Abstract

We investigate the so-called “Kaluza-Klein regularisation” procedure in supersymmetric extensions of the standard model with additional compact dimensions and Scherk-Schwarz mechanism for supersymmetry breaking. This procedure uses a specific mathematical manipulation to obtain a finite result for the scalar potential. By performing the full calculation, we show that the finiteness of this result is not only a consequence of the underlying supersymmetry, but also the result of an implicit fine-tuning of the coefficients of the terms that control the ultraviolet behaviour. The finiteness of the Higgs mass at one-loop level seems therefore to be an artefact of the regularisation scheme, and quadratic divergences are expected to reappear in higher orders of perturbation theory.
1 Introduction

Perturbative calculations in quantum field theories are usually plagued by the appearance of divergences. The program of regularisation and renormalisation has shown, however, that these divergences do not lead to inconsistencies (in the absence of quantum anomalies), but that they can be absorbed in a redefinition of the physical input parameters. In the worst case (as for quartic and quadratic divergences) one has to deal with a very strong dependence of the parameters on the ultraviolet cut-off procedure. The knowledge of physics in the far ultraviolet region has to be under control to have a meaningful understanding of the parameters of the theory even at very low energies. In situations where various different mass scales appear one then has to deal with the so-called hierarchy problems: why is one scale small compared to the other? In the standard model of particle physics this concerns the (mass)$^2$ of the Higgs boson (quadratically divergent) and the vacuum energy (quartically divergent). The cosmological constant and the Higgs mass are therefore naturally expected to be of the order of the ultraviolet cutoff, e.g. the Planck scale. Of course, it is not inconsistent to make an ad hoc choice of small values for these quantities, but this would require an extreme fine-tuning of the input parameters that has to be repeated order by order in perturbation theory. Thus the presence of quartic and quadratic divergences appears highly undesirable from a theoretical point of view.

The above theoretical arguments were the main motivation to propose the supersymmetric extension of the $SU(3) \times SU(2) \times U(1)$ standard model of strong and electroweak interactions. Supersymmetric field theories are free of quartic and quadratic divergences (provided some additional constraints are fulfilled in the presence of abelian $U(1)$ gauge groups), as a result of a cancellation between bosonic and fermionic degrees of freedom. In the calculation of the vacuum energy e.g. we find that the coefficient of the quartically divergent term is proportional to the number of bosons minus the number of fermions while the coefficient of the quadratically divergent piece is given by the supertrace of (mass)$^2$, $\text{STrM}^2$. Both of these vanish in the supersymmetric limit. In fact, supersymmetry does even more than this due to the appearance of so-called non-renormalisation theorems: corrections to the formerly divergent quantities vanish identically in perturbation theory. In Nature however, supersymmetry is broken. Still, there exists the notion of softly broken supersymmetry (e.g. spontaneous breakdown) where quadratic divergences for the Higgs mass are absent and the highest degree of divergence (for field dependent quantities) is logarithmic. Quadratic divergences are cutoff due to the presence of new physical states: supersymmetric partners of the standard model particles. $M_{\text{susy}}$, the supersymmetry breakdown scale (representative for the mass of the supersymmetric partners) provides us with a physical cutoff for the potentially quadratically divergent Higgs (mass)$^2$, leading to a finite correction of order $M_{\text{susy}}^2$. The quadratic divergence is recovered in the limit $M_{\text{susy}} \to \infty$. Thus if supersymmetry is supposed to solve the hierarchy problem, we expect supersymmetric partners in the TeV region (and not in the Planck mass region) to provide this physical cutoff. Softly broken supersymmetry with $M_{\text{susy}}$ of the order of the weak scale does therefore provide a solution to (at least the technical aspect of) the hierarchy problem of the Higgs mass. In the present paper we adopt the definition of soft supersymmetry breaking terms as defined in ref. [1]. It will be these specific soft terms we will have in mind whenever we will refer to soft breaking terms. With these terms one can show that the corresponding theory does not suffer from quadratic divergences for scalar masses. This is true to all orders in perturbation theory and is not just a reflection of the tuning of a one loop result.

