Evaluation of measurement uncertainty for a high-precision angle comparator with a vacuum preloaded structure

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Abstract

Angle measurement is widely used in various fields of science and technology. With the development of technology, the measurement uncertainty of angle metrology becomes more and more demanding. To achieve high-precision calibration of a high-precision angle comparator with a sub-arc-second level, a method of angle position error calibration and measurement uncertainty evaluation based on no material reference was established. Firstly, the structure of the angle comparator with a vacuum preloaded air bearing driven by an ultrasonic motor drive and the construction of the whole calibration system are briefly introduced. Then, the basic principle, algorithm and error separation principle of angle calibration based on the optical angle measurement method were studied, and the test process is introduced. Finally, the measurement results and error sources were analyzed, the measurement uncertainty model was established and the measurement uncertainty was evaluated. The experimental results show that the high-precision angle comparator with a vacuum preloaded structure has an angle position error of 0.12" and a measurement uncertainty of 0.05" (\(k = 2\)). Through many experiments, it is shown that the measurement system has a stable high-precision calibration capability with a sub-arc-second level for circular division artifacts.

Keywords: angle calibration, complete closure method, error analysis, uncertainty evaluation

(Some figures may appear in colour only in the online journal)
1. Introduction

The angle is one of the most basic geometric quantities and is used in almost all fields of science and technology. With the development of science and technology, especially the development of nanometer processing and measurement technology, angle measurement technology is more and more widely used in various fields, such as industry, scientific research and national defense. Over the past 20 years, the accuracy of angle measurement has reached ten times more than that of the past. The measurement and calibration of the sub-arc-second angle is of great significance to the development of ultra-precision machining and equipment, precision detection instruments, optical engineering, basic physics, astronomy, aerospace, etc [1–4].

In the development and production of high-end equipment, the measurement accuracy of an angle faces higher and higher requirements, and has the tendency to extend from the second level to the sub-arc-second level. The angle calibration objects include an ultra-precision turntable, precision dividing head, precision dividing table, industrial robot joints and other devices, among which the dividing turntable for ultra-precision machine tools and the angle reference turntable for high-precision testing instruments have reached the requirements for accurate measurement of angular positioning at the sub-arc-second level [5–8]. The verticality of the orthogonal axis of ultra-precision machine tools, testing instruments and other high-end process equipment, as well as the inspection and correction requirements of high-precision circular grating have also reached the sub-arc-second level.

With the development of science and technology, traditional mechanical angle measurement and calibration tools, such as measuring blocks, measuring sticks and dividing disks, have long been unable to meet the needs of modern science and technology industry development. Instead, a new angle comparator [9, 10] combined with advanced multi-disciplinary technologies, such as optics, mechanics, electronics and computers, has appeared. In recent years, national metrology institutes, including the Scientific and Technological Research Council of Turkey (TUBITAK) [11], the National Institute of Standards and Technology (NIST) [12], Physikalisch-Technische Bundesanstalt (PTB) [13], Istituto Nazionale di Ricerca Metrologica (INRIM) [14], the National Metrology Institute of Japan (AIST) [15] and the National Metrology Institute of Korea (KRISS) [16], have conducted in-depth research on the high-precision sub-second angle comparator and used it as the national standard for international angle comparison [17]. The angle measurement level of the developed countries in Europe, especially PTB, has been at the forefront of the world, obtaining 0.001″ high angle measurement uncertainty at a certain angle range through a combination of the most advanced ultra-precision turntable and shear calibration method for the second-level angle measurement [18]. Although the uncertainty is still at the laboratory research stage, is not really practical and will be implemented only within a small range, it still represents the highest level of angle measurement ability in the world. In addition, the circle indexing error can also reach 0.1″. Developed countries in Asia, such as Japan and South Korea, are currently following the research direction of Europe. In the research and development of an ultra-precision turntable for sub-arc-second angle measurement, they have made certain progress and have also developed angle grating calibration methods, such as Self-A.

There are many methods to detect the circular indexing error, which differ in preciseness, accuracy, measurement difficulty and calculation work. At present, the commonly used methods of circle indexing error detection in the world can be divided into two categories: the normal angle method and the comparison method [19, 20]. The internationally recognized complete closure method based on the circle closure principle is the most precise and accurate, but its algorithm is complex, and its detection and calculation work is heavy. In addition, there are many factors, such as internal design, external factors [21], eccentricity error and rotary accuracy [22], affecting the measurement uncertainty in the measurement process. To achieve higher detection accuracy, it is extremely demanding on the environment. Therefore, it is difficult to obtain an angular calibration accuracy with a very low measurement uncertainty, and it is necessary to fully grasp the main sources of measurement errors and precisely control the errors [23, 24].

