The local wavelet-based organization index – Quantification, localization and classification of convective organization from radar and satellite data

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Abstract
We present a revised local wavelet-based organization index (LW), which is applied to both synthetic fields and meteorological fields of precipitation and brightness temperature for different types of convective organization. The LW consists of three components that describe the scale (LW_{sc}), quantify the intensity (LW_{in}), and analyze the anisotropy (LW_{ai}) of the convection. It is based solely on 2D wavelet decomposition and does not require a clustering algorithm or thresholding. The great advantages of LW are that it localizes the organization in space, is a universally applicable measure of organization and can be calculated with little effort. A comparison with other organizational metrics shows that LW better describes the structure and organization of convection. We analyze the LW components in the vicinity of severe weather reports related to single-cell storms, supercells or multi-cell storms. Composites thereof reveal that large hail and heavy precipitation are particularly local events that lead to small-scale and very intense precipitation. Especially in the case of hail, LW_{ai} shows the structure of a hook echo, which is expected, since hail events are usually associated with supercells. In relation to wind gusts, precipitation is rather large-scale and linearly oriented, since strong wind gusts are often a consequence of linearly organized systems such as squall lines or derechos. The brightness temperature analysis provides similar results to those of the rain rates. Structures like the typical radar hook echo are not visible in the satellite data, but overshooting tops caused by strong updraughts are recognized as small-scale, intense regions. The three LW components are used to classify showers, thunderstorms and precipitation/hail events with the help of a neural network, where showers are mainly classified by their scale characteristics, and precipitation/hail events by their pronounced intensity.

KEYWORDS
convective organization, radar composites, satellite composites, storm structure, wavelet-based organization index
1 | INTRODUCTION

Convection and the degree of convective organization play an important role in the global circulation regarding moisture fluxes, energy and momentum transport (Stevens and Bony, 2013). In particular, the representation of convection and its organization in climate or weather prediction models is the subject of current research (e.g., Mapes and Neale, 2011; Birch et al., 2015; Badlan et al., 2017; Moncrieff et al., 2017; Senf et al., 2018; Senf et al., 2019). Therefore, the quantification of convective organization becomes important.

Convection may be organized into mesoscale structures by upper-level troughs, fronts or waves like the African Easterly Waves and Madden–Julian Oscillation (e.g., Ricard et al., 2012; Cannon et al., 2018; Xia et al., 2018). Also orography (Imamovic et al., 2019), surface heterogeneity (Lee et al., 2019) and/or cold pools (Haerter et al., 2019) influence the structure of convection. The interaction between convection and wind shear is well known (Rotunno–Klemp–Weisman theory; Rotunno and Klemp, 1982; Weisman and Rotunno, 2004) and leads to different convective storm types. Low-shear conditions result in less organized short-lived local storms such as pulse storms and single cells. Slightly increased wind shear may result in longer-lasting multicell storms, while strong wind shear fosters the evolution of long-lasting storm modes such as supercells or mesoscale convective systems (MCSs; Houze, 2004). All these storm types can become severe and damaging. Local, slow-moving single cells and multicells predominately produce strong precipitation, supercells are often responsible for large hail events, funnels or even tornadoes, and MCSs typically come along with severe wind gusts and/or heavy rainfall (Galli et al., 2008; Thompson et al., 2012; Groenemeijer et al., 2017).

To measure the aggregation and organization of convection, several convective aggregation/organization indices have been developed during recent years (Table 1). Most measurements have been based on clustering algorithms of different variables such as brightness temperature, outgoing long-wave radiation, vertical wind velocity, precipitation or integrated water vapour. Different (arbitrary) thresholds are used in the object-based analysis to distinguish between convective and non-convective pixels (Table 1). Since each individual index measures the organization in a different way (e.g., using the distance between the objects, their size or their form), Pscheidt et al. (2019) recommend the use of several convective organization indices. A few indices depend on the grid structure or are limited to specific regions. Especially the indices which are based on vertical wind velocity are not useful to diagnose convective organization from observable quantities such as satellite or radar data, because convective updraughts are smaller in scale than their corresponding clouds and rain rates. All object-based indices measure only the averaged convective organization over the selected domain and are not able to localize different organization forms over different regions of the domain.

Recent studies show that spatial structures of clouds and precipitation fields can be successfully analyzed using wavelets (e.g., Yano et al., 2001a; 2001b; Yano and Jakubiak, 2016; Weniger et al., 2017; Kapp et al., 2018; Klein et al., 2018). A variety of properties, namely scale, total energy and direction, may be assessed with a wavelet-based organization index (WOI), which was introduced by Brune et al. (2018). The components of WOI characterize spatially averaged measures of scale, intensity and anisotropy. Brune et al. (2018) show that WOI performs well in a case-study and may be used to distinguish between unorganized and organized convection.

In Brune et al. (2020) a local WOI (LWOI) is used to investigate the scale (LWOIsc) and intensity (LWOIsi) of convection over different parts of the tropical Atlantic. The great advantage of LWOI over convolutional indices is the localization of different organization structures. One drawback of LWOI_sc is that its definition requires the separation of scales into small and large scales, which introduces a certain arbitrariness. Another shortcoming of LWOI is that the discrete 2D wavelet decomposition is not invariant under rotation. This led to an anisotropy component LWOI_ii which was too sensitive to small changes in rotation, and hence is not recommended as an indicator of convective organization.

Several approaches have recently been used in the literature to classify the type of weather event. Garcia et al. (2015) use a scattering wavelet transform to classify 1,239 images from a single radar into the hand-labelled classes “rain”, “shower”, “organized storm” and “unorganized storm”. A similarly hand-labelled set of 260 precipitation forecasts was used for pixel-by-pixel classification into “scattered” and “continuous” precipitation by Hamidi et al. (2020). These authors relied on texture indicators and a random forest algorithm. Jergensen et al. (2020) considered a very large dataset of 123,387 weather events observed in the contiguous United States. They fed up to 341 model variables into several machine-learning algorithms to predict supercells, disorganized precipitation and quasi-linear convective systems.

