Achieving Adversarial Robustness via Sparsity

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Abstract
Network pruning has been known to produce compact models without much accuracy degradation. However, how the pruning process affects a network’s robustness and the working mechanism behind remain unresolved. In this work, we theoretically prove that the sparsity of network weights is closely associated with model robustness. Through experiments on a variety of adversarial pruning methods, we find that weights sparsity will not hurt but improve robustness, where both weights inheritance from the lottery ticket and adversarial training improve model robustness in network pruning. Based on these findings, we propose a novel adversarial training method called *inverse weights inheritance*, which imposes sparse weights distribution on a large network by inheriting weights from a small network, thereby improving the robustness of the large network.

1 Introduction
It is widely recognized that deep neural networks (DNNs) are usually over-parameterized, and network pruning has been adopted to remove insignificant weights from a large neural network without hurting the accuracy. Despite its success, pruning strategies have been rarely discussed in the adversarial learning setting where the network is trained against adversarial examples, and the robustness of the network is as important as accuracy.

It is unclear what pruning methods are effective and which factors are critical for retaining model robustness. Believing that the inherited model weights may not be effective in preserving network accuracy, Ye et al. (2019) and Liu et al. (2019) propose a concurrent adversarial training and weight pruning framework to seek a compressed robust model. Gui et al. (2019) further incorporates pruning and several other techniques into a unified optimization framework to preserve high robustness while achieving a high compression ratio. However, the conventional three-stage ‘training–pruning–fine-tuning’ pipeline has not been closely examined in the adversarial context. More crucially, it is unclear which components in the network pruning methods are critical to preserving model performance. To this end, we design a comprehensive set of experiments to answer these questions.

Despite some adversarial pruning methods that have been proposed, there is still a lack of theoretical foundation to explain the working mechanism behind those methods. In fact, there are seemingly contradictory opinions on the robustness of pruned networks: Madry et al. (2018) suggests network capacity is crucial to robustness, and a wider network is more likely to obtain higher accuracy and robustness than a simple network. In contrast, Guo et al. (2018) theoretically proves that an appropriately higher weight sparsity implies stronger robustness on naturally trained models. Other theories such as the ‘Lottery Ticket Hypothesis’ (Frankle and Carbin 2019) hold true in the adversarial learning context. As we verify that ‘Lottery Ticket Hypothesis’ (Frankle and Carbin 2019) point out that, a subnetwork extracted from a large network can always achieve comparable performance with the original one in the natural setting. However, it remains unknown if the hypothesis holds true for adversarially robust networks. We are motivated to explore how adversarial pruning affects the intrinsic characteristics of the network and its impact on model robustness.

In this study, we find that the robustness of the model improves as its weights become sparser. We show that weights sparsity not only includes the traditional $L_0$-sparsity, i.e., the number of parameters retained, but also a weight distribution closer to zero, represented generally by the $L_p$ norm of weights. These forms of sparsity can lead to robustness improvement, which is verified theoretically and experimentally. By extensive experiments on a variety of state-of-the-art pruning methods, models, and datasets, we also demonstrate that a pruned network inheriting weights from a large robust network has improved robustness than a network with the same structure but randomly initialized weights. Moreover, weight inheritance implicitly produces sparser weights distributions on adversarially pruned models.

Inspired by the connection between model sparsity and robustness, we propose a new adversarial training strategy called *Inverse Weights Inheritance*: by inheriting weights from a pruned model, a large network can achieve higher robustness than being adversarially trained from scratch. The pruned model can be the ‘winning ticket’ of the large network, as we verify that ‘Lottery Ticket Hypothesis’ (Frankle and Carbin 2019) holds true in the adversarial learning context. The performance results of our proposed training strategy corroborate that sparse weights and high capacity are not contradictory, but contribute joint efforts to model robustness.

The contributions of the paper can be summarized as fol-
with $\epsilon$ (PGD) as a universal ‘first-order adversary,’ and optimizes adversarial examples (Kurakin, Goodfellow, and Bengio 2017). Adversarial training and its variants are proposed to improve network robustness against adversarial examples (Kurakin, Goodfellow, and Bengio 2017). Madry et al. (2018) motivates projected gradient descent (PGD) as a universal ‘first-order adversary,’ and optimizes the saddle point formulation to train a robust network. Goldblum et al. (2020) observes robustness can transfer between networks by knowledge distillation, and such transfer can even improve the robustness of the student network. Following the convention, we adopt $L_\infty$-PGD attack (Madry et al. 2018), i.e., the strongest attack utilizing the local first-order information of the network, both in adversarial training strategy and the robustness evaluations.

