Study of passive seismic tomography with various grid by using Matlab

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Abstract. Tomography technique is one method in geosciences adapted from medical field like CT Scan and MRI. In medical field, tomography is used to describe human’s body condition but in geoscience it is used to describe subsurface of the earth, like passive seismic tomography. Subsurface description like velocity data that expressed in tomogram is a result of inversion from travel time and ray tracing data. Tomography is done in various grid model (regular and irregular grid). Irregular grid give a better tomogram result, it is caused by it does not need big data of hypocenter or station. The distribution of grid also showed the good propagation model of ray tracing that influenced the tomogram result.

1. Introduction
Describing or modelling the subsurface is almost the purpose of all geophysical method. In geophysics, modelling the subsurface is important to know physics parameterization of the earth, then it can be used in industry, research, and human safety from natural disaster like earthquake and volcano eruption. Tomography is a reliable way that can be applied in geophysics to get better subsurface model. One example, tomography can be used in passive seismic (include in seismological field), a part of geophysical way that learn about earthquake. Earthquake parameter usually used to describing the subsurface are Primary and Secondary wave of earthquake. Both of them have their own physical parameter. Primary wave evidence related to fluid existence, and Primary to Secondary Ratio related to gas existence. Beside that, by tomography, subduction model from certain place in the earth can be known. It has utilize to analyse potential of hazard in that place.

Tomography method in geophysics, is adopted from medical field, like CT Scan and MRI. In medical field, human’s body is an object that described by x-ray that passed on it to the detector. It can be used to detect variety of diseases and conditions. It also can scanning fast, painless, noninvasive and accurate [1]. Then with certain process, human’s body condition can be described to analyze by a doctor (Figure 1).

In Seismology, earthquake source (hypocenter) deliver the energy in the form of Primary and Secondary wave to the earth’s surface because plate tectonic movements. It acts like X-Ray in medical field. Then, Primary and Secondary wave is accepted by receiver in the surface (detector in CT Scan or MRI). From travel time information of Primary and Secondary wave with initial model of their velocity, subsurface condition can be described (Figure 2).
Based on Figure 2, the length of the ray should be known. It depends on the grid chosen by the researcher. The number of grids is determined amount of the total ray of seismic wave then it will determine the travel time too. In general, the shape of grid is square or called a regular grid. It makes calculation the ray’s length easier, but sometimes cannot give a good description in a complex model. On the other hand, there is irregular grid (triangle, hexagon, voronoi, etc) that can give a better model than regular grid. In this research, various grid will be used to tomography process to know the tomogram result of each grid. Matlab is chosen as a language program to execute the process because it has some advantages like the simplicity of the listing program, good visualization, etc.
2. Methodology
Tomography method used delay time between observed and calculation time to describe the subsurface by velocity determination. For initial model, AK 135 Table is usually used. Grid size also determine the tomography result (Figure 3), especially in ray tracing calculation.

![Figure 3. Sketch of Tomography method. T and R are Transmitter and Receiver that illustrated as hypocenter and the station.](image)

Based on Figure 3, it can be known that if the grid size is changed, the number of the grid also change and finally tomography result will follow to. In Figure 3, grid 1 to 3, grid 4 to 6 and 7 to 9 assumed have velocity model $V_a, V_b$ and $V_c$ then the travel time calculation ($t$) from T to R is defined as:

$$t = t_1 + t_{12} + t_{15} + t_6$$  \hspace{1cm} (1)

$$t = \frac{r_1}{v_a} + \frac{r_{12}}{v_a} + \frac{r_{15}}{v_b} + \frac{r_6}{v_b}$$  \hspace{1cm} (2)

While, travel time observation obtained from the seismogram in the station (R). Equation (2) is applied for one event earthquake. Delay time is obtained by calculate the differences between travel time calculation and travel time observation. If there are more than one earthquake or more than one station for example, by using inversion technique, velocity in each grid can be obtained (Equation (3)).

$$\begin{bmatrix}
\Delta t_1 \\
\Delta t_2 \\
\vdots \\
\Delta t_N
\end{bmatrix} =
\begin{bmatrix}
r_1 & r_{12} & 0 & 0 & r_{15} & 0 & 0 & 0 & 0 \\
 r_1 & r_2 & r_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 r_1 & 0 & 0 & r_4 & r_5 & r_6 & \cdots & \cdots & \cdots
\end{bmatrix}
\begin{bmatrix}
1/v_1 \\
1/v_2 \\
1/v_3 \\
1/v_4 \\
1/v_5 \\
1/v_6 \\
1/v_7 \\
1/v_8 \\
1/v_9
\end{bmatrix}$$

Furthermore, in Equation (3), $\Delta t_i$ matrix is called data or $d_i$, $r_i$ is called Kernell matrix (G) and $1/v_i$ is called parameter model m that will be looking for, so Equation (3) can be rewritten as $d = Gm$.

