Electroproduction of the $N^*(1535)$ nucleon resonance in QCD

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Following the 12 GeV upgrade, a dedicated experiment is planned with the Hall B CLAS12 detector at Jefferson Lab, with the aim to study electroproduction of nucleon resonances at high photon virtualities up to $Q^2 = 12$ GeV$^2$. In this work we present a QCD-based approach to the theoretical interpretation of these upcoming results in the framework of light-cone sum rules that combine perturbative calculations with dispersion relations and duality. The form factors are thus expressed in terms of the $N^*(1535)$ light-front wave functions at small transverse separations, called distribution amplitudes. The distribution amplitudes can therefore be determined from the comparison with the experimental data on form factors and compared to the results of lattice QCD simulations. The results of the corresponding next-to-leading order calculation are presented and compared with the existing data. We find that the form factors are dominated by the twist-four distribution amplitudes that are related to the $P$-wave three-quark wave functions of the $N^*(1535)$, i.e. to contributions of orbital angular momentum.

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I. INTRODUCTION

It is generally accepted that studies of baryon form factors at large momentum transfer $Q^2$ give access to the light-front wave functions at small transverse separations between the constituents, called hadron distribution amplitudes (DAs), although perturbative QCD factorization [1–3] does not seem to be applicable for realistic $Q^2$ accessible in current or planned experiments. The problem is that the leading contribution involves two hard gluon exchanges and is suppressed by the small factor $(\alpha_s/\pi)^2 \sim 0.01$ compared to the “soft” (end-point) contributions which are subleading in the power counting in $1/Q^2$ but do not involve small coefficients. Hence the collinear factorization regime is approached very slowly. Model calculations suggest that “soft” contributions play the dominant role at present energies. Taking into account soft contributions is challenging because they involve a nontrivial overlap of nonperturbative wave functions of the initial and the final state hadrons, and are not factorizable, i.e. cannot be simplified further in terms of simpler quantities.

In this situation the question what exactly do we learn from the studies of form factors is far from trivial. One existing description is to introduce more complicated, transverse-momentum dependent (TMD) quark distributions, taking advantage of Sudakov suppression of large transverse separations, following the technique suggested initially by Li and Sterman [4] for the pion form factor. Another approach that we advocate in this work, is to calculate the soft contributions to the form factors as an expansion in terms of nucleon DAs of increasing twist using dispersion relations and duality. This method is known as light-cone sum rules (LCSRs) [5] and provides one with the most direct relation of the hadron form factors and DAs that is available at present, with no other nonperturbative parameters.

The LCSR approach has been used successfully for the calculations of pion electromagnetic and also weak $B$-decay form factors, see Refs. [6–8] for several recent state-of-the-art calculations. The LCSRs for baryon form factors are more complicated and recent. The first application was for the nucleon electromagnetic form factors in Refs. [9, 10]. Several further studies aimed at finding an optimal nucleon interpolation current [10,11] and extending this technique to other elastic or transition form factors of interest. LCSRs for the axial nucleon form factor were presented in [10,12,13] for the scalar form factor in [14] and tensor form factor in [15]. A generalization to the full baryon octet was considered e.g. in [16]. Application of the same technique to $N\gamma\Delta$ transitions was suggested in [12,17] and to pion production at threshold in [18]. LCSRs for weak baryon decays $\Lambda_b \to p,\Lambda\ell\nu\bar{\nu}$ etc. were studied in [19–22], etc. In the early work only the leading order (LO) contributions to the coefficient functions in the LCSRs have been taken into account. The first complete next-to-leading order (NLO) analysis was done for the electromagnetic nucleon form factors in Ref. [23] and the results appear to be consistent with the constraints on nucleon DAs from lattice calculations [24]. The picture emerging from these studies suggests that the momentum fraction distribution of the valence quarks in the proton is rather broad, with $\sim 40\%$ of the momentum carried by the $u$-quark that carries proton helicity, and approximately symmetric to the interchange of the remaining quarks.

Our study is motivated by the dedicated experiment planned with the Hall B CLAS12 detector at Jefferson Lab following the 12 GeV upgrade, with the aim to study electroproduction of nucleon resonances at high photon virtualities up to $Q^2 = 12$ GeV$^2$ [25]. The corresponding form factors can be calculated using the LCSR machinery in terms of the DAs of nucleon resonances. Turning this relation around, information on the DAs of resonances can be extracted from the comparison of the LCSR calculations with the experimental data on form factors and compared to the constraints that can come, eventually, from lattice QCD simulations. This pro-
gram was suggested in Ref. [26] and an exploratory study was made there for the particular case of electroproduction of the lowest negative parity \(N^*(1535)\) resonance. In our paper we elaborate on this proposal. Learning about quark distributions in nucleon resonances is an exciting possibility, since existing QCD calculations of resonance properties, e.g. on the lattice, rarely go beyond the mass spectrum.

The case of \(N^*(1535)\) is special because the classification and the structure of the light-front wave functions for the states with opposite parity is almost identical. Hence the LCSRs for the corresponding electroproduction form factors are very similar to the LCSRs for electromagnetic nucleon form factors. In particular the NLO expressions derived in Ref. [23] can be overtaken with relatively minor modifications. A detailed analysis of these NLO LCSRs is the main goal of this work.

The presentation is organized as follows. The electroproduction form factors are introduced and the structure of the corresponding analysis form factors are introduced and the structure of the light-cone region is the main goal of this work.

The case of \(N^*(1535)\) can be expressed in terms of the form factors [27]:

\[
\langle N^*(P')|j^e_{\mu}|N(P)\rangle = \bar{u}_{N'}(P')\gamma_\mu \Gamma_\nu u_{N}(P),
\]

where \(q = P' - P\) is the momentum transfer. In what follows we use the standard notation \(Q^2 = -q^2\). The helicity amplitudes \(A_{1/2}(Q^2)\) and \(S_{1/2}(Q^2)\) for the electroproduction of \(N^*(1535)\) can be expressed in terms of the form factors [27]:

\[
A_{1/2} = e B \left[ Q^2 G_1(Q^2) + m_N(m_{N^*} - m_N) G_2(Q^2) \right],
\]

\[
S_{1/2} = \frac{e B C}{\sqrt{2}} \left[ (m_N - m_{N^*}) G_1(Q^2) + m_N G_2(Q^2) \right].
\]

Here \(e = \sqrt{4\pi\alpha}\) is the elementary charge and \(B, C\) are kinematic factors defined as

\[
B = \sqrt{\frac{Q^2 + (m_{N^*} + m_N)^2}{2m_N^2(m_{N^*} - m_N)}},
\]

\[
C = \sqrt{1 + \frac{(Q^2 - m_{N^*}^2 + m_N^2)^2}{4Q^2m_N^2}}.
\]

The basic object of the LCSR approach to baryon form factors [8-10] is the correlation function

\[
T_\nu(P, q) = i \int dx e^{-iqx} \langle N^*(P')|T\{j_\nu(x)\eta_N(0)\}|0\rangle
\]

in which \(j\) represents the electromagnetic (or weak) probe and \(\eta_N\) is a suitable local operator with nucleon quantum numbers. The \(N^*\) resonance is explicitly represented by its state vector \(\langle N^*(P')|\), see a schematic representation in Fig. 1. The LCSR is obtained by comparing (matching) two different representations for the correlation function. On the one hand, when both the momentum transfer \(q^2 = -Q^2\) and the momentum \(P^2 = (P' - q)^2\) flowing in the \(\eta_N\) vertex are large and negative, the main contribution to the integral comes from the light-cone region \(x^2 \to 0\) and can be studied using the operator product expansion (OPE) of the time-ordered product \(T\{j(x)\eta_N(0)\}\). The singularity at \(x^2 \to 0\) of a particular contribution is governed by the twist of the relevant composite operator whose matrix element \(\langle N^*|\ldots|0\rangle\) is related to the \(N^*\) DA. On the other hand, one can represent the answer in form of the dispersion integral in \(P^2\) and define the nucleon contribution by the cutoff in the quark-antiquark invariant mass, the so-called interval of duality \(s_0\) (or continuum threshold). The main role of the interval of duality is that it does not allow large momenta \(|k^2| > s_0\) to flow through the \(\eta_N\)-vertex; to the lowest order \(\langle\alpha^2\rangle\) one obtains a purely soft contribution to the form factor as a sum of terms ordered by twist of the relevant operators and hence including both the leading- and the higher-twist nucleon DAs. Note that the contribution of higher-twist DAs is suppressed by powers of the continuum threshold (or by powers of the Borel parameter after applying the usual QCD sum rule machinery), but not by powers of \(Q^2\), the reason being that soft contributions are not constrained to small transverse separations.

