Analyses of residual accelerations for TianQin based on the global MHD simulation

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Abstract.
TianQin is a proposed space-based gravitational wave observatory. It is designed to detect the gravitational wave signals in the frequency range of 0.1 mHz – 1 Hz. At a geocentric distance of 10^5 km, the plasma in the earth magnetosphere will contribute as the main source of environmental noises. Here, we analyze the acceleration noises that are caused by the magnetic field of space plasma for the test mass of TianQin. The real solar wind data observed by the Advanced Composition Explorer are taken as the input of the magnetohydrodynamic simulation. The Space Weather Modeling Framework is used to simulate the global magnetosphere of the earth, from which we obtain the plasma and magnetic field parameters on the detector’s orbits at $\varphi_s = 0^\circ$, 30$^\circ$, 60$^\circ$ and 90$^\circ$, where $\varphi_s$ is the acute angle between the line that joins the sun and the earth and the projection of the normal of the detector’s plane on the ecliptic plane. We calculate the time series of the residual accelerations and the corresponding amplitude spectral densities on these orbit configurations. We find that the residual acceleration produced by the interaction between the TM’s magnetic moment induced by the space magnetic field and the spacecraft magnetic field ($a_{M1}$) is the dominant term, which can approach $10^{-15}$ m/s$^2$/Hz$^{1/2}$ at $f \lesssim 0.2$ mHz for the nominal values of the magnetic susceptibility ($\chi_m = 10^{-5}$) and the magnetic shielding factor ($\xi_m = 10$) of the test mass. The ratios between the amplitude spectral density of the acceleration noise caused by the space magnetic field and the preliminary goal of the inertial sensor are 0.34 and 0.07 at 1 mHz and 10 mHz, respectively. We discuss the further reduction of this acceleration noise by decreasing $\chi_m$ and/or increasing $\xi_m$ in the future instrumentation development for TianQin.
Keywords: gravitational waves, space plasma, test mass, acceleration noise

1. Introduction
Since the direct detection of the gravitational waves (GWs) from the merger of a pair of stellar mass black holes (GW150914) by the two advanced detectors of the Laser Interferometer Gravitational-Wave Observatory (LIGO) [1], more than ten GW events have been detected by advanced LIGO and advanced Virgo [2, 3, 4, 5, 6, 7]. Recently, KAGRA [8] has joined the ground-based GW detector network. Due to the unshieldable impacts from the environment, namely seismic noise and gravity gradient noise, it is very difficult for these terrestrial laser interferometers to detect GWs with the frequencies lower than 10 Hz. However, in the low frequencies, there are rich sources of GWs that can be used to study the fundamental physics, astrophysics and cosmology [9]. The aim of space-borne laser interferometers is to explore the GWs in the millihertz range (0.1 mHz–1 Hz). Several projects, e.g., LISA [10], TianQin (TQ) [11], DECIGO [12], ASTROD-GW [13], g-LISA [14], Taiji (ALIA despoped) [15] and BBO [16] have been proposed and are currently under different stages of study and development.

Both LISA and TQ have three drag-free spacecraft which compose a nearly equilateral triangular constellation. Different from LISA, TQ’s spacecraft will be deployed in a geocentric orbit with an altitude of $1 \times 10^5$ km from the geocenter, which makes the distances between each pair of spacecraft $\approx 1.7 \times 10^5$ km [11]. The normal of the detector’s plane formed by the three spacecraft points toward the candidate ultracompact white-dwarf binary RX J0806.3+1527 [17]. The three spacecraft are interconnected by infrared laser beams and form up to three Michelson-type interferometers. The heterodyne transponder-type laser interferometers are used to measure the displacements of the test masses (TMs) to the accuracy of $10^{-12}$ m/Hz$^{1/2}$ in millihertz. The disturbance reduction system is designed to reduce the non-conservative acceleration of each TM (Pt-Au alloy, $m = 2.45$ kg) down to $10^{-15}$ m/s$^2$/Hz$^{1/2}$ in millihertz [11]. The nominal orbit and a set of alternatives for TQ have been optimized such that the stability of the orbits, in terms of the variations of arm lengths, breathing angles and relative range rates, can meet the requirements imposed by the long range space laser interferometry [18]. The response of TQ as a Michelson interferometer and its sensitivity curve have been given [19]. Its science objectives involving astrophysical sources [20, 21, 22, 23] and cosmological sources [24, 25] are currently under intensive investigations.

