Mixed-ADC Massive MIMO

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Abstract

Motivated by the demand for energy-efficient communication solutions in the next generation cellular network, a mixed receiver architecture for massive multiple input multiple output (MIMO) systems is proposed, which differs from previous works in that herein one-bit analog-to-digital converters (ADCs) partially replace the conventionally assumed high-resolution ADCs. The information-theoretic tool of generalized mutual information (GMI) is exploited to analyze the achievable data rates of the proposed system architecture and an array of analytical results of engineering interest are obtained. For deterministic single input multiple output (SIMO) channels, a closed-form expression of the GMI is derived, based on which the linear combiner is optimized. The analysis is then extended to ergodic fading channels, for which both lower and upper bounds of the GMI are obtained. Impacts of dithering and imperfect channel state information (CSI) are also investigated, and it is shown that dithering can remarkably improve the system performance while imperfect CSI only introduces a marginal rate loss. Finally, the analytical framework is applied to the multi-user access scenario, and the corresponding numerical results demonstrate that the mixed system architecture with a relatively small number of high-resolution ADCs is able to achieve a large fraction of the channel capacity without output quantization.

Index Terms

Analog-to-digital converter, dithering, generalized mutual information, massive MIMO, mixed architecture, multi-user access.

I. INTRODUCTION

The prosperity of mobile Internet calls for new technologies to meet the exponential increase in demand for mobile data traffic. In recent years, a heightened attention has been focused on massive multiple input multiple output (MIMO) systems, in which each base station (BS) is equipped with hundreds of antennas and serves tens or more users simultaneously [1], [2]. Because the large number of BS antennas can effectively average out noise, fading and to some extent, noncoherent interference,
massive MIMO achieves significant gains in both energy efficiency and spectral efficiency, and thus are envisioned as a promising key enabler for the next generation cellular network [3], [4].

Thus far, most of the literature on massive MIMO assume perfect hardware implementation. However, this assumption is not well justified, since the hardware cost and circuit power consumption scale linearly with the number of BS antennas and thus soon become economically unbearable unless low-cost, energy-efficient hardware is deployed which however easily suffers from impairments. Assuming an additive stochastic impairment model, the authors of [5] examined the impact of hardware impairments on both energy efficiency and spectral efficiency of massive MIMO systems. The authors of [6] obtained scaling law that describes how fast the tolerance levels of impairments increase with the number of BS antennas while reaping much of the performance gain promised by massive MIMO. The authors of [7] examined the accuracy of widely used additive or multiplicative stochastic impairment models by providing a more accurate hardware-specific deterministic model and performing comparative numerical studies.

Among various sources of hardware impairment, low-resolution analog-to-digital converters (ADCs) have attracted ubiquitous attention due to their favorable property of low cost, low power consumption and feasibility of implementation [8], [9]. For Nyquist-sampled real Gaussian channel, the authors of [10] established some general results regarding low-resolution output quantization, showing that for a quantizer with $Q$ bins, the capacity-achieving input alphabet should be discrete and need not have more than $Q$ mass points. The authors of [11] designed a modified minimum mean square error (MMSE) receiver for MIMO systems with output quantization and proposed a lower bound to the capacity. In [12], the authors investigated a practical monobit digital receiver paradigm for impulse radio ultra-wideband (UWB) systems. Recently, the authors of [13] examined the impact of one-bit quantization on achievable rates of massive MIMO systems with both perfect and estimated channel state information (CSI). The authors of [14] addressed the high signal-to-noise (SNR) capacities of both single input multiple output (SIMO) and MIMO channels with one-bit quantization, demonstrating that for a SIMO channel, the high SNR capacity increases linearly with the logarithm of the number of receive antennas, and that for an MIMO channel, the high SNR capacity is lower bounded by the rank of the channel.

Despite of its great superiority in deployment cost and energy efficiency, one-bit quantization generally has to tolerate large rate loss, especially in the high SNR regime [14], thus highlighting the indispensability of high-resolution ADC for digital receiver. Besides, the great overhead of pilot-aided channel estimation under one-bit quantization is also a big concern [12], [13], [15]. Thus motivated by such consideration, in this paper we propose a mixed receiver architecture for massive MIMO systems.
in which one-bit ADCs partially, but not completely, replace conventionally assumed high-resolution ADCs. This architecture has the potential of allowing us to remarkably reduce the hardware cost and power consumption while still maintain a large fraction of the performance gains promised by massive MIMO.

For such mixed-ADC massive MIMO systems, though the channel capacity is still the maximization of mutual information between the channel input and the quantized channel output vector, from an engineering perspective, however, the mutual information maximization problem appears to be a formidable task, and unsatisfactory in providing engineering insights, especially considering the high-dimensional channel law and the mixed nature of the channel output vector. Recognizing this challenge, we take an alternative route and seek to characterize the achievable data rates specified to certain encoding/decoding scheme. To this end, we exploit the information-theoretic tool of generalized mutual information (GMI) [16], [17] to address the achievable data rates of our proposed system architecture. As a performance measure for mismatched decoding, GMI has proved convenient and useful in several important scenarios such as fading channels with imperfect CSI at the receiver [17], channels with transceiver distortion [18], [19] and analysis of bit-interleaved coded modulation [20].

Exploiting a general analytical framework developed in [18], we obtain a series of analytical results. First, we consider a deterministic SIMO channel where the BS is equipped with $N$ antennas but only has access to $K$ pairs\(^1\) of high-resolution ADCs and $(N - K)$ pairs of one-bit ADCs, and derive a closed-form expression of the GMI for general ADC assignment and linear combiner. This enables us to optimize the linear combiner design and further explore the asymptotic behaviors of the GMI in both low and high SNR regimes that in turn help us suggest a plausible ADC assignment scheme. The corresponding numerical results indicate that even with a small number of high-resolution ADCs, our system architecture already achieves a substantial fraction of the channel capacity without output quantization, thus verifying the effectiveness of the mixed architecture. The analysis is also extended to the scenario of ergodic fading channels where, instead of directly working with the exact GMI, we derive lower and upper bounds of the GMI, which are shown to be very tight in numerical results.

Then, we examine the benefit of dithering. Dithering has been proved of great effectiveness in enhancing the performance of signal parameter estimation involving low-resolution quantization, e.g., [21]. Generally speaking, uniform dithering leads to the best convergence rate amongst a broad class of dither distributions [22], but Gaussian dithering on the other hand has its advantage of analytical convenience. Therefore we propose a simple but effective dithering scheme which injects additional Gaussian noise into an one-bit ADC provided that the receive SNR at the corresponding antenna

\(^1\)A pair of ADCs quantize the I and Q components of an antenna, respectively.
exceeds a prescribed threshold $T_{\text{opt}}$, so that the receive SNR after dithering is pulled back to $T_{\text{opt}}$. Numerical results show that this dithering scheme achieves remarkable rate gain, especially for the case of small $K$.

Subsequently, we evaluate the robustness of the mixed architecture against imperfect CSI. We only utilize the high-resolution ADCs to perform channel estimation, and thus the deduced estimation error is Gaussian distributed, allowing us to analytically characterize the resulting GMI. Following an argument similar to that in the ergodic fading channel scenario, we derive lower and upper bounds for the GMI. Numerical results show that the lower and upper bounds are again very tight and that there is only a marginal rate loss due to imperfect CSI.

Finally, we apply our analysis to the multi-user access scenario. Numerical results reveal that the mixed system architecture with a small number of high-resolution ADCs achieves a large fraction of the channel capacity without output quantization, provided that the multi-user system is properly loaded.

In summary, the proposed mixed architecture strikes a reasonable and attractive balance between cost and spectral efficiency, for both single-user and multi-user scenarios, and for both perfect CSI and imperfect CSI. Thus we envision it as a promising receiver paradigm for energy-efficient massive MIMO systems.

The remaining part of this paper is organized as follows. Section II outlines the system model. Adopting GMI as the performance metric, Section III establishes the theoretical framework for deterministic SIMO channels, based on which the optimal linear combiner design and the asymptotic behaviors of the GMI at both low and high SNR regimes are explored. Besides, extension of the theoretical framework to ergodic fading channels is also investigated therein. Then, Section IV and Section V evaluate the effects of Gaussian dithering and imperfect CSI on the system performance, respectively. Furthermore, Section VI extends the theoretical framework to the multi-user access scenario. Numerical results are given in Section VII to corroborate the analysis. Finally, Section VIII concludes the paper. Auxiliary technical derivations are archived in the appendix.

**Notation:** Throughout this paper, vectors and matrices are given in bold typeface, e.g., $\mathbf{x}$ and $\mathbf{X}$, respectively, while scalars are given in regular typeface, e.g., $x$. We use $||x||_1$ and $||x||$ to represent the 1-norm and 2-norm of vector $x$, respectively, and let $\mathbf{X}^*$, $\mathbf{X}^T$ and $\mathbf{X}^H$ denote the conjugate, transpose and conjugate transpose of $\mathbf{X}$, respectively. Normal distribution with mean $\mu$ and variance $\sigma^2$ is denoted by $\mathcal{N}(\mu, \sigma^2)$, while $\mathcal{CN}(\mu, C)$ stands for the distribution of a circularly symmetric complex Gaussian random vector with mean $\mu$ and covariance matrix $C$. Superscripts $R$ and $I$ are used to indicate the real and imaginary parts of a complex number, respectively, e.g., $x = x^R + i \cdot x^I$, with $i$ being the
imaginary unit. We use $\text{sgn}(x) = \text{sgn}(x^R) + i \cdot \text{sgn}(x^I)$ to denote the indicator function, and $\log(x)$ to denote the natural logarithm of positive real number $x$.

