On the Value of Retransmissions for Age of Information in Random Access Networks without Feedback

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Abstract—We focus on a slotted ALOHA system without feedback, in which nodes transmit time-stamped updates to a common gateway. Departing from the classical generate-at-will model, we assume that each transmitter may not always have fresh information to deliver, and tackle the fundamental question of whether sending stale packets can be beneficial from an age-of-information standpoint. Leaning on a signal-flow-graph analysis of Markov processes the study reveals that, when packets can be lost due to channel impairments, retransmissions can indeed lower AoI for low generation rates of new information, although at a cost in terms of throughput.

I. INTRODUCTION

Age of information (AoI) has emerged as a key parameter for the design of internet of things (IoT) systems where nodes report the status of a monitored process to a central gateway [1]. Originally introduced in [2], the metric offers a measure of the freshness of the available knowledge, capturing the time elapsed since the generation of the last available update. In spite of its simplicity, AoI has been shown to effectively describe the fundamental trade-offs in many machine-type and cyber-physical applications [3], [4], and an increasing attention has recently been devoted to characterize its behavior in large-scale wireless networks.

In this perspective, first important results have been obtained focusing on grant-free (random) access medium sharing strategies [5] which are commonly employed in commercial IoT solutions [6]. Assuming a generate-at-will policy, in which a device can collect new information on the process right before sending a packet, the performance of ALOHA [7], [8] as well as of more advanced variations of the strategy [9]–[11] has been extensively studied. Moreover, significant improvements have been demonstrated in the presence of feedback, allowing each device to adapt its transmission probability to the AoI level [12], [13]. In all cases, for slotted ALOHA the access policy maximizing throughput also entails the minimum AoI.

Inspired by practical considerations, we depart from these assumptions and consider a slotted ALOHA system in which transmitters may not always have fresh information to deliver. The setting is especially relevant for applications in which an IoT device receives readings from a sensor whenever these become available, and cannot generate fresh data at will. Moreover, we focus on wireless links without feedback, commonly encountered in IoT networks, e.g. LoRaWAN [6], to reduce terminal complexity and energy consumption.

In this setup, we tackle the fundamental and open problem of whether sending a packet multiple times can be useful from an AoI perspective. In fact, the possibility for a device to perform re-transmissions when fresher information is not yet available triggers a non-trivial trade-off, aiming to increase the possibility for the update to be delivered and reduce AoI, yet injecting more traffic onto the random access channel. To characterize this, we focus on three access policies: a plain throughput maximization approach, a reactive strategy in which a device transmits only when it has fresh information, and a retransmission-based policy, where novel and stale packets are sent with different probabilities. We derive closed-form expressions for throughput and average AoI for all the solutions, and identify some fundamental insights. Specifically, we show that for a pure collision channel retransmissions are never convenient in terms of AoI. Conversely, when packets can be lost due to channel impairments (captured by a simple on-off fading model [14]), the transmission of stale information can be beneficial for sufficiently low generation rates of fresh data. In this case, we provide simple exact formulations that capture such region, as well as the access probabilities that optimize AoI. Notably, we prove that information freshness can be improved at the expense of throughput, revealing what, to the best of our knowledge, is the first example of such a trade-off in plain slotted ALOHA schemes.

II. SYSTEM MODEL AND PRELIMINARIES

We focus on a system in which $n$ devices (transmitters) share a wireless channel to send packets to a common destination (gateway). Time is slotted, and the duration of a slot allows transmission of a single packet. Each device receives input from a sensor, which monitors a process. Processes are assumed i.i.d., and a sensor provides to its device a new reading at the beginning of a slot with probability $\alpha$. The transmitter stores the update until the next one is generated, thereby having at any time a single packet ready for delivery to the gateway, containing the last produced sensor reading.

Medium access is performed via slotted ALOHA [15]. Specifically, at any slot a node sends a packet with probability $\pi$ if new sensor data has just been generated, whereas the transmission is performed with probability $\pi_s$ whenever the available reading is not fresh. No feedback is provided by the

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gateway, so that the behavior of a node does not depend on the outcome of previous transmissions.

