From dark matter to neutrinoless double beta decay

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Associated with two TeV-scale leptoquark scalars, a dark matter fermion which is the neutral component of an isotriplet can mediate a testable neutrinoless double beta decay at one-loop level. The dark matter fermion with determined mass and spin-independent scattering can be verified by the future dark matter direct detection experiments. We also discuss the implications on neutrino masses and baryon asymmetry.

PACS numbers: 95.35.+d, 14.60.Pq, 14.80.Sv

I. INTRODUCTION

A process of neutrinoless double beta decay requires a lepton number violation of two units. Such lepton number violation can have various origins [1–3]. For example, the standard picture of the neutrinoless double beta decay is realized by the electron neutrino with a tiny Majorana mass. The smallness of neutrino masses can be understood by the seesaw mechanism [4–7] or some loop and chirality suppression factors [8–11]. Neutrino physics may be related to other interesting topics in particle physics and cosmology such as dark matter [11, 12] and inflation [13]. In the case that the neutrino masses are smaller than the eV order, the standard picture of the neutrinoless double beta decay is indirectly related to the dark matter particle [11, 12] and then result in a standard neutrinoless double beta decay at one-loop level. The dark matter fermion with determined mass and spin-independent scattering can be verified by the future experiments since it has a determined spin-independent cross section about $10^{-46}$ cm$^2$, besides a spin-dependent cross section smaller than $O(10^{-46}$ cm$^2$).

II. THE MODEL

For simplicity, we will not write down the full Lagrangian. Instead, we only give the following terms relevant to our demonstration,

\[
\mathcal{L} \supset -h_i \bar{q}_L^i \gamma_5 i \tau_2 T_L \eta - f_i \bar{l}_L^i \xi d_R^i - \lambda (\xi^\dagger \eta)^2 + \text{H.c.}
\]

where we have defined the $[SU(2)]$-triplet fermion:

\[
T_L(1,3,0) = \begin{bmatrix} \frac{1}{\sqrt{2}} T^0_L & T^+_L \\ T^-_L \end{bmatrix},
\]

and the $[SU(2)]$-doublet leptoquark scalars:

\[
\eta(3,2,-\frac{1}{6}) = \begin{bmatrix} \eta^+ \frac{1}{2} \\ \eta^- \frac{1}{2} \end{bmatrix}, \quad \xi(3,2,-\frac{1}{6}) = \begin{bmatrix} \xi^+ \frac{1}{2} \\ \xi^- \frac{1}{2} \end{bmatrix},
\]

besides the SM quarks and leptons:

\[
q_L(3,2,\frac{1}{6}) = \begin{bmatrix} u_L^i \\ d_L^i \end{bmatrix}, \quad d_R(3,1,-\frac{1}{3}) \]

\[
l_L(1,2,-\frac{1}{2}) = \begin{bmatrix} \nu_L^i \\ e_L^i \end{bmatrix}
\]

It should be noted that our model respects a $Z_2$ discrete symmetry under which only the fermion triplet $T_L$ and the leptoquark $\eta$ carry an odd parity.

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III. DARK MATTER FERMION

Although the charged and neutral components of the fermion triplet $T_L$ have a same mass at tree level, i.e.
\[
\mathcal{L} \supset -M_T^0 T_L^\pm - \frac{1}{2} M_T^{0 \bar{T}_L T^0} \text{ with }
\]
\[
T^\pm = T_L^\pm + (T_L^c)^c, \quad T^0 = T_L^0 + (T_L^c)^c,
\]
the one-loop electroweak corrections will make the charged $T^\pm$ to be heavier than the neutral $T^0$ \[14\],
\[
M_{T^\pm} - M_{T^0} \approx \frac{g^2}{4\pi} m_W \sin^2 \frac{\theta_W}{2} = 166 \text{ MeV}. \quad (6)
\]
So, the neutral $T^0$ can keep stable and leave a relic density to the present universe, if it is lighter than the leptoquark $\eta$.

