Prediction and control of drop formation modes in microfluidic generation of double emulsions by single-step emulsification

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Abstract

Hypothesis: Predicting formation mode of double emulsion drops in microfluidic emulsification is crucial for controlling the drop size and morphology.

Experiments and modelling: A three-phase Volume of Fluid-Continuum Surface Force (VOF–CSF) model was developed, validated with analytical solutions, and used to investigate drop formation in different regimes. Experimental investigations were done using a glue-free demountable glass capillary device with a true axisymmetric geometry, capable of readjusting the distance between the two inner capillaries during operation.

Findings: A non-dimensional parameter ($f$) for prediction of double emulsion formation mode as a function of the capillary numbers of all fluids and device geometry was developed and its critical values were determined using simulation and experimental data. At $\log f > 5.7$, drops were formed in dripping mode; the widening jetting occurred at $5 < \log f < 5.7$; while the narrowing jetting was observed at $\log f < 5$. The $f$ criterion was correlated with the ratio of the break-up length to drop diameter. The transition from widening to narrowing jetting was achieved by increasing the outer fluid flow rate at the high capillary flow regime.

\[ f = \frac{D_{\text{orf}}}{D_N} \times \frac{10^{0.254}}{C_{a_2} \cdot C_{a_3}^{0.567} \cdot C_3^{1.12}} \]

Keywords: Droplet microfluidics, Dripping regime, Widening jetting, Narrowing jetting, Core-shell droplets, Double emulsions, Dripping-to-jetting transition, Velocity profile.

Abbreviations: CNC, computer numerical control; CSF, continuum surface force; DC, Dow Corning; I.D., internal diameter; O.D., outer diameter; OTMS, octadecyltrimethoxysilane; PDMS, polydimethylsiloxane; PVA, polyvinyl alcohol; VOF, volume of fluid.

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1. Introduction

Core-shell capsules attract great interest for potential applications ranging from controlled release of drugs and nutrients [1–4], encapsulation of fragrances [5], to CO2 capture and energy storage [6–8]. The conventional fabrication strategies for core-shell capsules such as complex coacervation [9], internal phase separation [10], layer-by-layer electrostatic deposition [11], interfacial polymerisation [12], and spray drying [13] often require multi-stage processing and are associated with low encapsulation efficiency, low reproducibility and a lack of control over the size of the capsules and the shell thickness. In addition, the synthesised particles are usually highly polydispersed [14].

Microfluidic emulsification is a promising strategy for production of monodispersed double emulsions with controlled droplet size and morphology. Double emulsion drops can be created through single-step or two-step emulsification methods, using flow-focusing [15,16], cross-flow [17,18] or co-flow [19,20] droplet makers. Utada et al. [21] developed a three-phase 3D glass capillary device for single-step generation of double emulsions by combining co-flow and counter-current flow focusing flow patterns. The device consists of two tapered round capillaries, coaxially aligned within a square capillary and separated at some distance from each other, Fig. 1a and b. The device provides precise control over the droplet size and shell thickness. The wettability of each capillary can be independently modified, which is advantageous compared to polydimethylsiloxane (PDMS) devices that suffer from poor wetting [22]. However, the distance between the round capillaries is fixed during operation and the device is difficult to clean after use.

The drop formation in capillary devices occurs in two distinct instability modes, absolute and convective [23]. Dripping is the result of an absolute instability: the perturbations that lead to drop pinch-off grow at a fixed spatial location and at a frequency that is intrinsic to the system, leading to monodispersed drops. Jetting occurs due to convective instability, which is associated with the advection of the perturbations along the interface of the jet, which causes random variation in the pinch-off location. Therefore, the drops generated in jetting mode are polydispersed [23,24]. The mechanism of drop formation in each mode is governed by the interaction between viscous, inertial, interfacial, and gravitational forces [25]. However, the effect of gravity is negligible when the drop diameter is in the micrometre range [26,27].

In dripping mode, the interfacial force is dominant and the inertial forces are negligible. The drops are formed in the vicinity of the injection nozzle (in co-flow geometry) or within one orifice downstream of the orifice of the collection capillary (in flow-focusing geometry) once the viscous force, exerted by the continuous phase, exceeds the pinning force arising from the interfacial tension [27,29].

