Sensitivity analysis of geometric error for a novel slide grinder based on improved Sobol method and its application

Jihui Han1,2 · Liping Wang1,2,3 · Fengju Ma1,2,4 · Ziyang Ge3 · Dong Wang1,2 · Xuekun Li1,2

Received: 20 April 2022 / Accepted: 9 July 2022 / Published online: 20 July 2022
© The Author(s), under exclusive licence to Springer-Verlag London Ltd., part of Springer Nature 2022

Abstract
In order to improve both the accuracy and efficiency of grinding slide, this paper designs a novel grinder with dual-lead-dual-head. Since the geometric error is one of the major contributors causing machine inaccuracies, the sensitivity analysis is performed to identify the critical geometric error terms, and an application to accuracy self-test is also given. The volumetric model of the grinder with 38 geometric errors is built using the homogeneous transformation matrix (HTM) and multi-body system (MBS) theory. An improved Sobol method with quasi-Monte-Carlo algorithm is utilized to perform the global sensitivity analysis (GSA) in the entire workspace. The particular sensitivity analysis is further carried out on the basis of the machining characteristics of a typical slide. All sensitivity analysis results are validated through the error compensation simulations. Besides that, some discussions are given to determine the total critical errors of the grinder considering both entire workspace and particular machining requirements. Finally, the pitch and yaw errors between the dual-grinding-head are investigated, and based on the sensitivity analysis results, a quick accuracy self-test approach is proposed to reduce the measurement load in practice.

Keywords Slide grinding · Geometric error · Sensitivity analysis · Improved Sobol method · Accuracy self-test

1 Introduction
Slide is the critical part of a linear rolling guide, and the accuracy of the slide directly determines the motion smoothness and carrying ability of the linear motion axis, which further affects the performance of many types of equipment, such as CNC machine tools and coordinate measuring machines (CMMs) [1, 2]. Therefore, machining slide with high quality has always been an important goal in the industry.

Grinding is necessary for slide to achieve high accuracy; therefore, a grinder with satisfactory performance is an essential basis. Most conventional slide grinders only have one set of motion axes and one grinding head, which needs repeated clamping to complete the grinding of different surfaces of slide, and further leads to clamping errors and reduces efficiency [3, 4]. On the other hand, the translational motion axes of these grinders are general ball screw drive systems, which only have one pitch and cannot improve the mechanical resolution and motion speed simultaneously, resulting in the contradiction between precision and efficiency [5–7]. With the rapidly growing demand for grinding accuracy and efficiency, it is significant to design a novel slide grinder to solve the above problems.

After the configuration design of a machine tool, an important task is to identify the critical geometric error terms among a large number of terms, which is the basis to guide the following manufacturing and assembly, and sensitivity analysis is necessary in this process [8]. Local sensitivity analysis (LSA) is widely used to quantify the sensitivity, and it is also known as the one-at-a-time (OAT) approach. LSA determines the sensitivity of the model output to the input parameter by calculating the partial derivative, but it examines only one parameter and takes the central value
of the other parameters to evaluate the amount of variation in the output results. Although LSA is fast and easy to implement with a small amount of data, it is not applicable to nonlinear models [9]. Moreover, LSA requires prior measurement of all geometric errors of the machine tool or at least the assumptions to determine the error value, thus resulting in a deficient sensitivity analysis [10]. In contrast, the global sensitivity analysis (GSA) approach aims to determine the global response (averaged over the variations of all the parameters) by searching within a specific region. It is not limited to one single point but explores the whole input space, which can overcome the major limitation of LSA to test the total effect of multiple geometric error terms on the model output, such as the Morris method [11], but the method provides only semi-quantitative information without detecting and capturing interactions between parameters and nonlinearities [12, 13]. The Sobol method is another way to complete GSA, and it provides detailed information about the interactions of input parameters [14–17]. However, the normal Sobol method utilizes the pure Monte Carlo (MC) algorithm to compute the sensitivity index of each error term [18, 19]. The base sample matrices created by the pseudo-random sequence of the method show poor uniformity, thus reducing the accuracy and efficiency of sensitivity analysis. Obviously, a better method is needed to carry out GSA to deal with complex and nonlinear geometric relations. In addition, most works of sensitivity analysis are used to determine critical error terms, beyond that, more applications could be investigated considering the characteristics of the machine tool. In this paper, a novel grinder with dual-lead-dual-head is firstly designed to ensure the grinding accuracy and efficiency of slide. Furthermore, an improved Sobol method is presented to perform the GSA for the grinder. Considering both the entire workspace and particular machining situations of slide, the total critical error terms are investigated. Finally, based on the sensitivity analysis results, an application to accuracy self-test is proposed to reduce the measurement load for this grinder with dual-head.

The remaining parts of the paper are organized as follows: Sect. “2” presents the novel dual-lead-dual-head structure; Sect. “3” describes the volumetric error model of the grinder; Sect. “4” investigates the critical geometric error terms by applying the improved Sobol method and presents the simulation verification results; Sect. “5” gives the application to accuracy test; Sect. “6” states the conclusion.

2 Design of the novel slide grinder

The structure diagram of a typical slide is shown in Fig. 1, and the corresponding whole machining process is listed in Table 1. The top/side datum surfaces and the ball grooves are the most important parts requiring high accuracy (i.e., 1–3 µm), thus, these three parts need fine grinding, which is also the final procedure for machining slide.

