FUNDAMENTAL PLANES AND THE BARLESS $M_{bh}\sigma$ RELATION FOR SUPERMASSIVE BLACK HOLES

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ABSTRACT

The residuals about the standard $M_{bh}\sigma$ relation correlate with the effective radius, absolute magnitude, and Sérsic index of the host bulge, giving rise to an apparent black hole "fundamental plane." However, we show that the elliptical galaxies do not define such a plane. Instead, it is a handful of barred galaxies, which are shown to systematically deviate from the $M_{bh}\sigma$ relation by $\delta \log M_{bh} \approx -0.5$ to $-1.0$ dex (their $\sigma$-values are too large) and generate much of the aforementioned three-parameter correlations. Removal of the seven barred galaxies from the Tremaine et al. set of 31 galaxies gives a "barless" $M_{bh}\sigma$ relation with an intrinsic scatter of 0.17 dex (vs. 0.27 dex for the 31 galaxies) and a total scatter of 0.25 dex (vs. 0.34 dex for the 31 galaxies). Furthermore, removal of the barred galaxies, or all the disk galaxies, from an expanded and updated set of 40 galaxies with direct black hole mass measurements gives a consistent result, such that $\log (M_{bh}/M_\odot) = (8.25 \pm 0.05) + (3.68 \pm 0.25) \log (\sigma/200 \text{ km s}^{-1})$. In addition, the barless $\sigma$-$L$ relation for galaxies with direct black hole mass measurements is found to be consistent with that from the SDSS sample of early-type galaxies, and the barless $M_{bh}\sigma$ relation, the $M_{bh}-n$ relation, and the $K$-band $M_{bh}-L$ relation are all shown to yield SMBH masses less than $2-4 \times 10^9 M_\odot$.

Subject headings: black hole physics — galaxies: bulges — galaxies: fundamental parameters — galaxies: structure

1. INTRODUCTION

Tight correlations between supermassive black hole (SMBH) masses and large-scale properties of their host bulges are interesting for two obvious reasons. They enable us to predict SMBH masses in thousands of galaxies where the black hole's sphere of influence is highly unresolved, and they provide clues to the physical processes responsible for the coevolution of the black hole and the host bulge. Recent endeavors have advocated relations involving not one but two bulge parameters, and thereby claims of "fundamental planes," akin to the fundamental plane for elliptical galaxies (Djorgovski & Davis 1987; Dressler et al. 1987). The existence of such SMBH fundamental planes would imply that current theories for the $M_{bh}\sigma$ relation (Ferrarese & Merritt 2000; Gebhardt et al. 2000) or the $M_{bh}-L$ relation (McLure & Dunlop 2002; Marconi & Hunt 2003; updated in Graham 2007), which do not include a third parameter, are incomplete.

This article investigates the fundamental planes for SMBHs involving the parameters $M_{bh}$, $R_e$, and $\langle \mu \rangle_e$ (Barway & Kembhavi 2007), $M_{bh}$, $R_e$, and $\sigma$ (Marconi & Hunt 2003; de Francesco et al. 2006; Aller & Richstone 2007; Hopkins et al. 2007), and for the first time $M_{bh}$, $\sigma$, and $n$. In § 2, it is explained why a previous claim for a small "total" scatter (0.19 dex) about the $M_{bh}-R_e-\langle \mu \rangle_e$ plane was the result of a miscalculation. In § 3, it is revealed that the galaxies that deviate from the $M_{bh}\sigma$ relation, giving rise to the $M_{bh}\sigma-R_e$, $M_{bh}\sigma-L$, and $M_{bh}\sigma-n$ relations with less scatter than the $M_{bh}\sigma$ relation, are predominantly barred galaxies. A "barless" $M_{bh}\sigma$-relation, and an elliptical-only $M_{bh}\sigma$-relation, are subsequently constructed in § 4 and found to heavily nullify the evidence for fundamental planes for SMBHs and their host bulges.

Given the recent discussion in the literature about biases in the $M_{bh}\sigma$ and/or $M_{bh}-L$ relations, and also in the local sample of galaxies with direct SMBH mass measurements, these concerns are explored here. In § 5, a "barless" $\sigma$-$L$ relation is constructed and shown to be equal to that obtained using SDSS early-type galaxy data, thereby easing concerns that the local sample of galaxies with direct SMBH mass measurements may be biased with respect to the greater population (e.g., Yu & Tremaine 2002; Bernardi et al. 2007). Furthermore, in § 6 the $K$-band $M_{bh}-L$ relation and the barless $M_{bh}\sigma$ relation are shown to yield consistent results, with neither giving SMBH masses greater than $\sim 4 \times 10^9 M_\odot$.

2. THE $(M_{bh\sigma},\langle \mu \rangle_e, R_e)$ PLANES

Barway & Kembhavi (2007, hereafter BK07) made the interesting claim that a combination of two photometric parameters, namely the effective radius $R_e$ and the mean effective surface brightness $\langle \mu \rangle_e = -2.5 \log \langle I \rangle_e$, can be used to predict SMBH masses with a greater degree of accuracy than single quantities such as luminosity or velocity dispersion.

A tight relation exists between black hole mass, $M_{bh}$, and the luminosity, $L$, of the host bulge. The luminosity can of course be expressed in terms of two other parameters, because $L = 2\pi R_e^2 \langle I \rangle_e$, where $\langle I \rangle_e$ is the average intensity within the effective half-light radius, $R_e$, of the bulge. One question of interest is whether the scatter about the $M_{bh}\sigma$ relation can be reduced by allowing the exponents on $\langle I \rangle_e$ and $R_e$ to deviate from their 1:2 ratio, and, importantly, whether this results in less scatter than the other competing relations. Given the small scatter about the fundamental plane (involving $R_e$, $\langle \mu \rangle_e = -2.5 \log \langle I \rangle_e$, and $\sigma$) and the tight relationship between $M_{bh}$ and $\sigma$ (Merritt & Ferrarese 2001; Tremaine et al. 2002), one may indeed expect a well-defined plane using the parameters $R_e$, $\langle \mu \rangle_e$, and $M_{bh}$.

2.1. Barway & Kembhavi Data

This section examines the claim of BK07 that the total root mean square (rms) scatter in the log $M_{bh}$ direction, when using $\langle \mu \rangle_e$ and log $R_e$ as predictor quantities of $M_{bh}$, is 0.25 dex (0.19 dex when excluding the outlier NGC 4742).
A simple linear, \( Y = A + BX \), ordinary least-squares (OLS) regression analysis, OLS(\( Y \mid X \)), is performed with \( Y = \log M_{bh} \) and \( X = \log R_e + b(\mu) \). Solving for the parameters \( A, B, \) and \( b \), this non-symmetrical regression gives the smallest rms residual in the \( \log M_{bh} \) direction, which is what one wants when using the \( M_{bh}-(\mu)-R_e \) plane to predict \( M_{bh} \) in other galaxies (see Feigelson & Babu 1992). The data for \( \log M_{bh}, \log R_e, \) and \( \langle \mu \rangle_e \) have been taken from Table 1 in BK07. Due to the absence of reported errors on the quantities \( \log R_e \) and \( \langle \mu \rangle_e \) in BK07, measurement errors are not included in the regression, and consequently no attempt is made to quantify the intrinsic scatter. Parameter uncertainties are derived here using a bootstrap sampling of the data points (i.e., sampling with replacement from the original sample) to produce 1000 Monte Carlo samples from which 1000 optimal fits are derived. This provides a histogram of each parameter from which one can compute the central 68.3% width, which is used as the 1σ uncertainty.

