Using the Magnitude-Squared Coherence for Determining Order-Chaos Transition in a System Governed by Logistic Equation Dynamics

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Abstract

This paper is devoted to show the results obtained by using the magnitude-squared coherence for determining order-chaos transition in a system described by the logistic equation dynamics. For determining the power spectral density of a chaotic finite-duration discrete-time sequence the Welch average periodogram method was used. This method has the advantage that can be applied to any stationary signal by using the discrete Fourier transform (DFT) representation of a discrete-time series which allows an effective computation via fast Fourier transform (FFT) algorithm, and that can be applied to a discrete-time series shorter than that required by nonlinear dynamical analysis methods. The estimate of the Inverse Average Magnitude-Squared

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Coherence Index (IAMSCI) for each discrete-time series in the set obtained from the logistic mapping was calculated. The control parameter, \( r \), ranges in the interval \([2.8,4]\) for producing the discrete-time series set. When the condition \( r \geq 3.57 \) is satisfied, each discrete-time series exhibited a positive value for IAMSCI estimate, indicating a high level of coherence loss of the signal and corresponding to a chaotic behavior of the dynamical system. Its effectiveness was demonstrated by comparing the results with those obtained by calculating the largest Lyapounov exponent of the time series set obtained from the logistic equation.
I. INTRODUCTION

Nonlinear dynamical methods for analyzing a discrete-time series have been widely used in several fields of scientific research. For characterizing a scalar discrete-time series metric tools of nonlinear dynamical methods, such as dimensions, exponents, and entropies are calculated. Applying these tools to embed a discrete-time series in a phase space to obtain a phase-space portrait of the attractor of the dynamical system is required. In order to make the discrete-time series embedding defining the two embedding parameters which are the embedding dimension, $D_e$, and the time delay, $\tau$, is required. Several methods have been proposed for determining adequate values of these parameters [4,22,13,10,7,15,11]. When the dimensionality, $m$, of the dynamical system is known, it is directly considered that $D_e = m$, but it is not the case when we only know a scalar discrete-time series obtained by measuring one of the accessible variables of the state space of a dynamical system. The embedding dimension is usually estimate in accordance with Takens theorem, i.e., $D_e > 2 * D_2$, where $D_2$ is the correlation dimension of the attractor in the reconstructed phase space and it is required to be also estimate before $D_e$ be known. After determining the two embedding parameters, $D_e$ and $\tau$, one proceeds to calculate estimates of the metric tools. Two important requirements must be considered at this point. One of these is related to the stationarity of the discrete-time series and the other one is related to the length of the data point sequence. Nonlinear dynamical methods applying requires that a discrete-time series to be analyzed be a stationary free-noise sequence during the observation time. For calculating the correlation dimension, $D_2$, and the largest Lyapounov exponent, $\lambda_1$, Eckmann and Ruelle [7] have suggested a certain minimal value for the length of the experimental data points sequence in order for the results obtained by applying nonlinear dynamical analysis be reliable. It is recognized by several authors that the application of nonlinear dynamical methods implies a high computational cost. In this sense, therefore, any method that allows to obtain any information about the behavior of a dynamical system and that can operate over a relatively short discrete-time series, acquires an important practical value.
For complementing those methods of nonlinear dynamical analysis we shall now try to demonstrate that introducing the concept of magnitude-squared coherence is possible to quantifying the order-chaos transition in a dynamical system described by logistic mapping dynamics. This method is based on the discrete Fourier transform representation of a finite-duration discrete-time series and has the advantage that can be applied to a relatively shorter discrete-time series than that required by nonlinear dynamical methods. The magnitude-squared coherence has been used by several authors for measuring constancy of phase between two or more signals at one frequency [23,18,24,8,20,16]. Besides, according to literature the transition from a regular to a chaotic behavior has been studied in many dynamical systems, including the biological type. One of these is a fluid flow, where by calculating nonlinear dynamical parameters the order-chaos transition can be determined.

