Direct Photon Production in Pion-Proton Collisions

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Abstract. In this article we calculate the contribution of the higher-twist Feynman diagrams to the large-$p_T$ inclusive single direct photon production cross section in $\pi p$ collisions in case of the running coupling and frozen coupling approaches within holographic QCD. We determined the structure of infrared renormalon singularities of the higher-twist subprocess cross section. Also compared and analyzed the resummed higher-twist cross sections with the ones obtained in the framework of the frozen coupling approach.

1. Introduction
Quantum chromodynamics (QCD) is widely believed to be the fundamental theory of the strong interactions. One of the main goal in QCD is to describe the structure and dynamics of hadrons at the amplitude level. The hadronic distribution amplitude in terms of quark and gluon degrees of freedoms plays an important role in QCD process predictions.

The AdS/CFT model has led to important insights into the properties of QCD. AdS/CFT duality also gives accurate predictions for hadron spectroscopy and a description of the quark structure of hadrons which has scale invariance also dimensional counting at short distances and together with color confinement at large distances. Based on the AdS/CFT correspondence where these are many significant theoretical advances using this theory as application, interesting and important attempts have been made to understand non-perturbative aspects of QCD under the name “holographic QCD”. A holographic model based on a truncated AdS space can be used to obtain the hadronic spectrum of light $q\bar{q}$, $qqq$ and $gg$ bound states showed by Brodsky et al. [1].

The higher-twist effects was calculated within the frozen coupling constant approach in the References [2, 3, 4, 5, 6].

In this study, we apply the running [7] and frozen coupling approaches in order to compute the effects of the infrared renormalons on the direct photon production at $\pi p$ collision in holographic QCD. This approach was also employed previously [8, 9, 10, 11, 12, 13] to calculate the inclusive meson production in $pp$ and $\gamma\gamma$ collisions.

We are interested in the calculation and analysis of the higher-twist effects on the dependence of the pion distribution amplitude in direct single photon production at $\pi p$ collision the considered approaches within holographic QCD. In this respect, the contribution of the higher-twist Feynman diagrams to a single direct photon production cross section has been computed by using various pion distribution amplitudes from holographic QCD.

The rest of the present paper is organized as follows. The formulae for the calculation of the contributions of the higher-twist and leading-twist diagrams are provided in Section 2. Some
formulae and analysis of the higher twist effects on the dependence of the pion distribution amplitudes by the running coupling constant approach is presented in Section 3, and the numerical results of the cross section and discussion of the dependence of the cross section on the pion distribution amplitudes and conclusions are presented in Section 4.

2. HIGHER TWIST CONTRIBUTIONS TO INCLUSIVE DIRECT PHOTON PRODUCTION

The higher-twist Feynman diagrams for the direct photon production in the pion-proton collision \( \pi p \rightarrow \gamma X \) are shown in Fig. 1. The amplitude for this subprocess is found by means of the Brodsky-Lepage formula [14]:

\[
M(\hat{s}, \hat{t}) = \int_0^1 dx_1 \int_0^1 dx_2 \delta(1 - x_1 - x_2) \Phi_M(x_1, x_2, Q^2) T_H(\hat{s}, \hat{t}; x_1, x_2),
\]

where \( T_H \) is the sum of the graphs contributing to the hard-scattering part of the subprocess.

As higher-twist subprocess which contribute to \( \pi p \rightarrow \gamma X \), we take \( \pi q_N \rightarrow \gamma q \) where \( q_N \) is a constituent of the initial nucleon target. In this study, for \( \pi^+ p \rightarrow \gamma X \) and for \( \pi^- p \rightarrow \gamma X \) processes we take, subprocesses as \( \pi^+ d_N \rightarrow \gamma u, \pi^- u_N \rightarrow \gamma d \), respectively.
Differential higher-twist cross section of the main process $\pi p \rightarrow \gamma X$ we can write formal analogy with deep-inelastic scattering in this form:

$$E\frac{d\sigma}{d^3p}(\pi p \rightarrow \gamma X) = \int_0^1 dx \delta(\hat{s} + \hat{t} + \hat{u})\hat{s}G_{q/p}(x, Q^2)\frac{1}{\pi}\frac{d\sigma}{dt}(\pi q_N \rightarrow \gamma q),$$

(2)

where $G_{q/p}(x, Q^2)$ is the quark distribution function inside a proton.

