Evidence for the MSW effect

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Abstract. Recent solar and reactor neutrino data have convincingly established that electron neutrinos and antineutrinos are subject to flavour transitions driven by neutrino masses and mixing. In addition, such data can be used to prove that the interaction of neutrinos in matter modifies the flavour transition pattern with respect to the case of propagation in vacuum, as predicted long ago by Mikheyev, Smirnov and Wolfenstein (MSW). We present a brief review of how the current evidence for MSW solar neutrino transitions has developed in recent years, and how it has been strengthened by the latest reactor neutrino data presented at the Neutrino 2004 Conference.

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1. Introduction and notation

The Cl [1], Ga [2, 3], Super-Kamiokande (SK) [5]–[7] and Sudbury Neutrino Observatory (SNO) [8]–[11] solar neutrino experiments have established that the deficit of the observed solar $\nu_e$ flux...
with respect to expectations [12, 13] implies new neutrino physics beyond the standard model. In particular, the charged and neutral current (CC and NC) data from SNO have proven the occurrence of $\nu_e$ transitions into a different active state $\nu_a$ with high statistical significance [11].

Such transitions were predicted long ago as a consequence of possible flavour oscillations [14] in the $\nu_e \rightarrow \nu_a$ channel ($\nu_a$ being a linear combination of $\nu_\mu$ and $\nu_\tau$) driven by nonzero $\nu$ squared mass difference and mixing angle parameters ($\delta m^2, \theta_{12}$) [15]. We now know that the $(\nu_\mu, \nu_\tau)$ combination orthogonal to $\nu_a$ is probed by atmospheric $\nu$ oscillations [16], with different parameters ($\Delta m^2, \theta_{23}$) [17]. We also know that the third mixing angle $\theta_{13}$, needed to complete the $3 \times 3$ mixing matrix, is bound to be rather small ($\sin^2 \theta_{13} < \text{few %}$) by additional reactor results [17, 18], and we can set it to zero to a good approximation for our purposes (unless otherwise noticed). An updated overview of such a ‘standard’ neutrino oscillation scenario can be found in [19].

The first (2003) results from the Kamioka Liquid scintillator AntiNeutrino Detector (KamLAND) [20] have provided a beautiful and crucial confirmation of the solar $\nu_e$ oscillation picture through a search for long-baseline oscillations of reactor $\bar{\nu}_e$'s. Also the recent KamLAND data released at the Neutrino 2004 Conference [21, 22] clearly indicate that the observed disappearance of $\bar{\nu}_e$ in KamLAND is consistent with the same region of the $(\delta m^2, \theta_{12})$ parameter space which explains the solar neutrino deficit, the so-called large mixing angle (LMA) solution.

In the LMA parameter range, flavour transitions between $\nu_e$ and $\nu_a$ should be significantly affected by the neutrino interaction energy difference $V = V_e - V_a$ arising in solar (and possibly Earth) background matter [23, 24],

$$V(x) = \sqrt{2} G_F N_e(x),$$  

(1)

where $N_e$ is the electron number density at the point $x$. The associated flavour change, known as Mikheyev–Smirnov–Wolfenstein (MSW) effect [23], should occur adiabatically [25] in the solar matter, for LMA parameters (see [26] for a recent discussion).

In the context of the Hamiltonian ($\mathcal{H}$) evolution of $2\nu$ active flavours, the MSW effect enters through a dynamical term $\mathcal{H}_{\text{dyn}}$ in matter, in addition to the kinetic term $\mathcal{H}_{\text{kin}}$ in vacuum:

$$i \frac{d}{dx} \begin{pmatrix} \nu_e \\ \nu_a \end{pmatrix} = \begin{pmatrix} \mathcal{H}_{\text{dyn}} + \mathcal{H}_{\text{kin}} \\ 0 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_a \end{pmatrix},$$  

(2)

where

$$\mathcal{H}_{\text{dyn}} = \frac{V(x)}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$  

(3)

and

$$\mathcal{H}_{\text{kin}} = \frac{\delta m^2}{4E} \begin{pmatrix} -\cos 2\theta_{12} & \sin 2\theta_{12} \\ \sin 2\theta_{12} & \cos 2\theta_{12} \end{pmatrix},$$  

(4)

$E$ being the neutrino energy.

