Research on the Response Relation of Acoustic Logging in Fractured Reservoirs

Song Hu\textsuperscript{1,a}, Anni Li\textsuperscript{2,*}, Xiaoting Zhang\textsuperscript{2,b}, Wujun Jin\textsuperscript{1,c} and Jing Lu\textsuperscript{1,d}

\textsuperscript{1}State Key Laboratory of Shale Oil and Gas Enrichment Mechanisms and Effective Development, SINOPEC, Beijing, 10083, China
\textsuperscript{2}College of Geophysics, China University of Petroleum, Beijing, 102249, China

*Corresponding author e-mail: 233310826@qq.com, a3262899205@qq.com,
b1912620847@qq.com, c251482911@qq.com, dlujing.syky@sinopec.com

Abstract. Based on Hudson equivalent medium model and Schoenberg linear sliding model theory, a fracture equivalent medium model suitable for sonic logging is proposed, and the equivalent elastic parameter expression of the inclined fracture is derived. Using the established equivalent medium model, the effects of fracture density, width, dip angle and fillings on formation P-wave velocity and anisotropy information obtained from sonic logging were studied, which provides a theoretical basis for evaluating fractured formations using acoustic logging data.

1. Introduction
Fractured oil and gas reservoirs have complex storage space types and percolation physical characteristics, multiple fracture shapes and types of fillers. Acoustic logging is an important method for detecting and evaluating fractured formations. Hudson assumes that the fractures in the medium are isolated oblate spheroids, the elastic wavelength is larger than the fracture size, and the expression of effective elastic modulus is proposed under the condition that the bulk modulus and shear modulus of the contained material are smaller than the background rock. The fracture theory assumes that the fluid is confined in disconnected fractures, which is only applicable to isolated fractures, and has no effect on dry fractures [1]. However, this method has many parameters and is relatively complicated to operate. Schoenberg et al. Proposed the theory of linear sliding, ignoring the fracture shape and its microstructure, assuming that the fracture is an infinitely thin plane or stratum, and that it meets the linear sliding boundary condition [2]. The theory considers that the flexibility change caused by cracks is additional flexibility, and its matrix is equivalent to the difference between the flexibility coefficient matrix of the background medium and the flexibility coefficient matrix caused by cracks [3]. However, the linear sliding theory cannot accurately determine the compliance value of fracture media in practical applications.

Based on the theory of fracture equivalent media, combined with Hudson model and Schoenberg linear sliding model, an equivalent medium model for acoustic logging fractures is constructed. The expression of the equivalent elastic modulus of the formation at the angle of the fracture is derived. Through theoretical derivation, the effects of fracture density, width, inclination and filler on the longitudinal and transverse velocities of sonic logging are analyzed. The evaluation of fractured formations provides a theoretical basis.
2. Acoustic logging fracture equivalent medium model

If there are directional cracks in the medium, the equivalent medium elastic parameter matrix of the fracture is equivalent to the elastic parameters of the background isotropic medium \( C_b \), plus the effect of the elastic parameters of the fracture medium \( C_f \) [5]. Based on the Hudson equivalent theory, the superimposed linear sliding model can make up for the shortcomings of using the linear sliding model or Hudson model alone, so that the fracture flexibility value has practical physical significance and value [6]. The crack normal weakness \( \Delta N \) and crack tangential weakness \( \Delta T \) are:

\[
\Delta N = \frac{\lambda + 2\mu}{\mu} U_{33e} \tag{1}
\]

\[
\Delta T = U_{11e} \tag{2}
\]

Eventually established crack equivalent HIT (Horizontal Transversely Isotropy) elastic matrix:

\[
C_{mm} = C_b - C_f
\]

\[
= \begin{bmatrix}
(\lambda + 2\mu)(1 - \Delta N) & \lambda(1 - \Delta N) & \lambda(1 - \Delta N) & 0 & 0 & 0 \\
\lambda(1 - \Delta N) & (\lambda + 2\mu)(1 - \chi^2\Delta N) & \lambda(1 - \chi\Delta N) & 0 & 0 & 0 \\
\lambda(1 - \Delta N) & \lambda(1 - \chi\Delta N) & (\lambda + 2\mu)(1 - \chi^2\Delta N) & 0 & 0 & 0 \\
0 & 0 & 0 & \mu & 0 & 0 \\
0 & 0 & 0 & 0 & \mu(1 - \Delta T) & 0 \\
0 & 0 & 0 & 0 & 0 & \mu(1 - \Delta T)
\end{bmatrix} \tag{3}
\]

Where,

\[
\chi = \frac{\lambda}{\lambda + 2\mu} \tag{4}
\]

3. Research on P-wave velocity response of sonic logging

The staggered-grid finite difference method is used to simulate the borehole sound field in the fracture equivalent medium model, and the relationship between the P-wave velocity and the physical properties of the fracture can be studied. The influence of cracks on elastic parameters is mainly related to the density, shape, filling and inclination of cracks. Using the proposed fracture equivalent medium model, the effects of the P-wave velocity of a fractured formation on the density, width, inclination and filling of the fracture in the formation are studied respectively. Formation parameters are density with 2650 kg/m\(^3\), P-wave velocity with 4000 m/s, S-wave velocity with 2138 m/s, respectively.
### 3.1. Influence of fracture density on formation P-wave velocity

As shown in Figure 1, the curve shows the relationship between the longitudinal and transverse wave velocities and the changes in fracture density. The crack inclination angle is 0° (it is horizontal crack). In the figure, the abscissa is the crack density and the ordinate is the wave velocity, where \( V_{ph} \) is horizontal longitudinal wave velocity, \( V_{pv} \) is vertical longitudinal wave velocity, \( V_s \) is shear wave velocity. Besides, from the figures, we can get the critical values of the fracture density on wave velocity are 0.0001 for gas filling, 0.001 for oil filling and 0.005 for water filling, respectively.

