The Effect of Deformation on the Twist Mode

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Abstract

Using $^{12}$C as an example of a strongly deformed nucleus we calculate the strengths and energies in the asymptotic (oblate) deformed limit for the isovector twist mode operator $[rY^{(1)}l]_{\lambda=2}^{\lambda=2}t_+$ where $l$ is the orbital angular momentum. We also consider the $\lambda = 1$ case. For $\lambda = 0$, the operator vanishes. Whereas in a $\Delta N = 0$ Nilsson model the summed strength is independent of the relative $P_{3/2}$ and $P_{1/2}$ occupancy when we allow for different frequencies $\omega_i$ in the x, y, and z directions there is a weak dependency on deformation.
I. INTRODUCTION

In previous works we showed that at the $\Delta N = 0$ Nilsson level the summed strengths for the various isovector dipole modes $rY^1_t$, $[rY^1 s]_t^\lambda$, and $[rY^1 l]_t^\lambda$ were independent of deformation, or more precisely of the valence occupancy. This is in striking contrast to the scissors mode excitation induced by the operator $L_\pi - L_\nu$. For the latter operator, the summed strength for $^{12}$C increased by a factor of three in going from the spherical limit to the $\Delta N = 0$ asymptotic (oblate) limit. Likewise, the spin magnetic moment dipole strength shows a great sensitivity to deformation. The strength is quite large in the spherical limit but vanishes in the asymptotic (oblate) limit. The $\Delta N = 1$ strength for the stretched state in $^{12}$C will obviously depend on the details of the $P_{3/2}$ and $P_{1/2}$ occupancy. You cannot excite the stretched $4^-$ state in $^{12}$C by exciting a $P_{1/2}$ nucleon to the 1s-0d shell. Hence the summed strength is proportional to the $P_{3/2}$ occupancy.

In this work we remove the $\Delta N = 0$ restrictions and by using deformed harmonic oscillator wavefunctions in order to obtain the deformation dependence i.e. the dependence on $\delta$ of the above dipole modes. We will especially focus on the ‘twist mode’ an M2 orbital mode which has been given a picturesque geometric description by Holzwarth et. al [1]. The problem of separating the M2 orbital strength from the M2 spin strength has been discussed by Richter [2] in the context of a detailed calculation by Drozdz et. al [3]. We shall consider also consider the mode $[rY^1 l]_t^\lambda=1$. Note that the operator $[rY^1 l]_t^\lambda=0$ is zero.

II. THE CALCULATION FOR $^{12}$C

We are considering the asymptotic oblate limit for $^{12}$C. The occupied states can be described by the number of quanta in the x,y, and z directions ($N_x, N_y, N_z$) these are $(0,0,0)$ (1,0,0) and (0,1,0). The wave functions are

$$(0,0,0) \frac{N_{(0,0,0)}}{\sqrt{b_z b_y b_x}} e^{x(\frac{-x^2}{2b_x^2} - \frac{y^2}{2b_y^2} - \frac{z^2}{2b_z^2})}$$

$$(1,0,0) \frac{N_{(1,0,0)}}{\sqrt{b_z b_y b_x}} x e^{x(\frac{-x^2}{2b_x^2} - \frac{y^2}{2b_y^2} - \frac{z^2}{2b_z^2})}$$

where

$N_{(0,0,0)} = 1/\pi^{3/2}$

$N_{(1,0,0)} = 2/\pi^{3/2}$ etc

In the above $b_x$ is the oscillator length parameter $b_x^2 = \hbar/m\omega_x$ e.t.c.

Except for the fact that we now use deformed oscillator wave functions the calculations proceed in the same manner as described in the previous publications [4,5]
III. PREVIOUS RESULTS

In the $\Delta N = 0$ calculations we found that the summed strength for the electric dipole operator $rY^1t_+$ was $4\pi SUM/b^2 = 27$. For the spin dipole the sum was 20.25 which was shown to be $27 \vec{s} \cdot \vec{s}$ where of course $\vec{s} \cdot \vec{s} = 3/4$ for a single nucleon. For the orbital dipole $[Y1\vec{l}]^\lambda t_+$ the summed strength (including a sum over $\lambda$) was 48 which could be understood as the dipole value multiplied by $\vec{l} \cdot \vec{l}$. For the p shell the ordinary dipole contribution is 24 and $\vec{l} \cdot \vec{l}$ is two - this explains the result of 48.

