Bound on R-parity violating couplings from nonleptonic B decay

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Abstract

The effective Hamiltonian for the R-parity violating couplings induced nonleptonic decay $B^- \to K^- K^0$ is calculated including leading log QCD corrections running from $m_{\tilde{q}}$, the mass of the up-type squark, to $m_b$. Experimental upper limit of this decay is used to give the bound on the R-parity violating couplings $\lambda''$. 
1 Introduction

In the supersymmetric generalizations of the standard model, there is a kind of couplings which has no standard model analogue. These novel couplings, called the R-parity (or the matter parity) violating couplings \[1, 2\], are not forbidden by some fundamental principles. In a variety of supersymmetric models \[3\], due to additional horizontal symmetries which are broken at some scale between the supersymmetry breaking and the Planck scales, the R-parity violating couplings are bounded to be small \[4\]. However, these kinds of theoretical bounds are quite model dependent. They depend not only on the choices of the horizontal symmetries but also on the concrete power counting rules enforced on the superfields \[4\]. Some model dependent cosmological constraints, e.g. \[\lambda'' \ll 10^{-7}\] given by the requirement that at the grand unification scale baryogenesis does not get washed out, can be also evaded \[5\].

Lacking reliable predictions for these R-parity violating couplings, it becomes important to determine or bound them from experiments. In the literature some phenomenological analyses have been given based on a variety of experiments, from proton decays \[6\] to the precise measurements of the electroweak interactions at LEP \[7\] (see also \[8\]). Because these measurements are performed at different scales and the QCD couplings between these scales are not small, it is necessary to normalize the bounds at a same scale to extract informations at high energy scales (e.g. the grand unification or the Planck scale). This is essential to have a better understanding on the origins of these couplings. Note that in some newly performed renormalization group analyses \[9\], the running of the R-parity violating couplings has been carried out between \(m_Z\) and the grand unification scale.

Recently, it has been pointed out that nonleptonic decays of the B mesons, such as \(B^- \to K^- K^0\) whose decay rate in the standard model is quite small, can also provide useful bound on the baryon-number nonconserved R-parity violating couplings \(\lambda''\)'s \[10\]. In the present work, we also concentrate on this process. We will calculate the leading log QCD corrections to the effective Hamiltonian, sum up the large logs of the form \(\alpha_s \log(m_q/m_b)\)
(here $\tilde{q}$ is an up-type squark) using the renormalization group equations. We perform the hadronic analyses using the perturbative quantum chromodynamics (PQCD) method \cite{11,12} and then use the current upper limit of the branching ratio to bound $\lambda''$s.

2 Effective Hamiltonian with Leading Log Corrections

The superpotential for the R-parity violating couplings which do not conserve baryon-number is

$$W = \lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k,$$

where $\bar{U}$ and $\bar{D}$ are right-handed superfields and $i, j, k$ are generation indices. These superfields are defined in the basis in which the quarks are in their mass eigenstates. We consider only the lightest charge-$\frac{2}{3}$ right-handed squark (denoted as $\tilde{q}$) and we will return to this point later. Following the effective field theory description, $\tilde{q}$ is integrated out at the scale of $m_{\tilde{q}}$ and the results are two 4-quark operators \cite{3}:

$$O_1 = (\bar{d}_R \gamma_\mu s_R)(\bar{s}_R \gamma_\mu b_R),$$

$$O_2 = (\bar{s}_R \gamma_\mu s_R)(\bar{d}_R \gamma_\mu b_R).$$

Then the effective Hamiltonian is written as

$$\mathcal{H} = \frac{\lambda''_{pbs} \lambda''_{qds}}{2m_{\tilde{q}}^2} (C_1(\mu)O_1 + C_2(\mu)O_2).$$

The Wilson coefficients $C_1(\mu)$, $C_2(\mu)$ are calculated at the scale of $\tilde{q}$ mass as

$$C_1(m_{\tilde{q}}) = 1, \quad C_2(m_{\tilde{q}}) = -1.$$

The leading log QCD corrections are characterized by the anomalous dimensions for these two operators. Calculating the Feynman diagrams in Fig.1, we get the following anomalous dimension matrix:

$$\gamma = \frac{g_5^2}{8\pi^2} \begin{pmatrix} -1 & 3 \\ 3 & -1 \end{pmatrix}. $$


