A NOTE ON THE VALUES OF THE WEIGHTED \( q \)-BERNSTEIN POLYNOMIALS AND MODIFIED \( q \)-GENOCCHI NUMBERS WITH WEIGHT \( \alpha \) AND \( \beta \) VIA THE \( p \)-ADIC \( q \)-INTEGRAL ON \( \mathbb{Z}_p \)

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Abstract. The rapid development of \( q \)-calculus has led to the discovery of new generalizations of Bernstein polynomials and Genocchi polynomials involving \( q \)-integers. The present paper deals with weighted \( q \)-Bernstein polynomials and \( q \)-Genocchi numbers with weight \( \alpha \) and \( \beta \). We apply the method of generating function and \( p \)-adic \( q \)-integral representation on \( \mathbb{Z}_p \), which are exploited to derive further classes of Bernstein polynomials and \( q \)-Genocchi numbers and polynomials. To be more precise we summarize our results as follows, we obtain some combinatorial relations between \( q \)-Genocchi numbers and polynomials with weight \( \alpha \) and \( \beta \). Furthermore we derive an integral representation of weighted \( q \)-Bernstein polynomials of degree \( n \) on \( \mathbb{Z}_p \). Also we deduce a fermionic \( p \)-adic \( q \)-integral representation of product weighted \( q \)-Bernstein polynomials of different degrees \( n_1, n_2, \ldots \) on \( \mathbb{Z}_p \) and show that it can be written with \( q \)-Genocchi numbers with weight \( \alpha \) and \( \beta \) which yields a deeper insight into the effectiveness of this type of generalizations. Our new generating function possess a number of interesting properties which we state in this paper.

1. Introduction, Definitions and Notations

The \( q \)-calculus theory is a novel theory that is based on finite difference re-scaling. First results in \( q \)-calculus belong to Euler, who discovered Euler’s Identities for \( q \)-exponential functions and Gauss, who discovered \( q \)-binomial formula. The systematic development of \( q \)-calculus begins from F. H. Jackson who 1908 reintroduced the Euler Jackson \( q \)-difference operator (Jackson, 1908). One of important branches of \( q \)-calculus is \( q \)-type special orthogonal polynomials. Also \( p \)-adic numbers were invented by Kurt Hensel around the end of the nineteenth century and these two branches of number theory jointed with the link of \( p \)-adic \( q \)-integral and developed. In spite of their being already one hundred years old, these special numbers and polynomials, for instance \( q \)-Bernstein numbers and polynomials, \( q \)-Genocchi numbers and polynomials and etc. are still today enveloped in an aura of mystery within the scientific community. The \( p \)-adic integral was used in mathematical physics, for instance, the functional equation of the \( q \)-zeta function, \( q \)-stirling numbers and \( q \)-Mahler theory of integration with respect to the ring \( \mathbb{Z}_p \) together with Iwasawa’s \( p \)-adic \( q \)-L functions. Professor T. Kim [29], also studied on \( p \)-adic interpolation.
functions of special orthogonal polynomials. In the last ten years, the $q$-
Bernstein polynomials and $q$-Genocchi polynomials have attracted a lot of interest
and have been studied from different angles along with some generalizations and
modifications by a number of researchers. By using the $p$-adic invariant $q$-integral
on $\mathbb{Z}_p$, Professor T. Kim in [26], constructed $p$-adic Bernoulli numbers and polyno-
mials with weight $\alpha$. After Seo and first author in [9], extended Kim’s method for
$q$-Genocchi numbers and polynomials and also they defined $q$-Genocchi numbers
and polynomials with weight $\alpha$ and $\beta$. Our aim of this paper is to show that a
fermionic $p$-adic $q$-integral representation of product weighted $q$-Bernstein polyno-
mials of different degrees $n_1, n_2, \cdots$ on $\mathbb{Z}_p$ can be written with $q$-Genocchi numbers
with weight $\alpha$ and $\beta$.