As mentioned in the “Introduction,” most conventional slide grinders only have one set of motion axes (i.e., X-axis, Y-axis, and Z-axis) and one grinding head. When using this
kind of grinder in fine grinding procedures, repeated clamping of slide is necessary between grinding each datum surface and ball grooves. Obviously, the clamping error is unavoidable and would heavily reduce the grinding accuracy. In order to solve this problem, a novel slide grinder is designed, and the grinder has two symmetrical motion systems and two grinding heads, as shown in Fig. 2. One grinding head can be used to grind the top/side datum surfaces, and another head can be used to grind the ball grooves; therefore, the slide only needs once clamping before grinding, which completely eliminates the error caused by repeated clamping.

All translational axes of the grinder would utilize ball screw drive systems; however, conventional ball screw drive systems only have one fixed pitch, and the contradiction between motion speed and mechanical resolution always exists. For example, a large pitch could improve the motion speed, but the mechanical resolution would be reduced. On the basis of the grinder structure,
A new feed system is further designed to simultaneously ensure high motion speed and high mechanical resolution, and the structure diagram is shown in Fig. 3. The feed system mainly consists of a main screw, two nuts, two worktables, and several slides. The main screw has a large pitch to achieve high motion speed, and the first nut is especially machined with thread on its outer surface. The second nut is a rotation nut to match the first nut, and the pitch of the thread is small. During the motion process, the fast feed can be ensured through the main screw with a large pitch, and the accurate feed with high mechanical resolution can be realized through the nuts with a small pitch; therefore, the contradiction between motion speed and mechanical resolution can be solved. In addition, compared with other macro-micro feed systems [20–22], the two worktables of the proposed feed system share the same guide rails, which improve the stiffness of the whole system.

Although the new feed system could improve the accuracy and efficiency of the grinder in theory, it has more components than regular ball screw drive system, and further leads to more difficulties for geometric error analysis, such as more complex models and more error terms. In subsequent sections, the modeling and sensitivity analysis of geometric error would be investigated.

3 Geometric error modeling

Geometric error modeling is an important basis for carrying out precision design and error compensation. Based on the geometric error model, sensitivity analysis can be further implemented to clarify the interrelationship between geometric error terms and search for critical errors in the design phase.

The designed grinder comprises a bed, X-axis, a worktable, and two symmetrical motion axes, and each set of motion axes contains $Y_1$, $Y_2$, $Z_1$, $Z_2$, and a grinding head. Each axis has six geometric error terms, including linear and angular errors. For the translational axis, the error terms comprise one positioning error, two straightness errors, and three angular errors (denoted as pitch, yaw, and roll). The rotation axis includes three linear error terms, corresponding to one axial error and two radial errors, and three angular error terms. In addition, eight squareness errors between axis pairs are considered.
Considering the symmetrical structure of the grinder, only one set of motion axes needs to be analyzed, and it means a total of 38 geometric errors, which are labeled in Table 2. δ(j) denotes the positional error for the j-axis direction along the i-axis; ε(j) refers to the angular error, where j-axis shows the rotation axis and i-axis shows the direction of error (i = x, y, z; j = x, y1, y2, z1, z2). The kinematic motion chain with error terms is shown in Fig. 4, where the grinding head is at the end of the kinematic chain. According to the theory of multi-body system (MBS), the kinematic coordinate relationship between two adjacent bodies can be represented by a 4 × 4 homogeneous transformation matrix (HTM), and the transformation matrix between two adjacent bodies can be divided into four parts: a position transformation matrix \( \mathbf{T}_y^p \), where notation i is the lower-order body of j; a position error transformation matrix \( \Delta \mathbf{T}_y^p \) (listed in Table 3); a motion transformation matrix \( \mathbf{T}_y'^p \); a motion error transformation matrix \( \Delta \mathbf{T}_y'^p \) (listed in Table 4).

The topological structure of the grinder including a workpiece branch and a tool branch is established as shown in Fig. 5. Each rigid body is represented by a number, where the bed is set as inertial reference frame and expressed as the “0” body, “1” represents the worktable, “2” represents the workpiece, “3” represents the Z1-axis, “4” represents the Z2-axis, “5” represents the Y1-axis, and “6” represents the Y2-axis. Each rigid body has its own local coordinate system. The local coordinate systems of the end of bodies “2” and “6” are tool and workpiece coordinates, respectively.

Suppose \( \mathbf{P}_i \) denotes the grinding point in the tool coordinate, and \( \mathbf{P}_w \) represents the grinding point in the workpiece coordinate as

\[
\mathbf{P}_i = [P_{ix} P_{iy} P_{iz} P_{iw}]^T \tag{1}
\]

\[
\mathbf{P}_w = [P_{wx} P_{wy} P_{wz} P_{iw}]^T \tag{2}
\]

