Status of Three Flavor Baryon Chiral Perturbation Theory

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Abstract. I review the present status of three flavor baryon chiral perturbation theory in the heavy fermion formalism. It is argued that precise calculations have to include all terms quadratic in the quark masses. As examples, I consider the chiral expansion of the octet baryon masses, the baryon magnetic moments and kaon photoproduction off nucleons.

1 Introduction

Chiral perturbation theory (CHPT) with nucleons is by now in a fairly mature status, see for example Bernard’s talk at this workshop. The extension to the three flavor case is important for a variety of reasons. First, there exists a large body of interesting phenomenology to study, like e.g. hyperon radiative and nonleptonic weak decays, the baryon octet magnetic moments or kaon photoproduction off nucleons. Second, and more important, are three questions which can not be addressed in the two flavor case:

− The splittings in the baryon octet and the deviations from the octet Goldberger–Treiman relations allow to extract information about the quark mass ratios \( \frac{m_s - \hat{m}}{m_d - m_u} \) (with \( \hat{m} \) the average light quark mass) and \( m_s / \hat{m} \).
− Precise calculations can shed light on the question why flavor SU(3) works so well in some cases and less well in others, which ultimately helps to understand the quark model.
− We can study in great detail the long distance contributions to the nonleptonic weak interactions of the Standard Model.

As will become clear in what follows, we are still far away from precisely answering these questions. I will outline some recent developments based on complete calculations to a given order in the low energy expansion (the small momenta and meson masses underlying this expansion will be denoted collectively by \( q \) in what follows). After discussing some technical aspects, I will focus on three different observables. This is certainly subjective and should not be considered exhaustive. I will also stick to the conventional scheme having only the Goldstone boson and the baryon octet as active degrees of freedom. For an early review on three flavor baryon CHPT, including also the spin–3/2 decuplet, see Jenkins and Manohar (1992).
2 Generating functional and effective Lagrangian

The interactions of the Goldstone bosons with the ground state baryon octet states are severely constrained by chiral symmetry. The generating functional for Green functions of quark currents between single baryon states, $Z[j, \eta, \bar{\eta}]$, is defined via

$$
\exp\{iZ[j, \eta, \bar{\eta}]\} = N \int [dU][dB][d\bar{B}] \exp i \left[ S_M + S_{MB} + \int d^4x \langle \bar{\eta}B \rangle + \langle B\eta \rangle \right],
$$

with $S_M$ and $S_{MB}$ denoting the mesonic and the meson–baryon effective action, respectively, to be discussed below. $\eta$ and $\bar{\eta}$ are fermionic sources coupled to the baryons and $j$ collectively denotes the external fields of vector ($v_\mu$), axial–vector ($a_\mu$), scalar ($s$) and pseudoscalar ($p$) type. These are coupled in the standard chiral invariant manner. In particular, the scalar source contains the quark mass matrix $M$, $s(x) = M + \ldots$. Traces in flavor space are denoted by $\langle \ldots \rangle$. The underlying effective Lagrangian can be decomposed into a purely mesonic ($M$) and a meson–baryon ($MB$) part as follows (I only consider processes with exactly one baryon in the initial and one in the final state)

$$
\mathcal{L}_{\text{eff}} = \mathcal{L}_M + \mathcal{L}_{MB}
$$

subject to the following low–energy expansions

$$
\mathcal{L}_M = \mathcal{L}_M^{(2)} + \mathcal{L}_M^{(4)} + \ldots, \quad \mathcal{L}_{MB} = \mathcal{L}_{MB}^{(1)} + \mathcal{L}_{MB}^{(2)} + \mathcal{L}_{MB}^{(3)} + \ldots
$$

where the superscript denotes the chiral dimension. The pseudoscalar Goldstone fields ($\phi = \pi, K, \eta$) are collected in the $3 \times 3$ unimodular, unitary matrix $U(x)$,

$$
U(\phi) = u^2(\phi) = \exp\{i\phi/F_0\}
$$

with $F_0$ the pseudoscalar decay constant (in the chiral limit). Under SU(3)$_L \times$SU(3)$_R$, $U(x)$ transforms as $U \to U' = LUR^\dagger$, with $L, R \in$ SU(3)$_L,R$. The meson Lagrangian with the external fields coupled in a chiral invariant manner is standard and will not be discussed further, see Gasser and Leutwyler (1985). The effective meson–baryon Lagrangian starts with terms of dimension one,

$$
\mathcal{L}_{MB}^{(1)} = \langle \bar{B} [i\nabla, B] \rangle - m \langle \bar{B} B \rangle + \frac{D}{2} \langle \bar{B} \{\gamma_5, B\} \rangle + \frac{F}{2} \langle \bar{B} [\gamma_7, B] \rangle,
$$

with $m$ the average octet mass in the chiral limit. The $3 \times 3$ matrix $B$ collects the baryon octet,

$$
B = \begin{pmatrix}
\Sigma^0 + \frac{1}{\sqrt{2}} \Lambda & \Sigma^+ & p \\
\Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & \Lambda \\
\Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda
\end{pmatrix}.
$$
Under $SU(3)_L \times SU(3)_R$, $B$ transforms as any matter field,

$$B \to B' = K B K^\dagger,$$

with $K(U, L, R)$ the compensator field representing an element of the conserved subgroup $SU(3)_V$. $\nabla_\mu$ denotes the covariant derivative,

$$[\nabla_\mu, B] = \partial_\mu B + [\Gamma_\mu, B]$$

and $\Gamma_\mu$ is the chiral connection,

$$\Gamma_\mu = \frac{1}{2} [u^\dagger (\partial_\mu - i r_\mu) u + u (\partial_\mu - i l_\mu) u^\dagger].$$

