\( \gamma \gamma \to \pi \pi, KK \): leading term QCD vs handbag model

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Abstract

The "handbag" model was proposed as an alternative, at the present day energies, to the leading term QCD predictions for some hard exclusive processes. The recent precise data from the Belle Collaboration on the large angle cross sections \( \gamma \gamma \to \pi \pi, KK \) allow a check of these two approaches to be performed.

It is shown that the handbag model fails to describe the data from Belle, while the leading term QCD predictions are in reasonable agreement with these data.

I. The leading term QCD predictions

The leading contributions to the hard kernels for these amplitudes at large \( s = W^2 = (q_1 + q_2)^2 \) and fixed c.m.s. angle \( \theta \) were first calculated in \cite{1} for symmetric meson wave functions, \( \phi_M(x) = \phi_M(1-x) \), and later in \cite{2} (BC in what follows) for arbitrary wave functions. Two typical diagrams are shown in fig.1. The main features of these cross sections are as follows (everywhere below we follow mainly the definite predictions from BC in \cite{2}).

The helicity amplitudes look as:

\[
A_{\lambda_1 \lambda_2}^{(lead)}(s, \theta) = \frac{64 \pi^2}{9s} \alpha s \frac{f_K^2}{f^2} \int_0^1 dx_s \phi_K(x_s) \int_0^1 dy_s \phi_K(y_s) T_{\lambda_1 \lambda_2}(x_s, y_s, \theta).
\]

The hard kernels \( T_{\lambda_1 \lambda_2}(x_s, y_s, \theta) \) are:

\[
T_{++} = T_{--} = (e_s - e_u)^2 \frac{1}{\sin^2 \theta} \frac{A}{D},
\]

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obtained by evident replacements. The pion wave function is symmetric, symmetry breaking effects. Besides, even the part of \( \phi \) wave function. The above expressions are given for \( \nu \) wave functions, \( f \). Predictions of two frequently used models for \( \phi \) weakly sensitive to the form of \( \phi \) to \( \gamma \gamma \) value of \( T = 130 \) MeV \( \pi \) u \( \leftrightarrow \) \( (\pi \leftrightarrow \pi) \) is nearly independent of \( \theta \) and, as emphasized in [1], is \( 2 \) lead \( \sum s, \theta \equiv |A|_\lambda, \lambda_2^2 | \) is the leading contribution to \( d\sigma(\pi^+\pi^-) \) can be written as:

\[
\frac{s^3}{16\pi\alpha^2} \frac{d\sigma(\gamma\gamma \rightarrow \pi^+\pi^-)}{d|\cos\theta|} = \frac{|\Phi^{(\text{eff})}(s, \theta)|^2}{\sin^4\theta} = \frac{|s F_\pi^{(\text{lead})}(s)|^2}{\sin^4\theta} |1 - v(\theta)|^2,
\]

where \( F_\pi^{(\text{lead})}(s) \) is the leading term of the pion form factor [3]:

\[
|s F_\pi^{(\text{lead})}(s)| = \frac{8\pi \tau_s}{9} \left[ f_\pi \int_0^1 \frac{dx}{x} \phi_\pi(x, \bar{p}) \right]^2,
\]

and \( v(\theta) \) is due to the \( \sim AC \) term in eq.(2). Below we will compare the predictions of two frequently used models for \( \phi_\pi(x) : \phi^{(\text{asy})}(x) = 6x(1-x) \) and \( \phi^{(\text{CZ})}(x, \mu_o) = 30x(1-x)(2x-1)^2, \mu_o = 0.5 \) GeV [4]. While the numerical value of \( |s F_\pi^{(\text{lead})}(s)| \) is highly sensitive to the form of \( \phi_\pi(x) \), the function \( v(\theta) \) is nearly independent of \( \theta \) at \( |\cos\theta| < 0.6 \) and, as emphasized in [1], is weakly sensitive to the form of \( \phi_\pi(x) \). For the above two very different pion wave functions, \( v(\theta) \approx 0.12 \).
The recent data from Belle \cite{5} for \((\pi^+\pi^-)\) and \((K^+K^-)\) agree with \(\sim 1/\sin^4\theta\) dependence at \(W \geq 3\ GeV\), while the angular distribution is somewhat steeper at lower energies. The energy dependence at \(2.4\ GeV < W < 4.1\ GeV\) was fitted in \cite{5} as: 
\[ \sigma_0(\pi^+\pi^-) = \int_0^{0.6} dc(d\sigma/d|c|) \sim W^{-n} \], \(n = (7.9 \pm 0.4 \pm 1.5)\) for \((\pi^+\pi^-)\), and \(n = (7.3 \pm 0.3 \pm 1.5)\) for \((K^+K^-)\). However, the overall value \(n \simeq 6\) is also acceptable, see fig.2.