According to the topological structure of the grinder, the actual movement of the grinding head can be calculated through the product of the error transformation matrices as

\[
\mathbf{P}_{\text{actual}} = [\Delta \mathbf{T}_{02}]^{-1} \Delta \mathbf{T}_{04} \Delta \mathbf{T}_{46} \mathbf{P}_i
\]

\[
\Delta \mathbf{T}_{02} = \mathbf{T}^p_{01} \Delta \mathbf{T}^p_{01} \mathbf{T}^0_{01} \Delta \mathbf{T}^0_{01} \mathbf{T}^y_{12} \Delta \mathbf{T}^y_{12} \mathbf{T}^y_{12} \Delta \mathbf{T}^y_{12} \mathbf{T}^y_{12}
\]

\[
\Delta \mathbf{T}_{04} = \mathbf{T}^p_{03} \Delta \mathbf{T}^p_{03} \mathbf{T}^0_{03} \Delta \mathbf{T}^0_{03} \mathbf{T}^3_{34} \Delta \mathbf{T}^3_{34} \mathbf{T}^3_{34}
\]

\[
\Delta \mathbf{T}_{46} = \mathbf{T}^p_{45} \Delta \mathbf{T}^p_{45} \mathbf{T}^0_{45} \Delta \mathbf{T}^0_{45} \mathbf{T}^6_{56} \Delta \mathbf{T}^6_{56} \mathbf{T}^6_{56}
\]

Similarly, the ideal movement of the grinding head can also be expressed by the product of the transformation matrix without considering the 38 geometric errors as

\[
\mathbf{P}_{\text{ideal}} = [\mathbf{T}_{02}]^{-1} \mathbf{T}_{06} \mathbf{P}_i
\]

\[
\mathbf{T}_{02} = \mathbf{T}^p_{01} \mathbf{T}^0_{01} \mathbf{T}^y_{12} \mathbf{T}^y_{12}
\]

\[
\mathbf{T}_{06} = \mathbf{T}^p_{03} \mathbf{T}^0_{03} \mathbf{T}^3_{34} \mathbf{T}^3_{34} \mathbf{T}^6_{56} \mathbf{T}^6_{56}
\]
Based on Eq. 3 and Eq. 4, the comprehensive volumetric error can be obtained by comparing the actual and ideal positions of the grinding point, as shown in Eq. 5.

$$E = \begin{bmatrix} E_X, E_Y, E_Z \end{bmatrix} = P_{\text{actual}} - P_{\text{ideal}}$$

where $E_X$, $E_Y$, and $E_Z$ are the components of volumetric error $E$ in the $X$, $Y$, and $Z$ directions, respectively. Volumetric error can be expressed by the function of error terms as:

$$E = f(\delta_i(j), \epsilon_i(j), S)$$

with $i = x, y, z; j = x_1, y_1, z_1, z_2$

where $\delta_i(j)$ denotes the positional errors, $\epsilon_i(j)$ denotes the angular errors, and $S$ denotes the squareness errors.

Finally, the scale of volumetric error $E_T$ can be expressed as

$$E_T = \sqrt{E_X^2 + E_Y^2 + E_Z^2}$$

The volumetric error model derived in this section is the prerequisite and basis for the subsequent geometric error sensitivity analysis.

4 Sensitivity analysis of geometric error

4.1 Sensitivity analysis approach based on the improved Sobol method

In this study, a complex dual-lead-dual-head slide grinder is investigated, and the coupling effect and randomness of geometric errors are to be expected. Therefore, a global method that provides more detailed results is needed. In order to meet this demand, the Sobol method, which is a global sensitivity analysis method, is able to examine the model inputs comprehensively. Based on the normal Sobol method \cite{18, 19}, an improved one is utilized to complete the GSA of the slide...
The improved Sobol method is provided in Fig. 6. As can be seen, the established volumetric error model can be expressed as

\[ Y = f(e) \]

Table 3 Position and position error transformation matrices for the slide grinder

| Axis   | Position transformation matrix | Position error transformation matrix |
|--------|--------------------------------|-------------------------------------|
| X-axis | \( T_{01}^p = I_{4\times4} \) | \( \Delta T_{01}^p = I_{4\times4} \) |
| Z_1-axis | \( T_{01}^p = I_{4\times4} \) | \( \Delta T_{01}^p = I_{4\times4} \) |
| Z_2-axis | \( T_{34}^p = I_{4\times4} \) | \( \Delta T_{34}^p = I_{4\times4} \) |
| Y_1-axis | \( T_{45}^p = I_{4\times4} \) | \( \Delta T_{45}^p = I_{4\times4} \) |
| Y_2-axis | \( T_{56}^p = I_{4\times4} \) | \( \Delta T_{56}^p = I_{4\times4} \) |

where \( Y \) is defined as a scalar output, and vector \( e \) summarizes the independent model input parameters \( e_1, \ldots, e_k \). \( e_i \) and \( e_j \) represent the \( i \)th and \( j \)th model input parameters, respectively. In this study, \( \mathbb{R}^k \) represents the spatial workspace domain; therefore, all 38 error terms are considered.

The core concept of the Sobol method is to decompose the variance of output \( Y \) into conditional terms as

\[
V_i = \mathbb{V}_e_i[\mathbb{D}_{e_j}(Y | e_i)]
\]

\[
V_{ij} = \mathbb{V}_{e_i e_j}[\mathbb{D}_{e_{ij}}(Y | e_i, e_j)] - V_i - V_j
\]

where notation \( e_{ij} \) denotes the vector of all input parameters except \( e_i \). \( V_i \) and \( V_j \) represent the variance of input parameters \( e_i \) and \( e_j \), respectively. \( V_{ij} \) represents the variance contributed by the interaction of parameters \( e_i \) and \( e_j \). \( \mathbb{D}[-] \) denotes the expectation operator, \( \mathbb{V}[-] \) denotes the variance operator.

