Modelling redshift-space distortion in the post-reionization HI 21-cm power spectrum

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ABSTRACT

The post-reionization HI 21-cm signal is an excellent candidate for precision cosmology, this however requires accurate modelling of the expected signal. Sarkar et al. (2016) have simulated the real space HI 21-cm signal and have modelled the HI power spectrum as $P_{\text{HI}}(k) = b^2 P(k)$ where $P(k)$ is the dark matter power spectrum and $b(k)$ is a (possibly complex) scale dependent bias for which fitting formulae have been provided. This paper extends these simulations to incorporate redshift space distortion and predict the expected redshift space HI 21-cm power spectrum $P_{\text{HI}}^s(k, k_3)$ using two different prescriptions for the HI distributions and peculiar velocities. We model $P_{\text{HI}}^s(k, k_3)$ assuming that it is the product of $P_{\text{HI}}(k) = b^2 P(k)$ with a Kaiser enhancement term and a Finger of God (FoG) damping which has $\sigma_p$ the pair velocity dispersion as a free parameter. Considering several possibilities for the bias and the damping profile, we find that the models with a scale dependent bias and a Lorentzian damping profile best fit the simulated $P_{\text{HI}}^s(k, k_3)$ over the entire range $1 \leq z \leq 6$. The best fit value of $\sigma_p$ falls approximately as $(1 + z)^{-m}$ with $m = 2$ and 1 respectively for the two different prescriptions. The model predictions are consistent with the simulations for $k < 0.3 \text{Mpc}^{-1}$ over the entire range $1 \leq z \leq 6$. Moreover, for the monopole $P_{\text{HI}}^s(k_3)$ at large $k$, and the fit is restricted to $k < 0.15 \text{Mpc}^{-1}$.

Key words: methods: statistical, cosmology: theory, large scale structures, diffuse radiation

1 INTRODUCTION

After the Cosmic Microwave Background (CMB) radiation, the cosmological 21-cm background radiation is one of the most interesting observational frontiers. This originates from the spin-flip transition in the ground state of neutral hydrogen (HI). The cumulative redshifted 21-cm emission from all the HI sources forms a diffused background radiation. A statistical detection of the intensity fluctuations in this 21-cm background provides us a unique way of quantifying the source clustering (Bharadwaj et al. 2001; Bharadwaj & Sethi 2001). This technique, widely known as 21-cm intensity mapping, provides a three dimensional view of large-scale structures in the Universe and makes it possible to survey large volumes of space using currently functioning and upcoming radio telescopes (Bharadwaj & Pandey 2003; Bharadwaj & Ali 2005; Wyithe & Loeb 2008). In the post-reionization era ($z \leq 6$), the 21-cm signal is mostly unaffected by the complex reionization processes and the 21-cm power spectrum is proportional to the underlying matter power spectrum (Wyithe & Loeb 2009). A detection of the Baryon Acoustic Oscillations (BAO) in the 21-cm power spectrum can place tight constraints on the equation of state of dark energy (Wyithe et al. 2008; Chang et al. 2008; Seo et al. 2010; Masui et al. 2010). An accurate measurement of the 21-cm power spectrum can also provide independent estimates of the different cosmological parameters (Loeb & Wyithe 2008; Bharadwaj et al. 2009) without reference to the BAO. Pourtsidou (2016) have investigated the possibility of testing Einstein’s general theory of relativity (GR) and the standard cosmological model via the $E_G$ statistic (Zhang et al. 2007) using 21-cm intensity mapping.

Several 21-cm intensity mapping experiments like BAOBAB (Pober et al. 2013), BINGO (Battye et al. 2012), CHIME (Bandura et al. 2014), the Tianlai project (Chen 2012; Chen et al. 2016), GBT-HIM (Chang & GBT-HIM Team 2016), SKA1-MID/SUR (Bull et al. 2015) have been planned to cover the intermediate-redshift range $z \sim 0.5 - 2.5$. Their primary goal is to measure the comoving scale of BAO around

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the onset of acceleration at \( z \sim 1 \). Efforts are also underway to observe the 21-cm intensity fluctuations around \( z \sim 3.35 \) using OWFA (Subrahmanya et al. 2017). On the other hand, the upgraded GMRT (uGMRT; Gupta 2017) and the upcoming SKA2 (Santos et al. 2015) promises to cover a large redshift range.

The 21-cm signal is intrinsically very weak, and it is very important to correctly model the expected signal in order to make realistic predictions for the various upcoming experiments (e.g. Bull et al. 2015; Sarkar et al. 2017). Modelling is also important in order to correctly interpret the 21-cm signal once it is detected.

There has been considerable work to model the post-reionization HI distribution and the expected 21-cm signal using semi-numerical prescriptions coupled with N-body simulations (Bagla et al. 2010; Khandai et al. 2011; Guha Sarkar et al. 2012; Seehars et al. 2016). In subsequent works, semi-numerical prescriptions have also been coupled with hydrodynamic simulations (Davé et al. 2013; Villaescusa-Navarro et al. 2014; Kim et al. 2017; Villaescusa-Navarro et al. 2016). A number of analytical frameworks have also been developed (Marín et al. 2010; Padmanabhan et al. 2017; Castorina & Villaescusa-Navarro 2017; Pénin et al. 2017).

In a recent work (Sarkar et al. 2016, hereunder Paper I), we have used a semi-numerical technique coupled with N-body simulations to model the HI distribution in the redshift range 1 \( \leq z \leq 6 \). We have quantified the evolution of the HI bias across this range for \( k \) values in the range 0.04 \( \leq k/\text{Mpc}^{-1} \leq 10 \), and we provide polynomial fitting formulas for the bias across this \( k \) and \( z \) range. Paper I and most of the other works discussed above are however limited by the fact that they have modelled the real space HI distribution ignoring the redshift space distortion (RSD) introduced by peculiar velocities. It is important to note that RSD plays an important role in the 21-cm intensity mapping signal (Bharadwaj & Ali 2004). In this paper we have extended the work presented in Paper I to include peculiar velocities and use this to model the RSD in the 21-cm power spectrum.

The RSD in galaxy redshift surveys has proven to be an important field of study (see Hamilton 1998 for a review). It offers a distinct way of estimating the cosmological parameters and/or any departure from the standard theory of structure formation (Cole et al. 2005; Parkinson et al. 2012; Song et al. 2014; Li et al. 2016). This also gives an independent estimate of the cosmological expansion rate (Guzzo et al. 2008; Kazin et al. 2012) and can also help to break the degeneracy between various models of modified gravity (Johnson et al. 2016; de la Torre et al. 2016). We expect to get similar information from the study of RSD effects in 21-cm intensity mapping experiments (Raccanelli et al. 2015; Santos et al. 2015; Bull et al. 2015).

In Paper I we have used N-body simulations and a semi-numerical technique, originally proposed by Bagla et al. (2010), to simulate the real space HI distribution and model the real space HI power spectrum \( P_H(k) \) over the redshift range 1 \( \leq z \leq 6 \). In the present work we have used the same simulations with the addition that we have used the peculiar velocities from the N-body simulations to map the HI distribution to redshift space. In this work we model the RSD effect in the redshift space HI power spectrum \( P_H(k, l) \), and use this to study its various moments - namely the monopole \( P_0(k) \) and the quadrupole \( P_2(k) \).