The optimal (B-band) solution using all 18 data points from BK07 is

\[
\log \left( \frac{M_{bh}}{M_\odot} \right) = (8.18 \pm 0.09) + (3.15 \pm 0.33) \log (R_e/3 \text{ kpc}) - (0.90 \pm 0.18)(\langle \mu \rangle_e - 21.0). \tag{1}
\]

The total scatter in the \( \log M_{bh} \) direction is 0.32 dex. However, the total scatter in the \( M_{bh}-\sigma \) relation for this same galaxy set is 0.31 dex.

BK07 performed an additional analysis, excluding NGC 4742, whose SMBH mass derivation has not yet appeared in a refereed paper (see Tremaine et al. 2002) and may therefore potentially be erroneous. This galaxy also appeared as a clear outlier in their data. This does not necessarily mean that the data point is in error; it may simply be a 3σ event, or the distribution of residuals may perhaps not be “normal.” Robust statistics requires that underlying data points not bias an analysis. No single data point from a distribution should have the ability to significantly alter the result of an analysis dictated by the remaining population. The optimal relation after excluding NGC 4742 is given by the expression

\[
\log \left( \frac{M_{bh}}{M_\odot} \right) = (8.21 \pm 0.07) + (3.23 \pm 0.26)(\log R_e/3 \text{ kpc}) - (1.01 \pm 0.13)(\langle \mu \rangle_e - 21.0), \tag{2}
\]

with a total scatter of 0.25 dex. The low value of the 0.19 dex reported by BK07 appears to have arisen by dividing their scatter in the log \( R_e \) direction (0.061) by the coefficient in front of the log \( M_{bh} \) term in their equation (3) (which is their fitted plane). However, this approach overlooks the three-dimensional nature of the plane and consequently results in an overestimation of the plane’s ability to predict black hole masses. Computing the rms offset between the black hole mass lists in Table 1 of BK07 (excluding NGC 4742) and the values predicted from their plane (their eq. [3]), which can be rewritten as log \( M_{bh}/M_\odot \) = 8.28 + 3.13(\log R_e/3 \text{ kpc}) - 0.97(\langle \mu \rangle_e - 21), one obtains a total scatter in the log \( M_{bh} \) direction of 0.27 dex, not 0.19 dex, which is greater than the value of 0.25 dex obtained above using equation (2).

Although the claim in BK07 appears misplaced, based on an erroneous treatment of the data, the idea tested there is a valid one. In an effort to improve the \( M_{bh}-(\mu)-R_e \) plane’s reliability for predicting black hole masses, it is noted here that three of the galaxies used by BK07 are known disk galaxies, or at least not regular elliptical galaxies. M32 may be a stripped S0 galaxy (Bekki et al. 2001; Graham 2002), while NGC 2778 is a disk galaxy (Rix et al. 1999), as is NGC 4564 (Trujillo et al. 2004; see also Fig. 6 in Graham & Driver 2007a). Consequently, the effective radii and mean surface brightnesses that have been used for these three galaxies do not pertain to their bulges. IC 1459 is also excluded here due to the order of magnitude uncertainty on its SMBH mass (Cappellari et al. 2002). Excluding these four galaxies, plus NGC 4742, gives, from a reduced sample of only 13 galaxies, a total scatter of 0.28 dex. However, the total scatter in the \( M_{bh}-\sigma \) relation for this cleaned galaxy set is 0.27 dex. The \( M_{bh}-\sigma \) relation therefore appears more competitive than the \( M_{bh}-(\mu)-R_e \) plane.

This sample size is obviously small, which therefore makes it hard to reach reliable conclusions. The \( M_{bh}-(\mu)-R_e \) plane is thus investigated further with a larger galaxy sample in the following subsection.

2.2. Marconi & Hunt Data

Instead of using the B-band data in BK07, the scatter about the \( M_{bh}-(\mu)-\log R_e \) plane is explored here using the larger, homogeneous, K-band data set from Marconi & Hunt’s (2003; hereafter MH03) “Group 1” galaxies. Using the minor-to-major axis ratio, \( b/a \), of the GALFITted (Peng et al. 2002) Sérsic bulge component (A. Marconi & L. K. Hunt 2007, private communication), MH03’s tabulated major-axis effective radii, \( R_{e,\text{maj}} \), have been converted into a geometric mean radius \( R_e = (R_{e,\text{maj}}^2 b/a)^{1/2} \), which is used here. Although MH03 did not report any values for \( \langle \mu \rangle_e \), they can be derived from the expression

\[
\langle \mu \rangle_e = m_{\text{tot}} + 2.5 \log \left( 2\pi R_{e,\text{maj}}^2 b/a \right),
\]

where \( m_{\text{tot}} \) is the apparent magnitude of the bulge (obtained from the absolute magnitude and distance in Table 1 in MH03). These values are shown in Table 1.

Four of the five galaxies which were excluded at the end of §2.1 are in MH03’s “Group 1” list. They are again excluded here for the same reasons. Doing so, one obtains from the remaining 23 galaxies, using the \( M_{bh} \) values given in MH03,

\[
\log \left( \frac{M_{bh}}{M_\odot} \right) = (7.92 \pm 0.12) + (2.24 \pm 0.37)(\log R_e/3 \text{ kpc}) - (0.54 \pm 0.14)(\langle \mu \rangle_e - 17.5), \tag{4}
\]

with which a total scatter in the log \( M_{bh} \) direction of 0.33 dex. The coefficients in equation (4) are consistent with an \( M_{bh}-(L,R_e^2) \) plane, and the total scatter in the \( M_{bh}-L \) relation for these galaxies (0.34 dex) is comparable. Moreover, the total scatter in the \( M_{bh}-\sigma \) plane for this same galaxy set is 0.28 dex. We therefore conclude that the \( M_{bh}-(\mu)-\log R_e \) plane is not warranted.

The following section explores the \( M_{bh}-\sigma \) plane and other planes involving \( M_{bh} \), \( \sigma \), and some third parameter.

3. THE \( (M_{bh}-\sigma)-X \) PLANE

MH03 explored the addition of \( \log R_e \) to the \( M_{bh}-\sigma \) relation to create a “fundamental plane for SMBHs.” From their 27 “Group 1” galaxies, they constructed a relation between \( M_{bh} \) and \( R_e \sigma^2 \) (proportional to the virial bulge mass), which resulted in an intrinsic dispersion \( ^2 \) (total scatter) of 0.25 (0.30) dex in the
TABLE 1

| Galaxy   | Type   | $b/a$ | $R_e$ | $\langle m \rangle_{K}$ |
|----------|--------|-------|-------|------------------------|
| GC 4258  | Sb, bar| 1.00  | 0.7   | 15.54                  |
| GC 4486  | E      | 0.87  | 0.66  | 15.26                  |
| GC 5115  | S0     | 0.38  | 2.9   | 16.49                  |
| GC 4649  | E      | 0.80  | 7.2   | 17.08                  |
| GC 3031  | Sp     | 0.76  | 3.0   | 16.83                  |
| GC 4374  | (M84)  | 0.91  | 7.8   | 17.35                  |
| GC 221   | (M32)  | 0.71  | (0.20)| (15.30)                |
| GC 5128  | S0     | 0.85  | 3.3   | 16.68                  |
| GC 4697  | E      | 0.58  | 6.9   | 18.18                  |
| IC 1459  | E      | 0.75  | 7.1   | 16.95                  |
| GC 9525  | S0     | 0.48  | 6.7   | 17.19                  |
| GC 2787  | S0, bar| 0.70  | 0.27  | 14.41                  |
| GC 4594  | Sp     | 0.63  | 4.0   | 16.21                  |
| GC 3608  | E      | 0.82  | 3.9   | 17.44                  |
| GC 3245  | S0     | 0.61  | 1.0   | 15.32                  |
| GC 4291  | E      | 0.77  | 2.0   | 16.22                  |
| GC 3377  | E      | 0.51  | 3.9   | 17.91                  |
| GC 4473  | E      | 0.57  | 2.1   | 16.41                  |
| Cygnus A | E      | 0.86  | 28.7  | 18.78                  |
| GC 4473  | E      | 0.80  | 5.8   | 16.82                  |
| GC 4258  | E      | 0.57  | 12.8  | 18.18                  |
| NGC 6251 | E      | 0.82  | 10.0  | 17.06                  |

