Two-loop contribution to the pion transition form factor vs. experimental data

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We present predictions for the pion-photon transition form factor, derived with the help of light-cone sum rules and including the main part of the NNLO radiative corrections. We show that, when the Bakulev-Mikhailov-Stefanis (BMS) pion distribution amplitude is used, the obtained predictions agree well with the CELLO and the CLEO data. We found that no model distribution amplitude can reproduce the observed \( Q^2 \) growth of the new BaBar data, though the BMS model complies with several BaBar data points.

1. Form factor \( F^\gamma \gamma^\pi \) in QCD, collinear factorization

This transition form factor describes the process \( \gamma^\ast(q_1)\gamma^\ast(q_2) \rightarrow \pi^0(p) \) and is given by the following matrix element: \(-q_1^2 \equiv Q^2 > 0, -q_2^2 \equiv q^2 \geq 0\)

\[
\int d^4x e^{-iq_1 \cdot z} \langle 0 \mid T \{j_\mu(z)j_\nu(0)\} \mid 0 \rangle =
\]

\[
i\epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \cdot F^{\gamma^\ast \gamma^\pi} (Q^2, q^2). \tag{1}
\]

For sufficiently large photon momenta \( Q^2, q^2 \gg m^2 \) (where the hadron scale is set by the \( \rho \)-meson mass \( m_\rho \)), all binding effects can be accumulated into a universal (twist-two) pion distribution amplitude (DA), so that, on account of collinear factorization, one obtains the form factor as a convolution:

\[
F^{\gamma^\ast \gamma^\pi} (Q^2, q^2) = T(Q^2, q^2, \mu_F^2; x) \otimes \varphi^{(2)}_\pi (x; \mu_F^2) + O (Q^{-4}) \tag{2}
\]

While the pion DA \( \varphi^{(2)}_\pi \), which represents a parameterization of the pion matrix element at the (low) factorization scale \( \mu_F^2 \), cannot be calculated from first principles and has to be modeled within some nonperturbative approach, the amplitude \( T \), describing the hard parton subprocesses, can be calculated in QCD perturbation theory. On the other hand, also the evolution of the pion DA with \( \mu_F^2 \) is controlled by perturbatively calculable evolution kernels and associated anomalous dimensions. Hence, in leading order (LO) of the strong coupling and at leading twist two one has

\[
F^{\gamma^\ast \gamma^\pi} = \int_0^1 dx \frac{N_f}{Q^2 x + q^2 x} \varphi^{(2)}_\pi (x) + O (Q^{-4}) \tag{3}
\]

with \( N_f = \frac{N_c - 4}{3} f_\pi \) and \( \bar{x} = 1 - x \). The pion DA \( \varphi^{(2)}_\pi \) is defined as

\[
\langle 0 | \bar{q}(z) \gamma_\mu \gamma_5 C(z, 0) q(0) | \pi(P) \rangle \bigg|_{z^2 = 0} =
\]

\[
i P \int dx e^{ix (z \cdot P)} \varphi^{(2)}_\pi (x, \mu_F^2), \tag{4}
\]

where \( C(z, 0) = \mathcal{P} \exp \left( ig \int_0^z A_\mu(\tau) d\tau^\mu \right) \) is a path-ordered exponential to ensure gauge invariance, and can be reconstructed from its moments, e.g., within the framework of QCD sum rules using either local [1] or nonlocal condensates [23].

The radiative corrections to the hard amplitudes \( T \) [Eq. (2)] in next-to-leading order (NLO),...
encapsulated in $T_1$, have been computed in \cite{4}. More recently, the $\beta$-part of the next-to-next-to-leading order (NNLO) amplitude $T_2$, i.e., $\beta_0 \cdot T_3$, was also calculated \cite{5}.

2. Light Cone Sum Rules for the process

\[ \gamma^\ast (Q^2) \gamma (q^2 \approx 0) \rightarrow \pi^0 \]

Experimentally, the transition form factor was measured by different Collaborations \cite{6,7,8} when one photon is quasi real ($q^2 \to 0$). This kinematics requires the modification of the standard factorization formula Eq. \((2)\) in order to take into account the long-distance interaction, i.e., the hadronic content of the photon (below the effective threshold $\rho$ of vector mesons, viz., the “physical” spectral density $\rho$ contains the partonic part and is accumulates the hadronic content of the photon (below the effective threshold $s_0$) in terms of the form factors of vector mesons, viz.,

\[ \rho_{ph}(Q^2, s) = \sqrt{2} F_V F^{\gamma^\ast \gamma \pi}(Q^2) \cdot \delta(s - m_V^2), \quad (6) \]