In space, the terrestrial noises (e.g., seismic noise and gravity gradient noise) are much lower than those on the ground. Instead, the space plasma will serve as the main source of environmental noises. One example is the dispersion effect induced by the space plasma when the laser beams propagate from one spacecraft to the other [26]. It can cause time-varying optical paths and time delays, hence affects the displacement measurement accuracy. Taking the typical magnitudes of electron number density and magnetic field in the earth magnetosphere as $1–10$ cm$^{-3}$ and $1–10$ nT, respectively, at the altitude of TQ’s spacecraft, one can see that the dominating factor of the dispersion is from the former.

On the other hand, the magnetic field plays a central role in the formation of the structures in heliophysics, such as the photosphere and corona of the Sun [27, 28], Parker spiral field lines [29], the boundary of the heliosphere [30], and the magnetosphere of the Earth, Jupiter and Saturn [31, 32]. For LISA, it has been shown that the magnetic field of space plasma is the main source of the non-conservative forces acting on the TMs housed in the spacecraft through the interactions with its magnetic remanence and residual charges [33, 34]. Evaluating the impacts of these residual accelerations for TQ is the focus of this work.

In Fig. 1, TQ’s orbit (red circle) will pass through different background environments, including solar wind, bow shock, magnetosheath, magnetopause, lobes of magnetosphere, magnetotail and so on. There are abundant phenomena of space plasma at this orbit altitude, which cover three spatial scales: the global scale, the magnetohydrodynamic (MHD) scale and...
the plasma scale. In the global scale, the earth magnetosphere is formed by the interaction between the solar wind and the magnetosphere. The variations of the parameters (e.g., velocity, magnetic field) in solar wind can change the shape and position of the bow shock and the magnetopause \[35\]. In the MHD scale, there are instabilities, such as Kelvin-Helmholtz instability at the magnetopause \[36\]. In the plasma scale, the turbulence in solar wind and magnetosheath \[37, 38, 39\] and the plasma waves in solar wind and magnetosphere \[40, 41, 42\] are ubiquitous. Besides, the disturbances from the sun (e.g., coronal mass ejections, coronal shocks \[43, 44, 45\]) and the magnetosphere (e.g., magnetic storms, magnetic reconnections \[46, 47, 48\]) take place occasionally. All these phenomena can lead to the variations of the electron number density and the magnetic field, which in turn lead to the fluctuations of the dispersion along the optical paths and the non-conservative forces on the TMs.

The rest of the paper is organized as follows. Section 2 gives the formalism that is subsequently used to analyze the acceleration noises of TMs induced by the magnetic field of the space plasma. In section 3, we use a well established MHD model (Space Weather Modeling Framework; SWMF \[49\]) to simulate a magnetospheric environment that TQ’s spacecraft may encounter. In section 4, based on the simulated magnetic field, we calculate the acceleration noises and the associated amplitude spectral densities (ASDs) at four typical values of \(\varphi_s\), and discuss its reduction in the parameter space of \(\xi_m - \chi_m\). The paper is concluded in section 5.

2. Acceleration noise analysis
In the magnetic field of space plasma, there are two kinds of non-conservative forces acting on the TMs. The first one is the magnetostatic force produced by the interaction between the TM with a magnetic moment and the background magnetic field. The second one is the Lorentz force that produced by the movement of the TM with residual charges in this magnetic field.

2.1. Magnetostatic force
The magnetostatic force on the TM with a magnetic moment \((M_{tm})\) in the background magnetic field \((B)\) can be written as:

\[
F = \nabla(M_{tm} \cdot B). \tag{1}
\]

Here, \(B\) is composed of the space magnetic field \(B_{sp}\) and the spacecraft magnetic field \(B_{sc}\) inherited from the payloads (e.g., permanent magnet used in attitude control or laser frequency stabilization), i.e., \(B = B_{sp} + B_{sc}\). \(M_{tm}\) is composed of the remanent magnetic moment \(M_r\) and the inductive magnetic moment \(M_i\): \(M_{tm} = M_r + M_i\). \(M_i\) can be induced both by \(B_{sp}\) and \(B_{sc}\),

\[
M_i = \frac{\chi_m V_{im} (B_{sp} + B_{sc})}{\mu_0} = M_{isp} + M_{isc}, \tag{2}
\]

where, \(\mu_0\) is the vacuum magnetic permeability, \(\chi_m\) is the magnetic susceptibility, and \(V_{im}\) is the volume of the TM. Inserting \(M_{tm}\) and Equation (2) into Equation (1), the acceleration can be expressed as follows:

