Surviving opinions in Sznajd models on complex networks

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1 Abstract

The Sznajd model has been largely applied to simulate many sociophysical phenomena. In this paper we applied the Sznajd model with more than two opinions on three different network topologies and observed the evolution of surviving opinions after many interactions among the nodes. As result, we obtained a scaling law which depends of the network size and the number of possible opinions. We also observed that this scaling law is not the same for all network topologies, being quite similar between scale-free networks and Sznajd networks but different for random networks.

2 Introduction

Von Neumann and Ulam introduced the concept of cellular automata in the early 1950’s. Ever since, this concept has attracted continuing interest and has been subject to deep mathematical and physical analysis. A good deal of the popularity of cellular automata arises from their simplicity and potential to model many complex systems. The applications of this theory range from modelling biological pattern formation [1] to sociophysical phenomena [2].

In sociophysics, attention has been focused on modeling social phenomena, such as elections and propagation of information [2]. With this respect, particularly successful models have been developed by Sznajd-Weron [3], Deffuant et. al [4] and Krause and Hegselmann [5], which differ as for their definitions but tend to produce similar results.

Among those three models, the one developed by Sznajd is the most appropriate for simulation in networks and lattices, because it considers just the interactions between the nearest neighbors. The Sznajd model has been developed based on Ising model and been successfully applied to model sociological and economics systems [6]. In this paper we simulate the Sznajd model on networks with different topologies. The results turned out to be dependent on the network topology only for small values of the ratio between the number of possible opinions and the network size.
3 Sznajd model on complex networks

Complex networks are formed by a set of nodes \((i = 1, 2, \ldots, N)\), which are linked one another through edges. In biological networks, for example, proteins or genes can be linked according to their interactions [7]. In case of social networks, one can represent connections as defined by human relations such as friendship [8], relations between jazz musicians [9], collaborations in scientific researchers networks[10] and intermarriage between families [11].

To explain real network topologies, some models of complex networks have been developed, including random graphs [12, 13], small-world networks [14], scale-free networks [15] and, more recently, Sznajd complex networks [16].

As complex networks offer a more structured and realistic topology than regular lattices, the simulation of sociophysics models in these networks can produce more accurate results [17]. Stauffer et. al. simulated the Deffuant et. al. model in scale-free networks [18], obtaining the result that different surviving opinions in scale-free networks depend on the network size and the number of possible opinions. Moreover, Bernardes et. al [19], developed a model using the Sznajd model on a scale-free network which reproduced Brazilian election results. It was shown that the distribution of number of votes per candidate follows a hyperbolic law, in agreement with real elections results. Thus, simulation of sociophysical models on complex networks structures can reproduce real phenomena and suggest insights on the rules that guide de distribution of opinion dynamics.

3.1 Models of networks

The random graph is the simplest complex network model. The number of nodes on random graphs is constant and new edges randomly connect them with probability \(p\). Thereby, the distribution of connections in the network will follow the Poisson distribution. Thus, the node degree, given by the number of connections of each node, of the majority of network nodes will be next to the average.

In the case of scale-free networks, recently added nodes have greater probability to connect with those which have more connections. This process, called preferential attachment, generates a heterogeneous network, where most nodes will have a few number of connections, while a few nodes will have high number of connections. The connectivity distribution, \(P(k)\), for this network follows a power law distribution, \(P(k) \sim k^{-\gamma}\), where \(k\) is the number of links.

The Sznajd network is constructed by considering the geographical distribution of nodes and the evolution of Sznajd dynamics with feedback (contras) [16]. In this way, the network starts with a constant number of nodes distributed in a box of side \(L\) at random. Then, nodes distant less than \(d_{\text{max}}\) are connected in order to obtain the network \(U\) as shown in Figure 1(a). Then, the edges of this network are activated with uniform probability \(p\), thus obtaining a network \(K\), as shown in Figure 1(b). At each step, an edge \((i, j)\) is sampled at random from the initial network \(U\). If the edge is present in the network \(K\), all neighbors of \(i\) and \(j\) are identified in \(U\) and connected in \(K\). Otherwise, all neighbors of \(i\) and \(j\) are disconnected in \(K\). Another edge of \(U\) is sampled with probability \(q\) and the respective edge in \(K\) receives the contrary value to the current dominant opinion in the network. After stabilization, the geographical Sznajd network
is obtained and it is observed the presence of network communities, formed by sets of densely connected nodes.

Examples of random network, scale-free networks and Sznajd networks are presented in Figure 2. In our simulation, we used these three topologies to analyze the distribution of surviving opinions while varying the size of the network and the number of possible opinions.

