Savings of reinforcement in concrete continuous beam in calculation with redistribution of moments

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Abstract. Compare calculation data for continuous beam based on elastic system and with account of moment redistribution. Determine reinforcement savings based on the calculation data. Design model enabling the calculation based on elastic system and allowing for its transformation with account of moment redistribution. Identification of locations from where plastic hinges originate and determination of permanent moment values therein. The fracturing probability in the aforementioned cross-sections has been identified. Verification identified fulfillment of deformation conditions. It was demonstrated that implementation of the design model for continuous beam with account of moment redistribution results in reinforcement saving. In calculations with moment redistribution the saving of principal reinforcement in central spans of continuous beam comes up to 13 % as compared to calculation based on elastic scheme.

1. Introduction
Rational use of metal is an overriding priority in design and production of reinforced concrete products. Also, expenses in production of flexural reinforced concrete elements may be reduced by exclusion of reinforced concrete, which is virtually not feasible. It is evident that the rational use of metal may be attained through application of advanced design methods accounting for modification of the structure design models for the near-limit loads.

Hence, in one-piece, statically indeterminate structures fracture may occur only after the loss of geometric stability of the total system with formation of plastic hinges. A plastic hinge promotes development of deflections and increase of stresses in compressed area in the course of reinforcement flow, i.e. when \( \sigma_y = \sigma_y \) and \( \sigma_b < R_b \) [1].

Following the formation of a plastic hinge, while the load is increasing, the bending moment redistribution between specific cross-sections will take place. Taking account of such redistribution provides for rational selection of reinforcement by means of standard welded steel fabric and frameworks in those locations where, according to calculations based on elastic scheme, various bending moments will arise.

It was found out that calculations by equated moments of deflection as compared to elastic scheme calculations may yield savings of reinforcement steel up to 20-30% [2,3].

At the same time, change of moments of deflection in the cross-sections affects the magnitude of the load causing formation of the first and the last plastic hinges, along with the width of crack opening in the first plastic hinge.
In calculations made by traditional methods recommended by the current codes and standards the cross-sections are selected exclusively by elastic scheme. However, the advanced software complexes provide for calculation of deflection moments with due consideration of redistribution of moments.

In this connection, we review the task of calculating reinforced concrete structure by elastic scheme with account of moment redistribution [4]. Based on calculation outputs, we need to identify the cost effect of implementing such approach to calculation of flexural elements.

Sequence of task solution:
1. Development of design model providing for calculation by elastic scheme and its transformation with account of moment redistribution to identify locations of plastic hinge formation and determination of permanent moment values therein.
2. Inspection of the structure for cracking, crack opening and deformations.
3. Comparison of elastic scheme calculation outputs with account of moment redistribution.

2. Development of design model providing for calculation by elastic scheme and its transformation with account of moment redistribution

Generally, in most cases the floors are designed for the uniformly distributed load, while diagram extremum of deflection moments occur on supports and spans. Given the maximal values of moments are identified on supports, the system is restrained and in case of restraint loss it still remains geometrically stable (being partially hinge-supported). On the contrary, in case of maximal moments being on the span or if there is no moment at least on one of the supports, the system is poorly restrained, due to which in case of hinge insertion the system becomes geometrically unstable. Based on the aforementioned assumptions, insertion of a plastic hinge close to the supports would be practical. Given the generation of deformation in a plastic hinge at constant deflection moment in the cross-section, the hinge may be modeled by integration of displacements with turn allowance, but at constant moment. It is well known that the load-carrying capacity of a statically indeterminate structure is independent of correlation between support and span moments or of the sequence of plastic hinge formation; it is only required that equilibrium equations are observed.
For the ultimate equilibrium conditions, i.e. possibility of plastic hinge formation and development of sufficient local deformations when the structure reaches an ultimate equilibrium, the following requirements should be met:

1) shear of compressed area or reinforced concrete crushing by the main compressive stresses shall not be caused by a structure fracturing;
2) cross-sections where formation of plastic hinges is assumed shall be reinforced, given that $x/h_0 \leq 0.35$;
3) reinforcement bars with yield line or welded steel fabric made of common reinforcement wire shall be used.

