1. Editor’s note

This issue contains, in addition to the usual contents, a special festive announcement: A book. This book by Banakh and Zdomsky is announced in §2.3 below, and seems to be the first in a planned series by these authors. We believe that the book will become a cornerstone in many future mathematical investigations, in particular in the field of infinite-combinatorial topology. The book’s preliminary version is available online, as seen in the announcement, and the readers of the SPM Bulletin are encouraged to take a look and make comments.

Zdomsky has also made two detailed contributions to this issue. This is the ideal form of a contribution to the SPM Bulletin, and we urge all contributors to consider this possibility from time to time.

Finally, the BEST 2005 Conference is getting close – have a look at the corresponding announcement.

Contributions to the next issue are, as always, welcome.

Boaz Tsaban, tsaban@math.huji.ac.il
http://www.cs.biu.ac.il/~tsaban
2. Research announcements

2.1. On subclasses of weak Asplund spaces. Assuming the consistency of the existence of a measurable cardinal, it is consistent to have two Banach spaces, $X, Y$, where $X$ is a weak Asplund space such that $X^*$ (in the weak* topology) is not in Stegall’s class, whereas $Y^*$ is in Stegall’s class but is not weak* fragmentable.

http://www.ams.org/journal-getitem?pii=S0002-9939-04-07744-5

Ondrej F. K. Kalenda and Kenneth Kunen

2.2. The number of translates of a closed nowhere dense set required to cover a Polish group. For a Polish group $G$ let $\text{cov}_G$ be the minimal number of translates of a fixed closed nowhere dense subset of $G$ required to cover $G$. For many locally compact $G$ this cardinal is known to be consistently larger than $\text{cov}(\text{meager})$ which is the smallest cardinality of a covering of the real line by meagre sets. It is shown that for several non-locally compact groups $\text{cov}_G = \text{cov}(\text{meager})$. For example the equality holds for the group of permutations of the integers, the additive group of a separable Banach space with an unconditional basis and the group of homeomorphisms of various compact spaces.

http://arxiv.org/abs/math.LO/0409110

Arnold W. Miller and Juris Steprans

2.3. More on convexity numbers of closed sets in $\mathbb{R}^n$. The convexity number of a set $S \subseteq \mathbb{R}^n$ is the least size of a family $\mathcal{F}$ of convex sets with $\bigcup \mathcal{F} = S$. $S$ is countably convex if its convexity number is countable. Otherwise $S$ is uncountably convex.

Uncountably convex closed sets in $\mathbb{R}^n$ have been studied recently by Geschke, Kubiś, Kojman and Schipperus. Their line of research is continued in the present article. We show that for all $n \geq 2$, it is consistent that there is an uncountably convex closed set $S \subseteq \mathbb{R}^{n+1}$ whose convexity number is strictly smaller than all convexity numbers of uncountably convex subsets of $\mathbb{R}^n$.

Moreover, we construct a closed set $S \subseteq \mathbb{R}^3$ whose convexity number is $2^{\aleph_0}$ and that has no uncountable $k$-clique for any $k > 1$. Here $C \subseteq S$ is a $k$-clique if the convex hull of no $k$-element subset of $C$ is included in $S$. Our example shows that the main result of the above-named authors, a closed set $S \subseteq \mathbb{R}^2$ either has a perfect 3-clique or the convexity number of $S$ is $< 2^{\aleph_0}$ in some forcing extension of the universe, cannot be extended to higher dimensions.

http://www.ams.org/journal-getitem?pii=S0002-9939-04-07685-3

Stefan Geschke

2.4. A new book: Coherence of Semifilters. The book is devoted to studying the (sub)coherence relation on semifilters, that is families of infinite subsets of $\mathbb{N}$, closed under taking almost supersets. On the family of ultrafilters the coherence relation was introduced in eighties by Andreas Blass, who formulated his famous principle, the Near Coherence of Filters (NCF), that found many non-trivial applications in various fields of mathematics.

In the book the (sub)coherence relation is treated with help of cardinal functions defined on the lattice $\text{SF}$ of semifilters. Endowed with the Lawson topology the lattice
SF becomes a supercompact topological space. It can be interesting for topological algebraists because any reasonable binary operation on natural numbers induces a right-topological operation on SF in the same way as it does on the Stone-Cech compactification $\beta\mathbb{N}$.

A preliminary version of the book is available online:

http://www.franko.lviv.ua/faculty/mechmat/Departments/Topology/booksite.html

Taras Banakh and Lubomyr Zdomsky

3. Characterization of topological spaces with (strictly) $o$-bounded free topological group

This announcement is devoted to the problem of characterization of Tychonov spaces $X$ such that the corresponding free (abelian) topological group $F(X)$ ($A(X)$) is [strictly] $o$-bounded posed in [3], see [3] or [7] for corresponding definitions. All topological spaces are assumed to be Tychonov.

