Numerical Analysis of Blasting Effect on Concrete

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Abstract. In this study, the blasting effect on concrete is numerically simulated by the 2-D multi-material Eulerian method. Through operator splitting of the governing equations into Lagrangian and Eulerian steps, the Eulerian method is discussed in detail. For the material interface and transport of the mixed Eulerian cells, a modified Young’s interface reconstruction algorithm is proposed. The simulation results agree with the experimental data, indicating that the model and algorithm presented in this paper are valid and the numerical method can be used for engineering design.

1. Introduction

Concrete as an important engineering material is broadly used in many military structures, such as airport runways and underground fortifications. Therefore, the study of the destruction of the concrete structures under blasting loading is important. With the rapid development of computer techniques and numerical methods, numerical simulation is now widely used in the study of this type of projects in addition to theoretical researches and experimental investigations. In this paper, explosion simulations by the 2-D multi-material Eulerian method are studied. The numerical algorithms and solving procedures are discussed in detail. The explosion in the concrete is numerically simulated by the Eulerian method and compared with experimental data. This work enhances our understanding of the mechanisms of the blasting effect on concrete.

2. Governing Equations

In order to simplify the computational model, some basic assumptions are made. These assumptions are continuum of material, local thermodynamic equilibrium, uniform and isotropic material, ideal plasticity, and small deformation.

2.1. Conservation Equations

Governing equations of hydrodynamic model consist of mass conservation, momentum conservation, and energy conservation, which are [1]:

\[
\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho + \rho \mathbf{\nabla} \cdot \mathbf{u} = 0
\]  

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\[ \rho \left( \frac{\partial u}{\partial t} + u \cdot (\nabla u) \right) = \nabla \cdot \sigma \]  \hspace{1cm} (2) \\
\[ \rho \left( \frac{\partial e}{\partial t} + u \cdot \nabla e \right) = \sigma : \dot{\varepsilon} \]  \hspace{1cm} (3)

where \( t \) is time, \( u \) is the velocity, \( \sigma \) is the Cauchy stress tensor, \( \dot{\varepsilon} \) is the strain rate tensor, \( \rho \) is the density, and \( e \) is the specific internal energy.

2.2. Constitutive Relation

Within the range of elasticity, deviatoric stresses are defined by the general Hooke principle

\[ \nabla \cdot S = \dot{\varepsilon} + \Omega \cdot \dot{\varepsilon} + \dot{\varepsilon} \cdot \Omega \]

\[ = 2G \left[ \dot{\varepsilon} - (\text{tr} \dot{\varepsilon}) I \right] + \Omega \cdot \dot{\varepsilon} + \dot{\varepsilon} \cdot \Omega \]

where, the strain rate tensor and spin rate tensor are

\[ \dot{\varepsilon} = \frac{1}{2} (u \nabla + \nabla u) \]  \hspace{1cm} (5) \\
\[ \Omega = \frac{1}{2} (u \nabla - \nabla u) \]  \hspace{1cm} (6)

3. Numerical Methods

In the Eulerian frame, the computational domain is discretized to some rectangular cells. All the physical quantities, such as pressure, density, specific internal energy, velocity, deviatoric stress, and others, are defined at the center of each cell. The artificial viscosity is defined at the midpoint of the cell boundary.

The governing equations have the general form

\[ \frac{\partial \phi}{\partial t} + u \cdot \nabla \phi = H \]

where \( \phi \) represents \( \rho, u, \) and \( e; H \) is the source term.

In this study, the operator split method is adopted. Eq.(7) can be splitting to the following two equations

\[ \frac{\partial \phi}{\partial t} = H \]  \hspace{1cm} (8) \\
\[ \frac{\partial \phi}{\partial t} + u \cdot \nabla \phi = 0 \]  \hspace{1cm} (9)

where Eq.(8) is referred to as the ‘Lagrangian’ phase and Eq.(9) is the ‘Eulerian’ phase. In Lagrangian phase, the mesh is allowed to distort with the material, and the changes in velocity and internal energy due to the pressure and deviatoric stress terms are calculated. In the second Eulerian phase or remap phase, transport of mass, internal energy and momentum across cell boundaries are computed. This may be treated as the remapping the displaced mesh at the end of phase 1 back to the spatially-fixed Eulerian frame.
4. Multi-Material Interface Reconstruction Algorithm

In the Eulerian method, because the materials flow across the fixed Eulerian cells, it must occur mixed cells with two or three materials. For the mixed cell with two materials, the well-known Young’s interface reconstruction algorithm is adopted, which uses a straight line to represent the interface. The slope of the line is determined by the material distribution in the surrounding eight cells, and the position of the line is determined by the portion of volume in the cell.

