Adiabatic two-photon quantum gate operations using a long-range photonic bus

Anthony P Hope\(^1\), Thach G Nguyen\(^1\), Arnan Mitchell\(^1\) and Andrew D Greentree\(^2\)

\(^1\)ARC Centre of Excellence for Ultrahigh bandwidth Devices for Optical systems (CUDOS) and School of Electrical and Computer Engineering, RMIT University, Melbourne, Australia
\(^2\)Chemical and Quantum Physics, School of Applied Sciences, RMIT University, Melbourne, Australia

E-mail: anthony.hope@student.rmit.edu.au

Received 16 July 2014, revised 22 December 2014
Accepted for publication 2 January 2015
Published 4 February 2015

Abstract

Adiabatic techniques have much potential to realize practical and robust optical waveguide devices. Traditionally, photonic elements are limited to coupling schemes that rely on proximity to nearest neighbour elements. We combine adiabatic passage with a continuum based long-range optical bus to break free from such topological restraints and thereby outline a new approach to photonic quantum gate design. We explicitly show designs for adiabatic quantum gates that produce a Hadamard, 50:50 and 1/3:2/3 beam splitter, and non-deterministic controlled NOT gate based on planar thin, shallow ridge waveguides. Our calculations are performed under conditions of one and two-photon inputs.

Keywords: quantum computation, silicon photonics, adiabatic transport, quantum information processing, two photon, single photon, gates

1. Introduction

Integrated optics is becoming one of the most important platforms for the production of compact, scalable, linear optical quantum devices [1]. Much of this progress derives from the use of laser-defined waveguides in glass or polymer, which enables compact three-dimensional waveguide geometries to be designed and rapidly prototyped [2]. While these devices are clearly important for scientific applications, they are not compatible with standard lithographic processes and are limited in the topologies that can be considered. Direct write waveguides rely on serial material modification which places limitations on the complexity of the waveguide system designs that can be achieved. As a result, most geometries rely on evanescent coupling of nearest neighbours. It has recently been shown that long range coupling can be achieved through utilization of lateral leakage radiation in thin, shallow ridge silicon photonic waveguides [3]. This paper shows that the ability to break free from the limitations of simply connected waveguide topologies offers new opportunities for the realization of complex, multi-port quantum gates.

One of the most fundamental elements required for integrated optical devices, especially quantum devices, is the beamsplitter. This is the essential element for any interferometer, and can also be used (with trivial phase control) to effect a Hadamard rotation [4]. In the two-photon subspace, the beamsplitter can show the Hong–Ou–Mandel effect [5], one of the clearest non-trivial experiments to highlight the fundamental differences between classical and quantum optics. In integrated optics, a beamsplitter is typically realized through the use of a directional coupler. This is a device where two waveguides are brought into close proximity so that evanescent coupling causes population to tunnel between the waveguides. Truncating the device to the appropriate length then effects the desired beam splitting ratio. Although, in principle, it should be relatively easy to build such devices; in practice, any lack of control in the actual waveguide size leads to a lack of control in the evanescent tunnelling, and hence the length of device required to achieve a particular beam splitting ratio will be effectively unknown. A common solution is to post-select devices from a suite of similar devices, or alternatively, phase shifting elements such as heaters, can be used to fine tune and reconfigure devices [6].
Adiabatic passage promises a solution to issues of device variability that require post-fabrication tuning. This is because adiabatic evolution goes as the ratio of tunnel matrix elements, rather than the absolute value of those elements. The tradeoff is that adiabatic devices are typically longer than their non-adiabatic counterparts, and whether the rewards of seeking an adiabatic versus non-adiabatic device are justified depends on the degree of device control required and the available footprint. Adiabatic methods for transport of population between states of the kind we envisage here began with the stimulated Raman adiabatic passage (STIRAP) protocol [7, 8], where robust transfer of population between atomic energy levels is effected by laser control, and the all-spatial variant that is sometimes called coherent tunnelling adiabatic passage (CTAP) [9–13]. There are also related schemes considering the adiabatic conversion of frequency modes of light [14].

Here we explore theoretically the potential for effecting geometric gates via long-range CTAP in thin, shallow ridge silicon-on-insulator (SOI) waveguides, shown schematically in figure 1(a). Specifically, we demonstrate several important gate designs including 50:50 beamsplitter and 1/3:2/3 beamsplitter, using a spatial version of the method outlined by Unanyan et al (USB) [15]. Our calculations are performed for both one and two photon input states. Further, we concatenate these devices to show an adiabatic non-deterministic controlled NOT (CNOT) gate, following the approach described by Ralph et al [38]. Our scheme utilizes a long-range common bus mode present in thin, shallow ridge waveguide devices [3]. This common bus mode provides a significant and new opportunity to develop planar geometries which are nonetheless not restricted to linear nearest-neighbour coupling. In this way, we see our approach as being more amenable to mass production, especially CMOS compatible fabrication, than truly three-dimensional approaches such as those described in, for example, [2, 16, 17].

