A Review of Block Designs for Test Treatments – Control(s) Comparisons

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ABSTRACT

In practice there may arise experimental situations where it is desired to compare several treatments called the test treatments to a standard treatment called control. The main interest here lies in making test treatment-control comparison with as much precision as possible and comparison within the test treatments are of less importance. For example in agricultural experiments, the aim of the experimenter is to test a set of new varieties of a crop with an already existing variety and to determine which of the varieties perform better in comparison to the existing variety. Balanced Treatment Incomplete Block (BTIB) designs have been defined for this situation. The designs are balanced with respect to test treatment-control comparisons. The concept of BTIB is further extended to define Balanced Two Disjoint Sets of Treatments (BTDT) designs when there are more than one control. Some methods of constructing these designs are presented here. Some class of row-column designs, which are balanced for test treatments vs. control comparisons, referred to as the Balanced Treatment vs. Control Row-Column (BTCRC) designs are also described when heterogeneity is to be eliminated in two directions.

Key words: Balanced Treatment Incomplete Block (BTIB) design, Balanced Two Disjoint Sets of Treatments (BTDT) design, Balanced Treatment vs. Control Row-Column (BTCRC) design.

INTRODUCTION

Designs are usually characterized by the nature of grouping of experimental units and the procedure of random allocation of treatments to the experimental units. Block (Row-Column) designs are useful in experiments requiring elimination of heterogeneity in one (two) direction. These designs are useful in agricultural experiments for situation where the experimenter is interested in making comparison of all possible treatment pairs. The design adopted should be such that it allows these comparisons to be made with as high a precision as possible. However, in practice there may arise experimental situations where it is desired to compare several treatments called test treatments to standard treatment called control. For example, in screening experiments or in the beginning of a long-term experimental investigation, where it is initially desired to determine the relative performance of new test treatments with respect to the control. Let there be v treatments (such as new types of hybrid varieties, method of cultivation, pesticides, herbicides etc.) and an existing (old) one is to be replaced by one of these newer kinds. In such situations, the experimenter is not interested in making comparisons among all the treatments, but the main interest is to compare the new (test) treatments with the old (control) treatment and thus a higher precision is desired for these estimates.

The earliest work on this problem was carried out by Dunnett (1955). He posed (but did not solve) the problem of optimally allocating experimental units to control and test treatment so as to maximize the probability associated with the joint confidence statement concerning the many-to-one comparison between the mean of the control treatment and the mean of the test treatments. This optimal allocation problem was solved by Bachhofer and his coworkers (1969, 1970, 1981).

In all the above papers, it was assumed that a Completely Randomized (CR) design has been used. However, many practical situations may require the blocking of experimental units in order to cut down on bias and improve the precision of the experiment. If the block size is large enough to accommodate one replication of all the test treatments and additional control
treatments as well, then the design of experiment can be carried out using the optimal allocations described in Bechhofer (1969) and Bechhofer and Nocturne (1970) with only the usual modifications.

2. Balanced Incomplete Block Designs for Comparing Test Treatments with Control

The situation that commonly occurs in practice is when the block size is less than the total number of treatments. Bechhofer and Tamhane (1981) introduced a general class of incomplete block designs appropriate for the problem called Balanced Test Treatment Incomplete Block (BTIB) designs. The designs are balanced with respect to test treatment-control comparisons.

Let the treatments be denoted by 0,1,...,v with 0 denoting the control treatment and 1,2,...,v denoting the v ≥ 2 test treatments. Let k < v + 1 be the size of each block and b the number of blocks for experimentation. If treatment i is assigned to the hth plot of jth block (0 ≤ i ≤ v, 1 ≤ h ≤ k, 1 ≤ j ≤ b), then the usual additive linear model is,

\[ y_{ijh} = \mu + \alpha_i + \beta_j + e_{ijh}, \]

where \( y_{ijh} \) denote the corresponding response variable, \( \mu \) is the grand mean, \( \alpha_i \) is \( i \)-th treatment effect and \( \beta_j \) is \( j \)-th block effect. The \( e_{ijh} \) are assumed to be iid \( N(0, \sigma^2) \) random variables.

