Study of wear particles formation at single asperity contact: An experimental and numerical approach

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ABSTRACT

Wear particles generated due to the relative sliding of the roll and the sheet metal in the roll bite are one of the main factors that contaminate the surface of a cold rolled sheet metal. The details of the wear mechanisms in the contact of individual roll-sheet asperities define the total amount of wear particles generated. In this paper, a micro-mechanical experimental approach is coupled with material point method (MPM) scratch simulations to study the friction and wear behavior of a single roll asperity sliding through a sheet metal. Micro-tribology experiments in the form of single asperity scratch testing showed that ploughing is the dominant wear mechanism in lubricated conditions, while wedge forming was the main wear mechanism in the absence of lubricant. The beneficial influence of chrome plating the rolls on wear particles formation is found to stem from its interaction with the lubricant as the same influence was not observed in non-lubricated test conditions. MPM single asperity simulations revealed that almost all the frictional resistance arises from deforming the substrate in the case of an interfacial shear strength corresponding to the lubricated contact. In contrast, MPM showed that, in unlubricated sliding, the frictional resistance is primarily due to shearing of the adhesion junction and tearing the deforming body. Furthermore, using the degree of wear of the scratch experiments as a benchmark, a critical plastic strain needed to produce wear particles was found to be between 3 to 4 for the investigated interstitial-free steel.

1. Introduction

Surface cleanliness is an important measure of the quality of a cold rolled sheet metal, particularly in applications such as outer body parts of the automotive industry, where high requirements are put on the surface quality. The generation of wear particles (also known as iron fines) is one of the major factors that contaminate the sheet surface and cause degradation of its surface quality. Wear particles are generated during cold rolling due to the relative sliding between the roll and the sheet in the roll bite. Wear particles are mainly generated from the sheet in the rolling process [1,2]. In most rolling processes, the sheet is much softer than the roll. Some of the wear particles are removed by the coolant/emulsion used in the rolling process. However, those that remain on the sheet surface can cause problems in downstream processes such as annealing, galvanizing and forming. In addition, they can adversely affect filtration processes. It is therefore of paramount importance to reduce wear particles generation during cold rolling operations. To achieve this and in order to optimally tailor the surface quality of cold rolled sheet metals, a detailed understanding of the underlying micro-scale physical damage mechanisms at the roll-sheet metal interface is necessary.

The macro-scale rolling process parameters together with the surface properties of the roll and the sheet define the contact conditions at micro-scale. Micro-scale is where the actual wear particles generation takes place. At micro-scale, the hard roll asperities indent and plough through the soft sheet surface. This may lead to strip wear and subsequently the generation of wear particles, depending on the wear regime [3]. A study of the contact between a single hard roll asperity and a flat soft sheet is, therefore, essential in order to obtain a complete insight on the macro-scale wear properties of the cold rolling tribological system. This can be combined with rolling process parameters, material properties and roughness properties of both the roll and the strip to tailor the rolling process with respect to surface quality. The aim of the current work is, hence, to study, experimentally and numerically, the wear behavior of a single roll asperity sliding against a flat soft sheet.

Scratch experiments are commonly used to investigate the friction...
and wear phenomena at single asperity interactions. In scratch experiments, a sharp tip moves across the surface under a controlled load and speed. However, the amount of information that can be obtained from such scratch experiments is limited. One of the important qualities which is difficult to determine experimentally but can be easily deduced from computer simulations is the effect of asperity geometry on wear. Numerical modelling offers a fast, predictive capability to study the influence of several parameters such as asperity geometry, sliding speed and/or temperature on the wear behavior of single asperity contacts. This cannot be easily realized in the experiments. Nevertheless, modelling single-asperity contacts that involve the formation of wear particles poses many challenges.

Analytical models are attractive due to their simplicity. However, they have several limitations. No complete analytical model of the scratching of a flat surface by an indenter has yet been developed due to the geometric complexities inherent in three-dimensional plasticity solutions. Analytical models simplify the geometry and material behavior to make it easy for analysis. Many of the analytical models involve a simplified two-dimensional analysis [4]. Also, wear of brittle materials, which might involve brittle fracture, is typically not modelled in these plasticity-based models. Furthermore, modelling the generation of the wear track includes displacement and removal of material in three dimensions (3D), so modelling this process will require modelling to be done in three dimensions. Although some work has been done in this area, analytical models are complex and ultimately inadequate to capture the complex three-dimensional nature of abrasive wear [5].

In recent years, there have been attempts to develop a numerical model to study single-asperity wear, such as scratching simulations and formation of chips in grinding, machining and micro-milling. Several authors used a mesh-based finite element method (FEM) [6–11] and mesh-less continuum methods such as smooth particle hydrodynamics (SPH) [12,13] and material point method (MPM) [14,15] to simulate scratch experiments in three dimensions. Using mesh based techniques such as FEM has proven to be an unsuitable method for modelling wear as it involves large deformations and chip (wear particles) formation [11]. FE methods have extreme mesh distortion problems arising from large deformations in simulating abrasive wear that involves cutting. Although remeshing is possible, it leads to large computational overhead and convergence is not guaranteed. Particle based methods such as SPH have an advantage over mesh-based methods, in particular that arbitrary large deformation can be handled easily with no need for remeshing. However, classical SPH suffers from tensile instability [16]. Hence, if traditional SPH is used to simulate solids, a high and physically incorrect artificial viscosity is required to damp strong oscillatory modes and stabilize the system. A total Lagrangian SPH (TLSPH) addresses the rank deficiency and tensile instability problems by the use of artificial particle velocity damping and a hourglass control scheme similar to FEM, as described by Ganzenmüller [17]. Nevertheless, TLSPH introduces another constraint, because it utilizes a constant reference undeformed configuration to calculate the gradients and perform integration of stresses. This in turn limits the magnitude of deformation which can be simulated, as the difference between the reference configuration and the current configuration must not become too large. Although updates of the reference configuration can be performed, this approach is computationally expensive.

