Mixed Phase within the Multi-polytrope Approach to High Mass Twins.

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Received 26 April 2016; Revised 6 June 2016; Accepted 6 June 2016

We present a multi-polytrope approach to describe high-mass twins fulfilling chiral effective field theory estimations of the neutron star equation state and test it against the appearance of mixed phases at the hadron-quark interface. In addition, we discuss astrophysical applications of this method and expected future measurements that shall further constrain neutron star matter and the understanding of the QCD phase diagram.

Keywords: Compact stars, polytropes, phase transitions, QCD phase diagram

1 INTRODUCTION

Neutron stars (NS) are superdense compact objects with central densities of the order of several times the nuclear saturation density $n_0 = 0.15\text{fm}^{-3}$, the mean density of atomic nuclei. Hence, exotic states of matter could exist in the interior of NSs. The elucidation of the interior composition of compact stars is of special importance when considering the QCD phase diagram. In case of a strong first-order phase transition from hadronic matter into any type of exotic matter, like quark deconfinement, the so-called High Mass Twin (HMT) phenomenon (D. E. Alvarez-Castillo & Blaschke, 2013, 2015b; D. E. Alvarez-Castillo, Kaltenborn, & Blaschke, 2016; Benic, Blaschke, Alvarez-Castillo, Fischer, & Typel, 2015; D. Blaschke, Alvarez-Castillo, & Benic, 2013; Kaltenborn, Bastian, & Blaschke, 2017) predicts particular characteristics of macroscopic observables in compact stars: disconnected sequences (families) in the mass-radius diagram featuring compact star branches with overlapping ranges in the gravitational mass $M$ but with different ranges of radii so that the radius difference at equal mass can vary from one half to a couple of kilometers, depending on the model description. HMTs allow for a resolution of several problems in the description of dense nuclear matter and its relation to compact stars: the masquerade effect, the reconfinement EoS case and
the hyperon puzzle (D. Blaschke & Alvarez-Castillo, 2016). Moreover, HMTs lead to prediction of various astrophysical phenomena like energetic emissions that can potentially be detected. Furthermore, the recently detected gravitational wave emissions from the fusion of compact objects in binaries brings up the possibility of probing neutron star interiors. In this respect, NS-NS-merger calculations as performed, e.g., in (Bauswein, Stergioulas, & Janka, 2014; Hanauske et al., 2017) represent an important tool for understanding these mergers.

The aim of this work is to show that within the multi-polytrope approach for the compact star equation of state (EoS)—see (D. E. Alvarez-Castillo & Blaschke, 2017 and references therein)—yields HMTs which are in wide limits robust against the appearance of pasta phases at the interface where a strong first order phase transition occurs. Without loss of generality, the multi-polytrope EoS consists of several pieces that fulfill \( P_i(n) = \kappa_i n^{\Gamma_i} \), where \( P \) is the pressure and \( n \) the baryonic density. The coefficients \( \kappa_i \) and \( \Gamma_i \) serve to describe the state of matter in each density region. As shown in (D. E. Alvarez-Castillo & Blaschke, 2017) this approach is in accordance with the scheme of Hebeler et al. (Hebeler, Lattimer, Pethick, & Schwenk, 2013) that constrains the high-density EoS with the compact star maximum mass constraint \( M_{\text{max}} \geq 2 M_\odot \) and at subnuclear densities is in accordance with EoS constraints from chiral effective field theory, see figure 1.

The first necessary criterion for HMTs to appear is the so called Seidov constraint (Seidov, 1971):

\[
\frac{\Delta \varepsilon}{\varepsilon_{\text{crit}}} \geq \frac{1}{2} + \frac{3}{2} \frac{P_{\text{crit}}}{\varepsilon_{\text{crit}}} \quad (1)
\]

The corresponding star sequences are obtained by solving the Tolman-Volkoff-Oppenheimer (TOV) equations that describe a static, non-rotating, spherically symmetric star (Oppenheimer & Volkoff, 1939; Tolman, 1939)

\[
\frac{dP(r)}{dr} = -\frac{G(\varepsilon(r) + P(r))(M(r) + 4\pi r^3 P(r))}{r(r - 2GM(r))}, \quad (2)
\]

\[
\frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r), \quad (3)
\]
and by considering $P(r = R) = 0$ and $P_c = P(r = 0)$ as boundary conditions for a compact star with mass $M$ and radius $R$, respectively.

By increasing the chosen central pressure $P_c$ up to a value for which the maximum mass is reached, the complete compact star sequence is determined. Furthermore, the enclosed baryonic mass can be obtained by integrating

$$\frac{d N_B(r)}{dr} = 4\pi r^2 \left(1 - \frac{2GM(r)}{r}\right)^{-1/2} n(r).$$

This quantity is of particular importance because it plays an important role in the dynamics of compact star evolution scenarios, like mass gain by accretion leading to a transition from a pure hadronic star into a hybrid star configuration (Bejger, Blaschke, Haensel, Zdunik, & Fortin, 2017). In addition, a pure hadronic star whose central density is near the phase transition value could spin down into the hybrid twin configuration, a process expected to conserve the baryonic mass.