**Definition 2.1**: [Bechhofer and Tamhane (1981)]. For given \( (v, b, k) \), consider a design with the incidence matrix \( N = ((n_{ij})) \), where \( n_{ij} \) is the number of replications of the \( i \)-th treatment in the \( j \)-th block. Let \( \lambda_{ii'} = \sum n_{ij} \) denote the total number of times the \( i \)-th treatment appears with the \( i' \)-th treatment in the same block over the whole design (\( i \neq i' \); 0 ≤ i, \( i' \) ≥ v). Then the necessary and sufficient conditions for a design to be BTIB are

\[ \lambda_{01} = \lambda_{02} = \ldots, = \lambda_{0v} = \lambda_0 \ (\text{say}) \]
\[ \lambda_{12} = \lambda_{13} = \ldots, = \lambda_{v-1,v} = \lambda_1 \ (\text{say}) \]

In other words, each test treatment must appear with (i.e. in the same block) the control treatment the same number of times \( (\lambda_0) \) over the design, and each test treatment must appear with every other test treatment same number of times \( (\lambda_1) \) over the design. As a consequence of this definition of BTIB design, \( N \) is the \((v+1) \times b \) incidence matrix, given as

\[ N = \left[ \begin{array}{c} N_1 \\ n \end{array} \right], \]

where \( N_1 \) is a \((v \times b) \) incidence matrix pertaining to \( v \) test treatments and \( n \) is a \((1 \times b) \) incidence matrix pertaining a control treatment. The vector of replication is \( \left[r_1, r_0 \right] \) and vector of block sizes is \((k_1, \ldots, k_b) \), \( r \) and \( r_0 \) being the replication number of test treatments and control treatment respectively. Therefore the information matrix is given by

\[ C = \left[ \begin{array}{c} r_1 N \end{array} \right] - \left[ \begin{array}{c} N_1 \\ n \end{array} \right] K^{-1} \left[ \begin{array}{c} N_1' \\ n' \end{array} \right]. \]

2.1 Construction of (BTIB) designs

In this section, some methods of constructing BTIB designs for comparing a set of test treatment to a control treatment are given. The designs are balanced with respect to test treatments-control treatment comparisons according to Definition 2.1.

**Method 2.1.1**: Consider a BIB design with parameters \( (v, b, r, k, \lambda) \), then adding one control in each block of this design results in a reinforced BIB design or a BTIB design with parameters \( v, b, r_0 = k, k_0 = 1 \) and \( \lambda_1 = 1 \).

**Example 2.1.1**: Consider a BIB with parameters \( v = 7, b = 7, r = 3, k = 3, \lambda_1 = 1 \). By adding the control treatment (0) to each block, following BTIB design is obtained:

\[
\begin{bmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
2 & 3 & 4 & 5 & 6 & 7 & 1 \\
4 & 5 & 6 & 7 & 1 & 2 & 3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
Method 2.1.2: Start with a BIB design with \( t > v \) treatments in \( b \) blocks. Replace the treatments \( v + 1, v + 2, \ldots, t \) to zeros. A BTIB design with parameters \( v, b, r, r_0 = r(t-v), k, \lambda_0 = (t-v)\lambda \) and \( \lambda_1 = \lambda \) is obtained.

Example 2.1.2: Consider the following BIB design with parameters \((7, 3, 1)\):

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
7 & 1 & 2 & 3 & 4 & 5 & 6 \\
3 & 4 & 5 & 6 & 7 & 1 & 2 \\
\end{array}
\]

Replacing the 6’s and 7’s by zeros, the following BTIB design with parameters \(v=5, b=7, r=3, r_0=6, k=3, \lambda_0=2 \) and \( \lambda_1=1 \) is obtained:

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & 0 & 0 \\
0 & 1 & 2 & 3 & 4 & 5 & 0 \\
3 & 4 & 5 & 0 & 0 & 1 & 2 \\
\end{array}
\]

If treatment 5 is also replaced then the following BTIB design would be obtained with parameters \(4, 7, 3, 9, 3, \lambda_0=3, \lambda_1=1 \).

