A Cascade PI-SMC Method for Matrix Converter-Fed BDFIM Drives

Hui Wang, Xida Chen, Graduate Student Member, IEEE, Xun Zhao, Hanbing Dan, Mei Su, Member, IEEE, Yao Sun, Member, IEEE, Fan Zhang, Marco Rivera, Senior Member, IEEE, and Patrick Wheeler, Fellow, IEEE

Abstract—This article proposes a cascade proportional-integral second-order sliding mode control (PI-SMC) method for matrix converter-fed doubly fed induction machine (BDFIM). The PI-SMC method consists of a PI speed controller with an inner-loop second-order sliding mode control (SMC). The PI controller provides a reference to the inner-loop second-order SMC with constraints according to the system requirements in terms of maximum current and speed. As the discontinuous sign function in the traditional SMC is replaced by the continuous supertwisting function, the chattering problem is eliminated for inner-loop second-order SMC. Moreover, the inner-loop second-order SMC has robustness and interference immunity against nonlinear disturbances of the BDFIM. Simulation and experimental results demonstrate the excellent robust tracking performance of the proposed PI-SMC method.

Index Terms—Brushless doubly fed induction machine (BDFIM), cascade proportional-integral (PI)-sliding mode control (SMC) method, matrix converter.

I. INTRODUCTION

THE brushless doubly fed induction machine (BDFIM) [1]–[3] is a type of ac induction machine which has been developed from two cascaded asynchronous motors. Compared to the doubly fed induction machine (DFIM), the BDFIM inherits the advantages of the doubly fed machine and has higher reliability as it cancels the slip rings or brushes. Moreover, the BDFIM has a wider range of speed regulation. Therefore, the BDFIM has received extensive attention from researchers [4], [5] and has a broad range of potential applications in the field of variable-speed drive applications [6] and wind power generation [7].

As a promising alternative to the traditional back-to-back converter, matrix converter is employed for the BDFIM drive system in wind power generation application [8], [9]. Since matrix converter [10]–[12] can realize direct ac–ac conversion without using an intermediate dc link, this feature of matrix converter will bring several advantages to the matrix converter-fed BDFIM drive systems, such as high-power density and long lifetime.

Many control strategies have been proposed for BDFIM drive system, such as scalar control [6], [13], [14], direct torque control (DTC) [15], [16], model predictive control (MPC) [5], [9], and vector control [7], [8], [17]–[21]. Several scalar control methods have been proposed for the BDFIM system, such as open-loop control [13], closed-loop frequency control [14], and phase angle control [6]. However, the scalar control relies on the static analysis of the BDFIM, which makes it only fit for industrial applications with low-performance requirements. In [15] and [16], the DTC method for the brushless doubly fed induction generator (BDFIG) system is proposed. The proper switching vector is chosen by analyzing the effects of different switching vectors on flux and torque. The DTC method has a simple structure and fast dynamic response. However, the torque fluctuation is a disadvantage of this technique. The MPC method has advantages such as conceptual simplicity and easiness of implementation, fast dynamic response, easy inclusion of nonlinearities and constraints of the system, and the flexibility to include other system requirements in the controller [22]. In [9], a predictive torque control strategy has been proposed for controlling the flux and torque of a matrix converter-fed BDFIG. The system model is used to predict the system variable and evaluate the switching vector. The optimal switching vector with the minimum cost function value is applied during the next sample period. However, the unmatched model parameters will lead to inaccurate prediction of the motor behavior and deteriorate the performance and stability of the MPC method. The vector control methods based on different flux orientation frames are proposed [8], [17]–[19], [21]. Boger et al. [23] developed the dynamic vector model of the BDFIM with an arbitrary pole number referred to the rotor’s shaft position. The BDFIM

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model is built-in power winding (PW) and control winding (CW) synchronous reference frames related to each pole-pair distribution. The vector control technique implementation is complex for the rotor flux-oriented control method in [17]. The combined magnetizing flux-oriented control for the cascaded DFIM is studied in [18]. The rotor current vector is oriented into an artificial magnetizing flux reference frame based on the two stator currents measurements. However, this control method does not lead to natural decoupled active and reactive power control. Thus, the PW flux-oriented vector control strategy [19] is proposed based on a unified reference-frame model in [24]. It is simple to be implemented. Meanwhile, the natural decoupled active and reactive power control is obtained. Thus, the PW flux-oriented vector control method is usually adopted [8], [21]. In [8], the cascaded double closed-loops control method is adopted for a matrix converter-fed BDFIM system, including one fast inner current control loop and one outer flux/speed control loop. However, the inner current loop plant of the BDFIM has a complex nonlinear structure and is susceptible to various disturbances. Thus, the robust controller is required for the inner-loop.

Since the sliding mode control (SMC) [25] has the advantages of easy decoupling, disturbance rejection, and parameter variation insensitivity, it has been applied as an alternative robust solution for the induction motor (IM) drive applications [26], [27]. For traditional back-to-back inverter-fed BDFIM drive system, the integral SMC method is employed in the outer-loop speed controller [20]. However, the traditional SMC has the phenomenon of “chattering” because the switch does not have the ideal switching characteristics, which will affect the control effect of the system to a certain extent. Therefore, a second-order SMC based on supertwisting is considered a more suitable method for practical applications due to its continuity and robustness. The second-order SMC eliminates the chattering problem without affecting the tracking accuracy and robustness.

