Timed Epistemic Knowledge Bases for Social Networks
(Extended Version)

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We present an epistemic logic equipped with time-stamps in the atoms and epistemic operators, which allows to reason not only about information available to the different agents, but also about the moments at which events happen and new knowledge is acquired or deduced. Our logic includes both an epistemic operator and a belief operator, which allows to model the disclosure of information that may not be accurate.

Our main motivation is to model rich privacy policies in online social networks. Online Social Networks (OSNs) are increasingly used for social interactions in the modern digital era, which brings new challenges and concerns in terms of privacy. Most social networks today offer very limited mechanisms to express the desires of users in terms of how information that affects their privacy is shared. In particular, most current privacy policy formalisms allow only static policies, which are not rich enough to express timed properties like “my location after work should not be disclosed to my boss”. The logic we present in this paper enables to express rich properties and policies in terms of the knowledge available to the different users and the precise time of actions and deductions. Our framework can be instantiated for different OSNs, by specifying the effect of the actions in the evolution of the social network and in the knowledge disclosed to each agent.

We present an algorithm for deducing knowledge, which can also be instantiated with different variants of how the epistemic information is preserved through time. Our algorithm allows to model not only social networks with eternal information but also networks with ephemeral disclosures. Policies are modelled as formulae in the logic, which are interpreted over timed traces representing the evolution of the social network.

1 INTRODUCTION

Online Social Networks also known as Social Networking Sites, like Facebook [1], Twitter [5] and Snapchat [4] have exploded in popularity in recent years. According to a recent survey [14] nearly 70% of the Internet users are active on social networks. Some concerns, including privacy, have arisen alongside this staggering increase in usage. Several studies [13, 15–17] report that privacy breaches are growing in number. Currently, the most popular social networks do not offer mechanisms that users can use to guarantee their desired privacy effectively. Moreover, virtually all privacy policies are static and cannot express timing preferences, including referring to temporal points explicitly and policies that evolve in time.
In [20] we presented a first attempt to express dynamic privacy policies by introducing an explicit learning operator and time intervals in the semantics of policies. A major restriction of the logic in [20] is that time itself was not a present in the logic, so one cannot express different instants at which an event happens and at which knowledge is acquired. We solve here this problem by enriching the logic with explicit time instants, both in the atoms and in the epistemic operators. In the resulting logic, one can refer to the instant at which some knowledge is inferred, for example about the knowledge of another agent at another instant. The expressive power of the new logic allows to derive the learning operator from the time-stamped knowledge.

Second, we equip the logic with belief operators, with the only restriction that agents cannot believe in something that they know is false. This allows the instantiation of the framework to OSNs where gossiping is allowed (consider for example the recent interest in studying fake news in Facebook\(^1\)). Analogous to the learning operator, we derive the accept operator as the moment in which an agent start believing in something.

Third, we introduce the notion of extended knowledge bases which allow to answer queries of (temporal) epistemic formulas against the knowledge acquired during a sequence of events. The algorithm uses epistemic deductive reasoning starting from a set of axioms, which include the corresponding conventional axioms of epistemic reasoning populated for all instants. Depending on the desired instantiation of the framework, the axiom of perfect recall or weaker versions of it can be instantiated which allows to model knowledge acquisition in eternal OSNs like Facebook and ephemeral OSNs like Snapchat. When weaker versions are used, one can derive operators that capture when an agent stops knowing something (the forget operator) or stops believing in something (the reject operator).

Using extended knowledge bases we define the semantics of the logic used to construct dynamic privacy policies. This provides the means for a particular OSNs to enforce privacy policies, for example by blocking an event that would result in a violation of a user’s policy.

We illustrate with examples how our rich logic allows to express privacy policies and how agents can infer knowledge in different instantiations of the framework.

The rest of the paper is organized as follows. Section 2 presents the framework and Section 3 introduces the logic $\mathcal{KBL}_{RT}$. Section 4 shows how to express privacy policies using $\mathcal{KBL}_{RT}$. Section 4.0.1 shows how different existing OSNs can be modeled within our framework, including Snapchat. Finally, Section 5 presents related work and Section 6 includes concluding remarks.

2 A TIMED PRIVACY POLICY FRAMEWORK

We introduce a formal privacy policy framework for OSNs where properties regarding users knowledge and beliefs as well as time can be expressed. Our framework extends the framework in [20], in which temporal

\(^1\)There is a growing interest in the detection of the spread of fake news in OSNs:
- https://www.theguardian.com/media/2016/nov/17/Facebook-fake-news-satire
- https://www.theguardian.com/technology/2016/nov/29/Facebook-fake-news-problem-experts-pitch-ideas-algorithms
- https://www.theguardian.com/media/2016/nov/20/barack-obama-Facebook-fake-news-problem
properties were expressed using temporal operators ☐ and ◻. Our solution allows to describing properties at concrete moments in time. We also introduce a modality for beliefs which allows to distinguish between information that might be inaccurate.

Our framework, called $\mathcal{PPPRT}$, consists of the following components:

1. A timed epistemic logic, $\mathcal{KBLRT}$, where modalities and predicates are timestamped.
2. Extended social graphs, called social network models, which describe the state of the OSN. These graphs contain the users or agents in the system and the relations between them, and their knowledge and beliefs. We use $\mathcal{SNRT}$ to denote the universe of social network models, and use the notion of trace of social network models to describe the evolution of the system.
3. A parameter $\omega \in \mathbb{N}$ which determines for how long users remember information, e.g., 5 seconds, 24 hours or indefinitely. Users can acquire new beliefs that challenge their current knowledge and beliefs, which requires a resolution. We illustrate this by a parameter $\beta$, with two possibilities conservative or susceptible, which specifies how users behave when they learn new beliefs which are inconsistent with their current beliefs.
4. A privacy policy language based on the previous logic.

3 A TIMED KNOWLEDGE BASED LOGIC

$\mathcal{KBLRT}$ is a knowledge based first order logic which borrows modalities from epistemic logic [10], equips modalities and predicates with time-stamps, and allows quantifiers over time-stamps.

3.1 Syntax

Let $\mathcal{T}$ be a vocabulary which consists of a set of predicate symbols, function symbols, and constant symbols. Predicate and functions symbols have some implicit arity. We assume an infinite supply of variables $x, y, \ldots$. Terms of the elements of $\mathcal{T}$ can be built as $s ::= c \mid x \mid f(\overrightarrow{s})$ where $\overrightarrow{s}$ is a tuple of terms respecting the arity of $f$.

Let $\mathbb{T}$ denote a set of time-stamps, which is required to be a non-Zeno totally ordered set, that is, there is a finite number of instants between any two given instants. We use time-stamps to mark pieces of information or to query the knowledge of the agents at specific points in time. We consider $\mathcal{Ag}$ be a set of agents, $\mathcal{D}$ a set of domains, and use $\mathcal{EVT}$ for set of events that can be executed in a social network. For instance, in Facebook, users can share posts, upload pictures, like comments, etc. The set of events that users can perform depends on the social network. Similarly, we use $\mathcal{C}$ and $\mathcal{\Sigma}$ to denote special sets of predicate symbols that denote connections (between agents) and permissions. We introduce the syntax of $\mathcal{KBLRT}$ as follows:

Definition 3.1 (Syntax of $\mathcal{KBLRT}$). Given agents $i, j \in \mathcal{Ag}$ a time-stamp $t \in \mathbb{T}$, an event $e \in \mathcal{EVT}$, a variable $x$, a domain $D \in \mathcal{D}$, predicate symbols $c_c^t(i, j), a_a^t(i, j), p^t(\overrightarrow{s})$ where $c \in \mathcal{C}$ and $a \in \mathcal{\Sigma}$, the syntax of
the real-time knowledge-based logic $\mathcal{KBL}_{RT}$ is inductively defined as:

$$\varphi ::= \rho \mid \varphi \land \varphi \mid \neg \varphi \mid \forall t \cdot \varphi \mid \forall x : D \cdot \varphi \mid B^t_t \varphi$$

$$\rho ::= c^i_t(i,j) \mid c^a_t(i,j) \mid p^j(#, s) \mid occurred^t(e)$$

Given a nonempty set of agents $G \subseteq Ag$, the additional epistemic modalities are defined as:

$$S^i_t G \varphi \equiv \bigvee_{i \in G} K^i_t \varphi,$$

$$E^i_t G \varphi \equiv \bigwedge_{i \in G} K^i_t \varphi.$$ 

The epistemic modalities stand for: $K^i_t \varphi$, agent $i$ knows $\varphi$ at time $t$; $S^i_t G \varphi$, someone in the group $G$ knows $\varphi$ at time $t$. We use the following notation as syntactic sugar $P^i_a(i,j,t) \triangleq c^a_t(i,j) \land p^j(#, s)$, meaning that “agent $i$ is permitted to execute action $a$ to agent $j$”. For example, $p^Alic_{Bob} friendRequest$ means that Bob is allowed to send a friend request to Alice at time 5. We will use $\mathcal{F}_{\mathcal{KBL}_{RT}}$ to denote the set of all well-formed $\mathcal{KBL}_{RT}$ formulae. The syntax introduces the following novel notions that have not been considered in other formal privacy policies languages such as [8, 11, 18, 20].

