Optimization Scheme of Hydraulic Fracturing Simulation Experiments using Mixed-level Uniform Design Method Based on the PKN model

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Abstract. This paper aims at revealing the hydraulic fracturing effect of hydrodynamic influencing factors of fracturing fluid by laboratory experiments. The experimental design for huge number of hydrodynamic influencing factors with many levels, is oftentimes incorporated with the conventional methods, i.e., Based on the PKN model, injection rate, viscosity, and density (of the fracturing fluid) and their parameter values (levels) were considered for the construction of an optimal mixed-level design table U* 12(4×6), which constructed the optimization scheme. The experiments results were subjected to multiple regression analysis to establish optimal fitting formulas for the relationship of single factor or multi-factor with rock fracturing value. Compared with other experimental designs, experimental number of U* 12(4×6) yielded only half the number of experiments in the orthogonal design L24(4×6) and presented better distribution uniformity of experimental points than U12(4×6). This paper provides a fast and effective experimental scheme for the effect analysis, as well as a substantially accurate method for in-situ stress measurement.

Keywords: Hydraulic Fracturing Simulation Experiments, Effect Analysis, Uniform Design Method, Mixed-Level

1 Introduction

Hydraulic fracturing is an important method for measurement of in-situ stress in the surrounding rock of coal mines, hydropower stations, bridges, and tunnels, and it is also an effective way for increase of petroleum and gas production [1-4]. This method employs a simple and rapid test process, as well as a simple analysis scheme of data processing, although several factors may affect the rock fracturing value during measurement [5-7]. The error is mainly derived from: (1) deformation of the drill pipe, drilling [8, 9] and the packer [10] in the hydraulic fracturing test system; (2) difference in eigenvalue determination method in data analysis [9]; (3) different types of fracturing fluid and their hydrodynamic factors [11-14].

A number of researchers have devoted their work for determining the influence of fluid mechanics factors of fracturing fluid on rock fracture. Ito and Hayashi [11] and Chang et al. [13] asserted that increasing the injection rate of the fracturing fluid with respect to flow rate, viscosity, and density, enhances the tensile strength of the rock. Zhou et al. [12] and Zhang [14] conducted hydraulic
fracturing chamber tests using different density mud media and found that the variation yields a significant impact on rock fracturing value. Matsunaga et al. [15] and Ishida et al. [16] carried out experiments and verified the impact of the fracturing fluid’s viscosity on rock fracturing value. Wang and Song [10] and Zhou et al. [12] used clear water as fracturing fluid to analyze the error of stress measurement caused by the effect of liquid compressibility on system flexibility. Javadpour et al. [17] calculated apparent liquid permeability (ALP) in the shale matrix, and used ALP to study the effects of slip-corrected matrix permeability on the induced fracture network.

In summary, the factors affecting the hydraulic fracturing effect mainly include flow velocity, viscosity, density, and compressibility of the fracturing fluids. However, due to the many influencing factors and their parameter values, and these experiments are destructive to the testing core, that is, the core may no longer be reusable with cracks, which infers a rather complicated and costly experimental design. As an experimental design for space filling, the uniform design method relatively describes a test point spread evenly throughout the test range, and thus, has been proposed from the perspective of uniformity [18-21]. In this work, we selected factors of the fracturing fluids, i.e., fluid pressure, density, and density, combined with horizontal pressure, to form a mixed-level uniform design table. The table is utilized for the conduct of hydraulic fracturing simulation experiments, for which the number of experiments is reduced under the premise of ensuring the effect of the experiments and of significantly improving the experiments efficiency.

2 Influencing Factors of Fracturing Fluids

The test borehole considered in the measurement of hydraulic fracturing in-situ stress was generally vertical borehole mainly affected by the maximum and minimum horizontal principal stresses. On this account, the fracturing crack was perpendicular to the minimum horizontal principal stress plane and thus, was a vertical crack [22, 23]. In this paper, the Perkins-Kern-Nordgren (PKN) classical mechanical model [24, 25] was employed for analyzing the influence of fluid mechanics on the fracturing crack and its fracturing pressure. Without regard to the compression properties of the fracturing fluid, Nordgren [25] derived the fluid’s continuity equation in the crack as:

\[
\frac{\partial q}{\partial x} + q \cdot \frac{\partial q}{\partial t} = 0
\]  

