Identification of the nonlinearity in mass-spring system via experimental method

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Abstract. Engineering structures operate with a nonlinear dynamics behaviour at certain amplitude of input range which will produce noticeable changes and unpredictable effects. In this paper, the nonlinear property of a spring is investigated. The simplified mass-spring system is developed which consists of two springs attached on a thin plate structure. The translational degree of freedom is considered in the experimental and a preferential direction on the length direction of the spring is investigated. The linear property is first investigated via impact testing for the acceleration measurement. Subsequently, the spectral testing with different amplitude of periodic chirp excitations is carried out to identify the nonlinear property of the springs. The nonlinear dynamics behaviour of the springs are identified from the significant changes of natural frequencies from the frequency response function of the system.

1. Introduction

Structural modelling, computing and testing are very crucial when designing, constructing and maintaining the mechanical systems and engineering structures. However, the approximation approach by assuming the linear model is mostly applied in structural dynamics analysis [1-3]. However, the linear model is consequently invalid and the nonlinear effects are compulsory to take into account when these structures are subjected to dynamics loading with large displacement amplitudes [4-6].

The input energy and the system property are the main factors for the occurrence of nonlinear behaviour of engineering systems and structures [7-8]. Effective investigation is required before the nonlinear dynamic behaviour can be included in the numerical calculations since it is an indicator for the development of structural damage under dynamic excitation.

For example, in the operational of the aerospace flight, the aeronautical fuel consumption reduction can be achieved by increasing the wing aspect-ratio to enhance the flight efficiency while reducing emissions. The improvement in the lift-to-drag ratio however will result in higher deflections which can subsequently cause the nonlinear aero-elastic behaviour [9-11]. Several factors that contribute to the nonlinear behaviour in the mechanical systems are such as large displacement of the system, crack initiation and development, as well as the looseness and presence of friction characteristics of structural joints [12-14].

Nonlinear system identification can be identified numerically and experimentally. The identification in structural dynamics is classified into several categories, namely linearisation, time and frequency domain methods, time-frequency analysis and modal methods [15].
The structural nonlinear restoring forces can be identified using only partial measurements of the structural responses. The method combines the identification of locations of structural nonlinearities by the introduction of equivalent linear structural system for an original nonlinear system. Then, a power series polynomial is used to approximate structural nonlinear restoring force using the extended Kalman filter to obtain the unknown coefficients of the power series polynomials [16].

A method for detection and characterisation of nonlinear behaviour from a single frequency response function has been presented by [17]. A set of nonlinearity indexes is derived using normalised single mode FRFs together with the Hilbert transform to describe the nonlinear behaviour. This method is capable while being robust in the presence of measured noise.

The backbone curves can be used to identify the linear natural frequencies and the nonlinear parameters. The experimental backbone curve is estimated from the resonant decay data measurement. This method can identify the dynamic behaviour of structures containing discrete nonlinear stiffness in large displacements regimes [18]. On top of that, Londoño et al. applied the approach that is based on estimations of the instantaneous frequency and the envelope amplitude of a decaying response, following a tuned steady-state oscillation of the system, to extract the backbone curves of lightly damped nonlinear systems [19].

Meanwhile, Noël and Kerschen utilized the applied excitations and the corresponding response time series, without knowing the information about the system to identify the nonlinear restoring force which is a direct indicator of the extent of the nonlinearity. Estimation is based on a standard least-square techniques to identify the coefficient of power series polynomial [12].

In modal analysis, the equation of motion for the dynamic response of a linear model to describe the structural motion is a second-order differential equation as in equation (1).

\[ M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = F(t) \]

where \( M, C \) and \( K \) are the mass, damping and stiffness matrices. These matrices are a square \( n \times n \) system. \( \dot{x}, \ddot{x} \) and \( x \) are the \( n \times 1 \) acceleration, velocity and displacement vectors, respectively. \( F(t) \) is an external force vector [20], [21]. For a nonlinear structure, the equation of motion [16], [22] consists of a nonlinear parameter as shown in equation (2).

\[ M\ddot{x}(t) + C\dot{x}(t) + Kx(t) + K_{NL} = F(t) \]

where \( K_{NL} \) is a non-linear stiffness term. For a hardening or softening nonlinear restoring force spring with cubic nonlinearities, the equation of motion is known as Duffing’s equation [23], [24], [25] as shown in equation (3).

