Memristive electromagnetic induction effects on Hopfield neural network

Chengjie Chen · Fuhong Min · Yunzhen Zhang · Bocheng Bao

Received: 15 July 2021 / Accepted: 14 September 2021 / Published online: 18 October 2021
© The Author(s), under exclusive licence to Springer Nature B.V. 2021

Abstract Due to the existence of membrane potential differences, the electromagnetic induction flows can be induced in the interconnected neurons of Hopfield neural network (HNN). To express the induction flows, this paper presents a unified memristive HNN model using hyperbolic-type memristors to link neurons. By employing theoretical analysis along with multiple numerical methods, we explore the electromagnetic induction effects on the memristive HNN with three neurons. Three cases are classified and discussed. When using one memristor to link two neurons bidirectionally, the coexisting bifurcation behaviors and extreme events are disclosed with respect to the memristor coupling strength. When using two memristors to link three neurons, the antimonotonicity phenomena of periodic and chaotic bubbles are yielded, and the initial-related extreme events are emerged. When using three memristors to link three neurons end to end, the extreme events owning prominent riddled basins of attraction are demonstrated. In addition, we develop the printed circuit board (PCB)-based hardware experiments by synthesizing the memristive HNN, and the experimental results well confirm the memristive electromagnetic induction effects. Certainly, the PCB-based implementation will benefit the integrated circuit design for large-scale Hopfield neural network in the future.

Keywords Dynamical effect · Electromagnetic induction · Hopfield neural network (HNN) · Hardware experiment · Memristor · Neuron

1 Introduction

Memristor, a known nonlinear circuit element, is defined by Leon O. Chua for describing the relationship between flux and charge [1]. In virtue of the quasi-static expansion of Maxwell’s equations, an electromagnetic field interpretation of this unique relationship has been presented. Till now, the memristor has been applied in wide scientific domains due to its distinct natures, such as nanoscale dimension [2], nonlinearity [3, 4], synaptic plasticity effect [5]. In
neuroscience, from the point of view of electricity, we know that one neuron can be seen as a multichannel input- and output-signal processor or a non-autonomous nonlinear system that can modulate the external stimulus and thereby responses to it [6, 7]. Also, synapse can be seen as a two-port memristive device to connect two systems so as to realize the complex memory transmission characteristic [8]. Thus, memristor-based neurons or neural networks are now playing a vital role in neuromorphic computation and brain-like applications [9–11].

In the past few years, availing of the memristor to express the electromagnetic induction that induced by membrane potential or electromagnetic radiation has been a hot topic. In [11], Ma et al. thought that due to the transformation of intercellular and extracellular ion concentration or the differences of spatial distribution of ions, membrane potential of a neuron would be waved, and thereby, the time-changing electromagnetic flows were induced, the effects of which could be imitated by a flux-controlled memristor coupling with a neuron. On account of these, some memristive neuron models were proposed, from which mode transition or selection [6, 12], synchronous behaviors [13, 14], spatiotemporal patterns [15, 16], and coexisting modes [17] are uncovered profoundly. For some examples, in [6], to describe the membrane potential of Hindmarsh–Rose (HR) neuron model under the electromagnetic induction, Lv et al. constructed a memristive HR neuron model, where different electric modes of bifurcation, spiking, and chaotic bursting state were observed. In [16], based on a FitzHugh–Nagumo (FHN) neuron model, Takembo et al. constructed an n-neuron FHN chain network model under electromagnetic radiation, and the dynamical simulations proved that the function of the brain may be impaired when it was driven to the external electromagnetic environments with strong radiation intensities.

Many researchers not only concentrate on the dynamical effects of a single neuron but also explore interactions of neurons in a network. In neural network, electromagnetic induction flows can be induced when membrane potential differences are existed between each two interconnected neurons, whose effects are equivalent to the bi-directional induced currents emerged by a flux-controlled memristor linking each two neurons [8, 18, 19]. Accordingly, to pay attention to the electromagnetic induction effects on a unified network is a burning question.

Manifold dynamics of biological neurons and neural networks are concerned for further understanding the complex nonlinear structures and functional behaviors of brain [11, 20, 21]. Different from biological neurons, conductance-independent artificial neural network has received more and more attention for its high degree of flexibility and practicability. Hopfield neural network (HNN) is a classical neural network possessing simple algebraic expression but can display complex dynamical states, which has been widely applied in numerous domains [22–24]. Because the dynamical behaviors are closely related to its applications, over the past years, a mass of modified HNN models has been proposed, including fractional-order HNN model [23], time-delayed HNN models [25], and hidden HNN model [26], and multiple dynamical characterizations have been revealed accordingly.

By contrast, memristive HNN model brings some new views for cognizing the brain, and it has got long-term attention by scholars. Because some properties of memristor bear striking resemblance to synaptic plasticity of neurons, by replacing the resistive weight with the memristive synaptic weight, some memristive HNN models were established to achieve the variable connection weight for neurons [26, 27]. Followed by, in recent years, considering the complex electromagnetic environment, a neural network under electromagnetic radiation was reported [28, 29]. When considering the membrane potential difference between two interconnected neurons in HNN, a memristive HNN model with the electromagnetic induction was raised. Moreover, the authors of the manuscript [30, 31] discussed a memristive HNN model with two neurons under the action of electromagnetic induction, where coexisting behaviors triggered by different initial conditions were revealed and were validated by hardware experiments. Successively, the authors of the manuscript [32] focused on the initial sensitive dynamics in a memristive HNN model with three neurons when only considering the electromagnetic induction flows induced by the membrane potential difference between two neurons. Based on some commercially discrete components, analog implementations are all developed for the above-mentioned memristive HNN models [26–32]. Of course, based on the digitally circuit-implemented platforms such as DSP [33] and FPGA [34, 35], the memristive HNN model can
also be fabricated physically to verify the numerical simulations.

Nevertheless, we are ignorant of the electromagnetic induction on HNN induced by membrane potential differences of multiple neurons. Accordingly, it is necessary to establish a unified memristive HNN model to express the electromagnetic induction effects, which has not been reported until now.

In this paper, based on the hyperbolic-type memristor, a unified memristive HNN model is presented. For simplicity, a classical tri-neuron HNN model is taken as an example, based on which memristor-coupled HNN model with three cases is considered in succession. Interesting dynamical effects and intricate dynamical evolutions are uncovered. The main contributions for this paper are threefold. (1) A unified memristive HNN model is presented, and its boundedness is proved theoretically. (2) Multiple dynamical methods are employed to numerically reveal the bifurcations and coexisting attractors’ behaviors, which is helpful to mimic the real dynamical behaviors of collective neurons and to cognize the brain. (3) PCB-based memristive HNN circuit experiments are developed, and the results well confirm these dynamical effects.

The remaining contents are listed as follows. In Sect. 2, a unified memristive HNN model is established, and its boundedness is proved. In Sect. 3, memristive electromagnetic induction effects on HNN are numerically revealed. In Sect. 4, an electronic neuron circuit platform is built, and the dynamical effects are validated. And lastly, we summarize our work in Sect. 5.

