The kinetic theory to problem of attenuation transversal sound waves in metal

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Abstract

Earlier electron influence on sound absorption in metal on basis of the assumption of deformation of Fermi–surfaces under the influence of a sound wave was considered. In the present work other approach to this problem will be considered. Our approach based on the account of dynamic (kinetic) interaction of electronic gas with lattice fluctuations. The analysis of influence of electric field on process of attenuation of sound fluctuations is carried out. It is shown that in the long-wave limit this influence is essential.

Key words: collision degenerate plasmas, Vlasov—Boltzmann equation, attenuation coefficient.

PACS numbers: 52.25.Dg Plasma kinetic equations, 52.25.-b Plasma properties, 05.30 Fk Fermion systems and electron gas.

Introduction

Electron influence on sound absorption in metal was considered on basis of the assumption of deformation of Fermi–surfaces under the influence of a sound wave \[1\]. However process of change of Fermi–surfaces is caused by interaction of electronic gas with lattice and inevitably depends on characteristics of this interaction. This process cannot be strictly speaking to be considered in static approach. Dynamical and kinetical processes should be considered at the analysis of formation of Fermi–surfaces at propagation of a sound wave in metal. In the present work the approach to this problem will be considered, based on the account of dynamic (kinetic) interaction of electronic gas with lattice fluctuations.

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Let us consider propagation of the transversal sound in the isotropic metal. As well as in the previous works, we will assume, that interaction of a sound wave with electron conductivity not renders appreciable influence on speed of a sound wave. Our purpose there will be a question consideration how electrons conductivity of metal influence on process of attenuation of the sound wave.

The problem of attenuation of a sound wave in metal was considered in works 

1. Coefficient of attenuation of a sound wave

The transversal sound wave creates a field of velocities in metal

\[ u = u_0 e^{i(\omega t - kr)} , \quad ku = 0 , \quad \omega = s_{tr} k . \]

Here \( s_{tr} \) is the velocity of transversal sound oscillations, \( k \) is the wave vector.

Let us choose an axis \( x \) along a direction of distribution of a sound wave \( k = k(1, 0, 0) \), and an axis \( y \) along a direction of velocityd \( u \). Then \( u = u_y (0, 1, 0) \), where

\[ u_y = u_y (x, t) = u_0 e^{i(kx - \omega t)} . \]

The kinetic Vlasov–Boltzmann equation with integral of collisions of relaxation type for electrons will have the following form

\[ \frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} + e \mathbf{E} \frac{\partial f}{\partial \mathbf{p}} = \nu (f_{eq} - f) , \tag{1.1} \]

where \( f_{eq} \) is the equilibrium Fermi–distribution in degenerate plasmas.

Equilibrium distribution corresponds to a condition of electronic gas which has zero velocity comparatively of lattice. We will consider a case of isotropic metal with undisturbed spherical Fermi–surface. The transversal sound wave leads to distortion of lattice, that in turn can lead to distortion equilibrium Fermi–surface of metal. In the given work we will not to be consider this effect. Thus, in the considered case we have

\[ f_{eq} (\mathbf{v}) = \Theta (\mathcal{E}_F - \mathcal{E} (\mathbf{v} - \mathbf{u})) . \]
Here $\mathcal{E}(v - u)$ is the electron energy,

$$\mathcal{E}(v - u) = \frac{m(v - u)^2}{2},$$

$\Theta(x)$ is the unit step of Heaviside,

$$\Theta(x) = 1, \quad x > 0; \quad \Theta(x) = 0, \quad x < 0,$$

quantities $e$ and $m$ are charge and mass of electron correspondingly.

At such statement of this problem we consider, that electrons at scattering to pass in equilibrium condition with the lattice which possesses the local speed $u = u(x, t)$.

At such approach to this problem there is no necessity to enter "fictitious forces" [5], responsible for interaction electronic gas with lattice fluctuations.

At considered statement of the problem electric field $\mathbf{E}$ has only one nonzero to a component $E_y = E_y(0, 1, 0)$, where

$$E_y = E_y(x, t) = E_0 e^{i(kx - \omega t)}.$$

Because of an electroneutrality the volume density of a charge of ions (lattice) equals $-enu$, where $n = \text{const}$ is the electron concentration, the current density is equal $-en$. Then the equation on the field looks like

$$\frac{\partial^2 E_y(x, t)}{\partial x^2} = -\frac{4\pi i\omega}{c^2} \left[ j_y(x, t) - enu_y(x, t) \right]. \tag{1.2}$$

Here $j_y(x, t)$ is the electron current density along axis $y$,

$$j_y = e \int v_y f \frac{2d^3p}{(2\pi\hbar)^3}.$$

In linear approach the distribution electron function in the field sound wave we will search in the form

$$f = f_0 - \frac{\partial f_0}{\partial \mathcal{E}} \psi, \quad \mathcal{E} = \mathcal{E}(v) = \frac{mv^2}{2}. \tag{1.3}$$

Here $f_0$ is the absolute Fermian,

$$f_0 = f_0(\mathcal{E}) = \Theta(\mathcal{E}_F - \mathcal{E}(v)).$$
The energy flux (stream), a transferable by sound wave is equal

\[ I = \frac{\rho u_0^2 s_{tr}}{2}, \]

where \( \rho \) is the density of metal.

Because of attenuation of a sound wave

\[ I = I_0 e^{-\Gamma e x}, \]

where \( \Gamma \) is the coefficient of attenuation.

The coefficient of attenuation \( \Gamma_e \) is defined by following expression

\[ \Gamma_e = \frac{Q_e}{I}. \] (1.4)

Here \( Q_e \) is the density of energy dissipation of a sound wave.

The dissipation is caused by nonlinearity of fluctuations of a lattice and interaction of sound fluctuations of electronic component and field generated by a electromagnetic wave. The quantity \( Q_e \) is calculated as follows

\[ Q_e = -F_y u_y. \] (1.5)

Here force \( F_y \) is caused by two different factors.

First, on the charged ions operate the electric force induced by electric currents \( \sigma E \). Here \( \sigma \) is the volume density of a charge of a lattice. Because of a metal electroneutrality it is had \( \sigma = -ne \).

