Carbon Sequestration Model Based on Multiple Linear Regression

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Abstract. Proper deforestation promotes the development of man and nature, so it is necessary to design models to calculate the amount of carbon sequestered and further determine the value of forests. According to the formula for calculating the carbon sequestration of forest and its affiliated forest products, we use the intensity of human disturbance as a single variable to construct a differential equation for the relationship between forest and wood product accumulation. Combining five major indicators such as rainfall, temperature, and tree species, we constructed a multiple linear regression model for carbon sequestration using the data obtained for fifteen forests worldwide. As a result, we established the relationship between carbon sequestration and the influencing factors, learned the significant effects of tree species and artificial disturbance intensity on carbon sequestration, and further established the optimal forest plan.

Keywords: Carbon Sequestration, Multiple linear regression.

1. Introduction
   A forest management plan can effectively improve the efficiency of carbon stock and give an important way to solve the global warming problem.
   Our goal is to establish a complete evaluation system, involving ecological, economic, and social aspects, to assess and analyze the forest value quantitatively.
   Through the assessment system, choose the right forest management program to maximize the utilization of forest value, achieving effective balance in social, economic and ecology.

2. Model Establishment and Solution
2.1. Carbon Sequestration Model
2.1.1. Overview of the Carbon Sequestration
   To model carbon sequestration, we took carbon stock as the dependent variable. It is found through the literature that cutting down trees should be purposeful, planned, and respectful of science. Trees need to be cut down when their growth density is affected by high density or when they reach maturity. In terms of climate, it is more environmentally beneficial to use wood from the forest than not to cut trees, because the wood being used, even if it is converted into a product, still fixes the carbon in the body for a long time without being released. Therefore, we divide the carbon stock into forest carbon stock and wood product carbon stock, calculate the carbon stock quantitatively, introduce the logistic model to correct the effect of logging intensity of the independent variable, and finally build the carbon sink model.

2.1.2. Carbon storage calculation of forest
   Forests need to convert inorganic carbon dioxide to organic carbon during photosynthesis, which makes a large amount of carbon stored in forest trees. Therefore, the calculation of carbon stock needs to target the above-ground part of the tree layer, ignoring the effects of different management practices on shrubs, grasses, and their below-ground carbon stocks. According to the literature, we found that there are two methods to estimate forest carbon stock: the biomass expansion factor method and the other is the cumulative method (IPCC method). Here we choose the biomass expansion factor method for calculation.
The biomass expansion factor method uses the biomass conversion factor BEF to convert the storage volume in the second resource type into the biomass expansion factor method (BEF: Mean values of stand biomass to wood volume ratio) to estimate forest carbon stock. Here the total area is taken as 1 hm$^2$.

Next, we need to calculate the formula for biomass per unit volume on the mainland, and accumulation volume is equivalent to volume.

Aboveground biomass $W_{ij}$ of species $j$ in forest type $i$:

$$W_{ij} = A_i \cdot W_{pij} = A_i \cdot X_{pij} \cdot BEF_{ij} \cdot SVD_{ij}$$  \hspace{1cm} (1)

Notation:
- $A_i$: Area of the forest type $i$
- $W_{pij}$: Biomass per unit area of tree species $j$, forest type $i$
- $X_{pij}$: Stock per hectare of tree species $j$, forest type $i$
- $BEF_{ij}$: Biomass expansion coefficient of tree species $j$, forest type $i$
- $SVD_{ij}$: Wood density of the tree species $j$, forest type $i$

Therefore, we can obtain the total equation for forest carbon storage.

$$C_b = W_{total} \cdot O$$  \hspace{1cm} (2)

$C_b$, $W_{total}$, $O$ are respectively the carbon storage, biomass, and the biomass and carbon conversion coefficient of plants. Studies show that the carbon content of broad-leaved trees, coniferous trees, and other tree species is not very different, with an average of 0.44. Therefore, 0.44 is adopted in this paper as the conversion coefficient of tree biomass and carbon.

### 2.1.3. Carbon storage calculation of wood forest products

The main constituent elements of wood are carbon (50%), hydrogen (6%), and oxygen (43%), which means that half of the weight of dry wood is carbon. Hence, the carbon content is 50%, and the basic density of wood has individual anisotropy. However, wood products' annual production and annual consumption are usually measured in terms of total volume, without distinguishing between species.