Two types of jetting mode have been observed in capillary devices, narrowing and widening [30]. Narrowing jetting is associated with formation of a relatively thin and long jet that eventually breaks into small drops; this mode occurs when the viscous force generated by the continuous phase is significantly greater than the interfacial force. On the other hand, widening jetting occurs when the inertial forces of the jet dominate the interfacial tension force. The widening shape of the jet has been attributed to the deceleration of the jet [31].

The formation of single emulsion drops in two-phase glass capillary devices is well characterised [27,30–33]. In three-phase glass capillary devices, inner and outer drops can be formed in the same or different modes and thus, versatile drop morphologies can be achieved. Due to complexity of the coaxial jet

Nomenclature

| Symbol | Definition |
|--------|------------|
| C'     | constant, -|
| Ca     | capillary number, -|
| D      | diameter, m|
| D1     | inner droplet diameter, m|
| D2     | outer droplet diameter, m|
| Df     | drop generation frequency, s⁻¹|
| D'     | volume fraction, -|
| Dc     | break-up length, m|
| n      | unit normal, -|
| P      | pressure, Pa|
| Q      | volume flow rate, m³ s⁻¹|
| r      | radial distance, m|
| r_c    | radius of collection capillary, m|
| t      | time, s|
| V      | velocity, m s⁻¹|
| X      | axial distance, m|
| x      | direction (coordinates), m|
| z      | diameter ratio, -|
| κ      | curvature of interface, m⁻¹|
| μ      | dynamic viscosity, kg (m s)⁻¹|
| ρ      | density, kg m⁻³|
| σ      | interfacial tension, N m⁻¹|

Subscripts

1. inner phase
2. middle phase
3. outer phase
1.2 between inner and middle phase
2.3 between middle and outer phase
co outer capillary
ci injection capillary
dispersed dispersed phase
ef effective
j jet
N injection nozzle
orif orifice

Volume of fluid–continuum surface force model
Glass capillary device

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Number of the inner fluid. The drop size was reduced by reducing the distance between the two inner capillaries and the minimum drop size was achieved when the distance between the capillaries was zero. © 2017 The Authors. Published by Elsevier Inc. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/).
pinch-off, only a few studies have been carried out on the double emulsion formation in three-phase capillary devices and the prediction of drop sizes in dripping and jetting modes [21,29,34,35]. Utada et al. [21] studied the formation of core-shell drops in both dripping and widening jetting modes and found that double emulsions were not formed when the outer drops were formed in dripping mode and the inner drops in jetting mode. Although double emulsion formation by narrowing jetting has been reproduced numerically [29,35], it has not yet been observed experimentally in glass capillary devices. In addition, the variation of the distance between the inner capillaries changed the drop formation mode in numerical simulations [29], but no experimental evidence has been provided so far. Lee and Weitz [34] observed the formation of multi-core double-emulsion drops in capillary devices in an inertia-induced dripping-to-widening jetting transition mode at relatively high flow rates of the inner and middle fluids. To the best of our knowledge, no parameter capable of predicting the drop formation regime in capillary microfluidics has been developed so far.

In this study, a novel dimensionless criterion, \( \log \zeta \), that can be used for prediction of drop formation mode in three-phase glass capillary devices, was introduced and its critical values at which transitions occur were determined using a large amount of simulation and experimental data. Different transitions of drop formation modes were observed experimentally and a high ability of \( \log \zeta \) to accurately predict the drop formation behaviour was confirmed. In addition, a glue-free, detachable and reusable three-phase glass capillary device was fabricated, with the ability to adjust the distance between the opposing inner capillaries during operation. The device consists only of round capillaries and has a true axisymmetric geometry, unlike conventional capillary devices consisting of round inner capillaries placed within a square outer capillary, which have a quasi-axisymmetric geometry. The device was used to reveal the significant impact of the distance between the two tapered inner capillaries on droplet formation behaviour.

2. Materials and methods

2.1. Governing equations and numerical method

2.1.1. Governing equations

A two-dimensional incompressible axisymmetric Volume of Fluid-Continuum Surface Force (VOF-CSF) [36,37] numerical model has been developed to simulate double emulsion formation. The governing mass, Eq. (1), and momentum, Eq. (2), balance equations are:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho V_i)}{\partial x_i} = 0
\]

\[
\frac{\partial (\rho V_i)}{\partial t} + \frac{\partial (\rho V_i V_j)}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \mu \frac{\partial V_i}{\partial x_j} \right) + F_b
\]

where \( V \) and \( P \) are the velocity and pressure, respectively, while \( t \), \( \mu \), and \( \rho \) are the time, dynamic viscosity and density. \( F_b \) is a source term which includes interfacial force, \( F_n \) and gravitational force. Since the drop diameter is in the order of micrometres, the gravitational force is negligible and \( F_b \approx F_n \) [26,27]. In the VOF method, a momentum balance equation is solved for all the phases and the advection of interface is calculated by solving a transport equation, Eq. (3), for volume fraction, \( f' \):

\[
\frac{\partial f'}{\partial t} + \frac{\partial (f' V_i)}{\partial x_i} = 0
\]