II. SYSTEM MODEL

We start by focusing on a single-user system, where a single-antenna user communicates with an $N$-antenna BS. In this paper, we consider a narrow-band channel model, for which the frequency-flat fading channel\(^2\) $h$ is chosen according to $\mathcal{CN}(0, I)$ and is fixed throughout the transmission of the codeword. Moreover, the realization of the channel is assumed to be perfectly known by the BS and thus is deemed as deterministic in the subsequent analysis. The received signal at the BS can be expressed as

$$y^l = hx^l + z^l, \quad \text{for } l = 1, 2, ..., L,$$

where $x^l$ is the complex signal transmitted at the $l$-th symbol time, $z^l \sim \mathcal{CN}(0, \sigma^2 I)$ models the independent and identically distributed (i.i.d.) complex Gaussian noise vector, and $L$ is the codeword length.

In practice, the received signal at each antenna is quantized by a pair of ADCs, one for each of the in-phase and quadrature (I/Q) branches, so that further signal processing can be performed in the digital domain. Despite of this, most of the literature on receiver design assume ADC with virtually infinite precision for the tractability of analysis. For a large BS antenna array, however, this assumption is no longer justified since the cost and energy consumption scale linearly with the number of BS antennas, which will soon become the system bottleneck. Therefore, we propose a mixed architecture in which only $2K$ high-resolution ADCs are available and all the other $2(N - K)$ ADCs are with only one-bit resolution\(^3\). We further let the I/Q outputs at each antenna be quantized by two ADCs of the same kind. Thus the quantized output is

$$r^l_n = Q_n(y^l_n) = \begin{cases} h_n x^l + z^l_n, & \text{if } \delta_n = 1, \\ \text{sgn}(h_n x^l + z^l_n), & \text{if } \delta_n = 0, \end{cases}$$

for $l = 1, ..., L$, $n = 1, ..., N$. Here $\delta_n \in \{0, 1\}$ is the binding indicator: $\delta_n = 1$ means that the ADCs corresponding to the $n$-th antenna are high-resolution, whereas $\delta_n = 0$ indicates that they are with one-

\(^2\)Throughout this paper we focus on a narrow-band channel model with frequency-flat fading, similar to those considered in, e.g., [5] [6] [7] [13] [14] [19], among others. Wideband channel models include frequency-selective fading, which can still be treated using the general framework of GMI, and will be treated in a future work; a further discussion is in Section VIII.

\(^3\)Note that an one-bit ADC is particularly cost-efficient to implement in hardware, say, using a polarity detector [12]. Furthermore, the analytical approach we adopt in this work, based on the general framework in [18], can be readily extended to other types of ADCs.
bit resolution. Note that here we assume sufficiently high resolution, so that the residual quantization noise is negligible, for $\delta_n = 1$.

To make the expression compact, we introduce $\bar{\delta}_n \triangleq 1 - \delta_n$ and rewrite (2) as

$$r_l^n = \delta_n \cdot (h_n x_l + z_l) + \bar{\delta}_n \cdot \text{sgn}(h_n x_l + z_l).$$

Then, we define a binding vector $\delta \triangleq [\delta_1, ..., \delta_N]^T$, which follows the subsequent restriction

$$\|\delta\|_1 = \sum_{n=1}^{N} \delta_n = K,$$

and should be optimized according to the channel state $h$ so that the limited number of high-resolution ADCs will be well utilized to enhance the system performance.

For transmission of rate $R$, the user selects a message $m$ from $M = \{1, 2, ..., [2^{LR}]\}$ uniformly randomly, and maps the selected message to a transmitted codeword, i.e., a length-$L$ complex sequence, $\{x_l(m)\}_{l=1}^{L}$. In this paper, we restrict the codebook to be drawn from a Gaussian ensemble; that is, each codeword is a sequence of $L$ i.i.d. $\mathcal{CN}(0, \mathcal{E}_s)$ random variables, and all the codewords are mutually independent. Such a choice of codebook satisfies the average power constraint $\frac{1}{L} \sum_{l=1}^{L} \mathbb{E}[|x_l(m)|^2] \leq \mathcal{E}_s$.

We define the SNR as $\text{SNR}_d = \mathcal{E}_s/\sigma^2$, and let $\sigma^2 = 1$ thereafter for convenience.

As is well known, without receiver distortion, the Gaussian codebook ensemble together with nearest-neighbor decoding achieves the capacity of the SIMO channel\(^4\) $C(N, N, 0) = \log(1 + \|h\|^2 \text{SNR}_d)$, as the codeword length $L$ grows without bound. With one-bit quantization, the channel capacity $C(N, K, N - K)$ is generally less than $C(N, N, 0)$ due to information loss during quantization.

\(^4\)For an $N$-antenna SIMO channel, we let $C(N, K_1, K_2)$ denote its capacity when equipped with $K_1$ pairs of high-resolution ADCs and $K_2$ pairs of one-bit ADCs, where $0 \leq K_1, K_2, K_1 + K_2 \leq N$. Particularly, for our proposed system architecture, we have $K_1 = K$ and $K_2 = N - K$; for the case of only $K$ pairs of high-resolution ADCs, we have $K_1 = K$ and $K_2 = 0$, and thus discard the outputs of $N - K$ antennas.
As discussed in the introduction, the evaluation of $C(N, K, N - K)$ is a formidable task. In the following, we adopt the nearest-neighbor decoding rule at the decoder, and leverage the general framework developed in [18] to investigate the GMI of our proposed system architecture. The GMI acts as an achievable rate and thus also a lower bound of $C(N, K, N - K)$. To this end, we introduce a linear combiner to process the channel output vector, as illustrated in Figure 1. Thus the processed channel output is

\[
\hat{x}^l = w^H r^l,
\]

for $l = 1, ..., L$, where $w$ is designed according to the channel state $h$ and the binding vector $\delta$.

With nearest-neighbor decoding, upon observing $\{\hat{x}^l\}_{l=1}^L$, the decoder computes, for all possible messages, the distance metrics

\[
D(m) = \frac{1}{L} \sum_{l=1}^L |\hat{x}^l - ax^l(m)|^2, \quad m \in M,
\]

and decides the received message as the one that minimizes (6). The scaling parameter $a$ is selected appropriately for optimizing the decoding performance.

III. GMI AND OPTIMIZATION OF COMBINER

A. GMI of the Proposed System Framework

From now on, we suppress the time index $l$ for notational simplicity. To facilitate the exposition, we summarize (3) and (5) as

\[
\hat{x} = w^H r \triangleq f(x, h, z),
\]

where $f(\cdot)$ is a memoryless nonlinear distortion function that incorporates the effects of output quantization as well as linear combining, and maps the triple $(x, h, z)$ into the processed output $\hat{x}$. Although $\delta$ and $w$ are made invisible in the function $f(\cdot)$ since they are both determined by $h$, we need to keep in mind that $f(\cdot)$ implicitly includes $\delta$ and $w$.

We apply the general framework developed in [18] to derive the GMI of our proposed system architecture. Particularly, conditioned on $w$ and $\delta$, the GMI takes the following form analogous to [18, Equation (89)]; that is,

\[
I_{\text{GMI}}(w, \delta) = \sup_{a \in \mathbb{C}, \theta < 0} \left( \theta \mathbb{E}[|f(x, h, z) - ax|^2] - \frac{\theta \mathbb{E}[|f(x, h, z)|^2]}{1 - \theta |a|^2 \mathbb{E}_s} + \log(1 - \theta |a|^2 \mathbb{E}_s) \right),
\]

where the expectation is taken with respect to $x$ and $z$. Then employing the similar procedure as [18, Appendix C], we have the conditional GMI given as follows.
Proposition 1: With Gaussian codebook ensemble and nearest-neighbor decoding, the GMI conditioned on $w$ and $\delta$ is given as

$$I_{\text{GMI}}(w, \delta) = \log \left( 1 + \frac{\kappa(w, \delta)}{1 - \kappa(w, \delta)} \right),$$

where the parameter $\kappa(w, \delta)$ is

$$\kappa(w, \delta) = \frac{|E[f^*(x, h, z) \cdot x]|^2}{E_s E[|f(x, h, z)|^2]}.$$  

The corresponding optimal choice of the scaling parameter $a$ is

$$a_{\text{opt}}(w, \delta) = \frac{E[f(x, h, z) \cdot x^*]}{E_s}.$$  

We note that the expectation operation above is taken with respect to $x$ and $z$.

An immediate observation concludes that $\kappa(w, \delta)$ is the squared correlation coefficient of channel input $x$ and the processed output $f(x, h, z)$, and thus is upper bounded by one, from Cauchy-Schwartz’s inequality. Moreover, $I_{\text{GMI}}(w, \delta)$ is a strictly increasing function of $\kappa(w, \delta)$ for $\kappa(w, \delta) \in (0, 1)$. Therefore, in the following, we will seek to maximize $\kappa(w, \delta)$ by choosing well designed linear combiner $w$ and binding vector $\delta$. To proceed, we introduce the following lemmas, which help us derive the closed-form expression of $\kappa(w, \delta)$.