**Time-stamped Predicates.** Time-stamps are explicit in each predicate, including connections and actions. A time-stamp attached to a predicate captures those moments in time when that particular predicate holds. For instance, if Alice and Bob were friends in a certain time period, then the predicate $\text{friend}_t(Alice, Bob)$ is true for all $t$ falling into the period, and false for all $t$ outside. This can be seen as the valid time in temporal databases [22].

**Separating Knowledge and Belief.** Not all the information that users see in a social network is true. For instance, Alice may tell Bob that she will be working until late, whereas she will actually go with her colleagues to have some beers. In this example, Bob has the (false) belief that Alice is working.

Traditionally, in epistemic logic, the knowledge of agents consists on true facts. Potentially false information is regarded as beliefs [10]. The set of axioms $SS$ characterise knowledge and the axioms of $KD45$ characterise belief [10]. For $\mathcal{KBL}_{RT}$ we combine both notions in one logic. In the following section we describe how to combine these two axiomatisations based on the results proposed by Halpern et al. in [12].

**Time-stamped Epistemic Modalities.** Time-stamps are also part of the epistemic modalities $K$ and $B$. Using time-stamps we can refer to the knowledge and beliefs of the agents at different points in time. For example, the meaning of the formula $B^t_T \text{loc}^{19:00}(Alice, work)$ is that Bob believes at 20:00 that Alice’s location at 19:00 is work.

**Occurrence of Events.** It is important to be able to determine when an event has occurred. Such an expressive power allows users to define policies that are activated whenever someone performs an undesired event. Examples of these policies are: “if Alice unfriends Bob, she is not allowed to send Bob a friend request” or “if a Alice denies an invitation to Bob’s party, then she cannot see any of the pictures uploaded during the party.”

Here we introduce $occurred^t(e)$ to be able to syntactically capture the moment when a specific event $e$ occurred. A similar predicate was introduced by Moses et al. in [6] for analysing communication protocols.

### 3.2 Semantics
3.2.1 Real-Time Social Network Models. We introduce formal models which allow us to reason about specific social network states at a given moment in time. These models leverage the information in the social graph [9]—the core data model in most social networks [2, 3, 7]. Social graphs include the users (or agents) and the relationships between them. Moreover, in our models we include a knowledge base for each agent, and the set of privacy policies that they have activated. We reuse the models defined for the previous version of this framework [20]. Nevertheless, the expressiveness of the privacy policies that can be enforced in $\text{PPF}_{RT}$ have substantially increased (see Section 4).

**Definition 3.2 (Social Network Models).** Given a set of formulae $F \subseteq \mathcal{F}_{KB_{L_{RT}}}$, a set of privacy policies $\Pi$, and a finite set of agents $Ag \subseteq \mathcal{U}$ from a universe $\mathcal{U}$, a social network model (SNM) is a tuple $\langle Ag, A, KB, \pi \rangle$, where

- $Ag$ is a nonempty finite set of nodes representing the agents in the social network;
- $A$ is a first-order structure over the SNM. As usual, it consists of a set of domains, and a set relations, functions and constants interpreted over their corresponding domain.
- $KB : Ag \rightarrow 2^F$ is a function retrieving a set of knowledge of an agent—each piece with an associated time-stamp. The set corresponds to the facts stored in the knowledge base of the agent; we write $KB_i$, for $KB(i)$;
- $\pi : Ag \rightarrow 2^\Pi$ is a function returning the set of privacy policies of each agent; we write $\pi_i$, for $\pi(i)$.

In Def. 3.2, the shape of the relational structure $A$ depends on the type of the social network under consideration. We represent the connections—edges of the social graph—and the permission actions between social network agents, as families of binary relations, respectively $\{C_i\}_{i \in C} \subseteq Ag \times Ag$ and $\{A_i\}_{i \in \Sigma} \subseteq Ag \times Ag$ over the domain of agents. We use $\{D_i\}_{i \in \mathcal{D}}$ to denote the set of domains. The set of agents $Ag$ is always included in the set of domains. We use $C$, $\Sigma$ and $\mathcal{D}$ to denote sets of connections, permissions and domains, respectively.

3.2.2 Evolution of Social Network Models. The state of a social network changes by means of the execution of events. For instance, in Facebook, users can share posts, upload pictures, like comments, etc. The set of events that users can perform depends on the social network. We denote the set of events that can be executed in a social network as $EVT$. We use traces to capture the evolution of the social network. Each element of the trace is a tuple containing: a social network model, a set of events, and a time-stamp.

**Definition 3.3 (Trace).** Given $k \in \mathbb{N}$, a trace $\sigma$ is a finite sequence

$$\sigma = \langle (SN_0, E_0, t_0), (SN_1, E_1, t_1), \ldots, (SN_k, E_k, t_k) \rangle$$

such that, for all $0 \leq i \leq k$, $SN_i \in SN_{RT}$, $E_i \subseteq EVT$, and $t_i \in T$.

We define $T_\sigma = \{ t \mid (SN, E, t) \in \sigma \}$ to be the set of all the time-stamps of $\sigma$. We impose some conditions to traces so that they accurately model the evolution of social networks. We say that a trace is well-formed if it satisfies the following conditions:
Ordered time-stamps. Time-stamps are strictly ordered from smallest to largest.

Accounting for Events. The definition has to account for events being explicit in the trace. Let $\rightarrow$ be a transition relation defined as $\rightarrow \subseteq SN_{RT} \times EVT \times T \times SN_{RT}$. We have $(SN_1, e, t, SN_2) \in \rightarrow$ if $SN_2$ is the result of the set of events $E \in EVT$ happening in $SN_1$ at time $t$. Note that we allow $E$ to be empty, in which case $SN_2 = SN_1$. We will use the more compact notation of $SN_1 \xrightarrow{E,t} SN_2$ where appropriate.

Events are independent. For each $e \in EVT$ the set of events $E$ must only contain independent events. Two events are independent if, when executed sequentially, the execution order does not change their behaviour. Consider the events: post(Charlie, Bob,"London") (Charlie shares a post containing Bob’s location), and friendRequest(Alice, Charlie) (Alice sends a friend request to Charlie). Independently of the order in which the previous events are executed the resulting SNM will have a new post by Bob, and Charlie will receive a friend request from Alice. On the other hand, consider now: post(Charlie, Bob,"London") and disallowLoc(Bob) (Bob activates a privacy policy which forbids anyone to disclose his location). In this case, if post(Charlie, Bob,"London") is executed first, the resulting SNM will contain the post by Charlie including Bob’s location. However, if disallowLoc(Bob) occurs before post(Charlie, Bob,"London"), Charlie’s post would be blocked—since it violates Bob’s privacy policy. These two events are not independent.

More formally,

**Definition 3.4.** Given two events $e_1, e_2 \in EVT$, we say that $e_1$ and $e_2$ are independent iff for any two traces $\sigma_1$ and $\sigma_2$

\[
\sigma_1 = SN_1^0 \xrightarrow{\{e_1\},t} SN_1^1 \xrightarrow{\{e_2\},t'} SN_2^1 \\
\sigma_2 = SN_2^0 \xrightarrow{\{e_2\},t} SN_2^1 \xrightarrow{\{e_1\},t'} SN_2^2
\]

it holds $SN_1^0 = SN_2^0$ and $SN_1^2 = SN_2^2$.

We can now provide a formal definition of well-formed SNM traces.

**Definition 3.5 (Well-Formed Trace).** Let

$$\sigma = ((SN_0, E_0, t_0), (SN_1, E_1, t_1), \ldots, (SN_k, E_k, t_k))$$

be a trace. $\sigma$ is well-formed if the following conditions hold:

1. For any $i, j$ such that $0 \leq i, j \leq k$ and $i < j$, it is the case that $t_i < t_j$.
2. For all $i$ such that $0 \leq i \leq k - 1$, it is the case that $SN_i \xrightarrow{E_{i+1}, t_{i+1}} SN_{i+1}$.
3. For all $e_1, e_2 \in E_i$ for $0 \leq i \leq k$, $e_1$ is independent from $e_2$.

We will use $TCS$ to refer to the set of all well-formed $P^P_F_{RT}$ traces. In order to be able to syntactically refer to the previous or next social network model, given a concrete time-stamp, we assume that there exist the functions predecessor (pred) and next (next). $\text{pred} : T \rightarrow T$ takes a time-stamp and returns the previous time-stamp in the trace. Since the set of time-stamps is non-Zeno it is always possible to compute the
previous time-stamp. Analogously, \( \text{next} : T \to T \) takes a time-stamp and returns the next time-stamp in the trace. In \( \langle (SN_0, E_0, t_0), (SN_1, E_1, t_1), (SN_2, E_2, t_2) \rangle \), \( \text{pred}(t_1) = t_0 \) and \( \text{next}(t_1) = t_2 \). We define predecessor of the initial time-stamp to be equal to itself, i.e., \( \text{pred}(t_0) = t_0 \). Similarly, next of the last time-stamp of the trace is equal to itself, i.e., \( \text{next}(t_2) = t_2 \).