Van den Bleek and Schouten [4] determined the order-chaos transition in a fluidized bed by calculating both correlation dimension, $D_2$, and Kolmogorov entropy, $K$, as a function of the Reynolds number and the superficial gas velocity, indicating two different regimes, order and chaos, and the intermediate region. In a previous work [9] the DFT representation for qualitatively describing the order-chaos transition in a dynamical system is used. Now we want to make the description of order-chaos transition in a dynamical system governed by the logistic equation dynamics by applying the Welch average periodogram method for calculating the power spectral density and from this, to introduce an inverse average magnitude-squared coherence index for indicating quantitatively order-chaos transition in the dynamical system indicated.

The application of magnitude-squared coherence (MSC) method is reported by several authors. NV Thakor et al [23] presented the power spectral density analysis of electrocardiogram (ECG) waveforms as well as isolated QRS complexes and episodes of noise and artifacts. The power spectral analysis showed that the QRS complex could be separated
from other interfering signals. A bandpass filter that maximizes the signal-to-noise ratio would be of use in many monitoring instruments. They calculated the coherence function and, from that, the signal-to-noise ratio. Upon carrying out this analysis on experimentally obtained ECG data, they observed that a bandpass filter with a center frequency of 17 Hz and a Q factor of 5 yields the best signal-to-noise ratio. They estimated the coherence function as given by

$$C_{xy}^2 = \frac{|G_{xy}(f)G_{xy}(f)^*|^2}{G_{xx}(f)G_{yy}(f)}$$  \hspace{1cm} (1)$$

where $G_{xx}(f)$ is the estimates of the auto-power spectra of the QRS complex, $G_{yy}(f)$ is the complete ECG cycle, and $G_{xy}(f)$ is the cross power spectrum estimate. For calculating the power spectra they applied the Fourier transform of the windowing signal. To improve the resolution of the FFT algorithm and the power spectra, they filled the original data sets with zeros and formed new 1024 point data sets. S Narayanaswamy et al [18] developed three signal processing tools to identify three postulated mechanisms of isolated or single premature ventricular contraction (PVC) generation. Two hours of continuous ECG recording were digitally obtained from several patients with frequent PVC. They applied the magnitude-squared coherence (MSC) spectrum between the sinus intervals $x[n]$ and PVC intervals $y[n]$ as given by

$$MSC(f) = \frac{|S_x(f)S_y(f)^*|}{\sqrt{S_{xx}(f)G_{yy}(f)}}$$  \hspace{1cm} (2)$$

where $S_x(f)$ and $S_y(f)$ are the Fourier spectra of $x[n]$ and $y[n]$ respectively. $S_y(f)^*$ is the complex conjugate of $S_y(f)$, and $S_{xx}(f)$ and $S_{yy}(f)$ are the auto-power spectra of $x[n]$ and $y[n]$ respectively. The power spectral density of the PVC interval series is defined as

$$P(k) = |X(k)|^2$$  \hspace{1cm} (3)$$

where $X(k)$ is the Fourier transform of $x[n]$. In this work, details of the used method are not showed, but they used both the power spectral density of the PVC signal and the MSC estimates of the PVC and sinus intervals.
EG Lovett y KM Ropela [17] showed that magnitude-squared coherence (MSC) of intracardiac electrogram may be used to quantify what may be termed organization. Unfortunately, MSC computations are not meaningful from the surface due to the high correlation among measured body surface potentials. They showed that average magnitude-squared auto-bi-coherence (AMSABC) provides an organizational measure of cardiac rhythms from a single surface lead. They showed that AMSABC also discriminates three ventricular tachyarrhythmia. In this work, rather than measuring consistency of phase between two signals at one frequency (as in MSC), measuring consistency between two frequencies in one signal (as in MSABC) were considered. Haris J. Sih et al [20] applied the MSC function for analyzing the spatial organization of epicardial mapping in the frequency domain. They defined the magnitude-squared coherence (MSC) as a frequency domain measure of the phase consistency between two signals, \( x[n] \) y \( y[n] \), as given by

\[
MSC_{xy}(f) = \frac{|S_{xy}(f)|^2}{S_{xx}(f).S_{yy}(f)}
\]  