Lemma 1: For zero-mean real Gaussian random variables $S$ and $T$ with covariance matrix $K$, letting $\phi_{0,K}(s, t)$ denote their joint probability density function (PDF) and $\rho$ represent their correlation coefficient, we have

$$E[\text{sgn}(S) \cdot \text{sgn}(T)] = \frac{2}{\pi} \arcsin(\rho).$$

Proof: Applying [23, Proposition 2], we obtain the following relationship,

$$\int_0^\infty \int_0^\infty \phi_{0,K}(s, t) dtds = \frac{1}{4} + \frac{1}{2\pi} \arcsin(\rho).$$

Noticing the symmetry of $\phi_{0,K}(s, t)$, it is straightforward to verify that

$$E[\text{sgn}(S) \cdot \text{sgn}(T)] = \int_{st>0} \phi_{0,K}(s, t) dtds - \int_{st<0} \phi_{0,K}(s, t) dtds = 0.$$  

$$= 2 \int_{st>0} \phi_{0,K}(s, t) dtds - 1$$  

$$= 4 \int_0^\infty \int_0^\infty \phi_{0,K}(s, t) dtds - 1$$  

$$= \frac{2}{\pi} \arcsin(\rho).$$

We note that $E[\text{sgn}(S) \cdot \text{sgn}(T)]$ is independent of the variances of $S$ and $T$ but is solely determined by their correlation coefficient $\rho$. ■
Lemma 2: For independent complex Gaussian random variables \( S \sim \mathbb{C}N(0, \sigma_s^2) \) and \( T \sim \mathbb{C}N(0, \sigma_t^2) \), we have

\[
\mathbb{E}[S^* \cdot \text{sgn}(S + T)] = \mathbb{E}[S \cdot \text{sgn}^*(S + T)] = \sigma_s^2 \sqrt{\frac{4}{\pi(\sigma_s^2 + \sigma_t^2)}}.
\] (15)

Proof: With some manipulation, we have

\[
\mathbb{E}[S^* \cdot \text{sgn}(S + T)] = \mathbb{E}[S^R \cdot \text{sgn}(S^R + T^R)] + \mathbb{E}[S^I \cdot \text{sgn}(S^I + T^I)] + i \cdot \mathbb{E}[S^R \cdot \text{sgn}(S^I + T^I)] - i \cdot \mathbb{E}[S^I \cdot \text{sgn}(S^R + T^R)]
\]

\[
= \sigma_s^2 \sqrt{\frac{2}{\pi(\sigma_s^2/2 + \sigma_t^2/2)}} + \sigma_s^2 \sqrt{\frac{2}{\pi(\sigma_s^2/2 + \sigma_t^2/2)}} + i \cdot 0 - i \cdot 0
\]

\[
= \sigma_s^2 \sqrt{\frac{4}{\pi(\sigma_s^2 + \sigma_t^2)}}.
\] (16)

where (a) follows from [18, Equation (19)], the independence between \( S^R \) and \( S^I + T^I \), as well as between \( S^I \) and \( S^R + T^R \).

Proposition 2: Given \( w \) and \( \delta \), for (10) in Proposition 1, we have

\[
\kappa(w, \delta) = \frac{w^H R_{rx} R_{rx}^H w}{\mathcal{E}_s w^H R_{rr} w},
\] (17)

where \( R_{rx} \) is the correlation vector between \( r \) and \( x \), with its \( n \)-th element being

\[
(R_{rx})_n = h_n \mathcal{E}_s \left( \delta_n + \bar{\delta}_n \cdot \sqrt{\frac{4}{\pi(|h_n|^2 \mathcal{E}_s + 1)}} \right), \quad n = 1, \ldots, N,
\] (18)

and \( R_{rr} \) is the covariance matrix of \( r \), with its \( (n, m) \)-th entry being

\[
(R_{rr})_{n,m} = \begin{cases} 
1 + \delta_n \cdot |h_n|^2 \mathcal{E}_s + \bar{\delta}_n, & \text{if } n = m, \\
h_n h_m^* \mathcal{E}_s \left[ \delta_n \delta_m + \bar{\delta}_n \bar{\delta}_m \cdot \sqrt{\frac{4}{\pi(|h_n|^2 \mathcal{E}_s + 1)}} + \bar{\delta}_n \delta_m \cdot \sqrt{\frac{4}{\pi(|h_m|^2 \mathcal{E}_s + 1)}} \right] + \\
\bar{\delta}_n \bar{\delta}_m \cdot \frac{4}{\pi} \arcsin \left( \frac{(h_n h_m^*)^r \mathcal{E}_s}{\sqrt{|h_n|^2 \mathcal{E}_s + 1} \sqrt{|h_m|^2 \mathcal{E}_s + 1}} \right) + i \cdot \arcsin \left( \frac{(h_n h_m^*)^r \mathcal{E}_s}{\sqrt{|h_n|^2 \mathcal{E}_s + 1} \sqrt{|h_m|^2 \mathcal{E}_s + 1}} \right), & \text{if } n \neq m.
\end{cases}
\] (19)

The corresponding optimal choice of the scaling parameter \( a \) in (11) is

\[
a_{\text{opt}}(w, \delta) = \frac{1}{\mathcal{E}_s} w^H R_{rx}.
\] (20)

Proof: See Appendix IX-A.
B. Optimization of Linear Combiner

In the previous subsection, the conditional GMI of our proposed system architecture is derived, as a function of $h$, $w$ and $\delta$. In this subsection, we turn to the optimization of $w$ such that the GMI is maximized for given $h$ and $\delta$. The subsequent proposition summarizes our result.

**Proposition 3:** For given $h$ and $\delta$, the optimal linear combiner $w$ takes the following form

$$w_{opt} = R_{rr}^{-1}R_{rx}, \tag{21}$$

which is in fact a minimum mean square error (MMSE) combiner that minimizes the mean squared estimation error of $x$ upon observing $r$. The corresponding $\kappa(w, \delta)$ is

$$\kappa(w_{opt}, \delta) = a_{opt}(w_{opt}, \delta) = \frac{1}{E_s}R_{rx}^HR_{rr}^{-1}R_{rx}. \tag{22}$$

**Proof:** Noticing that $R_{rr}$ is a positive semidefinite Hermitian matrix, from (17) we have

$$\kappa(w, \delta) = \frac{1}{E_s}w^H R_{rx} R_{rr} w = \frac{1}{E_s}w^H R_{rr}^{1/2}R_{rx}^{-1/2}R_{rx} R_{rr}^{-1/2}w = \frac{1}{E_s}w^H R_{rr}^{1/2}R_{rx}^{-1/2}R_{rx} R_{rr}^{-1/2}w \leq \frac{1}{E_s}w^H R_{rr}^{1/2}R_{rx}^{-1/2}R_{rx} R_{rr}^{-1/2}w = \frac{1}{E_s}w^H R_{rx} R_{rr}^{-1/2}R_{rx} R_{rr}^{-1/2}w = \frac{1}{E_s}w^H R_{rx} R_{rr}^{-1/2}R_{rx} R_{rr}^{-1/2}w, \tag{23}$$

where the inequality follows from Cauchy-Schwartz’s inequality, and the equality holds if and only if $w^H R_{rx} R_{rr}^{-1/2}R_{rx} R_{rr}^{-1/2}w = 1$, i.e., $w_{opt} = R_{rr}^{-1}R_{rx}$. $\blacksquare$

The subsequent corollary demonstrates that our proposed system architecture achieves better performance than the system that only employs $K$ pairs of high-resolution ADCs.

**Corollary 1:** Suppose that the high-resolution ADCs are assigned to the antennas with the strongest $K$ link magnitude gains, and denote the corresponding binding vector as $\delta'$. Then, the following relationship

$$I_{GMI}(w_{opt}, \delta') > C(N, K, 0) \tag{24}$$

holds, where $C(N, K, 0) = \log(1 + \sum_{n=1}^{N} \delta'_n \cdot |h_n|^2 E_s)$ is the capacity of the SIMO channel equipped with only $K$ pairs of high-resolution ADCs.

**Proof:** Provided that the high-resolution ADCs are assigned according to $\delta'$, by specifying $w_n = \delta'_n \cdot h_n$, $n = 1, ..., N$, it is straightforward to verify that $I_{GMI}(w, \delta') = C(N, K, 0)$. Since this choice of $w$ is not the optimal one, we have $I_{GMI}(w_{opt}, \delta') > I_{GMI}(w, \delta')$ and (24) follows. $\blacksquare$

When $K = N$, i.e., all the $2N$ ADCs are high-resolution, we have the following corollary of Proposition 3.
**Corollary 2:** For the special case of $K = N$, the optimal linear combiner (21) reduces to a maximum ratio combiner (MRC). Thus in this case, the GMI coincides with the channel capacity $C(N, N, 0)$.

**Proof:** For the special case of $K = N$, i.e., $\delta = 1$, (18) reduces to $R_{rx} = \mathcal{E}_s h$, and (19) reduces to $R_{rr} = I + \mathcal{E}_s h h^H$. Then, the optimal combiner (21) turns out to be an MRC, since

$$w_{opt} = R_{rr}^{-1} R_{rx} = \left[ I - \frac{\mathcal{E}_s h h^H}{1 + \mathcal{E}_s h h^H} \right] \mathcal{E}_s h = \frac{\mathcal{E}_s}{1 + \mathcal{E}_s \| h \|^2} h,$$

where (a) is obtained by applying Sherman-Morrison formula [24]. Consequently, it is straightforward to verify that the effective SNR in (9) is

$$\frac{\kappa(w_{opt}, \delta)}{1 - \kappa(w_{opt}, \delta)} = \| h \|^2 \mathcal{E}_s,$$

thus completing the proof. □

**C. Asymptotic Behaviors of $I_{GMI}(w_{opt}, \delta)$**

In the previous subsection, the optimal linear combiner for our proposed framework is derived. Thus we are ready to examine its asymptotic performance at both low and high SNR regimes. Letting SNR tend to zero, we have the following corollary.