The volume fraction gives the portion of the cell which is filled with either phase, where

\[
f' = 0 \quad \text{the cell is filled with dispersed phase fluid} \\
0 < f' < 1 \quad \text{the interface exists in the cell} \\
f' = 1 \quad \text{the cell is filled with continuous fluid}
\]

The \( \mu \) and values in Eq. (2), were calculated using Eqs. (5) and (6), respectively:

Fig. 1. Glass capillary devices used in this study along with corresponding micrographs of double emulsion formation: (a) A photograph and (b) scheme of the standard device with a square outer capillary attached onto the microscope slide; (c) A photograph and (d) scheme of a glue-free device with a round outer capillary and a milled polyacetal holder. Supplementary movie S1.
\[ \mu = \rho \mu_{\text{continuous}} + (1 - \rho) \mu_{\text{dispersed}} \]  

(5)

\[ \rho = \rho_{\text{continuous}} + (1 - \rho) \rho_{\text{dispersed}} \]  

(6)

The CSF method was used to calculate \( F_b \) in Eq. (2) at the interface:

\[ F_b = \sigma \kappa \nabla f \]  

(7)

where \( \sigma \) is the interfacial tension and \( \kappa \) is the local curvature of the interface, given by:

\[ \kappa = \nabla \cdot \mathbf{n} \]  

(8)

and \( \mathbf{n} \) is the unit normal, given by Eq. (9):

\[ \mathbf{n} = \frac{\nabla f}{|\nabla f|} \]  

(9)

2.1.2. Numerical method

An unsteady-flow pressure-based segregated algorithm, established in finite-volume-based commercial software Ansys Fluent v. 14.0, was used to solve the governing equations. The discretised moment equation was approximated using a second order upwind scheme. The PRESTO scheme was used to interpolate the pressure term. Although the PRESTO scheme is computationally expensive, it directly calculates the pressures at cell faces and avoids interpolation errors. The pressure and velocity were coupled using the SIMPLE scheme. The interface interpolation was obtained by the Geo-Reconstruct algorithm. A variable-time-step method based on the maximum Courant number of 0.35 was used to reduce the computational cost. To evaluate grid independence, five meshes with a resolution of 8 \( \mu \), 4 \( \mu \), 3 \( \mu \), 2 \( \mu \), and 1 \( \mu \) were constructed. The difference in simulation results obtained with the resolutions of 1, 2, and 4 \( \mu \) was negligible. Therefore, a mesh size of 2 \( \mu \) was used around the injection orifice and inside the collection tube where the drops were formed, while a coarser mesh was used for the rest of the domain. Further details on the numerical model can be found elsewhere [29,38].

2.1.3. Dimensionless numbers

Non-dimensional numbers used in this study are the capillary numbers of inner, \( C_{a1} \), middle, \( C_{a2} \), and outer phase, \( C_{a3} \), given by:

\[ C_{a1} = \frac{\mu_1 V_1}{\sigma_{12}} \quad C_{a2} = \frac{\mu_2 V_2}{\sigma_{23}} \quad C_{a3} = \frac{\mu_3 V_3}{\sigma_{23}} \]

where \( V_1 = 4Q_1/(\pi D_{ci}^2) \), \( V_2 = Q_2/(D_{ci}^2 - \pi D_{co}^2/4) \), and \( V_3 = Q_3/(D_{co}^2 - \pi D_{co}^2/4) \). Subscripts 1, 2, and 3 stand for the inner, middle and outer phase, respectively, \( D_{ci} \) is the outer diameter of the injection capillary, and \( D_{co} \) is the internal height of the square capillary, Fig. 1b.