On the other hand on a lattice operates force of a friction from the party of electronic gas \( \mathbf{F}_e \). At each scattering of electrons, having velocity \( \mathbf{v} \), on the average the momentum \( m(\mathbf{u} - \mathbf{v}) \) is transferred to a lattice. Considering, that force \( \mathbf{F}_e \) has only one the nonzero component, directed along the axis lengthways \( y \), we have

\[ F_e = -\nu m \int (u_y - v_y) f \frac{2d^3p}{(2\pi \hbar)^3} = -\nu u_y m \int f \frac{2d^3p}{(2\pi \hbar)^3} + \]

\[ +\nu m \int v_y f \frac{2d^3p}{(2\pi \hbar)^3} = -\nu u_y mn + \nu mn \bar{v}_y. \]
Here it has been considered, that the numerical density (concentration) of electrons is equal

\[ n = \int f d\Omega_F = \int f \frac{2d^3p}{(2\pi\hbar)^3}, \]

also the average quantity of \( y \)-components of electron velocity along an axis \( y \) is entered,

\[ \bar{v}_y = \frac{1}{n} \int v_y f \frac{2d^3p}{(2\pi\hbar)^3}. \]

Thus, force of the friction is equal

\[ F_y = -\nu m u_y - en E + \nu mn \bar{v}_y. \]

Therefore the density of energy dissipation of a sound wave is the following

\[ Q_e = [en E + \nu mn (u_y - \bar{v}_y)] u_y. \]

It is obvious, that the velocity of of electron current and average quantity \( y \)-component of electron velocity are connected among themselves by equalities \( j_y = en \bar{v}_y \) and \( \bar{v}_y = (1/en) j_y \).

Let us consider further, that

\[ j_y = j_y^0 e^{i(kx-\omega t)}, \quad \bar{v}_y = \bar{v}_y^0 e^{i(kx-\omega t)}. \]

Let us make averaging on time (see, for example, \([2]\)) equalities (1.5). Passing to the real variables, we receive

\[ Q_e = \frac{-1}{2} \text{Re}(F_y u_y^*) = \frac{1}{2} \text{Re} \left\{ u_y^* [en E + \nu mn (u_y - \bar{v}_y)] \right\} = \]

\[ = u_0 \frac{1}{2} \left\{ en \text{Re} E_0 + \nu mn(u_0 - \text{Re} \bar{v}_y^0) \right\} = \]

\[ = \frac{u_0}{2} [en \text{Re} E_0 + \nu mn(u_0 - \text{Re} \bar{v}_y^0)]. \] (1.6)
Here the top index "asterisk" means complex conjugation. According to (1.4) and (1.6) coefficient of attenuation of a sound wave it is equal

$$\Gamma_e = \frac{en \, \text{Re} \, E_0 + \nu \eta n(u_0 - \text{Re} \, \bar{v}_y^0)}{\rho u_0 s_{tr}}. \quad (1.7)$$

2. The solution of system of the equations and electric current density

For the solution of the equation (1.1) we search according to (1.3). Transform (1.3), we receive, that in the case of degenerate plasmas in metal distribution function in linear approach it is searched in the form

$$f = f_0(\mathcal{E}) + \delta(\mathcal{E}_F - \mathcal{E}) \psi, \quad (2.1)$$

where $\psi = \psi(x, \mathbf{v}, t)$ is the new unknown function.

After linearization of locally equilibrium distribution function on quantity of speed $\mathbf{u}$ we receive

$$f_{eq} = f_0(\mathcal{E}) + mv_y u_y \delta(\mathcal{E}_F - \mathcal{E}). \quad (2.2)$$

By means of (2.1) and (2.2) the equation (1.1) it will be transformed to the following equation

$$\frac{\partial \psi}{\partial t} + v_x \frac{\partial \psi}{\partial x} + \nu \psi = eE_y v_y + \nu mv_y u_y. \quad (2.3)$$

In the right part of the equation (1.2) there is a density of current, which taking into account equality (2.1) it is equal

$$j_y = e \int v_y f \frac{2d^3 p}{(2\pi \hbar)^3} = e \int v_y \psi \delta(\mathcal{E}_F - \mathcal{E}) \frac{2d^3 p}{(2\pi \hbar)^3}. \quad (2.4)$$

From the equation (2.3) we find function

$$\psi = v_y \frac{eE_0 + \nu mu_0}{\nu - i\omega + ikv_x} e^{i(kx - \omega t)}. \quad (2.5)$$
Let us substitute expression (2.5) in the previous expression (2.4) for electric current density. Then for current density it is received following expression

\[ j_y = \frac{2e\mu^3 (eE_0 + \nu m u_0) e^{i(kx - \omega t)}}{(2\pi \hbar)^3} \int \frac{v_y^2 \delta(\mathcal{E}_F - \mathcal{E})}{\nu - i\omega + ikv_x} d^3v. \]  \hspace{1cm} (2.6)

Integral from (2.6) is equal

\[ \int \frac{v_y^2 \delta(\mathcal{E}_F - \mathcal{E})}{\nu - i\omega + ikv_x} d^3v = \frac{2i\pi v_F^2}{mk_Fq^3} \varphi(q, y). \]

Here

\[ q = \frac{k}{k_F}, \quad \Omega = \frac{\omega}{k_F v_F}, \quad y = \frac{\nu}{k_F v_F}, \quad \frac{q}{y} = \frac{k}{\nu v_F}, \]

\[ \varphi(q, \Omega, y) = q(\Omega + iy) - \frac{1}{2} [q^2 - (\Omega + iy)^2] \ln \frac{\Omega + iy - q}{\Omega + iy + q}. \]

The density of longitudinal current now equals

\[ j_y = i \frac{2e\mu_F (eE_0 + \nu m u_0) e^{i(kx - \omega t)}}{(2\pi \hbar)^2 q^3} \varphi(q, \Omega, y). \]  \hspace{1cm} (2.7)

Let us consider that fact, that velocity of the sound is much less than electron velocity on Fermi’s surfaces: \( s_{tr} \ll v_F \). Then the quantity \( \Omega \) is small parameter

\[ \Omega = \frac{\omega}{k_F v_F} = \frac{k s_{tr}}{k_F v_F} = q \varepsilon_1, \]

where \( \varepsilon_1 \) is so small parameter,

\[ \varepsilon_1 = \frac{s_{tr}}{v_F} \ll 1. \]

Taking into account this fact the expression for \( \varphi(q, \Omega, y) \) becomes simpler and looks like

\[ \varphi(q, y) = qyi - \frac{1}{2} (y^2 + q^2) \ln \frac{iy - q}{iy + q} = \]
\[= qyi - \frac{1}{2}(y^2 + q^2) \ln \frac{y + iq}{y - iq} = qyi - \frac{i}{2}(y^2 + q^2) \arg \frac{y + iq}{y - iq} =
\]

\[= i[qy - (q^2 + y^2) \arctg \frac{q}{y}],
\]

or

\[\varphi(q, y) = i\varphi_0(q, y),
\]

where

\[\varphi_0(q, y) = qy - (q^2 + y^2) \arctg \frac{q}{y}.
\]

Now expression for density of an electric current (2.7) becomes simpler, and its amplitude it is equal

\[j^\circ_y = -\frac{2ep_F(eE_0 + \nu mu_0)}{(2\pi\hbar)^2q^3} \varphi_0(q, y).
\]

(2.8)

3. The electric field

Let us return to the equation (1.2) on electric field.