### 3.2.3 Modelling knowledge

It is not a coincidence that \( \mathcal{KB}_T \) formulae look very similar to those of the language \( \mathcal{L}_n \) originally defined for epistemic logic [10]. We would like to provide users in our system with the same notion of knowledge. Traditionally, in epistemic logic, the way to model and give semantics to \( K_i \phi \) is by means of an undistinguishability relation which connects all world that an agent considers possible [10]. In particular, when talking about traces of events the framework used is Interpreted Systems (ISs). In ISs traces are called runs. A run describes the state of the system at any point in (discrete) time. An IS \( I \) is composed by a set of runs and a undistinguishability relation \( (\sim) \)—for each agent \( i \)—which models the states of the system that agents consider possible at any point in time. Determining whether an agent \( i \) knows a formula \( \phi \) (written in the language of epistemic logic), for a run \( r \) at time \( m \) is defined as follows:

\[
(I, r, m) \models K_i \phi \iff (I, r', m') \models \phi \text{ for all } (r, m) \sim_i (r', m')
\]

where \( (r, m) \) and \( (r', m') \) represent states of \( I \).

Additionally, Fagin \textit{et al.} proposed an alternative encoding to answer epistemic queries from a knowledge base consisting in a set of accumulated facts [10][Section 7.3]. Let \( kb \) be an agent representing a knowledge base that has been told the facts \( \langle \psi_1, \ldots, \psi_k \rangle \) for \( k \geq 1 \) in run \( r \) at time \( t \). It was shown in [10][Theorem 7.3.1] that the following are equivalent:

1. \( (I^{kb}, r, m) \models K_{kb} \phi \).
2. \( M^n_{\psi} \models K_{kb} (\psi_1 \land \ldots \land \psi_k) \Rightarrow K_{kb} \phi \).
3. \( (\psi_1 \land \ldots \land \psi_k) \Rightarrow \phi \) is a tautology.

where \( I^{kb} \) is an IS which models the behaviour of \( kb \) and \( M^n_{\psi} \models \phi \) means that \( \phi \) is valid in the Kripke models with an accessibility relation that is reflexive \( (r) \), symmetric \( (s) \) and transitive \( (t) \). The previous theorem holds not only for a system consisting in a single knowledge base, but systems including several knowledge bases.

This way of modelling knowledge is very suitable for our social network models. As mentioned earlier, in a social network model the users’ knowledge is stored in their knowledge base. Therefore, by using the equivalence in [10][Theorem 7.3.1] we can determine whether a user knows a formula \( \phi \) from the conjunction of all the formulae it has been told, formally, \( \land_{\psi \in KB} \psi \Rightarrow \phi \).

However, as mentioned earlier, \( \mathcal{KB}_T \) is not the same language as \( \mathcal{L}_n \). Therefore we cannot directly apply [10][Theorem 7.3.1] to determine whether a user knows a fact \( \phi \). In the following we described an extended knowledge base which supports all the components of \( \mathcal{KB}_T \).

### 3.2.4 Extended Knowledge Bases

An Extended Knowledge Base (EKB) consists in a collection \( \mathcal{KB}_T \) formulae without quantifiers. All domains in a SNM are finite at a given point in time—the might
grow as events occur. On the one hand, regular domains, i.e., \( D^t \) in \( \mathcal{A} \) are assumed to be finite. Therefore, they can be easily unfolded as a finite conjunction. On the other hand, the time-stamps domain—though infinite in general—for a given trace \( T_\sigma \) will be finite because traces are finite. Hence EKBs can be populated with the explicit time-stamps values that moment in time. Later in this section, we introduce some axioms which will define how time-stamps are handled. In what follows we introduce the axioms EKBs use to handle knowledge and belief.

**Derivations in EKBs.** The information stored in an agent’s EKBs along a trace determines her knowledge. At a concrete moment in time, an agent’s EKB contains the explicit knowledge she just learnt. New knowledge can be derived from the explicit pieces of information in agents’ EKBs. Derivations are not limited to formulae of a given point in time, but also can use old knowledge. A *time window*, or simply, window, determines how much old knowledge is included in a derivation. We write \( \Gamma \vdash (\varphi, w) \) to denote that \( \varphi \) can be derived from \( \Gamma \) given a window \( w \).

We provide a set of deduction rules, \( DR \), of the form

\[
\Gamma \vdash (\varphi, w') \quad \Gamma \vdash (\psi, w) \quad \overset{\text{DR}}{\Rightarrow} \quad \Gamma \vdash (\psi', w)
\]

meaning that, given the set of premises \( \Gamma, \psi \) can be derived with a window \( w \) from \( \varphi \) in a window \( w' \).

**Definition 3.6.** A *timed derivation* of a formula \( \varphi \in \mathcal{F}_{KB} \) given a window \( w \in \mathbb{N} \), is a finite sequence of pairs of formulae and windows, \( \mathcal{F}_{KB} \times \mathbb{N} \), such that \( (\varphi_1, w_1), (\varphi_2, w_2), \ldots, (\varphi_n, w_n) = (\varphi, w) \) where each \( \varphi_i \), for \( 1 \leq i \leq n \), follows from previous steps by an application of a deduction rule of \( DR \) which premises have already been derived, i.e., it appears as \( \varphi_j \) with \( j < i \), and \( w_j \leq w_i \).

In what follows we present the concrete derivation rules that can be used in EKBs to derive knowledge. We define deduction rules based on well studied axiomatisations of knowledge and belief together with rules to deal with knowledge propagation.

**Knowledge and Belief in EKBs.** In EKBs knowledge and belief coexist. So far, the definition of knowledge that we provided in the previous section only takes into account the axioms for knowledge. In particular, the axiomatisation \( S5 \). Thus, EKBs can use any of the \( S5 \) axioms to derive new knowledge from the conjunction of explicit facts in the knowledge base.

Fagin *et al.* provided an axiomatisation for belief [10], the \( KD45 \) axiomatisation. It includes the same set of axioms as \( S5 \)—replacing \( K_i \) by \( B_i \)—except for the axiom \( K_i \varphi \Rightarrow \varphi \) (A3). The difference between knowledge and belief is that believes do not need to be true—as required by A3 in knowledge. The requirement is that an agent must have *consistent* beliefs. It is encoded in the following axiom \( \neg B_i \bot \) (axiom D).

We can summarise the last two paragraphs as follows: Whenever a formula of the form \( K_i \varphi \) is encountered, axioms from \( S5 \) can be applied to derive new knowledge, and if the formula is of the form \( B_i \varphi \) axioms from \( KD45 \) can be used instead. Nonetheless, we are missing an important issue: How do knowledge and belief
Knowledge axioms

| A1 | All tautologies of first-order logic |
| A2 | $K^t_i \varphi \land K^t_i (\varphi \implies \psi) \implies K^t_i \psi$ |
| A3 | $K^t_i \varphi \implies \varphi$ |
| A4 | $K^t_i \varphi \implies K^t_i K^t_i \varphi$ |
| A5 | $\neg K^t_i \varphi \implies K^t_i \neg K^t_i \varphi$ |

Belief axioms

| K | $B^t_i \varphi \land B^t_i (\varphi \implies \psi) \implies B^t_i \psi$ |
| D | $\neg B^t_i \perp$ |
| B4 | $B^t_i \varphi \implies B^t_i B^t_i \varphi$ |
| B5 | $\neg B^t_i \varphi \implies B^t_i \neg B^t_i \varphi$ |

Knowledge-Belief axioms

| L1 | $K^t_i \varphi \implies B^t_i \varphi$ |
| L2 | $B^t_i \varphi \implies K^t_i B^t_i \varphi$ |

Table 1. EKB axioms for a trace $\sigma$ for each $t \in T_\sigma$.

relate to each other? To answer this question we use two axioms proposed by Halpern et al. in [12]: (L1) $K_i \varphi \implies B_i \varphi$ and (L2) $B_i \varphi \implies K_i B_i \varphi$.

L1 expresses that when users know a fact they also believe it. It is sound with respect to the definition of both modalities, since knowledge is required to be true by definition (recall axiom A3). This axiom provides a way to convert knowledge to belief. L2 encodes that when agents believe a fact $\varphi$ they know that they believe $\varphi$. Thus adding an axiom which introduces knowledge from belief—more precisely, introduces knowledge about the beliefs.

