Conventional nuclear effects on generalized parton distributions of trinucleons

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Abstract

The measurement of nuclear Generalized Parton Distributions (GPDs) will represent a valuable tool to understand the structure of bound nucleons in the nuclear medium, as well as the role of non-nucleonic degrees of freedom in the phenomenology of hard scattering off nuclei. By using a realistic microscopic approach for the evaluation of GPDs of $^3$He, it will be shown that conventional nuclear effects, such as isospin and binding ones, or the uncertainty related to the use of a given nucleon-nucleon potential, are rather bigger than in the forward case. These findings suggest that, if great attention is not paid to infer the properties of nuclear GPDs from those of nuclear parton distributions, conventional nuclear effects can be easily mistaken for exotic ones. It is stressed therefore that $^3$He, for which the best realistic calculations are possible, represents a unique target to discriminate between conventional and exotic effects. The complementary information which could be obtained by using a $^3H$ target is also addressed.
I. INTRODUCTION

The measurement of Generalized Parton Distributions (GPDs) \cite{1}, parametrizing the non-perturbative hadron structure in hard exclusive processes, represents one of the challenges of nowadays hadronic Physics (for reviews, see, e.g., \cite{2, 3, 4, 5, 6}). GPDs enter the long-distance dominated part of exclusive lepton Deep Inelastic Scattering (DIS) off hadrons. Deeply Virtual Compton Scattering (DVCS), i.e. the process $eH \rightarrow e'H'\gamma$ when $Q^2 \gg m^2_H$, is one of the the most promising to access GPDs (here and in the following, $Q^2$ is the momentum transfer between the leptons $e$ and $e'$, and $\Delta^2$ the one between the hadrons $H$ and $H'$) \cite{1, 7}. Relevant experimental efforts to measure GPDs are taking place, and a few DVCS data have been already published \cite{8, 9}. The issue of measuring GPDs for nuclei has been addressed in several papers. In the first one \cite{10}, it was shown that the knowledge of GPDs would permit the investigation of the short light-like distance structure of nuclei, and thus the interplay of nucleon and parton degrees of freedom in the nuclear wave function. In inclusive DIS off a nucleus with four-momentum $P_A$ and $A$ nucleons of mass $M$, this information can be accessed in the region where $A x_{Bj} \simeq \frac{Q^2}{2M\nu} > 1$, being $x_{Bj} = Q^2/(2P_A \cdot q)$ and $\nu$ the energy transfer in the laboratory system. In this region measurements are very difficult, because of vanishing cross-sections. As explained in \cite{10}, the same physics can be accessed in DVCS at much lower values of $x_{Bj}$. In Ref. \cite{11} it has been shown that, for finite nuclei, the measurement of GPDs would provide us with peculiar information about the spatial distribution of energy, momentum and forces experienced by quarks and gluons inside hadrons. This argument has been retaken and confirmed recently in Ref. \cite{12}. DVCS has been extensively discussed for different nuclear targets. Impulse Approximation (IA) calculations, supposed to give the bulk of nuclear effects at $0.05 \leq A x_{Bj} \leq 0.7$, have been performed for the deuteron \cite{13} and for spinless nuclei \cite{14}, in particular for $^4He$ \cite{15}, for which an experiment is going on at JLab \cite{16}. For nuclei of any spin, estimates of GPDs have been provided and prescriptions for nuclear effects have been proposed in \cite{17}. Analyses of nuclear DVCS beyond IA, with estimates of shadowing effects and involving therefore large light-like distances and correlations in nuclei have been also performed \cite{18, 19}. While several studies have shown that the measurement of nuclear GPDs can unveil crucial information on possible medium modifications of nucleons in nuclei \cite{20, 21}, great attention has to be paid to avoid to mistake them with conventional nuclear effects. To this respect, a special
role would be played by few body nuclear targets, for which realistic studies are possible and exotic effects, such as the ones of non-nucleonic degrees of freedom, not included in a realistic wave function, can be disentangled. To this aim, in Ref. [22], a realistic IA calculation of the quark unpolarized GPD $H^3_q$ of $^3$He has been presented. The study of GPDs for $^3$He is interesting for many aspects. In fact, $^3$He is a well known nucleus, and it is extensively used as an effective neutron target: the properties of the free neutron are being investigated through experiments with nuclei (the measurement of neutron GPDs using nuclei has been discussed in Ref. [23]), whose data are analyzed taking nuclear effects properly into account. For example, it has been shown, firstly in [24], that unpolarized DIS off trinucleons ($^3H$ and $^3$He) can provide relevant information on PDFs at large $x_{Bj}$, while it is known since a long time that its particular spin structure suggests the use of $^3$He as an effective polarized neutron target [25, 26, 27, 28]. Polarized $^3H$ will be therefore the first candidate for experiments aimed at the study of spin-dependent GPDs of the free neutron. In Ref. [22], the GPD $H^3_q$ of $^3$He has been evaluated using a realistic non-diagonal spectral function, so that momentum and binding effects are rigorously estimated. The scheme proposed in that paper is valid for $\Delta^2 \ll Q^2, M^2$ and it permits to calculate GPDs in the kinematical range relevant to the coherent, no break-up channel of deep exclusive processes off $^3$He. In fact, the latter channel can be hardly studied at large $\Delta^2$, due to the vanishing cross section [18]. Nuclear effects are found to be larger than in the forward case and to increase with $\Delta^2$ at fixed skewedness, and with the skewedness at fixed $\Delta^2$. In particular the latter $\Delta^2$ dependence does not simply factorize, in agreement with previous findings for the deuteron target [13] and at variance with prescriptions proposed for finite nuclei [17].

