The Model Technology of Stepping Motor Driving System

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Abstract. This paper proposes the dynamic characteristics of load motion have great influence on the performance of stepping motor. The dynamic model of stepping motor driving system is established on basis of least-squares, the system identification experiment is carried out on an actual Numerical Control Plate Printer drive system, and the dynamic parameters of the system were identified with MATLAB. The results have a good practical value to the optimal control of stepping motor.

1. Introduction
Stepping motors are widely adopted in mechatronics systems, and their starting characteristics play key role in controlling system. Although the research on stepping motor motion control has reached great achievements, the traditional control theory can’t effectively deal with the uncertain information of stepping motor servo system, such as the inertia, damping and elasticity of the load [1]. It is difficult to achieve satisfactory results in the control of stepping motor servo system using classical theory. Therefore, the study of load performance parameters is significant for improving the performance of stepping motor to the maximum extent.
This paper studied the dynamics model of stepping motor load system. The dynamic parameters of the system were identified with MATLAB. The mathematical model established may be used to provide a theoretical basis for the speed control of stepping motor.

2. Establishment of the mathematical model
Stepping servo system model includes two parts: stepping motor model and mechanical driving system model. They are connected by torque balance. The stepping motor model is related to the specific control method used. Generally, many domestic and international studies of the stepping motor speed control only consider its mathematical model, and seldom involve the influence of relevant parameters of the driving mechanical system [2]. However, only understanding the mathematical model of the entire system can the system be analysed, can the performance of the system be improved, can the motion of the system be predicted, and can the optimal speed control of the system be implemented [3]. This paper focuses on the mechanical driving system model. In order to make the electromechanical coupling model as close to the actual system as possible, factors such as inertia, viscous damping and elasticity from the motor shaft to the end of the mechanical chain are retained in the model process to reflect electromechanical coupling effects better.

2.1. System composition
Generally speaking, the mechanical transmission system for stepping motors is usually a deceleration system, mainly including gear reduction, synchronous belt deceleration, screw deceleration and steel wire deceleration.
2.2. Dynamic model of the system

In most textbooks, the electromechanical drive system shown in Fig.1 is simplified to the dynamic calculation model shown in Fig.2. According to the law of rotation, the acceleration torque on the motor shaft should be

\[ T_a = J \frac{d\omega}{dt} \]

\[ T_a = T_m - T_{el}. \]

where \( J = J_m + J_{el} \)

This can be rewritten as

\[ T_m = (J_m + J_{el}) \frac{d\omega}{dt} + T_{el}. \tag{1} \]

where

- \( T_m \) - the calculated torque of the motor, N·m;
- \( T_{el} \) - the equivalent load torque converted to the motor shaft, N·m;
- \( J \) - the equivalent moment of inertia converted to the motor shaft, N·m²;
- \( J_m \) - the moment of inertia of the rotor, N·m²;
- \( J_{el} \) - the equivalent moment of inertia converted from the inertial load to the motor shaft, N·m²;
- \( \omega \) - the angular velocity of the motor, rad/s [4].

2.3. Differential model of the system

Equation (1) does not consider the influence of the damping and elasticity on the system dynamic characteristics, which is the focus of this paper. In the modeling, on the one hand, the torsion stiffness of the motor shaft to the pulley and the output stiffness and damping of the pulley to the load are considered; on the other hand, the load torque is ignored.
The synchronous belt system transmits the torque generated by the stepping motor to the load, which drives the load to operate according to the required conditions. With the torque provided by the stepping motor as the input and the angular acceleration of the load as the output, the differential equation of the system may be established,

\[ J\ddot{\theta} + B\dot{\theta} + K\theta = T_m \quad (2) \]

Where, \( J \) and \( T_m \) is the same as the formula (1);
\( \theta \) - the angle of rotation of the big pulley (or load), rad;
\( B \) - the viscous damping coefficient of the system, N·m·S·rad\(^{-1}\);
\( K \) - the elastic coefficient of the system, N·m·rad\(^{-1}\).

The product of the synchronous belt tension and the small pulley radius is used as the input instead of the electromagnetic torque of the stepping motor; the tangential acceleration of the load measured by the acceleration sensor is used as the output, and the formula (2) is written as

\[ JS + BS + KS = F \cdot r \cdot R \quad (3) \]

Where, \( S \) - the arc coordinate of the point where the radius of the load is the largest;
\( F \) - the tension of the synchronous belt, N;
\( r \) - the radius of the small pulley, mm;
\( R \) - the radius of the load, mm.

Among them, \( J \) is obtained by calculation; \( B \) and \( K \) are parameters to be identified.