2 Memristor-coupled HNN model

In this section, availing of an example of the HNN model and a threshold hyperbolic-type memristor model, we construct a unified memristive HNN model to express the electromagnetic induction effects. Besides, the uniform boundedness of the presented model is proved theoretically.

2.1 An example of the HNN model

The mathematical model of a Hopfield neural network (HNN) with \( n \) neurons is generally described as

\[
\dot{X} = -X + W \tanh(X) + I
\]

where \( X = [x_1, x_2, \ldots, x_n]^T \) represents the \( n \)-neuron membrane potentials, \( W \) is an \( n \times n \) synaptic weight matrix, and \( I = [i_1, i_2, \ldots, i_n]^T \) is an external current matrix.

The tri-neuron HNN has been widely studied. Thus, an example of HNN, with a \( 3 \times 3 \) asymmetric synaptic weight matrix, can be referred to [36].

Utilizing the weight matrix (2) and taking no account of the external currents, the numerical simulations of the HNN model are shown in Fig. 1. One can be seen from Fig. 1a, the phase portraits of two symmetric period-1 limit cycles initiated from the initials \((-0.01, 0, 0)\) (red) and \((0.01, 0, 0)\) (blue) are coexisting in the \( x_1 - x_2 - x_3 \) phase space. Besides, as shown in Fig. 1b, two attracting domains depicted by local attraction basins are located in \( x_1(0) - x_2(0) \) initial plane, where ‘LP1’ and ‘UP1’ represent the lower period-1 and upper period-1 behaviors, respectively. As a result, this HNN model takes on bistable period-1 behaviors.

2.2 Unified memristive HNN model

Referring to [18], a monotone, differentiable, and threshold memristor is used to express the electromagnetic induction, whose mathematical form is written as

\[
\begin{align*}
I_M &= kG(\phi)V_M = k \tanh(\phi)V_M \\
\dot{\phi} &= f(V_M, \phi) = V_M - \phi
\end{align*}
\]

where \( k, G(\phi) = \tanh(\phi), V_M, \) and \( I_M \) stand for the memristor coupling coefficient, memductance function, input voltage of memristor, and output current of memristor, respectively. In this expression, \( V_M \) and \( I_M \) stand for the membrane potential difference between two interconnected neurons and the induced current flowing through the memristor.
Using one flux-controlled memristor model to link each two neurons bidirectionally and taking no account of the external currents, a unified memristive HNN mathematical model with $n$ neurons and $n$ memristor arrays can be established as

$$\begin{align*}
    \dot{X} &= -X + W \tanh(X) + KV_M \tanh(\Phi) \\
    \dot{\Phi} &= V_M - \Phi
\end{align*}
$$

(4)

where $K$ and $V_M$ denote the memristor coupling strength matrix and the membrane potential difference matrix. For $n = 3$, the two matrices can be denoted as

$$K = \begin{pmatrix}
    k_1 & 0 & -k_3 \\
    -k_1 & k_2 & 0 \\
    0 & -k_2 & k_3
\end{pmatrix},$$

$$V_M = \begin{pmatrix}
    V_{M1} & 0 & 0 \\
    0 & V_{M2} & 0 \\
    0 & 0 & V_{M3}
\end{pmatrix}$$

(5)

where $V_{M1} = x_1 - x_2$, $V_{M2} = x_2 - x_3$, and $V_{M3} = x_3 - x_1$.

Note that, the hyperbolic-type memristor is used to express the electromagnetic induction induced by the membrane potential difference between two interconnected neurons. Thus, in (4), the $KV_M \tanh(\Phi)$ term can be regarded as the induction current of the memristor. Besides, $\Phi = (\varphi_1, \varphi_2, \varphi_3)^T$ represents a magnetic flux matrix in the memristor array.

To intuitively express the electromagnetic induction flows induced by potential differences between the interconnected neurons, the abridged general view of the connection topology for the memristive HNN model with three neurons is depicted in Fig. 2, where the two-way induction currents are flowing through one memristor to mimic the electromagnetic induction flows. Therefore, each memristor is used to link two neurons bidirectionally, and six induction currents $\pm I_{M_i}$ $(i = 1, 2, 3)$ are yielded thereby.

In this paper, memristor is used to express the electromagnetic induction induced by membrane potential difference between two neurons. In neuroscience, as a matter of fact, a memristor can also be employed to express the synaptic plasticity of neurons, i.e., to replace the resistive weight with the memristive synaptic weight, and to express the electromagnetic induction induced by the external electromagnetic radiation or the inner membrane potential of neurons. To sum up, several examples of three expressions of the memristive HNN models are listed in Table 1.

One can be seen from Table 1 that due to the different nonlinear properties of memductance function, various memristor models can be adopted to construct the memristive HNN model. Besides, when using a memristor to express the synaptic plasticity effect, the memristive synaptic weight is a scalar, and when using a memristor to express the
electromagnetic induction effect, the physical direction of the induction currents flowing through the memristor is a vector. In this paper, we use the non-ideal memristors to connect neurons end to end; thus, the induction currents are the bi-directional vectors. In summary, the connection topologies for the six memristive HNN models involved in [26–31] are drawn in Fig. 3, where the memristors in the memristive HNN models are connected in different ways.

2.3 Model uniform boundedness

Boundedness is a vital property of a nonlinear dynamical system. In this paper, for \( n = 3 \), uniform boundedness of the model (4) is deduced in theory, proving that all the motions, including chaotic motions, are trapped into a bounded region.

1. Basic Definition of Uniform Boundedness: Consider a general nonlinear dynamical system as

\[
\dot{x} = h(t, x)
\]

where \( h: \mathbb{R}_+ \times B \rightarrow \mathbb{R}^n \) is continuous, and \( B \subset \mathbb{R}^n \) is a domain that contains the origin.

**Definition 1** [37]: The solutions of system (6) are uniformly bounded if there exists a positive constant \( c_1 \), independent of \( t_0 \geq 0 \), and for every \( c_2 \in (0, c_1) \), there is \( h_1 = h_1(c_2) > 0 \), independent of \( t_0 \), such that

\[
\|x(t_0)\| \leq c_2 \Rightarrow \|x(t)\| \leq h_1, \ \forall t \geq t_0
\]

2. Uniform Boundedness Analysis: Denote \( Y = [X, \Phi] \) and take

\[
A = \begin{pmatrix}
-I & 0 \\
B & -I
\end{pmatrix}
\]

\[
g(Y) = \begin{pmatrix}
W \tanh(X) + KV_M \tanh(\Phi) \\
0
\end{pmatrix}
\]

where \( I \) is a unit matrix, \( A \) is the linearized matrix of (4), and \( B \) can be regarded as the linearized matrix of the state equation \( \Phi \) against the state variable \( X \), which is a \( 3 \times 3 \) matrix denoted as

\[
B = \begin{pmatrix}
1 & -1 & 0 \\
0 & 1 & -1 \\
-1 & 0 & 1
\end{pmatrix}
\]
Then, the memristive HNN model (4) is rewritten by
\[ \dot{Y} = AY + g(Y) \]  \( (10) \)

For the initial condition \( Y(t_0) \), by the variation of parameters formula, any solution \( Y(t) \) of system (10) can be written as
\[ Y(t) = Y(t_0)e^{A(t-t_0)} + \int_{t_0}^{t} e^{A(t-s)}g(Y(s))ds \]  \( (11) \)

It is easy to know that all the characteristic roots of constant matrix \( A \) have negative real parts, so there exist positive constants \( L \) and \( \alpha \), if there is a constant \( D \) with \( |g(Y)| \leq D \), then for \( t \geq t_0 \), we have
\[ |Y(t)| = L|Y(t_0)|e^{-\alpha(t-t_0)} + LD/\alpha \]  \( (12) \)

Therefore, it is concluding that for the tri-neuron memristor-coupled HNN model, the model (4) is uniform boundedness.

