THE 331 MODEL WITH RIGHT-HANDED NEUTRINOS

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Abstract

We explore some more consequences of the SU(3)\textsubscript{L} \times U(1)\textsubscript{E} electroweak model with right-handed neutrinos. By introducing the mixing angle, the exact physical eigenstates for neutral gauge bosons are obtained. Because of the mixing, there is a modification to the $Z^1$ coupling proportional to $\sin \theta$. The data from the $Z$-decay allows us to set the limit for $\sin \theta$ as $0.0017$ and $0.00132$. From the neutrino neutral current scatterings, we estimate a bound for the new neutral gauge boson $Z^2$ mass in the range $300 \text{ GeV}$, and from symmetry-breaking hierarchy a bound for the new charged and neutral (non-Hermitian) gauge bosons $Y$, $X^0$ are obtained.

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I. Introduction

In the Standard Model (SM) [1], each generation of fermions is anomaly-free. This is true for any extensions of the SM as well, including the popular Grand Unified models [2]. In these models, therefore, the number of generations is completely unrestricted on theoretical grounds.

Recently, an interesting class of models has been proposed [3] in which each generation is anomalous but different generations are not exact replicas of one another, and the anomalies cancel when a number of generations are taken into account, and to be a multiple of three. The most economical gauge group which admits such fermion representations is SU(3)C × SU(3)L × U(1)Y, and it has been proposed by Pisano, Pleitez, and Frampton [4] (for further work on this model, see Refs. [5,6]). The original model did not have right-handed neutrinos, but recently we have included them in a non-trivial way in an interesting modification of the model [7]. We have pointed out that this model is simpler than the Pisano-Pleitez-Frampton (PPF) model, since fewer Higgs multiplets are needed in order to allow the fermions to gain masses and to break the gauge symmetry.

II. The 331 model and Yukawa interactions

Like the PPF model, our model is also based on the gauge group

\[ SU(3)_C \times SU(3)_L \times U(1)_Y \]  \hspace{1cm} (1)

This model deals with nine leptons and nine quarks. There are three left- and right-handed neutrinos \((\nu; \; \nu)\), three charged leptons \((e; \; \nu)\), four quarks with charge \(2/3\), and five quarks with charge \(-1/3\).

Under the gauge symmetry (1), the three lepton generations transform as

\[ f^a_L \equiv \begin{pmatrix} \ell^a_L \\ \nu^a_L \end{pmatrix} \hspace{1cm} (1;3; \; 1=3; \; 1=3; \; 1=3; \; 1=3; \; 1=3; \; 1=3) \hspace{1cm} (2) \]

\[ \ell^a_R \equiv \begin{pmatrix} \ell^a_R \\ c^a_R \end{pmatrix} \hspace{1cm} (1;1; \; 1) \]

Finally, our conclusions are summarized in the last section.

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where \( a = 1, 2, 3 \) is the generation index.

Two of the three quark generations transform identically and one generation (it does not matter which one) transforms in a different representation of the gauge group (1):

\[
\begin{align*}
Q_{3L} &= \mathbb{P} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\
Q_{3L} &= \mathbb{P} \begin{pmatrix} u_{3L} \\ d_{3L} \end{pmatrix} (3; 3; 1 = 3);
\end{align*}
\]

Here it can easily be checked that all gauge anomalies cancel with the above choice of gauge quantum numbers. Fermion mass generation and symmetry breaking can be achieved with just three \( SU(3)_L \) triplets. We define them by their Yukawa Lagrangians as follows:

\[
L_{Yuk} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} Q_{3L} T_R + 2 i \bar{Q}_{3L} d_{3R} + H C \chi
\]

where

\[
\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \mathbb{P} \begin{pmatrix} u_{3L} \\ d_{3L} \end{pmatrix} (3; 3; 1 = 3);
\]

If gets the vacuum expectation value (VEV):

\[
h \begin{pmatrix} 0 \\ 1 \end{pmatrix} = (0; 0; 1 = \mathbb{P} \mathbb{Z});
\]

then the exotic \( 2/3 \) and \( -1/3 \) quarks gain masses and the gauge symmetry is broken to the SM gauge symmetry:

\[
\begin{align*}
SU(3)_C &\to SU(3)_L U(1)_k \\
# h &i \\
SU(3)_C &\to SU(2)_L U(1)_k;
\end{align*}
\]

where \( Y = 2N \mathbb{P} \mathbb{Z}_8 = 3 (8 = \text{diag}(1; 1; 2) = \mathbb{P} \mathbb{Z}) \). Note that \( Y \) is identical to the standard hypercharge of the SM. Electroweak symmetry breaking and ordinary fermion mass generation are achieved with two \( SU(3)_L \) triplets; which we define through their Yukawa Lagrangians as follows:

\[
L_{Yuk} = \begin{pmatrix} 3a Q_{3L} u_{3R} + 4i a Q_{3L} d_{3R} + H C \chi \\
L_{Yuk} = a Q_{3L} d_{3R} + 2i a Q_{3L} u_{3R} + G_{ab} f^{ab} + G_{ab}^{a^b} f^{abc} (f_{ab}^{c})_k (f_{cb}^{b})_k + H C \chi
\]
where
\[ A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 3 \\ 1 & 3 & 1 & 3 \\ 1 & 3 & 1 & 3 \end{pmatrix}; \]
\[ B = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}; \]
\[ C = \begin{pmatrix} 1 & 3 & 2 & 3 \\ 1 & 3 & 1 & 3 \\ 1 & 3 & 1 & 3 \end{pmatrix}; \]
\[ D = \begin{pmatrix} 1; 3; 2 = 3 \\ 1; 3; 1 = 3 \end{pmatrix}; \]
\[ E = \begin{pmatrix} 8 \\ 8 \\ 8 \end{pmatrix}; \]
\[ F = \begin{pmatrix} 8 \\ 8 \\ 8 \end{pmatrix}; \]

We require the vacuum structure of

\[ h_i^\pm = (0; u=\frac{P^+}{2}; 0); h_i^0 = (v=\frac{P^-}{2}; 0; 0): \]

The last term in Eq. (7) gives the 3 3 antisymmetric mass matrix, which has eigenvalues 0, M, M. Hence, one of the neutrinos does not gain mass and the other two are degenerate, at least at the tree level [3]. It is easy to see that this term gives interactions which directly contribute to lepton-number violation processes such as neutrinoless double beta decay (00) and neutrino oscillations. The VEV h_i will generate masses for the three charged leptons, two up-type, one down-type quarks and two of the neutrinos will gain degenerate Dirac masses with one necessarily massless, while VEV h_i will generate masses for the remaining quarks. The VEVs h_i and h_i also give the electroweak gauge bosons masses and results in the symmetry breaking:

\[
\begin{align*}
SU(3)_C & \to SU(3)_{\text{L}} \times U(1)_{\text{Y}} \\
\# h_i & \\
SU(3)_C & \to SU(2)_{\text{L}} \times U(1)_{\text{Y}} \\
\# h_i; h_i & \\
SU(3)_C & \to U(1)_{\text{Y}} \\
\end{align*}
\]

Here the electric charge is defined:

\[ Q = \frac{1}{2} + \frac{1}{6} \frac{P^+}{2} s + N: \]

III. Gauge bosons

The gauge bosons of this theory form an octet \( W^a \) associated with SU(3)_L, an octet \( G^a \) (gluons) with SU(3)_C and a singlet \( B \) associated with U(1)_Y. It is easy to see that the massless \( G^a \) gauge bosons associated with SU(3)_C group decouple from the neutral gauge boson mass matrix. Since that reason, we neglect terms which contain the \( G^a \) gauge bosons in the covariant derivative. The gauge boson mass matrix arises from the Higgs boson kinetic term:

\[ L_{\text{kinetic}} = (D \bar{\psi} (\partial D + \bar{\psi}) (\partial D + \bar{\psi}) + (D \bar{\psi} (\partial D + \bar{\psi}) + (D \bar{\psi} (\partial D + \bar{\psi}) : \]

The covariant derivatives are

\[ D = \partial + ig A^a W^a + ig_N B; \]

\[ \text{where} \]

\[ X^a = \begin{pmatrix} \frac{1}{2} \end{pmatrix} W^a \]
where \( (a=1,\ldots,8) \) are the SU(3) \(_L\) generators, and \( q = \frac{q}{2} = 3 \) diag(1;1;1) are defined such that \( \text{Tr}(a^b) = 2 \) and \( \text{Tr}(9^9) = 2 \), and \( N \) denotes the \( N \) charge for three Higgs multiplets.

The non-Hermitian gauge bosons

\[
P = \begin{bmatrix} W_1^+ & W_2^+ \end{bmatrix}, \quad P = \begin{bmatrix} 2Y \end{bmatrix}, \quad P = \begin{bmatrix} X^0 \end{bmatrix}, \quad P = \begin{bmatrix} W_3^+ \end{bmatrix}
\]

have the following masses [3]:

\[
M^2_W = \frac{1}{4} g^2 (u^2 + v^2) ;
M^2_Y = \frac{1}{4} g^2 (v^2 + !^2) ;
M^2_X = \frac{1}{4} g^2 (u^2 + !^2);
\]

(14)

We assume \( h_i h_i \) such that \( M_W \leq M_X \leq M_Y \). This statement is very important, because the new gauge bosons must be sufficiently heavy to keep consistency with low energy phenomenology.

As the triplet scalar \( \phi \) acquires a VEV, the symmetry SU(3) \(_L\) \(_U\) \(_L\) breaks down to SU(2) \(_L\) \(_U\) \(_L\) \(_U\). By matching the gauge coupling constants at the SU(3) \(_L\) \(_U\) \(_L\) \(_U\) breaking, the coupling constant of SU(3) \(_L\) \(_U\) \(_L\) \(_U\), \( g^0 \), is given by

\[
\frac{1}{g^2} = \frac{1}{3g^2} + \frac{6}{g^2}.
\]

(15)

Eq. (14) may be satisfied by a \( 3 \times 3 \) mixing angle \( \theta \)

\[
g^0 = \frac{P}{3g \cos \theta} = \frac{1}{3} gN \sin \theta.
\]

