The effects of electron-electron interactions on the spin transport dynamics of a two-dimensional electron gas

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Abstract

Spin transport properties of a spin-polarized two-dimensional electron gas are studied in the presence of electron-electron interactions. Longitudinal and transverse spin diffusion coefficients are calculated with the quantum transport equation which includes many-particle effects in the random phase approximation. We find that the e-e scattering, which does not contribute to charge drift mobility, has a significant contribution to the spin diffusion. Thus, e-e interaction causes non-negligible effects on the operations of proposed devices dependent on spin transport in semiconductor heterostructures.

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I. INTRODUCTION

Spin dependent electronic transport phenomena in semiconductor nanostructures are extensively studied in recent years, with motivations to develop novel electronic and optical devices based on carrier spin dynamics such as spin transistors, polarization dependent optical modulators, and even quantum computers. Recent experimental progress in transporting electronic spins over the distance of 100 µm and injecting spins from ferromagnetic materials to compound semiconductors has been accelerating the prospect of these devices.

Charge transport is extensively studied to understand the operations of devices. In the conventional devices where the electron distribution is spatially homogeneous and the electrons move in the same direction irrespective of spin directions, the charge transport is expressed in terms of drift mobilities. When an electronic band is parabolic and umklapp processes are negligible, electron-electron (e-e) scatterings do not directly affect mobilities since the e-e scattering conserves the total momentum of the system. It influences the charge transport only indirectly by modifying momentum distribution functions and screening other scattering processes of electrons. Mobility is determined by e-phonon and e-ionized impurity scatterings. In high-quality, modulation-doped heterostructures of III-V semiconductors, electron mobility increases as temperature decreases and reaches the maximum value limited by the scattering with remote dopants.

In contrast to the charge transport, spin-up electrons move against spin-down electrons or vice versa in the spin transport. Thus, in addition to e-ionized impurity scattering, we expect that the Coulomb interaction between spin-up and -down electrons also contributes to the spin transport. (The Coulomb interaction between two electrons with the same spin direction does not directly influence the spin transport as it conserves the spin current.) As we show later, the e-e interaction is as important as the e-ionized impurity scattering to the spin transport.

The spin transport is also influenced by the many-particle effects of interacting electrons. An electronic band dispersion is renormalized, giving the effective mass of an electron somewhat different from the band effective mass, and the bare Coulomb interaction is screened, leading to the e-e interaction with a finite range. These many-particle effects affect collision dynamics and modify transport coefficients.

The many-particle effects also have consequences on spin dynamics itself. The spin axis of a noninteracting particle precesses around external magnetic field \( \mathbf{B}_{ex} \). In the interacting system, each electron feels effective magnetic field \( \mathbf{B}_{mol}(r) \) due to the molecular field of surrounding electrons. The effective field is related (almost proportional) to the local spin density and parallel to its direction as explicitly shown in our analysis below. Thus in a spin polarized electron gas, a spin axis precesses around \( \mathbf{B}_{ex} + \mathbf{B}_{mol} \). This induces a “spin-rotation term” in the equation of motion for the spin current, leading to a anomalous spin diffusion coefficients and spin waves, which were experimentally observed in \( ^3\text{He}, \text{ } ^3\text{He}-^4\text{He} \) mixture, and in atomic Hydrogen.

In this paper we present a theory and numerical calculations of spin diffusion coefficients in a two-dimensional electron gas (2DEG) interacting through the Coulomb force. In our previous publication we adopted the Hartree-Fock approximation (HFA) to incorporate the many-particle effects in the lowest order. Here we adopt the random phase approximation (RPA) to improve the treatment on many-particle aspects of electrons and obtain more
reliable numerical results. We compare the contributions from $e-e$ scattering and $e$-ionized impurity scattering to the spin diffusion in semiconductor heterostructures and discuss their relative importance.

