A REMARK ON THE ISOMORPHISM CONJECTURES
CRICHTON OGLE AND SHENKUI YE

Abstract. We show that for various natural classes of groups and appropriately defined $K$- and $L$-
theoretic functors, injectivity or bijectivity of the assembly map follows from the Isomorphism Conjecture
being true for acyclic groups lying within that class.

Statement of results. We will use the setup of [DL], with which we assume familiarity. For a discrete
group $G$, let $H^*_G(-;K^t)$ denote the equivariant homology theory associated to the topological $K$-theory
$\text{Or}(G)$-spectrum $K^t$. Let $E_{\text{Fin}}(G)$ be the space classifying proper actions of $G$ (it is uniquely character-
ized up to equivariant homotopy by the property that the fixed-point set $E_{\text{Fin}}(G)^H$ is non-empty and
contractible for every finite subgroup $H \leq G$). The Baum-Connes conjecture (as reformulated in [DL])
asserts that the assembly map

$$H^*_G(E_{\text{Fin}}(G);K^t) \to K^*_t(C^*_r(G))$$

(1)

is an isomorphism for all $*$, where the groups on the right are the topological $K$-groups of the reduced
$C^*$-algebra of $G$. We will write BC for the Baum-Connes Conjecture, MBC resp. EBC for the statement
the assembly map in (1) is a monomorphism resp. epimorphism, and $R$-BC (resp. $R$-MBC resp. $R$-EBC)
for the conjecture that the Baum-Connes assembly map becomes an isomorphism (resp. monomorphism
resp. epimorphism) after tensoring both sides of (1) with $R$ with $R \subseteq \mathbb{Q}$. Finally $R$-BC($G)$ resp. $R$-MBC($G)$
resp. $R$-EBC($G)$ will denote the conjecture that $R$-BC resp. $R$-MBC resp. $R$-EBC holds for a particular
group $G$. Let $\mathcal{G}$ be the class of all groups. Given a subclass $\mathcal{C} \subseteq \mathcal{G}$, we say that $R$-IC, $R$-EC, or $R$-MC
holds over $\mathcal{C}$ if the conjecture is true for all groups in $\mathcal{C}$. The subclasses of interest here are: i) $\mathcal{T} \mathcal{F} \subset \mathcal{G}$
consisting of all torsion-free discrete groups and ii) $\mathcal{F} \mathcal{L} \subset \mathcal{T} \mathcal{F}$ the subcollection of groups $G$ for which
$BG \simeq X$ a finite complex (called $\mathcal{F} \mathcal{L}$ groups). Recall that a group $G$ is acyclic if the reduced homology
group $\tilde{H}_i(G) = 0$ for all $i$.

Theorem 1. Let $R$ be a subring of $\mathbb{Q}$. Let $\mathcal{C} = \mathcal{G}, \mathcal{T} \mathcal{F}$ or $\mathcal{F} \mathcal{L}$. If $R$-BC($G)$ holds true for all acyclic
groups in $\mathcal{C}$, then $R$-MBC is true for all groups in $\mathcal{C}$.

The assembly map considered above is a special case of a much more general construction. For suitably
defined functors $F$ on the class $\mathcal{G}$ of discrete groups, one has an assembly map

$$HF_*(G) \to F_*(G)$$

(2)

and the Isomorphism Conjecture (IC) [DL] asserts that this map is an isomorphism, where $HF_*(\_)$
denotes the appropriate homology theory associated to $F$. Following the definitions given above, the
Epimorphism Conjecture (EC) resp. Monomorphism Conjecture (MC) for the theory being considered states
that the assembly map in (2) is a monomorphism resp. epimorphism. Again, given a subring $R \subseteq \mathbb{Q}$,
the conjecture $R$-IC resp. $R$-EC resp. $R$-MC is the conjecture that the assembly map is an isomorphism
resp. monomorphism after tensoring with $R$, with the appendage “($G)$” indicating the conjecture for a particular
group $G$.

