Perturbations of the spherically symmetric collapsar in the relativistic theory of gravitation: axial perturbations. I

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Abstract

Horizon solutions for the axial perturbations of the spherically symmetric metric are analyzed in the framework of the relativistic theory of gravitation. The gravitational perturbations cannot be absorbed by the horizon that results in the excitation of the new type of the normal modes trapped by the Regge-Wheeler potential. The obtained results demonstrate testable differences between the collapsar and the black hole near-horizon physics.

1 Introduction

Relativistic theory of gravitation (RTG) treats the gravitation as a symmetric second-rank tensor field on the Minkowski background spacetime [1]-[4]. Owing to such tensor nature, the field acts likewise a curved effective Riemannian spacetime as it takes a place in the Einstein’s theory of gravitation (theory of general relativity, GR). However, the bi-metric nature of the RTG requires a nontrivial inclusion of the Minkowski metric into field equation that violates the gauge-symmetry. As a result, the non-zero graviton’s mass appears in the theory as well as there is an additional gauge-fixing condition eliminating the undesirable graviton’s spin-states. The non-zero graviton’s mass $m_g < 10^{-66} \text{ g}$ does not contradict the modern astrophysical data [5] though its value can be essentially below this threshold [6] (graviton mass bound on the Solar System dynamics is $10^{-54} \text{ g}$ [7]).

The extremely small but nonzero mass of graviton excludes singularities from the RTG due to strong repulsion at small distances (or strong field strengths). As a result, the black holes (i.e. field singularities covered by the pure space-time horizons) are not possible in the theory (see, for example [8]). However, the
field equations allow the existence of the extremely dense and compact objects
formed at the final stage of the collapse (so called collapsars). Collapsar has a
radius, which is very close to the Schwarzschild one (see [9]), but its further con-
traction is not possible due to strong repulsion induced by the massive gravitons.
Therefore the collapsar has a surface in contrast to the black hole and falling
on this surface can entail the intensive bursts (most probably gravitational [9]).

The modern observational data suggesting the existence of the black-hole-
like objects do not allow distinguishing the GR’s black hole from the RTG’s
collapsar. Moreover, the existence of the latter has not been proved even the-
tically. First step in this direction is to prove stability of the spherically
symmetric metric in the framework of the RTG that has been made in [9] for
the approximated metric of Refs. [2, 8]. The main problem here is the bi-metric
character of the equations that troubles the calculations even for the simplest
field configurations. As a result, the approximations are required already at the
initial calculational stages.

In this work we consider the axial perturbations of the spherically symmetric
metric in the framework of the RTG. The rigid analysis is not possible even
in this simple case and we need using the near-horizon approximation. The
results show the essential difference in the near-horizon physics between the
GR and the RTG. The horizon reflects (not absorb) perturbations that allows
the gravitational modes trapped between the horizon and the Regge-Wheeler
potential barrier. This means that the scattering properties of the collapsar
differ from those of the black hole. As a result, the basic properties of the RTG
can be testable in the feature astronomical observations of the extra-compact
objects.

2 First-order perturbations of the spherically symmetric metric in the RTG

The Logunov’s equations for the massive gravitational field can be expressed
through the metric of the effective Riemannian spacetime [1]-[4]:

$$G^\mu_\nu - \frac{m^2}{2} \left( \delta^\mu_\nu + g^\mu_{\lambda\gamma_{\lambda\nu}} - \frac{1}{2} \delta^\mu_{\nu} g^{\kappa_{\lambda\gamma_{\kappa\lambda}}} \right) = -\frac{8\pi\kappa}{c^4} T^\mu_\nu, \quad (1)$$

$$D_\mu \tilde{g}^{\mu\nu} = 0,$$

where $\tilde{g}^{\mu\lambda}$ is the metric tensor of the effective Riemannian spacetime, $\tilde{g}^{\mu\nu} = \sqrt{-g} g^{\mu\nu}; \gamma^{\mu\nu}$ is the metric tensor of the background Minkowski spacetime, $G^\mu_\nu$ is the Einstein tensor, $T^\mu_\nu$ is the matter energy-momentum tensor, $D_\mu$ is the covariant derivative in the Minkowski spacetime; $m^2 = (m_g c / \hbar)^2$, $m_g$ is the graviton mass; $\kappa$ is the Newtonian gravitational constant. The second equation excluding unwonted spin-states for the gravitational field resembles the so-called
harmonic conditions introduced by Fock for the isolated gravitational systems
(see, for example, [10]).
Let us write the spherically symmetric static interval of the effective Riemannian spacetime in spherical coordinates:

