Detection of the Inductive Magnetic Field of Submarine with Upper Arc Sensor Array based on Integral Equation Method

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Abstract. In order to effectively solve the problem of the accuracy of the magnetic field measurement of submarine upper vault sensor, an integral equation method based on the magnetic field detection method of submarine upper vault was proposed. By analyzing the simulation data of the dome magnetic sensor, the analysis solution and the integral equation solution of the ellipsoid shell of the ship model are compared, so as to verify the accuracy of the integral equation method and provide a reference for the magnetic field measurement accuracy of the submarine upper part.

1. Introduction
In recent years, with the emergence of "quiet" submarine, its noise suppression has greatly reduced the detection range of passive and active acoustic detection equipment. Moreover, acoustic detection is less reliable and accurate than magnetic detection in identifying, locating and tracking submarines, and the role of magnetic detection is becoming more and more important [1]. At present, the magnetic stealth index of submarine in service only assists the vertical component of fixed magnetic field under water, which is far from meeting the need of three-dimensional magnetic protection under water and in air. In order to deal with the threat of airborne magnetic exploration, we need to bring the magnetic field above the submarine and high altitude into the assessment range, which puts forward new requirements for the future demagnetization service of submarine. However, the existing submarine demagnetization station has arc-shaped dome above it, and its magnetic sensor laying is not strictly symmetrical, which requires higher magnetic field precision. Moreover, as the magnetic sensor above the vault is not easy to adjust and maintain, it is necessary to check the measured data of the sensor to ensure the accuracy of the data so as to provide support for the later calculation of high-altitude magnetic field.

The magnetic field generated in the surrounding space by objects with regular shape, such as long and straight cylinders, spheres and ellipsoids, can be calculated with analytic solutions, while for objects with very complex shapes, such as ship hulls, there is no ready-made analytic formula for the calculation of their magnetic fields [2]. It is a new subject to solve the magnetic field problem of complex objects by numerical calculation in recent years. The generation of mesh discrete model is a very important step in the modeling and calculation of ship induced magnetic field based on integral equation method [3]. Nevertheless, in order to achieve a certain calculation accuracy, the number of subdivision units of ships often reaches more than a million, which makes the consumption of computer memory and CPU time very large, so such a huge numerical solution system may be difficult to be used in the data processing of ship magnetic field measurement. With the development of computer technology, parameterized modeling and batch processing provided by TrueGrid[4] enable users to build discrete model of different ship entity grid with a little modification of some defined geometric parameters, without the need to
repeat tedious and boring modeling work, thus saving a lot of time. The application of TrueGrid has
greatly promoted the development of numerical modeling technology of ship magnetic field based on
integral equation method. It is possible to use TrueGrid software to generate discrete model of ship
model grid and to use integral equation method for simulation calculation.

2. Theoretical model of integral equation method

In general, the source region is usually divided into many small blocks, and the material parameters and
magnetization in each block can be considered as a constant. In this way, a source region with
continuously changing parameters is replaced by a number of discrete small block units with different
material parameters [5]. An example of three-dimensional field subdivision diagram, with n subdivision
units.

In the boundary value problem of a constant magnetic field, a scalar magnetic potential can be used
$\phi$ to describe the magnetic field strength $H$, if it contains only magnetized material and no conduction
current, when $J = 0$, $\nabla \times H = 0$. At this point, the magnetic field intensity generated by the magnetized material is

$$H_m = -\frac{1}{4\pi} \int \left[ \nabla \cdot \left( \frac{1}{p - p'} \right) \right] dV = -\frac{1}{4\pi} \int \left[ \nabla \cdot \left( \frac{\vec{p} - \vec{p}'}{|p - p'|^3} \right) \right] dV$$

(2.1)

For a three-dimensional field, let $\vec{M} = M_x \vec{j} + M_y \vec{j} + m \vec{k}$, then

$$H_m = -\frac{1}{4\pi} \int \left[ \frac{x - x'}{|x - x'|^3} M_x + \frac{y - y'}{|y - y'|^3} M_y + \frac{z - z'}{|z - z'|^3} M_z \right] dV$$

(2.2)

Among them, those with ' are the source point coordinates, and those without' are the field point
coordinates.

If there is a field point in space, then the magnetic field strength $H$ at point $P(x, y, z)$ can be
considered to be caused by two field sources:

(1) The field intensity (including geomagnetic field) generated by the space current at Point $P(x, y, z)$;

(2) Field intensity generated by magnetization of magnetic material at Point $P(x, y, z)$;

Then $H = H', + H_m$, as long as the spatial current distribution is known, it can be obtained $H'$. However, due to the current magnetic material magnetization is not clear, that is $M_x \cdot M_y \cdot M_z$ are
unknown, so $H_m$ is unknown. Therefore, any field point's $H'$ cannot be obtained directly.

