TWO–SPIN ASYMMETRY OF HEAVY QUARKONIA PRODUCTIONS AT RHIC ENERGY

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ABSTRACT

Hadroproductions of charmonium and bottomonium \( \chi_J \) in polarized \( pp \) collisions at RHIC energies are studied. Two–spin asymmetries \( A_{\chi J}^{LL}(pp) \) for these reactions are very good parameters for examining the polarized gluon distributions in a proton.

1 Introduction

Relativistic Heavy Ion Collider (RHIC)\(^\dagger\) which is designed to have a beam polarization of about 70 % with a luminosity of \( 2 \times 10^{32} \text{ cm}^{-2} \text{ sec}^{-1} \) at an energy of \( \sqrt{s} = 50 – 500 \text{ GeV} \) will give us fruitful informations on the spin structure of proton. Recent measurements of the spin–dependent proton structure function \( g_1^p(x, Q^2) \) by EMC\(^\ddagger\) and SMC Collaboration\(^\S\), which are far from the Ellis–Jaffe prediction\(^\dagger\), confirmed again the suggestion that very little of the proton spin is carried by quarks. Many ideas have been proposed so far to solve this problem. One of the interesting approach is that the gluon plays a crucial role on the nucleon spin structure\(^\dagger\). But the magnitude \( \Delta G \) and distribution \( \delta G(x) \) of the gluon polarization inside a proton is not known yet. In order to improve our understanding of the nucleon structure, it is necessary to measure \( \Delta G \) and \( \delta G(x) \) in other reactions which are sensitive to the gluon polarization.

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Among various reactions, the $\chi_J$-productions in hadron-hadron collisions are interesting candidates. In this paper, we are concerned with the $\chi_J$-hadroproductions at low-$p_T$ regions. The cross sections of these processes are expected to depend sensitively on the shape of polarized gluon distributions because the gluon-gluon fusion is the dominant mechanism at the lowest order of QCD (see Figure 1). Although these productions have been carefully studied so far by several people in order to extract unpolarized gluon distributions $G(x)$ and test the perturbative QCD\cite{6}, we are interested in the behavior of the polarized gluon distributions $\delta G(x)$.

So far, various types of the polarized gluon distribution functions $\delta G(x)$ have been proposed: some of them have large $\Delta G(\simeq 5-6)$\cite{7,8,9,10} and others have small $\Delta G(\lesssim 2-3)$\cite{8,9,11,12}, where $\Delta G = \int_0^1 \delta G(x) dx$. Here we take the following typical types of $x\delta G(x)$ to examine the effect on the $\chi_J$-hadroproductions:

(a) MTY model \cite{10}:
\[
x\delta G(x, Q^2 = 10 GeV^2) = 9.52x^{0.4}(1 - x)^{17},
\]
with $\Delta G(Q^2_{SMC}) = 5.32$,

(b) Cheng–Lai model \cite{8}:
\[
x\delta G(x, Q^2 = 10 GeV^2) = 3.34x^{0.31}(1 - x)^{5.06}(1 - 0.177x),
\]
with $\Delta G(Q^2_{SMC}) = 5.64$,

(c) BBS model \cite{11}:
\[
x\delta G(x, Q^2 = 4 GeV^2) = 0.281 \left\{ (1 - x)^4 - (1 - x)^6 \right\}
+ 1.1739 \left\{ (1 - x)^5 - (1 - x)^7 \right\},
\]
with $\Delta G(Q^2_{SMC}) = 0.53$,

(d) no gluon polarization model:
\[
x\delta G(x, Q^2 = 10 GeV^2) = 0 , \text{ with } \Delta G(Q^2_{SMC}) = 0 ,
\]
where $Q^2_{SMC}$ is taken to be 10GeV$^2$. Among these distributions, $\Delta G$ of types (a) and (b) are large while those of types (c) and (d) are small and zero, respectively. The $x$-dependence of $x\delta G(x, Q^2)$ at $Q^2 = 10\text{GeV}^2$ and $\delta G(x, Q^2)/G(x, Q^2)$ which are evolved up to $Q^2 = M_{\chi_j(c\bar{c})}^2$ and $M_{\chi_j(b\bar{b})}^2$ by the Altarelli–Parisi equations, is depicted in Figures 2(A), (B) and (C), respectively. Most people\cite{7} who assume large $\Delta G$ take type (b) as the $x\delta G$. As shown in Figure 2(A), the $x\delta G(x)$ of type (b) has a peak...
at $x \approx 0.05$ and gradually decreases with increasing $x$ while that of (a) has a sharp peak at $x \approx 0.04$ and rapidly decreases with $x$. It is remarkable that the distribution of type (a) is consistent with the experimental data of not only two–spin asymmetries $A_{LL}^{\pi^0}(p \bar{p})$ for the inclusive $\pi^0$–production[10, 13] but also recent $A_{LL}^{\gamma \gamma}(pp)$ for the inclusive multi–$\gamma$ pair production[14] measured by the E581/704 Collaboration at Fermilab by using longitudinally polarized proton (antiproton) beams and longitudinally polarized proton targets, even though it has large $\Delta G (= 5.32)$ in a proton. Here we examine above four types of polarized gluon distribution functions to discuss low–$p_T$ $\chi_J$–hadroproductions, though the type (b) has been already ruled out by the data on two–spin asymmetries[15].