However, the axioms from $S5$, $KD45$ and L1, L2 need to be adapted to $\mathcal{KBL}_{RT}$ syntax—which is the type of formulae supported by EKBs. In particular, the modalities need a time-stamp. All these axiomatisations are defined for models which represent the system in a concrete time. That is, given the current set of facts that users have, they can apply the axioms to derive new knowledge (at that time). To preserve this notion we will simply add the time-stamp $t$ to all modalities. Intuitively, it models that if users have some knowledge at time $t$ they can derive knowledge using the previous axioms, and, this derived knowledge, belongs to the same time $t$. Table 1 shows the complete list of axioms that can be applied given a trace $\sigma$ for each time-stamps $t \in T_\sigma$.

In order for these axioms to be used in timed derivations we now express them as deduction rules as shown in Table 2. Since all derivations are performed for the same $t$ they all share the same window $w$.

As mentioned earlier, an EKB models the knowledge and beliefs of a user at a given moment in time. We encode this notion by adding an explicit $K^t_i$ to every formula in a user’s EKB. Formally, we say that users in a trace $\sigma$ are self-aware iff for all $t \in T_\sigma$ if $\varphi \in EKB^t_i \sigma$ then $\varphi = K^t_i \varphi'$. By assuming this property we can syntactically determine the time $t$ when some knowledge $\varphi$ enters an EKB. In what follows we assume that all well-formed traces are composed by self-aware agents.
Example 3.7. Consider the following EKB from a trace $\sigma$ of an agent $i$ at time $t$.

$$K_i^t(\forall t' \cdot \forall j: Ag^{t'} \cdot event^{t'}(j, pub) \implies loc^{t'}(j, pub))$$

$$K_i^t(event^t(Alice, pub))$$

$EKB_i^{[t]}$

In this EKB $i$ can derive using the axioms in Table 1 that Alice’s location at time $t$ is a pub, i.e., $loc^t(Alice, pub)$. Here we show the steps to derive this piece of information. We recall that quantifiers are unfolded when added to the knowledge base. Therefore, given $T_\sigma = \{t_0, t_1, \ldots, t\}$

$$K_i^t\forall j: Ag^t \cdot event^t(j, pub) \implies loc^t(j, pub) \land$$

$$K_i^t\forall j: Ag^t \cdot event^t(j, pub) \implies loc^t(j, pub) \land$$

$$\cdots$$

$$K_i^t\forall j: Ag^t \cdot event^t(j, pub) \implies loc^t(j, pub)$$

where each of these are also syntactic sugar, for instance, given $Ag^t = \{j_1, j_2, \ldots, j_n\}$ for $n \in \mathbb{N}$

$$K_i^t event^t(j_1, pub) \implies loc^t(j_1, pub) \land$$

$$K_i^t event^t(j_2, pub) \implies loc^t(j_2, pub) \land$$

$$\cdots$$

$$K_i^t event^t(j_n, pub) \implies loc^t(j_n, pub)$$

The predicate $event^t(j, pub)$ means that $j$ attended an event at time $t$ in a $pub$. The predicate $loc^t(j)$ means that $j$'s location is a $pub$. Thus, the implication above encodes that if $i$’s knows at $t$ that if an agent is attending an event in a pub at time $t$, her location will be a pub. Moreover, $i$ knows at time $t$ that Alice is attending an event at the pub, $event^t(Alice, pub)$. As mentioned earlier, in epistemic logic, knowledge is required to
be true. Therefore, \( \text{event}(\text{Alice}, \text{pub}) \) must be a true predicate. From this we can infer that \( \text{Alice} \in \text{Ag}^t \). Because of this, \( K^t_i \text{event}(\text{Alice}, \text{pub}) \implies \text{loc}(\text{Alice}, \text{pub}) \) must also be present in \( \text{EKB}_i^{\sigma[t]} \). Applying A2 to \( K^t_i \text{event}(\text{Alice}, \text{pub}) \) and the previous implication we can derive \( K^t_i \text{loc}(j, \text{pub}) \) as required. □

**Handling time-stamps.** In EKBs users can also reason about time. For instance, if Alice learns Bob’s birthday she will remember this piece of information, possibly, forever. Nonetheless, this is not always true, some information is transient, i.e., it can change over time. Imagine that Alice shares a post including her location with Bob. Right after posting, Bob will know Alice’s location—assuming she said the truth. However, after a few hours, Bob will not know for sure whether Alice remains in the same location. The most he can tell is that Alice was a few hours before in that location or that he believes that Alice’s location is the one she shared. We denote the period of time in which some piece of information remains true as duration.

Different pieces of information might have different durations. For example, someone’s birthday never changes, but locations constantly change. Duration also depends on the OSN. In Snapchat messages last 10 seconds, in WhatsApp status messages last 24 hours and in Facebook posts remain forever unless a user removes them. Due to these dependences we do not fix a concrete set of properties regarding time-stamps. Instead we keep it open so that they can be added when modelling concrete OSNs in \( \mathbb{P} \mathbb{P} \mathbb{F} \mathbb{R} \mathbb{T} \). Concretely, the parameter \( \omega \) introduced at the beginning of Section 2 corresponds to information duration for a particular OSN modelled in \( \mathbb{P} \mathbb{P} \mathbb{F} \mathbb{R} \mathbb{T} \).

Using the window \( w \)—from timed derivations, see Def. 3.6—we define the following deduction rule encoding knowledge propagation. Given \( t, t' \in T_\sigma \) where \( t < t' \):

\[
\Gamma \vdash (K^t_i \phi, w - (t' - t)) \\
\Gamma \vdash (K^{t'}_i \phi, w)
\]

KR1

The intuition behind KR1 is that \( w \) is consumed every time knowledge is propagated. Imagine that Alice knows at time 1 the formula \( \phi \), \( K^1_{Alice} \phi \). Using KR1 in a derivation would allow us to derive, for instance, that she knows \( \phi \) at a later time, e.g., at time 5, that is, \( K^5_{Alice} \phi \). Note that this derivation requires \( w \) to be at least 4, since Alice’s knowledge of \( \phi \) is propagated 4 units of time. As usual in when using this type rules, derivations can be described forwards or backwards. In the latter, the derivation starts with the conclusion of the rule—that is, we want to derive—and reduce the value of \( w \) every time we access old knowledge. In the former, we start from the premise of the rule and we increase \( w \) accordingly to derive the conclusion. The intuition and ways of deriving knowledge using KR1 are better illustrated with an example.

**Example 3.8.** Consider the sequence of EKBs in Fig. 1 of an agent \( i \) from a trace \( \sigma \) where \( T_\sigma = \{0, \ldots, 4\} \). In this example we show the purpose of the window \( w \) when making derivations. Note that it is not possible to derive Alice’s location only from the set of facts in a single knowledge base at a time \( t \). Instead it is required to combine knowledge from different knowledge bases. We use the knowledge recall rule with different windows to access previous knowledge. As mentioned earlier, intuitively, \( w \) determines for how
long agents remember information. Therefore, it is required to find an appropriate value for \( w \) that includes the sufficient knowledge—from all moments in time—to perform the derivation. In the figure, the red square marks the accessible knowledge for \( w = 2 \) and the blue square for \( w = 3 \).

As can be seen in the trace, in order for \( i \) to derive \( \text{event}^3(Alice, pub) \) she needs to combine knowledge from \( EKB_i^{0[0]} \) and \( EKB_i^{0[3]} \). Let \( EKB_i^{w} = \bigcup_{r \in T_i} EKB_i^{r[i]} \). First, we show how to construct a proof forwards, i.e., starting from the premises and a window of 0, move forward—by increasing the size of \( w \)—until the inference can be performed. In particular, we show that \( EKB_i^{w} + (K_i^{3} \text{loc}^3(Alice, pub), w) \) for \( w \in \mathbb{N} \). Let us start by applying the rule \text{PREMISE} with \( w = 0 \),

\[
EKB_i^{0} + (K_i^{0} \text{event}^3(Alice, pub)) \implies \text{loc}^3(Alice, pub, 0)
\]

Recall that the quantifiers in the example are just syntactic sugar. They are replaced when added to the EKB. Now we use KR1 to combine this knowledge with knowledge at time 3. In other words, we propagate knowledge from time 0 (\( K_i^{0} \)) to time 3 (\( K_i^{3} \)).

\[
\begin{align*}
EKB_i^{0} + (K_i^{0} \text{event}^3(Alice, pub)) & \implies \text{loc}^3(Alice, pub, 0) \\
(KR1) \quad EKB_i^{0} + (K_i^{3} \text{event}^3(Alice, pub)) & \implies \text{loc}^3(Alice, pub, 3)
\end{align*}
\]

As stated in the rule the window has been increased by 3, since \( 0 = 3 - (3 - 0) \). We apply \text{PREMISE} again to obtain \( EKB_i^{0} + (K_i^{3} \text{event}^3(Alice, pub), 3) \). As in Example 3.7 by applying A2 in the previous statements we derive \( EKB_i^{0} + (K_i^{3} \text{loc}^3(Alice, pub), 3) \). This proof shows that \( i \) knows Alices location provided that agents remember information during 3 units of time.

