Modelling and Simulation on Multibody Dynamics for Vehicular Cold Launch Systems Based on Subsystem Synthesis Method

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Abstract. The fast simulation of the vehicular cold launch system (VCLS) in the launch process is an essential requirement for practical engineering applications. In particular, a general and fast simulation model of the VCLS will help the designer to obtain the optimum scheme in the initial design phase. For these purposes, a system-level fast simulation model was established for the VCLS based on the subsystem synthesis method. Moreover, a comparison of the load of a seven-axis VCLS on the rigid ground through both theoretical calculations and experiments was carried out. It was found that the error of the load of the rear left outrigger is less than 7.1%, and the error of the total load of all the outriggers is less than 2.8%. Moreover, time taken for completion of the simulation model is only 9.5 min, which is 5% of the time taken by conventional algorithms.

1. Introduction
The typical vehicular cold launch system (VCLS) is the SS-25 developed by Russia. In this system, the missile is ejected by compressed gas from an external gas generator, followed by ignition of the missile. In the existing VCLSs, the carrier vehicle is usually a multi-axis vehicle that has many independent pneumatic suspensions (IPS). For practical applications, it is essential to perform a fast simulation of the VCLS in the launch process. A system-level fast simulation model (SFSM) is especially suitable to help designers obtain the optimum scheme in the initial design phase. Therefore, establishing a feasible SFSM for VCLS is the focus of this paper.

The VCLSs are very complex and consist of many components interconnected by hinges and elastic elements. Thus, the VCLS is characterized by complex constraints and changing structure [1], and it is difficult to meet the requirements of high speed simulations for such a system by using the classical modelling approaches. However, the subsystem synthesis method [2-5], proposed by Kim et al., provides an efficient way to analyze multibody vehicle dynamics. The computational efficiency of this method is greatly improved with the increase in the number of subsystems and the subsystem degree of freedom.

Thus, in this paper, a SFSM based on the subsystem synthesis method will be established, where the absolute coordinates are chosen to define the system configuration, and the Lagrange’s equation of the first kind is used to establish the governing equation. Taking the Euler parameters as the generalized coordinates, the subsystem synthesis method will be derived. Subsequently, a seven-axis
VCLS will be taken as an example to compare the theoretical results with the experimental results in order to verify the accuracy and efficiency of the SFSM.

2. Mathematical Modelling

2.1. Subsystem Structure of the VCLS

The VCLS consists of many components such as the carrier vehicle, the lifting device, the canister, the missile, IPS, and so on. These components are interconnected by hinges and elastic elements. The simplified schematic diagram of a typical VCLS is shown in figure 1.

![Schematic diagram of VCLS](image)

Figure 1. Schematic diagram of VCLS

The main assumptions are as follows:

- All hinge constraints are ideal and holonomic.
- The missile slides in the canister without jumping.
- Apart from the lifting auxiliary bracket, the deformations of other bodies need not be considered. The flexible bodies undergo small deformations, so the floating frame of reference formulation can be used to describe the deformations. Based on experience, the 20 lowest natural frequencies and corresponding modes of vibration of the flexible bodies are used.
- The interior ballistic pressure in the launch canister is simplified as the thrust \( F_t \) acting on the bottom of the missile, and the function of the rubber vesicle is modelled as the adjunctive force \( F_a \) acting on the canister.

The model of the VCLS based on the subsystem synthesis method is shown in figure 2. The IPS is simplified into subsystems, and the chassis is taken as a virtual base body which is the reference body to define kinematic relationships between bodies in the IPS.

![Subsystems structure of VCLS](image)

Figure 2. Subsystems structure of VCLS
2.2. Main System Equations of Motion
The rigid-flexible coupling dynamic equations of motion [6-9] are usually described as:

\[
\begin{bmatrix}
M & C_q^T \\
C_q & 0
\end{bmatrix}
\begin{bmatrix}
\dot{q} \\
\dot{\lambda}
\end{bmatrix} =
\begin{bmatrix}
Q_e + Q_r - D\dot{q} - Kq \\
\gamma - a\dot{\lambda} - \beta\dot{\lambda}
\end{bmatrix}
\]  

(1)

where \(M\) is the mass matrix, \(D\) is the proportional damping matrix, \(K\) is the stiffness matrix, \(Q_e\) is the vector of generalized forces, \(Q_r\) is a quadratic velocity vector, \(C_q\) is the Jacobian matrix associated with the constraint equations \(C(q,t) = 0\), and \(\lambda\) is the Lagrangian multiplier vector. With the constraints differentiated twice with respect to time, Eq. (1) is modified based on the Baumgarte’s Constraint Violation Stabilization Method [10] (CVSM).

