Research on Optimizing Model of Product Availability in Warranty Period

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Abstract. In order to improve the availability of products during the warranty period, a maintenance strategy combining incomplete preventive maintenance with two different degrees of maintenance is adopted to solve the problem of long downtime which is caused by failure of products during the warranty period. Based on the theory of incomplete maintenance, the availability model is established. Through an example, it is proved that the model can improve the availability of warranty products, and the scope of application is given. Sensitivity analysis is conducted for some factors affecting availability.

1. Introduction

For products that deteriorate over time, the failure rate of products can be affected by preventive maintenance. Usually, preventive maintenance includes fault detection, maintenance, regular recovery, condition-based maintenance and other types of work. Reasonable preventive maintenance can reduce the number of product failures during the warranty period, reduce the downtime caused by repairs during the entire warranty period, and improve product availability. In the existing warranty strategy, the maintenance strategy including preventive maintenance has been gradually applied to the warranty strategy of many products.

During the warranty period, many scholars have conducted in-depth research on how to make preventive maintenance decisions. However, in the current literature on the maintenance strategy model, preventive maintenance is mostly a single degree of maintenance work. Zhou divided the warranty period into several stages, and proposes that preventive maintenance with different intervals in different stages can solve the problem of poor product performance in the later period of the warranty due to the decline of failure rate \cite{1}. Liao carried on the periodic maintenance during the warranty period, and established cost model. Maintenance categories were divided into three types, and different maintenance types were performed with different probabilities \cite{2}. Park proposed the maintenance strategy of preventive maintenance with constant maintenance degree during the warranty period and minimal maintenance or replacement maintenance after product failure. By determining the optimal preventive maintenance interval, the maintenance cost was minimized \cite{3}. Cheng put forward the maintenance strategy under the condition of delaying the first preventive
maintenance for products that deteriorate with time, and established the corresponding maintenance cost model [4]. Wu studied the problem of periodic preventive maintenance for products that deteriorated over time [5].

For some large-scale equipment users, products need to be maintained in the use process, so users usually have a certain amount of maintenance force. However, for the new products during the warranty period, there is still a certain gap between the user and the manufacturer, due to the influence of human resources and maintenance technology. The level of preventive maintenance of products by users is low, and the effect of product performance recovery is not very good. There may be high failure rate, frequent failure, long downtime and other phenomena. Therefore, it is a good way to assist users in preventive maintenance work. However, in the current literature, preventive maintenance is assumed to be the maintenance with the same degree of maintenance. There is no research on the maintenance model involving the combination of two incomplete preventive maintenance with different degree of maintenance. In this paper, we propose a maintenance strategy that combines incomplete preventive maintenance with two different levels of maintenance. The availability during the warranty period is taken as the research objective, and the corresponding availability model is established.

2. Warranty strategy
On the basis of periodic preventive maintenance, the manufacturer is introduced to carry out preventive maintenance, and the manufacturer replaces the user for several preventive maintenance. Due to the different maintenance capabilities of users and manufacturers, the recovery degree of products after preventive maintenance is also different. The manufacturer can carry out preventive maintenance once, twice and many times. In this paper, the manufacturer carries out a preventive maintenance as a new warranty strategy to study. During the preventive maintenance interval, minimal maintenance is performed after product failure.

In order to distinguish preventive maintenance between manufacturers and users, incomplete maintenance theory is introduced. According to the recovery of the product after maintenance, the degree of maintenance can be divided into three categories: minimum maintenance, complete maintenance and incomplete maintenance. Assuming that $\delta$ represents a repair factor, $\delta \in [0,1]$. In the case of minimum maintenance ($\delta = 1$), each repair restores the project to its pre-failure level. In the case of complete maintenance ($\delta = 0$), each repair restores the project to a new product. In the case of incomplete maintenance [6], $0 < \delta < 1$, the degree of recovery is between the two. In this study, preventive maintenance is incomplete maintenance. In the modeling of the impact of incomplete maintenance on failure rate, it is usually assumed that incomplete maintenance reduces the virtual service life of products. In this hypothesis, preventive maintenance can not only reduce the service life of the product in the preceding preventive maintenance interval, but also reduce the whole service life before the preventive maintenance. The latter is adopted in this paper. Proportional Age Setback model is used in this hypothesis [7], and the same hypothesis is used in other literatures [8, 9].