In this work, we review the current evidence for $\mathcal{H}_{\text{dyn}} \neq 0$, i.e., for the occurrence of dynamical MSW effects in the solar matter, induced by the interaction energy $V$ in equation (1).
In order to quantify such evidence, we introduce [27, 28] a free parameter $a_{\text{MSW}}$ which modulates the overall amplitude of the dynamical term $\mathcal{H}_{\text{dyn}}$ through the substitution

$$V \rightarrow a_{\text{MSW}} \cdot V,$$

whenever matter is traversed by neutrinos (both in the Sun and in the Earth). This parameter formally interpolates between the case of no matter effect,

$$a_{\text{MSW}} = 0 \quad \text{(no MSW effect)},$$

and the case of standard matter effects,

$$a_{\text{MSW}} = 1 \quad \text{(standard MSW effect)}.$$  

We will show that the case $a_{\text{MSW}} = 0$ is ruled out, the data globally prefer $a_{\text{MSW}} \sim 1$, and that the previously large uncertainties in the allowed range of $a_{\text{MSW}}$ are significantly reduced by including the recent (2004) KamLAND data.

In the following, we discuss the evidence for the MSW effect in historical perspective, by showing the impact of four data sets which, in combination with previous results from the Cl, Ga and SK experiments [1]–[3], [5, 6] (as well from the CHOOZ reactor experiment [18]), have been crucial to test and constrain the MSW effect: the SNO-I data in 2001–2002 [8]–[10], the first KamLAND data in 2002 [20], the SNO-II data in 2003 [11] and the KamLAND data presented at the Neutrino 2004 Conference [21, 22]. As a useful complement to such a historical overview, the reader is referred to [19] for a detailed global analysis of neutrino oscillation data.

2. The impact of solar and reactor data during the period 2001–2003

Figure 1, based on the works [28]–[30], shows the dramatic progress in constraining the $2\nu$ oscillation parameters ($\delta m^2, \theta_{12}$) during the years 2001–2003. This progress is relevant also for the emerging evidence in favour of the MSW effect, as discussed below.

2.1. Converging towards the LMA MSW solution

For many years, the physics of the MSW effect has been mostly investigated in the limit of small $\theta_{12}$ (see, e.g., the review in [31]), for at least two reasons. The first was the theoretical prejudice in favour of small neutrino mixing, in analogy with the quark case. The second was the fact that, for small $\theta_{12}$, the MSW effect can become dramatically large; in particular, when $V$ is such that the diagonal elements of $\mathcal{H}$ vanish, the Hamiltonian becomes maximally off-diagonal in the flavour basis, thus inducing large flavour transition amplitudes—despite the smallness of $\theta_{12}$. This so-called ‘resonance’ condition can be easily realized in the Sun (where $V$ spans several orders of magnitude from the core to the photosphere [12]) and predicts strong distortions in the neutrino energy spectrum. Figure 1(a) shows the $2\nu$ fit to 2001 solar neutrino data (Cl + Ga + SK), prior to SNO-I data [29, 32], where mixing angle values as small as $\tan^2 \theta_{12} \sim 10^{-4}$–$10^{-3}$. New Journal of Physics 6 (2004) 139 (http://www.njp.org/)
Figure 1. Results of global analyses of solar and terrestrial neutrino experiments during the years 2001–2003. (a) All solar neutrino data (Cl + Ga + SK) before SNO (2001), (b) including SNO-I data (2001–2002), (c) including first KamLAND data (2002), (d) including SNO-II data (2003). Based on [28]–[30].
At the turn of the millennium, however, the increasing evidence for no spectral distortions in the SK solar neutrino experiment [5, 6] started to decrease the likelihood of small mixing solutions to the solar neutrino problem, in favour of large mixing ones. At about the same time, theoretical prejudices against large mixing had to face the increasing evidence in favour of $\theta_{23} \sim \pi/4$ from atmospheric neutrino data [16]. Eventually, small solar neutrino mixing was ruled out in 2001–2002 by the data from the SNO experiment (SNO-I phase [9]), which strongly enhanced the tension between total rate and spectrum shape data at small $\theta_{12}$ (see, e.g., [29, 32, 33]).

