The Characteristics of Debye Effect Magnetic Field Under Acoustic Excitation

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Abstract. Debye proposed the generation of electric potential difference under acoustic excitation in electrolyte solution because of ion separation. However, the ion separation can also induce magnetic field, which has always been neglected by researchers. This paper builds a theoretical model of Debye effect magnetic field, and a multi-physics computation method to study the magnetic field. According to the computation results, an acoustic excitation with frequency at 70Hz and amplitude at 0.1m/s can generate magnetic field at the level of one pico-Tesla, which can be detected nowadays.

1. Introduction
Debye effect, as called Electro-Acoustic effects or Ultrasonic Vibration Potentials, was firstly proposed by Debye in 1933. He predicted the generation of electric potential difference induced by ionization in electrolyte solution [1]. This phenomenon was originally utilized for determination of electrolytic ions mass with acoustic excitation, thus merely electric field force and friction force were taken into consideration.

The kinetic properties of ions were modified then. JJ Herms raised extra electrophoretic force, relaxation force, pressure gradient force and diffusion force on ions because of ionic atmosphere behavior as well as convection and diffusion effect [2]. A.L. Virovlyanskii introduced a Langevin random force to describe particle thermal motion [3-4]. JD Peddle corrected the ions kinetic equation with the addition of Lorentz force under the geomagnetic field [5]. O'Brien studied the inverse Debye effect by introducing the linear system theory [6].

The application of Debye effect is broad as well. The original usage of Debye effect is to measure the mass of ions in the electrolyte solution. Since then, Debye effect is widely used in the field of electrochemistry. Zana Raoul takes Debye effect into the determination of ionic partial volumes, which indicate the construction of hydration shell of ions [7]. DeLacey Emma H. B. extends Debye effect into dilute suspensions of colloidal, and measure its dielectric response and conductivity [8]. Stoimenova Maria explains the low frequency anomalies in colloidal electro-optic through Debye effect [9]. Muñoz Victor finds that the decays of electromagnetic waves in electron–positron plasmas are also related to
Debye effect [10]. Beveridge Andrew C raised a concept of polarization current in Debye effect, and uses this current in the field of imaging and medicine [11].

These modifications and applications described Debye effect more precisely, but they paid more attention on the analysis of electric potential difference. The generation of magnetic field has always been neglected. The lack of weak magnetic field sensors is the main reason for that. However, advanced atomic magnetometry can detect a magnetic signal lower than one pico-Tesla [12]. This huge progress in detection technology guarantees our research into Debye effect magnetic field phenomenon.

This paper focuses on the mechanism of Debye effect magnetic field induced by acoustic excitation in the electrolyte solution. Comparing with predecessors’ studies, we have done more progressive work in the following areas:

- We build the theoretical model of Debye effect magnetic field. Based on that, we analyze the mechanism and characteristics of the magnetic field.
- We utilize multi-physics simulation method to calculate the magnetic field under acoustic excitation. This method can provide more intuitive understanding of magnetic field. More detailed features can be analyzed so that more application field can be proposed.

2. Theoretical Formulation of Magnetic Field

Ions in electrolyte solution are surrounded by abundant water molecules. When disturbed by acoustic sound waves, all the particles will be vibrating in the solution, including ions and neutral water molecules. It is a reasonable assumption that the acoustic sound wave is a diverging spherical wave, spreading averagely along all directions, when the scale of acoustic source is much smaller than the scale of the survey region.

\[ v_0 = (u_0 / r) e^{-j(\omega t - kr)} \]  

(1)

Particles’ movement caused by stimulation source is shown as equation (1). Neutral water molecules follow this rule to move. Comparing with neutral water molecules, ions have more complicated behaviors because of carrying charges. These behaviors do not simply fit equation (1), on the contrast, they require modification on basis of Debye effect theory. Specifically, ions are influenced by electric field force, friction force, Lorentz force, concentration gradient force, pressure gradient force, relaxation force and dielectrophoretic force [13], as shown in equation (2). We use table 1 to explain the meaning of each force.

\[ m_i \frac{dv_i}{dt} = e_i E - \alpha_i (v_i - v_0) + e_i v_i \times B_e - \frac{kT}{n_i} \nabla n_i + V_s s_0 \frac{dv_0}{dt} - \frac{e_i E \kappa a}{6\pi \eta_0} - e_i |e_i e_2| q \kappa E \frac{|e_i e_2| q \kappa E}{3DKT[1 + q^{(3/2)}(1 + io\theta)^{1/2}]} \]  

