Using shortcut to adiabatic passage for the ultrafast quantum state transfer in cavity QED system

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We propose an alternative scheme to implement the quantum state transfer between two three-level atoms based on the invariant-based inverse engineering in cavity quantum electronic dynamics (QED) system. The quantum information can be ultrafast transferred between the atoms by taking advantage of the cavity field as a medium for exchanging quantum information speedily. Through designing the time-dependent laser pulse and atom-cavity coupling, we send the atoms through the cavity with a short time interval experiencing the two processes of the invariant dynamics between each atom and the cavity field simultaneously. Numerical simulation shows that the target state can be ultrafast populated with a high fidelity even when considering the atomic spontaneous emission and the photon leakage out of the cavity field. We also redesign a reasonable Gaussian-type wave form in the atom-cavity coupling for the realistic experiment operation.

I. INTRODUCTION

For various applications ranging from quantum storage and quantum communication \cite{1,2}, reliable quantum state transfer (QST) between two qubits has become an essential ingredient in the quantum information processing \cite{3}. Therefore, there is much interest in the QST for recent years \cite{4,5}. Experimentally, QST has been demonstrated with superconducting phase qubits and transmon qubits in the cavity quantum electronic dynamics (QED) system \cite{6,7}. Several theoretical proposals based on the technique of adiabatic passage have been applied in the implementation of the QST, and these adiabatic passage techniques include the stimulated Raman adiabatic passage (STIRAP) \cite{8,9} and the fractional stimulated Raman adiabatic passage (f-STIRAP) \cite{10,11}. For example, Ammit-Tulab et al. \cite{12} used the techniques of STIRAP and f-STIRAP to successfully transfer the quantum state between the Λ-type atoms and the photons. Although the adiabatic passage techniques are robust against the fluctuations of experimental parameters, the operation time needed to complete the QST is rather long in most cavity QED systems. However, the direct atom-photon interactions typically decays with the evolution time leading to the limitation for the perfect QST by adopting the adiabatic passage techniques.

Recently, Chen and Muga \cite{16} achieved the fast population transfer within two internal states of a single Λ-type atom by the invariant-based inverse engineering, and only two resonant laser pulses were used. The invariant-based inverse engineering combines the advantages of resonant pulses whose operation time is short and adiabatic technique which is robust against the variation of parameters, which has been used for different systems \cite{15,28}, except for the cavity QED system. The ultrafast population transfer between two or more atoms is a fundamental operation for scalable quantum information processors. However, previous studies based on the invariant-based inverse engineering focus on the ultrafast population transfer within two internal states of a single atom, and has not been devoted to the ultrafast population transfer between two or more atoms. Here, the major obstacle for the ultrafast population transfer between two or more atoms through the invariant-based inverse engineering exists in finding an appropriate medium that can be used to exchange energy or information with the atoms speedily.

To improve the efficiency of QST based on the traditional adiabatic passage in cavity QED system, we propose an alternative scheme based on the invariant-based inverse engineering to implement the ultrafast quantum state transfer between two Λ-type atoms in this paper. We take advantage of the cavity field as a medium for exchanging information between the atoms speedily, which is very different from that in Ref. \cite{16} where the population transfer is confined in two internal states of a single atom. Consider the photon leakage out of the cavity, the atoms are sent through the cavity with a short time interval, which suffer the opposite variation tendency in the time-dependent laser pulse and atom-cavity coupling, i.e., two processes of the invariant-based inverse engineering between each single Λ-type atom and the cavity field happen simultaneously. We find that the operation time needed to complete the QST is short enough before a photon leaks out of the cavity, and this is an obvious improvement compared with the QST based on the traditional adiabatic passage, resulting in the target state with a high fidelity even when taking the system’s decoherence into consideration, including the atomic spontaneous emission and the photon leakage out of the cavity. We also redesign the shape from a sinusoidal-wave form to a Gaussian-wave form in the time-dependent atom-cavity coupling strength for the realistic experiment. Compared with the traditional QST proposals, our improvement in the QST based on the invariant-based inverse engineering is very feasible with the current cavity.

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QED technology, and the present idea can be generalized to the QST models among three and more atoms.