The characterization we present here involves the concepts of selection principles and the Menger game on a uniform space. A uniform space $(X, U)$ is said to have the property $\bigcup_{\text{fin}} (O, X)$, where $X \in \{O, \Omega, \Gamma\}$, if for every sequence $(u_n)_{n \in \omega}$ of open uniform covers of $X$ there exists a sequence $(v_n)_{n \in \omega}$ such that $\{\cup v_n : n \in \omega\} \in X$ and each $v_n$ is a finite subfamily of $u_n$. Next, the Menger game on a uniform space $(X, U)$ is obtained from the Menger game on the corresponding topological space (see [5] for its definition) by restriction of the choice of the first player to open uniform covers of $X$. For a topological (uniform) space $X ((X, U))$ we shall simply write $II \uparrow M(X)$ ($II \uparrow M(X, U)$) in place of “the second player has a winning strategy in the Menger game on $X ((X, U))$.

Let $X$ be a Tychonov space. Recall from [2] that the uniformity $U$ on $X$ is called universal, if it generates the topology of $X$ and contains all uniformities on $X$ with this property. Throughout this section the universal uniformity of a topological space $X$ will be denoted by $U(X)$. We are in a position now to present the above mentioned characterizations.

**Theorem 3.1.** Let $X$ be a Tychonov space. Then the following conditions are equivalent:

1. $F(X)$ is (strictly) $o$-bounded;
2. $F(X)^n$ is (strictly) $o$-bounded for all $n \in \mathbb{N}$;
3. $A(X)$ is (strictly) $o$-bounded;
4. $A(X)^n$ is (strictly) $o$-bounded for all $n \in \mathbb{N}$;
5. $(X, U(X))$ has the property $\bigcup_{\text{fin}} (O, O)$ ($II \uparrow M(X, U(X))$).

In addition, the property $\bigcup_{\text{fin}} (O, \Gamma)$ is a counterpart of itself.

**Theorem 3.2.** Let $X$ be a Tychonov space and $U$ be a natural uniformity on $A(X)$. Then the following conditions are equivalent:

1. $(F(X), U^*)$ has the property $\bigcup_{\text{fin}} (O, \Gamma)$;
2. $(F(X)^n, U^*)$ has the property $\bigcup_{\text{fin}} (O, \Gamma)$ for all $n \in \mathbb{N}$;
(3) \((A(X), U')\) has the property \(\bigcup_{\text{fin}} (\mathcal{O}, \Gamma)\);
(4) \((A(X)^n, U')\) has the property \(\bigcup_{\text{fin}} (\mathcal{O}, \Gamma)\) for all \(n \in \mathbb{N}\);
(5) \((X, U(X))\) has the property \(\bigcup_{\text{fin}} (\mathcal{O}, \Gamma)\).

The following statement i crucial.

**Lemma 3.3.** Let \(X\) be a Lindelöf space. Then \(X\) has the property \(\bigcup_{\text{fin}} (\mathcal{O}, \mathcal{X})\) if and only if so is the uniform space \((X, U(X))\), where \(\mathcal{X} \in \{\mathcal{O}, \Omega, \Gamma\}\).

In addition, the second player has a winning strategy in the Menger game on \(X\) if and only if he has a winning strategy in this game on \((X, U(X))\).

Combining Lemma 3.3 with Theorems 3.1 and 3.2, we obtain the following important corollary

**Corollary 3.4.**
1. The properties \(\bigcup_{\text{fin}} (\mathcal{O}, \mathcal{X})\) as well as the property \(II \uparrow M(X)\) are \(M\)-invariant within the class of Lindelöf topological spaces, where \(\mathcal{X} \in \{\Omega, \Gamma\}\).
2. If \(X\) has one of the above properties and \(X^n\) is Lindelöf for all \(n \in \omega\), then so is every \(A\)-equivalent to \(X\) space \(Y\).
3. If \(X\) is hereditarily Lindelöf and has one of the above properties, then so is every \(M\)-equivalent to \(X\) space \(Y\).

We refer the reader to [8] for corresponding definitions.

As it was shown in [9], it is consistent with ZFC that each topological space with the property \(\bigcup_{\text{fin}} (\mathcal{O}, \mathcal{O})\) is \(\bigcup_{\text{fin}} (\mathcal{O}, \Omega)\). Therefore, Corollary 3.4 implies that it is consistent that the property \(\bigcup_{\text{fin}} (\mathcal{O}, \mathcal{O})\) is \(M\)-invariant within the class of Lindelöf spaces.

**Problem 3.5.** Is it consistent that the property \(\bigcup_{\text{fin}} (\mathcal{O}, \mathcal{O})\) is not \(M\)-invariant within the class of Lindelöf spaces?

Lubmoyr Zdomsky

4. **An equivalent of SPM Bulletin 2’s Problem of the month**

A family \(\mathcal{F}\) of infinite subsets of a countable set \(C\) is said to be a semifilter, if it is closed under taking supersets and finite modifications of its elements.