Given the continuity and formation of a plastic hinge on support, the value of deflection moment shall be set.

In most cases, in design of a statically indeterminate structure any software complex will assign maximal forces, by which the engineer will select reinforcement. It seems impossible to change the values of moments calculated to structural mechanics rules. However, timely introduction of a plastic hinge into design model may limit the subsequent increase of the moment [4].

Naturally, the original design model will change, and subsequent calculations will be made with due consideration of the changes.
As an example, we will review calculation by conventional algorithm of continuous beam with five equal spans of 6m, cross-section height of 0.4m and the width of 0.2 m each. The beam is made of concrete B30 and reinforcement A400. The beam is stressed by uniformly distributed load of 25 kN/m. Load distribution of the beam is presented on figure 1, a.

Design model is presented on figure 1, b as an integration of specific elements to be calculated by software complex.

By virtue the system symmetry, figure 1 c and d show half-diagrams of shears and deflection moments acting in beam cross-sections. Reinforcement assigned in compliance with provided calculation is presented on figure 1, e and f.

However, design model may be setup in a different way. A plastic hinge splits up the continuous system to separate elements, while the continuous system node where a plastic hinge is planned to be formed, is split up to the nodes , in which the resultant elements terminate along with a support node.

As the nodes at the ends of resultant elements may have small turning angles owing to plastic hinges, and a support node does not have a turning angle relative to support, in case of plastic hinge formation in one node a group of nodes is involved in the design model.

There is one more option of design model setup. A plastic hinge splits up the continuous system to separate elements, while the continuous system node where a plastic hinge is planned to be formed, is split up to the nodes , in which the resultant elements terminate along with a support node. This can be explained by the fact that the nodes at the ends of resultant elements have small turning angles owing to plastic hinges while support node does not have a turning angle relative to support.

The distance between the formed nodes is assumed negligibly small, and given the displacements of all nodes are integrated, this distance will not affect the values of derived forces. A scheme of support connections and integration of displacements are shown on figure 2, c and d.

When a plastic hinge is formed, a moment shall have a constant value, therefore hinge modelling is implemented with allowance for displacement and integration of the node group displacements.

The value of crack opening in the first plastic hinges is determined by balanced moment which should be at least 70 % of the moment of rupture.

Balancing of moments shall be performed with application of support moment as well as the span moment.

Then, for example, the moment on the second support \( M_{sup2} \) with account of a span moment \( M_{span2} \) on condition that \( M_{sup1} = 0 \) and \( x/h_0 = 0.215 \) will come up to:

\[
M_2 = \frac{2}{3} \left( \frac{1}{2} M_{sup2} - M_{span2} \right) = -78.3 \text{kNm} > 0.7 M_{sup2} = -66.3 \text{kNm}
\]

(1)

Given that \( x/h_0 = 0.149 \), the moment on the third support will come up to:

\[
M_3 = \frac{1}{2} M_{sup3} - \frac{1}{2} M_{span3} = -56.3 \text{kNm} > 0.7 M_{sup3} = -49.8 \text{kNm}
\]

(2)

As a result, we obtain a balanced moment diagram. The ultimate value of redistributed moment is not specified; however, calculation by ultimate states of the second group is mandatory.

3. Checkup by crack generation, opening and deformations.

Checkup by crack generation is conducted with account of reliability factor [6].

As an example, we will review the most typical cross-section 2-2 (figure 1, f).
Once we determine the center of gravity and the moment of cross-section inertia, we calculate the plastic moment of resistance and the moment of crack generation with account of inelastic deformations of tensional concrete. As a result, we have:

\[ M_{cr} = 78.3 \text{kNm} > M_{11} = 14.44 \text{kNm}. \]  

(3)

On this condition, the cracks will generate.