The Mandelstam invariant variables for higher-twist subprocess $\pi q_N \rightarrow \gamma q$ are

$$\hat{s} = (p_1 + p_\gamma)^2 = (p_2 + p_\pi)^2, \quad \hat{t} = (p_\gamma - p_\pi)^2, \quad \hat{u} = (p_1 - p_\pi)^2.$$  

(3)

The subprocess cross section for $\pi q_N \rightarrow \gamma q$ have the form [15]:

$$\frac{d\sigma}{d\hat{s}}(\hat{s}, \hat{t}, \hat{u}) = \frac{16\pi^2\alpha_E C_F}{9} [D(\hat{s}, \hat{u})]^2 \left[ \frac{1}{\hat{u}^2} + \frac{1}{\hat{s}^2} \right],$$

(4)

here $D(\hat{s}, \hat{u})$ in the framework of the frozen coupling constant approach takes the form:

$$D(\hat{s}, \hat{u}) = e_1 \hat{u} \alpha_s(Q_1^2) \int_0^1 dx \left[ \frac{\Phi_1(x, Q_1^2)}{x(1-x)} \right] + e_2 \hat{s} \alpha_s(Q_2^2) \int_0^1 dx \left[ \frac{\Phi_3(x, Q_3^2)}{x(1-x)} \right].$$

(5)

where $Q_1^2 = (1 - x)\hat{s}$ and $Q_2^2 = -x\hat{u}$ are represent the momentum squared carried by the hard gluon in Fig.1, $e_1$ and $e_2$ is the charge of quarks and $C_F = \frac{4}{3}$. For the case $\pi^- p \rightarrow \gamma X$, $e_1 = -\frac{1}{3}$ and $e_2 = \frac{2}{3}$, and for $\pi^+ p \rightarrow \gamma X$, $e_1 = \frac{2}{3}$ and $e_2 = -\frac{1}{3}$.

In this calculations we use pion distribution amplitudes predicted by AdS/CFT [16, 17, 18], the pQCD evolution [19],

$$\Phi^{hol}(x) = \frac{4}{\sqrt{3}\pi}f_\pi \sqrt{x(1-x)},$$

$$\Phi^{hol}_{SBGL}(x) = \frac{A_1 k_1}{2\pi} \sqrt{x(1-x)} \exp \left( -\frac{m^2}{2k_1^2 x(1-x)} \right), \quad \Phi_{asy}(x) = \sqrt{3}f_\pi x(1-x),$$

(6)

where $f_\pi$ is the pion decay constant, respectively.

The final form for the higher-twist differential cross section for direct photon production in the process $\pi p \rightarrow \gamma X$ is written as

$$E\frac{d\sigma}{d^3p}(\pi p \rightarrow \gamma X) = \frac{s}{s + u} x G_{q/p}(x, Q^2) \frac{16\pi\alpha_E C_F}{9} [D(\hat{s}, \hat{u})]^2 \left[ \frac{1}{\hat{u}^2} + \frac{1}{\hat{s}^2} \right].$$

(7)

The main problem, is extracting the higher-twist corrections to the direct photon production cross section. One can also interested consider the comparison of higher-twist corrections with leading-twist contributions. For the leading-twist subprocess in the direct photon production, we take as subprocess quark-antiquark annihilation $q\bar{q} \rightarrow g\gamma$. The corresponding cross section for subprocess $q\bar{q} \rightarrow g\gamma$ is

$$\frac{d\sigma}{dt}(q\bar{q} \rightarrow g\gamma) = \frac{8}{9}\pi\alpha_E \alpha_s(Q^2) \frac{e_1^2}{s^2} \left( \frac{\hat{t}}{\hat{u}} + \frac{\hat{u}}{\hat{t}} \right).$$