2.2. Materials

Glycerol was supplied by Fisher Scientific (UK) and used as a viscosity modifier. Polyvinyl alcohol (PVA, \( M_w = 13,000\text{–}23,000 \text{ g mol}^{-1} \), 87–89% hydrolysed) was purchased from Sigma-Aldrich (UK) and used as a hydrophilic stabiliser. Dow Corning 749 Fluid (DC 749) was supplied by Univar (UK) and used as a lipophilic surfactant. Polydimethylsiloxane (PDMS) fluid (Dow Corning 200/10cs fluid) was obtained from VWR (UK) and used as the main constituent in the middle phase. 2-[methoxy(polyethylene oxide)propyl]trimethoxysilane and octadecyltrimethoxysilane (OTMS) were purchased from Fluorochem, Ltd. (UK) and used for hydrophilic and hydrophobic treatment of glass capillaries. The Millipore Milli-Q Plus water purification system was used to produce water for preparing all aqueous solutions. The density, dynamic viscosity and interfacial tension were measured using pycnometer, capillary viscometer, and Krüss DSA-100 pendant drop tensiometer, respectively.

2.3. Droplet production

Fabrication of microfluidic devices: Two different kinds of three-phase glass capillary device were used in this study: standard and novel. In a standard device, inner capillaries were manually centred within a square capillary and fixed on a microscope slide using a two-part epoxy (Devcon 5-Minute Epoxy®) [29,38]. In this device, the capillaries are fixed in place and the distance between inner capillaries cannot be changed. This fabrication process is tedious and the device is difficult to clean; after several experiments, when the capillaries become fouled, a new device must be made. Hence, a novel glue-free separable device was designed and manufactured. The device has an adjustable distance between inner capillaries, which can be changed during operation and can be dismantled, cleaned and reassembled. The device is easier to fabricate since all the capillaries are placed into an engineered holder to avoid the manual centring. The holder was made of acetal copolymer with a density of 1.41 g/cm³, and manufactured by Computer Numerical Control (CNC) machining using a fully automated HAAS Super Mini Mill machine (HAAS Automation, Norwich, UK), Fig. 1c. The holder is similar to the 3D-printed holder developed by Martino et al. [39], but CNC milling offers a wide range of materials for manufacturing and provides a superior surface quality. First, round borosilicate glass capillaries of 1/0.58 mm (O.D./I.D.) (Intracel, UK) were pulled using a Flaming/Brown microipette puller (P-97, Sutter Instrument Co., Linton Instrumentation, Norfolk, UK) to obtain injection and collection capillary tubes. The pulled capillaries were polished to the desired tip size using an abrasive paper (Black Ice Waterproof T402 Paper, Alpine Abrasives, UK) under visual control with the aid of a Narishige Model MF-830 microforge (Japan). A borosilicate round capillary of 2.156 mm (O. D./I.D.) (World Precision Instruments, UK) was then placed in the holder and used as the outer capillary. Then, the injection and collection capillaries were inserted into the outer capillary at both ends. Finally, Omnifit connectors were attached to the holder and used for introduction of the three fluids into the device, Fig. 1c.

Double emulsion formation: Three gas-tight glass syringes with Luer-lock fitting (VWR Catalyst Company, UK) were filled with the three fluids and placed on Harvard Apparatus 11 Elite syringe pumps. The inner phase was 5–85 wt% aqueous glycerol solution, the middle phase was 2 wt% DC 749 in PDMS fluid, and the outer phase was an aqeous solution containing 40–70 wt% glycerol and 2 wt% PVA. The fluids were delivered into the device using polyethylene medical tubing of 1.52/0.86 mm (O.D./I.D.) (Fisher Scientific, UK). Double emulsion drops were formed upon breakup of the coaxial jet composed of inner and middle fluids in the collection tube, Fig. 1b. The drop generation was monitored and recorded using Phantom V9.0 high-speed camera (Vision Research, UK) attached to an inverted microscope (XDS-3, GX Microscopes, UK) and interfaced to a PC computer. The ImageJ program developed at the National Institute of Health was used to analyse the captured frames.

