Design and fatigue analysis of diagonally reinforced structures with a negative Poisson’s ratio lattice material

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Abstract

In this paper, a typical negative Poisson’s ratio lattice structure was studied and several diagonally reinforced 3D lattice structures were designed and analysed using the finite element method. On this basis, the fatigue life of the unreinforced and reinforced structures was calculated using SIMULIA Fe-safe software. Meanwhile, the fatigue failure process of the lattice structure was systematically simulated and quantitatively analysed by combining the multiaxial fatigue damage model. Results show that the enhancement design from the cell structure can provide an anti-diagonal shear enhancement effect. The structure A, B, and C can decrease the maximum von Mises stress by 95.8, 97.1, and 94.66%, and increase the compressibility by 80%, 56%, and 127%, respectively. The structure A enhancement yields the best overall performance in terms of structural stress, compressibility, and negative Poisson’s ratio properties enhancement. Compared with the unreinforced structure, the lifetime distribution of the structure A reinforced structure changes in both position and level. The overall lifetime has been improved from 10^{2.813} of the unreinforced structure to 10^7 of the reinforced structure. Quantitative calculation of the fatigue damage is consistent with the fatigue life prediction results, which further validate the effectiveness of the diagonal enhancement method and the enhancement structure of the negative Poisson’s ratio lattice structure.

Nomenclature

| Symbol | Description |
|--------|-------------|
| \(A_a\) | stress amplitude [MPa] |
| \(D_e\) | amount of elastic damage [-] |
| \(D_p\) | amount of elastic damage [-] |
| \(D_{total}\) | amount of total damage [-] |
| \(E\) | modulus of elasticity [MPa] |
| \(H\) | single layer height [mm] |
| \(K\) | material strength coefficient [MPa] |
| \(N\) | number of cycles [-] |
| \(n\) | material hardening index [-] |
| \(R_v\) | the stress triaxiality [-] |
| \(S\) | overall cross-sectional area [mm\(^2\)] |
| \(s_0\) | material constant [-] |

Greek Symbols

| Symbol | Description |
|--------|-------------|
| \(\theta\) | cell-angle [°] |
| \(\rho_x\) | density of unenhanced structure [kg/m\(^3\)] |
\[ \rho_s \] material density \([\text{kg/m}^3]\)
\[ \rho_d \] relative density \([\text{kg/m}^3]\)
\[ \sigma_{m} \] stress amplitude \([\text{MPa}]\)
\[ \sigma_{u} \] strength limit \([\text{MPa}]\)
\[ \sigma_{\text{mean}} \] average stress \([\text{MPa}]\)
\[ \varepsilon_{pa} \] equivalent plastic strain amplitude \([-]\)
\[ \varepsilon_{pa}^2 \] plastic strain amplitude \([-]\)

**Abbreviations**

NPR negative Poisson’s ratio
GCS gyroid cellular structure
SLM selective laser melting
BCC body-centred cubic structure
SAH star-arrow honeycomb
SSH star-shaped honeycomb
PPR positive Poisson’s ratio

### 1. Introduction

Demand for lightweight and multifunctional materials in modern industrial applications is increasing since traditional metal materials can no longer meet the needs of practical engineering structures [1]. In this context, lightweight lattice structural materials with high porosity, especially porous materials with a negative Poisson’s ratio (NPR), are widely used in the aerospace, automotive industry and biomedical and other fields due to their unique deformation characteristics, good impact and absorption properties, and better mechanical properties, such as specific stiffness and specific strength [2].

In the aviation industry, 3D lattice materials are often used to make aircraft walls and floors to reduce the weight of the aircraft. And these components are often subjected to loads, such as human footsteps or external impacts. Parts such as crash beam energy-absorbing boxes, foot pedals and car body in the automotive industry are also subjected to periodic compressive loads. In biomedical engineering, vascular stents, artificial bones and other components made of 3D lattice materials inevitably bear fatigue loads due to their participation in functional activities [3–5]. Due to the subjected cyclic compression loads during service, shear zone along the diagonal direction is usually formed in the structure coinciding with the maximum shear stress surface, leading to premature failure of the lattice material.

Currently, typical negative Poisson’s ratio structures include inner concave hexagonal structures, star structures, and chiral structures. The first macroscopic NPR structure which had a microscopic cell with an inner concave hexagonal structure was manufactured by Lakes [6] in 1987. Using Monte Carlo method, Wojciechowski [7] first studied the microscopic NPR (isotropic) model through computer simulations. Later, Wojciechowski [8] studied and solved the two-dimensional isotropic system exactly in the static limit with theoretical analysis method. The star structure was first proposed and studied in depth by Theocaris et al [9], who used the finite element method of homogenization theory to analyze the structure and showed that the equivalent Poisson’s ratio of the structure is negative when the fiber composite had a star row or concave polygon cross section. Grima and Evans [10] and Ishibashi and Iwata [11] proposed the lattice rotation theory in explaining the mechanism of the negative Poisson’s ratio effect in crystalline materials. They considered that negative Poisson’s ratio characteristic could be achieved through rationally constructing the crystal structure.

