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Ultra-sensitive passive wireless sensor exploiting high-order exceptional point for weakly coupling detection

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Abstract
Since the quantum concept of parity-time (PT) symmetry has been introduced into the conventional inductor–capacitor resonance, strategies based on exceptional points (EP) based strategies redefine our understanding of sensitivity limitation. This considerable enhancement of sensitivity originated in exploration of the non-Hermitian physics in photonics, acoustics and electronics, which exhibits a substantial application to the miniaturization of implanted electronic sensors in medicine field. By continuously accessing the EP, the spectral response of reader follows a dependency of \( \Delta \omega \sim \kappa^2/3 \) to a weakly coupling rate (\(|\kappa| \approx 0\)), which may approach the theoretical limit of sensitivity in a second-order EP system. In this paper, we experimentally demonstrate a high-order (higher than second-order) PT symmetric system for weak coupling detection, in which a third-order EP can be employed to fulfill the sensitivity of \( \Delta \omega \sim \kappa^{1/2} \). Particularly, we introduce the incoming wave as an effective gain to balance the loss and obtain a pair of purely real eigenfrequencies. There are absence of imaginary parts despite corresponding real parts shifts dramatically by using a neutral resonator, without a broadening of the reflection spectrum so that maintaining a high resolution on the sensitivity. This work may reveal the physical mechanics of a small perturbation at a high-order EP and promote applications in implanted medicine devices.

1. Introduction

Wireless sensors (WS) can be used to continuously operate in harsh environments or human bodies where wired connections are unable to. The first compact passive WS was proposed by Collins in 1967, which utilized a spiral inductor (L) and a pressure-sensitive capacitor (C) to realize an intraocular pressure sensor implanting in the eye [1]. Recently, WS based on LC-resonance have experienced a rapid expansion in the last few decades with many industrial and medical applications, including radio-frequency identification [2], ocular diagnostics [3] and physiological monitoring [4–9]. Generally, the working mechanism of these WS is based on detecting concomitant resonance frequency shifts caused by the variation of L or C, which will be easily affected by external factors such as pressure [1, 10, 11], strain [4] and humidity [12]. Besides, when a read-out coil is very close to a sensor coil, the change of the read-out coil spectral response \( \Delta \omega \) induced by the variation of coupling rate \( \kappa \) between them through inductive coupling will also emerge. However, limited by the depth and small dimensions of an implanted sensor, the coupling \( \kappa \) is unable to induce the resonant frequency shifts \( \Delta \omega \) to a critical value for measuring. A strategy to amplify the response \( \Delta \omega \) need to be proposed from a weakly coupled version to dramatically enhance the sensitivity of the reader.
Recent advances in the fields of non-Hermitian physics and parity-time (PT) symmetry [13–21] reveal that enhanced sensitivity has been realized in optics and photonics near to the PT phase transition point (exceptional point, EP) [22–39], where the eigenvalues and corresponding eigenvectors simultaneously coalesce. Such singularities are distinct features in non-Hermitian systems, which are non-conservative systems because they exchange energy with the surrounding environment [23]. In addition to ultrasensitive sensors, recently several research groups have explored many intriguing phenomena of non-Hermitian optics and photonics near EP including coherent perfect absorption [40, 41], light stops [42], anisotropic [43], Voigt [44] and vector [45] EP to mention just a few examples. Inspired by these schemes, other PT-symmetric sensors have been reported recently in acoustics [46], optomechanics [47] and electronics [9, 11, 12, 48–57]. As a specific example of a class of implanted WS, the second-order EP-locked (EP2) scheme [9] has recently been shown to exhibit a remarkable performance in a weakly coupled region. Moreover, it is clear that the sensitivity of the system will increase with the order of EP increasing, which has led to the tremendous interest in developing systems with high-order (higher than second-order) EP [28, 32, 58–61]. Hence, whether a combination of high-order EP and WS is highly desirable in weakly coupled detection application?

