Turbulent free convection over a horizontal heated plate in an open top cavity

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Abstract. An experimental study of the thermal field established, over a horizontal heated plate (1000x500x12 mm$^3$) in an open top cavity, of aspect ratio $\Gamma = 2$ is presented. The working fluid was air at atmospheric pressure and the imposed temperature conditions correspond to a Rayleigh number $Ra = 4.1 \times 10^8$, based on the cavity height and the temperature difference across it. Schlieren visualization was used to identify characteristic structures developing over the heated plate. Local temperature measurements along the vertical axis over the plate centre were accomplished using a fine thermocouple ($d = 0.025$ mm). Distributions of mean and rms temperatures are presented along with the corresponding skewness and flatness distributions. The exponential form of the PDF temperature distributions observed in the present experiments, as in several experiments in hard turbulence, is compatible with the intermittent presence of fluid structures with different origin and past history. Temperature PDFs have been approximated by mixtures of weighed Gaussian distributions to illustrate this concept.

1. Introduction
Thermal free convection over a horizontal, solid, hot boundary through an initially quiescent fluid remains a challenging problem in heat transfer. In contrast to the problem of free convection along a vertical plate, the presence of a bounding wall normal to the direction of buoyancy leads to flow and thermal fields that cannot be intuitively deduced, but are the result of a delicate balance depending on fluid properties and initial and boundary conditions. Small irregularities may have a significant impact on the development of instabilities and the resulting flow pattern. Since the pioneering work of Townsend [1], the problem of convection above horizontal heated surfaces in significantly larger enclosures, approximating a vast expanse of stationary fluid at laboratory scale, has been studied quite extensively [2-5]. Visualization experiments have provided insight on the resulting flow pattern [6-7]. Boundary layers (both viscous and thermal), have been recognized as playing a key role in convective turbulence, whereas the presence of moving thermal plumes, the jet like upward motion of hotter fluid, is probably the most characteristic flow attribute associated with horizontal plate free convection. According to Howard [8], the conduction layer grows by diffusion, becomes unstable, and erupts resulting in the release of a thermal. More recently, Theerthan and Arakeri [9, 10], based on experimental observation, introduced a model assuming that the near wall convection can be modelled as a periodic array of steady, laminar line plumes.

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During the last two decades several investigators have studied the Rayleigh – Bénard (RB) system over cylindrical or rectangular convection cells, in which very well-defined boundary conditions can be realized experimentally, allowing the use of a variety of fluids and pressure conditions to cover a large range of Prandtl and Rayleigh numbers. These efforts besides providing significant insight on horizontal plate free convection, have led to the identification of the hard turbulence regime developing in a RB cell for high Ra numbers \( (Ra > 10^7) \) \([11, 12]\), depending also on Pr number and the aspect ratio of the cell \([13]\). This regime is characterized by an exponential probability distribution function of the temperature fluctuations at the centre of the convection cell, whereas in the soft turbulence regime \( (Ra < 10^7) \) the corresponding distribution is almost Gaussian. Another characteristic of hard turbulence is the superposition of fluctuations on an average baseline which is associated with a large-scale mean circulating flow that spans the entire space of the convection cell \([14, 15]\) alternating its direction over large time periods \([16, 17]\).

The present work contributes to the experimental investigation of thermal convection over a heated plate in an open top cavity. A key objective of the study was to provide information whether attributes corresponding to the state of hard turbulence, observed in R-B convection cells, can be identified in an open top cavity. Visualization results provide information on the structural development of the flow and temperature fields. Temperature measurements along the centerline over the plate are presented demonstrating the mean and fluctuating characteristics of the thermal field. The compatibility of the exponential tails of the temperature PDFs with the concept of the intermittent presence of fluid structures with different past history is illustrated and discussed.

2. Experimental facility
A stainless steel, non-magnetic, horizontal plate \((1000 \times 500 \times 12 \text{ mm}^3)\), heated by ten uniformly spaced and independently controlled electrical resistors attached to its lower side, was used as heating surface. Temperature uniformity on the plate was achieved by adjusting the power supply to each resistor. The plate was encased flush in an insulating box minimizing heat losses from its lower surface and side edges. The plate temperature distribution, measured at a 1 mm depth by means of ten type J thermocouples along its long axis, was uniform to within 0.5% of \( \Delta \), the temperature difference between plate surface and ambient air.