A brief outline of the paper follows. We present the redshift space HI simulations in Section 2, and the simulated redshift space HI power spectrum \( P_H(k, l) \) is presented in Sub-Section 2.1. In Section 3 we outline the different models which we have considered for \( P_H(k, l) \), the \( \chi^2 \) minimization used to fit the models to the simulations is also discussed here. We present the Results in Section 4, and Section 5 presents Summary and Discussion.

We have adopted the best fit cosmological parameters from Planck Collaboration et al. (2015).

## 2 SIMULATING HI DISTRIBUTION IN REDSHIFT-SPACE

We first simulate the post-reionization HI distribution in real space. Here we follow the methodology which has been extensively described in Paper I. We use a gravity only Particle Mesh N-body code (Bharadwaj & Silkant 2004) to generate snapshots of the dark matter distribution at the desired redshifts. The simulations contain \([1,072]^3\) dark matter particles in a \([2,144]^3\) regular cubic grid of spacing 0.07 Mpc with a total simulation volume (comoving) of \([150.08 \text{ Mpc}]^3\). The grid spacing limits the mass resolution of our simulations to \(10^8 M_\odot\). Our simulations span the redshift range \( z = 1 \) to 6 with redshift interval \( \Delta z = 0.5 \).

We use the “Friends-of-Friends” (FoF), (Davis et al. 1985) algorithm to locate the haloes in the dark matter distribution. We have used a linking length \( l_f = 0.2 \) (in the unit of mean inter particle separation) and set a criterion that a halo must contain at least 10 dark matter particles and hence we can resolve haloes with mass \(10^9 M_\odot\) or larger in our simulations. Our halo mass resolution is consistent with a recent study (Kim et al. 2017) which shows that for \( z > 0.5 \), we require a dark matter halo resolution better than \(~ 10^{10} h^{-1} M_\odot\) to ensure that the predicted 21-cm brightness temperature fluctuations are well converged.

The Inter Galactic Medium (IGM) is highly ionized in the post-reionization era (Fan et al. 2006a,b). The bulk of the HI at these redshifts is believed to reside in self-shielded gas clouds that appear as Damped Lyman-\( \alpha \) systems (DLAs) in quasar spectra (Storrie-Lombardi & Wolfe 2000; Prochaska et al. 2005; Zalar et al. 2013). These clouds are further believed to be associated with galaxies (Haehnelt et al. 2000) which are hosted in dark matter haloes. HI therefore is a tracer of the galaxy distribution. Bouché et al. (2005) measured the clustering of DLAs at \( z \sim 3 \) in a numerical simulation and found that DLAs occupy moderate mass haloes with an upper limit of log(\( M_h/M_\odot \)) \( \sim 11.1 \). Again a cross correlation study between DLAs and Lyman Break Galaxies (LBGs) at \( z \sim 3 \) suggests that DLAs preferably reside in haloes having mass \( M_h \) in the range \( 10^9 < M_h/M_\odot < 10^{11} \) (Cooke et al. 2006) which is in agreement with the findings of Pontzen et al. (2008) based on numerical simulations. However, in a similar work with large volume simulations, Cen (2012) found that DLAs prefer to reside in relatively more massive haloes (\( 10^{10} < M_h/M_\odot < 10^{12} \) at 1.6 \( < z < 4 \)) where galactic winds control their kinematics. A cross-correlation analysis of DLAs and the Lyman-\( \alpha \)
forest at \(z \sim 3\) (Font-Ribera et al. 2012) suggests that the DLAs favour relatively massive haloes with mass \(M_h = 6 \times 10^{11} M_\odot\). Note that this mass limit has been calculated by assuming a relation between the DLAs cross-section and the host halo mass, and DLAs can be hosted by the smaller mass haloes if the slope of this relation is steep enough.

In this work we assume that the HI resides solely in the dark matter haloes, and the HI mass \(M_{HI}\) in a halo depends only on the halo mass independent of the environment of the halo. We expect \(M_{HI}\) to increase with \(M_h\), however, observations at very low redshift suggests that the massive elliptical galaxies or galaxy clusters contain very little HI (e.g. see Serra et al. 2012, and references therein). This indicates that haloes with mass greater than a maximum value, \(M_{max}\), will contain little or no HI. Further, we do not expect haloes with masses below a lower limit \(M_{min}\) to contain HI as they would not be able to shield the neutral gas from the harsh ionizing background radiation. Based on these arguments, Bagla et al. (2010) have proposed three schemes for populating the simulated haloes with HI. In Paper I and our current work we have used the third scheme of Bagla et al. (2010) to populate the haloes. This scheme considers a redshift dependent relation between the \(M_h\) and circular velocity \(v_{circ}\)

\[
M_h \simeq 10^{10} M_\odot \left( \frac{v_{circ}}{60 \text{ km s}^{-1}} \right)^3 \left( \frac{1 + z}{4} \right)^{-\frac{3}{2}}.
\]

(1)

It is assumed that only haloes with a minimum circular velocity \(v_{circ} = 30 \text{ km s}^{-1}\) will host HI, which sets the lower mass limit \(M_{min}\) and \(v_{circ} = 200 \text{ km s}^{-1}\) defines the upper mass limit \(M_{max}\).

The HI mass in a halo is related to the halo mass as

\[
M_{HI}(M_h) = \begin{cases} 
  f_3 \frac{M_h}{M_{min}} & \text{if } M_{min} \leq M_h \\
  0 & \text{otherwise}
\end{cases}
\]

(2)

where \(f_3\) is a free parameter which controls the total HI content in our simulations. The choice of \(f_3\) does not affect the results of this work, and we have used \(f_3\) such that \(\Omega_{HI}\) in our simulations remains fixed at a value \(\sim 10^{-3}\). Here we have placed all the HI at the halo center of mass. This is a reasonably good assumption in real space as the intensity mapping experiments do not have adequate angular resolution to resolve the HI distribution within individual galaxies at high redshifts. We refer to this method as ‘HC’ and Paper I is entirely based on this method.

Our simulations have a fixed halo mass resolution of \(M_h = 10^9 M_\odot\) and at \(z > 3.5\), \(M_{min}\) falls below this mass resolution. We have used \(M_{min} = 10^9 M_\odot\) for \(z > 3.5\) in the HI assignment scheme (eq. 2). For \(z \leq 3.5\), \(M_{min}\) is above our halo mass resolution and we can fully resolve the smallest haloes where HI resides. The effect of choosing \(M_{min} = 10^9 M_\odot\) for \(z > 3.5\) has been discussed in Paper I.

The steps described till now yield the real space HI distribution which was analyzed in Paper I. We now map this HI distribution to redshift space using the line of sight component of the peculiar velocity \(\mathbf{v}\). For a distant observer along the \(z\)-axis, the halo position in redshift space \(\mathbf{s}\) is related to its real position as

\[
\mathbf{s} = \mathbf{x} + \frac{(\mathbf{v} \cdot \hat{z}) \hat{z}}{a H(a)},
\]

(3)

where \(a\) and \(H(a)\) are the scale factor and Hubble parameter respectively.