Note.—Col. (3): Spheroidal component’s minor-to-major axis ratio, $b/a$, for the galaxies from A. Marconi & L. K. Hunt (2003, private communication). Col. (4) and (5): Geometric mean radii $R_e = (R_{e, maj} \times R_{e, min})^{1/2}$ in kpc and mean $K$-band effective surface brightnesses (eq. [3]) in mag arcsec$^{-2}$ for the spheroidal component. A bracketed entry indicates that no $K$-band bulge/disk decomposition was performed.

log $M_{bh}$ direction. Allowing the exponents on the $R_e$ and $\sigma$ terms to vary independently, Hopkins et al. (2007) used the same 27 Group 1 galaxies from MH03, along with some updated measurements, to report that

$$\log(M_{bh}/M_\odot) = (8.33 \pm 0.06) + (0.43 \pm 0.19) \log(R_e/3 \text{ kpc}) + (3.00 \pm 0.30) \log(\sigma/200 \text{ km s}^{-1}),$$

with an intrinsic scatter of 0.21 dex (and a total scatter of 0.30 dex; P. F. Hopkins 2007, private communication). For comparison, the $M_{bh}-\sigma$ relation in Tremaine et al. (2002) has an intrinsic (total) scatter of 0.27 (0.34) dex. It therefore appears that the introduction of a third parameter to the standard $M_{bh}-\sigma$ relation may reduce the scatter, as MH03 and later Hopkins et al. (2007) show that it does. Here, it is investigated which third parameter is optimal.

From Graham & Driver (2007a, hereafter GD07) the total scatter about the log-quadratic $M_{bh}$-$n$ relation is reported to be 0.31 dex. This highlights the strong connection between $M_{bh}$ and the radial structure in the stellar distribution of the host bulge (see also Graham et al. 2007, their § 1), and hence the need to advance beyond $R^{1/4}$ models and their associated luminosity/mass-dependent biases (e.g., Trujillo et al. 2001; Brown et al. 2003). This section explores whether the scatter about the $M_{bh}$-$\sigma$ relation is best reduced through the addition of the host bulge’s log $R_e$, K-band magnitude, or $\log n$.

The largest homogeneous sample of galaxies with published $M_{bh}$ and $n$ values is that in GD07. One of the strengths of the $M_{bh}$-$n$ relation is that photometrically uncalibrated images can be used. While this means that the GD07 galaxies do not have magnitudes, they do have bulge $R_e$ values and bulge-to-total ratios that can, when needed, be applied to the galaxy $M_K$ values in MH03. The Sèrsic indices from GD07 pertain to the major axis. It is perhaps worth noting that from very early on, it was known that the major and minor axes need not and do not have the same Sèrsic profile shape (Caon et al. 1993). In the presence of ellipticity gradients, the Sèrsic index will vary with position angle (Ferrari et al. 2004), and the value obtained from a symmetrical two-dimensional fit with a single Sèrsic index will match neither the major- nor minor-axis value. Moreover, the random viewing angles at which spheroids are viewed will also introduce scatter to the $M_{bh}$-$n$ relation (and the $M_{bh}$-$\sigma$ relation if the bulges are triaxial).

The Sérsic indices and SMBH masses for the 27 galaxies tabulated in GD07 are used here, along with the (geometric mean) effective radii (Table 1), K-band magnitudes from MH03, and the central velocity dispersions from Ferrarese & Ford (2005, hereafter FF05). NGC 6251 and NGC 7052 had a different distance in GD07 and MH03, and have had their $R_e$ and $M_K$ values adjusted to match the distances in GD07. Although MH03 did not include or model NGC 1399 (Houghton et al. 2006), a velocity dispersion $\sigma = 344$ km s$^{-1}$ (HyperLeda$^3$) has been adopted, along with the Sèrsic index, $R_e$ value, and $B$-band magnitude from D’Onofrio et al. (1994), adjusted to a distance of 20 Mpc (Tonry et al. 2001), and using $b/a = 0.94$ (NED) and $B-K = 4.14$ (Buzzoni 2005). The bulge parameters from Graham (2002) are used for NGC 221, along with a Johnson $R - K$ color of 2.34 (Buzzoni 2005).

This leaves two galaxies (NGC 2778 and NGC 4564) which have $R_e$ and $M_K$ values pertaining to the galaxy rather than the bulge in MH03. From the analysis in GD07, $R_{e, maj}$ equals 0.25 and 0.31 kpc, and a $B/T$ ratio of 0.21 and 0.24 has been adopted for each galaxy, respectively.

For these 27 galaxies, one obtains an $M_{bh}$-$\sigma$ relation similar to that reported in Tremaine et al. (2002); it is such that

$$\log(M_{bh}/M_\odot) = (8.09 \pm 0.07) + (4.08 \pm 0.40) \log(\sigma/200 \text{ km s}^{-1}),$$

where $\Delta$, the total rms scatter in the $M_{bh}$ direction, equals 0.31 dex. The $M_{bh}$-$\sigma$-$L$ plane for these galaxies is given by

$$\log(M_{bh}/M_\odot) = (8.13 \pm 0.07) - (0.11 \pm 0.05)(M_K + 24) + (3.34 \pm 0.48) \log(\sigma/200 \text{ km s}^{-1}),$$

$\Delta = 0.27$ dex.

See http://leda.univ-lyon1.fr.
The $M_{bh}\sigma-R_e$ plane is
\[
\log(M_{bh}/M_\odot) = (8.15 \pm 0.06) + (0.28 \pm 0.12) \log(R_e/3 \text{ kpc}) \\
+ (3.65 \pm 0.32) \log(\sigma/200 \text{ km s}^{-1}), \\
\Delta = 0.26 \text{ dex},
\] (7)

while the $M_{bh}\sigma-n$ plane is
\[
\log(M_{bh}/M_\odot) = (7.98 \pm 0.05) + (1.11 \pm 0.32) \log(n/3) \\
+ (2.72 \pm 0.52) \log(\sigma/200 \text{ km s}^{-1}), \\
\Delta = 0.23 \text{ dex}.
\] (8)

Given that the best performer is the $M_{bh}\sigma-n$ plane, the residuals about the $M_{bh}\sigma$ relation are plotted in Figure 1 versus the bulge Sérsic index. From Figure 1a, it is clear that a trend will still persist after the exclusion of the five galaxies whose SMBH sphere of influence is not resolved (according to Table II from FF05). On the other hand, Figure 1b reveals that the trend is caused by (some of) the disk galaxies. This intriguing aspect is explored further in the following section.

4. THE BARLESS $M_{bh}\sigma$ RELATION

4.1. Graham & Driver Data

As can been seen in Figure 1, much of the trend is due to some five data points from the small bulges of disk galaxies. While these five bulges have small ($R_e < 1 \text{ kpc}$) effective radii, some of the other disk galaxies have comparable radii but do not deviate from the $M_{bh}\sigma$ relation. These five systems have SMBH masses $\sim 0.5 \text{ dex}$ below the best-fitting $M_{bh}\sigma$ relation. Intriguingly, all of these five disk galaxies have been identified in the literature as containing bars. They are the Milky Way (e.g., López-Corredoira et al. 2007, their Fig. 5), NGC 1023 (Debattista et al. 2002, their Fig. 4b; Sil’chenko 1999), NGC 2778 (weak bar peaks at $5^\circ$), Rest et al. 2001, their Fig. 8; Trujillo et al. 2004), NGC 2787 (Erwin et al. 2003, their Fig. 1), and NGC 3384 (Busarello et al. 1996, their Fig. 7; Cappellari & Emsellem 2004; Erwin 2004; Meusinger & Ismail 2007).