This study presents a revised version of the local wavelet-based organization index, denoted as LW. The shortcomings of WOI and LWOI have been addressed, and the index has been brought to a maturity that allows convective organization to be studied in terms of scale, intensity, and orientation. We demonstrate the performance of LW on synthetic fields and show the advantages of LW over the conventional object-based organization...
### Table 1

Overview of some convective organization measures and their references

| Name | Variable and threshold | Reference |
|------|------------------------|-----------|
| AF1  | Ascending fraction, Vertical velocity at 500 hPa | Bao et al. (2017) |
| AF2  | Ascending fraction, Upward mass-weighted vertical integral of vertical velocity | Bao et al. (2017) |
| ARH  | Averaged relative humidity, Vertical-averaged relative humidity | Bao et al. (2017) |
| CAI  | Convective Aggregation Index, >99th percentile precipitation | Pendergrass et al. (2016) |
| COP  | Convective organization potential, Outgoing long-wave radiation | White et al. (2018) |
| H    | Information entropy of convective occurrence, Brightness temperature < 245 K and < 205 K inside cores | Sullivan et al. (2019) |
| Iorg | Organization Index, Vertical velocity > 1 m·s⁻¹ at 730 hPa | Tompkins and Semie (2017) |
| Iorg | Organization Index, Hourly averaged vertical velocity > 0.5 m·s⁻¹ at 500 hPa | Cronin and Wing (2017) |
| Iorg | Organization Index, Cloud-top temperature < 235 K | Cronin and Wing (2017) |
| Iorg | Organization Index, Outgoing long-wave radiation < 173 W·m⁻² | Wing et al. (2018) |
| Ishape | 2D shape index, Radar reflectivity ≥ 30 dBZ | Pscheidt et al. (2019) |
| Ishape | 2D shape index, Brightness temperature ≤ 240 K | Pscheidt et al. (2019) |
| LWOI | Local Wavelet-based Organization Index, Rain rate | Brune et al. (2020) |
| MICA | Morphological Index of Convective Aggregation, Brightness temperature < 240 K | Kadoya and Masunaga (2018) |
| ROME | Radar Organization Metric, Radar reflectivity > 40 dBZ | Retsch et al. (2020) |
| Sₚ(P) | Zonal convective clustering, Precipitation between 6°S and 6°N | Popp and Bony (2019) |
| SCAI | Simple Convective Aggregation Index, Brightness temperature < 240 K | Tobin et al. (2012; 2013) |
| SCAI | Simple Convective Aggregation Index, 95th percentile daily precipitation | Bao et al. (2017) |
| SCAIP | Simple Convective Aggregation Index Precipitation, Precipitation > 1.49 mm·day⁻¹ | Holloway (2017) |
| SF   | Subsiding fraction, Large-scale vertical velocity at 500 hPa | Coppin and Bony (2015) |
| WOI  | Wavelet-based Organization Index, Rain rate | Brune et al. (2018) |
| α    | Degree of aggregation, Integrated water vapour variance | Lebsock et al. (2017) |

indices. Then LW is used to study the local convective organization of showers and thunderstorms. We combine the convective organization from a dense radar network (RADKLIM) and high-resolution brightness temperatures with about 26,000 damaging severe weather reports and nearly 150,000 convective event reports over Germany over almost two decades (2001–2018) and investigate which degrees of convective organization are associated with damage reports. We also apply LW to the brightness temperatures and show that the applicability of LW is not limited to rain rates. Finally, we classify the type of weather event based on LW via a neural network.

This article is structured as follows. In Section 2 we describe the radar and satellite data, as well as the reports on weather and severe weather. Section 3 gives an overview of the methods including Dual-tree complex
wavelet transforms and the revised LW. At the beginning of Section 4 we apply LW on constructed fields and compare LW to other convective organization indices. Then we study case-studies of a supercell and a squall line and analyze the general spatial structure of different convective weather events with help of LW composites. At the end of Section 4 we use machine learning to classify these events solely on the basis of LW. We conclude the study with a summary and additional remarks in Section 5.

2 | DATA

2.1 | RADKLIM dataset

The nationwide calibrated rain rate composite RADKLIM (Winterrath et al., 2018) was published by the German meteorological service, Deutscher Wetterdienst (DWD), and is based on the RADOLAN method which combines radar data with rain gauge measurements. The reflectivity of all 17 C-band radars in Germany is corrected for radar-specific errors such as clutter, reduction effects, shading or increasing scan volumes (Winterrath et al., 2019). The application of these climatological corrections ensures that RADKLIM is a consistent dataset of radar-based precipitation estimates. Figure 1(a) shows the radar positions between 2001 and 2018. The radar composite (dark grey area) covers Germany and surrounding countries on a 1×1 km grid and is available every 5 min from 01 January 2001 to 31 December 2018. Some radar locations changed during the period (e.g., Emden, Frankfurt, Flechtdorf). The domain we use consists of 805×1,005 grid points (light grey area in Figure 1a), which is padded with zeros at the boundaries to obtain an array of size 1,024×1,024 for the wavelet transform (Section 3.3). RADKLIM contains missing values due to radar failure, maintenance and software updates or artefacts such as beam blockage. However, thanks to the quality control and high temporal and spatial resolution of the rain rates, the dataset should be suitable to study the convective organization over the last two decades.

2.2 | Satellite dataset

We also use half-hourly infrared brightness temperatures merged from the European (METEOSAT), Japanese (GMS, MTSat-1R, Himawari-8) and U.S. geostationary satellites (GOES) between 2001 and 2018. Details on the freely available dataset1 are provided in Janowiak et al. (2017). The dataset covers the region between 60°S and 60°N with a horizontal resolution of approximately 4 km. The data contain missing values due to zenith angle corrections, especially at high latitudes. Beam blockages behind deep vertical clouds with high cloud tops in the Northern Hemisphere result in missing values north of the intense convection. These missing values are set to a distance-weighted average to get a complete dataset, which is required to perform the wavelet transform. Other issues such as the parallax effect are not considered in this study. Bieliński (2020) showed that the parallax effect is about 8–10 km in Northern Germany for deep convective clouds, which corresponds to a shift of only one or two pixels in

1https://disc.gsfc.nasa.gov/datasets/GPM_MERGIR_1/summary; accessed 12 February 2021.
### Table 2

Total number of rain showers and thunderstorms reported by manned weather stations (SYNOP) and all severe weather reports of severe precipitation and hail events in ESWD between 01 January 2001 and 31 December 2018

| Class        | Controlled | Database | Description                                      |
|--------------|------------|----------|--------------------------------------------------|
| Shower       | 169,371    | SYNOP    | Rain shower at SYNOP station of DWD              |
| Thunderstorm | 32,004     | SYNOP    | Audible thunder at SYNOP station of DWD          |
| PRECIP       | 3,693      | ESWD     | Report of severe precipitation (e.g., flooding)  |
| HAIL         | 1,987      | ESWD     | Report of large hail (≥ 2 cm) or hail accumulation |
| precip_hail  | 5,680      | ESWD     | PRECIP+HAIL                                      |

*Note: Only quality- and time-controlled ESWD reports are listed. SYNOP reports are assumed to be exact in place and time. Precipitation and hail events together form the precip_hail class.*

the satellite image. After downloading the data, we select a box covering longitudes 0.00–18.61 °E and latitudes 40.00–58.60 °N. The extracted domain of 512 × 512 grid points covers Central Europe and the wavelet transform can be directly applied to this quadratic domain.

