Research on the current situation of water resources based on mathematical model

Cuiping Ren *, Qian Cheng
School of General Education, Xi’an Eurasia University, Beijing, China

* Corresponding author e-mail: 497527964@qq.com

Abstract. The deteriorating environment of human existence, especially the lack of water resources, has seriously affected the development of the country and the survival of mankind. Firstly, this paper design the great influential basic index of water resources and identify the basic index of controllable factors, then analyse the relationship among different index, set up the dynamic model of water use rate of a certain region and annual per capita surplus amount of water resources. Secondly, we choose India as the research object of the mission, From the perspective of the physical and economic analysis of the causes of water shortage. Factors according to the environment of the selected area development, population, GDP the reuse water utilization model and the annual per capita amount of surplus, forecasts for the region over the next 15 years the situation of water resource and the advantages and disadvantages. Lastly, we give the policy interventions to the government about the water resources.

Keywords: Water resource utilization; Water resources surplus; Intervention plan; The ecological system.

1. Introduction
According to figures released by the United Nations, global water use increased six times in the 20th century, the growth rate is twice the population growth. The United Nations educational, scientific and cultural organization says that the fresh water resources are totally adequate. But about 1/5 of the world population has no access to the safe drinking water and 40% of people lack basic sanitation because of the uneven distribution and poor management and environmental changes and inadequate infrastructure, etc. At present, there is already a shortage of fresh water in the Middle East, Africa, central Asia and parts of Latin America. The crisis of water scarcity has aroused world-wide attention and becomes a problem beyond politics and boundaries.

Our task is to develop a model for the water resources measurement, then analyze the reasons and effects of water scarcity.

2. Model Construction

2.1. Impact indicators
(1) The calculation of capitation water consumption per year
\[ d_{i1} = \alpha_{i1} \omega P_t + \alpha_{i2} (1 - \omega) P_t \]  

(1)

\( \alpha_{i1} \) stands for urban annual domestic water consumption per person in this area, \( \alpha_{i2} \) stands for rural annual domestic water consumption per person in this area, \( \omega \) stands for the proportion of urban population, \( P_t \) stands for gross population, \( t \) stands for time variable.

(2) The calculation of annual irrigation water consumption

\[ d_{i2} = \beta_t F_t \]  

(2)

\( \beta_t \) stands for irrigation water consumption per hectare in this area, \( F_t \) stands for the amount of arable land in this area (unit: hectare), and \( t \) stands for time variable.

(3) The calculation of annual industrial water consumption

\[ d_{i3} = \varepsilon_t G_t \]  

(3)

\( \varepsilon_t \) stands for industrial water consumption per trillion Yuan of GDP, \( G_t \) stands for the total GDP (trillion Yuan) of this area, \( t \) stands for time variable. Industrial GDP is relevant stable in the whole GDP, and it doesn’t produce variability except the financial crisis. So, it is very reasonable to use annual industrial GDP value to measure annual industrial water consumption.

(4) The calculation of evaporation per year

\[ d_{i4} = E_t \]  

(4)

\( E_t \) stands for annual evaporation in the area.

(5) The calculation of the water reused:

\[ L_t = \sum_{i=1}^{3} \phi_i d_{ii} \]  

(5)

\( \phi_i \) stands for the treatment rate of domestic sewage of \( d_{ii} \).

2.2. Mathematical Model of Water Resource Utilization Rate

Through the construction of above basic index, we can develop the mathematical model of the water resource utilization rate of this nation.

\[ \eta_t = \frac{Q_t}{M_t} \]  

(6)

\( Q_t \) represents the total water consumption of the \( t \) th year, which removes the water consumption of repeating used from the total consumption of water for domestic use, water for irrigation, and water for industrial use, namely.
\[ Q_t = \sum_{i=1}^{4} d_{ii} - L_t \]  

\[ M_t \] represents the total available quantity of fresh water, which consists of the total annual runoff and technical collection for water resources but eliminates the polluted water, namely.

\[ M_t = H_t + S_t - X_t \]  

\[ H_t \] is the average of the total annual runoff for the past 3 years, \( S_t \) represents indirect water resources collected by technical methods (e.g., seawater desalting and rainwater harvesting), \( X_t \) represents the unavailable water resource due to environmental issues and represents the time variable.

\[ H_t = \frac{1}{3} \sum_{i=1}^{4} H_i \]  

\[ S_t = \mu_t O_t + \mu_2 N_t \]  

\( O_t \) is available water quantity for years, \( N_t \) is total annual precipitation, \( \mu_t \) is the desalination rate, \( \mu_2 \) is the rainwater collection rate.

