Wide-Baseline Relative Camera Pose Estimation with Directional Learning

Kefan Chen*  Noah Snavely  Ameesh Makadia
Google Research  Google Research  Google Research
chenkefan950518@gmail.com  snavely@google.com  makadia@google.com

Abstract

Modern deep learning techniques that regress the relative camera pose between two images have difficulty dealing with challenging scenarios, such as large camera motions resulting in occlusions and significant changes in perspective that leave little overlap between images. These models continue to struggle even with the benefit of large supervised training datasets. To address the limitations of these models, we take inspiration from techniques that show regressing keypoint locations in 2D and 3D can be improved by estimating a discrete distribution over keypoint locations. Analogously, in this paper we explore improving camera pose regression by instead predicting a discrete distribution over camera poses. To realize this idea, we introduce DirectionNet, which estimates discrete distributions over the 5D relative pose space using a novel parameterization to make the estimation problem tractable. Specifically, DirectionNet factorizes relative camera pose, specified by a 3D rotation and a translation direction, into a set of 3D direction vectors. Since 3D directions can be identified with points on the sphere, DirectionNet estimates discrete distributions on the sphere as its output. We evaluate our model on challenging synthetic and real pose estimation datasets constructed from Matterport3D and InteriorNet. Promising results show a near 50% reduction in error over direct regression methods. Code will be available at https://arthurchen0518.github.io/DirectionNet.

1. Introduction

Estimating the relative pose between two images is fundamental to many applications in computer vision such as 3D reconstruction, stereo rectification, and camera localization [20]. For calibrated cameras, relative pose is synonymous with the essential matrix, which encapsulates the projective geometry relating two views. Prevailing approaches recover the global model from corresponding points [31, 21, 43] within an iterative robust model fitting process [16, 66]. Recent progress has introduced deep-learned modules that can replace components of this classic pipeline [64, 52, 11, 10, 54, 45, 49, 3]. While this class of techniques has been extensively analyzed [51], well-known failure cases include where feature detection or matching is difficult, such as low image overlap, large changes in scale or perspective, or scenes with insufficient or repeated textures.

In these cases, it is natural to consider if supervised deep learning can address this basic task of essential matrix estimation, given its success in a variety of challenging computer vision problems. Specifically, can we train a deep neural network to represent the complex function that directly maps image pairs to their relative camera pose? Such a model would provide an appealing alternative to formulations that are sensitive to correspondence estimation performance.

Unfortunately, evidence suggests designing regression models for pose estimation is challenging, and in fact finding a parameterization of the motion groups effective in deep learning models is still an active research topic [34, 68, 46]. Not surprisingly, the initial works exploring relative pose regression (e.g. [39, 48]) are not conclusively successful in the difficult scenarios described above.

In this work we introduce a novel deep learning model for relative pose estimation, focused on the challenging wide-baseline case. Conceptually, our model generates a discrete probability distribution over relative poses, and the final pose estimate is taken as the expectation of this distribution. Our method is inspired by works that show estimating a discrete distribution, or a heatmap over a quantized output space, consistently outperforms direct regression to the continuous output space. This observation has arisen in various applications, including estimating 2D and 3D keypoint locations [57, 33, 59] and estimating periodic angles [26].

However, it is currently unclear if this idea translates directly to complex higher-dimensional output spaces. Relative camera pose lives in a five dimensional space, so predicting a discrete distribution would require $O(N^5)$ storage in the output space alone [36]! Given that representing such large spaces is currently intractable for neural networks at

*Work done while Kefan was a member of the Google AI Residency program (g.co/airesidency).
any reasonable resolution, how can we effectively apply this concept to the relative pose problem?

To this end, we introduce a novel formulation that builds upon the idea of estimating discrete probability distributions on the 5D relative pose space. We propose two key components to execute this idea effectively:

1. A parameterization of the motion space that factorizes poses as a set of 3D direction vectors. A non-parametric differentiable projection step can map these directions to their closest pose.

2. **DirectionNet**, a convolutional encoder-decoder model for predicting sets of 3D direction vectors. The network outputs discrete distributions on the sphere $S^2$, the expected values of which produce direction vectors.

Our core contributions are in recognizing that incorporating a dense structured output in the form of a discrete probability distribution can improve wide-baseline relative pose estimation, and in introducing a technique to execute this idea efficiently. The attributes of this approach that help make it effective include (1) DirectionNet is fully-convolutional and as such does not utilize any fully-connected regression layers, and (2) it allows for additional supervision as both the dense distribution and final estimated pose can be supervised.

DirectionNet is deployed in two stages, wherein the relative rotation is estimated first, followed by the relative translation direction. This allows us to de-rotate the input images after the first stage, which reduces the complexity of the translation estimation task.

DirectionNet is evaluated on two difficult wide-baseline pose estimation datasets created from the synthetic images in InteriorNet [28], and the real images in Matterport3D [5]. DirectionNet consistently outperforms direct regression approaches (for the same pose representation as well as numerous alternatives), as well as classic feature-based approaches. This illustrates the effectiveness of estimating discrete probability distributions as an alternative to direct regression, even for a complex problem like relative pose estimation. Furthermore, these results validate that a supervised data-driven approach for wide-baseline pose estimation can succeed in cases that are extreme for traditional methods.

2. **Related Work**

Feature matching–based methods are still prevalent for the relative pose problem, yet suffer under large motions that yield unreliable correspondence (see [51] for a survey). Recent works deploy deep learning in subproblems such as feature detection [54, 11], filtering or reweighting outliers [64, 52]. Differentiable versions of consensus methods like RANSAC have also been proposed [49, 3]. These still rely on sufficiently accurate matches, an uncertain prospect in wide-baseline settings.

Many deep regression methods have addressed 3D object pose recovery from a single image [40, 56, 60, 29, 58]. For our task of relative pose estimation, [39] adopts a Siamese convolutional regression model to directly estimate relative camera pose from two images. [12] proposes a relative pose layer atop Siamese camera localization towers. [48] introduces a model for uncalibrated cameras that regresses to a fundamental matrix via an intermediate representation of camera intrinsics, rotation, and translation. Related to these efforts, [9] and [42] proposed deep convolutional networks for homography estimation. Despite targeting various tasks, most of the above methods share a common architecture design—a convolutional network culminating with fully connected layers for regression. A related problem to ours is camera re-localization which estimates pose from a single image in a known scene [23, 17]. Our setting is quite different as we try to recover the relative camera pose from two images in a previously unvisited scene.

