Laser-modified Angular Distribution of Muon Decay

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We show theoretically that the angular distribution of decay rate of muon can be changed dramatically by embedding the decaying muon in a strong linearly polarized laser field. Evaluating the S-matrix elements taking all electronic multiphoton processes into account. The results suggest the muon may have internal structures instead structureless as in the standard model.

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Introduction.- Recent advances in the generation and control of ultra-intense laser fields [1] paved the way for a number of spectacular applications ranging from particles accelerations [2, 3] and the generation of X-ray pulses [4] to laser-driven nuclear reaction [5], and laboratory astrophysical and high-energy processes [6]. In addition to laser-induced phenomena, modifications of the properties of elementary particles due to the presence of a strong light field are enjoying much of attention. To name but one, Chelkowski et al. [7] demonstrated theoretically that upon the ionization and dissociation of muonic molecular ions in superintense laser fields (with intensities $I \sim 10^{21} \text{Wcm}^{-2}$) the recolliding ions can ignite a nuclear reaction with sub-laser-cycle precision and serve hence as precursors for laser-assisted nuclear processes. A crucial assumption in this kind of studies is that the participating elementary particles are stable during the collision process. The purpose of this work is to point out that the particles field-free life time may change dramatically due to the presence of a strong electromagnetic field (field amplitude $\sim 10^6 \text{Vcm}^{-1}$). To accomplished such a process in experiment, we may shed a laser beam on the beam of muon. The strong field may also be used to affect the decay of other unstable particles, and reveal new aspects of the decay mechanism. This aspect of the laser-matter interaction is of a prime importance, e.g. when considering strong-field assisted collisions.

Specifically, we consider the modification of the decay life time of muons due to the presence of a strong laser field. As well known, muons have played a crucial role in the development and assessment of the standard model [8] and the field-free muon decay was the first from all leptonic processes to be investigated in full details [9].

The field-free muon lifetime may be modified by a number of factors: E.g., Czarnecki et al. [12] investigated the modifications of the $\mu^+$ lifetime due to muonium ($\mu^+e^-$) formation and other medium effects, whereas Vshivtsev and Éminov [15] studied the influence of a weak field on $\tau_\mu$. In recent years, few investigations of the muon or muonium-laser interactions were carried out: Chu et al. studied the laser excitation of the muonium $1S \rightarrow 2S$ transition [14] whereas Nagamine et al. [15] reported on an ultraslow $\mu^+$ generation upon laser ionization of thermal muonium. Our focus in this work is on the decay of the muon into an electron, a muon-neutrino $\nu_\mu$, and an electronic antineutrino $\bar{\nu}_e$ in a strong laser field. Here we note that present-day laser sources produce intensities of $10^{18} \text{Wcm}^{-2}$ or higher in which case the averaged quiver energy of the electron in the laser field may well exceed its rest energy [15] necessitating thus a full relativistic treatment.

Theoretical formulation.- We assume the decay of a muon to occur in the presence of a monochromatic, linearly polarized, spatially homogeneous laser field. The final state electron is treated relativistically. The electromagnetic field is described by the classical four-potential (unless otherwise stated, we use natural units in which $\hbar = c = 1$) $A(x) = a \cos(k \cdot x)$ that satisfies the Lorenz condition. The constant four vector $a = (0, E_0/\omega)$, where $E_0$ is the amplitude of the electric field strength of laser. The wave four vector $k = (\omega, k)$ follows from the laser frequency $\omega$ and wave number $k$. The $S$-matrix elements for the laser-assisted $\mu^-$ decay reads [9, 11]
\[ S_{fi} = -i \frac{G}{\sqrt{2}} \int \overline{\psi}_{\nu_\mu} \gamma_\lambda (1 - \gamma_5) \psi_{\mu} |\overline{\psi}_e \gamma^\lambda (1 - \gamma_5) \psi_{\nu_e}| d^4x. \] (1)

