Reliability of the belt conveyor bed when restoring failed roller supports

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Abstract. The reliability of modern belt conveyors, whose length reaches tens of kilometers, is primarily determined by the reliability of the roller supports that support the belt and ensure its movement. As they wear out, some roller bearings fail and need to be repaired or replaced. The dynamics of the number of working roller supports is determined by the system of Kolmogorov equations. Their solution allows us to find the probabilities of finding the system in states with a different number of working elements. The article finds probabilities for two cases. In the first case, when restoring, only one roller support is repaired each time. In the second case, all roller supports are repaired or replaced. In the case of sequential recovery, the mathematical expectation of the number of properly working roller supports may be less than the total number by several units. There are always elements that need to be repaired. If the recovery rate of the elements is many times higher than the failure rate, the mathematical expectation of the number of properly operating roller supports is less than the total number of roller supports by less than one, during most of the time all roller supports are serviceable. In the case of simultaneous recovery of elements, an equally high level of reliability is achieved even with comparable failure and recovery rates. The results obtained can be used to determine the necessary reserve of spare structural elements and to plan the maintenance of conveyors.

1. Introduction

The increase in the population causes an increased demand for the consumption of mineral resources, which in turn leads to the intensification of the extraction of minerals from the subsoil [1]. For the constantly growing mining industry, it is necessary to provide it with high-performance equipment. It should be borne in mind that mining equipment must have not only high productivity, but continuity of action. In the technological schemes of mining, it is necessary to: continuous supply of ore to the processing plant [2, 3]; continuous preparation of the backfill mixture and its supply to the treatment space [4, 5] for the safe conduct of underground work [6]. Uninterrupted supply of minerals and disposal of waste from mining and related industries becomes more efficient and less costly when using continuous mechanisms [7].

Such high-performance mechanisms of continuous action are pipeline and conveyor transport. The conveyor, in contrast to pipeline transport, is distinguished by its simple design, high operational reliability and the ability to transport most types of both piece and bulk cargo [8]. Higher requirements are imposed on mixtures transported by means of a pipeline. Mixtures with a large filler ensure high
strength of the created artificial mass, but significantly complicates the transport of such mixtures in underground conditions [9]. In this regard, the advantage of conveyor transport is undeniable. For the efficient operation of conveyors, it is necessary to provide an increased service life of individual units and mechanisms [10] by ensuring the required quality [11].

2. Problem statement

The reliability of the conveyor and the durability of its structural elements determine its profitability and efficiency [12]. The analysis of equipment reliability is an urgent and demanded task in practice, solved by modern researchers [13-15].

The conveyor is a remanufactured and repaired machine [16]. The probability of recovery at a given time and the mean recovery time are some of the indicators of reliability. At present, the calculation of reliability indicators can be carried out analytically or using new CAD tools [17, 18] or simulation methods [19-21].

As shown in [22, 23], the reliability of the conveyor train is determined by the reliability of the roller supports. In terms of reliability, the rollers are connected in series. Let us assume that it is possible to replace failed rollers without stopping the conveyor. To determine the required number of spare rollers, for planning repair work, it is necessary to know the probability of failure of a certain number of them within a certain period of time, during which they cannot be restored. The aim of the work is to study the dynamics of the number of working roller supports in the case when the failed ones can be repaired or replaced during the operation of the conveyor. The service life of modern conveyor rollers does not exceed 1500-2000 hours [24]. At the same time, repair or replacement of rollers is a complex technical procedure required a certain lengthy preparation. Therefore, the recovery time of the rollers may differ from the time of their work by only one order of magnitude.

Let us introduce some notation used in the theory of reliability. Suppose we have n rollers. The probabilities of failure-free operation of each of them during time t are denoted by $p_1(t)$, $p_2(t)$, ..., $p_n(t)$.

3. Scheme with alternate recovery of elements

Let us consider a model within which failed roller supports can be restored and put back into operation. In this case, the behavior of the system is described by the Kolmogorov system of equations.

