Effects of pseudoscalar condensation on the cooling of neutron stars

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Abstract

In this work we consider the effect that the appearance of pseudoscalar condensates in a neutron star can have on its cooling rate. We make no particular assumption on the origin and characteristics of these possible condensates and only assume that in regions where the pseudoscalar density varies the propagation of photons is governed by modified Maxwell-Chern-Simons electrodynamics. We find that this gives non-trivial reflection coefficients between regions of different pseudoscalar density and may affect very substantially the star cooling rate. While quantitative results do depend on precise details that can only be answered once a proper equation of state is determined, the general trend is quite universal and serious consideration should be given to this possibility.

1 Introduction

Interest on possible Lorentz and CPT invariance violation \cite{1} was aroused after the seminal paper \cite{2} where modified electrodynamics with an additional Chern-Simons (CS) parity-odd term including a constant CS four-vector was considered. From the analysis of the radiation from distant radio galaxies it was shown that there is no signal of such a violation at Hubble scales. Nevertheless, there are some areas where modified Maxwell-Chern-Simons (MCS) electrodynamics (also known as Carroll-Field-Jackiw electrodynamics \cite{2}) may be relevant.

There is evidence of some sort of parity breaking taking place in peripheral heavy ion collisions \cite{3} accounted for by the so-called Chiral Magnetic Effect (CME) \cite{4}. Spontaneous parity violation might also occur for sufficiently large values of the baryon density \cite{5}. Recently several experiments in heavy ion collisions have also indicated an abnormal yield of lepton pairs \cite{6,7} and it was conjectured that the effect may be another manifestation of local parity breaking in colliding nuclei \cite{8,9}. Furthermore, even though there is no direct evidence of the existence of axions (see however \cite{10}) it has been speculated that parity odd regions may occur after condensation of nearly massless axion-like fields \cite{11} – \cite{14} at space scales comparable with star or galaxies sizes, in particular, near (or inside of) very dense stars \cite{15}.
In particular, when dealing with neutron stars one could contemplate phenomena such as pion condensation \cite{16} or the occurrence of a disoriented chiral condensate \cite{17}. We will generically refer to this phenomenon as pseudoscalar condensation as the consequences will be independent of the precise mechanism triggering the appearance of local parity breaking.

A lot of our knowledge about neutron stars comes from the study of pulsars, i.e. rotating neutron stars with a magnetic field $B \leq O(10^{13})$ G \cite{18}. Nowadays there are various mechanisms proposed to explain physics of these object, including the flux-conserving compression of the magnetic field of a pre-supernova star \cite{18} or the self-exciting dynamo scenario \cite{19}. The ferromagnetism of dense matter itself is a possible source of the permanent magnetic field \cite{20}. However models of pulsars should not only explain the surface magnetic field but also should give an explanation for the very long decay time of these astrophysical objects, which can be $10^9$ y or even longer.

Models should in particular describe the cooling rate of these objects. In \cite{21} a predicted decay time was obtained from a model of color superconductivity and in a recent paper \cite{22} it was shown how the crust cooling may depend on the presence and properties of nuclear matter. In \cite{23}, \cite{24} one may review the differences between the theoretical predictions and experimental data. All in all, it seems fair to conclude that we still do not have a good understanding of the slow cooling of neutron stars.

In the near future a number of new astrophysical instruments will provide various information on neutron stars (see the recent review \cite{25}). In particular, the Neutron Star Interior Composition Explorer (NICER) \cite{26} to be launched in 2016 is expected to discover tens of thousands of neutron stars and help understanding their core phenomena.

The purpose of the present study is to propose a mechanism whereby pseudoscalar condensation may play a crucial role in the cooling mechanisms of neutron stars. Interestingly enough the mechanism relies on basic properties of quantum mechanics and electrodynamics alone.

It is quite natural to expect that if pseudoscalar condensation occurs several domains will be generated inside the neutron star. In the regions separating phases with different values for the pseudoscalar condensate the pseudoscalar density will vary rapidly between the two extreme values (one of them being possibly zero, i.e. no parity breaking at all). A region with constant pseudoscalar density has a vanishing CS four-vector and its electromagnetic properties are described by usual electrodynamics. On the other hand in the intermediate region the pseudoscalar density is space dependent and thus gives a non-zero CS vector. In this region the relevant lagrangian to describe its electromagnetic properties is the MCS lagrangian.

