Recursive Multistage Estimator for Bearings only Passive Target Tracking in ESM EW Systems

A. Jawahar* and S. Koteswara Rao

School of Electrical Sciences, KLEF, KL University, Vaddeswaram - 522 502, Guntur, Andhra Pradesh, India; jawaharannabattula@gmail.com, rao.sk9@gmail.com

Keywords: Bearing only Tracking, Maximum Likelihood Estimator, Multistage Estimator, Passive Target Tracking, Pseudo Linear Estimator

Abstract

Background/Objectives: A recursive multistage estimator for underwater target applications in Electronic Support Measures (ESM)/Electronic Warfare (EW) systems is presented in this paper. Methods/Statistical Analysis: The technique of recursive multistage estimator involves Maximum Likelihood Estimator (MLE) and Pseudo Linear Estimator (PLE), which is converted to sequential processing to suit real time applications. MLE is chosen for the purpose of estimation of target motion parameters as it follows Cramer Rao lower bound. Monte Carlo simulation is carried out and the efficiency of the algorithm is tested against two typical scenarios. Findings: From the results, MLE with PLE algorithm is recommended as it follows Cramer Rao lower bound. Application/Improvement: The algorithm is found to be accurate for passive target applications and can be used in the field of Navy.

1. Introduction

Passive mode of surveillance is gaining importance in maritime environment compared to active mode. In typical scenarios, it is essential for the observer not to disclose its location and it is possible by using passive sensors only. The Electronic Support Measures (ESM) provide instantaneous bearing of emitter. Let the target velocity not is varying during the analysis. The observer processes the corrupt bearing measurements to find the target motion parameters. Thus, the process is nonlinear.

The process is observable when observer maneuvers. Maximum Likelihood Estimator (MLE) is used in sequential processing. This sequential processing represents all covariance matrix elements recursively by calculating only incremental measurements. MLE requires some initial estimate as bearing measurements only are available. The target can be ship or aircraft moving at a distance of 15 Km to 250 Km and at a speed of 15 knots to 600 knots.

For bearings only target tracking, Branko Ristic, Sanjeev Arulapalam and Niel Gordon have suggested in their book ‘Beyond the Kalman Filter’ to use number of filters in parallel with different initializations to cover the range and speed. In this paper, the author would like to use Pseudo Linear Estimator’s (PLE) to generate initial estimates for MLE. PLE is developed from Least square estimator is recursive and no initialization. As PLE generates bias in the estimates, it is used for only initialization of MLE. The accuracy of this algorithm is improved by adaptively calculating the measurement variance and then simultaneously using this with the measurement. This work generates accurate and fast convergent results and provides good insight into bearing only tracking problem. MLE is tested using Monte-Carlo simulation for few scenarios. The results of two scenarios considered for analysis are presented.

2. Mathematical Modeling

The symbols used in mathematical modeling have their usual meaning. Let $X_s(k)$ be a target state vector given by

