Optimization-Based Inverse Identification of the Parameters of a Concrete Cap Material Model

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Abstract. Issues concerning the advanced numerical analysis of concrete building structures in sophisticated computing systems currently require the involvement of nonlinear mechanics tools. The efforts to design safer, more durable and mainly more economically efficient concrete structures are supported via the use of advanced nonlinear concrete material models and the geometrically nonlinear approach. The application of nonlinear mechanics tools undoubtedly presents another step towards the approximation of the real behaviour of concrete building structures within the framework of computer numerical simulations. However, the success rate of this application depends on having a perfect understanding of the behaviour of the concrete material models used and having a perfect understanding of the used material model parameters meaning. The effective application of nonlinear concrete material models within computer simulations often becomes very problematic because these material models very often contain parameters (material constants) whose values are difficult to obtain. However, getting of the correct values of material parameters is very important to ensure proper function of a concrete material model used. Today, one possibility, which permits successful solution of the mentioned problem, is the use of optimization algorithms for the purpose of the optimization-based inverse material parameter identification. Parameter identification goes hand in hand with experimental investigation while it trying to find parameter values of the used material model so that the resulting data obtained from the computer simulation will best approximate the experimental data. This paper is focused on the optimization-based inverse identification of the parameters of a concrete cap material model which is known under the name the Continuous Surface Cap Model. Within this paper, material parameters of the model are identified on the basis of interaction between nonlinear computer simulations, gradient based and nature inspired optimization algorithms and experimental data, the latter of which take the form of a load-extension curve obtained from the evaluation of uniaxial tensile test results. The aim of this research was to obtain material model parameters corresponding to the quasi-static tensile loading which may be further used for the research involving dynamic and high-speed tensile loading. Based on the obtained results it can be concluded that the set goal has been reached.

1. Introduction

Continuous and extensive use of concrete for the purpose of building the new structures currently leads to the efforts of refine the design of concrete structures through the computer numerical simulations based on the finite element method [1-3]. These efforts related to the design of safer, more durable and more economically efficient concrete structures, however, require the involvement of
tools of the advanced numerical analysis. The involvement of tools of the advanced numerical analysis especially means that it is necessary to consider the nonlinear behaviour of concrete within the context of computer simulations intended for the analysis and design of concrete structures. Current computational systems based on the finite element method, which include for example the programs ANSYS [4], LS-Dyna [5] or Atena [6], offer a relatively large amount of material models which are suitable for describing the nonlinear behaviour of concrete. These nonlinear material models of concrete may find the use in the static numerical simulations but also in the dynamic numerical simulations and a number of publications is devoted to their description and practical application [7-10]. However, the biggest problem related to the use of the nonlinear material models of concrete is often the inability to properly define the values of the material parameters (constants) because they can often be obtained only on the basis of the special tests of concrete. Some parameters have even only purely mathematical meaning and defining of their values is not, therefore, so easy. However, the correct definition of parameter values of the used material model is extremely important for the correct description of the behaviour of concrete within the computer simulation. Solution of the mentioned problem is currently possible with the use of so-called the inverse analysis [11, 12].

Inverse analysis, otherwise known as inverse identification, allows to find such parameter values of the used nonlinear concrete material model wherein the resulting response of the structure obtained from the computer simulation is very similar to the experimentally measured response of the concrete structure. The principle of the inverse analysis is based on a combination of numerical and experimental analysis with optimization algorithms, methods or procedures. Currently, the most used methods for the inverse parameter identification of nonlinear material models of concrete are methods based on the exercise of artificial neural networks [13]. A very powerful tool in the field of the inverse analysis is also the optiSLang program [14] which includes a variety of optimization algorithms suitable for the optimization-based inverse identification of the material parameters [15, 16].

This paper is focused on the optimization-based inverse identification of the parameters of a concrete cap material model which is known under the name the Continuous Surface Cap Model and which is implemented in an explicit finite element solver LS-Dyna [17]. Within this paper, parameters of the material model are identified on the basis of interaction between nonlinear computer simulations, gradient based and nature inspired optimization algorithms implemented in the optiSLang program and experimental data, the latter of which take the form of a load-extension curve obtained from the evaluation of unconfined uniaxial tensile test results.

2. Unconfined uniaxial tensile tests
The process of the optimization-based inverse parameter identification performed in this paper required the experimental data. For this purpose, the experimental data obtained from the evaluation of unconfined uniaxial tensile test results that were made within [18] were used. Used experimental data took the form of a load-extension curve which described the nonlinear behaviour of concrete test specimens during the unconfined uniaxial tensile loading. The load-extension curve is shown in figure 1.