Let us substitute in this equation expression of electric field, density of current and field of electron velocity. It is as a result received the following equation

\[-k^2E_0 + \frac{4\pi i\omega}{c^2} j^\circ_y = \frac{4\pi i\omega nu_0}{c^2},
\]

or, by mean (2.8),

\[-E_0 \left(k^2 + \frac{8\pi i\omega e^2p_F}{c^2(2\pi\hbar)^2q^3} \varphi_0(q, y) \right) =
\]

\[= \frac{4\pi i\omega nu_0}{c^2} \left(1 + \frac{2\nu mp_F}{(2\pi\hbar)^2nq^3} \varphi_0(q, y) \right).
\]

(3.1)

We transform the equation (3.1) to the form

\[-E_0 k_F^2 \left(q^2 + \frac{3i\Omega_y^2\varepsilon_0^2}{2q^2} \varphi_0(q, y) \right) =
\]

\[= \frac{4\pi i\omega nu_0}{c^2} \left(1 + \frac{3y}{2q^3} \varphi_0(q, y) \right).
\]

(3.2)
where
\[ \varepsilon_0 = \frac{v_F}{c}, \quad \Omega_p = \frac{\omega_p}{k_F v_F}, \quad \omega_p = \sqrt{\frac{4\pi e^2 n}{m}}, \quad n = \frac{k_F^3}{3\pi^2}, \]

\( \omega_p \) is the frequency of plasmas (Langmuir).

Let us result numerical calculations for three typical metals with the parameters taken from resulted below the table.

| material   | sound velocity, cm/sec | plasma frequency, 1/sec | Fermi' velocity, cm/sec | wave Fermi number, 1/cm |
|------------|------------------------|-------------------------|--------------------------|-------------------------|
| Potassium  | 2 \cdot 10^5           | 6.5 \cdot 10^{15}      | 8.52 \cdot 10^7         | 7.4 \cdot 10^4          |
| Gold       | 1.74 \cdot 10^5        | 1.37 \cdot 10^{16}     | 1.4 \cdot 10^8          | 9.2 \cdot 10^7          |
| Argentum   | 2.6 \cdot 10^5         | 0.96 \cdot 10^{15}     | 1.39 \cdot 10^8         | 6.9 \cdot 10^6          |

From equation (3.2) we obtain
\[ E_0 = -ieu_0 \frac{4k_F^2 s_{tr}}{3\pi c^2} \cdot 2q^3 + 3y\varphi_0(q,y) \frac{2q^4 + 3i\varepsilon_0 q}{2q^4 + 3i\varepsilon_0 q}, \]

(3.3)

or short
\[ E_0 = -ieu_0 \frac{4k_F^2 s_{tr}}{3\pi c^2} \cdot D, \]

where
\[ D = \frac{2q^3 + 3y\varphi_0(q,y)}{2q^4 + 3i\varepsilon_0 q}. \]

(3.4)

Here
\[ \varepsilon = \Omega_p^2 \varepsilon_0 \varepsilon_1 = \left( \frac{\omega_p}{k_F v_F} \right)^2 \left( \frac{v_F}{c} \right)^2 \left( \frac{s_{tr}}{u_F} \right) = \left( \frac{\omega_p}{k_F c} \right)^2 \left( \frac{s_{tr}}{u_F} \right). \]

From resulted above the table it is easy to receive following values of small parameters
for gold
\[ \varepsilon_0 = 4.67 \cdot 10^{-3}, \quad \varepsilon_1 = 1.2 \cdot 10^{-3}, \quad \varepsilon = 1.25 \cdot 10^{-10}, \]
for potassium
\[ \varepsilon_0 = 2.8 \cdot 10^{-3}, \quad \varepsilon_1 = 2.4 \cdot 10^{-2}, \quad \varepsilon = 5.3 \cdot 10^{-10}, \]
for argentum

$$\varepsilon_0 = 4.6 \cdot 10^{-3}, \quad \varepsilon_0 = 1.9 \cdot 10^{-3}, \quad \varepsilon = 1.7 \cdot 10^{-9}.$$  

Let us take advantage of equality (3.4) and we will allocate the real and imaginary part at fraction $D$

$$D = \frac{2q^4[2q^3 + 3y\varphi_0(q, y)]}{4q^8 + 9\varepsilon^2\varphi_0^2(q, y)} - i\frac{3\varepsilon\varphi_0(q, y)[2q^3 + 3y\varphi_0(q, y)]}{4q^8 + 9\varepsilon^2\varphi_0^2(q, y)}. \quad (3.5)$$

Therefore the quantity $E_0$ equals

$$E_0 = -eu_0 \frac{4k_F^2 s_{tr}}{3\pi c^2} \cdot \frac{(2q^3 + 3y\varphi_0(q, y))(3\varepsilon\varphi_0(q, y) + 2q^4 i)}{4q^8 + 9\varepsilon^2\varphi_0^2(q, y)}. \quad (3.6)$$

From here we receive the real part of this quantity

$$\text{Re } E_0 = -eu_0 \frac{4k_F^2 s_{tr}}{3\pi c^2} \cdot \frac{(2q^3 + 3y\varphi_0(q, y))3\varepsilon\varphi_0(q, y)}{4q^8 + 9\varepsilon^2\varphi_0^2(q, y)}.$$

**4. Coefficient of attenuation of a sound wave**

We note that according to (3.6) we have

$$eE_0 + \nu mu_0 = \nu mu_0 \left(1 + \frac{eE_0}{\nu mu_0}\right) = \nu mu_0 \left(1 - i\frac{4e^2 s_{tr} k_F^2}{3\pi \nu mc^2}D\right) =$$

$$= \nu mu_0 \left(1 - i\frac{\varepsilon}{y}D\right).$$

Hence, according to (2.8) for amplitude of density of an electric current it is received

$$j_y = -\frac{2evu_0 mp_F}{(2\pi \hbar)^2 q^3} \left(1 - i\frac{\varepsilon}{y}D\right) \varphi_0(q, y).$$