Formula (3) is called differential model, which reflects the dynamic characteristics of the system through the mathematical connection between the function and the derivative function [5].

### 2.4. Difference model of the system

In the field of engineering technology, \( n \)-th order differential equation is usually described the dynamic characteristics of a system. Modern system identification methods are closely related to the development and application of computers. When signals are processed with computer, the input and output continuous signals are first sampled. That is, the continuous signals are converted into discrete time series (or dynamic data), so that the mathematical model described by a difference equation is easily established by computer. If the input signal is \( u(k) \) and the output signal is \( y(k) \), the differential model of the controlled system is described by a difference model as:

\[ y(k) + a_1y(k-1) + \cdots + a_ny(k-n) = b_1u(k) + b_2u(k-1) + \cdots + b_nu(k-n) \quad (4) \]

Equation (4) can be rewritten as

\[ y(k) = -\sum_{i=1}^{n} a_i y(k-i) + \sum_{i=1}^{n} b_i u(k-i) \quad (5) \]

Equation (5) is a deterministic ARMA model. Because of the existence of equation error \( e(k) \), equation (5) is written as

\[ y(k) = -\sum_{i=1}^{n} a_i y(k-i) + \sum_{i=1}^{n} b_i u(k-i) + e(k) \quad (6) \]

As mentioned above, the input signal of this system is the tension of the synchronous belt, and the output signal is the acceleration of the load. Therefore, the question is how to estimate the parameters of the model from the input sequence \( \{u(k)\} \) and the output sequence \( \{y(k)\} \).

Suppose that the measurement is repeated \( N \) times, a ternary equation may be listed by equations (6):
The equation above is differential model of the system, which provides the input and output data for identification experiments. The form of a matrix is given by.

\[ Y = \Phi \Theta + \varepsilon \]  

Where \( \Theta^T = [a_1, a_2, b_0] \)

\[
\Phi(y, u) = \begin{bmatrix}
-y(n) & -y(n-1) & u(n+1) \\
-y(n+1) & -y(n) & u(n+2) \\
& \vdots & \vdots \\
-y(n+N-1) & -y(n+N-2) & u(n+N)
\end{bmatrix}
\]

\[
Y = \begin{bmatrix}
y(n+1) \\
y(n+2) \\
\vdots \\
y(n+N)
\end{bmatrix}
\]

\[
\varepsilon = \begin{bmatrix}
e(n+1) \\
e(n+2) \\
\vdots \\
e(n+N)
\end{bmatrix}
\]

Where \( \Theta \) is the parameter vector, and \( \Phi \) is the observation matrix.

\( a_1, a_2 \) and \( b_0 \) are the coefficients identified. Ignoring the equation error \( \varepsilon \), equation (7) is written as

\[
\begin{aligned}
y(n+1) &= a_1 y(n) - a_2 y(n-1) + b_0 u(n+1) \\
y(n+2) &= -a_1 y(n+1) - a_2 y(n) + b_0 u(n+2) \\
&\vdots \\
y(n+N) &= -a_1 y(n+N-1) - a_2 y(n+N-2) + b_0 u(n+N)
\end{aligned}
\]

Equation (9) is ARMA model of the system, which reflects the dynamic characteristics of the system through dynamic data [6]. A linear least-squares estimator of the model parameter may be inferred from the equation

\[
\hat{\Theta} = (\Phi^T \Phi)^{-1} \Phi^T Y
\]

According to equation (3), the observation matrix of the system identification parameter estimation formula is transformed as follows:

\[
\Phi(y, u) = \begin{bmatrix}
-y(n) & -y(n-1) & F(n+1) \cdot r \cdot R \\
-y(n+1) & -y(n) & F(n+2) \cdot r \cdot R \\
& \vdots & \vdots \\
-y(n+N-1) & -y(n+N-2) & F(n+N) \cdot r \cdot R
\end{bmatrix}
\]

Elements in the matrix shown in equation (11) are the input and output data to be used in system identification.

3. Experimental design and data processing

3.1. Formation of the experimental system

The experimental test system consists mainly of a resistance strain gauge, a bridge, an acceleration sensor, a charge amplifier, an oscilloscope, a dynamic strain gauge, a signal collection card and a computer, shown in Fig. 1.

The main purpose of this paper is to identify the dynamic parameters of stepping motor drive system. The resistance strain gauge, the bridge and the dynamic strain gauge are combined to complete the measurement of the system input signal. The measurement result of the acceleration sensor represents the tangential acceleration of the load circumference when Numerical Control Plate Printer works, namely the system output signal. Because the stepping angle of stepping motor is 0.9° and we only study the speed problem when the stepping motor starts, the value measured by the acceleration sensor is very small, which needs to be amplified by the charge amplifier. The signal collection card collects the input and output data from the dynamic strain gauge and the charge amplifier and sends them to
the computer, which converts the data into a MAT file that MATLAB can read. The oscilloscope is used to record the shape of the input and output curves online for qualitative analysis of the system.