To obtain a high robust tracking performance, this article proposes a cascade proportional-integral (PI)-second-order SMC for the matrix converter-fed BDFIM drive system. In this method, the outer-loop PI controller tracks the reference rotor speed, and the inner-loop SMC controller regulates the CW current. The main advantages of this method are that the outer loop controller provides constraints on the SMC through maximum current and speed limits and the second-order SMC controller achieves fast and fixed convergence times for the current control loop. Meanwhile, the parameter variations of the BDFIM are considered in the design of the inner-loop SMC controller. The proposed PI-SMC method in this article has been described in [28], more theoretical details and experimental results are added in this article.

The rest of this article is organized as follows. In Section II, the mathematical model and operation principle of matrix converter are described first, and the mathematical model of BDFIM drives is presented. Section III shows the SMC of the matrix converter-fed BDFIM. Then in Sections IV and V, the simulation and experiment for matrix converter-fed BDFIM drives are carried out to verify the proposed PI-SMC method. Finally, Section VI concludes the findings.

II. MATHEMATICAL MODEL OF THE MATRIX CONVERTER-FED BDFIM

A. Mathematical Model and Operation Principle of the Matrix Converter

The basic structure of the matrix converter-fed BDFIM is shown in Fig. 1. The relationship between the output and the input voltage of the matrix converter is shown as follows:

$$
\begin{bmatrix}
  v_{oa} \\
  v_{ob} \\
  v_{oc}
\end{bmatrix} =
\begin{bmatrix}
  S_{ka} & S_{kb} & S_{kc} \\
  S_{ka} & S_{kb} & S_{kc} \\
  S_{ka} & S_{kb} & S_{kc}
\end{bmatrix}
\begin{bmatrix}
  v_{ia} \\
  v_{ib} \\
  v_{ic}
\end{bmatrix}
$$

(1)

where $v_{oa}(y \in \{a, b, c\})v_{ob}(y \in \{a, b, c\})v_{oc}(y \in \{a, b, c\})$ is the output voltage of the matrix converter and $v_{ia}(y \in \{a, b, c\})v_{ib}(y \in \{a, b, c\})v_{ic}(y \in \{a, b, c\})$ is the input voltage of the matrix converter. Each group of switching state needs to meet $S_{xa} + S_{xb} + S_{xc} = 1$ $S_{xa} + S_{xb} + S_{xc} = 1$, $(x \in \{A, B, C\})$. The modulation method based on mathematical construction [29] is used in this article.

The switching functions can be replaced by the corresponding duty cycles as the switching frequency is much higher than the input–output frequencies. Thus, (1) can be written as follows:

$$
\begin{bmatrix}
  v_{oa} \\
  v_{ob} \\
  v_{oc}
\end{bmatrix} = M
\begin{bmatrix}
  v_{ia} \\
  v_{ib} \\
  v_{ic}
\end{bmatrix}
$$

(2)

where $M$ is the duty cycle matrix and is expressed as

$$
M = 
\begin{bmatrix}
  m_{11} & m_{12} & m_{13} \\
  m_{21} & m_{22} & m_{23} \\
  m_{31} & m_{32} & m_{33}
\end{bmatrix}
= 
\begin{bmatrix}
  m_{11}' + x + D m_{12}' + y + D m_{13}' + z + D \\
  m_{21}' + x + D m_{22}' + y + D m_{23}' + z + D \\
  m_{31}' + x + D m_{32}' + y + D m_{33}' + z + D
\end{bmatrix}.
$$

(3)
Define the transition modulation matrix \( M'' \) as follows:

\[
M'' = \begin{bmatrix}
m'_{11} & m'_{12} & m'_{13} \\
m'_{21} & m'_{22} & m'_{23} \\
m'_{31} & m'_{32} & m'_{33}
\end{bmatrix}
\]

\[
= K \begin{bmatrix}
\cos(\omega_0 t) & \cos(\omega_0 t - \theta_1) & \cos(\omega_0 t - \theta_1 - 2\pi/3) \\
\cos(\omega_0 t - 2\pi/3) & \cos(\omega_0 t - \theta_1 - 2\pi/3) & \cos(\omega_0 t - \theta_1 + 2\pi/3) \\
\cos(\omega_0 t + 2\pi/3) & \cos(\omega_0 t - \theta_1 + 2\pi/3) & \cos(\omega_0 t - \theta_1)
\end{bmatrix}^T
\]

where \( \omega_0, \theta_1, \) and \( K \) represent input angular frequency, input power factor angle, output angular frequency, and modulation index, respectively. Besides, to meet nonnegative constraints of the duty cycles, \( x, y, z \) needs to satisfy the following equation:

\[
\begin{align*}
x & \geq -\min(m''_{11}, m''_{21}, m''_{31}) \\
y & \geq -\min(m''_{12}, m''_{22}, m''_{32}) \\
z & \geq -\min(m''_{13}, m''_{23}, m''_{33})
\end{align*}
\]

(5)

To make the algorithm as simple as possible, the boundary values in (5) are usually adopted. At the same time, the modulation matrix \( M \) also needs to satisfy that the sum of elements in each row is 1. Therefore, an offset \( D \) is superimposed on each row in \( M \), and \( D \) can be selected as \( D = [1 - (x + y + z)] / 3 \), \( D \geq 0 \).

