Interpretation of the First Data on Central Au+Au Collisions at 
$\sqrt{s} = 56$ and 130 A GeV

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Abstract

We compare three semi-microscopic theories to the first data on particle production in central Au+Au collisions taken at RHIC by the PHOBOS collaboration as well as to existing data on central Pb+Pb collisions taken at the SPS by the NA49 collaboration. LEXUS represents the SPS data quite well but not the RHIC data, whereas the wounded nucleon model does the opposite. The collective tube model fails to describe any of the data. This suggests a transition in the dynamics of particle production between $\sqrt{s} = 17$ and 56 A GeV as one goes from the SPS to RHIC.

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The first data from the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory has been presented by the PHOBOS collaboration \[1\]. Their result is that the numbers of electrically charged hadrons per unit of pseudo-rapidity, \(dN_{ch}/d\eta\), produced in the 6% most central Au+Au collisions at \(\sqrt{s} = 56\) and 130 A GeV and averaged over the interval |\(\eta\)| < 1, are 408\(\pm\)12(stat)\(\pm\)30(syst) and 555\(\pm\)12(stat)\(\pm\)35(syst), respectively. Pseudo-rapidity is defined as \(\eta = \frac{1}{2} \ln \frac{(1 + \cos \theta)}{(1 - \cos \theta)}\), where the angle \(\theta\) is measured with respect to the beam axis. The previous maximum energy for heavy ion collisions was \(\sqrt{s} = 17\) A GeV for Pb+Pb collisions at the SPS at CERN. Particle production in a high energy heavy ion collision is one of the fundamental observables. In this paper we report on a comparison of three semi-microscopic theories with both the RHIC and the SPS data in an attempt to understand the basic dynamics of these collisions. These theories are (1) a Linear EXtrapolation of Ultrarelativistic nucleon-nucleon Scattering to nucleus-nucleus collisions (LEXUS) \[3\], (2) the Wounded Nucleon Model (WMN) \[5\], and (3) the Collective Tube Model (CTM) \[4\]. We refer to these as semi-microscopic theories because they are based on input from nucleon-nucleon collisions but are not computed with QCD. Below we briefly describe these theories; for details the reader should consult the original papers.

The LEXUS assumes that the nucleons follow straight-line trajectories, striking nucleons from the other nucleus that lie in their path and interacting with them exactly as in free space. Hadrons are produced in every nucleon-nucleon collision according to the parametrization

\[
N_{ch}^{NN}(s) = 1.568 \left( \sqrt{s} - M_{\text{min}} \right)^{3/4} / s^{1/8} \sim 1.568 s^{3/4} \quad (1)
\]

with \(\sqrt{s}\) being the center-of-mass energy in that nucleon-nucleon collision and is measured in GeV. With \(M_{\text{min}} = 2m_N + m_{\pi}\) this simple function represents particle production in nucleon-nucleon collisions up to \(\sqrt{s} = 62\) GeV, excluding single diffractive events, as well as proton-antiproton collisions at 200 GeV. It is also known that in nucleon-nucleon collisions the hadrons are produced with a Gaussian rapidity distribution centered at mid-rapidity and with a dispersion given by the formula
Unlike pseudo-rapidity, rapidity requires knowledge of the mass of the particle and is defined as 

\[ y = \frac{1}{2} \ln \left[ \frac{(E + p_z)}{(E - p_z)} \right], \]

where \( E \) is the energy and \( p_z \) is the momentum along the beam axis. When the mass goes to zero \( \eta \) and \( y \) coincide; for pions their difference is typically very small. As a nucleon cascades through the other nucleus it loses energy, and this is taken into account via an evolution equation which is solved numerically. All parameters in LEXUS are fit to nucleon-nucleon data and nothing should be adjusted to fit nucleus-nucleus data. The above information is folded together with a constant inelastic nucleon-nucleon cross section \( \sigma_{\text{inel}} \) and with a realistic density distribution for the colliding nuclei.

