Study on the Establishment and Optimization of Sandcastle Infrastructure Model

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Abstract. Sand castle was created by the tourists on the beach for fun, then how to set up a stable sand castle under the condition of waves and tidal waves to get the honor of "sandcastles builder" has been their main concern. In this paper, considering the formation principle of sandcastle and the effect of the uncertain factors and the effect on the stability of the sand castle, the sandcastle infrastructure model is established and the 3D geometric model of the sand pile is optimized step by step. We start with sand heap discretization, the impact process of waves and tides is regarded as the external force acting on the sand, and the stress analysis shows that there are three kinds of stress conditions: no external force, one external force and two external forces. Then from the point to surface, the stress energy of the whole layer of sand is analyzed according to the two aspects of energy transfer dissipation and internal force failure dissipation. The optimal bearing capacity plane is approximately pentagon by annealing algorithm. Finally, from the surface to the solid, taking the self-weight of the sand pile, the effective external force action and the seawater erosion action as the constraint conditions, the optimal tilt angle of the plane stack is calculated to be 36.87° by measuring the risk degree of the overlaying slope Angle. Therefore, the oblique pentagonal body with an inclination angle of 36.87° is relatively stable.

1. Introduction
Building sandcastles is a pleasure created by beachgoers on the casual beach, who use tools, toys and imagination to assemble simple piles of sand into complex architectural models which are known as sandcastles. The sandcastle is usually formed by cutting and molding on a pile of single, indescribable wet sand dunes, gradually forming an identifiable three-dimensional geometry as part of its infrastructure, and then sketching different architectural shapes on this basis. But inevitably, with the waves’ joint and the rising tides will erode the sandcastle. Influenced by the geometry of the sandcastle's foundation, even sandcastles of the same size and the same distance from the beach respond differently to waves and tides.
2. Stress Analysis of Sand Grains

2.1. Analysis of External Force
Due to the complicated mechanism of the tide shore lapping and sand-piling, to further simplify the model, we regard the surf and tidal shore lapping process as an external force $F_0$ acting on the sea surface of the sandcastle.

2.2. Analysis of Internal Forces
The sand grains standing beside the beach all the year round are often eroded by the seawater, so we consider that in a short time, the seawater does not react to the sand grains by chemical erosion, and the water content in the sand grains gap is evenly distributed and remains unchanged.

In fact, the sand pile is composed of the sand particles, the water and air in its interstitial phase. This paper uses the concept of "circular approximation", Two adjacent sand grains are taken as an example for stress analysis, equivalent to consider these two solid material in bending ball water - the contact boundary condition of gas, close contact between grains of sand, and the main components of the sand is $SiO_2$, which is a kind of hydrophilic material. Therefore, according to the phase equilibrium theory can draw interface, under such condition, due to the gap between the difference of hydraulic pressure and atmospheric pressure makes a matrix suction between sand, as shown in Fig 1(a), among them, $r_1$ says radius of curvature of concave to the water phase, which describes the ball tends to the trend of the water phase interface, $r_2$ says convex to the radius of curvature of the water phase. It shows the increasing trend of water phase pressure, and they control the change of water pressure from opposite aspects.

Fig 1. Stress between grains of sand

Consider the stress balance of the free body in the horizontal direction, as shown in Fig 1(b), the free body is affected by three forces: the surface tension ($T_s$) of the water at the interface, represented by $r_1$, is the energy on per unit length required when the water interface is damaged, which is a constant value under the condition of constant temperature, and generates a positive horizontal force. The surface tension ($T'_s$) at the interface represented by $r_2$, produces negative horizontal force. The two forces acting on the interface. The contact angle $\alpha$ is the angle between the tangent line on the gas-water interface and the straight line on the water-solid interface.

The projection of surface tension in the positive horizontal direction is:

$$F_1 = 4r_3 T'_s \sin \alpha$$

The projection of surface tension in the positive horizontal direction is:
\[ F_2 = -4r_1 T_s \sin \alpha \] (2)

The difference between water pressure and air pressure, that is, the projection of matric suction \( F_i \) is expressed as:

\[ F_3 = 4r_1 r_2 F_i \sin \alpha \] (3)

From the equilibrium of three forces, the matric suction is:

\[ F_i = T_s \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \] (4)

Let the particle radius be \( R \), and according to the approximate expression relation proposed by Dallavalles. In this paper introduced the filling angle to describe the relation among \( R \), \( r_1 \), \( r_2 \). As shown in Fig3, we can obtain the following relation formula:

\[ r_1 = R \left( \frac{1}{\cos \theta} - 1 \right), r_2 = R \tan \theta - r_1 = R \left( \tan \theta - \frac{1}{\cos \theta} + 1 \right), 0 \leq \theta \leq 85^\circ \] (5)

