Optimal Design of Brake Disc Structures for Brake Squeal Suppression

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Abstract. Aiming at the brake squeal problem of automobile disc brakes, an optimization design method of brake disc structure based on the weighted brake squeal tendency coefficient is proposed. This method is based on the brake squeal complex modal finite element model of a certain disc brake. Based on the validity of the model verified by the bench test, the single-sided disc surface height of the brake disc, the height of the radiating rib, the elastic modulus and the disc are selected. The four key structural parameters of the cap height are used as design variables. Taking the weighted braking squeal tendency coefficient proposed in this paper as the optimization target, the response surface method and the central composite test design are combined to construct a weighted braking squeal tendency coefficient response surface model, and use multi-island genetic algorithm to optimize the model. The results show that the optimization design method proposed in this paper can greatly improve the optimization efficiency while effectively reducing the screaming tendency of the disc brake in the full frequency band, so as to achieve the purpose of improving the NVH performance of the disc brake and improving the comfort of the car.

Keywords. Brake disc, brake squeal, complex eigenvalue method, response surface method, optimal design.

1. Introduction

Brake squeal is a high frequency noise with a frequency of 1-16k Hz generated by the brake during braking [1]. Brake squeal not only has a serious impact on the performance of the vehicle such as safety and comfort, but also causes acoustic pollution to the vehicle environment and the surrounding environment [2]. With the rapid development of new energy vehicles, after the vibration noise of traditional internal combustion engines is eliminated, the brake squeal noise generated by the working vibration of the brake system will gradually come to the fore. Therefore, the brake squeal problem is one of the major problems that need to be solved in the current automotive industry [3].

From the current research work, domestic and international researchers have done a lot of research on the brake squeal problem. Haruhisa Baba [4] found that the possibility of brake squeal occurring at specific unstable frequencies could be reduced by breaking the symmetry of the brake disc caps. In their study, Guan Di Hua [5] found and verified that the generation of brake squeal could be suppressed by changing the parameters of key components of disc brakes. Zhao Duo et al. [6] found the key component parameters affecting brake squeal based on sensitivity analysis and optimised the disc structure from the perspective of changing the depth of the v-groove of the disc, which has certain significance for the study of the optimisation method of brake squeal.
Although a great deal of research has been done on the issue of brake squeal both at home and abroad, most of their research has been conducted with a relatively single variable and most of them have used the squeal tendency of a particular instability frequency to evaluate the effect of brake squeal suppression, which does not reflect the overall noise reduction effect over the full range of brake squeal frequencies. This is because when modifications are made to the brake structure, the unstable frequencies may be shifted and new unstable frequencies with a greater tendency to squeal may be created. Most of their final research results only identified an improved solution or combination of design variables for optimal squeal performance, and failed to achieve an optimal design for brake squeal for all design parameters or key design parameters of the disc brake system or key components. Therefore, based on the above background, this paper proposes a brake disc structure optimisation design method for brake squeal suppression. The method is based on a complex modal finite element model of a disc brake with brake squeal, and four key structural parameters, namely the height of the unilateral disc surface of the brake disc, the height of the heat sink rib, the modulus of elasticity and the height of the disc cap, are selected as design variables in this paper, and the weighted brake squeal tendency coefficient proposed in this paper is used as the optimisation objective. Based on the response surface model, a multi-island genetic algorithm is used to optimise the design of the brake disc structure. The research in this paper is of great importance in improving the brake squeal performance of disc brakes by optimising the key structural parameters of the brake disc.

2. Disc Brake Complex Modal Theory Analysis

Due to the nonlinear, nonconstant and transient nature of the braking scream, the conventional linear damping system cannot be decoupled by the real modal matrix and can only be obtained by applying the superposition method through the real modal theory, which is known as the complex modal solution [7].

The kinetic equations of the braking system are:

\[ M \ddot{x} + C \dot{x} + (K + \mu K_f)x = 0 \]  \hspace{1cm} (1)

where: \( M \) is the mass matrix; \( C \) is the damping matrix; \( K \) is the stiffness matrix of the frictionless system; \( K_f \) is the stiffness matrix of the system under frictional contact conditions; \( \mu \) is the friction coefficient; \( x \) is the system vibration displacement vector.