Deep learning for ego-motion estimation or visual odometry is an active area which includes supervised methods such as [61] (depth supervision), and many self-supervised methods (see [6] for a survey). In general, these systems make design choices specific for small-baselines and video sequences, such as using frame-to-frame image reconstruction losses [67, 65, 63], or training with more than two frames [62, 13, 67]. In contrast, our focus is on learning to estimate relative pose for wide-baseline image pairs.

Probabilistic deep models have been used to capture uncertainty in pose predictions. In [8, 37] they estimate the distribution of 6D object poses to tackle shape symmetries and ambiguities, while [50, 19] uses directional statistics to model object rotations and regress the parameters of the probability distributions. [4] adopts mixtures of von Mises-Fisher and quasi-Projected Normal distributions to represent point sets. The multi-headed approach in [47] combines multiple predictions into a mean pose and associated covariance matrix. In contrast to most of these techniques, ours is a discrete representation and is not tied to any choice of parametric probability distribution model.

3. **Method**

We now outline our method for estimating relative pose from image pairs. The relative pose between two views is specified by a 3D rotation $R \in SO(3)$ ($R \in \mathbb{R}^{3 \times 3}$, $R^\top R = I$, $\det(R) = 1$), and a translation $t \in \mathbb{R}^3$. Without additional assumptions, the relative translation can only be recovered up to a scale factor, so we adopt the normalization $\|t\| = 1$; equivalently, $t$ is restricted to the 2-sphere, $t \in S^2$. Our task is to estimate the relative pose in $SO(3) \times S^2$.

In principle, we desire a model that estimates a discrete distribution over the pose space $SO(3) \times S^2$. This requires discretizing this five-dimensional space at a reasonable resolution, which is computationally infeasible in deep networks.
We can map $u$ to a probability distribution with a spherical metric (geodesics). However, it requires a non-convex optimization: the argmax operator identifies the most probable direction from a distribution, but is not differentiable and its precision is limited by the grid resolution. An alternative is the Fréchet mean [2], which is appealing since it defines a spherical centroid using the natural metric (geodesics). However, it requires a non-convex optimization and the solution is not necessarily unique. Instead, we choose the alternative of taking the expected value of the distribution.

### 3.1. Directional parameterization of relative pose

Parameterizing $\text{SO}(3) \times S^2$ requires a choice for $\text{SO}(3)$. We choose a simple over-parameterization of $\text{SO}(3)$, splitting the rotation matrix $R \in \text{SO}(3)$ into its component vectors $R = [x \ y \ z]$, where each component is itself a unit vector in $\mathbb{R}^3$. The relative pose between images, $(R, t) \in \text{SO}(3) \times S^2$, can then be specified by the four direction vectors $x, y, z, t \in S^2$. See Section 4.3 for a discussion and justification of our parameterization.

### 3.2. Estimating 3D directions

The relative pose task is now one of estimating a set of direction vectors $x, y, z, t \in S^2$. Making the simplifying assumption of independence of these vectors, our approach is to (1) predict a probability distribution over the space of possible directions ($S^2$) for each vector, and (2) extract the direction prediction from each distribution. It is important to note that representing functions on the sphere and integrating them requires careful consideration in the context of neural networks where the data representation is restricted to regular grids. The details of our approach are below.

### Spherical distributions.

We represent functions on $S^2$ with a 2D equi-rectangular projection indexed by spherical coordinates $(\theta_i, \phi_j)$. Our discretization follows [25]: $\theta_i$ is the angle of colatitude ($0 \leq i < H$, $\theta_i = \frac{(2i+1)\pi}{2H}$), $\phi_j$ is the azimuth ($0 \leq j < W$, $\phi_j = \frac{2\pi j}{2W}$), and $H \times W$ is the grid resolution. Let $u(\theta_i, \phi_j)$ denote the unnormalized output of a network, and $f(x) = \ln(1 + e^x)$ be the softplus function. We can map $u$ to a probability distribution with a spherical normalization:

$$
P(\theta_i, \phi_j) = \frac{f(u(\theta_i, \phi_j))}{\sum_{i=0}^{H-1} \sum_{j=0}^{W-1} f(u(\theta_i, \phi_j)) \sin(\theta_i)},$$  

(1)

where the $\sin(\theta_i)$ in the normalizing term comes from the area element on $S^2$.

### Spherical expectation.

The $\text{argmax}$ operator identifies the most probable direction from a distribution, but is not differentiable and its precision is limited by the grid resolution. An alternative is the Fréchet mean [2], which is appealing since it defines a spherical centroid using the natural metric (geodesics). However, it requires a non-convex optimization and the solution is not necessarily unique. Instead, we choose the alternative of taking the expected value of the distribution. In the continuous case, we define the expected value of a random variable $X$ on $S^2$ with PDF $p_X$ as $E[X] = \int_{\rho \in S^2} \rho \cdot p_X(\rho) d\rho$. In the discretization of the sphere introduced above, this becomes

$$E[X] = \sum_{i=0}^{H-1} \sum_{j=0}^{W-1} \rho(\theta_i, \phi_j) P(\theta_i, \phi_j) \sin(\theta_i),$$  

(2)

where $\rho(\theta_i, \phi_j)$ is the 3D unit vector corresponding to spherical point $(\theta_i, \phi_j)$. The expected value $v = E[X], v \in \mathbb{R}^3$ can be projected to the sphere in a straightforward manner:

$$\hat{v} = \frac{v}{\|v\|^2}.$$

### 3.3. DirectionNet for relative pose estimation

DirectionNet maps image pairs to sets of unit direction vectors following the steps outlined above. We adopt an encoder-decoder style architecture that learns a cross-domain...
mapping from two images to a spherical representation (see Figure 2(a)). We describe two ways that DirectionNet can be instantiated for relative pose estimation.

