Research Article

Bid Evaluation for Major Construction Projects Under Large-Scale Group Decision-Making Environment and Characterized Expertise Levels

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ARTICLE INFO

Article History
Received 26 Jun 2020
Accepted 28 Jul 2020

Keywords
Bid evaluation
Expert classification
Consensus reaching processes
ELECTRE III
Multi-attribute group decision-making

ABSTRACT

Rapid growth and development of civil engineering in recent years inspire building enterprises to concentrate on construction contractor selection for achieving more construction quality and lower construction cost. The existing studies generally regard the process of selecting the best contractor as a multi-criteria decision making problem. Few research studies addressed the contractor selection problem in the context of large-scale group decision making, which is common in practical scenarios in terms of major construction projects as a number of experts with diverse backgrounds are usually involved. On this basis, we establish a contractor selection framework under large-scale group decision making environment, which covers expert classification, consensus reaching process, collective decision matrix generation, and the ranking-oriented decision making method. We cluster expert group with K-means clustering method based on expertise levels, which are depicted by six features generated with an expertise identification approach. The consensus model manages consensus reaching process from both intra- and inter-layers and takes into account the interactions between them. After reaching agreements among experts, this paper utilizes the concept of proportional hesitant fuzzy linguistic term set to assemble intra-subgroup assessments for the reduction of information loss or distortion. Then, an aggregation process carries on as to gather subgroup assessments in which the subgroup weights are derived from their cluster centers and sizes in the use of the TOPSIS method. Finally, the well-established decision making tool integrating qualitative and quantitative criteria, ELECTRE III, is adapted to elicit the ranking of bidders. An illustrative study and a comparative analysis are performed to demonstrate the feasibility and effectiveness of the established multi-criteria group decision making approach.

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1. INTRODUCTION

In recent years, rapid industrialization and urbanization have greatly stimulated the growth and development of the civil engineering industry, implying more and more attention has been paid to the investigation of construction contractor selection issues. In a bid to strengthen project competitiveness as well as reduce construction costs, many countries ask project owners to use competitive bidding to select the best contractor strategically. However, past experience has shown that the traditional pattern of awarding the low-price compliant bid generally results in delayed completion, increased costs, poor construction quality, and even serious disputes with employers [1,2]. Therefore, it has been a general trend for bid evaluation to incorporate multiple core attributes with various multi-criteria (group) decision making (MCDM/MCGDM) tools.

There exists considerable amount of MCDM/MCGDM methods having been applied for bid evaluation [3–6], in which only a few experts are involved. However, for major construction projects, the employers may prefer to large-scale expert groups consisting of more than 20 experts for contractor selection with a view to reducing evaluation risks, motivating the focus of our paper to concentrate on solving bid evaluation problems under large-scale group decision making (LSGDM) context. The growth of LSGDM is stimulated by the rapid development and innovation of social networks, which provides an available information exchange platform for a tremendous number of stakeholders to participate in the decision making process with modest cost and time spent [7,8]. The popularity of LSGDM has attracted the attention of lots of scholars [9–12]. These researches divided LSGDM problems into four important phases, considered as expert classification, consensus
reaching processes (CRPs), collective decision matrix generation, and decision making method. This paper is also structured in line with the four phases to select the best contractor, which are described as follows:

- **Aiming at reducing the dimension of experts and gathering homogeneous experts together**, expert classification helps manage the preference information and reach consensus easier, and is generally conducted with clustering analysis [9]. It is worthy of mentioning that expertise level is an important label attached with experts, which usually indicates the reliability of their assessments in decision making process. Based on that, a renew expert classification approach that considers expert expertise is required to make bid evaluation problem flexible. In addition, the K-means clustering paradigm is utilized to group experts for its efficiency and simplicity of implementation. Its time complexity is considered as linear in the number of data points, indicating the K-means clustering can save considerable time for bid evaluation process.

- **As for CRPs in LSGDM context**, it aims at reaching an agreement with all experts before making decisions. It is one of the most important topics in LSGDM research because failing to achieve consensus in decision making will result in waste of resources [13]. However, the large number of involved experts and different cultures, educational backgrounds or personal interest preferences make this process difficult. Thus, expert classification is usually carried out in advance to help manage the assessment information and reach consensus. In this study, two levels of consensus are generated after expert classification, i.e., the level of consensus within a sub-group and the level of consensus of a sub-group to the global group, named as the intra consensus and inter consensus in this study, correspondingly [14]. Obviously, the two levels of consensus are interdependent to each other. Consensus adjustment on one level of consensus could impact another level, which leads to serious difficulties to CRPs. In order to solve this gap, a novel consensus model is necessary to detect and manage noncooperative behaviors in consensus where both levels of consensus are guaranteed.

- **After conducting the clustering analysis**, gather of decision information with various information fusion approaches outputs the collective opinions, which is a fundamental but important process in decision-making problems. Like CRPs, the generation of group opinions involves two layers: intra and inter layers. Attaching with similar expertise levels, the experts gathered into the same subgroup are treated as equal importance. Therefore, the notion of proportional hesitant fuzzy linguistic term sets (PHFLTSs) [15] provides a sound idea to gather individual assessments on intra layer in a bid to reduce information loss or distortion. Having obtaining the subgroup assessments represented by PHFLTSs, the information collection on inter layer can be conducted with an aggregation process.

- **As far as decision making method is concerned**, many studies in various fields [16–18] have shown that Elimination et Choix Traduisant la Réalité-III (ELECTRE-III) is a remarkable decision making approach. This outranking method allows imprecise, indeterminate, and uncertain criteria inherent to depict complex human decision process [19,20], and prevents compensations across criteria [21]. Both qualities are desirable for major construction projects because of the characteristics of complexity and uniqueness in which any shortcomings of contractors could lead to failure in the whole project.

Meanwhile, the complexity and variability of evaluation environment for major projects and the limitations of human subjective cognitive capacity normally cause spiraling uncertainty [22], compelling us to use linguistic variables for qualitative evaluations. However, it is necessary to point out that experts often struggle to elicit their preferences using single linguistic term due to the complexity of bid evaluation, time pressures, and partial lack of information. To provide rich and flexible expert expressions, this study introduces generalized comparative linguistic expressions (GCLEs) that allows experts to hesitate over several possible linguistic values [23,24]. This expression form can be further transformed into hesitant fuzzy linguistic term set (HFLTS) by means of context-free grammars [23,25], which has achieved wide applications in recent years [26,27].

The remainder of this paper is structured as follows. Section 2 briefly reviews previous studies on bid evaluation and LSGDM problems. A brief literature review reveals limitations of existing researches related to the problem under investigation and inspires the introduction of the main contributions of this study. Section 3 introduces some involved concepts in this paper as well as their associated properties. In section 4, we describe our proposal adopted to rank the optional bidders in major construction projects under LSGDM environment where collective decision matrix generation, expert classification, CRPs, and decision making methods are taken into consideration. The proposed approach is then applied to a bid evaluation problem regarding the reconstruction of settlement houses in section 5 in order to demonstrate its flexibility and practicality. Section 6 provides a comparison analysis to highlight the advantages of this presentation. Finally, Section 7 covers the conclusions of this paper and points out future research directions.

2. RELATED WORK

As mentioned previously, in this study, the bid evaluation problems are discussed under LSGDM environment, in which four significant phases are involved: expert classification, CRPs, collective decision matrix generation, and decision making method. Therefore, the presented literature review would be carried out in line with these phases, followed by a summarization of main contributions.

