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Dynamics of Middle East Respiratory Syndrome Coronavirus (MERS-CoV) involving fractional derivative with Mittag-Leffler kernel

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Abstract

Since 2012, the Middle East has seen a steady rise in the Middle East Respiratory Syndrome Coronavirus (MERS-CoV). A fractional derivative of the non-singular Mittag-Leffler type is used in this research to conduct a mathematical analysis of the dynamics of MERS-CoV infection transmission. The dynamics of such a disease with an additional degree of freedom and non-singular behavior are discovered through the use of the aforementioned fractional operator, and this is one of the important components of our prepared paper. Using the concept of fixed point theory, the existence and uniqueness of solutions are demonstrated. The stability analysis is also tested with the help of the Ulam–Hyers approach, respectively. The numerical solution has been conducted by using the fractional Adams–Bashforth scheme. In the numerical simulation, all classes are demonstrated through the graphical presentation regarding the changing values of fractional-order at time t. The results at various fractional-order laying between (0,1] are drawn with the help of Matlab. We also provide a comparison of the proposed approach with that of the Caputo operator. The outcomes that were achieved illustrate that the considered scheme is highly methodical and very efficient compared to the Caputo fractional operator.

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1. Introduction

Medical researchers in different countries have discovered different types of coronavirus cases. MERS-CoV is one of these viruses, which was discovered in Saudi Arabia in 2012 [1–3], mostly in camels from Middle Eastern countries. This virus is transmitted as an infection and enters through very close interaction, which is tested in the human population [4,5]. About 150 deaths and six hundred infected cases have been reported by MERS-CoV since 2012. The death percentage is about 29% that is occurred in Middle Eastern countries like Saudi Arabia, UAE, and Iraq [6], the death rate is higher when compared to the COVID-19. This infection is a zoonotic type spread by cross-transmission between camels, bats, and other animals and humans.

It is found that the MERS-CoV transmission is mainly due to riding camels or other animals. The infection spread rate is very small from camels to humans; nearly twenty-six percent of infected people are compared to the human population,
which is about seventy-four percent. To control the spread of this disease, a broad spectrum of mathematical models are used by different scholars to investigate it and also give some predictions for the present as well as the future. The easiest way is the mathematical modeling of such dynamics that provides mathematical expressions in the form of differential equations. Such mathematical equations are studied theoretically and numerically, which gives more knowledge about the past and predicts future expected results. Assire et al. [7], first formulated the largest human–human transmission of the MERS-CoV, whereas the indirect camel–human transmission was investigated by Zumla et al. [8]. They also found that taking some unprocessed camel’s milk in the deserts of the Middle East and also dairy products is a cause of infection.

In this article we have considered a modified biological model of [3], this system consist of six classes: susceptible $S(t)$, exposed $E(t)$, infectious and symptomatic $I(t)$, infectious and asymptotic $A(t)$, and hospitalized $H(t)$ and recovery class $R(t)$.

\[
\begin{align*}
\frac{dS}{dt} &= bN - \frac{\beta SI}{N} - \frac{\beta dHS}{N} - \pi_0 S, \\
\frac{dE}{dt} &= \frac{\beta SI}{N} + \frac{\beta HdS}{N} - (\alpha + \pi_0)E, \\
\frac{dI}{dt} &= \alpha \gamma E - (\lambda_1 + \lambda_1)I - (\pi_0 + \pi_1)I, \\
\frac{dA}{dt} &= \alpha(1 - \gamma)E - (\pi_0 + \pi_2)A, \\
\frac{dH}{dt} &= \lambda_0 I - \lambda_1 H - \pi_0 H, \\
\frac{dR}{dt} &= \lambda_1 I + \lambda_1 H - \pi_0 R,
\end{align*}
\]

(1)

dependent on the initial conditions

\[
\{S_0, E_0, I_0, A_0, H_0, R_0\} \geq 0.
\]

(2)

where the description of parameters in the model (1) is: $\beta$ shows transmission of human–human /time, $d$ shows approximate transmissibility $\pi$, rate of transmission from $E$ to $I$ is represented by $\alpha$. Further, $\gamma$ is proportion of the progression from $E \rightarrow I$, similarly $(1 - \gamma)$ represents progression rate to $A$, $\lambda_0$ represents average rate of transmission from $I$ to $H$ and $\lambda_1$ is recovery rate irrespective of being hospitalized, $\lambda_1$ is rate of recovery of a hospitalized patient.

Over the last few decades, fractional calculus (FC) has attracted much interest among researchers, and it has been used in various fields of science and engineering. The dynamics of infectious disease is the focus of this paper, and the well-known result of such dynamics is the commonly used integer order for differential and integral equations [9–11]. To understand the information with better accuracy and precision, fractional differential equations (FDEs) have been extensively used over the last few decades in bioengineering, social sciences, and other applied sciences [12–14]. It is difficult for the non-linear FDEs to obtain their general solutions, so researchers introduced various methods to approximate solutions for nonlinear systems with their analytical and semi-analytical versions [15–17] whereas for a qualitative investigation of fractional operators, interested readers are referred to [18,19] and their cited references. In 1980 Adomian introduced the best decomposition technique to tackle non-linear equations analytically. Further, this method gradually became a tool for obtaining semi-analytical or approximate solutions to various systems arising in applied science. The methods widely used to solve mathematical models are the homotopy perturbation method, LADM, and others, for details, see [20–22]. These methods are used to control linear and non-linear FDEs as well. To date, several other techniques for analyzing FDEs or systems of equations have been proposed, including the residual power technique, the double Laplace technique, and other approaches [23–25].

