Direct $CP$ violation from isospin symmetry breaking effects in PQCD

Gang Lü$^1$, Qin-Qin Zhi$^1$

$^1$College of Science, Henan University of Technology, Zhengzhou 450001, China

We investigate the direct $CP$ violation for the decay process of $\bar{B}_s \to P(V)\pi^0$ ($P,V$ refer to the pseudoscalar meson and vector meson, respectively) via isospin symmetry breaking effects from the $\pi^0 - \eta - \eta'$ mixing mechanism in PQCD factorization approach. Isospin symmetry breaking arises from the electroweak interaction and the u-d quark mass difference by the strong interaction which are known to be tiny. However, we find that isospin symmetry breaking at the leading order shifts the $CP$ violation due to the new strong phases.

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I. INTRODUCTION

The measurement of $CP$ violation is an important area in understanding Standard Model (SM) and exploring new physics signals. Cabibbo-Kobayashi-Maskawa (CKM) matrix due to the quark flavour mixing provides us the weak phases by the amplitudes from the tree and penguin contributions. The weak phase associated with the strong phase is responsible for the source of the CP asymmetry. The strong phase comes from the dynamics of QCD and the other mechanism.

The hadronic matrix elements of the nonleptonic weak decay is known to be associated with the strong phase. The factorization method can be estimate the power contribution in the limit of $1/m_b$ ($m_b$ refers to b quark mass) in B meson decay process. Bases on the QCD correction and taking into account transverse momenta, PQCD factorization method is applied to deal with the decay amplitude related with the hadronic matrix elements, which safely avoids the divergence by introducing the Sudakov factor. The decay amplitude can be written as the convolution of the meson wave functions and the hard kernel, which show the contributions of the non perturbative and the perturbative parts, respectively. In this paper, we will calculate the decay amplitude to make further investigation on the CP violation in the framework of the PQCD factorization.

Isospin symmetry plays an important part in the weak decay process of meson. We can infer sum rule associated with the isospin symmetry to form a triangular shape on a complex plane for the decay amplitude. One can eliminate uncertainty from the penguin diagram by the isospin analysis in B decays. Isospin symmetry breaking via $\rho-\omega$ mixing produces the strong phase to lead to the large CP violation in the three bodies decay process. Isospin symmetry is approximate symmetry due to identical u and d quark masses in Standard Model (SM). The pseudoscalar mesons $\pi^0$ are mixing from the isospin symmetry breaking within QCD. Isospin symmetry breaking

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* Email: ganglv66@sina.com
† Email: zhiqinqin11@163.com
plays a significant role for the decays of $B \rightarrow \pi \pi$, which break the triangle relationship in the framework of generalized factorization [10]. The $\pi^0$-$\eta$-$\eta'$ mixing is discussed by the model-independent way in $B \rightarrow \pi \pi$ decay process using flavor SU(3) symmetry [11]. The quark-flavor mixing produces the $\pi^0$-$\eta$-$\eta'$ mixing due to the isospin symmetry breaking [12]. Recently, isospin symmetry breaking is discussed by incorporating the Nambu-Jona-Lasinio model in a generalized multiquark interaction scheme [13]. However, one can find that the research rarely pay attention to the CP violation from the effect of isospin symmetry breaking via $\pi^0$-$\eta$-$\eta'$ mixing. The strong phase may be introduced to affect the value of CP violation accordingly which is similar with the contribution from the isospin symmetry breaking by the $\rho$-$\omega$ mixing [8, 9].

The remainder of this paper is organized as follows. In Sec. II we present the form of the effective Hamiltonian. In Sec. III we give the calculating formalism of CP violation from isospin symmetry breaking in $\bar{B}_s \rightarrow P(v)\pi^0$. Input parameters are presented in Sec. IV. We present the numerical results in Sec. V. Summary and discussion are included in Sec. VI. The related function defined in the text are given in the Appendix.

II. THE EFFECTIVE HAMILTONIAN

With the operator product expansion, the effective weak Hamiltonian can be written as [14]

$$\mathcal{H}_{\Delta B=1} = \frac{G_F}{\sqrt{2}} [V_{ub}V_{ud}^*(c_1O_1^u + c_2O_2^u) - V_{tb}V_{td}^* \sum_{i=3}^{10} c_iO_i] + H.C.,$$

(1)

where $G_F$ represents Fermi constant, $c_i$ ($i=1,...,10$) are the Wilson coefficients, $V_{ub}$, $V_{ud}$, $V_{tb}$ and $V_{td}$ are the CKM matrix elements. The operators $O_i$ have the following forms:

$$O_1^u = \bar{d}_\alpha \gamma_\mu (1 - \gamma_5) u_\beta \bar{u}_{\beta'} \gamma_\mu (1 - \gamma_5) b_\alpha,$$

$$O_2^u = \bar{d}_\gamma \gamma_\mu (1 - \gamma_5) u_\alpha \bar{u} \gamma_\mu (1 - \gamma_5) b,$$

$$O_3 = \bar{d}_\gamma \mu (1 - \gamma_5) b \sum_{q'} \bar{q}' \gamma_\mu (1 - \gamma_5) q'_\alpha,$$

$$O_4 = \bar{d}_\alpha \gamma_\mu (1 - \gamma_5) b \sum_{q'} \bar{q}' \gamma_\mu (1 - \gamma_5) q'_\alpha,$$

$$O_5 = \bar{d}_\gamma \mu (1 - \gamma_5) b \sum_{q'} \bar{q}' \gamma_\mu (1 + \gamma_5) q'_\alpha,$$

$$O_6 = \bar{d}_\alpha \gamma_\mu (1 - \gamma_5) b \sum_{q'} \bar{q}' \gamma_\mu (1 + \gamma_5) q'_\alpha,$$

$$O_7 = \frac{3}{2} \bar{d}_\gamma \mu (1 - \gamma_5) b \sum_{q'} e_{q'} \bar{q}' \gamma_\mu (1 + \gamma_5) q'_\alpha,$$

$$O_8 = \frac{3}{2} \bar{d}_\alpha \gamma_\mu (1 - \gamma_5) b \sum_{q'} e_{q'} \bar{q}' \gamma_\mu (1 + \gamma_5) q'_\alpha,$$

$$O_9 = \frac{3}{2} \bar{d}_\gamma \mu (1 - \gamma_5) b \sum_{q'} e_{q'} \bar{q}' \gamma_\mu (1 - \gamma_5) q'_\alpha,$$
\[ O_{10} = \frac{3}{2} i \alpha \gamma_\mu (1 - \gamma_5) h_\beta \sum_{q'} e_{q'} \bar{q}_{3} \gamma^\mu (1 - \gamma_5) q', \]

\[ \text{(2)} \]

where \( \alpha \) and \( \beta \) are color indices, and \( q' = u, d \) or \( s \) quarks. In Eq. (2) \( O_1^u \) and \( O_2^u \) are tree operators, \( O_3 - O_6 \) are QCD penguin operators and \( O_7 - O_{10} \) are the operators associated with electroweak penguin diagrams.