Here, the analysis of Ref. [22] is extended into various directions. The main point of the paper will be to stress that the properties of nuclear GPDs should not be trivially inferred from those of nuclear parton distributions. After a fast summary of the formalism of Ref. [22], in the third section of the paper a detailed study of the flavor dependence of the nuclear effects, which is due to the fact that $^3$He is non isoscalar, is carried on; a serious warning concerning the possibility to use momentum distributions instead of spectral functions is motivated; the dependence on the choice of the nucleon-nucleon potential used to estimate the nuclear GPDs is shown. In the following section, the information which could be obtained by using a $^3H$ target is addressed. Eventually, conclusions are drawn in the fifth section.
II. FORMALISM

The definitions of GPDs of Ref. [2] is used. For a spin 1/2 hadron target, with initial (final) momentum and helicity \( P(P') \) and \( s(s') \), respectively, the GPDs \( H_q(x, \xi, \Delta^2) \) and \( E_q(x, \xi, \Delta^2) \) are defined through the light cone correlator

\[
F_{s's}(x, \xi, \Delta^2) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P's' | \bar{\psi}_q \left( -\frac{\lambda n}{2} \right) \gamma_\mu \psi_q \left( \frac{\lambda n}{2} \right) | Ps \rangle = H_q(x, \xi, \Delta^2) \frac{1}{2} \bar{U}(P', s') \gamma_\mu U(P, s) + E_q(x, \xi, \Delta^2) \frac{1}{2} \bar{U}(P', s') i\sigma^{\mu\nu} n_\mu \Delta_\nu U(P, s),
\]

where \( \Delta = P' - P \) is the 4-momentum transfer to the hadron, \( \psi_q \) is the quark field and \( M \) is the hadron mass. It is convenient to work in a system of coordinates where the photon 4-momentum, \( q^\mu = (q_0, \vec{q}) \), and \( \bar{P} = (P + P')/2 \) are collinear along \( z \). The skewedness variable, \( \xi \), is defined as

\[
\xi = -\frac{n \cdot \Delta}{2} = -\frac{\Delta^+}{2P^+} = \frac{x_{Bj}}{2 - x_{Bj}} + O \left( \frac{\Delta^2}{Q^2} \right), \tag{2}
\]

where \( n \) is a light-like 4-vector satisfying the condition \( n \cdot \bar{P} = 1 \). (Here and in the following, \( a^\pm = (a^0 \pm a^3)/\sqrt{2} \). In addition to the variables \( x, \xi \) and \( \Delta^2 \), GPDs depend, on the momentum scale \( Q^2 \). Such a dependence, not discussed here, will be omitted. The constraints of \( H_q(x, \xi, \Delta^2) \) are:

i) the “forward” limit, \( P' = P \), i.e., \( \Delta^2 = \xi = 0 \), yielding the usual PDFs

\[
H_q(x, 0, 0) = q(x); \tag{3}
\]

ii) the integration over \( x \), yielding the contribution of the quark of flavour \( q \) to the Dirac form factor (f.f.) of the target:

\[
\int dx H_q(x, \xi, \Delta^2) = F_{1q}^q(\Delta^2); \tag{4}
\]

iii) the polynomiality property [2], involving higher moments of GPDs, according to which the \( x \)-integrals of \( x^n H_q \) and of \( x^n E_q \) are polynomials in \( \xi \) of order \( n + 1 \).

In Ref. [29], an expression for \( H_q(x, \xi, \Delta^2) \) of a given hadron target, for small values of \( \xi^2 \), has been obtained from the definition Eq. (11). The approach has been later applied in Ref. [22] to obtain the GPD \( H_3^q \) of \(^3\text{He}\) in IA, as a convolution between the non-diagonal
spectral function of the internal nucleons, and the GPD $H_q^N$ of the nucleons themselves. Let me recall the main formalism of Ref. [22], which will be used in this paper. In the class of frames discussed above, and in addition to the kinematical variables $x$ and $\xi$, already defined, one needs the corresponding ones for the nucleons in the target nuclei, $x'$ and $\xi'$. The latter quantities can be obtained defining the “+” components of the momentum $k$ and $k + \Delta$ of the struck parton before and after the interaction, with respect to $\bar{P}^+$ and $\bar{p}^+ = \frac{1}{2}(p + p')^+$:

$$k^+ = (x + \xi)\bar{P}^+ = (x' + \xi')\bar{p}^+ ,$$  \hspace{1cm} (5)$$

$$k^+ + \Delta = (x - \xi)\bar{P}^+ = (x' - \xi')\bar{p}^+ ,$$ \hspace{1cm} (6)

so that

$$\xi' = - \frac{\Delta^+}{2\bar{p}^+} ,$$ \hspace{1cm} (7)$$

$$x' = \frac{\xi'}{\xi}x ,$$ \hspace{1cm} (8)

and, since $\xi = -\Delta^+/(2\bar{P}^+)$, if $\bar{z} = p^+/P^+$

$$\xi' = \frac{\xi}{\bar{z}(1 + \xi) - \xi} .$$ \hspace{1cm} (9)

