Deviations in Representations
Induced by Adversarial Attacks

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Abstract
Deep learning has been a popular topic and has achieved success in many areas. It has drawn the attention of researchers and machine learning practitioners alike, with developed models deployed to a variety of settings. Along with its achievements, research has shown that deep learning models are vulnerable to adversarial attacks. This finding brought about a new direction in research, whereby algorithms were developed to attack and defend vulnerable networks. Our interest is in understanding how these attacks effect change on the intermediate representations of deep learning models. We present a method for measuring and analyzing the deviations in representations induced by adversarial attacks, progressively across a selected set of layers. Experiments are conducted using an assortment of attack algorithms on the CIFAR-10 dataset—with plots created to visualize the impact of adversarial attacks across different layers in a network. Code is available at https://github.com/dstein64/adv-deviations.

1 Introduction
It has been shown that carefully crafted adversarial instances can deceive deep learning models. This prompted further research to (1) develop new attack techniques, (2) defend against instances created with such methods, and (3) explore and characterize the properties of adversarial attacks. Here we’re interested in the latter task, seeking to understand how adversarial instances effect change on the hidden layers of their target networks.

Our Contribution We present a method for measuring and analyzing the deviations in representations induced by adversarial attacks. Experiments are conducted with an assortment of attack algorithms, using two separate distance metrics for measuring deviations in representations. Our plots show the transitional effect on representations induced by adversarial attacks.

2 Preliminaries
There are various ways to generate adversarial instances. A typical approach starts with an input $x$ and synthesizes a small additive perturbation $\Delta x$ for which $x + \Delta x$ would deceive a neural network—whereas a human would not perceive much difference between $x$ and $x + \Delta x$.

Here we briefly cover the three adversarial attack methods that we incorporate into our experiments. In addition to the $x$ and $\Delta x$ notation mentioned earlier, $y$ represents a ground truth class label and $J$ is a loss function for a neural network—which has already been trained in this scenario.

Fast Gradient Sign Method (FGSM) (Goodfellow, Shlens, and Szegedy 2015) generates an adversarial perturbation $\Delta x = \epsilon \cdot \text{sign}(\nabla_x J(x, y))$, which is in the approximate direction of the loss function gradient. The sign function transforms each element to $-1, 0,$ or $1$, with the result then scaled by $\epsilon$. Thus, the $L_\infty$ norm of $\Delta x$ is bounded by $\epsilon$.

Basic Iterative Method (BIM) (Kurakin, Goodfellow, and Bengio 2017) applies FGSM iteratively. At each step, $x_{t+1} = x_t + \alpha \cdot \text{sign}(\nabla_x J(x_{t+1}, y))$, with initialization $x_0 = x$. On each iteration, the $L_\infty$ norm of the latter addition term is bounded by $\alpha$, and clipping at each step can be used to constrain the final $x_{\text{adv}}$ to an $\epsilon$-neighborhood of $x$.

Carlini & Wagner (CW) (Carlini and Wagner 2017) constructs an adversarial perturbation using gradient descent to solve $\Delta x = \arg\min_\delta \{\|\delta\|_p + c \cdot f(x + \delta)\}$. Function $f$ is one for which $f(x + \delta) \leq 0$ if and only if the attack is successful on the target classifier; their $f_6$ formulation was found most effective. Positive constant $c$ can be determined using binary search. A box constraint on $x + \delta$ is satisfied by clipping or change of variables. $p$ specifies which norm is used.

3 Method
To inspect the deviations in representations induced by adversarial attacks, we start with a dataset of original (non-attacked) instances, and an attacked version of the same dataset. For the neural network that’s targeted—or some other network, e.g., if the context is transferability—we select a set of checkpoints, locations in the network for which we’ll extract representation vectors.

By passing an original instance $x$ through the network, we extract representation vector $\phi_i$ at checkpoint $i$. We repeat this for the adversarial counterpart $x_{\text{adv}}$, extracting $\phi'_i$. Next, we measure the distance $d(\phi_i, \phi'_i)$ between the vectors. While we don’t specify a particular distance function $d$, our experiments use Euclidean distance and cosine distance.