The first-order indices for single input \( S_i \) can be obtained by dividing the model output variance \( \mathbb{V}(Y) \) in Eq. 9 as:

\[
S_i = \frac{V_i}{\mathbb{V}(Y)} = \frac{\mathbb{V}_e_i[\mathbb{D}_{e_j}(Y | e_i)]}{\mathbb{V}(Y)}
\]

The first-order Sobol index is typically used to identify the most influential parameters. However, the output variance \( \mathbb{V}(Y) \) is the sum of variances contributed by input parameter \( e_i \), including interactions with other parameters. It might be involved in interactions with other parameters if it is not varied alone in the model. To comprehensively analyze the

Table 4 Motion and motion error transformation matrices for the slide grinder

| Axis   | Motion transformation matrix | Motion error transformation matrix |
|--------|-----------------------------|-----------------------------------|
| X-axis | \( T_{01}^m = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \) | \( \Delta T_{01}^m = \begin{bmatrix} 1 - \varepsilon_1(x) & \varepsilon_1(x) & \delta_1(x) \\ \varepsilon_1(x) & 1 - \varepsilon_1(x) & \delta_1(x) \\ -\varepsilon_1(x) & \varepsilon_1(x) & 1 - \delta_1(x) \\ 0 & 0 & 0 & 1 \end{bmatrix} \) |
| Z_1-axis | \( T_{03}^m = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \) | \( \Delta T_{03}^m = \begin{bmatrix} 1 - \varepsilon_3(z_1) & \varepsilon_3(z_1) & \delta_3(z_1) \\ \varepsilon_3(z_1) & 1 - \varepsilon_3(z_1) & \delta_3(z_1) \\ -\varepsilon_3(z_1) & \varepsilon_3(z_1) & 1 - \delta_3(z_1) \\ 0 & 0 & 0 & 1 \end{bmatrix} \) |
| Z_2-axis | \( T_{34}^m = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \) | \( \Delta T_{34}^m = \begin{bmatrix} 1 - \varepsilon_2(z_2) & \varepsilon_2(z_2) & \delta_2(z_2) \\ \varepsilon_2(z_2) & 1 - \varepsilon_2(z_2) & \delta_2(z_2) \\ -\varepsilon_2(z_2) & \varepsilon_2(z_2) & 1 - \delta_2(z_2) \\ 0 & 0 & 0 & 1 \end{bmatrix} \) |
| Y_1-axis | \( T_{45}^m = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \) | \( \Delta T_{45}^m = \begin{bmatrix} 1 - \varepsilon_5(y_1) & \varepsilon_5(y_1) & \delta_5(y_1) \\ \varepsilon_5(y_1) & 1 - \varepsilon_5(y_1) & \delta_5(y_1) \\ -\varepsilon_5(y_1) & \varepsilon_5(y_1) & 1 - \delta_5(y_1) \\ 0 & 0 & 0 & 1 \end{bmatrix} \) |
| Y_2-axis | \( T_{56}^m = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \) | \( \Delta T_{56}^m = \begin{bmatrix} 1 - \varepsilon_6(y_2) & \varepsilon_6(y_2) & \delta_6(y_2) \\ \varepsilon_6(y_2) & 1 - \varepsilon_6(y_2) & \delta_6(y_2) \\ -\varepsilon_6(y_2) & \varepsilon_6(y_2) & 1 - \delta_6(y_2) \\ 0 & 0 & 0 & 1 \end{bmatrix} \) |
The coupling effect of various parameters, an additional sensitivity measure that includes high-order interaction effects needs to be addressed. The total-order Sobol index is, therefore, defined as

\[
S_{Ti} = \frac{D_{e_i}(Y \mid e_{-i})}{\mathbb{V}(Y)} \approx 0
\]

When \( S_{Ti} \approx 0 \), it can be concluded that \( e_i \) can be fixed arbitrarily within its input range without affecting the output variance \( \mathbb{V}(Y) \). Because of this, total-order indices are particularly beneficial in the context of factor-fixing [12, 13]. It has been applied to distinguish between influential and non-influential model inputs, thus reducing the dimensionality of the uncertain space. Here, all sensitivity indices add up to 1 and they are all non-negative. If all 38 parameters have the same influence, each parameter can only contribute 1/38 of the variance. If few parameters are dominant to the output \( Y \), the contribution of the remaining parameters is even smaller. The most straightforward way to compute the Sobol index is through the Monte Carlo estimation. Fourier Amplitude Sensitivity Test (FAST) can also be employed to calculate Sobol index [23]. It consists of converting the input variable into a periodic function of a single frequency variable, and then proceeding to sample the model and analyze the sensitivity of input variables with Fourier analysis in the frequency domain. However, FAST is sensitive to the characteristic frequencies attributed to the input variable and is not straightforward as the Monte Carlo estimation [24].

