**Low x and diffraction: theory**

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**Abstract.**  
Some outstanding issues in high energy scattering are discussed. Particular emphasis is placed on recent developments concerning the next-to-leading log corrections to the BFKL equation.

1. Introduction  

The working group was concerned with high-energy or small-$x$ phenomena in QCD. In deep inelastic scattering at moderate $x = Q^2/W^2$ it is customary to think in the Breit Frame, in which the proton is moving very fast. For large $Q^2$, the short distance partonic valence structure of the proton is being probed. Striking a single valence parton very violently tends to break up the proton, hence the adjective deep-inelastic applied to this scattering. At small $x$ (less than about $10^{-3}$) the struck partons no longer carry significant amounts of the protons momentum and one may think of the photon as probing the radiation field of gluons and $q\bar{q}$-pairs surrounding the valence partons. One now has the possibility of striking a “wee lump” of this field without disturbing the integrity of the bound state of the proton. This is diffraction, and the HERA experiments have observed that indeed this possibility occurs about 10% of the time even for $Q^2$ as large as 800 GeV$^2$ (for a review of the experimental situation in diffraction see [1]).

At small-$x$ it becomes useful to switch frames to the proton’s rest frame. The photon fluctuates into a partonic system (a $q\bar{q}$ dipole to lowest order in $\alpha_s$) a large distance ($d \propto 1/x$) upstream of the stationary proton. On the timescale of the interaction with the proton, interactions within this partonic system are “frozen”. Diffractive scattering corresponds to the case in which there is no net transfer of colour between the proton and the partonic fluctuation of the photon which constitute eigenstates of diffraction. The probability of the dipole scattering is directly proportional to its transverse area. Hence, for small dipoles one expects small cross sections with the proton, since all the colour of the dipole is contained within a small transverse area, the colour field of the proton, which contains mainly long-distance fluctuations, appears transparent to it: this phenomena has become known as *colour transparency*. The longitudinally polarised photon is more inclined to fluctuate in a small symmetric system (quark and anti-quark
carry roughly the same light-cone momentum fraction of the photon, \( z \approx 1/2 \) whereas the transversely polarised photon can split either into a large asymmetric or a small symmetric system. Although the large asymmetric system \( (z \ll 1/2) \) is much less likely, if produced it scatters from the protons colour field with a much greater likelihood, so that both types of dipole contribute equally (to leading twist) in the diffractive cross section.

As \( x \to 0 \) this colour transparency picture must ultimately breakdown since the growth of the number of partons must reach the level where the impulse approximation breaks down and partons in the proton begin to recombine with each other. Exactly where in \( x \) this transition to a new regime happens, at a given \( Q^2 \), is not known at present (although a recent model has this saturation picture built in \([2]\)). Once the parton picture has broken down it is appropriate to pursue the problem using the classical field language (see \([3]\) and references therein).

2. Issues in diffraction

The major issue in diffraction is that of universality of diffractive phenomena. Regge factorisation remains the main (imperfect) tool to study this. In principle we have four types of diffractive (rapidity gap) “experiment” to compare with one another: 1) \( \gamma^*P \to X + Y \), 2) \( \gamma_{\text{direct}}P \to \text{jets} + Y \), 3) \( \gamma_{\text{resolved}}P \to X(\text{jets}) + Y \) 4) \( PP \to \text{jets} \) or \( W + \text{gap} \) (or gaps). In 1-3 \( X, Y \) are hadronic systems separated by a rapidity gap. The H1 QCD analysis of diffractive DIS events \([4]\), which require a gluon-dominated Pomeron to reproduce the positive scaling violations observed in 1), appear to do a reasonable job on 2) but overshoot 3) \([5]\) and fail completely to predict the Tevatron data 4). In the latter a very low fraction of diffractive events are observed (low gap survival probability) and a quark-dominated Pomeron seems to be preferred (see \([4]\) and references therein). This non-universality is almost certainly connected with the secondary interactions of “passenger” partons in the co-moving systems (different in each case) which act to fill-in the gap of the primary diffractive scatter. Levin and collaborators have an eikonal model of multiple interactions which they have applied to the problem of gap survival probability. They find broad agreement with the Tevatron dijets data (see e.g. \([6]\)). However, it is clear that a systematic study addressing the question of gap survival probability and the breakdown of universality of diffractive exchanges is urgently needed.

