Constant Time-Headway Spacing Policy with Limited Communication Range for Discrete Time Platoon Systems

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Abstract: We present a study of the scaling properties of the interconnection of n agents (e.g. vehicles) through an r-lookahead network. These networks are considered as a possible implementation for vehicle platooning, although we do not make any assumptions on what the agents represent, and we assume them to be linear time invariant (LTI) discrete time systems, locally controlled by an LTI controller. In particular, we show that the r-lookahead topology gives rise to dynamics which can be studied from the roots of polynomials with transfer functions as their coefficients. Through numerical simulations, we study aspects relating the use of lookahead measurements and their effect on the value of a time headway constant needed for the scalability property known as string stability.

Keywords: Discrete-time systems, Platooning, String stability, Interconnected systems, Linear control systems

1. INTRODUCTION

The study of unidirectionally interconnected vehicular agents has been of interest for many years (Middleton et al., 2010; Knorn et al., 2016; Stüdli et al., 2017; Feng et al., 2019). Applications can be found in such diverse areas as irrigation channels (Cantoni et al., 2007), supply chains (Perea et al., 2000), harmonic oscillators (Yu et al., 2015) and vehicle platooning (Flores et al., 2018; Darbha et al., 2018).

The unidirectionality and low information flow between agents is known to enable instances for the amplification of disturbances along the string of agents. Topologies that allow to compensate for this phenomenon are said to achieve String Stability. Many results focus on achieving string stability for homogeneous agents, (Flores et al., 2018, ?), whilst other results extend the discussion to heterogeneous agents, (Bian et al., 2019; Rodonyi, 2019). Most of the results have been derived for the case where the agents are continuous time systems. Recently, Vargas et al. (2018) considered extensions of String Stability results for the discrete time case, motivated by the fact that Cooperative Adaptive Cruise Control Systems (CACC) rely on wireless communications that can be usually more easily treated in a discrete time setting.

A possible analysis approach for continuous time interconnected agents in a chain is to make use of the Laplace transform in the time domain and the Z-transform in the spatial domain to define a string stability transfer function as

\[ \Gamma_i(s) = \frac{y_i(s)}{y_{i-1}(s)} \]

where \(y_i\) is the position of the \(i\)-th agent.

In this work, we aim to extend results from Konduri (2017); Bian et al (2019) where, for the continuous time case, extending the range of communications and their impact on the needed time headway for string stability were studied. In both works, the agents considered had fixed simple dynamics, and it was assumed that the agents had measurements of the position, velocity and acceleration of the nearest \(r\) predecessors ahead of them. Our contribution is to perform a first approach to the discrete time case with general models for the agents (without specifying their nature a priori), and only assuming that they can measure the output, and the rate of change of the outputs, of the \(r\) nearest predecessors. The latter is a key departure from the analysis of continuous time systems in a platooning setting. We provide formulae for the resulting dynamics in a general setting of the interconnection and we also consider the use of a constant time-headway spacing policy in a platooning setting.

Notation: In this work all matrix entries not explicitly stated are assumed to be zero and, for simplicity in the exposition, all transfer functions will have omitted arguments (the frequency domain variable \(z\)) unless needed.
2. PRELIMINARIES

We will consider \( n \) agents, which could be vehicles, with the same frequency domain description given by

\[
Y_i = H(U_i + D_i),
\]

where \( Y_i \) is the Laplace transform of the output of the \( i \)-th agent \((y_i(k))\), \( D_i \) is the Laplace transform of its input disturbance, \( H \) is a transfer function representing the agent LTI dynamics and \( U_i \) is the Laplace transform of its control action.

For the considered setting, the control signals \( U_i \) at every agent are defined as

\[
\begin{align*}
U_1 &= 0, \\
U_2 &= k(Y_1 - Y_2 - \Delta_2), \\
U_i &= k \left( \sum_{k=2}^{i-1} B_{k-2} Y_{k-1} + A Y_{i-1} - Y_i - \Delta_i \right), \quad 2 < i \leq r, \\
U_i &= \left( \sum_{k=0}^{r-2} B_k Y_{i-k-2} + A Y_{i-1} - Y_i - \Delta_i \right), \quad r < i \leq n,
\end{align*}
\]

where \( A \) and \( B_i, i = 0, \ldots, r - 2, \) are proper and stable transfer functions, \( K \) is a controller that stabilizes \( H \) in closed loop and \( \Delta_i \) reflects a certain reference policy or desired inter-agent spacing. Note that \( \Delta_i \) could be a constant but it could also contain other terms such as the rate of change of the \( i \)-th agent in platooning applications. In such cases, it would be possible that \( A \) and \( B_i \) have to be modified accordingly.