By introducing the coupling coefficient $C_{xx}$, $C_{xy}$, $C_{xz}$, $C_{yx}$, $C_{yy}$, $C_{yz}$, $C_{zx}$, $C_{zy}$, $C_{zz}$ then, the theory
of integral equation method can be written, because it can be known that $\vec{M} = \mathcal{J} \hat{H}$ and $H_{mx}$, $H_{my}$, $H_{mz}$ can be
written as

$$H_{mx} = C_{xx} \mathcal{J} \hat{H}_x + C_{xy} \mathcal{J} \hat{H}_y + C_{xz} \mathcal{J} \hat{H}_z$$

(2.3)

$$H_{my} = C_{yx} \mathcal{J} \hat{H}_x + C_{yy} \mathcal{J} \hat{H}_y + C_{yz} \mathcal{J} \hat{H}_z$$

(2.4)

$$H_{mz} = C_{zx} \mathcal{J} \hat{H}_x + C_{zy} \mathcal{J} \hat{H}_y + C_{zz} \mathcal{J} \hat{H}_z$$

(2.5)

Therefore, if A is the field point and B is the source point, the magnetic field generated by the field
point at the source point can be written as

$$H_{ax} = C_{ax,by} \mathcal{J} \hat{H}_x + C_{ax,by} \mathcal{J} \hat{H}_y + C_{ax,hc} \mathcal{J} \hat{H}_z$$

(2.6)

$$H_{ay} = C_{ay,by} \mathcal{J} \hat{H}_x + C_{ay,by} \mathcal{J} \hat{H}_y + C_{ay,hc} \mathcal{J} \hat{H}_z$$

(2.7)

$$H_{az} = C_{ax,by} \mathcal{J} \hat{H}_x + C_{ax,by} \mathcal{J} \hat{H}_y + C_{ax,hc} \mathcal{J} \hat{H}_z$$

(2.8)

The component expressions of contrast $H_{mx}$, $H_{my}$, $H_{mz}$, and can be obtained

$$C_{ax,by} = \frac{1}{4\pi} \int \frac{3(x - x_b)^2}{|x - x_b|^5} dV dx dy dz$$

(2.9)
\[ C_{av,by} = \frac{1}{4\pi} \iint_{v} \frac{3(x_a - x_b)(y_a - y_b)}{r_a^3 - r_b^3} dxdydz \quad (2.10) \]

\[ C_{av,ba} = \frac{1}{4\pi} \iint_{v} \frac{3(z_a - z_b)}{r_a^3 - r_b^3} dxdydz \quad (2.11) \]

\[ C_{av,ab} = \frac{1}{4\pi} \iint_{v} \frac{3(y_a - y_b)}{r_a^3 - r_b^3} dxdydz \quad (2.12) \]

\[ C_{av,by} = \frac{1}{4\pi} \iint_{v} \frac{3(y_a - y_b)^2}{r_a^3 - r_b^3} dxdydz \quad (2.13) \]

\[ C_{av,bz} = \frac{1}{4\pi} \iint_{v} \frac{3(z_a - z_b^2)}{r_a^3 - r_b^3} dxdydz \quad (2.14) \]

\[ C_{av,ba} = \frac{1}{4\pi} \iint_{v} \frac{3(z_a - z_b)}{r_a^3 - r_b^3} dxdydz \quad (2.15) \]

\[ C_{av,ab} = \frac{1}{4\pi} \iint_{v} \frac{3(y_a - y_b)}{r_a^3 - r_b^3} dxdydz \quad (2.16) \]

\[ C_{av,bz} = \frac{1}{4\pi} \iint_{v} \frac{3(z_a - z_b^2)}{r_a^3 - r_b^3} dxdydz \quad (2.17) \]

According to \( H = \bar{H} + \bar{H} \), then have

\[ C_{j,1x,1x}H_{1x} + C_{j,1y,1y}H_{1y} + C_{j,1z,1z}H_{1z} + \ldots + C_{j,ix,ix}H_{ix} + C_{j,iy,iy}H_{iy} + C_{j,iz,iz}H_{iz} = H_{jx} \quad (2.18) \]

\[ C_{j,1x,1x}H_{1x} + C_{j,1y,1y}H_{1y} + C_{j,1z,1z}H_{1z} + \ldots + C_{j,jx,jx}H_{jx} + C_{j,jy,jy}H_{jy} + C_{j,jz,jz}H_{jz} = H_{jx} \quad (2.19) \]

\[ C_{j,1x,1x}H_{1x} + C_{j,1y,1y}H_{1y} + C_{j,1z,1z}H_{1z} + \ldots + C_{j,jx,jx}H_{jx} + C_{j,jy,jy}H_{jy} + C_{j,jz,jz}H_{jz} = H_{jx} \quad (2.20) \]