The SVD variation. Here DirectionNet will produce four vectors \( \{ \hat{v}_x, \hat{v}_y, \hat{v}_z, \hat{v}_t \} \) which are the predictions of the directional pose components \( \{x, y, z, t\} \). Due to the simplifying assumptions earlier, in general \( [\hat{v}_x, \hat{v}_y, \hat{v}_z] \notin SO(3) \) as orthogonality is not guaranteed. However, we can project \( M = [\hat{v}_x, \hat{v}_y, \hat{v}_z] \) to SO(3) with orthogonal Procrustes [55], which is a differentiable procedure [59] and provides the optimal projection to SO(3) by Frobenius norm:

\[
R = U \text{diag}(1, 1, \det(UV^T))V^T,
\]

where \( U \Sigma V^T = M \) is the SVD of \( M \) [55].

The Gram-Schmidt variation. For \( [\hat{v}_x, \hat{v}_y, \hat{v}_z] \in SO(3) \), the last column \( \hat{v}_z \) is fully constrained: \( \hat{v}_z = \hat{v}_x \times \hat{v}_y \). Thus, we only need DirectionNet to produce three vectors \( \{\hat{v}_x, \hat{v}_y, \hat{v}_t\} \) and obtain a rotation by a partial Gram-Schmidt projection of \( \hat{v}_z \) and \( \hat{v}_y \) as in [68].

Both the SVD and Gram-Schmidt projections have been shown recently to reach state-of-the-art performance for different 3D rotation estimation tasks, especially when predicting arbitrary (large) rotations. See [27] and [68] for the analysis. Although overall performance between the two is similar, SVD is shown to be slightly more effective, with a possible explanation being that SVD finds a least-squares projection while Gram-Schmidt is greedy. Our experimental findings confirm the analysis.

3.4. Two-stage model with derotation

Intuitively, we expect that learning camera translation should be easiest when the data never exhibit rotational motion. Hence, we propose estimating camera pose sequentially: (1) A DirectionNet (denoted DirectionNet-R) estimates the relative rotation between input images, (2) this rotation is used to derotate the input images, and (3) a second DirectionNet (denoted DirectionNet-T) predicts translations from the rotation-free image pair. Figure 1 illustrates the stages of this process.

However, when the relative rotation between the cameras is large, derotating one image relative to the other could result in projecting most of the scene outside the camera’s field of view. To limit this effect we find that it is helpful to project both input images to an intermediate frame using half of the estimated rotation with a larger FoV and proportionally increased resolution. The derotation implementation involves a homography transformation \( H_r = K' \rho^{-1} K^{-1} \), where \( K \) is the matrix of camera intrinsics for the input image, \( K' \) is the intrinsics for the desired derotated image, and \( \rho \) is the half-rotation of the estimated rotation \( R \). Setting \( K' = K \) maintains the field of view (FoV) and resolution of the input image after derotation. We use \( H_{r^{-1}} \) and \( H_r \) to project input images \( I_0 \) and \( I_1 \), respectively, onto the “middle” frame. The output FoV and resolution are controllable parameters of the model. To mitigate the effect of rotation errors on translation prediction, we developed a rotation augmentation scheme with random perturbations (see appendix C).

3.5. Network architecture

The DirectionNet architecture is illustrated in Figure 2(a). Notably, we eschew the skip connections common to similar convolutional architectures ([41, 53]) since they would not reflect the correct spatial associations between the planar and spherical topologies (observed in [15]).

The image encoder embeds image pairs into \( \mathbb{R}^{512} \) using a Siamese architecture. A Siamese branch consists of a 7×7, stride-2 convolution followed by a series of residual blocks\(^2\), each of which downsamples by 2. The outputs are concatenated in the channel dimension, and after two

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\(^2\)All residual blocks consist of two bottleneck blocks [22] with 3×3 convolutions, batch normalization, and leaky ReLU pre-activations.
more residual blocks, a global average pooling produces the embedding.

The **spherical decoder** maps embeddings to spherical distributions by repeatedly applying bilinear upsampling and residual blocks. We use spherical padding before upsampling to ensure adjacent pixels at the boundaries reflect the correct neighbors on the sphere (see Figure 2(b)). The final outputs, of size $64 \times 64 \times k$, are interpreted as $k$ spherical distributions, and are subsequently mapped to $k$ direction vectors following equations 1 and 2. In total, a single DirectionNet contains $\approx 9M$ parameters. Note that we could also consider a Spherical CNN decoder [7, 14]. While such models are equivariant to 3D rotations, the benefit in our setting would be limited since the image encoder is not rotation equivariant. Hence, we opt for the computational efficiency of 2D convolutions.

### 3.6. Loss terms and model training

In this section, we describe the loss terms and training strategy for our model. We let $P(\theta_i, \phi_j)$ denote a single distribution generated by DirectionNet, and let its corresponding ground truth distribution be $P^*(\theta_i, \phi_j)$. The ground truth distributions are constructed using the von Mises-Fisher distribution as described in Sec. 4.1. With a slight abuse of notation we will denote with $E[P]$ the expected value of a distribution and is computed according to equation 2. One appealing property of DirectionNet is that both the output direction vectors as well as their corresponding dense probability distributions can be supervised.

The **direction loss** is the negative cosine similarity between two 3D vectors:

$$L_\Diamond(p_1, p_2) = -\frac{p_1^T p_2}{\|p_1\|\|p_2\|}. \quad (4)$$

We introduce two loss terms to supervise distributions. First, the **distribution loss** provides dense supervision on the equirectangular distribution grid:

$$L_D(P_1, P_2) = \frac{1}{HW} \sum_{i=0}^{H-1} \sum_{j=0}^{W-1} (P_1(\theta_i, \phi_j) - P_2(\theta_i, \phi_j))^2 \sin(\theta_i), \quad (5)$$

Second, the **spread loss** penalizes the spherical “variance” [38] to encourage unimodal and concentrated distributions:

$$L_\sigma(p) = 1 - \|p\|. \quad (6)$$

The full loss for a single predicted direction vector combines the three individual losses:

$$L(P, P^*) = L_\Diamond(E[P], E[\hat{P}]) + \lambda_D L_D(P, P^*) + \lambda_\sigma L_\sigma(E[P]). \quad (7)$$

See appendix E for an analysis of the individual loss terms. By far the most impactful loss on performance is the distribution loss $L_D$.