Remark 1. Having \( A \neq 0 \) for all \( z \) implies that the \( i \)-th agent has available information from its immediate neighbour with index \( i - 1 \). In the platooning setting this implies that the vehicle senses its distance from the nearest front neighbour, i.e. its predecessor.

Having \( B_i \neq 0 \) for all \( z \) implies that the \( i \)-th agent also has information from the agents up to the one with index \( i = 1 \) if \( i \leq r \) or up to the agent with index \( i = r \) otherwise. In a platooning setting this would correspond with the availability at the \( i \)-th agent of the lead vehicle information if \( i \leq r \) or of the \((i-r)\)-th agent otherwise. The latter would imply that there exists a limitation on the range of the inter-agent communications, which is a reasonable assumption in real applications.

The input of the controller is then a weighted sum of the outputs of the predecessor within the communication reach.

We will now consider as an illustrative example the case \( r = 2 \) and \( \Delta_i = 0 \) for all \( i \). That is, the agents possess information of their two nearest predecessors. Moreover, since the dynamics are LTI, we will consider zero initial conditions for the agents, that is \( y_i(0) = 0 \), for all \( i \).

Now, the dynamics of the agents can be written in matrix form as

\[
\begin{bmatrix}
\dot{Y}
\end{bmatrix} = (I - KHG)^{-1} HD.
\]

where \( \begin{bmatrix} Y_1 & \cdots & Y_n \end{bmatrix}^T, D = [D_1 \cdots D_n]^T, I \) is the \( n \times n \) identity matrix and \( G \in \mathbb{C}^{n \times n} \) is given by

\[
G = \begin{bmatrix}
0 & -1 & & \\
1 & B_0 & A & -1 \\
& \ddots & \ddots & \ddots \\
& & 1 & B_0 & A & -1
\end{bmatrix}.
\]

Now, in order to obtain closed form expressions for every element of the matrix \((I - KHG)^{-1} HD\) we can write \((I - KHG)^{-1}\) in the following way

\[
(I - KHG)^{-1} = \begin{bmatrix}
-T^{-1} & \zeta_{n-1} & \zeta_{n-1}^{-1} \\
-KH(1 + Be_k) \Phi^{-1} & \zeta_{n-1} & \zeta_{n-1}^{-1}
\end{bmatrix} \Phi^{-1},
\]

where \( \zeta_{n-1} \in \mathbb{R}^{n-1} \) is a vector of zeros, \( e_k \in \mathbb{R}^{n-1} \) is a canonical vector of \( \mathbb{R}^{n-1} \) (only the \( k \)-th entry is non zero and equal to 1) and \( \Phi \) is the \( n - 1 \times n - 1 \) lower triangular matrix

\[
\Phi^{-1} = \begin{bmatrix}
T^{-1} & \dots & \dots & \zeta_{n-1}^{-1} \\
-T & \zeta_{n-1} & \zeta_{n-1}^{-1} & \zeta_{n-1}^{-1} \\
& \ddots & \ddots & \ddots \\
& & -B_0 & A & -1 \\
& & f_1,1 & \ddots & \ddots \\
& & \ddots & \ddots & \ddots \\
& & \ddots & \ddots & \ddots \\
& & \ddots & \ddots & \ddots \\
& & f_{n-1,1} & \cdots & \cdots \\
& & f_{n-1,n-2} & \cdots & f_{n-1,n-1}
\end{bmatrix}
\]

with \( T \), the usual complementary sensitivity function of the local closed loops, given by

\[
T = \frac{KH}{1 + KH}.
\]

