The Minimal Phantom Sector of the Standard Model: 
Higgs Phenomenology and Dirac Leptogenesis

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Abstract

We propose the minimal, lepton-number conserving, SU(3)_c × SU(2)_L × U(1)_Y gauge-singlet, or phantom, extension of the Standard Model. The extension is natural in the sense that all couplings are of O(1) or forbidden due to a phantom sector global U(1)_D symmetry, and basically imitates the standard Majorana seesaw mechanism. Spontaneous breaking of the U(1)_D symmetry triggers consistent electroweak gauge symmetry breaking only if it occurs at a scale compatible with small Dirac neutrino masses and baryogenesis through Dirac leptogenesis. Dirac leptogenesis proceeds through the usual out-of-equilibrium decay scenario, leading to left and right-handed neutrino asymmetries that do not fully equilibrate after they are produced. The model contains two physical Higgs bosons and a massless Goldstone boson. The existence of the Goldstone boson suppresses the Higgs to bb branching ratio and instead the Higgs bosons will mainly decay to invisible Goldstone and/or to visible vector boson pairs. In a representative scenario, we estimate that with 30 fb^{-1} integrated luminosity, the LHC could discover this invisibly decaying Higgs, with mass \sim 120 GeV. At the same time a significantly heavier, partner Higgs boson with mass \sim 210 GeV could be found through its vector boson decays. Electroweak constraints as well as astrophysical and cosmological implications are analysed and discussed.

1 Introduction

The Standard Model (SM) has just two openings where renormalisable operators can be added which couple SU(3)_c × SU(2)_L × U(1)_Y gauge singlet fields to SM fields. One place is the super-renormalisable Higgs mass term \[ H \], the other place is the lepton-Higgs Yukawa interaction \[ L \tilde{H} \]. What would happen if we filled in these gaps?

There is no physical evidence, as yet, to suggest that B − L (baryon – lepton number) is not a good symmetry of nature. The SM preserves B − L, so we will choose to extend the SM in a B − L preserving way. However, overwhelming evidence supporting small, non-zero
neutrino masses does exist \cite{2}. We are therefore led to build a model with Dirac masses for the neutrinos and see if it is possible to create the observed baryon asymmetry of the Universe within this set up. Ideally, we should also strive to build a natural model, both in the ’t Hooft sense and the aesthetic sense. In particular, Yukawa couplings should be either $O(1)$ or strictly forbidden.

Following this approach, we augment the SM with two $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge singlet fields, a complex scalar $\Phi$ and a Weyl fermion $s_R$. These fields will provide the link between the SM and a phantom gauge singlet sector.

$$-\mathcal{L}_{\text{link}} = \left( h_\nu \bar{t}_L \cdot \bar{H} s_R + \text{H.c.} \right) - \eta H^\dagger H \Phi^* \Phi,$$  \hspace{1cm} (1)

where $h_\nu$ and $\eta$ are dimensionless couplings of $O(1)$ in line with our naturalness criterion. The field $H$ (or $\bar{H} = i \sigma_2 H^*$) is the standard $SU(2)_L$ Higgs doublet\footnote{In our notation is $H^T = (H^+, H^0)$ and $\bar{H}^T = (H^{0*}, -H^-)$.} responsible for spontaneous electroweak, $SU(2)_L \times U(1)_Y$, gauge symmetry breaking. Note that $s_R$ must carry lepton number $L = 1$.

In this form, the model is incomplete because neutrinos would have large, electroweak-scale masses. However, we have so far ignored the purely phantom, gauge singlet sector of the model. Here we add a phantom right-handed neutrino $\nu_R$, and $s_L$ the partner of the Weyl field $s_R$. These fields will also carry lepton number. The fermionic part of the phantom sector therefore contains

$$-\mathcal{L}_p = h_p \Phi \bar{s}_L \nu_R + M \bar{s}_L s_R + \text{H.c.}$$  \hspace{1cm} (2)

where $h_p$ is a general complex Yukawa coupling of $O(1)$. Other possible lepton number conserving terms,

$$\bar{t}_L \bar{H} \nu_R + M' \bar{s}_L \nu_R + \Phi \bar{s}_L s_R + \text{H.c.} + \ldots,$$  \hspace{1cm} (3)

are forbidden when imposing a global $U(1)_D$ symmetry, under which only the fields,

$$\nu_R \rightarrow e^{i\alpha} \nu_R, \quad \Phi \rightarrow e^{-i\alpha} \Phi,$$  \hspace{1cm} (4)

transform non-trivially. This choice for the phantom sector is purely motivated by the need for the simplest model leading to small, Dirac neutrino masses. A crucial point to notice here is that the spontaneous symmetry breaking of the global $U(1)_D$ will trigger consistent electroweak gauge symmetry breaking through the last term in $\mathcal{L}_{\text{link}}$ provided that $\langle \Phi \rangle \equiv \sigma \sim v$, with $v$ being the vacuum expectation value (vev) of $H$.

The model

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{link}} + \mathcal{L}_p,$$  \hspace{1cm} (5)

can be trivially embedded into Grand Unified Theories (GUTs) and can be supersymmetrized. Part of the model was first presented in the literature by Roncadelli and Wyler \cite{3}, who were motivated by the need for a model with naturally small Dirac neutrino masses \cite{4}.
Notice that its structure is different (and much simpler) than the ones exploited recently by [5, 6, 7], though the latter are supersymmetric.

We will confine our discussion to a three generation neutrino model. We will therefore add three generations of the $s_R$, $s_L$ and $\nu_R$. For simplicity we will consider just one copy of the complex scalar $\Phi$. This is the Minimal Phantom Sector of the SM consistent with naturally small Dirac neutrino masses, and as we shall see shortly, provides an explanation for the baryon asymmetry of the Universe and has Higgs phenomenology strikingly different to that of the SM.

2 Neutrino Masses

In this section we briefly repeat the main points of [3]. Notice that we are free to work in the basis where the Dirac mass matrix $M$ in (5) is diagonal. After spontaneous $U(1)_D$ symmetry breaking, the Lagrangian (5) results in Dirac-neutrino effective mass terms of the form, $\nu'_L m_{\nu} \nu'_R + s'_L m_N s'_R$, where up to terms $O(M^{-2})$, we obtain

$$m_{\nu} = -m \hat{M}^{-1} m_p, \quad m_N = \hat{M},$$

with $m = h_{\nu} v$ and $m_p = h_{\nu} \sigma$ being $3 \times 3$ matrices and with neutrino mass eigenstates

$$\nu'_L = \nu_L - m \hat{M}^{-1} s_L, \quad s'_L = \hat{M}^{-1} m^\dagger \nu_L + s_L$$

$$\nu'_R = \nu_R - m_p^\dagger \hat{M}^{-1} s_R, \quad s'_R = \hat{M}^{-1} m_p \nu_R + s_R.$$  \hspace{1cm} (7)

Bold face letters denote $3 \times 3$ matrices and column vectors. From (6) we obtain a typical seesaw spectrum with light and heavy Dirac neutrinos with masses $m_{\nu}$ and $\hat{M}$, respectively.

The physical neutrino masses will then result from the final rotation $\hat{m}_{\nu} = A^\dagger m_{\nu} B$ where $A, B$ are unitary matrices. In a basis where the charged lepton Yukawa couplings, $h_e$ are diagonal, the matrix $A$ will just be the usual PMNS matrix [8], measured by neutrino flavour oscillation experiments. Minimal mass matrices $m_{\nu}$ for Dirac neutrinos have been recently classified in [9] and can be exploited to shed light onto the connection between CP-violation and Dirac leptogenesis that follows in the next section. The reader has possibly already realized that the model in (5) is a simple realization of the Froggatt-Nielsen [10] mechanism, usually invoked to generate large hierarchies in quark masses. In this model neutrino masses are generated by the Feynman diagram in Fig. 1 and are given by

$$M_{ij} = g_{ij} e^{a_i + b_j},$$

(8)

Figure 1: The diagram responsible for light Dirac neutrino masses in the model [4].
with $\epsilon = \sigma/M$ and $a_i + b_j = 1$, corresponding to the difference between the $U(1)_D$-charges of the left and the right handed neutrino. The matrix $g_{ij}$ is a general matrix, in our case a product of the matrices $h_\nu v$ and $h_p$. In contrast with the quark case where $\epsilon$ naturally explains the large hierarchy in quark masses, here the smallness of $\epsilon$ explains the relative smallness of neutrino masses. Assuming that the Yukawa couplings are perturbative and of order one, with $v = 175$ GeV and $m_\nu = 0.05$ eV we find the ratio $\epsilon = \sigma/M$, needs to be

$$\epsilon \simeq 3 \times 10^{-13}.$$  \hfill (9)

In the next section we will see that it is possible to achieve this in a model consistent with our naturalness criterion and successful baryogenesis [11]. Significantly, in this model there is no neutrinoless double beta decay, however, we will later examine the consequences of the phantom sector for Higgs searches at the LHC.