\[
a_M = \frac{1}{m} \nabla[(M_r + M_{isp} + M_{isc}) \cdot (B_{sp} + B_{sc})] = \frac{1}{m} \nabla(M_r \cdot B_{sp} + M_r \cdot B_{sc} + \frac{\chi_m V_{im}}{\mu_0} B_{sp}^2 + \frac{2\chi_m V_{im}}{\mu_0} B_{sp} \cdot B_{sc} + \frac{\chi_m V_{im}}{\mu_0} B_{sc}^2), \tag{3}
\]

where \(m\) is the mass of the TM, \(B_{sc} = |B_{sc}|\), \(B_{sp} = |B_{sp}|\). Based on the vector operation rules, the first term in the second line of Equation (3) can be expanded as:

\[
\nabla(M_r \cdot B_{sp}) = (M_r \cdot \nabla)B_{sp} + (B_{sp} \cdot \nabla)M_r + M_r \times (\nabla \times B_{sp}) + B_{sp} \times (\nabla \times M_r). \tag{4}
\]
According to Maxwell Equations, $\nabla \times \mathbf{B}$ can be induced by the electric current density $\mathbf{j}$ and the displacement current density $\varepsilon_0\partial\mathbf{E}/\partial t$, where $\varepsilon_0$ is the vacuum electric permittivity. Since the TM is encapsulated by the house of the inertial sensor in the disturbance reduction system, the electric current on the TM can be ignored. Therefore, Equation (4) can be written as:

$$\nabla \left( \mathbf{M}_r \cdot \mathbf{B}_{sp} \right) = \left( \mathbf{M}_r \cdot \nabla \right) \mathbf{B}_{sp} + \mathbf{M}_r \times \left( \frac{\varepsilon_0 \mu_0 \partial \mathbf{E}_{sp}}{\partial t} \right) + \mathbf{B}_{sp} \times \left( \nabla \times \mathbf{M}_r \right). \quad (5)$$

Similarly, the second term in Equation (3) can be expanded as Equation (5) with $\mathbf{B}_{sp}$ and $\mathbf{E}_{sp}$ replaced by $\mathbf{B}_{sc}$ and $\mathbf{E}_{sc}$. And the fourth term in Equation (3) can be expanded as:

$$\nabla \left( \mathbf{B}_{sp} \cdot \mathbf{B}_{sc} \right) = \left( \mathbf{B}_{sp} \cdot \nabla \right) \mathbf{B}_{sc} + \left( \mathbf{B}_{sc} \cdot \nabla \right) \mathbf{B}_{sp} + \mathbf{B}_{sp} \times \left( \frac{\varepsilon_0 \mu_0 \partial \mathbf{E}_{sc}}{\partial t} \right) + \mathbf{B}_{sc} \times \left( \nabla \times \mathbf{M}_r \right). \quad (6)$$

According to $\nabla (uv) = u\nabla v + v\nabla u$, where $v$ and $u$ are scalars, the third and fifth term in Equation (3) can be written as:

$$\nabla (\mathbf{B}_{sp}^2) = 2\mathbf{B}_{sp} \nabla \mathbf{B}_{sp}, \quad (7)$$

$$\nabla (\mathbf{B}_{sc}^2) = 2\mathbf{B}_{sc} \nabla \mathbf{B}_{sc}. \quad (8)$$

Insert Equations (5)–(8) into Equation (3), we get:

$$a_{M} = \frac{1}{m} [(\mathbf{M}_r \cdot \nabla) + (2\mathbf{M}_{isp} \cdot \nabla)] \mathbf{B}_{sc} + \frac{1}{m} [(\mathbf{M}_r \cdot \nabla) + (2\mathbf{M}_{isc} \cdot \nabla)] \mathbf{B}_{sp} + \frac{1}{m} (\mathbf{M}_r + 2\mathbf{M}_{isp}) \times \left( \frac{\varepsilon_0 \mu_0 \partial \mathbf{E}_{sp}}{\partial t} \right) + \frac{1}{m} (\mathbf{M}_r + 2\mathbf{M}_{isc}) \times \left( \frac{\varepsilon_0 \mu_0 \partial \mathbf{E}_{sc}}{\partial t} \right) + \frac{1}{m} 2\mathbf{M}_{isp} \nabla \mathbf{B}_{sp} + \frac{1}{m} 2\mathbf{M}_{isc} \nabla \mathbf{B}_{sc}. \quad (9)$$

