Can a Measurement of the $B_s - \bar{B}_s$ Mass Difference Establish the CKM Paradigm?

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Abstract

We present a reanalysis of the allowed region in the $\rho - \eta$ plane of the CKM matrix, which follows from our present knowledge of the theoretical and experimental parameters associated with quark mixing and CP violation. Besides providing updated expectations for the angles of the unitarity triangle, this reanalysis predicts a range of allowed values for the $B_s - \bar{B}_s$ mass difference $\Delta m_s$. We argue that while values of $\Delta m_s \lesssim 10(ps)^{-1}$ could be consistent with a non-CKM origin for CP violation in the neutral Kaon system, larger values for $\Delta m_s$ would provide strong support for the CKM paradigm.
It has been realized for a long time \[1\] that a measurement of the \(B_s - \overline{B_s}\) mass difference \(\Delta m_s\) (or equivalently, of the \(B_s - \overline{B_s}\) mixing parameter \(x_s = \Delta m_s \tau_{B_s}\)) would add important information to our knowledge of the CKM mixing matrix. The equivalent parameters for the \(B_d - \overline{B_d}\) complex are now relatively well established, through measurements of the time integrated mixing probability at the \(\Upsilon(4S)\) - sensitive to \(x_d\) - and by direct analysis at LEP of the time dependence of this mixing - sensitive to \(\Delta m_d\). A recent summary analysis of all these measurements by Forty \[2\] - using the world average value for the \(B_d\) lifetime, \(\tau_{Bd} = 1.61 \pm 0.09(\text{ps})\) \[3\] - yields the values:

\[
\Delta m_d = 0.496 \pm 0.032 \text{ (ps)}^{-1}, \quad x_d = 0.78 \pm 0.05.
\]

Because the mass differences \(\Delta m_d\) and \(\Delta m_s\) arise theoretically from identical box graphs, save for the interchange of \(d \leftrightarrow s\) quarks, they are simply interrelated \[1\]. Indeed, apart from SU(3)-breaking factors associated with evaluating \(\Delta B_d = 2\) and \(\Delta B_s = 2\) quark operators between \(B_d, \overline{B_d}\) and \(B_s, \overline{B_s}\) states, respectively, the ratio of these mass differences measures simply the ratio of two CKM matrix elements:

\[
\frac{\Delta m_s}{\Delta m_d} = \left\{ \frac{M_{B_s} B_s f_{B_s}^2}{M_{B_d} B_d f_{B_d}^2} \right\} \frac{|V_{ts}|^2}{|V_{td}|^2} \equiv \xi_s^2 \frac{|V_{ts}|^2}{|V_{td}|^2}. \tag{1}
\]

The quantity in the curly bracket, which we have denoted by \(\xi_s^2\), is expected to be of order unity. It has been calculated theoretically recently by various groups using lattice QCD \[4\] and from QCD sum rules \[5\]. In what follows, we shall employ the value that Forty \[2\] uses:

\[
\xi_s^2 = 1.3 \pm 0.2.
\]

This is consistent with the recent theoretical results quoted and also is in line with the value adopted by Ali and London for this quantity in their recent analysis \[6\]. Using the standard Wolfenstein \[7\] expansion of the CKM matrix, one has, to a good approximation, \(|V_{ts}| \simeq |V_{cb}|\). Since this latter matrix element is now reasonably well determined \[8\], \(|V_{cb}| = 0.0378 \pm 0.0026\), one sees that a measurement of \(\Delta m_s\) provides direct information on \(|V_{td}|\). Alternatively, without appealing to a direct value for \(|V_{cb}|\), but instead using the Wolfenstein parametrization of the CKM matrix, a measurement of \(\Delta m_s\) along with a knowledge of \(\Delta m_d\) fixes an allowed region in the \(\rho - \eta\) plane:

\[
\Delta m_s = \Delta m_d \frac{\xi_s^2}{\lambda^2} \left[ \frac{1}{(1 - \rho)^2 + \eta^2} \right], \tag{2}
\]

\[
\lambda = \frac{|V_{ts}|}{|V_{td}|}.
\]
where $\lambda = 0.221 \pm 0.002$ [9] is the sine of the Cabibbo angle.

There have been recent interesting attempts at LEP to obtain some direct information on $\Delta m_s$ by searching for two frequency components in the proper-time distribution of tagged $B^0$ decays, with results from OPAL [10] and ALEPH [11] being presented at the Glasgow conference. The ALEPH result provides a particularly strong bound on $\Delta m_s$:

$$\Delta m_s > 6 \text{ (ps)}^{-1} \quad (95\% \text{ C.L.}),$$

from which, by fluctuating up $1\sigma$ the values of $\Delta m_s$ and $\xi^2_s$ given, one can infer an upper bound constraint in the $\rho - \eta$ plane:

$$\sqrt{(1 - \rho)^2 + \eta^2} < 1.61.$$ 

This bound turns out to be rather close to, or even to somewhat restrict, the "allowed" region in the $\rho - \eta$ plane determined by various recent analysis of constraints on the CKM matrix [6] [8] [12]. However, since the allowed region encompasses values for the above square-root which are as low as 0.6 - 0.8, it appears that, at present, a very large range for $\Delta m_s$ (and thus also for $x_s$) is permitted. Nevertheless, as we will demonstrate below, trying to obtain better bounds on $\Delta m_s$ (and certainly measuring this parameter) can provide important insights for the CKM paradigm.