And then, the relationship between matric suction and filling angle can be obtained:

\[ F_i = \frac{T_s \cos \theta (\sin \theta + 2 \cos \theta - 2)}{R (1 - \cos \theta) (\sin \theta + \cos \theta - 1)} \] (6)

When considering the construction model, we assume that the water content between the sand grains in the structure is saturated and unchanged, which is equivalent to the fixed contact Angle of the sand grains. Therefore, we regard the force \( F_i \) in the sand grains as a constant, and the destroyed energy of the matrix force is also fixed as a constant.

It is known that the radius of sand grains is 0.2063\( \text{mm} \), the surface tension of seawater at 20\( ^\circ \text{C} \) is about 72.25\( \text{mN/m} \), and the filling angle of saturated water is 45\( ^\circ \). Therefore, the matric suction of the particles was calculated about 247.639\( \text{mpa} \).

3. Analysis of Sand Pile Energy

According to the above analysis of forces, it can be known that the sand is affected by external and internal forces at the same time. When the waves or tides come ashore, regardless of the remaining force of the waves and ebb, there is an external force on the upper surface of the sand pile for oblique upward transmission. Because of the same force and the energy transmission direction, we use the "slicing method" to analyze the energy dissipation of the model layer by layer and then stack it layer by layer to obtain the energy consumption efficiency of the entire model.

3.1. Analysis of Horizontal Energy Consumption of Sand pile

We takes the \( k \)th middle layer section for consideration. As shown in fig2, when an external force acts on a certain sand, the direction of the external force is the \( y \) axial direction, and the \( y \) direction perpendicular to the axis is the \( x \) axial direction, the transmission plane coordinate system is established. The force and energy will be transmitted along the upper half plane of the axis, which is represented on the diagram as particles transmitting force and energy to the three particles above the \( x \). To further simplify the model, we only consider the force and energy transmission process in the direction of the external force.
Fig 2. Schematic diagram of stress analysis of sand grains: Geometry as an example, non-final sandcastle foundation model

Therefore, according to the plane arrangement of sand particles, it can be shown that there are three kinds of forces on the plane that can transfer particles:

Steady-state particles (1st particle in the figure): Particles without force and energy transfer are only bonded to adjacent sand particles by the internal force. The set of all steady-state particles is recorded as $A$;

Semi-steady-state particles (2nd particles in the figure): Particles in the direction of force and energy transmission can only carry force and energy to the adjacent sand particles along the transmission axis direction. The set of all semi-steady-state particles is recorded as $B$;

Unsteady particles (3rd particles in the figure): When there is no nearby sand particle that can transmit force along the direction of the transmission axis, and it can transmit nearby particles, the particle will decompose the force and energy to the upper surface of the transmission axis coordinate system. Two particles carry force and energy, and form a new coordinate system of the transmission axis. The set of all non-steady-state particles is recorded as $C$;

At the same time, we consider that in the transmission process, each time there is an energy decay process, for the convenience of calculation, we set the constant coefficient $\delta$. In addition, based on the force analysis of the sand particles, it can be known that the matrix suction force of the bonding particles along the energy transmission direction is destroyed during the transmission process, and each piece of destruction energy $\varepsilon$ is set to be constant.

We consider each layer as a $m \times n$ 0-1 matrix $P_k (k = 1, 2, ..., l)$. When there are sand particles in the $i$th position of the $j$th row, $p_{i,j+1,k} = 1$ and vice versa, $p_{i,j+1,k} = 0$. We suppose that the initial energy transferred is the initial energy which is harvested by the sand particles exposed to the impact of the seawater. Based on the above analysis, we obtain the energy harvest of the particles in the $i$th and $j$th positions of the $k$th layer:

$$e_{i,j,k} = \begin{cases} 0, & p_{i,j+1,k} \in A \\ \delta e_{i-1,j-1,k} + \varepsilon, & p_{i,j+1,k} \in B \\ \delta (e_{i-1,j-1,k} + e_{i-1,j,k}) + 2\varepsilon, & p_{i,j+1,k} \in C \end{cases}$$

Then the total capacity of the $k$th layer is $Energy_k = \sum_{i=1}^{m} \sum_{j=1}^{n} e_{i,j,k}$
3.2. Analysis of Vertical Energy Consumption

The vertical stacking structure of the sand pile is related to the inclination angle of the sand beach. Meanwhile, we can determine the difference in the number of sand particles in each layer by the inclination angle $\beta$, that is, the size, and stack each layer to obtain the entire three-dimensional geometric model.