Recently, Donnachie and Landshoff \([8]\) presented an analysis of a wide range of HERA data (including all high energy proton and charm structure function data) which seemed to require a second Pomeron with a much larger intercept (around 1.4). However, it is not clear how the inclusion of this additional Pomeron should, or would, affect the total and elastic cross-section fits. (it depends on what one assumes for the coupling of this secondary Pomeron as \( Q^2 \to 0 \)).

We now have very good high precision data on the inclusive and diffractive cross sections. However, many models with different dynamical assumptions produce
adequate fits to data. Essentially the reason for this is that it is easy to tune the input parameters of a model to fit slowly varying function of $x$ and $Q^2$. In principle each model should be able to provide predictions for $F_L, F_2^{c\bar{c}}, F_L^D$ and $F_2^{D,c\bar{c}}$. Of course, if $F_L$ and $F_L^D$ were to be measured, for example by lowering the HERA beam energy, this would constrain the possible dynamics considerably. It is encouraging that there are plans and good prospects of making a good measurement $R_D = F_D^L/F_D^T$ at nuclear HERA (see [9] for a summary of the possibilities of nuclear HERA).

Recently it has become fashionable to present the data on $F_2$ using the so-called Caldwell plot (see [1]). From this plot it is observed that $dF_2/d\ln(Q^2/Q_0^2)$ plotted as a function of $x$ has a maximum around $x \approx 10^{-3}$. This has been presented as evidence for a lack of gluons at small-$x$. However as $x$ decreases the typical $Q^2$ in each bin is decreasing and it could be that what we are seeing is a breakdown of the parton picture at small $Q^2$. The saturation model of Wusthoff and Golec-Biernat [2] is designed to take this observed feature into account and work is under way to apply the same model to the diffractive data.

2.1. Exclusive processes

In order to be sure that it is safe to use perturbative QCD, it is necessary to isolate those diffractive process in which only small dipoles contribute, these exclusive processes are known as hard diffractive, e.g. exclusive dijet production, deeply virtual Compton scattering or heavy vector meson production. At small-$x$ all such processes are governed by the exchange of two gluons in the $t$-channel in a colour-singlet configuration. It was recently realised that the off-diagonal nature of the amplitude means that one is probing new non-perturbative information about the proton’s field and it is necessary to replace the gluon density with an off-diagonal generalization (which involves gluon operators sandwiched between different quantum states) in expressions for the cross sections of exclusive processes (see [10] for a review and references). In heavy vector meson production in DIS this off-diagonality arises from the need to convert a space-like photon virtuality into a time-like vector meson mass. The evolution equations governing the evolution of these new distributions in different regimes, as well as various definitions for them, are discussed in detail in [10].

The exclusive electro- and photoproduction of (both heavy and light) vector mesons is now a mature field both experimentally and theoretically (for a recent review see [11]) Recent interesting developments include the measurements of the ratios $R_\phi/R_\rho$ and $R_{J/\psi}/R_\rho$ versus $t$, at high-$|t|$. These measurements probe short distance part of the wavefunctions of the vector mesons for the first time. It also appears that there is evidence for s-channel helicity non-conservation in $\rho$-production (see [1, 12]).
3. Next-to-leading order BFKL

In this section I discuss some of the recent developments in small-x QCD arising from the completion of the next-to-leading corrections to the BFKL equation by Fadin and Lipatov [13] (see also [14]). There have been many papers this year on this subject. I will attempt to summarise some of the issues here.

First of all we recall some results from leading order [15] (many aspects of the problem are discussed in the recent textbook by Forshaw and Ross [16]). The BFKL equation is most safely applied to the high-energy scattering of two small systems (one usually talks of ‘onia’, a bound state of a heavy quark and antiquark) in the Regge limit $s \gg |t|, m^2_{\text{onia}}$. To the leading-logarithmic accuracy in energy, onia-onia cross sections involve a four-point function for a gluon ladder in the t-channel convoluted with impact factors for each onia at the bottom and top of the ladder. By performing the convolution with one of the onia one has the unintegrated gluon structure function of this onia, $f(x, k^2_t)$, convoluted with the remaining impact factor. The ladder is calculated in the so-called multi-Regge kinematics (MRK), i.e. each cell in the ladder is treated in the Regge limit, and the longitudinal Sudakov components of the vertical lines are strongly ordered. These components correspond to the light-cone momentum fractions of the external onia carried by the gluon concerned. This generates a logarithm in energy for each power in $\alpha_s$. The BFKL equation is an integral equation for $f$ which sums up all the leading-log parts of all such gluon ladders, i.e. all terms of order

$$\sum_n (\alpha_s \ln s/s_0)^n.$$  \hfill (1)