3. Results and discussion

3.1. Validation of numerical model

The developed numerical model was validated with several analytical models in terms of the inner and outer drop sizes, \( D_1 \) and \( D_2 \), respectively. In the dripping mode, the numerical \( D_2 \) values were validated using analytical models developed by Erb et al. [28]
and Utada et al. [21]. Erb et al.’s analytical model is given by Eq. (10) [28]:

\[
D_2^4 + \Gamma (Q_3 + Q')D_2^2 - [D_{\text{orif, ef}}^2 + \Gamma D_{J2}(Q_3 + Q')]D_2^2 - \Gamma Q'^2 D_{\text{orif, ef}}^2 D_2 + \Gamma Q' D_{\text{orif, ef}} D_{J2} = 0
\] (10)

where

\[
\Gamma = 120 \mu_2 / [\pi (D_{J2} \sigma_{23} + D_1 \sigma_{12})]
\]

\[
D_{J2} = D_{\text{orif}} \sqrt{(Q_1 + Q_2)/(Q_1 + Q_2 + Q_3)}
\]

\[
D_{J1} = D_{J2} \sqrt{Q_1/(Q_1 + Q_2)}
\]

\[
Q' = Q_2 / \left[1 - (Q_1/(Q_1 + Q_3))^{2/3}\right]
\]

\[D_2 = kD_{J2}\] (11)

where \(k = (3\pi/2m)^{1/3}\) and \(D_{J2}\) can be calculated using Eq. (12):

\[
Q_1 + Q_2 = \frac{D_{J2}^2}{D_{\text{orif}}^2 - D_{J2}^2}
\] (12)

The proportionality constant \(k\) in Eq. (11) depends on the maximum instability (in), which is a function of the viscosity ratio, \(\mu_{\text{dispersed}}/\mu_3\) [40,41], where \(\mu_{\text{dispersed}}\) is the effective viscosity of the mixture of inner and middle phases. In current simulations, since the shell was relatively thick and the middle phase was much more viscous than the inner phase (\(\mu_2 \gg \mu_3\)), \(\mu_{\text{dispersed}}\) was approximated with the viscosity of pure middle phase, \(\mu_3\), and \(n = 0.55\). In jetting mode, \(D_2\) can be predicted using Eq. (13) developed by Utada et al. [21]:

\[
D_2 = \left(\frac{3C(Q_1 + Q_2)}{8\pi} \frac{D_{J2} \mu_3}{\sigma_{23}}\right)^{1/3}
\] (13)

where \(C\) is a constant, which depends on the viscosity ratio, \(\mu_2/\mu_3\) [42]. In this work, \(C = 79.4\) and \(D_{J2}\) can be estimated by solving Eq. (14) for \(\alpha (\alpha = D_{J2}/D_{\text{orif}})\) [21]:

\[
\frac{Q_1 + Q_2}{Q_3} = \frac{\mu_2}{\mu_3} \left(1 - \alpha^2\right) + 2 \frac{\alpha^2}{1 - \alpha^2}
\] (14)

For both regimes, \(D_1\) can be calculated from \(D_2\) using the mass balance equation:

\[
D_2 = \left(\frac{Q_1 + Q_2}{Q_1}\right)^{1/3}
\] (15)

Fig. 2 shows the validation of the present numerical model with existing analytical models in both dripping (green region) and jetting (yellow region) regimes. A very good agreement between the numerical and analytical results was achieved in both regimes, implying the high accuracy of the developed numerical model in predicting flow behaviour in the device. The slight discrepancy between the numerical result and Erb et al.’s model in the dripping regime at \(Q_3/(Q_1 + Q_2) < 1.44\) can be attributed to the variation of \(D_{\text{orif, ef}}\) with \(Q_3\), because the break-up length is a function of \(Q_3\). However, an average \(D_{\text{orif, ef}}\) value of 408 \(\mu\)m (1.36Dorif) was used for the validation. The available analytical models are valid only in dripping or jetting regime. No existing analytical model can predict the drop size in the dripping-to-jetting transitional regime, at \(Q_3/(Q_2 + Q_3) > 5\) and 12 (the blue region in Fig. 2).
profile changes from a saddle shape at $X = 0$ to parabolic at $X = 3.85D_{\text{orf}}$. In addition, almost the same velocity profile with a small decrease in the centreline velocity was observed until the end of the computational domain.