The “plus” spinor projection \([\Lambda_3]\) involving the “plus” component of the electromagnetic current can be parametrized in terms of two invariant functions

\[
\Lambda_+ T_+ = p_+ \left\{ m_N A(Q^2, P^2) + \frac{1}{\Lambda} B(Q^2, P^2) \right\} N^+(P),
\]

where \(Q^2 = -q^2\) and \(P^2 = (P - q)^2\). The correlation functions \(A(Q^2, P^2)\) and \(B(Q^2, P^2)\) can be calculated in QCD in terms of \(N^*\) DAs for sufficiently large Euclidean momenta.
\( Q^2, -P'^2 \gtrsim 1 \text{ GeV}^2 \) using OPE. Schematically,

\[
A(Q^2, P'^2) = \sum_k \int [dx] a_k(Q^2, P'^2, x, \mu_F^2) F_k(x, \mu_F^2),
\]

\[
B(Q^2, P'^2) = \sum_k \int [dx] b_k(Q^2, P'^2, x, \mu_F^2) F_k(x, \mu_F^2),
\]

where the sum goes over all existing DAs, \( F_k \in \{ V_k, A_k, T_k, S_k, P_k \} \) defined in Eq. (21), the integration goes over quark momentum fractions and \( \mu_F \) stands for the factorization scale. The coefficient functions \( a_k(Q^2, P'^2, x, \mu_F^2) \) and \( b_k(Q^2, P'^2, x, \mu_F^2) \) are known to the NLO accuracy for twist-three and twist-four DAs \[23\], and to leading order (LO) for twist-five and twist-six. In principle this expansion also contains contributions of four-particle DAs with an additional gluon, five-particle with two gluons or a quark-antiquark pair, etc. Such contributions start at twist-four and they are not included in the present calculation because the corresponding DAs are very poorly known (see, however, Ref. [28]). It turns out that the coefficient functions are the same for the states with negative and positive parity, \( N^*(1535) \) and the nucleon, if the definitions are chosen as explained in Appendix A. Thus we are able to use the NLO expressions for the electromagnetic nucleon form factors obtained in [23] with trivial modifications, e.g. replacing nucleon mass \( m_N \) by \( m_{N^*} \). One difference is that, because of the larger mass, corrections of the type \( m_{N^*}^2/Q^2 \) become much larger and numerically significant. For this reason in this work we use complete expressions for the LO coefficient functions from Ref. [10] rather than the corresponding expressions from Ref. [28] where the expansion in powers of \( m_{N^*}^2/Q^2 \) was truncated to match the accuracy of the calculated NLO corrections.

The results of the QCD calculation in Euclidean region can be presented in the form of a dispersion relation

\[
A^{\text{QCD}}(Q^2, P'^2) = \frac{1}{\pi} \int_0^\infty ds \frac{1}{s - P'^2} \text{Im} A^{\text{QCD}}(Q^2, s) + \ldots
\]

\[
B^{\text{QCD}}(Q^2, P'^2) = \frac{1}{\pi} \int_0^\infty ds \frac{1}{s - P'^2} \text{Im} B^{\text{QCD}}(Q^2, s) + \ldots
\]

where the ellipses indicate possible subtractions. The same correlation functions can be written in terms of physical spectral densities that contain a nucleon (proton) pole at \( P'^2 \to m_{N^*}^2 \), nucleon resonances and the continuum. The nucleon contribution is, obviously, proportional to the electroproduction form factors of interest, whereas for higher mass states one can use quark-hadron duality:

\[
A^{\text{phys}}(Q^2, P'^2) = \frac{2\lambda_1 F_{1}(Q^2)}{m_{N^*}^2 - P'^2} + \frac{1}{\pi} \int_{s_0}^\infty ds \frac{1}{s - P'^2} \text{Im} A^{\text{QCD}}(Q^2, s) + \ldots
\]

\[
B^{\text{phys}}(Q^2, P'^2) = \frac{\lambda_1 F_{2}(Q^2)}{m_{N^*}^2 - P'^2} + \frac{1}{\pi} \int_{s_0}^\infty ds \frac{1}{s - P'^2} \text{Im} B^{\text{QCD}}(Q^2, s) + \ldots
\]

where \( s_0 \approx (1.5 \text{ GeV})^2 \) is the interval of duality (also called continuum threshold). Matching the two above representations and making the Borel transformation that eliminates subtractions constants

\[
\frac{1}{s - P'^2} \to e^{-s/M^2}
\]

one obtains the sum rules

\[
\frac{2\lambda_1 N^2 G_{1}(Q^2)}{m_{N^*} m_{N}} = \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{e^{\left(m_{N^*}^2-s\right)/M^2}}{s-P'^2} \text{Im} A^{\text{QCD}}(Q^2, s),
\]

\[-2\lambda_1 N^2 G_{2}(Q^2) = \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{e^{\left(m_{N^*}^2-s\right)/M^2}}{s-P'^2} \text{Im} B^{\text{QCD}}(Q^2, s).
\]

The dependence of the Borel parameter \( M^2 \) is unphysical and has to disappear in the full QCD calculation. It can be used to estimate theoretical uncertainties.

### III. NUMERICAL ANALYSIS

Main nonperturbative input to the LCSRs for electroproduction form factors is provided by the DAs of nucleon resonances that can be parameterized by two normalization constants \( f_{N^*}, \lambda_{N^*} \) and a set of shape parameters \( \varphi_{nk}, \eta_{nk} \) corresponding to contributions of local operators of increasing dimension, see Eqs. (22), (23). The dependence of the form factors on these parameters is linear so that the results can conveniently be presented as

\[
G_1(Q^2) = \frac{\lambda_{1} N^*}{\lambda_1} \left\{ g_{10}^{00}(Q^2) + g_{10}^{10}(Q^2) \eta_{10} + g_{11}^{11}(Q^2) \eta_{11} + f_{11}^{11}(Q^2) \varphi_{11} + \ldots \right\}
\]

and similarly

\[
G_2(Q^2) = \frac{\lambda_{1} N^*}{\lambda_1} \left\{ g_{20}^{00}(Q^2) + g_{20}^{10}(Q^2) \eta_{10} + g_{21}^{11}(Q^2) \eta_{11} + f_{21}^{11}(Q^2) \varphi_{11} + \ldots \right\}
\]

where the ellipses stand for the contributions of second-order polynomials in the leading-twist DAs \[22\], terms in \( \varphi_{20}, \varphi_{21}, \varphi_{22} \). The coefficient functions \( f_{1,2}^{nk}(Q^2) \) and \( g_{1,2}^{nk}(Q^2) \) are given by very cumbersome analytic expressions \[23\] and depend implicitly on the masses of \( N^* \) and the nucleon, the continuum threshold \( s_0 \), Borel parameter \( M^2 \), QCD coupling \( \alpha_s(\mu_F) \) and the factorization scale \( \mu_F \). Note that, e.g., \( f_{1}^{10}(Q^2) \) includes the sum of contributions of the asymptotic leading-twist DA and the corresponding Wandzura-Wilczek terms in the higher-twist DAs, see Appendix A and Ref. [23] for more details. Note also that the DA \( \Xi_{A,2} \) corresponding to the \( \xi_z = 2 \) component of the light-front three-quark

