Abstract

The classical limit for generalized partition functions is obtained using coherent states. In this framework it is presented a general procedure to obtain all the corrections to the classical limit. In particular, the first and second order quantum corrections are worked out explicitly, and the classical limit for the Tsallis thermostatistics is determined. The results of this work generalize the ones obtained by E. Wigner (Phys. Rev. 40 (1932) 749) for usual statistical mechanics.

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I. INTRODUCTION

Long-range interactions are present in many physical contexts. The nature of these interactions can be connected to spatial and temporal dependencies. For instance, anomalous diffusion [1–4], astrophysics with long-range (gravitational) interactions [5–9], some magnetic systems [10–12], some surface tension questions [13,14]. In such cases we have a nonextensive behavior. This fact indicates that usual thermodynamics and statistical mechanics deserves some changes. Thus, it is important to generalize the concepts based in the usual thermodynamics. In this direction, it has been recently analyzed the Legendre transform structure [15,16] and stability conditions [16] in a very general context. The present work is devoted for kind of study. More precisely, the purpose of this work is to obtain the classical limit and its quantum corrections for arbitrary partition functions.

It is important to obtain the classical limit and its corrections because, in some cases, it is easier to calculate the classical partition function than the quantum one. This problem was solved first for the usual statistical mechanics by Wigner [17] using the Wigner functions, and by Kirkwood [18] employing the Bloch equations (see also reference [19,20]). The procedure employed by Kirkwood can not be applied easily for generalized partition functions. On the other hand, the procedure used by Wigner can be in principle extended for arbitrary partition functions, but in this work we prefer to employ coherent states [21]. By using coherent states we classify and give the prescription to calculate all the possible corrections for the classical partition function. Employing this prescription we obtain explicitly the first two corrections.

The general results obtained in this work can be in principle applied to an arbitrary thermostatistics. In particular, they are used in the context of Tsallis thermostatistics. This thermostatistics was proposed in order to cover nonextensive systems (long-range microscopic memory, long-range forces, fractal space time). Thus, the Tsallis thermostatistics [22] has interesting proprieties and has been applied in many situations, like for instance, self-gravitating systems [23,24], two-dimensional-like turbulence [25], Lévy-like [26,30] and
correlated-like \cite{31-34} anomalous diffusion, solar neutrino problem \cite{35}, linear response theory \cite{36} and magnetic systems \cite{37-40}.

II. QUANTUM CORRECTIONS

In this work we consider a partition function for a very general statistical mechanics,

\[ Z = Tr f(\hat{H}), \]

(1)

where \( Tr \) represents the trace, \( \hat{H} \) is the Hamiltonian of the system, and \( f(x) \) is the generalization of the exponential function of the usual thermostatistics.

To calculate the classical limit of (1) and its corrections it is convenient to employ coherent states. In particular, two basic properties of the coherent states are important for our calculations. The first one is the definition of coherent states, namely

\[ \hat{a}_n | z > = z_n | z >, \]

(2)

where

\[ \hat{a}_n = \left( \sqrt{\frac{m_n \omega}{2\hbar}} \hat{q}_n + \frac{i}{\sqrt{2m_n \omega \hbar}} \hat{p}_n \right), \]

(3)

\[ z_n = \left( \sqrt{\frac{m_n \omega}{2\hbar}} q_n + \frac{i}{\sqrt{2m_n \omega \hbar}} p_n \right), \]

(4)

and \( | z > = \prod_{n=1}^{N} | z_n > \). In this expression for \( | z > \), \( N \) is the dimension of the configuration space, and each \( | z_n > \) is normalized, \( < z_n | z_n > = 1 \). Moreover, \( q_n \) and \( p_n \) are respectively the classical values for coordinate and momentum \( n \). The other property is the overcompleteness relation,

\[ 1 = \int \prod_{n=1}^{N} \frac{dp_n dq_n}{2\pi \hbar} | z > < z |. \]

(5)