We train our two-stage pipeline sequentially. We first train DirectionNet-R, which takes the source and target image pair $(I_s, I_t)$ as input and produces three direction vectors $[\hat{v}_x, \hat{v}_y, \hat{v}_z]$, which are mapped onto $SO(3)$. The complete training loss for DirectionNet-R is the sum of individual losses for the three estimated directions $L_R = L(P_x, P_{\hat{x}}) + L(P_y, P_{\hat{y}}) + L(P_z, P_{\hat{z}})$. After training, DirectionNet-R is frozen and its predictions are used for derotating the inputs of DirectionNet-T. DirectionNet-T takes the input pair $(H_T^T(I_0), H_T(I_1))$ and outputs a translation direction $\hat{v}_t$. Since the translation is represented with a single unit vector, DirectionNet-T is trained with the loss $L_T = L(P_t, P_{\hat{t}})$.

### 4. Experiments

To evaluate our method on challenging data exhibiting a wide range of relative motion, we generate image pairs from existing panoramic image collections. Image pairs are generated by sampling pairs of panoramas from a common scene, then projecting them to overlapping planar perspective views. By varying the camera viewing angles we create a dataset with varied relative poses and overlap. Specific details of each dataset are provided below. Unless specified otherwise, our training image pairs have a resolution of $256 \times 256$ and a 90° FoV. In all cases, we generate 1M training pairs and $\sim 1K$ test pairs. **Note that there is no overlap between the train and test scenes.** See appendix B for more details.

**InteriorNet** [28] is a synthetic dataset with 560 scenes with panoramas rendered along smooth camera trajectories which we sample at random strides. InteriorNet-A is constructed to have rotations up to 30°, while InteriorNet-B has varied FoV (60° to 90°) and rotations up to 40°.

**Matterport3D** [5] contains 10K real panoramas captured from locations $\sim 2.25m$ apart covering 90 scenes. Matterport-A is constructed to have rotations up to 45°, and Matterport-B up to 90°. These are more challenging than InteriorNet due to a wider baseline and smaller overlap.

#### 4.1. Training details

We train with loss weights $\lambda_\sigma = 0.1$, $\lambda_D = 8 \times 10^7$, and Adam [24] with a learning rate of 1e-3 and a batch size of 20. The ground truth distributions are generated by the von Mises-Fisher distribution with concentration $\kappa = 10.0$ ($\kappa$ is analogous to $\frac{1}{\sigma^2}$ in a Gaussian distribution). The derotated images for InteriorNet have a 90° FoV, while the derotated Matterport3D images have a 105° FoV and an increased resolution of $344 \times 344$ to compensate for the larger rotations.

#### 4.2. Baselines

We now introduce the baselines. Full details can be found in appendix D.
**Table 1. Quantitative results on the Matterport datasets.** We report the mean and median angular error in degrees, as well as average rank of each method over all test pairs. Rotation (\(R\)) and translation (\(t\)) shown separately.

|                | Matterport-A |                | Matterport-B |                |
|----------------|--------------|----------------|--------------|----------------|
|                | \(R\)        | \(t\)          | \(R\)        | \(t\)          |
|                | mean (\(^\circ\)) | med (\(^\circ\)) | rank | mean (\(^\circ\)) | med (\(^\circ\)) | rank | mean (\(^\circ\)) | med (\(^\circ\)) | rank |
| DirectionNet   |              |                |              |                |
| 9D             | 3.96         | 2.28           | 2.76         | 14.17          | 6.46           | 3.29 | 13.60         | 3.54           | 2.89 |
| 6D             | 4.30         | 2.22           | 2.79         | 16.37          | 7.07           | 3.29 | 14.85         | 3.69           | 3.45 |
| 9D-Single      | 4.55         | 3.11           | 3.83         | 21.65          | 10.53          | 4.71 | 13.37         | 4.00           | 2.85 |
| Quat.          | 23.32        | 23.00          | 8.25         | 39.85          | 21.26          | 3.54 | 21.26         | 8.90           | 3.44 |
| Regression     |              |                |              |                |
| Bin&Delta     | 6.93         | 4.71           | 5.28         | 22.84          | 10.16          | 3.73 | 31.54         | 22.98          | 6.45 |
| Spherical     | 10.68        | 7.98           | 6.79         | 40.09          | 22.85          | 6.36 | 32.94         | 20.56          | 4.29 |
| 6D            | 5.73         | 3.66           | 3.79         | 35.75          | 21.89          | 6.32 | 18.23         | 7.69           | 4.29 |
| Quat.         | 15.40        | 12.66          | 6.86         | 41.57          | 21.47          | 7.18 | 28.38         | 19.23          | 6.19 |
| SIFT          | 25.55        | 5.63           | 7.71         | 35.53          | 14.84          | 6.20 | 36.58         | 10.54          | 8.13 |
| LMedS         | 19.33        | 6.66           | 7.31         | 45.04          | 29.78          | 8.08 | 31.30         | 9.55           | 7.47 |

**Table 2. Performance of learned feature-based methods on the Matterport A and B datasets.**

|                | Matterport-A |                | Matterport-B |                |
|----------------|--------------|----------------|--------------|----------------|
|                | \(R\)        | \(t\)          | \(R\)        | \(t\)          |
|                | mean (\(^\circ\)) | med (\(^\circ\)) | rank | mean (\(^\circ\)) | med (\(^\circ\)) | rank | mean (\(^\circ\)) | med (\(^\circ\)) | rank |
| SuperGlue (indoor) [54] | 8.34         | 5.22           | 21.08        | 11.86          |
| D2-Net [11]   | 18.79        | 7.12           | 41.58        | 25.56          |
| DirectionNet-9D | 3.96         | 2.28           | 14.17        | 6.46           |
| SuperGlue (indoor) [54] | 13.23        | 7.19           | 29.90        | 13.28          |
| D2-Net [11]   | 21.03        | 7.74           | 43.82        | 27.30          |
| DirectionNet-9D | 13.60        | 3.54           | 21.26        | 8.90           |

**DirectionNet variations.** We consider multiple variants of our full two-stage model with intermediate derotation. **DirectionNet-9D** projects three direction vectors onto \(SO(3)\) using SVD for the rotation estimation, while **DirectionNet-6D** uses a partial Gram-Schmidt projection [68] (refer to Sec. 3.3 for details). To understand the importance of derotation, we also consider a single-stage version without derotation (**DirectionNet-9D-Single**) which estimates four directions from a single DirectionNet module.