In the HC method we have assumed that the HI in a halo moves with the mean velocity of the host halo. In this method, in addition to placing the entire HI at the halo center of mass, we have also assigned the mean velocity of the host halo to the HI. This method ignores the motion of the HI within the halo. While the intensity mapping experiments lack the angular resolution to resolve the HI distribution within individual galaxies, they do have the frequency resolution required to make out the motion of the HI within individual haloes. Further, the effect of these motions may also extend to large length-scales through the Finger of God (FoG) effect (Jackson 1972). It is therefore desirable to include the velocity dispersion of the HI within the individual haloes. We presently have very little or no knowledge of how the HI is distributed within the haloes at high redshifts. Low redshift observations suggest that the HI is largely contained in the disks of spiral galaxies. On the other hand, there can be more than one galaxy in a halo. A detailed modelling of the galactic HI disks is beyond the scope of this paper, and we have not attempted this here. In the present work we have used a simple method to account for the HI velocity dispersion within the individual haloes. We have uniformly distributed the HI content of a halo among all the particles that constitute the halo. Further, the HI is assumed to move with the same velocity as the corresponding particle. We refer to this as the halo-particle ‘HP’ method. While neither the HC method nor the HP method is expected to give a very realistic picture of the HI distribution, it would still be quite reasonable to treat the two methods as respectively providing the lower and upper limits to the velocity dispersion effects. Both these methods are however expected to incorporate the coherent motions of the HI reasonably well.

Before proceeding to the redshift space power spectrum, it is important to note that the real space HI distribution and consequently the real space HI power spectrum differs between HC and HP methods. This difference is minimum at \(z = 3\) where the real space power spectra differ by \(\lesssim 1\%\) at \(k \lesssim 1\,\text{Mpc}^{-1}\) and \(\lesssim 10\%\) at \(k \lesssim 3\,\text{Mpc}^{-1}\). The difference increases at higher \((z \gtrsim 4)\) and lower \((z \lesssim 2)\) redshifts where it hovers between \(\lesssim 3\%\) at \(k \sim 1\,\text{Mpc}^{-1}\) and \(\lesssim 10\%\) at \(k \lesssim 3\,\text{Mpc}^{-1}\). This difference is maximum at \(z = 1\) where the real space power spectra differ by \(\lesssim 10\%\) at \(k \lesssim 0.4\,\text{Mpc}^{-1}\) and \(\lesssim 15\%\) at \(k \lesssim 1\,\text{Mpc}^{-1}\). These differences are relatively small given our current lack of understanding about how the HI is distributed at these redshifts. Throughout the present work, for the real space power spectra, we have ignored these small differences between the HC and HP methods and have used the results from Paper I to model the values of the HI bias parameters.

We have considered five statistically independent realizations of the simulation to estimate the mean and variance for all the results presented here.

2.1 The redshift space HI power spectrum

The real space HI power spectrum \(P_{HI}(k)\), studied in Paper I, is isotropic \(P_{HI}(k) = P_{HI}(k_\parallel)\). The peculiar velocity introduces an anisotropy along the line of sight direction. We quantify the HI distribution in redshift space in terms of the power spectrum, \(P_{HI}(k, \mu) \equiv P_{HI}(k_\parallel, k_\perp)\). Here, \(k_\parallel\) is the

\[
P_{HI}(k, \mu) \equiv P_{HI}(k_\parallel, k_\perp) = \int_0^\infty P_{HI}(k_\parallel, k_\perp) \, dk_\perp.
\]

We have considered five statistically independent realizations of the simulation to estimate the mean and variance for all the results presented here.
Figure 1. The red solid contours show the simulated redshift space HI power spectrum \( P_{\text{HI}}(k, k_{\parallel}) \) at six different redshifts, while the blue dotted contours show the real space counterpart. Both the power spectra are calculated using the HC method. The contour values increase inwards.

The simplest model for redshift space distortion

\[
P_{\text{HI}}(k, k_{\parallel}) = b^2 (1 + \beta^2) \sigma_b^2 \left( \frac{k_{\parallel}}{D_{\text{FoG}}(k_{\parallel})} \right) P(k) D_{\text{FoG}}(k_{\parallel}, \sigma_b). \tag{4}
\]

combines the Kaiser enhancement \((1 + \beta^2) \sigma_b^2\) with an independent small-scale suppression \(D_{\text{FoG}}(k_{\parallel}, \sigma_b)\) to account for the FoG effect. Such models have been extensively used (Peacock 1992; Park et al. 1994; Peacock & Dodds 1994; Ballinger et al. 1996) for the power spectrum of a variety of other tracers such as galaxies. Here \(\beta = f/b\) is the redshift distortion parameter with \(f\) being the logarithmic growth rate and \(b\) the linear scale-independent bias.
Models which incorporate dispersion models. A motivation for a derivation). In this paper we also been used (Bharadwaj Hatton & Cole 1999). Figure 2. As above are known as “dispersion models”. Kaiser enhancement along with a small-scale damping assumption of an exponential pairwise velocity distribution with a scale independent width leads to a Lorentzian profile in Fourier space (Davis & Peebles 1983; Hamilton 1998; Hatton & Cole 1999; White 2001; Seljak 2001). A motivation for the Gaussian damping profile can be found in Bharadwaj (2001). The square of a Lorentzian profile has also been used (Cole et al. 1995). Models which incorporate the Kaiser enhancement along with a small-scale damping as above are known as “dispersion models”.

In Paper I, we have modelled the HI distribution through a linear, complex, scale dependent bias \(\tilde{b}(k)\). The real space HI power spectrum \(P_{\text{HI}}(k)\) is related to the dark matter power spectrum through \(P_{\text{HI}}(k) = b^2(k)P(k)\), where \(b(k) = |\tilde{b}(k)|\) is the modulus of the complex bias. The real part of the complex bias \(b_r(k) = \text{Re}[\tilde{b}(k)]\) quantifies the cross-correlation between the HI and the dark matter density fields, and \(P_r(k)\), the HI-dark matter cross-correlation power spectrum can be expressed as \(P_r(k) = b_r(k)P(k)\). The cross-correlation can be equivalently expressed in terms of the stochasticity parameter \(r = b_r/b\). In this paper we use the scale dependent bias \(b(k)\) and the stochasticity parameter \(r(k)\) to quantify the clustering of the HI relative to dark matter fluctuations.

Paper I presents a detailed study of how \(b(k)\) and \(b_r(k)\) vary across the range \(z \leq 6\) and \(0.04 \leq k/\text{Mpc}^{-1} \leq 10\). The bias is found to be scale independent at large scales (small \(k\)) and exhibit scale dependence at small scales (large \(k\)). The value of the bias is found to decrease with decreasing \(z\). The stochasticity \(r(k)\) is found to be close to unity at all scales for \(z > 2\). At lower redshifts \((z \leq 2)\), \(r \sim 1\) at large scales, however \(r < 1\) at small scales. Paper I also presents polynomial fitting formulas in \(k\) and \(z\) for both \(b(k)\) and \(b_r(k)\). These formulas can be used to calculate \(b(k)\) and \(r(k)\) across the entire \(k\) and \(z\) range considered in the present paper. As discussed in Section 2, there is a small difference in \(P_{\text{HI}}(k)\) (also \(b(k)\) and \(r(k)\)) between the HC and the HP methods. In the present work we have ignored this and used the fitting formulas for the HI bias from Paper I for the rest of our analysis.

