AN ALGORITHM TO GENERATE CLASSICAL SOLUTIONS
OF STRING EFFECTIVE ACTION

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ABSTRACT

It is shown explicitly, that a number of solutions for the background field equations of the string effective action in space-time dimension $D$ can be generated from any known lower dimensional solution, when background fields have only time dependence. An application of the result to the two dimensional charged black hole is presented. The case of background with more general coordinate dependence is also discussed.
Recently, there has been tremendous interest in the search for nontrivial classical solutions of the background field equations of string theories. Solutions corresponding to black holes, strings, branes and monopoles have been reported in the literature. An interesting aspect of these solutions is that, many of them can be shown to correspond to exact conformal field theory. For example, black hole in two space-time dimensions correspond to the coset $SL(2,R)/U(1)$ in conformal field theory and has recently been studied extensively by formulating it as a gauged Wess-Zumino-Witten (WZW) model.

Two dimensional charged black hole as well as three dimensional charged black string have also been formulated as gauged WZW models.

The low energy string effective action was studied recently from a different angle by Meissner and Veneziano. They showed that the effective action in space-time dimension $D = d+1$ is invariant under an $O(d,d)$ group of symmetry transformations when graviton, dilaton and second rank antisymmetric tensor $B$ are the only nonvanishing background fields and depend only on time coordinate. Low energy string effective action, as well as the corresponding field equations have been written down in manifestly $O(d,d)$ invariant form. Therefore, given a solution of the background field equations, new solutions can be obtained by $O(d,d)$ transformations. Application of the $O(d,d)$ transformation in the context of two dimensional charged black hole was done by two of the present authors in Ref. 5. In Ref. 13, the idea of $O(d,d)$ invariance was generalized to the case when background fields have more than just time dependence. It was shown that, the string effective action has an $O(\tilde{d},\tilde{d})$ invariance if the background is independent of $\tilde{d}$ number of coordinates. In Ref. 14 interesting applications of $O(d,d)$ transformations were presented.
and in Ref. 15 the idea of \( O(d,d) \) invariance was generalized to heterotic strings.

In this paper, we study the field equations and effective action of Refs. 11 and 13. First for only time dependent case of Ref. 11, we show that, a number of solutions of the graviton, dilaton and second rank antisymmetric tensor background field equations, of the low energy string effective action, can be generated in space-time dimension \( 'D' \) from a given classical solution in any lower dimension. We apply these results to the two dimensional charged black holes \(^5,^6\). We then discuss the case when background fields have more than just time dependence.\(^{13}\) The key to generate these solutions is a proper parametrization of the \( D \)-dimensional metric and antisymmetric tensor in terms of the lower dimensional ones.

Our starting point is the genus zero low energy effective action for closed (super) strings in the limit when string tension \( \alpha' \rightarrow 0 \). Restricting to the graviton, dilaton and antisymmetric tensor field, this action in \( 'D' \) space-time dimensions \( D = d + 1 \) is written as \(^{11}\),

\[
S = \int d^Dx \sqrt{-\text{det}G} \ e^{-\phi} [V - R^{(D)} - G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho}] \tag{1}
\]

where \( \phi \) is the dilaton field, \( G_{\mu\nu} \) is the \( D \)-dimensional metric and \( H_{\mu\nu\rho} \) is the field strength for the antisymmetric tensor field \( B_{\mu\nu} \):

\[
H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \text{cyclic}. \tag{2}
\]

\( V \) in Eq.(1) contains cosmological constant as well as the dilaton potential. For many examples of interest \( V \) is just a constant \(^{5,12}\).

As in Refs. 11 and 12, we now investigate the solutions of the field equations of the action (1) when \( G \) and \( B \) are functions of time only. In this case, gauge symmetries of the action, Eq.(1), allow \( G \) and \( B \) to be always brought in the form \(^{11}\):
where $G(t)$ and $B(t)$ are $d \times d$ matrices. The action (1) for this case can be rewritten as

$$S = \int dt \sqrt{\det G} e^{-\Phi} \left[ V - 2 \partial_0^2 (\ln \sqrt{\det G}) - (\partial_0 (\ln \sqrt{\det G}))^2 + \frac{1}{4} Tr(\partial_0 G)(\partial_0 G^{-1}) ight. \\ \left. + (\partial_0 \phi)^2 + \frac{1}{4} Tr(\partial_0 B)(\partial_0 B) \right].$$

(4)