By applying the QCD renormalization group equations

$$\mu \frac{d}{d\mu} C_i(\mu) = \sum_j (\gamma^T)_{ij} C_j(\mu),$$

we can get the coefficients of the two operators at some low energy scale \( \mu \) (here \( \mu \sim m_b \)) from those at the scale \( \mu = m_{\tilde{q}} \). In the case that \( \tilde{q} \) is lighter than the top quark,

$$C_1(m_b) = \frac{1}{2}(C_1(m_{\tilde{q}}) + C_2(m_{\tilde{q}}))\xi^{6/23} + \frac{1}{2}(C_1(m_{\tilde{q}}) - C_2(m_{\tilde{q}}))\xi^{-12/23},$$

$$C_2(m_b) = \frac{1}{2}(C_1(m_{\tilde{q}}) + C_2(m_{\tilde{q}}))\xi^{6/23} - \frac{1}{2}(C_1(m_{\tilde{q}}) - C_2(m_{\tilde{q}}))\xi^{-12/23},$$

where \( \xi = \alpha_s(m_{\tilde{q}})/\alpha_s(m_b) \). Otherwise threshold effects of the top quark should be accounted for by two step running:

$$C_1(m_t) = \frac{1}{2}(C_1(m_{\tilde{q}}) + C_2(m_{\tilde{q}}))\eta^{6/21} + \frac{1}{2}(C_1(m_{\tilde{q}}) - C_2(m_{\tilde{q}}))\eta^{-12/21},$$

$$C_2(m_t) = \frac{1}{2}(C_1(m_{\tilde{q}}) + C_2(m_{\tilde{q}}))\eta^{6/21} - \frac{1}{2}(C_1(m_{\tilde{q}}) - C_2(m_{\tilde{q}}))\eta^{-12/21},$$

with \( \eta = \alpha_s(m_{\tilde{q}})/\alpha_s(m_t) \), and

$$C_1(m_b) = \frac{1}{2}(C_1(m_t) + C_2(m_t))\eta_2^{6/23} + \frac{1}{2}(C_1(m_t) - C_2(m_t))\eta_2^{-12/23},$$

$$C_2(m_b) = \frac{1}{2}(C_1(m_t) + C_2(m_t))\eta_2^{6/23} - \frac{1}{2}(C_1(m_t) - C_2(m_t))\eta_2^{-12/23},$$

where \( \eta_2 = \alpha_s(m_t)/\alpha_s(m_b) \). Note that by taking \( m_{\tilde{q}} = 100,300 \) or \( 500 \) GeV, the corrected Wilson coefficient \( C_1 \) at the scale of \( m_b \) is 1.34, 1.45 or 1.50, which shows large QCD corrections.

## 3 PQCD Calculations of the Decay Rate

Low energy hadronic transitions can be analyzed in the framework of PQCD \cite{11,12}. In the present case, the operator \( O_2 \) which gives the non-factorizable contributions has the same Wilson coefficient as that of \( O_1 \) (up to a minus sign). Thus it is reasonable to use the formalism of Simma and Wyler \cite{12} where no explicit use of the factorization hypothesis is
needed. Following this description we take the interpolating field of the B meson as

$$\psi_B = \frac{1}{\sqrt{2} \sqrt{3}} \frac{I_c}{I_B(x)\gamma_5(\not\!p - m_B)}, \quad (14)$$

and those of the $K^-$ and $K^0$ mesons as

$$\psi_{K^-} = \frac{1}{\sqrt{2} \sqrt{3}} \frac{I_c}{I_K-(y)\gamma_5(\not\!p - m_B)}, \quad (15)$$

$$\psi_{K^0} = \frac{1}{\sqrt{2} \sqrt{3}} \frac{I_c}{I_K0(z)\gamma_5(\not\!p - m_B)}. \quad (16)$$

Here $I_c$ is an identity in the color space. We have made the approximation $m_K = 0$.

The decay amplitudes can be calculated from Fig 2. With the insertions of the operator $O_1$, the calculations of the four diagrams give:

$$\mathcal{A}_a^1 = \int_0^1 [dx][dy][dz] \text{Tr} \left[ \frac{i}{\not\!p_b - m_b} \left( \frac{i}{2} \gamma_\alpha g s \right) \psi_B \left( \frac{i}{2} \gamma_\alpha g_s \right) \psi_{K^-} \gamma_\mu P_R \right] \text{Tr} \left[ \psi_{K^0} \gamma_\mu P_R \right] \frac{i}{l_g},$$

$$\mathcal{A}_b^1 = \int_0^1 [dx][dy][dz] \text{Tr} \left[ \psi_B \left( \frac{i}{2} \gamma_\alpha g_s \right) \psi_{K^-} \gamma_\mu P_R \right] \text{Tr} \left[ \psi_{K^0} \left( \frac{i}{2} \gamma_\alpha g_s \right) \frac{i}{\not\!p_d} \gamma_\mu P_R \right] \frac{i}{l_g},$$