During the warranty period, when the interval of preventive maintenance changes, the number of preventive maintenance of products and the time of preventive maintenance of manufacturers will also change. Under this warranty strategy, the interval of preventive maintenance and the time of preventive maintenance of the manufacturer should be taken as decision variables, and the corresponding availability model should be established.

3. Establishment of availability model

3.1. Symbols and hypothesis

- $T_W$: Warranty Period
- $T_0$: Preventive maintenance interval
- $m$: Maintenance level
\( \delta(m) \): Repair factors for preventive maintenance

\( T_{p1} \): Time for preventive maintenance by the user

\( T_{p2} \): Time for preventive maintenance by the manufacturer

\( T_f \): Maintenance time after product failure

\( \lambda(t) \): Initial failure rate of product

\( D(T_0, N) \): Expectation of product downtime during warranty period

\( A(T_0, N) \): Expected warranty availability

### 3.2. Cost model

During the warranty period \( T_W \), the product is periodically preventive maintenance for the interval \( T_0 \). Preventive maintenance is incomplete maintenance, and repair factor \( \delta(m) \) is related to the degree of maintenance \( m \) as follows [10, 11]: 

\[
\delta(m) = (1 + m)e^{-m}.
\]

If preventive maintenance is carried out at \( T \) time, \( T^- \) indicates the virtual service life before maintenance and \( T^+ \) indicates the virtual service life after maintenance. Product failure rate after maintenance: 

\[
\lambda(T^+) = \lambda[\delta(m)^* T^-].
\]

Assuming that the failure rate of products obeys Weibull distribution with shape parameter \( \beta \) and scale parameter \( \alpha \), 

\[
\lambda(t) = (\beta/\alpha) \times (t/\alpha)^{\beta-1}.
\]

During the warranty period, \( M \) times preventive repairs are carried out altogether. Suppose that in preventive maintenance work, the manufacturer carries out the \( N \) th order and the user carries out the remaining \((M-1)\) times, where \( N \in [1, M] \). When \( N \) changes, it has different effects on the failure rate of products during the warranty period, and the downtime caused by the fault is affected. The degree of incomplete preventive maintenance carried out by users and manufacturers is \( m_1 \) and \( m_2 \) \((m_2 > m_1)\).

During the warranty period, after the \( k \) th preventive maintenance \((k \in [0, M])\), the product failure rate \( \dot{\lambda}_k(t) \) changes as follows:

a. When \( k=0 \),

\[
\dot{\lambda}_k(t) = \dot{\lambda}(t)
\]

b. When \( 1 \leq k \leq N \),

\[
\dot{\lambda}_k(t) = \dot{\lambda}(t-(\delta(m_1)T_0 + \delta^2(m_1)T_0 + \ldots + \delta^k(m_1)T_0))
\]

\[
= \dot{\lambda}(t- \sum_{i=1}^{k} \delta^i(m_1)T_0)
\]

c. When \( k=N \),

\[
\dot{\lambda}_k(t) = \dot{\lambda}(t-(\delta(m_2)T_0 + \delta(m_2)\delta(m_1)T_0 + \ldots + \delta(m_2)\delta^{k-1}(m_1)T_0))
\]

\[
= \dot{\lambda}(t- \sum_{i=0}^{k-1} \delta^i(m_1)T_0)
\]

d. When \( N+1 \leq k \leq M \),

...
\[ \lambda_t(t) = \lambda(t - (\delta(m_1)T_0 + \delta^2(m_1)T_0 + \ldots + \delta^{N-1}(m_1)T_0) \\
+ (\delta(m_2)\delta^{N-1}(m_1)T_0 + \delta(m_2)\delta^{N-2}(m_1)T_0 + \ldots + \delta(m_2)\delta^{1}(m_1)T_0))) \]  
\[ = \lambda(t - \sum_{j=1}^{k} \delta^j(m_j)T_0 - \delta(m_2) \sum_{j=k-N}^{k-1} \delta^j(m_j)T_0) \]  

(4)

