Coulomb vs. physical string tension on the lattice

Giuseppe Burgio, Markus Quandt, Hugo Reinhardt, and Hannes Vogt
Institut für Theoretische Physik, Auf der Morgenstelle 14, 72076 Tübingen, Germany
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We investigate the precise relationship between the Coulomb and the physical (Wilson) string tension on the lattice, as the former is generally known to give an upper bound for the latter. We give evidence that the two string tensions are in a one to one correspondence at zero temperature, while they become unrelated at finite temperatures. More precisely, we show that the standard lattice calculations of the Coulomb gauge confinement scenario are always tied to the spatial string tension, which is known to survive the deconfinement phase transition and to cause screening effects in the quark-gluon plasma. Our analysis is based on the identification and elimination of center vortices which allows to control the physical string tension and study its effect on the Coulomb gauge observables. We also show how alternative definitions of the Coulomb potential may sense the deconfinement transition, although a true static Coulomb gauge order parameter for the phase transition is still elusive on the lattice.

I. INTRODUCTION

Recent years have seen a rising interest in Coulomb gauge investigations of Yang-Mills theories in general, and in the Hamiltonian formulation in particular [1–4]. Once Weyl-gauge is implemented to eliminate the $A_0(x)$ components of the gauge fields, the Hamilton operator and the Gauß’s law constraint are invariant under the residual time-independent gauge transformations and, moreover, only depend on the remaining space-like gauge fields and momenta $A^a(x)$, $\Pi^a(x)$. In Abelian theories, the transversal part of these vector fields is gauge-independent and thus physical, so that Coulomb gauge can be seen as a physical gauge that eliminates all non-physical (gauge-dependent) degrees of freedom. In non-Abelian theories, this is no longer strictly true, but the elimination of the longitudinal degrees of freedom still resolves Gauß’ law and provides a formulation in terms of the transversal field $A^a_\perp$, $\Pi^a_\perp$ alone, in which studies of the Yang-Mills ground state might be more natural.

The resolution of Gauß’ law incorporates the constraint in the Hamiltonian and circumvents the explicit construction of the physical Hilbert space [5], resulting in

$$H = H_G + H_C ,$$

$$H_G = \int d^3x \left[ \frac{1}{2} \mathcal{J}^{-1}[A] \Pi^a_\perp \mathcal{J}[A] \Pi^a_\perp + \frac{1}{4} F_{ij}^a F_{ij}^a \right],$$

$$H_C = \frac{g^2}{2} \int d^3(x,y) J^{-1}[A] \rho^a(x) \mathcal{J}[A] \hat{F}^{ab}(x,y) \rho^b(y) ,$$

where $\mathcal{J}[A]$ is the determinant of the Faddeev-Popov operator, i.e. the inverse Coulomb ghost propagator

$$(\hat{G}^{-1})^{ab}(x,y) = (-\partial_\mu \tilde{D}^{ab}_\mu) \delta(x-y) ,$$

while the Coulomb Hamiltonian $H_C$ describes the self-interaction of non-abelian color charges with density

$$\rho^a(x) = \psi^\dagger(x) T^a \psi(x) - f^{abc} A^b_i(x) \Pi^c_i(x)$$

through the non-abelian Coulomb kernel

$$\hat{F}^{ab}(x,y) = \int d^3 z G^{ac}(x,z)(-\partial^2 z) G^{cb}(z,y) .$$

The first term on the right-hand side of Eq. (5) is the matter charge density, which for the pure Yang-Mills case should be understood as an external source, while the second part is the dynamical charge density of the non-abelian gauge field. In the abelian theory the latter would, of course, be absent and Eq. (6) becomes the ordinary Coulomb kernel, i.e. the Green’s function of the Laplacian $\hat{F}(x,y) = (4\pi|x-y|)^{-1}$. From Eq. (6) one can define the non-abelian color Coulomb potential, i.e. the Coulomb energy density for a pair of static quark-antiquark color charges separated by a distance $x$,

$$V^{ab}_C(p) = g^2 \int d^3x e^{-ip\cdot x} \langle \hat{F}^{ab}(x,0) \rangle .$$

In a seminal paper [19] Zwanziger, extending ideas first put forward by Gribov [20], showed how such Coulomb potential gives a natural upper bound to Wilson’s physical potential [21]. In other words, the presence of Coulomb confinement is a necessary condition for the physical confinement mechanism to take place in Yang-Mills theories. These results are based on Gribov’s intuition that the Yang-Mills dynamics must be restricted to the first Gribov region, where the Fadddeev-Popov operator in Eq. (4) is positive definite. Further signatures of this idea are the infra-red (IR) divergence of the Coulomb gauge ghost form factor and the emergence of an IR scale in the gluon dispersion relation [20, 22].