This paper is organized as follows. We first provide a brief introduction to thin, shallow ridge SOI waveguides, with emphasis on their effective refractive index and coupling to slab modes. As will be shown, control of both the magnitude and the sign of the coupling between waveguides and the slab can be achieved by the position of the waveguide relative to the slab mode. A change in the sign of the coupling leads to symmetry breaking mechanisms that are essential for USB-style geometric gates. With this understanding, we generate an effective Hamiltonian that can be used to effect arbitrary geometric gate sequences, and in particular we describe methodologies to realize an adiabatic power splitter, a Hadamard gate and 1/3:2/3 beamsplitter via the USB approach. After demonstrating one-qubit gates, we show the extension to two-photon gates, in particular showing that the well-known Hong–Ou–Mandel effect is preserved under conditions of adiabatic passage. Finally, we show the full state evolution for a non-deterministic linear optical controlled gate operating in the coincidence basis. This non-trivial two-photon, two-qubit entangling gate is completely simulated across eight optical modes (seven waveguide modes plus one bus mode), each of which can potentially have either 0, 1 or 2 photons.

2. Adiabatic evolution with thin, shallow ridge SOI waveguides

Thin, shallow ridge SOI waveguides when operating in TM polarization can exhibit lateral leakage behaviour [18]. Photons in a TM guided mode can leak into a lateral unguided TE slab mode, propagating at a specific angle to the waveguide axis due to polarization conversion at the ridge side-walls [19]. This TE slab mode can act as a bus mode to allow long-range communication between isolated waveguides [3, 20].

When the silicon slab is terminated, the continuum of TE slab radiation is discretised into discrete TE slab modes. On careful selection of the slab width, one of these TE slab modes can be phase-matched to the TM guided mode. Thus, the photons from the guided TM mode can couple to this TE slab mode. One particular method for varying the strength of the coupling between the guided TM mode and TE slab mode is by varying the relative location of the ridge waveguide on the slab [3, 20]. This technique opens up a new class of coupler that can enable interactions between multiple, well separated waveguides simultaneously, which is not possible in a traditional planar evanescent arrangement and has recently been proposed for CTAP devices [3]. While here we only consider interactions mediated via a single, discrete slab mode for clarity, generalizing our method so that coupling is via continuum states should be possible following the methods in [21, 22].

We consider a discrete silicon slab supporting a laterally defined bus mode $|0\rangle$ with propagation constant $\beta_0$ and $N$ forward propagating waveguide modes $|i\rangle$ of $\beta_i = k_0 n_i$ with effective index $n_i$, where $k_0 = 2\pi/\lambda_0$ is the free space wavevector for wavelength $\lambda_0$. Under these definitions, we may write down the system using a tight-binding Hamiltonian in
second quantized form as:

\[ H(z) = \beta_0 a_0^{\dagger} a_0 + \sum_{i=1}^{N} \beta_i a_i^{\dagger} a_i + \Omega_i a_i^{\dagger} a_i + h.c., \]  

where \( a_i (a_i^{\dagger}) \) is the photon annihilation (creation) operator acting on mode \(|i\rangle\) for \( i = 0 \ldots N \). Each waveguide is mutually isolated by separating an appropriate distance, ensuring there is no appreciable evanescent coupling, so that the waveguides only communicate through the common bus. The strength of this coupling (\( \Omega_j \)) is controlled by translating the waveguides laterally across the slab and this response is sinusoidal due to the nature of the bus mode [3, 20]. The coupling of a single waveguide to the bus is \( \Omega(z) = \Omega_{\text{max}} \sin[\beta x(z)] \), where \( x \) is the lateral waveguide location, assuming isolation occurs in the centre of the slab. The lateral waveguide dimension in turn varies as a function of the propagation dimension, \( z \), which is the mechanism to effect the adiabatic passage. The relationship between the lateral (\( x \)) and forward propagation (\( z \)) dimensions is controlled so that the couplings are varied adiabatically. The maximum coupling (\( \Omega_{\text{max}} \)) available depends on the waveguide dimensions [19, 23] and can be calculated as the overlap integral between the bare bus TE and waveguide TM modes. This magnitude is represented as the imaginary effective index of a TM–TE coupled mode on an open slab [19], or in the discrete case by observing the level of mode splitting throughout translation [3]. In the case where the slab width increases to accommodate additional waveguides, it is expected that this coupling will decrease as the maximal overlap of the single bus mode at any point decreases, resulting in longer devices. This scalability is of importance when considering more general Morris-Shore type devices [24, 25].