Among them \( j=1,2 \ldots n \), transfer the term to arrange

\[ C_{j,1x,1x}H_{1x} + C_{j,1y,1y}H_{1y} + C_{j,1z,1z}H_{1z} + \ldots + (C_{j,jx,jx} - 1)H_{jx} + C_{j,jy,jy}H_{jy} + C_{j,jz,jz}H_{jz} = H_{jx} \quad (2.21) \]

\[ C_{j,jx,jx}H_{jx} + C_{j,jy,jy}H_{jy} + C_{j,jz,jz}H_{jz} = H_{jx} \quad (2.22) \]

\[ C_{j,jx,jx}H_{jx} + C_{j,jy,jy}H_{jy} + C_{j,jz,jz}H_{jz} = H_{jx} \quad (2.23) \]

Written it into matrix form \( AH = B \), where
Where $H$ is the required magnetic field strength at the center of gravity of each subdivision element, and $C$ is the coupling coefficient of the integral equation method.

Assume that the initial susceptibility of each subdivision unit in the source region is respectively $\chi_1$, $\chi_2$, ..., $\chi_n$, the magnetic field at the center of each subdivision unit can be obtained $H_x$, $H_y$, $H_z$ by solving the equations, and then the magnetization curve of the magnetic material can be checked. The actual susceptibility of the material under the magnetization state can be obtained $\chi_1$, $\chi_2$, ..., $\chi_n$. If $\max|\chi_i^{(p)} - \chi_i^{(p+1)}| < \varepsilon, i = 1, 2, ..., n$, then the assumed susceptibility is considered to be consistent with the reality. Otherwise, use $\chi_i^{(p+1)}$ instead of $\chi_i^{(p)}$, solve the system again until $\max|\chi_i^{(p)} - \chi_i^{(p+1)}| < \varepsilon$ [6].

### 3. Simulation analysis of integral equation method

According to the theoretical analysis of the integral equation method above, the ship's three-component induced magnetic field can be obtained by calculating the coupling coefficient between each subdivision element and analyzing the magnetization curve after the vessel is subdivided into grids. Next, MATLAB is used to carry out simulation analysis on the calculation results of integral equation method.

The ship model of the simple submarine is equivalent to the ellipsoid shell, the long half axis of the ellipsoid shell is 2 meters, the short half axis is 1 meters, the shell thickness is 0.1 meters, the susceptibility is 150. The ship model is meshed and subdivided to obtain each subdivision unit (Figure 1). Since the dome magnetic probe is distributed at five different heights (Figure 2), the ellipsoid shell and integral equation method are used to simulate the induced magnetic field at the sensor's position respectively.

![Figure 1 Submarine model grid discrete model based on TrueGrid](image-url)
According to the scheme of ship model magnetic field measurement, the number of longitudinal measurement points of ship model magnetic field is 40, and the number of transverse measurement points is 5. When the longitudinal measurement range is -3.9m ~ 3.9m, the spacing is 0.2m, the longitudinal measurement range is -2m ~ 2m, the spacing is 1m, and the measurement height is the elevation of the sensor on the vault(4.2m, 5.1m, 5.8m, 4.5m, 2.8m). Under the condition that the external magnetic field is only added with horizontal component, the three components of the induced magnetic field of the ship model are obtained. By comparing the integral equation method with the analytic solution of ellipsoid shell, the errors of each component are obtained. Because space is limited, only the Z component is selected here for comparison (Figure 3). The integral equation method above the ship keel is selected to calculate the solution and the analytic solution of the ellipsoid shell (Figure 4). The Z component comparison is drawn as follows.
Figure 4: Comparison of Z components of the magnetic field on the keel.

By comparing the calculated solution of the integral equation method with the analytic solution of the ellipsoid shell, it can be seen that the calculated solution and the analytic solution have a high consistency. The relative root-mean-square errors of the three components are 0.81%, 1.04% and 2.15% respectively, which have good engineering practicability.

4. Conclusion
Based on the actual degaussing station, this paper compares the errors of the solution calculated by the integral equation method and the analytic solution of the ellipsoid shell by meshing the ship model. The simulation results show that the method of integral equation is very accurate in calculating the three components of the induced magnetic field of a ship model. It is of great practical value to detect the measuring accuracy of the magnetic sensor of the vault and provides a reference for the measuring accuracy of the magnetic field above the submarine.

Acknowledgments
This work was supported in part by the National Natural Science Foundation of China under Grant (51377165).

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