The WNM defines a nucleon to be wounded the first time it undergoes an inelastic collision with a nucleon from the other nucleus. A wounded nucleon is assumed to produce 1/2 of the average charged hadrons in a nucleon-nucleon collision at the same energy. Once it is wounded it cannot produce any more, although it can strike an unwounded nucleon and that one can produce particles. The total number of charged hadrons produced by \( n_P \) wounded projectile nucleons and \( n_T \) wounded target nucleons is

\[ N_{\text{ch}}(n_P, n_T) = \frac{n_P + n_T}{2} N_{\text{ch}}^{NN}(s). \]

These hadrons are assumed to be distributed in rapidity in a Gaussian way, centered at the nucleon-nucleon rest frame, and with a dispersion given by eq. (2). There is no energy loss assigned to the nucleons as they strike and wound other nucleons. Otherwise the geometrical folding to compute the number of wounded nucleons is standard and is done in exactly the same way as LEXUS.

The CTM describes a nucleus-nucleus collision as a set of aligned tube-tube collisions. One tube is taken from the projectile nucleus and one from the target. The cross sectional area of the tubes is \( \sigma_{\text{inel}} \). If one tube contains \( n_P \) and the other tube \( n_T \) participants then the center-of-mass energy available for particle production is

\[ s(n_P, n_T) = 4n_Pn_Tp_{\text{cm}}^2 + (n_P + n_T)^2m_N^2, \]
where $p_{cm}$ is the beam momentum of an individual nucleon in the nucleus-nucleus frame. The number of charged hadrons produced in this tube-tube collision is the same as that produced in an elementary nucleon-nucleon collision with the same available energy (baryon masses subtracted). That is, eq. (1) is applied with $M_{\text{min}} = (n_P + n_T)m_N + m_\pi$. This means that knowledge of particle production in nucleon-nucleon collisions at energies much higher than 200 GeV is required for RHIC! There is no experimental information on nucleon-nucleon collisions above $\sqrt{s} = 62$ GeV; higher energies should be measured in the future at RHIC. There is data on proton-antiproton collisions from the UA5 collaboration at CERN [6]. The average multiplicity, exclusive of single diffractive events, may be represented by the function

$$N_{\text{ch}}^{P\bar{P}}(s) = 22 + 1.7 \ln \left( \frac{\sqrt{s}}{200} \right) + 5.1 \ln^2 \left( \frac{\sqrt{s}}{200} \right)$$

in the range $200 \leq \sqrt{s} \leq 900$ GeV. (The best fit would give the multiplicity at 200 GeV as 21.4, not 22, but the latter number is chosen to match on continuously with the parametrization of eq. (1); it is still within the error bars of UA5.) The CTM seems somewhat ambiguous when it comes to describing the rapidity distribution of the produced hadrons. The most sensible approach is to assume it is a Gaussian peaked at the center-of-mass frame of the tube-tube system, that is, at $y_{cm}(n_P, n_T)$ determined by

$$\sqrt{s(n_P, n_T)} \sinh (y_{cm}(n_P, n_T)) = (n_P - n_T)p_{cm}.$$  

The dispersion of the Gaussian is assumed to be of the same form as in eq. (2), namely

$$D^2(n_P, n_T) = \ln \left( \frac{\sqrt{s(n_P, n_T)}}{(n_P + n_T)m_N} \right) = D^2_{NN} + \frac{1}{2} \ln \left( \frac{4n_Pn_T}{(n_P + n_T)^2} \right).$$

For collisions between equal size nuclei the last factor is on average zero. As with the WNM the geometrical folding to compute the number of projectile and target nucleons is done the same way as with the LEXUS.

At this point it may be worth pointing out that the LEXUS, the WNM, and the CTM all reproduce nucleon-nucleon collisions by construction. It is only the extrapolation to
nucleus-nucleus collisions that is different. Since the same nucleon-nucleon input and the same geometrical folding for nucleus-nucleus collisions is used in all three theories, any differences can only arise because of the different dynamics assumed as described above. In particular there are no free parameters in any of these theories. One caveat is the numerical value of the inelastic nucleon-nucleon cross section $\sigma_{inel}$. For the SPS energy range and below the total and elastic cross sections are relatively constant at 40 and 8 mb, respectively. For the theories considered here it makes the most sense to exclude the single diffractive part of the inelastic cross section. The LEXUS uses 24 mb, corresponding to hard inelastic collisions \[\text{[3]}\]. Above the SPS energy range the total and elastic cross sections rise to about 50 and 10 mb, respectively, at the full RHIC energy of $\sqrt{s} = 200$ GeV. In all of the calculations presented in this paper we use 30 mb for the inelastic cross section in the WNM and the CTM because that is what the original authors of those models used. We have verified by direct calculation that the dependence of $dN_{ch}/dy$ on the value chosen for this cross section is negligible, typically a few percent.