Working to leading-log accuracy does not allow the scale of $\alpha_s$ or the minimal longitudinal energy scale $s_0$ to be determined. By taking account all such graphs to leading-log accuracy, it is seen that the t-channel gluons Reggeize, and one has Reggeon-Reggeon gluon (RRg) vertices in the cross-rungs. The kernel of the BFKL equation is essentially democratic in its choice of momentum scales: it is conformally invariant, i.e. does not change under $k^2_t \to 1/k^2_t$, (hereafter I will drop the subscript $t$). As a result the typical transverse scales are determined only by the external particles and the distribution in $k^2$ diffuses away from the ends of the ladder. This conformal invariance is possible only to leading order in logarithms since the running of the QCD coupling, which comes in at next to leading-order, explicitly breaks this scale invariance.

Because of the conformal invariance, the BFKL equation can be solved by taking Mellin moments of $f$:

$$f(s, k^2) = \int_{C_\omega} \frac{d\omega}{2\pi i} \int_{C_\gamma} \frac{d\gamma}{2\pi i} \left( \frac{s}{s_0} \right)^\omega \left( \frac{k^2}{k^2_0} \right)^\gamma f(\omega, \gamma),$$  \hfill (2)

where the Mellin space solution is

$$f(\omega, \gamma) \propto \frac{1}{\omega - \alpha_s \chi(\gamma)}.$$  \hfill (3)
If one expands the $\gamma$-integral about the saddle point in the kernel, $\gamma_s = (1/2, 0)$, it contains a pole at $\omega = \bar{\alpha}_s \chi(\gamma) = 4 \ln 2 \bar{\alpha}_s$ with $\bar{\alpha}_s = 3 \alpha_s / \pi$. This leads to the famous rise in energy of the unintegrated structure function:

$$f(s, k^2) \propto \left( \frac{k^2}{k_0^2} \right)^{\frac{1}{2}} \left( \frac{s}{s_0} \right)^{4 \ln 2 \bar{\alpha}_s}.$$ \hspace{1cm} (4)

This corresponds to the lowest energy eigenvalue of a Schrödinger equation in $k^2$, with $\nu = 0$ ($\gamma = 1/2 + i\nu$):

$$H^{(1)}(q) = \epsilon^{(1)}(\nu) \psi(\nu)(q)$$ \hspace{1cm} (5)

with $\epsilon^{(1)}_\nu$ equal to (minus) the BFKL kernel, $K(\nu) = \bar{\alpha}_s \chi(\gamma) = \bar{\alpha}_s [2 \psi(1) - \psi(1/2 + i\nu) - \psi(1/2 - i\nu)]$. The eigenfunctions are given by

$$\psi(\nu)(k^2) = \frac{(k^2)^{i\nu}}{\sqrt{2\pi^2 k^2}}.$$ \hspace{1cm} (6)

To next-to-leading log accuracy in energy one needs to sum all graphs which contains pieces which are one power down in $\alpha_s$, i.e. of order

$$\sum_n \alpha_s (\alpha_s \ln s/s_0)^n.$$ \hspace{1cm} (7)

These include all the previous gluon ladders evaluated in the quasi-multi-Regge kinematics (QMRK) in which one of the strong orderings is relaxed; the original graphs, evaluated in MRK, with running coupling at one loop, or NLO corrections to the RRg vertices, or NLO gluon reggeization, or an entirely new set of graphs, with real-quark-emission insertions on the horizontal rungs (for more details see [17]).

In the Hamiltonian formalism at next-to-leading order the Hamiltonian becomes $H = H^{(1)} + H^{(2)}$ where the action of the next-to-leading part, $H^{(2)}$, on the leading-order eigenfunctions is given by

$$H^{(2)}(q) = [K_{SI}(\nu) + K_r(\nu, q)] \psi(\nu)(q)$$ \hspace{1cm} (8)

