Perfection of the methodology for calculating the keyway broach through the experimental data of photomechanics

O Zhed¹,*, M Kozochkin² and K Kazymaeva¹

¹ Peoples Friendship University of Russia (RUDN University), 6 Miklukho-Maklaya str, Moscow, 117198, Russian Federation
² Moscow State University of Technology "STANKIN" (MSUT "STANKIN"), 1 Vadkovsky side-str, Moscow, 127055, Russian Federation

*zhed-ov@rudn.ru

Abstract. In work by the method of photomechanics, the stress-strain state of the models of the keyway broach is evaluated. Models are made of optically sensitive organic glass of E2 grade. The aim of the research is to improve the method of calculating the keyway broach based on experimental data obtained by the photomechanics method. The effect of the radius of the transitional fillet on the stress concentration in the models of new and reconditioned keyway broaches is studied.

1. Introduction

According to the classical technique, the evaluation of the strength of the keyway broach is performed by the calculation of its body for breaking along the cavity of the first cutting tooth or the section of the shank, and does not provide for checking the strength of the tooth [1]. The cases of failure of the broach (figure 1) due to breakage of the teeth are the most common. Therefore, the main purpose of experimental studies is to evaluate the strength of its teeth. The work was carried out in the photomechanics laboratory of the Academy of Engineering RUDN University.

Figure 1. Spline keyway broach teeth: a - normal; b – worn-out.

Geometric parameters of the model of the keyway broach are taken from the working drawing in scale 1:1. The difference concerned the radius of the fillets at the base of the teeth, due to the fact that the effect on the stress state in the calculated cross sections of the broach was investigated for the radii 3; 4 and 5 mm transitional fillet from the front surface to the base of the teeth (figure 2, a). The wear of the broach teeth was modeled in connection with its recesses over the entire period of operation and its influence on the change in the strength of the broach teeth. Two broach models identical in geometric parameters are produced except for the active section of the periphery of the teeth along their pitch (figure 2, b).
2. Method of photo-optical experiment

In the model, the size of the rear tooth surface is half the size, and was 3.75 mm, for the new broach was equal to 7.5 mm. Taring of the optical properties of the model material was performed on disks made of the material of the broach models. The optical constant of material E2 is \( \sigma_{0} \), or 13.8 kg/cm\(^2\) or 1.38 MPa/cm\(^2\). For carrying out the physical experiment, a loading device was developed and manufactured (figure 3).

The model of broach 1 is supported by platform 2, made of transparent optically insensitive plexiglass [2]. On the platform, two brackets 3 and 5 are mounted, which are rigidly connected to it by four bolts 13 and 14. In the bracket 5 are mounted load screws 11, by means of which a resultant \( R_i \) of cutting forces is created on each tooth. The force from the screws 11 during their screwing is transmitted through the disk optical microdynamometers 7 to the teeth of the broach model. In the bracket 3 there are three optical microdynamometers 4 in which the reaction of the support \( P_{yi} \) is modeled from the vertical composing \( P_{zi} \) of the resultant cutting force. The screws 6 align the position of each microdynamometers 4 so that the initial optical contact with the bar 9 takes place.

The broach model is pressed against the support bar 9 from the opposite side of the microdynamometers 4. A sliding frictional force \( F_{fr} \) arises between the broach model and the bar 9 from the normal component \( P_{zi} \) of the resultant cutting force \( R_i \). The friction of rolling force \( R_{fr} \) between the bar 9 and the support discs 4, due to its negligible magnitude, is neglected. The thrust force \( P_{th} \) of the machine stem must overcome the horizontal component \( P_{zi} \) of the cutting force and the frictional force \( F_{fr} \). In the loader, the support bar moves under the action of frictional force, which creates a reference reaction, which is registered by an optical microdynamometer 15. The screw 10 performs the function of adjusting the position of the support bar 9, making contact with the support 3 through the disc 15. The pulling force is transmitted to the broach through the gripper 16, and its total
value \( \Sigma P_{\mu} \) is recorded by the readings of a pair of microdynamometers 8 (\( \Sigma P_{\mu} = P_{\mu 1} + P_{\mu 2} \)). Screws 12 perform the regulation of the value of the pulling force.

