Calibration of Gamma-Ray Burst Luminosity Correlations Using Gravitational Waves as Standard Sirens

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Abstract

Gamma-ray bursts (GRBs) are a potential tool to probe the high-redshift universe. However, the circularity problem has encouraged people to find model-independent methods to study the luminosity correlations of GRBs. Here, we present a new method that uses gravitational waves (GWs) as standard sirens to calibrate GRB luminosity correlations. For the third-generation ground-based GW detectors (i.e., Einstein Telescope—ET), the redshifts of GW events accompanying electromagnetic counterparts can reach out to \( \sim 4 \), which is more distant than type Ia supernovae (\( z \gtrsim 2 \)). The Amati relation and Ghirlanda relation are calibrated using the mock GW catalog from ET. We find that the 1\( \sigma \) uncertainty of intercepts and slopes of these correlations can be constrained to less than 0.2\% and 8\% respectively. Using calibrated correlations, the evolution of the dark energy equation of state can be tightly measured, which is important for discriminating dark energy models.

\textit{Key words:} cosmological parameters – gamma-ray burst: general – gravitational waves

1. Introduction

Gamma-ray bursts (GRBs) are some of the most energetic phenomena in our universe (Kumar & Zhang 2015; Wang et al. 2015). Their high luminosity makes them detectable out to high redshifts. Therefore, GRBs are promising tools to probe the high-redshift universe: including the cosmic expansion and dark energy (Dai et al. 2004; Ghirlanda et al. 2006; Liang & Zhang 2006; Schaefer 2007; Wang et al. 2007; Kodama et al. 2008, the star formation rate (Totani 1997; Bromm et al. 2002; Wang \& Dai 2009; Wang 2013), the reionization epoch (Barkana \& Loeb 2004; Totani et al. 2006), and the metal enrichment history of the universe (Wang et al. 2012; Hartoog et al. 2015). Among them, the \( \gamma \)-ray burst correlations (for reviews, see Wang et al. 2015; Wang et al. 2019; Dainotti \& Del Vecchio 2017; Dainotti \& Amati 2018; Dainotti et al. 2018) are most widely studied, which can not only shed light on the radiation mechanism of GRBs, but also provide a promising tool to probe the cosmic expansion and dark energy (Wang et al. 2015; Dainotti \& Del Vecchio 2017). These correlations can be divided into three categories, such as prompt correlations, afterglow correlations, and prompt-afterglow correlations. The prompt correlations mainly include the Amati correlation (Amati et al. 2002; Amati 2006), the Ghirlanda correlation (Ghirlanda et al. 2004a), the Liang-Zhang correlation (Liang \& Zhang 2005), the Yonetoku correlation (Wei \& Gao 2003; Yonetoku et al. 2004), and the \( L_{\text{iso}} - \tau_{\text{lag}} \) correlation (Norris et al. 2000). Afterglow correlations contain only parameters in the afterglow phase, such as the Dainotti correlation \( (L_X(T_a) - T_X^\text{iso}) \) (Dainotti et al. 2008), the \( L_X(T_a) - T_X^\text{iso} \) and \( L_0(T_a) - T_0^\text{iso} \) correlations (Ghisellini et al. 2009), and the \( L_0(\text{iso}) - \tau_{\text{iso}} \) correlation (Oates et al. 2012). Prompt-afterglow correlations connect plateaus and prompt phases, referring to the \( E_{\text{iso}} - \tau_{\text{iso}} \), \( E_{\text{iso}} - \tau_{\text{iso}} \), \( L_{\text{iso}} - \tau_{\text{iso}} \), \( L_{\text{iso}} - \tau_0^\text{iso} \) correlation (Dainotti et al. 2011), \( L_{\text{iso}} - E_{\text{iso}} \), \( \tau_0^\text{iso} \) correlation (Li et al. 2015), and so on.