2.1. Expert Classification in LSGDM

Expert classification is of essential for LSGDM problems and often conducted with clustering methods, which has been researched by many scholars. For example, Liu et al. [28] constructed a partial binary tree DEA-DA cyclic classification model to achieve the accurate multiple groups’ classification of experts, with which for each interest group, group members with different interest preferences can be distinguished and distributed to the appropriate groups. Xu et al. [29] sorted the experts in LSGDM in accordance with their social network relationships with the Louvain algorithm, which was based on the idea of modularity in a social network and the
connection between decision experts. Fuzzy c-means clustering algorithm is adopted by Palomares et al. [30] to classify experts on the basis of fuzzy preference relations. The memberships of experts to clusters in fuzzy c-means clustering were also used as an indicator to detect subgroup noncooperative behaviors. Gou et al. [31] employed a similarity degree-based clustering algorithm to group experts based on double hierarchy hesitant fuzzy linguistic term sets (DHHFLTSs) with a predefined classification threshold. A new hierarchical clustering procedure integrating three-dimensional gray relational analysis was developed by Zhu et al. [9] to divide the experts considering double information: preference information and reference information.

However, the following limitations exist in these studies:

- The existing expert classification methods in LSGDM problems were principally targeted at gathering experts with similar assessment preference information. There has no paper sorted experts according to their expertise levels.

- Known for its efficiency and simplicity of implementation, K-means clustering analysis has not been applied by these studies, resulting in tedious calculation.

### 2.2. CRPs in LSGDM

As a significant topic in LSGDM context, considerable papers have paid more and more attention to CRPs [32–36]. Palomares et al. [30] detected and managed individual and subgroup noncooperative behaviors in consensus utilizing fuzzy c-means clustering algorithm. The model was complemented with a visual analysis tool based on self-organizing maps, which facilitates the monitoring of the process performance across the time. The consensus model proposed in [13] was targeted to detect overconfidence behaviors in consensus where a group consensus index considering both the fuzzy preference values and self-confidence was presented to measure the consensus level among experts. Xu et al. [29] proposed a consensus-based non-cooperative behavior management model in LGEDM that considered expert trust relations obtained according to the expert social network relationships and preference risks. Tang et al. [14] carried out CRPs from both levels of intra consensus and inter consensus, and generated different feedback suggestions in line with four consensus scenarios (high-high, high-low, low-high, low-low). Li et al. [37] constructed a consensus model based on personalized individual semantics in which the feedback recommendation considered two opposing consensus groups with respective acceptable and unacceptable consensus defined by consensus measure phase.

Nevertheless, these studies on CRPs in LSGDM are restricted to the following issues and challenges:

- Most of these consensus models did not generate feedback suggestions considering intra consensus and inter consensus levels at the same time, resulting in incompletely agreed group preference information.

- Tang et al. [14] established the CRPs from both levels of intra consensus and inter consensus, but ignored the interdependence between them. The feedback strategies managed intra and inter noncooperative behaviors separately.

### 2.3. Collective Decision Matrix Generation in LSGDM

There are two phases involved in collective decision making generation, subgroup weighting and information fusion. Many studies consider the subgroup weights based on the majority principle which only concerns the size of subgroups with the assumption that the experts in the same cluster are of equal importance [13]. Some integration methods also consider the preference information to determine subgroup weights. Wang et al. [11] developed a hybrid weight scheme to obtain the cluster weight vector, which took both the subgroup size and the variance into consideration. Ma et al. [38] employed cluster size and cluster reliability that was obtained on the basis of cluster similarity and cluster certainty as measure indicators of the hybrid cluster weight. As for information fusion, on intra layer, Wang et al. [11] applied weighted average operator to aggregate individual preference information based on cloud model. Some papers replaced the information fusion process with cluster centers. Zhu et al. [9] considered clustering centers gained by a heuristic algorithm as the core opinions of clusters and to represent the assessment of subgroups. At inter layer, the information fusion of subgroup assessments to generate global collective opinions was usually built on aggregation operators in paramount literature [8,11,39].

However, the research on collective decision making generation is limited to the following issues and challenges:

- No paper took into consideration the expertise levels of subgroups that indicates assessment reliability in subgroup weighting process, which tended to obtain unreasonable outcomes.

- In these studies, the fusion of individual assessments from a subgroup was conducted with aggregation operators or represented by cluster centers, which sacrificed partly preference information. Moreover, these approaches usually required to generate expert individual weights, which is a tricky task in LSGDM environment because of the huge size of expert team.

### 2.4. Decision Making Method for Bid Evaluation

Attributed to the rapid growth and development of civil engineering industry, a considerable number of decision making methods have been applied into bid evaluation. Juan et al. [40] proposed a housing refurbishment contractor evaluation model based on QFD with which residents can select an optimal refurbishment contractor according to requirements. Ko et al. [41] proposed the evolutionary fuzzy neural inference model (EFNIM) to develop a Sub-contractor Performance Evaluation Model (SPEM) using AI techniques. Vahdani et al. [42] proposed a new fuzzy compromise solution method for bid evaluation based on traditional VIKOR method considering the uncertainty of bid evaluation. Cheng [43] used the complicated hesitant fuzzy power Bonferroni mean operator for autocratic decision making using group recommendations (ADMUGRs) for bidders selection. The criterion system established in [2] including quantitative and qualitative criteria. The authors weighted the criteria with AHP method and utilized PROMETHEE technique to
select of the most efficient cultural heritage contractor. Padhi and Mohapatra [44] solved the bid evaluation problem by establishing a model that combined fuzzy AHP and the simple multi-attribute ranking technique (SMART); they applied this model to the Rural Development Department of the government of Orissa. However, these existing methods fail to consider the following points:

- Only a few experts were taken into consideration in these methods, which may be inappropriate for major construction projects. The employers generally prefer to large-scale expert groups for contractor selection for major projects in a bid to reduce evaluation risk.
- These methods allow compensations across criteria, indicating the bad performance on a criterion that may precipitate construction accidents can be compensated by other criteria, which flunks to conform the requirements and characteristics of construction projects.
- As a renowned fuzzy-linguistic approach-based tool and decision model, GCLEs have not been introduced to characterize the uncertainty of evaluation environment and expert knowledge.

### 2.5. A Summary of Main Contributions

Built on the analysis presented previously, the corresponding solutions to the pointed issues and challenges are offered by this subsection that are considered as the main contributions of the current paper.

- This paper focuses on bid evaluation of major construction projects under LSGDM environment. To obtain appropriate expert subgroups, a classification method based on K-means clustering paradigm is presented to sort the global expert group in the light of expert expertise levels that are depicted with a subjective- and objective-integrated identification approach.
- A consensus model guiding at reaching agreement among experts integrates both levels of consensus, intra consensus and inter consensus. In addition, the feedback mechanism in this consensus model is able to manage the intra and inter noncooperative behaviors simultaneously in order to take interdependence between them into account.
- A subgroup weight generation approach integrating subgroup sizes and expertise levels is proposed, followed by a PHFLTS-based individual information fusion process to reduce the sacrifice of preference information.
- An ELECTRE III based MCGDM outranking approach is developed to nominate the best contractor in a more reliable manner. The application of ELECTRE III introduces imprecise, indeterminate and uncertain criteria as well as prevents the compensations across criteria, both of which promote the elicitation of correct final results.
- Given the fact that most opinion holders are more comfortable using words rather than numbers to describe probabilities, the GCLEs, which are renowned fuzzy-linguistic approach-based tools and decision models, have been applied to characterize the uncertainty of major projects and experts’ knowledge.

