Quantum Imaging Exploiting Twisted Photon Pairs

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Quantum correlation of two-photon states is utilized to suppress the environmental noise in imaging down to the single-photon level. However, the size of the coherence area of the photon pairs limits the applications of quantum imaging based on spatial correlations. Here, a quantum imaging scheme exploiting twisted photon pairs with tunable spatial-correlation regions to circumvent this limitation is proposed. A bulk-density coincidence is employed to enhance the imaging signal. Specifically, a re-scaled image signal is introduced, which is immune to the background intensity distribution profile of the photon pulse. A destructive interference between the anti-bunched photon pair and bunched photon pair in the imaging process is revealed. This work can pave a way for twisted-photon-based quantum holography and quantum microscopy.

1. Introduction

The quantum correlation of photon pairs in time and space offers a great advantage in quantum imaging[1–3] and 3D structure tomography[4–6] down to the single-photon level. The correlated-photon imaging utilizing two-photon entanglement[7] inspired streams of research in density-change-sensitive ghost imaging[8–15] and phase-resolved quantum imaging.[16,17] In addition to quantum imaging, the two-photon Hong–Ou–Mandel (HOM) interference, which is sensitive to the spatial phase-amplitude structure of input single photons, has also been exploited for the hologram of single photons[18] and high-dimensional quantum photonics engineering.[19]

Spatial correlation is essential for correlated imaging. For a regular Gaussian photon pair from spontaneous parametric down-conversion (SPDC) processes, the size of its coherence area $A_C = \pi R^2 / k_0^2 \sigma^2$, [20] which is determined by the beam waist $\sigma$ of the pump beam, the center wave vector $k_0$, and the propagating distance $R$, limits the applications of quantum correlated imaging in many cases. A larger coherence area can be obtained by increasing the propagating distance $R$. However, this will attenuate the field strength significantly and reduce the signal-to-noise ratio (SNR). The tremendous advances in engineering complex optical fields open the possibility to develop photonic technologies for quantum imaging via precise manipulation of the transverse spatial properties of photons.[21–25] Here, we propose a quantum imaging scheme by exploiting twisted photon pairs with tunable coherence regions.

Twisted photons[26] carrying quantized orbital angular momentum (OAM)[27–29] stimulated much interest in high-dimensional quantum communication[30–35] and quantum computation[36–39] beyond the polarization, momentum, and spectral degrees of freedom. Quantum imaging and remote sensing exploiting the continuous spatial correlation of twisted photon pairs will be another important topic of interest.[40–41] Usually, the field strength of a twisted photon is of a donut shape in the transverse plane. The radius of the donut (i.e., the coherence region of a twisted photon pair) can be tuned by varying the OAM quantum number. Thus, signal photons can be concentrated on the target regions to avoid SNR degradation. However, the nonuniform background intensity distribution profile of the photon pulse will hamper the imaging performance. How to resolve this difficulty remains elusive.

Our proposed quantum imaging is based on HOM interference with coincidence measurements between a bucket detector and an image sensor as shown in Figure 1. Different from previous works sensing the intensity or phase change due to a quasi-transparent object,[17,41] our imaging system aims to probe the texture by extracting the spatially varying phase imprinted on the photon during the reflection. The axial symmetry of twisted photons leads to completely destructive interference between the anti-bunched photon pair and bunched photon pair in the imaging process. This interference results in an effect that the texture information cannot be extracted via photon density measurements directly, but can be obtained from the quantum correlation of the two output photons. We also show that by retrieving the texture image via our introduced re-scaled signal, the influence of the photon density profile on quantum correlated imaging can be significantly eliminated. Our work constitutes valuable resources not only for quantum imaging or remote sensing but also for studying the unique quantum statistical properties of twisted photon pairs.[42]

2. Hong–Ou–Mandel Interference of Twisted Photon Pairs

Our imaging scheme is based on HOM interferometer.[43] Here, we give a general quantum theory of HOM interference, specifically for 3D-structured photons. For two photons propagating in different directions, a linearly polarized photon pair can be
coherent-state input pulses. The texture of the object is imprinted on the



Figure 1. Schematic of the setup for our Hong–Ou–Mandel (HOM)-based quantum imaging via twisted photon pairs. The two photons generated in the spontaneous down conversion processes propagate in different paths labeled A and B, respectively. The texture of the object is imprinted on the phase $\Phi(x, y)$ of the photon in path A during the reflection. The information about the object is extracted via the HOM interference, that is, the coincidence of the photon number (obtained from the bucket detector) at one output port of the beam splitter (BS) and the photon number density (measured via the image sensor) at the other port.