3 Baryogenesis

3.1 Dirac Leptogenesis

Although $B - L$ is preserved exactly in this model, we will see that baryogenesis through (Dirac) leptogenesis is still possible [11]. Just as in the SM [14], in this model the combination $B + L$ is anomalous and at low temperatures $B + L$ violation proceeds through tunnelling and is un-observably small. At higher temperatures, close to and above the critical temperature for electroweak symmetry breaking, $T_c \gtrsim 150$ GeV, thermal fluctuations allow field configurations to pass over the ‘sphaleron barrier’, leading to rapid $B + L$ violation [15].

It is important to note that the rapid $B + L$ violating processes do not directly affect right-handed gauge singlet particles. Large Yukawa couplings between the SM quarks and charged leptons will however tend to equilibrate asymmetries in the left and right sectors of the model, depleting an overall ‘right-handed’ $B + L$ as an overall ‘left-handed’ $B + L$ is depleted via ‘sphaleron effects’.

The crucial idea behind Dirac leptogenesis (or Dirac neutrinogenesis) is that the small effective Yukawa couplings between the SM Higgs and the left and right handed neutrinos could prevent asymmetries in the neutrino sector of the model from equilibrating. Therefore, even in a model conserving total lepton number, a left-handed $B - L$ asymmetry could be produced at the same time as an opposite, right-handed $B - L_{\nu_R}$ asymmetry. The left-handed $B - L$ asymmetry would then lead to an overall baryon asymmetry just as in Majorana leptogenesis. Clearly, for this to work the effective neutrino Yukawa couplings must be small enough to keep the left and right lepton asymmetries from equilibrating until (at least) after the electroweak phase transition, when the sphaleron processes linking the baryon and lepton asymmetries would have dropped out of thermal equilibrium. This mechanism even works when the initial, overall $B = L = 0$. It is especially interesting to note that this mechanism links the baryon asymmetry directly to the smallness of the Dirac neutrino masses.

At temperatures above $T_c$, when sphaleron and other SM processes can maintain most SM species in thermal equilibrium it is possible to derive relations amongst the chemical potentials of the various particle species [16]. Since we demand the right handed neutrinos
be out of thermal equilibrium we can ignore their contribution for the moment, leading to the usual relation between baryon and lepton number used in Majorana leptogenesis

\[ Y_B = \frac{28}{79} (Y_B - Y_{L_{SM}}), \]

where \( Y_X \) refers to the ‘asymmetry in X’ to entropy ratio and \( L_{SM} \) refers to the lepton number held in SM particles (not including the right handed neutrinos). If we now consider the case where, initially, the total \( B - L \) was zero, we have the relation

\[ Y_B - Y_{L_{SM}} - Y_{L_{\nu_R}} = 0, \]

which in conjunction with equation (10) leads to

\[ Y_B = -\frac{28}{79} Y_{L_{\nu_R}}, \]

showing that just over a quarter of the right handed neutrino asymmetry is converted into a baryon asymmetry.

### 3.2 CP-violation

As is well known, in order to generate a particle-antiparticle asymmetry in the early universe, the three Sakharov criteria must be fulfilled \([17]\). Particularly relevant to this model are the requirements for a departure from thermal equilibrium and CP-violation.

In analogy with conventional leptogenesis, in this model, CP-violation and a departure from thermal equilibrium could arise during the decays of the heavy Dirac \( S \equiv s_L + s_R \) particles. In particular, CP-violation would originate through the interference between a tree level decay and a 1-loop self-energy diagram as shown in Fig. 2. If we define a ‘right’ CP-asymmetry as

\[ \delta_{R_i} = \frac{\sum_k \left( \Gamma(S_i \rightarrow \nu_{Rk} \Phi) - \Gamma(\bar{S}_i \rightarrow \bar{\nu}_{Rk} \Phi^*) \right)}{\sum_j \Gamma(S_i \rightarrow \nu_{Rj} \Phi) + \sum_l \Gamma(S_i \rightarrow L_l H)}, \]

Figure 2: Feynman diagrams for the decay of the heavy gauge singlet \( S_i \) into a \( \nu_{Rk} \) and a \( \Phi \). The CP-asymmetry in this decay is due to the interference between the tree-level diagram (a) and the 1-loop self-energy diagram (b).
where $\Gamma(S_i \rightarrow \nu R_k \Phi)$ is the rate of the decay process $S_i \rightarrow \nu R_k \Phi$ etc. $L_l$ and $H$ represent lepton and SM Higgs SU(2)$_L$ doublets respectively. In addition, unitarity and CPT conservation provide the following useful relation

$$\sum_j \Gamma(S_i \rightarrow \nu R_j \Phi) + \sum_l \Gamma(S_i \rightarrow L_l H) = \sum_{j'} \Gamma(\bar{S}_i \rightarrow \bar{\nu} R^{*}_{j'} \Phi) + \sum_{l'} \Gamma(\bar{S}_i \rightarrow \bar{L}_{l'} H^*) ,$$

which leads to the relation $\delta_{L_i} = -\delta_{R_i}$, where $\delta_{L_i}$ is defined in analogy with equation (13).

We also define right and left branching ratios as

$$B_{R_i} = \frac{\sum_k \left( \Gamma(S_i \rightarrow \nu R_k \Phi) + \Gamma(\bar{S}_i \rightarrow \bar{\nu} R_k \Phi^*) \right)}{\sum_j \Gamma(S_i \rightarrow \nu R_j \Phi) + \sum_l \Gamma(S_i \rightarrow L_l H)} ,$$

and $B_{L_i} = 2 - B_{R_i}$.

In the limit that the $M_i$ are hierarchical, the lepton asymmetry is generated via the decay of the lightest $S$ and $\bar{S}$. In this case, the CP-asymmetry (13) is given by

$$\delta_{R_1} \simeq \frac{1}{8\pi} \sum_j \frac{M_1}{M_j} \frac{\text{Im} \left[ (h_p h_p^\dagger)_{1j} (h_{\nu} h_{\nu}^\dagger)_{jj} \right]}{(h_p h_p^\dagger)_{11} + (h_{\nu} h_{\nu}^\dagger)_{11}} .$$

It should be noted that the structure of the amplitudes leading to CP-violation in this scenario are similar to the self-energy contribution to the CP-asymmetry in Majorana leptogenesis. Therefore, in the limit of quasi-degenerate masses for the $S_i$, a resonant enhancement of $\delta_R$ should be possible [18, 19]. We will however leave this case to be considered elsewhere. Using equation (6), (16) can be rewritten

$$\delta_{R_1} \simeq -\frac{1}{16\pi} \frac{M_1}{v \sigma} \frac{\text{Im} \left[ (X^\dagger \hat{m}_2 \nu)_{11} \right]}{(X^\dagger \hat{m}_2 \nu)_{11} + (W^\dagger \hat{m}_\nu W)_{11}} .$$

