Electric polarizabilities of proton and neutron and the relativistic center-of-mass coordinate

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Abstract

We argue that the relativistic correction $\delta \mathbf{R}_{\text{c.m.}}$ to the center-of-mass vector can lead to the approximate equality of the proton and neutron electric polarizabilities in the quark model. The explicit form of $\delta \mathbf{R}_{\text{c.m.}}$ depends only on the non-relativistic potential between quarks. In particular, this correction is the same for the potential generated by Lorentz-vector and -scalar interactions.

1 Introduction

The electric polarizability $\bar{\alpha}$ and magnetic polarizability $\bar{\beta}$ are fundamental structure constants of the nucleon having a direct relation to the internal dynamics of the particle. Their magnitudes depend not only on the quantum numbers of the constituents, but also on the properties of the interaction between these constituents. The prediction of the electromagnetic polarizabilities and the comparison with experimental data may serve as a sensitive tool for tests of hadron models.

The quantities $\bar{\alpha}$ and $\bar{\beta}$ can be obtained from the amplitude for low-energy Compton scattering off the nucleon. In the lab frame, this amplitude up to $O(\omega^2)$ terms reads

\begin{equation}
T = T_{\text{Born}} + \bar{\alpha}\omega_1\omega_2\epsilon_1 \cdot \epsilon_2^* + \bar{\beta}(\mathbf{k}_1 \times \epsilon_1) \cdot (\mathbf{k}_2 \times \epsilon_2^*),
\end{equation}

where $\omega_i$, $\mathbf{k}_i$, and $\epsilon_i$ are the energy, momentum, and polarization vector of the incoming ($i = 1$) and outgoing ($i = 2$) photons ($\hbar = c = 1$). The contribution $T_{\text{Born}}$ corresponds to the amplitude for Compton scattering off a point-like particle with the spin, mass, charge, and magnetic moment of the nucleon.

The result of the most recent and most precise experimental investigation \cite{3, 4}, obtained by Compton scattering off the proton below pion threshold, is shown in the first line of Table \ref{table:1}. For the neutron the same method is extremely difficult \cite{3, 4}: (i) Quasi-free Compton scattering from neutrons bound in the deuteron has to be carried out leading to sizable corrections at low energies. (ii) The Born amplitude $T_{\text{Born}}$ is very small for the neutron. Therefore, the corresponding interference term is also small and, hence, cannot be used as is done in case of the proton. (iii) The low-energy differential cross section is less sensitive to the electric polarizability in case of the neutron as compared to the proton.

In order to avoid these difficulties, the differential cross section for electromagnetic scattering of neutrons in the Coulomb field of heavy nuclei \cite{5}

\begin{equation}
\frac{d\sigma_{\text{pol}}}{d\Omega} = \pi M p (Ze)^2 \text{Re} a \left\{ \bar{\alpha}_n \sin \frac{\theta}{2} - \frac{e^2 \kappa_n^2}{2M^3} \left(1 - \sin \frac{\theta}{2}\right) \right\}
\end{equation}

(2)
has been investigated in a series of narrow-beam neutron transmission experiments. In (2)
p is the neutron momentum, \( -a \) the nuclear amplitude, \( \kappa_n \) the neutron anomalous magnetic moment, \( M \) the neutron mass, and \( e \) the proton charge. Of these experiments only the Oak Ridge experiment [4] has a reasonable precision. This result is cited in the second line of Table 1 where the necessary correction for the second (Schwinger) term in the braces of (2) has been carried out. This Schwinger-term correction was disregarded in the original evaluation of the experiment [4] and reads \(+e^2\kappa_n^2/4M^3 = +0.62\) [3].