This paper is organized as follows: In Sec. II, our calculation scheme is briefly described. In Sec. III the numerical results for the spin transport coefficients are shown with detailed discussions. Conclusions are given in Sec. IV. The differences in mathematical expressions from our previous calculation are summarized in Appendix.

II. THEORY

Our theoretical model is already presented in our previous publication\cite{ref1} and we describe it only briefly here. Several relevant expressions arising from the RPA will be given in the appendix. We consider two-dimensional electrons interacting through the Coulomb interactions. Other interactions such as electron-phonon, electron-ionized impurity or the exchange interaction of electrons with magnetic impurities are not included in the present study. We assume that electronic spins are polarized in the growth direction, $z$-axis (normal to the 2D plane), in the quasi-thermal equilibrium, and a direction of local spin polarization can be slightly tipped away from $z$-axis by applying r.f. field or by external spin injection. The total sheet density of spin-up and spin-down electrons, $n(\mathbf{r}, t) = n_+(\mathbf{r}, t) + n_-(\mathbf{r}, t)$, is assumed to be spatially and temporally constant, and the spin density, $M(\mathbf{r}, t) = n_+(\mathbf{r}, t) - n_-(\mathbf{r}, t)$, varies slowly on a hydrodynamic scale.\cite{ref1} The spin polarization is given by $P = (n_+ - n_-)/(n_+ + n_-)$. Spin relaxation is not included in the present analysis.

A moderate external magnetic field, $B_{\text{ex}}$, can be applied perpendicular to the 2D plane. We assume that the field is so weak that it only affects spinor space, leaving electron orbital motion intact. Otherwise the orbital motion is quantized into the Landau levels, and the system turns into the quantized Hall state, to which our present theory does not apply. The estimation in our earlier work\cite{ref1} shows that the weak field condition is satisfied when $B_{\text{ex}} < 1$ T. When this condition is satisfied, the external field can trivially be erased from the theory by using a rotating frame.

The quantum transport equation for the electron distribution functions $\underline{n}_k$ is given by

$$\frac{\partial \underline{n}_k(\mathbf{r}, t)}{\partial t} + \frac{1}{2} \left\{ \frac{\partial \underline{\varepsilon}_k(\mathbf{r}, t)}{\partial \underline{p}}, \frac{\partial \underline{n}_k(\mathbf{r}, t)}{\partial \underline{r}} \right\} - \frac{1}{2} \left\{ \frac{\partial \underline{n}_k(\mathbf{r}, t)}{\partial \underline{r}}, \frac{\partial \underline{\varepsilon}_k(\mathbf{r}, t)}{\partial \underline{p}} \right\} + \frac{i}{\hbar} \left[ \underline{\varepsilon}_k(\mathbf{r}, t), \underline{n}_k(\mathbf{r}, t) \right] = \left( \frac{\partial \underline{n}_k(\mathbf{r}, t)}{\partial t} \right)_{\text{coll.}}, \quad (2.1)$$

where the electron energy $\underline{\varepsilon}_k$ and the distribution function $\underline{n}_k$ are $2 \times 2$ matrices in spinor space, denoted by underlining. The diagonal components of $\underline{n}_k$ represent the population of spin-up and spin-down electrons and its off-diagonal elements represent the couplings between spin-up and spin-down states. The transport equation is similar to the Boltzmann equation except that there are anticommutators in the drift terms (the second and the third term on the left) and a commutator in the last term on the left. This term in the commutator is called the “spin-rotation term”, representing the effect of the external and molecular fields.

