Off-shell OPERA neutrinos

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Abstract

In the OPERA experiment, superluminal propagation of neutrinos can occur if one of the neutrino masses is extremely small. However the effect only has appreciable amplitude at energies of order this mass and thus has negligible overlap with the multi-GeV scale of the experiment.
1 Introduction

Recently the OPERA collaboration reported a measurement of the average time taken for neutrinos (ν_µ up to % level contamination) created at CERN (CN) to arrive at the Gran Sasso Laboratory (GS) compared to the time taken travelling at the speed of light in vacuo (c). They found an early arrival time of approximately δt = 60 ns, which corresponds, at a significance of 6.0σ, to faster-than-light travel with speed (v − c)/c = 2.48 × 10^{-5} [1].

Since their announcement, a large number of papers have written that variously seek to explain it with or without new physics, question the experimental setup, or point out difficulties with new physics explanations (a selection is [2, 3, 4, 5, 6]).

Here, we work entirely within the framework of standard relativistic quantum field theory and within the Standard Model (suitable modified to allow for neutrino masses and mixing matrix). We note that effective superluminal propagation of ν_µ (both for individual events and, as we will later see, on average), is possible if the mass m of one of the mass-eigenstates is extremely small, so small in fact that space-like propagation can take place with appreciable probability even over the L = 730 km distance between CN and GS.

To see that this is possible in principle, consider a neutrino of such a mass created at CN at time x^0 = 0 and position x = 0, and arriving at GS with space-time coordinates (y^0, y) = (L − δt, L). To describe this we should use the Feynman propagator for spinors. However the neutrino beam has average energy ⟨E⟩ = 17 GeV [1]. The neutrinos are created left handed (by weak decay of mesons in the decay tunnel) and being ultra-relativistic, will stay that way to very good approximation throughout their flight. Therefore we need only the left handed component, which effectively reduces the propagator to that of a scalar particle:

\[ S(y - x) = \lim_{\epsilon \to 0} \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(y-x)}}{p^2 - m^2 + i\epsilon} \]

(Here p^µ is the four-momentum.) When the interval s := (x^0 − y^0)^2 − (x − y)^2 is negative as is measured by OPERA, i.e. is space-like, then we have that

\[ S(y - x) = \frac{im}{4\pi^2\sqrt{-s}} K_1(m\sqrt{-s}), \]

where K_1 is a modified Bessel function. For large value of its argument it decays exponen-

\footnote{We can neglect the two transverse spatial coordinates. From hereon we work in units with \( \hbar = c = 1 \), translating back as necessary.}
tially as:

\[ K_1(m\sqrt{-s}) \sim \sqrt{\frac{\pi m}{2}} \frac{1}{(-s)^{1/4}} e^{-m\sqrt{-s}}. \]  

(3)

On the other hand for small values, \( K_1 \) diverges:

\[ K_1 \sim \frac{2i}{\pi m \sqrt{-s}} \]  

(4)

We see that if we choose

\[ m \sim \frac{1}{\sqrt{-s}} \approx \frac{1}{\sqrt{2L\delta t}} = 3.8 \times 10^{-11} \text{eV}, \]  

(5)

then a substantial fraction of the neutrinos can propagate superluminally.

Of course it is not the case here that the neutrinos are really tachyonic. The effect occurs because the neutrino remains off shell. The Feynman propagator arises from the Lorentz covariant time-ordered expectation of the \( \nu_\mu \) and \( \bar{\nu}_\mu \) fields

\[
S(y - x) = \langle 0 | T \nu(x) \bar{\nu}(y) | 0 \rangle \\
= \theta(x^0 - y^0) \langle 0 | \nu(x) \bar{\nu}(y) | 0 \rangle - \theta(y^0 - x^0) \langle 0 | \bar{\nu}(y) \nu(x) | 0 \rangle.
\]  

(6)

The propagator arises as an intermediate step in a chain of reactions, for example being created at CN through the decay \( \pi^+ \rightarrow \mu^+ \nu_\mu \), and absorbed at GS through \( \nu_\mu n \rightarrow \mu^- p \).