In sharp contrast to this, only one of the other eight disk galaxies (NGC 4258; van Albada 1980) is classified in NED as having a bar; therefore, if any of the other seven disk galaxies do possess a bar, it must be weak. If the probability of a disk galaxy having a bar is equal to the probability of not having a bar, then the distribution in Figure 1 has a 1 in 1024 chance of occurring. If 75% of disk galaxies have bars (e.g., Eskridge et al. 2000; see also Knapen et al. 2000 and Marinova & Jogee 2007), then the observed distribution has less than a 1 in 10,000 likelihood of occurring by chance.

In Figure 2, the barred galaxies can be seen to be largely responsible for the reduced scatter when going from the $M_{bh}\sigma$ relation to the $M_{bh}\sigma-R_e$, $M_{bh}\sigma-L$, and $M_{bh}\sigma-n$ planes. In other words, these galaxies deviate from the $M_{bh}\sigma$ relation. It is therefore of interest to rederive the $M_{bh}\sigma$ relation excluding those galaxies with bars. For the 21 nonbarred galaxies in GD07, the barred galaxies have an offset in the log $M_{bh}$ direction of $0.5-0.8 \text{ dex}$.

In passing, it is noted that a prolate bulge, even in the absence of a bar, will have a smaller effective radius when viewed along its major axis (e.g., Lanzoni & Ciotti 2003). A more detailed investigation of the above galaxies could therefore include how the measured size, magnitude, concentration of the spheroidal component, and velocity dispersion change with the orientation of the bulge and bar.

Although Figure 2 might appear to hint that the barred galaxies have smaller effective radii than the nonbarred disk galaxies, a Kolmogorov-Smirnov test reveals no significance (even at the 1 $\sigma$ level) that the cumulative distribution function for the barred and unbarred disk galaxies' effective radii may be different. Similarly, Student's $t$-test reveals no significant difference between the means of each distribution, with only an 86% ($\lesssim 1.5 \sigma$) probability of difference.

It is pertinent to ask whether the inclusion of an additional parameter to the above barless $M_{bh}\sigma$ relation is warranted. The answer appears to be “no.” Reductions of not more than 0.01 dex are achieved through the addition of either $R_e$, $L$, or $n$. This implies that a fundamental plane for SMBHs is not appropriate; if it were, it would apply equally to galaxies with and without bars.

4.1.1. Musings

A bar may result in the fueling of a SMBH (e.g., Wyse 2004; Ohta et al. 2007), perhaps eventually bringing its mass in line with the $M_{bh}\sigma$ relation. However, the large ratio of barred galaxies to active galaxies in the universe today seems to argue
against this as a common phenomenon, as does the incidence of bars in Seyfert and normal disk galaxies (Mulchaey & Regan 1997; Ho et al. 1997; although see Crenshaw et al. 2003). It may, however, be that some other physical property, such as nuclear disks or kinematically decoupled cores, is influencing the measurements. Bar instabilities are believed to lead to the formation of pseudobulges. Such evolution may have resulted in (pseudo)bulges with an increased velocity dispersion and luminosity but a relatively anemic SMBH (unless it also grew during the formation of the pseudobulge). If the barred galaxies do indeed have discrepantly low SMBH masses rather than high \( \sigma \)-values, they should also appear as systematic outliers in the \( M_{\text{bh}}-L \) diagram. Similarly, if bar dynamics deviate significantly from the (sometimes assumed axisymmetric, rather than triaxial) stellar orbits used to constrain the black hole mass, then this may also explain the apparently offset nature of the barred galaxies. While a larger galaxy sample with accurate bulge magnitudes would be beneficial, the current barred galaxy sample is not systematically offset in the \( M_{\text{bh}}-L \) diagram (Fig. 3), suggesting that the issue may reside with the velocity dispersions rather than the black hole masses.

We must consider whether bar dynamics bias the measurement of bulge velocity dispersions in real galaxies. The noncircular (streaming) motions of stars in bars obviously deviate from those of the random motions of the bulge stars and may potentially interfere with the central velocity dispersion measurements. An alignment of the (radial orbits in the) bar with our line of sight may result in such a scenario. Indeed, the barred galaxy NGC 3384 is highly inclined, and its bar is closely aligned with the projected minor axis (Busarello et al. 1996; Erwin 2004). The Milky Way’s bar is also pointed toward us, with an offset of only \( \sim 20 \) degrees (Gerhard 2002). In the case of NGC 1023, a quick visual inspection reveals disk and bar position angles of 80° and 72°, respectively, suggesting that the inclination of this galaxy does not result in us looking down the length of the bar. However, Debattista et al. (2002) revealed, after deprojecting this galaxy, that it has a strong bar whose position angle is offset by 102° from the galaxy’s (projected) major axis. That is, we are in fact looking down the length of the bar of this galaxy. A full treatment of each of the barred galaxies is beyond the scope of this paper; it is noted, however, that the (projected) bar position angles can appear more aligned with the (projected) major axis than they are in reality, as is the case with NGC 1023.

4.2. Tremaine et al. Data

Removing the seven barred galaxies (the six mentioned above plus NGC 4596) from the Tremaine et al. (2002) sample of 31 galaxies (with the SMBH mass for NGC 821 updated with the value in Richstone et al. 2007), application of Tremaine et al.‘s regression technique gives

\[
\log(M_{\text{bh}}/M_\odot) = (8.21 \pm 0.05) + (3.89 \pm 0.26) \log(\sigma/200 \text{ km s}^{-1}),
\]

with an intrinsic scatter of 0.17 dex (cf. 0.27 dex using the original 31 galaxies) and a total scatter of 0.25 dex (cf. 0.34 dex using the original 31 galaxies). This is in agreement with equation (9), which used a slightly different sample and velocity dispersions from FF05.

Construction of a barless \( M_{\text{bh}}-\sigma-M_B \) plane, with the absolute \( B \)-band magnitudes \( M_B \) taken from Tremaine et al. (2002), has the same scatter as the barless \( M_{\text{bh}}-\sigma \) relation. Furthermore, the intrinsic scatter from equation (10) is smaller than the value of 0.21 dex reported in Hopkins et al. (2007). It seems reasonable to conclude that previous claims for the existence of “fundamental planes for SMBHs” have been influenced by the presence of barred galaxies.

Using the Tremaine et al. (2002) sample and performing a regression that minimizes the residuals in the log \( \sigma \) direction, rather than the log \( M_{\text{bh}} \) direction, the intercept and slope of the barless \( M_{\text{bh}}-\sigma \) relation are 8.21 and 4.05, respectively. A symmetrical regression will therefore have a slope around \( (3.89 + 4.05)/2 = 3.97 \). Using the symmetrical bisector linear regression routine BCES from Akritas & Bershady (1996), one obtains

\[
\log(M_{\text{bh}}/M_\odot) = (8.20 \pm 0.05) + (3.95 \pm 0.25) \log(\sigma/200 \text{ km s}^{-1}).
\]

This expression should be preferred when trying to understand the physical processes responsible for the correlation (see Feigelson & Babu 1992), although this point is perhaps moot given the consistency of equation (11) with equations (9) and (10).

