CONDITIONS FOR THE OCCURRENCE OF MEAN-MOTION RESONANCES IN A LOW MASS PLANETARY SYSTEM

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Abstract. The dynamical interactions that occur in newly formed planetary systems may reflect the conditions occurring in the protoplanetary disk out of which they formed. With this in mind, we explore the attainment and maintenance of orbital resonances by migrating planets in the terrestrial mass range. Migration time scales varying between $\sim 10^6$ yr and $\sim 10^3$ yr are considered. In the former case, for which the migration time is comparable to the lifetime of the protoplanetary gas disk, a 2:1 resonance may be formed. In the latter, relatively rapid migration regime commensurabilities of high degree such as 8:7 or 11:10 may be formed. However, in any one large-scale migration several different commensurabilities may be formed sequentially, each being associated with significant orbital evolution. We also use a simple analytic theory to develop conditions for first order commensurabilities to be formed. These depend on the degree of the commensurability, the imposed migration and circularization rates, and the planet mass ratios. These conditions are found to be consistent with the results of our simulations.

1 Introduction

The increasing number of extrasolar multi-planet systems have stimulated studies of their origin, evolution and stability. An important feature in planetary system evolution is the occurrence of mean-motion resonances, which may relate to conditions at the time of or just after the process of formation. There are some well known examples of systems such as Gliese 876 (Marcy et al. 2001), HD 82943 (Mayor et al. 2004) and 55 Cancri (McArthur et al. 2004) involving planets with masses in the Jovian range. More recently a system of three superearths

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orbiting HD40307 has been announced for which the period ratios are not strictly commensurable but are close to a 2:1 commensurability (Mayor et al. 2008).

It is therefore important to establish the main features of the evolution of low mass planets embedded in a gaseous disc and in particular to determine the types of resonant configurations that might arise when a pair of such planets evolves together. The disc-planet interaction naturally produces orbital migration through the action of tidal torques (Goldreich & Tremaine 1980, Lin & Papaloizou 1986) which in turn may lead to an orbital resonance in a many planet system (e.g. Snellgrove, Papaloizou & Nelson 2001, Lee & Peale 2002). This is because it is expected that two planets with different masses will migrate at different rates. This has the consequence that their period ratio will evolve with time. In the situation where the migration is such that the orbits converge, they tend to enter and become locked in a mean-motion resonance (e.g. Nelson & Papaloizou 2002, Kley, Peitz & Bryden 2004) and then subsequently migrate together.

In the simplest case of nearly circular and coplanar orbits the resonances that are formed are the first-order resonances which occur at locations where the ratio of the two orbital periods can be expressed as \((p + 1)/p\), with \(p\) being an integer.

Papaloizou & Szuszkiewicz (2005) performed a recent analytic and numerical study of the formation of first order commensurabilities by a system of two planets in the earth mass range migrating in a laminar disc. Here we extend these studies using \(N\)-body simulations to a larger range of migration rates and commensurabilities and compare the numerical work to analytically derived conditions for particular commensurabilities to form. We begin by deriving the analytic criteria and go on to present the results of simulations to which they are reconciled. We consider migration time scales ranging between \(\sim 10^3\) yr and \(\sim 10^6\) yr. For the longest time scales, 2:1 commensurabilities may be set up while for the shortest, commensurabilities as high as 11:10 have been found. Finally we summarize our results.

2 Basic Equations

The equations of motion for a system of \(N\) planets \((i = 1, 2, 3, \ldots, N)\) moving in a fixed plane, about a dominant central mass, under a general Hamiltonian \(H\) may be written in the form (see e.g. Papaloizou 2003, Papaloizou & Szuszkiewicz 2005)

\[
\dot{E}_i = -n_i \frac{\partial H}{\partial \lambda_i} \quad (2.1)
\]

\[
\dot{L}_i = -\left( \frac{\partial H}{\partial \lambda_i} + \frac{\partial H}{\partial \varpi_i} \right) \quad (2.2)
\]