We show now that a window smaller than 3 makes this derivation impossible. Also we construct the proof backwards, i.e., starting from the conclusion we try to prove the required premises. Let us take a largest window smaller than 3, \( w = 2 \). We try to show that \( EKB_i^{0} + (K_i^{3} \text{loc}^3(Alice, pub), 2) \). In order to prove we need to show the following:

\[
\begin{align*}
(EKB_i^{0} + K_i^{3} \text{event}^3(Alice, pub), 2) \quad EKB_i^{0} + (K_i^{3} \text{event}^3(Alice, pub)) & \implies \text{loc}^3(Alice, pub, 2) \\
\text{(A2)} \quad EKB_i^{0} + (K_i^{3} \text{loc}^3(Alice, pub), 2) & \implies \text{loc}^3(Alice, pub, 2)
\end{align*}
\]

The first premise, \( EKB_i^{0} + K_i^{3} \text{event}^3(Alice, pub), 2) \), trivally follows by \text{PREMISE}. For the second premise we first try move one step back using KR1, i.e.,

\[
EKB_i^{0} + (K_i^{3} \text{event}^3(Alice, pub)) \implies \text{loc}^3(Alice, pub, 1),
\]

since there is no knowledge at time 2, the previous statement cannot be proven. We apply again KR1 obtaining

\[
EKB_i^{0} + (K_i^{3} \text{event}^3(Alice, pub)) \implies \text{loc}^3(Alice, pub, 0).
\]

Similarly, this statement cannot be proven. Note that the window is 0. Intuitively, it means that we have access all knowledge that \( i \) remembers. Therefore, we have reach a dead end in our proof tree. KR1 could be applied again, which would give

\[
EKB_i^{0} + (K_i^{3} \text{event}^3(Alice, pub)) \implies \text{loc}^3(Alice, pub, -1).
\]
Therefore, we cannot simply increase the window during the derivation. In particular, we propagate beliefs as long as they are inside the window \( w \) independently of the propagation rule of the agent, both algorithms will try to propagate the accumulated set of beliefs as long as they are inside the window \( w \). Formally we define this set as \( \Psi_c = \{ B^t_i \varphi \mid B^{t-n}_i \varphi \in EKB^t_i, \exists t \in T_\sigma \text{ such that } t_n - t > w \text{ and } B^t_i \varphi \} \).

**Belief propagation.** Beliefs cannot be propagated as easily as knowledge. The reason is that, at some point in the future, new beliefs—contradicting the current set of beliefs of an agent—can enter her EKB. Therefore, we cannot simply increase the window during the derivation. In particular, we propagate as long as beliefs are consistent.

As mentioned earlier, agents can be **conservative** or **susceptible**. It is specified in the parameter \( \beta \) of the framework (see Section 2). Conservative agents reject any new beliefs that contradict their current set of beliefs. On the contrary, susceptible agents always accept new beliefs and remove the old ones that contradict them. Here we present two different belief propagation algorithms which describe how agents behave when faced with a new belief which is contradictory.

Consider a trace \( \sigma \) where \( T_\sigma = \{ t_0, \ldots, t_{n-1}, t_n \} \). Let \( EKB^t_i = \bigcup_{t \in \{ t_0, \ldots, t_k \}} EKB^t_i \) for \( k \in \mathbb{N} \). Independently of the propagation rule of the agent, both algorithm will try to propagate the accumulated set of beliefs as long as they are inside the window \( w \).

**Conservative agents.** Conservative agents never change their beliefs. Therefore, when they start believing some fact, they will not accept its negation. Let \( B^t_i \varphi \) be a belief that enters an agent’s EKB at time \( t_n \). A conservative agent \( i \) updates her EKB as follows:

\[
EKB^t_i = \begin{cases} 
EKB^t_i \cup \Psi_c \text{ if } (EKB^t_i \cup \Psi_c \not\vdash B^t_i \varphi, w) \\
EKB^t_i \cup \Psi_c \cup \{B^t_i \varphi\} \text{ otherwise} 
\end{cases}
\]

Intuitively, if the new belief is contradictory to the current set of beliefs of \( i \) it will not be added to her EKB.

**Susceptible agents.** Susceptible agents always accept new beliefs. If new beliefs are contradictory to what they used to believe, they, simply, reject their old beliefs. Or, in other words, these beliefs are not propagated. However, sometimes agents might need to choose which old beliefs they reject. For example, consider that \( B^t_i \varphi \) and \( B^t_i (\neg \varphi \vee \neg \psi) \). At time 2 an \( B^t_i \psi \) enter \( i \)'s EKB. Since \( i \) is susceptible accepts the new belief but now she needs to choose whether to propagate \( B^t_i \varphi \) or \( B^t_i (\neg \varphi \vee \neg \psi) \). But not both, since it will create inconsistent beliefs. To address this issue we assume that there exists a total order among all beliefs of an agent \( \Delta = \{ \beta_0, \ldots, \beta_n \} \).
for \( n \in \mathbb{N} \) where \( \beta_j < \beta_k \) if \( j < k \). Assume now that a new belief \( B^t_i \varphi \) a susceptible agent’s EKB at time \( t_n \).

We define the set of propagated beliefs for susceptible agents \( \Psi_s \subseteq D \) as largest set of beliefs such that the following conditions hold:

1. \( EKB^s_i[\tau_{n-1}] \cup \Psi_s \not\vdash B^t_\varphi \)
2. If \( \beta_j \not\in \Psi_s \) then \( (EKB^s_i[\tau_{n-1}] \cup \{ \beta_k \in \Psi_s | k > j \} \cup \{ \beta_j \}) \vdash B^t_\varphi \).

Given the above, a susceptible agent updates her EKB as follows,

\[
EKB^s_i[\tau_n] = EKB^s_i[\tau_{n-1}] \cup \Psi_s \cup \{ B^t_\varphi \}
\]

Consistency is guaranteed by definition (see item (1) in the conditions for \( \Psi_s \)), since the rule states that any belief that contradicts \( \varphi \) will not be propagated.

**Example 3.9.** At 20:00 Bob receives a Facebook message from Alice telling him that she is at work. That is,

\[
EKB^s_{bob}[20:00] = \{K_{bob}^{20:00}B_{bob}^{20:00}loc_{20:00}(Alice, work)\}.
\]

At 22:00 Bob checks his Facebook timeline, and he sees a post of Charlie—who is a coworker of Alice—at 20:00 saying that he is with all his coworkers in a pub having a beer. Assuming that at 22:00 Bob still remembers his belief from 20:00—i.e., the time window is larger than 2 hours for Facebook—this new information creates a conflict with Bob’s beliefs. Note that information from Charlie’s post is also concerned as belief since there is no way for Bob to validate it. Depending on the type of agent that Bob is the will be two possible updates in Bob’s EKB. If Bob is a conservative agent, then \( EKB^{s_{22:00}}_{bob} = \{K_{bob}^{22:00}B_{bob}^{22:00}loc_{22:00}(Alice, work)\} \). Meaning that the new belief is rejected. Bob will continue believing that Alice is at work. On the other hand, if Bob is a susceptible agent, he will add this new believe to his EKB, i.e., \( EKB^{s_{22:00}}_{bob} = \{K_{bob}^{22:00}B_{bob}^{22:00}loc_{22:00}(Alice, work)\} \). For all \( t \) such that \( 20:00 < t < 22:00 \) Bob believes that Alice’s location at 20:00 is a pub—due to belief propagation. And, after 22:00, this belief does not propagate to avoid contradictions. \( \square \)

### 3.2.5 Semantics of \( \mathcal{KBL}_{RT} \) (RTKBL)

The semantics of \( \mathcal{KBL}_{RT} \) formulae is given by the following satisfaction relation \( \models \).

**Definition 3.10 (Satisfaction Relation).** Given a well-formed trace \( \sigma \in TCS \), agents \( i, j \in Ag \), a finite set of agents \( G \subseteq Ag \), formulae \( \varphi, \psi \in \mathcal{F}_{\mathcal{KB}_{RT}} \), \( m \in C \), \( n \in \Sigma \), \( o \in D \), a variable \( x \), an event \( e \in EVT \), and a time-stamp \( t \), the satisfaction relation \( \models \subseteq TCS \times \mathcal{F}_{\mathcal{KB}_{RT}} \) is defined as shown in Fig. 2.

