Social inequality: from data to statistical physics modeling

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Abstract. Social inequality is a topic of interest since ages, and has attracted researchers across disciplines to ponder over it origin, manifestation, characteristics, consequences, and finally, the question of how to cope with it. It is manifested across different strata of human existence, and is quantified in several ways. In this review we discuss the origins of social inequality, the historical and commonly used non-entropic measures such as Lorenz curve, Gini index and the recently introduced $k$ index. We also discuss some analytical tools that aid in understanding and characterizing them. Finally, we argue how statistical physics modeling helps in reproducing the results and interpreting them.

1. Introduction
Repeated social interactions produce spontaneous variations manifested as inequalities at various levels. With the availability of huge amount of empirical data for a plethora of measures of human social interactions makes it possible to uncover the patterns and look for the reasons behind socio-economic inequalities. Using tools of statistical physics, researchers are bringing in knowledge and techniques from various other disciplines [1], e.g., statistics, applied mathematics, information theory and computer science to have a better understanding of the precise nature (both spatial and temporal) and the origin of socio-economic inequalities that is prevalent in our society.

Socio-economic inequality [2, 3, 4, 5, 6] is concerned with the existence of unequal opportunities and rewards for various social positions or statuses in a society. Structured and recurrent patterns of unequal distributions of goods, wealth, opportunities, and even rewards and punishments are the key features, and measured as inequality of conditions, and inequality of opportunities. The first one refers to the unequal distribution of income, wealth and material goods, while the latter refers to the unequal distribution of ‘life chances’ across individuals, as

* Dedicated to the memory of Prof. J.-I. Inoue
is reflected in education, health status, treatment by the criminal justice system, etc. Socio-economic inequality is held responsible for conflict, war, crisis, oppression, criminal activity, political unrest and instability, and affects economic growth indirectly [7]. Usually, economic inequalities have been studied in the context of income and wealth [8, 9, 10]. However, it is also measured for a plethora of quantities, including energy consumption [11]. The inequality in society [12, 13, 14, 15] is an issue of current focus and immediate global interest, bringing together researchers across several disciplines – economics and finance, sociology, demography, statistics along with theoretical physics (See e.g., Ref. [9, 16, 17]).

Socio-economic inequalities are quantified in numerous ways. The most popular measures are absolute, as quoted with a single number, in terms of indices, e.g., Gini [18], Theil [19], Pietra [20] indices. The alternative measure approach is relative, using probability distributions of various quantities, but the most of the previously mentioned indices can be computed once one has the knowledge of the distributions. What is usually observed is that most quantities display broad distributions, usually lognormals, power-laws or their combinations. For example, the distribution of income is usually an exponential followed by a power law [9, 21].

In one of the popular methods of measuring inequality, one has to consider the Lorenz curve [22], which is a function that represents the cumulative proportion $X$ of ordered (from lowest to highest) individuals in terms of the cumulative proportion of their sizes $Y$. $X$ can represent income or wealth of individuals. But it can as well be citation, votes, city population etc. of articles, candidates, cities etc. respectively. The Gini index ($g$) is defined as the ratio between the area enclosed between the Lorenz curve and the equality line, to that below the equality line. If the area between (i) the Lorenz curve and the equality line is represented as $A$, and (ii) that below the Lorenz curve as $B$ (See Fig. 1), the Gini index is $g = A/(A+B)$. It is an useful measure to quantify socio-economic inequality. Ghosh et al. [23] recently introduced the ‘$k$ index’ (where ‘$k$’ symbolizes for the extreme nature of social inequalities in Kolkata), which is defined as the fraction $k$ such that $(1-k)$ fraction of people or papers possess $k$ fraction of income or citations respectively [24].

When the probability distribution is described using an appropriate parametric function, one can derive these inequality measures as a function of those parameters analytically. In fact, several empirical evidence have been reported to show that the distributions can be put into a finite number of types. Most of them turn out to be a of mixture of two distinct parametric distributions with a single crossover point [24].

This review is organized as follows: Sec. 2 discusses the evolutionary view of socio-economic inequalities and Sec. 3 discusses the basic and most popular quantities which are used for measuring inequalities. Next, in Sec. 4 we discuss the most realistic scenario in which probability distributions have to fitted to more than a single function, and how to measure inequalities from them. Further, we discuss if inequalities are natural, in context of the recent works of Piketty in Sec. 5, followed by a section on statistical physics modeling in Sec. 6. Finally, we summarize in Sec. 7.