The mathematical construction-based modulation method can obtain the duty cycles of nine switches directly in the matrix converter, which is simple and can achieve the maximum voltage transfer ratio of 0.866.

**B. Mathematical Model of the BDFIM Drives**

The BDFIM consists of three windings: PW, CW, and rotor winding (RW). The power stator side is usually directly connected with the grid side voltage, so the rotating speed of the stator magnetic field on the power side is known and remains unchanged. Therefore, the \( d-q \) reference frame based on the PW flux orientation is selected. The voltage and current variables of PW, CW, and RW are represented as \( x_p, x_c, \) and \( x_r \), respectively. These vectors are transformed from their reference frames to the \( d-q \) reference frame using the following corresponding equations:

\[
\begin{align*}
x_p'^d \ &= \ e^{-j\omega_0 f_p t} x_p^{abp} \\
x_p'^q \ &= \ e^{-j(\omega_p - (p_p + p_c)) t} x_p^{abp} \\
x_p'^r \ &= \ e^{-j(\omega_r - p_r t)} x_p^{abp}
\end{align*}
\]

(6)

where \( \omega_p \) and \( \omega_c \) are the angular frequencies of PW and CW, respectively; \( p_p \) and \( p_c \) are pole pairs of PW and CW, respectively; \( \omega_r \) is the angular speed of the rotor. To simplify the expressions, the vector superscripts \( dq \) is removed.

Since the rotor resistance is small to be ignored and the rotor flux is nearly zero, using the synchronous rotating coordinate system of power stator as reference coordinate system, the simplified mathematical model of the BDFIM [30]
is derived as follows:

\[
\begin{align*}
\psi_p & = R_p i_p + \frac{d\psi_p}{dt} + j\omega_p \psi_p \\
\psi_r & = R_r i_r + \frac{d\psi_r}{dt} - j\omega_p \psi_r \\
0 & = R_c i_c + \frac{d\psi_c}{dt} - j\omega_r \psi_c \\
\psi_r & = L_r i_r + M_p i_p + M_i i_c \\
\psi_p & = L_p i_p + M_p i_c \\
\psi_c & = L_c i_c + M_c i_r \\
T_e & = \frac{3v_{pq} p_c}{2(L_r L_p - M^2_p)} M_p M_c \psi_{pm} \psi_{cm} + \frac{3L_r v_{pq}^2}{2(L_r L_p - M^2_p)} \omega_p \\
J \frac{d\omega_r}{dt} & = T_e - T_L - B \omega_r
\end{align*}
\]

(7)

where \( v_p, v_c, i_p, i_c, i_r, \psi_p, \psi_c, \psi_r \), are the PW stator voltage vector, CW stator voltage vector, PW stator current vector, CW stator current vector, rotor current vector, PW stator flux vector, CW stator flux vector and rotor flux vector in the \( d-q \) rotating coordinate system; \( R_p, R_c, R_r, L_p, L_c, \) and \( L_r \) are the PW stator resistance, CW stator resistance, rotor resistance, PW stator inductance, CW stator inductance, and rotor inductance; \( M_p \) is the coupling inductance between the PW stator and RW; \( M_c \) is the coupling inductance between the CW stator and RW; \( T_e \) is the electromagnetic torque; \( j \) is the conjugate symbol; \( \Im \) is a symbol to obtain the imaginary part of vector; \( J \) and \( B \) are the moments of inertia and friction coefficient; \( T_L \) is the load torque; \( Q_{pw} \) is the reactive power. When the rotor resistance and the rotor flux are not considered, according to (7), the voltage equation of the BDFIM can be expressed as follows:

\[
\begin{align*}
\frac{di_{cd}}{dt} & = -\frac{\sigma_1 R_c i_{cd}}{\sigma_2} + \frac{\sigma_1}{\sigma_2} v_{cd} - \omega_c i_{cq} \\
\frac{di_{cq}}{dt} & = -\frac{\sigma_1 R_c i_{cq}}{\sigma_2} + \frac{\sigma_1}{\sigma_2} v_{cq} + \omega_c i_{cd} + D_1
\end{align*}
\]

(8)

where \( \psi_{pm} \) is the amplitude of the PW stator flux vector, \( \sigma_1 = L_p L_r - M^2_p, \sigma_2 = L_c L_p L_r - M^2_p L_c, \) and \( D_1 = M_p M_c \psi_{pm}/\sigma_2 \).

**III. SMC OF THE MATRIX CONVERTER-FED BDFIM**

The SMC diagram for the matrix converter-fed BDFIM is shown in Fig. 2. The control method adopts a double closed-loop control structure. The outer loops use the conventional PI control strategy to track the reactive power of the PW and the reference rotor speed. The output of the outer loop controller is the reference stator current of the inner loop control. To improve the robustness and interference immunity ability of the nonlinear inner-loop dynamics, a second-order SMC method is introduced as the current loop to obtain the reference voltage of the CW and the duty cycles of the matrix converter.