II. INVARIANT-BASED INVERSE ENGINEERING IN THE CAVITY QED SYSTEM

A. Invariant dynamics between a single Λ-type atom and the cavity mode

We generalize the dynamics of invariant-based engineering to the cavity QED system containing a Λ-type atom and the cavity mode. In the atom-cavity system, as shown in Fig. 1, the single Λ-type atom has an excited state $|e\rangle$ and two ground states $|f\rangle$ and $|s\rangle$. The transition $|e\rangle \leftrightarrow |f\rangle$ is resonantly driven by the laser pulse with the time-dependent Rabi frequency $\Omega(t)$ and the transition $|e\rangle \leftrightarrow |s\rangle$ resonantly couples to the cavity mode with the time-dependent coupling coefficient $g(t)$. $\Omega(t)$ and $g(t)$ are assumed to be real in the following for simplicity. The state vector $|M,N\rangle$ denotes that the atom is in the state $|M\rangle (M = f,s,e)$ and there are $N \ (N = 0,1,2,\ldots)$ photons in the cavity field. Note that the total excitation of the atom-cavity system is conserved during the state evolution. In the interaction picture, the time-dependent Hamiltonian $H(t)$ under the rotating-wave approximation is block diagonal in the $(N+1)$-excitation subspace $\{|f,N\rangle,|e,N\rangle,|s,N\rangle\}$. The vector $|s,0\rangle$ here is not coupled to any other ones, therefore, one can thus restrict the problem to the projection of the Hamiltonian in the single-excitation subspace $\{|f,0\rangle,|e,0\rangle,|s,1\rangle\}$ as following (\(\hbar = 1\)):

$$H(t) = \begin{pmatrix}
0 & \Omega(t) & 0 \\
\Omega(t) & 0 & g(t) \\
0 & g(t) & 0 
\end{pmatrix}. \tag{1}
$$

The instantaneous eigenstates of $H(t)$ are $|n_0(t)\rangle = |\cos \theta, 0, -\sin \theta\rangle^T$ and $|n_{\pm}(t)\rangle = \frac{1}{\sqrt{2}}[|\sin \theta, \pm 1, \cos \theta\rangle]^T$, with the corresponding eigenvalues $E_0 = 0$ and $E_{\pm} = \pm \omega$, where $\theta = \arctan(\Omega(t)/g(t))$ and $\omega = \sqrt{\Omega^2(t) + g^2(t)}$. When the adiabatic condition $|\dot{\theta}| \ll |\omega|$ is satisfied and $\Omega(t)$ and $g(t)$ are applied through a counter-intuition passage as that in the STIRAP, we can adiabatically transfer the population from the initial state $|f,0\rangle$ to the final state $|s,1\rangle$ along the dark state $|n_0(t)\rangle$. The key point here is to speed up the population transfer $|f,0\rangle \rightarrow |s,1\rangle$ in the atom-cavity system by using the dynamics of invariant-based inverse engineering. For the Hamiltonian in Eq. (1) that possesses the SU(2) dynamical symmetry, an invariant Hermitian operator can be employed to construct the invariant dynamics for our atom-cavity system with a time-independent expectation value $\langle I(t) \rangle$, which satisfies $10,30$:

$$i \frac{\partial}{\partial t} I(t) - [H(t), I(t)] = 0, \tag{2}$$

where the complex conjugate $I^\dagger(t) = I(t)$. The invariant $I(t)$ is given by $10$:

$$I(t) = \mu \begin{pmatrix}
0 & \cos \gamma \sin \beta - i \sin \gamma \\
\cos \gamma \sin \beta & 0 & \cos \gamma \cos \beta \\
i \sin \gamma & \cos \gamma \cos \beta & 0
\end{pmatrix}, \tag{3}$$

where $\mu$ is an arbitrary constant with units of frequency to keep $I(t)$ involving the energy dimension. The eigenstates of $I(t)$ with the corresponding eigenvalues $\lambda_0 = 0$ and $\lambda_{\pm} = \pm 1$ are respectively:

$$|\Phi_0(t)\rangle = \begin{pmatrix}
\cos \gamma \cos \beta \\
-i \sin \gamma \\
-\cos \gamma \sin \beta
\end{pmatrix}, \tag{4}$$

and

$$|\Phi_{\pm}(t)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix}
\sin \gamma \cos \beta \pm i \sin \beta \\
i \cos \gamma \\
-\sin \gamma \sin \beta \pm i \cos \beta
\end{pmatrix}. \tag{5}$$