Concrete test specimens used within the original unconfined uniaxial tensile tests performed in the context of [18] had dimensions 305 x 60 x 19 mm³ (length x width x depth for the critical cross-sectional area). The test specimens were manufactured and then they were subjected to the concrete hardening process which lasted 28 days. The uniaxial compression strength of the 28 days old and hardened concrete was 44 MPa. The maximum aggregate size used was 10 mm. The experimental data were measured in 85 mm length of each test specimen. During the unconfined uniaxial tensile loading, concrete test specimens were stretched at a quasi-static constant loading velocity.
It is clear from figure 1 that during the unconfined uniaxial tensile loading, the concrete test specimens first exhibited linearly elastic behaviour before reaching the maximum tensile force (i.e. the maximum tensile load capacity of the concrete test specimens). After reaching the maximum tensile force, the concrete test specimens began to show signs of tensile strain softening. The tensile strain softening of the concrete began to assert itself as a result of the damage to the concrete test specimens as the ultimate strength of the concrete in uniaxial tension was reached.

![Experimentally-measured load-extension curve.](image)

Figure 1. Experimentally-measured load-extension curve.

3. Computer simulations

The process of the optimization-based inverse parameter identification performed in this paper further demanded performing of nonlinear computer simulations. For this purpose, the simplified computational model of the unconfined uniaxial tensile test was created in LS-Dyna software which is based on an explicit finite element method and in which the nonlinear computer simulations were performed.

3.1. The computational model

Against the real unconfined uniaxial tensile test, the computational model of this test created within this paper was very simplified. Only the critical part of the test specimen was modelled. This means that the finite element model of the test specimen took the form only of the measured part of the test specimen with the length of 85 mm. Explicit 3-D structural finite elements were used for creation of the finite element model. In terms of boundary conditions, supports were not applied within the context of the finite element model. However, linearly increased vertical displacements over time were prescribed for nodes of both bases of the finite element model, see figure 2. These displacements simulated the axial stretching of the model at a constant loading velocity. The nonlinear material behaviour of the model was modelled through the Continuous Surface Cap material model [19, 20].

The mentioned simplifications introduced into the computational model of the unconfined uniaxial tensile test were acceptable because length and cross-sectional area of the finite element model corresponded to lengths and critical cross-sectional areas of the concrete test specimens for which the experimental data were measured. The simplifications were also acceptable from the vantage point of damage. In real tests the damage occurred always just on the measured length of the test specimen in the place of the critical cross-sectional area. It follows that in terms of the damage, it was necessary to construct a finite element model at least as the measured part of the test specimen with the critical cross-sectional area. This necessity was met within the context of this paper.

The computational model used within this paper is shown in figure 2.
3.2. Nonlinear material model of concrete
In this paper, nonlinear material behaviour was included into the computational model through the material model which is known under the name the Continuous Surface Cap Model [19, 20]. This material model is a part of the material model library which is implemented in LS-Dyna software.

Theoretical background of the Continuous Surface Cap Model is based on a yield surface which is defined as a function of three stress invariants according to the equation [21, 22]:

\[
Y(I_1, J_2, J_3) = J_2 - 9(I_1) J_3^2 F_f^2(I_1) F_c(I_1, \kappa)
\]

where \( I_1 \) is the first invariant of the stress tensor, \( J_2 \) and \( J_3 \) are invariants of the deviatory stress tensor (second and third), \( 9(I_1) \) is the Rubin strength reduction factor and \( \kappa \) is the cap hardening parameter. The yield surface is composed of two parts. These being the shear failure surface \( F_f(I_1) \) and the hardening compaction surface \( F_c(I_1, \kappa) \). The expression of the shear failure surface is given by equation:

\[
F_f(I_1) = \alpha - \lambda \exp^{-\beta I_1} + \theta I_1
\]

where \( \alpha, \beta, \lambda, \) and \( \theta \) are the material constants which are usually determined on the basis of the triaxial compression tests. The hardening compaction surface is defined by equations:

\[
F_c(I_1, \kappa) = 1 - \frac{(I_1 - L(\kappa))^2}{(X(\kappa) - L(\kappa))^2} \text{ for } I_1 > L(\kappa)
\]

\[
F_c(I_1, \kappa) = 1 \text{ for } I_1 \leq L(\kappa)
\]
in a context with equations:

\[ L(\kappa) = \kappa \quad \text{for} \quad \kappa > \kappa_0 \quad (5) \]
\[ L(\kappa) = \kappa_0 \quad \text{for} \quad \kappa \leq \kappa_0 \quad (6) \]
\[ X(\kappa) = L(\kappa) + RF_j(I_j) \quad (7) \]

where \( R \) is the cap aspect ratio. Within the yield surface, the shear failure surface and hardening compaction surface are combined using a multiplicative formulation which allows their combination to be continuous and smooth at their intersection.