**Corollary 3:** As $\mathcal{E}_s \to 0$, for given $\delta$ we have

$$I_{GMI}(w_{opt}, \delta) = \sum_{n=1}^{N} \left( \delta_n + \bar{\delta}_n \cdot \frac{2}{\pi} \right) |h_n|^2 \mathcal{E}_s + o(\mathcal{E}_s).$$

See Appendix IX-B for its proof. Comparing with $C(N, N, 0)$ in the low SNR regime, i.e., $C(N, N, 0) = \sum_{n=1}^{N} |h_n|^2 \mathcal{E}_s + o(\mathcal{E}_s)$, we conclude that part of the achievable rate is degraded by a factor of $\frac{2}{\pi}$ due to one-bit quantization. The expression (27) also suggests that, in the low SNR regime, high-resolution ADCs should be assigned to the antennas with the strongest $K$ link magnitude gains.

For the high SNR case, the subsequent corollary collects our results.

**Corollary 4:** As $\mathcal{E}_s \to \infty$, for given $\delta$ we have the effective SNR in (9) as

$$\frac{\kappa(w_{opt}, \delta)}{1 - \kappa(w_{opt}, \delta)} = \| p \|^2 \mathcal{E}_s + \frac{[4 + O(1/\mathcal{E}_s)]q^H B^{-1} q}{\pi - [4 + O(1/\mathcal{E}_s)]q^H B^{-1} q},$$

with $p$, $q$, and $B$ given in (65) and (69). As a result, $I_{GMI}(w_{opt}, \delta)$ scales as

$$I_{GMI}(w_{opt}, \delta) = 2 \log \| p \| + \log(\mathcal{E}_s) + O(1/\mathcal{E}_s).$$

Besides, for the special case of pure one-bit quantization, i.e., $K = 0$, we get

$$\lim_{\mathcal{E}_s \to \infty} I_{GMI}(w_{opt}, \delta) = \log \left( 1 + \frac{4q^H B^{-1} q}{\pi - 4q^H B^{-1} q} \right),$$

where $B$ is also given by (69) but ignore the $O(1/\mathcal{E}_s)$ terms.
The proof is given in Appendix IX-C. An observation from (28) indicates that the contributions of high-resolution ADCs and one-bit ADCs in the high SNR regime are separate, as the first term corresponding to high-resolution ADCs increases linearly with $E_s$, whereas the second term coming from one-bit ADCs tends to be a positive constant independent of $E_s$. Comparing with Corollary 3, we infer that one-bit ADCs are getting less beneficial as the SNR grows large, as will be validated by numerical study in Section VII. In addition to these, (29) suggests for high SNR that, high-resolution ADCs should be assigned to antennas with the strongest $K$ link magnitude gains.

For the special case of pure one-bit quantization, (30) indicates that the corresponding GMI approaches a finite limit, and thus the rate loss due to one-bit quantization is substantial. This is much different from the conclusion we get in the low SNR regime, where one-bit quantization degrades the achievable rate only by a factor of $\frac{2}{\pi}$. The reason underlying this phenomenon is the lack of diversity in the high SNR regime, and thus further enhancing the SNR does not help in improving $I_{GMI}(\mathbf{w}_{opt}, \delta)$ whenever $E_s$ is sufficient large.

D. Extension to Ergodic Fading Channel Scenario

Although our analysis thus far has been specified to the deterministic channel scenario, the analytical framework developed can be extended to the the random channel scenario. To this end, we assume that the fading process $\{h^t\}$ is ergodic and that its realization is known by the BS. A well-known example is the block fading channel model with i.i.d. realization in each coherence interval.

Since the channel vector $\mathbf{h}$ varies over time now, $\mathbf{w}$ and $\delta$ in this situation shall be designed based on the instantaneous channel realization. Then the GMI becomes

$$I_{GMI} = \sup_{a \in \mathbb{C}, \theta < 0} \left( \theta E_{x,z} \mathbb{E}_{x,z,h} [ |f(x, h, z) - ax|^2] \frac{\theta E_{x,z,h} [ |f(x, h, z)|^2]}{1 - \theta |a|^2 E_s} + \log(1 - \theta |a|^2 E_s) \right).$$

Notice that it shares the same nominal form as (8) except that the expectation here is over $x$, $z$, and $h$. Recognizing the difficulty of this optimization problem, we turn to evaluate lower and upper bounds of $I_{GMI}$, and arrive at the following proposition. Numerical results will be given in Section VII to verify the tightness of the lower and upper bounds.

**Proposition 4:** For the ergodic fading channel scenario, lower and upper bounds of $I_{GMI}$ are given by

$$I_{GMI}^{lower} = \log \left( 1 + \frac{\kappa_{lower}}{1 - \kappa_{lower}} \right), \quad \kappa_{lower} = \frac{1}{E_s} \mathbb{E}_h [ R_{rx} R_{rr}^{-1} R_{r2} ].$$

Here for simplicity we consider a fixed value of $a$ in the nearest neighbor decoding metric. Allowing $a$ to vary based on $h^t$ may result in some performance improvement especially when $N$ is not too large, and will be investigated in a future work.
I_{GMI}^{\text{upper}} = \mathbb{E}_h \left[ \log \left( 1 + \frac{\kappa(w_{\text{opt}}, \delta)}{1 - \kappa(w_{\text{opt}}, \delta)} \right) \right], \quad (33)

respectively, where \( \kappa(w_{\text{opt}}, \delta) \) is given by (22).

Proof: Following the similar procedure as [18, Appendix C], we obtain \( \kappa \) in this situation,

\[ \kappa = \frac{\mathbb{E}_x, z, h | f^*(x, h, z) \cdot x|^2}{\mathbb{E}_s \mathbb{E}_x, z, h | f(x, h, z)|^2}, \quad (34) \]

which shares exactly the same form as (10), except that the expectation is taken over \( x, z, \) and \( h \). The maximization of \( \kappa \) shall be accomplished by optimizing the linear combiner. Therefore by specifying \( w \) to be designed according to (21), we get a lower bound of the optimal \( \kappa \), since this design is just one of the feasible options and thus is not necessarily optimal; that is

\[ \kappa_{\text{lower}} = \frac{\mathbb{E}_h \mathbb{E}_x, z, h | f^*(x, h, z) \cdot x| h|^2}{\mathbb{E}_s \mathbb{E}_h \mathbb{E}_x, z, h | f(x, h, z)|^2 | h|^2} = \frac{\mathbb{E}_h \mathbb{E}_h | w^H R_{xx} |^2}{\mathbb{E}_s \mathbb{E}_h | w^H R_{rr} w |} = \frac{1}{\mathbb{E}_s} \mathbb{E}_h | R_{xx} R_{rr}^{-1} R_{rx} |, \quad (35) \]

where the last equation comes from (21). Consequently, we obtain the lower bound of \( I_{GMI} \) as given by (32).

To prove (33), we first rewrite (31) as

\[ I_{GMI} = \sup_{a \in \mathbb{C}, \theta < 0} \mathbb{E}_h \left( \theta \mathbb{E}_{x,z} | f(x, h, z) - a x |^2 | h | - \frac{\theta \mathbb{E}_{x,z} | f(x, h, z) |^2 | h |}{1 - \theta |a|^2 \mathbb{E}_s} \right) + \log(1 - \theta |a|^2 \mathbb{E}_s). \quad (36) \]

Then, to derive the upper bound we simply exchange the order of supremum operation and the expectation over \( h \). This leads to

\[ I_{GMI} \leq \mathbb{E}_h \left( \sup_{a \in \mathbb{C}, \theta < 0} \left( \theta \mathbb{E}_{x,z} | f(x, h, z) - a x |^2 | h | - \frac{\theta \mathbb{E}_{x,z} | f(x, h, z) |^2 | h |}{1 - \theta |a|^2 \mathbb{E}_s} \right) + \log(1 - \theta |a|^2 \mathbb{E}_s) \right). \quad (37) \]

Consequently, (33) follows directly from (8) and the corresponding analysis results established for the deterministic channel scenario.

IV. PERFORMANCE IMPROVEMENT VIA DITHERING

In the previous section, we derive the optimal linear combiner and explore the asymptotic behaviors of \( I_{GMI}(w_{\text{opt}}, \delta) \) in both low and high SNR regimes. As will be revealed by the corresponding numerical study in Section VII, increasing SNR may indeed degrade the GMI when the SNR exceeds a certain threshold that depends on a collection of system parameters. In this situation, Gaussian noise, as a special type of dither, can expand the effective bits of one-bit ADCs and this diversity helps reduce the estimate bias [22]. In general, uniform dithering is asymptotically optimal [22], but its analysis is rather complicated. Therefore, to provide insight, we adopt Gaussian dithering and investigate its impact on the system performance. Here, we only focus on the deterministic channel scenario for concision, and the extension to ergodic fading channels is straightforward.
We consider a dithering strategy, which injects additional Gaussian noise into the antenna output before quantization when the corresponding pair of ADCs are one-bit and the receive SNR of the antenna, $|h_n|^2E_s$, exceeds a prescribed threshold $T$. The power of the injected Gaussian noise is adjusted so that the resulting receive SNR of this antenna is pulled back to $T$. Accordingly, we rewrite (2) as

$$r_n = \begin{cases} h_n x + z_n, & \text{if } \delta_n = 1, \\ \text{sgn}(h_n x + z_n), & \text{if } \delta_n = 0, \ |h_n|^2E_s \leq T, \\ \text{sgn}(h_n x + z_n + z^d_n), & \text{if } \delta_n = 0, \ |h_n|^2E_s > T, \end{cases}$$

(38)

where the Gaussian dither $z^d_n \sim \mathbb{C}N(0, |h_n|^2E_s/T - 1)$ is independent of $z_n$ such that $z_n + z^d_n \sim \mathbb{C}N(0, |h_n|^2E_s/T)$. Since high SNR is always favorable for high-resolution ADC, we do not inject Gaussian dither into antennas with high-resolution ADCs.