(16)

As in the SM we put \( g^0 = g \tan \theta \); hence we finally get

\[
gN = \frac{g}{3} (2 \sin \theta \ (M_Z)) = \frac{g}{3} \sin \theta \ (M_Z);
\]

(17)

In this model, \( \sin^2 \theta < \frac{3}{4} \), while in the minimal version [3]

\[
\sin^2 \theta < \frac{1}{4}.
\]

The neutral (Hermitian) gauge bosons have the \( 3 \times 3 \) mass matrix \( M^2 \)

\[
M^2 = \begin{bmatrix} 0 & u^2 + v^2 & \frac{\sqrt{2}}{2} (u^2 + v^2) \\
\frac{\sqrt{2}}{2} (u^2 + v^2) & \frac{1}{3} (u^2 + v^2) + \frac{2}{3} (v^2) & \frac{2}{3} (u^2 + v^2 + 2v^2) \\
\frac{2}{3} (u^2 + v^2) & \frac{2}{3} (u^2 + v^2 + 2v^2) & \frac{1}{2} (u^2 + v^2 + 2v^2)
\end{bmatrix}
\]

(18)

\[
M^2 = \begin{bmatrix} 0 & U^2 & V^2 \\
U^2 & \frac{1}{3} (u^2 + v^2) + \frac{2}{3} (v^2) & \frac{2}{3} (u^2 + v^2 + 2v^2) \\
V^2 & \frac{2}{3} (u^2 + v^2 + 2v^2) & \frac{1}{2} (u^2 + v^2 + 2v^2)
\end{bmatrix}
\]

(20)
with the notation $t = g_W = g$. This mass matrix can be diagonalized to obtain the eigenstate fields.

We can identify the photon field $A$ as well as the massive bosons $Z$ and $Z^0$:

$$
A = s_W W^3 + c_W \theta \left( \frac{t_W}{3} W^8 + 1 \right) \frac{t_W}{3} B A; \\
Z = c_W W^3 + s_W \theta \left( \frac{t_W}{3} W^8 + 1 \right) \frac{t_W}{3} B A; \\
Z^0 = 1 \left( \frac{t_W}{3} W^8 + \frac{t_W}{3} B \right)
$$

(21)

where the mass-squared matrix for $Z; Z^0$ is given by

$$
M^2 = \begin{pmatrix}
M_Z^2 & M_{ZZ^0}^2 \\
M_{ZZ^0}^2 & M_{Z^0}^2
\end{pmatrix}
$$

with

$$
M_Z^2 = \frac{g^2}{4c_W^2} (u^2 + v^2) = \frac{M_W^2}{c_W^2}; \\
M_{ZZ^0}^2 = \frac{g^2}{4c_W^2} \left( \frac{3}{4} \frac{4}{3} \right) u^2 \left( 1 + \frac{2\theta^2}{c_W^2} \right); \\
M_{Z^0}^2 = \frac{g^2}{4(3 \frac{4}{3} \theta)} \left( \frac{4}{3} \frac{4}{3} \right) \left( 1 + \frac{2\theta^2}{c_W^2} \right)^2
$$

(22)

(23)

Here we use the following notations: $s_W \sin \theta; c_W \cos \theta$ and $t_W \tan \theta$. From Eq.(23) we see that, the limit for $M_{ZZ^0}$ is

$$
M_{ZZ^0}^2 \to \frac{1}{3} \frac{2\theta^2}{c_W^2} \frac{M_Z^2}{M_{Z^0}^2};
$$

(24)

Diagonalizing the mass matrix gives the mass eigenstates $Z^1$ and $Z^2$ which can be taken as mixtures,

$$
Z^1 = Z \cos Z^0 \sin; \\
Z^2 = Z \sin + Z^0 \cos
$$

(25)

The mixing angle is given by

$$
\tan^2 = \frac{M_{Z^1}^2 M_{Z^2}^2}{M_{Z^1}^2 M_{Z^2}^2};
$$

(26)
where $M_{Z^1}$ and $M_{Z^2}$ are the physical mass eigenvalues

$$
M_{Z^1} = \frac{1}{2} M_{Z^0}^2 + 2 \left[ \left( M_{Z^0}^2 - M_Z^2 \right)^2 + 4 M_{Z^0}^2 \right]^{-1/2}, \quad (27)
$$

$$
M_{Z^2} = \frac{1}{2} M_{Z^0}^2 + 2 \left( M_Z^2 + \left[ (M_{Z^0}^2 - M_Z^2)^2 + 4 M_{Z^0}^2 \right]^{-1/2} \right); \quad (28)
$$

From Eq.(22) we see that $u^2 = v^2 (1 - 2 \sin^2 \theta)$. Here $W^Z$ and $Z^1$ correspond to the Standard Model charged and neutral gauge bosons, and there are new gauge bosons $Y$, $X^0$, and $Z^2$. A fit to precision electroweak observables gives a limit of mixing angle (see below) of $0.0021$ and from the symmetry-breaking hierarchy $\mu, \nu$, Eq. (14) and Eq. (23) give us

$$
M_{Y^+} = \frac{3}{2} \frac{4 \sin^2 \theta}{2} M_{Z^2} ; \quad 0.72 M_{Z^2} : \quad (29)
$$

IV. Charged and neutral currents

The interactions among the gauge bosons and fermions are read off from

$$
L_F = N \phi \left( (Q_{1L} + \tilde{Q}_{1N}) \right) + \bar{\nu}_e \left( (\tilde{Q}_{1L} + \tilde{Q}_{1N}) \right) \nu_e, \quad (30)
$$

where $R$ represents any right-handed singlet and $L$ any left-handed triplet or antitriplet.