So far our discussion only considered the presence of a finite number of fields. The situation becomes much more complicated in the framework of string theories and/or models of Kaluza Klein compactification where (from the D=4 dimensional field theory point of view) we are dealing with infinitely many fields, and as a consequence with infinitely many new ways to cutoff integrals and fine-
tune parameters. In a given theory it is therefore important to control the ultraviolet sensitivity of the parameters. We have to understand whether a given value of a physical parameter is the result of the specific choice of the cut-off or has a more general validity.

Some classes of string theories are believed to be finite in perturbation theory. In the known cases, however, supersymmetry again plays a crucial role in the discussion. A physical cutoff would therefore be either the string tension (i.e. higher excitations of the string) or the scale $M_{\text{susy}}$ (i.e. supersymmetric partners). In the present paper we shall not address the case of string theories, but the simpler situation of compactified higher dimensional field theories known under the name of Kaluza-Klein theories. Naively, such theories show an ultraviolet behaviour worse than the corresponding four-dimensional ($D = 4$) theories and some amount of supersymmetry is needed again to arrive at ultraviolet insensitivity.

The investigation of the present paper was initiated by claims in the literature [2, 3, 4] of an amazing ultraviolet softness of such Kaluza-Klein theories. The starting point is represented by supersymmetric theories in $D > 4$ dimensions (e.g. N=1 in D=5) with specific hard (not soft in the sense discussed above) breaking terms. From our previous discussion we would therefore expect the appearance of e.g. quadratic divergences for scalar mass terms. According to the claim of [2, 3, 4] these theories give a finite result. This claim is based on the inspection of a one-loop contribution to the vacuum energy. This in fact means that this theory has a better ultraviolet behaviour than the underlying unbroken supersymmetric theory.

We find this a rather miraculous result that deserves further clarification. After all we know that quadratic divergences can be regularised to give a finite result at one loop level and the finite result will rely heavily on the details of the regularisation procedure. The observed miraculous behaviour could thus be the consequence of a special choice of the cut-off procedure. The calculational basis of the claims is a particular mathematical procedure in evaluating the vacuum energy of the D=5 dimensional Kaluza-Klein theory with hard supersymmetry breaking terms. The expression of the vacuum energy is an infinite sum of infinite integrals that are highly divergent. The aforementioned procedure exchanges the sum and the integral to lead to a finite result (this regularisation procedure is called “KK-regularisation”). So far there has been no calculation that goes beyond the use of this procedure and that would clarify the detailed physical assumptions employed in such a regularisation mechanism. It was argued [2, 3, 4] that it must be the underlying (broken) supersymmetry that leads to this result. Part of the cancellation is in fact due to Supersymmetry. But since the outcome of this calculation is a better ultraviolet behaviour than that of unbroken supersymmetric theory, a more detailed calculation seems to be required.

In the present work we perform such a calculation and we are able to clarify some of the points raised:

- The finite result requires a cancellation in bosonic (and fermionic) sectors respectively (apart from other cancellations due to supersymmetry).

- The regularisation procedure assumes a cancellation between physics above the momentum cutoff ($\Lambda$) and physics below this cutoff.

- The ultraviolet finiteness of vacuum energy at one loop level appears as a result of a specific fine-tuning of parameters in the “KK-regularisation”.

