Exact Type IIB Superstring Backgrounds

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Abstract

We obtain a family of type IIB superstring backgrounds involving Ramond-Ramond fields in ten dimensions starting from a heterotic string background with vanishing gauge fields. To this end the global $SL(2,\mathbb{R})$ symmetry of the type IIB equations of motion is implemented as a solution generating transformation. Using a geometrical analysis we show that the type IIB backgrounds obtained are solutions to all orders in $\alpha'$. 

\textsuperscript{*}Short term visitor.
There has been a resurgence of interest in string theory over the last one year in the context of the duality symmetries. These symmetries have assumed a central role in establishing a web of interconnections amongst disparate string theories in diverse dimensions. They include the T-dualities [1] which are realized perturbatively in the string loop expansion and the conjectured non perturbative S-duality [2, 3, 4]. The latter relates strong and weak coupling phases and electric and magnetic charged states. In the quantum theory, they are conjectured to arise as broken subgroups of continuous global symmetries of the classical effective field theory but are discrete gauge symmetries of the full string theory. As an example we have the toroidal compactification of a heterotic string theory to four dimensions [3]. It has been shown that there exists a continuous global classical $SL(2, R)$ symmetry which is broken to the discrete S-duality group $SL(2, Z)$ in the quantum theory. This relates electric charged states of the fundamental string spectrum to the magnetic charged solitons. An analogous situation exists for type IIB superstring theory in ten dimensions [5, 6]. There we have likewise a global $SL(2, R)$ symmetry of the classical effective field theory in ten dimensions which is broken in the quantum string theory to the discrete S-duality group $SL(2, Z)$. However, unlike the heterotic case the S-duality for the type IIB case relates electric and magnetic charged states of gauge fields arising from the two different sectors NS-NS and R-R of the superstring spectrum.

In the perturbative approach to the background field analysis in string theories, the well known recipe is to construct classical solutions and consider quantum corrections in the world sheet sigma model perturbation expansion, treating the various background fields as couplings [7]. Two dimensional conformal invariance then requires the beta functions for these couplings to vanish leading to the background field equations of motion to various orders in the sigma model coupling $\alpha'$. The higher order contributions to the equations of motion become significant in the regime of strong fields and gravitational singularities and may be computed perturbatively. Such an approach is however unsuitable for type II string backgrounds. The bosonic excita-
tions in type II string spectrum arise from both the NS-NS and the R-R sectors [8]. The sigma model coupling of the R-R backgrounds are complicated, involving spin fields which depend on the ghosts. This introduces the complex process of picture changing and break the separate superconformal invariances of the matter and the ghost sectors. Thus higher order $\alpha'$ computations for the type II case is an extremely nontrivial exercise [8, 9]. There exists however certain classes of string backgrounds, at least for the bosonic and heterotic strings, for which the leading order equations of motion are exact as all higher order contributions are vanishing [10, 11, 13, 12, 14]. A large class of such backgrounds are ones with a covariantly constant null killing vector [10, 12]. These include the K-models with plane wave backgrounds as an explicit example. For the latter backgrounds, assuming certain specific ansatz for the form of the field strength of the antisymmetric tensor and the dilaton, it is possible to show from purely geometrical arguments that all higher order terms in the equations of motion are vanishing [15]. This obviates any requirement for a higher order perturbative analysis in the world sheet sigma model expansion. These plane waves may also be considered as heterotic string backgrounds with vanishing space time fermions and gauge field [11, 13, 16].

In this article we obtain a class of all order ($\alpha'$) type IIB backgrounds in ten dimensions with nontrivial R-R fields but vanishing five form field strength. We start from an exact plane wave background embedded in a heterotic string theory. Utilising the global $SL(2, R)$ symmetry of the type IIB equations of motion as a solution generating transformation [3], we generate a type IIB background. We subsequently use geometrical arguments to show that all higher order contributions to the background field equations of motion are identically zero for the type IIB case also. In this way we are able to construct a family of $SL(2, R)$ invariant, exact (to all orders in $\alpha'$) type IIB superstring backgrounds, completely avoiding the complex issues of world sheet couplings of the R-R fields. Our results acquire added significance as the plane waves are special cases of the K-models [12]. Certain K-models for heterotic and type
II strings with vanishing R-R fields, known as *chiral plane waves*, are connected to extremal electric and magnetic charged black holes [14]. These are known to be exact (in $\alpha'$) and when embedded in ten dimensional supergravity they preserve half of the space-time supersymmetry.

