The Gravity Probe B electrostatic gyroscope suspension system (GSS)

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Abstract

A spaceflight electrostatic suspension system was developed for the Gravity Probe B (GP-B) Relativity Mission’s cryogenic electrostatic vacuum gyroscopes which serve as an indicator of the local inertial frame about Earth. The Gyroscope Suspension System (GSS) regulates the translational position of the gyroscope rotors within their housings, while (1) minimizing classical electrostatic torques on the gyroscope to preserve the instrument’s sensitivity to effects of General Relativity, (2) handling the effects of external forces on the space vehicle, (3) providing a means of precisely aligning the spin axis of the gyroscopes after spin-up, and (4) acting as an accelerometer as part of the spacecraft’s drag-free control system. The flight design was tested using an innovative, precision gyroscope simulator Testbed that could faithfully mimic the behavior of a physical gyroscope under all operational conditions, from ground test to science data collection. Four GSS systems were built, tested, and operated successfully aboard the GP-B spacecraft from launch in 2004 to the end of the mission in 2008.

Keywords: general relativity, inertial instrumentation, gyroscope

1. Introduction

Together with the SQUID Readout Electronics (SRE), the Telescope Readout Electronics, and the Experiment Control Unit, the Gyroscope Suspension System (GSS) form the set of
electronics instrumentation needed to operate the cryogenic Science Instrument Assembly (SIA) aboard the Gravity Probe B (GP-B) spacecraft (Tumeaure et al. 1996). The primary role of the GSS is to electrostatically suspend the science gyroscope rotors within their housing cavities so that the rotors may spin freely with minimal friction and torque and thus be capable of performing their role as stable indicators of the local inertial frame about the spacecraft.

Each gyroscope consists of a 38 mm diameter, 63.5 g, fused quartz sphere rotor that is spherical to 1 part in $10^6$ (50 nm). Each rotor is coated with a uniform layer of niobium metal, a low-temperature superconductor. The rotor is suspended electrostatically in a spherical cavity in a two part fused quartz housing via voltages provided through six dish-shaped electrodes located on the housing walls, arranged in opposing pairs along three orthogonal axes. The gyroscope is spun up to approximately 4000 RPM (about 70 Hz) via a tangential gas jet which flows in a channel in the housing wall (see figure 1). To activate the superconducting coating, the gyroscope is cooled to 1.8 K in a superfluid liquid helium dewar. The gyroscope spin axis is determined by measuring the coupling of the London moment (dipole moment) of the superconducting, spinning rotor to a superconducting coil on the parting plane of the clamshell-like gyroscope housing using a SQUID magnetometer. Further details of the design and operation of the gyroscope can be found in Buchman et al. (2015) and the cryogenic features of the SIA in Everitt et al. (2015).

To perform its mission, the GSS must satisfy a number of core requirements:

1. Suspend the gyroscope rotor reliably over eight orders of force magnitude.
2. Operate compatibly with the SQUID readout system.
3. Minimize electrostatic torques during science data collection.
4. Apply greatly enhanced and controlled torques to the rotor for calibration and rotor spin axis alignment after initial spin-up.
5. Act as the accelerometer sensor component of the drag-free translation control.

A summary of the architecture and electronics design developed to meet these requirements are discussed here in detail along with the ground-based measurements of the as-built system performance that gave assurance the GSS would perform as desired on-orbit.

**Figure 1.** The Gravity Probe B gyroscope. Photograph of the rotor and housing.
2. The physics of electrostatic suspension

Electrically, the GP-B gyroscope is a system of eight conductors: one rotor with a metallic coating; six electrodes; and one ground plane that covers the inner surface of the housing cavity not occupied by the electrodes or other features, such as the spin-up channel; see figures 1 and 2. In this system, only the rotor is permitted to move and it should not make physical contact with the other electrodes or housing once it is suspended.

The total electrical energy stored in the collection of capacitors formed by conductors in the gyroscope is given by the expression:

$$U = \frac{1}{2} \sum_{m=1}^{8} C_{mm} V_m^2 + \frac{1}{4} \sum_{m=1}^{8} \sum_{n=1}^{8} C_{mn} (V_m - V_n)^2. \quad (1.1)$$

The first summation is over all the self-capacitances of the system. The second summation includes the cross capacitance terms. Note that when:

$$n = m, \ (V_n - V_m) = 0, \quad (1.2)$$

so self-capacitance does not contribute to the sum. The factor of 1/4 in takes into account the fact that the summation counts each cross capacitance twice.

Through the principle of virtual work, the force on the rotor can be determined by computing the gradient of the potential,

$$F = -\nabla U = -\frac{1}{4} \sum_{m=1}^{8} \sum_{n=1}^{8} V C_{mn} (V_m - V_n)^2 \bigg|_{V_{\text{const}}}. \quad (1.3)$$

The conductor self-capacitance is not a function of position, and thus its self-energy term does not contribute to the net force. The voltages of the six suspension electrodes and one ground plane are controlled by the suspension electronics, which are all referenced to the suspension single-point ground. In a similar way, the torque on the rotor can be computed by considering the virtual work performed to rotate the rotor around a particular axis.

The suspended rotor is insulated by a vacuum gap, so its induced potential is a function of the potentials of the seven controlled conductors and the set of individual conductor-to-conductor capacitances. The rotor potential can be found by computing the total charge
induced on the rotor by the other potentials in the system and then solving for the rotor potential:

\[ Q_R = C_{RR} V_R + \sum_{m \neq R} C_{mR} (V_R - V_m), \]  

(1.4)

\[ V_R = \frac{Q_R + \sum_{m=1}^{6} C_{mR} V_m + C_{GP} V_{GP}}{C_{RR} + \sum_{m=1}^{6} C_{mR} V_m + C_{GP}}, \]  

(1.5)

where \( Q_R \) is the rotor charge, \( C_{GP} \) is the ground plane to rotor capacitance, and \( V_{GP} \) is the ground plane voltage which is nominally zero. The rotor charge is actively controlled while suspended via a photoemission system where ultraviolet light is used to liberate electrons from the rotor and housing coatings and a bias electrode is used drive electrons to or from the rotor as a function relative potential of the rotor and the bias electrode (Buchman et al 1998).

The rotor-electrode capacitance is more complicated than a parallel plate model since the electrode surfaces are formed by the curved surfaces of nearly concentric spheres. The capacitance between the rotor and a single electrode has been modeled by Wu (1993), and for small displacements compared with the nominal rotor-electrode gap for the positive \( x \)-axis is:

\[ C_x = \frac{\varepsilon_0 A_e}{d_0} \left( \frac{1}{1 - \alpha \xi} \right) \left[ 1 + \beta \frac{y^2 + z^2}{(1 - \alpha \xi)^2} + 2\beta^2 \left( \frac{y^2 + z^2}{1 - \alpha \xi} \right)^2 \right], \]  

(1.6)

where \( A_e \) is the area of the electrode, a spherical cap with a half-angle of \( \theta_e = 28.7^\circ \); \( d_0 \) is the rotor-electrode gap, 32 \( \mu \)m; and \( \xi = x/d_0 \) is the normalized position of the rotor along the \( x \)-axis. The expressions for the \( y \)- and \( z \)-axes are of similar form. The geometric constants can be calculated as follows:

\[ \alpha = 0.939 = \frac{\sin^2 \theta_e}{2(1 - \cos \theta_e)}, \]
\[ \beta = 0.056 = \alpha (1 - \alpha). \]  

(1.7)

For the \( x \)-axis under consideration, the capacitance is primarily a function of the displacement along the \( x \)-axis, however, it contains cross terms for displacements along the other two orthogonal axes. For displacements up to 25% of the nominal rotor electrode gap, 8 \( \mu \)m, the contribution of these off-axis terms amounts to less than 1% of the total capacitance. As such it can be safely ignored and can be simplified for most applications to:

\[ C_x = \frac{\varepsilon_0 A_e}{d_0} \left( \frac{1}{1 - \alpha \xi} \right) \]  

(1.8)

which evaluates to approximately 72 pF when the rotor is centered in the housing. Equation (1.3) can now be combined with the capacitance expression (1.8) to compute, for example, the net force on the rotor along the \( x \)-axis. The expressions for the \( y \)- and \( z \)-axes are similar.

\[ F_x = -\nabla U = -\frac{1}{4} \sum_{m=1}^{8} \sum_{n=1}^{8} (V_m - V_n) \frac{\partial C_{mn}}{\partial x}, \]  

(1.9)
In this derivation, the contribution of the ground plane was ignored; the symmetry of the ground plane makes this net force small and it does not contain a first-order dependence on rotor position. Equation (1.10) is the fundamental voltage-to-force relationship for the gyroscope.