The remanent magnetic moment $\mathbf{M}_r$ is approximated as uniform here. Since the house of the TM can provide magnetic and electric shielding, so that each magnetic and electric field in Equation (9) needs to be divided by a shielding factor $\xi_m$ ($\xi_e$). Thus, Equation (9) can be reorganized as the following six terms:

$$\begin{align*}
a_{M1} &= \frac{1}{m\xi_m} \left( (\mathbf{M}_r + 2\mathbf{M}_{isp}) \cdot \nabla \right) \mathbf{B}_{sc}, \\
a_{M2} &= \frac{1}{m\xi_m} \left( (\mathbf{M}_r + 2\mathbf{M}_{isc}) \cdot \nabla \right) \mathbf{B}_{sp}, \\
a_{M3} &= \frac{1}{m\xi_e} (\mathbf{M}_r + 2\mathbf{M}_{isc}) \times \frac{\varepsilon_0 \mu_0 \partial \mathbf{E}_{sp}}{\partial t}, \\
a_{M4} &= \frac{2}{m\xi_m} \mathbf{M}_{isp} \nabla \mathbf{B}_{sp}, \\
a_{M5} &= \frac{1}{m\xi_e} (\mathbf{M}_r + 2\mathbf{M}_{isp}) \times \frac{\varepsilon_0 \mu_0 \partial \mathbf{E}_{sc}}{\partial t}, \\
a_{M6} &= \frac{2}{m\xi_m} \mathbf{M}_{isc} \nabla \mathbf{B}_{sc}. \end{align*} \quad (10)$$

We consider $\mathbf{E}_{sc}$ and $\mathbf{B}_{sc}$ are invariant in time. Thus, $a_{M5} = 0$, $a_{M6}$ is a constant acceleration which is not concerned in this work.
2.2. Lorentz force

Energetic charged particles, such as solar energetic particles (SEPs) and galactic cosmic rays (GCRs), can penetrate the shields and make the TMs charged \[50\]. The interaction between the resulting charges and the background magnetic field can lead to Lorentz force. The acceleration caused by Lorentz force on the TM with charge \((q)\) in space magnetic field \((B_{sp})\) is as follows:

\[
a_L = \frac{1}{m} qv \times B_{sp},
\]

where \(v\) is the velocity of the TM. Similar to Equation (10), \(a_L\) has been divided by \(\xi_m\).

2.3. Evaluating residual accelerations

As shown in Equation (10), \(a_{M1}\) and \(a_{M2}\) can be simplified as the form \((A \cdot \nabla)B\), here \(A\) and \(B\) are vectors. The spatial variation of the direction of \(B_{sp}\) at the length scale of TM is ignored. For \((A \cdot \nabla)B\), we have

\[
|\nabla (A \cdot \nabla)B| = \left| \left( A_x \frac{\partial B}{\partial x} + A_y \frac{\partial B}{\partial y} + A_z \frac{\partial B}{\partial z} \right) \right| = |(A \cdot \nabla)B| e_B = |A| |\nabla B| \cos \theta \leq |A| |\nabla B|,
\]

where \(e_B\) is the unit vector of \(B\). Equation (12) is used here to estimate the maximum of \(a_{M1}\) and \(a_{M2}\).

The magnetic field of the permanent magnet onboard in the spacecraft is simplified as a dipole field \(B_{sc}\) which, at the location of the TM, is

\[
B_{sc} = -\frac{\mu_0}{4\pi} (M_{sc} \cdot \nabla) \frac{r}{r^3} \approx 2 \frac{\mu_0 M_{sc} r}{4\pi r^4},
\]

where the distance between the TM and the permanent magnet \(r = 0.5\) m. \(M_{sc} = 1\) A m\(^2\) \[33\]. The gradient of \(B_{sc}\) is

\[
|\nabla B_{sc}| = \left| \nabla \left( \frac{\mu_0 M_{sc} r}{4\pi r^4} \right) \right| \approx \frac{3 B_{sc}}{r}.
\]

In the following calculations, the mass of the TM \(m = 2.45\) kg, the side length of the cubic TM is 5 cm, \(|M_r| = 20\) nA m\(^2\). The shielding factors \(\xi_m\) and \(\xi_e\) are both set to be 10 as fiducial values \[52\]. The magnetic susceptibility \(\chi_m = 10^{-5}\), which can be taken as the upper bound for the Pt-Au alloy TMs of TQ \[11\].

3. MHD simulation

3.1. Global magnetosphere model and input data

We adopt the Space Weather Modeling Framework (SWMF) \[49,53\] to simulate the distributions of the parameters (e.g., \(B_{sp}\)) in the space region enclosing the TQ’s orbit (Fig. 1). The SWMF can simulate the interaction between the solar wind and the magnetosphere of the earth. It has been used in the studies of magnetospheric physics \[47,35,32\], and has been thoroughly validated \[54,55,56\].