Figure 2. a) Transformation of the two coordinate frames at the beam splitter. These two coordinates corresponding to the two optical paths A and B are co-moving with the two photons. The input photonic modes are denoted by annihilation operators $\hat{a}$ and $\hat{b}$ and the output modes are denoted by $\hat{a}$ and $\hat{c}$. b) Imaging via Mach–Zehnder interferometer with coherent-state input pulses. The texture of the object is imprinted on the phase $\Phi(x, y)$ in one optical channel and extracted via photon number density measurements directly.

The HOM interference is essentially described by the input–output relations at the beam splitter (see Figure 2a)\[47\]

$$\hat{c}_k = \left( R_k \hat{a}_k + T_k \hat{b}_k \right)$$

(4)

$$\hat{b}_k = \left( T_k \hat{a}_k + R_k \hat{b}_k \right)$$

(5)

with $\hat{k} = (k_x, -k_x, k_z)$. In the frame co-moving with photon, the y-component of wave vector changes sign under a reflection (Supporting Information)\[49\] This leads to an important effect that the sign of the quantum number of photonic OAM is changed (i.e., $m \rightarrow -m$) under every reflection (Supporting Information)\[49\].

In the following, we focus on 50 : 50 beam splitter with $T_k = 1/\sqrt{2}$ and $R_k = i/\sqrt{2}$ for paraxial quasi-single-frequency photons. Here, we only consider the HOM interference of two photons of the same polarization. Our formalism can be generalized to the cases for photons with different polarizations straightforward\[50,51\].

The coincidence probability after the beam-splitter

$$P_{cd}^{(2)} = \int \int d\vec{r} \int dr' \langle \Psi_{\text{out}} | \hat{\psi}_{r}^+ (\hat{r}) \hat{\psi}_{r'}^+ (\hat{r}') \hat{\psi}_{r}^- (\hat{r}) \hat{\psi}_{r'}^- (\hat{r}') | \Psi_{\text{out}} \rangle$$

(6)

can be obtained from the output state (Supporting Information)

$$\langle \Psi_{\text{out}} | = \frac{1}{2} \int \int dr \int d\vec{r} \int d\vec{r}' \left[ \tilde{\xi}(\hat{r}, \hat{r}', t) \hat{\psi}_{\hat{r}}^+ (\hat{r}) \hat{\psi}_{\hat{r}'}^+ (\hat{r}') + i \tilde{\xi}(\hat{r}, \hat{r}', t) \hat{\psi}_{\hat{r}}^- (\hat{r}) \hat{\psi}_{\hat{r}'}^- (\hat{r}') \right]$$

(7)

with $\tilde{\xi}(\hat{r}, \hat{r}', t) = \tilde{\xi}(\hat{r}, \hat{r}', t) - \tilde{\xi}(\hat{r}', \hat{r}, t)$ and $\tilde{\xi}'(\hat{r}, \hat{r}', t) = \tilde{\xi}(\hat{r}, \hat{r}', t) - \tilde{\xi}(\hat{r}', \hat{r}, t)$ for quasi-1D photon pairs, the WPF is axially symmetric, that is, $\tilde{\xi}(\hat{r}, \hat{r}', t) = \tilde{\xi}(\hat{r}', \hat{r}, t)$. Input two photons with an exchange-symmetric WPF $\tilde{\xi}(\hat{r}, \hat{r}', t) = \tilde{\xi}(\hat{r}', \hat{r}, t)$ will lead to vanishing $\tilde{\xi}'(\hat{r}, \hat{r}', t)$ in the output state. Thus, two photons always come out from the same port due to destructive HOM interference. Input two photons with an exchange-antisymmetric WPF $\tilde{\xi}(\hat{r}, \hat{r}', t) = -\tilde{\xi}(\hat{r}', \hat{r}, t)$ lead to constructive HOM interference. Two photons always come out from different output ports resulting in a HOM peak\[47\].

