Reexamine the nuclear chiral geometry from the orientation of the angular momentum

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Abstract

The paradox on the previous interpretation for the nuclear chiral geometry based on the effective angle has been clarified by reexamining the system with the particle-hole configuration $\pi (1h_{11/2})^{1}\otimes \nu (1h_{11/2})^{-1}$ and rotor with deformation parameter $\gamma = 30^\circ$. It is found that the paradox is caused by the fact that the angular momentum of the rotor is much smaller than those of the proton and the neutron near the bandhead. Hence, it does not support a chiral rotation interpretation near the bandhead. The nuclear chiral geometry based on the effective angle makes sense only when the angular momentum of the rotor becomes comparable with those of the proton and the neutron at the certain spin region.

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Chirality is a topic of general interest in the sciences, such as chemistry, biology, and physics. An object or a system is chiral if it is not identical to its mirror image, and cannot be superposed on its mirror image through any combination of rotations and translations.

The phenomenon of chirality in nuclear physics was initially introduced by Frauendorf and Meng in 1997 [1] for a fast rotating nucleus with triaxially deformed shape and high-$j$ valence particle(s) and valence hole(s). In that circumstances, the collective angular momentum is favor of aligning along the nuclear intermediate axis that provides the largest moment of inertia, while the angular momentum vectors of the valence particles (holes) align along the short (long) axis. Such arrangement makes the three angular momenta perpendicular to each other and form either a left- or a right-handed system. Reversing the direction of the component of the angular momentum on one of principal axes changes the chirality of the system. This phenomenon appears in the body-fixed reference frame where the spontaneous breaking of the chiral symmetry happens. In the laboratory reference frame, however, due to the quantum tunneling of the total angular momentum between the left- and right-handed system, the broken chiral symmetry is restored. Then, the chiral doublet bands, i.e., a pair of nearly degenerate $\Delta I = 1$ bands with the same parity, are expected to be observed [1].

After the pioneering work on the chirality in nuclei [1], the chiral symmetry in atomic nuclei has become one of the most intriguing phenomena that has attracted significant attentions and intensive studies from both experimental and theoretical sides in the last two decades. On the experimental side, the chiral doublet bands were first observed in four $N = 75$ isotones in 2001 [2]. So far, more than forty pairs of chiral doublet bands candidates have been reported in the $A \sim 80, 100, 130, \text{ and } 190$ mass regions. For recent reviews, see Refs. [3–9]. With the prediction [10] and confirmation [11] of the multiple chiral doublets ($M\chi D$) in a single nucleus, the investigation of the chirality continue to be one of the hottest topic in nuclear physics [12–25].

As demonstrated in Refs. [1, 26], the chirality of nuclear rotation results from not only the static (the triaxial shape) but also the dynamics (the angular momentum) properties of the nucleus. This is quite different from the chirality in chemistry, which is of static nature that characterizes just the geometrical arrangement of the atoms. Hence, it is of importance to examine the angular momentum geometry in order to verify whether the pair of nearly degenerated doublet bands are chiral doublet bands or not. To achieve this goal, one can investigate: (1) the angular momentum components of the rotor, the particle(s),
and the hole(s) along the three principal axes (e.g., in Refs. [1, 16, 17, 24, 27–33]); (2) the distributions of the angular momentum components on the three intrinsic axes (K plot) (e.g., in Refs. [16, 24, 29–31, 34, 35]); (3) the effective angles between the angular momenta of the rotor, the particle(s), and the hole(s) (e.g., in Refs. [36–38]); (4) the orientation parameter of the system (e.g., in Refs. [23, 36, 37]); (5) the distributions of the tilted angles of the angular momentum in the intrinsic frame (azimuthal plot) (e.g., in Ref. [35]); etc.

It is known now that chiral rotation (or static chirality) can exist only above a certain critical frequency [23, 27, 28, 39]. Namely, at low spin the chiral vibrations, understood as the oscillation of the total angular momentum between the left- and the right-handed configurations in the body-fixed frame, exists. This suggests that the orientation of the angular momenta of the rotor, the particle(s), and the hole(s) are planar at the bandhead of the chiral bands. However, it is noted that the effective angles between any two of the three angular momenta are closed to 90° as shown for the yrast band of 126Cs [38] (see also Fig. 1). This is the paradox which motivates us to reexamine the angular momentum geometries of the rotor, the particle(s), and the hole(s) in the chiral doublet bands.