As for the absolute normalization, the \((\pi^+\pi^-)\) data are fitted \cite{5} with:
\[ |\Phi_{\pi}(s, \theta)| = (0.503 \pm 0.007 \pm 0.035)\ GeV^2 \]. This value can be compared with: \(0.88 \cdot |F_\pi^{(CZ)}(s)| \simeq 0.40\ GeV^2\), obtained by using \(\phi_\pi(x) = \phi_\pi^{(CZ)}(x, \mu_0)\) in eq.(5). It is seen that there is a reasonable agreement. At the same time, using \(\phi_\pi(x) = \phi_{\pi}^{(asy)}(x)\) one obtains much smaller value: \(0.88 \cdot |F_\pi^{(asy)}(s)| \simeq 0.13\ GeV^2\). So, for the pion wave function \(\phi_\pi(x)\) close to \(\phi_{\pi}^{(asy)}(x)\) the leading term calculation predicts the cross section which is \(\simeq 15\) times smaller than the data. It seems impossible that, at energies \(s = 10 - 15\ GeV^2\), higher loop or power corrections can cure so large difference. 

b) The SU(3)-symmetry breaking, \(d\sigma(K^+K^-) \neq d\sigma(\pi^+\pi^-)\), originates not only from different meson couplings, \(f_K \neq f_\pi\), but also from symmetry breaking effects in normalized meson wave functions, \(\phi_K(x) \neq \phi_\pi(x)\). These two effects tend to cancel each other, when using for the K-meson the wave function \(\phi_K(x_s, x_u)\) proposed in \cite{6} (see \cite{7} for a review). So, instead of the naive prediction \(\simeq (f_K/f_\pi)^4 \simeq 2.3\) from \cite{1}, the prediction of BC for this

\footnote{Clearly, in addition to the leading terms \(A^{(lead)}\) given by eqs.(1-5), this experimental value includes also all loop and power corrections \(\delta A\) to the \(\gamma\gamma \rightarrow \pi^+\pi^-\) amplitudes \(A = A^{(lead)} + \delta A\). These are different, of course, from corrections \(\delta F_\pi\) to the genuine pion form factor \(F_\pi = F_\pi^{(lead)} + \delta F_\pi\). So, the direct connection between the leading terms of \(d\sigma(\pi^+\pi^-)\) and \(|F_\pi|^2\) in eq.(4) does not hold on account of corrections.}

\footnote{A similar situation occurs in calculations of charmonium decays. \(Br(\chi_o \rightarrow \pi^+\pi^-)\) and \(Br(\chi_2 \rightarrow \pi^+\pi^-)\) calculated with \(\phi_\pi(x) = \phi_{\pi}^{(asy)}(x)\) are \(\simeq 20 - 25\) times smaller than the data, while the use of \(\phi_\pi(x) = \phi_{\pi}^{(CZ)}(x, \mu_o)\) leads to values in a reasonable agreement with the data, see \cite{4},\cite{7}.}

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ratio is close to unity, and this agrees with the recent data from Belle [5]:

\[
\frac{\sigma_o(\gamma\gamma \rightarrow K^+K^-)}{\sigma_o(\gamma\gamma \rightarrow \pi^+\pi^-)} = \begin{cases} (f_{K}/f_{\pi})^4 \simeq 2.3 & \text{Brodsky, Lepage [1]} \\ \simeq 1.06 & \text{Benayoun, Chernyak [2]} \\ (0.89 \pm 0.04 \pm 0.15) & \text{Belle [5]} \end{cases}
\]