Theorem 2. Let $F_*(G) = L_*^{<\infty}(\mathbb{Z}[G])$, with $HF_*(G) := H^*_G(E_{\text{Fin}};L_*^{<\infty}(\mathbb{Z}))$ the equivariant ho-
mology theory associated to the algebraic $L$-theory $\text{Or}(G)$-spectrum $L_*^{<\infty}(\mathbb{Z})$. Let $\mathcal{C} = \mathcal{G}, \mathcal{T} \mathcal{F}$ or $\mathcal{F} \mathcal{L}$.
Fix $R \subseteq \mathbb{Q}$. If $\frac{1}{2} \in R$ and $R$-IC($G)$ is true for the functor $F$ for all acyclic groups in $\mathcal{C}$, then $R$-IC holds
for $F$ over $\mathcal{C}$. If $\mathcal{C} \subseteq \mathcal{T} \mathcal{F}$, the implication holds without restriction on $R$. In particular, the Novikov
Conjecture holds for all groups in $\mathcal{C}$ if the assembly map for $F$ is a rational isomorphism for all acyclic
$G \in \mathcal{C}$.

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Let $KH(S)$ denote the homotopy $K$-theory spectrum of the discrete ring $S$, as defined by Weibel in [CW].

**Theorem 3.** Let $F_*(G) = KH_*(\mathbb{Z}[G])$, with $HF_*(G) := H^G_*(E_{F\in}; KH_*(\mathbb{Z}))$. Let $\mathcal{C} = \mathcal{G}, \mathcal{T}F$ or $\mathcal{F}L$. Let $R$ be a subring of $\mathbb{Q}$. If $R$-IC holds for $F$ for all acyclic groups in $\mathcal{C}$, then $R$-IC holds for $F$ over $\mathcal{C}$.

For ordinary algebraic $K$-theory, a slightly weaker result can be shown.

**Theorem 4.** For a discrete ring $S$, set $FS_*(G) = K_*(S[G])$, with $HFS_*(G) := H^G_*(E_{F\in}; K(S))$. Let $\mathcal{C} = \mathcal{G}$ or $\mathcal{T}F$ and $R$ an arbitrary subring of $\mathbb{Q}$.

1. If $Q$-IC holds for $F \mathbb{Z}$ for all acyclic groups in $\mathcal{C}$, then $Q$-MC holds for $F \mathbb{Z}$ over $\mathcal{C}$.
2. Let $S$ be a regular ring containing the rationals $\mathbb{Q}$. If $R$-IC holds for $FS$ for all acyclic groups in $\mathcal{C}$, then $R$-MC holds for $FS$ over $\mathcal{C}$.
3. Let $S$ be a regular ring. If $R$-IC holds for $FS$ for all acyclic groups in $\mathcal{F}L$, then $R$-MC holds for $FS$ over $\mathcal{F}L$.

**Proof of Theorems 1, 2, and 3.** The proof in all cases is based on the method of [JOR], §6.5. For any discrete group $G$, a classical construction allows us to embed $G$ in an acyclic group $A(G)$ (its acyclic envelope), with the inclusion $i_G : G \rightarrow A(G)$ being functorial in $G$. Now the variation of the Kan-Thurston construction detailed in [JB] produces a group $T(G)$ together with a surjective homomorphism $p_G : T(G) \rightarrow G$ inducing an homology equivalence. The association $G \rightarrow T(G)$ is functorial in $G$; moreover $T(G)$ lies in the Waldhausen-Cappell class $\mathcal{E}$ consisting of those groups which can be constructed from free groups by i) amalgamated free products, ii) HNN extensions, and iii) taking direct limits. Additionally, as shown in [JB], starting with a group $G' \in \mathcal{E}$, the acyclic envelope $A(G')$ can be formed so as to remain inside of $\mathcal{E}$. In the case $\mathcal{C} = \mathcal{G}$ or $\mathcal{T}F$, $A(T(G))$ will denote Block’s construction of this envelope. Let $A_1 = G \times A(T(G))$, $A_2 = A(T(G))$. There are inclusions

$$(3) \quad T(G) \hookrightarrow A_1, \quad g \mapsto (p_G(g), i_{T(G)}(g)),$$

$$(4) \quad T(G) \hookrightarrow A_2, \quad g \mapsto i_{T(G)}(g).$$

Let $A_3 = A_1 \xrightarrow{T(G)} A_2$.

