Femtoscopy with unlike particles

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The possibilities of unlike particle correlations for a study of space–time characteristics of particle production are demonstrated. The correlation data from heavy ion collisions is discussed, particularly - in terms of the transport RQMD and hydrodynamic models. An attention is paid to the relative space-time asymmetries in the production of different particle species, measured as a ratio of the correlation functions corresponding to different directional selections of the relative momentum vector. Being sensitive to the relative time delays and to the intensity of collective flows, these asymmetries can become a promising tool in a study of the deconfinement transition and the formation of quark-gluon plasma.

I. INTRODUCTION

The momentum correlations of particles at small relative velocities are widely used to study space-time characteristics of the production processes, so serving as a correlation femtoscope. Particularly, for non-interacting identical particles, like photons, this technique is called intensity or particle interferometry. In this case the correlations appear solely due to the effect of quantum statistics (QS) [1,2]. This effect has a deep analogy in astronomy [3], where it leads to the two–photon space–time correlations and allows one to measure the angular radii of stars by studying the dependence of the two-photon coincidence rate on the distance between the detectors (HBT effect [4]). In particle physics, the QS interference was first observed as an enhanced production of the pairs of identical pions with small opening angles (GGLP effect [1]). Later on, similar to astronomy, Kopylov and Podgoretsky [2] suggested to study the interference effect in terms of the correlation function. In a series of papers, they settled the basics of correlation femtometry. Particularly, they clarified the role of the space–time parameters studying their effect on the directional and velocity dependence of the correlation function in various physical situations.

The effect of QS is usually considered in the limit of a low phase-space density such that the possible multi-particle effects can be neglected. This approximation seems to be justified by present experimental data which does not point to any spectacular multi-boson effects neither in single-boson spectra nor in two-boson correlations. These effects can however clearly manifest themselves in some rare events (e.g., those with large pion multiplicities) or in the eventually overpopulated regions of momentum space; see, e.g., [5,6] and references therein.

The momentum correlations of particles emitted at nuclear distances are also influenced by the effect of particle interaction in the final state (FSI) [7,8]. It should be emphasised that, depending on the characteristic space–time separation of the particle emitters, both the Coulomb and strong FSI can significantly influence the shape of the correlation function. Thus the effect of the Coulomb interaction dominates the correlations of charged particles at very small relative momenta (of the order of the inverse Bohr radius of the two-particle system), respectively suppressing or enhancing the production of particles with like or unlike charges. As a result, the correlation function of two charged particles emitted at large relative distances in their c.m.s. is mainly determined by the Coulomb interaction; it is increasingly sensitive to these distances with the increasing particle masses and charges, i.e. with the decreasing Bohr radius of the pair. Regarding the effect of the strong FSI, it is quite small for two pions, while for two nucleons it is often a dominant one due to the very large magnitude of the s-wave singlet scattering length of about 20 fm. Though the FSI effect complicates the correlation analysis, it is an important source of information allowing to

1Note that though both the KP and HBT methods are based on the QS interference, they represent just orthogonal measurements [3]. The former, being the momentum-energy measurement, yields the space-time picture of the source, while the latter does the opposite. In particular, the HBT method provides the information on the characteristic relative three-momenta of the emitted photons and so, when divided by the mean detected momentum, on the angular size of a star but, of course, - no information on the star radius or its lifetime.

2This effect has no analogy in the space–time correlations in astronomy, where the particles are emitted and detected at macroscopic distances and so, their mutual interaction is absent in principle.
• perform the correlation femtometry with unlike particles [3];
• get information on the strong scattering amplitudes that are hardly accessible by other means;
• access the relative space–time asymmetries in particle production (e.g., relative time delays) studying directional asymmetries of the unlike particle correlations [3].

II. FORMALISM

Following Kopylov and Podgoretsky, we define the two-particle correlation function $R(p_1, p_2)$ as a ratio of the differential two-particle production cross section to the reference one which would be observed in the absence of the effects of QS and FSI. In heavy ion collisions, one can neglect kinematic constraints and most of the dynamical correlations and construct the reference distribution by mixing the particles from different events, normalising the correlation function to unity at sufficiently large relative velocities. As usual, we assume that the correlation of two particles emitted with a small relative velocity is influenced by the effects of their mutual QS and FSI only and that the momentum dependence of the one-particle emission probabilities is inessential when varying the particle four-momenta $p_1$ and $p_2$ by the amount characteristic for the correlation due to QS and FSI (smoothness assumption). Clearly, the latter assumption, requiring the components of the mean space-time distance between particle emitters much larger than those of the space-time extent of the emitters, is well justified for heavy ion collisions.