Following the approach of [20], the most general $h_{\nu}$ and $h_p$ can be parameterised in the following way

$$h_{\nu} = \frac{1}{v} A D \sqrt{m_{\nu}} W D \sqrt{M} ,$$

$$h_p = \frac{1}{\sigma} D \sqrt{M} X^\dagger D \sqrt{m_{\nu}} B^\dagger ,$$

where $W$ and $X$ are general $3 \times 3$ matrices satisfying the condition $W X^\dagger = 1$ and $D \sqrt{Z} = +\sqrt{Z}$ for the diagonal matrix $Z$. This allows $\delta_{R_1}$ to be written as

$$\delta_{R_1} \simeq -\frac{1}{16\pi} \frac{M_1}{v \sigma} \frac{\text{Im} \left[ (X^\dagger \hat{m}_2 \nu)_{11} \right]}{(X^\dagger \hat{m}_2 \nu)_{11} + (W^\dagger \hat{m}_\nu W)_{11}} .$$

It is now straightforward to show, in analogy with [21], that $|\delta_{R_1}|$ is bounded from above by

$$|\delta_{R_1}| \lesssim \frac{1}{16\pi} \frac{M_1}{v \sigma} \left( m_{\nu_3} - m_{\nu_1} \right) .$$

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Again, we should stress that this bound could be violated grossly when the $M_i$ are nearly degenerate, and that this caveat should be noted when this bound is used later. As we seek the most minimal model, without additional flavour symmetries in the phantom sector, we will not consider this resonant scenario here.

3.3 Out of equilibrium decays

In the early Universe, if the expansion rate is faster than the interaction rate of a particle species then this species can become decoupled from the thermal bath. This statement can be quantified by considering the ratio of the particle interaction rate $\Gamma(T)$, to the Hubble expansion rate $H(T)$. If this ratio is less than 1, then the species evolves out of thermal equilibrium \[22\].

If we require successful Dirac leptogenesis, the distributions of left and right handed neutrinos should be prevented from coming into thermal equilibrium from the moment a $B - L_{\nu_R}$ asymmetry is created until after the electroweak phase transition, when the rate for $B + L$ violating processes will be much smaller than the expansion rate of the universe.

Processes leading to left-right equilibration include $L H \leftrightarrow \Phi \nu_R$ mediated by the $s$-channel exchange of an $S_1$, $L \bar{\nu}_R \leftrightarrow H \Phi$ and $L\Phi \leftrightarrow \nu_R H$ mediated by the $t$-channel exchange of an $S_1$. Approximately, at high temperatures these processes have a rate

$$\Gamma_{L \leftrightarrow R}(T) \sim \frac{|h_{\nu}|^2 |h_p|^2}{M_1^4} T^5,$$

which should be compared to the Hubble parameter in the relevant radiation dominated era

$$H(T) = \sqrt{\frac{8 \pi^3 g_\ast}{90}} \frac{T^2}{M_P},$$

where $g_\ast \simeq 114$ is the effective number of relativistic degrees of freedom in the SM plus 3 $\nu_R$ and 1 complex $\Phi$, and $M_P = 1.2 \times 10^{19}$ GeV is the Planck mass. The strongest constraint will come from the highest temperatures when $T \simeq M_1$, i.e. those at which the asymmetry is generated;

$$\frac{|h_{\nu}|^2 |h_p|^2}{M_1} \lesssim \frac{1}{M_P} \sqrt{\frac{8 \pi^3 g_\ast}{90}}.$$

To more accurately consider this constraint we need to solve the appropriate Boltzmann equations (discussed next). The dominant contribution to left-right equilibration will come from the inverse decay and subsequent decay of a real $S_1$ or $\bar{S}_1$.

In this model, a $L_{\nu_R}$ asymmetry can be generated via the standard out-of-equilibrium decay scenario, in analogy with various GUT baryogenesis scenarios and Majorana leptogenesis \[12\] \[13\]. In the following, we will consider the asymmetry to be generated solely during the decays of the lightest $S$. Pre-existing asymmetries from, for example, the decays of the heavier $S$s will be treated as initial conditions.

We will assume at $T \gtrsim M_1$ the abundance of $S_1$ and $\bar{S}_1$ is thermal (we will relax this assumption later), and the number density of $S_1$, $n_{S_1} \simeq n_{\bar{S}_1} \simeq n_\gamma$, where $n_\gamma$ is the number
density of photons. As the Universe expands and cools the number density of the $S_1$ ($\bar{S}_1$) must rapidly decrease if they are to remain in thermal equilibrium below $T \approx M_1$. If the interactions allowing this (primarily $S_1$ ($\bar{S}_1$) decays) are slow compared to the expansion rate of the Universe then the $S_1$ and $\bar{S}_1$ abundances will depart from their thermal equilibrium values. When the $S_1$ ($\bar{S}_1$) eventually decay, the rates of back-reactions such as inverse decays will be suppressed by the relatively low temperature $T \ll M_1$ and the resulting $L_{\nu_R}$ asymmetry will be \[22\]

$$Y_{L_{\nu_R}} \equiv \frac{n_{L_{\nu_R}}}{s} \approx \frac{\delta_R n_{S_1}}{g_* n_\gamma} \equiv \frac{\delta_R}{g_*}. \quad (25)$$

We can define a parameter $K$ such that

$$K \equiv \frac{\Gamma(S_1 \rightarrow \nu_R \Phi) + \Gamma(S_1 \rightarrow LH)}{H(T = M_1)} = \left[ (h_p h_p^\dagger)_{11} + (h_\nu h_\nu^\dagger)_{11} \right] \frac{M_P}{16\pi M_1} \sqrt{\frac{90}{8\pi^3 g_*}}, \quad (26)$$

where $K \ll 1$ signifies that $S_1$ is completely out of thermal equilibrium at $T = M_1$; the ‘drift and decay’ limit. We can re-cast this constraint in terms of ‘effective neutrino masses’ \[23\] to make the connection with light neutrino data more transparent. Defining the effective neutrino mass as

$$\tilde{m} \equiv \left[ (h_p h_p^\dagger)_{11} + (h_\nu h_\nu^\dagger)_{11} \right] \frac{v \sigma}{M_1} = K v \sigma \frac{16\pi}{M_P} \sqrt{\frac{8\pi^3 g_*}{90}}, \quad (27)$$

we see that $K < 1$ is satisfied for $\tilde{m} < m_*$ where

$$m_* = v \sigma \frac{16\pi}{M_P} \sqrt{\frac{8\pi^3 g_*}{90}}. \quad (28)$$

The connection between $\tilde{m}$ and the physical light neutrino masses is clearly model dependent, and most applicable when $(h_p h_p^\dagger)_{11} \approx (h_\nu h_\nu^\dagger)_{11}$.

Finally, we can also introduce an efficiency parameter $\kappa$, such that the $L_{\nu_R}$ yield is given by

$$Y_{L_{\nu_R}} = \frac{\delta_R \kappa}{g_*}. \quad (29)$$

For an initially thermal population of $S_1$ and when $K \ll 1$ the efficiency $\kappa \approx 1$.