According to Lewis Riesenfeld theory $31$, the general solution of the Schrödinger equation with respect to the instantaneous eigenstates of $I(t)$ can be written as:

$$|\Psi(t)\rangle = \sum_{m=0,\pm} C_m e^{i\alpha_m} |\Phi_m(t)\rangle, \tag{6}$$

where $C_m$ is a time-independent amplitude and $\alpha_m$ is the Lewis-Riesenfeld phase with the following form:

$$\alpha_m(t') = \int_0^{t'} dt \langle \Phi_m(t)|[i \frac{\partial}{\partial t} - H(t)]|\Phi_m(t)\rangle, \tag{7}$$

where $t'$ is the total interaction time. And the time-dependent parameters $\gamma(t)$ and $\beta(t)$ should satisfy the following auxiliary equations $16$:

$$\dot{\gamma} = \Omega \cos \beta - g \sin \beta, \tag{8}$$

$$\dot{\beta} = \tan^{-1}(g \cos \beta + \Omega \sin \beta), \tag{9}$$

where the dot represents a time derivative.
B. Fast population transfer for the atom-cavity system

The explicit expressions of $\Omega(t)$ and $g(t)$ to be designed can be inversely derived from Eqs. (8) and (9):

$$\Omega = \dot{\beta} \cot \gamma \cos \beta - \dot{\gamma} \sin \beta,$$
$$g = \dot{\beta} \cot \gamma \sin \beta + \dot{\gamma} \cos \beta.$$  \hspace{1cm} (10) \hspace{1cm} (11)

For a single-mode driving, the atom-cavity system is assumed to be prepared in one of the eigenstates $|\Phi_m(t)\rangle$ of $I(t)$ initially, then the atom-cavity system is driven along this instantaneous eigenstate $|\Phi_m(t)\rangle$ without worrying about its transition to the other eigenstates, while the adiabatic condition is unnecessary here. To achieve the fast population transfer from the atom to the cavity field, the feasible parameters $\gamma(t)$ and $\beta(t)$ can be chosen as $[16]$:

$$\gamma(t) = \epsilon,$$  \hspace{1cm} (12)
$$\beta(t) = \pi t / 2 f_t,$$  \hspace{1cm} (13)

leading to:

$$\Omega(t) = (\pi / 2 f_t) \cot \epsilon \cos(\pi t / 2 f_t),$$  \hspace{1cm} (14)
$$g(t) = (\pi / 2 f_t) \cot \epsilon \sin(\pi t / 2 f_t),$$  \hspace{1cm} (15)

where $\epsilon$ is small value to be chosen later and $\epsilon \neq 0$.

However, when the initial state is $|f, 0\rangle$, the atom-cavity model is essentially a multi-mode driving rather than a single-mode driving, meaning the time-dependent wave function $|\Psi(t)\rangle = \chi_1(t)|f, 0\rangle + \chi_2(t)|s, 0\rangle + \chi_3(t)|s, 1\rangle$ involves contributions stemming from all the eigenstates of the invariant $I(t)$ $[16, 23]$, where $|\chi_1(t)|^2 + |\chi_2(t)|^2 + |\chi_3(t)|^2 = 1$, $|\Psi(0)\rangle = |f, 0\rangle$, and $|\Psi(t_f)\rangle = |s, 1\rangle$. The superposition of $|\Phi_0(0)\rangle$ and $|\Phi_{\pm}(0)\rangle$ corresponds to the initial bare state $|\Psi(0)\rangle$, while the superposition of $|\Phi_0(t_f)\rangle$ and $|\Phi_{\pm}(t_f)\rangle$ corresponds to the final bare state $|\Psi(t_f)\rangle$. The present setup based on the multi-mode driving provides the ultrafast population transfer with less intensities of energy than that based on the single-mode driving. The coefficients $\Omega(t)$ and $g(t)$ are chosen the same as those in the situation of the single-mode driving, corresponding to Eqs. (10) and (11) respectively. Thus, the population in the initial state $|f, 0\rangle$ can be fast transferred to that in the final state $|s, 1\rangle$. Note that the final state $|s, 1\rangle$ contains only one photon in the field which contains the coded information we want to transfer, therefore, we should design an appropriate time interval between two atoms, and make the second atom obtain the coded information from the cavity field before the photon leaks out of the cavity.

