The Grey Forecasting Model for the Medium-and Long-Term Load Forecasting

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Abstract. Load forecasting is an important part of system planning. In this paper, the medium - and long-term load forecasting method is studied, and the grey prediction is selected as the prediction method. Through the establishment of the grey prediction model and the posterior difference method to test the accuracy of the model, the gray forecasting model for the medium - and long-term load forecasting is presented, and the feasibility of this method is verified by an example.

1. Introduction
The load forecasting plays an important role in power system development planning. The scientific load forecasting can make the resource allocation, network layout and operation more reasonable. Therefore, the load forecasting is an important link with system planning.

The load prediction of power system is to use the systematic methods to deal with the past and future loads and to determine the load value at a certain moment or at some specific moment in the future in a certain precision sense, with fully considering the operating characteristics of important system, capacity increase decisions and natural conditions [1].

The rational choice of load forecasting can use electricity for the more efficient planning; it can arrange the generator sets to start and stop reasonably, so that the maintenance of the units can be in a regular and planned way; It is good for the rational distribution of energy and energy conservation;

It plays a planning role in the construction of power grids and has a very important influence on the safe development of power system.

The load prediction is to find the changing rules through observing the load changes, and to establish the corresponding model to predict the load at a certain time in the future.

Common load forecasting methods as followed:
1) Time series method; 2) Expert system method [2, 3]; 3) Grey system theory [4]; 4) Wavelet analysis prediction technology [5, 6]; 5) Fuzzy load forecasting; 6) Neural networks theory.

All kinds of methods have their own characteristics. For example, the modeling process of time series method is relatively complex, which requires relatively high theoretical knowledge, and the model requires relatively high stability of the original time series. The expert system method is prone to human error in the process of prediction and it cannot be applied for other systems. Artificial neural network has a good applicability in short-term load prediction [7]. Grey mathematical theory is to enhance the regularity of data by means of grey information and corresponding methods. The grey prediction is to use the generate and transform of the load data to make it a regular generating sequence, then establishing a corresponding model and carrying out the load prediction.
In a word, the grey system theory is selected for the medium-and long-term load forecasting.

2. Data generation of grey prediction model

The grey system carries out a series of generation processing on the original data so that the irregular data become regular, and then builds the model. A common method used in the power load prediction is the generation of gray system, including the accumulation generation, the reduction generation, the mean generation, the grade-ratio generation, and the whitening function generation of grey number, etc. [8].

2.1 The accumulation generation

\( \{x(0)(k)\} \) is denoted as the original sequence: 
\[
x(0)(k) = (x(0)(1), x(0)(2), \ldots, x(0)(n))
\]

Then add it up, 
\[
x(1)(k) = \sum_{m=1}^{k} x(0)(m) \quad k = 1, 2, \ldots, n
\]

A new number sequence is as follows:
\[
x(1)(k) = (x(1)(1), x(1)(2), \ldots, x(1)(n))
\]

\( \{x(1)(k)\} \) is a form of one-time accumulation Generating for \( \{x(0)(k)\} \), which is denoted as 1-AGO. The new series will show stronger regularity after accumulation.

2.2 The generation of subtraction

The convergent Generating method is denoted as IAGO.

Marking the \( \{x^{(r)}(k)\} \) as the r-generating sequence, and performing the i times subtraction to it, then denote it as i-IAGO. And The result is defined as \( a^{(i)} \). Then the following relations are as followed. 0 times of reduction, which is denoted as 0-IAGO, and its formula is as follows:

\[
a^{(0)}(x^{(r)}(k)) = x^{(r)}(k)
\]

Notes: 0 times of reduction is equivalent to no reduction, and the result is the original value.