Since the friction between the brake disc blocks makes the stiffness matrix asymmetric, the solution of the above equation has complex characteristic roots [8]. The characteristic equation of equation (1) is given by:

\[ \lambda^2 M + \lambda C + (K + \mu K_f)\psi = 0 \]  \hspace{1cm} (2)

where: \( \lambda \) and \( \psi \) are the eigenvalues and their corresponding eigenvectors. \( \lambda = a + bi \), \( a \) is the real part of the eigenvalue and \( b \) is the imaginary part of the eigenvalue. If \( a \) greater than 0, the brake system shows a negative damping effect, at which point the brake system is a divergent unstable system; therefore the positive or negative real part can be used to characterize the stability of the brake system and thus predict the tendency of the brake system to generate squeal noise [9].

3. Disc Brake Squeal Complex Modal Finite Element Model

3.1. Brake Finite Element Model

As shown in table 1, a disc brake is used as the research object of this paper, and a simplified finite element model of the disc brake is established in Ansys Workbench, which consists of three parts: the ventilated brake disc, the friction lining and the brake backing plate. A hexahedral dominant meshing method is used for meshing, and the working area where the brake disc and the brake block are in contact is re-meshed using the face mapping method to exclude the influence of non-hexahedral cells.
on the complex modal simulation analysis, so that a pure hexahedral mesh can be generated for this area.

### Table 1. Brake finite element model.

| Brake finite element model | Schematic diagram of constraint, boundary and load |
|---------------------------|---------------------------------------------------|
| ![Brake finite element model](image1.png) | ![Schematic diagram](image2.png) |

#### 3.2. Material Property Settings

Material properties and contact relationships are detailed in tables 2 and 3.

### Table 2. Material property values taken.

| Part Name       | Material          | Density/(kg.m-3) | Modulus of elasticity/MPa | Poisson's ratio |
|-----------------|-------------------|------------------|---------------------------|-----------------|
| Brake discs     | gray cast iron    | 7 190            | 122 000                   | 0.230           |
| Brake block backplate | 45# steel     | 7 800            | 197 000                   | 0.300           |
| Friction liner  | NAO               | 2 615            | 8 600                     | 0.330           |

### Table 3. Contact relationships.

| Serial number | Parts in contact with each other | Contact relationship | Interaction in Workbench |
|---------------|----------------------------------|----------------------|--------------------------|
| 1             | Brake discs - friction linings    | Frictional contact   | Frictional Bond          |
| 2             | Friction Lining - Brake Backing Plate | Binding contact   |                          |

#### 3.3. Constraints, Boundary and Load Condition Settings

As shown in table 1, only the freedom of rotation of the brake disc around the central axis is retained and the friction lining and the brake block backing plate are bonded together by high temperature pressing, i.e. the brake block can be considered as one piece. The direction of the centre axis of the brake disc is defined as the Z direction, and the two degrees of freedom of displacement of the brake backing plate in the X and Y directions are constrained. The brake block is pressed against the brake disc at a given contact pressure of 1 MPa and the disc is rotated at a constant speed of 2 rad/s. The mutual contact surfaces of the two parts are directly unlubricated and the coefficient of friction is 0.44.

#### 3.4. Results of Complex Modal Calculations

The results of the complex modal analysis of the brake system in the initial state are shown in figure 1. The brake system produced four unstable frequencies, which are 4764.3 Hz, 6147.7 Hz, 7496.5 Hz, and 10143 Hz, respectively, and it is calculated that the real part value of the unstable frequency 10143 Hz is the largest and the complex modal damping ratio is the largest at 6147.7 Hz, therefore, at these two unstable frequencies where brake squeal is most likely to occur.

#### 3.5. Bench Test Verification

For the studied disc brake, the brake squeal bench test was carried out under four different operating conditions, and the test results shown in figure 2 were obtained. The disc brake mainly has four frequency bands of squeal instability frequency, at frequencies of 4700 Hz, 6100 Hz, 7500 Hz, and near 10100 Hz, respectively, where the noise near 6100 Hz and near 10100 Hz The highest number of occurrences were 80 and 88, respectively, and the highest sound pressure level was reached at 103 dB near 10100 Hz.
Figure 1. Calculated results of the complex mode in the initial state.

Figure 2. Graph of brake squeal noise bench test results.