The SWMF is integrated by several modules including Solar Corona, Inner Heliosphere, Global Magnetosphere, Inner Magnetosphere, etc. In this work, we requested our simulation on the Community Coordinated Modeling Center (CCMC) using SWMF/Block-Adaptive-Tree-Solarwind-Roe-Upwind-Scheme (BATS-RUS), coupled with the Rice Convection Model (RCM; \[57,58\]) and the Fok Radiation Belt Environment model (RBE; \[59,60,61\]). The inputs of the
simulation are the real time plasma data that observed by the Advanced Composition Explorer (ACE; 62) with a 1-minute cadence, in the time range from 2008-05-01 00:00 UT to 2008-05-04 24:00 UT. The input parameters are shown in Fig. 2 which contain ion number density ($n_i$), temperature ($T$), velocity of plasma, and the magnetic field of solar wind. Besides, we derived the solar wind dynamic pressure $P_{dyn}$ during the time range of the input, and it is shown in the bottom panel of Fig. 2. The simulation domain is defined as $-250 \text{ RE} < x < 33 \text{ RE}$ and $|y| = |z| < 48 \text{ RE}$ in the Geocentric Solar Magnetospheric (GSM) coordinates, considering the geocentric distance of each TQ’s spacecraft is $10^5 \text{ km} \approx 16 \text{ RE}$. On the orbit (e.g., the red circle in Fig. 1), the finest grids are in the vicinity of the near-tail and the dayside magnetopause with a resolution of 0.25 RE, and the rest has a resolution of 0.5 RE. The outputs contain the plasma parameters (number density of ion $n_i$, electron $n_e$, pressure $P$, bulk flow velocity $v_x$, $v_y$, $v_z$), magnetic field ($B_x$, $B_y$, $B_z$), and electric current ($J_x$, $J_y$, $J_z$) on the grid of the simulation domain. Note that these parameters are in the GSM coordinates which need to be converted to the Geocentric Solar Ecliptic (GSE) coordinates when calculating the acceleration noises in section 4.

3.2. Relative positions

The TQ detector’s plane facing the reference source is approximately perpendicular to the ecliptic plane. The global geometric structure of the bow shock and the magnetopause are quasi-axisymmetric along the Sun-Earth line ($-x$ axis in the GSE coordinates). The angle $\phi_s$ between the direction from the sun to the earth and the projection of the normal of the detector’s plane on the ecliptic plane ranges from 0 to 360° annually, with $\phi_s = 120.5°$ at the spring equinox 19. During one revolution of the TQ spacecraft around the Earth (3.65 days), $\phi_s$ can be approximately regarded as a constant. In order to describe the relative position of the TQ detector’s plane and the geometric structure of the earth magnetosphere conveniently, we transform $\phi_s$ to its associated acute angle $\varphi_s$ in the GSE coordinates. Taking the year 2008 as an example, numerical calculation based on JPL DE421 is adopted to evaluate the time-varying $\varphi_s$ which is shown as the solid lines in Fig. 3. The spring and autumn equinoxes, the summer and winter solstices are shown as red pluses. The observation windows of TQ is $2 \times 3$ months in one sidereal year 11. $\varphi_s$ in observation windows (thick yellow lines around summer and winter solstices) and non-observation windows (thin dark blue lines around spring and autumn equinoxes) range from 0° to 75.5° and from 14.5° to 90°, respectively.

In section 4, we study the acceleration noises in four representative relative positions with $\varphi_s = 0°$, 30°, 60°, and 90° which are marked as grey hexagons in Fig. 3. The hexagons on the yellow lines are in the observation window of TQ. The electron number densities for these four cases are given in Fig. 4 in which the simulation result for 2008-05-01 20:00 UT is used. Here $\xi$ is along the intersection of the detector’s plane and the ecliptic plane, $\zeta$ is perpendicular to the ecliptic plane. The red circle represents the orbit of the spacecraft. The black crosses located at $(\xi = 15.7, \zeta = 0)$ RE mark the initial phase for one of the spacecraft. Two distinct boundaries around 10–20 RE can be seen: the outer one is the bow shock, the inner one is the magnetopause. For $\varphi_s = 0°$, the bow shock and magnetopause are approximately axisymmetric, the bow shock is slightly larger than 20 RE, while the magnetopause is slightly larger than 10 RE. For $\varphi_s = 30°$, 60° and 90°, the spacecraft will pass through the solar wind, the bow shock, the magnetosheath, the magnetopause, the lobes of magnetosphere and the magnetotail. We can see that as $\varphi_s$ decreasing the orbit will gradually shrink into the magnetosheath.