we can obtain numerical values of \( c_i \). When \( c_i (m_b) \) \( [6] \),

\[ c_1 = -0.2703, \quad c_2 = 1.1188, \]

\[ c_3 = 0.0126, \quad c_4 = -0.0270, \]

\[ c_5 = 0.0085, \quad c_6 = -0.0326, \]

\[ c_7 = 0.0011, \quad c_8 = 0.0004, \]

\[ c_9 = -0.0090, \quad c_{10} = 0.0022. \]

\[ \text{(3)} \]

III. \( CP \) VIOLATION FROM ISOSPIN SYMMETRY BREAKING EFFECTS

A. Formalism

It is convenient to introduce isospin vector triplet \( \pi_3 \), isospin scalar \( \eta_n \) and isospin scalar \( \eta_s \) which can be distinguished by including strange quark or not. The \( SU(3) \) singlet \( \eta_0 \) and octet \( \eta_8 \) can be well described by the translation \( \eta_n = \sqrt{\frac{2}{3}} \eta_0 + \sqrt{\frac{1}{3}} \eta_8 \) and \( \eta_s = \sqrt{\frac{1}{3}} \eta_0 - \sqrt{\frac{2}{3}} \eta_8 \). The states of \( \pi_3, \eta_n \) and \( \eta_s \) are identified by \( \pi_3 = \frac{1}{\sqrt{2}} |u\bar{u} - d\bar{d}| \), \( \eta_n = \frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}| \) and \( \eta_s = |s\bar{s}| \) obtained from the quark model, respectively. The physical meson states can transforms from the \( \pi_3, \eta_n \) and \( \eta_s \) by unitary matrix \( U \) \[12\]:

\[
\begin{pmatrix}
\pi^0 \\
\eta \\
\eta'
\end{pmatrix} = U(\varepsilon_1, \varepsilon_2, \phi) \begin{pmatrix}
\pi_3 \\
\eta_n \\
\eta_s
\end{pmatrix},
\]

\[ \text{(4)} \]

where

\[
U(\varepsilon_1, \varepsilon_2, \phi) = \begin{pmatrix}
1 & \varepsilon_1 + \varepsilon_2 \cos \phi & -\varepsilon_2 \sin \phi \\
-\varepsilon_2 - \varepsilon_1 \cos \phi & \cos \phi & -\sin \phi \\
-\varepsilon_1 \sin \phi & \sin \phi & \cos \phi
\end{pmatrix},
\]

\[ \varepsilon_1, \varepsilon_2 \propto O(\lambda), \lambda \ll 1 \] and the next lead order terms are neglected. In the isospin limit of \( \varepsilon_1 \to 0, \varepsilon_2 \to 0 \), we can find that the formula is expressed as the the \( \eta - \eta' \) mixing in Eq. [11]:

\[
\begin{pmatrix}
\eta \\
\eta'
\end{pmatrix} = U'(\phi) \begin{pmatrix}
\eta_n \\
\eta_s
\end{pmatrix} = \begin{pmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{pmatrix} \begin{pmatrix}
\eta_n \\
\eta_s
\end{pmatrix},
\]

\[ \text{(6)} \]
where \( \phi = 39.3^\circ \pm 1.0^\circ \) which is the mixing angle \(^{15}\). The \( \eta \) and \( \eta' \) mixing depend on the quark flavor basis \( \eta_n \) and \( \eta_s \). The relevant decay constants can be calculated by the replacements \( \pi \rightarrow \eta_n \) and the parameters \( m_0^{n\pi} \) and \( m_0^{s\pi} \) have been defined \(^{15}\):

\[
\begin{align*}
\langle 0|\bar{\eta}n\gamma^\mu\gamma_5n|\eta_n(P)\rangle &= \frac{i}{\sqrt{2}}f_nP^\mu, \\
\langle 0|\bar{s}s\gamma^\mu\gamma_5s|\eta_s(P)\rangle &= if_sP^\mu.
\end{align*}
\]

The chiral scale parameter \( m_0 = \frac{M^2}{m_{u_1} + m_{d_2}} \) and the chiral enhancement factors \( m_0^{n\pi} \) and \( m_0^{s\pi} \) have been defined \(^{15}\):

\[
\begin{align*}
m_0^{n\pi} &= \frac{1}{2m_n} [m_n^2 \cos^2 \phi + m_{n'}^2 \sin^2 \phi - \frac{\sqrt{2}}{f_n} (m_{n'}^2 - m_n^2) \cos \phi \sin \phi], \\
m_0^{s\pi} &= \frac{1}{2m_s} [m_s^2 \cos^2 \phi + m_s^2 \sin^2 \phi - \frac{\sqrt{2}}{f_s} (m_s^2 - m_n^2) \cos \phi \sin \phi],
\end{align*}
\]

and the parameters \( m_0^i \ (i = \eta, \eta', \eta_n, \eta_s) \) are defined as \(^{15}\):

\[
m_0^{\eta, \eta'} = m_0^\eta = \frac{m_n^2}{(m_u + m_d)}, \quad m_0^{n, s\pi} = \frac{2M^2}{m_s - m_n^2}.
\]

One can understand that isospin breaking comes from the electroweak penguin contribution and \( u - d \) quark mass difference in stand model. We can calculate the isospin breaking correction by chiral perturbative theory which induces the \( \pi^0 - \eta - \eta' \) mixing. To the leading order of isospin breaking, the physical eigenstate \( \pi^0 \) can be written as

\[ |\pi^0> = |\pi_3> + \varepsilon|\eta> + \varepsilon'||\eta'>>, \]

which \( \varepsilon = \varepsilon_2 + \varepsilon_1 \cos \phi, \varepsilon' = \varepsilon_1 \sin \phi \). \( \pi_3 \) refer to the isospin \( I = 1 \) component in the triplet. We use the values of \( \varepsilon = 0.017 \pm 0.002, \varepsilon' = 0.004 \pm 0.001 \) \(^{12}\).

For the \( B_s \) meson function, we use the model \(^{15, 18}\):

\[
\phi_B(x, b) = N_B x^2 (1 - x)^2 \exp[-\frac{M_{B_s}^2 x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2],
\]

where the normalization factor \( N_B \) is dependent of the free parameter \( \omega_b \). \( b \) is the conjugate variable of the parton transverse momenta \( k_T \). \( M_{B_s} \) refers to the mass of the \( B_s \) meson. For the \( B_s \) meson, one can obtain the value of \( \omega_b = 0.50 \pm 0.05 \) from the light cone sum rule \(^{19}\).