In [8] it was shown that it is also possible to use experimental data on quasi-free Compton scattering on neutrons bound in deuterons at energies between \( \pi \) meson threshold and the \( \Delta \) peak to extract the electric polarizability of the neutron. This method has successfully been tested for the proton [3, 11, 12] and applied to the neutron [4, 10], leading to the result listed in the third line of Table 1.

| \( \bar{\alpha}_p \) | \( 12.2 \pm 0.3 \text{(stat)} \pm 0.4 \text{(syst)} \pm 0.3 \text{(model)} \) | Compton [3, 11] |
| \( \bar{\alpha}_n \) | \( 12.6 \pm 1.5 \text{(stat)} \pm 2.0 \text{(syst)} \) | Coulomb [4] |
| \( \bar{\alpha}_n \) | \( 12.5 \pm 1.8 \text{(stat)} \pm 1.1 \text{(syst)} \pm 1.1 \text{(model)} \) | Compton [4] |

From Table 1 we obtain the following result for the difference between the electric polarizabilities of proton and neutron

\[
\bar{\alpha}_p - \bar{\alpha}_n = -0.3 \pm 1.8
\]

which apparently is compatible with zero.

The electromagnetic polarizabilities of hadrons have been calculated in many different models. Though much effort has been devoted to these model calculations, all of them can not be considered as completely satisfactory. In particular, there is a problem in explaining within a non-relativistic quark model that the electric polarizabilities of proton and neutron are equal to each other as suggested in (3). It was derived many years ago [2, 11, 12] that \( \bar{\alpha} \) can be represented as a sum

\[
\bar{\alpha} = \frac{2}{3} \sum_{k \neq 0} \frac{|\langle k |D|0 \rangle|^2}{E_k - E_0} + \Delta \alpha = \alpha_0 + \Delta \alpha ,
\]

where \( D \) is the internal electric dipole operator, \( |0 \rangle \) and \( |k \rangle \) are the ground and excited states in terms of internal coordinates, \( E_k \) and \( E_0 \) are the corresponding energies. Note that \( \mathbf{P} |0 \rangle = \mathbf{P} |k \rangle = 0 \), where \( \mathbf{P} \) is the operator of total momentum of the particle. The term \( \Delta \alpha \) in \( \bar{\alpha} \) has a relativistic origin and its leading term is equal to

\[
\Delta \alpha = \frac{Z e^2 r_E^2}{3M} ,
\]

where \( Z \) and \( M \) are the particle electric charge number and mass, and \( r_E \) is the electric radius defined through the Sachs form factor \( G_E \). The quantity \( \alpha_0 \) calculated in a non-relativistic quark model without taking into account relativistic corrections leads to the same magnitude
of $\alpha_\circ$ for proton and neutron. Since $\Delta \alpha$ is equal to zero for the neutron but gives a significant contribution to $\bar{\alpha}$ for the proton, one can naively expect that

$$\bar{\alpha}_p - \bar{\alpha}_n = \Delta \alpha_p = 3.8 \pm 0.1. \quad (6)$$

Comparing (3) with (6) we would come to the conclusion that the experimental and predicted differences between the electric polarizabilities of proton and neutron deviate from each other by $2.3\sigma$. In the following it will be shown that this discrepancy may be connected with hitherto unknown relativistic corrections to $\alpha_\circ$ which are of the same order as $\Delta \alpha$. The relativistic corrections to $\alpha_\circ$ in (4) come from corrections to the wave functions, to the energies of the ground and excited states, and a correction to $D$. The relativistic correction to the electric dipole operator is connected with the appropriate relativistic definition of the center-of-mass coordinate \cite{13,14,15}. The neglect of this correction leads to an incomplete expression for $\bar{\alpha}$, and, as it has been shown in \cite{15}, the missing piece can be very essential.

In the present paper we show that the difference $\delta \alpha = (\alpha_\circ)_p - (\alpha_\circ)_n$ is due to the correction to the electric dipole operator only. By carrying out model calculations we show that $\delta \alpha$ can strongly compensate $\Delta \alpha$, thus removing the contradiction between (3) and (6) mentioned above.