By redefining the dilaton field:

$$\Phi = \phi - \ln \sqrt{\det G},$$

(5)

one can write Eq. (4) as

$$S = \int dt \ e^{-\Phi} [V + \dot{\Phi}^2 - \frac{1}{4} Tr(\dot{G}^{-1} \dot{G})^2 + \frac{1}{4} Tr(\dot{G}^{-1} \dot{B})^2].$$

(6)

Equations of motion for these fields

$$\left( \dot{\Phi} \right)^2 - \frac{1}{4} Tr[(\dot{G}^{-1} \dot{G})(\dot{G}^{-1} \dot{G})] + \frac{1}{4} Tr[(\dot{G}^{-1} \dot{B})(\dot{G}^{-1} \dot{B})] - V = 0,$$

(7)

$$\left( \dot{\Phi} \right)^2 - 2 \ddot{\Phi} + \frac{1}{4} Tr[(\dot{G}^{-1} \dot{G})(\dot{G}^{-1} \dot{G})] - \frac{1}{4} Tr[(\dot{G}^{-1} \dot{B})(\dot{G}^{-1} \dot{B})] - V + \frac{\partial V}{\partial \Phi} = 0$$

(8)

$$-\dot{\Phi} \dot{G} + \ddot{G} - \dot{G} \dot{G}^{-1} \dot{G} - \dot{B} \dot{G}^{-1} \dot{B} = 0$$

(9)

and

$$-\dot{\Phi} \dot{B} + \ddot{B} - \dot{B} \dot{G}^{-1} \dot{G} - \dot{G} \dot{G}^{-1} \dot{B} = 0,$$

(10)

where Eqs. (8), (9) and (10) follow from the variations with respect to the fields $\Phi$, $G_{ij}$ and $B_{ij}$ of action (6). Equation (7), which is obtained directly from the variation of the action
(1), with respect to $G_{00}$, is also called the "zero energy condition"\textsuperscript{11}. It has also been pointed out in Ref. 11, that Eqs.(7)-(10) are the complete set of equations of motion for this set of fields. In Ref. 16, these equations were solved for arbitrary $D$ when $\mathcal{G}$ is diagonal and $\mathcal{B}$ is absent. For $D = 2$, the solution of Ref. 16 is identical to the uncharged black hole solution\textsuperscript{2,3}, when roles of space and time are interchanged. In Ref.5, two of us obtained a solution of Eqs. (7)-(10) in the presence of off-diagonal terms in the metric $\mathcal{G}$ and vanishing $\mathcal{B}$, for $D = 3$. There we showed that, this solution can be interpreted as two dimensional charged black hole solution upon compactification of one of the space dimensions and by interchanging the roles of space and time as in Ref. 16.

Now we show that, a number of solutions of Eqs.(7)-(10) for $d$-dimensional metrics $\mathcal{G}$ and $\mathcal{B}$ can be obtained from a given solution of similar equations for $\hat{d}$ ($\equiv d - p$)-dimensional metrics $\hat{\mathcal{G}}$ and $\hat{\mathcal{B}}$. To obtain solutions for $\mathcal{G}$ and $\mathcal{B}$ from $\hat{\mathcal{G}}$ and $\hat{\mathcal{B}}$, we parametrize the $d \times d$ matrices $\mathcal{G}$ and $\mathcal{B}$ as,

\begin{align}
\mathcal{G} &= \begin{pmatrix} \hat{\mathcal{G}}(t) & \frac{1}{4} \hat{\mathcal{G}}(t) b^T \\
\frac{1}{2} b \hat{\mathcal{G}}(t) & \psi + \frac{1}{4} b \hat{\mathcal{G}}(t) b^T \end{pmatrix}, \quad \mathcal{B} = \begin{pmatrix} \hat{\mathcal{B}}(t) & \frac{1}{4} \hat{\mathcal{B}}(t) b^T \\
\frac{1}{2} b \hat{\mathcal{B}}(t) & \psi + \frac{1}{4} b \hat{\mathcal{B}}(t) b^T \end{pmatrix},
\end{align}

where $\hat{\mathcal{G}}(t)$ and $\hat{\mathcal{B}}(t)$ are $\hat{d} \times \hat{d}$ matrices, $\psi$ and $b$ are respectively $p \times p$ and $p \times \hat{d}$ matrices and $\psi$ is nonsingular. Constant metric $\mathcal{G}$ of the form in Eq.(11) has been used previously for string compactifications\textsuperscript{17}. For our case however, we note that, all elements of $\mathcal{G}$ and $\mathcal{B}$ in Eqs.(11) have nontrivial time dependence. We now make a simplifying assumption that $\psi$ and $b$ are constant matrices. From Eq.(11) one obtains,