### 2.3 ERA-Interim

Convection in Central Europe is often embedded in a southwesterly flow, with the consequence that convective cells move to the northeast (e.g., Weijenborg et al., 2015). For the analysis of cell composites, it is necessary to rotate the convective cells along the movement direction, which is often given by the main synoptic flow. We use the 10 m and 500 hPa wind field of the ERA-Interim reanalyses (Dee et al., 2011) from the European Centre for Medium-Range Weather Forecasts (ECMWF) to adjust the cells along the wind shear vector between the 500 hPa level and the surface. ERA-Interim is freely available and has a horizontal resolution of approximately 80 km (0.75° × 0.75°). The global ERA-Interim dataset covers the complete period between 2001 and 2018 with a temporal resolution of six hours (0000, 0600, 1200, 1800 UTC). In between (0300, 0900, 1500, 2100 UTC), short-term forecasts are provided.

### 2.4 SYNOP stations

The DWD observes the present weather every hour according to the convention of the World Meteorological Organization WMO (1992) at its synoptic stations in Germany. These so-called SYNOP reports report on the current weather situation and the weather within the last hour. Report of special events such as thunderstorms are given immediately. We use SYNOP reports at 210 manned stations shown in Figure 1(b). Because these are official reports at fixed stations, we assume that the SYNOP reports have no errors in either place or time. Until the end of 2018, some of the manned stations were replaced by automatic stations. Reports of automatic stations are not considered due to different shower and thunderstorm criteria. Between 2001 and 2018 there were 169,371 showers reported at manned stations, and thunderstorms occurred 32,004 times (Table 2).

### 2.5 European Severe Weather Database (ESWD)

Additionally, we use reports on convective storms from the quality-controlled European Severe Weather Database (ESWD). The ESWD database contains only those significant weather events that threaten people or may cause damage. Incoming reports are plausibility checked (QC0+), confirmed and documented by a reliable source such as storm spotters (QC1) or when extra work has been performed to validate the report (QC2). Details on the database are given in Dotzek et al. (2009) and Groenemeijer and Kühne (2014).

The database has been in existence since 2008, but many reports have been added retroactively. Our dataset consists of 23,790 reports over Germany until 31 December 2018 including 6,447 heavy precipitation events (PRECIP) and 4,111 hail events with hail diameter ≥ 2 cm (HAIL). Other reports such as severe wind or tornado reports do not necessarily come along with rainfall or low brightness temperatures and are discarded in this study. Each precipitation and hail report comprises information on the longitude, latitude and time, precisely to the minute. The reports may be displaced in space and/or time. Thus, most reports contain information on time uncertainty. To

2https://www.ecmwf.int/en/forecasts/datasets/reanalysis-datasets/era-interim; accessed 12 February 2021.

3https://www.eswd.eu/; accessed 12 February 2021.
METHODS

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Dual-tree complex wavelet transforms (DTCWT)

The essential improvement of the LW compared to the LWOI results from the change of the wavelet transform, which for LW is the so-called Dual-tree complex wavelet transforms (DTCWT). Like the discrete wavelet transform (DWT) used for LWOI, the DWT consists of projecting a two-dimensional field onto a series of new basis functions with limited support in space and frequency. These so-called daughter wavelets are obtained from a single mother wavelet \( \psi(x,y) \) via shifts \( (\psi_0(x,y) = \psi(x - u_0, y - u_0)) \), scaling \( (\psi_1(x,y) = \psi(2^{-j}x, 2^{-j}y)) \) and changes in orientation. The DWT’s poor directional properties stem from its use of only three distinct orientations, namely 0°, 90° and ±45° (Figure 2a). We therefore replace the real-valued \( \psi \) by a complex function \( \psi_c = \psi_r + i \psi_i \) according to Kingsbury (1999), where \( \psi_r \) and \( \psi_i \) are essentially phase-shifted versions of each other. As shown in Figure 2b, the resulting daughter wavelets have six well-defined directions \{15°, 45°, 75°, 105°, 135°, 165°\}. Combined with the slightly modified treatment of the diagonal directions as introduced by Kingsbury (2006), the wavelet transform is almost invariant to rotation. For an introduction and a comprehensive discussion of the benefits of the DTCWT, the reader is referred to Selesnick et al. (2005).

We use the redundant version of the DTCWT, where the daughter wavelets are shifted to any discrete location \((x,y)\) for all directions \(d \in \{1, \ldots, 6\}\) and scales \(j \in \{1, \ldots, J\}\), since we want to analyze the local degree of convective organization at each grid point. We denote the squared modulus of the corresponding complex coefficient as local spectral energy \(E_{j,d}(x,y)\). Nelson et al. (2018) extended the theory of locally stationary wavelet processes (Eckley et al., 2010) to the case of the DTCWT and formulated the necessary bias correction for the local energies, which removes unwanted overemphasis on the very large scales. The complete redundant DTCWT including the bias correction is implemented in the dualtrees R-package (Buschow et al., 2020).

The revised local wavelet-based organization index (LW)

Like its predecessor, LW has three components representing the dominant scale, LWsc, the intensity, LWin, and the anisotropy, LWai. Let \( E(x,y) = \sum_{j=1}^{J} \sum_{d=1}^{6} E_{j,d}(x,y) \) be the total bias-corrected energy at a given grid point. We define the normalized energy of scale \(j\) as \( e_j(x,y) = \frac{1}{E_j(x,y)} \sum_{d=1}^{6} E_{j,d}(x,y) \). Analogously, the normalized energy of direction \(d\) is \( e_d(x,y) = \frac{1}{E_d(x,y)} \sum_{j=1}^{J} E_{j,d}(x,y) \).