2 Two–spin asymmetry $A_{LL}$

Let us start by defining an interesting parameter called a two–spin asymmetry for $\chi_J$–hadroproductions $A_{LL}^{\chi_J}$, which is given by

$$A_{LL}^{\chi_J} = \frac{[d\sigma_{++} - d\sigma_{+-} + d\sigma_{-+} - d\sigma_{--}] + [d\sigma_{++} + d\sigma_{+-} + d\sigma_{-+} + d\sigma_{--}]}{d\sigma_{\chi_J}}. \tag{5}$$

Here $d\sigma_{+-}$, for instance, denotes that the helicity of one beam particle is positive and the one of the other is negative. Although several people have investigated $A_{LL}^{\chi_J}$ as a function of $\sqrt{s}$ [16], the experimental data are available more easily as a function of the longitudinal fraction of produced particle $x_L$. Then we investigate how the $x_L$–dependence of $A_{LL}^{\chi_J}$ for $\chi_J$–hadroproductions in polarized $pp$ collisions are affected by polarized gluon distribution functions.

Since the dominant source is the gluon–gluon fusion as shown in Figure 1, the reaction is very sensitive to polarized gluon distributions in a proton. The $\chi_1$ state cannot be produced by this reaction due to Yang’s theorem. The spin–dependent and spin–independent subprocess cross sections $\Delta \hat{\sigma}_{\chi_J}$ and $\hat{\sigma}_{\chi_J}$ for the subprocess $G + G \rightarrow \chi_{0,2}$ are straightforwardly calculated as [17]

$$\Delta \hat{\sigma}_{\chi_0} = \frac{12\pi^2\alpha_s^2|R_{11}|^2}{M_{\chi_0}^2} \frac{M_{\chi_0}^2}{s} \delta(1 - \frac{M_{\chi_0}^2}{s}), \quad \hat{\sigma}_{\chi_0} = \frac{12\pi^2\alpha_s^2|R_{11}|^2}{M_{\chi_0}^2} \frac{M_{\chi_0}^2}{s} \delta(1 - \frac{M_{\chi_0}^2}{s}) \tag{6}$$

for $\chi_0$–productions and

$$\Delta \hat{\sigma}_{\chi_2} = -\frac{-16\pi^2\alpha_s^2|R_{11}|^2}{M_{\chi_2}^2} \frac{M_{\chi_2}^2}{s} \delta(1 - \frac{M_{\chi_2}^2}{s}), \quad \hat{\sigma}_{\chi_2} = \frac{16\pi^2\alpha_s^2|R_{11}|^2}{M_{\chi_2}^2} \frac{M_{\chi_2}^2}{s} \delta(1 - \frac{M_{\chi_2}^2}{s}) \tag{7}$$
for $\chi_2$–productions. Here $R'_1$ is the derivative of the P–state wave function at the origin. The spin–dependent and spin–independent differential cross sections in terms of $x_L (= 2p_L/\sqrt{s})$ of produced $\chi_J (J = 0, 2)$ are

$$
\frac{d\Delta \sigma_{\chi_J}}{dx_L} = \Delta C_{\chi_J} \frac{\pi^2 a_s^2 |R'_1|^2}{M_{\chi_J}^7} \frac{\tau}{\sqrt{x_L^2 + 4\tau}} \delta G(x_a, Q^2) \delta G(x_b, Q^2), \quad (8)
$$

$$
\frac{d\sigma_{\chi_J}}{dx_L} = C_{\chi_J} \frac{\pi^2 a_s^2 |R'_1|^2}{M_{\chi_J}^7} \frac{\tau}{\sqrt{x_L^2 + 4\tau}} G(x_a, Q^2) G(x_b, Q^2) \quad (9)
$$