A. **Current Loop Design**

According to the nonlinear system shown in (8), in order to achieve zero steady-state error tracking of \( i_{cd}' \) and increase the
The degree of freedom in system bandwidth adjustment, the sliding mode surface is selected as

\[ S_d = X_1 + \lambda_1 X_2 \]

where \( X_1 = i_{cd} - \hat{i}_{cd} \), \( X_2 = \int (i_{cd} - \hat{i}_{cd}) dt \), \( \lambda_1 \) is the sliding mode coefficient to adjust the current loop bandwidth. When the sliding occurs, i.e., \( S_d = 0, \dot{S}_d = 0 \), the steady-state error \( \delta_{cd} \) can be expressed as follows:

\[ X_1(t) = X_1(0)e^{j\omega t}. \]

The control law of the SMC is selected as shown in the following equation:

\[
\begin{align*}
\dot{v}_{cd}^e &= v_{cd}^e + v_{cd}^n \\
\dot{v}_{cd}^e &= \left( \frac{1}{\sigma_1} \frac{R_c}{\sigma_2} i_{cd} + \omega_c i_{cq} + \frac{di_{cd}^*}{dt} - \lambda_1 X_1 \right) \sigma_2 \\
\dot{v}_{cd}^n &= -c_1 |S_d|^2 \text{sign}(S_d) - c_2 \int \text{sign}(S_d) dt
\end{align*}
\]

where \( v_{cd}^e \) is the equivalent control and \( v_{cd}^n \) is a super-twist control item, \( c_1 \) and \( c_2 \) are the positive constants, respectively.

Considering that the high-frequency noise can be presented in the system, it is not easy to obtain the derivative of \( \dot{i}_{cd}^* \). In order to solve this problem, this solution uses a second-order sliding mode differentiator as shown in the following equation to obtain the derivative of \( \dot{i}_{cd}^* \). Supposing \( \dot{i}_{cd}^* \) contains Langberg measurement noise, and \( (di_{cd}^*(dt) < L \), then

\[
\begin{align*}
\dot{z}_0 &= v_{cd}^e - \beta_0 L^{1/3} |z_0 - i_{cd}^*|^{2/3} \text{sign}(z_0 - i_{cd}^*) + z_1 \\
\dot{z}_1 &= v_1 - \beta_1 L^{1/2} |z_0 - \theta_k|^1/2 \text{sign}(z_0 - \theta_k) + z_2 \\
\dot{z}_2 &= -\beta_2 L \text{sign}(z_2 - v_1)
\end{align*}
\]

In (12), the parameter selection method in terms of \( \beta_0, \beta_1, \beta_2 \) can refer to [31]. In the absence of input noise, the following equation is true after a finite time transient process:

\[
\begin{align*}
\dot{z}_0 &= \dot{i}_{cd}^* \\
\dot{v}_0 &= \dot{z}_1 = \frac{di_{cd}^*}{dt},
\end{align*}
\]

B. Speed Loop Design

The outer speed loop adopts the conventional PI controller. According to (8), the mathematical model of the BDFIM can be expressed as

\[
\frac{d\omega_r}{dt} = \frac{3M_p M_r (p_p + p_c) \psi_{pm}^2}{2(L_p L_r - M_p^2)} i_{cq} - T_L - B_s \omega_r
\]

and the item \( B_s \omega_r \) can usually be treated as the disturbance part. The factor of \( i_{cq} \) is represented by \( k_{oa} \), then (14) can be simplified as

\[
\frac{d\omega_r}{dt} = k_{oa} i_{cq}.
\]

Assuming the inner loop’s bandwidth is ten times the bandwidth of the outer loop, the speed equation (15) is regarded as a first-order speed system. The control block diagram of the outer loop is shown in Fig. 3, and the PI controller with active damping item [32] is applied in this article

\[
i_{cq}^* = k_p (\omega_r^* - \omega_r) + k_i \int (\omega_r^* - \omega_r) dt - k_a \omega_r.
\]

The closed-loop transfer function of the system is obtained as follows:

\[
\Phi(s) = \frac{k_{oa} (k_p s + k_i)}{s^2 + k_{oa} (p_p + p_c) s + k_{oa} k_i}
\]

where \( V_{dc} \) represents the dc-side voltage, and \( \alpha \) is the angle between the reference voltage vector \( V_{ref} \) and the voltage basis vector \( V_1 \).
In this article, the parameter of the active damp item is selected as \( k_a = k_p \). The closed-loop transfer function of the speed control loop is designed as an inertial link in a first-order system with \( \omega_{ac} \) as the bandwidth
\[
\Phi(s) = \frac{\omega_{ac}}{s + \omega_{ac}}.
\] (18)

According to (17) and (18), the parameter of the PI controller is calculated as \( k_p = \omega_{ac} / k_{iq} \), \( k_i = \omega_{ac}^2 / k_{iq} \), and \( k_a = \omega_{ac} / k_{iq} \).

C. Robust Analysis and Proof of Stability

Considering the physical limitation of the system in practice and its behavior during the operation, such as eddy current loss and parameter uncertainties, the second-order SMC method is applied to the current inner loop to improve the system stability and antidisturbance ability [31].