We can interpret this as the \( \nu_\mu \) being created with energy-momentum \( p^\mu = \langle E \rangle (1, v) \). The faster-than-light propagation occurs because the field operators are not localised but have a spread of order the Compton wavelength \( 1/m \), which in our case is large enough to stretch from CN to GS.

Since we know that \( y^0 - x^0 > 0 \), i.e. that the neutrino arrives in GS after being created at CN, the first term currently plays no rôle. The potential problems with causality and Lorentz covariance arise when we view the propagation in a frame moving at speed \( v_F \) sufficiently fast that the event at GS happens before the one at CN: \( y^0 = \gamma_F (y^0 - v_F y) = \gamma_F y^0 (1 - vv_F) < 0 \) (where the dilation factor \( \gamma_F = 1/\sqrt{1 - v_F^2} \)). Since the Feynman propagator is Lorentz covariant, it still takes the same form, but in this frame it is the first term in (6) that now operates. It describes a superluminal antineutrino \( \bar{\nu}_\mu \) travelling from GS to CN. Energy and momentum conservation are preserved in the chain of reactions, and

\[ ^2 \text{It follows that the constraints noted in ref. } \]  

\[ ^2 \] do not apply: kinematically electron-positron bremsstrahlung would here require momentum transfer with constituents of rock, and is just one of many diagrams heavily suppressed by the Fermi constant and the requirement that the neutrino remains off shell.
moreover we see that the neutrino energy has also reversed sign: 
\[ E' = \gamma_F E_0 (1 - \nu v_F) < 0. \]

Therefore we have a consistent interpretation in this new frame: a \( \bar{\nu}_\mu \) is created at GS with energy \( -E' \) and momentum pointing from GS to CN, through the process \( n \rightarrow p \mu^- \bar{\nu}_\mu \), and absorbed at CN through the process \( \bar{\nu}_\mu \pi^+ \rightarrow \mu^+ \). The kinematics of these processes are allowed because the anti-neutrino’s 4-momentum is space-like.

This is of course just the standard argument due to Feynman with small adaptations to this novel context, and demonstrates that the Feynman propagator allows an interpretation consistent with Lorentz covariance. However, there still remains an issue of causality. Indeed, in the new frame it seems the neutron at GS has somehow to know to decay beforehand so that the antineutrino can later be absorbed by the pion produced at CN.

Once we have understood the process in more detail quantum mechanically, we can see how this issue with causality is resolved. We will return to this in the conclusions.

While this paper was being prepared, ref. [4] appeared, where similar ideas are proposed as an explanation for the OPERA measurement. In fact, if we adapt this observation to the setup in the OPERA experiment, we can see that the effect vanishes or at best is far too small to explain the measurement.

The reason our setup is not yet the pertinent one is that it is not true that the neutrinos are created at an exact time. By Heisenberg’s uncertainty principle, such a particle would have an infinite spread in energy. In sec. 2 we argue that the spread in energy for each neutrino’s wave packet is \( 1/\tau \sim 0.2 \) eV about its central value \( E_0 \). In the ensemble of neutrinos that make up the neutrino bunches, these central values are spread over a wide range [1, 7] but it is clear nevertheless that \( E_0 \) is set at a much larger scale (GeV to multi-GeV).