For these purposes, it is useful to present here a reanalysis of the constraints on the CKM matrix elements. Basically, three measurements fix the allowed region in the $\rho - \eta$ plane: those of $\epsilon$, the CP violating parameter inferred from neutral Kaon decays; the value of $\Delta m_d$ (or $x_d$), characterizing $B_d - \overline{B}_d$ mixing; and the ratio of $|V_{ub}|/|V_{cb}|$, obtained from studying semileptonic B decays near the end-point region of the electron spectrum. To complete the analysis, however, further experimental and theoretical information is needed. To translate the experimental value of $\epsilon$ into a constraint on $(\rho, \eta)$, one needs to know $|V_{cb}|$ and the top quark mass $m_t$, as well as have a theoretical estimate of the relevant $K_0 - \overline{K}_0$ matrix element (which is quantified in terms of the parameter $B_k$). For $B_d - \overline{B}_d$ mixing, besides needing values for $|V_{cb}|$ and $m_t$, the corresponding $B_d - \overline{B}_d$ matrix element needed requires a knowledge of $f_{B_d}B_d$. Finally, $|V_{ub}|/|V_{cb}|$ can be extracted directly from the data. However doing so necessitates some model input, and the uncertainties in the models significantly expand the experimental error.
As the formulas and procedures for relating $\epsilon$, $\Delta m_d$ and $|V_{ub}|/|V_{cb}|$ to allowed regions in the $\rho - \eta$ plane are fairly standard \cite{13}, we shall not repeat them here. Rather, we give in Table 1 a summary of the values we have used in our analysis. We include in the table two values of $|V_{ub}|/|V_{cb}|$, one where uncertainties due to model dependences are taken into account and the other where this ratio is extracted using a particular partonic model which we favor - the ACM model \cite{18}. Fig. 1 displays our results, with the cross-hatched area giving the region in the $\rho - \eta$ plane defined by the three intersecting 1$\sigma$ bands, using the value of $|V_{ub}|/|V_{cb}|$ having the larger uncertainty due to model dependence. If instead, one uses the ACM model value for $|V_{ub}|/|V_{cb}|$, the resulting overlap region - shown as the dashed swath - is much more restricted already. Expanding the errors slightly and following some rather standard procedures, one arrives at the combined allowed 1$\sigma$ region in the $\rho - \eta$ plane shown in Fig. 2.

Having determined the allowed region for $(\rho, \eta)$, it is straightforward to deduce the parameter range allowed for various quantities of experimental interest. Principal among these are the $\alpha$, $\beta$, $\gamma$ angles of the unitary triangle, whose values determine the size of the CP violating asymmetries in neutral B decays to CP self-conjugate states \cite{19}. Fig. 3 displays the combined allowed range for $\sin(2\beta)$ versus $\sin(2\alpha)$ which follow from our analysis. Similarly, one can deduce from Fig. 2 and Eq. (2) an allowed range for $\Delta m_s$. This range, as alluded to earlier, is quite large:

$$6.2 \text{ (ps)}^{-1} < \Delta m_s < 22.7 \text{ (ps)}^{-1}.$$ \label{eq:range}

This does not change much even if one restricts oneself only to the region allowed by the ACM model:

$$6.6 \text{ (ps)}^{-1} < (\Delta m_s)_{ACM} < 22.7 \text{ (ps)}^{-1}.$$ \label{eq:acm_range}

The situation is radically different, however, if one presumes that the $\epsilon$-parameter typifying CP violation in the Kaon system - has a non CKM origin, as for example it does in the superweak model \cite{20}. In this case, one would still have a quark mixing matrix, but this matrix rather than being unitary,

\footnote{To obtain an estimate of this range, we fluctuate the value of the quantity $(\Delta m_d \xi_s^2/\lambda^2)$ from Eq. 2 by $\pm 1\sigma$, then combine with the allowed values of $(\rho, \eta)$ from Fig. 2 with errors added in quadrature.}
would be orthogonal. In the standard Wolfenstein parameterization \cite{4} that we are using, this corresponds to having $\eta \equiv 0$. Although here $\epsilon$ provides (by assumption) no constraint, both $B_d - \overline{B}_d$ mixing and the ratio $|V_{ub}|/|V_{cb}|$ determine a corresponding allowed region for the remaining free parameter $\rho$. For the choice of parameters given in Table 1, the allowed values for $\rho$ for the most probable solution - corresponding to the overlapping segment on the real axis in Fig. 1 - is given by

$$\rho = -0.33 \pm 0.08 \quad (\eta = 0).$$

In this circumstance, the predicted range for $(\Delta m_s)$ is considerably narrower:

$$6.0 \, (ps)^{-1} < \Delta m_s < 9.2 \, (ps)^{-1} \quad (\eta = 0).$$

Indeed, since the allowed value for $\Delta m_s$ in this case is almost entirely dominated by the allowed range for $|V_{ub}|/|V_{cb}|$, one can write the approximate equation

$$\Delta m_s \simeq \Delta m_d \frac{\xi^2}{\lambda^2} \frac{1}{(1 + |V_{ub}|^2/|V_{cb}|^2)^2} \quad (\eta = 0). \quad (3)$$