Figure 1(b) shows the combination of all solar neutrino experiments as of 2002 (Cl + Ga + SK + SNO-I) [29], which allows only solutions at relatively large values of $\theta_{12}$. At 99% CL, the figure shows a best-fit solution at $\delta m^2 \sim \text{few} \times 10^{-5}$ eV$^2$ (called LMA), where MSW effects are expected to modulate the energy profile of the solar neutrino survival probability [26, 34]. However, figure 1(b) also shows other (non-LMA) solutions at lower $\delta m^2$, where the observability of the MSW effect is more problematic. For example, for $\delta m^2$ as low as a few $\times 10^{-10}$ eV$^2$, matter effects are dominant in principle inside the Sun (since $V \gg \delta m^2 / E$, i.e., $|H_{\text{dyn}}| \gg |H_{\text{kin}}|$); in practice, however, they simply inhibit flavour transitions up to the Sun’s surface (since $H \sim H_{\text{dyn}}$ is approximately flavour-diagonal inside the Sun), so that observable flavour transition effects are actually dominated by vacuum oscillations outside the Sun, where $H \sim H_{\text{kin}}$ (see [35] and references therein).

Therefore, from the solar neutrino data available in 2002 we have learned that MSW effects should be sought at large, and not small, values of the mixing angle $\theta_{12}$, and most likely in the so-called LMA region. However, there was still room for other regions of the mass-mixing parameter space where, from the phenomenological viewpoint, MSW effects would be unobservably small, i.e., where $a_{\text{MSW}} \simeq 0$ would be allowed. It is thus fair to say that the 2002 solar neutrino data did not provide compelling evidence in favour of matter effects in the Sun.

At the end of 2002, however, the allowed parameter space in figure 1(b) was dramatically reduced by the first evidence for $\bar{\nu}_e$ disappearance in the long-baseline reactor experiment KamLAND [20]. Figure 1(c) shows that the combination of all solar + reactor 2002 data uniquely selects the LMA region and splits it into (at least) two subregions, conventionally called LMA-I (at best fit, $\delta m^2 \sim 6 \times 10^{-5}$ eV$^2$ and $\sin^2 \theta_{12} \sim 0.3$) and LMA-II (at somewhat higher $\delta m^2$), see e.g. [30]. The reduction of the parameter space around the LMA region(s) is very important, when combined with the SK+SNO preference for $P_{ee} \sim 1/3 < 1/2$, where $P_{ee}$ is the average $\nu_e$ survival probability in the SK–SNO energy range [29, 32].

Indeed, for typical LMA parameters and for $a_{\text{MSW}} = 1$ (standard matter effects), MSW transitions occur adiabatically in the Sun [23, 25], leading to a survival probability of the form (up to residual Earth matter effects):

$$P_{ee} \simeq \cos^2 \theta'_{12} \cos^2 \theta_{12} + \sin^2 \theta'_{12} \sin^2 \theta_{12} \quad (a_{\text{MSW}} = 1),$$

where $\theta'_{12}$ is the rotation angle which diagonalizes $H$ at the $\nu_e$ production point in the solar core. On the other hand, for hypothetically zeroed matter effects ($a_{\text{MSW}} = 0$), one would get an energy-independent form for $P_{ee}$ in the LMA region,

$$P_{ee} \simeq 1 - \frac{1}{2} \sin^2 2\theta_{12} \quad (a_{\text{MSW}} = 0),$$

as originally suggested by Gribov and Pontecorvo [36] prior to the MSW papers [23] (see also [37, 38]).

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In the SK and SNO energy range (approximately $E > 5$ MeV), the above two expressions lead to comparable results in the second octant of the mixing angle ($\theta_{12} > \pi/4$), but differ considerably in the first octant, where $P_{ee}(\alpha_{\text{MSW}} = 1) < 1/2$, while $P_{ee}(\alpha_{\text{MSW}} = 0) > 1/2$. Therefore, within the LMA region, proving the occurrence of MSW effects in the Sun is equivalent to proving that $P_{ee} < 1/2$, which can only happen if the second octant ($\theta_{12} \geq \pi/4$) is ruled out. The 2002 SK+SNO data favoured $P_{ee} \sim 1/3$ [29], but the case $P_{ee} = 1/2$ (as well as $\theta_{12} = \pi/4$) was not convincingly ruled out; indeed, figure 1(c) shows that, combining all 2002 solar+reactor data, the LMA-I solution is still marginally compatible with maximal mixing. The discrimination of the cases $\alpha_{\text{MSW}} = 0$ and $\alpha_{\text{MSW}} = 1$ thus required a more precise determination of the experimental-to-theoretical ratio of CC to NC events in SNO, which is independent of standard solar model predictions, and directly measures the average of $P_{ee}$ over the SNO and SK energy response functions with appropriate thresholds [29], [39]–[41].