The system architecture and optimal linear combiner developed in Section III still apply directly. We only need to make some modifications about $R_{rx}$ in (18) and $R_{rr}$ in (19): for any $n \in \{1, 2, ..., N\}$, whenever $\delta_n = 0$ and $|h_n|^2E_s > T$, we make the following substitution,

$$|h_n|^2E_s + 1 \rightarrow |h_n|^2E_s(1 + 1/T),$$

(39)

in (18) and (19). The optimal threshold $T_{opt}$ depends on several parameters, including the SNR $E_s$, number of BS antennas $N$, and number of one-bit ADCs $2(N - K)$.

Noting that we mainly focus on the situation with relatively small $K$, the dependence of $T_{opt}$ on $K$ is actually negligible. Nevertheless, the optimization of $T$ is still analytically intractable, and thus we perform a numerical search. To be specific, for any given SNR $E_s$ and number of BS antennas $N$, we look for the optimal threshold $T_{opt}$ for $K = 0$ through Monte Carlo simulation, and then use $T_{opt}$ to evaluate the performance gain with $K \geq 1$. Numerical results will be given in Section VII.

V. Training and Effect of Imperfect CSI

Our results derived thus far are based on the perfect CSI assumption. In practice, however, CSI is typically absent for both the transmitter and the receiver, and thus some kind of channel estimation has to be performed. In general, the solution involved quantization is much complicated and thus makes it extremely difficult to evaluate its performance analytically. Therefore, to evaluate the robustness of our system architecture to imperfect CSI, we start by only utilizing the high-resolution ADCs to perform the channel estimation.

Specifically, we estimate the channel vector in a round-robin manner, by which we link the $K$ pairs of high-resolution ADCs to the first $K$ antennas and estimate the corresponding channel coefficients $h_1, ..., h_K$ at the first symbol time; at the next symbol time, we turn the $K$ pairs of high-resolution
ADCs to the next \(K\) antennas and estimate \(h_{K+1}, \ldots, h_{2K}\), and so on. Thus the training phase lasts about \(N/K\) symbol times. An MMSE estimator is adopted at the BS, and thus without loss of generality, we can decompose \(h_n\) as

\[
h_n = \hat{h}_n + \tilde{h}_n, \quad n = 1, \ldots, N,
\]

where \(\hat{h}_n \sim \mathcal{CN}(0, 1 - \sigma^2_{\text{I}})\) is the estimated channel coefficient, and \(\tilde{h}_n \sim \mathcal{CN}(0, \sigma^2_{\text{I}})\) accounts for the independent estimation error. Accordingly, we define the MSE of the channel estimation as \(\text{MSE}_t = \sigma^2_{\text{I}}\).

In this situation, the linear combiner \(w\) and the binding vector \(\delta\) should be designed based on the channel estimate \(\hat{h}\). Besides, we rewrite \(f(x, h, z)\) as \(f(x, \hat{h}, \hat{h}, z)\) in order to incorporate the effect of channel estimation. Then with some modification, our analysis developed in Section III-D still applies for the imperfect CSI case. To proceed, we have

\[
I_{\text{GMI}}^{\text{im}} = \sup_{a \in \mathbb{C}, \theta < 0} \left( \theta \mathbb{E}[|f(x, \hat{h}, \hat{h}, z) - ax|^2] - \frac{\theta \mathbb{E}[|f(x, \hat{h}, \hat{h}, z)|^2]}{1 - \theta |a|^2 \mathbb{E}_s} + \log(1 - \theta |a|^2 \mathbb{E}_s) \right),
\]

which obeys an analogous form as (31), except that the expectation here is taken with respect to \(x, \hat{h}, \hat{h}, \) and \(z\). Exploiting a similar argument as that in the proof of Proposition 4, we arrive at the following proposition.

**Proposition 5:** For ergodic fading channels with imperfect CSI, a lower bound of \(I_{\text{GMI}}^{\text{im}}\) is

\[
I_{\text{GMI}}^{\text{im}, l} = \log \left(1 + \frac{\kappa_{\text{im}, l}}{1 - \kappa_{\text{im}, l}}\right), \quad \kappa_{\text{im}, l} = \frac{1}{\mathbb{E}_h} \mathbb{E}_h \left[ (R_{\text{rx}}^{\text{im}})^H (R_{\text{rr}}^{\text{im}})^{-1} R_{\text{rx}}^{\text{im}} \right];
\]

and an upper bound takes the following form

\[
I_{\text{GMI}}^{\text{im}, u} = \mathbb{E}_h \left[ \log \left(1 + \frac{\kappa(w_{\text{opt}}^{\text{im}}, \delta)}{1 - \kappa(w_{\text{opt}}^{\text{im}}, \delta)}\right) \right], \quad \kappa(w_{\text{opt}}^{\text{im}}, \delta) = \frac{1}{\mathbb{E}_s} \mathbb{E}_h \left[ (R_{\text{rx}}^{\text{im}})^H (R_{\text{rr}}^{\text{im}})^{-1} R_{\text{rx}}^{\text{im}} \right]
\]

Here, \(R_{\text{rx}}^{\text{im}} = \mathbb{E}_h[R_{\text{rx}}]\) and \(R_{\text{rr}}^{\text{im}} = \mathbb{E}_h[R_{\text{rr}}]\) can be evaluated by numerical integration. \(w_{\text{opt}}^{\text{im}}\) and \(\kappa(w_{\text{opt}}^{\text{im}}, \delta)\) also come from (21) and (22), but we need to replace \(R_{\text{rx}}\) with \(R_{\text{rx}}^{\text{im}}\), and replace \(R_{\text{rr}}\) with \(R_{\text{rr}}^{\text{im}}\).

**VI. EXTENSION TO MULTI-USER SCENARIO**

In this section, the BS serves \(M\) single-antenna users simultaneously. For simplicity, we focus on the deterministic channel case\(^6\), and the channel matrix between the users and the BS is denoted by \(H \overset{\Delta}{=} [h_1, \ldots, h_N] \in \mathbb{C}^{M \times N}\), whose elements are i.i.d. \(\mathcal{CN}(0, 1)\), i.e., \(h_n \overset{\Delta}{=} [h_{1n}, \ldots, h_{Mn}]^T\) collecting

\(^6\)Further issues such as ergodic fading channels with or without channel estimation error can also be straightforwardly analyzed from the framework established for the single-user scenario, and thus are omitted due to space limitation.
The channel coefficients related to the $n$-th antenna at the BS. There are still only $K$ pairs of high-resolution ADCs available at the BS. Thus we rewrite the quantized output at the $n$-th antenna, with user $j$ considered, as

$$y_{n}^{mu} = \delta_n \left( h_{jn}x_j + \sum_{i \neq j}^{M} h_{in}x_i + z_n \right) + \bar{\delta}_n \cdot \text{sgn} \left( h_{jn}x_j + \sum_{i \neq j}^{M} h_{in}x_i + z_n \right),$$

where $x_i \sim \mathcal{C}\mathcal{N}(0, \mathcal{E}_s)$ denotes the i.i.d. coded signal dedicated to the $i$-th user, and $\sum_{i \neq j}^{M} h_{in}x_i + z_n$, being correlated across the antenna array, summarizes the co-channel interference and noise for the considered user $j$. For a fair comparison, the SNR in this situation is defined as $\text{SNR}_{d} = M\mathcal{E}_s$, reflecting the total transmit power from all users.

Following a similar derivation procedure as that constructed in Section III, we get the GMI of the considered user. The proof is omitted for concision.

**Proposition 6:** For given $\mathbf{H}$ and $\delta$, when treating other users’ signals as noise, the GMI of user $j$ is

$$I_{\text{GMI}}^{mu} = \log \left( 1 + \frac{\kappa^{mu}}{1 - \kappa^{mu}} \right),$$

where the parameter $\kappa^{mu}$ is

$$\kappa^{mu} = \frac{1}{\mathcal{E}_s}(\mathbf{R}_{tx}^{mu})^{H}(\mathbf{R}_{tr}^{mu})^{-1}\mathbf{R}_{tx}^{mu}.$$ (46)

$\mathbf{R}_{tx}^{mu}$ is the correlation vector between $\mathbf{r}^{mu}$ and $x_j$, with its $n$-th entry given as

$$\left( \mathbf{R}_{tx}^{mu} \right)_n = h_{jn}\mathcal{E}_s \left[ \delta_n + \bar{\delta}_n \cdot \sqrt{\frac{4}{\pi \left( \| \mathbf{h}_n \|^2 \mathcal{E}_s + 1 \right)}} \right], \quad n = 1, \ldots, N, \quad (47)$$

and $\mathbf{R}_{tr}^{mu}$ is the covariance matrix of $\mathbf{r}^{mu}$, with its $(n,m)$-th entry being

$$\left( \mathbf{R}_{tr}^{mu} \right)_{n,m} = \begin{cases} \delta_n \| \mathbf{h}_n \|^2 \mathcal{E}_s + \bar{\delta}_n, & \text{if } n = m, \\
\mathbf{h}_n^{T}\mathbf{h}_m^{*}\mathcal{E}_s \left[ \delta_n \delta_m + \bar{\delta}_n \bar{\delta}_m \cdot \sqrt{\frac{4}{\pi \left( \| \mathbf{h}_n \|^2 \mathcal{E}_s + 1 \right)}} + \bar{\delta}_n \delta_m \cdot \sqrt{\frac{4}{\pi \left( \| \mathbf{h}_n \|^2 \mathcal{E}_s + 1 \right)}} \right] + \bar{\delta}_n \delta_m \cdot \frac{4}{\pi} \text{arcsin}\left( \frac{\mathbf{h}_n^{T}\mathbf{h}_m^{*}\mathcal{E}_s}{\sqrt{\| \mathbf{h}_n \|^2 \mathcal{E}_s + 1}} \right) \right] + \\
i \cdot \text{arcsin}\left( \frac{\mathbf{h}_n^{T}\mathbf{h}_m^{*}\mathcal{E}_s}{\sqrt{\| \mathbf{h}_n \|^2 \mathcal{E}_s + 1}} \right), & \text{if } n \neq m. \end{cases}$$ (48)

In the multi-user scenario, there is no clear clue about how to assign the high-resolution ADCs. To obtain some hint, we explore the asymptotic behavior of Proposition 6 in the low SNR regime. This leads to the corollary below.