We now begin with a description of the plane wave string background for the heterotic case with vanishing gauge fields and briefly describe the geometrical arguments [15] to show that the leading order equations of motion are exact. These solutions may also be trivially embedded in type II string theories. The general form of a plane wave metric is as follows [15]

$$ds^2 = -2dudv + dx^i dx_i + F(u, x)du^2,$$  

where $u, v$ are light cone coordinates and $x^i$ (where $i = 2...9$) are the transverse coordinates. In our notation greek indices run over (0 to 9) and the roman indices over the transverse coordinates (2 to 9). The $v$ isometry of the metric leads to a null killing vector of the form $l^\mu = (0, 1, 0, ......., 0)$. For the metric in eqn. (1), the only non-zero connections are $\Gamma^i_{uu}$, $\Gamma^v_{uu}$ and $\Gamma^v_{ui}$. Using these, it is possible to show that $l^\mu$ is covariantly constant i.e. $D_\mu l^\nu = 0$. As the $G_{uv}$ component is a constant, we also have $D_\mu l^\nu = 0$. The metric may then be expressed in a closed form in terms of the killing vector $l^\mu$ as, $G_{\mu\nu} = \eta_{\mu\nu} + Fl_{\mu}l_{\nu}$, with the inverse metric as $G^{\mu\nu} = \eta^{\mu\nu} - Fl^\mu l^\nu$. Now the Riemann curvature tensor for this background may be calculated and shown to be as follows,

$$R_{\lambda\mu\nu\kappa} = 2 l_{[\lambda}[\partial_{\mu]}\partial_{\nu]}Fl_{\kappa]}.$$  

Employing the killing equation for $l^\mu$, it is straightforward to show that it is orthogonal to $R_{\lambda\mu\nu\kappa}$ on all the indices. The Ricci tensor for the metric in eqn. (1) is $R_{\mu\nu} = -\frac{1}{2}(\partial^2 F)l_\mu l_\nu$. From the form of the inverse metric we have $G^{uu} = 0$ and as $F$ is independent of $v$, then $\partial^2 F = \partial^2_i F$ where $\partial^2_i = \partial^i \partial_i$ and $i$ runs over the transverse coordinates $x^i$. It is evident that when $F$ is a harmonic function of the transverse coordinates, the metric (1) is a solution of the vacuum Einstein equation $R_{\mu\nu} = 0$. 

4
It is now straightforward to include the antisymmetric tensor and the dilaton. We choose the dilaton \( \phi \) to be any arbitrary function of \( u \) and the 3-form field strength for the antisymmetric tensor as

\[
H_{\lambda\mu\nu} = A_{ij}(u) \ l_{[\lambda} D_{\mu} x^i D_{\nu]} x^j,
\]  

(3)

The only independent tree level equation of motion for the background fields now is

\[
R_{\mu\nu} - \frac{1}{4} H_{\mu\alpha\beta} H_{\nu}^{\alpha\beta} - 2 D_\mu D_\nu \phi(u) = 0.
\]  

(4)

As \( D_\mu \phi \) is proportional to \( l_\mu \) from the Killing equation for the scalar dilaton field \( \phi(u) \), all terms in (4) are proportional to \( l_\mu l_\nu \). Thus (4) is satisfied provided we have

\[
\partial^i \partial_i F(u, x) + \frac{1}{18} A_{ij}(u) A^{ij}(u) + 4 \partial^2 \phi(u) = 0
\]  

(5)

So we may choose \( A_{ij}(u) \) and \( \phi(u) \) arbitrarily and solve for \( F(u, x) \). To satisfy eqn (5), \( F(u, x) \) must be a quadratic function of \( x^i \). There is a large class of such solutions, as we may also add to \( F(u, x) \) any solution of the homogeneous equation.