To define the index of local scales LWsc, we place \( e_j \) along a line so that the position of the energy of the \( j \)th scale is \( u_j \) and calculate the centre of mass. With the use of the so-called central scale, as introduced by Buschow et al. (2019), it is no longer necessary to define in advance which scales are considered large or small. By using the central scale, LWsc now also reacts to the redistribution of energy within the small or large scales, which was not the case with LWOIs.

Let us now extend the idea of a central scale to two dimensions and define an index for the anisotropy LWai(x,y). We place the energy of the six directions \( e_{i,d}(x,y) \) at equal distances along a circle with radius 1 (Figure 3). The vector pointing from the centre of the circle to the centre of mass has two components, LWai(x,y) and LWri(x,y). The degree of anisotropy at location \((x,y)\) is defined by the distance to the centre \( \sqrt{LW_{ai} + LW_{ri}} \). The angle \( \theta = \arctan 2(LW_{ai}, LW_{ri}) \) allows us to estimate the dominant direction at the location.

The intensity component LWin(x,y) is based on the total spectral energy \(E(x,y)\) and mainly related to the total variability of the variable in space. Its definition is the same...
as in Brune et al. (2020). In summary, the components of LW are defined as follows:

$$\text{LW}_{\text{sc}}(x, y) = \frac{1}{J - 1} \left( -1 + \sum_{j=1}^{J} e_j(x, y) \cdot j \right),$$  

$$\text{LW}_{\text{in}}(x, y) = 1 - \exp \{-E(x, y)\},$$  

$$\text{LW}_{\text{ai}}(x, y) = \sqrt{\text{LW}_{\text{u}}(x, y)^2 + \text{LW}_{\text{v}}(x, y)^2}. \quad (3)$$

All three LW components are normalized to the interval $[0, 1]$. Their calculation is included in the latest version of the calcWROI-package (Brune et al., 2019).

### 3.3 LW calculation of RADKLIM and satellite data

The two-dimensional wavelet transformation is applied to quadratic fields of $2^n \times 2^n$, $n \in \mathbb{N}$ with periodic boundaries. The non-rectangular domain of the radar composite (Figure 1a) allows neither mirroring of the data at the boundaries as done in Brune et al. (2018) nor a linear filtering at the boundaries as used in Brune et al. (2020).

For RADKLIM, we add zeros around the $805 \times 1,005$ grid point domain to obtain a $1,024 \times 1,024$ field. Missing values in RADKLIM are set to 0. This may introduce additional gradients in the rain rates at the edge of a radar or around regions with missing data and, since wavelet transformation is sensitive to such gradients, the results may be distorted at the boundaries. For this reason, reports that are close to the edge of the radar composite are not considered in the later analysis. We calculate LW over the complete domain at each time step with a shower or thunderstorm at a SYNOP station or a precipitation or hail report in the ESWD data. In regions where rain rates are below 0.1 mm·hr$^{-1}$, the components of LW are set to ‘not available’. To ensure that the radar image is directly linked to the report, we select the closest radar time step. Since RADKLIM is available every 5 min, the time shift between the reports is not more than 3 min.

For brightness temperatures we calculate LW directly on a square grid with $512 \times 512$ points. The region relevant for us is located at the centre of the domain and is therefore surrounded by many grid points, so the influence of the boundaries is small. In the absence of clouds, the brightness temperature essentially measures the temperature of the ground. This leads to strong signals from regions with high surface temperature gradients, for example, in the Alps or between land and sea. We therefore follow Sullivan et al. (2019) and introduce a threshold of 245 K to remove the higher surface temperatures, that is, brightness temperatures above 245 K are set to 245 K before applying the wavelet transform. After applying the wavelet transform, pixels with a brightness temperature of 245 K are marked as ‘not available’ and are not used for further analysis. Similar to RADKLIM, we choose the closest time step of the satellite data, resulting in a time offset of at most 15 min.

### 3.4 Composites

The composites are calculated for an environment of 25 grid points in each direction from the location of the SYNOP or severe weather report. The resulting cell composites consist of $51 \times 51$ grid points (i.e., approximately $51 \times 51$ km for the radar composites and $204 \times 204$ km for the satellite composites) with the report location centred in the middle. This area should cover the region of main convective activity, for example, strong updraughts with heavy rainfall or deep convective clouds with overshooting tops. Situations where not all surrounding 25 grid points are within the radar coverage are not included in the analysis. We rotate the individual images along the wind shear vector between 500 hPa and 10 m at the central grid point. We assume that the convective storms are embedded in the mean synoptic flow, which is represented by the direction of the wind shear (Kirkpatrick et al., 2007). Thus the images are rotated so that the wind shear vector points from left to right. Note that the direction of movement of a convective cell may be rotated by up to 20°–30° with respect to the direction of the wind shear. Maddox (1976) and Davies and Johns (1993) noted that supercells, in particular, have their own dynamics and their direction of motion can deviate to the right as well as to the left from
the direction of the shear vector. Deviations from our simple assumption that the cells move along the direction of the 500 hPa wind shear will introduce noise into the composites, which will only reduce the significance in the statistical tests.

We test the hypothesis that the mean value of the time series at a particular pixel is equal to the mean over all pixels and all 12 classes shown in Table 2. Thus Student t-tests were performed at each location individually; regions where the null hypothesis was not rejected at \( p = 1\% \) are marked accordingly in the plots of our composites (Section 4.3). For \( \text{LW}_{\text{in}} \), which has a strongly non-normal distribution due to the point-mass at 1, the tests were performed on the logarithm of the time series.

\( \text{LW}_{\text{ai}} \) represents a special case; because we are primarily interested in the existence of coherent directional structures in the data, we first average the two directional components \( \text{LW}_u \) and \( \text{LW}_v \) (cf. Figure 3) over time and then calculate the composite of \( \text{LW}_{\text{ai}} \) according to Equation (3). Due to the quadratic form of the definition, the result is not identical to the average over the instantaneous values of \( \text{LW}_{\text{ai}} \). By first averaging \( \text{LW}_u \) and \( \text{LW}_v \), we allow randomly oriented objects to average out, leaving us with a composite representing the degree to which a specific direction dominates the local structure. This is perfectly analogous to the difference between the average absolute wind speed and the absolute value of the average wind vector. These directions are well defined due to the shear-based rotation of the individual images. To assess significance, we test \( \text{LW}_u \) and \( \text{LW}_v \) individually and visually mark regions where one or both of the hypothesis tests rejected the null hypothesis at \( p = 1\% \).