Recently, the material point method has been used to numerically study micro-machining processes [18] and single-asperity ploughing [14,15,19]. MPM can handle problems involving arbitrarily large deformations and material detachment such as abrasive wear and machining. Mishra et al. [14,19] studied friction during the ploughing of a soft metallic sheet by a rigid indenter of spherical and elliptical geometries using MPM. They compared the numerical results in terms of depth of groove and friction coefficient with experiments and reported a good match between both the model and measured friction coefficients and wear depth. Leroch et al. [18] implemented MPM to model three-dimensional micro-milling considering both mechanical deformation and heat conduction. They studied the dependency of tool forces with uncut chips thickness and compared the MPM results with FEM simulations as well as the experimental data and found good agreements. In another recent study, Varga et al. [15] performed MPM simulations of scratch experiments at high temperatures to assess the wear mechanisms and the extent of wear of a hot rolled steel sliding against a rigid wall during transporting and compared their results with experiments. They reported a good agreement between the experiments and the simulations.

MPM combines both a background mesh and meshless concepts and can accurately model solid plastic flow and fluid behavior. It uses particles, which contain all history dependent information such as stress, strain and momentum. Furthermore, in contrast with the traditional particle methods such as SPH, it uses an auxiliary background grid to compute stresses and strain rates. Information between the particles and the grids is exchanged by using relatively simple basis functions. MPM can be considered as a true particle method, because the background grid is used as a computational scratch pad and is deleted at the end of each time step. The state of the system is then advanced by moving the particles only. A detailed computation procedure of the MPM can be found elsewhere [18].

In this study, the mechanics determining the tribological behavior (wear) of a single-asperity sliding against a flat sheet has been investigated and compared experimentally and numerically. Single-asperity scratch experiments have been carried out for this purpose and the material point method has been implemented to numerically model the scratching process in three dimensions. The effect of lubrication and degree of penetration on the degree of wear has been studied. Moreover, the correlation between tribological parameters, friction coefficient and wear mechanisms has been investigated.

2. Materials and methods

2.1. Experimental procedure

Scratch experiments have been carried out to simulate a single roll asperity sliding on a flat sheet. The experiments were performed using conical indenters with a hemispherical tip, which represent a rollasperity, sliding on a polished flat sheet surface. The indenters are made of a medium-alloyed cold work tool steel (Uddevholm Rigor®) that has similar composition as the commonly used work roll material. The radius of the hemispherical tip of the indenters is 225 μm. A hot rolled and pickled Titanium-stabilized interstitial-free (Ti-IF) steel strip (50 mm x 50 mm x 3 mm) was used as a substrate. This steel grade is extensively used in exterior automotive body part applications and has been observed to be critical with respect to wear particles formation. The strip samples and the tip of the indenters were polished to a mirror like surface finish (Sq < 20 nm). The tips of the indenters were examined under microscopy at high magnifications to ensure there are no irregularities on the surface. The experiments were conducted under lubricated and dry conditions. A palm-based fully formulated industrial cold rolling oil was used for the lubricated scratch experiments.

The experiments were conducted using a multi-purpose tribometer from Bruker (UMT Tribolab). The experiments were performed using a constant normal load, ranging between 2 and 20 N. The strip samples and the indenters were degreased and cleaned in isopropanol prior to the experiments. For the lubricated experiments, a film of 1 g/m² oil was applied on the strip surface. The amount of oil was determined by measuring the weight of the strip sample before and after applying the lubricant. The load was applied by moving the indenter downwards until the desired scratch load is reached. Once the desired load is achieved, the indenter is moved horizontally in the scratching direction at a constant speed (1 mm/s) under load-controlled conditions. The length of the scratches is 15 mm.

In order to investigate the influence of interfacial shear strength at the contact interface on the transition of wear modes and the rate of
material removal, the scratch experiments were carried out in four different distinct contact conditions: (i) using uncoated indenter in dry condition, (ii) using uncoated indenter in lubricated condition, (iii) using hard chrome plated indenter in dry condition, and (iv) using hard chrome plated indenter in lubricated condition. A total of 10 scratches were made (i.e. at 2 N, 4 N, ..., 20 N) for each contact condition. The experiments were performed at room temperature. The normal and tangential forces were recorded during the experiments. Afterwards, the tips of the indenters were examined by microscopy to check for material transfer and damage. The wear grooves of the scratches were analyzed using a non-contact three-dimensional optical profiler (S neox 3D Optical Profiler from Sensofar) and an optical microscopy (Keyence VHX-5000). The three-dimensional (3D) height profile of the scratches was measured using confocal microscopy. The wear tracks were aligned so that the scratch direction becomes parallel to the Cartesian coordinate using a Canny edge detection method and a Hough transform in Matlab. Next, tilt correction and shifting the reference plane to z = 0 was carried out by employing the flat section (away from the wear track) which is unaffected by the scratch experiments as a reference. The average two-dimensional (2D) wear groove profile for each scratch was determined by calculating the average profile of the middle 12 mm section of the wear track. The uneven portion at the beginning and the end of the scratches was not used to calculate the average 2D wear profile. The volume of material removed as wear particles was evaluated by calculating the degree of abrasive wear $\beta$. The degree of abrasive wear, which is a measure of the amount of material removed as a result of the abrasive wear, is defined as the ratio of groove area ($A_g$) to shoulder area ($A_s$). This is schematically illustrated in Fig. 1. $d_{wa}$ is calculated as follows:

$$d_{wa} = \frac{(A_g - A_s)}{A_s}$$  \hspace{1cm} (1)