In Ref. [22], a convolution formula for $H_q^3$ has been derived in IA, using the standard procedure developed in studies of DIS off nuclei [31, 32, 33]. It reads:

$$H_q^3(x, \xi, \Delta^2) \approx \sum_N \int dE \int d\vec{p} \left[ P^3_N(\vec{p}, \vec{p} + \vec{\Delta}, E) + O(\vec{p}^2/M^2, \vec{\Delta}^2/M^2) \right] \times \frac{\xi'}{\xi} H_N^N(x', \xi', \Delta^2) + O(\xi^2) .$$ \hspace{1cm} (10)

In the above equation, $P^3_N(\vec{p}, \vec{p} + \vec{\Delta}, E)$ is the one-body non-diagonal spectral function for the nucleon $N$, with initial and final momenta $\vec{p}$ and $\vec{p} + \vec{\Delta}$, respectively, in $^3He$:

$$P^3_N(\vec{p}, \vec{p} + \vec{\Delta}, E) = \frac{1}{(2\pi)^3} \frac{1}{2} \sum_{R,s} \langle \vec{P} M | (\vec{P} - \vec{p})S_R, (\vec{p} + \vec{\Delta})s \rangle \langle (\vec{P} - \vec{p})S_R, \vec{p}s | \vec{P} M \rangle \times \delta(E - E_{\min} - E^*_R) ,$$ \hspace{1cm} (11)

and the quantity $H_N^N(x', \xi', \Delta^2)$ is the GPD of the bound nucleon $N$ up to terms of order $O(\xi^2)$ (note that in its definition use has been made of Eqs. (7) and (8)).
The delta function in Eq (11) defines $E$, the removal energy, in terms of $E_{\text{min}} = |E_{3He}| - |E_{2H}| = 5.5$ MeV and $E^*_R$, the excitation energy of the two-body recoiling system. The main quantity appearing in the definition Eq. (11) is the overlap integral

$$\langle \vec{P} M | \vec{P}_R S_R, \vec{P}s \rangle = \int d\vec{y} e^{i\vec{p} \cdot \vec{y}} \langle \chi^s, \Psi^S_R(\vec{x}) | \Psi^M_3(\vec{x}, \vec{y}) \rangle,$$  

(12)

between the eigenfunction $\Psi^M_3$ of the ground state of $^3$He, with eigenvalue $E_{3He}$ and third component of the total angular momentum $M$, and the eigenfunction $\Psi^S_R$, with eigenvalue $E = E_{\text{min}} + E^*_R$ of the state $R$ of the intrinsic Hamiltonian pertaining to the system of two interacting nucleons [34]. As discussed in Ref. [22], the accuracy of the calculations which will be presented, since a NR spectral function will be used to evaluate Eq. (10), is of order $O \left( \frac{\vec{p}^2}{M^2}, \frac{\Delta^2}{M^2} \right)$, or, which is the same, $\vec{p}^2, \Delta^2 << M^2$. While the first of these conditions is the usual one for the NR treatment of nuclei, the second forces one to use Eq. (10) only at low values of $\Delta^2$, for which the accuracy is good enough. The interest of the present calculation is indeed to investigate nuclear effects at low values of $\Delta^2$, for which measurements in the coherent channel may be performed. The main emphasis of the present approach, as already said, is not on the absolute values of the results, but in the nuclear effects, which can be estimated by taking any reasonable form for the internal GPD. Taking into account that

\[ z - \frac{\xi}{\xi'} = z - [\tilde{z}(1 + \xi) - \xi] = z + \xi - \frac{p^+}{P^+} (1 + \xi) = z + \xi - \frac{p^+}{P^+}, \]

(13)

Eq. (10) can be written in the form

$$H^3_q(x, \xi, \Delta^2) = \sum_N \int_x^1 \frac{dz}{z} h^3_N(z, \xi, \Delta^2) H^N_q \left( \frac{x}{z}, \xi, \Delta^2 \right),$$

(14)

where the off-diagonal light cone momentum distribution

$$h^3_N(z, \xi, \Delta^2) = \int dE \int d\vec{p} P^2_N(\vec{p}, \vec{p}' + \Delta) \delta \left( z + \xi - \frac{p^+}{P^+} \right)$$

(15)

has been introduced. As it is shown in Ref. [22], Eqs. (14) and (15) or, which is the same, Eq. (10), fulfill the constraint $i) - iii)$ previously listed. The constraint $i)$, i.e. the forward limit of GPDs, is verified by taking the forward limit ($\Delta^2 \to 0, \xi \to 0$) of Eq. (14), yielding the parton distribution $q_3(x)$ in IA: [31, 32, 38]:

$$q_3(x) = H^3_q(x, 0, 0) = \sum_N \int_x^1 \frac{dz}{z} f^3_N(z) q_N \left( \frac{x}{z} \right).$$

(16)
In the latter equation,

\[ f_3^N(z) = h_3^N(z, 0, 0) = \int dE \int d\vec{p} P_3^N(\vec{p}, E) \delta \left( z - \frac{p^+}{P^+} \right) \]  

(17)

is the forward limit of Eq. (15), i.e. the light cone momentum distribution of the nucleon \( N \) in the nucleus, \( q_N(x) = H_3^N(x, 0, 0) \) is the distribution of the quark of flavour \( q \) in the nucleon \( N \) and \( P_3^N(\vec{p}, E) \), the \( \Delta^2 \rightarrow 0 \) limit of Eq. (14), is the one body spectral function.