(1)

where \( q(x, t) \) is the volume flow of the fluid through the crack cross-section, \( q(x, t) \) is the fluid loss volume flow per unit crack length, and \( A(x, t) \) is the crack cross-sectional area. In the absence of fluid leakage, the crack length \( L \), the local crack width \( w \), and the pore pressure \( P_w \) are calculated as follows [25, 26]:

\[
L = 0.68 \left[ \frac{GQ^2}{(1-\nu)\mu^2} \right]^{1/4} t^{1/4}
\]  

(2)

\[
w = 2.5 \left[ \frac{(1-\nu)\mu Q}{Gh} \right]^{1/4} t^{1/4}
\]  

(3)

\[
P_w = 2.5 \left[ \frac{G\mu Q}{(1-\nu)h^2} \right]^{1/4} t^{1/4}
\]  

(4)

Where \( G \) is the rock shear modulus; \( \nu \) is the rock poisson ratio; \( h \) is the crack length; \( Q \) is the injection rate, and \( \mu \) is the fracturing fluid viscosity. It could be inferred that the hydrodynamic factors affecting the length and width of the crack were mainly the injection rate and viscosity of the fracturing fluid.

3 Optimization Design of Experimental Scheme

The simulation experiments herein referred to the hydraulic fracturing test process, in which the
hollow rock column test method was employed to simulate the hydraulic fracturing in-situ stress measurement process [7, 13, 27].

3.1 Hydraulic Fracturing Simulation Experimental Scheme

In the simulation experiments, the fracturing fluids used include clean water, hydraulic oil, CMC aqueous solution, and mud medium [28, 29], and the mud medium had density and viscosity that could be proportioned according to the experimental requirements. Results of the theoretical analysis of PKN model pointed that mud as the fracturing fluid medium in the simulation test necessitates three hydrodynamic factors, such as injection rate, density, and viscosity, and a factor of loading axial compression. The levels of each factor are shown in Table 1.

Table 1. Experimental factors and their levels

| Factor                  | Level | Unit   | Parameter value |
|-------------------------|-------|--------|-----------------|
| Injection rate          | 6     | MPa/s  | 0.02; 0.05; 0.09; 0.17; 0.35; 0.48 |
| Density                 | 4     | mPa/s  | 1.0; 1.1; 1.2; 1.3 |
| Viscosity               | 4     | g/cm³  | 70; 130; 170; 280 |
| Axial compression       | 4     | MPa    | 1.2; 2.4; 3.6; 4.8 |

3.2 Mixed-Level Uniform Design Table

Due to the huge number of parameters and their levels, the mixed-level uniform design was employed as the experimental scheme. Through the Data Processing System software [30], there were 12 experiments accommodated for 3 factors (fracturing fluid density, viscosity, and loading axial pressure) with 4 levels and 1 factor (injection rate) with 6 levels. The maximum number of iterations was set to 1000 for the constructed optimal mix-level uniform design table U* 12(4³×6) in Table 2.

From the table, the values under columns x₁, x₂, x₃, and x₄ were combined to obtain the discrepancy D, which represents the discrepancy of uniformity. Where x₁, ..., x₉ are n homogeneously dispersed points in Cⁿ; x = (x₁, ..., x₉) ∈ Cⁿ is a vector in the matrix; v(x) = x₁, ..., x₉ is the volume of the rectangle [0, x]; nᵢ ∈ (x₁, ..., x₉) is the number of points falling in [0, x]; nᵢ/n is the percentage of points falling in the rectangle [0, x] [30]. Here, a smaller D implies better uniformity of the experimental design [18, 19]. Manipulating Eq. (5) yields D* of 0.1713 for U* 12(4³×6).