As a framework for our calculation and to motivate these statements we consider one particular example [5] of the models using “KK regularisation” and analyse this regularisation scheme which
involves the summing up of the effects of an infinite tower of Kaluza Klein states associated with the extra (compact) dimension. The model we consider is representative for the class of models considering “KK regularisation” and we only use it for illustrative purposes. To begin with we will only sum up the effects on the scalar potential of a finite number of Kaluza Klein states and then consider the limit when this number is infinite. This approach has the advantage of showing rather explicitly the origin of the cancellation of the ultraviolet divergences of the scalar potential. We argue that there exists such a cancellation which takes place separately for the bosons and for the fermions, therefore is not related to the presence of supersymmetry (whether broken or not). Rather we show that this is due to a subtle cancellation between the quadratic terms of a finite number of states in the Kaluza Klein tower against the logarithmic contribution of such states of mass larger than the momentum cut-off of the one-loop integrals responsible for the radiative corrections to the scalar potential. In a sense this can also be viewed as a fine-tuning of the coefficients of the $\Lambda$ dependent terms controlling the ultraviolet behaviour, such that the sum of these terms is zero when the number of Kaluza Klein states increases to infinity. As usual (broken) supersymmetry does play a role in the cancellation of quartic and (even part of) the quadratic divergences, as a consequence of an equal number of bosonic and fermionic degrees of freedom. But as we just mentioned, the finite character of the scalar potential (and soft masses) is traced back to individual cancellations for bosons and for fermions respectively due to Kaluza Klein states of mass larger than $\Lambda$, the momentum cut-off. Thus the mechanism is not related to the way supersymmetry is broken. The finite character of the potential is the result of a mathematical manipulation and is not necessarily supported by an underlying physical mechanism or a symmetry principle.

In the next section we address the corrections to the scalar potential of the model we investigate, and refer the reader to [5] for a more detailed description. In Section 3 we discuss the ultraviolet behaviour of the scalar potential. Our conclusions are presented in the last section.

2 The scalar potential

The model which provides the framework of our investigation considers a one-loop expansion of the scalar potential, whose radiative corrections are accounted for by Kaluza Klein modes of the top quark, associated with the extra spatial dimension considered. These states fall in $N=2$ multiplets, consequence of the enhanced supersymmetry in the 5D “bulk” (broken on the 4D boundary to $N=0$ by an orbifold-like $S_1/(Z_2 \times Z'_2)$ compactification). The scalar potential of for the Higgs field $\phi$ is the focus of our investigation, has a rather generic structure with similar expressions found in [2, 3, 4], and is used to illustrate our analysis. It has the structure

$$V(\phi) = \frac{1}{2} Tr \sum_{k=-l}^{l} \int \frac{d^4 p}{(2\pi)^4} \ln \frac{p^2 + m^2_{Bk}(\phi)}{p^2 + m^2_{Fk}(\phi)}$$

with the Trace over the top hypermultiplet of fixed k level contributing a factor of $(4N_c)$ where $N_c$ is the number of colours.

In a 4D effective field theory approach, the tower of Kaluza-Klein states should be truncated to a mass scale which should be of the order of the momentum cut-off scale of the loop integral. This means that, at least intuitively, there should be a correlation between the number of states in the tower and the momentum cut-off. To keep our analysis general we first consider the case of an arbitrarily fixed number ($l$) of Kaluza-Klein modes, which will prove very useful for gaining an insight into the physical
meaning and consequences of the regularisation procedure of summing over an infinite tower of Kaluza Klein states, as explored in [3, 4, 5, 6].

The field dependent boson and fermion masses in (1) are given by

\[ m_{F_k}(\phi) = \frac{2k}{R} + m_t(\phi) = \frac{2}{R}(k + \omega), \quad \omega = \frac{m_t(\phi)R}{2} \]  

and

\[ m_{B_k}(\phi) = \frac{2k+1}{R} + m_t(\phi) = \frac{2}{R}(k + \omega'), \quad \omega' = \omega + \frac{1}{2} \]  

and where following [5] we consider any integer values of \( k \) between \(-\infty < k < \infty\). The use of these mass formulas is again done to illustrate our analysis; other similar possibilities [2, 3] may be explored, following this approach. The mass splitting between fermions and bosons, following the susy breaking via a Scherk-Schwarz mechanism, is therefore encoded in the difference \( \omega' - \omega \) which for this model is positive. The sign of the scalar potential and also of its second derivative (at \( \phi = 0 \)) will crucially depend on the sign of this difference (for this see eq.(1)), with implications for electroweak symmetry breaking mechanism induced by radiative corrections. Thus, the existence of this mechanism is not really a prediction of this class of models, but can rather be traced back to the type of Scherk-Schwarz boundary conditions one chooses for the fermions and for the bosons respectively, leading in some cases to the presence [3] or the absence [4] of the electroweak symmetry breaking.[3]

The zero mode fermion in (2) identified with the top quark has a mass given by [5]

\[ m_t(\phi) = \frac{2}{\pi R} \arctan \left( \frac{\pi R \phi}{2} \right) \]  

This expression will not be used in our analysis and we present it only for illustrative purposes.

