On the Doppler effect for photons in rotating systems

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Abstract
The analysis of the Doppler effect for photons in rotating systems, studied using the Mössbauer effect, confirms the general conclusions of a previous paper dedicated to experiments with photons emitted/absorbed by atoms/nuclei in inertial flight. The wave theory of light is so deeply rooted that it continues to be applied to describe phenomena in which the fundamental entities at work are discrete (photons). The fact that the wave theory of light can describe one aspect of these phenomena should not overshadow two issues: the corpuscular theory of light, first applied to the Doppler effect for photons by Schrödinger in 1922, is by far more complete since it describes all of the features of the studied phenomena; the wave theory can only be used when the number of photons at work is statistically significant. This disregarding of basic methodological criteria may appear to be a minor fault. However, the historical development of quantum physics shows that the predominance of the wave theory of radiation, beyond its natural application domain, has hampered the reorientation toward the photon description of the underlying phenomena.

Keywords: Doppler effect, Mössbauer effect, relativity

1. Introduction
In a previous paper, the Doppler effect for photons emitted/absorbed by atoms/nuclei in inertial flight has been discussed from Einstein’s proposal of 1907 to recent, sophisticated, experiments [1]. The focus was on the use of the wave theory of light and on the disregarding of the photon treatment, firstly suggested by Schrödinger in 1922, and based on energy and momentum conservation.

It seems to be worth extending the previous analysis to experiments with photons emitted/absorbed by nuclei without recoil (the Mössbauer effect) on rotating devices. The Mössbauer effect was discovered in 1958. The experiments exploiting this effect, aimed at verifying predictions of special or general relativity, began in the 1960s. Mössbauer-type experiments have been superseded in accuracy by experiments with unstable particles...
Nevertheless, Mössbauer-type experiments in accelerated systems maintain their conceptual interest. Recently, the old experiments of the 1960s have been critically revisited [2] and repeated [3]. These recent studies claim that the re-elaborated original experimental data obtained by Kundig [4] along with the new data, show that the relativistic effect in rotor experiments is accompanied by a new effect whose physical origin has to be ascertained [5]. Since we are interested in the confrontation between the wave and corpuscular theories of light in the description of the relativistic effect in rotor experiments, we will not discuss the claim for the new effect further.

This paper completes the previous one; it may be of some interest to university teachers and researchers.

2. Theoretical and experimental background of rotor experiments

2.1. Theoretical background

As in the case of photons emitted/absorbed by atoms/nuclei in inertial motion, the theoretical background for rotor experiments can be traced back to Einstein’s work. In his paper on general relativity, Einstein used, among others, the following argument for illustrating the necessity for general covariance of natural laws (K is an inertial frame, K’ a frame rotating with constant angular velocity with respect to K):

...let us imagine two clocks of identical constitution placed, one at the origin of coordinates, and the other at the circumference of the circle, and both envisaged from the ‘stationary’ system K. By a familiar result of the special theory of relativity, the clock at the circumference—judged from K—goes more slowly than the other, because the former is in motion and the latter at rest. An observer at the common origin of coordinates, capable of observing the clock at the circumference by means of light, would therefore see it lagging behind the clock beside him. As he will not make up his mind to let the velocity of light along the path in question depend explicitly on the time, he will interpret his observations as showing that the clock at the circumference ‘really’ goes more slowly than the clock at the origin. So he will be obliged to define time in such a way that the rate of a clock depends upon where the clock may be [6, p 152].

This is the first appearance of the idea of comparing, at least in a ‘gedanken experiment’, the readings of two clocks placed on a rotating disc at different distances from its centre.

Some years later, in an appendix to his volume on special and general relativity, this idea was developed further. Einstein considered two identical clocks on a rotating disc, one at the centre and the other at a distance r from it. The rotating reference frame is denoted by K’ and the inertial (laboratory) frame K:

A clock, at a distance r from the center of the disc, has a velocity with respect to K given by

\[ v = \omega r \]  

where \( \omega \) is the angular velocity of the disc \( K' \) with respect to \( K \). If \( v_0 \) is the number of ticks of the clock per unit time (rate of the clock) with respect to \( K \) when the clock is at rest, then the rate of the clock (\( \nu' \)) when it moves with respect to \( K \) with velocity \( v \), but at rest with respect to the disc, will be given by, [...], the relation
or, with a good approximation by

\[ v = v_0 \left( 1 - \frac{1}{2} \frac{v^2}{c^2} \right). \] (3)

This expression may be written in the form

\[ v = v_0 \left( 1 - \frac{1}{c^2} \frac{\omega^2 r^2}{2} \right). \] (4)

If we denote by \( \varphi \) the potential difference of the centrifugal force between the position of the clock and the centre of the disc, i.e. the work, taken as negative, that must be done against the centrifugal force on a unit mass for transporting it from the position of the clock on the rotating disc to the centre of the disc, then we shall have:

\[ \varphi = -\frac{\omega^2 r^2}{2}. \] (5)

\[ v = v_0 \left( 1 + \frac{\varphi}{c^2} \right). \] (6)

Therefore from this relation we see at first that two identical clocks will have different rates when at different distances from the center of the disc. This result is correct also from the point of view of an observer rotating with the disc. Since, from the point of view of the disc, a gravitational field of potential \( \varphi \) appears, the result we have obtained will be valid in general for gravitational fields. Furthermore, we can consider an atom emitting spectral lines as a clock; therefore the following statement will hold: \textit{the frequency of the light absorbed or emitted by an atom depends on the gravitational field in which the atom finds itself} [7, p 88–89, emphasis in original].

The above passages by Einstein can reasonably be considered as the conceptual source of rotor experiments.

