One-particle properties of deformed $N \approx 28$ odd-$N$ nuclei with weakly-bound or resonant neutrons

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Abstract

Possible deformation of odd-$N$ nuclei with $N \approx 28$ towards the neutron drip line is investigated using the Nilsson diagram based on deformed Woods-Saxon potentials. Both weakly-bound and resonant one-particle levels are properly obtained by directly solving the Schrödinger equation in mesh of space coordinate with the correct boundary condition. If we use the same diffuseness of the potential as that of $\beta$-stable nuclei, the energy difference between the neutron $2p_{3/2}$ and $1f_{7/2}$ levels becomes very small or the $N=28$ energy gap almost disappears, as the binding energies of those levels approach zero. This suggests that the ground states of those neutron drip line nuclei are likely to be deformed. In particular, the spin-parity and the magnetic moment of the ground state of odd-$N$ nuclei, $^{43}$S$^{27}_{27}$ and $^{45}$S$^{29}_{29}$, are examined. Moreover, it is suggested that in $^{39}$Mg$^{27}_{27}$ lying outside the drip line the lowest resonant state may have $5/2^-$, if the $N=28$ energy gap almost vanishes.

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I. INTRODUCTION

The study of the properties of nuclei far from the line of $\beta$ stability is presently one of the most active and challenging topics in nuclear structure. In particular, some neutron-rich nuclei with the traditional magic number $N=20$ are now known to be deformed. It is theoretically expected that $N=28$ may also no longer be a magic number in some neutron-drip-line nuclei due to the behavior of weakly-bound neutrons with $\ell=1$ relative to that with $\ell=3$, though very few experimental data are presently available for pinning down the change of this magic number. Keeping a model as simple as possible, in the present work we examine the change of the $N=28$ energy gap towards the neutron drip line and its consequence, which is related to the characteristic feature of the weakly-bound and resonant one-particle levels with small $\ell$ values. This origin of the change of nuclear shell structure is quite different from the change related to the tensor force between protons and neutrons using harmonic-oscillator wave functions. The latter change of the neutron shell-structure depends on the number of protons which occupy specific orbits and, furthermore, the effect of weakly-bound neutrons is not taken into account.

The nucleus $^{40}_{12}\text{Mg}_{28}$ is known to lie inside the drip line, while $^{39}_{12}\text{Mg}_{27}$ lies outside [1]. It may still take a time to obtain spectroscopic informations on those neutron-rich Mg isotopes. Whether or not a given neutron-drip-line nucleus is deformed depends also on the preference by the proton number. In Ref. [2] studying the $2n$ separation energy $S_{2n}$ as a function of the number of neutrons, it is pointed out that the S isotope exhibits a pronounced change of slope around $N=26$, which is in contrast to the Ca isotope that is known to be spherical. It is in general very useful to obtain the spin-parity of low-lying states of odd-$N$ nuclei, which may directly provide the information on the neutron shell structure. In this sense, it is very interesting and important to pin down the spin-parity of neutron-rich nuclei, $^{43}_{16}\text{S}_{27}$ and $^{45}_{16}\text{S}_{29}$. We refer to Ref. [3] for the present status of the study of the $N=28$ shell closure.

In the present work we investigate the bound and resonant one-particle levels as a function of deformation, using the parameters which may correspond to nuclei, $^{45}_{16}\text{S}_{29}$ and $^{39}_{12}\text{Mg}_{27}$. The change of neutron shell structure at spherical as well as deformed shapes in the system with weakly-bound or resonant neutrons is studied. Calculated values of magnetic dipole moment for the neutron-rich odd-$N$ S isotope are presented as an additional information, since the measured magnetic moment gave a clear spin-parity assignment of $3/2^+$ to the ground state.
of $^6_{17}C_{11}$ with one weakly-bound neutron $^4_6$ and $3/2^-$ to that of $^{33}_{12}Mg_{21}$ $^5$. The model used here is basically the same as that employed in Ref. $^6$. It is the author’s hope that the Nilsson diagrams of the present paper may be useful as a basic help material in the interpretation of new experimental data on neutron drip line nuclei with $N \approx 28$.

In Sec. II the model is briefly described and discussed, while numerical results are presented in Sec.III. Conclusions and further discussions are given in Sec.IV.

