Testing supersymmetric Higgs inflation with non-Gaussianity

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We investigate multi-field signatures of the nonminimally coupled supersymmetric Higgs inflation-type cosmological scenario, focusing on the two-field Higgs-lepton inflation model as a concrete example. This type of inflationary model is realized in a theory beyond the Standard Model embedded in supergravity with a noncanonical Kähler potential. We employ the backward δN formalism to compute cosmological observables, including the scalar and tensor power spectra, the spectral indices, the tensor-to-scalar ratio and the local-type nonlinearity parameter. The trajectory of the inflaton is controlled by the initial conditions of the inflaton as well as by the coefficients in the Kähler potential. We analyze the bispectrum of the primordial fluctuations when the inflaton trajectory deviates from a straight line, and obtain constraints on the noncanonical terms of the Kähler potential using the Planck satellite data. Our analysis represents a concrete particle phenomenology-based case study of inflation in which primordial non-Gaussianities can reveal aspects of supergravity.

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I. INTRODUCTION

In cosmology, the precision of measurements has dramatically improved in the last decade or so. The recent Planck satellite experiments of the cosmic microwave background (CMB), for example, indicate that the scalar spectral index ns, the tensor-to-scalar ratio r and the local-type nonlinearity parameter fNL are in the following windows [1–3]:

\[ n_s = 0.9603 \pm 0.0073 \quad (68\% \, \text{C.L.}), \]
\[ r < 0.12 \quad (95\% \, \text{C.L.}), \]
\[ f_{NL} = 2.7 \pm 5.8 \quad (68\% \, \text{C.L.}) \quad (1) \]

Eventually, these data are to be accounted for by a model of the Universe based on, ideally, a well-motivated theory of particle physics. The leading account of the early Universe in agreement with present observational data is inflationary cosmology, which emerged as a solution to the flatness, horizon, and monopole problems of the standard Big Bang cosmology. Currently, inflationary model building is somewhat postmodernistic — there are a plethora of toy models inspired by string theory and M-theory, among others, and many of them can be adjusted to fit the data. Future observation could change this situation, however, as measurements with increasing accuracy are expected to put many models under pressure.

In order to build a realistic cosmological scenario beyond inflationary toy models, supersymmetric extension of the Standard Model provides a technically natural and phenomenologica well-motivated framework. A consistent scenario of cosmology needs to be compatible with physics at low energies, including particle phenomenology at collider scales, and thus must incorporate the Standard Model in some form. Moreover, if the energy scale of inflation turns out to be as high as \( H \approx 10^{14} \, \text{GeV} \) \( (H \) is the Hubble parameter) as implied \(^1\) by the BICEP2 experiments \(^7\), it is plausible that supersymmetry plays some role in the physics of inflation. Recently there has been a keen interest in the Standard Model Higgs inflation model \(^8\,9\), in which the gravitationally coupled Higgs field is identified as the inflaton. A supersymmetric version of the Higgs inflation model was implemented first in the next-to-minimal supersymmetric Standard Model (NMSSM) \(^10\,13\). Subsequently, various other models — based on the supersymmetric Pati-Salam model \(^14\), the supersymmetric grand unified theory \(^15\), the supersymmetric B-L model \(^16\) and the supersymmetric seesaw model \(^17\,19\) — were proposed. In contrast to the Standard Model Higgs inflation model, these supersymmetric models necessarily involve multiple scalar fields participating in the dynamics of inflation. The effects of multiple fields, so far, have not been studied in full detail, due to the complexities pertaining to the larger degrees of freedom.

In this paper we discuss non-Gaussianities of the primordial fluctuations in these supersymmetric Higgs inflation models. It is well known that single-field inflation typically predicts primordial fluctuations of Gaussian spectrum; hence detection of sizeable non-Gaussianities would be a strong evidence for multi-field inflation. Since present observation of cosmological parameters is all consistent with the prediction of single-field inflation \(^2\), we shall take a modest approach and start from a single-field limit, that is, inflation with a straight inflaton trajectory. We then analyze how the prediction for the bispectrum

\(^1\) Presuming that the observed B-mode polarization results from the primordial tensor mode fluctuations. See also \(^4\,5\).
changes as the trajectory deviates from a straight line. For the sake of concreteness, we consider the inflationary model based on the supersymmetric seesaw model, which is dubbed the supersymmetric Higgs-lepton inflation (HLI) model [17,19]. Also, we focus on the two-field case for simplicity. To compute the fluctuation spectrum of the inflationary model, we use the backward formulation [23,24] of the δN-formalism [25,26]. We find by numerical computations that the bispectrum of the inflationary model is susceptible to change of the inflaton trajectory, a fact known in generic cases, see e.g. [29,32]. Since the shape of the trajectory depends on a parameter of the Kähler potential in the class of inflationary models we consider, constraints on the Kähler potential are obtained from the experimental bounds of non-Gaussianities [1]. While the details can be model-dependent, the generic features of the outcome should be common in similar models. To illustrate another example of supersymmetric Higgs inflation, we comment on the NMSSM-based model in Appendix [3].

Non-Gaussianities have been studied extensively in various multi-field inflationary models. In the literature, studies of inflationary models with nontrivial field-space resulting from nonminimal coupling include [32,34]. Multi-field studies of supergravity-based inflationary toy models similar to ours in spirit include [35,37].

The rest of this paper is organized as follows. In Sec. II we illustrate the HLI model which is our main focus. In Sec. III we give a brief review of the backward δN-formalism and define quantities describing cosmological observables. The numerical results are shown in Sec. IV and observational constraints on the parameter space are also discussed there. We conclude in Sec. V with comments. Some formulae of the δN-formalism are collected in Appendix A and the NMSSM-based supersymmetric Higgs inflation model is described briefly in Appendix B.

II. INFLATIONARY MODEL

In this paper we consider a model of inflation described by the Lagrangian density 2

\[ \mathcal{L} = \sqrt{-g} \left[ \frac{1}{2} \mathcal{R} - \frac{1}{2} G_{IJ} g^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J - V(\phi^I) \right], \]

(2)

where \( g_{\mu\nu} \) is the spacetime metric (we consider the flat Friedmann-Lemaître-Robertson-Walker background metric), \( \mathcal{R} \) the Ricci scalar of the spacetime and \( g \equiv \det g_{\mu\nu} \). We have two real scalar fields \( \phi^I \equiv s \) and \( \phi^2 \equiv h \). The indices are \( \mu, \nu, \cdots = 0, 1, 2, 3 \) and \( I, J, \cdots = 1, 2 \). We will be interested in the special form of the field space metric \( G_{IJ} \) given by

\[ G_{ss} = \frac{12}{72} (1 + 2\xi s^2 - \xi h^2) \frac{\phi^2}{\Phi^2}, \]

\[ G_{sh} = G_{hs} = \frac{\xi h(1 - v^2 s^2)}{\Phi^2}, \]

\[ G_{hh} = \frac{6\xi^2 h^2 + \Phi}{\Phi^2}, \]  

(3)

where \( \xi, v \) (Greek letter upsilon) are real parameters and

\[ \Phi = 1 - \frac{1}{6} s^2 + \frac{1}{12} v s^4 + \xi h^2. \]  

(4)

The Christoffel symbol on the field space is

\[ \Gamma^I_{11} = \frac{\phi^2}{\Phi^2} \left[ s^2 - 12 (1 + 6\xi h^2) + (1 - v^2 s^2) \right], \]

\[ \Gamma^I_{12} = \frac{\xi h}{\Phi^2} + \frac{1}{6C}, \quad \Gamma^I_{22} = -\frac{1}{6C}, \]

\[ \Gamma^I_{12} = -\frac{\phi^2}{\Phi^2} \frac{\xi h}{C}, \quad \Gamma^I_{12} = \frac{1 - \phi^2}{\Phi^2} \frac{s^2}{C}, \]

\[ \Gamma^I_{22} = \frac{12}{12h} (1 - \xi h^2) + s^2 - 12 + (s^2 - 24) v^2 s^2 \]

\[ = 1 - 2v s^2 + (1 + 6\xi)(1 - 2v s^2) h^2 + \frac{1}{12} v s^4. \]  

(6)

The scalar curvature of the field space is

\[ R = -\frac{1}{3} \frac{(1 + 6\xi)\phi^2}{3C^2}. \]  

(7)

Note that the Riemann curvature is written by the scalar curvature as \( R^{IJKL} = \frac{1}{2} (\delta^K_L G_{IJ} - \delta^K_I G_{LJ}) \) in two dimensions.

