Speed of gravity and gravitomagnetism

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Abstract

A $v_J/c$ correction to the Shapiro time delay seems verified by a 2002 Jovian observation by VLBI. In this Essay, this correction is interpreted as an effect of the aberration of light in an optically refractive medium which supplies an analog of Jupiter’s gravity field rather than as a measurement of the speed of gravity, as it was first proposed by other authors. The variation of the index of refraction is induced by the Lorentz invariance of the weak gravitational field equations for Jupiter in a uniform translational slow motion with velocity $v_J = 13.5 \, \text{km/s}$. The correction on time delay and deflection is due not to the Kerr (or Lense-Thirring) stationary gravitomagnetic field of Jupiter, but to its Schwarzschild gravitostatic field when measured from the barycenter of the solar system.

PACS numbers: 04.80.-y, 04.80.Cc, 04.25.Nx, 95.30.Sf

Keywords: Lorentz transformation, invariants, gravitomagnetic field, speed of gravity, aberration of light
INTRODUCTION.

In general relativity, the causal propagation of gravity in vacuum, in the sense that it cannot travel faster than light in vacuum, usually follows from the hyperbolic form of the Einstein field equations in empty spacetime. Moreover, a general purely geometrical proof of this fact is given in [1] and it does not seem to be a direct consequence of the hyperbolic character of Einstein’s theory.

In spite of that, three years ago Kopeikin [2] suggested a solar system time delay test from which he concluded that the speed of gravity $c_g$ could be directly measured and hence compared with the speed of light $c$ in vacuum. His final result was $c_g = (1.06 \pm 0.21) c$.

A more detailed description of this solar system time delay test is the following: In Ref. [2] an extra time delay of the quasar light was predicted, caused by the passage of Jupiter by the quasar QSO J0842+1835 at a separation of only 3.7 arcminutes (14 Jovian radii) on September 8, 2002, which was measured by advanced VLBI and recorded by eleven radio antennae [3], an impressive observational feat. The observation was conducted by the National Radio Astronomical Observatory (USA) and the Max Planck Institute for Radio Astronomy (EU).

The extra time delay is the total time delay, due to the difference in the arrival time of two light rays leaving the quasar simultaneously, minus the classical Shapiro time delay [4] caused by the Schwarzschild gravitational field of a static mass at rest. As Jupiter’s gravity is very weak, the Jovian time delay (or deflection) observation measurement required remarkable accuracy: few picoseconds (or about 10 $\mu$arcsec). The order of magnitude of the time delay prediction is a delay of 115 psec (deflection of 1190 $\mu$arcsec) for the radial (static) term, and a time delay of 4.8 psec (deflection of 51 $\mu$arcsec) for the transverse (velocity) term at the point of the closest approach. Both these predicted theoretical values have been successively reduced from Ref. [2] to [3-5].

In a series of papers the authors of [2, 3-5, 6, 7, 8] have argued that the additional terms in the time delay depend upon the propagation speed of gravity, giving the aforementioned final result $c_g = (1.06 \pm 0.21) c$. Actually, the authors of [2, 3-5, 6, 7, 8] use a kind of vector-tensor theory instead of the Einstein’s general relativity which is a pure tensor theory. So far, different criticisms from various authors [9, 10] have inhibited the acceptance of this interpretation of the experiment although, seemingly, they have not ruled it out.
The main purpose of this Essay is to show an altogether new approach which, in our opinion, completely invalidates the claim made in [6, 8]. In these works it was argued that, working in the linear weak field approximation, a gravitomagnetic field appears after a passive Lorentz transformation from the static rest frame of Jupiter to the barycenter of the solar system has been applied. In [6, 8], the gravitomagnetic field of Jupiter is the cause of the extra delay of the quasarlight by dragging it in the direction of motion of Jupiter.

Here, we will show that a intrinsic gravitomagnetic field does not appear after making the Lorentz transformation. Hence, the extra time delay effect is only a test of the Lorentz invariance of the weak gravity equations and it is actually a fine measurement with VLBI of the aberration of light already observed by Bradley in 1728 using a telescope.

CRITERION FOR AN INTRINSIC GRAVITOMAGNETIC FIELD.

At first sight, the linearized theory of general relativity or gravitomagnetism may be thought of as that phenomenon by which the spacetime geometry and its curvature change. This change is due to mass-energy density and mass-energy currents relative to other mass-energy.