The concept of the fractional-order derivative was introduced by Riemann–Liouville, and Caputo later modified it. The primary fractional operators are based on the power-law kernel and must satisfy certain classical requirements, such as index and classical mechanical laws, and singularity. More complex operators, such as power-law operators, cannot be handled. Because singularity cannot mathematically describe objects, it was difficult for researchers to describe several naturally occurring phenomena with singularity. Therefore, in this situation, two important developments in FC have been driven and converted the singular into non-singular, namely, Caputo–Fabrizio (CF) [26] and Atangana–Baleanu (AB) [27] in the fractional derivative. These derivatives have non-singular type kernels and have no power law distribution. Pandey et al. used CF operator to study the transmission of coronavirus in India [28], Alderremy et al. used the Legendre spectral method and fuzzy AB operator to investigate a mathematical model of the Corona Virus [29]. A nonlinear mathematical model for COVID-19 has been studied using CF and AB fuzzy operators [30]. Another SEIR compartmental model of Coronavirus transmission has been analyzed by Panwar et al. using CF and AB operators [31]. Some other applications of the nonsingular fractional operators are listed in [32–35]. Specifically, the AB fractional operators have been used in various dynamical systems [36,37].

In light of the preceding literature, we investigate the fractional MERS-CoV model (1) in the sense of the $\text{ABC}$ operator, which has very valuable. The present work consists of theoretical, analytical, and also numerical results of studying the dynamics of MERS-CoV and how to get free from it in the community. The novelty and the key goal of this article are...
to find the dynamical behavior of such a disease on an extra degree of choice lying between 0 and 1 and non-singular behavior through ABC fractional operator. Our suggested model in ABC operator sense with order $0 < h \leq 1$ is as follows:

$$\begin{align*}
ABC D^h_{\text{IS}} &= hN - \frac{\beta IS}{N} - \frac{\beta dHS}{N} - \pi_0 S, \\
ABC D^h_{\text{IS}} &= \frac{\beta IS}{N} + \frac{\beta dHS}{N} - (\alpha + \pi_0) E, \\
ABC D^h_{\text{I}} &= \alpha \gamma E - (\lambda_0 + \lambda_1) I - (\pi_0 + \pi_1) I, \\
ABC D^h_{\text{A}} &= \alpha (1 - \gamma) E - (\pi_0 + \pi_2) A, \\
ABC D^h_{\text{H}} &= \lambda_0 I - \lambda_1 H - \pi_0 H, \\
ABC D^h_{\text{R}} &= \lambda_1 I + \lambda_2 R - \pi_0 R,
\end{align*}$$

with initial values

$$S(0) = S_0 > 0, \quad E(0) = E_0 > 0, \quad I(0) = I_0 > 0, \quad A(0) = A_0 > 0, \quad H(0) = H_0 > 0, \quad R(0) = R_0 > 0.$$ 

This article is arranged as follows. Section 2, is related to some basic definitions from FC. In Section 3, we prove the existence and uniqueness of the solution to the proposed model. The UH stability is also checked for the system in the same section. In Section 4, we construct an algorithm for the considered system and find an approximate solution to the model. For numerical simulation, we have used Matlab software to plot the obtained solution. In Section 7, we conclude our work.

2. Preliminaries

Here, we include some important and useful results related to FC and non-linear analysis [27,35].

**Definition 2.1.** The ABC derivative with order $h \in (0, 1]$ of a function $\gamma(t) \in H^1[0, T]$ is

$$ABC D^h_{\text{I}}(\gamma(t)) = \frac{\mathbb{N}(h)}{1 - h} \int_0^t E_h \left[ -\frac{h}{1 - h} \left( t - \varphi \right)^h \right] \frac{d\gamma(\varphi)}{d\varphi} d\varphi,$$

where $\mathbb{N}(h)$ denotes a normalization function with $\mathbb{N}(0) = \mathbb{N}(1) = 1$ and the ML function $E_h$ is given as

$$E_h(y) = \sum_{k=0}^{\infty} \frac{y^k}{\Gamma(hk + 1)}.$$

**Definition 2.2.** Suppose the function $\gamma \in L^1(0, T)$ and the LHS of fractional integral with order $h \in (0, 1]$ in ABC sense is

$$ABC I^h_{\text{I}}(\gamma(t)) = \frac{1 - h}{\mathbb{N}(h)} \gamma(t) + \frac{h}{\mathbb{N}(h)} \frac{1}{\Gamma(h)} \int_0^t (t - \varphi)^{h-1} \gamma(\varphi) d\varphi, \quad t > 0.$$ 

**Lemma 2.2.1** ([27]). The solution for problem:

$$ABC D^h_{\text{I}}(\gamma(t)) = \Psi(t),$$

$$\gamma(0) = \gamma_0,$$

is given as

$$\gamma(t) = \gamma_0 + \frac{1 - h}{\mathbb{N}(h)} \Psi(t) + \frac{h}{\mathbb{N}(h)} \frac{1}{\Gamma(h)} \int_0^t (t - \varphi)^{h-1} \Psi(\varphi) d\varphi.$$

3. Basic reproductive number

The threshold amount that decides if an epidemic appears or if the infection dies out is called the basic reproduction number $R_0$. It depicts the expected average number of new infections spread directly and indirectly by a single infectious agent when introduced into a fully susceptible population. We use the method of Driessche and Watmough [39,40] to find the basic reproduction number for the proposed model.
Fig. 1. The plots show sensitivity analysis of the basic reproductive number $R_0$.