### 3.2. Effect of crack width

As shown in Figures 2 (a) to 2 (c), Curves show the relationship between P-wave and S-wave velocities and crack widths. (The crack width here refers to the crack aspect ratio) At this time, set the inclination angle of the crack to 0° (Horizontal crack). In the figure, the abscissa indicates the crack width, and the ordinate indicates the wave velocity, where \( V_{ph} \) is the horizontal P-wave velocity, \( V_{pv} \) is the vertical P-wave velocity, \( V_s \) is the S-wave velocity. From the figures, we can get the critical values of the crack density on wave velocity are 0.02 for gas filling, 0.2 for oil filling and 0.8 for water filling.
3.3. Influence of crack dip

Figure 3. Curve of relationship between wave velocity and fracture inclination at different fillers, (a) Gas filling; (b) oil filling; (c) water filling;

As shown in Figures 3 (a) to 3 (c), the curve shows the relationship between the longitudinal and transverse wave velocities and the inclination of the fracture. In the figure, the abscissa is the inclination of the fracture, and the ordinate is the wave velocity, where $V_{ph}$ is the horizontal P-wave velocity, $V_{pv}$ is the vertical P-wave velocity, $V_s$ is the shear wave velocity. At this time, the crack density is set to 0.1.

Table 1. Critical value of the effect of crack inclination on wave velocity

| Lithology | Filler    | Crack density | Crack aspect ratio |
|-----------|-----------|---------------|--------------------|
| shale     | Gas filling | 0.1           | 0.0001             |
| shale     | Oil filling | 0.1           | 0.002              |
| shale     | Water filling | 0.1          | 0.008              |

Table 1 shows the critical value of the influence of the crack inclination angle on the wave velocity under different crack fillings. In the above three cases, as the inclination of the fracture increases, the P-wave velocity decreases first and then increase. When the inclination is 45°, the horizontal P-wave velocity and the vertical P-wave velocity are the same. As shown in the figure, the patterning shows a symmetrical relationship, that is, the vertical P-wave velocity at the inclination angle of 0° and the horizontal P-wave velocity at the inclination angle of 90° are the same, and vice versa. The shear wave velocity increases gradually with the increase of the fracture inclination angle from 0° to 90°. When the aspect ratio of the fracture is greater than the critical value in the table, the effect of the inclination of the fracture on the wave velocity becomes more obvious.

4. Conclusion

Combining the Hudson model and the linear sliding model, the equivalent elastic modulus expression of the fracture medium in the sonic logging environment is derived, and the elastic parameter expression of the inclined cracks are realized by Bond transformation. The finite difference numerical algorithm is used to simulate the acoustic wave of the fractured formation. The logging response is affected by parameters such as the density, aspect ratio, inclination and filling of fractures in the formation. Comparing and summarizing the response results, the following conclusions can be obtained:

1) For horizontal fractures, as the fracture density increases, the P-wave velocity gradually decreases, and the vertical P-wave velocity decreases at a faster rate than the horizontal P-wave velocity; the shear wave velocity does not change with the change of the aspect ratio of the fracture; The longitudinal wave anisotropy coefficient gradually increases, and the degree of formation anisotropy is getting stronger and stronger.

2) With the increase of the crack width, the P-wave velocity gradually decreases, and the vertical P-wave velocity decreases faster than the horizontal P-wave velocity. The shear wave velocity does not
change with the change of the aspect ratio of the crack. The longitudinal wave anisotropy coefficient gradually increases, and the degree of formation anisotropy is getting stronger and stronger.

(3) As the inclination of the fracture increases, the P-wave velocity decreases first and then increase. When the inclination is 45°, the vertical P-wave velocity and the horizontal P-wave velocity are equal. The shear wave velocity gradually increases as the inclination of the fracture increases from 0° to 90°. The longitudinal wave anisotropy gradually increases, and the formation anisotropy becomes stronger and stronger.

(4) For different crack fillers, compared with oil-water-wet fractures, the longitudinal wave velocity in gas-containing dry fractures is significantly reduced, and the shear wave velocity is not significantly changed, and the degree of anisotropy of gas-containing fractures is much greater than that of oil-water fractures degree.

References
[1] Hudson J.A. Wave speeds and attenuation of elastic waves in material containing cracks. Geophysical Journal of Royal Astronauts Society, 1981, 64 (1): 133-150.
[2] Schoenberg M. and Sayers C.M. Seismic anisotropy of fractured rock. Geophysics, 1995, 60 (1): 204-211.
[3] Schoenberg M. and Douma J. Elastic wave propagation in media with parallel fractures and aligned cracks. Geophysical Prospecting, 1988, 36 (6): 571–590.
[4] Gong Dan, Zhang Chengguang. Research on Response Characteristics of Acoustic Logging in Fractured Tight Sandstone Reservoir, Journal of Oil and Gas Technology, 2013, 357): 82-86.
[5] Hudson J A. Overall properties of a cracked solid [J]. Mathematical Proceedings of the Cambridge Philosophical Society, 1980. 88 (2): 371-384.
[6] Schoenberg M, Elastic wave behavior across linear slip interfaces [J]. The Journal of the Acoustical Society of America, 1980. 68 (5): 1516-1521.