Dark Halo or Bigravity?

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Abstract. Observations show that about the 20% of the Universe is composed by invisible (dark) matter (DM), for which many candidates have been proposed. In particular, the anomalous behavior of rotational curves of galaxies (i.e. the flattening at large distance instead of the Keplerian fall) requires that this matter is distributed in an extended halo around the galaxy. In order to reproduce this matter density profiles in Newtonian gravity and in cold dark matter (CDM) paradigm (in which the DM particles are collisionless), many ad-hoc approximations are required. The flattening of rotational curves can be explained by a suitable modification of gravitational force in bigravity theories, together with mirror matter model that predicts the existence of a dark sector in which DM has the same physical properties of visible matter. As an additional result, the Newton constant is different at distances much less and much greater than 20 kpc.

1 Introduction

Cosmological observations show the Universe to be nearly flat, i.e. the energy density is very close to the critical one: $\Omega_{\text{tot}} \simeq 1$. We can separate the different contributions present in the total density. Mainly, we can write $\Omega_{\text{tot}} = \Omega_M + \Omega_A$, in which we distinguish the contributions due to matter ($\Omega_M = 0.24\pm0.02$) and to the dark energy (cosmological term, $\Omega_A = 0.76\pm0.02$). In particular $\Omega_M = \Omega_B + \Omega_D$, where $\Omega_B = 0.042\pm0.005$ and $\Omega_D = 0.20\pm0.02$ are, respectively, the baryonic and dark matter components [1].

Let us focus on the dark matter problem. CMD model is in agreement with experimental results on Cosmic Microwave Background (CMB) and Large Scale Structure (LSS) that confirm the presence of the dark matter component. Many DM candidates have been proposed in the literature as axions ($m \sim 10^{-5}$ eV), neutralinos ($m \sim$ TeV), wimpzillas ($m \sim 10^{14}$ GeV) and so on. However, the question why the fractions of the baryonic and dark components are so close, $\Omega_D/\Omega_B \sim 5$, remains unresolved.

A most convincing proof for DM comes from rotational curves of galaxies and cluster dynamics. In the Newtonian picture, at large distance from the center of a galaxy one expects that the velocity along a circular orbits behaves as $v(r) \propto 1/\sqrt{r}$ (also known as Keplerian fall). This is due to the fact that the gravitational potential of a galaxy outside the inner core (bulge) is $\phi \simeq GM/r$, where $G$ is the Newton constant. Instead, one observes that in these regions $v$ is approximately constant. In order to explain this anomalous behavior, without modifying the Newtonian paradigm, one has to suppose the existence of dark matter distributed in a extended spherically symmetric halo around the galaxy, according to the “isothermal” mass distribution profile $\rho(r) \propto (1+(r/a)^2)^{-1}$, where $a$ is a scale radius. Since gravity is universal between visible
and dark matter, a point-like source of both types of matter generates a potential

$$\phi(r) = \frac{G}{r}(M_1 + M_2)$$

(1)

where $M_1$ and $M_2$ are, respectively, the visible and dark components. CDM model assumes that the DM particles are collisionless. Due to this property, N-body numerical simulations tend actually to predict a different behavior: the DM distribution has a cusp profile $\rho(r) \propto 1/r^\alpha$, with $\alpha = 1 \pm 1.5$ \cite{23}. For many galaxies these cusp profiles do not reproduce the observed rotational curves as well as the isothermal profile.

An alternative possible candidate is the Mirror matter \cite{4}. According to this model our Universe is made of two similar gauge sectors. In other words, in parallel to our sector of the ordinary particles (O-) and interactions described by the Standard Model, there exists a hidden sector (M-) that is an exact duplicate of ordinary sector in which particles and interactions have exactly the same characteristics, and the two sectors are connected by the common gravity (see for review \cite{5}). Therefore, if the mirror sector exists, then the Universe should contain along with the ordinary particles (electrons, nucleons, photons, etc.) also their mirror partners with exactly the same masses (mirror electrons, mirror nucleons, mirror photons, etc.). Mirror matter, invisible in terms of ordinary photons, can naturally constitute dark matter. One should stress that the fact that O- and M-sectors have the same microphysics, does not imply that their cosmological evolutions should be the same too. In fact, if mirror particles had the same temperature in the early universe as ordinary ones, this would be in immediate conflict with Big Bang Nucleosynthesis (BBN). The BBN limit on the effective number of extra neutrinos implies that the temperature of the mirror sector $T'$ must be at least about 2 times smaller than the temperature $T$ of the ordinary sector, which makes mirror baryons viable candidate for dark matter. In particular, the mirror dark matter scenario would give the same pattern of LSS and CMB as the standard CDM if $T'/T < 0.2$ or so \cite{6}. In addition, the baryon asymmetry of the Universe can be generated via out-of-equilibrium $B - L$ and $CP$ violating processes between ordinary and mirror particles \cite{7} whose mechanism could explain the intriguing puzzle of the correspondence between the visible and dark matter fractions in the Universe, naturally predicting the ratio $\Omega_D/\Omega_B \sim 1/10$ \cite{8}.