The \( \pi, \eta \) and \( \eta' \) are all pseudoscalar mesons and have the similar wave functions. The expressions of amplitudes can be obtained by the replacements

\[
\phi_\pi \rightarrow \phi_\eta, \quad \phi_\pi^p \rightarrow \phi_\eta^p, \quad \phi_\pi^t \rightarrow \phi_\eta^t, \quad r_\pi \rightarrow r_\eta.
\]

In this paper, we will use those distribution amplitudes \(^{15}\):

\[
\phi_\pi^A(x) = \frac{3f_\pi}{\sqrt{6}} x (1 - x) [1 + 0.44 C_2^{3/2}(t)],
\]

(13)
\[ \phi^P(x) = \frac{f_\pi}{2\sqrt{6}} [1 + 0.43C_2^{1/2}(t)], \]
\[ \phi_T^A(x) = -\frac{f_\pi}{2\sqrt{6}} [C_1^{1/2}(t) + 0.55C_3^{1/2}(t)], \]
\[ \phi_K^A(x) = \frac{3f_K}{\sqrt{6}} x(1-x)[1 + 0.17C_1^{3/2}(t) + 0.2C_2^{3/2}(t)], \]
\[ \phi_K^P(x) = \frac{f_K}{2\sqrt{6}} [1 + 0.24C_2^{1/2}(t)], \]
\[ \phi_K^T(x) = -\frac{f_K}{2\sqrt{6}} [C_1^{1/2}(t) + 0.35C_3^{1/2}(t)], \]
\[ \phi_\phi = 3\frac{f_\phi}{\sqrt{6}} x(1-x)[1 + 0.18C_2^{3/2}(t)], \]

where \( t = 2x - 1 \). \( f_{P(V)} \) are the decay constants of scalar (vector) mesons, respectively. Gegenbauer polynomials are defined as:

\[
\begin{align*}
C_1^{1/2}(t) &= t, & C_1^{3/2}(t) &= 3t \\
C_2^{1/2}(t) &= \frac{1}{2} (3t^2 - 1), & C_2^{3/2}(t) &= \frac{3}{2} (5t^2 - 1), \\
C_3^{1/2}(t) &= \frac{1}{2} t(5t^2 - 3).
\end{align*}
\]

**B. Calculation details**

In the framework of PQCD, we can calculate the decay amplitude for the decay process \( \bar{B}_s \to P(V)\pi^0 \). Next, we take the decay process \( \bar{B}_s \to K^0\pi^0 \) as example for study of the \( \eta - \eta' - \pi^0 \) mixing. According to the analysis of Hamiltonian, depending on CKM matrix elements \( V_{ub}V_{ud}^* \) and \( V_{tb}V_{td}^* \), the decay amplitude \( A \) of \( \bar{B}_s \to K^0\pi^0 \) in PQCD can be written as

\[ \sqrt{2} A(\bar{B}_s \to K^0\pi^0) = V_{ub}V_{ud}^* T_1 - V_{tb}V_{td}^* P_1, \]

where \( T_1 \) and \( P_1 \) are the amplitudes form tree and penguin contributions, respectively. The tree level amplitude \( T_1 \) can be given as

\[ T_1 = f_\pi F_{LL, B_s \to K}^{LL} [a_2] + M_{LL, B_s \to K}^{LL} [C_2], \]

where the \( f_\pi \) refers to the decay constant of \( \pi \) meson. The penguin level amplitude \( P_1 \) can written as

\[ P_1 = f_\pi F_{LL, B_s \to K}^{LL} \left[ -a_4 - \frac{3}{2}a_7 + \frac{3}{2}a_9 + \frac{1}{2}a_{10} \right] + f_\pi F_{LL, B_s \to K}^{SP} \left[ -a_6 + \frac{1}{2}a_8 \right] + M_{LL, B_s \to K}^{LL} \left[ -C_3 + \frac{3}{2}C_8 + \frac{1}{2}C_9 + \frac{1}{2}C_{10} \right] + f_{BS, F_{ann}}^{LL} \left[ -a_6 + \frac{1}{2}a_8 \right] + f_{BS, F_{ann}}^{SP} \left[ -a_4 + \frac{1}{2}a_{10} \right] + M_{ann}^{LL} \left[ -C_3 + \frac{1}{2}C_9 \right] + M_{ann}^{LR} \left[ -C_5 + \frac{1}{2}C_7 \right]. \]

The individual decay amplitudes in the above equations, such as \( f_\pi F_{LL, B_s \to K}^{LL}, f_\pi F_{LL, B_s \to K}^{SP}, M_{LL, B_s \to K}^{LL}, f_{BS, F_{ann}}^{SP}, f_{BS, F_{ann}}^{LL}, \)
Due to the CKM matrix elements, we can express the decay amplitude as following:

\[ \sqrt{2} A(\bar{B}_s \to \eta K^0) = A(\bar{B}_s \to \eta_s K^0) \cos \phi - A(\bar{B}_s \to \eta_c K^0) \sin \phi, \quad (24) \]

\[ \sqrt{2} A(\bar{B}_s \to \eta' K^0) = A(\bar{B}_s \to \eta_s K^0) \sin \phi + A(\bar{B}_s \to \eta_c K^0) \cos \phi. \quad (25) \]

Due to the CKM matrix elements, we can express the decay amplitudes as following:

\[ \sqrt{2} A(\bar{B}_s \to \eta K^0) = V_{ub} V_{ud}^* T_n - V_{tb} V_{td}^* P_n. \quad (26) \]

\[ \sqrt{2} A(\bar{B}_s \to \eta' K^0) = V_{ub} V_{ud}^* T_s - V_{tb} V_{td}^* P_s. \quad (27) \]