The electron (quasiparticle) energy $\underline{\varepsilon}_k$ is composed of a kinetic energy term, a coupling to the external field and a self-energy caused by the presence of other electrons. In our
earlier work we adopted the HFA to calculate an electron self-energy since the simple analytic expressions of the HFA considerably reduces the load of numerical calculations. But the HFA does not describe the many-particle properties of 2DEG appropriately: The energy dispersion of a quasiparticle is not given properly with a vanishing quasiparticle effective mass at Fermi surface due to the infinite range of the bare Coulomb interaction. Since the effective mass is one of the important parameters in our theory, a certain degree of arbitrariness exists in our previous calculations. And since the HFA does not include correlation terms, the self-energy for a spin-up electron does not contain the contribution from spin-down electrons. This leads us to an unrealistic situation when the spin polarization is high, i.e., provided that we prepared 2DEG with all spins pointing downward, and we put a spin-up electron into this 2DEG, then the energy dispersion of this spin-up electron remains that of a noninteracting electron in spite of the presence of many spin-down electrons. This prevented us from calculating the diffusion coefficients of highly polarized 2DEG in our earlier work.

In the present study we improve our calculation by adopting the RPA. It takes into account exchange and correlation (though not completely), and we can correct the problems of the HFA mentioned above. The RPA self-energy for an electron gas is discussed in many text books (e.g., see Ref. [14] and [15]), and its detailed expressions for a two-dimensional, multi-band electron system are given by Vinter [16] and DasSarma et. al. [17]. We use the standard RPA self-energy without vertex corrections. The Matsubara function (imaginary time Green’s function) for the self-energy, which is a $2 \times 2$ matrix in spinor space, is written as

$$\hbar \Sigma^{RPA}_{\alpha,\beta}(k, ik_m) = -\frac{1}{\beta V} \sum_q \sum_{i q_n} V^{RPA}_{\alpha,\beta}(-q, -i q_n) G^0_{\alpha,\beta}(k + q, i k_m + i q_n), \quad (2.2)$$

where $\alpha$ and $\beta$ are indices for spin, $G^0_{\alpha,\beta}$ is the noninteracting electron propagator. ($G^0_{\alpha,\beta}$ is finite only when $\alpha = \beta$.) The RPA screened Coulomb interaction $V^{RPA}_{\alpha,\beta}$ satisfies the Dyson equation,

$$V^{RPA}_{\alpha,\beta}(q, iq_n) = V_{\alpha,\beta}(q) + \sum_{\delta,\delta'} V_{\alpha,\delta}(q) P^0_{\delta,\delta'}(q, iq_n) V^{RPA}_{\delta,\beta}(q, iq_n), \quad (2.3)$$

where $V_{\alpha,\beta}(q)$ is the bare Coulomb interaction. The lowest order polarization (bubble diagram) is finite only when $\delta = \delta'$, given by

$$P^0_{\delta,\delta}(q, iq_n) = \frac{1}{\beta V} \sum_p \sum_{i p_m} G^0_{\delta,\delta}(p, i p_m) G^0_{\delta,\delta}(p + q, i p_m + i q_n). \quad (2.4)$$

The real-time functions are obtained by the analytic continuation after the momentum integration and the summation over Matsubara frequencies. The quasiparticle energy dispersion is obtained from the real part of the self-energy following the prescription suggested by Rice [18].

In Fig. 1 we show energy-momentum dispersions of spin-up and -down electrons in 2DEG with spin-polarizations $P = 0.1 - 0.9$ and with total electron sheet density at $2 \times 10^{11}$ cm$^{-2}$. The electron energy is measured relative to the zero of the noninteracting electron dispersion. One of the important many-particle effects in the interacting electrons
is the reduction of one-electron energy (Bandgap renormalization). Since \( n_+ > n_- \), the renormalization is larger in the spin-up electron. The energy differences between the spin-up and the spin-down electron increase with spin polarizations. When \( P < 0.5 \), the energy dispersions of both spin-up and spin-down electrons are well approximated by parabolic band dispersions. We obtained the quasiparticle effective mass \( m^* \) between 1.11\( m \) and 1.19\( m \) depending on spin polarizations. (\( m \) is a band effective mass. We used \( m = 0.067m_0 \) for the conduction band of GaAs.) In numerical computations of transport coefficients, we have used the parabolic band approximation for the energy dispersions.

