Dirac sea effects in $K^+$ scattering
from nuclei

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Abstract

The ratio $R_T$ of $K^+-^{12}C$ to $K^+-d$ cross sections has been calculated microscopically using a boson-exchange $KN$ amplitude in which the $\sigma$ and $\omega$ mesons are dressed by the modifications of the Dirac sea in nuclear matter. In spite of the fact that this dressing leads to a scaling of the mesons effective mass in nuclear matter, the effect on the $R_T$ ratio is found to be weak.
These last years, it has been shown that the in-nuclear-medium modifications of the $NN$ excitations of the Dirac sea might be important in the understanding of some phenomena like, for example, the quenching of the Coulomb sum rule in electron-nucleus scattering\cite{1, 2}.

It has also been shown that these modifications lead to an increase of the proton radius and to a decrease of the $\omega$ effective mass in nuclear matter\cite{1}, both phenomena often invoked\cite{3, 4, 5} to explain the discrepancies between the conventional nuclear physics calculations and the experimental data in $K^+$-nucleus scattering.

The purpose of the present work was to investigate quantitatively in what extent the $K^+$-nucleus cross sections are sensitive to these modifications of the Dirac sea.

We have analyzed these effects on the ratio $R_T$ of $K^+ - ^{12}C$ to $K^+ - d$ total cross sections.

$$R_T = \frac{\sigma_{tot}(K^+ - ^{12}C)}{6 \cdot \sigma_{tot}(K^+ - d)} \tag{1}$$

As emphasized by many authors, this ratio is less sensitive to experimental and theoretical uncertainties than, for example, differential cross sections, and thus more transparent to the underlying physics. The ratio $R_T$ has been
calculated from 400\,MeV/c to 900\,MeV/c.

In our calculation, we have used the same basic ingredients as in ref\cite{5}: the $K^+$-nucleus optical potential has been built by folding the density-dependent $K^+$-nucleon t-matrix by the nuclear densities, these nuclear densities have been deduced from electron scattering experiments, and the KN amplitude has been calculated using the Bonn boson exchange model\cite{6}. The $K^+$-nucleus total cross-section has been deduced from the forward $K^+$-nucleus elastic scattering amplitude using the optical theorem.

Since in isoscalar nuclear matter the main part of the KN interaction comes from $\sigma$ and $\omega$ exchange, we have considered only the dressing of these two mesons, the remaining part being kept as in free space. Moreover, since the $K^+$-nucleus forward amplitude is dominated by forward ($q \simeq 0$) KN scattering processes, the modification of the Dirac sea polarization will be included at the one-loop approximation.

In nuclear matter, the $NN$ excitations of the Dirac sea are modified by the surrounding medium. The main effect of the nuclear medium on the nucleon states is the modification of the Dirac spinor which takes the same
form as in free space but with an effective mass:

\[ M^*(\rho) = M + \Sigma_S(\rho) \]  \hspace{1cm} (2)

where \( \Sigma_S(\rho) \) is the scalar part of the nucleon self-energy at density \( \rho \). Up to now, this self-energy has not been unambiguously determined, but it is commonly admitted as the most realistic that, at saturation, the effective mass would be approximately 15% smaller than in free space\[^7\]. Thus, we have used here a density dependence in this way:

\[ M^*(\rho) = M(1 - 0.15 \frac{\rho}{\rho_0}) \]  \hspace{1cm} (3)

and we have verified that smooth variations from this linear dependence don’t change significantly the results.

Since the parameters of the KN interaction have been determined\[^6\] in order to reproduce the KN data in free space, the polarization of the Dirac sea in free space has already been taken into account in the KN amplitude, at least in average. Therefore, in nuclear matter, this free-space polarization of the Dirac sea must be subtracted from the in-medium one. Moreover, in order to eliminate divergent terms, a renormalization procedure has to be applied. It has been done as usual\[^1, 2, 8\] by requiring that, in free space, the three- and four-\( \sigma \) vertices vanish at zero momenta and the corrections to the
σ and ω masses and wave functions are zero at $q^2 = \mu^2$ ($\mu$ is a renormalization scale which disappears here when we substract the polarization in free space).