2.2. Experimental

At the end of the 1950s, Rudolf Mössbauer discovered that a substantial fraction of \( \gamma \) photons emitted or absorbed by nuclei in crystalline solids are emitted/absorbed without energy loss due to recoil of the emitting nucleus: the recoil energy is taken up by the crystal as a whole (in the form of translational energy) instead of exciting the quantized lattice vibrations. An elementary presentation of the effect can be found in Mössbauer’s Nobel Lecture [8]; in [9] Mössbauer recalled the path to the discovery. The basic formulas can be found in the appendix. Mössbauer’s discovery was so surprising that, as recalled by Harry Lipkin:

Rudolf Mössbauer’s first paper was followed by an avalanche of papers, many of which simply reproduced Mössbauer’s experiment, obtaining exactly the same results without adding anything new, and were published in refereed journals as original results. This unique phenomenon reflected the general feeling in the nuclear physics community that this experiment must be wrong and that it was necessary to correct this error by doing the experiment right [10, p 4].
The effect was unexpected despite the fact that the theory which explained it had been available for many decades. As Lipkin puts it:

That photons could be scattered by atoms in a crystal without energy loss due to recoil was basic to all work in X-ray diffraction and crystallography. All the quantitative calculations, including the definition and evaluation of the Debye–Waller factor, were well known. But nobody interpreted this as a probability that a photon could be scattered by an atom in a crystal without energy loss due to recoil. The X-ray physicist worked entirely with the wave picture of radiation and never thought about photons. The Debye–Waller factor written as \( \exp\left\langle -\frac{k^2 x^2}{2}\right\rangle \) clearly described the loss of intensity of coherent radiation because the atoms were not fixed at their equilibrium positions and their motion introduced random phases into the scattered wave. Nobody noted that scattering the X-rays involved a momentum transfer and that coherence would be destroyed if there was any energy loss in the momentum transfer process. They did not see that the Debye–Waller factor also could be interpreted as the probability that the scattering would be elastic and not change the quantum state of the crystal [10, p 4, my italics].

The crucial point of Lipkin’s comment is that the deep rooted wave picture of radiation made it difficult to switch over to a photon picture of the phenomenon and, consequently, reinterpret the Debye–Waller factor in terms of the probability of a photon emission without energy loss due to recoil. Also the neglect of the photon description of the emission/absorption of radiation by nuclei, discussed in section 5, has contributed to hampering the reorientation toward a photon picture of the studied phenomena.

The Mössbauer effect has been used widely in many fields of physics, chemistry and biophysics, starting with the study of nuclear hyperfine interactions and testing some predictions of special and general relativity.

Around 1960, several groups began to test time dilation and gravitational red-shift by using the Mössbauer effect, among them Robert Vivian Pound and Glen A Rebka at Harvard and John Paul Schiffer and his coworkers at Harwell. These two teams were primarily concerned themselves with gravitational red-shift. A historical reconstruction of these research projects can be found in [11]. Pound and Rebka realized that the emitter’s and absorber’s temperatures, if different, may overshadow the gravitational effect [12]. They found that the measured effect—the shift of the emission and absorption lines as a function of the sample’s temperature—could be explained by a second order Doppler effect due to the mean square velocity of the emitting/absorbing nuclei. Pound and Rebka did not provide details of calculations; these were developed soon after by Sherwin [13].

These results were preliminary to the famous paper on gravitational red-shift [14]. Pound and Rebka used the 14.4 keV transition of \( ^{57}\text{Fe} \). The emitter and the absorber were placed on a vertical line 22.6 m apart. The experimental set up was arranged in the same building used 60 years before by Erwin Hall for studying the free fall of bodies. In Hall’s words: ‘the exact place of my experiments was the axis of the enclosed, isolated tower of the Jefferson Physical Laboratory of Harvard University’ [15, p 245]. Pound and Rebka corroborated the predicted red-shift with an accuracy of 10%. According to Pound and Rebka, the data previously obtained by Schiffer and coworkers are unreliable owing to the neglect, among other aspects, of the temperature effect: ‘Our experience shows that no conclusion can be drawn from the experiment of Cranshaw et al.’ [14, p 340]. Five years later, Pound and Snider reached an accuracy of 1% [16]. Pound and Rebka spoke of the ‘apparent weight of photons’ (the title of their paper), thus suggesting, in some way, that they were studying the free fall of photons as, 60 years before, Hall studied the free fall of bodies. They quoted the paper in which Einstein
showed that if a body absorbs radiation energy this energy not only increases its inertial mass but also its gravitational mass, thus preserving the identities of the two [17]. In his arguments, Einstein also used the assumption that an acceleration field is (locally) equivalent to a gravitational field. Pound and Snider chose a neutral title (the ‘Effect of gravity on gamma radiation’). They stressed their theoretical disengagement by writing:

> It is not our purpose here to enter into the many-sided discussion of the relationship between the effect under study and general relativity or energy conservation. It is to be noted that no strictly relativistic concepts are involved and the description of the effect as an ‘apparent weight’ of photons is suggestive. The velocity difference predicted is identical to that which a material object would acquire in free fall for a time equal to the time of flight [16, p 788].1

And, just a few lines above:

> The present experiment makes no direct determinations of either frequency or wavelength. The determination of a compensating source velocity is an exact operational description of the experiment.

In a recent paper, the interpretations of the red-shift experiments in static gravitational fields have been discussed in detail [18].

### 3. The rotor experiments of the 1960s

Consider an emitter and an absorber of γ photons without recoil placed at two different distances from the centre of a rigid rotating system (rotor): the γ photons passing through the absorber are detected by a counter at rest in the laboratory.