(4)

where \( S_{xy}(f) \) is the cross-power spectrum, and \( S_{xx}(f) \) and \( S_{yy}(f) \) are the respective auto-power spectra, MSC is dimensionless and can vary from 0 to 1. A MSC value of 1 at some frequency would indicate a linear relationship or perfect coherence between the two signals at that frequency, while a value of 0 would indicate no relationship or perfect incoherence between the two signals at that frequency. MSC is sensitive to noise and interference, whereas uncorrelated noise in either of the two signals decreases MSC, and correlated interference increases MSC. While the relationship between MSC and correlated interference is difficult to characterize, the influence of noise on MSC can be derived for two real signals, \( u(t) \) and \( v(t) \), equal to ideal signals, \( x(t) \) and \( y(t) \), plus random, independent, zero-mean noise processes, \( n_1(t) \) and \( n_2(t) \). It is easy to show that

\[
MSC_{uv} = \frac{MSC_{xy}(f)}{1 + \frac{1}{SNR_x(f)} + \frac{1}{SNR_y(f)} + \frac{1}{SNR_x(f)SNR_y(f)}}
\]  

(5)

where \( MSC_{uv} \) is the magnitude-squared coherence between \( u(t) \) and \( v(t) \), \( MSC_{xy} \) is the magnitude-squared coherence between \( x(t) \) and \( y(t) \), \( SNR_x(f) \) is the frequency-dependent
signal-to-noise ratio between \( x(t) \) and \( n_1(t) \), and \( SNR_y(f) \) is the frequency-dependent signal-to-noise ratio between \( y(t) \) and \( n_2(t) \). As expected, as the signal-to-noise ratios increase, \( MSC_{uv} \) approaches to \( MSC_{xy} \). \( MSC \) can be estimated by replacing \( S_{xy}(f) \), \( S_{xy}(f) \), and \( S_{xy}(f) \) with respective estimates by using the Carter method. This method uses average Fast Fourier Transform of weighted overlapped segments of equal length from the two signals to form the spectral estimates. That is, \( MSC \) is estimated as given by

\[
MSC_{xy}(f) = \frac{|\sum_{i=1}^{N} X_i(f)Y_i^*(f)|^2}{|\sum_{i=1}^{N} X_i(f)X_i^*(f)\sum_{i=1}^{N} Y_i(f)Y_i^*(f)|}
\]  

(6)