In this study, the improved Sobol method based on quasi-random sequence is used to create the training sample matrices, which is different from the normal pure Monte Carlo method based on pseudo-random sequence. Quasi-random sequence for training samples provides a better convergence in the input space, leading to a better predictive quality. Here, the two-dimensional quasi-random sequence and pseudo-random sequence on the
interval \([0, 1]^2\) are taken as examples to compare their distribution uniformity as shown in Fig. 7. The final number of sampling points is \(N = 2^9\). It can be clearly seen that quasi-random sequences have greatly improved over pseudo-random sequences with the development of statistics. Furthermore, the comparison of statistical properties between quasi-random sequence and pseudo-random sequence is illustrated in Table 5. As can be seen, quasi-random sequence presents a better performance than pseudo-random sequence for all probabilistic distributions, indicating the improved Sobol method using the quasi-random sequence possesses higher accuracy and faster convergence speed than the normal Sobol method using the pseudo-random sequence. Therefore, the improved Sobol method would be used to identify highly sensitive geometric error terms. Finally, the estimators for \(S_i\) and \(S_r\) in Eqs. 10 and 11 can be written as:

\[
\mathbb{V}_{e_i}[D_{x_i}[Y | e_j]] = \frac{1}{N} \sum_{m=1}^{N} f(B)_m \left( f(A)^{i}_m \right) - f(A)_m
\]  

(12)
ing errors along all the linear errors except the positioning errors are set to adopting the average value.

The ISO230-6 specifies body diagonal displacement tests of motion space, which allows evaluation of the volumetric errors with different compensation ways. For the comprehensive volumetric model over the entire workspace, $\delta_y(y_1)$ and $\delta_z(z_1)$ have the largest first-order sensitivity values, and other critical geometric error terms are $\epsilon_x(x), \delta_y(y_1), \delta_z(z_1), S_y, S_z$.

To verify the results of the sensitivity analysis, the geometric errors are compensated in three different ways to investigate their influences. In the validation analysis, the middle number in the positive error range was taken as a fixed value of the error. The first way is to reduce the values of critical geometric error terms to half of the original values and maintain the original values of the other geometric error terms; the second way is to maintain the values of the critical geometric error terms and reduce half the other geometric error terms from their original values; the final way is to reduce half all the geometric errors from their original values.

As shown in Fig. 10, the bar plot illustrates the average volumetric errors with different compensation ways. Based on the results, the volumetric errors are improved by 38.2%, 10.2%, and 49.5% by compensating the critical error terms, other geometric error terms, and all error terms. Obviously, the volumetric errors in the entire workspace can be greatly improved by only compensating the critical error terms, which indicate the effectiveness of the error sensitivity analysis in the entire workspace.

### 4.2 Global sensitivity analysis results considering the entire workspace

The theoretical results of the sensitivity analysis concerning volumetric error over the entire workspace are visualized in Fig. 9. From a total of 38 geometric error terms, the 10 geometric error terms with the largest sensitivity indices were selected to create the histogram, as the rest of the error terms have little effect on the machining accuracy. As can be seen from Fig. 9, sensitivity indices of first-order and total-order are close to each other, which shows a slight interaction between any two arbitrary error terms. Therefore, the first-order sensitivity indices will be mainly used to distinguish the critical error terms. Among all geometric error terms of the grinder, those with first-order sensitivity indices greater than 0.08 are considered as critical sensitive error terms.

For the comprehensive volumetric model over the entire workspace, $\delta_y(y_1)$ and $\delta_z(z_1)$ have the largest first-order sensitivity values, and other critical geometric error terms are $\epsilon_x(x), \delta_y(y_1), \delta_z(z_1), S_y, S_z$.

To verify the results of the sensitivity analysis, the geometric errors are compensated in three different ways to investigate their influences. In the validation analysis, the middle number in the positive error range was taken as a fixed value of the error. The first way is to reduce the values of critical geometric error terms to half of the original values and maintain the original values of the other geometric error terms; the second way is to maintain the values of the critical geometric error terms and reduce half the other geometric error terms from their original values; the final way is to reduce half all the geometric errors from their original values.

As shown in Fig. 10, the bar plot illustrates the average volumetric errors with different compensation ways. Based on the results, the volumetric errors are improved by 38.2%, 10.2%, and 49.5% by compensating the critical error terms, other geometric error terms, and all error terms. Obviously, the volumetric errors in the entire workspace can be greatly improved by only compensating the critical error terms, which indicate the effectiveness of the error sensitivity analysis in the entire workspace.
4.3 Particular sensitivity analysis results considering machining characteristics

Besides the error sensitivity analysis in the entire workspace, a particular sensitivity analysis is further investigated considering the machining characteristics of a typical slide. In this subsection, the HIWIN HGH20CA [26] was targeted as the object of the sensitivity analysis, and the parameters of the slide are outlined in [26]. The test positions of the surface are uniformly selected as shown in Fig. 11. Red points and purple points illustrate the test points of the side datum surface and the top datum surface, respectively. Considering the structure of the slide and its corresponding machining characteristics, the volumetric error terms along the Y- and Z-directions, respectively, have a significant impact on the machining accuracy when machining the top datum surface and the side datum surface. It is therefore important to analyze the sensitivity indices for these two directions.