2.2. Computational method

The MPM simulations were implemented using an open source particle simulation code LAMMPS and the smooth mach dynamics package by Ganzenmüller [20], which was later extended by Mishra et al. [14] to incorporate the interfacial shear strength at the indenter-substrate interface and a dislocation based material model for the substrate material deformation. The MPM results were analyzed using OVITO, a visualizing software [21]. The scratches were simulated using a hemispherical rigid indenter, with the same radius as the experiments (225 $\mu$m), sliding on a deforming substrate. Fig. 2 displays a snapshot of the simulation setup. The deforming substrate is created by placing particles in a half cylindrical block with a diameter of 0.9 mm and a length of 1.8 mm. The outer most particles of the curved side of the cylinder are fixed in all directions. A periodic boundary condition is applied in the sliding direction. The indenter is modelled and imported as an STL file, which consists of a triangular mesh. The diameter of particles (deforming substrate) and the mesh size of the indenter was set to 5 $\mu$m. The interaction between the indenter surface and the deforming substrate is assumed to be purely repulsive and is given by the Hertz contact theory. A Coulomb friction is assumed at the contact interface. The length of the scratches was set to 1.4 mm to reduce the computation time, in contrast to 15 mm in the experiments. The simulations were executed under load-controlled conditions using similar constant normal loads as the scratch experiments. The indenter is moved down to the substrate until the desired load is reached and then moved in the sliding direction at a constant speed of 0.5 m/s. Since the experiments were performed at room temperature, the temperature was set to 20 °C in the simulations.

The indenter, which represents a roll asperity, is assumed to be rigid as the hardness of the rolls is generally much harder than the sheet material. The deforming substrate represents the sheet metal. A modified Bergstrom-van Liempt equation [22,23] is used as a constitutive equation for the deforming substrate. It describes the von Mises flow stress $\sigma_f$ as a function of strain $\varepsilon$, strain rate $\dot{\varepsilon}$ and temperature $T$ as follows:

$$\sigma_f = \sigma_s + \alpha G b \left( \frac{U}{\Omega} (1 - e^{-i\Omega}) + \rho_0 e^{-i\Omega} \right)^{1/2} + \sigma_0 \left(1 + \frac{k T}{2 G m} \frac{\ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)}{\Omega} \right)^p$$ \hspace{1cm} (2)

where $\sigma_s$ is the initial static stress, $\alpha$ is a crystallographic constant, $G$ is the elastic shear modulus, $b$ is the burgersvector, $U$ is the combined rate of annihilation and immobilization of mobile dislocations (measure of immobility), $\Omega$ is the remobilization probability, $\rho_0$ is the density of immobile dislocations, $\sigma_0$ is the maximum dynamic stress, $k$ is the Boltzmann’s constant, $4G0$ is the activation energy, $\dot{\varepsilon}_0$ is the initial strain rate and $p$ is the dynamic stress power. The material parameters of the Ti-If steel used in the current study are provided in Table 1.

One parameter which is varied in the simulations, although the actual value is not precisely known in the experiments, is the shear strength of the contact interface. What is measured in the scratch experiments is the overall frictional force, which is composed of the shearing resistance of the interface and the resistance to the plastic deformation. Ideally, the shear strength of the contact interface should be measured in a separate experiment and used as an input for the scratch simulations. However, measuring the shear strength of the contacting interface is very challenging. The plastic deformation component of friction should be eliminated to experimentally characterize the interfacial shear strength. This requires preparing molecularly smooth surfaces. For lubricated contacts, the boundary shear strength is typically measured by sliding a smooth sphere on flat, cylinder on flat, or orthogonal contacting cylinders using flamed glass or mica sheet substrates [24-26]. In the current work, a different approach was chosen. The interfacial shear strength was estimated by running a series of simulations with several Coulomb friction ($\mu$) values (0, 0.2, 0.4, 0.6 and 0.8) for the different loads. The shear strength of the interface at that load is then mapped by taking the $\mu$ value which gives the same overall frictional force as the corresponding experiment. Besides, varying the interfacial shear strength in the simulations (by varying $\mu$) provides a detailed insight of its influence on wear mechanisms. A summary of the scratch experiment and simulation parameters is provided in Table 2.

The average 2D scratch profile and the degree of wear of MPM simulations were calculated the same way as the experiments. Further, the average steady state friction forces were calculated by excluding the running in period. The friction coefficient, wear groove depth and degree of wear of MPM simulations were compared with the experimental results.

3. Results and discussion

3.1. Scratch experiments

3.1.1. Friction coefficient

The average steady state apparent friction coefficient of the scratch experiments is plotted Fig. 3. The friction curves of the individual scratches can be found in Fig. S1 of the supplementary document. The apparent coefficient of friction ($\mu_{ap}$) is defined as the ratio of the measured tangential force to the applied normal force. The $\mu_{ap}$ value shows dependence on the normal load. $\mu_{ap}$ increases slightly with the increase of the normal load for the lubricated scratch experiments. In contrast, in the dry scratch experiments, the friction resistance increases...
dramatically at low loads (8 N for the uncoated indenter and 6 N for the Cr plated indenter). This dramatic increase of friction coefficient can be related to the transition of the wear mechanism of the scratch experiments from ploughing to wedge forming [27]. The microscopic examination of the indenters after the scratch experiments (Fig. 4) and the morphology of the wear tracks (see Fig. 5 and Fig. S2) confirm the transition of the wear mechanism of the scratch experiments from ploughing to wedge forming.

Different degrees of fluctuation in the friction curves were observed depending on the lubrication condition and the normal load (see also Fig. S1). The apparent friction coefficient is most stable in the lubricated scratch experiments and at very low loads of the dry experiments (<6 N). The friction curves exhibited a large fluctuation for the dry experiments at high loads. Although slight fluctuations are observed for the lubricated experiments at higher loads, the magnitude of fluctuation is much smaller than the dry ones. The scratch experiments performed using chrome coated indenters in the lubricated condition showed lower \( \mu_{ap} \) values, particularly at higher loads (>16 N), than the equivalent scratches done using uncoated indenters. The opposite effect was observed for the dry scratch experiments, i.e. the experiments done using chrome coated indenters showed a higher average \( \mu_{ap} \) and a transition to unstable friction at lower load (6 N) than the uncoated ones (8 N), see Fig. 3.

