Aerodynamic Shape Optimization Design of Wing-Body Configuration Using a Hybrid FFD-RBF Parameterization Approach

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Abstract. Aerodynamic shape optimization design aiming at improving the efficiency of an aircraft has always been a challenging task, especially when the configuration is complex. In this paper, a hybrid FFD-RBF surface parameterization approach has been proposed for designing a civil transport wing-body configuration. This approach is simple and efficient, with the FFD technique used for parameterizing the wing shape and the RBF interpolation approach used for handling the wing body junction part updating. Furthermore, combined with Cuckoo Search algorithm and Kriging surrogate model with expected improvement adaptive sampling criterion, an aerodynamic shape optimization design system has been established. Finally, the aerodynamic shape optimization design on DLR F4 wing-body configuration has been carried out as a study case, and the result has shown that the approach proposed in this paper is of good effectiveness.

1. Introduction

With the rapid development in the computer science and technology, numerical optimization has been widely used as a powerful tool in research and development of aircraft industries. Aerodynamic shape optimization design is of the applications aiming at improving the efficiency of an aircraft by redesigning its aerodynamic shape. There are several key components that significantly influence the utilization and result of the aerodynamic shape optimization design, including computational fluid dynamics (CFD) flow solver for aerodynamic performance evaluation, surface parameterization technique for shape definition and manipulation, and the optimization algorithms for obtaining an optimum solution. By now Reynolds Averaged Navier-Stokes equations (RANS equations) CFD solvers have been a relatively mature technique in aircraft engineering research and development. Thus the other two components, surface parameterization technique and optimization algorithm have been the bottlenecks in efficient aerodynamic shape optimization design.

The surface parameterization not only determines whether the design space of an aerodynamic optimization design contains an optimum solution, but also determines that whether this solution can be obtained with an acceptable computational cost. Due to the limit of computers, researchers in the period from 1970s to 1990s tend to use 2D parameterization method to modify wing sections instead of a three-dimensional wing for designing the wing shape. Parameterization methods handling wing sections are, for example, PARSEC [1], Hicks-Henne bump function [2]. Then some interpolation or
spline based methods would be used for combining the several wing sections into a three-dimensional wing shape. Kulfan proposed the CST (class function/shape function transformation) technique [3] for aircraft component parameterization, and titled it as a universal parametric geometry representation method. However, CST cannot be used for modification of a specified initial aerodynamic shape, for it cannot represent a specified shape absolutely exactly. With the development in geometry technique and computer performance, after late 1990s, researchers have begun to use direct three-dimensional surface parameterization methods for aerodynamic shape optimization design. Hicken and Zingg used a patched B-spline surfaces approach for shape modification in aerodynamic shape design [4]. Yamazaki et al. used physics-based direct-manipulation FFD method for geometry parameterization of both airfoils and wings [5]. Samareh used a NURBS based approach to parameterize the shape perturbation of the wing geometry while using FFD to modify the planform [6]. Though there are quite a number of surface parameterization methods that can be applied in the three-dimensional aerodynamic shape design, the discussion about handling the junction part between the wing and the body in aerodynamic optimization design of wing-body configuration is relatively rare. Nakahashi et al. have proposed a surface parameterization method which can handle wing-body junction by neighbor-to-neighbor searching and geodesic distance calculating [7]. However within Nakahashi’s method, when the wing geometry is modified, the fuselage geometry would also be deformed obviously, which is often unacceptable in practical aerodynamic shape design.

In this paper, we have proposed an efficient surface parameterization method using an hybrid FFD-RBF parameterization approach, which can handle the junction part between the wing and body by interpolation based updating. In the implementation of this method, FFD technique [8] has been used for parameterizing the wing shape, including wing shape deformation and geometric twist. After the wing shape is modified, including wing rudder section deformed, the junction part between the wing and the body would be updated by a RBF based interpolation method to ensure preservation of original geometry continuity.

Apart from the surface parameterization method, optimization algorithm is another key component in aerodynamic shape optimization problem. With the development in computers, nature-inspired heuristic optimization algorithms such as genetic algorithm (GA) and particle swarm optimization (PSO) have already been widely used in aerodynamic shape optimization, related work can be found in literatures [9,10]. Nature-inspired algorithms have many advantages. One key advantage is their ability to handle non-smooth functions since most nature-inspired algorithms are gradient-free methods. Though there may not be guarantee for global optimality due to the stochastic nature of these algorithms, they can find global optimal solutions in practice. In this paper we use a novel optimization algorithm – Cuckoo Search (CS) as the optimum searching engine. Recent studies have shown that Cuckoo Search algorithm can be more efficient than Genetic Algorithm and Particle Swarm Optimization, with better optimized results and a relatively small number of function evaluations [11].