From equations (1), (2) and (3) we find

\[ V(\phi) = \frac{\eta R}{4} \sum_{k=-l}^{l} \int_0^{\Lambda} d\rho \rho^3 \ln \frac{\rho^2 + \pi^2(k + \omega')^2}{\rho^2 + \pi^2(k + \omega)^2}, \quad \Lambda = \frac{\pi R \Lambda}{2} \]  

where the constant in front of \( V \) is given by

\[ \eta = \frac{4N_c}{2\pi^5} \]  

with \( \Lambda \) as the momentum cut-off of the loop-integral in (1).

Having truncated the tower of Kaluza-Klein states to a finite number, one can safely commute the sum with the integral in eq.(3) without affecting its ultraviolet behaviour. We therefore choose to keep two different cut-off’s, one for the Kaluza-Klein sum (represented by \( l \), see below) and one for the loop-integral (\( \Lambda \) or equivalently, \( \Lambda \)). Strictly speaking the number of Kaluza-Klein states should however be restricted to those states whose mass is below the momentum cut-off of the loop integral, condition which would lead to

\[ m_{B_k,F_k} \approx \frac{2k}{R} \leq \Lambda \quad \Rightarrow \quad \Lambda \approx \pi l \]  

where \( l \) stands for the Kaluza Klein state of the largest mass. Later on we will consider the limit \( l \to \infty \) with \( \Lambda (\Lambda) \) fixed to analyse the physical implications and mathematical subtleties of the “Kaluza-Klein regularisation”.

---

3This also depends on which zero modes we identify with the Standard Model particles.
As a result of summing up the effects of a finite number of Kaluza-Klein states under the momentum integral in eq. (1), we obtain a result with three contributions under the loop-integral. Accordingly, the potential can be written as a sum of three terms, with the \( \phi \) dependence hidden in \( \omega \) and \( \omega' \). Thus

\[
V(\phi) = V_0(\phi) + V_1(\phi) + V_2(\phi)
\]

where we have

\[
V_0(\phi) = \frac{\eta}{R^4} \int_0^X d\rho 2 \rho^3 \ln \left( \frac{\cosh(2\rho) - \cos(2\pi\omega')}{\cosh(2\rho) - \cos(2\pi\omega)} \right)
\]

and

\[
V_1(\phi) = \frac{\eta}{R^4} \int_0^X d\rho 2 \rho^3 \ln \frac{\rho^2 + \pi^2(l + \omega')^2}{\rho^2 + \pi^2(l + \omega)^2} + (l \rightarrow -l)
\]

and finally

\[
V_2(\phi) = \frac{\eta}{R^4} \int_0^X d\rho 2 \rho^3 \ln \left[ \frac{\Gamma(l \pm \omega' \pm i\rho/\pi)}{\Gamma(l \pm \omega \pm i\rho/\pi)} \right]
\]

where under the last integral the symbol \( [\Gamma(l \pm \omega' \pm i\rho/\pi)] \) stands for a product of \( \Gamma \)'s with all possible (four) combinations of plus and minus signs. It is useful for later reference to mention that the numerators (\( \omega' \) dependent) of the integrands of \( V_0 \), \( V_1 \), \( V_2 \) correspond to bosonic degrees of freedom, while the denominators correspond to the contribution (\( \omega \) dependent) of the fermions. Details of the exact calculation leading to this result will be given elsewhere.

In the following we analyse each of the three contributions to the scalar potential \( V \) and investigate their physical implications. \( V_0 \) is part of the scalar potential corresponding to the result of reference which is finite and ultraviolet insensitive, as we discuss below. The expression of \( V_0 \) simply corresponds to summing over an infinite number of Kaluza Klein states in eq. (1) and is therefore Kaluza-Klein-level independent. It was assumed that this is the Kaluza-Klein regularised part of the scalar potential.