However, there is a circularity problem when treating GRBs as relative standard candles. The problem arises from the derivation of quantities like luminosity \( L_{\text{iso}} \), isotropic energy \( E_{\text{iso}} \), and collimation-corrected energy \( E_* \), which are dependent on luminosity distance \( d_L \) in a fiducial cosmology. For instance, the \( d_L \) in a flat \( \Lambda \)CDM model can be expressed as

\[
\begin{align*}
  d_L(z) &= \frac{c(1 + z)}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m(1 + z')^3 + (1 - \Omega_m - \Omega_L)(1 + z')^2 + \Omega_L}} \\
  &= \frac{d_L(z)}{\Omega_m(1 + z')^3 + (1 - \Omega_m - \Omega_L)(1 + z')^2 + \Omega_L}. 
\end{align*}
\]

Therefore, it is inappropriate to use the model-dependent luminosity correlations to study cosmology models in turn. Several approaches have been proposed to overcome the problem (Wang et al. 2015; Dainotti \& Del Vecchio 2017). One method is to fit the cosmological parameters and luminosity correlation simultaneously (Ghirlanda et al. 2004b; Li et al. 2008). Another method is to calibrate the correlations using type Ia supernovae (SNe Ia; Liang et al. 2008) or observational Hubble Data (Amati et al. 2018). This is based on the principle that objects of the same redshift should have the same luminosity distance. Wang (2008) pointed out that the GRB luminosity correlations calibrated by SNe Ia are no longer completely independent of the SNe Ia data points. Consequently, the GRB data cannot be combined with the SNe Ia data set directly to constrain cosmological parameters. Furthermore, high-redshift SNe Ia can hardly be found and the furthest SN Ia yet seen is GND12Col with \( z = 2.26^{+0.04}_{-0.05} \) (Rodney et al. 2015), while the redshift of GRBs can be up to 9.4 (Cucchiara et al. 2011). Moreover, there are many systematic uncertainties for SNe Ia, such as dust in the light path (Avgoustidis et al. 2009; Hu et al. 2017), possible intrinsic evolution of SN luminosity, magnification by gravitational lensing (Holz 1998), peculiar velocity (Hui \& Greene 2006), and so on. These processes will degrade the usefulness of SNe Ia as standard candles.
Here, we come up with the idea to calibrate GRB luminosity relations using gravitational wave (GW) standard sirens. The detection of GW170817 accompanied by electromagnetic counterparts heralds the new era of gravitational-wave multimessenger astronomy (Abbott et al. 2017). Schutz (1986) first pointed out that the waveform signals from inspiralling compact binaries can be used to determine the luminosity distance to the source, serving as a standard siren. This kind of standard siren is a self-calibrating distance indicator, which only relies on the modeling of the two-body problem in general relativity (Sathyaprakash et al. 2010). The detected BNS and BH–NS merger events can reach up to \( z \sim 4 \) to the farthest by the Einstein Telescope (ET; Abernathy et al. 2011; Li 2015; Cai & Yang 2017), going beyond the redshift limitation of SNe Ia. For the third-generation detectors, such as the ground-based ET (Abernathy et al. 2011), the space-based Big Bang Observer (BBO; Cutler & Holz 2009), and the Deci-Hertz Interferometer Gravitational wave Observatory (DECIGO; Kawamura et al. 2011), smaller distance uncertainty will be achieved than with Advanced LIGO and Virgo (Abbott et al. 2017).

The paper is organized as follows. In Section 2, we introduce the procedure of constructing the mock GW catalog. The calibration of GRB luminosity correlations with GW standard sirens is illustrated in Section 3. A summary of our results and future outlooks is provided in Section 4.