The interactions among the charged vector fields with leptons are

$$
L_L^{cc} = \frac{1}{2} \left( \bar{\nu}_e \left( W^+ + \frac{W}{2} \right) + \frac{e}{2} \right) Y^0 + H. c. ; \quad (31)
$$

For the quarks we have

$$
L_q^{cc} = \frac{1}{2} \left( \left( u_{3L} \; d_{3L} + \bar{u}_{3L} \; \bar{d}_{3L} \right) \bar{W}^+ + \left( T_L \; d_{3L} + u_{3L} \; d_{0L} \right) Y^+ + \left( u_{3L} \; T_L \; d_{0L} \; d_{0L} \right) X^0 + H. c. \right); \quad (32)
$$

We can see that the interactions with the $Y^+$ and $X^0$ bosons violate the lepton number (see Eq. (31)) and the weak isospin (see Eq. (32)).

The electromagnetic current for fermions is the usual one

$$
Q_{\text{ef}} = f A ; \quad (33)
$$

where $f$ is any fermion with $Q_f = 0$; $1/2 = 3$; $1 = 3$ and the electric charge $e$ is identified as follows

$$
e = g \sin \theta_w ; \quad (34)
$$
The neutral current interactions can be written in the form

\[ L^{Nc} = \frac{g}{2Q_W} f \left[ a_{1L}(f)(1_5) + a_{1R}(f)(1 + 5) \right] E Z^1 + f \left[ a_{2L}(f)(1_5) + a_{2R}(f)(1 + 5) \right] E Z^2 \]  

(35)

The couplings of fermions with \( Z^1 \) and \( Z^2 \) bosons are given as follows:

\[ a_{1L,R}(f) = \cos \left[ T^3(f_{L,R}) \right] Q(f) \]
\[ + \frac{c^2}{2} \frac{3N(f_{L,R})}{(3 + 4s_W^2)^{1/2}} \frac{(3 + 4s_W^2)^{1/2}}{2c_{Q_W}} Q(f) \sin \left[ T^3(f_{L,R}) \right] \]

(36)

where \( T^3(f) \) and \( Q(f) \) are, respectively, the third component of the weak isospin and the charge of the fermion \( f \). Note that for the exotic quarks, the weak isospin is equal to zero. Eqs. (36) are valid for both left- and right-handed currents. Since the value of \( N \) is different for triplets and antitriplets, the \( Z^2 \) coupling to left-handed ordinary quarks is different for the third family, and thus flavor changing.

We can also express the neutral current interactions of Eq. (35) in terms of the vector and axial-vector couplings as follows:

\[ L^{Nc} = \frac{g}{2Q_W} f \left[ g_{1V}(f) - g_{1A}(f) \right] E Z^1 + f \left[ g_{2V}(f) - g_{2A}(f) \right] E Z^2 \]  

(37)

The values of these couplings are:

\[ g_{1V}(f) = \cos \left[ T^3(f_L) \right] 2s_W^2 Q(f) \]
\[ + \sin \left[ T^3(f_L) \right] \frac{c^2}{2} \frac{3N(f_L) + t_{Q_W}^2 N(f_R)}{(3 + 4s_W^2)^{1/2}} \frac{(3 + 4s_W^2)^{1/2}}{2c_{Q_W}} Q(f) \sin \left[ T^3(f_L) \right] \]

(38)

\[ g_{1A}(f) = \cos \left[ T^3(f_L) \right] \]
\[ + \sin \left[ T^3(f_L) \right] \frac{c^2}{2} \frac{3N(f_L) + t_{Q_W}^2 N(f_R)}{(3 + 4s_W^2)^{1/2}} \frac{(3 + 4s_W^2)^{1/2}}{2c_{Q_W}} Q(f) \sin \left[ T^3(f_L) \right] \]

(39)

\[ g_{2V}(f) = \cos \left[ T^3(f_L) \right] 2s_W^2 Q(f) \]
\[ + \sin \left[ T^3(f_L) \right] \frac{c^2}{2} \frac{3N(f_L) + t_{Q_W}^2 N(f_R)}{(3 + 4s_W^2)^{1/2}} \frac{(3 + 4s_W^2)^{1/2}}{2c_{Q_W}} Q(f) \sin \left[ T^3(f_L) \right] \]

