 Thickness dependence of the critical current density in superconducting films: a geometrical approach

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Abstract

We analyze the influence of the magnetic field generated by the supercurrents (self-field) on the current density distribution by numerical simulations. The thickness of the superconducting film determines the self-field and consequently the critical current density at zero applied field. We find an equation, which derives the thickness dependence of the critical current density from its dependence on the magnetic induction. Solutions of the equation reproduce numerical simulations to great accuracy, thus enabling a quantification of the dependence of the self-field critical current density with increasing film thickness. This result is technologically relevant for the development of coated conductors with thicker superconducting layers.
A decrease of the critical current density $J_c$ with increasing film thickness $d$ was observed in thin superconducting films of YBa$_2$Cu$_3$O$_{7-\delta}$ on both single-crystalline and polycrystalline substrates. Possible explanations are a degradation of the film or a change in the defect structure with thickness, but also the self-field of the sample causes a significant reduction if $J_c$ depends on the low magnetic induction $B$ generated by the transport current$^6$. Identifying the true cause of the decrease is vital for the development of coated conductors, which must be grown to higher thicknesses to enhance their performance.

We employ numerical calculations of the current density distribution in a thin film to derive a practical approximation allowing the evaluation of the self-field depression of $J_c$ for a given $J_c(B)$ dependence. Therefore, it is essential to distinguish between the intrinsic $J_c(B)$ of the material, i.e., the (local) dependence of the critical current density on the magnetic induction $B$ in the film, and the average critical current density as a function of the external applied field $J_c(H)$, which is determined in a transport measurement of the critical current. Note, that $J_c(B)$ provides information on pinning in the material and bridges theory and experiment.

The procedure we employ is similar to the iterative algorithm used by Rostila et al.$^6$ At the beginning the external applied field determines the starting current density distribution $J_c(B(x, y) = \mu_0 H)$, which is constant across the cross-section of the film. After calculating the self-field distribution $B_{sf}(x, y)$ in the sample the current densities are updated to $J_c(B(x, y) = \mu_0 H + B_{sf}(x, y))$, which results in a new self-field distribution. This step is iterated until current density and magnetic induction satisfy the (arbitrary) material law $J_c(B)$ at every position in the film. For comparison with experiment $J_c(H)$ is computed by calculating the average current density flowing through the conductor at a range of external applied fields.

Earlier$^7$ we used this procedure to extrapolate the power-law $J_c(B) \propto B^{-\alpha}$ dependence, which is commonly observed in thin films of YBCO at high fields, to the lowest fields and showed that despite the divergence of $J_c(B)$ the film carries a finite critical current at zero applied field because of the self-field of the sample. This result (see Fig. 1) is particularly suited for our purpose, because the clear deviation between $J_c(B)$ and $J_c(H)$ at low fields emphasizes important features of the $J_c(H)$ curve: Starting from zero applied field, $J_c(H)$ remains approximately constant at the self-field critical current density $J_{sf} \equiv J_c(H = 0)$ as long as the applied field is negligible compared to the self-field of the sample. At high
FIG. 1. Approximations made to connect $J_{sf}(d)$ and $J_c(B)$. The simulations of $J_c(H)$ (solid line) are carried out assuming $J_c(B) \propto B^{-0.5}$ (dotted line) and result in a finite $J_{sf}$ at zero applied field. The simulated $J_c(H)$ remains approximately at this value up to applied fields of $B_f$, which is found by intersecting $J_{sf}$ (broken line) and $J_c(B)$, and approaches $J_c(B)$ above this field.

applied fields, on the other hand, the transport current alters the field distribution only marginally and $J_c(H)$ is identical to $J_c(B)$. The applied field at which $J_c(H)$ becomes field dependent and approaches $J_c(B)$, can be estimated by comparison to the self-field of the film. We use $H_t = B_t / \mu_0 = \gamma J_{sf} d / \pi$, which is equivalent to the field scale of thin films, but we replace the constant $J_c$ of the Bean model by the average critical current density at zero applied field and include a factor $\gamma$, which is approximately 1.2 as determined graphically from Fig. [1] or by least squares fitting (see below).

Because the sheet current density $J_{sf} d$ controls the self-field of the sample, the thickness dependence $J_{sf}(d)$ can be related to $J_c(B)$ by making the following approximations. We neglect any field dependence of $J_c(H)$ up to applied fields $H = H_t$, which leads to

$$J_c(H = 0) = J_c(H = H_t),$$

and assume that $J_c(H)$ and $J_c(B)$ are identical at applied fields above $H_t$:

$$J_c(H = H_t) = J_c(B = B_t).$$
Combining both equations

\[ J_c(H = 0) = J_c(B = B_f) \]  

(3)

and inserting the definition of \( B_f \) results in the implicit equation

\[ J_{sf} = J_c(B = \mu_0 \gamma J_{sf} d / \pi), \quad \gamma \approx 1.2. \]  

(4)

A graphical representation of the derivation is depicted in Fig. 1 and \( \gamma \) is determined by intersecting \( J_{sf} \) with \( J_c(B) \). The final result Eqn. 4 allows us to calculate the self-field critical current density as a function of thickness \( J_{sf}(d) \) for any given \( J_c(B) \).