The same analysis method and evaluation criteria as for the comprehensive volumetric error are used to analyze the sensitivity of the error terms in the Y- and Z-directions. The sensitivity analysis result for the Y-direction is illustrated in Fig. 12. The sensitivity values of $\delta_z(y_1), \delta_y(y_2)$, and $S_{y2y}$ are the largest, and other critical errors terms are $\delta_z(x), \delta_y(z_1), \epsilon_x(z_2), S_{y1z1}, S_{y2z2}$. The sensitivity analysis result for the Z-direction is illustrated in Fig. 13. The sensitivity values of $\delta_z(z_1), \delta_z(y_1)$, and $\epsilon_z(y_2)$ are the largest, and other critical error terms are $\delta_z(x), \epsilon_x(z_1), \epsilon_x(z_2), S_{y2z1}, S_{y2z2}$.

The validation is conducted to confirm the sensitivity analysis results, as illustrated in Figs. 14 and 15. Based on the results, the volumetric errors along Y-direction are improved by 35.7%, 10.8%, and 49.7% by compensating the critical error terms, other geometric error terms and all error terms.

The volumetric errors along Z-direction are improved by 37.5%, 11.1%, and 49.2%. Obviously, the volumetric errors considering the machining characteristics can be greatly improved by only compensating the critical error terms, which indicate the effectiveness of the particular error sensitivity analysis considering the machining characteristics of a typical slide.

4.4 Investigation of the total critical error terms

Considering above global and particular sensitivity analysis results, the total critical error terms for this novel grinder are statistically captured by calculating the weighted Euclidean norm as:

$$||r|| = \sqrt{\sum_{i=1}^{n} w_i(t_i)^2}$$  \hspace{1cm} (15)

$$t = (t_1, t_2, \ldots, t_n)$$

where $\sum_{i=1}^{n} w_i = 1$, $\forall w_i \in [0, 1]$. $t_i$ is the sensitivity index value obtained from corresponding global and particular sensitivity analysis, $|| \cdot ||$ denotes the weighted Euclidean norm with weight $w_i$.

Since this grinder is designed for machining slide, the weights for particular sensitivity analysis results (i.e., Y- and Z- direction) are set slightly higher than the sensitivity analysis result over entire workspace, as

$$w = \left( \begin{array}{ccc} 3/10 & 7/20 & 7/20 \end{array} \right)$$  \hspace{1cm} (16)

The analysis result of 10 largest weighted norm values is visualized in Fig. 16 using a circular bar plot. Based on the result, it can be confirmed that the sensitivity values...
of $\delta_z(z_1), \delta_z(y_1), S_{y_1}, S_{z_1}$ are the largest; therefore, these four error terms should be given priority to ensure the initial accuracy of the grinder. Other total critical error terms are $S_{y_2}, \delta_z(y_2), \delta_z(z_1), \delta_z(z_2), \delta_z(y_2)$. A compensation validation is further performed to confirm that the obtained total critical error terms can characterize both the entire workspace and particular machining situations. Figure 17 shows the volumetric error by different compensation ways of the total critical error terms for the entire workspace. Figures 18 and 19 illustrate the volumetric error along the $Y$- and $Z$-directions by different compensation ways of the total critical error terms considering the machining characteristics of typical slide. In all three cases, although the accuracy improvement of compensating total critical error terms is not as good as compensating the critical errors obtained from respective sensitivity analysis, the values are all greater than compensating the other remaining error terms. These results demonstrate that the obtained total critical error terms can describe both the entire workspace and particular machining situations.

![Simulation result of the volumetric error over the entire workspace by different compensation ways](image)

Fig. 10 Simulation result of the volumetric error over the entire workspace by different compensation ways
5 Application to accuracy self-test

Based on the above sensitivity analysis results, accuracy design can be carried out to ensure that the grinder has high accuracy in the initial stage. Although the accuracy of the grinder could meet requirements in initial stage, it is unable to guarantee that the accuracy still maintains at the acceptable tolerance range after a long-term operation. In order to ensure consistent accuracy for mass slide production, it is necessary to perform periodic accuracy test and maintenance. Traditionally, all critical error terms should be tested one by one, but there are 76 error terms in total with 20 critical error terms for the novel grinder with symmetrical structure, which means a heavy measuring load in practice. In addition, among all error terms, posture errors are much more difficult to be measured than positional errors.

![Test positions selected on the top (left) and side (right) datum surfaces of a HIWIN HGH20CA slide](image)

![Bar plots showing the geometric error sensitivity indices of typical slide surface in the Y-direction](image)
In order to solve the above problem, a fast accuracy self-test approach is proposed based on the structural characteristics of the grinder and the error sensitivity analysis results in this section.

5.1 Posture error of the dual-head

For the normal geometric sensitivity analysis, the error of translational axis consists of three positional errors and three angular errors, and they are not discussed separately. Since the novel grinder does not have a rotation axis, the posture error of grinding head, such as the pitch error shown in Fig. 20, is only affected by the angular geometric error of the motion axis. Hence, if the posture of the grinding head deviates angularly from its initial resting plane, it is caused by angular or squareness error. As a result, volumetric error can be divided into two parts as illustrated in Fig. 21: positional error vector \( P_{\text{err}} \) and posture error vector \( \theta_{\text{err}} \), as

\[
P_{\text{err}} = f[\delta_i(j)]
\]

\[
\theta_{\text{err}} = f[\varepsilon_i(j), S]
\]

with \( i = x, y, z \) and \( j = x, y_1, y_2, z_1, z_2 \)

where \( \delta_i(j) \) denotes the positional errors, \( \varepsilon_i(j) \) denotes the angular errors, and \( S \) denotes the squareness errors. According to the analysis results in Sects. “4-4.4,” only three total critical error terms are angle-independent, which means that the posture error would be heavily affected by the total critical error terms. In turn, the posture error could be used to evaluate the variations of critical error terms in a large extent, which provides a possible way to achieve fast accuracy test.

