Cosmology on a Three-Brane

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Abstract

In this paper a general solution is found for a five dimensional orbifold spacetime that induces a $k = 0$ cosmology on a three-brane. Expressions for the energy density and pressure on the brane in terms of the brane metric are derived. Given a metric on the brane it is possible to find five dimensional spacetimes that contain the brane. This calculation is carried out for an inflationary universe and for a metric that corresponds to a radiation dominated universe in standard cosmology. It is also shown that any $k = 0$ cosmology can be embedded in a flat five dimensional orbifold spacetime and the equation of the three-brane surface is derived. For an inflationary universe it is shown that the surface is the usual hyperboloid representation of de Sitter space, although it is embedded in an orbifold spacetime.
Introduction

Over the last few years there has been a great deal of interest in the possibility that we live on a three dimensional brane embedded in a higher dimensional space. Horava and Witten [1] showed that in the strongly coupled limit $E_8 \times E_8$ heterotic string theory can be viewed as an 11 dimensional theory on the orbifold $R^{10} \times S^1/Z_2$ with gravitons propagating in the bulk and super Yang-Mills fields confined to the two ten-branes that form the boundary of the spacetime. Recently a novel solution to the hierarchy problem was proposed [2, 3, 4] by considering the possibility that $n$ of the compactified dimensions are ‘large’. It was shown that the effective 4 dimensional Planck mass $M_{pl}$ is related to the 5 dimensional Planck mass $M$ via $M_{pl}^2 \sim M^{2+n}V(n)$, where $V(n)$ is the volume of the compactified space. Thus, if the extra dimensions are large enough it is possible to have a small $M$ (even on the order of a TeV) with $M_{pl} \sim 10^{19}$ Gev. Therefore, the hierarchy problem can be solved by reducing the 5 dimensional Planck mass. For $M \sim 1$ Tev they found that $n > 2$ to avoid conflicts with observations if the standard model fields are confined to a three brane while gravity propagates in the bulk. For $n = 2$ astrophysical constraints force $M \gtrsim 100$ Tev [5, 6] and the size of the dimensions is constrained to be $\lesssim 5 \times 10^{-5}$ mm.

Two other interesting possibilities were recently suggested by Randall and Sundrum [7, 8]. In their first model they considered a five dimensional spacetime with the fifth dimension, labeled by $w$, compactified on $S^1$ with $-w_c \leq w \leq w_c$ and with the orbifold symmetry $w \sim -w$. The brane at $w = 0$ is a domain wall with positive tension and the brane at $w = w_c$ is a domain wall with negative tension. They showed that mass scales on the negative tension brane can be severely suppressed, leading to a solution of the hierarchy problem. Of course, this assumes that we live on the negative tension brane. It was shown by Shiromizu, Maeda, and Sasaki [9] that the effective Einstein field equations on the negative tension brane involve a negative gravitational constant which means that gravity would be repulsive instead of attractive. However, they showed that one does recover the correct Einstein equations in the low energy limit on the positive tension brane. More recently, it has been shown [10] that the problem with the negative tension brane may disappear if the extra dimension is stabilized by a radion field. In the second scenario of Randall and Sundrum we live on the positive tension brane and the negative tension brane is moved off to infinity. Thus, in this scenario the extra dimension is infinite in extent. As usual the fields of the standard model live on the brane and gravity lives in the bulk. They showed that there is a single gravitational bound state confined to the brane that corresponds to the graviton. They also showed that, even though the extra dimension is infinite, the effective gravitational interaction on the brane is that of a four dimensional spacetime with some very small corrections.