Predicates of type \( occ_{i}(e) \) are true if the event \( e \) is part of the events that occurred at time \( t \) in the trace. \( \forall t \) quantifies over all the time-stamps in the trace \( T_\sigma \), which, as mentioned earlier, is a finite set. For the remaining domains, \( \forall x : D \) , the substitution is carried out over the elements of the domain at a concrete time \( t \). Remember that each individual domian \( D \) always contains a finite set of elements. However, the same domain at different points in time, e.g., \( D \) and \( D' \), for any \( t \neq t' \) might contain different number of elements. When checking connections \( c_{n}(i,j) \) and actions \( a_{n}(i,j) \) at time \( t \), we check whether the
Thus, \[ \sigma \models \text{occurred}'(e) \] iff \((SN, E, t) \in \sigma \) such that \(e \in E\)

\[ \sigma \models \neg \varphi \] iff \(\sigma \not\models \varphi\)

\[ \sigma \models \varphi \land \psi \] iff \(\sigma \models \varphi \) and \(\sigma \models \psi\)

\[ \sigma \models \forall t \cdot \varphi \] iff for all \(v \in T_{\sigma}, \sigma \models \varphi[v/t]\)

\[ \sigma \models \forall x : D^t \cdot \varphi \] iff for all \(v \in D^t_{\sigma[t]}, \sigma \models \varphi[v/x]\)

\[ \sigma \models c^m_n(i, j) \] iff \((i, j) \in C^m_n[i]\)

\[ \sigma \models a^m_n(i, j) \] iff \((i, j) \in A^m_n[i]\)

\[ \sigma \models p^t \exists \] iff \(p^t \exists \in KB^t_{\sigma[t]}\)

\[ \sigma \models K^t_i \varphi \] iff \(\bigcup\{r | r < t, r \in T_{\sigma} \} KB^t_i(r) \vdash (\varphi, \omega)\)

\[ \sigma \models B^t_i \varphi \] iff \(\bigcup\{r | r < t, r \in T_{\sigma} \} KB^t_i(r) \vdash (B^t_i \varphi, \omega)\)

Fig. 2. Satisfaction relation for \(KB_{\mathcal{L}_T}\)

The intuition behind the previous definitions is better illustrated in the following example.

**Example 3.11 (Snapchat).** In this example we model the OSN Snapchat. In Snapchat there are two main events that users can perform: i) Connect through a friend relation; ii) share timed messages (which last up to 10 seconds and can include text and/or a picture) with their friends. Fig. 3 shows an example trace for Snapchat. The trace consists of: A common set of agents \(Ag = \{Alice, Bob, Charlie\}\). Since \(Ag\) does not change we avoid using the superindex indicating the time-stamp of the domain. Three SNMs \(SN_0, SN_7\) and \(SN_{15}\). The subindex of the SNMs indicates their time-stamp.

At time 0, Alice and Bob are friends, i.e., \(friendship^0(Alice, Bob)\). This is represented by including the pair \((Alice, Bob)\) in the relation \(Friendship^0\), drawn in the picture as an arrow between Alice and Bob in \(SN_0\). This relation between Alice and Bob does not change in \(\sigma\). Moreover, in \(SN_0\), Alice can send a friend request to Charlie. It is depicted as an outgoing dashed arrow from Alice’s node to Charlie’s. Thus, \(\sigma \models f^0_A \text{friendRequest}\) holds. Moreover, Alice knows that there is a picture of Bob at the pub, \(picture^0(Bob, pub)\). On the other hand, she believes that his location is the pub, \(loc^0(Bob, pub)\). The reason for
this is because she cannot verify that the picture has not been modified or she cannot precisely identify the location. However, the existence of picture$^0$(Bob, pub) can be verified since it is a picture that Alice can see in the OSN.

At time 7, Alice sends a friend request to Charlie. Though in this paper we do not discuss modelling the behaviour of events, we assume for this example that Alice can perform this event since he is explicitly permitted.$^2$ After the execution of the event both agents know friendRequest$^7$(Alice, Charlie). Note that this event produces knowledge. This is because the agents can verify that the friend request has occurred.

Lastly, at time 15, Charlie accepts Alice’s request and Bob shares a picture at work. Note that these two events are independent. If they were to be executed sequentially, independently of their order, the final SNM would always be equal to SN$_{15}$. After Bob’s accepting Alice’s request (Alice, Charlie) $\not\in$ FriendRequest$^7$[15], and (Alice, Charlie) $\in$ Friendship$^\sigma$[15]. That is, Alice cannot send more friend requests to Charlie, and now they have become friends. Furthermore, both, Alice and Bob know that Bob shared a picture at work. Note

---

$^2$We refer the reader to [19] for the definition of operational semantics rules for $PP$ which describe the behaviour of the events in the OSN.
that, in this case, Bob also knows that his location is work. Nevertheless, Alice believes it. The reason is that, unlike Bob, Alice cannot confirm that Bob’s location is work.

We mentioned that Snapchat messages last for up to 10 seconds. Let us assume w.l.o.g. that all messages last 10 seconds, i.e., $\omega = 10$. Given that, in $\sigma$, Alice remembers Bob picture from 0 to 10. That is,

$$\sigma \models \forall t : 0 \leq t \leq 10 \implies K_{\text{Alice}}^{t} \text{ picture}(Bob, pub)$$

Similarly, her belief about Bob location, $\text{picture}(Bob, pub)$, vanishes at time 10. Note also that, when Charlie accepts Alice’s friend request, he still knows (or remembers) that Alice sent it. In Snapchat friend request are permanent, but in $\mathcal{PPF}_{RT}$ we can choose whether friend request disappear after a few seconds. It can be done by requiring that the agent knows that a friend request occurred in order to accept it. In such a case, in $\sigma$, after time 18 Charlie would not be able to accept Alice’s request.

As mentioned earlier, the purpose of $\mathcal{PPF}_{RT}$ main goal is expressing privacy policies. They can be expressed as a formulae in $\mathcal{KBL}_{RT}$. Let $Ag$ and $Locs$ be domains of agents and locations, respectively. For the purpose of this example we assume that they do not change and that is why we do not specify the superindex. Alice, who likes keep her weekends private can write the following policy $\forall i: Ag \land \forall l: Locs \land \forall t \cdot \text{weekend}(t) \implies (\forall t' : t' > t \implies K_{i}^{t'} \text{ picture}(Alice, l))$. In English it means “Nobody can know Alice location during the weekend”.

### 3.3 Properties of the framework

In this section we present interesting properties of our framework, together with a set of novel derived operators not present in traditional epistemic logics.

To Learn or not to learn; To believe or not to believe. In $\mathcal{KBL}_{T}$ we introduced the learning modality $L_{i} \varphi$ [20]. It stands for $i$ learnt $\varphi$ at a moment $t$ when the formula is being evaluated. Here $L_{i} \varphi$, or more precisely, $L_{i}^{t} \varphi$ becomes a derived operator.

We say that an agent has learnt $\varphi$ at time $t$, if she knows $\varphi$ at time $t$ but she did not know it for any previous timestamp. Formally, we define it in terms of $K_{i}^{t} \varphi$ as follows

$$L_{i}^{t} \varphi \triangleq K_{i}^{t} \varphi \land \forall t' : t' < t \land \neg K_{i}^{t'} \varphi.$$  

We can also model when users start to believe something, or accept a belief. To model this concept we define the acceptance operator. As before, it can be expressed using $B_{i}^{t}$

$$A_{i}^{t} \varphi \triangleq B_{i}^{t} \varphi \land \forall t' : t' < t \land \neg B_{i}^{t'} \varphi.$$  

Analogously we can express when users forget some knowledge or when they reject a belief. Intuitively, an agent forgets $\varphi$ at time $t$ if she knew it in the previous timestamp, i.e., $\text{pred}(t)$—recall the definition of $\text{pred}$ in Section 3.2.2—and in $t$ she does not know $\varphi$, and, analogously, for reject. Formally,

$$F_{i}^{t} \varphi \triangleq K_{i}^{\text{pred}(t)} \varphi \land \neg K_{i}^{t} \varphi.$$  

We prove this property in the following lemma.

Temporal modalities. The traditional temporal modalities $\Box$ and $\Diamond$ can easily be defined using quantification over timestamps as follows:

$$
\Box \varphi(t) \triangleq \forall t \cdot \varphi(t)
$$
$$
\Diamond \varphi(t) \triangleq \exists t \cdot \varphi(t)
$$

where $\varphi(t)$ is a formula $\varphi$ which depends on $t$.

How long do agents remember? Agents remember according to the length of the parameter $\omega$. It can be seen as the size of their memory. Intuitively, increasing agents memory could only increase her knowledge.

We prove this property in the following lemma.

**Lemma 3.12 (Increasing window and knowledge).** Given $\sigma$, $t \in \sigma$ and $w, w' \in \mathbb{N}$ where $w \leq w'$.

If $EKB_i^{\sigma[t]} \vdash (K_i^t \varphi, w)$, then $EKB_i^{\sigma[t]} \vdash (K_i^t \varphi, w')$.

**Proof.** Assume $EKB_i^{\sigma[t]} \vdash (K_i^t \varphi, w)$. By Definition 3.6, there exists a derivation $(\varphi_1, w_1, (\varphi_2, w_2) \ldots (\varphi_n, w_n))$ for $n \in \mathbb{N}$ such that $(\varphi_n, w_n) = (\varphi, w)$. Let $\alpha = w' - w$, since $w \leq w'$ it follows that $\alpha \geq 0$.