**Discrete pose representation alternatives.** To evaluate our choice of representation for the discretized pose space, we consider multiple alternatives: **Bin&Delta** [35] is a hybrid model combining a coarse rotation classification (over clustered quaternions) with a refinement regression network. To understand if decoupling a 3D rotation into a set of direction vectors is necessary, we introduce **DirectionNet-Quat** which estimates a discrete distribution over the space of unit quaternions at resolution \(32^3\). The spherical decoder is replaced by a 3D volumetric CNN decoder. **3D-RCNN** [26] estimates individual (Euler) angles with a discrete distribution over the quantized circle. 3D-RCNN consistently under-performed Bin&Delta, see appendix D.

**Regression baselines.** We evaluated multiple direct pose regression baselines: **Spherical regression** [30] uses a novel spherical exponential activation for regression to \(n\)-spheres; **6D** [68] regresses 6D outputs followed by a Gram-Schmidt projection for rotations; **Quaternion** regresses a unit quaternion for the rotation. This is what is used in the camera pose regression modules from PoseNet [23] and [39]. The multiple regression baselines share the same image encoder architecture as our DirectionNet.

**Parametric probabilistic pose.** **vM** [50] estimates individual (Euler) angles by directly regressing the continuous parameters of a von-Mises distribution (location and concentration) on the circle. We were unable to train vM successfully on all datasets (see appendix D).

**Feature-based baselines.** We consider two versions of the classic correspondence-based pipeline, SIFT features [32] with robust LMedS [66] (**SIFT+LMedS**) or with RANSAC [16] (**SIFT+RANSAC**). We also consider pipelines with learned components such as **SuperGlue** [54] and **D2-Net** [11].

Note, the differences in most baselines are in the rotation representation. Unless specified otherwise, the baselines predict a unit-normalized 3D vector for the camera translation direction.

**Evaluation metrics.** We report geodesic errors for both rotations and translation directions, separately. Additionally, we rank each method on every test pair, reporting the mean rank across examples (1 is the best possible rank, 10 is the worst possible rank).
which indicates our choice of the lower-dimensional spher-
very high, local feature based techniques are still superior.
models require just
requirements limits performance, whereas our directional
sis that the smaller resolution allowed by its
Quat baseline predicts a distribution over the half hyper-
quantized output space in some way, namely Bin&Delta [35],
Single consistently outperform alternatives which also use a
for estimating discrete distributions over relative camera
pose? Both DirectionNet-6D/9D and DirectionNet-9D-
Single consistently outperform alternatives which also use a
quantized output space in some way, namely Bin&Delta [35],
DirectionNet-Quat, and 3D-RCNN [26]. Our DirectionNet-
Quat baseline predicts a distribution over the half hyper-
sphere in $S^3$. Its poor performance supports our hypothe-
thesis that the smaller resolution allowed by its $O(N^3)$ space
requirements limits performance, whereas our directional
models require just $O(N^2)$ space.

Feature-based approaches. In this work our aim is to un-
derstand if relative pose regression can be improved using
a discrete pose distribution representation, especially in the
difficult wide-baseline setting. Thus, our primary experi-
mental analysis is a comparison of different regression and
probabilistic techniques. For completeness, we also evaluate
feature-based approaches. Unsurprisingly, the feature-based
methods have different performance characteristics com-
pared to the learned direct methods. For example, for those
image pairs where feature extraction and matching are likely
successful (e.g. high overlap), the estimated motion based
on SIFT features is consistently better than any learned tech-
nique (Fig. 3-b). We note that in the best cases the SIFT
methods both regularly reach sub-1° errors in relative ro-
tation estimation, while all of the learning methods rarely
reach errors that low. However, our dataset construction
intentionally includes a large fraction of large-motion pairs
likely to have low overlap, and this drives down the over-
all performance of these methods (Fig. 3-a). In addition
to classic feature-based methods, we compare with learned
descriptors and matching pipelines. Table 2 shows results
using pretrained models for SuperGlue [54] and D2-Net [11].
SuperGlue has slightly better performance than DirectionNet
on mean rotation error for Matterport-B, but in general Di-
rectionNet outperforms the learned feature-based methods.
This is not surprising since our datasets include many image
pairs where keypoint detection and matching can be difficult.

4.3. Analysis

Table 1 reports the quantitative results of the different
methods on the Matterport datasets (see Table 3 for results
on the InteriorNet datasets). We begin by considering the
two main questions for analyzing our approach.

Can relative camera pose be better learned using a dis-
crete pose distribution versus direct regression? We observe
that our DirectionNet outperforms all baselines on each met-
ric for each dataset. A particularly illustrative comparison
is DirectionNet-6D vs 6D regression, where for the same
pose representation, prediction via a discrete distribution is
consistently better. We do not see the same improvement
for DirectionNet-Quat over Quaternion regression, however,
which indicates our choice of the lower-dimensional spher-
ical/directional representation is important (see discussion
below). Finally, we observe that predicting a parametric
probabilistic representation of pose does not help (vM [50]
performed 5x worse than DirectionNet, see appendix D).

Is our directional representation better than alternatives
for estimating discrete distributions over relative camera
pose? Both DirectionNet-6D/9D and DirectionNet-9D-
Single consistently outperform alternatives which also use a
quantized output space in some way, namely Bin&Delta [35],
DirectionNet-Quat, and 3D-RCNN [26]. Our DirectionNet-
Quat baseline predicts a distribution over the half hyper-
sphere in $S^3$. Its poor performance supports our hypothe-
sis that the smaller resolution allowed by its $O(N^3)$ space
requirements limits performance, whereas our directional
models require just $O(N^2)$ space.