The inverse \( \Phi^{-1} \) is also a lower triangular matrix with constant diagonals, whose elements are given by

\[
f_{i,j} = \begin{cases}
F_{i-j} & \text{if } i \geq j \\
0 & \text{if } i < j
\end{cases}
\]

and the sequence \( \{F_k\} \) is such that \( F_0 \) corresponds to the main diagonal and \( F_1 \) to the first sub-diagonal, etc. As \( \Phi \) only has three non-zero diagonals, it is straightforward to note that the sequence \( \{F_k\} \) satisfies the recursion

\[
F_{k-1} = ATF_{k-2} + B_0TF_{k-3},
\]

with initial conditions \( F_0 = T \) and \( F_1 = AT^2 \). We can now solve the recursion (14) using any method. Using the \( Z \)-transform we have that

\[
F_k = \alpha_1 \lambda_1^{k-1} + \alpha_2 \lambda_2^{k-1},
\]

where \( \lambda_1, \lambda_2 \) are the roots of \( \lambda^2 - AT\lambda - B_0T = 0 \), that is

\[
\lambda_{1,2} = \frac{AT \pm \sqrt{(AT)^2 + 4B_0T}}{2},
\]

and

\[
\alpha_1 + \alpha_2 = T,
\]

\[
\alpha_1 \lambda_1 + \alpha_2 \lambda_2 = AT.
\]

Solving for \( \alpha_1 \) yields
\[ \alpha_1 \sqrt{(AT)^2 + 4B_0 T} + \frac{AT^2}{2} - \frac{T \sqrt{(AT)^2 + 4B_0 T}}{2} = AT^2 \]

and therefore
\[ \alpha_{1,2} = \frac{T}{2} \pm \frac{AT^2 \sqrt{(AT)^2 + 4B_0 T}}{2} \]

It is to be expected that all the terms with radicals disappear in the final expressions for \( F_n \), since we are just inverting a matrix with rational functions. However, the roots of the polynomial describing the recursion given in (14) contain implicit information about the limiting behaviour of \( F_k/F_{k-1} \), much like for the Fibonacci sequence. For example, if \( |\lambda_2| < 1 \) on the unit circle, we should have
\[ \lim_{k \to \infty} \left| \frac{F_k}{F_{k-1}} \right| = |\lambda_1| = \frac{|AT + \sqrt{(AT)^2 + 4B_0 T}|}{2} \]

Remark 2. In platooning applications, it is common to consider the study of the inter-vehicle spacings errors \( \epsilon_i(t) = y_i(t) - y_i(t) - \delta_i, i > 1 \), where \( \delta_i \) is the desired inter-vehicle spacing. The frequency domain of transfer functions \( F_k \) can be easily built from the formulae for \( Y \) in (6), already obtained.

Therefore, it is of interest to study the behaviour of \( F_k \) and \( F_{k-1} - F_k \) as \( k \) grows large. For platooning applications, it would be especially important to characterise conditions for the parameters that will ensure that the sequence of transfer functions \( F_k \) is bounded uniformly with \( k \) under certain norms.

3. ARBITRARY COMMUNICATION RANGE

It is well known that having information from the leader is enough to ensure good scalability in platooning applications (see for example Seiler et al. (2004)). It would then make sense, in the case that the communication range is restricted, to have every agent reaching for the agents closest to the leader that it can. We consider now the block matrix

\[ G = \begin{bmatrix} G_0 & G_2 & G_1 \end{bmatrix} \]

where

\[ G_0 = \begin{bmatrix} 0 & 1 & B_0 & A & -1 & B_0 & B_1 & A & -1 \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \end{bmatrix} \]

is an \((r+1) \times (r+1)\) matrix, \( G_1 \) is the \((n-r-1) \times (n-r-1)\) lower triangular Toeplitz matrix with non zero diagonals given by the vector \((-1, A, B_r-2, B_{r-3}, \ldots, B_0)\) (that is, the main diagonal is all \(-1\), the first sub-diagonal is all \(A\), and so on.) and \( G_2 \) is given by

\[ G_2 = \begin{bmatrix} 0 & B_0 & B_1 & \cdots & A \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ 0 & \cdots & 0 & B_0 \\ 0 & \cdots & 0 & 0 \end{bmatrix} \]

(n - r - 1 rows).

4. PLATOONING AND THE CONSTANT TIME HEADWAY SPACING POLICY

In 1D platooning applications, when the leader state is not available to the followers, the spacing policy is relaxed, in order to avoid scalability issues such as string stability. This is a key compromise between safety and performance, as the relaxation will decrease the usage of the capacity of the traffic network. Each follower will keep a desired distance to its predecessor given by a fixed distance \( \delta \) (now assumed to be equal for all agents) plus a distance proportional to its instantaneous speed, through the use of a time headway constant \( h \).

For example, in Fig. 1, we can see a graphical representation of the agents in a platoon configuration. In such case, the signal \( \psi_i \) represents the measured distance to the \((i-1)\)-th agent made by the \(i\)-th agent. The signal \( \ell_i \) represents the measured distance to the \((i-r)\)-th agent made by the \((i-1)\)-th agent.

