In-medium $\pi\pi$ Correlation Induced by Partial Restoration of Chiral Symmetry

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We show that both the linear and the non-linear chiral models give an enhancement of the $\pi\pi$ cross section near the $2\pi$ threshold in the scalar-iso-scalar ($I=J=0$) channel in nuclear matter. The reduction of the chiral condensate, i.e., the partial chiral restoration in nuclear matter, is responsible for the enhancement in both cases. We extract an effective 4-$\pi$-nucleon vertex which is responsible for the enhancement but has not been considered in the non-linear models for in-medium $\pi\pi$ interaction. Relation of this vertex and a next-to-leading order terms in the heavy-baryon chiral lagrangian, $L_{\pi N}^{(2)}$, is also discussed.

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Exploring possible evidence of partial restoration of chiral symmetry in hot and/or dense nuclear medium is an intriguing subject in hadron physics (see e.g., [1,2]). In particular, the softening of the scalar-iso-scalar ($I=J=0$) fluctuation in matter could be a distinct signal of the partial restoration of chiral symmetry, as was originally pointed out in [3].

In a recent paper [4], Shimizu and two of the present authors have shown, using the linear $\sigma$ model (LoM), that the partial restoration of chiral symmetry in nuclear matter leads to an enhanced spectral function near the $2\pi$ threshold in the $\sigma$-channel. (The similar effect in hot hadronic plasma has been considered before [5,6].) They also suggested that the enhancement of the cross section in the reaction $\pi A \rightarrow \pi^+\pi^-A'$ near $2\pi$ threshold reported by the CHAOS collaboration [7], which was originally motivated by a possibility of a strong $\pi\pi$ correlation in matter [8], may be interpreted as an evidence of the partial restoration of chiral symmetry.

In fact, the state-of-the-art calculations using the non-linear chiral lagrangian together with $\pi N$ many-body dynamics do not reproduce the cross sections in the $I=0$ and $I=2$ channels simultaneously [9]. In those calculations, the final state interactions of the emitted two pions in nuclei give rise to a slight enhancement of the cross section in the $I=0$ channel, but is not sufficient to reproduce the experimental data. This indicates that some additional mechanism such as the partial restoration of chiral symmetry may be relevant for explaining the data. Such effect can be readily incorporated in the LoM as adopted in [10], while it is not obvious how to take into account the effect in the non-linear chiral models.

The purposes of the present paper are twofold: Firstly, we show that the near-threshold enhancement in the $I=J=0$ channel in nuclear matter takes place irrespective of the representations (linear or non-linear) of chiral symmetry, if the effect of partial restoration of chiral symmetry in the medium is properly incorporated. Secondly, we show that $L_{\pi N}^{(2)}$, which is a next-to-leading order term in the non-linear chiral lagrangian in the heavy-baryon formalism, gives relevant vertices for the near-threshold enhancement in the $I=J=0$ channel. Such vertices have not been taken into account in previous calculations of the in-medium $\pi\pi$ scattering in the non-linear lagrangian approaches.

Let us first consider the $\pi\pi$ scattering in nuclear matter in the SU(2) LoM to demonstrate the essential idea of the near-threshold enhancement. The following argument is slightly different from the original one [1] but is easy to be generalized to the non-linear case. The lagrangian adopted in [9] is the standard Gell-Mann-Levy model with a minor modification

\begin{equation}
L = \frac{1}{4} \text{Tr}[\partial M \partial M^\dagger - \mu^2 M M^\dagger - \frac{2\lambda}{4!}(M M^\dagger)^2 - \hbar (M + M^\dagger)] + \bar{\psi}(i\gamma^5 \cdot \vec{\pi})\psi + \cdots,
\end{equation}

where $M = \sigma + i\vec{\tau} \cdot \vec{\pi}$, $M_5 = \sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}$, $\psi$ is the nucleon field, and $\text{Tr}$ is for the flavor index $[11]$. In this model, the difference of the $\pi\pi$ scattering in the vacuum and that in nuclear matter stems from the nucleon-loops which modify the self-energies and vertices of $\pi$ and $\sigma$. The effect of the nucleons in nuclear matter on the low-energy pions are suppressed due to chiral symmetry. Therefore, one-loop contributions leading to the substantial modifications of the in-medium $\pi\pi$ scattering come mainly from the diagrams shown in Fig.1. Here, Fig.1(a) and Fig.1(b) are the medium modifications of the $\pi\pi$ vertex, respectively. Those mean-field contributions can be incorporated by making the following replacement in the $\pi\pi$ amplitude in the vacuum;