4. Sensitivity analysis

This section deals with the sensitivity analysis for some parameters used in the considered model (1). From this analysis, we understand the parameters that have a great effect on the basic reproductive number. We used the developed scheme given by [41]. Sensitivity index of basic reproductive number $R_0$, is given by $\Delta_{h}^{R_0} = \frac{\partial R_0}{\partial h} R_0$ where $h$ is parameter. The graphical analysis of the sensitivity of $R_0$ is given in Fig. 1. In the surface plots, we observe that the basic reproduction number is very sensitive to some parameters.
5. Theoretical results of MERS-CoV model in fractional perspective

5.1. Existence and uniqueness results via ABC fractional operator

Here, we present the existence, uniqueness of the solution and also the stability of the proposed system (3) with the use of the theory of fixed points. Therefore, we consider the model as

\[
\begin{align*}
\text{ABC} D_t^\alpha S(t) &= \varphi_1(t, S) = bN - \beta \frac{SI}{N} - \beta d \frac{SH}{N} - \pi_0 S, \\
\text{ABC} D_t^\alpha E(t) &= \varphi_2(t, E) = \beta \frac{IN}{N} + \beta d \frac{EH}{N} - (\alpha + \pi_0) E, \\
\text{ABC} D_t^\alpha I(t) &= \varphi_3(t, I) = \alpha \gamma E - (\lambda + \lambda_1) I - (\pi_0 + \pi_1) A, \\
\text{ABC} D_t^\alpha A(t) &= \varphi_4(t, A) = \alpha (1 - \gamma) E - (\pi_0 + \pi_2) A, \\
\text{ABC} D_t^\alpha H(t) &= \varphi_5(t, H) = \lambda_2 I - \lambda_3 I - \pi_0 H, \\
\text{ABC} D_t^\alpha R(t) &= \varphi_6(t, R) = \lambda_1 I + \lambda_2 H - \pi_0 R.
\end{align*}
\]

(9)

We can write the model (3) in following form

\[
\text{ABC} D_t^\alpha \Psi(t) = \mathcal{U}(t, \Psi(t)),
\]

\[
\Psi(0) = \Psi_0,
\]

(10)

where

\[
\Psi := (S, E, I, A, H, R)^T,
\]

\[
\Psi_0 := (S_0, E_0, I_0, A_0, H_0, R_0)^T,
\]

\[
\mathcal{U}(t, \Psi(t)) := \mathcal{U}(S, E, I, A, H, R, t)^T, \quad i = 1, 2, 3...6,
\]

(11)

here, \((\cdot)^T\) is transpose of vector. Applying Lemma 2.2.1 to the system (10) is

\[
\Psi(t) = \Psi_0 + \frac{1 - \gamma}{N(t)} \mathcal{U}(t, \Psi(t)) + \frac{h}{N(t) \Gamma(\alpha)} \int_0^t (t - \varphi)^{\alpha-1} \mathcal{U}(\varphi, \Psi(\varphi)) d\varphi.
\]

(12)
Further, consider the Banach space $B = C([0, T], \mathbb{R})$, for function $\psi \in B$, whose norm is denoted as $\|\psi\| = \sup_{t \in [0, T]} |\psi(t)|$. In particular, $\|\psi\| = \sup_{t \in [0, T]} (|\psi| + |\psi| + |\psi| + |\psi| + |\psi| + |\psi|)$.

Now, we study existence result for model (3) by utilizing the “Schauder’s FPT”.

**Theorem 5.1.** Suppose a continuous function $\bar{u} \in B$ and $\exists$ constant $\kappa > 0$, $\exists |\bar{u}(t, \psi(t))| \leq \kappa(1 + |\psi|)$, $\forall$ $t \in [0, T]$ and $\psi \in B$. Then is at least one solution provided

$$\Delta_1 = \left(\frac{(1 - 3\kappa)\Gamma(h)\kappa + kT^h}{N(h)^\Gamma(h)}\right) < 1.$$  

Moreover, the solution is continuous and unique for every $t \in [0, T]$.

**Proof.** It is obvious that solution to considered system (3) is solution of the integral Eq. (12). Let operator $\mathcal{S} : B \to B$ is

$$\mathcal{S}(t) = t \psi + \frac{1}{N(h)}(\bar{u}(t, \psi(t)) + \frac{h}{N(h)^\Gamma(h)} \int_0^t (t - \varphi)^{h-1}\bar{u}(\varphi, \psi(\varphi))d\varphi.$$  

In order to prove that the operator is bounded and nonnegative it is sufficient to verify that $(\mathcal{S}) \subseteq \mathcal{R}_0$, for all $t \in [0, T]$. Let the convex ball which closed and bounded as well, is $\mathcal{R}_0 = \psi \in B : |\psi| \leq \varphi, \varphi > 0$ with $\varphi \geq \frac{\Delta_1}{1-\Delta_1},$ where

$$\Delta_2 = |\psi| + \frac{1}{N(h)}\kappa + \frac{T^h}{N(h)^\Gamma(h)}\kappa.$$  

Then, we have

$$|\psi(t)| \leq |\psi| + \frac{1}{N(h)}|\bar{u}(t, \psi(t))| + \frac{h}{N(h)^\Gamma(h)} \int_0^t (t - \varphi)^{h-1}|\bar{u}(\varphi, \psi(\varphi))|d\varphi,$$

$$|\psi(t)| \leq |\psi| + \frac{1}{N(h)}\kappa(1 + |\psi|) + \frac{h}{N(h)^\Gamma(h)} \int_0^t (t - \varphi)^{h-1}\kappa(1 + |\psi|)d\varphi.$$  

And $\psi \in \mathcal{R}_0$, we obtain

$$|\psi(t)| \leq |\psi| + \frac{1}{N(h)}\kappa + \frac{T^h}{N(h)^\Gamma(h)}\kappa + \left[1 - \frac{1}{N(h)}\kappa + \frac{T^h}{N(h)^\Gamma(h)}\kappa\right]|\varphi|,$$

$$\Delta_2 + \Delta_1|\varphi| \leq |\varphi|.$$

Since $(\mathcal{S}_0) \subseteq \mathcal{R}_0$, then $(\mathcal{S})$ is uniformly bounded.