The correlation function is then given by a square of the properly symmetrized Bethe-Salpeter amplitude in the continuous spectrum of the two-particle states, averaged over the four-coordinates $x_i = \{t_i, r_i\}$ of the emitters and over the total spin $S$ of the two–particle system [3]. After the separation of the unimportant phase factor due to the c.m.s. motion, this amplitude depends only on the relative four-coordinate $x = \{t, r\} = x_1 - x_2$ and the generalised relative momentum $\tilde{q} = q - P(qP)/P^2$, where $q = p_1 - p_2$, $P = p_1 + p_2$ and $qP = m_1^2 - m_2^2$; in the two-particle c.m.s., $P = 0$, $\tilde{q} = \{0, 2k^\ast\}$ and $x = \{t^\ast, r^\ast\}$. At equal emission times of the two particles in their c.m.s. ($t^\ast = t_1^\ast - t_2^\ast = 0$), the latter amplitude coincides with a stationary solution $\psi_{\tilde{k}_0}^{S}(r^\ast)$ of the scattering problem having at large distances $r^\ast$ the asymptotic form of a superposition of the plane and outgoing spherical waves (the minus sign of the vector $k^\ast$ corresponds to the reverse in time direction of the emission process). The Bethe-Salpeter amplitude can be usually substituted by this solution (equal time approximation) [3] so that $R(p_1, p_2) = \sum_S \rho_S(\langle|\psi_{\tilde{k}_0}^{S}(r^\ast)|^2\rangle)_S$. Here the averaging is done over the four–coordinates of the emitters at a given total spin $S$ of the two–particles, $\rho_S$ is the corresponding population probability, $\sum_S \rho_S = 1$. For unpolarised particles with spins $s_1$ and $s_2$ the probability $\rho_S = (2S + 1)/\{(2s_1 + 1)(2s_2 + 1)\}$. Generally, the correlation function is sensitive to particle polarisation. For example, if two spin-1/2 particles are emitted with polarisations $\mathcal{P}_1$ and $\mathcal{P}_2$ then $\rho_0 = (1 - \mathcal{P}_1 \cdot \mathcal{P}_2)/4$ and $\rho_1 = (3 + \mathcal{P}_1 \cdot \mathcal{P}_2)/4$.

For non-interacting identical particles, the QS symmetrization: $\psi_{\tilde{k}_0}^{S}(r^\ast) \rightarrow [\psi_{\tilde{k}_0}^{S}(r^\ast) + (-1)^S \psi_{\tilde{k}_0}^{S}(r^\ast)]/\sqrt{2}$ leads to the characteristic feature of the correlation function - the presence of the interference maximum or minimum at small relative momenta $|q| = |p_1 - p_2|$ with the width reflecting the inverse space-time extent of the production region. For example, assuming that for a fraction $\lambda$ of the pairs, the particles are emitted independently according to one–particle amplitudes of a Gaussian form characterised by the space–time dispersions $r_0^2$ and $\tau_0^2$ while, for the remaining fraction $(1 - \lambda)$ related to very long–lived sources ($\eta, \eta', K^0_s, \Lambda, \ldots$), the relative distances $r^\ast$ between the emitters in the pair c.m.s. are extremely large, one has $R(p_1, p_2) = 1 + \lambda \sum_S \rho_S(-1)^S \exp(-r_0^2q^2 - \tau_0^2q_0^2)$, where $\sum_S \rho_S(-1)^S = (-1)^{2s}/(2s + 1)$ for initially unpolarised spin–s particles.