In what follows, we will, for all of the functors considered above, write $HF_*(G)$ for $H^G_*(E_{F\in}; F)$, where $\mathbb{F}$ denotes the Or$(G)$-spectrum associated to $F$. There is a homomorphism of sequences where the vertical arrows are given by assembly:

$$\ldots \xrightarrow{\phi^*_{n+1}} HF_{n+1}(A_3) \xrightarrow{\phi^*_n} HF_n(T(G)) \xrightarrow{\phi^*_n \oplus \phi^*_{n+1}} HF_n(A_1) \oplus HF_n(A_2) \xrightarrow{\phi^*_{n+1}} HF_{n+1}(A_3) \xrightarrow{\phi^*_n} HF_{n-1}(T(G)) \xrightarrow{\phi^*_n \oplus \phi^*_{n+1}} HF_{n-1}(A_1) \oplus HF_{n-1}(A_2) \xrightarrow{\phi^*_n \oplus \phi^*_{n+1}} HF_n(A_1) \oplus HF_n(A_2) \xrightarrow{\phi^*_n} HF_{n-1}(A_1) \oplus HF_{n-1}(A_2) \xrightarrow{\phi^*_n \oplus \phi^*_{n+1}} HF_n(A_1) \oplus HF_n(A_2) \xrightarrow{\phi^*_n \oplus \phi^*_{n+1}} \ldots$$

As noted in [MV], the space $E_{F\in}(A_3)$ is equivalent (up to equivariant homotopy) to the homotopy push-out of the diagram

$$A_3/T(G) \times E_{F\in}(T(G)) \xrightarrow{A_3/A_1 \times E_{F\in}(A_1)} A_3/A_2 \times E_{F\in}(A_2)$$

by which one may derive the exactness of the top sequence for coefficients in any Or$(A_3)$-spectrum. The commutativity of the diagram, as well as the exactness of the bottom row, is the point that needs to be verified. We consider first the case $\mathcal{C} = \mathcal{G}$ or $\mathcal{T}F$ for the functor $F_*(G) = K^1_*(C_*(G))$; here exactness of the bottom row follows by the results of Pimsner [MP], while the commutativity of the diagram has been shown by Oyono-Oyono [OO]. As noted in [JB], the result of [MP] implies $\phi^*_T$ is an isomorphism. By the same reasoning, $\phi^*_{T}$ is an isomorphism, and $\phi^*_{n}$ is an isomorphism by hypothesis. The five-lemma then implies $\phi^*_{1}$ must be an isomorphism as well.

For a $\mathbb{Z}$[Or$(G)$]-module $M$ and $G$-CW complex $X$, denote by $H^*_{Or}(X; M)$ the Bredon homology of $X$ with coefficients $M$. Since the groups in $\mathcal{E}$ are torsion-free, the family of finite subgroups of $A_1$ is the
same as that of $G$. Taking $M = \pi_i(\mathbb{K}^{\text{top}})$, one has isomorphisms
\[
H^{\text{Or}}_{n+1}(A_1)(E_{F,\infty}(A_1); M) \cong H^{\text{Or}}_{n+1}(E_{F,\infty}(G) \times E(A(T(G)) \times M) \\
\cong H^{\text{Or}}_{n+1}(E_{F,\infty}(G) \times BA(T(G)) \times M) \\
\cong H^{\text{Or}}_{n+1}(E_{F,\infty}(G); M).
\]
By the equivariant Atiyah-Hirzebruch spectral sequence (cf. \[DL\]), there is an isomorphism
\[
H^{\text{Or}}_{n+1}(E_{F,\infty}(A_1); \mathbb{K}^{\text{top}}) \cong H^{\text{Or}}_{n}(E_{F,\infty}(G); \mathbb{K}^{\text{top}}), \quad n \in \mathbb{Z}.
\]
Therefore, the inclusion map $G \to A_1$ induces an injection
\[
\text{Ker}(H^{\text{Or}}_{n+1}(E_{F,\infty}(G); \mathbb{K}^{\text{top}}) \to K_n(C^*_r(A_1))) \subset \text{Ker}(H^{\text{Or}}_{n+1}(E_{F,\infty}(A_1); \mathbb{K}^{\text{top}}) \to K_n(C^*_r(A_1))).
\]
This implies that the assembly map $H^{\text{Or}}_{n+1}(E_{F,\infty}(G); \mathbb{K}^{\text{top}}) \to K_n(C^*_r(G))$ is injective, which completes the proof of Theorem 1 for $R = \mathbb{Z}$. Tensoring with any ring flat over $\mathbb{Z}$ yields the same result for all $R \subset \mathbb{Q}$.

