Shortcuts to adiabaticity for a qubit using detuning control

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Abstract. We derive shortcuts to adiabaticity for a two-level system using the detuning as sole control parameter while the Rabi frequency is kept fixed. We work in the adiabatic basis while we also employ a time-rescaling. The control function that we use is the time-derivative of the magnetic field angle relative to the z-axis. We find that a constant control attains adiabatic passage with perfect fidelity only at particular durations, which we call resonances. We also find that, if we use a control with appropriate on-off modulation, a perfect adiabatic passage is achieved for all durations exceeding the bound $\pi/\Omega$, with $\Omega$ being the constant Rabi frequency.

1. Introduction

During the last years, quantum science and technology have witnessed an unprecedented progress. At the heart of this second quantum revolution lies the problem of efficiently controlling qubits, the fundamental quantum cells. Among the various methods developed the previous years, adiabatic rapid passage (ARP) stands out as one of the most successful, due to its robustness against system uncertainties. The main drawback of this adiabatic technique is that it requires long times, something which degrades the fidelity in the presence of undesirable interactions with the environment.

Various methodologies have been developed to speed up quantum dynamics, while maintaining the adiabatic character of the process. The main idea of these Shortcuts to Adiabaticity (STA) is to bring the quantum system to the same state with the long adiabatic procedure but only at the final time, while following a shorter path in between. STA have been applied to various problems in many modern research areas of physics. For the case of a qubit (a two-level system) [1], both the detuning and the Rabi frequency are used to accelerate the adiabatic process.

The more general control setting of a qubit, in which both the detuning and the Rabi frequency can be exploited as control parameters, has a broad spectrum of applications. Nevertheless, for some crucial applications in quantum information processing applies a more strict setting where only the detuning can change in time and serves as the control variable,
while the Rabi frequency is fixed to a constant value [2, 3]. A particular example which can be formulated as the adiabatic control problem of a qubit in this stricter setting, is the problem of designing a CZ quantum gate [3].

In this work we consider the problem of adiabatically controlling a two-level system under this constrained framework. First we use a constant control signal and find that adiabatic passage with perfect fidelity is obtained only at particular times, where the spin-flip probability is zero. These times correspond to a series of resonant STA. We next apply a control with on-off modulation and find that, by properly selecting the modulation parameters, a perfect adiabatic passage is achieved for all durations exceeding the bound \( \pi/\Omega \), with \( \Omega \) being the constant Rabi frequency.

2. Resonant shortcuts

The Hamiltonian of a qubit with detuning \( \delta(t) \) and constant Rabi frequency \( \Omega \) is

\[
H(t) = \frac{\delta(t)}{2} \sigma_z + \frac{\Omega}{2} \sigma_x,
\]

where \( \sigma_x, \sigma_z \) are the Pauli matrices. The polar angle of the magnetic field, which can also be used at the control function, is

\[
cot \theta(t) = \delta(t)/\Omega.
\]

In the conventional adiabatic rapid passage, the detuning has initially a large negative value, with corresponding polar angle \( \theta(0) = \theta_i \approx \pi \). The system starts from the spin-down state, which coincides with an eigenstate of the instantaneous Hamiltonian. It is then linearly varied with time, until a large positive final value is obtained at \( t = T \), corresponding to a polar angle \( \theta(T) = \theta_f \approx 0 \). If the change is slow enough, the system remains during the whole process in the same eigenstate of the instantaneous Hamiltonian, which at the final time coincides with the spin-up state. Since at the boundary values of time the state during the adiabatic evolution coincides with the original spin up/down states, a perfect population inversion is implemented. Adiabatic rapid passage exhibits a remarkable robustness to moderate changes of parameters, but requires slow changes of the control variables so the system can follow the instantaneous eigenstates, a feature which may considerably reduce the final fidelity under the influence of undesirable environmental interactions. Here we find detuning controls \( \delta(t) \), which can also be expressed in terms of the polar angle \( \theta(t) \), that drive the system to the target state bypassing the adiabatic eigenstates at intermediate times.

Following Ref. [3], we use the adiabatic basis consisting of the instantaneous eigenvectors of \( H \), and apply a time rescaling from \( t \) to \( \tau \) using the expression \( d\tau = \Omega dt/\sin \theta \). The probability amplitudes \( b = (b_1, b_2)^T \) of the two adiabatic levels obey the following Schrödinger equation (\( \hbar = 1 \)) with a transformed Hamiltonian

\[
\frac{db}{d\tau} = H'_{ad} b, \quad H'_{ad} = \frac{1}{2} \sigma_z + \frac{1}{2} \frac{d\theta}{d\tau} \sigma_y.
\]