Yang et al [12, 13] investigated the compression-compression fatigue behavior and fatigue mechanism of a gyroid cellular structure (GCS), a typical porous Triply Periodic Minimal Surface Structure, and found that both cyclic ratchet effect and fatigue damage phenomena were responsible for the failure of GCS during fatigue tests. Under compression fatigue loading, most of the failure samples exhibited nearly 45° fracture bands along the diagonal surface. Kolken et al [14, 15] found that the failure form of the structure changed from the layer-by-layer collapse to densification and shear band generation as the relative density of NPR materials increased in compression-compression fatigue failure experiments. Kadkhodapour et al [16] studied the deformation and failure mechanism of porous Titanium biomaterials prepared by selective laser melting (SLM) using the finite element method and found that the diamond lattice structure with dominant bending structures exhibited a 45° shear band failure mechanism. The same phenomenon was also found through experiments and numerical simulations by Bill et al [17]. Diagonal shear failure under compressive loading has emerged as one of the
obstacles that limit the rapid development of lightweight lattice structures. Therefore, to solve the premature failure problem of diagonal fatigue shear of lattice structures, it is of great significance to explore the causes of compression-induced shear band damage and establish design methods for the structure enhancement.

Different improvement methods were studied and proposed for the problem of shear band damage in the diagonal direction of lattice materials. The first method assumed enhancing the strength of the structure by improving the nature of the matrix material itself. Qi et al. [18] investigated the effect of SLM process conditions on the material strength and found that the strut diameter monotonically increased with laser power, while the porosity was the largest at an intermediate laser power. The laser scanning speed yielded thickening of struts only at slow speeds, while the porosity was the largest at intermediate speeds. Experimental data of Wu et al. [19] showed that lattice structures fabricated by SLM exhibited poor endurance ratios with significant 45° shear band damage, while the microstructure transformed from brittle $\alpha'-\text{martensite}$ to tough $\alpha + \beta$ mixed phases after hot isostatic pressing; the tougher $\alpha + \beta$ mixture could blunt the fatigue cracks and decrease the yield strength from 143 to 100 MPa, and the fatigue strength and endurance ratio were improved. Song et al. [20] investigated the failure mechanism of the composite lattice structure using multi-scale experimental analysis methods and finite element simulations. They found that the average modulus and strength of a nickel-plated polymer fine crystal composite, Ni@PMLs, increased by 68.3% and 34.9%, respectively, compared with polymer lattice materials. The average specific strength of the lattice almost reached the upper limit of the boundaries of conventional metal/polymer foam and natural porous materials, improving the structural resistance to diagonal shear failure. Xu [21] designed and proposed novel dual-scale hybrid mechanical metamaterials consisting of different cell lattice scales, such as simple cubic structure, body-centered cubic structure (BCC), and face-centered cubic structure. The numerical results showed that the problem of diagonal directional damage under compressive loading conditions was improved. Still, improving the properties of matrix materials can enhance the overall structural strength to some extent, but the structural configuration and material category requirements limit this scenario.

The second approach is to perform a variable-density design, allocating more material where the strength of the lattice structure is low. This effective distribution of material, i.e., variably density, is better than fixed density because they have a better material distribution [22]. Qi et al. [23] showed by numerical simulations that using a tapered beam instead of a uniform beam in the structure enhanced the strength at the lattice structure nodes, changing the diagonal direction failure to the layer-by-layer collapse, thus improving the overall structural strength. Maskery et al. [24] varied the relative density of each layer while keeping the relative density, mass, and volume of the overall structure of the SLM AI-Si10-Mg lattice structure constant. The results of the quasi-static loading experiments showed that, compared to damage along the 45° shear zone at a constant density structure, the variable-density structure collapsed in a layer-by-layer sequence, progressing from low to high density regions. The complex density variation of the lattice structure was more likely to eliminate undesirable failure forms in the lattice structure than the linear density variation. Maskery et al. [25] further used experimental and theoretical analyses to investigate the effect of density gradient on the compression crushing behavior and mechanical properties of two types of lattice structures, BCC and reinforced BCC. They found that the energy absorbed by the fixed-density structure was higher than that of the variable-density structure at lower strains. When the strain exceeded 0.52, the energy absorbed per unit volume was about 114% more for the variable-density lattice structure than for the fixed-density lattice structure. Zhang et al. [26] conducted three-point bending experiments of supported two-dimensional hexagonal lattice honeycomb structures with fixed and variable densities, showing that the flexural stiffness of the variable-density structure was 37% higher compared with the fixed-density structure, confirming that the variable-density design was a more desirable lattice structure design method. Yin et al. [27] designed a variable-density diamond lattice structure with better compression properties using a bionic research approach. The simulation results showed that the variable-density structure with a decreasing density of filled cells centered on the stress point exhibited better compressive properties, and the maximum stress of the material decreased from 23.65 to 18.34 MPa for the same density, which improved the overall strength of the structure. Czarnecki [28] proposed an algorithm of designing optimal distribution of isotropic characteristics to minimize the compliance. He transmitted a given surface loading to a given support, reducing the auxiliary problem to an unconstrained problem of nonlinear programming. Czarnecki and Wawruch [29] used both mathematical and numerical methods to perform optimal isotropic design of auxetic materials. They found that isotropic composite materials forming the bodies of extremely high stiffness exhibit negative Poisson’s ratio in large subdomains, which further indicated the significance of the auxetic material in modern structural design. However, due to the special properties of some lattice structures and the complexity of the 3D molding process, the use of variable-density structure design is limited in specific applications.