The constraint \( ii \), i.e. the \( x \)-integral of the GPD \( H_q \), is also fulfilled. By \( x \)-integrating Eq. (14), one obtains:

\[ \int dx H_3^q(x, \xi, \Delta^2) = \sum_N F_q^N(\Delta^2) F_3^N(\Delta^2) = F_3^q(\Delta^2) . \]  

(18)

Eventually the polynomiality, condition \( iii \), is formally fulfilled by Eq. (10).

In the following, \( H_3^q(x, \xi, \Delta^2) \), Eq. (10), will be evaluated in the nuclear Breit Frame. The non-diagonal spectral function Eq. (11), appearing in Eq. (10), will be calculated along the lines of Ref. [35], by means of the overlap Eq. (12), which exactly includes the final state interactions in the two nucleon recoiling system [34]. The realistic wave functions \( \Psi_M^3 \) and \( \Psi_R^3 \) in Eq. (12) have been evaluated using the AV18 interaction [36]. In particular \( \Psi_M^3 \) has been developed along the lines of Ref. [37]. The same overlaps have been already used in Ref. [22, 38].

The other ingredient in Eq. (10), i.e. the nucleon GPD \( H_q^N \), has been modelled in agreement with the Double Distribution representation [30], as described in [39]:

\[ H_q^N(x, \xi, \Delta^2) = \int_{-1}^{1} d\tilde{x} \int_{-1+|\tilde{x}|}^{1-|\tilde{x}|} \delta(\tilde{x} + \xi \alpha - x) \tilde{\Phi}_q(\tilde{x}, \alpha, \Delta^2) d\alpha , \]  

(20)

using the factorized ansatz:

\[ \tilde{\Phi}_q(\tilde{x}, \alpha, \Delta^2) = h_q(\tilde{x}, \alpha) \Phi_q(\tilde{x}) F_q(\Delta^2) . \]  

(21)

The expressions for the functions \( h_q(\tilde{x}, \alpha) \) and \( \Phi_q(\tilde{x}) \) can be found in [22]; I recall here that the \( F_q(\Delta^2) \) term in Eq. (21), i.e. the contribution of the quark of flavour \( q \) to the nucleon
form factor, has been obtained from the experimental values of the proton, $F^p_1$, and of the neutron, $F^n_1$, Dirac form factors. For the $u$ and $d$ flavors, neglecting the effect of the strange quarks, one has

$$F_u(\Delta^2) = \frac{1}{2} [2F^p_1(\Delta^2) + F^n_1(\Delta^2)],$$
$$F_d(\Delta^2) = 2F^n_1(\Delta^2) + F^p_1(\Delta^2).$$

(22)

The contributions of the flavors $u$ and $d$ to the proton and neutron f.f. are therefore

$$F^p_u(\Delta^2) = \frac{4}{3} F_u(\Delta^2),$$
$$F^p_d = -\frac{1}{3} F_d(\Delta^2),$$

(23)

and

$$F^n_u(\Delta^2) = \frac{2}{3} F_d(\Delta^2),$$
$$F^n_d = -\frac{2}{3} F_u(\Delta^2),$$

(24)

respectively.

For the numerical calculations, use has been made of the parametrization of the nucleon Dirac f.f. given in Ref. [40]. I stress again that the main point of the present study is not to produce realistic estimates for observables, but to investigate and discuss nuclear effects, which do not depend on the form of any well-behaved internal GPD, whose general structure is safely simulated by Eqs. (20) – (21). In Ref. [22] it has been shown that the described formalism reproduces well, in the proper limits, the IA results for nuclear parton distributions and form factor. In particular, in the latter case, the IA calculation reproduces well the data up to a momentum transfer $-\Delta^2 = 0.25$ GeV$^2$, which is enough for the aim of this calculation. In fact, the region of higher momentum transfer is not considered here, being phenomenologically not relevant for the calculation of GPDs entering coherent processes.