Another important issue in aerodynamic shape optimization design on complex shapes is the application of surrogate model. In aerodynamic shape optimization design, especially when the problem is high-dimensional, the optimization process would require enormous number of function evaluation. To remedy this, Kriging surrogate model has been used for approximation of the aerodynamic parameters with expected improvement adaptive sampling criterion. With integration of the hybrid FFD-RBF geometry parameterization with CS algorithm and Kriging model, an aerodynamic shape optimization design system has been established and tested on a case of DLR F4 wing-body model.

2. Hybrid FFD/RBF Surface Parameterization Approach

The success of an aerodynamic shape optimization design largely depends on the choice of design variables, which would be represented in terms of efficient geometrical parameterization. In this paper, the FFD technique has been chosen for surface parameterization in aerodynamic shape design. When handling wing-body design, especially when the wing root section is deformed during optimization,
the junction part between wing and body has to be updated or reconstructed in order to preserve the original connectivity. An RBF based interpolation method has been adopted in this paper for updated the surface of the wing root junction area with preservation of surface connectivity when the wing root section has been changed in optimization process. The DLR F4 wing-body configuration would be redesigned in this paper using the hybrid FFD-RBF surface parameterization approach proposed in this paper as a study case.

2.1. Free Form Deformation Surface Parameterization

FFD is suitable for the parameterization of arbitrary three-dimensional spatial geometrical property, regardless of the representation of the objective. Without the need to fit or interpolate the original shape, the objective geometry could be parametrically deformed by using FFD with the conservation of its original smoothness and continuity.

The procedures of using the FFD technique for parametrically deforming an objective geometry (the wing of DLR F4 wing-body configuration in this paper) are as follows. At first, a parallelepiped box frame, also called FFD control frame, is established around the wing as shown in Figure 1. Then the Cartesian coordinates of the surface points of the wing shape are transformed into local coordinates that are defined by the vertices of the FFD control frame using the following formula:

\[ \bar{x}(s,t,u) = \sum_{l=0}^{l_m} \sum_{j=0}^{j_n} \sum_{k=0}^{k_n} \left[ B_l^j(s)B_m^k(t)B_n^k(u) \right] \cdot \bar{P}_{l,j,k} \]  

where \( \bar{x} \) is the position vector of a surface point of the wing shape, and \((s,t,u)\) are the corresponding local coordinates of the point, which is defined inside the FFD control frame by formula (1). \( \bar{P}_{l,j,k} \) is the position vector in the Cartesian system of every vertices of the FFD control frame, with \( l, m, n \) being the order of the FFD control frame along each of the three directions. Furthermore \( B_l^j(s), B_m^k(t) \) and \( B_n^k(u) \) are the Bernstein basis functions, defined as follows:

\[ B_l^j(s) = \frac{(l)!}{(l-j)!} s^j (1-s)^{l-j} \]

\[ B_m^k(t) = \frac{(m)!}{(m-k)!} t^k (1-t)^{m-k} \]

\[ B_n^k(u) = \frac{(n)!}{(n-k)!} u^k (1-u)^{n-k} \]  

Fig. 1 The FFD Control Frame around the Wing
2.2. RBF based Interpolation Approach Handling the Wing Root Junction Part

In this paper, the FFD technique is used for parameterizing the wing surface only, as in fig 2. FFD frame would only determine the deformation of the wing, the mesh color of which is red in fig 2. The wing body junction area, the mesh color of which is green, would not be influenced by the FFD frame.

After the wing has been deformed, the wing root section has been changed, thus the intersection between the wing and the body, together with the wing root junction area, have to be updated or reconstructed to adjust to the deformed wing shape. Here we adopt an RBF interpolation based method to calculate the displacements of every mesh point of the wing root junction part after the wing deformation.

Here we use the root section of the deformed wing and the outer boundary of the wing root junction part (green in fig 3 ) as the set of interpolation centers to realize the total interpolation process so as to update the mesh points on the wing body junction part after wing deformation. Fig 3 illustrates the different parts of the wing body surface, which are, wing mesh parameterized by FFD, body mesh which is fixed during the design process, and the wing body junction part area which is to be updated by RBF interpolation.