where \( X_i(f) \) and \( Y_i(f) \) are the FFT of the \( i \)-th weighted segments from \( x(t) \) and \( y(t) \), respectively, \( N \) is the total number of segments, and \((*)\) denoted the complex conjugate. From this method of estimation, one can interpret \( MSC \) as a measure of the phase consistency between the two signals. At a single frequency \( f_1 \), the cross-power term in the numerator for one segment can be interpreted as a vector with amplitude \(|X_i(f_1)||Y_i(f_1)|\) and phase \( \angle X_i(f_1) - \angle Y_i(f_1) \). Therefore, the sum of the cross-power terms in the numerator of the \( MSC \) estimate is equivalent to the vector sum of the cross-power vectors for each segment. If \( \angle X_i(f_1) - \angle Y_i(f_1) = \angle X_j(f_1) - \angle Y_j(f_1) \) for all \( i \) and \( j \), that is, if a constant phase relationship from one segment to next exists between \( x(t) \) and \( y(t) \) at \( f_1 \), the \( N \) vector terms are all aligned and the vector sum is at a maximum, as is the \( MSC \) estimate. If the terms are not constant from segment to segment, the \( N \) vector terms do not align, the vector sum in the numerator is not at a maximum, and the \( MSC \) estimate must be less than unity at that frequency. As with all methods of spectrum estimation, the \( MSC \) estimate has statistical bias and variance, which have been explicitly calculated for the case of Gaussian stationary signals. Another form of bias, known as a bias due to misalignment, has also been shown to exist for this estimator. As an example, for two signals, \( x(t) \) and \( y(t) = b * x(t-D) + n(t) \), where \( n(t) \) is uncorrelated noise, it has been demonstrated that the estimate of the true \( MSC \) spectrum will be degraded by a factor of \((1 - |D|/T)^2\) where \( T \) is the time duration of one windowed segment of data. For example, if \( T \) were approximately 0.25s, a relative delay of 0.1s between the two signals could decrease the \( MSC \) estimate to one-third the
true value. It is noted that these results are derived for two linearly related signals, and the effects of this bias due to misalignment on two nonlinearly related signals are yet to be determined. With various types of mapping data, as those of cardiology, of neurology, of geophysics, etc, multiple signals are recorded simultaneously, and often, it is necessary to preserve the physical location of these recordings relative to the underlying activity. Under this conditions, \( n \) signals, \( x_1(t), x_2(t), \ldots, x_n(t) \), are arranged in a \( 1-\), \( 2-\), or \( D- \) array, or map. By choosing some meaningful reference signal, \( X_r(t) \), which can be one of the \( n \) signals in the map, a corresponding map of MSC spectra can be formed. That is, what we have is a \( n-MSC \) spectra set: \( MSC_{x_1x_r}(f), MSC_{x_2x_r}(f), \ldots, MSC_{x_nx_r}(f) \), arranged in the same \( 1-, 2-, \) or \( D- \) map. In the case where the reference is one of the \( n \) signals in the map, the \( r- \)th MSC spectrum, \( MSC_{x_rx_r}(f) \), is unity for all frequencies. It is noted that the map of MSC spectra may indicate how the \( n \) signals relate to the reference, but they do not imply any relationship between one signal of the \( n \) signals with any other of the \( n \) signals. Different maps of MSC spectra can be generated using different references. A logical and concise method for displaying these MSC spectra is problematic as pointed out by many authors. To simplify the data, one can reduce each MSC spectrum to a single number, for example, by averaging MSC over frequency.

Finally, we refer to the work done by D Liberati et al [16], where the joint use of total and partial coherence between pairs of EEGs simultaneously recorded in a standard set is shown to enhance what is caused direct correlation between cortical subsystems and what is instead related to the spread of the electromagnetic field. A multi-variable AR approach is employed in the computation, giving results even for a very short time window, thus allowing coherence to be investigated at the main cortical latencies of evoked potentials. In particular, when a combined visual and somatosensory stimulation is applied, cortical interactions are captured in the frequency domain. In this work, the total squared coherence \( k \), as a function of the frequency \( f \) is defined, for every couple \( j, h \) of the channels, as the squared cross-spectrum \( P_{jh} \), normalized to the product of the norm of each auto-spectrum \( P_{jj} \) and \( P_{hh} \) as given by
\[ k_{jh}(f) = \frac{|P_{jh}(f)|^2}{|P_{jj}(f)| \cdot |P_{hh}(f)|} \quad (7) \]

where \( k_{jh} \) is a real function, ranging from 0 to 1, indicating the amount of correlation between the signals \( j \) and \( h \) as a function of the frequency \( f \). In this work we pretend to demonstrate that it is possible to apply the average Welch periodogram for estimating the power spectral density of a chaotic signal and to calculate the magnitude-squared coherence for determining quantitatively the order-chaos transition in a dynamical system described by the logistic mapping dynamics. The fundamental advantage of this method upon the others is that it can operate on a relatively shorter discrete-time series. By estimating the largest Lyapounov exponent was able to evaluate those results obtained here applying an inverse average magnitude-squared coherence index, defined by averaging \( MSC \) estimate over the frequency.

