Vibration energy harvesting via parametrically-induced bistability

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Abstract. The dynamic response to white Gaussian noise of a bistable non-linear vibration energy harvester based on the repulsive electrostatic interaction between a microcantilever and an electrode has been theoretically studied. The cantilever-electrode system can be brought from a linear regime characterized by a quadratic potential, when cantilever is far from the electrode, to a non-linear bistable regime characterized by a quartic potential, when both elements are close enough. This distance parameter, which is commonly used to tune bistability, is unusually used here also to inject the energy to the system in the form of displacement noise. Thus, the widening and shifting to the low-frequency region of the response spectrum as well as the enhancement of the $\text{rms}$ out-of-plane vibration of the cantilever are both demonstrated through this parametrically-induced bistability.

1. Introduction

Energy Harvesting (EH) activity in the scientific community experienced a fast growth since approximately ten years ago. During this period, several transduction strategies have been successfully tested to convert ambient energy in the form of mechanical vibrations [1], temperature gradients [2] or electromagnetic radiations [3], [4] into useful electrical energy. However, Vibration Energy Harvesting (VEH) is playing with the advantage that the energy source, mechanical vibrations, are almost permanently available in a wide variety of situations such as industrial environments with machinery, transport vehicles, domestic environments with appliances and civil infrastructures.

On the other hand, MEMS technology has played a key role on keeping unaltered, or even enhancing, the VEH performance in terms of harvested power density, when volume of the VEH is unavoidably reduced following the miniaturizing trends imposed by ICTs evolution, since in many cases harvested power scales more slowly than volume.

However, when VEH devices are scaled down to micrometer dimensions, their efficiency decreases due to a mismatch in the frequency of the harvester response and the environmental vibrations, and due to the narrow bandwidth of the harvester’s spectral response. To address this problem, the use of the chip substrate as inertial mass to reduce the resonance frequency of the harvester was tested in a CMOS-MEMS electrostatic implementation [5] and by using the AlN piezoelectric technology [6]. Alternatively, non-linear techniques have been also proposed, which widen the spectral bandwidth and
improve the low-frequency response [7], [8], [9], [10]. A common approach is to non-linearize an initially linear mechanical element by making use of magnetic [7], electrostatic [10] or strain [11] interactions, which results in a bistable system with an energy barrier that can be tuned by the design. The device is then typically evaluated by measuring the broadband response to a white Gaussian displacement noise, which directly excites the transducer.

In this work we theoretically demonstrate that a wideband low-frequency response can also be obtained by injecting displacement noise through the bistability tuning parameter. Thus, the energy enters the system through a parametric excitation that tunes the system stochastically from a monostable to a bistable regime, rather than applying the noise to a bistable system in which the parameters are constant.

2. The Non-linear Energy Harvesting Device description

To demonstrate this concept, we use a previously developed electrostatic nonlinear VEH [10], which is based on a microcantilever with a trapped charge, as is shown schematically in Figure 1(left).

2.1. Geometry

A commercial Si$_3$N$_4$ triangular cantilever and a counter electrode (CE), both of which contain a permanent charge, $q$, of several fC, are placed at a controllable distance $d_0$ in the in-plane $y$ direction. Tip ends of the cantilever and the CE and their trapped charges are precisely aligned in all three directions (in-plane, $x$, $y$ and out-of plane, $z$).

![Diagram of cantilever and counter electrode](image)

**Figure 1.** Schematic of the cantilever-based NLVEH (left) and potential energy (right) as calculated for three cantilever-electrode distances. For long distances ($d_0=10\mu$m) the potential is quadratic and monostable, while for short distances ($d_0<3\mu$m) it is quartic and bistable [10].