II. MODEL

Though a considerable amount of publications are available for the study of possibly deformed $N \approx 28$ nuclei towards the neutron drip line, to the author’s knowledge no Hartree-Fock (HF) or Hartree-Fock-Bogoliubov (HFB) calculation for deformed nuclei is presently available which is carried out by integrating the Schrödinger equation in mesh of space coordinate with proper asymptotic behavior for large $r$ values. This way of estimating one-particle levels is of essential importance, when we are interested in the phenomena, in which resonant and/or weakly-bound one-particle levels having small $\ell$-values as the major component of wave functions are playing an important role. Even if the technique to carry out that kind of deformed HF or HFB calculations is established, such a calculation gives one self-consistent deformation which depends on the effective interaction used, and it is not easy to find out how general the obtained results are. Moreover, the formulation of traditional HF calculations is not directly applicable for studying unbound nuclei such as $^{39}_{25}Mg$.

In the analysis of experimental data on nuclei away from the stability line with $N \approx 20$ and $N \approx 28$ the prediction by the traditional shell model with harmonic-oscillator wave-functions are so far often used as a convenient reference. The systematic shell-model calculation, in which weakly-bound particles are properly treated, is indeed available $^7$. However, the complicated calculations have so far been carried out only for very limited isotopes and certainly not for neutron-drip-line nuclei with $N \approx 20$ and $N \approx 28$. If some nuclei are deformed or very soft in deformation, the shell-model wave-functions become too complicated to extract the physics in a simple intuitive terminology. For example, the interesting quantities such as one-particle energies and nuclear shape are not directly obtained from shell model calculations.
Considering how useful the Nilsson diagram has been in providing the basis for the classification of experimental data on stable deformed nuclei \[8\], in the present work we apply the model and idea presented in Ref. \[6\] to the study of neutron-drip-line nuclei with \(N \approx 28\). In order to examine one-particle resonant states in the unbound nucleus \(^{39}\text{Mg}\), in any case we have to assume a one-particle potential. We use the parameters of Woods-Saxon potentials taken from the standard ones \[9\] for stable nuclei except for the depth, \(V_{WS}\). Namely, the diffuseness, the strength of spin-orbit potentials and the radius parameter are taken from those on p.239 of Ref. \[9\]. A slightly larger diffuseness might be more appropriate for presently studied nuclei considering the contribution by weakly-bound neutron(s) to the self-consistent potential. However, we note that the major part of the nuclear potential is provided by well-bound nucleons and, moreover, a larger diffuseness leads to the degeneracy of the 2p\(3/2\) and 1f\(7/2\) levels already at a larger binding energy than the one obtained in the present work. This is because for a given binding energy one-particle wave functions with small \(\ell\) values can more easily extend to the outside of the potential, if the potential surface is softer or more diffuse.

The coupled equations derived from the Schrödinger equation are solved in coordinate space with the correct asymptotic behavior of wave functions for \(r \to \infty\), both for bound \[10\] and resonant \[11, 12\] levels. The solution obtained in this way is totally independent of the upper limit of radial integration, \(R_{\text{max}}\), if both the potential and the coupling term are already negligible at \(r = R_{\text{max}}\).

One-particle resonance is obtained if one of calculated eigenphases \[13\] increases through \(\pi/2\) as energy increases. This is the definition of one-particle resonance in a deformed potential \[11, 12\], which is adopted in the present work. The definition is a natural extension of the definition of one-particle resonance for spherical potentials in terms of phase shift \[13\]. In the numerical examples presented in this work where \(A \approx 40\), for example, one-particle resonant levels with \(\Omega^\pi = 1/2^-\) and \(3/2^-\) can hardly be obtained for \(\varepsilon_\Omega > 1.5\) MeV if the main component of the wave functions inside the potential has \(\ell = 1\). In contrast, well-defined resonant levels with \(\Omega^\pi = 1/2^-\) and \(3/2^-\) can be obtained up till several MeV when the major component inside the potential has \(\ell \geq 3\). See the dotted and dashed curves for \(\varepsilon_\Omega > 0\) in Figs. 1 and 2.
III. NUMERICAL RESULTS