4 | RESULTS

4.1 | LW for idealized fields

We start with an investigation of the behaviour of the three LW components – scale (\( \text{LW}_{\text{sc}} \)), intensity (\( \text{LW}_{\text{in}} \)) and anisotropy (\( \text{LW}_{\text{ai}} \)) – for idealized images. Figure 4 shows different constructed objects that represent changes in size, intensity and anisotropy. All images have a size of \( 64 \times 64 \) grid points. The images are padded with zeros to obtain an array of size \( 512 \times 512 \). The LW components as shown in Figure 4d,h,l are spatial averages over the grid points with rain.

A single pixel with rain is the smallest possible object in the domain and is characterized by \( \text{LW}_{\text{sc}} \approx 0 \). \( \text{LW}_{\text{sc}} \) increases with the size of the object and is around 0.5 for an object of size \( 16 \times 16 \). For an object that covers a quarter of the total domain, \( \text{LW}_{\text{sc}} \) reaches 0.7. The intensity is around 0.15 for all different object sizes, because the rain rates are a constant 10 mm-hr\(^{-1}\) inside the squares and 0 mm-hr\(^{-1}\) outside. Anisotropy is close to 0 for a very small square, since the object is almost circular. The horizontal and vertical gradients along the sides of larger squares induce a slight increase in the anisotropy index \( \text{LW}_{\text{ai}} \).

If we increase the rain rates for all pixels inside the square from 1 to 32 mm-hr\(^{-1}\), the intensity component \( \text{LW}_{\text{in}} \) increases monotonically from 0 for weak intensities up to 0.8 for strong rainfall. The reason for this increase is that \( \text{LW}_{\text{in}} \) is sensitive to spatial gradients, not the strength of the rainfall itself. Scale and anisotropy remain constant, if the size of the squares does not change.

To investigate the behaviour of \( \text{LW}_{\text{ai}} \), we reduce the width of the object continuously from 32 to 1 grid point. The intensity remains at 10 mm-hr\(^{-1}\). The scale in the \( x \)-direction changes from large to very small, which results in a decreasing \( \text{LW}_{\text{sc}} \). The gradients between no rain and 10 mm-hr\(^{-1}\) along the left and right side of the objects remain the same, so that \( \text{LW}_{\text{in}} \) is almost constant. The anisotropy component \( \text{LW}_{\text{ai}} \) for the large square is relatively small (below 0.2). Reducing the extent in the \( x \)-direction leads to higher anisotropy. For a rectangle of size \( 16 \times 32 \), \( \text{LW}_{\text{ai}} \) is around 0.4; the anisotropy of a sharp line is even higher (0.6).

Conventional convection indices such as the simple convective aggregation index (SCAI; Tobin et al., 2012) or the convective organization potential (COP; White et al., 2018) cannot characterize individual objects as shown in Figure 4. Nevertheless, in order to be able to compare LW with SCAI and COP, we have calculated spatial averages of the LW components for eight different cases from Tobin et al. (2012) and White et al. (2018). The information from the different LW components turns out to be very useful, because LW allows us to distinguish between different structures that have the same SCAI and COP. Details of this comparison are given in the Appendix. On the basis of these encouraging findings, the convective organization of real cases using LW is analyzed in detail below.

4.2 | LW of a supercell and a squall line

We now analyze the spatial structure of convection and its organization during two cases: an isolated supercell west of Bonn in western Germany on 09 June 2014 at 1240 UTC, and a powerful squall line in northwestern Germany on 12 July 2010 at 1230 UTC (Figure 5). The supercell developed within a high-energy air mass and a heavily sheared environment ahead of the 2014 Pentecostal storm over western Germany (Barthlott et al., 2017; Mathias et al., 2017). Details of the squall line and its development can be found in Uebel and Bott (2015).
The supercell on 09 June 2014 is characterized by a sharp and intense hook echo with rain rates above 150 mm·hr$^{-1}$. Markowski (2002) provided a review on hook echo structures in radar observations. The hail report was located inside the hook in the region with the strongest rainfall. Rain intensity decreased rapidly from the location of the report to a distance of 10–15 km, where rain rates became more uniform and were only about 10 mm·hr$^{-1}$. LW$_{sc}$ is about 0.30 around the hook echo and indicates a large amount of small-scale variability in the rain field near the hook. Outside the area with strongest precipitation the scale increases to about 0.55 at the outer boundary of the supercell, because rainfall there was more stratiform. The supercell produced very intense rain rates, which results in a high LW$_{in}$ of nearly 1. Slightly less intense precipitation was identified only 20–25 km away from the report with LW$_{in} \approx 0.90$, since the spatial gradients between no rain and rain rates of about 10 mm·hr$^{-1}$ were smaller than inside the hook. LW$_{ai}$ increases with the distance from the hook, because small-scale structures in the centre lead to a more isotropic pattern (LW$_{ai} \approx 0.25$). The stratiform rainfall at the edge of the supercell was more orientated with LW$_{ai} \approx 0.50$. The direction follows the hook echo and is parallel to the shear vector in the stratiform region and crosses the wind vector in the centre.

The intensity of the precipitation along the squall line was slightly weaker (100 mm·hr$^{-1}$) than in the supercell. The highest intensities were located along a line perpendicular to the shear vector. The rainfall behind the squall line was less intense and more uniform (high LW$_{sc}$). Local precipitation maxima along the squall line are characterized as smaller in scale, because the spatial variation was
FIGURE 5  (a, b) Observed rain rate by RADKLIM, (c, d) LWsc, (e, f) LWin and (g, h) LWai of (a, c, e, g) an isolated supercell west of Bonn in western Germany on 09 June 2014 at 1240 UTC and (b, d, f, h) a squall line in northwestern Germany on 12 July 2010 at 1230 UTC. The images are centred around a large hail report in the supercell case and a severe wind gust in the squall line case. Grey circles represent the distance from the centre every 5 km. All the images contain 51 × 51 pixels, which correspond to 51 × 51 km². The images are rotated along the wind shear vector between 500 hPa and surface from the ERA-Interim reanalyses (grey arrow in (a, b); the distance between two circles corresponds to 2.5 m s⁻¹). Numbers at bottom left show minimum, maximum and spatial mean of the variables. White lines in (g, h) represent the preferred direction obtained from the wavelet spectrum at every second pixel particularly large in this area. LWin is maximal along the squall line due to the strong gradients between no rainfall in front of the squall line and very high intensities at the leading edge. In the region 20 to 25 km behind the squall line, the stratiform precipitation was weaker and characterized by LWin of about 0.75. We find a distinct orientation along the squall line perpendicular to the wind shear. LWai of up to 0.65 detects the most linear segments at the leading edge.

