On the nuclearity of certain Cuntz-Pimsner algebras

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Abstract

In the present paper, we give a short proof of the nuclearity property of a class of Cuntz-Pimsner algebras associated with a Hilbert \( \mathcal{A} \)-bimodule \( \mathcal{M} \), where \( \mathcal{A} \) is a separable and nuclear \( C^* \)-algebra. We assume that the left \( \mathcal{A} \)-action on the bimodule \( \mathcal{M} \) is given in terms of compact module operators and that \( \mathcal{M} \) is direct summand of the standard Hilbert module over \( \mathcal{A} \).

1 Introduction

M.V. Pimsner introduced in his seminal paper [15] a new family of \( C^* \)-algebras \( \mathcal{O}_\mathcal{M} \) that are naturally generated by a Hilbert bimodule \( \mathcal{M} \) over a \( C^* \)-algebra \( \mathcal{A} \). These algebras generalise Cuntz-Krieger algebras as well as crossed-products by the group \( \mathbb{Z} \). In Pimsner’s construction \( \mathcal{O}_\mathcal{M} \) is given as a quotient of a Toeplitz like algebra acting on a concrete Fock space associated to \( \mathcal{M} \). An alternative abstract approach to Cuntz-Pimsner algebras in terms of \( C^* \)-categories is given in [8, 12, 16] (for the notion of \( C^* \)-category see [10]).

In the present note, we give a short proof of nuclearity property of the Cuntz-Pimsner algebra (cf. Theorem 2.7) associated with a full Hilbert bimodule \( \mathcal{M} \) with faithful left action, satisfying the following additional properties:

(i) The coefficient \( C^* \)-algebra \( \mathcal{A} \) is nuclear and separable.

(ii) The left \( \mathcal{A} \)-action is given in terms of compact module operators and is non-degenerate.

(iii) \( \mathcal{M} \) is a direct summand in the standard Hilbert module \( \mathcal{H}_\mathcal{A} \) over \( \mathcal{A} \).

Nuclearity of Cuntz-Pimsner algebras has been discussed recently in the concrete Toeplitz algebra setting (cf. [13, 11]). The paper by Kumjian is in a certain sense complementary to ours. In fact, he considers left \( \mathcal{A} \)-actions that have a trivial intersection with the compact module operators. Our proof uses the alternative approach in [8]. In particular, we analyse the structure of some spectral subspaces \( \mathcal{O}_\mathcal{M}^k \), \( k \in \mathbb{N}_0 \), that are associated to a natural circle action. An important step in the proof is to recognize the structure of \( \mathcal{O}_\mathcal{M}^1 \) as an imprimitivity \( \mathcal{O}_\mathcal{M}^0 \)-bimodule. In this way we can apply a result by Brown, Green and Rieffel to the corresponding stabilizations and consider, roughly speaking, the Cuntz-Pimsner algebra as a crossed-product of the zero-grade spectral subspace \( \mathcal{O}_\mathcal{M}^0 \), which is shown to be a nuclear \( C^* \)-algebra.
2 Basic definitions and the main theorem

Let \( A \) be a \( C^* \)-algebra and \( M \) a Hilbert \( A \)-module. We denote by \( \mathcal{L}(M) \) the \( C^* \)-algebra of adjointable, right \( A \)-module operators on \( M \) and by \( \mathcal{K}(M) \subseteq \mathcal{L}(M) \) the (closed) ideal of compact operators generated by the maps

\[
\theta_{\psi,\psi'} \in \mathcal{L}(M), \quad \psi, \psi' \in M, \quad \text{with} \quad \theta_{\psi,\psi'}(\varphi) := \psi \langle \psi', \varphi \rangle, \quad \varphi \in M, \quad (1)
\]

where \( \langle \cdot, \cdot \rangle \) is the \( A \)-valued scalar product defined on \( M \).

We denote by \( O_A \) the standard (countably generated) Hilbert \( A \)-module of sequences \( (A_n)_n \) such that \( \sum_n A_n^*A_n \) converges in \( A \) (cf. [4, Example 13.1.2 (c)]). It is well-known that the \( C^* \)-algebra \( \mathcal{K}(O_A) \) of compact, right \( A \)-module operators on \( O_A \) is isomorphic to \( K \otimes A \), where \( K \) is the \( C^* \)-algebra of compact operators over a separable Hilbert space. The multiplier algebra of \( \mathcal{K}(O_A) \) is isomorphic to \( \mathcal{L}(O_A) \). We will regard the standard module \( O_A \) as a Hilbert \( A \)-bimodule with the obvious left \( A \)-module action.