In summary, from 2002 solar + reactor data we have learned that the solar neutrino deficit is explained by neutrino flavour transitions with mass-mixing parameters in the LMA region. In this region, the SNO and SK experiments can prove the occurrence of MSW effects in the Sun by showing that the energy-averaged survival probability $\langle P_{ee} \rangle$ is smaller than $1/2$ (which can only happen if $\theta_{12} < \pi/4$). In 2002, however, the emerging SNO+SK indications in favour of $\langle P_{ee} \rangle < 1/2$, although intriguing, were not compelling in our opinion [27].

2.2. Proving MSW effects in the LMA solution

In 2003, the SNO experiment finally provided compelling evidence for $\langle P_{ee} \rangle < 1/2$ through the so-called phase-II data, which allowed a better statistical separation of CC and NC events [11, 42]. In particular, a model-independent combination of SNO+SK data provided the following upper bound [28]:

$$\langle P_{ee} \rangle < 0.43 \ (3\sigma).$$

(10)

The above model-independent limit is crucial to our discussion, and its derivation deserves some comments. As already mentioned, it has been shown in [40] (see also [29]) that the SK and SNO experiments probe the same energy-averaged $\nu_e$ survival probability $\langle P_{ee} \rangle$ to a good accuracy, provided that the detector thresholds are appropriately chosen. For the SNO-II kinetic energy threshold ($T_{\text{SNO}} = 5.5$ MeV) and energy resolution [11], we find that the equivalent SK threshold is $E_{\text{SK}} \simeq 7.8$ MeV in total energy [27]. For equalized thresholds, the SK elastic scattering (ES) flux and the SNO NC and CC fluxes are linked by the exact relations [40]

$$\Phi_{ES}^{\text{SK}} = \Phi_B [\langle P_{ee} \rangle + r(1 - \langle P_{ee} \rangle)],$$

(11)

$$\Phi_{CC}^{\text{SNO}} = \Phi_B \langle P_{ee} \rangle,$$

(12)

$$\Phi_{NC}^{\text{SNO}} = \Phi_B,$$

(13)

where $r = 0.154$ is the ratio of (properly averaged) $\nu_{\mu,\tau}$ and $\nu_e$ CC cross-sections, and $\Phi_B$ is the true $^8$B neutrino flux from the Sun. From the above equations, one can (over)constrain both $\Phi_B$ and $\langle P_{ee} \rangle$ in a truly model-independent way, namely, without any prior assumption about the energy profile of $P_{ee}$ or about $\Phi_B$ predictions in standard solar models (SSMs).
Figure 2. Isolines of the energy-averaged solar \( \nu_e \) survival probability \( \langle P_{ee} \rangle \) with standard MSW effects (\( a_{\text{MSW}} = 1 \)) and with no matter effects (\( a_{\text{MSW}} = 0 \)). The grey region is allowed by the current SK + SNO combination (dominated by SNO-II data). No such region exists in the absence of MSW effects. See also [28].

Using the above set of equations one gets

\[
\Phi_B = (5.5 \pm 1.2) \times 10^6 \text{ cm}^{-2} \text{s}^{-1} (3\sigma),
\]

in good agreement with SSM predictions [12, 13], and

\[
\langle P_{ee} \rangle = 0.31^{+0.12}_{-0.08} (3\sigma),
\]

from which equation (10) is derived. The above 3\( \sigma \) limits on \( \langle P_{ee} \rangle \) are in very good agreement with the ‘3\( \sigma \) range’ obtained by naively triplicating the errors of the SNO CC/NC flux ratio, which is a direct measurement of \( \langle P_{ee} \rangle \): \( \Phi_{\text{CC}}^{\text{SNO}} / \Phi_{\text{NC}}^{\text{SNO}} = 0.306 \pm 0.105 (3\sigma) \) [11]. However, as emphasized in [42], the errors of the CC/NC ratio are not normally distributed, and should not be used in fits. Conversely, our bounds in equation (15) are statistically safe and well-defined [28].