According to (8) and (11), considering the parameters variation, the unmodeled dynamics and any other external disturbances, the current differential equation of the BDFIM with disturbance is shown in the following equation:
\[
di{t\hat{i}}_c = \begin{bmatrix} \hat{\sigma}_1 & \hat{\sigma}_2 \end{bmatrix} \begin{bmatrix} \hat{R}_c & 0 \\ 0 & \hat{R}_c \end{bmatrix} \frac{\hat{i}_c}{\hat{\sigma}_2} - \omega_{ac} \hat{i}_q,
\]
(19)

where \( \hat{\sigma}_1 \) includes the unmodeled dynamics and any other disturbances. The upper notation “\( \hat{\cdot} \)” represents the nominal value, after that, \( \hat{\sigma}_1 \) and \( \hat{\sigma}_2 \) are shown as:
\[
\begin{cases} \hat{\sigma}_1 = \sigma_1 + \Delta\sigma_1 = L_p \hat{L}_r - \hat{M}_p^2 \\ \hat{\sigma}_2 = \sigma_2 + \Delta\sigma_2 = \hat{L}_c \hat{L}_r - \hat{M}_c^2 \hat{L}_p - \hat{M}_p^2 \hat{L}_c \end{cases}
\] (20)

The parameter variation is shown in the following equation:
\[
\begin{bmatrix} \hat{L}_r = L_r + \Delta L_r, \hat{L}_p = L_p + \Delta L_p, \hat{L}_c = L_c + \Delta L_c \\ \hat{M}_p = M_p + \Delta M_p, \hat{M}_c = M_c + \Delta M_c \end{bmatrix}
\] (21)

where \( \Delta L_r, \Delta L_p, \Delta L_c, \Delta M_p, \Delta M_c \) are the parametric errors, \( \Delta\sigma_1 \) and \( \Delta\sigma_2 \) in (20) denote the parameter error caused by the variation of parameters \( L_r, L_p, L_c, M_p, M_c \), and are shown as follows:
\[
\Delta\sigma_1 = \Delta L_r L_p + \Delta L_p L_c + \Delta L_c L_p - 2M_p \Delta M_p - \Delta M_p^2 \\
\Delta\sigma_2 = \Delta L_r L_p + \Delta L_p L_c + \Delta L_c L_p + \Delta L_c L_r + \Delta L_r L_p + \Delta L_p \Delta L_c + \Delta L_c \Delta L_p + \Delta L_r \Delta L_c - 2L_p \Delta M_p + \Delta M_c - 2L_c \Delta M_p + L_c \Delta M_p - M_c \Delta M_P.
\] (22)

For ease of analysis, the errors in (19) can be integrated into the form shown in the following equation:
\[
d = \frac{\hat{\sigma}_1 \hat{\sigma}_2 - \hat{\sigma}_1 \hat{\sigma}_2}{\hat{\sigma}_2 \hat{\sigma}_2} \hat{i}_c + \frac{\hat{\sigma}_1 \hat{\sigma}_2 - \hat{\sigma}_1 \hat{\sigma}_2}{\hat{\sigma}_2 \hat{\sigma}_2} \hat{v}_c + \hat{\xi}.
\] (23)

In practice, the matrix converter-fed BDFIM needs to meet certain physical constraints in operation. The following assumptions can be made.

1) The capacity of the matrix converter is limited, that is, the output current and voltage are limited.

2) The disturbance of the BDFIM is bounded. The parameter variation \( \Delta L_r, \Delta L_p, \Delta L_c, \Delta M_p, \) and \( \Delta M_c \) are bounded.

According to the above assumption, the total disturbance is bounded as \( |d| \leq \varepsilon \). \( \varepsilon \) is a known positive constant, which represents the upper limit of the known current inner loop error.

According to (9) and (19), the first-order derivative of the sliding surface \( S_{sl} \) is derived as
\[
S_{sl} = \dot{X}_1 + \lambda_1 X_1 = \dot{X}_1 + \lambda_1 X_1 + d
\] (24)

where \( \dot{X}_1 \) is the nominal value of \( X_1 \). Substituting (11) into (24), the closed-loop system is derived as
\[
\begin{bmatrix} \dot{S}_{sl} = -c_1 |S_{sl}|^{1/2} \text{sign}(S_{sl}) + \nu + d \\
\dot{\nu} = -c_2 \text{sign}(\nu) \end{bmatrix}
\] (25)

To prove the stability of the system, the Lyapunov function is selected as [31]
\[
V = 2c_2 |S_{sl}| + \frac{1}{2} \nu^2 + \frac{1}{2} \left( |S_{sl}|^{1/2} \text{sign}(S_{sl}) - \nu \right)^2
\] (26)

where \( \nu^T \) and \( P_d \) are expressed as
\[
\nu^T = \begin{bmatrix} \text{sign}(S_{sl}) |S_{sl}|^{1/2} \nu \end{bmatrix}
\] (27)

\[
P_d = \frac{1}{2} \begin{bmatrix} 2c_2 + c_1^2 & -c_1 \\ -c_1 & 2 \end{bmatrix}.
\] (28)

From (26), it can be derived that
\[
\dot{V} = -\frac{1}{|S_{sl}|^{1/2}} \nu^T Q \nu + \frac{d}{|S_{sl}|^{1/2}} \nu^T \zeta
\] (29)

where \( Q \) and \( \nu^T \) can be expressed as
\[
Q = \frac{c_1}{2} \begin{bmatrix} 2c_1 + c_1^2 & -c_1 \\ -c_1 & 1 \end{bmatrix}, \quad \nu^T = \begin{bmatrix} \left( 2c_2 + c_1^2 \right)/2 \\ c_1/2 \end{bmatrix}.
\] (30)