II. METHODS AND MATERIALS

A. Obtaining the discrete-time series set

The logistic equation is an one-dimensional mapping of the real axis in the interval \((0,1)\) to itself, that is a prototype of a nonlinear dynamical system widely used [6] and can be formulated as

\[ x[n+1] = rx[n](1 - x[n]), \quad (8) \]

and it can also be formulated as

\[ x[n+1] = r^n(x[1] - \sum_{k=0}^{n} r^{-k+1}x[k+1]); \quad (9) \]

and with the initial condition \( x[1] \) specified, one can obtained a time sequence of a given length by a recursive action. The character of the behavior of a data sequence obtained with the iterative process can be modified by conveniently selecting a value of the control parameter \( r \). In this work a value of the initial condition \( x[1] = 0.65 \) was taken, and the
interval \( r \in [2.8, 4.0] \) was selected. A set of evenly spaced values of \( r \), with a step \( \delta = 0.01 \), was taken to produce a family of 120 discrete-time series, each containing \( N = 2^{10} \) samples. The building up of the histogram for each time series allows to have immediately a simple statistical characterization of this time series. In order to make easier further computations data points were organized into a rectangular matrix containing 120 columns, each of which is a time series with the length \( N = 1024 \) data points.

B. Determining the largest Lyapounov exponent \( \lambda_1 \)

Lyapounov exponents spectrum deals with average exponential divergence or convergence of two neighbor orbits in the phase space. It is common to order the spectrum from largest to shortest exponents: \( \lambda_1, \lambda_2, \lambda_3, \ldots \). A system containing one or more positive exponents is to be defined as chaotic. For calculating the Lyapounov exponents spectrum some algorithms have been developed, according to the particular situation, being one of the most used algorithms that reported by Wolf et al [25]. From the exponents spectrum, the largest exponent, \( \lambda_1 \), decides the behavior of the dynamical system. For calculating the largest exponent the algorithm proposed by Rosenstein et al [19] can be referred too. Details of these algorithm can be found in referred papers. We determined the largest Lyapounov exponent, \( \lambda_1 \), for each time series in the set, using a professional software [21]. The selection of the time delay, \( \tau \), and the embedding dimension, \( D_e \), required for reconstructing the phase space is part of the problem and it is necessary to consider some selecting criteria. This selection can be made by different methods [4,22,13,10,7,15,11]. For selecting \( \tau \) we may take into account: (a) optimal filling of the phase space [5], (b) by determining the position of the first local minimum of the data autocorrelation function [1], (c) by taking the first minimum in the mean mutual information plot [12], (d) based on the optimal tradeoff between redundance and the irrelevance [19], and by analyzing the differential equations of the system if they are given [26]. For selecting \( D_e \) it may be indicated: (a) by applying the Grassberger-Procaccia method, which allows to obtain simultaneously both correlation
dimension and embedding dimension [13], (b) by applying the false nearest neighbors method [19,3], and (c) by applying geometric considerations [15]. In this work, a time series obtained from the logistic equation was analyzed by using the method (a) for \( \tau \) determination and by taking a value \( D_e = 3 \), which satisfies the Grassberger-Procaccia algorithm condition \( D_e > 2D_2 \), and Takens theory [13,22], being \( D_2 = 0.5 \) a value reported by Hoyer et al [14] for this time series type. Literature refers [13,22] that for selecting the embedding dimension as

\[
D_e \geq D_2 + 1, \tag{10}
\]

it is sufficient to have a good representation for the attractor in the phase space, reconstructed in delayed coordinates from a scalar time series observed. Any reconstruction made with a value of the embedding dimension less than the minimal value will produce a projection, in that dimension, of the original attractor, and therefore it will be more difficult to understand that projection or it will produce false results when an estimation of a nonlinear dynamical parameter is to be made. It is important to take a value for embedding dimension close to \( 2D_2 + 1 \) in order to avoid the computational cost be excessively high.