## 2 Correction to the center-of-mass vector

Let us consider a system of spin-$1/2$ particles having the masses $m_i$ and spin operators $s_i$. The Hamiltonian of the system $H$, accounting for the first relativistic correction, is the sum of the non-relativistic Hamiltonian $H_{nr}$ and the Breit Hamiltonian $H_B$, where

$$H_{nr} = \sum_i \frac{p_i^2}{2m_i} + \sum_{i>j} V_{ij}(r_{ij}). \quad (7)$$

Here $r_{ij} = r_i - r_j$ and $V_{ij}(r)$ are some potentials. The explicit form of the Breit Hamiltonian $H_B$ depends on the Lorentz structure of interaction, namely, whether it is Lorentz-scalar or Lorentz-vector (see \cite{16} and references therein). As in most models, we assume that the potential $V(r)$ contains a short-range Coulomb-type term $V_C(r)$ due to single-gluon exchange corresponding to vector interaction, and a long-range confining potential $V_{conf}(r)$. The Lorentz structure of the latter is not well known. We show that it makes no difference in the form of $\delta R_{c.m.}$.

The center-of-mass vector $R_{c.m.}$ should satisfy the following relations \cite{13}:

$$[R^j_{c.m.}, P^k] = i\delta^{jk},$$

$$[R_{c.m.}, H_{nr} + H_B] = i\frac{P}{M + H} \approx i\frac{P}{M} \left(1 - \frac{H_{nr}}{M}\right), \quad (8)$$

where $M$ and $P$ are the total mass and the total momentum of the system, respectively. Using the explicit form of the Breit Hamiltonian it is easy to check that in both cases, i.e. Lorentz-scalar and Lorentz-vector interactions, the vector $R_{c.m.}$ for the system of particles
The non-relativistic Hamiltonian $H$ has the form

$$H_{c.m.} = \mathbf{R} + \delta \mathbf{R}_0 + \delta \mathbf{R}_s, \quad \mathbf{R} = \frac{1}{M} \sum_i m_i \mathbf{r}_i,$$

$$\delta \mathbf{R}_0 = \frac{1}{2M} \sum_i \left\{ \mathbf{r}_i - \mathbf{R}, \frac{\mathbf{p}_i^2}{2m_i} + \frac{1}{2} \sum_{j \neq i} V_{ij}(\mathbf{r}_i - \mathbf{r}_j) \right\},$$

$$\delta \mathbf{R}_s = \frac{1}{2M} \sum_i \left( \frac{\mathbf{p}_i}{m_i} \times \mathbf{s}_i \right),$$

(9)

where $M = \sum m_i$, and where the notation $\{a, b\} = ab + ba$ is used. Thus, the relativistic correction $\delta \mathbf{R}_0$ is expressed via the non-relativistic potential of the interaction between the constituents.

Let us consider proton and neutron. In this case all potentials are equal to each other, i.e. $V_{ij} = V$. Let the vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ correspond to two $u$-quarks in the proton and two $d$-quarks in the neutron, and $\mathbf{r}_3$ correspond to the $d$-quark in the proton and the $u$-quark in the neutron, and $m_1$ to the mass of the identical quarks ($u$ quarks in the proton and $d$-quarks in the neutron), and $m_3$ to the mass of the third quark. We pass to the Jacobi variables:

$$\rho = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2), \quad \lambda = \frac{1}{\sqrt{6}}(\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3), \quad \mathbf{R} = \frac{1}{M}(m_1 \mathbf{r}_1 + m_1 \mathbf{r}_2 + m_3 \mathbf{r}_3),$$

(10)

where $M = 2m_1 + m_3$. Then the momentum operators are

$$\mathbf{p}_\rho = \frac{1}{\sqrt{2}}(\mathbf{p}_1 - \mathbf{p}_2), \quad \mathbf{p}_\lambda = \frac{\sqrt{6}}{2M}(m_3 \mathbf{p}_1 + m_3 \mathbf{p}_2 - 2m_1 \mathbf{p}_3), \quad \mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3.$$  