A further breakdown of the orbital dipole into the $\lambda$ components is the second work on this subject \[4,5\] that gave the following results both in the spherical and asymptotic (oblate) limits.

| $\lambda$ | $4\pi SUM$ |
|-----------|-------------|
| 0         | 0           |
| 1         | 18          |
| 2         | 30          |

For $\lambda = 0$ the operator simply vanishes. It is proportional to $\vec{r} \cdot \vec{l}$ and one does not have any component of the angular momentum along the radial direction. Note that the $\lambda = 2$ to $\lambda = 1$ strength is in the ratio $(2\lambda + 1)_{\lambda=2}/(2\lambda + 1)_{\lambda=1} = 30/18$.

We use the results in this section as taking off points. In the next section, we introduce explicit deformation effects coming from the fact that $\omega_x$ and $\omega_z$ (or $b_x$ and $b_z$) are different. Of course the results that we obtain should reduce to the ones in this section when the frequencies in the x, y, and z direction are taken as equal to each other.

IV. DEFORMED OSCILLATOR MODEL WITH MOTTELSON CONDITIONS:

We use the Mottelson conditions to get the deformation parameters for $^{12}\text{C}$. They are

$$\Sigma_x \omega_x = \Sigma_y \omega_y = \Sigma_z \omega_z$$

(1)

where $\Sigma_x = \text{Sum}(N_x + 1/2)$ where for a given state $N_x$ is the number of quanta in the x direction etc. The occupied states $(N_x, N_y, N_z)$ are (0,0,0), (1,0,0), and (0,1,0) giving values of $\Sigma_x = 10$, $\Sigma_y = 10$, and $\Sigma_z = 6$. This yields the correct result that $^{12}\text{C}$ is oblate and strongly deformed. We also have

$$\hbar \omega_x = \hbar \omega_y = \frac{6}{10} \hbar \omega_z$$

(2)
We also introduce the deformation parameter $\delta$ (which Bohr and Mottelson called $\delta_{\text{OSC}}$) defined by

\begin{equation}
(h\omega_x)^2 = (h\omega_0(\delta))^2(1 + \frac{2}{3}\delta) \tag{3}
\end{equation}

\begin{equation}
(h\omega_z)^2 = (h\omega_0(\delta))^2(1 - \frac{4}{3}\delta) \tag{4}
\end{equation}

The volume conservation condition is

\begin{equation}
\bar{h}\omega_x\bar{h}\omega_y\bar{h}\omega_z = (\bar{h}\bar{\omega}_0)^3 \tag{5}
\end{equation}

which leads to

\begin{equation}
\bar{h}\omega_0(\delta) = [(1 + \frac{2}{3}\delta)^2(1 - \frac{4}{3}\delta)]^{-1/6}\bar{h}\bar{\omega}_0 \tag{6}
\end{equation}

We choose $\bar{h}\bar{\omega}_0$ (which is independent of $\delta$) to be 15 MeV. We then get the following values

\begin{align*}
\bar{h}\omega_0 &= 15.966 MeV \tag{7} \\
\bar{h}\omega_x &= 12.651 MeV \tag{8} \\
\bar{h}\omega_z &= 21.086 MeV \tag{9}
\end{align*}

The oscillator length parameters have the following values

\begin{align*}
b_x^2 &= 3.2775 \tag{10} \\
b_z^2 &= 1.9665 \tag{11} \\
b_0^2 &= 2.7643 \tag{12}
\end{align*}

V. DISCUSSION OF RESULTS

The results for the energies and excitation strengths for the $\lambda = 2$ twist mode are given in Table I, and for the corresponding $\lambda = 1$ mode in Table II. We see that the main effect of deformation on the $\lambda = 2$ twist mode is not so much to increase the overall strength, but rather to redistribute it. It is true that when we go from the spherical limit to the asymptotic deformed limit the strength, or more precisely $4\pi \text{strength}/b_0^2$ increases from 30 to 38.784, a 30 percent rise but this is small compared to the corresponding factor of three increase for the scissors mode. However, as shown in Table I, whereas in the spherical oscillator limit all the strength would be at an excitation energy of $\bar{h}\bar{\omega}_0 = 15$ MeV, we now have the $\Delta N = 1$ strength split into three parts at 12.651, 21.086, and 29.251 MeV. There is also some $\Delta N = 3$ strength at 46.388 MeV and 54.823 MeV. For the $\lambda = 1$ mode there is a similar redistribution of strength, as seen in Table II.
VI. EXPRESSIONS IN TERMS OF THE DEFORMATION PARAMETER $\delta$