In Fig. 1 one-particle energies as a function of axially-symmetric quadrupole deformation are plotted, in which $A=45$ and $V_{WS} = -41$ MeV are used. The $V_{WS}$ value is approximately equal to the value for $^{45}S$ estimated from Eq. (2-182) of Ref. [9]. Considering that the measured neutron separation energy of $^{45}S$ is 2.21 MeV, the level occupied by the 29th neutron has an approximately right binding-energy in both spherical and deformed cases. In Fig. 1 one-particle resonant levels connected to the $1f_{5/2}$ level at 1.90 MeV for $\beta=0$ are plotted for all deformations as far as the resonance can be obtained in terms of eigenphase. On the other hand, one-particle levels connected to the $1g_{9/2}$ level are shown only around the spherical shape, except the $\Omega^{-}=1/2^+$ and $3/2^+$ levels on the prolate side.

It is seen that at $\beta=0$ the $2p_{3/2}$ level lies 2.27 MeV higher than the $1f_{7/2}$ level. The energy gap of 2.27 MeV is much smaller than that for $N=28$ stable nuclei and may not be large enough to have $N=28$ as a magic number. For comparison, the energy difference between $1f_{7/2}$ and $1d_{3/2}$, which characterizes the magic number $N=20$ in Fig.1, is 4.92 MeV. According to our experience in the analysis of stable nuclei in terms of Nilsson diagrams, the ground state has a clear tendency to having the deformation where the local density of one-particle levels is minimum (or very small) for a given particle-number. It may further help to realize the deformation if some down-sloping one-particle levels around the Fermi level are occupied compared with the configuration for the spherical shape. If we apply this rule, it is seen from Fig. 1 that the ground state of $N\approx 28$ nuclei with $S_n$ of a few MeV, namely $^{46}_{16}S_{27}$ and $^{45}_{16}S_{29}$, may be prolate-deformed with $\beta=0.3-0.5$ rather than a spherical shape. If so, the ground state of $^{43}S$ with $N=27$ will have either $1/2^-$ (dotted curve in Fig. 1) or $5/2^-$ (dot-dashed curve in Fig. 1) while the ground state of $^{45}S$ with $N=29$ may have either $7/2^-$ (dot-dot-dashed curve in Fig. 1) or $1/2^-$, since $\Gamma^\pi = \Omega^\pi$ is expected for respective band-head states in the region of $\beta=0.3-0.5$ examining the one-particle wave functions in the Woods-Saxon potential. These values of spin-parity are quite different from those for the spherical shape, for which one expects $7/2^-$ for the nucleus with $N=27$ and $3/2^-$ for the nucleus with $N=29$. Indeed, for example, the ground-state spin of all known $N=27$ isotones with even $Z\geq 18$ is $7/2^-$, indicating that those nuclei are not far from being spherical.

Next, we estimate possible values of magnetic dipole moment. Either the $p_{3/2}$ or $f_{7/2}$ neutron in the seniority-one state produces the magnetic dipole moment of $(1/2)g_{eff}^\pi$, which
is equal to $-1.34 \mu_N$ if we use $g_s^{eff} = (0.7)g_s^{free}$. Indeed, the measured magnetic moment of the ground state ($I^\pi=7/2^-$) of $^{47}$Ca is $-1.38(3) \mu_N$ \[14\], while the preliminary value of the ground state ($I^\pi=3/2^-$) of $^{49}$Ca is $-1.38(6) \mu_N$ \[15\]. The magnetic moment of deformed nuclei is calculated using Eqs. (4-86), (4-87) and (4-88) of Ref. \[8\], when the matrix elements of $\ell_\nu$ and $s_\nu$ are obtained from one-particle wave-functions in the deformed potential (see Eqs. (5-86) and (5-87) of Ref. \[8\]). As an example of magnetic moments of deformed neutron-rich nuclei, we may mention that the measured magnetic moment of the ground state ($I^\pi=3/2^-$) of $^{33}$Mg is $-0.7456(5) \mu_N$ \[5\]. On the other hand, the value estimated in the present way is $(-0.88)-(-0.75) \mu_N$ for $\beta=0.3$ and $g_s^{eff}=(0.7-0.6)g_s^{free}$, identifying the $I^\pi=3/2^-$ state as the band-head state of the neutron [330 1/2] configuration (with the calculated decoupling parameter $a = -3.2$).