\[
\begin{bmatrix}
M & C_q^T \\
C_q & 0
\end{bmatrix}
\begin{bmatrix}
\dot{q} \\
\dot{\lambda}
\end{bmatrix} =
\begin{bmatrix}
Q_e + Q_r - D\dot{q} - Kq \\
\gamma - a\dot{\lambda} - \beta\dot{\lambda}
\end{bmatrix}
\]  

(2)

where \(\alpha\) and \(\beta\) are the parameters of the CVSM, usually \(1 \leq \alpha = \beta \leq 50\). In this paper, the preferred parameters are \(\alpha = \beta = 2\), \(\gamma\) is the right term of the acceleration constraint equation.

2.3. Subsystem Synthesis Method

2.3.1. Relative Cartesian Coordinates Formulation. In order to obtain the effective mass matrices and force vectors of the IPS, motions of the bodies in the IPS must be represented in the body-fixed frame of the chassis. For this reason, relative Cartesian coordinates are used as shown in figure 3.

![Figure 3. Relative Cartesian coordinate kinematics.](image)

The generalized coordinates of the body \(i\) can be represented by the kinematic of the chassis and the relative kinematic of body \(i\) with respect to the chassis [3].

\[
r_i = r_e + A_ir_{io}
\]

(3)

\[
\omega_i = A_i^{\omega} + \omega_{io}
\]

(4)

where \(r_i\) and \(r_e\) are the position vector of the body \(i\) and the chassis, respectively, \(A_i\) is the orientation matrix of the chassis, and \(r_{io}\) is the relative position vector of the body \(i\) with respect to the chassis, and it is expressed in the body-fixed frame of the chassis. The notation (\(\cdot\))\(^*\) indicates that the quantity (\(\cdot\)) is expressed in the body-fixed frame of the chassis. \(\omega_i\) is the angular velocity vector of the body \(i\), and the notation (\(\cdot\))\(^*\) indicates that the quantity (\(\cdot\)) is expressed in the body-fixed frame. \(A_{io}\) is the relative orientation of the body with respect to the chassis, \(\omega_{io}\) is the angular velocity of the chassis, and \(\omega_{io}\) is the relative angular velocity of the body \(i\) with respect to the chassis.

When the Euler parameter \(\mathcal{A}\) is used to describe the orientation, Eq. (4) is modified as:
\[
\dot{A}_i = R_i^T R_i \dot{A}_i + L_i^T L_o \dot{A}_o
\]

where the matrices \( R \) and \( L \) are two semi-transformations. Their product is the orientation matrix.

\[
A = RL
\]

The velocity of the body \( i \) is obtained by differentiating Eq. (3) with respect to time as:

\[
\dot{r}_i = \dot{r}_o + A_\theta \dot{r}_o
\]

where the notation \( (\cdot) \) refers to the skew symmetric matrix made of the elements of vector \( (\cdot) \). For convenience, the following compact velocity vectors are employed.

\[
y_i = \begin{bmatrix} \dot{r}_i \\ \dot{A}_i \end{bmatrix}, \quad y_o = \begin{bmatrix} \dot{r}_o \\ \dot{A}_o \end{bmatrix}, \quad y_{\omega} = \begin{bmatrix} \dot{r}_{\omega} \\ \dot{A}_{\omega} \end{bmatrix}
\]

Then, the following relative velocity relationship expressed by the Euler parameter is obtained.

\[
y_i = E_{\omega o} y_o + G_{\omega o} y_{\omega}
\]

where

\[
E_{\omega o} = \begin{bmatrix} I & -2A_\theta \dot{r}_o L_o \\ 0 & R_i^T R_i \end{bmatrix}, \quad G_{\omega o} = \begin{bmatrix} A_o & 0 \\ 0 & L_i^T L_o \end{bmatrix}
\]

The relative acceleration between the body \( i \) and the chassis is obtained by differentiating Eq. (9) with respect to time as:

\[
\ddot{y}_i = E_{\omega o} \dot{y}_o + G_{\omega o} \ddot{y}_o + h_{\omega o}
\]

where \( h_{\omega o} \) is the velocity coupling term.