**Corollary 5:** When $\mathcal{E}_s \to 0$, for given $\mathbf{H}$ and $\delta$, we have the GMI of user $j$ as

$$I_{\text{GMI}}^{mu} = \sum_{n=1}^{N} \left( \delta_n + \bar{\delta}_n \cdot \frac{2}{\pi} \right) \cdot |h_{jn}|^2 \mathcal{E}_s + o(\mathcal{E}_s).$$ (49)
The proof procedure is virtually the same as Appendix IX-B and thus is omitted for concision. We notice that $I_{\text{GMI}}^{\text{mu}}$ behaves analogously with $I_{\text{GMI}}$ in the low SNR regime, which is foreseeable since the system is now noise-limited. Since the sum GMI now equals
\[
\sum_{n=1}^{N} \sum_{j=1}^{M} (\delta_n + \bar{\delta}_n \cdot \frac{2}{\pi}) \cdot |h_{jn}|^2 \mathcal{E}_n + o(\mathcal{E}_n),
\]
it suggests that the $K$ pairs of high-resolution ADCs may be assigned to the antennas with the maximum $\sum_{j=1}^{M} |h_{jn}|^2$.

The asymptotic behavior of $I_{\text{GMI}}^{\text{mu}}$ in the high SNR regime is analytically intractable, and thus there is no generally convincing assignment scheme for the multi-user scenario. For this reason, we consider two heuristic ADC assignment schemes in the numerical study. That is,

- Scheme #1: high-resolution ADCs are assigned to antennas with the maximum $\sum_{j=1}^{M} |h_{jn}|^2$.
- Scheme #2: high-resolution ADCs are assigned randomly.

The corresponding numerical results will be given in the next section to examine the performance of both assignment schemes.

VII. NUMERICAL RESULTS

In this section we validate our previous analysis with numerical results. Unless otherwise specified, we assign the high-resolution ADCs to the antennas with the strongest $K$ link magnitude gains, as suggested by Corollary 3 and 4.

A. GMI for Deterministic Channel with Perfect CSI

Figure 2 and Figure 3 display the GMI\(^7\) of our proposed system architecture for $N = 100$ and $N = 400$, respectively. In both figures, the solid curves represent the GMI of our proposed system architecture, the dashed lines correspond to the channel capacity with $N$ pairs of high-resolution ADCs, i.e., $C(N, N, 0) = \log(1 + \| \mathbf{h} \|^2 \text{SNR}_d)$, and the dash-dot curves are $C(N, K, 0) = \log(1 + \sum_{n=1}^{N} \delta_n \cdot |h_n|^2 \text{SNR}_d)$, i.e., the channel capacity with only $K$ pairs of high-resolution ADCs. Several observations are in order. First, both figures clearly show that the gain of deploying more high-resolution ADCs decreases rapidly, and thus a small number of high-resolution ADCs actually achieves a large fraction of $C(N, N, 0)$. For example, our proposed system architecture with 10 pairs of high-resolution ADCs attains 84% of $C(100, 100, 0)$ when $\text{SNR}_d = 0\text{dB}$, and it achieves 91% of $C(100, 100, 0)$ when $K$ increases to 20. Moreover, both figures indicate that one-bit ADCs are less beneficial when the SNR grows large, but still significantly improve the system performance in the low to intermediate SNR regime.

\(^7\)In the deterministic channel scenario, the GMI and the channel capacity are both averaged over a large number of realizations of the channel. The same averaging is also adopted in the numerical study for Gaussian dithering as well as the multi-user scenario.
On the other hand, Figure 4 and Figure 5 account for the effects of SNR on the GMI, with special focus on small $K$. For the special case of pure one-bit quantization ($K = 0$), an interesting observation shows that $I_{\text{GMI}}(w_{\text{opt}}, \delta)$ increases first but then turns down as the SNR grows large. Besides, as predicted by Corollary 4, $I_{\text{GMI}}(w_{\text{opt}}, \delta)$ asymptotically approaches a positive limit illustrated by the dashed lines in both Figures. The reason underlying this phenomenon is the lack of diversity at high
SNR when only one-bit quantization is adopted across the antenna array. By introducing noise into the one-bit quantizers, we are equivalently expanding the family of one-bit ADCs, and this diversity is beneficial [22]. Moreover, although the limit of $I_{\text{GMI}}(w_{\text{opt}}, \delta)$ as $\text{SNR} \to \infty$ seems insensitive to the number of BS antennas $N$, deploying more antennas at the BS does substantially improve the performance in the low to intermediate SNR regime.
With an additional pair of high-resolution ADCs, however, the GMI of our proposed framework is always nondecreasing with \( \text{SNR}_d \) and increases linearly with respect to \( 10 \log_{10}(\text{SNR}_d) \) in the high SNR regime, even though there seems to exist a flat segment for \( 0 \text{dB} \leq \text{SNR}_d \leq 5 \text{dB} \) when the number of BS antennas is large enough, e.g., \( N = 400 \). This indicates that the positive impact of high SNR on high-resolution quantization generally overwhelms the negative effect of high SNR for
one-bit quantization of multiple observations, except for the flat segment where these two effects are balanced. In addition, even though the rate loss due to pure one-bit quantization is significant in the high SNR regime, the GMI in the low SNR regime closely approaches those of $K > 0$, as predicted by Corollary 3 and Corollary 4.
Fig. 10. GMI of the proposed system architecture with imperfect CSI: impact of $\text{MSE}_t$, $N = 100$, $K = 20$.

Fig. 11. GMI of the considered user $j$ in multi-user scenario, $N = 100$, $M = 10$.

B. GMI for Ergodic Fading Channel with Perfect CSI

Here we examine the tightness of the lower and upper bounds derived in Proposition 4. To this end, we let the channel state in each coherence interval be i.i.d. $\mathcal{CN}(0, I)$. Numerical results are given by Figure 6. Benefiting from the favorable properties of large random matrices, the lower and upper bounds turn out to be virtually coinciding with each other.
C. Performance Gain of Gaussian Dithering

Here we examine the performance gain of Gaussian dithering. For given number of BS antennas $N$ and SNR $\mathcal{E}_s$, we optimize the threshold $\mathcal{T}$ assuming $K = 0$, and then take the resulting $T_{\text{opt}}$ to evaluate the performance gain with $K \geq 1$. Figure 7 and Figure 8 collectively illustrate the results.

When the number of BS antennas $N$ is taken to be 100, Figure 7 indicates that Gaussian dithering
achieves promising improvement of the GMI, especially for the case of $K = 0$. Increasing either $K$ or SNR$_d$, the benefit of dithering decays gradually, since the contribution of high-resolution ADCs tends to be dominating. We notice that the maximum gain for $K = 1$ and $N = 100$ is about 0.3 bit/s/Hz. Deploying more antennas as well as one-bit ADCs at the BS, we see that Gaussian dithering achieves more prominent performance gain, as can be observed from Figure 8, where a 0.55 bit/s/Hz rate gain is achieved for $K = 1$ and $N = 400$.

**D. Impact of Channel Estimation Error**

In the following, we turn to examine the impact of imperfect CSI on the performance. Numerical results are given by Figure 9 assuming that the MSE of the channel estimation is MSE$_t = -10$dB, which indicates that the gap between lower and upper bounds is still virtually negligible. Moreover, though there is a noticeable rate loss due to estimation error, a small number of high-resolution ADCs still achieves much of $C(100,100,0)$. As a numerical evidence, when SNR$_d = 0$dB, we have $I_{\text{lim}}^{\text{GMI}}/C(100,100,0) \approx 81\%$ for $K = 10$ and $I_{\text{lim}}^{\text{GMI}}/C(100,100,0) \approx 87\%$ for $K = 20$. Besides, Figure 10 accounts for the impact of MSE$_t$ on the performance. From the figures, we conclude that our proposed system architecture enjoys good robustness against imperfect CSI.

**E. GMI for the Multi-user Scenario**

Now, we examine the feasibility of our proposed system architecture in the multi-user scenario. As aforementioned, elements of the channel matrix are i.i.d. $\mathcal{CN}(0,1)$ and the SNR is defined as SNR$_d = M\varepsilon$. Figure 11 collects the result, where the solid curves correspond to Scheme #1 suggested by Corollary 5, the dash-dot curves are obtained by random assignment per Scheme #2, and the dashed lines refer to the per-user capacity with $N$ pairs of high-resolution ADCs, i.e., $\frac{1}{M} \log \det (I + \varepsilon_sHH^H)$.

We notice that though Scheme #1 is only analytically validated for the low SNR case, it does achieve better performance than Scheme #2. For the special case of $K = N$, it is well known that the linear MMSE receiver is suboptimal for MIMO channel [27], and thus we observe a distinguishable gap between the per-user capacity and the GMI. Most importantly, similar to the conclusion we obtained for the single-user scenario, here a small number of high-resolution ADCs also attain a large fraction of the channel capacity with $N$ pairs of high-resolution ADCs. For example, when SNR$_d = 0$dB and $N = 100$, Scheme #1 with $K = 10$ achieves 76% of the per-user capacity with $N$ pairs of high-resolution ADCs, and this number rises to 80% when we have $K = 20$.

