Investigation of the stability of spatial modes in quarter-disk resonators

M S Dunaevskiy1,* and P A Alekseev1

1 Ioffe Institute, Saint-Petersburg, 194021, Russia

* e-mail: Mike.Dunaevskiy@mail.ioffe.ru

Abstract. In this work, the stability of short-wavelength spatial modes in quarter-disk cavities was studied. It was found that in such resonators, short-wave modes (with wavelengths $\lambda$ smaller than the radius $R$ of the quarter-disk $\lambda<<R$) in space form figures similar to folded twice $m$-polygons. The $m$-order of allowed stable modes decreases rapidly with an increase in the $\delta/R$ parameter of deviation from the ideal quarter-disk shape. Unlike half-disk resonators, quarter-disk resonators with $\delta/R>0.1$ lack total internal reflection modes.

1. Introduction
Two-dimensional disk-shaped optical resonators can support the so-called whispering gallery modes and have a high quality factor [1]. However, disk-shaped resonators do not allow directional light output, which limits their use in light-emitting devices. The study of asymmetric resonant cavities and the search for the optimal cavity geometry, which allows combining high quality factor and directional radiation output, is an urgent task today. One of the rather simple geometries that allows one to obtain a directed radiation output is either a half-disk [2] or a quarter-disk resonator [3]. It is worth noting that often such resonators are obtained by cleaving the whole disk-shaped resonators. In this paper, we will consider resonators slightly smaller than the quarter-disk. These resonators are partially chaotic resonators, that is, they can support both stable modes and chaotic modes. A detailed study of such resonators has not yet been performed by other authors, i.e., the study performed in this paper contains some novelty. The study of possible stable and unstable short-wave modes in quarter-disk resonators will be performed using geometric optics methods. In this work, we will use for the analysis the methods of geometric optics, which are in good agreement with experiment in the case when the wavelength $\lambda$ is smaller than the characteristic size of the resonator (in our case, this is the radius $R$ of curvature of a quarter-disk) $\lambda<<R$. We will consider in this work only this case ($\lambda<<R$) and will call it the short-wave approximation. The types of stable modes and the shape of the beam of rays forming stable modes will be determined using these methods.

2. Object and methods
In this work, a resonator similar in shape to a quarter-disk was investigated. Figure 1 shows this type of resonator, it is formed by an arc of a circle of radius $R$, as well as X- and Y-chords, indented $\delta$ from the center. This shape can be achieved by two cleavages (along X and along Y) of the initially disk-shaped resonator [3]. In this paper, the deviation parameter $\delta$ is chosen the same for both the X- and
Y-chords. Here, this was done just for simplicity, but in further works it is supposed to perform an analysis of mode stability for two independent values of deviation ($\delta_x$ and $\delta_y$).

Calculations in the framework of geometric optics were carried out using the method of Poincare diagrams (PSOS-Poincare section of surface). The PSOS-diagram method allows one to study the stability of modes in 2D-resonators. The PSOS-diagrams are constructed in the following way: (i) the optical ray is launched into the resonator and the following are then monitored: (ii) the positions of the points on the resonator boundary $s_i$ in which the ray undergoes reflections, and (iii) the angles of reflection from the resonator boundaries $\theta_i$. Then, for the whole set of rays in the resonator, a diagram is constructed in the coordinates $(\sin(\theta_i); s_i)$ or $(\theta_i; s_i)$ — this is the PSOS-diagram [4]. Stable trajectories in the PSOS-diagram will correspond to closed figures, and unstable trajectories will correspond to the “dust” of random points called “chaotic sea” [5]. Using the PSOS-diagram method, it is possible to determine in the 2D-resonator the presence of stable modes and the so-called “chaotic” modes. Chaotic modes correspond to trajectories of rays that do not close and, starting from a certain moment, move randomly in the resonator.

![Figure 1. The geometric scheme of the investigated resonators.](image)

Using the method of PSOS-diagrams allows not only to determine the class of stable modes in the resonator under study, but also to distinguish among them modes with total internal reflection (TIR-modes). On the PSOS-diagrams, TIR-modes corresponds to “figures” entirely placed above the TIR-angle $\theta_{\text{TIR}}=\arcsin(1/n)$. Also, with the help of PSOS-diagrams, it is possible to determine the light emission regions at the cavity boundary, as well as the characteristic divergence angles of light emission for non-TIR modes.