Theoretically, various approaches have been developed extensively to investigate the chiral doublet bands. For example, the particle rotor model (PRM) [1, 29, 30, 34, 40], the titled axis cranking (TAC) model [27, 28, 39, 41], the TAC plus random-phase approximation (RPA) [42], the collective Hamiltonian method [43, 44], the interacting boson-fermion-fermion model [45], and the angular momentum projection (AMP) method [35, 46–48]. In this work, the PRM will be used. The basic microscopic inputs for PRM can be obtained from the constrained covariant density functional theory (CDFT) [10, 11, 19, 21, 22, 33, 49]. PRM is a quantal model consisting of the collective rotation and the intrinsic single-particle motions, which describes a system in the laboratory reference frame. The total Hamiltonian is diagonalized with total angular momentum as a good quantum number. The energy splitting and quantum tunneling between the doublet bands can be obtained directly. Hence, it is straightforward to be used to investigate the angular momentum geometries of the chiral doublet bands.

The detailed formalism of PRM can be found in Refs. [1, 29, 30, 34, 40]. In the calculations, a system of one $h_{11/2}$ proton particle and one $h_{11/2}$ neutron hole coupled to a triaxial rigid rotor with quadruple deformation parameters $\beta = 0.23$ and $\gamma = 30.0°$ are taken as the example to illustrate the angular momentum geometry. In addition, the irrotational flow
type of moments of inertia $J_k = J_0 \sin^2(\gamma - 2k\pi/3)$ ($k = 1, 2, 3$) with $J_0 = 30 \hbar^2$/MeV are used.

The effective angle $\theta_{pn}$ between the proton ($\mathbf{j}_p$) and neutron ($\mathbf{j}_n$) angular momenta is defined as [37],

$$\cos \theta_{pn} = \frac{\langle \mathbf{j}_p \cdot \mathbf{j}_n \rangle}{\sqrt{\langle \mathbf{j}_p^2 \rangle \langle \mathbf{j}_n^2 \rangle}},$$

(1)

and similarly for $\theta_{Rp}$, $\theta_{Rn}$, $\theta_{Ip}$, $\theta_{In}$, and $\theta_{IR}$. Here, the subscripts $p$, $n$, $R$, and $I$ denote the proton, the neutron, the rotor, and the total spin, respectively, and $|\rangle$ is the wave function of the yrast or yrare bands. In geometry, any three vectors lie in a planar only when the sum of any two angles between the vectors equals the other one or the sum of the three angles equals $360^\circ$.

In Fig. 1, the obtained effective angles $\theta_{pn}$, $\theta_{Rp}$, $\theta_{Rn}$, $\theta_{Ip}$, $\theta_{In}$, and $\theta_{IR}$ as functions of spin for the yrast and yrare bands are presented. The dashed-dotted lines at $I = 8$, 13, and 15-17ℏ label the bandhead, the onset of aplanar rotation, and the static chirality, respectively, which are based on the Figs. 2 and 3 showing later.

From Fig. 1(a), it is observed that the effective angles $\theta_{pn}$, $\theta_{Rp}$, and $\theta_{Rn}$ are about $120^\circ$ around $I = 0\hbar$, i.e., the angular momenta $\mathbf{j}_p$, $\mathbf{j}_n$, and $\mathbf{R}$ have to cancel out to obtain the total spin zero. The sum of the three effective angles equals to $\sim 360^\circ$, i.e., the three angular momenta indeed lie in a plane.

The three effective angles gradually decrease with spin and drop to $\sim 90^\circ$ at the bandhead ($I = 8\hbar$), which leads to the conclusion that the angular momenta $\mathbf{j}_p$, $\mathbf{j}_n$, and $\mathbf{R}$ are nearly mutually perpendicular to each other in Ref. [38]. This is the paradox with respect to the understanding of chiral vibration near the bandhead.

At the static chiral region ($15 \leq I \leq 17\hbar$), the three effective angles of the doublet bands are rather similar. Note that the values of these three effective angles are about $70^\circ$, a bit far from $90^\circ$. It seems that the aplanar rotation at this spin region is less than that near the bandhead. This is also contradiction with our empirical understanding for the static chirality and need to be solved.