From here for average quantity of electron velocity it is had

$$\bar{v}_y = -\frac{2\nu u_0 mp_F}{n(2\pi \hbar)^2 q^3} \left(1 - i\frac{\varepsilon}{y}D\right) \varphi_0(q, y). \quad (4.1)$$
Let us return to the formula (1.7) for coefficient of attenuation and we will present it in the form

$$\Gamma_e = \frac{Q_1 + Q_2}{\rho u_0 s_{st}},$$

(4.2)

where

$$Q_1 = en \, \text{Re} \, E_0, \quad Q_2 = \nu mn (u_0 - \text{Re} \, \bar{v}_0^c).$$

For expression $Q_1$ according to (3.3) – (3.5) it is had

$$Q_1 = -u_0 \nu nm \cdot 4e^2 s_{st} k_F^2 D_1 = -u_0 \nu nm \cdot \frac{\varepsilon}{y} D_1,$$

where

$$D_1 = \frac{(2q^3 + 3y \varphi_0(q))3\varepsilon \varphi_0(q, y)}{4q^8 + 9\varepsilon^2 \varphi_0^2(q, y)}.$$

Let us pass to a finding $Q_2$. According to (4.1) it is found

$$Q_2 = \nu mn u_0 \left[1 + \frac{2\nu m p_F \varphi_0(q, y)}{n(2\pi \hbar)^2 q^3} \left(1 - \frac{\varepsilon}{y} \text{Re}(iD)\right)\right].$$

Let us transform the previous expression to the form

$$Q_2 = \nu mn u_0 \left[1 + \frac{3\varphi_0(q, y)y}{2q^3} \left(1 - \frac{\varepsilon}{y} D_1\right)\right].$$

The coefficient of attenuation of a sound wave according to (4.2) is equal

$$\Gamma_e = \frac{\nu mn}{\rho s_{st}} \left[ -\frac{\varepsilon}{y} D_1 + 1 + \frac{3y}{2q^3} \varphi_0(q, y) \left(1 - \frac{\varepsilon}{y} D_1\right)\right] =$$

$$= \frac{\nu mn}{\rho s_{st}} \left(1 - \frac{\varepsilon}{y} D_1\right) \left(1 + \frac{3y}{2q^3} \varphi_0(q, y)\right).$$

(4.3)

Let us transform the formula (4.3) to the form

$$\Gamma_e = \frac{\nu mn}{\rho s_{st}} K(q, y),$$

where $K(q, y)$ is the dimensionless coefficient of attenuation,

$$K(q, y) = \left(1 - \frac{\varepsilon}{y} D_1\right) \left(1 + \frac{3y}{2q^3} \varphi_0(q, y)\right).$$

(4.4)
Let us present this coefficient in an explicit form

\[ K(q, y) = \left( 1 - \frac{3\varepsilon^2\varphi_0(q, y)}{y} \cdot \frac{2q^3 + 3y\varphi_0(q, y)}{4q^8 + 9\varepsilon^2\varphi^2_0(q, y)} \right) \left( 1 + \frac{3y}{2q^3}\varphi_0(q, y) \right). \] (4.5)

From expression (4.5) it is visible, that the dimensionless coefficient of attenuation consists of two components

\[ K(q, y) = K_1(q, y) + K_2(q, y). \]

Here \( K_1(q, y) \) is the contribution to coefficient of attenuation which brings electric field,

\[ K_1(q, y) = -\frac{3\varepsilon^2\varphi_0(q, y)}{y} \cdot \frac{2q^3 + 3y\varphi_0(q, y)}{4q^8 + 9\varepsilon^2\varphi^2_0(q, y)}, \]

\( K_2(q, y) \) is the contribution to coefficient of attenuation which brings electronic friction with a crystal lattice,

\[ K_2(q, y) = 1 + \frac{3y}{2q^3}\varphi_0(q, y) \left( 1 - \frac{3\varepsilon^2\varphi_0(q, y)}{y} \cdot \frac{2q^3 + 3y\varphi_0(q, y)}{4q^8 + 9\varepsilon^2\varphi^2_0(q, y)} \right). \]

According to (4.5) dimensionless components of coefficient of attenuation are connected by equality

\[ K_2(q, y) = 1 + \frac{3y}{2q^3}\varphi_0(q, y) [1 + K_1(q, y)]. \]

Investigating behavior of dimensionless factor \( K(q, y) \), it is possible to find out, that with the big accuracy coefficient \( K(q, y) \) it is possible to replace by coefficient

\[ K_0(q, y) = 1 + \frac{3y}{2q^3}\varphi_0(q, y). \]

Really, we form function of relative errors

\[ Er(q, y) = \frac{K(q, y) - K_0(q, y)}{K(q, y)} \cdot 100\%. \]
It is easy to see, that

$$Er(q, y) = \frac{K_1(q, y)}{1 + K_1(q, y)} \cdot 100\%.$$ 

Let us underline, that all plots in the present work are constructed for gold.

From Figs. 1 and 2 it is visible, that the coefficient $K_0(q, y)$ effectively approximates coefficient $K(q, y)$ at all values of wave numbers. Comparison of plots on Fig. 1 and 2 shows, that quantity of the deviation of coefficient $K_0(q, y)$ from $K(q, y)$ decreases at transition of dimensionless frequency of collisions from $10^{-4}$ to $10^{-3}$. More below we investigate is more thin structure of dimensionless coefficient, representing the sum of coefficients $K_1(q, y)$ and $K_2(q, y)$.

Fig. 1. Relative deviation of dimensionless coefficient $K_0(q, y)$ from coefficient $K(q, y)$. Curves of 1 and 2 correspond to values of frequency $y = 0.0001$ and 0.0002.
Fig. 2. Relative deviation of dimensionless coefficient $K_0(q, y)$ from coefficient $K(q, y)$. Curves of 1 and 2 correspond to values of frequency $y = 0.001$ and $0.002$. 
Fig. 3. Coefficient of attenuation of the transversal sound wave. Curves of $1, 2, 3$ correspond to values dimensionless frequency of electron collisions $y = 0.001, 0.002, 0.005$.

Fig. 4. Coefficient of attenuation of the transversal sound wave. Curves of $1, 2, 3$ correspond to values dimensionless frequency of electron collisions $y = 10^{-5}, 10^{-4}, 10^{-3}$. 
On Figs. 3 and 4 dependence of dimensionless coefficient on quantity wave number at the fixed values of dimensionless frequency of electron collisions is presented. Plots on Figs. 3 and 4 show monotonous increase of coefficient of attenuations from zero to unit. On Fig. 5 dependence of coefficient of attenuations from dimensionless frequency of collisions at the fixed values dimensionless wave number is presented. Plots show monotonous decrease of coefficient of attenuations with growth of frequency of collisions.