Consider now the following derivation where the same deduction rules as in the previous derivation has been applied, and $\alpha$ is added to each $w_i, (\varphi_1, w_1 + \alpha) (\varphi_2, w_2 + \alpha) \ldots (\varphi_n, w_n + \alpha)$. We show now that, if $(\varphi_1, w_1) (\varphi_2, w_2)$ using a deduction rule $R$ then $(\varphi_1, w_1 + \alpha) (\varphi_2, w_2 + \alpha)$ can also be derived using $R$, for all $R$ in Table 2. We split the proof in derivation rules which, copy, reduce or introduce $w$.

- Rules that copy $w$. These are, A2, A3, A4, A5, K, B4, B5, L1 and L2. If $w \in \mathbb{N}$ given that $\alpha \geq 0$ it trivially follows that $w + \alpha \in \mathbb{N}$ which complies with the conditions of any of these rules. In this case $w = w'$ therefore the same applies to $w'$.
- Rules that reduce $w$. This is, KR1. In this case $w < w'$. In order for the derivation to be correct both $w$ and $w'$ are in $\mathbb{N}$. Since $\alpha \in \mathbb{N}$, it follows that $w + \alpha$ and $w' + \alpha$ are in $\mathbb{N}$. Moreover, since $\alpha$ is a constant it also follows that $w - w' = (w + \alpha) - (w' + \alpha)$. From the previous statement we conclude that the same window increase is required and, therefore the same derivation is performed.
- Rules that introduce $w$. These are, A1, D and Premise. No conditions are imposed in the value of $w$ in order to apply these rules. Therefore, if they can be applied with window $w$ since $\alpha \geq 0$ they can also be applied with window $w + \alpha$.

We can characterise how long agents remember information depending on the parameters $\omega$ and $\beta$ of the framework. We differentiate for how long agents remember knowledge or beliefs since the parameter $\beta$ might influence it.

**Lemma 3.13 ($\omega$ knowledge monotonicity).** Given $\sigma$ and $t \in T_\sigma$. If $K_i^t \varphi \in EKB_i^{\sigma[t]}$ then for all $t' \in T_\sigma$ such that $t \leq t' \leq t + \omega$ it holds $\sigma \models K_i^{t'} \varphi$.
PROOF. Assume $K_i^{t} \varphi \in \text{EK}_i^{\sigma[t]}$. By premise we can derive $(K_i^{t} \varphi, 0)$. Let $t' = t + \omega$, by applying KR1 we can derive $(K_i^{t'} \varphi, \omega)$. By $t' = t + \omega$, we conclude $\sigma \models K_i^{t'} \varphi$. Given the above and by Lemma 3.12, for all $t' \leq t'' < t + \omega$ it always possible to derive $(K_i^{t''} \varphi, \omega)$.

This parameter gives us a lot of flexibility in modeling agents’ memories. By choosing $\omega = \infty$ we can model agents with perfect recall, i.e., agents that never forget. Or we can also $\omega = 0$, i.e., agents who do not remember anything.

When $\beta$ = conservative memories about beliefs behave in the same way as knowledge.

**Lemma 3.14 (ω CONSERVATIVE BELIEF MONOTONICITY).** Given $\sigma$ and $t \in T_\sigma$. If $B_i^t \varphi \in \text{EK}_i^{\sigma[t]}$ and $B_i^{t'} \varphi \notin \text{EK}_i^{\sigma[t]}$, then for all $t' \leq t + \omega$ the following holds $\sigma \models \neg K_i^{t'} \varphi \implies B_i^{t'} \varphi$.

**Proof.** The condition $B_i^t \varphi \in \text{EK}_i^{\sigma[t]}$ and $B_i^{t'} \varphi \notin \text{EK}_i^{\sigma[t]}$ ensures that $B_i^{t'} \varphi$ was not propagated. By definition $\Psi_c$ contains the beliefs that are to be propagated from $\text{pred}(t)$ to $t$ up to the parameter $\omega$. There are three cases in which the EKB can be updated: i) No new information enters EKB; ii) New knowledge enters EKB; iii) A new belief enters the EKB. We show that the lemma holds for the three previous cases.

i) It trivially follows by the definition of belief propagation and $\Psi_c$.

ii) Assume $\sigma \models \neg K_i^{t'} \varphi$ for all $t' \leq t + \omega$. Then for any new knowledge $\psi$ that enters i’s EKB it holds $\psi \neq \neg \varphi$, otherwise it holds $\sigma \models K_i^{t'} \neg \varphi$ thus deriving a contradiction. Hence, by definition of belief propagation it follows that $B_i^{t'} \varphi \in \Psi_c$ for all $t' \leq t + \omega$ and, consequently, $\sigma \models \neg K_i^{t'} \neg \varphi \implies B_i^{t'} \varphi$.

iii) Let $B_i^{t'} \psi$ the belief to be introduced for any $t' \leq t + \omega$. By the definition of belief propagation, if $\text{EK}_i^{\text{pred}(t')} \cup \Psi_c \cup \{B_i^{t'} \psi\} \models B_i^{t'} \neg \varphi$ then $\text{EK}_i^{t'} = \text{EK}_i^{\text{pred}(t')} \cup \Psi_c$ and, since $B_i^{t'} \psi \notin \Psi_c$ and $B_i^{t'} \psi \notin \text{EK}_i^{\text{pred}(t')}$, it follows $B_i^{t'} \psi \notin \text{EK}_i^{t'}$. Hence, $B_i^{t'} \varphi$ is propagated. Therefore, it holds that $\sigma \models B_i^{t'} \varphi$. Otherwise, $\text{EK}_i^{t'} = \text{EK}_i^{\text{pred}(t')} \cup \Psi_c \cup \{B_i^{t'} \psi\}$, since $B_i^{t'} \varphi \in \Psi_c$ it holds that $\sigma \models B_i^{t'} \varphi$.

On the contrary, susceptible agents can reject a belief when exposed to new contradictory beliefs. Therefore, the duration of their beliefs can be limited by an event introducing new beliefs in the EKBs.

**Lemma 3.15 (ω SUSCEPTIBLE BELIEF MONOTONICITY).** Given $\sigma$ and $t, t' \in T_\sigma$ such that $t < t'$ and $t' - t \leq \omega$. If $B_i^t \varphi \in \text{EK}_i^{\sigma[t]}$, $B_i^{t'} \neg \varphi \in \text{EK}_i^{\sigma[t']}$, $B_i^{t'} \varphi \notin \text{EK}_i^{\text{pred}(t')}$ and $B_i^{t'} \neg \varphi \notin \text{EK}_i^{\text{pred}(t')}$, then for all $t'' \in T_\sigma$ such that $t \leq t'' \leq t'$ it holds $\sigma \models \neg K_i^{t''} \neg \varphi \implies B_i^{t''} \varphi$.

**Proof.** The proof of Lemma 3.14 can easily be adapted for this case by considering only all timestamps $t''$ such that $t \leq t'' \leq t' \leq t + \omega$.

Refined versions of the $\beta$ can be considered. Here we only studied the two extreme approaches. It is also possible to consider different $\omega$ for different pieces of information. In any case, the results of the previous lemmas are general enough to capture these modifications of the framework.
\[ \sigma \models_C \forall x. \delta \quad \text{iff} \quad \text{for all } v \in D_o, \sigma \models_C \delta[v/x] \]
\[ \sigma \models_C \llbracket \neg \alpha \rrbracket_i^x \quad \text{iff} \quad \sigma \models \neg \alpha \]
\[ \sigma \models_C \llbracket \varphi \implies \neg \alpha \rrbracket_i^s \quad \text{iff} \quad \sigma \models \varphi \implies \neg \alpha \]

**4 WRITING PRIVACY POLICIES**

In this section we provide a language for writing privacy polices, \( \mathcal{PPL}_{RT} \). In a nutshell \( \mathcal{PPL}_{RT} \) is a restricted version of \( \mathcal{KBL}_{RT} \) wrapped with \( \llbracket /rrbracket \) to indicate the owner of the policy \( i \) and its starting time \( s \).

**Definition 4.1 (Syntax of \( \mathcal{PPL}_{RT} \)).** Given agents \( a, b \in Ag \), a nonempty set of agents \( G \subseteq Ag \), timestamps \( s, t \), a variable \( x \), relation symbols \( c_t\) for (a, b), \( a_t\) for (a, b), \( p\) for \( c_m \), and a formula \( \varphi \in \mathcal{F}_{\mathcal{KBL}_{RT}} \), the syntax of the real-time privacy policy language \( \mathcal{PPL}_{RT} \) is inductively defined as:

\[ \begin{align*}
\delta & ::= \delta \land \delta \mid \forall x. \delta \mid \llbracket \neg \alpha \rrbracket_i^x \mid \llbracket \varphi \implies \neg \alpha \rrbracket_i^s \\
\alpha & ::= \alpha \land \alpha \mid \forall x. \alpha \mid \exists x. \alpha \mid \psi \mid \gamma' \\
\psi & ::= c_t^m(a, b) \mid a_t^m(a, b) \mid \text{occurred}^t(e) \\
\gamma' & ::= K_t^i y \mid B_t^i y \\
\gamma & ::= y \land y \mid \neg y \mid p\left(\overline{\overline{s}}\right) \mid y' \mid \forall x. y
\end{align*} \]

We will use \( \mathcal{T}_{\mathcal{PPL}_{RT}} \) to denote the set of all privacy policies created according to the previous definition. To determine whether a policy is violated in an evolving social network, we formalise the notion of conformance for \( \mathcal{PPL}_{RT} \).

