Field-induced axion decay $a \to e^+e^-$ in plasma

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Abstract

The axion decay $a \to e^+e^-$ is investigated in the presence of a plasma and an external magnetic field. The results demonstrate a strong catalyzing influence of medium. The axion lifetime in the magnetic field of order $10^{15}$ G and at the temperature of order $10^4$ MeV is reduced to $10^4$ s.

1 Introduction

The Peccei-Quinn (PQ) symmetry $U_{PQ}(1)$, with its accompanying axion, continues to be the most attractive solution to the strong CP problem in QCD. At present axions are of great interest not only in theoretical aspects of elementary particle physics, but in some astrophysical and cosmological applications as well. Although the original axion is excluded experimentally, modified PQ models with very light and weakly coupled "invisible" axions are still tenable. Invisible axion models are classified into two types depending on whether or not they have a direct coupling to leptons. In KSVZ model axions have only induced coupling to leptons. In DFSZ model axions couple to leptons at tree level.

At present axions are strongly constrained by astrophysical and cosmological considerations which leave a rather narrow window:

\[ 10^{-5} \text{ eV} \lesssim m_a \lesssim 10^{-2} \text{ eV} \]  \hspace{1cm} (1)

in which they can exist and provide a significant fraction or all of the cosmic dark matter. The extremely small value of the axion mass $m_a$ and, consequently, a gigantic axion lifetime in vacuum:

\[ \tau^{(0)} \sim 6.3 \cdot 10^{42} \text{ s} \left( \frac{10^{-2} \text{ eV}}{m_a} \right)^6 \left( \frac{E_a}{1 \text{ MeV}} \right) \]  \hspace{1cm} (2)

does not leave any hope to observe $a \to \gamma\gamma$ decay in laboratory conditions.
It is known that an external electromagnetic field can affect substantially decays of very light, weakly interacting particles. The examples of the strong catalyzing influence of the field on processes allowed in vacuum are radiative decays of the massive neutrino \( \nu \rightarrow \nu'\gamma \) \[12\] and the axion \( a \rightarrow \gamma\gamma \) \[13\]. One of the results obtained in \[12, 13\] is that the external field removes the main suppression caused by the smallness of the decaying particle mass. To illustrate this for \( a \rightarrow \gamma\gamma \) decay \[13\] we give the comparison of the decay probability \( W^{(F)} \) induced by the field with the vacuum one \( W^{(0)} \):

\[
\frac{W^{(F)}}{W^{(0)}} \simeq 10^{33} \left( \frac{10^{-2}\text{eV}}{m_a} \right)^4 P(\chi),
\]

where \( P(\chi) \) is some function of the field dynamical parameter.

Another special feature of the external field influence is that the field can open novel channels forbidden in vacuum. In particular, the axion decay into electron-positron pair \( a \rightarrow e^+e^- \) is opened due to a specific kinematics of charged particles in the field. We have studied this decay in two classes of invisible axion models \[14, 15\]. It was shown \[14\] that the axion lifetime in DFSZ model is reduced to \( 10^5 \text{ s} \) in the case of the decaying axion energy \( \sim 1 \text{ MeV} \) in the magnetic field of strength \( \sim 10^{15} \text{ G} \) while for KSVZ axions the lifetime becomes even of order seconds \[15\] for the above mentioned parameters.

However considering axions effects in astrophysical and cosmological environments it is important to take into account not only the magnetic field influence, as it was done in \[14, 15\], but the plasma one as well. The most physically realistic situation is that when from both components of the active medium the plasma dominates. So, the magnetic field \( B \) is relatively weak

\[
eB \ll \mu^2, T^2 \tag{3}
\]

(where \( \mu \) and \( T \) are the electron chemical potential and temperature, respectively) and a great number of the Landau levels is excited. Whereas the condition \( T^2 \gg eB \) is fulfilled, the magnetic filed is strong enough, \( eB \gg m_e^2 \), in comparison with the known Schwinger value \( B_s = m_e^2/e \simeq 4.41 \cdot 10^{10} \text{ G} \). Possible mechanisms to generate fields as strong as \( B \sim 10^{15} - 10^{17} \text{ G} \) in astrophysical objects \[16, 17\] and in the early Universe \[18\] are widely discussed.

In this paper we investigate the field-induced axion decay into electron-positron pair \( a \rightarrow e^+e^- \) in the plasma.

