2 day mini-workshop:

Axion Cosmology

Yukawa Institute for Theoretical Physics, Kyoto University
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Exploring the string axiverse and parity violation in gravity with gravitational waves

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Menu

1. Traces of the string axiverse

2. Axion dark matter

3. Dynamical Chern-Simons gravity

4. Parametric Resonance

5. Estimations

6. Conclusion
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Traces of String Axiverse

- String theory gives a lot of massive pseudo-scalar fields (Axion).
  
  A. Arvanitaki, et al (2010),
  P. Svrcek and E. Witten (2006)

- Their mass is $10^{-33} \sim 10^{-10}$ eV. Its range is the very wide.

- It is indicated that the axion can behave as the cold dark matter.
  
  W. Hu, R. Barkana, and A. Gruzinov (2000)

- The string axiverse gives the parity-violated interactions as follows,
  $\Phi \tilde{F} F$, $\Phi \tilde{R} R$. In this talk, we concentrate on the modified gravity.
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**Axion Dark Matter**

- The cold dark matter is the dust which is pressureless.

- The occupation number of the state of the axion dark matter is estimated by

\[
\frac{N}{\Delta x^3 \Delta p^3} \sim \frac{n}{p^3} = \frac{\rho_{DM}}{m p^3} \sim 10^{43} \left( \frac{\rho_{DM}}{0.3 \text{ GeV/cm}^3} \right) \left( \frac{10^{-10} \text{eV}}{m} \right)^4.
\]

A. Khmelnitsky & V. Rubakov, 2014

- The oscillation of the axion as a classical field is given by

\[
\Phi(t) \sim \Phi_0 \cos(mt).
\]

\[
v \sim 10^{-3}, \ E \simeq m + \frac{1}{2}mv^2 \sim m
\]

→ Energy density: \( T_{00} = \rho_{DM} = \frac{1}{2} \dot{\Phi}^2 + \frac{1}{2} m^2 \Phi^2 \sim \frac{1}{2} m^2 \Phi_0^2 \)

\[
\Phi_0 \simeq 2.1 \times 10^7 \text{eV} \left( \frac{10^{-10} \text{eV}}{m} \right) \sqrt{\frac{\rho}{0.3 \text{ GeV/cm}^3}}
\]

Pressure: \( p_{DM} = \frac{1}{2} \dot{\Phi}^2 - \frac{1}{2} m^2 \Phi^2 \sim -\frac{1}{2} m^2 \Phi_0^2 \cos 2mt \)

- The period of the oscillation of the axion can be estimated as \( 1/m \). Then, by the average under the long time scale \( T \gg 1/m \), the pressure of the oscillating axion can be regarded as zero. Then, the oscillating axion behaves as the CDM.
Size of Axion Dark Matter

- In space, the matters are collected by the gravitational forces.
  - It appears the equilibrium between the characteristic pressure of matters and the gravitational force induced by matters themselves.

  - This equilibrium is expressed by the size of the halo of matters.

  - Jeans length of the axion dark matter is given by
    \[
    r_J = 4.3 \times 10^{-3} \text{pc} \times \left(\frac{0.3 \text{ GeV/cm}^3}{\rho}\right)^{1/4} \left(\frac{10^{-10} \text{ eV}}{m}\right)^{1/2}
    \]

- Jeans length is the maximum size which the matters can exist stably in space.
  - In this talk, we assume the axion dark matters have the enough energy density and behave as the classical oscillating field.

  - In low energy scale, the potential of the axion can be approximated by
    \[V \sim \frac{1}{2}m^2\Phi^2.\]
Menu

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Dynamical Chern-Simon Gravity

- The action is given by

\[ S = \int d^4x \sqrt{\left| g \right|} \left[ \kappa R + \frac{1}{4} \alpha \Phi \tilde{R} = \frac{1}{2} \left( \overline{\nabla} \mu \Phi \right) \left( \overline{\nabla}^\mu \Phi \right) - V(\Phi) \right] \]

\[ \alpha = \sqrt{\frac{\kappa}{2}} \ell^2 \quad \tilde{R} = \frac{1}{2} \epsilon^{\gamma \delta \rho \sigma} R^\alpha_{\beta \rho \sigma} R^{\beta}_{\alpha \gamma \delta} \equiv \tilde{R}^\alpha_{\beta \gamma \delta} \]

- Dynamical or non-dynamical
  → This is switched by the existence of the dynamical term of the axion field.

- Because the axion field is the pseudo scalar, this interaction term induces the circular polarized gravitational waves.

- The upper bound of the coupling constant \( \ell \) is given as follows,
  \[ \ell \sim 10^8 \text{km} \]
  in non dynamical theory by Y. Ali-Haimoud & Y. Chen (2011).
  in dynamical theory by Y. Nakamura et. al. (2018).