In the warranty strategy, the minimum maintenance after product failure can be considered as non-homogeneous Poisson process (NHPP) during the preventive maintenance interval. Therefore, the downtime caused by repairing maintenance is as follows:

\[ T_{TP} = T_f * (\int_0^{T_0} \lambda_0(t)dt + \int_{T_0}^{2T_0} \lambda_1(t)dt + \ldots + \int_{(N-1)T_0}^{NT_0} \lambda_{N-1}(t)dt + \int_{MT_0}^{T} \lambda_M(t)dt) \]

\[ = T_f * (\sum_{k=0}^{M-1} \int_{kT_0}^{(k+1)T_0} \lambda_k(t)dt + \int_{MT_0}^{T} \lambda_M(t)dt) \]  

(5)

Under this strategy, the total preventive maintenance time is:

\[ T_{TP} = T_{p1} * (M - 1) + T_{p2} \]  

(6)

Formulas (5) and (6) show that the downtime caused by product maintenance during the warranty period is as follows:

\[ D(T_0, N) = T_{TP} + T_{TP} \]

\[ = T_{p1} * (M - 1) + T_{p2} + T_f * (\sum_{k=0}^{M-1} \int_{kT_0}^{(k+1)T_0} \lambda_k(t)dt + \int_{MT_0}^{T} \lambda_M(t)dt) \]  

(7)

Therefore, the product availability during the warranty period is as follows:

\[ A(T_0, N) = \frac{T_w - D(T_0, N)}{T_w} \]  

(8)

The steps of solving the availability optimization model are as follows:

1. During the warranty period \( T_w \), the preventive maintenance interval \( T_0 \) is taken as 0.01 \( T_w \) step in \([0, T_w]\). Determine the total number of preventive maintenance \( M \) for each \( T_0 \), \( M = \left\lceil \frac{T_w}{T_0} \right\rceil \) ("\( \lceil \cdot \rceil \" denotes tail removal and rectification); (2) Calculate the failure rate of the product during each preventive maintenance interval; (3) Use the availability model to calculate the value of \( A(T_0, N) \); (4) Comparing the availability \( A(T_0, N) \) corresponding to each \( T_0 \) and \( N \), the maximum value is the optimal value of availability.

4. Example analysis

For a certain type of dump truck during the warranty period, regular preventive maintenance is usually carried out to reduce the number of breakdowns and downtime during the warranty period. Through years of experience in the use of the same type of dump truck, users have already possessed certain maintenance capabilities. In the early stage of use, there is still a gap between the user's maintenance ability and the manufacturer's. Therefore, preventive maintenance is carried out by using the two levels of strength of the manufacturer and the user respectively. Minimum maintenance after failure. During the warranty period, the manufacturer provides a preventive maintenance service for users.
The dump truck's warranty period is $T_w = 5$ years. According to expert evaluation, the maintenance degree of preventive maintenance of the vehicle by users and manufacturers is $m_1 = 1$ and $m_2 = 2$, respectively. The remaining parameters are shown in Table 1.

| parameter | parameter value |
|-----------|-----------------|
| $\delta(m_1)$ | 0.74 |
| $\delta(m_2)$ | 0.41 |
| $T_{p_1}$ | 6 days |
| $T_{p_2}$ | 15 days |
| $T_f$ | 3 days |
| $\beta$ | 3 |
| $\alpha$ | 480 |

By calculating the model with MATLAB software, it can be concluded that when the interval of preventive maintenance is $0.17T_w$ and the manufacturer carries out preventive maintenance for the third time, the highest availability is 95.77% during the warranty period. Preventive maintenance is carried out only by users, with the highest availability of 95.44% at intervals $0.13T_w$, while preventive maintenance is carried out only by manufacturers, with the highest availability of 95.02% at intervals $0.16T_w$.

By comparison, the availability of the new strategy is 0.33% and 0.75% higher than that of the original preventive maintenance carried out by the user or the manufacturer alone.

5. Result analysis
In the current warranty period, some users have introduced manufacturers to carry out preventive maintenance in order to improve product availability, but the timing of preventive maintenance is mostly decided by experience or man-made, lacking reasonable basis. In order to illustrate the effect of preventive maintenance time on product availability, two sets of data, $T_0=0.13T_w$ and $T_0=0.17T_w$, are selected from the results to illustrate. As shown in Table 2.

| $N$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-----|---|---|---|---|---|---|---|
| $A(0.13T_w, N)$ | 95.19% | 95.40% | 95.53% | 95.59% | 95.54% | 95.41% | 95.16% |
| $A(0.17T_w, N)$ | 95.29% | 95.63% | 95.77% | 95.68% | 95.34% | - | - |

When $T_0=0.13T_w$, seven times preventive maintenance were carried out. Only when the manufacturer carries out preventive maintenance 3th, 4th or 5th will the product availability be greater than that of preventive maintenance independently carried out by the user; when $T_0=0.17T_w$, five times preventive maintenance will be carried out altogether. Only when the manufacturer carries out preventive maintenance 2th, 3th and 4th can the product availability be greater than that of preventive maintenance independently carried out by the user. Therefore, if the manufacturer's choice of
preventive maintenance time is not appropriate, it will reduce the availability. So correct method must be used to make decision to determine the optimal maintenance plan.

