillaton (since we are dealing only with a real scalar field) or a Bose-Einstein condensate. Second, we identify the ejected scalar field with the halo of this galaxy.

In a realistic model the metric and fields should depend on time and a complete study would involve numerical calculations within and beyond General Relativity. One alternative is to study the behavior of the galaxy numerically with all the hypotheses stated above. This procedure has the advantage that we do not need to eliminate any a
priori consideration, but we can lose some important physical information inside of the numbers we obtain. Other alternative is to perform some approximations on the metric and fields and find exact solutions. These approximations could also lead us to lose crucial information of the system but we can keep physics under control. In our opinion, both procedures must be developed in order to have a better understanding of the hypothesis.

In this work we will adopt the second alternative, i.e. we will build a toy model for case stated above in purely geometrical terms and considering only the final stage of the collapse. We let for a future work the dynamical evolution. To start with, we support our toy model on the numerical results studied in. First, since the time-dependence of the metric in an oscillation is quite small, we suppose then that the center of this toy galaxy is an oscillation which oscillates coherently but considering a static metric. This is an approximation because neither the galactic nuclei nor the oscillation are expected to be static. However, for the purposes of this analytic work, we suppose that the dynamics of the oscillation can be frozen in time in a way we explain below. Second, we do not expect the scalar halo to possess the same properties than the collapsed oscillation; in some sense, they must be different. Thus, we will consider the scalar halo as another scalar field. Third, baryonic matter is considered to lie in the galaxy. We suppose that only the baryonic matter at the galaxy center and the bulge will contribute essentially to the curvature of the space-time. This baryonic component is assumed to be dust. As in previous works, we let the luminous matter around the galaxy as test particles, i.e. they do not essentially contribute to the curvature of the space-time.

III. THE ANALYTICAL SOLUTION

The Lagrangian density we have to start with reads \( \mathcal{L} = R + \mathcal{L}_{SF O} + \mathcal{L}_{SF H} + \mathcal{L}_{dust} \), being \( R \) the scalar curvature and the former terms correspond to the Lagrangian of the scalar field oscillation (SFO), the scalar field halo (SFH) and dust, which should play the role of a galactic bulge. For the scalar objects, the following expressions are available

\[
\mathcal{L}_{SF O} = -\frac{1}{2}(\partial_{\mu} \Phi \partial^{\mu} \Phi) - V(\Phi), \quad \mathcal{L}_{SF H} = -\frac{1}{2}(\partial_{\mu} \psi \partial^{\mu} \psi) - U(\psi)
\]

Here \( \Phi \) is the scalar field corresponding to the oscillation and \( V(\Phi) \) its potential of self-interaction. \( \psi \) is the scalar field describing the scalar field halo and \( U(\psi) \) its respective scalar potential. As a consequence of the non-coupling of the fields in the total Lagrangian, the Einstein’s equations we have to start with reads \( G_{\mu\nu} = \kappa_0 [T^{SF O}_{\mu\nu} + T^{SF H}_{\mu\nu} + T^{dust}_{\mu\nu}] \), with

\[
T^{SF O}_{\mu\nu} = \partial_{\nu} \Phi \partial_{\mu} \Phi - 1/2g_{\mu\nu} [\partial^\tau \Phi \partial_{\tau} \Phi + 2V(\Phi)]
\]

\[
T^{SF H}_{\mu\nu} = \partial_{\nu} \psi \partial_{\mu} \psi - 1/2g_{\mu\nu} [\partial^\tau \psi \partial_{\tau} \psi + 2U(\psi)]
\]

\[
T^{dust}_{\mu\nu} = du_{\mu}u_{\nu}
\]