- The ghost modes exist in this theory. We assume this theory as the effective theory of gravity.
Equations of Motion of dCS Gravity

- The action of the dynamical Chern-Simons gravity is

\[
S = \kappa \int d^4x \sqrt{-g} R + \frac{1}{4} \alpha \int d^4x \sqrt{-g} \Phi \tilde{R} R
\]

\[
- \frac{1}{2} \int d^4x \sqrt{-g} [g^{\mu\nu} (\nabla_{\mu} \Phi)(\nabla_{\nu} \Phi) + 2V(\Phi)]
\]

- The equations of motion are derived by

\[
G_{\mu\nu} + \frac{\alpha}{\kappa} C_{\mu\nu} = \frac{1}{2\kappa} T_{\mu\nu}
\]

for the gravitational field and

\[
C^{\mu\nu} \equiv (\nabla_{\alpha} \Phi) \epsilon^{\alpha\beta\gamma}(\mu \nabla_{\gamma} R^\nu_{\beta}) + (\nabla_{\alpha} \nabla_{\beta} \Phi) \tilde{R}^{\beta(\mu\nu)\alpha}
\]

\[
\nabla_{\mu} \nabla^{\mu} \Phi - \frac{dV(\Phi)}{d\Phi} = -\frac{\alpha}{4} \tilde{R} R
\]

for the axion field.
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Parametric Resonance

- **The Swings**
  - The parameters vary at roughly twice the natural frequency of the arms of the swing, the amplitude of it will grow.

- The parametric resonance is induced by Mathieu equation.
  \[
  \ddot{x}(t) + \omega_0^2 \left( 1 - f_0 \sin(\omega_p t) \right) x(t) = 0
  \]

- The properties of the Mathieu equation are noted.
  - The (first) resonance frequency is given by \( \omega_p \approx 2\omega_0 \)
  - The growth rate of the amplitude is given by \( \Gamma_{\text{max}} = \frac{1}{4} |f_0| \omega_0 \)

- Enhanced GW

\[ h \propto e^{\Gamma \eta} \]
Gravitational Wave in dCS Gravity

- We choose the ansatz of the spacetime below,
  \[ ds^2 \simeq a(\eta)^2(-d\eta^2 + \delta_{ij}dx^i dx^j + h_{ij}dx^i dx^j) \]

- For the interest in the circular polarization, we take the circular polarization basis which are defined by
  \[ h_{ij}^{TT} = \sum_A h_A e_i^A_\nu \]
  \[ e_i^R(n) = \frac{1}{\sqrt{2}}(e^+_i(n) + ie^\times_i(n)) \quad h_R = \frac{1}{\sqrt{2}}(h_+ - ih_\times) \]
  \[ e_i^L(n) = \frac{1}{\sqrt{2}}(e^+_i(n) - ie^\times_i(n)) \quad h_L = \frac{1}{\sqrt{2}}(h_+ + ih_\times) \]

Then, the equations of the gravitational wave is derived by
  \[ \left( a^2 - \epsilon_A \Phi'^\frac{\alpha}{\kappa} k \right) h''_A + \left(2aa' - \epsilon_A \Phi''\frac{\alpha}{\kappa} k \right) h'_A + \left(a^2 - \epsilon_A \Phi'\frac{\alpha}{\kappa} k \right) k^2 h_A = 0 \]

- We can neglect the effect of the expansion of space in the time scale when the gravitational waves pass Galaxy. Then, the scale factor can be estimated as
  \[ a(\eta) \simeq 1. \] Then, we can get the solution of the axion field, \[ \Phi \simeq \Phi_0 \cos(m\eta). \]

- Finally, the equations of the gravitational wave is derived as
  \[ h''_A + \frac{\epsilon_A \delta \cos(m\eta)}{1 + \epsilon_A \frac{k}{m} \delta \sin(m\eta)} k h'_A + k^2 h_A = 0 \]
  \[ \epsilon_A \equiv \begin{cases} 1 & : A = R \\ -1 & : A = L \end{cases} \quad \delta \equiv \frac{\alpha}{\kappa} m^2 \Phi_0 \]
Growth of Amplitudes of GWs

- These plots show the growth of the amplitude in the circular polarization basis. We used the values below:
  \[
  \ell = 10^8 \text{ km}, \quad m = 10^{-10} \text{ eV}, \quad \rho = 0.3 \times 10^6 \text{ GeV/cm}^3
  \]
  \[
  \delta \simeq 0.02
  \]

There is the difference between the growth of the amplitude in R- and L-circular polarization basis. In these conditions, the amplitude of \( h_R \) becomes \( 10^4 \) times bigger than the amplitude of \( h_L \). So, the fully circular polarized wave appears.
Strength of Circular Polarization

- This plot shows the time evolution of the strength of the circular polarization. We used the same values before slide.