We will assume that the agents start at rest in the desired formation (when all the agents have zero velocity) and have the following control signals in the frequency domain.
we have that the current control scheme is equivalent to the interconnection studied in the previous section with
\[ A = \frac{1 - \eta W}{W}, \quad B_0 = \eta \frac{W}{W}, \quad B_i = \eta \frac{1 - W}{W}, \quad (38) \]
for \( i = 1, \ldots, r - 2. \)

\textbf{Remark 4.} For achieving the main goal of platooning, that is, every agent reaching the speed of the leader, while maintaining the desired inter-vehicle spacings, it is required that the product \( HK \) possesses two integrators (see for instance Seiler et al. (2004)). We will assume that this is indeed the case.

5. CONDITIONS FOR STRING STABILITY

The following Bode-like integral Lemma, taken from Seron et al. (2012), is key in determining the aspects of the local closed loops at each agent that could produce disturbance amplifications.

\textbf{Lemma 5.} Let \( T \) be a real rational scalar function of \( z \in \mathbb{C} \). Suppose that \( T(1) = 1 \) and also that \( T \) is stable. Then
\[ \int_0^\pi \log |T(e^{j\theta})| \frac{d\theta}{1 - \cos(\theta)} \geq \pi T'(1). \]

Lemma 5 can be used to establish that the complementary sensitivity function \( T \), defined in (12), satisfies \( ||T||_{\infty} > 1 \), since the product \( HK \) possesses two integrators, as stated in Remark 4 (Vargas et al, 2018).

According to results in Konduri (2017); Bian et al (2019) the string stability of the \( r \)-lookahead interconnection is completely determined by the spectral radius of the polynomial in (29). In general we cannot obtain analytical expressions for the roots of this polynomial, however, we can make some initial simple observations.

\textbf{Proposition 6.} The interconnected system (6) is string unstable for any arbitrary but finite communication range \( r \) if \( ||B_0T||_{\infty} > 1 \).

\textbf{Proof.} \( B_0T \) corresponds to the constant term of the polynomial in (29), which also corresponds to the product of all the roots of the polynomial (recall Vieta’s formulas). If there exists any \( \theta \in [0, 2\pi] \) such that \( |B_0(e^{j\theta})T(e^{j\theta})| > 1 \), we must have that the product of the roots of the polynomial is greater than 1 at said frequency, which implies string instability.

Although this result is straightforward, it provides an initial design restriction of the filters used to interconnect the agents. In particular, we have that \( B_0 = \eta / W \). Both \( \eta \) and \( h \) can be selected to ensure that \( ||B_0T||_{\infty} < 1 \).

6. NUMERICAL EXAMPLES

We will consider a collection of agents modelled by double integrator dynamics, that is, in (1)
\[ H = \frac{1}{(z - 1)^2}, \quad (39) \]
and the control structure and spacing policy (30)-(33) with local controllers given by

\[ U_2 = K \left( Y_1 - Y_2 - \delta - h \left( Y_2 - \frac{Y_1}{z} \right) \right), \quad (30) \]
\[ U_1 = K \left( \eta \left( Y_1 - Y_i - (i - 1)\delta - h \sum_{k=2}^{i} \left( Y_k - \frac{Y_k}{z} \right) \right) \right) + (1 - \eta) \left( Y_{i-1} - Y_i - \delta - h \left( Y_i - \frac{Y_i}{z} \right) \right), \quad (31) \]
for \( 3 < i \leq r \), and
\[ U_i = K \left( \eta \left( Y_{i-r} - Y_i - (i - 1)\delta - h \sum_{k=i-r}^{i-1} \left( Y_k - \frac{Y_k}{z} \right) \right) \right) + (1 - \eta) \left( Y_{i-1} - Y_i - \delta - h \left( Y_i - \frac{Y_i}{z} \right) \right), \quad (32) \]
for \( 2 < r < i \leq n \), and
\[ U_i = K \left( \eta Y_{i-r} - \eta h \sum_{k=i-r}^{i-1} \left( Y_k - \frac{Y_k}{z} \right) \right) + (1 - \eta) \left( Y_{i-1} - \left( 1 + h \frac{z-1}{z} \right) Y_i \right), \quad (33) \]
Recalling Fig. 1, each agent is using the measurements \( \psi_i \) and \( \ell_i \) in order to build the input to the controller \( K \) by comparing them to the desired inter-vehicle spacings and the leader-follower spacings. If we consider the transfer function
where $h$ is the time headway constant for the used spacing policy. Since the product $HKW$ has two integrators, Lemma 5 implies that $\|T\|_\infty > 1$. In particular, for the data above $\|T\|_\infty \approx 1.856$.