The first paper on this topic—by Hay, Schiffer, Cranshaw and Egelstaff—was published in 1960 [19]. The emitting and absorbing nucleus was 57Fe and the maximum rotation speed was 500 revolutions s\(^{-1}\).

The authors wrote:

> In an adjoining paper an experiment is described in which the change of frequency in a photon passing between two points of different gravitational potential has been measured. Einstein’s principle of equivalence states that a gravitational field is locally indistinguishable from an accelerated system. It therefore seemed desirable to measure the shift in the energy of 14 keV gamma rays from \(^{57}\)Fe in an accelerated system

[...]

The expected shift can be calculated in two ways. One can treat the acceleration as an effective gravitational field and calculate the difference in potential between the source and absorber, or one can obtain the same answer using the time dilatation of special relativity.

The authors speak of ‘the change of frequency in a photon passing between two points of different gravitational potential’: another way of expressing the idea of ‘the apparent weight

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1 For the sake of brevity, the authors did not report the following, simple, calculation. If the source is at the eight \(h\) from the ground and the absorber on the ground, the energy mismatch between the emitter and absorber is \(E_\text{emission}(gh/c^2)\) where \(E_\text{emission}\) is the energy that the photon must have in order to be absorbed by the absorber. Then, in order to absorb the photon emitted by the emitter, the absorber must be moved toward the emitter with the velocity \(gh/c\); this is the velocity change that a free falling body would acquire during a time interval equal to the time of flight of the photon \(h/c\). In fact: \(\Delta v = g\Delta t = gh/c\).
of photons’ of Pound and Rebka. Only the theoretical descriptions are referred to, without any calculations or commentary. The questions not asked by that paper were raised by Champeney and Moon [20]. They write:

Reporting a test of the effect of circular motion on the resonant frequency of gamma-ray transition in $^{57}$Fe, Hay, Schiffer, Cranshaw and Egelstaff point out that one can treat the acceleration as an effective gravitational field and calculate the frequency shift from the difference of potential between source and absorber, or one can obtain the same answer by using the time dilatation of special relativity.

For their arrangement, with the source at the center and the absorber at the periphery of the rotating system, the same result also follows from the argument that since source and absorber have relative velocity $v (\ll c)$ in a direction perpendicular to the line joining them, there exists a transverse Doppler effect giving a fractional frequency shift $v^2/2c^2$.

It is perhaps surprising that the na"ive use of this formula, without any account being taken of acceleration, should give the correct answer; an indication of the subtleties that may be involved is obtained by considering source and absorber to move on the same circle, e.g. at opposite points on the periphery. Their pseudo-gravitational potentials are equal, so are their time-dilatations, yet their relative velocity is $2v$.

Since in this laboratory we were undertaking a ‘source at center’ experiment similar to that of Hay et al., we also decided to make an experimental test of the ‘peripherally opposite’ arrangement [20, p 350].

With respect to the paper by Hay et al., the wave Doppler effect is also considered. This leads to an apparent puzzle since the experiment with an emitter and absorber at the opposite ends of a diameter shows that there is no frequency shift, while a transverse Doppler frequency shift should be expected. Champeney and Moon stress ‘the subtleties involved’ in this null result; however, but they do not discuss this ‘subtle’ point further.

In a subsequent paper, Champeney, Isaak and Khan reported more accurate measurements taken with the source at the centre and the absorber at the tip of the rotor [21]. They also reported on an experiment with the source at the tip and the absorber at the centre which confirmed the asymmetry of the phenomenon, due to the position interchange between the source and absorber. Champeney, Isaak and Khan outlined the basic points of a theoretical treatment based on the wave Doppler effect and, in commenting on the agreement between theory and experiment, the authors stressed that this agreement implies that the acceleration does not influence the rate of nuclear clocks. They also showed that the same result may be obtained by applying the time dilation of moving clocks. Strangely enough, they did not stress that the formula [21, p 584] for the Doppler effect, written in the laboratory inertial reference system, easily explains the ‘subtle point’ which one of the authors was referring to in [20].

The experiments by Kundig [4] were, according to the critical analysis by [2], more accurate. In Kundig’s experiments, the emitter was at the rotor centre and the absorber at 9.3 cm away from it; the rotor was rotated up to 35000 revolutions min$^{-1}$ (this maximum rotation speed was, more or less, a common value in the experiments of the 1960s); furthermore, the resonance condition was achieved by moving the emitter periodically toward/away from the absorber. As in the preceding papers, the theoretical descriptions referred to—without developing them—were time dilation, the wave Doppler effect and general relativity (through the weak equivalence principle). However, in Kundig’s case, the conceptual background is
more intricate. In fact, in addition to time dilation, the wave Doppler effect and the weak equivalence principle, Kundig also refers to the clock paradox, considered as a peculiar effect of accelerated systems. As he puts it: 'The frequency shift in a rotating system might be described as the transverse Doppler effect for accelerated systems, also known as the 'clock paradox' [4, p 2371']. Furthermore, in commenting on the description of the rotor experiment using the weak equivalence principle, he writes: 'We thus see that the transverse Doppler effect and the time dilatation produced by gravitation appears as two different modes of expressing the same fact, namely that the clock which experiences acceleration is retarded compared to the clock at rest [4, p 2372]'.

4. Review of theoretical descriptions

In the experimental papers discussed above, the theoretical descriptions are only referred to, with the noticeable exception of [21]. In this section, we shall present 'a rational reconstruction' of these theoretical descriptions and discuss them with regard to the confrontation of the corpuscular and wave descriptions of radiation.