\begin{align}
\mathcal{G}^{-1} &= \begin{pmatrix} \hat{\mathcal{G}}^{-1} + \frac{1}{4} b^T \psi^{-1} b & -\frac{1}{2} b^T \psi^{-1} \\
-\frac{1}{2} \psi^{-1} b & \psi^{-1} \end{pmatrix},
\end{align}

and

\begin{align}
\det \mathcal{G} &= (\det \hat{\mathcal{G}})(\det \psi).
\end{align}
Since we have assumed that $\psi$ and $b$ are constant matrices, we have from Eqs.(11) and (12):

\[
\dot{\mathcal{G}} = \begin{pmatrix} \dot{\mathcal{G}} & \frac{1}{2} \dot{\mathcal{G}} b^T \\ \frac{1}{2} b \mathcal{G} & \frac{1}{4} b \mathcal{G} b^T \end{pmatrix}, \quad \dot{\mathcal{B}} = \begin{pmatrix} \dot{\mathcal{B}} & \frac{1}{2} \dot{\mathcal{B}} b^T \\ \frac{1}{2} b \mathcal{B} & \frac{1}{4} b \mathcal{B} b^T \end{pmatrix},
\]

(14)

and

\[
\ddot{\mathcal{G}} = \begin{pmatrix} \ddot{\mathcal{G}} & \frac{1}{2} \ddot{\mathcal{G}} b^T \\ \frac{1}{2} b \ddot{\mathcal{G}} & \frac{1}{4} b \ddot{\mathcal{G}} b^T \end{pmatrix}, \quad \ddot{\mathcal{B}} = \begin{pmatrix} \ddot{\mathcal{B}} & \frac{1}{2} \ddot{\mathcal{B}} b^T \\ \frac{1}{2} b \ddot{\mathcal{B}} & \frac{1}{4} b \ddot{\mathcal{B}} b^T \end{pmatrix},
\]

(15)

and

\[
(\mathcal{G}^{-1}) = \begin{pmatrix} (\mathcal{G}^{-1}) & 0 \\ 0 & 0 \end{pmatrix}.
\]

(16)

We now use equations (11)-(16) to simplify the background field equations (7)-(10). Using (12) and (14) one can show that,

\[
Tr[(\mathcal{G}^{-1}\dot{\mathcal{G}})(\mathcal{G}^{-1}\dot{\mathcal{G}})] = Tr[(\dot{\mathcal{G}}^{-1}\dot{\mathcal{G}})(\dot{\mathcal{G}}^{-1}\dot{\mathcal{G}})]
\]

(17)

and

\[
Tr[(\mathcal{G}^{-1}\dot{\mathcal{B}})(\mathcal{G}^{-1}\dot{\mathcal{B}})] = Tr[(\dot{\mathcal{G}}^{-1}\dot{\mathcal{B}})(\dot{\mathcal{G}}^{-1}\dot{\mathcal{B}})].
\]

(18)

Therefore, Eq.(7) can be rewritten as,

\[
(\dot{\Phi})^2 - \frac{1}{4} Tr[(\dot{\mathcal{G}}^{-1}\dot{\mathcal{G}})(\dot{\mathcal{G}}^{-1}\dot{\mathcal{G}})] + \frac{1}{4} Tr[(\dot{\mathcal{G}}^{-1}\dot{\mathcal{B}})(\dot{\mathcal{G}}^{-1}\dot{\mathcal{B}})] - V = 0.
\]

(19)

Similarly, Eq.(8) can be rewritten as

\[
(\dot{\Phi})^2 - 2\dot{\Phi} + \frac{1}{4} Tr[(\dot{\mathcal{G}}^{-1}\dot{\mathcal{G}})(\dot{\mathcal{G}}^{-1}\dot{\mathcal{G}})] - \frac{1}{4} Tr[(\dot{\mathcal{G}}^{-1}\dot{\mathcal{B}})(\dot{\mathcal{G}}^{-1}\dot{\mathcal{B}})] - V + \frac{\partial V}{\partial \Phi} = 0.
\]

(20)