Here \( G = (1.16637 \pm 0.00002) \times 10^{-11} MeV^{-2} \) is the constant of the weak interaction, and \( x \) stands for the spatial coordinates. \( \psi_{\nu_\mu}, \psi_{\nu_e}, \psi_{e}, \) and \( \psi_{\nu_e} \) are respectively wave functions of the muon, the muonic neutrino, the electron, and the electronic antineutrino. The neutrinos are treated as massless particles describable by Dirac spinors [18]; the minor finite-mass effects can be included as done in Ref. [11]. For the description of the laser-dressed states we note the following. The muon, due to its large mass, is much less influenced by the laser than the electron (for the laser intensities considered here). The state of the electron in the laser field is represented by the Dirac-Volkov function (normalized in a large volume \( V \)) [19]. For a linearly polarized field it has the form

\[ \psi_e(x) = \left[ 1 + \frac{e\mathbf{k} \cdot \mathbf{q}}{2(k \cdot p)} \cos(k \cdot x) \right] \frac{\psi_e}{\sqrt{2E}} \exp \left[ -q \cdot x - \frac{e(a \cdot p)}{k \cdot p} \sin(k \cdot x) \right]. \] (2)

where \( e \) is the electron charge, \( p \) is the four-momentum of electron for laser free, and \( q = p - \frac{e^2}{2(k \cdot p)} \mathbf{F} k = (E, q) \) can be viewed as the (time) averaged four-momentum of the electron in the presence of the laser field, with \( \mathbf{A}^2 \) being the square of the four-potential averaged in a laser cycle.

\[ S_{fi} = -i \frac{G}{\sqrt{2}} \frac{m_\mu m_e}{2E_{\nu_\mu} E_{\nu_e}} \frac{1}{V^2} \sum_l [\overline{\psi}_{\nu_\mu} \gamma_\lambda (1 - \gamma_5) u_\mu] [\overline{\psi}_e f^\lambda v_{\nu_e}] \delta(P - q - k_{\nu_\mu} - k_{\nu_e} - lk), \] (3)

where \( E_{\nu_\mu}, E_{\nu_e}, \) and \( E_{\nu_e} \) are respectively the energies of \( \mu, \nu_\mu, \) and \( \nu_e \). Furthermore, \( P, k_{\nu_\mu}, \) and \( k_{\nu_e} \) are the four-momenta of \( \mu, \nu_\mu, \) and \( \nu_e \). \( u_\mu, u_{\nu_\mu}, \) and \( u_{\nu_e} \) are respectively the free Dirac spinors of them. In Eq. (3) \( l \) is the number of photons \( f^\lambda = (\Delta_0 \gamma^\lambda + \Delta_1 \delta k^\gamma^\lambda)(1 - \gamma_5) \), with \( \Delta_0 = J_l(D) \) and \( \Delta_1 = \frac{J_l(D)}{2(a \cdot p)} \), where \( D = -\frac{e^2}{k \cdot p} \).

\[ \frac{dW}{d\Omega} = \sum_{l=-\infty}^{\infty} \frac{G^2 \pi}{96 \pi^3} \int_{m_{\mu} + \omega}^{m_{\mu} + \omega} \frac{dE}{E} \left[ \frac{Q^2}{E} \right] [\Delta_0^2 (Q^2 (P \cdot p) + 2 (Q \cdot P) (Q \cdot p)] + 2 \Delta_0 \Delta_1 (a \cdot p) [Q^2 (k \cdot p)] \\
+ 2 (Q \cdot k) (Q \cdot P)] - 2 \Delta_1 \Delta_0 (k \cdot p) [Q^2 (a \cdot P) + 2 (Q \cdot a) (Q \cdot P)] - 2 \Delta_1^2 a^2 (k \cdot p) [Q^2 (k \cdot P) + (Q \cdot k) (Q \cdot P)]. \] (4)

Here we introduced the photon-number-resolved decay rate \( W_l \) and the momentum \( Q = P - q - lk \).

**Results and discussion.** Now we discuss the numerical result of the laser modified decay rate. The origin of the coordinate system is chosen to be on the muon (before decay), the z-axis is set along the direction of the electric-
The decay rate is enlarged around \( \theta = 90^\circ \) in the presence of a Nd:YAG laser (\( \hbar \omega = 1.17 \ eV \)) with an electric field amplitude of \( 10^7 \ V/cm \). This suggests the photons are directly interacting with the components in the muon, and affect the angular distribution of the final state electron.

In summary we provided theoretical evidence that the decay of a muon could be modified by a linearly polarized laser field. It suggests that muon may have structures, and indicates an approach to study the structure by applying a strong laser background. This effect deserves further investigation both experimentally and theoretically.

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[25] We note that Eq.\( \{4\} \) does not account for the influence of radiative corrections which are (in absence of the laser) small in comparison to the modifications brought about by the laser field.