The volume occupied by representations can differ at separate layers of a neural network. Therefore, it’s important to normalize our distance measurements prior to comparing them across checkpoints. We follow the approach of Mahendran and Vedaldi Section 4. Our earlier formulation $d(\phi_i, \phi'_i)$ becomes $d(\phi_i, \phi'_i)/N_i$. Normalization constant $N_i$...
(for checkpoint $i$) is the average pairwise distance between representation vectors—computed using $d$—across a sample of instances.

We calculate the normalized distances across all network checkpoints for the full dataset of original and attacked instances (limited in our experiments to include only pairings where the attack was successful). This provides a sample of deviations at each checkpoint, for which the distributions can be analyzed. We use violin plots, but other techniques could also provide insight. The method is illustrated in Figure 1.

### 4 Experiments

#### 4.1 Experimental Settings

Our experiments utilized the CIFAR-10 dataset [Krizhevsky 2009], comprised of 60,000 32×32 RGB images—with a designated split into 50,000 training images and 10,000 test images—across 10 classes. Using the training data, we fit a neural network classifier that follows the kungliu ResNet-18 architecture [11,173,962 parameters]. With pixel values scaled by $1/255$ to be between zero and one, the network was trained for 100 epochs using Adam [Kingma and Ba 2015] for optimization, with the training data augmented via (1) random horizontal flipping and (2) random crop sampling on images padded with four pixels per edge. The resulting model had accuracy of 92.67% on the test data.

We use 10 checkpoints throughout the model for extracting representations. Figure 2 displays the model architecture, highlighting the locations of the numbered checkpoints. The shape and dimensionality of representations at each checkpoint are shown in Table 1.

#### Adversarial Attacks

We utilized the cleverhans library [Papernot et al. 2018] to generate untargeted adversarial attacks for the 9,267 correctly classified test images.

| Checkpoint(s) | Shape          | Dimensionality |
|---------------|----------------|----------------|
| 1             | $3 \times 32 \times 32$ | 3,072          |
| 2, 3          | $64 \times 32 \times 32$ | 65,536         |
| 4             | $128 \times 16 \times 16$ | 32,768         |
| 5             | $256 \times 8 \times 8$ | 16,384         |
| 6             | $512 \times 4 \times 4$ | 8,192          |
| 7             | 512             | 512            |
| 8, 9, 10      | 10              | 10             |

Attacks were conducted with the FGSM, BIM, and CW algorithms. The attacked images were clipped between zero and one for all attacks, and quantized to 256 discrete values—whereby each image could be represented in 24-bit RGB space—for FGSM and BIM. We did not quantize the CW-attacked images, as doing so would essentially revert the attack.

For FGSM, $\epsilon$ was set to $3/255$ for a maximum perturbation of three intensity values out of 255 for each pixel on the unnormalized data. Model accuracy was 16.80% on the 9,267 attacked images from the test dataset. That is, the attack succeeded at a rate of 83.20%.

Our attacks generated with BIM used 10 iterations with $\alpha = 1/255$ and $\epsilon = 3/255$. This limits the per-step maximum perturbation to one unnormalized intensity value per pixel, ultimately clipped to a maximum perturbation of three intensity values. The resulting model accuracy on the attacked images was 0.29% (i.e., an attack success rate of 99.71%).

We used CW with an $L_2$ norm distance metric along with

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1This differs in filter counts and depth from the ResNet-20 architecture that was used for CIFAR-10 in the original ResNet paper [He et al. 2016].

2The original CW paper [Carlini and Wagner 2017] addresses the issue with a greedy search process that restores the attack quality one pixel at a time—an approach that we did not utilize.
the package’s default parameters—five binary search steps, a learning rate of 0.005, and up to 1,000 iterations. Accuracy after attack was 0.00%, a perfect attack success rate.

Figure 3 shows examples of randomly selected CIFAR-10 images and their adversarially perturbed counterparts.