Since the disturbance of the current inner loop is bounded, it can be shown that [33]
\[
\dot{V} \leq -\frac{1}{|S_{sl}|^{1/2}} \nu^T Q \nu + \frac{d}{|S_{sl}|^{1/2}} \nu^T \zeta
\] (31)

where
\[
\zeta = \frac{c_1}{2} \begin{bmatrix} 2c_2 + c_1^2 - \left( \frac{4c_1}{c_1} + c_1 \right) \nu - (c_1 + 2\nu) \end{bmatrix}
\] (32)

When \( \nu \) is positive and \( \dot{\nu} \) is negative, the system satisfies the asymptotic stability condition, if \( \dot{V} > 0 \). Therefore, by setting the values of \( c_1 \) and \( c_2 \), the system is stable under bounded noise interference. The choice of \( c_1 \) and \( c_2 \) is shown in the following equation:
\[
\begin{bmatrix} c_1 > 2\varepsilon \\ c_2 > c_1 \frac{5c_1 \varepsilon + 4\varepsilon^2}{2(c_1 - 2\varepsilon)} \end{bmatrix}
\] (33)

Therefore, the second-order SMC of \( i_{cd} \) is designed according to the theory of SMC, and its robustness is proved by mathematical method. In the same theory, the second-order SMC of \( i_{cq} \) can be designed in the same way.
TABLE I
SIMULATION PARAMETERS

| Symbol | Parameter | Value |
|--------|-----------|-------|
| $P_p$ | the number of pole pairs of the PW | 1     |
| $P_r$ | the number of pole pairs of the CW | 3     |
| $R_p$ | PW stator resistance (Ω) | 0.40355 |
| $R_c$ | CW stator resistance (Ω) | 0.5470 |
| $R_t$ | rotor resistance (Ω) | 0.785244 |
| $L_p$ | PW stator inductance (mH) | 253.3 |
| $L_c$ | CW stator inductance (mH) | 129.6 |
| $M_p$ | PW excitation inductance (mH) | 247.4 |
| $M_c$ | CW excitation inductance (mH) | 123.7 |

Fig. 4. Simulation results of the PI-SMC method: the reference rotor speed is changed from 200 to 400 r/min at $t = 1$ s; the load torque is changed from 0 to 30 N·m at $t = 3$ s. (a) Rotor speed. (b) Reactive power. (c) Electromagnetic torque. (d) CW dq-axis stator current.

IV. SIMULATION RESULTS

To verify the correctness of the proposed method, numerical simulations are performed on MATLAB/Simulink. The parameters of BDFIM are shown in Table I.

Fig. 4 shows the simulation results of the PI-SMC method. It can be seen that the actual speed of the BDFIM reaches a steady state within 1 s when the reference rotor speed changes. The steady-state error of speed is approximately zero and the reactive power fluctuates slightly and keeps to zero rapidly.

Fig. 5. Simulation results with parameter errors: ±20% $M_p$; the reference rotor speed is changed from 200 to 600 and 1000 r/min at $t = 1$ and 4 s. (a) Rotor speed. (b) Reactive power. (c) Electromagnetic torque. (d) CW dq-axis stator current.

When the load torque changes abruptly from 0 to 30 N·m, the rotor speed has a drop, but recovers in 1 s. The reactive power is almost maintained at zero.

IV. SIMULATION RESULTS

To verify the correction of the proposed method, numerical simulations are performed on MATLAB/Simulink. The parameters of BDFIM are shown in Table I.

Fig. 4 shows the simulation results of the PI-SMC method. It can be seen that the actual speed of the BDFIM reaches a steady state within 1 s when the reference rotor speed changes. The steady-state error of speed is approximately zero and the reactive power fluctuates slightly and keeps to zero rapidly.

When the load torque changes abruptly from 0 to 30 N·m, the rotor speed has a drop, but recovers in 1 s. The reactive power is almost maintained at zero.

To simplify the analysis, the variations of parameters $M_p$ and $M_c$ are considered. The simulation results under parametric error ±20% $M_p$ and ±20% $M_c$ are presented in Figs. 5 and 6 to validate the robustness of the proposed method. The steady and dynamic control performances of the proposed method are different when the parameter $M_p$ has ±20% error. Nonetheless, the tracking performances of the rotor speed and CW stator current are both satisfied with quick tracking processes. When the parameter $M_c$ has ±20% error, the tracking performances of the rotor speed and CW stator current are both acceptable as shown in Fig. 6. In conclusion, when the parametric error exists, the waveforms in steady and dynamic states are affected to some extent. However, the system stability under variable parameters is guaranteed with suitable parameters $c_1$ and $c_2$.

V. EXPERIMENTAL VERIFICATION

To verify the correctness of the proposed cascade PI-SMC method for the matrix converter-fed BDFIM drive, an experimental prototype is built. The experimental system arrangement is shown in Fig. 7. The root means square value of input line-to-line voltage is 200 V. An 11-kW three-phase IM is mechanically coupled to the 15-kVA BDFIM and driven by a matrix converter. The matrix converter-fed BDFIM drive system consists of four parts: control platform, clamp circuit,
Fig. 8. Experimental results of the cascade PI-SMC method when the reference rotor speed is 400 r/min. (a) Experimental waveforms of PI-SMC method. (b) Reference and real values of rotor speed. (c) Reference and real values of dq-axis CW stator current.