Before discussing the case of the type IIB backgrounds with nontrivial R-R fields, we first discuss the higher order sigma model contributions to the tree level equation (4) for the heterotic background \[13\]. From the tensor structure of the equation (4), it is evident that the higher order terms are all second rank tensors constructed from powers of the Riemann tensor \( R_{\lambda\mu\nu\kappa} \), the 3-form field strength \( H_{\lambda\mu\nu} \), the dilaton \( \phi(u) \) and their covariant derivatives. The first such class of higher order terms are the ones which involve a single Riemann tensor. This has the explicit form, \( D^\lambda D^\nu R_{\lambda\mu\nu\kappa} \), which may be expressed in terms of the covariant derivatives of the Ricci tensor \( R_{\mu\nu} \) through the Bianchi identities. As \( F \) must be a quadratic function of \( x^i \) cf. eqn (5),

\[
D^\lambda D^\nu R_{\lambda\mu\nu\kappa} = 0.
\]

From the expression of the Riemann tensor (2) in terms of the null killing vector \( l^\mu \), it may be shown that all second rank tensors constructed from more than one \( R_{\lambda\mu\nu\kappa} \) involve a contraction of the killing vector \( l^\mu \) with itself or with \( R_{\lambda\mu\nu\kappa} \). Thus
they are all identically zero, as \( l^\mu \) is orthogonal to \( R_{\lambda \mu \nu \kappa} \) and is also a null vector. For the case when \( l^\mu \) is contracted on a covariant derivative, using the fact that it is covariantly constant, it may be shown that

\[
 l^\rho D_\rho (D......D R_{\lambda \mu \nu \kappa}) = \mathcal{L}_l (D......D R_{\lambda \mu \nu \kappa}),
\]

(6)

where \( \mathcal{L}_l \) denotes a Lie derivative along the killing vector field and hence vanishes. Furthermore we have from (3) that \( H_{\lambda \mu \nu} l^\nu = 0 \) and employing similar arguments as above it may be shown that all second rank tensors constructed from more than two \( H_{\lambda \mu \nu} \) and their derivatives vanish. Similarly all terms in one \( R_{\lambda \mu \nu \kappa} \) and one or more \( H_{\lambda \mu \nu} \) or \( D_\mu \phi \) are identically zero. The only remaining higher order terms are of the form \((D......DH)^2\). For this, notice that \( l_\lambda D_\mu x^i D_{\nu j} x^j \) is covariantly constant. So all derivatives of \( H_{\lambda \mu \nu} \) acts on \( A_{ij}(u) \) which results in more \( l_\mu \) and hence this term vanishes. So the background fields satisfying eqn. (4) are exact to all orders in \( \alpha' \).

After presenting the arguments that the backgrounds defined by equations (1), (3) and \( \phi(u) \), are all order solutions (in \( \alpha' \)), of the heterotic string equations of motion, we now proceed to the type IIB case with non-trivial R-R fields. A type IIB superstring background consists of the following fields, the string frame metric \( G_{\mu \nu} \), two 3-form field strengths \( H^{(k)}_{\lambda \mu \nu} \) where \( k = (1,2) \), two scalars \( \chi \) and \( \phi \) from the NS-NS and R-R sectors respectively and a 5-form field strength \( F_{\lambda \mu \nu \rho \sigma} \). The two scalars \( \chi \) and \( \phi \) may be combined to form a complex scalar \( \lambda = \chi + ie^{-\phi} \). So we may consider the heterotic background defined by \( \phi(u) \), (1) and (3) to be a special case of a type IIB background which has \( H^{(2)}_{\lambda \mu \nu} = 0 \), \( \chi = 0 \) and \( F_5 = 0 \). As shown in [5, 6], type IIB strings in \( D = 10 \) has a global \( SL(2,R) \) symmetry at the level of the equations of motion [5, 7]. This acts on the type IIB background fields as follows:

\[
 G'_{\mu \nu} = | c\lambda + d | G_{\mu \nu},
\]

(7)

\[
 \lambda' = \frac{a\lambda + b}{c\lambda + d},
\]

(8)

and

\[
 H'^{(k)}_{\lambda \mu \nu} = \Lambda H^{(k)}_{\lambda \mu \nu},
\]

(9)
where $\Lambda$ is an $SL(2, R)$ matrix such that

$$\Lambda = \begin{pmatrix} d & c \\ b & a \end{pmatrix},$$

(10)

with $ad - bc = 1$.

