A Dark Side of Neutrino Mass

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We propose a simple scenario that directly connects the dark matter (DM) and neutrino mass scales. Based on an interaction between the DM particle $\chi$ and the neutrino $\nu$ of the form $\chi\nu\nu/\Lambda^2$, the DM annihilation cross section into the neutrino is determined and a neutrino mass is radiatively induced. Using the observed neutrino mass scale and the DM relic density, the DM mass and the effective scale $\Lambda$ are found to be of the order MeV and GeV, respectively. We construct an ultraviolet-complete toy model based on the inverse seesaw mechanism which realizes this potential connection between DM and neutrino physics.

INTRODUCTION

The Standard Model (SM) is not able to explain the existence of Dark Matter (DM) in the universe as well as the finite masses of neutrinos. Experimentally, both phenomena are firmly established. The two neutrino mass-squared differences are very well measured in neutrino oscillation experiments\(^1\). Together with the upper limit on the sum of the neutrino masses, $\sum m_\nu \lesssim 0.66$ eV, derived from cosmological observations\(^2\), they imply that the heaviest active neutrino has a mass of 0.05 to 0.22 eV. While the DM mass is largely unconstrained, the crucially important DM relic abundance is very well measured at $\Omega h^2 = 0.12$\(^2\).

Connections between DM physics and the origin and size of the neutrino masses have been proposed in the literature in the context of radiative neutrino mass models, for example in Refs.\(^3\)\(^4\), where the neutrino mass is induced radiatively with DM particles and heavy neutrinos in the loop. In these models, the neutrino mass scale depends on the DM and heavy neutrino masses as well as various coupling constants. This implies that the DM mass can not be uniquely determined given the observations, unless other model parameters are fixed. An alternative scenario was proposed in Refs.\(^5\)\(^6\). Similar to our case, it connects neutrino physics with an MeV scale DM particle, although the underlying model is quite different.

In this work, we propose a simple scenario that connects the DM particle and neutrino mass scales. Starting with an effective operator 6-dimensional operator

$$\frac{\chi\nu\nu}{\Lambda^2}, \quad (1)$$

where $\chi$ refers to a gauge singlet Majorana DM particle, while $\nu$ is the SM neutrino\(^7\). Here and in the following, we use the two-component Weyl spinor notation for all fermionic fields. We implicitly assume that $\chi$ is odd under a $Z_2$ symmetry to ensure its stability. Assuming that this operator is the only one coupling DM to SM particles, the DM annihilation cross section times the DM relative velocity $\sigma v_{\text{rel}}$ is approximated by $\sigma v_{\text{rel}} \approx m_\chi^2/\langle \pi \Lambda^4 \rangle$. This implies a DM relic abundance of $\Omega h^2 \approx 8.2 \times 10^{-10} \text{GeV}^{-2}/\langle \sigma v_{\text{rel}} \rangle$. On the other hand, the neutrino receives a radiative mass by contracting two $\chi$ fields in the interaction operator, $m_\nu \approx 55 m_\chi^4/\langle \pi^2 \Lambda^2 \rangle$.

Using the experimental data on the DM relic abundance and the light neutrino mass scale, the DM mass $m_\chi$ and the scale $\Lambda$ of the interaction operator can be determined easily,

$$m_\chi \approx 0.4 \text{ MeV} \left( \frac{m_\nu}{0.1 \text{ eV}} \right)^{1/2} \left( \frac{\Omega h^2}{0.12} \right)^{1/4}, \quad (2)$$

$$\Lambda \approx 1.5 \text{ GeV} \left( \frac{m_\nu}{0.1 \text{ eV}} \right)^{1/4} \left( \frac{\Omega h^2}{0.12} \right)^{3/8}. \quad (3)$$

Naturally, $m_\chi$ and $\Lambda$ are of the order MeV and GeV, respectively. The effective operator scale $\Lambda$ is far below the electroweak (EW) scale, which is why the operator is not invariant under the SM gauge group. It also naturally implies the existence of at least one more particle lighter than the EW scale in order to obtain the interaction operator $\chi\nu\nu/\Lambda^2$. We proceed by constructing a possible model that realizes the previous operator in two steps: firstly by discussing an effective Lagrangian, and then a possible fully ultraviolet (UV)-complete toy model.