Note that the first term in Eq.(5) is of dimension one since $[i\nabla, B] - m B = O(q)$, Gasser et al. (1988). The lowest order meson–baryon Lagrangian contains two axial–vector coupling constants, denoted by $D$ and $F$. It is important to note that to leading order, no symmetry–breaking terms appear. The dimension two and three terms have been enumerated by Krause (1990). Treating the baryons as relativistic spin–1/2 fields, the chiral power counting is no more systematic due to the large mass scale $m$, $\partial_\mu B \sim m B \sim \Lambda_\chi B$. This problem can be overcome in the heavy mass formalism proposed in Jenkins and Manohar (1991). I follow here the path integral approach developed in Bernard et al. (1992). Defining velocity–dependent spin–1/2 fields by a particular choice of Lorentz frame and decomposing the fields into their velocity eigenstates (sometimes called ‘light’ and ‘heavy’ components),

$$H_v(x) = \exp\{imv \cdot x\} P^+_v B(x), \quad h_v(x) = \exp\{imv \cdot x\} P^-_v B(x),$$

$$P^\pm_v = \frac{1}{2} (1 \pm \frac{v}{v^2}) \quad v^2 = 1,$$

the mass dependence is shuffled from the fermion propagator into a string of $1/m$ suppressed interaction vertices. In this basis, the three flavor meson–baryon action takes the form

$$S_{MB} = \int d^4x \left\{ H_v^a A^{ab}_v H^b_v - \bar{h}_v^a C^{ab}_v h^b_v + \bar{h}_v^a B^{ab}_v H^b_v + \bar{H}_v^a \gamma_0 B^{ab}_v \gamma_0 h^b_v \right\},$$

with $a, b = 1, \ldots, 8$ flavor indices. The $8 \times 8$ matrices $A$, $B$ and $C$ admit low energy expansions, see Müller and Meißner (1997). Similarly, one splits the baryon source fields $\eta(x)$ into velocity eigenstates,

$$R_v(x) = \exp\{imv \cdot x\} P^+_v \eta(x), \quad \rho_v(x) = \exp\{imv \cdot x\} P^-_v \eta(x),$$

and shift variables

$$h_v^a = h_v^a - (C^{ac}_v)^{-1} (B^{cd}_v H^d_v + \rho_v^c),$$

so that the generating functional takes the form

$$\exp[iZ] = N \Delta_h \int [dU][dH_v][dH_v] \exp\{iS_M + iS_{MB}'\}$$
in terms of the meson–baryon action $S'_{MB}$,

$$S'_{MB} = \int d^4 x \bar{H}_a^v (A^{ab} + \gamma_0 [B^{ac}]^\dagger \gamma_0 [C^{cd}]^{-1} B^{db}) H_b^v$$

$$+ \bar{H}_a^v (R_c^e + \gamma_0 [B^{ac}]^\dagger \gamma_0 [C^{cd}]^{-1} \rho_e^d) + (\bar{R}_e^a + \bar{\rho}_e^c [C^{cb}]^{-1} B^{eb}) H_a^v .$$

(15)

The determinant $\Delta_h$ related to the ‘heavy’ components is identical to one. The generating functional is thus entirely expressed in terms of the Goldstone bosons and the ‘light’ components of the spin–1/2 fields. The action is, however, highly non-local due to the appearance of the inverse of the matrix $C$. To render it local, one now expands $C^{-1}$ in powers of $1/m$, i.e. in terms of increasing chiral dimension,

$$[C^{ab}]^{-1} = \frac{\delta^{ab}}{2m} - \frac{1}{(2m)^2} \left\{ \langle \lambda_a^\dagger [iv \cdot \nabla, \lambda_b^\dagger] \rangle + D\langle \lambda_a^\dagger \{ S \cdot u, \lambda_b^\dagger \} \rangle 
+F\langle \lambda_a^\dagger [S \cdot u, \lambda_b^\dagger] \rangle \right\} + O(q^2) ,$$

(16)

with $S_\mu$ the Pauli–Lubanski spin vector and $u_\mu \sim i \partial_\mu \phi_0 + \ldots$. To any finite power in $1/m$, one can now perform the integration of the ‘light’ baryon field components $N_v$ by again completing the square,

$$H_a^v = [T^{ac}]^{-1} (R_c^e + \gamma_0 [B^{ac}]^\dagger \gamma_0 [C^{cd}]^{-1} \rho_e^d)$$

$$T^{ab} = A^{ab} + \gamma_0 [B^{ac}]^\dagger \gamma_0 [C^{cd}]^{-1} B^{db} .$$

(17)

Notice that the second term in the expression for $T^{ab}$ only starts to contribute at chiral dimension two (and higher). In this manner, one can construct the effective meson–baryon Lagrangian with the added virtue that the $1/m$ corrections related to the Lorentz invariance of the underlying relativistic theory are correctly given. To end this section, I give the chiral dimension $D$ for processes with exactly one baryon line running through the pertinent Feynman diagrams,

$$D = 2L + 1 + \sum_{d=2,4,6,\ldots} (d-2) N_d^M + \sum_{d=1,2,3,\ldots} (d-1) N_d^{MB} \geq 2L + 1$$

(18)

with $L$ denoting the number of (meson) loops, and $N_d^M$ ($N_d^{MB}$) counts the number of mesonic (meson–baryon) vertices of dimension $d$ (either a small momentum or meson mass). This means that tree graphs start to contribute at order $q$ and $L$–loop graphs at order $q^{(2L+1)}$. Consequently, the low energy constants (LECs) appearing in $L_c^{(2)}_{MB}$ are all finite.