(2)

| Force                        | Expression                                      | Declaration                                      |
|------------------------------|-------------------------------------------------|-------------------------------------------------|
| Electric Field Force         | \( \vec{F}_{\text{Electric}} = e_i \vec{E} \)   | \( e_i \) is the charge, \( E \) is the electric field. |
| Friction Force               | \( \vec{F}_{\text{Friction}} = \alpha_i (\vec{v}_i - \vec{v}_0) \) | Where \( \alpha_i \) is the friction coefficient. |
| Lorentz Force                | \( \vec{F}_{\text{Lorentz}} = e_i \vec{v}_i \times \vec{B}_e \) | \( B_e \) is geomagnetic field.                  |
| Concentration Gradient Force | \( \vec{F}_{\text{Diffusion}} = -\frac{kT}{n_i} \nabla n_i \) | \( k \) is Boltzmann constant, \( T \) is the thermodynamic temperature. |
| Pressure Gradient Force      | \( \vec{F}_{\text{Convection}} = -V_s s_0 \frac{dv_0}{dt} \) | \( s_0 \) is the density of the solution. \( V_i \) stands for the effective volume. |
| Relaxation Force             | \( \vec{F}_{\text{Relaxation}} = -e_i |e_i e_2| q \kappa E \frac{|e_i e_2| q \kappa E}{3DKT[1 + q^{(3/2)}(1 + io\theta)^{1/2}]} \) | \( D \) is the dielectric constant of solution, \( \kappa^{-1} \) is Debye shielding distance, \( \omega \) is the frequency of the perturbation. |
To simplify the expression, we use a coefficient to represent the relationship between relaxation force, dielectrophoresis force and the electric field intensity, considering its linear relationship.

\[
C_i = -\frac{e\kappa a_i}{6\pi \eta_0} - e_i \frac{|e_i e_j| q\kappa}{3DkT[1 + q^{(1/2)}(1 + i\omega\theta)^{(1/2)}]}
\]  

(3)

The difference between ions and neutral water molecules results in a hysteresis of ions. In the meantime, the difference between various ions, especially in mass, effective volume and friction coefficient, also results in velocity disparity. The dynamic ion separation can ultimately generate electromagnetic field. The electromagnetic field can be described through Maxwell equations.

\[
\begin{align*}
\nabla \cdot \mathbf{E} &= \frac{\sum n_i e_i}{\varepsilon_0 \varepsilon_r} \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \cdot \mathbf{H} &= 0 \\
\n\nabla \times \mathbf{H} &= \sum n_i e_i \mathbf{v}_i + e_i \varepsilon_r \frac{\partial \mathbf{E}}{\partial t}
\end{align*}
\]  

(4)

To solve the magnetic field, we still need one more equation to describe the characteristics of ion concentration. There are several ways to settle the change of concentration. For example, Hypothesis of quasi-electric neutrality suggests that \( n_i \approx n_0 \), considering the small perturbations situation. Molecular kinetics theory suggests that concentration can be quantified through Vlasov equation [4]. Here we introduce the equation of continuity to describe the mass conservation of ions. It is always workable under system of classical physics.

\[
\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_i) = 0
\]  

(5)

Combine equation (1)-(4), the magnetic field is solvable. Considering the nonlinearity in the equation system, we use numerical computation method, which will be discussed below, to study the characteristics quantificationally.

3. Simulation and Results Analysis
We employ COMSOL Multiphysics to solve the equations. All the parameters in the equations are set as the Table 2 shows. All the relative parameters and variables are calculated under SI unit.

| Parameter | Value | Unit |
|-----------|-------|------|
| \( e_0 \) | \( 1.60219 \times 10^{-19} \) | C |
| \( a_{Na} \) | \( 1.92271 \times 10^{-12} \) | kg/s |
| \( a_{Cl} \) | \( 3.41182 \times 10^{-12} \) | kg/s |
| \( B_e \) | \( (3.5 \times 10^{-5}, -2 \times 10^{-6}, 3.3 \times 10^{-5}) \) | T |
| \( k \) | \( 1.38065 \times 10^{-21} \) | J/K |
| T | 298 | K |

(6)
Excitation source velocity is directly given under the diverging spherical wave hypothesis. Ions’ dynamics equation and continuity equation are solved in the PDE model, which allows us to write in partial differential equation in arbitrary formulation. Maxwell equation are solved in the AC/DC model, including electrostatic model and magnetic model. We have made a quasi-static hypothesis to simplify the calculation, since the magnetic field can be regarded as quasi-static under kHz level frequency. The coupling relationship in the electromagnetic field can be weaken under this assumption, and such simplifications can promote its stability and convergence performance. The coupling relationship in the equation system is demonstrated in figure 2.

