Two-dimensional direct numerical simulation of bubble cloud cavitation by front-tracking method

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Abstract. Unsteady bubble cloud cavitation phenomenon caused by negative pressure pulse has been treated numerically by applying a front tracking method. The behaviour of bubble cloud expanding and contracting is evaluated by tracking the motion of all bubble interfaces. Numerical investigation demonstrates that: (1) In the collapsing of bubble cloud micro liquid jets toward the inner bubbles are formed while the outer layer bubbles contract extremely, and then a high impact pressure is released when the inner central bubble contacts to its minimum. (2) The oscillation of bubble cloud depends upon the void fraction greatly. In the case of high void fraction, the frequency of cloud oscillation is lower than that of individual bubble and the decay of the oscillation becomes much slowly also.

1. Introduction
Cavitation is an important phenomenon often observed in variety of hydraulic systems such as nozzles and hydraulic machineries, where small vapour bubbles are formed when the surrounding liquid pressure drops sufficiently low in some region of relative high velocity [1]. Corresponding to the variation of surrounding pressure these bubbles expand and contract in a very short time and the impact pressure released at bubble collapsing may reach to several thousand atmospheric pressures [2]. Experimental investigations has been demonstrated that when a cloud of bubbles collapse coherently it results in much greater material damage and much strong noise compared to cumulative effect resulted by collapsing of the individual bubbles made up the cloud [3]. That is to say, impact pressures released at collapsing of bubble cloud are much higher than that released at collapsing of one single bubble. In order to utilize the high cavitation impact actively high-speed water jets have been developed and widely applied [4, 5]. Recently, active utilisations of cavitation extended to various fields such as ultrasound medical applications and drag delivery for cancer therapy [6]. However, the precise physical phenomena involved in cloud cavitation have not yet been completely identified. For clarifying the mechanism of cavitation phenomena it is important to understand the collapse and interaction of multiple bubbles. Generally, two approaches are developed to treat the dynamics of bubble cloud. One deals with the macroscopic motions of bubbles by using the continuum two-fluid method, in which the motions of liquid and bubbles are treated separately by interactively solving two sets of governing equations derived for liquid and gas phases [7, 8]. Usually, the volume change of bubbles is described by the Rayleigh-Plesset equation or similar ones and the exchanges of mass, momentum, and energy are estimated explicitly as transfer terms. However, the transfer terms depends
upon the estimation of the interaction between the liquid and bubbles, which is difficult since there is not a general physical model describing the interaction of liquid and bubbles [9]. The other one is to analyze the microscopic motions of individual bubble directly by tracking the motion of sharp bubble surfaces [10]. For the high requirement of computation resource for solving Navier–Stokes equations, the direct numerical simulations of cavitation bubbles are very limited. Popinet & Zaleski [11] studied the effects of viscosity on creation of re-entrant jets during collapse of a cavitation bubble near a rigid boundary. Dabiri et al. [12] studied the deformation of a cavitation bubble in shear flows.

Under this background, this work makes a try to treat bubble cloud cavitation phenomenon. Focused on the mutual interaction of bubbles cavitation of a bubble cloud caused by negative pressure pulse is simulated numerically by applying the front tracking method. By tracking the motion of all bubble interfaces the behavior of bubble cloud expanding and contracting is simulated and the coherent structure of bubble oscillation is then analyzed.

2. Method of DNS simulation and computational setup of bubble cloud

Cavitation phenomena of bubble cloud are concerned in consideration of mutual interactions among multi bubbles. So, the behaviour of a bubble cloud responding to the action of a negative pressure pulse simulated directly by applying the front tracking method developed by Tryggvason et al. [13-15]. The density field is expressed uniformly by using a Heaviside function defined along fluid interface, which is calculated by tracking the marker points on the front. The working fluids are treated as viscous ones and the surface tension acted on bubble interfaces is counted. In order to simplify the complex cavitation phenomena the following assumptions are adopted. (1) The compressibility of liquid phase is negligible compared to that of the gas phase including in bubbles. (2) Bubbles are consisted of non-condensable gas, which obeys the perfect gas law. (3) Pressure inside a bubble is uniformly distributed. (4) Coalescence and fragmentation of bubbles do not happen in the present process. With above assumptions one set of flow governing equations is written for both liquid and gas phases involved, and the phase boundary is treated as an imbedded interface by adding the appropriate source terms to the conservation laws. These source terms are in the form of delta-functions localized at the interface and are selected in such a way to satisfy the correct matching conditions at the phase boundary. For both of the incompressible liquid and compressible gas phases, the momentum conservation equation is given as follows:

\[
\frac{\partial (\rho u)}{\partial t} + \nabla \cdot (\rho uu) = -\nabla p + \nabla \cdot \left[ \mu (\nabla u + \nabla^T u) \right] + \rho F_b + F_s
\]  