The two-field Lagrangian [2] with the field space metric [3] is obtained from supergravity with a particular type of (noncanonical) Kähler potential. This class of cosmological scenario includes those based on the NMSSM [10,11], the supersymmetric Pati-Salam model [13], the supersymmetric grand unified theory [15], and the supersymmetric seesaw model [17,19]. The form of the potential \( V(\phi^I) \) depends on details of each phenomenological setup. In this paper we focus on the HLI model based on the supersymmetric seesaw. Below in this section we review the construction of this model [17,19]. For comparison, we sketch the model based on the NMSSM in Appendix B.

A. Supersymmetric seesaw model

The supersymmetric seesaw model is an extension of the minimal supersymmetric Standard Model (MSSM)
along the up type Higgs doublet-lepton doublet ($L$-$H_u$) parameters and $\phi$ is fixed to $\phi = 3.696 \times 10^{-3}$ by the condition that in the single-field limit the amplitude of the curvature perturbation corresponding to $N_e = 60$ e-folds is Planck-normalized $A_s = 2.215 \times 10^{-9}$ [2]. The red curves are the inflaton trajectories with initial conditions $s_{\text{init}} = 0, \dot{s}_{\text{init}} = 0$ at $h = h_{\text{init}} = 21.99$ (this value of $h_{\text{init}}$ corresponds to $N_e = 60$ e-folds in the single-field limit); the initial value for $h$ is determined by the slow-roll equation of motion. On each panel the point $(s, h) = (0.2199)$ is marked with a black dot. On the left panel ($v = 0$), the flat regions on the sides represent negative $V(\phi^i)$ which are considered unphysical. For small values of $v$ a trajectory can reach the supersymmetric vacuum $(s, h) = (0, 0)$ only when the initial conditions are fine-tuned ($s_{\text{init}} = 1.617 \times 10^{-11}, \dot{s}_{\text{init}} = 0$ for the yellow dashed curve). For generic initial conditions the inflaton will fall into either of the $V(\phi^i) < 0$ regions (so does the red curve in the case of $s_{\text{init}} = 0, \dot{s}_{\text{init}} = 0$). When $v = 0.055$ (center) the potential is stabilized in the $s$-field direction. The orange dotted curve that makes a mild turn corresponds to $s_{\text{init}} = 1.0 \times 10^{-5}, \dot{s}_{\text{init}} = 0$. When $v = 0.1$ (right), the trajectories are more convergent. Two trajectories [initial conditions $(s_{\text{init}}, \dot{s}_{\text{init}}) = (0, 0)$ and $(1.0 \times 10^{-5}, 0)$] are shown, but they are almost indistinguishable.

by adding a right-handed neutrino superfield $N_R^c$. Its simplest version is described by the superpotential

$$ W = W_{\text{MSSM}} + \frac{1}{2} M N_R^c N_R^c + y_D N_R^c LH_u, \quad (8) $$

where $y_D$ is the Dirac Yukawa coupling, $M$ the seesaw mass parameter and

$$ W_{\text{MSSM}} = \mu H_u H_d + y_u u^c Q H_u + y_d d^c Q H_d + y_e e^c L H_d, \quad (9) $$

with the MSSM superfields $Q, u^c, d^c, L, e^c, H_u$ and $H_d$. In [3] $\mu$ is the MSSM $\mu$-parameter and $y_u, y_d, y_e$ are the Yukawa couplings. Assuming odd R-parity for $N_R^c$, the superpotential [5] preserves the R-parity. For generation of the small nonvanishing (left-handed) neutrino masses by the seesaw mechanism [38%-40], the Dirac Yukawa coupling $y_D$ and the right-handed neutrino mass $M$ in [8] must satisfy the seesaw relation

$$ m_\nu = \frac{y_D^2 \langle H_u \rangle^2}{M}, \quad (10) $$

where $m_\nu$ is the left-handed neutrino mass and $\langle H_u \rangle \approx 174$ GeV is the Higgs vacuum expectation value at low energies. Evaluating the neutrino mass by $m_\nu^2 = \Delta m_{32}^2 = 2.44 \times 10^{-3}$ eV$^2$ [41], we find

$$ y_D = \left( \frac{M}{6.13 \times 10^{14} \text{GeV}} \right)^{1/2}. \quad (11) $$

B. Higgs-lepton inflation

The HLI model assumes that slow-roll takes place along the up type Higgs doublet-lepton doublet ($L$-$H_u$) D-flat direction of the supersymmetric seesaw model. Parametrizing this direction using a superfield $\varphi$ as

$$ L = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi \\ 0 \end{pmatrix}, \quad H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \varphi \end{pmatrix}, $$

the superpotential becomes, ignoring $Q, u^c, d^c, e^c$, and $H_d$ that do not play any role during inflation,

$$ W = \frac{1}{2} MN_R^c N_R^c + \frac{1}{2} y_D N_R^c \varphi^2. \quad (13) $$

This is embedded in supergravity with the Kähler potential (in the superconformal framework) $K = -3\Phi$, where the real function $\Phi$ is chosen to be

$$ \Phi = 1 - \frac{1}{3} \left( |N_R^c|^2 + |\varphi|^2 \right) + \frac{1}{4} \gamma (\varphi^2 + \text{c.c.}) + \frac{1}{3} v |N_R^c|^4, \quad (14) $$

with $\gamma, v \in \mathbb{R}$. The term proportional to $\gamma$ violates the R-parity (which is benign [18]) and the one proportional to $v$ represents a higher dimensional term that controls the inflaton trajectory. For simplicity, we consider only one generation of the right-handed neutrinos [3]

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3 It is straightforward to extend this model to the phenomenologically realistic cases of 2 or 3 generations of the right-handed neutrinos [18].
components), the scalar-gravity part of the Lagrangian reads

$$\mathcal{L}_J = \sqrt{-g_J} \left[ \frac{1}{2} \Phi R_J - \frac{1}{2} g_J^{\mu \nu} \partial_\mu h \partial_\nu h - \frac{1}{2} \kappa g_J^{\mu \nu} \partial_\mu \phi s \partial_\nu s - V_J \right],$$

(15)

where

$$\kappa = 1 - 2v^2,$$

$$\xi = \frac{1}{4} \gamma - \frac{1}{6},$$

and $\Phi$ is given by (4). For $v \neq 0$ the Kähler metric is nontrivial. The potential is found to be

$$V_J = \frac{1}{4} y_D^2 h^2 s^2 + \frac{(2\sqrt{2} M s + y_D h^2)^2}{16(1-2v^2)}$$

$$- \frac{1}{8} \left( \sqrt{2} M s + 3\gamma y_D h^2 - \frac{v x (y_D h^2 + 2\sqrt{2} M s)}{1-2v^2} \right)^2$$

$$- \frac{3\gamma h^2 (2\gamma - 1)}{12 + \frac{v x}{1-2v^2}}.$$  

(17)

The Lagrangian (15) involves nonminimal coupling of the scalar fields to gravity (the subscript $J$ stands for the Jordan frame). Upon Weyl rescaling of the metric one may go to the Einstein frame in which the scalars are minimally coupled to gravity. The resulting Lagrangian is the one we saw at the beginning (2) with the scalar potential in the Einstein frame given by $V(\phi^I) = \Phi^{-2} V_J$.