c) The leading terms in the cross sections for neutral particles are much smaller than for charged ones. For instance, it was obtained by BC that the ratio \(d\sigma(\text{lead})(\pi^0\pi^0)/d\sigma(\text{lead})(\pi^+\pi^-)\) varies from \(\simeq 0.07\) at \(\cos \theta = 0\) to \(\simeq 0.04\) at \(\cos \theta = 0.6\), while the ratio: \((K^0\bar{K}^0)(\text{lead})/(\pi^0\pi^0)(\text{lead}) \simeq 1.3 \cdot (4/25) \simeq 0.21\). So, one obtains for the cross sections \(\sigma_{\text{lead}}(K^0K^0)/\sigma_{\text{lead}}(\pi^0\pi^0)\) integrated over \(0 \leq |\cos \theta| \leq 0.6\) for charged particles and over \(0 \leq \cos \theta \leq 0.6\) for neutral ones: \(\sigma_{\text{lead}}(K^0K^0)/\sigma_{\text{lead}}(\pi^0\pi^0) \simeq 0.005\).

It is seen that the leading contribution to \(\sigma_o(K^0K^0)\) is very small. This implies that it is not yet dominant at present energies \(W^2 < 16\text{ GeV}^2\). In other words, the amplitude \(A(\gamma\gamma \rightarrow K^0K^0) = a(s, \theta) + b(s, \theta)\) is dominated by the non-leading term \(b(s, \theta) \sim g(\theta)/s^2\), while the formally leading term \(a(s, \theta) \sim C_0f_{BC}(\theta)/s\) has so small coefficient \(C_0\) that \(|b(s, \theta)| > |a(s, \theta)|\) at, say, \(W^2 < 12\text{ GeV}^2\). Therefore, there has no meaning to compare the leading term prediction of BC (i.e. \(d\sigma(K^0K^0)/d\cos \theta \sim |a(s, \theta)|^2/s \sim |f_{BC}(\theta)|^2/W^6\) at \(s \rightarrow \infty\)) for the energy and angular dependence of \(d\sigma(K^0K^0)\) with the recent data from Belle [8]. Really, the only QCD prediction for \(6\text{ GeV}^2 < W^2 < 12\text{ GeV}^2\) is the energy dependence: \(d\sigma(K^0K^0)/d\cos \theta \sim |b(s, \theta)|^2/s \sim |g(\theta)|^2/W^{10}\), while the angular dependence \(|g(\theta)|^2\) and the absolute normalization are unknown. This energy dependence agrees with the recent results from Belle [8], see fig.4.

II. The handbag model predictions

The hand-bag model [9] (DKV in what follows) is a part of a general ideology which claims that present day energies are insufficient for the leading terms QCD to be the main ones. Instead, the soft nonperturbative contributions are supposed to dominate the amplitudes. The handbag model represents applications of this ideology to description of \(d\sigma(\gamma\gamma \rightarrow MM)\). It
Figure 2: The cross sections $d\sigma/d\cos\theta$, integrated over the c.m. angular region $|\cos\theta| < 0.6$, together with a $W^{-6}$ dependence line \cite{5}: a) $\gamma\gamma \to \pi^+\pi^-$, b) $\gamma\gamma \to K^+K^-$, c) the cross section ratio, the solid line is the result of the fit for the data above 3 GeV, the errors indicated by short ticks are statistical only.
Figure 3: a) the overall picture of the handbag contribution, b) the lowest order Feynman diagram for the light cone sum rule

assumes that the above described hard contributions really dominate at very high energies only, while the main contributions at present energies originate from the fig.3a diagram. Here, two photons interact with the same quark only, and these "active" $q\bar{q}$-quarks carry nearly the whole meson momenta, while the additional "passive" $q'\bar{q}'$ quarks are "wee partons" which are picked out from the vacuum by soft non-perturbative interactions. It was obtained by DKV that the angular dependence of amplitudes is $\sim 1/\sin^2 \theta$, while the energy dependence is not predicted and is described by some soft form factors $R_M(s)$, which are then fitted to the data. Because the "passive" quarks are picked out from the vacuum by soft forces, these soft form factors should be power suppressed at large $s$: $R_M(s) \leq 1/s^2$, in comparison with the leading meson form factors, $F_M(s) \sim 1/s$.

a) As for the flavor and charge dependence, the predictions of the handbag model look as follows:

$$\frac{d\sigma(\gamma\gamma \rightarrow \pi^0\pi^0)/d \cos \theta}{d\sigma(\gamma\gamma \rightarrow \pi^+\pi^-)/d \cos \theta} \equiv \frac{\langle \pi^0\pi^0 \rangle}{\langle \pi^+\pi^- \rangle} = \frac{1}{2} \left| \frac{e_u^2 R_{u\rightarrow u}(s) + e_d^2 R_{d\rightarrow d}(s)}{e_u^2 R_{u\rightarrow d}(s) + e_d^2 R_{d\rightarrow u}(s)} \right|^2 = \frac{1}{2}.$$  