\[
\begin{align*}
    ds^2 &= U(r) \, dt^2 - V(r) \, dr^2 - W(r)^2 \, (d\theta^2 + \sin^2 \theta \, d\phi^2), \tag{2}
\end{align*}
\]

and normalize lengths to the Schwarzschild radius \( r_s = 2\kappa M / c^2 \) \((M\) is the collapsar mass\). The approximated solution of Eqs. (1) for metric (2) gives an essential deviation from the Schwarzschild solution in the vicinity of \( r_s \) \([2, 8]\):

\[
\begin{align*}
    ds^2 &= \frac{\epsilon}{2} \, dt^2 - \frac{r^2 \, dr^2}{2 \Delta(r)} - r^2 \, (d\theta^2 + \sin^2 \theta \, d\phi^2), \tag{3}
\end{align*}
\]

where \( \epsilon = \frac{1}{2} \left( \frac{2\kappa M m_g}{hc} \right)^2 \), \( \Delta(r) = r^2 - r \), and we have to use \( r_l = r_s + \chi \) instead of the Schwarzschild radius. \( \chi \) is approximately proportional to \( \epsilon \): \( r_l \approx r_s (1 + 1.65\epsilon) \) for \( \epsilon \in [10^{-51}, 10^{-42}] \) \([9]\). \( \epsilon \) is extremely small for the stellar objects \((\approx 10^{-44} \) for \( M = 10M_\odot \) and maximum estimation for \( m_g \)). However, its small but non-zero value causes an irresistible repulsion from the Schwarzschild sphere (or rather from the sphere of \( r_l \) radius) due to \( U \neq 0 \) when \( V \to \infty \) \([8, 9]\). In the form of Eq. (3) the spherically symmetric metric is stable \([9]\). But this statement is not decisive for the general metric including both Eq. (3) (in the vicinity of \( r_l \)) and Schwarzschild (out of \( r_l \)) cases. To join both solutions we use the numerical consideration of Ref. \([9]\). Fig. 1 shows the dependence of \( U \) and \( V \) coefficients of interval (2) for the Schwarzschild (black lines) and the Logunov (blue line and blue point, see Eq. (3)) metrics. Red curve corresponds to the numerically obtained metric. At last, the green curves result from the approximated matched metric:

\[
\begin{align*}
    ds^2 &= \left( \frac{\epsilon}{2} + \frac{\Delta(r)}{r^2} \right) \, dt^2 - \frac{r^2 \, dr^2}{\Delta(r)} - r^2 \, (d\theta^2 + \sin^2 \theta \, d\phi^2), \tag{4}
\end{align*}
\]

which reproduces the Schwarzschild asymptotic and simultaneously agrees with the numerical solution in the vicinity of \( r_l \). The main requirements for Eq. (4) are i) to replace \( r_s \) by \( r_l \) and ii) to consider only \( r > r_l \). Also, it is necessary to point at the absence of \( \frac{1}{2} \) in \( V(r) \) (in comparison with Eq. (3)).

To analyze the axial perturbations of the metric (4) we shall use the methods of Ref. \([11]\). Let us consider the perturbed metric:
Figure 1: V (left) and U (right) functions of the interval for the Schwarzschild’s (black) and the Logunov’s (blue line and blue point) metrics. Red curves correspond to the numerically calculated metric. Green curves result from the approximated matched metric.