C. Estimating the IAMSCI using the Welch average periodogram

It is highly meaningful when dealing with a chaotic system keeping in mind that local sensitivity to a small error is the hallmark of such a system. Such dynamical system as the one described by the logistic equation produces a discrete-time non-chaotic series when the control parameter \( r \) value satisfies the condition \( r < 3.57 \). For such a condition a discrete-time series generated by the recursive process does not almost change if the initial condition is modified and there exists phase consistency between the two discrete-time series at a given frequency. However, when the control parameter value is greater than 3.57, a discrete-time series generated by the logistic equation does meaningfully change if the initial condition is slightly modified and phase consistency between the two discrete-time series at
a given frequency, in general, must be lost. This effect may be quantified by applying an inverse average magnitude-squared coherence index. In order to apply the Welch average periodogram method for determining the power density spectrum $S_x[k]$ of a discrete-time series one follows the following steps:

(a) Decompose the sequence of $N (= 1024)$ data points in $M (= 9)$ segments, each one with the same length $L (= 256)$, and which may be overlapped $P$ samples. The usual value $P = 0.625 \times L$ was chosen.

(b) Calculate the DFT representation by FFT algorithm for each segment as given by $X_m[k] = \text{fft}(x_m[n])$, where $m = 1, 2, \ldots, M$, and $n = 0, 1, 2, \ldots, L - 1$, and fft represents a function in MatLab for calculating the DFT of a given signal. For each segment a L-FFT was calculated, being L the number of points in the sequence $X_m[k]$.

(c) Calculate the periodogram for each segment $m$ as given by

$$P_m[k] = \frac{1}{L} |X_m[k]|^2, \quad m = 1, 2, \ldots, M \quad (11)$$

(d) Finally, calculate the average periodogram as given by

$$S_x[k] = \frac{1}{M} \sum_{m=1}^{M} P_m[k] \quad (12)$$

where $k = 0, 1, 2, \ldots, L - 1$. For a short discrete-time series a rectangular window is recommended. For determining the magnitude-squared coherence sequence the procedure described above is applied to a second signal $y[n]$ obtaining the average periodogram $S_y[k]$, and the average cross-periodogram $S_{xy}[k]$ is also calculated. The magnitude-squared coherence sequence was calculated as given by

$$MSC_{xy}[k] = \frac{S_{xy}[k] \ast S_{xy}^*[k]}{S_x[k] \ast S_y[k]} \quad (13)$$

The $MSC_{xy}[k]$ sequence describes the phase consistency between the two signals at a given frequency $k$. The mean value of the sequence, $\langle MSC_{xy}[k] \rangle$, may be used as an index
of global coherence between the two signals. When dealing with a chaotic discrete-time series this index must tend to zero, and when dealing with a deterministic discrete-time series this index must tend toward one. However, it was found that a better index may be defined as given by

\[ f = 10 \times \log_{10} \frac{1}{\langle MSC_{xy}[k] \rangle}. \]  \hspace{1cm} (14)

It makes easier to compare the results obtained here with those obtained by applying the largest Lyapounov exponent estimate.

III. RESULTS

Figures, from 1 to 4, show a sample of four representative time series and their corresponding magnitude-squared coherence spectra. In order to give a better view of each time series it was only considered 128 data points of each discrete-time series in its plotting, but it was completely considered when its corresponding magnitude-squared coherence was calculated. In either case, the particular values of \( r \) parameter were indicated. Figure 5 depicts results of plotting the inverse average magnitude-squared coherence \( \left( 1/\langle MSC_{xy}[k] \rangle \right) \), calculated for each time series in the set, as a function of the control parameter \( r \). And figure 6 shows the results of plotting the inverse average magnitude-squared coherence index, in decibels \( (f = 10 \times \log_{10}(1/\langle MSC_{xy}[k] \rangle)) \), calculated for each time series in the set, as a function of the control parameter \( r \) too. Figure 7 shows results of plotting the largest Lyapounov exponent estimate, \( \lambda_1 \), calculated for each time series in the set, as a function of the control parameter \( r \). This reference serves as a control for the discussion of the results obtained by applying the inverse average magnitude-squared coherence index method.