The total frictional resistance in the scratch experiments is made up of two components: (i) the adhesion component, which is the force needed to shear the boundary layer in lubricated contacts and/or the welded junctions formed by adhesion in dry contacts, and (ii) the ploughing component, which is the force needed to push waves of plastically deformed material ahead of the indenter [4,27]. The contribution of each component to the overall friction force depends on the lubrication condition, the geometry of the indenter, the degree of

Table 1
Bergstrom-van Liempt material parameters for the investigated Ti-IF steel.

| Parameter | Value            | Parameter | Value            |
|-----------|------------------|-----------|------------------|
| \( \sigma_1 \) | 58.52 MPa        | \( \sigma'_0 \) | 1229 MPa         |
| \( \alpha \) | 0.82             | \( \rho_0 \) | 1.0x10^{12}      |
| \( G \) | 7.9x10^{8}       | \( k \) | 8.62x10^{-5} eV  |
| \( b \) | 2.48x10^{-10}    | \( \Delta G_0 \) | 0.97             |
| \( U \) | 2.04x10^{7}      | \( \dot{\epsilon}_0 \) | 4.63x10^{10} s^{-1} |
| \( \Omega \) | 7.5              | \( p \) | 3.38             |

Table 2
Scratch experiment and MPM simulation parameters.

| Parameter                   | Value                                      |
|-----------------------------|--------------------------------------------|
| Indenter radius             | 225 \( \mu \) m                          |
| Sliding speed               | 1 mm/s (experiments), 0.5 m/s (MPM simulations) |
| Normal load                 | 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 N     |
| Coulomb friction used in MPM simulations | 0.0, 0.2, 0.4, 0.6, 0.8      |
| Scratch length              | 15 mm (experiments), 1.4 mm (MPM simulations) |
| MPM particle size           | 5 \( \mu \) m                             |
penetration and the bulk properties of the materials in contact. In frictionless contacts, the entire frictional resistance emanates from the deformation component. In the absence of deformation, the frictional force arises solely from the mechanical shear strength of the sliding interface. The presence or absence of a lubricant is theoretically expected to have little or no effect on the deformation component of the frictional resistance [28, 29]. Another factor that affects the friction and wear behavior of the scratch experiments is the relative shear strength of the contact interface and the bulk shear strength of the deforming material. Two cases can happen depending on these values: (i) slip occurs at the interface when the interface shear strength is smaller than the shear strength of the softer counterpart, and (ii) shearing happens in the bulk of the softer counterpart when the interface shear strength is larger than the shear strength of the softer counterpart. Wear particles can form in case of the latter.

In the lubricated scratch experiments, a boundary layer prevents direct metal to metal contact and slip occurs at the contact interface. Hence, the friction resistance is predominantly due to the deformation component. As the normal load is increased, the indentation depth, and consequently, the deformation component of friction increase. As the load is increased further, the lubricant starts to fail locally and some surface irregularities may penetrate and tear through the boundary film. Eventually, metal to metal adhesion may take place, which may lead to material transfer. Material transfer was observed on the tip of the indenters after the experiments (Fig. 4). Nonetheless, the friction curves show only small fluctuations. The friction coefficient fluctuation in

| Load (N) | Uncoated indenter - dry | Uncoated indenter - lubricated | Cr plated indenter - dry | Cr plated indenter - lubricated |
|----------|-------------------------|-------------------------------|--------------------------|--------------------------------|
| 2        | ![Image](image1.png)     | ![Image](image2.png)         | ![Image](image3.png)     | ![Image](image4.png)           |
| 4        | ![Image](image5.png)     | ![Image](image6.png)         | ![Image](image7.png)     | ![Image](image8.png)           |
| 6        | ![Image](image9.png)     | ![Image](image10.png)        | ![Image](image11.png)    | ![Image](image12.png)          |
| 8        | ![Image](image13.png)    | ![Image](image14.png)        | ![Image](image15.png)    | ![Image](image16.png)          |
| 12       | ![Image](image17.png)    | ![Image](image18.png)        | ![Image](image19.png)    | ![Image](image20.png)          |
| 16       | ![Image](image21.png)    | ![Image](image22.png)        | ![Image](image23.png)    | ![Image](image24.png)          |
| 20       | ![Image](image25.png)    | ![Image](image26.png)        | ![Image](image27.png)    | ![Image](image28.png)          |

Fig. 4. Microscopic image of the indenters after the scratch experiments. The direction of sliding is from top to bottom.
Lubricated contacts corresponds to the formation of chips [3]. This happens because the reduced amount of adhesion between the strip and the indenter prevents the build-up of a lump and facilitates formation of a loose chip (see for example Fig. S3).

No clear difference was observed in terms of $\mu_{ap}$ values for the lubricated scratch experiments performed using the uncoated & Cr plated indenters at low loads ($< 16$ N). However, the scratch experiments carried out using Cr plated indenters exhibited lower friction at high normal loads, see Fig. 3 and Fig. S1. Furthermore, reduced material pickup was observed on the tip of the Cr plated indenters after the experiments compared to the uncoated ones (Fig. 4). It is known that the durability of a boundary lubricant is influenced by the bonding strength of the lubricant to the lubricated surfaces and by the rate of material transfer between rubbing surfaces [25]. Weakly adhered lubricants are easily removed by asperity contacts from the contacting interface, and hence, the lubricant fails. On the other hand, a boundary lubricant that adsorbs strongly on a solid surface will hold under extreme conditions and provide lubrication to high contact pressures. The lower friction coefficient of the lubricated experiments using Cr plated indenter at higher loads suggests that the chromium/chromium oxide layer promotes the formation of a strong tribolayer by reacting with the lubricant additives which prevented direct metal-metal contact to higher contact pressures and/or reduces adhesion between the contacting interfaces [30].