Here, $e_u = 2/3$, $e_d = e_s = -1/3$ are the quark charges, while the form factor $R_{u\rightarrow d}(s)$ corresponds to the active $u$-quark and passive $d$-quark, etc. The isotopic symmetry implies that: $R_{u\rightarrow u} = R_{u\rightarrow d} = R_{d\rightarrow u} = R_{d\rightarrow d}$. Unfortunately, the cross section $d\sigma(\gamma\gamma \rightarrow \pi^0\pi^0)/d \cos \theta$ is not measured up to now.

As for the $SU(3)$-symmetry breaking effects, it seems clear that it is harder for soft interactions with the scale $\sim \Lambda_{QCD}$ to pick out from the vacuum a heavier $s$s-pair, than the light $u$- or $d$-pairs. So: $R_{u\rightarrow s}(s)/R_{s\rightarrow u}(s) = (1 - 2\Delta)$, $\Delta > 0$. (The same inequality $\Delta > 0$ follows from the fact that the

\[4\] The effect due to $m_s \neq 0$ of the hard quark propagating between two photons in fig.3 is small and can be neglected, see the Appendix.
heavier s-quark carries the larger mean fraction of the K-meson momentum, ⟨x_s⟩ > 0.5. Therefore, the handbag model predicts:

\[
\frac{(K_S K_S)}{(K^+ K^-)} = \frac{1}{2} \left| e_s^2 R_{s-d}(s) + e_d^2 R_{d-s}(s) \right|^2 = \frac{2}{25} \left| 1 - \Delta \right|^2 > 0.08.
\]

b) To obtain more definite predictions, in comparison with DKV [9], we have directly calculated the soft contributions of the handbag model to γγ → ππ, KK.

Below we present in a short form the explicit calculation of the handbag diagram contribution to the amplitude A(γγ → π+π−), the results for K+K− and KSKS are then obtained by trivial replacements.

To deal with such soft contributions we use the method of "the light cone sum rules" [10], [11]. One pion with the momentum p1, p22 = m2 ≈ 0 is replaced by the interpolating current Aμ = ωμγ5d with the momentum \( \overline{p}_1 \), \( \overline{p}_1^2 \) ≠ 0, while the second pion π(p2) stays intact, see fig.3b. The amplitude A is calculated which is:

\[
\mathcal{A} = (q_1 - q_2) \mu \langle \pi(p_2)|A_\mu(0)|\gamma(q_1), \gamma(q_2) \rangle.
\]

Its discontinuity in \( \overline{p}_1^2 \) has the form:

\[
(1/\pi) \Delta \mathcal{A} = f_\pi (q_1 - q_2) \overline{p}_1 \delta(\overline{p}_1^2) A(\gamma\gamma \rightarrow \pi^+\pi^-) \theta(\overline{p}_1^2 - s_\pi) \mathcal{A}^\text{(cont)},
\]

where \( (q_1 - q_2) \overline{p}_1 = (u - t)/2 \) and \( s_\pi \simeq 0.7 \text{ GeV}^2 \) is the pion duality interval. The sum rule is written for the quantity

\[
\left[ f_\pi (u - t)/2 \right]^{-1} \int_0^{s_\pi} d\overline{p}_1^2 \exp\left\{-\overline{p}_1^2/M^2\right\} \left(1/\pi\right) \Delta \mathcal{A} = A(\gamma\gamma \rightarrow \pi^+\pi^-).
\]

When calculating this quantity from the fig.3b diagrams, one proceeds first in the Euclidean region \( \overline{s} = -s = -(q_1 + q_2)^2 > 0 \), and the final result is then continued analytically into the physical region \( s > 0 \). The calculation here is simple and straightforward, we note only that the leading twist wave function \( \phi_\pi(x) \) is used for \( \pi(p_2) \). One obtains:

\[
A_{\text{handbag}}(\gamma\gamma \rightarrow \pi^+\pi^-) = 16\pi \alpha \left( e_s^2 R_{u-d}(s) + e_d^2 R_{d-u}(s) \right) (1 + e_\mu^{(1)} e_\mu^{(2)}),
\]

\[
R_{u-d}(s) = \frac{\omega}{2} \int_0^1 dz \phi_\pi(x_u = 1 - \omega z, x_d = \omega z) \exp\left\{-\frac{z}{(1 - \omega z)(1 + s_\pi/\overline{s})} \frac{s_\pi}{M_o^2}\right\},
\]