Some aspects of the cosmology of branes embedded in higher dimensional spacetimes have been examined. Binétruy, Deffayet, and Langlois (BDL) [11] showed that the scale
factor $a_0$ on a brane with flat spatial sections satisfies

$$\frac{\ddot{a}_0}{a_0} + \frac{\dot{a}_0^2}{a_0^2} = -\frac{k_{(5)}^4}{36}\rho(\rho + 3p)$$

(1)

where $k_{(5)}$ is the five dimensional Einstein constant. I have dropped a term in this equation proportional to $T_{55}$ by taking the bulk to be empty. As they pointed out, this equation is similar to one of the equations from standard cosmology except that the source terms are nonlinear. This leads to nonstandard cosmological evolution. Csáki, Graesser, Kolda, and Terning [12] then showed that one can get standard evolution at late times if the brane has a cosmological constant $\Lambda_b$. Note that $\Lambda_b$ is different than the bulk cosmological constant, as it is confined to the brane. If $\Lambda_b >> \rho, p$ the right hand side of (1) gives the usual source terms plus nonlinear corrections that will become small at late times. One remaining difference between this scenario and the standard one is that equation (1) is second order. In the standard scenario the evolution is governed by a first order equation and the initial condition $a(t = 0) = a_0$ (if there is an initial singularity $a_0 = 0$) is sufficient to determine the solution for a given equation of state in a $k = 0$ universe. On the other hand, the $k = 0$ cosmology on a brane depends on $\dot{a}(t = 0)$ as well. Specific five dimensional and three-brane cosmologies have also been discussed by various authors [13, 14, 15, 16, 17, 18, 19].

In this paper a general solution is found for a five dimensional orbifold spacetime that gives a $k = 0$ cosmology on a three-brane. The energy density and pressure on the brane are written in terms of the metric on the brane. It is shown that it is possible to find five dimensional spacetimes that contain a brane with a given metric. This procedure is carried out for an inflationary universe and for a metric that correspond to a radiation dominated universe in standard cosmology. It is also shown that any $k = 0$ cosmology can be embedded in a five dimensional flat orbifold spacetime and the embedding equation is found. For an inflationary universe it is shown that the surface is the usual hyperboloid representation of de Sitter space, although it is embedded in an orbifold spacetime.

**Cosmology in the Five Dimensional Space-time**

Here we consider a five dimensional spacetime with $w$ labeling the extra dimension. The fifth dimension is taken to be compactified on $S^1$ with $-w_c \leq w \leq w_c$. An orbifold $S^1/Z_2$ is produced by identifying $w$ and $-w$, so that the range of $w$ can be taken to be $0 \leq w \leq w_c$. The spacetime contains a three-brane at $w = 0$ and at $w = w_c$. These branes will, in general, have non-zero surface energies and pressures. We will take the brane at $w = 0$ to be the brane we live on and the brane at $w = w_c$ will be a hidden brane.