\begin{equation}
\langle \sigma \rangle_0 \rightarrow \langle \sigma \rangle = \Phi(\rho) \langle \sigma \rangle_0,
\end{equation}

where $\langle \cdot \rangle_0$ ($\langle \cdot \rangle$) denotes the vacuum (nuclear matter) expectation value, and $\rho$ ($\rho_0$) is the baryon density (nuclear
The in-medium $\pi\pi$ scattering amplitude in the tree level is written as $T_{\text{tree}}(s,t,u) = \delta^{ab}\delta^{cd}A(s,t,u) + \text{(crossings)}$ with

$$A(s,t,u) = \frac{-\lambda}{3}s - m_s^2,$$

where $m_s^*$ is the in-medium mass defined by $m_s^* = \lambda(\sigma)^2/3 + m_s^2$. The scattering amplitude after the projection to $I = J = 0$ channel reads

$$T_{\text{tree}}(s) = \frac{1}{2}\int_{-1}^{+1} d(\cos \theta) \left[3A(s) + A(t) + A(u)\right].$$

The tree-level amplitude generally breaks unitarity because the imaginary part is not incorporated. A simplest way to restore unitarity is to start with a trivial identification $T_{\text{tree}}(s) \approx T_{\text{tree}}(s)$, while the unitarity condition $\lambda(T^\dagger T) = \lambda(T^\dagger T)$ determines the imaginary part as $\text{Im} T^{-1} = \Theta(s)/2 = -\theta(s - 4m_s^2)/2\pi\sqrt{1 - 4m_s^2/s}$. Here, $\Theta(s)$ is the phase-space volume of the 2$\pi$ intermediate state. Thus one arrives at the unitarized amplitude

$$T(s) = \frac{1}{T_{\text{tree}}^{-1}(s) - i\Theta(s)/2}.$$ (5)

The in-medium $\pi\pi$ cross section is obtained by multiplying the flux factor: $\sigma_{\pi\pi}(s;\rho) \propto |T(s)|^2/s$. Shown in Fig.2 is the cross section for the $I = J = 0$ channel in arbitrary unit. One can see the enhancement near 2$\pi$ threshold associated with the decrease of the chiral condensate $\langle \sigma \rangle$. The parameters in the lagrangian are chosen so that $m_\pi=140$MeV, $\langle \sigma \rangle=\lambda=93$MeV and $m_s^*(\rho = 0) = 550$MeV (which corresponds to $\lambda/4\pi = 7.8$) are reproduced. We have checked that for the mass $m_\pi^*(\rho = 0) = 750$MeV ($\lambda/4\pi = 15.0$) does not show qualitative change. The scattering lengths for $(I = 0, J = 0)$ and $(I = 2, J = 0)$ channels are well reproduced in these parameters. In Fig.3, the in-medium cross section relative to its vacuum value defined below is shown:

$$R = \frac{\sigma_{\pi\pi}(s;\rho)}{\sigma_{\pi\pi}(s;\rho = 0)}.$$ (6)

As is anticipated, a large enhancement near 2$\pi$ threshold can be seen in Fig. 3. However, to make a realistic comparison of our schematic calculation with the experimental data, one needs to incorporate the amplitudes describing the elementary processes of the pion production and absorption in the reaction $\pi A \rightarrow \pi A'$. This is beyond the scope of this work.

The near-threshold enhancement of $T(s)$ and $R$ in eqs. (6) takes place in association with partial restoration of chiral symmetry in nuclear matter. In fact, as $\langle \sigma \rangle$ decreases, $m_\pi^*$ approaches $2m_\pi$, which implies that $T_{\text{tree}}^{-1}(s \approx 2m_\pi)$ tends to be suppressed. Hence the $s$-dependence of the full inverse amplitude $T^{-1}(s)$ just above the 2$\pi$ threshold is governed by the imaginary part $\Theta(s)/2$, which causes the near-threshold enhancement of $T(s)$. This is consistent with the observation made in Ref. [3], where the spectral function in the $\sigma$-channel rather than the $T$-matrix is shown to have a near-threshold enhancement by the same mechanism [13].