Implementing the transformations (7), (8), and (9) we generate a nontrivial type IIB background with R-R fields starting from the trivial type IIB configuration defined by (1), (3) and $\phi(u)$. Explicitly we have

$$G'_{\mu\nu}(u, x) = f(u)G_{\mu\nu}(u, x),$$

(11)

where $f(u) = [d^2 + c^2 e^{-2\phi(u)}]^{\frac{1}{2}}$ and

$$\chi' = \frac{iae^{-\phi} + b}{ice^{-\phi} + d},$$

(12)

with $\chi' = \chi' + ie^{-\phi}$ and $\lambda = ie^{-\phi}$. So we have the final expressions for the type IIB scalars as

$$\chi'(u) = \frac{1}{f(u)^2}[db + ac e^{-2\phi}],$$

(13)

$$\phi'(u) = \phi(u) + 2 \ln f(u).$$

(14)

For the 3-form field strength $H^{(k)}$, $k = 1, 2$ we have

$$H^{(1)}_{\lambda\mu\nu} = dH^{(1)}_{\lambda\mu\nu},$$

(15)

and

$$H^{(2)}_{\lambda\mu\nu} = bH^{(2)}_{\lambda\mu\nu}.$$  

(16)

The new metric is given as follows

$$ds^2 = -2f(u)dudv + f(u)dx^i dx_i + K(u, x)du^2,$$

(17)

where $K(u, x) = f(u)F(u, x)$. A rescaling $f(u)du = dU$ of the metric leads to the general form for the plane wave metric

$$ds^2 = -2dUdv + \tilde{f}(U)dx^i dx_i + \tilde{K}(U, x)dU^2.$$  

(18)
Dropping the tilde and rewriting $U$ as $u$ in (18) we have

$$ds^2 = -2dudv + f(u)dx^i dx_i + K(u, x)du^2.$$  \hspace{1cm} (19)

We now show that the type IIB background generated in equations (13)-(19), is also an all order solution to the type IIB equations of motion. To the lowest order, the type IIB equations of motion are those of the $N = 2, D = 10$ chiral supergravity in ref. [17, 6]. They consist of equations for the scalar fields $\phi$ and $\chi$, second rank tensors $G_{\mu\nu}$ and $B^{(k)}_{\mu\nu}$ and a fifth rank antisymmetric tensor equation for the self dual five form field strength $F_{\mu\nu\rho\sigma\kappa}$. In studying the all order solutions for the type IIB equations, the possible corrections to all these equations must be considered. It is also noticed that all the gauge fields, i.e. $B^{(k)}_{\mu\nu}$, and $D_{\mu\nu\rho\sigma\kappa}$ can appear in the higher order terms only as the corresponding gauge invariant field strengths. As a result we consider the higher order terms in these equations obtained from combinations of the following quantities; $R_{\mu\nu\rho\sigma}$, $H^{(k)}_{\mu\nu\rho}$, $D_{\mu}\phi$, $D_{\mu}\chi$, $F_{\mu\nu\rho\sigma\kappa}$ and their covariant derivatives. In our case we choose $F$ to be zero, which is obtained by setting the four-form field $D_{\mu\nu\rho\sigma} = 0$ in its definition, together with the form of $H^{(k)}$ in eqn. (15) and (16).

As earlier, the $v$ independence leads to a null killing vector $l^\mu$. From (19) the only non-zero components of the inverse metric are $G^{uv} = -1$, $G^{vv} = -K$ and $G^{ii} = f(u)^{-1}$. Using these, it may be shown that the only non-zero components of the Christoffel connections are $\Gamma^{v}_{uu}, \Gamma^{v}_{uu}, \Gamma^{v}_{ui}$, and $\Gamma^{v}_{ii}$. This leads to the null killing vector being covariantly constant i.e. $D_\mu l^\nu = 0$. We also have $D_\mu l_\nu = 0$.