1 reduction, which is denoted as 1-IAGO, and its formula is:

\[
a^{(1)}(x^{(r)}(k)) = a^{(0)}(x^{(r)}(k)) - a^{(0)}(x^{(r)}(k - 1))
\]

i times reduction, which is denoted as I-IAGO, and its formula is:

\[
a^{(i)}(x^{(r)}(k)) = a^{(i-1)}(x^{(r)}(k)) - a^{(i-1)}(x^{(r)}(k - 1))
\]

Because \( a^{(i)}[x^{(r)}(k)] = x^{(r-i)}(k) \), and when \( i=r \), there is

\[
a^{(r)}[x^{(r)}(k)] = x^{(r-r)}(k) = x^{(0)}(k)
\]

When doing the r-AGO to the r-IAGO, the original data can be obtained \( r^{(0)} \).

2.3 The mean generation

There are two types of mean generation: adjacent mean generation and non-adjacent mean generation. Even if there is an original sequence: \( \{x\} = [x(1), x(2), \ldots, x(n)] \).Marking the generated value of point \( k \) as \( z(k) \), and:

\[
z(k) = 0.5x(k) + 0.5x(k - 1)
\]

It's called \( z(k) \) as the adjacent mean generation value.
Either it is the original sequence: \( \{x_1, x_2, \ldots, x_{k-1}, \Phi(k), x_{k+1}, x_n\} \). Here the \( \Phi(k) \) is the hole, and marking the generated value of point \( k \) as \( z(k) \), and now
\[
z(k) = 0.5x(k-1) + 0.5x(k+1)
\]
(7)

\( z(k) \) is called as the non-adjacent mean generated value.

3. Grey prediction model modeling
Grey system modeling is a mathematical model established by generating and transforming the original sequences that show few characteristics of the system or have uncertain factors.

3.1 Modeling mechanism of grey prediction model
Grey theory builds differential equation model by grey number, which is called GM. The GM \((1,1)\) model is commonly used in the load forecasting, it is suitable for the exponential growth of data. It is the data proved that the prediction of model needs less amount of data, less amount of calculation and the prediction is precision. It is essentially to be a raw data with the accumulation generation processing, having on the sequence of the discipline, convenient to the rest of the model.

Marking the \( x^{(0)} \) as the modeling sequence of GM \((1,1)\): \( x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\} \).

Marking the \( x^{(1)} \) as the first-order accumulation generated sequence of \( x^{(0)} \):
\[
x^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\}
\]
(8)
\[
x^{(1)}(1) = x^{(0)}(1)
\]
\[
x^{(1)}(k) = \sum_{m=1}^{k} x^{(0)}(m) \quad k = 1, 2, \ldots, n
\]
(9)

Marking the \( z^{(1)} \) as the mean sequence of \( x^{(1)} \):
\[
z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1) \quad k=2,3,\ldots,n
\]
(10)

Then, the definition of GM \((1,1)\), it is the grey differential equation of GM \((1,1)\) is:
\[
x^{(0)}(k) + az^{(1)}(k) = u
\]
(11)

Where, \( a \) is the development coefficient; \( u \) is the grey action; \( z^{(1)}(k) \) is the white background value sequence.

GM \((1,1)\) grey differential equation corresponds to the following albino differential equation
\[
\frac{dx^{(1)}}{dt} + ax^{(1)} = u
\]
(12)

Where, \( a \) and \( u \) are parameters, denoted as \( P = [a, u]^T \), and according to the least square method, the solution of \( y_n = BP \) is: \( P = [a, u]^T = (B^T B)^{-1} B^T y_n \).

The equation is the matrix identification formula for parameters \( a \) and \( u \) of GM \((1,1)\). Actually, \((B^T B)^{-1} B^T\) is the generalized inverse of the data matrix \( B \). And then:
\[
B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} = \begin{bmatrix} -0.5[x^{(1)}(1) + x^{(1)}(2)] & 1 \\ -0.5[x^{(1)}(2) + x^{(1)}(3)] & 1 \\ \vdots & \vdots \\ -0.5[x^{(1)}(n-1) + x^{(1)}(n)] & 1 \end{bmatrix}
\]
(13)
\[ \begin{align*}
\mathbf{y}_n &= [x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)]^T \\
B &= \text{data matrix, } \mathbf{y}_n = \text{data vector, and } P = \text{parameter vector.}
\end{align*} \] (14)