At this point, a natural question to ask is whether the near-threshold enhancement obtained in LoSMe arises also in the non-linear models. Furthermore, if it is the case, what kind of vertices in the non-linear chiral lagrangian are responsible for the enhancement? To study these problems, we start with the standard polar parameterization of the chiral field, $M = \sigma + iT \cdot \vec{\pi} = \langle \sigma \rangle + S$ with $U = \exp(i\vec{\tau} \cdot \vec{\phi}/f_\pi^*)$. Here, $\langle \sigma \rangle$ is the sigma condensate in nuclear matter as before, while $f_\pi^*$ is a would-be "in-medium pion decay constant" taken as an arbitrary constant for the moment. Putting the parameterization into eq.(4) and making suitable redefinition of the nucleon field from $\psi$ to $N$, one obtains the non-linear form of $\langle \sigma \rangle$,

$$\mathcal{L} = \frac{1}{2}(\partial S)^2 - m_\sigma^2 S^2 - \frac{\lambda}{6}\langle \sigma \rangle S^3 - \frac{\lambda}{4!}S^4$$

$$+ \frac{\langle \sigma \rangle + S}{4} \text{Tr}[i\partial U\partial U^\dagger] + \frac{\langle \sigma \rangle + S}{4} h \text{Tr}[U^\dagger U]$$

$$+ \mathcal{L}_{\pi\pi N}^{(1)} = gS\bar{N}N,$$ (7)

with

$$\mathcal{L}_{\pi\pi N}^{(1)} = \bar{N}(i\partial + i\psi + i\vec{\psi}\gamma_5 - m_\pi^*\gamma_5)N.$$ (8)

where $(\psi, \rho_\mu) = (\xi\partial_\nu \xi^\dagger \pm \xi^\dagger \partial_\nu \xi)/2$, and $m_N^* = g(\sigma)$. $\langle \sigma \rangle$ is to be determined by minimizing the effective potential in nuclear matter calculated with eq.(8), or equivalently by the condition $\langle S \rangle = 0$. In the mean-field level, the density dependence of $\langle \sigma \rangle$ is dictated solely by the Yukawa coupling, $S\bar{N}N$.

It is worth emphasizing that $f_\pi^*$, which has been left undetermined, should be chosen as $f_\pi^* = \langle \sigma \rangle$ to impose proper normalization of the pion field in nuclear matter. This is in fact crucial for the linear and non-linear representations to give equivalent results in nuclear matter.

In-medium $\pi\pi$ amplitude $A(s)$ in the tree level obtained from (6) reads

$$A(s) = \frac{s - m_s^2}{\langle \sigma \rangle^2} - \frac{(s - m_s^2)^2}{\langle \sigma \rangle^2} \frac{1}{s - m_s^2}.$$ (9)

The first term in (9) comes from the contact $4\pi$ coupling generated by the expansion of the second line in (8) with the coefficient proportional to $1/\langle \sigma \rangle^2$. On the other hand,
the second term in (i) is from the contribution of the scalar meson $S$ in the $s$-channel.

Of course, $A(s)$ in eq. (11) and its unitarized amplitude $T(s)$ are exactly the same with eq. (1) and eq. (3), respectively, since what we have done is simply the field redefinition. Nevertheless, we have two observations from the chiral invariant decomposition eq. (11):

(i) If $m_{\pi}^* = \lambda \langle \sigma \rangle_0^2 / 3 + m_{\pi}^2$ is finite and decreases as density increases, the near-threshold enhancement occurs exactly in the same way as the previous case shown in Fig.2 and Fig.3.

(ii) Even in the limit where $m_{\pi}^*$ is infinitely large ($\lambda \to \infty$) and hence the second term of eq. (11) is negligible, the first term signals the enhanced attraction of the $\pi\pi$ interaction as long as $\langle \sigma \rangle$ decreases in nuclear matter.

The second observation is demonstrated in Fig.4 where the in-medium $\pi\pi$ cross section in the $I = J = 0$ channel is calculated with the unitarized amplitude obtained only from the first term of (i). Although the enhancement is smaller than that of $\sigma \cdot \rho M$, there is still a large enhancement in the narrow range of the $\pi\pi$ invariant mass near $2\pi$ threshold. One should note here that as $\langle \sigma \rangle$ decreases, the convergence of the chiral expansion of $A(s)$ by $s/\langle \sigma \rangle$ becomes less reliable. Therefore, Fig.4 with only the leading term in the chiral expansion should be taken as a qualitative one.