In this paper, a novel design based on a thrust bearing with a vacuum preloaded structure has been applied to an angle comparator for a simpler system with sufficient stability and precision. To evaluate the measuring ability of the angle comparator, an angle calibration system based on an autocollimator and an indexing table is built. This system analyzed the measurement algorithm of the circle indexing error based on no material reference through the complete closure and least square method, and discussed the influence of the testing environment, instrument and equipment installation error on the measurement results and the suppression method. Through the evaluation of measurement uncertainty, it is proved that the angle comparator has high testing ability.

2. Measurement system

2.1. Angle comparator

Figure 1 shows a schematic diagram of the angle comparator. Its internal structure is mainly composed of an air-bearing casing, an air-bearing rotor, a working table, a ceramic ring, four ultrasonic motors, a divided circle, two reading heads and a support.

To obtain stable and high-precision positioning accuracy, a simple supporting system with higher stiffness was designed. The supporting system is composed of a thrust air bearing and a radial air bearing. Among them, the thrust air bearing (figure 2) is preloaded with vacuum to stabilize the air clearance and improve the stiffness. The outstanding advantage of using a vacuum preloaded structure is to reduce the error transfer and error accumulation of the supporting system’s mechanical assembly. At the same time, the supporting system has no drift, reduces the number of parts, improves the dynamic characteristics of the system and ensures the precise rotation.
of the shafting. The experimental results show that the axial
stiffness of the turntable is 400 N µm⁻¹, the maximum load is
80 kg and the rotary accuracy is better than 50 nm.

The driving control system is composed of a ceramic ring,
four ultrasonic motors, a divided circle and two reading heads.
Four ultrasonic motors are uniformly distributed around the
ceramic ring, and the working table is driven by friction to
achieve nano-level movement. Figure 3 is a block diagram
of the angle comparator control system, which is composed
of a SPiiPlus universal driver interface (UDhp4422N01N,
ACS™), a computer installed with real-time motion control
software (SPiiPlusSC HP0800004BAY5A, ACS™), a motor
driver (AB2, Nanomotion™), four ultrasonic motors (HR2,
Nanomotion™) and a dual readout head encoder (ERP880,
Heidenhain™). Four HR2 ultrasonic motors, which have a
theoretical resolution of 10 nm (with an AB2 driver), are
connected in parallel to synchronize the movement of each
motor. And the ultrasonic motors drive the working table in
the tangential direction with the friction force between the
motor driving heads and ceramic ring. A multi-mode control
strategy based on a linear extended state observer is proposed,
through the control mode, the position data of the two reading
heads are averaged to improve the feedback accuracy, and the
fast angle positioning and high stable accuracy is obtained.
The sampling rate and update rate of the servo loop are both
20 kHz. The Heidenhain ERP880 round grating used by the
rotary table includes 180000 1 Vpp signal periods in a 360°
range, and through the digital subdivision of 2¹², a control
resolution of about 0.0018″ can be obtained.

2.2. Construction of the measurement system

The basic working principle of the system: the aim of the
angle measurement is to compare the measured angle with the
reference angle to accurately measure the angle deviation data
between the two angles, and then process the data and remove
the measurement error through a certain algorithm.

In addition to the self-developed angle comparator, the
instrument constituting the hardware platform of the meas-
urement system also includes an indexing table, an autocol-
limator, an optical polygon, a clamp, an adjustable plate and
a granite base, etc. The technical parameters of the main
devices of the measurement system are shown in table 1. The
installation of the main devices of the measurement system
is shown in figure 4. The indexing table, optical polygon and
autocollimator are installed on the granite base, and a protec-
tive cover is installed to prevent the influence on the detec-
tion results from vibration, airflow and light. The axis of the
optical polygon should be parallel to the axis of the rotary
center of the indexing table. The axis of the clamp (optical
polygon center) should coincide with the rotary center of the
indexing table to control the deviation range. The autocol-
limator is adjusted so as to make the center of the parallel
beam coincide with that of the optical polygon working face,
(2)\[ \delta \alpha_i = \sum_{k=1}^{n} (\delta \alpha_i - \delta \beta_k) \]

(3)\[ \sum_{k=1}^{n} \delta \beta_k = 0, \quad i = 1, \ldots, n; \sum_{i=1}^{n} \delta \alpha_i = 0, \quad k = 1, \ldots, n. \]