After analysis, the above results are consistent with the actual situation. At the beginning of the warranty period, the failure rate of the product is still at a relatively low level and the product has a high reliability. If preventive maintenance is carried out by the manufacturer at this time, the effect of reducing the failure rate of the product is not obvious, which has little effect on the total number of product failures during the warranty period. In the later period of the warranty period, due to the approaching of the product warranty period, many failures have occurred, and then the manufacturer carries out preventive maintenance. Preventive maintenance, the remaining period is relatively short, even if it can reduce the number of product failures in the remaining period, but for the entire warranty period caused by failure downtime has little impact.

6. Sensitivity analysis
In order to analyze the influence of different parameters on the strategy, the failure rate function and the repair factor of preventive maintenance are used as variables to carry out sensitivity analysis. For convenience of expression, the strategy of preventive maintenance by manufacturer and user is recorded as strategy A, and the original strategy of preventive maintenance by user independently is recorded as strategy B.

a. The scale parameter $\alpha$ of failure rate is analyzed as variable, and the results are shown in Table 3.

| $\alpha$ | strategy A | strategy B | Optimal strategy |
|----|----------|----------|------------------|
| 480 | 95.77%  | 95.44%  | A                |
| 530 | 96.31%  | 96.11%  | A                |
| 580 | 96.68%  | 96.64%  | A                |
| 630 | 96.94%  | 97.02%  | B                |
| 680 | 97.13%  | 97.29%  | B                |
| 730 | 97.27%  | 97.50%  | B                |

From the results in Table 3, we can see that with the increase of product failure rate, the advantage of Strategy A decreases. When the failure rate increases to a certain value, the advantage of Strategy A disappears, and Strategy B can improve the availability of products during the warranty period. The usability trend is shown in Figure 1.

![Figure 1. Trends in product availability under different $\alpha$ conditions.](image)

We can see that if new strategies are needed to improve product availability, product failure rate must meet certain conditions.
b. The repair factor was used as a variable for sensitivity analysis. In order to facilitate the analysis, the values $\delta(m_2)$ will be taken at different maintenance levels, $\delta(m_1)$ is unchanged, the ratio of $\delta(m_2)/\delta(m_1)$ will change, and the remaining parameters will remain unchanged. The calculation results are shown in Table 4.

| $\delta(m_2)$ | $\delta(m_1)$ | $\delta(m_2)/\delta(m_1)$ | Strategy A | Strategy B | Improvement Value |
|--------------|--------------|--------------------------|-----------|-----------|------------------|
| 0.41         | 0.74         |                          | 95.77%    | 95.44%    | 0.33%            |
| 0.20         | 0.27         |                          | 96.22%    | 95.45%    | 0.77%            |
| 0.09         | 0.12         |                          | 96.41%    | 95.57%    | 0.84%            |
| 0.02         | 0.03         |                          | 96.52%    | 95.62%    | 0.90%            |

From the results in Table 4, we can see that with the increasing of preventive maintenance degree of manufacturers, $\delta(m_2)/\delta(m_1)$ decreasing gradually, and the advantage of strategy A increases continuously.

7. Conclusions and next research directions
In this paper, on the premise of the existing warranty strategy, without changing the preventive maintenance cycle, we propose two different levels of maintenance of the incomplete preventive working group of the warranty strategy, and establish the corresponding availability model. Moreover, the calculation method of preventive maintenance interval and the time of preventive maintenance by manufacturers are given. This method enhances the scientificity of warranty decision-making and is conducive to improving the reliability of products during the warranty period. Through sensitivity analysis, the influence of different factors on the strategy is obtained. However, this paper only considers the product availability during the warranty period, without considering the warranty cost. In the next step, we should combine the warranty cost and reliability to make a multi-objective comprehensive decision.

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