In the discussion that follows we will adopt two separate notations. When we consider only the one-photon subspace, we will use the compact notation of defining the basis states by the position of the photon, i.e. we define \(|i\rangle \equiv a_i^{\dagger} |\emptyset\rangle\), where \(|\emptyset\rangle\) is the vacuum state of the system. However, when we deal with two-photon states, we will define the states by the occupation numbers of each mode, for example the state \(|0110\rangle \equiv a_1^{\dagger} a_2^{\dagger} a_3 |\emptyset\rangle\). All of our simulations use a tight binding approach to solving the spatially varying Hamiltonian, and do not assume that the adiabatic limit is achieved.

Here we wish to inject light into port [3] and arrive in an even superposition of both [1] and [2]. The position of each waveguide is selected to provide ideal initial CTAP conditions \([\Omega_1(0) = 0, \Omega_2(0) = \Omega_{\text{max}}] \). By translating the waveguides linearly across the slab, as illustrated in figure 2(a), the couplings are varied sinusoidally to effect the counter-intuitive pulse sequence. In particular, we have \( \Omega_3(z) = \Omega_{\text{max}} \sin \left( \pi/z_{\text{max}} \right) \) and \( \Omega_1(z) = \Omega_2(z) = \Omega_{\text{max}} \cos \left( \pi/z_{\text{max}} \right) \) shown in figure 2(b). The sin/cos coupling scheme has the nice property that the adiabaticity is constant throughout the protocol [30–32]. As \( \Omega_1 \) and \( \Omega_2 \) remain identical, an even power split arrives in each waveguide with the populations throughout the protocol described in figure 2(c). Combining the predicted paths, calculated population values and expected Gaussian mode profile of the waveguides gives a more visual representation of this transfer (figure 2(d)).

This one input-two output device is equivalent to a Y-splitter but without the conventional restrictions of close proximity or ordering of the waveguides. There is therefore enhanced flexibility with the bus approach than more conventional approaches. This technique can also be extended to distribute population evenly across many waveguides in a method akin to that in [17]. The overall device length required to perform a successful adiabatic passage depends on \( \Omega_{\text{max}} \).

The waveguide dimensions specified in [3] provided a coupling length of 150 \( \mu \)m for a single waveguide and bus. It was shown that device lengths of \( z_{\text{max}} \geq 2 \text{ mm} \) were required to successfully achieve robust adiabatic transport between two waveguides using the long-range bus, without significant population of the bus. This implies that to achieve successful CTAP behaviour the total device length must be at least longer than 15 coupling lengths. The magnitude of maximum coupling \( \Omega_{\text{max}} \) can be enhanced through waveguide engineering [23, 33]. Increasing the available coupling will decrease the coupling length and hence reduce the absolute device length \( z_{\text{max}} \) required to maintain adiabaticity, although such optimization is not critical to explain our concepts.

4. Controlled-ratio beamsplitters

The method of power splitting can be modified to effect robust quantum gates via the USB method [15, 34]. In this process, a double application of the power division is applied, with a change in the sign of the coupling applied between the first and second applications of the splitting. Because of the standing wave nature of the bus mode, the coupling between the waveguide and the slab varies sinusoidally with the waveguide position. The sinusoidal variation means that the sign of the coupling reverses in each standing-wave period. By ensuring that the forward and backward adiabatic passage crosses periods with the appropriate signs, the necessary symmetry breaking that is at the heart of the USB process can be achieved by waveguide translation alone.

In the one-photon subspace, the quantum gate is specified without loss of generality to act on qubit subspace of [1] and
with $|3\rangle$ as an auxiliary mode, explained below. We assume that the system is initialized in an arbitrary superposition $|\psi\rangle = \eta_1 |1\rangle + \eta_2 |2\rangle$, and for convenience we express the qubit in the dark/bright state basis

$$|D\rangle = \frac{\Omega_2 |1\rangle - \Omega_1 |2\rangle}{\sqrt{\Omega_1^2 + \Omega_2^2}}, \quad |B\rangle = \frac{\Omega_1 |1\rangle + \Omega_2 |2\rangle}{\sqrt{\Omega_1^2 + \Omega_2^2}}.$$  

Note that provided the ratio of the couplings remains constant, the compositions of the dark and bright states will not change. Ideally $|D\rangle$ remains completely isolated from $|0\rangle$, while $|B\rangle$ can be influenced using CTAP. The counter-intuitive sequence transfers $|B\rangle \rightarrow |3\rangle$, instigating a phase reversal of $|B\rangle$ before returning it will alter the superposition, performing a rotation of the single qubit. The magnitude of this rotation is set by $\alpha = \Omega_2/\Omega_1$, and the net effect of a double application of the adiabatic passage is the gate:

$$G = \frac{1}{1 + \alpha^2} \begin{bmatrix} \alpha^2 - 1 & -2\alpha \\ -2\alpha & 1 - \alpha^2 \end{bmatrix}.$$  

We illustrate the operation of this gate with three examples.

