Research on Creep Damage and Life Forecast of Rod Fastening Rotor Based on Damage Mechanics

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Abstract. Rod fastening rotor (RFR) is the core part of gas turbine, the degradation of RFR has great effect on gas turbine’s performance. One of the main reasons which cause the RFR performance degradation is creep damage, while few studies have been carried out in terms of it so far. In order to analyze the influence of preload on virtual material parameters, a dynamic model of RFR considering interface contact effect was built. Then equivalent stiffness of RFR and elements was analyzed as well. Furthermore, creep damage of elements under higher stress were analyzed with damage mechanics to get their influence on the total damage. Likewise, RFR were analyzed with damage mechanics to get the connection between the total damage and rupture life. The results showed that connection between the total damage and rupture life was a complicated, non-linear process. Moreover, the rods of turbine and combustion chamber were the biggest influencing factors. The results of this dissertation can be a support for structural design and life prediction of RFR.

1. Introduction
RFR is composed of wheel disks which compressed by rods, so the structure of it is more complicated than integral rotor. RFR is the core part of gas turbine, marine engine, etc. Furthermore, it always works in high temperature and high rotating speed environment for a long time, so performance degradation will generate easily. According to the statistics of Siemens, 58% malfunction of gas turbine is from RFR [1]. Therefore, studying the connection between damage and rupture life has significant importance for ensuring gas turbine running safety in a long-term.

RFR works in harsh conditions, so it is easy to generate creep damage. However, the theory for studying damage and rupture life of rotor is almost linear cumulative damage theory which is very inaccurate. At present, Damage Mechanics is applied to many domains, such as fatigue damage,
ductile damage, creep damage, etc [2]. 600MW turbine rotor with Damage Mechanics model considering multiaxial stress has been researched in order to get the nonlinear cumulative damage model [3]. Linear cumulative damage theory was combined with Damage Mechanics theory to study rotor stress field with finite element analysis, the result was more accurate [4]. Creep damage of 2Cr11NiMoVNbNB steel with creep experimental data has been studied to get damage rule and relationship between stress and strain when steel occur to fracture [5]. Creep damage and anti-cracking characteristics of turbine rotor has been studied in order to indicate connection between fracture parameters and propagation rate of creep crack [6]. Calculation method of turbine rotor creep life was proposed to calculate unsteady state creep stress and steady state creep stress with finite element program [7]. High-temperature creep of turbine rotor has been analyzed with finite element program to get temperature field, stress field and strain field [8].

The above research objects are all about solid rotor. But RFR includes many elements, so the total creep damage of RFR also contain different creep damage of elements, while few studies have been carried out in terms of it so far. Therefore, building creep damage analyzing model of RFR and indicating connection between element damage and the total damage of RFR has important significance for ensuring gas turbine running safety. This paper built a dynamic model of RFR considering interface contact effect to get equivalent stiffness of RFR. What’s more, creep damage of combustion chamber, rods of turbine and the total rotor was analyzed in order to indicate connection among the total damage, element damage and rupture life.

2. Rod fastening rotor model considering interface contact effect

2.1. Virtual Material Method

Virtual Material Method proposes an isotropic virtual material hypothesis-based analytical dynamical modeling method for immovable joint surface in order to enhance the modeling precision of ensemble machine tools. Some analytical expressions of elastic modulus, poisson ratio, thickness and density was put forward for Virtual Material to simulate interface contact effect of joint surface [9].

Analytical expressions of virtual material elastic modulus, poisson ratio, thickness and density will be got as the following [10]:

\[ E_c = \frac{2^{0.5D}D\psi^{1-0.5D}}{3\pi \sqrt{\pi}} EG^{1-D} a_L^{0.5D} (a_c^{-0.5} - a_L^{-0.5}) \]  \hspace{1cm} (1)

\[ \nu_c = \frac{(1+\mu')E^*}{G^*} - 1 \]  \hspace{1cm} (2)

\[ h_c = h_1 + h_2 \]  \hspace{1cm} (3)

\[ \rho_c = \frac{\rho_1 h_1 + \rho_2 h_2}{h_1 + h_2} \]  \hspace{1cm} (4)

Where \( D \) is fractal dimension, \( \psi \) is a parameter decided by \( D \), \( E^* \) is equivalent elastic modulus, \( G \) is fractal length scales, \( a_L \) is the largest contact area, \( a_c \) is critical contact area, \( \mu' \) is
equivalent poisson ratio, $E^*$ is dimensionless virtual material elastic modulus, $G_s^*$ is dimensionless virtual material shear modulus, $\rho_1, \rho_2$ are joint wheel disc density, $l_1, l_2$ are joint wheel disc thickness.

2.2. Contact model between wheel discs of rod fastening rotor

According to the real project, material of RFR is GH4169 whose density is $7850 kg/m^3$, elastic modulus is $205 GPa$, yield stress is $950 MPa$, poisson ratio is $0.3$.