with $x_a$ and $x_b$ being the momentum fraction in a proton and given as

$$x_a = \frac{x_L + \sqrt{x_L^2 + 4\tau}}{2}, \quad x_b = \frac{-x_L + \sqrt{x_L^2 + 4\tau}}{2}, \quad \tau = \frac{M_{\chi_0, 2}^2}{s}, \quad (10)$$

where $\Delta C_{\chi_J}$ and $C_{\chi_J}$ are 12 ($-16$) and 12 ($16$) for $\chi_0 (\chi_2)$–hadroproductions, respectively. By using these cross section formulas, the explicit form of two–spin asymmetries $A_{LL}^{\chi_0(2)} (pp)$ can be written as

$$A_{LL}^{\chi_0(2)} (pp) = (-) \frac{\delta G(\frac{x_L + \sqrt{x_L^2 + 4\tau}}{2}, Q^2)}{G(\frac{x_L + \sqrt{x_L^2 + 4\tau}}{2}, Q^2)} \frac{x_L + \sqrt{x_L^2 + 4\tau}}{2}, \quad \delta G(x, Q^2). \quad (11)$$

It is remarkable that at low–$p_T$ regions, $A_{LL}^{\chi_0(2)} (pp)$ which is originated mainly from gluon–gluon fusion of Figure 1, is directly proportional to polarized gluon distributions $\delta G(x)$. Therefore, $A_{LL}^{\chi_0(2)} (pp)$ sensetively depends on $\delta G(x)$. Namely, $A_{LL}^{\chi_0(2)} (pp)$ at low–$p_T$ regions is a very good physical parameter to test the behavior of $\delta G(x)$. To derive the differential cross sections in Eqs.(8) and (9), the higher order QCD corrections, the quarkonium binding effect and the corrections due to relativistic effects of the quarkonium should be taken into account. However, these uncertainties are generally included in so–called “K–factor”, and they would cancel if those corrections contribute to the denominator and numerator of Eq. (11) alike.

As the PHENIX detector which is to work at RHIC is dedicated to have a special device for detecting leptons and $\gamma$’s, the observable signal for $\chi_0$–productions is expected to be two–muon decays of $3S_1$ coming from radiative decays $\chi_{0, 2} \rightarrow 3S_1 + \gamma$. Since the differential cross sections $d\Delta \sigma_{\chi_2}/dx_L$ and $d\sigma_{\chi_2}/dx_L$ are almost equal to $d\Delta \sigma_{\chi_0}/dx_L$ and $d\sigma_{\chi_0}/dx_L$, respectively, the $\chi_2$ is easier to be detected because it has a larger branching ratio for the radiative decay than that of the $\chi_0$.

Using the polarized gluon distributions of four types given in Eqs.(1)–(4) and taking $Q^2$ as $M_{\chi_2}^2$, we estimate the $A_{LL}^{\chi_2} (pp)$ as a function of $x_L$ for $\chi_2$–productions of not only the charmonium $\chi(c\bar{c})$ but also the bottomonium $\chi(b\bar{b})$ states at relevant
RHIC energies, $\sqrt{s} = 50, 200$ and $500\text{GeV}$. Although the cross section of $\chi(b\bar{b})$ is smaller than that of $\chi(c\bar{c})$ about by $10^{-3}$, $\chi(b\bar{b})$ has different values of $x_a$ and $x_b$ given in Eq. (10) from those of $\chi(c\bar{c})$ productions, and so we can obtain not only $Q^2$–dependence but also $x$–dependence of the polarized gluon distributions by examining $\chi(b\bar{b})$ states together with $\chi(c\bar{c})$ states at the same time. We show the results of $A_{LL}^{\chi_2}$ calculated for (A) $c\bar{c}$ and (B) $b\bar{b}$ states at $\sqrt{s} = 50$, 200 and $500\text{GeV}$ in Figures 3, 4 and 5, respectively.

In the case of $\chi_2(c\bar{c})$, there is a tendency that the smaller $\sqrt{s}$ (namely $\tau$) is, the larger $A_{LL}^{\chi_2(c\bar{c})}$ become. In particular, for $\sqrt{s} = 50\text{GeV}$, it might be easy to examine not only the magnitude $\Delta G$ but also the behavior $\delta G(x)$ of the gluon polarization. In addition, $A_{LL}^{\chi_2(c\bar{c})}(pp)$ at $\sqrt{s} = 200$ and $500\text{GeV}$ is of use to know information on polarized gluon distributions, provided experiments are carried out with the high precision.