The Continuous Surface Cap Model allows within its formulation to take into account the effect of strain rate on the resulting stress state. However, this model capability can be neglected during the calculations. The response of the model is then always quasi-static and independent of computational time. Due to the mentioned facts, it is clear that the material model can be used in dynamic, but also in quasi-static or static, computer simulations. The fact, that the material model allows to calculate the quasi-static response, was used within this paper.

### Table 1. The identified material parameters of the Continuous Surface Cap Model.

| Material parameter | Parameter description                                      | Unit |
|--------------------|----------------------------------------------------------|------|
| RO                 | Mass density, \( \rho \)                                | Mg/mm\(^3\) |
| E                  | Young’s modulus, \( E \)                                | MPa  |
| PR                 | Poisson’s ratio, \( \nu \)                              | -    |
| ALPHA              | Triaxial compression surface constant term, \( \alpha \) | MPa  |
| THETA              | Triaxial compression surface linear term, \( \theta \)  | -    |
| LAMDA              | Triaxial compression surface nonlinear term, \( \lambda \) | MPa  |
| BETA               | Triaxial compression surface exponent, \( \beta \)      | MPa\(^{-1}\) |
| ALPHA1             | Torsion surface constant term, \( \alpha_1 \)          | -    |
| THETA1             | Torsion surface linear term, \( \theta_1 \)            | MPa\(^{-1}\) |
| LAMDA1             | Torsion surface nonlinear term, \( \lambda_1 \)        | -    |
| BETA1              | Torsion surface exponent, \( \beta_1 \)                | MPa\(^{-1}\) |
| ALPHA2             | Triaxial extension surface constant term, \( \alpha_2 \) | -    |
| THETA2             | Triaxial extension surface linear term, \( \theta_2 \) | MPa\(^{-1}\) |
| LAMDA2             | Triaxial extension surface nonlinear term, \( \lambda_2 \) | - |
| BETA2              | Triaxial extension surface exponent, \( \beta_2 \)     | MPa\(^{-1}\) |
| R                  | Cap aspect ratio, \( R \)                              | -    |
| X0                 | Cap initial location, \( X_0 \)                        | MPa  |
| W                  | Maximum plastic volume compaction, \( W \)             | -    |
| D1                 | Linear shape parameter, \( D_1 \)                      | MPa  |
| D2                 | Quadratic shape parameter, \( D_2 \)                   | MPa\(^2\) |
| B                  | Ductile shape softening parameter, \( B \)             | -    |
| GFC                | Fracture energy in uniaxial stress, \( G_{fc} \)        | N/mm |
| D                  | Brittle shape softening parameter, \( D \)             | -    |
| GFT                | Fracture energy in uniaxial tension, \( G_{ft} \)       | N/mm |
| GFS                | Fracture energy in pure shear stress, \( G_{fs} \)      | N/mm |

The Continuous Surface Cap material model is implemented in LS-Dyna software in two modifications, specifically as the general version *MAT_CSCM and the modified version.
*MAT_CSCM_CONCRETE [17]. The general version *MAT_CSCM of the material model was used in calculations performed within this paper. In order to obtain the most realistic response of the model, a total of 25 parameters (constants) of the material model version *MAT_CSCM were identified. However, the material parameters $G$ (shear modulus) and $K$ (bulk modulus), which originally belonged between identified parameters, were replaced by parameters $E$ (Young’s modulus) and $PR$ (Poisson’s ratio, $\nu$) due to their dependency on these parameters according to the equations:

$$G = \frac{E}{2(1+\nu)}$$

(8)

$$K = \frac{E}{3(1-2\nu)}$$

(9)

Descriptions and used units of mentioned 25 identified material parameters are given in table 1 [17].

4. Optimization-based inverse parameter identification

In this paper, the optimization-based inverse parameter identification was performed using the optiSLang program and consisted of three steps:

1. Sensitivity analysis
2. Global optimization
3. Local optimization

4.1. Sensitivity analysis

The sensitivity analysis [23, 24] formed the first step of the whole material parameter identification process. Within the context of this first step the sensitivity of the input variable data to the defined reference response was analyzed. The input variable data were represented by individual material parameters that were to be identified and that formed so-called design vector. Reference response was represented by individual points lying on the used experimentally-measured load-extension curve. The major goal of the sensitivity analysis was to reduce the number of identified material parameters in the design vector to the necessary minimum. Another goal was to modify the range of variability for the individual material parameters contained in the design vector.