### 3.3. Prediction of double emulsion formation mode

Each drop generation mode can be characterised with a certain range of $D_2/L_2$ values, where $L_2$ is the break-up length, i.e., the distance from the orifice of the injection capillary to the point of drop pinch-off. In dripping mode, $D_2$ is similar to $D_{\text{orf}}$, while $L_2$ is very small, thus $D_2/L_2$ is relatively large ($D_2/L_2 > 0.3$). In narrowing jetting mode, $D_2$ is small and $L_2$ is large and consequently, $D_2/L_2$ is small ($D_2/L_2 < 0.1$). In widening jetting, $D_2$ is larger than in narrowing jetting and dripping mode, but $L_2$ is much larger than in dripping mode and thus, $D_2/L_2$ is between the limits for the dripping and narrowing jetting mode ($0.1 < D_2/L_2 < 0.3$). The drop formation mode can be altered by changing fluid properties and flow rates [29], which can be expressed in terms of $C_{a1}$, $C_{a2}$, and $C_{a3}$. Moreover, the variations in $D_{\text{orf}}$ can impact the drop formation regime with higher $D_{\text{orf}}$ values favouring the dripping mode [29]. The numerical results collected over a wide range of fluid properties, flow rates and device geometries were analysed to develop an empirical parameter, $\zeta$, that can be used for prediction of the double emulsion formation regime:

$$\zeta = \left[ \frac{C_{a1}^{0.254}}{C_{a2}^{0.507} C_{a3}^{1.12}} \right] \frac{D_{\text{orf}}}{D_2}$$

where $\zeta$ can be calculated from Eq. (16) or measured from $D_2/L_2$ using the equation: $\zeta_{\text{measured}} = 5.7 \times 10^6 (D_2/L_2)^{1.35}$. The comparison of $\zeta$ values calculated from Eq. (16) and measured from the numerical $D_2/L_2$ values is shown in Fig. 4.

Good agreement between the measured and calculated $\zeta$ values was achieved. For log $\zeta > 5.7$ (green region), double emulsion was formed in dripping mode, while log $\zeta < 5.7$ (yellow region) was
associated with the jetting mode. The widening jetting regime was observed for $5 \times \log \zeta < 5.7$, while the narrowing jetting mode occurred at $\log \zeta < 5$. The transition between the regimes was not sharp but occurred gradually near the critical $\log \zeta$ values, which might cause inaccurate regime predictions near the critical $\zeta$ values. The ability of $\log \zeta$ to predict the drop formation regime was tested experimentally in Figs. 5 and 6. In most cases, $\log \zeta$ calculated from Eq. (16) was able to accurately predict the drop formation regime. In addition, $\log \zeta$ was calculated for the numerical data points in Fig. 2 and the results are shown in Fig. S1 in the supplementary material. The dripping-to-jetting transition occurred at $\log \zeta = 5.7$. The $D_2/L_2$ values were plotted on the same graph and

![Fig. 5](image)

**Fig. 5.** The dripping-to-widening jetting transformation achieved by increasing $C_{a2}$ at: $C_a1 = 0.1$, $C_a3 = 1.5 \times 10^{-3}$, $\rho_1 = 1225 \text{ kg/m}^3$, $\rho_2 = 940 \text{ kg/m}^3$, $\rho_3 = 1118 \text{ kg/m}^3$, $\mu_1 = 109 \text{ mPa s}$, $\mu_2 = 10.4 \text{ mPa s}$, $\mu_3 = 33 \text{ mPa s}$, $\sigma_{12} = 29.9 \text{ mN/m}$, $\sigma_{23} = 31.8 \text{ mN/m}$, $D_{mot} = 327 \mu$m, and $D_n = 50 \mu$m. $\log \zeta$ was calculated using Eq. (16). The scale bars are 400 $\mu$m.

![Fig. 6](image)

**Fig. 6.** The widening-jetting-to-narrowing-jetting transition achieved by increasing $C_{a3}$ at: $C_a1 = 1.3$, $C_a2 = 2.4 \times 10^{-3}$, $\rho_1 = 1225 \text{ kg/m}^3$, $\rho_2 = 940 \text{ kg/m}^3$, $\rho_3 = 1118 \text{ kg/m}^3$, $\mu_1 = 109 \text{ mPa s}$, $\mu_2 = 10.4 \text{ mPa s}$, $\mu_3 = 33 \text{ mPa s}$, $\sigma_{12} = 29.9 \text{ mN/m}$, $\sigma_{23} = 31.8 \text{ mN/m}$, $D_{mot} = 327 \mu$m and $D_n = 50 \mu$m. $\log \zeta$ was calculated using Eq. (16). The scale bars are 500 $\mu$m.

the dripping-to-jetting transition occurred at $D_2/L_2 = 0.3$, as predicted. It should be highlighted that Eq. (16) was developed for primary drops and does not take into account the formation of satellite drops.