One can see from (4) and (17) that the shape of the potential $V(\phi^I)$ is controlled by the four parameters $M$, $y_D$, $\gamma$ (or $\xi$) and $v$, among which $y_D$ is determined by the seesaw relation (11) from $M$. Moreover, the amplitude of the curvature perturbation (we use the Planck normalization $A_s = 2.215 \times 10^{-9}$) provides constraints on the shape of the potential; we use this condition to fix the value of $\xi$ for a given number of e-folds $N_e$ (see Sec. II C below). Thus, when $N_e$ and $M$ are given, the potential depends only on $v$. In Fig. 1 we depict the shape of the potential in the Einstein frame $V(\phi^I)$ when $M = 1$ TeV, $N_e = 60$, and $v$ is varied as $v = 0, 0.055, 0.1$. For large values of $v$, the $s$-field becomes massive and the inflaton trajectory is forced to lie along the $s = 0$ direction. This feature is used in the previous studies of the supersymmetric Higgs inflation type scenarios [10,13,17,19] where only single-field inflation was considered. For smaller $v$, an inflaton trajectory is curved (see Fig.1) and the single-field inflation picture breaks down. While it is possible to consider the single-field case by introducing a large enough quartic Kähler term, it would certainly be important to investigate what will happen to the cosmological observables when $v$ is smaller and the multi-field effects are not negligible. Before start studying the multi-field case in Sec. III, we shall briefly review the prediction of this model in the single-field limit.

C. Prediction in single-field limit

This type of inflationary model has been analyzed in detail in the single-field limit [11,12,14,15,17,19] (see also [10,13]). Here we summarize the prediction of the HLI model. When the $s$-field is stabilized at $s = 0$, the potential (17) dramatically simplifies and the Lagrangian (15) becomes

$$\mathcal{L} = \sqrt{-g} \left[ \frac{1}{2} \Phi R - \frac{1}{2(1 + \xi h^2)^2} g^{\mu \nu} \partial_\mu h \partial_\nu h - \frac{y_D^2 h^2}{16(1 + \xi h^2)^2} \right].$$

(18)

This is the Lagrangian of the nonminimally coupled $\lambda \phi^4$ model [49], which includes the nonminimally coupled Standard Model Higgs inflation model [3,9] as a special case. Note that the Einstein frame Lagrangian (18) is also [10,13].

| $M$ (GeV) | $y_D$ | $N_e = 50$ | $N_e = 60$ | $N_e = 70$ |
|-----------|-------|-----------|-----------|-----------|
| $10^3$    | $1.277 \times 10^{-6}$ | 0.001588   | 0.003696  | 0.005957  |
| $10^4$    | $4.039 \times 10^{-6}$ | 0.03333    | 0.04612   | 0.05956   |
| $10^6$    | $4.039 \times 10^{-5}$ | 0.7862     | 0.9490    | 1.122     |
| $10^{12}$ | 0.04039 | 868.4     | 1031      | 1194      |

TABLE I: The seesaw mass $M$, the Dirac Yukawa coupling $y_D$ and the nonminimal coupling parameter $\xi$. These quantities are related one-to-one to the seesaw relation (11) and the Planck normalization of the curvature perturbation.
can be obtained directly by Weyl-transforming the Jordan frame Lagrangian

\[ \mathcal{L}_J = \sqrt{-g_J} \left[ \frac{1}{2} \mathcal{R}_J - \frac{1}{2} g_J^{\mu \nu} \partial_\mu h \partial_\nu h - \lambda h^4 \right]. \]  

(19)

where we identify \( \lambda = \frac{y_D^2}{16} \).

The single-field model \cite{18} contains two real parameters \( \xi \) and \( y_D \). These are not independent, given that the amplitude of the primordial curvature perturbation is normalized as \( A_s = 2.215 \times 10^{-9} \) \cite{2}. The model is then parametrized by (say) \( y_D \) only, for a given number of e-folds \( N_e \). The value of \( \lambda = \frac{y_D^2}{16} \) depends on phenomenological setup underlying the inflationary model. In the case of Standard Model Higgs inflation the parameter \( \lambda \) is the Higgs self-coupling, \( \lambda \sim \mathcal{O}(1) \) in the low energies, which gives \( \xi \sim 10^3 - 10^4 \) (with the renormalization group effects taken into account). In the HLI model, in contrast, there is no severe experimental constraints on the Dirac Yukawa coupling \( y_D \), and correspondingly the value of the nonminimal coupling parameter can be \( \xi \ll \mathcal{O}(1) \). In Table I we list the values of \( M \), \( y_D \), and \( \xi \) for \( N_e = 50, 60, \) and 70 e-foldings.

Once the shape of the single-inflaton potential in the Einstein frame is determined, the slow-roll paradigm gives prediction for the spectra of the primordial fluctuations. The values of the scalar spectral index \( n_s \) and the tensor-to-scalar ratio \( r \) in the single-field case is shown in Fig. 2 for given values of the e-folding number \( N_e \) and the seesaw mass parameter \( M \) (recall that \( M \) and \( y_D \) are related by \cite{11}). The uppermost points \((M = 589 \text{ GeV}, 346 \text{ GeV}, 220 \text{ GeV})\) correspond to minimal coupling, \( \xi = 0 \). Also shown on the background are the 68\% and 95\% confidence level (CL) contours from the Planck (Planck+WP+highL, red) \cite{1} and the BICEP2 (Planck+WP+highL+BICEP2, blue) \cite{2} experiments. An interesting feature of the HLI model is that observation of the tensor-to-scalar ratio gives the seesaw scale \( M \). Small values of \( r \) (the observation by the Planck satellite) indicate large seesaw scale, whereas large \( r \) implied by the BICEP2 experiments small seesaw scale. It has been also pointed out that if the underlying theory is the type III seesaw mechanism, the parameter region favored by BICEP2 falls into an interesting mass range that can be searched by the Large Hadron Collider (LHC) at 14 TeV run \cite{19}. The HLI model also has other salient features. It is based on the well-motivated supersymmetric seesaw model of particle physics, naturally explaining the small neutrino masses through the seesaw mechanism \cite{38,10}; the unitarity problem of the Standard Model Higgs inflation \cite{50,55} is alleviated as the coupling \( \xi \) can be small; leptonogenesis can be implemented. The model can be tested by the future ground-based, satellite and collider experiments, and thus the prediction of the model deserves careful study. In the next section, we explain the \( \delta N \)-formalism on which our numerical study of the two-field inflation model is based.

### III. \( \delta N \)-FORMALISM

In this section we collect elements of cosmological perturbation theory that are needed for our numerical study. Our computation of various cosmological observables is based on the \( \delta N \)-formalism \cite{20,28}, which has become standard for studying multi-field inflation. The \( \delta N \)-formalism is particularly powerful for analyzing the super-horizon evolution of the curvature perturbation, which is our main focus. We essentially follow the notations of \cite{20,21} (also \cite{22,50}).

#### A. Backward formalism

We shall start by writing the background Klein-Gordon equation and the Friedmann equation in the following forms:

\[ \frac{d^2 \phi^I}{dN^2} + \Gamma^I_{JK} \frac{d\phi^J}{dN} \frac{d\phi^K}{dN} + \left( 3 + \frac{1}{H} \frac{dH}{dN} \right) \frac{d\phi^I}{dN} + \frac{G^{IJ} \partial_I V}{H^2} = 0, \]  

(20)

\[ H^2 = \frac{1}{3} \left( V + \frac{2}{3} H^2 G_{IJ} \frac{d\phi^I}{dN} \frac{d\phi^J}{dN} \right), \]  

(21)

where \( I = 1, 2 \). We have chosen the e-folding number \( N \) defined by \( dN = Hdt \) as the time variable. The e-folding number \( N_e \) in the previous section is \( N_e = N_{\text{end of inflation}} - N_s \), where \( N_s \) is the e-folding number at the horizon exit of the CMB scale. The metric \( G_{IJ} \) and the Christoffel symbol \( \Gamma^I_{JK} \) of the field space are given by \cite{3} and \cite{6}. Renaming the field value and the field velocity as

\[ \varphi^I_\tau \equiv \phi^I, \quad \varphi^I_\tau \equiv \frac{d\phi^I}{dN} = \frac{d\varphi^I_\tau}{dN}, \]  