For the typical case of a centered and uncharged rotor \((\bar{x}, \bar{y}, \bar{z} = 0, V_0 = 0)\), the force on the rotor is proportional to the difference in squares of the electrode voltages, \(D_i\), as seen in equation (1.10).

\[ F_i \propto (V_{i+}^2 - V_{i-}^2) \approx D_i; \quad i \in \{x, y, z\}. \quad (1.11) \]

For a given force, this equation does not have a unique solution for the choice of suspension voltages. However, given a certain \(D_i\) requirement—a net force requirement—a constraint in the form of a sum of the squares of the voltages, \(S_i\), can be chosen to fully specify the voltages on an electrode axis.

\[ S_i \equiv (V_{i+}^2 + V_{i-}^2); \quad i \in \{x, y, z\}. \quad (1.12) \]

\(S_i\) can be viewed as the common-mode pull on the rotor from the two opposing electrodes and is referred to as ‘preload’ force or ‘preload’ acceleration, by analogy to mechanical spring preload. As will be seen in section 3, there are practical constraints on the choice of \(S_i\) in this system.

### 3. Requirements

The GP-B was designed to measure precession of the local inertial frame with respect to the frame of the distant stars to an accuracy of better than \(4.4 \times 10^{-15}\)° s\(^{-1}\) (0.5 mas yr\(^{-1}\)), as indicated by the time histories of the orientations of the gyroscopes’ spin axes over the course of the mission. Disturbance torques from the suspension system are a significant fraction of the overall GP-B error budget. The physics for torque generation are similar to the physics for force generation outlined above. However, here we are interested in the virtual change in potential with respect to the rotation, not translation:

\[ \tau_\phi = -\frac{\partial U}{\partial \phi} = -\left[ \sum \frac{1}{2} V_i^2 \frac{\partial C}{\partial \phi} \right]_{V_i = \text{const}}, \quad (2.1) \]

where \(\phi\) is the angular rotation about a body axis of interest. More detail on the computation of the support-dependent torques can be found in Keiser (1985). It is clear from this relationship that lowering the suspension voltages has a direct effect on lowering torques, thus the control system must be designed to minimize the suspension voltages and suspension centering forces to be consistent with the disturbance environment.
3.1 Performance requirements

The GP-B Error Tree tracks 157 error sources from all aspects of the experiment (Keiser et al 2003), of which approximately $2.6 \times 10^{-15} \text{ s}^{-1}$ (0.3 mas yr$^{-1}$) are assessed to be suspension-dependent. Of this set, four suspension-dependent terms are particularly significant ($>2 \times 10^{-16} \text{ s}^{-1}$) and are the chief drivers of the suspension system design, as outlined in table 1 (Brumley 2004).

Of these, the gravity-gradient acceleration induced drift is a function of forces that must be applied to keep the rotor centered; it is a consequence of the particular disturbance environment and cannot be changed by modifying the suspension parameters. However, the remaining three support-dependent terms depend on the sum of the square of the voltages, $S_i$, on a particular axis, and can be changed by modifying suspension parameters. As a group, they depend on the dc (zero-frequency) and roll-frequency components of $S_i$ and the dc and roll-frequency components of the rotor miscentering. In the GP-B configuration, the $x$- and $y$- suspension axes are oriented at 45° to the space vehicle roll axis; the $z$-axis is perpendicular to the roll axis. Applying the torque relationship for the specific electrode axis configuration and expected rotor asymmetry, a suspension voltage-centric set of derived requirements can be

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**Table 1.** Primary support-dependent drift rates ($\text{° s}^{-1}$).

| Drift component name | North–south drift rate | East–west drift rate |
|----------------------|-------------------------|----------------------|
| 1 Preload imbalance at SV roll frequency | $1.1 \times 10^{-15}$ | $1.1 \times 10^{-15}$ |
| 2 Static preload magnitude, $S_x, S_y, S_z$ | $6.3 \times 10^{-16}$ | $6.3 \times 10^{-16}$ |
| 3 Gravity gradient acceleration perpendicular to SV roll axis | $7.0 \times 10^{-16}$ | $9.7 \times 10^{-17}$ |
| 4 Static preload magnitude, $S_x, S_y, S_z$, together with rotor miscentering at SV roll frequency | $1.8 \times 10^{-16}$ | $2.2 \times 10^{-16}$ |
| Root-sum-square (RSS), $\text{° s}^{-1}$ | $1.5 \times 10^{-15}$ | $1.3 \times 10^{-15}$ |

**Table 2.** Derived suspension system specifications to meet science mission drift requirements

| Derived requirement | Specification |
|---------------------|---------------|
| 1 Average $S_i$ on any axis | $\{S_x, S_y, S_z\} < 0.08 \text{ V}^2$ |
| 2 Differences in $S_i$ between axes | $0.5(S_x+S_y)-S_z < 0.04 \text{ V}^2$ |
| 3 Roll frequency variation in mismatch of $S_i$ along different axes | $S_x(\text{roll})-S_y(\text{roll}) < 1.7 \times 10^{-6} \text{ V}^2$ |
| 4 Roll frequency variation in $S_i$ along any axis | $S_{x,y,z}(\text{roll}) < 1.7 \times 10^{-5} \text{ V}^2$ |
| 5 Twice roll frequency variation in $S_i$ along any axis | $S_{x,y,z}(2\times\text{roll}) < 0.05 \text{ V}^2$ |
| 6 Average miscentering parallel to the SV roll axis | $R < 0.6 \text{ um}$ |
| 7 Sinusoidal miscentering perpendicular to the SV roll axis at roll frequency | $R(\text{roll}) < 0.3 \text{ nm}$ |
| 8 Sinusoidal miscentering perpendicular to the SV roll axis at roll $\pm 2 \times$ orbit frequency (gravity gradient) | $R(\text{roll} \pm 2 \times \text{orbit}) < 3 \text{ nm}$ |

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formed, as outlined in table 2 (Brumley 2004). Note that space vehicle roll frequency is 12.9 mHz (77.5 s period); orbit frequency is 0.17 mHz (96.7 min period).

The specifications that involve constraints on $S_i$ (items 1 through 5) drive stability requirements on the electronics package. During normal operation, the preloads are set to a constant value; any variation in these voltages will be due primarily to thermal or radiation induced effects or non-uniform component aging. Specification items 6 through 8 are a function of the rotor’s motion in the housing and are set by the design of the control law. The averaging period here is one year, the nominal duration of the science mission data-gathering interval.

3.2. Functional requirements

As noted above, the GSS must satisfy a number of core requirements in addition to minimizing electrostatic torques to perform its mission successfully:

1. Operate over eight orders of force magnitude; the same system must be able to suspend the gyroscopes on Earth as well as generate minimal disturbances at the $10^{-8}$ g ($10^{-7}$ m s$^{-2}$) level during data collection.
2. Suspend the gyroscopes reliably. The system must never let a spinning rotor touch the housing. There is sufficient mechanical energy in a rotor spinning at $\sim 70$ Hz to effectively destroy the gyroscope in such an event.
3. Operate with minimal interference to the SQUID readout system. The SQUID magnetometers are very sensitive to stray ac electrostatic fields and EM radiation, thus the suspension system must not interfere with these sensors during ground and on-orbit operation.
4. Apply controlled torques to the rotor for calibration and initial rotor spin axis alignment.
5. Act as an accelerometer as a part of the ‘drag-free’ translation control system to further minimize classical torques on the rotors.

A successful GSS design must combine the performance and functional requirements into a spaceflight compatible electronics instrumentation package.

4. Control system design

The voltage-to-force relationship given by equation (1.10) presents a number of practical difficulties for the control system designer:

1. The force is a strong, nonlinear function of the electrode voltages.
2. The force is a strong, inverse nonlinear function of the position of the rotor in the housing.
3. The rotor potential is a function of all of the electrode voltages; this requires putting some constraints on the choice of the particular voltages used.
4. The rotor must be positioned throughout the housing for calibration and spin-up, so control system operation around a single, linearized operating point is not viable.

A key control design task is to invert these nonlinearities, if possible, to present a linear or near-linear force-to-voltage relationship to the control law.