In calculation by crack opening and deformation, we assume reliability factor for stress load is equal to one. Then the bending moment \( M_{2,n} = 65.25 \text{kNm} \).
In this example, we do not make distinction between long-term and short-term loads, i.e. we assume that the total load is a long-term load. Considering the above, we will have the height of compressed 0.139m, reduced inertia moment without account of tensile concrete $J_{red} = 0.0007267 \text{m}^4$ with $\alpha_{s_2} = \alpha_{s_1} = 13.636$.

Calculation by crack opening with account of reinforcement value calculated by the first group of ultimate states with assumed factor $\varphi_{ult, long-term} = 1.4$ for a long-term effective load resulted in:

$$a_{crc, long-term} = 0.324 \text{mm} > a_{crc, ult, long-term} = 0.3 \text{mm}, \quad a_{crc, short-term} = 0.231 \text{mm} < a_{crc, ult, short-term} = 0.4 \text{mm}$$

(4)

The conditions are not met; however, there are options to account for some factors:

- given the account for the work of tensile concrete restrained between two rows of principle reinforcement and assuming that inertia moment factor is equal to $\alpha_{s_2} = 16.571$, then $J_{red} = 0.0008304 \text{m}^4$. The diagram of deflected mode of a flexible beam with cracks and tensile reinforcement arranged in two rows for cross-section 2-2 is presented on figure 3a.

Considering the above, we have:

$$a_{crc, long-term} = 0.283 \text{mm} < a_{crc, ult, long-term} = 0.3 \text{mm}, \quad a_{crc, short-term} = 0.202 \text{mm} < a_{crc, ult, short-term} = 0.4 \text{mm}$$

(5)

In this case, the condition is met.

- in case of arrangement of selected valves in one row, which is possible at the beam width $b = 0.2 \text{m}$ (figure 3, b), then with account of $\alpha_{s_2} = \alpha_{s_1} = 13.636$ we will have $J_{red} = 0.0008295 \text{m}^4$.

The diagram of deflected mode of tensile beam with cracks at such arrangement of tensile reinforcement is given on figure 3c.

Then:

$$a_{crc, long-term} = 0.285 \text{mm} < a_{crc, ult, long-term} = 0.3 \text{mm}, \quad a_{crc, short-term} = 0.204 \text{mm} < a_{crc, ult, short-term} = 0.4 \text{mm}.$$  

(6)

In this case, the condition is also met.

- considering that the load is always recognized as long-term and short-term and assuming that a long-term load is maximum 94 %, then compressed area will come to 0.139m, reduced inertia moment without account of tensile concrete $J_{red} = 0.0007267 \text{m}^4$ with $\alpha_{s_2} = \alpha_{s_1} = 13.636$. Under such conditions, the stresses in reinforcement will come to $\sigma_s = 263 \text{MPa}$.

Then:

$$a_{crc, long-term} = 0.296 \text{mm} < a_{crc, ult, long-term} = 0.3 \text{mm}, \quad a_{crc, short-term} = 0.231 \text{mm} < a_{crc, ult, short-term} = 0.4 \text{mm}.$$  

(7)

In this case, the condition is met as well.

Hence, following the review of suggested options, we may meet the conditions for crack opening.

Checkup by deformations, i.e. by flexure, was conducted with reliability factor for the load equal to one.

Due to the hinged bearing on the first and the last supports and the largest moments in the first spans in continuous beam the curvature and deflection in the extreme spans are also higher than in the middle.

Due to this, it is practical to compare the deflection in the terminal span with the maximum permissible value, in our case it is $f_{ult} = 0.03 \text{m}$.
The maximal deflection was calculated by conventional design model presented on figure 4.a, without account of cracks $f_{k, \text{elast}} = 5.12 \text{ mm}$ with deflection in the adjoining span $f_{k, \text{flexure}} = 0.168 \text{ mm}$. Similarly, a maximal deflection $f_{k, \text{elast}} = 5.97 \text{ mm}$ in the absence of a rundown was calculated by design model, figure 4. b without account of cracks.