The contributions of \( T_n, P_n, T_s \) and \( P_s \) can written:

\[ T_n = f_n F_{B_s \to K}^{LL} [a_2] + M_{B_s \to K}^{LL} [C_2], \quad (28) \]

\[ P_n = f_n F_{B_s \to K}^{SP} \left[ a_0 - \frac{1}{2} a_8 \right] + f_{B_s} F_{ann}^{LL} \left[ a_4 - \frac{1}{2} a_{10} \right] \\
+ f_{B_s} F_{LL}^{LL} \left[ 2a_4 + a_4 - 2a_5 - \frac{1}{2} a_7 + \frac{1}{2} a_9 - \frac{1}{2} a_{10} \right] \\
+ M_{B_s \to K}^{LL} \left[ C_3 + 2C_4 + \frac{1}{2} C_9 - \frac{1}{2} C_8 - \frac{1}{2} C_10 \right] \\
+ M_{ann}^{LL} \left[ 1C_3 - \frac{1}{2} C_9 \right] + M_{ann}^{LR} \left[ C_5 - \frac{1}{2} C_7 \right], \quad (29) \]

for the formula of \( \sqrt{2} A(\bar{B}_s \to \eta K^0) = V_{ub} V_{ud}^* T_n - V_{tb} V_{td}^* P_n \), and

\[ T_s = 0, \quad (30) \]

\[ P_s = f_s F_{B_s \to K}^{LL} \left[ a_3 - a_5 + \frac{1}{2} a_7 - \frac{1}{2} a_9 \right] + f_K F_{B_s \to \eta_s}^{LL} \left[ a_4 - \frac{1}{2} a_{10} \right] \\
+ f_K F_{B_s \to \eta_s}^{SP} \left[ a_6 - \frac{1}{2} a_8 \right] + M_{B_s \to K}^{LL} \left[ C_4 + C_4 - \frac{1}{2} C_8 - \frac{1}{2} C_10 \right] \\
+ M_{B_s \to \eta_s}^{LL} \left[ C_5 - \frac{1}{2} C_9 \right] + M_{ann}^{LL} \left[ C_5 - \frac{1}{2} C_7 \right] + M_{ann}^{LR} \left[ C_5 - \frac{1}{2} C_7 \right], \quad (31) \]

for the formula of \( \sqrt{2} A(\bar{B}_s \to \eta' K^0) = V_{ub} V_{ud}^* T_s - V_{tb} V_{td}^* P_s \).
\[ C_{2i} + \frac{1}{3}C_{2i-1} \text{ and } a_{2i-1} = C_{2i-1} + \frac{1}{3}C_{2i} \ (i = 3, \ldots, 5). \]

We can express
\[ T_2 = T_n \cos \phi - T_s \sin \phi, \quad P_2 = P_n \cos \phi - P_s \sin \phi, \quad T_3 = T_n \sin \phi + T_s \cos \phi, \quad P_3 = P_n \sin \phi + P_s \cos \phi. \]

Hence, the amplitudes of T and P from the decay process of \( \bar{B}_s \to K^0\pi^0 \) with \( \eta - \eta' - \pi^0 \) mixing can be written as:
\[ T = T_1 + T_2 + T_3, \quad P = P_1 + P_2 + P_3. \]

We can see that the above formula without \( \eta - \eta' - \pi^0 \) mixing is reduced to
\[ T = T_1, \quad P = P_1. \]

The relevant weak phase \( \phi \) and strong phase \( \delta \) are obtained as following
\[ re^{i\delta}e^{i\phi} = \frac{P}{T} \times \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*}. \]

The parameter \( r \) represents the absolute value of the ratio of penguin and tree amplitudes:
\[ r \equiv \left| \frac{K^0\pi^0|H^T|\bar{B}_s^0}{K^0\pi^0|H^T|\bar{B}_s^0} \right|. \]

The strong phase associated with \( r \) can be given
\[ re^{i\delta} = \frac{P}{T} \times \left| \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*} \right| = r \cos \delta + ir \sin \delta, \]
where
\[ \left| \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*} \right| = \sqrt{[\rho(1-\rho)-\eta^2/2+\eta^2]}(1-\lambda^2/2)(\rho^2+\eta^2). \]

The \( CP \) violation, \( A_{CP} \), can be written as
\[ A_{CP} \equiv \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} = \frac{-2r\sin \delta \sin \phi}{1 + 2r \cos \delta \cos \phi + r^2}. \]

**IV. INPUT PARAMETERS**

The CKM matrix, which elements are determined from experiments, can be expressed in terms of the Wolfenstein parameters \( A, \rho, \lambda \) and \( \eta \) [20]:
\[
\begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & \lambda & \lambda \lambda^2 (\rho - i \eta) \\
-\lambda & 1 - \frac{1}{2} \lambda^2 & \lambda \\
\lambda \lambda^2 (1 - \rho - i \eta) & -\lambda \lambda^2 & 1
\end{pmatrix},
\]
FIG. 1: The direct $CP$ violation as a function of $\rho$ and $\eta$ without isospin symmetry breaking from the CKM matrix element for the decay process of $\bar{B}_s \to K^{0}\pi^{0}$. The horizontal axis and vertical axis refer to the values of $\rho$ and $\eta$, respectively.

where $O(\lambda^4)$ corrections are neglected. The latest values for the parameters in the CKM matrix are [21]:

$$\lambda = 0.22506 \pm 0.00050, \quad A = 0.811 \pm 0.026,$$

$$\bar{\rho} = 0.124^{+0.019}_{-0.018}, \quad \bar{\eta} = 0.356 \pm 0.011.$$  \hspace{1cm} (41)

where

$$\bar{\rho} = \rho(1 - \frac{\lambda^2}{2}), \quad \bar{\eta} = \eta(1 - \frac{\lambda^2}{2}).$$ \hspace{1cm} (42)

From Eqs. (41) (42) we have

$$0.109 < \rho < 0.147, \quad 0.354 < \eta < 0.377.$$ \hspace{1cm} (43)

The other parameters are given as following [20]:

$$f_\pi = 0.131\text{GeV}, \quad f_K = 0.160\text{GeV},$$

$$m_{B_s^0} = 5.36677, \quad f_{B_s} = 0.23,$$

$$f_n = 0.1742, \quad f_s = 0.1391,$$

$$m_\pi = 0.13957\text{GeV}, \quad m_W = 80.385\text{GeV},$$

$$m_t = 173021\text{GeV}, \quad m_b = 4.8\text{MeV}.$$ \hspace{1cm} (44)

V. NUMERICAL RESULTS

The CP violation depends on the weak phase from the CKM matrix elements and the strong phase associated with QCD. The CKM matrix elements depend on the parameters of $A$, $\rho$, $\lambda$ and $\eta$. We find that the results are less reliant on $A$ and $\lambda$ in the course of calculation. Hence, we present the $CP$ violation from the weak phases associated with the $\rho$ and $\eta$ in the CKM matrix elements while the $A$ and $\lambda$ are assigned for the central values. In
TABLE I: The CP asymmetry of $B_s$ decay mode via isospin symmetry and isospin symmetry breaking via $\eta - \eta' - \pi^0$ mixing. The rate of increase are defined $|x_2| - |x_1| \times 100\%$, where $x_1, x_2$ represent the results from isospin symmetry and isospin symmetry breaking, respectively. The fluctuation numerical values refer to the contribution of the limiting parameters from the CKM matrix elements.