In this paper, we experimentally demonstrate ultra-sensitive frequency response in passive WS for the small coupling rate (|κ| ≈ 0) near to a third-order EP (EP3) for weak coupling detection. Inspired by the EP2 scheme, we design a three-resonance system based on kHz inductor–capacitor (LC) circuit that emerges an EP3 with suitable separation between the resonators. Both theoretical and experimental results show that the spectral response of the reader system Δω near the EP3 follows a dependency of Δω ∼ κ−1/2. Compared to a recently proposed EP2 scheme (figure 1(a)), in that the spectral sensitivity follows Δω ∼ κ−1/3, EP3 scheme defines more competitive in practice. Using an additional detector coil as a sensor coil weakly coupled with the relay coil (neutral resonator) of the system, purely real eigenfrequencies shifts (red and blue shifts) away from EP3 can be clearly observed simultaneously (figure 1(b)). We also consider a higher-order EP scheme for WS and the theoretical prediction results show an enhancement of sensitivity with increasing the order of the strong coupled resonators in a sensor system.

2. Theoretical and experimental results

First, let us consider a sensing scheme based on a third-order PT symmetric reader system consisting of three coupled resonators (transmitter, relay and receiver coils), as shown in figure 1(a). With respect to an input port with forward wave s1+ = S1+e−iωt and backward wave s1− = −s1+ + $\sqrt{2}\gamma a_1$ connected to the transmitter coil, the coupled mode equations are given by [54, 62, 63]:

$$\begin{align*}
\frac{da_1}{d\tau} &= (-\omega_0 - \gamma) a_1 - i\mu a_2 + \sqrt{2}\gamma a_1, \\
\frac{da_2}{d\tau} &= -i\omega_0 a_2 - i\mu a_1 - i\mu a_3, \\
\frac{da_3}{d\tau} &= (-\omega_0 - \gamma) a_3 - i\mu a_2,
\end{align*}$$

(1)

where $|a_1|^2$, $|a_2|^2$ and $|a_3|^2$ correspond to the energy stored in transmitter, relay and receiver coils, respectively. $\omega_0$ is the resonant frequency of the resonant coil. $\mu$ is normalised coupling rate between the nearest-neighboring coils. $\gamma$ signifies the coupling rate from transmitter/receiver to port 1/2 (input/output port) in vector network analyzer (VNA). By putting zero backward wave $s_{1-} = 0$ into equation (1), an equivalent eigenfrequency problem in solving the perfect absorption (zero reflection) states can be described as follows [40, 54, 63]:

$$\begin{align*}
H \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} &= \omega \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix},
\end{align*}$$

(2)

where the effective Hamiltonian (H) is

$$H = \begin{pmatrix} \omega_0 + i\gamma & \mu & 0 \\ \mu & \omega_0 & \mu \\ 0 & \mu & \omega_0 - i\gamma \end{pmatrix}.$$

(3)

By solving |ωI − H| = 0, where I is an identical matrix, the corresponding eigenfrequencies are as follows:

$$\begin{align*}
\omega_1 &= \omega_0, \\
\omega_{2,3} &= \omega_0 \pm \sqrt{2\mu^2 - \gamma^2}.
\end{align*}$$

(4)
Figure 1. Comparison of sensing between an EP2 scheme (a), (c) and an EP3 scheme (b), (d). Schematics diagrams of the third-order (a) and second-order (b) PT-symmetric reader systems consisting of transmitter, relay and receiver coils, which are analogous to electronic molecules with gain, neutral and loss properties, respectively. The inductive coupling between the nearest-neighboring coils is $\mu(s)$, where $s$ is distance between the two coils. VNA inputs forward waves to transmitter through port 1 with coupling $\gamma$, so as the receiver and port 2 in VNA. (a) A detector coil is coupled weakly ($|\kappa| \approx 0$) with transmitter and receiver coils of this system in EP2 ($\mu = \gamma$) condition simultaneously. (b) A detector coil (yellow) is coupled weakly ($|\kappa| \approx 0$) with the relay coil of this system in EP3 condition ($\mu = \sqrt{2}\gamma/2$). Parameters to be detected can be obtained by monitoring the reflection coefficient of these reader systems. Analytical solutions for the real (top) and imaginary (bottom) parts of the eigenfrequencies near EP2 (c) and EP3 (d), respectively. It is clear that frequency response in reflection spectra and purely real eigenfrequencies splitting with EP3 scheme (red and green) is larger than that of EP2 (orange).