Figure 2 illustrates the impact of the above bounds through isolines of \( \langle P_{ee} \rangle \) in the \( (\delta m^2, \sin^2 \theta_{12}) \) parameter range relevant to the LMA region, for both \( a_{\text{MSW}} = 1 \) (left panel) and \( a_{\text{MSW}} = 0 \) (right panel) [28]. The superposed grey region is allowed by the 3\( \sigma \) model-independent bounds in equation (15). No such region exists in the case \( a_{\text{MSW}} = 0 \), which is therefore rejected.
at the $3\sigma$ level (at least). In the standard MSW case (left panel), the allowed region appears to confine $\delta m^2$ below $\sim 2 \times 10^{-4}$ eV$^2$ and $\sin^2 \theta_{12}$ below $\sim 0.4$. In particular, the SNO-II data significantly contribute to reject maximal mixing ($\sin^2 \theta_{12} = 1/2$) and to reduce the likelihood of the ‘high $\delta m^2$’ LMA-II parameter region, as also evident in the global fit of figure 1(d).

In conclusion, from the combination of 2003 solar and reactor data—including the crucial contribution of SNO-II data—we have obtained the first compelling evidence in favour of MSW effects, by ruling out the null hypothesis of no matter effects in the Sun ($\alpha_{\mathrm{MSW}} = 0$). The evidence is closely related to the evidence for $\langle P_{ee} \rangle < 1/2$ in SK and SNO (figure 2), and to the confinement of the LMA solution(s) in the first octant, $\theta_{12} < \pi/4$ (figure 1(d)). The reader is referred to [28] for further details.

### 3. The impact of KamLAND 2004 data

At the Neutrino 2004 Conference, the KamLAND Collaboration has released new reactor neutrino data, which not only confirm disappearance of electron neutrino flavour, but also show the first imprint of energy spectrum distortions due to an oscillatory pattern [21, 22]. These new spectral data have greatly reduced the multiplicity of solutions and the uncertainties on $\delta m^2$ as compared with previous KamLAND-only fits [20], especially in the region of relatively high $\delta m^2$ (LMA-II and higher), where the KamLAND event energy spectrum would be undistorted. This fact helps the observability of MSW effects in the Sun, which would vanish at relatively high $\delta m^2$ (i.e., for $H_{\text{kin}} \gg H_{\text{dyn}}$) in the LMA region. The 2004 KamLAND data, in combination with solar neutrino data, are thus expected to enhance the evidence for MSW effects that emerged [27] from 2003 data.\(^1\)

In our analysis of the KamLAND 2004 data, we fit the 13-bin energy spectrum reported in [22] with the same statistical procedure as in [30], but taking into account the new estimate of the accidental and cosmogenic backgrounds [22] as in [19], and the updated detector energy resolution, average reactor fuel composition, and unoscillated reactor neutrino flux distribution from [22]. A more accurate (event-by-event) likelihood analysis could be performed if the KamLAND collaboration would disclose further (but currently unpublished) information about time variations of reactor neutrino fluxes and of the detector lifetime, as well as about the energy and time tagging of each event, as emphasized in [43, 44].\(^2\)

Figure 3 shows our analysis of the updated 2004 solar neutrino data (including the SSM input from [13]) at 99.73% CL for 2 d.o.f. (degrees of freedom), superposed to our fit to 2004 KamLAND data at the same CL. The combined solar + KamLAND region (shown at four different CLs) is in good agreement with the one shown in the official KamLAND 2004 analysis [22], as well as in other recent papers [19, 46]. The global best fit $[(\delta m^2/\text{eV}^2, \sin^2 \theta_{12}) = (8.3 \times 10^{-5}, 0.29)]$ is controlled by the intersection of the KamLAND LMA-I horizontal band with the ‘vertical’ solar allowed region. The ‘LMA-II’ KamLAND-only solution is borderline (CL $\simeq 3\sigma$) and may marginally appear (as in [22]) or marginally disappear (as in figure 3) at $3\sigma$, depending on small variations in the analysis. The separate solar and KamLAND best-fit $\delta m^2$ values are not coincident, the latter $\delta m^2 = 8.3 \times 10^{-5}$ eV$^2$ being a factor 1.5 higher than the former $\delta m^2 = 5.5 \times 10^{-5}$ eV$^2$) in our analysis. Such small mismatch [22], although not