\[ g_{\mu\nu} = \begin{pmatrix}
U(r), & 0, & 0, & \zeta \omega(t, r, \theta) \times W(r)^2 \sin^2 \theta, \\
0, & -V(r), & 0, & \zeta q_2(t, r, \theta) \times W(r)^2 \sin^2 \theta, \\
0, & 0, & -W(r)^2, & \zeta q_3(t, r, \theta) \times W(r)^2 \sin^2 \theta, \\
\zeta \omega(t, r, \theta) \times W(r)^2 \sin^2 \theta, & \zeta q_2(t, r, \theta) \times W(r)^2 \sin^2 \theta, & \zeta q_3(t, r, \theta) \times W(r)^2 \sin^2 \theta, & -W(r)^2 \sin^2 \theta
\end{pmatrix}, \tag{5}\]

where \( \zeta \) is the small expansion parameter defining the perturbation amplitude; \( \omega(t, r, \theta), q_2(t, r, \theta) \) and \( q_3(t, r, \theta) \) are the perturbation functions defining their form.

Let us restrict oneself to the first-order perturbations, i.e. omit all higher than linear on \( \zeta \) terms in the expansion of (5). Additionally we shall suppose the harmonic law for the time-dependence of the perturbations:

\[ \omega(t, r, \theta) = \tilde{\omega}(r, \theta) \exp(-i\sigma t), \tag{6} \]
\[ q_2(t, r, \theta) = \tilde{q}_2(r, \theta) \exp(-i\sigma t), \]
\[ q_3(t, r, \theta) = \tilde{q}_3(r, \theta) \exp(-i\sigma t), \]

where \( \sigma \) is some number (complex in general case).
Second equation of (1), Eqs. (4-6) immediately lead to the gauge-fixed condition:

\[
\begin{align*}
\frac{6\bar{q}_3 \cos(\theta) (r^2 \epsilon + 2\Delta)}{\sin(\theta)} + \bar{q}_2 \left[ 2\Delta \left( 3r \epsilon + \frac{4\Delta}{r} \right) + \frac{d\Delta}{dr} \left( r^2 \epsilon + 4\Delta \right) \right] + (7) \\
2 \left( r^2 \epsilon + 2\Delta \right) \left[ \Delta \frac{\partial \bar{q}_2}{\partial r} + \frac{\partial \bar{q}_3}{\partial \theta} \right] + 4ir^4\bar{\omega}\sigma = 0.
\end{align*}
\]

It is clear that for real perturbation functions and not pure imaginary \(\sigma\), Eq. (7) requires \(\bar{\omega} = 0\) (see Appendix).

Governing equations for the axial perturbations can be obtained from examining the [2, 4]- and [3, 4]-components of the first Logunov’s equation (1). It is convenient to use the new function:

\[
Q (r, \theta) = \left[ \frac{\partial \bar{q}_3}{\partial r} - \frac{\partial \bar{q}_2}{\partial \theta} \right] \Delta \sin^3 \theta.
\]

Then examining the corresponding components of the Logunov’s equations results in:

\[
\frac{(2\Delta + \epsilon r^2)^2}{r^2 \Delta \sin^3 \theta} \frac{\partial Q}{\partial \theta} = \bar{q}_2 \left[ 2\epsilon \cos^2 \theta r^2 \Delta \left( \epsilon r + \frac{2\Delta}{r} \right)^2 + 4r^2 \Delta \epsilon^2 \sigma^2 \right] + \\
\bar{q}_2 \epsilon \left[ 8r^3 \left( 1 + 3\Delta - r^3 \right) + 5 - 6r + 2r^4 \sigma^2 \right] - \bar{q}_2 \epsilon^2 \left[ r \left( 8r \Delta^2 + 3 \right) + 2\epsilon r^4 \Delta \right],
\]

\[
\frac{\Delta}{Q r^3 \sin^3 \theta} \left[ \frac{\partial}{\partial r} \left( \frac{Q^2 r^2}{\Delta} \right) + 4Q \frac{\partial Q}{\partial r} \right] = 4\bar{q}_3 \left[ \sin^2 \theta \left( \epsilon^2 r + \frac{2\epsilon \Delta}{r} \right) - r \sigma^2 \right].
\]