\[
\dot{\lambda}_i = \frac{\partial H}{\partial L_i} + n_i \frac{\partial H}{\partial E_i} \quad (2.3)
\]

\[
\dot{\varpi}_i = \frac{\partial H}{\partial L_i}. \quad (2.4)
\]
Here the orbital angular momentum of planet \( i \), which has reduced mass \( m_i = m_{i0} M / (M + m_{i0}) \), with \( m_{i0} \) being the actual mass, is \( L_i \) and the orbital energy is \( E_i \). For motion around a central point mass \( M \) we have

\[
L_i = m_i \sqrt{GM a_i (1 - e_i^2)} \quad \text{and} \quad (2.5)
\]

\[
E_i = -GM m_i a_i / 2a_i. \quad (2.6)
\]

Here \( M_i = M + m_{i0} \), \( a_i \) denotes the semi-major axis and \( e_i \) the eccentricity of planet \( i \).

The mean longitude of planet \( i \) is \( \lambda_i = n_i (t - t_{0i}) + \varpi_i \), where \( n_i = \sqrt{GM_i / a_i^3} \) is its mean motion, with \( t_{0i} \) denoting its time of periastron passage and \( \varpi_i \) the longitude of periastron.

In this paper we consider \( N = 2 \) and arrange the two planets such that \( i = 1 \) denotes the outer planet and \( i = 2 \) denotes the inner planet. We remark that the above formalism does not incorporate disk tides. However, forces resulting from these can be added in separately (see below).

### 2.1 Coordinate system

We adopt Jacobi coordinates (Sinclair 1975, Papaloizou & Szuszkiewicz 2005) for which the radius vector \( \mathbf{r}_2 \), is measured from \( M \) and that of the outer planet, \( \mathbf{r}_1 \), is referred to the centre of mass of \( M \) and the inner planet \( i = 2 \). The required Hamiltonian correct to second order in the planetary masses is given by

\[
H = \frac{1}{2} (m_1 |\mathbf{r}_1|^2 + m_2 |\mathbf{r}_2|^2) - \frac{GM_1 m_1}{|\mathbf{r}_1|} - \frac{GM_2 m_2}{|\mathbf{r}_2|} - \frac{G m_1 m_2}{|\mathbf{r}_{12}|} \cdot \frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{|\mathbf{r}_1|^3}. \quad (2.7)
\]

Here \( M_1 = M + m_1, M_2 = M + m_2 \) and \( \mathbf{r}_{12} = \mathbf{r}_2 - \mathbf{r}_1 \).

The Hamiltonian may quite generally be expanded in a Fourier series involving linear combinations of the three angular differences \( \lambda_i - \varpi_i, i = 1, 2 \) and \( \varpi_1 - \varpi_2 \).

Near a first order \( p + 1 : p \) resonance, we expect that both \( \phi_1 = (p + 1) \lambda_1 - p \lambda_2 - \varpi_2 \), and \( \phi_2 = (p + 1) \lambda_1 - p \lambda_2 - \varpi_1 \), will be slowly varying. Following standard practice (see e.g. Papaloizou 2003 and Papaloizou & Szuszkiewicz 2005) only terms in the Fourier expansion involving linear combinations of \( \phi_1 \) and \( \phi_2 \) as argument are retained because only these are expected to lead to large long-term perturbations. In general there are an infinite number of such terms that need to be considered. However, in the limit of small eccentricities only terms that are first order in the eccentricities need to be retained. This approximation is valid when the circularization times are small enough to ensure that the eccentricities remain small. This situation is realized for realistic examples of low mass protoplanets migrating in protoplanetary discs (Papaloizou & Szuszkiewicz 2005).