In addition to the above two methods, the strength of the lattice structure can also be improved by changing the cell configuration through topology optimization. Mohsen et al. [30] proposed a new periodic NPR lattice structure based on the conventional re-entrant honeycomb structure. The results of quasi-static compression experiments and finite element analysis showed that the structure and its geometry (e.g., re-entrant angle) were more important than the structural materials. Shen et al. [31] used an explicit dynamic finite element method to
optimize the traditional re-entrant hexagonal structure and proposed a model of the re-entrant annular structure. The simulation results showed that the NPR effect of the specimen improved with the curvature of the re-entrant annulus at low and medium velocity impacts from 10 to 50 m s\(^{-1}\). At a 50 m s\(^{-1}\) impact load, the introduction of the annular structure reduced the maximum peak stress of the structure by more than 40%. Wang [32] designed a star-arrow honeycomb (SAH) based on the star-shaped honeycomb (SSH). The theoretical analysis and finite element numerical simulation results revealed that, at a certain relative density, the absorbed energy per unit mass of SAH is 21.11% larger than that of SSH under the impact velocity of 1 m s\(^{-1}\), and 3.99% larger for the impact velocity of 70 m s\(^{-1}\). In other words, SAH exhibited better energy absorption under low-velocity impact. The same conclusion was reached by Wei et al. [33] using the same method to investigate the compression performance of star-triangular structures and SSH. Hu et al. [34] theoretically analyzed the NPR effect and the crushing stress of the re-entrant hexagonal honeycomb, showing that the NPR effect increased with the increase of the cell-wall angle and the decrease of the cell-wall length ratio. Crushing velocity significantly influenced the honeycomb’s NPR effect at the early stage of crushing. However, this influence almost vanished when the overall strain was larger than about 0.2. Schultz et al. [35] optimized the structural parameters of the porous honeycomb structure with the in-plane impact energy as the optimization objective, showing that a slight tensile expansion structure with inclined honeycomb walls could yield the maximum in-plane energy absorption. Yang et al. [36] investigated the compression characteristics of a 3D concave lattice structure with NPR characteristics and pointed out that the NPR of 3D concave lattice structures decreased and the specific strength increased by more than 4 times by decreasing the angle of concave struts or increasing the length of vertical and re-entrant struts. Ingr ole et al. [37] designed a novel in-plane performance-enhanced tensile expansion structure and compared the deformation and failure modes of honeycomb, re-entrant auxetic, auxetic-strut, and two different auxetic-strut/honeycomb hybrid lattice structures by experimental and FEM numerical analyses. The newly designed auxetic-strut structure exhibited compressive strength 60% higher than the re-entrant auxetic structure and 300% higher than the honeycomb structure. Besides, the total energy absorbed by the auxetic-strut design was 170% higher than that of the honeycomb and 30% higher than that of the re-entrant auxetic models. The specific energy absorbed by auxetic-strut structure was about 13% higher than that of the other traditional structures. In addition, aiming at the problem of the lattice structure damage due to insufficient strength of nodes, Xu et al. [38] established a new type of self-balancing nodes by adding bridging and strengthening rivets, which could effectively control delamination while significantly reducing the eccentric moment at the nodes. The experimental results showed that the ultimate load capacity of the self-balancing node was increased from 65 to 125 kN, and the maximum displacement was reduced from 12.36 to 7.67 mm compared with the traditional node connection method.

Conclusively, improving the strength of the lattice structure by topology optimization can effectively enhance the overall strength, and it has wider adaptability and is not limited by the cells’ arrangement compared with the first two improvement methods. Although studies on the corresponding optimization design for different lattice structures exist in the literature, including the tensile expansion structure with NPR, previous research mainly focused on the deformation characteristics, energy absorption mechanism, and impact performance enhancement of different structures. Few studies specifically deal with enhancement methods and enhanced structural design of NPR lattice structures for diagonal shear fatigue damage. At the same time, the fatigue damage evolution process of the lattice structure is still unclear, which further hinders in-depth investigation on the design of the NPR lattice structure against diagonal shear fatigue and is very unfavorable to the improvement of the overall service life of the NPR lattice structure.

