Semiclassical dynamics of horizons in spherically symmetric collapse

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We study a semiclassical description of the spherically symmetric gravitational collapse with a massless scalar field. An effective scenario provided by holonomy corrections from loop quantum gravity is applied to the homogeneous interior spacetime. The classical singularity that arises at the final stage of our collapsing system, is resolved and replaced by a bounce. Our main purpose is to investigate the evolution of trapped surfaces during the semiclassical collapse. We find that there exists a small threshold mass for the collapsing cloud in order for horizons to form. By employing suitable matching conditions at the boundary shell, quantum gravity effects are carried out to the exterior region, leading to an effective Vaidya geometry. In addition, the effective mass loss emerging in this model predicts an outward energy flux from the collapse interior.

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I. INTRODUCTION

There are two main aspects related to the final state of gravitational collapse of a star. The first is the singularity formation, in the sense that as the radius of the star vanishes the matter energy density diverges at its centre. The second is the evolution of the horizon during the collapse. In the latter, if trapped surfaces form as the collapse proceeds, then the final singularity will be covered by the horizon and a black hole can form. Otherwise, if such trapped surfaces do not form as the collapse evolves, the radial null geodesics emerging from the singularity can reach the distant observer and hence, singularity will be naked [1,4].

It is believed that the singularity problem will be overcome in a quantum theory of gravity. Loop quantum gravity (LQG) [5–7], as a non-perturbative and background independent approach of quantum gravity, provides a fruitful ground to investigate the removal of singularities [8]. Results from the symmetry reduction of LQG (known as loop quantum cosmology (LQC)) lead to the conclusion that the spacetime singularities is resolved in quantum gravity [9]. Within this context, the status of the classical singularities that arise at the late time stages of the spherically symmetric gravitational collapse, has been studied in LQG [10–15]. Therein, different matter such as a standard scalar field [10–12] or a tachyon field [13,14] have been considered as the collapsing matter source. By employing the quantum gravity effects (such as the inverse triad correction imported from LQG), it was shown that the geometry of spacetime near the classical singularity is regular. Furthermore, some novel features, such as evaporation of horizons in the presence of quantum gravity effects was studied in Refs. [10,12]. In addition, it was shown in Ref. [11] that (inverse triad modifications) quantum gravity effects predict a critical threshold scale for the horizon formation which may lead to the formation of very small non-singular astrophysical black holes.

Improved LQC dynamics with a massless scalar field has been developed in the last few years [17,18] (see also Ref. [19]). Concerning the physical implications of the singularity resolution in this model, it was shown that the classical big bang singularity is resolved and replaced with a quantum bounce in LQC [17,18]. In view of these elements, it is expected that the singularity arising at the end state of the gravitational collapse would be also resolved and replaced by a bounce. However, the question we address is how loop quantum effects can indeed affect on the emergence of trapped surfaces in this model.

A trapped surface in classical general relativity is defined as a compact 2-dimensional smooth space-like submanifold of spacetime such that the families of outgoing as well as ingoing future-pointing null normal geodesics are contracting [2]. When quantum geometry becomes relevant, the geodesic description stop and the classical statements, based on the properties of geodesics on a differential geometry, are not valid anymore and has to be replaced by something more appropriate from quantum theory. However, in order to extract physical information out of the theory it is of interest to employ the effective theory of LQG. Quantum dynamics of LQC, including holonomy corrections, can be approximated by an effective continuous equations of motion, which results in an effective theory of LQC [18]. The effective theory shares the form of the classical theory, but contains correction terms from the quantum theory. Furthermore, in the semiclassical regime, this scenario agrees with classical general relativity. Within such a scenario, the classical concepts (such as singularities and trapped surfaces), looks promising to be developed in quantum theory.

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To that purpose, we apply the recent results of the effective theory of LQC in the resolution of the singularities related to the gravitational collapse of a star. More precisely, we consider in this paper, a spherically symmetric framework for gravitational collapse, whose matter content includes a scalar field. We study a semiclassical scenario, provided by the holonomy corrections from LQC, for our collapsing model; classical singularity is resolved in this model and is replaced by a bounce. Then, by considering the physical conditions for the trapped surface formation, we investigate how the semiclassical corrections can affect the final state of the collapse. We are mainly interested in whether or not the final bounce at the collapse end state can be observed by the distant observer.

The context of this paper is organised as follows. In section II we provide the background scenario, by introducing the choice of spacetime geometry for the collapsing system as well as the matter source. In particular, we consider a flat Friedmann-Lemaitre-Robertson-Walker (FLRW) interior spacetime to be matched to the generalised Vaidya geometry at the boundary of matter. The matter source is considered to be a homogeneous and massless scalar field. In section III we study the semiclassical scenario for the interior spacetime by employing the holonomy corrections imported from LQC. In this section, we investigate how quantum effects influence the evolution of the trapped surfaces as the collapse evolves. In section IV by employing the matching conditions at the boundary of the collapsing cloud, we study the exterior geometry for the semiclassical interior collapse; depending on the initial conditions of the collapse, we have scenarios where the final bounce is visible to a distant observer or is covered by a non-singular black hole horizon. Finally, we present the conclusion of our results in section V.

