Spiral instability and screech effect in supersonic jet flows

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Abstract. The present paper is devoted to the effect of the discrete tone sound, which is generated by gaseous jet flows. It is known in scientific literature as the effect of jet screech. Thanks to the works by A. Powell, the modern theory explains this effect as the interaction between the jet shock waves and the vortical structures, which are developed at the jet boundary. In the recent work by I. Menshov, I. Semenov, and I. Akhmedyanov (Doklady Akademii Nauk, 420, 3, 2008) by means of the method of numerical modeling it was shown that there exist a correlation between the quantitative characteristics of the screech and the spiral instability developing in the jet flow, which gave occasion to the authors to doubt the validity of Powell’s theory. In the present paper the analytical analysis of the stability of an axisymmetric jet flow is carried out. The results obtained provide solid grounds in the favor of the hypothesis of spiral instability as the basic mechanism of the jet screech.

1. Introduction

The present paper addresses the compressible unsteady flow in supersonic underexpanded jets. This flow is typically accompanied by rather strong noise that results from a complicated flow structure downstream the nozzle exit. It basically includes the flow features such as shocks, vortices, laminar-turbulent transition, and eventually the formation of a turbulent plume. Aeroacoustics of jet flows is still problematic: the questions like ”where are basic noise sources located” or ”which mechanism is responsible for one or another type of sound produced by the jet” are far from their ultimate answers.

This study concerns one, but important aspect of jet aeroacoustics that is known in literature as jet screech. This phenomenon is typically appears in supersonic underexpanded jets as strong sound at a discrete frequency. Its amplitude can amount up to 40-50% higher than the amplitude of the broadband noise.

Due to A. Powell a theory of the jet screech exists that explains appearance of this sound as the result of the interaction between roll-up vortices developing at the jet boundary and shock-cell structures Powell (1953). Because of instability, a chain of vortex rings is formed at the jet boundary. Traveling downstream, these vortices grow in size and, passing over the
system of shock waves, interact with them. As a result, periodic acoustic pulses are generated in the ambient medium. According to Powell’s theory, these very pulses account for the observed discrete sound.

In the recent work Menshov, Semenov & Akhmedyanov (2008), it was established that the experimentally observed screech frequencies are closely correlated with the corresponding frequencies of three-dimensional helical instability. These results will be demonstrated in the next section. The helical instability of jet flows is a well-known fact. Norum (1983) noted that, with increase in the jet intensity, the initial predominantly axisymmetric flow pattern changes and the flow velocity vector acquires a considerable angular component, which was indicative of the presence of a helical motion.

The real flow in a jet has a very complicated structure resulted from the superposition of many factors, such as viscid/inviscid interaction, instabilities, interaction between shocks and vortex formations, turbulence. By using experiments, it is difficult to extract a separate structure from the whole flow pattern and link it with appropriate sound. The same situation occurs when we use precise direct numerical simulations to study jet flows and mechanisms of sound generation. As closer to reality we are, as harder to trace the basic mechanism of a sound, say screech; many separate details overshadow the pattern as a whole.

The present paper starts with the results of simple numerical simulation that clearly show a definite link between the screech characteristics and the characteristics of spiral instability. Then we discuss the results of the linear stability analysis for a model axisymmetric jet flow, compare frequencies of dominant unstable modes with those of the screech, and study numerically the nonlinear phase of the instability development. The results obtained give us definite reasons to put forward an alternative to Powell’s theory hypothesis of the jet screech mechanism. Jet screech is caused by spiral instability that change the flow to the shape of a rapidly rotating drill. An angular component appears in the jet flow, while the flow character changes from near-rectilinear to helical. In other words, we say that screech nature is essentially the same as that of the sound produced by a rapidly rotating in air drill.

2. Results of numerical modelling

We intentionally used the idealized physical model based upon the Euler equations, which do not take into account dissipative effects. Of course, this model does not involve many real factors, but it allows us to concentrate on the physical phenomenon itself without influence of other factors. Making the model coarser we can have a deeper insight into the nature of the phenomenon. In some sense, the physical (and numerical) model can be used as a filter that removes minor factors leaving the object to be studied refined.

We consider the problem of a gas jet emitted from a circular nozzle into an ambient resting medium with pressure $P_a$ and density $\rho_a$. The exit section diameter is $D = 7.8$ mm, and the nozzle wall thickness at the exit is $L = 1.5$ mm. The jet flow regime is defined by two non-dimensional parameters: $NPR = P_0/P_a = 5$ and $P_e/P_a = 1.73$, where $P_0$ is the total pressure (stagnation pressure) and $P_e$ is the pressure at the nozzle exit. The ambient density $\rho_a = 1.17$ kg/m$^3$ and the nozzle exit density $\rho_e = 3.225$ kg/m$^3$. These values correspond to the conditions of the experiment Khalil (2002) that will be referred below for comparisons. The Mach number at the nozzle exit $M_e = 1.33$, while the Mach number of the isentropically totally expanded jet is $M_j = 1.7$.