being \( d \) the density of the dust and \( u^\alpha \) the four velocity of its particles. Since the scalar field \( \Phi \) is time-dependent oscillating coherently around the minimum of its scalar potential, we can write \( \Phi = P(r)\cos(\omega t) \) (see 12). In order to handle Einstein’s equations we average them during the period of a scalar oscillation, that is, we take \( <G_{\mu\nu} = \kappa_0 [T^{SF O}_{\mu\nu} + T^{SF H}_{\mu\nu} + T^{dust}_{\mu\nu}] > \). This procedure gives the lowest order approximation for an oscillation and we are left with time-independent differential equations (12); in the notation of reference (12) where the metric functions are written as \( g(r, t) = g_0(r) + g_1(r)\cos(\omega t) + ... \) we are looking only for \( g_0 \) terms of the metric and fields, which are the dominant ones. Besides of Einstein’s equations there are two Klein-Gordon (KG) equations for the scalar fields,

\[
\Phi^{\mu\nu}_{;\mu} + \frac{dV}{d\Phi} = 0, \quad \psi^{\mu\nu}_{;\mu} + \frac{dU}{d\psi} = 0
\]

The KG on the left depends on time, but it can be shown that one can factorize an overall cosine-term and then the resulting differential equation is time-independent (12).

Then, starting from a spherically symmetric space-time, using the harmonic maps ansatz we were able to find a solution of the system. In few words, the main idea behind the harmonic maps ansatz is the reparametrization of the metric functions with convenient auxiliary functions which will obey a generalization of the Laplace equation, along with some consistency relationships; the latter are usually quite difficult to fulfill, and great care and intuition must
be taken in order to get a system of equations both workable with and interesting enough \cite{17}. The exact solution of the averaged Einstein’s field equations is

$$\psi = \sqrt{\frac{v_a^2}{2\kappa_0}} \ln(r^2 + b^2) + \psi_0$$

$$U(\psi) = \frac{2v_a^2}{(1 - v_a^2)\kappa_0} \exp\left(-\sqrt{\frac{2\kappa_0}{v_a^2}(\psi - \psi_0)}\right)$$  \hspace{1cm} (6)

for the scalar field $\psi$. The scalar field $\Phi$ for the oscillatlon we obtained is

$$V(\Phi(r)) = -\frac{1}{4} \frac{\omega^2 (r^2 + b^2)^{-\frac{v_a^2}{2}} r P^2}{(r - 2M) B_0} + \frac{1}{2} \frac{(1 + v_a^2) r^2 + b^2 (1 - v_a^2)}{(r^2 + b^2)^2 \kappa_0 r^2 (1 - v_a^2)}$$

$$+ \frac{1}{2} \left(\frac{v_a^2 + 1}{2}\right) r^4 + M v_a^2 \left(1 + 2 v_a^2\right) r^3 - 2b^2 \left(1 + 2 v_a^2\right) r^2 + 5 v_a^2 M b^2 r - b^4$$

$$\kappa_0 r^2 (1 - v_a^2) (r^2 + b^2)^2$$  \hspace{1cm} (7)

where the function $P$ is given up to quadratures

$$P = \int \sqrt{-\frac{2}{(r - 2M) \kappa_0 r^2} + \frac{2}{(r^2 + b^2)^2 \kappa_0 r^2 (1 - v_a^2) (r^2 + b^2)^2}} dr$$  \hspace{1cm} (8)

being $v_a$ an asymptotic value of the tangential velocity of the test particles in our toy galaxy. The density of the dust is given by

$$d = \frac{1}{\kappa_0 r^2} - \frac{1}{2} \frac{\omega^2 (r^2 + b^2)^{-\frac{v_a^2}{2}} r P^2}{B_0 (r - 2M)}$$

$$\frac{(1 - v_a^2) r^4 - v_a^2 M (3 - 2 v_a^2) r^3 + 2b^2 (1 - v_a^2) r^2 + v_a^2 M b^2 r + b^4}{\kappa_0 r^2 (1 - v_a^2) (r^2 + b^2)^2}$$  \hspace{1cm} (9)