4.3  Composites of weather events

Figure 6 shows the composites of LW calculated from radar data for SYNOP showers and SYNOP thunderstorms as well as ESWD PRECIP and ESWD HAIL events. All plots are oriented such that the direction of the wind shear is from left to right (arrows in Figure 5). We find that the scale of the rain rates is generally smallest near the report for all LWsc composites. This is in line with the results presented in Steiner et al. (1995); in the small-scale convective core (i.e., the centre of the SYNOP shower or thunderstorm) the spatial gradient in precipitation intensity is very large, while outside, in the stratiform area, the precipitation echoes are larger in scale and spatially more homogeneous.

For SYNOP showers and thunderstorms, the area with minimum scale is slightly shifted towards the wind shear. Thus, assuming that the shear vector roughly indicates the direction of cell motion, the spatial scale of a shower or thunderstorm event is smallest right at the beginning of the event. After the onset of the first precipitation, the structures in the radar image become somewhat larger (left region of the composite). SYNOP showers and thunderstorms reveal larger scales than ESWD HAIL and PRECIP with weaker gradients over the area. Thus rain rates related to SYNOP shower and thunderstorm reports act on larger scales and are more uniform in a range of 25 km around the report. This pattern is possibly due to the fact that SYNOP showers and thunderstorms usually come along with relatively uniform rain rates.

In contrast, LWsc is much smaller near the ESWD HAIL and PRECIP events and increases strongly with distance from the observation point. Large hailstones and heavy precipitation usually develop only locally in a severe thunderstorm, for example in a prominent hook echo of a supercell (Figure 5) or within a powerful updraught of a multicell. Another indicator for the occurrence of hail in the vicinity of a supercell is the large-scale precipitation field to the left of the shear vector at a distance of 10 to 20 km from the report (e.g., Kumjian, 2011).

The intensity of rain rates (LWin) is of course highest in the immediate vicinity of ESWD HAIL and PRECIP events. With increasing distance from the event, the rain rate decreases evenly in all directions. This is consistent with the results of Lochbihler et al. (2017). They also found that precipitation intensities rapidly decrease radially from the centre of the event. Since the significance tests were performed against all four classes and thus also against the
FIGURE 6  From top to bottom, composites of LW_{sc}, LW_{in}, LW_{ai} and the preferred local direction calculated from radar data for shower and thunderstorms of SYNOP stations, and damage reports of precipitation (PRECIP) and hail events (HAIL) in ESWD. The centre of each composite is marked by a black cross and the radius of each circle is 25 km. Solid and dashed contours correspond to values above and below the overall average of the respective variable. The hypothesis that the local values differ from this overall average was rejected at \( p = 1\% \) in the grey regions. For LW_{ai}, the light grey region indicates that at least one directional component was significant; both were significant in the dark grey zone (Section 3.4).

many showers (Table 2), the rain rates are also significant for thunderstorms.

The strong anisotropy (LW_{ai}) in the ESWD HAIL composite extends from the large-scale precipitation area to approximately the observation site and somewhat beyond. The predominant direction is from large-scale precipitation to the hail observation site. Although the complete hook curvature cannot be traced exactly, the anisotropy pattern is significant and fits to the structures described in Markowski (2002). For SYNOP thunderstorms and showers, the anisotropy is lower, so the directional information in the wavelet spectrum is less meaningful. Low anisotropy suggests that the rain rates related to SYNOP showers and thunderstorms have a more rounded shape.
and, accordingly, rainfall rates do not have a specific direction (case-study in Brune et al., 2018).

The satellite-based composites in Figure 7, which represent both a larger area (100 km versus 25 km) and a different physical quantity (clouds instead of precipitation), reveal a number of interesting differences and similarities. LWsc shows, as with precipitation rates, that the brightness temperature structures around ESWD PRECIP and especially HAIL reports are lower than for SYNOP thunderstorms. We attribute this to the fact that some precipitation and most hail events are associated with strong updraughts, which can be seen in satellite images as overshooting convective cloud tops (e.g., Bedka, 2011; Punge et al., 2017). The overshooting tops lead to small-scale temperature variations at the upper edge of the cloud, which are detected by LWsc. Showers are generally smaller in scale and show almost no spatial variance in LWsc, because the corresponding satellite images consist of isolated pixels with brightness temperatures below the defined 245 K threshold.

The highest LWin values occur in the case of ESWD PRECIP and HAIL when the brightness temperatures are particularly low. The maxima are shifted about 20 to 30 km from the centre to the right, seen from the shear vector. The shifted position of the LWin maximum could detect the inflow line, which is typically found in this region (Weaver et al., 1994; Mazur et al., 2009; Bedka et al., 2010). For SYNOP thunderstorms and especially showers, LWin does not vary in space. This could be related to the fact that in showers the threshold value of 245 K is only
slightly undercut (Pscheidt et al., 2019) and therefore the spatial variance in brightness temperature is lower than for hail events.

The most striking structures in anisotropy can be found in the brightness temperatures of the powerful updrafts for ESWD HAIL events. As in the rain rate analysis, \( L_{W_{ai}} \) is largest to the left of the shear vector. In the case of satellite data, the typical radar hook echo is of course not visible, instead we find a tongue with significantly increased \( L_{W_{ai}} \) values extending from the centre to the right side of the shear vector. This tongue is also clearly visible in the direction of the wavelet spectrum. Responsible for this pattern could be the already mentioned inflow line, which is more linear in its orientation at the top compared to the circular and therefore rather isotropic overshoot.

### 4.4 Classification of shower and thunderstorms using machine learning

In order to quantify the link between LW and the occurrence of reported weather events, we now attempt to predict the class of the observed event based on the three LW components, averaged over the circular environments shown in Figures 6 and 7. Due to their relatively small number of cases, as well as similarities in the respective LW composites, we combine the two ESWD classes HAIL and PRECIP into a single “precip_hail” class.