We will show that the heat transfer mechanism inside the star is severely modified by the appearance of these layers where MCS applies. Namely there is a strong boundary effect for outgoing photons which are responsible for energy transfer from the core to the outer parts of the star. It turns out that such an effect reduce an energy flow from the core and may significantly slow down the cooling of the core and neutron star entirely.

## 2 Theoretical Model

In ordinary stars it takes a long time for radiation to reach surface. For instance, in the Sun the photon diffusion time scale is $\sim 10^5$ years \cite{27}. Neutron star are much
denser \((\rho \sim \rho_0)\), so the mean free path of photons is very small (a full picture about the structure of neutron stars one may find in [28]). Taking this into account we will assume that every volume element in the star re-emits photons in all directions.

In [28] one may find the structure of the inner layers of neutron star as commonly accepted. We shall assume that the baryon density in some regions inside the star is high enough for pseudoscalar condensation to take place. However it is unlikely that the pseudoscalar condensate takes the same value everywhere and one would naturally expect domains and transition regions. These domains are characterized by approximately constant values of the condensate \(a_i\). Of course \(a = 0\), i.e. no parity breaking at all is also a possibility and at the very least a transition region should exist with the more superficial parts where \(a = 0\).

Figure 1: A sketch of possible domains with pseudoscalar condensates inside a neutron star

These various condensates cannot just change discontinuously on the boundary, there should be some shell or layer, were the value for the \(a\) condensate goes from the values \(a_+\) to \(a_-\) or to zero eventually. We shall assume a linear interpolation for \(a\) between the two limiting values. We will also assume that the thickness of this shell is much larger than the wavelength of the photons propagating inside the star.

Within these boundary layers the behavior of the photons can be described by Carroll-Field-Jackiw model, or MCS electrodynamics, with the Lagrange density

\[
\mathcal{L} = -\frac{1}{4} F^{\alpha\beta} (x) F_{\alpha\beta} (x) - \frac{1}{4} F^{\mu\nu} (x) \tilde{F}_{\mu\nu} (x) a (x)
\]

where \(A_\mu (x)\) and \(a(x)\) stand for the vector and effective background pseudoscalar fields respectively, \(\tilde{F}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}\) is a dual field strength. The classical background is different for different regions. In the area outside the boundary layers (regions 1 and 3) \(a(x) = \) constant. In this case the second term in Lagrangian does not give any corrections to field equation and one recovers standard electrodynamics. The only nontrivial area to describe is region 2. We assume that in this layer \(a\) changes linearly from \(a_-\) to \(a_+\) across the gap. Then inside region 2 the relevant pseudoscalar background can be locally described by

\[
a(x) = \zeta_\lambda x^\lambda \left[ \theta (\zeta \cdot (x - x_-)) - \theta (\zeta \cdot (x - x_+)) \right]
\]

in which a fixed constant four vector \(\zeta^\mu\) with mass dimension is used as an argument. \(\zeta_\lambda\) is actually proportional to the local gradient of the pseudoscalar condensate. Since we assume that the wave function of the photons is considerably shorter that the thickness of the layer, we will eventually take the width of the latter to infinity to simplify our calculations. In this case Lorentz invariance would be violated in the Minkowski half space.
We build polarization vectors using the projector on the plane transverse vectors \( k_\mu, \zeta_\nu \) \cite{29}.

\[
S_{\lambda}^\nu \equiv \delta_\lambda^\nu D + k^\nu k_\lambda \zeta^2 + \zeta^\nu \zeta_\lambda k^2 - \zeta \cdot k (\zeta_\lambda k^\nu + \zeta^\nu k_\lambda); \quad D = (\zeta \cdot k)^2 - \zeta^2 k^2 = \frac{1}{2} S_{\nu}^\nu. \tag{3}
\]