C. Ultrafast QST between two $\Lambda$-type atoms

As emphasized in the previous section, we send two $\Lambda$-type atoms through the cavity with a special time interval $\Delta T$, as plotted in Fig. 2.

FIG. 2: Experimental setup for the ultrafast QST between two $\Lambda$-type atoms through the cavity field. The atoms are denoted by atom 1 and atom 2 respectively. Each solid arrow represents the trajectory of each atom entering the cavity. The laser field is confined within the red circle area. The special value $\Delta T$ is chosen so that atom 2 can quickly obtain the coded information from the cavity field when atom 1 interacts with the cavity field. The dash line represents the reference coordinate axis.

If the initial state for the atom-cavity system is $|fs, 0\rangle$ (the symbol $|fs, 0\rangle$ represents that atom 1 is in $|f\rangle$ state, atom 2 is in $|s\rangle$ state, and the cavity field is in the vacuum state), the atom-cavity system will evolve in the single-excitation Hilbert space: $\Gamma_{nf} \equiv \{|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle, |\phi_4\rangle, |\phi_5\rangle\}$, with

$$|\phi_1\rangle = |fs, 0\rangle, |\phi_2\rangle = |es, 0\rangle,$$
$$|\phi_3\rangle = |ss, 1\rangle, |\phi_4\rangle = |se, 0\rangle, |\phi_5\rangle = |sf, 0\rangle.$$ \hspace{1cm} (16)

The main task here is to fast transfer the population from the state $|\phi_1\rangle$ to $|\phi_5\rangle$, leaving the cavity field in the vacuum state. According to invariant dynamics of the multi-mode driving between a single $\Lambda$-type atom and the cavity mode in the previous section, we design a time-dependent laser pulse to drive the atoms within the interaction time $0 \leq t \leq t_f$ ($t_f = 0.5us$) simultaneously. For the experimental setup in Fig. 2, due to the relative position between the cavity and laser fields (as plotted in Fig. 3), we can divide the whole atom-cavity-atom interaction process into three subprocedures when the atoms pass through the cavity and laser fields, i.e., $O_1$ ($-0.5us \leq t \leq 0$), $O_2$ ($0 \leq t \leq 0.5us$), and $O_3$ ($0.5us \leq t \leq 1us$). Therefore, the interaction Hamiltonian $H_{tot}$ of the atom-cavity system, under the rotating-wave approximation, is:

$$H_{tot} = \sum_{i=1}^{2} |\Omega_i(t)|f_i\langle e| + gi(t)|e\rangle_i(s|a + H.c.|.$$  \hspace{1cm} (16)

For the subprocedure $O_1$ ($-0.5us \leq t \leq 0$), atom 1 is sent through the cavity and arrives at the center of the cavity field when $t = 0$, while atom 2 is sent through the cavity with a time delay $\Delta T = 0.5us$ and arrives at the center of the laser field when $t = 0$. Consider the initial state of the atom-cavity system is $|fs, 0\rangle$, the transition $|f\rangle \leftrightarrow |e\rangle$ in atom 1 is forbidden due to the absence of the laser field, and the transition $|s\rangle \leftrightarrow |e\rangle$ atom 2 is also forbidden due to the absence of the photon in the cavity field. Therefore, when $t = 0$, the atom-cavity system keeps in the state $|\psi(0)\rangle = |fs, 0\rangle$.

For the subprocedure $O_2$ ($0 \leq t \leq 0.5us$), atom 1
encounters the cavity and laser fields with the respective forms:

\[ g_1(t) = \left( \pi / 2 t_f \right) \cot \epsilon \cos(\pi t / 2 t_f), \quad \Omega_1(t) = \left( \pi / 2 t_f \right) \cot \epsilon \sin(\pi t / 2 t_f), \]

while atom 2 encounters the cavity and laser fields with the respective forms:

\[ g_2(t) = \left( \pi / 2 t_f \right) \cot \epsilon \cos \left[ \pi (t - \Delta T) / 2 t_f \right], \quad \Omega_2(t) = \left( \pi / 2 t_f \right) \cot \epsilon \sin \left[ \pi (t + \Delta T) / 2 t_f \right]. \]

Then the atom-cavity system’s evolution becomes:

\[ |\psi(t)\rangle = \sum_{k=1}^{5} D_k(t) |\phi_k\rangle, \]

where \( D_k(t) \) is the time-dependent coefficient for the state \( |\phi_k\rangle \).