### 3. PRELIMINARIES

Bid evaluation is a process where several invited experts identify the best bidder from some optional candidates. In general, there are some main differences between traditional bid evaluation and bid evaluation for major construction projects. First, traditional bid evaluation only involves a limited number of experts since superfluous experts usually require lots of cost and time. However, for major construction projects, it is necessary to assemble the opinions from a large-scale expert group that consists of more than 20 individuals. Second, complexity and uncertainty exist in major construction projects, thereby motivating the bid evaluation process to adopt fuzzy theory and linguistic values for expert evaluation. The involved basic notations and concepts are given as below:

For major construction projects, the inherent complexity and uncertainty make experts struggle to evaluate objects using a single linguistic term. Therefore, the concept of GCLEs and are introduced into this paper. The transformation rules between GCLEs and HFLTSs are given as below:

**Definition 1.** [45] Let $E_{G^{ij}}$ be a function that transforms the linguistic expression $ll \in S^l_H$ obtained by context-free grammar $G^l_H$ into HFLTS. $S$ is the linguistic term set used by $G^l_H$, and $S^l_H$ is the expression domain generated by $G^l_H$. $E_{G^{ij}}: S^l_H \rightarrow H$. The linguistic expressions generated by $G^{ij}_l$ using the production rules will be transformed into HFLTS by the following transformations:

$$E_{G^{ij}}(s_i) = \{ s_i \} \in S$$

$$E_{G^{ij}}(at \ most \ s_i) = \{ s_i \} \in S \ and \ s_i \leq s_j$$

$$E_{G^{ij}}(lower \ than \ s_i) = \{ s_i \} \in S \ and \ s_i < s_j$$

$$E_{G^{ij}}(at \ least \ s_i) = \{ s_i \} \in S \ and \ s_i \geq s_j$$

$$E_{G^{ij}}(greater \ than \ s_i) = \{ s_i \} \in S \ and \ s_i > s_j$$

$$E_{G^{ij}}(between \ s_i \ and \ s_j) = \{ s_i, s_j \} \in S \ and \ s_i \leq s_k \leq s_j$$

**Definition 2.** [46] Suppose two HFLTS assessments $H_{S_1} = \{ s_{\sigma_1}, ..., s_{\sigma_l} \}$ and $H_{S_2} = \{ s_{\sigma_2}, ..., s_{\sigma_l} \}$ defined on $S = \{ s_{\sigma_1}, ..., s_{\sigma_l} \}$. The distance measure between these two HFLTSs can be defined as

$$D(H_{S_1}, H_{S_2}) = \left( \frac{1}{l} \sum_{l=1}^{l} \left( \frac{|s_{\sigma_1} - s_{\sigma_2}|}{2\tau} \right)^2 \right)^{1/2}.$$
A large-scale expert group is involved in bid evaluation processes for major construction projects. The introduction of our proposal is essential to elicit the final outcomes. As mentioned above, the aggregation of expert opinions is critical to achieving the final outcomes.

**Definition 4.** Suppose two PHFLTSs $P_{hs}^1$ and $P_{hs}^2$, and their corresponding proportional information are denoted as $P_1 = (p_1^1, \ldots, p_1^l, \ldots, p_1^n)$ and $P_2 = (p_2^1, \ldots, p_2^l, \ldots, p_2^n)$, respectively. The distance measure between these two PHFLTSs can be defined as

$$
D(P_{hs}^1, P_{hs}^2) = \frac{1}{2\tau} \left| \sum_{i=-\tau}^\tau NS(s_i) p_1^i - \sum_{i=-\tau}^\tau NS(s_i) p_2^i \right|,
$$

where $NS(s_i)$ is the numerical scale of $s_i$, $NS(s_i) = 1$.

As mentioned above, a large-scale expert group is involved in bid evaluation processes for major construction projects. The aggregation of expert opinions is essential to elicit the final outcomes.

**Definition 5.** Suppose a set of PHFLTSs $P_{hs} = \{P_1^1, \ldots, P_1^l, \ldots, P_1^n\}$, and the corresponding proportion information is denoted as $P_1 = (p_1^1, \ldots, p_1^l, \ldots, p_1^n)$. $W = (w_1, \ldots, w_r, \ldots, w_n)$ are the associated weights such that $0 \leq w_i \leq 1$ and $\sum_{i=1}^n w_i = 1$. The hesitant fuzzy linguistic weighted average (HFLWA) operator is also defined as a PHFLTS

$$
HFLWA(P_1^1, \ldots, P_1^l, \ldots, P_1^n) = (p_1^1, \ldots, p_1^l, \ldots, p_1^n),
$$

where $p_i^j$ is computed by

$$
p_i^j = \sum_{i=1}^n w_i \cdot p_i^j.
$$

For more details regarding the PHFLTS, please refer to Refs. [15,48].

### 4. BID EVALUATION APPROACH FOR MAJOR CONSTRUCTION PROJECTS IN LSGDM CONTEXT

In this section, we introduce our proposal of a ELECTRE III-based LSGDM approach considering expertise levels to solve bid evaluation problems for major construction projects. The proposal is structured in line with collective decision matrix generation, expert classification, consensus model, and decision making method. The solution process follows the scheme presented in Figure 1.

The following notations are used to denote the sets and variables in the LSGDM problem, which are used throughout this paper.

- $C = \{c_1, \ldots, c_r, \ldots, c_m\}$: the set of $m$ assessment criteria, where $c_j$ denotes the $j$th criterion, $j = 1, 2, \ldots, m$.
- $Q = \{q_1, \ldots, q_l, \ldots, q_n\}$: the set of $n$ optional alternatives, where $q_l$ denotes the $l$th alternative, $l = 1, 2, \ldots, n$.
- $E = \{e_1, \ldots, e_k, \ldots, e_t\}$: the set of $t$ invited experts, where $e_k$ denotes the $k$th expert, $k = 1, 2, \ldots, t$.
- $W = (w_1, \ldots, w_p, \ldots, w_m)$: the weight vector corresponding to $C = \{c_1, c_2, \ldots, c_m\}$.
- $S = \{s_{-\tau}, \ldots, s_p, \ldots, s_\tau\}$: the linguistic term set used by this study, which is composed of $2\tau$ linguistic terms.

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**Figure 1** | The contractor selection framework.
4.1. Expert Classification

1) Elicitation of expertise features

Sellak et al. [49] proposed an expert expertise identification approach with three relevant features, hesitancy, interest, and preference that are derived objectively from the expert decision information. Built on that, this paper improves the expertise identification approach by adding another three features, considered as consensus, experience, and subjective assessment. Among these features, consensus, and experience are objectively derived from the assessment matrices and background information, while the feature of subjective assessment is obtained in accordance with provided expert working files by bid evaluation expert database.

As mentioned previously, the novel fuzzy-linguistic approach-based tools, GCLEs can be further transformed into HFLTSs by means of context-free grammars, which are given as Definition 1.

With the converted HFLTS evaluation matrices $H_{k}^{ij} = [H_{k}^{ij}]_{mn}$, the expertise features can be elicited as below:

- **Hesitancy**

  It is convincing that high-expertise experts would assess objectives with high confidence levels, implying less hesitancy is reflected in their HFLTS evaluation matrices. Thus, hesitancy is identified as expertise features can be elicited as below:

  **Hesitancy on element level:**

  \[
  H\Delta_{k} = \left| H_{k}^{ij} - 1 \right| \frac{2\tau - 1}{2\tau - 1},
  \]

  where $|H_{k}^{ij}|$ denotes the cardinality of $H_{k}^{ij}$ and indicates a uncertainty degree caused by hesitancy. Obviously, a larger cardinality $|H_{k}^{ij}|$ implies a higher hesitant degree.