With a help of the latter equality one can find that

\[
S^\mu_\lambda \varepsilon^\lambda_{\nu\rho\beta} \zeta^\alpha k^{\beta} = D \varepsilon^\mu_{\nu\rho\beta} \zeta^\alpha k^{\beta}. \tag{4}
\]

Then to our purpose it is convenient to introduce two orthonormal, one-dimensional, Hermitian projectors

\[
\pi_{\pm}^\mu_\nu \equiv \frac{S^\mu_\nu}{2D} \pm i \frac{1}{2} \varepsilon^\mu_{\nu\rho\beta} \zeta_\alpha k_\beta D^{-\frac{1}{2}} = (\pi_{\pm}^{\mu_\nu})^* = (\pi_{\pm}^{\mu_\nu}) (D > 0). \tag{5}
\]

A couple of chiral polarization vectors for the MCS field can be constructed out of constant tetrades \( \varepsilon_\nu \)

\[
\varepsilon_{\pm}^\mu (k) \equiv \pi_{\pm}^\mu \varepsilon_\nu. \tag{6}
\]

Their properties were thoroughly described in \cite{29}.

In order to obtain the normal modes of propagation of the MCS field, let us introduce the kinetic \( 4 \times 4 \) Hermitian matrix \( K \) with elements

\[
K_{\lambda\nu} \equiv g_{\lambda\nu} (k^2 - m^2) + i\varepsilon_{\lambda\rho\beta} \zeta^\alpha k^\beta; \quad K_{\lambda\nu} = K_{\nu\lambda}^*. \tag{7}
\]

We obtain the general solution for \( \zeta \cdot x < 0 \) from the relations (5), (6)

\[
K_{\mu\nu} \varepsilon_{\pm}^\nu (k) = \left[ \delta^\mu_\nu (k^2 - m^2) + \sqrt{D} \left( \pi^\mu_\nu - \pi^\mu_{-\nu} \right) \right] \varepsilon_{\pm}^\nu (k) = \left( k^2 - m^2 \pm \sqrt{D} \right) \varepsilon_{\pm}^\nu (k). \tag{8}
\]

If we consider small area, which does not feel the curvature of the shell\(^1\), we may choose for simplicity the first coordinate along the local radius of curvature of the bubble and assume the first boundary \( x_+ \) to be located at \( x_1 = 0 \). In this particular case

\[
a(x) = \zeta x_1 \theta(x_1), \tag{9}
\]

where we have now assumed that the thickness of the boundary is much larger than the characteristic photon wave length and accordingly we have taken \( x_+ \to \infty \). This approximation makes the calculation simpler as it allows to decouple the effect of the two successive interfaces.

Then the wave equation reads

\[
\Box A^\nu + m^2 A^\nu + \zeta \varepsilon^{1\nu\sigma\rho} \theta(x_1) \partial_\sigma A_\rho = 0 \tag{10}
\]

and the corresponding dispersion relations are

\[
\begin{align*}
k_{CS}^{1L} &= k_1^0 = \sqrt{\omega^2 - m^2 - k_{\perp}^2}; \\
k_{CS}^{1+} &= \sqrt{\omega^2 - m^2 - k_{\perp}^2 - \zeta x \sqrt{\omega^2 - k_{\perp}^2}}; \\
k_{CS}^{1-} &= \sqrt{\omega^2 - m^2 - k_{\perp}^2 + \zeta x \sqrt{\omega^2 - k_{\perp}^2}}.
\end{align*} \tag{11}
\]

\(^1\)Again, this is a good approximation for photons whose wavelength is much shorter than the characteristic sizes involved.
where the index ‘0’ labels the medium with the usual dispersion relation. This expression describes three different physical polarizations in the MCS medium, i.e. for \( x_1 > 0 \). Of course the standard Maxwellian behavior is recovered by setting \( \zeta = 0 \) in all expressions, which corresponds to the region \( x_1 < 0 \), where there is no Lorentz symmetry breaking.

We can now describe the propagation of photons inside every of three regions indicated in Fig. 1. Regions 1 and 3 correspond to taking \( \zeta = 0 \) above while \( \zeta \neq 0 \) in the intermediate transition region 2. The question now is what happens on the boundaries of these regions. As it is manifest in the previous equations, different regions lead to different dispersion relations. As a consequence non-trivial reflection and transmission coefficients between the different regions appear. This issue was first discussed in [30] by two of the present authors.