Figure 3. (a) True rotation magnitude (°) vs error (°). The scatter
plot shows that our model is robust in the presence of large relative
rotations. (b) Median error (°) vs. overlap (%). As image overlap
decreases from 90% to 20%, the median test errors of our method
increases much slower than the SIFT+LMedS. When overlap is
very high, local feature based techniques are still superior.

Quantitative Results. Table 2 shows results
using pretrained models for SuperGlue [54] and D2-Net [11].
SuperGlue has slightly better performance than DirectionNet
on mean rotation error for Matterport-B, but in general Di-
rectionNet outperforms the learned feature-based methods.
This is not surprising since our datasets include many image
pairs where keypoint detection and matching can be difficult.

Generalization. To demonstrate the generalization ability
of our model, we train DirectionNet-9D on InteriorNet-A
(synthetic) and test it on Matterport-A (real). The mean
and the median errors of the rotation are 8.42° and 5.13°,
and 20.71° and 8.60° for translation. Even without any fine-
tuning on real data our approach still outperforms most
baselines which had the benefit of training on Matterport-
A. To test the model’s performance on outdoor scenes, we
trained on a subset of KITTI [18]. DirectionNet gives 9.19°
mean rotation error and 19.36° translation error while the
best baseline gives 13.44° and 22.53° for rotation and trans-
lation respectively. See Table 4.

5. Conclusion

The results presented above tell a consistent story. Mod-
els that regress relative pose directly from wide-baseline
image pairs can be improved by estimating a discrete prob-
ability distribution in the pose space. Our approach
effectively executes this idea by operating on a factorized
pose space that is lower dimensional than the 5D pose space
Figure 4. **Qualitative evaluation on Matterport-B.** Any point in one image plane corresponds to a ray shooting from the optical center, which could be projected to the other image plane as the *epipolar line*. (a) We draw a number of points detected by SIFT in different colors on each target image $I_t$, and (b) show their corresponding epipolar lines on the source image using the ground truth pose, (c) visualizations from our DirectionNet-9D, (d) Bin & Delta, (e) spherical regression, (f) 6D regression, (g) SIFT+LMedS. Most examples demonstrate some of the most difficult scenarios, such as drastic change in viewpoint and significant occlusion. The last two rows show that SIFT+LMedS can outperform the others in the case of smaller motions for which the feature-based approach can find reliable feature correspondences.
and suitable for discretized outputs. Evaluated on challenging synthetic and real wide-baseline datasets, DirectionNet generally outperforms regression models, parametric probabilistic models, and alternative discretization schemes.

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Appendix

A. Spherical Padding

We propose using spherical padding in our decoder network to reflect the correct topology on a spherical representation (See Figure 5 for our motivation).

Figure 5. Discrete distributions on the sphere (left) are represented internally as equirectangular grids (right). Although pixels A and B are adjacent on the sphere, as are C and D, they are not adjacent in the grid. Our spherical padding (shown in Fig. 2b, page 4) corrects for this.

B. Dataset Generation

Since large-scale wide-baseline stereo datasets are difficult to acquire, we create our datasets from corpora of panoramic scene captures by taking pairs of panoramas that observe overlapping parts of scenes, sampling camera look-at directions for each panorama using a heuristic that ensures image overlap, and projecting the panoramas to perspective views with a given field of view. Figure 6(a) illustrates this process. The look-at direction, $\ell_1$, for the first camera is uniformly sampled from a band around the equator, which is bounded in colatitude $\theta \in [-\frac{\pi}{4}, \frac{\pi}{4}]$ and azimuth angle $\phi \in [0, 2\pi)$. The look-at direction, $\ell_2$, for the second camera is uniformly sampled from a circular cone centered at the direction $\ell_1$ so that the magnitude of rotations are uniformly distributed in each of our datasets. The limit in latitude prevents the cameras from looking only at the ceiling or floor, which are relatively textureless for many scenes. The aperture of the cone can be adjusted to vary the amount of overlap between image pairs while maintaining variability in the relative camera orientations. Each camera rotation matrix is then constructed from the appropriate look-at vector and the world up vector.

In Figure 3(b) in the main paper, we show results on the Matterport-B test set grouped by overlap percentage between the input images. Matterport3D panoramas contain depth channels which allows us to calculate the overlap percentage between the input image pair as

$$O(I_0, I_1) = \min \left( \frac{|I_0 \cap I_1|}{|I_0|}, \frac{|I_0 \cap I_1|}{|I_1|} \right) \tag{8}$$

Figure 6. (a) We randomly sample perspective images from pairs of panoramas by picking the look-at directions $\ell_1$ and $\ell_2$ of the source and target cameras based on a heuristic (left). The red boundaries overlaid on the spherical images (center) match the output perspective views (right). (b) Any point $x$ in one image plane corresponds to a ray shooting from its optical center $o$, which represents all possible 3D locations of $x$ in the world. The projection of this ray into the second image plane forms a line called the *epipolar line*, shown in purple in the figure. The 2D point in the second image corresponding to $x$ must lie on this line.

C. Training details

We implemented our model in Tensorflow [1]. The model was trained asynchronously on 40 Tesla P100 GPUs. A single DirectionNet has approximately 9M parameters. Our full model DirectionNet-9D/6D, which consists of two DirectionNets, contains in total 18M trainable parameters. Each net was trained for 3M steps.

**Rotation Perturbation.** To improve the robustness of DirectionNet-T to rotation estimation errors, we apply data augmentation to its input by perturbing the rotations used for derotation. Given $R \in SO(3)$, we perturb it by randomly sampling three unit vectors no further than 15° away from the component vectors of $R$ and projecting the result back onto $SO(3)$. We perturb the estimated rotation from DirectionNet-R before derotating the input images for DirectionNet-T. This perturbation is critical to performance. Without it, the translation error is 4° worse on InteriorNet, and much worse when the rotation range is large (MatterportB).
D. Relative Pose Baselines

We now provide additional details of the baselines including the ones not in the main paper.