Following the procedure described above and expanding to first order in the eccentricities, the Hamiltonian may be written in the form \( H = E_1 + E_2 + H_{12} \),
where
\[ H_{12} = -\frac{Gm_1m_2}{a_1} (e_1C_1 \cos(\phi_2) - e_2C_2 \cos(\phi_1)), \] (2.8)
with
\[ C_1 = \frac{1}{2} \left( x \frac{d(b_{1/2}^{(p)}(x))}{dx} + (2p + 1)b_{1/2}^{(p)}(x) - 4x\delta_1^{p} \right) \] and (2.9)
\[ C_2 = \frac{1}{2} \left( x \frac{d(b_{1/2}^{(p+1)}(x))}{dx} + 2(p + 1)b_{1/2}^{(p+1)}(x) \right). \] (2.10)
Here \( b_{1/2}^{(p)}(x) \) denotes the usual Laplace coefficient (e.g. Brouwer & Clemence 1961) with the argument \( x = a_2/a_1 \) and \( \delta_1^{p} \) denotes the Kronecker delta. We have also replaced \( M_i \) by \( M \).

2.2 Behaviour near a resonance with disk tides incorporated

The governing equations for motion near to the \( p + 1 : p \) resonance are to lowest order in the eccentricities
\[
\frac{1}{2} \frac{d}{dt} \frac{C_1}{a_1 \sqrt{GMa}_1} e_1 \sin \phi_2 - \left[ \frac{e_1^2}{t_{c1}} \right]
\] (2.11)
\[
\frac{1}{2} \frac{d}{dt} \frac{C_2}{a_1 \sqrt{GMa}_2} e_2 \sin \phi_1 - \left[ \frac{e_2^2}{t_{c2}} \right]
\] (2.12)
\[
\dot{n}_1 = \frac{3n_1(p + 1)Gm_2}{a_1 \sqrt{GMa}_1} (C_1e_1 \sin \phi_2 - C_2e_2 \sin \phi_1) + \left[ \frac{n_1}{t_{mig1}} + \frac{3n_1e_1^2}{t_{c1}} \right]
\] (2.13)
\[
\dot{n}_2 = -\frac{3n_2pGm_1}{a_1 \sqrt{GMa}_2} (C_1e_1 \sin \phi_2 - C_2e_2 \sin \phi_1) + \left[ \frac{n_2}{t_{mig2}} + \frac{3n_2e_2^2}{t_{c2}} \right].
\] (2.14)

Here the terms on the right-hand sides enclosed in square brackets give the contributions arising from disk tides. These have been discussed elsewhere (see e.g. Papaloizou & Szuszkiewicz 2005). The circularization and migration times for planet \( i \) are \( t_{c_i} \) and \( t_{mig_i} \) respectively. Note that the migration time is defined here as the time for the mean motion to change by a factor of \( e \) as a result of disk torques. The terms \( \propto e_i^2/t_{c_i} \) in (2.13) and (2.14) account for the orbital energy dissipation occurring as a result of circularization at the lowest order in \( e_i \) which it appears.

The other terms on the right-hand sides of (2.11)-(2.14) are derived from the Hamiltonian system (2.1)-(2.4) using the Hamiltonian (2.8).

2.3 Migration maintaining resonance

When the two planets migrate together maintaining resonance, we expect and find solutions for which \( e_1 \) and \( e_2 \) are either actually nearly constant or more generally
very nearly constant in a time average sense while the ratio $n_1/n_2$ is maintained very close to $p/(p + 1)$. When considering time averages, we here consider the average to be taken over a time long compared to the characteristic orbital period but short enough that the semi-major axes and tidal time scales may be considered constant. Using angle brackets to denote such a time average and setting $(e_1\dot{e}_1) = (e_2\dot{e}_2) = 0$ in (2.11) and (2.12) and taking the result of dividing the time averaged (2.13) by the time averaged (2.14) gives three equations from which $(e_i\sin(\phi_i))$, $i = 1, 2$ may be determined together with a constraint on the mean square eccentricities and migration times that does not depend on $\phi_i$. Proceeding in this way we obtain

\[
\langle e_1 \sin \phi_2 \rangle = -\frac{e_1^2 a_1 \sqrt{GMa_1}}{Gm_2 C_1 t_{c1}} \tag{2.15}
\]