According to the specific situation of the calibrated instrument, suppose the measurement step number \( n \) is pre-selected, then the basic indexing angle is \( 2\pi/n \) and the number of the observation is also \( n \), the normal angle sequence is

\[ \varphi_i = \frac{2\pi}{n}, \quad i = 1, 2, 3, \ldots, n. \]  

(1)

After the mutual comparison of the working angle between the indexing table and the angle comparator, \( n \) deviation measurement results are obtained

\[ \theta_i = \alpha_i - \beta_i = (\varphi_i + \delta \alpha_i) - (\varphi_i + \delta \beta_i) = \delta \alpha_i - \delta \beta_i \]  

(2)

here, \( \alpha_i \) is the angle of the indexing table; \( \beta_i \) is the angle of the angle comparator; \( \delta \alpha_i \) is the deviation of angle \( \alpha_i \) to the constant angle; and \( \delta \beta_i \) is the deviation of angle \( \beta_i \) to the constant angle.

After \( n \) times of dislocation comparison, \( n^2 \) deviation measurement results can be obtained. For the evaluation of the measurement results, through the method of arranging and comparing each other, the errors of the indexing table and angle comparator are separated, respectively, and the separated errors only represent the deviations of the indexing table and angle comparator at the fixed angle position. The key of this method is the complete closure principle; that is, the circle closure error is zero. There is

\[ \sum_{k=1}^{n} \delta \beta_k = 0, \quad i = 1, \ldots, n; \sum_{i=1}^{n} \delta \alpha_i = 0, \quad k = 1, \ldots, n. \]

It can be seen from the above analysis that the algorithm can eliminate the influence of the reference angle error, can greatly improve the accuracy of the test and can achieve the circular indexing error of the indexing table and angle comparator at the same time. In addition, to obtain a higher accuracy, it is theoretically necessary to increase \( n \), i.e. 'the multiple number of the observation for the small constant angle'. At this time, the measurement cost also increases to a quadratic power, and the measurement times, time and data processing will increase significantly.

3. Process

The experiment object is a high-precision angle comparator. A complete set of the system is placed in a granite base instrument installation, as shown in figure 4. The rotary center axes of the optical polygon and indexing table are parallel to that of the angle comparator. The axis of the clamp coincides...
Figure 5. A schematic diagram of the circular dividing error detection.

Figure 6. The experimental process.

Table 2. The experiment parameters.

| Parameters                      | Symbol | Specification |
|---------------------------------|--------|---------------|
| Measuring interval             | $\alpha$ | 30°           |
| Number of observations         | $n$    | 12            |
| Acquisition time               | $T$    | 10 s          |
| Sampling frequency             | $f$    | 1 s$^{-1}$    |
| Permissible value of closure   | $\theta$ | 0.1″          |
| error                           |        |               |
| Permissible value of standard  | $\sigma$ | 0.05″         |
| deviation                       |        |               |

Figure 7. The position error of the angle comparator.
with the rotary center of the indexing table, and the deviation is no more than 0.02 mm.

The test method adopts the whole combination arrangement comparison method. The test flow is shown in figure 6, and the experiment parameters are shown in table 2.

4. Results and discussion

4.1. Calibration results

The angle comparator (designed precision: 0.2′) was calibrated by the established calibration system, and the possible influencing factors were strictly controlled during the test, including temperature fluctuation (<0.3 °C), humidity fluctuation (±1%), the flatness error of the reflecting surface (better than 50 nm), tower difference (<35′), etc. A total of 12 groups of 144 data were obtained in the experiment. After data processing, the calibration results of the angle comparator were obtained (figure 7), and it can be seen that the angle position error reached 0.12′.

To evaluate the long-term running stability of the measurement system under certain environmental conditions, ten repeated tests were conducted in 5 d. The test results are shown in figure 8. It can be seen from the figure that the average angle position error of the angle comparator is 0.18′, and the maximum value of the standard deviation of the angle position repeatability is only 0.035′, indicating that the measurement system has the stability to meet the long-term running
In addition, the repeatability can also be used to verify the reliability of subsequent measurement uncertainty evaluation, because the repeatability is due to a variety of measurement errors in the measurement process.

4.2. Environmental monitoring results

In the process of calibration, the errors caused by the autocolimator were mainly composed of two aspects, namely the value error and the quantization error (resolution), which were mainly affected by environmental factors. In the case of general precision measurement, the calibration value given by the calibration certificate is usable; however, to obtain higher uncertain measurement results, it is necessary to monitor the measurement environment.