3 Memristive electromagnetic induction effects

In this section, three cases are classified in succession, including the memristive HNN model with \( M_1 \), the memristive HNN model with \( M_1 \) and \( M_2 \), and the memristive HNN model with three memristors, corresponding to the values of matrix \( K \). On account of the example of the HNN model, memristive electromagnetic induction effects are thoroughly revealed by multiple numerical methods.
3.1 *Case I*: Memristive HNN model with $M_1$

Firstly, one memristor $M_1$ that connects neurons 1 and 2 is taken into account, the memristor coupling strength matrix in this case is denoted as

$$K = \begin{pmatrix} k_1 & 0 & 0 \\ -k_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (13)$$

where $k_1$ stands for the memristor coupling strength between neurons 1 and 2. The detailed expressions of the memristive HNN model for *Case I* can be written as

$$\begin{align*}
\dot{x}_1 &= -x_1 + 3.8 \tanh(x_1) - 1.9 \tanh(x_2) + 0.7 \tanh(x_3) + k_1 \tanh(\varphi_1)(x_1 - x_2) \\
\dot{x}_2 &= -x_2 + 2.8 \tanh(x_1) + \tanh(x_3) - k_1 \tanh(\varphi_1)(x_1 - x_2) \\
\dot{x}_3 &= -x_3 - 6.6 \tanh(x_1) + 1.3 \tanh(x_2) \\
\dot{\varphi}_1 &= x_1 - x_2 - \varphi_1 \\
\dot{\varphi}_2 &= x_2 - x_3 - \varphi_2 \\
\dot{\varphi}_3 &= x_3 - x_1 - \varphi_3 
\end{align*} \quad (14)$$

The connection topology for the *Case I* is depicted in Fig. 4, and the two-way induction current flowing through $M_1$ is used to mimic the electromagnetic induction flow induced by neurons 1 and 2.

Stability of the equilibrium point is also a vital property for the HNN dynamical system, which is closely related to its application [38]. Setting the left side of (4) to zero and configuring the equilibrium points as $P(\eta_1, \eta_2, \eta_3, \eta_{\varphi 1}, \eta_{\varphi 2}, \eta_{\varphi 3})$, then there are $\eta_3 = -6.6 \tanh(\eta_1) + 1.3 \tanh(\eta_2)$, $\eta_{\varphi 1} = \eta_1 - \eta_2$, $\eta_{\varphi 2} = \eta_2 - \eta_3$, and $\eta_{\varphi 3} = \eta_3 - \eta_1$.

Accordingly, the equilibrium points $P$ are able to be solved as long as two variables $\eta_1$ and $\eta_2$ are determined. Two transcendental equations with respect to two parameters are confirmed as

$$\begin{align*}
H_1 &= -\eta_1 - \eta_2 + 6.6 \tanh(\eta_1) - 1.9 \tanh(\eta_2) + 1.7 \\
H_2 &= -\eta_2 + 2.8 \tanh(\eta_1) + a - k_1(\eta_1 - \eta_2) \tanh(\eta_1 - \eta_2) 
\end{align*} \quad (15)$$

where $a = \tanh[-6.6 \tanh(\eta_1) + 1.3 \tanh(\eta_2)]$.

Due to the difficulty of obtaining the arithmetic solutions, a graphic analysis method is employed to obtain the analytical solutions of (15) using MATLAB platform. When the memristor coupling intension $k_1$ is set as 0.12 and 0.18, respectively, the equilibrium points $P$ can be determined by examining the intersections of functions, as shown in Fig. 5.

As observed from Fig. 5, when $\eta_1$ and $\eta_2$ are in the regions $[-0.6, 0.6]$ and $[-0.9, 0.7]$, the black $H_1$ curve with determined function remains unchanged, but the red and blue $H_2$ curves involve two different values of $k_1$. Therefore, three examined intersections including one zero equilibrium point $P_0$ as well as two nonzero equilibrium points $P_1$ and $P_2$ can be precisely calculated, respectively.

Availing of the $K$ matrix in (13), when the Jacobian matrix of the model (4) in *Case I* is deduced, the equilibrium points, eigenvalues, and stabilities for two representatives $k_1 = 0.12$ and $k_1 = 0.18$ can be acquired. Two zero eigenvalues at $P_0$ are both calculated as $-1$, $-1$, $-1$, $1.0124$, $-0.1062 \pm j2.0600$. Besides, nonzero eigenvalues at $P_1$ and $P_2$ for $k_1 = 0.12$ are calculated as $-1$, $-1$, $-0.9765$, $-0.9637$, $0.4442 \pm j1.2647$ and $-1$, $-1$, $-1.0201$, $-0.9645$, $0.4973 \pm j1.2445$.

![Fig. 4](image)

**Fig. 4** The connection topology for the memristor-coupled HNN model with $M_1$

![Fig. 5](image)

**Fig. 5** For $k_1 = 0.12$ and $k_1 = 0.18$, two parameters $\eta_1$ and $\eta_2$ achieved by examining the intersections of functions in (15)
respectively, and the ones at $P_1$ and $P_2$ for $k_1 = 0.18$ are calculated as $-1, -1, 0.4356 \pm j1.2685$, $-0.9642 \pm j1.2396$ and $-1, -1, -1.0295$, $-0.9651, 0.5089 \pm j1.4838$, respectively. These equilibrium points all behave unstable saddle-foci (USF), indicating that the spiral chaotic attractors can be formed theoretically according to Shil’nikov theorem [39].

Numerical simulation methods are valid to disclose the dynamical behaviors. When selecting four sets of initial conditions, the max-spike bifurcation diagrams of $x_2$ and the first three Lyapunov exponents (LEs) are plotted in Fig. 6a and b, respectively. Note that, ODE23 (built-in MATLAB) is employed to simulate the phase portraits, bifurcation diagrams, and local attraction basins. Besides, the Wolf-based algorithm is adopted to simulate the LE spectra in the whole paper.