If $K > 1$, Dirac leptogenesis can still be successful if the CP-asymmetry $\delta_R$ is large enough. However, the simple estimate of the lepton asymmetry, equation \[22\] will no longer be valid, and the Boltzmann equations (BEs) should be solved. There are 4 coupled Boltzmann equations relevant to this scenario, two for the $S_1$ total abundance and asymmetry, one for the asymmetry in $L_L$ and one for the asymmetry in $L_{\nu_R}$. Lepton number conservation means that only three of the asymmetry BEs are independent, we choose to eliminate the equation for $L_{\nu_R}$. The derivation of the BEs has been extensively covered in the literature, see for example \[23, 24, 25\], so we will just write down the set of simplified equations for this
leptogenesis efficiency in scenarios with zero initial abundance of $S$ asymmetry is predominantly determined by processes at $T < T_{\text{eq}}$. The BEs read

$$\frac{d\eta_{\Sigma S_1}}{dz} = \frac{z}{H(z = 1)} \left[ 2 - \frac{\eta_{\Sigma S_1}}{\eta_{\text{eq}}^S} + \delta_R \left( \frac{3\eta_{\Delta L}}{2} + \eta_{\Delta S_1} \right) \right] \Gamma^{D1},$$

$$\frac{d\eta_{\Delta S_1}}{dz} = \frac{z}{H(z = 1)} \left[ \eta_{\Delta L} - \frac{\eta_{\Delta S_1}}{\eta_{\text{eq}}^S} - B_R \left( \frac{3\eta_{\Delta L}}{2} + \eta_{\Delta S_1} \right) \right] \Gamma^{D1},$$

$$\frac{d\eta_{\Delta L}}{dz} = \frac{z}{H(z = 1)} \left\{ \left[ \delta_R \left( 1 - \frac{\eta_{\Sigma S_1}}{2\eta_{\text{eq}}^S} \right) - \left( 1 - \frac{B_R}{2} \right) \left( \eta_{\Delta L} - \frac{\eta_{\Delta S_1}}{\eta_{\text{eq}}^S} \right) \right] \Gamma^{D1} - \left( \frac{3\eta_{\Delta L}}{2} + \eta_{\Delta S_1} \right) \Gamma^W \right\} \tag{30}$$

where $\eta_{\Sigma S} = (n_S + n_\bar{S})/n_\gamma$, $\eta_{\Delta S} = (n_S - n_\bar{S})/n_\gamma$, $z = M_1/T$ and

$$\Gamma^{D1} = \frac{1}{n_\gamma} \left[ \Gamma(S_1 \rightarrow \nu_R \Phi) + \Gamma(S_1 \rightarrow LH) \right] g_{S_1} \int \frac{d^3p}{(2\pi)^3} \frac{M_1}{E_{S_1}} e^{-E_{S_1}/T} ,$$

$$= \left[ \Gamma(S_1 \rightarrow \nu_R \Phi) + \Gamma(S_1 \rightarrow LH) \right] \frac{z^2}{2} K_1(z) \tag{31}$$

where $g_{S_1} = 2$ is the number of internal degrees of freedom of $S_1$, $E_{S_1} = \sqrt{p^2 + M_1^2}$ and $K_1(z)$ is a 1st order modified Bessel function. Decays and inverse decays of $S_1$ and $\bar{S}_1$ are included through terms proportional to $\Gamma^{D1}$. Notice that these terms also include the most important CP-violating $2 \leftrightarrow 2$ scattering contribution coming from the subtraction of real intermediate states from the process $LH \leftrightarrow \nu_R \Phi$ [24]. This subtraction is necessary to ensure unitarity and CPT are respected by avoiding double counting of processes in the BEs.

$\Gamma^W$ parameterises the ‘wash-out’ due to processes of higher order in the Yukawa couplings which will tend to equilibrate the $L$ and $\nu_R$ asymmetries, after the above subtraction of possible real intermediate states has been carried out. These process will be predominantly mediated by the off-shell exchange of $S_{1,2,3}$ and will therefore be highly model dependent [23, 25]. For small Yukawa couplings where $|h_p|^2 \simeq |h_{p'}|^2 < 1$, the processes $L \bar{\nu}_R \leftrightarrow H \Phi$ and $L \Phi \leftrightarrow \nu_R H$ will be negligible compared to decays and inverse decays. The contribution from the off-shell process $LH \leftrightarrow \nu_R \Phi$ is bounded from above by $\Gamma^{D1}$ for the region around $z \sim 1$, if the $S_i$ have a reasonably large hierarchy in mass. We will therefore make the conservative approximation that $\Gamma^W = \Gamma^{D1}$.

Other $2 \leftrightarrow 2$ processes, for example those involving an $S_1$ in the initial or final state, have been neglected since their main contribution is at $T \gtrsim M_1$ where they would act to help create an initially thermal population of $S_1$ [23]. These processes would tend to increase the leptogenesis efficiency in scenarios with zero initial abundance of $S_1$, when $K \ll 1$. The bounds derived later will depend on the leptogenesis efficiency at large values of $K \gg 1$, therefore we expect these processes to have a negligible impact. The same point can be made with regard to thermal effects, which would kinematically block the decays of $S_1$ at temperatures $T \gtrsim M_1$ [25]. As we consider scenarios in the large $K$ regime, the baryon asymmetry is predominantly determined by processes at $T \lesssim M_1$, leading to only small finite temperature corrections to our $T = 0$ estimates.
Clearly, our treatment of the BEs is approximate. We have also neglected lepton flavour effects, coming from the charged lepton Yukawa couplings and the neutrino Yukawa couplings. These effects could be large for scenarios with \( M_1 < 10^{12} \text{ GeV} \), when the charged lepton Yukawa couplings are in thermal equilibrium, or when there is only a mild hierarchy in the \( M_i \)'s. These effects could lead to differences in the final baryon asymmetry of up to an order of magnitude, however they are highly model dependent and would require a study beyond the scope of this paper. Since the purpose of our discussion here is to provide an existence proof and a reasonable estimate of the baryon asymmetry in very general scenarios our approach is expected to be accurate enough.

After solving the BEs we can still parameterise the baryon asymmetry using the efficiency \( \kappa \) defined in equation (29). \( \kappa \) will depend on the values of \( M_1, (h_1 h_\nu)_{11} \) and \( (h_p h_\nu^c)_{11} \). Fig. 3 shows the dependence of \( \kappa \) on \( K \), for various initial conditions and various values of \( B_R \). In cases with no initial abundance of \( S_1 \) we see that the maximal efficiency \( \kappa \sim 1 \) is indeed reached at \( K \sim 1 \).

With either an initially thermal abundance of \( S_1 \), or no initial \( S_1 \), the behaviour of \( \kappa \) for large \( K \gg 1 \) is the same, and for \( K \gtrsim 20 \) is well fitted by the power-law

\[
\kappa \simeq \frac{0.12}{K^{1.1}} = 6.4 \times 10^{-17} \left( \frac{\sigma}{m} \right)^{1.1}.
\] (32)

Although much larger CP-asymmetries are required to produced the observed baryon asymmetry, this large \( K \), or ‘strong wash-out’ regime clearly has the advantage of being insensitive to initial conditions. Fig. 3 also shows the behaviour of \( \kappa \) for differing \( B_R \) (or effectively the ratio \( h_\nu : h_p \) for small \( \delta_R \)). \( B_R = 1.98 \) or \( B_R = 0.02 \) corresponds to a factor of 10 difference between \( h_\nu \) and \( h_p \). We see that as \( B_R \) departs from 1 the efficiency for large \( K \) increases slightly. This effect is due to the less efficient wash-out via inverse-decays in these cases, as

Figure 3: (Left panel) Leptogenesis efficiency, \( \kappa \) defined in (24), versus \( K \), for thermal and non-thermal initial abundance of \( S (\bar{S}) \) and for various \( B_R \). (Right panel) Area in the \( M_1, (h_1 h_\nu)_{11} \) parameter space allowed by successful baryogenesis when \( (h_1 h_\nu)_{11} = (h_p h_\nu^c)_{11} \) and \( \sigma = v = 175 \text{ GeV} \).
can be seen in the BEs (30) where the second term in the equation for $\eta_{\Delta L}$ responsible for the wash-out of the asymmetry via inverse decays is also dependent on $B_R$.

On the right hand side of Fig. 3 we show the area in the $M_1$, $(h^\dagger h)_{11}$ parameter space which is allowed by successful baryogenesis when $(h^\dagger h_{\nu})_{11} = (h^\dagger h^\dagger_p)_{11}$ and $\sigma = v = 175$ GeV. The points plotted correspond to numerical solutions of the BEs with the CP-asymmetry set to the maximum allowed by the bound (21), where the final lepton asymmetry would result in a baryon asymmetry equal to or exceeding the observed one. On the plot we also superimpose $\tilde{m}$ iso-contours.