The caret R-package (Kuhn, 2020) allows us to easily implement and test a variety of classifier algorithms. In a first step, the labelled dataset is separated into 70% training data, 15% test data for model selection and 15% validation data which is used to assess the final model. Each model is fitted to the training data for a range of appropriate meta parameters in a ten-fold cross-validation procedure. The best version of each model is selected based on the optimal value of the Cohen’s Kappa (Cohen, 1960), that is, the improvement in accuracy over a random classification. Next, each optimized model predicts the probabilities of the three classes in the test data.

Based on these preliminary tests with a variety of linear and nonlinear classifiers (including random forests, linear and quadratic discriminant analysis and others), we select a simple feed-forward neural network with nine nodes in a single hidden layer as the best algorithm for our task. In this simple nonlinear model, nine different intermediate values are computed as linear combinations of the input variables and normalized to unit sum via a soft-max activation function. These nine “nodes” are again linearly re-combined and transformed in the same way to produce probabilities for the three classes. In our case, the optimal weights are found via the Broyden–Fletcher–Goldfarb–Shanno algorithm.

| Brier skill scores of the neural network classifier | BSS  |
|---------------------------------------------------|------|
| Everything (radar) | 0.44 |
| Everything (satellite) | 0.21 |
| No intensity | 0.18 |
| No scale | 0.34 |
| No anisotropy | 0.43 |

Note: The top two rows correspond to models using all three indicators from radar or satellite data. The following three rows are based on radar data, where one of the three LW components was left out.

The performance of our selected classifier is summarized by Brier skill scores in Table 3; the climatological frequencies of the classes in the test data serve as the reference forecasts. When all three radar-based LW components are used as input, the model achieves a 44% improvement over the climatology. The skill decreases by roughly 50% when the satellite-based LW is used instead. An even more severe drop in performance occurs when the intensity information is withheld. The scale component appears to be the second most relevant predictor; anisotropy contributes only very little to the overall skill.

Figure 8 shows the predicted class probabilities, averaged over (a) all \( L_{W_{ai}} \) and (b) all \( L_{W_{in}} \), together with the occurrence of the three classes in the validation dataset. We find that SYNOP thunderstorms live on larger scales than SYNOP showers, the predicted division line sloping towards smaller scales at higher intensities. Damaging precipitation and hail (ESWD precip_hail) are exclusively predicted in a small region of intermediate scales and extreme intensities (\( L_{W_{in}} > 0.75 \)), although some instances of this class are scattered across the entire point cloud. Larger values of \( L_{W_{ai}} \) lead to a preference for showers even at larger scales.

In order to quantify the impact of each individual predictor, we construct partial dependency plots (Figure 9). This simplified visualization confirms that SYNOP showers and thunderstorms are well separated by both intensity and scale while the ESWD “precip_hail” class can only be detected by its intensity. As expected, the anisotropy has only marginal influence on the class probabilities.

For a more detailed look at the final model’s behaviour, we consider several scalar forecast attributes for each of the three classes (Table 4). Unsurprisingly, the majority class “SYNOP shower” was slightly over-forecast (Bias > 1) while having the best hit rate as well as the fewest false alarms. The “ESWD precip_hail” class has the best overall proportion correct due to many correct negatives. Conversely, SYNOP thunderstorms appear to be hardest to classify and exhibits the worst performance metrics.
FIGURE 8  Distribution of SYNOP shower, SYNOP thunderstorm and ESWD precipitation or hail events in (a) scale-intensity space and (b) scale-anisotropy space. Thin contours indicate the predicted probabilities of the three classes (starting at \( p = 0.5 \)), and thick lines mark the decision boundaries of the neural network classifier.

FIGURE 9  Partial dependency plots for the neural network classifier. Each predictor is set to a series of fixed values, leaving the remaining test data unchanged. The predicted class probabilities are averaged over all samples and plotted as a function of the fixed predictor value.

|        | PC  | CSI | Bias | FAR | Hit-rate | ETS |
|--------|-----|-----|------|-----|----------|-----|
| Shower | 0.88| 0.85| 1.06 | 0.10| 0.95     | 0.48|
| Thunderstorm | 0.85| 0.44| 0.83 | 0.32| 0.56     | 0.35|
| Precip_hail | 0.97| 0.45| 0.85 | 0.33| 0.57     | 0.43|

TABLE 4  Proportion correct (PC), critical success index (CSI), bias, false alarm ratio (FAR), hit-rate and equitable threat score (ETS) of the neural network classifier.

5 | SUMMARY AND CONCLUDING REMARKS

In this study the convective organization was investigated with the revised local wavelet-based organization index, LW. First, the performance of LW was tested for different types of objects such as squares and lines of different sizes and intensities. The three components LW_{sc}, LW_{in} and LW_{ai}, which describe the scale, intensity and anisotropy of convection, are able to recognize the structure of the tested forms. A comparison of the spatially averaged LW with the conventional organization indices SCAI (Tobin et al., 2012) and COP (White et al., 2018) shows that LW, thanks to its manifold information, is able to distinguish between forms
of organization that look the same in the eyes of SCAI and COP.

After these first encouraging results, LW was applied to real cases. The databases for our analysis are 1 × 1 km radar data and 4 × 4 km satellite observations of brightness temperatures between 2001 and 2018 over Germany. We relate them to convective reports such as showers or thunderstorms reported by weather stations and to severe weather reports such as precipitation or hail provided by the European Severe Weather Database.

Snapshots of two case-studies show that the structures of a supercell are much smaller, can sometimes be more intense and are more isotropic than squall lines. LW is able to identify the small region with hail. LW also detects the larger scale and stronger anisotropy in the stratiform part of the supercell in contrast to the small scale and very intense hook echo. The linear structure of the squall line is characterized by large anisotropy, which is also clearly found in the direction of the wavelet spectrum.

The analysis of cell composites of the radar-based rain rates confirmed the results from the snapshots: hail events, which mostly occur in combination with supercells, as well as heavy precipitation are very local phenomena (small scale, low LWsc). Both types show very large spatial variability as they are very intense (high LWin) and isotropic (higher LWai). For hail reports the wavelet spectrum follows the direction of the hook echo.

Since our LW analysis is not based on any threshold values and the only requirement is a complete spatial field of any variable, LW is universally applicable. To illustrate this, we have also investigated the structure of brightness temperatures. The LW analysis led to similar results. In particular, the small-scale overshooting tops in the region of the large anvil could be identified on the basis of scale, intensity and isotropy.