Therefore, we can obtain the total equation for carbon storage of wood forest products.

$$P_b = V \cdot d \cdot \alpha$$  \hspace{1cm} (3)

$P_b$ stands for forest product volume, $V$ stands for basic density and $\alpha$ stands for carbon content. The following table is used for the values of basic density and carbon content.

| Forest product          | Industrial logs | Sawn wood | Artificial board | Paper and cardboard |
|-------------------------|-----------------|-----------|------------------|---------------------|
| $d$                     | 0.486           | 0.486     | 0.486            | 0.670               |
| $\alpha$               | 0.496           | 0.496     | 0.496            | 0.466               |

### 2.1.4. Modification of logistic model

In studying trees and forest products, we found that the relationship was consistent with the predator competition model. Then we use the logistic Stee model to modify it as follows. Model original condition:

Let $x(t)$ be the population size at time $t$, and $x(t)$ is continuously differentiable.

The growth rate of the population $r$ is a decreasing function about $x$, and $S_m$ is the maximum number of populations that can be accommodated by natural conditions.

The variation of the population size is closed, and each individual's fertility and mortality are the same.

We can obtain the differential equation from the initial conditions with the following equation.
We can derive the system of differential equations as follows.

\[
\begin{align*}
\frac{dx}{dt} &= r \left(1 - x/x_m\right)x \\
\frac{dx_1}{dt} &= \left[r - \frac{r}{S_m}(1 - kh)x - h\right]x \\
\frac{dx_2}{dt} &= hx \\
x_1(0) &= x_0 \\
x_2(0) &= x_0
\end{align*}
\]

(4)

We set the initial value of \( x_0 \) as 30 and solve this differential equation. The result is shown below:

\[
\begin{align*}
x_1 &= -\frac{\sigma_1(1250h - 250)}{\sigma_1 - 1} \\
x_2 &= 1250h(\log(\sigma_2 eht - et/5\sigma_1) - ht) - 1250h \log(\sigma_2 - \sigma_1)
\end{align*}
\]

(6)

\( x_1 \) is the forest stock (volume). \( x_2 \) is the volume of wood products; \( h \) is the intensity of artificial harvesting; \( k \) represents the promotion effect of appropriate artificial felling on the growth of new trees, taken as 0.5; \( S_m \) is the maximum number of populations that natural conditions can accommodate.

2.1.5 Model formula presentation

Through the above analysis and proof, we can establish the equation of the carbon sequestration model as follows.

\[
F_{\text{target}} = A \cdot x_1 \cdot \text{BEF} \cdot \text{SVD} \cdot O_a + x_2 \cdot \text{SVD} \cdot O_b
\]

(7)

At the same time, we fit the recent data on the volume of forest stock and wood products of the world’s major forests to our modified logistic model of forest stock and wood product volume to solve for a set of artificial disturbance intensities \( h \) (normalized) and obtain a better solution for \( h = 0.3385 \) by the particle swarm algorithm. This indicates that appropriate artificial disturbance of the forest is beneficial for increasing carbon sequestration.

2.2 Regression analysis of carbon sequestration

2.2.1 Overview of the regression analysis

In this paper, multiple linear regression analysis was used to analyze the dependent variable carbon sink and the six independent variables one by one, and the regression equation was finally obtained. Then, the standardized regression coefficients are analyzed to obtain the relationship between the weights of the six indicators and the dependent convenience. At this point, we can prove that "the amount of carbon sequestration changes over time". Here we collected indicators from 15 forests worldwide and performed regression analysis.

The four major indicators with positive significance were selected. Here we collected indicators from 15 forests around the world and performed regression analysis.
Figure 1 Carbon flow diagram

cited: rainfall, tree species, the intensity of artificial intervention, and area.
(1) Area: Through the analysis of the bar graph, the larger the area, the more carbon is sequestered.
(2) Rainfall: By using inverse logistic model, where there is a maximum value point that makes the maximum amount of carbon sequestered.
(3) Artificial intervention: We can consider this aspect similar to rainfall.
(4) Tree species: A comparison of carbon sequestered by the four major tree species can be obtained. This results in our optimal forest management plan.

2.2.2 Measure indicators

For the dependent variable of carbon sequestration, we will divide it into three directions: artificial, natural and semi-artificial, and semi-natural factors, and analyze its relationship with the dependent variable quantitatively, with the artificial direction being disturbance intensity, the natural direction being light, average annual rainfall, and temperature, and the semi-artificial and semi-natural direction being forest area and main dominant tree species for regression analysis.

Artificial disturbance intensity (ADI): It is an explicitly artificial variable that is more abstract, and we can normalize it to obtain simulated values of disturbance intensity. Thus, disturbance intensity brings human felling in the forest and forest cultivation and management into the regression analysis, reflecting the importance of forest management plans.

Light intensity (LI): This is a natural variable, which is an effective catalyst for photosynthesis in the forest, so it will play an important role in promoting the process of carbon sequestration.