Network Pruning Methods. Network pruning methods related to this paper can be divided into two categories: structured pruning and unstructured pruning. Structured pruning prunes a network at the level of filters (Lang 2018; Li et al. 2017; Luo, Wu, and Lin 2017); channels (Liu et al. 2017) or columns (Wen et al. 2016), depending on their respective importance. The importance of a filter or a channel can be determined by the norm of the weights (Li et al. 2017) or the channel scaling factor (Ye et al. 2018; Liu et al. 2017) (sometimes the scaling factor in batch normalization layers). The unstructured pruning (LeCun, Denker, and Solla [1990], Hassibi and Stork [1993]) prunes at the level of individual weight according to the Hessian matrix of the loss function. Han et al. (2015) proposes to prune weights with small magnitude, and the compression ratio is further enhanced in Han, Mao, and Dally (2016) by quantization and Huffman coding. By incorporating non-negative stochastic gates, Louizos, Welling, and Kingma (2018) turns network pruning into an optimization problem with $L_0$-norm regularization. We pick representative structured and unstructured pruning methods to implement in our experiments.

2 Related Work

Adversarial Training. Adversarial training and its variants are proposed to improve network robustness against adversarial examples (Kurakin, Goodfellow, and Bengio 2017). Madry et al. (2018) motivates projected gradient descent (PGD) as a universal ‘first-order adversary,’ and optimizes the saddle point formulation to train a robust network. Goldblum et al. (2020) observes robustness can transfer between networks by knowledge distillation, and such transfer can even improve the robustness of the student network. Following the convention, we adopt $L_\infty$-PGD attack (Madry et al. 2018), i.e., the strongest attack utilizing the local first-order information of the network, both in adversarial training strategy and the robustness evaluations.

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Network Pruning in Adversarial Context. Network pruning in adversarial context has been recently discussed in search of small and robust models (Wang et al. 2018; Zhao et al. 2018; Ye et al. 2019; Sehwag et al. 2019). Several frameworks (Rakin et al. 2019; Madaan, Shin, and Hwang 2019; Gui et al. 2019) have been proposed to adversarially train a neural network while constraining its size by pruning and/or quantization. However, these works do not answer which pruning factors are important for robust networks, nor which pruning methods are effective.

The ‘lottery ticket hypothesis’ (Frankle and Carbin 2019) shows the existence of a sparse subnetwork (or ‘winning ticket’) in a randomly initialized network that can reach comparable performance with the large network. Nevertheless, Ye et al. (2019) argues against the existence of ‘winning ticket’ in adversarial settings. On the other hand, Cosentino et al. (2019) manages to acquire adversarial winning tickets of simple models without harming model robustness. Li et al. (2020) further proposed an optimized learning rate schedule to boost the searching performance of lottery tickets, while demonstrating why Ye et al. (2019) fails to find them.

Liu et al. (2019) claims that for network pruning in the natural setting, weights inherited by unstructured and predefined structured pruning may not be useful, as it may trap the pruned network to bad local minima. We show with experiments that weight inheritance improves the robustness in the adversarial setting, which we conjecture that this is because the inverse weights inheritance embrace larger networks during training, which can help jump out of local minima and achieve better generalization performance. Hein and Andriushchenko (2017) proposes a formal guarantee of adversarial robustness in terms of the local Lipschitz constant. By building a bridge between the local Lipschitz constant and weight sparsity, Guo et al. (2018) considers that an appropriately higher weight sparsity on naturally trained networks implies higher robustness. Dinh et al. (2020) also finds an adversarially trained network with sparser weights distribution tends to be more robust, such as EnResNet20 (Wang, Shi, and Osher 2019). Different from compression, Dhillon et al. (2018) proposes dynamic sparsity as an approach to improve robustness. By supplementing the concept of ‘sparsity,’ we found empirical evidence of the link between robustness and sparsity, as well as training strategies to boost robustness.
3 Study of Robustness and Network Pruning

In this section, we theoretically prove that sparser weights distribution indicates an improved level of robustness. In the theoretical deduction, we assume DNNs with ReLU activation functions, but the conclusion can be generalized to a variety of models, as we verify by experiments.