III. DISCUSSION OF CONVENTIONAL NUCLEAR EFFECTS

In this section, some conventional nuclear effects on the GPDs of $^3He$ will be discussed. The aim is that of avoiding to mistake them for exotic ones in possible measurements of nuclear GPDs, and to stress the relevance of experiments using $^3He$ targets.
As already done in Ref. [22], the full result for $H_3^q$, Eq. (10), will be compared with a prescription based on the assumptions that nuclear effects are neglected and the global $\Delta^2$ dependence is described by the f.f. of $^3\text{He}$:

$$H_3^{3,(0)}(x, \xi, \Delta^2) = 2H_3^{3,p}(x, \xi, \Delta^2) + H_3^{3,n}(x, \xi, \Delta^2),$$

(25)

where the quantity

$$H_3^{3,N}(x, \xi, \Delta^2) = \tilde{H}_N^N(x, \xi)F_3^q(\Delta^2)$$

(26)

represents effectively the flavor $q$ GPD of the bound nucleon $N = n, p$ in $^3\text{He}$. Its $x$ and $\xi$ dependences, given by $\tilde{H}_N^N(x, \xi)$, are the same of the GPD of the free nucleon $N$ (represented by Eq. (20)), while its $\Delta^2$ dependence is governed by the contribution of the flavor $q$ to the $^3\text{He}$ f.f., $F_3^3(\Delta^2)$. The effect of nucleon motion and binding can be shown through the ratio

$$R_q(x, \xi, \Delta^2) = \frac{H_3^q(x, \xi, \Delta^2)}{H_3^{3,(0)}(x, \xi, \Delta^2)},$$

(27)

i.e. the ratio of the full result, Eq. (10), to the approximation Eq. (25). The latter is evaluated by means of the nucleon GPDs used as input in the calculation, and taking $F_u^3(\Delta^2) = \frac{10}{3}F_{ch}^3(\Delta^2)$, and $F_d^3(\Delta^2) = -\frac{4}{3}F_{ch}^3(\Delta^2)$, where $F_{ch}^3(\Delta^2)$ is the f.f. which is calculated within the present approach, by means of Eq. (18). The coefficients $10/3$ and $-4/3$ are chosen assuming that the contribution of the valence quarks of a given flavour to the f.f. of $^3\text{He}$ is proportional to their charge. The ratio Eq. (27) shows nuclear effects in a very natural way. As a matter of facts, its forward limit yields an EMC-like ratio for the parton distribution $q$ and, if $^3\text{He}$ were made of free nucleon at rest, it would be one. This latter fact can be realized by observing that the prescription Eq. (25) is obtained by placing $z = 1$, i.e. no convolution, into Eq. (10). One should note that the prescription suggested in Ref. [17] for finite nuclei, assuming that the nucleus is a system of almost free nucleons with approximately the same momenta, has the same $\Delta^2$ dependence of the prescription Eq. (25).

In Figs. 1 to 7, results will presented concerning: A) flavor dependence of nuclear effects; B) binding effects; C) dependence on the nucleon-nucleon potential.
A. Flavor dependence of nuclear effects

In the upper panel of Fig. 1, the ratio Eq. (27) is shown for the $u$ and $d$ flavor, in the forward limit, as a function of $x_3 = 3x$. The trend is clearly EMC-like. It is seen that nuclear effects for the $d$ flavour are very slightly bigger than those for the $u$ flavour. The reason is understood thinking that, in the forward limit, the nuclear effects are governed by the light cone momentum distribution, Eq. (17): no effects would be found if such a function were a delta function, while effects get bigger and bigger if its width increases. In the lower panel of the same figure, the light cone momentum distribution, Eq. (17), for the proton (neutron) in $^3He$ is represented by the dashed (full) line. The neutron distribution is slightly wider than the proton one, meaning that the average momentum of the neutron in $^3He$ is a little larger than the one of the proton [32, 34, 41]. Since the forward $d$ distribution is more sensitive than the $u$ one to the neutron light cone momentum distribution, nuclear effects for $d$ are slightly larger than for $u$, as seen in the upper panel of the same figure.

In Fig. 2, the same analysis of Fig. 1 is performed, but at \( \Delta^2 = -0.25 \text{ GeV}^2 \) and \( \xi_3 = 3\xi = 0.2 \). In this case, nuclear effects are governed by the non-diagonal light cone momentum distribution, Eq. (15), shown in the lower panel of the figure. In this case, the difference between the neutron and proton distributions is quite bigger than in the forward case, governing the difference in the ratio Eq. (27) for the two flavors, which is of the order of 10 \%, as it is seen in Fig. 3.

From Figs. 1-3 three main conclusions can be drawn. 1) First of all, if one infers properties of nuclear GPDs thinking to those of nuclear PDs, conventional nuclear effects as big as 10 \% can be easily lost, or mistaken for exotic ones. 2) Secondly, this behavior is a typical conventional effect, being a prediction of IA in DIS off nuclei. If a 10 \% effect would be observable in experimental studies of nuclear GPDs, the presence of such a flavor dependence, or its absence, would be clear signatures of the reaction mechanism of DIS off nuclei. Its presence would mean that the reaction involves essentially partons inside nucleons, whose dynamics is governed by a realistic potential in a conventional scenario; on the contrary, its absence would mean that, in a different, exotic scenario, other degrees of freedom have to be advocated. 3) Eventually, it is clear that, for this kind of studies, $^3He$ is a unique target, for which experiments are worth to be done: the flavor dependence cannot be investigated with isoscalar targets, such as $^2H$ or $^4He$, while for heavier nuclei calculations cannot be
performed with comparable precision.

**B. Binding effects**

In the previous section it has been explained how Eq. (14) takes into account properly the nucleon momentum and energy distributions through a non-diagonal spectral function. In the following, the performances will be studied of three approximations with increasing complexity of the full result, defined considering: i) binding and momentum effects simulated by a rescaling of variables; ii) binding effects neglected by using a momentum distribution; iii) average binding effects evaluated within a simple model of the spectral function.

i) Binding and momentum effects simulated by a rescaling of variables.