(22)

the background equations of motion become

\[ F_1^I \equiv \frac{d\varphi^I_\tau}{dN} = \varphi^I_\tau, \]  

\[ F_2^I \equiv \frac{2}{6 - G_{IJ} \varphi^J_\tau} \]  

(23)

\[ H^2 = \frac{2V}{6 - G_{IJ} \varphi^J_\tau \varphi^I_\tau}. \]  

(24)

Here, \( V^I \equiv G^{IJ} \partial_J V \) and \( D \) denotes the covariant derivative in the field space, i.e. \( D \varphi^I_\tau = d\varphi^I_\tau + \Gamma^I_{JK} \varphi^J_\tau d\varphi^K_\tau \). Following \cite{20,22}, we introduce a compact notation,

\[ X^a = X^I_\tau. \]  

(25)
The slow-roll parameters are defined as
d\frac{\ddot{H}}{H^2} = -\frac{1}{H}\frac{dH}{dN}, \quad \eta \equiv \frac{\dot{\epsilon}}{cH}, \quad (26)

(the dot denotes derivative with respect to the cosmic time) and we assume $\epsilon, |\eta| < 1$ during inflation and max\{\epsilon, |\eta|\} $= 1$ at the end of inflation. Temporal violation of the slow-roll approximation is possible as long as the number of e-foldings is not affected significantly, but we do not need to consider such cases in our study.

Based on the $\delta N$-formalism, the curvature perturbation $\zeta$ is related to the difference of the number of e-foldings between an initial flat hypersurface and a final uniform energy density hypersurface. We take the initial flat hypersurface to be at the Hubble exit time (i.e. $N = N_e$) and the final time to be $N = N_c$ after which the background trajectories converge. The curvature perturbation, then, will remain constant for $N > N_c$. Thus we find

$$\zeta(N_c) \approx \delta N(N_c, \varphi(N_e)) = N_a^* \delta \varphi_a^* + \frac{1}{2} N_{ab}^* \delta \varphi_a^* \delta \varphi_b^* + \cdots, \quad (27)$$

where $\delta \varphi_a^* \equiv \delta \varphi_a(N_e)$ is the perturbation evaluated at the initial flat hypersurface and $N_a^* \equiv DN/\partial \varphi^a$ etc.

The perturbations of the scalar fields on the constant energy density hypersurface, i.e. in the $N = \text{constant}$ gauge, are given by

$$\delta \varphi^a(\lambda, N) \equiv \varphi^a(\lambda + \delta \lambda, N) - \varphi^a(\lambda, N), \quad (28)$$

where $\lambda$'s are the $2n - 1$ integration constants for an $n$-component scalar field (we consider $n = 2$), parameterizing the initial values of the fields \[20\, \[21\]. We will be interested in cosmological observables up to the bispectrum of the curvature perturbation. For our purposes it is convenient to decompose $\delta \varphi^a$ into the first- and second-order quantities as

$$\delta \varphi^a = \delta \varphi^a + \frac{1}{2} \delta \varphi^a. \quad (29)$$

Perturbing the set of the first-order equations of motion \[23\] we obtain

$$\frac{d}{dN} \delta \varphi^a(N) = P^a_b(N) \delta \varphi^b(N), \quad (30)$$

and

$$\frac{d}{dN} \delta \varphi^a(N) = P^a_b(N) \delta \varphi^b(N) + Q^a_{bc}(N) \delta \varphi^b(N) \delta \varphi^c(N). \quad (31)$$

The explicit forms of $P^a_b, Q^a_{bc}$ are given in Appendix A.1

We may write down formal solutions of \[30\] and \[31\] as

$$\delta \varphi^a(N) = \Lambda^a_b(\lambda, N_e) \delta \varphi^b(N_e), \quad (1)$$

$$\delta \varphi^a(N) = \int_{N_e}^N dN' Q^b_{cd}(N, N') \delta \varphi^c(N') \delta \varphi^d(N'), \quad (2)$$

where $\Lambda^a_b$ satisfies

$$\frac{d}{dN} \Lambda^a_b(N, N_e) = P^a_c(N) \Lambda^c_b(N, N_e), \quad (3)$$

and $\Lambda^a_b(N, N_e) = \delta^a_b$. Here we have chosen $\lambda^a = \varphi^a(N_e)$ so that we have $\delta \varphi^a(N_e) = \delta \varphi^a$. Thus the second-order perturbation vanishes at $N = N_e$.

Now if we take $N_F$ to be some time later during the scalar dominant phase, the curvature perturbation is rewritten as follows:

$$\zeta(N_e) \approx \delta N(N_e, \varphi(N_F)) = N_a^* \delta \varphi^a + \frac{1}{2} N_{ab}^* \delta \varphi^a \delta \varphi^b + \cdots \quad (4)$$

Comparing with \[27\] we have

$$N_a^* = N_a^F \Lambda^b_a(N_F, N_e) \quad (35)$$

and

$$N_{ab}^* = N_{ab}^F \Lambda^c_a(N_F, N_e) \Lambda^d_b(N_F, N_e) + 2 \int_{N_e}^{N_F} dN' \varphi^c(N') \Lambda^d_a(N', N_e) \Lambda^e_b(N', N_e). \quad (36)$$

It is convenient to introduce a quantity $\Theta$ defined by

$$\Theta^a(N) \equiv \Lambda^c_a(N, N_e) A^{ab} N_b^*, \quad (37)$$

where $A^{ab}$ is the normalization factor of the two point correlation function $\langle \delta \varphi^a \delta \varphi^b \rangle$ including the slow-roll corrections \[24\, \[57\, \[58\]. The definition and the explicit forms of $A^{ab}$ are given in Appendix A.1 Then $N_a(N)$ and $\Theta^a(N)$ satisfy the following equations:

$$\frac{d}{dN} N_a(N) = -N_b(N) P^a_b(N), \quad (38)$$

$$\frac{d}{dN} \Theta^a(N) = P^a_b(N) \Theta^b(N). \quad (39)$$

Following the prescription of \[20\], we first solve the first equation of \[38\] backward until $N = N_e$, with the initial conditions $N_a(N_F) = N_{ab}^F$. Then, with the initial conditions $\Theta^a(N_e) = A^{ab} N_b^*$, we solve the second equation of \[38\] forward until $N = N_F$.

The explicit expressions for $N_a^F$ and $N_{ab}^F$, which are presented in Appendix A.2 may be obtained by using the fact that the uniform energy density hypersurface is

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4 These slow-roll parameters are convenient for multi-field inflation. In the single-field limit they are related to $\epsilon_V \equiv M_P^2 (V'/V)^2/2, \eta_V \equiv M_P^2 V''/V$ by $\epsilon = \epsilon_V$ and $\eta = 4\epsilon_V - 2\eta_V$. 
equivalent to the constant Hubble hypersurface on the super-horizon scales \( [20] \) (see also \([32, 59]\)),

\[
H(\varphi^a(N_F + \zeta(N_F))) = H(\varphi^a(N_F)),
\]

(39)

where \( \varphi^a \) are the background trajectories. Note that \( N_F \) is a uniform energy density hypersurface and we neglect the later evolution of the curvature perturbations \([20]\).

### B. Cosmological observables

Using the backward formalism, one can compute various cosmological observables. Here we give the expressions for the scalar and tensor power spectra, the scalar and tensor spectral indices, the tensor-to-scalar ratio, and the nonlinearity parameter \([20, 23, 56]\).