Now we present continuity of $\mathcal{S}$. For this consider $\{\psi_n\}$ is sequence of solution, such that $\mathcal{S}_n \to \mathcal{S}_n$ as $n \to \infty$, then for each $t \in [0, T]$, we obtain

$$|\mathcal{S}(\psi_n) - \mathcal{S}(\psi)| \leq \frac{1}{N(h)}\kappa|\bar{u}(\psi_n(t)) - \psi(t, \psi(t))| + \frac{h}{N(h)^\Gamma(h)} \int_0^t (t - \varphi)^{h-1}|\bar{u}(\varphi, \psi(\varphi)) - \bar{u}(\varphi, \psi(\varphi))|d\varphi,$$

$\leq \frac{1}{N(h)}\kappa||\bar{u}(\varphi, \psi_n(\varphi)) - \bar{u}(\varphi, \psi(\varphi))|| + \frac{T^h}{N(h)^\Gamma(h)}||\bar{u}(\varphi, \psi_n(\varphi)) - \bar{u}(\varphi, \psi(\varphi))||.$

Thus from the continuity of function $\bar{u}$, we have

$$|\mathcal{S}(\psi_n) - \mathcal{S}(\psi)| \to 0 \quad \text{as} \quad n \to \infty.$$

It shows that, $\mathcal{S}$ is a continuous function in $\mathcal{R}_0$. Further, we present that $(\mathcal{S})$ is relatively compact.

Next, we show $\mathcal{S}$ to be an equi-continuous on $\mathcal{R}_0$. For this consider, $\psi \in \mathcal{R}_0$ and $t_1, t_2 \in [0, T]$ with $t_1 < t_2$. Then, we have

$$\|\mathcal{S}(\psi) - \mathcal{S}(\psi(t_1))\| \leq \frac{1}{N(h)}\kappa|\bar{u}(t_2, \psi(t_2)) - \bar{u}(t_1, \psi(t_1))| + \frac{h}{N(h)^\Gamma(h)} \int_0^{t_2} (t_2 - \varphi)^{h-1} - \int_0^{t_1} (t_1 - \varphi)^{h-1}|\bar{u}(\varphi, \psi(\varphi))|d\varphi,$$

$$\leq \frac{1}{N(h)}\kappa|\bar{u}(t_2, \psi(t_2)) - \bar{u}(t_1, \psi(t_1))| + \frac{h}{N(h)^\Gamma(h)} \int_0^{t_2} \kappa(1 + |\psi|)(t_2^h - t_1^h).$$
We see that R.H.S \( \| \mathcal{J} \varphi(t_2) - \mathcal{J} \varphi(t_1) \| \rightarrow 0 \) as \( t_2 \rightarrow t_1 \). Therefore from the theorem of “Arzela-Ascoli”, the operator \( \mathcal{J} \mathcal{B}_o \) is compact, so \( \mathcal{J} \) is completely continuous. We obtained that the suggested model (3) has at least one solution. □

Now, we show uniqueness of the considered system (3), the system has a unique solution under the assumption

\[
0 \leq \left[ 1 - \frac{1}{N(h)} \varrho - \frac{h T h}{N(h) \Gamma(h)} \varrho \right].
\] (17)

Let there is another solution such as \( S_1, \mathbb{E}_1, I_1, A_1, \mathbb{H}_1, \mathbb{R}_1 \), it holds that

\[
S(t) - S_1(t) = \frac{1 - h}{N(h)} (\varphi_1(t, S) - \varphi_1(t, S_1)) + \frac{h}{N(h) \Gamma(h)} \int_0^t (\varphi_2(\varphi, S) - \varphi_2(\varphi, S_1)) d\varphi,
\] (18)

using norm to Eq. (18), we have,

\[
\| S - S_1 \| = \left\| \frac{1}{N(h)} \left( \varphi_1(t, S) - \varphi_1(t, S_1) \right) + \frac{h}{N(h) \Gamma(h)} \int_0^t (\varphi_2(\varphi, S) - \varphi_2(\varphi, S_1)) d\varphi \right\|,
\]

\[
\leq \frac{1}{N(h)} \varrho \| S - S_1 \| + \frac{h}{N(h) \Gamma(h)} \| S - S_1 \|.
\] (19)

Thus, gives that

\[
\| S - S_1 \| \left[ 1 - \frac{1}{N(h)} \varrho - \frac{h T h}{N(h) \Gamma(h)} \varrho \right] \leq 0.
\] (20)

This implies \( S = S_1 \), if the inequality (17) holds. Similarly, for the remaining classes \( \mathbb{E} = \mathbb{E}_1, I = I_1, A = A_1, \mathbb{H} = \mathbb{H}_1 \) and \( \mathbb{R} = \mathbb{R}_1 \). Thus, we say that solution of the model under study is unique.