Throughout our analysis, we consider a collision channel model with erasures. Accordingly, a sent packet reaches the gateway with probability $1 - \varepsilon$, or is completely erased with probability $\varepsilon \geq 0$, bringing no power contribution to the destination.\(^1\) The number of incoming data units at the gateway over a slot follows thus a binomial distribution of parameters $\left(n, (1 - \varepsilon) \rho\right)$, where

$$\rho := \alpha \pi_t + (1 - \alpha) \pi_s \quad (1)$$

summarizes the probability for a transmitter to access the channel. Collisions are regarded as destructive, so that no information can be retrieved from slots which see at the receiver the superposition of two or more unerased packets, while data units over singleton slots are always correctly decoded. Under these assumptions, a transmitter accessing the channel successfully delivers its update with probability

$$\omega = (1 - \varepsilon)(1 - \rho(1 - \varepsilon))^{n-1}.$$  

Accordingly, the aggregate throughput, i.e. the average number of packets decoded per slot,\(^2\) evaluates to

$$S = n\rho \omega \simeq n\rho(1 - \varepsilon) e^{-n\rho(1 - \varepsilon)} \quad (2)$$

where the approximation relies on the well-known limit

$$\lim_{n \to \infty} (1 - a/n)^{n-1} = e^{-a}$$

and becomes very tight already for moderately small values of $n$. For convenience, we shall refer to the quantity $n\rho(1 - \varepsilon)$, describing the average number of non-erased packets incoming at the receiver over a slot, as the channel load. Along this line, we recall that throughput is maximized at a load of $1$ [pkt/slot].

In this setting, we are interested in evaluating the ability of the medium access strategy to maintain an up-to-date view at the gateway of the status of the monitored processes. To this aim, we consider AoI as a proxy for information freshness. We assume each packet to have a time stamp denoting the generation of the update, at least equal to the one slot needed for transmission. The process $\Delta(t)$ is ergodic (see Remark 1), and we will focus in the remainder of our study on its stationary behavior, considering the average AoI for a device

$$\bar{\Delta} := \mathbb{E}\left[\Delta(t)\right].$$

**Notation:** Throughout our discussion, we refer to a random variable (r.v.) and its realizations with capital and lowercase letters, respectively. For a non-negative discrete r.v. $A$ with alphabet $\mathbb{N}^0$, we denote the probability mass function (PMF) as $p_A(a)$, and consider its probability generating function

$$G_A(x) := \mathbb{E}\left[x^A\right] = \sum_{a=0}^{\infty} p_A(a) x^a. \quad (3)$$

From the definition in (3), the first and second order moment of $A$ readily follow as [17]

$$\mathbb{E}[A] = G_A'(1), \quad \mathbb{E}[A^2] = G_A''(1) + G_A'(1) \quad (4)$$

where $G_A'(x)$ and $G_A''(x)$ denote the first and second order derivative of the probability generating function.

Finally, for a finite-state Markov chain (MC), we denote the one-step transition probability between states $i$ and $j$ as $q_{ij}$.

**Signal Flow Graphs:** The notion of signal flow graph (SFG), originally introduced by Shannon in 1942 [18], denotes a directed weighted graph in which each vertex $v$ is associated to a variable. The value taken by the variable is in turn given by the sum of the variables of all vertices emitting an edge towards $v$, each multiplied by the corresponding edge weight. As such, a SFG offers a simple graphical representation of a system of linear equations, and serves as basis for a number of useful mathematical tools (see, e.g., [19]). Among them, Mason’s gain formula [20] allows to derive the direct dependency between two variables – also known as transfer function – through a simple visual inspection of the graph, i.e., properly identifying paths and cycles connecting the vertices.