In Fig. 1 we plot the predictions for $dN_{ch}/dy$ from the three theories and the available data. The first panel is for the 5% most central Pb+Pb collisions at the SPS with $\sqrt{s} = 17$ A GeV. The data from NA49 contains identified electrically charged kaons and pions and is truly the rapidity density. The next three panels are for the 6% most central Au+Au collisions at RHIC with $\sqrt{s} = 56$, 130 and 200 A GeV. The data presented by PHOBOS does not identify the particles and is $dN_{ch}/d\eta$. There is a Jacobian relating the rapidity and pseudo-rapidity distributions. For a hadron of mass $m$ and momentum $p$ emerging at 90 degrees with respect to the beam, $y = \eta = 0$, the relationship is

\[
\frac{dN_{ch}}{d\eta}(\eta = 0) = v \frac{dN_{ch}}{dy}(y = 0)
\]

where $v = p/\sqrt{p^2 + m^2}$ is the velocity of that particle. If most of the charged particles are pions with an average transverse momentum of 3 times their mass we have $v = 0.95$. The two data points from PHOBOS plotted in Fig. 1 are the numbers quoted at the beginning of the paper multiplied by 1.05 to convert pseudo-rapidity to rapidity density. This is a small
effect and does not actually affect any conclusions, assuming that pions are indeed the most abundant charged hadrons.

The ordering of the three theories is easily understood. The LEXUS produces more particles than the WNM because in the latter theory a nucleon, once wounded, cannot itself produce any more particles. On the contrary, in the LEXUS a struck nucleon loses momentum but continues to produce particles on every subsequent collision. The CTM produces fewer particles than the WNM as a consequence of the fact that particle production increases more slowly than $\sqrt{s}$. For example, for a collision with $n_P = n_T \equiv n$ the ratio of particles produced by the WNM relative to the CTM is $\sqrt{n}$ if eq. (1) is used, and is even greater if eq. (5) comes into play.

The LEXUS represents the NA49 data very well, but predicts about 60% more particles than is measured by PHOBOS at both energies. The WNM represents the PHOBOS data reasonably well but predicts only about 70% of the particles observed by NA49. The CTM predicts far too few particles at all of these energies. What interpretation can we give to these results? One obvious possibility is that particle production at 17 A GeV is dominated by incoherent nucleon-nucleon interactions, but as the energy rises to 56 A GeV destructive interference plays an increasing role resulting in a reduction in the number of particles produced. This is not the only interpretation one might give but it is the most obvious one in the context of these three theories. A caveat is that once produced, the particles (whether they initially be considered quarks and gluons or hadrons) can interact with one another and change the total particle number. Generally one expects that the number of observed particles can be higher than originally produced, not lower, on account of the nondecrease of entropy. Even if that occurs it cannot change our conclusion. If extra particle production was invoked such as to make the WNM model agree with the NA49 data then it would predict too many particles compared to PHOBOS.

It will be very interesting to identify the charged hadrons measured by PHOBOS. It will also be very interesting to discover whether any other observables display a similar change in going from SPS to RHIC energies. The HIJING model [7] has already been compared to
the PHOBOS data in their paper [8] and comparisons to other theories will surely follow. With RHIC operational the game is afoot at last!

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FIGURE CAPTIONS

Figure 1: The charged hadron rapidity distributions for central Pb+Pb collisions (SPS) and Au+Au collisions (RHIC) at the indicated energies. The SPS data is from NA49 [2] and the RHIC data is from PHOBOS [1]. The solid (top) curves are LEXUS, the dashed (middle) curves are the wounded nucleon model, and the dot-dashed (bottom) curves are the collective tube model.
$Pb+Pb \ (\sqrt{s} = 17 \ A GeV)$

$Au+Au \ (\sqrt{s} = 56 \ A GeV)$

$Au+Au \ (\sqrt{s} = 130 \ A GeV)$

$Au+Au \ (\sqrt{s} = 200 \ A GeV)$