\footnote{This approach was used in [11] for the calculation of the B-meson decays into two baryons. The kinematics is Minkowskian in these decays, as in \( \gamma\gamma \rightarrow \pi^+\pi^- \).}
where $\omega = s\pi/(\bar{s} + s\pi)$. $\epsilon^{(1)}$ and $\epsilon^{(2)}$ are the polarization vectors of two photons, $M_\phi^2$ is the optimal value of $M^2$ (typically, $M_\phi^2 \approx m_\rho^2 \approx 0.6\text{GeV}^2$), and $R_{d\to u}(s)$ is obtained from $R_{u\to d}(s)$ by the replacement: $\phi_\pi(x_u = 1 - \omega z, x_d = 1 - \omega z, x_u = \omega z)$.

It is seen from eq.(7) that the handbag amplitude is independent of the scattering angle $\theta$, in contradiction with the DKV result \[9\] about $\sim 1/\sin^2 \theta$ dependence. As for the energy dependence, using $\omega z$, $x$ and $\pi$$^0$ scattering angle $\phi$ corresponding values for $\phi_\pi(x)$ for $\phi_\pi(x_u, x_d) = 0$ get:

\[
\sim \text{leading behavior}
\]

Similarly, for $\phi_\pi(x_u, x_d)$ with $\phi_\pi(x_u, x_d)$ at $x \ll 1$, where $C_\pi = \text{const}$, one has: $A_{\text{handbag}}(\gamma\gamma \to \pi^+\pi^-) \sim R(s) \sim (s\pi/s)^2$, as expected.

For comparison with eq.(4), let us define the effective “handbag form factor” $\Phi_{\pi}(s)$ for $\pi^+\pi^-$ as:

\[
\frac{s^3}{16\pi\alpha^2} \frac{d\sigma^{\text{bb}}(\gamma\gamma \to \pi^+\pi^-)}{d|\cos \theta|} \equiv |\Phi_{\pi}(s)|^2,
\]

Its numerical value depends strongly on the form of the amplitude $\phi_\pi(x)$, see eq.(7). For $\phi_\pi(x) = \phi_{\text{asy}}(x) = 6x(1-x)$, $|\Phi_{\pi}(s)|$ is: $0.029\text{GeV}^2$ at $s = 6\text{GeV}^2$, $0.016\text{GeV}^2$ at $s = 10\text{GeV}^2$, and $0.009\text{GeV}^2$ at $s = 16\text{GeV}^2$. The corresponding values for $\phi_{\pi}(\gamma \to K\pi)$ are respectively: $0.19\text{GeV}^2$, $0.09\text{GeV}^2$ and $0.05\text{GeV}^2$. It is seen that, even for $\phi_{\pi}(\gamma \to K\pi)$, $|\Phi_{\pi}(s)|$ is small in comparison with the experimental value: $|\Phi_{\pi}^{\text{eff}}(s, \theta)| \approx \text{const} = 0.50\text{GeV}^2$, see eq.(4), and, besides, decreases strongly with increasing energy. So, even at available energies, these soft form factors $R_i(s)$ “honestly” decay with the non-leading power, $R_i(s) \sim 1/s^2$, and do not imitate the leading behavior $\sim 1/s$.

Neglecting terms $O(M_K^2/s)$, one obtains for $A_{\text{handbag}}(\gamma\gamma \to K^+K^-)$ the same eq.(7), with $\phi_\pi(x_u, x_d) \to \phi_\pi(x_u, x_d) \exp\{M_K^2/M^2\}$ and $s\pi \to s\pi$. Using for $\phi_\pi(x_u, x_d)$ the wave function $\phi_\pi(\gamma \to K\pi)$ proposed in \[6\], one then obtains:

\[
\Delta = \frac{1}{2} \left( 1 - \frac{R_{u\to d}(s)}{R_{s\to u}(s)} \right) \approx 0.25.
\]

