Inconsistency of QED in the Presence of Dirac Monopoles II.

Hong-Jian He $^{a,b}$ Zhaoming Qiu $^b$ Chia-Hsiung Tze $^a$

$^a$ Department of Physics and Institute for High Energy Physics
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061-0435, U.S.A.

$^b$ China Center of Advanced Science and Technology (World Laboratory)
P. O. Box 8730, Beijing 100080, P.R. China

Abstract

We enlarge the local gauge invariance of QED from $U(1)_A$ to $U(1)_A \times U(1)_\Theta$ by introducing another unphysical pure gauge field $\Theta$ with an independent, unphysical gauge coupling $\tilde{e}$. This pure gauge field can be gauge-transformed away and the resulting theory is identical to standard QED. We then re-examine the Dirac quantization condition (DQC) for point monopoles and find that two essentially different DQCs can be derived. One DQC involves a gauge coupling $e$ in the $U(1)_A$ group and the other only the unphysical gauge coupling $\tilde{e}$ in the $U(1)_\Theta$ group. The unique physically consistent solution of these two DQCs is a vanishing magnetic charge, which implies that no Dirac monopole exists in nature.

$^1$Work supported in part by the U.S. Department of Energy under grant DEFG05-92ER40749.

$^2$Present address of H.J. He.
In this second of a series of papers\textsuperscript{[1]} devoted to point monopoles in QED, we present an alternative proof of the inconsistencies of QED in the presence of Dirac monopoles. We noticed that the singular Dirac string\textsuperscript{[2]} in the monopole gauge potential is purely a gauge-artifact. It is just the gauge freedom which allows us to arbitrarily move the string around without any physical effect, provided that a consistent condition — Dirac quantization condition (DQC)\textsuperscript{[2]} is satisfied. By introducing another unphysical pure gauge field into QED, we find it possible to attribute part of the singularities to this pure gauge field and thus the corresponding DQC involves the unphysical gauge coupling associated with this pure gauge field. So the physically consistent solution to both the original DQC and this new DQC can only be a vanishing magnetic charge. In Sec.1, a generalized QED Lagrangian with an enlarged local gauge symmetry $U(1)_A \times U(1)_\Theta$ is proved to be identical to standard QED up to the quantum-field-theory-level. Of course, the gauge coupling associated with this pure gauge field in the $U(1)_\Theta$ group is shown to be entirely arbitrary. Two independent DQCs are carefully derived in Sec.2 and some conclusions are given in Sec.3.

1. A generalized QED Lagrangian and the Ward-Takahashi identities

An Abelian or non-Abelian global symmetry can always be localized by introducing an \textit{unphysical pure gauge field}, which has no kinetic term and can be gauge-transformed away. A dynamical gauge field is only a natural generalization and at present its existence can be determined only by experiments. A pure gauge field is \textit{sufficient and necessary} to insure the ordinary local gauge invariance. This may be why without discovering the corresponding dynamical gauge fields we have observed a lot of global symmetries (such as the lepton and baryon numbers conservations) which had been independently tested at different local places. Besides the electric charge conservation, standard QED has an extra global $U(1)$ symmetry which is the electron number conservation. In the following we shall localize this extra global $U(1)$ symmetry by introducing a pure gauge field. One should notice that only the physical gauge coupling associated with a dynamical gauge field can be related to its global charge and the unphysical gauge coupling associated with the pure gauge field has nothing to do with the global charge since it is non-observable and the corresponding pure gauge field can be completely gauge-transformed away.

But when including monopoles, we should carefully distinguish two essentially different situations. In the Dirac monopole case, a singular gauge transformation must be allowed in order
to arbitrarily move the Dirac string and thus make it non-observable as desired. This singular $U(1)$ gauge transformation (which is usually called as an "extended" gauge transformation\cite{3}) can thus arbitrarily change the pure gradient part of the monopole’s gauge field or even entirely transform it away while leaving the physical magnetic field invariant. This is in sharp contrast with the case of the spatially extended ’t Hooft-Polyakov monopole\cite{4} which, as finite energy solution to the spontaneously broken gauge theories, is naturally singularity-free at the beginning. All allowed regular gauge transformations cannot rid of the pure gauge field (or even change their homotopy class). Furthermore any singular gauge transformation which transforms the pure gauge field away must be forbidden since it leaves a vanishing magnetic field.