### 3 Entrance to the boundary layer

We now consider photons created inside a given domain. We assume that the thickness of the shell (region 2 in Fig. 1) is much larger than a typical wavelength and a mean free path. Under this assumptions (10) works.

Photons fall on the boundary from a region where ordinary electrodynamics holds and attempt to penetrate in one governed by MCS electrodynamics. Matching conditions for this problem were discussed in detail in [31]. To understand which photons penetrates into the shell it is worth to recall the discussion. Solution of (10) can be found by using Fourier transformation over all components but \( x_1 \) and may be written in the form

\[
\tilde{A}_\nu = \begin{cases} 
\tilde{u}_{\nu-}(\omega, k_2, k_3)e^{ik_1^0x_1} + \tilde{u}_{\nu+}(\omega, k_2, k_3)e^{-ik_1^0x_1}, & x_1 < 0; \\
\sum_A \left[ \tilde{v}_{\nu A \to}(\omega, k_2, k_3)e^{ik_1^C A x_1} + \tilde{v}_{\nu A \to}(\omega, k_2, k_3)e^{-ik_1^C A x_1} \right], & x_1 > 0.
\end{cases}
\]  

(12)

The first index of \( \tilde{v} \) denotes the corresponding component of \( A_\nu, \nu = 0, 2, 3 \), the second index \( A \) stands for the polarizations \( L, +, - \) and the arrows \( \to, \leftarrow \) point out the direction of particle propagation. It is necessary to distinguish among the various polarizations because they obey different dispersion relations. Note that we contemplate the possibility of photons developing an effective mass \( m \) even if this is not explicitly written in the lagrangian.

Furthermore, equation (10) in a half-space \( x_1 > 0 \) enforce \( v \) to satisfy the following conditions

\[
\begin{align*}
\tilde{v}_{2\to} &= \frac{k_2k_3+i\omega\sqrt{\omega^2-k_2^2}}{\omega^2-k_3^2}\tilde{v}_{3\to}; \\
\tilde{v}_{2\to} &= \frac{k_2k_3-i\omega\sqrt{\omega^2-k_2^2}}{\omega^2-k_3^2}\tilde{v}_{3\to}; \\
\tilde{v}_{0\to} &= -\frac{\omega k_3-i\omega\sqrt{\omega^2-k_2^2}}{2(\omega^2-k_3^2)}\tilde{v}_{3\to}; \\
\tilde{v}_{0\to} &= -\frac{\omega k_3+i\omega\sqrt{\omega^2-k_2^2}}{2(\omega^2-k_3^2)}\tilde{v}_{3\to}; \\
\tilde{v}_{2\leftarrow} &= \frac{k_2^2}{k_3^2}\tilde{v}_{3\leftarrow}; \\
\tilde{v}_{2\leftarrow} &= \frac{-\omega k_3}{k_3^2}\tilde{v}_{3\leftarrow}.
\end{align*}
\]  

(13)

Thus we have the solutions in both half-spaces, and we should now match them on the boundary. If we believe that all contribution to the vector field \( A \) are continuous, the integration over \( x_1 \) from \( -\varepsilon \) to \( \varepsilon \) will give us the next relations [30]

\[
\begin{align*}
\tilde{u}_{0\to}^{(L)} &= \frac{\omega^2}{\omega^2-k_2^2}\tilde{u}_{0\to} + \frac{\omega k_3}{\omega^2-k_3^2}\tilde{u}_{3\to} + \frac{\omega k_3}{\omega^2-k_3^2}\tilde{u}_{2\to}; \\
\tilde{u}_{0\to}^{(\pm)} &= -\frac{k_2^2}{2(\omega^2-k_3^2)}\tilde{u}_{0\to} - \frac{\omega k_3\pm ik_3\sqrt{\omega^2-k_2^2}}{2(\omega^2-k_3^2)}\tilde{u}_{3\to} - \frac{\omega k_3\pm ik_3\sqrt{\omega^2-k_2^2}}{2(\omega^2-k_3^2)}\tilde{u}_{2\to}.
\end{align*}
\]  