A prescription for nuclear GPDs has been proposed, based on a rescaling of the variables [17]:

\[
H^{3,(1)}_u(x, \xi, \Delta^2) = F^{3}_u(\Delta^2) \frac{dx_N}{dx} \theta(|x_N| \leq 1) [ZH^u(x_N, \xi_N, 0) + NH^d(x_N, \xi_N, 0)]
\]  (28)

for the \( u \) flavour; the analogous expression for the \( d \) flavour is obtained by isospin symmetry. In the above equation, one has, for \( ^3He \)

\[
x_N = 3x \frac{1 + \xi_N}{1 + \xi},
\]  (29)

and

\[
\xi_N = \frac{3\xi}{(A-1)\xi - 1},
\]  (30)

while \( F^{3}_u(\Delta^2) \) is the flavour \( u \) contribution to the \( ^3He \) f.f., to be fixed by experimental data. The reliability of the approximation Eq. (28) can be established studying the ratio:

\[
R^{(1)}_q(x, \xi, \Delta^2) = \frac{H^{3}_q(x, \xi, \Delta^2)}{H^{3,(1)}_q(x, \xi, \Delta^2)},
\]  (31)

where the numerator is given by the full result, Eq. (14), and the denominator by Eq. (28). This comparison permits to estimate to what extent Eq. (28) describes effectively the nucleon motion and binding. Such a prescription has been indeed used to parametrize nuclear GPDs for estimates of DVCS cross sections and asymmetries for finite nuclei. To have a consistent check, the denominator in Eq. (31), i.e. Eq. (28), has been evaluated through the model for \( H^{N}_q \) used in the full calculation, together with the \( u \) contribution to
the $^3He$ f.f. previously used. Results are presented in Fig. 4, where the ratio Eq. (31) is shown for the $d$ flavor, in the forward limit and for $\Delta^2 = -0.25$ GeV$^2$ and $\xi_3 = 0.2$. It is clearly seen that, while the approximation Eq. (28) differs from the full result by at most 5 % in the forward limit, yielding something similar to an EMC-like effect, it differs systematically by more than 10 % for all the values of $x_3$. In general, the prescription Eq. (28) does not describe effectively the conventional nuclear effects, not even in a rough way, and one should not use it, at least for light nuclei.

ii) Binding effects neglected by using a momentum distribution.

In the previous section, it has been shown that IA leads to nuclear effects governed by a one body non-diagonal spectral function. This means that overlap integrals involving excited states with a given excitation energy, $E^*_R$, of the two body recoiling system, have to be evaluated. When a momentum distribution is used instead of a spectral function, not only the IA, but also another approximation, the so called “closure approximation”, has been used: an average excitation energy, $\bar{E}^*$, has been inserted in the expression of the delta function appearing in the definition of the spectral function Eq. (11), so that the completeness of the two body recoiling states can be used [32]:

$$P^A_N(\vec{p}, \vec{p} + \vec{\Delta}, E) \simeq \sum_M \sum_s \langle \vec{P}^M | a_{\vec{p} + \vec{\Delta}, s} a^\dagger_{\vec{p}, s} | \vec{P} M \rangle \delta(E - E_{\text{min}} - \bar{E}^*)$$

$$= n(\vec{p}, \vec{p} + \vec{\Delta}) \delta(E - E_{\text{min}} - \bar{E}^*),$$  

(32)

and the spectral function is approximated by a one-body non diagonal momentum distribution times a delta function defining an average value of the removal energy. Whenever the momentum distribution is used instead of the spectral function, in addition to the IA the above closure approximation has been used assuming $\bar{E}^* = 0$, i.e., binding effects have been completely neglected. The difference between the full calculation and the one using the momentum distribution, for the ratio Eq. (27), is shown in Fig. 5. It is seen that, while the difference is a few percent in the forward limit, it grows in the non-forward case, becoming an effect of 5 % to 10 % between $x = 0.4$ and 0.7. From this analysis one should draw three main conclusions, basically the same arisen in the study of the flavor dependence: 1) the size of nuclear effects found for GPDs is bigger than that found in inclusive DIS; 2) if the conventional binding were not taken into account, a possible 10 % effect, found experimentally, could be mistaken for an exotic effect; 3) $^3He$ is a unique target to study the binding effects, since the realistic evaluation of non diagonal spectral functions is a challenging task.
for heavier targets.

iii) Average binding effects evaluated within a simple model of the spectral function.

A little more refined calculation can be performed using, in Eq. (32), an average value of the removal energy, instead of the minimum one. A calculation has been performed assuming, as average values of the excitation energy, the values calculated by means of the spectral functions corresponding to the AV18 interaction. The obtained values are in agreement with the ones listed in Ref. [41]. In Fig. 6 it is shown that the situation, if compared with the previous calculation, performed using a momentum distribution only, improves a little. In any case, the difference becomes negligible in the forward limit, in agreement with the findings of Ref. [32], while it keeps being sizable in the non-forward one.