5. Coefficient of attenuations of sound wave in long–wave cases
Let us expand into series at small $q$ expression for $\varphi_0(q, y)$

$$\varphi_0(q, y) = qy - (q^2 + y^2) \left( \frac{q}{y} - \frac{q^3}{3y^3} + \frac{q^5}{5y^5} - \frac{q^7}{7y^7} + \cdots \right).$$

From here we obtain

$$\varphi_0(q, y) = -\frac{2q^3}{3y} + \frac{2q^5}{15y^3} - \frac{2q^7}{35y^5} + \frac{2q^9}{63y^7} \cdots =$$

$$= -\frac{2q^3}{3y} \left[ 1 - \frac{q^2}{5y^2} + \frac{3q^4}{35y^4} - \frac{q^6}{21y^6} \right]$$

and

$$2q^3 + 3y\varphi_0(q, y) = \frac{2q^5}{5y^2} - \frac{6q^7}{35y^4} + \frac{2q^9}{21y^6} \cdots =$$

$$= \frac{2q^5}{5y^2} \left[ 1 - \frac{3q^2}{7y^2} + \frac{5q^4}{21y^4} - \cdots \right].$$

Herefore the second bracket from (4.3) at small $q$ has the following expansion

$$1 + \frac{3y}{2q^3} \varphi_0(q, y) = \frac{q^2}{5y^2} - \frac{3q^4}{35y^4} + \frac{q^6}{21y^6} - \cdots =$$

$$= \frac{q^2}{5y^2} \left[ 1 - \frac{3q^2}{7y^2} + \frac{5q^4}{21y^4} - \cdots \right].$$

Let us return to equality (3.5) and we will consider the denominator $4q^8 + 9\varepsilon^2 \varphi_0^2(q, y)$. We have at small $q \to 0$

$$4q^8 + 9\varepsilon^2 \varphi_0^2(q, y) = q^6 \left( q^2 + \varepsilon^2 / y^2 \right).$$

Let us enter critical value of dimensionless wave number $q_0$, such, that

$$q_0 = \frac{\varepsilon}{y}.$$

This critical wave number breaks an interval of values of the dimensionless wave number on two interval: the first interval $I_1 = \{ 0 < q < q_0 \}$ and the second interval $I_2 = \{ q > q_0 \}$. 
Let us estimate typical values of critical wave number for gold. We take two values of dimensionless frequency of collisions $y = 10^{-3}$ and $y = 10^{-4}$. Corresponding values of critical wave number are equal

$$q_0 = 1.25 \cdot 10^{-7}, \quad \text{and} \quad q_0 = 1.25 \cdot 10^{-6}.$$

From Fig. 6 it is visible, that the critical wave number is equal in accuracy $q = q_0 = 1.25 \cdot 10^{-6}$.

On Fig. 6 dependence of coefficients $K_1(q, y)$ and $K_2(q, y)$ from wave number at the fixed value of frequency of electron collisions is presented. From Fig. 6 it is visible, that graphics of coefficients $K_1(q, y)$ and $K_2(q, y)$ are crossed in the point $q_0$. The point crossings $q_0$ of graphics is the critical value of wave number. At change wave number from zero to $q_0$ the attenuation coefficient is defined by the electric field (the coefficient $K_1(q, y)$), and at $q > q_0$ the attenuation coefficient is defined by friction of electrons on the crystal lattice (the coefficient $K_2(q, y)$).

In the first interval the dominant member of the denominator of fraction $D_1$ is $9\varepsilon^2\varphi_0^2(q)$, in second interval the dominant member is $4q^8$. Let us underline, that in the first interval the dominant contribution to attenuation coefficient brings electric field (the interval of $0 < q < q_0$ on Fig. 6 see), i.e. composed

$$K_1(q, y) = -\frac{3\varepsilon^2\varphi_0(q, y)}{y} \cdot \frac{2q^3 + 3y\varphi_0(q, y)}{4q^8 + 9\varepsilon^2\varphi_0^2(q, y)}.$$

In the second interval the dominant contribution to attenuation coefficient brings the electronic friction on the crystal lattice (the interval $q > q_0$ on Fig. 6 see), i.e. composed

$$K_2(q, y) = 1 + \frac{3y}{2q^3\varphi_0(q, y)} \left(1 - \frac{3\varepsilon^2\varphi_0(q, y)}{y} \cdot \frac{2q^3 + 3y\varphi_0(q, y)}{4q^8 + 9\varepsilon^2\varphi_0^2(q, y)}\right).$$

Let us begin with the first interval. We will find asymptotics of attenuation coefficient in the case $0 < q \ll q_0$. We will take advantage of the
Fig. 6. The contribution to attenuation of the transversal sound wave. Curves of 1 and 2 according correspond to electric field and friction of electrons on crystal lattice, $y = 10^{-4}$. At $q < q_0$ the contribution to attenuation is defined by electric field, and at $q > q_0$ the contribution to attenuation is defined by an electronic friction.

previous decomposition. At small $q \to 0$ for fraction $D_1$ we have

$$D_1 = -\frac{\varepsilon q^2}{5y^3} \cdot \frac{\left(1 - \frac{q^2}{5y^2} + \frac{3q^4}{35y^4} - \cdots\right)\left(1 - \frac{3q^2}{7y^2} + \frac{5q^4}{21y^4} + \cdots\right)}{q^2 + \frac{\varepsilon^2}{y^2}} \left(1 - \frac{2q^2}{5y^2} + \cdots\right)$$

$$= -\frac{\varepsilon q^2}{5y^3} \cdot \frac{1}{\varepsilon^2 + q^2} = -\frac{\varepsilon q^2}{5y} \cdot \frac{1}{\varepsilon^2 + q^2 y^2} + \cdots.$$
From this equality we obtain that

$$\frac{\varepsilon}{y} D_1 = -\frac{q^2}{5y^2} \cdot \frac{1 - \frac{22q^2}{35y^2} + \cdots}{1 + \frac{q^2y^2}{\varepsilon^2} + \cdots} =$$

$$= -\frac{q^2}{5y^2} \cdot (1 - \frac{22q^2}{35y^2} + \cdots) \left(1 - \frac{q^2y^2}{\varepsilon^2} + \cdots\right) =$$

$$= -\frac{q^2}{5y^2} \left[1 - \left(\frac{22}{35y^2} + \frac{y^2}{\varepsilon^2}\right)q^2\right].$$

We note that $y^2/\varepsilon^2 = 1/q_0^2$, $q_0 \ll y$, therefore in the considered interval

$$\frac{22}{35y^2} + \frac{y^2}{\varepsilon^2} = \frac{22}{35y^2} + \frac{1}{q_0^2} = \frac{1}{q_0^2}.$$