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TABLE I: Parameters of the $N^*(1535)$ distribution amplitudes at the scale $\mu^2 = 2$ GeV$^2$. For the lattice results [24] only statistical errors are shown. The set of parameters indicated as LCSR (1) corresponds to the fit to the form factors $G_1(Q^2)$ and $G_2(Q^2)$ extracted from the measurements of helicity amplitudes in Ref. [29] adding the errors in quadrature. The set of parameters indicated as LCSR (2) is obtained from the fit to helicity amplitudes including all available data at $Q^2 \geq 1.7$ GeV$^2$ [29, 32].

| Method  | $\chi_1^N/\chi_d^N$ | $f_{N^*/\chi_d^N}$ | $\varphi_{10}$ | $\varphi_{11}$ | $\varphi_{20}$ | $\varphi_{21}$ | $\varphi_{22}$ | $\eta_{11}$ | $\eta_{10}$ | Reference |
|---------|-----------------------|----------------------|----------------|----------------|---------------|---------------|---------------|-------------|-------------|-----------|
| LCSR (1) | 0.633 | 0.027 | 0.36 | -0.95 | 0 | 0 | 0 | 0.00 | 0.94 | this work |
| LCSR (2) | 0.633 | 0.027 | 0.37 | -0.96 | 0 | 0 | 0 | -0.29 | 0.23 | this work |
| LATTICE | 0.633(43) | 0.027(2) | 0.28(12) | -0.86(10) | 1.7(14) | -2.0(18) | 1.7(26) | - | - | [24] |

wave function does not contribute to the LCSRs for our choice of the nucleon interpolating current.

Calculations in this work are done for the "standard" choice of the specific LCSR parameters: continuum threshold $s_0 = (1.5 \text{ GeV})^2$, Borel parameter $M^2 = 2 \text{ GeV}^2$ and factorization (and renormalization) scale $\mu_F^2 = 2 \text{ GeV}^2$. The dependence on these parameters is rather mild; in particular varying the Borel parameter in the range $1.5 - 2$ GeV$^2$ induces an overall variation of form factor of the order of 10% so that, e.g., the ratio $G_2/G_1$ is largely unchanged.

The resonance mass corrections enter the LCSRs in a complicated way, as terms in $m_{N^*}^2/Q^2$ and $m_{N^*}^2/s_0$. The latter ones do not decrease at large momentum transfers and in ideal case have to be resummed to all orders. The corresponding expression exists for the LO LCSR [10] but not for the NLO corrections. In order to minimize this mismatch we have rescaled the $O(\alpha_s)$ contributions calculated in [23] by the ratio of the corresponding LO terms calculated with account for $m_{N^*}^2$ corrections and putting $m_{N^*}^2$ to zero. For the numerically important contributions this rescaling corresponds to a reduction of the NLO correction by $10-20\%$.

Existing information on the DAs of negative parity resonances is very scarce. The results of the recent lattice calculation [24] are presented in Table 1. The most interesting feature of these results is that the corrections to the asymptotic leading twist DAs have alternating signs for the lattice states with increasing mass. In particular the twist-three DA of $N^*(1535)$ has a very small value at the origin and is approximately antisymmetric with respect to the exchange of the two valence quarks forming a scalar "diquark", whereas the DA of $N^*(1535)$ is symmetric and similar in shape to the nucleon DA, see Fig. 9 in Ref. [24]. These results are still exploratory and have to be taken with caution because identification of lattice states with particular physical resonances is not obvious and requires further study. Even with this uncertainty, the lattice values are very helpful as knowing the order of magnitude of the parameters allows one to establish a hierarchy of different contributions to the LCSR.

As an illustration, the NLO LCSR result for the form factors at $Q^2 = 2$ GeV$^2$ normalized to the dipole formula

$$D(Q^2) = \frac{1}{(1 + Q^2/a)^2}, \quad a = 0.71 \text{ GeV}^2 \quad (13)$$

can be written as follows:

$$G_1^{\text{NLO}}(Q^2) = \frac{\lambda_1^N}{\lambda_d^N} \left[ 0.666 - 2.18\eta_{10} + 0.86\eta_{11} 
- 0.69 f_{N^*} - 1.76 f_{N^*}\varphi_{10} + 1.05 f_{N^*}\varphi_{11} 
+ 1.3 f_{N^*}\varphi_{20} + 0.66 f_{N^*}\varphi_{21} - 0.06 f_{N^*}\varphi_{22} \right],$$

$$G_2^{\text{NLO}}(Q^2) = \frac{\lambda_1^N}{\lambda_d^N} \left[ -0.466 + 1.84\eta_{10} + 0.06\eta_{11} 
- 0.82 f_{N^*} - 1.06 f_{N^*}\varphi_{10} - 1.08 f_{N^*}\varphi_{11} + 2.6 f_{N^*}\varphi_{20} + 1.5 f_{N^*}\varphi_{21} + 0.39 f_{N^*}\varphi_{22} \right]$$

where we use a notation $\tilde{f}_{N^*}$ for the ratio of twist-three and twist-four couplings

$$\tilde{f}_{N^*} = \frac{\tilde{f}_{N^*}}{\lambda_d^N} = 0.027(2) \quad (14)$$

For comparison, the similar decomposition of the form factors for the LO LCSR [20] for the same value $Q^2 = 2$ GeV$^2$ reads

$$G_1^{\text{LO}}(Q^2) = \frac{\lambda_1^N}{\lambda_d^N} \left[ 0.816 - 2.02\eta_{10} + 0.88\eta_{11} 
- 0.59 f_{N^*} - 1.60 f_{N^*}\varphi_{10} + 1.19 f_{N^*}\varphi_{11} 
+ 1.26 f_{N^*}\varphi_{20} + 0.70 f_{N^*}\varphi_{21} + 0.12 f_{N^*}\varphi_{22} \right],$$

$$G_2^{\text{LO}}(Q^2) = \frac{\lambda_1^N}{\lambda_d^N} \left[ -0.466 + 1.84\eta_{10} + 0.06\eta_{11} 
- 1.19 f_{N^*} - 0.78 f_{N^*}\varphi_{10} + 3.82 f_{N^*}\varphi_{11} + 2.9 f_{N^*}\varphi_{20} + 1.6 f_{N^*}\varphi_{21} + 0.28 f_{N^*}\varphi_{22} \right]$$

so that the NLO corrections are significant.

For convenience we provide a simple parametrization for the coefficient functions $f_{1,2,12}^{\text{LO}}$ as functions of $Q^2$ in Appendix B. This parametrization was obtained for the region of momentum transfers $2 \text{ GeV}^2 < Q^2 < 12 \text{ GeV}^2$ and should not be used outside this interval. In particular we found that the mass corrections $m_{N^*}^2/Q^2$ become very large for $Q^2 < 2 \text{ GeV}^2$ so that the LCSRs become unstable (and not reliable). In general, different contributions to the LCSRs are distinguished by their $Q^2$-dependence so that one needs a sufficient lever arm in $Q^2$ to determine several of them simultaneously.
Since the existing data for $Q^2 \geq 1.5 - 2$ GeV$^2$ are very limited, we put in this work all second-order coefficients in the leading-twist DAs to zero, $\varphi_{20} = \varphi_{21} = \varphi_{22} = 0$, used central lattice values for $f_{N^*}$ and $\lambda_N^{1\kappa}$, and constrained $\varphi_{10}$, $\varphi_{11}$ to the lattice values within the given error bars. In this way we are left, essentially, with two free parameters — $\eta_0$ and $\eta_1$. We expect that much more data will become available after the 12 GeV upgrade at Jefferson Lab where a dedicated experiment is planned to study electroproduction of nucleon resonances at high photon virtualities up to $Q^2 = 12$ GeV$^2$ [25].