The procedures for updating the wing body junction part are illustrated as below. First, after the wing has been deformed as the design variables changes, the displacements of the interpolation centers, \( \Delta X_j \), \( X_j \subseteq (B_{wr} \cup B_{j}) \) are calculated, where \( X_j \) are the positions of the interpolation centers, \( B_{wr} \) and \( B_{j} \) are the point set on wing root section and the junction part outer boundary respectively. Since that the updated junction part mesh have to match the fixed junction part boundary and the deformed wing root section, so that the RBF interpolation function is subject to:

\[
\Delta X_j = \sum_{i=1}^{n} \gamma_i \phi(\|X_{ij} - X_{cj}\|)
\]  

(3)

Where \( X_{ij} \) and \( X_{cj} \) are the positions of mesh points in the set of interpolation centers, the number of which is \( n \). \( \phi(\cdot) \) is the radial basis function, which is chosen as follows in this paper:

\[
\phi(\xi) = \begin{cases} 
(1 - \xi)^2 & \xi \leq 1 \\
0 & \xi > 1
\end{cases}, \quad \xi = \frac{\|X - X_{cj}\|}{r}
\]

(4)

Where \( r \) is the support radius which is specified in the interpolation process, and \( \|\cdot\| \) is the Euclidian distance in three dimensional space. \( \gamma_i \) in formula (3) is the interpolation parameters, which are to be computed by solving the linear system dominated by formula (3). After the interpolation parameters have been obtained, then the RBF interpolation function has been defined, as follows:

\[
\Delta X = \sum_{i=1}^{n} \gamma_i \phi(\|X - X_{ci}\|)
\]

(5)

The displacements of the mesh points of wing root junction area can be computed by the formula above. Then the new position of a mesh point on the wing body junction part area after the wing deformation could be computed by:

\[
X_{new} = X_{old} + \sum_{i=1}^{n} \gamma_i \phi(\|X_{old} - X_{ci}\|)
\]

(6)
3. Cuckoo Search Algorithm

Cuckoo Search algorithm (CS) is one of the novel nature-inspired based algorithms developed by Yang and Deb[12] in 2009. Its basic methodology is to mimic the brood parasitism behavior of some cuckoo species. This algorithm is further enhanced by Lévy flight [13] based on Lévy distribution, which can generate solutions both locally and at far field, and thus has been proven to be a more efficient searching strategy than those using Gaussian isotropic random walks. Recent studies have shown that CS algorithm has better efficiency compared with the GA and PSO [11].

In the implementation of Cuckoo Search algorithm, a balanced combination of a local random walk and the global explorative random walk can ensure that the entire design space can be explored by both local and global search capabilities. The switching between local and global searching is essentially controlled by a switching parameter \( p_a \). The local random walk can be denoted as:

\[
\hat{x}_i^{t+1} = x_i^t + \alpha s \odot H(p_a - \varepsilon) \odot (x_j^t - x_i^t)
\]  

where \( x_j^t \) and \( x_i^t \) are two different solutions selected by random permutation, \( H(u) \) is a Heaviside function and \( \odot \) represents the entry-wise multiplication operator. Here \( \varepsilon \) is a random number drawn from a uniform distribution, and \( s \) is the step size. Technically speaking, this step can also become a global search step if the step size is large enough by controlling the scaling factor \( \alpha \).

The variations of the step length of random walks based on a Gaussian distribution are typically small. In contrast, the steps of the Lévy flight usually differ significantly from each other. In Cuckoo Search algorithm, the global random walk is implemented by using Lévy flights strategy:

\[
\hat{x}_i^{t+1} = x_i^t + \alpha s \odot \text{Le}^\text{vy} (\beta) \quad 0 < \beta \leq 2
\]
Where \( \alpha > 0 \) is a scaling factor of the step size, and is related to the scales of the particular characteristics of the problem. \( \text{Le}^\text{vy}(\beta) \) denotes the Lévy distribution, given in terms of a simple power law formula [14].

4. Aerodynamic Performance Computation by RANS Solver

The aerodynamic performance of the wing-body configuration is evaluated by solving Reynolds Averaged Navier-Stokes (RANS) equations solver using multi-block structured grid with cell center scheme. The spatial discretization scheme chosen is 2nd order Roe scheme and the turbulence model is Spalart-Allmaras turbulence model. A approximation factor decomposition approach has been adopted to accelerate the convergence.

For validating the accuracy of the solver, simulations on the DLR F4 wing-body model have been carried out and the results have been compared with the experimental data. The test conditions are: Mach=0.75, Reynolds Number=3.0E6, and Left Coefficient=0.5. The computational mesh size is 1.2E6 in mesh point’s number. The computational results of the pressure coefficient distribution of three sections are compared with the experimental data, which are shown in Figure 8. Considering the mesh size is relatively small in optimization process, which requires a large number of CFD evaluations, the computed pressure coefficient distribution obtained by the RANS solver matches well to the experimental data along the wing span, which shows that the solver adopted in this paper has good accuracy for evaluating the aerodynamic performance.

