Determining the (non-)membership degrees in the range (0,1) independently of the decision-makers for bipolar soft sets

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ABSTRACT
Bipolar soft sets, which are a generalization of soft sets, are a very useful mathematical model for performing complex data analysis correctly since each parameter also takes into account the NOT parameter. However, in bipolar soft sets, we can express membership degrees as 0 or 1, and we cannot process data about non-membership degrees. In all proposed mathematical models, the task of expressing a (non-)membership degree in the range (0, 1) focuses on the decision-maker. In this paper, these values in (0, 1) were tried to be determined in an objective way independent of the decision-maker. For this purpose, the concepts of (NOT) bipolar relational membership degree and (NOT) bipolar relational non-membership degree have been proposed. Moreover, using these concepts, a decision-making algorithm focusing on selecting the best object is proposed. Finally, a comparison is given for the proposed algorithm by emphasizing some important points.

1. Introduction

In order to structure a data analysis in a near-ideal way, it is necessary to be able to express the uncertain data very well [1,2]. The inadequacy of classical mathematics in this regard has been a motivation source for researchers in the creation of different mathematical models. In this sense, the fuzzy set theory [3], one of the first proposed mathematical models, was put forward by Zadeh [3]. Thus, membership degrees expressed as 0 or 1 in classical mathematics can be expressed in the range [0, 1]. This very important advantage has allowed it to be a mathematical model that is still widely used to express uncertain situations even today [4–10]. Atanassov [11,12], who thinks that it may be useful to express the membership degrees as well as the non-membership degrees in order to analyze complex data more easily, introduced intuitionistic fuzzy sets to the literature. Since intuitionistic fuzzy sets are a generalization of fuzzy sets, there are still many application areas based on intuitionistic fuzzy sets [13–18]. In the following years, soft sets [19] were proposed by Molodtsov with the contribution of a parameterization tool in order to better express the decision-making processes for uncertainty problems. The contribution of the parameterization tool enabled the expression of objects corresponding to each parameter. In this way, the important contribution of the parameterization tool, which is one of the most important shortcomings for all mathematical modelling built before it, is better understood. Moreover, soft sets have been successfully applied in many fields such as the theory of measurement, game theory, Riemann Integration, smoothness of functions, and so on. In addition, with the contribution of the parameterization tool, the application areas of soft sets continue to increase rapidly in expressing uncertain situations [20–30].

More complex environments may be encountered in the analysis of uncertain data. For example, in some uncertainty problems, considering the NOT parameter of each parameter may provide a better approach to the decision-making process between objects. For this type of uncertainty problems, bipolar soft set theory was proposed by Shabir and Naz [31]. In bipolar soft sets, the data analysis is also made for the negative parameters of each parameter, and the decision-making process is structured more accurately. A soft set structure can also be considered for negative parameters, so a bipolar soft set is a mathematical model that can express two different soft sets together. This makes bipolar soft sets a generalization of soft sets. The interest in bipolar soft sets, which can express two different soft sets as a single mathematical model, continues to increase today [32–42]. However, one of the biggest inadequacies of this mathematical model is to express the membership degrees of objects as 0 or 1. Another inadequacy is that there is no configuration about the non-membership degree of objects. Researchers have proposed many mathematical models for these inad-
With the development of the theory [43–48], however, in all these proposed mathematical models, it is left to the decision-maker to express the (non-)membership degrees of the objects. This situation can directly affect the decision-making process. Because it is very difficult for decision-makers to accurately express the (non-)membership degrees, some technical formulations have been proposed to objectively determine the membership degrees in the range (0, 1). The most important reason for this is that there are many numbers in this range. In order to manage the decision-making process more accurately, it is necessary to act independently from the decision-maker as much as possible. In this paper, some technical formulations have been proposed to objectively determine the membership degrees in (0, 1) for bipolar soft sets. Moreover, in order to determine these values correctly, it is sufficient for the decision-maker as much as possible. In this paper, some technical formulations have been proposed to objectively determine the membership degrees in (0, 1) for bipolar soft sets. Moreover, in order to determine these values correctly, it is sufficient for the decision-maker to express membership degrees as 0 or 1. In this way, decision-making processes for uncertainty problems can be better managed, largely independent of decision-makers.

The presentation of the rest of this paper is structured as follows. In the second section, the framework of fuzzy sets, intuitionistic fuzzy sets, soft sets and bipolar soft sets are introduced. In the third section, for bipolar soft sets, the concepts of (NOT) bipolar relational membership degree and (NOT) bipolar relational non-membership degree have been proposed to objectively determine the (non-)membership degrees of the objects in (0, 1) independently of the decision-maker. Moreover, some properties of these concepts are examined in detail. In the fourth section, a decision-making algorithm based on the concepts given in the previous section is proposed. In the fifth section, other decision-making approaches available in the literature for bipolar soft sets were analyzed and compared with the proposed algorithm. Then, some important points are highlighted for the proposed algorithm. The final section consists of the conclusion of the paper.

2. Preliminaries

In this section, some theories from the literature are reminded to support the concepts expressed in the rest of the paper. Some supporting information is also provided.

Throughout this paper, \( U = \{u_1, u_2, \ldots, u_m\} \) is an initial universe, \( E = \{e_1, e_2, \ldots, e_n\} \) is a set of parameters and \( 2^U \) is the power set of \( U \).