Furthermore, employing the previous notation, we use \( \hat{A} \) to represent an operator and \( A \) to represent its classical analog.
To obtain the corrections to the classical partition function we basically expand \( f(\hat{H}) \) around its classical value, \( f(H) \). More precisely, we write \( \hat{H}(\hat{p}_n, \hat{q}_n) \) as \( \hat{H}(p_n + \delta \hat{p}_n, q_n + \delta \hat{q}_n) \) and express \( f(\hat{H}) \) as a power series in \( \delta \hat{H} \),

\[
f(H + \delta \hat{H}) = f(H) + \frac{df}{dH} \delta \hat{H} + \frac{1}{2} \frac{d^2 f}{dH^2} (\delta \hat{H})^2 + \ldots ,
\]

where

\[
\delta \hat{H} = H(p_n + \delta \hat{p}_n, q_n + \delta \hat{q}_n) - H(p_n, q_n),
\]

\[
\hat{H} = H(\hat{p}_n, \hat{q}_n) = \sum_{n=1}^{N} \frac{\hat{p}_n^2}{2m_n} + V(\hat{q}_n),
\]

\( \delta \hat{q}_n = \hat{q}_n - q_n \) and \( \delta \hat{p}_n = \hat{p}_n - p_n \). Note that in the present analysis we are considering a Hamiltonian that depends only on the spatial coordinates and its corresponding momenta.

By using the relation (5) we can now express the partition function as

\[
Z = \int \prod_{n=1}^{N} \frac{dp_n dq_n}{2\pi \hbar} < z \mid f(H + \delta \hat{H}) \mid z > ,
\]

and to calculate it we employ the expansion presented above. The first term of this expansion gives the classical partition function,

\[
Z_0 = \int \prod_{n=1}^{N} \frac{dp_n dq_n}{2\pi \hbar} f(H) .
\]

The corrections to (10) can be classified using a simple but important result, i.e.

\[
< z \mid \delta \hat{X}_1 ... \delta \hat{X}_u \mid z > \propto \hbar^{u/2} ,
\]

where \( \delta \hat{X}_n \) can be \( \delta \hat{q}_n \) or \( \delta \hat{p}_n \). In fact, the expression (11) comes directly from the definition of \( \delta \hat{q}_n \) and \( \delta \hat{p}_n \),

\[
\delta \hat{q}_n = \sqrt{\frac{\hbar}{2m_n \omega}} (\delta \hat{a}_n^\dagger + \delta \hat{a}_n) \quad (12)
\]

and

\[
\delta \hat{p}_n = i \sqrt{\frac{m_n \omega \hbar}{2}} (\delta \hat{a}_n^\dagger - \delta \hat{a}_n) ,
\]
where $\delta \hat{a}_n = \hat{a}_n - z_n$ and $\delta \hat{a}_n^\dagger = \hat{a}_n^\dagger - z_n^*$. Furthermore, when $u$ is an odd number a direct calculation leads to $< z | \delta \hat{X}_1...\delta \hat{X}_u | z >= 0$. Therefore, the corrections to (10) are proportional to integer powers of $\hbar$. Thus, the partition function can be written as

$$Z = Z_0 + \hbar Z_1 + \hbar^2 Z_2 + ... .$$

(14)

For instance, the first and second quantum corrections, $\hbar Z_1$ and $\hbar Z_2$, comes respectively from the second and fourth order expansions in $\delta \hat{q}_k$ and $\delta \hat{p}_k$.

Let us now proceed to calculate the first quantum correction. First we note, by means of equation (2), (12) and (13), that

$$< z | \delta \hat{H} | z > = \frac{\hbar \omega}{2} \sum_{n=1}^{N} \left[ \frac{1}{2} + \frac{1}{2m_n\omega^2} \frac{\partial^2 V}{\partial q_n^2} \right] + O(\hbar^2)$$

(16)

and

$$< z | (\delta \hat{H})^2 | z > = \frac{\hbar \omega}{2} \sum_{n=1}^{N} \left[ \frac{p^2_n}{m_n} + \frac{1}{m_n\omega^2} \left( \frac{\partial V}{\partial q_n} \right)^2 \right] + O(\hbar^2).$$

(17)

It follows directly from (3), (9), (14), (16) and (17) that

$$Z_1 = \frac{\omega}{4} \int \prod_{n=1}^{N} \frac{dq_n dp_n}{2\pi \hbar} \sum_{j=1}^{N} \left\{ \left( \frac{df}{dH} + \frac{p^2_j}{m_j} \frac{d^2 f}{dH^2} \right) + \frac{1}{m_j\omega^2} \left[ \frac{\partial^2 V}{\partial q_j^2} \frac{df}{dH} + \left( \frac{\partial V}{\partial q_j} \right)^2 \frac{d^2 f}{dH^2} \right] \right\}.$$ 

