Search for a light extra gauge boson in Littlest Higgs model at a linear collider

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Littlest Higgs model predicts some extra particles beyond the Standard Model. Among them, an extra neutral gauge boson $A_H$ is lightest and its mass could be a few hundred GeV. We study production and decay of $A_H$ at future $e^+e^-$ linear collider and compare them with those of $Z'$ bosons in supersymmetric (SUSY) $E_6$ models. We find that, if the extra gauge boson mass is smaller than $\sqrt{s}$ of the linear collider, the forward-backward asymmetries of $b$- and $c$-quarks at the $A_H$ pole differ significantly from those given by the $Z'$ bosons, and are useful to test the littlest Higgs model and SUSY $E_6$ models.

Since such an experimental signature of the $A_H$ boson is quite similar to a $Z'$ boson in models which have an extra $U(1)$ gauge symmetry, it is very important to identify the models if an extra neutral gauge boson is discovered in hadron collider experiments such as Tevatron Run-II or LHC. The discovery limit of the $Z'$ boson at the LHC experiment is expected to be roughly 3 TeV $\sqrt{s}$. However, it is hard to test the $Z'$ models at the hadron colliders. On the other hand, an $e^+e^-$ linear collider can play a complementary role for such purpose. In this paper, we would like to study production and decay of $A_H$ in the littlest Higgs model at a future $e^+e^-$ linear collider. In particular, we compare the experimental signatures of $A_H$ and those of $Z'$ boson in supersymmetric $E_6$ models, and examine a possibility to distinguish these models in the linear collider experiments. In our study, we assume that Tevatron or LHC discovers a certain $Z'$ boson whose mass is smaller than $\sqrt{s}$ of the linear collider. Then we can study the $Z'$ boson at the linear collider by tuning the $e^+e^-$ beam energy at the peak of $Z'$ resonance. We will show that the forward-backward asymmetry of $c$-quark is useful to test if the $Z'$ boson is $A_H$ in the littlest Higgs model or one of the SUSY $E_6$ models.

Since the littlest Higgs model has the gauge symmetry $[SU(2) \times U(1)]^2$ at high energy scale, there are two $SU(2)$ gauge bosons $W_1, W_2$ and $U(1)$ gauge bosons $B_1, B_2$. The global $SU(5)$ symmetry breaking at the scale $\Lambda$ induces the following mixing among them:

\[
\begin{pmatrix}
W^a
\hline
W'^a
\end{pmatrix} = \begin{pmatrix}
s_\theta & c_\theta \\
-c_\theta & s_\theta
\end{pmatrix} \begin{pmatrix}
W_{1a} \\
W'_{2a}
\end{pmatrix},
\]

\[
\begin{pmatrix}
B
\hline
B'
\end{pmatrix} = \begin{pmatrix}
s_{\theta'} & c_{\theta'} \\
-c_{\theta'} & s_{\theta'}
\end{pmatrix} \begin{pmatrix}
B_1 \\
B_2
\end{pmatrix},
\]

where $W^a$ and $B$ are $SU(2)_L$ and $U(1)_Y$ gauge bosons in the SM, respectively. They are massless at this stage while $W'^a$ and $B'$ are massive. Note that we use the shorthand notation $s_\theta \equiv \sin \theta$, $c_\theta \equiv \cos \theta$, $s_{\theta'} \equiv \sin \theta'$, $c_{\theta'} \equiv \cos \theta'$. After the electroweak symmetry is broken, the massless gauge bosons will acquire masses and...
be mixed with $W'^a$ and $B'$. The mass eigenstates for charged and neutral gauge bosons are obtained by introducing unitary matrices $U_W$ and $U_N$ as:

$$
\begin{pmatrix}
W_L \\
W_H
\end{pmatrix} = U_W \begin{pmatrix}
W \\
W'
\end{pmatrix}, \quad (A_L, Z_L, A_H, Z_H)^T = U_N (B, W^3, B', W'^3)^T
$$

The explicit expressions of the unitary matrices $U_W, U_N$ and the mass eigenvalues of gauge bosons can be found in ref. [5].

The interaction of $A_H$ to a fermion $f$ in the SM is described by the following Lagrangian [5]:

$$
\mathcal{L} = -\frac{g_Y}{2s_Yc_Y} Q_{f\alpha}^{A_H} T^{\alpha\gamma\mu}_f A_H \gamma^\mu f_\alpha A_H, \quad (5)
$$

where $\alpha(L, R)$ denotes the chirality of fermion $f$.

We summarize the charge $Q_{f\alpha}^{A_H}$ for the fermion $f_\alpha$ in Table II which is obtained by taking account of the $[SU(2) \times U(1)]^Z$ anomaly cancellation without introducing any chiral fermions beyond the SM.