Contributions \( V_1 \) and \( V_2 \) to the scalar potential are each vanishing in the limit of an infinite number of Kaluza-Klein states, \( V_1(l \rightarrow \infty) = 0 \), and \( V_2(l \rightarrow \infty) = 0 \) while keeping \( X \) fixed, consequence of the vanishing of their integrands respectively. It is however instructive for our purposes to explicitly compute \( V_1 \) and \( V_2 \) in the general case (of finite \( l \)) to understand the physical implications of taking the limit \( l \rightarrow \infty \) (\( X \) fixed). This procedure will also clarify whether the breaking of supersymmetry by a Scherk-Schwarz-like boundary condition quoted as the origin of obtaining an ultraviolet finite result for \( V \) in the limit \( l \rightarrow \infty \), plays actually any role in the cancellation of ultraviolet (quadratic and logarithmic) terms of the scalar potential to give a finite result.

### 3 Ultraviolet dependence of the scalar potentials

To make our discussion more quantitative, we start with the analysis of \( V_0 \). After performing the integral for \( V_0 \), one finds

\[
V_0(\phi) = \frac{3\eta}{R^4} \sum_{k=0}^{\infty} \frac{\cos(2\pi\omega(2k + 1))}{(2k + 1)^5} \left[ 1 - e^{-2\pi(2k+1)} \sum_{m=0}^{3} \frac{(2\pi(2k+1))^m}{m!} \right]
\]

where the factor within curly braces is equal to unity in the large \( \Lambda \) limit, to leave only the potential of \( V_0 \). Of the latter, only the first (small \( k \)) terms significantly contribute to \( V_0 \). In (12) we used
\( \omega' - \omega = 1/2 \), which has implications for the sign of the potential (eq.(11)) and electroweak symmetry breaking as a result of the relation \( \omega' > \omega \). Further analysis of this potential and its phenomenological implications have been discussed in [3].

The result of integrating \( V_1(\phi) \) of eq.(11) is given, irrespective of the dependence \( m_t(\phi) \), by the following expression

\[
V_1 = \left( V_1(\omega') + \Delta \right) - \left( V_1(\omega) + \Delta \right) = V_1(\omega') - V_1(\omega)
\]  

(13)

where we used the following notation

\[
V_1(\omega) = \frac{\eta}{2R^4} \left\{ \pi^2(l + \omega')^2 \Lambda^2 - \pi^4(l + \omega')^4 \ln \left[ 1 + \frac{\Lambda^2}{\pi^2(l + \omega')^2} \right] + \Lambda^4 \ln \left( \pi^2(l + \omega')^2 + \Lambda^2 \right) + (l \to -l) \right\}
\]  

(14)

with terms obtained by the substitution \( l \to -l \) to account for modes moving in the opposite direction in the compact dimension.

Eq.(13) shows explicitly how some ultraviolet divergent terms accounted for by \( \Delta \) cancel between the bosonic and fermionic contributions, to give \( V_1(\omega') \) as the difference between a (remaining) bosonic part which we denote by \( V_1(\omega') \) and a fermionic one, \( V_1(\omega) \). One sees for example in the expression of \( V_1 \) how \( \Lambda^4 \) terms (included in \( \Delta \)) are cancelled due to the equal number of bosonic and fermionic degrees of freedom, consequence of the initial presence of (now broken) supersymmetry. Additional terms \( \Lambda^2 \) and \( \Lambda^4 \ln(1 + \Lambda^2/\pi^2(l \pm \omega')^2) \) are however present in (14) for the bosons with similar counterparts for the fermions, and their overall coefficient (at fixed \( l \)) is non-zero. For example \( \Lambda^2 \) term has an overall coefficient proportional to \( (\omega'^2 - \omega^2) \). The absence of such terms in the final result for \( V \) in the limit \( l \to \infty \) has been assumed [4, 5] to be due to the so-called “soft nature” of breaking supersymmetry by a Scherk-Schwarz mechanism which would preserve a cancellation between bosonic and fermionic contributions. We argue that this seems not be the case, as terms like \( \Lambda^2 \) and \( \Lambda^4 \ln(1 + \Lambda^2/\pi^2(l \pm \omega')^2) \) do not cancel exactly between the bosons and the fermions, for any value, whether finite or infinite of the number \( l \) of Kaluza Klein states that we summed over.