4.3. Marconi & Hunt Data

The analysis in Hopkins et al. (2007) used the 27 “Group 1” galaxies from MH03. In an effort to better understand the result in Hopkins et al. (2007), the original data from MH03 are analyzed here. Given that MH03’s \( M_{\text{bh}} \) and \( \sigma \)-values for the Milky Way and M31 were not in dispute (only their \( K \)-band magnitudes were somewhat in doubt), these two galaxies have been included here with the 27 “Group 1” galaxies.

The residuals, in the log \( M_{\text{bh}} \) direction, about the \( M_{\text{bh}}-\sigma \) relation are shown in Figure 4a for the above 29 galaxies. The trend between the residuals and the effective radii of the host spheroids does indeed appear to suggest the need for a fundamental plane—type relation, akin to that proposed by MH03 and later by Hopkins et al. (2007) and Aller & Richstone (2007) using a sample of 23 galaxies. However, once one identifies the (five) barred galaxies in the above sample, the evidence for such a plane is reduced. The three galaxies with the largest negative residual are barred galaxies.

The three galaxies with the highest positive residuals in Figure 4a (two of which still seem to advocate the need for a “fundamental plane”) are, in order of increasing \( R_e \), the radio galaxy Centaurus A, the Seyfert galaxy NGC 5252 at \( \sim 100 \) Mpc, and Cygnus A at a distance of 240 Mpc. The SMBH mass estimate for Centaurus A that was used by MH03 and used in Figure 4 has since been revised downward, however, by more than a factor of 2 (Marconi et al. 2006), and therefore Cent A is...
now known not to be an outlier. Due to their distances and somewhat disturbed morphology, none of these galaxies were fitted with a Sérsic profile by GD07, nor were they included in Tremaine et al. (2002). While one can conclude that (some) barred galaxies deviate significantly from the $M_{bh}$–$R_e$ relation, the inclusion of NGC 5252 and Cygnus A may present some evidence in favor of a fundamental plane for SMBHs.

If a fundamental plane for black holes does exist, demonstrating its existence with an elliptical-only sample would help eliminate concerns that unrelated processes pertaining to bars and disks are misleading us. Figure 4b shows the residuals about the $M_{bh}$–$R_e$ relation for the 17 elliptical galaxies from the sample of 29. One can immediately see that there is as yet no convincing evidence for an elliptical galaxy SMBH fundamental plane involving $M_{bh}$, $\sigma$, and $R_e$. Given the obvious need for more data, the following section introduces and uses new SMBH data obtained after 2003.

### 4.4. Additional Data

Since MH03, additional galaxies have had their SMBH masses measured. These are provided in Table 2, along with galaxies from MH03 for which some updates have become available, giving a total of 40 galaxies with direct SMBH mass measurements. An additional 15 galaxies with somewhat uncertain SMBH mass estimates (see FF05) are listed in Table 3. Although these are not used here, they are given in order to provide an awareness of further galaxies that may be useful in the future.

Using (1) the (updated) data for the 27 “Group 1” galaxies from MH03 (except for IC 1459 and NGC 4594, whose $M_{bh}$ values are somewhat uncertain), plus (2) MH03’s ten “Group 2” galaxies (minus NGC 1068, NGC 4459, and NGC 4596, for which the SMBH mass estimates are also not secure), plus (3) the nine new galaxies in Table 2 (excluding NGC 2748, for which there is no published velocity dispersion), gives a total sample of (25 + 7 + 8) = 40 galaxies, from which an updated $M_{bh}$–$R_e$ diagram has been constructed (Fig. 5). For these galaxies, the $M_{bh}$–$\sigma$ relation and the $M_{bh}$–$R_e$ plane are given in Table 4, along with the associated total scatter. For the full data set, one obtains an $M_{bh}$–$\sigma$ relation in good agreement with Tremaine et al. (2002). One also has $M_{bh} \propto \sigma^{2.25 \pm 0.28}$, $R_e^{0.43 \pm 0.11}$, which is in agreement with the result in Hopkins et al. (2007).

However, from Figure 5 one can clearly see that many of the barred galaxies deviate from the $M_{bh}$–$R_e$ relation and are obviously responsible for some of the perceived need for a fundamental plane. If these galaxies had $R_e$ values that were smaller than any of the other bulges, then one could argue that the presence of the bar may have nothing to do with their displacement from the $M_{bh}$–$R_e$ relation, and that a “fundamental plane” is needed. That is, if the dynamic range in $R_e$ of the nonbarred galaxies was relatively small compared to the full sample, then this may curtail the emergence of a fundamental plane relation for the nonbarred galaxies. However, this is not the case, as small spheroids from nonbarred galaxies exist which do not deviate from the $M_{bh}$–$R_e$ relation. The only nonbarred galaxy with a notable negative $\delta \log M_{bh}$ residual in Figures 5 and 6 is the LINER galaxy NGC 3998 (de Francesco et al. 2006). As remarked by Fisher (1997), this galaxy has a very steep central velocity dispersion profile, dropping from $\sim 320 \text{ km s}^{-1}$ at $r = 0''$ to $\sim 160 \text{ km s}^{-1}$ at $r = 4''$ (270 pc). A velocity dispersion of 210 (or 250) km s$^{-1}$ for this galaxy would result in a zero (or 1 $\sigma$) residual about the $M_{bh}$–$R_e$ relation.

By removing the 11 barred galaxies from the sample of 40, one obtains the “barless” $M_{bh}$–$R_e$ relation given in Table 4. The vertical residuals about this relation are shown in Figure 6a, along with the offsets of the barred galaxies relative to this $M_{bh}$–$R_e$ relation defined by the nonbarred galaxies. While seven of the ten barred galaxies with known $R_e$ values are responsible for much of the trend between the $M_{bh}$–$\sigma$ residuals and $R_e$, the nonbarred galaxies do still reveal a trend. Indeed, from Table 4, one can see that the nonbarred galaxies favor a fundamental plane relation. However, note that the removal of just two galaxies (NGC 3998 and Cygnus A) from the sample of 29 nonbarred galaxies leaves the coefficient in front of the log ($R_e/3$) term inconsistent with a value of zero at a significance of less than 2 $\sigma$. It is disconcerting that just a couple of points are responsible for the apparent plane. The bulk of the data do not suggest the need for a fundamental plane.

As noted previously, to be certain that a three-parameter fundamental plane is required to describe the connection between SMBHs and their host spheroids, one would ideally like to use a sample of elliptical galaxies. This would ensure that the “plane” is not a by-product of additional physical mechanisms or biases related to the presence of a disk and/or bar. Using the 19 elliptical galaxies from the sample of 40 galaxies, one has

$$\log(M_{bh}/M_{\odot}) = (8.25 \pm 0.05) + (3.68 \pm 0.25) \log(\sigma/200 \text{ km s}^{-1}),$$

with a total scatter of 0.24 dex. Exclusion of the single data point for Cygnus A, the galaxy with the greatest residual offset in Figure 6b, reduces the total scatter to 0.18 dex. This is the same scatter as that about the best-fitting $M_{bh}$–$\sigma$–$R_e$ plane to this set of 18 elliptical galaxies. The elliptical galaxies therefore do not provide substantial support for the existence of an $M_{bh}$–$R_e$ fundamental plane for SMBHs (see Table 4). When using all 19 elliptical galaxies, the 2 $\sigma$ uncertainty on the coefficient in front of the $R_e$ term ranges from $-0.08$ to 0.55. This parameter is inconsistent with a value of zero at only the 1.4 $\sigma$ level. Moreover, removing just one data point (Cygnus A) reduces the coefficient in front of the $R_e$ term to 0.09 $\pm 0.11$.