Information on the electrocouplings of nucleon resonances at large momentum transfers is obtained by studying electroproduction of $\pi$ and $\eta$ mesons in the respective resonance region [29,32]. The results are usually presented for the helicity amplitudes, and in earlier work only the larger one, $A_{12}(Q^2)$, was studied for large momentum transfers. The latest study [29] also includes the results on $S_{12}(Q^2)$ up to $Q^2 = 4.16$ GeV$^2$ allowing us to extract from these data the Dirac-like and Pauli-like transition form factors $G_1(Q^2)$ and $G_2(Q^2)$ [1] that are more relevant for QCD studies. In this extraction we assumed that the errors for helicity amplitudes given in Ref. [29] are uncorrelated and added them in quadrature. The results are shown in Fig. 2 and Fig. 3 on the right panels; it is seen that the Pauli-like form factor changes sign and becomes negative at large $Q^2$, although the errors are quite large.

Two different LCSR fits of the experimental data are shown in Figs. 2, 3. The difference is that in Fig. 2 the fit is done to the form factors extracted from the data on helicity amplitudes reported in Ref. [29], and in Fig. 3 we make a fit to the data on helicity amplitudes $A_{12}(Q^2)$ and $S_{12}(Q^2)$ themselves including all existing data for $Q^2 \geq 1.7$ GeV$^2$. In the second case the fit is driven by the data [30,32] on $A_{12}(Q^2)$ that have smaller errors and not entirely consistent with [29], so that a worse description of the form factors in this fit is not a surprise. The corresponding parameters are listed in Table 1.

Because of the small value of the leading twist normalization constant suggested by lattice calculations [14], the results for $A_{12}(Q^2)$ and $G_1(Q^2)$ prove to be almost insensitive to the leading twist DA of the $N^*(1535)$ resonance and are domi-
nated by the twist-four contributions corresponding to the P-wave parts of the three-quark light-front wave functions (see Appendix A). Moreover, sensitivity of the results to the shape parameters of the twist-four DAs, \( \eta_{10} \) and \( \eta_{11} \), is rather mild, cf. two last columns in the first and the second line in Table 1. Thus \( A_{12}(Q^2) \) and \( G_1(Q^2) \) are both sensitive mostly to the ratio of the normalization constants \( \lambda^N_+ / \lambda^N_+ \), which we fix to the lattice value 0.633 [24]. The LCSR predictions for these observables are very stable, and the agreement of the existing data with the normalization suggested by lattice calculations is encouraging.

The \( S_{12}(Q^2) \) amplitude and especially the Pauli-like form factor \( G_2(Q^2) \) are much more sensitive to the nonperturbative input and in particular to the shape parameters of the twist-four DAs, compare Fig. 2 and Fig. 3. Also the leading-twist contributions play some role in this case because of strong cancellations. More precise data and a larger interval in \( Q^2 \) are needed to make this comparison quantitative.

IV. CONCLUSIONS AND OUTLOOK

In this work we argue that the LCSR approach can provide one with quantitative information on the wave functions of nucleon resonances at short distances. The basic idea behind this technique is that soft Feynman contributions to the form factors are calculated in terms of small transverse distance quantities using dispersion relations and duality. The form factors are thus expressed in terms of light-front wave functions at small transverse separations, called DAs, without additional parameters. Alternatively, the distribution amplitudes can be extracted from the comparison with the experimental data on form factors and compared to the results of lattice QCD simulations or other nonperturbative approaches based on, e.g., QCD sum rules or Dyson-Schwinger equations. The results of the corresponding NLO calculation for the particular case of the \( N^+(1535) \) resonance are presented and compared with the existing data. We find that the form factors are dominated by twist-four DAs that are related to the \( P \)-wave three-quark wave functions, i.e., to the distribution of orbital angular momentum.

Interestingly enough the LCSRs have the same form for spin-1/2 resonances of both parities so that apart from the (calculable) effects of resonance mass corrections the difference in observed form factors of, say, \( N^+(1535) \) and \( N^+(1650) \) can be attributed to the difference in the wave functions, which is of major interest. The differences between nucleon elastic form factors and electroexcitation of the Roper resonance can be studied in a similar manner; however, it is likely that in the latter case interpretation of the results may require a better understanding and more sophisticated models of twist-five DAs than are available at present.

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Appendices

Appendix A: Light-front wave functions and DAs of \( J^P = \left( \frac{1}{2} \right)^- \) nucleon resonances

In the light-front description [3] a hadron is represented by the superposition of Fock states with different number of partons. Restricting ourselves to the three-quark (valence) components we view, e.g., the proton with positive helicity as a superposition of states with different values of the quark orbital angular momentum projection on the direction of motion, \( \ell_z = -1, 0, 1, 2 \),

\[
|N\uparrow\rangle = \sum_{\ell_z} |N\uparrow\rangle_{\ell_z} \, . \tag{A.1}
\]

A nonzero value of \( \ell_z \) accounts for the mismatch between the proton helicity and the sum of helicities of the valence quarks \( \lambda_i \) so that \( 1/2 = \lambda_1 + \lambda_2 + \lambda_3 + \ell_z \). The four different contributions can be written in terms of six independent scalar light-front wave functions as [33–35].

\[
|N\uparrow\rangle_{\ell_z = 0} = \frac{\epsilon^{ijk}}{\sqrt{6}} \int \frac{[dx]dk_{12}}{x_1 x_2 x_3} \left[ \psi_{N, i:1}(1, 2, 3) + i\epsilon^{\alpha\beta}k_1^\perp \psi_{N, i:2}(1, 2, 3) \right] b_{u_1}^{i\dagger} (1) \left( b_{u_2}^{j\dagger} (2) b_{d_3}^{k\dagger} (3) - b_{d_2}^{j\dagger} (2) b_{u_3}^{k\dagger} (3) \right) |0\rangle ,
\]

\[
|N\uparrow\rangle_{\ell_z = 1} = \frac{\epsilon^{ijk}}{\sqrt{6}} \int \frac{[dx]dk_{12}}{x_1 x_2 x_3} \left[ k_1^\perp \psi_{N, i:1}(1, 2, 3) + k_2^\perp \psi_{N, i:2}(1, 2, 3) \right] \left( b_{u_1}^{i\dagger} (1) b_{u_2}^{j\dagger} (2) b_{d_3}^{k\dagger} (3) - b_{d_2}^{j\dagger} (2) b_{u_3}^{k\dagger} (3) \right) |0\rangle ,
\]

\[
|N\uparrow\rangle_{\ell_z = -1} = \frac{\epsilon^{ijk}}{\sqrt{6}} \int \frac{[dx]dk_{12}}{x_1 x_2 x_3} \left[ k_1^\perp \psi_{N, i:1}(1, 2, 3) \right] \left( b_{u_1}^{i\dagger} (1) b_{u_2}^{j\dagger} (2) b_{d_3}^{k\dagger} (3) - b_{d_2}^{j\dagger} (2) b_{u_3}^{k\dagger} (3) \right) |0\rangle ,
\]

\[
|N\uparrow\rangle_{\ell_z = 2} = \frac{\epsilon^{ijk}}{\sqrt{6}} \int \frac{[dx]dk_{12}}{x_1 x_2 x_3} \left[ k_1^\perp k_2^\perp \psi_{N, i:1}(1, 2, 3) \right] \left( b_{u_1}^{i\dagger} (1) b_{u_2}^{j\dagger} (2) b_{d_3}^{k\dagger} (3) - b_{d_2}^{j\dagger} (2) b_{u_3}^{k\dagger} (3) \right) |0\rangle . \tag{A.2}
\]

Here \( b_{u, d}^{i\dagger} (1) \) etc. are creation operators for the quarks of specific flavor with positive \( \uparrow \) or negative \( \downarrow \) helicity; the argument (1) stands for the dependence on longitudinal momen-
tum fractions and transverse momenta of the given quark, i.e. $u_{+i}(1) = u_{+i}(x_1, k_1^\perp)$, and so on. We use the notation for transverse momenta

$$k^\perp = k_x^\perp + ik_y^\perp, \quad \bar{k}^\perp = k_x^\perp - ik_y^\perp.$$  \tag{A.3}