Let $p$ be a fixed odd prime number. Throughout this paper we use the following
notations. By $\mathbb{Z}_p$ we denote the ring of $p$-adic rational integers, $\mathbb{Q}$ denotes the field
of rational numbers, $\mathbb{Q}_p$ denotes the field of $p$-adic rational numbers, and $\mathbb{C}_p$
denotes the completion of algebraic closure of $\mathbb{Q}_p$. Let $\mathbb{N}$ be the set of natural numbers and
$\mathbb{N}^* = \mathbb{N} \cup \{0\}$. The $p$-adic absolute value is defined by $|p| = \frac{1}{p}$. In this paper we
assume $|q - 1|_p < 1$ as an indeterminate. In [23-25], let $UD(\mathbb{Z}_p)$ be the space of
uniformly differentiable functions on $\mathbb{Z}_p$. For $f \in UD(\mathbb{Z}_p)$, the fermionic $p$-adic
$q$-integral on $\mathbb{Z}_p$ is defined by T. Kim:

$$ I_{-q}(f) = \int_{\mathbb{Z}_p} f(\xi) \, d\mu_{-q}(\xi) $$

$$ = \lim_{N \to \infty} \frac{1}{[p^N]_{-q}} \sum_{\xi=0}^{p^N-1} q^\xi f(\xi) (-1)^\xi. $$

For $\alpha, k, n \in \mathbb{N}^*$ and $x \in [0,1]$, T. Kim et al. defined weighted $q$-Bernstein
polynomials as follows:

$$ B_{k,n}^{(\alpha)}(x, q) = \binom{n}{k} [x]^k_{q^\alpha} [1-x]^{n-k}_{q^{-\alpha}}, \text{ (for detail, see [3, 27, 33, 34]).} $$

In (1.2), we put $q \to 1$ and $\alpha = 1$, $[x]_{q^\alpha}^k \to x^k$, $[1-x]_{q^{-\alpha}}^{n-k} \to (1-x)^{n-k}$ and we obtain the classical Bernstein polynomials (see [11, 22]),
where, $[x]_q$ is a $q$-extension of $x$ which is defined by

$$ [x]_q = \frac{1 - q^x}{1 - q}, \text{ (see [2-28, 32-34]).} $$

Note that $\lim_{q \to 1} [x]_q = x$.

In previous paper [8], for $n \in \mathbb{N}^*$, modified $q$-Genocchi numbers with weight $\alpha$
and $\beta$ are defined by Araci et al. as follows:

$$ g_{n+1,q}(x) = \int_{\mathbb{Z}_p} q^{-\beta \xi} [x + \xi]_q^n \, d\mu_{-q^\alpha}(\xi) $$

$$ = \frac{[2]_{q^\alpha}}{[\alpha]_q^n (1 - q)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{\alpha \ell} \frac{1}{1 + q^{\alpha \ell}} $$

$$ = [2]_{q^\alpha} \sum_{m=0}^{\infty} (-1)^m [m + x]_q^n. $$
In the special case, $x = 0$, $g_{n,q}^{(\alpha,\beta)}(0) = g_{n,q}^{(\alpha,\beta)}$ are called the $q$-Genocchi numbers with weight $\alpha$ and $\beta$.

In [8], for $\alpha \in \mathbb{N}^*$ and $n \in \mathbb{N}$, $q$-Genocchi numbers with weight $\alpha$ and $\beta$ are defined by Araci et al. as follows:

(1.4) $g_{n,q}^{(\alpha,\beta)}(0) = 0$, and $g_{n,q}^{(\alpha,\beta)}(1) + g_{n,q}^{(\alpha,\beta)} = \begin{cases} [2]_q^{\beta}, & \text{if } n = 1, \\ 0, & \text{if } n > 1. \end{cases}$

In this paper, we obtained some relations between the weighted $q$-Bernstein polynomials and the modified $q$-Genocchi numbers with weight $\alpha$ and $\beta$. From these relations, we derive some interesting identities on the $q$-Genocchi numbers with weight $\alpha$ and $\beta$.

2. ON THE $q$-GENOCCHI NUMBERS AND POLYNOMIALS WITH WEIGHT $\alpha$ AND $\beta$

By the definition of $q$-Genocchi polynomials with weight $\alpha$ and $\beta$, we easily get

$$g_{n+1,q}^{(\alpha,\beta)}(x) = \int_{\mathbb{Z}_p} q^{-\beta \xi} [x + \xi]_{q^\alpha}^n d\mu_{-q^\alpha}(\xi)$$

$$= \int_{\mathbb{Z}_p} q^{-\beta \xi} \left([x]_{q^\alpha} + q^{\alpha x} [\xi]_{q^\alpha}\right)^n d\mu_{-q}(\xi)$$

$$= \sum_{k=0}^{n} \binom{n}{k} [x]_{q^\alpha}^{n-k} q^{\alpha kx} \int_{\mathbb{Z}_p} q^{-\beta \xi} [\xi]_{q^\alpha}^k d\mu_{-q}(\xi)$$

$$= \sum_{k=0}^{n} \binom{n}{k} [x]_{q^\alpha}^{n-k} q^{\alpha kx} g_{k+1,q}^{(\alpha,\beta)} \frac{1}{k+1}.$$