Method 2.1.3: Suppose that for given \((v, k)\) there is a design \(D_1\) with \(\lambda_0 > 0\). Then new design \(D_2\) for the same \((v, k)\) can be obtained by taking a “complement” of \(D_1\) in the following way. Separate the blocks of \(D_1\) in different sets so that each block in given set has zero assigned in an equal number of plots (0 times, 1 time etc.). For example, consider the above design the blocks of which can be separated into three sets as follows:

\[
D_1 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 2 & 3 & 0 & 0 \\
4 & 3 & 3 & 4 & 1 & 2 \\
\end{bmatrix}
\]

For each set of \(D_1\) write its “complementary” set (with zero assign in the same number of plots) which will result in a BTIB design. These complementary sets for above are

\[
D_2 = \begin{bmatrix}
1 & 1 & 2 & 0 & 0 & 0 \\
2 & 3 & 3 & 1 & 1 & 2 \\
3 & 4 & 4 & 2 & 4 & 4 \\
\end{bmatrix}
\]

This is a BTIB design with parameters \(4, 7, 4, 5, 3, \lambda_0=2, \lambda_1=2 \).

3. Balanced Block Designs for Comparing Two disjoint Sets of Treatments

Section 2 gives the concept of BTIB design for comparing a set of treatments to single control treatment. But there are situations in which more than one control treatment is required. The problem is that there are two sets of treatments [Majumdar (1988), Jaggi (1992), Jaggi et al. (1996)]. One set \(T\) of cardinality \(v_1\), containing test treatments denoted by \(1,2,\ldots,v_1\) and the second set \(S\) of cardinality \(v_2\), containing standard or control treatments denoted by \((v_1+1),\ldots,v\), where \(v = v_1 + v_2 \geq 4\) and \(T \cap S = \phi\). The primary interest of experimenter is to estimate the contrast \((\tau_t - \tau_s)\) with as much precision as possible, where \(t \in T\) and \(s \in S\). The comparisons between the treatments within the set are not of interest. The concept of BTIB design is extended to compare a set of test treatments to a set of control treatments. The designs for comparing two disjoint sets of treatment are called as Balanced Two Disjoint Sets of Treatments (BTDT) designs. The two sets of treatments are disjoint in the sense that there are no common treatments between the two sets.

These designs are balanced with respect to test treatments-control treatments comparisons. Let \(N\) be a \((v_1 + v_2) \times b\) incidence matrix, given as

\[
N = \begin{bmatrix} N_1 \end{bmatrix}
\]

where \(N_1\) is a \((v_1 \times b)\) incidence matrix pertaining to \(v_1\) test treatments and \(n\) is a \((v_2 \times b)\) incidence matrix pertaining to \(v_2\) control treatments. Also \(r = \left( r_1 \begin{bmatrix} 1 \end{bmatrix}_{v_1}, r_2 \begin{bmatrix} 1 \end{bmatrix}_{v_2} \right)\) and \(k = (k_1, \ldots, k_b)'\), \(r_1\) and \(r_2\) being the replication number of test treatments and control treatments respectively and \(k\) is the vector of block
sizes. Therefore, the information matrix is given by
\[
C = \begin{bmatrix}
   r_1I_{v_1} & 0 \\
   0 & r_c I_{v_2}
\end{bmatrix} - \begin{bmatrix}
   N_1 \\
   n
\end{bmatrix}K^{-1}\begin{bmatrix}
   N_1' \\
   n'
\end{bmatrix}
= \begin{bmatrix}
   r_1I_{v_1} - N_1K^{-1}N_1' \\
   -nK^{-1}N_1' & r_c I_{v_2} - nK^{-1}n'
\end{bmatrix}
\]

3.1 Construction of BTDT Design

Method 3.1.1: This method is developed by the Jaggi in 1992. Let \( N_1 \) be the incidence matrix of a partially balanced incomplete block (PBIB) design with two associate classes and with parameters \( v_1, b_1, k_1, \lambda_{11}, \lambda_{12}, m, n \), and \( N_2 \) be the incidence matrix of another PBIB design with the same association scheme \((m,n)\) and parameter \( v_2, b_2, k_2, \lambda_{21}, \lambda_{22}, m, n \).