In Fig. 6a, with the increase in memristor coupling strength $k_1$ in the region [0.08, 0.18], the memristive HNN model behaves as globally periodic states when selecting two sets of initial conditions ($\pm 0.4, 0, 0, 0, 0$). By contrast, the memristive HNN model has a
reverse period-doubling bifurcation route to chaos when considering two other sets of initial conditions. Taking the memristive HNN model with the initial conditions \((0.5, 0, 0, 0, 0, 0)\) as an example, its orbit starts with period-1, enters into chaos at \(k_1 = 0.109\) via chaos crisis, and then degrades into period-6 at \(k_1 = 0.144\) and period-3 at \(k_1 = 0.179\) successively via reverse period-doubling bifurcation. In addition, some periodic windows and chaos crisis scenarios can be also found in the chaotic regions. Furthermore, there are at least three different attractors’ states coexisting in the memristive HNN model for a determined memristor coupling strength, demonstrating that the multistable patterns appear in Case I [30].

For \(k_1 = 0.12\), the phase portraits initiated by four sets of initial conditions are plotted in Fig. 7. The results show that multiple attractors with different locations and topological structures coexist in the phase space, including lower period-1 limit cycle, upper period-1 limit cycle, period-8 limit cycle, and spiral chaotic attractor. The local attraction basins can be used to better explore the influences of initial conditions on the model (4) in Case I. Two representative examples for \(k_1 = 0.12\) and \(k_1 = 0.18\) with \((x_3(0), x_4(0), x_5(0), x_6(0)) = (0, 0, 0, 0)\) are shown in Fig. 8a and b, where seven colors represent different types of attractors. Here, LP1, UP1, P02, P03, P04, P08, and CH represent lower period-1, upper period-1, period-2, period-3, period-4, period-8, and chaos, respectively, indicating the coexisting multistable patterns. Observed from Fig. 8a, four types of attractors are revealed when \(k_1 = 0.12\) is fixed, and orange, blue, and banded yellow regions dominate the initial plane.

In addition, the riddled basins of attraction are displayed in small regions, implying that the model in this case is sensitive to the initial conditions, and the extreme events are yielded [40–44]. As shown in Fig. 8b, when \(k_1\) increases to 0.18, complex stability evolutions happen, leading to that period-2 with the riddled domain is embedded in period-4. As a result, the memristive HNN model displays coexisting multistable patterns related to the initial conditions.

It should be noted that the extreme events are related to many contexts such as tsunamis, earthquakes, tornadoes, market crashes, and human brain seizures [40]. For a dynamical system, it can be defined as a recurrent and rare event on which an appropriate variable exhibits an unusual behavior [41]. Therefore, an extreme event can be effectively exhibited by the riddled basin of attraction in such a dynamical system [42–44].

### 3.2 Case II: Memristive HNN model with \(M_1\) and \(M_2\)

Two memristors \(M_1\) and \(M_2\) used to link three neurons are taken into account in Case II. Hence, the memristor coupling strength matrix can be denoted as

\[
K = \begin{pmatrix}
  k_1 & 0 & 0 \\
- k_1 & k_2 & 0 \\
0 & - k_2 & 0 \\
\end{pmatrix}
\]  

where \(k_1\) and \(k_2\) are two parameters representing two different memristor coupling strengths. The detailed expressions of the memristive HNN model for Case II can be expressed as

\[
\begin{align*}
\dot{x}_1 &= - x_1 + 3.8 \tanh(x_1) - 1.9 \tanh(x_2) \\
&\quad + 0.7 \tanh(x_3) + k_1 \tanh(\varphi_1)(x_1 - x_2) \\
\dot{x}_2 &= - x_2 + 2.8 \tanh(x_1) + \tanh(x_3) + k_2 \tanh(\varphi_2) \\
&\quad (x_2 - x_3) - k_1 \tanh(\varphi_1)(x_1 - x_2) \\
\dot{x}_3 &= - x_3 - 6.6 \tanh(x_1) + 1.3 \tanh(x_2) \\
&\quad - k_3 \tanh(\varphi_2)(x_2 - x_3) \\
\dot{\varphi}_1 &= x_1 - x_2 - \varphi_1 \\
\dot{\varphi}_2 &= x_2 - x_3 - \varphi_2 \\
\dot{\varphi}_3 &= x_3 - x_1 - \varphi_3
\end{align*}
\]  

The connection topology for the Case II is depicted in Fig. 9, and the two-way induction currents flowing
through $M_1$ and $M_2$ are used to mimic the electromagnetic induction flows induced by neurons 1 and 2 as well as neurons 2 and 3. To investigate the bifurcation scenarios with these two parameters, the two-dimensional bifurcation diagram is plotted in the regions $k_1 = [0.08, 0.18]$ and $k_2 = [-0.005, 0.065]$ under the initial conditions $(-0.4, 0, 0, 0, 0, 0)$, as shown in Fig. 10a. When setting four representative values of $k_1$ as 0.08, 0.11, 0.15, and 0.18, respectively, the one-dimensional bifurcation diagrams are plotted with respect to $k_2$, as shown in Fig. 10b. Notably, the two-dimensional parameter plane is based on the ODE23 (built-in MATLAB) algorithm and painted by different colors according to the periodicities of the membrane potential $x_3$ [17, 45].

As can be seen from Fig. 10a, there exist eight types of attractors. Here, $P_0$, $P_1$, $P_2$, $P_4$, $P_8$, $P_{12}$, $P_{16}$, and $CH$ represent stable point, period-1, period-2, period-4, period-8, period-12, period-16, and chaos, respectively. The period-8, period-16, chaos, and period-12 are embedded in successive, resulting in the occurrence of the marvelous bifurcation structure.

Glanced in Fig. 10b, the phenomena of antimonotonicity appear distinctly. For fixed $k_1 = 0.08$, when increasing $k_2$ in the region $[-0.005, 0.065]$, the orbit of the memristive HNN model in Case II begins with period-1 goes into period-2 and period-4 via the forward period-doubling bifurcations, then degrades into period-2 and period-1 via the reverse period-doubling bifurcations, and finally settles down to stable point at $k_2 = 0.0614$. When increasing $k_1$ from 0.08 to 0.18, the period-4, period-8, period-12, and chaos bubbles are formed, respectively. Thus, the forward and reverse period-doubling bifurcation routes are obviously visible in bifurcation processes for the model (4), which are affected by two memristor coupling strengths.

For $k_1 = 0.17$ and $k_2 = 0.005$ with fixed $(x_3(0), x_4(0), x_5(0), x_6(0)) = (0, 0, 0, 0)$, Fig. 11 depicts the local attraction basin in the $x_1(0)-x_2(0)$ initial plane.

For $k_1 = 0.17$ and $k_2 = 0.005$, the phase portraits initiated by five different sets of initial conditions

Fig. 11 For $k_1 = 0.17$ and $k_2 = 0.005$, local attraction basin in the $x_1(0)-x_2(0)$ initial plane

Fig. 12 For $k_1 = 0.17$ and $k_2 = 0.005$, the phase portraits initiated by five different sets of initial conditions
local attraction basins related to the coexisting multiple patterns in Case II. Here, P01, P02, P03, P05, and CH represent period-1, period-2, period-3, period-5, and chaos, respectively. There are five colors distributing in the $x_1(0) - x_2(0)$ initial plane, and the purple and yellow regions are relatively sparse. What’s more, the red region is riddled and mixed in the blue region, which means that the extreme events also exist in Case II.