Fig. 9. Harmonic spectra of PW and CW current when the reference rotor speed is 400 r/min. (a) PW stator current $i_{pa}$. (b) CW stator current $i_{oa}$.

Table II

| Motor1-BDFIM | Motor2-IM |
|--------------|-----------|
| Type         | ZWSF250L-8 | Type         | YZPFM180L-8 |
| Rated Power  | 15kVA      | Rated Power  | 11kW        |
| Rated Voltage| 400V       | Rated Voltage| 380V        |
| Rated Current| 17.3A      | Rated Current| 24A         |
| Speed Range  | 0-1450 r/min| Speed Range  | 0-1450 r/min|
| Pole Pairs   | 3          | Polar Pairs  | 4           |
| PW stator resistance $R_p$ | 0.40355 Ω | Stator Resistance $R_s$ | 0.3 Ω |
| CW stator resistance $R_c$ | 0.5470 Ω | Rotor Resistance $R_r$ | 0.317065 Ω |
| Rotor Resistance $R_t$ | 0.785244 Ω | Stator Inductance $L_s$ | 0.055165 H |
| $(L_p+L_c)/L_t$ | 0.0178 H | Rotor Inductance $L_r$ | 0.055165 H |
| $(M_p/M_c)$ | 0.0824 H | Mutual Inductance $L_{mr}$ | 0.0498 H |

drive system, and IGBT module. The control platform mainly consists of a DSP (TMS320F28335) and field-programmable gate array (FPGA, EP2C8T144C8N). The drive system is utilized to convert the acquired duty cycle into high or low levels to drive the IGBTs of the matrix converter. The clamp circuit is equipped to prevent damage to the matrix converter caused by issues such as commutation failure or shut down. The precise values of parameters $L_p$, $L_c$, $L_t$, $M_p$, and $M_c$ are unknown. The data relationship between the inductors is obtained through a simple measurement. The motor specifications of BDFIM and IM are shown in Table II.

To verify the steady-state performance and dynamic performance of the proposed PI-SMC method under the condition of unknown parameters, the following tests are carried out.
A. Steady-State Performance Test

Fig. 8(a) shows the experimental waveforms of PW stator phase voltage $v_{pa}$, PW stator current $i_{pa}$, and CW stator current $i_{oa}$ when the reference speed is set as 400 r/min for the matrix converter-fed BDFIM. The PW stator phase current is approximately sinusoidal and in phase with the PW stator phase voltage. Thus, the reactive power is nearly zero. The data in Fig. 8(b) and (c) are obtained through the digital-to-analog converter of the control platform. The rotor speed tracks the reference speed well, as shown in Fig. 8(b). The steady-state error of rotor speed is $\pm 3$ r/min after filtering sampling noise. The real values of dq-axis CW stator current are stable with small fluctuations and track the reference value well, as shown in Fig. 8(c). The proposed PI-SMC method still has a perfect steady-state performance in the case of uncertain motor specific parameters.

The harmonic spectra of PW and CW current when the reference rotor speed is 400 r/min are shown in Fig. 9. The total harmonic distortion (THD) value of PW and CW current is 8.68% and 6.78%, respectively. The distortion of PW and CW current is generated due to the device voltage drop, commutation dead time, and other nonlinear factors.

B. Dynamic Performance Test

Fig. 10 shows the experimental waveforms of PW stator phase voltage $v_{pa}$, PW stator current $i_{pa}$, and CW stator current $i_{oa}$, when the reference rotor speed changes from 200 to 400 r/min. The enlarged drawing of the red dotted boxes in four different states is given. The PW stator current and CW stator current increase to provide larger electromagnetic torque when the reference rotor speed increases. Then PW stator current and CW stator current are stable after the reference rotor speed keeps constant. As shown in Fig. 11(a), when the reference rotor speed changes, the real rotor speed can track the reference value accurately with the proposed cascade PI-SMC method. The real values of the dq-axis CW stator current track the reference value in the dynamic process, as shown in Fig. 11(b), which shows a good dynamic performance.

Fig. 12 shows the experimental waveforms of PW stator phase voltage $v_{pa}$, PW stator current $i_{pa}$, and CW stator current $i_{oa}$ when the load torque changes from 0 to 5 N·m and changes back to 0 N·m with a constant reference rotor speed. The enlarged drawing of the red dotted boxes in four different states is given. The PW stator current and CW stator current increase or decrease when the load torque increases or decreases. As shown in Fig. 13, a few adjustments occur in the real rotor speed and dq-axis CW stator current when the load torque changes. Then the rotor speed recovers to its original value in a short time. The proposed cascade PI-SMC method has a good antidisturbance interference ability and achieves good dynamic performance.
VI. CONCLUSION

In this article, a cascade PI-SMC method is proposed for a matrix converter-fed BDFIM drive system. The proposed method consists of a PI speed controller and an inner-loop second-order SMC. The outer loop adopts the PI controller with the active damping to track the reference value of the rotor speed and reactive power. The inner loop uses the continuous supertwisting SMC to control the CW current of the brushless doubly fed motor, which eliminates the chattering problem. The proposed cascade PI-SMC method has a good antidisturbance interference ability. Meanwhile, it has excellent robust tracking performance. The experimental results verify the effectiveness and correction of the proposed cascade PI-SMC method.