4.1. Doppler effect for waves

Let us start with the general formula for the relativistic Doppler effect valid for both acoustic and light signals [22, 23]:

$$\frac{\omega_a}{\omega_e} = \frac{1 - (v_a/V) \cos(\vec{V}, \vec{v}_a)}{1 - (v_e/V) \cos(\vec{V}, \vec{v}_e)} \sqrt{1 - v_e^2/c^2} \sqrt{1 - v_a^2/c^2}. \tag{7}$$

The reference system is the one in which the medium is at rest; $\vec{V}$ is the signal velocity; and $\vec{v}_e$ and $\vec{v}_a$ are the emitter and absorber velocities. Equation (7) is obtained by assuming that either the source emits signals of ideally null duration at a specified time interval or a periodic wave. In the first case, the phenomenon’s period is the time interval between two consecutive signals; in the latter, it is the wave period. In the case of light in a vacuum, the reference system is an arbitrary inertial system and equation (7) assumes the form:

$$\frac{\omega_a}{\omega_e} = \frac{1 - (v_a/c) \cos(\vec{c}, \vec{v}_a)}{1 - (v_e/c) \cos(\vec{c}, \vec{v}_e)} \sqrt{1 - v_e^2/c^2} \sqrt{1 - v_a^2/c^2}. \tag{8}$$

This equation is particularly useful when dealing with laboratory experiments in which both the emitter and absorber are in motion. On the other hand, it easily yields the standard formula for the Doppler effect in which only the relative velocity between the emitter and absorber appears. For instance, in the reference system of the absorber $\vec{v}_a = 0$; then equation (8) reduces to:

$$\frac{\omega_a}{\omega_e} = \sqrt{1 - v_e^2/c^2} \sqrt{1 - v_a^2/c^2}. \tag{9}$$

Since, in this case, $\vec{v}_e$ is the velocity of the emitter with respect to the absorber, equation (9) shows that the Doppler effect for light in a vacuum depends only on the relative velocity between the emitter and absorber.

In the case of rotor experiments, since both cosines are null in the reference frame of the laboratory, equation (8) assumes the simple form:

$$\frac{\omega_a}{\omega_e} = \frac{\sqrt{1 - v_e^2/c^2}}{\sqrt{1 - v_a^2/c^2}} \sqrt{1 - \Omega^2 R_e^2/c^2} \sqrt{1 - \Omega^2 R_a^2/c^2}. \tag{10}$$

where $\Omega$ is the rotor angular velocity and $R_e, R_a$ are the distances from the rotor centre of the emitter and absorber, respectively. This equation shows immediately that the two frequencies
are equal if \( R_a = R_e \), i.e. if the emitter and absorber are at the opposite ends of a diameter. For small velocities \( (\Omega R \ll c) \) equation (10) can be approximated by:

\[
\frac{\omega_a - \omega_e}{\omega_e} \approx \frac{1}{2c^2} \Omega^2 \left( R_a^2 - R_e^2 \right).
\]

(11)

The above description, developed by treating the gamma radiation as an electromagnetic wave, explains the experimental results. However, the Mössbauer effect deals with the emission/absorption of photons by nuclei without recoil. This notwithstanding, we would like to carry on with a wave description; then, we have to argue as follows. It is true that nuclei emit/absorb photons; however, since we are dealing with a statistically significant number of photons emitted or absorbed (see section 6 below), we can treat them as an electromagnetic wave and apply to it the standard equations for the Doppler effect in a vacuum. As far as absorption is concerned, we know that this is possible only if the frequency of the emitted wave, measured in the reference frame of the emitter, is the same as that measured in the reference frame of the absorber; therefore, we must compensate for the frequency deficiency or excess by moving the absorber in a suitable way toward or away from the emitter; this works.

4.2. Time dilation

This description is based on the assumption that nuclei can be conceived as clocks. If \( t_0 \) is the fundamental period of a nucleus-clock at rest in the laboratory, the same period as judged by a nucleus-clock in flight with instantaneous velocity \( v_e \) will be \( t_0 \sqrt{1 - v_e^2/c^2} \). Similarly, for the nucleus-clock in flight with instantaneous velocity \( v_a \), the period will be \( t_0 \sqrt{1 - v_a^2/c^2} \). Then the numbers of strokes per unit time will be related by the formula:

\[
\nu_e = \frac{v_e}{\sqrt{1 - v_e^2/c^2}}; \quad \nu_a = \frac{v_a}{\sqrt{1 - v_a^2/c^2}}.
\]

(12)

Then:

\[
\frac{v_a}{v_e} = \frac{\sqrt{1 - v_e^2/c^2}}{\sqrt{1 - v_a^2/c^2}} = \frac{\sqrt{1 - \Omega^2 R_e^2/c^2}}{\sqrt{1 - \Omega^2 R_a^2/c^2}}
\]

(13)

which coincides with equation (10). In spite of its simplicity, this derivation is open to criticism. In fact, it is questionable that nuclei (or atoms) can be considered as clocks. A clock is made up of a frequency standard, a counting device and a display. In this context, we can forget the display and the counting device. But what about the frequency standard of a nucleus (or atom)? If we consider a single nucleus, it should be clear that it cannot be considered as a clock: it emits or absorb photons. On the other hand, if we consider a statistically significant number of photons emitted by nuclei, then we could describe the emitted photons as a wave: in this case, the frequency standard is the frequency of the emitted/absorbed wave related to the photon energy by the relation \( v = E/h \). In the case under discussion, the number of photons used in the experiments are statistically significant; a wave description is then possible. However, the wave emitted by the nuclei of the emitter leads us immediately to the previous case of the Doppler effect: the ‘time dilation explanation’ coincides with that of the Doppler effect for waves; therefore, it is not a different explanation. In any case, the ‘time dilation explanation’ stresses the fact that the transverse Doppler effect allows the measurement of the time dilation factor \( \sqrt{1 - v^2/c^2} \).