Equation (4) indicates that all three eigenfrequencies merge at $\omega_0$, providing the critical value condition of $\mu = \sqrt{2}\gamma/2$ is satisfied, thus EP3 is obtained.

When a detector coil (exotic resonator) is coupled with the relay coil of this system in EP3 condition, equation (1) can become:

$$\frac{d\alpha_1}{d\tau} = \left(-i\omega_0 - \sqrt{2}\mu\right)\alpha_1 - i\mu\alpha_2 + \sqrt{2}\sqrt{2}\mu\alpha_3$$

$$\frac{d\alpha_2}{d\tau} = -i\omega_0\alpha_2 - i\mu\alpha_1 - i\kappa\alpha_3 - i\kappa\alpha_4$$
Figure 2. Comparison of sensitivities of reader systems. Analytical (deep color solid line) and approximate (light color dashed line) results for frequency response as a function of coupling $\kappa$ near EP2, EP3, EP4 and EP5 on a logarithmic scale, with the slope of $2/3$, $1/2$, $2/5$ and $1/3$, respectively. Approximate results represent the corresponding series expansion truncated to the first order. For analogical reasoning, we can predict that a spectral response of the reader system near EP$_n$ will follow the relationship $\sim \kappa^{2/(n+1)}$. The higher the order $n$, the higher the sensitivity of EP$_n$ scheme.

\[
\frac{da_1}{dr} = \left(-i\omega_0 - \sqrt{2}\mu\right) a_3 - i\mu a_2,
\]
\[
\frac{da_2}{dr} = -i\omega_0 a_3 - i\kappa a_2,
\]

(5)

where $|a_3|^2$ represents the energy stored in detector coil. The eigenfrequencies around the EP3 are derived from equation (4) as follows:

\[
\Delta \omega_{1,2,3,4} = \pm \sqrt{\left(\kappa^2 \pm \sqrt{\kappa^2 (\kappa^2 + 8\mu^2)}\right)/2},
\]

(6)

The above eigenfrequencies can potentially mask the hypersensitive resonance shift, corresponding to a deep reflection ($R$) dip for reflection spectrums obtained by $R = |s_1^- / s_1^+|$. We also study an analogous non-Hermitian sensing scheme of reader system in EP2 condition ($\mu = \gamma$), as is shown in figure 1(b). Similarly, the dependence of the purely real eigenfrequency on $\kappa$ around this EP2 is

\[
\Delta \omega' = \frac{2\kappa^2}{3^{1/3} \left(9\kappa^2 \mu + \sqrt{3}\sqrt{-8\kappa^6 + 27\kappa^4 \mu^2}\right)^{1/3}} + \left(9\kappa^2 \mu + \sqrt{3}\sqrt{-8\kappa^6 + 27\kappa^4 \mu^2}\right)^{1/3}/3^{2/3}.
\]

(7)

Figures 1(c) and (d) plot the theoretical values of complex eigenfrequencies as a function of $\kappa$ for the parameter setting $\mu = 1$, respectively. It is clear that the third-order PT symmetric system demonstrates giant frequency splitting near EP3 (red and pink) compared with that with EP2 (orange). For comparison, we also present these results on the same figure. The Newton–Puiseux series expansion of equation (6) can be found as follows [28, 54]:

\[
\Delta \omega = 2^{1/4} \sqrt{\mu \kappa} + O (\kappa) + \cdots,
\]

(8)

where O stands for higher-order term. Consequently, the dependence of the frequency shift is expected to approximately follow

\[
\Delta \omega = 2^{1/4} (\mu \kappa)^{1/2}.
\]

(9)