To simplify the form of the metric we can use a Gaussian normal coordinate system based on the brane at $w = 0$. In this coordinate system $g_{55} = 1$ and $g_{5\mu} = 0$ ($\mu =$
0, 1, 2, 3). Assuming that the three dimensional spatial sections are flat gives
\[ ds^2 = -b^2(t, w)dt^2 + a^2(t, w)[dx^2 + dy^2 + dz^2] + dw^2. \]  
(2)
Since it is always possible to write a two dimensional metric in conformally flat form we can take the metric to be
\[ ds^2 = c(t, w)[dw^2 - dt^2] + a^2(t, w)[dx^2 + dy^2 + dz^2]. \]  
(3)
Transforming to retarded and advanced coordinates \( u = t - w \) and \( v = t + w \) gives
\[ ds^2 = -c(u, v)dudv + a^2(u, v)[dx^2 + dy^2 + dz^2] \]  
(4)
The Einstein field equations \( R_{kl} = \lambda g_{kl} \) in the bulk give
\[ c(\partial_u^2 a) - (\partial_u c)(\partial_u a) = 0, \]  
(5)
\[ a(\partial_u \partial_v a) + 2(\partial_u a)(\partial_v a) = -\frac{1}{4}\lambda c a^2, \]  
(6)
\[ c(\partial_v^2 a) - (\partial_v c)(\partial_v a) = 0, \]  
(7)
and,
\[ 3c^2(\partial_u \partial_v a) + ac(\partial_u \partial_v c) - a(\partial_u c)(\partial_v c) = -\frac{1}{2}\lambda c a^3, \]  
(8)
where \( \lambda = -2/3\Lambda(5) \) and \( \Lambda(5) \) is the five dimensional cosmological constant. First consider the case \( \lambda = 0 \). The above equations are trivial to integrate. The general solution for \( \partial_a a, \partial_v a \neq 0 \) is
\[ a(u, v) = [f(u) + g(v)]^{1/3}, \]  
(9)
and
\[ c(u, v) = c_1 f'(u)g'(v)[f(u) + g(v)]^{-2/3}; \]  
(10)
where \( f(u) \) and \( g(v) \) are arbitrary functions of \( u \) and \( v \) respectively and \( c_1 \) is an arbitrary constant. If \( \partial_u a = \partial_v a = 0 \) then
\[ a = a_1 \quad c(u, v) = f(u)g(v). \]  
(11)
where \( a_1 \) is a constant. Since all of these solutions correspond to Minkowski spacetime on the brane and in the bulk they will not be discussed further. If \( \partial_v a = 0 \) and \( \partial_u a \neq 0 \) then
\[ a(u) = h(u) \quad c(u, v) = c_2 h'(u)k(v). \]  
(12)
A similar result holds if \( \partial_v a = 0 \) and \( \partial_u a \neq 0 \).
Transforming (9) and (10) back to \( (t, w) \) coordinates and imposing the orbifold condition \( w \sim -w \) gives
\[ ds^2 = |\dot{f}(u)\dot{g}(v)|[f(u) + g(v)]^{-2/3}(dw^2 - dt^2) + [f(u) + g(v)]^{2/3}(dx^2 + dy^2 + dz^2) \]  
(13)
where \( u = t - |w| \) and \( v = t + |w| \) and I have absorbed some constants into the definition of \( t \) and \( w \). Since the sign of \( c_1 \) cannot be absorbed the coefficients \( g_{tt} \) and \( g_{ww} \) should have a ± sign. I have forced the correct spacetime signature by using absolute values instead and by restricting \( f \) and \( g \) to be positive. Note that \( f \) and \( g \) interchange roles as we cross the brane at \( w = 0 \). Transforming \((12)\) back to \((t, w)\) coordinates gives
\[
ds^2 = |\dot{h}(u)|k(v)(dw^2 - dt^2) + h^2(u)(dx^2 + dy^2 + dz^2)
\]
(14)
where \( k \) has been taken to be positive. This metric corresponds to a flat five dimensional spacetime since the Riemann tensor vanishes. However, the geometry induced on the brane will not, in general, be flat. For notational simplicity the metric in both cases will be written as
\[
ds^2 = n^2(t, |w|)[dw^2 - dt^2] + a^2(t, |w|)[dx^2 + dy^2 + dz^2]
\]
(15)
where \( n \) and \( a \) are given in \((13)\) or in \((14)\).

Now consider \( \lambda \neq 0 \). Here \( \lambda \) will be taken to be positive, as in the Randall-Sundrum model. Eliminating \( c(u, v) \) from \((5)\) and \((7)\) and integrating gives
\[
a(u, v) = H[f(u) + g(v)]
\]
(16)
where \( H, f(u), \) and \( g(v) \) are arbitrary functions. Solving for \( c(u, v) \) from \((5)\) and \((7)\) gives
\[
c(u, v) = \alpha f'(u)g'(v)H'[f(u) + g(v)]
\]
(17)
where \( \alpha \) is an arbitrary constant. Substituting \( a(u, v) \) and \( c(u, v) \) into \((8)\) gives
\[
HH'' + 2(H')^2 = -\frac{1}{4}\lambda\alpha H^2 H'.
\]
(18)
A first integral of this equation is
\[
H^2 H' + \frac{1}{16}\lambda\alpha H^4 = c_1
\]
(19)
where \( c_1 \) is an arbitrary constant. The solution is
\[
\tan^{-1}[\beta H] - \frac{1}{2}\ln \left[ \frac{\beta H + 1}{\beta H - 1} \right] = c_2 - 2(f(u) + g(v))
\]
(20)
where \( \beta = (\frac{\alpha\lambda}{16c_1})^{1/4} \) and \( c_2 \) is an arbitrary constant. Unfortunately \( H \) cannot be found explicitly from the above solution. To simplify the problem consider the case \( c_1 = 0 \). The solution then simplifies to
\[
H = \left[ c_2 + \frac{\alpha\lambda}{16}(f(u) + g(v)) \right]^{-1}.
\]
(21)
Absorbing \( \alpha\lambda/16 \) and \( c_2 \) into the functions \( f(u) \) and \( g(v) \) gives
\[
ds^2 = \frac{16}{\lambda} \frac{f'(u)g'(v)}{[f(u) + g(v)]^2}[dt^2 - dw^2] + \frac{1}{[f(u) + g(v)]^2}[dx^2 + dy^2 + dz^2]
\]
(22)
where \( u = t - |w| \) and \( v = t + |w| \) as before.
Cosmology on the Brane I: \( \lambda = 0 \)