(14)
\begin{align}
\begin{cases}
\tilde{u}^{(L)}_{2\rightarrow} &= -\frac{k_2^2}{\omega^2-k^2_\perp} \tilde{u}_{2\rightarrow} - \frac{\omega k_2}{\omega^2-k^2_\perp} \tilde{u}_{0\rightarrow} - \frac{k_2 k_3}{\omega^2-k^2_\perp} \tilde{u}_{3\rightarrow}; \\
\tilde{u}^{(\pm)}_{2\rightarrow} &= \frac{\omega^2-k^2_\perp}{2(\omega^2-k^2_\perp)} \tilde{u}_{2\rightarrow} + \frac{\omega k_2 \mp i k_3 \sqrt{\omega^2-k^2_\perp}}{2(\omega^2-k^2_\perp)} \tilde{u}_{0\rightarrow} + \frac{k_2 k_3 \mp i \omega \sqrt{\omega^2-k^2_\perp}}{2(\omega^2-k^2_\perp)} \tilde{u}_{3\rightarrow}. 
\end{cases} 
\end{align}
(15)

\begin{align}
\begin{cases}
\tilde{u}^{(L)}_{3\rightarrow} &= -\frac{k_3^2}{\omega^2-k^2_\perp} \tilde{u}_{3\rightarrow} - \frac{\omega k_3}{\omega^2-k^2_\perp} \tilde{u}_{0\rightarrow} - \frac{k_2 k_3}{\omega^2-k^2_\perp} \tilde{u}_{2\rightarrow}; \\
\tilde{u}^{(\pm)}_{3\rightarrow} &= \frac{\omega^2-k^2_\perp}{2(\omega^2-k^2_\perp)} \tilde{u}_{3\rightarrow} + \frac{\omega k_3 \mp i k_2 \sqrt{\omega^2-k^2_\perp}}{2(\omega^2-k^2_\perp)} \tilde{u}_{0\rightarrow} + \frac{k_2 k_3 \mp i \omega \sqrt{\omega^2-k^2_\perp}}{2(\omega^2-k^2_\perp)} \tilde{u}_{2\rightarrow}. 
\end{cases} 
\end{align}
(16)

where \( \tilde{u}_{\nu\rightarrow} = \sum_{A=L,\pm} \tilde{u}^{(A)}_{\nu\rightarrow} \). Each component of the incoming amplitudes has its own transmission coefficient \([30]\),

\[ \tilde{v}_{\nuA\rightarrow} = \frac{2k^0_1}{k^0_1 + k^{CS}_1 A} \tilde{u}^{(A)}_{\nu\rightarrow}. \]
(17)

Then we take the initial amplitude, and using eqs. (14-17) we find the amplitudes of the transmitted waves with polarizations \( L, +, - \)

\[ \mathcal{T}^\pm = \frac{1}{1 + \sqrt{1 \mp \zeta}}, \]
(18)

Longitudinally polarized photons (should they be present) are not affected by the change in the medium.

Figure 2: The geometry of photon propagation. \( \vec{n} \) is a normal vector. In the left region (region 1) the pseudoscalar condensate takes a constant value. Region 2 (right) is assumed to be described by MCS electrodynamics describing a varying pseudoscalar condensate as befits a transition region.

The direction of outgoing photons corresponds to the angle \( \beta \) (Fig. 2). After the propagation through the boundary, their direction will be changed (angle \( \alpha \)) accordingly to their polarization of the falling particles. We decompose \( \vec{k} = k_n \vec{n} + k_\perp \vec{T} \) (Fig. 2). From \([31]\) we know that \( k_\perp \) remains the same after crossing the boundary. This means that the trajectory of the particle and normal vector lie in one plane. However, \( k_n \) changes. We introduce the new variable \( k^{CS}_n \) to describe the normal component of \( \vec{k} \) after crossing the boundary. For \( x_1 < 0 \) one has

\[ \cos \beta = \frac{k^0_1}{\omega}, \]
(19)