The GM (1,1)’s prediction model can be obtained by solving the differential equation in the bleaching form as follows:

\[ \hat{x}^{(0)}(k+1) = (x^{(0)}(1) - \frac{u}{a})e^{-ak} + \frac{u}{a}, \quad k=1,2,\ldots,n \] (15)

Doing the cumulative subtraction generates reduction to the \( \hat{x}^{(0)}(k) \), and getting the predicted results of \( \hat{x}^{(0)}(k) \). Then, the prediction model of GM (1,1) is:

\[ \hat{x}^{(1)}(k + 1) = \hat{x}^{(1)}(k + 1) - \hat{x}^{(1)}(k) = (x^{(0)}(1) - \frac{u}{a})(1-e^{a})e^{-ak} \quad k = 1, 2, \ldots n \] (16)

### 3.2 The test of grey prediction model

In this paper, the posterior difference method is used to test the accuracy of the GM (1,1) models. Based on residuals (absolute errors) \( \varepsilon \), the posterior difference test examines the probability of the points in small residuals and the variance index of prediction errors according to the absolute residuals of each cycle. The specific steps are as follows

Marking the \( x^{(0)} \) as the original sequence: \( x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)) \), \( \hat{x}^{(0)}(k) \) is the predicted value sequence, \( k = 1, 2, \ldots, n \).

And, \( \bar{x} = \frac{1}{n} \sum_{k=1}^{n} x^{(0)}(k) \) is the average value of the original data \( x^{(0)}(k) \); \( S_1^2 = \frac{1}{n} \sum_{k=1}^{n} (x^{(0)}(k) - \bar{x})^2 \) is the variance of \( x^{(0)}(k) \); \( \bar{\varepsilon} = \frac{1}{n} \sum_{k=1}^{n} \varepsilon^{(0)}(k) \) is the average value of \( \varepsilon^{(0)}(k) \); \( S_2^2 = \frac{1}{n} \sum_{k=1}^{n} (\varepsilon^{(0)}(k) - \bar{\varepsilon})^2 \) is the variance of \( \varepsilon^{(0)}(k) \); \( C = \frac{S_2}{S_1} \) is the posterior difference ratio;

\[ P = \{\varepsilon^{(0)}(k) - \bar{\varepsilon} < 0.6745S_1\} \] is the small error probability.

In the prediction, \( C \) is the smaller, the better, and \( P \) is the bigger, the better. According to the sizes of \( P \) and \( C \), the prediction accuracy can be divided into four grades, and the standards are shown in Table 1.

| Table 1. Prediction accuracy criteria |
|--------------------------------------|
| level 1 | Good (level 1) | Qualified (level 2) | Barely (level 3) | Unqualified (grade 4) |
|---------|----------------|---------------------|-----------------|----------------------|
| P       | >0.95          | >0.8                | >0.7            | ≤0.7                 |
| C       | <0.35          | <0.45               | <0.5            | ≥0.65                |

### 3.3 The specific steps of grey prediction

(1) Getting original sequence: \( x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)) \); (2) Calculating the accumulated generated value \( x^{(1)}(k) \) of the original sequence according to formula (9); (3) Calculating a and \( u \) parameters according to the differential equation (12); (4) The GM (1,1) prediction model \( \hat{x}^{(1)}(k + 1) \) is obtained by using formula (15). The results \( \hat{x}^{(0)}(k + 1) \) is obtained by progressive reduction of the GM (1,1) model; (5) Calculating the errors and judging the accuracy of prediction.
4. Examples of load forecasting

4.1 The medium-and long-term load forecast data
This example will use the gray theory to predict the power load of a city. The city has many mineral resources, mining industry and mineral development enterprises gather, and the electricity consumption is relatively large, so the factors affecting the load have great uncertainty. The load changes from 2001 to 2011 are shown in Table 2.