\(^1\) It should also be noted that the recent KamLAND data can help to improve the upper bounds on $\theta_{13}$, see [19].

\(^2\) We also mention that background estimates are currently being revised by the KamLAND Collaboration, and that minor changes in the best-fit values of $\delta m^2$ and $\sin^2 \theta_{12}$ can be expected accordingly [45].
In conclusion, the combination of solar and reactor neutrino data (as available at the Neutrino 2004 Conference) leads to a very satisfactory situation: KamLAND and solar data are highly consistent with each other, and select a single LMA solution in the oscillation parameter space, where the null hypothesis of no MSW effects in the Sun is ruled out. In the next section we go beyond the test of the null hypothesis, and quantify how well the magnitude of the MSW contribution to the hamiltonian ($H_{\text{dyn}}$) is determined by experimental data.

4. Bounds on the size of MSW effects: evolution from 2002 to 2004

According to equations (3) and (4) the size of the dynamical (MSW) and kinematical (vacuum) terms in the neutrino propagation Hamiltonian is governed by the prefactors $V$ and $\delta m^2/E$, respectively. The relative ratio $V/(\delta m^2/E)$ is small for KamLAND neutrinos propagating in the Earth, while it can be of $O(1)$ for solar neutrinos propagating in the Sun (within LMA parameters).

**Figure 3.** Our global analysis of solar and KamLAND data in the $2\nu$ oscillation case, including the new results presented at the Neutrino 2004 Conference [4, 21, 22], as well as the updated standard solar model input from [13].

alarming from a statistical viewpoint, governs the deviation of the best-fit value of $a_{\text{MSW}}$ from unity, as discussed in the next section.
This means that KamLAND data are crucial to fix the kinematical part of the Hamiltonian ($H_{\text{kin}}$), while only solar neutrinos can really test the dynamical part ($H_{\text{dyn}}$). So far, we have shown that the combination of all data constrains the mass-mixing parameters of $H_{\text{kin}}$ around a single ‘LMA point’ (figure 3), and that the null case $H_{\text{dyn}} = 0$ is ruled out at $>3\sigma$ (figure 2).

However, one can do better than just switching the MSW effect on ($a_{\text{MSW}} = 1$) and off ($a_{\text{MSW}} = 0$). By treating the parameter $a_{\text{MSW}}$ (equation (5)) as a continuous variable, one can check whether the ‘solar + KamLAND’ combination of data can constrain matter effects in the Sun to have the right size [$a_{\text{MSW}} \sim O(1)$]. We have thus performed global analyses with ($\delta m^2$, $\sin^2 \theta_{12}$, $a_{\text{MSW}}$) unconstrained, including solar and reactor data as available in 2002 [27], 2003 [28] and 2004 (this work). The results are marginalized with respect to ($\delta m^2$, $\sin^2 \theta_{12}$), in order to obtain the relevant bounds on $a_{\text{MSW}}$. In a sense, such bounds tell us how well the Fermi constant is ‘measured’ through MSW effects ($V = \sqrt{2} G_F N_e$) on solar neutrino flavour transitions, i.e., $a_{\text{MSW}}$ can be interpreted as

$$a_{\text{MSW}} = \frac{G_F (\nu_\odot)}{G_F}.$$

Figure 4 shows the statistical $\Delta \chi^2$ variations as a function of $a_{\text{MSW}}$, derived by our analysis of 2002 data (including SNO-I and first KamLAND results) [27], 2003 data (including SNO-II results) [28] and 2004 data (including the latest KamLAND results [22]). The horizontal dotted lines at $\Delta \chi^2 = n^2$ correspond to $n-\sigma$ bounds ($n = 1, 2, 3$ and 4). CHOOZ data [18] are also included, being relevant in setting upper limits to $\delta m^2$ (from reactor data only) in the 2002 and 2003 fits; they are currently less relevant, since KamLAND 2004 data place an upper bound on $\delta m^2$ by themselves. The three curves are discussed below in historical sequence.