Preparing the system with all population initially in $|1\rangle$, the waveguides are translated to provide counter-intuitive multiple waveguide population transfer, the sidebar to the right shows the magnitude of the sinusoidal coupling between the waveguides and the slab mode, based on the modal overlap. (b) Magnitude of coupling terms using the provided waveguide geometries. (c) Modal population of all states throughout system evolution shows 50:50 power division to output coupling between the waveguides and the slab mode, based on the modal overlap. (b) Magnitude of coupling terms using the provided waveguide geometries. (c) Modal population of all states throughout system evolution shows 50:50 power division to output coupling between the waveguides and the slab mode, based on the modal overlap. (b) Magnitude of coupling terms using the provided waveguide geometries. (c) Modal population of all states throughout system evolution shows 50:50 power division to output coupling between the waveguides and the slab mode, based on the modal overlap. (b) Magnitude of coupling terms using the provided waveguide geometries. (c) Modal population of all states throughout system evolution shows 50:50 power division to output coupling between the waveguides and the slab mode, based on the modal overlap. (b) Magnitude of coupling terms using the provided waveguide geometries. (c) Modal population of all states throughout system evolution shows 50:50 power division to output coupling between the waveguides and the slab mode, based on the modal overlap.

Other gate operations occur when $\alpha \neq 1$. The available gates are confined to rotations in the canonical $X-Z$ plane for the qubit defined across modes $|1\rangle$ and $|2\rangle$. If we define the trajectory of the state with greatest coupling to be linear, i.e. so that the coupling between this waveguide and the slab is sinusoidal and maximal, then it follows that the other must trace out a curved trajectory. So without loss of generality, assuming $\Omega_1 > \Omega_2$, we have

$$\Omega_1(z) = \Omega_{\text{max}} \sin \left( \beta_0 x_1(z) \right),$$  

$$\Omega_2(z) = \Omega_{\text{max}} \sin \left( \beta_0 x_2(z) \right) = \alpha \Omega_1,$$  

$$\therefore x_2(z) = \frac{\sin^{-1}(\alpha \Omega_1/\Omega_{\text{max}})}{\beta_0}.$$  

The Hadamard gate is a commonly used quantum information primitive, and is equivalent (up to phase) to a conventional beamsplitter. Preparing the system entirely in $|1\rangle$ a successful Hadamard operation will result in the state being transformed to $(1/(\sqrt{2})(|1\rangle + |2\rangle))$. Using (3), a value of $\alpha = \tan(\pi/8) \approx 0.4142$ provides this behaviour. The waveguide trajectories required to provide this value of $\alpha$ are shown in figure 3(a), with the resulting coupling scheme in figure 3(b). Evolving this system adiabatically results in the expected Hadamard operation as demonstrated in figures 3(e) and (f).

Another important beamsplitter is the 1/3:2/3 beamsplitter. Beamsplitters with this splitting ratio form the basis for non-deterministic linear optical quantum computing [38, 39]. A suitable two port 1/3:2/3 beam splitter is designed with $\alpha = 0.5176$ and operation is shown in figures 3(g) and (h).
5. Two-photon operation

There are few studies that have explicitly considered adiabatic passage of more than one particle \[40 - 42\], without invoking some mean-field or other effective treatment (as in for example \[43, 44\]). We are not aware of any previous works that consider adiabatic multi-particle gates such as we describe here, and hence some explanation of the two-photon gate operation is required.

The one-photon calculations described above are indistinguishable from the results that would be obtained for a purely classical modal analysis. Although the operation of the adiabatic gates on two-photon states is exactly what should be predicted from an equivalent conventional device, the microscopic details of how the adiabatic device achieves two-photon interactions are interesting and non-trivial.