(8)

The leading-twist cross section for production of direct photon is

$$\Sigma_{LT} = E\frac{d\sigma}{d^3p}(\pi p \rightarrow \gamma X) = \int_0^1 dx_1 \int_0^1 dx_2 \delta(\hat{s} + \hat{t} + \hat{u})G_{q/M}(x_1, Q_1^2)G_{q/p}(x_2, Q_2^2)\hat{s} \frac{d\sigma}{dt}(q\bar{q} \rightarrow g\gamma),$$

(9)

where

$$\hat{s} = x_1x_2s, \quad \hat{t} = x_1t, \quad \hat{u} = x_2u.$$
3. HIGHER TWIST EFFECTS WITHIN HOLOGRAPHIC QCD AND THE ROLE INFRARED RENORMALONS

The main aim in our study is the calculation of integral in (5) by the running coupling constant approach within holographic QCD and the extraction of pure contribution of higher-twist and renormalon effects to the cross section. The renormalization scale according to Figs.1(a) and 1(b) should be chosen equal to $\mu_{R_1}^2 = (1-x)\hat{s} = \lambda_1\hat{s}$, whereas in Figs.1(c) and 1(d) $\mu_{R_2}^2 = -x\hat{u} = \lambda_2\hat{u}$, which directly we obtain from Feynman diagram. The integral in Eq.(5) in the framework of the running coupling approach takes the form

$$D(\mu_{R_2}^2) = \int_0^1 \frac{\alpha_s(\mu_{R_1}^2)\Phi_M(x, \mu_{R_1}^2)dx}{x(1-x)} + \int_0^1 \frac{\alpha_s(\mu_{R_1}^2)\Phi_M(x, \mu_{R_2}^2)dx}{x(1-x)}. \quad (10)$$

The $\alpha_s(\lambda\mu_{R_0}^2)$ has the infrared singularity for $\lambda = 0$ and so the integral (10) diverges. Thus for the regularization of the integral, we express the running coupling at scaling variable $\alpha_s(\mu_{R_0}^2)$ with the aid of the renormalization group equation in terms of the fixed one $\alpha_s(Q^2)$. The solution of renormalization group equation for the running coupling $\alpha \equiv \alpha_s/\pi$ has the form

$$\frac{\alpha(\lambda)}{\alpha} = \left[ 1 + \frac{3\beta_0}{4\ln \lambda} \right]^{-1}. \quad (11)$$

Then, for $\alpha_s(\lambda Q^2)$, we get

$$\alpha(\lambda Q^2) = \frac{\alpha_s}{1 + \ln \lambda/t}, \quad (12)$$

where $t = 4\pi/\alpha_s(Q^2)\beta_0 = 4/\alpha_0\beta_0$.

If we insert Eq.(12) into Eq.(10), then we obtain

$$D(\hat{s}, \hat{u}) = e_1\hat{u} \int_0^1 dx \frac{\alpha_s(\mu_{R_1}^2)\Phi_M(x, Q_1^2)}{x(1-x)} + e_2\hat{s} \int_0^1 dx \frac{\alpha_s(\mu_{R_2}^2)\Phi_M(x, Q_2^2)}{x(1-x)}$$

$$= e_1\hat{u}\alpha_s(\hat{s}) t_1 \int_0^1 dx \frac{\Phi_M(x, Q_1^2)}{x(1-x)(t_1 + \ln \lambda_1)} + e_2\hat{s}\alpha_s(-\hat{u}) t_2 \int_0^1 dx \frac{\Phi_M(x, Q_2^2)}{x(1-x)(t_2 + \ln \lambda_2)}, \quad (13)$$

where $t_1 = 4\pi/\alpha_s(\hat{s})\beta_0$ and $t_2 = 4\pi/\alpha_s(-\hat{u})\beta_0$.