The four dimensional spacetime on our brane will be described by the metric

\[
ds^2 = -|f\dot{g}|[f + g]^{-2/3}dt^2 + [f + g]^{2/3}(dx^2 + dy^2 + dz^2)
\]

or by

\[
ds^2 = -|h|kdt^2 + h^2(dx^2 + dy^2 + dz^2)
\]

where \( f = f(t) \), \( g = g(t) \), \( h = h(t) \), and \( k = k(t) \). The energy-momentum tensor on the brane will have the form

\[
T^m_n = \frac{\delta(w)}{n_0} \text{diag}(-\rho, p, p, p, 0)
\]

where \( n_0 = n(t, 0) \). It can easily be calculated using the jump conditions that follow from the Einstein field equations [20]. BDL [11] have found that

\[
[\partial_w a] = -\frac{k_5^2}{3} \rho
\]

and

\[
[\partial_w n] = \frac{k_5^2}{3} (3p + 2\rho)
\]

where \( k_5 \) is the five dimensional Einstein constant, and \([\ ]\) indicates the jump in a quantity, i.e.

\[
[\partial_w a] = \partial_w a(0^+) - \partial_w a(0^-).
\]

For the metric given in (13) the energy density and pressure on our brane are given by

\[
\rho = \frac{2[f + g]^{-2/3}[f - \dot{g}]}{k_5^2 |f\dot{g}|},
\]

and

\[
p = \frac{[f + g]^{1/3}}{k_5^2 |f\dot{g}|} \left[ \frac{\dot{g}}{\dot{f}} - \frac{\ddot{f}}{f} \right] - \frac{1}{3} \rho.
\]

For the metric given in (14) we find that

\[
\rho = \frac{6}{k_5^3 |k\dot{h}|} \text{sign}(\dot{h})
\]

and

\[
p = \frac{1}{k_5^2} [k|\dot{h}|]^{-1/2} \left[ \frac{k}{\dot{h}} \dot{\dot{h}} - \frac{k}{\dot{h}} \right] - \frac{1}{3} \rho.
\]
Thus given \( f \) and \( g \) or \( h \) and \( k \) we can find the metric and the energy-momentum tensor on the brane. For example if \( f(t) = g(t) \) we find that

\[
\text{ds}^2 = -d\tau^2 + \tau(dx^2 + dy^2 + dz^2)
\]

(33)

where \( d\tau = |\dot{f}|(2f)^{-1/3}dt \). This is the spacetime for a radiation dominated universe in the standard scenario. However, as can be seen from (29) and (30) the pressure and energy density on the brane vanish! Note that this is consistent with equation (1) found by BDL. It is also important to note that Minkowski space is also a solution for the spacetime on the brane if \( \rho = p = 0 \). Surprisingly, flat spacetime also arises with \( \rho = -3p = (4/k^2)\text{sign}(\dot{h}) \) if \( f + g = \text{constant} \). It is important to note that the energy densities and pressures above arise in the five dimensional Einstein equations.