#### 1. Power spectra

In momentum space the two-point correlator of the curvature perturbation is written as

\[
\langle \zeta_{k_1} \zeta_{k_2} \rangle = (2\pi)^3 \delta^3 (k_1 + k_2) P_{\zeta}(k).
\]

(40)

The power spectrum of the scalar perturbation is given by

\[
P_S = \frac{k^3}{2\pi^2} P_{\zeta}(k),
\]

(41)

and in the \( \delta N \)-formalism it is expressed as \([23, 58]\)

\[
P_S = \left(\frac{H_*}{2\pi}\right)^2 A_{ab}^* N_a^* N_b^*,
\]

(42)

where the explicit form of \( A_{ab}^* \) is given in Appendix A.1. Similarly, the power spectrum of the tensor perturbation is

\[
P_T = \frac{k^3}{2\pi^2} P_h(k),
\]

(43)

where \( P_h(k) \) is given by the two-point correlator of the tensor perturbation

\[
\langle h_{ij}(k_1) h^{ij}(k_2) \rangle = (2\pi)^3 \delta(k_1 + k_2) P_h(k).
\]

(44)

In the \( \delta N \)-formalism,

\[
P_T = 8 \left(\frac{H_*}{2\pi}\right)^2 [1 - (1 + \alpha)\epsilon]_s,
\]

(45)

where \( \alpha \equiv 2 - \ln 2 - \gamma_{EM} \simeq 0.7296 \), with \( \gamma_{EM} \simeq 0.5772 \) the Euler-Mascheroni constant.

#### 2. Spectral indices

The spectral index for the scalar perturbation is

\[
n_s - 1 = \frac{D \ln P_S}{d \ln k} \simeq \frac{D \ln P_S}{d N},
\]

(46)

where we used \( d \ln k = d \ln a H \simeq d \ln a = d N \) to obtain the last expression. Similarly, the tensor spectral index is

\[
n_t = \frac{D \ln P_T}{d \ln k} \simeq \frac{D \ln P_T}{d N} \simeq -2\epsilon - \frac{(1 + \alpha)\epsilon}{1 - (1 + \alpha)\epsilon}.
\]

(47)

It is implicit that these quantities are evaluated at \( N = N_* \).

#### 3. Tensor-to-scalar ratio

The tensor-to-scalar ratio is defined by

\[
r \equiv \frac{P_T}{P_S},
\]

(48)

and using \([42] \) and \([49] \), we have

\[
r = \frac{8}{A_{ab}^* N_a^* N_b^*} [1 - (1 + \alpha)\epsilon]_s.
\]

(49)

#### 4. Nonlinearity parameter

The nonlinearity parameter \( f_{NL} \) is a measure of non-Gaussianities in the primordial density fluctuations, defined by the bispectrum, i.e. the three-point correlation function of the curvature perturbation

\[
\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = (2\pi)^3 \delta^3 (k_1 + k_2 + k_3) B_{\zeta}(k_1, k_2, k_3).
\]

(50)

We will be focusing on the so-called local-type nonlinearity parameter defined through the ratio of the bispectrum and the power spectrum as

\[
B_{\zeta}(k_1, k_2, k_3) = \frac{6}{5} f_{NL} \left\{ P_{\zeta}(k_1) P_{\zeta}(k_2) + 2 \text{ perms} \right\}.
\]

(51)

The local-type non-Gaussianity is generated by nonlinear interactions after the horizon exit \([60, 63]\). There are other types of non-Gaussian profiles that can be generated in different mechanisms (see e.g. \([64]\)).

The local-type non-linearity parameter \( f_{NL} = f_{NL}^{(4)} \) (we will omit ‘local’ hereafter) is conveniently computed using the \( \delta N \)-formalism \([27]\) and its leading contribution (the scale-independent part) is

\[
f_{NL} \simeq f_{NL}^{(4)} = \frac{5}{6} A_{ab}^* N_a^* N_b^* (A_{cd}^* N_c^* N_d^*)
\]

(52)

(the superscript ‘(4)’ denotes the the scale-independent part in the convention of \([65, 66]\)). Other (scale-dependent) parts are subleading and will be neglected.
FIG. 3: The amplitude of the scalar perturbation $A_s$ as a function of $v$ (left). The initial condition for the $s$-field is chosen as $s_{\text{init}} = 0, 1.0 \times 10^{-7}, 1.0 \times 10^{-6}$ and $1.0 \times 10^{-5}$. The initial conditions for $s_{\text{init}}, h_{\text{init}}$ and $\dot{h}_{\text{init}}$ are the same as in Fig.4. The green-shaded region is the Planck constraints [1] $A_s = (2.23 \pm 0.16) \times 10^{-9}$. The panel on the right shows the contour plot for $-10^{-5} \leq s_{\text{init}} \leq 10^{-5}$ and $0.06 \leq v \leq 0.07$. The parameter region within the Planck constraints is shaded red.

IV. NUMERICAL RESULTS

We present the numerical results of our analysis in this section. Before going into details, we comment on our strategy and the method. Our analysis is based on the $\delta N$-formalism as outlined in the previous section $^5$. We investigate the parameter space only in the vicinity of the single-field limit of the model; for this purpose we choose the initial conditions for the inflaton to realize nearly straight trajectory. We also use the value of the parameter $\xi$ which is fixed by the normalization of the scalar power spectrum $A_s$ in the single-field limit; this leads to slight inconsistency of the parameter choice as $A_s$ also changes as other parameters ($v$ and the initial $s$-field value $s_{\text{init}}$) are varied. We shall however see that this is minor as the change of $A_s$ is less significant than that of the nonlinearity parameter $f_{\text{NL}}$. Using a fixed value of $\xi$ is also convenient for observing the overall behavior of the cosmological parameters without introducing complexities.

A. Generic features

1. Procedure and model parameters

In the numerical study below we use the e-folding number (between the horizon exit of the CMB scale and the end of inflation characterized by max$(\epsilon, |\eta|) = 1$) $N_c = 60$ and the seesaw mass scale $M = 1$ TeV (we also made computation for other parameters; see comments below). These are in the parameter range that is interesting in both cosmology and in particle phenomenology. For the value of the nonminimal coupling parameter $\xi$ we use $\xi = 3.696 \times 10^{-3}$, which is determined by the Planck normalization $A_s = 2.215 \times 10^{-9}$ of the scalar power spectrum (we use the Planck + WP best fit value [1]). The Dirac Yukawa coupling $y_D$ is fixed by the seesaw relation [1]. We shall vary the parameter $v$ below to study its effects.

2. Initial condition dependence

As shown in Fig.1 the trajectory of the inflaton is sensitive to the initial conditions of the field dynamics when the value of $v$ is small. Since our focus is on the prediction of the model as the trajectory deviates from a straight line, we shall specify the initial values as follows. We assume that the initial value of $s$ to be small and the initial $h$ is determined by the single-field value, e.g. $h_{\text{init}} = 21.99$ for $N_c = 60$. As any light field has quantum fluctuations of the order of the Hubble parameter during inflation, we expect small but nonzero values of initial $s$. Assuming that the $s$-field is light, the size of the fluctuations is

$$\langle (\Delta s_{\text{can}})^2 \rangle \approx \langle G_{11}(\Delta s)^2 \rangle \approx \frac{H^2}{(2\pi)^2},$$

(53)

where $s_{\text{can}}$ is the canonically normalized $s$-field in the Einstein frame. The Hubble parameter at the horizon

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$^5$ We checked our numerical code on simple two-field models with separable potentials, including [23],[29].
FIG. 4: The nonlinearity parameter $f_{NL}$ as a function of $v$ (left). The initial conditions for the $s$-field are chosen as $s_{\text{init}} = 0$, $1.0 \times 10^{-7}$, $1.0 \times 10^{-6}$ and $1.0 \times 10^{-5}$. The initial conditions for $s_{\text{init}}$, $h_{\text{init}}$ and $\dot{h}_{\text{init}}$ are the same as in Fig.1. The green-shaded region corresponds to the Planck constraints $f_{NL}^{\text{local}} = 2.7 \pm 5.8$. The right panel shows the contour plot of $f_{NL}$ for $-10^{-5} \leq s_{\text{init}} \leq 10^{-5}$ and $0.06 \leq v \leq 0.07$, with the red-shade indicating the parameter region allowed by Planck.