The presence of a complex bias \(\tilde{b}\), or equivalently the stochasticity \(r\), modifies the Kaiser term to \((1 + 2r\beta^2 + \beta^4\mu^4)\) (see Appendix A for a derivation). In this paper we have used

\[
P_{\text{HI}}(k_{\perp}, k_{\parallel}) = b^2(1 + 2r\beta^2 + \beta^4\mu^4) P(k)D_{\text{FoG}}(k_{\parallel}, \sigma_p),
\]

where \(P(k)\) is the real space dark matter power spectrum from the simulations in Paper I, and \(b, \beta\) and \(r\) are three scale dependent parameters whose values change with redshift (Paper I). Here we have considered three different cases, namely:

(A) scale dependent bias and stochasticity as determined in Paper I. This essentially treats the bias as a complex quantity.

**Figure 2.** The red solid contours show the simulated redshift space HI power spectrum \(P_{\text{HI}}(k_{\perp}, k_{\parallel})\) for the HP method at six different redshifts, while the blue dotted contours show the real space counterpart. The contour values increase inwards.
provides the best fit values of $\sigma_p$ at different redshifts for models A1 and B1 respectively (Table 2). The solid red line and the dotted blue line show the predictions of eq. (6). The above two curves correspond to the HP method while the two curves below correspond to the HC method.

(B) scale dependent bias as determined in Paper I with $r = 1$. This is equivalent to a real bias $b = |b|$. 
(C) scale independent bias $b = b_0$ with $r = 1$. Here the bias only evolves with $z$ but has no $k$ dependence, and $b_0$ is the constant term in the polynomial which quantifies the $k$ dependence in Paper I. The value of $b_0$ essentially corresponds to a large length-scales where we have a nearly scale independent bias (Paper I).

Here we have also considered three different damping profiles, namely:

1. Lorentzian, $D_{RSC}(k, \sigma_p) = (1 + \frac{1}{2}k^2\sigma_p^2)^{-1}$
2. Gaussian, $D_{RSC}(k, \sigma_p) = \exp\left(-\frac{1}{2}k^2\sigma_p^2\right)$
3. Lorentzian squared, $D_{RSC}(k, \sigma_p) = (1 + \frac{1}{2}k^2\sigma_p^2)^{-2}$

We therefore have nine possible combinations A1, A2,..., C2 and C3 which we have considered in this paper. All the different models have only one free parameter $\sigma_p$. Note that, $\sigma_p$ here is in units of comoving Mpc and we can equivalently use $[\sigma_p a H(a)]$ in units of km/s. In order to determine how well these models are able to capture the anisotropy in the simulated $P_{HI}^s(k, k_1, k_2)$ (using both the HC and HP methods), for each model we fit the simulated $P_{HI}^s(k, k_1, k_2)$ to determine the best fit value of the parameter $\sigma_p$. The corresponding goodness of fit for all the models is provided in Table 1. While nearly all the models work well at small $k$, they all exhibit very significant deviations from the simulations at large $k$. Further, these deviations are also seen to increase at lower redshifts. These deviations at large $k$ severely influence the fitting procedure, and we find that it is advantageous to exclude the large $k$ values for fitting the simulated $P_{HI}^s(k, k_1, k_2)$. The values of $(k, k_1)$ were restricted to be within $(0.9, 1.1, 1.5, 1.7, 2.0)$ Mpc$^{-1}$ for the redshifts $(1, 1.5, 2, 2.5, 3)$ respectively.

4 RESULTS

In this work we have modelled the anisotropy in the simulated $P_{HI}^s(k, k_1, k_2)$ using the nine models mentioned above (eq. (5)). Each model has a single free parameter $\sigma_p$ to fit. We perform a $\chi^2$ minimization with respect to $\sigma_p$ to determine the value that best fits the simulated $P_{HI}^s(k, k_1, k_2)$. Table 1 shows the goodness of fit which is quantified by the reduced chi square $\chi^2/N$ where $N$ is the degree of freedom. We consider models with $\chi^2/N \sim 1$ acceptable fits to the simulated $P_{HI}^s(k, k_1, k_2)$, and models with $\chi^2/N$ considerably larger than unity are rejected.

We first discuss our results for the HC method. Considering the $\chi^2/N$ values in Table 1, we find that in the redshift range $z = 2 - 3$ and possibly at 3.5, nearly all the models fit the simulated $P_{HI}^s(k, k_1, k_2)$ reasonably well. At higher redshifts ($z \geq 3.5$) we see that the constant bias models (C1, C2 and C3) fail to fit the simulations. We also see that at lower redshifts ($z < 2$), the constant bias models, particularly with the Gaussian (C2) and the Lorentzian squared (C3) damping profiles, do not provide a good fit. On the other hand, the scale dependent complex bias models (A1, A2 and A3) and the scale dependent real bias models (B1, B2 and B3) provide reasonably good fits to the simulated $P_{HI}^s(k, k_1, k_2)$ through nearly the entire $z$ range considered here. However, at $z \geq 5.5$, the $\chi^2/N$ values are greater than $1$ for the scale dependent complex and real bias models. Now we discuss the results for the HP method. Almost all the above conclusions hold true for the HP method. However, for $z = 2 - 4$, models A2 and B2 show large $\chi^2/N > 1$ and hence do not give a good fit to the simulated $P_{HI}^s(k, k_1, k_2)$. At $z \geq 5.5$, the $\chi^2/N \sim 1$ (less than the values obtained for the HC method) indicative of a better fit to the simulations.

Considering the $\chi^2/N$ values in the entire $z$ range, we find that the Lorentzian damping profile provides the best fit compared to the two other profiles. The Lorentzian squared profile performs better than the Gaussian profile, but it has slightly higher $\chi^2/N$ values compared to the Lorentzian profile at some of the redshifts. All the three damping profiles, combined with the scale dependent real and complex bias models, work almost equally well in the range $z = 2 - 5$ for the HC method. In case of the HP method, the Lorentzian and the Lorentzian squared profiles combined with the scale dependent real and complex bias models are comparably good in the entire redshift range. The Gaussian damping profile show somewhat larger $\chi^2/N$ at the intermediate redshifts for the HP method.

We can broadly summarize that the Lorentzian damping profile coupled with the scale dependent complex bias and the scale dependent real bias (models A1 and B1) both work nearly equally well and they best fit the simulated power spectrum over the entire $z$ range considered here. We mostly focus on these two models in the subsequent discussion of this paper.

Table 2 provides the best fit values of $\sigma_p$ at different $z$ for the two models A1 and B1 for the two different HI mass assignment methods. Here $\sigma_p$ is in units of comoving Mpc, or equivalently $[\sigma_p a H(a)]$ in units of km/s. For the HC method, we see that the best fit values of $\sigma_p$ decrease with increasing redshift and we have $\sigma_p \sim 0$ for $z \geq 5$ for Model A1 and for $z > 5$ for Model B1 which implies that the PoG effect is not very prominent at the higher redshifts. This also indicates
Table 1. This tabulates the minimum value of the reduced \( \chi^2/N \) for the different models, here \( N \) is the degrees of freedom. The \( \chi^2/N \) values outside (inside the brackets) are related to the HC (HP) method.