According to the aforementioned analysis, Fig. 12 shows the phase portraits under five sets of initial conditions $(0.4, 0, 0, 0, 0, 0), (-0.4, 0, 0, 0, 0, 0), (0.7, 0, 0, 0, 0, 0), (0.2, 0, 0, 0, 0, 0), \text{and} (-1, 0, 0, 0, 0, 0)$ for the model (4) in Case II. Five different types of attractors are triggered, including period-1, period-2, period-3, period-5, and chaos.

3.3 Case III: Memristive HNN model with $M_1$, $M_2$, and $M_3$

In this case, three memristors from $M_1$ to $M_3$ are used to link three neurons end to end. Three memristor coupling strengths from $k_1$ to $k_3$ are set as three adjusting parameters. Figure 2 gives the connection topology for the memristor-coupled HNN model with $M_1$, $M_2$, and $M_3$.

Considering the special situation in Case III that memristive electromagnetic inductions between three neurons are the same, i.e., $k_1 = k_2 = k_3 = k$, thereby $k$ is chosen as a single adjustable parameter in Case III.
The memristive HNN model can be rewritten as
\[
\begin{cases}
    \dot{X} = -X + W \tanh(X) + kAV_M \tanh(\Phi) \\
    \dot{\Phi} = V_M - \Phi
\end{cases}
\]
where \( k \) is the single adjustable parameter, and \( A \) is a constant matrix
\[
A = \begin{pmatrix}
1 & 0 & -1 \\
-1 & 1 & 0 \\
0 & -1 & 1
\end{pmatrix}
\]

Note that the other matrixes in (18) are exactly the same as those used in (4). Besides, the detailed expressions of the memristive HNN model for Case III under this special situation can be described as
\[
\begin{cases}
    \dot{x}_1 = -x_1 + 3.8 \tanh(x_1) - 1.9 \tanh(x_2) + 0.7 \tanh(x_3) \\
    + k \tanh(\varphi_1)(x_1 - x_2) - k \tanh(\varphi_3)(x_3 - x_1) \\
    \dot{x}_2 = -x_2 + 2.8 \tanh(x_1) + \tanh(x_3) + k \tanh(\varphi_2) \\
    (x_2 - x_3) - k \tanh(\varphi_1)(x_1 - x_2) \\
    \dot{x}_3 = -x_3 - 6.6 \tanh(x_1) + 1.3 \tanh(x_2) \\
    + k \tanh(\varphi_3)(x_3 - x_1) - k \tanh(\varphi_2)(x_2 - x_3) \\
    \dot{\varphi}_1 = x_1 - x_2 - \varphi_1 \\
    \dot{\varphi}_2 = x_2 - x_3 - \varphi_2 \\
    \dot{\varphi}_3 = x_3 - x_1 - \varphi_3
\end{cases}
\]

Three sets of initial conditions are set as \((0.01, 0, 0, 0, 0, 0), (-0.01, 0, 0, 0, 0, 0), \) and \((-0.07, 0, 0, 0, 0, 0)\), respectively. When increasing \( k \) from 0 to 0.04, the bifurcation plots with respect to \( k \) are drawn in Fig. 13. As can be seen, the phenomena of cascaded chaotic bubbles, chaos crisis scenarios, and coexisting bifurcations emerge in the memristive HNN model. Specifically, when setting the initial conditions as \((-0.07, 0, 0, 0, 0, 0)\), the forward and reverse period-doubling bifurcation routes are also visible in bifurcation processes. Accordingly, when we endow four sets of parameter \( k \) for the model (4) in Case III, the phase portraits of chaotic attractor, periodic limit cycles, and stable point are plotted in Fig. 14.

In this special situation of Case III, to uncover the initial-dependent dynamics, the value of \( k \) is kept as 0.005. The local attraction basin is plotted in Fig. 15, and the coexisting tri-stable patterns are displayed, such as the blue period-1 (P01), light cyan period-2 (P02), and red chaos (CH). Note that all the red regions are fully mixed and riddled in the light cyan periodic regions, meaning the occurrence of the rare, recurrent, and irregular dynamics of the extreme events.

It is necessary to point out that using numerous simulation methods, the rich and complex dynamical behaviors can be numerically revealed in the memristive HNN model as the aforementioned memristor coupling strengths for each case are chosen. In other words, the dynamical effects of bistability, multistability, and extreme events can be clearly observed from the local basins of attraction, and the dynamical effects of antimonotonicity can be viewed from the bifurcation diagrams. Summarily, the dynamical effects on the memristive HNN model given in (4) are listed in Table 2. As can be seen, when the number of memristors increases from zero to two, the number of coexisting attractors goes from two to four and to five. However, when the memristive HNN model involves three memristors with the aforesaid special situation, the coexisting tri-stable patterns with the extreme events appear distinctly. Besides, the antimonotonicity behavior displays in Case II, which is related to two memristor coupling strengths. Notably, when three memristors utilized in the memristive HNN model are endowed with three different memristor coupling strengths, some more intriguing and intricate dynamical effects need to be further investigated.

### 4 PCB-based analog circuit validation

Electronic neuron circuit is nowadays regarded as an excellent artificial block to implement the VLSI
applying in neuromorphic computing [46]. It can be achieved in three ways, namely analog, digital, and hybrid analog/digital circuits [47–50]. In this section, the memristive HNN model is implemented in analog circuit, and the PCB-based hardware experiments are carried out to validate the memristive electromagnetic induction effects.

4.1 Circuit synthesis for the memristive HNN model

Based on the unified memristive HNN model given in (4), an electronic neuron circuit can be implemented, and its circuit equations are described as

\[
\begin{align*}
RC \frac{dv}{dt} &= -v + R_W \tanh(v) + R_K v_{VM} \tanh(v_{\phi}) \\
RC \frac{dv_{\phi}}{dt} &= v_M - v_{\phi}
\end{align*}
\]  \tag{21}

where \(v\) and \(v_{\phi}\) are \(n \times 1\) voltage variable matrixes, and the integrating time constant \(\tau = RC = 10\, \text{k}\Omega \times 10\, \text{nF} = 0.1\, \text{ms}\). In addition, for \(n = 3\), on account of the synaptic weight matrix \(W\), memristor coupling strength weight \(K\), and matrix \(V_M\) in (5), three \(3 \times 3\) resistance arrays are presented as

| Cases of memristive HNN model | Theoretical resistances for \(R_W\) and \(R_K\) |
|------------------------------|-----------------------------------------------|
| An example of the HNN model  | \(R_W = R/|w_{ij}|, W = 1, \ldots, 7\)        |
| Case I: Memristive HNN model with \(M_1\) | \(R_{k_1} = R/k_1 = 83.33\, \text{k}\Omega\) |
| Case II: Memristive HNN model with \(M_1\) and \(M_2\) | \(R_{k_1} = R/k_1 = 58.82\, \text{k}\Omega, R_{k_2} = R/k_2 = 2000\, \text{k}\Omega\) |
| Case III: Memristive HNN model with three \(M\) | \(R_k = R/k = 2\, \text{M}\Omega, 500\, \text{k}\Omega, 333.33\, \text{k}\Omega,\) and \(250\, \text{k}\Omega\) |
\[
R_W = \begin{pmatrix}
\frac{R}{R_1} & -\frac{R}{R_2} & \frac{R}{R_3} \\
\frac{R}{R_4} & 0 & \frac{R}{R_5} \\
-\frac{R}{R_6} & \frac{R}{R_7} & 0
\end{pmatrix}
\]