Although the integral (13) is still divergent, but this expression can be transformed to more useful form as making the change of variable $z = \ln \lambda$, which gives

$$D(\hat{s}, \hat{u}) = e_1\hat{u}\alpha_s(\hat{s}) t_1 \int_0^1 \frac{\Phi_M(x, Q_1^2)dx}{x(1-x)(t_1 + z_1)} + e_2\hat{s}\alpha_s(-\hat{u}) t_2 \int_0^1 \frac{\Phi_M(x, Q_2^2)dx}{x(1-x)(t_2 + z_2)}. \quad (14)$$

We can calculate (14) by applying the integral representation of $1/(t + z)$ [20, 21]:

$$\frac{1}{t + z} = \int_0^\infty e^{-(t+z)u} du, \quad (15)$$

gives

$$D(\hat{s}, \hat{u}) = e_1\hat{u}\alpha_s(\hat{s}) t_1 \int_0^1 \int_0^\infty \frac{\Phi_x(x, Q_1^2)e^{-(t_1+z_1)u}dudx}{x(1-x)} +$$

$$+ e_2\hat{s}\alpha_s(-\hat{u}) t_2 \int_0^1 \int_0^\infty \frac{\Phi_x(x, Q_2^2)e^{-(t_2+z_2)u}dudx}{x(1-x)}. \quad (16)$$
For $\Phi^{bol}(x)$ the Eq.(16) is written as

$$D(\hat{s}, \hat{u}) = \frac{16f_\pi e_1 \hat{u}}{\sqrt{3} \beta_0} \int_0^\infty du e^{-t_1 u} B \left( \frac{1}{2}, \frac{1}{2} - u \right) + \frac{16f_\pi e_2 \hat{s}}{\sqrt{3} \beta_0} \int_0^\infty du e^{-t_2 u} B \left( \frac{1}{2}, \frac{1}{2} - u \right),$$

$$D(\hat{s}, \hat{u}) = \frac{4\sqrt{3}f_\pi e_1 \hat{u}}{\beta_0} \int_0^\infty du e^{-t_1 u} \left[ \frac{1}{1-u} \right] + \frac{4\sqrt{3}f_\pi e_2 \hat{s}}{\beta_0} \int_0^\infty du e^{-t_2 u} \left[ \frac{1}{1-u} \right].$$

where $B(\alpha, \beta)$ is Beta function and $u$ is Borel parameter.

4. NUMERICAL RESULTS AND CONCLUSIONS

We discuss the numerical results for higher-twist and renormalon effects with higher-twist contributions calculated in the context of the running and frozen coupling approaches on the

Figure 2. Higher-twist $\pi^+ p \rightarrow \gamma X$ direct photon production cross section $(\Sigma^{HT}_\gamma)^0$ as a function of the $p_T$ transverse momentum of the photon at the c.m. energy $\sqrt{s} = 200$ GeV.

Figure 3. Higher-twist $\pi^+ p \rightarrow \gamma X$ direct photon production cross section $(\Sigma^{HT}_\gamma)^{res}$ as a function of the $p_T$ transverse momentum of the photon at the c.m. energy $\sqrt{s} = 200$ GeV.

Figure 4. Ratio $(\Sigma^{HT})^{res}/(\Sigma^{HT}_\gamma)^0$, in the process $\pi^+ p \rightarrow \gamma X$, where higher-twist contribution are calculated for the photon rapidity $y = 0$ at the c.m.energy $\sqrt{s} = 200$ GeV as a function of the photon transverse momentum, $p_T$.