However, in contrast to the collisionless CDM, mirror baryons obviously constitute collisional and dissipative dark matter. Therefore, one should expect that mirror matter undergoes a dissipative collapse and thus in the galaxies it is distributed in a similar manner as the visible matter instead of producing extended quasi-spherical CDM halos. Indeed even the hidden sector undergoes a dissipative collapse as the visible sector that follows an exponential profile $\rho(r) \propto e^{-r/r_m}$. In this way the distribution of the dark matter is more compact in the center of the galaxy and is not extended as the CDM halo.

Since gravity is universal between the two sectors, this mirror dark matter hypothesis gets into difficulties to explain the flat rotational curves of galaxies. However we can suppose that each sector has its own gravity and that mixing term produces a suitable modification of gravity at large distance\footnote{The particle mixing phenomena between ordinary and mirror sectors were discussed in the literature for photons \cite{9}, neutrinos \cite{10}, neutrons \cite{11}, etc., as well as possible common gauge interactions between two sectors \cite{12}. The mixing between the ordinary and mirror gravitons was first discussed in our recent papers \cite{19,20}.}. In particular we show that the interaction term allows us to obtain a massive graviton and leads to a modified potential. A test mass of type 1 at distance $r$ from the origin in which there is a sources of both types of matter $(M_1$ and $M_2)$, instead of $M_1$, feels a potential

$$\phi(r) = \frac{G}{2r}(M_1 + M_2) + \frac{Ge^{-r/r_m}}{2r}(M_1 - M_2),$$

(2)

where $G$ is the Newton constant and $r_m$ is the range of the massive graviton. Notice that at small distance $r \ll r_m$ the test mass interacts only with $M_1$ through the ordinary Newton potential, whereas, at large distance $r \gg r_m$ the test mass interact with the sum of the two kinds of matter $(M_1 + M_2)/2$. This result, together with the mirror matter hypothesis, enables us to reproduce the observed rotational curves of galaxies.
2 The Model

Let us consider a theory with two dynamical metrics $g_{1,2\mu\nu}$, each of them interacting with its own matter. The total action contains two Hilbert-Einstein actions and a mixed term $\mathcal{V}$:

$$S = \int d^4x \left[ \sqrt{g_1} \frac{M_1^2}{2} R_1 + \mathcal{L}_1 + \sqrt{g_2} \frac{M_2^2}{2} R_2 + \mathcal{L}_2 - \mu^4 (g_1 g_2)^{1/4} \mathcal{V}(g_1, g_2) \right],$$

where $M_{1,2}$ are the Planck masses (in general different) and $\mathcal{L}_{1,2}$ are the corresponding matter Lagrangians (respectively, ordinary matter or type 1 and dark matter or type 2). The action describes a more generic bigravity theory, that is the simplest case of the multigravity theory which considers $N$ metrics interacting each other through a mixing term (see for review \[13\]). The interaction term breaks down the invariance under the diffeomorphism group $D_1 \otimes D_2$ to a diagonal diffeomorphism $D_{1+2}$. The two metrics $g_1$ and $g_2$, in the flat (Minkowski) space approximation, can be written as $g_{1,2\mu\nu} \simeq \eta_{1,2\mu\nu} + h_{1,2\mu\nu}/M_{1,2}$. The mixed term in \[3\] can induce the non diagonal rank 1 mass matrix between two gravitons $h_1$ and $h_2$ which has one massless and one massive eigenstates:

$$\begin{cases} h_{\mu\nu} = \cos \theta h_{1\mu\nu} + \sin \theta h_{2\mu\nu} \\ \tilde{h}_{\mu\nu} = -\sin \theta h_{1\mu\nu} + \cos \theta h_{2\mu\nu} \end{cases}$$

where $\theta$ is a mixing angle: $\tan \theta = M_2/M_1$. In the case $M_1 = M_2$, i.e. $\theta = \pi/4$, the rotation \[4\] reduces to even and odd combinations of $h_{1,2\mu\nu}$. The massless state $h_{\mu\nu}$ is the ordinary graviton that exhibits a Newtonian potential $\sim 1/r$ universally coupled with both matters.