### 2.1 Renormalization to third order

Before discussing some specific examples, let me turn to some more theoretical aspects, i.e. the problem that to one loop in the chiral expansion divergences appear. The divergence structure of the one–loop generating functional to order
$q^3$ has been worked out, see Müller and Meißner (1997). It extends previous work by Ecker (1994) for the pion–nucleon Lagrangian to the SU(3) case. While Ecker’s method can also be used in SU(3), the fact that the baryons are in the adjoint representation of SU(3) whereas the nucleons are in the fundamental representation of SU(2), complicates the calculations considerably. In fig. 1 the various contributions to the one–loop generating functional together with the tree level generating functional at order $\hbar$ are shown. The solid (dashed) double lines represent the baryon (meson) propagator in the presence of external fields. Only if one ensures that the field definitions underlying the mesonic and the baryon-meson Lagrangian match, the divergences are entirely given by the irreducible self–energy ($\Sigma_1$) and the tadpole ($\Sigma_2$) graphs. The explicit calculations to extract the divergences from $\Sigma_1, \Sigma_2$ are given in Müller and Meißner (1997). The generating functional can be renormalized by introducing the following counterterm Lagrangian

\[
L^{(3)\text{ct}}_{MB} = \frac{1}{(4\pi F_0)^2} \sum_{i=1}^{102} d_i \bar{H}^{abv}(x) O_i^{bcv}(x) H^{cav}(x),
\]

with $'a, b, c'$ SU(3) indices and the field monomials $O_i^{bcv}(x)$ are of order $q^3$. The dimensionless LECs $d_i$ are decomposed as

\[
d_i = d_i^r(\mu) + (4\pi)^2 \beta_i L(\mu),
\]

with

\[
L(\mu) = \frac{\mu^{d-4}}{(4\pi)^2} \left\{ \frac{1}{d-4} - \frac{1}{2} \left[ \log(4\pi) + 1 - \gamma \right] \right\}.
\]

Here, $\mu$ is the scale of dimensional regularization, $\gamma$ the Euler–Mascheroni constant and the $\beta_i$ are dimensionless functions of the axial couplings $F$ and $D$ that cancel the divergences of the one–loop functional. They are tabulated in Müller and Meißner (1997) together with the $O_i^{bcv}(x)$. These 102 terms constitute a complete set for the renormalization with off–shell baryons. As long as one is only interested in Greens functions with on–shell baryons, the number of terms can be reduced considerably making use of the baryon equations of motion. Also, many of these terms involve processes with three or more mesons. So for calculations of kaon–nucleon scattering or kaon photoproduction off nucleons, many of these terms do not contribute (or only start to contribute at higher orders).

![Fig. 1. Contributions to the one–loop generating functional at order $\hbar$.](image)
At present, only a few of the finite $d_i^q(\mu)$ have been determined. There are two main directions to extend these investigations. First, a systematic effort to pin down as many LECs as possible is needed and, second, the divergences at order $q^4$ should be extracted.

### 2.2 The fourth order Lagrangian

To fourth order in small momenta, the effective Lagrangian has not been worked out in full detail. Here, I will collect the pieces necessary to discuss the scalar sector and the baryon magnetic moments to that order. Consider first the scalar sector. There are three explicit symmetry breaking terms at dimension two,

$$L_{MB}^{(2)} = b_D \langle \overline{\chi}_+ B \rangle + b_F \langle \overline{\chi}_+ B \rangle + b_0 \langle \overline{\chi}_+ B \rangle,$$

making use of the Cayley–Hamilton identity. The $e_i$ have dimension $(\text{mass}^{-3})$.

It is important to note that some of the $e_i$ simply amount to quark mass renormalizations of some of the dimension two LECs. To be specific, one can absorb the effects of $e_5$ and $e_6$ in $b_F$ and $b_0$, respectively, as follows

$$b_F \to b_F - e_5 \langle \chi_+ \rangle, \quad b_0 \to b_0 - e_6 \langle \chi_+ \rangle.$$