Given that each neutrino’s energy is thus sharply peaked we can already expect that the effect must be heavily suppressed. From [5], the evanescent part of the propagator appears dominated by energies of order \( \mathcal{O}(E) \) – indeed we will see that it is a manifestation of quantum mechanical tunnelling, requiring energy to be less than \( m \); this mismatch with the energies in the wavepacket \( \sim E_0 \pm 1/\tau \), ensures that any remaining effect is consigned to any small tail in the wavepacket that reaches down to these small energies. For example, if we assume that the wavepacket is a Gaussian of width \( 1/\tau \) then this supplies a suppression factor

\[
\sim \exp -\tau^2 E_0^2 \sim \exp -\tau^2 \langle E \rangle^2 \sim \frac{1}{10^{10^{10}}}. \tag{7}
\]

Alternatively, a wave packet with a lower cutoff \( > m \) on the neutrino energy would eliminate
the effect entirely. Furthermore, the space-like neutrino then has to be detected at Gran Sasso, which requires overlap of (2) with the typical wavefunction for a detected neutrino. Such a wavefunction has again an order GeV central energy and spread $\sim 1/\tau$. (At the most optimistic we can note that a lower bound is set by the lowest threshold energy for detection of $\nu_\mu$ via charge current interactions in the Gran Sasso Laboratory. This is for the process $\nu_\mu n \rightarrow \mu^- p$, thus $E_0 > m_\mu + \frac{1}{2}m_\mu^2/m_n = 112$ MeV.) This results in multiplying by another suppression factor $\sim \exp(-\tau^2 E_0^2)$ from the tail of its distribution.

One might try to resuscitate the proposal by bringing $1/\tau$ and $E_0$ closer to each other. The relevant $E_0$ could be as low as the threshold energy of 112 MeV and perhaps we have missed something in our estimate of the energy spread $1/\tau$ in the neutrino wave packet and the true spread is far larger. If we for the moment take this optimistic view, we note that we still cannot fit the data. The dominance of the very low energy scale is not seen at OPERA: they repeated the analysis concentrating on only those $\nu_\mu$ charged current events occurring in the OPERA target (where reliable energies could be measured). They split this sample into bins of nearly equal statistics, taking events with energy higher or lower than 20 GeV. With a significance of greater than 3$\sigma$, they still see a superluminal velocity in the higher energy sample of the same magnitude (within errors) [1].

Another way one might try nevertheless to use this effect to explain the OPERA measurement is to boost (1). In other words, we note that the neutrinos are neither created at an exact time nor at an exact location; in reality we need to integrate over position space terms that supply the neutrino with the appropriate ultra relativistic momentum. One could then hope that the behaviour (3) would be the correct one for directions transverse to the neutrino’s momentum, corresponding to small deficits in the energy required by the ultra relativistic on-shell energy-momentum relation. The amplitude would effectively take the form of the kilometres-wide wave function assumed in ref. [5], where the OPERA result is then explained without any adjustable parameters by the off-centre detection of these wave packets. However we will see that this set-up does not result in such transverse evanescence.

Although neutrino species oscillate into one another, we can ignore this effect in the present paper. The known mass-squared differences [8] show that only one mass eigenstate can be as light as [5]. Therefore the effect we are looking for is entirely due to evanescence
in this mass eigenstate. The only effect of mixing is the multiplicative inclusion of mixing angles at the beginning where the CN $\nu_\mu$ converts to this state, and at the end where it reverts to $\nu_\mu$, as detected in GS.

The structure of the paper is as follows. In the next section (sec. 2) we estimate the initial spread in energy of the neutrino wavefunction at CN and the effective further decoherence due to its detection at GS. In sec. 3, we construct the initial wavefunction and the amplitude for neutrinos as seen at the Gran Sasso Laboratory, and draw out the piece relevant for this effect, confirming and extending the arguments given above. Finally, in sec. 4, we present our conclusions and also argue why these results ensure there is no violation of causality.

2 Estimating coherence

We have already noted that we cannot assume that the neutrino is created at a definite time. Likewise however, it is not true that each neutrino has been created with a definite energy. Such a particle would be completely delocalised in time, so the question of when it arrives in Gran Sasso would become meaningless. We therefore have no choice but to take into account the shape of its wavepacket, at least in its gross details, since the results will depend on this.