As for $\chi_2(b\bar{b})$–productions, the dependence of $A_{LL}^{\chi_2(b\bar{b})}(pp)$ on $\sqrt{s}$ has a similar tendency as that of $A_{LL}^{\chi_2(c\bar{c})}(pp)$. However, the behavior of $A_{LL}^{\chi_2(b\bar{b})}(pp)$ calculated at $\sqrt{s} = 50\text{GeV}$ by using our $\delta G(x)$ (type (a)) differs from that at the other $\sqrt{s}$. This is because for small $x_L (\lesssim 0.3)$, $x_a$ and $x_b (= x_a - x_L)$ are taken to be about 0.2–0.4 and hence $\delta G(x_a)/G(x_a)$ and $\delta G(x_b)/G(x_b)$ have around the minimal value as shown in Figure 2(C). Accordingly, Figure 3(B) indicates that it is rather difficult to distinguish our large $\delta G(x)$ (type (a)) from no polarization $\delta G(x) = 0$ (type (d)) in the region $x_L \lesssim 0.3$. At other $\sqrt{s}$, i.e. $\sqrt{s} = 200$ and $500\text{GeV}$, since the difference of $A_{LL}^{\chi_2(b\bar{b})}$ for any type of polarized gluon distributions is relatively larger than that of $A_{LL}^{\chi_2(c\bar{c})}$, it might be useful to examine the magnitude and the $x$–dependence of $\delta G$, though the differential cross section of $\chi_2(b\bar{b})$ is smaller than that of $\chi_2(c\bar{c})$. Both $A_{LL}^{\chi_2(c\bar{c})}$ and $A_{LL}^{\chi_2(b\bar{b})}$ for the no gluon polarization (type (d)) are almost zero in Figures (4), (5) and (6).

We find that the low–$p_T$ $\chi_2$–productions allow to give the clear test as a probe of the magnitude and the $x$–dependence of the gluon polarization because $A_{LL}^{\chi_2}$ for those reactions is directly proportional to $\delta G(x)/G(x)$. We have not taken up here the large–$p_T$ $\chi J$–hadroproductions though they are also interesting for getting the informations on polarized gluon [8].

3 Summary

We have calculate the two–spin asymmetries for the low–$p_T$ productions for $\chi_0$ and $\chi_2$ of the $c\bar{c}$ state and the $b\bar{b}$ state. The experimental data on charmonium and bottomonium $\chi J$–productions in polarized $pp$ collisions at RHIC will be very helpful to put constraints on the magnitude and the behavior of the gluon polarization,
and improve our understanding of the proton structure. In particular, the low–$p_T$ productions for $\chi_2$ can give a very useful information on polarized gluon distributions because the two–spin asymmetry for these reactions is simply proportional to $\delta G(x)/G(x)$. We hope that our predictions will be tested in forthcoming RHIC spin experiments.

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Figure captions

**Fig. 1:** A diagram of the dominant process for low–$p_T$ productions of $pp \to \chi_J X$ at lowest order QCD.

**Fig. 2:** The $x$–dependence of (A) $x\delta G(x, Q^2)$ at $Q^2 = 10\text{GeV}^2$, (B) $\delta G(x, Q^2)/G(x, Q^2)$ at $Q^2 = M_{\chi_2(c\bar{c})}^2$ and (C) $\delta G(x, Q^2)/G(x, Q^2)$ at $Q^2 = M_{\chi_2(b\bar{b})}^2$ for various types (a)–(d) given by Eqs.(1)–(4).

**Fig. 3:** Two–spin asymmetries (A) $A_{LL}^{\chi_2(c\bar{c})}(pp)$ and (B) $A_{LL}^{\chi_2(b\bar{b})}(pp)$ for $\sqrt{s} = 50\text{GeV}$, calculated with various types of $\delta G(x)$, as a function of longitudinal momentum fraction $x_L$ of $\chi_2(c\bar{c})$ and $\chi_2(b\bar{b})$. The solid, dashed, small–dashed and dash–dotted lines indicate the results using types (a), (b), (c) and (d) in Eqs.(1), (2), (3) and (4), respectively.

**Fig. 4:** Two–spin asymmetries (A) $A_{LL}^{\chi_2(c\bar{c})}(pp)$ and (B) $A_{LL}^{\chi_2(b\bar{b})}(pp)$ for $\sqrt{s} = 200\text{GeV}$. Various lines represent the same as in Figure 3.

**Fig. 5:** Two–spin asymmetries (A) $A_{LL}^{\chi_2(c\bar{c})}(pp)$ and (B) $A_{LL}^{\chi_2(b\bar{b})}(pp)$ for $\sqrt{s} = 500\text{GeV}$. Various lines represent the same as in Figure 3.
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