Different from the 1D case, the HOM interference of 3D twisted photon pairs becomes a little bit complicated. We now input an entangled twisted photon pair with WPF

$$\tilde{\xi}(\hat{r}, \hat{r}', t) = N \tilde{\eta}(\hat{r}, \hat{r}') e^{i m (\phi - \phi') + e^{-i m (\phi - \phi')}}$$

(8)
where the integer \( m \) denotes the OAM quantum number of each photon, \( \eta(r) \) characterizes the shape of each photon pulse and is usually independent on azimuthal angle \( \phi \), and \( N \) is the normalization factor. An HOM interference dip will be obtained for both the exchange-symmetric WPF \( \tilde{\xi}(r, r', \hat{t}) \) and the exchange-antisymmetric WPF \( \tilde{\xi}(r, r', \hat{t}) \) due to the fact \( \tilde{\xi}(r', r, \hat{t}) = \tilde{\xi}(r, r', \hat{t}) \) (note the sign change in \( m \) by \( \hat{r} \)). For photon pairs with WPF

\[
\tilde{\xi}(r, r', \hat{t}) = N \eta(r, \hat{t}) \eta(r', \hat{t}) \left[ e^{im(\phi + \phi')} - e^{-im(\phi + \phi')} \right] \tag{9}
\]

a HOM interference peak will be obtained,\(^{[15]}\) since \( \tilde{\xi}(r', \hat{r}, \hat{t}) = -\tilde{\xi}(r, \hat{r}, \hat{t}) \). More details can be found in the Supporting Information. We also note that the coincidence probability \( \delta \) could be continuously tuned not only by the optical path difference of the two photons but also by the phase of a photon in the transverse plane. This will pave a new way in engineering the quantum coherence of two-photon states.

### 3. HOM-Based Quantum Imaging

Originally, the HOM interference was explored to measure the time delay \textit{i.e.}, optical path difference between the two incident photons.\(^{[41]}\) Later on, it has been explored for various applications, such as quantum-optical coherence tomography,\(^{[41]}\) photon indistinguishability testing,\(^{[32]}\) quantum state engineering,\(^{[15,51]}\) as well as quantum imaging.\(^{[17,18]}\) We now apply the HOM interferometer to detect the texture of an object with twisted photon pairs. As shown in Figure 1, the photon in path-A is reflected once by the object. The texture of the object is imprinted on the wave packet function of this photon by adding a spatially varying phase factor \( e^{i\Phi(x, y)} \) in the WPF \( \tilde{\xi}(r, r', \hat{t}) \). After the HOM interference, this phase factor directly enters the WPF of output photons at both output ports as shown in Equation (7).

In experiments, we can measure the photon number density at each output port, such as

\[
n_s(r, t) = \langle \Psi_{\text{out}} | \hat{\psi}_d^\dagger(r_a) \hat{\psi}_d(r_a) | \Psi_{\text{out}} \rangle \tag{10}
\]

via a single-photon detector array, a CCD camera, or any other image sensor. Intuitively, we would expect to extract the phase factor \( \Phi(x, y) \) directly from the photon-number density \( n_s(r, t) \).\(^{[13]}\) However, this cannot be done in HOM-based imaging with twisted photon pairs as explained in the example.

The essence of the HOM interference lies in the beamsplitter-generated quantum entanglement between the two output photons. We now utilize the quantum correlation function \( \langle \Psi_{\text{out}} | \hat{\psi}_d^\dagger(r_a) \hat{\psi}_d^\dagger(r_b) \hat{\psi}_d(r_b) \hat{\psi}_d(r_a) | \Psi_{\text{out}} \rangle \) to extract the spatially dependent phase \( \Phi(x, y) \). To obtain a larger signal and to speed up the imaging process, we perform the following coincidence signal detection

\[
\langle \tilde{C}_d(r, \hat{t}) \rangle = \int dr' \langle \Psi_{\text{out}} | \hat{\psi}_d^\dagger(r_a) \hat{\psi}_d^\dagger(r_b) \hat{\psi}_d(r_b) \hat{\psi}_d(r_a) | \Psi_{\text{out}} \rangle \tag{11}
\]

As shown in Figure 1, a bucket detector is employed to collect the photon number \( \int dr' \langle \hat{\psi}_d(r_a) \hat{\psi}_d(r_b) | \Psi_{\text{out}} \rangle \) at port-c, and an image sensor is used to measure the photon number density \( n_s(r, t) \) at port-d. The quantum imaging of the texture of the object is achieved via the coincidence signal \( \langle \tilde{C}_d(r, \hat{t}) \rangle \).