In this paper, we will consider Hilbert \( A \)-modules \( M \) which are direct summands of \( O_A \). This implies that \( M \) is finitely or countably generated. Conversely by Kasparov stabilization every countably generated Hilbert \( A \)-module is a direct summand of \( O_A \). Moreover, if \( A \) is unital, then also every algebraically finitely generated Hilbert \( A \)-module is a direct summand of \( O_A \). The left \( A \)-action on \( M \) is given by a \( * \)-homomorphism \( \alpha: A \rightarrow \mathcal{L}(M) \). In the present paper, we will assume that \( \alpha \) is faithful (in the sequel, we will identify elements of \( A \) with their image in \( \mathcal{L}(M) \)) and with image contained in \( \mathcal{K}(M) \). We will also assume that \( \alpha \) is non-degenerate in the sense that

\[
A \cdot \mathcal{K}(M) := \text{closed span} \{ AT \mid A \in A, T \in \mathcal{K}(M) \} = \mathcal{K}(M). \quad (2)
\]

Note that when \( M \) is algebraically finitely generated, then \( \mathcal{L}(M) = \mathcal{K}(M) \), thus every left \( A \)-action is given by compact module operators.

We denote by \( O_M \) the Cuntz-Pimsner algebra associated with the Hilbert bimodule \( M \) (cf. [15]). Recall that \( O_M \) is generated as a \( C^* \)-algebra by \( M \) and \( A \) satisfying the relations

\[
\psi^* \psi' = \langle \psi, \psi' \rangle, \quad \psi, \psi' \in M \quad (3)
\]

\[
A \psi := \alpha(A) \psi, \quad A \in A. \quad (4)
\]

Note that (3) implies \( \psi' \psi^* \varphi = \theta_{\psi',\psi}(\varphi), \psi', \psi^*, \varphi \in M \). Therefore one has the natural identification

\[
\theta_{\psi',\psi} = \psi' \psi^*. \quad (5)
\]

Moreover, if \( M^r := \mathcal{M} \otimes_A \cdots \otimes_A \mathcal{M}, r \in \mathbb{N}, \) is the \( r \)-fold tensor product with coefficients in \( A \), then there is an identification

\[
M^r \simeq \text{closed span} \{ \psi_1 \cdots \psi_r \in O_M \mid \psi_k \in M, k = 1, \ldots, r \}. \quad (6)
\]

There is a natural action of the circle \( T := \{ z \in \mathbb{C} \mid |z| = 1 \} \) on the Cuntz-Pimsner algebra given by

\[
\delta: T \rightarrow \text{Aut} \, O_M, \quad \delta_z(\psi) := z \psi, \quad z \in T, \, \psi \in M. \quad (7)
\]

We denote by

\[
O_M^k := \left\{ T \in O_M \mid \delta_z(T) = z^k T, \quad z \in T \right\}, \quad k \in \mathbb{Z},
\]

the spectral subspaces associated to the circle action. In particular, \( O_M^0 \) is the closed span of elements of the form \( \psi' \psi^*, \psi, \psi' \in M^r, r \in \mathbb{N} \). From Eq. (2) we have that

\[
A \cdot M := \text{closed span} \{ A \psi \mid A \in A, \psi \in M \} = M. \]
(Use the fact that every $\psi \in \mathcal{M}$ is of the form $\psi = T\psi'$ for some $T \in \mathcal{K}(\mathcal{M})$, $\psi' \in \mathcal{M}$, cf. [5, Lemma 1.3]). Therefore $A \cdot \mathcal{O}_\mathcal{M}^0 = \mathcal{M}^r$, $r \in \mathbb{N}$, and

$$A \cdot \mathcal{O}_\mathcal{M}^0 = \text{closed span } \{ AT \mid A \in \mathcal{A} , T \in \mathcal{O}_\mathcal{M}^0 \} = \mathcal{O}_\mathcal{M}^0 .$$ (9)

In order to discuss universality properties of the Cuntz-Pimsner algebra we give the following definitions (cf. [8, §2])

**Definition 2.1** Let $\mathcal{A} \subset \mathcal{B}$ be an inclusion of $\mathcal{C}^*$-algebras. A Hilbert $\mathcal{A}$-bimodule in $\mathcal{B}$ is a closed vector space $\mathcal{M} \subset \mathcal{B}$ satisfying

(i) $A\psi \in \mathcal{M}$, $\psi A \in \mathcal{M}$ and $\psi^*\psi' \in \mathcal{A}$ for every $A \in \mathcal{A}$, $\psi, \psi' \in \mathcal{M}$.