### 2 Matrix Element

The axion decay \( a \rightarrow e^+e^- \) is described by two diagrams in Fig.1 where solid double lines imply the influence of the magnetic field on the electron wave functions and undulating double lines imply the influence of medium on the photon propagator. The diagram (a) describes this process in the model \[8\] with a direct axion-fermion coupling:

\[
\mathcal{L}_{af} = -ig_{af} \left( \bar{f} \gamma_5 f \right) a, \tag{4}
\]
where \( g_{af} = C_f m_f / f_a \) is a dimensionless coupling constant; \( f_a \) is the Peccei-Quinn symmetry breaking scale; \( C_f \) is a model-dependent factor; \( m_f \) is the fermion mass (electron in our case); \( f \) and \( a \) are the fermion and axion fields, respectively.

The diagram (b) describes the decay via a photon intermediate state. The effective axion-photon coupling can be presented in the form:

\[
L_{a\gamma} = \bar{g}_{a\gamma} \left( \partial_\mu A_\nu \right) \tilde{F}_{\nu\mu} a,
\]

where \( A_\mu \) is the four potential of the quantized electromagnetic field, \( \tilde{F} \) is the dual external field tensor; \( \bar{g}_{a\gamma} \) is an effective coupling in the presence of the magnetic field with the dimension \((\text{energy})^{-1}\) [20]:

\[
\bar{g}_{a\gamma} = g_{a\gamma} + \frac{\alpha}{\pi} \sum_f \frac{Q_f^2 g_{af}}{m_f} (1 - J).
\]

Here, \( g_{a\gamma} \) corresponds to the well-known \( a\gamma\gamma \) coupling in vacuum (diagram (a) in Fig. 2) with a constant \( g_{a\gamma} = \alpha \xi / 2\pi f_a \) [4], where \( \xi \) is the model-dependent parameter. The second term in (6) is the field-induced contribution to the effective coupling \( \bar{g}_{a\gamma} \) which comes from a diagram (b) in Fig. 2:

\[
J = \left( \frac{4}{\chi_f} \right)^{2/3} \pi^{2/3} \int_0^\infty f(\eta) \sin^{-1/3} \phi d\phi,
\]

\[
f(\eta) = i \int_0^\infty du \exp \left\{ -i \left( \eta u + \frac{u^3}{3} \right) \right\},
\]

\[
\eta = \left( \frac{4}{\chi_f \sin^2 \phi} \right)^{2/3}.
\]

Here, \( f(\eta) \) is the Hardy-Stokes function and \( \chi_f \) is the dynamic parameter:

\[
\chi_f^2 = \frac{e_f^2 (qFFq)}{m_f^6},
\]

where \( e_f = eQ_f, e > 0 \) is the elementary charge, \( Q_f \) is a relative electric charge of a loop fermion; \( q = (E_a, q) \) is the four-momentum of the axion. As it was pointed in [20] this term gives a contribution comparable with \( g_{a\gamma} \) for the fermions with \( \chi_f \gg 1 \) only.

The decay probability of particles in the magnetic field depends on invariant field parameters. In a general case of arbitrary values of particles energies and the magnetic field strength two independent field invariants \( (e^2 (FF))^{1/2} \) and \( (e^2 (qFFq))^{1/3} \) exist. Therefore the decay probability is a function of these parameters. In the case of a relatively weak magnetic field, when a great number of the Landau levels is excited, and ultrarelativistic particles the invariant \( (e^2 (qFFq))^{1/3} \) occurs the largest one. So, the decay probability depends actually on the dynamic parameter \( \chi_f \) [4] only. It means that we can perform
calculations in a crossed field ($\mathbf{E} \perp \mathbf{B}$, $E = B$) with zeroth field invariants ($FF$) and ($\mathbf{F} \mathbf{F}$).

The matrix element of $a \to e^+e^-$ decay corresponding to the diagrams in Fig. 1 is a sum:

$$S = S^{(a)} + S^{(b)},$$

(8)

where $S^{(a)}$ corresponds the diagram (a) in Fig. 1:

$$S^{(a)} = \frac{g_{ae}}{\sqrt{2E_aV}} \int d^4x \bar{\psi}(p, x) \gamma_5 \psi(-p', x) e^{-iqx},$$

and $S^{(b)}$ describes the contribution from the diagram (b):

$$S^{(b)} = \frac{g_{eg}}{\sqrt{2E_aV}} \int d^4x \bar{\psi}(p, x) \gamma h \psi(-p', x) e^{-iqx},$$

$$h_\alpha = -ie(q^\alpha FG(q))_\alpha = -ieq_\mu \tilde{F}_\mu G_{\nu\alpha}(q).$$