We suppose a variation in the polar angle from \( \theta(\tau = 0) = \theta_i \) to \( \theta(\tau = T') = \theta_f \) with \( \theta_i > \theta_f \), where note that the prime characterizes the corresponding duration in rescaled time. Observe that for \( d\theta/d\tau = -u < 0 \) with \( u \) constant, \( H'_{ad} \) is also constant and Schrödinger’s equation can be readily solved. Starting from the adiabatic state \( b(0) = (1, 0)^T \), the system performs a STA if it returns to this state at the final time \( \tau = T' \), while in the traditional adiabatic passage the system should closely track the adiabatic state during the whole time interval. During this time \( T' \) the polar angle of the magnetic field in the original frame changes from \( \theta_i \) to \( \theta_f \), thus
Figure 1. For the case of the first resonant shortcut $k = 1$ with duration $T_1 \approx 1.195\pi/\Omega$ we display (a) the detuning $\delta(t)$, (b) the polar angle $\theta(t)$ of the magnetic field, (c) the trajectory (magenta) of the Bloch vector in the original basis, as well as the corresponding variation of the polar angle $\theta$ (black), (d) the trajectory (magenta) of the Bloch vector in the adiabatic basis, returning to north pole after traversing a cycle, while the magnetic field (black) is constantly up.

$$\theta_i - \theta_f = -\int_0^{T'_{k}} \theta' d\tau = uT'$$ with $u$ constant. These conditions are both satisfied for the resonant values

$$u_k = \frac{\theta_i - \theta_f}{2k\pi} \sqrt{1 - \left(\frac{\theta_i - \theta_f}{2k\pi}\right)^2}, \quad T'_k = 2k\pi \sqrt{1 - \left(\frac{\theta_i - \theta_f}{2k\pi}\right)^2},$$ (4)

where $k = 1, 2, \ldots$.

The dependence of angle $\theta$ on the original time $t$ is easily found to be

$$\theta_k(t) = \cos^{-1}(\cos \theta_i + u_k \Omega t).$$ (5)

When expressed in original time $t$, times $T'_k$ become

$$T_k = \frac{\cos \theta_f - \cos \theta_i}{u_k} \cdot \frac{1}{\Omega}.$$ (6)
Figure 2. (a) Modulated control $u(\tau)$ with duration $T' = 1.5\pi$ in rescaled time. (b) Polar angle $\theta(t)$ in original time $t$, where the total duration is $T = 2t_1 + t_2 \approx 1.108\pi/\Omega$. (c) Trajectory (magenta) of the Bloch vector in the original basis, as well as corresponding variation of the polar angle $\theta$ (black). (d) Trajectory (magenta) of the Bloch vector in the adiabatic basis, returning to north pole, while the magnetic field (black) is constantly up.

In the symmetric case $\theta_f = \pi - \theta_i$, where $\cos \theta_f = -\cos \theta_i$, if we use a shifted time $t_s = t - t_k/2$, so $\theta_k(t_s) = \cos^{-1}(u_k\Omega t_s)$ and $\theta_k(t_s = 0) = \pi/2$, we find the detuning to be

$$\delta(t_s) = \frac{u\Omega t_s}{\sqrt{1 - (u\Omega t_s)^2}}\Omega,$$

for $-T_k/2 \leq t_s \leq T_k/2$. In Fig. 1 we present an example with a detuning variation from $-10\Omega$ to $10\Omega$, same as in [3], corresponding to $\theta_f = \tan^{-1}(1/10)$, $\theta_i = \pi - \theta_f$.

3. Generalized resonant shortcuts

We can find STA of any duration larger than the bound $\pi/\Omega$, if we apply as $u(\tau)$ an on-off pulse-sequence, see for example the simplest on-off-on sequence shown in Fig. 2(a). We can select the characteristics of this control function (durations $\tau_1$ and $\tau_2$ of on and off pulses, maximum value $u$), such that to drive our system back to the $(1\ 0)^T$ adiabatic state at a desired duration $\tau = T'$. Using the fact that the Hamiltonian $H'_{ad}$ is piecewise constant and the property $\sigma_e\sigma_f = \delta_{ef}I + i\epsilon_{efg}\sigma_g$ of the Pauli matrices, where the indices $e, f, g$ can be $x, y$ or $z$,
this condition can be translated to the following equation
\[
\cos \frac{\omega \tau_1}{2} \cos \frac{\tau_2}{2} - n_z \sin \frac{\omega \tau_1}{2} \sin \frac{\tau_2}{2} = 0,
\]
where
\[
\omega = \sqrt{1 + u^2}, \quad n_z = \frac{1}{\sqrt{1 + u^2}}.
\]
(8)
The variation of the polar angle during the on segments of the control, where \( u \) is constant, is \( \theta_i - \theta_f = 2u\tau_1 \), while the duration of the full sequence is \( T' = 2\tau_1 + \tau_2 \). Combining these relations we find
\[
\tau_1 = \frac{\theta_i - \theta_f}{2u}, \quad \tau_2 = T' - \frac{\theta_i - \theta_f}{u}.
\]
(10)
Using these expressions in Eq. (8), we end up with a transcendental equation for the amplitude \( u \). Once this equation is numerically solved, the durations \( \tau_1, \tau_2 \) can be easily obtained. Then, the durations in time \( t \) can be calculated. In Fig. 2 we display an example with \( T' = 1.5\pi \). Note that, as the total duration \( T' \) increases, it turns out that it is necessary to use pulse sequences with more switchings, so the corresponding transcendental equations have a solution.

4. Conclusion
We found novel STA for adiabatic rapid passage in a qubit with only detuning control. The results of the current study may be applied in quantum information, in the design of CZ quantum gates, as well as to other research areas which employ adiabatic rapid passage.

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