The braking system instability frequencies calculated by the complex modal simulation were compared with the screaming frequencies of the bench test, and the results are shown in table 4. As can be seen from table 4, the four screaming frequencies in the bench test were accurately predicted with an error of no more than 2%, and the possibility of the brake screaming occurring near the instability frequencies of 6147.7 Hz and 10143 Hz predicted by the complex modal simulation was the highest. This is consistent with the actual bench test results in which the occurrence of noise in these two frequency bands is greater, which indicates that the accuracy of the established complex modal finite element model meets the requirements and is accurate and valid.

| Result          | Unstable frequency (Hz) |
|-----------------|-------------------------|
| Bench test      | 4700 6100 7500 10100    |
| Simulation      | 4764.3 6147.7 7496.5 10143 |
| Inaccuracies    | 1.4% 0.8% 0.05% 0.4%    |

### Table 4. Braking squeal complex modal simulation results.

4. Optimal Design of Brake Disc Structure for Brake Screaming

4.1. Selection of Design Variables

The brake disc is in direct contact with the friction lining during braking, generating friction and is one of the key components of a disc brake. In order to control the variables, considering that changes in the radius of the brake disc will cause corresponding changes in the structural dimensions of the brake block, this paper selects the height of the unilateral disc surface of the brake disc, the height of the heat dissipation rib, the modulus of elasticity and the height of the disc cap as the design variables for the optimisation design of this paper, and the initial values of each design variable are detailed in table 5.

| Design variables | Description                           | Initial value |
|------------------|---------------------------------------|---------------|
| X1               | Brake disc single side disc height (mm) | 8             |
| X2               | Heat sink rib height (mm)              | 14            |
| X3               | Modulus of elasticity (10^10pa)        | 12.2          |
| X4               | Disc cap height (mm)                   | 16            |

4.2. Selection of Brake Disc Optimization Objectives Based on Weighted Brake Squeal Propensity Coefficient

There are three main indicators available for evaluating brake squeal in disc brakes: 1. the complex modal damping ratio, 2. the maximum value of the real part of the complex eigenvalue of the system,
and 3. the minimum value of a specific modal frequency isolation range. All three of these are based on specific brake squeal instability frequencies, which are not fully and reliably represented in the entire frequency range of 1-16kHz for focused control of brake squeal instability frequencies in disc brakes, and for global control of the frequency range in which brake squeal may occur. The weighted brake squeal tendency coefficient (WBSTC) is defined as follows.

\[ WBSTC = \frac{A_m}{B_m} \times W_1 + \frac{A_n}{B_n} \times W_2 + (\sum_{i=1}^{j} \frac{A_i}{B_i} - \frac{A_m}{B_m} - \frac{A_n}{B_n}) \times W_3 \]  

(3)

where, \( A_m, B_m \) are the maximum value of the real part of the unstable frequency of the brake system and its corresponding imaginary part, \( A_n, B_n \) are the values of the real part and imaginary part of the unstable frequency corresponding to the maximum ratio of the real part to the imaginary part of the unstable frequency of the brake system, \( j \) is the number of unstable frequencies, \( i \) is a positive real number, \( i = 1, 2, 3, ..., j \), \( A_i \) is the real part of the complex eigenvalue of the brake system, is the imaginary part of the \( B_i \) complex eigenvalue of the brake system and \( A_i > 0, B_i > 0, W_1, W_2, W_3 \) is the weighting factor, the values of the three factors are 0.45, 0.45, 0.1 respectively.

4.3. Response Surface Modeling and Error Analysis

In the analysis of brake squeal complex mode, the weighted brake squeal tendency coefficient is an implicit function of the design variables, and there is no explicit mathematical function expression between them. Therefore, this paper introduces the response surface method into the optimal design of brake structure for brake squeal, and obtains the function relationship between the weighted brake squeal tendency coefficient and each design variable, which effectively solves this problem [10].

Seventeen sample points were obtained by applying the central composite test design method in Design Expert software, and the sample points were substituted into the disc brake squeal complex modal finite element model built in the previous section, and the weighted brake squeal tendency coefficients were solved and calculated for each sample point separately, and the specific calculation results are detailed in table 6.