4. Results

4.1. Space plasma and magnetic field

On the spacecraft orbit, the values of the space plasma and magnetic field parameters are obtained from the interpolation of the ones on the 3D grid of the simulation domain. Here, the
The dotted blue, orange and green lines represent the x, y and z components of \( \mathbf{v} \), \( \mathbf{B}_{sp} \), \( \mathbf{J} \) and \( \mathbf{E}_{sp} \) on a spacecraft’s orbit are shown in Fig. 5. The dotted blue, orange and green lines represent the x, y and z components of \( \mathbf{v} \), \( \mathbf{B}_{sp} \), \( \mathbf{J} \) and \( \mathbf{E}_{sp} \), respectively. Note that \( \mathbf{E}_{sp} \) is calculated from \( -\mathbf{v} \times \mathbf{B}_{sp} \) as mentioned in section 2.3.

Take \( \varphi_s = 90^\circ \) as an example, combining Fig. 4 and Fig. 5, we can see that \( n_i \) in the solar wind is lower than that in the magnetosheath, but larger than that in the magnetosphere. Meanwhile, the absolute value of \( v_x \) is obviously larger than that of \( v_y \) and \( v_z \), the absolute value of \( \mathbf{B}_{sp} \) is smaller than those in the magnetosheath and magnetosphere. In the magnetosheath, the absolute values of \( n_i \) and \( \mathbf{B}_{sp} \) are larger than those in the solar wind, because that the magnetosheath is the downstream of the bow shock, \( n_i \) and \( \mathbf{B}_{sp} \) can be enhanced by the shock [63, 45]; and the fluctuations of all these parameters are larger than those in the solar wind and the magnetosphere. In the magnetosphere, \( n_i \) is lower than these in the solar wind and the magnetosheath, the absolute values of \( \mathbf{B}_{sp} \) is larger than that in the solar wind. In the magneto-tail, the x component of \( \mathbf{B}_{sp} \) reverses and the absolute value of \( \mathbf{J} \) becomes larger, since there is a cross-tail current sheet in the magnetotail [64]. All these features are consistent with the observations [55, 65]. As \( \varphi_s \) decreases, the TQ spacecraft will spend more time in the magnetosheath and the time series of these parameters become more fluctuated.

4.2. Residual accelerations

According to Equation (10) and (11) and \( \mathbf{B}_{sp} \) and \( \mathbf{E}_{sp} \) on the orbit, we can get the time series of the acceleration noises shown in Fig. 6. Since \( \mathbf{a}_{M1}, \mathbf{a}_{M2}, \mathbf{a}_{M3} \) and \( \mathbf{a}_L \) are directly related to \( \mathbf{B}_{sp} \) and \( \mathbf{E}_{sp} \) in \( \mathbf{a}_{M4} \) is calculated by \( \mathbf{B}_{sp} \), the acceleration noises share some similarities to that of \( \mathbf{B}_{sp} \) in relative amplitudes. The smaller \( \varphi_s \) is, the larger the fluctuations of the acceleration noises are, because that TQ spacecraft will spend more time in the magnetosheath. The amplitudes of \( \mathbf{a}_{M1}, \mathbf{a}_{M2}, \mathbf{a}_{M3}, \mathbf{a}_{M4} \) and \( \mathbf{a}_L \) are in the orders of \( 10^{-14}, 10^{-20}, 10^{-29}, 10^{-23} \) and \( 10^{-18} \) m/s\(^2\), respectively. \( \mathbf{a}_{M1} \) is the dominant acceleration noise caused by the space plasma.

There is a six orders of magnitude difference between \( \mathbf{a}_{M1} \) and \( \mathbf{a}_{M2} \). Since the magnitude of \( M_i + 2M_{\text{Le}} \) and \( M_i + 2M_{\text{sp}} \) are similar, this large difference is mainly due to the difference between the values of \( \nabla \mathbf{B}_{xc} \) and \( \nabla \mathbf{B}_{sp} \). For the MHD simulation, the ratio between displacement current density and electric current density can be approximated as follow:

\[
\frac{\varepsilon_0 \partial \mathbf{E}_{sp}/\partial t}{\mathbf{J}} \sim \frac{E/T}{c^2 B/L} \sim \frac{v \times BL/T}{c^2 B} \sim \frac{v^2}{c^2},
\]

where \( T \), \( L \) and \( v \) are the typical time, length, and velocity of MHD bulk flow in the magnetosphere and the solar wind, respectively. Here \( v \sim 10^2 \text{ km/s} \) in the magnetosphere or the solar wind at 1 AU. From Equation (15), we can see that the displacement current density is much lower than the electric current density. While for the electromagnetic (EM) waves in plasma, which are ubiquitous in heliophysics [66, 39, 67], the displacement current density and the electric current density are approximately on the same order of magnitude. However, these EM waves can not be revealed in the MHD simulation. The impact of EM waves in plasma, especially on \( \mathbf{a}_{M3} \), will be subjected to our future investigations.