The velocity profile at the nozzle exit is taken in the form $U(r) = 0.5U_j\{1 + \tanh[\theta(R/r - r/R)]\}$, where $U_j$ is the core velocity, $\theta$ is a variable parameter that controls the width of the shear layer. In calculations this parameter was chosen as $\theta = 25/4$.

We use the Godunov method to solve numerically the system of governing equations. Time integration is performed using the explicit/implicit, matrix-free LU-SGS scheme (Menshov...
The scheme is absolutely stable, which makes it possible to perform calculations with a reasonably chosen time step not limited by the Courant-Friedrichs-Levy (CFL) stability condition. At the same time, the scheme minimizes the dissipative error introduced by the implicit component.

The computation domain consists of two subdomains. The first one that adjoins the nozzle exit section is the main region. It extends at a distance of $17D$ from the jet axis in the radial direction and $25D$ from the nozzle exit in the longitudinal direction. The second subdomain is a buffer zone, which adjoins the former subdomain (being located downstream of it) and is intended for suppressing disturbances and eliminating undesirable reflections from the outer boundary of the computation domain. The second subdomain has a length of $30D$.

The grid of the main region consists of 480 cells in the longitudinal, 208 cells in the radial, and 112 cells in the angular directions. In the radial direction, the grid is refined near the axis, so that the typical cell size in the jet region is $0.014D$. In the longitudinal direction, the cell size is $0.04D$ (in the region $0 < x/D < 12$). The grid of the buffer zone is much coarser and consists of 25 cells along the axis, which are generated so as to provide a smooth transition from the baseline to the buffer grid. The time integration step $\delta = 0.5mks$, which corresponds to a CFL value of about 50. This choice turned out to be optimal, since it made possible calculations with a single LU-SGS iteration.

Let us consider some results of this numerical simulations (see also Menshov, Semenov & Akhmedyanov (2008)). Fig. 1 shows the instantaneous isosurface of density for a value of $\rho = 1.7kg/m^3$, which is intermediate between $\rho_a$ and $\rho_e$. This surface can be thought as a kind of visualization of the jet boundary. As can be seen, the axisymmetric jet flow pattern breaks up at a distance of three to four shock cells (are not shown in the figure; the cell length is about $2D$) and the jet acquires a typical helical drill-like shape.

![Figure 1. Instantaneous isosurface for $\rho = 1.7kg/m^3$ and the pressure distribution in the near field of the ambient gas.](image)

This shape is unsteady. Fig. 2 represents the pressure field in the jet cross-section at a distance of $13D$ from the nozzle exit at several successive moments. In these patterns, the trace of the aforementioned isosurface is depicted by a gray curve. One can see that the shape of the isosurface remains unchanged, and only rotates in the space at a constant angular velocity. Thus, it behaves as the surface of a rigid rotating drill. The rotation frequency, which can readily be determined from Fig. 2, amounts to 10.58 kHz. Note that this value is close to the fundamental screech frequency that is estimated in experiments by a value of 13.1 kHz. Thus, we may conclude that the axisymmetric jet structure is destroyed due to development of the helical instability, and the jet takes the shape of a rapidly rotating drill. An angular component appears in the jet flow, while the flow character changes from near-rectilinear to helical.

What happens in the ambient gas when the helical instability arises? Obviously, the effect is the same as that of a solid with a similar surface rotating at a frequency of 10580 rpm. In
other words, if we could imagine that the jet core seen in Fig. 1 is replaced with a similar solid drill rotating at the indicated frequency, the behavior of the ambient medium would not change much.

A rapidly rotating drill produces discrete sound, at a frequency equal to the frequency of rotation. The nature of this sound is clear: due to its asymmetry, the cross-section of the rotating drill executes periodic upward and downward motions leading to periodic compressions and rarefactions in the ambient gas. The mechanism of the supersonic jet screech generation by our opinion is the same.