(ii) For any $A \in \mathcal{A}$ with $A\psi = 0$, $\psi \in \mathcal{M}$, one has $A = 0$.

We say that $\mathcal{M}$ is full if for every $A \in \mathcal{A}$ there are $\psi, \psi' \in \mathcal{M}$ such that $A = \psi^*\psi'$.

Now, there is a natural identification

$$\mathcal{M}\mathcal{M}^* := \text{closed span } \{ \psi' \cdot \psi^* \mid \psi, \psi' \in \mathcal{M} \subset \mathcal{B} \} \simeq \mathcal{K}(\mathcal{M}) ,$$

hence $\mathcal{M}\mathcal{M}^*$ may be regarded as a $\mathcal{C}^*$-subalgebra of $\mathcal{B}$. We say that $\mathcal{M}$ has support $\mathbb{1}$ if there exists a sequence $\{ \psi_n \} \subset \mathcal{M}$ such that $\{ U_N := \sum_{n=1}^N \psi_n^*\psi_n \} \subset \mathcal{M}$ is an approximate unit for $\mathcal{B}$ (recall that by assumption the $\mathcal{C}^*$-algebras are separable.)

**Remark 2.2** Note that if $\mathcal{M} \subset \mathcal{B}$ has support $\mathbb{1}$ and if there are elements $A \in \mathcal{A}$, $T \in \mathcal{M}\mathcal{M}^*$ satisfying $A\psi = T\psi$ for every $\psi \in \mathcal{M}$, then

$$T = A.$$ (10)

In fact, since $T\psi_n = A\psi_n$ for every $n \in \mathbb{N}$, we have $TU_N = AU_N$ for every $N \in \mathbb{N}$ and therefore

$$T = \lim N TU_N = \lim N AU_N = A .$$

The following result is just a translation of [15, Theorem 3.12] in terms of Hilbert bimodules in $\mathcal{C}^*$-algebras. We note explicitly that Eq. (10) is equivalent to condition (4) in the above-cited theorem.

**Proposition 2.3** Let $\mathcal{A} \subset \mathcal{B}$ be an inclusion of unital $\mathcal{C}^*$-algebras, $\mathcal{M} \subset \mathcal{B}$ a full Hilbert $\mathcal{A}$-bimodule in $\mathcal{B}$ with support $\mathbb{1}$. Then there is a canonical morphism $\mathcal{O}_\mathcal{M} \to \mathcal{B}$.

**Remark 2.4** Examples of the above universality property can be found in the Cuntz-Pimsner algebra itself. In fact, if $\mathcal{M}$ is algebraically finitely generated and $\mathcal{A}$ is unital, then it follows from the definition of the Cuntz-Pimsner algebra that $\sum_n \psi_n^*\psi_n = 1$ for every finite set $\{ \psi_n \}$ of (normalized) generators of $\mathcal{M}$. If $\mathcal{M}$ is countably generated as a right $\mathcal{A}$-module, then there are elements $\{ \psi_n \}_{n=1}^\infty \subset \mathcal{M}$ such that $U_N := \sum_{n=1}^N \psi_n^*\psi_n$ is an approximate unit for $\mathcal{K}(\mathcal{M})$, hence also for $\mathcal{A}$ which may be regarded as a $\mathcal{C}^*$-subalgebra of $\mathcal{K}(\mathcal{M})$ (see for example [9] or p. 266 in [8]). Finally, since $\mathcal{O}_\mathcal{M}$ is generated as a $\mathcal{C}^*$-algebra by $\mathcal{M}$ and $\mathcal{A}$, we conclude that $\{ U_N \} \subset \mathcal{M}$ is an approximate unit for $\mathcal{O}_\mathcal{M}$, so that $\mathcal{M}$ has support $\mathbb{1}$ in $\mathcal{O}_\mathcal{M}$.