(1)

where

\[
F_s = \sigma \kappa \gamma \delta (x - x_f) dA
\]  

(1.1)

Here, \( u \) is the velocity, \( p \) is the pressure, and \( \rho \) and \( \mu \) are the discontinuous density and viscosity fields, respectively. \( F_b \) denotes the body force and \( F_s \) does the surface tension acted on the interface. The subscript \( f \) denotes the front. \( \delta \) is a high-order delta-function constructed by repeated multiplication of one-dimensional delta functions. \( \kappa \) is the mean curvature of the front. \( \gamma \) is a unit vector normal to the front. Formally, the integral is over the entire front segment, thereby adding the delta-functions together to create a force that is concentrated at the interface, but smooth along the front. \( x \) is the point at which the equation is evaluated and \( x_f \) is the position of the front segment.

Concerning the mass conservation, for the continuous incompressible liquid phase the following equation is supplemented.

\[
\nabla \cdot u = 0
\]  

(2)

When combined with the above momentum equation these equation leads to an elliptic equation for the liquid pressure, which is linked to the bubble inner pressure across the interface. Therefore mass conservation of the gas phase contained in bubbles needs to be concerned to determine the pressure
inside a bubble. For the gas inside a bubble it density varies greatly with surround liquid pressure, and thus the following equation is adopted.

\[ \nabla \cdot \mathbf{u} = -\frac{1}{\rho} \frac{D\rho}{Dt} \]  

(3)

For the convenience of numerical treatment the gas included in bubbles is assumed to be non-condensable one that conforms to the adiabatic law, and the equation of state is written as

\[ \frac{p}{p_0} = \left( \frac{\rho}{\rho_0} \right)^\gamma \]  

by neglecting the variation of temperature. Then, the density variation is related to pressure variation and the mass conservation for the gas phase is rewritten to the following form.

\[ \nabla \mathbf{u} = -\frac{1}{\gamma \rho} \frac{Dp}{Dt} \]  

(4)

where \( \gamma \) is the ratio of specific heats taken to be 1.4, and \( D/Dt \) represent the Lagrangian derivative.

As shown in Fig. 1, an arbitrarily arranged bubble cloud in an open rectangular channel of liquid is concerned. In order to induce the bubble cloud to cavitation a negative pressure pulse given by the following sine waveform is acted on the bubble cloud.

\[ p_m = \begin{cases} p_{L0}[1 + \cos(2\pi t/T_0)]/2, & (t \leq T_0) \\ 0, & (t > T_0) \end{cases} \]  

(5)

where \( p_{L0} \) denote the initial liquid pressure around the cloud and \( T_0 \) the duration time of the pressure pulse. Then, the boundaries in the horizontal and vertical directions are set to be a liquid boundary of given pressure. The boundaries in the span-wise direction are thought to be periodic ones. For convenience of computation above equations are solved in non-dimensional way and all variables are normalized as follows.

\[ x^* = \frac{x}{R_0}, \quad t^* = \frac{t}{R_0 \left( \frac{\rho_0}{p_0} \right)^{1/2}}, \quad u^* = \frac{u}{v \left( \frac{\rho_0}{p_0} \right)^{1/2}}, \quad \rho^* = \frac{\rho}{\rho_0}, \quad p^* = \frac{p}{p_0} \]  

(6)

As an important physical parameter of bubbly mixture the void fraction of gas phase is often used to express the amount of gas per unit volume [16]. Usually, it is defined by the total volume of bubbles in a given domain. However, differing to a uniform bubbly flow a bubble cloud occupies only its local area rather the whole domain. Geometrically, the size of a bubble cloud is determined by the envelope surface of all the outer layer bubbles, and then the void fraction of a bubble cloud is defined as follows.

Figure 1. Sketch of computational domain
where \( I(x) \) is an indicator function whose value is given to be 1.0 in the interior of all the bubbles and 0 elsewhere, and \( x \) denotes the coordinates of small volume \( dV \) taken within the cloud. \( R \) denotes the equivalent radius of a bubble cloud and \( \alpha \) does the void fraction of the cloud. For convenience the superscript * denoting dimensionless parameters are omitted hereafter.

3. Results and discussions

Figure 2 shows, as an example, the cavitating behaviour of a bubble cloud composed of seven same bubbles at a sequence of time. At the initial state shown in Fig. 2 (a) the void fraction and the radius of the bubble cloud are evaluated to be that \( \alpha_0 = 0.51 \) and \( R_{C0} = 6.5 \), where \( R_{C0} \) is also normalized by the initial bubble radius \( R_0 \). Figure 2 (b) shows the state of bubble cloud expanding, where the bubble surface moving velocity demonstrated by vectors is relatively small and the cloud is approximately reaching to its biggest size. Figure 2 (c) shows a moment that the outer layer bubbles begin to contract although the inner bubble is still in expanding. Figure 2 (d) shows the state that the inner bubble begins to contract also under the action of outer bubbles. Figure 2 (e) shows the collapsing of outer layer bubbles where re-entry jets towards to the inner bubble are formed although the inner bubble is still in contracting. Figure 2 (f) shows the moment that the inner bubble gets into collapsing (contracting to the smallest one) where a strong impulse pressure high as 35 times of the initial pressure is demonstrated. From the figure we may see that strong re-entry jets are generated in the outer layer bubbles while they are closing to be broken.