In this paper, a typical NPR lattice structure was studied, and three diagonally reinforced 3D lattice structures were designed using the ABAQUS finite element explicit dynamics method. The structural reinforcement form with the best compressibility and NPR characteristics was determined through the static structural stress and deformation analyses. On this basis, the fatigue life of reinforced and unreinforced lattice structures was calculated and compared using SIMULIA Fe-safe software (Fe-safe). Combined with the established fatigue damage constitutive model, fatigue failure and damage processes of the lattice structure were systematically simulated, and the fatigue damage evolution law of the lattice structure was revealed. Moreover, the effectiveness of the enhanced structure against the fatigue shear zone suggested in this paper was also determined.

### 2. Geometric model and the finite element analysis method

#### 2.1. Geometric model

A typical re-entrant hexagonal lattice structure was investigated. The parameters of this cell structure are shown in figure 1(a), where \( a \) is the length of bending cell-wall, \( b \) is the length of the connecting cell-wall, \( h \) is the length of the supporting cell-wall, and \( \theta \) is the length of the cell element angle. The cell element angle is 10\(^\circ\), the bending cell-wall and the connecting cell-wall are 2.5 mm, the supporting cell-wall is 1.25 mm, and the beam diameter is
1.2 mm. The cell structure is cylindrical, with an overall height of 37.5 mm and a cross-sectional diameter of 25 mm, as shown in figure 1(c). The units of the physical dimensions in figure 2 are mm.

2.2. Finite element analysis method
Metal materials under external load undergo elastic deformation and plastic deformation until fracture failure. To analyze the stress distribution on the surface of the NPR lattice structure under compressive loading, a finite element model using an explicit code of the nonlinear finite element software package ABAQUS is established. The isotropic plasticity material model is chosen for commercially pure titanium (CP-Ti), which meet the von Mises yield criterion. For the fatigue life calculation, Fe-safe software that works based on the stress and number of cycles curve (S-N curve) of the matrix material is employed. Since ABAQUS can only qualitatively display the damage, the secondary development of ABAQUS is performed by using Fortran subroutines to realize the quantitative calculation of the fatigue damage.

2.3. Boundary conditions
Due to the high ductility, CP-Ti is chosen as the matrix material in this study. The material density is 4510 kg m$^{-3}$, the modulus of elasticity is 105 GPa, the Poisson’s ratio is 0.33, the yield strength is 312 MPa, and the strength limit is 473 MPa at room temperature. Experimentally measured stress-strain curves for CP-Ti can be found in the literature [39]. The bottom of the lattice structure is a fixed end, and the top is set as a mobile end. Since the dominant mode of lattice structures in the aerospace, automotive industry and biomedical field is compression, the compression-compression fatigue behaviour of porous structures was evaluated in many studies [3–5, 13–15]. According to reference [10] and the ISO 13314 Standard, a displacement load of 15 mm ($U_1 = 0, U_2 = 15, U_3 = 0$) was applied on the top of the lattice structure in this paper, as shown in figure 2. $U_1$, $U_2$ and $U_3$ represent displacements in X, Y and Z directions, respectively. The relationship between strain and stress for CP-Ti is shown in figure 3.
The above FEM method of using ABAQUS software to simulate the quasi-static compression process of a three-dimensional lattice structure has been verified by experimental results in reference [13, 30, 37]. All cells are meshed using a B31 beam. Considering the mesh density will have a certain influence on the calculation results, a mesh sensitivity test has been performed before the final calculation. In the test, five sets of grids are employed to discretize the mesh of the unreinforced structure, with grid numbers of $2.8 \times 10^4$, $3.4 \times 10^4$, $4.7 \times 10^4$, $6.0 \times 10^4$, and $7.1 \times 10^4$, and the corresponding minimum grid sizes of 0.25, 0.2, 0.15, 0.12, and 0.10 mm, respectively. Results show that when the grid number reaches $6.0 \times 10^4$, its further increase to $7.1 \times 10^4$ yields the change rates of maximum Mises stress and apical displacement of 0% and 1.16%, respectively. Therefore, the mesh number of all models in this paper exceeds $6.0 \times 10^4$ to ensure the calculation speed and accuracy of the calculation results.

Besides, since the focus of this paper is to explore the structure enhancement design method and fatigue damage evolution process of NPR lattice structures against diagonal shear, the manufacturing defects of lattice structures, which include geometry imperfections, voids and cracks are not considered in the simulation.