**Figure 3.** Comparison of the numerically calculated and measured sound pressure spectra.

Fig. 3 compares the results of numerical modeling to the experimental data Khalil (2002) for the frequency spectra of the pressure at a point of observation ($r = 6D$ and $\varphi = 40^\circ$). The experiments were carried out at Nagoya University (Fluid Dynamics Lab of the Aerospace Engineering Department). As can been seen, the results are in good qualitative agreement. The calculations capture the fundamental, secondary, and even tertiary frequencies of the helical instability (10.58, 21.16, and 31.74 kHz) that coincide with the experimental screech frequencies (13.1 and 26.2 kHz) to within 20%. The screech amplitudes are quantitatively different, and broadband sound is found to be somewhat underestimated. This may be explained by the physical model used (the Euler equations) that does not take into account important factors.
Nevertheless, what we used so simple model allowed us to purify the phenomenon of spiral instability and find out its close relation to the jet screech.

3. Linear stability analysis

In this section the spiral instability is investigated by the method of linear stability analysis. We consider a model jet that is represented by infinite, uniform in x-direction, and axisymmetric parallel flow with a radial distribution of velocity in the form given in Section 1. Pressure is constant and equals to the ambient pressure. The temperature distribution is related to velocity by the Busemann-Crocco law, and the density distribution is determined from the equation of state.

This class of solutions of the Euler equations depends on 3 parameters, which are the density $\rho_j$ and the velocity $U_j$ at the axis of symmetricity, and $\theta$ that characterizes steepness of the velocity profile. We chose the first two, $\rho_j$ and $U_j$ so that maximally approach experimental conditions mentioned in Sect. 1. This is a supersonic nozzle with a designed Mach number $Me = 1.33$ and $D = 7.8mm$. The jet flow regime of this facility depends on the value of $NPR = P_0/P_a$ and the total temperature $T_0$. For the model jet flow to be studied we chose $\rho_j$ and $U_j$ equal to the values of the perfectly expanded jet. The total temperature is fixed, $T_0 = 295K^\circ$, which corresponds to the experiment. Therefore, the class of solutions we investigate is completely determined by two parameters: the nozzle pressure ratio $NPR$ and the steepness parameter $\theta$.

For the above flow model, we carry out the linear analysis of stability with respect to small perturbations in the form of normal harmonics $f = f_0(r) \exp[i(\lambda t + m\varphi + kx)]$, where $f_0(r)$ is the perturbation amplitude, $m$ is the azimuthal wave number (positive integer), $k$ is the longitudinal wave number, and $\lambda$ is the complex frequency: $Re(\lambda)$ is the azimuthal phase frequency, and $Im(\lambda)$ is the growth rate factor.

The linear problem for amplitudes is reduced to solving the system of two equations with respect to amplitudes of the radial velocity perturbations and pressure $(u_0, p_0)$:

$$\frac{dp_0}{dr} = -iu_0R(r)\sigma$$
$$\frac{du_0}{dr} = \frac{m^2}{r^2R(r)\sigma} - \frac{\sigma}{R(r)C^2} p_0 + \left[\frac{k}{\sigma \frac{dU(r)}{dr} - \frac{1}{r}}\right] u_0$$

where $\sigma = \lambda + kU(r)$ is the Doppler frequency, $C$ is the speed of sound, and $\gamma = 1.4$ is the specific heat ratio. The capital letters here designate parameters of the base jet flow.

The problem consists in finding such values of the parameter $\lambda$, for which the solution of Eqs. (1) and (2) in the region of $0 \leq r \leq \infty$ satisfies the boundary conditions $u_0(0) = 0$ and $u_0(\infty) = 0$, $p_0(\infty) = 0$. Thus, the problem is reduced to searching for an eigenvalue (complex frequency) for the differential semi-infinite boundary value problem (1) and (2). The presence of a nonzero imaginary part of this value indicates the existence of unstable modes. The spectrum of the problem is symmetric with respect to the real axis; i.e., if as a result of the numerical solution, the negative value of the growth rate factor $Im(\lambda)$ is obtained, its positive analogue also should exist.

The numerical solution of the eigenvalue problem (1)-(2) is fulfilled in three stages. At the first stage, we find analytically the asymptotic solutions near zero and at infinity. At the second stage, the problem is reduced to a finite interval with the help of the obtained asymptotic solutions. At the third stage, the iterative numerical solution of the problem is carried out on a sequence of extending intervals up to the convergence in the value of $\lambda$ is reached. For details we refer to the paper Menshov & Nenashev (2011).
The results of the linear stability analysis show the existence of the spiral instability. A typical pattern of this instability is shown in Fig. 4. In this figure an instant surface of a constant value of the pressure perturbation is shown for two time moments. The modes shown correspond to the wave numbers $m = 0, 1, \text{ and } 2$, respectively from left to right. The unstable mode $m = 0$ manifests itself in the form of vortex rings moving downstream. Higher modes ($m = 1$ and $m = 2$) manifest themselves as thin vortex tubes - ”straps” surrounding the jet core along the spiral.