| X1 | X2 | X3  | X4  | WBSTC |
|----|----|-----|-----|-------|
| 1  | 8  | 16.3545 | 12.2 | 16 | 0.00736507 |
| 2  | 8  | 14 | 14.2518 | 16 | 0.0149698 |
| 3  | 8  | 14 | 12.2 | 13.3091 | 0.00883441 |
| 4  | 7.2 | 12.6 | 13.42 | 14.4 | 0.0121498 |
| 5  | 7.2 | 12.6 | 10.98 | 14.4 | 0.0137407 |
| 6  | 8  | 14 | 10.1482 | 16 | 0.0118651 |
| 7  | 9.34543 | 14 | 12.2 | 16 | 0.00979336 |
| 8  | 8  | 14 | 12.2 | 16 | 0.00891734 |
| 9  | 8.8 | 12.6 | 10.98 | 17.6 | 0.00725793 |
| 10 | 8.8 | 15.4 | 10.98 | 14.4 | 0.00677264 |
| 11 | 8  | 11.6455 | 12.2 | 16 | 0.0152627 |
| 12 | 7.2 | 15.4 | 13.42 | 17.6 | 0.00913756 |
| 13 | 8  | 14 | 12.2 | 18.6909 | 0.00735644 |
| 14 | 8.8 | 12.6 | 13.42 | 17.6 | 0.0119282 |
| 15 | 7.2 | 15.4 | 10.98 | 17.6 | 0.0105847 |
| 16 | 6.65457 | 14 | 12.2 | 16 | 0.0126725 |
| 17 | 8.8 | 15.4 | 13.42 | 14.4 | 0.0145468 |

Considering that a total of four design variables were selected in this paper, all quadratic terms, primary terms and interactions between the design variables were considered when establishing the
response surface, and the backward method was used to ignore design variables and interactions with insignificant effects in the quadratic regression analysis\textsuperscript{[11]}, and the final approximate function obtained is:

\[
WBSTC = 0.01 - 6.649 \times 10^4 \cdot x(1) - 2.65 \times 10^3 \cdot x(2) + 1.422 \times 10^3 \cdot x(3) - 2.732 \times 10^4 \cdot x(4) + 1.146 \times 10^3 \cdot x(1) \cdot x(2) + 1.663 \times 10^3 \cdot x(1) \cdot x(3) - 2.217 \times 10^4 \cdot x(1) \cdot x(4) + 8.092 \times 10^4 \cdot x(2) \cdot x(4) - 9.387 \times 10^4 \cdot x(3) \cdot x(4) + 1.881 \times 10^4 \cdot x(1) \cdot x(1) + 3.99 \times 10^4 \cdot x(2) \cdot x(2) + 8.616 \times 10^4 \cdot x(3) \cdot x(3) - 4.485 \times 10^4 \cdot x(4) \cdot x(4)
\]

After the response surface model was obtained, its accuracy needed to be tested to ensure the accuracy and validity of the fitted model. The results of the test are shown in Table 7. The model has a high F value as well as a very low P value, indicating that the model has good significance. The correlation coefficient reflects the degree of approximation of the regression model, and the closer its value is to 1, the better the correlation between the test and predicted values. The correlation coefficient for this model was 0.9838, indicating a high degree of agreement between the test and predicted values.

| Response surface model | Correlation coefficient | F-value | P-value |
|------------------------|-------------------------|---------|---------|
| WBSTC                  | 0.9838                  | 14.02   | 0.0257  |

4.4. Optimal Design
In order to improve the brake squeal performance of disc brakes, this paper takes the weighted brake squeal tendency coefficient reduction to the minimum as the optimization objective and establishes the optimization mathematical model shown in the following equation.

\[
\min f(X) = WBSTC \\
\text{s.t. } WBSTC > 0 \\
X_{cl} \leq X_c \leq X_{cu} \\
c = 1, 2, 3, 4
\]

where, \(X\) is the design variable, \(WBSTC\) is the weighted brake squeal propensity coefficient, \(X_{cu}\), \(X_{cl}\) is the lower and upper limits of the design variable, respectively, and the range of each design variable is shown in Table 8.

| Design variables | Initial value | Lower limit | Upper bound |
|------------------|--------------|-------------|-------------|
| X1               | 8            | 7.2         | 8.8         |
| X2               | 14           | 12.6        | 15.4        |
| X3               | 12.2         | 10.9        | 13.5        |
| X4               | 16           | 14.4        | 17.6        |

The optimization model is built in Isight software, the initial values and the range of values of each design variable are set according to Table 8, and the optimization calculation is carried out using a multi-island genetic algorithm, and the system is solved and calculated 1000 times to finally obtain a set of optimized solutions, and the optimization results are shown in Table 9, and it can be seen from the optimization results that the weighted brake squeal tendency coefficient is reduced from 0.009566 to 0.004607 after optimization. The squeal performance of the system is improved substantially.
Table 9. Contrast values of design variables and objective functions before and after optimization.