**Remark 2.5** The $\mathcal{C}^*$-algebra $\mathcal{K}$ of compact operators over a separable Hilbert space is clearly a Hilbert $\mathcal{K}$-bimodule with left and right actions defined by multiplication and scalar product $(V, V') := V^*V'$, $V, V' \in \mathcal{K}$. Let $\mathcal{M}$ be a Hilbert $\mathcal{A}$-bimodule. We consider the external tensor product of Hilbert bimodules $\mathcal{M} \hat{\otimes} \mathcal{K}$, which has a natural structure of Hilbert $(\mathcal{A} \otimes \mathcal{K})$-bimodule, and denote by $\mathcal{O}_{\mathcal{M} \hat{\otimes} \mathcal{K}}$ the associated Cuntz-Pimsner algebra. Then, there is a natural identification $\mathcal{O}_{\mathcal{M} \hat{\otimes} \mathcal{K}} \simeq \mathcal{O}_\mathcal{M} \otimes \mathcal{K}$ defined by the map $\psi \hat{\otimes} V \mapsto \psi \otimes V, \psi \in \mathcal{M}, V \in \mathcal{K}$. 

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**Proposition 2.6** Let $\mathcal{M}$ be a Hilbert $\mathcal{A}$-bimodule satisfying properties (i)-(iii) of the introduction and let $\mathcal{O}_\mathcal{M}$ be the corresponding Cuntz-Pimsner algebra. Then the zero grade $C^*$-algebra $\mathcal{O}^0_{\mathcal{M}}$ (cf. [3]) is nuclear.

**Proof:** First we prove that every $\mathcal{K}(\mathcal{M}^r), r \in \mathbb{N}$, is a nuclear $C^*$-algebra. Since $\mathcal{M}$ is countably generated (or algebraically finitely generated if $\mathcal{A}$ is unital), we conclude that $\mathcal{M}^r$ is countably generated (or algebraically finitely generated if $\mathcal{A}$ is unital). By Kasparov stabilization, we obtain that $\mathcal{M}^r$ is a direct summand of $\mathcal{O}_\mathcal{K}$. This implies that $\mathcal{K}(\mathcal{M}^r)$ is a corner of the nuclear $C^*$-algebra $\mathcal{K}(\mathcal{O}_\mathcal{K}) \simeq \mathcal{A} \otimes \mathcal{K}$ and, therefore, $\mathcal{K}(\mathcal{M}^r)$ is nuclear. Now, for every $r \in \mathbb{N}$ there is an embedding

$$i_r: \mathcal{K}(\mathcal{M}^r) \to \mathcal{L}(\mathcal{M}^{r+1}) \quad i_r(T) \psi \varphi := (T\psi)\varphi,$$

$\psi \in \mathcal{M}^r$, $\varphi \in \mathcal{M}$ (in the usual tensor notation the embedding is given simply by $i_r(T) := T \otimes 1$, where $1$ is the identity of $\mathcal{L}(\mathcal{M})$). Next we show that if the image of the left $\mathcal{A}$-action is contained in $\mathcal{K}(\mathcal{M})$, then $i_r(T) \in \mathcal{K}(\mathcal{M}^{r+1})$. First put $T := \psi_1 \psi_2^*$, $\psi_1, \psi_2 \in \mathcal{M}^r$ so that, $i_r(T)\psi \varphi = \psi_1 \psi_2^* \psi \varphi$. By [3] Lemma 1.3, there is a decomposition $\psi = \psi_0 A_0$ for some $A_0 \in \mathcal{A}$, $\psi_0 \in \mathcal{M}^r$. Moreover, since the left $\mathcal{A}$-module action is given by compact operators, there exist $\varphi_0, \varphi_1 \in \mathcal{M}$ with $A_0 = \varphi_0 \varphi_1^*$, hence $i_r(T) = \psi_0 \varphi_0 \varphi_1^* \psi_2^*$ is an element of $\mathcal{K}(\mathcal{M}^{r+1})$. Therefore the zero grade algebra $\mathcal{O}^0_{\mathcal{M}}$ is an inductive limit

$$\mathcal{O}^0_{\mathcal{M}} = \lim_{\rightarrow} (\mathcal{K}(\mathcal{M}^r), i_r).$$

Since every $\mathcal{K}(\mathcal{M}^r)$ is nuclear, we conclude that $\mathcal{O}^0_{\mathcal{M}}$ is nuclear.

