Lorentz-violating neutral-pion decays in isotropic modified Maxwell theory

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We consider an extension of the Standard Model with isotropic nonbirefringent Lorentz violation in the photon sector and restrict the discussion to the case of a “fast” photon with a phase velocity larger than the maximum attainable velocity of the quarks and leptons. With our conventions, this case corresponds to a negative Lorentz-violating parameter $\kappa$ in the action. The decay rate of a neutral pion into two nonstandard photons is calculated as a function of the 3-momentum of the initial pion and the negative Lorentz-violating parameter $\kappa$ of the final photons.

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1. Introduction

In this short paper, we calculate the decay rate of a particular Lorentz-violating process, neutral-pion decay into two photons, which can be tested by modeling and observing extensive air showers in the upper Earth’s atmosphere.

2. Theory

We restrict our attention to the strong and electromagnetic interactions of the Standard Model. The corresponding theory is a vector-like (nonchiral) $SU(3) \times U(1)$ gauge theory. The action density of this gauge theory is now augmented by a single Lorentz-violating CPT-even dimension-4 photonic term with a dimensionless real coupling constant $\kappa$. The Lorentz-violating parameter $\kappa$ is taken to be negative and to have a very small magnitude (a possible underlying theory has been suggested in Ref. 5).

A photon of 3-momentum $\vec{k}$ then has a modified dispersion relation,

$$\left[ \omega(\vec{k}) \right]^2 = \frac{1 - \kappa}{1 + \kappa} c^2 |\vec{k}|^2, \quad (2.1)$$
and modified polarization 3-vectors,
\[ \vec{e}^{(1)}(\vec{k}) = \sqrt{\frac{1}{1 + \kappa R \hat{k} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}} , \]
(2.2a)
\[ \vec{e}^{(2)}(\vec{k}) = \sqrt{\frac{1}{1 + \kappa R \hat{k} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}} , \]
(2.2b)
with the $3 \times 3$ rotation matrix $R \hat{k}$ which transforms the unit column 3-vector $(1, 0, 0)^T$ to the unit column 3-vector $\hat{k} \equiv \vec{k}/k \equiv |\vec{k}|/|\vec{k}|$. See Ref. 4 for further details. In addition, we employ the Minkowski metric $g_{\mu\nu}(x) = \eta_{\mu\nu} \equiv \text{diag}(+1, -1, -1, -1)$ and, from now on, use natural units with $\hbar = c = 1$.

Incidentally, the velocity $c$ appearing on the right-hand side of (2.1) corresponds to the maximum attainable velocity of the quarks and leptons. See also the last paragraph of Sec. 4 for a brief discussion of the theory considered.

3. Calculation
In the theory as outlined in Sec. 2, we study the decay of a neutral pion ($\pi^0$) of mass $M > 0$ into two nonstandard photons ($\tilde{\gamma}$), with energies and 3-momenta denoted as follows:
\[ \pi^0(E, \vec{q}) \to \tilde{\gamma}(\omega, \vec{k}) + \tilde{\gamma}(\omega', \vec{k}') , \]
for on-shell energies $E(\vec{q}) = \sqrt{|\vec{q}|^2 + M^2}$ and $\omega(\vec{k}) \geq 0$ from (2.1). This Lorentz-violating decay process has already been discussed qualitatively in Ref. 6. A later heuristic discussion in terms of effective-mass-squares has been given in App. A of Ref. 8.

Lorentz-violating decays have been discussed extensively in Ref. 7. The general expression for the neutral-pion decay parameter $\gamma$ is then as follows (see, e.g., Sec. 3.6 of Ref. 9):
\[ \gamma(\vec{q}) = 2 E(\vec{q}) \Gamma(\vec{q}) \]
\[ = \frac{1}{2} \frac{1}{(2\pi)^2} \int \frac{d^3k}{2\omega(k)} \int \frac{d^3k'}{2\omega(k')} \delta^4(q - k - k') |M|^2 , \]
(3.2)
with the symmetry factor $1/2$ and the matrix element $M$. In a Lorentz-invariant theory, $|M|^2$ is a scalar and the right-hand side of (3.2) is manifestly Lorentz invariant, so that $\gamma$ becomes independent of $\vec{q}$ and is called the decay constant.

Following the discussion of Secs. 4.5 and 7.1 in Ref. 9 we now take the effective pion-photon-photon interaction to be given by
\[ \mathcal{L}_{\text{eff}} = \alpha C \phi \epsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta} , \]
(3.3)
with the neutral-pion pseudoscalar field $\phi$, the Maxwell field strength $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, and a coupling constant $C$ of mass dimension $-1$. A straightforward but tedious calculation of the Lorentz-violating decay parameter (3.2) then gives

$$\gamma(\vec{q}) = \frac{1}{2} \frac{1}{8\pi} \left(8\alpha C\right)^2 \frac{1}{2} M^4 g\left(\frac{q}{M}, \kappa\right),$$

(3.4)
in terms of a dimensionless function $g$ of the 3-momentum modulus $q \equiv |\vec{q}|$ in units of $M$ and the Lorentz-violating parameter $\kappa$. The calculated function $g(q/M, \kappa)$ has the following properties:

$$g(q/M, 0) = 1,$$

(3.5a)

$$g(0, \kappa) = \frac{\sqrt{1 - \kappa^2}}{(1 - \kappa)^3} = 1 + 3\kappa + O(\kappa^2),$$

(3.5b)

$$g(q/M, \kappa) = 0, \quad \text{for } \sqrt{q^2 + M^2} \geq E^{(\text{cutoff})},$$

(3.5c)

with cutoff energy

$$E^{(\text{cutoff})} = \sqrt{\frac{1 - \kappa}{-2\kappa}} M \sim \frac{M}{\sqrt{-2\kappa}}.$$  (3.6)

The last approximate expression for the cutoff energy in (3.6) agrees with the kinematic result of Ref. 6.