It should be clear that here we are not discussing the energy spectrum of the ensemble of neutrinos in the beam, which is very broad, but rather the inherent quantum mechanical uncertainty in the momentum of a given neutrino when it is created at CN and any further inherent uncertainty imposed on it by being measured at GS.

We follow closely the analysis in ref. [6]. We start at CERN with a proton bunch extracted from the Super Proton Synchotron. The energy uncertainty inherent in an individual proton wavefunction can certainly can be no smaller than that set by $\Delta t \approx 5$ ns, the smallest time features in the proton bunch [1]. This corresponds to $\Delta E = 1/\Delta t \approx 1.3 \times 10^{-7}$ eV.

However, even if the proton beam is coherent at this level, it suffers decoherence on its way to becoming the $\nu_\mu$ beam. Firstly, the protons impact the graphite target, producing the mesons (mostly pions) that will decay to muon neutrinos. Initially these mesons are in a quantum state together with the other products of the collision (including various nuclei), however they then suffer decoherence from thermalisation in a hot target, both directly and
also through their quantum mechanical coupling to the decay products. Assuming a target temperature of, say 300°C, this limits the energy-momentum resolution to $k_B T \sim 0.05$ eV.

Finally the mesons decay in the decay tunnel and here further decoherence takes place, again through coupling to the decay products (in this case the muon). Consider for example the decay of a $\pi^+$ to $\mu^+ \nu_\mu$. The resulting quantum state takes the form:

$$\psi_\pi(q) \int_{\text{phase space}} M(p,q) \left| \nu_\mu(p) \right> \left| \mu^+(q-p) \right>, \quad (8)$$

where $\psi_\pi(q)$ is the wave function of the erstwhile pion, $q$ and $p$ are 3-momenta, and $M$ stands for the matrix element for the decay. The muon is absorbed by a combination of rock, a Hadron stop and two muon detectors [1, 9]. This allows the experimenters to measure the transverse coordinates of the proton beam spot when it hits the target to a precision of $\sim 50 - 90 \mu m$ [9], however it is reasonable to assume that the rock itself localises the muons at the $\mu m$ level (similar to emulsion — see below), even if this is not recorded. This corresponds to a momentum decoherence of order 0.2 eV/c. Through (8) this decoherence is transferred to the neutrino.

We conclude that the chief limiting factor on the coherence of the initial neutrino wave packet is through the decay of the mesons and results in a wave packet with an energy spread of $1/\tau \sim 0.2$ eV.

For muon neutrinos that interact in the Gran Sasso detector, the impact spot is discernible in the emulsion at the $\mu m$ level [10]; we can expect similar localisation in the rock in front of the detector for the external events. Therefore the act of measurement results in an effective energy spread in the wave packet of similar size to that in the initial packet.

3 Tunnelling to Gran Sasso

We can thus regard our neutrino as being created at position $x = 0$ at an uncertain time centred around $t = 0$, with energy localised to $E = E_0$ with an accuracy $1/\tau$. Let us model the shape as a Gaussian. Then we have for the initial wavefunction:

$$\Psi(0, t) \propto \exp \left\{ -\frac{t^2}{2\tau^2} - iE_0 t \right\}. \quad (9)$$

The amplitude to find the neutrino at Gran Sasso at time $t_2 = y^0$ is then

$$A(t_2) \propto \int_{-\infty}^{\infty} dt \Psi(0, t) S(t_2 - t, L), \quad (10)$$
where $S$ is the Feynman propagator. Since we are dealing with a situation where we know the neutrino arrives at Gran Sasso, we will choose the normalization factor so that the probability distribution reflects this:

$$\int_{-\infty}^{\infty} d t_2 |A(t_2)|^2 = 1. \quad (11)$$

We can now make a further simplification. We can drop the transverse momentum integrations in (1) since these will only result in $\sim 1/L^2$ losses that are scaled away when we normalise. Therefore, substituting (1) and using (9), we have