Given the small sample sizes involved, it may be premature to completely rule out the existence of a fundamental plane for SMBHs. Some may object to the removal of outlying data points,
clipping of distributions is a somewhat common practice these outcomes hinge on outlying data points. Indeed, for similar reasons, a certain degree of caution must be associated with any conclusion sets have been provided. Most, however, would acknowledge that

| Galaxy                  | Type | Distance (Mpc) | $\sigma$ (km s$^{-1}$) | log $R_e$ (kpc) | $M_{bh}$ (10$^6 M_{\odot}$) |
|-------------------------|------|----------------|------------------------|----------------|-----------------------------|
| Cygnus A                | E    | 240            | 270                    | 1.49           | 26.0$^{+7.0}_{-5.0}$ (2)    |
| Cen A                   | S0   | 4.2            | 150                    | 0.56           | 1.1$^{+0.0}_{-0.1}$ (10)    |
| NGC 221                 | S0   | 0.8            | 75                     | $-0.98$ (6)    | 0.025$^{+0.001}_{-0.005}$   |
| NGC 821                 | E    | 24.1           | 200 (5)                | 1.30           | 0.85$^{+0.11}_{-0.06}$ (11) |
| NGC 2778                | S0   | 22.9           | 175                    | $-0.60$ (2)    | 0.14$^{+0.08}_{-0.09}$      |
| NGC 3379                | E    | 10.6           | 206                    | 0.46           | 1.4$^{+2.0}_{-0.6}$ (12)    |
| NGC 4342                | S0   | 17.0 (1)       | 251 (5)                | $-0.36$        | 3.3$^{+1.9}_{0}$            |
| NGC 4374                | E    | 18.4           | 296                    | 0.91           | 4.64$^{+3.46}_{-0.73}$ (13) |
| NGC 4564                | S0   | 15.0           | 162                    | $-0.50$ (2)    | 0.56$^{+0.03}_{-0.08}$      |
| NGC 5252                | S0   | 94.4 (2)       | 190                    | 0.98           | 9.7$^{+4.6}_{0}$            |
| NGC 6251                | E    | 101 (2)        | 290                    | 1.02           | 5.8$^{+1.8}_{0}$            |
| NGC 7052                | E    | 60.0 (2)       | 266                    | 1.00           | 3.4$^{+1.3}_{0}$            |

**Updated Marconi & Hunt (2003) Data**

**New Galaxies with SMBH Measurements**

| Galaxy                  | Type | Distance (Mpc) | $\sigma$ (km s$^{-1}$) | log $R_e$ (kpc) | $M_{bh}$ (10$^6 M_{\odot}$) |
|-------------------------|------|----------------|------------------------|----------------|-----------------------------|
| NGC 1300                | SBBc | 19.3$^{+1.2}_{-1.1}$ | 229 (5)                | $-0.28$ (7) | 0.68$^{+0.65}_{-0.03}$ (14) |
| NGC 1399                | E    | 20.0 (3)       | 317                    | 1.09 (4)       | 12$^{+0.6}_{-0.13}$ (15)    |
| NGC 2748                | Sbe  | 23.8$^{+0.7}_{-0.6}$ | ...                     | ...            | 0.45$^{+0.10}_{-0.07}$ (14) |
| NGC 3227                | SB   | 17.0$^{+0.7}_{-0.6}$ | 160                    | $-0.57$        | 0.15$^{+0.05}_{-0.06}$ (16) |
| NGC 3998                | S0   | 14.1 (3)       | 305                    | $-0.16$        | 2.2$^{+1.7}_{-0.8}$ (17)    |
| NGC 4151                | SBab | 13.9           | 156 (5)                | $-0.23$ (8)    | 0.45$^{+0.05}_{-0.06}$ (18) |
| NGC 4435                | SB0  | 16.0           | 157                    | $-0.07$        | < 0.075 (19)                |
| NGC 4486a               | E    | 17.0 (1)       | 110                    | $-0.39$ (9)    | 0.13$^{+0.10}_{-0.07}$ (20) |
| NGC 7582                | SBab | 22.4           | 156                    | ...            | 0.55$^{+0.20}_{-0.19}$ (21) |

**Note.**—Unless otherwise specified, the distances, velocity dispersions, and effective radii of each galaxy come from the reference that provides the SMBH mass. Both $M_{bh}$ and $R_e$ have been adjusted to the distance given in col. (2).

**References.**—(1) Jerjen et al. 2004; (2) Graham & Driver 2007a; (3) Toney et al. 2001; (4) D’Onofrio et al. 1994; (5) HyperLeda; (6) Graham 2002; (7) Aguerri et al. 2001; (8) Virani et al. 2000; (9) Kormendy et al. 2005; (10) Marconi et al. 2006; (11) Richstone et al. 2007; (12) Shapiro et al. 2006; (13) Maciejewski & Binney 2001; (14) Atkinson et al. 2005; (15) Houghton et al. 2006; (16) Davies et al. 2006; (17) de Francesco et al. 2006; (18) Onken et al. 2007; (19) Coccato et al. 2006; (20) Nowak et al. 2007; (21) Wold et al. 2006.

which is why equations using both complete and adjusted data sets have been provided. Most, however, would acknowledge that a certain degree of caution must be associated with any conclusion that hinges on outlying data points. Indeed, for similar reasons, 3 $\sigma$ clipping of distributions is a somewhat common practice these days. One thing that is clear is that the biasing presence of disk (especially barred) galaxies appears responsible for much of the alleged evidence for requiring a fundamental plane for SMBHs.

One should not use either the barless $M_{bh}$-$\sigma$ relation nor the standard $M_{bh}$-$\sigma$ relation for barred galaxies because the resultant SMBH mass estimates may be in error (too high) by $0.5 \sim 1.0$ dex. One should also not apply an $M_{bh}$-$\sigma$-$R_e$ fundamental plane in the

| Galaxy                  | Reference                  | Note                              |
|-------------------------|----------------------------|-----------------------------------|
| Abell 1836              | Dalla Bontà et al. (2007)  | no refereed publication          |
| Abell 3565              | Dalla Bontà et al. (2007)  | no refereed publication          |
| Cireonus                | Greenhill et al. (2003)    | poorly known disk inclination     |
| IC 1459                 | Cappellari et al. (2002)   | gas/stellar dynamics differ       |
| NGC 1068               | Huré (2002); Lodato & Bertin (2003) | disk model uncertain          |
| NGC 4041               | Marconi et al. (2003)      | disk might be dynamically decoupled |
| NGC 4303               | Pastorini et al. (2007)    | poorly known disk inclination     |
| NGC 4350               | Pignatelli et al. (2001)   | possible SMBH, high $M_{bh}/M_{sdyn}$ |
| NGC 4459               | Sarzi et al. (2001)        | poorly known disk inclination     |
| NGC 4486B              | Kormendy et al. (1997)     | possible SMBH, $M_{bh}/M_{sdyn}$ $\sim 0.09$ |
| NGC 4594               | Kormendy (1988)            | no three-integral model           |
| NGC 4596               | Sarzi et al. (2001)        | poorly known disk inclination     |
| NGC 4945               | Greenhill et al. (1997)    | no 2D velocity field              |
| NGC 5055               | Biax-Ouellette et al. (2004) | possibly no black hole         |
| NGC 7332               | Häring & Rix (2004)        | no refereed publication          |
hope of accounting for barred galaxies, because such a plane will introduce a bias to the nonbarred galaxies.

5. THE \( \sigma-L \) RELATION AND SAMPLE BIAS

There has been some concern recently that the \( M_{\text{bh}}-\sigma \) and/or \( M_{\text{bh}}-L \) relations may be biased, and that they are not consistent with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other. Lauer et al. (2007), Bernardi et al. (2007), and Graham (2007, his Appendix A) have reported a slight difference with each other.