The friction coefficient of the dry scratch experiments at very low loads ($< 6$ N) stays stable for the whole wear track, see Fig. S1. The main wear mechanism at these loads is ploughing. This higher $\mu_{ap}$ compared to the lubricated scratch experiments at these loads (Fig. 3) can be explained by the higher shear strength of the dry contact interface. As indicated in the work of Challen & Oxley [4], a rapid increase in the $\mu_{ap}$ is observed when there is a transition from ploughing to wedge forming, which can happen when the interfacial shear strength is sufficiently large. In addition, unstable friction behavior with large fluctuations is observed at higher loads. The main cause of this fluctuation is lump growth and detachment. As the scratch proceeds, some material is pushed sideways by ploughing and a lump builds up in front of the indenter. The frictional resistance increases steadily as the lump accumulation progresses. Once the lump reaches a certain critical height, it is detached and gross sliding occurs at the bottom of the lump, which is indicated by a sharp drop in the $\mu_{ap}$ value. At this point, a stable transferred layer is formed on the indenter and remains there; no new wear debris is formed in the following sliding process. This was confirmed by performing multiple scratches with an indenter at the same load (see Fig. S4). A similar observation is reported in Ref. [3]. The critical length of the scratch for the lump to build up and detach may vary depending on the normal load and the local property of the substrate [28]. The critical length of the scratch for a lump to build up decreases as the normal load increases. This is clearly visible in the sliding distances of the friction curves to reach a maximum value in Fig. S1 (b, d) and on the wear track morphologies in Fig. S2 (c, d). Chrome plating the indenter did not have the same advantage in the dry scratch experiments as in the lubricated ones. In fact, dry scratch experiments performed using chrome coated indenters decreased the critical load of the transition from ploughing to wedge forming wear mode, see Figs. 3 and 5. This suggests that the unlubricated Cr plated surface-steel interface has a higher shear strength than the steel-steel contact and its favored performance in the lubricated experiments originates from its reaction with the lubricant and/or additives.

![Fig. 5. Confocal microscopy images of the representative wear tracks of the scratch experiments. The length of the scratches is 15 mm and the direction of sliding is from left to right.](image-url)
3.1.2. Wear mechanisms

The wear track morphology of the scratches can provide important information about the wear behavior. The height profiles of some of the scratches are given in Fig. 5. The wear track morphology of all the scratches is provided in Fig. S2 of the supplementary document. The scratch morphologies of the lubricated experiments are relatively smooth at all loads, whereas two distinct regions can be distinguished on the scratch morphology of the dry ones. While smooth scratches are observed at very low loads (< 6 N), a non-uniform wear with rough scratches indicating lump growth and detachment are seen at high loads. The non-uniform wear with rough scratches at high loads may arise from the dominant wear mechanism (i.e. wedge forming), which is not necessarily uniform, the stiffness of the system or the inhomogeneity of the material.

The abrasive wear behavior is controlled by the hardness, load and shape of the abrasive as well as the shear strength at the contact interface. Based on Challen & Oxley [4] slip-line theory and experimental results, Hokkirigawa & Kato [27] presented an abrasive wear mode diagram with three possible wear modes as a function of the non-dimensional parameters, degree of penetration ($d_p$) and interfacial shear factor ($f$). The degree of penetration is a measure of the ‘sharpness’ of asperities. It is defined as the ratio of the depth of the indentation to the half width of indentation in the sliding direction. Interfacial shear factor is defined as the ratio of the shear strength of the interface to the bulk shear flow stress of the deforming material. Its value ranges between 0 ≤ $f$ ≤ 1. The three possible abrasive wear modes are ploughing, wedge forming and cutting. In ploughing, the material from the groove is primarily pushed to the ridges without any actual material removal. In pure cutting all the material is removed as a wear debris without any side ridges. In wedge forming, a lump is formed in front of the indenter and wear particles are formed from the growth and detachment of lumps.

Both the lubricated and dry scratch experiments showed nearly ideal ploughing at low loads. In the lubricated scratch experiments, increasing the load led to a gradual transition of the wear mechanism from ploughing to predominantly wedge forming. This means that there is a local lubricant failure at some locations which is reflected by the material pickup seen on the indenters, see Fig. 4. The effect of a lubricant is to reduce $f$ so that ploughing wear is maintained at higher degrees of penetration. Furthermore, in the lubricated experiments, the Cr coating played an important role in terms of increasing the critical degree of penetration of the transition from ploughing to wedge forming by delaying local lubricant failure to higher contact pressures and/or reducing adhesion between the indenter and the strip. These wear behaviors are reflected on the wear tracks of the scratch experiments (Fig. 5 and Fig. S2) and the microscopic images of the indenters after the experiments (Fig. 4). In the dry scratch experiments, increasing the load was accompanied by a rapid transition of the wear mechanism from ploughing to wedge forming, which is clearly visible on the wear track morphology (Fig. S2) and in the friction curves (Fig. S1). This transition occurred at 8 N and 6 N load for the uncoated and Cr plated indenters respectively, which correspond to degrees of penetration of 0.17 and 0.15. The degree of penetration of the scratches was calculated instead of using measured height profiles. This was done to avoid measurement errors and after checking the measured scratch width values match with the calculated ones with a margin of error of ±12%. $d_p$ is calculated as a function of the applied normal load by assuming the front half of the indenter is in contact (see Fig. 2) and the mean contact pressure is equal to the indentation hardness of the deforming material (Ti-IF steel). An indentation hardness value of 91.5 ± 1.5 HV0.5 was measured from 10 Vickers hardness tests.

3.1.3. Degree of wear

In the scratch experiments, only part of the material displaced by the groove is removed as wear debris to cause material loss. The remaining material is displaced by plastic flow to form piled up ridges at the sides of the grooves. It is important to know the proportion of (groove) material removed to predict the quantity of wear particles generated. The relative amount of the material removed from the wear track can be described by the degree of wear ($d_w$). $d_w$ is defined as the ratio of the volume of material on the shoulders to the volume of material on the grooves. The details of degree of wear calculation from an average 2D scratch profile are provided in section 2.1.

Fig. 6 displays the degree of wear of the scratch experiments as a function of the degree of penetration. The value of $d_w$ is higher for the dry scratch experiments and its value increases with the increase of the degree of penetration. The $d_w$ of the dry experiments that are in wedge forming wear mode range between 0.15 to 0.31. Whereas, $d_w$ was less than 0.15 at all the applied normal loads in the lubricated scratch experiments. The $d_w$ of the scratch experiments when using Cr plated indenters was higher in the dry experiments and slightly lower in the lubricated ones compared to those using the uncoated indenters. This is consistent with the friction behavior (see Fig. 3).