2. Construction of GW Standard Sirens

2.1. Redshift Distribution

In order to construct a mock GW catalog, we need to consider the redshift distribution of the sources, which satisfies the following expression

\[ P(z) \propto \frac{4\pi d_c^2(z)R(z)}{H(z)(1+z)}, \]

where \( d_c(z) \) is the comoving distance of the source. The time evolution of the NS–NS merger rate \( R(z) \) is given by (Schneider et al. 2001)

\[ R(z) = \begin{cases} 1 + 2z, & z \leq 1 \\ \frac{1}{2}(5 - z), & 1 < z < 5 \\ 0, & z \geq 5. \end{cases} \]

The NS–NS merger rate at redshift \( z \) is \( \dot{n}(z) = n_0 \cdot R(z) \) and the merger rate today is about \( n_0 = 1.54^{+3.2}_{-1.22} \times 10^{-6} \text{Mpc}^{-3} \text{yr}^{-1} \) (Abbott et al. 2017).

2.2. Simulation of Luminosity Distances

It is necessary to define the total mass \( M_{\text{phys}} = m_1 + m_2 \), symmetric mass ratio \( \eta = \frac{m_1}{m_2} \), and chirp mass \( M_{\text{c,obs}} = M_{\text{c,phys}}^{3/5} \) before our analysis, given binary component masses \( m_1 \) and \( m_2 \). The observed chirp mass is related to physical chirp mass via \( M_{\text{c,obs}} = (1 + z) M_{\text{c,phys}} \). Similarly, the observed total mass is \( M_{\text{obs}} = (1 + z) M_{\text{phys}} \).

2.2.1. Frequency Domain Waveform and Fourier Amplitude

The response of the detector \( h(t) \) is a linear combination of two components

\[ h(t) = F_+(\theta, \phi, \psi)h_+(t) + F_\times(\theta, \phi, \psi)h_\times(t), \]

where \( F_+ \) and \( F_\times \) are the antenna pattern functions of the detector, \( \psi \) is the polarization angle, and \((\theta, \phi)\) is the location of the source on the sky. The antenna pattern functions of ET are

\[
F_+(\theta, \phi, \psi) = \frac{\sqrt{3}}{2} \left[ \frac{1}{2} (1 + \cos^2 \theta) \cos 2\phi \cos 2\psi - \cos \theta \sin 2\phi \sin 2\psi \right],
\]

\[
F_\times(\theta, \phi, \psi) = \frac{\sqrt{3}}{2} \left[ 1 \right] (1 + \cos^2 \theta) \cos 2\phi \sin 2\psi + \cos \theta \sin 2\phi \cos 2\psi \]

The other pattern functions are \( F_+(\theta, \phi, \psi) = F_+(\phi, \psi) \) and \( F_\times(\theta, \phi, \psi) = F_\times(\phi, \psi) \) respectively. The Fourier transform of time domain waveform \( h(t) \) is given by

\[
H(f) = A f^{-7/6} \exp(i(2\pi f t_0 - \pi/4 + 2\psi(f/2) - \phi(2, 0))),
\]

where

\[
A = \frac{1}{\Delta f} \sqrt{\frac{F_+^2(1 + \cos^2 \psi)^2}{2} + \frac{F_\times^2}{\pi^2} \frac{\pi^{-7/6}}{96} M_{\text{c,obs}}^{5/6}},
\]

is the Fourier amplitude. The post-Newtonian formalism of the GW waveform phase up to 3.5 PN is used (Blanchet et al. 2002) and the expressions of functions \( \psi \) and \( \phi(2, 0) \) can be found in Arun et al. (2005) and Zhao et al. (2011).

The component masses of binary neutron stars are randomly sampled in \([1, 2] M_{\odot}\), while for neutron star–black hole systems, the component mass of the black hole is uniform in \([3, 10] M_{\odot}\) (Fryer & Kalogera 2001; Li 2015; Cai & Yang 2017). The beaming angle of \( \gamma \)-ray bursts are randomly sampled in interval \([0^\circ, 20^\circ]\). Since the GW signal-to-noise ratio \( (S/N) \) in Section 2.2.2 is independent of the waveform phase, the \( \psi \) and \( \phi(2, 0) \) are not considered here.