Based on equation (9), the sensitivity of this reader system can be expressed as $\Delta \omega / \kappa = 2^{1/4} (\mu / \kappa)^{1/2}$. For $\mu = 1$, this sensitivity becomes $2^{1/4} / \kappa^{1/2}$, thus $\Delta \omega / \kappa > 1$ can be obtained when $\kappa$ is very small, which means enhanced sensitivity. Besides, we consider a model where the coupling exists between the detector and both of the resonators in the EP3 system simultaneously, and the coupling between the detector coil and any coil in the EP3 system is supposed to $\kappa$. Using methods in reference [28, 54], we can still derive equation (9), namely a spectral response of the reader near EP3 following $\sim \kappa^{1/2}$. Under this circumstance, this EP3 WS still works normally.

Similarly, the approximate result of equation (7) can be derived as follows:

\[
\Delta \omega' = (2\mu)^{1/3} \kappa^{2/3}.
\]

(10)
Figure 3. (a) A schematic diagram of a wireless-sensor system based on third-order PT symmetry for implementing at EP3. It consists of transmitter, relay and receiver coils. A source coil is coupled to a transmitter coil at a rate $\gamma$, transferred across relay coil before being delivered to the load coil at the same rate $\gamma$. And the transmitter-relay as well as relay-receiver resonant coupling rate is $\mu$. For $\gamma = \sqrt{2}\mu$, all three eigenfrequencies coalesce at the resonant frequency and the system exhibits EP3. A resonant detector coil is coupled with the relay coil of the system in EP3 condition, with the coupling rate $\kappa$.

(b) Photograph of our sample. (c) Measured (blue dots) and calculated (green lines) reflection spectra as a function of frequency for the proposed wireless sensor under different values of $\kappa$ (or separation distance $d$) in the weak-coupling region. Note that a clear frequency splitting emerges with smaller $\kappa$ (or larger $d$).

We find a good agreement between analytical (deep color solid line) and approximate (light color dashed line) solutions. The observed linear slope of 1/2 (yellow) and 2/3 (grey) in the corresponding logarithmic plot shown in figure 2 affirm the behavior of square-root and three-second power, respectively. Compared with equations (9) and (10), for $\mu = 1$, we can also observe $\Delta\omega/\Delta\omega' = 1/(2^{1/12} \kappa^{3/6}) > 1$ with small $\kappa$. It means the frequency response of EP3 is more significant than that of EP2. Similarly, we can derive a spectral response of the reader near EP4 and EP5 following $\sim\kappa^{2/(4+1)}$ (brilliant blue) and $\sim\kappa^{2/(5+1)}$ (light green), respectively. For analogical reasoning, we can predict that a spectral response of the reader near EP$n$ will follow $\sim\kappa^{2/(n+1)}$. The use of higher order EP will further amplify the frequency response to coupling, leading to even greater sensitivity.

Figure 3(a) shows the schematics of the third-order PT symmetric WS system, corresponding sample photo is shown in figure 3(b). All coils are arranged coaxially towards. The transmitter, the relay, the receiver and the detector coils are equidistant resonant coils (LC tanks) composed of wire-wound inductors and film capacitors. They are made of identical Litz wire whose width is about 2.36 mm ($0.078 \text{ mm} \times 400$) and fixed on the polymethyl methacrylate hollow box. It has each side length of 12 cm, with thickness is 3.06 cm and empty center section outer diameter of 6 cm. The film capacitors have fixed values of about
4.7 nF, and they are loaded on each resonant coil in parallel with the adjustable capacitors ranging from 10–100 pF. By carefully tuning the adjustable capacitors, the three resonant coils have nearly the same resonance frequency at \( f_0 = \omega_0/2\pi = 113.3 \) kHz.