![Fig. 5.—\( M_{\text{bh}}-\sigma \) diagram for 40 galaxies (see § 4.4). The 11 barred galaxies are denoted with a cross.](image)

![Table 4: SMBH Mass-Spheroid Relations](image)

| Sample | Relation | \( \Delta \) (dex) | \( \Delta' \) (dex) |
|--------|----------|-------------------|-------------------|
| 40 galaxies | \( 8.13 \pm 0.06 + (3.92 \pm 0.27) \log (\sigma/200) \) | 0.38 | 0.35 |
| 29 nonbarred | \( 8.26 \pm 0.06 + (3.67 \pm 0.19) \log (\sigma/200) \) | 0.30 | 0.25 |
| 19 elliptical | \( 8.25 \pm 0.05 + (3.68 \pm 0.25) \log (\sigma/200) \) | 0.24 | 0.18 |

**Note:**—The total scatter \( \Delta \) is given rather than the (smaller) internal/intrinsic scatter, as the latter quantity depends on the measurements errors that one assigns. The final column shows the total scatter \( \Delta' \) after removing just two data points (Cygnus A and NGC 3998).

Applying the regression analysis scheme from Tremaine et al. (2002; see also Novak et al. 2006) to minimize the scatter in the log \( \sigma \) direction, the optimal \( \sigma-L \) relation is

\[
\log \sigma = 2.23 \pm 0.03 - (0.092^{+0.018}_{-0.012})(M_R + 21),
\]

which is shown in Figure 7. The parameter uncertainties have been estimated from a Monte Carlo bootstrap analysis. Although MH03 note that the \( M_R \) value for M31 may be in error, excluding it from the regression has no effect on equation (13). However, the extreme outlying point NGC 4342, the smallest and faintest spheroid from MH03’s sample after M32, is excluded from this regression.

The reason for constructing an \( R_e \)-band relation was to allow a comparison with the result from Tundo et al. (2007, their eq. [4]), which is a SDSS \( r' \)-band \( \sigma-L \) relation for early-type SDSS galaxies, the majority of which presumably do not have bars. Using \( r' - R_e = 0.24 \) (Fukugita et al. 1995), Tundo et al.’s (2007) expression is such that \( \log \sigma = 0.27 - 0.092 M_R = 2.20 - 0.092(M_R + 21) \), which is in remarkable agreement with equation (13). Therefore, it is not yet established that the local sample of galaxies with direct SMBH mass measurements is biased.

6. \( M_{\text{bh}}-\sigma \) VERSUS \( M_{\text{bh}}-L \)

Given that the local (predominantly inactive) sample of galaxies with direct SMBH mass measurements appears to be unbiased with respect to the greater population, it is appropriate to...
re-examine whether the (barless) $M_{\text{bh}}$-$\sigma$ and $M_{\text{bh}}$-$L$ relations predict different SMBH masses. Indeed, it has been claimed that these relations are not consistent with each other, in the sense that massive galaxies are predicted to have more massive SMBHs when using the $M_{\text{bh}}$-$L$ relation, with values up to $10^{10}\,M_\odot$ (Lauer et al. 2007).

This discrepancy is investigated here by first looking at the upper extremity of the $M_{\text{bh}}$-$\sigma$ relation. At 400 km s$^{-1}$, it turns out that both the old $M_{\text{bh}}$-$\sigma$ relation, as given by Tremaine et al.'s (2002) regression of $\log M_{\text{bh}}$ on $\log \sigma$, and the new relation (eq. [12]) predict the same black hole mass, $2.3 \pm 1.5 \times 10^9\,M_\odot$. A value of 400 km s$^{-1}$ is used here due to the rapid decline in the number density of systems with higher velocity dispersions (Sheth et al. 2003; Bernardi et al. 2006). This upper black hole mass agrees well with that from the $M_{\text{bh}}$-$\sigma$ relation in GD07, where $M_{\text{bh,upper}} = 1.2^{+0.6}_{-0.4} \times 10^9\,M_\odot$, implying an upper (1 $\sigma$) SMBH mass limit of $\sim 4 \times 10^9\,M_\odot$.

For the (K-band) $M_{\text{bh}}$-$L$ relation, $\log (M_{\text{bh}}/M_\odot) = (8.29 \pm 0.08) - (0.37 \pm 0.04)(M_K + 24)$ (Graham 2007), to predict a more massive black hole than $2.3 \times 10^9\,M_\odot$, requires a spheroid with $M_K < -26.9$ mag. While many galaxies are close to this limit (a determination made using the K-band magnitudes for the 102 brightest cluster galaxies [BCGs] in Stott et al. [2008]), only two are brighter (after the small adjustment of 0.1 mag when switching from $H_0 = 70$ to 73 km s$^{-1}$ Mpc$^{-1}$). Furthermore, from the (corrected) SDSS $r'$-band BCG magnitudes in both Desroches et al. (2007, their Fig. 9) and Liu et al. (2008, their Fig. 13), we see that the brightest magnitudes truncate at $M_K \sim -24.2$ mag. Using $r' - K = 2.8$, this corresponds to a $K$-band magnitude of $-27.0$ mag and an ($M_{\text{bh}}$-$L$)-derived SMBH mass of $2.5 \times 10^9\,M_\odot$. It therefore appears that the $M_{\text{bh}}$-$L$ relation does not predict higher SMBH masses than the $M_{\text{bh}}$-$\sigma$ relation. The near-infrared analysis by Batcheldor et al. (2007) also supports this picture, but see the cautionary note in Lauer et al. (2007, their Appendix B). The $M_{\text{bh}}$-$M_{\text{bh}}$ diagram from Lauer et al. (2007, their Fig. 2) is at odds with the above result. However, note that no non-BCG in Lauer et al. (2007) has a magnitude brighter than NGC 6876 at $M_K = -23.49$ mag ($H_0 = 73$). Assuming a $V - K$ color of 3.22 for elliptical galaxies (Buzzoni 2005), this magnitude corresponds to $M_K = -26.71$ mag, giving a black hole mass of $2.0 \times 10^9\,M_\odot$, which is consistent with the upper bound from the $M_{\text{bh}}$-$\sigma$ relation.

To try and resolve the issue with the BCGs in Lauer et al. (2007), their $M_{\text{bh}}$-$M_{\text{bh}}$ diagram is reproduced here after applying a number of updates. First, the new $M_{\text{bh}}$-$\sigma$ relation (eq. [12]) is applied to the velocity dispersions tabulated in Lauer et al. (2007). Second, the above-mentioned $K$-band $M_{\text{bh}}$-$L$ relation is used (and $V - K = 3.22$ applied). As detailed in Graham (2007), this updated relation benefits from a number of factors, including (1) the identification of lenticular galaxies previously treated as elliptical galaxies, (2) the fact that it was constructed in the near infrared rather than the $B$ or $V$ band, rendering the magnitudes less prone to biases from dust attenuation and young stellar populations, and (3) the performing of a careful Sérsic bulge plus exponential disk decomposition.

Graham (2007) showed that a symmetrical regression of the $M_{\text{bh}}$ masses and $K$-band magnitudes produces a relation with a slope of $-0.40$ (i.e., $M_{\text{bh}} \propto L_\odot^{1.100}$). Given that the $K$-band stellar mass-to-light ratio is roughly constant for elliptical galaxies (Chabrier 2003; Bruzual & Charlot 2003), this roughly corresponds to a constant black hole to stellar mass ratio—not that dissipative processes must a priori produce such a relation. Importantly, however, this linear correlation is consistent with the expectations of dry merging in which the luminosity and black hole mass increase in step. This $K$-band relation is therefore applicable at the high-luminosity end, where large elliptical galaxies are believed to have formed via dry merging.