| Design variables | Initial value | Optimization value |
|------------------|---------------|--------------------|
| X1               | 8             | 7.98               |
| X2               | 14            | 12.71              |
| X3               | 12.2          | 13.00              |
| X4               | 16            | 16.278             |
| WBSTC            | 0.009566      | 0.004607           |

The time required to optimize the design by applying the response surface model consists of two main times: one is the time required to build the response surface model, which requires 17 times of complex modal analysis, each time requires 2h, a total of 34h; the other is the time required for 1000 iterations of the optimization search, which takes 0.2h, a total of 34.2h for one design; the time required to optimize the design by directly applying the finite element model is mainly the time for optimisation iteration, which requires 1000 times of complex modal analysis, takes a total of 1500 h. Comparing the time of the two optimisation designs, the efficiency of the response surface model is improved by \((1500-34.2)/1000 = 97.7\%\). Therefore, the method proposed in this paper can greatly improve the efficiency of the solution.

In order to check the correctness of the optimisation results, the braking squeal complex modal finite element model was modified according to the values of the optimised design variables, and the weighted braking squeal tendency coefficient of 0.004538 was obtained through the complex modal analysis, and the error of the response surface model calculation was only 1.52\%, indicating that the response surface model was valid. The modal vibration pattern and the distribution of the complex eigenvalue of the unstable frequency of the system before and after optimisation were compared, and the results are shown in Table 10 and Figure 3.

Table 10. Comparison of unstable frequency mode vibration patterns of the system before and after optimization.

| Pre-optimization | Unstable frequency (Hz) | Modal vibration pattern | Post-optimization | Unstable frequency (Hz) | Modal vibration pattern |
|------------------|-------------------------|-------------------------|------------------|-------------------------|-------------------------|
|                  |                         | 4764.3                  | 6147.7           | 7496.5                  | 10143                   |
|                  |                         |                         |                  |                         |                         |
|                  | Modal vibration pattern | /                       |                  |                         |                         |
|                  | Unstable frequency (Hz) | 1930.3                  | 4703.9           | 6097.5                  | /                       |
|                  |                         |                         |                  |                         |                         |
|                  | Modal vibration pattern |                         |                  |                         |                         |
Figure 3. Comparison results of system instability frequency before and after optimization.

From table 10 and figure 3, we can see that: 1) Before optimization, the brake system generated four unstable frequencies, namely 4764.3Hz, 6147.7Hz, 7496.5Hz and 10143Hz, of which the unstable frequency 10143Hz has the largest real part value and 6147.7Hz has the largest complex modal damping ratio. Hz and 10143Hz are suppressed, the real part of the complex eigenvalue of the unstable frequency is reduced by half, the unstable frequency 7496.5Hz disappears after the optimized design, and the new unstable frequency 1930.3Hz appears, but the real part of its complex eigenvalue is very small; 2) The results of the complex modal calculation show that the weighted braking squeal tendency coefficient of the optimized system is reduced from 0.009566 to 0.004538, a reduction of 52.6% compared to the pre-optimisation period, indicating that the squeal tendency of the brake system is greatly reduced and the NVH performance is significantly improved after the optimisation design.

5. Conclusions and Outlook
A disc brake squeal complex modal finite element model was established in Workbench, and on the basis of bench tests to verify the validity of the model, the central composite test design and the response surface method were combined to carry out an optimised design of the ventilated brake disc structure for brake squeal, establishing a response surface with the key structural parameters of the brake disc as design variables and the weighted brake squeal tendency coefficient as the optimisation target. The response surface model with the key structural parameters of the brake disc as the design variables and the weighted brake squeal tendency coefficient as the optimization target was established, which greatly simplified the calculation workload and improved the calculation efficiency by 97.7%. The optimisation results show that the weighted brake squeal tendency coefficient is reduced from 0.009566 to 0.004538 after optimisation, a reduction of 52.6%, indicating that by optimising the design with the weighted brake squeal tendency coefficient as the optimisation target, the disc brake can be effectively controlled at the most likely unstable frequency of brake squeal and the full range of possible frequencies, significantly improving the brake squeal performance over the full frequency range. The global control significantly improves the brake squeal performance of the disc brake in the full frequency range.

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