EFFECTIVE LAGRANGIAN

The natural scale of the operator\(^1\) is GeV and in order to discuss a possible SM effective model we have to introduce another light particle that connects the DM sector with the SM. In addition, we assume that the only source of lepton number violation (LNV) is situated in the hidden sector and which is generating the DM Majorana mass. We do not specify this source of LNV but it could for example result from a seesaw-like mechanism in the hidden sector. Note that one has to make sure that in the UV-complete theory, the hidden sector does

\(^1\) We neglect the flavour structure of the three neutrinos and work with one Majorana neutrino field with mass scale $m_\nu \approx 0.1$ eV.
not couple to the SM directly, i.e. it has to go through the DM particle $\chi$. Therefore, any other effective operators have to conserve lepton number which for example forbids the Weinberg operator $LHLH$.

We introduce a complex scalar $\Phi$ with two units of lepton number, $L(\Phi) = 2$ which connects the DM and SM sectors, 

$$\mathcal{L} \supset c_2 \Phi\chi + \frac{\Phi^* LHLH}{\Lambda^2} + h.c..$$

(4)

Here, $L$ and $H$ are the SM lepton and Higgs boson doublets, respectively. Choosing $L(\chi) = -1$, the Lagrangian conserves lepton number. After integrating out $\Phi$ and EW breaking, $H = (0, v)^T$, one obtains

$$\mathcal{L} \supset \frac{\chi \nu \nu^*}{\Lambda^2} + h.c.,$$

(5)

where $\Lambda = \Lambda_x m_\chi/\sqrt{c_2 v}$.

Due to the Majorana nature of the DM particle $\chi$, the light neutrino $\nu$ will obtain a loop-induced Majorana mass becomes

$$m_\nu = \frac{m_x^2}{2\pi^2\Lambda^2} \left(6 \ln \frac{m_x}{\mu} - 1\right),$$

(6)

using the dimensional regularization scheme with modified minimal subtraction\(^2\), renormalized at the scale $\mu$. We take $\mu$ to be the neutrino mass $m_\nu$, with the incoming momentum $p$ set to zero.

On the other hand, the relic abundance of $\chi$ is determined by the same effective operator\(^3\). The DM annihilation cross section reads, up to $v^2_{\text{rel}}$,

$$\sigma_{\text{rel}} = \frac{m_\chi^2}{\pi \Lambda^2} \left(1 + \frac{1}{2} v^2_{\text{rel}}\right).$$

(7)

The DM relic density can then be obtained from the thermally averaged annihilation cross section $\langle \sigma v_{\text{rel}} \rangle$ as for example described in Ref. [7].

Given the observed neutrino mass scale and the DM relic density, the DM mass $m_\chi$ is around the sub-MeV scale while $\Lambda \approx 2$ GeV, as estimated in the previous section. The general relation between the model parameters $m_\chi$, $\Lambda$ and the observables $m_\nu$, $\Omega h^2$ is shown in Fig. 2. The red curve denotes the observed relic abundance $\Omega h^2 = 0.12$ while the blue (purple) line corresponds to the upper (lower) limit on the heaviest active neutrino mass. The fact that $\Lambda$ is much smaller than the EW scale justifies the explicit EW symmetry breaking of Eq. (5) and it implies the existence of the light particle $\Phi$ in this scenario.

**UV-COMPLETE TOY MODEL**

As a final step, we construct a UV-complete toy model that in turn generates the effective Lagrangian and the low energy DM-neutrino interaction, as shown in Fig. 3. The corresponding Lagrangian reads

$$\mathcal{L} \supset \frac{c_1}{2} (\Phi_x + \langle \Phi_x \rangle) \chi \chi + c_2 \Phi \chi \chi + c_3 \Phi^* \xi \xi$$

$$+ yLHN - m_{\chi} \Phi_x \Phi_x^* - m_{\phi} \Phi \Phi - m_N \xi + h.c.,$$

(8)

where $\Phi_x$ and $\Phi$ are scalar fields with lepton number $L = 2$, $N$ and $\xi$ are heavy Dirac neutrinos with opposite $L$. The vacuum expectation value (VEV) $\langle \Phi_x \rangle$ of $\Phi_x$ generates the DM mass $m_\chi = c_1 \langle \Phi_x \rangle$. In principle, $\Phi_x$ could be very heavy compared to $\langle \Phi_x \rangle$. For instance, $\Phi_x$ may couple to another scalar $\phi$ such that

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\(^2\) As we shall see later, this is well justified in the UV-complete model which has exactly same loop structure.