| decay mode        | isospin symmetry | $\eta - \eta' - \pi^0$ mixing | the rate of increase |
|-------------------|------------------|--------------------------------|---------------------|
| $\bar{B}_s \to K^0 \pi^0$ | $53.43^{+2.26}_{-2.14}$ | $78.78^{+4.71}_{-4.51}$ | $47.45^{+2.47}_{-2.65}$ % |
| $\bar{B}_s \to K^0* \pi^0$ | $-23.58^{+1.12}_{-1.22}$ | $-38.00^{+1.57}_{-1.67}$ | $61.15^{+1.05}_{-1.20}$ % |
| $\bar{B}_s \to \phi \pi^0$ | $8.32^{+0.48}_{-0.47}$ | $-11.67^{+0.62}_{-0.69}$ | $40.26^{+17.19}_{-14.70}$ % |
| $\bar{B}_s \to \pi^0 \eta$ | $-8.76^{+0.28}_{-0.25}$ | $16.90^{+1.13}_{-1.00}$ | $92.92^{+19.70}_{-16.45}$ % |
| $\bar{B}_s \to \pi^0 \eta'$ | $27.43^{+1.09}_{-1.05}$ | $6.76^{+0.43}_{-0.39}$ | $-75.36^{+0.57}_{-0.49}$ % |

In our numerical calculations, we let $(\rho, \eta)$ vary from the limiting values $(\rho_{\text{min}}, \eta_{\text{min}})$ to $(\rho_{\text{max}}, \eta_{\text{max}})$, respectively. For the decay channel of $\bar{B}_s \to K^0 \pi^0$, we show the $CP$ violation as a function of $\rho$ and $\eta$ without and with the isospin symmetry breaking from the CKM matrix element in Fig.1 and Fig.2, respectively. One can see that the $CP$ violation is enhanced in the present of the weak phase associated with the CKM matrix elements and the strong phase from the isospin symmetry breaking. The numerical results are shown for the decay $\bar{B}_s \to P(V)\pi^0$ from the isospin symmetry and the isospin symmetry breaking in table.1. We find the $CP$ violation can be shifted via $\pi^0-\eta-\eta'$ mixing for the isospin symmetry breaking. One can find that the rate of increase of the $CP$ violation can reach 92.92 % for the decay process of $\bar{B}_s \to \pi^0 \eta$. In table.1, we also find the rate of increase of the $CP$ violation is larger in $\bar{B}_s \to \pi^0 \eta(\prime)$ decay process comparing with the other channels we are considering. It is intelligible that the final states include the $\eta$ or $\eta'$ meson. The isospin symmetry breaking changes the sign of the $CP$ violation, such as from $8.32$ to $-11.67$ for the channel of $\bar{B}_s \to \phi \pi^0$, and from $-8.76$ to $16.90$ from the process of $\bar{B}_s \to \pi^0 \eta$.
VI. SUMMARY AND CONCLUSION

In this paper, we study the $CP$ violation for the decay process of $\bar{B}_s \to P(V)\pi^0$ in Perturbative QCD. It is found that the $CP$ violation can be shifted via $\eta - \eta' - \pi^0$ mixing from the isospin symmetry breaking. The $CP$ violation arises from the weak phase difference in CKM matrix and the strong phase difference. The maximum $CP$ violation reaches 78.78% for the decay mode $\bar{B}_s \to K^0\pi^0$ via $\eta - \eta' - \pi^0$ mixing. The rate of increase of the $CP$ violation is larger for the decay process of $\bar{B}_s \to \pi^0\eta'$ than other decay channels. For the decay process $\bar{B}_s \to \phi\pi^0$ and $\bar{B}_s \to \pi^0\eta$, the isospin symmetry breaking changes the sign of the $CP$ violation.

In the calculation of $CP$ violation parameters, there are some uncertainties in our work. Generally, power corrections beyond the heavy quark limit give the major theoretical uncertainties. We discussed that the error in this article mainly comes from the following aspects. The first uncertainty refers to hadronic parameters, such as the decay constants and the wave function of the $B_s$ meson. The second error comes from the CKM matrix elements. The last error arises from the choice of the high order corrections.

In order to achieve the required energy and luminosity requirements, the Large Hadron Collider (LHC), which has currently started at CERN, has been upgraded many times. The LHC Run I data started in 2010. The peak instantaneous luminosity documentary during Run I was $8.0 \times 10^{32}$ cm$^{-2}$s$^{-1}$. The center-of mass energy was primarily $\sqrt{s} = 7$ TeV and was raised to 8 TeV in 2012 [22]. This was followed by the first long shutdown period (LS1), which was devoted to upgrades essential for increasing beam energy to $\sqrt{s} = 13$ TeV centre of mass energy and peak instantaneous luminosity $1.7 \times 10^{34}$ cm$^{-2}$s$^{-1}$ [23, 24]. In the following years, there are two primary detector (CMS and ATLAS) upgrades happening after Run II and Run III. Phase-I and II upgrade prepares for an instantaneous luminosity of $2 - 3 \times 10^{34}$ cm$^{-2}$s$^{-1}$ and $5 - 7 \times 10^{34}$ cm$^{-2}$s$^{-1}$ [25], respectively. With a series of modifications and upgrades, the LHC gives access to high energy frontier at TeV scale and an occasion to further improve the consistency test for the CKM matrix. The production rates for heavy quark flavors will be great at the LHC, and the $b\bar{b}$ production cross section will be of the order of 0.5 mb, providing as many as $10^{12}$ bottom events per year [22, 26]. The heavy quark physics is one of the major topics of LHC experiments. Especially, the LHCb experiment exploits amounts of $b$ mesons, produced in proton-proton collisions at the LHC to search for $CP$ violation. Recently, LHCb Collaboration presents observation of the decay $B^0_s \to \phi\pi^+\pi^-$ meson. Obtaining more data from LHC, it is possible to make further analysis for $CP$ violation of $B^0_s$ decays [27]. We expect our results is valuable for measurement of $CP$ violation of $B^0_s$ decays in the following LHCb experiments.