3. Design of diagonally enhanced lattice structures

Literature search reveals that structure A [40], structure B [41], and structure C [42] showed better performance in improving the strength of the lattice structure. Structure A adds auxiliary parts to the nodal position of the original structure, thus reducing the stress concentration at the nodal position and improving the NPR in the compression direction. Structure B is the same structure nested in the original structure to achieve structural enhancement. Structure C combines a re-entrant hexagonal structure and a diamond structure, with four reinforcing walls embedded in each unit to achieve structural strength enhancement. To explore the effectiveness of different reinforcement forms on the NPR lattice structure against diagonal shear fatigue failure, we used structure A, B, and C methods to design the reinforced structure for the re-entrant hexagonal NPR structure. The cell structures of the three reinforced structures are shown in figure 4. The units of the physical dimensions in figure 4 are mm. The number of cells of each reinforced structure is $5 \times 8$, and the connection of two cell layers constitutes the third cell layer, and the specific structure is shown in figure 5.

To avoid the effect of the density variations on the compression resistance of the lattice structure, we kept the relative density of the lattice structure constant in all three enhanced structure designs. The specific design method is as follows: since all the structures consist of 8 layers of cells, one of the layers can be taken for calculation, as shown in figure 6. $S$ represents the overall cross-sectional area of one layer of structural cells, $H$ represents the single layer height, $\theta$ denotes the cell-angle (as shown in figure 1 (a)), $s$, $s_1$, $s_2$, and $s_3$ denote the re-entrant hexagonal honeycomb, structure A, B, C single rod, respectively, and $d$, $d_1$, $d_2$, and $d_3$ denote the diameter of the re-entrant hexagonal honeycomb, structure A, B, C single rod, respectively.

Figure 3. Relationship between strain and stress of CP-Ti.
The cell volume $V_1$ of the unenhanced structure be calculated as follows:

$$V_1 = SH$$  \hfill (1) 

Assuming that the losses at the connections in figure 2(a) are not counted, the volume of material can be calculated as:
where \( n_1 \) is the number of connected cell-wall, \( n_2 \) is the number of bent cell-wall, and \( n_3 \) is the number of supported cell-wall.

This gives the density of the unenhanced structure:

\[
\rho_u = \frac{V_2}{V_1}
\]

where \( \rho_u \) is the material density.

The relative density of the lattice structure is defined as the ratio of the density of the structure to the density of the structural material. Thus, the relative density can be defined as follows:

\[
\rho_d = \frac{\rho_u}{\rho_s} = \frac{V_2}{V_1} \frac{(n_1a + n_2b + n_3h)s}{SH}
\]

The relative densities of the three reinforced structures can be calculated in the same way, as shown in equations (5)–(7).

\[
\rho_{d1} = \frac{n_1(a - \tan \theta) + n_2\left(b - \frac{1}{\cos \theta} + 1\right) + n_3(h - 0.5 \tan \theta)}{SH} s_1
\]

(5)

\[
\rho_{d2} = \frac{n_1(a + 1) + n_2\left(3b + a - \frac{1}{\cos \theta} - 0.25\right) + n_3(h + 0.5)}{SH} s_2
\]

(6)

\[
\rho_{d3} = \frac{n_1(a + 1) + n_2\left(b + 2\left(\frac{a}{2}\right)^2 + \left(\frac{b}{\cos \theta}\right)^2 + n_3h\right)}{SH} s_3
\]

(7)

Keeping the relative densities of the three reinforced and unreinforced structures the same in the design process, it yields:

\[
\rho_d = \rho_{d1} = \rho_{d2} = \rho_{d3}
\]

From equation (8), the cross-sectional areas of the rods of the three reinforced structures are:

\[
s_1 \approx s, \quad s_2 \approx s/4, \quad s_3 = s/4
\]
From this, the single-rod diameters for each of the three reinforced structures can be calculated as follows:

\[ d_1 \approx d = 1.2\text{mm}, \quad d_2 = d/2 = 0.6\text{mm}, \quad d_3 = d/2 = 0.6\text{mm} \]

4. Results and discussion

4.1. Static load strength analysis

Figure 7 shows the compressive stress contour of the unreinforced structure and the three reinforced structures of structure A, B, C under a 15 mm-displacement load at the top of the structure. The lack of unit cells around the periphery of each structure with its interconnection leads to a low strength around the structure. However, in practice, the structure is applied over a large area, and insufficient peripheral strength can be considered equivalent to the edge effect, which has less impact on the whole structure. Therefore, higher stresses around each structure are ignored in the analysis.