**Definition 2.1 ([3]):** A fuzzy set \( F \) over \( U \) is a set defined by \( \mu_F : U \rightarrow [0, 1] \). \( \mu_F \) is called the membership function of \( F \), and the value \( \mu_F(u) \) is called the membership degree of \( u \in U \). Thus, a fuzzy set \( F \) over \( U \) can be represented by

\[
F = \{ (u, \mu_F(u)) : u \in U \}
\]  

**Definition 2.2 ([12]):** An intuitionistic fuzzy set \( I \) on \( U \) can be defined by \( \mu_I : U \rightarrow [0, 1] \) and \( \nu_I : U \rightarrow [0, 1] \) such that \( 0 \leq \mu_I(u) + \nu_I(u) \leq 1 \); \( \forall u \in U \). Here, \( \mu_I(u) \) and \( \nu_I(u) \) is the membership degree and non-membership degree of \( u \in U \), respectively. Thus, an intuitionistic fuzzy set \( I \) over \( U \) can be represented by

\[
I = \{ (u, \mu_I(u), \nu_I(u)) : u \in U \}
\]  

Clearly, when \( \nu_I(u) = 1 - \mu_I(u), u \in U \), the \( I \) becomes a fuzzy set. Then, an indeterminacy degree of \( u \) to \( X \) is determined by \( \pi_I(u) = 1 - (\mu_I(u) + \nu_I(u)) \).

State that the set of all the intuitionistic fuzzy sets over \( U \) will be denoted by \( IF(U) \).

**Example 2.1:** Let

\[
U = \{ u_1 : \text{purple colored pencil}, u_2 : \text{pink colored pencil}, u_3 : \text{green colored pencil}, u_4 : \text{orange colored pencil}, u_5 : \text{blue colored pencil}, u_6 : \text{brown colored pencil} \}
\]

be the set of pencils in a colored pencil box. Then, the intuitionistic fuzzy set \( I \) on \( U \) can be written as:

\[
I = \{ (u_1, 1, 0), (u_2, 0.55, 0.14), (u_3, 0, 0), (u_4, 0.35, 0.5), (u_5, 0, 1), (u_6, 0.2, 0.45) \}
\]

or

\[
I = \{ (u_1, 1, 0), (u_2, 0.55, 0.14), (u_4, 0.35, 0.5), (u_5, 0, 1), (u_6, 0.2, 0.45) \}
\]

For example, when \( I \) for “pink colored pencil” is examined, the membership degree is 0.55 and the non-membership degree is 0.14. Therefore, the indeterminacy degree for “pink colored pencil” to \( I \) is \( \pi_I(u_2) = 1 - (\mu_I(u_2) + \nu_I(u_2)) = 1 - (0.55 + 0.14) = 0.31 \).

**Definition 2.3 ([19]):** A mapping \( S : E \rightarrow 2^U \) is called a soft set over \( U \) and denoted by \( (S, E) \). Thus, a soft set \( (S, E) \) over \( U \) can be represented by

\[
(S, E) = \{ (e, S(e)) : e \in E, S(e) \in 2^U \}
\]  

**Definition 2.4 ([49]):** The NOT set of \( E \), denoted by \( \overline{E} \), is defined by \( \overline{E} = \{ e \in E \} \), where \( \overline{e} = \overline{e} \in \overline{E} \).

**Definition 2.5 ([31]):** Let \( (\Phi, E), (\Psi, \overline{E}) \) be two soft sets over \( U \) such that \( \Phi(e) \cap \overline{\Psi(e)} = \emptyset \); \( \forall e \in E \). Then, a triplet \( (\Phi, \Psi, E) \) is called a bipolar soft set over \( U \).

State that the set of all the bipolar set over \( U \) will be denoted by \( B(U) \).

**Definition 2.6 ([50]):** Let \( (\Phi, \Psi, E) \in B(U) \). Then, \( (\Phi, \Psi, E) \) can be represented by

\[
(\Phi, \Psi, E) = \{ (e, \Phi(e), \Psi(\overline{e})) : e \in E, \overline{e} \in \overline{E}; \Phi(e), \Psi(e) \in 2^U \}
\]
Example 2.2: Let \( U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\} \) be the set of some personnel working in a private equity-owned company and \( E = \{e_1 : hardworking, e_2 : determined, e_3 : honest, e_4 : successful\} \) be the set of desired parameters from personnel. In this case, \( \neg E = \{-e_1 : lazy, e_2 : unstable, e_3 : not honest, e_4 : unsuccessful\} \) is the set of undesirable parameters in personnel. If \( \Phi(e_1) = \{u_2, u_3, u_6\}, \Phi(e_2) = \{u_5, u_7\}, \Phi(e_3) = \{u_1, u_3, u_8\}\) and \( \Psi(e_1) = \{u_4, u_6, u_7\} \) and \( \Psi(e_2) = \{u_3, u_5, u_6\}, \Psi(e_3) = \{u_1, u_2, u_5\} \); then the bipolar soft set \((\Phi, \Psi, E)\) on \( U \) can be written as

\[
(\Phi, \Psi, E) = \begin{cases}
(e_1, \{(u_2, 1, 0), (u_3, 1, 0), (u_6, 1, 0)\}), \\
(e_2, \{(u_1, 1, 0), (u_5, 1, 0), (u_7, 1, 0)\}), \\
(e_3, \{(u_2, 1, 0), (u_3, 1, 0), (u_6, 1, 0)\}), \\
(e_4, \{(u_1, 1, 0), (u_5, 1, 0), (u_6, 1, 0)\}), \\
(e_5, \{(u_1, 1, 0), (u_2, 1, 0), (u_5, 1, 0)\})
\end{cases}
\]

or

\[
(\Phi, \Psi, E) = \begin{cases}
(e_1, \{(u_2, 1, 0), (u_3, 1, 0), (u_6, 1, 0)\}), \\
(e_2, \{(u_1, 1, 0), (u_5, 1, 0), (u_7, 1, 0)\}), \\
(e_3, \{(u_2, 1, 0), (u_3, 1, 0), (u_6, 1, 0)\}), \\
(e_4, \{(u_1, 1, 0), (u_5, 1, 0), (u_6, 1, 0)\}), \\
(e_5, \{(u_1, 1, 0), (u_2, 1, 0), (u_5, 1, 0)\})
\end{cases}
\]

which describes the personnel parameters working in the company.