On a hinge insertion, deflection increased by 16.6 % as compared to continuous structure. In this connection, it is necessary to determine how deflection values will change with regard to selected design models, figure 1a and figure1d, i.e. to identify the presence or absence of deflection.

For this purpose we will review design models presented on figures 4.a and 4.b, as appropriate. The a.m. models are notable for availability of adjacent spans or their replacement by a hinge with a moment of deflection applied at the end of the beam, matching the support moment occurring in the continuous beam.

\[ M_{\text{span}, n} = 58.5 \text{ kNm}, \quad M_{\text{on}2, n} = 78.9167 \text{ kNm}. \]

Percentage of reinforcement as per design model (figure 4.a) in cross-section 1-1 $\mu_1 = 0.009595$ and $\mu'_1 = 0.003549$, in cross-section 2-2 $\mu_2 = 0.012314$ and $\mu'_2 = 0.005435$.

On assumption that the total load is of short-term nature, according to [1] $\varepsilon_{\text{bl, red}} = 0.0015$, then $\alpha_1 = 13.6364$ and $\alpha_2 = 16.5708$. With such conditions we will have in the span $y_{c1} = x_{m1} = 0.148 \text{ m}$ and $J_{\text{red1}} = 0.000754 \text{ m}^4$, on support $y_{c2} = x_{m2} = 0.1581 \text{ m}$ and $J_{\text{red2}} = 0.000918 \text{ m}^4$.

The curvature in the span will amount to \( \left( \frac{1}{r_{l/m}} \right) = 0.00635 \frac{1}{\text{m}} \) and on support \( \left( \frac{1}{r_{l/h}} \right) = 0.00704 \text{ m} \).

According to [2, 4] deflection value will amount to:

\[ f_{k, \text{short-term}} = 0.0212 \text{ m} < f_{k, \text{alt}} = 0.03 \text{ m} \]  \hspace{1cm} (8)

If we assume that the total load of long-term exposure, then according to [1] $\varepsilon_{\text{bl, red}} = 0.0034$, then $\alpha_1 = 30.9091$ and $\alpha_2 = 37.5606$. With such conditions we will have in the span $y_{c1} = x_{m1} = 0.1885 \text{ m}$ and $J_{\text{red1}} = 0.001364 \text{ m}^4$, on support $y_{c2} = x_{m2} = 0.1966 \text{ m}$ and $J_{\text{red2}} = 0.001676 \text{ m}^4$. 

\[ $\mu_1 = 0.0009595$ \]

\[ $\mu'_1 = 0.003549$ \]

\[ $\mu_2 = 0.012314$ \]

\[ $\mu'_2 = 0.005435$ \]

\[ $\alpha_1 = 13.6364$ \]

\[ $\alpha_2 = 16.5708$ \]

\[ $\varepsilon_{\text{bl, red}} = 0.0015$ \]

\[ $\varepsilon_{\text{bl, red}} = 0.0034$ \]
The curvature in the span will amount to \( \left( \frac{1}{r} \right)_m = 0.00663 \frac{1}{m} \) and on support \( \left( \frac{1}{r} \right)_s = 0.00728 \frac{1}{m} \).

According to [6, 7] deflection value will amount to:

\[
f_{a,\text{long-term}} = 0.0221 \, m < f_{ult} = 0.03 \, m
\]

(9)

We have discovered that a long-term load exposure is the most unfavorable, as in this case the largest deflections have been observed.

In this connection, our further calculations of deflections for various design models will be made for continuous load exposures.

As an example, let us determine the value of beam deflection by design model as per figure 4.b. the bending moment and percentage of reinforcement are similar to design model as per figure 4.a.

In the first span, cross-section 1-1  \( M_{span,1} = 58.5 \, kNm \) and \( \mu_{s1} = 0.009595 \), \( \mu'_{s1} = 0.003549 \).

On support, cross-section 2-2 - \( M_{sup,2} = 78.9167 \, kNm \) and \( \mu_{s2} = 0.012314 \), \( \mu'_{s2} = 0.005435 \).