The exit of the CMB scale is determined by amplitude of the curvature perturbation and the tensor-to-scalar ratio as

$$H \approx \frac{\pi}{M_p} \sqrt{\frac{r A}{2}} \approx 0.5 \times 10^{-4} M_p,$$

where we used $r \approx 0.1$. Combining this with (53) we evaluate the natural initial values for the $s$-field to be in the range

$$-\Delta s \leq s_{\text{init}} \leq \Delta s,$$

where

$$\Delta s \cong \frac{H}{2\pi} \sqrt{1 + \xi h} \approx 10^{-5} M_p,$$

in the case of $M = 1 \text{ TeV}$ and $N_e = 60$. We thus consider $s_{\text{init}}$ in the range $-10^{-5} M_p \leq s_{\text{init}} \leq 10^{-5} M_p$ in the following analysis.

The velocity of the $s$-field is set to be zero, $\dot{s}_{\text{init}} = 0$, and the velocity of the $h$-field is determined by the single-field slow-roll equation of motion, i.e.

$$3H \dot{h} + \frac{\partial V(s = 0, h)}{\partial h} = 0.$$

3. Kähler parameter dependence

Our Kähler potential (14) (see also (4)) includes two tuneable parameters $\gamma$ and $v$. The former is determined by $\xi$ through (10). We vary $v$ and investigate how the observables change. Obviously, one can see from (4) that there is no effects of $v$ when $s = 0$; in this case the model becomes the nonminimally coupled $\lambda \phi^4$ model which we illustrated in Sec. III C. The effects of $v$ become important when the inflaton trajectory deviates from $s = 0$.

As shown on the left panel of Fig. 1, for very small values of $v$ the initial value for $s$ needs to be fine-tuned to some non-zero value in order for the inflaton trajectory to reach the supersymmetric vacuum $(s, h) = (0, 0)$ (we see in the expression (17) that the potential $V(\phi^c)$ is not symmetric in $s$; thus a trajectory with the initial conditions $s_{\text{init}} = 0$ and $\dot{s}_{\text{init}} = 0$ does not necessarily come straight down to the supersymmetric vacuum). For larger values of $v$, the potential is stabilized in the direction of $s$ and thus the danger of the trajectory falling into an unphysical vacuum ceases to bother us. However, a curved trajectory generally results in cosmological parameters outside the observational constraints. For even larger values of $v$ the inflaton trajectory becomes insensitive to the initial conditions and the prediction of the model converges to that of single-field inflation. As we start from the single-field limit (large enough $v$) that agrees with observations and tune $v$ to lower values, the prediction of the model goes outside the observational bound at some value of $v$. This transition takes place around $v \sim 0.0607$, for the $M = 1 \text{ TeV}$ and $N_e = 60$ case that we consider. While it is certainly possible that there may be islands in the parameter space that give observables in agreement with the current data, the analysis as prescribed above gives reasonable constraints on the Kähler potential in the vicinity of the straight trajectory background solutions.
go out of the observational bounds. For larger value, the predicted value of $A_s$ becomes larger). This can be understood as an effect of the isocurvature mode: the curvature perturbation at superhorizon scales is source by the isocurvature mode. The conversion of power from the isocurvature mode to the curvature mode takes place when the trajectory is curved. As a consequence, the curvature perturbation becomes larger at the end of inflation than at the horizon exit, and this enhancement is more efficient if the inflaton makes a sharp turn (i.e. for larger $|s_{\text{init}}|$). Due to the quantum fluctuations, uncertainty of $\Delta s_{\text{init}} \sim 10^{-5}$ is expected. This means that fine-tuning of the initial condition for $s_{\text{init}}$ to be less than $10^{-5}$ is unnatural. We thus conclude that the constraints $A_s = (2.23 \pm 0.06) \times 10^{-9}$ (Planck) give $\nu \gtrsim 0.06767$. The Planck + WP constraints $A_s = 2.196(\pm 0.051) \times 10^{-9}$ give a milder bound, $\nu \gtrsim 0.06827$.

2. Scalar bispectrum

Now we turn our attention to the nonlinearity parameter $f_{\text{NL}}$. Since the main contribution comes from the scale-independent part of the local-type bispectrum, we consider the $f_{\text{NL}}^{(4)}$ given by the expression (52). The numerical results are shown in Fig. 4. The left panel shows $f_{\text{NL}}$ for the same parameter choice as in the scalar power spectrum case above, namely $0.06 \leq \nu \leq 0.074$ and $s_{\text{init}} = 0, 10^{-7}, 10^{-6}$ and $10^{-5}$. The nonlinearity parameter becomes large as $\nu$ is decreased, similarly to the $A_s$ case above. However, $f_{\text{NL}}$ is more susceptible than $A_s$ to the multi-field effects and goes outside the Planck constraints at a larger value of $\nu$. Both the enhancement in $A_s$ and the generation of $f_{\text{NL}}$ are due to interaction at superhorizon scales. However, as pointed out e.g. in [30, 33] the primary contribution to $f_{\text{NL}}$ comes from the change of the curvature-isocurvature transfer function $T_{\mathcal{S}S}$ (proportional to $N_{ab}$), whereas the growth of $A_s$ is caused by $T_{\mathcal{S}S}$ itself ($\sim N_a$). This observation justifies the procedure of our analysis — the value of $\xi$ was fixed by $A_s$, so when $A_s$ changes $\xi$ needs to be readjusted; the above finding indicates that such readjustment is not necessary within the parameter range where only $f_{\text{NL}}$ changes significantly.

The right panel of Fig. 4 shows a contour plot in the corresponding parameter region. It indicates that large non-Gaussianities are obtained within some islands of the parameter space. This is in agreement with our understanding that the local-type non-Gaussianities are generated at superhorizon scales by nonlinear interactions and there is a trade-off between generation of a sizeable isocurvature mode and efficient conversion of it into the curvature mode [29, 30].

3. Tensor power spectrum

FIG. 5: The amplitude of the tensor perturbation $A_t$ as a function of $\nu$, for the initial conditions $s_{\text{init}} = 0, 10^{-7}, 10^{-6}$ and $10^{-5}$. The initial conditions for $s_{\text{init}}$, $h_{\text{init}}$ and $\dot{h}_{\text{init}}$ are the same as in Fig. 4. The tensor mode does not interact outside the horizon and hence is insensitive to the change of the background trajectory.

B. Numerical results for cosmological parameters

In this subsection we describe the behavior of cosmological parameters as the values of $s_{\text{init}}$ and $\nu$ are varied.

1. Scalar power spectrum

The scalar power spectrum [42] may be written as

$$P_S = A_s \left( \frac{k}{k_0} \right)^{n_s - 1 + \frac{1}{2} \frac{d n_s}{d \ln k} \ln \frac{k}{k_0}} \cdots,$$

where $A_s$ is the normalized amplitude at the pivot scale $k = k_0$ and $n_s$ is the scalar spectral index that will be discussed later. This $A_s$ is to be compared with the observational constraints [1]

$$A_s \times 10^9 = 2.23 \pm 0.16 \quad \text{(Planck)},$$

$$= 2.196^{+0.051}_{-0.060} \quad \text{(Planck + WP)},$$

at $k_0 = 0.05 \text{Mpc}^{-1}$. In Fig. 3 we show our numerical results for the scalar power spectrum [42]. The panel on the left shows the values of $P_S \approx A_s$ for different initial conditions $s_{\text{init}} = 0, 10^{-7}, 10^{-6}, 10^{-5}$ and for the Kähler potential parameter $0.06 \leq \nu \leq 0.074$. We have chosen $M = 1 \text{TeV}$ and $N_a = 60$. The green-shaded region indicates the Planck constraints of [59]. The right panel shows a contour plot in the $s_{\text{init}}$-$\nu$ plane. The red-shaded color indicates the allowed parameter region within the Planck constraints [59].

