Retrieval of Research-level Mathematical Information Needs: A Test Collection and Technical Terminology Experiment

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Abstract

In this paper, we present a test collection for mathematical information retrieval composed of real-life, research-level mathematical information needs. Topics and relevance judgements have been procured from the on-line collaboration website MathOverflow by delegating domain-specific decisions to experts on-line. With our test collection, we construct a baseline using Lucene’s vector-space model implementation and conduct an experiment to investigate how prior extraction of technical terms from mathematical text can affect retrieval efficiency. We show that by boosting the importance of technical terms, statistically significant improvements in retrieval performance can be obtained over the baseline.

1 Introduction

Since their introduction through the Cranfield experiments (Cleverdon, 1960; Cleverdon, 1962; Cleverdon et al., 1966a; Cleverdon et al., 1966b), test collections have become the foundation of information retrieval (IR) evaluation.

Recent interest in Mathematical information retrieval (MIR) has prompted the construction of the NTCIR Math IR test collection (Aizawa et al., 2013). Like many general-purpose, domain-specific IR test collections, the NTCIR collection is composed of broad queries intended to test systems over a wide spectrum of query complexity.

In this paper we present a test collection composed of real-life, research-level mathematical topics and associated relevance judgements procured from the online collaboration web-site MathOverflow. The resulting test collection contains 160 atomic questions - material derived from 120 MathOverflow discussion threads.

Topics in our test collection capture specialised information needs that are complex to resolve and often demand collective effort from multiple domain experts. For example:

The "most symmetric" Mukai-Umemura 3-fold with automorphism group $PGL(2, \mathbb{C})$ admits a Kaehler-Einstein metric according to Donaldson’s result. On the contrary, there are some arbitrarily small complex deformations of the above 3-fold which do not admit Kaehler-Einstein metrics, as shown by Tian. All examples considered by Tian seem to have no symmetries at all. Is it possible to find similarly arbitrarily small complex deformations with $\mathbb{C}^*$-action and which do not admit any Kaehler-Einstein metric?

Due to their specialised nature, our topics have a relatively small number of relevant documents. Fortunately, there is precedent of this from IR tasks such as QA (Ishikawa et al., 2010) and known-item search (Craswell et al., 2003).

With our test collection, we construct a baseline using Lucene’s default implementation of the vector space model (VSM). Additionally, we conduct an experiment designed to investigate the hypothesis that technical terms in mathematics have elevated retrieval significance.

Information in mathematics is communicated by defining, manipulating and otherwise operating on mathematical structures and objects which can be instantiated in the mathematical discourse. In this sense, technical terminology in mathematics has an elevated role. This hypothesis stems from the observation that the mathematical discourse is dense with named mathematical objects, structures, properties and results.

2 Adapted from MathOverflow post 68096, http://mathoverflow.net/questions/68096/
In the next section, we present our test collection and discuss the procedure for its construction from crowd-sourced expertise on MathOverflow. In section 3, we discuss related material in the literature and compare it to our work. Our experimental setup and results are discussed in section 4, with a brief summary of our work presented in section 5.

2 The Test Collection

The main motivation behind this work comes from our long-term goal to develop and evaluate MIR models intended to satisfy research-level mathematical information needs. Evaluation is an important final step in the development of IR models and is preconditioned on the availability of a test collection.

A test collection is a resource composed of (1) a document collection (or corpus) with uniquely identifiable documents (e.g., scientific papers, news articles), (2) a set of topics from which search queries can be produced and (3) a set of relevance judgements: pairs connecting individual topics to documents (in the corpus) known to satisfy the corresponding information need.

General-purpose MIR test collections, such as the one produced for NTCIR-10 (Aizawa et al., 2013), are expected to contain both broad and narrow topics capturing a wide range of retrieval complexity. In contrast, we require a collection of topics characterised by a higher lower bound on topic complexity with individual topics capturing highly-specialised, real-world information needs.