Table 2. For models A1 and B1 (for the two HI assignment methods), this presents the best fit values of \( \sigma_p \) along with the 1 – \( \sigma \) uncertainty intervals at different redshifts.

that the FoG damping can safely be ignored for \( z > 5 \), and the anisotropy in \( P_{\text{HI}}(k_\perp, k) \) can be adequately modelled by only using the modified Kaiser term with a scale dependent complex or real bias if one uses the HC method. For the HP method, the best fit values of \( \sigma_p \) also decrease with increasing redshift. However, unlike the HC method, here the best fit values of \( \sigma_p \) are non zero even at the highest redshift. The \( \sigma_p \) values are larger than the HC method and this indicates that the HP method takes into account the FoG effect due to the velocity dispersion inside the halos which is ignored by the HC method.

Figure 3 shows the best fit values of \( \sigma_p \) as a function of redshift for the two models A1 and B1. For both the models \( \sigma_p \) shows a nearly parabolic \( z \) dependence for \( z < 5 \) (\( z \leq 6 \)) for the HC (HP) method. We have used the following functional form to fit the \( z \) dependence of \( \sigma_p \),

\[
\sigma_p(z) = \sigma_p(0) (1 + z)^{-m} \exp \left[ -\frac{z}{z_p} \right],
\]

where \( \sigma_p(0) \), \( m \) and \( z_p \) are three fitting parameters. Considering the HC method, we have carried out a rough fitting for Model A1 (B1) with the values of the parameters \( \sigma_p(0) = 11.0 \) (13.4) Mpc, \( m = 1.9 \) (2.0) and \( z_p = 11.0 \) (11.5) which works in the range 1 \( \leq z \leq 5 \). The same fitting for the
HP method yields $\sigma_z(0) = 9.12$ (10.8) Mpc, $m = 1.15$ (1.24) \text{ and } z_p = 12.0$ (12.0) for Model A1 (B1) which works in the range $1 \leq z < 6$. We see that the values of $\sigma_p$ are slightly larger for Model B1 as compared to Model A1, however it is not clear if the differences are statistically significant. The $\sigma_p$ values are also higher for the HP method which again indicates that the FoG damping increases when the velocity dispersion inside the haloes is taken into account. However, $\sigma_p$ falls slowly with redshift for the HP method in comparison to the HC method. Considering the HC method, our fit fails for $z \geq 5$ where the $\sigma_p$ values lie below the fit and are consistent with zero. On the other hand, the fit works almost for the entire $z$ range for the HP method.

It is convenient to decompose the anisotropic redshift space HI power spectrum $P_{HI}^s(k_\parallel, k_\perp)$ into angular multipoles (Hamilton 1992; Cole et al. 1994)

$$P_{HI}^s(k, \mu) = \sum_\ell \mathcal{L}_\ell(\mu) P^s_\ell(k),$$

where $\mathcal{L}_\ell(\mu)$ are the Legendre polynomials and $P^s_\ell(k)$ are the different angular moments of the power spectrum. The angular moments $P^s_\ell(k)$ are functions of a single variable $k$ and therefore are relatively easy to visualize and interpret. Only the even moments survive in the flat sky approximation. Further, in linear theory (i.e. with just the Kaiser enhancement) only the first three even moments $\ell = 0$ (monopole), 2 (quadrupole) and 4 (hexadecapole) are non-zero. Here we have calculated the first three even moments from the simulated $P_{HI}^s(k_\parallel, k_\perp)$.

The hexadecapole ($P^s_4(k)$) is found to be rather noisy (large cosmic variance), and we have included results only for the monopole $P^s_0(k)$ and the quadrupole $P^s_2(k)$. We have compared these to the predictions of the different models in order to analyze how well the models are able to reproduce the simulated HI power spectrum.

Considering both the HC and HP methods, Figure 4 shows the dimensionless power spectra $\Delta^P_\ell(k) = k^3 P^s_\ell(k)/2\pi^2$ and $\Delta^P_2(k) = k^3 P^s_2(k)/2\pi^2$. The real space HI power spectrum $\Delta^P_0(k) = k^3 P^s_0(k)/2\pi^2$ is also shown for reference. Unlike the real space HI power spectrum $P(k)$ which evolves as $(1 + z)^2$ in the linear regime, we see that the real space HI power spectrum shows a very weak redshift evolution (see Paper I for a detailed discussion). However, we see that both $\Delta^P_0(k)$ and $\Delta^P_2(k)$ evolve with redshift, the effect being relatively more pronounced for $\Delta^P_2(k)$. We first consider the large length-scales (small $k$) where the results from the HC and HP methods are indistinguishable. We see that both $\Delta^P_0$ and $\Delta^P_2$ grow with decreasing $z$. This growth is due to the RSD which occurs here primarily due to the coherent flows. This component of RSD is expected to be identical in both the methods which is why their results are indistinguishable. The monopole, which has contributions from both $P_{HI}^s(k)$ and the RSD, does not show very significant evolution at large $z$ ($z > 3$) and shows a modest growth only at low $z$ ($z < 3$). This can be explained by noting that the bias $b(z)$ increases with $z$ and has value $b(z) > 2$ for $z > 3$ whereby $\beta$, which determines the relative contribution from the RSD (eq. 5), has a very small value at large $z$. The quadrupole, which arises entirely due to the RSD, appears to evolve through the entire $z$ range.

Considering the small length-scales (large $k$), we find that the results from the two methods are different. The RSD here is primarily due to the FoG suppression arising form the velocity dispersion which is different for the two methods. The HP method has an enhanced FoG suppression and the monopole $\Delta^P_0(k)$ is smaller than that for the HC method. Further, for both the methods the $z$ evolution is opposite to that seen at small $k$, and the value of $\Delta^P_0(k)$ is found to decrease with decreasing $z$. We can explain this by noting that the velocity dispersion and the FoG suppression increases with decreasing redshift (Figure 3). For both the methods, the quadrupole is negative at large $k$. This essentially arises due to the FoG elongation of structures along the line of sight direction. We find that the $k$ value corresponding to this transition decreases with decreasing $z$. Further, these $k$ values are relatively larger for the HC method where the FoG is restricted to relatively smaller length-scales.