\[
\langle e_2 \sin \phi_1 \rangle = -\frac{e_2^2 a_1 \sqrt{GMa_2}}{Gm_1 C_2 t_{c2}} \tag{2.16}
\]

together with the constraint

\[
\frac{e_1^2}{t_{c1}} + \frac{e_2^2}{t_{c2}} m_2 n_1 a_1 - \left( \frac{e_1^2}{t_{c1}} - \frac{e_2^2}{t_{c2}} \right) f = \left( \frac{1}{t_{mig1}} - \frac{1}{t_{mig2}} \right) \frac{f}{3}, \tag{2.17}
\]

where $f = m_2 a_1 / ((p + 1)(m_2 a_1 + m_1 a_2))$ and for ease of notation we have suppressed the angle brackets enclosing $e_i$ which is now implicitly assumed to be a time averaged value. We remark that (2.17) has already obtained from general considerations (Papaloizou & Szuszkiewicz 2005). However, (2.15) and (2.16) have not.

### 2.4 A condition for a $p+1:p$ commensurability to be maintained

We may use (2.15) and (2.16) to express $e_i^2$ in terms of $(e_i \sin \phi_i)$ in the constraint (2.17). Using Schwartz’s inequality and applying the condition $\langle \sin^2 \phi_i \rangle \leq 1$ results in an inequality that must be satisfied if the resonance is to be maintained. This inequality can be written in the form

\[
\frac{p^2 n_2 m_2}{(p + 1)^2 M} \left( \frac{1 - f}{m_2 C_1^2 t_{c1}} + \frac{m_1 a_1^2 C_2^2 t_{c2}}{M a_2^2} \right) \geq \left( \frac{1}{t_{mig1}} - \frac{1}{t_{mig2}} \right) \frac{f}{3}. \tag{2.18}
\]

For the simple example where $m_1 \gg m_2$ is in a prescribed slowly shrinking circular orbit and controls the migration ($t_{mig2} \gg t_{mig1}$) the relation (2.18) simplifies to the form

\[
m_1^2 \frac{a_2}{M^2} \geq \left( \frac{a_2}{3pa_1 n_1 n_2 t_{mig1} t_{c2} C_2^2} \right). \tag{2.19}
\]

We further remark that when it is satisfied, the maximum $\langle \sin^2 \phi_i \rangle$ has to exceed the ratio of the right-hand side to the left-hand side of the inequality (2.18).
2.5 Resonance overlap

Because it is found that both $C_1$ and $C_2$ increase with $p$, while $f$ decreases with $p$, the inequality (2.18) indicates that for given planet masses the maintenance of resonances with larger values of $p$ is favoured. However, the maintenance of resonances with large $p$ may be prevented by resonance overlap and the onset of chaos. Resonance overlap occurs when the difference of the semi-major axes of the two planets is below a limit that, in the case of two equal mass planets, has a half-width given by Gladman (1993) as

$$\frac{\Delta a}{a} \sim 2 \frac{2}{3p} \approx 2 \left( \frac{m_{\text{planet}}}{M_*} \right)^{2/7},$$

(2.20)

with $a$ and $m_{\text{planet}}$ being the mass and semi-major axis of either planet respectively. Thus for a system consisting of two equal four-Earth-mass planets orbiting a central solar mass we expect resonance overlap for $p > 8$. Conversely we might expect isolated resonances in which systems of planets can be locked and migrate together if $p < 8$. But note that the existence of eccentricity damping may allow for somewhat larger values of $p$ in some cases. In this context the inequality (2.18) also suggests that resonances may be more easily maintained for lower circularization rates. However, this may be nullified for large $p$ by the tendency for larger eccentricities to lead to greater instability. Note also that higher order commensurabilities may also be generated in such cases and these are not covered by the theory described above.