In the context of Markov processes, SFGs provide a convenient approach to compute the moments of recurrence times. In particular, given a finite-state MC, an associated SFG can be constructed mapping each state into a vertex, and adding a directed edge with weight $x q_{ij}$, where $x$ is a dummy variable, whenever a one-step transition between states $i$ and $j$ is possible. In this case, the transfer function of the SFG between a generic vertex $v$ and an absorbing one $w$ (i.e., a vertex with no outgoing edges), corresponds to the probability generating function of the r.v. describing the absorption time for the chain in state $w$ when starting from $v$ [19].

\(^1\) In spite of its simplicity, the model has been shown to capture well the fundamental trade-offs of slotted ALOHA channels, see, e.g. [16].

\(^2\) We note that this standard definition of throughput simply measures the channel utilization efficiency irrespective of the transmitted information. In other words, the metric is not influenced by whether the same packet is received multiple times due to retransmissions performed by a node.
III. AVERAGE AOI ANALYSIS

With reference to Fig. 1, let us denote by $Y$ the *inter-refresh* time, i.e., the number of slots that elapse between two successive refreshes of the AoI value for a transmitter of interest. It is important to remark that this interval does not generally coincide with the time separating two successive receptions at the gateway of packets from the device. Indeed, the AoI is refreshed only upon decoding of a status update containing a time stamp that is more recent than what already available. In other words, receiving retransmitted copies containing the same reading does not improve the perception of the tracked process. Let us further indicate by $Z$ the r.v. capturing the value of the current AoI right after a refresh has taken place.

Following this notation, focus on a generic time $t$, and denote by $Y(t)$ the duration of the inter-refresh period such observation instant falls into. Recalling that the AoI value grows linearly starting from the value at which it was last reset, under the condition $Z = z$, we can write $\Delta(t) = z + u$, where $u \in [0, Y(t)]$ is the time elapsed between the start of the inter-refresh period and $t$. Observe now that the probability for the observation time $t$ to belong to an interval of duration $y$ slots can be expressed as

$$P\{Y(t) = y\} = \frac{y p_Y(y)}{\sum y' p_Y(y')} = \frac{y p_Y(y)}{E[Y]}.$$  

Moreover, conditioned on $Y(t)$, the observation instant follows a uniform distribution within the inter-refresh interval. Combining these results, we have

$$E[\Delta(t) | Z = z] =$$

$$= \sum_{y=1}^{\infty} E[\Delta(t) | Z = z, Y(t) = y] P\{Y(t) = y\}$$

$$= \sum_{y=1}^{\infty} \int_{0}^{y} z + u \, du \cdot \frac{y p_Y(y)}{E[Y]} = z + \sum_{y=1}^{\infty} \frac{y^2 p_Y(y)}{2 E[Y]}.$$  

(5)

Finally, observing that $Y$ and $Z$ are independent, and removing the conditioning on the latter from (5), a compact expression for the average AoI is obtained in the form:

$$\bar{\Delta} = E[Z] + \frac{E[Y^2]}{2 E[Y]}.$$  

(6)

The characterization of $\bar{\Delta}$ requires then the calculation of the first and second order moments of two relevant stochastic processes.

*Average AoI refresh value, $E[Z]$*: Let us start by considering the statistics of $Z$, and focus first on a slot over which the AoI for the transmitter of interest is refreshed. In this case, two events can occur:

(a) the metric is reset to 1 slot. This is the case if the sender has a fresh update to send, accesses the channel, and the delivery succeeds, i.e., with probability $\alpha \pi_f \omega$;

(b) conversely, the instantaneous AoI is reset to $k > 1$ slots if and only if three conditions are met: i) a new reading was produced $k$ slots ago and fed to the transmitter, but not immediately delivered to the gateway (probability $\alpha (1 - \pi_f \omega)$); ii) $k - 2$ further slots have elapsed without a new readings being generated and without the latest update reaching the gateway (probability $[(1 - \alpha)(1 - \pi_s \omega)]^{k-2}$; and iii) no new reading is acquired in the current slot, but the stale one is transmitted and decoded (probability $(1 - \alpha)\pi_s \omega$).