We see that as the parameter $\nu$ is tuned to a smaller value, the predicted value of $A_s$ will become larger and go out of the observational bounds. For larger $|s_{\text{init}}|$, the constraints on $\nu$ becomes tighter (the lower bound for $\nu$ becomes larger). This can be understood as an effect of the isocurvature mode: the curvature perturbation at
at the pivot scale $k_0 = 0.05$ Mpc [2],
\[ \mathcal{P}_T = A_\nu \left( \frac{k}{k_0} \right)^{n_s + 1} \frac{d}{\ln k} \ln \left( \frac{k}{k_0} \right) \cdots . \] (60)

The change of the initial value $s_{\text{init}}$ and the shift of $\nu$ has no effects on $A_\nu$. This is expected, since the tensor mode fluctuations are generated inside the horizon and do not interact once they exit the horizon.

Thus, the behavior of the tensor-to-scalar ratio $r$ is entirely determined by that of the scalar amplitude $A_\nu$ (Fig. 6). Consequently, the multi-field effects only lower the tensor-to-scalar ratio $r$.

4. **Scalar spectral index**

In Fig. 7 we show the behavior of the scalar spectral index $n_s$. The behavior agrees well with our understanding that the curvature perturbation is sourced by the isocurvature perturbation at superhorizon scales, and this effect shifts the spectral index. Comparing with Fig. 4 we see that the Planck constraints on $f_{\text{NL}}$ impose a more stringent bound on $\nu$ than $n_s$. The contour plot on the right panel shares some similarity with the $f_{\text{NL}}$ case; this is attributed to the fact that the shift of $n_s$ is controlled by the derivative of the transfer function $T_{\text{RS}}$ [30, 33].

5. **Tensor spectral index**

Finally, we show the tensor spectral index $n_t$ in Fig. 8. As the tensor fluctuations do not interact with the scalar mode and hence freeze once they exit the horizon, it is insensitive to the change of the inflaton trajectory. Thus, the value of $n_t$ does not depend either on $\nu$ nor on $s_{\text{init}}$.

C. **Constraints on the Kähler potential from non-Gaussianity**

We studied above the cosmological observables of the two-field HIL model in the phenomenologically interesting case of the seesaw mass scale $M = 1$ TeV and the e-folding number $N_e = 60$. For the initial values of the two-field $s_{\text{init}}$ parameter, then the scalar spectral index and the scalar power spectrum deviate from the observationally supported single-field values as we vary the Kähler potential parameter $\nu$ from above. Within the range of the parameters we have searched, we did not see significant change in the tensor power spectrum and the tensor spectral index. Putting the value of the tensor-to-scalar ratio $r$ aside, the observational constraints for the scalar spectral index $n_s$ and the local-type nonlinearity parameter $f_{\text{NL}}$ thus give constraints on the $\nu$ parameter. The Figs. 3, 4, 7 show that among them the nonlinearity parameter puts the most stringent bound, $\nu \gtrsim 0.06925$. The large nonlinearity parameter at small values of $\nu$ is understood as a consequence of the nontrivial inflaton trajectory. Generation of non-Gaussianities involves several competing effects. It is known that large non-Gaussianities can be generated when the trajectory makes a mild turn after spending sufficient e-foldings on an unstable potential. In fact there are many studies in
the literature (especially before the Planck satellite mission) in search of inflationary model generating large non-Gaussianities. While non-Gaussianities in the primordial fluctuations are still elusive, our case study above shows that the observational bounds are useful in constraining the model parameters.

We have also analyzed the model for various other parameter values. For example, in the case of $M = 10 \, \text{TeV}$ and the same value of the e-folding $N_c = 60$ we have $\xi \approx 0.04612$ from the Planck normalization of the curvature perturbation in the single-field limit. We obtained similar constrains on the $v$ parameter: the 68% bounds on the scalar spectral index giving $v \gtrsim 0.01257$ and the 68% bounds on $f_{\text{NL}}$ giving $v \gtrsim 0.01246$. The scalar power spectrum changes at a smaller value of $v$ and the tensor power spectrum and the tensor spectral index barely change. Thus we conclude $v \gtrsim 0.01257$, the strongest bound put by the scalar spectral index. We find similar features in other parameter values.

V. CONCLUSION

In this paper we analyzed the multi-field dynamics of the supersymmetric Higgs inflation type cosmological models that are implemented in (beyond) the Standard Model embedded in supergravity with a noncanonical Kähler potential. We studied the two-field HLI model based on the supersymmetric seesaw model in detail, and studied how the cosmological observables constrain the model parameters. Realistic particle theory-based inflationary models commonly involve multiple flat directions, and in such models understanding of multi-dimensional inflaton dynamics is crucial. In the recent Planck satellite experiment, no sign of non-Gaussianities was detected \cite{3}. However, as we have illustrated in this paper, the upper bound of the nonlinearity parameter $v$ can be useful for obtaining information on the inflaton trajectory; in our case, this led to the constraints on the Kähler potential.

The main focus of the present paper was the local-type nonlinearity parameter $f_{\text{NL}}$, that is, the bispectrum of the primordial density fluctuations. While $f_{\text{NL}}$ is relatively a clean signal of multi-field inflation, it is certainly not the only parameter that characterizes multi-field inflation. While more dependent on details of the phenomenological setup, constraints from the isocurvature
modes are also a rich source of information. In models of inflation where the neutrino sector is involved (such as the HLI model), the baryon number density through possible leptogenesis may also provide further constraints on the model (as discussed in the line of \[70, 71\]). Furthermore, higher order non-Gaussianities such as the trispectrum will naturally give richer information on the multi-field dynamics, although regarding the present status of these parameters \[3\], reliable constraints may not be obtainable in the near future.

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Appendix A: Expressions for coefficients in the backward $\delta N$ formalism

1. $P^a_b$, $Q^a_{bc}$ and $A^{ab}$

The expressions for $P^a_b$ and $Q^a_{bc}$ can be obtained by performing the derivatives of the background equations of motion with respect to $\lambda$. The results are as follows:

\[
P_{11}^1 = 0, \quad P_{11}^2 = \delta^1_l,
\]
\[
P_{21}^1 = -\frac{\nabla J \nabla^I V}{H^2} + \frac{(\nabla^I V)(\nabla_J V)}{H^2 V} - R^I_{JKL} \varphi^J_2 \varphi^L_2,
\]
\[
P_{21}^2 = G_{JK} \varphi^K_2 \varphi^J_2 - \frac{V}{H^2} \delta^J_l + \frac{\nabla^I V}{V} G_{JK} \varphi^K_2,
\]
\[
Q_{11}^1 = -R^I_{JKL} \varphi^J_2, \quad Q_{11}^{12} = 0,
\]
\[
Q_{11}^{12} = \frac{\nabla_J R^I_{MKL}}{H^2} \varphi^J_2 M - \frac{\nabla_J \nabla^I V}{H^2 V} + \frac{2 (\nabla_J V)(\nabla_K V)}{H^2 V} + \frac{(\nabla^I V)(\nabla_K \nabla_J V)}{H^2 V},
\]
\[
Q_{21}^{12} = -2 R^I_{JKL} \varphi^J_2 + \frac{\nabla_J \nabla^I V}{V} G_{KL} \varphi^L_2,
\]
\[
Q_{21}^{11} = -2 R^I_{JKL} \varphi^J_2 + \frac{\nabla_J \nabla^I V}{V} G_{JL} \varphi^L_2 - \frac{(\nabla^I V)(\nabla_J V)}{V^2} G_{JL} \varphi^L_2,
\]
\[
Q_{21}^{22} = 2 G_{JL} \varphi^J_2 \varphi^L_2 - G_{JK} \varphi^J_2 \varphi^L_2 + \frac{\nabla^I V}{V}.
\]

The quantity $A^{ab}$ is defined via the two-point correlation function of field perturbations, $\delta \varphi^a$, as follows:

\[
\langle \delta \varphi^a \delta \varphi^b \rangle = A^{ab} \left( \frac{H_*}{2\pi} \right)^2.
\]