The final loading of the model was carried out on a PPU-7 polarization projection unit [3]. The model under load was maintained, and then a digital camera was taken (figure 4).

![Photograms of loaded models](image)

**Figure 4.** Photograms of loaded models: a, c - a new broach; b, d - a reconditioned broach; a, b - shot in white light, polarized in a circle; c, d - mercury lamp, polarized in a monochromatic light, circularly polarized.

On these photograms a constant value of the resultant cutting force \( R_i \) was assigned, which corresponded to two bands in the center of the disk optical dynamometers with \( d = 0.8 \) cm. A resultant \( R_i \) at an angle \( \beta = 24^\circ \) to the base of the broach model is plotted (table 1), which corresponds to the ratio of the cutting force components \( P_{yi}/P_{zi} = 0.4 \), and according to the data of [1] is the most loaded variant of the possible ones.

Optical microdynamometers were manufactured of epoxy compound ED6 MTGFA with an optical constant \( \sigma_0^{1.0} = 19.35 \) kg/cm/band. The price of one band in disks with a diameter \( d = 0.8 \) cm is equal to 6.1 kgf, and in disks of thrust with \( D = 1.6 \) cm, respectively 12.2 kgf. Photograms were shot in a direct stream of monochromatic light from a mercury lamp, and in white light, polarized in a circle. The areas of the colour photogram, painted in black, indicate an isotropic zone or zero point from which the counting of bands is performed.

To determine the value of a given force in a particular dynamometer, the order of the band at the center of this disk was determined. The accuracy of the removal of this experimental information was within 10%. The pulling force is recorded by two disks 8 with the order of the band \( n_b = 1.6 \) in each (figure 4), which is: \( \Sigma P_{\mu} = 38.9 \) kgf. Let’s compare the value of the total pulling force \( \Sigma P_{\mu} \) with the total horizontal cutting force on the three teeth of the new broach model \( \Sigma P_{zi} = 33.7 \) kgf (table 1 and figure 5).
The difference between the pulling force and the horizontal total cutting force should be equal to the frictional force arising in the guide support and, based on the calculations, is: $F_{fr} = \Sigma P_{ti} - \Sigma P_{zi} = 5.2 \, \text{kgf}$. The pulling force $\Sigma P_{ti}$ overcomes the total cutting force $\Sigma P_{zi}$ and frictional force $F_{fr}$ in the support guide of the broach base. The value of this frictional force is created by the total normal component $\Sigma R_{yi}$ from the total resultant $\Sigma R_i$ cutting forces on the three teeth of the broach. On the photogram (figure 5) we have $F_{fr} = 6.1 \, \text{kgf}$, since in the center of the disc 15 there is only one band. The difference in the determination of the frictional force according to the methods described is about 15%.

Let’s consider one more variant of an estimation of the friction force value in the directing support of the broach model. The total normal component $\Sigma P_{yi}$ of cutting forces is fixed by three disk optical dynamometers 4. All these optical dynamometers recorded the same level of bands about $n_b = 1.1$. Therefore, for the three disks, we have a total normal support reaction: $\Sigma P_{y_i, sup} = 3 \sigma_0 n_b d / 8 = 20 \, \text{kgf}$. It is possible to equate the results by different schemes of its determination. On the basis of this, we write the equality: $F_{fr} = 5.2 = f \Sigma P_{y_i, sup}$, and solve it with respect to the coefficient of friction, i.e. $f = 0.26$. We obtained a coefficient of sliding friction between the broach model made of plexiglas and a steel floating support.

All the calculated and experimental data of the calculated cross sections of the models of the examined broaches are summarized in table 1.