2. Evolutionary view of socio-economic inequality

The human species is known to have lived their life as hunter gathers for more than 90% of its existence. It is widely believed that those early societies were egalitarian, still seen in the lifestyle of various tribes like the !Kung people of the Kalahari [25]. Early hunter-gatherer societies are even believed to have championed sex equality [26]. Traditionally having a few possessions, they were semi nomadic in the sense they were moving periodically. Having hardly mastered farming and lived as small groups, the mere survival instinct was driving them to overlook individual interests. They were sharing what they had, so that all of their group members were healthy and strong, be it food, weapons, property, or territory [27]. With the advent of agricultural societies, elaborate hierarchies were created, with less stable leadership in course of time. These
Figure 1. Schematic representations for Lorenz curve, Gini index $g$ and $k$ index. The dashed line depicts the Lorenz curve and the solid line represents perfect equality. The area enclosed between the equality line and the Lorenz curve is $A$ and that below the Lorenz curve is $B$. The Gini index is given by $g = A / (A + B)$. The $k$ index is given by the abscissa of the intersection point of the Lorenz curve and $Y = 1 - X$ (adapted from Ref. [24]).

evolved into clans or groups led mostly by family lines, which eventually developed as kingdoms. In these complex scenarios, new strategies for hoarding surplus produce of agriculture or goods were adapted by the chiefs or kings, predominantly for survival in times of need, and this lead to the concentration of wealth and power (see Ref. [28] for models with savings). Along with the advancement of technologies, intermediate mechanisms helped in wealth multiplication. This completed the transition from egalitarianism to societies having competition and the inequality paved way for the growth of chiefdoms, states and industrial empires [29, 6].

3. Basics and generic properties of inequality measures

In this section, we formally introduce the measures to quantify the degree of social inequality, namely, Lorenz curve, Gini index and $k$ index.

The Lorenz curve shows the relationship between the cumulative distribution and the cumulative first moment of $P(m)$:

$$X(r) = \int_{m_0}^{r} P(m) dm, \quad Y(r) = \frac{\int_{m_0}^{r} m P(m) dm}{\int_{m_0}^{\infty} m P(m) dm}. \tag{1}$$

The set $(X(r), Y(r))$ defines the Lorenz curve, assuming $P(m)$ to be defined in $[m_0, \infty)$. Fig. 1 shows the typical behavior of Lorenz curve. The Lorenz curve gives the cumulative proportion $X$ of ordered individuals (from lowest to highest) holding the cumulative proportion $Y$ of wealth. Lorenz curve, Gini index etc. were historically introduced in the context of income/wealth. So, let us call $X$ as individuals and $Y$ as ‘wealth’, but in principle the attributes $X$ and $Y$ can be
any of the combinations like article/citations, candidate/vote, city/population, student/marks, company/employee etc. Hence, when all individuals take the same amount of wealth, say \( m' \), we have \( P(m) = \delta(m - m') \), with \( m_0 < m' < \infty \), and one obtains

\[
X(r) = \int_{m_0}^{r} \delta(m - m') dm = \Theta(r - m'), \quad Y(r) = \frac{\int_{m_0}^{r} m \delta(m - m') dm}{\int_{m_0}^{\infty} m \delta(m - m') dm} = \frac{m' \Theta(r - m')}{m'} = X(r).
\]

where \( \Theta(x) \) is a step function defined by

\[
\Theta(x) = \begin{cases} 
1, & x \geq 1, \\
0, & x < 1.
\end{cases}
\]

Thus \( Y = X \) is the ‘perfect equality line’ (see Fig. 1), where \( X \) fraction of people takes \( X \) fraction of wealth in the society. On the other extreme, if the total wealth in the society of \( N \) persons is concentrated to a few persons, \( P(m) = (1 - \varepsilon) \delta_{m,0} + \varepsilon \delta_{m,1} \), with \( \varepsilon \sim \mathcal{O}(1/N) \) and with the total amount of wealth is normalized to 1, we get \( X(r) = 1 - \varepsilon + \varepsilon \delta_{r,1} \) and \( Y(r) = \delta_{r,1} \). Hence, \( Y = 1 \) iff \( X = r = 1 \) and \( Y = 0 \) otherwise, and the Lorenz curve is given as ‘perfect inequality line’ \( Y = \delta_{X,1} \) where \( \delta_{x,y} \) is a Kronecker’s delta (see Fig. 1).