II. GRAVITATIONAL COLLAPSE WITH A SCALAR FIELD

The physical model we consider is that of a collapsing spherical body (star), whose matter is described by a homogeneous massless scalar field. The matter is confined to a spherically symmetric region, whose coordinates are considered as \((t, r, \theta, \phi)\). In order to describe the whole spacetime structure, we assume that the spacetime is splitted into two parts: The interior region which constitutes the matter field; and the exterior region which must be matched to the interior one at the boundary with the coordinate radius \(r = r_0\).

The geometry in the interior region can be modeled by a metric of the FLRW family [10, 13]:

\[
g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) \left( dr^2 + r^2 d\Omega^2 \right),
\]

where \(d\Omega^2\) is the standard line element on the unit two sphere. We can identify every shell with its coordinate radius \(r\). The physical radius of such a shell is given by

\[
R(t, r) := a(t)r,
\]

called the area radius. In light of the canonical analysis, which differentiates \(r\) and \(t\), it is a good idea to fix the coordinate \(r\) and regard \(R(t, r)\) as a function on the gravitational phase space.

The corresponding classical Hamiltonian constraint for the interior geometry is provided by [9]

\[
C = -\frac{3}{4\pi G \gamma^2} c^2 |p| + C_{\text{matt}},
\]

where \(c := \gamma \dot{a} < 0\) and \(p := a^2\) are, respectively, conjugate connection and triad satisfying the non-vanishing Poisson bracket \(\{c, p\} = 8\pi G \gamma / 3\); moreover, \(G\) is the Newton constant, and \(\gamma \approx 0.23\) is the Barbero-Immirzi dimensionless parameter. The interior matter content is assumed to be a massless scalar field whose Hamiltonian reads

\[
C_{\text{matt}} = pV = \frac{\pi^2}{|p|^{3/2}},
\]

where \(V = |p|^{3/2}\) being the volume of the fiducial cell [9]. For a massless scalar field \(\phi\), the energy density \(\rho\) and the pressure \(p\) coincide, and can be expressed in terms of the matter dynamical variables as \(\rho = p = \pi^2 / 2|p|^{3/2}\). Notice that, since the scalar field \(\phi\) does not enter in the expression of the constraint equation \((2.3)\), its momentum \(\pi_\phi\) is a constant of motion; the matter field \(\phi\) and its conjugate momentum \(\pi_\phi\) which satisfy the Poisson bracket \(\{\phi, \pi_\phi\} = 1\) coordinatize the two dimensional matter phase space.

Solving for the Hamiltonian constraint \((2.3)\), the corresponding Einstein’s equations for the interior region can be presented [4] as

\[
8\pi G \rho = \frac{F_r}{R^2 R_r}, \quad 8\pi G p = -\frac{\dot{F}}{R^2 R_r}, \quad \dot{R}^2 = \frac{F}{R},
\]

where a ‘dot’ and ‘, r’ denote the differentiation with respect to the proper time \(t\), and to the coordinate \(r\), respectively. The mass function \(F(t, r)\) is the total gravitational mass within the shell labelled by \(r\).

In order to investigate the geometry of trapped surfaces inside the star, it is convenient to study the behaviour of the radial null geodesics emerging from the interior spacetime. The interior metric \((2.1)\) can be recast into the double null form [20] as

\[
g_{\mu\nu} dx^\mu dx^\nu = -2d\xi^+ d\xi^- + R^2 d\Omega^2,
\]

where \(\xi^\pm\) null coordinates. Then, the radial null geodesics are then obtained by solving \(g_{\mu\nu} dx^\mu dx^\nu = 0\) with the condition \(d\Omega^2 = 0\). From here we deduce that there exist two kinds of null geodesics, corresponding to \(\xi^+ = \text{const.},\) and \(\xi^- = \text{const.}\). We can compute the expansion parameters \(\theta_\phi\) for these geodesics, which measure whether the bundle of null rays normal to the sphere...
is diverging ($\theta_+ > 0$) or converging ($\theta_- < 0$) \[20\]. Introducing a new parameter $\Theta(t, r) := \theta_+ \theta_-$, we have that

$$\Theta = \frac{8}{R^2} \left( \hat{R}^2 - 1 \right).$$  \hspace{1cm} (2.7)

Then, the spacetime is said to be respectively, trapped, untrapped or marginally trapped, depending on

$$\Theta(t, r) > 0, \quad \Theta(t, r) < 0, \quad \Theta(t, r) = 0.$$ \hspace{1cm} (2.8)

The third case in Eq. (2.8) characterises the outermost boundary of the trapped region, namely, the “apparent horizon” which corresponds to the equation $\hat{R}^2 = 1$. The Eq. (2.7) is also to be thought of as a function of the phase space, for every fixed shell $r$. Specifically, we will be most interested in the boundary shell, $r = r_b$, which bounds the support of matter. In that case, we define

$$\Theta_b(t) := \Theta(t, r_b) = \frac{8}{a^2} \left( a^2 - \frac{1}{a^2} \right).$$ \hspace{1cm} (2.9)

