On Osmotic Pressure in Hyperelastic Biphasic Fiber-Reinforced Articular Cartilage

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Articular cartilage belongs to the group of connective tissues and ensures a low-friction performance of the human musculoskeletal system. It is a complex multiphase material with two main components: a collagen fiber-reinforced porous solid and a liquid phase. The mechanical behavior of articular cartilage is decisively determined by the interaction between the highly anisotropic, inhomogeneously distributed collagen fiber network, the isotropically distributed proteoglycans and the interstitial fluid. The electrically negatively charged proteoglycans generate an electrical gradient that creates an osmotic negative pressure on the fluid which causes swelling of the cartilage. This leads to mechanically prestressed collagen fibers resulting in a decrease of total pressure occurring inside the cartilage. An increasingly common joint disease is osteoarthritis (OA), which severely impairs everyday life, especially in the elderly population. It is characterized by a loss of matrix stiffness as well as by a reduced tension stiffness and viscoelasticity of the collagen fibers. The aim of the developed model is to describe the previously mentioned material properties of cartilage as precisely as possible. The biphasic model uses the Theory of Porous Media for homogenization and contains an incompressible poroelastic solid matrix reinforced with collagen fibers and an incompressible pore fluid. By including the occurring osmotic pressure, the correct initial stress state of the imaged configuration is considered.

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1 Introduction

Osteoarthritis (OA) is a degenerative joint disease which mainly affects articular cartilage. By changing the structure of the proteoglycans, the osmotic pressure is reduced, leading to more abrasion and finally to a defibration of the collagen fibers [1, 2]. This in turn causes a reduced tensile strength and viscoelasticity in fiber direction. Frictionless performance can no longer be guaranteed, which ultimately leads to a loss of function of the joint. OA is one of the most common musculoskeletal conditions and it is expected that the numbers will further increase due to obesity and increasing life expectancy [3]. Since modern treatments mainly treat symptoms or involve surgeries, a more precise description of the behaviour of healthy and damaged cartilage is needed to counteract at an early stage. To this end, a hyperelastic biphasic model is developed. Using non-invasive imaging, the initial symptoms will be recorded and examined in order to assess the development of the disease through simulations. This opens up the possibility for patients to receive specific treatments to contain the progression of the disease.

Articular cartilage is an intricate multiphase material consisting of a porous solid saturated with liquid and reinforced with collagen fibers. The mechanical properties mainly result from the interaction of the isotropically distributed proteoglycans and the extremely anisotropic, inhomogeneously distributed collagen fiber network. The proteoglycans are bound in the solid matrix and are characterized by electrically charged ions. The electrical gradient between these ions and the electrolytes in the liquid produces an osmotic negative pressure. The solid phase swells until the inner chemical equilibrium is established and causes a certain prestressing of the collagen fibers. It should be noted that this prestress is not caused by any load but occurs continuously inside the cartilage.

2 Numerical model

The numerical model describing the material behaviour of articular cartilage given here is based on the one presented in [4–6]. In it, the Theory of Porous Media (TPM, see e.g. [7, 8]) is utilized for homogenization. TPM is based on the mixture theory where the individual constituents are statistically distributed yielding a homogenized model. The total body \( \varphi \) consists of \( \kappa \) different phases \( \varphi^\alpha \):

\[
\varphi = \sum_{\alpha=1}^{\kappa} \varphi^\alpha \quad \text{with} \quad \kappa \in \{S,F\},
\]

with \( \varphi^S \) being the porous solid phase and \( \varphi^F \) the interstitial fluid phase. The proteoglycans in the solid phase have an excess of fixed electrically charged ions and thus generate an electrical gradient. In order to compensate for this, the electrolytes in
the liquid generate an osmotic pressure, which leads to swelling of the solid phase and places the collagen fibers under tensile stress. In [9], the significant influence of this phenomenon on cartilage behaviour was demonstrated. In order to include all relevant parts, the Helmholtz free energy $\psi$ of the solid phase consists of a part for the isotropic matrix $\psi_{IM}^S$, a part representing the fiber network $\psi_{FN}^S$ and finally the part for the osmotic strain energy $\psi_{OP}$, cf. [9]:

$$\psi^S = \psi_{OP}^S(J_S) + (1-\nu)\psi_{IM}^S(J_S, I_1) + \nu\psi_{FN}^S(C_S, M)$$

with

$$\psi_{OP}^S = R\theta_{\psi OS}^S U_J^S \left[ \frac{2c_m}{c_m^r} - \sqrt{\frac{4(c_m^r)^2 + (c_m^f)^2}{c_m^r}} + \sinh\left(\frac{c_m^f}{2c_m^r}\right) \right],$$

$$\psi_{IM}^S(J_S, I_1) = \frac{1}{\rho_{\psi OS}} \left[ U(J_S) + \frac{1}{2} \mu^S (I_1 - 3) \right],$$

$$\psi_{FN}^S(C_S, M) = \frac{1}{\rho_{\psi OS}} \int_{\Omega} \rho(M) \left[ k_1 \frac{1}{2k_2} \left( \exp[k_2(I_4 - 1)^2] - 1 \right) \right] \mathcal{H}(I_4 - 1) d\Omega.$$

In order to enable sample-specific modeling, the local orientation and distribution of the collagen fibers is determined experimentally. This is done by means of diffusion tensor MRI and yields the angular density $\rho(M)$, where $M$ is the direction vector of the fibers.

The osmosis occurring inside the cartilage does not only lead to a term for the energy function but also to a second aspect. Due to the osmotic pressure, the in-vivo and in-vitro state of cartilage is subject to a constant, chemically induced prestressed state. As a result, the actual configuration measured with DT-MRI is not stress-free, but prestressed, even for the unloaded case. In continuum mechanics, however, the usual starting point is from a stress free reference configuration. Therefore, the correct initial stress state under imaged configuration is determined by applying a backward displacement method [9].

### 3 Results

The constitutive model is implemented in the finite element program FEAP (University of California at Berkeley, CA, USA). To solve the governing equation system Taylor Hood elements with quadratic shape functions are used for the displacement $u$ and linear shape functions for the fluid pore pressure $p$. In [9], the test setup of a stress-relaxation test under unconfined compression is described. It is discovered that the prestressed state has a significant influence on the behaviour. The prestressing leads to the fact that even under compressive load the deep zones of the cartilage are partly still under tension. If the osmotic influences are neglected, the cartilage shows a worse behaviour and is exposed to greater compressive stresses, some of which remain even after unloading. These results underline the significant influence of the osmotically prestressed condition on cartilage behaviour. The prestressing counteracts the hydrostatic pressure caused by the load and thus the total pressure in the cartilage is reduced.

### 4 Conclusion

The complex material behaviour of articular cartilage is described by a two-phase model. The non-linear, anisotropic behaviour and the mechanical influences of the osmotic pressure were taken into account. Also, the depth-dependent fiber composition was determined by DT-MRI, enabling sample-specific modeling. The correct initial stress state under imaged configuration was determined by applying an iterative inverse method. Overall, the model is able to capture the behaviour of articular cartilage. It was also demonstrated that prestressing has a positive influence on the behaviour and therefore, has to be considered for an accurate representation of cartilage behaviour.

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