For screening and verifying the environmental influence on the autocolimator, a continuous monitoring acquisition test method was adopted, which conducted a synchronous monitoring test on the temperature and humidity of the test environment, jitter and drift of the autocolimation. The sampling interval of the test environment was 10 min, and the sampling interval of the autocolimation was 10 s. The monitoring test curves, as shown in figures 9 and 10, are obtained.

Figure 9 shows the monitoring results of ambient temperature and humidity. It can be seen from the figure that during the test, the temperature showed an upward trend. Over 2 h, the temperature rose by 0.3 °C and remained stable at 20.3 °C for 50 min. The humidity showed an oscillating trend and was maintained within (51 ± 1)% for 2 h. The comprehensive environment was good, and there was no obvious interference from environmental factors that greatly affected the measurement results.

Figure 10 shows the monitoring results of the autocolimation. It can be seen from the figure that the indicated value of the autocolimator can remain within ±0.01″ for 40 min, ±0.02″ for 70 min and no more than 0.06″ drifting for 120 min. One test is about 15 min, and after each test, the autocolimator will be cleared, so that the autocolimator error caused by environmental stability is less than 0.02″. This not only proves the stability of the environment, but also proves the stability of the autocolimator’s error. The stability of the laboratory conditions ensures the stability of the system; once the conditions change, the measurement ability of the system will be decreased.

5. Evaluation of measurement uncertainty

5.1. Modeling

The angle position error can be expressed as

\[
\Delta = \max \left( \frac{1}{n} \sum_{i=1}^{n} \alpha_{ij} - \frac{1}{n} \sum_{i=1}^{n} \alpha_{ij} \right) - \min \left( \frac{1}{n} \sum_{i=1}^{n} \alpha_{ij} - \frac{1}{n} \sum_{i=1}^{n} \alpha_{ij} \right)/n
\]

(4)

where, \( n \) is the number of the measurement series; \( i \) is the position number of the indexing table; \( j \) is the position number of the angle comparator; and \( \alpha_{ij} \) is the deviation obtained by comparison at the positions \( i \) and \( j \).

Therefore, the uncertainty model can be expressed as

\[
u_{c} = \sqrt{v^{2}(A) + u^{2}(A)} = \sqrt{\sum_{i=1}^{m} c_{i}^{2}u_{i}^{2}}
\]

(5)

here, \( u_{i} \) is the standard uncertainty component; and the sensitivity coefficient \( c_{i} = c(A) = \sqrt{n-1}/n \).
(1) The error of the autocollimator mainly consists of two parts: the indication error and the quantification error. Between them, the uncertainty caused by the indication error can be evaluated by class B. The indication error given by the calibration certificate is calculated according to the assumption of uniform distribution. The uncertainty caused by the quantization error can also be evaluated by class B, and the standard uncertainty component can be calculated according to the resolution of the autocollimator and the same assumption is subject to uniform distribution. The uncertainty caused by the autocollimator can be calculated by considering that the two components are not related to each other.

(2) The repeatability of the measured instruments is an important factor that influences the measurement accuracy. Its influence degree depends on the measurement procedures, personnel, equipment, environment, etc. Under the conditions of the consistent procedures, e.g., personnel, environment, most of the influence is determined by the characteristics of the instruments. Therefore, in a measurement process, the above factors should be as consistent as possible. The repeatability of the standard angle measuring instrument is usually verified by a double observation method, and the measuring interval is distributed \( n \) positions in the whole circumference. The repeatability is

\[
u_2 = s_r = \sqrt{\frac{\sum_{i=1}^{n} (\alpha_i - \alpha'_i)^2}{2n}}
\]

(6)

here, \( \alpha_i \) is the measured value of the \( i \)th adjacent working angle in the first circle; and \( \alpha'_i \) is the measurement value of the \( i \)th adjacent working angle in the second circle.

(3) The autocollimator needs to be used together with the reflector. When the autocollimator is aligned with different reflecting positions, the flatness error of the reflecting surface of the reflector will affect the angle measurement. The principle is shown in figure 11. Assume that the flatness of the middle area of the working surface of the polyhedral edges is ideal to be an arc, then

\[R = \frac{4H^2 + H'^2}{8h} \approx \frac{H'^2}{8h}\]

(7)

here, \( R \) is the radius of the curvature of the working surface; \( H \) is the width of the working surface; and \( h \) is the flatness error of the working face.