In a final step, we used the organization and structure metrics LWsc, LWin and LWai to classify showers, thunderstorms and severe precipitation/hail events. We averaged the LW components for each single composite and fitted a variety of classifier algorithms to our data. In cross-validation, a neural network using all three radar-based LW components delivers the best results (BSS ≈ 0.44 compared to climatology). The winning classifier has learned that showers and thunderstorms act on small and large scales, respectively, whereas precipitation/hail events are mainly distinguished by their extreme intensity. Anisotropy has very little influence on the classification. The skill for the satellite-based LW decreases by almost 50%, indicating that the structure of different precipitation events is more discerned based on radar data.

LW also has some limitations: for the calculation of the wavelet spectrum some technical requirements have to be met. The domain should preferably be a square, have a size of 2^n × 2^n, n ∈ N with periodic boundary conditions, and should not contain missing values. These criteria are usually not all met at the same time, so compromises have to be made. The boundary conditions for the radar data had no effect in this study, because the observations were mostly far enough inside the domain. More problematic were the missing values in the brightness temperatures due to beam blockage, which were given a spatial average value. These problems could be avoided, for example, by choosing a model area near the Equator or calculating LW for the model output of numerical weather forecasting systems or reanalyses. However, missing values are also a problem for other organization indices.

The mathematical background of the wavelet transformation is not trivial. Therefore, this paper does not deal with wavelet theory in detail, but rather aims to show the benefits of wavelet spectra. Information on scale, intensity, and anisotropy can be obtained from any two-dimensional field with a single wavelet transformation. To provide easy access to LW and to apply LW to your own datasets, the R-package calcWOI is provided free.

LW is not only a supplement to the many different organizational indices, but combines many features of the individual metrics in the literature. We do not need cluster analysis or well-considered thresholds, but can apply LW directly to two-dimensional fields of any variable. This makes LW universally applicable. The strength of LW, which distinguishes it from all others, is the exact localization of the organization. LW is the first index that displays convective organization on a 2D map.

In this study we identified significant patterns in the radar and satellite observations using LW such as hook echoes, inflow lines, overshooting tops or linear structures. It would be interesting to see whether these results could be confirmed in other studies based on larger datasets and in other regions. LW offers a great opportunity to study climatologies of convective organization. This could lead to new parametrizations of convection and its organization to improve representation in weather and climate prediction models.

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APPENDIX A

COMPARISON: LW VERSUS SCAI AND COP

The results for the idealized images are encouraging in the sense that the index behaves as one would expect. We are extending our investigation to more realistic and compare LW with the organization indices SCAI and COP. In Figure A1 we reproduce four snapshots of different organization obtained from brightness temperatures in Tobin et al. (2012) (their figure 2) and four constructed snapshots from White et al. (2018) (their figure 4). All grey pixels in Figure A1 represent deep convective clouds, defined by brightness temperatures below 240 K. White pixels are assigned to the non-convective environment. SCAI and COP suggest the highest degree of convective aggregation (SCAI = 5.42, COP = 0.52) in situation (a). Higher SCAI values and a lower COP indicate lower convective organization in (d) and (c). The scattered cells in scene (b) are unorganized (SCAI = 16.42, COP = 0.19). All four synthetic fields in Figure A1e–h are not distinguishable in
FIGURE A1  Snapshots of (a)–(d) segmented domains, reproduced from figure 2 in Tobin et al. (2012), and (e)–(h) four synthetic fields, reproduced from figure 5e–h in White et al. (2018). Corresponding SCAI and COP values are shown above each scene.

| Scene | Pixels | SCAI | COP | LW_{sc} | LW_{in} | LW_{ai} | LW_{u} | LW_{v} |
|-------|--------|------|-----|---------|--------|--------|-------|-------|
| (a)   | 22     | 5.42 | 0.52| 0.35    | 0.27   | 0.34   | -0.05 | 0.07  |
| (b)   | 21     | 16.42| 0.19| 0.36    | 0.24   | 0.25   | 0.00  | 0.12  |
| (c)   | 33     | 10.02| 0.31| 0.36    | 0.28   | 0.34   | -0.14 | 0.09  |
| (d)   | 27     | 7.00 | 0.44| 0.32    | 0.26   | 0.32   | 0.01  | 0.06  |
| (e)   | 84     | 11.11| 0.51| 0.44    | 0.23   | 0.15   | 0.05  | 0.05  |
| (f)   | 76     | 11.11| 0.45| 0.45    | 0.23   | 0.15   | 0.05  | 0.05  |
| (g)   | 108    | 11.11| 0.51| 0.45    | 0.23   | 0.15   | -0.03 | 0.06  |
| (h)   | 100    | 11.11| 0.45| 0.45    | 0.23   | 0.15   | -0.00 | 0.06  |

Note: Bold numbers represent maxima (minima for SCAI), italic numbers represent minima (maxima for SCAI) overall eight scenes.

Table A1 compares SCAI and COP with spatial averages of LW for the eight different scenes. Scenes (a)–(d) are generally characterized as smaller in scale, more intense and more anisotropic than the constructed images (e)–(h). Obviously, the absolute number of connected convective pixels in scenes (a)–(d) is much smaller than in (e)–(h) with resulting lower LW_{sc}. The intensity LW_{in} measures the horizontal gradients of brightness temperature. The spatial variance in scenes (a)–(d) is higher, due to the higher number of edges in relation to the total number of convective pixels. In contrast, there is no spatial variance inside the constructed squares in (e)–(h) with consequential smaller LW_{in} values. In the real world we expect high spatial variance in the radar and satellite data. In comparison to SCAI and COP, LW_{ai} considers...
the spatial structures and needs no thresholding. The anisotropy in the constructed cases (e)–(h) is lower than in (a)–(d), because the spatial extent in zonal and meridional directions of the squares is equal.

SCAI and COP reveal no differences between scenes (e) and (g), although there are two big objects in (g), while the size of the squares in (e) is similar. Only the LW index is able to differentiate between these scenes. Large objects in (g) lead to higher LW\textsubscript{ac} values and the regular structured squares in (e) are more isotropic, while both large squares in the right half of scene (g) could be interpreted as an interrupted meridional squall line. The north–south orientation can also be seen in the increased LW\textsubscript{v} component. LW indicates larger and more anisotropic objects in scene (h) compared to scene (f), while the degree of convective organization is the same in the eyes of SCAI and COP.