Considering the monopole $P^s_0(k)$, the upper panels of Figure 5 show a comparison of the model predictions against the values obtained from the simulations for the HC method. For convenience, the values of $P^s_0(k)$ have been normalized using the predictions for Model B1. The lower panels of Figure 5 show the same quantities for the HP method. We see that the results from the two methods are qualitatively similar over a large portion of the $k$ range across all the redshifts shown here, and as noted earlier the results are nearly indistinguishable at small $k$ ($< 0.2 \text{Mpc}^{-1}$). At $z = 1$, models A1 and B1 match the simulated values for $k < 0.3 \text{Mpc}^{-1}$ for the HC method while this range increases to $k < 0.6 \text{Mpc}^{-1}$ for the HP method. For both the methods, the deviations of A1 and B1 from the simulated values lie within 10% across the entire fitting range. All the other models deviate from the simulations at a smaller $k$ value ($k \sim 0.2 \text{Mpc}^{-1}$). The deviations from the simulations are also larger compared to models A1 and B1. The behaviour at $z = 2$ for the HC method is similar to that seen at $z = 1$. Models A1 and B1 match the simulations up to $k < 0.5 \text{Mpc}^{-1}$, whereas all the other models match the simulations for a smaller $k$ range ($k < 0.4 \text{Mpc}^{-1}$). For models A1 and B1, the deviations from the simulations are within 10% across the entire fitting range, whereas we find up to ~15% deviations for all the other models. In contrast, for the HP method Model B3 provides a better fit compared to the models A1, B1 and all the other models. Models A1 and B1 match the simulated values within $k \sim 0.2 \text{Mpc}^{-1}$ while this extends to $k < 0.4 \text{Mpc}^{-1}$ for Model B3. The deviations from the simulations are within 10% across the entire fitting range for all three of these models. At $z = 3$, in case of the HC method almost all the models agree well with the simulations for $k < 0.3 \text{Mpc}^{-1}$. Further, the deviations from the simulations are within 15% through the entire $k$ range. For the HP method, Model B2 matches the simulations for a larger $k$ range ($k < 0.5 \text{Mpc}^{-1}$) compared to the other models which deviate after $k \sim 0.2 \text{Mpc}^{-1}$. However, the deviations from the simulations are within 10% for all the models barring B2 and C1. At $z = 4$ the models with a scale independent bias all fail to match the simulated monopole, and for the monopole we focus only on the models with a $k$ dependent bias, either complex or real. Considering the HC method, the predictions of all these models are indistinguishable across the entire $k$ range irrespective of the damping profile, and they match the simulations for $k < 0.3 \text{Mpc}^{-1}$. Considering the HP method which has a larger velocity dispersion, the predictions depend on the damping profile and...
shows any of the subsequent figures. We find that the HC and HP methods as indicated in the figure. Note that the quadrupole \( P_{2}(k) \) becomes negative at large \( k \), this transition is marked by a circle (triangle) for the HC (HP) method. For the HC method this transition occurs at \( k > 2 \text{ Mpc}^{-1} \) for \( z > 3 \).

Model B2 fits the simulations over a relatively large \( k \) range \( k < 0.6 \text{ Mpc}^{-1} \) whereas the other damping profiles work well within \( k < 0.3 \text{ Mpc}^{-1} \). For both the methods, the deviations from the simulations are within \( \sim 10\% \) over the entire fitting range. The predictions at \( z = 5 \) are very similar to those at \( z = 4 \) and these have not been shown separately in Figure 5 and any of the subsequent figures. We find that the models with a scale dependent bias are able to match the simulations at \( k < 0.3 \text{ Mpc}^{-1} \) and \( k < 0.4 \text{ Mpc}^{-1} \) for the HC and HP methods respectively. For the HC method the predictions for the different damping profiles are indistinguishable, whereas there are small differences (which get smaller with increasing \( z \)) for the HP method.

Figure 6 shows \( P_{3}(k) \), the results are normalized by Model B1. We see that at \( z = 1 \), models A1 and B1 perform better than the other models in matching the simulations for both the HC and HP methods. The match extends to \( k < 0.5 \text{ Mpc}^{-1} \) and \( k < 0.3 \text{ Mpc}^{-1} \) for the HC and HP methods respectively, and the deviations are within 20\% at \( k \lesssim 0.5 \text{ Mpc}^{-1} \) for the HP method. For both the methods,
these models do not match the simulated quadrupole beyond this $k$ range, and some of the simulated values are in excess of the range considered in the figure. At $z = 2$ we see that all the models with $k$ dependent bias match the simulations up to $k < 0.5\,\text{Mpc}^{-1}$ for the HC method while only Model B3 is able to cover the same range for the HP method where the other scale dependent bias models match the simulations only to $k < 0.3\,\text{Mpc}^{-1}$. Surprisingly Model C1 matches the simulations for a large $k$ range ($k < 0.7\,\text{Mpc}^{-1}$) for the HC method, however this is reduced to ($k < 0.3\,\text{Mpc}^{-1}$) for the HP method. For both the methods, across the entire fitting range the deviations from the simulated values are within 30% for models A1 and B1, while all the other models show somewhat larger deviations. At $z = 3$ for the HC method we see that all the models match the simulations for $k < 0.3\,\text{Mpc}^{-1}$, and for models A1 and B1 the deviations lie within 20% across the entire $k$ range while these are somewhat larger for the other models. For the HP method Model B2 matches the simulations for a relatively larger $k$ range ($\lesssim 0.6\,\text{Mpc}^{-1}$) while this is restricted to $k < 0.3\,\text{Mpc}^{-1}$ for the other models. All the models deviate significantly from the simulations at large $k$. At $z = 4$ for both the methods the models match the simulated values for a very limited range $k < 0.15\,\text{Mpc}^{-1}$ and differ significantly beyond this. At $z > 4$ the results are very similar to those for $z = 4$ and we have not shown these here.

Considering the ratio $P_2^s(k)/P_2^b(k)$, the upper (lower) panels of Figure 7 show the simulated values along with the model predictions and also the linear theory prediction for the HC (HP) method. The linear theory prediction is computed using just the Kaiser enhancement term with the scale dependent real bias $b(k)$. We see that, at $z = 1$ models A1 and B1 match the simulations for the entire fitting range for the HC method while this range shrinks to $k < 0.6\,\text{Mpc}^{-1}$ for the HP method. Considering the HC method, models B2 and B3 match the simulations for $k < 0.4\,\text{Mpc}^{-1}$ and $k < 0.7\,\text{Mpc}^{-1}$ respectively, while, for the HP method, they match the simulations up to $k \sim 0.2\,\text{Mpc}^{-1}$ and $k \sim 0.3\,\text{Mpc}^{-1}$ respectively. Model C1 and the linear theory prediction deviate from the simulations before $k \sim 0.3\,\text{Mpc}^{-1}$ ($k \sim 0.2\,\text{Mpc}^{-1}$) for the HC (HP) method. At $z = 2$, models A1 and B1 match the simulations for $k < 0.7\,\text{Mpc}^{-1}$ for the HC method and this range increases to $k \sim 0.3\,\text{Mpc}^{-1}$ for the HP method. All the other models deviate from the simulations at a smaller $k$ value. The linear theory predictions agree with the simulations for $k < 0.4\,\text{Mpc}^{-1}$ and $k < 0.3\,\text{Mpc}^{-1}$ for the HC and HP methods respectively. At $z = 3$, for the HC method, all the models with a scale dependent bias are in reasonable agreement with the simulations over the entire $k$ range while models with a scale independent bias deviate at large $k$. For the HP method Model B2, which works best, matches the simulations for $k < 0.7\,\text{Mpc}^{-1}$ while this range is limited to $k \sim 0.3\,\text{Mpc}^{-1}$ for all the other models. For both the methods, the linear theory agrees with the simulations for $k < 0.5\,\text{Mpc}^{-1}$. At $z = 4$, considering both the HC and HP methods, the models with a scale dependent bias match the simulations for $k < 0.3\,\text{Mpc}^{-1}$. However, for both the methods, Model C1 which has a scale independent bias and the linear theory prediction agree with the simulations for $k < 0.4\,\text{Mpc}^{-1}$. At $z > 4$, for both the HC and HP methods, all the model predictions along with the linear theory match the simulations for a very limited $k$ range ($k < 0.15\,\text{Mpc}^{-1}$). The results are nearly similar to those at $z = 4$ and these have not been shown here.