Sensitivity analysis was carried out via the statistical method known as the Latin Hypercube Sampling (LHS) method [14]. Based on this method, a total of 300 random realizations of the design vector were generated. The generated amount of random realizations sufficiently covered the design space.

The results produced by the sensitivity analysis indicated that only 11 identified material parameters out of the total of 25 exerted major influence on the resultant form of the numerically-simulated load-extension curve. Therefore, for the subsequent global and local optimization the original design vector, which contained all 25 identified material parameters given in table 1, was reduced to the design vector which contained only mentioned 11 parameters exerted major influence on the resultant form of the load-extension curve. After the reduction the design vector acquired the following form:

$$X_{\text{reduced}} = \{E, ALPHA, THETA, LAMDA, BETA, THETA2, BETA2, D2, D, GFT, GFS\}^T$$

(10)
In the reduced design vector $X_{\text{reduced}}$ you can see the mentioned 11 material parameters.

The values of the pre-optimized material parameters and objective function ($EUCLID\_NORM$) obtained from the best random realization generated by LHS method are given in table 2.

| Material parameter | Unit       | Sensitivity analysis (LHS method) | Global and local optimization (NLPQL and PSO methods) | Global optimization (NLPQL method) | Local optimization (PSO method) |
|--------------------|------------|----------------------------------|-----------------------------------------------------|----------------------------------|----------------------------------|
|                    |            | Pre-optimized values              | Deterministic values Optimized values Optimized values |
| $RO$              | Mg/mm$^3$  | $2.329 \times 10^{-9}$          | $2.400 \times 10^{-9}$ - -                             |
| $E$               | MPa        | $31600$                          | -                                                    | $31600$                          | $30862$                          |
| $PR$              |            | $0.1855$                         | $0.1500$                                           | -                                | -                                |
| $ALPHA$           | MPa        | $15.509$                         | -                                                   | $15.509$                         | $15.432$                         |
| $THETA$           |            | $0.3432$                         | -                                                   | $0.3216$                         | $0.3186$                         |
| $LAMDA$           | MPa        | $10.388$                         | -                                                   | $10.388$                         | $10.257$                         |
| $BETA$            | MPa$^1$    | $2.185 \times 10^{-2}$          | -                                                   | $2.189 \times 10^{-2}$          | $2.197 \times 10^{-2}$          |
| $ALPHA_1$         |            | $0.7441$                         | $0.6500$                                           | -                                | -                                |
| $THETA_1$         | MPa$^1$    | $1.148 \times 10^{-3}$          | $0.700 \times 10^{-3}$ - -                           |
| $LAMDA_1$         |            | $0.1908$                         | $0.1600$                                           | -                                | -                                |
| $BETA_1$          | MPa$^1$    | $6.224 \times 10^{-2}$          | $4.500 \times 10^{-2}$ - -                           |
| $ALPHA_2$         |            | $0.6184$                         | $0.5800$                                           | -                                | -                                |
| $THETA_2$         | MPa$^1$    | $7.995 \times 10^{-4}$          | -                                                   | $7.995 \times 10^{-4}$          | $8.514 \times 10^{-4}$          |
| $LAMDA_2$         |            | $0.1645$                         | $0.1200$                                           | -                                | -                                |
| $BETA_2$          | MPa$^1$    | $7.112 \times 10^{-2}$          | -                                                   | $7.112 \times 10^{-2}$          | $7.264 \times 10^{-2}$          |
| $R$               |            | $5.2871$                         | $4.7000$                                           | -                                | -                                |
| $X_0$             | MPa        | $104.584$                        | $95.000$                                           | -                                | -                                |
| $W$               |            | $6.614 \times 10^{-2}$          | $4.000 \times 10^{-2}$ - -                           |
| $D_1$             | MPa        | $2.849 \times 10^{-4}$          | $2.000 \times 10^{-4}$ - -                           |
| $D_2$             | MPa$^2$    | $3.445 \times 10^{-7}$          | -                                                   | $3.445 \times 10^{-7}$          | $3.424 \times 10^{-7}$          |
| $B$               |            | $81.988$                         | $100.000$                                          | -                                | -                                |
| $GFC$             | N/mm       | $4.6363$                         | $2.6000$                                           | -                                | -                                |
| $D$               |            | $0.2678$                         | -                                                   | $0.1638$                         | $0.1000$                         |
| $GFT$             | N/mm       | $4.974 \times 10^{-2}$          | -                                                   | $4.798 \times 10^{-2}$          | $4.827 \times 10^{-2}$          |
| $GFS$             | N/mm       | $4.116 \times 10^{-2}$          | -                                                   | $6.343 \times 10^{-2}$          | $6.284 \times 10^{-2}$          |
| $EUCLID\_NORM$    | kN         | $0.34427$                        | -                                                   | $0.280801$                       | $0.252410$                       |