The variation of degree of wear as a function of the degree of penetration depends on the specific material under investigation, the lubrication condition, and the type of active abrasive wear mode [31]. The degree of wear in pure ploughing is 0 as all the material from the groove is pushed to the ridges without any actual material removal. In pure cutting, all the material is removed as a wear debris without any side ridges ($d_w = 1$). Although theoretically no wear is assumed to occur in ploughing regime as all the material from the groove is pushed to the ridges without any actual material removal, wear on a much-reduced scale can occur at the interface. Similarly, not all the material is removed as wear debris in cutting wear mode. Kato and co-workers [3, 32,33] have experimentally studied the transitions in wear modes and the degree of wear in single asperity scratch experiments. They reported typical values of $d_w < 0.15$ for ploughing, $0.2 < d_w < 0.8$ for wedge forming, and $0.8 < d_w < 0.95$ for cutting. The strength of adhesion between the contacting bodies plays an important role in wedge forming wear mode. Lubrication decreases the degree of wear by lowering the interfacial shear strength, and consequently, the strain and damage on the contacting surfaces. Hence, when wedge forming occurs in dry and lubricated contacts, the degree of wear is expected to be smaller for lubricated cases.

Ploughing is the dominant wear mechanism in the lubricated experiments. As the degree of penetration increases, the contribution of ploughing decreases while material removal by wedge forming becomes more pronounced. The material extrudes from the sides of the contact giving rise to flakes which subsequently break off and produce wear...
particles (see Fig. S3), similar to that reported in Ref. [34]. On the other hand, wedge forming is the main wear mechanism in the dry scratch experiments and the material from the groove is removed by shearing of the wedge. However, due to the ductility of the material (i.e. Ti-IF steel), the chips do not break off but remain next to the track, resulting in lower experimental value of \( d_w \) compared to the ones in the literature. Similar observations are reported in Ref. [35].

According to literature [3,32,33], the degree of wear is expected to increase from 0 to a maximum value following an S-curve depending on the scratch hardness of the material. The S-curve was not observed in the current experiments. This is ascribed to the high ductility and work hardening property of the Ti-IF steel. Hokkirigawa et al. [33] showed that increasing ductility increases the critical degree of penetration of the transition from ploughing to wedge forming/cutting and it decreases the degree of wear within cutting regime. Significant work hardening can be expected at the vicinity of the scratch where the actual deforming and tearing takes place. Challen et al. [36] studied the effect of strain hardening on the critical degree of penetration of chip formation (abrasive) wear. They demonstrated that the critical degree of penetration of the transition of wear modes increases with an increase in the rate of hardening. Moreover, past research [37,38] have shown that work hardening influences not only the wear regime transition boundaries but also spreads the plastic zone further from the indenter reducing the pile-up around the indenter and making chip formation more difficult.

### 3.2. MPM simulations

#### 3.2.1. Friction coefficient

The abrasive wear behavior (i.e. the wear mode and the degree of wear) at single asperity contact is highly influenced by the shear strength at the contact interface and the sharpness of the indenter [27]. MPM scratch simulations were executed at several loads and at several interfacial shear strength values for a given normal load. This allows to study the influence of degree of penetration (asperity sharpness) and interfacial shear strength on the apparent friction coefficient as well as on the wear behavior of scratch simulations. The interfacial shear strength \( \tau \) in the MPM simulations was varied by changing the Coulomb friction \( \mu \). The relationship between the Coulomb friction and the interfacial shear strength is given by:

\[
\tau = \mu P
\]  

Where, \( P \) is the nominal contact pressure.

Typical friction curves of the scratch simulations at several interfacial shear strength values and the corresponding scratch morphologies...
are displayed in Fig. 7. While the friction curves are smooth at low \( \mu \) values, they exhibited fluctuation for larger values of \( \mu \) and/or when particle detachment occurs. The deformation takes place mostly by ploughing at low loads, as evidenced by the ridges to the side of the scratch as material is displaced (Fig. 7c). A wedge is formed ahead of the indenter at higher loads, which is manifested by the lump at the end of the scratches, and chips being formed in some cases. The volume of the lump ahead of the indenter increases with the increase of interfacial shear strength.

The relationship between the interfacial shear strength and the steady state apparent friction coefficient of the scratch simulations is shown in Fig. 8. The apparent friction coefficient increases as the normal load and the interfacial shear strength increase. The increase of the apparent friction coefficient with the normal load can be attributed to the increase of degree of penetration, which in turn increases the deformation component of friction due to the increased strain and strain hardening experienced by the substrate [14]. The apparent friction coefficient increases almost linearly at low interfacial shear strength and low loads. However, a slight deviation from the linear behavior is observed once particle detachment starts to occur at higher loads. Earlier studies on the contribution of interfacial shear strength to the overall friction force pointed out that the deformation and the adhesion component of friction seem to be simply additive only when one term is relatively small compared with the other one [26]. The simulation results clearly showed that the interfacial shear strength plays a vital role in the apparent friction value of the scratch experiments.

It has been shown that the shear strength of a boundary layer varies with the contact pressure [24,26]. To directly compare the friction and wear behavior of the scratch experiments and the numerical scratch simulations, the interfacial shear strength value at the given contact pressure must be known. However, measuring the interfacial shear strength of a lubricant and dry contact interfaces is very difficult. Besides, when calculating the commonly used non-dimensional parameter interfacial shear factor \( f \), the effective shear strength of the deforming material should be considered. Since the material in the present study strain hardens, the shear strength of the material does not remain constant and its value depends on the accumulated strain. This requires the average flow stress to be obtained from stress-strain curve of the deforming material. Considering the above factors, classical Coulomb friction (\( \mu \)) is used in this work to describe the shear strength of the contact interface. Since the shear strength of the contact interface was not experimentally measured, the \( \mu \) of the scratch experiments was estimated by equating the measured apparent friction coefficient of the experiments to that of the simulations at the given normal load. More specifically, the Coulomb friction of MPM simulations for direct comparison with the experiments was estimated so that it yields the same \( \mu_{ap} \) value as the experimental scratch experiments for the given load and lubrication condition. This was achieved by linear interpolation of the MPM simulation results in Fig. 8 to that of the measured \( \mu_{ap} \) (Fig. 3).