We are interested in finding the overall flux of outgoing photons from region 1 to region 2 in Fig. 2. To do this we first consider a small volume near the boundary.
We assume that it radiates uniformly in all directions. For us is important the flux of energy which propagates outwards so we are interested only in upper half-sphere in Fig. 3. We assume that this small volume radiates with a certain frequency $N_\Omega^\omega$ per solid angle. In order to find the total luminosity we should integrate over the solid angle, frequency and surface of the layer. The last integration would be the same for the case with or without pseudoscalar condensation

$$L \propto \int_0^{\infty} d\omega N_\Omega^\omega \int_{\max(\frac{\pi}{2}, \arccos(\frac{\zeta}{\omega}))}^{\frac{\pi}{2}} d\beta \Sigma^+ (\beta, \omega, \zeta) + \int_0^{\infty} d\omega N_\Omega^\omega \int_0^{\frac{\pi}{2}} d\beta \Sigma^- (\beta, \omega, \zeta)$$

(20)

Here one can see that integration over angles in the first term begins from the value $\cos \beta = \zeta/\omega$. This value comes from kinematical condition of the positive polarization. It is easy to see from (18), that for $T +$ in denominator we have a negative value under the root if $\zeta > k_1$. Physically it means, that for falling photons with $k_1 < \zeta$ it is kinematically forbidden to convert into positive polarization photons in the medium with a linearly varying pseudoscalar field. For negative polarization there is no restriction, in (11) $k_1$ is positive for any values of $\zeta$ and as a result, we do not see any special limits of integral for negative polarization.

We assume that the medium has a temperature $T$ and radiates as a black body $N_\Omega^\omega \propto \omega^3/(e^{\omega T}/(e^{\omega T} - 1))$.

We will compare this value with the luminosity of the same volume of neutron star without any parity breaking, i.e. without any boundary effects. Let us call this last value $L_0$

$$L_0 \propto 2 \int_0^{\infty} d\omega N_\Omega^\omega \int_0^{\frac{\pi}{2}} d\beta$$

(21)

where the factor two stands just for two usual photon polarizations. Finally, we can plot a graph which demonstrate the effect of the intermediate shell on the luminosity. From Fig. 4 one can see that the effect of changing the dispersion relation across the boundary between region 1 with $a = \text{constant}$ and region 2 where $a$ depends linearly in $x_1$ is very noticeable. At large values of $\zeta$ (compared with temperature) most photons are reflected from the boundary and accordingly the energy flux of the domain decreases dramatically.
Figure 4: Relative difference between the outgoing energy flow for the two cases (with and without pseudoscalar condensate)

4 Escaping from the boundary layer

After escaping the first region photons appear, if they are not reflected, in the intermediate shell where MCS electrodynamics is at work. To leave this medium and gain access to another domain where \( a = \) constant (possibly zero) photons have to pass through one more boundary. This corresponds to the boundary between regions 2 and 3 in Fig. 1. We use the same technique as in the previous section. Figs. 2 and 3 still apply but reversing the areas where ordinary and MCS electrodynamics apply. In this case

\[
\frac{k_{n}^{CS}}{k_{\perp}} = \cot(\alpha); \quad \frac{k_{n}}{k_{\perp}} = \cot(\beta). \quad (22)
\]

Furthermore we know that for spatial CS vector there are two transversal polarizations in the MCS medium with the dispersion relations

\[
k_{n}^{CS} = \sqrt{\omega^2 - k_{\perp}^2 \mp \zeta \sqrt{\omega^2 - k_{\perp}^2}} \quad (23)
\]

or, since we consider photons and \( k_{n} = \sqrt{\omega^2 - k_{\perp}^2} \),

\[
k_{n}^{CS} = \sqrt{k_{n}^2 \mp \zeta k_{n}}; \quad (24)
\]

and

\[
\frac{k_{n}}{k_{\perp}} = \cot(\alpha); \quad \frac{\sqrt{k_{n}^2 \mp \zeta k_{n}}}{k_{\perp}} = \cot(\beta). \quad (25)
\]

Using the results obtained in [31], one may find the transmission coefficient of outgoing particles for every polarization

\[
\mathcal{T}^{A} = \frac{2k_{1A}^{CS}}{k_{1A}^{CS} + k_{1}^{0}}. \quad (26)
\]