2.2.2. The S/N and Estimated Error

A GW signal is claimed to be detected only when combined \( S/N \geq 8 \) for a single detector network (Sathyaprakash et al. 2010). For ET, the combined \( S/N \) is

\[ \rho = \sqrt{\sum_{i=1}^{3} (\rho^{(i)})^2}, \]

where

\[
\rho^{(i)} = \sqrt{\langle H^{(i)}, H^{(i)} \rangle}. \]

and the bracket is defined by

\[ \langle a, b \rangle = 4 \int_{f_{\text{min}}}^{f_{\text{max}}} a(f)b^{*}(f) + a^{*}(f)b(f) \frac{df}{S_n(f)}, \]

where \( S_n(f) \) is the one-sided noise power spectrum density (PSD), which determines the performance of a GW detector.
We take the noise PSD of ET to be

\[
S_b(f) = S_0 \left[ x^{p_1} + a_1 x^{p_3} + a_2 \left( 1 + b_1 x^1 + b_2 x^2 + b_3 x^3 + b_4 x^4 + b_5 x^5 + b_6 x^6 \right) \right],
\]

as in Zhao et al. (2011), where \( x \equiv f/200 \, \text{Hz} \) and \( S_0 = 1.449 \times 10^{52} \, \text{Hz}^{-1} \). The parameters \( p_1, a_1, b_1, b_2, b_3, b_4, b_5 \) and \( c_1, c_2, c_3, c_4 \) are also provided in Zhao et al. (2011). The upper cutoff frequency \( f_{\text{max}} \) is twice the orbit frequency at the last stable orbit, \( f_{\text{max}} = 2f_{L, \text{SO}} = 2/(6^{3/2} 2\pi M_{\odot}) \). The lower cutoff frequency is \( f_{\text{lower}} = 1 \, \text{Hz} \).

At every simulated redshift, the fiducial value of the luminosity distance \( d_L^{\text{fid}} \) is calculated according to Equation (1). Then we simulate the \( \ln d_L^{\text{mea}} \) to be a Gaussian distribution centered around \( \ln d_L^{\text{fid}} \) with standard deviation \( \sigma_{\ln d_L} \).

\[
\ln d_L^{\text{mea}} = \mathcal{N}(\ln d_L^{\text{fid}}, \sigma_{\ln d_L}).
\]

The fiducial cosmology is flat \( \Lambda \)CDM cosmology with \( \Omega_m = 0.308, H_0 = 67.8 \, \text{km s}^{-1} \, \text{Mpc}^{-1} \) (Ade et al. 2016) when calculating \( d_L^{\text{fid}} \).

The Fisher matrix \( \Gamma_{ij} \) is widely used to estimate the errors in the measured parameters (Zha & Wen 2018),

\[
\Gamma_{ij} = \left( \frac{\partial H}{\partial \eta_i} \frac{\partial H}{\partial \eta_j} \right),
\]

where \( \eta_i \) denotes the parameters on which the waveforms depend, namely \( \ln M_c, \ln \eta, t_0, \theta_0, \cos i, \psi, \ln d_L \). Then the estimated error \( \sigma_{\eta_i} \) of parameter \( \eta_i \) is \( \Gamma^{-1/2} \). However, for simplicity of calculation, we follow Cai & Yang (2017) and take the distance uncertainty \( \sigma_{d_L} \) to be \( 2d_L/\rho \), allowing for the correlation between \( d_L \) and \( i \). When the additional error \( \sigma_{d_L}^{\text{lens}} \) due to the weak lensing is taken into account, the total uncertainty is

\[
\epsilon_{d_L} = \sqrt{\left( \sigma_{d_L}^{\text{inst}} \right)^2 + \left( \sigma_{d_L}^{\text{lens}} \right)^2} = \sqrt{\left( \frac{2d_L}{\rho} \right)^2 + (0.05 z d_L)^2}.
\]