Case (A):
If \( \frac{\lambda_{11}}{k_1 + v_2} + \frac{\lambda_{12}}{k_2} = \frac{\lambda_{12}}{k_1 + v_2} + \frac{\lambda_{22}}{k_2} = \lambda \), then
\[
N = \begin{bmatrix}
   N_1 & N_2 \\
   I_{v_1} & I_{v_2}
\end{bmatrix}
\]
is the incidence matrix of a BTDT design for comparing \( v_1 \) test treatment to \( v_2 \) control treatments with parameters \( v = v_1 + v_2, b = b_1 + b_2 \).

Case (B):
If \( \frac{\lambda_{11}}{k_1 + v_2} + \frac{\lambda_{21}}{k_2 + v_2} = \frac{\lambda_{12}}{k_1 + v_2} + \frac{\lambda_{22}}{k_2 + v_2} = \lambda \), then
\[
N = \begin{bmatrix}
   N_1 & N_2 \\
   I_{v_1} & I_{v_2}
\end{bmatrix}
\]
is the incidence matrix of a BTDT design for comparing a set of \( v_1 \) test treatments to a set of \( v_2 \) control treatments with parameters \( v = v_1 + v_2, b = b_1 + b_2 \).

Example 3.1.1: Consider a semi-regular GD design (SR6) with parameters \( v_1 = 6, b_1 = 9, r_1 = 3, k_1 = 2, m = 2, n = 3, \lambda_{11} = 0, \lambda_{12} = 1 \) and another regular GD design (R94) with \( v_1 = 6, b_2 = 6, r_2 = 4, k_2 = 4, m = 2, n = 3, \lambda_{21} = 3, \lambda_{22} = 2 \), both the designs having the same association scheme. The design obtained by taking the blocks of these two designs together and augmenting two new treatments \( (v_2 = 2) \) in each block of first design, gives following BTDT design with the parameters \( v = 8, b = 15, r = 7, r_0 = 9, k = 4 \):

\[
(1,2,7,8); (1,2,4,6); (3,4,6,2); (4,5,1,3); (5,4,7,8); (1,4,7,8); (3,6,7,8); \]

\[
(1,2,7,8); (1,2,4,6); (3,4,6,2); (4,5,1,3); (5,4,7,8); (1,4,7,8); (3,6,7,8);
\]

Method 3.1.2: This method of construction of design for making comparisons between two sets of treatments is derived from the use of variance balanced block designs with equal and unequal replications and equal and unequal block sizes. Using this method, design with equal as well as unequal block sizes, and equal and unequal replications can be constructed.

Consider any variance balanced block design with unequal replication numbers and unequal block sizes. The design obtained by deleting one block from this design is a BTDT design. Then the information matrix \( C \) is

\[
C = \begin{bmatrix}
   (v-1)I_{v-1} - \begin{bmatrix}
   0 & -1 \\
   v & k_0
\end{bmatrix}I_{v-1}I_{v-1} & 0 & 0I_{v-1}I_{v-1} \\
   -vI_{v-1}I_{v-1} & 0I_{v-1}I_{v-1}
\end{bmatrix}
\]

Example 3.1.2: Consider the following variance balanced design with unequal replication and unequal block sizes and with parameters \( v = 6, b = 11, r = [41_{4}^{I} 51_{2}^{I}], k = [41_{2}, 21_{0}^{I}] \)

\[
1 1 1 1 2 2 3 3 4 4 5 \\
2 2 5 6 5 6 5 6 5 6 6 \\
3 3 \\
4 4
\]
The design obtained by deleting the first block i.e. \((1, 2, 3, 4)\) is a BTDT design with unequal replication numbers, \(v_1 = k_1 = 4\), \(v_2 = v - k_1 = 2\), \(b = 10\), \(r = [31_4 \ 51_2]\), \(k = [4 \ 2]_9\).

4. Row-Column Designs for Comparing Test Treatments with a Control

This section considers the designs for comparing several treatments with a control when heterogeneity is to be eliminated in two directions. Row-column designs, which are balanced for test treatments vs. control comparisons referred to as the Balanced Treatment vs. Control Row-Column (BTCRC) designs has been proposed by Majumdar and Tamhane (1996).