Since we are mainly interested in the trapped surfaces eventual formation due to the gravitational collapse of the interior spacetime, we assume that the star is not trapped from the initial configuration at $t_0$; in other words, $\Theta(t_0, r) < 0$ for all shells $0 < r < r_b$. At this classical level, it is possible to solve the Hamilton equation for the field $\phi$, analytically, where $\phi = \phi_{\text{eff}} / |p|^{3/2}$. We then get

$$\phi = \pm \sqrt{\frac{3}{16\pi G}} \ln \frac{|p|}{|p_0|} + \phi_0.$$ \hspace{1cm} (2.10)

Here $(\phi_0, p_0)$ are integration constants describing the initial conditions for the collapsing star at the $t = t_0$ space slice. We will see on the next section that, for $\phi \to \infty$, it is $p = 0$, i.e., the volume of the interior region vanishes; but since the matter is contained in such region, the energy density of the cloud diverges, producing a physical singularity. We will see that once quantum corrections are taken into account, the situation changes completely.

To model the exterior geometry, we choose a metric of the Vaidya family\[1\]. Written in advanced Eddington-Finkelstein coordinates $(v, r_v, \theta, \phi)$, it has the form \[21\]

$$g_{\mu\nu}dx^\mu dx^\nu = -(1 - 2M(v)G/r_v) dv^2 - 2dvdr_v + r_v^2d\Omega^2,$$ \hspace{1cm} (2.11)

where $M(v)$ is a generic function of $v$, which is fixed by matching the Eq. (2.11) with Eq. (2.1) at the boundary $r = r_b$ (for a discussion, see \[10\] \[13\]). The matching condition is given by matching the area radius at the boundary $\Sigma$ \[4\]:

$$r_v(v) = R(r_b, t) = r_b a(t),$$ \hspace{1cm} (2.12)

together with the first and second fundamental forms

$$\left( \frac{dv}{dt} \right)_\Sigma = \frac{R_r + r_b \dot{a}}{1 - \frac{F}{R}},$$ \hspace{1cm} (2.13)

$$F(t, r_b) = 2M(r_b, v)G,$$ \hspace{1cm} (2.14)

$$GM(r_v, v)_v = \frac{F}{2R} + r_v^2 \dot{a} \ddot{a}.$$ \hspace{1cm} (2.15)

It should be noted that the singularity formation at $a = 0$ is independent of these matching conditions.

Matching the exterior generalised Vaidya family to the interior spacetime plays two important roles in a collapsing process: Firstly, it allows the matter to be radiated away as collapse evolves; and secondly, it enables the study of formation and evolution of horizon during the collapse. The second aspect is particularly important; indeed, formation of a black hole as the end state of a collapsing star indicates that there exists a moment when an apparent horizon develops inside the cloud, and hence all the matter content collapses inside such horizon. Otherwise, if the end state is not a black hole, it means that trapped surfaces never develop at any stage of the collapse, and hence apparent horizons never form inside the star. In this paper we are mainly concerned with whether or not trapped surfaces can form in the interior region, once quantum gravity corrections are taken into account.

### III. SEMICLASSICAL SCENARIO

On this section, we discuss the quantum gravity induced corrections to the classical evolution presented in previous section. To do this, we follow an effective scenario provided by the holonomy corrections imported from LQG for the spherically symmetric model \[21\].

In LQC, the algebra generated by the holonomy of phase space variables $c$ is just the algebra of the almost periodic function of $c$, i.e., $e^{iuc/\mu}$ (where $\mu$ is assigned as the kinematical length of the square loop since its order of magnitude is similar to that length), which together with $p$, constitutes the fundamental canonical variables in quantum theory \[29\]. This consists in replacing $c^2$ by a $\sin^2(\mu c)/\mu^2$ term in Eq. (2.3); hence we have \[18\] \[23\]

$$C_{\text{eff}} = -\frac{3}{4\pi G \mu^2 \gamma} \sqrt{|p|} \sin^2(\mu c) + C_{\text{mat}}.$$ \hspace{1cm} (3.1)

The dynamics of the fundamental variables is then obtained by solving the system of Hamilton equations; i.e.,

$$\dot{p} = \{ p, C_{\text{eff}} \} = -\frac{8\pi G \gamma}{3} \frac{\partial C_{\text{eff}}}{\partial c},$$ \hspace{1cm} (3.2)

$$= \frac{2\sqrt{|p|}}{\gamma \mu} \sin(\mu c) \cos(\mu c),$$

together with the vanishing Hamiltonian constraint (3.1), which subsequently results in a modified Friedmann

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\[1\] This is a generalization of Schwarzschild metric which accounts for possible matter emissions, and realizes the astrophysical realistic case of a star surrounded by a radiating zone \[27\].
The classical Friedmann equation corresponds to the last relation in the classical Einstein’s field equation \( H^2 = F/R^2 \). Consequently, since, in the semiclassical regime the Friedmann equation is modified as Eq. (3.3), this might lead to the modifications of the mass function in Eq. (2.5). In other words, we can introduce an effective mass function \( F_{\text{eff}} \) corresponding to the modified Friedmann equation (3.3) as

\[
F_{\text{eff}} = \frac{8\pi G}{3} \rho_{\text{eff}} R^3.
\]