One should also keep in mind that, in general, for $^3$He, which is a loosely bound system, binding effects are smaller than for heavier nuclei. Moreover, for the latter systems, for which realistic spectral functions are not available, the evaluation of the average removal energy is not easy and it is affected by theoretical uncertainties. Therefore the conclusions of the previous subsection, concerning the importance and relevance of binding effects in studies of nuclear GPDs, are confirmed.

C. Dependence on the nucleon-nucleon potential

In Fig. 7, the difference is shown between the full calculation, Eq. (27), evaluated with the AV18 interaction [36], and the same quantity, evaluated by means of the AV14 one. It is seen that there is basically no difference in the forward limit, confirming previous findings in inclusive DIS [38], while a sizable difference is seen in the non-forward case (preliminary results of this behavior have been accounted for in a talk at a Conference [22]). From these analyses the same conclusions of the previous two subsections can be drawn: 1) properties of nuclear GPDs should not be naively inferred by those of nuclear PDs; 2) if conventional effects were not properly evaluated they could be mistaken for exotic ones in the analysis of the data; 3) $^3$He, for which the best realistic calculations are possible, is a unique target to study these effects. We note on passing that a difference between observables evaluated using AV18 and AV14 potentials is not easily found, in particular in inclusive DIS.
IV. GPDS FOR THE $^3H$ TARGET

In the perspective of using $^3H$ targets after the 12 GeV upgrade of JLab \cite{42}, it is useful to address what could be learnt from simultaneous measurements with trinucleon targets, $^3He$ and $^3H$.

The procedure proposed firstly in Ref. \cite{24} for the unpolarized DIS to extract, with unprecedented precision, the ratio of down to up quarks in the proton, $d(x)/u(x)$, at large Bjorken $x$, is extended here to the case of the GPDs of trinucleons. To minimize nuclear effects, the following “super-ratio”, a generalization of the one proposed in Ref. \cite{24}, can be defined

$$S_{qq'}(x, \xi, \Delta^2) = \frac{R^H_q(x, \xi, \Delta^2)}{R^T_{q'}(x, \xi, \Delta^2)}, \quad (33)$$

where the ratio

$$R^A_q(x, \xi, \Delta^2) = \frac{H^A_q(x, \xi, \Delta^2)}{Z_A H^p_q(x, \xi, \Delta^2) + N_A H^n_q(x, \xi, \Delta^2)}, \quad (34)$$

has been introduced for $^3He$ ($A = H$) and $^3H$ ($A = T$), with $q = u, d$, $Z_A(N_A)$ the number of protons (neutrons) in the nucleus $A$, and $H^N_q(x, \xi, \Delta^2)$ the GPD of the quark $q$ in the nucleon $N = p, n$. Now, using the isospin symmetry of GPDs \cite{4}, we can call

$$H_u(x, \xi, \Delta^2) = H^p_u(x, \xi, \Delta^2) = H^n_d(x, \xi, \Delta^2), \quad (35)$$

$$H_d(x, \xi, \Delta^2) = H^p_d(x, \xi, \Delta^2) = H^n_u(x, \xi, \Delta^2), \quad (36)$$

so that Eq. (33) is given, for example for $q = d$ and $q' = u$, by the simple relation

$$S_{du}(x, \xi, \Delta^2) = \frac{H^H_d(x, \xi, \Delta^2)}{H^T_u(x, \xi, \Delta^2)}, \quad (37)$$

a quantity in principle observable.

In the IA approach discussed here, using Eq. (14) to calculate the nuclear GPDs, one has therefore

$$S_{du}(x, \xi, \Delta^2) = \frac{\int_x^1 \frac{dz}{z} \left\{ h^H_p(z, \xi, \Delta^2) H_d \left( \frac{x}{z}, \xi, \Delta^2 \right) + h^H_n(z, \xi, \Delta^2) H_u \left( \frac{x}{z}, \xi, \Delta^2 \right) \right\}}{\int_x^1 \frac{dz}{z} \left\{ h^T_p(z, \xi, \Delta^2) H_d \left( \frac{x}{z}, \xi, \Delta^2 \right) + h^T_n(z, \xi, \Delta^2) H_u \left( \frac{x}{z}, \xi, \Delta^2 \right) \right\}}, \quad (38)$$

where $h^{H(T)}_{p(n)}(z, \xi, \Delta^2)$ represents the light cone off diagonal momentum distribution for the proton (neutron) in $^3He$ ($^3H$). If the Isospin Symmetry were valid at the nuclear level, one
should have $h_p^H(z, \xi, \Delta^2) = h_n^T(z, \xi, \Delta^2)$, and $h_n^H(z, \xi, \Delta^2) = h_p^T(z, \xi, \Delta^2)$, so that the ratio Eq. (38) would be identically 1. From the analysis of Section III and the Figures 1 and 2 it is clear anyway that these relations are only approximately true, and some deviations are expected.

In Fig. 8, the super-ratio $S_{du}(x, \xi, \Delta^2)$, Eq. (33), evaluated by using the AV18 interaction for the nuclear GPDs in Eq. (14), taking into account therefore the Coulomb interaction between the protons in $^3He$ and a weak charge independence breaking term, is shown for different values of $\Delta^2 \leq 0.25 \text{ GeV}^2$ and $\xi$. While it is seen that, as expected, $S_{du}(x, \xi, \Delta^2)$ is not exactly 1 and the difference gets bigger with increasing $\Delta^2$ and $\xi$, for the low values of $\Delta^2$ and $\xi$ relevant for the present investigation of GPDs, such a difference keeps being a few percent one.