Based on this, the overall probability to observe a reset of the AoI over a time slot, denoted for convenience as $\zeta$, is obtained by summing the contributions of the described events, leading to

$$\zeta = \alpha \omega \left( \pi_f + \sum_{k=2}^{\infty} \pi_s (1 - \alpha)^{k-1} (1 - \pi_f \omega)(1 - \pi_s \omega)^{k-2} \right)$$

$$= \alpha \omega p \cdot [\alpha + (1 - \alpha)\pi_s \omega]^{-1}.$$  

If we now recall that the PMF $p_Z(z)$ captures the probability for $\Delta(t)$ to be reset to $z$ given that a refresh has taken place, its expression follows by normalizing the probabilities of events (a) and (b) to $\zeta$:

$$p_Z(z) = \frac{\alpha \omega}{\zeta} \begin{cases} 
\pi_f & z = 1 \\
\pi_s (1 - \alpha)^{k-1} (1 - \pi_f \omega)(1 - \pi_s \omega)^{k-2} & z > 1
\end{cases}$$

Accordingly, the first order moment required for the calculation of (6) can be derived after simple algebraic manipulations:

$$E[Z] = \sum_{z=1}^{\infty} z p_Z(z) = 1 + \frac{\pi_s (1 - \alpha)(1 - \pi_f \omega)}{\rho [\alpha + (1 - \alpha)\pi_s \omega]}.$$  

(7)

*Statistical moments of $Y$*: Let us then focus on the statistics of the inter-refresh time, and consider the MC reported in Fig. 2, which usefully characterizes the behavior of a transmitter from an AoI perspective. Transitions take place at each slot, across three possible states. Specifically, the chain enters state $R$ whenever the transmitter delivers a packet that triggers a reset of the AoI value at the gateway. On the other hand, state $D$ is visited when the time-stamp of the process reading available at the node matches the one already delivered to the receiver. Note that, in these conditions, a successful packet transmission from the node to the gateway would not lead to an AoI refresh. Conversely, the MC transitions into state $F$ when the transmitter has been fed with fresher information, which would lead to a reset of $\Delta(t)$ if sent and successfully decoded. The transition probabilities among states can readily be computed for the presented system model. For example, the chain will remain in $R$ if a new reading is generated, sent and delivered over the slot, i.e. with probability $q_{R,R} = \alpha \pi_f \omega$. A jump from $R$ to $D$, instead, takes place with probability $q_{R,D} = 1 - \alpha$, corresponding to having no fresher information available at the transmitter compared to the receiver’s picture. Similarly, consider state $F$. From here, the chain moves into $R$ whenever an update is delivered from the node, i.e. with probability $q_{F,R} = \rho \omega$. Conversely, no jump to state $D$ is possible, since the transmitter does have innovative information for the gateway. The remaining transition probabilities are reported for completeness in Fig. 2.
Leaning on the presented MC, the inter-refresh time can be computed as the recurrence time for \( R \), i.e. the number of slots between two successive visits to the state. To this aim, recalling the discussion of Sec. II, we consider the SFG associated to the Markov process, reported in Fig. 3. For convenience, we split \( R \) into two states: \( R' \) (with only outgoing transitions from \( R \)) and \( R'' \) (collecting all of \( R \)'s incoming edges). In this configuration, the transfer function between the two states captures the probability generating function of the absorption time into \( R'' \) when starting from \( R' \), which corresponds exactly to the sought inter-refresh period. A direct application of Mason’s gain formula [20] provides then the generating function
\[
G_Y(x) = \frac{\alpha \pi f \omega \cdot x - \alpha \omega (\pi f - \rho) \cdot x^2}{1 - (2 - \alpha - \rho \omega) \cdot x + (1 - \alpha)(1 - \rho \omega) \cdot x^2}.
\]

In turn, the first and second order moments of the r.v. \( Y \) can be derived computing the derivatives of \( G_Y(x) \) as highlighted in (4), leading after few simple manipulations to the expression
\[
\frac{\mathbb{E}[Y^2]}{2\mathbb{E}[Y]} = \frac{n}{5} + \frac{1}{\alpha} - \frac{1}{2} - \frac{1}{\alpha + (1 - \alpha) \pi f \omega}.
\]  