This describes an effective geometry on which the phase space trajectories are considered to be classical, whereas the matter content is assumed to be modified by quantum gravity effects. In the classical limit, as \( \rho_{\text{eff}} \to \rho \), the effective mass function reduces to the classical one \( F = (8\pi G/3)\rho R^3 \). The Eq. (3.4) shows that, if \( \rho_0 < \rho < \rho_{\text{cr}} \) on the interior region, the effective mass function remains finite during the collapse. It is convenient to rewrite the Eq. (3.4) as

\[
F_{\text{eff}} = F \left( 1 - \frac{F^2}{F_{\text{cr}}^2} \right),
\]

in which, for a massless scalar field we have used

\[
\frac{\rho}{\rho_{\text{cr}}} = \frac{F^2}{F_{\text{cr}}^2}, \quad (3.6)
\]

where \( F_{\text{cr}} := \frac{8\pi G\rho_{\text{cr}}^2}{3}\sqrt{\rho_{\text{cr}}/3\sqrt{2}} = \text{const} \). The Eq. (3.5) shows that, mass function \( F \) changes in the interval \( F_0 < F < F_{\text{cr}} \) along with the collapse dynamical evolution. Nevertheless, if on the initial condition the mass function starts with \( F < R \), then as the scale factor decreases, the mass function also decreases and becomes zero at the bounce; consequently, there will be no trapped surfaces forming.

To discuss the dynamics of the trapped surfaces in the semiclassical regime herein, particular importance is played by the function \( \Theta_b \) defined in (2.9). Therein, by replacing \( \dot{a}/a \) with the effective Friedmann equation (3.3) we get

\[
\Theta_b(t) = \frac{64\pi G}{3} \rho \left( 1 - \frac{\rho}{\rho_{\text{cr}}} \right) - \frac{8}{a^2 r_b^2} \quad (3.7)
\]

We assume that the cloud is initially untrapped, at the initial space slice where \( \rho \ll \rho_{\text{cr}} \), thus, \( \Theta_b(t=0) < 0 \). Now, we can study the behaviour of the effective \( \Theta_b \) as a function of energy density \( \rho \). Let us rewrite Eq. (3.7) by setting \( X := \rho/\rho_{\text{cr}} \)

\[
\Theta_b(X) = AX (1 - X) - BX^{1/3}, \quad (3.8)
\]

where \( A := (64\pi G/3)\rho_{\text{cr}} \) and \( B := 8(2\rho_{\text{cr}}/\pi^2)^{1/3}/r_b^2 \) are constants. Behaviour of \( \Theta_b \) with respect to \( X \), for the different choices of the initial conditions, is given by figure 2. Therein, the solid curves represent the trajectories provided by the semiclassical gravitational collapse; whereas the dashed curve shows the classical trajectories which coincides with the semiclassical ones for \( X < 1 \). The equation for the apparent horizon on the effective geometry can be obtained by equating (3.8) to zero. Then, we get

\[
X^2 (1 - X)^3 - \left( \frac{B}{A} \right)^3 = 0 \quad (3.9)
\]

Solving this equation, we obtain the values of energy density at which apparent horizons form. This corresponds to the intersections of the \( \Theta_b \) curve with the horizontal axe in figure 2. Therefore, depending on the initial conditions, in particular on the choice of the \( r_0 \), three cases can be evaluated, which correspond to no apparent horizon formation, one and two horizon formations. Notice that, denoted by a dashed curve, always only one horizon can form classically. Let us to be more precise as follows.
The total mass $m$ inside the collapsing star is given by the integral of the energy density on a sphere of radius $R_b = ar_b$; hence we have that $m = (2\pi/3a_0^3)\pi^2 r^3_b$. Thus, we can translate the condition $r_b \geq r_\ast$, on the radius, into a condition on the mass of star. Let us define a $m_\ast$ to be the mass of a star with the shell radius $r_\ast$:

$$m_\ast := \frac{25}{48a_0^3} \left( \frac{5}{\pi G} \right) \frac{\phi}{\rho_\ast}.$$  \hfill (3.13)

Therefore, in order for trapped surfaces to form, the mass, $m$, of a collapsing star must be bigger than the threshold mass $m_\ast$. In other words, the minimum mass of the final black hole must be bigger than the threshold mass, $m \geq m_\ast$. This gives a threshold mass for the micro black holes, which are expected to form e.g., in the primordial universe, as well as in the LHC [25].

IV. FINAL OUTCOMES OF THE SEMICLASSICAL COLLAPSE

So far, we have analysed the interior spacetime of the collapse in the presence of the quantum gravity effect. This quantum effect is expected to be carried out to the exterior geometry by using the matching conditions on the boundary $r_b$ of two regions. In the following we will focus on the main physical consequences that can emerge from this scenario in order to predict the possible exterior geometry for the collapse. To this aim, we will consider two cases: First, we will investigate the fate of the collapsing star whose mass is less than the threshold mass $m_\ast$. Secondly, we will study the exterior geometry for the case in which the mass of the star is bigger than the threshold mass whereby a black hole can form.