In this section we first discuss QED without Dirac monopoles. Consider the following generalized QED Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \gamma_\mu D^\mu \psi - m \bar{\psi} \psi \quad (1)$$

with

$$D_\mu = (\partial_\mu - ie A_\mu - i\tilde{e} \partial_\mu \Theta),$$

$$F_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

$$\bar{A}_\mu \equiv A_\mu + \partial_\mu \Theta,$$

where $\bar{A}_\mu$ is only a notation in which the coefficient of $\partial_\mu \Theta$ is arbitrary but can always be chosen to be unity since the unphysical $\Theta$ field has no kinetic term and can be arbitrarily rescaled without any physical effect. The above QED Lagrangian has a larger local symmetry $U(1)_A \times U(1)_{\Theta}$, i.e. it is invariant under the following two kinds of independent gauge transformations:

(i). The $U(1)_A$ gauge transformation

$$\psi'(x) = e^{-i\zeta(x)} \psi(x), \quad \bar{\psi}'(x) = \bar{\psi}(x) e^{i\zeta(x)};$$

$$A'_\mu = A_\mu - e^{-1} \partial_\mu \zeta(x),$$

$$\Theta' = \Theta. \quad (3)$$

(ii). The $U(1)_{\Theta}$ gauge transformation

$$\psi'(x) = e^{-i\eta(x)} \psi(x), \quad \bar{\psi}'(x) = \bar{\psi}(x) e^{i\eta(x)};$$

$$A'_\mu = A_\mu,$$

$$\Theta' = \Theta - \tilde{e}^{-1} \eta(x). \quad (4)$$
Eq.(4) clearly shows that the unphysical pure gauge field can be completely gauge-transformed away and thus our generalized QED Lagrangian simply reduces to the standard QED Lagrangian. Actually the standard QED is in the "unitary gauge" of eq.(1), in which the pure gauge field has been transformed away. Here it is clear that the gauge coupling $e$ and $\bar{e}$ belong to the two direct product group $U(1)_A$ and $U(1)_\Theta$ respectively, and thus are independent of each other.

From (1) and (2), the definition of the magnetic field is

$$\vec{B} = \vec{\nabla} \times \vec{A} = \vec{\nabla} \times \vec{A}.$$  (5)

The nonintegrable phase factor is now expressed as

$$P(x_2, x_1; C) = \exp \left[ ie \int_{x_1}^{x_2} A_\mu dx^\mu + i\bar{e} \int_{x_1}^{x_2} \partial_\mu \Theta dx^\mu \right].$$  (6)

When doing quantization, we need two gauge-fixing terms for two gauge groups $U(1)_A$ and $U(1)_\Theta$ respectively, i.e.

$$L_{gf} = -\frac{1}{2\xi_A} F_1(A)^2 - \frac{1}{2\xi_\Theta} F_2(\Theta)^2.$$  (7)

For example, the gauge-fixing functions $F_1(A)$ and $F_2(\Theta)$ can be chosen as

$$F_1(A) = \partial^\mu A_\mu, \quad F_2(\Theta) = \partial^2 \Theta.$$  (8)

Now we derive some new Ward-Takahashi (WT) identities for the $U(1)_\Theta$ gauge group. By introducing the external sources $J_\mu A^\mu + \bar{I}\psi + \bar{\psi} I$ in the generating functional for Green functions and doing a $U(1)_\Theta$ gauge transformation, we can easily re-derive the following generating equation

$$(\xi_\Theta \bar{e})^{-1} \partial^4 \Theta + \bar{e}^{-1} \frac{\delta \Gamma}{\delta \Theta} = i \left[ \frac{\delta \Gamma}{\delta \psi} \psi + \bar{\psi} \frac{\delta \Gamma}{\delta \bar{\psi}} \right].$$  (9)

From (9) we get the following two WT identities

$$i\tilde{D}^{-1}(k) = -\xi_\Theta ^{-1} k^4,$$

$$i(p'_\mu - p_\mu) \Lambda^\mu(p', p) = iS^{-1}(p') - iS^{-1}(p),$$  (10)

where

$$i\tilde{D}^{-1}(k) = \int_{FT} \delta^2 \Gamma / \delta \Theta(y) \delta \Theta(x), \quad \bar{e}(p'_\mu - p_\mu) \Lambda_\mu(p', p) \equiv \int_{FT} \delta^2 \Gamma / \delta \bar{\psi}(z) \delta \bar{\psi}(y) \delta \Theta(x).$$  (11)
(\int F T \) denotes the Fourier transform) and \( S(p) \) is the full fermion propagator. Also we can easily find that
\[
\int F T \delta^3 \Gamma/[\delta \psi(z) \delta \bar{\psi}(y) \delta A^\mu(x)] = e \Lambda_\mu(p', p).
\]