If we take the most representative natural scenario, namely $(h^\dagger h_{\nu})_{11} = (h^\dagger h^\dagger_p)_{11} \approx 1$ and the reasonable assumption that $\tilde{m} = 0.05$ eV for hierarchical light neutrinos we can use the large $K$ fit to the efficiency (32) and the bound on the CP-asymmetry (21) in conjunction with the value of the observed baryon asymmetry to set an approximate lower limit on $\sigma$. We find that unless $\sigma \gtrsim 0.1$ GeV the $L_{\nu R}$ asymmetry produced is insufficient to explain the observed baryon asymmetry. Notice that this bound depends on several assumptions, in particular that the heavy $S_i$ are hierarchical in mass. Furthermore, the requirement that the universe reheats enough to thermally produce the $S_i$ at the end of inflation leads to an upper bound on $M_1$ of the order of $T_{RH}$, the reheating temperature. This leads to the approximate upper bound $\sigma \lesssim 2$ TeV ($T_{RH}/10^{16}$ GeV).

In summary the scale of the spontaneous symmetry breaking of $U(1)_D$ is bounded,

$$0.1 \text{ GeV} \lesssim \sigma \lesssim 2 \text{ TeV} \left( \frac{T_{RH}}{10^{16} \text{ GeV}} \right) ,$$

and we can therefore conclude that an electroweak scale $\sigma$ is both natural and compatible with successful Dirac leptogenesis.

4 The Higgs sector

4.1 The potential

The complete potential of the neutral scalar fields under consideration reads,

$$V(H, \Phi) = \mu_H^2 H^\dagger H + \mu_\Phi^2 \Phi^\dagger \Phi + \lambda_H (H^\dagger H)^2 + \lambda_\Phi (\Phi^\dagger \Phi)^2 - \eta H^\dagger H \Phi^\dagger \Phi ,$$

where all the parameters are real. For the following, we denote $H \equiv H^0$. Notice that linear or trilinear terms do not appear thanks to the phantom $U(1)_D$ symmetry. After $U(1)_D$ is spontaneously broken, the field $\Phi$ develops a vev $\sigma$, which through the link $\eta$-term in (33), forces the Higgs field $H$ to also develop a vev $v$, triggering the “observed” electroweak $SU(2)_L \times U(1)_Y$ symmetry breaking. Expanding the fields around the minimum we obtain,

$$H = v + \frac{1}{\sqrt{2}} (h + iG) , \quad \Phi = \sigma + \frac{1}{\sqrt{2}} (\phi + iJ) .$$

Notice that the limit $\mu_H \to 0$ is attainable and causes no problem for electroweak symmetry breaking. However, we cannot justify a possible absence of the $\mu_H$-term from (33) by using symmetry or other arguments.
While the Goldstone boson $G$ is eaten by the gauge bosons, the same is not true for the remaining massless Goldstone boson $J$. Furthermore, the fields $h$ and $\phi$, under the influence of the $\eta$-term, mix and become two physical massive Higgs fields $H_i, i = 1, 2$ with masses $m_{H2} > m_{H1},$

$$
\begin{pmatrix}
H_1 \\
H_2
\end{pmatrix} = O
\begin{pmatrix}
h \\
\phi
\end{pmatrix}
with
O = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix},
$$
(36)

and mixing angle

$$
\tan 2\theta = \frac{\eta v \sigma}{\lambda_\phi \sigma^2 - \lambda_H v^2}.
$$
(37)

The limits $v \gg \sigma$ and $v \ll \sigma$ lead to the usual SM scenario with an isolated hidden sector. However, bear in mind that these limits require an un-naturally small $\eta$ and will not be simultaneously compatible with neutrino masses and baryogenesis as advocated in the previous section. The case $\eta \ll 1$, with phenomenology resembling that of the SM, looks like an unnatural corner of the parameter space from the perspective of the Minimal Phantom SM. However, the exact limit $\eta = 0$ is preserved under radiative corrections. The most interesting scenario is the most “natural” one when we require small Dirac neutrino masses and leptogenesis: $\tan \theta \sim 1$. If the phantom sector is responsible for electroweak symmetry breaking then the natural choice of parameters, when taking into account the positivity constraint $\lambda_H \lambda_\phi > \eta^2 / 4$, is

$$
\lambda_H \sim \lambda_\phi \sim \eta \sim 1, \quad \tan \theta \sim 1, \quad \tan \beta \equiv v / \sigma \sim 1.
$$
(38)

Under these conditions, succesful EW symmetry breaking happens only when $\mu_\Phi^2 < 0$. This naturalness condition is supported by an enhanced symmetry $\Phi \leftrightarrow H$ of the potential which is broken however by the Yukawa couplings and the $U(1)_Y$. The Higgs potential of has been studied in in the context of Majoron models where $\sigma$ is an arbitrary vev. This study was mainly confined to LEP collider signatures. It is therefore interesting to update the phenomenology of this Higgs sector after the LEP era and in light of the forthcoming LHC experiments and the condition.

From we see that $h = O_{11} H_1$ and therefore the couplings of Higgs bosons $H_i$ to fermions and gauge bosons will be reduced by a factor $O_{11}$ relative to their corresponding SM ones. It is almost obvious from that $H_1$ will couple to the “invisible” massless Goldstone pair $J J$. The situation is completely different to the SM where the $H \rightarrow bb$ mode dominates for relatively light Higgs masses $\lesssim 160$ GeV. Here we find that in this mass range, the decay rate $H_i \rightarrow J J$ relative to $H_i \rightarrow bb$ reads as,

$$
\begin{align*}
\frac{\Gamma(H_1 \rightarrow J J)}{\Gamma(H_1 \rightarrow bb)} &= \frac{1}{48} \left(\frac{m_{H1}}{m_b}\right)^2 \tan^2 \beta \tan^2 \theta, \\
\frac{\Gamma(H_2 \rightarrow J J)}{\Gamma(H_2 \rightarrow bb)} &= \frac{1}{48} \left(\frac{m_{H2}}{m_b}\right)^2 \tan^2 \beta \cot^2 \theta.
\end{align*}
$$
(39-40)

\[\text{At this stage this has to be put in by hand i.e. we cannot offer a radiative mechanism to explain this.}
\]Dynamical EW symmetry breaking by fourth generation condensates has been discussed in.
Figure 4: Dominant branching ratios of the two Higgs bosons decaying to invisible $JJ$, and to SM particles $bb$, $WW$, $ZZ$ and $tt$ as a function of the Higgs boson mass $H_1$ (left panel) and $H_2$ (right panel) and $\theta = \pi/4$. In every case the mass of the partner Higgs boson is displayed in the upper horizontal axis for comparison. The shaded area is excluded by LEP.

Therefore, from (38,39,40) we see that a light Higgs boson will decay dominantly to invisible $JJ$ as long as it is heavier than 60 GeV. The existence of the massless Goldstone boson $J$ diminishes the Higgs boson decay into a $bb$ pair. LHC experimenters should be aware of this situation which arises in a very simple and natural extension of the SM! On the other hand, the Higgs decays to SM-vector bosons and fermions respectively, as

$$\Gamma(H_i \rightarrow VV) = \Gamma(H_i \rightarrow VV)|_{SM} \times O_{11}^2,$$

$$\Gamma(H_i \rightarrow ff) = \Gamma(H_i \rightarrow ff)|_{SM} \times O_{11}^2,$$  \hspace{1cm} (41) \hspace{1cm} (42)

with $V = Z$ or $W$. Analogous formulae are valid for the cross sections $\sigma(e^+e^- \rightarrow V^* \rightarrow VH_i)$ and $\sigma(pp \rightarrow V^* \rightarrow VH_i)$. In the year 2001, the LEP collaboration presented bounds on an “invisible” Higgs boson mass \[31\]. Following this analysis, certain Higgs boson mass values are excluded as a function of the parameter

$$\xi_i^2 \equiv \frac{\sigma(e^+e^- \rightarrow HZ)}{\sigma(e^+e^- \rightarrow HZ)|_{SM}} \times \text{Br}(H \rightarrow \text{invisible}) = O_{11}^2 \times \text{Br}(H \rightarrow \text{invisible}).$$  \hspace{1cm} (43)

