Equivalence of Covariant and Light-Front Perturbation Theory

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ABSTRACT

Light-Front Field Theory (LFFT) is a good candidate to describe bound states. In LFFT covariance is non-manifest. Burkardt and Langnau[1] claim that, even for scattering amplitudes, rotational invariance is broken. We will take a different path of obtaining rules for light-front time-ordered diagrams[2]. Covariance depends on the choice of a regulator $\alpha$. We need to apply some regularisation scheme to render physical amplitudes finite. This is done by applying minus regularisation[3]. In this process all ambiguities related to the regulator $\alpha$ are removed. Therefore there is equivalence between the perturbative expansions of covariant and LFFT.

1 Introduction

Covariant Field Theory (CFT) has been very successful in describing elementary particles but has not produced a convenient framework to describe bound states of elementary particles. However, Hamiltonian field theories seem to be good candidates to explain properties of bound states. In a Hamiltonian framework the initial conditions are specified on some surface. The Hamiltonian then gives the evolution of the system in time. Already in 1949, Dirac pointed out that there are several possible choices for the surface of quantisation. One of these surfaces is the light-front. There are a number of advantages to LFFT over quantisation on, e.g., the equal-time plane. In LFFT there can be no creation/annihilation of massive particles from/to the vacuum. This reduces the number of time-ordered diagrams. However, we have to add so-called instantaneous terms for every fermion line, because we use on-shell spinors. For a number of reasons, quantisation on the light-front is very complicated. In Naive Light-Cone Quantisation (NLCQ) some problems are not satisfactorily solved. Still, NLCQ rules have been constructed for light-front time (lime) ordered diagrams. Inspired by Ligterink[2] we will construct rules for lime-ordered diagrams in another way. Ligterink derived rules for lime-ordered diagrams by integrating covariant Feynman diagrams over light-front time. For some types of diagrams the integrals diverge. So, only upon regularisation these integrals, the relation becomes definite.

In LFFT, or any other Hamiltonian theory, covariance is not manifest. Burkardt and Langnau[1] claim that in NLCQ rotational invariance is broken. They fix rotational invariance by introducing non-covariant counterterms. Transverse divergences are dealt with using dimensional regularisation. Instead, we will use the method of
minus regularisation\[3\]. In this method longitudinal and transverse divergences are treated in the same way. Another advantage of minus regularisation is that the ambiguity, caused by integration over light-front time, is removed. This means that LFFT is equivalent to CFT in perturbation theory and that our method will yield covariant physical amplitudes.

## 2 Calculation of the fermion self energy

As an example we will show equivalence for one diagram in the Yukawa model. Our light-front coordinates are defined as: \( k^\pm = (t^0 \pm k^3)/\sqrt{2} \). The self interaction of a fermion with mass \( m \) and momentum \( q \) via a boson with mass \( l \) is given by

\[
\begin{align*}
\text{fermion self energy} & = -\int \frac{d^2k^\perp dk^+dk^-}{4k^+(q^+ - k^+)} \frac{k^-\gamma^+ + k^+\gamma^- - k^\perp\gamma^\perp + m}{(k^- - \frac{k^+ + m^2 - i\epsilon}{2k^+}) (q^- - k^- - \frac{(q^+ - k^+)^2 + l^2 - i\epsilon}{2(q^+ - k^+)})} \\
\end{align*}
\]

The integral (1) is not defined and we insert the following regulator to eliminate the pole at infinity.

\[
\alpha(k^+) \frac{k^-\gamma^+ + k^+\gamma^- - k^\perp\gamma^\perp + m}{1 + i\delta q^+k^- + 1 - \alpha(k^+)}
\]

(2)

It is convenient to take \( \alpha(k^+) = 0 \) for \( k^+ < q^+ \) and \( \alpha(k^+) = 1 \) for \( k^+ > q^+ \) to simplify the contour integration. After taking the limit \( \delta \to 0 \) we find

\[
\begin{align*}
\text{fermion self energy} & = \int d^2k^\perp \int_0^{q^+} \frac{dk^+}{4k^+(q^+ - k^+)} \frac{m^2 + k^+ + l^2 + (q^+ - k^+)^2}{2q^+ - m^2 + k^+} \\
\end{align*}
\]

The instantaneous part containing an extra factor depending on the regulator. At this level equivalence between covariant and LFFT is ambiguous because it depends on the regulator \( \alpha \). We will see that this dependence is removed naturally upon using minus regularisation.
3 Minus regularisation

The propagating and the instantaneous contributions suffer from both longitudinal and transverse divergences. If these divergences are treated in different ways we should not be surprised that it is hard to recover covariance. However, we will use the minus regularisation scheme[3] which treats these divergences on the same footing. It removes the lowest orders in the Taylor expansion of the amplitude. We differentiate the diagram with respect to the external energy until the integration is finite. After integrating over the internal momenta we integrate the result as many times with respect to the external energy as we have differentiated before. The propagating part of the fermion self energy (4) contains a term proportional to $k_\perp^2/2k^+$ which has to be differentiated twice. The operation we perform is then

$$\int_{q_{\perp}^+}^{q^+} dq_0 \int_{q_{\perp}^+}^{q^+} dq'' \int d^2 k_\perp \int_0^{q^+} dk^+ \left( \frac{\partial}{\partial q''_{\perp}} \right)^2$$

(6)

The other terms in the numerator of (4) must only be differentiated once. Otherwise the subtracted terms would not be local and would not correspond to counterterms in the Lagrangian. Apparently, we need to discriminate between different parts of the same diagram. The instantaneous diagram (5) has to be differentiated once to remove the singularity. Since the integrand is independent of the energy $q^-$ the differentiation kills the integrand. Therefore the $\alpha$ dependence is lost.

4 Conclusions

At the level of the unregularised diagrams equivalence is obscured by longitudinal divergences. These divergences are dealt with using a regulator $\alpha$. Upon using minus regularisation the $\alpha$ dependence, and therefore the ambiguity, is removed. Since the minus regularisation is a linear operation it commutes with the expansion of the covariant diagram in time-ordered diagrams. Therefore, the regularised perturbation series are equivalent. Our procedure can be generalised to any diagram in the Yukawa model. Minus regularisation removes local terms in the series of time-ordered diagrams. Those local terms are the diagrams containing the regulator. So, equivalence is restored and therefore LFFT will yield covariant physical amplitudes.

[1] M. Burkardt and A. Langnau, Rotational invariance in light-cone quantization, Physical Review D44 (1991) 3857.

[2] N. E. Ligterink and B. L. G. Bakker, Equivalence of light-front and covariant field theory, Physical Review D52 (1995) 5954.

[3] N. E. Ligterink and B. L. G. Bakker, Renormalization of light-front Hamiltonian field theory, Physical Review D52 (1995) 5917.