(22a)

\[
R_K = \begin{pmatrix}
\frac{R}{R_{k1}} & 0 & -\frac{R}{R_{k3}} \\
-\frac{R}{R_{k1}} & \frac{R}{R_{k2}} & 0 \\
0 & -\frac{R}{R_{k2}} & \frac{R}{R_{k3}}
\end{pmatrix}
\]

(22b)

\[
v_{VM} = \begin{pmatrix}
v_{VM1} & 0 & 0 \\
0 & v_{VM2} & 0 \\
0 & 0 & v_{VM3}
\end{pmatrix}
\]

(22c)

where \( R_W \) represents the determined resistance array, \( R_K \) represents the adjustable resistance array whose values change with different cases, and \( v_{VM} \) represents the voltage matrix of membrane potential difference. Notice that, because the resistances are positive in the real circuit, the negative signs in (22a–22c) are adjusted by changing the connection way of the circuit. According to (22a–22c), the resistances in \( R_W \) are configured as \( R_1 = R/3.8 = 2.6316 \text{ k}\Omega, R_2 = R/1.9 = 5.2632 \text{ k}\Omega, R_3 = R/0.7 = 14.2857 \text{ k}\Omega, R_4 = R/2.8 = 3.5714 \text{ k}\Omega, R_5 = R/1 = 10 \text{ k}\Omega, R_6 = R/6.6 = 1.5152 \text{ k}\Omega, \) and \( R_7 = R/1.3 = 7.6923 \text{ k}\Omega, \) respectively.

Circuit schematic is designed synthetically using Multisim 12.0 software, the screenshot of which can be seen in Fig. 16, where three cases of circuit modules can be controlled by six-pin DIP switch with \( S_1 \sim S_3 \) keys. Therefore, the states of these three keys for the electronic neuron circuit and theoretical resistances for \( R_K \) are summarized in Table 3. Besides, three different functional circuits denoted by fourteen hierarchical blocks (HBs) are shown in Fig. 16, where six HBs from ‘ – T1’ to ‘ – T6’ represent the hyperbolic tangent function circuit modules with negative output, five HBs from ‘ – H1’ to ‘ – H5’ represent the inverting operation circuits, and three HBs from ‘I1’ to ‘I3’ represent the subtraction operation circuits. Notice that, global connectors are employed to connect the common port for simplifying the circuital connection.

4.2 PCB-based hardware circuit validation

The designed PCB is made using the Altium designer 10.0 software. The off-the-shelf commercial components contain bipolar junction transistor MPS2222, operational amplifier TL082CP, analog multiplier AD633JNZ, chip resistor, precision potentiometer, and monolithic ceramic capacitor. And the phase portraits are captured by Tektronix digital oscilloscope in the X–Y mode.

The photograph of the electronic neuron circuit is displayed in Fig. 17, where the HNN circuit module is in the left, and six same and independent \(-\text{tanh}()\) modules are in the right dotted box. ‘\( S \)’ is a six-pin DIP switch that is used to control the circuit connection states. For instance, when \( S_1 \) and \( S_2 \) are on and \( S_3 \) is off in this figure, the circuit is in the state of Case II.
Fig. 18 The phase portraits captured by adjusting the states of keys and the resistances $R_k$. 

(a) experimental resistance $R_1 = 83.33 \, \text{k}\Omega$ for Case I 

(b) experimental resistances $R_{k1} = 58.82 \, \text{k}\Omega$ and $R_{k2} = 2012 \, \text{k}\Omega$ for Case II

Fig. 19 When $S_1 \sim S_3$ are all on, experimental phase portraits and photograph a four captured phase portraits by setting $R_{k1} = R_{k2} = R_{k3} = 2000 \, \text{k}\Omega$, 600 $\text{k}\Omega$, 430 $\text{k}\Omega$, and 250 $\text{k}\Omega$, respectively. b photograph for PCB-based hardware circuit linked with a digital oscilloscope
On account of Table 3, when three keys are all off, the example of the HNN model given in (1) can be denoted by a third-order analog circuit. Besides, when Case I and Case II are classified, the adjustable resistance arrays $R_K$ are configured as

$$R_K = \begin{pmatrix} \frac{R}{R_{k1}} & 0 & 0 \\ -\frac{R}{R_{k1}} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(23a)

$$R_K = \begin{pmatrix} \frac{R}{R_{k1}} & 0 & 0 \\ -\frac{R}{R_{k1}} & \frac{R}{R_{k2}} & 0 \\ 0 & -\frac{R}{R_{k2}} & 0 \end{pmatrix}$$

(23b)

respectively, where $R_{k1}$ and $R_{k2}$ are two adjustable resistances for each case. When configuring the two resistances in turn, the phase portraits for the first two cases are experimentally captured, as shown in Fig. 18. Note that due to the difficulty in achieving specific capacitor initial voltages in analog circuit, power supply should be on-and-off switched to endow the initial conditions in the real circuit [31, 51]. In Fig. 18, three types of coexisting attractors for Case I and three types of coexisting attractors for Case II can be captured. Thus, the initial-related coexisting behaviors can be availably realized by circuit simulations [27, 30].

Furthermore, when three keys are all on, the adjustable resistance array $R_K$ is configured as (22b). For the special situation in Case III, the adjustable resistances need to be the same, i.e., $R_k = R_{k1} = R_{k2} = R_{k3} = R/k_1 = R/k_2 = R/k_3$. Thus, the experimental phase portraits can be captured and shown in Fig. 19.

As can be seen, dynamical evolutions from chaos to period-2, to period-1, and to stable point can be readily observed, and the experimental results validate the simulation results given in Fig. 14. Besides, the photograph of PCB-based hardware circuit for Case III is shown in Fig. 19b, in which the chaotic attractor is cropped by Tektronix digital oscilloscope.

Due to the parasitic resistances and inner interferences in the real circuit, PCB-based experiment results are consistent with the numerical results basically though possessing some errors. In general, we can say that the electronic neuron circuit for implementing the memristive HNN model well validates the memristive electromagnetic induction effects on HNN. And in the next step, we may try to use simple activation function in HNN or find some alternative schemes for the complex activation functions so as to investigate the unified HNN model with $n$ neurons.