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Hui Wang received the B.S. degree in automation, the M.S. degree in electrical engineering, and the Ph.D. degree in control science and engineering from Central South University, Changsha, China, in 2008, 2011, and 2014, respectively.

Since 2016, he has been with the School of Automation, Central South University. His research interests include matrix converters, dc/dc converters, and solid-state transformers.

Xida Chen (Graduate Student Member, IEEE) received the B.S. degree in electrical engineering and automation from the Changsha University of Science and Technology, Changsha, China, in 2017. He is currently pursuing the Ph.D. degree in control science and engineering with Central South University, Changsha.

His research interests include matrix converters and model predictive control for power converters.

Xun Zhao was born in Heilongjiang, China, in 1996. He received the B.S. degree in electrical engineering from Central South University, Changsha, China, in 2018. He is currently pursuing the M.S. degree in electrical engineering.

His research interests include modeling and control of power electronics converters.

Hanbing Dan (Member, IEEE) was born in Hubei, China, in 1991. He received the B.S. degree in automation and the Ph.D. degree in control science and engineering from Central South University, Changsha, China, in 2012 and 2017, respectively.

He was a Visiting Researcher with the Faculty of Engineering, University of Nottingham, Nottingham, U.K., during 2017. He is currently an Associate Professor with the School of Automation, Central South University. His research interests include power converter, motor control, model predictive control, and fault diagnosis.

Mei Su (Member, IEEE) was born in Hunan, China, in 1967. She received the B.S. degree in automation, the M.S. degree in automation, and the Ph.D. degree in control science and engineering from Central South University, Changsha, China, in 1989, 1992, and 2005, respectively.

She is currently a Full Professor with the School of Automation, Central South University. Her research interests include matrix converter, adjustable speed drives, and wind energy conversion system.

Dr. Su is currently an Associate Editor of the IEEE Transactions on Power Electronics.

Yao Sun (Member, IEEE) was born in Hunan, China, in 1981. He received the B.S., M.S., and Ph.D. degrees from Central South University, Changsha, China, in 2004, 2007, and 2010, respectively.

He is currently a Professor with the School of Automation, Central South University. His research interests include matrix converter, microgrid, and wind energy conversion system.

Fan Zhang was born in Hunan, China, in 1994. He received the B.S. degree in electrical engineering and the automatization and the master’s degree in electric engineering from Central South University, Changsha, China, in 2016 and 2019, respectively.

He is currently an Engineer with the GAC Research and Development Center, Guangzhou, China.

Marco Rivera (Senior Member, IEEE) received the B.Sc. degree in electronics engineering and the M.Sc. degree in electrical engineering from the Universidad de Concepcion, Concepcion, Chile, in 2007 and 2008, respectively, and the Ph.D. degree from the Department of Electronics Engineering, Universidad Tecnica Federico Santa Maria, Valparaíso, Chile, in 2011, with a scholarship from the Chilean Research Fund CONICYT.

From 2011 and 2012, he held a post-doctoral position and worked as a part-time Professor of Digital Signal Processors and Industrial Electronics with Universidad Tecnica Federico Santa Maria. He is currently an Associate Professor with the Faculty of Engineering, Universidad de Talca, Curico, Chile. He is the Head of the Laboratory of Energy Conversion and Power Electronics (LCEEP) and the Technology Center for Energy Conversion (CTCE), Chile. His research interests include matrix converters, predictive and digital controls for high-power drives, four-leg converters, renewable energies, and the development of high-performance control platforms based on field-programmable gate arrays.

Prof. Rivera was awarded a scholarship from the Marie Curie Host Fellowships for early stage research training in electrical energy conversion and conditioning technology at University College Cork, Cork, Ireland, in 2008. He was awarded the Chilean Academy of Science Doctoral Thesis Award (Premio Tesis de Doctorado Academia Chilena de Ciencias) in 2012, for the best Ph.D. thesis published in 2011, selected from among all national and international students enrolled in any exact or natural sciences program in Chile and he was awarded as an Outstanding Engineer in 2015.

Patrick Wheeler (Fellow, IEEE) received the B.Eng. (Hons.) and Ph.D. degrees in electrical engineering for his work on matrix converters from the University of Bristol, Bristol, U.K., in 1990 and 1994, respectively.

In 1993, he moved to the University of Nottingham, Nottingham, U.K., and worked as a Research Assistant with the Department of Electrical and Electronic Engineering. In 1996, he became a Lecturer with the Power Electronics, Machines and Control Group, University of Nottingham, where he has been a Full Professor since January 2008. He was the Head of the Department of Electrical and Electronic Engineering, University of Nottingham, from 2015 to 2018. He is currently the Head of the Power Electronics, Machines and Control Research Group and the Global Director of the Institute of Aerospace Technology, University of Nottingham, and was the Li Dak Sum Chair Professor of Electrical and Aerospace Engineering. He has authored or coauthored 750 academic publications in leading international conferences and journals.

Dr. Wheeler is the Vice-President of the IEEE PELS and was an IEEE PELS Distinguished Lecturer from 2013 to 2017.