If material surface roughness is $0.8 \mu m$, joint surface virtual material thickness $h_1 = h_2 \approx 0.5 mm$ [11]. The thickness of virtual material layers can be got as:

$$h = h_1 + h_2 = 1 mm$$

The virtual material thickness between wheel discs is $1 mm$, as shown in Figure 1.

![Figure 1. Virtual material layer](image)

By the above equation (1)(2)(3)(4), material parameters of virtual material with different preload can be calculated, as shown in Table.1.

| Preload of each rod (kN) | Elastic modulus (GPa) | Poisson ratio | Density (kg/m$^3$) |
|--------------------------|-----------------------|---------------|-------------------|
| 1000                     | 93.792                | 0.270         | 7800              |

2.3. Research on equivalent bending stiffness of rod fastening rotor

The study object of this paper is 300MW gas turbine rotor designed by Dong Fang Turbine Company Limited. Figure 2 shows the model of RFR. RFR is composed of compressor, combustion chamber and turbine. Compressor and turbine consist of 17 wheel discs and 12 tied rods. Turbine consists of 4 wheel discs and 12 tied rods. Combustion chamber is between the compressor and turbine. The wheel discs on compressor and turbine are connected with 12 tied rods on which the pre-load is applied, many contact interfaces exist between wheel discs. Physical parameters of RFR are showed in table 2.
RFR is composed of compressor, combustion chamber and turbine, and its total stiffness can be cascaded with the three parts. So the total stiffness can be got as the following:

$$K_{rot} = (\frac{1}{K_{com}} + \frac{1}{K_{tur}} + \frac{1}{K_{axl}})^{-1} \quad (5)$$

Compressor and turbine are consist of wheel discs, virtual material layers and rods. Wheel discs are cascaded with virtual material layers and they are all parallel with rods. Therefore, the stiffness of compressor and turbine can also be got:

$$K_{com}(K_{tur}) = (\frac{n_d}{K_d} + \frac{n_r}{K_r})^{-1} + K_r \quad (6)$$

Analytical expressions of the part and total stiffness is shown as the following [12]:

(1) Twisting stiffness of wheel discs

The twisting moment of wheel discs is $M$ and strain energy is $U_d$, the equation of twisting stiffness is:

$$K_d = \frac{M^2}{2U_d} \quad (7)$$

(2) Twisting stiffness of virtual material layers

With calculation of $E_c$, the stiffness of virtual material layers can be got as the following:

$$K_c = \frac{E_c I_{pc}}{I_c} \quad (8)$$
(3) Twisting stiffness of rods
The distribution of rods is shown in Figure 3, so the equation of twisting stiffness is:

\[ K_{rt} = 3n_r \left( \frac{r_p}{l_r} \right)^2 K_{rbl} \]  (9)

![Figure 3. Distribution of rods](image)

(4) Twisting stiffness of combustion chamber
The twisting stiffness combustion chamber is given as:

\[ K_{axl} = \frac{EI_{axl}}{l_{axl}} \]  (10)

Where \( K_{rot}, K_{com}, K_{tur}, K_{axl} \) are twisting stiffness of RFR, compressor, turbine and combustion chamber. \( K_d, K_c, K_r \) are twisting stiffness of wheel discs, virtual material layers and rods of compressor and turbine. \( K_{axl}, K_{rbl} \) are axial stiffness and twisting stiffness of each rod. \( n_d, n_c, n_r \) are numbers of wheel discs, virtual material layers and rods. \( l_d, l_r, l_{axl} \) are length of wheel discs, rods of turbine and combustion chamber. \( r_p \) is pitch radius of rods.

By the equations (5)–(10), some stiffness of RFR can be calculated, as shown in table 3.

### Table 3. Twisting stiffness of RFR

| Position          | Wheel disks | Rods (turbine) | Rods (compressor) | Virtual material layers | Combustion chamber | RFR   |
|-------------------|-------------|----------------|-------------------|-------------------------|-------------------|-------|
| Stiffness (N/m)   | 4.32E11     | 1.23E6         | 9.39E4            | 1.04E13                 | 5.82E10           | 6.28E9 |

3. Research on creep damage of rod fastening rotor

3.1 Creep damage cumulative model
Creep damage cumulative model was proposed by Lermaitre[13,14], as the following equation:
\[ D_c = 1 - (1 - R_v (1 + \alpha) \left( \frac{\sigma_{eq}}{\lambda} \right)^{\frac{1}{1 + \alpha}} ) t \]  

(11)

Where \( \alpha, \lambda, r \) are material parameters.

Multiaxial factor is defined as the following:

\[ R_v = \frac{2}{3} (1 + \nu) + 3(1 - 2\nu) \left( \frac{\sigma_{eq}}{\sigma_{eq}} \right)^2 \]  

(12)

Where \( \sigma_{eq} \) is hydrostatic stress, \( \sigma_{eq} \) is equivalent stress, \( \nu \) is poisson ratio.

If element has no damage, \( t = 0, D_c = 0 \). If element is ruptured completely, \( t = t_r, D_c = 1 \).