2.2. Potential energy

The strength of the repelling Coulomb force mutually exerted between the trapped charges of same sign is adjustable by varying $d_0$, which tunes the potential energy landscape of the cantilever from a quadratic potential (for large $d_0$) to a quartic potential (small $d_0$). In the latter case the system is bistable, with two symmetric potential wells that are separated by a potential barrier, as is shown in Figure 1(right). For the particular case of our previous work [10], an as measured implanted charge of 10fC is inducing a potential barrier of approximately 100fJ between two wells separated by a distance of around 3\mu m.
3. Dynamic response to noise
In our previous work [10], we demonstrated the existence of an optimum fixed distance, \(d_{\text{opt}}\), for which, at a given input noise intensity, the rate of the random transitions maximizes and an optimum in the harvester’s energy output is achieved [7,10]. Here we demonstrate that a similar optimum can be achieved when displacement noise is applied to the distance parameter, \(d_0\). Figure 2 shows a simulation of the time evolution of the cantilever deflection when white Gaussian displacement noise (100 nm standard deviation) is applied around the working point defined by the static distance \(d_0\).

![Figure 2](image1.png)

**Figure 2.** Time evolution of the cantilever out-of-plane deflection, \(z\), when the cantilever-electrode distance is modulated by a 100 nm (standard deviation) white Gaussian displacement noise around a static distance \(d_0=3.5\,\mu\text{m}\) (black), \(d_0=2.8\,\mu\text{m}\) (red) and \(d_0=1\,\mu\text{m}\) (blue).

![Figure 3](image2.png)

**Figure 3.** Frequency response of the cantilever as obtained from the FFT of the time series of figure 2 corresponding to the optimum distance (red curve, \(d_0=2.8\,\mu\text{m}\)) and the well trapping distance (blue curve, \(d_0=1\,\mu\text{m}\)).
Then, the working point $d_0$ is varied from a monostable regime with $d_0=3.5 \ \mu m$ (black curve) to a deep in the bistable regime at $d_0=1 \ \mu m$ (blue curve). In between, an optimum distance is found at $d_0=2.8 \ \mu m$ (red curve), in which inter-well transitions occur.

In Figure 3, it is shown that an appropriate tuning of $d_0$, which corresponds to the optimum working point ($d_0=2.8 \mu m$) results in a broadband response at a lower frequency, when compared for instance to the case of short working point distance ($d_0=1 \mu m$), where the cantilever is trapped into on the potential wells.

To demonstrate the resulting enhancement on the harvested energy, the $\text{rms}$ value of the cantilever tip deflection is plotted as a function of $d_0$ in Figure 4. A peak observed at $d_0=2.8 \ \mu m$, which marks a local optimum working point for the parametrically excited energy harvester, corresponds to the maximized inter-well transition regime (red curves in Figures 2 and 3). However, it is worth to notice that, additionally to this local maximum at $d_0=2.8 \ \mu m$, the vibration energy of the cantilever is also maximized in the deep bistable regime, as it is reflected by the increase of the out-of-plane $\text{rms}$ deflection value produced when the working point is approaching $d_0=0.5 \ \mu m$. This second local optimum, which in this case becomes an absolute maximum, demonstrates that a tuning range for the energy enhancement wider than the first traditional one ($d_0=2.8 \ \mu m$) is also available.

![Figure 4](image_url)

**Figure 4.** The $\text{rms}$-deflection of the cantilever tip measured at different static working point distances $d_0$, when white Gaussian displacement noise (100 nm standard deviation) is applied.

4. Conclusions
A theoretical analysis of the dynamic response to white Gaussian noise of a microcantilever based bistable non-linear vibration energy harvester has been presented. An unusual mechanism to introduce the energy to the system based on the injection of displacement noise through the bistability tuning parameter has been proposed. The widening and shifting to the low frequency band of the harvester spectral response and the enhancement of the $\text{rms}$ vibration amplitude has been demonstrated. Additionally to the conventional optimum tuning value that produce an enhancement of the harvested energy, a new and wider optimum enhancement range is also found at the deep bistable regime.

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