(18)

This expression can be written in a more compact form. In fact, after some derivations we verify that

$$Z_1 = \frac{\omega}{4} \int \prod_{n=1}^{N} \frac{dq_n dp_n}{2\pi \hbar} \sum_{j=1}^{N} \left[ \frac{\partial}{\partial p_j} \left( p_j \frac{df}{dH} \right) + \frac{1}{m_j\omega^2} \frac{\partial}{\partial q_j} \left( \frac{\partial V}{\partial q_j} \frac{df}{dH} \right) \right].$$

(19)

Furthermore, the expression (19) is more convenient than (18) for future discussions.
The calculation of the second quantum correction, $h^2Z_2$, is similar to the previous one. In fact, to obtain the analogous formula to (12),

$$< \delta X_n \delta X_k \delta X_s \delta X_u > = -m_n^{-\sigma_n/2}m_k^{-\sigma_k/2}m_s^{-\sigma_s/2}m_u^{-\sigma_u/2}(i\omega)^{-\sigma_n+\sigma_k+\sigma_s+\sigma_u}/2 \times \sigma_n[\sigma_s\delta_{nk} + \sigma_k(\delta_{ku}\delta_{sn} + \delta_{nu}\delta_{ks})],$$

(20)
it is necessary only the equations (2), (12) and (13). In the expression (20) it was used a compact notation suggested in the Eq. (11) because there are sixteen combinations of $\delta q_j$ and $\delta p_j$. In this notation $\delta X_j = h^{1/2}i(1-\sigma_j)/2(m_j\omega)^{-\sigma_j/2}(\delta a_j^+ + \sigma_j\delta a)$ is equal to $\delta q_j$ when $\sigma_j = 1$ and equal to $\delta p_j$ when $\sigma_j = -1$. By using the expressions (15) and (20) we can obtain after some calculation the contributions proportional to $h$ and $h^2$ in $< z | (\hat{H})^n | z >$, i.e.

$$< z | \delta \hat{H} | z > = \frac{\hbar \omega}{2} \sum_{n=1}^N \left[ \frac{1}{2} + \frac{1}{2m_n\omega^2} \frac{\partial^2 V}{\partial q_n^2} \right] + \frac{\hbar^2}{4\omega^2} \sum_{nk} \frac{1}{8m_nm_k} \frac{\partial^4 V}{\partial q_n^2 \partial q_k^2} + O(h^3),$$

(21)

$$< z | (\delta \hat{H})^2 | z > = \frac{\hbar \omega}{2} \sum_{n=1}^N \left[ \frac{p_n^2}{m_n} + \frac{1}{m_n\omega^2} \left( \frac{\partial V}{\partial q_n} \right)^2 \right] + \frac{\hbar^2}{4} \sum_{j,k=1}^N \left\{ \frac{1}{\omega^2m_jm_k} \left[ \frac{\partial V}{\partial q_j} \frac{\partial^3 V}{\partial q_j^2 \partial q_k} \right] + \frac{1}{\omega^4} \frac{\partial^2 V}{\partial q_j^2} \right\} + O(h^3),$$

(22)

$$< z | (\delta \hat{H})^3 | z > = \frac{3\hbar^2}{4} \sum_{j,k=1}^N \left\{ \frac{1}{\omega^2m_jm_k} \left[ \frac{1}{2} \left( \frac{\partial V}{\partial q_j} \right)^2 \frac{\partial^2 V}{\partial q_j^2} + \frac{\partial V}{\partial q_j} \frac{\partial V}{\partial q_k} \frac{\partial^2 V}{\partial q_j \partial q_k \partial q_j} \right] + \frac{p_j^2}{2m_jm_k} \frac{\partial^2 V}{\partial q_k^2} - \frac{p_jp_k}{3m_jm_k} \frac{\partial^2 V}{\partial q_j \partial q_k} \right\} + O(h^3),$$

(23)

and

$$< z | (\delta \hat{H})^4 | z > = 3\hbar^2 \sum_{j,k=1}^N \left\{ \frac{1}{4\omega^2m_jm_k} \left( \frac{\partial V}{\partial q_j} \right)^2 \left( \frac{\partial V}{\partial q_k} \right)^2 + \frac{p_j^2}{2m_jm_k} \left( \frac{\partial V}{\partial q_k} \right)^2 + \omega^2 \frac{p_j^2}{2m_j} \frac{p_k^2}{2m_k} \right\} + O(h^3).$$