We focus on nonlinear DNNs with ReLU activation functions for classification tasks as an example to study the connection between sparsity and robustness. Let us consider a multi-layer perceptron \( g(\cdot) \) trained with labeled training datasets \( \{(x_i, y_i)\} \). Each layer of the network is parameterized by a weight matrix \( W_d \in \mathbb{R}^{n_d \times n_d} \) and \( w_k = W_d[:; k] \) represents the weights associated with the \( k \)-th class in the final layer. \( \sigma \) denotes the ReLU function. Then the prediction scores of \( x_i \), for class \( k \) can be denoted as

\[
g_k(x_i) = w_k^T \sigma (W_d^{T} \sigma (... \sigma ( W_1^T x_i))))
\]

(1)

Let \( \tilde{y} = \arg \max_{k \in \{1, \ldots, c\}} g_k(x) \) denote the class with the highest prediction score. Assuming the classifier is Lipschitz continuous, the local Lipschitz constant of function \( g_k(x) - g_k(x') \) over the neighborhood of \( x \) is defined as \( L_{k,x} = \max_{x \in B_p(x, R)} \| \nabla g_k(x) - \nabla g_k(x') \| \), where \( B_p(x, R) \) denotes a ball centered at \( x \) with radius \( R \) under \( L_p \) norm. Previous works (Hein and Andriushchenko 2017; Guo et al. 2018) have associated robustness with the local Lipschitz constant by the following theorem:

**Theorem 1** (Hein and Andriushchenko 2017; Guo et al. 2018) Let \( \tilde{y} = \arg \max_{k \in \{1, \ldots, c\}} g_k(x) \) and \( p + \frac{1}{q} = 1 \).

For any perturbation \( \delta_x \in B_p(0, R) \), \( p \in \mathbb{R}^+ \) and a set of Lipschitz continuous functions \( \{g_k : \mathbb{R}^n \rightarrow \mathbb{R}\} \), the classification decision on \( x' \) will not change with

\[
\|\delta_x\|_p \leq \min \left\{ \min_{k \neq \tilde{y}} \frac{g_{\tilde{y}}(x) - g_k(x)}{L_{k,x}^k} \right\},
\]

where \( L_{k,x}^k = \max_{x' \in B_p(x, R)} \| \nabla g_k(x') - \nabla g_k(x') \| \).

Eqn. (2) has clearly depicted the relation between robustness and the local Lipschitz constant — a smaller \( L_{k,x}^k \) represents a higher level of robustness as a larger distortion can be tolerated without changing the prediction. Guo et al. (2018) further gives the relation between the local Lipschitz constant and the weights. We further deduce that the relation satisfies the following theorem:

**Theorem 2** (The robustness and weights distribution of ReLU networks.) Letting \( \frac{1}{p} + \frac{1}{q} = 1 \), for any \( x \in \mathbb{R}^n \), \( k \in \{1, \ldots, c\} \) and \( q \in \{1, 2\} \), the local Lipschitz constant of function \( g_{\tilde{y}}(x) - g_k(x) \) satisfies

\[
L_{k,x}^k \leq \|w_{\tilde{y}} - w_k\|_q \sum_{j=1}^{d-1} (\|W_j\|_p).
\]

(3)

Note that the local Lipschitz constant is upper bounded by the product of the \( L_p \)-norm of the weights matrices. That is to say, if \( \|W_j\|_p \) is small, \( L_{k,x}^k \) is constrained to be small, leading to a higher level of robustness. The proof of Thm. 2 is omitted here due to space constraint and we refer readers to the supplementary document for the detailed proof.

We have at least two interpretations of Thm. 2: if we let \( p = 0 \), Eqn. (3) is bounded by the number of non-zero weights of the model, and hence the higher the proportion of non-zero weights, the more robust the model is. On the other hand, a smaller value of \( \|W_j\|_p \) suggests the distribution of weights is closer to zero. This indicates that if a model has a weights distribution closer to zero, it may be more robust than other models with the same structure. We will respectively show how the two points are supported by the experimental results.