\[ M\ddot{x}(t) + C\dot{x}(t) + Kx(t) + K_{NL}x^3(t) = F(t) \]

The force-deflection characteristic for the hardening spring bends to reduce the displacement with a positive value for the parameter \( K_{NL} \) while the softening spring has a negative value for the parameter \( K_{NL} \) corresponding to allow for more displacement as the force increases. In this paper, methodology for detecting linear and nonlinear behaviour of a mass-spring system is described. The fundamental property of frequency response function (FRF) which will be exploited for the identification is a frequency-amplitude dependence respectively.
2. Methodology

2.1. Experimental Set-up

The experimental work consists of measuring the response of the single degree-of-freedom mass-spring system at selected point. The experimental setup is shown in Figure 1. The system consists of a single plate connected by two springs with equal stiffness property, which makes the equilibrium condition of the plate in the horizontal position. The springs were attached to the strings and the fixed base creating the system in free-free boundary conditions.

An electromagnetic shaker provides excitation at the centre of the plate in the vertical direction. The plate was excited symmetrically, therefore this setup is restricted to the study of axisymmetric rigid body modes of the plate that correspond exactly to the stiffness of the spring. An accelerometer was mounted on the plate to measure acceleration response of the mass-spring system.

![Figure 1. The modal experimental setup.](image)

In this study, the experimental method is based on the comparison between linear and nonlinear identifications of the mass-spring system. The significance of the first part of this methodology is to identify the natural frequency, while the second part will be the nonlinear identification in more details. All the equipment involved in the experimental work are listed in Table 1.

| Description       | Brand and model          |
|-------------------|--------------------------|
| Accelerometer     | DYTRAN 3032A             |
| Force transducer  | PCB PIEZOTRONICS 208C03  |
| Hammer            | DYTRAN 5800              |
| Shaker            | DATA PHYSICS V4/T4       |

2.2. Measurement Techniques

Note that the experiments were carried out to verify the linear and nonlinear experimental identifications, hence the knowledge of the applied force and acceleration response of the system are required. For linear case, modal testing was performed by using the impact hammer excitation to quantify the nonlinear natural frequency as well as the amplitude of the frequency response function, an electromagnetic shaker was used with five different excitation values. While running each test, the acceleration response was measured using a number of accelerometers.
The spring used in this study is a conventional industrial spring. The specification of the spring is listed in Table 2. In order to obtain the stiffness of the spring, manual laboratory measurement was conducted from the elongation of the spring due to the weight’s load.

**Table 2.** Property of the spring.

| Parameter       | Value |
|-----------------|-------|
| Stiffness (N/mm)| 0.105 |
| Length (mm)     | 19    |
| Diameter (mm)   | 8     |

3. Results and Discussion

3.1. Linear identification of the mass-spring system

The linear identification was carried out on the basis of experimentally determined frequency response function (FRF) characteristics of free vibration response signals measured from the test system. Conventional modal testing using impact hammer was used in this section to identify the linear modal properties. Error! Reference source not found. shows the FRF plot for the mass-spring system. It shows that this system has a single natural frequency of 5.31 Hz. The coherence plot shown in Figure 3 drops in the anti-resonance region and justifies a high confidence output for this measurement.

![Figure 2. FRF plot by impact testing](image1)

![Figure 3. Coherence function](image2)

For linear identification in the second method, the FRF plot was obtained by exciting the system using burst random type-excitation. The detail parameters for the burst random experiment are given in Table 3. The bandwidth range is between 0 and 10 Hz. The excitation direction is in negative Z direction.

**Table 3.** Experimental parameters for the burst random excitation.

| Parameter                  | Value          |
|----------------------------|----------------|
| Excitation type            | Burst random   |
| Bandwidth (Hz)             | 10             |
| Spectral line              | 32             |
| Resolution                 | 0.31           |
| Voltage (V)                | 0.5            |
| Excitation DOF / direction | 1 / -Z         |
Error! Reference source not found. shows the FRF plot from shaker excitation and associated coherence function is shown in Figure 5. The average recording is ten measurements. It can be seen that between 0 to 10 Hz, this single degree-of-freedom system comprises single natural frequency which is 5.31 Hz. From both results, it is proof that this mass-spring system has a linear natural frequency of 5.31 Hz. In the next section, the nonlinear identification will take place.