For $\xi^2 = 1$, LEP excludes Higgs boson masses up to its kinematical limit, $m_H \leq 114.4$ GeV. This bound changes only slightly in our case. To make the above discussion more concrete we focus on a representative example with the natural choice of parameters given in \[38\], i.e. $\theta = \beta = \pi/4$ and all couplings equal to one. In this case the production cross sections and branching ratios to gauge bosons, given by \[41\], amount to half of the SM predictions.
Thus $\xi_i^2 \sim 1/2$ for $\text{Br}(H \to JJ) \simeq 100\%$ which is the case for Higgs masses in the region $70 \lesssim m_{H_1} \lesssim 160$ GeV. This is nicely summarised in Fig. 4 where we plot the branching ratios for both Higgs bosons, $H_1$ and $H_2$, decaying into invisible Goldstone bosons $J$ and to other SM-like particles. In general there are two more “invisible” decay “leaks” not depicted in Fig. 4: the first is $H_2 \to H_1 H_1$ which is kinematically forbidden for our choice of $\theta = \pi/4$ and the other is $H_i \to \nu \nu$ which is proportional to the neutrino masses and is therefore negligible. It turns out that for $m_{H_1} \lesssim 160$ GeV the LEP parameter $\xi^2 \gtrsim 0.4$ and therefore LEP excludes a light invisible Higgs boson with a mass $m_{H_1} < \sim 110$ GeV. This also sets a lower bound on the partner Higgs boson mass, $m_{H_2} > \sim 191$ GeV, which is now forced to decay only to visible particles $WW, ZZ, tt$. A search for the latter would follow the SM-type plan, looking for $qqH_2 \to qqWW^{(*)}$ or $gg \to H_2 \to WW^{(*)}, ZZ^{(*)}$, modes at the Tevatron and LHC. Regarding these channels, the only difference here is that production cross sections and decays are reduced by half.

It is apparent from Fig. 4, that there is a mass region

$$110 \lesssim m_{H_1} \lesssim 160 \text{ GeV},$$

where $H_1$ decays to invisible Goldstone bosons with $\text{Br}(H_1 \to JJ) > 90\%$. The question is how can we identify this invisible Higgs boson at the LHC? This question has been studied extensively in the literature $[32, 33, 34]$. The purely invisible Higgs $H_1$ can be searched for at the LHC through the $Z + H_1$ and/or the W-boson fusion channels. Our analysis closely follows the results of $[34]$ for the $Z(\to l^+l^-) + H_1$ production mode, where we multiply their $S/\sqrt{B}$ by a factor of $1/2$ because of $(41)$. We find that for an LHC integrated luminosity of $30$ fb$^{-1}$ the signal significance for the invisible $H_1$ with a mass of $120$ ($140$) $[160]$ GeV is $4.9\sigma (3.6\sigma) [2.7\sigma]$ respectively. Although these results refer to the case where $\theta = \pi/4$ the situation is rather generic in the region of $(38)$. Although the above analysis for the Higgs boson decay to invisible is very important to identify the nature of the phantom sector one may also identify the light Higgs boson when $m_{H_1} < \sim 140$ GeV through the conventional $H_1 \to \gamma\gamma$. We have not made a detailed study for this mode and we believe that it is worth further investigation. For $m_{H_1} > 160$ GeV, $H_1$ decays mainly to $WW$ and $ZZ$ since the $\text{Br}(H \to JJ) \simeq 1/13$ is suppressed. Notice however that $m_{H_1} \gtrsim 200$ GeV is rather disfavoured by the EW data as we will see shortly.

In conclusion, the phantom sector of $(34)$ allows for Higgs decays into invisible Goldstone scalars and to visible gauge bosons. A situation may arise where LHC experimenters could detect $H_1$ with $m_{H_1} \lesssim 120$ GeV through the invisible $Z(\to l^+l^-) + H_1$ mode and $H_2$ with $m_{H_2} \simeq 200$ GeV through its production and decays in association with gauge bosons.

4.2 The $\rho$-parameter and other observables:

In a model with only Higgs doublets and singlets, the tree level value for the electroweak parameter, $\rho \equiv m_W^2/M_Z^2 \cos^2 \theta_W$, is automatically equal to one without further adjustment of the parameters of the theory. The correction to the parameter $\rho$, denoted by $\Delta \rho$, appears at one loop level. For the model at hand, the phantom singlet $\Phi$ will affect gauge boson loops through its $\eta$-mixing term with the observable Higgs field $H$. Then it is straightforward to
calculate the Higgs contribution to $\Delta \rho$ \[35\]. It reads,

$$
\Delta \rho^H = \frac{3G_F}{8\sqrt{2}\pi^2} \sum_{i=1}^{2} O_{i1}^2 \left[ m_W^2 \ln \frac{m_{H_i}^2}{m_{W}^2} - m_Z^2 \ln \frac{m_{H_1}^2}{m_{W}^2} \right],
$$

(45)

where $O$ is the orthogonal matrix in \[30\]. We can establish a useful connection between this formula and the SM one. Note that from the similarity condition of the rotation matrix, $O^T m^2 O = \text{diag}(m_{H1}^2, m_{H2}^2)$, with $m$ being the $2 \times 2$ Higgs mass matrix, we read the following identity,

$$
\sum_{i=1}^{2} m_{H_i}^2 O_{i1}^2 = 4\lambda_H v^2 \equiv m_H^2,
$$

(46)

where $m_H$ is the SM Higgs boson mass expression. It is easy now to simplify our expression for $\Delta \rho$, by Taylor expanding (45) around $m_{Hi}^2$.

$$
\sum_{i=1}^{2} O_{i1}^2 f(m_{H_i}^2) = \sum_{i=1}^{2} O_{i1}^2 \left[ f(m_{H_i}^2) + (m_{H_i}^2 - m_{H}^2) f'(m_{H}^2) + \ldots \right],
$$

(47)

where $f(x)$ is a continuous function and $f'(x)$ denotes its derivative with respect to $m_{H_i}^2$. Using (46) and the orthogonality condition $O^T O = 1$, the second term in (47) vanishes identically, leading to

$$
\Delta \rho^H = \frac{3G_F}{8\sqrt{2}\pi^2} \left[ m_W^2 \ln \frac{m_{H_1}^2}{m_{W}^2} - m_Z^2 \ln \frac{m_{H_1}^2}{m_{W}^2} \right],
$$

(48)

which is just the SM Higgs contribution to $\Delta \rho$. One arrives at the same conclusion for the $S, T$ and, $U$ parameters. Assuming that the Higgs contributions to the non-oblique corrections follow the same pattern, we can use the electroweak constraint on the SM Higgs boson mass, $m_H < 194$ GeV at 95% C.L \[36\] in order to set constraints on the Higgs boson masses and mixing angle of this model. Thus, comparing (45) with (48) we arrive at

$$
\cos^2 \theta \log(m_{H_1}^2) + \sin^2 \theta \log(m_{H_2}^2) < \log(194^2 \text{ GeV}^2) \quad (\text{at 95\% C.L.}).
$$

(49)

In the case of our working example $\theta = \pi/4$, this translates into $m_{H1} m_{H2} < 194^2 \text{ GeV}^2$, e.g, $m_{H1} \lesssim 115$ GeV and $m_{H2} \lesssim 327$ GeV. These bounds have to be combined with the LEP bounds on the Higgs masses derived in the previous section.

It is apparent that the inclusion of the phantom singlet field $\Phi$, does not affect the GIM mechanism \[37\] which is responsible for the absence of tree level flavour changing neutral currents (FCNC). One may think of Higgs mediated contributions to rare B-decays at loop-level. The most striking one would have been: $B$\,(or $Y$)-meson decays to invisible, $B \rightarrow JJ$. Alas, the amplitude for this decay is proportional to $\sum_i O_{i1}O_{i2}$ which vanishes because the matrix $O$ is orthogonal. This is a kind of GIM mechanism suppression in the Higgs sector. Other Higgs mediated contributions to observables like $B \rightarrow \mu^+\mu^-$ or to the muon anomalous magnetic moment, $g - 2$, will follow the SM prediction thanks to the relation (47) and the bound on the light Higgs mass (44). In conclusion, the minimal singlet phantom sector does not change the FCNC predictions for processes existing in the SM.
Cosmological and astrophysical constraints

Let us finally address the implications for cosmology and astrophysics. The presence of the massless Goldstone boson, \( J \), has interesting consequences which need to be analysed.