### 3.4. Transitions of drop generation modes

In this section, the effect of $C_{a1}$, $C_{a2}$, and $C_{a3}$ on the transitions of drop generation regimes has been investigated experimentally. The inner phase was 85 wt% aqueous solution of glycerol. The middle phase consisted of 2 wt% DC 749 in PDMS fluid, and the outer phase was a mixture of 70 wt% glycerol and 2 wt% PVA in water. Core-shell drops produced using this formulation were stable for at least several hours after production [8,43]. Fig. 5 shows the transition from the dripping to widening jetting achieved by increasing $Q_{a2}$ and thereby $C_{a2}$ at constant values of $C_{a1}$ and $C_{a3}$. An increase in $C_{a2}$ from $0.7 \times 10^{-4}$ to $3.6 \times 10^{-4}$ (Fig. 5a-d) caused an increase in $D_2$ from 361 to 504 $\mu$m and a decrease in $D_1$ from 266 to 245 $\mu$m, which resulted in 2.7 times thicker shell. The reduction in $D_1$ was due to higher shear force exerted by the middle phase on the inner liquid, which led to faster
generation of inner drops. According to the mass balance equation: 
\[ Q_1 = f_1 r_D^3 / 6 \]
higher drop generation frequency, \( f_1 \), at 
\[ Q_1 = \text{constant results in smaller } D_1. \]
An increase in \( D_2 \) can be attributed to the larger inertia of the middle phase at higher \( C_2 \) and introduction of a higher amount of the middle phase into the outer drop prior to drop pinch-off [29].

In Fig. 5a–c, the outer drops were formed in dripping mode (\( \log L > 5.7 \)), but in Fig. 5d, the value of \( \log L \) was at the boundary between dripping and widening jetting. At \( C_2 = 4.1 \times 10^{-4} \) (Fig. 5e), the outer drops were formed in the widening jetting regime (\( \log L = 5.6 \)); two smaller inner drops were encapsulated within each outer drop and the diameter of the outer drops was larger than the diameter of the collection tube. At \( C_2 = 6.7 \times 10^{-4} \) (Fig. 5f), the outer drops were formed by pinching of a long, wide jet (\( D_2/L_2 < 0.3 \)), while the inner drops were formed in the dripping mode. At \( C_2 < 0.7 \times 10^{-4} \), a thin oil shell burst during drop formation and the inner phase leaked into the continuous phase. At \( C_2 > 7.9 \times 10^{-4} \), the jet diameter reached the orifice diameter causing wetting of the orifice and the formation of double emulsion drops failed.

The dripping-to-narrowing-jetting transition could not occur by changing \( Q_2 \) at constant \( C_3 \) and \( C_1 \). However, the widening-to-narrowing jetting transition, which was not reported earlier, was achieved by gradually increasing \( C_1 \) at relatively high inner phase flow rate.

Fig. 6 shows the transition from widening to narrowing jetting achieved by increasing \( C_3 \) at constant \( C_1 \) and \( C_2 \), starting from a \( C_3 \) value which was 13 times larger than in Fig. 5. At low \( C_3 \) (Fig. 6a and b), both inner and outer drops were formed in the widening jetting mode (\( \log L = 5.4–5.5 \)). The drops were large and the jets occupied a significant portion of the tube cross section. At high \( C_3 \) (Fig. 6c and d), both inner and outer drops were formed in the narrowing jetting mode (\( \log L = 4.7–4.8 \)). For intermediate \( C_3 \) values (Fig. 6e–d), the regime was not clearly defined and \( \log L \) was close to the critical value of 5.0 at which the transition from widening to narrowing jetting occurs (\( \log L = 4.9–5.2 \)). In Fig. 6c, monodisperse core-shell drops with thin shells were formed at relatively high flow rates of the inner and outer phases.

By increasing \( C_3 \) from 4.6 \( \times \) 10\(^{-3} \) to 9.2 \( \times \) 10\(^{-3} \), the length of the jet decreased due to increasing shear exerted by the outer fluid, causing a faster jet pinch-off and formation of smaller drops.

At the transition between the widening and narrowing jetting (\( \log L \approx 5 \), Fig. 6d), the inner drops reached the minimum size and the neck of the middle phase between the first and second inner drop was stable, causing the formation of dual-core double-emulsion drops with tightly packed core drops. Interestingly, two core drops were also formed at the transition between dripping and widening jetting (Fig. 5e), but the outer drops were much larger. Finally, the increase in \( C_3 \) from 15.3 \( \times \) 10\(^{-3} \) to 21.5 \( \times \) 10\(^{-3} \) (Fig. 6d–f) resulted in shear-driven stretching of the inner phase to an increasingly long and narrow jet, which led to small drops.