For the $\lambda = 2$ (TWIST) mode up to second order in $\delta$, the values of $4\pi/b^2$ summed strengths for $\delta = -0.55814$ are for $\Delta N = 1$

$$30(1 - \frac{2}{15}\delta + \frac{133}{360}\delta^2) = 35.685$$  \hspace{1cm} (13)

and for $\Delta N = 3$

$$\frac{33}{2}\delta^2 = 5.140$$  \hspace{1cm} (14)

Note that for $\lambda = 2$, $\Delta N = 1$ there are terms linear in $\delta$ but not for $\lambda = 2$, $\Delta N = 3$. We find that at this order the $\Delta N = 1$ result is good (35.685 compared to exact value of 35.558) but for the $\Delta N = 3$ result we would need to consider higher order terms for such a large deformation. (5.140 compared to exact value of 3.226)

For the $\lambda = 1$ mode up to second order in $\delta$ we obtain

$$18(1 + \frac{35}{72}\delta^2) = 20.725$$  \hspace{1cm} (15)

$$\frac{21}{2}\delta^2 = 3.271$$  \hspace{1cm} (16)

In contrast to the $\lambda = 2$ case, for $\lambda = 1$ there are no terms linear in $\delta$. Again for the $\Delta N = 1$ the expansion is good (20.725 compared to exact value of 20.156) but for the $\Delta N = 3$ result we would need to consider higher order terms for such a large deformation. (3.271 compared to exact value of 2.751)

VII. COMPARISON WITH OTHER MODE - ENERGY WEIGHTED SUM RULE

The twist excitation operator is $[rY^1\hat{l}]^{\lambda=2}t$. The scissors mode operator is $(\hat{L}_\pi - \hat{L}_\nu)$. The latter mode will not be excited unless there are open shells of both neutrons and protons. The twist mode on the other hand can be excited in a closed shell nucleus. Indeed measurements attempting to separate the orbital M2 from the spin M2 in $^{48}$Ca and $^{90}$Zr were performed by P. von Neumann-Cosel et. al. [6] This already indicates that the twist mode is less sensitive to deformation than the scissors mode. The scissors mode depends on deformation for its very existence. As just mentioned whereas in $^{12}$C when we go from the spherical limit to the asymptotic (oblate) limit the summed scissors mode strength increase by a factor of three whereas the twist mode strength increases by only 30 percent.

We next compare the twist mode to the ordinary dipole mode $rY^1t$. For the latter mode even with deformation there will be no $\Delta N = 3$ excitation for our simple deformed...
oscillator hamiltonian $P^2 + \frac{1}{2}m\omega_x x^2 + \frac{1}{2}m\omega_y y^2 + \frac{1}{2}m\omega_z z^2$. This is because the dipole operator components are basically $x$, $y$, and $z$ and as such can only excite one quantum.

Whereas in the case of axial symmetry the twist mode is split into three $\Delta N = 1$ parts and two $\Delta N = 3$ parts, the ordinary dipole is split only into two parts at excitation energies of $\hbar\omega_x$ and $\hbar\omega_z$, with twice the strength in the former.

The energy weighted strength for the ordinary dipole does not change as we go from spherical to the asymptotic (oblate) limit. This is because the dipole operator components $x$, $y$, and $z$ commute with the potential energy term $\frac{1}{2}m\omega_x x^2 + \frac{1}{2}m\omega_y y^2 + \frac{1}{2}m\omega_z z^2$. Furthermore the ordinary strength does not change either. We can see this by the fact that the ordinary strength is proportional to $\frac{1}{\omega_x} + \frac{1}{\omega_y} + \frac{1}{\omega_z}$. When we calculate the EWSR we get rid of these energy factors.

The Energy Weighted Strengths for the twist mode and corresponding $\lambda = 1$ mode on the other hand are much greater in the deformed limit than in the spherical limit. For $\lambda = 2$ the corresponding values are 854.136 MeV and 450 MeV, while for $\lambda = 1$ they are 516.622 MeV and 270 MeV. The reason for this is that the operator $[rY^1\ell^\lambda]$ does not commute with the potential energy terms unless all three frequencies $\omega_x$, $\omega_y$, and $\omega_z$ are the same.