Therefore finally we have

$$\frac{\varepsilon}{y} D_1 = -\frac{q^2}{5y^2} \left[1 - \frac{q^2y^2}{\varepsilon^2}\right] = -\frac{q^2}{5y^2} \left(1 - \frac{q^2}{q_0^2}\right).$$

From the formula (4.1) for average electron velocity at small values of wave number we receive

$$\text{Re} \bar{v}_y^0 = -\frac{2\nu u_0 m p_F}{n(2\pi\hbar)^2 q^3} \left(1 - \frac{\varepsilon}{y} D_1\right) \varphi_0(q, y),$$

or

$$\text{Re} \bar{v}_y^0 = \frac{4\nu u_0 m p_F}{3ny(2\pi\hbar)^2} \left(1 - \frac{q^2}{5y^2} + \frac{3q^4}{35y^4} + \cdots\right) \left(1 - \frac{\varepsilon}{y} D_1\right).$$

From here we find that

$$\text{Re} \bar{v}_y^0 = u_0 \left(1 - \frac{q^2}{5y^2} + \frac{3q^4}{35y^4} + \cdots\right) \left(1 - \frac{\varepsilon}{y} D_1\right).$$

Further we receive that

$$\text{Re} \bar{v}_y^0 = u_0 \left(1 - \frac{q^2}{5y^2} + \frac{3q^4}{35y^4} + \cdots\right) \left(1 + \frac{q^2}{5y^2} - \frac{q^4}{5\varepsilon^2} + \cdots\right),$$

from which

$$\text{Re} \bar{v}_y^0 = u_0 \left(1 - \frac{q^4}{5\varepsilon^2} + \cdots\right).$$
From here it is visible, that on absolute value in the limit at $q \to 0$
average electron velocity tends to velocity of the lattice

$$\lim_{q \to 0} \text{Re} \bar{v}_y^o = u_0.$$ 

Now we will find the coefficient $K(q, y)$. According to (4.4) it is had

$$K(q, y) = \left(1 - \frac{q^4}{5\varepsilon^2} - \cdots \right) \left(\frac{q^2}{5y^2} - \frac{3q^4}{35y^4} + \frac{q^6}{21y^6} - \cdots \right) =$$

$$= \frac{q^2}{5y^2} - \frac{3q^4}{35y^4} + \cdots = \frac{q^2}{5y^2} \left[1 - \frac{3q^2}{7y^2} + \cdots \right] .$$

So, at small values of wave number the attenuation coefficient is equal

$$\Gamma_e(q, y) = \frac{\nu nm}{\rho_{str}} \left(\frac{q^2}{5y^2} - \frac{3q^4}{35y^4} + \cdots \right) =$$

$$= \frac{\nu nm q^2}{5\rho_{str} y^2} \left(1 - \frac{3q^2}{7y^2} + \cdots \right) ,$$

or, with dimensional parameters,

$$\Gamma_e(k, \nu) = \frac{nmv_F^2 k^2}{5\rho \nu s_{tr}} \left(1 - \frac{3v_F^2 k^2}{7\nu^2} \right) .$$

From here in square-law approach (on $k$) $\Gamma_e(k) = \Gamma_0 k^2 \ (k \to 0)$, where

$$\Gamma_0 = \frac{nmv_F^2}{5\rho \nu s_{tr}} = \frac{mv_F^2 k_F^3}{15\pi^2 \rho_{str} \nu} .$$

Let us estimate one after another this quantity for gold. The mass of electron is equal $m = 9 \cdot 10^{-28} \ gr$, the gold density is equal $\rho = 19.32 gr/cm^3$, we consider, that $\nu \sim 10^{13} 1/sec$, other data we take from resulted above the table. We receive, that

$$\Gamma_0 = \frac{9 \cdot 10^{-28} (1.4 \cdot 10^8)^2 (8.2 \cdot 10^7)^3}{15\pi^2 (19.3) 10^{13} (1.7 \cdot 10^5)} \sim 2 \cdot 10^{-9} \frac{1}{cm} .$$

On Fig. 7 the behaviour of factors $K_1(q, y)$ and $K_2(q, y)$ is presented. We will notice, that in the range of wave numbers, bigger critical, the coefficient
$K_1(q, y)$ has the unique maximum, and then quickly decreases to zero. From Fig. 8 it is visible, that the maximum quantity decreases with growth of frequency of electron collisions.

We investigate separately behaviour of dimensionless coefficient $K_1(q, y)$ and $K_2(q, y)$ at $q \to 0$.

Fig. 7. Dependence of dimensionless coefficients of attenuation $K_1(q, y)$ (the curve 1) and $K_2(q, y)$ (the curve 2) on dimensionless frequency of collisions, $y = 0.0001$. 
Fig. 8. Dependence of dimensionless coefficient of attenuation $K_1(q, y)$ on dimensionless collision frequency. Curves 1, 2, 3 correspond to values of frequency $y = 0.0001, 0.000105, 0.00011$.

Let us begin with the first factor. We will present it in the form

$$K_1(q, y) = -6\frac{\varepsilon^2}{y}\varphi_0(q, y)q^3\frac{1 + \frac{3y}{2q^3}\varphi_0(q, y)}{4q^8 + 9\varepsilon^2\varphi_0^2(q, y)}.$$ 

We note that

$$\varphi_0^2(q, y) = \frac{4q^6}{9y^2} \left[1 - \frac{2q^2}{5y^2} + \frac{37q^4}{175y^4} - \cdots \right].$$

Not resulting calculation, we receive, that

$$K_1(q, y) = \frac{q^2}{5y^2}\left[1 - \left(\frac{y^2}{\varepsilon^2} + \frac{3}{7y^2}\right)q^2 + \cdots \right] = \frac{q^2}{5y^2}\left(1 - \frac{y^2}{\varepsilon^2}q^2\right),$$
or,

\[
K_1(q, y) = \frac{q^2}{5y^2} \left(1 - \frac{q^2}{q_0^2}\right), \quad q \to 0.
\]

This expression in accuracy coincides with expression for coefficient 
\(K(q, y)\), a definiendum (5.1).

For finding asymptotics of the second coefficient we will take advantage of the formula

\[
K_2(q, y) = 1 + \frac{3y}{2q^3} \varphi_0(q, y)(1 + K_1(q, y)).
\]

We obtain that

\[
K_2(q, y) = \frac{q^4}{5y^2} \left(\frac{y^2}{\varepsilon^2} + \frac{1}{5y^2}\right) = \frac{q^4}{5\varepsilon^2}, \quad q \to 0.
\]

Thus, at small values of wave number the contribution to attenuation coefficient it is defined by presence of electric field.