Therefore, we obtain the following Theorem:

**Theorem 1.** For $n, \alpha, \beta \in \mathbb{N}^*$, we have

(2.1) $g_{n,q}^{(\alpha,\beta)}(x) = q^{-\alpha x} \sum_{k=0}^{n} \binom{n}{k} q^{\alpha kx} g_{k,q}^{(\alpha,\beta)} [x]_{q^\alpha}^{n-k}$.

Moreover,

(2.2) $g_{n,q}^{(\alpha,\beta)}(x) = q^{-\alpha x} \left(q^{\alpha x} g_q^{(\alpha,\beta)} + [x]_{q^\alpha}\right)^n$,

by using the umbral (symbolic) convention $(g_q^{(\alpha,\beta)})^n = g_{n,q}^{(\alpha,\beta)}$.

By expression of (1.3), we get

$$g_{n+1,q}^{(\alpha,\beta)}(1-x) \frac{1}{n+1} = \int_{\mathbb{Z}_p} q^\beta [1-x + \xi]_{q^{-\alpha}}^n d\mu_{-q^{-\alpha}}(\xi)$$

$$= \frac{[2]_q^{-\beta}}{(1-q^{-\alpha})^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{-\alpha l (1-x)} \frac{1}{1+q^{-\alpha l}}$$

$$= (-1)^n q^{\alpha n - \beta} \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{-\alpha lx} \frac{1}{1+q^{-\alpha l}}$$

$$= (-1)^n q^{\alpha n - \beta} g_{n+1,q}^{(\alpha,\beta)}(x) \frac{1}{n+1}.$$
Consequently, we obtain the following Theorem:

**Theorem 2.** The following
\[ g_{n+1,q}^{(\alpha,\beta)} (1 - x) = (-1)^n q^{\alpha n - \beta} g_{n+1,q}^{(\alpha,\beta)} (x) \]
is true.

From expression of (2.2) and Theorem 1, we get the following Theorem:

**Theorem 3.** The following identity holds
\[ g_{0,q}^{(\alpha,\beta)} = 0, \quad q^{-\alpha} \left( q^\alpha g_q^{(\alpha,\beta)} + 1 \right)^n + g_{n,q}^{(\alpha,\beta)} = \begin{cases} [2]_{q^\beta}, & \text{if } n = 1, \\ 0, & \text{if } n > 1, \end{cases} \]
with the usual convention about replacing \( g_q^{(\alpha,\beta)} \) by \( g_{n,q}^{(\alpha,\beta)} \).

For \( n, \alpha \in \mathbb{N} \), by Theorem 3, we note that
\[
q^{2\alpha} g_{n,q}^{(\alpha,\beta)} (2) = \left( q^\alpha \left( q^\alpha g_q^{(\alpha,\beta)} + 1 \right) + 1 \right)^n
= \sum_{k=0}^{n} \left( \begin{array}{c} n \\ k \end{array} \right) q^{k\alpha} \left( q^\alpha g_q^{(\alpha,\beta)} + 1 \right)^k
= \left( q^\alpha g_q^{(\alpha,\beta)} + 1 \right)^0 + n q^\alpha \left( q^\alpha g_q^{(\alpha,\beta)} + 1 \right)^1
+ \sum_{k=2}^{n} \left( \begin{array}{c} n \\ k \end{array} \right) q^{k\alpha} \left( q^\alpha g_q^{(\alpha,\beta)} + 1 \right)^k
= n q^{2\alpha} [2]_{q^\beta} - q^\alpha \sum_{k=0}^{n} \left( \begin{array}{c} n \\ k \end{array} \right) q^{nk} g_{k,q}^{(\alpha,\beta)}
= n q^{2\alpha} [2]_{q^\beta} + q^\alpha g_{n,q}^{(\alpha,\beta)}, \text{ if } n > 1.
\]