Suppose \(v \geq 2\) treatments, labeled \(1, 2, \ldots, v\) are to be compared with a control, labeled 0, in a row-column design with \(a \geq 2\) rows and \(b \geq 2\) columns. Assume that only one treatment is applied in each of the \((a, b)\) plots. Let \(y_{ijk}\) be the observation on the \(i\)th treatment applied in the \(j\)th row and \(k\)th column (\(0 \leq i \leq v\), \(1 \leq j \leq a\), \(1 \leq k \leq b\)).

Fixed-effects additive linear model assumed is as follows:

\[
y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \epsilon_{ijk},
\]

where \(\mu\) is the grand mean, \(\alpha_i\) is the \(i\)th treatment effect, \(\beta_j\) is the \(j\)th row effect, \(\gamma_k\) is the \(k\)th column effect and \(\epsilon_{ijk}\) are uncorrelated random error with zero mean and constant variance \(\sigma^2\).

Here again the interest is to estimate the treatments vs. control contrasts \(\alpha_0 - \alpha_i\) \(i = 1, \ldots, v\). Let the row-column design have \(m_{ij}\) incidences of the \(i\)th treatment in the \(j\)th row and \(n_{ik}\) incidences of the \(i\)th treatment in the \(k\)th column \((0 \leq i \leq v\), \(1 \leq j \leq a\), \(1 \leq k \leq b\)\). Let \(M = \{m_{ij}\}\) and \(N = \{n_{ij}\}\) denote the row and column incidence matrices, respectively. Further, let \(r_i = \sum_{j=1}^{a} m_{ij} = \sum_{k=1}^{b} n_{ij}\) be the number of replications of the \(i\)th treatment, \(r = (r_0, r_1, \ldots, r_v)\) and

\[
\mu_{i} = \sum_{j=1}^{u} m_{ij} \quad \text{and} \quad \nu_{i} = \sum_{k=1}^{b} n_{ik}
\]

Define

\[
\lambda_{i} = \frac{1}{ab} \left[ a \mu_{i} + b \nu_{i} - rr_i \right]
\]

**Definition 4.1:** The necessary and sufficient conditions for a row column design to be BTCRC is that \(\lambda_{01} = \lambda_{02} = \ldots, = \lambda_{0v} = \lambda_0\) (say), \(\lambda_{12} = \lambda_{13} = \ldots, = \lambda_{v-1,v} = \lambda_1\) (say).

4.1 Construction of BTCRC Designs

**Method 4.1.1:** [Notz (1985)]. Start with a Latin Square of order \(w \geq v\) and replace symbols \(v+1, \ldots, w\) by the symbol 0 (control). The resulting design is a BTCRC with parameters \(v, a = b = w, r = w, r_0 = (w-v)w, \mu_{i0} = v, \mu_{0i} = (w-v)w, \lambda_0 = w-v, \lambda_1 = 1\).

**Example 4.1.1:** Consider the 5 x 5 Latin Square Design. Replacing symbols 4 and 5 to 0, the following design with three test treatments is obtained:

\[
\begin{bmatrix}
1 & 2 & 3 & 0 & 0 \\
2 & 3 & 0 & 0 & 1 \\
0 & 0 & 1 & 2 & 3 \\
0 & 1 & 2 & 3 & 0
\end{bmatrix}
\]

Here \(\mu_{ii'} = \nu_{ii'} = 5\). This design is row as well as column balanced with \(\lambda_0 = 2\) and \(\lambda_1 = 1\).

**Method 4.1.2:** Construct a Pseudo-Youden design (PYD) introduced by Cheng (1981). Thus a BTCRC design can be constructed from a PYD in \(w\) symbols by changing symbols \(v+1, \ldots, w\) to 0.

**Example 4.1.2:** Consider a 6 x 6 PYD for 9 treatments. Replacing symbols 7, 8 and 9 by 0’s, the following BTCRC design for six test treatments with \(\mu_0 = 7\) and \(\nu_{i0} = 8\) and \(\mu_{ii'} = 3, \nu_{ii'} = 2\) for \(1 \leq i \neq i' \leq 6\) is obtained:
The design has \( \lambda_0 = 7/6, \lambda_1 = 7/18 \).

**Method 4.1.3:** The transversal of a Latin Square of order \( v \) is a set of \( v \) cells such that each row, column and symbol is represented exactly once in this set [Hedayat and Seiden (1974)]. Changing all symbols in a transversal to 0, a BTCRC design with \( a = b = v, r_1 = \ldots = r_v = v-1 \) and \( r_0 = v \) is obtained.