If the optical axis of the autocollimator deviates from the center of the working face at \( \delta H \), then the angular error of the measurement interval will be \( \delta \alpha \), i.e.

\[\delta \alpha = \frac{\delta H}{R} \rho = \frac{8h\delta H}{H^2} \rho \]

(8)

here, \( \rho = 206\ 265'' \).

From the above analysis, it can be seen that reducing the optical axis deviation of the autocollimator from the working face at \( \delta H \) and the working face flatness error \( h \) will reduce the impact on the measurement results, and it is more convenient to reduce \( \delta H \) than to reduce \( h \).

(4) The principle of uncertainty caused by the non-perpendicularity of the reflection surface to the base plane is shown in figure 12. When there are the installation of tilt or the position error, the error of perpendicularity generated by the mirror reflector to the installation support is \( \varphi \), the optical axis of illumination parallel to the supporting surface is equivalent to the \( a'b'ML \) reflector that forms an \( \varphi \) angle with the \( a'b'ML \) plane and the measured angle will be \( \varphi' \). Therefore, the working angle error generated by the verticality error is

\[\delta \alpha = \sin \alpha \frac{\sin^2 \varphi}{\cos \varphi} \rho \]

(9)

here, \( \alpha \) is the measurement interval; \( \varphi \) is the verticality error (tower difference) between the edge working face and the positioning face; and \( \rho = 206\ 265''/\text{rad} \).

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**Table 3. The uncertainty budget.**

| \( u_i \) | Source of uncertainty | Parameters/(\%) | Type | Distribution | Standard uncertainty/(\%) | \( c_i \) | \( v_i \) |
|---|---|---|---|---|---|---|
| \( u_1 \) | Indication error of autocollimator | \( \pm 0.05 \) | B | U | 0.029 | 0.035 | 50 | 88 |
| \( u_2 \) | Quantification error of autocollimator | 0.05 | B | U | 0.02 | 50 |
| \( u_3 \) | Repeatability of angle comparator | 0.02 | A | N | 0.02 | 0.276 | 23 |
| \( u_4 \) | Flatness error of reflecting surface | 50 nm | B | U | 0.035 | 50 |
| \( u_5 \) | Tower difference | 35 | B | U | 0.002 | 50 |
| \( u_6 \) | Closure error | 0.1 | B | U | 0.058 | 50 |

Uncertainty of synthetic standard measurement: \( u_c = 0.022'' \)

Extended uncertainty: \( U = 0.05'' \) \((k = 2)\)
According to the simulation calculation (figure 13), the measurement error can be reduced by controlling the tower difference and reducing the measurement step. At the measuring step of 30”, the tower difference is controlled within 30” and the influence is less than 0.003”.

(5) Closure error is caused by the change in the ambient temperature and self-collimating instrument drift. In the whole combined test process, the error back to zero is strictly controlled below 0.1”, and the standard uncertainty can be obtained by assuming that the error follows the rectangular distribution.

5.3. Uncertainty calculation

The calculation of uncertainty components can be carried out according to the evaluation criteria of the uncertainty and in combination with the analysis of the influencing factors. The calculation results, as shown in table 3, are presented without going into detail here.

6. Conclusions

This paper introduced an angle comparator with high precision, which was composed of vacuum preloaded air floating support, an ultrasonic motor friction drive and feedback control of a double readout head encoder. It had the advantages of high resolution, high precision and simple structure etc, and could build an angle calibration system to obtain the testing ability of the angle comparator. Moreover, the error elimination principle of the circle indexing error measurement algorithm based on no material reference was systematically analyzed, and the high-precision circle indexing error calculation model was obtained. Finally, for the calibration of the angle comparator and the precision index to be achieved, the elimination principle of the circle indexing error measurement process were discussed, and the corresponding suppression method was developed. By fully analyzing the error factors affecting the measurement uncertainty, the error control of the measurement process was limited, which provided theoretical support for ensuring high-precision measurement and further optimization. The angle position error of the angle comparator was calibrated through the constructed calibration system, the angle position error of the self-developed angle comparator reached 0.12” and the measurement uncertainty was only 0.05” (k = 2). Even in long-term operation, the angle comparator could still maintain an average angle position error of 0.18”, while the maximum of the repetitive standard deviation of the angle position was only 0.035”, 70% of the U, indicating that the evaluation of the measurement uncertainty was credible and the measurement system had reliable operation stability. And it could provide calibration accuracy with a sub-arc-second level for circular division artifacts.

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