We are interested only in transversal polarizations (we deal with photons, however, the longitudinal one does not feel the boundary anyway), so in our terms we write,

\[
\mathcal{T}_{\mp} = \frac{2k_{n}^{CS}}{k_{n}^{CS} + k_{n}} = \frac{2 \cot(\alpha)}{\cot \alpha + \cot(\beta)} \quad (27)
\]
For our purposes it is necessary to express $\Sigma^\pm$ as a function of $\beta$ that means expressing $\alpha$ in terms of $\beta$. For constant $a(x)$, $\omega = |\vec{k}|$ and one can use $k_n = \omega \cos \alpha$. So, (22) gives

$$\cot \alpha = \cot \beta \frac{\omega \cos \alpha}{\sqrt{\omega^2 \cos^2 \alpha + \zeta \omega \cos \alpha}}.$$  \hspace{1cm} (28)

Where as usual the $\mp$ stands for different polarizations.

Solving this equation one can find the expression for $\cot \beta$ for different polarization and the value of $\Sigma^\pm$,

$$\Sigma^\pm(\beta, \zeta, \omega) = \frac{2 \cot \beta}{\cot \beta + \frac{\pm \zeta + \sqrt{\zeta^2 + 4 \omega^2 \cot^2 \beta (1 + \cot^2 \beta)}}{\sqrt{4 \omega^2 (1 + \cot^2 \beta) - 2 \zeta^2 + 2 \zeta \sqrt{\zeta^2 + 4 \omega^2 \cot^2 \beta (1 + \cot^2 \beta)}}}.$$ \hspace{1cm} (29)

Using this formula we may find for any angle $\beta$ the fraction of incoming photons succeeding in crossing the boundary at $x_+$ to the second region with constant $a$ (that could be zero, as previously stated). Like in the previous section we consider a small volume which radiates at certain frequency $N_{\Omega}^\omega$ in a unit of solid angle and write total luminosity

$$L \propto \int_0^\infty d\omega N_{\Omega}^\omega \int_0^{\frac{\pi}{2}} d\beta \Sigma^+(\beta, \omega, \zeta) + \int_0^\infty d\omega N_{\Omega}^\omega \int_0^{\frac{\pi}{2}} d\beta \Sigma^-(\beta, \omega, \zeta) \hspace{1cm} (30)$$

It is worth commenting on the integration region of both terms. Looking at the expression (22) one may see that for negative polarization there are no any restriction on the kinematics of photon. This fact means we should integrate over all energies of outgoing photons. In case of positive polarization $\omega$ cannot be less than $\zeta$.

Like in previous section, to show the qualitative effect of pion condensate we assume that the whole medium has a temperature $T$ and radiates as a black body. One can see that for $\zeta > T$ there is a clear effect on the flux of outgoing energy

$$\text{Figure 5: Relative difference between outgoing energy for two cases (with and without pseudoscalar condensate)}$$

which should slow down the cooling of the domain. Combined with the decreasing of energy flux of the other boundary of the shell, we obtain a very strong effect which can slow down the cooling of some regions inside the star. If we neglect thermal capacity of the shell\(^2\) then we may plot a graph (Fig. 6) showing the combined

\(^2\)We assume that the thickness of the shell is much smaller linear sizes of a typical domain where $a = \text{constant}$.  

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effect of two boundaries of the domain. One may see that the effect is substantial: the total luminosity decreases by 10 times. Of course, we are speaking only about small regions inside the star. The effect on the total rate of cooling should not be so dramatic. In fact its precise magnitude does depend on many parameters that we do not and are of astrophysical nature such as the size, distribution and number of domains, magnitude of gradients in the intermediate shells. However the existence of parity breaking areas in the inner layers of star should slow down the cooling for sure.

5 Photon decay

Now let us discuss another phenomena, which may give contribution to the flux of outgoing particles, namely, the possibility of photon decay in a volume where $a \neq \text{constant}$. For neutron stars the importance of this phenomenon is probably small but it is interesting on its own nevertheless.