We now turn to the determination of the stress concentration coefficient $\alpha_\sigma$, which is calculated from the relations $\alpha_\sigma = \sigma_{max} / \sigma_{nom}$ or $\alpha_\tau = \tau_{max} / \tau_{nom}$, where $\sigma_{max}$ and $\tau_{max}$ are the maximum values of local stresses in the investigated zone, $\sigma_{nom}$ and $\tau_{nom}$ are design stresses in the investigated zone, determined from the dependence of the resistance of materials. The maximum stresses in each section are determined by the formula: $\sigma_{max} = n_{max} \cdot \sigma_0$, where $n_{max}$ is the maximum order of the band, $\sigma_0$ is the price of the band of the model.
Table 1. Experimental and calculation data of cutting force components and geometric parameters of the calculated sections of the broach models.

| Parameters     | Broach model | New | Reconditioned |
|----------------|--------------|-----|---------------|
| $r_i$ (mm)     |              | 3   | 4             |
| $n_{max}$ (stripes) |            | 4.4 | 3.7           |
| $\beta_{ri}$ (degrees) |           | 24  | 22            |
| $H_{ri}$ (mm)  |              | 9.5 | 9.0           |
| $L_{ri}$ (mm)  |              | 12.9| 11.9          |
| $P_{Zri}$ (kgf)$^a$ |           | 11.1| 11.3          |
| $P_{Yri}$ (kgf)$^b$ |           | 5.0 | 4.6           |
| $P^*_{Zri}$ (kgf)$^c$ |         | 9.5 | 9.8           |
| $P^*_{Yri}$ (kgf)$^d$ |         | 7.7 | 7.3           |

$a$ $P_{Zri}=R_{ri}\cos\beta_i$ - the horizontal component of the cutting forces, coinciding with the thrust direction.

$b$ $P_{Yri}=R_{ri}\sin\beta_i$ - the vertical component of cutting forces, perpendicular to the guide support.

$c$ $P^*_{Zri}=R_{ri}\cos(\beta_i+\gamma)$ - the projection of the resultant $R_{ri}$ on the direction perpendicular to the front surface of the tooth.

$d$ $P^*_{Yri}=R_{ri}\sin(\beta_i+\gamma)$ - the projection of the resultant $R_{ri}$ on the front surface of the tooth.

Let's consider two variants of calculation of nominal stresses. The first variant is connected with the evaluation of the strength of the active section of the body of the keyway broach passing through the point with the maximum level of the contour stresses in the transitional fillet of the first tooth. The second variant relates to the evaluation of the strength of the tooth of the broach, which is under the action of cutting forces, which subject the broach tooth to bending and compressive stresses.

We apply the first variant of definition of nominal stresses on strength of active section of the keyway broach body. Nominal stresses are determined by the formula: $\sigma_{nom}=P_t/F$, where pulling force $P_t=P_{Zri}+F_{fr\ ri}$, $F_{fr\ ri}=f\cdot P_{Yri}$ . The area of the calculated cross section is $F=1.5\,cm^2$, where $t=0.5\,mm$ is the thickness of the model and $h$ is the height of the calculated cross section (figure 2, a).

The step-by-step calculations for all cross sections considered are summarized in table 2.

Table 2. A summary table of the stress concentration coefficients $\alpha_{\sigma_i}$ ($\beta = 24^\circ$).

| Parameters of the broach model | $\sigma_{nom}^{ri}$ (kg/cm$^2$) | $\sigma_{max}^{ri}$ (kg/cm$^2$) | $\alpha_{\sigma_i}^{ri}$ |
|-------------------------------|-------------------------------|-------------------------------|------------------------|
| $r_i$ (mm)                    | 3                             | 4                             | 5                      |
| I variant - in the cross-section of the keyway broach model $r_i/H_{ri}$ |                               |                               |                        |
| New                           | 8.1                           | 8.1                           | 121.4                  |
| Reconditioned                 | 8.1                           | 8.0                           | 220.8                  |
| II variant - in the calculated section of the base of the model tooth $r_i/L_{ri}$ |                               |                               |                        |
| New                           | 53.6                          | 62.6                          | 121.4                  |
| Reconditioned                 | 168.0                         | 154.6                         | 220.8                  |