To consider the background field equations of motion we now proceed to compute the Riemann curvature tensor for the metric (19) of the type IIB background generated by us. Once again it is possible to write the new metric (19) in a closed form in terms of the null killing vector as $G_{\mu\nu} = M_{\mu\nu} + K(u, x)l_\mu l_\nu$. Where $M_{\mu\nu}$ is a $10 \times 10$ symmetric matrix with the only non-zero components being $M_{ij} = f(u)\delta_{ij}$ and $M_{uv} = -1$. Employing this closed form expression we calculate the Riemann
tensor to be
\[ R_{\lambda\mu\nu\kappa} = R^{(1)}_{\lambda\mu\nu\kappa}(u) + R^{(2)}_{\lambda\mu\nu\kappa}(u, x), \]  
where we have
\[ R^{(1)}_{\lambda\mu\nu\kappa}(u) = R^{(M)}_{\lambda\mu\nu\kappa}, \]  
with \( R^{(M)}_{\lambda\mu\nu\kappa} \) being the Riemann tensor for the metric \( M_{\mu\nu} \) and
\[ R^{(2)}_{\lambda\mu\nu\kappa}(u, x) = 2 l_\lambda \partial_{\mu\nu} K K l_\kappa. \]  
Notice that \( R^{(2)} \) in eqn. (22) is exactly of the same form as that of the heterotic case (or the type II case with vanishing R-R fields) in eqn. (2). However now we have an extra contribution \( R^{(1)} \) which cannot be expressed in a closed form in terms of the killing vector \( l^\mu \). This is because unlike the heterotic case, \( M_{\mu\nu} \) is a function of \( u \) and not equal to the flat diagonal metric \( \eta_{\mu\nu} \). We are, however, able to prove that the type IIB background in eqns. (13-19) is an all order (in \( \alpha' \)) solution by purely geometrical arguments.

It is apparent that the only non-zero independent component of \( R_{\lambda\mu\nu\kappa} \) is \( R_{uiui} \), the reason being that \( M_{ij} = f(u)\delta_{ij} \) and the only non-zero component of the killing vector \( l_\nu \) is \( l_u = -1 \). In the same way the only nonzero independent fully contravariant component is \( R^{vi vi} \) because \( G^{ij}(i \neq j) = G^{ui} = G^{vi} = 0 \). Similarly we may show that the only nonzero component of the Ricci tensor is \( R_{uu}, R^u_u \) and \( R^{vv} \). From the form of the inverse metric \( G^{\mu\nu} \) it may be shown that it is impossible to construct any non-zero scalar from \( R_{uu} \) by contraction, hence \( R = 0 \). The type IIB dilaton \( \phi' \) in eqn. (14) is also a function of \( u \) only, as in the heterotic case and the type IIB 3-form field strengths \( H^{(k)}_{\lambda\mu\nu} \) are of the same form as in the heterotic case in eqn. (3). From (3) it may also be shown that the only non-zero independent component of the field strength \( H_{\lambda\mu\nu} \) is \( H_{uij} \). In the subsequent discussion we drop the primes on the type IIB fields defined in eqns. (13-19).
We now proceed to present geometrical arguments to show that the backgrounds
defined in eqns. (13-19) are solutions to all orders (in $\alpha'$) of the type IIB equations
of motion. As earlier, we now consider the higher order terms in the equations of
motion which are second rank tensors. These correspond to the equations for $G_{\mu\nu}$
and $B_{\mu\nu}^{(k)}$. Terms involving just one Riemann tensor has the form $D\lambda D\nu R_{\lambda\mu\nu\kappa}$. Notice
that, the $SL(2,R)$ transformation cf. eqn. (11) and the subsequent rescaling in $u$
does not affect the quadratic $x^i$ structure of $F(u,x)$. So the final $K(u,x)$ in eqn. (19)
is also a quadratic function of $x^i$. Using these we have

$$D^u D^u R_{uiui} = D^i D^i R_{uiiu} = D^u D^i R_{uiiu} = D^i D^u R_{uiiu} = 0,$$  \hspace{1cm} (23)

which implies

$$D\lambda D\nu R_{\lambda\mu\nu\kappa} = 0.$$  \hspace{1cm} (24)