**Example 4.1.3:** Consider the following Latin Square of order 4 with a transversal parenthesized:

\[
\begin{bmatrix}
1 & 2 & 3 & 4 \\
3 & 4 & 1 & 2 \\
4 & 3 & 2 & 1 \\
2 & 1 & 4 & 3 \\
\end{bmatrix}
\]

Replacing the parenthesized treatments by 0 gives the following BTCRC design with \( \mu_{ii'} = 2, \mu_{i0} = v_i = 3 \):

\[
\begin{bmatrix}
1 & 2 & 0 & 4 \\
3 & 4 & 1 & 0 \\
0 & 3 & 2 & 1 \\
2 & 0 & 4 & 3 \\
\end{bmatrix}
\]

This design has \( \lambda_0 = 3/4, \lambda_1 = 7/16 \).

**Method 4.1.4:** Two transversals in a Latin Square of order \( v \) are called parallel if they have no cell in common. Suppose such parallel transversals are identified. First apply Method 4.1.3 to obtain a \( v \times v \) BTCRC design using the first transversal. Then take horizontal and vertical projections (Hedayat and Seiden 1974) of the second transversal, and add a 0 to complete the design.

**Example 4.1.4:** Consider the following 4 x 4 Latin Square with two parallel transversals, one parenthesized and other square-bracketed:

\[
\begin{bmatrix}
4 & 0 & 0 & 6 & 0 & 5 \\
3 & 1 & 2 & 0 & 0 & 0 \\
2 & 5 & 1 & 3 & 6 & 4 \\
0 & 3 & 6 & 2 & 5 & 0 \\
0 & 6 & 0 & 4 & 1 & 3 \\
5 & 0 & 4 & 0 & 2 & 1 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 2 & 3 & 4 \\
3 & 4 & 1 & 2 \\
4 & 3 & 2 & 1 \\
2 & 1 & 4 & 3 \\
\end{bmatrix}
\]

Replace the parenthesized transversals by 0's to obtain the following BTCRC design using Method 4.1.3:

\[
\begin{bmatrix}
0 & 2 & 0 & 4 \\
3 & 4 & 1 & 0 \\
0 & 3 & 0 & 1 & 2 \\
2 & 0 & 4 & 0 & 3 \\
1 & 4 & 2 & 3 & 0 \\
\end{bmatrix}
\]

This design has \( \lambda_0 = 34/25 \) and \( \lambda_1 = 14/25 \).

**Method 4.1.5:** This method of Kiefer (1975) can be used to construct large BTCRC designs from smaller ones as shown with the following example:

**Example 4.1.5:** BTCRC design with 4 test treatments with 9 x 9 array can be constructed as follows:

\[
\begin{bmatrix}
\{d_1, d_2'\} \\
\{d_2, d_3\} \\
\end{bmatrix}
\]

where \( d_1 \) is 6 x 6 BTCRC design with \( v = 4 \) obtained from a Latin square of order 6 by changing symbols 5 and 6 to 0.

\[
d_1 = \begin{bmatrix}
0 & 1 & 2 & 3 & 4 & 0 \\
0 & 0 & 1 & 2 & 3 & 4 \\
4 & 0 & 0 & 1 & 2 & 3 \\
3 & 4 & 0 & 0 & 1 & 2 \\
2 & 3 & 4 & 0 & 0 & 1 \\
1 & 2 & 3 & 4 & 0 & 0 \\
\end{bmatrix}
\]

\( d_2 \) is BTCRC design belonging to the “Euclidean family” of Hedayat and Majumdar (1988).