We test Eqn. 4 by comparing its solution for a certain \( J_c(B) \) to simulations of \( J_{sf}(d) \) on a 100 \( \mu \)m wide film having a thickness between 100 nm and 3 \( \mu \)m (the typical thickness range in experiments). Inserting, for example, the power-law

\[ J_c(B) = J_1 \left( \frac{B}{B_1} \right)^{-\alpha}, \]  

(5)
where \( J_1 \) is the current density at \( B = B_1 \), generates a power-law also in the thickness dependence:

\[
J_{sf}(d) = J_1^{1/(1+\alpha)} \left( \frac{\mu_0 \gamma d}{B_1 \pi} \right)^{-\alpha/(1+\alpha)}.
\] (6)

Figure 2 compares simulations and Eqn. 6. We find by least squares fitting \( \gamma \approx 1.2 \), in agreement with the above result. Note further, that this pre-factor represents only a constant vertical shift in Fig. 2 and that the dependence on the thickness, which is in excellent agreement with the numerical simulations, is thus entirely the result of Eqn. 4.

After confirming the analytical solution for a power-law \( J_c(B) \) we can analyze experimental data. Fits to \( J_{sf}(d) \) curves are, for example, available from Ijaduaola et al.\(^5\), who finds \( J_{sf}(d) \propto d^{-0.4} \) in all but the thinnest film. According to Eqns. 5 and 6 the exponent \( \alpha \) of \( J_c(B) \) translates into an exponent of \( \alpha/(1+\alpha) \) in the thickness dependence. We infer \( \alpha = 0.6 \), which is reasonably close to \( \alpha \approx 5/8 \) determined in measurements of \( J_c(H) \) at applied fields much above the self-field, where \( J_c(B) \) and \( J_c(H) \) are identical. The \( J_{sf}(d) \) depression observed in this work can therefore be explained without any additional assumption by continuing the power-law \( J_c(B) \) down to the low self-fields of the sample.

Another popular parametrization of \( J_c(B) \) is a generalized form of the Kim model

\[
J_c(B) = \frac{J_0}{(1 + B/B_0)^\alpha},
\] (7)

which reproduces a power-law if \( B \gg B_0 \) and is constant if \( B \ll B_0 \). The maximum current density of \( J_c(B = 0) = J_0 \) limits the self-field of the sample to about \( 1.2 \mu_0 J_0 d/\pi \), which is roughly 14 mT at 3 \( \mu \)m (the thickest film of the simulations) if we assume \( J_0 = 10^{10} \) Am\(^{-2} \). Thus we expect that the self-field, which increases linearly with thickness, significantly affects \( J_{sf}(d) \) only if it is comparable to or exceeds \( B_0 \). Calculations for three different values of \( B_0 \) are given in Fig. 3 and show the expected behavior: The self-field of the thinnest sample is approximately half of \( B_0 = 1 \) mT and causes a noticeable \( J_c \) decrease, whereas the self-field of the sample is insignificant and \( J_c(d) \) approximately independent of thickness, if \( B_0 = 100 \) mT. The intermediate \( B_0 = 10 \) mT shows a transition from the first to the second case; in the thinnest films \( J_{sf}(d) \) is nearly constant and the depression evolves as the thickness (and the self-field) increases. The (numerical) solution of Eqn. 4 accounts for all three situations and matches perfectly the simulations using again \( \gamma \approx 1.2 \).
FIG. 3. Same as in Fig. 2 but for a generalized Kim-model. Depending on the relative strength of $B_0$ to the self-field of the sample, $J_{sf}(d)$ is almost constant if $B_0 \gg B_f$ (squares) or decays rapidly if $B_0 \ll B_f$ (circles). Equation 4 fully accounts for this behavior.

We demonstrated how to calculate $J_{sf}(d)$ if a theoretical $J_c(B)$ is available, but our result is also technologically relevant for investigating the reason of the $J_{sf}$ decrease in coated conductors with thicker superconducting layers. For such an analysis $J_c(H)$ should be measured on the thinnest sample, where substrate interface effects do not play a role anymore. From this data $J_c(B)$ is determined down to inductions of about $B_f$ of the thinnest film. The remaining uncertainty in $J_c(B)$ (we suggest to assume a constant $J_c(B)$ at inductions below $B_f$ if no theoretical model is available) will introduce only a minor error in calculating $J_{sf}(d)$ for thicker films, provided they carry higher sheet current densities and thus elevate $B_f$ above the value of the thinnest film.

We wish to emphasize, that the effect of the self-field is not an alternative explanation for a thickness dependent critical current density and will always be present, if the critical current density depends on the magnetic induction. A depression of $J_{sf}(d)$ significantly different from that calculated from $J_c(B)$ is thus indicative of an additional mechanism, such as a change in the material or pinning properties with thickness, determining the performance of the thicker conductor. If, however, the calculated and the measured $J_{sf}(d)$ are similar, the self-field controls the thickness dependence. In this case an improvement in
pinning in the entire sample is necessary to enhance $J_c(B)$ and to improve the conductor.

In summary we have derived a practical equation, which bridges theory and experiment by relating the (local) dependence of the critical current density on the magnetic induction $J_c(B)$ to the thickness dependence of the average critical current density at zero applied field $J_{sf}(d)$. Solutions of the equation correctly predict the thickness dependence for two common $J_c(B)$ models and excellent agreement is reached by fitting a single pre-factor. Thus the influence of $J_c(B)$ on the measured $J_{sf}(d)$ in transport experiments can be quantified, which is technologically important, because it allows investigating the depression of the critical current density observed in coated conductors when they are grown to higher thicknesses.

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