During the expansion of the Universe, a critical temperature is reached below which the \( U(1)_D \) symmetry is spontaneously broken. As we have already explained in the previous section, the field \( \Phi \) then develops a non-vanishing vev and its real part, \( \phi \), mixes with the real part of the SM Higgs field, giving rise to two scalar Higgs mass eigenstates, \( H_i \). On the other hand, \( J \) survives as the massless Goldstone boson. From (34,35,36) \( H_i \) couples to the Goldstone pair \( JJ \) as

\[
- \mathcal{L}_J \supset \frac{(\sqrt{2}G_F)^{1/2}}{2} \tan \beta O_{i2} m_{H_i}^2 H_i JJ .
\]

The Goldstone bosons are then kept in equilibrium via reactions of the sort \( JJ \leftrightarrow f\bar{f} \), mediated by \( H_i \). However, since the amplitudes for these processes are suppressed due to a GIM-like mechanism which stems from the orthogonality condition, \( \Sigma_i O_{i1} O_{i2} = 0 \), of the matrix \( O \), \( J \) falls out of equilibrium before the QCD phase transition and remains as an extra relativistic species thereafter.

The presence of relativistic particles, apart from neutrinos, is strongly constrained by Big Bang Nucleosynthesis (BBN), since they alter the predicted abundances for the light elements. Namely, additional relativistic particles would increase the expansion rate of the Universe, leading to a larger neutron-to-proton ratio and therefore a larger \( ^4\text{He} \) abundance. The allowed number of extra relativistic degrees of freedom is usually parameterised by the effective number of neutrino species, \( N_{\text{eff}} = 3 + \Delta N_\nu \). Observations of the primordial \( ^4\text{He} \) abundance, combined with the CMB determination of the baryon-to-photon ratio yield \( N_\nu = 3.24 \pm 1.2 \) at 90\% CL [38, 39], and a similar upper bound on \( N_\nu \) can be derived from analysis of the CMB and large scale structure [40]. Notice that this does not pose a problem for \( J \)s. Since they decoupled at a temperature above the QCD phase transition, when \( \gamma_\ast \lesssim 60 \), their temperature at BBN, \( T_J \), is smaller than that of neutrinos and photons, \( T \). Namely \( (T_J/T)^4 \lesssim (10.75/60)^{1/3} \). This is equivalent to an increase in the number of neutrino species of just \( \Delta N_\nu = 4/7 (T_J/T)^4 \lesssim 0.06 \), well in agreement with the above-mentioned constraints.

On the other hand, \( J \) also induces the decay of heavy neutrinos into the lightest one, \( \nu_H \rightarrow J \nu_L \). Their interaction is described by the following effective Lagrangian,

\[
\mathcal{L}_{J\nu\nu} \supset \frac{\hat{m}_\nu}{\sigma} i \bar{\nu} \gamma_5 \nu J ,
\]

with \( \hat{m}_\nu \) a diagonal matrix which contains the physical neutrino mass eigenstates. Since the decay does not include any photon in the final state, some potential cosmological problems associated with radiative neutrino decays (e.g., contributions to the diffuse photon background and distortions to the cosmic microwave background black body spectrum) are avoided. However, limits on the non-radiative decay of neutrinos (usually expressed as upper bounds on the \( J\nu\bar{\nu} \) coupling) can be derived from solar neutrino observations \( (g_{J\nu\bar{\nu}}^2 \lesssim 10^{-5}) \).
meson decays \( (g_{J\nu\bar{\nu}}^2 \lesssim 10^{-4}) \) \([42]\), as well as from preventing overcooling in supernovae (which exclude a range around \( g_{J\nu\bar{\nu}}^2 \lesssim 10^{-10} \), although the bounds are model dependent) \([43]\).

As we can read from (51), in the present model \( g_{J\nu\bar{\nu}} = m_\nu/\sigma \). For natural (electroweak scale) values of \( \sigma \), and with \( m_\nu \lesssim 1 \text{ eV} \) we obtain \( g_{J\nu\bar{\nu}}^2 \lesssim 10^{-22} \), thus fulfilling all the aforementioned constraints.

Alternatively, a calculation of the heaviest ‘light’ neutrino lifetime, in the case of hierarchical neutrino masses, yields

\[
\frac{\tau_\nu}{m_\nu} = \frac{16\pi}{g_{J\nu\bar{\nu}}^2 m_\nu^2} \sim \mathcal{O}(10^{13}) \text{s/eV},
\]

where \( \sigma \sim 100 \text{ GeV} \) and \( m_\nu \sim 0.05 \text{ eV} \), has been used. A similar value is obtained in the opposite limit, when neutrino masses are quasi-degenerate. The heaviest ‘light’ neutrino is therefore extremely long-lived and escapes all the constraints on neutrino decays.

Finally, notice that the Goldstone boson \( J \) couples very weakly to electrons, through one loop diagrams, with strength \( g_e J \simeq G_F m_e^2 / \sigma \), and it does not affect the evolution of stars \([44]\) if \( g_e J \lesssim 10^{-12} \) which implies that \( \sigma \gtrsim 5 \times 10^{-17} \text{ GeV} \). Also, the emission of a Goldstone pair mediated by (virtual) Higgses is negligible in our model, once more due to the orthogonality condition of \([50]\), and the strong constraints on the Higgs couplings, obtained from studies on star evolution \([55]\), are trivially fulfilled.

### 6 Conclusions

We have proposed the minimal, lepton number conserving, \( \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y \) gauge-singlet, or phantom extension to the Standard Model leading to naturally small Dirac masses for the neutrinos and baryogenesis through Dirac leptogenesis. The extension is natural in the sense that all couplings are either of \( \mathcal{O}(1) \) or strictly forbidden.

Spontaneous breaking of a global, phantom sector \( \text{U}(1)_D \) symmetry will trigger electroweak symmetry breaking. The scale of this phantom sector symmetry breaking is constrained to be around the electroweak scale by the simultaneous requirement of successful Dirac leptogenesis and small light neutrino masses. In this model, small Dirac neutrino masses arise through a mechanism very similar to the standard Majorana see-saw. The model can also be viewed as a very simple Froggatt-Nielsen scenario of the sort usually invoked to generate large hierarchies in the quark masses.

Baryogenesis through Dirac leptogenesis occurs naturally in this model since the small effective Yukawa couplings of the left and right-handed neutrinos prevent the left and right neutrino asymmetries from equilibrating once they are created. The initial neutrino asymmetry is created via the out of thermal equilibrium decays of the heavy Dirac particles \( S_i \) and \( \bar{S}_i \), in analogy with conventional Majorana leptogenesis.

A Davidson-Ibarra-like bound on the CP-asymmetry in the \( S_i, \bar{S}_i \) decays exists when their masses are hierarchical. This bound, in conjunction with information on the efficiency of leptogenesis extracted from the solution of the Boltzmann equations allows us to place
a lower bound on the vev of the phantom sector, SM gauge-singlet $\Phi$, such that the asymmetry created in Dirac leptogenesis is enough to explain the observed baryon asymmetry of the Universe. Assuming a hierarchical light neutrino spectrum and a hierarchical $S_1$ mass spectrum, we find that

$$\sigma \gtrsim 0.1 \text{ GeV}.$$  \hspace{1cm} (53)

Making the further assumption that leptogenesis must proceed after the thermal production of the $S_1$ following a period of inflation leads us to an approximate upper bound on $M_1$ and therefore $\sigma$

$$\sigma \lesssim 2 \text{ TeV} \left( \frac{T_{RH}}{10^{16} \text{ GeV}} \right).$$  \hspace{1cm} (54)

Thus we find that an electroweak scale $\sigma$ is simultaneously compatible with both light neutrino data and successful Dirac leptogenesis. Significantly, an electroweak scale $\sigma$ is also required by our naturalness criterion, since the mixing of the $\Phi$ and the SM Higgs is expected to be maximal.