Next let us briefly review the supersymmetric $E_6$ models to fix our notation [4]. Since the rank of $E_6$ is six, it has two $U(1)$ factors besides the SM gauge group which arise from the following decompositions:

$$
E_6 \supset SO(10) \times U(1)_\psi \supset SU(5) \times U(1)_X \times U(1)_{\psi}. \quad (6)
$$

An additional $Z'$ boson in the electroweak scale can be parametrized as a linear combination of the $U(1)_\psi$ gauge boson $Z_\psi$ and the $U(1)_X$ gauge boson $Z_X$ as:

$$
Z' = Z_\psi \cos \beta_E + Z_X \sin \beta_E. \quad (7)
$$

There are four $Z'$ models, which are called $\chi, \psi, \eta$ and $\nu$ models, corresponding to the different value of the mixing angle $\beta_E$. The interaction of $Z'$ boson with the fermion $f$ is described as:

$$
\mathcal{L} = -\frac{g_E}{\sqrt{2}} Q_{f\alpha}^{Z'} f_\alpha A_H \gamma^\mu f_\alpha A_H \gamma^\mu f_\alpha A_H \gamma^\mu \quad (8)
$$

where $g_E$ denotes the extra $U(1)$ gauge coupling constant.

The extra $U(1)$ charge $Q_{f\alpha}^{Z'}$ for the SM quarks and leptons in four $Z'$ models are summarized in Table III.

It should be noted that not only in the littlest Higgs model but also in the SUSY $E_6$ models, the extra neutral gauge boson can mix with the SM $Z$ boson after the electroweak symmetry breaking. Such mixing is, however, severely constrained from the experimental data of the electroweak precision measurements at the $Z$-pole [10], and we assume that the mixing is small enough to neglect in our study.

![FIG. 1: The peak cross section of $e^+e^-\rightarrow \mu^+\mu^-$ at the $A_H$ or $Z'$ pole in the littlest Higgs model (solid line) and SUSY $E_6$ models as functions of $A_H$ (or $Z'$) mass. The mixing angle $\theta'$ in the littlest Higgs model is fixed by $\tan \theta' = 0.5$.](image)

![TABLE II: The hypercharge $Y$ and the extra U(1) charge $Q_E$ of SM quarks and leptons in SUSY $E_6$ models. The value of extra U(1) charge follows the hypercharge normalization.](table2)

| Field | $Y$ | $Q$ | $Q_E$ |
|-------|-----|-----|------|
| $u_R$ | $2/3$ | 1 | -1 |
| $e_R$ | -1 | 1 | -1 |
| $L$ | -1/2 | 3 | 1 |
| $d_R$ | -1/3 | -3 | 1 |

![TABLE I: Charge $Q_{f\alpha}^{E_6}$ for quarks and leptons. $L$ and $Q$ denote the SU(2)$_L$ doublet lepton and quark, respectively. The charges are determined to satisfy the [SU(2) x U(1)]$^Z$ anomaly free condition without introducing any chiral fermions beyond the SM.](table1)

| Field | Charge | $L$ | $Q$ | $Q_{f\alpha}^{E_6}$ |
|-------|--------|-----|-----|------------------|
| $u_R$ | $2/3$ | 1 | -1 |
| $e_R$ | -1 | 1 | -1 |
| $L$ | -1/2 | 3 | 1 |
| $d_R$ | -1/3 | -3 | 1 |

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where $\Gamma_\ell$ is the partial decay width of $V \to f \bar{f}$ and $\Gamma_V$ is the total decay width of $V$. The coupling $g_\alpha^f(\alpha = L, R)$ in (10) follows the normalization

$$L = -g_\alpha^f \frac{\Gamma_\alpha^f}{\Gamma_V} f_\alpha V_\mu.$$

We note that $\sigma_{\text{peak}}^f$ does not depend on the gauge coupling ($g_Y$ in the littlest Higgs model, $g_E$ in SUSY $E_6$ models) because it is canceled in (11). In Fig. 1 we show the peak cross section of $e^+e^- \to \mu^+\mu^-$ in the littlest Higgs model and SUSY $E_6$ models as a function of extra gauge boson mass $m_{A_H}(m_{Z'})$. The prediction of the littlest Higgs model is shown for $\tan \theta' = 0.5$ as an example. The peak cross section in the littlest Higgs model is roughly a few hundred pb, which is a few times larger than those in SUSY $E_6$ models, so the cross section measurement seems to be useful to test the models at the linear collider with $100 fb^{-1}$ integrated luminosity.

![Graph showing peak cross section of $e^+e^- \to \mu^+\mu^-$](image)

**FIG. 2:** The peak cross section of $e^+e^- \to \mu^+\mu^-$ in the littlest Higgs model (solid line) and SUSY $E_6$ models as functions of the mixing angle $\theta'$. The mass of $A_H$ (or $Z'$) is fixed at 750 GeV.