Early suspension system designs for GP-B from the 1970s to the mid-1990s solved some of these problems by driving the electrodes with a high-impedance current source rather than a voltage source (VanPatten 1995). An ac current source connected to a variable capacitor
acts as an implicit feedback control system to change the voltage on the plates as needed and to keep current at its commanded value, thus effectively removing the inverse position nonlinearity (NL). Assuming negligible rotor charge, the force relationship now takes the following form (Nikirk 1973):

\[
F_i = \frac{\alpha (I_{c+}^2 - I_{c-}^2)}{2d_0 C_0 (1 + \gamma)^2 \omega^2},
\]

\[
= \frac{2\alpha I_p I_c}{d_0 C_0 (1 + \gamma)^2 \omega^2}; \quad I_{c+} = I_p + I_c, \quad I_{c-} = I_p - I_c,
\]

(4.1)

where \(I_{c+}\) and \(I_{c-}\) are the currents into the \(x\) electrodes. \(C_0\) is nominal rotor-electrode capacitance for a centered rotor, 72 pF; \(\gamma\) is the ratio of the stray capacitance to ground to the electrode-rotor capacitance; and \(\omega\) is the excitation frequency. The transfer function can be linearized by introducing a common-mode preload current, \(I_P\), and a differential-mode control current, \(I_C\), to the electrode pair. \(I_P\) is a fixed value based on the maximum force required along the axis and \(I_C\) is computed by the control law.

The first suspension systems were implemented completely in analog electronics using a 20 kHz, 3-phase current drive applied to the three pairs of electrodes. The rotor position was sensed using a capacitance bridge with a 1.0 MHz 3-phase sense signal superimposed on the electrode pairs.

These ‘ac-drive’ suspension systems were effective tools during the development of the gyroscope and other flight hardware. However, they suffered from a number of drawbacks, including:

1. **Readout interference.** Given the close proximity of the suspension electrodes and readout pickup loop on the parting plane of the gyroscope housing, the ac drive and high-frequency position bridge sense signals were found to seriously interfere with the operation with the SQUID magnetometer essential to the spin axis readout system.

2. **Field emission rotor charging.** The required ac carrier for the current drive system necessarily presents peak voltages 1.41 times the RMS value needed produce a commanded force. During laboratory testing, electric fields on the order of \(2 \times 10^7 \text{ V m}^{-1}\) are required to suspend the rotor on Earth. Under these very high field conditions, small imperfections in the electrode or rotor coatings can lead to a field emission current which changes the rotor potential. In the worst case, arcing may occur which would cause the suspension system to fail.

In response to these issues a ‘dc-drive’ laboratory suspension system was developed by Wu in 1992 (Wu 1993), made possible in part by advances in semiconductor electronics and microprocessor technology. In the dc case, slowly varying voltages are directly applied to the electrodes via six low-impedance amplifiers. The position and voltage nonlinearities are computed in and explicitly compensated for by the digital control algorithm. It is important to note that the force-to-voltage mapping is underdetermined; there are three force constraints, one for each axis as required by the controller, and one constraint that rotor potential be known so forces can be computed in equation (1.10). In practice the rotor potential is set to suspension ground. In the nominally centered, uncharged case, this constrains the sum of the electrode voltages are equal to zero per equation (1.5).

The inverse nonlinearity (INL) is implemented as follows, again with a preload voltage, \(V_P\), and a force command, \(\pm Ku\), on opposing electrodes:
The magnitude of the preload voltages can be chosen to minimize the maximum electric field between the rotor and the electrodes for a given operating condition, e.g. laboratory test, rotor spin-up, or science data collection. Note that two degrees of freedom exist in the choice of the preload voltages in equation (4.4); this choice determines steady-state voltages applied to the electrodes.

The $M$ terms are modulation terms; they are either all set to unity for pure dc operation, or to a sinusoid where $f_s$ is 20 Hz. This permits three-phase modulation of the drive voltages. Unlike the ac case, this is not done as part of a constant-current drive system, but to move the drive voltages away from dc so as to minimize the low-frequency amplifier noise being applied to the gyroscope. By setting all three preload voltages, $V_P$, to the same value, the three-phase modulation keeps the rotor potential at 0 V and eliminated the special axis where, in the dc case, it has twice the preload than the other two, further reducing torques on the rotor.

Based on this force-to-voltage mapping, a linear control system can be designed to control the position of the rotor in the housing. Figure 3 shows a single axis of the position control loop for the flight suspension system.

Here, $r$ is the position of the rotor along the axis and $CMD$ is the desired (commanded) position of the rotor in the housing. The gyroscope dynamics are that of an inertial mass:

\[
V_x = M_x(V_{Px} + K_x u_x)\left(1 - \alpha x\right);
\]
\[
V_y = M_y(V_{Py} + K_y u_y)\left(1 - \alpha y\right);
\]
\[
V_z = M_z(V_{Pz} + K_z u_z)\left(1 - \alpha z\right);
\]

\[
K_n = \frac{1}{4V_{IN}}\left(\frac{2d^2}{\varepsilon_0 nA_c}\right); \quad n \in \{x, y, z\},
\]

\[
V_{Px} + V_{Py} + V_{Pz} = 0,
\]

\[
M_x \in \left\{1, \sqrt{2} \sin\left(2\pi f_s t\right)\right\},
\]
\[
M_y \in \left\{1, \sqrt{2} \sin\left(2\pi f_s t + 2\pi/3\right)\right\},
\]
\[
M_z \in \left\{1, \sqrt{2} \sin\left(2\pi f_s t + 4\pi/3\right)\right\}.
\]
\[ \ddot{x} = \frac{1}{m} f(t), \]  
\[ (4.6) \]

where \( m \) is the rotor mass, 63.5 g. As seen in section 1, the axis-to-axis coupling though the capacitance cross-terms are negligible, so each axis is modeled as a simple, uncoupled inertial mass. The voltage-to-force NL is modeled by equation (1.10); the computed force-to-voltage INL is implemented by equation (4.2) through (4.4); and the dynamics of the controller, \( C \), is chosen to meet the performance requirements for rotor positioning, control effort, and other system metrics. The control loop model contains terms for other error sources, such as voltage amplifier errors, \( \delta V \); position measurement errors, \( \delta P \); other forces on the rotor in the housing frame, \( F_{\text{EXT}} \); and the motion of the housing/space vehicle with respect to the rotor, \( R \). The rotor position, \( r \), is measured using a capacitance bridge operating at 34.1 kHz at 10 mV excitation.

The key design choices required for this control system are (1) the particular form of the controller, \( C \), and (2) the choice of the preload voltages, \( V_P \). These choices are driven by mission requirements.

### 4.1. Gyroscope position controller

Low-torque operation requires the control authority (voltage and bandwidth) of the controller to be minimized. The disturbance environment, however, is characterized by random micrometeoroid strikes that can apply impulsive disturbance forces as large as 10,000 times the nominal acceleration environment. A fixed authority control scheme that can handle these impulses produces gyroscope disturbance torques far in excess of mission requirements.

An adaptive, linear quadratic estimator Authority-on-Demand (AOD) control algorithm was developed to meet the high dynamic range requirements for the electrostatic suspension in the science configuration (Eglington 2000, Franklin et al 2014). The controller architecture is novel in that it uses estimates of the plant’s state, rather than estimates of the plant’s parameters, as inputs for adaptation. During operation, this allows the algorithm to dynamically increase its control authority and bandwidth to the level needed to respond to large impulsive disturbances while continuously minimizing control authority during nominal operation to reduce residual torques on the rotor. The net result of this scheme is that the control system performs like a high-bandwidth and high-authority controller with respect to disturbances that move the rotor far from its commanded position; but performs like a low-bandwidth, low-authority control system with respect to gyroscope disturbance torques. This algorithm was implemented in a radiation-hardened digital computer based on a British Aerospace RAD6000 processor, a space-qualified PowerPC derivative, running at 16.368 MHz within the suspension electronics package. To achieve maximum performance from the processor, a single-thread, non-preemptive real-time operating system was developed for this application. During peak demand periods (i.e. during spin-up operations), the processor is loaded to 96% of its capacity; science operation requires approximately 65% of capacity. The code for this application is small, however, only occupying 500 kbytes of memory.