(40)

\[ g_{2A}(f) = \cos \left[ T^3(f_L) \right] \]
\[ + \sin \left[ T^3(f_L) \right] \frac{c^2}{2} \frac{3N(f_L) + t_{Q_W}^2 N(f_R)}{(3 + 4s_W^2)^{1/2}} \frac{(3 + 4s_W^2)^{1/2}}{2c_{Q_W}} Q(f) \sin \left[ T^3(f_L) \right] \]
The values of $g_{V}$; $g_{A}$ and $g_{S}$; $g_{A}$ are listed in Tables 1 and 2, where the third generation is assumed to belong to the triplet. To get some indication as to why the top quark is so heavy, we have to treat the third generation differently from the first two as in Refs [4] and [3].

| Table 1: The $Z^1$ $ff$ couplings in the 331 model with right-handed neutrinos. |
|---|---|---|
| $f$ | $g_{LV}(f)$ | $g_{A}(f)$ |
| e; i; | $(1/2 + 2s_{W}^{2})(\cos(3/4s_{W}^{1})^{1/2})$ | $\cos(3/4s_{W}^{1})^{1/2}$ |
| e; i; | $\frac{1}{2}(\cos + \sin(3/4s_{W}^{2}1/2))$ | $\frac{1}{2}(\cos + \sin(3/4s_{W}^{2}1/2))$ |
| t | $\frac{1}{2} + \frac{4s_{W}^{2}}{3}$ | $\frac{1}{2} + \frac{4s_{W}^{2}}{3}$ |
| b | $(1/2 + 2s_{W}^{2})$ | $(1/2 + 2s_{W}^{2})$ |
| u; c | $(1/2 + 2s_{W}^{2})$ | $(1/2 + 2s_{W}^{2})$ |
| d; s | $(1/2 + 2s_{W}^{2})$ | $(1/2 + 2s_{W}^{2})$ |
| T | $(1/2 + 2s_{W}^{2})$ | $(1/2 + 2s_{W}^{2})$ |
| $d_{1}^{0}$ | $(1/2 + 2s_{W}^{2})$ | $(1/2 + 2s_{W}^{2})$ |

| Table 2: The $Z^2$ $ff$ couplings. |
|---|---|---|
| $f$ | $g_{LV}(f)$ | $g_{A}(f)$ |
| e; i; | $(1/2 + 2s_{W}^{2})(\sin + \cos(3/4s_{W}^{2}1/2))$ | $(1/2 + 2s_{W}^{2})(\sin + \cos(3/4s_{W}^{2}1/2))$ |
| e; i; | $\frac{1}{2}(\sin + \cos(3/4s_{W}^{2}1/2))$ | $\frac{1}{2}(\sin + \cos(3/4s_{W}^{2}1/2))$ |
| t | $\frac{1}{2} + \frac{s_{W}^{2}}{3}$ | $\frac{1}{2} + \frac{s_{W}^{2}}{3}$ |
| b | $(1/2 + 2s_{W}^{2})$ | $(1/2 + 2s_{W}^{2})$ |
| u; c | $(1/2 + 2s_{W}^{2})$ | $(1/2 + 2s_{W}^{2})$ |
| d; s | $(1/2 + 2s_{W}^{2})$ | $(1/2 + 2s_{W}^{2})$ |
| T | $(1/2 + 2s_{W}^{2})$ | $(1/2 + 2s_{W}^{2})$ |
| $d_{1}^{0}$ | $(1/2 + 2s_{W}^{2})$ | $(1/2 + 2s_{W}^{2})$ |

We can realize that in the limit $\theta = 0$ the couplings to $Z^1$ of the ordinary leptons and quarks are the same as in the SM. Furthermore, the electric charge defined in Eq. [34] agrees with the SM. Because of this, we can test the new phenomenology beyond the SM. In this model, the exotic quarks carry electric charges 2/3 and -1/3, respectively, similarly to ordinary quarks. Consequently, the exotic quarks can mix with the ordinary ones. This type of mixing gives the flavor changing neutral currents (FCNCs). These FCNCs will be induced due to breakdown of the GIM mechanism. This type of situation has been discussed previously and bounds on the mixing strengths can be obtained from the non-observation of FCNC's in the experiments beyond those predicted by the SM [10].
In the PPF model, the coupling strength of $Z^2$ to quarks is much stronger than that of leptons due to the factor $1 - \frac{1}{4}s^2$. Therefore, low-energy experiments such as neutrino-nucleus scattering and atomic parity violation measurements would be useful to further constrain the model [3]. However, from Tables it is easy to see that this does not happen in our model.

In our model, the interactions with the heavy charged and neutral (non-Hermitian) vector bosons $Y^+; X^0$ violate the lepton number and the weak isospin. Because of the mixing, the mass eigenstate $Z^1$ now picks up flavor-changing couplings proportional to $\sin^2$. However, since $Z^0$ mixing is constrained to be very small, evidence of 3-3-1 FCNCs can only be probed indirectly at present via the $Z^2$ couplings.

V. Constraints on the $Z$ $Z^0$ mixing angle and the $Z^2$ mass

There are many ways to get constraints on the mixing angle and the $Z^2$ mass. Below we present a simple one. A constraint on the $Z$ $Z^0$ mixing can be followed from the $Z$-decay data. Hence we now calculate a $Z$ width in this model.