### 2.2.3. The Predicted Event Rates

Abernathy et al. (2011) predicted event rates in ET, which is expected to observe \( \mathcal{O}(10^3 \sim 10^5) \) BNS merger events and \( \mathcal{O}(10^3 \sim 10^5) \) BH–NS events per year. However, this prediction is very uncertain. Li (2015) expected that only a small fraction (\( \sim 10^{-3} \)) of GW detections are accompanied by observed GRBs. Therefore, we typically construct a catalog of 1000 BNS events in our simulation. Besides, when the ratio between NS–BH and BNS events is assumed to be 0.03 as predicted by the Advanced LIGO-Virgo network (Abadie et al. 2010; Li 2015; Cai & Yang 2017), 30 NS–BH events are included in the mock catalog. These simulated events can reach out to a redshift \( z \sim 4 \). Figure 1 shows the \( d_L - z \) diagram of our mock GW catalog.

### 3. Calibration of GRB Luminosity Correlations

The GRB samples used for calibrating the Amati relation \((E_{\text{iso}} - E_p)\) and the Ghirlanda relation \((E_l - E_p)\) are taken from Wang et al. (2016) and Wang & Dai (2011) respectively.

The energy spectrum of GRBs is modeled by a broken power law (Band et al. 1993),

\[
\Phi(E) = \begin{cases} 
\left( \frac{E}{100 \, \text{keV}} \right)^{\alpha} \exp(-(2 + \alpha)E/E_{p, \text{obs}}), & E \leq \frac{\alpha - \beta}{2 + \alpha} E_{p, \text{obs}} \\
\left[ \frac{\alpha - \beta}{2 + \alpha} E_{p, \text{obs}} \right]^{\alpha - \beta} \exp(\beta - \alpha) \left( \frac{E}{100 \, \text{keV}} \right)^{\beta}, & \text{otherwise},
\end{cases}
\]

where the typical spectral index values are taken to be \( \alpha = -1.0 \) and \( \beta = -2.2 \) if they are not given in the references.

For each GRB in the sample, the fluence \( S \) have been corrected to \( 1 - 10,000 \, \text{keV} \) energy band with \( k \)-correction (Bloom et al. 2001),

\[
S_{\text{holo}} = S \times \frac{\int_{E_{\text{min}}^{10}}^{E_{\text{max}}^{10}} E \Phi(E) dE}{\int_{E_{\text{min}}^{10}}^{E_{\text{max}}^{10}} E \Phi(E) dE},
\]

where \( E_{\text{min}}^{10} \) and \( E_{\text{max}}^{10} \) are detection thresholds of the observing instrument. The isotropic energy \( E_{\gamma, \text{iso}} \) and collimation-corrected energy \( E_{\gamma} \) are

\[
E_{\gamma, \text{iso}} = \frac{4 \pi d_L^2 S_{\text{holo}}}{1 + z},
\]

and

\[
E_{\gamma} = \frac{4 \pi d_L^2 S_{\text{holo}} F_{\text{beam}}}{1 + z},
\]

respectively, in which \( F_{\text{beam}} = 1 - \cos \theta_{\text{jet}} \) is the beaming factor for jet opening angle \( \theta_{\text{jet}} \). The luminosity distance \( d_L \) of low-redshift GRBs is derived from GW standard sirens using the linear interpolation method (Wang et al. 2016), which is
independent of cosmology models
\[
\ln d_L = \ln d_{L,i}^{GW} + \frac{z_{i+1} - z_i}{z_{i+1} - z_i} (\ln d_{L,i}^{GW} - \ln d_{L,i}^{GW}).
\] (19)

The 1σ error can be obtained by
\[
\sigma_{\ln d_L}^2 = \left( \frac{z_{i+1} - z_i}{z_{i+1} - z_i} \right)^2 \sigma_{\ln d_{L,i}^{GW}}^2 + \left( \frac{z - z_i}{z_{i+1} - z_i} \right)^2 \sigma_{\ln d_{L,i+1}^{GW}}^2,
\]
where \( \epsilon_{\ln d_{L,i}^{GW}} \equiv \epsilon_{d_{L,i}^{GW}} / d_{L,i}^{GW} \) is the distance uncertainty of the \( i \)th GW event (the mock GW catalog has been sorted by redshift before interpolation).