  **Hesitancy on alternative level:**

  \[
  H\Delta_{k} = \frac{\sum_{i=1}^{n} HD_{k}^{ij}}{n}.
  \]

  **Hesitancy on criterion level:**

  \[
  H\Delta_{k} = \frac{\sum_{i=1}^{m} HD_{k}^{ij}}{m}.
  \]

Hesitancy on overall level:

\[
H\Delta_{k} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} HD_{k}^{ij}}{mn}.
\]

Obviously, both $H\Delta_{k}^{ij}$ and $H\Delta_{k}$ assume values within the unit interval $[0, 1]$. Reversing the hesitancy degree, the confidence denoted as expertise feature score $f_{k}^{1}$ is defined as:

\[
f_{k}^{1} = 1 - H\Delta_{k}.
\]

- **Interest**

  An expert with excellent profession and knowledge structure on a criterion is supposed to unmistakably discern among the set of alternatives under this criterion. The capability of distinguishing diverse alternatives against one criterion is mirrored by interest feature, another feature for expertise measure, which is usually calculated in accordance with the following principles:

  **Distinguishable assessments:** This principle discerns alternative set under criteria by the heterogeneity of alternative assessment information, represented by the central values of HFLTS assessments in this paper. For a given HFLTS assessment $H_{k}^{ij} = \{s_{\ell_{i}}\}_{\ell_{i}=1}^{n}$, the central value is computed by:

  \[
  CV_{k}^{ij} = \begin{cases} 
  s_{\ell_{i}} & \text{if} \quad l = L \\
  \frac{\sum_{l=1}^{L} s_{\ell_{i}}}{\ell_{i} - \ell_{i} + 1} & \text{if} \quad L > 1
  \end{cases}
  \]

  With the defined central values, the distinguishability ratio $DR_{k}^{ij}$ indicating the heterogeneity of HFLTS assessments given by $e_{k}$ for distinct alternatives under criterion $c_{j}$ is calculated as:

  \[
  DR_{k}^{ij} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \delta_{i}^{j+1} (e_{k}, c_{j})}{\sum_{i=1}^{n} (i - 1)},
  \]

  where $\delta$ is a operator defined for comparing any two HFLTS central values:

  $\delta_{i}^{j+1} (e_{k}, c_{j}) = \begin{cases} 
  0 & \text{if} \quad CV_{k}^{ij} = CV_{k}^{(j+1)} \\
  1 & \text{otherwise}
  \end{cases}$

  **Overlapping assessments:** This principle focuses on the occupation ratio of the uniform linguistic terms in HFLTS assessments to discern two alternatives under one criterion. The overlap ratio of $e_{k}$ under criterion $c_{j}$, denoted as $\Theta R_{k}^{ij}$, is obtained by:

  \[
  \Theta R_{k}^{ij} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \Theta_{i}^{j+1} (e_{k}, c_{j})}{\sum_{i=1}^{n} (i - 1)},
  \]

  where $\Theta$ is an operator for comparing two HFLTS assessments based on the uniform linguistic terms in them.
Finally, the interest feature score $U_i(q_i)$ integrating the distinguishability and overlap ratios across all evaluation criteria are calculated as below:

$$f_k^2 = \frac{\sum_{j=1}^{m} DR_k^j \cdot w_j + (1 - \sigma OR_k^j) \cdot (1 - HD_k^j)}{2m}.$$

### Preference

Corresponding to the feature of interest, the preference feature pays attention to the capability of distinguishing diverse criteria against one alternative, which is also evaluated by the principles of distinguishable assessments and overlapping assessments.

The distinguishability ratio of $e_k$ against criteria under alternative $q_i$, denoted as $DR_k^j$, is calculated as

$$DR_k^j = \frac{\sum_{j=1}^{m} \sum_{m-j}^{m-1} \theta_j^{j+1}(e_k, q_i)}{\sum_{j=1}^{m} (j - 1)}.$$

where $\delta$ is an operator that compares the central values of any two HFLTSs, defined as

$$\theta_j^{j+1}(e_k, q_i) = \begin{cases} 0 & \text{if } C^{q_i}_{k(e)} = C^{q_{i+1}}_{k(e)}, \\ 1 & \text{otherwise.} \end{cases}$$

The overlap ratio of $e_k$ under $q_i$, denoted as $\sigma OR_k^j$, is calculated as

$$\sigma OR_k^j = \frac{\sum_{j=1}^{m-1} \sum_{m-j}^{m-1} \theta_j^{j+1}(e_k, q_i)}{\sum_{j=1}^{m} (j - 1)}.$$

where $\Theta$ is an operator to compare any two HFLTSs directly as

$$\Theta^{j+1}_i(e_k, q_i) = \begin{cases} 1 & \text{if } L(HS_k^q) \geq L(HS_k^q_{i+1}) \text{ and } U(HS_k^q) \leq U(HS_k^q_{i+1}) \text{ and } U(HS_k^q_{i+1}) > L(HS_k^q) \text{ and } U(HS_k^q) \geq L(HS_k^q_{i+1}) \text{ and } \theta_j^{j+1}(e_k, q_i) \\ 0.5 & \text{if } U(HS_k^q) > L(HS_k^q_{i+1}) \text{ and } U(HS_k^q) \geq L(HS_k^q_{i+1}) \text{ and } \theta_j^{j+1}(e_k, q_i) \\ 0 & \text{otherwise.} \end{cases}$$

where $U(\cdot)$ (respectively, $L(\cdot)$) denotes the index of the upper (respectively, lower) linguistic term of an HFLTS assessment.

Finally, the interest feature score $f_k^2$ integrating the distinguishability and overlap ratios across all evaluation criteria are calculated as below:

The overall preference score at alternative level $P_k^q$ is given as:

$$P_k^q = \frac{\sum_{j=1}^{n} p_k^q}{\sum_{j=1}^{m} \sum_{m-j}^{m-1} \sigma OR_k^j \cdot (1 - HD_k^j)}.$$
• Experience

An experienced expert is able to give more reliable assessments in most cases, implying experience should be regarded as a critical factor for the identification of expertise levels. It is worthy of mentioning that the experts are required to provide relevant background information in advance in which the number of times that he/she has previously participated in bid evaluation \( s_h \), as well as his/her work years \( t_h \) is given. Finally, this paper elicit the fifth expertise feature \( f_k^5 \) as

\[
f_k^5 = \frac{\theta s_k + (1 - \theta) t_k}{\sum_{k=1}^{n} \theta s_k + (1 - \theta) t_k},
\]

where \( \theta \) indicates the importance of \( s_k \) in experience feature elicitation.

• Subjective assessment

The five features identified previously are objectively derived from the evaluation opinions and background information of experts. In a bid to obtain accurate and comprehensive expertise, this paper also introduces a subjective indicator, namely subjective assessment, as the sixth expertise feature. It is worthy of noting that sound supervision mechanisms have been established by bid evaluation expert databases today. After bid evaluation, the employers are often inspired to eliminate each expert according to their performance in bid evaluation and submit the elimination results to bid evaluation expert databases to form expert working files, which provides a significant resource for this paper to elicit the feature of subjective assessment.

Generally, the bid evaluation committee needs to recommend a sophisticated expert as committee director to control bid evaluation procedure. In possession of great profession, the recommended director is selected to assess the expertise of bid evaluation committee with real values in the unit interval \([0, 1]\) according to their working files. The assessment results are considered as the feature scores of subjective assessment.

2) K-means clustering paradigm

The collected six-dimensional expertise data of expert group forms the set of \( F \) that consists of \( t \) data points. Having obtained multi-dimension expertise information, the next task is to divide experts into distinct expertise subgroups. Since the efficiency of K-means clustering paradigm, this paper utilizes this method to classify experts into distinct expertise subgroups. Since the efficiency of K-means clustering paradigm, this paper utilizes this method to classify experts into distinct expertise subgroups.

\[
d(F_{x_k}, F_h) = \sqrt{\sum_{p=1}^{6} (f_{x_k}^p - f_h^p)^2}
\]

Step 3: Calculate the mean of the formed cluster \( C_h \) \((l = 1, 2, \cdots, K)\) in Step 2 as the new cluster centers

\[
C_h^p = \frac{1}{\#C_h} \sum_{i \in C_h} f_i^p, \quad p = (1, 2, \cdots, 6)
\]

where \( \#C_h \) represents the number of data points contained in cluster \( C_h \).

• Step 4: Repeat Steps 2-3 until the centers of clusters remain unchanged.

4.2. Consensus Reaching Process

The CRPs aim at guiding GDM processes towards agreed solutions. The lack of CRPs may lead to conflict assessments among experts, which leads to unfair bid evaluation. Therefore, a consensus model based on two levels of consensus is presented in this study. The entire expert group having been clustered into several subgroups in previous phase, the consensus management is also divided into two levels, intra consensus and inter consensus. In this consensus model, the intra consensus is manipulated first to reach agreement within subgroups; then this model focuses on inter consensus to adjust conflict subgroup opinions with intra consensus satisfied. For clear description, this subsection would depict the consensus model in the light of consensus measurement and feedback mechanism.