1. $Z$ decay modes

The tree-level expression for the partial width for the $Z \to ff$ where $f = e, \mu, \tau, u, d, ...$ is given by [11,20]:

$$\Gamma_{\text{tree}}(Z \to ff) = \frac{G_F}{6} \frac{M_W^3}{2} N_C^f \left[ (g_{1V}(f))^2 + (g_{1A}(f))^2 \right];$$  \hspace{1cm} (38)

where $N_C^f$ is the color factor. From Eq. (38) we get

$$\Gamma_{\text{tree}}(Z \to ll) = \frac{G_F}{6} \frac{M_W^3}{2} N_C^l \sin^2 \frac{1}{2} \left( \cos \theta \sqrt{\frac{3}{4}} + \frac{1}{4} + \frac{1}{2} \frac{2^2}{s^2} \right);$$  \hspace{1cm} (39)

where $f = e, \mu, \tau$.

To get results consistent with experiments, the QCD and electroweak radiative corrections have to be included. The weak radiative corrections that depend upon the assumptions of the electroweak theory and on the value of the $M_{\text{top}}$ and $M_{\text{Higgs}}$ are accounted for by absorbing them into the coupling, which are then called the effective coupling $g_{1V}$ and $g_{1A}$. Then Eq. (39) becomes [11,20]:

$$\Gamma_{\text{tree}}(Z \to ff) = \frac{G_F}{6} \frac{M_W^3}{2} N_C^f \left[ (g_{1V}(f))^2 + (g_{1A}(f))^2 \right] (1 + Q_{\text{ED}})(1 + Q_{\text{CD}});$$  \hspace{1cm} (40)

where $f = e, \mu, \tau, Q_{\text{ED}} = 3 Q_f^2 = 4$ and $Q_{\text{CD}} = 0$ for leptons and $Q_{\text{CD}} = (s = ) + 1.409 (s = )^2$ for quarks, $s$ being the strong coupling constant at $= M_Z$. Here $[20] \frac{1}{2} = M_W = M_{\text{Z}^0}$, and hence from Eqs. (14,27) we can get its explicit expression, and see that $s^2$ depends on the VEVs and $s_W$. In the limit $s^2$ high, i,
we have $I = 1$. We will ignore the effects due to the combination of mixing and radiative corrections since both are very small, i.e.,

$$g_{1V}(f) = \cos \theta_W g_{1V}^\text{SM}(f) + \sin \theta_W \frac{c_W^2}{(3/4 g_W^2)^{1/2}} (G_N (f_L) + t_W^2 N (f_R))$$

$$g_{1A}(f) = \cos \theta_W g_{1A}^\text{SM}(f) + \sin \theta_W \frac{c_W^2}{(3/4 g_W^2)^{1/2}} (G_N (f_L) + t_W^2 N (f_R))$$

The effective coupling constants depend on the fermion $f$ and on the renormalization scheme [14, 15, 16]:

$$g_{1V}^\text{SM}(f) = \frac{q}{1 + (T_{3L} (f)) 2Q (f) \frac{s_W^2}{2}}; g_{1A}^\text{SM}(f) = \frac{q}{1 + T_{3L} (f)}.$$  

For the case $f = b$, where additional vertex corrections are important, one must replace [14] by $b = 1 (1 + \frac{4}{3} \frac{1}{3})$ and $s_W$ by $s_q (1 + \frac{2}{3} \frac{1}{3})$. Here $s_q$ is the effective $s_{w}^2$ [13, 15]: $s_q^2 = (1 + \frac{2}{3} \frac{1}{3}) s_W^2$, and $s_q^2$ is defined by [14, 15]:

$$s_q^2 = \frac{Q}{2 G_F M_{Z}^2}$$

By assuming the masses of all the ordinary fermions except the top quark to be much lighter than the mass of the Z boson and the masses of the exotic fermions to be much heavier than the mass of the Z boson, the total width of the Z boson is given as

$$\text{total} = (Z! \text{ all}) = \frac{G_F}{6} M_Z^3 \frac{\cos^2 \theta_W}{2^{1+n}} \frac{1}{\cos^2 \text{SM}_{total}}$$

$$+ 3 \sin 2 \theta W + \frac{3}{2} \frac{4 g}{4 g_W^2} + \frac{1}{13} \frac{D}{4 \frac{3}{4 g_W^2}} + \frac{E}{13} \frac{A}{4 \frac{3}{4 g_W^2}}$$

$$+ \frac{F}{12} \frac{A}{4 \frac{3}{4 g_W^2}} + O (\sin^2 \theta W);$$

where

$$D = 5 \frac{20}{3} s_W^2 + \frac{272}{9} s_W^4; E = 3 \frac{20}{3} s_W^2 + \frac{128}{9} s_W^4;$$

$$F = 33 \frac{332}{3} s_W^2 + \frac{1808}{9} s_W^4; G = \frac{3}{18} \frac{4 g_W^2}{\beta (2 s_W^2)} \frac{4 g_W^2}{\beta (3 s_W^2)} \frac{4 g_W^2}{\beta (3 s_W^2)}$$

and

$$\text{total} = \frac{X}{\text{SM}} \frac{g_{1V}^\text{SM}(f) + g_{1A}^\text{SM}(f)}{f} (1 + \frac{\frac{2}{3} \frac{1}{3}}{\text{QED}}) (1 + \frac{\frac{2}{3} \frac{1}{3}}{\text{QCD}}).$$
We get then the ratio