Besides, the source and the load coils are non-resonant coil, made of identical Litz wire whose width is about 2.82 mm (0.07 mm \( \times \) 714) and fixed on transmitter and receiver coils, respectively. They are connected to port 1 and port 2 of the VNA (Keysight E5071C), respectively. A harmonic wave at a frequency \( \omega \) is coupled to the transmitter coil at a rate \( \gamma \), transferred across relay coil before delivered to the load coil at the same rate \( \gamma \). The transmitter-relay resonant coupling rate as well as the relay-receiver resonant coupling rate is \( \mu \). The following fitting parameters are set as \( \gamma = 8.00 \) kHz and \( \mu = 5.65 \) kHz to satisfy EP3 condition of \( \gamma = \sqrt{2}\mu \). The relay coil is subjected to the coupling rate of \( \kappa \) to be zero with the reader system in EP3 condition. Figure 3(c) plot the calculated (dots) and experimental (lines) reflection coefficient versus frequency with different coupling rate \( \kappa \) for the proposed third-order PT symmetry. When the value of \( \kappa \) increases from zero to 0.1 kHz, the separation distance \( d \) between the reader system and the detector (center to center) reduces from about 25 cm to 18.2 cm. It is indeed that there is nearly zero reflection at \( f_0 \) at EP3 (\( \kappa = 0 \)) with about \(-60 \) dB signal power. Below \(-60 \) dB, the noise and residual crosstalk contributions will be more significant relative to other sources of measurement errors. With the increasing of \( \kappa \), the same magnitude of blue and red shifts of the frequency resonance (reflection dip) are observed simultaneously, corresponding to purely real parts of eigenfrequencies in figure 1(d), which coincides with the theoretical predictions.

We also present the results for the frequency shifts near EP3 and EP2 on the same figure, in which we have normalized \( \Delta f \) by selecting the ratio of \( \Delta f/\Delta f_{\text{max}} \). In this case, the comparison of frequency response plot between EP3 and EP2 scheme is significantly clear. As can be observed from figure 4, The observed linear slopes of 1/2 (red) and 2/3 (grey) in the logarithmic plot affirm square-root and three-second power behaviors of calculated (lines) and experimental (dots) results, respectively. Inset shows experimental (dots) and fitting (line) results of coupling \( \kappa \) as a function of coupling distance \( d \), where the fitting result is \( \kappa = 1.86 \exp(-0.16d) \).

For analogical reasoning, under ideal circumstance, when \( t \) independent resonators are coupled with the system in EP3 condition with coupling rate \( \kappa \), the effective coupling rate is \( \kappa' = \sqrt{t\kappa} \). Using methods in references \([28, 54]\), we can predict that a generalization of spectral response of the reader near EP3 will follow \((2t)^{3/4}(\mu\kappa)^{1/2}\). The application of such techniques to draw near the theoretical limits of sensitivity for weak coupling remains an important direction in the long term.
3. Conclusion

In conclusion, we have demonstrated ultra-sensitive sensing in passive WS system for weakly coupled detection in the condition of EP3. With regard to our theoretical models in third-order PT symmetry, when an external resonator is coupled to the neutral resonator, the reflection dip matches the purely real eigenfrequencies (red or blue shifts) of the system, not broadening the reflection spectrum and causing large noise. The frequency response is further amplified as $\Delta \omega \sim \kappa_{1}^{1/2}$ near EP3, better performance than that with $\Delta \omega \sim \kappa_{2}^{2/3}$ near EP2. The frequency response is further amplified as $\Delta \omega \sim \kappa_{1}^{1/2}$ near EP3, better performance than that with $\Delta \omega \sim \kappa_{2}^{2/3}$ near EP2. The designed kHz LC system with balanced gain and loss verified our predictions experimentally and demonstrated that the eigenfrequency bifurcation close to EP3 amplifies more 1.8 times in the sensitivity than that using EP2. Further, the theoretical prediction results show considerable improvement of the sensitive property in a higher-order EP as the spectral response follows a dependency of $\Delta \omega \sim \kappa_{2}^{2/(n+1)}$. We envision that such new higher-order PT symmetric systems will enable a superior sensing capability, which can also be extended to other microwave, millimeter wave and terahertz wireless systems.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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References