The addition of the phantom sector scalar $\Phi$, which mixes with the SM Higgs, introduces an additional massive Higgs boson. After the breaking of the global $U(1)_D$, we are also left with a massless Goldstone boson, $J$. This Goldstone boson couples to the two physical Higgs bosons introducing an additional, invisible decay mode for the Higgs $H_i \to JJ$. This decay mode suppresses the branching ratio of Higgs to $bb$ and instead both Higgs bosons decay dominantly to invisible $JJ$ and/or to vector boson pairs. We discuss in detail a natural scenario with a representative mixing angle ($\theta = \pi/4$) and estimate that with $30 \text{ fb}^{-1}$ of integrated luminosity the LHC could find the invisible Higgs with a mass 120 GeV, at a significance of $4.9 \sigma$. In addition, at the same time a significantly heavier, partner Higgs could be found with a mass 200 GeV through its vector boson decays. Interestingly, electroweak constraints suggest an upper limit of $\sim 250$ GeV for the mass of the Higgs bosons.

The model passes relevant FCNC constraints thanks to a GIM-like mechanism. Cosmological bounds on the number of relativistic species at BBN are also fulfilled, due to the Goldstone boson decoupling before the QCD phase transition. Finally, astrophysical constraints on the Goldstone couplings from neutrino decays and stellar evolution are trivially satisfied.

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References

[1] B. Patt and F. Wilczek. [arXiv:hep-ph/0605188]. Singlet extensions of the Higgs sector of the SM have been considered previously in e.g., E. D. Carlson and L. J. Hall, Phys. Rev. D 40 (1989) 3187; G. Jungman and M. A. Luty, Nucl. Phys. B 361 (1991) 24.
See for example, M. Maltoni, T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. 6 (2004) 122, and references therein.

M. Roncadelli and D. Wyler, Phys. Lett. B 133 (1983) 325.

Other non-supersymmetric models for naturally small neutrino masses exist, see for instance, D. Chang and R. N. Mohapatra, Phys. Rev. Lett. 58 (1987) 1600; E. Ma and P. Roy, Phys. Rev. D 52 (1995) 4780.

H. Murayama and A. Pierce, Phys. Rev. Lett. 89 (2002) 271601; B. Thomas and M. Toharia, Phys. Rev. D 73 (2006) 063512.

S. Abel, A. Dedes and K. Tamvakis, Phys. Rev. D 71 (2005) 033003; S. Abel and V. Page, JHEP 0605 (2006) 024.

M. Boz and N. K. Pak, Eur. Phys. J. C 37 (2004) 507.

Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28 (1962) 870; B. Pontecorvo, Sov. Phys. JETP 26 (1968) 984 [Zh. Eksp. Teor. Fiz. 53 (1967) 1717].

C. Hagedorn and W. Rodejohann, JHEP 0507 (2005) 034.

C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B 147 (1979) 277.

K. Dick, M. Lindner, M. Ratz and D. Wright, Phys. Rev. Lett. 84 (2000) 4039.

A. Y. Ignatiev, N. V. Krasnikov, V. A. Kuzmin and A. N. Tavkhelidze, Phys. Lett. B 76 (1978) 436; M. Yoshimura, Phys. Rev. Lett. 41 (1978) 281 [Erratum-ibid. 42 (1979) 746]; S. Dimopoulos and L. Susskind, Phys. Rev. D 18 (1978) 4500.

M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986) 45.

G. 't Hooft, Phys. Rev. Lett. 37 (1976) 8.

V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B 155 (1985) 36.

J. A. Harvey and M. S. Turner, Phys. Rev. D 42 (1990) 3344.

A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. 5 (1967) 32 [JETP Lett. 5 (1967 SOPUA,34,392-393.1991 UFNAA,161,61-64.1991) 24].

J. Liu and G. Segre, Phys. Rev. D 48 (1993) 4609; M. Flanz, E. A. Paschos and U. Sarkar, Phys. Lett. B 345 (1995) 248 [Erratum-ibid. B 382 (1996) 447]; L. Covi, E. Roulet and F. Vissani, Phys. Lett. B 384 (1996) 169.

A. Pilaftsis, Phys. Rev. D 56 (1997) 5431; A. Pilaftsis and T. E. J. Underwood, Nucl. Phys. B 692 (2004) 303;

J. A. Casas and A. Ibarra, Nucl. Phys. B 618 (2001) 171.
[21] S. Davidson and A. Ibarra, Phys. Lett. B 535 (2002) 25.
[22] E. W. Kolb and M. Turner, *The Early Universe*, Perseus Books, 1999.
[23] W. Buchmüller, P. Di Bari and M. Plumacher, Annals Phys. 315 (2005) 305.
[24] E. W. Kolb and S. Wolfram, Nucl. Phys. B 172 (1980) 224 [Erratum-ibid. B 195 (1982) 542].
[25] G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, Nucl. Phys. B 685 (2004) 89.
[26] R. Barbieri, P. Creminelli, A. Strumia and N. Tetradis, Nucl. Phys. B 575 (2000) 61; E. Nardi, Y. Nir, E. Roulet and J. Racker, JHEP 0601 (2006) 164; A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada and A. Riotto, JCAP 0604 (2006) 004; A. Abada, S. Davidson, A. Ibarra, F. X. Josse-Michaux, M. Losada and A. Riotto, arXiv:hep-ph/0605281.
[27] A. Pilaftsis, Phys. Rev. Lett. 95 (2005) 081602; A. Pilaftsis and T. E. J. Underwood, Phys. Rev. D 72 (2005) 113001.
[28] C. T. Hill, M. A. Luty and E. A. Paschos, Phys. Rev. D 43 (1991) 3011.
[29] A. S. Joshipura and S. D. Rindani, Phys. Rev. Lett. 69 (1992) 3269; A. S. Joshipura and J. W. F. Valle, Nucl. Phys. B 397 (1993) 105.
[30] Y. Chikashige, R. N. Mohapatra and R. D. Peccei, Phys. Rev. Lett. 45 (1980) 1926.
[31] [LEP Higgs Working for Higgs boson searches Collaboration], arXiv:hep-ex/0107032.
[32] S. G. Frederiksen, N. Johnson, G. L. Kane and J. Reid, Phys. Rev. D 50 (1994) 4244.
[33] R. M. Godbole, M. Guchait, K. Mazumdar, S. Moretti and D. P. Roy, Phys. Lett. B 571 (2003) 184.
[34] H. Davoudiasl, T. Han and H. E. Logan, Phys. Rev. D 71 (2005) 115007.
[35] See for example, P. Ramond, *Journeys Beyond the Standard Model*, Frontiers in Physics, 2004.
[36] For a recent analysis see J. Erler, hep-ph/0604035.
[37] S. L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D 2 (1970) 1285.
[38] R. H. Cyburt, B. D. Fields, K. A. Olive and E. Skillman, Astropart. Phys. 23 (2005) 313.
[39] B. Fields and S. Sarkar, arXiv:astro-ph/0601514.
[40] M. Cirelli and A. Strumia, arXiv:astro-ph/0607086; S. Hannestad and G. G. Raffelt, arXiv:astro-ph/0607101.
[41] J. F. Beacom and N. F. Bell, Phys. Rev. D 65 (2002) 113009. A. Bandyopadhyay, S. Choubey and S. Goswami, Phys. Lett. B 555 (2003) 33.

[42] D. I. Britton et al., Phys. Rev. D 49 (1994) 28; V. D. Barger, W. Y. Keung and S. Pakvasa, Phys. Rev. D 25 (1982) 907; G. B. Gelmini, S. Nussinov and M. Roncadelli, Nucl. Phys. B 209 (1982) 157.

[43] Y. Farzan, Phys. Rev. D 67 (2003) 073015.

[44] D. S. P. Dearborn, D. N. Schramm and G. Steigman, Phys. Rev. Lett. 56 (1986) 26.

[45] S. Bertolini and A. Santamaria, Phys. Lett. B 220 (1989) 597.