One may see that, due to the relation $q_0 = \mathbf{v}q$ (following from the equality $qP = 0$), strongly correlating the energy difference $q_0$ with the projection of the three–momentum difference $\mathbf{q}$ on the direction of the pair velocity $\mathbf{v} = \mathbf{P}/P_0$, the correlation function at $\nu \tau_0 > r_0$ substantially depends on the direction of the vector $\mathbf{q}$ even in the case of a

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3 Besides the events with a large phase-space density fluctuations, this assumption may be not justified also in low energy heavy ion reactions when the particles are produced in a strong Coulomb field of residual nuclei. To deal with this field a quantum adiabatic (factorisation) approach can be used [1].

4 For non-interacting particles, the non–symmetrized Bethe-Salpeter amplitude reduces to the plane wave $e^{i\phi/2} \equiv e^{-i k^\ast \cdot r^\ast}$ which is independent of the relative time in the two-particle c.m.s. and so, coincides with the corresponding equal–time amplitude. For interacting particles, the equal time approximation is valid on condition $|t^\ast| \ll m_2 r_0^2$ for sign($t^\ast$) = ±1 respectively. This condition is usually satisfied for heavy particles like kaons or nucleons. But even for pions, the $t^\ast = 0$ approximation merely leads to a slight overestimation (typically < 5%) of the strong FSI effect and, it doesn’t influence the leading zero–distance ($r^\ast \ll |a|$) effect of the Coulomb FSI.
spherically symmetric spatial form of the production region. Generally, the directional and velocity dependence of the correlation function can be used to determine both the duration of the emission process and the form of the emission region \[1\], as well as - to reveal the details of the production dynamics (such as collective flows; see, e.g. \[2\]). For this, the correlation functions are often analysed in terms of the out (x), side (y) and longitudinal (z) components of the relative momentum vector \( \mathbf{q} = (q_x, q_y, q_z) \) \[3\]: the out and side denote the transverse, with respect to the reaction axis, components of the vector \( \mathbf{q} \), the out direction is parallel to the transverse component of the pair three–momentum. Note that for unlike particles, the relative momentum \( q \) has to be substituted by the generalised one, \( \tilde{q} \), vanishing at equal particle velocities.

It is well known that particle correlations at high energies usually measure only a small part of the space–time emission volume, being only slightly sensitive to its increase related to the fast longitudinal motion of the particle sources. In fact, due to limited source decay momenta \( p^{(s)} \) of few hundred MeV/c, the correlated particles with nearby velocities are emitted by almost comoving sources and so - at nearby space–time points. In other words, the maximal contribution of the relative motion to the correlation radii in the two–particle c.m.s. is limited by the moderate source decay length \( \tau p^{(s)}/m \). The dynamical examples are sources–resonances, colour strings or hydrodynamic expansion. To substantially eliminate the effect of the longitudinal motion, the correlations can be analysed in terms of the invariant variable \( q_{inv} \equiv Q = (-\mathbf{q}^2)^{1/2} = 2k^* \) and the components of the momentum difference in pair c.m.s. \( (\mathbf{q}^\ast \equiv Q = 2k^*) \) or in the longitudinally comoving system (LCMS) \[5\]. In LCMS each pair is emitted transverse to the reaction axis so that the generalised relative momentum \( \tilde{q} \) coincides with \( \mathbf{q}^\ast \) except for the component \( q_z = \gamma \mu q^\ast_z \), where \( \gamma \mu \) is the LCMS Lorentz factor of the pair.

### III. FEMTOMETRY WITH UNLIKE PARTICLES

The complicated dynamics of particle production, including resonance decays and particle rescatterings, leads to essentially non–Gaussian tail of the \( r^* \)--distribution. Therefore, due to different \( r^* \)--sensitivity of the QS, strong and Coulomb FSI effects, one has to be careful when analysing the correlation functions in terms of simple models. Thus, the QS and strong FSI effects are influenced by the \( r^* \)--tail mainly through the suppression parameter \( \lambda \) while, the Coulomb FSI is sensitive to the distances as large as the pair Bohr radius \( |a| \). For \( \pi\pi \), \( \pi K \), \( \pi\pi \), \( \pi K \) and \( pp \) pairs, \( |a| = 387.5, 248.6, 222.5, 109.6, 83.6 \) and \( 57.6 \) fm, respectively. Clearly, the usual Gaussian parametrisations of the components of the vector \( r^* \) may happen to be insufficient in the case of charged particle correlations, leading to inconsistencies in the treatment of QS and FSI effects (the Coulomb FSI contribution requiring larger effective radii). These problems can be at least partially overcome with the help of transport code simulations accounting for the dynamical evolution of the emission process and providing the phase space information required to calculate the QS and FSI effects on the correlation function.