1) Consensus measurement for intra consensus

The consensus measurement for intra consensus in this paper is based on similarity among experts. For subgroup \( C_h \), with the calculation of similarity among experts given in the last subsection, we can obtain pairwise similarities matrices between experts; then the consensus model elicit consensus degrees of expert \( c_k \) on element level, alternative level, and overall level as below:

• Element level:

\[
CO_k^{ij} = \frac{1}{\#C_h} \sum_{b \in C_h} S_k^{ij}
\]

• Alternative level:

\[
CO_k^i = \frac{1}{m} \sum_{j=1}^{m} CO_k^{ij}
\]

• Overall level:

\[
C_h^o = \frac{1}{\#C_h} \sum_{k \in C_h} CO_k^o
\]

Overall consensus degree for subgroup \( C_h \):

\[
CO_h = \frac{1}{\#C_h} \sum_{k \in C_h} CO_h
\]
2) Feedback mechanism for intra consensus

If the obtained overall consensus degree for subgroup \( C_h \), \( CO_h \), reaches the consensus threshold \( \gamma \) fixed a priori by the group, then the bid evaluation can move onto the next step. Otherwise, feedback mechanism is required for conflict adjustments to gain agreed group decision matrix. Usually, the feedback mechanism contains identification and direction phases.

1) Identification rules

The identification rules are targeted to find noncooperative behaviors in subgroups. It is worthy of noting that several rounds of adjustments are needed to reach agreed group decision matrices. In the current round \( r \), the identification process is implemented with the following three steps [47].

(a) IR for experts:

\[
IRE = \{ e_k | CO_{k(r)} < \gamma, k \in C_h \}.
\]

(b) IR for alternatives:

\[
IRA_k = \{ q_i | e_k \in IRAECO_{ij(r)} < \gamma \}.
\]

(c) IR for criteria:

\[
IRC_k = \{ q_i | e_k \in IRAECO_{ik} \in IRAEMin \{ CO_{ik(r)} \} \}.
\]

2) Direction rules

In order to keep experts in control of their own preferences and preserve their sovereignty throughout the CRPs, this paper develops a feedback-based adjustment model to update expert decision matrices in subgroups until the conflict opinions are removed. Worthy of mentioning, the individuals assembled into the same subgroup should be regarded as equal importance because they are in possession of indistinguishable expertise. Thus, this study can merge individual HFLTS assessments held in subgroups with the notion of PHFLTSs given in Definition 3. With collective assessments of subgroup \( C_h \), \( P^h \), this adjustment model revises the assessments of identified experts, \( HS^h \), according to the following direction rules:

1) If \( E(HS^h_k) < E(P^h) \), then expert \( e_k \) should increase his/her assessment,

2) If \( E(HS^h_k) > E(P^h) \), then expert \( e_k \) should decrease his/her assessment,

3) If \( E(HS^h_k) = E(P^h) \), then expert \( e_k \) should remain his/her assessment;

where the expected values of \( HS^h \) and \( P^h \) are calculated as follows:

\[
E(HS^h_k) = \frac{1}{\|HS^h_k\|} \sum_{i \in HS^h_k} N(s) \quad E(P^h) = \sum_{i=-\tau}^{\tau} N(s) p_i^h.
\]

3) Consensus measurement for inter consensus

As mentioned previously, because of the identical individual importance in a subgroup, the collective assessment of the subgroup can be represented with a PHFLTS. In this sense, the consensus measurement for inter consensus is defined as the similarities among these subgroup collective PHFLTS assessments. The similarity between collective PHFLTS assessments of subgroup \( C_h \) and \( C_d \) is denoted as the negation of their distance that has been defined in Definition 4,

\[
S^h_{in} = 1 - D\left( P^h, P^d \right).
\]

Correspondingly, in line with consensus measurement for intra consensus, this paper obtains the consensus degree of subgroup \( C_h \) at element level \( CO^h \), alternative level \( CO^h \), and overall level \( CO_h \), which are omitted to avoid wordiness. Finally, the consensus degree on entire group is elicited as

\[
CO = \frac{1}{K} \sum_{h=1}^{K} CO_h.
\]

4) Feedback mechanism for inter consensus

The identification rules for inter consensus aiming to detect conflict subgroup opinions are identical as that for intra consensus. Given the interactions between intra consensus and inter consensus, the feedback mechanism in this study revises the noncooperative subgroup \( P^h \) by the following direction rules:

1) If \( P^l < P^h \), then subgroup \( C_h \) should increase group assessment by amplifying its smallest HFLTS assessment,

2) If \( P^l > P^h \), then subgroup \( C_h \) should decrease group assessment by reducing its biggest HFLTS assessment,

3) If \( P^l = P^h \), then subgroup \( C_h \) should remain the HFLTS assessments in it;

where \( P^l \) is the collective assessment of the entire group, which is obtained with the HFLWA operator defined in Definition 5. And the comparison law for PHFLTSs based on expected values and variances is given as

1) If \( E(\hat{s}_1) < E(\hat{s}_2) \), then \( \hat{s}_1 < \hat{s}_2 \).

2) If \( E(\hat{s}_1) = E(\hat{s}_2) \), then: a) if \( Var(\hat{s}_1) > Var(\hat{s}_2) \), then \( \hat{s}_1 < \hat{s}_2 \); b) if \( Var(\hat{s}_1) = Var(\hat{s}_2) \), then \( \hat{s}_1 = \hat{s}_2 \), where \( \hat{s}_1 \) and \( \hat{s}_2 \) are two PHFLTSs, the variances of PHFLTS assessment \( P_{lis} \) are calculated as follows:

\[
Var(P_{lis}) = \sum_{i=\tau}^{\tau} (N(s_i) - E(P_{lis}))^2 p_i.
\]

Obviously, the direction rules in our proposal reach appreciable consensus levels in views of both intra consensus and inter consensus. Take case 1 \( (P^h < P^l) \) as an example, amplifying its smallest HFLTS assessment not only increases the collective assessment of subgroup \( C_h \) to improve the inter consensus levels, but also further increases the intra consensus level of \( C_h \), because the HFLTS assessments in it become more closer. In this sense, the consensus model in this study is remarkable for simultaneous consideration of intra consensus and inter consensus in an accurate and reliable manner.

4.3. Collective Decision Matrix Generation

1) Subgroup weight generation

As noted previously, the expert importance is mainly determined by his/her expertise level, which inspires us to generate subgroup
weights based on the obtained subgroup clustering centers that represent the overall expertise levels of subgroups. In addition, it is plain that large-size subgroups are supposed to have more significant effects on decision making process. Constructed on the distance of cluster centers from the positive ideal solution (PIS) and the negative ideal solution (NIS), TOPSIS method has been a widely-applied decision making tool considering its simplicity and practicability. In this method, this paper is utilized to elicit subgroup weights considering subgroup sizes and expertise levels by the following steps:

- **Step 1:** Determine the best and worst feature score corresponding to each dimension of expertise level, forming the PIS \( \mathcal{P} = (f^1, f^2, \ldots, f^5) \) and NIS \( \mathcal{N} = (f^1, f^2, \ldots, f^5) \) and strong preference (P). Their definitions are as follows:

\[
\text{indeterminate and uncertain criteria and rejects the compensation by the following steps:}
\]

- **Step 2:** Calculate the distance of each cluster center \( \mathcal{S}_h = (C^1_h, C^2_h, C^3_h, C^4_h, C^5_h, C^6_h) \) from the PIS and NIS as follows:

\[
d^+_h = d(\mathcal{P}, \mathcal{S}_h) = \sqrt{\sum_{h=1}^{6} (f^h - C^h)^2},
\]

\[
d^-_h = d(\mathcal{N}, \mathcal{S}_h) = \sqrt{\sum_{h=1}^{6} (f^h - C^h)^2}.
\]

- **Step 3:** Calculate the relative closeness of each cluster center to the ideal solution as below:

\[
RC_h = d^-_h / (d^-_h + d^+_h), \quad h = 1, 2, \ldots, K.
\]

- **Step 4:** Determine the cluster weights combining \( RC_h \) and subgroup size \( N_h \) by a normalized process.

\[
w_h = \frac{RC_h \times N_h}{\sum_{h=1}^{K} RC_h \times N_h}.
\]

2) **Information fusion for individuals and subgroups**

Gathering individuals with indistinguishable expertise level together, then the subgroups can merge individual HFLTS assessments held in them into PHHTLS under the assumption that the experts are of equal priority is satisfied. This manner for information fusion effectively avoids information loss or distortion and is conducive to accuracy collective opinion generation. After fusing individual preference information in subgroups, the next phase is to aggregate the obtained PHHTLS subgroup assessments to generate collective decision matrix generation, which is conducted with the HFLWA operator defined in Definition 5.