The massive state $\tilde{h}_{\mu\nu}$ in turn can have a Lorentz breaking (LB) mass pattern \[14\]

$$\mathcal{L}_{\text{mass}} = \frac{M^2}{2} \left( m_6^2 \tilde{h}_{00}^2 + 2m_1^2 \tilde{h}_{0i}^2 - m_3^2 \tilde{h}_{ij}^2 + m_5^2 \tilde{h}_{ii}^2 - 2m_4^2 \tilde{h}_{00} \tilde{h}_{ii} \right),$$

(0 and $i = 1, 2, 3$ are the time and space indices, respectively) that can induce, for a suitable combination of the masses $m_i$'s \[14\], a Yukawa term in the potential $\sim (1/r) e^{-r/r_m}$ in weak field limit approximation.

From these solutions, in general it follows that a test particle of ordinary matter (type 1) feels a static potential induced by a point-like source containing the mass fractions $M_1$ and $M_2$ of the two types of matter, as

$$\phi(r) = \frac{G}{2r} \left( M_1 + M_2 \right) + \xi \frac{G e^{-r/r_m}}{2r} \left( \tan^2 \theta M_1 - M_2 \right),$$

where $G = 1/(8\pi(M_1^2 + M_2^2))$ and $\xi$ and $r_m$ are parameters that depend on the pattern of the masses in \[3\]. The first term is mediated by massless gravity and the second term by the massive one. The symmetric case, i.e. $M_1 = M_2$, allow us to simplify the potential \[5\] in the following form

$$\phi(r) = \frac{GM_1}{r} \left( \frac{1 + \xi e^{-r/r_m}}{2} \right) + \frac{GM_2}{r} \left( \frac{1 - \xi e^{-r/r_m}}{2} \right),$$

that shows directly the modification with respect to the Newtonian law \[11\]. Let us distinguish two cases: the mass term \[5\] is Lorentz invariant or Lorentz breaking. In the first case the only consistent mass term (without ghosts) is Pauli-Fierz type \[14\] (i.e. $m_0 = 0$ and $m_{1,2,3,4} = m_1$). In this case in \[5\] one has $\xi = 4/3$ and therefore a deviation from the General Relativity prediction for the light bending in gravitational field \[2\] also noted as van Dan-Veltmann-Zakharov discontinuity (vDVZ).

\[2\] In this case one can define the discontinuity parameter $\delta = 1 + (\sin^2 \theta)/3$. Experimental limits on the post-Newtonian gravity \[17\] requires $\delta = 1.0000 \pm 0.0001$, therefore $\theta \simeq 10^{-2}$. As a consequence the Lorentz invariant case $M_1 = M_2$ is excluded.
In the second case, in general the masses in (5) are different. The discontinuity is absent if, in the mass term, $m_0 = 0$, $m_1 \neq m_4$ and/or $m_2 \neq m_3$. However, the bigravity (3) produces a mass term with $m_1 = 0$ and therefore does not give a Yukawa potential [18]. The addition of a third auxiliary metric $g_3$, which works as a bridge between $g_1$ and $g_2$ and decouples with a Planck mass $M_5 \gg M_{1,2}$, allows us to have an effective bigravity theory with $m_1 \neq 0$ [19]. In this case the massive gravity introduces a Yukawa potential (with range $1/(\sqrt{3}m_4)$) without vDVZ discontinuity (i.e. $\xi = 1$) as in (2). This potential, in the mirror matter model, can explain the rotational curves of galaxies with the condition $M_2 \simeq 10M_1$ and $r_m \simeq 20$ kpc (i.e. $\Omega_D/\Omega_M \simeq 10$).

3 Rotational Curves of the Galaxies

The galaxy rotational curves describe the velocity of stars and interstellar gases as a function of the distance $r$ from the center. For the sake of simplicity we apply our model to disk galaxies, where most of the matter (about 2/3) is concentrated in the inner region, called bulge. Indeed, one can suppose that the matter density along the profile follows approximately the luminosity, which decreases exponentially with $r$ moving out from the center. In a spherically symmetric approximation the visible matter has an exponential distribution depending only on $r$, i.e. $\rho(r) = \rho_0 e^{-r/r_0}$, where $\rho_0$ is the density in the central region and $r_0$ is the size of the bulge. The Newtonian theory, that takes into account only visible baryonic mass, does not explain the flattening of the rotational curves at large distance and in general needs to introduce an extended halo composed by dark matter with different density profile with respect to the visible one.