The relic density is determined by the annihilations of the neutral $T^0$ and the charged $T^\pm$ \[14\]. The effective cross section contains two parts,
\[
\langle \sigma v_{\text{rel}} \rangle = \langle \sigma v_{\text{rel}} \rangle_g + \langle \sigma v_{\text{rel}} \rangle_Y, \quad (7)
\]
where the first term is from the gauge interactions \[14\] ,
\[
\langle \sigma v_{\text{rel}} \rangle_g \approx \frac{37g^4}{192\pi} \frac{1}{M_{T^0}^2}, \quad (8)
\]
while the second term is from the Yukawa interactions,
\[
\langle \sigma v_{\text{rel}} \rangle_Y \approx \frac{(h^1 h^*_{\eta})^2}{12\pi} \frac{M_{T^0}^4}{[M_{T^0}^2 + M_{\eta}^2]^2}. \quad (9)
\]
Here $v_{\text{rel}}$ is the relative velocity between the two annihilating fermions in their cms system, while $M_\eta$ is the mass of the leptoquark $\eta$. If the neutral $T^0$ accounts for the dark matter relic density \[16\], it should have a determined mass about $M_{T^0} = 2.4 \text{ TeV} \[14\] \[15\] in the absence of the Yukawa contribution \[19\]. The dark matter mass will increase if the Yukawa contribution is taken into account. Actually, we find
\[
\frac{\langle \sigma v_{\text{rel}} \rangle_Y}{\langle \sigma v_{\text{rel}} \rangle_g} \approx \frac{16 (h^1 h^*_{\eta})^2}{37g^4} \frac{M_{T^0}^4}{[M_{T^0}^2 + M_{\eta}^2]^2} \ll 1 \quad \text{for } h^1 \lesssim \mathcal{O}(1), \quad M_{T^0}^2 \ll M_{\eta}^4. \quad (10)
\]
So, the dark matter mass can be fixed by
\[
M_{T^0} = 2.4 \text{ TeV}. \quad (11)
\]

At one-loop level, the scattering of the dark matter fermion $T^0$ on the nucleons $N$ can have a spin-independent cross section \[14\] :
\[
\sigma_{SI}^N = \frac{g^4}{256\pi^3} \left( 1 + \frac{m_N^2}{m_T^2} \right)^2 \times \frac{f^2_{N}}{m_W^2} \text{ with }
\]
\[
f_N = \sum_{q=u,d,s} f_{Tq}^{(N)} + \frac{2}{27} \sum_{q=c,b,t} f_{Tq}^{(N)}. \quad (12)
\]
By inputting \[17\]
\[
g^4 = 32G_F^2 m_W^4 = 0.182, \quad m_W = 80.398 \text{ GeV}, \quad m_p = 938.3 \text{ MeV}, \quad m_n = 939.6 \text{ MeV}, \quad (13)
\]
and \[18\]
\[
f_{Tq}^{(p)} = 0.020, \quad f_{Tq}^{(d)} = 0.026, \quad f_{Tq}^{(p)} = 0.118, \quad f_{Tq}^{(p)} = f_{Tq}^{(p)} = f_{Tq}^{(p)} = 1 - \sum_{q=u,d,s} f_{Tq}^{(N)} = 0.836, \quad (14)
\]
we can read
\[
\sigma_{SI}^N \approx 10^{-45} \text{ cm}^2 \quad \text{for } M_H = 126 \text{ GeV}. \quad (15)
\]
So, the spin-independent scattering can be verified by the future dark matter direct detection experiments such as the superCDMS experiment \[19\].

We can also have a spin-dependent dark-matter-nucleon scattering through the tree-level exchange of the leptoquark $\eta$. The relevant axial-vector interactions are given by
\[
\mathcal{L} \supset - \frac{h^1 h^*_{\eta}}{16 M_{\eta}^2} (T^0_\mu \gamma^\mu \gamma^5 T^0_\nu) (\bar{d}_i \gamma_\mu \gamma^5 d_j + \bar{u}_i \gamma_\mu \gamma^5 u_j) + \text{H.c.}, \quad (16)
\]
from which we can write the effective dark-matter-nucleon interaction to be
\[
\mathcal{L} \supset - a_N (T^0_\mu \gamma^\mu \gamma^5 T^0_\nu) (\bar{N} s_i^{(N)} N) \quad \text{with } a_N = \frac{1}{8 M_{\eta}^2} \left( |h_1|^2 [\Delta_u^{(N)} + \Delta_d^{(N)}] + |h_2|^2 \Delta_s^{(N)} \right). \quad (17)
\]
Here $s_i^{(N)}$ is the spin of the nucleon $N$, and $\Delta_q^{(N)}$ are extracted from the data on polarized deep elastic scattering. The spin-dependent cross section quoted by the experiments can be calculated by \[20\]
\[
\sigma_{SD}^N = \frac{6}{\pi} \mu_r^2 a_N^2 \quad \text{with } \mu_r = \frac{M_{T^0} m_N}{M_{T^0} + m_N}. \quad (18)
\]
By taking \[18\]
\[
\Delta_u^{(p)} = \Delta_d^{(n)} = 0.78, \quad \Delta_d^{(p)} = \Delta_u^{(n)} = -0.48, \quad \Delta_u^{(n)} = -0.15, \quad (19)
\]
and assuming
\[
M_{\eta} = 3 M_{T^0}, \quad (20)
\]
we can obtain
\[
\sigma_{SD}^N \leq \mathcal{O}(10^{-46} \text{ cm}^2) \quad \text{for } h_1, h_2 \lesssim \mathcal{O}(1). \quad (21)
\]
which is far below the experimental limits \[21\].
### IV. Neutrinoless Double Beta Decay