The Coulomb friction of the scratch experiments estimated from MPM simulations are presented in Fig. 9. The \( \mu \) of the lubricated scratch experiments remained very low (\( \mu < 0.05 \)) at all loads. Despite the material transfer seen to the indenter at higher loads, the lubricant plays a very important role in reducing adhesion and keeping the interface shear stress low. The positive influence of Cr plating in reducing friction for the lubricated experiments is evident at higher loads.

In dry contacts, the shear strength of the contact interface is expected to be very close to the shear strength of the softer counterpart [33]. This relationship is manifested in the measured friction values of the dry scratch experiments. The \( \mu \) of the dry scratch experiments reached the theoretical maximum value of \( 1/\sqrt{3} \) (according to von Mises yield criterion of the workpiece material) in some cases. In those cases, the interface shear strength exceeds the shear strength of the strip material. As a result, the strip material welds firmly on the indenter surface and tearing takes place on the strip. The MPM simulation results revealed that nearly all the frictional energy is dissipated in the plastic work of the deforming surface in the lubricated scratch experiments. By contrast, the frictional resistance in the dry scratch experiments is primarily due to shearing of the adhesion junction and from tearing the deforming substrate.

3.2.2. Wear profile & degree of wear

Agreement between the experiments and the simulations can be judged by comparing the wear groove profiles and the degree of wear values at the same load. Fig. 10 presents a comparison of the wear grooves of the scratch experiments and MPM scratch simulations. The MPM simulations overestimated the scratch width and depth as well as the material displaced to the sides as a consequence of plastic deformation. This overestimation implies that the investigated Ti-IF steel is in reality less ductile than considered in the MPM simulations. The reason for the overestimation is likely caused by the use of the bulk material behavior (constitutive equation) in the MPM scratch simulations, which may be different from the surface behavior on the top few micrometers. Another reason could be that the severe strain hardening under the complex stress state in the vicinity of the scratches might not be properly
taken into account in the constitutive model. Regardless, MPM simulations capture junction growth very well, see for example Fig. 9b. The real area of contact, and as a result, the indentation depth in the scratch experiments increases as the interface shear strength is increased for a given normal load, for instance in the absence of a lubricant [39].

The degree of wear results of MPM simulations are presented in Fig. 11. Compared with the experimental results (Fig. 6), it can be clearly seen that the MPM simulations underestimate the degree of wear. This underestimation is also reflected in the shoulders of the wear grooves (Fig. 10). There is no physically-based damage model in the MPM simulations. Material detachment is simply taken into account by using a connectivity criteria between neighboring particles [20]. The interaction between the MPM particles is lost as the separation between neighboring particles exceeds the defined kernel radius. This connectivity criteria might be unsatisfactory when it comes to predicting the mechanical behavior of ductile materials experiencing damage in compression. In order to study the variation of degree of wear in MPM simulations and compare with the experiments, a physically-based material removal criterion should be defined to allow debris separation from the deforming substrate. The following section discusses wear particle formation criteria proposed in literature and the approach followed in this study.

3.2.3. Wear particle formation criterion

A wear particle separation criterion should consider damage mechanisms and damage evolution until ultimate debris formation. Some of the physical quantities used as a damage criterion include effective plastic strain, strain energy density, fracture energy, shear failure criterion, displacement, stress, and adiabatic shearing [11]. Several authors have proposed, for ductile materials, whether a wear debris particle is produced and how much material is removed depends on the accumulated plastic shear strain (also known as ratchetting failure) [40–43]. Other researchers examined the relationship between the amount of material removed and the degree of penetration in abrasive wear based on the concept of low cycle fatigue [44–49]. In the former case, it is proposed that material is detached and wear particles are produced only when the accumulated surface plastic strain produced by the asperity interaction exceeds a critical value comparable to the strain to failure in a monotonic test. The assumption in the latter case is that the behavior of the material that experiences deformation in abrasive wear resembles failure of material during low cycle process: if the plastic strain amplitude is less than the limit plastic strain, then the material does not fail and no material removal occurs. Only when some critical amount of accumulated strain is reached can the material be removed. Some authors [40,42] argue that these two failure modes are competitive, so that the actual failure corresponds to the one which would occur first. Note that both theories are for abrasive wear processes involving multiple loading cycles. In the current study, however, a single loading cycle is considered.

Although there seems to be no consensus on the underlying principle of failure mechanism for abrasive wear processes involving multiple loading cycles, and while it is not the focus of the current study, both approaches consider abrasive wear as a plastic strain controlled process. Hence, critical equivalent plastic strain $\varepsilon_c$ can be used as a criterion to define wear particle formation. More specifically, a wear particle will be formed if the equivalent plastic strain $\varepsilon$ exceeds the critical value $\varepsilon_c$. The criterion for failure can then be defined as $\varepsilon / \varepsilon_c \geq 1$. In this work, it was attempted to estimate the critical plastic strain ($\varepsilon_c$) needed for the material under investigation (Ti-IF steel) by relating the degree of wear in the scratch experiments and the plastic strain produced by asperity interaction in MPM simulations. The analysis consists of the following steps. First, MPM particles whose value exceeds the defined $\varepsilon_c$ are considered to be removed as wear particles and deleted in post processing. This is schematically illustrated in Fig. 12. Next, the average
cross-section of the scratch wear track is determined. Finally, the degree of wear is calculated using equation (1).