Let us consider the second variant of determining the nominal stresses for the strength of the active section of the body of the broach tooth in the plane perpendicular to its front surface and passing through the point with the maximum level of contour stresses in the transitional fillets of the tooth.
calculate the nominal stresses, we use the compound resistance formula for a straight cantilever beam (figures 6, 7):

\[
\sigma_{\text{nom}} = \frac{R_i}{t} \left( \frac{\sigma H \cos(\beta + \gamma)}{L_i^2} - \frac{\sin(\beta + \gamma)}{L_i} \right),
\]

where \( \gamma \) is the front angle of the broach tooth; \( R_i \) is the resultant of the cutting forces on the \( i \)-th tooth, which, taking into account the angle \( \beta \), is projected onto the front surface by the quantity \( P^* Y_{ri} \) and causes a stressed state of compression in the calculated section.

Figure 6. Scheme for determining the calculated cross-section at the base of the tooth.

Figure 7. Diagram of real and nominal stresses in the calculated section of the X-X tooth with radius 3 mm of the new model.

The projection of the resultant \( R_i \) on a direction perpendicular to the front surface creates a force \( P^* Z_{ri} \), which bends the pulling tooth and causes tension in the calculated section of the tooth base; \( L_i \) is the width of the dangerous section at the base of the tooth passing through the point \( \sigma_{\text{max}} \) on the transition fillet. All the linear and angular values of \( H_{ri} \), \( \beta \), \( L_{ri} \) entering the formula are measured on photograms and substituted using the scale factor. We present the results of calculations for the second variant in table 2.

According to the data in table 2, graphical dependences of the concentration coefficient \( \alpha_{\sigma} \) on the relative geometric parameters of the calculated section \( r_i / H_{ri} \) (I variant) and \( r_i / L_{ri} \) (II variant) are presented in figure 8.

Figure 8. Dependence of the stress concentration coefficient \( \alpha_{\sigma}^{ri} \) on the geometric parameters of the broach model: a - \( r_i / H_{ri} \) (I variant); b - \( r_i / L_{ri} \) (II variant); 1 – new, 2 – reconditioned.

The graphical dependences in figure 8 indicate that the concentration of stresses in the transition fillets increases with decreasing the radius of curvature of the fillet at the base of the broach tooth, and this fact is confirmed for all design variants. Comparison of the absolute values of the concentration coefficients for different design variants is incorrect, because the same maximum stress level for a
particular tooth is compared with the nominal stresses in different sections. One of the calculated sections (the body of the broach) is permanently for the entire service life of the tool.

This is the first design variant, when the rated stresses does not depend on the wear of the tooth. The calculated cross-section for the first tooth's hollow is constant for the entire life of the instrument.

In the second variant the other design section (the base of the tooth) changes in the direction of decrease during the life of the tool. In connection with this, in addition to the maximum and nominal stresses, the concentration coefficient for the second design variant also varies within a narrow range.

In this case, the stress intensity in the calculated section will be greater for a smaller value of the concentration coefficient for the same radius of fillet curvature. In addition, based on this, for the calculated broach point with the same maximum level of contour stresses, a design variant of the section with a higher level of rated stresses should be adopted, i.e. the worn-out broach tooth. The results of the research showed that the base of the tooth of the reconditioned broach, which has the minimum radius of the transitive fillet, is most heavily loaded, so this variant should be taken as a calculated one.

The performed studies showed that the endurance of broaches depends not only on the maximum stresses, but also on their distribution into the interior of the part, i.e. from stress gradients in the most stressed zone [4]. The stress gradient is the intensity of their variation in a specific direction with respect to the contour of the part and is expressed by the dependence: \( G = \frac{\partial \sigma}{\partial x} \) (MPa/mm). The role of gradients consists in the action of stresses exceeding a certain limit on vacancies located in front of the radiative crack, in which, owing to stress concentration, a zone of plastic deformations is formed. Obviously, when the gradient is reduced, a further colony of vacancies is involved in the process of crack formation.