The obvious odd-even staggering behaviors of $\theta_{pn}$ at $I \geq 20\hbar$ and of $\theta_{Rp/Rn}$ at $I \geq 21\hbar$ indicate a strong signature splitting of a principle axis rotation.

For the effective angles with respect to the total spin, $\theta_{Ip/In}$ are smaller than $90^\circ$ at the whole spin region, which implies that $\mathbf{j}_p$ and $\mathbf{j}_n$ align toward the total spin. At $I \geq 13\hbar$, they
FIG. 1: The effective angles $\theta_{pn}$, $\theta_{Rp}$, $\theta_{Rn}$, $\theta_{Ip}$, $\theta_{In}$, and $\theta_{IR}$ as functions of spin for the yrast and yrare bands.

do not vary much. For $\theta_{IR}$, it is larger than 90° for the yrast band below the bandhead. This means that the $R$ anti-aligns along the total spin. The decreasing of $\theta_{IR}$ indicates that the role of the rotor becomes more and more essential. Meanwhile, the differences of $\theta_{Ip/In}/\theta_{IR}$ between the doublet bands become smaller with spin. At $I = 15-17\hbar$, they are almost the same. At the high spin region ($I > 20\hbar$), $\theta_{Ip/In}/\theta_{IR}$ show small staggering behaviors.

To solve this paradox, we first reexamine the energy spectra of the chiral doublet bands in Fig. 2(a). Similar results has already been presented in Refs. [1, 16, 24, 31], but here lower spin (from $0\hbar$) ones will be focused. At $I \leq 8\hbar$, the energies of the doublets decrease with spin, since the collective rotations have not yet started. In the shown figures, the dashed-dotted line at $I = 8\hbar$ is plotted to label this bandhead position. At the intermediate spin region (around $I = 15-17\hbar$), near energy degeneracies of doublets are found. To show this more clearly, the energy difference between the doublet bands $\Delta E(I) = E_{yrare}(I) - E_{yrast}(I)$ is shown in Fig. 2(c). One sees that it decreases first and then increases. At $I = 15-17\hbar$,
it is the smallest, corresponding to the best degeneracy and static chirality (marked by a shadow). At high spin region \( I \geq 18\hbar \), it shows an odd-even staggering behavior, caused by the signature splitting of the principal axis rotation.

![Energy spectra and rotational frequencies](image)

**FIG. 2:** (a) Energy spectra as functions of spin for the yrast and yrare bands. (b) The extracted rotational frequencies as functions of spin. (c) Energy difference between the doublets. (d) The normalized orientation parameter calculated by Eq. (2).

From the energy spectra, the rotational frequencies \( \hbar \omega(I) = E(I) - E(I - 1) \) are extracted \([50]\) and shown in Fig. 2(b). It is seen that the \( \hbar \omega \) increases with spin.

Below the bandhead \( I < 8\hbar \), \( \hbar \omega \) is negative. This indicates the angular momentum of the rotor anti-aligns along the spin, which is consistent with the results of \( \theta_{IR} \) shown in Fig. 1(b).

At the bandhead, \( \hbar \omega \) is near zero. The collective rotation is just starting and rather small.

At \( I = 13\hbar \), a kink appears. As discussed in Ref. [1], this is the evidence of the onset of the aplanar rotation. A dashed-dotted line is plotted to label this position. Note that the spin region \( 8 \leq I < 13\hbar \) from the bandhead to the kink are usually called as chiral vibration region, which in fact is a planar rotation \([1, 23, 27, 28]\).
At $I = 15-17\hbar$, the spin region of the best degeneracy of the doublets, the $\hbar\omega$ of the doublet bands are very similar. This gives a hint that the angular momentum geometries of the doublets are similar.