Youngs’ algorithm only determines the location of material interfaces in mixed cells. It does not take into account the sequence of transported materials, which has to be addressed in the Eulerian method. Here we propose a criterion to determine the transportation sequence [2].

In this criterion, the material occupation numbers for material $k$ in the cells to the left and right of the current cell, $IL_k$ and $IR_k$, are assigned to 0 if material $k$ is absent and 1 if it is present, respectively. Then the volume fractions of material $k$ in the cells to the left and right of the current cell, $VL_k$ and $VR_k$, are calculated, where $0 \leq VL_k \leq 1$ and $0 \leq VR_k \leq 1$. Based on the values of $IL_k$, $IR_k$, $VL_k$ and $VR_k$, the variables $L_k$ and $R_k$ are calculated by

$$L_k = IL_k \cdot \text{Sign}(VR_k - VL_k)$$

$$R_k = IR_k \cdot \text{Sign}(VR_k - VL_k)$$

For all the possible combinations of $L_k$ and $R_k$, the distribution of material $k$ can be categorized into five configurations as shown in Table.1.

| $L_k$ | $R_k$ | configuration |
|------|------|--------------|
| 1    | 0    | 1            | a             |
| 2    | 1    | 1            | b             |
| 3    | -1   | -1           | c             |
| 4    | -1   | 0            | d             |
| 5    | 0    | 0            | e             |

When the configuration of a specific material is known, its transportation priority can be determined based on the continuous principle. The transportation priority is $a > b > c > d > e$. For example, if species A belongs to the configuration b whereas species B belongs to the configuration c, the transport of species A is preferentially considered than species B.

For the mixed cell with three materials, they can be arranged according to the volume ratio first. Then the material with the minimum volume ratio is deleted. The mass, momentum, and energy are...
added to the material with the maximum volume ratio. Thus the question is reduced to a mixed cell with two materials, and it can be treated by the improved Youngs’ interface reconstruction algorithm described above.

5. Numerical Simulation of Blasting Effect on Concrete

5.1. The Computational Model
The physical model of the explosion field is shown in Fig.2. In the simulation, the explosive field is reduced to axial symmetry. The left boundary is axial symmetry boundary, while the rest are nonreflecting boundaries. The size of the computational region is as follows: \( L=60\text{cm} \), \( H_1=60\text{cm} \), and \( H_2=160\text{cm} \). The charge is TNT, with the mass, size and height as shown in Table 2 [3].

![Figure 2. The physical model of explosion field](image)

Table 2. The mass, size and height of the charge

| Mass of explosive(kg) | Size of explosive(mm) | \( W \)(m) |
|-----------------------|-----------------------|-------------|
| 1                     | 0.326                 | \( \Phi64.5\times56.5 \) | 0.28       |
| 2                     | 0.326                 | \( \Phi64.5\times56.5 \) | 0.215      |
| 3                     | 0.326                 | \( \Phi59.5\times66.7 \) | 0.198      |
| 4                     | 0.326                 | \( \Phi64.5\times56.5 \) | 0.18       |

5.2. Constitutive Model
We take air as an ideal gas, with the equation of state

\[
P = (k - 1) \rho \cdot e
\]

where \( k \) is the isentropic index of the air, \( k=1.4 \).

The JWL equation of state for the detonation products is

\[
P = A(1 - \frac{\omega}{R_1 V})e^{-R_1 V} + B(1 - \frac{\omega}{R_2 V})e^{-R_2 V} + \frac{\omega E}{V}
\]

where \( V \) is the relative volume. \( A, B, R_1, R_2, \omega \) are known constants.
Table 3. Parameters of explosive and JWL equation of state

| ρ (kg·m⁻³) | P_CJ/Pa | D_CJ/(m·s⁻¹) | A/ Pa | B/ Pa | R₁ | R₂ | ω | E₀/(KJ/g) |
|------------|---------|---------------|-------|-------|-----|----|----|-----------|
| 1560       | 2.225×10¹⁰ | 6700          | 6.5×10¹¹ | 9.253×10⁹ | 4.2 | 1.1 | 3.4 | 5.45      |

For concrete, the HJC constitutive model [4] is adopted. The normalized equivalent stress is defined as

$$\sigma^* = \left[ A(1-D) + Bp^* C \right](1 + C \ln \dot{\varepsilon}^*)$$

(14)

where $D$ is the damage ($0 \leq D \leq 1$); $p^* = p/f_c$ is the normalized pressure, and $f_c$ is the quasi-static uniaxial compressive strength; $\dot{\varepsilon}^* = \dot{\varepsilon}/\dot{\varepsilon}_0$ is the dimensionless strain rate; $A$, $B$, $N$, $C$ are material constants.