The quantum transport equation eq. (2.1) is solved by the Chapman-Enskog expansion to the lowest order using variational solutions.[13] The final expression of the equation for the spin current is obtained by multiplying the transport equation by the particle velocity and integrating over the momentum as.[13]

\[
\frac{\partial J_{\sigma,i}(r, t)}{\partial t} + \hat{e}_\sigma \frac{\partial M(r, t)}{\partial r_i} A(k_{F\pm}) + M(r, t) \frac{\partial \hat{e}_\sigma}{\partial r_i} B(k_{F\pm}) + \mu_{SR} M \times J_{\sigma,i} + \gamma_{gy} B_{ex} \times J_{\sigma,i} \nonumber
\]

\[
= -\frac{1}{\tau_\parallel} J_{\sigma,||,j} - \frac{1}{\tau_\perp} J_{\sigma,\perp,j}, \tag{2.5}
\]

where \( J_{\sigma,j} \) is a spin current representing the flow of the spin component \( \sigma \) in the spatial direction \( j = x, y, z \), \( \hat{e}_\sigma \) is a unit vector in the direction of spin polarization, and \( M = M \hat{e}_\sigma \). The factor \( \mu_{SR} \), which is a function of \( k_{F\pm} \), represents the contribution of the molecular field, and \( \gamma_{gy} = -g\mu_B/\hbar \) is a gyromagnetic ratio. The coefficients \( A(k_{F\pm}) \), \( B(k_{F\pm}) \) and \( \mu_{SR} \) will be given in the appendix.

III. RESULTS AND DISCUSSION

A. Spin diffusion coefficients

We show the results of numerical calculations for spin diffusion coefficients. Since a spin current is a tensor defined by the two vectors; the direction of spatial flow and the direction of spin, we should distinguish longitudinal (\( D_\parallel \)) and transverse (\( D_\perp \)) spin diffusion coefficients. (See Fig. 2) In the longitudinal spin diffusion, consider the system where the direction of spin polarization is aligned in \( z \) everywhere but its magnitude has gradient in the direction \( x \). Then the longitudinal spin current flows in the spatial direction \( x \) whose spin direction is in \( z \), parallel to the spin polarization. While in the transverse spin diffusion, consider the system where the magnitude of spin polarization is the same everywhere but its direction is tilted gradually from \( z \) to \( x \) by a small angle when we go from \( x = x_0 \) to \( x_0 + dx \). In this case the transverse spin current flows in the spatial direction \( x \) with the spin direction in \( x \), perpendicular to the spin polarization. These two diffusions show different magnitudes and temperature dependence.[20]

The temperature dependence of \( D_\parallel \) and \( D_\perp \) is plotted in Fig. 3 and 4, respectively, with \( 0.01 < P < 0.5 \). We calculate in the degenerate region, \( T < 20 \) K. The Fermi temperature is 83 K in the unpolarized 2DEG with the sheet density \( 2 \times 10^{11} \) cm\(^{-2} \) (Fermi wave number \( k_F = 0.112 \) nm\(^{-1} \)). The magnitudes of the diffusion coefficients are somewhat larger than those calculated previously in Ref. [11] but their temperature dependencies are identical. We believe
that the present results calculated with the RPA should be more accurate in predicting the transport coefficients. As shown in Fig. 3, $D_\parallel$ increases at low temperatures and diverges as $T^{-2}$ when $T$ approaches zero. A close inspection shows it deviates slightly from $T^{-2}$ dependence (bulging downward). Actually we find $D_\parallel \propto (E_F/k_B T)^2/\ln(E_F/k_B T)$ as expected of the “normal” transport coefficient in 2D degenerate Fermions. (The logarithmic correction factor is characteristic of a 2D system.) While in Fig. 4, $D_\perp$ increases in the higher temperatures in a similar manner as $D_\parallel$, but it departs from $T^{-2}$ dependence and becomes constant as $T \to 0$ at the larger $P$.