### 3.1. The \( E_{\text{iso}}-E_p \) Correlation

We parameterize the Amati relation \((E_{\text{iso}}-E_p \) correlation; Amati et al. 2002) as
\[
\log_{10} \frac{E_{\gamma,\text{iso}}}{1 \text{erg}} = a + b \log_{10} \left[ \frac{E_{p,\text{obs}}(1 + z)}{300 \text{ keV}} \right],
\] (20)

where \( E_{p,\text{obs}}(1 + z) \) is the cosmological rest-frame peak energy of GRB. The Markov chain Monte Carlo algorithm is applied to constrain intercept \( a \), slope \( b \), and intrinsic scatter \( \sigma_{\text{int}} \) of the correlation. We use the python module emcee to carry out parameter fitting (Foreman-Mackey et al. 2013). The likelihood to fit the linear relation
\[
y = ax + b \] (D’Agostini 2005) is
\[
L = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi} \sqrt{\sigma_{\text{int}}^2 + \sigma_{yi}^2 + b^2 \sigma_i^2}} \exp \left[ -\frac{(y_i - a - bx_i)^2}{2(\sigma_{\text{int}}^2 + \sigma_{yi}^2 + b^2 \sigma_i^2)} \right],
\]
where \( \sigma_{y}^2 = \frac{\sigma_{y_0}^2}{(\ln 10 S_{\text{bolo}})^2} \).

### 3.2. The \( E_{\gamma}-E_p \) Correlation

The parameterization of the Ghirlanda relation \((E_{\gamma}-E_p \) correlation; Ghirlanda et al. 2004a) is
\[
\log_{10} \frac{E_{\gamma}}{1 \text{erg}} = a + b \log_{10} \left[ \frac{E_{p,\text{obs}}(1 + z)}{300 \text{ keV}} \right].
\] (23)

The likelihood function has the same form as that of the \( E_{\text{iso}}-E_p \) correlation, while the propagated uncertainties of \( y \equiv \log_{10} \frac{E_{\gamma}}{\text{erg}} \) are calculated from
\[
\sigma^2_{y} = \frac{\sigma_{\text{int}}^2}{(\ln 10 S_{\text{bolo}} F_{\text{beam}})^2} + \frac{\sigma_{y_0}^2}{(\ln 10 S_{\text{bolo}} F_{\text{beam}})^2}.
\]

Figure 2. Confidence contours (1σ, 2σ, and 3σ) and marginalized likelihood distributions for intercept \( a \) and slope \( b \) in the Amati relation.
The same procedure as handling the \( E_{\text{iso}} - E_p \) correlation is used to calibrate the \( E_{\text{iso}} - E_p \) correlation.

### 3.3. Results

With our mock GW catalog, the constraints on intercept \( a \) and slope \( b \) of the Amati relation is \( a = 52.93 \pm 0.04, b = 1.41 \pm 0.07 \), and \( \sigma_{\text{int}} = 0.39 \pm 0.03 \) (1\( \sigma \)), while for the Ghirlanda relation, \( a = 50.63 \pm 0.08, b = 1.50 \pm 0.12 \), and \( \sigma_{\text{int}} = 0.16 \pm 0.04 \) (1\( \sigma \)). The 1\( \sigma \), 2\( \sigma \), and 3\( \sigma \) confidence contours and marginalized likelihood distributions are shown in Figures 2 and 3 respectively. Wang et al. (2016) standardized Amati relation of the form \( \log_{10}(E_{\gamma, \text{iso}}/\text{erg}) = a + b \log_{10}[E_{p, \text{obs}}(1 + z)/\text{keV}] \) with the SNe Ia Union2.1 sample. Their fitting results are \( a = 48.46 \pm 0.03, b = 1.766 \pm 0.007 \) with \( \sigma_{\text{ext}} = 0.34 \pm 0.04 \). Amati et al. (2018) calibrated Amati relation of the form \( \log_{10}(E_p/\text{keV}) = q + m[\log_{10}(E_{\text{cal}}/\text{erg}) - 52], \) finding \( q = 2.06 \pm 0.03, m = 0.50 \pm 0.12, \) and \( \sigma_{\text{int}} = 0.20 \pm 0.01 \).