In work [5] on the determination of the statistical computational characteristics of the fatigue resistance, the influence of the relative gradient of the first principal stress on the magnitude of the endurance limit, determined in the probabilistic aspect through the general expression:

\[
\sigma_{ld} \cdot \alpha_{\sigma} = u + 10A - B \cdot \lg \left( \frac{L}{G} \right),
\]

where \( \sigma_{ld} \) is the nominal stress in the alternating regime, \( \alpha_{\sigma} \) is the coefficient of concentration, \( u \) - is the minimum limit of the endurance limits, \( A \) and \( B \) are the coefficients depending on the material properties, \( L \) is the geometric parameter for the keyway broach, \( G \) - relative gradient.

Let us consider a technique of definition of stress gradients according to polarization-optical researches on a flat model of the photogram of the keyway broach, shown in figure 9, a. Calculation of gradients is carried out on flat models of broaches in approximate and specified dependences. The approximate method takes into account that in the immediate vicinity of the tooth fillet contour one of the main stresses \( \sigma_l \) has a maximum value (this is the point \( \sigma_{max} = n_{max} = 6.6 \) bands) and simultaneously in the depth of the body at the base of the broach tooth \( \sigma_z = 0 \) at a small distance \( \Delta x \).
from the contour $\sigma_1 - \sigma_2 = \sigma_l$. The gradient can be calculated by the dependence $G = \partial \sigma_1 / \partial x = \sigma_0 (n_{max} - n_{\Delta x}) / \Delta x$ and the relative gradient is equal to $\tilde{G} = (n_{max} - n_{\Delta x}) / (n_{max} \cdot \Delta x)$, where $n_{max}$ and $n_{\Delta x}$ are the orders of the bands at points on the contour and at a distance $\Delta x$ from the contour of the model. This formula is true for the variants, when $\Delta x < r_i$.

There is a refined method for determining the stress gradients, based on the solution of problems from the theory of photomechanics about the principal stresses of the axis of symmetry with the Rapid method [3]. Here we confine ourselves to illustrating the final expression for the relative gradient, which was obtained by the refined method: $\tilde{G} = (1 - \Delta x / r_i - n_{\Delta x} / n_{max}) / \Delta x$. In figure 9, b shows the graphical dependence of the change in the relative gradients of fillets, which is constructed from the results of processing experimental data.

3. Results and discussions
The technique of physical experiment by the method of photomechanics for loading a model of a keyway broach made of optically sensitive organic glass of E2 grade was developed and implemented.

The effect of the radius of the transitional fillet on the stress concentration in the models of new and reconditioned keyway broaches is studied. According to the calculated and experimental information, the dependence of the concentration coefficient $\alpha_{\sigma l}$ of the maximum contour stresses in fillets on the model broach geometric parameters. The performed studies showed that the endurance of the broaches depends not only on the maximum stresses, but also on their distribution into the interior of the part, i.e. from stress gradients in the most stressed zone.

The received calculation and experimental information by the method of photomechanics on models can be recommended in the practice of calculations of keyway broaches and transferred to a real made of tool material with allowance for geometric and force similarity.

References
[1] Schegolev A V 1960 Designing broaches (Moscow: Mashgiz Publ) pp 1-352
[2] Rogov V A, Koshelenko A S, Zhed O V and Berdashev R S 2015 Investigation by the photomechanics method of the stress-strain state of the keyed broaches on flat optical models StIN 5 pp 11-18
[3] Koshelenko A S and Poznyak G G 2004 Theoretical Foundations and Practice of Photomechanics in Machine-building (Moscow: Granitsa Publ) pp 1-296
[4] Rogov V A, Koshelenko A S, Zhed O V and Predeina A I 2013 Stress concentration in the plane Russian Engineering Research 33 (7) pp 416-421
[5] Sorensen S V and Kogaev V P 1979 Probabilistic methods for strength calculation under variable loads, Mechanical fatigue in the statistical aspect (Moscow: Nauka Publ) pp 1-135