The range of operation of the digital control algorithm is shown in figure 4. The digital controller is able to operate over eight orders of magnitude of force disturbances. The primary science mode digital suspension operates over a specific force range from \( 10^{-7} \) to \( 10^{-2} \) m s\(^{-2} \) and a corresponding bandwidth range of 1.5–8 Hz, at a sample rate of 220 Hz. The spin-up controller has a fixed bandwidth with a 660 Hz sample rate and has been designed with sufficient control authority to reject the disturbances from the spin-up gas flow. This system is also able to suspend a gyroscope in the laboratory in 1 g conditions for performance testing, and can apply up to 1600 V to provide the required electric fields to suspend in 1 g. The ground test controller is also fixed bandwidth with a sample rate of 1320 Hz. Figure 5 shows
the effect of the AOD controller in operation; regardless of the size of the disturbance, the AOD controller varies its control authority to generate a near constant excursion for widely differing disturbance accelerations.

Computers are notoriously unreliable in a space radiation environment. Three all-analog backup control systems are also provided to suspend the gyroscope in the event of a computer fault. The spin-up backup controller is used during gas flow operations, and the two science mode backups are active during the remainder of the mission. All the analog controllers are robust proportional-derivative (PD) designs with an analog force-to-voltage NL inversion circuit with fixed preloads to allow good control over the range of motion of the rotor in the housing. The primary role of these controllers is to prevent the rotor from contacting the housing wall, and thus an integral term in the controller to minimize position error at low frequencies is not needed; this significantly simplifies the design of these compensators.

Figure 4. Range of operation of various suspension modes.

Figure 5. Simulated response of the AOD controller to micrometeoroid strikes.
An analog arbiter monitors computer health and rotor position and it will autonomously switch into the appropriate backup controller in the event of a computer fault or a large position excursion of the rotor. The analog ‘low backup’ controller uses the same minimum bandwidth and low preloads as the science digital system operating at the bottom of its adaptive range, with a specific force capability of $10^{-7}$ m s$^{-2}$ to minimally disturb the rotor when operating in this backup mode. The analog ‘high backup’ can handle the full impact of a meteoroid with momentum 1 kg m s$^{-1}$ the maximum the space vehicle can withstand without serious damage to the dewar or other critical subsystems.

During spin-up operations, 150 V peak and a control effort of 40 mN (0.6 m s$^{-2}$ specific force) are required to hold the rotor at its ideal position in the housing against the pressure of the spin-up gas, Helium flowing at 725 SCCM at a temperature of 6 K.

This architecture permits the controller to operate over the required eight orders of force magnitude, providing reliable suspension of the rotors via prime and backup systems, and minimize electrostatic torques on the rotor via the variable bandwidth adaptive control law.

5. Spin axis alignment

The suspension system, described above, is designed to keep the rotor suspended in the housing and to minimize torques that can induce any spin axis drift that may masquerade as a relativistic effect. However, a key functional requirement on the system is to align the spin axis to within 0.003° of the roll axis of the space vehicle to minimize torques caused by a misalignment of the spin and roll axes as well as to maximize the sensitivity of the SQUID readout system. The gas spin-up process was expected to place the spin axes of the gyros within 1° of the roll axis. The suspension system must generate controlled torques to bring the spin axes to within the 0.003° goal alignment.

After manufacture, rotors are not perfectly round and will also exhibit a centrifugal bulge of approximately 15 nm when spinning at 70 Hz. Though these effects are small in a gross mechanical sense, they do generate an aspherical shape that can be worked against to generate controllable suspension system-induced torques. These torques can be calculated directly from equation (2.1), which is, in general, a complicated function of rotor shape and its orientation in the housing. However, a few key approximations can be made which greatly simplify relationship.

First, the gyroscopes operate in a very low acceleration environment in orbit and thus, will not precess significantly ($\ll 0.01°$) over the mission. The rotor spin axis, to first order, is fixed in the housing frame. Second, the extreme roundness and substrate homogeneity of the rotors yield spheres with a very small inertial imbalance, $\Delta I/I < 10^{-5}$. Thus the rotor’s polhode period (a natural torque-free motion of the spin axis in the rotor body frame (Greenwood 1988)) will be on the order of 15–30 min. This is slow with respect to the rotor rotational period of 17 ms.

These two conditions permit the approximation that the spin axis of the gyroscope is both fixed in inertial space (no precession) and fixed in the rotor body axis frame (insignificant polhode over one spacecraft roll period, 77.5 s). The electrostatic torques can now be averaged over one revolution of the rotor. This spin averaging of the torques is equivalent to averaging the rotor’s shape over a revolution, which gives the following result:

$$
\bar{r}(\theta) = \frac{1}{2\pi} \int_{0}^{2\pi} (r(\theta, \phi)) \phi d\phi = n_0 + \sum_{j=1}^{\infty} n_j P_j(\cos \theta). 
$$

(5.1)
This transforms the arbitrarily-shaped rotor into a body of revolution about the spin axis as a sum of Legendre polynomials with shape coefficients $r_l$. The application of the torque relation, equation (2.1), shows that the electrostatic torques on such a body take on a particularly simple form. If the orientation of the body with respect to an electrode axis, $n$, is parameterized as a 323 Euler rotation with angles $(\alpha, \beta, \gamma)$, it is straightforward to show that rotations about the spin axis, $\gamma$, and about an electrode axis, $\alpha$, do not change the rotor/electrode capacitances along the axis of interest. Only changes in $\beta$, the angle between an electrode axis and the spin axis, have such an effect (Bencze 2006); see figure 6.

\[
\tau_n = \frac{\varepsilon_0 r_0^2}{2d_0^2} \sum_{l=1}^{\infty} \left[ V_{n+}^2 + (-1)^l V_{n-}^2 \right] \\
\times \eta \frac{d}{d\beta} P_l(\cos \beta) \left( \hat{e}_{n+} \times \omega \right).
\] (5.2)

The net effect of this electrostatic torque on the spin-averaged rotor causes the rotor to precess about the electrode axis of interest. For example, electrode voltages on the $x$-axis will cause the rotor to precess in a cone about the $x$-axis; the sign of precession (clockwise or counterclockwise) is a function of the underlying spin-averaged shape. The net torque on the rotor is the vector sum the torques about each of the three electrode axes.

Figure 7 depicts a map of relative magnitude and direction of the torque that results from voltages on the $x$-axis electrodes interacting with a dominant $r_2$ (oblateness) shape term. This map is a Mercator-type projection of the spherical housing. The $x+$ and $y+/y-$ electrodes are shown along the equator. The $z+/z-$ electrodes at the poles are thus spread out across the top and bottom of the map (the $x-$ electrode is not shown). The triangles depict the direction and relative magnitude of this oblateness torque on the rotor when the spin axis vector pierces the housing at various points. Considering only the $x$-axis, this diagram shows that there is no torque on the rotor if the spin axis is aligned with the electrode axes (centers of the electrodes), reaches a maximum 45° away from the electrode axes, and points in a direction
proportional to the cross product of the electrode and spin axes unit vectors. This tends to cause the rotor to precess about the electrode axis.

Similar relations hold for the $y$ and $z$ axes, and the net torque is the vector sum of the contributions of the three axes. The case of particular interest to GP-B is the torque situation in the vicinity of the nominal spin orientation of the rotor, depicted by $\omega$ in Figure 7. Here, for equal voltages on the $x$ and $y$ electrodes, the $x$ and $y$ electrodes axis torques are equal and opposite, and thus cancel. To generate a net torque, an imbalance in the electrode voltages must be introduced, which can be done within limits while maintaining suspension of the gyroscope.

From equation (5.2) it can be seen the torque components are either proportional to the sum or difference of the squares of the applied voltages depending on the harmonic order, $l$. Comparing this torque relation with the force relation, equation (1.10), it is clear that the odd-harmonic torques are proportional to the applied suspension force and are thus a non-controllable quantity from a spin axis alignment standpoint. These torques, however, are also small because expected rotor accelerations relative to the housing will be on the order of $10^{-7}$ m s$^{-2}$ on-orbit. But, the even-harmonic torques are proportional to the sum of the squares of the electrode voltages, and can be influenced by choice of the electrode preload voltages, $S_i$.

5.1. Spin axis orientation controller

Though the process of spin averaging significantly simplifies the torque relations noted above, they still are functions of the angles between the spin axis and electrode axes as well as the shape terms, $r_i$. These expressions can be further simplified by noting the geometrical symmetry of the gyroscope in the rotating spacecraft.

The orientation of the gyroscope housing in the satellite has been arranged such that the gyroscope will spin up within 1° of the satellite's roll axis, which is at a 45° angle with respect to the $x$- and $y$- electrode axes, and perpendicular to the $z$- electrode axis (see $\omega$ in Figure 7).