To examine the angular momentum coupling modes of the system, the normalized orientation parameter $o$ is calculated \cite{36, 37}:

$$o = \frac{\langle L | \mathbf{R} \cdot (\mathbf{j}_p \times \mathbf{j}_n) | L \rangle}{\sqrt{\langle L | j_p^2 | L \rangle}\sqrt{\langle L | j_n^2 | L \rangle}\sqrt{\langle L | R^2 | L \rangle}}, \quad (2)$$

with $\langle L | \mathbf{R} \cdot (\mathbf{j}_p \times \mathbf{j}_n) | L \rangle = |\langle + | \mathbf{R} \cdot (\mathbf{j}_p \times \mathbf{j}_n) | - \rangle |$ and $\langle L | j^2 | L \rangle = \frac{1}{2}[\langle + | j^2 | + \rangle + \langle - | j^2 | - \rangle]$ ($j$ denotes $\mathbf{j}_p$, $\mathbf{j}_n$, and $\mathbf{R}$). Here, $|+\rangle$ and $|-\rangle$ denote the wave functions of yrast and yrare bands, and $|L\rangle$ the wave function of left-handed state in the intrinsic frame. In classical mechanics, the normalized orientation parameter would vary between $o = 1$ for mutually perpendicular vectors and $o = 0$ for planar vectors \cite{36, 37}.

The result of the normalized orientation parameter was given for the static chirality \cite{36, 37} or for the bandhead \cite{23}. Here we present it for the whole spin region in Fig. 2(d).

At $I = 0$, $o = 0$. This indicates a planar angular momentum geometry and is consistent with the result that the effective angles $\theta_{pn}$, $\theta_{Rp}$, and $\theta_{Rn}$ are about $120^\circ$ (cf. Fig. 1(a)).

With the increase of spin, $o$ first increases and then decreases, corresponding to the appearance and disappearance of the aplanar rotation. It shows strong correlation with the energy difference $\Delta E$ of the doublet bands (cf. Fig. 2(c)). At $I = 15-17\hbar$, $o$ reaches to the maximal value, corresponding to the smallest $\Delta E$ and the static chirality. It is also noted that the maximal value of $o$ is not 1. This is consistent with the result that the effective angles $\theta_{Rp}$, $\theta_{Rn}$, and $\theta_{pn}$ are not $90^\circ$ at this spin region as shown in Fig. 1(a). Hence, one concludes that the angular momenta of the rotor, the proton particle, and the neutron hole are not ideally mutually perpendicular to each other at the static chiral region. Nevertheless, the aplanar angular momentum geometry at the static chiral region is better than that near the bandhead.

The angular momentum geometry can also be illustrated by its profile on the ($\theta, \varphi$) plane $\mathcal{P}(\theta, \varphi)$, i.e., the azimuthal plot \cite{35}. Here, ($\theta, \varphi$) are the tilted angles of the angular momentum with respect to the intrinsic reference frame. In the calculations, we choose 1, 2, and 3 axes as short ($s$), long ($l$), and intermediate ($i$) axes, respectively. Thus, $\theta$ is the angle between the angular momentum and the $i$-axis, and $\varphi$ is the angle between the projection of the angular momentum on the $s$-$l$ plane and the $s$-axis.
In Fig. 3, the obtained profiles $P(\theta, \varphi)$ are shown at $I = 8, 13, 15,$ and $20\hbar$ for the doublet bands. It is observed that the maxima of the $P(\theta, \varphi)$ always locate at $\varphi = 45^\circ$ for all cases, since the angular momentum has the same distributions along the $s$- and $l$- axes for the current symmetric particle-hole configuration with triaxial deformation $\gamma = 30^\circ$. In addition, the $P(\theta, \varphi)$ is symmetric with respect to the $\theta = 90^\circ$ line. This is expected since the broken chiral symmetry in the intrinsic reference frame has been fully restored in the PRM wave functions. Hence, in the following, only the value of the $\theta (\leq 90^\circ)$ is given when mentioning the position of the maximal $P(\theta, \varphi)$.