(24)

It is convenient now to group the terms of the previous expressions with the same power in $\omega$. Thus, the expression for $Z_2$ can be written as
\[ Z_2 = \int \prod_{n=1}^{N} \frac{dq_n dp_n}{2\pi \hbar} \left( \frac{1}{w^2 I_1 + I_2 + w^2 I_3} \right), \] (25)

where

\[ I_1 = \frac{1}{8} \sum_{j,k=1}^{N} \frac{1}{m_j m_k} \left\{ \left( \frac{1}{4} \frac{\partial^4 V}{\partial q_j \partial q_j \partial q_k \partial q_k} \right) \frac{df}{dH} + \frac{\partial V}{\partial q_j} \frac{\partial^3 V}{\partial q_j \partial q_j \partial q_k} + \frac{1}{4} \frac{\partial^2 V}{\partial q_j^2} \frac{\partial^2 V}{\partial q_k^2} + \frac{1}{2} \left( \frac{\partial^2 V}{\partial q_j \partial q_k} \right)^2 \right\} \frac{d^2 f}{dH^2} + \left[ \frac{1}{2} \left( \frac{\partial V}{\partial q_j} \right)^2 \frac{\partial^2 V}{\partial q_j^2} + \frac{\partial V}{\partial q_j} \frac{\partial V}{\partial q_j} \frac{\partial^2 V}{\partial q_k \partial q_k} \frac{\partial q_j \partial q_k}{\partial q_j \partial q_k} \right] \frac{d^3 f}{dH^3} + \left[ \frac{1}{4} \left( \frac{\partial V}{\partial q_j} \right)^2 \frac{\partial V}{\partial q_k} \right] \frac{d^4 f}{dH^4} \right\}, \] (26)

\[ I_2 = \frac{1}{8} \sum_{j,k=1}^{N} \left\{ \left( -\delta_{jk} + \frac{1}{2m_j} \frac{\partial^2 V}{\partial q_j^2} \frac{\partial q_j^2}{\partial q_k} \right) \frac{d^2 f}{dH^2} + \left[ \frac{p_j^2}{2m_j m_k} \frac{\partial^2 V}{\partial q_j \partial q_k} \right] \frac{d^3 f}{dH^3} + \left[ \frac{p_j^2}{2m_j m_k} \left( \frac{\partial V}{\partial q_k} \right)^2 \right] \frac{d^4 f}{dH^4} \right\}, \] (27)

and

\[ I_3 = \frac{1}{8} \sum_{j,k=1}^{N} \left\{ \left( \frac{1}{4} + \frac{\delta_{jk}}{2} \right) \frac{d^2 f}{dH^2} + \left[ \frac{1}{2m_j} \frac{\partial q_j^2}{\partial q_k} (\delta_{kk} + 2) \right] \frac{d^3 f}{dH^3} + \left[ \frac{p_j^2}{2m_j} \frac{p_k^2}{2m_k} \right] \frac{d^4 f}{dH^4} \right\}. \] (28)

As in the \( Z_1 \) case it is more convenient, for the following discussions, to write \( I_1 \), \( I_2 \) and \( I_3 \) in a more compact form,

\[ I_1 = \frac{1}{32} \sum_{j,k=1}^{N} \frac{\partial^3}{\partial q_j \partial q_j \partial q_k} \left( \frac{1}{m_j m_k} \frac{\partial V}{\partial q_j} \frac{df}{dH} \right), \] (29)

\[ I_2 = -\frac{1}{4} \sum_{k=1}^{N} \frac{1}{m_k} \left( \frac{\partial^2 V}{\partial q_k^2} \right) \frac{d^2 f}{dH^2} - \frac{1}{6} \sum_{j,k=1}^{N} \left[ \frac{p_j}{m_j m_k} \frac{\partial^2 V}{\partial q_j \partial q_k} + \frac{1}{m_j m_k} \left( \frac{\partial V}{\partial q_k} \right)^2 \right] \frac{d^3 f}{dH^3} - \frac{1}{8} \sum_{k=1}^{N} \left( \frac{p_k}{m_k} \frac{\partial V}{\partial q_k} \right)^2 \frac{d^4 f}{dH^4} + \frac{1}{8} \sum_{j,k=1}^{N} \frac{\partial}{\partial q_k} \frac{\partial^2 f}{\partial q_j \partial p_j} \left[ \frac{p_k}{m_k} \frac{\partial V}{\partial q_k} + \frac{p_j}{2m_j} \frac{\partial V}{\partial q_j} \right] \frac{d^2 f}{dH^2} \right\}, \] (30)