From Figure 7, the torque causes the spin axis to circulate in a counterclockwise direction about the $x$- electrode axis, and circulate the same manner for $y$ (not shown). In the spin-up direction, these two torques are of equal magnitude, but of opposite sign when $S_x = S_y$ and...
thus nominally cancel. Note that the combined torque direction is along a single line (vertical arrows in figure 7), and thus the spin axis can only be moved in one dimension in the housing frame. The satellite is rolling, however, about the line of sight to the guide star, so torque in the inertial frame rotates with the satellite. If net torque direction is projected onto a non-rotating plane perpendicular to the satellite roll axis, the equation of motion of this spin axis projection can be written as:

\[
\tau = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = K(t) \begin{bmatrix} \cos(\Omega_s t + \phi_s) \\ \sin(\Omega_s t + \phi_s) \end{bmatrix} (S_x - S_y),
\]

(5.3)

where \( \Omega_s \) and \( \phi_s \) are the satellite roll rate and initial phase; \( \dot{x} \) and \( \dot{y} \) are components of the projection of the small spin axis misalignment onto the perpendicular plane in the north–south and east–west directions. \( K(t) \) is a slowly varying gain dictated by the instantaneous spin-averaged shape; this term varies primarily at the polhode rate and can change sign.

From an automatic control system perspective, this is a linear, first-order system with a time-varying gain on the control input \( u = (S_X - S_Y) \). In principle, this system can be brought to the origin (to alignment) in an arbitrarily short time given a large enough control input. However, due to voltage limitations in the suspension system hardware, the maximum difference between \( S_i \) is limited to 10% of their nominal values.

During instrument calibration on orbit, a limited time has been allocated for initial gyroscope spin axis alignment, thus it is desirable to align the four gyroscope axes as quickly as possible. The linear form of equation (5.3), together with magnitude limits on control authority, suggests a ‘bang–bang’ control scheme, where the control input switches between positive and negative maximum depending on the sign of the error signal (Franklin et al 2014). Equation (5.3) gives torques in an inertial frame. However, the spin axis is sensed by the pickup loop in the rotating housing frame. Using the expression,

\[
\omega_H = \omega + \begin{bmatrix} 0 \\ -\Omega_s \end{bmatrix},
\]

(5.4)

that relates rates of change of a vector measured with respect to inertial an inertial frame to rates of change measured with respect to a rotating frame (Greenwood), the torque function in the housing frame becomes:

\[
\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}_H = \begin{bmatrix} 0 & \Omega_s \\ -\Omega_s & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}_H + K(t) \begin{bmatrix} 1 \\ 0 \end{bmatrix} (S_x - S_y),
\]

(5.5)

where the subscript \( H \) denotes that the vectors are referenced to the rotating housing frame axes. This system is a harmonic oscillator in the housing-fixed frame; the projection of the spin vector appears to circle about the roll axis at the satellite roll rate. The required switching surface to bring the state vector to the origin for this system is well known (Ryan 1982). Implementation of such a switching surface is computationally expensive and not necessary in this case; here the goal is to bring the spin axis to within 0.003° of the satellite roll axis, not to the absolute origin. On orbit, the expected convergence rate is approximately \( 3 \times 10^{-7} \)° per switch; this places the terminal limit cycle of this controller well inside the \( 3 \times 10^{-5} \)° final alignment requirement. If the plane containing the \( x \)- and \( y \)-electrode axes is taken as the switching line as shown in figure 8, the only required measurement is whether the spin axis is on the left or right side of this line. This state is easily inferred from measurements of the London Moment by the pickup loop/SQUID system. Expected on-orbit performance: for a gyro spinning at 70 Hz with a 15 nm centrifugal bulge and a control input of \( u = |S_X - S_Y| \sim 10^4 \) V^2 the expected alignment rates are on the order of 0.3°/day.
5.2. Flight performance

Figure 9 shows the orientation of the gyroscope in the inertial frame axis with respect to the mean line of sight to the guide star during the final alignment process. The goal is the red circular region near the origin. With this procedure, alignment rates on the order of $2 \times 10^{-7} \text{ s}^{-1}$ can be generated when driven by the high voltage amplifier applying 50 V common-mode on the electrode axes (coarse alignment), while rates of $8 \times 10^{-9} \text{ s}^{-1}$ with an
8 V drive from the low voltage amplifiers (fine alignment). The spiral shape of the predicted trajectory is expected because convenient, though sub-optimal, switching surface was used in the bang–bang controller: the pickup loop on the housing parting plane. The spiral is exaggerated in the on-orbit implementation because of an additional phase shift introduced by the on-orbit algorithm. To fit conveniently into real-time contact periods with the spacecraft, the spin axis orientation measurement is interleaved with the torque generation. During torque generation intervals, which can last up to 3 h at a stretch, the spin axis of the gyroscope moves but the switching surface that is used to generate the torques does not. This introduces an increasing phase error over time during the torque application, and thus exaggerates the spiral trajectory.

On-orbit it was found that the precession rate, though proportional to the preload, as expected, was 20% of the prelaunch estimate. This measurement implies that the rotors are effectively rounder than prelaunch estimates indicated. This observed spin axis alignment rate is a valuable calibration, as it allows sensitivity bounds to set on the disturbance torques that operate on the gyroscope during the science data-gathering phase of the mission. Higher preload voltages, in principle, can be used to align the spin axes more rapidly. However, the instrument team decided to be conservative with the voltage imbalance ratios to ensure that the alignment process did not cause any stability problems with the suspension control loops through enhanced cross-axis coupling from the preload imbalances. Roughly, one week in coarse alignment and one week in fine alignment mode was required to move the spin axes from their initial 0.05° offset to better than the required 0.003° goal orientation.

6. Drag-free sensor

As seen above, any suspension force has the potential to induce disturbance torques on the rotor. Drag-free control of the spacecraft is used to minimize support-induced forces on the gyroscopes, compensating for environmental forces on the spacecraft—such as atmospheric drag, solar wind and radiation pressure—by causing the vehicle to follow a near-perfect gravitational orbit through the use of a counterbalancing thrust. The system is designed to reduce environmental accelerations on the spacecraft that are transmitted to the rotor via the suspension system from $10^{-7}$ to $10^{-11}$ m s$^{-2}$, permitting the use of lower suspension voltages and lower bandwidth control laws to minimize disturbance torques.

In this drag-free system, the gyroscopes act as the sense accelerometers and the GSS sends the measurements to the space vehicle’s attitude and translation control (ATC) system (Dougherty et al 1995). The ATC uses these measurements to match the trajectory of the vehicle to that of the sense gyroscope. There are two operating modes of the drag-free system, one where the rotor is unsuspended and allowed to float freely, and another using a suspended rotor as an accelerometer.

6.1. Drag-free free-floating mode

In this mode, the suspension system of the drag-free reference gyroscope is placed in a standby mode and the rotor is allowed to freely move under the influence of gravity; during this time, the vehicle is maneuvered to keep the rotor at the center of the housing cavity. The GSS monitors the position of the gyroscope with respect to the housing and only allows rotor position excursions within a radius of 4 μm from the center of the 32 μm rotor/electrode gap prior to disabling the drag-free mode and re-centering the gyroscope electrostatically. At 70 Hz spin speed, there is sufficient mechanical energy in the rotor to destroy the gyroscope assembly should the rotor come into contact with the housing wall. Therefore, conservative
limits have been built into the suspension system to preclude this possibility during normal operation.

The ATC system flies the position of the inertial mass of the spacecraft, $\mathbf{R}$, with respect to the position of the gyroscope, $\mathbf{r}$, as shown in figure 10. The active control path is shown in thick lines (gyro suspension controller block is disabled). The GSS passes rotor position information to the ATC controller through the interface gain $K_1$. The controller is implemented as a 3-axis PID (proportional + integral + derivative) (Franklin et al 2014) controller with sufficient gain at the space vehicle roll frequency to meet its $1 \times 10^{-11}$ m s$^{-2}$ residual acceleration requirement. Calculations are performed in the NADIR frame and are rotated into the vehicle body frame to issue commands to the 16 cold gas thrusters aboard the spacecraft. The ATC control system is tuned to avoid any mechanical resonance modes of the spacecraft structure, all of which are expected greater than 1 Hz.

All gyroscopes are subject to a gravity gradient force because the center of mass of the space vehicle is approximately 23 cm away from the location of the first gyroscope; each subsequent gyroscope is 8.8 cm further from the vehicle mass center along the vehicle roll axis. The gravity-gradient acceleration is on the order of $4 \times 10^{-7}$ m s$^{-2}$ on the rotor closest to the vehicle mass center (Gyroscope 1) and increases linearly with distance to the other gyroscopes. Drag-free operation in effect moves the vehicle center of mass to the location of the drag-free gyroscope, thereby reducing the gravity-gradient acceleration on the other gyroscopes as well.