As shown in Fig. 22, the dark matter fermion $T^0$ associated with the leptoquark scalars $\eta^\pm$ and $\xi^\pm$ can mediate a one-loop diagram to generate a neutrinoless double beta decay. The effective operators are given by

$$\mathcal{L} \supset -G_{0\nu}(\bar{d_R}d_R)(\bar{e_L}e_L)(\bar{u_L}u_L)$$

$$= -\frac{1}{4}G_{0\nu}(\bar{e_L}e_L)(\bar{u_L}u_L) - (\bar{u_L}\sigma_{\mu\nu}d_R)(\bar{u_L}\sigma^{\mu\nu}d_R) + \text{H.c.},$$

with the coefficients:

$$G_{0\nu} = \frac{i}{4\pi^2} \frac{\lambda(h^*_1)^2 f_{11}^2}{M_\xi(M_\eta^2 - M_T^2)} \times \left( 1 - \frac{M_\eta^2}{M_\eta^2 - M_T^2} \ln \frac{M_\eta^2}{M_T^2} \right).$$

(23)

Here $M_\xi$ is the mass of the leptoquark $\xi$. We need to calculate the half-life of the neutrinoless double beta decay to compare with the experimental limits. For this purpose, we rewrite the low energy Lagrangian [22] to be

$$\mathcal{L} \supset \frac{G_\xi^2}{2m_p} [\bar{e}(1 + \gamma_5)e] \{\eta^{PS}[\bar{u}(1 + \gamma_5)d][\bar{u}(1 + \gamma_5)d]$$

$$+ \frac{1}{4} \eta^T[\bar{u}\sigma_{\mu\nu}(1 + \gamma_5)d][\bar{u}\sigma^{\mu\nu}(1 + \gamma_5)d] + \text{H.c.},$$

(24)

where the effective lepton number violating parameters $\eta^{PS}$ and $\eta^T$ are defined by

$$\eta^{PS} = \frac{m_p G_{0\nu}}{16G_\xi^2}, \quad \eta^T = \frac{m_p G_{0\nu}}{4G_\xi^2}.$$  

(25)

The experimental lower bound $T_{1/2}^{exp}(Y)$ on the half-life of a certain isotope $Y$ can constrain the effective lepton number violating parameters [22],

$$\eta = \frac{5}{8} \eta^{PS} + 3 \frac{8}{8} \eta^T \leq \eta^{exp}_{\xi} = 10^{-7} \zeta(Y) \sqrt{\frac{10^{24\text{ yrs}}}{T_{1/2}^{exp}(Y)}},$$

(26)

where the quantity $\zeta(Y)$ is an intrinsic characteristic of the isotope $Y$. By taking $G_F = 1.16637 \times 10^{-5}\,\text{GeV}^{-2}$ [17], $\zeta(76\,\text{Ge}) = 5.5$ [22] and $T_{1/2}^{exp}(76\,\text{Ge}) = 1.9 \times 10^{19}\,\text{yrs}$ [1], we can find

$$G_{0\nu} \leq 3.1 \times 10^{-18}\,\text{GeV}^{-5}. \quad (27)$$

To fulfill the above constraint, we can consider a proper parameter choice such as

$$G_{0\nu} = i \left( \frac{\lambda}{4} \right) \left( \frac{h^*_1}{T} \right)^2 \left( \frac{f_{11}}{T} \right)^2 \times \left( \frac{750\,\text{GeV}}{M_\xi} \right)^4 \times 3.0 \times 10^{-18}\,\text{GeV}^{-5} \text{ for } M_\eta = 3M_{T_0}. \quad (28)$$

### V. Implications on Neutrino Masses and Baryon Asymmetry

Any neutrinoless double beta decay processes will eventually result in a Majorana neutrino mass term according to the Schechter-Valle theorem [23]. For the present one-loop neutrinoless double beta decay, the leading neutrino masses should appear at five-loop level as shown in Fig. 3. If the interactions involving the second- and third-generation charged fermions are also taken into account, we can obtain a $3 \times 3$ neutrino mass matrix as below,

$$m_{\nu}^{3-\text{loop}} \sim \frac{4g^4}{(16\pi^2)^5} \lambda(f^{\,*}) \frac{M_{T_0}}{M_\eta^2} (f^{\,*})_T. \quad (29)$$

with $f^{\,*} = \text{diag}(m_d, m_s, m_b)$ being the down-type quark masses and $V$ being the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Our model can also generate the Majorana neutrino masses at three-loop level even if we don’t resort to the gauge interactions. The relevant diagram is shown in Fig. 3. We can estimate the three-loop neutrino masses to be

$$m_{\nu}^{3-\text{loop}} \sim \frac{4}{(16\pi^2)^3} \lambda(f^{\,*}) \frac{M_{T_0}}{M_\eta^2} (f^{\,*})_T. \quad (30)$$