We can now prove our main theorem.

**Theorem 2.7** Let $\mathcal{A}$ be a nuclear and separable $C^*$-algebra. Assume that $\mathcal{M}$ is a full and non-degenerate Hilbert $\mathcal{A}$-bimodule with faithful left $\mathcal{A}$-action and satisfying:

(i) $\mathcal{M}$ is a direct summand (as a right Hilbert $\mathcal{A}$-module) of the standard Hilbert module $\mathcal{O}_{\mathcal{K}}$ over $\mathcal{A}$.

(ii) The left $\mathcal{A}$-action on $\mathcal{M}$ is given in terms of compact module operators.

Then the Cuntz-Pimsner algebra $\mathcal{O}_\mathcal{M}$ is nuclear.

**Proof:** Consider the spectral subspace $\mathcal{O}^0_{\mathcal{M}} \subset \mathcal{O}_\mathcal{M}$ which was introduced in [3] and note that it has a natural structure as a $\mathcal{O}^0_{\mathcal{M}}$-bimodule. In fact, take left and right multiplication by elements of $\mathcal{O}^0_{\mathcal{M}} \subset \mathcal{O}_\mathcal{M}$ and define the $\mathcal{O}^0_{\mathcal{M}}$-valued scalar product by

$$\langle T, T' \rangle := T^* T' \quad T, T' \in \mathcal{O}^0_{\mathcal{M}}.$$

We show next that $\mathcal{O}^1_{\mathcal{M}}$ is a full Hilbert $\mathcal{O}^0_{\mathcal{M}}$-module: by Eq. [9] we have that $\mathcal{A} \cdot \mathcal{O}^0_{\mathcal{M}} = \mathcal{O}^0_{\mathcal{M}}$, so that any $T \in \mathcal{O}^0_{\mathcal{M}}$ can be written as $T = AT'$ for some $A \in \mathcal{A}$ and $T' \in \mathcal{O}^0_{\mathcal{M}}$. Since $\mathcal{M}$ is full, there are $\psi, \psi' \in \mathcal{M}$ such that $A = \psi^* \psi'$ and therefore

$$T = AT' = \psi^* \psi' T' = \langle \psi, \psi' T' \rangle \in \langle \mathcal{O}^1_{\mathcal{M}}, \mathcal{O}^1_{\mathcal{M}} \rangle.$$

Therefore $\mathcal{O}^1_{\mathcal{M}}$ is full as a Hilbert $\mathcal{O}^0_{\mathcal{M}}$-module.

Denote by

$$\mathcal{O} := \mathcal{O}^0_{\mathcal{M}}$$

the Cuntz-Pimsner algebra associated with the $\mathcal{O}^0_{\mathcal{M}}$-bimodule $\mathcal{O}^1_{\mathcal{M}}$. It is enough to show that $\mathcal{O}$ is nuclear since $\mathcal{O}$ is isomorphic the original Cuntz-Pimsner algebra $\mathcal{O}_\mathcal{M}$. In fact, $\mathcal{O}^1_{\mathcal{M}}$ is a Hilbert
$O_{M}^{0}$-bimodule in $O_{M}$ with support $1$ so that by Proposition 2.6 there exists a monomorphism $I: \mathcal{O} \hookrightarrow O_{M}$. Moreover, since $M \subset O_{M}^{1}$, it follows that $I$ is surjective (cf. [15 Theorem 2.5]).

To show the nuclearity of $\mathcal{O}$ we need to exploit the additional structure of the bimodule $O_{M}^{1}$. Note first that there is an isomorphism of Hilbert bimodules

$$\mathcal{O}_{M}^{1} \cong \mathcal{E}z,$$

where $\mathcal{E}$ denotes the external tensor product of Hilbert bimodules and $z$ is the $C^{*}$-algebra of compact operators over a separable Hilbert space. Then, $\mathcal{E}$ is an imprimitivity $B$-bimodule (cf. Remark 2.5). Since $B$ is a stable and separable $C^{*}$-algebra we obtain from Corollary 3.5 in [7] that there is an isomorphism of Hilbert bimodules

$$\beta_{0}: J \to B,$$

where $\mathcal{B}$ is considered as a bimodule over itself with multiplication as right action and the left action being specified by a suitable automorphism $\theta \in \text{Aut} B$. The isomorphism $\beta_{0}$ extends to an isomorphism of the corresponding Cuntz-Pimsner algebras. Moreover, $\beta_{0}$ also extends to an isomorphism of the associated multiplier algebras. Hence we have

$$\beta_{0}: \mathcal{O}_{J} \to \mathcal{O}_{B} \cong B \rtimes_{\theta} \mathbb{Z},$$

where for the last isomorphism with the crossed product we use the results in [15 Chapter 1].