$$X_s(k) = \begin{bmatrix} \dot{x}(k) & \dot{y}(k) & x(k) & y(k) \end{bmatrix}^T$$ (1)
The bearing \( B_a(k) \) is given by
\[
B_a(k) = \tan^{-1}\left( \frac{r_x(k)}{r_y(k)} \right)
\] (2)
The measured bearing, \( B_m(k) \) is given by
\[
B_m(k) = B_a(k) + \gamma(k)
\] (3)
where \( \gamma(k) \) is a zero mean Gaussian noise. And
\[
Z = \begin{bmatrix} B_m(1) & B_m(2) & \ldots & B_m(k) \end{bmatrix}^T
\] (4)
The likelihood function is given by \[1\]
\[
P(W|Z,h) \propto \exp\left[ -0.5(Z-h(X_i))^T W^{-1}(Z-h(X_i)) \right]
\] (5)
In the MLE approach
\[
\frac{\partial}{\partial X} \ln P_{\mathcal{Z}X} = 0
\] (6)
From equations (5) and (6),
\[
\frac{\partial}{\partial \gamma} \sum_{i=1}^{n} (B(i) - B_m(i)) \frac{\partial B(i)}{\partial (X_i)} = 0
\] (8)
\[
B_m - B = \frac{G - F}{r} = \frac{E}{r} \quad \text{(from Figure 1)}
\] (9)
\[
= \frac{r_y \sin B_m - r_x \cos B_m}{r}
\]
\[
= \left( (y + t\gamma - y_0) \sin B_m - (x + t\gamma - x_0) \cos B_m \right) / r
\] (10)
Also we have
\[
\frac{\partial B}{\partial (X_i)} = \frac{1}{r^2} \left[ \frac{r_y \partial r_x}{\partial X_i} - \frac{r_x \partial r_y}{\partial X_i} \right]
\] (11)
From (8), (10) and (11), we have
\[
\sum E \left[ \frac{r_y \partial r_x}{\partial X_i} - \frac{r_x \partial r_y}{\partial X_i} \right] = 0
\] (12)
After obtaining the first bearing, equation (12) can be written as
\[
\frac{E_1}{r^3(1)} \frac{r_y(1) \partial r_x(1)}{\partial \gamma} = 0
\] (13)
Where
\[
C(1,1) = k^2 t^2 \cos B_m(1) (y - k t y_0) + (1 - k)^2 t^2 \cos B_m(2) (y + (1 - k) t y_0) + ...
\]
\[
C(1,2) = -k^2 t^2 \sin B_m(1) (y - k t y_0) + (1 - k)^2 t^2 \sin B_m(2) (y + (1 - k) t y_0) + ...
\]
\[
C(1,3) = -k t \cos B_m(1) (y - k t y_0) + (1 - k) t \cos B_m(2) (y + (1 - k) t y_0) + ...
\]
\[
C(1,4) = k t \sin B_m(1) (y - k t y_0) + (1 - k) t \sin B_m(2) (y + (1 - k) t y_0) + ...
\]
\[
C(2,1) = -k^2 t^2 \cos B_m(1) (x - k t x_0) + (1 - k)^2 t^2 \cos B_m(2) (x + (1 - k) t x_0) + ...
\]
\[
C(2,2) = k^2 t^2 \sin B_m(1) (x - k t x_0) + (1 - k)^2 t^2 \sin B_m(2) (x + (1 - k) t x_0) + ...
\]
\[
C(2,3) = k t \cos B_m(1) (x - k t x_0) + (1 - k) t \cos B_m(2) (x + (1 - k) t x_0) + ...
\]
\[
C(2,4) = -k t \sin B_m(1) (x - k t x_0) + (1 - k) t \sin B_m(2) (x + (1 - k) t x_0) + ...
\]
\[
C(3,1) = C(1,3)
\]
\[
C(3,2) = C(1,4)
\]
\[
C(3,3) = \cos B_m(1) (y - k t y_0) + \cos B_m(2) (y + (1 - k) t y_0) + ...
\]
\[
C(3,4) = -\sin B_m(1) (y - k t y_0) - \sin B_m(2) (y + (1 - k) t y_0) + ...
\]
\[
C(4,1) = C(2,3)
\]
\[
C(4,2) = C(2,4)
\]
\[
C(4,3) = -\cos B_m(1) (x - k t x_0) - \cos B_m(2) (x + (1 - k) t x_0) + ...
\]
\[
C(4,4) = \sin B_m(1) (x - k t x_0) + \sin B_m(2) (x + (1 - k) t x_0) + ...
\]

2.1 Sequential Processing

Initially, the noise variance is assumed to be constant. Calculation of variance of 20 second bearing measurement process is elaborately discussed. Now RECURSIVE SUMS (CSUMS, DSUMS etc.) are used to convert it into sequential processing. The impact of all the bearing measurements on C and D matrices is in the form of RECURSIVE SUMS. For a new bearing measurement, only its corresponding calculations are done and subsequently updated to the RECURSIVE SUMS.

For example, after the arrival of first bearing measurement
\[
C(4,4) = \cos^2 B_m(1)/\sigma^2(1)
\]
Let
\[
KSUMS[1] = \cos^2 B_m(1)/\sigma^2(1)
\]
Then
\[
C(4,4) = KSUMS[1]
\]

The SUMS are updated after second measurement as
\[
ESUMS[1] = \left( \cos^2 B_m(1)/\sigma^2(1) \right) + \text{Previous ESUMS}[1]
\]

In the above fashion C and D matrices can be obtained.