3 Numerical simulations

In order to check the applicability of the inequality (2.18) we have performed numerical simulations of a pair of migrating planets using an $N$-body code. The approach is the same as that used by many authors (e.g. Snellgrove, Papaloizou & Nelson 2001; Nelson & Papaloizou 2002; Lee & Peale 2002; Kley, Peitz & Bryden 2004). The reader is referred to these papers for details. In particular, orbital migration and eccentricity damping that are presumed to result from interaction with the protoplanetary disk are modelled through the addition of appropriate acceleration terms to the equations of motion. We considered planets with masses $m_1 = 4M_\oplus$, and $m_2 = M_\oplus$ for a wide range of imposed migration and circularization rates. The planets were started in circular orbits with $m_1$ being at 0.34au and $m_2$ at 0.2au (note that these results can be scaled to apply to other initial radii by appropriate renormalization of space and time scales). Thus they begin just outside of the 2:1 resonance before commencing convergent migration. Low mass planets of the type we consider undergo type I migration (see Papaloizou & Terquem 2006 and references therein). On account of dependence on the equation of state and non linear effects, the precise rate that should be employed is uncertain even in a laminar disk as adopted here (e.g. Paardekooper & Papaloizou 2008). To survey a range of possibilities, the migra-
Table 1. Details of the simulations discussed in this paper. Column 1 gives the value of $f_{\text{tid}}$. Column 2 gives the run time in years. Column 3 lists the commensurabilities for which sustained trapping and joint migration was noted. The occurrence of such episodes was characterized by eccentricity boosts. Column 4 firstly indicates the first order commensurability of lowest degree estimated to be possible by use of the inequality (2.18) and secondly the same estimate assuming the maximum allowed $\langle \sin^2 \phi_i \rangle \leq 0.1$. All the simulation results are consistent with the inequality (2.18).

| $f_{\text{tid}}$ | Run time yr. | Commensurabilities | Lowest degree expected |
|------------------|-------------|--------------------|------------------------|
| 1                | 1278400     | 2:1 3:2            | 2:1, 2:1               |
| 2                | 638400      | 2:1 3:2            | 2:1, 2:1               |
| 4                | 638400      | 2:1 3:2            | 2:1, 2:1               |
| 16               | 492800      | 2:1, 3:2, 4:3      | 2:1, 2:1               |
| 32               | 345600      | 2:1, 3:2, 4:3      | 2:1, 2:1               |
| 64               | 123200      | 3:2, 4:3, 5:4      | 2:1, 2:1               |
| 128              | 57000       | 3:2, 4:3, 5:4, 6:5 | 2:1, 3:2               |
| 256              | 31150       | 4:3, 5:4, 6:5      | 2:1, 3:2               |
| 512              | 15550       | 4:3, 5:4, 6:5      | 3:2, 5:4               |
| 1024             | 7875        | 5:4, 6:5, 7:6      | 4:3, 7:6               |
| 2048             | 3906        | 6:5, 7:6, 8:7      | 5:4, 10:9              |
| 4096             | 1959        | 11:10              | 8:7, $p > 12$          |

Fig. 1. The evolution of the semi-major axes (left panel) and eccentricities (right panel) of the two planets are shown as functions of time for $f_{\text{tid}} = 1$. The planets migrate inwards while locked in commensurabilities. The early episode of high eccentricity corresponds to a 2:1 commensurability while the final one corresponds to a 3:2 commensurability. The inner planet has the larger eccentricity.
Fig. 2. As in Fig. 1 but for $f_{\text{tid}} = 64$. In this case the planets fail to lock into the 2:1 commensurability, showing only an eccentricity spike when the system passes through this commensurability. Subsequently the configuration becomes sequentially locked in the 3:2, 4:3 and 5:4 commensurabilities.