![Figure 4. FRF plot by burst random](image1)

![Figure 5. Coherence function](image2)

3.2. Nonlinear Identification
The sine chirp excitation was used to identify the nonlinear property of this mass-spring system. This selection is chosen because the level of force from low to high frequency can be controlled and the effect of leakage is non-existent providing the steady state response is achieved. The detail of parameters for the nonlinear identification testing is listed in Table 4. The process of underlying the linear behaviour that has been explained in Section 3.1 showed that the linear natural frequency is 5.31 Hz. Therefore, the sweep sine excitation is realized at a low excitation level over a small frequency range between 5 Hz and 6 Hz. The type for the periodic chirp sine testing is linear sweeping up direction. The sweeps were repeated for increasing excitation voltage in increment of 0.2 V.

| Table 4. Parameters for the nonlinear identification via sine chirp method. |
|---------------------------------------------------------------|
| Parameter                  | Value                      |
| Excitation type           | Periodic chirp             |
| Sweep direction           | Up                         |
| Frequency sweep (Hz)      | 5 to 6                     |
| Bandwith (Hz)             | 10                         |
| Spectral line             | 32                         |
| Resolution                | 0.31                       |
| Voltage (V)               | 0.2, 0.4, 0.6, 0.8 and 1.0 |
| Excitation DOF            | 1                          |
| Excitation direction      | -Z                         |
The frequency response function (FRF) for each voltage is shown in Figure 6. The variations between FRF for each excitation force are used as an indicator for the detection and characterisation of nonlinear property. The pattern shows that there is a distortion in the FRF plot where there is the natural frequency shifting and amplitude changing when the mass-spring system was excited, with a higher voltage excitation input to the plate.

![Nonlinear FRF plot by sweep sine excitation](image)

**Figure 6.** Nonlinear FRF plot by sweep sine excitation

To further evaluate the nonlinear behaviour, the natural frequency and the corresponding amplitude values for each voltage are extracted and listed in Table 5. For lower voltage excitation at 0.2 V and 0.4 V, the natural frequency is constant at 5.31 Hz with decreasing amplitude of the peak from 5.13 g/N to 4.87 g/N. However, the nonlinear behaviour becomes apparent where the natural frequency shifts to lower value at 5.00 Hz when the excitation level is between 0.6 V and 1.0 V. It can be seen that the displacement values, however increase from 4.90 g/N to 6.56 g/N. This result justifies that this mass-spring system has a softening stiffness characteristic, since the natural frequency reduces to lower value when higher voltage excites the system.
Table 5. Magnitude of voltage excitation and the natural frequency of the mass-spring system.

| Voltage (V) | Natural Frequency (Hz) | Amplitude (g/N) |
|-------------|------------------------|-----------------|
| 0.2         | 5.31                   | 5.13            |
| 0.4         | 5.31                   | 4.87            |
| 0.6         | 5.00                   | 4.90            |
| 0.8         | 5.00                   | 5.64            |
| 1.0         | 5.00                   | 6.56            |

4. Conclusion
A linear and nonlinear dynamics behaviour of the mass-spring system has been investigated in this paper. The natural frequency of this single degree-of-freedom system has been obtained by experimental modal analysis. The experimental set up was carefully designed for a high quality investigation of the nonlinear property of a spring. According to the results, the linear identification by impact testing using an impact hammer and burst random excitation testing using an electromagnetic shaker found that the natural frequency is 5.31 Hz. Then, for the nonlinear detection, periodic chirp sine sweep testing with several magnitudes of voltages between 0.2 V and 1.0 V was utilized as an excitation to the system. The results found that the impact testing via the impact hammer cannot capture the nonlinearity effect. This type of excitation has an impulsive with high peak level but low root mean square (r.m.s) level property to detect the nonlinear system. However, the spectral testing via the electromagnetic shaker showed the reduction of natural frequency of the system at higher level of excitation. Moreover, the spring behaves the softening dynamics behaviour. Finally, it can be concluded that the frequency response function measured at a higher excitation level found deviates from the frequency response function of the linear part of the system.

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