We apply excitation with the form as shown in equation (6). The frequency of excitation is 70Hz, and sound velocity in water is 1500m/s. The study region we choose is a sphere with radius 100 meters long, and the excitation source is settled in the centre of the region.

| $V_{Na}$ | $1.33355 \times 10^{-29}$ | m$^3$ |
|----------|--------------------------|------|
| $V_{Cl}$ | $7.45153 \times 10^{-29}$ | m$^3$ |
| $s_0$    | 1000                     | kg/m$^3$ |
| $C_{Na}$ | $5.69631 \times 10^{-19}$ | C    |
| $C_{Na}$ | $4.29867 \times 10^{-19}$ | C    |
| $n_0$    | 540                      | mol/m$^3$ |
\[
\begin{align*}
\mathbf{v}_{\alpha} &= 0.1 \cdot \frac{x}{x^2 + y^2 + z^2} \sin(\omega t + kr) \\
\mathbf{v}_{\beta} &= 0.1 \cdot \frac{y}{x^2 + y^2 + z^2} \sin(\omega t + kr) \\
\mathbf{v}_{\gamma} &= 0.1 \cdot \frac{z}{x^2 + y^2 + z^2} \sin(\omega t + kr)
\end{align*}
\]  
(6)

With such excitation, we analyze the motion of ions and the generation of electromagnetic field.

Figure 2 shows that the ion vibrates in the same frequency as the acoustic source, and the intensity decays with the distance. The concentration of ions also follows this frequency feature, but it is superimposed on a large average value (540 mol/m^3). Figure 3 shows the separation of different ions from the perspective of ion velocity and concentration. Figure 4 indicates that the range of electromagnetic field intensity. The electric field can reach a level of $10^{-7}$ N/C, and the magnetic field can reach a level of $10^{-12}$ Tesla. Figure 5 and Figure 6 demonstrate how the magnetic field change with time and space. These results prove the frequency feature more directly.
4. Conclusion

We build a theoretical model of Debye effect magnetic field. Based on the theory, we employ a multi-physics computation method to solve the nonlinear equations and study the characteristics of the magnetic field. The computation result indicate that tiny acoustic perturbation can lead to a measurable magnetic field. This phenomenon can be applied in the field of magnetic field detection in the future.
References

[1] P. Debye, “A Method for the Determination of the Mass of Electrolytic Ions,” The Journal of Chemical Physics, vol. 1, no. 1, pp. 13–16, Jan. 1933, doi: 10.1063/1.1749213.

[2] J. J. Hermans, “XXXV. Charged colloid particles in an ultrasonic field,” The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, vol. 25, no. 168, pp. 426–438, Mar. 1938, doi: 10.1080/14786443808562026.

[3] A. L. Virovlyanskii and A. N. Malakhov, “Kinetic description of Debye effect,” Radiophys Quantum Electron, vol. 25, no. 9, pp. 710–715, Sep. 1982, doi: 10.1007/BF01043647.

[4] A. L. VIROVLYANSKII, “Electric field of an arbitrarily moving electrolyte,” p. 4, 1981.

[5] J. B. Peddell and P. D. Loach, “MECHANISMS FOR ACOUS’FO-ELECTROKINETIC COUPLING,” p. 6, 1996.

[6] R. W. O’Brien, “The response of a colloidal suspension to an alternating electric field,” Advances in Colloid and Interface Science, vol. 16, no. 1, pp. 281–320, Jul. 1982, doi: 10.1016/0001-8686(82)85021-5.

[7] R. Zana and E. Yeager, “Ultrasonic vibration potentials and their use in the determination of ionic partial molal volumes,” J. Phys. Chem., vol. 71, no. 3, pp. 521–536, Feb. 1967, doi: 10.1021/j100862a010.

[8] E. H. B. DeLacey and L. R. White, “Dielectric response and conductivity of dilute suspensions of colloidal particles,” J. Chem. Soc., Faraday Trans. 2, vol. 77, no. 11, p. 2007, 1981, doi: 10.1039/f29817702007.

[9] M. Stoimenova, A. Alekov, and T. Okubo, “Relation of electro-acoustic effects to low frequency anomalies in colloidal electro-optics,” Colloids and Surfaces A: Physicochemical and Engineering Aspects, vol. 148, no. 1–2, pp. 83–86, Mar. 1999, doi: 10.1016/S0927-7757(98)00558-5.

[10] V. Muñoz and L. Gomberoff, “Electro-acoustic damping effects on the parametric decays of electromagnetic waves in electron–positron plasmas,” Physics of Plasmas, vol. 7, no. 12, pp. 4916–4922, Dec. 2000, doi: 10.1063/1.1322065.

[11] A. C. Beveridge, S. Wang, and G. J. Diebold, “Imaging based on the ultrasonic vibration potential,” p. 4, 2004.

[12] Edelstein A S, Burnette J, Fischer G A, etc., “Advances in magnetometry through miniaturization,” Journal of Vacuum Science & Technology A, vol. 26, pp. 757–762, 2008.

[13] E. Yeager and F. Hovorka, “Ultrasonic Waves and Electrochemistry. I. A Survey of the Electrochemical Applications of Ultrasonic Waves,” The Journal of the Acoustical Society of America, vol. 25, no. 3, pp. 443–455, May 1953, doi: 10.1121/1.1907062.