where $V$ stands for a $\rho$ or an $\omega$ meson, while $\rho_{PT}$ contains the partonic part and is based on Eq. \((2)\) via the relation $\rho_{PT}(Q^2, s) = \text{Im} \left[ (T \otimes \varphi_{\pi})(Q^2, -s) \right]$. Using quark-hadron duality in the vector channel, it is possible to express the pion-photon transition form factor $F^{\gamma^\ast \gamma \pi}(Q^2, 0)$ in terms of $\rho_{PT}$:

\[ F^{\gamma^\ast \gamma \pi}(Q^2) = \frac{1}{\pi} \int_{s_0}^{\infty} \frac{\text{Im} (T \otimes \varphi_{\pi})(Q^2, -s)}{s} ds + \frac{1}{\pi} \int_{0}^{s_0} \frac{\text{Im} (T \otimes \varphi_{\pi})(Q^2, -s)}{m^2_T} e^{(m^2_T - s)/M^2} ds. \quad (7) \]

Here $s_0 \simeq 1.5$ GeV$^2$ and $M^2$ denotes the Borel parameter in the interval $(0.5 - 0.9)$ GeV$^2$.

Partial results for $\rho_\pi^{(1)}$ at the NLO level have been presented in \cite{10}, while the general solution $\rho_\pi^{(1)}(Q^2, s) = \text{Im} \left[ (T_1 \otimes \psi_n)(Q^2, -s) \right]$ was recently obtained in \cite{11}:

\[ \rho_\pi^{(1)}(x; \mu^2) = C_F \left\{ -3 \left[ 1 - v^n(n) \right] + \frac{\pi^2}{3} \right\} \]

\[ -\ln^2 \left( \frac{x}{\mu^2} \right) + 2v(n) \ln \left( \frac{x^2 Q^2}{\mu^2} \right) \psi_n(x) \]

\[ -C_F 2 \sum_{i=0,2,\ldots} (G_{nl} + v(n) \cdot b_{nl}) \psi_i(x). \quad (8) \]

Here $\{\psi_n\}$ are the Gegenbauer harmonics which constitute the LO eigenfunctions of the Efremov-Radyushkin-Brodsky-Lepage (ERBL) evolution equation, with $v(n), \varphi^{(n)}(n)$ being the corresponding eigenvalues, whereas $G_{nl}$, $b_{nl}$ are calculable triangular matrices (see for details \cite{11}).

Predictions for $F^{\gamma^\ast \gamma \pi}(Q^2)$ at the NLO were given in \cite{12} for various pion DAs, notably for the asymptotic one, the CZ model \cite{1}, and the BMS model \cite{3}. We found that the result of the NLO processing of the CLEO data \cite{7}, the BMS bunch of DAs \cite{3}, and the most recent lattice estimates of the second moment of the pion DA are in good mutual agreement and inside the 1σ error ellipse.

The inclusion of the NNLO contribution to the main partial spectral density $\rho_\pi^{(2)}$, proportional to $\beta_0$, was realized in \cite{11} taking recourse to the results of \cite{6}. It turns out that it is negative and about $-7\%$ at small $Q^2 \sim 2$ GeV$^2$, decreasing rapidly to $-2.5\%$ at $Q^2 \geq 6$ GeV$^2$. The net result is a slight suppression of the prediction for the scaled form factor (see Fig. \cite{11}).

3. NNLO LCSR results vs. BaBar data

Very recently, the BaBar Collaboration published new results on the pion-photon transition form factor that cover a wide range of momenta $4 < Q^2 < 40$ GeV$^2$ with high precision \cite{5}. Surprisingly, their data exceed the asymptotic QCD prediction $\sqrt{2} F_{\pi}^\ast$ already at $\sim 10$ GeV$^2$ and continue to grow with $Q^2$ up to the highest measured momentum. This behavior would indicate that the $\gamma^\ast \gamma \to \pi$ process cannot be correctly described within the convolution scheme of QCD based on the collinear factorization. We argued

\footnote{The effect of a more realistic Breit-Wigner ansatz for the meson resonance in Eq. \((6)\) was taken into account in \cite{11}.}
in \cite{11} that the inclusion of the NNLO radiative corrections cannot reconcile the BaBar data with perturbative QCD for any pion DA that vanishes at the endpoints \( x = 0, 1 \)—see Fig. 1. Indeed, from Table 1 it becomes clear that also a wide pion DA, like the CZ model, cannot reproduce all BaBar data both in isolation or jointly with the CLEO data \cite{7}. We conclude:

![Figure 1](image-url)  

**Figure 1.** \( Q^2 F_{\gamma^* \pi}(Q^2) \) calculated with three different pion DAs: Asymptotic (lower solid line), BMS (shaded green strip), and CZ (upper solid red line). The BaBar data \cite{5} are shown as diamonds with error bars. The CELLO \cite{6} and the CLEO data \cite{7} are also shown. The displayed theoretical results include the NNLO \( \beta \) radiative corrections and the twist-four contributions. The horizontal dashed line marks the asymptotic QCD prediction \( \sqrt{Q^2} \).