4 Performance Evaluation

4.1 Implementation Details

In this part, we describe the implementation details in examining adversarially robust network pruning. To obtain objective results, we mostly follow the experimental settings in previous works (Liu et al. 2019; Yang et al. 2019; Ye et al. 2019; Zhang and Zhu 2019). Our experiments are carried out with PyTorch 1.0 on NVIDIA GeForce 2080 Ti GPUs.

Datasets and Networks. For the fairness of the results, we conduct experiments on CIFAR-10, Tiny-ImageNet, and CIFAR-100, which are representatives for small-scale datasets, large-scale datasets and datasets somewhere in between. Three state-of-the-art network architectures are chosen: VGG (Simonyan and Zisserman 2015), ResNet (He et al. 2016), and DenseNet (Huang et al. 2017) as the base large networks. A DenseNet-BC with depth 40 and growth rate \( k = 12 \) is also used.

One-Shot Pruning Methods. We pick four representative and intrinsically different pruning methods: Global Unstructured Pruning (GUP) (Frankle and Carbin 2019), Local Unstructured Pruning (LUP) (Han, Mao, and Dally 2016), Filter Pruning (FP) (Li et al. 2017) and Network Slimming (NS) (Liu et al. 2017). LUP and GUP are unstructured pruning, whereas FP and NS are structured pruning. Both GUP and NS prune globally according to the importance of weights or channels across all convolutional layers, while LUP and FP prune an identical percentage of weights or filters per layer locally. FP is a predefined pruning method while GUP, LUP and NS are automatic pruning methods where the structure is determined by the pruning algorithm at runtime.

We conduct these pruning methods in a one-shot manner that removes the parameters at one step, followed by post retraining to convergence. For all pruning methods, we implement each to achieve comparable performance with that reported in the current literature. For FP in ResNet, we conduct it on every two consecutive convolutional layers and skip the shortcuts according to (Luo, Wu, and Lin 2017), also it is not available on DenseNet as pruning one filter would lead to input channel changes in all subsequent layers (Li et al. 2017; Liu et al. 2019). For NS, the highest pruning ratio is selected according to the maximum channel pruning ratio to avoid the removal of layers (Liu et al. 2017).

Adversarial Training and Evaluation. We employ the widely used \( L_\infty \)-PGD adversary with \( \epsilon = 8/255 \), step size = \( 2/255 \) in our experiments. Following recent works (Guo et al. 2018), we employ the 1-step PGD to release the adversarial examples to the pre-trained model.
we set the start learning rate to be $10^{-2}$. On Tiny-ImageNet, we choose to report the performance where the sum of distortions remain high. In the natural setting, we show in the following that pruning can help the pruned network converge, we conduct a series of comparison experiments, as shown in Table 1. Since there is a tradeoff between accuracy and robustness, at a sparser weights distribution than models with the same structure.

**Stopping Criteria.** Typically, it is not well-defined how to train models to ‘full convergence’ when stepwise decay learning rate schedule is applied. Hence we adopt two stopping criteria indicating models have been sufficiently trained for ease of comparison. **Stop-E** denotes the network is trained for a fixed number of epochs. For CIFAR-10, CIFAR-100, and Tiny-ImageNet, we set the start learning rate to be $0.1, 0.1$, and $0.01$, respectively. The learning rate is divided by 10 for every $1/3$ of the total epochs. **Stop-C** monitors the validation loss changes to automatically adjust the learning rate. For example, if we define patience to be $10$ for adversarial training, and weight ratio to be $10^{-2}$, the learning rate only decays when the average validation loss does not decrease by more than $0.001\%$ for consecutive $10$ epochs. Models stop training after $2$ learning rate decays.

4.2 **Adversarial Network Pruning Improves Robustness by Imposing Higher Sparsity**

Although Thm. 2 establishes a preliminary link between sparsity and robustness, it does not tell us how to achieve sparsity and therefore robustness by the equation. An intuitive way is to prune a network to reduce the number of non-zero weights of the model, which is also done in (Guo et al. 2018) only in the natural setting. We show in the following that pruning also works in the adversarial setting. Beyond that, we found that adversarial retraining after pruning mostly improves robustness, at a sparser weights distribution than models with the same structure.