Under the laser pulse \( \Omega_1(t) \) and the atom-cavity coupling \( g_1(t) \), the transition \( |\phi_1\rangle \rightarrow |\phi_3\rangle \) for atom 1 quickly happens; at the same time, the transition \( |\phi_3\rangle \rightarrow |\phi_5\rangle \) for atom 2 also quickly happens under \( \Omega_2(t) \) and \( g_2(t) \). The whole transition \( |\phi_1\rangle \rightarrow |\phi_5\rangle \) process takes advantage of the cavity field as a medium for transporting the coded information between two atoms. The population in the initial state \( |\psi(0)\rangle = |sf, 0\rangle \) is transferred to that in the final state \( |\psi(t_f)\rangle = |sf, 0\rangle \). This process is very different from that in Ref. [10] where the population transfer is limited within two internal states of a single atom.

For the subprocedure \( O_2 \) (0.5 \( \mu s \leq t \leq 1 \mu s \)), the initial state for the atom-cavity system is \( |sf, 0\rangle \), which does not evolve with time \( t \) due to the absences of photons in the cavity field and the laser driving for atom 1 and atom 2, respectively.

If the initial state for the atom-cavity system is \( |ss, 0\rangle \), the whole system under the cavity and laser fields keeps on the state \( |ss, 0\rangle \) due to absence of photons in the cavity field. Therefore, the fast QST between two atoms is achieved leaving the cavity field in the vacuum state, i.e.,

\[ |s\rangle_2 \otimes (|x\rangle f_1 + y |s\rangle_1) \rightarrow (|x\rangle f_2 + y |s\rangle_2) \otimes |s\rangle_1 \] (the subscripts 1 and 2 respectively denote atom 1 and atom 2),

where \( x, y \) are the coded informations and \( |x|^2 + |y|^2 = 1 \).

As we can see from the above analysis, the time evolution of the initial state \( |ss, 0\rangle \) does not change during the whole operation, thus its transfer keeps 100\%. The only factor that affects the fidelity of our QST is the time evolution of the initial state \( |fs, 0\rangle \). Therefore, to verify the reliability of our QST between two atoms, we should consider the detailed characteristic parameter of the atom-cavity system in the subprocedure \( O_2 \) (0 \( \leq t \leq t_f \)).

Although the analytic solution for the coefficient \( D_k(t) \) is hard to be obtained, we numerically verify the fidelity \( F \) for the final state \( |\phi_5\rangle \) by setting the evolutive time \( t = 0.5 \mu s \) in Fig. 4, where \( F = |D_5(t)|^2 \). Interestingly, for the present multi-mode driving in the atom-cavity system, we find that there are several distinct values where the fidelity turns out to be close to unit, i.e., \( \epsilon = 0.1152 \) for \( N = 1; \epsilon = 0.0651 \) for \( N = 2 \) and so on. In the following, we choose the maximal value \( \epsilon = 0.1152 \) to satisfy \( F \approx 1 \). The behavior of \( F \) against \( \epsilon \) is oscillating, which is essentially caused by different Lewis-Riesenfeld phases generated in the transitions \( |\phi_1\rangle \rightarrow |\phi_3\rangle \) and \( |\phi_3\rangle \rightarrow |\phi_5\rangle \) for the eigenvectors of the invariant \( I(t) \). This result coincides with that in the single-atom system, meaning two atoms experience the similar invariant dynamics of two internal states in a single A-type atom [10]. We remark that the whole interaction time needed to complete the QST between two atoms is only 0.5 \( \mu s \), which is an improvement compared with the previous QST without using the dynamics of invariant-based engineering in the cavity QED system [4].