For $C = G$ or $T \mathcal{F}$, the proofs of Theorems 2 and 3 follow exactly the same line of reasoning, after applying the following modifications:

- In the case $F_s(G) = L_{s-\infty}^{-\infty}([\mathbb{Z}[G]]$), the exactness of the bottom row follows by the results of \[SC1\], the one complication being the possible existence of $UNil$-terms. These terms vanish when tensoring with any $R$ containing $\frac{1}{2}$, or in the case the groups in question are torsion-free. For this functor, the assembly map is an integral isomorphism for groups in the class $\mathcal{C}$ by \[SC1, SC2\].
- For $F_s(G) = K\pi_1(\mathbb{Z}[G])$, the corresponding results (exactness of bottom row and equivalence of assembly map for $\mathcal{C}$-groups) has been shown in \[BLJ\].
- In both cases we have functoriality with respect to arbitrary group homomorphisms, not just injective ones. The injection $G \to A_1$ of the first factor, the projection $A_1 \to G$ onto the first factor, and the naturality of the assembly map together allow us to conclude that $R$-IC for the group $A_1$ implies $R$-IC for $G$.

We next consider the smaller class $\mathcal{F}L$. In order to duplicate the above argument, the construction of the acyclic envelope requires modification, as Block’s construction does not preserve this class. Instead (as in \[JOR\]), we use Leary’s metric refinement of the Kan-Thurston construction \[IL\]. To any complex $X$ Leary associates a cubical CAT(0)-complex $C(X)$ together with a map $p_X : C(X) \to X$ which is an epimorphism on $\pi_1$ and an isomorphism in homology. The association $X \mapsto (C(X), p_X)$ is functorial in $X$; moreover if $X$ is finite, so is $C(X)$.

Let $G \in \mathcal{F}L$, and fix a finite basepointed complex $X_G$ with $X_G \simeq BG$. Let $\hat{X}_G$ denote the cone on $X_G$; then the canonical inclusion $X_G \to \hat{X}_G$ is covered by an inclusion of cubical CAT(0)-complexes $C(X_G) \hookrightarrow C(\hat{X}_G)$. Define the groups $A_i, 1 \leq i \leq 3$ by
\[
A_1 := G \times \pi_1(C(\hat{X}_G)); \\
A_2 := \pi_1(C(\hat{X}_G)); \\
A_3 := \pi_1(C(\hat{X}_G)) \cdot A_2
\]
where $\pi_1(C(X_G)) \hookrightarrow \pi_1(C(\hat{X}_G))$ is the inclusion of CAT(0)-group\[E\] corresponding to the inclusion $X_G \hookrightarrow \hat{X}_G$. Writing $L_{s-\infty}^{-\infty}([\mathbb{Z}[H]]$ as $L_s([\mathbb{Z}[H]]$ and $H_n(BH; L(\mathbb{Z}))$ simply as $H_n(BH)$, one has as before a commuting diagram of long-exact sequences with the vertical maps induced by assembly:

\[\cdots \to H_{n+1}(BA_3) \xrightarrow{\partial} H_n(B\pi_1(C(X_G))) \xrightarrow{\partial} H_n(BA_1) \oplus H_n(BA_2) \xrightarrow{\partial} H_n(BA_3) \xrightarrow{\partial} H_{n-1}(B\pi_1(C(X_G))) \xrightarrow{\partial} \cdots \]

\[\psi_n^1 \downarrow \quad \psi_n^2 \downarrow \quad \psi_n^3 \downarrow \quad \psi_n^{E-1} \downarrow \]

\[\cdots \to L_{n+1}(\mathbb{Z}[A_3]) \xrightarrow{\partial} L_n(\mathbb{Z}[\pi_1(C(X_G))]) \xrightarrow{\partial} L_n(\mathbb{Z}[A_1]) \oplus L_n(\mathbb{Z}[A_2]) \xrightarrow{\partial} L_n(\mathbb{Z}[A_3]) \xrightarrow{\partial} L_{n-1}(\mathbb{Z}[\pi_1(C(X_G))]) \xrightarrow{\partial} \cdots \]

\[\footnote{In the case of the reduced $C^*$-algebra, it is unknown in general whether the projection $A_1 \to G$ defines an appropriate element of $KK(C^*_r(A_1), C^*_r(G))$. If it does, then the stronger conclusions of Theorems 2 and 3 would apply as well to Theorem 1.}