In Table I the calculated magnetic dipole moments using deformed one-particle wave functions together with $g_R=0.35$ and $g_s^{eff} = (0.7)g_s^{free}$ are shown. Interestingly enough, calculated magnetic moments of $^{43}$S and $^{45}$S for the possible deformed shape are quite different from those for the spherical shape. This suggests that the measurement of magnetic moments of the odd-N S-isotope may not only fix the spin-parity but also tell whether or not the nuclei are deformed.

Using the intermediate-energy Coulomb excitation of a radioactive $^{44}$S beam, in Ref. \[16\] the reduced transition probability $B(E2;0^+_g.s. \rightarrow 2^+_1) = 314(88) e^2 fm^4$ with $E_x(2^+_1) = 1297(18)$ keV is obtained. The relatively large $B(E2)$ value is often quoted as an evidence for a deformed shape of the nucleus $^{44}$S. Furthermore, in Ref. \[17\] a level at 940 keV with a $B(E2)$ of 175(69) $e^2 fm^4$ in $^{43}$S is reported, also using the Coulomb excitation. If we assume that the Z=16 proton core is the same for $^{44}$S and $^{43}$S, the $B(E2)$ value is proportional to the Clebsch-Gordan coefficient squared, $[C(I_i \ 2 \ I_f ; K \ 0 \ K)]^2$, for E2 transitions within a band with a given K. Since the E2 excitation from the band-head state is here considered, the Coriolis perturbation may be neglected in the first approximation. Then, using the measured value of $B(E2)$ in $^{44}$S, $B(E2)$ values of $(I=K=5/2) \rightarrow (I=7/2,K=5/2)$ and $(I=K=1/2) \rightarrow (I=3/2,K=1/2)$ in $^{43}$S are estimated to be 150(42) and 126(35) $e^2 fm^4$, respectively. Both of these $B(E2)$ values lie within the experimental error of the measured value of 175(69) $e^2 fm^4$. Though the excitation energy of 940 keV seems to be a bit too large for the expected energy difference between the $(I=K=1/2)$ and $(I=3/2,K=1/2)$ states, the measured $B(E2)$ value in Ref. \[17\] does not really tell whether the possibly deformed ground state of $^{43}$S is
I^\pi = K^\pi = 5/2^- \text{ or } I^\pi = K^\pi = 1/2^-.

In Fig. 2 the bound and resonant one-particle levels as a function of deformation are plotted for A=39 and \( V_{WS} = -37 \text{ MeV} \). The parameters of the Woods-Saxon potential are chosen approximately for the unbound nucleus \(^{39}\text{Mg}\). Though the \( V_{WS} \) value estimated from Eq. (2-182) of Ref. \([9]\) is \(-38.3 \text{ MeV}\) for \(^{39}\text{Mg}\), a slightly shallower potential is used so that the one-particle level which the N=27th neutron will occupy is unbound for most of the likely deformation including spherical shape. The one-particle level scheme of Fig. 2 may still give an N=28 system (such as \(^{40}\text{Mg}\)) inside the neutron drip line, since the pair correlation between the last-occupied pair of neutrons may produce an extra binding energy to the even-N system.

The \( 2p_{1/2} \) level at \( \beta = 0 \) and the \( \Omega^\pi = 1/2^- \) level connected to the \( 2p_{1/2} \) level cannot be obtained as a resonant level and, thus, they do not appear in Fig. 2. Both the \( \Omega^\pi = 1/2^- \) and \( 3/2^- \) levels which are connected to the \( 2p_{3/2} \) level at \( \beta = 0 \) cannot exist as resonant levels for \( \epsilon_\Omega > 1.25 \text{ MeV} \), since the main component of the wave functions inside the potential has \( \ell = 1 \). In contrast, the \( \Omega^\pi = 1/2^- \) level connected to the \( 1f_{5/2} \) level at \( \beta = 0 \) can continue as a resonant level for \( \beta < 0.14 \) (or \( \epsilon_\Omega > 4.44 \text{ MeV} \)) on the prolate side, since the major component of the wave function inside the potential has then \( \ell = 3 \). For values of \( \beta \) larger than 0.14 the increasing portion of the \( \ell = 1 \) component of the wave function inside the potential prevents the eigenphase from reaching \( \pi/2 \).