and

\[ I_3 = \frac{1}{32} \sum_{j=1}^{N} \frac{\partial}{\partial p_j} \left[ \frac{p_j}{dH} \frac{d^2 f}{dH^2} + \sum_{k=1}^{N} \frac{\partial}{\partial p_k} \left( \frac{p_j p_k}{dH} \right) \right]. \] (31)

This general procedure can be extended to other orders in \( \hbar \) to obtains further quantum corrections, but we prefer to discuss the previous results.

In \( Z_1 \) and \( Z_2 \) are present terms which depends of the frequency \( \omega \) employed in the definition of the coherent states. Note that these terms are surface terms. However, a
classical limit can not depend on the particular choice of the coherent states basis. This apparent contradiction can be eliminated if we consider that in a macroscopic body the surface effects can be neglected when we compare them with the bulk contributions. By using this consideration we conclude that the $\hbar Z_1$ contribution to the partition function can be neglected, and only $I_2$ is relevant in the computation of $\hbar^2 Z_2$. Therefore, when these considerations are employed we verify directly that

$$Z = \int \prod_{n=1}^{N} \frac{dp_n dq_n}{2\pi \hbar} f(H) + \hbar^2 \int \prod_{n=1}^{N} \frac{dp_n dq_n}{2\pi \hbar} \left\{ -\frac{1}{4} \sum_{k=1}^{N} \frac{1}{m_k} \left( \frac{\partial^2 V}{\partial q_k^2} \right) \frac{d^2 f}{dH^2} 
- \frac{1}{6} \sum_{j,k=1}^{N} \left[ \frac{p_j p_k}{m_j m_k} \frac{\partial^2 V}{\partial q_j \partial q_k} + \frac{1}{m_j m_k} \left( \frac{\partial V}{\partial q_k} \right)^2 \frac{d^3 f}{dH^3} - \frac{1}{8} \sum_{k=1}^{N} \left( \frac{p_k}{m_k} \frac{\partial V}{\partial q_k} \right)^2 \frac{d^4 f}{dH^4} \right] \right\} + O(\hbar^2). \quad (32)$$

Note that, as it is expected, $f(H) = \exp(-\beta H)$ in the previous expression recovers the usual results \cite{17,19,20}.

III. THE CASE OF TSALLIS THERMOSTATISTICS

We apply now the previous findings for Tsallis thermostatistics \cite{22}. The entropy is defined as

$$S_q = Tr[\hat{\rho}(1 - \hat{\rho}^{q-1})] \cdot \left( q - 1 \right), \quad (33)$$

where $q$ is a real parameter and $\rho$ is the density matrix. Note that $q$ gives, essentially, the measure of the non-extensivity of (33). Furthermore, to thermal averages, one has to use the $q$-expectation value defined as

$$\langle \hat{A} \rangle_q = Tr(\hat{\rho}^q A), \quad (34)$$

for some relevant observable $\hat{A}$. Thus, the canonical distribution is given by

$$f(\hat{H}) = \left[ 1 - (1 - q)\beta \hat{H} \right]^{\frac{1}{1-q}}. \quad (35)$$

When $(1 - q)\beta E_n > 1$, where $E_n$ are the eigenvalue of $\hat{H}$, the probability is not be positive defined quantity. To avoid this behavior it is usually assumed that $f(\hat{H})$. In particular, the Tsallis thermostatistics reduces to the usual one in the limit $q \to 1$. 

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The substitution of (35) into the previous results gives the partition function for Tsallis thermostatistics up to $\hbar^2$ order. In this case we have

$$\frac{d^n f}{dH^n} = (-\beta)^n q(2q - 1)(3q - 2) \cdots [(n - 1)q - (n - 2)][1 - (1 - q)\beta H]^{\frac{nq - (n-1)}{1-q}}. \quad (36)$$

The factor $\beta^n$ above allow us to arrive at an important conclusion. In fact, in the limit $\beta \to 0$ the corrections to the classical limit disappear, i.e., the quantum corrections to the classical partition function are irrelevant in the high temperature limit. In this case we are supposing basically that the derivatives $[1 - (1 - q)\beta H]^{\frac{nq - (n-1)}{1-q}}$ in (32) do not diverge. Note that the above conclusion is a direct consequence of the fact that the dependence of the function $f(\hat{H})$ with $\beta$ and $\hat{H}$ appears only in the form $\beta \hat{H}$. Thus, the previous conclusion can be applied not only to the Tsallis thermostatistics but to a wide class of possible partition functions.