Also, using Eqs.(12), (14) and (15), Eq. (9) can be rewritten as a matrix equation:

\[
-\dot{\Phi} \begin{pmatrix} \dot{\mathcal{G}} & \frac{1}{2} \dot{\mathcal{G}} b^T \\ \frac{1}{2} b \dot{\mathcal{G}} & \frac{1}{4} b \dot{\mathcal{G}} b^T \end{pmatrix} + \begin{pmatrix} \dot{\mathcal{G}} & \frac{1}{2} \dot{\mathcal{G}} b^T \\ \frac{1}{2} b \dot{\mathcal{G}} & \frac{1}{4} b \dot{\mathcal{G}} b^T \end{pmatrix}
\]

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Equation (21) gives rise to four matrix-equations. However, it is recognized that all of them are same and can be written as a single matrix equation:

\[-\dot{\Phi} \dot{\hat{G}} + \ddot{\hat{G}} - \dot{\hat{G}} \hat{G}^{-1} \dot{\hat{G}} - \dot{\hat{B}} \hat{G}^{-1} \dot{\hat{B}} = 0.\] (22)

Eq.(10) can similarly be simplified to

\[-\dot{\Phi} \dot{\hat{B}} + \ddot{\hat{B}} - \dot{\hat{B}} \hat{G}^{-1} \dot{\hat{G}} - \dot{\hat{G}} \hat{G}^{-1} \dot{\hat{B}} = 0.\] (23)

We now use Eqs.(5) and (13) to write

\[\Phi = \hat{\phi} - \ln \sqrt{\det \hat{G}}\] (24)

where

\[\hat{\phi} = (\phi - \ln \sqrt{\det \psi}).\] (25)

Therefore we note that, by making a constant shift from \(\phi \rightarrow \hat{\phi}\), \(\Phi\) can also be thought of as the ”redefined” dilaton in \(\hat{d}\)-dimensions. Then we recognize that Eqs.(19), (20), (22) and (23) are in fact the background field equations in \(\hat{d}\)-dimensions. Our Eqs.(11) and (25) therefore provide a prescription for generating solution of the field equations for the \(d\)-dimensional fields \(G\) and \(B\) from the \(\hat{d}\)-dimensional ones, i.e. \(\hat{G}, \hat{B}\).

As an application of our result, we now consider the case of two dimensional charged black hole. We start with \(\hat{d} = 1\) so that \(\hat{G}, \hat{B}\) are single functions and a solution for Eqs.(19), (20), (22) and (23) is given by 16,

\[\hat{G} \equiv \hat{g} \tanh^2 t, \ \hat{B} = 0\] (26)
and
\[
\hat{\phi} \equiv -\ln (\cosh^2 t) + c_1,
\] (27)

where \( \hat{g} \) and \( c_1 \) are constants. Then, using Eq.(11), one can generate \((p+1)\)-dimensional solutions of the type,
\[
G = \begin{pmatrix}
\hat{g} \tanh^2 t & \frac{1}{2} b_i \hat{g} \tanh^2 t \\
\frac{1}{2} b_j \hat{g} \tan h^2 t & \psi_{ij} + \frac{1}{4} \hat{g} b_i b_j \tanh^2 t
\end{pmatrix}, \quad B = 0.
\] (28)

where \((i, j = 1, 2, \ldots p)\). Also from (25),
\[
\phi = -\ln (\cos h^2 t) + c_2,
\] (29)

where \( c_2 \) is a constant different from \( c_1 \). For \( b_i = 0, i > 1 \) and \( b_1 = b \) solution (28) for \( G \) implies that metric \( G \) in Eq.(3) is of the form:
\[
G = \begin{pmatrix}
-1 & 0 & 0 \\
0 & \hat{g} \tanh^2 t & \frac{1}{2} b \hat{g} \tanh^2 t \\
0 & \frac{1}{2} b \hat{g} \tan h^2 t & \psi_{11} + \frac{1}{4} b^2 \hat{g} \tanh^2 t
\end{pmatrix}.
\] (30)

Now, if third dimension is treated as a compact one, then \( G \) in Eq.(30) gives rise, in two space-time dimensions, to the metric \( \tilde{G}_{\mu\nu} \), gauge field \( \tilde{A}_\mu \) and Higgs field \( \tilde{\psi} \) as \( ^5 \)
\[
\tilde{G}_{\mu\nu} \equiv \begin{pmatrix}
-1 & 0 \\
0 & \hat{g} \tanh^2 t
\end{pmatrix},
\] (31)
\[
\tilde{A}_\mu \equiv (0, b \hat{g} \tanh^2 t)
\] (32)

and
\[
\tilde{\psi} \equiv \psi_{11} + \frac{1}{4} b^2 \hat{g} \tanh^2 t.
\] (33)