A. Outward flux of energy

For a collapsing star whose mass is less than $m_\ast$, we study the modified mass loss due to the semiclassical corrections to the interior spacetime. Let us designate the mass function at scales $\rho \ll \rho_\ast$, i.e., in the classical regime, as $F = (8\pi G/3)\rho R^3$, whereas for $\rho \approx \rho_\ast$ (in the semiclassical regime) we use $F_{\text{eff}}$ given by Eq. (3.5). Then, the mass loss, $\Delta F/F$, is provided by the following expression:

$$\frac{\Delta F}{F} = 1 - \frac{F_{\text{eff}}}{F} = \frac{\rho}{\rho_\ast}.$$ \hfill (4.1)

This equation can be interpreted as follows: If $\rho \ll \rho_\ast$, i.e., at the classical limit, then $\Delta F \approx 0$. However, when $\rho \rightarrow \rho_\ast$, i.e., at the bouncing point, we have $\Delta F/F \rightarrow 1$; this means that quantum gravity corrections to the interior region give rise to an outward flux of energy towards the bounce in the semiclassical regime.

It is worthwhile to mention that, within an inverse triad correction to the collapsing system (with a scalar...
field \[10\], or a tachyon field \[13\], as matter sources), the (quantum) modified energy density decreases as collapse evolves. Whence, as the collapsing cloud approaches the center (with a vanishing scale factor, where the classical singularity is located) the energy density reaches its minimum value, whereas the mass loss tends to one. In the model herein, as the gravitational collapse proceeds in the semiclassical regime, the energy density increases and reaches its maximum value \(\rho_{\text{cr}}\) at the bounce (with a finite non-zero volume). In this process, however, the mass loss tends to unit earlier, and before that the collapse reaches to the center of star.

Let us assume that the energy density flux is measured locally by an observer with a four-velocity vector \(\xi^\mu\). Then, the energy flux and the radiation energy density are measured in this local frame and given by \(\sigma \equiv T_{\mu\nu}\xi^\mu\xi^\nu\). Furthermore, the total luminosity for a radially moving observer with radial velocity \(\vartheta \equiv \xi^r = \frac{dr}{dt}\) at the radius \(r_c\), is given by \(L(v) = 4\pi r_c^2 \vartheta\) \[20\], so that:

\[
L(v) = -\frac{1}{(\dot{\vartheta} + \vartheta)^2} \frac{dM(v)}{dv}, \quad (4.2)
\]

where \(\dot{\vartheta} = (1+\vartheta^2-2M(v)/r_c)^{-1}\). As long as \(dM/dv \leq 0\), the total luminosity of the energy flux is positive; this indicates that there exist an energy flux radiated away from interior spacetime and reaching the distant observer. Moreover, from \(\xi_\nu \xi^\nu = -1\) and \(\xi^\vartheta = \xi^v = 0\), we have

\[
\frac{dv}{dt} = \xi^v = \frac{1}{\dot{\vartheta} + \vartheta}. \quad (4.3)
\]

Now, using Eqs. (4.2) and (4.3), and substituting \(M(v)\) with \(M = F(t,r_b)/2\) at the boundary \(\Sigma\) we obtain,

\[
L(v) = -\left(\frac{\dot{F}_{\text{eff}}}{2}\right) \frac{1}{(\dot{\vartheta} + \vartheta)}. \quad (4.4)
\]

For an observer at rest \((\vartheta = 0)\) and infinitely distant \((r_v \rightarrow \infty)\), the total luminosity of the energy flux \(L_\infty\), can be obtained by taking the limit of Eq. (4.4):

\[
L_\infty(t) = -\left(\frac{\dot{F}_{\text{eff}}}{2}\right). \quad (4.5)
\]

Using Eq. (3.4) in Eq. (4.5), we can estimate the time variation of the mass function as

\[
\dot{F}_{\text{eff}} = -8\pi GHR^3 \rho \left(1 - 3\frac{\rho}{\rho_{\text{cr}}}\right) \quad (4.6)
= 8\pi GHR^3 \left(\frac{8\pi G}{3}\right)^{\frac{1}{2}} \rho \left(1 - 3\frac{\rho}{\rho_{\text{cr}}}\right). \quad (4.7)
\]

Using the Raychaudhuri equation we can define the effective pressure for the massless scalar field as \[17, 19\]

\[
p_{\text{eff}} := \rho \left(1 - 3\frac{\rho}{\rho_{\text{cr}}}\right). \quad (4.8)
\]

Figure 3: Behaviour of the time derivative of the mass function, \(\dot{F}\), in the semiclassical regime (from Eq. (1.9) for the value of parameters \(G \approx c_{\text{light}} \approx 1\), and \(\pi_\rho \approx 1000\). The solid curve is for effective dynamics, whereas the dashed curve is for the classical counterpart.