5.2. Stability result

In this part, we discuss the Hyers-Ulam (HU) stability of solution of the proposed system. We give the definition of HU-stability in the context of our problem.

**Definition 5.2.** The system (3) is HU-stable if for every \( \epsilon > 0 \), and for the inequality:

\[
\left| \mathcal{D}^h_t \mathcal{J} \varphi(t) - \bar{\varphi}(t, \mathcal{J} \varphi(t)) \right| < \epsilon, \quad \forall t \in [0, T],
\]

there exists a unique solution \( \bar{\varphi} \in \mathcal{B} \) and \( \kappa > 0 \) such that

\[
\| \varphi - \bar{\varphi} \| < \kappa \epsilon.
\]

In the theorem presented below, we discuss the results of stability for the model (3) by perturbing the initial values.

**Theorem 5.3.** Suppose a function \( \bar{\varphi} \in \varphi \) and \( 3 \) constant \( \kappa > 0 \) \( \| \bar{\varphi}(t, \mathcal{J} \varphi(t)) - \bar{\varphi}(t, \mathcal{J} \varphi(t)) \| \leq \kappa \| \varphi - \bar{\varphi} \|, \forall t \in [0, T] \) and \( \mathcal{J} \varphi \) with \( 1 > \frac{(1-h) \Gamma(h) \kappa + k T h}{N(h) \Gamma(h)} \). Suppose \( \varphi \) and \( \bar{\varphi} \) be solution for model (10) and

\[
\mathcal{D}^h_t \bar{\varphi}(t) = \bar{\varphi}(t, \bar{\varphi}(t)),
\]

\[
\bar{\varphi}(0) = \mathbb{E}_0 + \epsilon \geq 0,
\] (21)

where

\[
\left\{ \begin{array}{l}
\mathbb{E}_0 + \epsilon = (S_0 + \epsilon, E_0 + \epsilon, H_0 + \epsilon, A_0 + \epsilon, \mathbb{R}_0 + \epsilon) \\
\bar{\varphi}(t, \bar{\varphi}(t)) = \mathcal{D}^h_t \bar{\varphi}(t) = (S, E, H, A, \mathbb{R})^T, \\
\end{array} \right.
\]

and for the inequality above equation holds as

\[
\| \varphi - \bar{\varphi} \| \leq \left[ 1 - \frac{(1-h) \Gamma(h) \kappa + k T h}{N(h) \Gamma(h)} \right]^{-1} |\epsilon|.
\] (23)

**Proof.** The solution of the models (10) and (21) are equivalent to Eq. (12) and

\[
\mathcal{J} \varphi(t) = \mathbb{E}_0 + \epsilon + \frac{1 - h}{N(h)} \bar{\varphi}(t, \mathcal{J} \varphi(t)) + \frac{h}{N(h) \Gamma(h)} \int_0^t (t - \varphi)^{-1} \bar{\varphi}(\varphi, \mathcal{J} \varphi) d\varphi,
\] (24)
respectively, so for each \( t \in [0, T] \), we get

\[
|\mathbf{y}(t) - \mathbf{\tilde{y}}(t)| \leq |\epsilon| + \frac{1 - h}{N(h)} \left[ |\mathbf{u}(t, \mathbf{y}(t)) - \mathbf{u}(t, \mathbf{\tilde{y}}(t))| + \frac{h}{N(h)} \mathbb{I}^\nu(h) \right]
\times \int_0^t (t - \varphi)^{\nu - 1} |\mathbf{u}(\varphi, \mathbf{y}(\varphi)) - \mathbf{u}(\varphi, \mathbf{\tilde{y}}(\varphi))| d\varphi
\leq |\epsilon| + \frac{1 - h}{N(h)} |\mathbf{y}(t) - \mathbf{\tilde{y}}(t)| + \frac{h}{N(h)} \mathbb{I}^\nu(h)
\times \int_0^t (t - \varphi)^{\nu - 1} |\mathbf{u}(\varphi) - \mathbf{u}(\varphi)| d\varphi
\leq |\epsilon| + \frac{1 - h}{N(h)} + \frac{T^\nu}{N(h)} \mathbb{I}^\nu(h) \mathbb{K} |\mathbf{y} - \mathbf{\tilde{y}}|.
\]

Thus, we get

\[ \|\mathbf{y} - \mathbf{\tilde{y}}\| \leq |\epsilon| + \left[ \frac{(1 - h)\mathbb{I}^\nu(h) + T^\nu h}{N(h)} \mathbb{K} \right] |\mathbf{y} - \mathbf{\tilde{y}}|.
\]

Hence,

\[ \|\mathbf{y} - \mathbf{\tilde{y}}\| \leq \left[ 1 - \frac{(1 - h)\mathbb{I}^\nu(h)K + KT^\nu h}{N(h)} \right]^{-1} |\epsilon|.
\]

Thus, the theorem is proved. \( \square \)