The solutions in Eqs.(31)-(33) can be interpreted as the two dimensional charged black hole solution \( ^5,^6 \) by changing the roles of space and time. In Ref. 5 it was also shown that
the solutions Eqs.(31)-(33), correspond to the coset of the type \([SL(2,R) \times U(1)]/U(1)\) in the conformal field theory, which is formulated as the gauged WZW model \(^6\). In general, for \(b_i \neq 0\), one has \(\tilde{G}_{\mu\nu}\) as in Eq.(31) together with \([U(1)]^p\) gauged fields,

\[
\tilde{A}_\mu \equiv (0, b_i \tilde{g} \tanh^2 t), \tag{34}
\]

and Higgs fields,

\[
\tilde{\psi}_{ij} = (\psi_{ij} + \frac{1}{4} \tilde{g} b_i b_j \tanh^2 t). \tag{35}
\]

By comparing with the gauged WZW model, it can be shown that the solution of the type given in Eqs.(29), (31), (34) and (35) corresponds to the coset \([SL(2,R) \times [U(1)]^p]/U(1)\) in conformal field theory.

We now discuss the generalization of these results to the case when background has more general coordinate dependence, but is independent of at least one of them. Following the second paper of Ref. 13, we split the space- time coordinates \(x^\mu\) in Eq.(1) into two sets \(y^m\) and \(\tilde{y}^\alpha\) \((1 \leq m \leq \tilde{d}, 1 \leq \alpha \leq D - \tilde{d})\) and consider backgrounds independent of \(y^m\). As in Ref. 13, we restrict to the backgrounds: \(G_{m\alpha} = 0, B_{m\alpha} = 0\). Then background fields \(G_{\mu\nu}, B_{\mu\nu}\) can be written in the form,

\[
G_{\mu\nu} = \begin{pmatrix} \tilde{G}_{\alpha\beta}(\tilde{y}) & 0 \\ 0 & G_{mn}(\tilde{y}) \end{pmatrix}, \quad B_{\mu\nu} = \begin{pmatrix} \tilde{B}_{\alpha\beta}(\tilde{y}) & 0 \\ 0 & B_{mn}(\tilde{y}) \end{pmatrix}. \tag{36}
\]

For this case, after an integration by parts, action (1) can be written as \(^{13}\):

\[
S = \int d^\tilde{d} \tilde{y} \int d^{(D-\tilde{d})} \tilde{y} \sqrt{-\text{det}(\tilde{G}(\tilde{y}))} e^{-\Phi} [V - \tilde{G}^{\alpha\beta} \partial_\alpha \Phi \partial_\beta \Phi - \frac{1}{4} \tilde{G}^{\alpha\beta} \text{Tr}(\partial_\alpha \tilde{G} \partial_\beta \tilde{G}^{-1})] \\
- \frac{1}{4} \tilde{G}^{\alpha\beta} \text{Tr}(\tilde{G}^{-1} \partial_\alpha B \tilde{G}^{-1} \partial_\beta B) - R^{(D-\tilde{d})}(\tilde{G}) - \frac{1}{12} \tilde{H}_{\alpha\beta\gamma} \tilde{H}^{\alpha\beta\gamma}] \tag{37}
\]
where \( \Phi = \phi - \ln \sqrt{\det \mathcal{G}(\tilde{y})} \). We now do a similar substitution as in Eq.(11), \( \text{i.e.} \)

\[
\mathcal{G} = \left( \begin{array}{c}
\frac{1}{2} \tilde{G}(\tilde{y}) \\
\psi + \frac{1}{4} b \tilde{G}(\tilde{y}) b^T
\end{array} \right), \quad \mathcal{B} = \left( \begin{array}{c}
\frac{1}{2} \tilde{B}(\tilde{y}) \\
\frac{1}{4} b \tilde{B}(\tilde{y}) b^T
\end{array} \right),
\]

where \( \tilde{G}(\tilde{y}) \) and \( \tilde{B}(\tilde{y}) \) are \( \hat{d} \times \hat{d} \) matrices and \( \psi, b \) are as defined earlier but \( \hat{d} \) is now such that, \( \hat{d} = \tilde{d} - p \). We now use Eq.(38) to rewrite the action (37) as,