This is a very general phenomenon of CHPT calculations in higher orders. For example, in $\pi\pi$ scattering there are six LECs at two loop order ($q^6$), but only two new independent terms $\sim s^3$ and $\sim s M_\pi^2$. The other four LECs make the $q^4$ counter terms $\ell_i$ ($i = 1, 2, 3, 4$) quark mass dependent. At this point, one has two options. One can either treat the higher order LECs as independent from the lower order ones or lump them together to minimize the number of independent terms. In the latter case, one needs to refit the numerical values of the lower dimension LECs. If one uses e.g. resonance saturation to estimate the LECs, one should work with the first option and treat all the $e_i$ separately from the $b_i$. Let me now turn to the magnetic moments, i.e. construct the terms involving electromagnetic field strength tensor. First, I need the pertinent terms of the lowest order chiral meson–baryon Lagrangian of dimension two,

$$L_{MB}^{(2)} = -\frac{i}{4m} b_6^F \langle \overline{B}[S^\mu, S^\nu][F^+_{\mu\nu}, B] \rangle - \frac{i}{4m} b_6^D \langle \overline{B}[S^\mu, S^\nu][F^+_{\mu\nu}, B] \rangle.$$
with \( F^\dagger_{\mu\nu} = -e(u^\dagger QF_{\mu\nu}u + uQF_{\mu\nu}u^\dagger) \), \( F_{\mu\nu} \) the conventional photon field strength tensor and \( Q = \text{diag}(2, -1, -1)/3 \) the quark charge matrix. It is straightforward to construct the terms contributing to the counterterm (tree) contributions with exactly one insertion from the dimension four effective Lagrangian. For simplicity, I consider the ones related to the explicit breaking of SU(3) due to the large strange quark mass. These have the form, Bos et al. (1997):

\[
\mathcal{L}^{(4)}_{MB} = -\frac{i\alpha_1}{4m} \langle \bar{B}[S^\mu, S^\nu] \{[F^+_{\mu\nu}, B], \chi^+\} \rangle - \frac{i\alpha_2}{4m} \langle \bar{B}[S^\mu, S^\nu] \{[F^+_{\mu\nu}, B], \chi^+\} \rangle
- \frac{i\alpha_3}{4m} \langle \bar{B}[S^\mu, S^\nu] \{[F^+_{\mu\nu}, B], \chi^+\} \rangle - \frac{i\alpha_4}{4m} \langle \bar{B}[S^\mu, S^\nu] \{[F^+_{\mu\nu}, B], \chi^+\} \rangle
- \frac{i\beta_1}{4m} \langle \bar{B}[S^\mu, S^\nu] B \rangle \langle \chi^+ F^+_{\mu\nu} \rangle.
\]

(26)

Here, \( \chi^+ \) is the spurion, \( \chi^+ = \text{diag}(0,0,1) \), i.e. a factor \( m_s \) has been pulled out and absorbed in the low–energy constants \( \alpha_1, \alpha_2, \alpha_3, \alpha_4 \) and \( \beta_1 \). The terms given in eq.(26) are of chiral dimension four since \( m_s = \mathcal{O}(q^2) \) and \( F_{\mu\nu} = \mathcal{O}(q^2) \). Of course, in general these five terms should be written in terms of the full quark matrix, but since \( m_s \gg m_d, m_u \), it is legitimate to neglect at this order the pionic contribution. There are two more terms which could contribute. These have the form

\[
\mathcal{L}^{(4')}_{MB} = -\frac{i\tilde{b}^F_6}{4m} \langle \chi^+ \rangle \langle \bar{B}[S^\mu, S^\nu] [F^+_{\mu\nu}, B] \rangle - \frac{i\tilde{b}^D_6}{4m} \langle \chi^+ \rangle \langle \bar{B}[S^\mu, S^\nu] \{F^+_{\mu\nu}, B\} \rangle.
\]

(27)

As before, these two terms obviously amount to quark mass renormalizations of \( b_6^{D,F} \), i.e. \( b_6^{D,F} \rightarrow b_6^{D,F} + \langle \chi^+ \rangle b_6^{D,F} \). Their contribution can therefore be absorbed in the values of the corresponding dimension two LECs. We note that the seven terms given in eqs.(26,27) have already been enumerated in Jenkins et al. (1993) (in other linear combinations). This is all the machinery needed for what follows.