The light-front wave functions $\psi^{(l)}_{N;i} (1, 2, 3)$ depend on momentum fractions $x_i$ and transverse momenta squared $|k_{\perp,i}|^2 = k_x^\perp k_y^\perp$ of all partons. The integration measure is chosen as \cite{28}

$$[dx] = \frac{1}{2(2\pi)^3} \prod_{k=1}^3 d^2 k^\perp \delta^{(2)} \left( \sum k_i^\perp \right) , \tag{A.4}$$

and

$$[dk_\perp] = \frac{1}{4(2\pi)^3} \prod_{k=1}^3 d^2 k^\perp \delta^{(2)} \left( \sum k_i^\perp \right) . \tag{A.5}$$

The proton light-cone DAs, in turn, are defined as matrix elements of gauge-invariant nonlocal operators with the three quark fields separated by a light-like distance. Standard decomposition \cite{30} involves 24 invariant functions:

$$4(0)\epsilon^{ijk} u_\alpha^i (a_1 n) u_\beta^j (a_2 n) d_\gamma^k (a_3 n) |N(P, \lambda)\rangle =$$

$$= S_{1}^{N} m_{N} C_{\alpha \beta} (\gamma_{5} u_{N}^{\perp}) + S_{2}^{N} m_{N} C_{\alpha \beta} (\gamma_{5} u_{N}^{\perp}) + P_{1}^{N} m_{N} (\gamma_{5} C)_{\alpha \beta} (u_{N}^{+}) + P_{2}^{N} m_{N} (\gamma_{5} C)_{\alpha \beta} (u_{N}^{-})$$

$$+ \frac{1}{2} V_{1}^{N} (\gamma \gamma_{5} C)_{\alpha \beta} (\gamma_{5} u_{N}^{\perp}) + \frac{1}{2} V_{2}^{N} (\gamma \gamma_{5} C)_{\alpha \beta} (\gamma_{5} u_{N}^{\perp}) + \frac{1}{2} V_{3}^{N} (\gamma_\perp C)_{\alpha \beta} (\gamma_\perp u_{N}^{\perp})$$

$$+ \frac{1}{2} A_{1}^{N} (\gamma \gamma_{5} C)_{\alpha \beta} (u_{N}^{+}) + \frac{1}{2} A_{2}^{N} (\gamma \gamma_{5} C)_{\alpha \beta} (u_{N}^{-}) + \frac{1}{2} A_{3}^{N} (\gamma \gamma_{5} C)_{\alpha \beta} (u_{N}^{\perp})$$

$$+ \frac{1}{2} T_{1}^{N} (i \gamma_{\perp} C)_{\alpha \beta} (\gamma_\perp u_{N}^{\perp}) + \frac{1}{2} T_{2}^{N} (i \gamma_{\perp} C)_{\alpha \beta} (\gamma_\perp u_{N}^{\perp}) + \frac{1}{2} T_{3}^{N} (i \gamma_{\perp} C)_{\alpha \beta} (\gamma_\perp u_{N}^{\perp})$$

$$+ \frac{1}{2} T_{4}^{N} (i \gamma_{\perp} C)_{\alpha \beta} (\gamma_\perp u_{N}^{\perp}) + \frac{1}{2} T_{5}^{N} (i \gamma_{\perp} C)_{\alpha \beta} (\gamma_\perp u_{N}^{\perp}) + \frac{1}{2} T_{6}^{N} (i \gamma_{\perp} C)_{\alpha \beta} (\gamma_\perp u_{N}^{\perp})$$

$$+ \frac{1}{2} m_{N} T_{7}^{N} (i \gamma_{\perp} C)_{\alpha \beta} (\gamma_\perp u_{N}^{\perp}) + \frac{1}{2} m_{N} T_{8}^{N} (i \gamma_{\perp} C)_{\alpha \beta} (\gamma_\perp u_{N}^{\perp}) , \tag{A.6}$$

In this expression $\alpha$, $\beta$, $\gamma$ are spinor indices, $n_{\mu}$ is an auxiliary light-like vector, $n^{\perp} = 0$,

$$P_{\mu} = P_{\mu} - \frac{m_{N}^{2}}{2} n_{\mu} , \quad p^{2} = 0 , \tag{A.7}$$

where $P_{\mu}$ is the proton momentum, $p^{2} = m_{N}^{2}$. Further, $u_{N}^{\perp} = \Lambda_{\perp} u_{N}(P, \lambda)$ where $u_{N}(P, \lambda)$ is the usual Dirac spinor in relativistic normalization, the projectors are defined as

$$\Lambda_{+} = \frac{\gamma_{\mu} n_{\mu}}{2 p_{n}} , \quad \Lambda_{-} = \frac{\gamma_{\mu} n_{\mu}}{2 p_{n}} ,$$

$$g_{\mu \nu}^{\perp} = g_{\mu \nu} - \frac{p_{\mu} n_{\nu} + p_{\nu} n_{\mu}}{p_{n}} \tag{A.8}$$

and $C$ is the charge-conjugation matrix. We use a shorthand notation $\sigma_{\perp \perp} \otimes \gamma_{\perp} = \sigma_{\perp \perp} n^{\mu} g_{\mu \nu}^{\perp} \otimes \gamma_{\alpha}$, etc. The invariant functions $F = V_{1}, A_{1}, T_{1}$ correspond to contributions of a given collinear twist and can be written as Fourier integrals

$$F(a_{j}, p_{n}) = \int [dx] e^{-i(p_{n})\sum_{i} x_{i} a_{i}} F(x_{i}) \tag{A.9}$$

where $F(x_{i})$ depend on the three valence quark momentum fractions $x_{i}$.

Using various symmetries these functions can be combined in 8 independent light-cone DAs \cite{30}. There exists a single DA for the leading twist-three \cite{33}

$$\langle 0| i^{ijk} \left( u_{i}^{+} (a_{1} n) C \gamma_{5} \gamma_{j} (a_{2} n) \right) \not{d}_{k}^{+} (a_{3} n) |N(P, \lambda)\rangle$$

$$= - \frac{1}{2} F_{n}(p_{n}) \not{d}_{k}^{+} (P) e^{-i(p_{n})\sum_{i} x_{i} a_{i}} \varphi_{N}(x_{i}) \tag{A.10}$$

such that \cite{37}

$$V_{1}(1, 2, 3) = \frac{1}{2} F_{2} \left[ \varphi_{N}(1, 2, 3) + \varphi_{N}(2, 1, 3) \right] ,$$

$$A_{1}(1, 2, 3) = \frac{1}{2} F_{2} \left[ \varphi_{N}(2, 1, 3) - \varphi_{N}(1, 2, 3) \right] ,$$

$$T_{1}(1, 2, 3) = \frac{1}{2} F_{2} \left[ \varphi_{N}(1, 2, 3) + \varphi_{N}(2, 3, 1) \right] . \tag{A.11}$$
and for twist-four there are three independent DAs [36]

\[ \langle 0 | e^{ijk} (u^+_1 (a_1 n) C \phi u^+_2 (a_2 n)) \gamma^\perp \gamma^\perp d^\perp_k (a_3 n) | N, i \rangle \]
\[ = - \frac{1}{4} (pm) \phi u^+_N (P) \int [dx] e^{-i (pn) \sum x_i a_i} \]
\[ \times \left[ f_N \Phi^N_{i,WW} (x_i) + \lambda^N_1 \Phi^N_1 (x_i) \right], \quad (A.12) \]

\[ \langle 0 | e^{ijk} (u^+_1 (a_1 n) C \phi u^+_2 (a_2 n)) \gamma^\perp \gamma^\perp d^\perp_k (a_3 n) | N, i \rangle \]
\[ = - \frac{1}{2} (pm) \phi m_N u^+_N (P) \int [dx] e^{-i (pn) \sum x_i a_i} \]
\[ \times \left[ f_N \Psi^N_{i,WW} (x_i) - \lambda^N_1 \Psi^N_1 (x_i) \right], \quad (A.13) \]