Average annual rainfall (AR): This is a natural variable. Rainfall plays a vital role in the transpiration and respiration of plants, so we chose the annual rainfall near the forest to represent the effect of this latitude on the dependent variable carbon sequestration.

Temperature (T): This is a natural variable. Low temperature affects the activities of photosynthetic and respiratory enzymes, and the net photosynthetic rate is lower at this time. So, we also included it in the index for regression analysis.

Forest area (FA): This is a semi-artificial and semi-natural variable. Forest area also plays an essential role as a carbon sequestration indicator in this integrated environment, so we also take it into account.

Main dominant tree species (DTS): This is a semi-human, semi-natural variable. We can select the tree species with the highest population density in the area as the dominant species and perform regression analysis based on population competition between dominant and inferior tree species.
### 2.2.3 Correlation test

**Table 2. Pearson Coefficient**

|            | Intensity of intervention | Temperature | Precipitation | Average |
|------------|---------------------------|-------------|---------------|---------|
| Fixed Carbon | 0.94                      | 0.36        | 0.54          | 0.80    |

The regression equation was first established to cascade the indicators with the dependent variable carbon sequestration, and the coefficients of each indicator were given as $a_1$ to $a_6$. The formula is as follows:

$$F_{carbon} = a_1 \cdot ADI + a_2 \cdot LI + a_3 \cdot AR + a_4 \cdot T + a_5 \cdot FA + a_6 \cdot DTS + \beta$$  \hspace{1cm} (8)

From the above table, we can see that the model equation is: (carbon sequestration in tons)

$$F_{carbon} = -1.489 \times 10^{11} + 5.355 \times 10^{10} ADI + 1.773 \times 10^7 LI + 8.615 \times 10^9 AR + 3.157 \times 10^4 FA + 2.368 \times 10^9 T + 4.199 \times 10^{10} DIS$$  \hspace{1cm} (9)

By analyzing the other data in the table above, we evaluate the model as follows.

The model $R^2$ was 0.912, implying that the mode of operation, light, average annual rainfall, area, temperature, and major tree species explained 91.2% of the variation in carbon sequestration.

The model passed the F-test ($F = 5.068, p = 0.020 < 0.05$), implying that at least one of the following factors influences carbon stocks.

**Table 3. Regression analysis**

|            | Non-standard | Standard | $p$ | VIF |
|------------|--------------|----------|-----|-----|
| Constant   | -1.489E11    | -        | 0.188 | -   |
| $ADI$ (MJ/m²) | 5.355E10    | 0.653    | 0.002 | 1.324 |
| $LI$ (mm)  | 1.773E7      | 0.184    | 0.455 | 2.094 |
| $FA$ (km²) | 8.615E9      | 0.425    | 0.049 | 3.36 |
| $T$ (c)    | 31572.575    | 0.716    | 0.003 | 1.073 |
| $DTS$      | 2.368E9      | 0.391    | 0.129 | 2.85 |
| Constant   | 4.199E10     | 0.473    | 0.035 | 2.918 |
| $R^2$      |              | 0.912    |      |     |
| $F$        | $F (6,8)=5.068, p=0.020$ |          |      |     |

The multicollinearity of the model is tested, and all the VIF values in the model are less than 5, implying that there is no covariance problem.

The D-W value is around 2, thus indicating that the model is not autocorrelated, there is no correlation between the sample data, and the model is better.

### 2.2.4 Analysis of related indicators

Because from the regression analysis above, the indicators with significant effects in the regression equation are: area, artificial disturbance intensity, tree species, and rainfall. Therefore, we focus on these four indicators below:

The main dominant tree species. We ranked the carbon sequestration indicators of four different forest systems and made graphs. Thus, it can be obtained that tropical rainforests are the best tree species for carbon sequestration, followed by evergreen broadleaf and deciduous broadleaf forests, respectively. In contrast, coniferous forests perform the weakest in the carbon sequestration cycle. (Figure 2(b))

Forest area. We grouped and ranked 15 groups of forests to conclude by the control variables method that under the same tree species, if the forest area is larger, its carbon sequestration capacity is higher (as shown in Figure 2(a)).
Artificial disturbance intensity: For the artificial disturbance intensity, we find that it is not in a perfectly positive relationship with the dependent variable but presents a functional relationship as shown below, with the functional equation:

\[ g(x) = \frac{1}{cx-1} (cx)^a \ln(cx + b) \]  

3. Conclusion

Using the formulas related to ecology, we can quantitatively determine the amount of carbon absorbed by the forest for subsequent analysis.

The assessment of forest value is multifaceted, and our dimensions have some limitations.

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