\[
h_{\omega o} = \begin{bmatrix} A_o \dot{\theta}_o \dot{r}_o^* + 2A_\theta \dot{\theta}_o \dot{r}_o^* \\ R_i^T R_i \dot{A}_i + L_i^T L_o \dot{A}_o \end{bmatrix}
\]

2.3.2. Subsystem Equations of Motion. According to the Eq.(9), the acceleration relationship between all the bodies in the IPS and the chassis can be expressed in the following compact form as:

\[
\ddot{y} = E_{\omega o} \ddot{y}_o + G_{\omega o} \dddot{y}_o + \dddot{h}_{\omega o}
\]

Based on the subsystem synthesis method, the effective mass matrix and force vectors \( [3] \) can be expressed as:

\[
\dot{\mathbf{M}}^e = \dot{\mathbf{M}}_{EE} - \mathbf{M}_{EG} \mathbf{M}_{\omega o} \mathbf{M}_{\omega o}^T \dot{\mathbf{M}}_{EE}
\]

\[
\dot{\mathbf{g}}^e = \dot{\mathbf{g}}_E - \mathbf{M}_{EG} \mathbf{M}_{\omega o} \mathbf{g}_o
\]

where

\[
\dot{\mathbf{M}}_{EE} = \dot{\mathbf{M}}_{\omega o} \dot{\mathbf{M}}_{\omega o}^T, \quad \dot{\mathbf{M}}_{EG} = \dot{\mathbf{M}}_{\omega o} \dot{\mathbf{M}}_{\omega o}^T, \quad \dot{\mathbf{M}}_{GG} = \dot{\mathbf{M}}_{\omega o} \dot{\mathbf{M}}_{\omega o}^T, \quad \dot{\mathbf{M}}_{EG} = \dot{\mathbf{M}}_{EG}, \quad \dot{\mathbf{g}}_E = \dot{\mathbf{g}}_E - \dot{\mathbf{M}}_{EG} \dot{\mathbf{g}}_o, \quad \dot{\mathbf{g}}_G = \dot{\mathbf{g}}_G - \dot{\mathbf{M}}_{GG} \dot{\mathbf{g}}_o
\]

\[
\mathbf{M}_{EE} = \bar{\mathbf{M}}_{EE}, \quad \mathbf{M}_{EG} = \bar{\mathbf{M}}_{EG}, \quad \mathbf{M}_{GG} = \bar{\mathbf{M}}_{GG}, \quad \mathbf{N}_{EG} = \bar{\mathbf{N}}_{EG}, \quad \mathbf{N}_{GG} = \bar{\mathbf{N}}_{GG}, \quad \mathbf{R}_E = \bar{\mathbf{R}}_E, \quad \mathbf{R}_G = \bar{\mathbf{R}}_G - \bar{\mathbf{M}}_{EG} \bar{\mathbf{g}}_o
\]

\[
\mathbf{M} \text{ is the constant mass and inertia matrix of the subsystem, and } \bar{\mathbf{g}} \text{ is the generalized composite force vector acting on all the bodies in the subsystem. It should be noted that the matrix } \mathbf{M} \text{ and vector } \bar{\mathbf{g}} \text{ are calculated in the inertial frame.}
\]

In order to obtain the expressions for the transform matrix \( \mathbf{N} \) and the matrix \( \mathbf{p} \), the acceleration in relative Cartesian coordinates is expressed in terms of acceleration of the independent generalized coordinates \( \dot{\theta} \) as:

\[
\ddot{y}_{\omega o} = \mathbf{N} \ddot{\theta} + \mathbf{p}
\]

The hinge constraint equations in the IPS can be expressed as:

\[
\mathbf{C}(\bar{\mathbf{z}}_{\omega o}) = \mathbf{0}
\]

where \( \bar{\mathbf{z}}_{\omega o} \) is the displacement vector of the composite relative Cartesian coordinates of all the bodies in the IPS.

The acceleration constraint equations can be obtained by differentiating Eq.(18) with respect to time twice as:
\[ C_{zio} \vec{y}_{io} = \gamma \]  

(19)

where \( C_{zio} \) is the associated Jacobian matrix of the subsystem constraint equations.

The L-U factorization is applied to the Jacobian matrix. Assuming that the constraints are independent, the Jacobian matrix has full row rank. Therefore, there is always at least one non-singular submatrix by applying column transformation to the Jacobian matrix.

\[ C_{zio} P_2 = [C_o \ C_v] \]

(20)

where \( P_2 \) is the column transform matrix.