It was shown in [32] that a photon of positive polarization may decay in presence of a gradient of the pseudoscalar field into an $e^+e^-$ pair. This process will suppress the number of outgoing photons with positive chirality and possibly increase the number of outgoing electrons, positrons and antineutrino due to the process

$$e^+ + n \rightarrow p^+ + \bar{\nu}_e.$$  \hspace{1cm} (31)

We will present here some calculations to understand what may be the quantitative effect of the photon decaying. The total decay width for high-energy photons with positive polarization in a linearly varying pseudoscalar background is [32]

$$\Gamma_+ \simeq \frac{\alpha \zeta}{3}.$$  \hspace{1cm} (32)

In order to evaluate an effect of such decaying we use the same model, as in previous section. We assume a layer at some radius of neutron star, where the CS vector is pointed along the radius. We also assume that the total flux of positive polarized photons outgoing from the layer is $N_0(\omega)$. After photons with positive polarization propagate inside the region governed by MCS electrodynamics their number should decrease

$$N(t, w) = N_0 e^{-\frac{\Gamma_+ t}{\gamma}},$$  \hspace{1cm} (33)
where \( \gamma = \frac{1}{1-v^2} \) stays for the Lorenz factor of the particle. Firstly, we will find this \( \gamma \) factor.

\[
v = \frac{|k_+|}{\omega} = \frac{\sqrt{\omega^2 - \zeta \omega}}{\omega}; \quad (34)
\]

\[
\gamma = \frac{1}{\sqrt{1 - v^2}} = \frac{\sqrt{\zeta}}{\omega}. \quad (35)
\]

And we get,

\[
N(t, \omega) = N_0 e^{-\frac{t \sqrt{\zeta}}{\omega}}. \quad (36)
\]

It is important to remind that there is a threshold for the described decay

\[
\omega \sim \frac{m_e^2}{\zeta}. \quad (37)
\]

We represent the effect for photon energies \( \omega \sim GeV \). In this case for decay \( \zeta \) should be \( \gtrsim \) keV. We show in the figure the dependence of number of "surviving" photons on the propagation time.

![Figure 7: Ratio between final number of particles, propagated through parity breaking area and the initial number.](image)

In Fig. 7 one can easily see that for \( t > 10^{-12}s \) almost all photons with positive chirality decays. This time (10^{-12}s) corresponds to a distance scale \( \sim 100 \mu m \), which is a quite small number. So we have to conclude that if inside the star there is a strong enough gradient of pseudoscalar background, photons with positive polarization will decay in \( e^+e^- \) giving an increase of lepton pairs going out from the star and suppressing number of photons with positive chirality.

### 6 Conclusions

In this work we have investigated how the appearance of a pseudoscalar condensate may change the cooling properties of neutron stars.

The relevance of the present study hinges on a number of hypothesis. First of all we have to assume that parity may be spontaneously broken due to the high density. Secondly we have to adopt that several domains of these characteristics
form in the central part of the star. At the very least there should be one domain surrounded by an external crust where parity is not broken. We also have to assume that the characteristic scale of these domains and also the intermediate transition regions are much larger than typical wave length of photons trying to escape from the star. This last assumption seems natural.

The appearance of a pseudoscalar condensate in nuclear matter at high baryon densities has not yet been observed but it is predicted theoretically in a solid way [9]. Then if the above hypothesis hold, the mechanism of cooling is seriously affected by the presence of a CS vector induced by a varying pseudoscalar condensate. There is little or none model dependence in the predictions of this phenomenon. The transition regions are described by Maxwell-Chern-Simons electrodynamics and their consequences can be worked out independently of the microscopic details of pion/axion condensation. However in order to get a numerical estimate of the modified cooling rate because the present mechanism would require a knowledge of the distribution of the different domains, at least in average.

Of course if parity is broken inside a neutron star other consequences should follow since this would undoubtedly modify the equation of state. There could also be birefringence [31] at the boundary layer and photon instability [32]. All taken together could help to detect parity breaking in dense stars.

Summarizing one can hope to get soon relevant information on the core processes in dense neutron stars from the NICER and other astrophysical observations in order to unravel footprints of parity breaking due to pseudoscalar condensation.

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