Basic parameters that characterize the behavior of unstable modes are the growth rate factor $-\Im(\lambda)$ and the azimuthal phase frequency $\Re(\lambda)$. In Fig. 5(a), we present the growth rate factor as a function of longitudinal wave number $k$ for the harmonics of various angular modes. There can be seen a finite interval of wave numbers $k$ where corresponding harmonics are unstable. Moreover, it should be noted that the local maximum exists in the distribution of $-\Im(\lambda)$ with respect to $k$. This indicates the existence of dominant harmonics - the harmonics with the wave number near the value of $k_{\text{max}}$ corresponding to the maximum value of the imaginary part of the eigenvalue $\lambda$ (Fig. 5(a)). One can suggest that the very dominant harmonics develop during the formation of the spiral instability in jet flows.

The dependence of the azimuthal phase frequency $\Re(\lambda)$ on the longitudinal wave number is shown in Fig. 5(b). It is nearly linear. Taking a value of $\Re(\lambda)$ that corresponds to $k_{\text{max}}$, we can estimate the phase frequency of the corresponding (dominant) spiral structure and, hence, the frequency of the sound generated by this structure. By our hypothesis, the very sound generated by the dominant harmonics is the jet screech. In this sense this (dominant) harmonics is referred below as screech harmonics.

How the phase frequency at a fixed wave number $k$ and the screech frequency depend on the jet velocity profile is illustrated in Fig. 6. Fig. 6(a) shows $F = \Re(\lambda)/(2\pi), \text{Hz}$ as a function of the mixing zone size $\theta$ for a fixed value of $k$ and different values of NPR. The screech frequency in the same dependence is given in Fig. 6(b) for $NPR = 2.9 \text{ and } 4.02$. Note that the fixed $k$ harmonic frequency tends to a constant value with increasing the parameter $\theta$, while the screech frequency is almost linearly increased.

The study of the dependence of the growth rate factor on the jet intensity determined by
Figure 5. (a) Growth factor $\text{Im}(\lambda)$ and (b) the azimuthal phase frequency $\text{Re}(\lambda)$ for different angular modes at the fixed flow mode $M_j = 1.22$, and $\theta = 6.25$. The vertical lines separate the region of dominant harmonics.

Figure 6. Dependence of frequency $F$ on the mixing zone size $\theta$ for the fixed value of the longitudinal wave number (a) $k$ and (b) $k = k_{\text{max}}$.

Figure 7. Comparison of results of the linear analysis (curve) with the data of experiments (points).

$NPR$ shows non-monotonic character of this distribution; when $\theta$ is fixed, $-\text{Im}(\lambda)$ gets a global maximum at a certain value of $NPR$. 
Thus, for each value of the parameter $\theta$, there is an $NPR$ value (together they determine the basic flow model) for which the screech frequency reaches maximum. With increase in $\theta$ (decrease in the size of the mixing zone) the screech frequency is also increased.

To relate the $NPR$ parameter and the mixing zone thickness $\theta$ for a real jet flow, we use several semi-empirical formulas and facts. First, we estimate the distance at which the instability starts to develop in one shock cell. The size of the shock cell $\Delta_0$ is related with $NPR$ as $\Delta_0 = 1.12D\sqrt{\Delta p} - 0.93$, where $\Delta p = p_0 - p_a$ in $kg/cm^2$. The experimental dependence of the size of the mixing zone $\theta$ on the distance downstream from the nozzle exit is given by Michalke (1984): $1/\theta = 0.06(x/D) + 0.04$.

In such a manner a value of $\theta$ can be estimated at which the spiral instability is expected to happen in a jet under given $NPR$. Choosing for these $\theta$ and $NPR$ the screech frequency, we can draw it as a function of $NPR$. This plot is shown in Fig. 7 together with screech frequencies measured in experiments Khalil (2002). In spite of rather rough estimations the results are found to be in reasonably good agreement with experimental observations, which argues for the hypothesis of the relation between the jet screech phenomenon and the flow spiral instability.

4. Non-linear stage of instability development

In this section, the non-linear stage of the instability development is numerically modeled by solving the full system of gas dynamics equations. The aim is to understand the scenario of the non-linear development of large vortical structures that appear in the jet flow due to instability and basic trends of their interaction with the base jet flow. In the scope of this paper we limit our consideration by only the axisymmetric case ($m = 0$ modes).