Let us consider the second interval of values of dimensionless wave number \(q > q_0\). This interval we will break into three intervals: \(q_0 \ll q \ll y, y \ll q \ll 1\) and \(q \sim 1\).

Let us begin with an interval \(q_0 \ll q \ll y\). In the considered interval

\[
K(q, y) = 1 + \frac{3y\varphi_0(q, y)}{2q^3} = \frac{q^2}{5y^2} \left(1 - \frac{3q^2}{7y^2} + \frac{5q^4}{21y^4} - \cdots\right).
\]

This formula means, that the damping coefficient is computed under the same formula, as in the previous case.

Let us observe the third case small \(q\): \(y \ll q \ll 1\). In this case

\[
\arctg \frac{q}{y} \approx \frac{\pi}{2},
\]

\[
\varphi_0(q, y) = qy - \frac{\pi}{2}(q^2 + y^2) = q^2 \left[\frac{y}{q} - \left(1 + \frac{y^2}{q^2}\right)\frac{\pi}{2}\right] \approx
\]

\[
\approx -\frac{\pi}{2}q^2.
\]

Hence, the dimensionless attenuation coefficient is equal

\[
K(q, y) = \left(1 + \frac{3y\varphi_0}{2q^3}\right) = 1 - \frac{3\pi}{4} \cdot \frac{y}{q}.
\]
The dimensional attenuation coefficient is equal
\[ \Gamma_e = \frac{\nu nm}{\rho str} \left( 1 - \frac{3\pi}{4} \cdot \frac{y}{q} \right). \]

Now we will observe the case, when \( q \sim 1 \). In this case
\[ \varphi_0(q, y) = q^2 \left[ \frac{y}{q} - \frac{\pi}{2} \left( 1 + \frac{y^2}{q^2} \right) \right] = -q^2 \left( \frac{\pi}{2} - \frac{y}{q} \right). \]

The dimensionless attenuation coefficient now is equal
\[ K(q, y) = 1 + \frac{3y\varphi_0}{2q^3} = 1 - \frac{3\pi}{4} \cdot \frac{y}{q} + \frac{3\pi}{4} \cdot \frac{\varepsilon^2}{yq^3} = 1 - \frac{3\pi}{4} \cdot \frac{y}{q}. \]

The dimensional attenuation coefficient is computed under the same formula, as in the previous interval.

On Fig. 9 the behaviour of function \( K(q, y)/q^2 \) depending on the wave number at various values of dimensionless electron collision frequencies is presented.

Graphics show dependence of coefficient \( \Gamma_0 \) from magnitude of a collision frequency of electrons. For comparison on Fig. 10 dependence of coefficient \( K_1(q, y) \) is shown at \( q \to 0 \) at various values of collision frequency of electrons. Let us note, that magnitudes \( K(q, y)/q^2 \) and \( K_1(q, y) \) decrease with growth of collision frequency.
Fig. 9. The behaviour of quantity $K(q, y)/q^2$. Curves 1, 2, 3 correspond to values of dimensionless of electron frequency of collisions $y = 0.001, 0.002, 0.003$. 
Fig. 10. The dependence of dimensionless coefficient of attenuation $K_1(q,y)$ on dimensionless frequency of collisions. Curves 1, 2, 3 correspond to values of frequency $y = 0.00105, 0.00110, 0.00115$. 
6. Comparison with Pippard’ results

Let us result of damping coefficient discovered by Pippard [5]

\[ \alpha_T = \frac{nm}{\rho_{st} \tau} \left[ \frac{2}{3} \frac{a^3}{(1 + a^2) \arctg a - a} - 1 \right]. \]

Here

\[ a = kl = k \tau v_F = \frac{q}{y}. \]

Let us this result to our denotations

\[ \alpha_T = \frac{\nu nm}{\rho_{st}} \left[ \frac{2q^3}{3y(y^2 + q^2) \arctg \frac{q}{y} - qy^2} - 1 \right] = \]

\[ = \frac{\nu nm}{\rho_{st}} \left[ - \frac{2q^3}{3y \varphi_0(q, y)} - 1 \right]. \]

We investigate the case of small values of the wave number. We have

\[ \alpha_T = \frac{\nu nm}{\rho_{st}} \left[ \frac{1}{1 - \frac{q^2}{5y^2} + \frac{3q^4}{35y^4}} - 1 \right] = \]

\[ = \frac{\nu nm}{\rho_{st}} \left[ \frac{q^2}{y^2} \frac{3q^4}{35y^4} + \frac{q^6}{21y^6} + \cdots \right]. \]

Therefore from here we obtain

\[ \alpha_T = \frac{\nu nmq^2}{\rho_{st}y^2} \left( 1 + \frac{4q^2}{35y^2} + \cdots \right). \]

This effect is comparable with our effect

\[ \Gamma_e(q, y) = \frac{\nu nmq^2}{5\rho_{st}y^2} \left( 1 - \frac{3q^2}{7y^2} + \cdots \right), \]

Both coefficients diminish at \( q \to 0 \) as \( q^2 \), but with various coefficients (Fig. 11) see.
Fig. 11. Comparison of dimensionless coefficients of attenuation at small values of dimensionless wave number. The curve 1 corresponds to coefficient $K(q, y)$, obtained in the present operation, the curve 2 corresponds to coefficient, discovered by Pippard’ (see [5]), $y = 0.001$.

7. Conclusion

In summary we will observe the case when the electric field misses. Then

$$Q_1 = 0, \quad \bar{v}_y^o = -\frac{2p_F
nu mu_0}{(2\pi \hbar)^2 q^3} \varphi_0(q, y),$$

$$\Gamma e = \frac{\nu nm}{\rho_{str}} \left(1 - \frac{1}{u_0} \mathrm{Re} \bar{v}_y^o \right).$$

Last expression will easily be converted to the form

$$\Gamma e = \frac{\nu nm}{\rho_{str}} K_0(q, y),$$

где

$$K_0(q, y) = 1 + \frac{3y}{2q^3} \varphi_0(q, y).$$

As it was already specified, coefficient $K_0(q, y)$, obtained without electrical field, the attenuation dimensionless coefficient $K(q, y)$ effectively approximates. However, the analysis carried out above shows, that for the thin
analysis of a damping coefficient its representation in the form of the total
\[ K(q, y) = K_0(q, y) + K_1(q, y) \]
is required.

The present work represents the kinetic approach to damping coefficient
investigation sound wave into degenerate plasma, somewhat alternative
to the approach Pippard' \[5\]. Investigation of coefficient is carried out
of attenuations of a sound wave on the basis of the kinetic (dynamic)
approach of interaction of degenerate electronic gas with oscillations of
lattice in given work.