After including the slow-roll corrections, the (1,1)-component of $A^{ab}$ is given by \[21\]

\[
\langle \delta \varphi^a_{1*} \delta \varphi^b_{1*} \rangle = \left( \frac{H_*}{2\pi} \right)^2 \left[ g^{11} - 2 \epsilon G_{IJ} + 2 \epsilon G_{IJ} M_{KL} G^{IJ} \right],
\]

where

\[
\epsilon_{KL} \equiv G_{KL} + G_{KM} G_{LN} - \frac{1}{3} R_{KMLN} \varphi^M_2 \varphi^N_2 - \frac{\nabla_K \nabla_L V}{3 H^2},
\]

and

\[
M_{KL} \equiv \frac{N_{K}^{1} N_{L}^{1}}{G_{AB} N_{A}^{1} N_{B}^{1}}.
\]

The other components of $A^{ab}$ can be obtained by considering the derivatives of the background equations of motion with respect to $\lambda$. Assuming that the slow-roll conditions are satisfied at the Hubble exit, i.e., at $N = N_*$, we find

\[
\delta \varphi^a_{1*} \approx \left[ \frac{(\nabla^I V)(\nabla_J V)}{V^2} - \frac{\nabla_J \nabla^I V}{V} \right] \delta \varphi^a_{1*} \equiv \Delta^a_{\lambda} \delta \varphi^a_{1*}.
\]
The resultant expressions for the components of $A^{ab}$, with slow-roll corrections, are found to be

$$A^{ij}_{11} = G^{ij} - 2e G^{ij} + 2\alpha G^{ij} \epsilon_{KL} \frac{G^{KC} G^{LD} N^1_{\alpha} N^1_{\beta}}{G^{AB} N^1_{\alpha} N^1_{\beta}},$$

$$A^{ij}_{12} = \Delta^i_K A^{i\mu}_{12} - \Delta^j_K A^{j\mu}_{12} + (A^{i\mu}_{12})^T,$$

$$A^{ij}_{22} = \Delta^i_K \Delta^j_K A^{i\mu}_{121}$$

where $T$ stands for the transpose.

2. $N_a^F$ and $N_a^G$

The explicit expressions of $N_a^F$ and $N_a^G$ are as follows:

$$N^1_j = \frac{\nabla_j V}{2\epsilon}, \quad N^2_j = \frac{G_{ij} \varphi_j^2}{2\epsilon(3-\epsilon)}, \quad (A9)$$

and

$$N^1_{ij} = \frac{\nabla_i \nabla_j V}{2\epsilon} - \frac{3}{4\epsilon^2(3-\epsilon)} (\nabla_i V)(\nabla_j V) \frac{\varphi^K_2}{\varphi^K_2},$$

$$N^2_{ij} = \frac{3}{4\epsilon^2(3-\epsilon)} \frac{G_{ik} \varphi^K_2}{2\epsilon} + \frac{\varphi^K_2}{2\epsilon} \frac{(\nabla_i V)(\nabla_j V) \frac{\varphi^K_2}{\varphi^K_2}}{2\epsilon} - \frac{3}{4\epsilon^2(3-\epsilon)} (\nabla_i V)(\nabla_j V) \frac{\varphi^K_2}{\varphi^K_2},$$

$$N^2_{ij} = \frac{3}{4\epsilon^2(3-\epsilon)} \frac{G_{ij} \varphi^K_2}{2\epsilon} + \frac{\varphi^K_2}{2\epsilon} \frac{(\nabla_i V)(\nabla_j V) \frac{\varphi^K_2}{\varphi^K_2}}{2\epsilon} - \frac{3}{4\epsilon^2(3-\epsilon)} (\nabla_i V)(\nabla_j V) \frac{\varphi^K_2}{\varphi^K_2},$$

$$N^2_{ij} = \frac{G_{ij}}{2\epsilon(3-\epsilon)} + \frac{3}{4\epsilon^2(3-\epsilon)} (\nabla_i V)(\nabla_j V) \frac{\varphi^K_2}{\varphi^K_2},$$

$$- \frac{G_{ik} \varphi^K_2 \varphi^K_2}{2\epsilon} \frac{(\nabla_i V) \frac{\varphi^K_2}{\varphi^K_2}}{2\epsilon} - \frac{G_{ij} \varphi^K_2 \varphi^K_2}{2\epsilon} \frac{(\nabla_i V) \frac{\varphi^K_2}{\varphi^K_2}}{2\epsilon}.$$ (A10)

**Appendix B: NMSSM Higgs inflation**

Higgs inflation based on the NMSSM is studied in [10–12]. The NMSSM is also an extension of the MSSM by a singlet field $S$ and the superpotential of its simplest version (the $Z_3$ invariant NMSSM) is (see [22] for a review)

$$W_{\text{NMSSM}} = W_{\text{MSSM}} + \lambda S H_u H_d + \frac{\rho}{3} S^3,$$ (B1)

where $W_{\text{MSSM}}$ is [9]. Since one of the main motivations for considering the NMSSM is to solve the MSSM $\mu$ problem by generating an effective $\mu$-term by the expectation value of the $S$ field, the $\mu H_u H_d$ term of [9] is usually not included in the NMSSM. The parameters $\lambda$ and $\rho$ in (B1) are not arbitrary but are constrained by the conditions that (i) $\langle S \rangle = 0$ is not the global minimum (leading to $\rho^2 < \lambda^2$), that (ii) there is no Landau singularity below the GUT scale (leading to $\lambda < 0.8$) and that (iii) $\mu_{\text{eff}}$ is in the electroweak scale ($\lambda$ not too small). If one gives up solving the MSSM $\mu$ problem and keeps the $\mu H_u H_d$ term of the MSSM, then $\lambda$ can be taken arbitrarily small and the NMSSM approaches the MSSM limit. In the NMSSM Higgs inflation model the nonminimal coupling $\xi = \frac{1}{4} - \frac{1}{2}$ (see also the main text) is related to $\lambda$ via the normalization of the curvature fluctuations. The value of $\xi$ can be $O(1)$ if $\lambda$ is allowed to be tuned small.

The NMSSM Higgs inflation model [10–12] is obtained by supergravity embedding with the Kähler potential $K = -3\Phi$ (we use the superconformal framework) with

$$\Phi = 1 - \frac{1}{3} (|S|^2 + |H_u|^2 + |H_d|^2 \cdots) + \frac{\gamma}{2} (H_u H_d^* + c.c.) + \frac{\upsilon}{3} |S|^4.$$ (B2)

The ellipsis represents the canonical Kähler terms for the other fields that are not relevant to the study of inflation. Setting the charged Higgs to be zero and assuming that the $H_u \cdot H_d$ D-flat direction and the singlet direction are parametrized by two real scalar fields $h, s$ as

$$H_u^0 = \frac{h}{2}, \quad H_d^0 = \frac{h}{2}, \quad S = \frac{s}{\sqrt{2}},$$ (B3)

the scalar-gravity part of the Lagrangian becomes [2] in the Einstein frame. The scalar potential is $V(\phi^i) = \Phi^{-2} V_j(s, h)$, with the Jordan frame potential in this case reads

$$V_j(s, h) = \left(\frac{2h^2 + \rho s^2}{4(1-2\upsilon s^2)} + \frac{\lambda^2}{4} s^2 h^2\right),$$

$$- \frac{3}{32} \left(\frac{\lambda^2 s^2}{4} + 2 \upsilon \frac{h^2 + \rho s^2}{12(1-2\upsilon s^2)}\right).$$ (B4)

There is a typo in (D4) of [12]. The function $\Phi$ takes the form of [1]. While [B4] differs from [17] in details, the overall shape of the potential is similar. In the Einstein frame, the Kähler potential parameter $v$ controls the stability of the potential $V(\phi^i)$ in the $s$-direction. Decreasing the value of $v$ enhances the multi-field effects, including the non-Gaussianities of the primordial fluctuations. It is thus possible to constrain the value of $v$ using the observational bounds of non-Gaussianities, as we did in the main text for the HLI model.
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