The different temperature dependence of $D_\parallel$ and $D_\perp$ is attributed to the phase space available in the collisions of two diffusion processes. In the longitudinal spin diffusion, a close inspection of the associated collision term shows that electrons in the vicinity of the Fermi surface within the width $k_B T$ only are allowed to participate in the collision due to the kinematics and the Pauli exclusion. Thus as the temperature is lowered, the number of electrons participate in the collision decreases, leading to the reduction of a collision rate. While in the transverse case, all electrons between the two Fermi surfaces of the spin-up and -down electrons participate in the collision. Thus the larger the spin polarization is, the more electrons are involved in the collision, leading to the finite and constant diffusion coefficients at $T \to 0$. This anomalous temperature dependence is observed experimentally in $^3$He-$^4$He system.

B. Electron-impurity scattering

In the previous subsection we have discussed the spin diffusion of 2DEG in the presence of $e-e$ scattering alone. The spin diffusion in actual samples of heterostructures and quantum wells (QWs) should also be affected by other scattering processes. In heterostructures of III-V semiconductors, a low temperature electron mobility, $\mu_e$, is dominated by the scattering with remote ionized impurities (dopants). We expect that the $e$-impurity scattering will also strongly influence the spin transport. We compare the spin diffusion coefficients limited by $e-e$ scattering which we have calculated above with those limited by $e$-impurity scattering, and show that the $e-e$ scattering bears substantial contribution to the spin transport in actual sample structures, although the $e-e$ scattering has only an indirect consequence to the charge transport. We do not give the explicit calculations of the spin diffusion including the $e$-impurity scattering but we estimate its magnitude from experimental carrier mobilities.

The highest mobility reported so far in GaAs/AlGaAs heterostructures exceeds $1 \times 10^7$ cm$^2$ V$^{-1}$ s$^{-1}$ below 1 K. The charge diffusion coefficient $D_e$ arising from $e$-impurity scattering is related to the mobility $\mu_e$ through the generalized Einstein’s relation. For a degenerate system, we have

$$\frac{eD_e}{\mu_e k_B T} = (1 + e^{-\mu/k_B T}) \log(1 + e^{\mu/k_B T}),$$

where $\mu$ is a chemical potential. In the high quality samples cited above, the charge diffusion coefficient $D_e$ is estimated to be $9 \times 10^4$ cm$^2$ s$^{-1}$ below 10 K. Assuming that the magnitude of the spin diffusion limited by $e$-impurity scattering is similar to the charge diffusion, we obtain the estimated value of $9 \times 10^4$ cm$^2$ s$^{-1}$ for the spin diffusion, which is, compared
with Fig. 3, close to the spin diffusion limited by $e$-$e$ scattering at the lowest temperatures. Thus the $e$-$e$ scattering has a significant contribution to the spin diffusion, comparable to the $e$-impurity scattering. In moderate-quality samples with $\mu \leq 1 \times 10^6$ cm$^2$ V$^{-1}$ s$^{-1}$, the relative weight of $e$-ionized impurity scattering should become larger than that of $e$-$e$ scattering.