5.2 The accuracy self-test approach

A sensitivity analysis is firstly performed for the posture error of the grinding head. It is assumed that one side of the grinding head does not have any posture errors from its initial static phase. The posture error between dual grinding heads around X-direction (i.e., pitch) is examined as shown in Fig. 22. The sensitivity indices are largest with the error terms of \( S_{y1z1}, S_{y1z2}, \varepsilon_x(z_2) \), and other critical errors terms are \( \varepsilon_x(z_1), \varepsilon_y(z_1), \varepsilon_y(y_2), S_{xy} \), which completely covers the seven critical posture error terms in Sects. 4-4.4. Meanwhile, the posture error between dual grinding heads around Z-direction (i.e., yaw) is examined. Fig. 23 shows that \( S_{y1z1}, S_{y1z2} \) have the largest sensitivity values, and other
critical error terms are $\varepsilon_z(x), \varepsilon_x(x), \varepsilon_z(y)\), $S_{xy}$, covering three critical posture error terms in Sects. 4.4.4. Considering the overlapping ratio, the pitch error shows a stronger correlation with the total critical error terms. Therefore, it is more effective to characterize the geometric accuracy by measuring the pitch error between the dual grinding heads.

Furthermore, a fast accuracy self-test approach is proposed, as shown in Fig. 24. First, the critical positional error terms (i.e., $\delta_z(z_1), \delta_y(y_1), \delta_z(y_1)$) and the pitch error between dual grinding heads are measured. If the critical positional error terms are larger than the tolerance value $T_c$, then the maintenance should be performed. On the other hand, only if the pitch error of the grinding heads is larger than the tolerance value $T_p$, other seven critical posture error terms should be measured. In this way, the initial measurement of 10 critical error terms is reduced
to 4. Also, since posture errors are much more difficult to be measured than positional errors, it saves great effort to use pitch error to represent seven critical posture errors.

Nowadays, all individual geometric error terms can be directly measured by using a 6D interferometer. However, the installation and adjustment of laser interferometry are relatively complicated and time-consuming. Several identification methods based on an interferometer have been developed to reduce the measurement difficulty, such as 22-, 14-, 15-, and 9-line methods. However, some deficiencies still exist in these identification methods. Therefore, efficiency can be a critical issue for both direct and indirect calibration methods using laser interferometers. In addition, a major concern is that most manufacturing factories are not equipped with adequate laser interferometers or other advanced measurement equipment due to the high cost of the
devices. To determine the error, they usually use traditional mechanical instruments such as gauge blocks and dial indicators to measure geometric errors one by one, which means heavy measuring load. All of these considerations led to the application of sensitivity analysis to reduce the measurement load in practice. From this perspective, the proposed self-test flow creates an automatic algorithm for engineers to check the geometric accuracy status of the grinder without measuring all the critical error terms in the beginning. The method can increase the effectiveness of the measurement process and reduce the costs involved in ensuring the consistent accuracy of mass-production slides.

Fig. 16 Circular bar plot showing the total critical errors of the grinder considering both entire workspace and particular machining requirements.
Fig. 17  Simulation result of the volumetric error by different compensation ways of the total critical error terms for the entire workspace
Fig. 18 Simulation result of the volumetric error along the Y-direction by different compensation ways of the total critical error terms considering the machining characteristics of typical slide.
Fig. 19 Simulation result of the volumetric error along the Z-direction by different compensation ways of the total critical error terms considering the machining characteristics of typical slide
Fig. 20  The pitch error of the grinding head

Fig. 21  Posture error caused by angular errors and squareness errors

\[ P_{\text{err}} = f[\delta_i(j)] \]

\[ \theta_{\text{err}} = f[\varepsilon_i(j), S] \]
Fig. 22 Bar plots showing the sensitivity indices of posture error between dual grinding heads around X-direction

Fig. 23 Bar plots showing the sensitivity indices of posture error between dual grinding heads around Z-direction
6 Conclusion

This paper designs a novel slide grinder with dual-lead-dual-head to improve both accuracy and efficiency, and the sensitivity analysis of geometric error is further investigated for the new structure. By utilizing the improved Sobol method, sensitivity analyses are conducted considering the entire workspace and typical machining characteristics, and corresponding critical error terms are obtained. Based on all analysis results for different cases, the total critical error terms are determined, and the results are validated through error compensation simulation. These analysis results could be used to help engineers with accuracy design. Moreover, an approach for accuracy self-test is proposed for the grinder, as an application of error sensitivity analysis. This approach reduces the measuring error terms from 10 to 4 in the initial stage, which improves the efficiency in practice.

Author Contributions  Methodology, validation, formal analysis, and writing-original draft preparation, J.H.; conceptualization, resources, funding acquisition, and writing-review and editing, D.W., X.L.; investigation and data curation, Z.G.; visualization, supervision, project administration, L.W. and F.M. All the authors have read and agreed to the published version of the manuscript.

Funding  This work was supported by the National Natural Science Foundation of China (Grant No. 52105520, No. 51975319), Beijing Municipal Natural Science Foundation (Grant No. 3214043), and Project of State Key Lab of Tribology of Tsinghua University (Grant No. SKLT2021D16).