5. Kriging Model with Expected Improvement Adaptive Sampling Criterion

The Kriging metamodel, originating from geostatistics, has widely been used in engineering optimization for its good accuracy and efficiency, the detailed description of using Kriging model in optimization design could be found in literatures [15]. The most powerful feature of the Kriging model is its capability for estimating the variance or error at a position in design space for approximation. Thus, in optimization using Kriging as approximation model, the uncertainty of probability of the optimum function value could be taken into consideration. With this methodology, the solution at which the global optimum most likely to occur would be determined, in other words, we want to obtain a solution with the largest probability of being the global optimum. The probability of being the global optimum is evaluated using the expected improvement (EI) criterion [16]. The expected improvement of Kriging model is defined as below:

\[
E[I(x)] = (f_{\min} - \hat{y})\Phi\left(\frac{f_{\min} - \hat{y}}{s}\right) + s\phi\left(\frac{f_{\min} - \hat{y}}{s}\right)
\]  

(9)
where the \( I(x) \) is the improvement of \( f_{\min} \), which is the minimum true objective function value obtained so far. And \( \Phi(*) \) is the normal cumulative distribution function, and \( \phi(*) \) is the normal probability density function. \( s \) is the root mean squared error (RMSE) predicted by the Kriging model. \( I(x) \) is defined as:

\[
I(x) = \max \left[ f_{\min} - \hat{y}(x), 0 \right]
\] (10)

Expected improvement criterion serves as an adaptive sampling infilling method, for additional sampling at the location where the global minimum most likely to occur.

With integration of the hybrid FFD-RBF parameterization approach in section 2, cuckoo search algorithm in section 3 and the Kriging model with expected improvement adaptive sampling criterion above, an aerodynamic shape optimization design system aiming at drag reduction of wing-body configuration has been established. The flowchart of the optimization design procedures are as follows:

![Flow Chart of Aerodynamic Optimization Design Process using Hybrid FFD-RBF Surface Parameterization, Krigin Metamodel and CS](image)

**Fig. 5** Flow Chart of Aerodynamic Optimization Design Process using Hybrid FFD-RBF Surface Parameterization, Krigin Metamodel and CS

### 6. Study Case Verification

A study case using the aerodynamic optimization design system established in this paper has been carried out on the DLR F4 wing-body model, the wing-body shape and the corresponding FFD control frame are as the figure 2 shows, the total number of the design variables is 38 (36 ffd control points’ vertical displacements together with 2 sections’ rotation). The objective function is drag reduction at fixed lift coefficient of 0.5, with the constraint that the wing volume should be no less than the initial one. The optimization model could be formulated as:
min \quad Cd
\nonumber
\text{s.t.} \quad Cl = 0.5
\nonumber
Volume_{\text{optimized}} \geq Volume_{\text{original}} \quad (11)

The Cuckoo Search Algorithm with a population size of 300 has been used with 100 generations to search the objective function minimum value solution and EI value maximum solution in every optimization cycle. The parameters in CS are $P_a = 0.5, \alpha = 0.01$ and $\beta = 1.5$. The comparison of the final optimized wing-body with the original DLR F4 is as the Table 1 shows. The comparisons of the wing upper surface pressure coefficient distribution and several wing sections pressure coefficient distribution are as the figure 6 to 7 show.

| Wing | Cd       | Cl/Cd  | $\Delta$Cd | $\Delta$Cl/Cd |
|------|----------|--------|-------------|---------------|
| Original | 300.2E-4 | 16.656 |             |               |
| Optimized | 286.3E-4 | 17.464 | 4.63%       | 4.85%         |

Fig. 6 Comparison of the Wing Upper Surface Pressure Coefficient Distribution of the Optimized and the Original.

Fig. 7 Comparison of the Wing Section Pressure Coefficient Distributions of the Optimized and the Original.
The comparisons above have shown that, after optimization design, the drag coefficient has been reduced 4.63% and the lift-drag-ratio has been improved 4.85% compared to the original one, while the lift coefficient is at fixed Cl=0.50. Figure 6 has shown that the strength of the shock wave has been alleviated apparently, though the shock wave has not been eliminated after optimization. Figure 7 (a) to (c) have shown that the pressure coefficient at the suction peaks have been reduced after optimization, which are all nearly below the line of Cp=-1.0.

7. Conclusion
A methodology that combines the FFD technique with RBF interpolation approach has been proposed as a hybrid FFD-RBF surface parameterization approach for wing-body configuration. The FFD technique is used for parametrically definition of the wing surface and the RBF approach is used for handling the updating the wing-body junction part while the wing has been deformed by interpolation. Together with a novel heuristic optimization algorithm – Cuckoo Search, and Kriging model with expected improvement adaptive sampling criterion, an aerodynamic design optimization study case has been carried out on a standard test model – DLR F4 wing-body. The result of the study case has shown that the hybrid approach proposed in this paper is of good feasibility and effectiveness of drag reduction, and the shock wave over the upper surface of the wing has been alleviated dramatically.

Further research will focus on shape optimization problems on much more complicated configurations with more engineering practical geometrical constraints, like a wing-body-nacelle-pylon configuration.

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