Improved mathematical model of particle trajectories in multimodal electrostatic separators

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Abstract. The multimodal electrostatic separator has been designed to facilitate the feasibility assessment of corona-, tribo-, or purely-electrostatic processes of selective sorting of the constituents of granular mixtures. The versatility of the separator has imposed the development of an equally-agile simulation program for modelling the particle behavior in each of the operational configurations of the machine. Thus, the paper discusses the numerical models that describe the various specific physical phenomena: corona-charging at the surface of the belt conveyor; induction charging of the conductive particles in contact with the grounded electrode; impact of particles with the rotating-roll high-voltage electrode. An example of conductive and insulating particles trajectories computation is given. The simulated results are compared with the experimental observations.

1. Introduction
The multimodal electrostatic separator has many advantages: robustness, low energy consumption, and high sorting efficiency, due to the fact that it enables the use of several mechanisms of electrical charging [1] – [4]: triboelectric effect, corona discharge and electrostatic induction. Nevertheless, the multitude of operational modes imposes a rigorous numerical modelling of the movement of particles in this separator [3], [8], taking into account several interrelated electrical and mechanical phenomena. The aim of this study is to introduce and validate such a numerical model, which takes into account various charging phenomena, such as corona-charging and electrostatic induction, and the different forces that act on charged or polarized particles in an electric field. The results of the numerical simulations performed with this model for the case of conductive particles of various sizes are presented and compared to the experimental observations made by high-speed camera.

2. Experimental set-up
The active zone of the novel multimodal separator, designed by the PPRIME Institute, and manufactured by CITF, Saint Cybardeaux, France, is presented in Figure 1. The mixed granular materials to be separated are deposited on the surface of a metal conveyor, with a length of 700 mm and a width of 70 mm, which also serves as grounded electrode. The active electrode of the multimodal separator is a stainless steel rotating cylinder (diameter 12 cm), connected to a high voltage supply (up to 30 kV). The conveyor and the rotating electrode are driven by 120 W electric motors, controlled by a variable speed drives. An electromagnetic vibratory feeder is employed to disperse the granular mixture as a monolayer of particles at the surface of the conveyor electrode. The choice of charger will depend mainly on the electrical properties of the materials to be separated. The distinction between these materials will be made with respect to their conductivity which defines their ability to keep the charge acquired by tribo-electric effect, by corona discharge or by electrostatic induction (Table 1). For each case, a specific charging device will be adapted to the separator.
Figure 1. Photograph of the multimodal metal-belt-conveyor electrode electrostatic separator for mixed granular materials; 1: metal-belt conveyor; 2: cylindrical rotating electrode; 3: electric motor.

Table 1. Different charging modes depending on the type of mixture

| Type of mixture       | Charging mechanism                          | Devices adapted to the separator                          |
|-----------------------|---------------------------------------------|-----------------------------------------------------------|
| Insulator / Insulator | tribo-electric effect [1],[5] and electrostatic induction | tribo-electric charger (fluidized bed, or vibratory plate or rotating cylinder) |
| Insulator/Conductive  | corona discharge and electrostatic induction [6],[7] | wire-type corona electrode connected to the high-voltage supply |
| Conductive/Conductive | electrostatic induction                     | rotating cylindrical electrode connected to the high-voltage supply |

Figure 2. Conductive particles used in the experiments (nickel-coated polypropylene spheres)

3. Charging processes

Particle charging varies depending on the shape and position of the particle in the electric field [11]. The formulas given below correspond to the case of spherical conductive particles which were the object of the simulation and experimentation work presented in this paper (Figure 2).

3.1 Corona-charging

The maximum charge of a spherical particle in contact with a plane electrode, immersed in a mono ionized electric field [10] is

\[ Q = 12\pi\varepsilon_0 E_0 R^2 \]  

(1)

where \( \varepsilon_0 = 8.854 \times 10^{-12} \) F/m; \( E_0 \): mono ionized electric field; \( R \): radius of the particle.

3.2 Electrostatic induction

A spherical particle in contact with a grounded plane, in a uniform electric field, acquires a charge of opposite sign to the active electrode and expressed as [9]:

\[ Q = \left(\frac{2}{3}\right)\pi^2\varepsilon_0 E_0 R^2 \]  

(2)

The charged particle is attracted to the high voltage electrode.
4. Modelling and numerical simulation of conductive particles trajectories

The conductive particles are charged by electrostatic induction and introduced by the conveyer into the electric field zone. The movement of the particles in this zone can be described by the equation:

$$m \frac{d\vec{v}}{dt} = \sum \vec{F}$$

(3)

where \( m \) and \( \vec{v} \) represent respectively the mass and the velocity of the particle and \( \sum \vec{F} \) is the sum of the forces acting on the moving particle.