It is in order here to emphasize the relation of the near-threshold enhancement and the softening of the $I = J = 0$ fluctuation in nuclear matter. Such fluctuation can be characterized by the complex pole of the unitarized amplitude $T(s)$ in eq. (3). For simplicity, let us take the case (ii) in the chiral limit, where the analytic solution can be written as $\sqrt{s_{\text{pole}}} = \sqrt{8\pi \langle \sigma \rangle} (1 - i)$. This complex pole corresponds to a broad $I = J = 0$ mode dynamically generated by the $\pi\pi$ rescattering. As $\langle \sigma \rangle$ is reduced in the medium, the pole moves toward the origin in the complex $\sqrt{s}$-plane (4) and thus causes an enhancement in the $\pi\pi$ cross section as shown in Fig.4. It is also true for the case (i) leading to Fig.2. Such connection between the moving pole in the complex plane and the near-threshold enhancement has been first pointed out in (4) using the Nambu-Jona-Lasinio model. Further study in the present context with finite $m_\pi$ will be discussed elsewhere (4).

Now, we turn to the question on the effective vertex responsible for the enhanced $\pi\pi$ attraction in nuclear matter in the non-linear chiral lagrangian. To study this, let us start with eq. (3) in the vacuum and take a heavy-scalar ($S$) limit and heavy-baryon ($N$) limit simultaneously. These limits can be achieved by $\lambda, g \to \infty$ with $g/\lambda$ and $\langle \sigma \rangle_0 = f_\pi$ being fixed. In this limit, the heavy scalar field $S$ may be integrated out and the following effective lagrangian is obtained:

$$\mathcal{L} = \left( \frac{f_{\pi}^2}{4} - \frac{g_f f_\pi}{2 m_\pi^2} \right) \langle \sigma \rangle_0 \left( \frac{\hbar}{f_\pi} \text{Tr}[\partial U \partial U^\dagger] + \frac{\text{Tr}[U^\dagger + U]}{f_\pi} \right) + \mathcal{L}_{\pi N}^{(1)} + \cdots,$$ (10)

where all the constants take their vacuum values: $f_\pi = \langle \sigma \rangle_0$, $m_{\pi}^2 = \lambda \langle \sigma \rangle_0^2 / 3 + m_{\pi}^2$, and $m_N = g_\sigma \langle \sigma \rangle_0$. Note that $g_f f_\pi / 2 m_\pi$ in front of $\langle \sigma \rangle_0$ in eq. (10) approaches a finite value $3g_f / 2 f_\pi$ in the heavy limit, thus it cannot be neglected. In (4), $\cdots$ denotes other higher dimensional operators which are not relevant for the discussion below.

In the uniform nuclear matter, $\langle \sigma \rangle_0$ in eq. (10) may be replaced by $\rho$. This leads to a reduction of the vacuum condensate; $f_\pi = \langle \sigma \rangle_0 \to \langle \sigma \rangle = \langle \sigma \rangle_0 (1 - g g_f / f_\pi m_\pi^2) = f_\pi'$. Then the proper normalization of the pion field in nuclear matter should be $\phi' = (\phi / f_\pi) \cdot f_\pi'$. The in-medium $\pi\pi$ scattering with this normalization exactly reproduces the first term in (i) as it should be. Therefore, the origin of the near-threshold enhancement in the heavy limit can be ascribed to the following new vertex:

$$\mathcal{L}_{\text{new}} = - \frac{3g_\pi}{2\lambda_f} \bar{N} N \text{Tr}[\partial U \partial U^\dagger].$$ (11)

Because this vertex is proportional to the scalar-isoscalar density of the nucleon, it affects not only the pion propagator but also the interaction among pions in nuclear matter. In Fig.5, $4\pi N N$ vertex generated by $\mathcal{L}_{\text{new}}$ is shown as an example. Note that the vertex in Fig.5 acts essentially the same as $\mathcal{L}_{\pi N}^{(1)}$ (5) in eq. (3) is the lowest order chiral lagrangian for the pion, $\mathcal{L}_{\pi N}^{(1)}$ (essentially the same as $\mathcal{L}_{\pi N}^{(2)}$ in (6)) is the $O(\langle \sigma \rangle)$ term with a single derivative. $\mathcal{L}_{\pi N}^{(2)}$ is an $O(\langle \sigma \rangle^2)$ term having the following structure (the complete structure is given in Appendix A of (6)),

$$\mathcal{L}_{\pi N}^{(2)} = \delta_5 \tilde{N}(u \cdot u) N + (c_2 - \frac{g_\pi^2}{8 m_\pi^2}) \tilde{N}(v \cdot u) N + c_1 N \text{Tr}(U^\dagger \chi + \chi U) + \cdots.$$ (13)

where $u^\mu = i \xi^\mu \partial_\mu \xi^\dagger$, $U = \xi^2 = \exp(i \pi^a \phi^a / f_\pi)$, $\nu_\mu$ is the four velocity of the nucleon, $g_\pi$ is the axial charge of the nucleon, and $\chi = h^a / f_\pi$ denotes the explicit symmetry breaking. Thorsson and Wirzba (6) have derived the in-medium chiral lagrangian from eq. (3) by taking the mean-field approximation for the nucleon field;

$$\langle \mathcal{L} \rangle = \left( \frac{f_{\pi}^2}{4} + \frac{c_3}{2} \right) \text{Tr}[\partial U \partial U^\dagger]$$
serve that the new vertex of the form \( \bar{c} \) respectively. We just quote here the numbers although they have only the Lorentz invariant terms because we have appear in both cases with the same sign. In eq.(10), we isospin symmetry in nuclear matter.