Thus, in a preliminary analysis of the NA49 correlation data from central \( Pb + Pb \) 158 AGeV collisions \[10\], we have used the events simulated with the RQMD v.2.3 code \[1\]. The correlation functions have been calculated using the code of Ref. \[8\], weighting the simulated pairs by squares of the corresponding wave functions. To account for a possible mismatch in \( \langle r^* \rangle \), the RQMD simulations have been done with the space–time coordinates of the emission points scaled by a factor of 0.7, 0.8 and 1. The \( Q \)--dependence of the correlation function was than fitted according to the formula \[10\] \( R(Q) = \text{norm} \cdot \text{purity} \cdot \text{RQMD}(|r^* \rightarrow \text{scale} \cdot r^*) + (1 \text{ – purity}) \), where the dependence on the scale parameter has been introduced using the quadratic interpolation of the points simulated at the three different scales. Based on a sample of 900k events, representing a quarter of the available statistics, the parameters fitted for the \( \pi^+\pi^- \), \( \pi^+p \) and \( \pi^-p \) systems are respectively: purity = 0.77±0.01, 0.76±0.03 and 0.74±0.04, scale = 0.78±0.01, 0.87±0.03 and 0.88±0.04. The values of the purity parameter are in reasonable agreement with the expected contamination from strange particle decays and particle misidentification. The fitted values of the scale parameter indicate that RQMD overestimates the distances \( r^* \) by 10–20\%. Similar overestimation has been also observed when comparing RQMD predictions with the NA49 data on \( pp \) and \( \pi^+\pi^- \) correlations \[18–20\]. Regarding the absolute estimates of \( \langle r^* \rangle \), they depend on the truncation of large distances. For example, for \( \pi^+\pi^- \) system, RQMD model yields 21 and 29 fm for the means truncated at 50 and 500 fm respectively. Scaling this numbers down by a factor of 0.78, one gets for the corresponding experimental truncated means 16.4 and 22.6 fm for \( r^* < 39 \) and 390 fm respectively.

Recently, there appeared data on \( pA \) correlation functions from \( Au + Au \) experiment E985 at AGS \[21\]. As the Coulomb FSI is absent in this system, one avoids here the problem of its sensitivity to the \( r^* \)--tail. Also, the absence of the Coulomb suppression of small relative momenta makes this system more sensitive to the radius parameters as compared with \( pp \) correlations \[22\]. Despite of rather large statistical errors, a significant enhancement is seen at low relative momentum, consistent with the known positive singlet and triplet \( pA s \)--wave scattering lengths (defined here as the corresponding amplitudes at \( k^* = 0 \)) of 2.3 and 1.8 fm respectively; the corresponding effective radii are 3.0 and 3.2 fm. In fact, using the analytical expression for the correlation function from Ref. \[8\] (originally derived}
for \( pn \) system), one gets a good fit of the combined (4, 6 and 8 AGeV) correlation function with the suppression parameter \( \lambda = 0.5 \pm 0.2 \) and the Gaussian radius \( r_0 = 4.5 \pm 0.7 \) fm. The \( \lambda \)–value is in agreement with the product of the reconstruction \( \Lambda_p \) purity (\( \sim 80\% \)) and the non–feed–down purity (70–75\%) (the feed-down from \( \Lambda^0 \) is estimated at 25–30\%). As for the fitted radius, it agrees with the radii of 3–4 fm obtained from \( pp \) correlations in heavy ion collisions at GSI, AGS and SPS energies.

There is now coming a high quality data on particle correlations from experiment STAR at RHIC [23]. Particularly, the \( \pi K \) correlations in all possible charge states are available from \( Au + Au \) collisions at 130 AGeV c.m.s. energy. They practically coincide for the same and opposite charges and show the mirror symmetry. This points to the same production mechanism of positive and negative pions (kaons). A preliminary analysis of this data within a static sphere model yields the radius of about 7 fm [23].