This leads, for the σ channel, to:

$$\Pi_\sigma(q) = \frac{3g^{2N}q^2}{2\pi^2}[3(M^* - M)^2 - 4(M^* - M)M - (M^* - M)^2 \int_0^1 \ln \frac{M^* - x(1-x)q^2}{M^2} \, dx - \int_0^1 (M^2 - x(1-x)q^2) \ln \frac{M^* - x(1-x)q^2}{M^2 - x(1-x)q^2} \, dx]$$

(4)

and for the ω channel to:

$$\Pi_\omega^{\mu\nu}(q) = (\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu})\Pi_\omega(q)$$

(5)

with

$$\Pi_\omega(q) = \frac{g^{2N}q^2}{\pi^2} \int_0^1 x(1-x)\ln \frac{M^* - x(1-x)q^2}{M^2 - x(1-x)q^2} \, dx$$

(6)

This modification of the Dirac sea polarization produces a displacement of the poles of the mesons propagators which occur now at $q_0$ such that $q_0^2 = m_i^2 + \Pi_i(q_0)$ where $i$ stands for σ or ω. If we define an effective mass for the mesons, $m_i^*(\rho)$ such that $m_i^*(\rho)^2 = q_0^2$, we can see on fig.1 that we obtain approximately the scaling law derived by Brown and Rho from general symmetry arguments:

$$\frac{m_\sigma^*(\rho)}{m_\sigma} \approx \frac{m_\omega^*(\rho)}{m_\omega} \approx M^*(\rho) / M$$

(7)
As it has already been pointed out by several authors[4, 5], a replacement of the \(\sigma\) and \(\omega\) mass in the calculation of the KN amplitude by these density dependent effective masses would lead to an improvement of the agreement with the experimental data in \(K^+\)-nucleus scattering (fig.2, curve b).

However, this approximation would be rather questionable here since the dominant contributions to the total \(K^+\)-nucleus cross-section come from forward \((q \simeq 0)\) and not from \(q \simeq q_0\) KN scattering, and, as we can see on table 1, \(\Pi_i(0)\) is very different from \(\Pi_i(q_0)\). If we use now, in the mesons propagator, the expressions obtained above for the polarizations (eq. 4 and 6), we obtain a ratio \(R_T\) (fig.2, curve c) very close to that calculated with the free-space KN amplitude (fig.2, curve a). Indeed, the modification of the polarization being much smaller in the forward direction than in the \(q \simeq q_0\) region, this result can be easily understood.

Thus, in spite of the fact that the modification of the polarization of the Dirac sea leads to a scaling of the \(\sigma\) and \(\omega\) effective mass in nuclear matter, we have found that its influence on the \(K^+\)-nucleus cross-section is weak and that it does not improve appreciably the agreement with experiment for the \(R_T\) ratio.
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Table 1: Modification of the vacuum polarization in nuclear matter (eq. 4 and 6) at $q = 0$ and $q = q_0$ (pole of the propagator) in the $\sigma$ and $\omega$ channels at saturation density.

| $i$ | $\Pi_i(0)$ ($MeV^2$) | $\Pi_i(q_0)$ ($MeV^2$) |
|-----|------------------------|--------------------------|
| $\sigma$ | 2.24 $10^4$ | $-1.22 \times 10^5$ |
| $\omega$ | 0. | $-2.25 \times 10^5$ |
Figure captions

fig.1: Relative variation of the effective mass for nucleons, $\sigma$ and $\omega$ mesons with nuclear density.

fig.2: Ratio $R_T$ of the $K^+ - ^{12}C$ and $K^+ - d$ total cross sections as a function of $p_{lab}$ calculated, curve (a): with the free-space $K^+N$ interaction, curve (b): with the effective mass approximation for the $\sigma$ and $\omega$ mesons, curve (c): with the expressions \[4\] and \[6\] for the polarization of the Dirac sea in the $\sigma$ and $\omega$ channels. The experimental points are taken from ref.\[11\] (circles), from ref.\[10\] (squares) and from ref.\[12\] (triangles).