3. Formulations for determining (non-)membership degrees in the range \((0, 1)\) for bipolar soft sets

In this section, some formulations are proposed to determine the (non-)membership degree in the range \((0, 1)\) of an object in bipolar soft sets independently from the decision-maker. In this way, it is aimed to prevent a possible error that may occur when expressing (non-)membership degrees by decision-makers in order to be able to analyze data well in environments where an uncertain is tried to be eliminated. Moreover, some important properties of these formulations are highlighted.

Bipolar soft sets, which is one of the mathematical models recommended for the better structure of more complex data sets in data analysis, is a generalization of soft sets. In this mathematical model, unlike soft sets, the NOT parameter of each parameter is taken into account. In this way, it is possible to model more complex data with a single set type. However, it is an important inadequacy to express the objects corresponding to the parameters as membership degrees of 0 or 1. Moreover, we do not have any information about the non-membership degrees for this set type. Many mathematical models have been constructed to overcome such inadequacies [43–48,51,52]. However, in all these constructed mathematical models, the task of expressing the membership degrees in the range \((0, 1)\) focuses on the decision-maker. We should consider a possible error problem when the decision-makers express these values. In this section, some concepts for these deficiencies are proposed. These concepts are for the determination of (non-)membership degrees in \((0, 1)\) based on the values 0 and 1 expressed by the decision-makers.

Definition 3.1: Let \((\Phi, \Psi, E) \in B(U)\). For \( u_k \in [U \setminus \Phi(e_i)] \cap [U \setminus \Psi(\neg e_i)] \) and \( u_j \in \Phi(e_j) \), the bipolar relational membership degree and bipolar relational non-membership degree of \( u_k \) to \( \Phi(e_i) \) is expressed by

\[
\mu_{\Phi(e_i)}(u_k) = \frac{1}{|\Phi(e_i)|(n-1)} \sum_{u_j \in \Phi(e_i)} \sum_{e \in E} \mu_{\Phi(e_i)}(u_k, u_j)
\]

where

\[
\mu_{\Phi(e_i)}(u_k, u_j) = \begin{cases}
1, & \mu_{\Phi(e_i)}(u_k) + \mu_{\Phi(e_i)}(u_j) = 2 \\
0, & \text{otherwise}
\end{cases}
\]

is a mapping given by \( \phi_{\Phi(e_i)} : [U \setminus \Phi(e_i)] \times \Phi(e_i) \rightarrow [0, 1] \). Then, the bipolar relational non-membership function is given as follows:

\[
\nu_{\Phi(e_i)}(u_k) = \frac{1}{|\Phi(e_i)|(n-1)} \sum_{u_j \in \Phi(e_i)} \sum_{e \in E} \nu_{\Phi(e_i)}(u_k, u_j)
\]

where

\[
\nu_{\Phi(e_i)}(u_k, u_j) = \begin{cases}
1, & \mu_{\Phi(e_i)}(u_k) = 0 \text{ and } \mu_{\Phi(e_i)}(u_j) = 1 \\
0, & \text{otherwise}
\end{cases}
\]

is a mapping given by \( \phi_{\Phi(e_i)} : [U \setminus \Phi(e_i)] \times \Phi(e_i) \rightarrow [0, 1] \). Here, \( \mu_{\Phi(e_i)} \) is the membership function of \( \Phi \) and \( |\Phi(p_i)| \) is the cardinality of \( \Phi(p_i) \).

Definition 3.2: Let \((\Phi, \Psi, E) \in B(U)\). For \( u_k \in [U \setminus \Phi(e_i)] \cap [U \setminus \Psi(\neg e_i)] \) and \( u_j \in \Psi(\neg e_j) \), the NOT bipolar relational membership degree and NOT bipolar relational non-membership degree of \( u_k \) to \( \Psi(\neg e_i) \) is expressed by

\[
\mu_{\Psi(\neg e_i)}(u_k) = \frac{1}{|\Psi(\neg e_i)|(n-1)} \sum_{u_j \in \Psi(\neg e_i)} \sum_{e \in E} \mu_{\Psi(\neg e_i)}(u_k, u_j)
\]

where

\[
\mu_{\Psi(\neg e_i)}(u_k, u_j) = \begin{cases}
1, & \mu_{\Psi(\neg e_i)}(u_k) = 0 \text{ and } \mu_{\Psi(\neg e_i)}(u_j) = 1 \\
0, & \text{otherwise}
\end{cases}
\]

is a mapping given by \( \phi_{\Psi(\neg e_i)} : [U \setminus \Psi(\neg e_i)] \times \Psi(\neg e_i) \rightarrow [0, 1] \). Here, \( \mu_{\Psi(\neg e_i)} \) is the membership function of \( \Psi \) and \( |\Psi(p_i)| \) is the cardinality of \( \Psi(p_i) \).
function is given as follows:

\[
(\Phi, \Psi, E) \rightarrow \Lambda_{\mu} (u_k, y_j) = \begin{cases} 
1, & \mu_{\Psi}(u_k) + \mu_{\Phi}(y_j) = 2 \\
0, & \text{otherwise}
\end{cases}, \quad \forall -e \in -E
\]