\[ \langle 0 | e^{ijk} (u^+_1 (a_1 n) C \phi u^+_2 (a_2 n)) \gamma^\perp \gamma^\perp d^\perp_k (a_3 n) | N, i \rangle \]
\[ = \frac{\lambda^N_2}{12} (pm) \phi m_N u^+_N (P) \int [dx] e^{-i (pn) \sum x_i a_i, \Xi^N (x_i), \quad (A.14) \]

\[
\begin{align*}
   f_N \varphi_N (x_1, x_2, x_3) &= -4 \sqrt{6} \int \left[ dk_1 x_3 m_N \right] \psi^{(0)}_{N/3} (1, 2, 3), \\
   \left[ \lambda^N_1 \Phi^N_1 + f_N \Phi^N_{i,WW} \right] (x_2, x_1, x_3) &= -8 \sqrt{6} \int \left[ \frac{dk_1}{x_3 m_N} \right] k^\perp_1 \cdot \left[ \bar{k}^\perp_1 \psi^{(1)}_{N/3} + \bar{k}^\perp_2 \psi^{(1)}_{N/2} \right] (1, 2, 3), \\
   \left[ \lambda^N_1 \Psi^N_1 - f_N \Psi^N_{i,WW} \right] (x_2, x_1, x_3) &= -8 \sqrt{6} \int \left[ \frac{dk_1}{x_3 m_N} \right] k^\perp_1 \cdot \left[ k^\perp_1 \psi^{(1)}_{N/1} + k^\perp_2 \psi^{(1)}_{N/2} \right] (1, 2, 3), \\
   \lambda^N_1 \Xi^N (x_1, x_2, x_3) &= -24 \sqrt{6} \int \left[ \frac{dk_1}{x_3 m_N} \right] k^\perp_1 \cdot \left[ k^\perp_1 \left( \psi^{(-1)}_N (1, 3, 2) - \psi^{(-1)}_N (1, 2, 3) \right) \right. \\
   & \quad \left. + \bar{k}^\perp_2 \left( \psi^{(-1)}_N (2, 3, 1) - \psi^{(-1)}_N (2, 1, 3) \right) \right] \quad (A.16)
\end{align*}
\]

so that DAs correspond to integrals over the light-front wave functions over transverse momenta, with some prefactors. One has to have in mind that these relations are somewhat schematic since transverse momentum integrals on the right-hand side (r.h.s.) are divergent and have to be regulated e.g. introducing a cutoff. In turn, the DAs are usually defined using dimensional regularization and the MS-subtraction so that a matching coefficient can be necessary. Also the wave function renormalization factors have to be added for the quark fields. The twist-four DAs include additional contributions from the four-particle Fock states with an extra gluon [28, 34]. If these contributions are taken into account, the four-particle quark-gluon nucleon DAs have to be added as well [28, 33].

The complete set of nucleon DAs carries the full information on the nucleon structure, in the same manner as the complete basis of light-front wave functions. In practice, however, both expansions have to be truncated and the usefulness of a truncated version, taking into account either the first few Fock states or a few lowest twist contributions, may depend on the concrete physics application.

The classification of the three-quark nucleon light-front wave functions in Eq. (A.2) can be overtaken for the negative parity isospin-1/2 resonances, e.g. N* (1535), without modification. The symmetry under parity transformation does not constrain the light-front wave functions but affects the relation between the wave functions of the states with opposite helicity in terms of the helicity-flipped quarks. The corresponding expressions can be worked out using the Jacobi-Wick transformation [41]

\[ \psi (N, \lambda) = \eta_N (-1)^{1/2-\lambda} (N, -\lambda) \quad (A.17) \]

where \( \tilde{\psi} \) is the parity transformation followed by a 180° rotation along the y-axis, and \( \eta_N \) is internal parity, \( \eta_N = 1 \) for the nucleon and \( \eta_N = -1 \) for \( N^* (1535) \). Thus \( k^\perp \tilde{\psi} \rightarrow -k^\perp \), \( |N, \uparrow \rangle \rightarrow \eta_N |N, \downarrow \rangle \), whereas for the quark states \( q = u, d \)
\[ b_{q_\uparrow}^{i | 0} \overset{0}{\rightarrow} b_{q_\downarrow}^{i | 0} \text{, but } b_{q_\downarrow}^{i | 0} \overset{0}{\rightarrow} -b_{q_\uparrow}^{i | 0} \]. Applying this transformation to both sides of Eq. (A.2) one obtains \[ 33 \]

\[ |N_\downarrow\rangle_{\text{ud}}^{\ell_z=0} = -\eta N \frac{e_{ijk}}{\sqrt{6}} \int \frac{[dx][dk_1]}{x_1 x_2 x_3} \left[ \psi_{N;3}^{(0)}(1, 2, 3) + i e^{\alpha \beta} \tilde{k}_{1 \alpha} \tilde{k}_{2 \beta} \psi_{N;2}^{(0)}(1, 2, 3) \right] b_{\alpha \beta}^{i}(1) \left( b_{d_1}^{i}(2) b_{d_2}^{i}(3) - b_{d_2}^{i}(1) b_{d_1}^{i}(3) \right) |0\rangle, \]

\[ |N_\downarrow\rangle_{\text{ud}}^{\ell_z=-1} = \eta N \frac{e_{ijk}}{\sqrt{6}} \int \frac{[dx][dk_1]}{x_1 x_2 x_3} \left[ \tilde{k}_{1 \beta} \psi_{N!;1}^{(1)}(1, 2, 3) + \tilde{k}_{2 \beta} \psi_{N!;2}^{(1)}(1, 2, 3) \right] b_{\alpha \beta}^{i}(1) \left( b_{u_1}^{i}(2) b_{u_2}^{i}(3) - b_{u_2}^{i}(1) b_{u_1}^{i}(3) \right) |0\rangle, \]

\[ |N_\downarrow\rangle_{\text{ud}}^{\ell_z=1} = \eta N \frac{e_{ijk}}{\sqrt{6}} \int \frac{[dx][dk_1]}{x_1 x_2 x_3} \left[ \tilde{k}_{1 \alpha} \tilde{k}_{2 \beta} \psi_{N!}^{(-1)}(1, 2, 3) \right] b_{\alpha \beta}^{i}(1) \left( b_{u_1}^{i}(2) b_{d_2}^{i}(3) - b_{d_2}^{i}(2) b_{u_1}^{i}(3) \right) |0\rangle, \]

\[ |N_\downarrow\rangle_{\text{ud}}^{\ell_z=-2} = -\eta N \frac{e_{ijk}}{\sqrt{6}} \int \frac{[dx][dk_1]}{x_1 x_2 x_3} \left[ \tilde{k}_{1 \beta} \tilde{k}_{2 \alpha} \psi_{N}^{(-2)}(1, 2, 3) \right] b_{\alpha \beta}^{i}(1) \left( b_{u_1}^{i}(2) b_{d_2}^{i}(3) - b_{d_2}^{i}(2) b_{u_1}^{i}(3) \right) |0\rangle. \] (A.18)

so that, e.g., for the $\ell_z = 0$ states

\[ \psi_{N;3}^{(0)}(1, 2, 3) \big|_{\text{ud}}^{\ell_z=0} = -\eta N \psi_{N;3}^{(0)}(1, 2, 3) \big|_{\text{ud}}^{\ell_z=0} \quad (A.19) \]