Here, $\psi(p, x)$ is the exact solution of the Dirac equation in the external crossed field [19]; $p = (E, \mathbf{p})$ and $p' = (E', \mathbf{p}')$ are the quasi-momenta of final electron and positron ($p^2 = p'^2 = m_e^2$); $(\gamma h) = \gamma_\mu h_\mu$, $\gamma_\mu$ are the Dirac $\gamma$-matrices. The condition of the relative weakness of the magnetic field, $eB \ll T^2, \mu^2$, means that from both components of the active medium the plasma determines basically the properties of the photon propagator $G_{\alpha\beta}$. The propagator can be presented as a sum of transverse and longitudinal parts:

$$G_{\alpha\beta}(q) = -i \left( \frac{\mathcal{P}_{\alpha\beta}^{(T)}}{q^2 - \Pi^{(T)}} + \frac{\mathcal{P}_{\alpha\beta}^{(L)}}{q^2 - \Pi^{(L)}} \right),$$

(9)

$$\mathcal{P}_{\alpha\beta}^{(T)} = -\sum_{\lambda=1}^{2} t^{(\lambda)}_\alpha t^{(\lambda)}_\beta,$$

$$\mathcal{P}_{\alpha\beta}^{(L)} = -l_\alpha l_\beta.$$

Here, $\Pi^{(T)}$ and $\Pi^{(L)}$ are the transverse and longitudinal eigenvalues of the polarization operator; $t^{(\lambda)}_\alpha (\lambda = 1, 2)$ and $l_\alpha$ denote the transverse and longitudinal photon polarization vectors:

$$t^{(1)}_\alpha = \frac{(qF)_\alpha}{\sqrt{(qFq)}}, \quad t^{(2)}_\alpha = \frac{\varepsilon_{\alpha\beta\mu\nu} t^{(1)}_\beta q_\mu u_\nu}{\sqrt{(uq)^2 - q^2}},$$

$$l_\alpha = \frac{q^2}{(uq)^2 - q^2} \left( u_\alpha - \frac{uq}{q^2} q_\alpha \right),$$

where $u_\alpha$ is the four-velocity of medium.

Being integrated over the variable $x$ the matrix element (8) is:

$$S = \frac{\delta^{(2)}(Q_\perp) \delta(kQ)}{\sqrt{2E_aV \cdot 2E'V \cdot 2EV}} \frac{(2\pi)^4}{\pi uz}$$

(10)
\[ \times U(p) \left[ g_{ae} \gamma_5 \left( \Phi(\eta) + \frac{ie \gamma^2}{2m^2_e} (\gamma F \gamma) \Phi'(\eta) \right) \\
+ \ g_{a\gamma} (\gamma h) \Phi(\eta) + \frac{ie \gamma^2 (\chi - \chi')}{m^2_e \chi_a} (\gamma F h) \Phi'(\eta) \\
- \ e \frac{\gamma^5 (\gamma F h)}{m^2_e} \Phi'(\eta) + \frac{m^2_e}{2} \frac{(\gamma k)(kh)}{(kp)(kp')} \eta \Phi(\eta) \right] U(-p'), \]
\[ u^2 = -e^2 a^2, \quad z = \left( \frac{\chi_a}{2 \chi \chi'} \right)^{1/3}, \]
\[ \chi^2 = \frac{e^2 (p FF p)}{m^6_e}, \quad \chi'^2 = \frac{e^2 (p' FF p')}{m^6_e}, \quad \chi_a = \frac{e^2 (q FF q)}{m^6_e}. \]

Here, \( F_{\mu\nu} = k_{\mu} a_{\nu} - k_{\nu} a_{\mu} \) \((k^2 = (ka) = 0)\) is the crossed field tensor; \( Q = q - p - p' \), \( Q_{\perp} \) is the perpendicular to \( k \) component \((Q_{\perp} k = 0)\). The bispinor \( U(p) \), which is normalized by the condition \( \bar{U} U = 2m_e \), satisfies the Dirac equation for the free electron \((\gamma p - m_e)U(p) = 0\). Finally, \( \Phi(\eta) \) is the Airy function:

\[ \Phi(\eta) = \int_0^\infty dt \cos \left( \eta t + \frac{t^3}{3} \right), \]
\[ \eta = z^2 (1 + \tau^2), \quad \tau = -\frac{e (p F q)}{m^2_e \chi_a}, \]

and \( \Phi'(\eta) = d\Phi(\eta)/d\eta \).