IV. DISCUSSION OF THE RESULTS

The application of the DFT representation of a chaotic-type signal gives satisfactory results for determining the order-chaos transition in a system described by the logistic equation
dynamics [9]. A critical value for control parameter $r$ is reported in literature beyond which a sequence produced by the logistic equation exhibits a chaotic behavior. This threshold value approaches $r \approx 3.57$. On the other hand, figures from 1 to 4 show the time domain representation of some of the time series and their corresponding magnitude-squared coherence spectrum. Comparing figures 1 ($r = 2.91$) with 2 ($r = 3.64$), 3 ($r = 3.75$), and 4 ($r = 3.84$), it can be deduced the bifurcation effect and the transition to chaos. Note that in figures 2, 3 and 4 the control parameter satisfies the condition $r > 3.57$, and it corresponds to a value for a positive largest Lyapounov exponent, as it can be estimated from figure 7. It may be observed in those cases that the magnitude-squared coherence spectrum exhibits values far away from one for several values of the frequency in the MSC spectrum. It makes the average magnitude-squared coherence to be less than one, corresponding to a situation for which the discrete-time series becomes incoherent and indicating in this case a chaotic behavior of the dynamical system. This situation can be observed in figures 5 and 6 where either the quantity $(1/⟨MSC_{xy}[k]⟩)$ as the inverse average MSC index, $(f = 10 \times log_{10}(1/⟨MSC_{xy}[k]⟩))$, are greater than 1 and 0, respectively, for $r > 3.57$. Applying the inverse average magnitude-squared coherence index (IAMSCI) to a discrete-time series in a family derived from an observable in a dynamical system is recommended for quantitative detecting the order-chaos transition, and this can be added to the metric tools of the nonlinear dynamical analysis for complementing. Any method that allows to evaluate the behavior of a dynamical system, and that can be applied to a relatively shorter discrete-time series, must be taken into account as an important practical method.

V. CONCLUSIONS

The application of the inverse average magnitude-squared coherence index (IAMSCI), $f$, to a discrete-time series obtained from the logistic equation can be done on a chaotic discrete-time series. This discrete-time series has the property that exhibits a high level of loss of coherence, and it can be detected by applying the IAMSCI method. This method
does not exert a strong requirement on the length of the experimental data sequence. If one deals with a nonlinear dynamical system which can modify its behavior between order and chaos, a time series produced by that system reflects this change, and the IAMSCI method applying to the discrete-time series allow to determine this change. Any method that permits some quantitative evaluation about the behavior of a dynamical system and that can work over a relatively short time series acquires a practical importance. Besides, this method does not require the phase space reconstruction of the attractor.

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VI. FIGURES
FIG. 1. Discrete-time series and its magnitude-squared coherence, for $r=2.91$

FIG. 2. Discrete-time series and its magnitude-squared coherence, for $r=3.64$
FIG. 3. Discrete-time series and its magnitude-squared coherence, for $r=3.75$

FIG. 4. Discrete-time series and its magnitude-squared coherence, for $r=3.84$
FIG. 5. Inverse average magnitude-squared coherence index

FIG. 6. Inverse average magnitude-squared coherence index, in dB
FIG. 7. Largest Lyapounov versus control parameter $r$
Discrete-time series, $r=2.91$

Magnitude-squared coherence
Discrete-time series, $r=3.64$

Magnitude-squared coherence
Discrete-time series, $r=3.75$

Magnitude-squared coherence
Discrete-time series, $r=3.84$

Magnitude-squared coherence
Control parameter, $r$

Inverse average msc index
Control parameter, $r$

Inverse average msc index, in dB
Control parameter, $r$

Largest Lyapunov exponent, LLE

Experimental value

Behaviour