For two photons across four modes (three waveguides + one bus mode), there are ten states that need to be considered. These, along with the couplings between the modes, are shown in figure 4. The states are: \[|0011\rangle, |0101\rangle, |0110\rangle, |1001\rangle, |1010\rangle, |1100\rangle, |0002\rangle, |0020\rangle, |0200\rangle, and |2000\rangle\], where as before the most significant digit denotes the number of photons in the bus mode, and the subsequent digits refer to the number of photons in waveguides one to three. The Hamiltonian of the two-photon states is spanned by a four-dimensional null space compraising the (unnormalized)
Figure 4. There are ten states involved in the two-photon, four mode Hamiltonian. This figure indicates the pertinent states and the strengths of the couplings between them.

vectors

\[ |D_{1}^{(2)}\rangle = -\frac{\Omega_{3}^{2} + \Omega_{2}^{2} + \Omega_{1}^{2}}{\sqrt{2}} |0011\rangle - \frac{\Omega_{2} + \Omega_{3}}{\sqrt{2}} |0101\rangle - \frac{\Omega_{2} + \Omega_{3}}{\sqrt{2}} |0110\rangle + |0200\rangle, \]

\[ |D_{2}^{(2)}\rangle = \frac{\Omega_{3}^{2}}{\sqrt{2}} |0011\rangle - \frac{\Omega_{1} + \Omega_{2}}{\sqrt{2}} |0101\rangle - \frac{\Omega_{1} + \Omega_{2}}{\sqrt{2}} |0110\rangle + |0200\rangle, \]

\[ |D_{3}^{(2)}\rangle = -\frac{\Omega_{3}^{2}}{\sqrt{2}} |0011\rangle + \frac{\Omega_{1}^{2}}{\sqrt{2}} |0101\rangle - \frac{\Omega_{1}^{2}}{\sqrt{2}} |0110\rangle + |0020\rangle, \]

\[ |D_{4}^{(2)}\rangle = -\frac{\Omega_{3}^{2}}{\sqrt{2}} |0011\rangle - \frac{\Omega_{3}^{2}}{\sqrt{2}} |0101\rangle + \frac{\Omega_{1}^{2}}{\sqrt{2}} |0110\rangle + |0020\rangle. \]

With this four-dimensional null space, it is difficult to gain insight into the exact properties of the null state for any given problem, but there are some important features that can be gleaned. Firstly, notice that there is no overlap with states with a single photon in the bus mode. This is desirable as it minimizes sensitivity to loss from this mode. There is, however, potential overlap with the bus mode from |\( D_{4}^{(2)} \rangle \). We have numerically confirmed that providing the system is initialized in either |0011\rangle, |0101\rangle or |0110\rangle, there is no population in |0000\rangle up to the limits of the adiabaticity of our numerical experiment, as seen for example in figure 5, indicating that |\( D_{4}^{(2)} \rangle \) is not populated during the adiabatic gate operation. The absence of population in the bus is important as it means that the adiabatic gate is indeed robust against loss from the bus mode.

Considering the 50:50 beamsplitter for the case of two indistinguishable input photons. The system is prepared in the state \( a_{1}^{1} a_{2}^{1} |\emptyset\rangle = |0110\rangle \). Using the same coupling scheme as in the one-photon case, namely

\[ \Omega_{1} = \Omega \sin(\pi z/\z_{max}), \]

\[ \Omega_{2} = \Omega \tan(\pi/8) \sin(\pi z/\z_{max}), \]

our results are shown in figure 5. Note the smooth, adiabatic evolution. In this case, the initial state |0110\rangle is transformed to an entangled state at the midpoint of the protocol, with non zero population in |0110\rangle, |0011\rangle, |0101\rangle, |0002\rangle, |0020\rangle and a very small contribution from |0200\rangle. This evolution should be contrasted with the corresponding case from the one-photon Hadamard interaction (figures 3(e) and (f)), where the bright state was entirely transported to the state |0001\rangle. At the midpoint, there is no change in |\Omega_{3}\rangle. This sign change leads to interference, with the net result that at the end of the protocol, the entangled state \((1/\sqrt{2})(|0000\rangle - |0020\rangle)\). This state is precisely the state that provides the expected two-photon Hong–Ou–Mandel response as measurement of the output ports will project the photons to be either both at waveguide 1, or both at waveguide 2.

6. CNOT operation

The CNOT gate is a fundamental entangling gate and popular choice as member of a universal gate set for quantum computing [45]. This is a two qubit gate, where the state of the target is flipped conditional on the state of the control qubit. One method to generate a scalable, but non-deterministic CNOT gate between individual photons is through combinations of linear optical elements (beamsplitters) [46]. Here we show the set of results when applying one particular implementation (the coincidence basis implementation) of a non-deterministic CNOT gate, based on 1/3:2/3 and 50:50 beamsplitters, following Ralph et al [38].