3.5. Effect of distance between inner capillaries on drop generation

Fig. 7 shows the effect of distance between the two orifices, \( \Delta L \), on the generation of double emulsion drops. \( Q_1 = 2 \) mL/h, \( Q_2 = 7 \) mL/h, \( Q_3 = 35 \) mL/h, \( \rho_1 = 1012 \) kg/m\(^3\), \( \rho_2 = 940 \) kg/m\(^3\), \( \rho_3 = 1197 \) kg/m\(^3\), \( \mu_1 = 1.2 \) mPa s, \( \mu_2 = 10.4 \) mPa s, \( \mu_3 = 7.9 \) mPa s, \( \sigma_{11} = 29.9 \) mN/m, \( \sigma_{12} = 31.8 \) mN/m, \( D_{orif} = 376 \) µm, and \( D_h = 50 \) µm. All the scale bars are 400 µm.

The effect of distance between the two inner capillaries, \( \Delta L \), on the generation of double emulsion drops. At \( \Delta L < -0.6 \), both inner and outer drops were formed at the tip of the injection tube. At \( \Delta L = -1 \), the double emulsion drops reached the minimum size, because the jet experienced the highest shear. At \( \Delta L > 0 \), the jetting of the middle phase and formation of outer drops with two inner drops [34,43]. However, both inner and outer drops were highly uniform. The formation and advection of inner drops can considerably destabilise the outer interface and suppress the middle phase jetting. At \( \Delta L/D_{orif} = -1 \), when the inner drops were formed relatively far from the outer interface, the middle phase jet was very stable. Therefore, a long middle phase jet was developed...
within the collection tube, which eventually broke up into large drops. When the inner drops were formed very close to the orifice, e.g., at $\Delta L/D_{c,d} = -0.2$, the growing inner drops strongly interacted with the outer interface. In this geometry, the outer interface was significantly deformed by the inner drops, which led to hydrodynamic instabilities in the jet and caused its early break-up into smaller drops. At $\Delta L/D_{c,d} = 0.2$ or 0.4, a very thin stream of the middle phase was formed at the entrance of the collection tube, due to the relatively long distance between the drop generation and the entry section of the tube. Further thinning of the middle phase stream around the injection tube at $\Delta L/D_{c,d} > 0.4$ caused interruption of the middle phase flow and double emulsion drops failed to form.

4. Conclusions

The developed Volume of Fluid-Continuum Surface Force (VOF-CSF) numerical model was successfully validated with existing analytical solutions [21,28]. The velocity profile at the entry section of the tube was flat in dripping and widening jetting mode, while a saddle-shaped profile was observed in narrowing jetting mode. The numerical results were used to develop a dimensionless parameter, $\zeta$, for prediction of drop formation mode, which is a function of the capillary numbers of the three fluids and the device geometry. For $\log \zeta > 5.7$, double emulsion drops were formed in dripping mode, the widening jetting was observed for $5 < \log \zeta < 5.7$, while the narrowing jetting occurred at $\log \zeta < 5$. The dimensionless criterion $\zeta$ was correlated with the ratio of the break-up length to drop diameter and tested against a large number of numerical and experimental results to show its high accuracy in predicting the drop formation mode.

Narrowing jetting reported in numerical simulation studies [29,35] was achieved starting from widening jetting by increasing the flow rate of the outer fluid at constant flow rates of the inner and middle phases. Near the transition between widening jetting and narrowing jetting, it was possible to generate monodispersed core-shell drops with thin shells at much higher flow rates of inner fluid than in the dripping mode. A glue-free reusable capillary device was used to change the separation distance between the capillary inlets during operation to manipulate the drop formation behaviour. The device consists of two coaxial round capillaries, and therefore, has a true axisymmetric geometry, which differs from the quasi-axisymmetric geometry of a recently fabricated reusable capillary device [44]. The size of core-shell drops was significantly affected by changing the distance between the two tapered capillaries. The minimum drop size was achieved when the orifice of the injection tube was placed exactly at the entry section of the collection tube. In future work, the capillary device will be equipped with a thermo-electric generator in order to study the effect of temperature on generation of double emulsion drops.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.jcis.2017.05.115.
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