It is apparent now that to construct second rank tensors it is required to contract
at least two indices of $R_{\lambda\mu\nu\kappa}$ with other $R$ or derivatives of $R$. Potentially non-zero
contributions to these terms may come from the contractions of covariant indices
$(u,i)$ and contravariant ones $(v,i)$. Contractions on derivatives have been shown to
be zero. Contractions on other $R$ requires at least a covariant index $v$ or contravariant
index $u$ which are unavailable. Hence we conclude that it is impossible to construct
non-zero second rank tensors from contraction of $R_{\lambda\mu\nu\kappa}$ and its derivatives. Hence all
such higher order contributions are vanishing.

We now focus on other higher order terms which are obtained from $D_\mu \chi(u)$,
$D_\mu \phi(u)$, $H_{(k)}$, and $R_{\lambda\mu\nu\kappa}$. As earlier, from the killing equation for the scalar fields $\chi$, 
$\phi$ we find that $D_\mu \phi$ is proportional to $l_\mu$. Also, from the killing equation, it maybe
shown that the killing vector $l_\mu$ is orthogonal to $R$ in all the indices. This shows that
terms involving $D_\mu R^{(k)}_{\lambda\mu\nu\kappa}$ are identically zero. Similarly terms of the form $D^\mu \phi R_{\lambda\mu\nu\kappa}$
are also zero.

For the terms obtained from more than two $H_{(k)}$ and their derivatives we have
the following observations. As $H_{\lambda\mu\nu}$ contains one killing vector, all terms in more
than two $H$ or its derivatives contains more than two killing vectors $l^\mu$. As we have only two free indices one of these must be contracted with another $H$ or a derivative. Thus all such terms vanish as $H^{(k)}_{\lambda\mu\nu} l^\lambda = 0$ and contraction on a derivative is equivalent to a Lie derivative in the direction of the killing vector of the tensor being considered, which is also zero. In a similar fashion we may show that terms involving $D\phi H$, $D\chi H$ and their derivatives are vanishing as $D_\mu \phi$ or $D_\mu \chi$ are proportional to $l_\mu$ which is orthogonal to $H_{\lambda\mu\nu}$ in all the indices.

Let us now consider second rank tensors which may be obtained from one or more $H$ and $R$ and their derivatives. As an example, consider $H_{\alpha\beta\gamma} D_\kappa R^{\beta\gamma\kappa}_{\lambda}$. This is non-zero for $\alpha = u$ or $(i, j)$. For $\alpha = u$, we have $(\beta, \gamma) = (i, j)$ because the only non-zero independent component of $H$ is $H_{uij}$. However the only contravariant transverse indices of non-zero $R$, available for contraction are as described earlier, are $(i, i)$ or $(j, j)$, hence the term is zero for $\alpha = u$. For $\alpha = (i, j)$, $(\beta, \gamma)$ must be $(u, j)$ and must be contracted with a contravariant index $u$ in $R$ which is unavailable. Hence the the term is also vanishing for $\alpha = (i, j)$. Exactly similar arguments may be used to show that terms of the generic form $(DH)R$ as well as terms like $(D...DH)^2$ are vanishing.

We have therefore examined all the possible second rank tensors, in more than two derivatives, constructed from $R$, $H^{(k)}$ and their covariant derivatives together with $D\phi$ and $D\chi$. To convince the reader that these are all the possible forms to be examined, we once again stress that only possible covariant tensor components are $D_u \phi$, $D_u \chi$, $R_{uiuj}$ and its covariant derivatives with respect to $D_u$ and $D_j$, $H^{(k)}_{uij}$ and its covariant derivatives with respect to $D_u$. One also has the corresponding contravariant tensor components. In addition we have the identities in equation (23). Using these it is possible to show that all second-rank tensors with higher derivatives vanish. A similar argument shows that all possible higher order corrections to the equations of motion for the scalars and to the self-duality condition for $F_{\mu\nu\rho\sigma\kappa}$ which appears as fifth rank fully antisymmetric tensors are also zero. Thus we have shown that the type IIB background obtained in eqns. (13-19) are exact to all orders in $\alpha'$. 