If an observer on the brane defined \( T^{(4)}_{\mu\nu} = k^{-2}G^{(4)}_{\mu\nu} \) the energy densities and pressures obtained would differ from the above expressions. Once we have \( f(t) \) and \( g(t) \), which specify the cosmology on the brane, we can extend the solution into the bulk by using (23) or (24). For example if \( f + g = \text{constant} \), which gives flat spacetime on the brane, then

\[
\text{ds}^2 = \dot{f}(t - |w|)^2(2dw^2 - dt^2) + dx^2 + dy^2 + dz^2.
\]

(34)

This is flat five dimensional spacetime. If \( f(t) = g(t) = t \) on the brane the bulk metric is given by

\[
\text{ds}^2 = -d\tau^2 + \tau(dx^2 + dy^2 + dz^2) + \frac{1}{\tau}dw^2,
\]

(35)

which corresponds to a five dimensional anisotropic Kasner cosmology. Of course, for other choices of \( f \) and \( g \) the bulk spacetime metric will depend on \( w \).

For the remainder of this section I will work with metrics of the form (14). As discussed earlier all metrics of this form correspond to a flat five dimensional spacetime. To see this let \( \bar{u} = h(u) \) and \( \bar{v} = k_1(v) = \int k(v)dv \) (for simplicity I have taken \( \dot{h} > 0 \)). In these coordinates the metric is given by

\[
\text{ds}^2 = -d\bar{u}d\bar{v} + \bar{u}^2(dx^2 + dy^2 + dz^2).
\]

(36)

The coordinate transformation that takes this to the flat spacetime metric

\[
\text{ds}^2 = -d\bar{u}d\bar{v} + dx^2 + dy^2 + dz^2
\]

(37)

is

\[
\bar{u} = \bar{u}
\]

\[
\bar{v} = \bar{v} + \bar{u}x^2
\]

(38)

\[
\bar{x}^k = \bar{u}x^k
\]
where \( \tilde{x}^2 = x^k x_k \). This transformation can easily be found by using the null geodesics as coordinate lines. Thus, the transformation from \((t, w, x^k)\) to \((\tilde{t}, \tilde{w}, \tilde{x}^k)\) is given by

\[
\tilde{t} = \frac{1}{2\lambda}[k_1(t + w) + (x^2 + \lambda^2)h(t - w)],
\]

\[
\tilde{w} = \frac{1}{2\lambda}[k_1(t + w) + (x^2 - \lambda^2)h(t - w)],
\]

\[
\tilde{x}^k = h(t - w)x^k,
\]

where \( \tilde{u} = \frac{1}{\lambda}(\tilde{t} - \tilde{w}) \), \( \tilde{v} = \lambda(\tilde{t} + \tilde{w}) \), and \( \lambda \) is a constant with dimensions of length. The orbifold in five dimensions is produced by identifying the points \( \tilde{x}^\mu(t, x^k, w) \) and \( \tilde{x}^\mu(t, x^k, -w) \), where \( \mu \) labels the five dimensional spacetime. The surface of the brane can therefore be parameterized by

\[
\tilde{t} = \frac{1}{2\lambda}[k_1(t) + (x^2 + \lambda^2)h(t)],
\]

\[
\tilde{w} = \frac{1}{2\lambda}[k_1(t) + (x^2 - \lambda^2)h(t)],
\]

\[
\tilde{x}^k = h(t)x^k,
\]

where \((t, x^k)\) are the coordinates on the surface. This surface can also be described by the equation

\[
\tilde{t}^2 - \tilde{w}^2 - \tilde{x}^2 - \tilde{y}^2 - \tilde{z}^2 = \left(\tilde{t} - \tilde{w}\right) \frac{k_1}{\lambda} \left[ h^{-1} \left(\frac{\tilde{t} - \tilde{w}}{\lambda}\right) \right].
\]

Now given any metric of the form

\[
ds_{(4)}^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)
\]

it is easy to see that \( h(t) = a(t) \) and \( k_1(t) = \int \frac{dt}{a(t)} \). Thus, any \( k = 0 \) cosmology can be embedded in a flat five dimensional orbifold spacetime. For example consider the inflationary cosmology with \( a(t) = e^{Ht} \). The equation of the surface is

\[
\tilde{t}^2 - \tilde{w}^2 - \tilde{x}^2 - \tilde{y}^2 - \tilde{z}^2 = -\frac{1}{H^2}.
\]

This is the usual hyperboloid representation of de Sitter space except that it is embedded in an orbifold spacetime.