(11)

The non-relativistic Hamiltonian $H_{nr}$ in terms of the Jacobi variables has the form

$$H_{nr} = \frac{\mathbf{p}_\rho^2}{2m_1} + \frac{\mathbf{p}_\lambda^2}{2m_\lambda} + V(\sqrt{2}\rho) + V(\sqrt{2}\xi) + V(\sqrt{2}\eta),$$

(12)

where $m_\lambda = 3m_1m_3/M$, $\xi = (\rho + \sqrt{3}\lambda)/2$, and $\eta = (-\rho + \sqrt{3}\lambda)/2$. In terms of Jacobi variables the spin-independent correction to the center-of-mass vector is

$$\delta \mathbf{R}_0 = \frac{1}{2M^2} \left( \{\rho, \mathbf{P} \cdot \mathbf{p}_\rho\} + \{\lambda, \mathbf{P} \cdot \mathbf{p}_\lambda\} \right)$$

$$+ \frac{1}{2\sqrt{6}m_1 M} \{\rho, \mathbf{p}_\lambda \cdot \mathbf{p}_\rho\} + \frac{\sqrt{6}}{8M^2} \left\{ \lambda, \frac{m_3 \rho_\rho^2}{m_1 \rho} + \frac{m_3 - 2m_1}{m_\lambda} \rho_\lambda^2 \right\}$$

$$+ \lambda \sqrt{6m_3} V(\sqrt{2}\rho) + \left[ \frac{\rho}{\sqrt{2}} + \frac{\sqrt{6}(m_3 - 2m_1)\lambda}{2M} \right] V(\sqrt{2}\xi)$$

$$+ \left[ \frac{-\rho}{\sqrt{2}} + \frac{\sqrt{6}(m_3 - 2m_1)\lambda}{2M} \right] V(\sqrt{2}\eta),$$

(13)

and the spin-dependent correction

$$\delta \mathbf{R}_s = \frac{\mathbf{P} \times \mathbf{S}}{2M^2} + \frac{\mathbf{p}_\rho \times (\mathbf{s}_1 - \mathbf{s}_2)}{2\sqrt{2}m_1 M} + \frac{\mathbf{p}_\lambda \times [m_3(\mathbf{s}_1 + \mathbf{s}_2) - 2m_1 \mathbf{s}_3]}{2\sqrt{6}m_1 m_3 M}.$$  

(14)
When \( m_1 = m_3 = m \), then \( m_\lambda = m \) and the expressions for \( \delta R_0 \) and \( \delta R_s \) become essentially simpler:

\[
\delta R_0 = \frac{1}{18m^2} \{ \{ \rho, P \cdot p_\rho \} + \{ \lambda, P \cdot p_\lambda \} \}
+ \frac{1}{6\sqrt{6}m^2} \left[ \{ \rho, p_\lambda \cdot p_\rho \} + \frac{1}{2} \{ \lambda, p_\rho^2 - p_\lambda^2 \} \right]
+ \frac{1}{6\sqrt{6}m} \left[ 2\lambda V(\sqrt{2}\rho) + \left( \sqrt{3}\rho - \lambda \right) V(\sqrt{2}\xi) + \left( -\sqrt{3}\rho - \lambda \right) V(\sqrt{2}\eta) \right],
\]

and

\[
\delta R_s = \frac{1}{6m^2} \left[ \frac{1}{3} P \times S + \frac{1}{\sqrt{2}} p_\rho \times (s_1 - s_2) + \frac{1}{\sqrt{6}} p_\lambda \times (s_1 + s_2 - 2s_3) \right].
\]