\(^3\) One has to include the contribution from $\chi^4 \chi^4 \nu^4 \nu^4 / \Lambda^2$, which involve a different chirality. The interference between the two different chirality contributions is tiny, being proportional to the very small neutrino mass $m_\nu$. 
This linear term will induce a small VEV of $\Phi$ as light neutrino mass will be $m$, which is exactly the inverse seesaw [8]. The resulting $\langle \Phi \rangle$ in the toy model.

In our letter we focus purely on the relation between the neutrino mass generation and the DM annihilation. As an outlook, we would like to comment on other potential signatures of the model. The DM annihilation cross section is $S$-wave dominated without velocity suppression. This implies a neutrino flux due to ongoing DM annihilation, for example, from the Galactic center. The DM mass and hence the energy scale of the neutrino flux is of order MeV, in the vicinity of the energy threshold of neutrino experiments such as Super-Kamiokande [14], KamLAND [15, 16], SNO [17] and Borexino [18]. See Ref. [19] for a recent study of MeV DM using neutrino experiments.

The fact that the effective scale $\Lambda$ is naturally of order GeV implies the existence of light exotic states. With regard to direct searches at colliders, it is difficult to make a general statement without fully specifying a UV-complete model. For a TeV scale neutrino $N$, $m_\Phi \approx$ GeV. The scalar $\Phi$ only couples indirectly through $N$ and is only by the DM particle $\chi$. Secondly, the heavy particles that are being integrated do not enter the $\chi$-loop, which radiatively induces the light neutrino mass. It is a distinctive feature of the model, setting it apart from the existing literature, for instance Refs. [11–13], where heavy particles exist inside loops that give rise to radiative neutrino masses. It is this feature that renders our model more predictive.

CONCLUSIONS

In this letter, we propose a simple scenario that directly connects the physics of DM and neutrino masses. The introduced operator $\chi\chi\nu\nu/\Lambda^2$ induces a radiative Majorana neutrino mass as well as leads to the DM annihilation. Given the observed DM density and the neutrino mass scale, the DM mass $m_\chi$ and the operator scale $\Lambda$ are uniquely fixed to be of order MeV and GeV, respectively.

The UV-complete toy model satisfies two very important requirements necessary for this mechanism to work: Firstly, LNV arises in the hidden sector, and it is mediated to the SM sector (including right-handed neutrinos)
hardly constrained by collider searches. If $\Phi$ couples to the SM Higgs via $\Phi^* \Phi H (H)$, invisible Higgs decays without phase space suppression would be generated. The mass range of the heavy quasi-Dirac neutrino $N$ (and $\xi$) is confined to be $100 \text{ GeV} \lesssim m_N \lesssim 100 \text{ TeV}$ with a relatively large heavy-light neutrino mixing. Therefore, the heavy neutrino production cross section could be sizeable at the LHC.

Finally, we would like to point out that the MeV scale DM particle can contribute to the entropy of the universe during the time of Big Bang Nucleosynthesis (BBN). In our scenario, DM is still in thermal equilibrium with neutrinos after they decouple from the thermal bath around the temperature $T = 2.3 \text{ MeV}$ [20]. The DM particles subsequently become non-relativistic and transfer entropy to the neutrinos, thereby re-heating the neutrino temperature with respect to that of photons. This leads to a larger number of relativistic degrees of freedom $N_\nu$ during the time of last scattering producing the cosmic microwave background (CMB). MeV scale Majorana DM coupling to the neutrinos will result in $N_\nu^{\text{BBN}} = 4$ and $N_\nu^{\text{CMB}} = 4.4$ [21]. This is consistent with the BBN observation, $1.8 < N_\nu^{\text{BBN}} < 4.5$ [1, 22], but it is in tension with $N_\nu^{\text{CMB}} = 3.36 \pm 0.34$ from the CMB data alone [2].

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