6. Numerical scheme

This section is devoted to present the numerical approximation of the solution of the considered system (3). There are several numerical methods in the literature which have been used for numerical solution of nonlinear fractional differential equations [42–45]. Here, we use the Adams–Bashforth method to study the numerical results of the proposed model. So we express the model (3) as

\[
\begin{align*}
S(t) - S_0 &= \frac{1 - h}{N(h)} \mathbb{I}_0^\nu P_1(t, S), \\
E(t) - E_0 &= \frac{1 - h}{N(h)} \mathbb{I}_0^\nu P_2(t, E), \\
I(t) - I_0 &= \frac{1 - h}{N(h)} \mathbb{I}_0^\nu P_3(t, I), \\
A(t) - A_0 &= \frac{1 - h}{N(h)} \mathbb{I}_0^\nu P_4(t, A), \\
H(t) - H_0 &= \frac{1 - h}{N(h)} \mathbb{I}_0^\nu P_5(t, H), \\
R(t) - R_0 &= \frac{1 - h}{N(h)} \mathbb{I}_0^\nu P_6(t, R),
\end{align*}
\]

which gives

\[
\begin{align*}
S(t) - S_0 &= \frac{1 - h}{N(h)} \mathbb{I}_0^\nu P_1(t, S) + \frac{h}{N(h)} \mathbb{I}^\nu(h) \int_0^t (t - \varphi)^{\nu - 1} P_1(S(\varphi), \varphi) d\varphi, \\
E(t) - E_0 &= \frac{1 - h}{N(h)} \mathbb{I}_0^\nu P_2(t, E) + \frac{h}{N(h)} \mathbb{I}^\nu(h) \int_0^t (t - \varphi)^{\nu - 1} P_2(E(\varphi), \varphi) d\varphi, \\
I(t) - I_0 &= \frac{1 - h}{N(h)} \mathbb{I}_0^\nu P_3(t, I) + \frac{h}{N(h)} \mathbb{I}^\nu(h) \int_0^t (t - \varphi)^{\nu - 1} P_3(I(\varphi), \varphi) d\varphi, \\
A(t) - A_0 &= \frac{1 - h}{N(h)} \mathbb{I}_0^\nu P_4(t, A) + \frac{h}{N(h)} \mathbb{I}^\nu(h) \int_0^t (t - \varphi)^{\nu - 1} P_4(A(\varphi), \varphi) d\varphi, \\
H(t) - H_0 &= \frac{1 - h}{N(h)} \mathbb{I}_0^\nu P_5(t, H) + \frac{h}{N(h)} \mathbb{I}^\nu(h) \int_0^t (t - \varphi)^{\nu - 1} P_5(H(\varphi), \varphi) d\varphi, \\
R(t) - R_0 &= \frac{1 - h}{N(h)} \mathbb{I}_0^\nu P_6(t, R) + \frac{h}{N(h)} \mathbb{I}^\nu(h) \int_0^t (t - \varphi)^{\nu - 1} P_6(R(\varphi), \varphi) d\varphi.
\end{align*}
\]

(25)

(26)
Suppose $\nu = \nu_0, \nu_1, \nu_2,$ and $\nu_3$ into the system (26) gives:

$$S(t_{n+1}) - S_0 = \frac{1 - h}{N(h)} p_1(S(t_n), t_n) + \frac{h}{N(h) \Gamma(h)} \sum_{p=0}^{\nu} \int_{t_p}^{t_{p+1}} (t_{n+1} - \nu)^{h-1} p_1(S(\nu), \nu) d\nu,$$

$$E(t_{n+1}) - E_0 = \frac{1 - h}{N(h)} p_2(E(t_n), t_n) + \frac{h}{N(h) \Gamma(h)} \sum_{p=0}^{\nu} \int_{t_p}^{t_{p+1}} (t_{n+1} - \nu)^{h-1} p_2(E(\nu), \nu) d\nu,$$

$$I(t_{n+1}) - I_0 = \frac{1 - h}{N(h)} p_3(I(t_n), t_n) + \frac{h}{N(h) \Gamma(h)} \sum_{p=0}^{\nu} \int_{t_p}^{t_{p+1}} (t_{n+1} - \nu)^{h-1} p_3(I(\nu), \nu) d\nu,$$

$$A(t_{n+1}) - A_0 = \frac{1 - h}{N(h)} p_4(A(t_n), t_n) + \frac{h}{N(h) \Gamma(h)} \sum_{p=0}^{\nu} \int_{t_p}^{t_{p+1}} (t_{n+1} - \nu)^{h-1} p_4(A(\nu), \nu) d\nu,$$

$$H(t_{n+1}) - H_0 = \frac{1 - h}{N(h)} p_5(H(t_n), t_n) + \frac{h}{N(h) \Gamma(h)} \sum_{p=0}^{\nu} \int_{t_p}^{t_{p+1}} (t_{n+1} - \nu)^{h-1} p_5(H(\nu), \nu) d\nu,$$

$$R(t_{n+1}) - R_0 = \frac{1 - h}{N(h)} p_6(R(t_n), t_n) + \frac{h}{N(h) \Gamma(h)} \sum_{p=0}^{\nu} \int_{t_p}^{t_{p+1}} (t_{n+1} - \nu)^{h-1} p_6(R(\nu), \nu) d\nu,$$

On applying the two step interpolation for $p_1(S(\nu), \nu), p_2(E(\nu), \nu), p_3(I(\nu), \nu), p_4(A(\nu), \nu), p_5(H(\nu), \nu)$ and $p_6(R(\nu), \nu)$ in the integral (27) on interval $[t_p, t_{p+1}]$, we obtain