### 2 HIGH MASS TWINS WITH MIXED PHASE FORMATION

Pasta phases can appear at the hadron-quark boundary. By choosing an EoS for the HMT’s (see D. E. Alvarez-Castillo & Blaschke, 2017) for details), the inclusion of such a mixed phase can be phenomenologically described by an interpolation. Here we adopt a parametrization of the EoS of the form (D. E. Alvarez-Castillo & Blaschke, 2015a):

$$\epsilon(p) = \epsilon_h(p)f_s(p) + \epsilon_q(p)f_s(p).$$

where $\epsilon_h(p)$ and $\epsilon_q(p)$ are the energy densities in the hadronic and quark matter phases, respectively. The resulting EoS are presented in figure 1 where the parameter $\Gamma_s$ determines the strength of the mix. The net effect is both smoothing the phase transition plateau with a non-zero slope resembling the $\Gamma_2 \neq 0$ multipolytrope case (see Read, Lackey, Owen, & Friedman, 2009; Zdunik, Bejger, Haensel, & Gourgoulhon, 2006) for examples of such configurations). Panel (c) of Figure 2 shows the high-mass part of the corresponding mass-radius sequences for the mixed phase parameters of table 1. One interesting effect of the mixed phase is the lowering of the speed of sound (panel (c)) at the critical density and resulting in a lowering of the maximum mass of the second branch. As a result, the baryonic mass difference for the twins is also reduced (see panel (d) in figure 2).

| TABLE 1 Parameter values for mixed phase sets with high density polytrope parameters $\Gamma_3 = 3.2$ and $\kappa_3 = 490.41$ MeV fm$^{3(1-\Gamma_3)}$. |
|-----------------|-----------------|-----------------|-----------------|
| $\Gamma_s$     | $M_{NS}^{max}$  | $M_{HS}^{max}$  | $M_{HS}^{max}$  |
| [MeV fm$^{-3}$] | [M$_{\odot}$]   | [M$_{\odot}$]   | [M$_{\odot}$]   |
| set $a$         | 0.0             | 2.176           | 2.054           | 2.050           |
| set $b$         | 6.0             | 2.088           | 2.039           | 2.017           |
| set $c$         | 9.0             | 2.064           | 2.035           | 2.009           |

### 3 CONCLUSIONS

The multi-polytrope approach to the HMTs proves to be an effective tool for exploring the compact star EoS through
diverse astrophysical phenomena. In this work we have found that the HMTs in this approach are robust against the appearance of pasta phases at the hadron-quark boundary. Due to the flexibility in the parameter variation of the multi-polytrope approach it is feasible to perform a Bayesian Analysis for parameter estimates fulfilling observational constraints, as presented in various works (D. Alvarez-Castillo et al., 2016; D. E. Alvarez-Castillo, Ayriyan, Blaschke, & Grigorian, 2015; Ayriyan et al., 2017; Ayriyan, Alvarez-Castillo, Blaschke, & Grigorian, 2016; Ayriyan, Alvarez-Castillo, Blaschke, Grigorian, & Sokolowski, 2015; D. B. Blaschke, Grigorian, Alvarez-Castillo, & Ayriyan, 2014; Raithel, Özel, & Psaltis, 2016, 2017). In addition, the recently deployed NICER detector is expected to measure the NS radius to an accuracy of 5% for the first three NS candidates, therefore being potentially able to elucidate HMTs. Moreover, gravitational wave signals from NS-NS mergers have been already studied in (Hanauske et al., 2017) by using a multi-polytrope description with generic temperature addition but so far not including the HMTs case. The observation of such events by advanced LIGO is expected soon. Further work in this direction is a natural step for testing the compact star EoS and for exploring the QCD phase diagram. The resulting study will serve as a guide for future gravitational signal detection of this process in interferometers like advanced LIGO or VIRGO. Last but not least, the SKA array in the southern hemisphere will gather pulsar data that shall constrain NS moments of inertia, masses and rotational values, all of them important quantities in the study of the NS EoS.
Acknowledgments

D.E.A-C. and S.T. received support from the Heisenberg-Landau programme. D.B. was supported in part by the MEPhI Academic Excellence Project under grant No. 02.a03.21.0005 and by the Polish National Science Centre (NCN) under grant number UMO - 2014/13/B/ST9/02621. D.E.A-C is grateful for support from the programme for exchange between JINR Dubna and Polish Institutes (Bogoliubov-Infeld programme). This research was supported in part by the ExtreMe Matter Institute EMMI at the GSI Helmholtzzentrum fuer Schwerionenphysik, Darmstadt, Germany.

Author contributions

All authors contributed equally to the elaboration of this article.

Financial disclosure

None reported.

Conflict of interest

The authors declare no potential conflict of interests.

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**How cite this article:** David Alvarez-Castillo, David Blaschke and Stefan Typel (2017), Mixed Phase Transition within the Multi-polytrope Approach to High Mass Twins.