1We will omit the index $\perp$ on transversal vector fields in the following.

2A unique elimination of all gauge copies requires an even further restriction to the so-called fundamental modular region, where the gauge functional only possesses absolute maxima.
The Gribov-Zwanziger confinement scenario has been investigated in detail on the lattice \[23-43\], indeed confirming the expected relationships between Coulomb gauge Greens-functions, Coulomb potential and confinement. All lattice investigations are, however, defined at fixed time slices, involving only the space components of the vector fields. At \( T = 0 \) the \( \mathcal{O}(4) \) rotational symmetry is unbroken and the restriction to spatial observables is irrelevant: a Coulomb gauge analysis is even more obfuscated at finite temperatures, due to the fixed finite length of the compactified time direction.

The main problem is that in any Euclidean-Lagrangian formalism static quantities should be extracted from correlators which extend along the time direction. (For instance, Polyakov loops are a most efficient way to determine the static inter-quark potential \[44\].) Lattice Coulomb gauge observables, on the other hand, are defined at fixed time slices, involving only the space components of the vector fields. At \( T = 0 \) the \( \mathcal{O}(4) \) rotational symmetry is unbroken and the restriction to spatial observables is irrelevant: a Coulomb gauge analysis of confinement on the lattice is fully valid. As tial observables is irrelevant: a Coulomb gauge analysis is even more obfuscated at finite temperatures, due to the fixed finite length of the compactified time direction.

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In this paper we give evidence that:

- on the lattice, the relationship between Gribov-Zwanziger and physical confinement disappears above the deconfinement phase transition;
- the reason for such failure lies in the one-to-one connection between the Coulomb string tension and the spatial Wilson string tension.

To test our assumption, we need to control the Wilson string tension, both overall and in the spatial directions separately. To do so, we adapt a method pioneered in Ref. \[46\] by either removing:

- all center vortices from the gauge field (full vortex removal);
- only vortices that pierce space-like Wilson loops (spatial vortex removal).

The rationale behind this strategy is clear: physical confinement should be caused by percolating center vortices piercing time-like Wilson loops \[27, 30, 37, 50\]. Removing spatial vortices only should therefore not affect the inter-quark potential. (In fact, Polyakov loops correlators only involve temporal links and thus remain exactly unaffected by such a procedure.) Any effect of spatial vortex removal on Coulomb gauge observables thus cannot be related to confinement and must, instead, be attributed to the disappearance of the spatial string tension. This would then be a direct proof that such observables predominantly see the spatial correlations in the gluon plasma rather than the confining force between static colour charges.

II. SETUP

A. Lattice setup

For our Coulomb gauge investigation we will employ the anisotropic Wilson action \[65, 66\] for the colour group \( SU(2) \) as proposed in Refs. \[35, 40, 41\]:

\[
S = \sum_x \left\{ \beta_s \sum_{j>i=1}^3 \left( 1 - \frac{1}{2} \mathrm{tr} [U_{ij}(x)] \right) + \beta_t \sum_{i=1}^3 \left( 1 - \frac{1}{2} \mathrm{tr} [U_{id}(x)] \right) \right\},
\]

where \( U_{ij}(x) \) is the standard plaquette. For each choice of \( \beta_s \neq \beta_t \) the spatial and temporal lattice spacings \( a_s \) and \( a_t \) have to be determined non-perturbatively, giving the renormalized anisotropy through the ratio \( \xi = a_s/a_t \).

The couplings are usually parameterized as \( \beta_s = \beta \gamma \) and \( \beta_t = \beta/\gamma \), where \( \gamma \) is the bare anisotropy that needs to be tuned with \( \beta \) in order to realize the desired \( \xi \). Tables for \( \xi \) and \( a_s \) at selected choices of \( \beta \) and \( \gamma \) can be found, for the colour group \( SU(2) \), in Ref. \[41\]. All simulations for which no value explicit for \( \xi \) is indicated have been performed in the isotropic case \( \xi = 1 \).