Inserting Eq. (4.8) in Eq. (4.7), the time derivative of the mass function can also be expressed as

\[
\dot{F}_{\text{eff}} = 8\pi G\rho R^3 \left(\frac{8\pi G}{3}\rho_{\text{eff}}\right)^{\frac{1}{2}} \rho_{\text{eff}}. \quad (4.9)
\]

The classical limit of Eq. (4.9) is

\[
\dot{F} = 8\pi G\rho R^3 \left(\frac{8\pi G}{3}\rho\right)^{\frac{1}{2}} = -8\pi G\rho R^2 \dot{R}, \quad (4.10)
\]

as expected from Eq. (2.5). The Eq. (4.9) shows that in late time stages of the gravitational collapse, the mass function decreases when the effective pressure becomes negative. In contrast, the classical system shows a continuous increase of the mass function, with an increasing (positive) pressure, pointing to a black hole end state. In figure 3 we have a graphical representation of Eq. (4.9) during the collapse. Therein, we have that the effective mass function’s time derivative, being initially positive (like its classical counterpart), starts to decrease until it becomes negative and then vanishes at the bounce. At this stage, as \(L_\infty \approx -\dot{F}_{\text{eff}}\), the luminosity positiveness is related to the mass loss occurring near the bounce. The existence of a negative \(\dot{F}_{\text{eff}}\), with some substantial variation in the vicinity of the bounce, points to the existence of an energy flux radiated away from the interior spacetime and reaching the distant observer \[20\].

Figure 3 presents the behaviour of the effective pressure (4.8) conveniently scaled with the critical density \(\rho_{\text{cr}}\). Therein we should stress that the effective pressure becomes super negative near the bounce and takes the value \(p_{\text{eff}}(\rho_{\text{cr}}) = -2\rho_{\text{cr}}\) at the bounce. This verifies the general understanding derived from the homogeneous and isotropic models indicating that, the singularity resolution is associated with the violation of energy con-
Figure 4: Behaviour of the effective pressure (from Eq. (4.8)) for the value of parameters $G = c_{\text{light}} = 1$, and $\pi_{\phi} = 10000$. At the bounce we have $p_{\text{eff}}(\rho_{cr}) = -2\rho_{cr}$. The solid curve denotes the semiclassical collapse, whereas the dashed curve presents the classical counterpart.

This suggests that quantum gravity provides a repulsive force at the very short distances [27].

B. Non-singular black hole formation

If the initial mass of the collapsing star is bigger than the threshold mass $m_*$, given by Eq. (3.13), then a black hole would form at the collapse final state. We will now analyse a possible prediction for the exterior geometry of the collapsing system in this case. The total mass measured by an asymptotic observer is given by $m_{\text{ext}} = m_M + m_\phi$, where $m_M$ is the total mass in the generalized Vaidya region, and $m_\phi = \int \rho dV$ is the interior mass related to the scalar field $\phi$. Since the matter related $m_M$ is not specified in the exterior Vaidya geometry in our model, we will focus on a qualitative analysis of behaviour of the horizon close to the matter shells.

From the matching conditions (2.12)-(2.15), we can get the information regarding the behaviour of trapping horizons in the exterior region. Indeed, when the relation $2M(v,r_v)G = r_v$ is satisfied at the boundary, trapped surfaces will form in the exterior region close to the matter shells. On the classical geometry, the boundary function, $F = 1 - 2M(v)G/r_v$, becomes negative in the trapped region and vanishes on the apparent horizon. Therefore, the equation for event horizon is given at the boundary of the collapsing body by $F |_{\Sigma} = 0$. Nevertheless, in semiclassical regime, the boundary function is expected to be modified by employing the matching conditions due to the fact that the interior spacetime was modified by the quantum gravity effects. Using the conditions (2.12) and (2.14), the classical function $F$ can be written as

$$F |_{\Sigma} = 1 - \frac{F}{R}, \quad \text{(4.11)}$$

where we have used $2M(v,r_v)G/r_v = F(t)/R(t)$ at the boundary surface $\Sigma$ with $r = r_b$. In the semiclassical regime, the mass function is modified as Eq. (3.4), whereby the boundary function (4.11) is modified as

$$F_{\text{eff}} = 1 - \frac{F}{R} \left(1 - \frac{F^2}{F_{\text{cr}}^2}\right), \quad \text{(4.12)}$$

at the boundary shell $r_b$. Eq. (4.12) shows that the quantum gravity induced effects lead to a modification of the boundary function by a cubic term $F^3$.

By substituting the classical mass function with $F = (8\pi G/3)\rho R^3$, we can rewrite the Eq. (4.12) as

$$F_{\text{eff}}(R) = 1 - \frac{C}{R^3} \left(1 - \frac{D}{R^6}\right), \quad \text{(4.13)}$$

where $C \equiv (4\pi G/3)\pi_{\phi}^2 r_b^6$, and $D \equiv \pi_{\phi}^2 r_b^6/(2\rho_{cr})$ are constants. Eq. (4.13) represents a non-singular, exotic black hole geometry. Notice that, this function for large values of $R$ tends to the classical limit:

$$F(R) = 1 - \frac{C}{R^3}, \quad \text{(4.14)}$$

which represents a classical singular black hole geometry [12]. In addition, as we expected, in the presence of a nonzero matter pressure (of the massless scalar field) at the boundary, the (homogeneous) interior spacetime is not matched with an empty (inhomogeneous) Schwarzschild exterior [11].