Now, the transform matrix \( N \) is obtained when the subsystem constraint equations are holonomic.

\[ N = P_2 \begin{bmatrix} -C_o^{-1}C_v & I \end{bmatrix} \]

(21)

The matrix \( p \) in Eq. (16) can be expressed as:

\[ p = P_2 \begin{bmatrix} -C_o^{-1}(q - \alpha \dot{q} - \beta C) & 0 \end{bmatrix} \]

(22)

As all the bodies are rigid in the subsystem, the preferred parameters of the CVSM are: \( \alpha = \beta = 40 \).

With the effective mass matrix and force from each of the subsystems, the following equation of motion for the chassis is obtained to solve for accelerations.

\[ (M^o + \sum_{i=1}^{n} M^{ei})\dot{q}^o = Q_e^o + Q_v^o + \sum_{i=1}^{n} g^{ei} \]

(23)

where \( M^o \) is the inertia matrix, \( Q_e^o \) and \( Q_v^o \) are the vectors of generalized force and quadratic velocity, respectively, and \( q^o \) refers to the generalized coordinates, where the superscript \( o \) represents the chassis.

3. Program Implementation

3.1. Static Analysis

The launch dynamics simulation generally begins with a static equilibrium state. In order to obtain the static equilibrium state, dynamics analysis is carried out by adding additional damping force to the system. Thus, the generalized external force is corrected to the following formula.

\[ F^o = F^o - Mq \text{ if } |q| > preci \]

(24)

where \( preci \) is a given precision.

3.2. Launch Dynamics Simulation

FORTRAN is used for the programming language. There are five main data structures defined in the program including body, hinge, mark, force and force element. These data structures are linked by a linked list for easy deletion and addition. Eq. (2) represents the stiff differential algebraic equations (SDAEs), and the \( L_1 \) condition number is approximately \( 5.0 \times 10^{18} \). The variable-step Gear implicit integral method is effective for handling the SDAEs, and the step size \( \Delta t \in [10^{-3}, 10^{-6}] \) is used in this work. The calculation flow chart of the VCLS is given in figure 4.
4. Simulation and Verification
In order to verify the above general algorithm for the VCLS shown in figure 1, a seven-axis VCLS is taken as an example. The seven-axis VCLS has 72 bodies including the flexible lifting auxiliary bracket, which can be divided into a main system comprised of 16 bodies and 14 subsystems. Each subsystem is comprised of 4 bodies. The seven-axis VCLS is supported by tires together with outriggers on the rigid ground, where the stiffness $K_r$ of the rigid ground is $3 \times 10^8$ N/m, and the damping $C_r$ of the rigid ground is $1 \times 10^6$ Ns/m.

For the convenient display of the results, the non-dimensional time $\tilde{t}$ is defined as:

$$\tilde{t} = \frac{t}{t_e}$$  \hspace{1cm} (25)

where $t$ is the current time, and $t_e$ is the length of the simulation time.

The non-dimensional force $\tilde{F}$ is defined as:

$$\tilde{F} = \frac{F}{M_0 g}$$  \hspace{1cm} (26)

where $F$ is the current force, $M_0$ is the total mass of the VCLS, and $g$ is the local gravitational acceleration (9.8 m/s$^2$).

A comparison of the theoretical and experimental results for non-dimensional force $\tilde{F}_l$ of the rear left outrigger in the model is carried out and the results are shown in figure 5. The comparison of the total non-dimensional force $\tilde{F}_{total}$ of all the outriggers is shown in figure 6. The error of $\tilde{F}_l$ is less than 7.1%, and the error of $\tilde{F}_{total}$ is less than 2.8%. Moreover, the time taken for completion of the SFSM is only 9.5 min, which is 5% of the completion time of the HHT-SI2 method [11] which takes approximately 180 min for the seven-axis VCLS. The above results illustrate that the mathematical model of VCLS is feasible and can be used for practical engineering applications.
5. Conclusion
In this paper, a SFSM was established based on the subsystem synthesis method, and a comparison of the load of a seven-axis VCLS on the rigid ground in theory and experiment has been carried out. It was found that the error of the load of the rear left outrigger is less than 7.1%, and the error of the total load of the outriggers is less than 2.8%. Moreover, the completion time is only 9.5 min, which is just 5% of the completion time of the HHT-SI2 method. Therefore, the SFSM is able to meet the requirements for practical engineering applications.

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Figure 5. Simulation results of the load of the rear left outrigger.
Figure 6. Simulation results of the total load of all outriggers.