It is easy to check that the neutrino masses dominated by the three-loop contribution can arrive at an acceptable level, i.e.

$$(m_{\nu}^{3-\text{loop}})_{ij} \sim \left( \frac{\lambda}{4} \right) \left( \frac{h^*_1}{T} \right)^2 \left( \frac{f_{11}}{T} \right) \left( \frac{f_{13}}{T} \right) \left( \frac{m_p}{4.2\,\text{GeV}} \right)^2 \times 0.8\,\text{eV} \text{ for } M_\eta = 3M_{T_0}. \quad (31)$$

We should keep in mind that the neutrino mass matrix is rank-1 and only has one nonzero eigenvalue so that it cannot explain the neutrino oscillation data. In other words, the neutrino masses should have additional sources such as the conventional seesaw. The neutrinoless double beta decay and other lepton number violating processes will wash out any lepton asymmetries until they go out of equilibrium at the
temperature estimated by

\[ \Gamma \sim G_{0\nu}^2 T^{11} = H(T) \Rightarrow \]

\[ T = 100 \text{GeV} \left( \frac{10^{-18} \text{GeV}^{-5}}{G_{0\nu}} \right)^{1/8} \left( \frac{g_*(T)}{100} \right)^{1/8}. \] (32)

Here

\[ H(T) = \left[ \frac{8\pi^3 g_*(T)}{90} \right]^{1/2} \frac{T^2}{M_{\text{Pl}}} \] (33)

is the Hubble constant with \( M_{\text{Pl}} \approx 1.22 \times 10^{19} \text{GeV} \) being the Planck mass and \( g_*(T) \) being the relativistic degrees of freedom. These lepton number violating processes will also wash out the existing baryon asymmetry in the presence of the \( SU(2)_L \) sphaleron processes \[25\]. The cosmological baryon asymmetry thus should be produced below the temperature \( T \approx 100 \text{GeV} \).

VI. CONCLUSION AND DISCUSSION

In this paper we have shown that the dark matter particle can be directly responsible for generating the neutrinoless double beta decay and that the neutrino masses will not exceed the eV order. The neutrinoless double beta decay can arrive at a testable level, while the accompanying neutrino masses will not exceed the eV order. The neutrinoless double beta decay and other lepton number violating processes will constrain the cosmological baryon asymmetry to produce at a scale below 100 GeV. The dark matter fermion with the determined mass and spin-independent scattering cross section can be verified by the future dark matter direct detection experiments.

The present model can be modified. For example, we can replace the \([SU(2)_L]\)-doublet leptoquarks \( \eta(3, 2, -\frac{1}{2}, -) \) and \( \xi(3, 2, -\frac{3}{2}, +) \) by two \( SU(2)_L \) triplets \( \Delta(3, 3, +\frac{1}{2}, -) \) and \( \Omega(3, 3, +\frac{3}{2}, +) \), or by one \( SU(2)_L \) triplet \( \Delta(3, 3, +\frac{1}{3}, -) \) and one \( SU(2)_L \) singlet \( \omega(3, 1, +\frac{1}{2}, +) \). Here and thereafter "+" and "-" denote the parity under the \( Z_2 \) discrete symmetry. Alternatively, the dark matter fermion may be a gauge-singlet fermion \( S_R(1, 1, 0, -) \). In this case, the leptoquarks should be (i) two \( SU(2)_L \) doublets \( \eta(3, 2, -\frac{1}{6}, -) \) and \( \xi(3, 2, -\frac{1}{6}, +) \), (ii) two \( SU(2)_L \) singlets \( \delta(3, 1, +\frac{1}{6}, -) \) and \( \omega(3, 1, +\frac{1}{6}, +) \), (iii) one \( SU(2)_L \) singlet \( \delta(3, 1, +\frac{1}{6}, +) \) and one \( SU(2)_L \) triplet \( \Omega(3, 3, +\frac{1}{3}, +) \).

Acknowledgement: This work is supported by the Sonderforschungsbereich TR 27 of the Deutsche Forschungsgemeinschaft.
FIG. 3: Three-loop neutrino mass generation.

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