Let for a system with k working elements (state k) the intensity of transitions to state $k - 1$, that is, the failure rate, is $\mu_k$, and the intensity of transitions to the state $k + 1$, that is, the intensity of restoration, is $\lambda_k$. Then the Kolmogorov equations [25] take the form

$$\begin{align*}
\frac{dp_\alpha(t)}{dt} &= -\lambda_\alpha P_\alpha(t) + \mu_\alpha P_{\alpha+1}(t) \\
\cdots \\
\frac{dp_k(t)}{dt} &= \lambda_{k-1} P_{k-1}(t) - \mu_k P_k(t) - \lambda_k P_k(t) + \mu_{k+1} P_{k+1}(t) \\
\cdots \\
\frac{dp_n(t)}{dt} &= \lambda_{n-1} P_{n-1}(t) - \mu_n P_n(t)
\end{align*}$$

(1)

We will be interested in the steady state of the system, in which all probabilities are constant, and their derivatives are equal to zero.
Then we obtain the chain of relations

\[ \mu_k p_i(t) = \lambda_{k-1} p_{i-1}(t), \]

where \( p_i \) – limiting values of probabilities \( p_i(t) \) at \( t \to \infty \). Now we can consistently express \( p_i \) from the system (1):

\[ p_{n-1} = \frac{\mu_n}{\lambda_{n-1}} p_n; \quad p_{n-2} = \frac{\mu_n}{\lambda_{n-1}} \frac{\mu_{n-1}}{\lambda_{n-2}} p_n; \quad \ldots; \quad p_k = \frac{\mu_n}{\lambda_{n-1}} \ldots \frac{\mu_{k+1}}{\lambda_k} p_n. \] (3)

Introducing for \( k = 0, 1, \ldots, n-1 \) the notation \( \theta_k = \frac{\mu_n}{\lambda_{n-1}} \ldots \frac{\mu_{k+1}}{\lambda_k} \); \( p_k = \theta_k p_n; \quad \theta_n = 1 \) and taking into account the requirement \( \sum_{i=0}^{n} p_i = 1 \), we obtain an expression for the required probabilities

\[ p_k = \frac{\theta_k}{\sum_{i=0}^{n} \theta_i} \] (4)

These expressions allow one to obtain various characteristics of the system under study. For example, the mathematical expectation of the number of working elements is

\[ M(k) = \sum_{i=0}^{n} i p_i = \sum_{i=0}^{n} \frac{i \theta_i}{\sum_{i=0}^{n} \theta_i}. \] (5)

Let us consider in more detail a particular case \( \lambda = \ldots = \lambda_n = \lambda \), \( \mu = \ldots = \mu_n = \mu \). This assumption seems realistic: with a large number of system elements, the failure of several of them has little effect on the rate of further failures; it can also be assumed that the recovery rates of failed elements are the same. It is also natural to accept that \( \lambda > \mu \): the characteristic recovery time of an element is, of course, much shorter than its service life. The typical values of \( \mu \) have order of magnitude \( 10^{-3} \text{h}^{-1} \) [24]; the typical values of \( \lambda \) are 1 or 2 orders more.
In this case $\theta_{n-1} = \frac{\mu}{\lambda}$, $\theta_{n-1} = \left(\frac{\mu}{\lambda}\right)^2$, ..., $\theta_{1} = \left(\frac{\mu}{\lambda}\right)^{n-1}$. Let’s replace approximately $\sum_{i=0}^{n} \theta_{i}$ by the sum of an infinitely decreasing geometric progression:

$$\sum_{i=0}^{n} \theta_{i} = 1 + \frac{\mu}{\lambda} + \left(\frac{\mu}{\lambda}\right)^2 + ... = \frac{1}{1 - \frac{\mu}{\lambda}} = \frac{\lambda}{\lambda - \mu}.$$  

(6)