Fig. 7 shows the amplitude spectral densities (ASDs) of the time series for \( \mathbf{a}_{M1}, \mathbf{a}_{M2}, \mathbf{a}_{M3}, \mathbf{a}_{M4} \) and \( \mathbf{a}_L \). The dashed horizontal lines (\( \sqrt{S_a} = 10^{-15} \text{ m/s}^2/\text{Hz}^{1/2} \)) mark the preliminary goal of the acceleration noise from the low-noise inertial sensor of TQ [11]. The largest ASDs of the acceleration noises are that of \( \mathbf{a}_{M1} \), while the second largest ASDs are that of \( \mathbf{a}_i \), which are about \( 10^{-16} \text{ m/s}^2/\text{Hz}^{1/2} \) at the lowest frequency (1/3.65 day). The ASDs of \( \mathbf{a}_{M2}, \mathbf{a}_{M3} \) and \( \mathbf{a}_{M4} \) are much lower in the whole frequency range and will be ignored in the following analysis.

The single power law is used here to fit the ASDs of \( \mathbf{a}_{M1} \) and \( \mathbf{a}_L \). The fitting results are shown as the red dashed lines in Fig. 7. We can see that the lower frequency parts of the best
fit spectra for \( a_{M1} \) approach \( 10^{-15} \text{ m/s}^2/\text{Hz}^{1/2} \) when \( f \approx 0.20, 0.11, 0.15 \) and \( 0.13 \) mHz for \( \varphi_s = 0^\circ, 30^\circ, 60^\circ \) and \( 90^\circ \), respectively. The corresponding spectral indexes for \( a_{M1} (a_L) \) are -0.699 (-0.731), -0.546 (-0.615), -0.695 (-0.621) and -0.681 (-0.561).

Since the ASD of the acceleration noise caused by the space magnetic field (\( \sqrt{S_{as}} \)) is dominated by \( a_{M1} \), we set \( \sqrt{S_{as}} = 10^{-15} \text{ m/s}^2/\text{Hz}^{1/2} \) at 0.2 mHz and simply approximate the spectral index as \( -2/3 \). The ratios \( \sqrt{S_{as}/S_a} \) are 0.34 and 0.07 at 1 mHz and 10 mHz, respectively, for the nominal values of the magnetic susceptibility (\( \chi_m = 10^{-5} \)) and the magnetic shielding factor (\( \xi_m = 10 \)) of the test mass.

Note that \( \xi_m \) and \( \chi_m \) are tuneable and set to be conservative values here. Therefore, the aforementioned residual accelerations from space plasma can be regarded as an upper bound. It will be further reduced by increasing \( \xi_m \) and/or decreasing \( \chi_m \). For example, shielding TMs with multiple shields and/or with novel materials can possibly increase \( \xi_m \) several times \( \footnote{Note that} \).

On the other hand, the ultra-low \( \chi_m \) material for fabricating the TMs can be achieved by alloying diamagnetic and paramagnetic materials with a certain proportion \( \footnote{Besides, \( \chi_m \) is expected to be further minimized by optimizing the fabrication process such as increasing the alloying homogeneity and avoiding the introduction of impurities. As a reference, Fig. \ref{fig:8} gives \( \sqrt{S_{as}/S_a} \) at 1 mHz in the parameter space of \( \xi_m - \chi_m \). The yellow lines mark the contours of \( \sqrt{S_{as}/S_a} = 10^{-1} \) and \( 10^{-2} \). We can see that \( \sqrt{S_{as}/S_a} \) can be readily suppressed below \( 10^{-1} \) by a mild adjust from the nominal values.} \) of the test mass.

\section*{5. Discussion and conclusions}
In this work, we study the acceleration noises caused by the magnetic field of space plasma for the test mass of TQ, which include the contributions from the magnetostatic force and Lorentz force. The SWMF is adopted to simulate the global structure of the earth magnetosphere. The solar wind conditions from the ACE observations with time resolution of 60 s are taken as inputs. The MHD simulation outputs the space plasma and magnetic field parameters which are then used to calculate the acceleration noises of the test mass on the orbit of TQ’s spacecraft at various time resolutions. It turns out that the acceleration noise produced by the interaction between the TM’s magnetic moment induced by the space magnetic field and the spacecraft magnetic field (\( a_{M1} \)) is the largest component with the ASD \( \sqrt{S_{as}} \approx 10^{-15} \text{ m/s}^2/\text{Hz}^{1/2} \) at \( f \lesssim 0.2 \) mHz for the nominal values of \( \xi_m \) and \( \chi_m \). \( \sqrt{S_{as}/S_a} \) are 0.34 and 0.07 at 1 mHz and 10 mHz, respectively. We further discuss the reduction of \( \sqrt{S_{as}} \) in the parameter space of \( \xi_m - \chi_m \) which can be considered as a reference of future instrumentation development for TQ.