To aid the controller in rejecting this significant disturbance, a gravity gradient feed-forward signal, scaled by the distance from the vehicle center of mass to the center of the gyroscope housing, is generated and added to the thrust commands sent to the space vehicle. In this case, the controller works primarily to reject the effects of vehicle resonance modes (minimal) and external disturbances from aerodynamic drag, solar pressure, and other environmental disturbances.

![Figure 10. Free-floating drag-free control topology. Active control path shown by thick lines.](image-url)
The main advantage of this mode is that the rotor is free-floating, and thus it represents the purest implementation of a drag-free system and, in principle, minimizes the torques on the rotors by removing all support forces—a great value for this experiment. However, this topology does have some particular disadvantages for GP-B. Because of the need to protect the spinning rotor, the drag-free system is only allowed to operate in a relatively narrow dead band around the center of the housing. This leads to more frequent drag-free shutdowns by the GSS when the vehicle encounters external forces that temporarily overwhelm the capability of the ATC system (approximately 5 mN per vehicle axis).

This mode also does not allow for acceleration bias compensation in the sensor since it relies on relative position of the rotor and housing as its input. Any force (due to an electrical patch charge, for example) between the rotor and housing will cause the rotor to accelerate toward the housing. In turn, the vehicle will accelerate to follow. To hold the rotor at a fixed position in the housing, the ATC system would need to apply a constant acceleration on the vehicle, which would change the orbit over time.

6.2. Drag-free accelerometer mode

In this alternate mode, the gyroscope is suspended in science configuration by the GSS and the drag-free system flies the vehicle to minimize the measured suspension forces on the gyroscope. The ATC system works to drive the measured three-axis (vector) control effort on the proof mass, \( u \), to zero, via three-axis translation control commands, \( U \), as shown in figure 11. The active control path is shown in thick lines (gyro suspension controller block is disabled). The GSS passes rotor control effort information to the ATC controller through the gain \( K_2 \); any accelerometer biases are removed on the translation control side of the interface. At a low frequency, near space vehicle roll rate, the transfer function from \( U \) to \( u \) is simply a constant and, in principle, a simple integral control is all that is required to minimize \( u \). The controller is implemented in the same structure as the suspension-off drag-free mode,
but with different coefficients. Gravity gradient feed-forward is again applied to compensate for known large disturbance acceleration.

The chief advantage of this mode is that the spinning gyroscope is always actively centered by the GSS, and thus is at minimal risk of contacting the housing wall. In addition, acceleration is measured directly and thus a post-measurement accelerometer bias adjustment can be readily made in the ATC system; this feature was exercised on-orbit as noted below.

The disadvantage to the science measurement is that the drag-free gyroscope is always suspended, and thus is subject to greater electrostatic torques than a free-floating gyroscope because of the active suspension. However, this is not a significant disadvantage, since three of the four gyroscopes in primary drag-free mode need to be suspended against the gravity gradient in any case, and the experiment error due to these residual forces has been shown to be well within the requirements.

6.3. Drag-free flight performance

The design of the space vehicle allows any gyroscope to act as the drag-free reference; on-orbit, the selection of which gyroscope to serve as the drag-free sensor was largely driven by practical, operational concerns. To minimize the overall residual gravity-gradient acceleration on the gyroscopes, Gyroscopes 2 or 3 (near the center of the linear array) are preferable. Gyroscope 4 is the furthest from the center of mass of the vehicle, and thus would require the most thrust—the most helium usage—to force the space vehicle to free-fall around its center. Gyroscope 2, early on, required some control system tuning to optimize suspension performance and thus was not used for drag-free control during testing. Thus, drag-free testing and optimization was done with Gyroscopes 1 and 3.

Figure 12. Example transition sequence from non-drag-free to backup drag-free to prime drag-free operation.
Both the drag-free control modes were tested on the vehicle. Figure 12 shows an example transition sequence from non-drag-free, to accelerometer drag-free to free-floating drag-free control. During these transitions, the gravity-gradient acceleration moves from the gyroscope to the space vehicle ATC system as the space vehicle is forced to free-fall about the drag-free sensor. In free-floating drag-free mode, the suspension control efforts are disabled, so the vehicle must fly around the position of the rotor. Additional control activity in the ATC system is seen during this period.

Though free-floating drag-free is the preferred operational mode, it was not used during the science data-gathering phase of the mission. After on-orbit calibration, it was found that some gyroscopes exhibited a small acceleration bias, up to 20 nN, that the free-floating drag-free system would track. Though this bias would have no immediate effect on the drift performance the gyroscopes, the ATC control action over time would slowly change the space vehicle’s orbit. The accelerometer mode system could properly compensate for these biases and showed acceptable performance for the mission during testing, thus it was selected as the baseline drag-free mode during science data collection.

Representative backup drag-free performance is shown in figure 13. The top curve shows the spectrum of the space vehicle translation control effort showing the twice-orbit frequency gravity gradient signature at $3.4 \times 10^{-4}$ Hz (red line); the peak at $1.7 \times 10^{-4}$ Hz is the suspension system reacting to the 10 nm rotor asphericity as modulated by the rotor’s polhode motion. The residual acceleration on the rotor is $4 \times 10^{-11}$ m s$^{-2}$ from less than 0.01 mHz to 10 mHz in inertial space (averaging period: 24 h). If the gyroscope is viewed as an accelerometer, the noise floor amplitude spectral density is $1.2 \times 10^{-9}$ m s$^{-2}$ Hz$^{-1/2}$. Performance of the gyroscopes as accelerometers is limited by noise introduced through coupling from the spacecraft’s pointing system and the low signal-to-noise ratio on the position sensing bridge, as required for compatibility with the SQUID readout system. A GPS receiver on board the spacecraft measures the position of the vehicle in orbit and is used, together with ground-based laser ranging data, to confirm that the resulting vehicle orbit is indeed drag-free. This
orbit data has been used to identify and remove force biases in both the ATC and GSS systems.

7. Flight GSS implementation

Photographs of the GSS flight units are shown in figure 14 and figure 15. The aft-mounted control unit (ACU) and aft power supply (APU) is shown on the left, and the forward-mounted quiet analog electronics is on the right. One pair of boxes is required for each gyroscope. During the mission, the temperature of the aft assembly ranges between 300 K and 320 K; the forward assembly ranges between 300 K and 320 K.

The forward suspension unit (FSU) resides in the spacecraft’s forward equipment enclosure, which is passively thermally controlled to a range of 300–320 K over the mission. The FSU contains the sensitive analog electronics that are used to sense the position of the rotor and apply suspension voltages. A simplified block diagram of this system is shown in figure 16. The main components of this system are described below.
7.1. Low voltage drive amplifier (gain = 5)

During nominal science operation, voltages are applied to the electrodes through six ±50 V low noise amplifiers each with a nominal gain of 5 with bandwidth 100 Hz. Each amplifier has an input filter set that allows two modes of operation: (1) constant gain from dc to 100 Hz, and (2) a ‘science band pass’ path that rolls off the gain below 1 Hz to a gain of 0.5. The ‘backup dc coupled’ path is provided to simplify the design and operation of the backup analog control loops. The band pass mode is used to limit the electrostatic force noise on the gyroscope by allowing only signals in the neighborhood of the 20 Hz ac science drive carrier to make their way to the gyroscope.

7.2. High voltage drive amplifier (gain = 200)

A ±1600 V set of amplifiers is used during ground test and gyroscope spin-up to provide enough electrostatic force on the gyroscopes to suspend them in a 1 g field or while spin-up gas is flowing in the channel. A 1 g ground suspension requires approximately 650 V to maintain suspension once centered, and roughly twice that to lift the rotor from the housing wall. Gas spin-up requires about 150 V per axis to hold the gyroscope in the spin-up channel. Bandwidth of this amplifier is 2 kHz and power draw during spin-up is 800 mW per channel. The outputs of the low and high voltage amplifiers are switched to the gyroscope by a high voltage RF vacuum relay, one per electrode. Switching time is approximately 1 ms, during which the rotor is not under active suspension control. During the mission, careful checks are made on vehicle activity prior to making the switch so as to ensure the gyro will not move significantly during this uncontrolled period.