Finally, plugging (7) and (8) into (6) leads to a simple closed form formulation of the average AoI for a transmitter:
\[
\bar{\Delta} = \frac{1}{2} + \frac{n}{5} + \frac{1}{\alpha} - \frac{\pi f}{\rho}.
\]  

Remark 1 (Ergodicity of \( \Delta(t) \)): Consider the imbedded MC capturing the current AoI at start of each slot. For this process, the transition probabilities follow from the system model, and the chain can easily be shown to be aperiodic and irreducible. Observe now that at each slot the AoI is reset to its minimum value with probability \( \alpha \pi f \omega \). The recurrence time for state 1 of the MC is thus geometric distributed, and has mean value \( 1/(\alpha \pi f \omega) > 0 \). Recalling that this is also the reciprocal of the stationary probability for that state, the chain admits a proper limiting distribution, and is thus ergodic.

IV. MINIMUM AOI UNDER DIFFERENT ACCESS POLICIES

The compact expression of the average AoI derived in (9) conveniently allows to capture the behavior of different transmission strategies. To delve into the role of retransmissions, we tackle in particular three relevant policies, all of which operate without feedback from the gateway.

A. Plain throughput optimization

As a starting point, let us focus on a plain slotted ALOHA scheme, which foresees nodes to access the channel at each slot with probability \( \pi_t \) to send the buffered packet. In other words, transmitters operate agnostically of the freshness of the available reading, using the same probability to perform the first delivery attempt of an update as well as to retransmit it over successive slots until a new one is generated. Note that such an approach represents a benchmark of practical relevance, epitomizing the typical operation of a random access system which does not cope with AoI.

In this case, \( \pi_f = \pi_s = \pi \), and \( \rho = \pi \), so that, from (9), \( \bar{\Delta} = n/S + 1/\alpha - 1/2 \). Accordingly, for any generation rate \( \alpha \), the access probability that maximizes throughput also optimizes \( \bar{\Delta} \). The minimum AoI follows then by setting \( \pi = [n/(1 - \varepsilon)]^{-1} \), obtaining via (2)
\[
\bar{\Delta}^* \simeq n \varepsilon + \frac{1}{\alpha} - \frac{1}{2}.
\]  

B. Reactive policy

A second relevant strategy foresees a node operate in a reactive fashion, accessing the channel with probability \( \pi_t \) only when new information is available. In our setting, this corresponds to setting \( \pi_s = 0 \), so that \( \rho = \alpha \pi_t \). This approach ensures that any packet decoded at the gateway is innovative, and triggers a refresh of the AoI. On the other hand, in the absence of retransmissions, the loss of a message (either due to an erasure or a collision) results in a more stale perception of the monitored process at the receiver. Incidentally, we note that the reactive policy can also be seen as a slotted ALOHA system with generate-at-will traffic model, in which each transmitter attempts delivery of a fresh packet at each slot with probability \( \rho \).

Accordingly, the average AoI obtained from (9) takes the well-known form \( \bar{\Delta} = n/S + 1/2 \), see, e.g., [7], [9]. Also in this case, the optimal behavior in terms of AoI coincides then with throughput maximization, obtained by setting \( \pi_t = \min \{1, [n/(n - 1 - \varepsilon)]^{-1} \} \) and leading to a minimum average age
\[
\bar{\Delta}^* \simeq \frac{1}{2} + \begin{cases} \frac{ne}{\alpha} & \alpha > [n/(n - 1 - \varepsilon)]^{-1} \\ \frac{1}{\alpha} & \alpha \leq [n/(n - 1 - \varepsilon)]^{-1} \end{cases}.
\]  

Fig. 2. Markov chain to track the behavior of a transmitter. The inter-refresh time corresponds to the recurrence time of state \( R \).