A Lorentz-covariant definition of the DA's of negative parity resonances involves some freedom. It is convenient to choose the definition in such a way that the coefficients functions in the OPE of currents \[ 4 \] are the same for states of both parities, and also the relations between different DA's imposed by QCD equations of motion remain the same. As noticed in Ref. (28), this can be achieved using invariant decomposition of the $N^*$ (1535) matrix element in terms of the $\gamma_5$-rotated quark fields

\[ 4(\gamma_5)_{\alpha \alpha'}(\gamma_5)_{\beta \beta'}(\gamma_5)_{\gamma' \gamma}(0) e^{ijk} u_{\alpha'}(a_1 n) u_{\beta'}(a_2 n) d_{\gamma'}(a_3 n) |N^*(P, \lambda)\rangle = S_{1}^{N^*} m_N C_{\alpha \beta} (\gamma_5 u_{\gamma'} N^*) + \ldots \] (A.20)

where the expression on the right hand side is the same as in Eq. (A.6) with obvious replacements $m_N \rightarrow m_{N^*}$ etc. Projecting out the $\gamma_5$ matrices we obtain

\[ 4(0) e^{ijk} u_{\alpha}(a_1 n) u_{\beta}(a_2 n) d_{\gamma}(a_3 n) |N^*(P, \lambda)\rangle = \]

\[ = S_{1}^{N^*} m_N C_{\alpha \beta} (u_{\gamma'} N^*) + S_{2}^{N^*} m_N C_{\alpha \beta} (u_{\gamma'} N^*) + P_{1}^{N^*} m_N C_{\alpha \beta} (\gamma_5 u_{\gamma'} N^*) + P_{2}^{N^*} m_N C_{\alpha \beta} (\gamma_5 u_{\gamma'} N^*) - V_{1}^{N^*} (\gamma \alpha \beta) (u_{\gamma'} \gamma) \gamma + \frac{1}{2} V_{2}^{N^*} (\gamma \alpha \beta) (u_{\gamma'} \gamma) \gamma + \frac{1}{2} V_{3}^{N^*} (\gamma \alpha \beta) (u_{\gamma'} \gamma) \gamma + V_{4}^{N^*} m_N C_{\alpha \beta} (\gamma_5 u_{\gamma'} N^*) - A_{1}^{N^*} (\gamma \alpha \beta) (u_{\gamma'} \gamma) \gamma \gamma + A_{2}^{N^*} (\gamma \alpha \beta) (u_{\gamma'} \gamma) \gamma \gamma + \frac{1}{2} A_{3}^{N^*} m_N C_{\alpha \beta} (\gamma_5 u_{\gamma'} \gamma) \gamma \gamma + T_{1}^{N^*} (i \sigma_\perp P C) (u_{\gamma'} \gamma) \gamma \gamma + T_{2}^{N^*} (i \sigma_\perp P C) (u_{\gamma'} \gamma) \gamma \gamma + T_{3}^{N^*} m_N (\gamma_5 u_{\gamma'} \gamma) \gamma \gamma + \frac{1}{2} m_N T_{4}^{N^*} (\gamma \alpha \beta) (u_{\gamma'} \gamma) \gamma \gamma + T_{5}^{N^*} m_N T_{6}^{N^*} (\gamma \alpha \beta) (u_{\gamma'} \gamma) \gamma \gamma + \frac{1}{2} m_N T_{7}^{N^*} (\gamma \alpha \beta) (u_{\gamma'} \gamma) \gamma \gamma, \] (A.21)

This expression replaces the decomposition (A.6) for the nucleon. Note that there are some minus signs and in particular all three leading twist DA's $V_1$, $A_1$ and $T_1$ are defined in our convention with a different sign as compared to the nucleon. As a consequence in the definition of leading-twist DA in terms of the chiral quark fields there is a minus sign as com-
pared to (A.10).

\[
(0|e^{ijk}(u_i^1(a_{1n})C\gamma_\mu u_j^2(a_{2n}))\gamma^\mu d_k^I(a_{3n})|N^*(P)) = \frac{1}{2} f_{N^*}(pm) \delta u_N^i(P) \int [dx] e^{-i(pm) \cdot \Sigma x, \alpha_i} \varphi_{N^*}(x_i),
\]

(A.22)

where, of course, \( P^2 = m_{N^*}^2 \), and the expressions for the invariant functions \( V_1^{N^*}, A_1^{N^*}, T^{N^*} \) in terms of \( \varphi_{N^*} \), are the same as for the nucleon, Eq. (A.11).

The twist-four DAs also acquire some signs \(^{[26]}\)

\[
(0|e^{ijk}(u_i^1(a_{1n})C\gamma_\mu u_j^2(a_{2n}))\gamma^\mu d_k^I(a_{3n})|N^*(P)) = \frac{1}{4} \delta u_N^i(P) \int [dx] e^{-i(pm) \cdot \Sigma x, \alpha_i} \times \left[ f_{N^*}\Phi_{N^*}^{WW}(x_i) + \lambda_N^i \Phi_{N^*}^N(x_i) \right],
\]

(A.23)

\[
\langle 0|e^{ijk}(u_i^1(a_{1n})C\gamma_\mu u_j^2(a_{2n}))\gamma^\mu d_k^I(a_{3n})|N^*(P) \rangle = -\frac{1}{4} \delta u_N^i(P) \int [dx] e^{-i(pm) \cdot \Sigma x, \alpha_i} \times \left[ f_{N^*}\Phi_{N^*}^{WW}(x_i) + \lambda_N^i \Phi_{N^*}^N(x_i) \right],
\]

(A.24)

\[
\langle 0|e^{ijk}(u_i^1(a_{1n})C\gamma_\mu u_j^2(a_{2n}))\gamma^\mu d_k^I(a_{3n})|N^*(P) \rangle = \frac{\lambda_N^i}{12} \delta u_N^i(P) \int [dx] e^{-i(pm) \cdot \Sigma x, \alpha_i} \Xi_{N^*}^N(x_i),
\]

(A.25)

where \( \Phi_{N^*}^{WW}(x_i) \) and \( \Psi_{N^*}^{WW}(x_i) \) are given by the same expressions in terms of the expansion of the leading-twist DA \( \varphi_{N^*}(x_i) \) as for the nucleon.

The price to pay for universality of correlation functions for positive and negative parities is that the relations between DAs and light-front wave functions in this convention acquire some signs as well,

\[
\begin{align*}
\varphi_{N^*}(x_1, x_2, x_3) &= +4\sqrt{\frac{3}{6}} \int \frac{dk_{\perp}}{x_1m_{N^*}} \psi_{N^*}^{(0)}(x_1, 1, 2, 3), \\
[\lambda_{N^*}^i \Phi_{N^*}^{WW} + f_N \Phi_{N^*}^{WW}](x_2, x_1, x_3) &= +8\sqrt{\frac{1}{6}} \left[ \frac{dk_{\perp}}{x_2m_{N^*}} \right] k_{\perp}^2 \cdot \left[ 2 \psi_{N^*}^{(1)}(x_2, 1, 2, 3) \right] - \left[ \frac{dk_{\perp}}{x_3m_{N^*}} \right] k_{\perp}^2 \cdot \left[ 2 \psi_{N^*}^{(1)}(x_3, 1, 2, 3) \right], \\
[\lambda_{N^*}^i \Psi_{N^*}^{WW} - f_N \Psi_{N^*}^{WW}](x_1, x_2, x_3) &= -8\sqrt{\frac{1}{6}} \left[ \frac{dk_{\perp}}{x_1m_{N^*}} \right] k_{\perp}^2 \cdot \left[ 2 \psi_{N^*}^{(1)}(x_1, 3, 2) - \psi_{N^*}^{(1)}(1, 2, 3) \right] + \left[ \frac{dk_{\perp}}{x_3m_{N^*}} \right] k_{\perp}^2 \cdot \left[ 2 \psi_{N^*}^{(1)}(2, 3, 1) - \psi_{N^*}^{(1)}(2, 1, 3) \right].
\end{align*}
\]

(A.26)

that have to be taken into account for the interpretation of the results.