$$A(t_2) \propto \int_{-\infty}^{\infty} dt \Psi(0,t) \int_{-\infty}^{\infty} dE dq \exp \left\{ -iEt_2 - \tau^2 \Delta E^2/2 + iqL \right\} \frac{E^2 - q^2 - m^2 + i\epsilon}{E^2 - q^2 - m^2 + i\epsilon}, \quad (12)$$

where $\Delta E = E - E_0$, and we have performed the $t$ integration. Now we do the momentum integration. Since we know that $L > 0$, the $i\epsilon$ prescription tells us to close the contour over the top. We obtain:

$$A(t_2) = N \int_{-\infty}^{\infty} dE \ e^{-iEt_2 - \tau^2 \Delta E^2/2} \frac{e^{ipL}}{p}, \quad (13)$$

where $N$ is the normalisation constant, where $p = \sqrt{E^2 - m^2}$ has either a vanishing positive imaginary part (the $i\epsilon$) for $|E| > m$, or $p = i\sqrt{m^2 - E^2}$ for $|E| < m$.

We see that the exponential decay in (3) indeed arises from energies of order $m$, as we claimed. We also confirm that the exponential decay component is suppressed into the far tail of the probability distribution by $\sim \exp -\tau^2 E_0^2$. The amount of evanescent component thus depends crucially on the unknown shape of this tail. If we had chosen a wave packet with a sharp cutoff at some realistic minimum energy, we would eliminate the evanescent contribution completely.

For completeness we note that (13) can be evaluated by the method of steepest descents. The dominant term comes from $E \approx E_0$ giving:

$$A(t_2)|_{\text{dominant}} \approx N \frac{\sqrt{2\pi}}{p_0 \tau} \exp \left\{ \frac{1}{2\tau^2} \left( t_2 - \frac{L}{v_0} \right)^2 + i(E_0 t_2 - p_0 L) \right\}. \quad (14)$$

(The approximation is valid providing $|t_2 - L/v_0| \ll \tau^2 p_0$.) Here $p_0 = \sqrt{E_0^2 - m^2}$ is the central momentum and $v_0 = p_0/E_0$ is the classical velocity. We recognise this as nothing but the expected result of propagating the wave packet to Gran Sasso without dispersion. The evanescent part can also be evaluated. Writing $E = m(1 - z)$ for small positive $z$ at the
top boundary, the integral can again be evaluated by steepest descents. Adding to this the term from the bottom boundary $E = -m(1-z)$, and similar size pieces from $E = \pm m(1+z)$ (which can again be evaluated by steepest descents), we get for the evanescent part

$$A(t_2)_{\text{evanescent}} \approx -2iN\sqrt{\frac{2\pi}{mt_2}}e^{-\tau^2E_0^2/2}\exp\left\{\frac{mL^2}{2t_2} - mt_2 + \frac{\pi}{4}\right\}. \quad (15)$$

This approximation is valid providing $mt_2 \gg 1$. We see again the damping by the tail of the wave function. We can now carry this through to a computation of the superluminal component of velocity. The evanescent term will provide one, but we do not present the computation since we have seen that it necessarily depends on a vanishingly small unknown quantity.

Finally, we address the question raised in the introduction: whether, after taking into account appropriate spatial dependence of $\Psi$ and spatial integration in (10), so as to incorporate the fact that the neutrino has some momentum $\tilde{p}_0$ (with associated small uncertainty) which is slightly larger in magnitude than the energetically allowed $p_0 = \sqrt{E_0^2 - m^2}$, the resulting amplitude could take the form similar to (14) in the direction of $\tilde{p}_0$ but with transverse evanescent tails corresponding to imaginary transverse momenta? Recall that such an amplitude would be very similar to the wave packet envisaged in ref. [5] where results consistent with the OPERA measurement were derived as a result of off-centre measurement of these wave packets.