Data availability  Not applicable.

Code availability  Not applicable.

Declarations

Ethics approval  Not applicable.
References

1. Rahmani M, Bleicher F (2016) Experimental and numerical studies of the influence of geometric deviations in the performance of machine tools linear guides. Procedia CIRP 41:818–823
2. Qibo F, Bin Z, Cunxing C, Cuiyang K, Yusheng Z, Fenglin Y (2013) Development of a simple system for simultaneously measuring 6dof geometric motion errors of a linear guide. Opt Express 21(22):25805–25819
3. Zae M, Oertli T, Milberg J (2004) Finite element modelling of ball screw feed drive systems. CIRP Ann 53(1):289–292
4. Vicente DA, Hecker RL, Villegas FJ, Flores GM (2012) Modeling and vibration mode analysis of a ball screw drive. Int J Adv Manuf Technol 58(1):257–265
5. Kopac J, Krajinik P (2006) High-performance grinding—a review. J Mater Process Technol 175(1–3):278–284
6. Rowe WB (2013) Principles of modern grinding technology. William Andrew
7. Wang D, Zhang S, Wang L, Liu Y (2021) Developing ball screw drive system of high speed machine tool considering dynamics. IEEE Trans Ind Electron
8. Wang L, Wang D, Wang B, Li W (2020) Development of an oscillating grinding machine tool based on error analysis. Sci China Technol Sci 63(6):912–922
9. Saltelli A, Aleksankina K, Becker W, Fennell P, Ferretti F, Holst N, Li S, Wu Q (2019) Why so many published sensitivity analyses are false: A systematic review of sensitivity analysis practices. Environ Model Softw 114:29–39
10. Fan J, Liu Q, Li W, Xue L, Li C (2020) Geometric error modeling and sensitivity analysis of cnc internal circular compound grinding machine. International Journal of Mechanical Engineering and Applications 8:118. https://doi.org/10.11648/j.ijmea.20200805.12
11. Morris MD (1991) Factorial sampling plans for preliminary computational experiments. Technometrics 33(2):161–174
12. Saltelli A, Tarantola S, Campolongo F, Ratto M (2004) Sensitivity analysis in practice: a guide to assessing scientific models, vol 1. Wiley Online Library
13. Saltelli A, Ratto M, Andres T, Campolongo F, Cariboni J, Gatelli D, Saisana M, Tarantola S (2008) Global sensitivity analysis: the primer. John Wiley & Sons
14. Guo S, Zhang D, Xi Y (2016) Global quantitative sensitivity analysis and compensation of geometric errors of cnc machine tool. Math Probl Eng 2016. https://doi.org/10.1155/2016/2834718
15. Zou X, Zhao X, Li G, Li Z, Sun T (2017) Sensitivity analysis using a variance-based method for a three-axis diamond turning machine. Int J Adv Manuf Technol 92:4429–4443. https://doi.org/10.1007/s00170-017-0394-y
16. Zou X, Zhao X, Wang Z, Li G, Hu Z, Sun T (2020) Error distribution of a 5-axis measuring machine based on sensitivity analysis of geometric errors. Math Probl Eng 2020. https://doi.org/10.1155/2020/8146975
17. Jiang X, Cui Z, Wang L, Liu C, Li M, Liu J, Du Y (2022) Critical geometric errors identification of a five-axis machine tool based on global quantitative sensitivity analysis. Int J Adv Manuf Technol. https://doi.org/10.1007/s00170-021-0369-8
18. Sobol’ IM (1990) On sensitivity estimation for nonlinear mathematical models. Matematicheskoe modelirovanie 2(1):112–118
19. Sobol’ IM (2001) Global sensitivity indices for nonlinear mathematical models and their monte carlo estimates. Math Comput Simul 55(1–3):271–280
20. Sun LN, Jie DG, Liu YJ, Chen ZC, Cai HG (2006) Investigation on a novel dual-grating macro-micro driven high speed precision positioning system for nems. In: 2006 1st IEEE International Conference on Nano/Micro Engineered and Molecular Systems, IEEE, pp 644–648
21. Yang C, Wang GL, Yang BS, Wang HR (2008) Research on the structure of high-speed large-scale ultra-precision positioning system. In: 2008 3rd IEEE International Conference on Nano/Micro Engineered and Molecular Systems, IEEE, pp 9–12
22. Zhang L, Gao J, Chen X, Chen Y, He Y, Zhang Y, Tang H, Yang Z (2018) Implementation and experiment of an active vibration reduction strategy for macro-micro positioning system. Precis Eng 51:319–330
23. Saltelli A, Bolado R (1998) An alternative way to compute fourier amplitude sensitivity test (fast). Comput Stat Data Anal 26(4):445–460
24. Saltelli A, Amoni P, Azzini I, Campolongo F, Ratto M, Tarantola S (2010) Variance based sensitivity analysis of model output, design and estimator for the total sensitivity index. Comput Phys Commun 181(2):259–270
25. 230-6 I (2002) Test code for machine tools, part 6: determination of positioning accuracy on body and face diagonals (diagonal displacement tests)
26. Hgh20ca. http://www.hiwin.sc.cn/HIWIN/HG/HG20CA.html. Accessed 30 Mar 2022