Ensuring the accuracy of modal control in systems with multicomponent actions

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Abstract. The problem of ensuring the accuracy for constant and harmonic input actions in systems with modal control is considered. An analytical method of combined formation of both the control accuracy and the type of transient process is proposed. The modal control possibilities for harmonic signals are shown, for which the synthesis method is usually applied to the desired frequency response. It also introduces a requirement of quality in terms of speed. This problem is being solved by drawing up an inequality system. These inequalities are illustrated graphically. Using a numerical example, the effectiveness of the proposed method for solving the problem is shown.

1. Introduction
The problem of meeting the requirements for the accuracy indicators of modal control systems has now become an independent problem due to the fact that the modal controller itself does not have the means to form these indicators [1-3].

In this regard, the main method for constructing high-precision systems with modal control is the introduction of a setting action into the observer's system in the form of an internal model using, in particular, the derivatives of the input signal [4-8].

However, independent synthesis of the modal controller and the internal model of the input signal often leads to undesirable changes in the transient regimes of the system. In particular, the oscillation can occur in the system while controlling in response to the step actions [9-10].

The introduction of a scaling coefficient is another method of ensuring that a given error value is achieved. It is worth paying attention to the fact that both of these methods are unable to provide the required static value for different types of effects: for example, constant and harmonic actions and also for harmonic signals with different frequencies, etc.

The proposed method, in which the required quality parameters are achieved by introducing additional output feedback and the free parameters required for the physical implementation of the specified interoperability condition, without the use of observation devices, allows the above disadvantages to be eliminated.

2. Method
The problem is to synthesize a modal control system with a given nature of the transient process and accuracy indicators determined by the location of the eigenvalues of the characteristic matrix; astaticism of the first order with respect to the setting action with the quality factor in speed $D$, maximum permissible relative errors $\delta_1$ and $\delta_2$ of reproduction of the amplitude of harmonic actions at frequencies $\omega_{1.g}$ and $\omega_{2.g}$. To solve the problem, a system with the structure shown in Fig. 1 is formed.
where \( B \) is the characteristic matrix, \( N, A \) are the control and output matrices, \( R \) is the controller, \( x \) is the vector of state coordinates, \( u \) is the control, \( y \) is the output coordinate, \( d \) and \( k \) are additional parameters, \( g \) is the setting action.

For the presented structure the equations of motion in Laplace transforms look like this:

\[
y(s) = A \cdot \left( s \cdot E - B + N \cdot d \cdot R + N \cdot d \cdot A \cdot \frac{k}{s} \right)^{-1} \cdot N \cdot d \cdot \frac{k}{s} \cdot g(s)
\]  

(1)

The characteristic equation for equation (1) is obtained in the following form:

\[
s^2 + s^{-1} \cdot \left( a_1 + d \cdot r_1 \right) + \cdots + s \cdot \left( a_n + d \cdot r_n \right) + k \cdot d = 0
\]  

(2)

The roots of the equation (2) depend on the variables \( r_1, \ldots, r_n, k, d \):

\[
s_1 \left( r_1, \ldots, r_n, k, d \right), \ldots, s_n \left( r_1, \ldots, r_n, k, d \right).
\]

Let us compose inequalities that reflect the accuracy requirements for various input actions and the nature of the transient process. Let us find \( n \) inequalities based on the regulation time requirements. It is useful to position some of the roots \( v \) at a sufficient distance from the dominant poles so that they are not affected by regulation time.

As the regulation time depending on the found roots can be much less than required, which will negatively affect the actuators, we will use a two-sided inequality:

\[
\begin{align*}
1.2 \cdot (-\Omega) & < s_1 \left( r_1, \ldots, r_n, k, d \right) < -\Omega; \\
1.2 \cdot (-\Omega) & < s_{n-1} \left( r_1, \ldots, r_n, k, d \right) < -\Omega; \\
& \vdots \\
 s_{n-\nu} \left( r_1, \ldots, r_n, k, d \right) & < q \cdot (-\Omega); \\
s_n \left( r_1, \ldots, r_n, k, d \right) & < q \cdot (-\Omega),
\end{align*}
\]  

(3)

where \( \Omega \) is the desired geometric mean root determined by the given regulation time, \( n \) is the order of the system, \( q \) is measure of root remoteness, \( \nu \) is the number of non-dominant roots.