### III. Analysis of Experiment Feasibility

To achieve the ultrafast QST between two A-type atoms, we have obtained the analytic expressions of the time-dependent laser pulse and the atom-cavity coupling strength, as depicted from Eqs. (17) to (20), the shapes of which are sinusoidal-wave forms. However, for the realistic experiment, it is more easy to obtain the atom-cavity coupling strength \( g(t) \) with the Gaussian-wave form. Therefore, we use the mathematical method of minimum quadratic fitting to redesign the wave forms for the parameters \( g_1(t) \) and \( g_2(t) \); then we get the redesigned parameters \( G_1(t) \) and \( G_2(t) \) with the Gaussian-
wave forms as following:

\[ G_1(t) = \epsilon' \exp \left( \frac{t^2}{\sigma^2} \right), \] (22)

\[ G_2(t) = \epsilon' \exp \left[ \frac{(t - \Delta T)^2}{\sigma^2} \right], \] (23)

where \( \epsilon' = 4.5 \times 2\pi MHz \) and \( \sigma = \sqrt{0.14}\mu s \), as plotted in Fig. 5(a).

To verify the feasibility of the redesigned atom-cavity coupling strength, we numerically simulate the population \( P_k \) of the state \( |\phi_k\rangle \) by combining Eqs. (18), (20), (22), and (23), in which \( P_k \) is defined as \( P_k = |D_k(t)|^2 \), as depicted in Fig. 6(a) and (b). It is obvious to see that the population \( P_k \) by taking the redesigned atom-cavity coupling strength \( G_i(t) (i = 1, 2) \) with the Gaussian-wave form agrees very well with that by taking the atom-cavity coupling strength \( g_i(t) \) with the sinusoidal-wave form, except for the slight deviations between the corresponding excited states.

To compare with the traditional QST between two atoms based on the adiabatic passage in the cavity QED system, we replace the present set of parameters \( G_i(t) \) and \( \Omega_i(t) (i = 1, 2) \) with the general set of parameters \( g_i(t) \) and \( \Omega_i(t) \) for the traditional QST between two atoms in the cavity QED system.

FIG. 5: (a) The green solid and red dash lines represent the atom-cavity coupling coefficients \( g_i(t) (i = 1, 2) \) with the sinusoidal-wave form and \( G_i(t) \) with the Gaussian-wave form, respectively. (b) The parameters of general adiabatic passages \( \Omega_i(t) \) and \( g_{1(2)}(t) \) for the traditional QST between two atoms in the cavity QED system.

FIG. 6: Time evolution of the population \( P_k \) \( (k = 1, 2, 3, 4, 5) \) with the set of parameters: (a) \( \{ \Omega_1(t), \Omega_2(t), g_1(t), g_2(t) \} \); (b) \( \{ \Omega_1(t), \Omega_2(t), G_1(t), G_2(t) \} \); (c) \( \{ \Omega_1(t), \Omega_2(t), g_{1(2)}(t) \} \).

\[ g_{1(2)}(t) \] and \( \Omega_i(t) \) are defined as

\[ g_{1(2)}(t) = g' \exp \left( \frac{(t - T_a/2)^2}{w_C^2} \right), \] (24)

\[ \Omega_1(t) = \Omega' \exp \left[ \frac{(t - T_a/2 - d)^2}{w_L^2} \right], \] (25)

\[ \Omega_2(t) = \Omega' \exp \left[ \frac{(t - T_a/2 + d)^2}{w_L^2} \right], \] (26)

where \( T_a = 10\mu s \), \( w_C = \frac{T_a}{6} \), \( w_L = \frac{T_a}{12} \), \( g' = 4.5 \times 2\pi MHz \), \( \Omega' = 0.3g' \), and \( d = \frac{T_a}{10} \), as plotted in Fig. 5(b). Based on the parameters \( g_{1(2)}(t) \) and \( \Omega_i(t) \), we numerically simulate the corresponding population \( P_k \) in Fig. 6(c). Apparently, the total operation time in Fig. 6(c) needed to complete the QST is rather longer than that in Fig. 6(a) or (b), and the population in Fig. 6(c) drops below 90%, which is far smaller than that in Fig. 6(a) or (b). This is an obvious improvement both in the operation time and the fidelity when we use the invariant-based inverse engineering to implement the QST in the cavity QED system. Furthermore, we also consider the
effect caused by the fluctuations of the redesigned parameters \( \epsilon' \) and \( \sigma \) on the fidelity \( F \) in Fig. 7, and the result shows that the fidelity keeps higher than 97% even when the fluctuations \( \delta \epsilon' \) and \( \delta \sigma \) are both 10%. Thus, the present QST using the redesigned parameters \( G_1(t) \) and \( G_2(t) \) with the Gaussian-wave forms is rather robust against the fluctuations of \( \epsilon' \) and \( \sigma \), and thus it is proved to be valid.