is a mapping given by \( e \Lambda_{\mu} : [U \setminus (\Psi(\neg e))] \times (\Psi(\neg e)) \rightarrow [0, 1] \). Then, the NOT bipolar relational non-membership function is given as follows:

\[
(\Phi, \Psi, E) \rightarrow \Lambda_{\nu} (u_k, y_j) = \begin{cases} 
1, & \mu_{\Psi}(\neg u_k) + \mu_{\Phi}(\neg y_j) = 1 \\
0, & \text{otherwise}
\end{cases}, \quad \forall -e \in -E - (\neg e)
\]

is a mapping given by \( e \Lambda_{\nu} : [U \setminus (\Psi(\neg e))] \times (\Psi(\neg e)) \rightarrow [0, 1] \). Here, \( \mu_{\Psi}(\neg \cdot) \) is the membership function of \( \Psi \) and \( \Psi(\neg e) \) is the cardinality of \( \Psi(\neg e) \).

**Proposition 3.1:** Let \((\Phi, \Psi, E) \in B(U)\). Then, we have \( e \Lambda_{\mu} (u_k, y_j) = e \Lambda_{\mu} (u_k, y_j) \) and \( -e \Lambda_{\mu} (u_k, y_j) = -e \Lambda_{\mu} (u_k, y_j) \) for all \( e \in E, -e \in -E \) and \( u_k, y_j \in U \). However, the similar situation may not be valid for \( e \Lambda_{\nu} \) and \( -e \Lambda_{\nu} \).

**Proof:** This proof is easily obtained from Definitions 3.1 and 3.2. \[ \blacksquare \]

**Example 3.1:** Let \( U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7 = m\} \) and \( E = \{(e_1, e_2, e_3, e_4, e_5, e_6, e_7)\} \). If \( \Phi(e_1) = \{u_1, u_3, u_7\}, \Phi(e_2) = \{u_2, u_6, u_7\}, \Phi(e_3) = \{u_1, u_2, u_5\}, \Phi(e_4) = \{u_4, u_5, u_6\}, \Phi(e_5) = \{u_3, u_5, u_7\} \) and \( \Psi(\neg e_1) = \{u_4, u_5, u_7\} \) and \( \Psi(\neg e_2) = \{u_1, u_5\}, \Psi(\neg e_3) = \{u_3, u_4\}, \Psi(\neg e_4) = \{u_1, u_2, u_7\}, \Psi(\neg e_5) = \{u_2, u_4\} \); then the bipolar soft set

\[
(\Phi, \Psi, E) = \begin{cases} 
\{\Phi(e_1) = \{u_1, u_3, u_7\}, \Phi(e_2) = \{u_2, u_6, u_7\}, \Phi(e_3) = \{u_1, u_2, u_5\}, \Phi(e_4) = \{u_4, u_5, u_6\}, \Phi(e_5) = \{u_3, u_5, u_7\}\} \\
\{\Psi(\neg e_1) = \{u_4, u_5, u_7\} \}
\end{cases}
\]
Moreover, since $\pi_{\Phi_1(\epsilon_1)}(u_6) = 1 - (\mu_{\Phi_1(\epsilon_1)}(u_6) + \nu_{\Phi_1(\epsilon_1)}(u_6)) = 1 - \frac{1}{2} = \frac{1}{2}$ and $\pi_{\Psi(\epsilon_1)}(u_6) = 1 - (\mu_{\Psi(\epsilon_1)}(u_6) + \nu_{\Psi(\epsilon_1)}(u_6)) = \frac{5}{12}$, the indeterminacy degrees of $u_6$ to $\Phi_1(\epsilon_1)$ and $\Psi(\epsilon_1)$ in $(\Phi, \Psi, E)$ are 2/3 and 7/12, respectively. Here, $\pi_{\Phi_1(\epsilon_1)}(u_6), \mu_{\Phi_1(\epsilon_1)}(u_6)$ and $\pi_{\Psi(\epsilon_1)}(u_6), \mu_{\Psi(\epsilon_1)}(u_6), \nu_{\Psi(\epsilon_1)}(u_6)$, $\nu_{\Phi_1(\epsilon_1)}(u_6)$ are the interdeterminacy degree, membership degree and non-membership degree of $\Phi_1(\epsilon_1)$ and $\Psi(\epsilon_1)$ for $e_1 \in E$, $e_\not\in \not\in E$, respectively.

Similarly, for $e_5$: Since $u_3, u_4 \in [(\cup \Phi(\epsilon_2)) \cup (\cup \Psi(\epsilon_2))]$, then
$$\left\{ \begin{array}{c} \forall_{\Phi_1(\epsilon_1)}(e_5) = 1/6, \forall_{\Psi_1(\epsilon_1)}(e_5) = 1/6 \text{ and } \forall_{\Psi_1(\epsilon_1)}(e_5) = 0, \forall_{\Phi_1(\epsilon_1)}(e_5) = 1/4 \text{ and } \forall_{\Phi_1(\epsilon_1)}(e_5) = 1/4, \forall_{\Psi_1(\epsilon_1)}(e_5) = 0, \forall_{\Psi_1(\epsilon_1)}(e_5) = 1/4 \text{ and } \forall_{\Psi_1(\epsilon_1)}(e_5) = 1/4. \end{array} \right.$$}