| The serial number | year | Electricity consumption | The serial number | year | Electricity consumption |
|-------------------|------|-------------------------|-------------------|------|-------------------------|
| 1                 | 2001 | 711.32                  | 7                 | 2007 | 1487.67                 |
| 2                 | 2002 | 724.92                  | 8                 | 2008 | 1917.54                 |
| 3                 | 2003 | 736.09                  | 9                 | 2009 | 2125.21                 |
| 4                 | 2004 | 813.43                  | 10                | 2010 | 2556.63                 |
| 5                 | 2005 | 1093.05                 | 11                | 2011 | 3026.98                 |
| 6                 | 2006 | 1294.43                 |                   |      |                         |

According to the data from 2001 to 2011 in Table 2, the curve is shown in Figure 1, which is the graph of the original data \( \{x^0(k)\} \) of electricity consumption. The figure shows that the annual load data from 2001 to 2008 have an overall growth trend, which meets the basic conditions of the gray prediction model.

4.2 The grey prediction simulation design
The GM (1,1) model was used for prediction, and the data were reduced to get the predicted data. After the gray prediction, a posterior check is carried out on the model. The specific error analysis is shown in Table 3.

| year | The original data | Forecast data | Absolute error | The relative error |
|------|------------------|---------------|----------------|--------------------|
| 2001 | 711.32           | 711.32        | 0              | 0.00%              |
| 2002 | 724.92           | 617.2         | 107.72         | -14.86%            |
| 2003 | 736.09           | 740.4         | 4.31           | 0.59%              |
| 2004 | 813.43           | 888.3         | 74.87          | 9.20%              |
| 2005 | 1093.05          | 1065.7        | 27.35          | -2.50%             |
| 2006 | 1294.43          | 1278.5        | 15.93          | -1.23%             |
| 2007 | 1487.67          | 1533.9        | 46.23          | 3.11%              |
| 2008 | 1917.54          | 1840.2        | 77.34          | -4.03%             |

The original data and prediction results are shown in Figure 2. The posterior check of model precision: the variance of the original data is 412.7; The residual variance is 55.92; The posterior difference ratio C is 0.1354<0.35. The small error probability P is 1>0.95.
It can be seen from the range of posterior difference ratio C and the small error probability P that the accuracy of GM (1,1) model established by load prediction in this region is level 1, so it is feasible to carry out load prediction by GM (1,1) model in this region. GM (1,1) is used to predict electricity consumption in 2009-2011, and the accuracy of the prediction model for future load is tested. The results are shown in Table 4.

| Year | The original data | Forecast data | Absolute error | The relative error | Prediction accuracy |
|------|------------------|---------------|----------------|-------------------|---------------------|
| 2009 | 2125.21          | 2207.7        | 82.49          | 3.88%             | 96.12%              |
| 2010 | 2556.63          | 2648.6        | 91.97          | 3.60%             | 96.4%               |
| 2011 | 3026.98          | 3177.6        | 150.62         | 4.98%             | 95.02%              |

The accuracy level of the GM (1,1) model of grey prediction is level 1, which is an effective model. It can also be seen from the error analysis table and load prediction data graph that the error of the medium-and long-term load prediction using this method is within the acceptable range, and the GM (1,1) model has a good effect on regional load prediction.

5. Conclusion
Grey prediction using the method of differential equation to describe the model, load data onto processing, it is not easy to find the law of its existence. We need to generate the data onto the grey prediction, after processing the data regularity will strengthen, using the least squares method to calculate the development coefficient and grey action, use the operation such as differential equation and b-b reduction bleaching finally get the GM (1, 1) model is used to predict. Although the established model can be used for prediction, the GM (1,1) model is not necessarily valid, and the accuracy of the model needs to be tested. The posterior error check is adopted here, and the posterior error ratio C and small error probability P are obtained through various calculations of residuals, so as to determine the accuracy of the model.

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