Now we concentrate on discussing the fidelity \( F \) in the presence of dissipation caused by the noisy environment, which includes the atomic spontaneous emission and the photon leakage out of the cavity. When taking the effect of decoherence into account, the master equation for the density matrix \( \rho(t) \) of the present atom-cavity system is expressed as:

\[
\dot{\rho} = -i[H_{\text{tot}}, \rho] - \frac{\kappa}{2}(a^\dagger a \rho - 2a^\dagger a \rho^\dagger + \rho a^\dagger a)
- \sum_{k=1}^{2} \sum_{m=s,f} \Gamma_{em}^k (S_{em}^k S_{me}^\dagger \rho S_{me}^k - 2S_{me}^k \rho S_{em}^k + \rho S_{em}^k S_{me}^k),
\]

where \( \Gamma_{em}^k \) is the atomic spontaneous emission rate from the excited state \( |e\rangle \) to the ground state \( |m\rangle \) of the \( k \)th atom, \( S_{em}^k = |e\rangle\langle m| \) and \( S_{me}^k = |m\rangle\langle e| \) in the \( k \)th atom. \( \kappa \) is the photon leakage rate. We assume \( \Gamma_{em} = \Gamma/2 \) for simplicity. In Fig. 8 we plot the fidelity \( F \) of the final state \( |sf, 0\rangle \) versus the dimensionless parameters \( \Gamma/g \) and \( \kappa/g \) via numerically solving the master equation (27) with the set of parameter \( \{ G_1(t), G_2(t), \Omega_1(t), \Omega_2(t) \} \). The result of Fig. 8 shows that the QST between two atoms based on the invariant-based engineering is insensitive to the photon leakage and the atomic spontaneous emission. This is because the present QST in the atom-cavity system based on the invariant-based engineering is largely sped up, in which the population from the initial state to the final state is fast enough before the photon leaks out of the cavity, and the total population for the excited states is rather small during the whole system evolution. But, the fidelity of our QST is more sensitive to the photon leakage than the atomic spontaneous emission because the population in the excited states is larger than that in the excited states which involve the atomic excited state during the whole system evolution, i.e., \( P_3 \gg P_2 + P_1 \) in Fig. 6 (a) and (b).

Finally, we present a brief discussion about the basic elements in the real experiment. The \( \Lambda \)-type atomic configuration can be achieved with the Cs atoms, in which the state \( |f\rangle \) corresponds to \( F = 4 \), \( m = 4 \) hyperfine state of \( 6^2S_{1/2} \) electronic ground state, \( |s\rangle \) corresponds to \( F = 3 \), \( m = 2 \) hyperfine state of \( 6^2S_{1/2} \) electronic ground state, and \( |e\rangle \) corresponds to \( F = 3 \), \( m = 3 \) hyperfine state of \( 6^2P_{3/2} \) electronic state. For the typical experimental parameters \( (g, \kappa, \Gamma)/2\pi = (750, 3.5, 2.62) \times \text{MHz} \), which have been reported in the recent cavity QED experiments \( 62, 63 \), we find the fidelity for the target state is still higher than 98.85% with the set of parameter \( \{ G_1(t), G_2(t), \Omega_1(t), \Omega_2(t) \} \). Therefore, the realization of the present QST scheme is very promising with the current technology.

FIG. 7: The fidelity \( F \) versus the fluctuations \( \delta \epsilon'/\epsilon' \) and \( \delta \sigma/\sigma \).

FIG. 8: The fidelity \( F \) versus the dimensionless parameters \( \Gamma/g \) and \( \kappa/g \).

IV. CONCLUSION

To conclude, we have proposed an alternative scheme to implement the ultrafast quantum state transfer between two \( \Lambda \)-type atoms. Compared with the traditional QST in cavity QED system, the present QST based on the invariant-based inverse engineering is an obvious improvement for both the operation time and the fidelity, in which the quantum information is fast transferred between the atoms by taking advantage of the cavity field as a medium. Through designing the time-dependent laser pulse and atom-cavity coupling, we obtain the target state with a high fidelity even when taking the atomic spontaneous emission and the photon leakage out of the cavity. We also redesign a Gaussian-type wave form in the atom-cavity coupling for the realistic experiment operation.
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