Integrate equation (11), \( t \) and \( D_c \) can be got as the following:

\[ t_r = \frac{1}{R_v (\alpha + 1)} \left( \frac{\sigma_{eq}}{\lambda} \right)^{-\alpha} \]  

(13)

\[ D_c = 1 - \left( 1 - \frac{t}{t_r} \right)^{\frac{1}{\alpha + 1}} \]  

(14)

The equation (13) and (14) is non-linear creep damage cumulative model which considering influence of equivalent stress. Therefore, it can calculate creep damage with multiaxial stress.

3.2. Element creep damage combined model in series or parallel

With equilibrium condition and equivalent strain principle[15], if \( D = 1 - \frac{E'}{E}, \text{ then } \sigma = E (1 - D) \varepsilon \).

Damage parameter D can be defined as\[16]:

\[ D = 1 - \frac{K'}{K} \]  

(15)

(1) If elements are in parallel, then:

Total displacement is \( L_p = l_1 = \cdots = l_i \), total stiffness is \( K_p = \sum_{i=1}^{n} k_i \), total force is \( F_p = \sum_{i=1}^{n} F_i = \sum_{i=1}^{n} k_i (1 - d_i) l \).

According to the above equations, total damage parameter in parallel is:

\[ D_p = \frac{\sum_{i=1}^{n} k_i d_i}{K_p} \]  

(16)

(2) If elements are in series, then:
Total displacement is \( L = \sum_{i=1}^{n} l_i \), total stiffness is \( K = \sum_{i=1}^{n} k_i \), total force is \( F_i = F_i = \cdots = F_i = K(1-D)L = k_i(1-d_i)l \).

According to the above equations, total damage parameter in series is:

\[
D_s = 1 - \frac{1}{K \sum_{i=1}^{n} \frac{1}{k_i(1-d_i)}}
\]  

(17)

3.3. Research on creep damage of rod fastening rotor

According to finite element analysis of RFR, the position of maximal stress is combustion chamber and rods of turbine, therefore, creep damage will generate more easily. Accordingly, this paper will research on creep damage of RFR caused by combustion chamber and rods of turbine. Equivalent stress and creep strain of RFR are shown in Figure 4 and Figure 5.

**Figure 4. Equivalent stress of rotor**

**Figure 5. Equivalent creep strain of rotor**

The creep damage of combustion chamber, rod of turbine and the total rotor can be calculated with the above equations.

1) Creep damage of rods of turbine and combustion chamber

According equations (11)–(14), creep damage can be got as the following.

Damage of Rods is: \( R_v = 1.46 \), \( t_r = \frac{1}{10.264} \left( \frac{\sigma_{eq}}{1575.48} \right)^{-8.596} \), \( D_c = 1 - \left( 1 - \frac{t}{t_r} \right)^{0.1724} \).

Damage of combustion chamber is: \( R_v = 1.41 \), \( t_r = \frac{1}{8.178} \left( \frac{\sigma_{eq}}{1575.48} \right)^{-8.696} \), \( D_c = 1 - \left( 1 - \frac{t}{t_r} \right)^{0.1724} \)

2) Creep damage of the total rotor when creep damage of rods is homogeneous

Only the creep damage of combustion chamber and rods of turbine is considered, furthermore, the creep damage of rods is homogeneous. As turbine and combustion chamber are in series, therefore, the creep damage of turbine can be calculated as the following:
According to above analysis, the total creep damage of rotor can be calculated as the following:

$$D_{rot} = D_{rot} = \sum_{i=1}^{n} \frac{k_{tur} d_{tur}}{K_{tur}}$$  \hspace{1cm} (18)

It is supposed that the creep damage of rods is homogeneous. Compare the nonlinear creep damage of combustion chamber, rods and the total RFR with Miner linear damage cumulative model[17], damage curves are shown in Figure 6.

Conclusion from the creep damage curves:

① When $t/t_r \leq 0.9$, creep damage increase equably as $t/t_r$ increase; when $t/t_r \geq 0.9$, creep damage increase rapidly as $t/t_r$ increase; when $t/t_r = 1$, creep damage reach 1, it means that RFR is fractured thoroughly.

② The total creep damage of RFR is decided by creep damage of combustion chamber and turbine, but bigger than them.

![Figure 6. Comparison of creep damage curves](image)

4. Conclusion

In this paper, nonlinear creep damage model was proposed for indicating connection between the total creep damage and rupture life against the disadvantages of current creep damage theory and life prediction. Furthermore, a dynamic model of RFR which has virtual material layer between wheel disks was built in order to calculate equivalent stiffness of RFR.

Results show that:

(1) The elastic modulus and poisson ratio of virtual material layer between wheel disks increase as preload increase, but the density is not change.
According to the analysis of creep damage with nonlinear creep damage model, damage parameter $D$ increase nonlinearity as cycle index. RFR will fracture thoroughly when $D=1$.

The total creep damage of RFR is decided by combustion chamber and rods of turbine, and the total creep damage is lager than creep damage of combustion chamber and turbine.

5. References

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