4.3. The general relativity treatment

The energy of a particle of mass \( M \) at rest in a constant gravitational field is given by [24, p 387]:
where $\phi$ is the Newtonian gravitational potential. If the particle is a nucleus in an excited state, equation (14) may be written as:

$$\mathcal{E} = (mc^2 + \Delta E)\sqrt{1 + \frac{2\phi}{c^2}}$$

(15)

where $m$ is the nucleus' mass in its fundamental state and $\Delta E$ is the energy difference between the two levels of the nuclear transition. Then, the energy difference between the two levels of the nuclear transition is modified by the gravitational potential by the term $\sqrt{1 + \frac{2\phi}{c^2}}$. Therefore, the angular frequency of the nuclear transition is given by:

$$\omega(\phi) = \omega_\infty \sqrt{1 + \frac{2\phi}{c^2}}$$

(16)

$\omega_\infty$ being the transition frequency without gravitational field. According to the weak equivalence principle, an acceleration field is locally indistinguishable from a gravitational field. Then, in a reference frame co-rotating with the rotor, the energy of a photon emitted by the emitter without recoil is given by:

$$\varepsilon_e = \Delta E\sqrt{1 - \frac{\Omega^2 R_e^2}{c^2}}$$

(17)

since $\phi = -1/2(\Omega^2 R_e^2)$ is the pseudo-gravitational potential due to acceleration. Analogously, the energy of the photon that can be absorbed by the absorber is given by:

$$\varepsilon_a = \Delta E\sqrt{1 - \frac{\Omega^2 R_a^2}{c^2}}$$

(18)

Therefore:

$$\frac{\varepsilon_a}{\varepsilon_e} = \frac{\sqrt{1 - \frac{\Omega^2 R_a^2}{c^2}}}{\sqrt{1 - \frac{\Omega^2 R_e^2}{c^2}}}$$

(19)

In the approximation of small velocities ($\Omega R \ll c$):

$$\frac{\varepsilon_a - \varepsilon_e}{\varepsilon_e} \approx \frac{1}{2} \frac{\Omega^2}{c^2} (R_e^2 - R_a^2).$$

(20)

If we compare this equation with equation (11), we see that they differ for a change in sign in the second member. We shall discuss this apparent contradiction in next section.

5. A straightforward explanation in terms of photons

In rotor experiments, photons are emitted by nuclei in the emitter and, in resonance conditions, they are absorbed by nuclei in the absorber. The macroscopic detector used for measuring the gamma radiation not absorbed by the absorber is a counter that counts the photons impinging on it. Therefore, we should look for a theoretical description based only on photons without any reference to waves. As first shown by Schrödinger [25], the emission of a photon by an atom or a nucleus can be appropriately described by writing down the conservation equations for energy and linear momentum. The Schrödinger treatment can be immediately extended to

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2 This treatment can also be found in [18, p 117]. A more general derivation can be found in [24, p 401–7].
In the laboratory’s (inertial) reference frame, the energy of the emitted photon is given by:

$$E_{\text{ph}} = E_{\text{ph}}^0 \sqrt{1 - \frac{v_1^2}{c^2} \cos \theta_1}$$  \hspace{1cm} (21)$$

with

$$E_{\text{ph}}^0 = \Delta E \left(1 - \frac{\Delta E}{2E_1}\right), \quad (v_1 = 0)$$  \hspace{1cm} (22)$$

where: \(\Delta E = (E_1 - E_2)\); \(E_1\) and \(E_2\) are the rest energy of the emitting particle before and after the emission, respectively; \(v_1\) the velocity of the particle before emission; \(\theta_1\) the angle between \(\vec{v}_1\) and the direction of the emitted photon; and \(E_{\text{ph}}^0\) is the photon energy when the emitting particle is at rest before the emission. Note that: (a) \(\Delta E\) is the energy difference between the two quantum energy levels of the transition; (b) both \(\Delta E\) and \(E_{\text{ph}}^0\) are relativistic invariants since they depend only on rest energies. In the case of absorption, the energy \(E_{\text{ph}}\) that a photon must have for being absorbed by an atom/nucleus is again given by equation (21), where, in this case:

$$E_{\text{ph}}^0 = \Delta E \left(1 + \frac{\Delta E}{2E_1}\right)$$  \hspace{1cm} (23)$$

is the energy of the photon absorbed by an atom/nucleus at rest before the absorption and, of course, now \(\Delta E = E_2 - E_1\).

Then, the Doppler effect for emitted photons is the result of the energy and momentum conservation: if the free emitting particle is at rest before emission, the energy of the emitted photon is always less than the energy difference \(\Delta E\) between the two energy levels of the quantum transition, owing to the fact that, during emission, the emitting particle recoils and gains kinetic energy. On the other hand, if the free emitting particle is in motion before emission, the energy of the emitted photon may be greater than \(\Delta E\), the extra energy coming from a decrease of the kinetic energy of the emitting particle. In general, and more precisely, the variation of the kinetic energy of the emitting particle due to emission is given by:

$$\Delta E_{\text{kinetic}} = (\gamma_2 E_2 - E_2) - (\gamma_1 E_1 - E_1).$$  \hspace{1cm} (24)$$

Since the energy of the emitted photon is \(E_{\text{ph}} = \gamma_1 E_1 - \gamma_2 E_2\) and the energy difference \(\Delta E\) between the two quantum levels is \((E_1 - E_2)\), equation (24) yields:

$$E_{\text{ph}} = \Delta E - \Delta E_{\text{kinetic}}$$  \hspace{1cm} (25)$$

where \(\Delta E_{\text{kinetic}} \neq 0\). Similar considerations can be developed for the energy that a photon must have to be absorbed by an atom or a nucleus. See also [1, p 1038–9].