Unfortunately, research-level mathematical information needs are hard to source from documents in a way that would not render them artificial. Furthermore, manual construction of topics and relevance judgements is unrealistic due to the large number of experts required to cover the various specialised sub-fields of mathematics. This, coupled with limited access to numerous MIR systems, makes TREC-like pooling (Harman, 1993; Voorhees and Harman, 2005) impractical.

We propose that topics and relevance judgements be procured from the on-line collaboration website MathOverflow (MO), an online QA site for research mathematicians. A user (information seeker) can post a question on the site, usually relating to a small niche field in mathematics. Colleagues can either post a candidate answer, comment on the question, comment on and/or upvote existing answers. Ultimately, the information seeker decides which answer satisfies the underlying information need by marking it as “accepted”.

Material on MO is closely aligned with our requirements. Specifically, Tausczik et al. (2014) and Martin and Pease (2013) agree that MO questions (information needs) arise from doing mathematics research and are novel to the mathematician involved. The authors conclude that, having been produced by experts, MO answers are authoritative and partially credit the website’s reward system for their strong reliability.

MO questions often have multiple sub-parts, which we refer to as micro-topics since they encode atomic information needs. Furthermore, information in MO questions is carried by two types of sentences: prelude sentences, which are used to set the mathematical context (introduce mathematical constructs and results) and query sentences, which transcribe the information need itself and are semantically bound to the accompanying prelude.

As the underlying document collection, we have used the Mathematical Retrieval Corpus (MREC)\(^3\) (Líška et al., 2011), which contains more than 439,000 mathematical publications, complete with mathematical formulae converted to machine-readable MathML. Similarly, we have made mathematical expressions in our topics accessible to MIR systems by converting all \LaTeX\ embedded in MO questions into MathML using the \LaTeXXML\ tool-kit.

For the purpose of constructing our test collection we have adopted a multi-step process. All steps in the process are systematically applicable regardless of the subject material of the topic being considered for inclusion. As such, our test collection can be as diverse, in terms of mathematical subject and sub-fields, as MathOverflow.

Table 1: MO post 14655, prelude and micro-topics

| Prelude | 1) Apparently, physicist can calculate the GW invariants of quintic CY 3-fold up to genus 51.  
|         | 2) For each genus \(g\), there is a lower bound \(d(g)\) such that for every \(d < d(g)\), all genus \(g\) degree \(d\) invariants of quintic are zero. |
| MT-1    | I am looking for a reference that has a table of these number for some low degrees (say up to degree 5) and low genera (at least until \(g \geq 3\)). |
| MT-2    | Where can I found this lower bound? |

\(^3\)version 2011.4.439
Decisions relating to relevance of material to a given topic (MO question) are delegated to experts on the website. However, the information seeker (MO user posting the question) remains the ultimate judge of relevance. This authority is typically exercised by either accepting an answer directly or, by explicitly commenting on the relevance of posted material.

In the first step, all MO discussion threads with at least one citation to the MREC in their accepted answer were collected. Each identified thread was examined by one of the authors for conformance to two ideal-standard criteria: (1) Useful MO questions should not be too broad or vague but rather express an information need that is clear and can be satisfied by describing objects or properties, stating conditions and/or producing examples or counter-examples. (2) MREC documents cited in MO accepted answers should address all sub-parts of the question in a manner that requires minimal deduction and do not synthesise mathematical results from multiple resources.

Subsequently, relevance of documents for each micro-topic is decided using two criteria: totality and directness. A cited resource is total if it contains all necessary information to derive the answer for the micro-topic and partial if it only addresses a special case. A cited resource is also said to be direct, if the answer can be derived with little intellectual effort from its text, or indirect if the same information requires considerable effort (such as mathematical deduction or reasoning) for the information seeker to reproduce.

Making these determinations involves matching the language of arguments and the symbolic context of the answer to the cited resource. As part of this step, we also examine the post-answer (PA) comments for expressions of confirmation of the usefulness of a cited resource from the information seeker.