Let us consider the substitutions

\[
\rho \rightarrow \frac{1}{2}\rho - \frac{\sqrt{3}}{2}\lambda, \quad \lambda \rightarrow \frac{\sqrt{3}}{2}\rho + \frac{1}{2}\lambda,
\]

\[
p_\rho \rightarrow \frac{1}{2}p_\rho - \frac{\sqrt{3}}{2}p_\lambda, \quad p_\lambda \rightarrow \frac{\sqrt{3}}{2}p_\rho + \frac{1}{2}p_\lambda.
\]

If \( m_1 = m_3 \), then the non-relativistic Hamiltonian (12) is invariant under this substitution and \( \delta R_0(P = 0) \rightarrow -\delta R_0(P = 0) \). In addition, the Hamiltonian (12) is invariant under the transformation \( \rho \rightarrow -\rho \) and \( \lambda \rightarrow -\lambda \). The properties of the operators with respect to the transformations (17) are useful for the selection rules for the matrix elements.

### 3 Charge radii

Let us consider the charge radii of the nucleon \( r_E^2 = \langle \sum e_i(r_i - R_{\text{c.m.}})^2 \rangle \), where \( e_i \) is equal to 2/3 for the \( u \)-quark and -1/3 for the \( d \)-quark:

\[
(r_E^2)_p = \frac{1}{3} \langle 0| \left[ 4(r_1 - R_{\text{c.m.}})^2 - (r_3 - R_{\text{c.m.}})^2 \right] |0\rangle,
\]

\[
(r_E^2)_n = -\frac{2}{3} \langle 0| \left[ (r_1 - R_{\text{c.m.}})^2 - (r_3 - R_{\text{c.m.}})^2 \right] |0\rangle.
\]

Here we used the symmetry of the wave function with respect to the permutation \( r_1 \leftrightarrow r_2 \).

The values of \( (r_E^2)_p \) and \( (r_E^2)_n \) were measured to be \( (r_E^2)_p = 0.74 \text{ fm}^2 \) and \( (r_E^2)_n = -0.119 \pm 0.003 \text{ fm}^2 \) [17]. Thus, \( \langle 0|(r_1 - R_{\text{c.m.}})^2|0\rangle > \langle 0|(r_3 - R_{\text{c.m.}})^2|0\rangle \). Due to the symmetry of \( H_{nr} \) with respect to the permutation \( r_1 \leftrightarrow r_3 \) for the case of equal quark masses, it is evident that \( (r_E^2)_n = 0 \) in the non-relativistic approximation. The relativistic correction to \( (r_E^2)_n \) comes from the correction to the center-of-mass vector and from the correction to the wave function:

\[
\delta(r_E^2)_n = \frac{2}{3} \langle 0| \left( \{ r_1 - r_3, \delta R_0 + \delta R_s \} \right) |0\rangle
- \frac{2}{3} \langle 0| \left[ (r_1 - R)^2 - (r_3 - R)^2 \right] G_0 H_B |0\rangle
- \frac{2}{3} \langle 0| H_B G_0 \left[ (r_1 - R)^2 - (r_3 - R)^2 \right] |0\rangle,
\]
where $G_0$ is the non-relativistic reduced Green function

$$G_0 = [\varepsilon_0 - H_{nr} + i0]^{-1}(1 - |0\rangle\langle 0|),$$ \hspace{1cm} (20)

and $\varepsilon_0$ is the ground state binding energy in the non-relativistic approximation.

Using the properties of $\delta R_0$ and $\delta R_n$ with respect to the transformations [17], it is easy to show that the contribution to $\delta(r^2_E)_n$ [19] from the correction to the center-of-mass vector, as well as the contribution of the spin-independent part of $H_B$, vanish. Due to parity conservation the contribution of the part of $H_B$ being linear in spin is also zero. The spin-spin part $H_B^{(ss)}$ of the Breit Hamiltonian gives a non-zero contribution to $\delta(r^2_E)_n$. This operator exists only for the Lorentz-vector part $V_v(r)$ of the potential:

$$H^{(ss)} = \sum_{i \neq j} \frac{1}{3m_i m_j} \Delta V_v(r_{ij}) (s_i \cdot s_j)$$

$$- \sum_{i \neq j} \frac{1}{6m_i m_j} [V''(r_{ij}) - V'(r_{ij})/r_{ij}] [3(s_i \cdot \hat{r}_{ij})(s_j \cdot \hat{r}_{ij}) - s_i \cdot s_j],$$ \hspace{1cm} (21)

where $\hat{r}_{ij} = r_{ij}/r_{ij}$. Substituting $V_v(r) = -2\alpha_s/3r$ and averaging over the spin part of the neutron wave function, we obtain

$$H^{(ss)} = \frac{2\pi \alpha_s}{9\sqrt{2}m^2} [\delta(\rho) - 2\delta(\xi) - 2\delta(\eta)].$$ \hspace{1cm} (22)

Using Jacobi variables, we can represent $\delta(r^2_E)_n$ as

$$\delta(r^2_E)_n = -\frac{1}{3} \langle 0| [\rho^2 - \lambda^2]G_0 H^{(ss)}|0\rangle - \frac{1}{3} \langle 0| H^{(ss)}G_0 [\rho^2 - \lambda^2]|0\rangle. \hspace{1cm} (23)$$

In order to calculate the matrix element (23) we follow the prescription of [18] and set $V(r) = Kr^2/2$ in the non-relativistic Hamiltonian $H_{nr}$. Then we obtain

$$H_{nr} = \frac{p^2}{2M} + \frac{p_\rho^2}{2m} + \frac{p_\lambda^2}{2m} + \frac{3K(\rho^2 + \lambda^2)}{2}.$$ \hspace{1cm} (24)

Thus, we have two independent oscillators with equal frequencies $\omega_\rho = \omega_\lambda = \sqrt{3K/m}$. Taking into account that $(\rho^2 - \lambda^2)|0\rangle$ is an eigenfunction of $H_{nr}$ (24) with the excitation energy $E - E_0 = 2\omega_\rho$, we have

$$\delta(r^2_E)_n = \frac{1}{3\omega_\rho} \langle 0| [\rho^2 - \lambda^2]H^{(ss)}|0\rangle = -\frac{\alpha_s}{3\sqrt{2\pi\omega_\rho m^3}}.$$ \hspace{1cm} (25)

Thus, we obtain the negative value for $(r^2_E)_n$. The magnitude of this quantity can also be made in agreement with the experimental value by taking the appropriate parameters.

\section*{4 Electric polarizability}

Using the Jacobi variables we obtain the operator of the internal dipole moment $D = e \sum e_i (r_i - R_{c.m.})$ for proton and neutron:

$$D_p = e \left( \sqrt{\frac{2}{3}} \lambda - \delta R_{c.m.} \right), \hspace{1cm} D_n = -e \sqrt{\frac{2}{3}} \lambda.$$ \hspace{1cm} (26)
If we neglect $\delta R_{c.m.}$ in $D_p$, we immediately obtain $(\alpha_0)_p = (\alpha_0)_n$ since $\alpha_0$ is quadratic in $D$ (see (4)). Therefore, the difference between $(\alpha_0)_p$ and $(\alpha_0)_n$ arises only due to the correction $\delta R_{c.m.}$ in $D_p$. The contribution to this difference being linear in $\delta R_{c.m.}$ reads:

$$\delta \alpha = (\alpha_0)_p - (\alpha_0)_n = e^2 \left( \frac{2}{3} \right)^{3/2} \left[ \langle 0| \chi G \delta R_{c.m.}|0 \rangle + \langle 0| \delta R_{c.m.} G \alpha |0 \rangle \right],$$  

(27)

where $G$ is the reduced Green function accounting for the first relativistic correction:

$$G = [E_0 - H_{nr} - H_B + i0]^{-1}(1 - |0\rangle \langle 0|).$$  

(28)