  → The horizontal axis shows the time $\eta$.

  → The vertical axis shows the frequency $k$ of the GWs.

  → The plotted function is defined by

    $$\text{parity}(\eta) \equiv \frac{|h_R|^2 - |h_L|^2}{|h_R|^2 + |h_L|^2}$$

    parity $\sim \pm 1$ : R or L

- The color shows the strength of the circular polarization.

  → At the resonance frequency, the GW is completely circular polarized.
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From the general theory of the parametric resonance, we can give the formulae for the estimations of the resonance frequency and the rate of the amplification in the length.

Resonance

\[ k_r = \frac{m}{2} \Rightarrow f_r = \frac{k_r}{2\pi} \approx 1.2 \times 10^4 \text{ Hz}\left(\frac{m}{10^{-10} \text{ eV}}\right) \]

Width of the resonance:

\[ \frac{m}{2} - \frac{m}{8} \delta \lesssim k_r \lesssim \frac{m}{2} + \frac{m}{8} \delta \quad \delta \equiv \frac{\alpha}{\kappa} m^2 \Phi_0 \quad \alpha = \sqrt{\frac{\kappa}{2}} \ell^2 \]

Amplification of the amplitude of GW

We can convert \( \Gamma \) into the length \( R_{\times 10} \) in which the amplitude of the GWs becomes 10 times bigger.

\[ R_{\times 10} \approx 5.2 \times 10^{-8} \text{ pc} \]

\[ \times \left(\frac{10^{-10} \text{ eV}}{m}\right)^2 \left(\frac{10^8 \text{ km}}{\ell}\right)^2 \sqrt{\frac{0.3 \text{ GeV/cm}^3}{\rho}} \]
Simple Estimation

- The length in which the GWs becomes $10$ times bigger is estimated as
  \[
  R_{\times 10} \simeq 5.0 \times 10^{-8} \text{ pc}
  \times \left( \frac{10^{-10} \text{ eV}}{m} \right)^2 \left( \frac{10^8 \text{ km}}{\ell} \right)^2 \sqrt{\frac{0.3 \text{ GeV/cm}^3}{\rho}}
  \]

- If the GWs which have the resonance frequency through in Galaxy for $10 \text{ kpc}$, the amplitude of the GWs become $10^{10^{12}}$ times bigger.

- This effect will be detected in the difference between the waveform from the interferometers and from the numerical template of the GWs in pure GR.

- If we found no difference in the waveform, we can give the constraint to the coupling constants or the energy density of the axion dark matter.

→ If you believe $\ell \sim 10^8 \text{ km}$, then $\rho$ will be constraint by $\rho \leq 10^{-26} \text{ GeV/cm}^3$.

→ If you believe $\rho \sim 0.3 \text{ GeV/cm}^3$, then $\ell$ will be constraint by $\ell \leq 10 \text{ km}$.
The **green region** is excluded by the observation.

The **blue line** shows the local dark matter density $0.3 \, \text{GeV/cm}^3$.

The **red line** represents the 10 times enhancement of GWs.

The **red dashed line** represents the 0.1 times enhancement.

The **red dotted line** represents the 0.01 times enhancement.
Sensitivity Curves and Axion Dark Matter

\[ f_R \approx 1.2 \times 10^4 \text{ Hz} \left( \frac{m}{10^{-10} \text{ eV}} \right) \]

These Sources are from “http://rhcole.com/apps/GWplotter/”.

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Conclusion

- String axiverse generates the axions which have the light mass and the dCS coupling between the axion field and the gravitational field.

- The coherently oscillating axion can behave as the cold dark matter well, so, through the above coupling, the monochromatic and parity violated gravitational waves are generally induced.

- This effect may use to detect the counterpart of the GR.
  → They might give the new constraint to the abundance of the axion dark matter or the CS-coupling.

- This analysis can connect the problems about the CDM and the modified gravity.
Related Researches

- Articles
  “Electromagnetic memory effect induced by axion dark matter”
  DY and Jiro Soda, Phys. Rev. D 96, 064005 (2017)

  “Exploring the string axiverse and parity violation in gravity with gravitational waves”,
  DY and Jiro Soda, International Journal of Modern Physics D Vol. 27, No. 9 (2018) 1850096

  “Electromagnetic waves propagating in the string axiverse”,
  DY and Jiro Soda, Progress of Theoretical and Experimental Physics, Volume 2018, Issue 4, 1 April 2018, 041E01

- Proceedings
  “Exploring String Axions with Gravitational Waves”,
  Jiro Soda and DY, Galaxies 5 (2017) no.4, 96