The calibrated GRB Hubble diagram is shown in Figure 4. The solid line in this figure is plotted based on the Planck15 cosmological parameters (Ade et al. 2016).

### 3.4. Constraining \( \Lambda \)CDM Model and Cosmological Applications

We combine the calibrated GRB data and SNe Ia from the Pantheon sample (Scolnic et al. 2018) to constrain the nonflat \( \Lambda \)CDM model. The nuisance parameters \( \{ \alpha, \beta, M_0, \Delta t \} \) of the SNe Ia lightcurve are fitted with cosmological parameters \( \{ \Omega_m, \Omega_k \} \) simultaneously with the following total likelihood function

\[
L \propto L_{\text{GRB}} \cdot L_{\text{SN}}.
\]

The likelihood function of GRB is given by

\[
L_{\text{GRB}} = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi} \sigma_{\mu_{\text{GRB},i}}} \exp \left[ -\frac{(\mu_{\text{th},i} - \mu_{\text{GRB},i})^2}{2\sigma_{\mu_{\text{GRB},i}}^2} \right],
\]

where the distance modulus uncertainty \( \sigma_{\mu_{\text{GRB}}} \) is

\[
\sigma_{\mu_{\text{GRB}}}^2 = \left( \frac{5}{2} \sigma_{\mu_{E_{\text{iso}, \text{iso}}}}^2 \right)^2 + \left( \frac{5}{2\ln 10} \sigma_{S_{\text{iso}}}^2 \right)^2.
\]
The Hubble constant $H_0$ in our fitting is fixed to the Planck15 (Ade et al. 2016) value. With the combined sample (GRBs + SNe), the best-fit values for the nonflat ΛCDM model are $\Omega_m = 0.33 \pm 0.04$ and $\Omega_\Lambda = 0.52 \pm 0.08$ with $1\sigma$ uncertainties. The constraints on $\Omega_m$, $\Omega_\Lambda$, and SNe Ia lightcurve parameters $\{\alpha, \beta, M_b^*, \Delta M\}$ are shown in Figure 5.

4. Summary

In this paper, we propose the calibration of the GRB luminosity relations using GW standard sirens. This method is model-independent and will overcome the circularity problem. The constraints for intercepts and slopes of the Amati relation and the Ghirlanda relation are $a = 52.93 \pm 0.04$, $b = 1.41 \pm 0.07$, $\sigma_{\text{int}} = 0.39 \pm 0.03$ ($1\sigma$) and $a = 50.63 \pm 0.08$, $b = 1.50 \pm 0.12$, $\sigma_{\text{int}} = 0.16 \pm 0.04$ ($1\sigma$), respectively, with our mock GW catalog. The performance of our method will improve
with the upgrade of GW detector’s sensitivity, especially with third-generation detectors ET (Abernathy et al. 2011), BBO (Cutler & Holz 2009), and DECIGO (Kawamura et al. 2011). GRBs serve as a complementary tool to other cosmological probes such as SNe Ia, BAO, and CMB. Besides, it plays a crucial role in constraining $w(z)$ especially at high redshifts (Wang et al. 2011), which may help us understand the nature of dark energy.

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