![Fig. 3: The azimuthal plots, i.e., profiles for the orientation of the angular momentum on the $(\theta, \varphi)$ plane, calculated at $I = 8, 13, 15,$ and $20\hbar$, for the yrast and yrare bands.](image)

For the bandhead $I = 8\hbar$, the angular momentum for yrast band mainly orientates at $\theta = 90^\circ$, namely, a planar rotation within the $s$-$l$ plane. The angular momentum for yrare band orientates at $\theta \sim 60^\circ$, in accordance with the interpretation of chiral vibration along the $\theta$ direction (i.e., with respect to the $s$-$l$ plane). For $I = 13\hbar$, the angular momenta orientate at $\theta \sim 70^\circ$ for yrast band and $\theta \sim 50^\circ$ for yrare band. Starting from this spin, the rotational mode of the yrast band changes from planar to aplanar rotation. This is consistent with the appearance of kink in the rotational frequency plot shown in Fig. 2(b). For $I = 15\hbar$, the $P(\theta, \varphi)$ of the yrast and yrare bands are rather similar, which demonstrates the occurrence
of static chirality. The angular momenta orientate at \( \theta \sim 45^\circ \) for both bands. For \( I = 20\hbar \), the static chirality disappears. The angular momentum for yrast band orientates to \( \theta \sim 20^\circ \), while that for yrare band to \( \theta \sim 30^\circ \). The small values of \( \theta \) correspond to the fact that the angular momentum has large component along the \( i \)-axis.

Therefore, from the investigations of the azimuthal plots in Fig. 3, we confirm that the rotational mode at bandhead is indeed a planar rotation. Then, how to understand the results that the effective angles \( \theta_{pn}, \theta_{Rp}, \) and \( \theta_{Rn} \) are about 90°? We turn to investigate the vector lengths of the angular momenta.

The angular momenta of the rotor, the proton particle, and the neutron hole are coupled to obtain the total spin as \( I = R + J \) with \( J = j_p + j_n \). As a consequence, \( I^2 \) can be decomposed as

\[
I^2 = R^2 + (j_p^2 + j_n^2) + 2R \cdot J + 2j_p \cdot j_n. \tag{3}
\]

The ratios \( \langle R^2 \rangle/\langle I^2 \rangle \), \( \langle j_p^2 + j_n^2 \rangle/\langle I^2 \rangle \), \( \langle 2R \cdot J \rangle/\langle I^2 \rangle \), and \( \langle 2j_p \cdot j_n \rangle/\langle I^2 \rangle \) (labeled as \( R_{R^2}, R_{j^2}, R_{R\cdot J}, R_{j_p \cdot j_n} \), respectively) as functions of spin for the doublet bands are calculated and shown in Fig. 4(a). Obviously, the sum of these four ratios are equal to 1.

From Fig. 4(a), it is seen that \( R_{j^2} \) decreases in a hyperbola-like behavior, since \( \langle j_p^2 + j_n^2 \rangle = j_p(j_p+1) + j_n(j_n+1) \) is a constant in the single-\( j \) shell model, while \( \langle I^2 \rangle = I(I+1) \) increases in term of \( I^2 \). For the others, the \( R_{R\cdot J} \) increases gradually, the \( R_{j_p \cdot j_n} \) first increases and then keeps nearly constant above the bandhead, and the \( R_{R^2} \) first decreases and then increases.

In detail, both the \( R_{R\cdot J} \) and the \( R_{j_p \cdot j_n} \) give negative contributions below the bandhead. At the bandhead, the \( R_{j_p \cdot j_n} \) is zero, and above the bandhead, its contribution to the total spin is rather small. For the \( R_{j^2} \), its contribution is much larger than 1 below the bandhead. At the chiral vibration region (\( 8 \leq I < 13\hbar \)), it still has a major contribution (\( \geq 40\% \)) to the total spin. At the static chiral region, its contribution is similar as those of \( R_{R^2} \) and \( R_{R\cdot J} \). However, beyond this region, it becomes much smaller than \( R_{R^2} \) and \( R_{R\cdot J} \). This is because the angular momentum of the rotor plays more and more essential roles than those of particle and hole as the spin increases. At the bandhead, the angular momentum of the rotor is rather small in comparison with those of particle and hole. As a result, although it is perpendicular to \( j_p \) and \( j_n \), it does not indicate a aplanar rotation and good chirality. Bear this in mind, the total angular momentum for the yrast band still lies in the \( s-l \) plane (cf. Fig. 3).
FIG. 4: (a) Ratios $\langle R_2^2 \rangle / \langle I^2 \rangle$, $\langle j_p^2 + j_n^2 \rangle / \langle I^2 \rangle$, $\langle 2R \cdot J \rangle / \langle I^2 \rangle$, and $\langle 2j_p \cdot j_n \rangle / \langle I^2 \rangle$ (labeled as $R_{R2}$, $R_j^2$, $R_{R \cdot J}$, $R_{j_p \cdot j_n}$, respectively) as functions of spin for the yrast and yrare bands. (b) Angular momentum vector projection along the total spin $I = \sqrt{\langle I^2 \rangle}$ of the rotor $R_I = \langle R \cdot I \rangle / \sqrt{\langle I^2 \rangle}$, the particles $J_I = \langle J \cdot I \rangle / \sqrt{\langle I^2 \rangle}$ and $j_I = \langle j_p \cdot I \rangle / \sqrt{\langle I^2 \rangle} = \langle j_n \cdot I \rangle / \sqrt{\langle I^2 \rangle}$ as functions of spin for the yrast and yrare bands.