Figure 2 (g) to (i) show the variation of bubble shapes in the process of rebounding where the inner bubble expanding nearly uniformly in all directions but the outer layer bubbles expand in the bilateral directions fast and they present to be horse-like shape instantaneously. The results demonstrate that the outer layer bubbles shields the inner bubble and get into collapsing earlier due to the effect of outside liquid pressure. As expected, re-entry jet occurs at the outer layer bubble surfaces and it works to promote subsequent collapsing of the inner bubble.

Figure 3 shows the variation of radii of all bubbles included in the cloud. The thick solid line presents the radius \( R_{b1} \) of the central bubble and the thin solid lines do the radii of outer layer bubbles numbered as 2 to 7. According to the result we know that the central bubble expands continually while the outer layer bubbles begin to contract. But the collapsing velocity of the inner bubble is much fast than that the outer layer bubbles. So, the central bubble contracts to the minimum size closing to breaking before the outer layer ones get into collapsing at the moment denoted by line A in Fig. 3 (b) and then rebound quickly. At the moment denoted by line B the inner bubble begins to contract again for the shielding effect of outer layer bubbles and then gets into its minimum state closing to breaking again at the moment denoted by line C. Therefor two large impact pressures are generated before and after the outer layer bubbles contacts to minimum ones as shown by the thick dashed line in Fig. 3 (b). The result demonstrates that the behaviour of bubble cloud cavitation is much more complex for the interaction of bubbles in different positions compared to the case of single bubble. The above result agrees qualitatively with the experimental results reported by N. Bremond et al. [17] although the arrangement of bubbles and the intensity of negative pressure pulse are different [18].

Corresponding to expanding and contracting of individual bubbles in a bubble cloud, the size and the void fraction of bubble cloud vary temporally. Figure 4 shows, as example, the oscillations of three different bubble clouds composed of the same bubbles distributed differently, where Case A denotes the bubble cloud shown in Fig. 2 and Case B and Case C do bubble clouds whose initial radius are increase to \( R_{C0} = 13.1 \) and \( R_{C0} = 23.5 \) respectively by increasing the distance among bubbles. Figure 3 (a) shows the variation of bubble cloud size and Fig. 3 (b) does the variation of void fraction responding to the given pressure pulse. According to Fig. 3 (a), we understand that the bubble cloud of Case A expands and contacts periodically at frequency lower than that of single bubble except a
disorder near \( t=38 \). This high frequency oscillation should be actually the effect of interaction between the outer layer bubbles and the inner bubble. Comparing the results of Case A, B and C we known that the frequency of bubble cloud oscillation becomes higher while the void fraction decreases corresponding to the increase of the distance among bubbles. In the Case C the oscillation of bubble cloud radius closes to that of an individual bubble since the interaction of surrounding bubbles becomes very weak. Comparing Fig. 3 (a) and (b) we understand that the void fraction of bubble cloud oscillates in a good periodic manner. The damping of bubble cloud oscillation becomes quicker with the decrease of initial void fraction since the resonance of bubbles become weaker gradually.

**Figure 2.** Expansion and collapse of a bubble cloud (RC0=6.5, \( \alpha_0=0.182 \)) responding to a negative pressure pulse: (a) to (i) flow distributions and the shape of bubbles at a sequence of times.

### 4. Conclusions
Direct numerical simulation of bubble cloud cavitation were performed by applying the front tracking method and the response of a arbitrarily distributed bubble cloud to a negative sinusoidal pressure pulse has been investigated numerically. The behaviour of bubble cloud expanding and collapsing
reproduced by numerical simulation qualitatively agrees with the experimental ones reported by N. Bremond et al. [17] and the capability of present method to treat critical bubble cloud cavitation has been demonstrated.

Numerical investigation demonstrates that: (1) In the collapsing of bubble cloud micro liquid jets toward the inner bubbles are formed while the outer layer bubbles contract extremely, and then a high impact pressure is released when the inner central bubble contacts to its minimum. (2) The oscillation of bubble cloud depends upon the void fraction greatly. In the case of high void fraction, the frequency of cloud oscillation is lower than that of individual bubble and the decay of the oscillation becomes much slowly also.

![Figure 3](image)

(a) Oscillation of bubble radius  
(b) Interaction of bubbles at cloud collapsing

**Figure 3.** Oscillation of bubble radii and impulse pressures generated at bubble collapsing

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