$$\begin{align*}
p_1(S(\nu), \nu) &\equiv p_1(S(t_p), t_p) (t - t_{p-1}) + \frac{p_1(S(t_{p-1}), t_{p-1})}{\Delta} (t - t_p), \\
p_2(E(\nu), \nu) &\equiv p_2(E(t_p), t_p) (t - t_{p-1}) + \frac{p_2(E(t_{p-1}), t_{p-1})}{\Delta} (t - t_p), \\
p_3(I(\nu), \nu) &\equiv p_3(I(t_p), t_p) (t - t_{p-1}) + \frac{p_3(I(t_{p-1}), t_{p-1})}{\Delta} (t - t_p), \\
p_4(A(\nu), \nu) &\equiv p_4(A(t_p), t_p) (t - t_{p-1}) + \frac{p_4(A(t_{p-1}), t_{p-1})}{\Delta} (t - t_p), \\
p_5(H(\nu), \nu) &\equiv p_5(H(t_p), t_p) (t - t_{p-1}) + \frac{p_5(H(t_{p-1}), t_{p-1})}{\Delta} (t - t_p), \\
p_6(R(\nu), \nu) &\equiv p_6(R(t_p), t_p) (t - t_{p-1}) + \frac{p_6(R(t_{p-1}), t_{p-1})}{\Delta} (t - t_p),
\end{align*}$$

which gives

$$S(t_{m+1}) = S_0 + \frac{1 - h}{N(h)} p_1(S(t_m), t_m) + \frac{h}{N(h) \Gamma(h)} \sum_{p=0}^{m} \left( \frac{p_1(S(t_p), t_p)}{\Delta} t_{p-1,h} + \frac{p_1(S(t_{p-1}), t_{p-1})}{\Delta} t_{p,h} \right),$$

$$E(t_{m+1}) = E_0 + \frac{1 - h}{N(h)} p_2(E(t_m), t_m) + \frac{h}{N(h) \Gamma(h)} \sum_{p=0}^{m} \left( \frac{p_2(E(t_p), t_p)}{\Delta} t_{p-1,h} + \frac{p_2(E(t_{p-1}), t_{p-1})}{\Delta} t_{p,h} \right),$$

$$I(t_{m+1}) = I_0 + \frac{1 - h}{N(h)} p_3(I(t_m), t_m) + \frac{h}{N(h) \Gamma(h)} \sum_{p=0}^{m} \left( \frac{p_3(I(t_p), t_p)}{\Delta} t_{p-1,h} + \frac{p_3(I(t_{p-1}), t_{p-1})}{\Delta} t_{p,h} \right),$$

$$A(t_{m+1}) = A_0 + \frac{1 - h}{N(h)} p_4(A(t_m), t_m) + \frac{h}{N(h) \Gamma(h)} \sum_{p=0}^{m} \left( \frac{p_4(A(t_p), t_p)}{\Delta} t_{p-1,h} + \frac{p_4(A(t_{p-1}), t_{p-1})}{\Delta} t_{p,h} \right),$$

$$H(t_{m+1}) = H_0 + \frac{1 - h}{N(h)} p_5(H(t_m), t_m) + \frac{h}{N(h) \Gamma(h)} \sum_{p=0}^{m} \left( \frac{p_5(H(t_p), t_p)}{\Delta} t_{p-1,h} + \frac{p_5(H(t_{p-1}), t_{p-1})}{\Delta} t_{p,h} \right),$$

$$R(t_{m+1}) = R_0 + \frac{1 - h}{N(h)} p_6(R(t_m), t_m) + \frac{h}{N(h) \Gamma(h)} \sum_{p=0}^{m} \left( \frac{p_6(R(t_p), t_p)}{\Delta} t_{p-1,h} + \frac{p_6(R(t_{p-1}), t_{p-1})}{\Delta} t_{p,h} \right),$$

where

$$t_{p-1,h} = \int_{t_p}^{t_{p+1}} (t - t_p) \delta(t_{m+1} - t)^{h-1} dt,$$
and
\[ I_{p,h} = \int_{t_p}^{t_{p+1}} (t - t_p)(t_{m+1} - t)^{h-1} dt. \]

Now, simplifying \( I_{p-1,h} \) and \( I_{p,h} \) as follows

\[ I_{p-1,h} = -\frac{1}{h} \left[ (t_{p+1} - t_{p-1})(t_{m+1} - t_p)h - (t_p - t_{p-1})(t_{m+1} - t_p)h \right] - \frac{1}{h(h-1)} \left[ (t_{m+1} - t_{p+1})^{h+1} - (t_{m+1} - t_p)^{h+1} \right], \]

and

\[ I_{p,h} = -\frac{1}{h} \left[ (t_{p+1} - t_p)(t_{m+1} - t_{p+1})h \right] - \frac{1}{h(h-1)} \left[ (t_{m+1} - t_{p+1})^{h+1} - (t_{m+1} - t_p)^{h+1} \right], \]

put \( t_p = wh \) then we easily find that

\[ I_{p-1,h} = \frac{h^{h+1}}{h(h+1)} \left[ (\sigma + 1 - p)^{h+1} - (\sigma - p)^{h}(\sigma - p + 1 + h) \right], \quad (30) \]

and

\[ I_{p,h} = \frac{h^{h+1}}{h(h+1)} \left[ (\sigma + 1 - p)^{h+1} - (\sigma - p)^{h}(\sigma - p + 1 + h) \right]. \quad (31) \]