\[
R_{331} = \left( \frac{Z!}{11} \right)_\text{total} = R_{1}^{\text{SM}} \cdot \frac{8}{1} \cdot 2 \tan^2 \theta + \frac{3}{4} \frac{4^2}{W} + \frac{1}{4} \left( 3 \cdot 4^2 \right)_{\text{SM}} \cdot \frac{1}{2} \tan^2 \theta + 1
\]

\[
\theta \text{ G} + \frac{3}{4} \frac{4^2}{W} + \frac{1}{11} \frac{4^2}{W} + \phi_{\text{CD}} \theta \text{ G} + \frac{4}{3} \frac{4^2}{W} \left( \frac{12}{A \times 0} + 0 \left( \tan^2 \theta \right) \right)
\]

where \( R_{1}^{\text{SM}} \) denotes the SM result: \( R_{1}^{\text{SM}} = 0.03362 \) for \([11,19]\), \( M_Z = 128.87 \), \( s(M_Z) = 0.118 \), and \([17]\) \( s(M_Z) = 0.2333 \). Taking the experimental result in \([11]\) \( (3.367 \pm 0.006) \% \), we obtain the limit for the mixing angle

\[
0.0021 \quad 0.00132
\]

As is known, recent results on left-right asymmetry \( A_{LR} \) at SLD \([14]\) and \( R_b \) \( (Z! bb) \) = \( (Z! \text{ hadrons}) \) measured at LEP \([16]\) indicate a possible disagreement at the 2 to 2.5 level with the SM prediction \( R^{\text{SM}}_b = 0.215 \) for \( M = 175 \text{ GeV} \). If confirmed, this could indicate new physics coupled in a different way to the third generation. Therefore, it is interesting to consider \( R_b \) in this model. After some manipulations we get

\[
R_{331}^{331} = \left( \frac{Z!}{bb} \right)_{\text{hadrons}} = R_{b}^{\text{SM}} \cdot \frac{8}{1} \cdot 2 \tan^2 \theta + \frac{3}{4} \frac{4^2}{W} + \frac{9}{9} \frac{4^2}{W} \cdot A_{b} \]

\[
\theta \text{ G} + \frac{3}{4} \frac{4^2}{W} + \frac{113}{4} \frac{4^2}{W} + \phi_{\text{CD}} \theta \text{ G} + \frac{12}{3} \frac{4^2}{W} \left( \frac{12}{A \times 0} + 0 \left( \tan^2 \theta \right) \right)
\]

where \( R_{b}^{\text{SM}} \) is the SM result \([13,18]\): \( R_{b}^{\text{SM}} = 0.215 \) and

\[
A_{b} = \frac{3}{2} \left( 1 + \frac{4}{3} \frac{s^2}{W} + \frac{8}{9} \frac{s^4}{W} \right); B = \frac{15}{2} \left( 14 \frac{s^2}{W} + \frac{44}{3} \frac{s^4}{W} \right);
\]

\[
C_{b} = \frac{33}{2} \left( 38 \frac{s^2}{W} + \frac{140}{3} \frac{s^4}{W} \right); F_{b} = \frac{15}{3} \left( 116 \frac{s^2}{W} + \frac{512}{9} \frac{s^4}{W} \right);
\]

Substituting Eq. (43) into Eq. (44) we get

\[
0.21495 \quad R_{b}^{331} \quad 0.21564
\]

This result still disagrees with the recent experimental value \( R_b = 0.2192 \pm 0.0018 \) measured at LEP \([14]\). We hope, however, with the inclusion of new heavy particle
loop effects like exotic quarks, Higgs scalars or of new box diagram s, this result will be improved and consistent with the experimental data (for recent works on this direction see [21]).

2. Neutrino-electron scattering

The motivation for focusing on the neutrino neutral current scatterings is the following: From the theoretical point of view these reactions are basic processes free from the complications of strong interactions and can be used to determine the parameters of the theories. We emphasize that in the PPF model, these processes are almost the same as in the SM (for this purpose only neutrino-nucleus scattering and atomic parity violation, etc, are suitable). The effective four-fermion interactions relevant to four-fermion neutral current processes, in this model, are presented as follows [22]:

\[
I_{\text{eff}}^e = \frac{G_F}{2} \left( L \right) \left[ C_L^e f \left( L \cdot g \right) f + C_R^e f \left( L \cdot g \right) f \right];
\]

where

\[
C_L^e = 2 \left( g_{1V} (\phi) + g_{1A} (\phi) \right) \left( g_{1V} (\phi) + g_{1A} (\phi) \right) + \frac{M_{Z}^2}{M_{Z'}^2} \left( g_{2V} (\phi) + g_{2A} (\phi) \right) \left( g_{2V} (\phi) + g_{2A} (\phi) \right);
\]

\[
C_R^e = 2 \left( g_{1V} (\phi) + g_{1A} (\phi) \right) \left( g_{1V} (\phi) + g_{1A} (\phi) \right) + \frac{M_{Z}^2}{M_{Z'}^2} \left( g_{2V} (\phi) + g_{2A} (\phi) \right) \left( g_{2V} (\phi) + g_{2A} (\phi) \right);
\]