IV. ACCESSING SCATTERING AMPLITUDES

In case of a poor knowledge of the two–particle strong interaction, which is the case for exotic systems like \((M = \text{meson)}\) \( MM, \ MA \) or \( \Lambda \Lambda \), the correlation measurements can be also used to study the latter. Particularly important is the experimental study of \( \Lambda \Lambda \) interaction, especially in view of a recent experimental indication on the enhanced \( \Lambda \Lambda \) production near threshold [24] and its possible connection with the 6-quark \( \mathbb{H} \) dibaryon problem.

On the absence of a noticeable FSI, the behaviour of the correlation function \( \mathcal{R} \) of two identical particles at small relative momenta \( Q = 2k^* \) in pair rest frame is determined by the symmetry requirement of QS; at \( Q \to 0 \) the contribution of the even total spin \( S \) (e.g., singlet part \( \mathcal{R}_S \)) is enhanced by a factor of 2 while that of the odd \( S \) (e.g., the triplet part \( \mathcal{R}_T \)) vanishes; the enhancement or dip width is inversely related to the effective radius \( r_0 \) of the emission region. In heavy ion collisions, \( r_0 \) can be considered much larger than the range \( d \) of the strong interaction potential. The FSI contribution is then independent of the actual potential form [25]. At small \( Q \), it is determined by the s-wave scattering amplitudes \( f^S(k^*) \). In case of \( |f^S|^2 > r_0 \), this contribution is of the order of \( |f^S/r_0|^2 \) and dominates over the effect of QS. In the opposite case, the sensitivity of the correlation function to the scattering amplitude is determined by the linear term \( f^S/r_0 \).

To demonstrate the possibilities of the correlation measurements of the scattering amplitudes, we have repeated the analysis of the NA49 \( \pi^+ \pi^- \) correlation function within the RQMD model, introducing the additional parameter sisca (strong interaction scale), redefining the original scattering length \( f_0 = 0.232 \) fm: \( f_0 \to \text{sisca} \cdot f_0 \). The introduction of this new scale lead to a substantial improvement of the fit quality and to a noticeable increase and decrease of the purity and \( r^* \)–scale parameters respectively: purity = 0.81 \pm 0.01, scale = 0.72 \pm 0.02. The parameter sisca = 0.63 \pm 0.08 appears to be significantly lower than unity, showing that the correlation data prefer the value of the s-wave \( \pi^+ \pi^- \) scattering length \( f_0 = 2a_0^0 + a_0^2 \) \( a_0^0, a_0^2 \) are the two–pion isosinglet and isotensor s-wave scattering lengths) by \( \sim 30\% \) lower than the present table value. To a similar shift (\( \sim 20\% \)) point also the recent BNL data on \( K_{44} \) decays [24]. These results are in agreement with the two–loop calculation in the chiral perturbation theory with a standard value of the quark condensate [27]. Comparing with the theoretical predictions, one should have in mind that they are subject to the electro–magnetic corrections on the level of several percent and that the correlation measurement underestimates \( f_0 \) by a few percent due to the use of the equal–time approximation.

As for the \( \Lambda \Lambda \) system, one can try to estimate the singlet \( \Lambda \Lambda \) s-wave scattering length \( f_0 \), fitting the recent NA49 data on \( \Lambda \Lambda \) correlations in \( Pb + Pb \) collisions at 158 AGeV [25]. Using the analytical expression for the correlation function from Ref. [22] (originally derived for \( nn \) system [8]), one gets \( \lambda = 0.9 \pm 0.6, r_0 = 1.5 \pm 0.3 \) fm and \( f_0 = 0.1 \pm 0.5 \) fm. Fixing the suppression parameter \( \lambda \) at a reasonable value of 0.5, one gets \( r_0 = 1.8 \pm 0.3 \) fm and \( f_0 = -2.6 \pm 2.6 \) fm. Though the fitting results are not very restrictive, they certainly exclude the possibility of a large positive singlet scattering length comparable to that for the two–nucleon system.