We first adversarially train each base network until reaching the state-of-the-art clean and adversarial accuracy, and then prune each network by different means. Although pruning shows a promising method to introduce sparsity, it does not end up in robust models each time. We hence impose adversarial retraining on pruned networks to enhance robustness. The results are provided in Table 1. Since there is a tradeoff between accuracy and robustness, we choose to report the performance where the sum of adversarial accuracy and clean accuracy is the highest. Distortion bound is also reported for a complete view. We refer readers to the supplementary material for further discussions on the results.

Most networks in Table 1 obtain higher accuracy and robustness than pruning without retraining, and a large proportion of them can achieve better performance than the base networks. Specifically, LUP and NS only suffer notable performance degradation at high pruning ratios, whereas GUP remains a remarkable high performance across all pruning ratios. FP cannot preserve network performance well.

To see whether the weights inherited from a large network help the pruned network converge, we conduct a series of comparison experiments, as shown in Table 2. Compared to FP, FP-rand initializes a small network with the same struc-

| Network       | $p\%$ | LUP    | GUP    | FP     | NS     |
|---------------|------|--------|--------|--------|--------|
| ResNet18      | 30   | 82.13/49.9/3.221 | 81.92/46.56/2.402 | 83.62/46.61/2.505 | 84.18/49.92/2.023 |
| (82.84/49.40/2.519) | 60   | 82.21/48.44/2.777 | 84.73/49.64/2.612 | 82.61/48.08/2.501 | 83.57/49.46/2.666 |
|               | 90   | 80.09/46.76/1.533 | 83.89/47.09/2.940 | 78.87/46.24/1.764 | -      |
| VGG16         | 30   | 79.81/43.17/1.982 | 80.43/44.24/1.630 | 77.05/43.91/3.002 | 80.10/43.81/2.991 |
| (78.57/44.68/3.471) | 60   | 78.78/43.30/2.136 | 80.26/43.51/2.275 | 77.13/44.21/2.032 | 79.56/44.29/2.607 |
|               | 90   | 72.14/41.98/2.510 | 79.83/44.36/2.501 | 69.38/41.20/2.270 | 79.54/43.76/2.443 |
| DenseNet-BC   | 30   | 74.42/43.76/1.525 | 74.68/43.40/2.928 | -      | 73.86/43.08/2.572 |
| (76.01/44.26/1.109) | 60   | 73.16/42.70/1.734 | 73.24/42.88/1.781 | -      | 66.33/37.54/1.059 |
|               | 90   | 63.15/36.68/2.000 | 65.19/36.85/1.784 | -      | -      |

| Network       | $p\%$ | LUP    | GUP    | FP     | NS     |
|---------------|------|--------|--------|--------|--------|
| ResNet18      | 30   | 42.72/14.87/2.356 | 43.18/15.82/2.713 | 42.68/14.91/3.300 | 41.89/15.23/2.397 |
| (41.94/14.43/2.594) | 60   | 42.26/15.51/2.022 | 42.80/16.12/3.272 | 40.88/15.87/1.172 | 37.92/14.11/3.250 |
|               | 90   | 40.32/16.11/2.581 | 42.21/17.26/2.797 | 36.79/14.43/1.819 | -      |
| DenseNet121   | 30   | 48.86/20.03/3.616 | 48.19/20.52/2.519 | -      | 46.43/19.71/1.597 |
| (49.48/19.65/1.922) | 60   | 48.63/19.98/2.300 | 48.96/19.92/1.984 | -      | 40.82/16.51/3.334 |
|               | 90   | 45.72/18.75/1.722 | 46.99/18.65/1.478 | -      | -      |
We also compare our conclusion with previous works and summarize the difference as follows. We find inherited weights by automatic pruning (LUP, GUP, NS) provide better initialization for small networks, while predefined pruning does not. Liu et al. (2019) argues that weights inherited from structured pruning have little impact on the performance of the pruned network. While the experiments on FP agree with the conclusion, that on NS does not. Wang et al. (2018) also suggests inherited weights are important to preserving network accuracy and robustness in adversarial settings, but they do not discuss the working mechanism behind.

