An Improved BEMD Method for Denoising the Phase-OTDR Signal

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Abstract. Empirical mode decomposition (EMD) is often used for the processing of non-linear and non-stationary signals, but it needs further improvement in some specific application scenarios. According to the characteristics of Phase-OTDR signal, this paper improves the two-dimensional empirical mode (BEMD) and uses it to denoise the Phase-OTDR signal. The improved method takes into account the inconsistency of the correlation between the time axis and the space axis, and uses the spatial resolution of the Phase-OTDR signal as the minimum interval to enter the envelope surface structure. At the same time, the boundary is smoothed during the reconstruction of the sub-envelope surface. Experiments on the noisy Phase-OTDR signal verified the effectiveness of the improved method in this paper.

1. Introduction

Currently Phase-OTDR distributed optical fiber sensing technology is often applied to the detection of distributed vibration signals. However, because of the acquisition environment and acquisition equipment, the original signal data collected by the Phase-OTDR sensor is relatively weak and accompanied by more noise, so denoising processing is necessary. The experimental analysis of the original signal data shows that the modified signal is a non-linear, non-stationary signal, and a linear filter is not suitable. In order to solve these problems, non-linear methods have been proposed. As a new signal decomposition technique, empirical mode decomposition (EMD) is used by Huang et al [1, 2]. Based on this, two-dimensional empirical mode decomposition (BEMD) has been developed [3, 4]. Compared with the traditional decomposition method, BEMD represents the signal as the expansion of the basis function related to the signal, and estimates it through screening and iterative processes. The BEMD process is completely data-driven and does not require any prior known basis, and is especially suitable for two-dimensional nonlinear and non-stationary signals. However, the vibration signal collected by the Phase-OTDR sensor has certain characteristics. It is not an image signal that is usually processed by the general BEMD algorithm. It contains spatial distribution information, and is two-dimensional similar to the image signal and not exactly the same because the distributed vibration signal is composed of the time axis and the space axis, there is a certain difference in the correlation between the two dimensions. In order to achieve a better denoising effect, the existing general BEMD method must be appropriately improved.
2. Improved BEMD Denoising Method

2.1. The General Process of BEMD

EMD is a process that decomposes a complex signal $s(t)$ into a series of inherent modal functions (IMFs) and residuals, while BEMD makes some appropriate improvements and changes based on the EMD idea. The main difference is the screening process of IMFs. The construction of the mean line is transformed into the mean surface construction. The general process of its decomposition follows the following steps [5]:

Step 1): Perform matrix processing on the data $O(x, y)$ to obtain two-dimensional data $O_{i,j}(x, y)$. Where $i$ is the number of layers of the IMF component obtained, $j$ is the number of iterations of the IMF component currently obtained, and the minimum value of $i$ and $j$ is 1.

Step 2): Find the maximum and minimum points in $O_{i,j}(x, y)$.

Step 3): The two-dimensional interpolation algorithm is used to fit the maximum and minimum points of the data to construct the upper and lower envelope surfaces and calculate the mean envelope surface.

Step 4): Subtract the mean envelope surface obtained in the previous step from the current IMF component, namely $O_{i,j}(x, y)$, to obtain $O_{i,j+1}(x, y)$.

Step 5): If $O_{i,j+1}(x, y)$ satisfies the termination condition for generating IMF, judge $O_{i,j+1}(x, y)$ as the $i$-th IMF, that is, IMF$i$; if the termination condition is not met, then Go to step 4) until the termination condition is met.

Step 6): Let $i = i + 1$, $O_{i,j}(x, y) = O_{i-1,j}(x, y) - IMF_{i-1}(x, y)$, then repeat steps 1) to 5), until the residual IMF has no extreme points, the BEMD decomposition process ends. Finally $O(x, y)$ is decomposed into $IMF_{i}(x, y)$, $i = 1, 2, ... n$, and the residual $R_n(x, y)$:

$$O(x, y) = \sum_{i=1}^{n} IMF_{i}(x, y) + R_n(x, y)$$  \hspace{1cm} (1)

It can be known from the above decomposition process that in the general BEMD process, the problem of inconsistency in the two dimensions is not considered. According to the characteristics of Phase-OTDR signal data, the envelope surface structure can be constructed with the spatial resolution of the collected data as the minimum interval, and the boundary effect [6] can be suppressed at the same time, so as to achieve the purpose of distinguishing the time dimension and the space dimension area and improving the denoising effect.

2.2. Improved BEMD Method

In view of the characteristics of Phase-OTDR signal data, this article mainly improves step 3) in the general BEMD process, that is, considers the issue of correlation and boundary effects in the process of constructing the envelope surface:

Let x be the spatial axis of the data, $0 \leq x \leq L$, y be the time axis of the data, $0 \leq y \leq T$. And the interval $x \in [a_i, a_i + S]$, $y \in [0, T]$ divides the data into a plurality of two-dimensional sub-intervals. Let $p = (x, y)$ represent a point in a two-dimensional space. Where $a_i \in [0, L]$, $S$ is the spatial resolution of Phase-OTDR signal data, $L$ is the spatial length of a single set of data, and $T$ is the sampling time of a single set of data.