**4.2. Global optimization**

The global optimization formed the second step of the whole material parameter identification process. Within this second step the optimized material parameter values were sought so that the result of computer simulation approximated the experimental data so well as possible. Defined objective function was used for the evaluation of global optimization results. Therefore, within the global optimization the optimized material parameter values were sought so that the value of defined objective function was minimized. Based on previous information, it is clear that the global optimization was based on minimizing the objective function [25]. The objective function defined for the purposes of this paper took the form of Euclidean norm which was, of course, minimized:
\[ EUCLID \_ NORM = \left( \sum_{i=1}^{n} (y_i - y_i^*)^2 \right)^{1/2} \rightarrow \min \] (11)

where, for \( y_i \), we substituted the force values obtained from the appropriate numerically-simulated load-extension curve at the certain deformations, and \( y_i^* \) was substituted with the force values obtained from the experimental load-extension curve at the same deformations.

As pointed out above, the global optimization involved only those material parameters that were part of the reduced design vector. The remaining parameters were defined by the constant values from their original range of variability which was obtained for each parameter on the basis of test calculations. The global optimization of the material parameter values was performed using the gradient based optimization method known as the Non-Linear Programming by Quadratic Lagrangian (NLPQL) [14]. NLPQL is a sequential quadratic programming method which solves problems with smooth continuously differentiable objective function and constraints. The algorithm of this method uses a quadratic approximation of the Lagrangian function and a linearization of the constraints. For the calculations performed via the NLPQL method, the best random realization acquired from the LHS method was used as the starting point.

The optimized values of the material parameters provided by the best generation of the NLPQL method are, together with the relevant minimum value of the objective function, given in table 2.

4.3. Local optimization

The local optimization formed the third step of the whole material parameter identification process. Within the context of this third step, aim, purpose and objective function are the same as in the case of the global optimization. The objective function was, of course, minimized again. The local optimization was performed in an effort to search the vicinity of the global minimum with a goal to try to refine the mentioned global minimum.

As in the global optimization case, the local optimization involved only material parameters that were part of the reduced design vector. The remaining parameters were defined by the same constant values as in the case of the global optimization. The local optimization of the material parameters was carried out using the nature inspired optimization method known as the Particle Swarm Optimization (PSO) method [14]. It is the method that is inspired by the behaviour of bird flocks during the searching of food. For the calculations performed via the PSO method, the best generation of the NLPQL method was used as the start point.

The optimized values of the material parameters provided by the best generation of the PSO method are, together with the relevant minimum value of the objective function, given in table 2.

It is clear from table 2 that the PSO method provided the most optimized material parameters because the value of the objective function was the smallest for this method. Figure 3 below compares the load-extension curve obtained via the computer simulation, in which we applied the most optimized parameter values of the Continuous Surface Cap Model from the PSO method, with the experimentally-measured load-extension curve. It is then obvious from the representation that the parameters of the Continuous Surface Cap Model were identified very accurately via the PSO method because the result of the computer simulation ensures a very good approximation of the experimental data.
5. Conclusions
In this paper, the material parameters of the Continuous Surface Cap Model were identified via optimization methods implemented in the optiSLang program. The optimization-based inverse parameter identification was carried out on the basis of an experimentally-measured load-extension curve obtained from the result evaluation of the unconfined uniaxial tensile tests which were performed on the specific concrete test specimens. The results obtained from the parameter identification process showed that the Continuous Surface Cap Model is able to describe the behaviour of a real concrete in unconfined uniaxial tension, and they also showed that we may obtain a very good approximation of the experimental data by computer simulation in any case in which we define the material parameters by the correct values. This claim is proved by the result of the computer simulation performed using the resulting identified parameter values of the material model from the PSO method. The mentioned result approximated the applied experimental data with a high degree of accuracy. Advantageously, the product of the parameter identification process performed in this paper can be exploited for further research concerning the nonlinear numerical response of concrete structures.

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