### 4.3 Lottery Tickets in Adversarial Settings

We seek that in a randomly-initialized large network, if a subnetwork exists achieving comparable robustness as the large one, which is also known as the ‘winning ticket’ in Frankle and Carbin (2019) in a natural setting. More specifically, we perform Alg. 1 to find out the ‘winning ticket’ in the adversarial setting. A discussion of hyperparameters can be found in the supplementary material.

The results on CIFAR-10 and CIFAR-100 are displayed in Table 3. We mark the results with comparable performance

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**Algorithm 1** Lottery Ticket in Adversarial Settings

**Input:** A large network \( f(x; \theta_0 \odot M_0) \) where \( x \) is the input, \( \theta_0 \) is the randomly initialized weights, and \( M_0 = 1^{[6]} \) denoting weight masks.

- Iterative pruning ratio \( p \% \).
- Pruning iteration \( K \).
- Training epochs \( N \) per pruning iteration.

**Output:** A winning ticket \( f(x; \theta_0 \odot M_K) \).

1. for \( k \in \{1, \ldots, K\} \) do
2. Conduct adversarial training on \( f(x; \theta_0 \odot M_{k-1}) \) for \( N \) epochs and obtain the network \( f(x; \theta_k \odot M_{k-1}) \).
3. Prune \( p \% \) weights from the current network and obtain a new weights mask \( M_k \).
4. Re-initialize weights of \( f \) as \( f(x; \theta_0 \odot M_k) \).
5. end for
6. return \( f(x; \theta_0 \odot M_K) \).
Table 3: Clean testing accuracy/adversarial testing accuracy (in %) of adversarially trained ‘winning ticket’. p% is the pruning ratio. ‘60 (20 × 3 iter)’ means iteratively remove 20% of the weights in each for 3 iterations to achieve a final pruning ratio of 60%. Each iteration of pruning is preceded by 1 epoch of training, and the total training epoch is 240. Accuracy and distortion bounds higher than the base model are in bold.

| Winning Tickets on CIFAR-10 and CIFAR-100 w/ Stop-E       | CIFAR-10 | CIFAR-100 |
|----------------------------------------------------------|----------|-----------|
| Network                                                  | p%       |           |
| ResNet18                                                 |          |           |
| 0 (baseline)                                             | 82.84/49.40 | 50.50/21.13 |
| 30 (30 × 1 iter)                                         | 84.29/45.54 | 50.62/21.72 |
| 60 (20 × 3 iter)                                         | 84.03/47.99 | 52.68/21.54 |
| 80 (20 × 4 iter)                                         | 81.41/48.19 | 52.23/20.80 |
| 90 (30 × 3 iter)                                         | 70.29/47.36 | 49.43/21.27 |
| 95 (31.7 × 3 iter)                                       | 70.29/39.35 |           |
| VGG16                                                    |          |           |
| 0 (baseline)                                             | 78.57/44.68 | 44.44/18.86 |
| 30 (30 × 1 iter)                                         | 80.90/45.58 | 42.21/19.16 |
| 60 (20 × 3 iter)                                         | 80.05/45.56 | 42.65/19.12 |
| 80 (20 × 4 iter)                                         | 79.30/45.16 | 45.90/18.93 |
| 90 (30 × 3 iter)                                         | 78.87/45.15 | 45.76/18.89 |
| 95 (31.7 × 3 iter)                                       | 68.48/40.10 |           |

explained by the more complicated decision boundary of a robust model (in theory, a model with a higher Rademacher complexity is needed to achieve adversarial robustness), and hence its ‘winning ticket’ requires a higher capacity.

To better understand lottery tickets in adversarial settings, we compare the weights distribution between one-shot pruned model and the winning ticket at the same pruning ratio. Fig. 3 illustrates the example of two models pruned at the same pruning ratio by GUP and Alg. [1] respectively on CIFAR10, with adversarial accuracy 47.09% versus 47.36% on ResNet18, and 44.36% versus 45.15% on VGG16, correspondingly. As we observe, whereas GUP models tend to have a flatter distribution which is consistent with Ye et al. [2019], the winning tickets have more near-zero valued weights, indicating a higher level of sparsity. Thus we conclude that it is able to achieve preferable adversarial robustness through the lottery tickets settings.