Inversion technique of Equation (3) can be looked in Equation (4):

$$m = (G^T G)^{-1}G^T d$$  \hspace{1cm} (4)

But, Kernell matrix in Equation (3) has many 0 elements. The matrix will be singular. To solve singular matrix, Equation (4) is modified as in Equation (5) [5].

$$m = G^T(GG^T + \lambda I)^{-1} d$$  \hspace{1cm} (5)
with a perturbation factor ($\lambda$) with the range between 0 and 1, Equation (5) can be solved. It is just a mathematical technique.

Equation (4) and (5) is commonly equation that used in inverse problem. That equation usually applied in regular grid approach like square form. It is very simple and easy to calculate. Sambdridge and Gudmundsson (1998) and Bolum et al (2000) opcit [6] proposed irregular grid in tetrahedral and Voronoi diagram (Figure 4). The inversion calculated in irregular mesh is more complex than regular mesh, but the result is more uniform in irregular mesh than in regular mesh. Irregular mesh also can describe complex geometrical challenge, system with significant heterogenities include reservoir with faults and fractures, geological strata that intersecting at arbitrary angles [7].

![Figure 4. Sketch of Tetrahedral (solid line) and Voronoi diagram (dashed line) [6]](image)

Irregular grid can increase the resolution and the realibility of tomographic result. It is because in regular grid, especially in borehole seismic study case, the distribution of soruces and receivers sometimes is irregular. With regular grid, of course it can not adequate for reconstruct the velocity field because the ray path will be linier dependent on the equation [8]. [9] used Frontal-Delaunay refinement and hill climbing type optimisation technique for get better modelling of the oceanic and atmospheric model communities. That algorithm produce very high quality multi resolution triangulation. Equivalent to the explanation before, [10] give statement that mesh generation or adaptive refinement is essential in computational modelling in various scientific and industrial fields. Low quality meshes can potentially lead larger numerical appoximation error.

3. Result and discussion
In this paper, tomography with regular and various irregular grid is used to get a description of tomography. The data is synthetic data. There are hypocenter, station and initial velocity model. In Figure 5, it can be seen regular grid, triangulation grid, voronoi and hexagonal grid.
Then, hypocenter, stations and velocity model is applied to the grid. Ray tracing calculated from hypocenter to station with certain velocity model. There is trial and error process to determine which ray tracing that good for the model. Length of ray in each grid then arranged in Kernel matrix. By inversion technique, velocity value in each grid is observed to be a model.

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Figure 5. (a) Regular grid; (b) triangulation grid; (c) voronoi grid; (d) hexagonal grid

Figure 6. (a) Ray tracing from hypocenter (yellow o) to the station (black ∇) and the tomography. Colorbar is Δv; (b) Ray hit count map in log10 of regular grid
Figure 7. (a) Ray tracing from hypocenter (yellow o) to the station (black ▼) and the tomography. Colorbar is Δv; (b) Ray hit count map in log10 of triangulation grid

Figure 8. (a) Ray tracing from hypocenter (yellow o) to the station (black ▼) and the tomography. Colorbar is Δv; (b) Ray hit count map in log10 of voronoi grid

Figure 9. (a) Ray tracing from hypocenter (yellow o) to the station (black ▼) and the tomography. Colorbar is Δv; (b) Ray hit count map in log10 of hexagonal grid

Ray hit count map in Figure 6 to 9 is a Quality Control of the tomography. If there are a lot of ray tracing in a grid, it will be darker than the others, like showed by red circle in each Figure, although not every grid that have much ray tracing will be expressed with black colour. At least there is indicated that dense of ray tracing expressed by black colour, the tomogram is reach good. Numbers of hypocenter, station, and the grid size determine the result of the tomogram. Based on the tomogram result in Figure 6 to 9, triangulation and voronoi grid have grid size more tight than others with 5 hypocenters and 11 stations for triangulation and 3 stations in voronoi. In regular grid, 15 hypocenters with 3 stations give
a good enough of tomogram result and in hexagonal grid there are 8 hypocenters with 3 stations, could not give a good enough of tomogram result yet, it is might caused by the grid size that too much wide in hexagonal grid. But, generally, from the various grid, irregular grid give a better tomogram result with the limited numbers of hypocenter or station, because ray tracing in irregular grid become different than in regular grid. In regular grid, ray tracing is connected by one edge of grid and others with constant distance and velocity differences. While, in irregular grid, ray tracing connected between one grid and it’s neighbour, so the ray tracing can be deflected irregularly with not consistent of velocity value. To solve this problem, many possibilities of ray tracing is needed to be done with consideration with Hyugens and Fermat’s Law.

4. Conclusion
Modelling tomography in passive seismic study case related to the number data of hypocenter or station data. Low number of data will give a bad result in tomography (tomogram) in regular grid. It is caused by low information that given to calculated. But, in irregular grid, the grid distribution, make it possible to give a better tomogram result although with low hypocenter or station information data.

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