The intensity \( E \) of the electrostatic field in the separation zone can then be calculated from:

$$\vec{E} = -\nabla \vec{V}$$

(4)

In this study, the solution of equation (4) is obtained using software specialized in solving partial differential equations by the finite element method (COMSOL). The distribution of the electric field between the high-voltage electrode and the grounded conveyer can be examined in Figure 3.

**Figure 3.** Graphic representation of the results of COMSOL computation of the electric field \( E \) in the active zone of the electrostatic separator.

During their movements in the electrostatic separation zone, the charged particles are subjected to the force of gravity, the electrostatic force, and the centrifugal force. The gravity force exerted on a particle of mass \( m \) is:

$$\vec{F}_g = \begin{bmatrix} 0 \\ mg \end{bmatrix}$$

(5)

where \( m \) represents the mass of the particle and \( g = -9.81 \) (m/s²) is the gravitational acceleration.

The air drag force is:

$$\vec{F}_d = \begin{bmatrix} \frac{1}{2} \rho \pi r_p^2 v_x \\ \frac{1}{2} \rho \pi r_p^2 v_y \end{bmatrix}$$

(6)

where \( v_x \) and \( v_y \) are the \( x \) and \( y \) component of particle velocity \( \vec{v} \); \( r_p \): radius of the spherical particle; \( C_f \): friction coefficient with air; \( \rho \): mass density of the particle.

The centrifugal force tends to detach the particle from the surface of the conveyer, as well as from the surface of the high-voltage rotating roll electrode:

$$\vec{F}_c = m \omega_j^2 r_j n_j$$

(7)

where \( j = c \) for the conveyer or \( j = e \) for the rotating roll; \( m \) is the particle mass (kg), \( \omega_j \) is the angular speed (rad/s) and \( r_j \) is the radius (m). The vector \( n_j \) represents the exterior normal vector on surfaces.

The electrostatic force applied to a charged particle in the inter-electrode region can be assessed by:

$$\vec{F}_e = Q \begin{bmatrix} E_x(x, y) \\ E_y(x, y) \end{bmatrix}$$

(8)

where \( Q \) is the charge of the particle and \( E_x (x, y) \); \( E_y (x, y) \) components of the electric field vector.
The conductive particles are charged by electrostatic induction under the effect of the field \( \vec{E} \) created by the rotating electrode; their charges take a sign opposite to that of the high voltage electrode. As soon as they enter the separation zone, their charges increase and the electrostatic force becomes larger than the forces holding them to the conveyor (image and gravitational forces). Before the lift off, due to the image force attraction, the electrostatic force is [11]:

\[
F_e = 0.832 \times Q \times \vec{E}
\]  

(9)

Once the particle is lifted, \( Q \) remains unchanged, and the electrostatic force is expressed as (8). If an impact with rotating electrode occurs, the charge will change and take an opposite sign (the sign of the rotating electrode).

5. Results and discussion

In this study, MATLAB code is used to simulate the behavior of conductive particles in uniform electric field. The simulation program takes into account several physical phenomena, such as forces described above. The computed trajectories are compared to experimental observations, obtained with a high-speed camera (SpeedSense VEO 340) the image acquisition frequency of which was set at 300 Hz. The trajectories displayed on Figure 4 show that smaller (lighter) particle take-off from the conveyor before the larger (heavier) one.

![Figure 4. Simulation and real trajectories for two spherical conductive particles (diameter, 2 mm, and mass, 0.005 g; diameter, 4 mm, and mass, 0.045 g), under the following conditions: high voltage, 24 kV; distance between the electrodes, 27 mm; position angle of the rotating electrode with respect to the vertical plane passing through the axis of the entraining drum of the conveyor, 32°); the take-off points are designated by a triangle.](image-url)
The lighter particle, under the action of an electrostatic force stronger than the gravitational force, will be attracted to the high-voltage electrode and collide with it. At the moment of impact, the particle will get charged by contact. As a consequence, its charge changes the sign and will be repulsed towards the conveyor. At the impact with the grounded electrode, the particle will be recharged by electrostatic induction, and attracted again towards the rotating high-voltage electrode. The particle will bounce several times between the two electrodes, as its charge will take an opposite sign and a new value after each impact.

The heavier particle is also attracted to the high-voltage electrode of opposite polarity, but it will not impact it. The simulation trajectories are similar to the real ones, which means that the numerical model is accurate enough.

6. Conclusions
The behaviour of charged particles in a multimodal electrostatic separator requires an advanced numerical model that describes their trajectories, taking into consideration several physical phenomena and the possible impact with the electrodes.

The numerical simulation of the trajectories gives the possibility of controlling the movements of the particles according to several parameters, including their size.

The experimental observation of the trajectories in the multimodal electrostatic separator using a high-speed camera validates the effectiveness of the numerical model employed for the simulation.

Acknowledgments
Part of this work was also sponsored by a FEDER-CPER program ELECTRINOV financed by European Union and Nouvelle Aquitaine Regional Council.

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