![Figure 8. The form of initial perturbations superimposed on the base flow.](image)

We consider the solution of 2D axisymmetric unsteady gas dynamics equations written in a short form as $L(q) = 0$, where $L$ is the differential operator of conservation of mass, momentum, and energy, $q$ is the conservative state vector, in the domain $D = \{(z, r) : 0 \leq z \leq L, 0 \leq r < \infty \}$ with periodical boundary conditions $q \big|_{z=0} = q \big|_{z=L}$. The initial data is taken in the form of weakly perturbed base flow of Section 2 ($q = q_0 + q'$) with $NPR = 2.9$ and $\theta = 5$. In accordance to the results of the linear stability analysis given above the screech mode for this case corresponds a wavenumber $k = 3.6$, and the complex eigenfrequency is $\lambda = 2.92760.7275i$. The corresponding eigenfunctions are shown in Fig. 8. These functions are used as initial perturbations ($q'$) with the amplitude $p'/p_0 = 10^{-4}$.

The solution method is the Godunov method based on the exact solution to the Riemann problem, implemented with the third order MUSCL-type cell interpolation scheme Van Leer.
Figure 9. Pressure disturbance vs time at different points of observation: 1 - region of the linear stage, 2 - metastable vortex structure ("vortex train"), 3 - interaction of vortex rings, 4 - formation of large vortex structures.

Figure 10. Numerical and theoretical data of the instability growth rate: disturbance pressure vs time (right), pressure amplitude vs time (left).

Figure 11. Regular metastable vortex structure - vortex train: disturbance density (left) and vorticity (right).

(1979). The base computational domain extends $2R$ in the radial direction, and $L = 3l$ in the longitudinal direction, where $l = 2\pi/k_{max}$ is the wavelength of the screech mode used as initial perturbation. The computational grid in the base domain assures a cell size of 0.005$R$ in both directions. In the region $r > 2R$ a buffer region is implemented to let acoustic waves go out.

The $m = 0$ instability mode is developed in the form of a set of equidistributed vortex rings surrounding the jet core flow. This process can be divided into 4 stages, as can be seen in Fig. 9. Here we show the disturbance pressure at an observation point as a function of time. The stages are indicated by digits.

The first stage is the linear growth of disturbances. Up to $t = 5$ (non-dimensional time), disturbances grow exponentially in accordance with the linear-stability theory. This can be seen from results shown in Fig. 10, where data of numerical calculations is compared with theoretical ones.

The linear stage results in the formations of a regular vortex structure that we name as vortex train. It consists of equidistributed ring-type vortices that move downstream with a velocity of
Figure 12. Vortex interplay - vortex ring leapfrog: disturbance density (left) and vorticity (right) for different time moments ($t = 12.74$, $t = 14$, $t = 15.19$, $t = 21$ from the top to the bottom, respectively).

about 50% of the maximal velocity of the base flow. The vortex train is demonstrated in Fig. 11 by the fields of disturbance density (left) and disturbance vorticity (right).

The vortex train is a metastable structure; its lifetime is of the order of the linear stage (see Fig. 9). Our hypothesis is that the vortex train is an exact solution of the Euler equations, and the first instability just transfer one exact solution (parallel axisymmetric jet) to another one (vortex train).

However, the vortex train is also unstable solution to the Euler equations. Due to secondary instability it decays (the third stage). This process is mostly defined by interplay of the vortex rings vortex ring leapfrog. It can be seen in Fig. 12, where density and vorticity fields are shown for different time moments.

The interplay of the vortex rings in the vortex train is accompanied by cascade capturing and merging vortex rings. As the result of this process vortical structures become large and eventually form one large toroidal-shaped vortex closed by a shock wave (the fourth stage, Fig. 12).

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References

Powell, A. 1953 On the mechanism of chocked jet noise. *Proc. Phy. Soc.* B66, 1039–1056.

Menshov, I., Semenov, I. & Akhmedyanov, I. 2008 Mechanism of Discrete Tone Generation in Supersonic Jet Flows. *Doklady Physics* 53, 278–282.

Norum, T. D. 1983 Screech Suppression in Supersonic Jets. *AIAA J.* 21, 235–240.

Khalil, M. H. 2002 On mixing enhancement and sound emission of high speed jet flows. PhD thesis, Nagoya University.

Menshov, I. & Nakamura, Y. 2004 Hybrid explicit-implicit, unconditionally stable scheme for unsteady compressible flows. *AIAA J.* 42, 551–559.

Menshov, I. & Nenashev, A. 2011 Spiral mode instability and the screech tone effect in supersonic jet flows *Doklady Physics* 56, 288–293.

Michalke, A. 1984 Survey on jet instability theory. *Prog. Aerospace Sci.* 21, 159–199.

Van Leer, B. 1979 Towards the ultimate conservative difference scheme V. A second order sequel to Godunovs method. *J. Com. Phys.* 32, 101–136.