\( X_t \) can be calculated by the following calculation procedure

\[ X_t = \sum_{i=1}^{4} (1 - \phi_{ii}) d_{ii} \]  

Having substituted the data into the mathematical model of water resource utilization rate [1], we can achieve the water resource utilization rate of the year. According to the index of the abundance degree of world water resources, when the resulting value is lower than 0.15, it means the area obtains rich water resources; when the resulting value falls between 0.15~0.25, it means the area has inadequate water resources; and when the resulting value is higher than 0.5, it means the area has water resource scarcity [2].

2.3. Mathematical model of per capita amount of water resources surplus

The model of water resources utilization rate is not enough to indicate the surplus of water resources in that area, thus we need to establish a model of per capita amount of water resources surplus.

\[ k_t = \frac{(H_t + S_t - X_t) - (\sum_{i=1}^{4} d_{ii} - l_i)}{P_t} \]  

\( k_t \) represents annual per capita amount of water resources surplus, meanwhile it is also the amount of untapped water resources.
2.4. Conclusion
India was chosen as the research object. India is exploited, and the exploitation of water resources is severely overwhelmed. This paper is going to discuss the reasons of India’s water scarcity from physical and economic perspectives. We use Logistic model to predict India’s forecast results in 15 years. In the assumption of the present increasing level of the science and technology and other factors (i.e. the increase of population and GDP and the increase of global warming and water contamination, etc.) and utilization of the water use rate model, the figures in India in the 15 years are shown below in Table 1.

### Table 1. Utilization ratio of water resources in 15 years and Per capita annual surplus

| Year | Water use rate | Annual Per capita surplus amount |
|------|----------------|----------------------------------|
| 2016 | 0.35           | 926.03                           |
| 2017 | 0.36           | 910.98                           |
| 2018 | 0.36           | 895.92                           |
| 2019 | 0.36           | 880.86                           |
| 2020 | 0.36           | 865.78                           |
| 2021 | 0.36           | 850.66                           |
| 2022 | 0.37           | 835.49                           |
| 2023 | 0.37           | 820.26                           |
| 2024 | 0.37           | 804.96                           |
| 2025 | 0.38           | 789.57                           |
| 2026 | 0.38           | 774.07                           |
| 2027 | 0.38           | 758.45                           |
| 2028 | 0.39           | 742.70                           |
| 2029 | 0.39           | 726.79                           |
| 2030 | 0.40           | 710.70                           |

![Utilization ratio of water resources in 15 years](image)

**Figure 1.** Utilization ratio of water resources in 15 years

Chapter 1 shows it more obvious that the situation of water consumption rate changing. The growth of population and the development of industry exacerbate their impacts on environment, and in the next 15 years the water scarcity will be increasingly severe because the whole water resources consumption rate is on the rise.
3. **Analysis of the situation**
India’s physical water scarcity is mainly reflected by inadequate per capita water resources and maldistribution of water resources. India has the sixth largest water resources in the world, however, it is a country with 1.2 billion people. Therefore, India is facing a massive threat of water scarcity, for its per capita water consumption is only 1000 m$^3$, which is far lower than the global per capita water consumption. On the other hand, India’s water resources are uneven distributed in terms of time and space.

India’s economic water scarcity is indicated from two aspects. The first aspect is its low level of technology, namely the water treatment facility is out of date, which leads to poor water quality. Other than that, sewage, chemical fertilizer and pesticide has polluted almost all rivers in India.

Another aspect is the low utilization rate, which leads to severe over-extraction of groundwater. Third, because of the limitation of technology, those fresh water collection methods which depend on technology such as seawater desalination and rainwater harvesting are limited as well.

4. **Intervention plan**
The reasons that are responsible for water scarcity in India are mainly uneven spatial and temporal distribution of water resources, large population (agricultural irrigation and industrial water use), low science and technology and the poor capacity of sewage treatment. We will deal with these reasons with Intervention Plan.

The first one is rational exploitation of the groundwater.

The second one is Inland waterway network planning, which can take after China’s the South-to-North Water Diversion Project.

As for water for industrial use, Enterprises should establish sewage treatment equipment, the industrial waste water should be discharged after treatment, which can reduce water pollution, or use the water after treatment for the second time in order to improve the utilization rate.

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