If one were to further restrict oneself to the region allowed by the ACM model, the above equation would narrow down the allowed range for $\Delta m_s$ to

$$5.9 \, (ps)^{-1} < (\Delta m_s)_{ACM} < 8.3 \, (ps)^{-1} \quad (\eta = 0).$$

One sees from this discussion, that if a unitary CKM matrix is the correct paradigm for CP violation, improved bounds on $\Delta m_s$ are unlikely to make a significant impact on the range of values allowed for the CKM matrix. However, conversely, a significant improvement of the ALEPH bound to $\Delta m_s \gtrsim 10 \, (ps)^{-1}$, which may be possible at the SLC \cite{22}, could serve to establish the CKM paradigm by excluding the possibility that $\eta$ vanishes. In fact, modest improvements in our understanding of the theoretical uncertainties in both $|V_{ub}|/|V_{cb}|$ and $\xi^2$ could bring the necessary lower bound on $\Delta m_s$ for these purposes down to about $8.5 \, (ps)^{-1}$.

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Table 1: Parameters used for the $\rho - \eta$ plane analysis

| Parameter | Value |
|-----------|-------|
| $|\epsilon| = (2.26 \pm 0.02) \times 10^{-3}$ | $m_t = 174 \pm 10^{\pm 10}$ GeV |
| $B_k = 0.825 \pm 0.035$ | $\sqrt{B_{dfBd}} = 180 \pm 30$ MeV |
| $\Delta m_d = (0.496 \pm 0.032) \text{ ps}^{-1}$ | $|V_{cb}| = 0.0378 \pm 0.0026$ |
| $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$ | $|V_{ub}/V_{cb}| = 0.082 \pm 0.006$ (ACM) |

Figure 1: Constraints on the $(\rho, \eta)$ plot
Figure 2: Allowed region in the $\rho - \eta$ plane
Figure 3: Allowed region of $\sin(2\beta)$ versus $\sin(2\alpha)$

References

[1] P. Krawczyk, D. London and H. Steger, Nucl. Phys B321, 1 (1989).

[2] R. Forty, to appear in the Proceedings of the International Conference on High Energy Physics (ICHEP94), Glasgow, Scotland, July 1994.

[3] P. Roudeau, to appear in the Proceedings of the International Conference on High Energy Physics (ICHEP94), Glasgow, Scotland, July 1994.

[4] For a recent review, see J. Shigemitsu, to appear in the Proceedings of the International Conference on High Energy Physics (ICHEP94), Glasgow, Scotland, July 1994.

[5] S. Narison, Phys. Lett. B322 (1994) 247; S. Narison and A. A. Pivovarov, Phys. Lett. B327 (1994) 341.

[6] A. Ali and D. London, to appear in the Proceedings of the International Conference on High Energy Physics (ICHEP94), Glasgow, Scotland, July 1994.
[7] L. Wolfenstein, Phys. Lett. 51 (1983) 1945.

[8] S. Stone, to appear in the Proceedings of the 1994 DPF Conference, Albuquerque, N. Mexico, August 1994.

[9] H. Leutwyler and M. Roos, Z. Phys. C25 (1984) 91.

[10] S. Komamiya, to appear in the Proceedings of the International Conference on High Energy Physics (ICHEP94), Glasgow, Scotland, July 1994.

[11] Y. B. Pan, to appear in the Proceedings of the International Conference on High Energy Physics (ICHEP94), Glasgow, Scotland, July 1994.

[12] J. Rosner, to appear in the Proceedings of the 1994 DPF Conference, Albuquerque, N. Mexico, August 1994.

[13] See for example, R. D. Peccei in the Proceedings of the Puri Winter School, Puri, India, January 1993.

[14] Particle Data Group: L. Montanet et al., Phys Rev. D50 (1994) 1173.

[15] F. Abe et al. (CDF Collaboration), Phys. Rev. D50 (1994) 2966.

[16] S. Sharpe, Nucl. Phys. B (Proc. Suppl.) 34 (1994) 403.

[17] This is our guesstimate of the value of this parameter coming from lattice QCD - for a compilation of the most recent results and a discussion, see [4].

[18] G. Altarelli et al., Nucl. Phys. B207 (1982) 365.

[19] See for example, Y. Nir and H. R. Quinn in B Decays, ed. S. Stone, p.362 (World Scientific, Singapore, 1992).

[20] L. Wolfenstein, Phys. Rev. Lett. 13 (1964) 562.

[21] F. James and M. Roos, Phys. Rev. D44 (1991) 299; F. James, Computer Phys. Communications 20 (1980) 29.

[22] M. Breidenbach, private communication.