Let $E \in \mathcal{K}$ be a minimal projection. Then using Eq. (11) we may define a monomorphism

$$\beta: O_{M}^{1} \to \mathcal{B} \text{ by means of } T \mapsto \beta_{0}(T \otimes E).$$

Note that the image of $O_{M}^{1}$ generates $\mathcal{B}$ as a $B$-bimodule and that (by universality) $\beta$ can be extended to a monomorphism

$$\beta: \mathcal{O} \to \mathcal{O}_{B},$$

where $\mathcal{O} := O_{O_{M}}^{0}$ was introduced in the beginning of the proof. Since $O_{M}^{0}$ is nuclear (cf. Proposition 2.6 we have that $\mathcal{B}$ is nuclear and the same is true for the crossed product $B \rtimes_{\theta} \mathbb{Z}$ (11 Theorem 15.8.2)). From Eq. (12) we obtain that $\mathcal{O}_{J}$ and $\mathcal{O}_{B}$ are nuclear $C^{*}$-algebras.

Finally, we turn our attention to the Cuntz-Pimsner algebra $\mathcal{O}$. We will conclude the proof by showing that this algebra has a corner of the nuclear algebra $O_{B}$: By Remark 2.5 we may identify $\mathcal{O}_{J} \cong \mathcal{O} \otimes \mathcal{K}$ and using (12) we conclude that $\{\beta_{0}(T \otimes V) \mid T \in \mathcal{O}, V \in \mathcal{K}\}$ is total in $\mathcal{O}_{B}$. Let us now consider the identity $1$ of the multiplier algebra $M(\mathcal{O})$. Then, $1 \otimes E \in M(\mathcal{O} \otimes \mathcal{K})$ and we define the projection $E_{\beta} := \beta_{0}(1 \otimes E) \in M(\mathcal{O}_{B})$. For $T \in \mathcal{O}$ we have that $\beta(T) = E_{\beta} \beta(T) E_{\beta}$. On the contrary, take $B = \sum_{i=1}^{n} \beta_{0}(T_{i} \otimes V_{i}) \in O_{B}, T_{i} \in \mathcal{O}, V_{i} \in \mathcal{K}, i = 1, \ldots, n$. Then

$$E_{\beta}BE_{\beta} = \sum_{i} E_{\beta} \beta_{0}(T_{i} \otimes V_{i}) E_{\beta} = \sum_{i} \beta_{0}(T_{i} \otimes EV_{i}E) = \beta_{0} \left( \left( \sum_{i} z_{i}T_{i} \right) \otimes E \right),$$

where $z_{i} \in \mathbb{C}$ are given by $EV_{i}E = z_{i}E$ (recall that $E$ is minimal). We conclude that $\mathcal{O}$ is isomorphic to the corner $E_{\beta}O_{B}E_{\beta}$. Thus, $\mathcal{O}$ is nuclear.

$\blacksquare$
3 Outlook

Cuntz-Pimsner algebras provide an important family of examples in the theory of operator algebras. Moreover, these algebras, which generalise Cuntz algebras, appear naturally in the extension of Doplicher-Roberts superselection theory (cf. [9]) to the case where the observable algebra has a nontrivial center $Z$ (see e.g. [11] [12] [3]). In this context the category of canonical endomorphisms is isomorphic to a category whose objects are free $Z$-bimodules. The Cuntz-Pimsner algebras associated to these $Z$-bimodules generate a $C^*$-algebra $F$ on which one can realise concretely the dual of a compact group (cf. [14] and references cited therein). The class of algebras considered in the present paper contain the Cuntz-Pimsner algebras that appear in this application. In certain special cases, tensor products of these algebras may also appear in concrete models.

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