A. The harmonic oscillator

To exemplify the application of the above expressions for a concrete case, we consider a harmonic oscillator whose Hamiltonian is $\hat{H} = \hat{p}^2/(2m) + m\omega^2\hat{x}^2$. For this one-dimensional case, (32) is reduced to

$$Z = \int \frac{dp \, dx}{2\pi \hbar} f(H) + \hbar^2 \int \frac{dp \, dx}{2\pi \hbar} \left\{ -\frac{\omega^2}{4} \frac{d^2 f}{dH^2} - \frac{1}{6} \left( \frac{\omega^2}{m} \rho^2 + m\omega^4 x^2 \right) \frac{d^3 f}{dH^3} - \frac{\omega^4}{8} x^2 \rho^2 \frac{d^4 f}{dH^4} \right\} + \mathcal{O}(\hbar^2), \quad (37)$$

where the derivatives are obtained through (36). In (37) we have to evaluate the integrals for two cases: $q < 1$ and $q > 1$.

Let us consider first the case of $q < 1$. Since the integrands appearing in (37) are composed of successive derivatives obtained from (36), the relevant integrals are of the form

$$\int dxdy \left[ 1 - (ax^2 + by^2) \right]^\sigma = \pi a^{-1/2} b^{-1/2}, \quad (38)$$

$$\int dxdyx^2 \left[ 1 - (ax^2 + by^2) \right]^\sigma = \pi a^{-3/2} b^{-1/2} \frac{1}{2(1+\sigma)(2+\sigma)}, \quad (39)$$
and

\[\int dx dy x^2 y^2 \left[ 1 - (ax^2 + by^2) \right]^{\sigma} = \pi a^{-3/2} b^{-3/2} \frac{1}{8(1 + \sigma)(2 + \sigma)(3 + \sigma)}, \quad (40)\]

with \( \sigma > -1 \) in order to have well defined integrals. The above integrals are more easily performed if the transformation \( x = a^{-1/2}r \cos \theta \) and \( y = b^{-1/2}r \sin \theta \) is employed. By substituting the resulting integrals in (37) we obtain for the partition function

\[Z = \frac{1}{2 - q} \frac{1}{\hbar \omega \beta} - \frac{\hbar \omega \beta}{24} + \mathcal{O}(\hbar^2). \quad (41)\]

One observes that in the limiting case \( q \to 1 \) we recover the usual result. Moreover, the second factor which accounts for the quantum corrections in the first order in \( \hbar \) is independent of \( q \) for the case of harmonic oscillator. It is important to stress that the above results are meaningful for \( q > 2/3 \) in order to achieve the convergence of all the integrals in (37). In the general case, the lower limit for \( q \) is fixed by the higher order derivative of \( f(H) \) present in the expansion for \( Z \).

The calculations for the case \( q > 1 \) are similar to the previous one and lead to the same expression for \( Z \), i.e., (41). However, in this case the upper limit is \( q < 2 \), and it is now fixed by the exponent of the first term in the expansion for \( Z \).

**IV. CONCLUSIONS**

In this paper we have presented for the first time a general procedure to obtain all the quantum corrections for generalized partition functions. As underlined before, this procedure is a generalization of the Wigner expansion for usual statistical mechanics. However, instead to use Wigner representation, we have used the coherent states representation, which is convenient for our purposes. The general formalism was used to explicitly evaluate the first quantum correction to the classical case. This result was applied to a non-extensive (Tsallis) thermostatistics. As an application we have considered the case of the harmonic oscillator. In particular, we have shown that the dominant term in the partition function
depends on \( q \). It is reduced to the usual one when \( q \to 1 \) as expected. Moreover, the first quantum correction for the partition function of the harmonic oscillator coming from the Tsallis thermostatistics is shown to be the same as the usual one. We have also determined the specific heat of the system for \( q > 1 \) and \( q < 1 \).

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