If we replace $G$ by its non-relativistic limit $G_0$ and use the symmetry with respect to the transformations (17), we obtain zero as the result for $\delta \alpha$. Therefore, it is necessary to take into account the corrections to the wave function and to the Green function due to the spin-dependent part of $H_B$. In addition, it is necessary to account for the term quadratic in $\delta R_{c.m.}$ in (4), and the second-order relativistic correction to $R_{c.m.}$. The calculation of the latter is a very complicated problem. Since we are only going to demonstrate the possible cancellation between $\delta \alpha$ and $\Delta \alpha$ (see (4)), we may simplify our problem and assume, in a spirit of the diquark model [18], that the relativistic effects reduce to the small difference between the masses $m_1$ and $m_3$. As a result, we use the non-relativistic Hamiltonian

$$H_{nr} = \frac{p^2}{2M} + \frac{p^2}{2m_1} + \frac{p^2}{2m_\lambda} + \frac{3K(\rho^2 + \lambda^2)}{2}$$  

(29)

and the expression (13) for $\delta R_0$. Remind that $M = 2m_1 + m_3$ and $m_\lambda = 3m_1m_3/M$. Thus, the corresponding frequencies are $\omega_\rho = \sqrt{3K/m_1}$ and $\omega_\lambda = \sqrt{3K/m_\lambda}$. We can fix the parameters of the model from the experimental values of $(r_E^2)_p$ and $(r_E^2)_n$. Using (10), (18) and (29) it is easy to find that

$$(r_E^2)_p = \frac{1}{m_1\omega_\rho} \left[ 1 + \frac{(m_3 - m_1^2)\omega_\rho}{m_3M\omega_\lambda} \right] \approx \frac{1}{m_1\omega_\rho},$$

$$(r_E^2)_n = -\frac{1}{2m_1\omega_\rho} \left[ 1 - \frac{(2m_1 - m_3)\omega_\rho}{m_3\omega_\lambda} \right] \approx -\frac{5(1 - x)}{6m_1\omega_\rho},$$  

(30)

where $x = m_1/m_3$. Substituting the experimental values of charge radii into (10), we obtain $x = 0.8$, and $m_1\omega_\rho = 6 \cdot 10^4$ MeV$^2$. Using the conventional value $m_1 = 330$ MeV, we have $\omega_\rho/m_1 = 0.6$. As a result of simple calculations we have in our model:

$$\Delta \alpha_p = \frac{e^2}{9m_1^2\omega_\rho}, \quad \Delta \alpha = \frac{-79e^2(1 - x)}{108m_1^2\omega_\rho} + \frac{53e^2}{1296m_1^4}.$$  

(31)

The second term in $\delta \alpha$ comes from the correction being quadratic in $\delta R_{c.m.}$ and numerically is much smaller than the first one. For the parameters of our model we obtain

$$\frac{\delta \alpha + \Delta \alpha_p}{\Delta \alpha_p} = -0.1,$$  

(32)

which leads to the approximate equality of the predicted proton and neutron electric polarizabilities in agreement with the experimental data.
Conclusion

Thus we have demonstrated that the equality of the proton and neutron electric polarizability can be explained in the frame of constituent quark model. Therefore, it is not necessary to deal with such ad hoc contributions (for constituent quark model) as mesonic currents. Though it has been shown in [13] that there are many sources of the relativistic corrections of the same order as $\Delta \alpha$, due to the isospin invariance it is crucial to take into account the correction to the center-of-mass vector. If this correction is neglected, the difference $(\alpha_o)_{p} - (\alpha_o)_{n}$ vanishes identically. The estimate for the relativistic corrections made above with the use of a toy model serves only as an illustration of the possibility to make the difference $(\bar{\alpha})_{p} - (\bar{\alpha})_{n}$ consistent with the experimental data at reasonable values of the parameters.

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