4.4. **Decision-Making Method**

**ELECTRE III** method is used to derive the final alternative ranking for bid evaluation problem because it considers the imprecise, indeterminate and uncertain criteria and rejects the compensation among criteria.

There are three outranking relations among alternatives in ELECTRE III that are indicated as indifference (I), weak preference (Q), and strong preference (P). Their definitions are as follows:

(a) \( |E(P^a_{H^i}) - E(P^b_{H^i})| \leq r_j \iff ab \),

(b) \( r_j < E(P^a_{H^i}) - E(P^b_{H^i}) \leq p_j \iff aQb \),

(c) \( E(P^a_{H^i}) - E(P^b_{H^i}) > p_j \iff aPb \),

where \( r_j \) and \( p_j \) represent the indifference threshold and preference threshold, respectively, of criterion \( c_j \).

Having obtained group decision matrix \( [P^a_{H}]_{m \times n} \) represented as PHHTLSs in the previous phase, this stage can elicit concordance index \( c_j(a, b) \) and discordance index \( d_j(a, b) \) under criterion \( c_j \), \( j = 1, 2, \ldots, 5 \), according to the following equations:

\[
c_j(a, b) =
\]

\[
d_j(a, b) =
\]

\[
where \( v_j \) is the vote threshold of criteria \( c_j \).

As for quantitative criteria \( c_6 \), bid price, we can calculate \( c_j(a, b) \) and \( d_j(a, b) \) in these cases as follows:

\[
c_j(a, b) =
\]

\[
d_j(a, b) =
\]

where \( g(a) \) and \( g(b) \) indicate the normalized criterion values of the bid price for bidders \( q_a \) and \( q_b \), respectively.

Our simulation obtains the overall concordance index \( c(a, b) \) by fusing concordance indexes under each criteria with weighted average operator.

\[
c(a, b) = \sum_{j=1}^{m} w_j c_j(a, b).\]
After calculating the overall concordance index \( c(a, b) \), the credibility index of alternative \( q_a \) outranking \( q_b \) is defined by

\[
S(a, b) = \begin{cases} 
    c(a, b), & \text{if } d_j(a, b) \leq c(a, b) \\
    c(a, b) \cdot \prod_{j \in C(a,b)} (1 - d_j(a, b)) / (1 - c(a, b)), & \text{otherwise},
\end{cases}
\]

where \( C(a, b) \) is the set of criteria satisfying \( d_j(a, b) > c(a, b) \). After gaining the credibility index among alternative under each criterion, a distillation procedure adapted by [19] is introduced to this paper to derive the final alternative ranking.

5. CASE STUDY

5.1. Implementation of the Proposed Method

Jingzhou flood-diversion area in Hubei lies an important part of the flood-control system that stores the superfluous flood water from upper Jingjiang river. This significant function requires this area to be equipped with sound safety facilities to ensure the safety of residents. However, local residents are suffering from serious housing condition problems. The existing settlement houses are of low construction quality, and not able to resist damaging weather like hail and thunderstorms. In addition, awful housing management has led the settlement houses to wasteyards, which exposes local people to hidden dangers. Finally, the local government determines to tender for the reconstruction of settlement houses. This section would apply the proposed methodology to solve the bid evaluation problem in a bid to select the best contractor for this reconstruction project.

A considerable number of contractors are attracted by the settlement house reconstruction project and submit their bid documents to the local government. For the significance of this reconstruction project, an expert group consisting of 36 individuals with diverse professional backgrounds is established to eliminate the performances of these bidders. After the primary filter five contractors \( Q = \{q_1, q_2, \cdots, q_5\} \) are admitted to bid evaluation stage according to criterion system \( C = \{c_1, c_2, \cdots, c_6\} \). The indications of these criteria are given as \( c_1 \): construction quality, \( c_2 \): experience, \( c_3 \): construction duration, \( c_4 \): management quality, \( c_5 \): finance, \( c_6 \): bid price. The bid prices of the five bidders are (i) 4,665,179 RMB, (ii) 4,546,240 RMB, (iii) 4,699,519 RMB, (iv) 4,495,201 RMB, and (v) 4,597,027 RMB, respectively. We then implement the proposed methodology by the following steps:

**Step 1:** Generate individual decision matrices. The performances on \( c_1 - c_5 \) of the five bidders are evaluated by the expert team with GCLEs. The linguistic term set used by this case study is

\[
S = \{s_{-3} = \text{very poor}, s_{-2} = \text{poor}, s_{-1} = \text{slightly poor}, s_0 = \text{medium}, s_1 = \text{slightly good}, s_2 = \text{good}, s_3 = \text{very good}\}.
\]

The GCLE decision matrix provided by expert \( e_1 \) is as Table 1 in Supplementary Information File.

**Step 2:** Transform the GCLEs into HFLTSs by the aforementioned context-free-grammar approach. The HFLTS information of \( e_1 \) is shown as Table 2. Table 2 attached in the supplementary information file gives the transformed HFLTSs of the remaining experts.

**Step 3:** The background information for each expert is given as Table 3.

Integrating the evaluation matrix and background information of each expert, we can obtain the normalized expert expertise level according to the proposed expertise identification approach as Table 4.

**Step 4:** In line with the acquired expertise features in Step 3, this step clusters the expert group into four subgroups. Table 5 gives the clustering analysis result of each subgroup.

**Step 5:** This step concerns the consensus reaching process, which includes intra consensus and inter consensus. With the developed

### Table 1 GCLE assessment information provided by \( e_1 \).

| \( q_1 \) | \( q_2 \) | \( q_3 \) | \( q_4 \) | \( q_5 \) |
|---|---|---|---|---|
| \( c_1 \) | Between slightly good and good | Less than slightly good | At least medium | At least medium | Greater than slightly good |
| \( c_2 \) | Greater than medium | At most slightly good | At most medium | Greater than slightly poor | Less than slightly good |
| \( c_3 \) | At least poor | Between slightly poor and medium | At most medium | At least medium | At most slightly good |
| \( c_4 \) | Between slightly good and very good | Greater than medium | Less than slightly good | At most slightly poor | At least slightly good |
| \( c_5 \) | At least medium | Between slightly poor and medium | Greater than slightly good | At least slightly good | Less than slightly poor |