In bigravity theory, in which we consider the mirror matter as dark matter candidate, a different explanation emerges (20). Let us assume the following hypotheses

- The ordinary and dark matters interact only via gravity, modified according to the bigravity model (2).
- The two types of matter have similar density profiles in the galaxy, i.e. exponential along the disk, $\rho(r) \propto \exp(-r/r_0)$.

The velocity of an object at distance $r$ is determined by equating the centrifugal acceleration with the radial component of gravitational acceleration $a(r)$ derived from the potential (2). For instance, from the gravitational field of point-like source of both types sitting in the center, we find

$$a(r) = G_N \left[ \frac{M_1 + M_2}{2r^2} + \frac{M_1 - M_2}{2r^2} \left( 1 + \frac{r}{r_m} \right) e^{-r/r_m} \right].$$

(8)

In order to obtain the total force on a star moving approximately along a circular orbit around the galaxy center we have to integrate (5) on the matter spatial distribution. Notice that for $r << r_m$, the influence of matter of type 2 on a particle 1 is negligible and the behavior is Newtonian. The behavior in the opposite limit $r >> r_m$ is also essentially Newtonian, though the test particle feels the presence of the total mass $M_1 + M_2$ and the effective Newton constant is $G/2$. In the region $r \sim r_m$ there is a significant deviation from the Newtonian theory due to the presence of matter of type 2, resulting in an enhancement of $v$. This result avoids the cusp problem which is present in the context of the CDM paradigm and reproduces a isothermal-like shape ($M_D/M_B \simeq 5$ and $\rho_{DM}(r) \propto (1+(r/r_0)^2)^{-1}$). For instance, let us consider a galaxy with $M_1 = 10^{11}M_\odot$ and the bulge size $r_0 \simeq 3$ kpc (e.g. the Milky Way). In Fig. 3 we compare the rotational curves fitted with the isothermal DM halo ($a \simeq 8$ kpc) in standard gravity and those fitted with the exponential DM profile in the bigravity theory ($r_0 = 5.4$ kpc for the invisible distribution). Notice that both curves have approximately the same behavior. In addition, we show the visible matter contribution to the velocity reproducing the Keplerian fall proportional to $1/r$.

The flat rotational curves can be reproduced varying the parameter $r_m$ and the mass ratio $M_2/M_1$. Fig. 2 shows the rotational curves for different $M_2/M_1$, with $r_m = 20$ kpc. Notice, that for $M_1 = M_2$ we obtain the Keplerian fall as we expect from the Newtonian potential (2).
Fig. 1. Rotational curves fitted by the isothermal DM halos in standard gravity (dashed) and by the exponential DM profile in the bigravity theory (solid) with $M_2/M_1 = 10$ and $r_m = 20$. Contribution of the visible matter to the velocity given by the dotted curve.

Fig. 2. The rotational curves for different $M_2/M_1$, with $r_m = 20$ kpc. Notice, that for $M_1 = M_2$ we obtain the Keplerian fall.

Incidentally, if both the sectors have the same content of matter the potential is indistinguishable from the standard Newton potential $\phi = G(M_1/r)$ generated by the visible matter.

4 Conclusions

If the Universe is made of two separate sectors, one visible and one hidden, each of them can have its own gravity and the two metrics can interact as in a bigravity pattern. This lead to a large distance modification of the gravitational force, though it remains Newtonian at small distances where each type of matter feels only itself, in agreement with the precision test of General Relativity in the Solar System.

The mirror matter as dark matter, together with Lorentz breaking bigravity can explain the flattening of rotational curves of galaxy at large distance. This model supposes that both dark matter and modification of gravity are present. The main advantage of this theory is that, due to the same property of both types of matter, the dark matter can have a mass distribution in galaxy similar to the visible one. This avoids the need to invoke the presence of extended halo distributions that are in conflict with numerical simulations in the CMD paradigm.

As an additional result, it interesting to note that the Newton constants for type 1 - type 1 attraction differs by a factor 2 for large and small distances: $G_N(r \ll r_m) = G$ and $G_N(r \gg$
In the general potential (6) the Newton constant can be measured as $G_N(r \ll r_m) = G[(1 + \xi \tan^2 \vartheta)/2]$ between type 1 - type 1 matter at small distance, while at large distance we have $G_N(r \gg r_m) = G$. Moreover, it is interesting to note that, if $\xi > 1$, at small distance $r \ll r_m$ one has antigravity between type 1 and type 2 objects.

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