Figure 7(a) shows that the unreinforced structure experiences large stress at the node position with a maximum of 473 MPa, reaching the strength limit of the material at service temperature. In the 45° diagonal direction of the structure, the nodes are located in the same line, and there is no rod between the nodes to facilitate stress transition to them. Therefore, numerous nodes of the unreinforced structure fail macroscopically as a whole fracture in the 45° diagonal direction. In the vertical direction, there are different rods between the nodes and the load-bearing nodes, and the connecting rods do not fail when the nodes fail, so they do not connect into a straight line and do not exhibit the phenomenon of vertical direction damage.

Besides, figure 7 demonstrates that under the same compressive load, the maximum von Mises stresses of structure A, B, and C are 19.42, 13.57, and 25.05 MPa, respectively. Compared with the unreinforced structure, the maximum von Mises stresses of the structure A, B, and C decrease by 95.8, 97.1, and 94.66%, respectively. Among them, structure A and B exhibit better enhancement than structure C in reducing the compressive stress of the lattice structure.

To analyse further the variation of stresses with time at key nodes inside the unreinforced and the three reinforced structures under the same compressive load, we take four points (numbers 1 ~ 4) from the diagonal position of each structure and four points (numbers 5 ~ 8) from the y-axis direction, as shown in figure8. The stress-strain curves are plotted for each structural key node, and the results are shown in figure 9. For the unreinforced structure, the stress at the diagonal position is larger than that in the y-axis direction, and the stress
at the nodes of diagonal positions is in the plastic stage, while the stress at the nodes in the y-axis direction is still in the elastic stage. The nodal stresses in the diagonal direction and the y-axis direction of the three reinforced structures are all in the elastic stage and show a better stress state. The stress state at each point in the diagonal direction and vertical direction of each structure indicates that structure A, B, and C all enhance the overall load-bearing capacity of the structure. The structural stresses are always in the elastic stage.

Figure 10 further shows the deformation of several structures. The maximum compression displacement of the unreinforced structure, structure A, B, and C are 6.33, 11.38, 9.86, and 14.36 mm, respectively, under the same compression load. Structure A, B, and C increase the compressibility of the unreinforced structure by 80, 56, and 127%, respectively. Among them, structure A and C exhibit a better compressibility enhancement effect.

Figure 11 illustrates the deformation process and stresses of each structure under compressive loading. The results show that the unreinforced, structure A, and B show good NPR properties in different compression stages. However, structure C is limited by the diamond structure. After certain compression, structure C first densifies and then forms a stable triangular structure that cannot be compressed further. At this time, it transforms into a structure with positive Poisson’s ratio (PPR) properties.

Figure 12 further shows the lateral deformation of each structure with the variation of longitudinal strain. The calculation point is located at the edge of the half height of the structure. The displacement of the point is output as a function of time. The horizontal displacement of the point is regarded as the lateral deformation of the structure and employed as the vertical coordinates in the figure. The abscissa of figure 12 is the longitudinal strain, which can be obtained from the ratio of the longitudinal displacement of the structure to the original height (37.5 mm). Obviously, the unreinforced, structure A and B exhibit NPR properties throughout the whole compression process, with a maximum transverse deformation of 0.42, 0.68, and 1.13 mm, respectively. The structure A and B form an increased NPR (maximum transverse deformation) of the structure by 61.9% and

Figure 9. Stress–strain relationships for diagonal and non-diagonal positions.
The structure C exhibits NPR properties at the beginning of compression and then changes to PPR at a 17% strain with a maximum transverse deformation of 0.58 mm. Thus, the structure A and B exhibit better NPR properties under the same compression load.

Summarizing stresses, strains, and distributions of different reinforced structures under the same compressive load, three reinforcement methods of structure A, B, and C decrease the maximum von Mises stress of the structure by 95.8, 97.1, and 94.66%, respectively. The three types of reinforcement increased the structure’s compressibility (maximum compression displacement) by 80, 56, and 127%, respectively.

Compared with structure B, structure A and C exhibited better compressible properties. Meanwhile, the NPR properties of structure C are worse than those of the other two enhanced structures because it changes from NPR to PPR properties in the compression process. Therefore, considering the performance of the three reinforcement methods in terms of structural stress, compressibility, and NPR properties, structure A method is finally chosen to enhance the NPR lattice structure.

4.2. Fatigue lifetime assessment
To verify the effectiveness of the selected structure A-reinforced structure further, we performed lifetime calculations for the unreinforced structure and structure A. Fe-safe software can predict the fatigue life of the material using its S-N curve. In section 4.1, von Mises stresses for each structure have been determined in static load calculations. Therefore, in this section, the ABAQUS calculation result file ‘.odb’ file from section 4.1 is imported into Fe-safe and a compressive load of $-3 \text{ mm}$, $-1 \text{ mm}$ and a frequency of 100 HZ is applied to predict the fatigue life of the nodal structure by the von Mises stresses obtained from the static load analysis.