VII. ACKNOWLEDGMENTS

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VIII. APPENDIX: RELATED FUNCTIONS DEFINED IN THE TEXT

The functions related with the tree and penguin contributions are presented for the factorization and non-factorization amplitudes with PQCD approach [5, 6, 15].
The hard scales \( t \) are chosen as

\[
\begin{align*}
t_a &= \max\{\sqrt{t} M_B, 1/b_1, 1/b_3\}, \\
t'_a &= \max\{\sqrt{t} M_B, 1/b_1, 1/b_3\}, \\
t_b &= \max\{\sqrt{t} M_B, \sqrt{|1 - x_1 - x_2|x_3 M_B, 1/b_1, 1/b_2\}, \\
t'_b &= \max\{\sqrt{t} M_B, \sqrt{|x_1 - x_2|x_3 M_B, 1/b_1, 1/b_2\}, \\
t_c &= \max\{\sqrt{1 - x_3 M_B}, 1/b_2, 1/b_3\}, \\
t'_c &= \max\{\sqrt{t} M_B, 1/b_2, 1/b_3\}, \\
t_d &= \max\{\sqrt{t} (1 - x_3) M_B, \sqrt{1 - (1 - x_1 - x_2)x_3 M_B, 1/b_1, 1/b_2\}, \\
t'_d &= \max\{\sqrt{t} (1 - x_3) M_B, \sqrt{|x_1 - x_2|(1 - x_3) M_B, 1/b_1, 1/b_2\}.
\end{align*}
\]

The functions \( h \) comprises the jet function \( S_t(x_i) \) arising from the threshold re-summation \[28\] and the propagator of virtual quark and gluon \[5, 6, 15\]. They are defined by

\[
\begin{align*}
h_c(x_1, x_3, b_1, b_3) &= \left[ \theta(b_1 - b_3) J_0(\sqrt{t} M_B, b_3) K_0(\sqrt{t} M_B, b_1) \right. \\
& \quad + \left. \theta(b_3 - b_1) J_0(\sqrt{t} M_B, b_1) K_0(\sqrt{t} M_B, b_3) \right] K_0(\sqrt{t} M_B, b_1) S_t(x_3), \\
h_n(x_1, x_2, x_3, b_1, b_2) &= \left[ \theta(b_2 - b_1) K_0(\sqrt{t} M_B, b_2) J_0(\sqrt{t} M_B, b_1) \right. \\
& \quad + \left. \theta(b_1 - b_2) K_0(\sqrt{t} M_B, b_1) J_0(\sqrt{t} M_B, b_2) \right] \\
& \quad \times \left\{ \begin{array}{ll}
\frac{i\pi}{2} H_0^{(1)}(\sqrt{t} x_3 M_B, b_2), & x_1 - x_2 < 0 \\
K_0(\sqrt{t} x_3 M_B, b_2), & x_1 - x_2 > 0
\end{array} \right., \\
h_n(x_2, x_3, b_2, b_3) &= \left( \frac{i\pi}{2} \right)^2 S_t(x_3) \left[ \theta(b_2 - b_3) H_0^{(1)}(\sqrt{t} M_B, b_2) J_0(\sqrt{t} M_B, b_3) \right. \\
& \quad + \left. \theta(b_3 - b_2) H_0^{(1)}(\sqrt{t} M_B, b_3) J_0(\sqrt{t} M_B, b_2) \right] H_0^{(1)}(\sqrt{t} x_3 M_B, b_2), \\
h_{na}(x_1, x_2, x_3, b_1, b_2) &= \frac{i\pi}{2} \left[ \theta(b_1 - b_2) H_0^{(1)}(\sqrt{t} x_2(1 - x_3) M_B, b_1) J_0(\sqrt{t} x_2(1 - x_3) M_B, b_2) \right. \\
& \quad + \left. \theta(b_2 - b_1) H_0^{(1)}(\sqrt{t} x_2(1 - x_3) M_B, b_2) J_0(\sqrt{t} x_2(1 - x_3) M_B, b_1) \right] \\
& \quad \times K_0(\sqrt{t} - (1 - x_1 - x_2) x_3 M_B, b_1), \\
h'_{na}(x_1, x_2, x_3, b_1, b_2) &= \frac{i\pi}{2} \left[ \theta(b_1 - b_2) H_0^{(1)}(\sqrt{t} x_2(1 - x_3) M_B, b_1) J_0(\sqrt{t} x_2(1 - x_3) M_B, b_2) \right. \\
& \quad + \left. \theta(b_2 - b_1) H_0^{(1)}(\sqrt{t} x_2(1 - x_3) M_B, b_2) J_0(\sqrt{t} x_2(1 - x_3) M_B, b_1) \right] \\
& \quad \times \left\{ \begin{array}{ll}
\frac{i\pi}{2} H_0^{(1)}(\sqrt{t} x_2 - x_1)(1 - x_3) M_B, b_1), & x_1 - x_2 < 0 \\
K_0(\sqrt{t} x_1 - x_2)(1 - x_3) M_B, b_1), & x_1 - x_2 > 0
\end{array} \right.,
\end{align*}
\]

where \( H_0^{(1)}(z) = J_0(z) + i Y_0(z) \).

The \( S_t \) re-sums the threshold logarithms \( \ln^2 x \) appearing in the hard kernels to all orders and it has been parame-
terized as

\[ S_t(x) = \frac{2^{1+2c} \Gamma(3/2 + c)}{\sqrt{\pi} \Gamma(1 + c)} [x(1 - x)]^c, \]  

(58)

with \( c = 0.4 \). In the nonfactorizable contributions, \( S_t(x) \) gives a very small numerical effect to the amplitude \[ 29 \]. Therefore, we drop \( S_t(x) \) in \( h_n \) and \( h_{na} \).

The evolution factors \( E_c^{(t)} \) and \( E_a^{(t)} \) are given by \[ 3, 6, 15 \]

\[
E_c(t) = \alpha_s(t) \exp[-S_B(t) - S_3(t)], \quad E'_c(t) = \alpha_s(t) \exp[-S_B(t) - S_2(t) - S_3(t)]\big|_{b_1=b_3},
\]

(59)

\[
E_a(t) = \alpha_s(t) \exp[-S_2(t) - S_4(t)], \quad E'_a(t) = \alpha_s(t) \exp[-S_B(t) - S_2(t) - S_3(t)]\big|_{b_2=b_3},
\]

(60)
in which the Sudakov exponents are defined as

\[
S_B(t) = s \left( x_1 \frac{M_{B_1}}{\sqrt{2}} b_1 \right) + \frac{5}{3} \int_{1/b_1}^t \frac{d\tilde{\mu}}{\tilde{\mu}} \gamma_q(\alpha_s(\tilde{\mu})),
\]

(61)

\[
S_2(t) = s \left( x_2 \frac{M_{B_2}}{\sqrt{2}} b_2 \right) + s \left( 1 - x_2 \right) \frac{M_{B_2}}{\sqrt{2}} b_2 + 2 \int_{1/b_2}^t \frac{d\tilde{\mu}}{\tilde{\mu}} \gamma_q(\alpha_s(\tilde{\mu})),
\]