Finally, the corresponding line element reads

$$ds^2 = -B_0 (r^2 + b^2)^{\frac{v_a^2}{2}} \left(1 - \frac{2M}{r}\right) dt^2 + \frac{A_0}{(1 - 2M/r)} dr^2 + r^2 d\Omega^2$$  \hspace{1cm} (10)

with $d\Omega^2 = d\theta^2 + \sin^2 \theta d\varphi^2$ and $M$ is a constant with the interpretation discussed below. This metric is singular at $r = 0$, but it has an event horizon at $r = 2M$. This metric does not represent a black hole because it is not asymptotically flat. Nevertheless, for regions where $r \ll b$ but $r > 2M$ the metric behaves like a Schwarzschild black hole. Inside of the horizon the pressure of the perfect fluid is not zero anymore, thus our toy model is valid only in regions outside of the horizon, where it could be an approximation of the galaxy. Metric (10) is not asymptotically flat, but it has a natural cut off when the dark matter density equals the intergalactic density as mentioned in \cite{14}.

**IV. PHYSICAL FEATURES OF THE MODEL**

The dust density and the oscillatlon scalar field potential depend on the value of the function $P$. We find that $P$ goes very rapidly to a positive constant value, which depends on the boundary conditions one imposes. Let its asymptotic value be $P_0$ when $r \gg 2M$ whose interpretation would be the asymptotic amplitude of $\psi$.

In order to understand the other parameters of the metric, let us proceed in the following way. It is believed that in a standard galaxy, the central object has a mass of $M \sim 2 - 3 \times 10^9 M_\odot \sim$some a.u. Far away from the center of the galaxy, say from 1pc up, the term $2M/r \ll 1$. In this limit metric (10) becomes

$$ds^2 = -B_0 (r^2 + b^2)^{\frac{v_a^2}{2}} dt^2 + A_0 dr^2 + r^2 d\Omega^2.$$  \hspace{1cm} (11)

This space-time is very similar to metric (18) of reference \cite{6}, but now with the potential
\[ U(\psi(r)) = \frac{2v_a^2}{\kappa_0 (1 - v_a^2)} \frac{1}{(r^2 + b^2)} \]  

being both solutions the same in the limit \( r \to \infty \). This implies that \( A_0 = 1 - v_a^4 \), recovering in this way the asymptotic results shown in [1]. Parameter \( b \) is related to parameter \( b \) of metric (21) in reference [3], where it acts as a gauge parameter. Of course this metric is only valid far away from the center of the galaxy. With parameter \( b \) it is now possible to fit quite well the rotation curves of spiral galaxies. Therefore metric (11) could not only represent the exterior part of the galaxy, but it could be a good approximation for the core part of it as well. Let us see this point.

The rotation curves \( v^{rot} \) seen by an observer at infinity for a spherically symmetric metric are given by \( v^{rot} = \sqrt{g_{tt,t}}/(2g_{tt}) \) [3]. For metric (10) such result reads

\[ v^{rot}(r) = \sqrt{\frac{v_a^2(r - 2M)r^2 + M(r^2 + b^2)}{(r - 2M)(r^2 + b^2)}} \]

formula that allows one to fit observational curves. In Figure 1 it is shown the change of the rotation curve when the value of \( M \) changes, and it is evident that such fact would affect only the kinematics in the central parts of the galaxy, exactly in the same way as the mass of the central object should do [3].

![Graph showing rotation curves](image)

FIG. 1. Rotation curves provided by the line element (11), for different values of the mass of the central object: \( M = 3, 5, 7, 9, 11 \times 10^6 M_\odot \). In the left side it is shown the rotation curve in the innermost part of the galactic nuclei, in the middle an intermediate radial distance an finally in the right hand side it is shown the properly called rotation curve associated to the dark matter.