With 2002 data, upper and lower bounds on $a_{\text{MSW}}$ are already visible. In particular, the hypothetical case of zeroed matter effects is already disfavoured at a formal $\sim 3.5\sigma$ level, thus providing an indirect indication in favour of matter effects in the Sun. However, the 2002 $\Delta \chi^2$ curve also presents several quasi-degenerate minima, and allows a $\pm 3\sigma$ range for $a_{\text{MSW}}$ which spans about three orders of magnitude. The width of this range can be understood by recalling the following facts [27]: (1) The LMA range of $\delta m^2$ constrained by solar neutrino data, which covers about one decade (figure 3), can be shifted up or down by shifting $a_{\text{MSW}}$ with respect to 1, since the LMA oscillation physics depends on $V/\delta m^2 \propto a_{\text{MSW}}/\delta m^2$. (2) The range of $\delta m^2$ constrained by KamLAND + CHOOZ 2002 data, which covers about two decades [30], is much less affected by $a_{\text{MSW}}$ variations. As a consequence, by appropriately shifting $a_{\text{MSW}}$, it is possible to overlap the reconstructed ranges of $\delta m^2$ from solar and from reactor data over about 1 + 2 decades. When the overlap sweeps through the degenerate $\delta m^2$ intervals allowed by KamLAND [30], the fit is locally improved, leading to a ‘wavy’ structure in $\Delta \chi^2$. In conclusion, although 2002 solar + reactor data strongly disfavoured $a_{\text{MSW}} = 0$ (zeroed matter effects) and provided a best fit close to $a_{\text{MSW}} = 1$ (standard matter effects), the presence of other local minima in the $\Delta \chi^2$ function, as well as the broad $3\sigma$ allowed range for $a_{\text{MSW}}$, did not provide a clear evidence of standard matter effects [27].

In 2003, the previous 2002 ‘indication’ in favour of matter effects has become a real ‘evidence’ [28] by means of the SNO-II data, as already mentioned in section 2. In fact, the 2003 curve in figure 4 puts both upper and lower bounds on $a_{\text{MSW}}$ at a safe CL ($>5\sigma$), and shows an intriguing (although weak) preference for $a_{\text{MSW}} \sim 1$. However, as for the 2002 curve, also

3 We refer the reader to [27, 28] for technical details of this kind of analysis.

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Figure 4. Bounds on $a_{\text{MSW}}$, considered as a continuous free parameter, from the combination of all solar, CHOOZ and KamLAND data, as available in the years 2002, 2003 and 2004 (Neutrino 2004 Conference). For the ‘2002’ and ‘2003’ curves, see also [27] and [28], respectively. Horizontal short dashed lines are drawn at the 1, 2, 3 and 4σ CL ($\Delta \chi^2 = 1, 4, 9$ and 16). A clear trend emerges in favour of MSW effects with standard magnitude ($a_{\text{MSW}} = 1$), and against the hypothetical case of no matter effects (now excluded at $>6\sigma$). In particular, using the Neutrino 2004 data, the case $a_{\text{MSW}} = 1$ is confirmed within a factor of $\sim 2$ uncertainty at 2σ (95% CL).

the 2003 allowed 3σ range of $a_{\text{MSW}}$ appears to be rather large and with several quasi-degenerate minima, basically governed by the multiplicity of KamLAND-only solutions (unchanged from 2002 to 2003). Therefore, although 2003 solar + reactor data provided clear evidence in favour of the MSW effect, they did not allow us to ‘measure’ the magnitude of MSW effects [28].