\[
d_2 = \begin{bmatrix}
1 & 0 & 3 & 4 & 2 & 0 \\
0 & 3 & 4 & 2 & 0 & 1 \\
4 & 2 & 0 & 0 & 1 & 3 \\
\end{bmatrix}
\]
And $d3$ is a $3 \times 3$ matrix of all 0's. Then design is,

$$
\begin{align*}
\{d_1 & d_2 \\
\{d_2 & d_3 \}
\end{align*}
\begin{bmatrix}
0 & 1 & 2 & 3 & 4 & 0 & 1 & 0 & 4 \\
0 & 0 & 1 & 2 & 3 & 4 & 0 & 3 & 2 \\
4 & 0 & 0 & 1 & 2 & 3 & 3 & 4 & 0 \\
3 & 4 & 0 & 0 & 1 & 2 & 4 & 2 & 0 \\
2 & 3 & 4 & 0 & 0 & 1 & 2 & 0 & 1 \\
1 & 2 & 3 & 4 & 0 & 0 & 0 & 1 & 3 \\
1 & 0 & 3 & 4 & 2 & 0 & 0 & 0 & 0 \\
0 & 3 & 4 & 2 & 0 & 1 & 0 & 0 & 0 \\
4 & 2 & 0 & 0 & 1 & 3 & 0 & 0 & 0
\end{bmatrix}
$$

This design has $\lambda_0 = 40/9$, $\lambda_1 = 14/9$.

### CONCLUSIONS

A general class of incomplete block designs that are appropriate for use in the comparison of test treatment-control problem have been described. These designs are referred as BTIB designs. BTIB designs are balanced with respect to test treatments-control treatment comparisons. The concept of BTIB designs is extended to compare a set of test treatment to a set of control treatments. The designs for comparing two disjoint sets of treatment are called as Balanced Two Disjoint Sets of Treatments (BTDT). A class of row-column designs, which are balanced for test treatments vs. control comparisons referred to as the Balanced Treatment vs. Control Row-Column (BTCRC) can be used for comparing several treatments with a control when heterogeneity is to be eliminated in two directions.

### REFERENCES

Bechhofer, R.E., and Nocturne, D.J.M., (1970). Optimal allocation of observations when comparing several treatments with a control (II): 2-Sided comparisons. Technometrics, 14: 423-436.

Dunnett, C.W., (1955). A multiple comparison procedure for comparing several treatments with a control. J. Am. Statist Assoc., 50, 1096-1121.

Hedayat, A. S., and Majumdar, D., (1988). Model robust optimal design for comparing test treatment with a control. J. Statist. Plann. Inference, 18: 25-33.

Hedayat, A. S., and Seiden, E., (1974). On the theory and application of sum composition of Latin squares and orthogonal Latin Squares. Pacific J. Math, 54: 85-112.

Hedayat, A. S., Jacoux, M. and Majumdar, D., (1988). Optimal designs for comparing test treatments with a control. Statist. Sc., 3: 462-491.

Jaggi Seema (1992). Study on optimality of one-way heterogeneity designs for comparing two disjoint sets of treatments. Ph.D Thesis, IARI, New Delhi.

Jaggi Seema, Gupta, V.K. and Parsad, R., (1996). A-efficient block designs for comparing two disjoint sets of treatments. Comm.Statist.:Theory and Method, 25(5): 967-983.

Jaggi Seema, Parsad, R. and Gupta, V.K., (1997). General efficiency balanced block designs with unequal block sizes for comparing two sets of treatments. Jour. Ind. Soc. Ag. Statistics, 50(1): 37-46.

Kiefer, J., (1975). Construction and optimality of generalized Youden designs. In: J. N. Srivastava ed., A survey of Statistical Design and Linear Models, north-Holland, Amsterdam, 333-353.

Majumdar, D., (1988). Optimal repeated measurements designs for comparing test treatments with a control. Comm. Statist. Theory Methods, 17: 3687-3703.
Majumdar, D. and Tamhane, A.C., (1996). Row-column designs for comparing treatments with control. J. Statist. Plann. and Inference, 49: 387-400.
Majumdar, D., (1986) Optimal designs for comparisons between two sets of treatments. J. Statist. Plann. Inference, 14: 359-372.
Pandey, A., (1993). Study of Optimality of Block Designs under mixed effects models. Ph.D Thesis, IARI, New Delhi.
Parsad., R., (1991). Studies on optimality of block designs with unequal block sizes for making test treatment control comparison under a heteroscedastic model. Ph.D. Thesis, IARI, New Delhi.
Ramana, D.V.V. (1995). Optimality aspects of designs for making test treatments-control treatments comparisons under fixed and mixed effects models. Ph.D Thesis, IARI, New Delhi.