C. Effective magnetic field

In a many-electron system an individual electron interacts with surrounding electrons through the Coulomb interaction. This effect is represented in the Fermi liquid theory that an electron is subject to a molecular (mean) field produced by surrounding electrons. Due to the exchange part of the interaction, the energy of an individual electron depends on the spin directions of itself and neighboring electrons. Thus the molecular field acts as an effective magnetic field $B_{mol}$ to the individual electron, and its spin precesses around $B_{mol}$. The consequence of $B_{mol}$ was first pointed out by Leggett and Rice that the effective transverse spin diffusion coefficients measured by spin echo experiments should depend on the spin-tipping angle by the initial r.f. pulse (Leggett-Rice effect). The anomalous behavior of the transverse spin diffusion and the spin waves discussed in our previous publication have its physical origin in the precession of flowing spins around the effective magnetic field. Consequences of the effective magnetic field can be seen explicitly in the last two terms on the left-hand side of eq. (2.5), $\mu_{SR} M \times J_{\sigma,i} + \gamma_g B_{ex} \times J_{\sigma,i}$. The second term expresses the spin precession around the external magnetic field, while the first term represents the spin precession around the effective magnetic field. The effective field is parallel to spin density $M$, and its magnitude depends on $\mu_{SR}$ and $M$. We should notice that the precession occurs not on the spin density but on the spin current $J_{\sigma}$, i.e., the precession occurs on an electron flowing into the region with its spin direction slightly tipped away from the local spin density. (If the direction of $M$ is constant throughout the system, the effective field $B_{mol}$ has no net effect.) In Fig. 5 we plot the magnitude of the effective field in terms of spin splitting energy, $\Delta E_{mol} = \hbar \mu_{SR} M$ [eV], (or the Larmor precession frequency) as a function of spin polarization $P = M/n$. The spin splitting shows a sublinear dependence on the spin polarization since $\mu_{SR}$ decreases with $P$.

IV. CONCLUSIONS

We have calculated spin diffusion coefficients of 2DES interacting through the Coulomb force. The improved approximation in the RPA leads to a more precise treatment of many-particle aspects of the electron properties compared with our earlier calculations using the HFA. The magnitudes of both the longitudinal and the transverse spin diffusion coefficients are larger than our previous calculations but their temperature dependencies are qualitatively similar to the previous ones.

Charge transport in heterostructures, at low temperatures, is dominated by the electron-ionized impurity scattering, while the $e$-$e$ scattering has little consequences. In contrast, as we have shown, the spin transport is largely affected by the $e$-$e$ scattering in high-quality samples. Thus we expect that the spin transport properties in semiconductor devices should
depend on spin-up and -down carrier distributions both in real and momentum space, in addition to the distributions of remote dopants. This fact should be taken into account in designing recently proposed spintronics devices.

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APPENDIX A: EXPRESSIONS IN RPA

The equations are shown in detail in our previous publication\textsuperscript{11} calculated in the HFA. In the present study, the mathematical expressions can be obtained by substituting the self-energy terms calculated in the RPA for those in the HFA. In contrast to the HFA self-energy, which gives an analytic expression, the RPA expressions require numerical integrations. We give the relevant expressions used in the present study below.

The quasiparticle energy $\varepsilon_0^0(p, \vec{r}, t)$ in the local equilibrium (Eq. (2.4) in Ref. \textsuperscript{11}) is obtained from the real part of the RPA self-energy following the prescription suggested by Rice,\textsuperscript{18}

$$\varepsilon_{p+}^0(r, t) = \varepsilon_{Kin} + \text{Re} \Sigma^RPA_{p+}(\vec{p}, \hbar \omega = \varepsilon_{Kin}; \vec{r}, t),$$

(A1)

where $+$ ($-$) for $(1,1)$ ($(2,2)$) component, respectively, and the off-diagonal elements are zero. $\varepsilon_{Kin} = p^2/2m$ is the noninteracting energy of an electron, and $\hbar \Sigma_{p+}^RPA(\vec{p}, \hbar \omega; \vec{r}, t)$ is obtained from the imaginary-time function in Eq. (2.2). In the equation of motion for spin current $J_{\sigma,j}$, the coefficient $A$ is a function of spin-up and spin-down electron densities (or the Fermi wave numbers) and is given by

$$A(k_{F\pm}, r, t) = \frac{1}{2m} \left( \frac{n_+(r, t)}{G_+} + \frac{n_-(r, t)}{G_-} \right),$$

(A2)

where $G_{\pm}^{-1} = G_{\pm}^{-1} - \alpha_{\pm}$ with

$$G_{\pm} = \frac{m_{\ast\pm}^2}{2\pi \hbar^2}, \quad \alpha_{\pm} = -\frac{\partial \varepsilon_{p+}^0(k_{F\pm}, \vec{r}, t)}{\partial n_+} - \frac{\partial \varepsilon_{p-}^0(k_{F\pm}, \vec{r}, t)}{\partial n_-}.$$