$$\mathcal{A}_c^1 = \int_0^1 [dx][dy][dz] \text{Tr} \left[ \psi_B \left( \frac{i}{2} \gamma_\alpha g_s \right) \psi_{K^-} \gamma_\mu P_R \right] \text{Tr} \left[ \frac{i}{\not\!p_s} \left( \frac{i}{2} \gamma_\alpha g_s \right) \psi_{K^0} \gamma_\mu P_R \right] \frac{i}{l_g},$$

$$\mathcal{A}_d^1 = \int_0^1 [dx][dy][dz] \text{Tr} \left[ \psi_B \left( \frac{i}{2} \gamma_\alpha g_s \right) \psi_{K^-} \left( \frac{i}{2} \gamma_\alpha g_s \right) \frac{i}{\not\!p_s} \gamma_\mu P_R \right] \text{Tr} \left[ \psi_{K^0} \gamma_\mu P_R \right] \frac{i}{l_g}. \quad (17)$$

where $[dx]$, $[dy]$, and $[dz]$ denote $(dx_1dx_2)$, $(dy_1dy_2)$ and $(dz_1dz_2)$, respectively, and

$$l_g = y_1p_- - x_1p_B, \quad p_b = x_2p_B - l_g, \quad p_d = z_2p_0 + l_g, \quad p_s = -z_1p_0 - l_g, \quad p_s = y_2p_- + l_g, \quad (18)$$

$p_-$ and $p_0$ being the momenta of the $K^-$ and the $K^0$ mesons, respectively. With the insertions of the operator $O_2$, the results are:

$$\mathcal{A}_a^2 = \int_0^1 [dx][dy][dz] \text{Tr} \left[ \frac{i}{\not\!p_b - m_b} \left( \frac{i}{2} \gamma_\alpha g_s \right) \psi_B \left( \frac{i}{2} \gamma_\alpha g_s \right) \psi_{K^-} \gamma_\mu P_R \psi_{K^0} \gamma_\mu P_R \right] \frac{-i}{l_g},$$

$$\mathcal{A}_b^2 = \int_0^1 [dx][dy][dz] \text{Tr} \left[ \psi_B \left( \frac{i}{2} \gamma_\alpha g_s \right) \psi_{K^-} \gamma_\mu P_R \psi_{K^0} \frac{i}{\not\!p_d} \gamma_\mu P_R \right] \frac{-i}{l_g},$$

$$\mathcal{A}_c^2 = \int_0^1 [dx][dy][dz] \text{Tr} \left[ \psi_B \left( \frac{i}{2} \gamma_\alpha g_s \right) \psi_{K^-} \gamma_\mu P_R \frac{i}{\not\!p_s} \left( \frac{i}{2} \gamma_\alpha g_s \right) \psi_{K^0} \gamma_\mu P_R \right] \frac{-i}{l_g},$$

$$\mathcal{A}_d^2 = \int_0^1 [dx][dy][dz] \text{Tr} \left[ \psi_B \left( \frac{i}{2} \gamma_\alpha g_s \right) \psi_{K^-} \left( \frac{i}{2} \gamma_\alpha g_s \right) \frac{i}{\not\!p_s} \gamma_\mu P_R \psi_{K^0} \gamma_\mu P_R \right] \frac{-i}{l_g}. \quad (19)
Performing the trace operations in both the spinor and the color spaces, we find that the contributions of $A_1^b$, $A_1^c$ vanish due to their color structures. The amplitudes $A_1^d$ and $A_2^d$ are proportional to $m_K$ and thus vanish under the approximation of $m_K = 0$. The amplitude $A_1^a$ is the same as $A_2^a$ up to a color factor. Denoting $A_a = C_1(m_b)A_1^a + C_2(m_b)A_2^a$, we get

$$A_a = -2 \int_0^1 dxdydz \frac{8g_s^2}{3\sqrt{6}} C_2(\mu)\phi_B(x)\phi_{K^-}(y)\phi_{K^0}(z) \frac{2x_1 - y_1 - 1}{x_1(2x_1 - x_1^2 - y_1)(y_1 - x_1)}, \quad (20)$$

$$A_b = -\int_0^1 dxdydz \frac{8g_s^2}{3\sqrt{6}} C_2(\mu)\phi_B(x)\phi_{K^-}(y)\phi_{K^0}(z) \frac{1}{x_1(y_1 - x_1)^2}, \quad (21)$$

$$A_c = \int_0^1 dxdydz \frac{8g_s^2}{3\sqrt{6}} C_2(\mu)\phi_B(x)\phi_{K^-}(y)\phi_{K^0}(z) \frac{2x_1 - y_1 - z_1}{x_1(x_1 - z_1)(y_1 - x_1)^2}, \quad (22)$$

$$A_d = 0. \quad (23)$$

Now the total decay amplitude reads

$$A \equiv \frac{\lambda_{qbs}^\prime \lambda_{qds}^\prime *}{2m_B^2} (A_a + A_b + A_c + A_d). \quad (24)$$