Fig. 3. As in Fig. 1 but for $f_{\text{tid}} = 1024$. In this case the planets fail to lock into the 2:1, 3:2 and 4:3 commensurabilities showing only eccentricity spikes when the system passes through these commensurabilities. Subsequently the configuration becomes sequentially locked in the 5:4, 6:5 and 7:6 commensurabilities.

tation and circularization rates we adopted were given by

$$t_{migi} = \frac{1.4 \times 10^7 M_\oplus}{3f_{\text{tid}} \ m_i} \ yr \quad (3.1)$$
Fig. 4. As in Fig. 1 but for $f_{\text{tid}} = 4096$. In this case the planets fail to lock into any commensurability until $p = 10$, corresponding to the 11:10 commensurability. Many eccentricity spikes occur as the system passes through lower degree commensurabilities.

and

$$t_{\text{ci}} = \frac{2.5 \times 10^4 M_{\oplus}}{f_{\text{tid}} m_i} \text{ yr},$$

respectively where $f_{\text{tid}}$ is a scaling factor. We have adopted $1 \leq f_{\text{tid}} \leq 4096$, giving migration time scales that range between $\sim 10^6$ yr and $\sim 10^3$ yr. In this way timescales that are both comparable to the gas disk lifetime, and significantly shorter are considered. The ratio $t_{\text{ci}}/t_{\text{migi}}$ has been chosen to be comparable to the disk aspect ratio squared for an aspect ratio $\sim 0.05$ as expected from theoretical work (see e.g. Papaloizou & Larwood 2000). We summarize the parameters and outcomes of the simulations in Table 1. In particular the commensurabilities for which sustained trapping and joint migration were observed are noted. As expected, it is found that lower degree commensurabilities are formed for smaller $f_{\text{tid}}$ and then become unsustainable if $f_{\text{tid}}$ is increased beyond a critical level. In general each simulation is found to involve the setting up of a sequence of commensurabilities each of which is disrupted after a significant contraction of the planetary orbits.

To illustrate this, the evolution of the semi-major axes and eccentricities of the two planets for $f_{\text{tid}} = 1$ are plotted in Fig. 1. In this case a 2:1 commensurability is first formed which is then disrupted after the planets have migrated inwards together contracting their orbits by $\sim 30\%$. This resonance is disrupted and a 3:2 commensurability is then formed. The evolution for $f_{\text{tid}} = 64$ is illustrated in Fig. 2. In this case the planets pass through the 2:1 commensurability. Subsequently the configuration sequentially forms 3:2, 4:3 and 5:4 commensurabilities. Each of these are associated with significant contractions of the orbits. The case with
$f_{\mathrm{tid}} = 1024$ is illustrated in Fig. 3. In this case the system passes through the 2:1, 3:2 and 4:3 commensurabilities. Subsequently 5:4, 6:5 and 7:6 commensurabilities are formed each of which is associated with a significant orbit contraction. The results for $f_{\mathrm{tid}} = 4096$ are shown in Fig. 4. No lasting commensurability is formed until $p = 10$. Again this commensurability survives while the orbits undergo a large contraction. This commensurability is of somewhat higher degree than would be expected from Gladman (1993). It is likely that it can survive on account of the eccentricity damping employed here.

In Table 1 we also indicate the first order commensurability of lowest degree that is estimated to be possible by use of the inequality (2.18) together with an estimate made assuming the maximum allowed $\langle \sin^2 \phi_i \rangle \leq 0.1$. All the simulation results are consistent with the inequality (2.18). If it is used directly, the lowest degree commensurability that is possible to form is, as expected, somewhat underestimated. However, if we assume the maximum allowed $\langle \sin^2 \phi_i \rangle \leq 0.1$, Table 1 indicates that the lowest degree commensurability that is possible to form is somewhat overestimated for the larger values of $f_{\mathrm{tid}}$.

4 Conclusion

We have studied the development of orbital resonances by migrating planets in the terrestrial mass range for migration time scales varying between $\sim 10^6$ yr and $\sim 10^3$ yr. In a typical simulation, a sequence of resonances occurred, each of which could survive significant orbital evolution before being lost. For the slowest migration rates considered, a 2:1 commensurability was able to form. At the fastest rates only commensurabilities with large $p$ persisted. We also found analytic conditions for first order commensurabilities to be formed. These were consistent with our numerical simulations.

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