Parametrization of the DAs of the resonances can be over- 

taken from that for the nucleon. The leading-twist DA \( \varphi_{N^*}(x_i, \mu) \) can be expanded in the set of orthogonal polynomials \( P_{nk}(x_i) \)

\[
\varphi_{N^*}(x_i, \mu) = 120x_1x_2x_3 \sum_{n=0}^{\infty} \sum_{k=0}^{n} \varphi_{nk}(\mu) P_{nk}(x_i),
\]

(A.27)

\[
\int [dx] x_1x_2x_3 P_{nk}(x_i) P_{nk'}(x_i) \propto \delta_{nn'} \delta_{kk'},
\]

such that the coefficients are renormalized multiplicatively to one-loop accuracy,

\[
\begin{align*}
f_{N^*}(\mu) &= f_{N^*}(\mu_0) \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{2/(3\beta_0)},
\varphi_{nk}(\mu) &= \varphi_{nk}(\mu_0) \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\gamma_{nk}/\beta_0}.
\end{align*}
\]

(A.28)

Here \( \beta_0 = 11 - \frac{2}{3}n_f \) is the first coefficient of the QCD \( \beta \)-

function and \( \gamma_{nk} \) are the anomalous dimensions. The double sum in Eq. (A.27) goes over a complete set of orthogonal polynomials \( P_{nk}(x_i), k = 0, \ldots, n, \) of degree \( n \):

\[
\begin{align*}
P_{00} &= 1, \\
P_{10} &= 21(x_1 - x_3), \\
P_{11} &= 7(x_1 - 2x_2 + x_3), \\
P_{20} &= \frac{63}{10}[3(x_1 - x_3)^2 - 3x_2(x_1 + x_3) + 2x_2^2], \\
P_{21} &= \frac{63}{2}(x_1 - 3x_2 + x_3)(x_1 - x_3), \\
P_{22} &= \frac{9}{5}[x_1^2 + 9x_2(x_1 + x_3) - 12x_1x_3 - 6x_2^2 + x_3^2].
\end{align*}
\]

(A.29)

e etc., and the corresponding anomalous dimensions are

\[
\begin{align*}
\gamma_{00} &= 0, \\
\gamma_{10} &= \frac{20}{9}, \\
\gamma_{11} &= \frac{8}{3}, \\
\gamma_{20} &= \frac{32}{9}, \\
\gamma_{21} &= \frac{40}{9}, \\
\gamma_{22} &= \frac{14}{3}.
\end{align*}
\]

(A.30)
The normalization condition (A.13) implies that $\varphi_{00} = 1$. In the main text we refer to the coefficients $\varphi_{nk}(\mu_0)$ with $n = 1, 2, \ldots$, as shape parameters. The set of these coefficients together with the normalization constant $f_N(\mu_0)$ at a reference scale $\mu_0$ specifies the momentum fraction distribution of valence quarks on the nucleon. They are related to matrix elements of local gauge-invariant three-quark operators and can be calculated, e.g., on the lattice [24, 26].

The twist-four DAs can be parameterized as [38]

$$
\Phi_4^{N^*}(x_i, \mu) = 24x_1x_2 \left\{ 1 + \eta_{10}(\mu) R_{10}(x_3, x_1, x_2) - \eta_{11}(\mu) R_{11}(x_3, x_1, x_2) \right\},
$$

$$
\Psi_4^{N^*}(x_i, \mu) = 24x_1x_3 \left\{ 1 + \eta_{10}(\mu) R_{10}(x_2, x_3, x_1) + \eta_{11}(\mu) R_{11}(x_2, x_3, x_1) \right\},
$$

$$
\Xi_4^{N^*}(x_i, \mu) = 24x_2x_3 \left\{ 1 + \frac{9}{4} \xi_{10}(\mu) R_{10}(x_1, x_3, x_2) \right\},$

(A.31)

where

$$
R_{10}(x_1, x_2, x_3) = 4 \left( x_1 + x_2 - \frac{3}{2} x_3 \right),
$$

$$
R_{11}(x_1, x_2, x_3) = \frac{20}{3} \left( x_1 - x_2 + \frac{1}{2} x_3 \right)
$$

and $\eta_{10}(\mu), \eta_{11}(\mu), \xi_{10}(\mu)$ are the new shape parameters. The corresponding one-loop anomalous dimensions are [38]

$$
\gamma_{10}^{(q)} = \frac{20}{9}, \quad \gamma_{11}^{(q)} = 4, \quad \gamma_{10}^{(\xi)} = \frac{10}{3}.
$$

(A.33)

For the twist-five DAs we take into account contributions of geometric twist-three and twist-four operators as explained in Ref. [23].

Note that the asymptotic DAs (at very large scales) for the nucleon and the resonances are the same:

$$
\varphi_{as}(x_i) = 120x_1x_2x_3, \quad \Phi_4^{as}(x_i) = 24x_1x_2,
$$

$$
\Phi_4^{WW,as}(x_i) = 24x_1x_2(1 + \frac{2}{3}(1 - 5x_3)),
$$

$$
\Psi_4^{WW,as}(x_i) = 24x_1x_3(1 + \frac{2}{3}(1 - 5x_2)),
$$

$$
\Xi_4(x_i) = 24x_2x_3, \quad \Psi_4^{as}(x_i) = 24x_1x_3.
$$

(A.34)

For completeness we also give here the definitions of the normalization constants in terms of matrix elements of local three-quark operators:

$$(0|\epsilon^{ijk}(u_iC\gamma_{\mu}u_j)(0)\gamma_5^{\mu}d_k(0)|N^*(P)) = f_N(\mu_0)\gamma_5^{\mu}u_{N^*}(P),$$

$$(0|\epsilon^{ijk}(u_iC\gamma_{\mu}u_j)(0)\gamma_5^{\mu}d_k(0)|N^*(p)) = \lambda_1^{N^*}m_{N^*}\gamma_5u_{N^*}(P),$$

$$(0|\epsilon^{ijk}(u_iC\sigma_{\mu\nu}u_j)(0)\gamma_5\sigma^{\mu\nu}d_k(0)|N^*(P)) = \lambda_2^{N^*}m_{N^*}\gamma_5u_{N^*}(P).$$

(A.35)

Appendix B: Parametrization of coefficient functions

For convenience we provide a simple parametrization for the coefficient functions $f_{1,2}^{nk}, g_{1,2}^{nk}$ appearing in [11], [12], for the range $2 < Q^2 < 12$ GeV$^2$:

$$
f_{1,2}^{nk}(Q^2) = D(Q^2) \sum_{p=0}^{4} b_{1,2}^{nk} \left( \frac{m_{N^*}^2}{Q^2} \right)^p,$$

$$
g_{1,2}^{nk}(Q^2) = D(Q^2) \sum_{p=0}^{4} a_{1,2}^{nk} \left( \frac{m_{N^*}^2}{Q^2} \right)^p,$$

(B.1)

where $D(Q^2)$ is the dipole form factor [13]. The coefficients $a_{1,2}^{nk}, a_{1,2}^{nk}, b_{1,2}^{nk}$ and $b_{1,2}^{nk}$ are collected in Table [11].

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| p   | $a_{p_{1,1}}^{00}$ | $a_{p_{1,2}}^{00}$ | $a_{p_{1,1}}^{11}$ | $b_{p_{1,1}}^{00}$ | $b_{p_{1,2}}^{10}$ | $b_{p_{1,1}}^{11}$ | $b_{p_{1,2}}^{21}$ | $b_{p_{1,1}}^{22}$ | $b_{p_{1,2}}^{22}$ |
|-----|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| 0   | 0.014791         | 0.251939         | 0.0256977        | 0.00716919       | 0.192078        | -0.0271761       | -0.340351       | -0.653521       | 0.00864077       |
| 1   | 0.773677         | -0.0864          | 0.646869         | -0.307557        | -1.94975        | -0.706607        | 4.61356         | 3.76777         | 0.112153         |
| 2   | -0.18913         | 5.62993          | 0.0535879        | -2.242848        | 0.246661        | 5.43478          | -1.81907        | -2.25019        | -0.258147        |
| 3   | 0.13253          | 0.13253          | 0.210365         | 0.763116         | 0.1675          | 2.84988          | 23.7579         | 19.1668         | 7.37946          |
| 4   | 0.763116         | 0.763116         | 0.1675           | 2.84988          | 23.7579         | 19.1668          | 7.37946         | 7.37946         | 7.37946          |

**Table II:** Coefficient functions in the LCSRs for $N^*(1535)$ production

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