For $e_5$: Since $u_3, u_4 \in [(\cup \Phi(\epsilon_2)) \cup (\cup \Psi(\epsilon_2))]$, then
$$\left\{ \begin{array}{c} \forall_{\Phi_1(\epsilon_1)}(e_5) = 1/6, \forall_{\Phi_1(\epsilon_1)}(e_5) = 1/6 \text{ and } \forall_{\Psi_1(\epsilon_1)}(e_5) = 0, \forall_{\Psi_1(\epsilon_1)}(e_5) = 1/4 \text{ and } \forall_{\Psi_1(\epsilon_1)}(e_5) = 1/4. \end{array} \right.$$}

Thus, the bipolar soft set $(\Phi, \Psi, E)$ can be revised as follows:

(i) $u_{a_1}, u_{a_2} \in \Phi(\epsilon)$ and $u_{a_2}, u_{a_3} \in \Psi(\not\in \epsilon)$. 
(ii) $u_{a_1}, u_{a_2} \in \Phi(\epsilon)$ and $u_{a_3} \in \Psi(\not\in \epsilon)$. 
(iii) $u_{a_1}, u_{a_2} \in \Phi(\epsilon)$ and $u_{a_3} \in \Psi(\not\in \epsilon)$. 
(iv) $u_{a_1}, u_{a_2} \in \Phi(\epsilon)$ and $u_{a_3} \in \Psi(\not\in \epsilon)$. 
(v) $u_{a_1}, u_{a_2} \in \Phi(\epsilon)$ and $u_{a_3} \in \Psi(\not\in \epsilon)$. 

**Proof:** (i) Since $u_{a_1} \in \Phi(\epsilon)$ and $u_{a_2} \in \Phi(\epsilon)$, then $\mu_{\Phi_1(\epsilon_1)}(u_{a_1}) = 1$ and $\mu_{\Phi_1(\epsilon_1)}(u_{a_2}) = 1$; respectively. Similarly, since $u_{a_3} \in \Psi(\not\in \epsilon)$ and $u_{a_3} \in \Psi(\not\in \epsilon)$, then $\mu_{\Psi_1(\epsilon_1)}(u_{a_3}) = 1$ and $\mu_{\Psi_1(\epsilon_1)}(u_{a_3}) = 1$; respectively. Thus, $\mu_{\Phi_1(\epsilon_1)}(u_{a_1}) = 1$ and $\mu_{\Phi_1(\epsilon_1)}(u_{a_2}) = 0$; respectively. Similarly, since $u_{a_3} \in \Psi(\not\in \epsilon)$ and $u_{a_3} \in \Psi(\not\in \epsilon)$, then $\mu_{\Psi_1(\epsilon_1)}(u_{a_3}) = 0$ and $\mu_{\Psi_1(\epsilon_1)}(u_{a_3}) = 0$; respectively. Thus, $\mu_{\Phi_1(\epsilon_1)}(u_{a_1}) = 0 = \mu_{\Phi_1(\epsilon_1)}(u_{a_2})$.

(ii) Since $u_{a_1} \in \Phi(\epsilon)$ and $u_{a_2} \in \Phi(\epsilon)$, then $\mu_{\Phi_1(\epsilon_1)}(u_{a_1}) = 1$ and $\mu_{\Phi_1(\epsilon_1)}(u_{a_2}) = 0$; respectively. Similarly, since $u_{a_3} \in \Psi(\not\in \epsilon)$ and $u_{a_3} \in \Psi(\not\in \epsilon)$, then $\mu_{\Psi_1(\epsilon_1)}(u_{a_3}) = 0$ and $\mu_{\Psi_1(\epsilon_1)}(u_{a_3}) = 0$; respectively. Thus, $\mu_{\Phi_1(\epsilon_1)}(u_{a_1}) = 0 = \mu_{\Phi_1(\epsilon_1)}(u_{a_2})$.

(iii) It can be proved similar way (iii).

(iv) Since $u_{a_1}, u_{a_2} \in [(\cup \Phi(\epsilon_2)) \cup (\cup \Psi(\epsilon_2))]$, then $u_{a_1}, u_{a_2} \in U \setminus \Phi(\epsilon)$ and $u_{a_1}, u_{a_2} \in U' \setminus \Psi(\not\in \epsilon)$. So, $\mu_{\Phi_1(\epsilon_1)}(u_{a_1}) = 0$ and $\mu_{\Phi_1(\epsilon_1)}(u_{a_2}) = 0$ and $\mu_{\Phi_1(\epsilon_1)}(u_{a_3}) = 0$ and $\mu_{\Phi_1(\epsilon_1)}(u_{a_3}) = 0$. Hence, $\mu_{\Phi_1(\epsilon_1)}(u_{a_1}) = 0$. 

(v) It can be proved similar way (iv).

**Proposition 3.3:** Let $(\Phi, \Psi, E) \in B(U)$. If $e_{\Lambda_{\mu_{\Phi_1(\epsilon_1)}}(u_{a_1}), e_{\Lambda_{\mu_{\Phi_1(\epsilon_1)}}(u_{a_2}), e_{\Lambda_{\mu_{\Phi_1(\epsilon_1)}}(u_{a_3})}}$ for all $e \in E$, $e \in \not\in E$ and $u_{a_1}, u_{a_2}, u_{a_3}, u_{a_3} \in U$; then any of the following items are provided:

(i) $u_{a_1}, u_{a_2} \in \Phi(\epsilon)$ and $u_{a_2}, u_{a_3} \in \Psi(\not\in \epsilon)$. 
(ii) $u_{a_1} \in \Phi(\epsilon)$ and $u_{a_2} \in \Psi(\not\in \epsilon)$. 
(iii) $u_{a_1} \in \Phi(\epsilon)$ and $u_{a_2} \in \Psi(\not\in \epsilon)$. 
(iv) $u_{a_1}, u_{a_2} \in [U \setminus \Phi(\epsilon)] \cap [U \setminus \Psi(\not\in \epsilon)]$. 
(v) $u_{a_1}, u_{a_2} \in [U \setminus \Phi(\epsilon)] \cap [U \setminus \Psi(\not\in \epsilon)]$. 