### 3 Kaon Photoproduction

Pion photo– and electroproduction in the threshold region has been studied intensively over the last few years by Bernard, Kaiser and myself (see Bernard’s contribution to these proceedings) with high precision data coming from MAMI, SAL and NIKHEF. In addition, at the electron stretcher ring ELSA (Bonn) ample kaon photoproduction data have been taken over a wide energy range. Only a small fraction of these data is published in Bockhorst et al. (1994), the larger fraction is still in the process of being analyzed. It therefore seems timely to study the reactions \( \gamma p \rightarrow \Sigma^+ K^0, \Lambda K^0 \) and \( \Sigma^0 K^+ \) in the framework of CHPT. This has been done in an exploratory study published in Steininger and Meißner (1997). It only has been done to third order in small momenta and thus it is obvious that one can not expect a high precision. However, before embarking on a full scale \( q^4 \) calculation, one first has to find out whether the strict CHPT
approach is at all applicable. Here, I will critically summarize the status of these
calculations, some more details are given by Steininger in these proceedings.
The threshold energies for these three processes are 1046, 1048 and 911 MeV, in
order. In the threshold region, for energies less than 100 MeV above the respective
threshold, it is advantageous to perform a multipole decomposition. It suffices
to work with S– and P–wave multipoles. The transition current for the process
\[ \gamma^*(k) + p(p_1) \rightarrow M(q) + B(p_2) \] (M = K\(^+\), K\(^0\), B = \(\Lambda\), \(\Sigma^0\), \(\Sigma^+\)) calculated to \(O(q^3)\) can be decomposed into Born, one–loop and counterterm contributions,
\[ T = T^{\text{Born}} + T^{\text{1–loop}} + T^{\text{c.t.}}, \] (28)
where the Born terms subsume the leading electric and the subleading magnetic
couplings of the photon to the nucleon/hyperon and \(\gamma^*\) denotes a real
\((k^2 = 0)\) or virtual \((k^2 < 0)\) photon. The calculation of the Born terms is
standard, for charged kaon production the SU(3) generalization of the Kroll–
Rudermann term gives the dominant contribution to the electric dipole amplitude.
Of particular interest is the observation that the leading P–wave multipoles for \(\Sigma^0 K^+\) production are very sensitive to the yet unmeasured magnetic
moment of the \(\Sigma^0\) because it is enhanced by the coupling constant ratio
\[ g_{pK\Lambda}/g_{pK\Sigma^0} = (D + 3F)/\sqrt{3}/(F - D) \approx -5. \] The one loop graphs are also easy
to calculate. Two remarks concerning these are in order. First, the SU(3) calculation
allows one to investigate the effect of kaon loops on the SU(2) predictions.
As expected, it is found that these effects are small, e.g. for neutral pion photoproduction off protons,
\[ E_{\pi^0,\text{thr}}^{K^+} = (\varepsilon FM_K^2)/(96\pi^2 F^3 M_K) = 0.14 \cdot 10^{-3}/M_{\pi^+}, \] which is just 1/10th of the empirical value and considerably smaller than the
pion loop contribution. This result is in agreement with the famous decoupling
theorem. In the chiral SU(2) limit, that is for a fixed strange quark mass, kaon
loop effects must decouple, which means that they are suppressed by inverse
powers or logs of the heavy mass, here \(M_K\). Second, the loop graphs give rise to
the imaginary part of the transition amplitude. Here, one encounters the standard
problem of CHPT, namely that at a given order the imaginary parts are
given to much less precision than the real ones due to the \(O(p^{2N})\) suppression
for \(N–\)loop graphs. One finds that these imaginary parts come out much too big,
which is caused by the pion loops. This can be understood by considering the
rescattering graph \(\gamma p \rightarrow \pi^+ n \rightarrow Y K^+\). By virtue of the Fermi–Watson
theorem, one finds \(\text{Im } E_{\pi^+} = \text{Re } E_{\pi^+} \cdot a_{\pi K} \cdot \text{PS} \), where \(\text{PS}\) denotes the phase space
allowed for the virtual pion and \(a_{\pi K}\) the \(\pi K\) scattering length in the respective
channel. Obviously, the initial charged pion photoproduction process is far away
from its threshold, out of the range of applicability of CHPT. In Steininger and Meißner (1997) these multipoles were thus taken from the existing multipole
analysis. Clearly, this needs refinement. At next order in the chiral expansion,
one has e.g. additional contributions from \(\pi^0\) and \(\eta\) rescattering graphs. Note
also that to this order, \(q^3\), the loop graphs are not finite but need standard renor-
malization. This can either be done by direct Feynman diagram calculation or
by using the general method described above (this particular example is worked
out in detail in Müller and Meißner (1997)). Finally, there are the counter terms.
Altogether, there are 13 various operators with unknown low-energy constants. One combination, $d_1 + d_2$, can be fixed from the nucleon axial radius. This also constrains the yet unmeasured $p \rightarrow \Lambda K^+$ transition axial radius,

$$\langle r^2 \rangle_{p \rightarrow \Lambda K^+} = \frac{3\sqrt{2}}{D + 3F}(d_1 + 3d_2) = 0.23 \ldots 0.70 \text{ fm}^2,$$  \hspace{1cm} (29)

To fix the other LECs, $d_3, \ldots, d_{13}$, resonance saturation including the baryon decuplet and the vector meson nonet was used. I now summarize the results for the various final states (photoproduction case).

$K^0 \Sigma^+$: All LECs are determined by resonance exchange. The total cross section has been calculated for the first 100 MeV above threshold. No data point exists in this range so far, but soon the new ELSA data should be available. The electric dipole amplitude is real at threshold, we have $E_{0+}^{\text{thr}}(K^0 \Sigma^+) = 1.07 \times 10^{-3}/M_\pi$.

We also have given a prediction for the recoil polarization at $E_\gamma = 1.26$ GeV (which is the central energy of the lowest bin of the not yet published ELSA data).

$K^+ \Lambda$: The total cross section from threshold up to 100 MeV above is shown in Fig.2a (left panel). The lowest bin from ELSA, see Bockhorst et al. (1994), $E_\gamma \in [0.96, 1.01]$ GeV, has $\sigma_{\text{tot}} = (1.43 \pm 0.14)$ $\mu$b, i.e. we slightly underestimate the total cross section. In Fig.2b (left), I show the predicted recoil polarization $P$ at $E_\gamma = 1.21$ GeV (which is higher in energy than our approach is suited for). Amazingly, the shape and magnitude of the data is well described for forward angles, but comes out on the small side for backward angles. Most isobar models, that give a descent description of the total and differential cross sections also at higher energies, fail to explain this angular dependence of the recoil polarization.