(62)

with the quark anomalous dimension \( \gamma_q = -\alpha_s/\pi \). Replacing the kinematic variables of \( M_2 \) to \( M_3 \) in \( S_2 \), we can get the expression for \( S_3 \). The explicit form for the function \( s(Q, b) \) is \[ 3, 6, 15 \]:

\[
s(Q, b) = \frac{A^{(1)}}{2\beta_1} \ln \left( \frac{\hat{q}}{b} \right) - \frac{A^{(1)}}{2\beta_1} \left( \hat{q} - \hat{b} \right) + \frac{A^{(2)}}{4\beta_1^2} \left( \frac{\hat{q}}{b} - 1 \right) - \left[ \frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1} \ln \left( \frac{e^{2\gamma_E-1}}{2} \right) \right] \ln \left( \frac{\hat{q}}{b} \right) + \frac{A^{(1)}\beta_2}{4\beta_1^2} \left[ \ln(2\hat{q}) + 1 - \frac{\ln(2\hat{b}) + 1}{b} \right] + \frac{A^{(1)}\beta_2}{8\beta_1^2} \left[ \ln^2(2\hat{q}) - \ln^2(2\hat{b}) \right],
\]

(63)

where the variables are defined by

\[
\hat{q} \equiv \ln[Q/(\sqrt{2}A)], \quad \hat{b} \equiv \ln[1/(bA)],
\]

(64)

and the coefficients \( A^{(i)} \) and \( \beta_i \) are

\[
\beta_1 = \frac{33 - 2n_f}{12}, \quad \beta_2 = \frac{153 - 19n_f}{24},
\]

\[
A^{(1)} = \frac{4}{3}, \quad A^{(2)} = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} n_f + \frac{8}{3} \beta_1 \ln(\frac{1}{2} e^{\gamma_E}),
\]

(65)

\( n_f \) is the number of the quark flavors and \( \gamma_E \) is the Euler constant. We will use the one-loop running coupling constant, i.e. we pick up the four terms in the first line of the expression for the function \( s(Q, b) \) \[ 3, 6, 15 \].

The \( LL, LR \) and \( SP \) refer to the contributions from \( (V - A)(V - A) \) operators, \( (V - A)(V + A) \) operators and \( (S - P)(S + P) \) operators, respectively. The form factor of \( B_s \rightarrow M_3 \) can be given \[ 3, 6, 15 \]:

\[
S_t(x) = \frac{2^{1+2c} \Gamma(3/2 + c)}{\sqrt{\pi} \Gamma(1 + c)} [x(1 - x)]^c,
\]
where the color factor $C_F = 4/3$ and $a_i$ represents the corresponding Wilson coefficients from different decay channels. $r_i = \frac{m_{0i}}{m_Z}$, where $m_{0i}$ refers to the chiral scale parameter.

- $(V - A)(V - A)$ operators:

$$f_{M_z} F_{B^+ \rightarrow M_3}^{LL}(a_i) = 8\pi C_F M_{B^+}^4 f_{M_z} \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 db_3 \phi_{B^+}(x_1, b_1) \left\{ a_i(t_a) E_c(t_a) \right\} \left\{ (1 + x_3) \phi_3^A(x_3) + r_3(1 - 2x_3)(\phi_3^P(x_3) + \phi_3^T(x_3)) \right\} h_c(x_1, x_3, b_1, b_3)$$

$$+ 2r_3 \phi_3^P(x_3) a_i(t'_a) E_c(t'_a) h_c(x_3, x_1, b_3), \quad (66)$$

- $(V - A)(V + A)$ operators:

$$F_{B^+ \rightarrow M_3}^{LR}(a_i) = -F_{B^+ \rightarrow M_3}^{LL}(a_i), \quad (67)$$

- $(S - P)(S + P)$ operators:

$$f_{M_z} F_{B^+ \rightarrow M_3}^{SP}(a_i) = 16\pi r_2 C_F M_{B^+}^4 f_{M_z} \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 db_3 \phi_{B^+}(x_1, b_1) \left\{ a_i(t_a) E_c(t_a) \right\} \left\{ \phi_3^A(x_3) + r_3(2 + x_3)\phi_3^P(x_3) - r_3 x_3 \phi_3^T(x_3) \right\} h_c(x_1, x_3, b_1, b_3)$$

$$+ 2r_3 \phi_3^P(x_3) a_i(t'_a) E_c(t'_a) h_c(x_3, x_1, b_1), \quad (68)$$

$$\text{where the color factor } C_F = 4/3 \text{ and } a_i \text{ represents the corresponding Wilson coefficients from different decay channels.}$$

- $(V - A)(V - A)$ operators:

$$M_{B^+ \rightarrow M_3}^{LL}(a_i) = 32\pi C_F M_{B^+}^4 / \sqrt{6} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 db_2 \phi_{B^+}(x_1, b_1) \phi_3^A(x_2)$$

$$\times \left\{ \left[ (1 - x_2) \phi_3^A(x_3) - r_3 x_3 (\phi_3^P(x_3) - \phi_3^T(x_3)) \right] a_i(t_b) E_c(t_b) \right\} \left[ h_n(x_1, 1 - x_2, x_3, b_1, b_2) + h_n(x_1, x_2, x_3, b_1, b_2) \right]$$

$$\times \left\{ -(x_2 + x_3) \phi_3^A(x_3) + r_3 x_3 (\phi_3^P(x_3) + \phi_3^T(x_3)) \right\} a_i(t'_b) E_c(t'_b), \quad (69)$$

- $(V - A)(V + A)$ operators:

$$M_{B^+ \rightarrow M_3}^{LR}(a_i) = 32\pi C_F M_{B^+}^4 / \sqrt{6} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 db_2 \phi_{B^+}(x_1, b_1)$$

$$\times \left\{ h_n(x_1, 1 - x_2, x_3, b_1, b_2) \left[ (1 - x_2) \phi_3^A(x_3) \left( \phi_3^P(x_2) + \phi_3^T(x_2) \right) \right] \right\} \left[ r_3 x_3 \left( \phi_3^P(x_2) - \phi_3^T(x_2) \right) \left( \phi_3^P(x_3) + \phi_3^T(x_3) \right) \right] \right\} a_i(t_b) E_c(t_b)$$

$$+ r_3 x_2 \phi_3^P(x_2) - \phi_3^T(x_2)) \left( \phi_3^P(x_3) - \phi_3^T(x_3) \right) \right\} a_i(t'_b) E_c(t'_b), \quad (70)$$

$$\text{where the color factor } C_F = 4/3 \text{ and } a_i \text{ represents the corresponding Wilson coefficients from different decay channels.}$$
• \((S - P)(S + P)\) operators:

\[
M^{\text{SP}}_{B_s \rightarrow M_3}(a_i) = 32\pi C_F M^2_{B_s}/\sqrt{6} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1) \phi_2^A(x_2) 
\]

\[
\times \left\{ \left[ (x_2 - x_3 - 1) \phi_3^A(x_3) + r_3 x_3 (\phi_3^P(x_3) + \phi_3^T(x_3)) \right] 
\times a_i(t_b) E'_c(t_b) h_a(x_1, 1 - x_2, x_3, b_1, b_2) + a_i(t'_b) E'_c(t'_b) 
\times [x_2 \phi_3^A(x_3) + r_3 x_3 (\phi_3^P(x_3) - \phi_3^T(x_3))] h_a(x_1, x_2, x_3, b_1, b_2) \right\}. \quad (71)
\]