When the \( 2p_{3/2} \) and \( 1f_{7/2} \) levels approach zero binding, those two levels become almost degenerate. See also Fig. 4 of Ref. \([6]\). In Fig. 2 the distance between the \( 2p_{3/2} \) and \( 1f_{7/2} \) resonant levels, namely the energy gap at N=28, is only 180 keV. The partial filling of the almost degenerate neutron \( 2p_{3/2} \) and \( 1f_{7/2} \) shells may suggest a deviation from the spherical shape. Moreover, the proton number \( Z=12 \) (Mg-isotope) in the region of stable nuclei is known to prefer prolate shape as seen from measured properties of \(^{24}\text{Mg}_{12}\) and \(^{25}\text{Mg}_{13}\). Then, the spin-parity of the lowest-lying one-particle resonant state of the unbound nucleus \(^{39}\text{Mg}\) is likely to be \( 5/2^- \) for a possible appreciable amount of prolate deformation, examining the level structure around N=27 in Fig. 2. Indeed, the spin-parity of the level occupied by the 27th neutron on the prolate side depends on the size of the N=28 energy gap at \( \beta = 0 \), though on the oblate side it is always \( \Omega^\pi = 1/2^- \). For example, for an appreciable amount of prolate deformation the level occupied by the 27th neutron may have \( \Omega^\pi = 1/2^- \) in the case that the \( 2p_{3/2} \) level lies appreciably higher than the \( 1f_{7/2} \) level (see Fig. 3). If the
lowest-lying one-particle resonant level in the possibly deformed (unbound) nucleus $^{39}$Mg is observed, the one-particle decay width will tell us whether the resonance has $I^\pi=5/2^-$ or $1/2^-$. At $\beta=0$ the $1f_{7/2}$ resonance at $\varepsilon(1f_{7/2}) = 0.26$ MeV has the calculated width of 0.8 keV, while the $2p_{3/2}$ resonance at $\varepsilon(2p_{3/2}) = 0.44$ MeV has the width of 890 keV. Since the major component of the $\Omega^\pi=5/2^-$ and $1/2^-$ resonant levels inside the potential on the prolate side has $\ell=3$ and $\ell=1$, respectively, the large difference of the widths of the $I^\pi=5/2^-$ and $1/2^-$ levels remains unchanged also for a moderate amount of prolate deformation.

Since some weakly-bound neutron orbits with $\Omega^\pi=1/2^-$ and $3/2^-$ are occupied in the even-even core of $^{39}$Mg, one may wonder whether the appropriate parameters of the Woods-Saxon potential given by the core in this case may be different from those of stable nuclei. In order to check the validity of our numerical results of $^{39}$Mg we have tried the numerical calculation by increasing the diffuseness parameter $a=0.67$ fm of the potential used in Fig. 2 to $a=0.75$ fm. Then, we find the one-particle resonance energies 0.14 and 0.37 MeV for the $2p_{3/2}$ and $1f_{7/2}$ neutron levels, respectively. Namely, the $2p_{3/2}$ resonant level lies now lower than the $1f_{7/2}$ resonant level by 230 keV. Nevertheless, in the deformation region of $0.25 < \beta < 0.5$ the negative-parity one-particle level scheme corresponding to $20<N<28$ remains nearly the same as in Fig. 2, because almost degenerate $2p_{3/2}$ and $1f_{7/2}$ levels strongly mix with each other as soon as deformation sets in. Consequently, the statements on the unbound nucleus $^{39}$Mg written in the previous paragraph are valid also for the larger diffuseness $a=0.75$ fm.

For reference, in Fig.3 the Nilsson diagram for the case in which all levels of the 1f-2p shell at $\beta=0$ are well bound is shown. The parameters of the Woods-Saxon potential are designed for the stable nuclei $^{52}$Cr$_{28}$. The energy distance between the $2p_{3/2}$ and $1f_{7/2}$ levels at $\beta=0$, namely the energy gap at $N=28$, is 4.03 MeV.