To understand what really happens we consider the limit of large \( l \) and fixed momentum cut-off \( \Lambda, \pi l \gg \Lambda \equiv \pi \Lambda R/2 \). In this limit the logarithm in the second term of the bosonic term \( V_1(\omega') \) can be expanded in a (rapidly convergent) power series, with the first term in the expansion to cancel the quadratic divergence \( \Lambda^2 \) of the first term in \( V_1 \); more explicitly, the second term in (14) which we denote as \( A(l, \omega') \) has the form

\[
A(l, \omega') \approx \frac{1}{2R^4} \left\{ -\pi^2(l + \omega')^2 \Lambda^2 - \frac{1}{2} \Lambda^4 + \frac{1}{3} \Lambda^6 (l + \omega')^2 + \cdots \right\}
\]  

(15)

and its quadratic term\(^4\) cancels the first term in \( V_1(\omega') \). This cancellation is independent of the presence of (exact or softly broken) supersymmetry, as it takes place separately for the bosons and for the fermions. The physical interpretation of this mathematical observation is that states of mass larger (!) than the momentum cut-off scale \( \Lambda \) of the loop integral \( (2l/R \gg \Lambda) \) cancel the contributions to the quadratic divergence of Kaluza-Klein states of level less or equal to \( l \) and this takes place separately for bosons and fermions, i.e. independent of the presence/absence of supersymmetry or of the way it is broken.

\(^4\)obtained by the replacement \( \omega' \to \omega \)

\(^5\)including bosonic and fermionic contributions.

\(^6\)The \( \Lambda^4 \) term in the expansion (14) is cancelled in \( V_1 \) after including both bosonic and fermionic contributions.
Summing over an infinite tower of Kaluza Klein states corresponds to the limit \( l \to \infty \) and in this case one finds that the quadratic dependence \((\bar{X}^2)\) in the bosonic part \(V_1(\omega')\) of (14) is lost, and a similar mechanism separately applies for the fermionic part, due to the aforementioned reasons, only to leave a \(\bar{X}^4\) dependence cancelled between the bosonic and fermionic parts. Indeed

\[
\lim_{l \to \infty} \frac{1}{2} \frac{\eta}{R^4} \left\{ \pi^2 (l + \omega')^2 \bar{X}^2 - \pi^4 (l + \omega')^4 \ln \left[ 1 + \frac{\bar{X}^2}{\pi^2 (l + \omega')^2} \right] + (l \to -l) \right\} = \frac{1}{2} \frac{\eta}{R^4} \bar{X}^4
\]

where the r.h.s. \(\bar{X}^4\) is cancelled between bosons and fermions due to the initial presence of supersymmetry. The vanishing of \(\bar{X}^2\) dependence can also be viewed as a fine-tuning of the \((l \text{ and } \omega'\text{ dependent})\) coefficients of the two terms (momentum cut-off dependent) in eq. (16) such that their sum only contains \(\bar{X}^4\) but no quadratic contributions. In the same limit \(l \to \infty\), the third term in (14) \(\bar{X}^4 \ln(\pi^2 (l \pm \omega')^2 + \bar{X}^2)\) is indeed cancelled between bosons and fermions. This last cancellation is in a sense a remnant of the initial presence of supersymmetry which ensured the matching of bosonic and fermionic degrees of freedom, and again brings in a fine-tuning to zero of the (logarithmic) coefficient of the ultraviolet term \(\bar{X}^4 [\ln(\pi^2 (l \pm \omega')^2 + \bar{X}^2) - (\omega' \to \omega)]\) when \(l \to \infty\).