$K^+ \Sigma^0$: The total cross section is shown in Fig.2a (right panel). It agrees with the two data points from ELSA. The recoil polarization at $E_\gamma = 1.26$ GeV is shown in Fig.2b (right). It has the right shape but comes out too small in magnitude. Nevertheless, one observes the important sign difference to the $K^+ \Lambda$ case, which is commonly attributed to the different quark spin structure of the $\Lambda$ and the $\Sigma^0$. Notice that this argument is strictly correct for massless quarks only. Here, it stems from an intricate interference of the complex S– and P–wave multipoles. In any case, one would like to have data closer to threshold and with finer energy binning to really test the CHPT scheme. Clearly, these results should only be considered indicative since one should include (a) higher order effects (for both the S– and P–waves), (b) higher partial waves and (c) has to get a better handle on the ranges of the various coupling constants. In addition, one would also need more data closer to threshold, i.e. in a region where the method is applicable. However, the results presented are encouraging enough to pursue a more detailed study of these reactions (for real and virtual photons) in the framework of “strict” chiral perturbation theory.
Fig. 2. Left panel: (a) Total cross section for $\gamma p \rightarrow K^+ A$ (solid line). The S–wave contribution is given by the dashed line. (b) Recoil polarization at $E_\gamma = 1.21$ GeV. Right panel: (a) Total cross section for $\gamma p \rightarrow K^+ \Sigma^0$. (b) Recoil polarization at $E_\gamma = 1.26$ GeV. The data are from Bockhorst et al. (1994).

4 Baryon Masses

The scalar sector of baryon CHPT is particularly interesting since it is sensitive to scalar–isoscalar operators and thus directly to the symmetry breaking of QCD. This is most obvious for the pion– and kaon–nucleon $\sigma$–terms, which measure the strength of the scalar quark condensates $\bar{q}q$ in the proton. Here, $q$ is a generic symbol for any one of the light quarks $u$, $d$ and $s$. Furthermore, the quark mass expansion of the baryon masses allows to gives bounds on the ratios of the light quark masses, see Gasser (1981). The most general effective Lagrangian to fourth order necessary to investigate the scalar sector consists of seven dimension two and seven dimension four terms with LECs plus some additional dimension two terms with fixed coefficients $\sim 1/m$. The dimension two terms with LECs fall into two classes, one related to symmetry breaking and the other are double–derivative meson-baryon vertices. The LECs related to the latter ones can be estimated with some confidence from resonance exchange. A method to estimate the symmetry breakers will be discussed below. Although the analysis of the octet baryon masses in the framework of chiral perturbation theory already has a long history, only recently the results of a calculation including all terms of second order in the light quark masses, $O(m_q^2)$, were presented in Borasoy and Meißner (1997). The calculations were performed in the isospin limit $m_u = m_d$ and the electromagnetic corrections were neglected. Previous investigations considered
mostly the so-called computable corrections of order $m_q^2$ or included some of the finite terms at this order. This, however, contradicts the spirit of CHPT in that all terms at a given order have to be retained. The quark mass expansion of the octet baryon masses takes the form

$$m = \hat{m} + \sum_q B_q m_q + \sum_q C_q m_q^{3/2} + \sum_q D_q m_q^2 + \ldots$$

modulo logs. Here, $\hat{m}$ is the octet mass in the chiral limit of vanishing quark masses and the coefficients $B_q, C_q, D_q$ are state-dependent. Furthermore, they include contributions proportional to some LECs which appear beyond leading order in the effective Lagrangian. In contrast to the $O(q^3)$ calculation, which gives the leading non-analytic terms $\sim m_q^{3/2}$, the order $q^4$ one is no longer finite and thus needs renormalization. Intimately connected to the baryon masses are the $\sigma$-terms (I only consider $\sigma_{\pi N}$ on what follows),

$$\sigma_{\pi N}(t) = \hat{m} \langle p' | \bar{u}u + \bar{d}d | p \rangle,$$

with $|p\rangle$ a proton state with four-momentum $p$ and $t = (p' - p)^2$ the invariant momentum transfer squared. A relation between $\sigma_{\pi N}(0)$ and the nucleon mass is provided by the Feynman–Hellmann theorem, $\hat{m} \left( \partial m_N / \partial \hat{m} \right) = \sigma_{\pi N}(0)$, with $\hat{m}$ the average light quark mass. Furthermore, the strangeness fraction $y$ and $\hat{\sigma}$ are defined via

$$y = \frac{2 \langle p | \bar{s}s | p \rangle}{\langle p | \bar{u}u + \bar{d}d | p \rangle} \equiv 1 - \frac{\hat{\sigma}}{\sigma_{\pi N}(0)}.$$