Then, for the probabilities of the system being in one state or another, we find

$$p_{n} = \frac{\lambda - \mu}{\lambda} ; \quad p_{n-1} = \frac{\lambda - \mu}{\lambda}, \frac{\mu}{\lambda}; \quad ...; \quad p_{1} = \frac{\lambda - \mu}{\lambda} \left(\frac{\mu}{\lambda}\right)^{n-1}.$$  

(7)

We now transform the numerator into (5):

$$\sum_{i=0}^{n} i \theta_{i} = n(n - 1) \cdot \frac{\mu}{\lambda} + (n - 2) \cdot \left(\frac{\mu}{\lambda}\right)^2 + ... + 1 \cdot \left(\frac{\mu}{\lambda}\right)^{n-1} =$$

$$n \left(1 + \frac{\mu}{\lambda} + \left(\frac{\mu}{\lambda}\right)^2 + ...\right) - \left(0 + 1 \cdot \frac{\mu}{\lambda} + 2 \cdot \left(\frac{\mu}{\lambda}\right)^2 + ...\right) \approx$$

$$n \sum_{i=0}^{\infty} \left(\frac{\mu}{\lambda}\right)^i = \sum_{i=0}^{\infty} i \left(\frac{\mu}{\lambda}\right)^i.$$  

(8)

Here the first sum is the already mentioned sum of an infinitely decreasing geometric progression, the second is also well known: $\sum_{i=0}^{n} i x^i = \frac{x}{(1-x)^2}$, so

$$\sum_{i=0}^{n} i \theta_{i} = n \frac{\lambda}{\lambda - \mu} - \frac{\mu}{\lambda} \left(1 - \frac{\mu}{\lambda}\right)^2 = \frac{\lambda}{\lambda - \mu} \left(n - \frac{\mu}{\lambda} - \lambda + \mu\right),$$

(9)

$$M(k) = n - \frac{\mu}{\lambda}.$$  

(10)

The formula (10) is valid by the condition $\left(\frac{\mu}{\lambda}\right)^n \approx 1$ , when the using of the simple result (6) for the infinitely decreasing geometric progression is correct. When $\lambda \gg \mu$ (recovery of elements is much faster than their failure) $M(k)$ differs from $n$ by less than one, almost all elements are turned on, the system works very reliably. If $\lambda$ and $\mu$ are comparable, $M(k)$ may be several units less than $n$.

4. Scheme with simultaneous recovery of elements

Let's consider a slightly different scheme that better reflects the real situation in technical systems. Let now $\mu_k$ – the intensity of transitions from state $k$ to state $k-1$, and $\lambda_k$ – the intensity of transitions from state $k$ directly to state $n$. This model describes a situation when all failed elements are restored.
during repair work. Let us rewrite the system of Kolmogorov equations:

\[
\frac{dp_k(t)}{dt} = -\lambda_k p_k(t) + \mu_{k+1} p_{k+1}(t) \\
\vdots \\
\frac{dp_n(t)}{dt} = -\lambda_n p_n(t) + \sum \lambda_{n+1} p_{n+1}(t) - \mu_n p_n(t).
\]  

(11)

The graph of the transitions between the levels for this case is presented on the fig. 2.

![Figure 2. Scheme with alternate recovery of elements](image)

In the stationary limit

\[
(\mu_k + \lambda_k) p_k = \mu_{k+1} p_{k+1}; (\mu_k + \lambda_{k+1}) p_{k+1} = \mu_{k+1} p_{k+2}; \quad p_{k-1} = \frac{\mu_k}{\mu_k + \lambda_k} p_k.
\]  

(12)

Let us again restrict ourselves to the case \( \lambda_1 = \ldots = \lambda_n = \lambda \), \( \mu_1 = \ldots = \mu_n = \mu \). In this case

\[
p_{k-1} = \frac{\mu}{\mu + \lambda} p_k.
\]  

(13)

and in all reasoning for the first case, you need to replace the factor \( \frac{\mu}{\lambda} \) by the factor \( \frac{\mu}{\mu + \lambda} \). Then we obtain:

\[
p_n = \frac{\lambda}{\mu + \lambda}; \quad p_{n-1} = \frac{\lambda}{\mu + \lambda} \cdot \frac{\mu}{\mu + \lambda}; \quad \ldots; \quad p_k = \frac{\lambda}{\mu + \lambda} \left( \frac{\mu}{\mu + \lambda} \right)^{n-k}; \quad \ldots
\]  