One can argue that it is possible for the presence of quadratic ultraviolet terms in \(V_1\) to be cancelled by similar terms which may be present in \(V_2\), to give - for any finite summation, up to level \(l\) - a finite, ultraviolet insensitive result. Here we show that this is not the case by using an (approximate) expression for \(V_2\) valid in the limit of large modulus of \(l + i\bar{X}\) which does not restrict the relative values of \(l, \bar{X}\), therefore various limits for these quantities may still be taken on the final result to find the ultraviolet behaviour of \(V_2\). After some algebra the integral of \(V_2\) can be written as

\[
V_2 \approx \frac{\eta}{R^4} \left\{ \pi^2 g(l, \omega')(l + \omega') \left[ \bar{X}^2 - \pi^2 (l + \omega')^2 \ln \left( 1 + \frac{\bar{X}^2}{\pi^2 (l + \omega')^2} \right) \right] + (\omega' \to -\omega') \right\} \\
+ \frac{\eta}{R^4} \frac{\omega \bar{X}^4}{2} \left\{ \ln \frac{\bar{X}^2 + \pi^2 (l + \omega')^2}{\bar{X}^2 + \pi^2 (l - \omega')^2} - \frac{4}{5} \frac{\bar{X}^5}{R^4} \left\{ \arctan \frac{\bar{X}}{\pi (l - \omega')} + \arctan \frac{\bar{X}}{\pi (l + \omega')} \right\} - (\omega' \to \omega) \right\} \\
+ \frac{\eta}{R^4} \frac{\bar{X}^3}{2} \left( l - \frac{1}{2} \right) \ln \left\{ \frac{\bar{X}^2 + \pi^2 (l \pm \omega')^2}{\bar{X}^2 + \pi^2 (l \pm \omega')^2} \right\}
\]

where \(g(l, \omega') = [10 - 15l + 6l^2 + 3(-5\omega' + 4l\omega' + 2\omega'^2)]/60\) and where the substitution \((\omega' \to \omega)\) only applies to terms in front of it, to give the (separate, \(\omega\) dependent) fermionic contribution. The last term in (17) contains both the fermionic and the bosonic contributions. Ultraviolet divergences of the type \(\bar{X}^2, \bar{X}^4, \bar{X}^6\) do cancel between the bosonic and fermionic contribution while computing \(V_2\), but these cancellations are *not* exact, with remaining (uncancelled) contributions (e.g. \(\bar{X}^2\)) vanishing separately for the bosons and fermions, respectively.

In the first curly braces of (17) quadratic terms \(\bar{X}^2\) due to states up to level \(l\) are cancelled for very large \(l\) by the first term in the (rapidly converging) power series of the logarithmic contribution. For \(l \to \infty\) the whole (bosonic) term in the first \{\} has no \(\bar{X}^2\) dependence, giving only a \(\bar{X}^4\) contribution

\footnote{The expression of \(V_2\) (thus of \(V\)) cannot be integrated analytically to obtain a transparent result for our purposes and this justifies the use of our approximation. In a different approach, one can first compute the integral of \(V\) and then sum over a finite tower of states. The summation is equally difficult to perform, with the exception of that for \(\bar{X}^2\) terms, which shows again that \(\bar{X}^2\) terms are present for any finite summation, for both bosons and fermions and do not cancel between these.}
cancelled by its fermionic counterpart due to the initial presence of supersymmetry. The cancellation of $\Lambda^2$ term takes place if the second term in the argument of the logarithm is smaller than unity, which is only possible for Kaluza Klein mass terms larger than the momentum cut-off of the loop-integral. This is true indeed in the limit $l \to \infty$ considered by “Kaluza Klein regularisation”. A similar mechanism also applies to the case of the fermionic contribution.