Figure 6. Same as Figure 5 for the quadrupole $P_2^s(k)$.
where the HC method suppresses the difference can be ignored on large scales. However, this is not true in redshift space where the HC method incorporates this component of the velocity dispersion possibly overestimates the FoG effect. It should be noted that the two methods are expected to have the same coherent flows and differ only in the velocity dispersion. The actual HI distribution is possibly somewhere in between. The differences between the two methods which we have considered. We interpret the results from the two methods as representing two limiting cases, and we expect the predictions for the actual HI distribution to lie somewhere in between. The differences between the two methods are apparent from Figure 1 and Figure 2 where the \( P_{HI}^s(k_{\perp}, k_{\parallel}) \) obtained from two methods is plotted against its real space counterpart at different redshifts.

We have modelled \( P_{HI}^s(k_{\perp}, k_{\parallel}) \) using the simple assumption (eq. (5)) that this can be obtained by multiplying the model predictions of Paper I \( (P_{HI}(k) = b^2P(k)) \) with a Kaiser enhancement term and a Finger of God (FoG) damping term. The various models considered here (Section 3) all have a single free parameter \( \sigma_p \) which is the pair velocity dispersion that appears in the FoG damping term. We have carried out a \( \chi^2 \) minimization with respect to \( \sigma_p \) in order to determine how well our models match the simulated \( P_{HI}^s(k_{\perp}, k_{\parallel}) \). Using the \( \chi^2/N \) values (Table 1) to estimate the goodness of fit for both the HI assignment methods, we find that models A1 and B1, both of which have a Lorentzian damping profile with a scale dependent bias (complex and real respectively), provide the best match to the simulations over the entire \( z \) range. The same two bias schemes combined with a Gaussian (A2, B2) and Lorentzian

Figure 7. The ratio \( P_{HI}^s(k)/P_0^s(k) \) for the HC and HP methods are shown in the upper and lower panels respectively. The blue circles (connected with blue solid line) and the vertical error bars, respectively, present the mean and the 1 − \( \sigma \) spread determined from the five independent realizations of the simulations. Predictions from different models and linear theory are shown for comparison. The shaded regions denote the \( k \) range which has not been used for the fitting. Note that the ratio has negative values beyond the sharp dip at large \( k \).

5 SUMMARY AND DISCUSSION

The post-reionization HI 21-cm signal, which is expected to be a pristine probe of the large scale structures in the Universe, is an excellent candidate for precision cosmology. This requires accurate and reliable modelling of the expected signal. In an earlier paper (Paper I) we have simulated the expected HI 21-cm power spectrum \( P_{HI}(k) \) in real space (as against redshift space) and used this to model the \( k \) dependence of the (possibly complex) bias \( b(k) \) over the redshift range \( 1 < z < 6 \). In this paper we have extended the earlier simulations to include the redshift space distortion (RSD) due to the peculiar motion of the HI, and we have used this to model the anisotropy of the redshift space HI 21-cm power spectrum \( P_{HI}^s(k_{\perp}, k_{\parallel}) \). Such modelling is important on two counts. This is first required to make accurate predictions for various instruments which aim to carry out precision cosmology observations using the HI 21-cm signal. Precise modelling is also required to correctly interpret the signal and extract the relevant astrophysical and cosmological informations once the HI 21-cm signal is measured.

Here we have used two separate methods, subsequent to the HI assignment scheme in eq. (2), to distribute HI in the haloes. In the first method we place the HI content of a halo at the halo center of mass and we refer to this as the HC method. The whole analysis of Paper I is based on the HC method. In the second method we distribute the HI content of a halo equally among all the member particles of the halo and this is referred to as the HP method. The real space clustering of HI in these two methods differ very little and the difference can be ignored on large scales. However, this is not true in redshift space where the HC method suppresses the FoG damping as it does not take into account the velocity dispersion within a halo. On the other hand, the HP method which incorporates this component of the velocity dispersion possibly overestimates the FoG effect. It should be noted that the two methods are expected to have the same coherent flows and differ only in the velocity dispersion. The actual HI distribution is possibly somewhere in between the two methods which we have considered. We interpret the results from the two methods as representing two limiting cases, and we expect the predictions for the actual HI distribution to lie somewhere in between. The differences between the two methods are apparent from Figure 1 and Figure 2 where the \( P_{HI}^s(k_{\perp}, k_{\parallel}) \) obtained from two methods is plotted against its real space counterpart at different redshifts.

We have modelled \( P_{HI}^s(k_{\perp}, k_{\parallel}) \) using the simple assumption (eq. (5)) that this can be obtained by multiplying the model predictions of Paper I \( (P_{HI}(k) = b^2P(k)) \) with a Kaiser enhancement term and a Finger of God (FoG) damping term. The various models considered here (Section 3) all have a single free parameter \( \sigma_p \) which is the pair velocity dispersion that appears in the FoG damping term. We have carried out a \( \chi^2 \) minimization with respect to \( \sigma_p \) in order to determine how well our models match the simulated \( P_{HI}^s(k_{\perp}, k_{\parallel}) \). Using the \( \chi^2/N \) values (Table 1) to estimate the goodness of fit for both the HI assignment methods, we find that models A1 and B1, both of which have a Lorentzian damping profile with a scale dependent bias (complex and real respectively), provide the best match to the simulations over the entire \( z \) range. The same two bias schemes combined with a Gaussian (A2, B2) and Lorentzian
squared (A3, B3) damping also work reasonably well, with the Lorentzian squared having lower $\chi^2/N$ compared to the Gaussian. In contrast, the models with a scale independent bias (C1, C2, and C3) fail to match the simulations. However, we note that there is a degradation in the goodness of fit with increasing $z$ at $z \geq 5$ for the HC method (Table 1), and our models do not provide a very good fit to the simulations at $z \geq 5$ even if we incorporate a scale dependent or real bias.

Considering the HC method, for models A1 and B1, we find that the best fit values of $\sigma_P$ (Table 2 and Figure 3) decrease approximately as $(1+z)^{-5}$ with increasing $z$ (eq. (6)), and the values of $\sigma_P$ are consistent with zero for $z > 5$. This essentially tells us that it is not necessary to include the FoG effect for modelling $P_{\text{HI}}(k_{\perp},k_{\parallel})$ at $z > 5$ if one uses the HC method. This is also clearly seen in the simulations (Figure 1) where there is no evidence for the FoG effect at $z \geq 5$. Considering the HP method, we see that the the best fit values of $\sigma_P$ (Table 2 and Figure 3) fall relatively slowly ($(1+z)^{-1}$) as compared to the HC method. Further, unlike in the HC method, $\sigma_P$ has a small but finite value even at the highest redshift. We may interpret this as arising from the velocity dispersion within a halo. This is expected to lead to an enhanced FoG suppression for the HP method as noticeable in Figure 2. In contrast to the HC method, for the HP method the $k$ dependent complex and real bias models provide a reasonably good fit to the simulations (Table 1) over the entire $z$ range.