\[\footnote{Leary shows that for any inclusion of complexes $X \hookrightarrow Y$, the resulting inclusion $C(X) \hookrightarrow C(Y)$ is isometric and that the image is a totally geodesic subcomplex of $C(Y)$, implying injectivity on $\pi_1$.} \]
Both $A_2$ and $\pi_1(C(X_0))$ are fundamental groups of finite cubical CAT(0)-complexes; it follows from the results of [BL2] that the assembly maps $\psi^3_*$ and $\psi^2_*$ are isomorphisms. Moreover, $HL_*(BA_1) \cong HL_*(BG)$, and so as before on has an identification of kernels

$$\ker(\psi^1_*) \cong \ker(HL_*(BG) \to L_*(\mathbb{Z}[G])).$$

which, together with the injectivity of $\psi^3_*$ yields an injection

$$\ker(HL_*(BG) \to L_*(\mathbb{Z}[G])) \cong \ker(\psi^1_*) \hookrightarrow \coker(\psi^3_{n+1}).$$

As all of the groups in the above diagram are objects in the category $\mathcal{FL}$, we arrive at the same conclusion as before. This completes the proof of Theorem 2. In the case of the reduced group $C^*$- algebra, the same argument for torsion-free groups applies, given that groups acting properly on cubical CAT(0)-complexes satisfy the Haagarup property [NR], and thus satisfy the Strong BC Conjecture by the work of Higson-Kasparov [HK], which completes the proof of Theorem 1.

Next we consider the statement of the third theorem when $\mathcal{C} = \mathcal{FL}$. For brevity, we say that $G$ satisfies condition $\mathcal{FCAT}$ if it acts properly, isometrically and cocompactly on a finite dimensional CAT(0)-space.

**Lemma 1.** Suppose $G$ satisfies $\mathcal{FCAT}$. Then the natural transformation of spectrum-valued functors $K(-) \to KH(-)$ from algebraic to homotopy $K$-theory induces a weak equivalence

$$K(\mathbb{Z}[G]) \xrightarrow{\sim} KH(\mathbb{Z}[G]).$$

**Proof.** For an arbitrary ring $A$, there exists a right half-plane spectral sequence [Thm 1.3, CW]:

$$E^1_{pq} := N^pK_q(A) \Rightarrow KH_{p+q}(A), \quad p \geq 0, q \in \mathbb{Z}.$$

For $A = \mathbb{Z}[G]$ and $p > 0$, the groups $N^pK_*(\mathbb{Z}[G])$ are summands of $K_*(\mathbb{Z}[G \times \mathbb{Z}^p])$. But if $G$ satisfies $\mathcal{FCAT}$, so does $G \times \mathbb{Z}^p$ for all $p \geq 0$. Again, by the main result of [BL2] and [CW], these summands identify isomorphically with the corresponding summands in the domain of the Farrell-Jones assembly map, where they vanish. Thus for such groups, $N^pK_*(\mathbb{Z}[G]) = 0$ for all $p > 0$, yielding the required isomorphism on homotopy groups in all degrees. □

Thus the Farrell-Jones assembly map for $KH(-)$ - which for torsion-free groups agrees with the classical assembly map $H_*(BG; \mathbb{K}(\mathbb{Z}) \to KH_*(\mathbb{Z}[G])$ - is an isomorphism for $G$ satisfying $\mathcal{FCAT}$. With this additional fact in hand, the proof of Theorem 3 is complete.

Unlike the reduced $C^*$ algebra, the full (or maximal) group $C^*$ algebra is functorial with respect to arbitrary group homomorphisms. On the other hand, for groups with property $T$, the Baum-Connes assembly map fails to be surjective, even rationally. Consequently, the methods of the previous two theorems imply

**Corollary 1.** There exist acyclic groups $G$ for which the assembly map

$$H_*^G(E_{\mathcal{F}in}(G); \mathbb{K}^1) \to K_*^1(C^*(G))$$

fails to be an isomorphism, even rationally.