The exact formula for $g(q/M, \kappa)$ is, at first, rather cumbersome but can be brought to a manageable form,

$$g(q/M, \kappa) = \begin{cases} \frac{\sqrt{1 - \kappa^2}}{(1 - \kappa)^3} \left[1 - \left(q/q_c\right)^2\right]^2 & \text{for } q < q_c, \\ 0 & \text{for } q \geq q_c, \end{cases}$$

(3.7a)

$$q_c = \sqrt{\frac{1 + \kappa}{-2\kappa}} M \sim \frac{M}{\sqrt{-2\kappa}}.$$  (3.7b)

Figure 1 gives a plot of the function $g(q/M, \kappa)$ for a relatively large absolute value of the negative Lorentz-violating parameter $\kappa$.

The final result (3.7) for the Lorentz-violating neutral-pion decay parameter (3.4) can be used in numerical simulations of extensive air showers, as discussed in Ref. 8.

4. Discussion

The first version of this paper dates from October, 2016. Since then, numerical simulations of extensive air showers have been performed, which include the effects of two Lorentz-violating decay processes in the theory considered, photon decay into an electron-positron pair as calculated in Ref. 3 and modified neutral-pion
Figure 1. Function $g(q/M, \kappa)$ from (3.7) for the neutral-pion decay parameter (3.4). The numerical value of the Lorentz-violating parameter $\kappa$ is taken to be $\kappa = -5 \times 10^{-5}$, which gives a cutoff momentum $q_c \sim 10^2 M$, according to (3.7). The shape of the function $g$ shown is close to the asymptotic shape $(1 - x^2)^2$, with definition $x \equiv q/q_c$.

decay into two photons as calculated in the present paper. Comparing the simulated values of the average atmospheric depth of the shower maximum $\langle X_{\text{max}} \rangle$ to the measured values from the Pierre Auger Observatory gives a new bound on the negative Lorentz-violating parameter $\kappa$, namely, $\kappa > -3 \times 10^{-19}$ (98% CL).

Remark that the numerical value of this new negative $\kappa$ bound is of the same order as the qualitative bound $0 \leq v_\gamma - v_\pi < 10^{-20}$ from Ref. 11. The bound of Ref. 11 relies, however, on a sharp kinematic cutoff of the standard neutral-pion decay rate and does not take possible photon-decay effects into account. The Lorentz violation considered in Ref. 11 does not trace back to a consistent theory of elementary particle interactions, whereas the bound of Ref. 10 follows from explicit decay rates calculated in standard quantum electrodynamics (QED) with a single Lorentz-violating term added to the photonic action.

Recall, finally, that the isotropic Lorentz-violating term in the photon sector can be moved into the fermion sector by an appropriate coordinate transformation; see App. B of Ref. 3 for details and further references. In fact, the Lorentz-violating parameter $\kappa$ measures the relative difference in the photon phase velocity and the maximum attainable velocity of the massive Dirac fermions considered (quarks and leptons), as clarified by Eq. (4) in Ref. 10.

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References

1. S. Chadha and H.B. Nielsen, “Lorentz invariance as a low-energy phenomenon,” *Nucl. Phys. B* **217**, 125 (1983).
2. V.A. Kostelecký and M. Mewes, “Signals for Lorentz violation in electrodynamics,” *Phys. Rev. D* **66**, 056005 (2002), arXiv:hep-ph/0205211.
3. F.R. Klinkhamer and M. Schreck, “New two-sided bound on the isotropic Lorentz-violating parameter of modified Maxwell theory,” *Phys. Rev. D* **78**, 085026 (2008), arXiv:0809.3217.
4. F.R. Klinkhamer and M. Schreck, “Consistency of isotropic modified Maxwell theory: Microcausality and unitarity,” *Nucl. Phys. B* **848**, 90 (2011), arXiv:1011.4258.
5. F.R. Klinkhamer and M. Schreck, “Models for low-energy Lorentz violation in the photon sector: Addendum to ‘Consistency of isotropic modified Maxwell theory’,” *Nucl. Phys. B* **856**, 666 (2012), arXiv:1110.4101.
6. S.R. Coleman and S.L. Glashow, “High-energy tests of Lorentz invariance,” *Phys. Rev. D* **59**, 116008 (1999), arxiv:hep-ph/9812418.
7. C. Kaufhold and F. R. Klinkhamer, “Vacuum Cherenkov radiation and photon triple-splittings in a Lorentz-noninvariant extension of quantum electrodynamics,” *Nucl. Phys. B* **734**, 1 (2006), arXiv:hep-th/0508074.
8. J.S. Diaz, F.R. Klinkhamer, and M. Risse, “Changes in extensive air showers from isotropic Lorentz violation in the photon sector,” *Phys. Rev. D* **94**, 085025 (2016), arXiv:1607.02090.
9. B. De Wit and J. Smith, *Field Theory in Particle Physics, Volume 1* (North-Holland Physics Publ., Amsterdam, 1986).
10. F.R. Klinkhamer, M. Niechciol, and M. Risse, “Improved bound on isotropic Lorentz violation in the photon sector from extensive air showers,” *Phys. Rev. D* **96**, 116011 (2017), arXiv:1710.02507.
11. E.E. Antonov, L.G. Dedenko, A.A. Kirillov, T.M. Roganova, G.F. Fedorova, and E.Y. Fedunin, “Test of Lorentz invariance through observation of the longitudinal development of ultrahigh-energy extensive air showers,” *JETP Lett.* **73**, 446 (2001).