Figure 13 shows the lifetime of the unreinforced structure. According to the calculated results, the maximum lifetime of the unreinforced structure is $10^7$ and the minimum lifetime is $10^7.813$. The minimum lifetime appears at the surface location, but the surface location does not completely show its true structural form due to its low strength and small impact on the structure’s overall strength. Therefore, in this paper, only the lifetime of the internal position is considered, and the intermediate section is taken for observation, as shown in figure 13. The results show that the lifetime at the node position is only $10^7.813$, while the lifetime of the support cell-wall and bending cell-wall can reach up to $10^{4.557} \sim 10^{5.952}$ and $10^7$, respectively. The lifetime at the node position is 100 times smaller than that at the non-node position, greatly limiting the overall lifetime of the structure.

The lifetime calculation of structure A using Fe-safe indicates ‘no damage’. From the S-N curve of the working principle of Fe-safe, the reason for no damage of structure A is analysed. The S-N curve of the CP-Ti matrix material is obtained from high-cycle fatigue tests results in literature [43]. In the test, two samples were conducted by electro-hydraulic servo fatigue testing machine at each stress level, and the normal pressure during the experiment was 10N. As specified in figure 14, when the stress is greater than 200 MPa, as point 1 in the
Figure 11. Deformation process of different structures.

figure, the lifetime of the material sharply decreases. However, when the maximum stress of the material is less than 60 MPa, as point 2 in the figure, the calculated lifetime is $10^7$. It can therefore be regarded as infinite. The maximum stress of structure A is 23.44 MPa, i.e., less than 60 MPa. At this time, the lifetime is calculated to be
Figure 12. Lateral deformation of each structure.

Figure 13. Lifetime of the unreinforced structure.

Figure 14. S-N curve of CP-Ti.
10^7. Thus, no damage before the expiration of its service life can be considered. Therefore, structure A successfully improves the fatigue life of the NPR lattice structure.

4.3. Fatigue damage process analysis

The fatigue life of the lattice structure was analyzed previously, but for the negative Poisson’s ratio diagonally enhanced lattice structure designed in this paper, the specific fatigue failure and damage evolution process are not clear, and they are difficult to characterize experimentally. To solve the above problems, the material fatigue damage process is described by constructing a damage evolution equation from the perspective of continuous damage mechanics. Meanwhile, the degree of material fatigue damage is quantitatively characterized by defining fatigue damage variables, to represent the strength degradation of the material visually.

The elastic and plastic fatigue damage equations are derived based on the second law of thermodynamics Clausius-Duhamel inequality [39], as shown in equations (9) and (10).

\[
\frac{dD_e}{dN} = \frac{2B_{-1}}{(S_0 + 1)} \left( \frac{\sigma_n}{1 - \sigma_m/\sigma_n} \right)^{2S_0+2}
\]

\[
\frac{dD_p}{dN} = 4 \left( \frac{K^2R_f}{2E\beta^2(1 - D_p)^2} \right)^{n_0} \left( \varepsilon_{pa} \right)^{2n_0 + 1}
\]

\(D_e\) is the amount of elastic damage, \(D_p\) is the amount of plastic damage, \(N\) is the number of cycles, \(K\) is the material strength coefficient, \(n\) is the material hardening index, \(\varepsilon_{pa}\) is the equivalent plastic strain amplitude, \(\sigma_m\) is the stress amplitude, \(\sigma_n\) is the strength limit, \(s_0\) is the material constant, \(E\) is the modulus of elasticity, and \(R_f\) is the stress triaxiality. \(B_{-1}\), \(S_0\), and \(\beta\) are the fatigue parameters of material, which is pointed out by chandrakanth et al [44]. The specific values of these parameters are derived from literature [39].

The total damage of the lattice structure is the sum of the elastic damage and plastic damage. By replacing the stress amplitude, average stress, and plastic strain amplitude in the uniaxial state with the corresponding values in the multiaxial state, the expression of fatigue damage in the multiaxial state can be obtained as follows:
where $D_{\text{total}}$ is the total damage, $A_a$ is the stress amplitude, $\sigma_{\text{mean}}$ is the average stress, and $\bar{\varepsilon}_{pa}$ is the plastic strain amplitude.

To obtain the coefficients in equation (11), we fitted the experimental stress-strain curve data measured at a strain rate of 0.001 s$^{-1}$ as the expression of $\varepsilon = K\varepsilon_{pa}^n$. The obtained result show that the material strength coefficient $K$ is 895, and material hardening index $n$ is 0.08627. Therefore, according to the fitting results in this article and the deduction from literature [39], the coefficients for the fatigue damage model of CP-Ti in equation (11) are obtained and displayed in table 1.

| $K$ (MPa) | $n$  | $B_{-1}$ | $S_0$ | $s_0$ | $b'$ |
|----------|------|----------|-------|-------|------|
| 895      | 0.086| $1.92 \times 10^{-15}$ | 0.66  | 3.98  | 3.16 |