[1] Collins C C 1967 IEEE Trans. Biomed. Eng. 14 74–83
[2] Hu X L, Aggarwal K, Yang M X, Parizia K B, Xu X Q, Akin D, Poon A S Y and Wong H S P 2017 Phys. Rev. Appl. 8 014031
[3] Kim J et al 2017 Nat. Commun. 8 18997
[4] Chen L Y, Tee B C, Chortos A L, Schwartz G, Tse V, Lipomi D J, Wong H S, McConnell M V and Bao Z 2014 Nat. Commun. 5 5028
[5] Boutry C M et al 2019 Nat. Biomed. Eng. 3 47–57
[6] Niu S et al 2019 Nat. Electron. 2 361–8
[7] Lin R, Kim H J, Achatvamishth S, Kurt S A, Tan S C C, Yao H, Tee B C K, Lee J K W and Ho J S 2020 Nat. Commun. 11 444
[8] Dautta M, Alshetaiwi M, Escobar A, Torres F, Bernardo N and Tseng P 2020 Adv. Electron. Mater. 6 1901311
[9] Dong Z, Li Z, Yang F, Qiu C-W and Ho J S 2019 Nat. Electron. 2 335–42
[10] Po Jui C, Rodger D C, Saati S, Humayun M S and Yu-Chong T 2008 J. Micro. Syst. 17 1342–51
[11] Chen P-Y, Sakhadari M, Hajizadeh M, Cui Q, Cheng M M-C, El-Ganainy R and Al A 2018 Nat. Electron. 1 297–304
[12] Zhou B-B, Deng W-J, Wang L-F, Dong L and Huang Q-A 2020 Phys. Rev. Appl. 13 064022
[13] Bender C M and Boettcher S 1998 Phys. Rev. Lett. 80 5243–6
[14] Rüter C E, Makris K G, El-Ganainy R, Christodoulides D N, Segev M and Kip D 2010 Nat. Phys. 6 192–5
[15] Konotop V Y, Yang J and Zeyl A D A 2016 Rev. Mod. Phys. 88 035002
[16] Feng L, El-Ganainy R and Ge L 2017 Nat. Photon. 11 752–62
[17] Zhao H and Feng L 2018 Natl. Sci. Rev. 5 183–99
[18] El-Ganainy R, Makris K G, Khajavikhan M, Musslimani Z H, Rotter S and Christodoulides D 2018 Nat. Phys. 14 11–9
[19] Liu Y, Hao T, Li W, Capmany J, Zhu N and Li M 2018 Light Sci. Appl. 7 38
[20] Özdemir Ş K, Rotter S, Nori F and Yang L 2019 Nat. Mater. 18 783–98
[21] Gupta S K, Zou Y, Zhu X Y, Lu M H, Zhang L J, Liu X P and Chen Y F 2019 Adv. Mater. 32 1903639
[22] Heiss W D 2012 J. Phys. A: Math. Theor. 45 444016
[23] Miri M A and Ali A 2019 Science 363 eaar7709
[24] Wiersig J 2014 Phys. Rev. Lett. 112 203901
[25] Liu Z P et al 2016 Phys. Rev. Lett. 117 110802
[26] Chen P Y and Jung J 2016 Phys. Rev. Appl. 5 064018
[27] Ren J, Hodaei H, Harari G, Hassan A U, Chow W, Soltani M, Christodoulides D and Khajavikhan M 2017 Opt. Lett. 42 1556–9
[28] Hodaei H, Hassan A U, Wittek S, Garcia-Gracia H, El-Ganainy R, Christodoulides D N and Khajavikhan M 2017 Nature 548 187–91
[29] Chen W, Kaya Özdemir Ş, Zhao G, Wiersig J and Yang L 2017 Nature 548 192–6
[30] Chen W, Zhang J, Peng B, Özdemir Ş K, Fan X and Yang L 2018 Photon. Res. 6 A23–30
[31] Zhong Q, Ren J, Khajavikhan M, Christodoulides D N, Ozdemir S K and El-Ganainy R 2019 Phys. Rev. Lett. 122 153902