Eqs. (9,10) can be reduced to a single equation:

\[
\frac{\partial}{\partial r} \left\{ \Delta \left[ \frac{\epsilon}{4r^3 \sin^3 \theta} \left( \frac{Q^2 r^2}{\Delta} \right) + 4Q \frac{\partial Q}{\partial r} \right] \right\} + \frac{\partial}{\partial \theta} \left\{ \frac{2(2\Delta + \epsilon r^2)^2}{\sin^3 \theta} \frac{\partial Q}{\partial \theta} \right\} = -\frac{Q}{\sin^3 \theta \Delta}.
\]

where

\[
\Psi (r, \theta) = 4\epsilon \cos^2 \theta r^2 \Delta \left( \epsilon r + \frac{2\Delta}{r} \right)^2 - 2\epsilon^2 \left[ r \left( 8r \Delta^2 + 3 \right) + 2\epsilon r^4 \Delta \right] + \\
8r^2 \Delta \sigma^2 + 2\epsilon \left[ 8r^3 \left( 1 + 3\Delta - r^3 \right) + 5 - 6r + 2r^4 \sigma^2 \right],
\]

\[
\Psi (r, \theta) = 4\epsilon \cos^2 \theta r^2 \Delta \left( \epsilon r + \frac{2\Delta}{r} \right)^2 - 2\epsilon^2 \left[ r \left( 8r \Delta^2 + 3 \right) + 2\epsilon r^4 \Delta \right] + \\
8r^2 \Delta \sigma^2 + 2\epsilon \left[ 8r^3 \left( 1 + 3\Delta - r^3 \right) + 5 - 6r + 2r^4 \sigma^2 \right],
\]

5
3 Horizon solution

Eq. (11) is too complicate to be solved directly and we have to made some approximations. Let us linearize this equation on $\epsilon$

\[
- \left( \frac{\partial}{\partial r} \frac{\Delta \partial Q}{\partial r \sigma^2 \sin^3 \theta} + \frac{\partial}{\partial \theta} \frac{\partial Q}{\partial \sigma^2 \sin^3 \theta} \right) + \epsilon \left( \frac{\partial}{\partial r} \frac{\Delta \partial Q}{\partial r^2} - 2 \frac{\partial^2 Q}{\partial r \partial \sigma^2 \sin^3 \theta} - 8 \sin^2 \theta \frac{\partial}{\partial r} \frac{\Delta \partial Q}{\partial \sigma^2 \sin^3 \theta} \right) + \epsilon \frac{r^4}{4 \sigma^2} \left[ \frac{\partial}{\partial r} \frac{\Delta \partial Q}{\partial r^2} - 2 \frac{\partial^2 Q}{\partial r \partial \sigma^2 \sin^3 \theta} - 8 \sin^2 \theta \frac{\partial}{\partial r} \frac{\Delta \partial Q}{\partial \sigma^2 \sin^3 \theta} \right]
\]

where $\Upsilon (r) = 24r^3 \Delta - 8r^3 (r^3 - 1) - 6r + 5 - 2r^4 \sigma^2$.

To separate the variables in Eq. (15) we consider the limit $\Delta \longrightarrow 0$ ("horizon" solution) and take into consideration only leading terms. This leads to:

\[
\frac{1}{\sin^3 \theta} \frac{\partial}{\partial r} \frac{\Delta \partial Q}{\partial r^2} + \frac{1}{\sigma^2 r^4} \frac{\partial}{\partial \theta} \frac{\partial Q}{\partial \sigma^2 \sin^3 \theta} = \epsilon \frac{r^4}{4 \sigma^2} \left[ 1 \frac{\partial}{\partial r} \frac{\Delta \partial Q}{\partial r^2} + \frac{5 - 8r^3 (r^3 - 1) - 6r - 2r^4 \sigma^2}{\sigma^2 r^6 \Delta} \frac{\partial}{\partial \theta} \frac{\partial Q}{\partial \sigma^2 \sin^3 \theta} \right] = \frac{Q}{\sin^3 \theta} \left( \frac{3 \epsilon}{2r^4 \sigma^2} - \frac{1}{\Delta} \right).
\]

To separate the variables in Eq. (16) we consider the limit $\Delta \longrightarrow 0$ ("horizon" solution) and take into consideration only leading terms. This leads to:

\[
r^4 \frac{\partial}{\partial r} \frac{Q \Delta}{\partial r^2 \Delta} + \Upsilon' (r) \frac{\sin^3 \theta}{\partial \sigma^2 \Delta} \frac{\partial}{\partial \theta} \frac{\partial Q}{\partial \sin^3 \theta} = 4r^4 \sigma^2 \frac{Q}{\epsilon \Delta},
\]

where $\Upsilon' (r) = \Upsilon (r) - 24r^3 \Delta - 4 \sigma^2 \Delta / \epsilon$.