Fig. 3. Signal flow graph corresponding to the Markov chain in Fig. 2.
C. Retransmission-based policy

Finally, let us consider a complete policy which allows a transmitter to send a packet multiple times, and tackle the fundamental question of whether, in the absence of feedback, retransmissions can be useful to reduce AoI.

To this aim, we start by observing how, (i) for any value of the overall transmission probability \( \rho \), the average age is minimized by picking the maximum possible value of \( \pi_t \). This is clearly highlighted in (9), where such a choice maximizes the absolute value of the AoI reduction term \( -\pi_t / \rho \), and reflects the benefit of being more aggressive in the transmission of fresh rather than stale information. Moreover, (9) shows that (ii) from an AoI perspective it is not convenient to operate the channel in congested conditions, i.e., with average channel load at the receiver larger than 1 [pkt/slot]. Indeed, it is easy to verify how the derivative of \( \Delta \) with respect to \( \rho \) is strictly negative for \( n \rho (1 - \varepsilon) > 1 \), as the negative effect of a drop in throughput more than counterbalances the age reduction brought by the addend \(-\pi_t / \rho\).

Combining these two remarks we can conclude that, whenever the overall generation rate of new updates suffices to saturate the channel, i.e., for \( \alpha > [n(1 - \varepsilon)]^{-1} \), no retransmissions shall be performed (\( \pi_s = 0 \)). In this regime the system behaves thus like the reactive policy, and \( \pi_t \) shall simply be set to maximize throughput, possibly dropping some newly generated packets.

Consider now the more interesting case \( \alpha \leq [n(1 - \varepsilon)]^{-1} \). From observation (i), a fresh update shall always be immediately transmitted, setting \( \pi_t = 1 \). Accordingly, taking the derivative of the average AoI with respect to the overall transmission probability \( \rho \), we get

\[
\frac{\partial \Delta}{\partial \rho} = \frac{1}{\rho^2} \left[ 1 + \left( n\rho - \frac{1}{1 - \varepsilon} \right) e^{\rho(1 - \varepsilon)} \right].
\]

For any value of \( \alpha \) and \( \varepsilon \), the function has a single zero for \( \rho \in [0, 1] \), corresponding to a minimum of \( \Delta \). Specifically, the optimal performance is obtained for an access probability

\[
\rho^* (\alpha, \varepsilon) = \frac{1 + W\left(-\left(1 - \alpha\right) e^{-1}\right)}{n(1 - \varepsilon)}
\]

(12)

where \( W(x) \) is the principal value of the Lambert W function, solving in \( w \) the equation \( we^w = x \geq 0 \). Finally, recalling the composition of \( \rho \) reported in (1), we infer that a node shall retransmit at each slot a stale packet with probability

\[\pi_s = \max\{0, (\rho^* - \alpha) / (1 - \alpha)\}\].

Summarizing, the optimal strategy is given as follows:

\[
\pi_t = \begin{cases} 
  1 & \alpha > [n(1 - \varepsilon)]^{-1} \\
  \frac{\alpha n(1 - \varepsilon)}{\alpha(1 - \varepsilon)} & \alpha \leq [n(1 - \varepsilon)]^{-1}
\end{cases}
\]

\[
\pi_s = \begin{cases} 
  0 & \alpha \geq \rho^* \\
  \frac{\rho^* - \alpha}{1 - \alpha} & \alpha < \rho^*
\end{cases}
\]

(13)

While the presented results lean on the approximation in (2), exact expressions can also be derived considering the binomial formulation of \( \omega \).

V. DISCUSSION AND RESULTS

The closed form result in (13) is reported graphically in Fig. 4, and calls for some interesting and non-trivial remarks.