The functions are related with the annihilation type process, whose contributions are:

• \((V - A)(V - A)\) operators:

\[
f_{B_s F_{\text{ann}}^{\text{LL}}(a_i)} = 8\pi C_F M^4_{B_s} f_{B_s} \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \left\{ a_i(t_c) E_a(t_c) 
\times \left[ (x_3 - 1) \phi_3^A(x_2) \phi_3^A(x_3) - 4r_3 x_3 \phi_3^P(x_2) \phi_3^P(x_3) 
+ 2r_3 x_3 \phi_3^P(x_2) (\phi_3^P(x_3) - \phi_3^T(x_3)) \right] h_a(x_1, 1 - x_2, x_3, b_2, b_3) 
\times \left[ x_2 \phi_3^A(x_2) \phi_3^A(x_3) + 2r_3 x_3 (\phi_3^P(x_2) - \phi_3^T(x_2)) \phi_3^P(x_3) 
+ 2r_3 x_2 (\phi_3^P(x_2) + \phi_3^T(x_2)) \phi_3^P(x_3) \right] a_i(t'_c) E_a(t'_c) h_a(1 - x_3, x_2, b_3, b_2) \right\}. \quad (72)
\]

• \((V - A)(V + A)\) operators:

\[
F_{\text{ann}}^{\text{LR}}(a_i) = F_{\text{ann}}^{\text{LL}}(a_i). \quad (73)
\]

• \((S - P)(S + P)\) operators:

\[
f_{B_s F_{\text{ann}}^{\text{SP}}(a_i)} = 16\pi C_F M^4_{B_s} f_{B_s} \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \left\{ [2r_3 x_3 (\phi_3^P(x_2) \phi_3^A(x_3) 
\times \left[ (1 - x_3) r_3 \phi_3^A(x_2) (\phi_3^P(x_3) + \phi_3^T(x_3)) \right] a_i(t_c) E_a(t_c) h_a(x_2, 1 - x_3, b_2, b_3) 
\times [2r_3 \phi_3^P(x_2) \phi_3^P(x_3) + r_2 x_2 (\phi_3^P(x_2) - \phi_3^T(x_2)) \phi_3^A(x_3) \right] \right\}. \quad (74)
\]
\( (V - A)(V - A) \) operators:

\[
M_{\text{ann}}^{LL}(a_i) = 32\pi C_F M_{B_s}^4 / \sqrt{6} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_2 db_3 \phi_{B_s}(x_1, b_1)
\times \left\{ h_{na}(x_1, x_2, x_3, b_1, b_2) \left[ -x_2 \phi_x^A(x_2) (x_2) \phi_3^A(x_3) - 4r_2 \phi_2^P(x_2) \phi_3^P(x_3)
+ (1 - x_3) \phi_2^T(x_2) + \phi_3^T(x_3) \right] a_i(t_d) E_a(t_d)
+ h_{na}(x_1, x_2, x_3, b_1, b_2) \left[ (1 - x_3) \phi_2^2(x_2) \phi_3^A(x_3)
+ (1 - x_3) \phi_2^T(x_2) + \phi_3^T(x_3) \right] a_i(t_d') E_a(t_d') \right\},
\]

(75)

\( (V - A)(V + A) \) operators:

\[
M_{\text{ann}}^{LR}(M_2, M_3, a_i) = 32\pi C_F M_{B_s}^4 / \sqrt{6} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_2 db_3 \phi_{B_s}(x_1, b_1)
\times \left\{ h_{na}(x_1, x_2, x_3, b_1, b_2) \left[ r_2 (2 - x_2) (x_2) + \phi_3^P(x_3) \phi_3^A(x_3)
- r_3 (1 + x_3) \phi_2^2(x_2) \phi_3^A(x_3)
+ h_{na}(x_1, x_2, x_3, b_1, b_2) \left[ r_2 (x_2) (\phi_2^P(x_2) + \phi_3^T(x_3) \phi_3^A(x_3)
+ r_3 (1 - x_3) \phi_2^2(x_2) \phi_3^A(x_3)
\right] a_i(t_d) E_a(t_d)
+ h_{na}(x_1, x_2, x_3, b_1, b_2) \left[ r_2 (x_2) (\phi_2^P(x_2) + \phi_3^T(x_3) \phi_3^A(x_3)
+ h_{na}(x_1, x_2, x_3, b_1, b_2) \left[ r_2 (x_2) (\phi_2^P(x_2) + \phi_3^T(x_3) \phi_3^A(x_3)
\right] a_i(t_d') E_a(t_d') \right\},
\]

(76)

\( (S - P)(S + P) \) operators:

\[
M_{\text{ann}}^{SP}(a_i) = 32\pi C_F M_{B_s}^4 / \sqrt{6} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_2 db_3 \phi_{B_s}(x_1, b_1)
\times \left\{ a_i(t_d) E_a(t_d) h_{na}(x_1, x_2, x_3, b_1, b_2) \left[ (x_3 - 1) \phi_2^2(x_2) \phi_3^A(x_3)
- 4r_2 \phi_2^P(x_2) \phi_3^P(x_3) + r_2 \epsilon_3 x_3 \phi_2^P(x_2) + \phi_3^T(x_2) \phi_3^P(x_3) - \phi_2^2(x_3)
+ r_3 (1 - x_3) \phi_2^P(x_2) - \phi_3^T(x_2) \phi_3^P(x_3) + \phi_3^T(x_3) \phi_2^2(x_3)
+ a_i(t_d') E_a(t_d') h_{na}(x_1, x_2, x_3, b_1, b_2) \left[ x_2 \phi_2^2(x_2) \phi_3^A(x_3)
+ x_2 \phi_3^T(x_2) \phi_3^A(x_3)
\right] a_i(t_d) E_a(t_d)
\right\},
\]

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