The total beam deflection will be the sum of deflection caused by uniform load distribution of and deflection caused by the moment on support.

\[
f_{\text{long-term}} = f_{\text{distr.}} + f_{\text{moment}}
\]

(10)

\[
f_{\text{distr.}} = \frac{q_n x \left( l^3 - 2l x^2 + x^3 \right)}{24 E_{b,\text{red}} J_{\text{red1}}}
\]

(11)

\[
f_{\text{moment}} = -\frac{M_{sup,2} x \left( l^2 - x^2 \right)}{6 E_{b,\text{red}} J_{\text{red2}} l^3}
\]

(12)

By substitution of values and term-by-term adding, we will come up with equation of total deflection. By solving the equation, we will have \( x = 2.7443 \, m \) and find the extremum:

\[
f_{b,\text{long-term}} = 0.0237 \, m < f_{ult} = 0.03 \, m
\]

(13)

That is, in absence of deflection in the next span, deflection \( f_{b,\text{long-term}} \) of a span with design model as per figure 4b will increase by 6.62% relative to \( f_{a,\text{long-term}} \) of a span with design model as per figure 4.a.

Now we determine a span deflection value with account of redistribution of moments by design model as per figure 4.c.

Deflection moment and percentage of reinforcement in the first span, cross-section 1-1, \( M_{1,n} = 63.9167 \, kNm \) and \( \mu_{s1} = 0.009595 \), \( \mu'_{s1} = 0.003549 \), on support, cross-section 2-2, \( M_{2,n} = 65.25 \, kNm \) and, \( \mu'_{s2} = 0.005435 \). Beam deflection will be the sum of deflection caused by uniform load distribution of and deflection caused by the moment on support:

\[
f_{\text{moment,c}} = -\frac{M_{2,n} x \left( l^2 - x^2 \right)}{6 E_{b,\text{red}} J_{\text{red2}} l^3}
\]

(14)
On support \( y_{c,2} = x_{m,2} = 0.188 \, \text{m} \) and \( J_{\text{red},2} = 0.001537 \, \text{m}^4 \). By substitution of values and term-by-term adding (11) and (14), we will come up with equation of total deflection. By solving the equation, we will have \( x = 2.7798 \, \text{m} \) and find the extremum:

\[
 f_{c,\text{long-term}} = 0.0253 \, \text{m} < f_{\text{ult}} = 0.03 \, \text{m}
\]  

(15)

Hence, deflection calculated by design model of figure 4.c, as related to deflection calculated by design model of Fig.4.a has increased by 14.16 %.

Actually, the hinge in design model implicitly presented on figure 2.a was introduced just to determine the forces with account of redistribution of moments, therefore the beam remains continuous and probability of hogging in the next span exists. With due consideration of hogging, increase of deflection by design model of figure 4.c relative to deflection by design model of figure 4.a, will come to merely 6.6 %, which satisfies checkup by deformations.

4. Comparison of results

To identify the cost savings, we performed comparison analysis of reinforcement consumption.

Total reinforcement of a five-span beam determined by design model in elastic system amounts to 217.63kg. Total reinforcement of a five-span beam determined by design model with account of redistribution of moments amounts to 212.33kg, which saves the reinforcement by 2 %.

Mass of the principal reinforcement of a five-span beam determined by elastic model amounted to 156.08kg, while the mass of reinforcement determined by design model with account of redistribution of moments amounted to 148.92kg thus totally reducing the mass of the product by 5 %.

For the multi-span systems, the most typical indicator will be the mass value of the principle reinforcement in the middle span.

By this indicator reinforcement, saving comes to 13%, given that calculation accounted for redistribution of moments.

5. Conclusions

Reduction of maximal values of bending moment in elastic system is possible with transformation of the system by accounting for redistribution of moments. Under such transformation, formation of cracks may occur, however, checkup conditions for crack opening and deformations are satisfied.

Application of design model for continuous beam with account of redistribution of moments will yield saving of the principal reinforcement in the central spans by 13 %.

References

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