The angular moments $P_2^\parallel(k)$ are relatively easy to visualize and interpret. Considering the monopole $P_0^\parallel(k)$ and quadrupole $P_2^\parallel(k)$, we have investigated how well our models match the simulations. Considering both the HC and HP methods, for $P_0^\parallel(k)$ we find that models A1 and B1 match the simulations at large scales ($k < 0.3 \text{ Mpc}^{-1}$) over the entire $z$ range, and the deviations are within 10–20% for larger $k$ values within the fitting range (Figure 5). The deviations are somewhat larger for the other models with a scale dependent bias, whereas the scale independent bias models show significantly larger deviations particularly at high $z$. For $P_2^\parallel(k)$, models A1 and B1 match the simulations (Figure 6) over a relatively large range ($k < 0.5 \text{ Mpc}^{-1}$ for the HC method and $k < 0.3 \text{ Mpc}^{-1}$ for the HP method) at low $z$ ($\lesssim 2$). The $k$ range shrinks to $k < 0.3 \text{ Mpc}^{-1}$ for the HP method and $k < 0.2 \text{ Mpc}^{-1}$ for the HC method at $z = 3$, and is even smaller ($k < 0.15 \text{ Mpc}^{-1}$) for both the methods at $z \geq 4$. We find that all the models considered here significantly underpredict the quadrupole at $z \geq 4$, and these deviations increase with increasing $z$.

The linear theory of redshift space distortion predicts (Hamilton 1998)

$$\frac{P_2^\parallel(k)}{P_0^\parallel(k)} = \frac{(4/3)\beta + (4/7)\beta^2}{1 + (2/3)\beta + (1/5)\beta^2}. \quad (8)$$

Measuring this ratio from observations of the HI 21-cm power spectrum holds the promise of determining $\beta = f(\Omega)/b$. Assuming that the bias $b$ is known, this can be used to determine $f(\Omega)$ the growth rate of density perturbations, which is a sensitive probe of cosmology. It is particularly important to model the ratio $P_2^\parallel(k)/P_0^\parallel(k)$ for precision cosmology. We find that the results for the two simulation methods are qualitatively very similar (Figure 7). The results from the two methods are indistinguishable at small $k$ ($\lesssim 0.2 \text{ Mpc}^{-1}$) where the ratio has a value $\sim 1$ or larger. The ratio drops at larger $k$ where the results from the two methods start to differ, and becomes negative at larger $k$. As noted earlier, the negative quadrupole is an outcome of the FoG elongation along the line of sight. We see that the same features can be identified in the results for both the methods, however they typically are shifted to smaller $k$ for the HP method due to the larger velocity dispersion in comparison to the HC method. This is particularly noticeable for the dip which corresponds to the transition to negative values.

Considering the HC method first, we find that linear theory (eq. (8)) with a scale dependent bias matches the simulations for $k < 0.3, 0.4, 0.7 \text{ Mpc}^{-1}$ at $z = 1, 2, 3$ respectively. This range increases to $k < 0.9, 0.7, 2.0 \text{ Mpc}^{-1}$ if we include a Lorentzian damping profile. The scale independent bias models do not provide a very good fit even with a damping profile. For $z = 0$ all the models are in agreement with the simulations for $k < 0.4 \text{ Mpc}^{-1}$, and they deviate for larger $k$. At $z = 5.6$ the models match the simulations for a very limited $k$ range $k < 0.15 \text{ Mpc}^{-1}$, and the models underpredict the ratio for larger $k$. For the HP method, we find that the linear theory agrees with the simulations for $k < 0.2, 0.3 \text{ Mpc}^{-1}$ at $z = 1, 2$ respectively. This range, however, increases to $k < 0.6, 1 \text{ Mpc}^{-1}$ with the inclusion of Lorentzian damping. At $z = 3, 4$ linear theory matches the simulations for $k < 0.5, 0.4 \text{ Mpc}^{-1}$ respectively. This $k$ range does not improve with the inclusion of any damping profile. The scale independent bias models fail to provide a good fit to the simulated ratio at these redshifts even with a damping profile. At $z = 5.6$ all the models match the simulations for a very limited $k$ range $k < 0.15 \text{ Mpc}^{-1}$, and the models underpredict the ratio for $k \lesssim 1 \text{ Mpc}^{-1}$.

In summary, we find that models with a scale dependent bias and a Lorentzian damping profile provide a good fit to the simulations for large length-scales at $1 \leq z \leq 4$. We also find that the complex nature of the bias is not important over the $k$ range considered here, and it suffices to consider a real bias with $r = 1$. While our models work reasonably well for $P_2^\parallel(k)$ over the entire $z$ range, the models underpredict $P_2^\parallel(k)$ for a considerable part of the $k$ range at $z \geq 4$. We note that the bias increases rapidly with $z$ (Paper I) resulting in a small value of $\beta$ at high $z$. The discrepancy in $P_2^\parallel(k)$ is possibly telling us that the simple Kaiser enhancement term (eq. (5)) which depends only on $\beta$ underpredicts the redshift space anisotropy at high $z$. The required modification cannot be modelled through a damping profile which reduces the quadrupole instead of enhancing it. More sophisticated modelling is possibly needed at high $z$, and we plan to address this in future work.

Simulating the post-reionization HI 21-cm signal requires relatively expensive high resolution cosmological simulations (Section 2) which can be run for a limited cosmological volume. Here we have provided models which can be used to compute $P_{\text{HI}}(k_{\perp},k_{\parallel})$ in the range $1 \leq z \leq 6$, and could possibly be extrapolated to lower redshifts $0 \leq z \leq 1$. The models considered here relate $P_{\text{HI}}(k_{\perp},k_{\parallel})$ to $P(k)$ which is the dark matter power spectrum. The latter can be computed using relatively inexpensive low resolution simulations covering larger cosmological volumes. Our models rely on a scale dependent bias and a Lorentzian damping pro-
file. Paper I provides fitting formulas which can be used to calculate the scale dependent bias, and eq. (6) of this paper presents a fitting formula which can be used to calculate $\sigma_P$ which is used in the Lorentzian profile. These can be used in eq. (5) to calculate $P_{HI}(k_x, k_y)$.

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APPENDIX A:

In real space (as against redshift space) we assume that \( \Delta_{HI}(k) \) which refers to the Fourier components of the HI density contrast is related to its matter counterpart \( \Delta(k) \) through a linear bias parameter \( \tilde{b}(k) \) whereby
\[
\Delta_{HI}(k) = \tilde{b}(k) \Delta(k) .
\] (A1)

Note that the value of \( \tilde{b}(k) \) may vary with \( k \) (scale dependent bias) and is, in general, complex (Paper I). We then have
\[
P_{HI}(k) = b^2(k) P(k)
\] (A2)

for the real space power spectra where \( b(k) \) is the modulus of \( \tilde{b}(k) \). Considering the HI-matter cross-power spectrum \( P_c(k) \) (Paper I) we have
\[
P_c(k) = b_r(k) P(k)
\] (A3)

where \( b_r(k) \) is the real part of \( \tilde{b}(k) \). We also have the stochasticity defined as \( r(k) = b_r(k)/\tilde{b}(k) \).

In the linear theory of density perturbations, the Fourier components of the HI density contrast in redshift space is given by
\[
\Delta_{HI}^r(k) = \Delta_{HI}(k) + f \mu^2 \Delta(k)
\] (A4)

where the second term in the R.H.S. incorporates the effect of peculiar velocities (Kaiser 1987). Using this to calculate the redshift space HI power spectrum, we have
\[
P_{HI}^r(k) = P_{HI}(k) + 2f \mu^2 P_c(k) + f^2 \mu^4 P(k) .
\] (A5)

This can be re-written in terms of the bias and the stochasticity as
\[
P_{HI}^r(k) = (b^2 + 2rb \mu^2 + f^2 \mu^4) P(k)
\] (A6)

which leads to the modified Kaiser term in eq. (5).