The damage $D$ is defined as

$$D = \int_0^\infty \frac{f^\rho d\tau}{f(P)}$$

(15)

where $f(P)$ is the plastic strain to fracture under a constant pressure, $P$. The specific expression is

$$f(P) = \varepsilon_p^f + \mu_p^f = D\left( P^* + T^* \right)^{D_3}$$

(16)

where $T^*$ is the normalized maximum tensile hydrostatic pressure, and $T^* = T/f_c$; $\dot{\varepsilon}^p$ is the equivalent plastic strain rate, which is

$$\dot{\varepsilon}^p = \left( \frac{2}{3} \varepsilon_{\rho,1}^p \varepsilon_{\rho,2}^p \right)^{1/2}$$

(17)

The hydrostatic pressure-volume relationship is shown in Eq.18, which is separated into three response regions.

$$P = \begin{cases} 
\mu K_{elast} = \mu \frac{P_{crush}}{\mu_{crush}} & P < P_{crush} \text{ (elastic region)} \\
P_{crush} + \frac{P_{lock} - P_{crush}}{\mu_{lock} - \mu_{crush}} (\mu - \mu_{crush}) & P_{crush} \leq P \leq P_{lock} \text{ (transition region)} \\
k_1 \bar{\mu} + k_2 \bar{\mu}^2 + k_3 \bar{\mu}^3 & P > P_{lock} \text{ (fully dense region)}
\end{cases}$$

(18)

where

$$\bar{\mu} = \frac{\mu - \mu_{lock}}{1 + \mu_{lock}}, \quad \mu = \frac{\rho}{\rho_0} - 1$$

(19)

For tensile pressure,

$$P = \begin{cases} 
\mu K_{elast} & \text{ (elastic region)} \\
\left[ (1-F) K_{elast} + F k_1 \right] \mu & \text{ (transition region)} \\
k_1 \mu & \text{ (fully dense region)}
\end{cases}$$

(20)

where
\[ F = \frac{\mu_{\text{max}} - \mu_{\text{crush}}}{\mu_{\text{lock}} - \mu_{\text{crush}}} \]  

(21)

The material constants for concrete can be seen in Ref. [4].

5.3. Analysis of Numerical Results
The images of the explosion process in concrete are shown in Fig.3 with the visualization software VISC2D [5]. When the charge explodes, the concrete moves up under the propellant effect of the detonation products until the formation of a funnel-shaped crater.

![Figure 3. Images of the explosion process in concrete](image)

Table 4 shows the comparison between numerical results and experimental data. In which, \( R \) is the radius of damage area, and \( H \) is the height of the funnel-shaped crater. It can be seen from this Table that the numerical results of \( R \) and \( H \) agree with the experimental ones, and \( R \) are smaller than the experimental data, while \( H \) is greater than experimental results.

| W(m) | \( R \) Experimental results | \( R \) Numerical results | \( H \) Experimental results | \( H \) Numerical results |
|------|-----------------------------|--------------------------|-----------------------------|--------------------------|
| 1    | 0.28                        | 0.44                     | 0.38                        | 0.43                     |
| 2    | 0.215                       | 0.41                     | 0.36                        | 0.35                     |
| 3    | 0.198                       | 0.394                    | 0.325                       | 0.32                     |
| 4    | 0.18                        | 0.385                    | 0.305                       | 0.30                     |

In the experiment, with the expansion of explosion cavity, cracks appear in the concrete medium, which does not occur in the simulation process. The main reason is likely that the elastic-plastic hydrodynamic model is used in the Eulerian method in this study. This problem will be further investigated in the future work.

6. Conclusions
A 2-D multi-material Eulerian method for elastic-plastic hydrodynamic problems and especially for explosion problems is studied. The blasting effect on concrete is numerically simulated by this Eulerian method. Our simulation results agree with the experimental data, indicating that the model and algorithm presented in this paper are valid and the numerical method can be used for engineering design.
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