V. CONCLUSIONS AND DISCUSSION

We considered a spherically symmetric and homogeneous spacetime for gravitational collapse whose matter content is a massless scalar field. The homogeneous interior is matched with an exterior Vaidya geometry. We subsequently studied the interior spacetime within the effective theory of loop quantum gravity, provided by the holonomy corrections to the dynamics of the collapse [18]. It was shown that loop quantum effects remove the classical singularity that arises at the end state of the collapse, and replace it by a bounce. Furthermore, we investigated the evolution of the trapped surfaces emerging from the semiclassical interior spacetime. The physical modifications related to the semiclassical regime provided three cases for the trapped surfaces formation, depending on the initial conditions of the collapsing star. In particular, our solutions showed that, if the initial mass of the collapsing cloud is less than a threshold mass, namely $m_*$, no horizon forms during the collapse, whereas for the mass equal and larger than the threshold mass, one and two horizons form, respectively. It is worthy to mention that, this scenario is qualitatively similar to the model previously predicted from an inverse triad correction [11].

The interior semiclassical effects is carried out to the exterior geometry by the matching conditions on the
boundary of two regions. Hence, an effective geometry emerged for the exterior metric which describes the physical consequences for the late time evolution of the collapse. In the case in which no horizon forms, we have shown that, as the collapse evolves, the energy density and the mass function increase towards the maxima \( \rho_{\text{cr}} \) and \( F_{\text{cr}} \), respectively. However, this energy growth is accompanied by the effective mass loss, induced by the quantum gravity effects. Therefore, in the late time stages of the collapse, it gives rise to an outward flux of energy from the interior semiclassical spacetime to the distant observer. Similar results was discussed for a scalar field collapse, where an inverse triad modification was employed [10]. However, the mass loss obtained therein, was characterized by a reduction of the energy density and the mass function towards the center of the star. It has been proposed that gamma-ray bursts may occur at the final state of the collapse [28]. In our model herein, although the effective theory is not accurate in a deep Planck regime, there might be some stages of the gravitational collapse where the effective theory becomes relevant. Therefore, from such a scenario it could be observed some of those outward flux of energy from gamma-ray bursters. In addition, in the cases in which one or two horizons form, the resulting exterior geometry corresponds to an exotic non-singular black hole which is different than the Schwarzschild spacetime [11, 12].

Modification to the mass function through the (interior) quantum gravity effects at the boundary of two regions, results in an effective exterior Vaidya geometry. More precisely, from Eq. (3.5) we have that \( F_{\text{eff}} = F (G_{\text{eff}} / G) \). Therefore, using Eq. (2.14), we get \( F_{\text{eff}} = 2MG_{\text{eff}} \), so that \( M_{\text{eff}} G = MG_{\text{eff}} \). Consequently, the Vaidya metric (2.11) takes the effective form:

\[
\tilde{g}_{\mu \nu} d\tau^\mu d\tau^\nu = - (1 - 2M(\nu)G_{\text{eff}} / r_{\nu}) d\tau^2 - 2d\nu dr_{\nu} + r_{\nu}^2 d\Omega^2, \tag{5.1}
\]

The effective Vaidya solution (5.1) shows that the semiclassical (interior) spacetime is manifest through a modification to the Newton constant \( G \) as \( G (1 - \rho / \rho_{\text{cr}}) \). Similar scenario was presented in the recent works [29, 31]. Therein, it was shown that, in situations where the interior spacetime undergoes a transition from positive to negative pressures, a quantum improvement to the Vaidya outgoing solution should be implemented. Nevertheless, this effective outgoing Vaidya solution, results in a modification of the Newton constant as \( G (r_{\nu}) \) being a scale dependent function [29]. The redefinition of the Newton constant in this context was introduced to model quantum effects and avoid the problem of unbound back scattered radiation [29]. Furthermore, it is seen from Eq. (5.1) that near the bounce, as \( \rho \rightarrow \rho_{\text{cr}} \), then \( 2M(\nu)G_{\text{eff}} / r_{\nu} \rightarrow 0 \). Hence, the (effective) exterior metric (5.1) along with the bounce, smoothly transforms to

\[
\tilde{g}_{\mu \nu} d\tau^\mu d\tau^\nu = - d\tau^2 - 2d\nu dr_{\nu} + r_{\nu}^2 d\Omega^2, \tag{5.2}
\]

which describes a Minkowski spacetime in retarded null coordinate. From the modified equation, corresponding to Eq. (2.14) in this case, it is seen that the (effective) Vaidya geodesic, which emerges from the bounce before it evaporates into free space, is null. It follows that the non-space-like trajectories can come out from the bounce and reach the exterior observers at the final stages of the gravitational collapse. Hence, the final bounce will be naked in the case in which the occurrence of trapped surfaces is avoided (cf. section IV A).

The qualitative picture that emerges from our toy model was influenced by a homogeneous interior spacetime. Nevertheless, in a realistic collapsing scenario one has to employ a more general inhomogeneous setting (see Ref. [32, 33] for recent development of techniques to handle inhomogeneous systems, which gives promising indications). Furthermore, the effective theory that lead to a modified homogeneous dynamics, for the interior spacetime, may also modify the spacetime inhomogeneous structure [34]. On the other hand, using the homogeneous techniques, only the interior spacetime carries quantum effects while the outside is still described by a generalized Vaidya metric of general relativity. Some quantum effects are transported to the outside by matching at the boundary surface, which then enter the Vaidya solution effectively through a non-standard energy-momentum tensor. This is still more indirect than considering a complete inhomogeneous quantization for the exterior region which may provide significant modifications to the spacetime structure. These effects may not be captured for a general Vaidya mass in a spacetime line element [35]. Nonetheless, it is difficult to calculate such effects, and we are still far from a complete picture.