Figure 12 and Figure 13 account for the impact of increasing the number of users on the achievable sum rates, focusing on $K = 20$ and $K = M$, respectively. Since the capacity without output
quantization is achieved by using MMSE-SIC receiver and the performance loss due to not applying SIC may be substantial for heavily loaded system, for a fair comparison, we introduce additional performance curves corresponding to MMSE receiver without output quantization. From the figures, we conclude that our proposed architecture achieves satisfactory performance even when the system is heavily loaded, by noticing that, for $K = 20$ and $M = N = 100$, our proposed architecture still attains 70% of that achieved by MMSE receiver without quantization.

VIII. CONCLUSION

The numerous BS antennas enable massive MIMO systems to achieve unprecedented gains in both energy efficiency and spectral efficiency, but on the other hand, make the hardware cost and circuit power consumption increase unbearably, demanding energy-efficient design of the transceivers. In this paper, we propose a mixed-ADC receiver architecture, and leverage the GMI to analytically evaluate its achievable data rates under various scenarios. The corresponding numerical study concludes that the proposed architecture with a small number of high-resolution ADCs suffices to achieve a significant fraction of the channel capacity without output quantization, for both single-user and multi-user scenarios. For practitioners, our approach provides a systematic way of designing energy-efficient massive MIMO receivers, once the system parameters are specified: for example, given $K$, one can quantify the receiver energy efficiency as the ratio between the GMI and the processing power consumption, and furthermore, optimize over $K$ in order to maximize the energy efficiency.

A number of interesting and important problems remain unsolved beyond this paper, such as designing the optimal ADC assignment scheme for any SNR, especially for the multi-user scenario; making full use of the available one-bit ADCs when acquiring the CSI; extending the analysis to more comprehensive hardware impairment models beyond ADC; among others. Besides, in order to make this approach effective for wideband channels which are more prevailing in the future communication systems, it is particularly crucial to extend the analysis to frequency-selective fading channels. When one adopts multi-carrier transceiver architectures like OFDM, since one-bit ADCs are applied in the time domain rather than the frequency domain, severe inter-carrier interference due to quantization is inevitable and thus the decoder needs to properly account for this, say, by using a vectorized nearest-neighbor decoding algorithm and evaluating the resulting GMI. This is feasible but beyond the scope of this paper, and will be treated in a future work.
IX. APPENDIX

A. Derivation of $\kappa(w, \delta)$

For given $w$ and $\delta$, we have

$$|\mathbb{E}[f^*(x, h, z) \cdot x]|^2 = |\mathbb{E}[w^T r^* x]|^2 = |w^T R^*_r x|^2 = w^H R_{rx} R_{rx}^H w,$$

(50)

where $R_{rx} \triangleq \mathbb{E}[r x^*]$ is the correlation vector between $r$ and $x$, whose $n$-th element is

$$(R_{rx})_n = \delta_n \cdot \mathbb{E}[x^* \cdot (h_n x + z_n)] + \tilde{\delta}_n \cdot \mathbb{E}[x^* \cdot \text{sgn}(h_n x + z_n)]$$

(a)

$$= \delta_n \cdot h_n \mathcal{E}_s + \tilde{\delta}_n \cdot h_n \mathcal{E}_s \sqrt{\frac{4}{\pi(|h_n|^2 \mathcal{E}_s + 1)}}$$

$$= h_n \mathcal{E}_s \left[ \delta_n + \tilde{\delta}_n \cdot \sqrt{\frac{4}{\pi(|h_n|^2 \mathcal{E}_s + 1)}} \right],$$

(51)

where (a) follows from Lemma 2.

On the other hand,

$$\mathbb{E}[|f(x, h, z)|^2] = \mathbb{E}[w^H r r^H w] = w^H R_{rr} w,$$

(52)

where $R_{rr} \triangleq \mathbb{E}[r r^H]$ is the covariance matrix of $r$. The diagonal elements of $R_{rr}$ are given by

$$(R_{rr})_{n,n} = \mathbb{E}[|\delta_n \cdot (h_n x + z_n) + \tilde{\delta}_n \cdot \text{sgn}(h_n x + z_n)|^2]$$

$$= \delta_n \cdot \mathbb{E}[|h_n x + z_n|^2] + \tilde{\delta}_n \cdot \mathbb{E}[|\text{sgn}(h_n x + z_n)|^2]$$

$$= \delta_n \cdot (|h_n|^2 \mathcal{E}_s + 1) + \tilde{\delta}_n \cdot 2$$

$$= 1 + \delta_n \cdot |h_n|^2 \mathcal{E}_s + \tilde{\delta}_n,$$

(53)

while the nondiagonal elements can be obtained by applying Lemma 1 and Lemma 2, as follows. Applying Lemma 2 we have

$$\mathbb{E}[y_n \cdot \text{sgn}^*(y_m)] = \mathbb{E}[(h_n x + z_n) \cdot \text{sgn}^*(h_m x + z_m)]$$

$$= \mathbb{E}[\mathbb{E}[(h_n x + z_n) \cdot \text{sgn}^*(h_m x + z_m)|x, z_m]]$$

$$= \mathbb{E}[h_n x \cdot \text{sgn}^*(h_m x + z_m)]$$

$$= h_n h_m^* \mathcal{E}_s \sqrt{\frac{4}{\pi(|h_m|^2 \mathcal{E}_s + 1)}},$$

(54)

and analogously we obtain

$$\mathbb{E}[^\text{sgn}(y_n) \cdot y_m^*] = h_n h_m^* \mathcal{E}_s \sqrt{\frac{4}{\pi(|h_m|^2 \mathcal{E}_s + 1)}},$$

(55)
Next, we turn to evaluate $\mathbb{E}[\text{sgn}(y_n) \cdot \text{sgn}^*(y_m)]$, which is

$$
\mathbb{E}[\text{sgn}(y_n) \cdot \text{sgn}^*(y_m)] = \mathbb{E}[\text{sgn}(y_n^R) \cdot \text{sgn}(y_m^R)] + \mathbb{E}[\text{sgn}(y_n^I) \cdot \text{sgn}(y_m^I)] - i \cdot \mathbb{E}[\text{sgn}(y_n^R) \cdot \text{sgn}(y_m^I)] + i \cdot \mathbb{E}[\text{sgn}(y_n^I) \cdot \text{sgn}(y_m^R)]
$$

$$
= \frac{2}{\pi} \arcsin(\rho_{y_n^R,y_m^R}) + \frac{2}{\pi} \arcsin(\rho_{y_n^I,y_m^I}) - i \cdot \frac{2}{\pi} \arcsin(\rho_{y_n^R,y_m^I}) + i \cdot \frac{2}{\pi} \arcsin(\rho_{y_n^I,y_m^R}),
$$

(56)

where the last equation follows from Lemma 1. To proceed, we need to evaluate some correlation coefficients, e.g., $\rho_{y_n^R,y_m^R}$, which is given as

$$
\rho_{y_n^R,y_m^R} = \frac{\mathbb{E}[y_n^R y_m^R]}{\sqrt{\mathbb{E}[(y_n^R)^2] \mathbb{E}[(y_m^R)^2]}} = \frac{\mathbb{E}[(h_n^R x^R - h_n^I x^I + z_n^R)(h_m^R x^R - h_m^I x^I + z_m^R)]}{\sqrt{\mathbb{E}[(h_n^R x^R - h_n^I x^I + z_n^R)^2] \mathbb{E}[(h_m^R x^R - h_m^I x^I + z_m^R)^2]}} = \frac{(h_n^R h_m^R + h_n^I h_m^I) \varepsilon_s}{\sqrt{|h_n|^2 |\varepsilon_s| + \frac{1}{2} \sqrt{|h_m|^2 |\varepsilon_s| + \frac{1}{2}}} = (h_n h_m^*)^R \varepsilon_s \sqrt{|h_n|^2 |\varepsilon_s| + 1 \sqrt{|h_m|^2 |\varepsilon_s| + 1}}.
$$

(57)

Similarly, we obtain

$$
\rho_{y_n^I,y_m^R} = \rho_{y_n^R,y_m^I}, \quad \rho_{y_n^I,y_m^I} = -\rho_{y_n^R,y_m^R} = \frac{(h_n h_m^*)^R \varepsilon_s}{\sqrt{|h_n|^2 |\varepsilon_s| + 1 \sqrt{|h_m|^2 |\varepsilon_s| + 1}}}. 
$$

(58)

Now, we can combine (56)-(58) and thus get $\mathbb{E}[\text{sgn}(y_n) \cdot \text{sgn}^*(y_m)]$ as

$$
\mathbb{E}[\text{sgn}(y_n) \cdot \text{sgn}^*(y_m)] = \frac{4}{\pi} \arcsin \left( \frac{(h_n h_m^*)^R \varepsilon_s}{\sqrt{|h_n|^2 |\varepsilon_s| + 1 \sqrt{|h_m|^2 |\varepsilon_s| + 1}}} \right) + i \cdot \frac{4}{\pi} \arcsin \left( \frac{(h_n h_m^*)^I \varepsilon_s}{\sqrt{|h_n|^2 |\varepsilon_s| + 1 \sqrt{|h_m|^2 |\varepsilon_s| + 1}}} \right),
$$

(59)

Further, from (54), (55) and (59), we obtain $(R_{rr})_{n,m}$, given as

$$
(R_{rr})_{n,m} = \delta_n \delta_m \cdot \mathbb{E}[y_n^* y_m^*] + \bar{\delta}_n \bar{\delta}_m \cdot \mathbb{E}[y_n^* y_m^*] + \delta_n \bar{\delta}_m \cdot \mathbb{E}[y_n^* y_m^*] + \bar{\delta}_n \delta_m \cdot \mathbb{E}[y_n^* y_m^*]
$$

$$
= h_n h_m^* \varepsilon_s \left[ \delta_n \delta_m + \delta_n \bar{\delta}_m \cdot \sqrt{\frac{4}{\pi (|h_n|^2 |\varepsilon_s| + 1)}} + \bar{\delta}_n \delta_m \cdot \sqrt{\frac{4}{\pi (|h_n|^2 |\varepsilon_s| + 1)}} \right] + 
$$

$$
\bar{\delta}_n \delta_m \cdot \frac{4}{\pi} \left[ \arcsin \left( \frac{(h_n h_m^*)^R \varepsilon_s}{\sqrt{|h_n|^2 |\varepsilon_s| + 1 \sqrt{|h_m|^2 |\varepsilon_s| + 1}}} \right) + ight.
$$

$$
\left. i \cdot \arcsin \left( \frac{(h_n h_m^*)^I \varepsilon_s}{\sqrt{|h_n|^2 |\varepsilon_s| + 1 \sqrt{|h_m|^2 |\varepsilon_s| + 1}}} \right) \right].
$$

(60)
Thus we conclude the proof.