The subroutine UVARM was compiled based on the Fortran language and embedded in ABAQUS for the fatigue damage model constructed above. It is assumed that there is no damage to the material until it is subjected to fatigue loading, and the damage reaches a critical value when material damage occurs, that is: $N = 0, D_{\text{total}} = 0$; $N = N_f$, and $D_{\text{total}} = 1$. Considering that the calculation results are abnormal when the damage value equals 1, the critical damage value is set to 0.99, and the range of fatigue damage is set to 0–0.99. When the value of fatigue damage is 0, it means that no damage occurs. When the value of fatigue damage is 0.99, the fatigue failure of the material occurs. At this time, the structure loses its load-bearing capacity, and cracks sprout and expand.

The whole calculation flow chart is shown in figure 15. First, the stress amplitude, average stress, and equivalent variation amplitude under the current cycle of each node are calculated in the model. Then, the damage increment at the current cycle step is calculated according to the fatigue damage equation, and the damage accumulation is performed. If a damage amount of 0.99 is reached, no further accumulation will take place, otherwise, go to the next cycle. Finally, the results are output, and the operation is finally finished.

Figure 16 shows the fatigue damage of the unreinforced structure, and figure 17 indicates the relationship of the fatigue damage amount with the number of cyclic loading at the location where 0.99 damage appears first. The results show that the unreinforced structure initially reaches the damage amount of 0.99 at the node location and then loses the load-bearing capacity, which is consistent with the predicted results of the fatigue life shown in figure 13. The nodal location of the unreinforced structure reached the damage threshold of 0.99 at $10^{3.5}$ (approximately 2000) cycles, and the structure has lost its load capacity. The remaining bar locations gradually fail as the number of cycles loaded increases to $10^4$ (10,000).

The fatigue life analysis results show that the unreinforced structure shows damage at $10^{2.813}$ (approximately 650) cycles, and structure A does not produce damage at $10^7$ cycles. Figure 18 shows the damage results of the reinforced structure after 10,000 loading cycles. The calculated fatigue damage of the reinforced structure is consistent with the fatigue life prediction results, which again confirms the reliability of the numerical simulation method in this paper.

**Table 1** Fatigue damage parameters of CP-Ti.

**Figure 16.** Fatigue damage contour of the unreinforced structure.
5. Conclusions

In this paper, a typical NPR lattice structure was studied, and several diagonally reinforced 3D lattice structures are designed using the finite element method. Through the static structure analysis, the stress, compressibility and NPR characteristics of different structures are compared, and the best form of the reinforced structure is determined. Based on the static stress analysis results, the fatigue life of the unreinforced and reinforced structures is calculated using Fe-safe software. At the same time, the fatigue failure and the damage process of the lattice structure are systematically simulated and analyzed by combining the fatigue damage constitutive model, determining the effectiveness of the reinforced structure proposed in this paper against the fatigue shear zone.

The main research findings of this paper are as follows:

(1) The enhancement design from the cell structure can provide an anti-diagonal shear enhancement effect. Keeping the relative density constant was used to design the reinforcement from the cell structure of the lattice structure. The structure A, B, and C can decrease the maximum von Mises stress by 95.8%, 97.1%, and 94.66%, and increase the compressibility by 80%, 56%, and 127%, respectively. The three enhancement methods alleviate the failure problem along the diagonal position and show better mechanical properties.

(2) Under the same compression load, the structure A and B always show better NPR properties at different compression stages, while the structure C, due to the limitation of the rhombus structure, changes from...
NPR to PPR properties after certain compression, exhibiting worse NPR properties. The structure A enhancement yields the best overall performance in terms of structural stress, compressibility, and NPR properties enhancement.

3) The results of fatigue life calculations based on Fe-safe software show that the location of the shortest lifetime of the unreinforced structure occurs in the nodes along the diagonal direction, with a minimum value of 650 cycles. Compared with the unreinforced structure, the lifetime distribution of the structure A reinforced structure changes in both position and level. The overall lifetime has been improved from $10^{2.813}$ of the unreinforced structure to $10^4$ of the reinforced structure.

4) Based on the fatigue damage equation, a quantitative calculation of the fatigue damage is performed to the NPR lattice structures for the first time. The results show that the structure loses its load-bearing capacity, which is manifested as a sudden failure. The reinforced structure can improve the problem of high stress at the node position and fatigue failure along the shear zone of the original structure by reducing the amount of fatigue damage and the damage initiation time of the structure.

The diagonal enhancement method of the NPR lattice structure and the enhancement structure proposed in this paper provide a technical basis for mitigating premature failure damage and improving the service life of the lattice structure.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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Declaration of interest statement

The authors declared that there is no conflict of interest.

Data availability

All data generated or analyzed during this study are included in this published article and the referenced papers.

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