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Appendix: calculation of integral

\[ J = \int \frac{v_y^2 \delta(\varepsilon_F - \varepsilon)}{\nu - i\omega + ikv_x} d^3v. \]

We calculate the integral \( J \). We note that

\[ \delta(\varepsilon_F - \varepsilon) = \delta\left(\frac{mv_F^2}{2} - \frac{mv^2}{2}\right) = \frac{2}{m} \delta(v_F^2 - v^2) = \frac{1}{mv_F} \delta(v_F - v). \]

Further we have

\[ J = \int \frac{v_y^2 \delta(\varepsilon_F - \varepsilon)}{\nu - i\omega + ikv_x} d^3v = \frac{1}{mv_F} \int \frac{v_y^2 \delta(v_F - v)d^3v}{\nu - i\omega + ikv_x}. \]

We will use the spherical coordinates

\[ v_x = v \cos \theta, \quad v_y = v \sin \theta \cos \xi, \quad v_z = v \sin \theta \sin \xi, \]

\[ d^3v = v^2 d\mu d\theta d\xi, \quad \mu = \cos \theta. \]

Therefore

\[ J = \frac{1}{mv_F} \int_{-1}^{1} (1 - \mu^2) d\mu \int_{0}^{2\pi} \cos^2 \xi d\xi \int_{0}^{\infty} \frac{v^4 \delta(v_F - v) dv}{\nu - i\omega + ikv \mu}. \]
\[ \frac{\pi v_F^3}{m} \int_{-1}^{1} \frac{(1 - \mu^2)d\mu}{\nu - i\omega + ikv_F\mu} = -i\frac{\pi v_F^2}{mk} \int_{-1}^{1} \frac{(1 - \mu^2)d\mu}{\mu - \frac{\omega + iv}{kv_F}} = \]
\[ = -i\frac{\pi v_F^2}{mk} \int_{-1}^{1} \frac{(1 - \mu^2)d\mu}{\mu - z/q}. \quad \text{(A.1)} \]

Here

\[ z = \Omega + iy = \frac{\omega + iv}{k_Fv_F}, \quad q = \frac{k}{k_F}, \quad \Omega = \frac{\omega}{k_Fv_F}, \quad y = \frac{\nu}{k_Fv_F}. \]

It is easy to see that

\[ \int_{-1}^{1} \frac{(1 - \mu^2)d\mu}{\mu - z/q} = -2zq + \left[ 1 - \left( \frac{z}{q} \right)^2 \right] \ln \frac{1 - z/q}{-1 - z/q} = \]
\[ = -\frac{1}{q^2} \left\{ 2zq - [q^2 - z^2] \ln \frac{z - q}{z + q} \right\} = \]
\[ = -\frac{1}{q^2} \left\{ 2q(\Omega + iy) - [q^2 - (\Omega + iy)^2] \ln \frac{\Omega + iy - q}{\Omega + iy + q} \right\}. \]

We denote

\[ J_0 = 2q(\Omega + iy) + [q^2 - (\Omega + iy)^2] \ln \frac{\Omega + iy - q}{\Omega + iy + q}. \]

Then

\[ \int_{-1}^{1} \frac{(1 - \mu^2)d\mu}{\mu - z/q} = -\frac{J_0}{q^2}. \quad \text{(A.2)} \]

Substituting (A.1) in (A.2), we gain, that the integral \( J \) is equal

\[ J = -i\frac{\pi v_F^2}{mk} \cdot \left( -\frac{J_0}{q^2} \right) = \frac{i\pi v_F^2}{mkFq^3}J_0, \quad \text{(A.3)} \]

or

\[ J = \frac{i\pi v_F^2}{mkFq^3} \left\{ 2q(\Omega + iy) - [q^2 - (\Omega + iy)^2] \ln \frac{\Omega + iy - q}{\Omega + iy + q} \right\}. \quad \text{(A.4)} \]
We consider the expression

\[ J_0 = 2q(\Omega + iy) - [q^2 - (\Omega + iy)^2] \ln \frac{\Omega + iy - q}{\Omega + iy + q}. \]  

(A.5)

We denote

\[ Z = \frac{\Omega + iy - q}{\Omega + iy + q}, \quad \ln Z = \ln |Z| + i \arg Z. \]

It easy see that

\[
\ln \frac{\Omega + iy - q}{\Omega + iy + q} = \ln \frac{y + i(q - \Omega)}{y - i(q + \Omega)} = \\
= \ln(y + i(q - \Omega)) - \ln(y - i(q + \Omega)) = \\
= \frac{1}{2} \ln \frac{(q - \Omega)^2 + y^2}{(q + \Omega)^2 + y^2} + i \left( \arctg \frac{q - \Omega}{y} - \arctg \frac{-q + \Omega}{y} \right) = \\
= W_0 + iW_1,
\]

where

\[ W_0 = \ln |Z| = \frac{1}{2} \ln \frac{(\Omega - q)^2 + y^2}{(\Omega + q)^2 + y^2}, \]

\[ W_1 = \arg Z = \left( \arctg \frac{q - \Omega}{y} + \arctg \frac{q + \Omega}{y} \right). \]

We separate in (2.13) real and imafginary parts

\[ J_0 = \text{Re} J_0 + i \text{Im} J_0 = J'_0 + iJ''_0, \]

where

\[ J'_0 = \text{Re} J_0 = 2q\Omega - (q^2 - \Omega^2 + y^2)W_0 - 2y\Omega W_1, \]

\[ J''_0 = \text{Im} J_0 = 2qy - (q^2 - \Omega^2 + y^2)W_1 + 2y\Omega W_0. \]

Finally, the real and imaginary parts of integral \( J \) are equal

\[
\text{Re} J = -\frac{\pi v_F^2}{mk_F q^3} \left[ 2qy - (q^2 - \Omega^2 + y^2) \left( \arctg \frac{q - \Omega}{y} + \arctg \frac{q + \Omega}{y} \right) + \\
+ \frac{y}{2} \ln \frac{(\Omega - q)^2 + y^2}{(\Omega + q)^2 + y^2} \right]
\]
and

\[ \text{Im } J = \frac{\pi v_F^2}{m k_F q^3} \left[ 2q\Omega - (q^2 - \Omega^2 + y^2) \frac{1}{2} \ln \frac{(\Omega - q)^2 + y^2}{(\Omega + q)^2 + y^2} \right. \\
\left. - 2y \left( \arctg \frac{q - \Omega}{y} + \arctg \frac{q + \Omega}{y} \right) \right]. \]