For a given Lorenz curve, the Gini index is defined by twice of area between the curve \((X(r), Y(r))\) and perfect equality line \( Y = X \) (shaded part ‘A’ in Fig. 1). It reads

\[
g = 2 \int_{0}^{1} (X - Y) dX = 2 \int_{r_0}^{\infty} (X(r) - Y(r)) \frac{dx}{dr} dr,
\]

where \( X^{-1}(0) = r_0, X^{-1}(1) = \infty \) should hold. Graphically (see Fig. 1), the Gini index is the ratio of the two areas (‘A’ and ‘B’), \( g = A/(A + B) \). Thus, the Gini index \( g \) is zero for perfect equality and unity for perfect inequality. The Gini index may be evaluated analytically when the distribution of population is obtained in a parametric way.

The recently introduced \( k \) index is the value of \( X \)-axis for the intersection between the Lorenz curve and a straight line \( Y = 1-X \). For the solution of equation \( X(r) + Y(r) = 1 \), say \( r_* = Z^{-1}(1), Z(r) \equiv X(r) + Y(r) \), the \( k \) index is given by \( k = X(r_*) \). Thus, the value of \( k \) index indicates that \( k \) fraction of people shares totally \((1-k)\) fraction of the wealth. Hence, the \( k \) index equals 1/2 for perfectly equal society, and 1 for perfectly unequal society. This is obviously easier to estimate by eyes in comparison with the Gini index (shaded area \( A \) in Fig. 1).

Apart from these indices, Pietra’s \( p \) index [20] and \( m \) or median index [30] has been used as inequality measures, and can be derived from the Lorenz curve. The \( p \) index is defined by the maximal vertical distance between the Lorenz curve and the line of perfect equality \( Y = X \), while the \( m \) index is given by \( 2m - 1 \) for the solution of \( Y(m) = 1/2 \).

### 4. Results for mixture of distributions

It is rather easy to perform analytic calculations for the \( g \) and \( k \) indices when the distribution of population are described by parametric distributions such as a uniform, power law and lognormal distributions. It is quite common to find that the probability distributions of quantities like wealth, income, votes, citations etc. fit to more than one theoretical function depending on the range. Formally, \( P(m) = F_1(m)\theta(m, m_\times) + F_2(m)\Theta(m - m_\times) \), with \( \theta(m, m_\times) \equiv \Theta(m) - \Theta(m - m_\times) \), where \( m_\times \) is the crossover point. The functions \( F_1(m) \) and \( F_2(m) \) are suitably normalized and computed for their continuity at \( m_\times \). In such cases, one can also develop a framework [24], using which it is reasonably straightforward to calculate Lorenz curve, Gini and \( k \)-index.
Figure 2. Left: Income distribution $p(w)$ for USA [31] for a few years, the data is rescaled by the average income $\langle w \rangle$. The lower income range fits to an exponential form $a \exp(-bw)$ with $b = 1.3$ while the high income range fits to a power law decay $cw^{-d}$ with exponent $d = 2.52$. Right: Same for Denmark [32]. The middle and upper ranges fit to different power laws $cw^{-d}$ with $d = 2.96$ and $c_1w^{-d_1}$ with $d_1 = 4.41$. Taken from Ref. [24].

4.1. Empirical data
The most well studied data in the context of socio-economic inequality is that of income. Incomes are re-calculated from the income tax data reported in the Internal Revenue Service (IRS) [31] of USA for 1996-2011. This data is used to compute the probability distribution $P(w)$ of income $w$ for each year. Similar data from Denmark were used from the years 2000-2012 [32]. The $g$ index is found to be around $0.54 - 0.60$ for USA and $0.34 - 0.38$ for Denmark, while the $k$-index is around $0.69 - 0.71$ for USA and $0.65 - 0.69$ for Denmark [24]. Fig. 2 shows both these data sets.

The data related to several socio-economic measures were analyzed. The (i) voting data of proportional elections from a few countries [33]), (ii) citations of different science journals and institutions, collected from ISI Web of Science [34], and (iii) population data for city sizes for Brasil [35], municipalities of Spain [36] and Japan [37] were analyzed. The data showed broad distributions of the above quantities, which well-fitted to (see Ref. [24] for details) (a) a single lognormal, (b) a single lognormal with a power law tail, (c) uniform with a power law tail, (d) uniform with a lognormal tail, (e) a mixture of power laws, (f) a single power law with a lognormal tail.