Clearly in order to investigate this we now need to keep the transverse momentum integrations in (1). The modified wave function and spatial integration incorporated in (10) results in a momentum integral in (12) of the form:

$$J(L) = \int d^3q \Phi(q) \frac{\exp\{iq \cdot L\}}{E^2 - q^2 - m^2 + i\epsilon}, \quad (16)$$

where $\Phi$, strongly peaked about $q = \tilde{p}_0$ incorporates the momentum dependence induced by the initial wave function.

Now we appeal to the Grimus-Stockinger theorem [11] which states that

$$J(L) = -\frac{2\pi^2}{L^3}\Phi(-pL/L)\exp\{ipL\} + O(L^{-3/2}), \quad (17)$$

if $p = \sqrt{E^2 - m^2}$ is real, and is $O(1/L^2)$ otherwise. We see that providing $E > m$ we get only the analogous dependence to (14). Once we integrate over energy in (12), the

\footnote{providing $\Phi$ satisfies the reasonable conditions that it is three-times continuously differentiable and first and second derivatives decrease at least as $1/q^2$ for $|q| \to \infty$.}
mismatch between $p_0$ and $\tilde{p}_0$ will result in suppression arising from the partial overlap of $\exp\left(-\frac{\tau^2}{2}E^2/\Delta E^2\right)\Phi\left(-pL/L\right)$, however we do not generate evanescent tails as a result of this mismatch. On the other hand, for $E < m$, $J$ decays faster in $L$, which we can associate with the evanescence. We see that even with this ‘boosted’ wave packet, it is still the case that the evanescent behaviour responsible for superluminal propagation, is restricted to the regime $E < m$.

4 Conclusions

We have seen that although superluminal propagation is possible over the 730 km from CERN to Gran Sasso if the mass of the lowest neutrino mass eigenstate is so small that it remains off shell, cf. (5), the effect does not survive projection on the relevant energy scales, being killed dead or by typically a huge suppression (7). This projection is required because one must integrate over the initial neutrino wave function which carries a very rapidly oscillating exponential, set by the multi-GeV energy of the created neutrino. One could reduce the suppression (i.e. increase the overlap with energy scale $m$) by concentrating on muon neutrinos with energies only just higher than the threshold energy of 112 MeV. This is still not enough: one would have to argue that the quantum-mechanical energy uncertainty in the neutrino wave packet was not of order 0.2 eV as we have argued but – implausibly – tens or hundreds of MeV. Even if one does this, the effect is still concentrated at energies of order $m$, resulting in the wrong energy dependence.

One could reduce the time for which the neutrino has to remain off shell by considering cases where it propagates on shell to a point near the detector and, due to interaction with the rock, is then kicked off shell for the remaining part of its journey. Note that an off-shell neutrino can be passed from this point to the detector in principle almost instantaneously. In this way we achieve the smallest negative interval $s$ from the instantaneous jump $(0, \delta t)$ from $(L-\delta t, L-\delta t)$ to the detector. In (5) we then get the larger mass $m \sim 1/\delta t \sim 10^{-8}$ eV [4]. This is still far too small to bridge the gap to the energy scales set by the experiment.

Although the OPERA measurements cannot be explained by assuming very low mass off-shell neutrinos, it is still the case that such neutrinos can propagate superluminally. How can this be reconciled with causality, in particular why does this not lead to faster-than-light communication? Note that we have shown that these neutrinos must carry energy $E$ less than $m$. Leaving aside practical issues involved in detecting such neutrinos, we note
that even in principle, a detector for such neutrinos would have to be restricted to using wavelengths larger than or of order their Compton wavelength \( \lambda = 1/m \), otherwise the observation itself would disturb the system too much – for example by pair creating the very neutrinos it was trying to measure. Thus we see that the restriction to \( E < m \) for superluminal neutrinos, which comes out from the detailed analysis, is in fact necessary to ensure that they cannot be detected with sufficient spatial resolution to allow faster than light signals.

Acknowledgments

The author thanks the STFC for financial support and Konstantin Kuzmin and Bartolome Alles Salom for useful conversations.

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