This selection for the local controllers, with $W$ as a factor in the denominator, is made by many works that consider a constant time headway spacing policy. In particular Knorn et al (2013) computes the infimal value of the constant $h$ for the continuous time case in order to achieve string stability of a nearest neighbour communication topology. The discrete time case was first reported in Vargas et al (2018). In particular, the infimal value of the time headway constant for string stability when $r = 1$, $h_{inf}$ is given by the largest root of

$$2h(1 + h) - c = 0,$$  \hspace{1cm} (41)

where $c$ is computed as

$$c = \sup_{\theta \in (0, \pi)} \left\{ \left( \|T(e^{j\theta})\|^2 - 1 \right) / (1 - \cos \theta) \right\}. \hspace{1cm} (42)$$

For the considered data we have that $h_{inf} \approx 3.3566$.

For the time simulations, we will focus on the signals $e_f(t) = y_{i-1}(t) - y_i(t) - h(y_i(t) - y_{i}(t-1))$, \hspace{1cm} (43)

that is, the inter-vehicle errors from the desired spacings.

6.1 Effect of the time headway constant

In Fig. 2 we have the Bode plots of the transfer function $T/W$ for different values of the time headway constant $h$. For $r = 1$ this is the transfer function that defines the string stability of the interconnection, as it straightforward to note that the polynomial (29) has a single root at $T/W$. As computed before we see that for values of $h$ greater than 3.3566 the magnitude peak of $T/W$ occurs at $\theta = 0$ and equals unity. Otherwise, the peak is greater than one and string instability will occur.

In Fig. 3 we see the time response of 50 agents when $r = 1$ and the leader travels at 1 unit per second. For $h = 3.8 > h_{inf}$, the inter-vehicle errors $e_f$ do not grow along the platoon. However, the opposite occurs when $h = 2.8 < h_{inf}$. Note that Blue corresponds to agents closer to the leader and Red corresponds to agents farther away from the leader.

6.2 Effect of increasing the communication range

As given by (16), the eigenvalues of the interconnection can be obtained explicitly. For $\eta = 0.3$ in Fig. 4 we can see the magnitudes of these eigenvalues for different values of the time headway constant $h$. We can observe that when $h = 3.1 < h_{inf}$ both eigenvalues have magnitude less than 1. Therefore, amplification of disturbances should no longer be observed.

In Fig. 5 we see the time response of 50 agents for the same leader trajectory as before when $h$ is fixed at $3.1 < h_{inf}$. For $r = 1$, as predicted before, the transient amplifies along the string. For $r = 2$ and $\eta = 0.3$, we have that the platoon becomes string stable. This highlights the potential for increasing the platoon performance in the inter-vehicle spacings, at the cost of increased communication range.

6.3 Effect of varying the parameter $\eta$

In Fig. 6 we observe the effect of the parameter $\eta$ with fixed $h = 3.2$ and $r = 3$ with $N = 100$. We can see that increasing $\eta$ has a negative impact on the string stability of the platoon. When $r \to \infty$ (that is, when every agent communicates with the leader) it is known that $\eta > \|T\|_\infty^{-1}$ is sufficient to ensure string stability, even with $h = 0$, (see for instance Seiler et al. (2004)). It is unclear what is the trade-off between $\eta$ and $h$ when $r$ is finite.

According to results from Bian et al (2019), increasing $r$ yields smaller necessary values of the time-headway for string stability. However, this result was obtained with every follower possessing measurements of the position, velocity and acceleration of its predecessors. In the current setting, it is not clear whether this same conclusion is true and more research is required.
7. CONCLUSION

We have studied a platooning problem with limited range communications, where the agents are modelled by discrete time LTI plants. By using numerical simulations and a direct method for obtaining the resulting dynamics of the interconnected system we have demonstrated that increasing the forward communication range of an agent may allow for the relaxation of the necessary time headway constant value for string stability. More work is needed to characterize the relationship between the increase in the range and the control scheme parameters. We noted that if the parameters are not selected properly, the range increase may deteriorate the string stability property. Future works will be directed at understanding this and related issues.

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