Substitution $Q (r, \theta) \equiv \Xi (r) \Theta (\theta)$ gives

\[
\frac{d^2 \Theta}{dt^2} - 3 \frac{\cos \theta}{\sin \theta} \frac{d \Theta}{dt} + n \Theta = 0,
\]
\[
\left[ r^2 (2r - 1) \right] \frac{d\Xi}{dr} + \left[ \frac{4\Delta}{\varepsilon} + \frac{2 + 4r (r + 1) - \frac{4}{r} - \frac{2}{r^2}}{\sigma^2} \right] n - \frac{4r^2 \sigma^2}{\varepsilon} - \frac{r^2 (2r - 1)^2}{\Delta} \Xi = 0, \tag{18}
\]

Eq. (17) is the well-known Gegenbauer’s equation [11] and can be solved through the Legendre’s functions. Then \( n = (l + 2)(l - 1) \), \( l \) is an integer (angular harmonic index). \( l = 1 \) corresponds to zero angular momentum.

Eq. (3) can be integrated immediately that results in
\[
\Xi (r) = C \Delta (r) (2r - 1) \left( \frac{2n(1/\varepsilon + 23/\sigma^2)}{} + \frac{\sigma^2}{\varepsilon} \right) r (-4n(12/\sigma^2 + 1/\varepsilon)) \right) \times \exp \left\{ \frac{\sigma^2 r (r + 1)}{\varepsilon} + \frac{(21\varepsilon + 5\varepsilon + 78\varepsilon r^2)}{3\varepsilon^3 \sigma^2} \right\}, \tag{19}
\]

where \( C \) is the constant of integration. Eq. (19) agrees with the assumption accounting for Eq. (16) because of \( \Delta \rightarrow 0 \) in the considered limit.

From Eq. (19) the radial perturbations have zero horizon asymptotic, i.e. the collapsar “surface” behaves like a perfectly rigid body. Physically this means that the collapsar’s radius, mass and form are not affected by such perturbations, i.e. such perturbations can not be absorbed by the collapsar. It results in the crucial difference between the RTG collapsar and the GR black hole (see below).

For \( \Delta \rightarrow 0 \) leading term of horizon solution is (decay of the perturbation amplitude as a result of \( \Delta \rightarrow 0 \) can be obtained also in the \( \varepsilon/\Delta \)-order, see Appendix) :
\[
\Xi_L (r) = C_L (r - 1), \tag{20}
\]

Eq. (20) has to be in accord with the Schwarzschild horizon solution [12, 13]:
\[
\Xi_S (r) = C_S (r - 1)^{-i\sigma} + c.c. \tag{21}
\]

Equalization of Eq. (20) and Eq. (21) as well as their derivatives results in the equations for \( C_L \) and the radial coordinate of the overlap region \( r_0 \):
\[
C_L = -\frac{\sigma}{r_0 - 1} \left\{ A_1 \sin [\sigma \ln (r_0 - 1)] - A_2 \cos [\sigma \ln (r_0 - 1)] \right\},
\]
\[
r_0 = 1 + \exp \left[ -\sigma^{-1} \arctan \frac{A_1 - A_2 \sigma}{A_2 + A_1 \sigma} \right], \tag{22}
\]

where \( A_1 = 2 \Re (C_S) \), \( A_2 = -2 \Im (C_S) \) and can be expressed through the amplitudes of ingoing and outgoing perturbation waves in the vicinity of the horizon:
\[ a_{\text{out}} = \frac{1}{2} (A_1 - iA_2), \]
\[ a_{\text{in}} = \frac{1}{2} (A_1 + iA_2). \]  

(23)

Reflection coefficient \( R \equiv a_{\text{out}} / a_{\text{in}} \) (\(|R| = 1\), that corresponds to nonabsorbing surface, see above), \( a_{\text{in}} \) is the amplitude of the ingoing wave transmitted through the Regge-Wheeler potential barrier [12]. If we neglect the momentum transmitted by the ingoing wave to the collapsar, it is possible to choose \( A_1 = 0 \) or \( A_2 = 0 \) (i.e. the phase shift due to reflection is equal to \( \pi \)).