Consequently, we state the following Theorem:

**Theorem 4.** For \( n \in \mathbb{N} \), we have
\[ g_{n,q}^{(\alpha,\beta)} (2) = n [2]_{q^\beta} + \frac{g_{n,q}^{(\alpha,\beta)}}{q^\alpha}. \]

From expression of Theorem 2 and (2.3), we easily see that
\[
(n + 1) q^{-\beta} \int_{\mathbb{Z}_p} q^{-\beta \xi} [1 - \xi]_{q^{-\alpha}}^n d\mu_{-q^\alpha} (\xi)
= (-1)^n q^{n\alpha - \beta} \int_{\mathbb{Z}_p} q^{-\beta \xi} [\xi - 1]_{q^n}^n d\mu_{-q^n} (\xi)
= (-1)^n q^{n\alpha - \beta} g_{n+1,q}^{(\alpha,\beta)} (-1) = g_{n+1,q}^{(\alpha,\beta)} (2).
\]

Thus, we obtain the following Theorem:

**Theorem 5.** The following identity
\[ (n + 1) q^{-\beta} \int_{\mathbb{Z}_p} q^{-\beta \xi} [1 - \xi]_{q^{-\alpha}}^n d\mu_{-q^\alpha} (\xi) = g_{n+1,q}^{(\alpha,\beta)} (2) \]
is true.
Let $n, \alpha \in \mathbb{N}$. By expression of Theorem 4 and Theorem 5, we get

\begin{equation}
(2.5) \quad (n + 1)q^{-\beta} \int_{\mathbb{Z}_p} q^{-\beta} [1 - \xi]_\alpha^n d\mu_{-q^\delta}(\xi) = (n + 1)q^{-\beta}[2]_{q^\delta} + q^{\alpha}g^{(\alpha, \beta)}_{n+1, q^{-1}}.
\end{equation}

For (2.5), we obtain corollary as follows:

**Corollary 1.** For $n, \alpha \in \mathbb{N}^*$, we have

\[\int_{\mathbb{Z}_p} q^{-\beta} [1 - \xi]_\alpha^n d\mu_{-q^\delta}(\xi) = [2]_{q^\delta} + q^{\alpha}g^{(\alpha, \beta)}_{n+1, q^{-1}}.\]

3. Novel identities on the weighted $q$-Genocchi numbers

In this section, we develop modified $q$-Genocchi numbers with weight $\alpha$ and $\beta$, namely, we derive interesting and worthwhile relations in Analytic Number Theory.

For $x \in \mathbb{Z}_p$, the $p$-adic analogues of weighted $q$-Bernstein polynomials are given by

\begin{equation}
B^{(\alpha)}_{k,n}(x) = \binom{n}{k} \left[ x \right]_q^{k} \left[ 1 - x \right]_q^{n-k}, \quad \text{where } n, k, \alpha \in \mathbb{N}^*.
\end{equation}

By expression of (3.1), Kim et. al. get the symmetry of $q$-Bernstein polynomials weight $\alpha$ as follows:

\begin{equation}
B^{(\alpha)}_{k,n}(x) = B^{(\alpha)}_{n-k,n}(1 - x, q^{-1}), \quad \text{for detail, see [27]).}
\end{equation}

Thus, from Corollary 1, (3.1) and (3.2), we see that

\[\int_{\mathbb{Z}_p} B^{(\alpha)}_{k,n}(\xi, q) q^{-\beta} d\mu_{-q^\delta}(\xi) = \int_{\mathbb{Z}_p} B^{(\alpha)}_{n-k,n}(1 - \xi, q^{-1}) q^{-\beta} d\mu_{-q^\delta}(\xi) = \left( \binom{n}{k} \sum_{l=0}^{k} \binom{k}{l} (-1)^{k+l} \int_{\mathbb{Z}_p} q^{-\beta} [1 - \xi]_{q^{-1}}^{n-l} d\mu_{-q^\delta}(\xi) \right).
\]