The coefficient $B$ is

$$B(k_{F\pm}, r, t) = \frac{\pi \hbar^2 n}{mm^*} + \frac{1}{m} \left( \frac{\Delta \varepsilon_{p+}(k_{F\pm}, \vec{r}, t)}{n_+ - n_-} + \frac{\Delta \varepsilon_{p-}(k_{F\pm}, \vec{r}, t)}{n_+ - n_-} \right),$$

(A3)

where $\Delta \varepsilon_p$ is defined as,

$$\Delta \varepsilon_p(r, t) = \frac{1}{2} \left( \varepsilon_{p+}^0(r, t) - \varepsilon_{p-}^0(r, t) \right).$$

The spin-rotation parameter $\mu_{SR}$ is obtained by the numerically integration;

$$\mu_{SR} = -\frac{1}{\pi \hbar n M^2} \int \frac{d^2k}{(2\pi)^2} k^2 \left( \varepsilon_{k+}^0 - \varepsilon_{k-}^0 \right) \Delta \varepsilon_k.$$  

(A4)

The collision terms are given by the same expressions as given in Ref. \textsuperscript{11} except that the quasiparticle energies in the RPA are used.
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FIGURES

FIG. 1. The inplane energy dispersions of interacting, spin-polarized 2D electrons with a total sheet density $2 \times 10^{11}$ cm$^{-2}$ calculated in the RPA. The energy is measured relative to the zero of a noninteracting electron dispersion. The Fermi surfaces for spin-up (spin-down) electrons are at 0.1127 nm$^{-1}$ (0.1115 nm$^{-1}$) for $P = 0.1$, 0.1373 nm$^{-1}$ (0.0793 nm$^{-1}$) for $P = 0.5$, and 0.1545 nm$^{-1}$ (0.0354 nm$^{-1}$) for $P = 0.9$, respectively.

FIG. 2. The definitions of a longitudinal spin diffusion coefficient $D_\parallel$ and a transverse spin diffusion coefficient $D_\perp$. See the text for details.

FIG. 3. The temperature dependence of the longitudinal spin diffusion coefficients $D_\parallel$ with $0.01 < P < 0.5$. The total electron sheet density is fixed at $2 \times 10^{11}$ cm$^{-2}$.

FIG. 4. The temperature dependence of the transverse spin diffusion coefficients $D_\perp$ with $0.01 < P < 0.5$. The total electron sheet density is fixed at $2 \times 10^{11}$ cm$^{-2}$.

FIG. 5. The magnitude of the effective magnetic field due to the molecular field expressed in terms of the spin splitting energy (left) or the Larmor precession frequency (right).
Fig. 1

\[ n = n_+ + n_- = 2.0 \times 10^{11} \text{ cm}^{-2} \]

- P = 0.1, Spin Down
- P = 0.5
- P = 0.9
- P = 0.1, Spin Up
- P = 0.5
- P = 0.9

Energy (x10^{-3} eV) vs. Wavenumber (nm^{-1})
Logitudinal spin diffusion

$P \parallel z$-axis

Magnitude of $P$ changes along $x$

Flow of spin component $\parallel P$

spin current $J_{\sigma_z,x}$

Transverse spin diffusion

$P$

Direction of $P$ changes along $x$

Flow of spin component $\perp P$

spin current $J_{\sigma_x,x}$
Fig. 3

$D_{//}$ vs $1/T^2$

$n = n_+ + n_- = 2.0 \times 10^{11}$ cm$^{-2}$
$D_\perp$ vs $1/T^2$

$n = n_+ + n_- = 2.0 \times 10^{11} \text{ cm}^{-2}$

Fig. 4
\[ n = n_+ + n_- = 2.0 \times 10^{11} \text{cm}^{-2} \]

**Fig. 5**