To perform the renormalization, we define
\[
\psi = Z_2^{1 \over 2} \psi_R, \quad \bar{\psi} = Z_2^{1 \over 2} \bar{\psi}_R, \quad A^\mu = Z_A^{1 \over 2} A^\mu_R, \quad \Theta = Z_\Theta^{1 \over 2} \Theta_R, \quad m = Z_m m_R, \quad e = Z_e e_R, \quad \bar{e} = Z_e \bar{e}_R, \quad \xi_A = Z_{\xi_A} \xi_{AR}, \quad \xi_\Theta = Z_{\xi_\Theta} \xi_{\Theta R}.
\]

Here the non-observable gauge coupling \( \bar{e} \) of the pure gauge field \( \Theta \) has an independent renormalization constant \( Z_{\bar{e}} \).

We rewrite (1) as
\[
L = Z_A^{-1 \over 4} F_R^{\mu \nu} F_R^{\mu \nu} + Z_2 \bar{\psi}_R(i \partial - Z_m m_R) \psi_R + Z_1 e_R A^\mu_R \bar{\psi}_R \gamma_\mu \psi_R + \bar{Z}_1 \bar{\psi}_R \partial^\mu \Theta_R \bar{\psi}_R \gamma_\mu \psi_R.
\]

Then we have
\[
Z_e = Z_1 Z_2^{-1 \over 2} Z_A^{1 \over 2}, \quad Z_{\bar{e}} = Z_1 Z_2^{-1 \over 2} Z_\Theta^{1 \over 2}.
\]

The WT identity (10) only requires that, after renormalization, \( Z_{\xi_\Theta} = Z_\Theta \), where either \( Z_\xi \) or \( Z_A \) but not both can be arbitrarily chosen. Since (10) shows that \( \bar{D}_{\mu \nu} \) has no loop correction at all, the most natural and simplest choice is
\[
Z_{\xi_\Theta} = Z_\Theta = 1.
\]

In general, we can choose \( Z_{\xi_\Theta} = Z_\Theta = 1 + (\text{arbitrary loop-order quantities}) \). The WT identity (11) and eqs.(12)(13) give \( Z_1 = \bar{Z}_1 = Z_2 \). So substituting this equation and (16) into (15) we get
\[
Z_e = Z_A^{-1 \over 2}, \quad Z_{\bar{e}} = Z_{\xi_\Theta}^{-1 \over 2} = 1.
\]

In consequence we prove that the renormalization for \( \bar{e} \) is actually arbitrary and may need no renormalization whatsoever. This is not surprising since for the product groups \( U(1)_A \times U(1)_\Theta \), the gauge coupling \( \bar{e} \) of \( U(1)_\Theta \) has nothing to do with the the physical coupling \( e \) of \( U(1)_A \).

Finally, we emphasize again that our above generalized QED is identical to standard QED, even up to loop-level. Clearly, the introduction of a pure gauge field which can be gauge-transformed away can have no physical effects.

2. Dirac quantization condition re-examined
Following our part-I we still work in the standard Dirac formulation\cite{5}. Let us consider a Dirac monopole $g$ with magnetic field $\vec{B}(x) = \frac{g}{r^2} \vec{r}$, where $r = |\vec{x}|$. The magnetic field is related to the monopole’s gauge potential by $\vec{B} = \vec{\nabla} \times \vec{A}$ which implies that $\vec{A}_\mu$ cannot be regular everywhere and must contain some singularities. Since the physical $\vec{B}$ field is regular everywhere except at the origin, in the standard Dirac formulation\cite{3}, the above definition is modified by adding the so-called Dirac string to cancel the singularities in $\vec{\nabla} \times \vec{A}$, so that the correct $\vec{B}$ field is obtained. Following the same steps as before, we obtain the two simplest Dirac solutions for $\vec{A}_\mu(\equiv A_\mu + \partial_\mu \Theta)$ with singular lines along the negative and positive $z$-axes, respectively:

\begin{align}
(\vec{A}_+)_t &= (\vec{A}_+)_r = (\vec{A}_+)_\theta = 0, \\
(\vec{A}_+)_\varphi &= \frac{g}{r} \frac{\pm 1 - \cos \theta}{\sin \theta}.
\end{align}

They are connected by the gauge transformation

\begin{equation}
\vec{A}_-^\mu = \vec{A}_+^\mu - \partial^\mu (2g\varphi).
\end{equation}