IV. CONCLUSIONS AND DISCUSSIONS

First, being inspired by the observation that the behavior of $S_{2n}$ of the S isotope for $N>26$ as a function of the number of neutrons exhibits a pronounced difference from that of the Ca isotope, the possible spin-parity and the magnetic moment of the nuclei $^{43}$S$_{27}$ and $^{45}$S$_{29}$ are estimated for an appreciable amount of prolate deformation. The study of the relevant Nilsson diagram indicates the possible prolate deformation $\beta=0.3$-0.5 for those
nuclei, applying the rule that a large one-particle level density around the Fermi level at the spherical point may lead to a possible deformation where the local one-particle level density is a minimum (or very small). For the possible prolate deformation; (a) The ground-state spin of \(^{43}\text{S}\) \(^{(45}\text{S})\) is either \(5/2^-\) or \(1/2^-\) \((7/2^-\) or \(1/2^-\)) in contrast to \(7/2^-\) \((3/2^-\)) for the ground state of \(N=27\) \((N=29)\) spherical nuclei; (b) As shown in Table I, the calculated magnetic moments of the ground states of \(^{43}\text{S}\) and \(^{45}\text{S}\) are clearly different from \((1/2)g_{\text{eff}}^s\) which is the value in the spherical limit of both \(N=27\) and \(N=29\). Thus, it is very interesting to experimentally obtain any of the spectroscopic informations, in order to find an evidence of the change of the magic number \(N=28\).

For the ground state of the nucleus \(^{44}\text{S}\) a subtle competition between the prolate and oblate deformations is previously obtained in the self-consistent mean field or more elaborate calculations by several theoretical groups. (See Ref. \[18\] and references quoted therein.) Examining the local density of one-particle levels around \(N=28\) in the Nilsson diagrams of Figs. 1, 2 and 3, it is seen that for a larger \(N=28\) shell gap at \(\beta=0\) such as in Fig. 3 the energy minima of prolate, spherical and oblate shapes may compete, while for a smaller \(N=28\) energy gap at \(\beta=0\) the oblate shape is likely to be unfavored. In presently available HF or HFB or more elaborate calculations the weakly-binding and resonant one-particle levels do not seem to be properly treated. Therefore, the size of the \(N=28\) shell gap and the shell structure of weakly-binding neutrons, which are obtained in those elaborate calculations, need to be checked in a more careful way, though some of the calculations certainly showed a reduction of the \(N=28\) spherical shell gap due to one or other reasons. On the other hand, protons are deeply bound in those neutron-rich nuclei and, thus, the deformation preferred by \(Z=16\) protons may be guessed, in the first approximation, from the Nilsson diagram obtained by using wave functions based on harmonic-oscillator wave functions. For example, examining Figs. 2 and 3 of Ref. \[18\], it is seen that the deformation preferred by \(Z=16\) protons may be prolate rather than oblate.

Secondly, the spin-parity of the ”ground” state (namely the lowest one-particle resonant state) of the unbound nucleus \(^{39}\text{Mg}_{27}\) is investigated and suggested to be \(5/2^-\) if the \(N=28\) energy gap almost collapses and the even-even core which provides the potential is moderately prolate-deformed. In contrast, the lowest one-particle resonant state of \(^{39}\text{Mg}\) may have \(1/2^-\) for a prolate-deformed core if the \(N=28\) energy gap at the spherical point is appreciable. The expected spin-parity for the deformed shape makes a contrast to \(7/2^-\) which
is expected for a spherical nucleus with N=27. The resonant state with 5/2− or 7/2− where the minimum \( \ell \) value of the wave-function components \( \ell_{\text{min}}=3 \) can easily be differentiated from the resonance with \( \ell_{\text{min}}=1 \) if the one-particle decay width of the resonance is observed.

In the present paper the possible many-body pair correlation is not included. However, we note that the spin-parity as well as the magnetic dipole moment of the ground state obtained in the present work will remain unchanged when one particle is replaced by one quasiparticle which comes from a mean-field approximation to the many-body pair correlation.