Nevertheless, it calls the attention in Figure 1 that there is a region where the velocity of test particles is near to zero, and in fact the rotation curve decays in a Keplerian way. Let us see what happens in such a region near the galactic center: the factor \((r^2 + b^2)^{v_a^2}\) in \( g_{tt} \) is almost 1 for \( r \sim 10pc \) for reasonable values \( b \sim kpc \) and \( v_a \sim 10^{-3} \), therefore the time-like geodesics are determined by the factor \( 1 - 2M/r \). Under such condition the contribution of \( \psi \) is accurately zero for an observer far away from the galaxy. The interesting situation comes after calculating the density of all the components to the central density, it reads

\[ \rho_{dust} + \rho_{SFO} + \rho_{SFH} = -1/2 \frac{M (r - 2M) (r^2 + b^2)^{v_a^2-2}}{r^6\kappa_0 (1 - v_a^4)} \left( -\frac{4r^6v_a^2}{B_0} + b^2 \left( 4 - 7v_a^2 + 2v_a^4 \right) r^4 + 3b^4 \left( 3 - v_a^2 \right) r^2 + 3b^6 \right) \]  

where the approximation \( v_a \ll 1 \) and \((r^2 + b^2)^{v_a^2} \sim 1 \) is considered to be valid for reasonable values of the parameters. Moreover, we can infer that the total energy density as seen by an observer far away from the center of the galaxy diverges when \( r \) approaches to zero and goes rapidly to zero as \( 1/r^2 \) when \( r \to \infty \). But an observer at infinity measures the mass of the central object, it has not the possibility to distinguish between the density of each component.

Now let us explore an ADM-like concept of mass associated to the central region; we stress that metric (10) is not asymptotically flat and therefore the ADM concept of mass strictly fails to be valid, so we recall that there is a region into the interval \( 2M \ll r < b \) geometrically almost flat, where we will define a useful infinity \( \infty_f \) that should serve to calculate the mass of the central configuration through the standard metric.
\[ ds^2 = -c^2 dt^2 + \frac{dr^2}{(1 - \frac{2m}{r})} + (1 - \alpha)r^2 d\Omega^2 \]  

where \( m = m(r) \) is interpreted as the mass function and \( \delta = \delta(r) \) as the gravitational potential. This form of the metric is convenient because in this coordinates \( m, \alpha = 4\pi r^2 \rho_T \), where \( \rho_T \) is the total density of the object. This interpretation is correct in regions where the space-time is almost flat, i.e. far away from the horizon \( r = 2M \).

Close to the horizon or inside of it, function \( m \) is a quantity that should be similar to the mass of the object, but it is not since it contains the contribution of all the components together; in this region, where the curvature of the space-time is huge, the volume element is different from \( 4\pi r^2 dr \). Furthermore, inside of the horizon we are not able to know the real physics of the object. On the other side, far away from the center of the toy galaxy, this function can be interpreted as the mass of an infinitesimal shell at radius \( r \). Anyway we will call function \( m \) the mass function everywhere. Thus, the ADM-like mass is obtained by \( M_{\text{ADM}} = \lim_{r \to \infty} m \). We perform the coordinate transformation \( \sqrt{A_0} r \to r, \sqrt{A_0} b \to b \) in order to compare metrics \([10]\) and \([14]\). We obtain

\[ ds^2 = -\frac{B_0}{A_0^2} \left( r^2 + b^2 \right)^{\frac{v^2}{2}} \left( 1 - \frac{2M\sqrt{A_0}}{r} \right) dt^2 + \frac{dr^2}{(1 - \frac{2M\sqrt{A_0}/r}{r})} + \frac{r^2}{A_0} d\Omega^2 \]  

with \( A_0 = 1 - v_0^4 \). Thus, \( M_{\text{ADM}} = \sqrt{1 - v_0^4} M \). Probably an observer at any huge \( r \) would see the mass \( M_{\text{ADM}} \) at the center of the galaxy.