The Schrödinger treatment can be applied directly to rotor experiments. For zero phonon emission or absorption, the recoil energy of the entire crystal is given by \(E_R = \Delta E^2 / 2Mc^2 \approx 0\) where \(M\) is, at least, the mass of the entire emitter/absorber^4. Then the energy of the emitted or absorbed photon, when the emitting/absorbing nucleus is at rest before emission/absorption, is simply \(\Delta E\). Therefore, since both \(\cos \theta_e\) and \(\cos \theta_a\) are zero in the laboratory reference frame, the energy of the photon emitted by the emitter is:

$$\varepsilon_e = \Delta E \sqrt{1 - \frac{v_e^2}{c^2}}$$  \hspace{1cm} (26)$$

^3 As shown in [1], the Schrödinger derivation was rapidly forgotten. When writing [1], I was not aware of the paper by Dragan Redžić on Schrödinger’s work [26]. I thank Dr Redžić for having pointed out my omission.

^4 At least, since the emitter and absorber are tightly bound to the rotor and the rotor to the laboratory.
and the energy that the photon must have for being absorbed by the absorber is:

\[ \epsilon_a = \Delta E \sqrt{1 - \frac{v_a^2}{c^2}}. \]  

(27)

If \( \Omega \) is the angular velocity of the rotor and \( R_e \) and \( R_a \) are the distances of the emitter and the absorber from the centre of the rotor, equations (26) and (27) become, in the approximation for low velocities:

\[ \epsilon_e \approx \Delta E \left(1 - \frac{\Omega^2 R_e^2}{2c^2}\right) \]  

(28)

\[ \epsilon_a \approx \Delta E \left(1 - \frac{\Omega^2 R_a^2}{2c^2}\right). \]  

(29)

Then:

\[ \frac{\epsilon_a - \epsilon_e}{\epsilon_e} \approx \frac{1}{2} \frac{\Omega^2}{c^2} (R_a^2 - R_e^2). \]  

(30)

This equation describes the experimental data. For instance, if \( R_a > R_e \), the energy that a photon must have to be absorbed by the absorber is smaller than the energy of the photon emitted by the emitter. In this case, to restore the resonance condition, the absorber must be moved away from the emitter, thus compensating for the energy mismatch by the first order Doppler effect.

Equation (30) coincides with equation (20) of the general relativity treatment; both differ from equation (11) of the wave treatment for a change in sign of the second member. We shall explain this apparent contradiction by comparing equation (30) with equation (11), i.e. by comparing the corpuscular description with that for the wave. A similar explanation holds for the comparison of the wave and general relativity treatments. In equation (30) the photon energies are measured in the laboratory reference frame: \( \epsilon_e \) is the energy of the photon emitted by the emitter and \( \epsilon_a \) is the energy that the photon must have to be absorbed by the absorber. Instead, in equation (11) the frequencies are measured in the reference frames of the emitter (\( \omega_e \)) and absorber (\( \omega_a \)). However, the predictions implied by the two equations are the same from the operational point of view. In fact, if \( R_a > R_e \), the angular frequency measured by the absorber in its reference frame is greater than that measured by the emitter in its reference frame. Then, to restore the resonance condition, the absorber must be moved away from the emitter, as required by equation (30). Of course, the two descriptions are also operationally equivalent for \( R_a < R_e \).

The equivalence of the two descriptions is only operational. The physics is different. In fact, the photon description deals with the physics of the emission and absorption process, while the wave one relies only on the frequency transformation due to the change of the reference frame. As Einstein phrased it in 1905:

In spite of the complete experimental confirmation of the theory applied to diffraction, reflection, refraction, dispersion, etc, it is still conceivable that the theory of light that operates with continuous spatial functions may lead to contradictions when it is applied to the phenomena of emission and transformation of light [27].

Within the wave description, the contradictions evoked by Einstein are turned away by simply ignoring the emission and absorption processes.

The photon description also explains the temperature effect discovered by Pound and Rebka. If, following [12], we assume that the velocity entering equation (30) is the mean nucleus velocity, the average value of \( \vec{v}_1 \cos \theta_1 \) is zero and there is no first order effect; only
the second order effect remains. Then, by expressing the mean square velocity in terms of the crystal temperature, we get the formula used by Pound and Rebka.

Schrödinger’s treatment can take also into account the dependence of the transition energy $\Delta E$ on the gravitational potential. In fact, it is sufficient to rewrite, for the emission case, equation (22) as follows:

$$E_p^0 \approx \Delta E \left(1 - \frac{\Delta E}{2E_1^0}\right) \left(1 + \frac{\phi}{c^2}\right)$$  \hspace{1cm} (31)

where $\phi$ is the gravitational potential and the $\approx$ sign is due to the approximation for small gravitational potential. An analogous correction must be made for equation (23) (the absorption case). Then, the Schrödinger treatment also describes the red-shift experiments by Pound and Rebka. In this case, it implies that the gravitational potential modifies the transition energy $\Delta E$ (the difference between the two quantum levels of the transition): there is no energy loss of a photon emitted upward or energy gain of a photon emitted downward; see also [18, p 117–8].

Finally, the general relativity description of rotor experiments can be reduced to Schrödinger’s treatment, if it is assumed—as implicitly or explicitly assumed in the papers of the 1960s—that the process of emission and absorption of a photon by an atom/nucleus in an accelerated system is the same as that in an inertial system. Then, it is sufficient to replace the gravitational potential $\phi$ in equation (31) by the pseudo-gravitational potential due to acceleration.

6. Discussion

In sections 4 and 5 it was shown that rotor experiments can be described by three different approaches: the wave theory of light, the corpuscular theory of light and general relativity. However, in section 5, it has been argued that the general relativity treatment can be included in Schrödinger’s treatment of the emission/absorption of a photon by an atom/nucleus. Therefore, only two descriptions are left: the wave and corpuscular descriptions.