Most of the finite temperature simulations have been performed on lattices of sizes \( V = N_t \times 3^2 \) with varying \( N_t \). (Deviations from this rule will be indicated explicitly in the data.) The gluon propagator, the ghost propagator and the Coulomb potential have been computed from 100 independent samples in double precision, while the precise determination of the string tension required up to \( 10^5 \) samples at large Creutz ratios.

B. Center vortex removal

To identify center vortices, we first fixed the MC-configurations to the direct maximal center gauge \[49\].

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\(^3\)A precise relationship between such gauge-fixed, so called P-
i.e. we maximized

\[ F[U] = \sum_{x,\mu} \text{tr} \left[ U_\mu(x)^2 \right] \] (9)

where \( \mu = 0, 1, 2, 3 \) for the full (standard) maximal central gauge and \( \mu = 1, 2, 3 \) for the maximal center gauge restricted to the space-like links ("spatial maximal center gauge"). For configurations which required subsequent Coulomb gauge fixing we stopped the center gauge fixing as soon as the functional value Eq. (9) changed by less than \( \epsilon = 10^{-12} \) within 100 iterations. For the measurements where no further gauge fixing was required we performed the center gauge fixing in single precision using \( \epsilon = 10^{-7} \). Center projected configurations are then obtained after center gauge-fixing by mapping the links to the closest center element:

\[ Z^{s/f}_\mu(x) = \text{sign tr} \left[ U_\mu(x) \right] I, \] (10)

where the index "s" and "f" stands for "spatial" and "full", respectively, with the index \( \mu = 1, 2, 3 \) in the former and \( \mu = 0, \ldots, 3 \) in the latter case. To create vortex free configurations, we follow Ref. [44] and define

\[ V^{s/f}_\mu(x) = Z^{s/f}_\mu(x) \cdot U_\mu(x), \] (11)

where \( \mu \) runs again over only spatial or all Lorentz indices, respectively.

C. Coulomb gauge

Since we want to investigate the effect of vortex removal and center projection on correlators in Coulomb gauge, we need to transform each of the configurations \{\( Z^1, Z^2, V^4, V^5 \)\} discussed above to Coulomb gauge. We employ a combination of simulated annealing and over-relaxation \[75, 76\], again adapting the CUDA code cuLGT [67]. For the center projected configurations we first had to apply a random gauge transformation, since the Coulomb FP-operator would otherwise be singular; the links in the center-projected, Coulomb gauge-fixed configurations are therefore no longer elements of \( \mathbb{Z}_2 \), but again of \( SU(2) \). After gauge-fixing, we calculated the ghost propagator

\[ G(p) = \frac{d(p)}{|p|^2} = \text{tr} \left\langle \left( -\hat{D} \cdot \nabla \right)^{-1} \right\rangle, \] (12)

or the ghost form factor \( d(p) \), and the Coulomb potential

\[ V_C(p) = g^2 \text{tr} \left\langle \left( -\hat{D} \cdot \nabla \right)^{-1} \left( -\nabla^2 \right) \left( -\hat{D} \cdot \nabla \right)^{-1} \right\rangle \] (13)

in momentum space, where \( \left( -\hat{D} \cdot \nabla \right) \) is the Faddeev-Popov operator. In position space we also used the alternative definition of the Coulomb potential,

\[ aV_C(|x - y|) = -\lim_{t \to 0} \frac{d}{dt} \log \left\langle \text{tr} \left[ P_t(x) P_t^\dagger(y) \right] \right\rangle = -\log \left\langle \text{tr} \left[ U_0(x) U_0^\dagger(y) \right] \right\rangle \] (14)

borrowed from Ref. [27, 38, 77]. Here, \( P_t(x) \) is a Polyakov line of length \( t \) starting at lattice site \( (0, x) \). Note that Eq. (13) depends only on spatial links, whereas Eq. (14) depends only on temporal links.