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TABLE I. Results for Operator $[rY^{1+}]^{\lambda=2}$ (Twist Mode)

a) Symbolic

| Excitation Energy | $4\pi$ Strength |
|-------------------|-----------------|
| $\hbar\omega_x$   | $\frac{114b_x^4 + 18b_x^2 - 66b_x^2b_z^2}{8b_x^2}$ |
| $\hbar\omega_z$   | $9b_z^2 + 6b_z^4$ |
| $2\hbar\omega_z - \hbar\omega_x$ | $\frac{6b_z^4 + 6b_z^4 + 12b_z^4}{4b_x^2}$ |
| $\Delta N = 1$ Sum | \(\frac{9b_x^2}{2} + \frac{6b_z^2}{2} + \frac{15b_z^4}{2b_x^2} + 6b_z^2\) |
| $2\hbar\omega_x + \hbar\omega_z$ | \(\frac{15b_x^2}{2} - 15b_x^2 + \frac{15b_z^4}{2b_x^2}\) |
| $2\hbar\omega_z + \hbar\omega_x$ | \(\frac{9b_x^2}{2} - 9b_z^2 + \frac{9b_z^4}{2b_x^2}\) |
| $\Delta N = 3$ Sum | \(-\frac{39b_x^2}{2} + 3b_z^2 + \frac{9b_z^4}{2b_x^2} + 12b_z^4\) |

b) Numerical (Deformed Oscillator)

| Excitation Energy (MeV) | $4\pi$ Strength/$b_0^2$ | Spherical limit ($4\pi$ Strength/$b_0^2$) |
|-------------------------|--------------------------|------------------------------------------|
| 12.651                  | 12.520                   | 9                                        |
| 21.086                  | 18.259                   | 15                                       |
| 29.521                  | 4.779                    | 6                                        |
| $\Delta N = 1$ Sum     | 35.558                   | 30                                       |
| 46.388                  | 2.372                    | 0                                        |
| 54.823                  | 0.854                    | 0                                        |
| $\Delta N = 3$ Sum     | 3.226                    | 0                                        |
TABLE II. Results for Operator $[rY^1][\lambda=1$

| Excitation Energy       | 4 $\pi$ Strength | $\hbar\omega$ |
|--------------------------|------------------|--------------|
| $\hbar\omega_x$         | $\frac{33b_x^2}{4} - \frac{3}{2}b_x^2 + \frac{9}{4}b_x^4$ | $\frac{3}{2}b_x^2$ |
| $\hbar\omega_z$         | $\frac{3}{4}b_x^4$ | $\frac{3}{2}b_x^4$ |
| $2\hbar\omega_z - \hbar\omega_x$ | $\frac{3}{2}b_x^2 + 3b_x^2 + \frac{4}{2}b_x^4$ | $3\frac{b_x^4}{b_x^2} + \frac{30}{4}b_x^2 + \frac{3}{2}b_x^2 + \frac{15}{4}b_x^4$ |
| $\Delta N = 1$ Sum      | $3\frac{b_x^4}{b_x^2} + \frac{30}{4}b_x^2 + \frac{3}{2}b_x^2 + \frac{15}{4}b_x^4$ | $6b_x^2 + 6\frac{b_x^4}{b_x^2} - 12b_x^2$ |
| $2\hbar\omega_x + \hbar\omega_z$ | $6b_x^2 + 6\frac{b_x^4}{b_x^2} - 12b_x^2$ | $9b_x^2 + \frac{9}{2}b_x^2 - 9b_x^2$ |
| $2\hbar\omega_x + \hbar\omega_x$ | $9b_x^2 + \frac{9}{2}b_x^2 - 9b_x^2$ | $-\frac{15}{2}b_x^2 - 3b_x^2 + \frac{9}{2}b_x^4 + 6\frac{b_x^4}{b_x^2}$ |

b) Numerical (Deformed Oscillator)

| Excitation Energy(MeV) | $4\pi$ Strength/$b_0^2$ | Spherical limit ($4\pi$ Strength/$b_0^2$) |
|-------------------------|--------------------------|------------------------------------------|
| 12.651                  | 9.675                    | 9                                        |
| 21.086                  | 5.928                    | 3                                        |
| 29.521                  | 4.553                    | 6                                        |
| $\Delta N = 1$ Sum      | 20.156                   | 18                                       |
| 46.388                  | 1.897                    | 0                                        |
| 54.823                  | 0.854                    | 0                                        |
| $\Delta N = 3$ Sum      | 2.751                    | 0                                        |
References

1. G. Holzwarth and G. Eckart, Z. Phys. A283 (1977) 219; Nucl. Phys. A 325 (1979) 1

2. A Richter, Progress in Particle and Nuclear Physics 44 (2000) 3

3. S. Drozdz, S. Nishizaki, J. Speth and J. Wambach, Phys. Rep. 197 (1990) 1

4. L. Zamick and N. Auerbach, Nuclear Physics A 658, (1999) 285

5. S.J.Q. Robinson, L Zamick, A. Mekjian, and N. Auerbach Phys. Rev C62,(2000) 017302

6. P. von Neumann-Cosel et. al., Phys. Rev. Lett. 82 (1999) 1105