The error in bearing is around 3–5 degree r.m.s. Let the measurement be available at 1 Hz rate for a period of 12 minutes. It is not possible to track the target using ESM measurements with the above mentioned order of noise. Hence, to reduce the errors, the measurements are averaged over a fixed period (i.e. 20 seconds). Similarly, the variance of measurements at one second for 20 seconds is used along with the averaged measurement in the filter equations. This leads to the auto editing of the
measurements that means if the measurement is good (i.e., variance of the error is less), more weightage to that measurement is given and vice versa. The solution is updated every 20 Seconds and extrapolated at the rate of one second. The stable PLE solution after first observer maneuver is used as MLE initial estimates. The state estimate is used to track the target.7,8.

4. Results

Simulation platform is synthesized to create target, observer and measurements. The position and velocity of target and observer are updated every second. Gaussian noise is added to these samples. The two typical scenarios with ship and aircraft as targets shown in Table 1 are considered. The preprocessing of bearing measurements over 20 seconds to minimize the variance in the errors. Figure 2 shows the manner of observer maneuver.

Let the observer performs ‘S’ maneuver and measurements are available continuously. The results implied that the target is observable only after the observer maneuver. In the Figures 3 and 4, R-error, C-error and S-error denote error in the range, course and speed estimates for two scenarios respectively. The target is tracked accurately from seventh minute.

Table 1. Target Scenarios

| Item Description          | Ship                     | Aircraft                     |
|---------------------------|--------------------------|------------------------------|
|                           | Scenario 1               | Scenario 2                   |
| Initial Range             | 18000 Mtrs               | 225000 Mtrs                  |
| Initial Bearing           | 200°                     | 130°                         |
| Target Speed              | 18 Kts                   | 600 Kts                      |
| Target Course             | 45°                      | 350°                         |
| Ownship Speed             | 20 Kts                   | 20 Kts                       |
| Error in Bearing (rms)    | 5°                       | 3°                           |

Figure 2. Observer maneuver.

Figure 3. Error in estimates for Scenario-1.
MLE for passive target tracking is presented in this paper. The concept of recursive SUMS is explored which is modified upon arrival of new bearing measurement which is contrast to conventional batch processing. The state estimate is used to calculate the target motion parameters at any instant and the SUMS are updated to obtain accurate solution at the next instant. MLE is recommended as an efficient algorithm for target tracking.

5. Conclusion

Figure 4. Error in estimates for Scenario-2.

MLE for passive target tracking is presented in this paper. The concept of recursive SUMS is explored which is modified upon arrival of new bearing measurement which is contrast to conventional batch processing. The state estimate is used to calculate the target motion parameters at any instant and the SUMS are updated to obtain accurate solution at the next instant. MLE is recommended as an efficient algorithm for target tracking.

6. References

1. Nardone SC, Lindgren AG, Gong KF. Fundamental properties and performance of conventional bearings only target motion analysis. IEEE Trans Automatic Control. 1984 Sep; 29(9):775–87.

2. Ristic B, Arulapalam S, Gordon N. Beyond the Kalman Filter-Particle filters for tracking applications. USA: Artech House; 2004.

3. Rao SK. Pseudo linear estimator for bearings only passive target tracking. IEEE Proc, Radar, Sonar, Navigation. 2001 Feb; 148(1):16–22.

4. Aidala VJ, Hammel SE. Utilization of modified polar coordinates for bearings only tracking. IEEE Trans Automatic Control. 1983 Mar; 28(3):283–94.

5. Song TL, Speyer JL. A stochastic analysis of a modified gain extended Kalman filter with applications to estimation with bearing only measurements. IEEE Trans Automatic Control. 1985 Oct; 30(10):940–9.

6. Grossman W. Bearings only tracking a hybrid coordinate system approach. J Guidance. 1994 May-Jun; 17(3):451–7.

7. Rao SK. Modified gain extended Kalman filter with application to bearings-only passive maneuvering target tracking. IET Proc, Radar, Sonar, Navigation. 2005 Sep; 152(4):239–44.

8. Safarinejadian B, Mozaffari M. A new Kalman filter based state estimation method for multi-input multi-output unit time-delay systems. Indian Journal of Science and Technology. 2013 Mar; 6(3):4205–12.