- **DirectionNet-Quat** directly generates a probability distribution over the half-hypersphere in $S^3$. In this case, the spherical decoder consists of 3D upsampling and 3D convolutional layers. Since the output is on a hypersphere, the discretization requires much higher resolution ($O(N^3)$) compared with our model ($O(N^2)$). DirectionNet-Quat generates output at $32^3$. We believe the limited resolution is partly responsible for the poor performance compared to DirectionNet-9D and -6D.

- **Bin&Delta** [35] adopts the Bin-Delta hybrid model that consists of a classification network which gives a coarse estimation of the rotation and a regression network that refines the estimate. The rotation space is discretized K-Means clustering on the training data (we use $K = 200$). We use the same encoder as ours and DirectionNet-T for the translation.

- **Spherical regression** [30] uses a novel spherical exponential activation on the $n$-sphere to improve the stability of gradients during training. The final outputs of the model are the absolute values of the coordinates of a unit vector in $\mathbb{R}^n$, along with $n$ classification outputs for their signs. We use the same encoder as ours followed by separate two-layer prediction networks (one for a quaternion representation of rotation and one for translation).

- **6D regression** uses the same image encoder as ours, followed by two fully connected layers with leaky ReLU and dropout, to produce a 6D continuous representation for the rotation and 3D for the translation. The 6D output is mapped to a rotation matrix with a partial Gram-Schmidt procedure; see [68] for details. This approach uses a continuous representation for 3D rotation, and consequently facilitates training.

- The **quaternion** regression baseline is implemented using same Siamese network as described in [39] without the spatial pyramid pooling layer, followed by fully connected layers to produce a 4D quaternion and a 3D translation. We normalize the quaternion and the translation during training, and use the same loss as suggested in the paper (L2). In our experiment, we weight the quaternion loss with $\beta = 10$, as in the original paper.

- **SIFT+LMedS** is a classic technique for recovering the essential matrix from correspondences. Local features are detected in images with SIFT, and subsequently matched across images. These feature matches are filtered with Lowe’s proposed distance ratio test. Given the remaining putative correspondences, least median of squares (LMedS) is used to robustly estimate the essential matrix, from which we can recover the rotation and normalized translation direction. We use the OpenCV implementation for all of these steps.

- **SIFT+RANSAC** is the same as SIFT+LMedS, with RANSAC instead of LMedS.

- **SuperGlue** [54] uses CNN and graph neural network to extract and match local features from images. D2-Net [11] trains a single CNN as a dense feature descriptor and a feature detector. Both methods require correspondence/depth supervision from real data, which is not available in our Matterport datasets. We ran pre-trained indoor SuperGlue (training code not available) and D2-Net with RANSAC.

Additional baselines. The following baselines are not presented in the main paper due to the limit of space. For reference, the mean rotation error of our DirectionNet-9D tested on Matterport-A is 3.96°, on Matterport-B is 13.60°.

- **vM** [50] provides a probabilistic formulation for the 2D pose by estimating parameters of von Mises distribution on a circle ($S^1$). We adapt this method to estimate the 3D rotation by producing three von Mises distributions representing the Euler angles. However, the training is hindered by the singularities known in the Euler angles representation[68]. The mean rotation error tested on Matterport-A is 20.28° (5x worse than DirectionNet-9D) and the training diverges on Matterport-B.

- **3D-RCNN**[26] uses a classification-regression hybrid model for 2D pose estimation by uniformly discretize the 2D circle into bins. This can be directly adapted to 3D rotation by estimating the three Euler angles. Due to the discontinuity of the Euler angles representation[68], the performance is poor compared with the similar hybrid **Bin&Delta** model. The mean rotation error tested on Matterport-A is 18.61° and the error on Matterport-B is 32.33°.

- **D2-Net**[11] trains a single CNN as a dense feature descriptor and a feature detector. Both methods require correspondence/depth supervision from real data, which is not available in our Matterport datasets. We ran pretrained indoor SuperGlue (training code not available) and D2-Net with RANSAC.

- **PoseNet[23]** relocates images in known scenes; we consider relative pose in scenes never seen during training. PoseNet regresses to a 3D position and quaternion, and this is similar to the **quaternion** regression baseline.
E. Additional Results and Discussion

DirectionNet consistently outperforms regression methods, showing the potential value in a fully convolutional model that avoids fully-connected regression layers and discontinuous parameterizations of pose. We show more comprehensive results to compare our model DirectionNet-9D with the baselines. Note that the spherical regression baseline generally has a higher error in rotation compared with the 6D regression method. Even though the spherical exponential activation does improve training the regression model, the 6D continuous rotation representation is still preferable to quaternions. Figure 7 and Figure 8 compare the error histogram distribution of our model with the best two regression baselines, the Bin&Delta and the 6D Regression. Figure 9 compare the error histogram distribution of ours with SIFT+LMedS. Note that SIFT+LMedS has a higher error histogram distribution of ours compared with the baselines. Note that the spherical regression baseline generally has a higher error in rotation compared with the 6D regression method.

To visualize results of the different methods, we select a few points detected by SIFT in image \( I_1 \) and draw their corresponding epipolar lines in image \( I_2 \) as determined by the estimated relative pose. Figure 6(b) illustrates the epipolar geometry. We show additional qualitative results to compare our primary model DirectionNet-9D with baselines representative of regression models and the classic method, see Figure 10 and 11. In Figure 12, we highlight scenarios where our method struggles, such as repeating or complex texture, scenes with few objects and minimal texture, or extreme motion between images.

Ablation study on loss terms. We study the effects of the loss terms by training the DirectionNet-9D on Matterport-A. The mean rotation error is 4.68° without the spread loss, 4.85° without direction loss, and 14.66° without distribution loss, compared with 3.96° with all losses. The distribution loss which provides the direct supervision on the output distribution plays the key role in the training, because we provide the prior knowledge on the distribution by generating the ground truth from von Mises-Fisher distribution on 2-sphere which resembles the spherical normal distribution. This shows evidence that distributional learning with dense supervision is advantageous to direct regression [57, 33]. Alternatively, the distribution loss could use the KL divergence to provide the prior knowledge on the distribution by generating the ground truth from von Mises-Fisher distribution on 2-sphere which resembles the spherical normal distribution.