There is a strong reason to consider with reservation the analysis near the horizon: we are considering a time averaged photograph of the space-time, which in the center would be strongly time dependent due to the presence of the oscillation, and the stationarity fails to be a good approximation, thus it is not possible to know under the present approach which are the exact roles played by the binding energy of the oscillation, the negative energy of the dust and the self-energy of the whole central system; such features could be known by performing the evolution of Einstein's equations and in fact represent a topic itself which will be discussed elsewhere \([10]\).

For this model, there is a contributor which does not fulfill the energy conditions, this is the reason to call it “toy model”, although an observer at infinity will see the sum of all energy densities of the components. In other words, the amount of matter, with negative or positive energy density, is the quantity which determines the value of \( m(r) \), the “black hole mass” concentrated at radius \( r \) of the toy galaxy, not the contributors separately. Only if the observer could measure the contributions of the energy density very close to the center of the galaxy, it could recognize them. At infinity, the observer will only measure \( M_{\text{ADM}} \), i.e. it will see a Black-Hole-like metric at the center of the galaxy which horizon lies at \( r = 2M_{\text{ADM}} \).

We present in Figure 2 the situation when parameter \( b \) takes different values, playing thus the role of the core radius in the usual dark halo hypothesis. Here we show that for an observer at infinity, the test particles close to the galaxy center behave as if there were a black hole of mass \( M_{\text{ADM}} \) in the center. The event horizon avoids that this observer could see inside of the horizon surface, the velocity of test particles is higher for closer test particles and the rotation velocity decays as \( 1/\sqrt{r} \) in this region. For particles far away from the center, the rotation curves behave just as the rotation curves given by the contribution of the dark matter halo in a typical galaxy.

![Circular velocity vs. Radial distance](image)

**FIG. 2.** We show the rotational curves when the parameter \( b = 1, 2, 3, 4, 5 \)kpc. It is evident that in such case the shape of the curve associated with the dark matter changes, but in the center remains unchanged.
In Figure 3 the fit of the curves is done using the observed rotation curves of some dwarf galaxies, whose dark matter contribution is extremely dominating and therefore are considered as the test of fire for a dark matter model in galaxies. In general, for disc galaxies, the fit of the rotation curves using this metric is analogous to that in reference [3]. It seems then that metric (10) is a good approximation for some late stadium of the space-time of a spiral galaxy; it is a good approximation of a "long exposition photograph" of a galaxy.

![Rotation curves of three dwarf galaxies are shown.](image)

FIG. 3. Rotation curves of three dwarf galaxies are shown. The units are as usual: kms$^{-1}$ in the vertical axis and Kpc in the horizontal one.

V. DISCUSSION

The configuration we found is as follows: in the innermost region of the toy galaxy, it lies a time averaged oscillaton and dust, providing the picture of an object very similar to a black hole for an observer far away. Moreover, the resting scalar field should accommodate in the outer parts. This picture is precisely very similar to the one required for a current galaxy, which center would be dominated by a supermassive object and it should posses a halo region dominated by the presence of dark matter. The corresponding line element (10) is free of naked singularities and is not asymptotically flat. Instead, it behaves exactly as a black hole near the coordinate center and it is asymptotically constructed to provide the flat curve condition [4].

The main difference between this results and previous research supporting the scalar field dark matter hypothesis in galaxies, is precisely that in this case we have been able to extend the solution towards the galactic core. We have shown two important statements: First, an averaged spherically symmetric source field exists that supports a real scalar field under the assumption of non asymptotic flatness. Second, a fully relativistic treatment is able to explain the kinematic of test particles in the whole background space-time of a galaxy at once.

Therefore, we believe that the metric presented here could be a first approximation of the galactic space-time provided the presence of coherent field (a Bose-Einstein condensate) instead of usual matter. Even when our solution accounts for negative energy densities of some components, observers at infinity are not able to know it. Furthermore, for such observers the test particles around the central object behave as if there would be a black hole of mass $M_{ADM}$.

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