We use pressure to consider its strength:

$$ P = \frac{F}{S_{\text{effect}}} \quad (8) $$

Among them, $S_{\text{effect}}$ is the effective area where the external force acts, $S_{\text{effect}} = S_{\text{real}} \sin \beta$, and $S_{\text{real}}$ is the real contact area between the geometry and the ocean tide.

At the same time, based on the Wiesenfeld’s sand pile model, it can be known that after the sand pile reaches the "critical" load-bearing state, the sand particles piled on the sand pile are in an integrated state, and the newly accumulated sand will disturb the sand pile due to gravity. The transmission in the whole sand pile makes the structure of the sand pile fragile, and eventually the sand pile collapses. Therefore, we need to judge the load-bearing capacity when considering the sand layer stacking process.

From the aspects of pressure and weight of sand discharge, this article has developed a measure of the degree of danger of stacking sand layers to help us determine the range of inclination angle.

$$ \zeta = k_1 \frac{1}{S_{\text{effect}}} + k_2 E \left( \frac{\sum_{j=1}^{n} w_k}{S_k} \right) + k_3 E \left( - \log \left( \frac{S_k}{S_1} \right) \right) \quad (9) $$

Assuming that $w_k$ is the weight of the $k$ layer. It can be seen from the figure5 that $\Delta x_i$ represents the most heterodyne value of the $k$ layer and $k + 1$ layer. $\text{eps}$ is the offset term. $k_1$ is the amount of external force applied. $k_2$ is the weight value for the load bearing size of the sand pile. The slower the slope, the smaller the value.

The second term is the average of the pressure generated by its own gravity on the next layer, and then even if the volume is certain, the slope is slower and the height is higher, but because the slope is very slow, the pressure on the next layer is very low, so it's still getting smaller and smaller.

The third term is the damage term of seawater to its horizontal layer, because the more the slope changes, the smaller the horizontal layer is, and the smaller the damage will be greater, so this number is getting larger and larger. $S_k$ is the area of the $k$ layer, and $S_1$ is the area of the bottom layer.

4. Optimization Model

4.1. Determination of Horizontal Optimal Cross Section

For the plane layer, the energy brought by external forces will destroy its stability, that is, the more total energy it obtains, the more easily its stacked structure will be destroyed. Therefore, in order to make its structure the most stable, the total energy of the plane layer is the least under the same external forces:

$$ \min_{\text{Energy}_k} = \sum_{i=1}^{m} \sum_{j=1}^{n} e_{i,j,k} \quad (10) $$

From the above equation, we can calculate the optimal plane layer under the condition of fixed sand grains as follows:
It can be seen that in the horizontal section, the plane which is approximately in the shape of a spindle has the optimal load-bearing capacity of applying pressure to the front.

4.2. Determination of the Optimal Vertical Inclination Angle

According to the risk coefficient measurement standard, in order to make the sand pile most stable, it is equivalent to making the sand pile least dangerous:

\[
\min \xi = \min \left[ k_1 \frac{1}{s_{\text{eff}}^t} + k_2 E \left( \sum_{k=1}^i w_k \frac{s_k}{s_i} \right) + k_3 E \left( -\log \left( \frac{s_i}{S_1} \right) \right) \right]
\] (11)

Finally, we selected the minimum risk as the optimal inclination Angle of the vertical structure:

\[
\beta = \arctan \frac{R}{\Delta x_i + \epsilon}
\] (12)

According to the optimal plane layer and the optimal slope Angle, the sand grain ratio of each layer is calculated, and the optimal model of sand castle is finally determined.

By fig4, we found that the smaller the cross section in the horizontal direction, the higher the risk, and that for the slope in the vertical direction, in the case of the total volume and the bottom area, there is a slope that is the best value of the damage and the best. Through the intersection of two intersections, we get the optimal slope 0.75, as a result, we get the angle \( \beta = \arctan(0.75) = 36.87^\circ \)

5. Conclusion

According to the actual situation, the sand pile is made up of many sand particles. Therefore, we discretize the sand pile at first in this paper, and regarding it as a structure composed of many highly rigid and frictional spherical particles of the same shape and size. For the sand castle construction model, we fix the number of sand particles in the sand pile and make an explanation that the condition that the
water content between the sand grains are saturated and unchanged. At this time, only the influence of geometry on the stability of the sand pile is taken into consideration.

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