III. RESULTS

A. Finite temperature in Coulomb gauge

As discussed in the introduction, a coherent picture of the Gribov-Zwanziger confinement mechanism emerges from lattice Coulomb gauge investigations at \( T = 0 \). As \( T \) is increased, however, propagators do not seem to show a significant sensitivity to the deconfinement phase transition, as can be seen in Figs. [1, 2, 3] for the gluon propagator, the ghost form factor and the Coulomb potential, respectively. Any deviation from the \( T = 0 \) case starts well above \( T = 1.5 T_c \), and then seems to enhance the non-trivial infrared behaviour rather than decaying towards the perturbative expectation, except for the ghost form factor in Fig. [2] whose IR exponent seems to decrease for \( T > T_c \). In particular, the Coulomb string tension extracted from the Coulomb potential in Fig. [3] persists above the deconfinement phase transition. See the figure captions for further details.

These unexpected results were, in fact, the initial motivation for the present work. The fundamental puzzle is how it comes about that finite temperature correlators on the lattice decouple from the critical behaviour at \( T_c \) expected from continuum investigations [13], while agreeing so well with the same expectations at \( T = 0 \)? Our working hypothesis is that it is the spatial rather than the temporal string tension which underlies the finite temperature lattice Coulomb gauge dynamics. Indeed, the spatial string tension is known to persist and even rise above \( T_c \), causing the strong correlations expected in the quark-gluon plasma. We have therefore decided to investigate this matter in more detail by going back to \( T = 0 \) and controlling the string tension via the removal of (all or only spatial) center vortices in MC configurations.
B. Vortex removal vs. Coulomb gauge

1. String tension

In a first step, we calculated the temporal and spatial string tensions through Creutz ratios [78], defined at distance $R$ as in Ref. [79]:

$$\chi(T + 0.5, R + 0.5) = - \log \frac{W(T + 1, R + 1)W(T, R)}{W(T + 1, R)W(T, R + 1)}.$$  

(15)

To reduce the statistical noise we used 5 steps of APE smearing [80] with $\alpha = 0.5$ for all links, or only for the spatial links (if only spatial vortices were removed); such procedure cannot of course be applied to the center projected links. As expected from the literature, we first checked that the string tension drops to zero after full vortex removal and, conversely, keeps its $SU(2)$ value after full center projection (see Fig. 4).

Next, we repeat this procedure (vortex removal and center projection) in the spatial directions only. For the resulting configurations, it is necessary to distinguish between the temporal $\chi(T, R)$ and the spatial Creutz ratios $\chi(R_1, R_2)$. As expected, the spatial string tension drops to zero after removing all spatial vortices, cf. Fig. 5. On the other hand, the temporal string tension measured from Polyakov loop correlators cannot change under spatial center projection or vortex removal, since both procedures do not affect the temporal links from which the Polyakov lines are built. This is an important observation, since a direct measurement of the temporal string tension turns out to be impossible: As illustrated in the histogram in Fig. 6, all space-time Wilson loops receive random sign flips through spatial vortex removal. The signal to noise ratio becomes thus hopeless; only the indirect argument through the Polyakov loops is applicable.

The spatial projection, with or without vortex removal, can further introduce gauge noise in the temporal links.
FIG. 3. Coulomb potential for various temperatures at $\beta = 2.49$ and anisotropy $\xi = 4$. The extrapolation to $p \to 0$ is compatible with a slight increase of the Coulomb string tension $\sigma_c$ at larger temperatures.

FIG. 4. Creutz ratios for full center vortex projection and removal at $\beta = 2.6$. The reference value was taken from Ref. [81]. Dotted lines are fits to the formula $\sigma + \frac{2k}{r}$, see Ref. [79] for further details.

if followed by a Coulomb gauge fixing. This makes a direct measurement of the temporal string tension through Creutz-ratios challenging, as can be seen from the large error bars arising at large distances $r$ in Fig. 5. As can also be seen from this figure, both string tensions still exceed the asymptotic $SU(2)$ reference string tension at distances as large as $r \sim 9$, where they either have not yet reached a plateau (spatial) or are disappearing in statistical noise (temporal). We do not have a clear explanation for this slow convergence.