Then the total cross-sections for elastic scattering processes are given, respectively,

\[
( e ) = \frac{2m_e E G_F^2}{6} \left[ \frac{M_{Z}^2}{M_{Z'}^2} \right] \left[ \frac{1}{2} \cos^2 \theta + \frac{1}{3} \frac{2s^2}{4s^2} \sin^2 2\theta \right] \left[ \frac{1}{2}\left( 1 + \frac{2s^2}{4s^2} \right)^2 + 4s^4 \right];
\]

\[
( e ) = \frac{2m_e E G_F^2}{6} \left[ \frac{M_{Z}^2}{M_{Z'}^2} \right] \left[ \frac{1}{2} \cos^2 \theta + \frac{1}{3} \frac{2s^2}{4s^2} \sin^2 2\theta \right] \left[ \frac{1}{2}\left( 1 + \frac{2s^2}{4s^2} \right)^2 + 4s^4 \right];
\]

In the above equations \( m_e \) is the mass of the electron and \( E \) is the energy of the incident (anti)neutrino. From Eq.(46) and Eq. (47) we get the ratio of the cross-sections:

\[
R = \frac{\left( e \right)}{\left( e \right)} = \frac{3\left( 1 + \frac{2s^2}{4s^2} \right)^2 + 4s^4}{\left( 1 + \frac{2s^2}{4s^2} \right)^2 + 12s^4} = 1.1425;
\]
which is the same as in the SM [24]. In this model, $\theta$ is a free parameter. However, we can follow Degrassi, Fanchiotti and Sidlin to put [20,23] $\theta = 1 + t$, where

$$t' = 0.0031 \frac{m_t}{100 \text{ GeV}}^2$$

Taking an average value for $m_t = 174 \text{ GeV}$ [19], the running $s_W^2 = 0.21$ and the experimental results on $-\frac{1}{\cos^2 \theta} = (1.17 \pm 0.206) \times 10^{42} \text{ cm}^2$= GeV and $-\frac{1}{\cos^2 \theta} = (1.8 \pm 0.2) \times 10^{42} \text{ cm}^2$= GeV given in [24], the allowed range of the new gauge boson masses are $M_{Z^2}$ = 250 GeV and 330 GeV, respectively. Thus Eq. (29) gives a limit for the masses of the gauge bosons $Y/X^0: M_Y = 180 \text{ GeV}$ and 230 GeV, respectively.

In our model, the free parameters are $\sin^2 \theta$ and $M_{Z^2}$, and which are constrained from experiment. $M_{Z^2}$ is related by Eq. (28) where $M_{Z^2} = M_W = \cos \theta$ is the prediction for the Z mass in the absence of mixing $\theta = 0$. It interesting to consider the special case $\theta = 0$. We have then $M_{Z^2} = M_Z$; $n = 1$ and $s_W^2 (M_{Z^2}) = s_W^2 (M_Z)$. From Eq. (46) and Eq. (47) we get bounds for the new gauge bosons $Z^2$ mass: $M_{Z^2} = 270$ GeV and 350 GeV, respectively. Thus, the only way to get a rigorous bound of $M_{Z^2}$ is through low energy processes as considered in this paper. Our bounds could be improved significantly with more precise data.

V I. Discussion

In this paper, we presented a further development of the 331 model with right-handed neutrinos. We have shown that this model has some advantages over the original 331 model. Firstly, in the Higgs sector, we need only three Higgs triplets for generating fermion and gauge bosons masses as well as for breaking the gauge symmetry. Moreover in the limit $\theta = 0$, all couplings of the ordinary fermions to $Z^1$ boson are the same as in the SM. In this model there is no limit for the Weinberg angle $\sin^2 \theta < \frac{9}{4}$.

In our model, the lepton number is violated both in the Higgs sector and in the heavy charged and neutral (non-Hermitian) vector bosons interactions. We also have flavor-changing neutral currents in the quark sector coupled to the new $Z^2$ boson. All the heavy bosons have masses depending on $h$ and this VEV is in principle, arbitrary.

Processes like neutrinoless double beta decay and neutrino oscillations ($\beta \beta; a \beta b$), etc, are typical ones in this model.

Finally, we emphasize again that experimental data from the $Z$-decay and $e^{-}e^{+}$ scattering processes allows us to estimate the mixing angle and the new gauge boson masses. To get stronger limits we have to consider other parameters such as the left-right cross section asymmetry, $N_{e}$, etc, and we will publish these results elsewhere.

To summarize, we have shown that because of the $Z^0$ mixing there is a modification to the $Z^1$ coupling proportional to $\sin \theta$, and the $Z$-decay gives $0.0021 \pm 0.000132$. The data from neutrino neutral current elastic scatterings shows that mass
of the new neutral gauge boson $M_{Z'}$ is in the range $300 \text{ GeV}$, and from the symmetry-breaking hierarchy we get: $M_{Y'} M_{X'} 0.072 M_{Z'} 220 \text{ GeV}$. We think that new physics can arise at not too high energies.

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