Besides, one can calculate also the $SU(3)$-symmetry breaking effects in $K^+K^-/(\pi^+\pi^-)$. For instance, with $M_\phi^2 \approx M_K^2 \approx 0.8\text{GeV}^2$, $s_K \approx 0.93\text{GeV}^2$, one obtains the prediction of the handbag model:

\[
\frac{(K^+K^-)_{hh}}{(\pi^+\pi^-)_{hh}} = \frac{R_{s\to u}(s)}{R_{d\to u}(s)} \left( 1 - \frac{8}{5}\Delta \right)^2 \approx 0.86^2 \approx 0.73.
\]
On the whole, the above described predictions of the handbag model for \((\pi^+\pi^-)\) and \((K^+K^-)\) disagree with the data \([8]\) both in the energy dependence and the angular dependence. Besides, even for the wide wave functions \(\phi_s^{(C\bar{Z})}(x, \mu_o)\) \([4]\) and \(\phi_K^{(C\bar{Z})}(x, \mu_o)\) \([6]\), the predictions of the handbag model for the absolute values of \(\sigma_o(\pi^+\pi^-)\) and \(\sigma_o(K^+K^-)\) are too small. For instance, one obtains from eqs.(7,8) that at \(s = 10\,\text{GeV}^2\) and \(\phi_\pi(x) = \phi_s^{(C\bar{Z})}(x, \mu_o)\), \(\sigma_o^{(hh)}(\pi^+\pi^-)\) is \(\simeq 40 - 50\) times smaller than the experimental value.

As for the handbag model predictions for \((K_SK_S)\), one has for the ratio in eq.(6) (see eq.(9)):

\[
\frac{(K_SK_S)^{hh}}{(K^+K^-)^{hh}} = \frac{2}{25} \left| \frac{1 - \Delta}{1 - \frac{3}{8}\Delta} \right|^2 \simeq 0.08 \left| 1.25 \right|^2 \simeq 0.12. \tag{11}
\]

\textbf{c)} The cross section \(d\sigma(K_SK_S)/d\cos\theta\) has been measured recently by the Belle collaboration \([8]\). The energy dependence at \(2.4\,\text{GeV} < W < 4.0\,\text{GeV}\) was found to be: \(\sigma_o(K_SK_S) \sim W^{-k}\), \(k = (9.93 \pm 0.44)\), see fig.4. This agrees with the qualitative QCD prediction, see above, that the formally leading (at sufficiently large \(s\)) term \(a(s, \theta) \sim C_o f_{BC}(\theta)/s\) in the amplitude \(A(\gamma\gamma \rightarrow K_SK_S) = a(s, \theta) + b(s, \theta)\), has a very small numerical coefficient \(C_o\), so that the non-leading term \(b(s, \theta) \sim g(\theta)/s^2\) is really larger at present energies. This is seen also from the absolute value of \(\sigma_o(K_SK_S)\) measured by Belle \([8]\), see fig.4. Even at the highest energy \(W \simeq 3.8\,\text{GeV}\), it is still above the value of \(\sigma_o^{(lead)}(K_SK_S)\), obtained by BC for the leading at \(s \rightarrow \infty\) term. The same can be seen, of course, from \(\sigma_o(K_SK_S)/\sigma_o(K^+K^-)\). While the value of \(\sigma_o^{(lead)}(K^+K^-)\) predicted by BC is in a reasonable agreement with the data at present energies \([4]\), the prediction of BC for the ratio of the leading at \(s \rightarrow \infty\) contributions: \(\sigma_o^{(lead)}(K_SK_S)/\sigma_o^{(lead)}(K^+K^-) \simeq 0.005\), is still below its values at present energies, see figs.(2b,4). In other words, for the formally leading term \(|a(s, \theta)|^2\) to be really dominant in \(d\sigma(K_SK_S)/d\cos\theta\), the energy \(W\) has to be increased. Only then one will see the behavior \(\sigma_o^{(lead)}(K_SK_S) \sim 1/W^6\).