The powerset \(\mathcal{P}(C)\) admits a natural structure of a compact space, and thus we can speak about topological properties of its subsets such as semifilters. In [9] the following characterization of the Hurewicz property was proven.

**Theorem 4.1.** A Lindelöf topological space \(X\) has the Hurewicz property if and only if for every countable large cover \(u\) of \(X\) the smallest semifilter \(\mathcal{F}\) on \(u\) containing the family \(\{\{U \in u : x \in U\} : x \in X\}\) is Hurewicz.

This characterization permits us to prove the subsequent reformulation of the problem whether the Hurewicz property implies the property \(S_{\text{fin}} (\Gamma, \Omega)\).

**Theorem 4.2.** The following conditions are equivalent:
1. The Hurewicz property implies \(S_{\text{fin}} (\Gamma, \Omega)\);
(2) For every Hurewicz semifilter $F$ on $\omega \times \omega$ such that $\{n\} \times \omega \subset^* F$ for every $F \in F$ there exists a sequence $(K_n)_{n \in \omega}$ of finite subsets of $\omega$ such that each element of the smallest filter containing $F$ meets $\bigcup_{n \in \omega} \{n\} \times K_n$.

Lubmoyr Zdomsky

5. Boise Extravaganza In Set Theory (March 25–27, 2005)

We are pleased to announce our fourteenth annual BEST conference. There will be 4 talks by invited speakers: Mirna Dzamonja (University of East Anglia, England); Greg Hjorth (UCLA); Paul Larson (Miami University); William Mitchell (University of Florida).

The talks will be held on Friday, Saturday and Sunday in the Department of Mathematics at Boise State University. BEST social events are planned as well.

The conference webpage at

http://math.boisestate.edu/ best/

contains the most current information including lodging, abstract submission, maps, schedule, etc. Anyone interested in giving a talk and/or participating should contact one of the organizers (Justin Moore and Bernhard Koenig) as soon as possible. You can reach both of us by writing to best@math.boisestate.edu

The conference is supported by a grant from the National Science Foundation, whose assistance is gratefully acknowledged.

Justin Moore

6. Problem of the Issue

Recall that $\leq^*$ is defined on $\mathbb{N} \mathbb{N}$ by: $f \leq^* g$ if $f(n) \leq g(n)$ for all but finitely many $n$. Let $b$ (the unbounding number) denote the minimal cardinality of an unbounded subset of $\mathbb{N} \mathbb{N}$ (with respect to $\leq^*$).

**Problem 6.1.** Is it provable that there is a set of reals of cardinality $b$ which satisfies $S_1(\Gamma, \Gamma)$.

The closest results we know of are the following. The answer is positive if we replace $b$ by $t$ (Scheepers [6]),\(^1\) or if we replace $S_1(\Gamma, \Gamma)$ by “$U_{\text{fin}}(\Gamma, \Gamma)+\text{no perfect subsets}”$ (Bartoszyński-Tsaban [1]). It is still open whether “$U_{\text{fin}}(\Gamma, \Gamma)+\text{no perfect subsets}”$ implies (and is therefore equivalent to) $S_1(\Gamma, \Gamma)$ [4].

7. Problems from earlier issues

In this section we list the still open problems among the past problems posed in the SPM Bulletin (in the section **Problem of the month**). For definitions, motivation and related results, consult the corresponding issue.

For conciseness, we make the convention that all spaces in question are zero-dimentional, separable metrizble spaces.

**Issue 1.** Is $\left(\frac{t}{\Gamma}\right) = \left(\frac{t}{\Gamma}\right)$?

**Issue 2.** Is $U_{\text{fin}}(\Gamma, \Omega) = S_{\text{fin}}(\Gamma, \Omega)$? And if not, does $U_{\text{fin}}(\Gamma, \Gamma)$ imply $S_{\text{fin}}(\Gamma, \Omega)$?

\(^1\)It is the tower number, see Issue 5 of the SPM Bulletin. $t \leq b$, but it is consistent that $t < b$. 

Issue 4. Does $S_1(\Omega, T)$ imply $U_{\text{fin}}(\Gamma, \Gamma)$?

Issue 5. Is $p = p^*$? (See the definition of $p^*$ in that issue.)

Issue 6. Does there exist (in ZFC) an uncountable set satisfying $S_1(\mathcal{B}_\Gamma, \mathcal{B})$?

Issue 8. Does $X \not\in \text{NON}(\mathcal{M})$ and $Y \not\in \text{D}$ imply that $X \cup Y \not\in \text{COF}(\mathcal{M})$?

Issue 9. Is $\text{Split}(\Lambda, \Lambda)$ preserved under taking finite unions?

Partial solution. Consistently yes (Zdomsky). Is it “No” under CH?

Issue 10. Is $\text{cov}(\mathcal{M}) = \mathfrak{d}$? (See the definition of $\mathfrak{d}$ in that issue.)

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