The completed test collection contains 160 micro-topics with 184 associated relevance judgements (involving 224 unique MREC documents) organised in 120 topics. Topic text in our collection is sentence tokenised, with relevance judgements being represented conceptually as tuples of the form:

$$ (\text{Topic ID}, \text{sentence ID}, \text{Micro-topic ID}, \text{relevant MREC document ID}) $$

From Table 2 we observe that the vast majority of topics (93.33%) have either 1 or 2 micro-topics, with the average being close to 1 (1.33). The majority of topics (97, 80.83%) have only one relevant document while a further 21 (17.5%) have two relevant documents. Two topics have more than 2 relevant documents: one with 3 and another with 4. In terms of micro-topics, this corresponds to 140 micro-topics (87.50%) with 1 relevant document, 17 (10.625%) with 2, 2 micro-topics (1.25%) with 3 and just one (0.625%) with 4 relevant documents.

### Table 2: Topic/Micro-topic break-down

| Micro-topics | 1 | 2 | 3 | ≥ 4 |
|-------------|---|---|---|----|
| Instances (topics) | 88 | 24 | 8 | 0 |
| Percentage | 73.33% | 20.00% | 6.67% | 0% |

3 Related Work

Test collections over scientific publications were first introduced for the Cranfield experiments (Cleverdon, 1960; Cleverdon, 1962; Cleverdon et al., 1966a; Cleverdon et al., 1966b). Despite criticism for sourcing queries from collection documents, the Cranfield experiments highlighted the importance of jointly reporting recall and precision, pioneered the practice of using authors and citations for augmenting relevance judgements and established the test collection paradigm.

Expert citations have already been exploited for procuring relevance judgements. For example, Ritchie et al. (2006) elicited relevance judgements for citations in papers accepted in a scientific conference from their authors and used these judgements as part of their test collection of scientific publications.

In terms of domain, our work is related to the NTCIR-10 Math IR test collection (Aizawa et al., 2013). Furthermore, the topics in our collection are analogous to those in the NTCIR full-text search, in the sense that they take the form of coherent text interspersed with mathematical expressions. Rather than being focused on accommodating information needs of varying complexity, however, our test collection has been designed to facilitate retrieval of highly specialised, mathematical information needs of uniformly high complexity.

Similar use of crowd-sourced expertise has been proposed in the context of QA. For example, Gyongyi et al. (2008), examined 10 months-worth of “Yahoo! Answers” material as part of an investigation of QA data, which was later used for the
the beginning of the section is achieved since no overlap beyond the prelude is introduced between queries generated for micro-topics attached to a given topic.

4.3 Systems

Using Lucene as the indexing and searching backend, we compare the performance of two retrieval methods. Underpinning both methods is Lucene’s default similarity (project, 2013), which is based on cosine similarity:

\[ \text{sim}(q,d) = \frac{V(q) \cdot V(d)}{|V(q)||V(d)|} \]

where \( V(q) \) and \( V(d) \) are weighted vectors for the query and candidate document respectively.

As a performance measure, we use mean average precision (MAP):

\[ \text{MAP}(Q) = \frac{1}{|Q|} \sum_{j=1}^{m} \frac{1}{m_j} \sum_{k=1}^{m_j} \text{Precision}(R_{jk}) \]

4.3.1 Baseline

Lucene’s VSM implementation with default TF-IDF weighting and scoring is used as the baseline. This is intended to emulate a general-purpose information retrieval scenario, which is the motivation behind the design of Lucene’s default configuration.

4.3.2 Boosted Technical Terms

The alternative model is designed to give more weight to technical terminology common to both documents and queries. In order to construct this model, all technical terms are extracted from the document collection using an implementation of the C-Value multi-word technical term extraction method (Frantzi et al., 1996; Frantzi et al., 1998). Given an input corpus, the C-Value method extracts multi-word terms by making use of a linguistic and a statistical component.

The linguistic component is responsible for eliminating multi-term strings that are unlikely to be technical terms through the use of a stop-list (composed of high-frequency corpus terms) and linguistic filters (regular expressions) applied on sequences of part-of-speech tags. The statistical component assigns a “termhood” score to a candidate term sequence based on corpus-wide statistical characteristics of the sequence itself and those of term sequences that contain it. The output of
the algorithm is a list of candidate technical terms in the corpus, ordered by their C-Value termhood score.