\[
S = \int d^{(D-\hat{d})} \tilde{y} \int d^{(D-\hat{d})} \tilde{y} \sqrt{-\det \tilde{\mathcal{G}(\tilde{y})}} e^{-\Phi} [V - \tilde{G}^{\alpha\beta} \tilde{\partial}_\alpha \Phi \tilde{\partial}_\beta \Phi - \frac{1}{4} \tilde{G}^{\alpha\beta} \text{Tr} \left( \tilde{\partial}_\alpha \tilde{\partial}_\beta \tilde{G} \right) - \frac{1}{4} \tilde{G}^{\alpha\beta} \text{Tr} \left( \tilde{B} \right) - \frac{1}{12} \tilde{H}^{\alpha\beta\gamma} \tilde{H}^{\alpha\beta\gamma}]
\]

We see that the action (37) has retained its form after the substitution (38) and therefore Eqs. of motion will also preserve their form. Hence, using Eq.(38), new higher dimensional solutions \( \mathcal{G} \) and \( \mathcal{B} \) can be generated from the known lower dimensional ones, \( \text{i.e.} \) \( \tilde{G} \) and \( \tilde{B} \) as before. However, unlike the case of only time dependent background, this has been demonstrated for metric and antisymmetric tensor of the block diagonal form only, \( \text{i.e.} \) \( G_{\alpha\mu} = B_{\alpha\mu} = 0 \)

To conclude, we have shown that a number of solutions (classified by constant parameters \( b_{ia} \) and \( \psi_{ij} \)), of the background field equations can be obtained in higher dimension from a given solution in any lower dimension. It will be interesting to generalize the results to heterotic string \(^{15}\).
References:

[1] G. W. Gibbons and K. Maeda, Nucl. Phys. B298, 741 (1988); C. G. Callan, R. C. Myers and M. J. Perry, Nucl. Phys. B311, 673 (1988/89); H. J. Vega and N. Sanchez, Nucl. Phys. B309, 552 (1988); 557 (1988).

[2] E. Witten, Phys. Rev. D44, 314 (1991); R. Dijgraaf, E. Verlinde and H. Verlinde, Institute for Advanced Study preprint IASSNS-HEP-91/22 (1991).

[3] G. Mandal, A. M. Sengupta and S. Wadia, Mod. Phys. Lett. A6, 1685 (1991); S. P. de Alwis and J. Lykken, Phys. Lett. B269, 264 (1991); A. Tseytlin, John Hopkins preprint JHU-TIPAC-91009; A. Giveon, LBL preprint LBL-30671.

[4] D. Garfinkle, G. T. Horowitz and A. Strominger, Phys. Rev. D43, 3140 (1991).

[5] S. P. Khastgir and A. Kumar, Mod. Phys. Lett. A6, 3365 (1991).

[6] N. Ishibashi, M. Li and A. R. Steif, Phys. Rev. Lett. 67, 3336 (1991).

[7] G. T. Horowitz and A. Strominger, Nucl. Phys. B360, 197 (1991); J. H. Horne, G. T. Horowitz and A. R. Steif, Univ. of California, Santa Barbara preprint UCSBTH-91-53, October 1991.

[8] J. H. Horne and G. T. Horowitz, Univ. of California, Santa Barbara preprint UCSBTH-91-39, July 1991.

[9] S. B. Giddings and A. Strominger, Phys. Rev. Lett. 67, 2930 (1991).

[10] J. A. Harvey and J. Liu, Phys. Lett. B268, 40 (1991).

[11] K. A. Meissner and G. Veneziano, Phys. Lett. B267, 33 (1991).

[12] K. A. Meissner and G. Veneziano, Mod. Phys. Lett. A6, 3397 (1991).

[13] A. Sen, Phys. Lett. B271, 295 (1991); A. Sen, Tata preprint, TIFR/TH/91-37, August
1991.

[14] M. Gasperini, J. Maharana and G. Veneziano, CERN Theory preprint CERN-TH-6214/91 (1991); M. Gasperini and G. Veneziano, CERN Theory preprint CERN-TH-6321/91 (1991).

[15] S. F. Hassan and A. Sen, Tata preprint, TIFR/TH/91-40, September 1991.

[16] G. Veneziano, Phys. Lett. B265, 287 (1991).

[17] A. Giveon, E. Rabinovici and G. Veneziano, Nucl. Phys. B322, 167 (1989).