**Definition 4.2 (Conformance Relation).** Given a well-formed trace \( \sigma \in TCS \), a variable \( x \), a timestamp \( s \), and an agent \( i \in Ag \), the conformance relation \( \models_C \) is defined as shown in Fig. 4.

The definition is quite simple, especially compared to that of conformance of \( \mathcal{PPL}_{T} \) [20]. If the policy is quantified, we substitute in the usual way. The main body of the policy in double brackets is dealt with by simply delegating to the satisfaction relation.

**4.0.1 Examples.**

**Example 4.3.** Assume Alice decides to hide all her weekend locations from her supervisor Bob. She has a number of options how to achieve this, depending on what the precise meaning of the policy should be.

If the idea she has is to restrict Bob learning her weekend location directly when she posts it, she can define

\[ \delta_1 = \forall t \cdot \llbracket \text{weekend}(t) \implies \neg K_{Bob}^t \text{location}(Alice, t) \rrbracket_i^{2016-04-16}_{Alice} \]

where the \( \text{weekend} \) predicate is true if the timestamp supplied represents a time during a weekend. This policy can be read as “if \( x \) is a time instant during a weekend, then Bob is not allowed to learn at \( x \) Alice’s location from time \( x \)."
This, however, is a very specialized scenario that captures only a small number of situations. Bob is, for example, free to learn Alice’s location at any point not during the weekend, or at any point during the weekend when Alice’s location is no longer up-to-date. Though there might be scenarios where this might be the desired behavior, we can define a policy that seems much closer to the intuitive meaning of learning someone’s location on a weekend. Consider

\[ \delta_2 = \forall t \cdot \left[ \text{weekend}(t) \implies \neg \exists t' \cdot (K_{Bob}^{t'} \text{location}(Alice, t)) \right]^{2016-04-16} \]

Here, Bob is not allowed to learn Alice’s location from a weekend, no matter when. If the policy does not get violated, then Alice’s weekend locations will be completely safe from Bob – on the social network, at least.

**Example 4.4.** One of the advantages of \(\mathcal{PP}_{\mathcal{LR}T}\) is the ability to distinguish between the original time of a piece of information and the time when it should be hidden. Suppose Diane activates the following policy:

\[ \delta_1 = \forall t \cdot \forall x: Ag^t \left[ \neg \text{friends}(Diane, x) \implies \neg \exists t' \cdot (K_x^{t'} \text{post}^{t'}(Diane)) \right]^{2016-05-28} \]

This aims to prevent anyone who is not a friend of Diane’s from learning any of her posts (here we assume that the \text{friends} connection is not reflexive for simplicity, otherwise the restriction would target Diane herself, too).

Though \(\delta_1\) may seem reasonable enough, it might be unnecessarily restrictive. Let us say there is another user, Ethan. Diane becomes friends with Ethan on May 31, so when her policy is already in effect. Should Ethan be able to learn about Diane’s posts from when they were not friends? Not according to \(\delta_1\), which says that no one, regardless of their relationship with Diane at the moment, is able to learn about her posts from when they were not friends.

Note that while this may indeed be the desired behaviour, it is, for example, not what happens on Facebook, where when two users become friends, they are free to access each other’s timeline including past events and posts. \(\mathcal{PP}_{\mathcal{LR}T}\) is expressive enough to model this behaviour as well. We can define:

\[ \delta_2 = \forall t \cdot \forall t' \cdot \forall x: Ag^t \left[ \neg \text{friends}(Diane, x) \land \neg \text{friends}(Diane, x) \implies \neg K_x^{t'} \text{post}^{t'}(Diane) \right]^{2016-05-28} \]

This policy precisely defines the point in time \text{from} when to hide information, \(y\), as well as the point in time \text{when} to hide it, \(y'\). It says, “if Diane is not friend with someone, then that someone cannot learn her posts, but only if they come from a time when they were not friends”. Note that \(\delta_2\) says nothing about users who are currently friends of Diane’s, which is different from \(\delta_1\) – here her friends can learn anything, including past posts from when they were not friends with her.

## 5 RELATED WORK

Specifying and reasoning about temporal properties in multi-agent systems using epistemic logic have been previously studied in a number of papers (e.g., in [10] for interpreted systems; see also [20] and references...
therein). More recently, Moses et al. have extended interpreted systems to enhance reasoning about past and future knowledge. In [6] they extend $K_t$ with a time-stamp $K_{i,t}$, allowing for the expression of properties such as “Alice knows at time 10 that Bob knew $p$ at time 1”, i.e., $K_{Alice, 10} K_{Bob, 1} p$. Though there are some similarities between the work by Moses et al. and ours, namely the use of time-stamps in the knowledge modality and the moving along a trace to place the evaluation in the “right” place, there are quite a few differences. First, we differ in the intended use of the logics: Moses et al. use time to model delays in protocols, whereas our main motivation is to provide a rich privacy policy language for OSNs. Besides, our logic includes belief and other operators not present in the timed versions introduced by them, and we have time-stamps associated with propositions. We claim, however, that $KBL_{RT}$ is at least as expressive as the logics introduced by Moses et al., though this would need to be formally proved and for that we would need to relate our (non-standard) semantics with interpreted systems. This is left as future work.

As mentioned in the introduction, our work solves the open issues and limitations described in our previous work [20]. In that paper we introduced $PPF_T$, a temporal epistemic framework for describing policies for OSNs. $PPF_T$ relies on the temporal epistemic logic $KBL_T$ that allows to express temporal constraints using the classical box and diamond temporal operators. Neither the policy language nor the underlying logic $KBL_T$ have a notion of time: this is only used at the semantic level, allowing to move along a social network model trace in order to interpret the temporal operators. $PPF_{RT}$ strictly extends $PPF_T$, so our work not only addressed the already identified limitations identified in [20] but also extends that work by allowing the definition of more modalities (e.g., forget, accept, belief) allowing for a more expressive policy language and underlying logic. These allow us to define policies for more complex OSNs, like Snapchat.

Besides the above, it is worth mentioning the work by Woźni & Lomuscio [23] where TCTLKD was presented. TCTLKD is a combination of epistemic logic, CTL, a deontic modality and real time. It is difficult to compare our logic $KBL_{RT}$ with TCTLKD as they use CTL, while we have time-stamps in the propositions. The models used to interpret formulae in TCTLKD are based on a semantics for a branching logic, being a combination of timed automata and interpreted systems plus an equivalence relation for modelling permission. Ours is based on (timed) social network models. Besides we can also reason about belief.

6 CONCLUSIONS
In this paper, we have presented a novel privacy policy framework based on a logic that offers explicitly support for expressing timestamps in events and epistemic operators. This framework extends [18, 21], which did not offer any support for time, and [20] which only had limited support due to the implicit treatment of time. Our framework is based on Extended Knowledge Bases (EKB). A query to an EKB starts by instantiating a number of epistemic axioms that handle knowledge, belief and time (the concrete axioms depend on the OSN instantiation). The deductive proof system give an algorithm to deduce the knowledge of agents acquired at each instant, and in turn a model checking algorithm for the logic and a check for privacy policy
violations. The explicit time-stamps allow to define learning and forget operators that capture when knowledge is acquired. Similarly, one can derive accept and reject operators that model when beliefs come into existence and are rejected.

The flexibility of the EKBs allows to model different kinds of OSNs in terms of how the actions affect knowledge and how this knowledge is preserved through time. We have also sketched how different existing OSNs can be modeled using this flexibility.

Two important avenues for future research are the following. First, many instantiations enable efficient implementations of checking privacy policy violations by exploiting whether events can affect the knowledge of the agents involved. Once the effect of the actions is fixed one can prove that a distributed algorithm guarantees the same outcome as the centralized algorithm proposed here. For example, tweets can only affect the knowledge of subscribers so all other users are unaffected. Second, once an effective system to check policy violations is in place, there are different possibilities that the OSN can offer. One is to enforce the policy by forbidding the action that the last agent executed, which would lead to the violation. Another can be the analysis of the trace to assign blame (and correspondingly, reputation) to the agents involved in the chain of actions. For example, the creator of a gossip or fake news may be held more responsible than users forwarding them. Even a finer analysis of controllability can give more powerful algorithms by detecting which agents could have prevented the information flow that lead to the violation. Finally, yet another possibility would be to remove past events from the history trace of the OSN creating a pruned trace with no violation. All these possibilities are enabled by having a formal framework like the one presented in this paper.

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