As shown in Table 3, each entry in the resulting list represents a single technical term (the class) and enumerates all forms of the candidate term as observed in the input corpus. In total, 3 million classes of technical terms have been detected in the MREC. Using Lucene’s positional indexing mechanism, we retrieved the position of each technical term (all forms), recorded its term frequency (TF) and produced a new technical term index. This technical term index contains 426 million tuple entries of the form

\[ \langle \text{class, form, MREC docid, TF, position of occurrence list} \rangle \]

The same re-indexing process is repeated for the queries and the result is stored in a separate query table (10,433 entries).

Subsequently, the indexed document and query term vectors were modified by (1) adding new tokens to represent technical term phrases and (2) re-attributing the TF of component terms to the term vector of the phrase.

Finally, the text for each MREC document and query is re-generated from the term vectors and stored in a “delta index”. At this stage, the number of technical term instances emitted is twice that recorded by the original term vector. This has the effect of boosting the significance of technical terms and phrases. An example of the application of this process, from original text to delta index generation is presented in Table 4. Rankings for the alternative model can be obtained by searching the delta index using the re-generated query.

### 5 Conclusions and Further Work

We have constructed a Math IR test collection for real-life, research-level mathematical information needs. As part of the work of constructing our test collection, we have developed a methodology for compiling domain-specific test collections that requires minimal expertise in the domain itself.

Using 160 micro-topics in our test collection, we have shown experimentally that the performance of VSM-based retrieval models with research mathematics can be improved by boosting the importance of technical terminology. Furthermore, our experimental work suggests that our test collection can be used to identify statistically significant differences between MIR systems. It is our intention to make our collection available to the IR community.

As part of on-going and future work, we will be incorporating additional retrieval models, such as the Okapi BM25, in our evaluation framework. In addition, we are looking into investigating the statistical properties of our test collection along the lines of Harman (2011) and Soboroff et al. (2001).

### Table 3: C-Value technical-term list entry

| Class/Form 1 | Form 2 | Form 8 | C-Value |
|--------------|--------|--------|---------|
| Riemannian manifold | Riemannian manifold | RIEMANNIAN MANIFOLDS | 13256.6 |

### Table 4: Example of re-attribution and delta index

| Original Text | Original Term vector | Technical terms | Re-Attributed Term Vector | Re-generated delta index text |
|---------------|----------------------|-----------------|---------------------------|-----------------------------|
| a Riemannian manifold is a smooth manifold | (a,2), (Riemannian,1),(manifold,2),(is,1),(smooth,1) | Riemannian manifold, smooth manifold | (a,2), (Riemannian,1),(manifold,1),(is,1),(smooth,1) | a a a a Riemannian manifold is a smooth manifold |

### Table 5: Difference in MAP performance between models (* statistically significant at \( \alpha = 0.05 \))

|          | Baseline | Tech-Term boosting | Difference |
|----------|----------|--------------------|------------|
| MAP      | 0.0002   | 0.0732             | 0.013* (17.7%) |

Although the choice of boosting factor 2 is arbitrary, our intention is to demonstrate the presence of a difference in retrieval efficiency, rather than optimising the effect of boosting.

### 4.4 Results

The MAP scores obtained for the models are presented in Table 5. We observe that the difference in MAP is in favour of the alternative model. This difference is statistically significant at \( \alpha = 0.05 \) using the Wilcoxon signed-rank test \( p < 0.05 \). Therefore, we have sufficient evidence to conclude that, in the context of the VSM, boosting technical terms improves retrieval efficiency of research mathematics.

When compared to MAP scores produced by the same systems in more traditional IR tasks, the scores in Table 5 may seem poor. We attribute this phenomenon to the fact that sense in written mathematics is communicated via a complex interaction of text and mathematical expressions and is thus hard to extract using shallow methods.
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