The canonical coincidence-based implementation requires six photonic modes, here encoded in the spatial modes available to the photons and shown in figure 6. Modes 1–3 correspond to the modes of the control. Mode 1 is the vacuum state for the control, mode 2 the control in the 1 state, and mode 3 the control in the 0 state. Mode 4 is the target 1 state, mode 5 the target 0 state and mode 6 the target vacuum state. There are then five full gate sequences,
which we denote $G_1$–$G_5$. $G_1$ and $G_5$ are 50:50 beam splitters, while $G_2$–$G_4$ are 1/3:2/3 beamsplitters. Our implementation requires eight modes, so in addition to the six modes discussed already, there is the bus mode (denoted by mode 0) that couples all of the modes via CTAP, and an auxiliary mode (mode 7) which plays the same role as the auxiliary mode for the one photon gate. Each gate works using the methods described above, with population adiabatically transferred from the interacting modes and auxiliary mode, via the bus mode. In the canonical CNOT gate, the 1/3:2/3 gates are performed in parallel, however in our case, due to the shared bus and auxiliary modes, all gates must be performed sequentially.

The results of performing the full CNOT gate operation on the appropriate starting states is shown in figure 7. The total state space for the problem with eight modes and up to two photons per mode has dimensionality 6561. However only 64 states actually participate in the problem, and in the adiabatic limit, only 49 of these will have non-zero population. Nevertheless, we do not label all of the occupied modes in the evolution shown in figure 7, instead only highlighting the starting states, with the final states given in table 1. The various output configurations are labelled as success or failure on the basis of whether they correspond to heralded success or failure of the non-deterministic gate. As expected, the table shows that the adiabatic passage CNOT gate operates in the same way as a conventional, coincidence-basis CNOT gate [38], with the correct state appearing with probability 1/9. Exactly the same heralding steps as are utilized in conventional linear optical implementations of the CNOT gate can be used in the adiabatic version.

### 7. Experimental considerations

We now turn our attention to some practical considerations around the manufacture of adiabatic gates via the methods outlined here. While we cannot treat every potential error mode, two of the most important are: photon loss from scattering off roughness in the slab and misalignment of the waveguides with respect to the slab mode. Photon loss is suppressed by the adiabatic operation of the gate [28], so here we briefly describe the effect of waveguide misalignment during fabrication. Here we may observe that our scheme is less robust than laser-defined protocols such as STIRAP, because in our case the tunnel matrix elements go like the sine of the position. Note that this also means that the bus approach is more robust than conventional waveguide adiabatic passage, where the tunnel matrix elements go as the exponential of the waveguide separation.
If we consider a silicon on insulator device operating at a free-space wavelength of 1.5 µm, fabricated via e-beam lithography, then the relative error between waveguides is likely to be insignificant. However the alignment of the waveguides with respect to the slab is limited by the fabrication accuracy, which is likely to be of order nm. From this viewpoint, we consider a lithographically-defined waveguide protocol to effect a particular gate, but an error that misaligns the waveguides in one axis, by an amount $\epsilon$. This error channel is potentially dangerous as the misalignment means that the ratio of the tunnel matrix elements, which is required for the adiabatic operation, will not be constant throughout the protocol. For a set of designed waveguide trajectories, $x_1(z), x_2(z)$ and $x_3(z)$, which are set to produce an ideal Hadamard operation (i.e. $\alpha = \tan(r/\pi)$), we have recalculated the gate fidelity, where the tunnel matrix elements have been replaced by

$$\Omega'_1(z) = \Omega \sin \left( \frac{\pi [x_1(z) + \epsilon]}{\epsilon_{\text{max}}} \right),$$  

$$\Omega'_2(z) = \Omega \sin \left( \frac{\pi [x_2(z) + \epsilon]}{\epsilon_{\text{max}}} \right),$$  

$$\Omega'_3(z) = \Omega \sin \left( \frac{\pi [x_3(z) + \epsilon]}{\epsilon_{\text{max}}} \right).$$

Under these conditions, with $\epsilon_{\text{max}} = 1$ mm, we retrieve the gate fidelity shown in figure 8. It is important to observe that the error is a smooth function of the misalignment, as is expected for an adiabatic process, and that fidelities of greater than 99.2% are achieved for $|\epsilon| < 10$ nm.

**8. Conclusions**

We have shown several designs based on adiabatic long-range couplers that can be useful in the distribution of power within integrated photonic circuits and have also demonstrated how this concept can be extended to quantum information processing specifically in the form of Hadamard and NOT gates, 1/3:2/3 splitters and describe how to perform an arbitrary $X$–$Z$ rotation on a photonic qubit. We extend this to demonstrate a two-photon Hong–Ou–Mandel effect, and show the design of a complete non-deterministic linear optical CNOT gate. The feasibility to realize these designs in a planar CMOS compatible platform is very attractive in regards to large scale integration, fabrication accuracy and circuit complexity. While this work focusses on thin, shallow ridge SOI waveguides, it can be applied to other high index contrast materials that exhibit an accessible long-range bus mode. This technique may be useful in realising more general Morris-Shore type devices where large scale integration is of interest. As discussed, this scalability can require an increase in the overall device length to accommodate additional waveguides, as the overlap between each waveguide to a single bus mode decreases with increased slab width. The adiabatic nature of

![Figure 8](image-url)
these devices results in robust and repeatable signal transfer which is insensitive to variations in the device length.