### Table 2 HFLTS assessment matrix of \( e_1 \).

| \( q_1 \) | \( q_2 \) | \( q_3 \) | \( q_4 \) | \( q_5 \) |
|---|---|---|---|---|
| \( c_1 \) | \( \{s_{-3}, s_{-2}, s_{-1}, s_0\} \) | \( \{s_0, s_1, s_2, s_3\} \) | \( \{s_0, s_1, s_2, s_3\} \) | \( \{s_{-3}, s_{-2}, s_{-1}, s_0\} \) |
| \( c_2 \) | \( \{s_{-3}, s_{-2}, s_{-1}, s_0\} \) | \( \{s_0, s_1, s_2, s_3\} \) | \( \{s_0, s_1, s_2, s_3\} \) | \( \{s_{-3}, s_{-2}, s_{-1}, s_0\} \) |
| \( c_3 \) | \( \{s_{-3}, s_{-2}, s_{-1}, s_0\} \) | \( \{s_0, s_1, s_2, s_3\} \) | \( \{s_0, s_1, s_2, s_3\} \) | \( \{s_{-3}, s_{-2}, s_{-1}, s_0\} \) |
| \( c_4 \) | \( \{s_0, s_1, s_2, s_3\} \) | \( \{s_{-1}, s_0\} \) | \( \{s_2, s_3\} \) | \( \{s_{-1}, s_0\} \) |
| \( c_5 \) | \( \{s_0, s_1, s_2, s_3\} \) | \( \{s_{-1}, s_0\} \) | \( \{s_2, s_3\} \) | \( \{s_{-1}, s_0\} \) |
consensus measurement paradigm, we obtain the intra consensus levels of the four clusters as 0.951, 0.926, 0.943, 0.932, which indicates all subgroups reach appreciable agreements compared with the consensus threshold 0.900. With regard to inter consensus levels of the four clusters that are 0.884, 0.905, 0.951, 0.952, respectively, some adjustments are required by subgroup $C_4$ to improve its inter consensus level. Based on the identification rules defined in Section 4.2, this step considers the assessment on alternative $q_5$ against criterion $c_4$, $P_{5,4}^5 = (0.0, 0.0, 0.111, 0.333, 0.333, 0.222)$ for further amendment. Comparing with the collective assessment of the entire group, $P_{5,4}^5 = (0.138, 0.146, 0.138, 0.220, 0.141, 0.144, 0.072)$, we should decrease the assessment $P_{5,4}^5$. The biggest HFLTS assessments on alternative $q_5$ in subgroup $C_4$ are provided by $c_1$ and $c_4$, denoted as $[s_1, s_2, s_3]$. After changing them into $[s_2, s_1, s_3]$, the inter consensus of this subgroup is increased to an acceptable level, 0.903.

**Step 6:** Having reaching appreciable consensus levels, the following task aims to generate subgroup weights. The PIS and NIS of subgroup cluster centers are $(0.88, 0.89, 0.92, 0.99, 0.82, 0.82)$ and $(0.64, 0.71, 0.74, 0.97, 0.27, 0.39)$, correspondingly. After calculation, the subgroup weight vector is denoted as $(0.0167, 0.159, 0.293, 0.531)$, with which this step can elicit the group collective opinions by using Definition 5. The results are listed as Table 6.

**Step 7:** Based on the group opinions obtained by the last step, this step focuses on elicitation of alternative ranking with the ELECTRE III. To achieve this goal, the current case assigns the thresholds as $r_j = 0.05$, $d_j = 0.15$, $v_j = 0.25$ to calculate concordance and discordance index between arbitrary two alternatives. Subsequently, considering criterion weight vector as $(0.20, 0.10, 0.20, 0.15, 0.15, 0.20)$, this study obtains credibility index matrix as Table 7. Ultimately, utilizing the distillation procedure where $\alpha = 0.3, \beta = -0.15$, we elicit the ranking of contractors as $q_1 > q_3 > q_5 > q_2 > q_4$.

### 6. Comparison Analysis

In this section, we compare our proposal with the existing approaches to further demonstrate the advantages of our proposal. The decision making process is to elicit alternative rankings according to collective decision matrix using various decision making methods. Built on that, this paper conducts the comparison analysis from the aspects of collective opinion generation and decision making methods.
Table 6 | The group collective opinions.

|     | \( q_1 \)          | \( q_2 \)          | \( q_3 \)          |
|-----|---------------------|---------------------|---------------------|
| \( c_1 \) | (0.069, 0.076, 0.076, 0.212, 0.162, 0.259, 0.147) | (0.127, 0.167, 0.162, 0.156, 0.103, 0.103, 0.250, 0.215, 0.161, 0.065) | | |
| \( c_2 \) | (0.100, 0.100, 0.100, 0.190, 0.237, 0.230, 0.043) | (0.106, 0.123, 0.106, 0.184, 0.099, 0.100, 0.099, 0.294, 0.236, 0.169, 0.002) | | |
| \( c_3 \) | (0.107, 0.141, 0.133, 0.179, 0.112, 0.205, 0.122) | (0.107, 0.147, 0.129, 0.192, 0.120, 0.133, 0.120, 0.209, 0.149, 0.201, 0.068) | | |
| \( c_4 \) | (0.052, 0.052, 0.063, 0.183, 0.184, 0.275, 0.191) | (0.154, 0.179, 0.179, 0.208, 0.096, 0.113, 0.096, 0.233, 0.188, 0.184, 0.090) | | |
| \( c_5 \) | (0.098, 0.114, 0.108, 0.201, 0.111, 0.202, 0.167) | (0.130, 0.178, 0.140, 0.160, 0.133, 0.146, 0.133, 0.210, 0.124, 0.183, 0.073) | | |
| \( q_4 \) | (0.108, 0.185, 0.179, 0.289, 0.131, 0.092, 0.014) | (0.124, 0.141, 0.120, 0.366, 0.100, 0.102, 0.047) | | |
| \( c_2 \) | (0.150, 0.217, 0.217, 0.302, 0.085, 0.027, 0.002) | (0.141, 0.194, 0.176, 0.197, 0.084, 0.148, 0.060) | | |
| \( c_3 \) | (0.124, 0.158, 0.127, 0.317, 0.137, 0.129, 0.008) | (0.160, 0.177, 0.161, 0.245, 0.141, 0.094, 0.022) | | |
| \( c_4 \) | (0.189, 0.231, 0.244, 0.281, 0.031, 0.025, 0) | (0.176, 0.181, 0.164, 0.252, 0.078, 0.111, 0.038) | | |
| \( c_5 \) | (0.116, 0.127, 0.127, 0.284, 0.186, 0.151, 0.010) | (0.018, 0.044, 0.027, 0.387, 0.157, 0.281, 0.088) | | |

Table 7 | Credibility index matrix.

|     | \( q_1 \) | \( q_2 \) | \( q_3 \) | \( q_4 \) | \( q_5 \) |
|-----|---------|---------|---------|---------|---------|
| \( q_1 \) | – | 1.000 | 1.000 | 1.000 | 0.958 |
| \( q_2 \) | 0.867 | – | 0.896 | 1.000 | 0.800 |
| \( q_3 \) | 0.677 | 1.000 | – | 1.000 | 0.800 |
| \( q_4 \) | 0.706 | 0.822 | 0.660 | 1.000 | – |

Table 8 | Clustering analysis result and collective opinion of each subgroup.

| Subgroup | Expert Element |
|----------|----------------|
| \( C_1 \) | \( e_1, e_2, e_4, e_6, e_3, e_8, e_3, e_14, e_15, e_16, e_17, e_20, e_22, e_23, e_25, e_31, e_33 \) |
| \( C_2 \) | \( e_3, e_4, e_8, e_13, e_14, e_15, e_16, e_17, e_20, e_22, e_23, e_25, e_31, e_33 \) |
| \( C_3 \) | \( e_9, e_{10}, e_{12}, e_{27}, e_{28}, e_{29}, e_{30}, e_{32} \) |
| \( C_4 \) | \( e_2, e_5, e_6, e_7, e_{11}, e_{18}, e_{19}, e_{21}, e_{23}, e_{34}, e_{35} \) |

6.1. Comparison Analysis for Collective Opinion Generation

This subsection compares the collective opinion generation approach in our proposal with that presented in [13]. For conscientious comparison of aggregation outcomes, this subsection replaces the cluster method in [13] with K-means paradigm and implements the following phases for collective opinion generation.

- **Step 1: Clustering experts based on similarity between preference information**

Liu et al. [13] clustered experts with the grey clustering algorithm based on similarity measures between experts to implement opinion clustering. As mentioned above, this step replaces the clustering algorithm with K-means cluster algorithm. The clustering analysis results is as Table 8.