It is possible to use the analogy with electromagnetism as in [11], however, we stress that apart from formal analogies, gravitomagnetism and the Maxwell-Lorentz electromagnetic theory are fundamentally different. The main difference is the equivalence principle: in freely falling frames it is possible to eliminate the local acceleration effects (first derivatives of the metric $g_{\alpha\beta}$) of a gravitational field. To characterize the electromagnetic field one must calculate the two Casimir invariants under the Poincaré group of the Faraday tensor $F_{\alpha\beta}$. They are the scalar, $-\frac{1}{2} F_{\alpha\beta} F^{\alpha\beta} = \|E\|^2 - \|B\|^2$, and the pseudoscalar, $\frac{1}{4} F_{\alpha\beta} \star F^{\alpha\beta} = E \cdot B$, where $\star$ is the dual operation. If we have only a charge $q$, in the rest frame $"r"$, then we have an electric field $E \neq 0$ but the magnetic one is $B = 0$ and the invariant $F_{\alpha\beta} \star F^{\alpha\beta}$ is zero. Therefore, even in other inertial frames where $B = -\gamma (v/c \times r) E \neq 0$ and $E \neq 0$, this pseudoinvariant will be zero. However, if in the rest frame we have a charge $q$ and a magnetic dipole $m$ then $F_{\alpha\beta} \star F^{\alpha\beta} \neq 0$ and this pseudoinvariant will be different from zero in any other inertial frame.

Something similar occurs in general relativity. Now, only the diagonal components of the Schwarzschild metric are non-vanishing when these are measured in the rest frame $"r"$. 
These components are the “gravitoelectric” $g_{00}$ and “space’s curvature” $g_{ij}$ potentials. However, if we consider a boosted inertial frame with velocity, $\mathbf{v} = v_i e^i$, $i = 1, 2, 3$, relative to the mass $M$, we will have the “gravitomagnetic” components $g_{0i} = -4GMv_i/rc^3$, proportional to the velocity of the source measured by an observer of the moving inertial frame. In this case we have an extrinsic gravitomagnetic field.

The metric for this observer reads at linear approximation as:

$$ds^2 = -(1 + \frac{2U}{c^2})c^2 dt^2 + (1 - \frac{2U}{c^2}) dx^i dx_i + 2g_{0i}c dt dx_i,$$

where $U = -GM/r$ is the Newtonian potential. In every spacetime we can always make the “gravitomagnetic” vector potential $\mathbf{A}$, defined by $g_{0i} = -\frac{2}{c^2} A_i$, different from zero, realizing a coordinate transformation. Hence, to study the intrinsic properties of a gravitational field one has to compute the invariants of the Riemann curvature tensor instead of analyzing the metric components in a particular coordinate system.

In gravitational theory, the invariants of the curvature tensor analogous to the electromagnetic ones are the Kretschmann invariant $R^\alpha\beta\mu\nu R^\alpha\beta\mu\nu$ and the pseudo-invariant $^*R^\alpha\beta\mu\nu R_{\alpha\beta\mu\nu} = ^*\mathbf{R} \cdot \mathbf{R}$. Hence, if one only considers a mass $M$ (the Schwarzschild solution) then, in the rest frame “$r$” of this mass, one has that the “gravitoelectric” components $^rR_{i0j0}$ are non-zero whereas $^*\mathbf{R} \cdot \mathbf{R}$ is zero.

Due to the Lorentz invariance of the weak gravitational equations, this also happens if the invariants are measured from other inertial frames where the “gravitomagnetic” components are $R_{i0j0} \neq 0$. Then at the lowest order, the pseudo-invariant is $^*\mathbf{R} \cdot \mathbf{R} \simeq \nabla^2 [\nabla(U/c^2) \cdot (\nabla \wedge \mathbf{A}/c^2)]$. For a static Schwarzschild metric boosted with velocity $\mathbf{v}$, we have $\mathbf{A} = -2U\mathbf{v}/c$. However, $^*\mathbf{R} \cdot \mathbf{R}$ is still zero:

$$^*\mathbf{R} \cdot \mathbf{R} \simeq \nabla^2 [\nabla(U/c^2) \cdot (\nabla \wedge U\mathbf{v}/c^3)] = 0.$$  

In this case, the “gravitomagnetic” components $R_{i0j0}$ just measure the effect of a (passive) local Lorentz transformation from the rest frame of the mass $M$ to a boosted frame on a static gravitational field which remains intrinsically unchanged. Hence, an intrinsic gravitomagnetic field does not appear in the boosted linear Schwarzschild metric (1).

However, if one considers a steady rotating source with mass $M$ and intrinsic angular momentum $\mathbf{S}$ (that is to say, Kerr solution, or its linear approximation: Lense-Thirring), the gravitomagnetic potential is $\mathbf{A} = G\mathbf{S} \wedge r/cr^3$, and both curvature invariants are non-zero in any inertial frame. The last sentence is the criterion of the existence of a intrinsic
gravitomagnetic field. Note that if this criterion is not satisfied then some confusion might appear concerning the gravitomagnetic properties of a gravitational field. For instance, a geodetic de Sitter effect might be misinterpreted as an intrinsic gravitomagnetic Lense-Thirring one due to the orbital angular momentum $L$ measured by an orbiting observer freely falling around a non-rotating mass.