Various studies [43,45,46,50–52] demonstrated that large strains are involved in wear particle formation and the critical strain needed for failure to occur depends on the specific material under consideration and the compressive stress conditions. Zum Gahr [52] performed scratch tests using pyramidal indenter on polished surfaces of different materials. He measured a true strain of at least 3 on the wear debris and worn surfaces of brass. Challen et al. [45,50] have shown that, for Al-Mg alloy, fracture occurs only when the effective shear strain put into the surface is higher than 10 from scaled up model asperity experiments, in which a hard wedge is indented into the surface of a softer specimen and slid along it. Moore and Douthwaite [51] reported very large strains in the range of 2.5–8 on worn surfaces of a copper-silver composite in abrasion tests. Yanyi et al. [46] performed wear tests in which a hard tool steel wedge was pressed against the periphery of a rotating bar of soft metal and found that a monotonic effective shear strain of 9.96 for aluminum and 5.19 for brass fit their experimental values. In a recent article, Tyfour et al. [41] reported an accumulated shear strain of 30 below the contact surface in sliding experiments in which a flat ended cylindrical indenter made of hot drawn low carbon steel was run against a rotating hardened steel disc. The reported high level of ductility and large strains are attributed to the high hydrostatic compression which clearly seen that at each atomistic stress. Such large plastic strains on the wearing surfaces are possible due to the high hydrostatic compression which greatly enhances ductility.

In the current work, the degree of wear values obtained in the scratch experiments are used as a benchmark to estimate the critical plastic strain needed to produce wear particles for the material under consideration (Ti-IF steel). To show the variation of the equivalent plastic strain (as a function of the normal load and friction coefficient (interfacial shear strength), a snapshot of MPM simulations for several loads are provided in Fig. 13. As expected, higher load and higher interfacial shear strength lead to higher strains in the sheet. Increasing the Coulomb friction from 0 to 0.8 increased the maximum plastic strain experienced underneath (and in the surrounding of) the scratch by three to six-fold, from 0.4 to 2.8 at 2 N and from 3.8 to 12.7 at 20 N normal load.

Analysis of the degree of wear was performed for several $\varepsilon_c$ values ranging from 2–8. The calculated degree of wear values for several $\varepsilon_c$ and $\mu$ values are presented in Fig. S5 of the supplementary material. It can be clearly seen that $\varepsilon_c$ of 2 greatly overestimates the degree of wear (Fig. S5a), whereas $\varepsilon_c$ values greater than 5 underestimate the degree of wear (Fig. S5 d-f). A critical plastic strain in the range of 3–4 gives a reasonable agreement with the experiments in terms of predicting the degree of wear (Fig. S5 b, c). Although the materials and loading conditions are different, these critical strain values are in the range reported by other researchers who have conducted scratch and abrasion tests [51,52] but lower than the values reported elsewhere for multiple loading cycles or scaled up model asperity experiments [41,45,46,50]. This suggests that failure occurs at lower critical strain in monotonic loading than repeated contacts. It must be noted that the occurrence of wear particle formation will depend on the magnitude of the plastic strain imparted to the deforming material and the magnitude of the corresponding hydrostatic stress. Such large plastic strains on the wearing surfaces are possible due to the high hydrostatic compression which greatly enhances ductility.

Although the trends are consistent between the scratch experiments and the MPM simulations, the absolute values of $d_w$ are not completely comparable. Only one scratch experiment was performed for each load and a scratch of length <1 mm was used to calculate $d_w$ in the case of MPM simulations in contrast to 12 mm in the experiments. More scratch experiments are necessary to conduct a statistically significant comparison. Moreover, the accuracy of MPM scratch simulations with regard to strain distribution can be improved further by refining the particle size at the expense of higher computational cost. One limitation of the MPM model is that the MPM particles do not stick to the indenter due to the absence of adhesion between the indenter and the substrate in the current contact implementation. In addition, one consequence of the failure introduced as post processing in the simulations is that the failed particles did not influence the deformation or friction coefficient. In reality, the failed particles are detached from the strip at the moment of failure, which are either removed from the contact interface as loose wear debris, stick to one of the contacting bodies, or stay in the contact and act as a third body abrasive. All these phenomena can influence the strain distribution under the sphere and the friction coefficient.

In summary, the current study provides a way to include the three-dimensional details of strain induced by asperity interaction in calculating the degree of wear in single asperity abrasive contacts. Early
studies that attempted to follow similar approach did not take into account the three-dimensional nature of the deformation zone [47,48]. They utilized either simplistic analytical equations [48] or two dimensional FEM simulations [47] to determine the average strain induced in the deformation zone. The MPM model can be extended to study the scratch behavior in a range of different geometries, sliding speed and temperature, representative of the typical rolling processes in the industry, which are too difficult to be realized in the experiments. The single asperity contact can be further extended to macro scale contact with multiple asperities using a statistical approach. This, in turn, can be combined with rolling parameters to predict strip surface cleanliness (i.e., strip wear rate) in cold rolling processes.

4. Conclusion

Scratch experiments and numerical modelling were combined to study the formation of wear particles at a single asperity contact. The lubricated scratch experiments were ploughing dominated, whilst wedge forming was the main wear mechanism in the dry scratch experiments for both uncoated and hard Cr plate indenter. The lubricated scratch experiments showed both lower friction and degree of wear compared to the unlubricated condition. The lubricated scratch experiments using Cr plate indenter exhibited reduced friction and material transfer at high loads. On the other hand, the scratch experiments performed using Cr plate indenter under dry conditions showed a transition to unstable sliding at lower normal load than the ones using uncoated indenter. The scratch experiments suggested that the beneficial influence of chrome plating on wear particles formation stems from its interaction with the lubricant as the same influence was not observed in the dry experiments.

MPM scratch simulations revealed that almost all the frictional resistance arises from deforming the substrate in the case of an interfacial shear strain corresponding to the lubricated contact. In contrast, the frictional resistance in the case of dry scratch experiments is primarily due to shearing of the adhesion junction and tearing the deforming body. Additionally, MPM simulations demonstrated that increasing the interfacial shear strain from 0 to 80% of the contact pressure increases the maximum plastic strain experienced underneath and in the surrounding of the scratch by a factor of three to six. Using the degree of wear of the scratch experiments as a benchmark, the critical plastic strain needed to produce wear particles was found out to be between 3 to 4 for the investigated Ti-IF steel.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

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