### 3 Decay Probability

After integration over the phase space of the \( e^+ e^- \) pair the decay probability is:

\[ W \approx \frac{g_{ae}^2 3^{5/3}}{16 \pi^3} \Gamma^4(2/3) \frac{m^2_e \chi_a^{2/3}}{E_a} \rho_1 (E_a, T, \mu) \]
\[ + \frac{g_{a\gamma}^2 (eB)^2}{36 \pi} \frac{E_a^3 \cos^2 \theta}{(E_a^2 - \mathcal{E}^2)^2 + \gamma^2 \mathcal{E}^4} \rho_2 (E_a, T, \mu), \]
\[ \rho_1 = \frac{2\pi}{3\sqrt{3} \Gamma^3(2/3)} \int_0^1 dx x^{1/3} (1 - x)^{1/3} (1 - n) (1 - \bar{n}), \]
\[ \rho_2 = 6 \int_0^1 dx x (1 - x) (1 - n) (1 - \bar{n}), \]
\[ n = \left( \exp \frac{x E_a - \mu}{T} + 1 \right)^{-1}, \quad \bar{n} = \left( \exp \frac{(1 - x) E_a + \mu}{T} + 1 \right)^{-1}, \]

where the variable \( x = E/E_a \) is the relative electron energy; \( n \) and \( \bar{n} \) are the Fermi-Dirac distributions of electrons and positrons, respectively. The functions \( \rho_{1,2} \) have a meaning...
of the average values of suppressing statistical factors and are, in general case, inside the interval $0 < \rho_{1,2} < 1$.

The second term in (12) describes the contribution of the longitudinal plasmon intermediate state only and has a resonant character at a particular energy of the decaying axion $E_a \sim \mathcal{E}$. This is due to the fact that the axion and the longitudinal plasmon dispersion relations always cross for a certain wave-number $k = \mathcal{E}$. The contribution of the transverse plasmon to the decay probability of $a \rightarrow e^+ e^-$ in the ultrarelativistic case is negligibly small. The dimensionless resonance width $\gamma$ in Eq. (12) is:

$$\gamma = \frac{\mathcal{E} \Gamma_L(\mathcal{E})}{q^2 Z_L},$$

where $\Gamma_L(\mathcal{E})$ is the total width of the longitudinal plasmon; $Z_L$ is the renormalization factor of the longitudinal plasmon wave-function:

$$Z_L^{-1} = 1 - \frac{\partial \Pi^{(L)}}{\partial q^2}.$$

Note that without the external field the plasmon decay into neutrino pair is kinematically allowed only. In the presence of the strong magnetic field ($\epsilon B \gg \alpha_3 \mathcal{E}^2$) the main contribution to the width $\Gamma_L(\mathcal{E})$ is determined by the process of the longitudinal plasmon absorption $\gamma_L e^- \rightarrow e^-$ which becomes possible in this kinematical region also.

Below we give the expressions for $\mathcal{E}^2$ and $\gamma$ in two cases:

i) degenerate plasma ($\mu \gg T$)

$$\mathcal{E}^2 \simeq \frac{4\alpha}{\pi} \mu^2 \left( \ln \frac{2\mu}{m_e} - 1 \right),$$

$$\gamma \simeq \frac{2\alpha}{3} \mu^2 \mathcal{E}^2. \tag{15}$$

ii) nondegenerate hot plasma ($T \gg \mu$)

$$\mathcal{E}^2 \simeq \frac{4\pi\alpha}{3} T^2 \left( \ln \frac{4T}{m_e} - 0.647 \right),$$

$$\gamma \simeq \frac{2\pi^2\alpha}{9} T^2 \mathcal{E}^2. \tag{16}$$

Considering possible applications of the result to cosmology we estimate the axion lifetime in a hot plasma. Under the early Universe conditions the hot plasma is nondegenerate one and the medium parameters $\rho_{1,2}$ are inside the interval $1/4 < \rho_{1,2} < 1$. With $\mathcal{E}^2$ and $\gamma$ (16) we obtain the following estimation for the axion lifetime (diagram (b) in Fig. 1) in the resonance region:

$$\tau(a \rightarrow \gamma_{pl} \rightarrow e^+ e^-) \simeq 6.1 \cdot 10^4 \text{s} \left( \frac{10^{-10}}{g_{a\gamma} \text{GeV}} \right)^2 \left( \frac{T}{10 \text{MeV}} \right)^2 \left( \frac{10^{15} \text{G}}{B} \right)^2. \tag{17}$$
It is interesting to compare Eq. (17) with the axion lifetime without taking into account the resonant contribution via plasmon

$$\tau(a \rightarrow e^+e^-) \simeq 3.4 \cdot 10^6 \text{s} \left( \frac{10^{-13}}{g_{ae}} \right)^2 \left( \frac{T}{10 \text{ MeV}} \right)^{1/3} \left( \frac{10^{15} \text{ G}}{B} \right)^{2/3}. \quad (18)$$

The expressions (17) and (18) demonstrate the strong catalyzing influence of medium, the plasma and the magnetic field, on the axion decay. Nevertheless, the axion lifetime is determined by the axion decay via the longitudinal plasmon in both invisible axion models [7, 8].

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Figure 1:

\[ a(q) + e(-p') \]

Figure 2:

\[ a + a F_{ext} \]

\[ a + a F_{ext} \]

\[ a \]