Observation 1: In the absence of erasures (\( \varepsilon = 0 \)), i.e. for a pure collision channel, retransmissions are never convenient, irrespectively of the generation rate \( \alpha \). Mathematically, the result follows by noting from (12) that \( \lim_{\alpha \to 0} \rho^*(\varepsilon) = 0 \), so that, from (13), \( \pi_s = 0 \) for any \( \alpha \). More insightfully, this can be explained focusing on the low channel load region, where retransmissions might play a role. In this regime, the effect of collisions becomes negligible, and, in the absence of erasures, sent packets will be correctly received. Therefore, for \( \varepsilon = 0 \), the delivery of a fresh update suffices to reset the AoI value, and additional transmissions do not bring improvement.

Observation 2: Conversely, for any \( \varepsilon > 0 \), sending multiple copies of the same packet in the absence of feedback becomes convenient for sufficiently low generation rates. Specifically, a reduction of the average AoI is achieved by properly tuning the access probability \( \pi_s \) for any \( \alpha < \rho^* \). The traffic threshold for triggering retransmissions, captured by (12), solely depends on the number of nodes and on the erasure probability. Remarkably, even for very low generation rates (\( \alpha \to 0 \)), \( \pi_s \) shall never exceed a maximum value of \( \rho^* \). Such an insight is of practical relevance, clarifying how nodes monitoring processes with sporadic changes shall always transmit fresh information, and select instead the channel access probability when stale packets are available based on the \((n, \varepsilon)\) pair.

Observation 3: From (12) we also note that, when \( \alpha < \rho^* \), AoI is minimized by operating the system so that the receiver experiences a channel load \( n\rho^* (1 - \varepsilon) < 1 \). This condition, however, entails a loss in throughput with respect to the maximum achievable performance, obtained for a channel load at the receiver of 1 [pkt/slot]. This result is particularly interesting, as it pinpoints the existence of a trade-off between throughput and AoI. In fact, for low generation rates, maintaining an up-to-date perception of the monitored sources at the gateway comes at the expense of a less efficient utilization of the channel.

To further elaborate on these key trends we report some numerical results, obtained unless otherwise specified in the

\[\text{Fig. 4. Access probabilities } \pi_t \text{ and } \pi_s \text{ minimizing } \Delta \text{ for the retransmission-based policy, reported against } n\varepsilon. \text{ Results obtained for } n = 1000, \varepsilon = 0.25.\]
setting \( n = 1000, \varepsilon = 0.25 \). As a starting point, Fig. 5 shows, for the three considered strategies, the average AoI normalized to the network population (i.e., \( \bar{\Delta}/n \)) against the overall average number of new updates generated in a slot (i.e., \( n\alpha \)). Notably, for high generation rates both the reactive and the retransmission-based policies attain a significant reduction of AoI with respect to what achieved via plain throughput optimization. Mathematically, this is confirmed noting that, for \( \alpha > [n(1-\varepsilon)]^{-1} \), the value of \( \bar{\Delta} \) in (10) \((n\varepsilon + 1/\alpha - 1/2)\) is always larger than the one of (11) \((n\varepsilon + 1/2)\), with the two values converging for \( \alpha \to 1 \). This result pinpoint the potential of strategies that account for the freshness of available updates, and stems from the fact that the basic throughput optimization policy may unnecessarily flood the channel with transmissions that would not refresh AoI.

On the other hand, a profoundly different trend emerges when terminals generate new updates less frequently. In this region, Fig. 5 reveals that the reactive policy starts to suffer, performing worse than the benchmark throughput maximization approach (see highlight box in the plot). For low values of \( \alpha \), a one-shot transmission of newly generated information may indeed be exceedingly conservative, leading to long update-less periods at the gateway whenever a packet loss is undergone. Under these conditions, the sporadic transmission of stale information may become useful, triggering refreshes of the AoI value without the need to wait for new information to be generated. The benefits of the retransmission-based strategy are in turn apparent, as the presented optimization of the channel access probability properly balances the transmission rate based on the availability of fresh updates at each sender.