Table 2. Table of values for the optimal mixed-level uniform experimental design U* 12(4³×6)
4 Analysis of Experimental Results

4.1 Single Factor Experimental Results

According to the viscosity experimental scheme in section 3.1, the CMC aqueous solutions with viscosities of 1, 70, 170 and 280 mPa·S were respectively subjected to fracturing simulation experiments, and the relationship between the fracture pressure and the viscosity of the aqueous solution was obtained. In Fig. 1, $\mu_i/\mu_1$ represents the viscosity ratio of different fracturing fluid to clean water; $T_i/T_1$ represents the ratio of pressure value of different density fracturing fluids to clean water.

\[
D(x_1, \ldots, x_n) = \inf_{x \in \mathbb{R}^n} \left\{ \frac{\mu_i}{\mu_1} - v(x) \right\}
\]

The relationship between viscosity ratio and pressure ratio is as follows:

\[
T_i/T_1 = 7E^{-0.6(\mu_i/\mu_1)^2} - 0.002(\mu_i/\mu_1) + 0.9935
\]

Comparing the rock fracture pressure values obtained by using different density mud media and fresh water as fracturing fluid, respectively, Fig. 2 can be obtained.

4.2 Multiple Regression Analysis of Uniform Design Experimental Results

Table 4 shows the effective rock fracturing value obtained by the hydraulic fracturing simulation experiment based on the optimal uniform design method.
Table 3. Optimal design table of values for the hydraulic fracturing simulation experiments

| No. | Fracturing pressure (MPa) | No. | Fracturing pressure (MPa) |
|-----|--------------------------|-----|--------------------------|
| 1   | 12.45                    | 7   | 11.83                    |
| 2   | 10.38                    | 8   | 11.94                    |
| 3   | 9.31                     | 9   | 12.13                    |
| 4   | 10.12                    | 10  | 12.85                    |
| 5   | 10.55                    | 11  | 13.18                    |
| 6   | 11.10                    | 12  | 13.12                    |

In this optimization experiment, \( y \) is the effective rock fracturing pressure value, \( x_1 \) is the fracturing fluid viscosity, \( x_2 \) is the fracturing fluid density, \( x_3 \) is the loading axial compression, and \( x_4 \) is the injection rate. Establishing a quadratic polynomial regression model between the effective fracture value \( y \) and the influencing factors \( (x_1, x_2, x_3, x_4) \), the corresponding multivariate polynomial regression equation is:

\[
y = 17.937 + 0.023x_1 - 10.266x_2 + 2.054x_3 + 1.598x_4 - 0.361x_1^2 - 0.067x_1x_4 + 18.535x_4^2 \quad (8)
\]

Eq. (11) yields a p-value = 0.000405, and p-value \( < 0.05 \) (significance level). In Fig. 3, Eq. (8) is considered in this paper as the optimal fitting formula for the rock fracturing value \( (y) \) and the multi-influence factors \( (x_1, x_2, x_3, x_4) \).

Fig 3. Plot comparison for the multivariate polynomial regression fittings

In summary, the regression models are represented by Eq. (8) reflected that the dependent variable \( y \) (the effective fracturing value of the rock) exhibits the same change trend with \( x_1 \) (fracturing fluid viscosity) and \( x_4 \) (injection rate), and a reverse change trend with \( x_2 \) (fracturing fluid density). More specifically, rock fracturing increases with increasing fluid viscosity or injection rate increases, and decreases with increasing density, which confirms the conclusion of the theoretical analysis in Section 2. Further, these results validate the suitability of the uniform design method for the simulation experiment of hydraulic fracturing with multi-impact parameters.

Conclusion

The mixed-level uniform design method was the suitable scheme employed for hydraulic fracturing simulation experiments. On the basis of reducing the number of experiments, such design also achieves better uniformity and test effect, which at this point, can be presumed to provide a fast and effective means for formulating a subsequent error correction equation and a compensation model for different fluid mechanics influencing factors, where an improved measurement accuracy of the hydraulic fracturing test method is desired. These insights are provided:

1) This study presented an optimization scheme for the hydraulic fracturing simulation experiments based on the mixed-level uniform design method, based on the characteristics of mud as fracturing fluid, mainly, with many fluid mechanics factors and levels. The design was utilized for construction of an experimental design that exhibited a simplified test process, saved time, and reduced implementation cost, which further significantly improved the efficiency of the simulation experiments.
(2) The effects of different hydrodynamic influencing factors on fracturing pressure were studied through the single factor and multifactor experimental results, the relationship between hydraulic influence factors and rock fracturing pressure are shown in Eqs. (6), (7) and (8). Moreover, their effective fracturing values were discussed, and the impact of theoretical analysis of the PKN model was confirmed.

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