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[1] S. W. Hawking, G. F. R. Ellis, The Large Scale Structure of Space-Time, (Cambridge University Press, 1974).
[2] R. Penrose, Phys. Rev. Lett. 14, 57 (1965).
[3] S. W. Hawking Proc. Roy. Soc. Lond. A 300, 187 (1967);
S. W. Hawking and R. Penrose Proc. Roy. Soc. Lond. A 314, 529 (1970).
[4] P. Joshi, Gravitational Collapse and Spacetime Singularities, (Cambridge University Press, 2007).
[5] A. Ashtekar, J. Lewandowski, *Background Independent Quantum Gravity: A Status Report*, Class. Quant. Grav. **21**: R 53 (2004), [arXiv: gr-qc/0404018).
[6] Carlo Rovelli, *Quantum Gravity*, (Cambridge University Press, Cambridge, England, 2004).
[7] Thomas Thiemann, *Introduction to Modern Canonical Quantum General Relativity*, (Cambridge University Press, Cambridge, England, 2007).
[8] Martin Bojowald, *Canonical Gravity and Applications: Cosmology, Black Holes, and Quantum Gravity* (Cambridge University Press, 2010).
[9] Abhay Ashtekar, Martin Bojowald, Jerzy Lewandowski, *Mathematical structure of loop quantum cosmology*, Adv. Theor. Math. Phys. **7**, 233 (2003).
[10] R. Goswami, P. S. Joshi and P. Singh, Phys. Rev. Lett. **96**, 031302 (2006).
[11] M. Bojowald, R. Goswami, R. Maartens, P. Singh, Phys. Rev. Lett. **95**, 091302 (2005).
[12] Benjamin K. Tippett and Viqar Husain, Phys. Rev. D **84**, 104031 (2011).
[13] Yaser Tavakoli, João Marto, Amir Hadi Ziaie, and Paulo Vargas Moniz, Phys. Rev. D **80**, 084002 (2009); M. Bojowald, T. Harada, R. Tubrewala, Phys. Rev. D **78**, 064057 (2008).
[14] L. Modesto, Phys. Rev. D **70**, 124009 (2004); [arXiv:gr-qc/ 0504043]; Int. J. Theor. Phys. **47**, 357 (2008).
[15] Yaser Tavakoli, joão Marto, Amir Hadi Ziaie, and Paulo Vargas Moniz, Gen. Rel. Gravitation **45**, 819-844 (2013).
[16] Abhay Ashtekar, Tomasz Pawłowski, and Parampreet Singh, Phys. Rev. D **73**, 124038 (2006).
[17] Abhay Ashtekar, Tomasz Pawłowski, and Parampreet Singh, Phys. Rev. D **74**, 084003 (2006).
[18] *Loop quantum cosmology: a status report*, Class. Quantum Grav. **28**, 213001 (2011).
[19] S. A. Hayward, Phys. Rev. D **53**, 1938 (1996).
[20] P. C. Vaidya, *The External Field of a Radiating Star In General Relativity*, Curr. Sci. 12 (1943) 183; P. C. Vaidya, *The Gravitational Field of a Radiating Star*, Proc. Indian Acad. Sci. A. 33 (1951) 264; P. C. Vaidya, *Newtonian Time In General Relativity*, Nature, 171 (1953) 260.
[21] Thomas Thiemann, *Quantum Spin Dynamics (QSD)*, Class. Quant. Grav. **15** 839 (1998).
[22] V. Taveras, IGPG preprint (2006).
[23] S. Hayward, Phys. Rev. D **49**, 6467 (1994); A. Ashtekar and B. Krishnan, Phys. Rev. Lett. **89**, 261101 (2002).
[24] R. Casadio and B. Harms, Int. J. Mod. Phys. A **17**, 4635 (2002).
[25] R. W. Lindquist, R. A. Schwartz, and C. W. Misner, Phys. Rev. B **137**, 1364 (1965).
[26] Parampreet Singh, *Effective State Metamorphosis in Semi- Classical Loop Quantum Cosmology*, Class. Quant. Grav. **22**, 4203 (2005).
[27] P. Joshi, N. Dadhich, R. Maartens, *Gamma-ray bursts as the birth cries of black holes*, Mod. Phys. Lett. A **15**, 991 (2000).
[28] F. Fayos and R. Torres, Class. Quantum Grav. **28** 105004 (2011).
[29] F. Fayos and R. Torres, Class. Quantum Grav. **21** 1351–1370 (2004).
[30] F. Fayos and R. Torres, Class. Quantum Grav. **22** 4335–4354 (2005).
[31] M. Bojowald, R. Swiderski, Class. Quantum Grav. **23**, 2129 (2006).
[32] M. Campiglia, R. Gambini and J. Pullin, Class. Quantum Grav. **23**, 3649 (2007).
[33] M. Bojowald, G. M. Paily, J. D. Reyes and R. Tubrewala, Class. Quantum Grav. **28**, 185006 (2011).