[32] Wang S, Hou B, Lu W, Chen Y, Zhang Z Q and Chan C T 2019 Nat. Commun. 10 832
[33] Hokmabadi M P, Schumer A, Christodoulides D N and Khajavikhan M 2019 Nature 576 70–4
[34] Park J-H, Ndao A, Cai W, Hsu L, Kodigala A, Lepetit T, Lo Y-H and Kanté B 2020 Nat. Phys. 16 462–8
[35] Huang Y, Shen Y and Vertonis G 2019 Opt. Express 27 37494–507
[36] Qin G-Q, Wang M, Wen J-W, Ruan D and Long G-L 2019 Photon. Res. 7 1440–6
[37] Wiersig J 2020 Photon. Res. 8 1457–67
[38] Grant M J and Digonnet M J F 2020 Opt. Lett. 45 6538–41
[39] Wang Q, Ding K, Liu H, Zhu S and Chan C T 2020 Opt. Express 28 1758–70
[40] Sun Y, Tan W, Li H Q, Li J and Chen H 2014 Phys. Rev. Lett. 112 143903
[41] Sweeney W R, Hsu C W, Rotter S and Stone A D 2019 Phys. Rev. Lett. 122 093901
[42] Goldzak T, Mailybaev A A and Moiseyev N 2018 Phys. Rev. Lett. 120 013901
[43] Ding K, Ma G, Zhang Z Q and Chan C T 2018 Phys. Rev. Lett. 121 083902
[44] Richter S et al 2019 Phys. Rev. Lett. 123 227401
[45] Wu T et al 2020 Phys. Rev. Lett. 124 083901
[46] Fleury R, Sounas D and Alu A 2015 Nat. Commun. 6 5905
[47] Djorwe P, Pennec Y and Djafari-Rouhani B 2019 Phys. Rev. Appl. 12 024002
[48] Sakhdari M, Hajizadegan M, Li Y, Cheng M M-C, Hung J C H and Chen P Y 2018 IEEE Sensors J. 18 9548–55
[49] Hajizadegan M, Sakhdari M, Liao S and Chen P Y 2019 IEEE Trans. Antennas Propag. 67 3233–4
[50] Zhang Y J, Kwon H, Miri M A, Kallos E, Cano-Garcia H, Tong M S and Alu A 2019 Phys. Rev. Appl. 11 044049
[51] Chen P-Y and El-Ganainy R 2019 Nat. Electron. 2 323–4
[52] Sakhdari M, Hajizadegan M, Zhong Q, Christodoulides D N, El-Ganainy R and Chen P Y 2019 Phys. Rev. Lett. 123 193901
[53] Xiao Z, Li H, Kottos T and Alu A 2019 Phys. Rev. Lett. 123 213901
[54] Zeng C, Sun Y, Li G, Li Y, Jiang H, Yang Y and Chen H 2019 Opt. Express 27 27562–72
[55] Assawaworrarit S and Fan S 2020 Nat. Electron. 3 273–9
[56] Sakhdari M, Hajizadegan M and Chen P-Y 2020 Phys. Rev. Res. 2 013152
[57] Wu L, Zhang B and Zhou J 2020 IEEE Trans. Power Electron. 35 12497–508
[58] Pan L, Chen S and Cui X L 2019 Phys. Rev. A 99 063616
[59] Zhong Q, Kou J, Ozdemir S K and El-Ganainy R 2020 Phys. Rev. Lett. 125 203602
[60] Ryu J, Gwak S, Kim J, Yu H-H, Kim J-H, Lee J-W, Yi C-H and Kim C-M 2019 Photon. Res. 7 1473–8
[61] Quiroz-Juárez M A, Perez-Leija A, Tschernig K, Rodríguez-Lara B M, Magaña-Loaiza O S, Busch K, Joglekar Y N and León-Montiel R D J 2019 Photon. Res. 7 862–7
[62] Assawaworrarit S, Yu X and Fan S 2017 Nature 546 387–90
[63] Zeng C, Sun Y, Li G, Li Y H, Jiang H T, Yang Y P and Chen H 2020 Phys. Rev. Appl. 3 034054