Graphical interpretation of the expression (3) is shown in Fig. 2.
To reproduce the setting action at the frequency $\omega_{z,g}$ with a given value of the static error $\delta_z$, we will introduce the inequalities:

$$\delta_z > 1 - W(\omega_{z,g}),$$

where $z$ is the sequence number of the action; $W(\omega_{z,g})$ is the modulus of the frequency function $W(j\omega)$ at the frequency $\omega_{z,g}$.

To the sought system of inequalities, we will add the requirement for the speed quality factor, the expression for which will be obtained from equation (2):

$$D_v = \frac{k \cdot d}{d \cdot \eta + 2}.$$  

The control points equivalent to inequalities (4) and (5) in the construction of the desired frequency response are shown in Fig. 3.

**Fig. 3. Equivalent control points that reflect accuracy requirements**

where $L_o(\delta_1)$ and $L_o(\delta_2)$ are gains, that provide setpoint values for static errors $\delta_1$ and $\delta_2$ at frequencies $\omega_{1,g}$ and $\omega_{2,g}$.

Combining equations (3), (4) and (5), we will obtain a system of inequalities, the solution of which will give us the desired values that provide the required quality indicators of the synthesized control system:

$$\begin{align*}
1.2 \cdot (-\Omega) &< s_1 (r_1, \ldots, r_n, k, d) \cdot s_r < -\Omega; \\
1.2 \cdot (-\Omega) &< s_{n+1} (r_1, \ldots, r_n, k, d) \cdot s_r < -\Omega; \\
\vdots &< \quad s_{n+1} (r_1, \ldots, r_n, k, d) \cdot s_r < q \cdot (-\Omega); \\
s_n (r_1, \ldots, r_n, k, d) &< q \cdot (-\Omega); \\
1 - W(\omega_{z,g}) &< \delta_z \\
D_v &< D_r,
\end{align*}$$

where $D_v$ – required quality value.

Thus, the solution to the problem of ensuring accuracy in terms of harmonic and monotonous setting influences with the simultaneous fulfillment of the requirements for the nature of the transient process and the regulation time was reduced to the fulfillment of a system of restrictions. In this case, it is allowed to obtain a set of acceptable solutions from which the subsequent choice can be made.

3. Example

Let us consider the following object:

$$B = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad N = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0 \end{bmatrix}. \quad (6)$$
For \( g(t) = \text{const} \) it is necessary to provide the first-order astaticism with a speed quality factor \( D_v \geq 30 \text{ s}^{-1} \); a harmonic signal \( g(t) = 0.5\sin (3t) \) needs to be processed with a static error \( \delta_1 = 0.01 \); \( g(t) = 0.5\sin (30t) \) needs to be processed with a static error \( \delta_2 = 0.1 \). For the control time \( t_r = 0.05 \text{ s} \), the geometric mean root \( \Omega \) is defined as:

\[
\Omega = \frac{T}{t_r} = \frac{4.8}{0.05} = 96,
\]

where \( T \) is a constant depending on the order of the desired polynomial.

Taking into account (3), (4) and (5) the system of inequalities for \( z = 1,2 \) and \( q=5 \) will take the following form:

\[
\begin{align*}
-115.2 &\leq s_1 \leq -96; \\
-115.2 &\leq s_2 \leq -96; \\
 s_3 &\leq -480; \\
 D_v &\geq 30; \\
 \delta_1 &\geq 1-W(\omega_{h,2}) \\
 \delta_2 &\geq 1-W(\omega_{d,2}).
\end{align*}
\]

4. Results
The solution of expression (8) is

\[
R = [650.722 \quad 3.167]; \quad k = 34559; \quad d = 836.129.
\]

The desired and actual values of the quality of the control system are shown in Table 1.

| Table 1. Comparison of quality indicators |
|-------------------------------------------|
| Value | Quality indicator |
| t_r, s | \( \delta_1 \) | \( \delta_2 \) | \( D_v, \text{ s}^{-1} \) |
| desired value | 0.05 | 0.01 | 0.1 | 30 |
| actual value | 0.044 | 0.001 | 0.07 | 53 |

The requirements for the quality indicators of the synthesized system are met, which confirms the effectiveness of the proposed method. The presence of astaticism is also illustrated in Fig. 4.

![Fig. 4. Transient processes at various setting actions](image-url)
5. Conclusion
This paper proposes an analytical method of ensuring the accuracy of modal control systems, using a structure with additional output feedback. At the same time, accuracy targets are provided for constant target effects, speed effects, and for harmonic signals.

A distinctive feature of the method is consistency i.e. compatibility of the requirements for the accuracy of regulation and the nature of the transient process. The specified compatibility is achieved by a general solution of a system of algebraic inequalities reflecting the listed requirements.

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