In practice, we cannot measure the true photon number density at a single point. Instead, we measure the accumulated signal at a finite small volume \( \Delta V \) determined by the pixel area and measuring time. Thus, the measured signal from the pixel labeled by X will be the mean value of the operator

\[
\tilde{C}_d(X) = \int_{\Delta V} dr \int dr' \langle \hat{\psi}_d^\dagger(r_a) \hat{\psi}_d^\dagger(r_b) \hat{\psi}_d(r_b) \hat{\psi}_d(r_a) | \Psi_{\text{out}} \rangle \tag{12}
\]

where the integral over r is limited in the small volume \( \Delta V \) corresponding to the pixel-X. This also removes the divergence in the density correlation at the same point \( \langle \Psi_{\text{out}} | \hat{\psi}_d^\dagger(r_a) \hat{\psi}_d(r_b) | \Psi_{\text{out}} \rangle \).\(^{[28]}\) The SNR for two-photon-state imaging after N independent measurements is defined as (Supporting Information)^{[15]}

\[
\text{SNR}_{\text{TPS}} = \frac{\sqrt{N} \langle \tilde{C}_d(X) \rangle}{\sqrt{\langle \tilde{C}_d^2(X) \rangle - \langle \tilde{C}_d(X) \rangle^2}} \tag{13}
\]

where \( \langle \tilde{C}_d(X) \rangle \) \( \propto 1 \), since it characterizes the probability of detecting a photon by pixel-X.

The object texture can also be extracted via the Mach–Zehnder interference with coherent-state laser pulses (see Figure 2b). The image signal is obtained by the pixelated photon number density operator \( D_d(X) = \int_{\Delta V} dr \langle \hat{\psi}_d^\dagger(r_a) \hat{\psi}_d(r_a) \rangle \). The SNR for coherent-state imaging is

\[
\text{SNR}_{\text{CS}} = \frac{\langle D_d(X) \rangle}{\sqrt{\langle \Delta D_d^2(X) \rangle}} = \frac{\langle D_d(X) \rangle}{\sqrt{\langle D_d(X) \rangle}} \tag{14}
\]

For a coherent-state pulse with a large photon number \( N \), the well-known \( \sqrt{N} \)-factor enhancement in \( \text{SNR}_{\text{CS}} \) will be obtained. Comparing \( \text{SNR}_{\text{TPS}} \) and \( \text{SNR}_{\text{CS}} \), we see that the quantum imaging based on two-photon HOM interference enhances the SNR slightly at the single-photon level. However, due to technological limitations, the dark-counting related noise dominates in quantum correlated imaging instead of the quantum noise in \( \text{SNR}_{\text{TPS}} \).\(^{[54]}\) In the following, we focus more on the theoretical exploration of exploiting twisted photon pairs in quantum imaging. Additionally, we show twisted photon pairs can be used for the quantum encryption of images via HOM interference.

### 4. Quantum Imaging with Twisted Photon Pairs

In Gaussian-type photon pairs, the energy of a photon is concentrated around the pulse center. The corresponding imaging SNR decreases with the distance to the pulse axis fast. This issue could be solved by exploiting twisted photon pairs with tunable coherence regions. We now apply our quantum imaging approach to specific twisted photon pairs. The input two photons in a product state are described by Equation (2) with WPF

\[
\tilde{\xi}(r, r', \hat{t}) = N \eta_m(r, z, \hat{t}) \eta_m(r', z', \hat{t}) e^{im(\phi - \phi')} \tag{15}
\]

Here, the two photons have opposite OAM quantum numbers. Only the photon in path-A has been reflected by the object. Thus,
the texture information of the object is imprinted on the WPF of this photon via the phase \( \Phi_1(\rho, \varphi) \) re-expressed in a cylindrical coordinate. Even for a product input state, the HOM interferometer generates quantum entanglement in the output photons, which plays an essential role in the imaging process.

The photon number density (10) measured from output port \( d \) is given by

\[
n_d(r, t) = \frac{1}{4} |\tilde{n}_m(\rho, z, t)|^2 \{ 4 + [ (I_1 - I_2) e^{i\Phi(\rho, \varphi)} + \text{c.c.} ] \} \tag{16}
\]