3. Results and discussion

Using the method of PSOS-diagrams, it was made a study of the stability of modes in quarter-disk resonators depending on the $\delta$ parameter of deviation from the shape of an ideal quarter-disk. In the obtained Poincare diagrams (see Figure 2 for $\delta/R=0.2$ cavity), one can observe both the “chaotic sea” and “islands” corresponding to stable modes. The stable mode rays trajectories form figures in real space that are double folded regular m-polygons (e.g. m=4 “double folded square” at Figure 2b). For convenience, these modes can be characterized by a number m that is the order of the corresponding polygon. In the case of modes with rational m, folded star-like figures are formed by rays. An example of such a mode is a mode with m=8/3 (see Figure 2c) which is stable in $\delta/R=0.2$ cavity. This mode has the shape of a folded octagram. As it can be seen from the PSOS-diagram (see Figure 2a) for a resonator with the deviation parameter $\delta/R=0.2$, only three modes m=2, m=8/3, and m=4 are stable. It
is worth noting that stable modes in the framework of geometric optics can be characterized by two main parameters: 1) the mode number \( m \) (associated with the mode shape in real space) and 2) the divergence angle \( \theta \). It should be noted also, that for each stable group of modes of order \( m \), one can determine from the PSOS-diagram the limiting divergence angle \( \theta_{\text{max}} \), which corresponds to the size of the stability island of the corresponding mode in the diagram. The divergence angle is an important characteristic that determines the angular divergence of the light emitted from the resonator.

![PSOS-diagram](image)

**Figure 2.** (a) – PSOS-diagram for a resonator with a deviation parameter \( \delta/R = 0.2 \), (b) – beam pattern corresponding to “folded square” mode \( m=4 \) (red areas on the PSOS-diagram), (c) – beam pattern corresponding to “folded octagram” mode \( m=8/3 \) (green areas on the PSOS-diagram).

The Poincare diagrams were calculated for three resonators with a \( \delta/R \) parameter of 0.1, 0.2 and 0.3. Table 1 provides information on the types of stable modes (order \( m \)) and limiting divergence angles. It can be seen that as the deviation \( \delta/R \) from the shape of the ideal quarter-disk increases, chaotization of high-order modes and part of low-order modes occurs. The most stable are the modes of order \( m=2 \) (“bouncing ball” mode) and \( m=4 \) (“folded square” mode). Also, our calculations show that with increasing \( \delta/R \), the divergence angles of the stable modes gradually decrease (see Table 1).

**Table 1.** Stable modes in quarter-disk resonators with a deviation parameter \( \delta/R \) of 0.1, 0.2, and 0.3.

| \( \delta/R = 0.1 \) | \( \delta/R = 0.2 \) | \( \delta/R = 0.3 \) |
|------------------|------------------|------------------|
| \( m \)         | \( \theta_{\text{max}} \) | \( m \)         | \( \theta_{\text{max}} \) | \( m \)         | \( \theta_{\text{max}} \) |
| 2               | 0.72             | 2               | 0.7             | 2               | 0.65             |
| 12/5            | 0.1              | -               | -               | -               | -               |
| 8/3             | 0.25             | 8/3             | 0.2             | -               | -               |
| 3               | 0.1              | -               | -               | -               | -               |
| 4               | 0.7              | 4               | 0.6             | 4               | 0.5              |
| 6               | 0.1              | -               | -               | -               | -               |
| 8               | 0.35             | -               | -               | -               | -               |
Low m-value modes in these cavities are not located near the curved edge like traditional whispering gallery modes, but are "shifted" away from the edge, which is consistent with experimentally observed results [6]. It should be noted also, that in the investigated quarter-disk resonators with $\delta/R>0.1$ there are no TIR-modes. This property qualitatively distinguishes quarter-disk resonators from half-disk resonators in which TIR-modes play an important role.

4. Conclusion

The calculations of the PSOS-diagrams indicate that in small quarter-disk resonators, with an increase in deviation parameter $\delta$, the number of stable modes decreases rapidly. One can say that with increasing $\delta$, the modes with a high m-order (traditional whispering gallery modes) are absorbed by chaotic sea. The m-orders of stable modes in quarter-disk resonators are different from those in the corresponding half-disk resonators. Unlike half-disk resonators, quarter-disk resonators with $\delta/R>0.1$ lack total internal reflection modes.

References

[1] McCall S L, Levi A F J, Slusher R E, Pearton S J and Logan R A 1992 Appl. Phys. Lett. 60, 20
[2] Alekseev P A, Dunaevskiy M S, Monakhov A M, Dudelev V V, Sokolovskii G S, Arinero R, Teissier R and Baranov A N 2018 Optics Express 26 14433
[3] Monakhov A M, Sherstnev V V, Astakhova A P, Yakovlev Yu P, Boissier G, Teissier R and Baranov A N 2009 Appl. Phys. Lett. 94 051102
[4] Nöckel J U and Stone A D 1997 Nature 385 45
[5] Cao H and Wiersig J 2015 Reviews of Modern Physics 87 61
[6] Dunaevski M S, Alekseev P A, Baranov A N, Monakhov A M, Teissier R, Arinero R, Girard P and Titkov A N 2013 Appl. Phys. Lett. 103 053120