(14)

For the mathematical expectation of the number of working elements, we obtain

\[
M(k) = n - \frac{\mu}{\lambda}.
\]  

(15)

Since \( \hat{\lambda} > \mu \), \( M(k) \) differs from \( n \) by less than one, which indicates the extremely reliable operation of the system. Here is the condition \( \left( \frac{\mu}{\lambda} \right)^n \) of course satisfied.
5. Example of practical calculation of statistical parameters of a belt conveyor with recoverable roller supports

When restoring the rollers one by one, two cases can be considered. To illustrate the case of comparable rates of failure $\mu$ and of the recovery $\lambda$ we take $\lambda = 2\mu$. Then $\sum_{i=0}^{n} \theta_i = 2$; and the probability of the system being at the upper level (all rollers are in working order) $p_n = \frac{1}{2}$. Probability of the conveyor operation with one broken roller support $p_{n-1} = \frac{1}{4}$, the likelihood of working with two broken rollers $p_{n-2} = \frac{1}{8}$, and so on. Then the probability of their failure for at most $i$ elements is

$$P(n - k \leq i) = 1 - \frac{1}{2^i}. \quad (16)$$

For example, for example, the probability of failure is no more than 9 rollers is $P(n-k \leq 9) = 1 - \frac{1}{1024}$, and the mathematical expectation of the number of serviceable rollers, according to (10), in this case is $M(k) = n - 1$.

Let now $\lambda \gg \mu$. Let’s take for illustration $\lambda = 10\mu$. Then $p_n = 0.9$, $p_{n-1} = 0.09$, $p_{n-2} = 0.009$. The probability of failure is not more than two rollers is $P(n-k \leq 2) = 0.999$; the probability of failure is no more than three -- $P(n-k \leq 3) = 0.9999$ and so on. The mathematical expectation of the number of serviceable elements is $M(k) = n - \frac{1}{9}$. Obviously, in the case when the failure rate is much less than the recovery rate of the elements, the system works with high reliability, and almost all the elements are included in the work.

Consider the same ratio of failure and recovery rates for simultaneous recovery of elements. Let us accept $\lambda = 2\mu$. Then $p_n = \frac{2}{3}$, $p_{n-1} = \frac{2}{9}$, $p_{n-2} = \frac{2}{27}$.

The probability of failure is not more than $i$ rollers is $P(n-k \leq i) = 1 - \frac{1}{3^{i+1}}$. The mathematical expectation of the number of serviceable elements is $M(k) = n - \frac{1}{2}$.

Let it now $\lambda = 10\mu$. Then $p_n = \frac{10}{11}$, $p_{n-1} = \frac{10}{11^2}$, $p_{n-2} = \frac{10}{11^3}$.

The probability of failure is not more than $i$ rollers is $P(n-k \leq i) = \frac{1}{11^{i+1}}$.

The mathematical expectation of the number of serviceable elements is $M(k) = n - \frac{1}{10}$. These and similar results can be used to assess the technical condition of the conveyor and the need for repairs [26].

6. Conclusions and discussion

In the article, the following characteristics of a system of $n$ independently operating roller supports are obtained: the probability of correct operation of any number $k$ of roller supports; mathematical expectation of the number of properly working rollers; the probability of failure of at most $i$ rollers. These probabilities can be used to estimate the number of spare rollers needed and to plan conveyor
maintenance work. In the case of alternate restoration of roller supports, a very high reliability of the conveyor operation is ensured if the intensity of the restoration of elements is many times higher than the intensity of their failures. In the case of simultaneous restoration of elements, a high level of reliability and failure-free operation of almost all roller bearings is achieved even with comparable rates of failure and restoration of elements.

The solution of such problems can be used for monitoring and planning repairs.

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