The inequality indices computed using a combination of fitting functions were compared with that computed directly from the empirical data.

5. Is wealth and income inequality natural?
One is often left wondering whether inequalities in wealth and income are natural. It has been shown using models [38] and their dynamics that certain minimal dynamics over a completely random exchange process and subsequent entropy maximization produces broad distributions. Piketty [39] argued recently that inequality in wealth distribution is quite natural. He pointed out that before the great wars of the early 20th century, the strong skewness of wealth distribution was prevailing as a result of a certain ‘natural’ mechanisms. The Great Depression that followed after the two great World Wars have helped in dispersion of wealth. This in turn brought the prevailing extreme inequality under check and subsequently gave rise to a sizable middle class. After analyzing very accurate data, he concluded that the world is currently
‘recovering’ back to this ‘natural state’, which is happening due to capital ownership driven growth of finance [12], which has been dominant over a labor economy, and this is simply the result of the type of institution and policies that are adopted by the society. His work raises quite fundamental issues concerning bot only economic theory but also the future of capitalism. It points out the large increases in the wealth/output ratio. According to standard economic theory, such increases are attributed to the decrease in the return to capital and an increase in the wages. However, the return to capital has not diminished, while the wages have. He has also prescribed the following: higher capital-gains and inheritance taxes, higher spending towards access to education, tight enforcement of anti-trust laws, corporate-governance reforms that restrict pay for the executives, and finally, the financial regulations which have been an instrument for banks to exploit the society. It is anticipated that all of these might be able to help reduce inequality and increase equality of opportunity. There is further speculation that this might be able to restore the shared and quick economic growth that characterized the middle-class societies in the mid-twentieth century.

6. Statistical physics modeling

One of the most efficient ways to model evolution of systems to broad distributions showing strong inequality, is by using the toolbox of statistical physics. Microscopic and macroscopic modeling helps in imitating real socio-economic systems.

There is a whole body of empirical evidence supporting the fact that a number of social phenomena are characterized by emergent behavior out of the interactions of many individual social components. Recently, the growing community of researchers have analyzed large-scale social dynamics to uncover certain ‘universal patterns’. There has also been an attempt to propose simple microscopic models to describe them, in the same spirit as the minimalistic models commonly used in statistical physics [16, 17].

6.1. Income & wealth distribution

In case of wealth distribution, the popular models are chemical kinetics motivated Lotka-Volterra models [40, 41, 42], polymer physics inspired models [43] and most importantly, models inspired by kinetic theory of gases [44, 45, 46, 28] (see Ref. [9] for details). The two-class structure [8] of the income distribution (exponential dominated low income and power law tail in the high incomes) is well understood to be a result of very different dynamics of the two classes. The bulk is described by a process which is more of a random kinetic exchange [45, 44], producing a distribution dominated by an exponential functional form. The dynamics is very simple, as described in the kinetic theory of gases [47]. The minimal modifications that one can introduce are additive or multiplicative terms.

Processes creating inequality involving uniform retention rates [48] or equivalently, savings [45] produce Gamma-like distributions. These models are defined as a microcanonical ensemble, with fixed number of agents and wealth. Here, the wealth exchanging agent retains a certain fraction (termed as ‘saving propensity’) of what they had before each trading process and randomly exchanges the rest of the wealth. When agents are assigned with the same value of the ‘saving propensity’ (as in Ref. [45]), it could not produce a broad distribution of wealth. What is important to note here is that the richest follow a different dynamic from the poor and thus heterogeneity in the saving behavior plays a crucial role. So, to obtain the power law distribution of wealth for the richest, one needs to simply consider that each agent is different in terms of how much fraction of wealth they will save in each trading [46], which is a very natural ingredient to assume, because it is quite likely that agents in a trading market think very differently from one another. In fact, with this small modification, one can explain the entire range of the wealth distribution [28]. These models, moreover, can show interesting characteristics if the exchange processes and flows are made asymmetric, e.g., put on directed networks [49].
variants of these models, results and analyses find possible applications in a variety of trading processes [9].