5 Conclusions

Using the threshold hyperbolic-type memristors to link the interconnected neurons, this paper presented a unified memristive HNN model to simulate electromagnetic induction effects. The uniform boundedness of the memristive HNN model was deduced in theory, proving that all the motions are trapped into a bounded region. With the consideration of three cases, the dynamical effects were revealed in succession using multifold dynamical analysis methods. In Case I, stability analysis proved that the equilibrium points are all unstable saddle-foci, and numerical simulations disclosed the coexisting bifurcation behaviors, the coexisting multistable patterns, and the extreme events herein. In Case II, the antimonotonicity phenomena appeared with the creation and annihilate of periodic and chaotic bubbles, which was controlled by two memristor coupling strengths. Besides, the extreme events with coexisting five-stable patterns were induced by the initial conditions. In the special situation of Case III, when three memristor coupling strengths were the same, the extreme event with complex riddled basins taken place distinctly, meaning that the memristive HNN model is increasingly sensitive to the initial conditions.

The electronic neuron circuit of the memristive HNN model was constructed by a PCB hardware platform, and the results beautifully validated the numerical simulations for three cases. Accordingly, the study of the memristive electromagnetic induction effects on HNN not only reveals the interactions of neurons, but also provides a potential application in the neuromorphic computation. Furthermore, we emphasize that based on the other examples of memristive HNN models, the electromagnetic induction effects may not the same and need to be explored in the future.

Acknowledgements This work was supported by the National Natural Science Foundation of China under Grant Nos. 61971228 and 51777016, the Natural Science Foundation of Henan Province under Grant No. 202300410351, and the Key Scientific Research of Colleges and Universities in Henan Province under Grant No. 21A120007.
 Availability of data and materials  The datasets supporting the conclusions of this article are included within the article and its additional files.

Declarations

Conflict of interest  The authors declare that they have no conflicts of interest. These authors contribute equally to this work.