(i) The combined effect of the negative NNLO pion DA radiative corrections and the Breit-Wigner ansatz for the meson resonances in Eq. (7) results in a moderate suppression of \( Q^2 F_{\gamma^* \pi}(Q^2) \) in the range of momentum transfers 10–40 GeV\(^2\) \cite{11}.

(ii) The hadronic content of the real photon is a twist-four contribution that is rapidly decreasing with increasing \( Q^2 \) and can, therefore, not be the origin of the enhancement of the scaled form factor measured by BaBar.

(iii) Within the QCD convolution scheme, all pion DA models (cf. the \( \chi^2 \) in Table 1), which have a convergent projection onto the Gegenbauer harmonics, and hence vanish at the endpoints 0, 1, are in conflict with the BaBar data for \( Q^2 F_{\gamma^* \pi}(Q^2) \) between 10 and 40 GeV\(^2\) (see Fig. 1), because in this range these data violate the collinear factorization formula per se.

**Table 1**  
\( \chi^2_{\text{val}} \) for Asymptotic (Asy), BMS, and CZ DAs

| Pion DA | BaBar and CLEO data | BaBar only |
|--------|----------------------|------------|
| Asy    | 11.5                 | 19.2       | 19.8 |
| BMS    | 4.4                  | 7.8        | 11.9 |
| CZ     | 20.9                 | 36.0       | 6.0  |

**4. Can the BaBar data be explained?**

There are no formal explanations of the BaBar data within the general framework of QCD at present—there are no reasons to adduce. However, some theoretical scenarios have already been proffered \cite{13,14,15}. We will discuss one class of such proposals based on the idea that the pion DA may be “practically flat”, hence violating the collinear factorization and entailing a (logarithmic) growth of \( Q^2 F_{\gamma^* \pi}(Q^2) \) with \( Q^2 \). One has \cite{14} \((\sigma = 0.53 \text{ GeV}^2; \varphi_\pi(x) = f_\pi)\):

\[
Q^2 F_{\gamma^* \pi} = \frac{\sqrt{2}}{3} \int_0^1 \frac{\varphi_\pi(x)}{x} \left[ 1 - e^{-x \frac{Q^2}{\Lambda^2}} \right] dx. \tag{9}
\]

Another option \cite{15} gives instead \((m \approx 0.65 \text{ GeV})\):

\[
Q^2 F_{\gamma^* \pi} = \frac{\sqrt{2}}{3} \int_0^1 \frac{\varphi_\pi(x, Q)}{x + \frac{m^2}{Q^2}} dx \tag{10}
\]

with \( \varphi_\pi(x, \mu_0) = f_\pi(N + (1 - N)6x\bar{x}), (N \approx 1.3) \). Eq. (10) can be compared with the available experimental data \cite{6,7,8} by parameterizing them via the phenomenological fit \((\Lambda = 0.9 \text{ GeV}, b \approx -1.4)\):

\[
Q^2 F_{\gamma^* \pi} = \frac{Q^2}{2\sqrt{2}f_\pi \pi^2} \left[ \frac{\Lambda^2}{\Lambda^2 + Q^2} + b \left( \frac{\Lambda^2}{\Lambda^2 + Q^2} \right)^2 \right]
\]

as shown in Table 2. The main message is that one cannot fit the CELLO/CLEO data and the BaBar data with the same accuracy simultaneously with both parameterizations. Now let us
push this point further and divide the BaBar data into two ‘experiments’ BaBar1 and BaBar2, as indicated graphically in Fig. 2. One sees from Table 3 in terms of a $\chi^2_{ndf}$ criterion that the flat-DA scenario cannot describe both BaBar ‘experiments’ simultaneously with the same accuracy. One could interpret this outcome as an indication for an intrinsic inconsistency in the analysis of the BaBar data that deserves further attention.

5. Conclusions

In conclusion, we have presented a calculation within the LCSR approach of the pion-photon transition form factor which includes the main NNLO radiative corrections and twist-four contributions. Our predictions are based on collinear factorization and agree with the data of the CELLO and the CLEO Collaborations but greatly disagree with the new high-$Q^2$ BaBar data. Our analysis shows that all pion DAs, which vanish at the endpoints $x = 0, 1$, cannot reproduce the observed growth of the scaled form factor above 10 GeV$^2$. On the other hand, flat pion DAs may describe this growth at high $Q^2$, but at the expense that they fail to comply with the whole set of the BaBar data and also with those of the CELLO and the CLEO Collaborations. Future experiments may clarify this situation.

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