6.2. Cities & firms
City [50] and firm sizes [51] consistently exhibit broad distributions with power law tails for the largest sizes, commonly known to be Zipf’s law.

Gabaix [52] showed that if cities grow randomly at the same expected growth rate and the same variance (Gibrat’s law [53]), the limiting distribution converges to the Zipf’s law. He proposed that growth ‘shocks’ are random and they impact utilities in both positive and negative way. A similar approach resulted in diffusion and multiplicative processes [54]. Shocks were also used to immitate sudden migration [55]. Simple economics arguments demonstrated that expected urban growth rates were identical irrespective of city sizes and variations were random normal deviates, resulting Zipf law with exponent unity.

6.3. Consensus
Consensus in social systems is an interesting topic, due to its dynamics. The dynamics of agreement and disagreement in a ‘society’ is complex, and statistical physicists working on opinion dynamics have been brave enough to model opinion states in a population and their dynamics that determine the transitions within such states. A huge body of old and recent literature [17, 16] discusses models that explain various social phenomena and the observed inequality in such instances of consensus formation.

6.4. Bibliometrics
The increasing amount of data produced from bibliometric tools have led to a better understanding of how researchers and their publications ‘interact’ with one another in a ‘social system’ consisting of articles and researchers. The patterns of citation distribution and growth are now well studied, and some of the most successful models have used statistical physics [56].

Statistical physics tools have aided in formulating these microscopic models, which are simple enough yet rich in terms of socio-economic ingredients. Toy models help in understanding the basic mechanism at play, and demonstrate the crucial elements that are responsible for the emergent distributions of income and wealth. A variety of models, ranging from zero-intelligence variants to the more complex agent based models (including those incorporating game theory) have been proposed over years and are found to be successful in interpreting the empirical results [9]. Simple modeling is also effective in understanding how entropy maximization produce distributions which are dominated by exponentials, and also explaining the reasons for aggregation at the high range of wealth, including the power law Pareto tail [8, 9].

7. Summary and discussions
Social inequalities are manifested in several forms, and are recorded well in history, being the reason of unrest, crisis, wars and revolutions. Traditionally a subject of study of social sciences, though scholars from different fields have been investigating the causes and effects from a sociological perspective, and trying to understand its consequence on the prevailing economic system. Reality is not as simple and pointing out the causes and the effects are much more complex.

Imagine a the world which is very equal, where it would have been difficult to compare the extremes, differentiate the good from the bad, hardly any leadership people will look up to, will lack stable ruling governments if there were almost equal number of political competitors, etc.

The recent concern about the increase of inequality in income and wealth, as pointed out from different measurements [39] has renewed the interest on this topic among the leading social
scientists across the globe. Society always had classes, and climbing up and down the social ladder \cite{57, 58} is quite difficult to track, until recent surveys which provide some insight into the dynamics. Several deeper and important issues of our society \cite{4} still need attention in terms of inequality research, and this can only be achieved by uncovering hidden patterns on further analysis of the available data.

Measurement of inequalities in society can be as simple as measuring zeroth order quantities as Gini index, to finding exact probability distributions. The complexity of the underlying problems have inspired researchers to propose multi-dimensional inequality indices \cite{59}, which serve well in explaining a lot of factors in a compact form.

As physicists, our interests are mostly concentrated on subjects which are amenable to modeling using macroscopic or microscopic frameworks. Tools of statistical physics can very well explain the emergence of broad distributions which are signatures of inequalities. The literature already developed, contains serious attempts to understand socio-economic phenomena, under Econophysics and Sociophysics \cite{60}. The physics perspective brings alternative ideas and a fresh outlook compared to the traditional approach taken by social scientists, and is reflected in the increasing collaborations between researchers across disciplines \cite{1}.

Acknowledgments
A.C. thanks V.M. Yakovenko for discussions, in the context of a more extensive material which is being written up. A.C. and B.K.C. acknowledges support from B.K.C.’s J. C. Bose Fellowship and Research Grant. J.I. was financially supported by Grant-in-Aid for Scientific Research (C) of Japan Society for the Promotion of Science (JSPS) No. 2533027803 and Grant-in-Aid for Scientific Research (B) of 26282089, Grant-in-Aid for Scientific Research on Innovative Area No. 2512001313. He also thanks Saha Institute of Nuclear Physics for their hospitality during his stay in Kolkata.

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