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TABLE I: Calculated magnetic dipole moments of the N=27th and 29th neutrons for the deformation $\beta = 0.35$ and 0.45 in Fig. 1. Values of $g_R=0.35$ and $g_{s}^{\text{eff}}=(0.7)g_{s}^{\text{free}}$ are used. Corresponding one-particle levels in Fig. 1 are identified from the tabulated values of $\beta$ and $\varepsilon_\Omega$ and noting $I^\pi = \Omega^\pi$. The first and second rows correspond to the N=29th neutron in Fig. 1 depending on $\beta$, while the third and fourth rows to the N=27th neutron.

| $I^\pi$ | $\mu (\beta = 0.35)$ ($\mu_N$) | $\varepsilon_\Omega (\beta = 0.35)$ (MeV) | $\mu (\beta = 0.45)$ ($\mu_N$) | $\varepsilon_\Omega (\beta = 0.45)$ (MeV) |
|---------|-------------------------------|--------------------------------------|-------------------------------|--------------------------------------|
| $7/2^-$ | $-0.765$                      | $-1.09$                              | $-0.765$                      | $-0.38$                              |
| $1/2^-$ | $+0.545$                      | $-1.21$                              | $+0.584$                      | $-1.61$                              |
| $5/2^-$ | $-0.580$                      | $-4.19$                              | $-0.600$                      | $-4.47$                              |
| $1/2^-$ | $+0.600$                      | $-3.81$                              | $+0.545$                      | $-4.83$                              |
**Figure captions**

Figure 1 : Neutron one-particle levels as a function of axially-symmetric quadrupole deformation. Parameters of the Woods-Saxon potential are designed approximately for the nucleus $^{45}_{16}$S$_{29}$. The diffuseness, the radius and the depth of the Woods-Saxon potential are 0.67 fm, 4.52 fm, and $-41.0$ MeV, respectively. The $\Omega^\pi = 1/2^-$ levels are denoted by dotted curves, the $3/2^-$ levels by dashed curves, the $5/2^-$ levels by dot-dashed curves and the $7/2^-$ levels by dot-dot-dashed curves, while positive-parity levels are plotted by solid curves. The neutron numbers 20 and 28, which are obtained by filling in all lower-lying levels, are indicated with circles. See the text for details.

Figure 2 : Neutron one-particle levels as a function of axially-symmetric quadrupole deformation. Parameters of the Woods-Saxon potential are designed approximately for the unbound nucleus $^{39}_{12}$Mg$_{27}$. The diffuseness, the radius and the depth of the Woods-Saxon potential are 0.67 fm, 4.31 fm, and $-37.0$ MeV, respectively. Neither the $2p_{1/2}$ level at $\beta = 0$ nor one-particle levels at $\beta \neq 0$ connected to the $2p_{1/2}$ level are obtained as one-particle resonant levels and, thus, are not plotted. See the text for details and the caption to Fig. 1.

Figure 3 : Neutron one-particle levels as a function of axially-symmetric quadrupole deformation. Parameters of the Woods-Saxon potential are designed approximately for the stable nucleus $^{52}_{24}$Cr$_{28}$. The diffuseness, the radius and the depth of the Woods-Saxon potential are 0.67 fm, 4.74 fm, and $-49.0$ MeV, respectively. See the caption to Fig. 1 and the text for details.
Neutron one-particle levels in Woods-Saxon potential

\[ \Omega_{\pi} = \frac{1}{2} \quad \Omega_{\pi} = \frac{3}{2} \quad \Omega_{\pi} = \frac{5}{2} \quad \Omega_{\pi} = \frac{7}{2} \]

\[ f_{\frac{7}{2}} \quad p_{\frac{3}{2}} \quad p_{\frac{1}{2}} \quad f_{\frac{5}{2}} \quad g_{\frac{9}{2}} \]

\[ \varepsilon_{\Omega} \text{ (MeV)} \]

quadrupole-deformation parameter \( \beta \)
Neutron one-particle levels in Woods-Saxon potential

\[ \Omega^\pi = \frac{1}{2}, \quad \frac{3}{2}, \quad \frac{5}{2}, \quad \frac{7}{2} \]

\[ f_{\frac{7}{2}}, \quad p_{\frac{3}{2}}, \quad d_{\frac{3}{2}}, \quad f_{\frac{5}{2}} \]

\( \varepsilon_\Omega (\text{MeV}) \)

quadrupole-deformation parameter \( \beta \)
Neutron one-particle levels in Woods-Saxon potential

- $\Omega^\pi = 1/2$
- $\Omega^\pi = 3/2$
- $\Omega^\pi = 5/2$
- $\Omega^\pi = 7/2$

Energy levels for different quantum numbers and deformation parameters.