The dissipationless merging of gas-free elliptical galaxies will evolve them to brighter magnitudes without increasing the stellar population’s metallicity or age, and therefore the stellar mass-to-light ratio will not increase above that of the more massive (and thus more metal-rich and older) progenitor galaxy. Some evidence for this can be seen in the flattening (at bright magnitudes) of the elliptical galaxy color-magnitude relation (e.g., Baldry et al. 2004, their Fig. 9; Ferrarese et al. 2006b, their Fig. 123; see also P. Côté et al. 2008, in preparation) and implies, in agreement with expectations, that the stellar $M/L$ ratio plateaus at a constant value once dry merging among luminous galaxies commences. The $V$-band $M_{\text{bh}}$-$L$ relation used by Lauer et al. (2007) has a slope of $-0.53$ (i.e., $M_{\text{bh}} \propto L_\odot^{1.32}$). Such a steep $M_{\text{bh}}$-$L$ relation, defined in part by galaxies that experienced a dissipational formation scenario, is conceptually at odds with the concept of dry merging and arguably not applicable to luminous elliptical galaxies. If there is a constant stellar $M/L$ ratio for luminous galaxies formed via dry mergers, then this relation implies that $M_{\text{bh}} \propto M_{\text{star}}^{1.32}$, which is at odds with the statement of Lauer et al. (2007) that the ratio of SMBH to stellar mass should be preserved for dry mergers. Use of the new $K$-band $M_{\text{bh}}$-$L$ relation effectively circumvents the issue of how the stellar mass-to-light ratio may change for dry mergers. The results of applying the new $K$-band $M_{\text{bh}}$-$L$ relation and the barless $M_{\text{bh}}$-$\sigma$ relation to predict the SMBH masses for Lauer et al.’s (2007) data are shown in Figure 8a.

While Lauer et al. (2007) correctly used bulge magnitudes for the disk galaxies in their Figure 2, these were obtained from $R^{1/4}$ bulge-plus-exponential disk decompositions. It is well known that because most bulges have a Sérsic (1963) $R^{1/n}$ light profile (Graham & Driver 2005) with $n < 4$ (e.g., Graham 2001; Balcells

6 While this is not ideal because galaxies with bars may have their SMBH masses overestimated, at the high-mass end we do not predict larger SMBH masses than the $M_{\text{bh}}$-$\sigma$ relation from Tremaine et al. (2002) on these same data.

7 The possibility of SMBH ejection from a system will alter this picture (e.g., Guo et al. 2008 and references therein).
et al. 2003; MacArthur et al. 2003), such an approach overestimates the flux (e.g., Brown et al. 2003). To account for this, the bulge-to-disk flux ratios from Graham & Driver (2007b, their Table 2) have been applied.8 This entailed reducing the S0 bulge magnitudes by 2.5 log (0.25/0.60) and the Sa-Sb bulge magnitudes by 2.5 log (0.17/0.33). The results of this are shown in Figure 8b. This resolves the conflict seen in Figure 8a at the low-mass end. While such a correction is valid in a statistical sense, individual galaxy corrections should ideally be applied, and this may well account for the increased scatter about the one-to-one line in Figure 8b.

Although BCGs tend to have \((M_{\text{bh}}-L)\) -derived black hole masses smaller than \(4 \times 10^9 M_\odot\), the Lauer et al. (2007) BCG magnitudes do tend to produce SMBH mass estimates that are roughly twice as large as those predicted from their velocity dispersions. From Liu et al. (2008, their Fig. 5), one can see that the stellar envelope that surrounds (some) BCGs becomes significant (albeit relative to an \(R_{1/4}^d\) model) at \(\mu_r \sim 23\) mag arcsec\(^{-2}\), while Gonzalez et al. (2005, their Fig. 3) indicate a value around \((23.5-25)\mu_r\) mag arcsec\(^{-2}\). These ranges are in agreement with the values seen in Seigar et al. (2007). This halo of stars is very likely due, at least in part, to stars that have been tidally stripped from galaxies within the cluster environment (e.g., Merritt 1985). As such, it pertains more to the cluster than the BCG, and should be excluded from measurements of the BCG luminosity.

To avoid the issue of the outer envelope, which was thought to occur at \(\mu_r \sim 25\) mag arcsec\(^{-2}\), Lauer et al.’s (2007) BCG magnitudes were obtained from \(R_{1/4}^d\) model fits to surface brightness profiles brighter than \(\mu_r = 23.74\) mag arcsec\(^{-2}\) (Graham et al. 1996, using \(r' - R_c = 0.24\)). The danger is that some of the outermost portion of the modeled light profile may have been elevated to a brighter level by the flux of the envelope. As the light profiles did not extend to large radii, the “break” in the profile, where the envelope starts to dominate, may have been missed. If so, such contamination would result in the best-fitting \(R_{1/4}^d\) model having an increased effective radius and a brighter total flux. Therefore, before concluding that a problem exists with the BCGs, it would be prudent to actually perform a galaxy/envelope decomposition (e.g., Seigar et al. 2007) of such systems (not just those in Lauer et al. 2007), enabling one to quantify how the envelope flux might be biasing the magnitudes. This is, however, beyond the scope of this paper. A second issue to bear in mind is that some BCGs are known to have blue cores, which results in a 0.5–1.0 mag offset from the red sequence (Bildfell et al. 2008). The use of near-infrared magnitudes, which minimizes biases from BCG star formation (e.g., O’Dea et al. 2008), is therefore preferable.

It is perhaps worth noting that a few galaxies are known to possess both a SMBH and a nuclear star cluster, leading one to wonder (1) whether the combined masses should be used, and (2) whether the “offset” barred galaxies have significant nuclear star clusters. As noted in GD07, the barred galaxy NGC 3384 has \(M_{\text{SC}}/M_{\text{bh}} \sim 2\). Using \(M_{\text{SC}} + M_{\text{bh}}\) rather than only \(M_{\text{bh}}\) would bring this galaxy back in line with the \(M_{\text{bh}}-\sigma\) relation. However, the barred galaxy NGC 1023 has an offset of nearly \(-0.7\) dex but \(M_{\text{SC}}/M_{\text{bh}} \sim 0.1\), while the barred galaxy NGC 2778 has no nuclear cluster but is offset by \(-0.9\) dex. Furthermore, the unbarred galaxy NGC 7457, which has no offset from the \(M_{\text{bh}}-\sigma\) relation, has \(M_{\text{SC}}/M_{\text{bh}} \sim 10\) (GD07). Nonetheless, a careful quantitative analysis of the nuclear structure of galaxies with SMBHs would be highly desirable; it would additionally enable one to explore the \(M_{\text{bh}}-\)central surface brightness relation proposed by GD07.

At 100 km s\(^{-1}\), the barless \(M_{\text{bh}}-\sigma\) relation yields masses (for unbarred galaxies) that are 67% higher than the expression in Tremaine et al. (2002). Past efforts to measure the SMBH mass function and mass density using the old \(M_{\text{bh}}-\sigma\) relation (see the roundup in Graham & Driver 2007b, their Table 3) may therefore need tweaking. Consequences for the \(M_{\text{SC}}-\sigma\) and \(M_{\text{SC}}-L\) relation involving nuclear star clusters (e.g., Ferrarese et al. 2006a; Balccells et al. 2007) are also deferred for elsewhere.

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![Figure 8](image-url)

Fig. 8. — (a) \(M_{\text{bh}}\) masses for the galaxies tabulated in Lauer et al. (2007) obtained using the new \(M_{\text{bh}}-\sigma\) relation (eq. [12]) and the \(M_{\text{bh}}-L\) relation from Graham (2007, his eq. [14]). (b) \(R_{1/4}^d\) bulge magnitudes of the disk galaxies adjusted as described in § 6. The BCGs and normal galaxies are denoted with open circles and filled circles, respectively.