When there are two theories that describe the same experiment, we must use some epistemological criteria to choose between them. In our case, it is sufficient to use a criterion that can reasonably be considered as widely accepted: between two theories choose the one that describes more features of the phenomenon under scrutiny, i.e. the theory that is more complete. The corpuscular treatment is, by far, the more complete one. In fact, it describes the process of emission and absorption of a photon by an atom/nucleus by focusing on the physical state of the emitting or absorbing particle: in doing this, it can take into account any possible dependence of the physical state of the emitting/absorbing particle on external physical agents such as, for instance, gravitational potential. The wave description ignores the emission and absorption processes with their dynamical implications and only answers the question: given two inertial reference systems, what is the relation between the frequencies of an electromagnetic wave measured in the two systems? Therefore, it is not surprising that its application to an experiment in which only discrete entities (photons) are at work obscures some fundamental features of the phenomenon. The reconstruction of Mössbauer’s discovery by Lipkin quoted above is, in this respect, paradigmatic: the deeply rooted wave theory of radiation in the description of the elastic scattering of x-rays by crystals rendered it difficult to reorient people’s minds toward a probabilistic interpretation in terms of photons of the Debye–Waller factor, necessary for the explanation of the Mössbauer effect.

The case of the Mössbauer effect is not the only one in which the wave description of radiation had to give way to the corpuscular one. Twentieth-century physics has recorded
several cases that have been, at the same time, fundamental landmarks in the development of quantum physics. It is worth recalling these steps. We have to begin with Planck’s derivation of the black body radiation law based on two breakthroughs: the use of the statistical approach to thermodynamics (summarized by Boltzmann’s formula for the entropy: $S = A \ln W$) and the hypothesis that, given $N$ harmonic resonators of frequency $\nu$ in thermal equilibrium inside a cavity, the energy should be attributed to them in terms of the basic quantity $h\nu$, where $h$ is a ‘constant of Nature’ (1900) [28]. In 1905, Einstein introduced the concept of light quantum as a heuristic tool that, incidentally, explained the fact that there is a frequency threshold in the photoelectric effect [27]. A year later, Einstein stressed that Planck’s derivation of the black body radiation law presupposes the idea of light quanta and the hypothesis that the energy of an oscillator is quantized [29]. Furthermore, Planck’s starting formula for the energy density in the cavity:

$$u(\nu) = \frac{8\pi \nu^2}{c^3} \langle U(\nu) \rangle$$  \hspace{1cm} (32)

(where $\langle U(\nu) \rangle$ is the average energy of the oscillator of frequency $\nu$) is derived by supposing that, classically, the oscillators continuously exchange energy with the cavity radiation. Then, the quantization of the term $\langle U(\nu) \rangle$ is in contradiction with the derivation of the equation containing it: a rigorous derivation of Planck formula implies the derivation of the term $8\pi \nu^2 / c^3$ independent of Maxwell’s theory.

Originally, Einstein’s light quanta had only energy $E = h\nu$; some ten years later, Einstein also endowed them with a linear momentum $h\nu / c$ [30]. The first, successful application of these two properties of light quanta was Schrödinger’s corpuscular treatment of the Doppler effect for photons (1922). Compton [31] and Debye [32], after about ten years of painstaking and frustrating use of the wave theory of radiation, explained the inelastic scattering of x-rays by switching to the photon description. Finally, in 1924, Bose outlined the main passages for a rigorous deduction of the black body radiation law by developing the statistics of light quanta contained in a cavity at thermal equilibrium [33].

The cases of Planck and Compton–Debye are characterized by the fact that the switch from the wave to the corpuscular theory of radiation was forced by the need to explain experimental results long since resistant to a wave description. They are clear examples of the tendency to maintain acquired theories as long as possible, in spite of the stubborn resistance of experimental results to be explained by them. In the case of the Doppler effect for photons, the switch to the photon description has been avoided by focusing only on the frequencies measured in the reference frames of the emitter and absorber. Usually, physicists stay clear from epistemological issues. Exceptions to this habit have been, in some cases, the origin of creative insights: Einstein’s special theory of relativity and the light quanta hypothesis are two examples; another example is given by the Bose statistics of light quanta.

Einstein’s formula $E = h\nu$ relating the energy of a light quantum to the frequency of a monochromatic electromagnetic wave and de Broglie’s relation $\lambda = h / p$ giving the wavelength to be associated with a particle (this relation also holds for light quanta) constitute the two pillars of the so-called wave–particle duality. We will not deal with the multifaceted question of whether the wave–particle duality is a real property of the physical world or the product of sedimentation of theoretical layers which are, perhaps, waiting for new insight. Instead, we shall discuss how a beautiful two slit (Fresnel prism) experiment with one photon at a time [34] can be described using the wave or the photon approach. In this experiment, the interaction of an incoming photon with the detector produces a localized bright spot. Only when the number of photons is statistically significant (about two thousands are enough), the interference pattern begins to appear as the ‘sum’ of the spots. Maxwell’s theory can be twisted to explain the appearance of localized spots by assuming, ad hoc, that the probability for a photon to hit
Table 1. Wave and quantum description of light interference.