**INTERPRETATION OF THE 2002 JOVIAN OBSERVATION.**

According to Einstein’s theory, a gravitational field is identified with the spacetime curvature. Also, in an analogous way, it can be considered as a retarder, a deflector and a lens simultaneously, i.e., as an inhomogeneous refractive medium of index $n(r)$ acting upon the propagation of the light rays along a trajectory between two points in spacetime. Moreover, an optical refractive medium causes a time delay (proportional to $n$), a deflection of the light rays (through the spatial gradient of $n$) and the appearance of lensed images. In general, these three actions are mathematically related through the phase or eikonal function and its successive derivatives. However, the first two effects can also take place in the Minkowski spacetime, when gravitation is neglected, due to the Doppler or the aberration special relativistic kinematical effects. In this last case both effects are measured at the same point by two different observers.

The effect of gravity is to make the medium optically denser in the vicinity of a mass and hence the coordinate speed of light diminishes as we approach the mass and we could say that the light is repelled by the mass. However, let us remark that the invariant speed of light measured by any observer will be always $c$, as in vacuum. For a frame at rest with a static mass, described by the linear Schwarzschild metric, the index is $r n(r) = 1 - 2U/c^2$. When it is measured from a boosted inertial frame, the index is

$$n(r) = \left[ r n(r) - \frac{v}{c} \right] \left[ 1 - \frac{r n(r)}{c} \right]^{-1}, \tag{3}$$

which at first order w.r.t. $v/c$ and $U/c^2$, gives the refractive index associated with the boosted linear Schwarzschild metric \cite{11}, (as in Ref. \cite{12}), which reads

$$n(r) = 1 - 2U/c^2 - 4(v/c)U/c^2, \tag{4}$$

where $n(r) > r n(r)$. 

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On the other hand, following [2, 5, 9, 13], the total (Shapiro plus extra) time delay caused by Jupiter’s gravity and measured by two VLBI antennae can be written as

\[ \Delta T = \frac{2GM_J}{c^3} \left( 1 - \frac{k \cdot v_J}{c} \right) \ln \frac{r_{1J} - K \cdot r_{1J}}{r_{2J} - K \cdot r_{2J}}, \]  

where \( k \) is the unit vector from the quasar to the barycenter of the solar system; \( M_J \) and \( v_J \) are the Jovian mass and velocity; \( r_{iJ} \) are the difference between the barycenter coordinates of the \( i \)-th VLBI station and Jupiter and, finally, \( K \) is given by \( k - \frac{1}{c} v_{TJ} \), where \( v_{TJ} = k \wedge (v_J \wedge k) \) is the transverse Jovian velocity in the plane of the sky which supplies the aberration of light. In (5) two different corrections \( \frac{1}{c} v_J \) appear, the first one in the logarithmic term and the second in the pre-factor. In the considered observation the main one is the correction, from \( k \) to \( K \) due to \( -\frac{1}{c} v_{TJ} \), in the argument of the logarithmic term.

All quantities are evaluated at the same time \( t \) of reception of the light on Earth.

Since the above formula (5) is obtained by means of a Lorentz transformation from the rest frame of Jupiter (where Shapiro’s formula is applied) to the barycenter frame of the solar system then, by the arguments exposed in this letter, an intrinsic gravitomagnetic field does not exist. In spite of that, in [2, 3, 5, 6, 7, 8] the gravity speed \( c_g \), due to Jupiter’s gravitomagnetic field, appears in the above formulas instead of the speed of light \( c \). We conclude that the right interpretation of the \( v_J/c \) correction terms in the time delay involves, as a result of a passive Lorentz transformation, the aberration of the speed of light rather than the speed of gravity.

There are two main statements about the gravitomagnetic field of Jupiter in [6, 8]. The first one appears in [6] and says: “Another general relativistic interpretation of the Jupiter-quasar experiment (apart from its association with the measurement of the speed of gravity) consists in the statement that the experiment has measured the magnitude of the gravitomagnetic field generated by the orbital motion of Jupiter”. The second statement appears in [8] and reads: ”Thus, in a moving frame, the translational motion of Jupiter produces the gravito-magnetic field which deflects the light by dragging it to the direction of motion of Jupiter. We can measure this gravito-magnetic dragging of light and express its magnitude in terms of the speed of characteristics of...”. However, we think that our above analysis shows that the interpretation given in the quoted statements cannot be applied to the 2002 Jovian observation.
ACKNOWLEDGMENTS

I wish to thank C. Dehesa and J. Martín for discussions on this subject and to A. San Miguel, M. del Mar San Miguel and F. Vicente for comments and a critical reading of the manuscript. A previous version of this work received an Honorable Mention in the 2004 Essay Competition of the Gravity Research Foundation (GRF) and it has appeared in the arXiv as gr-qc/0405123 before Ref. 6 was revised and published as 7. The author is currently partially supported by MCYT TIC2003-07020.

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