The temporal resolution of our simulation is 60 s (corresponding to a Nyquist frequency of 1/120 Hz), therefore the phenomena with dynamic frequencies higher than about \( 10^{-2} \) Hz will not be visible. It is certainly important and will be subjected to our future investigations to study the space magnetic field with the aid from high-temporal high-spatial resolution observations and simulations.

On the other hand, the results presented here are based on the simulation from the SWMF model. Thus, neither the phenomena in the plasma scale, such as plasma waves and turbulence \( \footnote{The temporal resolution of our simulation is 60 s (corresponding to a Nyquist frequency of 1/120 Hz), therefore the phenomena with dynamic frequencies higher than about \( 10^{-2} \) Hz will not be visible. It is certainly important and will be subjected to our future investigations to study the space magnetic field with the aid from high-temporal high-spatial resolution observations and simulations.} \), nor the associated residual accelerations can be revealed in the current work. Furthermore, in the solar wind inputs, \( P_{dyn} \) owns moderate values with the mean of \( \approx 2.1 \) (see the bottom panel of Fig. \ref{fig:2}). However, this value can be increased significantly in the eruption events, such as ICMEs, IP shocks, magnetic reconnections, etc. The impacts of these on the TM’s residual acceleration and the spacecraft per se will be followed up.

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**Figure 1.** The schematic view of TQ’s orbit (red circle) in the background of the global magnetosphere at $\varphi_s \approx 90^\circ$. Thin white lines are the laser beams that interconnect the spacecraft of TQ (filled yellow circles). Modified from [72].
Figure 2. The input parameters (ion number density $n_i$, temperature $T$, bulk flow $v$, space magnetic field $B_{sp}$) that are observed by the ACE. The solar wind dynamic pressure $P_{dyn}$ is derived from the observation inputs.
Figure 3. The time variation of $\varphi_s$ in the year 2008. The thick yellow lines represent the observation duration of TQ; the thin blue lines represent the non-observation duration of TQ. The red pluses mark the spring and the autumn equinoxes, the summer and the winter solstices. $\varphi_s$ on the grey hexagons are equal to $0^\circ$, $30^\circ$, $60^\circ$ and $90^\circ$, respectively.
Figure 4. The electron number densities on the TQ’s detector planes where $\varphi_s = 0^\circ$, $30^\circ$, $60^\circ$ and $90^\circ$, respectively. Black crosses mark the initial phase for one of the TQ’s spacecraft. White lines are the representative magnetic field lines in the simulation domain.
Figure 5. Distributions of ion number density $n_i$, bulk flow $v$, space magnetic field $B_{sp}$, electric current $J$ and electric field $E_{sp}$ on the orbit are in rows 1 to 5, respectively. The blue, orange and green lines in rows 2 to 5 represent the $x$, $y$, $z$ components of $v$, $B_{sp}$, $J$, and $E_{sp}$ in the GSE coordinates. Columns 1 to 4 denote the distributions of these parameters on the detector’s plane where $\varphi_s = 0^\circ$, $30^\circ$, $60^\circ$ and $90^\circ$. Note that the abscissa represents the orbital phase for one spacecraft in a 3.65-day circular orbit around the Earth.
Figure 6. Distributions of $a_{M1}$, $a_{M2}$, $a_{M3}$, $a_{M4}$ and $a_L$ at $\varphi_s = 0^\circ$, $30^\circ$, $60^\circ$ and $90^\circ$. 
Figure 7. Amplitude spectral densities of $a_{M1}$, $a_{M2}$, $a_{M3}$, $a_{M4}$ and $a_{L}$ at $\varphi_s = 0^\circ$, $30^\circ$, $60^\circ$ and $90^\circ$. The orange dashed horizontal lines mark the preliminary goal of the acceleration noise for TQ. The red dashed line mark the best fits of the ASDs in rows 1 and 5.
Figure 8. $\sqrt{S_{as}/S_a}$ at 1 mHz in the parameter space of $\xi_m - \chi_m$. The contours of $\sqrt{S_{as}/S_a} = 10^{-1}$ and $10^{-2}$ are shown as yellow lines.