From (3) and (4), we see that this can be regarded as a gauge transformation of $U(1)_A$ with

\begin{align}
A_+^\mu &= A_-^\mu - e^{-1} \partial^\mu \zeta(x), \\
\vec{A}_- &= \frac{g}{r} \frac{1 - \cos \theta}{\sin \theta} = \vec{A}_- - \frac{2g}{r \sin \theta} \hat{\varphi}, \\
\Theta_- &= \Theta_- = 0;
\end{align}

or, a gauge transformation of $U(1)_\Theta$ with

\begin{align}
\vec{A}_- &= \vec{A}_- = \frac{-g \cos \theta}{r \sin \theta} \hat{\varphi}, \\
\Theta_- &= \Theta_- = -e^{-1} \eta(x), \\
\theta_- &= -g \varphi = -\Theta_-.
\end{align}

Now we can repeat the three standard approaches given in part-I to derive the DQC by using the above two kinds of gauge potentials and their transformations in (20) and (21), respectively. Thus, from (20) we just obtain the ordinary DQC

\begin{equation}
eg \varphi = \frac{n}{2}, \quad (n = 0, \pm 1, \pm 2, \cdots);
\end{equation}

while from (21) we get an independent new DQC

\begin{equation}\tilde{\varphi} \varphi = \frac{k}{2}, \quad (k = 0, \pm 1, \pm 2, \cdots).
\end{equation}
which has a similar form to (22) but has a completely different physical meaning. Here an important observation is that in (23) the physical magnetic charge $g$ is constrained by the non-observable gauge coupling $\tilde{e}$ for $k \neq 0$. This is not surprising since the singular Dirac string is a pure gauge artifact and thus can be naturally attributed to an unphysical pure gauge field. It is easy to check that for the case of the 't Hooft-Polyakov monopole[4] the DQC (23) cannot be derived even if one introduces an extra unphysical $U(1)$ pure gauge field, since there is no singularity. Also the original consistent condition (22) is unnecessary and the electric charge is automatically quantized in the 't Hooft-Polyakov monopole case.

3. Inconsistency of QED in the presence of Dirac monopoles

In (22) and (23) the gauge couplings $e$ and $\tilde{e}$ belong to two direct product $U(1)$ groups respectively and thus are independent of each other as we pointed out before. There are actually two possible solutions to the original DQC (22): $g = 0$ with $n = 0$ and $g \neq 0$ with $n \neq 0$. However, in our new DQC (23) the only physically consistent solution is $g = 0$ with $k = 0$, which is also a possible solution to DQC (22). The nonvanishing solution $g \neq 0$ in (23) constrains the physical magnetic charge $g$ with unphysical coupling $\tilde{e}$ and thus can never be consistent as already analyzed in our part-I. Hence we conclude that the unique physically reasonable solution to both (22) and (23) is $g = 0$, which implies that no Dirac monopoles exist in the nature. Thus this alternative proof strengthens our conclusion in part-I from a different point of view. Other inconsistencies of Dirac monopoles are presented elsewhere[6].

Acknowledgement  H.J. He thanks Prof. Lay Nam Chang for discussions.

References

1. H.J. He, Z. Qiu, C.H. Tze, VPI-IHEP-93-12;

2. P.A.M. Dirac, Proc. Roy. Soc. A133(1931)60. An extension of this idea to include relativity is given in his another paper: Phys.Rev. 74(1940)817. After Dirac’s 1st paper, thousands of papers on this subject have since appeared. For a more detailed list, see, for example, Resource Letter MM-1: Magnetic Monopoles by A.S. Goldhaber, W.P. Trower in Am.J. Phys. 58, 429(1990).
3. See, for example, P. Goddard and D.I. Olive, Rep.Prog.Phys. 41(1978)1357.

4. G. 't Hooft, Nucl. Phys. B79, 276(1974);
   A.M. Polyakov, Sov. Phys. JETP Lett. 20, 194(1974).

5. For general reviews, see for examples: B. Zumino, "Recent developments in the theory of magnetically charged particles" in Strong and Weak Interactions—Present Problems, (Proc. 1966 Int. School of Physics 'Ettore Majorana'), p.711, ed. A. Zichichi, Academic Press, New York and London; P. Goddard and D.I. Olive, in Ref.[3].

6. H.J. He, Z. Qiu, C.H. Tze, VPI-IHEP-93-14;
   H.J. He, C.H. Tze, VPI-IHEP-93-08, 09, 10, 11.