Finally we consider the statement of Theorem 4. Here the results of Waldhausen [FW] produce a Mayer-Vietoris type of long-exact sequence which appears as the bottom row in the commuting diagram

\[
\begin{array}{cccccccc}
\vdots \\
HFS_{n+1}(A_1) & \xrightarrow{\partial} & HFS_n(T(G)) & \xrightarrow{\phi^1_n} & HFS_n(A_1) \oplus HFS_n(A_2) & \xrightarrow{\phi^2_n} & HFS_n(A_3) & \xrightarrow{\partial} & HFS_{n-1}(T(G)) \\
\downarrow \phi^3_{n+1} & & \downarrow \phi^2_n & & \downarrow \phi^1_n & & \downarrow \phi^0_n & & \downarrow \phi^3_{n-1} \\
K_{n+1}(S[A_3]) & \xrightarrow{\partial} & K_n(T(G)) \oplus N\mathfrak{u}_n(T(G), A_1, A_2) & \xrightarrow{\partial} & K_n(S[A_1]) \oplus K_n(S[A_2]) & \xrightarrow{\partial} & K_n(S[A_3]) & \xrightarrow{\partial} & K_{n-1}(S[T(G)]) \oplus N\mathfrak{u}_n(T(G), A_1, A_2) & \xrightarrow{\partial} & \vdots
\end{array}
\]

Assume that the Farrell-Jones assembly map is an isomorphism for any acyclic group. Then $\phi^2_n$ and $\phi^3_n$ are isomorphisms. When either $Q \subset S$, or $\mathbb{K}$ represents rationalized algebraic $K$-theory with $S = \mathbb{Z}$, the

\[\text{More precisely, one only needs } G \text{ to be in the class } \mathcal{B} \text{ as given in [Def. 1, BL2] for this Lemma to apply.}\]
Farrell-Jones assembly map is injective for any group in the Waldhausen-Cappell class $\mathcal{C}$ [BL1]. With $\phi^T_n$ injective, the map $\phi^1_n$ is injective by a diagram chase. This shows that the kernel
$$\text{Ker}(H_n^G(E_{\text{Fin}}(G); \mathbb{K}) \to K_n(S[G])) \subset \text{Ker}(\phi^1_n)$$
is trivial. When $G \in \mathcal{FL}$, we produce $A_1, A_2$ and $A_3$ using CAT(0) cubical complexes as before. The group $\pi_1(C(X_G))$ acts properly and cocompactly on the universal cover of $C(X_G)$, which is a CAT(0) cubical complex. According to a result of Wegner [CW], the Farrell-Jones conjecture is true for $\pi_1(C(X_G))$ with any coefficients. Using a similar diagram chasing, we see that $\text{Ker}(H_n^G(E_{\text{Fin}}(G); \mathbb{K}) \to K_n(S[G])) = 0$ in (3). The rational algebraic $K$-theory with $R = \mathbb{Z}$ is proved similarly, completing the proof of Theorem 4.

For a torsion-free acyclic group $A$, there are isomorphisms $H_n^A(E_{\text{Fin}}(A); \mathbb{F}) \cong H_n(BA; \mathbb{F}(A/e)) \cong H_n(pt; \mathbb{F}(A/e))$, where $e$ denotes the trivial subgroup of $A$. This implies that the assembly map is injective for a torsion-free acyclic group. Therefore

**Corollary 2.** Following Theorems 1 and 4,

1. The Baum-Connes conjecture is true for every torsion-free group if and only if the Baum-Connes assembly map is an epimorphism for every torsion-free group.
2. Let $R$ be a regular ring with $\mathbb{Q} \subset R$. The Farrell-Jones conjecture with coefficients in $R$ (resp. the rational Farrell-Jones conjecture with coefficients in $\mathbb{Z}$) holds for every torsion-free group if and only if the integral (resp. rational) assembly map is an epimorphism for every torsion-free group.
3. Let $R$ be a regular ring. The Farrell-Jones conjecture is true for every FL group (with coefficients in $R$) if and only if the assembly map is an epimorphism for every FL group (with coefficients in $R$).

**Remark 1.** It is currently unknown whether the original Baum-Connes Conjecture holds for CAT(0)-groups of the type considered by Bartels and Lück in [BL2]. However, based on the results of [GY] and [BCGNW], it seems plausible that similar results as those above can be obtained for the Coarse Baum-Connes Conjecture. We hope to address these issues more completely in future work.

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Department of Mathematics, The Ohio State University, Columbus, OH 43210,
E-mail address: ogle@math.ohio-state.edu

Department of Mathematics, National Univ. of Singapore, Singapore, ID 119076L,
E-mail address: yeshengkui@nus.edu.sg