I now discuss some of the results presented in Borasoy and Meißner (1997). Some more details are given by Borasoy in these proceedings. As stated before, there are ten LECs related to symmetry breaking. Since there do not exist enough data to fix all these, they were estimated by means of resonance exchange. To deal with such scalar-isoscalar operators, the standard resonance saturation picture based on tree graphs was extended to include loop diagrams. In contrast to the two-flavor case, the scalar mesons in SU(3) cannot explain the strength of the symmetry breakers because these mesons are not effective degrees of freedom parametrizing strong pionic/kaonic correlations. To be precise, the dimension two symmetry breakers can be estimated by performing a best fit to the baryon masses based on a $O(q^3)$ calculation, see Bernard et al. (1993). For scalar couplings of “natural” size, these values can not even be reproduced within one order of magnitude. One way to solve this problem, although it has its own conceptual difficulties, is to consider besides standard tree graphs with scalar meson exchange also Goldstone boson loops with intermediate baryon resonances (spin–3/2 decuplet and the spin–1/2 (Roper) octet) for the scalar–isoscalar LECs. In Borasoy and Meißner (1997) a consistent scheme to implement resonance exchange under such circumstances was developed. In particular, it avoids double-counting and abids to the strictures from analyticity. Within the one–loop approximation and to leading order in the resonance masses, the analytic pieces of the pertinent graphs are still divergent, i.e. one
is left with three a priori undetermined renormalization constants \( \beta_\Delta, \delta_\Delta \) and \( \beta_R \). These have to be determined together with the finite scalar meson–baryon couplings \( F_S \) and \( D_S \) and the octet mass in the chiral limit. Using the baryon masses and the value of \( \sigma_{\pi N}(0) \) as input, one can determine all LECs in terms of one parameter, \( \beta_R \). This parameter can be shown to be bounded and the observables are insensitive to variations of it within its allowed range. Furthermore, it was also demonstrated that the effects of two (and higher) loop diagrams can almost entirely be absorbed in a redefinition of the one loop renormalization parameters. Within this scheme, one finds for the octet baryon mass in the chiral limit \( \bar{m} = 770 \pm 110 \) MeV. The quark mass expansion of the baryon masses, in the notation of Eq.(30), reads

\[
\begin{align*}
m_N &= \bar{m} (1 + 0.34 - 0.35 + 0.24) , \\
m_A &= \bar{m} (1 + 0.69 - 0.77 + 0.54) , \\
m_\Sigma &= \bar{m} (1 + 0.81 - 0.70 + 0.44) , \\
m_\Xi &= \bar{m} (1 + 1.10 - 1.16 + 0.78) .
\end{align*}
\] (33)

One observes that there are large cancellations between the second order and the leading non–analytic terms of order \( q^3 \), a well–known effect. The fourth order contribution to the nucleon mass is fairly small, whereas it is sizeable for the \( \Lambda \), the \( \Sigma \) and the \( \Xi \). This is partly due to the small value of \( \bar{m} \), e.g. for the \( \Xi \) the leading term in the quark mass expansion gives only about 55% of the physical mass and the second and third order terms cancel almost completely. From the chiral expansions exhibited in Eq.(33) one can not yet draw a final conclusion about the rate of convergence in the three–flavor sector of baryon CHPT. Certainly, the breakdown of CHPT claimed by Gasser (1981) is not observed. On the other hand, the conjecture by Jenkins and Manohar (1992) that only the leading non–analytic corrections (LNAC) \( \sim m_q^{3/2} \) are large and that further terms like the ones \( \sim m_q^2 \) are moderately small, of the order of 100 MeV, is not supported. The chiral expansion of the \( \pi N \sigma \)–term shows a moderate convergence, i.e. the terms of increasing order become successively smaller,

\[
\sigma_{\pi N}(0) = 58.3 (1 - 0.56 + 0.33) \text{ MeV} = 45 \text{ MeV} .
\] (34)

Still, the \( q^4 \) contribution is important. For the strangeness fraction \( y \) and \( \hat{\sigma} \), one finds

\[
y = 0.21 \pm 0.20 , \quad \hat{\sigma} = 36 \pm 7 \text{ MeV} .
\] (35)

The value for \( \hat{\sigma} \) compares favourably with Gasser’s estimate, \( \hat{\sigma} = 33 \pm 5 \) MeV. Finally, a comment concerning the difference of the pion–nucleon \( \sigma \)–term at \( t = 0 \) and at the Cheng–Dashen point is in order. In Bernard et al. (1997) it was shown that the remainder \( \Delta_R \) not fixed by chiral symmetry, i.e. the difference between the on–shell \( \pi N \) scattering amplitude \( \bar{D}^+(0, 2M_\pi^2) \) and the scalar form factor \( \sigma_{\pi N}(2M_\pi^2) \), contains no chiral logarithms and vanishes simply as \( M_\pi^4 \) in the chiral limit. In addition, an upper limit was reported, \( \Delta_R \leq 2 \) MeV. While this is a small effect, it is an important ingredient in the analysis of the \( \sigma \)–term.
5 Magnetic Moments