Acknowledgments

Anthony Hope, Thach Nguyen and Arnan Mitchell acknowledge the support of the Australian Research Council (ARC) Centre of Excellence Funding (CE110001018), Anthony Hope acknowledges Robert and Josephine Shanks Scholarship, Thach Nguyen acknowledges support from the ARC APD fellowship (DP1096153), Andrew Greentree acknowledges the ARC for financial support (DP130104381) and useful conversations with Andon Rangelov and Alberto Peruzzo.

References

[1] O’Brien JL, Furusawa A and Vučočić 2009 Photonic quantum technologies Nature Photonics 3 687–95
[2] Meany T, Delanty M, Gross S, Marshall G D, Steel M J and Withford M J 2012 Classical interference in integrated 3D multiports Opt. Express 20 26895–905
[3] Hope A P, Nguyen T G, Greentree A D and Mitchell A 2013 Long-range coupling of silicon photonic waveguides using lateral leakage and adiabatic passage Opt. Express 21 22705–16
[4] Cerf N J, Adami C and Kwiat P G 1998 Optical simulation of quantum logic Phys. Rev. A 57 R1477–80
[5] Hong C K, Ou Z Y and Mandel L 1987 Measurement of incommensurate field shifts Phys. Rev. Lett. 59 2044–6
[6] Shadbolt P J, Verde M R, Peruzzo A, Politi A, Laing A, Lobino M, Matthews J C F, Thompson M G and O’Brien J L 2012 Nat. Photonics 6 45–9
[7] Gaubatz U, Rudecki P, Becker M, Schiemann S, Kühl M and Bergmann K 1988 Population switching between vibrational levels in molecular beams Chem. Phys. Lett. 149 463–8
[8] Král P, Thanopulos I and Shapiro M 2007 Colloquium: coherently controlled adiabatic passage Rev. Mod. Phys. 79 53–77
[9] Eckert K, Lewenstein M, Corbalán R, Birkl G, Ertmer W and Mompart J 2004 Three-level atom optics via the tunneling interaction Phys. Rev. A 70 023606
[10] Greentree A D, Cole J H, Hamilton A R and Hollenberg L C L 2004 Coherent electronic transfer in quantum dot systems using adiabatic passage Phys. Rev. B 70 235317
[11] Greentree A D, Devitt S J and Hollenberg L C L 2006 Quantum-information transport to multiple receivers Phys. Rev. A 73 032319
[12] Paspalakis E 2006 Adiabatic three-waveguide directional coupler Opt. Commun. 258 30–4
[13] Longhi S, Della Valle G, Ormigotti M and Laporta P 2007 Coherent tunneling by adiabatic passage in an optical waveguide system Phys. Rev. B 76 201101
[14] Suchowski H, Porat G and Arie A 2014 Adiabatic processes in frequency conversion Laser Photonics Rev. 8 333
[15] Unanyan R G, Shore B W and Bergmann K 1999 Laser-driven population transfer in four-level atoms: consequences of non-Abelian geometrical adiabatic phase factors Phys. Rev. A 59 2910
[16] Hill C D, Greentree A D and Hollenberg L C L 2011 Parallel interaction-free measurement using spatial adiabatic passage New J. Phys. 13 125002
[17] Rangelov A A and Vitanov N V 2012 Achromatic multiple beam splitting by adiabatic passage in optical waveguides Phys. Rev. A 85 055803
[18] Webster M, Palchek R, Mitchell A and Koch T 2007 Width dependence of inherent TM-mode lateral leakage loss in silicon-on-insulator ridge waveguides IEEE Photonics Technol. Lett. 19 429–31
[19] Nguyen T G, Tummidi R, Koch T and Mitchell A 2009 Rigorous modeling of lateral leakage loss in SOI thin-ridge waveguides and couplers IEEE Photonics Technol. Lett. 21 486–8
[20] Dalvand N, Nguyen T G, Tummidi R S, Koch T L and Mitchell A 2011 Thin-ridge silicon-on-insulator waveguides with directional control of lateral leakage radiation Opt. Express 19 5635–43
[21] Peters T and Halfmann T 2007 Stimulated Raman adiabatic passage via the ionization continuum in helium: experiment and theory Opt. Commun. 271 475–86
[22] Dreisow F, Szameit A, Heinrich M, Keil R, Nolte S, Tünnermann A and Longhi S 2009 Adiabatic transfer of light via a continuum in optical waveguides Opt. Lett. 34 2405–7