**Proof:** It is similar to Proposition 3.2.

**Remark 3.1:** Let $(\Phi, \Psi, E) \in B(U)$. Then,

(i) If $\mu_{\Phi_1(\epsilon_1)}(u_{a_1})$ is found close to 1(0); $\mu_{\Phi_1(\epsilon_1)}(u_{a_2})$ is found close to 0(1).

(ii) If $\mu_{\Phi_1(\epsilon_1)}(u_{a_1})$ is found close to 1(0); $\mu_{\Phi_1(\epsilon_1)}(u_{a_2})$ is found close to 0(1).

(iii) If $\mu_{\Psi_1(\epsilon_1)}(u_{a_1})$ is found close to 1(0); $\mu_{\Phi_1(\epsilon_1)}(u_{a_2})$ is found close to 0(1).

(iv) If $\mu_{\Psi_1(\epsilon_1)}(u_{a_1})$ is found close to 1(0); $\mu_{\Phi_1(\epsilon_1)}(u_{a_2})$ is found close to 0(1).

These situations are a direct consequence of Definitions 3.1 and 3.2. Hence, 0 $\leq \mu_{\Phi_1(\epsilon_1)}(u_{a_1}) + \mu_{\Phi_1(\epsilon_1)}(u_{a_2}) < 1$ and 0 $\leq \mu_{\Psi_1(\epsilon_1)}(u_{a_1}) + \mu_{\Psi_1(\epsilon_1)}(u_{a_2}) < 1$. 

4. A decision-making approach for bipolar soft sets

In this section, a decision-making algorithm based on the concepts given in the previous section is proposed for bipolar soft sets.

The decision-making algorithm is constructed using the concepts of (NOT) bipolar relational membership degree and (NOT) bipolar relational non-membership degree based on bipolar soft sets as follows:

Now, let’s give an example to show the principles and steps of Algorithm 1 in an uncertain environment:

Example 4.1: Suppose a person wants to buy a hybrid car. For this reason, he/she wants to go to a gallery and determine the most suitable cars for the parameter \( E = \{ e_1 : \text{air conditioned}, e_2 : \text{comfortable}, e_3 : \text{cheap}, e_4 : \text{simple}, e_5 : \text{luxury}, e_6 : \text{economic} \} \) he/she wants. Moreover, let \( U = \{ u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8 \} \) be the set of cars available in the gallery. In order to facilitate the determination of the most suitable car for this person; the gallery owner also evaluates the undesired parameters based on the desired parameters, i.e. \( \neg E = \{ \neg e_1 : \text{without air conditioning}, \neg e_2 : \text{comfortless}, \neg e_3 : \text{expensive}, \neg e_4 : \text{complex}, \neg e_5 : \text{standard}, \neg e_6 : \text{uneco-nomical} \} \). In this way, it is thought that it may be easier to eliminate some cars. As a result of these evaluations made by the gallery owner, the vehicles corresponding to each parameter is given as follows:

\[
\begin{align*}
\Phi(e_1) &= \{ u_1, u_3, u_5, u_6 \}, \\
\Phi(e_2) &= \{ u_2, u_5, u_6, u_7 \}, \\
\Phi(e_3) &= \{ u_1, u_4, u_7, u_8 \}, \\
\Phi(e_4) &= \{ u_1, u_4, u_5 \}, \\
\Phi(e_5) &= \{ u_2, u_3 \}, \\
\Phi(e_6) &= \{ u_2, u_4, u_6 \}, \\
\Psi(-e_1) &= \{ u_2, u_4, u_7 \}, \\
\Psi(-e_2) &= \{ u_3, u_4, u_7 \}, \\
\Psi(-e_3) &= \{ u_3, u_5, u_6 \}, \\
\Psi(-e_4) &= \{ u_2, u_3, u_6, u_7 \}, \\
\Psi(-e_5) &= \{ u_1, u_4, u_5, u_6 \}, \\
\Psi(-e_6) &= \{ u_1, u_3, u_7, u_8 \}.
\end{align*}
\]

Step 1: The evaluation results expressed by the gallery owner can be expressed with the help of a bipolar soft set as follows:

\[
(\Phi, \Psi, E) = \left\{ (e_1, \{ u_1, u_3, u_5, u_6 \}), \{ u_2, u_4, u_7 \}), \right. \\
\left. (e_2, \{ u_2, u_5, u_6, u_7 \}), \{ u_3, u_4, u_7 \}), \right. \\
\left. (e_3, \{ u_1, u_4, u_7, u_8 \}), \{ u_3, u_5, u_6 \}), \right. \\
\left. (e_4, \{ u_1, u_4, u_5 \}), \{ u_2, u_3, u_6, u_7 \}), \right. \\
\left. (e_5, \{ u_2, u_3 \}), \{ u_1, u_4, u_5, u_6 \}), \right. \\
\left. (e_6, \{ u_2, u_4, u_6 \}), \{ u_1, u_3, u_7, u_8 \}) \right\}
\]

Step 2: For \((\Phi, \Psi, E)\); since

\[
\begin{align*}
u_8 &\in \left( U \setminus \Phi(e_1) \right) \cap \left( U \setminus \Psi(-e_1) \right), \\
u_1 &\in \left( U \setminus \Phi(e_2) \right) \cap \left( U \setminus \Psi(-e_2) \right), \\
u_2 &\in \left( U \setminus \Phi(e_3) \right) \cap \left( U \setminus \Psi(-e_3) \right), \\
u_8 &\in \left( U \setminus \Phi(e_4) \right) \cap \left( U \setminus \Psi(-e_4) \right), \\
u_7, u_8 &\in \left( U \setminus \Phi(e_5) \right) \cap \left( U \setminus \Psi(-e_5) \right), \\
u_5 &\in \left( U \setminus \Phi(e_6) \right) \cap \left( U \setminus \Psi(-e_6) \right);
\end{align*}
\]

then

\[
\begin{align*}
\Psi_{u_1} (e_1) &= 3/20, \\
\Psi_{u_1} (e_1) &= 1/5, \\
\Psi_{u_1} (e_1) &= 1/3, \\
\Psi_{u_1} (e_1) &= 3/20, \\
\Psi_{u_1} (e_1) &= 1/3, \\
\Psi_{u_1} (e_1) &= 1/3, \\
\Psi_{u_1} (e_1) &= 3/20, \\
\Psi_{u_1} (e_1) &= 4/15, \\
\Psi_{u_1} (e_1) &= 1/5, \\
\Psi_{u_1} (e_1) &= 1/5, \\
\Psi_{u_1} (e_1) &= 7/20, \\
\Psi_{u_1} (e_1) &= 1/10, \\
\Psi_{u_1} (e_1) &= 7/20, \\
\Psi_{u_1} (e_1) &= 3/10, \\
\Psi_{u_1} (e_1) &= 3/20, \\
\Psi_{u_1} (e_1) &= 3/20, \\
\Psi_{u_1} (e_1) &= 1/5, \\
\Psi_{u_1} (e_1) &= 1/4, \\
\Psi_{u_1} (e_1) &= 2/15, \\
\Psi_{u_1} (e_1) &= 3/20, \\
\Psi_{u_1} (e_1) &= 1/5.
\end{align*}
\]

Step 3: For \( u_1 \); \( \bigcup \) \((u_1)\) is calculated as follows:

\[
\begin{align*}
\bigcup (u_1) &= \sum_{e \in E} \left[ \mu_{\Phi(e)} (u_1) - \nu_{\Psi(e)} (u_1) \right] \\
&= \sum_{e \in E} \left[ \mu_{\Phi(e)} (u_1) - \nu_{\Psi(e)} (u_1) \right] \\
&= \left[ (1 - 0) + (1/5 - 3/20) \right] \\
&= \left[ (1 - 0) + (1 - 0) \right] \\
&= \left[ 3/15 - 1/3 \right] + (1 - 0) + (1 - 0) \\
&= 1.184.
\end{align*}
\]

Similarly,

\[
\begin{align*}
\bigcup (u_2) &= 1.084, \\
\bigcup (u_3) &= -1, \\
\bigcup (u_4) &= 1.184, \\
\bigcup (u_5) &= 1.184, \\
\bigcup (u_6) &= 0, \\
\bigcup (u_7) &= -3.3, \\
\bigcup (u_8) &= 1.567.
\end{align*}
\]

Step 4: For \( \bigcup (u_k) = \max \{ \bigcup (u_k) : 1 \leq k \leq m \} = 1.567 \); by taking into account the desired and undesirable parameters, it is determined that the best hybrid car for the current person is \( u_8 \).
Algorithm 1 Determine the best object based on a bipolar soft sets.

Require: $U = \{u_1, u_2, \ldots, u_m, \ldots, u_n\}$, $E = \{e_1, e_2, \ldots, e_i, \ldots, e_n\}$, $1 \leq i \leq n$, $1 \leq k \leq m$ and $m, n \geq 2$

Step 1: Input the bipolar soft set $(\Phi, \Psi, E) \in B(U)$ as follows:

$$(\Phi, \Psi, E) = \{(e, \Phi(e), \Psi(\neg e)) : e \in E, \neg e \in E; \Phi(e), \Psi(e) \in 2^U\}.$$  

Step 2: Calculate all (NOT) bipolar relational membership degrees and (NOT) bipolar relational non-membership degrees using the $(\Phi, \Psi, E)$.

Step 3: Calculate the $\bigcup (u_k)$ of objects $u_k$:

$$\bigcup (u_k) = \sum_{e \in E} [\mu_{\Phi(e)}(u_k) - \nu_{\Phi(e)}(u_k)] - \sum_{e \in E} [\mu_{\Psi(\neg e)}(u_k) - \nu_{\Psi(\neg e)}(u_k)]$$

where

$$\mu_{\Phi(e)}(u_k) = \begin{cases} 1, & u_k \in \Phi(e) \\ \nu_{\Phi(e)}(u_k), & u_k \in [U \setminus \Phi(e)] \cap [U \setminus \Psi(\neg e)] \end{cases}$$

and

$$\nu_{\Phi(e)}(u_k) = \begin{cases} 0, & u_k \in \Phi(e) \\ \nu_{\Phi(e)}(u_k), & u_k \in [U \setminus \Phi(e)] \cap [U \setminus \Psi(\neg e)] \end{cases}$$

$$\mu_{\Psi(\neg e)}(u_k) = \begin{cases} 1, & u_k \in \Psi(\neg e) \\ \nu_{\Psi(\neg e)}(u_k), & u_k \in [U \setminus \Phi(e)] \cap [U \setminus \Psi(\neg e)] \end{cases}$$

$$\nu_{\Psi(\neg e)}(u_k) = \begin{cases} 0, & u_k \in \Psi(\neg e) \\ \nu_{\Psi(\neg e)}(u_k), & u_k \in [U \setminus \Phi(e)] \cap [U \setminus \Psi(\neg e)] \end{cases}.$$  

Here; $\nu_{\Phi(e)}$ and $\nu_{\Psi(\neg e)}$ are the non-membership functions of $\Phi$ and $\Psi$, respectively.

Step 4: Find $r$, for which $\bigcup (u_r) = \max \left\{ \bigcup (u_k) : 1 \leq k \leq m \right\}$.

5. A comparison

In this section, other decision-making approaches available in the literature for bipolar soft sets were analyzed and compared with the proposed algorithm.

In this paper, the issue of expressing the membership degrees and non-membership degrees of decision-makers in the range $(0, 1)$ for bipolar soft sets is discussed. It is quite a challenge for decision-makers to accurately express a (non-)membership degree in $(0, 1)$. In this case, it becomes difficult to make an accurate data analysis by relying on the data provided by decision-makers. For this reason, it is clear that the algorithm we propose in this section, based on the concepts proposed in the previous section, is better than the approaches given for bipolar soft sets constructed by considering (intuitionistic) fuzzy sets. Because these types of mathematical models are built in a way that focuses directly on decision-makers.

Here, decision-making algorithms given for bipolar soft sets and the approach proposed in this paper are compared. In this way, it is aimed to emphasize which algorithm is superior by using the available data for bipolar soft set theory. In the literature, there are two different approaches that are constructed independently of parametric weights from the proposed decision-making algorithms for bipolar soft sets. If we consider these approaches and the approach proposed in this study for the solution of the uncertainty problem given in Example 4.1, the order among the objects is as follows:

As can be seen from Table 1, it is clear that the decision-making algorithm proposed in this paper should be preferred in order to select the best object and to make a better distinction between objects. Therefore, we can say that the approach built on the basis of the data set, which the decision-makers expressed as 0 and 1, is quite effective in better structuring the decision-making processes for uncertainty problems.
Table 1. Consideration of Example 4.1 by some proposed decision-making algorithms for bipolar soft sets.

| Algorithm | Result of algorithm | The ranking order of objects |
|-----------|---------------------|-----------------------------|
| Algorithm 1 in [31] | \{u_1, u_2, u_3, u_4\} | \( u_7 < u_1 < u_5 = u_6 < u_4 = u_1 = u_5 = u_2 = u_3 \) |
| Algorithm in [53] | \{u_1, u_2, u_3, u_4\} | \( u_7 < u_1 < u_5 = u_6 < u_4 = u_1 = u_2 = u_3 = u_5 \) |
| Algorithm 1 in this paper | \{u_3\} | \( u_7 < u_1 < u_5 = u_6 < u_4 < u_7 < u_1 = u_5 < u_2 \) |

6. Conclusion

Soft sets, the first mathematical model built with the contribution of a parameterization tool, can model uncertainty problems quite successfully. Recently, increasing complex data analysis has increased the interest in bipolar soft sets, which is a generalization of soft sets, as it increases the need for more accurate modelling of uncertain data. Unlike soft sets, this mathematical model can better structure the decision-making process by taking into account a NOT parameter of each parameter. However, one of the most important inadequacies of bipolar soft sets is to be able to express the membership degrees of objects as 0 or 1. Moreover, the inability to process data about the non-membership degrees of objects is another major drawback of this set type. For such inadequacies, bipolar soft set theory is considered together with fuzzy sets and intuitionistic fuzzy sets, and more complex hybrid structures are constructed.

In these hybrid structures, since the task of expressing a (non-)membership degree in the range \((0,1)\) focuses on decision-makers, an approach to solving uncertainty with objective data is avoided. This means modelling the uncertainty problem by considering the risk of decision-makers making mistakes. In this paper, the concepts of (NOT) bipolar relational membership degree and (NOT) bipolar relational non-membership degree are proposed to overcome this important inadequacy. These concepts require the decision-makers to specify only 0 or 1 membership degrees in order to determine the (non-)membership degrees in \((0,1)\) objectively. In this way, the values in the desired range are determined in an objective way without the need for the decision-makers to determine the (non-)membership degrees in \((0,1)\). In addition, some properties based on these concepts were examined in the paper. Moreover, a decision-making algorithm (Algorithm 1) based on the proposed concepts is given and how it can be applied to an uncertainty problem is illustrated. Some points that need attention about the Algorithm 1 are as follows:

(i) Algorithm 1 aims to express the membership degree of an object in \((0,1)\) independently of the decision-maker for bipolar soft sets. In this respect, it is preferred more than other decision-making algorithms available in the literature for bipolar soft sets. Because a possible margin of error in the values expressed by the decision-makers is ignored.

(ii) Algorithm 1 is built considering bipolar soft set theory. Therefore, this algorithm can be improved by reconsidering many mathematical models such as bipolar N-soft set [51], fuzzy bipolar soft set [43], m-polar fuzzy bipolar soft set [47], multi-fuzzy bipolar soft sets [46], Fermatean fuzzy bipolar soft set [52], rough fuzzy bipolar soft set [45], rough Pythagorean fuzzy bipolar soft set [44].

Finally, some important points for Algorithm 1 are emphasized and compared with some approaches available in the literature for bipolar soft sets. The concepts are given for bipolar soft sets in this paper prove that there are no limitations on obtaining objective data independent of decision-makers. This situation is thought to be a source of motivation for future studies in this direction.

Compliance with ethical standards

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

Informed consent Informed consent was obtained from all individual participants included in the study, including their legal guardians.

Disclosure statement
No potential conflict of interest was reported by the author(s).

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