Comparison with previous results. Ye et al. [2019] argues against the existence of ‘winning ticket’ in adversarial settings. Nevertheless, through experiments we show that ‘winning ticket’ exists in adversarial settings and can be obtained efficiently with a few rounds of pruning and less retraining. Our conclusion is different mostly because we search ‘winning ticket’ by iterative global unstructured pruning as in Frankle and Carbin [2019], while Ye et al. [2019] uses a layer-wise pruning method. As indicated in Frankle and Carbin [2019], layers with fewer parameters may become bottlenecks under a layer-wise pruning method, and thus winning tickets fail to emerge. We also compare our work with Li et al. [2020], and find the few-shot pruning in Li et al. [2020] does not outperform iterative pruning results in our setting.

We also plot the results in Table 1 and Table 3 by showing the relation between the number of parameters of the pruned models against the adversarial accuracy in Fig. 1. By comparing with recent works including RobNet (Guo et al. 2020) and ATMC (Gui et al. 2019) utilizing the same training and testing metrics, which is PGD10 and PGD100, respectively, we demonstrate that our approach are able to acquire smaller networks with robustness comparable to the original dense models through adversarial network pruning, extensively effective under different current model structures among ResNet, VGG, and DenseNet.

4.4 Inverse Weights Inheritance
According to our experimental results in one-shot adversarial pruning, it seems that networks with smaller capacities (higher L0-sparsity) can also have an equivalent or even higher accuracy and robustness than large networks. This appears to be contradictory to the conclusion in Madry et al. [2018] that classifying examples in a robust way requires the model to have a larger capacity, as the decision boundary is more complicated. We ask the question that, can a network be sparse and have larger capacity at the same time? As we analyze, it is indeed possible to have such networks with superior performance.

Algorithm 2 Inverse Weights Inheritance (w/ Lottery Ticket)

Input: \{ f(x; θ₀ ⊕ M₀), p%, K, N \} same as in Alg. 1
Adversarial fine-tuning epochs \( N_f \).

Output: A robust network \( f(x; θ'') \).
1: Find the winning ticket \( f(x; θ₀ ⊕ M_K) \) by Alg. 1
2: Adversarially fine-tune the ‘winning ticket’ for \( N_f \) epochs, obtain a robust small network \( g(x; θ' ⊕ M_K) \).
3: Load the weights of the pruned network \( g \) to the corresponding place in the large network \( f \) and obtain \( f(x; θ' ⊕ M_K) \).
4: Re-initialize weights of \( f \) as \( f(x; θ'' ⊕ M_K + θ₀ ⊕ (M₀ - M_K)) \).
5: Train \( f \) until convergence and obtain \( f(x; θ'') \).
6: return \( f(x; θ'') \).
We introduce a new training strategy called inverse weights inheritance (IWI), which is inspired by Thm. 2 and adversarial network pruning results. By the strategy, a large network acquires sparse weights distribution by inheriting weights from a small robust network, which is pruned from the same large network in the first place and is adversarially trained. Alg. 2 gives an example of using the lottery ticket to obtain such a small network. For a fair comparison, we train the base networks with Stop-C and Stop-E (240 epochs) and report the one with higher performance. To train the large network with inherited weights, we first run Alg. 1 to obtain the ‘winning ticket’ and then train the ‘winning ticket’ (a small network) for 120 epochs. Then the weights of the trained ‘winning ticket’ and then train the ‘winning ticket’ (a small network) for 120 epochs. Then the weights of the trained ‘winning ticket’ are loaded back to the large network to train for another 45 epochs (Stop-E) or until convergence (Stop-C).

Details can be found in the supplementary material. We have also tried other methods, such as using an additional regularization term to impose sparsity in large networks, but it failed. Interested readers may refer to the supplementary material for more details.

5 Conclusion

We conduct comprehensive studies on adversarial network pruning. The contributions are three-fold: First, we give a new explanation on the connection between robustness and network sparsity, which is supported by much empirical evidence. Second, we demonstrate the efficacy of training network with robustness via our proposed algorithm including one-shot pruning and searching the ‘winning ticket.’ Third, we discover a new adversarial training strategy to achieve sparsity and large capacity at the same time for robustness.

Figure 4: Weights distribution of large networks trained by Inverse Weights Inheritance (IWI) and adversarially trained with random initialization (Baseline). Net XX% denotes a large network trained by inheriting the weights of a ‘winning ticket’ with XX% weights pruned.
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