An open top cavity of dimensions \(1000 \times 500 \times 500 \text{ mm}^3\) was formed over the plate with transparent glass windows as sidewalls, supported on an aluminum frame (figure 1). The experimental set up was arranged to facilitate visualization and positioning of temperature probes. The experimental conditions were set to approximate a plane solid boundary of uniform temperature below the flow region, which is confined by lateral boundaries of height \( L \), having an open top, purely convecting upper boundary, that maintained a constant temperature difference across the fluid. The temperature measurements in the fluid were accomplished using a 0.025 mm type E OMEGA, fast response thermocouple probe, with \( \pm 0.1 \degree \text{C} \) uncertainty. The sampling rate was constant at 50 Hz and 32768 samples were recorded for approximately 11 min at each measuring location. Measurements were conducted once the thermal

![Figure 1. The experimental apparatus and measuring system.](image)
field reached steady state. A schlieren system consisting of a hydrogen lamp, two parabolic mirrors (ø 20.32 cm) and a knife edge was used for visualization. Schlieren images, obtained with the beam aligned parallel to the plate short axis, were projected on a screen and captured with a photographic or a video camera. Further details on the experimental facility and technique can be found in [18].

3. Experimental Results and Discussion

3.1. Flow visualization
Video recordings of the flow field using schlieren visualization have been used to identify the characteristic structures developing over the heated plate, influencing the flow field global characteristics and turbulence. In figure 2, a typical set of successive video frames at 1/25 s are presented for the steady state free convection field established over the plate. Close to the plate a white line of varying thickness, extending in the horizontal direction, depicts the limits of the thermal layer, which are constantly rearranged over time. Several vertical or inclined lines starting at this level designate the presence of line plumes at various positions over the plate surface. A horizontal movement of the structures, in varying direction, is also noticeable. At the middle and upper part of the photos the characteristic orientation of the majority of the structures is in the vertical direction (or close to it). Mushroom type elliptical structures, indicating the front of developing plumes, are also observable close to the surface becoming weaker (and fewer) towards the upper part of the images. Characteristic events may be identified in these pictures. In frames a and b a developed plume is visible on the lower left corner becoming detached at frame c. The initiation of a plume, resulting in the development of a mushroom can be identified in the lower right part of frames d, e and f. The rearrangement of the thermal layer, observable in previous frames, results in an intense burst of events at the central area of the picture in frame k. In figure 3 and 4 the development of plumes and mushroom structures in the transient period during the establishment of the flow field are presented.

![Figure 2. Successive schlieren visualization video frames depicting flow patterns over a heated plate in an open top cavity, at steady state.](image-url)
Less energetic structures are also observable in the background. Summarizing the observations we may conclude that moving line plumes seem to be the predominant structures in the near wall field originating inside the conduction layer. The plumes are swept slowly horizontally, inclined forward in the direction of a global motion, which alternates randomly in time. Adjacent plumes may merge with one another, and then erupt as blobs of hot fluid. The structures originating close to the surface are travelling upwards, becoming weaker and losing gradually their identity at higher distance, participating also in an attenuated sweeping horizontal motion.

3.2. Mean and rms profiles

Point temperature measurements were obtained at regularly spaced locations along the vertical axis, over the centre of the plate. In all the experiments, the working fluid was air at atmospheric pressure. The presented temperature measurements refer to Rayleigh number, $Ra = 4.1 \times 10^8$ defined as

$$Ra = \frac{g \beta L^3}{\nu \alpha}$$

where $g$ is the acceleration due to gravity, $L$ is the height of the open – top cavity, $\Delta$ is the temperature difference between the plate surface and the ambient, measured at height $L$ ($\Delta = 53.91 \, ^\circ C$). The parameters $\beta$, $\nu$, and $\alpha$ are the isobaric thermal expansion coefficient, the kinematic viscosity and the thermal diffusivity of the convective fluid (air) respectively. The aspect ratio $\Gamma$, defined as the lateral extent of the cell divided by the height, $L$, of the open – top box was $\Gamma = 2$.

The mean temperature profiles, depicted in figure 5, are quite typical, demonstrating the three spatial regions expected in free convection over a horizontal plate. Close to the wall the temperature decreases linearly through the conduction layer, where due to the non-slip and zero permeability

![Figure 3. Development of an inclined plume forming a mushroom (heating up phase).](image)

![Figure 4. Large plume rising near the centre of the flat plate (heating up phase).](image)

![Figure 5. Mean (solid circles) and rms (open squares) temperature profiles along the central vertical axis.](image)
boundary conditions, heat is transferred solely by conduction. Outside this layer the presence of both conduction and convection through rapid mixing, characterize the transitional region, whereas far away from the plate the mean temperature gradient becomes negligible, indicating that convection is dominant. The thermal boundary layer thickness $\lambda_{th}$ is usually defined as the distance at which the extrapolation of the linear part close to the plate intersects the line of constant $T_{\infty}$ [13, 16, 17, 19]. The thermal layer thickness has been estimated as $\lambda_{th} = 2.10 \pm 0.05$ mm. The temperature profile is linear for about 50% of the thickness of the thermal layer, whereas at least half of the total temperature difference $\Delta$ is confined within its limits. In the present results, as in Puits et al. [20], the mean temperature profile attains a plateau close to $z/\lambda_{th} > 10$, whereas the results of Belmonte et al. [21] and Lui and Xia [22] attain plateau values significantly closer to the surface at $z/\lambda_{th} = 5$.

The distribution of the temperature fluctuations rms (also shown in figure 5) separates in two regions. The closest to the plate measurement is already of significantly high value although the boundary conditions impose a zero value on the surface. The fluctuations increase quickly with $z/\lambda_{th}$ reaching a maximum near the edge of the thermal boundary layer. The intermittent eruption of hot buoyant fluid, close to edges of the thermal layer, which is the most energetic mechanism dominating the flow, is probably responsible for the large values of turbulent intensity at this location. Further away from the wall, rms values decrease smoothly towards the centre of the open – top cell.

3.3. Skewness and Flatness

The dimensionless skewness and flatness factors are defined as

$$S = \frac{\langle (T - \langle T \rangle)^3 \rangle}{\langle (T - \langle T \rangle)^2 \rangle^{3/2}}$$  \hspace{1cm} (2)

and

$$F = \frac{\langle (T - \langle T \rangle)^4 \rangle}{\langle (T - \langle T \rangle)^2 \rangle^{2}}$$  \hspace{1cm} (3)

respectively, where $\langle \rangle$ is the averaging operator. These factors may be considered as measures of the signal probability density function (PDF) attributes. Skewness characterizes the symmetry of the distribution around the mean, with zero value denoting a perfectly symmetric distribution, whereas positive or negative values indicate skewed distributions with long tails towards positive or negative values respectively. Flatness is always positive, and characterizes the sharpness of the PDF. Small or large values indicate respectively flat or pointed distributions. Skewness and flatness values are usually assessed with reference to the corresponding values of a Gaussian PDF, which are 0 and 3 respectively [23, 24]. Skewness values departing from zero and flatness factors larger than 3, are associated with the presence of inhomogeneities and intermittent flow fields.

Figure 6. Skewness (solid circles) and Flatness (open squares) distributions along the central vertical axis (large scale trends depicted in inserts).
The skewness and flatness factors of the temperature measurements along the vertical axis, over the centre of the plate are presented in figure 6. In the conduction layer the values are close to 0 and 3 respectively indicating homogeneity. Outside the conduction layer the skewness and flatness factors increase rapidly through and outside the limits of the boundary layer and at a slower pace after $z/\lambda_{th} \approx 5-10$. Beyond $z/\lambda_{th} = 70$ both factors decrease steadily until around $z/\lambda_{th} = 125$, and further away from the plate they follow an oscillatory decrease, towards the open top of the cavity, to values close to those corresponding to a normal distribution.

3.4. Temperature PDF’s
Probability density function distributions of the temperature measurements time series at several locations over the centre of the plate, in the region $z/L = 10^{-4} - 0.24$, are presented in figure 7. In a previous work [18], the authors suggested that exponential tails in a PDF and values of skewness and flatness factors departing from the normal distribution values of 0 and 3, may be interpreted as the result of a probability distribution which is a weighed mixture of normal probability distributions. This concept is considered compatible with the intermittent presence of fluid structures with different past history, which did not had the time to become homogenized through diffusion. Following this line of thinking, we have approximated the measured PDF’s, in a least square sense, with a weighed mixture of six normal distributions with different mean values and variance. The results are depicted in figure 8 for most of the distributions of figure 7. In most of the cases only a few of the six distributions are visible, indicating that the rest had very low weighing factors to reach over the $10^{-1}$ threshold.

The first distribution refers to the closest to the wall measurement at $z/L = 10^{-4}$ ($z/\lambda_{th}=0.024$), within the conduction layer. The skewness factor has a low negative value indicating a longer low temperature tail. The flatness factor is close to 3 as in a Gaussian distribution. The PDF can be reasonably well reconstructed as a mixture of only two normal distributions of similar widths, with that corresponding to a higher mean temperature having a larger weighing factor. The high temperature distribution may be associated with the typical resident fluid whose temperature is varying following the changes of the temporal boundary layer thickness (due to a periodic accumulation of hot fluid and the sudden eruption of thermals). The lower temperature distribution may be reasonably related to the presence of colder fluid descending towards the surface. The second distribution, at $z/L = 0.0031$ ($z/\lambda_{th}=0.74$), corresponds to a location just past the linear increase of the mean temperature. The rms value is quite high as indicated by the width of the distribution. Skewness is increasing quickly with $z$ in this region although its value is still close to zero, whereas flatness is also close to 3. The PDF may be reconstructed as a mixture of two normal distributions, of wider extends than previously. Moreover in this case the larger weighing factor corresponds to the lower temperature distribution, indicating the increasing influence of colder descending fluid. At $z/L = 0.014$ ($z/\lambda_{th}=3.33$) the temperature rms values have started to decrease with $z$, whereas the skewness and the

![Figure 7. Temperature PDFs at various z/L along the central vertical axis.](image-url)
flatness factors have attained rather high values (1 and 4 respectively). The PDF at this location is reconstructed as a mixture of three Gaussian distributions, with weighing factors diminishing for distributions having mean values corresponding to higher temperatures (there is also a fourth Gaussian distribution in this mixture, with low weighing factor which we will assume negligible). We may assume that the lower temperature Gaussian corresponds to the typical properties of the resident fluid, or the colder fluid descending in this area slowly enough, so that it has the time to become homogenized with the resident fluid. The other two distributions may be associated with the influence of ascending structures. A comparatively small percentage of significantly hotter structures which travel quickly upwards driven by significant buoyant forces and a larger percentage of less hotter fluid ascending at a relatively reduced rate under the influence of reduced buoyancy forces. Further away from the surface at $z/L = 0.048$ ($z/\lambda_{th}=11.4$) the skewness and flatness factors have increased whereas the Gaussian mixture indicates that the variety of ascending structures properties has increased to four, probably due to the differentiation of their past history. At the last presented station $z/L = 0.048$ ($z/\lambda_{th}=11.4$) the trends have been somehow changed. Most of the Gaussian distributions consisting the mixture have closely spaced mean values and relatively narrow widths which are probably the result of homogenization through conduction and small scale mixing. The pronounced exponential tails of the overall PDF towards high temperature values have been retracted, resulting in an almost symmetric peak. Two or three Gaussian distribution with mean values corresponding to higher temperature values are still present, with significantly lower weighing factors, resulting in very high skewness and flatness factors. At larger distances from the plate (not presented here, see [18]) these secondary Gaussian distributions become weaker and the skewness and flatness factors start to decrease.

4. Conclusions
An experimental study of the thermal field established, over a horizontal heated plate in an open top cavity, of aspect ratio $\Gamma = 2$ and $Ra = 4.1 \times 10^8$, has been conducted by means of schlieren visualization and local temperature measurements along the vertical centreline. Characteristic features identified by the visualization include energetic plumes, mushrooms and undulations close to the heated surface and less intense rather vertically oriented structures at larger distances from it. The thermal layer thickness has been estimated as $\lambda_{th} = 2.10 \pm 0.05$ mm. Temperature PDFs have been approximated as weighed mixtures of Gaussian distributions and the results indicate that this concept can be possibly used to identify the intermittent presence of structures with different past histories. Within the boundary layer two Gaussian distributions seem to be sufficient to reproduce the measured PDF, designating the intermittent presence of boundary layer fluid and colder fluid descending from higher levels. Outside

![Figure 8](image-url)  

**Figure 8.** Temperature PDFs at various $z/L$ reconstructed as intermittent mixture of Gaussian processes.
the boundary layer the number of Gaussian PDFs sufficient to reproduce the measured PDF increases, indicating the intermittent presence of families of structures with different past histories. Far away from the plate the families of structures become more and more homogenized through conduction and small scale mixing. These observations are compatible with the statistical characteristics of the thermal field including the temperature rms and skewness and flatness factors.

5. References

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