![Original, FGSM, BIM, CW](image)

**Measuring Distances** As a next step we extract representations at the 10 selected checkpoints. This is conducted for all 9,267 test images that were correctly classified originally. We then repeat the extraction for the adversarially perturbed counterparts of these same images. With representations extracted for both original and adversarially attacked images, for each image we measure distances between the original representations and corresponding adversarial counterparts. For our experiments we use two metrics for measuring the distance between vectors \( u \) and \( v \), Euclidean distance \( \|u - v\| \) and cosine distance \( 1 - (u \cdot v)/(\|u\|\|v\|) \).

As we mentioned earlier, it’s important to normalize the measured distances. We calculate normalization constants across all \( \binom{9,267}{2} = 42,934,011 \) pairs, to serve as divisors for scaling the originally calculated distances. This is done separately for Euclidean and cosine metrics. That is, the originally measured Euclidean distances are normalized by scaling constants calculated using Euclidean distance, and the cosine distances are scaled with normalization constants determined using cosine distance.

**4.2 Results**

We just discussed the details of how we measured the distances between representations of original images and their adversarial counterparts. To analyze the results, violin plots were employed to visualize the distribution of distance measurements so that we could see how adversarial attacks induce deviations progressively throughout the network.

Figure 4 shows the plots, which were generated across the three adversarial attacks and two distance metrics described earlier. For each attack algorithm, our analysis is limited to the subset of images that could be successfully attacked from the 9,267 correctly classified test images—7,710 images for FGSM, 9,240 for BIM, and 9,267 for CW. Because we only consider successfully attacked images, the deviations at checkpoint 10 are constant—the distance between two non-equal one-hot vectors is always the same.

**5 Discussion**

**Trend** Something that stands out is that the average deviation in representations—induced by adversarially perturbing inputs—primarily increases as a function of depth (i.e., normalized distances tend to be larger for higher numbered checkpoints). This matches the general expectation we had prior to running the experiments. Interestingly, the pattern does not hold for checkpoint 8, where the average distance was lower than it was at checkpoint 7. Checkpoint 8 occurs after applying the linear portion of a densely connected layer, whereas the earlier checkpoints are chiefly positioned after the application of a convolution and ReLU activation function (the exceptions are checkpoint 1, which captures the input prior to subsequent network processing, and checkpoint 7, which follows a pooling operation).
Figure 4: Violin plots showing the distribution of deviations in representations across selected checkpoint layers, induced by adversarial attacks. The figure subplots each correspond to a specific distance metric (indicated by the leftmost labels) and a specific attack algorithm (indicated by the header labels). Distance measurements were normalized. The diamond-shaped markers show sample means. We generated the plots using `matplotlib` [Hunter 2007], with the default settings of 100 points for Gaussian kernel density estimation and Scott’s Rule for calculating estimator bandwidth.

**Attacks** Relative to the other attacks, it appears that BIM induces the smoothest transition in deviations from input to output. For FGSM there is a noticeable jump in normalized distances from checkpoint 8 to 9. A similar condition is present for CW, but occurs between checkpoint 9 and 10. For the configurations of attacks we considered, CW led to the smallest change in intermediate representations for original versus adversarially perturbed images.

**Distance Metrics** The first row’s plots, which use Euclidean distance, are similar to the plots on the second row, which use cosine distance. The average normalized distances are always higher for the Euclidean metric than for cosine, not counting checkpoint 10 where the means are the same. This might be related to the property that two non-equal vectors in the same direction have zero cosine distance between them, but positive Euclidean distance. For vectors of slightly different direction, cosine distance would be small, whereas Euclidean distance could be large.

**6 Conclusion**

We set out to inspect how adversarial perturbations impact neural network representations. Our proposed technique considers the normalized representation distances at selected checkpoints between original images and their adversarially perturbed counterparts. Experiments conducted on CIFAR-10 data using an assortment of attack algorithms allowed us to visualize the deviations in representations induced by adversarial attacks. We observed different behavior—in the way normalized distances transition from input to output throughout the network—across the configurations of attacks we considered.

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