As noted earlier, the AoI improvements exhibited by the considered strategies entail however a cost in terms of achievable throughput. This fundamental trade-off is explored in Fig. 6, which reports \( S \) against the overall average number of new updates generated in a slot when the schemes are operated to minimize AoI. By construction, the throughput-optimal policy offers the peak performance of a slotted ALOHA scheme \((S \simeq 0.36\ [\text{pkt/slot}])\) regardless of \( \alpha \) (dash-dotted line). Conversely, the purely reactive strategy (dashed line) sees a steady throughput reduction as \( \alpha \) decreases. Note indeed that a more sporadic update generation has the system operate at low channel loads, simply not sharing the channel as efficiently as a slotted ALOHA access could allow throughput-wise. More interestingly, let us focus on the retransmission-based solution. As discussed, for \( \alpha > \rho^* \), the strategy behaves as the reactive one. Instead, for lower generation rates, transmission of non-new updates is possible, tuning the access probability so that the system operates at a load at the receiver of \( n\rho^*(1-\varepsilon) > 1 \). Recalling the definition of \( \rho^* \) in (12), the undergone throughput loss can be readily computed taking the ratio of the value of \( S \) attained for this load to the peak value \( e^{-1} \) and obtaining:

\[
1 + W\left(-(1-\varepsilon)e^{-1}\right) \cdot e^{-W\left(-(1-\varepsilon)e^{-1}\right)}.
\]

Notably, the loss is independent of the number of terminals in the system, and is only driven by the erasure probability \( \varepsilon \). For instance, for \( \varepsilon = 0.25 \) (see Fig. 6), the retransmission-based policy attains \( \sim 88\% \) of the maximum possible throughput for low \( \alpha \). From (14), the loss becomes even sharper for lower values of \( \varepsilon \), e.g., with half of the peak throughput sacrificed to improve AoI for \( \varepsilon \sim 0.04 \).

To conclude, let us explore the impact of the erasure probability. To capture this aspect, we report in Fig. 7 the ratios of \( \bar{\Delta}^* \) obtained with the reactive and the retransmission-based policies to the one of the throughput maximization approach. The trends are drawn once more against \( n\alpha \), and results are shown for different values of \( \varepsilon \). For a pure collision channel (\( \varepsilon = 0 \)) a single line is reported (dash-dotted), as both strategies behave in the same way. More relevantly, a reduction of AoI of up to 30\% can be obtained by avoiding transmission of stale information. On the other hand, when channel impairments start to play a role, the performance of a purely reactive approach can plummet for low update generation rates. For example, an AoI more than 30\% higher than the one achieved with a simple throughput optimization approach is experienced for \( \varepsilon = 0.25 \) and low \( \alpha \). Instead, the retransmission-based approach enables consistent

\[4\] This is captured by (2), since for the reactive policy we have \( \pi_t = 0 \), and, for \( \alpha \leq [n(1-\varepsilon)]^{-1} \), \( \pi_t = 1 \), so that \( S \simeq n\alpha(1-\varepsilon)e^{-n\alpha(1-\varepsilon)} \).
AoI reductions under all conditions. From this standpoint, the importance of leaning on, and carefully selecting the access probability for retransmissions clearly emerges.

VI. CONCLUSIONS

In this paper we investigated the role played by retransmissions in terms of age of information in an IoT random access network without feedback. Focusing on a setting in which devices monitoring processes of interest may not always have fresh information to deliver, we considered a practically ALOHA-based channel access and studied three distinct transmission policies. First, a commonly employed throughput optimization approach, which disregards the freshness of packets available at the transmitter side. Second, a reactive solution, foreseeing devices to access the channel only when new information is generated. Finally, we proposed a scheme where a node can adapt its transmission probability based on the availability or not of fresh updates. In all cases, exact closed form expressions for throughput and average AoI were obtained following a signal flow graph analysis of Markov processes. The study revealed that, for a pure collision channel, transmission of stale information is never convenient in terms of AoI. Conversely, when messages may be lost due to channel impairments, allowing a terminal to transmit a packet multiple times (thus attempting delivery of stale information) can indeed be beneficial for sufficiently low generation rates. The achievable benefits in terms of AoI can be remarkable, yet come at the expense of a loss in aggregate throughput.

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