V. ACCESSING RELATIVE SPACE-TIME ASYMMETRIES

The correlation function of two non–identical particles, compared with the identical ones, contains a principally new piece of information on the relative space-time asymmetries in particle emission [10]. Since this information enters in the two–particle amplitude \( \psi_{\mathbf{k^*} \mathbf{r^*}}(\mathbf{r^*}) \) through the terms odd in \( \mathbf{k^*} \mathbf{r^*} \), it can be accessed studying the correlation functions \( \mathcal{R}_{+i} \) and \( \mathcal{R}_{-i} \) with positive and negative projection \( k_i^* \) on a given direction \( \mathbf{i} \) or, - the ratio \( \mathcal{R}_{+i}/\mathcal{R}_{-i} \). For example, \( \mathbf{i} \) can be the direction of the pair velocity or, any of the out (\( x \)), side (\( y \)), longitudinal (\( z \)) directions. Note that in the LCMS system, one has \( k_i^* = r_i \) except for \( r_x^* \equiv \Delta x^* = \gamma_t(\Delta x - v_x \Delta t) \), where \( \gamma_t = (1 - v_x^2)^{1/2} \) and \( v_x = P_x/P_0 \) are the pair LCMS Lorentz factor and velocity. One may see that the asymmetry in the out (\( x \)) direction depends on both space and time asymmetries (\( \Delta x \)) and (\( \Delta t \)). In case of a dominant Coulomb FSI, the
intercept of the correlation function ratio is directly related with the asymmetry \( \langle r^*_t \rangle \): \( R_{+t}/R_{-t} \approx 1 + 2\langle r^*_t \rangle /a \), where \( a = (\mu z_1 z_2 e^2)^{-1} \) is the Bohr radius of the two-particle system taking into account the sign of the interaction (\( z_i e \) are the particle electric charges, \( \mu \) is their reduced mass).

At low energies, the particles in heavy ion collisions are emitted with the characteristic emission times of tens to hundreds \( \text{fm}/c \) so that the observable time shifts should be of the same order \([11]\). Such shifts have been indeed observed with the help of the \( R_{+}/R_{-} \) correlation ratios for proton-deuteron systems in several heavy ion experiments at GANIL \([12]\) indicating, in agreement with the coalescence model, that deuterons are on average emitted earlier than protons.

For ultra-relativistic heavy ion collisions, the sensitivity of the \( R_{+}/R_{-} \) correlation ratio to the relative time shift \( \langle \Delta t \rangle \) (introduced \textit{ad hoc}) was studied for various two-particle systems simulated using the transport codes \([13]\). The scaling of the effect with the space-time asymmetry and with the inverse Bohr radius \( a \) was clearly illustrated. It was concluded that the \( R_{+}/R_{-} \) ratio can be sensitive to the shifts in the particle emission times of the order of a few \( \text{fm}/c \). Motivated by this result, the correlation asymmetry for the \( K^+K^- \) system has been studied in a two-phase thermodynamic evolution model and the sensitivity has been demonstrated to the production of the transient strange quark matter state even if it decays on strong interaction time scales \([12]\). The method sensitivity to the space-time asymmetries arising also in the usual multi-particle production scenarios was demonstrated for AGS and SPS energies using the transport code RQMD \([10,14]\). At AGS energy, the \( Au + Au \) collisions have been simulated and the \( \pi \pi \) correlations have been studied in the projectile fragmentation region where proton directed flow is most pronounced and where the proton and pion sources are expected to be shifted relative to each other both in the longitudinal and in the transverse directions in the reaction plane. It was demonstrated \([34]\) that the corresponding \( R_{+}/R_{-} \) ratios are sufficiently sensitive to reveal the shifts; they were confirmed in the directional analysis of the experimental AGS correlation data \([35]\).