Finally, we will show that it is possible to find a three brane metric and the equation of its surface given the energy density on the brane. Instead of working with the metric in the form (24) it is convenient to work in a coordinate system in which the metric takes the form

\[
ds_{(4)}^2 = -d\tau^2 + a^2(\tau)(dx^2 + dy^2 + dz^2)
\]
where $d\tau = \sqrt{k|h|} dt$. Equations (31) and (32) become

$$\rho = \frac{6}{k_{(5)}^2} \frac{da}{d\tau},$$

(45)

and

$$p = -\frac{2}{k_{(5)}^2} \left( \frac{da}{d\tau} \right)^{-1} \frac{d^2a}{d\tau^2} - \frac{2}{3} \rho.$$

(46)

Note that the above are independent of $k$ so that $k$ will remain arbitrary. Inverting (45) gives

$$a(\tau) = h_0 \exp \left[ \frac{k_{(5)}^2}{6} \int \rho(\tau) d\tau \right].$$

(47)

Note that $p$ is fixed once $h$ is known. For example if $\rho = \rho_0 = \text{constant}$ then $a(\tau) = \exp(H\tau)$ and $p = -\rho$ where $H = k_{(5)}^2 \rho_0 / 6$. The equation of the surface is given in (43).

**Cosmology on the Brane II: $\lambda > 0$**

From (22) the induced metric on the brane is

$$ds^2_{(4)} = \frac{16}{\lambda} \frac{\dot{f} \dot{g}}{(f + g)^2} dt^2 + \frac{1}{(f + g)^2} (dx^2 + dy^2 + dz^2)$$

(48)

and the energy density and pressure on the brane for $f + g > 0$ are given by

$$\rho = \frac{3\sqrt{\lambda}}{2k_{(5)}^2} \left[ \dot{g} - \frac{f}{\dot{g}} \right]$$

(49)

and

$$P = \frac{\sqrt{\lambda}}{2k_{(5)}^2 \sqrt{-\dot{f} \dot{g}}} \left[ 3(\dot{f} - \dot{g}) + \frac{1}{2} (f + g) \left( \frac{\ddot{g}}{\dot{g}} - \frac{\ddot{f}}{f} \right) \right].$$

(50)

Note that $\dot{f} \dot{g} < 0$ for the metric on the brane to have the correct signature and for the energy density and pressure to be real.

As an example consider the spacetime with $f(t) = 1/t$, $g(t) = t$, and $t > 0$. The metric on the brane is

$$ds^2_{(4)} = -d\tau^2 + \frac{1}{4} \sin^2 \left( \frac{\sqrt{\lambda} \tau}{2} \right) [dx^2 + dy^2 + dz^2]$$

(51)

where $\tau = \frac{1}{\sqrt{\lambda}} \tan^{-1}(t)$. This describes a cosmology in which the universe initially expands and then collapses. The energy density and pressure on the brane are given by

$$\rho = \frac{3\sqrt{\lambda}}{2k_{(5)}^2} \left[ \sin \left( \frac{\sqrt{\lambda} \tau}{2} \right) \right]^{-1}$$

(52)
and

\[ P = -\frac{2}{3} \rho. \quad (53) \]

It is important to note that the energy density and pressure found above arise in the five dimensional Einstein equations. If an observer on the brane defined \( T^{(4)}_{\mu\nu} = k^{−2} G^{(4)}_{\mu\nu} \) different results would be obtained.

**Conclusion**

A general solution was found for a five dimensional orbifold spacetime that induces a \( k = 0 \) cosmology on a three-brane. Expressions for the energy density and pressure on the brane where found in terms of the metric on the brane. It was shown that it is possible to find five dimensional spacetimes that contain the brane given the brane metric. This procedure was carried out for scale factors \( a(\tau) = e^{H\tau} \) and \( a(\tau) = \frac{\tau}{2} \). It was also shown that any \( k = 0 \) cosmology can be embedded in a flat five dimensional orbifold spacetime and the embedding equation was found. For an inflationary universe it was shown that the surface is the usual hyperboloid representation of de Sitter space, although it is embedded in an orbifold spacetime.

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