If \( A_1 = 0 \), then

\[ C_L = \frac{\sigma}{r_0 - 1} A_2 \cos [\sigma \ln (r_0 - 1)], \]
\[ r_0 = 1 + \exp \left[ -\sigma^{-1} \arctan (-\sigma) \right]. \]  

(24)

If \( A_2 = 0 \), then

\[ C_L = \frac{-\sigma}{r_0 - 1} A_1 \sin [\sigma \ln (r_0 - 1)], \]
\[ r_0 = 1 + \exp \left[ -\sigma^{-1} \arctan (\sigma^{-1}) \right]. \]  

(25)

It is clear that only Eq. (25) provides the overlap of the Logunov’s and the Schwarzschild’s regions in the low-frequency limit.

4 Perturbation modes

As it was mentioned, our analysis is based on the following main assumptions: 1) \( \sigma \) is not pure imaginary (see Appendix); 2) \( \Delta \to 0 \) and \( \Delta^2 = O (\epsilon^2) \), \( \epsilon \Delta = O (\epsilon^2) \). Only in this case the angular and the radial variables are separable in the vicinity of horizon. Such separability allows finding the horizon asymptotic of the axial perturbations, when \( \epsilon \neq 0 \). Such asymptotic differs radically from that in the Schwarzschild case: collapsar horizon is nonabsorbing.

As a result, there exist following main types of the perturbation modes: 1) unbounded modes: (i) propagating through the Regge-Wheeler potential barrier, escaping the vicinity of the collapsar after reflection from its surface and the subsequent backward propagation through the barrier; (ii) reflecting from the Regge-Wheeler potential barrier; 2) bounded modes, which are resonantly trapped between the collapsar surface and the Regge-Wheeler potential barrier.

The latter modes are most astonishing as the vicinity of collapsar behaves like resonator accumulating the gravitational perturbations with the resonant frequencies. As a consequence, the band-gaps in the frequencies of the reflected modes have to exist.

The trapped modes can be calculated on the simple basis. In the correspondence with the previous analysis the trapped mode has the zero amplitude on
Table 1: Frequencies of the trapped perturbations with different angular momentum, $\chi = 10^{-44}$.

| $L = 0$                  | $L = 1$                  | $L = 2$                  |
|--------------------------|--------------------------|--------------------------|
| 0.03504283094            | 0.07765951326            | 0.08629600222            |
| 0.1499033022             | 0.1588426895             |                          |
| 0.2218760915             | 0.2321809639             |                          |
| 0.2933223483             | 0.3052900260             |                          |
| 0.3642142227             | 0.3780251314             |                          |
| 0.434882047              | 0.450365174              |                          |
|                          | 0.5223159963             |                          |
|                          | 0.5938754078             |                          |

the horizon and the zero asymptotic far off the Regge-Wheeler potential. In the tortoise radial coordinate we have the usual stationary Schrödinger equation for the modes trapped between collapsar and the Regge-Wheeler potential formed by the curvature of the effective Riemannian spacetime [12]. $\sigma$ corresponding to the trapped modes are given in Table 1 (angular momentum is defined through $L = l - 1$).

5 Conclusion

Approximated horizon solutions for the axial perturbations of the spherically symmetric metric have been obtained in the RTG framework. It was shown, that there is no absorption of the perturbations by the horizon. The reflection of the gravitational waves forms the modes trapped between the horizon and the Regge-Wheeler potential barrier. Hence the collapsar can resemble rather the gravitational “resonator”or “mirror”than the “hole”.

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This work has been realized in the Maple 9 computer algebra system [14].