B. Asymptotic behavior of $I_{\text{GMI}}(w_{\text{opt}}, \delta)$ in the low SNR regime

For simplicity of exposition, we define

$$\mathbf{R}_{rx}^0 \triangleq \lim_{\mathcal{E}_s \to 0} \frac{1}{\mathcal{E}_s} \mathbf{R}_{rx}, \quad \mathbf{R}_{rr}^0 \triangleq \lim_{\mathcal{E}_s \to 0} \mathbf{R}_{rr}. \quad (61)$$

Then from (18) and (19), it is straightforward to derive

$$(\mathbf{R}_{rx}^0)_n = h_n \left[ \delta_n + \tilde{\delta}_n \cdot \frac{2}{\sqrt{\pi}} \right], \quad \mathbf{R}_{rr}^0 = \text{diag}(1 + \tilde{\delta}_1, \ldots, 1 + \tilde{\delta}_n, \ldots, 1 + \tilde{\delta}_N). \quad (62)$$

Thereby we examine the asymptotic behavior of $\kappa$ as $\mathcal{E}_s \to 0$, given as

$$\lim_{\mathcal{E}_s \to 0} \frac{\kappa(w_{\text{opt}}, \delta)}{\mathcal{E}_s} \overset{(a)}{=} \lim_{\mathcal{E}_s \to 0} \left( \frac{1}{\mathcal{E}_s} \mathbf{R}_{rx} \right)^H \mathbf{R}_{rr}^{-1} \left( \frac{1}{\mathcal{E}_s} \mathbf{R}_{rx} \right)$$

$$\overset{(b)}{=} \left( \lim_{\mathcal{E}_s \to 0} \frac{1}{\mathcal{E}_s} \mathbf{R}_{rx} \right)^H \left( \lim_{\mathcal{E}_s \to 0} \mathbf{R}_{rr}^{-1} \right) \left( \lim_{\mathcal{E}_s \to 0} \frac{1}{\mathcal{E}_s} \mathbf{R}_{rx} \right)$$

$$\overset{(c)}{=} \left( \mathbf{R}_{rx}^0 \right)^H \mathbf{R}_{rr}^{-1} \left( \mathbf{R}_{rx}^0 \right)$$

$$= \sum_{n=1}^{N} \left( \delta_n + \tilde{\delta}_n \cdot \frac{4}{\pi} \right) |h_n|^2$$

$$= \sum_{n=1}^{N} \left( \delta_n + \tilde{\delta}_n \cdot \frac{2}{\pi} \right) |h_n|^2, \quad (63)$$

where (a) follows from (22), (b) is obtained by applying the algebraic limit theorem since the limits of $\mathbf{R}_{rx}^0/\mathcal{E}_s$ and $\mathbf{R}_{rr}$ exist, while (c) comes from the fact that the inverse of a nonsingular matrix is a continuous function of the elements of the matrix, i.e., $\lim_{\mathcal{E}_s \to 0} \mathbf{R}_{rr}^{-1} = \left( \lim_{\mathcal{E}_s \to 0} \mathbf{R}_{rr} \right)^{-1}$ [25]. As a result, as $\mathcal{E}_s \to 0$, we have

$$\kappa(w_{\text{opt}}, \delta) = \sum_{n=1}^{N} \left( \delta_n + \tilde{\delta}_n \cdot \frac{2}{\pi} \right) |h_n|^2 \mathcal{E}_s + o(\mathcal{E}_s). \quad (64)$$

Noting that $\log(1 + x/(1 - x)) = x + o(x)$, as $x \to 0$, we immediately have (27).

C. Asymptotic behavior of $I_{\text{GMI}}(w_{\text{opt}}, \delta)$ in the high SNR regime

For simplicity of exposition, we rearrange $\mathbf{h}$ and stack the channel coefficients corresponding to the antennas equipped with high-resolution ADCs in the first $K$ positions of $\mathbf{h}$. Then, to proceed, we define

$$\mathbf{p} \triangleq \left[ h_1, \ldots, h_K \right]^T, \quad \mathbf{q} \triangleq \left[ |h_{K+1}|/|h_{K+1}|, \ldots, |h_N|/|h_N| \right]^T. \quad (65)$$

When $\mathcal{E}_s$ tends to infinity, we have

$$h_n \sqrt{\frac{4}{\pi (|h_n|^2 \mathcal{E}_s + 1)}} = \left[ \sqrt{4\mathcal{E}_s/\pi} + O(1/\sqrt{\mathcal{E}_s}) \right] \frac{h_n}{|h_n|}. \quad (66)$$
for $n = K + 1, \ldots, N$. Thus we are allowed to denote the deduced $R_{rx}$ for $E_s \to \infty$ as

$$R_{rx} = \begin{bmatrix} \frac{E_s p}{\sqrt{4E_s/\pi + O(1/\sqrt{E_s})}} \end{bmatrix}. \quad (67)$$

Besides, we denote by partitioned matrices $R_r$ and its inverse $R_r^{-1}$, i.e.,

$$R_r \triangleq \begin{bmatrix} A & U \\ U^H & B \end{bmatrix}, \quad R_r^{-1} \triangleq \begin{bmatrix} C & V \\ V^H & D \end{bmatrix}, \quad (68)$$

in which the invertible square matrices $A \in \mathbb{C}^{K \times K}$, $B \in \mathbb{C}^{(N-K) \times (N-K)}$ and the rectangle matrix $U \in \mathbb{C}^{K \times (N-K)}$ are taken to be

$$A = I + E_s p p^H, \quad U = (\sqrt{4E_s/\pi + O(1/\sqrt{E_s})}) p q^H,$$

$$(B)_{n,m} = \frac{4}{\pi} \left[ \arcsin \left( \frac{h_n^K h_m^K}{h_n^K h_m^K} \right) + i \cdot \arcsin \left( \frac{h_n^K h_m^K}{h_n^K h_m^K} \right) \right] + O(1/E_s). \quad (69)$$

Then, applying the Sherman-Morrison formula [24] and the inverse of partitioned matrix [26], we obtain

$$C = (A - UB^{-1}U^H)^{-1} = I - \frac{E_s \left[ \pi - (4 + O(1/E_s)) q^H B^{-1} q \right]}{\pi + E_s \left[ \frac{\pi}{\pi - (4 + O(1/E_s)) q^H B^{-1} q} \right]} \cdot p p^H,$$

$$V = -A^{-1}U(B - U^H A^{-1}U)^{-1} = -\frac{\left[ \sqrt{4\pi E_s + O(1/\sqrt{E_s})} \right] \cdot p q^H B^{-1}}{\pi + E_s \left[ \frac{\pi}{\pi - (4 + O(1/E_s)) q^H B^{-1} q} \right]} \cdot \|p\|^2,$$

$$D = (B - U^H A^{-1}U)^{-1} = B^{-1} + \frac{\left[ 4E_s + O(1) \|p\|^2 \cdot B^{-1} q q^H B^{-1} \right]}{\pi + E_s \left[ \frac{\pi}{\pi - (4 + O(1/E_s)) q^H B^{-1} q} \right]} \cdot \|p\|^2. \quad (70)$$

With all of these, we are ready to derive $\kappa(w_{opt}, \delta)$ as $E_s \to \infty$; that is,

$$\kappa(w_{opt}, \delta) = \frac{1}{E_s} R_{rx}^H R_r^{-1} R_{rx}$$

$$= E_s p^H C p + 2 \left( \sqrt{4E_s/\pi + O(1/\sqrt{E_s})} \right) p^H V q + (4/\pi + O(1/E_s)) q^H D q$$

$$= \frac{E_s \left[ \pi - (4 + O(1/E_s)) q^H B^{-1} q \right] \cdot \|p\|^2 + \left[ 4 + O(1/E_s) \right] q^H B^{-1} q}{\pi + E_s \left[ \frac{\pi}{\pi - (4 + O(1/E_s)) q^H B^{-1} q} \right]} \cdot \|p\|^2, \quad (71)$$

and subsequently we get the effective SNR as

$$\frac{\kappa(w_{opt}, \delta)}{1 - \kappa(w_{opt}, \delta)} = \|p\|^2 \cdot E_s + \frac{\left[ 4 + O(1/E_s) \right] q^H B^{-1} q}{\pi - \left[ 4 + O(1/E_s) \right] q^H B^{-1} q}. \quad (72)$$

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