| Maxwell                            | Quantum                          |
|------------------------------------|----------------------------------|
| $\lambda = c/\nu$                  | $\lambda = h/p = c/\nu = hc/E$   |
| Electric field $\vec{E}$           | Probability amplitude $\Psi = Ce^{i\phi}$ |
| $\vec{E} = \vec{E}_1 + \vec{E}_2$ | $\Psi = \Psi_1 + \Psi_2$        |
| Energy density $\propto E^2$       | Probability $\propto |\Psi|^2$ |

a point of the detector is proportional to the classical intensity predicted for that point. This ad hoc assumption, while confirming the vitality of Maxwell’s theory, is incompatible with its continuous nature. On the other hand, if the number of photons is statistically significant, Maxwell theory predicts the correct result, independent of the fact whether the photons are used one at a time or all at once. If someone shows us a picture of the experiment performed by [34], we could not say if that picture has been obtained by using, say 20000 photons one at a time or all at once. Instead, if we choose to describe the experiment using the photon concept, we can refer to the elementary treatment of the electrons’ interference given by Feynman [35, p 37–35]. The only thing we have to do is to replace electrons with photons: as recalled above, de Broglie’s formula is also valid for photons. Then, we shall find—as stressed by Feynman [35, p 37–36]—that the wave and particle descriptions have a common mathematical structure, shown in table 1.

This common mathematical structure explains why the wave description predicts the correct result when the number of photons is statistically significant. However, this common mathematical structure does not tell us why a statistically significant number of photons can be treated as a wave, independent of the fact that they are used one at a time or all at once. This unanswered question can be put in another way. All of our experimental and theoretical acquired knowledge tells us that light is composed of ‘a finite number of energy quanta which are localized at points in space, which move without dividing, and which can only be produced and absorbed as complete units’ as Einstein boldly predicted in 1905 [27, p 368]. Then, why is a wave description possible under the circumstances of sufficiently many photons? We do not know the answer: we only know that it works.

One further comment: the wave description of the two slit experiment cannot say which is the path of the energy between the two slits and the detector, just as in the quantum description we cannot say through which slit the photon has passed; this is another aspect of the common mathematical structure of the two descriptions.

7. Conclusions

The wave theory of light is so deeply rooted that it continues to be applied to describe phenomena in which the fundamental entities at work are discrete (photons): the Doppler effect for photons, studied in [1] and in this paper, is, from this viewpoint, paradigmatic. The fact that the wave theory of light can describe one aspect of these phenomena cannot overshadow two issues: the corpuscular theory of light, first applied to the Doppler effect for photons by Schrödinger in 1922, is far more complete since it describes all of the features of the studied phenomena; the wave theory can be used only when the number of photons at work is statistically significant. This disregard of basic methodological criteria may appear as to be a minor fault. However, the historic development of quantum physics shows that the predominance of the wave theory of radiation, beyond its natural application domain, has hampered the reorientation toward the photon description of the underlying phenomena.
Appendix. Basic formulas of the Mössbauer effect

In the case of emission of a photon by a nucleus in a crystal, the recoil energy $E_R$ delivered to the crystal can be written as:

$$E_R = E_{\text{transl}} + E_{\text{vib}}.$$  \hspace{1cm} (A.1)

The emission probability of a photon without excitation of lattice vibrations (zero phonon process, in the language of solid state physics) is given by the (reinterpreted) Debye–Waller factor $f$:

$$f = \exp \left[ -\frac{E_R}{k_B\Theta_D} \left( \frac{3}{2} + \frac{\pi^2 T^2}{\Theta_D^2} \right) \right]$$ \hspace{1cm} (A.2)

if we assume the Debye model for lattice vibrations and $T \ll \Theta_D$. $\Theta_D$ is the Debye temperature defined as $k_B\Theta_D = \hbar\omega_D$, $\omega_D$ being the maximum angular frequency of Debye model. From (A.2) we see that the emission probability of a photon without excitation of lattice vibrations increases with decreasing recoil energy and temperature and with increasing Debye temperature. As $T \to 0$, $f$ becomes constant and assumes the value:

$$f \approx \exp \left( -\frac{3E_R}{2k_B\Theta_D} \right).$$ \hspace{1cm} (A.3)

The zero phonon emission line has a natural line width (width at half height) given by $\Gamma_0 = \hbar/\tau_n$ where $\tau_n$ is the lifetime of the excited state of the nuclear transition. For the 14.4 keV transition of $^{57}\text{Fe}$, $\Gamma_0 = 4.7 \times 10^{-9}$ eV and the energy resolution is $\Gamma_0/E_\gamma = 3.26 \times 10^{-13}$. Now suppose that the zero phonon line emitted by the emitter is absorbed by the absorber. If we move the emitter toward/away from the absorber, the absorber will be driven off resonance when the two zero phonon lines of the emitter and absorber no longer overlap, due to the first order Doppler effect. Then, by plotting the measured absorption as a function of the emitter velocity, the absorption curve will have a half width of about $2\Gamma_0$. The velocity $v = c\Gamma_0/E_\gamma$ is the one required to go from the maximum of the measured absorption line to its half value. For the 14.4 keV transition of $^{57}\text{Fe}$, $v = 9.8 \times 10^{-2}$ mm s$^{-1}$.

Mössbauer obtained his first result with the 129 keV line of $^{191}\text{Ir}$ the properties of which are somewhat worse than those of the 14.4 keV transition of $^{57}\text{Fe}$: a larger natural line width ($4.6 \times 10^{-6}$ eV), lower energy resolution ($3.5 \times 10^{-11}$) and lower fraction of emitted photons without recoil owing to their larger energy (see equation (A.2)). The better performance of the 14.4 keV transition of $^{57}\text{Fe}$ is the reason why successive experiments have been performed using this transition.

If, for some reason, the energy of the photons emitted through a zero phonon transition is altered by an amount larger than the natural half width, these photons cannot be absorbed by the absorber. In order to restore resonance, the absorber, for instance, must be moved toward/away from the emitter with an appropriate velocity, thus compensating through the first order Doppler effect for the energy mismatch.

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