Because the leading at large \(s\) term in \(d\sigma(K_SK_S)/d\cos\theta\) has so small coefficient, this process is the ideal place for the handbag model to be applicable. As it is, the handbag diagram calculated above contributes to the non-leading term \(b(s, \theta) = \text{const}/s^2\). This agrees with the data in the energy dependence, but predicts the flat angular dependence \(d\sigma(K_SK_S)/d\cos\theta \sim \text{const}/W^{10}\), while the data prefer \(d\sigma(K_SK_S)/d\cos\theta \sim 1/(W^{10}\sin^4\theta)\) \([8]\). As for the absolute value of \(\sigma_o^{(hh)}(K_SK_S)\), one obtains from the above that at
Figure 4: The measured energy dependence of $\sigma_o(K_S K_S)$ \[8\]. The solid line is $\sim W^{-k}$, $k \approx 10$.

$s = 10 \text{ GeV}^2$ and even for the wide $\phi_\pi(x) = \phi_\pi^{CZ}(x, \mu_o)$ it is $\approx 10$ times smaller than the experiment, and more than two orders smaller for $\phi_\pi(x) = \phi^{(asy)}(x)$.

So, even in this process, the handbag contribution does not dominate. \[6\]

The measured ratio $(K_S K_S)/(K^+ K^-)$ decreases strongly with increasing energy, and becomes smaller than the lower bound 0.080 in eq.(6) at $W > 2.8 \text{ GeV}$ \[8\], see figs.(2b, 4). This is also in contradiction with the handbag model predictions, see eq.(6) and eq.(11).\[7\]

Conclusions

Our conclusion is that the leading term QCD predictions are in reasonable agreement with the recent data from Belle, but only for the wide pion and kaon wave functions, like $\phi_\pi^{(CZ)}(x)$. At the same time, the handbag model contradicts these data.

\[6\] One has to remember that there are also other contributions to the non-leading term $b(s, \theta)$ with the same energy dependence $\sim 1/s^2$ (and unknown angular dependence and absolute normalization), in addition to the handbag one.

\[7\] The ratio $\sigma_o(K_S K_S)/\sigma_o(K^+ K^-)$ of measured cross sections \[5\], \[8\] is: $\approx 0.030$ at $W = 3.2 \text{ GeV}$, and $\approx 0.015$ at $W = 3.8 \text{ GeV}$, see figs.(2b, 4).
Acknowlegements

I am grateful to A.E. Bondar and S.I. Eidelman for useful discussions about experimental results.

Appendix

Besides the above described $SU(3)$-symmetry breaking effects originating from the K-meson wave function, there is an additional handbag contribution to $K^+K^-$ from $m_s \neq 0$ in the hard quark propagator in fig.3b. It seems at the first sight that, relative to $A_{hb}(\pi^+\pi^-)$, it is additionally suppressed by the factor $O(M_K^2/s)$ and can be neglected in comparison with the corrections $O(M_K^2/s_K)$ from the $SU(3)$-symmetry breaking effects in the K-meson wave function $\phi_K(x)$ in eq.(7). Really, it gives also $O(M_K^2/s_K)$ correction. The reason is that this contribution includes the twist-tree wave function $\phi_P(x)$, originating from $\langle K|\pi(z)\gamma_5s(0)|0 \rangle$. It behaves as $\phi_P(x) \sim 1$ at $x \ll 1$ (compare with $\phi_K(x) \sim x \sim (s_K/s)$ in eq.(7)), so that this correction is also $\sim (M_K^2/s_K)(s/s_K) \sim (M_K^2/s_K)$. One obtains for this additional contribution $\delta A_{handbag}$ :

$$\delta A_{handbag}(\gamma\gamma \rightarrow K^+K^-) = 16\pi\alpha e_s^2 \frac{2M_K^2}{s} \frac{e^{(1)}_\mu e^{(2)}_\mu}{\sin^2 \theta} I_P,$$

$$I_P = \omega \int^1_0 \frac{dz \phi_P(\omega z)}{(1-\omega z)^2} \exp \left\{ -\frac{z}{(1-\omega z)(1+s_K/s)} \frac{s_K}{M_o^2} \right\}. \tag{12}$$

It is seen from comparison of eq.(7) and eq.(12) that the helicity structures are different, so that these two contributions do not interfere in the cross section. As for the numerical value of $\delta A_{handbag}$, using $\phi_P(x) = 1$ and $e_s^2 = 1/9$, one obtains that $|\delta A_{handbag}|^2$ is very small (about 100 times smaller than even $|A_{hb}(\gamma\gamma \rightarrow \pi^+\pi^-)|^2$ from eq.(7) with $\phi_\pi(x) = \phi^{(asy)}(x)$), and can be neglected.
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