7.3. Digital to analog converters (AD)

Six 16-bit digital-to-analog converters are used to send voltage commands to both the high and low voltage amplifiers. The update rate for the system is 220 Hz in science mode, 660 Hz during spin-up, and 1 320 Hz in ground test configuration.
7.4. Position bridges

A 3-phase, 34.1 kHz sinusoidal position bridge is used to measure the position of the rotor in the housing. The bridge sense signal amplitude is selectable between 10 mV for the science configuration and 100 mV for the spin-up and ground test. Measurement noise is 1 nm Hz$^{-1/2}$ and the bandwidth is approximately 1 kHz. A 16-bit successive-approximation analog-to-digital converter digitizes each bridge channel and this information is sent to the GSS processor.

7.5. Analog backup control: spin-up backup, science high backup, and science low backup

Three all-analog backup controllers are provided to cover all flight operational modes. These controllers are of PD type and are designed to center the gyroscope in the event of a failure of the computer in order to maintain positive control of the gyroscopes. During science data collection, two controllers are available: science mission high backup (SMHB) and science mission low backup (SMLB). The high backup controller is designed to have sufficient gain and bandwidth to allow re-centering of the gyroscope under a worst-case micrometeoroid strike; the design point was a micrometeoroid with momentum of 1 kg m s$^{-1}$ maximum. This controller can use most of the 50 V range of the low voltage amplifier and is needed primarily to insure gyroscope safety, but will subject the rotor to higher suspension forces and torques than the experiment error budget allows. The low backup controller is much less aggressive and is designed to emulate the normal science mission suspension voltages. In the low backup mode, forces and torques are significantly reduced, thus preserving the science mission configuration to a much larger extent while computer control is re-established. In spin-up mode, a similar controller is provided to hold the rotor in position in the spin-up channel while gas is flowing in the case that computer control fails during this critical phase.

7.6. Arbiter state machine

An autonomous state machine in the FSU is used to monitor the health of the digital suspension algorithm and to immediately switch to a backup system should a fault be detected. The arbiter monitors the health of the GSS processor via a ‘heartbeat’ signal that indicates the processor is functioning normally and the position of the rotor via radius-from-center position thresholds. Should either of these tests fail, the backup system is engaged. In science mode, control is passed initially to the high authority backup controller (SMHB). Once the gyro position is stable for 30 s, control is passed to the low authority (SMLB) controller where it remains until the computer is ready to take control. In the event of a position excursion greater than the position threshold (>10 μm), the high authority controller is used until the position settles again. Before control may be passed back to the flight computer, a handshake set of commands must be sent to the FSU to convince the arbiter that the computer is healthy and ready to take back control.

7.7. Flight computer/DSP

Each GSS employs a dedicated Lockheed-Martin Federal Systems (now BAE) RAD6000 processor which functions as the primary Command and Control Computer Assembly, running at 16.368 MHz. Onboard is 1 MB of EEPROM and 3 MB of static RAM. The processor communicates with the spacecraft via a Mil-Std-1553 serial link and with the GSS electronics via a parallel port on the processor card. This processor holds the GSS software and telemetry processing needed to receive commands and send measurements to the ground via the
spacecraft processor. The telemetry system continuously provides data at 2 s intervals of gyro position, control effort, suspension voltages applied, system state, analog monitor points, box temperatures, and other engineering information. Also available are high-speed snapshots consisting of 12 s of full rate suspension control loop measurements and commands.

7.8. Power supply

Mounted external to the ACU enclosure is the GSS power supply, a custom designed dc/dc converter set to supply the required voltages to the system. This system supplies single-string +3.3, +5.0, and ±15.0 V to the ACU as well as redundant +5, ±12, and high voltage (either ±800 or ±1200) to the FSU. In addition to supplying electronics power, the system also supplies voltages for survival heaters used when the boxes are unpowered. Normal operation naturally generates sufficient heat to eliminate their need during the in-orbit operational checkout and the science data-gathering phase of the mission.

8. Performance validation—GSS Testbed

One of the most challenging aspects of the GSS development program was to ensure that the suspension system would actually meet its performance requirements on orbit. It is not possible to use a real gyroscope to verify the performance of the flight suspension system prior to launch. In order to minimize suspension-induced drift on the rotor, the flight suspension system applies fractions of a volt to keep the rotor centered. On the ground such low voltages would have no measurable effect at all on a gyroscope. The flight system’s 1 g mode allows levitation of gyroscopes on the ground. Although this is useful as a ‘torture test’ of the hardware, it provides no particular insight into how the suspension system will perform with respect to the subtle science requirements. Although the dynamics of the gyroscope are identical on Earth and in orbit, the fact that the ground-based disturbance environment is $10^7$ times larger than the relevant on-orbit environment makes it impossible to use it to verify suspension system requirements directly.

The GSS/gyro system requirements are particularly difficult to verify for flight based only on analysis. Even at its simplest, the suspension system is a three-input, 6-output nonlinear controller connected to a nonlinear plant. The system’s overall performance is a complex function of the closed-loop behavior of the system interacting with the on-orbit disturbance environment.

In addition, the failure modes of subsystems within the suspension system must be considered. For instance, the high radiation environment encountered on orbit may cause the computer that is running the suspension algorithm to occasionally crash. When this happens, the system must sense that the computer has lost control of the gyroscope, using an analog state estimator, and switch to an analog backup mode. When the computer comes back online, the analog backup must gracefully transfer control back to the digital controller. All this happens while the surface of the rotor is moving at approximately 20 m s$^{-1}$ only 20 µm away from the housing wall. Given the complexity of this operation, it is vital not to depend only on analysis or simulation to verify performance. The real hardware, operating in a closed loop, should be used.

The GSS Testbed was developed to be an electromechanical surrogate gyroscope constructed to address this verification need. Its purpose is to provide a simulator with flight interfaces that behaves like a real gyroscope subject to any defined disturbance environment. To meet this goal, it should:
(1) Have the same inputs and outputs as a gyroscope.
(2) Respond according to the dynamics of a gyroscope.
(3) Have a disturbance environment that can be specified by the user.

As noted earlier, electrically, the gyroscope is a set of seven capacitors with vacuum dielectrics—6 electrodes and a ground plane—whose relative values change as a function of rotor position and orientation in the housing. The Testbed consists of six parallel-plate variable capacitors whose values are controlled by commanding the spacing between the plates. A digital algorithm calculates the modeled dynamics of a gyroscope, and a controller forces the plates to follow those dynamics. Since the suspension system is basically a capacitance control device, the Testbed’s fundamental interface to the suspension system is the capacitance between the plates; see figure 17.

Although the parallel plate capacitors are, physically, a very different geometry than a real gyroscope, it is still possible to present the correct set of capacitances to any desired accuracy. A gyroscope has a spherical geometry where the rotor has three degrees of freedom as it moves about the housing. However, the Testbed has six degrees of freedom since each of the electrode-rotor pairs can be independently controlled. This gives the Testbed the capability of faithfully reproducing the position-capacitance relationship of a real gyroscope.

8.1. Testbed design

The final mechanical design of a Testbed carriage is shown in figure 18; a description of the key components is given below.

8.1.1. Piezo actuator. Electrorestrictive actuators control the relative spacing of the disks. These are similar to the more familiar piezoelectric actuators in that they are crystals whose dimension changes as a function of the electric field across the crystal. Electrorestrictive
devices have less position hysteresis, however. These actuators are cylinders approximately 100 mm long and 6 mm in diameter. Their range of motion is 75 μm with a small-signal bandwidth of about 1 kHz.

These electrorestrictive actuators are the heart of a linear positioning fixture shown in figure 18 that controls the spacing between the two parallel plates containing the gyro and measurement electrodes. The mechanical structure’s lowest resonance is approximately 700 Hz.

8.1.2. Sensor disks. The electrode whose capacitance is being controlled must be connected to the flight suspension system and is consequently not available to the Testbed as a measurement of the capacitance between the disks. Therefore an additional set of measurement electrodes are provided for the Testbed control system’s use on each gyroscope electrode substrate. Clearly, for the Testbed to function properly it is necessary for the capacitance between measurement electrodes to correlate in a stable way with the capacitance between gyroscope electrodes. This calibration is done prior to connection to the GSS hardware.

The mechanical substrate for each sensor disk is a quartz cylinder with a thickness of 12 mm and a diameter of 50 mm. Each sensor disk must contain both gyroscope and measurement electrodes. The gyroscope electrode is deposited in the center and is designed to have approximately the same area as a real gyroscope’s electrode. This means that when the Testbed is implementing a given rotor displacement, the disk separation is roughly equal to the rotor-electrode spacing of a real gyroscope.