2. Ghost form factor

From the results above, it is obvious that the MC configurations after spatial vortex removal still exhibit temporal confinement but no spatial confinement. It is interesting to see how the Coulomb gauge correlators react to this change of physics in the underlying ensemble. As shown in Fig. 7, the ghost form factor is no longer compatible with a power law in the deep infrared, both after full and spatial vortex removal. As for the center projected configurations, a naive computation of ghost propagator is ill-defined because the Faddeev-Popov (FP) operator acquires additional zero modes from the center vortices which sit directly on the Gribov horizon. It is, however, possible to invert the FP operator in the subspace orthogonal to the kernel. The result is shown in Fig. 8, where we observe an enhancement in the mid-momentum regime and a suppression in the deep infrared as compared to the unprojected Coulomb gauge.

From these investigations, it is clear that the infra-red enhancement of the original Coulomb gauge form factor is in fact tied to the spatial string tension, as elimination

$^5$The $N_c^2 - 1$ constant zero modes are easy to take care of by restricting the calculation to momenta $p \neq 0$. 
3. Coulomb potential

The extrapolation of the Coulomb string tension $\sigma_C$ from the potential Eq. (13) is possible but suffers from large uncertainties for a variety of reasons. Estimates were given in Refs. [27, 29, 31, 34, 37, 41]. We follow the convention in the literature and plot in Fig. 9 the ratio $p^4 V_C(p)/(8\pi \sigma_W)$, since a linear rising Coulomb potential $V_C(r) = \sigma_C r$ at large distances translates into the Fourier transform $V_C(p) = 8\pi \sigma_C/p^4$ at very small momenta. As can be clearly seen from the plot, the Coulomb string tension $\sigma_C$ disappears after both full and spatial vortex removal. Since the latter case still contains the full temporal string tension, it is clear that the definition of the Coulomb string tension through Eq. (13) is directly related to the spatial string tension.

It is interesting to take Eq. (14) as an alternative definition of the Coulomb potential. From Ref. [27], this is known to allow for a better extrapolation of the Coulomb string tension while still vanishing after full vortex removal. In Fig. 10 we show Eq. (14) together with its full center-projected, full vortex removed and spatial center projected counterpart. The spatial vortex removed correlators are identical since Eq. (14) employs the temporal links $U_0(x)$ only. From the plot, it is clear that this definition of the Coulomb potential is indeed sensitive to the temporal string tension as it still raises linearly even after spatial center projection and vortex removal. Because of the distance to the Hamiltonian formulation, however, a single time slice in Eq. (14) may not be sufficient to have overlap with all the excited states. Indeed, Eq. (14) still does not offer a clear signal at $T_c$, i.e. it cannot fully discern spatial and temporal string tension. The very large value it assumes after spatial center gauge fixing and projection (see Fig. 10) is likely a related phenomenon, since a mixing of degrees of freedom obviously occurs. Modifications of Eq. (14) might offer better results, since correlators of longer open Polyakov lines could turn out to be closer to the finite temperature dynamics in Coulomb
FIG. 7. Ghost from factor before and after vortex removal at $\beta = 2.15$ and $\beta = 2.60$.

FIG. 8. Ghost form factor after center projection (and restriction to the non-zero subspace) at $\beta = 2.15$.

gauge. However, as the length of the line increases, the relationship with the original Coulomb potential becomes obfuscated. Thus, a static Coulomb gauge observable that can detect the deconfinement phase transition on the lattice remains somewhat elusive.

**IV. CONCLUSIONS**

In this paper we have investigated the relationship between spatial and Coulomb string tension as measured through the standard lattice definition of Coulomb gauge correlators. Such observables are made out of the space-like links at a fixed time slice and, as we have seen, can only be used for investigations at $T = 0$. As the temperature increases, temporal and spatial string tension decouple and we find that the dynamics of static Coulomb gauge observables are clearly dominated by the latter and not the former. This explains why, e.g., the Coulomb string tension from Eq. (13) persists above $T_c$, and the low-order lattice Green’s functions do not react to the loss of the temporal string tension at and above $T_c$.

We have attempted to improve this situation by means of an alternative definition of the Coulomb potential pioneered in Ref. [77]. While this observable is sensitive to the temporal string tension, it still cannot fully resolve the deconfinement phase transition, as this would require longer lines with temporal extensions comparable to the first excited states of the theory [27]. Such observables can, however, no longer be easily related to the static Coulomb potential. More refined lattice observables are clearly necessary, and they may be tested with the methods laid out in this paper.

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FIG. 9. Coulomb potential at $\beta = 2.15$.

FIG. 10. Coulomb potential from Eq. (14) at $\beta = 2.60$.

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