The analyses given above are independent of the choice of the wave functions. Below we will take the wave functions of the $B$ and $K$ mesons as [11, 12]

$$\phi_B(x) = \frac{f_B}{2\sqrt{3}} \delta(x_1 - \epsilon_B), \quad (25)$$

$$\phi_{K^-}(y) = \sqrt{3} f_K y_1 (1 - y_1), \quad (26)$$

$$\phi_{K^0}(z) = \sqrt{3} f_K z_1 (1 - z_1). \quad (27)$$

Now, after integrating over $[dx]$, $[dy]$ and $[dz]$, we get the final amplitudes:

$$A_a = -2 \left( -\frac{1}{2} + 2 \ln \frac{1 - 2\epsilon_B}{2\epsilon_B} - \ln \frac{1 - \epsilon_B}{\epsilon_B} + i\pi \right) G,$$

$$A_b + A_c = \frac{3}{2} G,$$  

where

$$G = \frac{8\pi\alpha_s(\mu)}{9\sqrt{2}\epsilon_B} C_2 f_B f_K^2. \quad (29)$$

The decay width of $B^- \to K^- K^0$ is then

$$\Gamma = \frac{|\lambda_{qbs}^\prime \lambda_{qds}^\prime *|^2 |A_a + A_b + A_c|^2}{64\pi m_B m_K^2}. \quad (30)$$

Taking the $B^-$ meson lifetime as $1.54 \times 10^{-12}$ s, the branching ratio of this decay can be easily found.
4 Result and Conclusion

The present experimental upper bound for the branching ratio of the decay $B^- \rightarrow K^- K^0$ is $5 \times 10^{-5}$ \cite{13}. This gives the constraints for the R-parity violated coupling $|\lambda'^u_{qbs} \lambda'^u_{qds}^*|$ depending on the mass of $\tilde{q}$ (see equations (8) - (13) and (28) - (30)). As we have stated previously, we have considered only the lightest up-type squark. When the masses of the supersymmetric partners of the up, the charm and the top quarks are comparable with one another, none of their contributions can be neglected. In this case, there exists the possibility of cancellations between the contributions from these up-type squarks, and what we get is the bound on $|\sum_{i=u,c,t} C^i_1 \lambda'^u_{ibs} \lambda'^u_{ids}^*/m_i^2|$.

The excluded region in the parameter space is shown in Fig.3. For the numerical estimations, we have used parameters as $\alpha_s(M_Z) = 0.117$, the hadronic scale $\mu = 1$GeV, and the decay constants are taken to be $f_B = 132$MeV and $f_K = 113$MeV. The value of $\epsilon_B$ characterizes most of the uncertainties in the PQCD calculations. Note that if we take $\epsilon_B = 0.05$, the bound without QCD corrections coincides with what is given in \cite{10}. Here we also give the bound by using $\epsilon_B = 0.065$ which is favored by the rare decay of $B \rightarrow K^* \gamma$ \cite{14}.

As a conclusion, the effects of QCD corrections to the effective Hamiltonian induced by the R-parity violating couplings are significant. The bound on a particular combination of these couplings has been upgraded by including the QCD corrections.

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Figure Captions:

Fig. 1 Diagrams in the calculation of the anomalous dimensions of the operators. The black blob represents the effective operator $O_1$ or $O_2$.

Fig. 1 Diagrams contributing to $B^- \rightarrow K^- K^0$ in PQCD. The black blob represents the effective operator $O_1$ or $O_2$.

Fig. 3 Bounds on $|\lambda''_{qbs} \lambda'_{qds}^*|$ from the experimental upper limit of $B^- \rightarrow K^- K^0$ as a function of $\bar{q}$ mass. The upper two lines represent results without QCD corrections, the dash-dotted line with $\epsilon_B = 0.05$, the dotted line with $\epsilon_B = 0.065$. The lower two lines represent results with QCD corrections, with the solid one for $\epsilon_B = 0.05$ and the dashed one for $\epsilon_B = 0.065$. 
Fig. 2