Take the radial basis function $\phi(r) = r$ to perform interpolation in each sub-interval, then for each two-dimensional sub-interval, there is a point set $Up$ with two-dimensional spatial data extreme points $P = \{p_j\} \subset \mathbb{R}^2$ and its The corresponding set of function values $F = \{f_j\} \subset \mathbb{R}$, find the interpolation implicit function mapping $s: \mathbb{R}^2 \rightarrow \mathbb{R}$ makes the following formula true:

$$s(p_j) = f_j, j = 1, 2, ..., N$$  \hspace{1cm} (2)
In order to obtain a smooth sub-envelope surface, the following formula is used to constrain the interpolation surface to minimize the surface energy $E$:

$$E = \int_{R^2} f_{xx}^2(p) + f_{yy}^2(p) + f_{xy}^2(p) \, dp$$  \hspace{1cm} (3)

With the constraint of formula (3), using the variational method [7] to solve formula (2), the following formula can be obtained:

$$s(p) = l(p) + \sum_{j=1}^{N} \lambda_j \phi(||p - p_j||)$$  \hspace{1cm} (4)

In the formula, $l$ is a low-order polynomial, $\lambda_j$ is a combination coefficient, and $||\cdot||$ are Euclidean norms. After adding the orthogonal condition, the sub-envelope surface interpolation result is solved for formula (4), and the construction of each sub-envelope surface is completed.

Based on the cubic spline function [8], the boundary smoothing is performed in the direction of the spatial axis at the interface of each sub-envelope surface: in the $y_{j}E \ [0, T]$ interval, let $t$ be a set of nodes in the sub-interval: $t_1 < t_2 < t_3 < \ldots < t_n$, and a set of corresponding function values: $y_1, y_2, y_3, \ldots, y_n$. Let the function $P_i(t)$ be a cubic polynomial on a subinterval, as shown in the following formula:

$$P_i(t) = a_i + b_i(t - t_i) + c_i(t - t_i)^2 + d_i(t - t_i)^3, i = 1, 2, \ldots, n - 1$$  \hspace{1cm} (5)

At the connection of the sub-envelope surfaces, the function values of the spatial axis direction, the first derivative and the second derivative are equal to achieve a smooth transition of adjacent sub-envelope surfaces:

$$\begin{align*}
P_i(t_{i+1}) &= P_{i+1}(t_{i+1}), i = 1, 2, \ldots, n - 1 \\
P'_i(t_{i+1}) &= P'_{i+1}(t_{i+1}), i = 1, 2, \ldots, n - 1 \\
P''_i(t_{i+1}) &= P''_{i+1}(t_{i+1}), i = 1, 2, \ldots, n - 1
\end{align*}$$  \hspace{1cm} (6)

Through the above process, the envelope surface construction dominated by the time axis correlation can be realized, and then the Phase-OTDR signal decomposition can be completed according to the general BEMD decomposition step. In the method based on BEMD denoising, the first inherent modal function $Imf_1(x, y)$ is often regarded as a direct removal of noise signals, and other components are denoised by setting thresholds. The soft threshold method [9] is used to process other components:

$$\tilde{Imf}_i(x, y) = \begin{cases} Imf_i(x, y), & \text{max}(|Imf_i(x, y)|) > T_i, \\
0, & \text{max}(|Imf_i(x, y)|) \leq T_i
\end{cases}$$  \hspace{1cm} (7)

In the formula, $T_i = C \sqrt{2E_i \ln(T_i)}$, $C$ is a constant and $E_i$ is the energy of the $i$-th order IMF. The final denoised signal is obtained:
\[
\tilde{O}(x, y) = \sum_{i=2}^{n} \text{Imf}_i(x, y) + R_n(x, y)
\]  \hspace{1cm} (8)

The above-mentioned improved BEMD denoising method for Phase-OTDR signals takes into account the inconsistency between the time dimension and the spatial dimension in the envelope surface construction, thereby making the BEMD denoising better applicable in this scenario.

3. Experiments and Results
In order to verify the effectiveness of the improved BEMD algorithm, a denoising comparison experiment was performed on the Phase-OTDR signal. A total of 100 sets of signal data were constructed, with a spatial resolution of 1m and a measurement length of 1000m. The number of samples in each group was 512. Gaussian white noise was added to the original signal, which kept the signal-to-noise ratio at 12dB after the noise was added. The denoising effect diagram is as follows:

![Denoising results comparison.](image)

Figure 1. Denoising results comparison.
Figure 1. *a* is the original data, Figure 1. *b* is the data after adding noise, Figure 1. *c* is the result of the general denoising method, and Figure 1. *d* is the result of the improved denoising method in this paper. It can be observed that the denoising results of the improved method in this paper are closer to the original data than the general method. In addition, the original data contains environmental noise, which is valid information in some application scenarios [10]. In general denoising methods, these environmental noises have not been well preserved, but in the improved method, the certain reservations.

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The three quantitative indicators used for the denoising judgment effect [11]: the signal-to-noise ratio SER, the mean square error MSE, and the correlation coefficient R are listed in Table 1:

|                | SER   | MSE   | R     |
|----------------|-------|-------|-------|
| General BEMD Denoising | 10.262 | 0.514 | 0.9729 |
| Improved BEMD Denoising | 13.031 | 0.232 | 0.9803 |

As can be seen from Table 1, for Phase-OTDR signals, the improved BEMD denoising method results in higher SER and R and lower MSE.

4. Conclusion
According to the characteristics of Phase-OTDR signal, this paper improves the general BEMD denoising method. Considering the inconsistency between the time dimension and the space dimension, the spatial resolution of the collected data is taken as the minimum interval to enter the envelope surface structure, and the boundary effect is suppressed. Experimental results on Phase-OTDR signals show that the method has better denoising effect in such application scenarios than the general BEMD method.

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