Identifying the secondary school students’ misconceptions about number

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Abstract. This study aims to identify students’ misconceptions about the conception of number and its operation, especially whole number and fractions. Data was collected from 31 students of 7th grade with knowledge test as much as 6 items. To further identify the students’ misconceptions that occurred in questions then researchers execute the interview from several students. Generally, the result of this study found that many students experience errors in the operation of whole number and fractions. In addition, students’ errors are also obtained in order to process the workings of whole number and fractions as well as change the fraction to decimals or percent. From the errors described above and based on the result of interview found the misconception that occurs in the students’ concept about negative number, fractions, and sorting in working the operation of whole number and fractions.

1. Introduction

Mathematics is a study of pattern and relationship that needs for understanding the concepts in next lessons; a way of thinking that provides people with strategies for organizing, analyzing, and synthesizing information. Often symbolizing a real-life problem reduces it to a well-known mathematical procedure, making the problem easier to solve; an art, characterized by order and internal consistency; a language that uses carefully defined terms and symbols that enhances our ability to communicate about science, real-life situations, and mathematics itself. Like any language, you need to understand the meaning of these words and when it is appropriate to use them; a tool, not just mathematicians use mathematics, but everyone in the course of daily life uses it. Seeing this will help children appreciate why they are learning mathematics. Besides, children will be able to use mathematics to solve both abstract and practical problems, just as mathematicians and other people do. Mathematics has become an essential part of our world, both in everyday life and in the workplace [1]. If seen from the above definition can be concluded that in general mathematics carrying the vision of the present and the future. Therefore, learning mathematics becomes very important in human life.

Misconceptions are considered as steps the students must go through, that must be controlled under a didactic point of view and that are not an obstacle for students’ future learning if they are bound to weak and unstable images of the concept; they represent an obstacle to learning if they are rooted in strong and stable models of a concept. Misconceptions are an image that worked well, has become inappropriate in a new situation and needs to be broadened for further use of the concept [2]. In other
words, these misconceptions cannot be avoided because they are an important part of the learning process and the result of something known not as a lack of knowledge.

It is important to establish the difference between an ‘error’ and ‘misconception’ as both seem to be equivalent regarding the incorrect result they produce. An error might be caused due to a misconception. Other factors may include carelessness, problems in reading or interpreting a question and lack of numbers knowledge. A misconception, on the other hand, is the result of a lack of understanding or in many cases misapplication of a ‘rule’ or mathematical generalization [3]. In line with that, Riccomini divides unsystematic and systematic errors [4], unsystematic errors are unintended, non-recurring wrong answers which learners can readily correct by themselves and systematic errors though, are recurrent wrong responses methodically constructed and produced across space and time. Systematic errors are symptomatic of a faulty line of thinking causing them referred to as a misconception.

Understanding about whole number and fractions be an important thing in learning mathematics. Research has revealed a strong predictive relation between early knowledge of whole number and fractions and later mathematics achievement [5-9]. In Indonesia, the concept of whole number and fractions is introduced to students in elementary school to secondary school (SMP) 7th grade. However, when students face problems that related to the problem, there are still many errors that occur [10, 11].

To solve a problem, a student must first understand the related concepts. When students experience misconceptions in learning a concept for the first time, then it will not only have an impact on when the student is learning the concept but will also result in further learning which is the development of the concept. Basic concepts are interpreted well by students will give influence in understanding higher level abstraction. Research on misconceptions experienced by students will help teachers for knowing the nature of a misconception and its source helps teachers to fathom ways of planning appropriate instruction that is beneficial to learners [12].

2. Method
This study aims to identify the misconceptions experienced by students in learning mathematics, especially about number. This research seeks to understand the phenomenon of what students experience holistically by describing in terms of words and language to a specific of natural context as well as utilizing various natural methods [13]. Therefore, method in this study is qualitative research.

Data was collected from 31 students 7th grade on one of the secondary school in Lembang. Students were given 6 items of questions that took from the book of Kurikulum 2013 revised edition 2017 by Kementerian Pendidikan dan Kebudayaan (Kemendikbud). Items consisted of several sub topic number i.e. 2 items of whole number and 4 items of fractions. Items was designed to identify misconceptions experienced by students on the fractional concept and procedures of operation for whole number and fractions. Furthermore, some students who had mistaken were interviewed on face-to-face.

3. Results and discussion
Students were given 6 items about whole number and fractions. The recapitulation of student answers for sub whole number (Table 1) is as follows:

| Table 1. Recapitulation of student answers for sub whole number. |
|---------------------------------------------------------------|
| Number of Students | Correct Answer | Wrong Answer | Correct Answer Percentage |
|--------------------|----------------|--------------|--------------------------|
| Question 1         | 31             | 14           | 17                       | 45.2%                                      |
| Question 2         | 31             | 7            | 24                       | 22.6%                                      |
| Total              | 62             | 21           | 41                       | 33.9%                                      |

Based on the recapitulation in Table 1 appears only 33.9% of students that able to answer sub material for whole number. One of the many mistakes’ students make namely error on sorting the operations for whole number. Error on sorting the operation for whole number in many responses, students do calculation procedures that match the sequence of context questions (figure 1).
Figure 1. Student answer A.

Student A gives the answer without going through proper procedure of operation. Student A on question number 1 should be first multiply $5 \times (-2)$ and result $-10$. As well for question number 2, student A should work on division $(-16) \div (-2)$ first instead of following the sequence of operations suitable the context of question. From these two statements, students generally still lack of understanding the rules of operation which if there is the question containing the addition, subtraction, multiplication, and division operations should be done first namely multiplication, division, addition, and subtraction.

For operation of negative number, many students lack of understanding the concept of negative number operation (figure 2).

Student C gives the answer $1 - (-10) = -9$, the rule on negative number operation for sum on which $a - (-b) = a + b$, thus the answer in question 1 should be 11. For question 2 on student B and D was found to be the same as that of C students who lack of understanding the rules on negative number operations. According to some students, if the sum $(-a) + b$ then result on the operation is added not deductible. In addition, there are also students who consider the multiplication operation on the negative number $a \times (-b)$ the concept will be equal to the sum when the larger positive number multiplied by the negative number will produce a positive number. This concept makes some student answers to be wrong even though the order of operation is correct.

As for sub fractions, the possible source of the problem relates to the basic concept of fractions, namely students' understanding about the meaning of fractions and the order as well as location of fractions associated with decimal or percent. Another problem is the operation of fractions. There is recapitulation for sub fraction as follows.

Table 2. Recapitulation of student answers for sub fractions

|                  | Number of Students | Correct Answer | Wrong Answer | Unanswered | Correct Answer Percentage |
|------------------|--------------------|----------------|--------------|------------|--------------------------|
| Question 3       | 31                 | 6              | 23           | 2          | 19.4%                    |
| Question 4       | 31                 | 15             | 15           | 1          | 48.4%                    |
| Question 5       | 31                 | 6              | 18           | 7          | 19.4%                    |
| Question 6       | 31                 | 12             | 14           | 5          | 38.7%                    |
| Total            | 31                 | 39             | 70           | 15         | 31.5%                    |

Based on the above table, students experienced the most error on question number 3 of 74.2%. The context of question number 3, students are asked the question for sorting the value of fractions, decimal, and percent from smallest to largest. There are some students who understand that the percent value in
the question is 30% greater than the decimal value of 0.55. From some interviews conducted, students are still a lot of confusion in representation the meaning of percentage.

The wrong perception about changing decimals or percentage to fractions experienced by many students approximately 54.84%. Most students have difficulty when converting fractions to decimals or percent (figure 3).

![Figure 3. Student answer E.](image)

Some of the errors described above can be concluded in general that students lack of understanding the concept of fractions, decimal, and percent thus for answer the question associated with them can be found students will experience a systematic error as long as the concept is still strong in cognitive structure of students. Similarly, with other problems related to fractions, there are still students who are less able to convert mixed fractions into fractions, applying the concept of Lowest Common Multiple (LCM) in daily life, and using the appropriate of algorithm in the question.

4. Conclusions

Many people in the field of education and psychology agree that the difficulty is associated with children’s whole number knowledge which represents numbers discretely and therefore may interfere with children’s construction of the concept of fraction and rational numbers that are ordered and continuous [14]. This indicates that there is a gap between the study of whole number with fraction that can cause students to experience disequilibrium or imbalance in their cognitive thinking.

The most common systematic error in sub whole number of 45% is concept in the operation of negative number. Students face many challenges as they confront new mathematical ideas, especially ideas that extend the scope of previously secure knowledge or require its modification. Learning about negative numbers is framed as the acquisition of a new discourse that is incommensurable with the old discourse of natural numbers [15]. Negative number which has different sign with positive number cannot be imagined in reality by students. The problem aroused in understanding the sign of number play a role in determining strategy which is used to solve the problem because the sign which opposite is more complex compared to the sign which is similar [16].

The procedure of whole number operation is also one of the misconceptions that occurs about 20%. Many students lack of understanding about order of operations in context of the given problem. Even though whole number arithmetic accuracy has repeatedly been shown to be related to fraction arithmetic accuracy, and early whole number arithmetic accuracy predicts later fraction arithmetic accuracy, even after controlling for other relevant variables [17-22] and whole number arithmetic accuracy predicts decimal arithmetic accuracy [21]. With the statement that, it is not surprising when students in this study has difficulty of 35% in working on the problem of fractional arithmetic.

The sub fraction of the systematic error that occurs a lot of 54.84% is the concept of converting fractions into other forms such as decimal. Decimals are another way of writing fractions. In fact, fraction may be represented in terms of tenths, hundredths, and thousandths instead of writing in the form of common fraction. It was found that one cannot understand decimals without having sense of fraction [22]. Understanding fractions and decimals is difficult because whole numbers are the most frequently and earliest experienced type of number, and learners must avoid conceptualizing fractions and decimals in terms of their whole-number components [23]. Besides that, for understanding fraction
and decimal arithmetic requires understanding of the fractions and decimals themselves; indeed, as will be seen, failure to grasp fraction and decimal arithmetic often reflects a more basic lack of understanding of the component fractions and decimals [22].

Children in the range of 11-13 age are in transition from concrete to abstract thinking. According to Piaget, the formal operational stage extends concrete operational thought. No longer is thought focused exclusively on tangibles; children are able to think about hypothetical situations. Reasoning capabilities improve, and children can think about multiple dimensions and abstract properties. Egocentrism emerges in adolescents’ comparing reality to the ideal; thus, they often show idealistic thinking [24]. Students in 7th grade are transition from elementary school into secondary school, when initially they think informal with concrete operation switch to formal operation which is more abstract [25]. It is not surprising that understanding the concept of whole number and fractions become obstacle for student. A process bridging those two matters will be very helpful for students to minimize the misconception occurred.

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