References

1. Chua, L.O.: Memristor—the missing circuit element. IEEE Trans. Circuit Theory 18(5), 507–519 (1971)
2. Kumar, S., Strachan, J., Williams, R.: Chaotic dynamics in nanoscale NbO2 Mott memristors for analogue computing. Nature 548, 318–321 (2017)
3. Li, C., Min, F.H., Li, C.B.: Multiple coexisting attractors of the serial-parallel memristor-based chaotic system and its adaptive generalized synchronization. Nonlinear Dyn. 94, 2785–2806 (2018)
4. Xie, W.L., Wang, C.H., Lin, H.R.: A fractional-order multistable locally active memristor and its chaotic system with transient transition, state jump. Nonlinear Dyn. 104, 4523–4541 (2021)
5. Miranda, E., Milano, G., Ricciardi, C.: Modeling of short-term synaptic plasticity effects in ZnO nanowire-based memristors using a potentiation-depression rate balance equation. IEEE Trans. Nanotechnol. 19, 609–612 (2020)
6. Lv, M., Wang, C.N., Ren, G.D., Ma, J., Song, X.L.: Model of electrical activity in a neuron under magnetic field effect. Nonlinear Dyn. 85(3), 1479–1490 (2016)
7. Bao, H., Chen, C.J., Hu, Y.H., Chen, M., Bao, B.C.: Two-dimensional piecewise-linear neuron model. IEEE Trans. Circuits Syst. II Exp. Briefs 68(4), 1453–1457 (2021)
8. Xu, Y., Jia, Y., Ma, J., Alsaadi, A., Ahmad, B.: Synchronization between neurons coupled by memristor. Chaos Solit. Fractals 104, 435–442 (2017)
9. Sun, J.W., Xiao, X., Yang, Q.F., Liu, P., Wang, Y.F.: Memristor-based Hopfield network circuit for reconstruction and sequencing application. AEU-Int. J. Electron. Commun. 134, 153698 (2021)
10. Hong, Q.H., Yan, R., Wang, C.H., Sun, J.R.: Memristive circuit implementation of biological nonassociative learning mechanism and its applications. IEEE Trans. Biomed. Circ. Syst. 14(5), 1036–1050 (2020)
11. Ma, J., Tang, J.: A review for dynamics in neuron and neuronal network. Nonlinear Dyn. 89(3), 1569–1578 (2017)
12. Ge, M.Y., Jia, Y., Xu, Y., Yang, L.J.: Mode transition in electrical activities of neuron driven by high and low-frequency stimulus in the presence of electromagnetic induction and radiation. Nonlinear Dyn. 91(1), 515–523 (2017)
13. Parastesh, F., Azarnoush, H., Jafari, S., Hatef, B., Perc, M., Repnik, R.: Synchronizability of two neurons with switching in the coupling. Appl. Math. Comput. 350, 217–223 (2019)
14. Fang, T.T., Zhang, J.Q., Huang, S.F., Xu, F., Wang, M.S., Yang, H.: Synchronous behavior among different regions of the neural system induced by electromagnetic radiation. Nonlinear Dyn. 98(17), 1267–1274 (2019)
15. Parastesh, F., Rajagopal, K., Alsaadi, F.E., Hayat, T., Pham, V.T., Hussain, I.: Birth and death of spiral waves in a network of Hindmarsh-Rose neurons with exponential magnetic flux and excitable media. Appl. Math. Comput. 354, 377–384 (2019)
16. Takembo, C.N., Mvogo, A., Fouda, H.P.E., Kofané, T.C.: Effect of electromagnetic radiation on the dynamics of spatiotemporal patterns in memristor-based neuronal network. Nonlinear Dyn. 95, 1067–1078 (2018)
17. Bao, H., Hu, A.H., Liu, W.B., Bao, B.C.: Hidden bursting firings and bifurcation mechanisms in memristive neuron model with threshold electromagnetic induction. IEEE Trans. Neural Netw. Learn. Syst. 31(2), 502–511 (2020)
18. Bao, H., Liu, W.B., Hu, A.H.: Coexisting multiple firing patterns in two adjacent neurons coupled by memristive electromagnetic induction. Nonlinear Dyn. 95, 43–56 (2019)
19. Li, R.H., Wang, Z.H., Dong, E.Z.: A new locally active memristive synapse-coupled neuron model. Nonlinear Dyn. 104, 4459–4475 (2021)
20. Ma, J., Yang, Z.Q., Yang, L.J., Tang, J.: A physical view of computational neurodynamics. J. Zhejiang Univ. Sci. A 20(9), 639–659 (2019)
21. Zhou, Q., Wei, D.Q.: Collective dynamics of neuronal network under synapse and field coupling. Nonlinear Dyn. (2021). https://doi.org/10.1007/s11071-021-06575-0
22. Yang, K., Duan, Q.X., Wang, Y.H., Zhang, T., Yang, Y.C., Huang, R.: Transiently chaotic simulated annealing based on intrinsic nonlinearity of memristors for efficient solution of optimization problems. Sci. Adv. 6(33), eaab9901 (2020)
23. Pu, Y., Yi, Z., Zhou, J.: Fractional Hopfield neural networks: Fractional dynamic associative recurrent neural networks. IEEE Trans. Neural Netw. Learn. Syst. 28(10), 2319–2333 (2017)
24. Cai, F.X., Kumar, Suhas., Vaerenbergh, T.V., et al.: Power-efficient combinatorial optimization using intrinsic noise in memristor Hopfield neural networks. Nat. Electron. 3(7), 409–418 (2020)
25. Wang, Z., Parastesh, F., Rajagopal, K., Hamarash, I.I., Hussain, I.: Delay-induced synchronization in two coupled chaotic memristive Hopfield neural networks. Chaos Solit. Fractals 134, 109702 (2020)
26. Pham, V.T., Jafari, S., Vaidyanathan, S., Volos, C., Wang, X.: A novel memristive neural network with hidden attractors and its circuitry implementation. Sci. China Tech Sci. 59(3), 358–363 (2016)
27. Lin, H.R., Wang, C.H., Hong, Q.H., Sun, Y.C.: A multistable memristor and its application in a neural network. IEEE Trans. Circuits Syst. II Exp. Briefs 67(12), 3472–3476 (2020)
28. Hu, X.Y., Liu, C.X., Liu, L., Ni, J.K., Yao, Y.P.: Chaotic dynamics in a neural network under electromagnetic radiation. Nonlinear Dyn. 91, 1541–1554 (2018)
29. Lin, H.R., Wang, C.H., Tan, Y.M.: Hidden extreme multistability with hyperchaos and transient chaos in a Hopfield neural network affected by electromagnetic radiation. Nonlinear Dyn. 99, 2369–2386 (2020)
30. Chen, C.J., Chen, J.Q., Bao, H., Chen, M., Bao, B.C.: Coexisting multi-stable patterns in memristor synapse-
coupled Hopfield neural network with two neurons. Nonlinear Dyn. 95(4), 3385–3399 (2019)
31. Chen, C.J., Bao, H., Chen, M., Xu, Q., Bao, B.C.: Non-ideal memristor synapse-coupled bi-neuron Hopfield neural network: Numerical simulations and breadboard experiments. AEUInt. J. Electron. Commun. 111, 152894 (2019)
32. Chen, M., Chen, C.J., Bao, B.C., Xu, Q.: Initial sensitive dynamics in memristor synapse-coupled Hopfield neural network. J. Electron. Inf. Technol. 42(4), 870–877 (2020)
33. Ma, C., Mou, J., Yang, F., Yan, H.Z.: A fractional-order hopfield neural network chaotic system and its circuit realization. EUR.‑Int. J. Electron. Commun. 111, 152894 (2019)
34. Rajagopal, K., Munoz-Pacheco, J.M., Pham, V.T., Hoang, D.V., Alsadawi, F.E., Alsadawi, F.E.: A Hopfield neural network with multiple attractors and its FPGA design. EUR.‑Phys. J. Spec. Top. 227, 811–820 (2018)
35. Yu, F., Zhang, Z.N., Shen, H., Huang, Y.Y., Cai, S., Jin, J., Du, S.C.: Design and FPGA implementation of a pseudorandom number generator based on a Hopfield neural network under electromagnetic radiation. Front. Phys. 9, 690651 (2021)
36. Zheng, P.S., Tang, W.S., Zhang, J.X.: Dynamic analysis of unstable Hopfield networks. Nonlinear Dyn. 61(3), 399–406 (2010)
37. Khalil, H.K.: Nonlinear systems, 3rd edn. Prentice-Hall, Upper Saddle River, NJ, USA (2002)
38. Chen, T.P., Amari, S.I.: Stability of asymmetric Hopfield networks. IEEE Trans. Neural Netw. Learn. Syst. 12(1), 159–163 (2001)
39. Silva, C.P.: Shirikov’s theorem-a tutorial. IEEE Trans. Circuits Syst. I, Fundam. Theory Appl. 40(10), 675–682 (1993)
40. Ansmann, G., Karnatak, R., Lehnhertz, K., Feudel, U.: Extreme events in excitable systems and mechanisms of their generation. Phys. Rev. E 88, 052911 (2013)
41. Karnatak, R., Ansmann, G., Feudel, U., Lehnhertz, K.: Route to extreme events in excitable systems. Phys. Rev. E 90, 022917 (2014)
42. Saha, A., Feudel, U.: Riddled basins of attraction in systems exhibiting extreme events. Chaos 28(3), 033610 (2018)
43. Mishra, A., Kingston, S.L., Hens, C., Kapitaniak, T., Feudel, U., Dana, S.K.: Routes to extreme events in dynamical systems: dynamical and statistical characteristics. Chaos 30(6), 063114 (2020)
44. Ray, A., Rakshit, S., Basak, G.K., Dana, S.K., Ghosh, D.: Understanding the origin of extreme events in El Niño southern oscillation. Phys. Rev. E 101, 062210 (2020)
45. Bao, H., Zhu, D., Liu, W.B., Xu, Q., Chen, M., Bao, B.C.: Memristor synapse-based Morris–Lecar model: Bifurcation analysis and FPGA-based validations for periodic and chaotic bursting/spiking firings. Int. J. Bifurcation Chaos 30(3), 2050045 (2020)
46. Ribar, L., Sepulchre, R.: Neuromodulation of neuromorphic circuits. IEEE Trans. Circuits Syst. I, Reg. Papers 66(8), 3028–3040 (2019)
47. Haghiri, S., Naderi, A., Ghanbari, B., Ahmad, A.: High speed and low digital resources implementation of Hodgkin-Huxley neuronal model using FPGA neurons. IEEE Trans. Circuits Syst. I, Reg. Papers 68(1), 275–287 (2021)
48. Jokar, E., Abolfathi, H., Ahmadi, A., Ahmadi, M.: An efficient uniform-segmented neuron model for large-scale neuromorphic circuit design: Simulation and FPGA synthesis results. IEEE Trans. Circuits Syst. I, Reg. Papers 66(6), 2336–2349 (2019)
49. Li, K.X., Bao, H., Li, H.Z., Ma, J., Hua, Z.Y., Bao, B.C.: Memristive Rulkov neuron model with magnetic induction effects. IEEE Trans. Ind. Informat. (2021). https://doi.org/10.1109/TII.2021.3086819
50. Lin, H.R., Wang, C.H., Chen, C.J., Sun, Y.C., Zhou, C., Xu, C., Hong, Q.H.: Neural bursting and synchronization emulated by neural networks and circuits. IEEE Trans. Circuits Syst. I, Reg. Papers (2021). https://doi.org/10.1109/TCSI.2021.3081150
51. Bao, B.C., Jiang, T., Wang, G.Y., Jin, P.P., Bao, H., Chen, M.: Two-memristor-based Chua’s hyperchaotic circuit with plane equilibrium and its extreme multistability. Nonlinear Dyn. 89(2), 1157–1171 (2017)

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.