11
To conclude, we have obtained a class of exact (to all orders in $\alpha'$) type IIB backgrounds with R-R fields dependent on the parameters of the $SL(2, R)$ group. We emphasise that such an exercise in the sigma model framework is highly nontrivial owing to the non standard forms of the couplings for the R-R fields cf. [8, 9]. However this issue could be completely bypassed through the geometrical analysis of the higher order terms in the equations of motion. The plane wave backgrounds considered here are similar in nature to a larger class of string backgrounds called K-Models, which have been discussed by Tseytlin et. al. [12] for heterotic and type II strings in the absence of R-R fields. These are distinguished by the presence of a null killing isometry such that the corresponding killing vector is covariantly constant. It would be interesting to show that the general type II K-models with non trivial R-R fields are also all-order solutions of the background field equations. We hope to report on this issue very soon.

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References

[1] For a review see A. Giveon, M. Porrati and E. Rabinovici, Phys. Rep. 244 (1994) 77; and references therein. See also E. Alvarez, L. Alvarez-Gaume and Y. Lozano, *An Introduction to T-duality in String Theory*, hep-th/ 9410237.

[2] A. Font, L. Ibanez, D. Luest and F. Quevedo, Phys. Lett. B 249 (1990) 35.

[3] A. Sen, Int. Jour. of Mod. Phys. A9 (1994) 3707.

[4] J.H. Schwarz, *String Theory Symmetries*, hep-th/ 9503127 and *Evidence for Nonperturbative Symmetries in String Theory*, Caltech Report No. Calt-68-95; E.Witten, Nucl. Phys. B 443 (1995) 85.

[5] C.M. Hull and P.K. Townsend, Nucl. Phys. B438 (1995) 109; Nucl. Phys. B451 (1995) 525; P. Aspinwall, *Some relationships Between Dualities in String Theory*, hep-th/ 9508154.

[6] E. Bergshoeff, C.M. Hull and T. Ortin, Nucl. Phys. B451 (1995) 547; E.Bergshoeff, *Duality Symmetries and Type II String Effective Actions*, Groningen Report No. UG-11/95, hep-th/ 9509145.

[7] For a review see A. Tseytlin, Int. J. Mod. Phys., A4 (1989) 1257, and references therein.

[8] D. Friedan, E. Martinec and S. Shenker, Nucl. Phys. B271 (1986) 93.

[9] J. Polchinski, Phys. Rev. Lett. 75 (1995) 4724; D. Polyakov, *R-R Dilaton Interaction in a Type IIB Superstring*, Rutgers Univ. Report No. RU-95-85, hep-th/ 9512028.

[10] A. Tseytlin, Nucl. Phys. B390 (1993) 193.

[11] E. Bergshoeff, R. Kallosh, and T. Ortin, Phys. Rev. D47 (1993) 5444; E. Bergshoeff, I. Entrop and R. Kallosh, Phys. Rev. D49 (1994) 6663.
[12] G. Horowitz and A. Tseytlin, Phys. Rev. D50 (1994) 5204; A.Tseytlin, Exact String Solutions and Duality, Imperial College Report No. Imperial/TP/93-94/46, hep-th/9407099.

[13] G. Horowitz and A. Tseytlin, Phys. Rev. D51 (1995) 2896.

[14] A. Tseytlin, Exact Solutions of Closed String Theory, Class. Quantum Grav. 12 (1995) 2365.

[15] G. Horowitz and A.R. Steif, Phys. Rev. Lett. 64 (1990) 260; Phys. Rev. D42 (1990) 1950.

[16] R. Guven, Phys. Lett. B191 (1987) 275.

[17] J.H. Schwarz, Nucl. Phys. B226 (1983) 269.