The magnetic moments of the octet baryons have been measured with high precision over the last decade. On the theoretical side, SU(3) flavor symmetry was first used by Coleman and Glashow (1961) to predict seven relations between the eight moments of the $p$, $n$, $A$, $\Sigma^\pm$, $\Sigma^0$, $\Xi^-$, $\Xi^0$, and the $\mu_{\Lambda \Sigma^0}$ transition moment in terms of two parameters. One of these relations is in fact a consequence of isospin symmetry alone. In modern language, this was a tree level calculation with the lowest order effective chiral meson–baryon Lagrangian of dimension two given in Eq.(25) and depicted in fig.3a. Given the simplicity of this approach, these relations work remarkably well, truly a benchmark success of SU(3). The first loop corrections arise at order $q^3$ in the chiral counting, see Caldi and Pagels (1974) (cf. fig.3b). They are given entirely in terms of the lowest order parameters from the dimension one (two) meson–baryon (meson) Lagrangian. It was found that these loop corrections are large for standard values of the two axial couplings $F$ and $D$. Caldi and Pagels (1974) derived three relations independent of these coupling constants. These are, in fact, in good agreement with the data. However, the deviations from the Coleman–Glashow relations get considerably worse. This fact has sometimes been taken as an indication for the breakdown of SU(3) CHPT. To draw any such conclusion, a calculation of order $q^4$ is mandatory. This was attempted by Jenkins et al. (1993), however, not all terms were accounted for. To be precise, in that calculation the contribution from the graphs corresponding to fig.3c-f were worked out. As pointed out by Meißner and Steininger (1997), there are additional one–loop graphs at $\mathcal{O}(q^4)$.
namely tadpole graphs with double–derivative meson–baryon vertices (fig. 3g) and diagrams with fixed $1/m$ and symmetry breaking insertions $\sim b_{D,F}$ from the dimension two Lagrangian, see fig. 3h,i. Some (but not all) of these contributions were implicitly contained in the work of Jenkins et al. (1993) as becomes obvious when one expands the graphs with intermediate decuplet states. However, apart from these decuplet contributions to the LECs (in the language used here), there are also important $t$–channel vector meson exchanges which are not accounted for if one includes the spin–$3/2$ decuplet in the effective theory and calculates the corresponding tree and loop graphs. In total, there are seven LECs related to symmetry breaking and three related to scattering processes (the once appearing in the class of graphs fig. 3g). These latter LECs can be estimated with some accuracy from resonance exchange. The strategy of Meißner and Steininger (1997) was to leave the others as free parameters and fit the magnetic moments. One is thus able to investigate the chiral expansion of the magnetic moments and to predict the $\Lambda\Sigma^0$ transition moment. The chiral expansion of the various magnetic moments thus takes the form

$$\mu_B = \mu_B^{(2)} + \mu_B^{(3)} + \mu_B^{(4)} = \mu_B^{(2)} (1 + \epsilon^{(3)} + \epsilon^{(4)})$$

with the result (all numbers in nuclear magnetons)

$$\begin{align*}
\mu_p &= 4.48 (1 - 0.49 + 0.11) = 2.79, \\
\mu_n &= -2.47 (1 - 0.34 + 0.12) = -1.91, \\
\mu_{\Sigma^+} &= 4.48 (1 - 0.62 + 0.17) = 2.46, \\
\mu_{\Sigma^-} &= -2.01 (1 - 0.31 - 0.11) = -1.16, \\
\mu_{\Sigma^0} &= 1.24 (1 - 0.87 + 0.40) = 0.65, \\
\mu_A &= -1.24 (1 - 0.87 + 0.37) = -0.61, \\
\mu_{\Xi^0} &= -2.47 (1 - 0.89 + 0.40) = -1.25, \\
\mu_{\Xi^-} &= -2.01 (1 - 0.64 - 0.03) = -0.65, \\
\mu_{\Lambda\Sigma^0} &= 2.14 (1 - 0.53 + 0.19) = 1.40,
\end{align*}$$

setting here the scale of dimensional regularization $\lambda = 800$ MeV. In all cases the $O(q^4)$ contribution is smaller than the one from order $q^3$ by at least a factor of two, in most cases even by a factor of three. Like in the case of the baryon masses, one finds sizeable cancellations between the leading and next–to–leading order terms making a precise calculation of the $O(q^4)$ terms absolutely necessary. In particular, the previously omitted terms with a symmetry-breaking insertion from Eq.(22) (fig.3h) turn out to be very important. One can predict the transition moment to be $\mu_{A\Sigma^0} = (1.40 \pm 0.01)\mu_N$ in agreement with the lattice gauge theory result of Leinweber et al. (1991), $\mu_{A\Sigma^0} = (1.54 \pm 0.09)\mu_N$. Jenkins et al. (1993) derived one relation amongst the magnetic moments independent of the axial couplings. This relation does not hold any more in the complete $O(q^4)$ calculation. To be specific, the graphs fig.3h with a scalar symmetry breaking insertion do not respect this relation (for details, consult the appendix in Meißner and Steininger (1997)). Of course, this is not quite the end of the story. What is certainly missing is a deeper understanding of the numerical values of the
6 Summary and outlook

A final conclusion on the convergence of three flavor baryon chiral perturbation theory can not yet be drawn. While the scalar sector still looks somewhat discouraging (which might be due to the model used to estimate the LECs), the first steps including electromagnetic probes like for the magnetic moments and kaon photoproduction off protons appear encouraging. From the long list of topics to be addressed I personally view three as most relevant: a) we should try to sharpen the calculations of the mass splittings in the baryon octet, b) repeat the analysis of the Goldberger–Treiman discrepancies (GTDs) and c) finally settle the old problem of the hyperon nonleptonic decays first addressed by Bijnens et al. (1985). Clearly, more precise data are mandatory, for example the present day knowledge on the NYK coupling constants is absolutely insufficient for extracting stringent bounds on $m_s/\hat{m}$ from the octet GTDs. I am hopeful that more and more precise data will come from ELSA, Jefferson Lab, MAMI and DAΦNE helping us to resolve all these puzzles.

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