Note that \( L^{(1)}_{\pi N} \) in [12] disappears in (14) because of the isospin symmetry in nuclear matter.

As is shown in [15], the coefficients \( c_1, c_2, c_3 \) are related to the \( \pi N \) sigma term, the iso-spin even S-wave \( \pi N \) scattering length, and the nucleon axial polarizability, respectively. We just quote here the numbers although they have large potential uncertainties: \( c_1 = -0.87 \text{GeV}^{-1} \), \( c_2 = 3.34 \text{GeV}^{-1} \), and \( c_3 = -5.25 \text{GeV}^{-1} \).

By comparing eq. (10) with eq. (13) or eq. (14), we observe that the new vertex of the form \( N \pi N \text{Tr} [\partial U \partial U^\dagger] \) appear in both cases with the same sign. In eq. (10), we have only the Lorentz invariant terms because we have integrated out only the scalar meson \( S \). On the other hand, eq. (14) contains more general \( O(3) \) invariant terms because it potentially contains the effect of heavy excited baryons [18].

Finally, let us comment on the recent analysis by Oset and Vicente Vacas on the \( \pi \pi \) scattering in nuclear matter using non-linear chiral lagrangian [21]. They adopted only \( L^{(2)}_{\pi N} + L^{(1)}_{\pi N} \) for the \( \pi N \) dynamics and concluded that the near-threshold enhancement is not obtained from the \( 4\pi - N \pi \) vertex generated by \( L^{(1)}_{\pi N} \). This observation is consistent with what we have mentioned before; the effect of \( L^{(1)}_{\pi N} \) disappears due to isospin symmetry and has no significance on the threshold \( \pi \pi \) correlation. Therefore, it is of great importance to make an extensive analysis of the \( \pi \pi \) interaction in nuclear matter with \( L^{(2)}_{\pi N} \).

In summary, we have shown that the near-threshold enhancement of the in-medium \( \pi \pi \) scattering in the \( I = J = 0 \) channel occurs both in the linear and non-linear representation of chiral symmetry. The enhancement is caused by the in-medium reduction of the chiral condensate associated with partial restoration of chiral symmetry. In fact, the decrease of the chiral condensate is shown to enhance the \( \pi \pi \) attraction in the \( I = J = 0 \) channel and simultaneously induce the softening of the scalar-iso-scalar fluctuations in nuclear matter. Also, we have identified an effective \( 4\pi \)-nucleon vertex responsible for the enhancement and discussed its relation to the next-to-leading order term \( L^{(2)}_{\pi N} \) in the non-linear chiral lagrangian in the heavy-baryon formalism. It is thus an intriguing problem to examine whether the softening of the scalar-iso-scalar fluctuation induced by \( L^{(2)}_{\pi N} \) manifests itself in explaining the CHAOS data in the non-linear lagrangian approach.

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FIG. 1. The medium modifications of (a) the $\sigma$ propagator and (b) the $\sigma\pi\pi$ vertex through the nucleon-loop in the mean-field approximation. The solid line with arrow, the dashed line and the double line represent the nucleon, $\pi$ and $\sigma$, respectively.

FIG. 2. In-medium $\pi\pi$ cross section in the $I=0$ channel for different values of $\Phi(\rho)$. The cross section is shown in the arbitrary unit (A.U.).

FIG. 3. $R$ (the in-medium cross section divided its vacuum value) in the $I=J=0$ channel.

FIG. 4. In-medium $\pi\pi$ cross section in the $I=J=0$ channel in the heavy $S$ limit where $m_\sigma^*$ is taken to be infinity. The cross section is shown in the arbitrary unit (A.U.) but in the same scale with Fig.2.

FIG. 5. An example of the new $4\pi$-N-N vertex generated by $L_{\text{new}}$ or by $L_{\pi N}^{(2)}$. The solid line with arrow and the dashed line represent the nucleon and pion, respectively.