where the overlap integrals \( I_1 = \int dr' |\tilde{n}_m(\rho', z', t)|^2 e^{-i\Phi(\rho', \varphi')} \) and \( I_2 = \int dr' |\tilde{n}_m(\rho', z', t)|^2 e^{-i\Phi(\rho, \varphi)} \) come from the first term (bunched photon pair) and third term (anti-bunched photon pair) of the output state (7), respectively. For an axisymmetric function \( \tilde{n}_m(\rho, z, t) \), we can prove \( I_1 = I_2 \) (Supporting Information); thus, the corresponding terms carrying texture information in \( n_d(r, t) \) cancel out. Interference between the anti-bunched pair (processes (a) and (b) in Figure 3) and bunched pair (process (c) in Figure 3) occurs. This destructive interference leads to a striking effect that the texture of the object cannot be extracted via simply measuring \( n_d(r, t) \). We note that this completely destructive interference is essentially due to the axial symmetry of twisted photons, and it is significantly different from the well-known interference resulting in the HOM dip or peak in \( P_{\text{out}}(2) \), which occurs only between the anti-bunched photons (the processes (a) and (b) in Figure 3). The numerical simulation of \( n_d(r, t) \) is shown by the top row in Figure 4. Only the donut-structure of twisted light has been observed. This also shows a fundamental departure from the interference of two coherent-state pulses in a Mach–Zehnder experiment, where the photon number density at the output port will be \( \propto \text{exp}(\text{imp} + i\Phi(x, y)) + \text{exp}(\text{imp}))^2 \).

The coincidence signal is given by

\[
\langle \hat{C}_d(r, t) \rangle = \frac{1}{4} |\tilde{n}_m(r, t)|^2 \{ 2 - (I_1 e^{i\Phi(\rho, \varphi)} + \text{c.c.} ) \} \tag{17}
\]

During the calibration by replacing the object with a mirror (i.e., \( \Phi = 0 \)), the optical paths of the two photons have been carefully matched and no coincidence signal will be obtained \( \langle \hat{C}_{id} \rangle = 0 \). The texture of the object introduces extra phase difference \( \Phi(\rho, \varphi) \) resulting in non-vanishing coincidence signal \( \langle \hat{C}_d(r, t) \rangle \). As shown by the middle row of Figure 4, the texture of the object looms up in \( \langle \hat{C}_d(r, t) \rangle \). By varying the OAM quantum number \( m \), the imaging region can be tuned gradually.

To remove the influence of the background density distribution profile of twisted photons, we introduce a re-scaled signal

\[
\tilde{S}_d(r, t) = \frac{\langle \hat{C}_d(r, t) \rangle - n_d(r, t)/2}{n_d(r, t)/2} = -\frac{1}{2} [ I_1 e^{i\Phi(\rho, \varphi)} + \text{c.c.} ] \tag{18}
\]

As shown by the bottom row of Figure 4, a texture image of the object with a much higher contrast is obtained. Due to the perfect destructive interference, no texture information of the object is contained in the photon number density \( n_d(r, t) \). The imaging signal has been completely extracted via \( \tilde{S}_d(r, t) \). In this work, only the quantum imaging with product-state photon pairs has been investigated. Our approach can be directly applied to the entangled twisted photon pairs, such as the two-photon states in Equations (8) and (9). Helical-phase-modified quantum coherence of entangled twisted photon pairs will manifest in the texture image as shown in our following work.

Our discovered destructive interference effect in the imaging process can be exploited for the quantum encryption of images with twisted photon pairs. Similar to our imaging process, we first encode an image in one of the photon’s phase and then perform a HOM interference. Based on our results, the information of the photon cannot be extracted via the photon number density at either output. Retrieval of the photo can only be achieved via the coincidence signal. In our theoretical simulation, we only take a twisted pair with WPF in a product form for simplicity. Two photons generated in an SPDC process are entangled in frequency degrees of freedom. However, the frequency entanglement does not change the main features of a HOM interference. In experiments, spectral filters can be used to ensure spectral indistinguishability of the two photons and to remove the frequency entanglement.

5. Discussion

We conduct a theoretical exploration of quantum imaging with twisted photon pairs in this letter. In our numerical simulation, diffraction-free Bessel pulses with micrometer-scale cross-sections have been taken. However, our quantum imaging approach is not limited to microscopy. Laguerre–Gaussian-mode twisted photons can be used for imaging a macroscopic object.
The advances in single-photon-level image sensors lay a solid hardware foundation for our proposed experiment. To suppress the imaging noise in experiments, we can replace the bucket detector with a superconducting nanowire single-photon detector with an extremely low dark-counting rate. The new generation of megapixel single-photon avalanche photodiode image sensors with smaller pitch size (< 10 μm) and higher frame rate and time resolution will enable more exciting applications of quantum imaging at the single-photon level.

Supporting Information
Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest
The authors declare no conflict of interest.

Data Availability Statement
The data that support the findings of this study are available from the corresponding author upon reasonable request.

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Hong–Ou–Mandel interference, orbital angular momentum of light, quantum correlation, quantum imaging, twisted photon pair

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