It is also noted that the $R_{R2}$ of the doublet bands are quite different at the chiral vibration region. This is attributed to that the angular momentum of the rotor lies mainly in the s-$l$ plane for the yrast band, while deviates from this plane for the yrare band in the chiral vibration region. Such differences cause the energies of the doublet bands are different as shown in Fig. 2(c). It also provides additional information that the static chirality is not realized yet.

From the above analysis, one concludes that the total spin below the static chiral region ($I < 15\hbar$) mainly comes from the proton and the neutron, in the static chiral region ($15 \leq I \leq 17\hbar$) also from the rotor, and beyond the static chiral region ($I > 17\hbar$) mainly from the rotor. The paradox is caused by the fact that the angular momentum of the rotor is much
smaller than those of proton and neutron near the bandhead. To show this more clearly, the projections of the rotor $R_I$ and the particles $J_I$ and $j_I$ along the total spin are calculated:

$$R_I = \frac{\langle \mathbf{R} \cdot \mathbf{I} \rangle}{\sqrt{\langle I^2 \rangle}}, \quad (4)$$

$$J_I = \frac{\langle \mathbf{J} \cdot \mathbf{I} \rangle}{\sqrt{\langle I^2 \rangle}}, \quad (5)$$

$$j_I = \frac{\langle j_p \cdot \mathbf{I} \rangle}{\sqrt{\langle I^2 \rangle}} = \frac{\langle j_n \cdot \mathbf{I} \rangle}{\sqrt{\langle I^2 \rangle}}. \quad (6)$$

Note that here $J_I = 2j_I$ and $R_I + J_I = \sqrt{\langle I^2 \rangle} = \sqrt{I(I + 1)}$. The obtained results are given in Fig. 4(b).

With the increase of spin, $R_I$ increases gradually. The $J_I$ as well as the $j_I$ increase slightly below the kink ($I \leq 13\hbar$), and keep nearly constant in the above ($I > 13\hbar$). Below the bandhead, $R_I$ contributes negatively as it anti-aligns along the total spin. At the bandhead, it is very small. Then it becomes gradually comparable with $j_I$, but is still smaller than $J_I$. At static chiral region, $R_I \approx J_I$. This is consistent with the result that the value of the $\theta$ for the maximal $\mathcal{P}(\theta, \varphi)$ is about $45^\circ$ (cf. Fig. 3). In this case, the energy difference between the doublet bands is the smallest. Beyond the static chiral region, $R_I$ becomes larger than $J_I$ and responsible for the increase of total spin, which results in a principal axis rotation along the $i$-axis. Therefore, with the increase of spin, the angular momentum of the rotor plays gradually more and more important roles than those of proton particle and neutron hole.

In summary, the paradox on the previous interpretation for the nuclear chiral geometry based on the effective angle has been clarified by reexamining the system with the particle-hole configuration $\pi(1h_{11/2})^1 \otimes \nu(1h_{11/2})^{-1}$ and rotor with deformation parameter $\gamma = 30^\circ$. According to the studies of normalized orientation parameter of the system and the azimuthal plot of the total angular momentum, we confirm that chiral rotation does indeed exist only at a certain high spin region. Further study for the angular momentum shows that the paradox is caused by the fact that the angular momentum of the rotor is much smaller than those of the proton and the neutron near the bandhead. Hence, it does not support a chiral rotation interpretation near the bandhead. The nuclear chiral geometry based on the effective angle makes sense only when the angular momentum of the rotor becomes comparable with those of the proton and the neutron at the certain spin region.

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