- **Step 2: Determination of subgroup weight**

Liu et al. [13] assumed that (1) Experts in the same cluster should be assigned the same weights; (2) Clusters that have large number of experts should be assigned larger weights based on the majority principle. Built on that, we can obtain the subgroup weight vector as \( (0.111, 0.389, 0.222, 0.278) \).

- **Step 3: Aggregation of preference information**

Liu et al. [13] used the weighted averaging operator for preference information aggregation. With the assumptions above, we can obtain the collective opinions as Table 9.

From this comparison analysis, we can highlight our proposal with the following advantages:

- This paper provides a novel and remarkable clustering indicator for expert classification under LSGDM environment. It is obvious that the clustering results in Table 5 are far different from Table 8, which means expertise level is a renew clustering indicator alien from preference information similarity. Concerning this novel clustering indicator in LSGDM process can help decision maker effectively take expert status, knowledge structures and education into account.

- Our proposal presents a subgroup weight determination approach considering both subgroup sizes and expertise levels, which is beneficial to obtain accurate results. However, it is found that there are significant differences between the weight vectors generated by the two compared approaches, indicating the majority principle-based subgroup weight determination approach in [13] tends to lead to unreliable results.
• The group opinions in our proposal are represented with PHFLTSs. Compared with aggregation operators that deduce a signal indicative assessment to represent the aggregated information, the two-dimension PHFLTS structure reserves all aggregated individual preference opinions and is capable to extract rational and accurate alternative ranking.

6.2. Comparison Analysis for Decision-Making Method

As a classical decision making tool, TOPSIS method is employed to confirm the flexibility of our proposal by implementing the following steps:

Step 1: Determine the PIS $f_j^+$ and NIS $f_j^-$ for each criteria by comparing the expected values and variances of the HFTLS possibility distributions presented in the collective decision matrix.

Step 2: Calculate the distance of each bidder from the PIS and NIS as follows:

$$d_i^+ = \sum_{j=1}^{n} w_j d\left(p_{ij}^c, f_j^+\right), \quad d_i^- = \sum_{j=1}^{n} w_j d\left(p_{ij}^c, f_j^-\right),$$

$$d\left(p_{ij}^c, f_j^+\right) = \frac{1}{2\tau} \left| \sum_{l=-\tau}^{\tau} N(s_l) p_{ij}^{c+} - \sum_{l=-\tau}^{\tau} N(s_l) p_{ij}^c \right|,$$

$$d\left(p_{ij}^c, f_j^-\right) = \frac{1}{2\tau} \left| \sum_{l=-\tau}^{\tau} N(s_l) p_{ij}^{c-} - \sum_{l=-\tau}^{\tau} N(s_l) p_{ij}^c \right|,$$

where $p_{ij}^{c+} = \left(p_{ij}^{c+}, \ldots, p_{ij}^{c+}, \ldots, p_{ij}^{c+}\right)$ and $p_{ij}^c = \left(p_{ij}^{c}, \ldots, p_{ij}^{c}, \ldots, p_{ij}^{c}\right)$ are the possibility distributions of $f_j^+$ and $f_j^-$, respectively.

Step 3: Calculate the relative closeness to the ideal solution as below:

$$RC_i = d_i^- / (d_i^- + d_i^+).$$

After calculation, we have $RC_1 = 0.8297, RC_2 = 0.4751, RC_3 = 0.5923, RC_4 = 0.3034$, and $RC_5 = 0.3974$. Consequently, the bidder ranking is $q_1 > q_3 > q_2 > q_5 > q_4$.

It can be observed that the TOPSIS method produces almost uniform bidder ranking with our proposition except for the order of alternatives $q_3$ and $q_5$, which conceivably stems from the features of ELECTRE III method. Observing the collective opinions, we find $q_2$ holds a slight dominance over $q_5$ on criteria $c_1 - c_4$ while is outranked by $q_5$ on criterion $c_5$. The higher overall performance of $q_2$ provides it a more prominent order in TOPSIS method. However, because of the vote threshold in ELECTRE III, the great deficiency of $q_2$ on criterion $c_5$ prevents it surmounting $q_5$.

7. CONCLUSION

This paper solves the MC(G)DM problems in bid evaluation under LSGDM environment by taking into consideration expert classification, CRPs, collective decision matrix generation, and decision making method with the following work involved:

• This paper clusters expert using K-means clustering analysis from the perspective of expert expertise features, which are identified as hesitancy, interest, preference, consensus, experience, and subjective assessment. Among these features, hesitancy, interest, preference, consensus are derived from individual assessment matrices while the features of experience and subjective assessment are induced from background information and working files of experts.

• The consensus model in this paper detects and manages noncooperative behaviors from both intra layer and inter layer, and considers the interaction between them. This model first manages consensus within sub-groups, i.e., intra consensus, by consensus measurement and the feedback-based adjustment mechanism. Afterwards, the inter consensus level is measured by similarity among the subgroup collective assessments that are represented as PHFLTSs. Then, an effective adjustment mechanism is proposed to improve inter-consensus level by identifying and revising deviant individual assessments in subgroups, which also further improve the consensus levels within subgroups.

• Because of the similar expertise levels attached with the individuals in the same cluster, PHFLTSs are used to gather their preference information to elicit subgroup assessment, which greatly decreases information loss or distortion compared with traditional aggregation process. In addition, a subgroup weight determination approach based on TOPSIS method considering both subgroup sizes and expertise levels is proposed to generate reliable subgroup weights.

• The collective assessments of entire group are obtained by HFLWA operator where subgroup weights are derived from cluster centers based on TOPSIS method. Then, ELECTRE III method integrating qualitative and quantitative criteria is adopted by the selection process to pander to the requirements and characteristics of bid evaluation, which are indicated as the imprecise, indeterminate and uncertain criteria as well as eradication of compensations across attributes. Both are conducive to extract more appropriate outcomes for bid evaluation problems.

Despite of the effectiveness and flexibility of this developed proposal, there are still issues remaining to be solved to further expand its application, which are described as below:

• Expert classification in this paper merely concerns the expertise features, neglecting the influence of decision attitudes on decision making problems. Therefore, a double-hierarchy expert clustering analysis that considers both expertise features

| Table 9 | Clustering analysis result and collective opinion of each subgroup. |
|--------|------------------|------------------|------------------|------------------|------------------|
|        | $q_1$    | $q_2$    | $q_3$    | $q_4$    | $q_5$    |
| $c_1$  | 1.03     | 0.83     | 0.60     | 0.40     | 0.19     |
| $c_2$  | 0.74     | 0.25     | 1.94     | 0.75     | 0.02     |
| $c_3$  | 0.61     | 0.07     | 0.26     | 0.26     | 0.33     |
| $c_4$  | 0.22     | 0.06     | 0.44     | 0.96     | 0.35     |
| $c_5$  | 0.69     | 0.08     | 0.31     | 0.07     | 0.60     |
and decision attitudes is required for further research to output more accurate classification results.

- Actually, the criterion systems associated with bid evaluation problem are generally complex where criteria are usually interdependent with each other. Thus, it is necessary to take interactions among criteria into consideration in further research, which has been supported by various tools such as the well-established Choquet integral [50,51] and the Bonferroni-type aggregation functions [52–54].

CONFLICT OF INTEREST

The authors declare they have no conflicts of interest.

AUTHORS’ CONTRIBUTIONS

All authors contributed to the work. All authors read and approved the final manuscript.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (grant nos. 71801175, 71871171, 71971182, and 71902041), Guangdong Basic and Applied Basic Research Foundation (grant no. 2020A1515011511), the Theme-based Research Projects of the Research Grants Council (Grant no. T32-101/15-R), the Fundamental Research Funds for the Central Universities (grant no. 2042018kf0006), the Ger/HKJRS project (grant no. G-CityU103/17), and the City University of Hong Kong SRG (grant no. 7004969).

Supplementary Material

Supplementary data related to this article can be found at https://doi.org/10.2991/ijcis.d.200801.002

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