It would be very interesting to measure this ratio experimentally. If strong deviations from this predicted behavior were observed, there would be a clear evidence that the description in terms of IA, i.e. in terms of the conventional scenario of partons confined in nucleons bound together by a realistic interaction, breaks down. In other words one could have a clear signature of possible interesting exotic effects. One should notice that the trend obtained for the ratio Eq. (33) is almost flat; this may have to do with the simple model used for the nucleon GPD in the convolution formula. The present analysis permits only to estimate the size of nuclear effects between the forward and the general case, and not to predict the $x$ behavior of the ratio with good precision. In any case, the possibilities offered by a $^3H$ target deserve more attention and will be discussed in more detail elsewhere.

V. CONCLUSIONS

In this paper, using a realistic microscopic calculation, some peculiar conventional nuclear effects on the unpolarized quark GPD $H_q^3(x, \xi, \Delta^2)$ for $^3He$ have been investigated. By studying the dependence of nuclear effects on the flavor, the nucleon binding and the nucleon-nucleon potential, the same three main conclusions have been obtained: 1) the size of nuclear effects found in inclusive DIS should not be naively used to estimate the one expected for GPDs; 2) if conventional nuclear physics is not taken into account properly, several experimentally observable effects, each of the order of 10 %, could be mistaken for exotic ones; 3) trinucleons represent unique targets to study these effects, since a comparably
realistic treatment for heavier targets is a challenging task. It has also been shown that the simultaneous use of a $^3He$ and $^3H$ target would help disentangling the conventional effects from the exotic ones.

The issue of applying the obtained GPDs to estimate cross-sections and to establish the feasibility of experiments, is in progress and will be presented elsewhere. In particular, the study of polarized GPDs will be very interesting, due to the peculiar spin structure of $^3He$ and its implications for the study of the angular momentum of the free neutron.

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Figure Captions

**Fig. 1**: Upper panel: the dashed (full) line represents the ratio Eq. (27), for the $u$ ($d$) flavor, in the forward limit. Lower panel: the dashed (full) line represents the light cone momentum distribution, Eq. (17), for the proton (neutron) in $^3He$.

**Fig. 2**: Upper panel: the dashed (full) line represents the ratio Eq. (27), for the $u$ ($d$) flavor, at $\Delta^2 = -0.25$ GeV$^2$ and $\xi_3 = 0.2$. Lower panel: the dashed (full) line represents the light cone off diagonal momentum distribution, Eq. (15), for the proton (neutron) in $^3He$, at $\Delta^2 = -0.25$ GeV$^2$ and $\xi_3 = 0.2$.

**Fig. 3**: The ratio of the ratios Eq. (27), for the $d$ to the $u$ flavor, at $\Delta^2 = -0.25$ GeV$^2$ and $\xi_3 = 0.2$ (full line) and in the forward limit (dashed line).

**Fig. 4**: The ratio Eq. (31), for the $d$ flavor, in the forward limit (full line) and at $\Delta^2 = -0.25$ GeV$^2$ and $\xi_3 = 0.2$ (dashed line).

**Fig. 5**: Upper panel: the ratio Eq. (27), in the forward limit, for the $d$ flavor, corresponding to the full result of the present approach (full line), compared with the one obtained using in the numerator the approximation Eq. (32) with $\bar{E}^* = 0$, i.e., using a momentum distribution instead of a spectral function (dashed line). Lower panel: the same as before, but evaluated at $\Delta^2 = -0.25$ GeV$^2$ and $\xi_3 = 0.2$.

**Fig. 6**: The same of Fig. 5, but using the approximation Eq. (32) with the values of $\bar{E}^*$ obtained from the spectral function corresponding to the AV18 interaction.

**Fig. 7**: Upper panel: the ratio Eq. (27), in the forward limit, for the $d$ flavor, corresponding to the full result of the present approach, where use is made of the AV18 interaction (full line), compared with the one obtained using in the numerator the AV14 interaction (dashed line): the two curves cannot be distinguished. Lower panel: the same, but evaluated $\Delta^2 = -0.25$ GeV$^2$ and $\xi_3 = 0.2$: now the curves are distinguishable.

**Fig. 8**: The ratio Eq. (38), in the forward limit (dot-dashed line), at $\Delta^2 = -0.15$ GeV$^2$ and $\xi_3 = 0.1$ (dashed line), and at $\Delta^2 = -0.25$ GeV$^2$ and $\xi_3 = 0.2$ (full line).
FIG. 1:
FIG. 2:
FIG. 3:
FIG. 4:
FIG. 5:

$R_d(x_3,0,0)$

$R_d(x_3,\xi_3=0.2,\Delta^2=-0.25 \text{ GeV}^2)$
$R_d(x_3; 0,0)$

$R_d(x_3; \xi_3 = 0.2, \Delta^2 = -0.25 \text{ GeV}^2)$

FIG. 6:
FIG. 7:
FIG. 8: