Transport Studies of Lattice Bosons: Paradigms for Fluctuating Superconductivity

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A strong periodic potential generally enhances the short wavelength fluctuations of a superfluid beyond the validity of standard continuum approaches. Here we report some recent results on hard core bosons on finite lattices. We find several interesting effects of the periodic potential on the ground state, vortex dynamics, and Hall conductivity. For example, the Magnus field on a vortex abruptly reverses direction at half filling. A rotating Bose condensate on an optical lattice may allow an experimental test of our results. Insight may also be gained towards about strongly fluctuating superconductors modelled by charge 2e lattice bosons.

Keywords: Boson Hall conductivity, vortex mass, quantum vortex liquid

1. Strongly Fluctuating Superconductors

"Conventional" superconductors undergo a pairing transition at $T_c$, which can be well described by BCS mean field theory. In general they have large superfluid density $n_s$, (e.g. in two dimensions $\hbar^2 n_s/m >> T_c$), and weak phase fluctuations. In contrast, under-doped high $T_c$ cuprates, small capacitance Josephson arrays, and disordered thin films, are characterized by low superfluid density. This enhances the role of phase fluctuations and vortex delocalization near $T_c$. Pairing correlations persist well above $T_c$, and "normal state" transport coefficients do not follow familiar Fermi liquid behavior.

To describe strongly fluctuating superconductors, it is natural to consider effective Hamiltonians of charge 2e bosons. Continuum and weakly interacting superfluids are well approximated by the Gross-Pitaevski (GP) equation. However, a strong periodic potential generally enhance the short wavelength fluctuations beyond the validity of the GP approach. Non uniform potentials are unavoidable in solid state superconductors. Nowadays, their effects can be systematically studied in cold atom condensates on optical lattices. At strong interactions and commensurate fillings, the superfluid is unstable toward charge-gapped Mott insulator phases, or "vortex condensates". The phase diagrams of lattice bosons have been studied extensively in recent years. However, little is known about their vortex dynamics and transport...
coefficients, especially in the strongly interacting regime. Here we report some recent results on hard core bosons on finite toroidal clusters. We find several interesting effects of the lattice on the ground state and Hall conductivity, which may have experimental implications.

1.1. The Model

The gauged quantum $XY$ model on a square lattice represents two dimensional hard core bosons in a perpendicular magnetic field:

$$
\mathcal{H} = -t \sum_{\langle ij \rangle} \left( e^{iqA_{ij}} S_i^+ S_j^- + \text{H.c.} \right) - 2 \sum_i \mu_i S_i^z. \tag{1}
$$

The local density fluctuations are given by $n_i = S_i^z + \frac{1}{2}$, and the superfluid order parameter is the magnetization in the $xy$ plane. The mean field superfluid transition temperature goes as $T_c = t n_b (1 - n_b)$, where $n_b = N_b / N$ is the filling fraction. An important distinction between hard core lattice bosons and continuum bosons, is the existence of a charge conjugation symmetry $C \equiv \exp \left( i \pi \sum_r S_r^x \right)$. $C$ transforms "particles" into "holes", i.e. $n_i \to (1 - n_i)$, and the Hamiltonian into

$$
C \mathcal{H}[\mathbf{A}, n_b] C = \mathcal{H}[\mathbf{A}, 1 - n_b], \tag{2}
$$

where $n_b = N_b / N$ is the filling fraction.

Fig. 1. Reversal of Hall conductivity and Magnus action of hard core bosons at half filling. (a) The zero temperature Hall conductivity given by the ground state Chern number of a 16 site square lattice on a torus. (b) Vortex drift directions (purple arrows) in the presence of a bias current (red arrows), for regions of lower (blue) and higher (green) boson density than half filling.
1.2. Hall Conductivity

A consequence of (2) is that the Hall conductivity is antisymmetric in \( n_b - 1/2 \):

\[
\sigma_H(n_b, T) = -\sigma_H(1 - n_b, T).
\]  

The temperature-dependent Hall conductance of the finite cluster is given by the thermally averaged Chern numbers.\(^{13}\) A zero temperature Hall conductance as a function of filling for \( N_\phi = 1 \) is plotted in Fig. 1a. At zero temperature, \( \sigma_H = N_b \) below half filling follows the Galilean invariant result \( \sigma_H \propto N_b/N_v \). At half filling, \( \sigma_H \) reverses sign as expected by (3).

Fig. 1a shows a dramatic effect of the lattice on the Hall coefficient: \( \sigma_H \) undergoes a sharp transition between \( \sigma_H > 0 \) (\( \sigma_H < 0 \)) just below (above) half filling. For hard core bosons \( \sigma_H(T, n_b) \) decreases with temperature, with a characteristic temperature scale which vanishes at half filling.\(^{12}\)

In terms of vortex dynamics, Hall conductivity inversion implies that vortices suddenly drift in opposite directions as density of bosons is varied near half filling (see Fig. 1b). We propose to try to observe such a dramatic effect for bosonic atoms on rotating optical lattices.\(^{10}\) As the density changes in space with the trapping potential, vortices on either side of the half filling separatrix are expected to flow in opposite directions relative the local superflow.

1.3. V-spins of Vortices at Half Filling

At half filling, vortices see no Magnus field, but instead they acquire spin half quantum numbers we denote by \('v'-spins\'). When the gauge field in Eq. (II) describes \( N_\phi \) flux quanta uniformly penetrating the torus, \( N_\phi \) vortices are inserted
into the ground state. At half filling \( n_b = 1/2 \), for any odd number of vortices \( N_\phi = 2m + 1, m = 0, 1, 2, \ldots \), all eigenstates are at least two-fold degenerate. We have proven\(^\text{12} \) that these doublets are associated with SU(2) algebra of local symmetry operators. The v-spin in the "z" direction measures a bipartite charge density wave in the vortex core, as depicted in Fig[2]\(^\text{2} \) V-spin interactions between vortices decay exponentially. V-spin excitations are expected to dominate the low temperature thermodynamics at low values of external magnetic field.

1.4. \textit{Vortex Mass, and Vortex Lattice Melting}

At half filling, a vortex hops on the dual lattice with half a flux quantum per plaquette. Its hopping rate \( t_v \) was fit to exact numerical eigenenergies of \( \mathcal{H} \). Our results for \( N = 20 \), show\(^\text{12} \) that at half filling, vortices are as 'light' as bosons, \( t_v \approx t \).

When multiple vortices are introduced by a magnetic field or rotation, they tend to localize in an Abrikosov lattice which coexists with superfluidity. In two dimensions the vortex lattice can melt by quantum fluctuations resulting in a non-superfluid Quantum Vortex Liquid (QVL). A system of interacting vortices can be mapped to the Boson Coloumb Liquid studied by Magro and Ceperly (MC)\(^\text{14} \). Using our values of \( t_v \), the critical vortex melting density was bounded by a surprisingly low vortex density,

\[
n_v^\text{cr} \leq \left( 6.5 - 7.9 \frac{V}{t} \right) \times 10^{-3} \text{ vortices per site.} \tag{4}
\]

This implies that a QVL is achievable at manageable rotation frequencies for cold atoms on optical lattices, and by moderate magnetic fields for Josephson junction arrays and cuprate superconductors.

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