The second term in the right hand side of equation (8) comes from the running of the coupling and explicitly breaks the conformal invariance. The first term corresponds to all the other corrections, it turns out that this piece is scale invariant, like the leading order piece denoted $\epsilon^{(1)}_\nu$ and leads to a shift in the the leading order eigenvalue which depends on the value of $\bar{\alpha}_s$ given by

$$\epsilon^{(1)}_\nu \rightarrow \epsilon_\nu = \epsilon^{(1)}_\nu \left( 1 - \frac{\bar{\alpha}_s C(\nu)}{4} \right)$$ \hspace{1cm} (9)

where the function $C(\nu)$ can be found in [13]. It turns out that if one concentrates on the saddle point method and expands about $\nu = 0$ then the shift of the power of energy is huge and negative for reasonable values of $\bar{\alpha}_s$:

$$\omega_p^{(2)} = 4 \ln 2 \bar{\alpha}_s (1 - 6.56 \bar{\alpha}_s)$$ \hspace{1cm} (10)
$$= \omega_p^{(1)} (1 - 2.366 \omega_p^{(1)}),$$ \hspace{1cm} (11)

and considerably reduces the strong rise with energy, $s^{\omega_p^{(1)}}$ found at leading order. For example, taking $\bar{\alpha}_s = 0.15$, a large leading-power $\omega_p^{(1)} = 0.4$ is reduced to $\omega_p^{(1+2)} = 0.02$. 

Ross [18] has pointed out that concentrating on the point at $\gamma_s = (1/2, 0)$ may be misleading, except for very small values of $\alpha_s \leq 0.05$. For larger values the saddle point on the real-$\gamma$ axis is replaced by two saddle points off-axis and that it is necessary to expand about these new complex conjugate saddle points to get a more accurate answer. This may be achieved by expanding the kernel to order $O(\nu^4)$. The saddle point method then gives a larger energy power of approximately $\omega_p^{(1+2)} = 0.02 + 0.09 = 0.11$. This result should also be taken with some caution since the eigenfunctions are now oscillatory, $\psi(q) \propto (q^2)^{(\nu_s)}$, and when folded in with the impact factors could lead to negative cross sections (in addition, it implies that the full power may not be seen until very high energies).

The running coupling piece of the next-to-leading order kernel is potentially even more troublesome as pointed out by Armesto, Bartels and Braun [19]. The corresponding part in the Hamiltonian is a potential piece proportional to $\beta_0 \ln k^2/\mu^2$. Since this piece can take on any value, the energy spectrum of the Hamiltonian is unbounded from below which implies an arbitrary large power of energy is possible (there is no rightmost singularity in the $j$-plane and therefore no intercept). Such extreme growth with energy is very worrying, however, as Armesto et al point out, it comes with a non-perturbative damping factor, $\exp(-1/\alpha_s b)$, which may lead to its suppression for the scattering of small objects. Assuming this to be the case the authors of [19] also reproduce the non-Regge type behaviour (a non-power-like behaviour in $s$) arising from the running coupling found in [20] (see also [21]), this behaviour also points to the restricted applicability of the whole formalism.

In practice we know that the unboundedness comes from the running coupling approaching the Landau pole. We know that there must be some regulation in this infra-red region which marks the transition between perturbative and non-perturbative QCD. The question which needs addressing is whether the way in which one implements this infra-red regularization [22, 23] makes a critical difference to the energy dependence. If the answer is yes, it would appear that non-perturbative physics is inextricably entangled with perturbative physics, even in idealised onia-onia scattering, and that one loses all predictability in perturbative QCD at high enough energies. If the answer is no, it may be possible to factorize all non-perturbative behaviour into boundary conditions and retain some predictability. This issue is intimately connected with the enhancement of the diffusion in $k^2$ caused by the running coupling. For a more detailed discussion of the effects of running coupling in the leading-order BFKL kernel see chapter (5) of [16].

On a more optimistic note, DIS at moderate $x$ is well-understood and it would be very surprising if it was impossible to approach the small-$x$ region in a controlled way. A formalism exists to do this which incorporates information from BFKL dynamics into the DGLAP [24] formalism by using resummed anomalous dimensions (see [13, 14, 25] and references therein). Thorne gave an interesting presentation on this issue, which included the possibility of an energy dependent coupling constant in the small-$x$ region this may lead to enlargement of the region of applicability of the DGLAP formalism [20].
One optimistic view of the large correction factor in equation (11) pointed out in the discussions is that what one is seeing is the first piece of some large all-orders effect which may be resummed. This all orders effect could be connected to coherence phenomena. Salam [27] gave a presentation in which double logarithms in $k^2$ are resummed and discussed the corresponding radical changes in the structure of the kernel (see [20] for more details).

As the summary above should indicate there are many issues that still need to be resolved in this fascinating area. It is clear from the discussions at Durham that the intense theoretical interest in this area is set to continue.

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