At SPS energy, the simulated central \( Pb + Pb \) collisions yield practically zero asymmetries for \( \pi^+\pi^- \) system while, for \( \pi^\pm p \) systems, the LCMS \( x \)- and \( t \)-asymmetries are \( \langle \Delta x \rangle = -6.2 \text{ fm} \), \( \langle \Delta t \rangle = -0.5 \text{ fm}/c \), \( \langle \Delta x^* \rangle = -7.9 \text{ fm} \) in the central rapidity window \([33]\) and, \( \langle \Delta x \rangle = -5.2 \text{ fm} \), \( \langle \Delta t \rangle = 2.9 \text{ fm}/c \), \( \langle \Delta x^* \rangle = -8.5 \) for the NA49 acceptance (shifting the rapidities into the forward hemisphere). Besides, \( \langle x \rangle \) increases with particle \( p_t \) or \( u_t = p_t/m \), starting from zero due to kinematic reasons. The asymmetry arises because of a faster increase with \( u_t \) for heavier particle. The non–zero positive value of \( \langle x \rangle = \langle r_T\hat{x} \rangle \) \((x = p_t/p_1 \) and \( r_T \) is the transverse radius vector of the emitter\)) and the hierarchy \( \langle x_x \rangle < \langle x_K \rangle < \langle x_p \rangle \) is a signal of a universal transversal collective flow. To see this, one should take into account that the mean thermal velocity \( \langle \beta^T \rangle \) as well as the mean particle transverse velocity \( \langle \beta_T \rangle \) are smaller for heavier particles. Thus, in the non–relativistic approximation, one has \( \beta_t = \beta_0 + \beta_0^T \) and \( \langle x \rangle = \langle r_T(\beta_0 + \beta_0^T \cos \phi)/\beta_t \rangle \approx \langle r_T\beta_0/\beta_t \rangle \), where \( \beta_0 \parallel r_T \) is the transversal flow velocity, \( \phi \) is the angle between the vectors \( \beta_0^T \) and \( \beta_0 \). As a result, in case of a locally equilibrated expansion process, one expects a negative asymmetry \( \langle x \rangle \equiv \langle x_1 - x_2 \rangle \) provided \( m_1 < m_2 \). Moreover, this asymmetry vanishes in both limiting cases: \( \beta_0 < \beta^T \) and \( \beta_0 \gg \beta^T \). These conclusions agree with the calculations in the longitudinal-boost invariant hydrodynamic model. Thus, assuming the linear transversal rapidity profile with \( y_t = 0.4 \) at the characteristic Gaussian transverse radius of 6 \( \text{fm} \) and the temperature of 140 MeV (corresponding to central \( Pb + Pb \) collisions at SPS energy) and, using Eq. (30) of Ref. \([20]\) to calculate \( \langle x \rangle \) as a function of \( u_T \), one confirms a faster rise of \( \langle x \rangle \) for heavier particles and gets \( \langle \Delta x \rangle = -3 \text{ fm} \) and \( \langle \Delta x^* \rangle = -4 \text{ fm} \) for \( \pi^\pm p \) systems. In fact, the NA49 data on \( R_{+x}/R_{-x} \) ratio for \( \pi^\pm p \) and \( \pi^-n \) systems show consistent mirror symmetric deviations from unity, their size of several percent and the \( Q \)-dependence being in agreement with RQMD calculations corrected for the resolution and purity. The predictions of the simple hydrodynamic model (assuming the universal freeze-out time) appear to be \( \sim 50\% \) too low.

Similar pattern of the correlation asymmetries has been reported also for \( \pi^\pm K^\pm \) and \( \pi^\pm K^\mp \) correlation function ratios in the out \( \langle x \rangle \) direction from experiment STAR at RHIC \([23]\). They seem to be in agreement with the hydrodynamic type calculations without any corrections, thus leaving a room for the additional space–time asymmetries (e.g., due to the time shifts).

\section{VI. CONCLUSIONS}

Analysing the experimental and simulated correlation data from heavy ion collisions, we have shown that unlike particle correlations can serve as an important femtometry tool complementary to the usual interferometry with identical particles. We have demonstrated that two–particle correlations provide a useful information on the strong scattering amplitudes that are hardly accessible by other means. We have shown that directional asymmetries of unlike particle correlations contain a principally new piece of information on the relative space-time asymmetries in particle emission. The data on correlation asymmetries in heavy ion collisions at AGS, SPS and RHIC appear to be in quantitative or qualitative agreement with transport or hydrodynamic calculations. Being sensitive to relative time
delays and collective flows, the correlation asymmetries can be useful to study the effects of the quark–gluon plasma phase transition. As for the detection of the unlike particles with close velocities, there is practically no problem with the two-track resolution since these particles, having either different momenta or different charge-to-mass ratios, have well separated trajectories in the detector magnetic field. For the same reason, however, a large momentum acceptance of the detector is required.

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