Other terms in (17) with $\Lambda^4, 5$ also vanish separately for the bosons and fermions respectively, in the limit $l \to \infty$, with fixed $\Lambda$. The vanishing in this limit of such ultraviolet terms containing powers of $\Lambda$ brings in an amount of fine-tuning (to zero) of their ($l$ dependent) coefficients. This explains the absence of these terms in $V_2$ (and thus in $V$) when $l \to \infty$. To conclude, just as in the case of $V_1$, (part of) the ultraviolet behaviour (e.g. $\Lambda^2$) of the bosonic (fermionic) part is separately cancelled by bosonic (fermionic) contributions respectively, due to Kaluza-Klein terms of mass larger than the momentum cut-off of the loop integral.

4 Conclusions and outlook

We have shown that the procedure of summing up an infinite tower of Kaluza Klein states contributing to the scalar potential of supersymmetric models, used in the literature as a regularisation method seems to have a rather unphysical meaning. Although some ultraviolet terms do cancel between fermionic and bosonic contributions due to the initial presence of supersymmetry, the cancellation of the (remaining) quadratic and logarithmic divergences in this procedure to lead to an ultraviolet finite potential takes place independently for bosons and fermions and is not a feature of softly broken supersymmetry, as initially thought. The cancellation is simply due to considering Kaluza Klein states of masses larger than the momentum cut-off of the one-loop integrals for the bosons and for the fermions respectively, leading us to a conclusion which questions the physical meaning of this regularisation procedure.

“Kaluza-Klein regularisation” employs an ad hoc choice for the cut-off of the quadratic divergence at one loop order, via a subtle mathematical procedure of summing over an infinite number of Kaluza Klein states before performing the one-loop momentum integral. Even so, in contrast to the situation in softly broken Susy, we will still have a strong ultraviolet sensitivity of the Higgs mass since the quadratic dependence has been cancelled at one loop only and will potentially reappear in higher orders in perturbation theory.

Other unanswered questions by this string-inspired “KK regularisation” come from string theory. Unless a full string model is built to justify the summation over an infinite tower of states, one would expect the presence of a (unique) cut-off in the theory below which all masses are situated, and this was not the case in the models addressed. In addition, any regularisation of string origin should also include the effects of possible winding modes with respect to the compact extra dimension, and it is not clear at all that these will render the scalar potential ultraviolet insensitive in a model embedded in a full string set-up. To exemplify, one can think of the heterotic string calculation of string thresholds to the gauge couplings. Such string threshold corrections may be viewed as the free energy of compactification, thus intuitively related to the scalar potential in our model. These corrections still exhibit (for two-torus compactification) a quadratic dependence on the compactification scale as well as on the string scale. For an effective field theory model the string scale is assimilated to the ultraviolet cut-off, thus an ultraviolet behaviour of the potential stronger than the (ultraviolet insensitive) finite one of the “KK regularisation” should then be expected.

\footnote{Note that the sum of quadratic terms in $V_1 + V_2$ does not vanish for any finite $l$.}
Acknowledgement We would like to thank Graham G. Ross and Stefan Groot Nibbelink for very useful discussions on this work. This work was supported by the University of Bonn under the European Commission RTN programmes HPRN-CT-2000-00131 and 00148.

References

[1] S. Ferrara, L. Girardello, F. Palumbo, Phys. Rev. D20 (1979) 403.
[2] I. Antoniadis, S. Dimopoulos, A. Pomarol, M. Quiros, Nucl. Phys. B544 (1999) 503.
[3] A. Delgado, A. Pomarol, M. Quiros, Phys. Rev. D60 (1999) 095008.
[4] N.Arkani-Hamed, L.J. Hall, Y. Nomura, D. Smith, N. Weiner, preprint hep-ph/0102090.
[5] R. Barbieri, L.J. Hall, Y. Nomura, preprint hep-ph/0011341.
[6] A. Delgado, M. Quirós, preprint hep-ph/0103058.
[7] L.Dixon, V.Kaplunovsky, J.Louis, Nuclear Physics B355 (1991) 649.
[8] S. Ferrara, C. Kounnas, D. Lust and F. Zwirner, Nucl. Phys. B365 (1991) 430.
[9] D. Ghilencea, H.-P. Nilles, work in preparation.