Appendix

Let’s be based on Eqs. (9,10), linearize theirs on $\epsilon$ and suppose that $\Delta = O(\epsilon)$ (or smaller). We keep the terms of $O(\epsilon/\Delta)$ then the resulting equation for the function $Q$ is:

$$
\frac{\partial}{\partial r} \left\{ \frac{\Delta}{r^4} \left[ \frac{\partial Q}{\partial r} \frac{\epsilon}{Q} \frac{\partial Q}{\partial r} \frac{Q^2 r^2}{\Delta} \right] \right\} + \frac{\Sigma \sigma^2 \sin^3 \theta}{r^4} \left( \frac{\partial^2 Q}{\partial \theta \partial r} \right) + \frac{Q \sigma^2}{\Delta} =

i \sin^3 \theta \frac{\partial^2 \omega}{\partial r \partial \theta} \left( 1 - \frac{\Sigma}{1 + \frac{\epsilon^2}{\Delta}} \right),
$$

(26)
where

$$\Sigma = \frac{1 + \frac{\varepsilon^2}{2} \frac{\Delta}{\Delta}}{\sigma^2 + \varepsilon \left(-8r^6(r^3-1)+5-6\right)}.$$ 

If \( \tilde{\omega} = 0 \) and \( \epsilon/\Delta \) is the term of order \( O(1) \) or higher then Eq. (26) results in:

$$
\left(1 + \frac{\varepsilon}{\Delta}\right) \frac{\partial Q}{\partial r} + \frac{1 + \frac{\varepsilon}{\Delta}}{\sigma^2 - \frac{\varepsilon}{\Delta^2}} \sigma^2 \sin^3 \theta \frac{\partial}{\partial \theta} \sin^3 \theta \left(\frac{\sigma^2}{\Delta} - \frac{\varepsilon}{\Delta^2}\right) Q = 0. \quad (27)
$$

After separation of the variables and keeping the leading terms we have for the radial function:

$$
\frac{d\Xi}{dr} + \Xi \left[\frac{\sigma^2}{\Delta} - n - \frac{\varepsilon}{\Delta} \left(1 + \frac{\sigma^2}{\Delta} + \frac{n}{2} \left(\frac{1}{2\sigma^2} - 1\right)\right)\right] = 0. \quad (28)
$$

Integrating this equation, neglecting \( \epsilon \) and keeping \( \epsilon/\Delta \) gives:

$$
\Xi = C_L (r - 1)^{-\sigma^2} r^\sigma^2 \exp \left[nr - \frac{\varepsilon}{\Delta} \left(1 + \sigma^2\right)\right], \quad (29)
$$

that demonstrates the strong near-horizon suppression of the perturbation amplitude. In the vicinity of some point, which is displaced from the horizon on \( \alpha = o(\epsilon) \), we have the leading term:

$$
\Xi = C_L (r - (1 + \alpha)). \quad (30)
$$

More rigorous analysis of the axial perturbations needs a numerical approach.

Stability of the metric under consideration needs some additional comments. Eqs. (19) and (29) admit the purely imaginary \( \sigma \). However in the Schwarzschild’s case, the time-growing ingoing near-horizon solution \((r - 1)^{-i\sigma} = (r - 1)^\gamma \to 0 \) for \( \gamma > 0 \) providing \( \exp(\gamma t) \to \infty \) can not be matched to the time-damped outgoing at infinity solution [15]. The RTG near-horizon solution has to be matched to the GR near-horizon solution. As the last doesn’t exist for the case of the ingoing perturbation with \( \sigma = i\gamma \) and \( \gamma > 0 \), the metric under consideration is stable against such perturbation. However, in contrast to the GR, there exist the near-horizon outgoing perturbations, which, in principle, allow matching with the Schwarzschild’s asymptotic at infinity even for \( \sigma = i\gamma \) (\( \gamma > 0 \)). When \( \tilde{\omega} = 0 \), such perturbations don’t permit the matching between the GR and the RTG solutions as long as \( \gamma \leq 1 \) (see Eq. (29)). Case of \( \gamma > 1 \) doesn’t provide the finite asymptotic on horizon. Thus, the stability of the metric against outgoing perturbation can be accepted as proved for \( \tilde{\omega} = 0 \).

The case of \( \tilde{\omega} \neq 0 \) needs an additional consideration, which is difficult in the RTG because it is not possible to reduce the problem to an analysis of the single perturbation function (see Eq. (26)).
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