[23] Tummidi R, Nguyen T G, Mitchell A and Koch T 2011 Group IV Photonics (GFP) 8th IEEE Int. Conf. (London) pp 104–6
[24] Morris J R and Shore B W 1983 Reduction of degenerate two-level excitation to independent two-state systems Phys. Rev. A 27 906
[25] Rangelov A A, Vitanov N V and Shore B W 2006 Extension of the Morin-Shore transformation to multilevel ladders Phys. Rev. A 74 053402
[26] Dreisow F, Ormigotti M, Szameit A, Heinrich M, Keil R, Nolte S, Tünnermann A and Longhi S 2009 Polychromatic beam splitting by fractional stimulated Raman adiabatic passage Appl. Phys. Lett. 95 261102
[27] Menchen-Enrich R, Llobera A, Cadarso V J, Mompert J and Ahufinger V 2012 Adiabatic passage of light in CMOS-compatible silicon oxide integrated rib waveguides IEEE Photonics Technol. Lett. 24 536–8
[28] Chung K, Karle T J, Rab M, Greentree A D and Tomljenovic-Hanic S 2012 Broadband and robust optical waveguide devices using coherent tunneling adiabatic passage Opt. Express 20 23108
[29] Circot C, Coda V, Rangelov A A, Neshet D N and Montemezzani G 2012 Planar achronic multiple beam splitter by adiabatic light transfer Opt. Lett. 37 3789–91
[30] Carroll C E and Hioe F T 1990 Analytic solutions for three-state systems with overlapping pulses Phys. Rev. A 42 1522–31
[31] Laine T A and Stenholm S 1996 Achronic processes in three-level systems Phys. Rev. A 53 2501–12
[32] Vaitkus J A and Greentree A D 2013 Digital three-state adiabatic passage Phys. Rev. A 87 063820
[33] Koshaba M, Kakihara K and Saitoh K 2008 Reduced lateral leakage losses of TM-mode like modes in silicon-on-insulator ridge waveguides Opt. Lett. 33 2008–10
[34] Kis Z and Renzoni F 2002 Qubit rotation by stimulated Raman adiabatic passage Phys. Rev. A 65 032318
[35] Greenstreet A D, Smith T B, de Echaniz S R, Durrant A V, Marangoz J P, Segal D M and Vuccaro J A 2002 Resonant and off-resonant transients in electromagnetically induced transparency: turn-on and turn-off dynamics Phys. Rev. A 65 053802
[36] Kasapi A 1996 Three-dimensional vector model for a three-state system J. Opt. Soc. Am. B 13 1347–51
[37] Vuccaro J A, Durrant A V, Richards D, Hopkins S A, Chen H X and Hill K E 1998 Stochastic wavefunction diagrams for electromagnetically induced transparency, inversionless gain and shelving J. Mod. Opt. 45 315–33
[38] Ralph T C, Langford N K, Bell T B and White A G 2002 Linear optical controlled-NOT gate in the coincidence basis Phys. Rev. A 65 062324
[39] Hofmann H F and Takeuchi S 2002 Quantum phase gate for photonic qubits using only beam splitters and postselection Phys. Rev. A 66 024308
[40] Bradly C J, Rab M, Greentree A D and Martin A M 2012 Coherent tunneling via adiabatic passage in a three-well Bose–Hubbard system Phys. Rev. A 85 053609
[41] Olsen M K 2014 Quantum dynamics and entanglement in coherent transport of atomic population J. Phys. B: At. Mol. Opt. Phys. 47 095301
[42] Longhi S 2014 Coherent transfer by adiabatic passage in two-dimensional lattices Ann. Phys., NY 348 161
[43] Graefe E M, Korsch H J and Witthaut D 2006 Mean-field dynamics of a Bose–Einstein condensate in a time-dependent triple-well trap: nonlinear eigenstates, Landau–Zener models, and stimulated Raman adiabatic passage Phys. Rev. A 73 013617
[44] Rab M, Cole J H, Parker N G, Greentree A D, Hollenberg L C L and Martin A M 2008 Spatial coherent transport of interacting dilute Bose gases Phys. Rev. A 77 061602R
[45] Barenco A, Bennett C H, Cleve R, DiVincenzo D P, Margolus N, Shor P, Sleator T, Smolin J A and Weinfurter H 1995 Elementary gates for quantum computation Phys. Rev. A 52 3457
[46] Knill E, Laflamme R and Milburn G J 2001 A scheme for efficient quantum computation with linear optics Nature 409 46