Figure 5. Ratio $(\Sigma^{HT})^{res}/(\Sigma^{HT}_\gamma)^0$, in the process $\pi^+ p \rightarrow \gamma X$, as a function of the $y$ rapidity of the photon at the transverse momentum of the photon $p_T = 15.5$ GeV/c, at the c.m. energy $\sqrt{s} = 200$ GeV.
dependence of the chosen pion distribution amplitudes in the direct photon production process, \( \pi p \rightarrow \gamma X \), within holographic QCD. For the numerical calculations, for \( \pi^+ p \rightarrow \gamma X \) process, we take subprocess as \( \pi^+ d_N \rightarrow \gamma u \). Inclusive direct photon production represents a significant test case in which higher-twist terms dominate those of leading-twist in certain kinematic domains. For the dominant leading-twist subprocess for the photon production, we take the quark-antiquark annihilation \( q\bar{q} \rightarrow \gamma g \). In the numerical calculations, for the quark distribution functions inside the pion and proton used as given in [22, 23], respectively. The results of our numerical calculations are displayed in Figs.2-5. Firstly, it is very interesting comparing the higher-twist cross sections obtained within holographic QCD with the ones obtained within perturbative QCD. In Fig.2 and Fig.3 we show the dependence of higher-twist cross sections \( (\Sigma^{HT}_\gamma)^0 \) (frozen cross section), \( (\Sigma^{HT}_\gamma)^{res} \) (resummed cross section) calculated in the context of the frozen and running coupling constant approaches as a function of the photon transverse momentum \( p_T \) for the pion distribution amplitudes \( \Phi^{hol}(x) \), \( \Phi^{asy}(x) \), \( \Phi^{VSBGL}(x) \), at \( y = 0 \). It is seen from Fig.2 and Fig.3 that the higher-twist cross section is monotonically decreasing with an increase in the transverse momentum of the photon. In the region 5 GeV/c < \( p_T < 95 GeV/c \) the resummed cross sections of the process \( \pi^+ p \rightarrow \gamma X \) decreases in the range between \( 2.256 \cdot 10^{-11} \mu b/GeV^2 \) to \( 5.331 \cdot 10^{-22} \mu b/GeV^2 \). In Fig.4-Fig.5 we show the dependence of the ratios \( (\Sigma^{HT}_\gamma)^{res}/(\Sigma^{HT}_\gamma)^0 \), in the process \( \pi^+ p \rightarrow \gamma X \) which are displayed as a function of \( p_T \) and \( y \) rapidity for \( \Phi^{hol}(x) \), \( \Phi^{asy}(x) \) and \( \Phi^{VSBGL}(x) \) pion distribution amplitudes. As shown in Fig.4, in the region 5 GeV/c < \( p_T < 95 GeV/c \), resummed cross section for \( \Phi^{hol}(x) \) is suppress by about two orders of magnitude relative to the frozen cross section for \( \Phi^{VSBGL}(x) \). In Fig.5 the dependence of ratio \( \left( \Sigma^{HT}_\gamma\right)^{res}_\hol\left/\left(\Sigma^{HT}_\gamma\right)^0_\hol\right. \) for \( \Phi^{hol}(x) \) distribution amplitude at the point \( y = -2 \) has a minimum, but for \( \Phi^{VSBGL}(x) \) pion distribution amplitude has a maximum.

We think that this feature of infrared renormalons may help the explain theoretical interpretations with future experimental data for the direct photon production cross section in the pion-proton collisions. Higher-twist cross section obtained in our study should be observable at hadron collider. The following results can be concluded from the experiments; the higher-twist contributions to single direct photon production cross section in the pion-proton collisions have important phenomenological consequences, the higher-twist photon production cross section in the pion-proton collisions depends on the form of the pion distribution amplitudes and may be used for future study. Also the contributions of renormalon effects within the framework holographic QCD in this process is essential and may help to analyse experimental results. Further investigations are needed in order to clarify the role of higher-twist effects in QCD. In hadron-hadron collisions, real photons at high transverse momentum can serve as a short distance in the probe of the incident hadrons. Especially, the future experimental measurements will provide further tests of the dynamics of large-\( p_T \) hadron production beyond the leading twist.

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