**Multimodal distribution on high uncertainty scenarios.** In rare scenarios, our model gives higher uncertainty and produces multimodal or even antipodal distributions. Based on our observations, this usually happens in certain scenes, for example, the scene structure exhibits some symmetry or repetitive textures and causes ambiguity in the direction of the motion from two images. (See Figure 13 and Figure 14 for more examples.)

**Outdoor scenes.** We used KITTI odometry [18] dataset (sequence 0-8 for train, 9-10 for eval) and sampled image pairs with a min rotation of 15° and translation of 10m (36K train pairs, 1K test pairs, mean translation ~15m). Table 4 shows generalization from MatterportA to KITTI (we cropped Matterport images to approximate the KITTI FoV). This is a hard generalization task as the distribution of relative poses in KITTI is extremely different from Matterport, yet fine-tuning with just 20% of data is on par with the local feature baselines, and strong results after retraining with 100% of the data indicates DirectionNet is also effective outdoors.

**RANSAC vs. LMedS.** We use the OpenCV library (findEssentialMat() and recoverPose()) to implement both base-

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Table 3. Quantitative results on the InteriorNet datasets. We report the mean and median angular error in degrees, as well as mean rank of each method over all test pairs. Rotation (\( R \)) and translation (\( t \)) shown separately.

| Method          | \( R \) mean (°) | \( R \) med (°) | \( R \) rank | \( t \) mean (m) | \( t \) med (m) | \( t \) rank |
|-----------------|------------------|----------------|--------------|------------------|----------------|-------------|
| DirectionNet    | 28.83            | 14.58          | 86.51        | 18.55            | 31.60          | 103.49      |
| SIFT            | 19.02            | 14.69          | 81.48        | 22.75            | 35.07          | 122.83      |
| Quat.           | 23.88            | 14.69          | 86.74        | 22.75            | 35.07          | 122.83      |

Table 4. Generalization to KITTI [15].

| Method          | \( R \) mean (°) | \( R \) med (°) | \( T \) mean (m) | \( T \) med (m) |
|-----------------|------------------|----------------|------------------|----------------|
| DirectionNet-9D (20%) | 10.50          | 9.21            | 26.74            | 15.67          |
| DirectionNet-9D (100%) | 9.19           | 6.31            | 19.36            | 11.71          |
| Regression 6D (100%) | 13.44          | 12.74           | 22.53            | 16.68          |
| SuperGlue (outdoor) | 16.35          | 11.53           | 24.24            | 17.15          |
| D2-Net | 24.07 | 5.18 | 34.36 | 14.05 |

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3Note, since we do not have ground truth point correspondences between images in our datasets, we cannot draw matching points on the two images for visualization.
lines by solving the essential matrix using the 5-point algorithm [44] from which we recover the pose. In the main paper, we showed that LMedS performs better than RANSAC in terms of errors in translation and median errors in rotation, but RANSAC has much lower mean errors in rotation on all datasets. Note that due to the nature of indoor images, a large portion of the feature correspondences may be co-planar (e.g. features on a wall or a floor). For RANSAC, we use the default parameters (threshold equals 1.0 and the confidence equals 0.999). Figure 15 shows that the design choice of robust fitting method doesn’t make a big overall difference in our experiments.

**Runtime performance.** DirectionNet-Single inference takes under 0.02 seconds with a TESLA P100.
Figure 7. Error histograms DirectionNet-9D vs. Bin&Delta. Top: rotation, bottom: translation.

Figure 8. Error histograms DirectionNet-9D vs. 6D Regression. Top: rotation, bottom: translation.

Figure 9. Error histograms DirectionNet-9D vs. SIFT+LMedS. Top: rotation, bottom: translation.
Figure 10. Additional qualitative results on Matterport-B.
Figure 11. Additional qualitative results on InteriorNet-A.
Figure 12. **Failure cases.** Our method could fail in cases such as repeating or complex textures, large textureless area or space with few objects, and extremely large motion.
Figure 13. **Multimodal prediction on the rotation.** This figure shows two cases in Matterport-B when the DirectionNet-R fails and gives very high uncertainty in the presence of repeating texture or extreme motion. $I_0$ and $I_1$ are input images. $P^x_{GT}$, $P^y_{GT}$, and $P^z_{GT}$ are the ground truth distributions correspond to $v_x$, $v_y$, and $v_z$ respectively. $P^x_{pred}$, $P^y_{pred}$, and $P^z_{pred}$ are the predictions corresponds to the three directions. The spherical distributions are illustrated as equirectangular heatmaps and the blue dot shows where the spherical expectation locates.

Figure 14. **Multimodal prediction on the translation.** This figure shows examples in Matterport-B when the DirectionNet-T produces multimodal distributions. $I_0$ and $I_1$ are original images and $H_R(I_1)$ is the input derotated image. $P_{GT}$ is the ground truth distribution of the translation and $P_{pred}$ is the prediction. The arcade scene in the first example has a symmetry, so it is ambiguous whether the camera is moving left or right. Thus, our model produces an antipodal distribution with almost equal uncertainty. The second example has similar symmetry because of the two identical glass windows, but we can figure out the motion from some inconspicuous clues such as the objects through the window. Thus, the model produces two modes but it is much more certain toward the correct one and the predicted direction is very close to the truth. The last two examples demonstrate another difficult scenario for the model to figure out the translation direction when the translation amount is tiny (3rd: large R and small T, 4th: tiny R and small T). The 3rd example is less ambiguous than the 4th, so the network gives high certainty at the correct mode. Note that even though the model is highly uncertain in the 4th case, the network still manages to produce a secondary mode at the correct location.
Figure 15. (a) We visualize a few examples and compare RANSAC with LMedS in different scenes. Note that the feature detected by SIFT often occurs co-planar due to the nature of the indoor scenes. (b) The error histogram shows that RANSAC and LMedS has similar performance on our dataset.