Next we show the peak cross section of $e^+e^- \to \mu^+\mu^-$ in the littlest Higgs model as a function of the mixing angle $\theta'$ in Fig. 2. The mass of $A_H$ is fixed at 750 GeV as an example. For comparison, we depict the predictions of SUSY $E_6$ models in the same figure. We can see in the figure that the peak cross section in the littlest Higgs model rapidly decreases around $\tan \theta' = 1.2$. This is because that the left- and right-handed electron couplings to $A_H$ are given by (see Table I)

$$Q_{eL}^A = \frac{1}{2} Q_{eR}^A = \frac{1}{2 s_{\theta'} c_{\theta'}} \left( -\frac{2}{5} + c_{\theta'}^2 \right),$$

and they diminish for $c_{\theta'}^2 \sim 2/5$, which corresponds to $\tan \theta' \sim 1.2$. Therefore, even if the result of peak cross section measurement is consistent with one of SUSY $E_6$ models, there is still a possibility of the littlest Higgs model, and we should find another observable to test the models.

The forward-backward (FB) asymmetry of the $e^+e^- \to f\bar{f}$ process does not have the $\theta'$ dependence. Using the couplings defined in (11), the FB asymmetry at the pole of $A_H(Z')$ can be expressed as

$$A_{\text{FB}}^f = \frac{3}{4} A^e A^f,$$

$$A^f = \frac{(g_{1f}^f)^2 - (g_{3f}^f)^2}{(g_{1f}^f)^2 + (g_{3f}^f)^2}.$$ (14)

When we write the right-handed coupling $g_R^f$ as

$$g_{1f}^f = r_f g_{1L}^f,$$ (15)

the asymmetry parameter $A^f$ is expressed as follows

$$A^f = \frac{1 - r_f^2}{1 + r_f^2}.$$ (16)

In the littlest Higgs model, the coupling $g_{1L}^f$ is replaced by $g_Y Q_{eL}^A (m_{Z'})/(2 s_{\theta'} c_{\theta'})$ (see eq. (13)), so the parameter $r_f$ for $f = \mu, \mu$ is independent of $\theta'$;

$$r_f = (r_e, r_\mu, r_d) = (2, 4, -2).$$ (17)

The FB asymmetry, therefore, is a good observable to compare the littlest Higgs model and the SUSY $E_6$ models. We show the FB asymmetry for the muon, $c$-quark and $b$-quark in the littlest Higgs model and the SUSY $E_6$ models in Fig. 3. The numbers of the asymmetries in each model are summarized in Table II. It is remarkable that the asymmetries in the $\psi$ model are zero because the extra U(1) charge assignments on the SM fermions are parity invariant. Beside on the $\psi$-model, the difference of predictions between the littlest Higgs model and the SUSY $E_6$ models are very clear in the $b$- and $c$-quark asymmetries. In the $b$-quark FB asymmetry, the littlest Higgs model predicts a positive value while the SUSY $E_6$ model is negative one. Especially, it is noticeable that there is no $c$-quark asymmetry in the SUSY $E_6$ models though the littlest Higgs model gives a 40% asymmetry. The reason why the FB asymmetry of $c$-quark vanishes in SUSY $E_6$ models is as follows. As shown in eq. (11), the FB asymmetry is given by the difference of the couplings between the left- and right-handed fermions to the $Z'$ boson. In SUSY $E_6$ models, both left- and right-handed $c$-quarks are embedded in the same multiplet, $10$ representation in SU(5), so that they have a common coupling which leads to no asymmetry. Fig. 3 tells us that the measurements of $b$- and $c$-quark asymmetries in a few % accuracy is enough to test if a $Z'$ boson is $A_H$ in the littlest Higgs model or one of the SUSY $E_6$ models.

In summary, we have studied a possibility to test the littlest Higgs model and the SUSY $E_6$ models through...
FIG. 3: The forward-backward asymmetry of muon (top), b-quark (middle) and c-quark (bottom) at the $Z'$ pole in the littlest Higgs model and SUSY $E_6$ models. The predictions of littlest Higgs model is shown by solid circles, while those of SUSY $E_6$ models are given by squares ($\chi$-model), diamonds ($\psi$-model), upward triangles ($\eta$-model) and downward triangles ($\nu$-model).

have same couplings to the $Z'$ boson due to the SU(5) GUT symmetry. The measurements of FB-asymmetries for heavy quarks are, therefore, very useful to test if the $Z'$ boson is $A_H$ in the littlest Higgs model or one of the SUSY $E_6$ models.

If the measurements of FB-asymmetries are consistent with the prediction of the littlest Higgs model, one should determine the couplings of fermion pairs to the $A_H$ boson for completeness. We briefly comment on this possibility before close our paper. From the data of $A_{FB}^{\mu}, A_{FB}^{b}, A_{FB}^{c}$, we can obtain the parameter $r_{e}, r_{u}, r_{d}$, respectively, where we assume that the couplings are universal for each generation. Taking account of the $r_f$-parameters, the absolute values of the left- and right-handed couplings can be extracted from the partial decay width of $A_H \rightarrow f\bar{f}$ (10). In order to fix the sign of the couplings, the interference effect between $A_H$ and the SM $Z$ boson in the cross section of $e^+e^- \rightarrow f\bar{f}$ at the off-resonance of $A_H$ should be measured very precisely.

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