Deposited around the periphery of the disk is the measurement electrode that is used by the Testbed hardware to detect the disk separation. After initial setup of the system, a set of calibration data is taken which determines the mapping function between measurement electrode capacitance and gyroscope electrode capacitance. Clearly the stability of this mapping function is of great importance. Careful mechanical design yield a mapping function that is stable to better than 10 nm. This is the accuracy of the Testbed positioning, and is well within requirements.

Temperature variation in the system is the dominant contributor to drift in the mapping function. The closed-loop positioning noise is approximately 10 pm Hz$^{-1/2}$. In order to minimize crosstalk between the two sets of electrodes, a shield was provided between the
gyroscope and measurement electrodes. The core of the shielding scheme is a ground plane that underlies the other electrodes and a choke ring between the opposing disks. In the end, the measured attenuation between the measurement electrode and the gyroscope electrode is approximately 180 dB at 34.1 kHz, the suspension system’s position measurement frequency; see figure 19.

Each actuator has one pair of sensor disks, and therefore one pair of gyroscope electrodes. One side represents a suspension electrode and is thus connected to the suspension system via a flight-like interface. The other side represents the rotor. The six rotor electrodes are connected together electrically with wires whose potential is allowed to float.

The Testbed must position the plates of the capacitors to very high precision. Suspension system requirements mandate that the rotor’s position must be controlled at the satellite roll frequency to better than 0.3 nm across the nominal 32 μm gap. This dictates a positioning precision of better than 1 part in 10⁶. In addition, the Testbed must function across the broad dynamic range corresponding to the suspension system’s different modes of operation in order to provide an accurate test of the flight suspension system. Therefore it inherits the requirement to function across an extremely broad dynamic range and must simulate the harsh 1 g dynamics through the gentle drag-free modes of operation. Although it technically is not necessary to model a 1 g gyroscope—real gyroscopes operating on Earth are available in the lab—it is desirable to include this function so that the Testbed’s performance can be compared to that of a real gyroscope. In so doing the Testbed serves to validate the gyroscope model.

8.1.3. Suspension system interface. This provides a plug-compatible with the interface to the flight probe. It connects the GSS Measure to the voltage the suspension system is applying to levitate the sphere. It must perform a high-impedance, low-noise measurement without affecting the operation of the capacitance bridge.

8.1.4. Plant model and controller. The hardware is controlled by a Texas Instruments TMS320C40 DSP that implements a model that transforms the natural dynamics of the actuators discussed above into the dynamics of a modeled gyroscope in the desired disturbance environment. Figure 20 shows a simplified block diagram of the suspension...
system and Testbed system working together. One iteration of the loop goes through the following steps:

1. The suspension system senses the rotor position via its capacitance bridge connected to the Testbed’s gyro electrodes. The GSS controller $C_{GSS}$, then chooses a force to apply to the rotor.

2. The suspension system’s force to voltage mapping (INL) chooses a set of voltages to apply to the gyroscope.

3. The Testbed senses those voltages via the interface box. It also senses the rotor displacement via the measurement electrodes. It uses this information to compute the force that the suspension system is applying to the rotor (the NL block).

4. This force is summed with whatever external force is desired and integrated to yield a calculated rotor trajectory.

5. The Testbed controller then makes the six actuators follow that trajectory. This is done via a standard feedback control system called the ‘Inner Loop’ in figure 19. Depending on the bandwidths required, it may be necessary to further shape the resultant transfer function using the transfer function $D_{TB}$.

8.2. GSS performance as measured by the Testbed

The GSS Testbed was used to test all operational modes of the GSS flight hardware and to evaluate how well the flight hardware met the science performance requirements.

Functionally, the GSS flight hardware was shown to operate properly in the science configuration and in spin-up modes with simulated gas flow. Transitions from the prime, to backup, to prime controller were exercised via simulated micrometeoroid impacts or simulated computer failures. The performance of the AOD controller was probed using external disturbances of varying magnitude. A stringent test was performed to suspend the Testbed gyro under a simulated 1 g field—laboratory test conditions—and was found to be nearly indistinguishable from a physical gyroscope.

Science level performance was also extensively tested. A disturbance model that included space vehicle translation and rotation, space environment (gravity gradient, micrometeoroid impacts), and worst-case models of the measured thermal sensitivities of the flight electronics at the science critical frequencies (dc, space vehicle roll, roll ±2×orbit frequency).
summary of the results is given in table 3; full details of the verification process are given in Brumley (2004). Overall, the GSS design met or exceeded its science requirements by a healthy margin, lending confidence to the belief that the GSS system would perform as expected on orbit.

Since the Testbed was focused specifically on gyro suspension dynamics, some aspects of the GSS operation that relied on interaction with other subsystems of the spacecraft—such as spin axis alignment (SQUID/SRE) and drag-free control (ATC)—were not verifiable directly on the Testbed. These other functions were verified via laboratory testing, system analyses, and integrated system tests during space vehicle assembly.

After launch, the Testbed was an invaluable aid during GSS turn-on and testing. During checkout, the Mission Operations team found that the suspension in the Science Mission AOD mode was failing and dropping into the backup modes. Analysis of snapshot data taken from the suspension system on the flight vehicle showed that there was a large force pulling the gyro off center, which overwhelmed the suspension mode. This force also appeared to be consistent with a very high rotor charge. The GSS has the ability to measure the rotor charge, but only in the digital modes. With the Testbed we were able to rapidly test his hypothesis and generate a simulation that matched the observed behavior with about 500 mV of charge on the rotor; then initial suspension system parameters were set to 0 V and it was expected that only 10–20 mV charge would be on the rotor after launch. Once this new charge estimate parameter was uploaded to the spacecraft, the digital suspension system worked as expected and the charge measurement system indicated 480 mV on the rotor. The charge was subsequently brought to zero using the UV charge control system. Without such a high fidelity simulator in the loop system, what was an overnight task would have ballooned into weeks of analysis in order to identify the root cause.

9. Conclusions

The GP-B GSS was one of the four major electronics instrumentation systems aboard the GP-B spacecraft and was responsible for the electrostatic suspension of the science gyroscopes. The flight GSS system was the result of 30 years of research and development work at

| Title | Specification | Achieved |
|-------|---------------|----------|
| 1     | \( \{S_x, S_y, S_z\} < 0.08 \text{ V}^2 \) | \(< 0.08 \text{ V}^2 \) |
| 2     | \( 0.5(S_x+S_y)-S_z < 0.04 \text{ V}^2 \) | \(< 0.001 \text{ V}^2 \) |
| 3     | \( S_x \text{ (roll)} - S_y \text{ (roll)} < 1.7 \times 10^{-6} \text{ V}^2 \) | \( 0.27 \times 10^{-6} \text{ V}^2 \) |
| 4     | \( S_{x,y,z} \text{ (roll)} < 1.7 \times 10^{-5} \text{ V}^2 \) | \( 0.27 \times 10^{-6} \text{ V}^2 \) |
| 5     | \( S_{x,y,z} \text{ (2 \times roll)} < 0.05 \text{ V}^2 \) | \(< 0.001 \text{ V}^2 \) |
| 6     | \( R < 0.6 \text{ \(\mu\)m} \) | \(< 0.03 \text{ \(\mu\)m} \) |
| 7     | \( R(\text{roll}) < 0.3 \text{ nm} \) | 0.034 nm |
| 8     | \( R(\text{roll} \pm 2 \times \text{orbit frequency (gravity gradient)} < 3 \text{ nm} \) | \(< 0.1 \text{ nm} \) |
Stanford University, and the system took full advantage of advancing control theory and digital technology in the 1990s to create a low-torque suspension system that would protect the gyroscope from impulsive disturbances in the form of micrometeoroid impacts, radiation-induced computer failures, and other environmental effects. In addition to suspension, the system was able to create controlled torques for the initial alignment of the gyroscopes’ spin axes after initial gas spin-up, and act as an accelerometer for the drag-free control system in conjunction with the spacecraft’s ATC System. To verify the flight performance of the GSS system, an innovative hardware in the loop Testbed was developed to act as a precision gyroscope surrogate that appeared as a physical gyroscope to the suspension system, but that could be controlled to simulate the on-orbit environment to assess both the functional and performance properties of the suspension system. With this Testbed and other ground testing, the GSS instrument was shown to meet or exceed all its flight requirements, and subsequently successfully performed all required operations on-orbit.

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