Finally, the solid curve in figure 4 shows the most recent bounds on $a_{\text{MSW}}$, as obtained by our analysis of the solar + reactor data, updated after the Neutrino 2004 Conference [4, 21, 22]. The null hypothesis ($a_{\text{MSW}} = 0$) is now rejected at $>6\sigma$. Moreover, the dramatic reduction of the multiplicity of KamLAND solutions (figure 3) leads to a corresponding reduction of the allowed range for $a_{\text{MSW}}$, which can now be ‘measured’ within a factor of $\sim 2$ (at the $\pm 2\sigma$ level) around the best fit, which is found at $a_{\text{MSW}} \simeq 1.6$. The slight deviation of the best fit from 1 reflects the slight mismatch of the best fit values of $\delta m^2$ from solar and KamLAND data separately.
(see previous section), which is traded for an increase of $a_{\text{MSW}}$, when this parameter is left free. Within the uncertainties, it can be claimed that the size of standard MSW effects in solar neutrino flavour transitions has been experimentally determined within a factor of $\sim 2$ at 95% CL. Neutrino oscillation data thus provide an alternative (although inherently coarse) way to probe the magnitude of standard electroweak interactions (equation (16)) through the MSW effect—a very satisfactory result.

A final remark is in order. We have assumed for simplicity 2ν flavour oscillations in the channel $\nu_e \rightarrow \nu_a$ ($\nu_a$ being a linear combination of $\nu_\mu$ and $\nu_\tau$) driven by the $(\delta m^2, \theta_{12})$ parameters. The $(\nu_\mu, \nu_\tau)$ combination orthogonal to $\nu_a$ is probed by atmospheric $\nu_\mu \rightarrow \nu_\tau$ oscillations, with different parameters $(\Delta m^2, \theta_{23})$. As far as the third mixing angle $\theta_{13}$ is zero (and $\delta m^2 / \Delta m^2 \ll 1$), the two oscillation channels are practically decoupled, and all our previous considerations hold without changes. However, for small but nonzero $\theta_{13}$, the 3ν survival probability slightly deviates from the 2ν case for both solar and KamLAND $\nu_e$ oscillations:

$$P_{3\nu} \approx (1 - 2 \sin^2 \theta_{13}) P_{2\nu}.$$

This deviation is expected to lead to a slight relaxation of all previous bounds. For instance, in testing the null hypothesis $a_{\text{MSW}} = 0$ (section 2.2), the minimum value of $(P_{ee})$ in the right panel of figure 2 can slightly decrease from 1/2 to $1/2 - \sin^2 \theta_{13}$. Since the 3σ upper bound on $\sin^2 \theta_{13}$ is at the level of a few per cent only (0.05–0.07, depending on details of global analyses, see e.g. [19, 47]), the relaxation of the upper bound on $(P_{ee})$ is marginal, and does not change our conclusions. In the more general case of variable $a_{\text{MSW}}$, we have not performed the 3ν generalization of the analysis presented in this section and in figure 4. Our educated guess is that an allowance for small values of $\theta_{13}$ should only slightly weaken—but should not spoil—the main results presented in this work.

5. Conclusions and prospects

In recent years, solar and reactor neutrino data have been shown to be consistent with (and to favour) MSW effects in the flavour evolution of solar neutrinos. We have witnessed the emergence of indications in favour of MSW effects in the Sun in 2002, of compelling evidence for such effects in 2003, and of a first ‘measurement’ of the magnitude of such effects (within a factor of $\sim 2$ at 95% CL) in 2004. In particular, the rejection of the null hypothesis of no MSW effect in 2003 (figure 2) and the increasingly stringent bounds on the size of the MSW dynamical term $\mathcal{H}_{\text{dyn}}$ in equation (5) during the years 2002–2004 (figure 4) show a dramatic progress in this sense. At the same time, figures 1 and 3 testify the progress in constraining the mass-mixing parameters which govern the kinematical term $\mathcal{H}_{\text{kin}}$ in equation (4).

We conclude by observing that, although MSW effects are an expected and unavoidable consequence of the standard theory of electroweak interactions, their basic confirmation in the current neutrino phenomenology represents an important and reassuring experimental accomplishment, which strengthen our confidence in the emerging picture of neutrino masses and mixings. Further confirmation of the MSW effects in the Sun might come from more precise solar and reactor data, as well as from possible future indications for slight MSW-induced distortions in the low-energy part of the SK and SNO spectra. Finally, in a more distant future, one might also hope to prove the occurrence of MSW effects in the Earth through very accurate determinations of day–night differences in the solar neutrino flux (not discussed in this work).
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