3.2 Simulations

The most widespread version of the Sznajd model uses a square lattice with two opinions, \( Q = \pm 1 \) \[20\], where each individual \( i \) (\( i = 1, 2, ..., N \)) has equal number of neighbors. The simulation of this model starts by distributing opinions at random on the lattice. Then, at each step, two neighbor pairs are selected and if they have the same opinion all their neighbors are made to agree with them. If the initial state of the system has more than half opinions as 1, the final consensus is reached with all individuals reaching this opinion. Thus, a phase transition is observed \[20\]. Nevertheless, such a model is very simple and cannot reproduce some real results. To overcome the limitations, improvements were added to the Sznajd model in order to consider more than two opinions \((q = 1, 2, \ldots, Q)\) as well as more sophisticated real topologies, including complex networks \[19, 21\].

The simulation of the Sznajd model on complex networks is similar to performed on lattices: a pair of neighboring nodes are chosen at random and checked if they have the same opinion. If they do, all their nearest neighbors assume that same opinion.

4 Results

For the three models of networks (i.e. random graph, scale-free network and Sznajd network), \( Q \) different opinions were randomly distributed initially among the nodes. The interaction between nodes proceeds by choosing uniformly a node and one of its neighbors at random. Nobody can convince anyone \[4, 5\] if the two opinions differ by more than \( |\epsilon| \); in our simulation we adopted \( \epsilon = 1 \). So, if two neighbor nodes have opinion \( q = 2 \), they can convince just their neighbor with opinions \( q = 1 \) or \( q = 3 \). This process was executed for several numbers of opinions, varying from 2 to \( Q \), and the number of surviving opinions was recorded at each step for different network sizes.

In Figure 3 it is shown the scaling behavior of the number of surviving opinions as a function of the \( Q \) possible opinions. In fact, if the number of people, \( N \), is much larger than the number of possible opinions, \( Q \), the number of surviving opinions, \( S \), will tend to agree with \( Q \), and no opinion will disappear during the process. However, when the number of opinions \( Q \) is much larger than the number of people \( N \), the number of surviving opinions \( S \) will become \( N \), i.e. each person keeps its own opinion. This relation can be mathematically expressed as

\[
\frac{S}{Q} = f\left(\frac{Q}{N}\right),
\]

where \( f \) is constant for \( Q \ll N \) and \( f = N/Q \) for \( Q \gg N \), valid for large \( Q, S \) and \( N \) \[22\].
5 Discussion

The obtained results are similar to those considering the Deffuant model on a scale-free network [18]. Therefore, the distribution of surviving opinions for Sznajd simulation yields two limits: (i) when there are many people and few opinions, all opinions have some followers and (ii) when there are few people and many opinions, each person will keep her/his own opinion. The interval between these two extremes follows a scaling law.

As shown in Figure 3, the scaling number of surviving opinions depends on the network topology. For random graph the scaling of surviving opinion is not well defined for $N \gg Q$. However, for scale-free network and Sznajd networks, the number of surviving opinions is well determined for larger networks. As social networks are not guided by random distribution of connections [23], the Sznajd model is more likely to reproduce dynamical behavior of opinions in real social networks. Note that the scale-free and Sznajd network models implied similar opinion dynamics, while the random model produced results which originates from Equation 1 for small values of $Q/N$.

6 Conclusion

In this paper we simulated the Sznajd model in three different network topologies, namely random, scale-free and Sznajd network models. Starting from a network with $N$ agents and varying the number of possible opinions from $2 \ldots Q$, we simulate the Sznajd interaction between the agents and calculated the number of surviving opinions after a large number of interactions. As result, we obtained that the number of surviving opinions undergoes two states: (i) when the number of agents is much larger than the number of opinions, the number of surviving opinions tends to be the same as for possible opinions, where all opinions have some followers; and (ii) when the number of possible opinions is much larger than the number of agents, the number of surviving opinions will tend to that number of agents, and each person will keep her/his own opinion.

The behavior described by Equation 1 is best fitted by scale-free networks and Sznajd networks, where the distribution of connections between nodes is not uniform. For random graphs, this behavior is valid just when $Q > N$. As real social networks are not described by random graphs, the Sznajd model is appropriate to model dynamical behavior of opinions on this kind of networks.

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Figure 1: (a) The construction of Sznajd network starts with a geographical distribution of nodes at random and linking them that are distant less than a defined euclidian distance $d_{max}$. (b) Then, each of the network links is activated with probability $p$. (c) Finally, the feedback Sznajd and contrary feedback are applied, thus resulting the Sznajd network.
Figure 2: The simulation of SznaJD model is carried out for three different topologies: (a) random graphs, (b) scale-free networks and (c) SznaJD networks.
Figure 3: Scaling of the number $S$ of surviving opinions for $N = 10, 100, 1000$ as a function of the number of possible opinions. We see at the right part of the graph, when $N \gg Q$, all person keep his/her own opinion. On the other hand, at the left hand side of the graph, $Q \gg N$, each opinion is shared by many people.