For $n, k \in \mathbb{N}^*$ and $\alpha \in \mathbb{N}$ with $n > k$, we obtain

\begin{equation}
(3.3) \quad \int_{\mathbb{Z}_p} B^{(\alpha)}_{k,n}(\xi, q) q^{-\beta} d\mu_{-q^\delta}(\xi) = \left( \binom{n}{k} \sum_{l=0}^{k} \binom{k}{l} (-1)^{k+l} \left( [2]_{q^\delta} + q^{\alpha}g^{(\alpha, \beta)}_{n-l+1, q^{-1}} \right) \right) = \begin{cases} [2]_{q^\delta} + q^{\alpha}g^{(\alpha, \beta)}_{n+1, q^{-1}}, & \text{if } k = 0, \\ [2]_{q^\delta} + q^{\alpha}g^{(\alpha, \beta)}_{n+l+1, q^{-1}}, & \text{if } k > 0. \end{cases}
\end{equation}
Let us take the fermionic $p$-adic $q$-integral on $\mathbb{Z}_p$ on the weighted $q$-Bernstein polynomials of degree $n$ as follows:

$$(3.4)\int_{\mathbb{Z}_p} B^{(\alpha)}_{k,n}(\xi, q) q^{-\beta \xi} d\mu_{-q^\alpha}(\xi) = \left( \begin{array}{c} n \\ k \end{array} \right) \sum_{l=0}^{n-k} \binom{n-k}{l} (-1)^l \frac{[2] q^\alpha + q^\alpha g^{(\alpha, \beta)}_{n+k+1,q}}{l+k+1},$$

where $k, l, n, \alpha, \beta \in \mathbb{N}$.

Consequently, by expression of (3.3) and (3.4), we state the following Theorem:

**Theorem 6.** The following identity holds

$$\sum_{l=0}^{n-k} \binom{n-k}{l} (-1)^l \frac{[2] q^\alpha + q^\alpha g^{(\alpha, \beta)}_{n+k+1,q}}{l+k+1} = \left\{ \begin{array}{ll}
[2] q^\alpha + q^\alpha g^{(\alpha, \beta)}_{n+1+q^\alpha}
 & \text{if } k = 0,
\sum_{l=0}^{k} \binom{k}{l} (-1)^{k+l} \left[2 q^\alpha + q^\alpha g^{(\alpha, \beta)}_{n+1+q^\alpha} \right]
 & \text{if } k > 0.
\end{array} \right.$$
Thus, we can obtain the following Corollary:

**Corollary 2.** For $n_1, n_2, k \in \mathbb{N}^\ast$ and $\alpha \in \mathbb{N}$ with $n_1 + n_2 > 2k$, we have

$$
\sum_{l=0}^{n_1 + n_2 - 2k} \binom{n_1 + n_2 - 2k}{l} (-1)^l \frac{g^{(\alpha, \beta)}_{l+2k+1, q}}{l+2k+1} = \begin{cases} 
[2]_q \beta + q^{\alpha-\beta} \frac{g^{(\alpha, \beta)}_{n_1+n_2+1, q^{-1}}}{n_1+n_2+1}, & \text{if } k = 0, \\
\sum_{l=0}^{2k} \binom{2k}{l} (-1)^{2k+l} \left( [2]_q + q^{\alpha-\beta} \frac{g^{(\alpha, \beta)}_{n_1+n_2+l+1, q^{-1}}}{n_1+n_2+l+1} \right), & \text{if } k \neq 0.
\end{cases}
$$

For $\xi \in \mathbb{Z}_p$ and $s \in \mathbb{N}$ with $s \geq 2$, let $n_1, n_2, \ldots, n_s, k \in \mathbb{N}^\ast$ and $\alpha \in \mathbb{N}$ with $\sum_{l=1}^{s} n_l > sk$. Then we take the fermionic $p$-adic $q$-integral on $\mathbb{Z}_p$ for the weighted $q$-Bernstein polynomials of degree $n$ as follows:

$$
\int_{\mathbb{Z}_p} B_{k,n_1}^{(\alpha)} (\xi, q) B_{k,n_2}^{(\alpha)} (\xi, q) \ldots B_{k,n_s}^{(\alpha)} (\xi, q) q^{-\beta \xi} d\mu_{-q^\alpha} (\xi)
= \prod_{i=1}^{s} \left( \frac{n_i}{k} \right) \int_{\mathbb{Z}_p} [\xi]_{q^\alpha}^{sk} \left[ 1 - \xi \right]_{q^{-\alpha}}^{n_1+n_2+\ldots+n_s-sk} q^{-\beta \xi} d\mu_{-q^\alpha} (\xi)
= \prod_{i=1}^{s} \left( \frac{n_i}{k} \right) \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{l+sk} \int_{\mathbb{Z}_p} q^{-\beta \xi} \left[ 1 - \xi \right]_{q^{-\alpha}}^{n_1+n_2+\ldots+n_s-sk} d\mu_{-q^\alpha} (\xi)
= \begin{cases} 
[2]_q \beta + q^{\alpha-\beta} \frac{g^{(\alpha, \beta)}_{n_1+n_2+\ldots+n_s+l+1, q^{-1}}}{n_1+n_2+\ldots+n_s+l+1}, & \text{if } k = 0, \\
\prod_{i=1}^{s} \left( \frac{n_i}{k} \right) \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{sk+l} \left( [2]_q + q^{\alpha-\beta} \frac{g^{(\alpha, \beta)}_{n_1+n_2+\ldots+n_s+l+1, q^{-1}}}{n_1+n_2+\ldots+n_s+l+1} \right), & \text{if } k \neq 0.
\end{cases}
$$

So from above, we have the following Theorem:

**Theorem 8.** For $s \in \mathbb{N}$ with $s \geq 2$, let $n_1, n_2, \ldots, n_s, k \in \mathbb{N}^\ast$ and $\alpha \in \mathbb{N}$ with $\sum_{l=1}^{s} n_l > sk$. Then we have

$$
\int_{\mathbb{Z}_p} q^{-\beta \xi} \prod_{i=1}^{s} B_{k,n_i}^{(\alpha)} (\xi) d\mu_{-q} (\xi)
= \begin{cases} 
[2]_q \beta + q^{\alpha-\beta} \frac{g^{(\alpha, \beta)}_{n_1+n_2+\ldots+n_s+l+1, q^{-1}}}{n_1+n_2+\ldots+n_s+l+1}, & \text{if } k = 0, \\
\prod_{i=1}^{s} \left( \frac{n_i}{k} \right) \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{sk+l} \left( [2]_q + q^{\alpha-\beta} \frac{g^{(\alpha, \beta)}_{n_1+n_2+\ldots+n_s+l+1, q^{-1}}}{n_1+n_2+\ldots+n_s+l+1} \right), & \text{if } k \neq 0.
\end{cases}
$$
From the definition of weighted \( q \)-Bernstein polynomials and the binomial theorem, we easily get

\[
\int_{\mathbb{Z}_p} q^{-\beta \xi} \prod_{k,n_1}^n \left( \sum_{d=1}^n (n_d - k) \right) (-1)^l \int_{\mathbb{Z}_p} q^{-\beta \xi} [\xi]^{sk+l} d\mu_{-q^\alpha} (\xi)
\]

\[
= \prod_{i=1}^s \left( \begin{array}{l} n_i \\ k \end{array} \right) \sum_{l=0}^{n_i+n_s-sk} \left( \sum_{d=1}^n (n_d - k) \right) (-1)^l \frac{g_i^{(\alpha,\beta)}}{l+sk+1}
\]

Therefore, from (3.6) and Theorem 8, we get interesting Corollary as follows:

**Corollary 3.** For \( s \in \mathbb{N} \) with \( s \geq 2 \), let \( n_1, n_2, ..., n_s, k \in \mathbb{N}^* \) and \( \alpha, \beta \in \mathbb{N} \) with \( \sum_{i=1}^s n_i > sk \). We have

\[
\sum_{i=0}^{n_1+n_2-n_sk} \left( \sum_{d=1}^n (n_d - k) \right) (-1)^l \frac{g_i^{(\alpha,\beta)}}{l+sk+1} = \left\{ \begin{array}{ll}
[2]_{q^\alpha} + q^{1-\beta} \frac{g_i^{(\alpha,\beta)}}{n_1+n_2+...+n_s+i-1}, & \text{if } k = 0,

\sum_{i=0}^{sk} \left( \begin{array}{l} sk \\ i \end{array} \right) (-1)^{sk+i} \left[ [2]_{q^\alpha} + q^{1-\beta} \frac{g_i^{(\alpha,\beta)}}{n_1+n_2+...+n_s+i-1} \right], & \text{if } k \neq 0.
\end{array} \right.
\]

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