Substituting Eqs. (30) and (31) into (29), we get

\[ S(t_{m+1}) = S_0 + \frac{(1 - h)}{N(h)} \left[ \phi_1(S(t_m), t_m) + \sum_{p=0}^{\sigma} \frac{\phi_1(S(t_{m-1}), t_{m-1})}{\Gamma(h+2)} \right] \times h^h \left[ (\sigma + 1 - p)^h(\sigma - p + 2 + h) - (\sigma - p)^h(\sigma - p + 2 + 2h) \right] \]

\[ - \phi_1(S(t_{m-1}), t_{m-1})h^h \left[ (\sigma + 1 - p)^{h+1} - (\sigma - p)^h(\sigma - p + 1 + h) \right], \quad (32) \]

\[ E(t_{m+1}) = E_0 + \frac{(1 - h)}{N(h)} \left[ \phi_2(E(t_m), t_m) + \sum_{p=0}^{\sigma} \frac{\phi_2(E(t_{m-1}), t_{m-1})}{\Gamma(h+2)} \right] \times h^h \left[ (\sigma + 1 - p)^h(\sigma - p + 2 + h) - (\sigma - p)^h(\sigma - p + 2 + 2h) \right] \]

\[ - \phi_2(E(t_{m-1}), t_{m-1})h^h \left[ (\sigma + 1 - p)^{h+1} - (\sigma - p)^h(\sigma - p + 1 + h) \right], \quad (33) \]

\[ I(t_{m+1}) = I_0 + \frac{(1 - h)}{N(h)} \left[ \phi_3(I(t_m), t_m) + \sum_{p=0}^{\sigma} \frac{\phi_3(I(t_{m-1}), t_{m-1})}{\Gamma(h+2)} \right] \times h^h \left[ (\sigma + 1 - p)^h(\sigma - p + 2 + h) - (\sigma - p)^h(\sigma - p + 2 + 2h) \right] \]

\[ - \phi_3(I(t_{m-1}), t_{m-1})h^h \left[ (\sigma + 1 - p)^{h+1} - (\sigma - p)^h(\sigma - p + 1 + h) \right], \quad (34) \]

\[ A(t_{m+1}) = A_0 + \frac{(1 - h)}{N(h)} \left[ \phi_4(A(t_m), t_m) + \sum_{p=0}^{\sigma} \frac{\phi_4(A(t_{m-1}), t_{m-1})}{\Gamma(h+2)} \right] \times h^h \left[ (\sigma + 1 - p)^h(\sigma - p + 2 + h) - (\sigma - p)^h(\sigma - p + 2 + 2h) \right] \]

\[ - \phi_4(A(t_{m-1}), t_{m-1})h^h \left[ (\sigma + 1 - p)^{h+1} - (\sigma - p)^h(\sigma - p + 1 + h) \right], \quad (35) \]
### Table 1

| Notation | Value | Notation | Value |
|----------|-------|----------|-------|
| b        | 0.09  | π₀       | 0.22  |
| d        | 0.002 | β        | 0.26  |
| α        | 0.05  | γ        | 0.065 |
| λ₀       | 0.64  | λ₁       | 0.04  |
| π₁       | 0.04  | π₂       | 0.02  |
| λ₂       | 0.12  |          |       |

### 6.1. Numerical simulation

Here we provide graphical representation using numerical simulations for verification. To obtain so, we consider some initial conditions for each compartment of our said problem (3) having fractional-order $h$ and parameters values as given in (see Table 1).

Figs. 2 and 3 depicts the behavior of all compartments under initial conditions $S₀ = 1000$, $E₀ = 800$, $I₀ = 600$, $A₀ = 500$, $H₀ = 400$ and $R₀ = 300$. The class $S(t)$ is growing due to declines in infections and an increase in recovered population, and $E(t)$ declines as its transfer to infectious and hospitalized recovered populations. Both classes converge to equilibrium at different fractional orders and different initial values. Also, in fractional orders, the dynamics in different classes are simulated, showing that they converge fast at low orders and slowly at higher orders. At any of the initial values, three of the human individual classes converge to an equilibrium point. The diseased class decreases, while the asymptomatic and hospitalized classes rise at first before stabilizing. We have also presented the dynamics of different classes for various orders $h$. The final subfigure (Fig. 1f) depicts the behavior of a recovered class in humans and its transfer.
Fig. 3. Simulation of the proposed model at different fractional $h$.

Fig. 3 shows the dynamics of the proposed system for Caputo and ABC operators. The susceptible class quickly increases in the case of ABC, which is better for humans to avoid infection. The exposed, symptomatic infected, and asymptotic infected classes decrease quickly in the case of ABC operators and attain stability rapidly. Similarly, the stability of the hospitalized and recovered class occurs faster for ABC than for the Caputo operator. Thus, the ABC operator provides better dynamics of the considered model than the Caputo operator.

7. Summary

In this paper, we investigated the MERS-CoV model using fractional-order derivative in the ABC sense. The results of fixed points have been used to study the existence and uniqueness of the solution to the proposed model. HU-stability has been derived for the considered model using the notions of functional analysis. The Adams–Bashforth technique has been employed for numerical approximation with two-step Lagrangian interpolation. Stability and convergence have been attained for the various compartments of the investigated system in the numerical simulation portion. The population decreases rapidly at low orders and gradually at higher orders over the simulation procedure. While the population in the growth process is attained at quickly at higher-order and slowly at low-order. Moreover, we have provided a comparison of the proposed approach with the Caputo operator. The simulation in Fig. 5 reveals that the ABC operator provides better dynamics due to the Mittag-Leffler kernel as compared to the Caputo operator. Therefore, the fractional differential and integral operators with Mittag-Leffler kernel predict the dynamics of different problems more accurately than those of integer order.
Fig. 4. Simulation of the proposed model with Caputo and ABC operators for $h = 0.97$.

Fig. 5. Simulation of the proposed model with Caputo and ABC operators for $h = 0.97$. 
CRediT authorship contribution statement

Tariq Mahmood: Conceptualization, Methodology, Software, Writing – original draft, Writing – review & editing. Fuad S. Al-Duais: Formal analysis, Methodology, Supervision, Writing – review & editing. Mei Sun: Conceptualization, Investigation, Supervision, Validation, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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