3.2. **Signals testing**

After the experimental device is connected, the oscilloscope is opened to analyse the waveform and voltage amplitude of the motor in different steps. Then adjust the amplification factor of the charge amplifier to 46.5, and finally adjust the parameters of the dynamic strain gauge to: filter 100HZ, gain 1/2, and supply voltage 8V.

Open the control program of the collection card, run the WT.EXT program and control the collection card to collect data. Two analogy channels were used in the experiment: the voltage on channel 0 reflects the acceleration of the load, and the voltage on channel 1 reflects the deformation of the belt. After setting the sampling frequency, sampling points and sampling mode, start the Numerical Control Plate Printer to collect data and store the collected data in the computer for processing.

3.3. **Experimental data processing**

The data processing of the stepping motor drive system mainly includes the fitting of the synchronous belt elastic coefficient, the input/output filtering and the system parameter identification. The processing of these data is carried out in the MATLAB environment, relying on the good openness, operational reliability and powerful computing functions of MATLAB software.

3.3.1 **Fitting of the synchronous belt elastic coefficient.** When the resistance of the strain gauge on the synchronous belt changes, the dynamic strain gauge indicates the voltage. Because the tension on the synchronous belt of the stepping motor drive system is used as the input signal in this experiment, the corresponding relationship between the voltage and the tension should be found. When the device is stationary, the dynamic strain gauge is calibrated to display the voltage for different tensions, as shown in Table 1.

| Tension/N | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 |
|-----------|---|----|----|----|----|----|----|----|
| Voltage/V | 0 | 0.13 | 0.25 | 0.47 | 0.52 | 0.54 | 0.85 | 0.95 |
| Tension/N | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 |
| Voltage/V | 1.1 | 1.25 | 1.4 | 1.55 | 1.7 | 1.8 | 1.95 | 2.1 |

Table 1 indicates that the change of the strain of the resistance strain gauge in the synchronous belt varies the tension when the device is stationary, that is, the relationship between tension and deformation. If it is fitted as a straight line, the slope of the straight line is the elastic coefficient of the synchronous belt.

If we use the MATLAB program [7]

\[
[a,S]=polyfit(x,y,n)
\]

plot(x,y)

A fitted polynomial can get.

When n=1,

\[
a = [0.1415,-0.0267]
\]

Variance: 0.1750

it can be seen that

\[
Y = 0.1415X - 0.0267.
\]
When \( n=2 \),
\[
a = [0.0005, 0.1339, -0.0089]
\]
Variance: 0.1707
it can be seen that
\[
Y=0.0005X^2+0.1339X
\]
Comparison of the first-order and the second-order fitting polynomials shows that the \( X \) term coefficients are not much different, and the \( X^2 \) term coefficient of the second-order fitting polynomial (only 0.0005) is negligible. Therefore, the first-order fitting can ensure sufficient accuracy, as illustrated in Fig. 3.
The elastic coefficient of the synchronous belt identified by the first-order fitting is 0.1415, which is essentially the proportional relationship between the voltage signal on the dynamic strain gauge and the tension of the synchronous belt, that is, the ratio of voltage to tension is equal to 0.1415.

3.3.2 Simulation based MATLAB. After running the WT.EXT program, the Numerical Control Plate Printer is started and the data is collected by the signal collection card. The collected data is directly converted to a MAT file for signal analysis processing in the MATLAB environment [7, 8].

![Figure 3. Linear fitting](image)

![Figure 4. A simulation model of the input and output signals](image)
Fig. 5. Comparison of 1000 Hz filtering of the input and the output graphics

Fig. 5 (a) shows the strain curve and the acceleration curve recorded by the oscilloscope, when the stepping motor runs 5 steps. It is observed that the signal is mixed with relatively regular high-frequency clutter. The calculated frequency of the clutter signal is about 1250 Hz, the cut-off frequency is selected as 1000 Hz, and the noise is filtered by a Butterworth low-pass filter. After 1000 Hz filtering, 10,000 of these points were observed, and the amplified waveform was shown in Fig. 5 (b).

3.3.3 Identification of the damping and elastic coefficients. The least squares parameter estimation formula shown in equation (10) is converted into a M file of MATLAB, then the system dynamic coefficients can be obtained.

| Serial number | Period    | Damping coefficient | Elastic coefficient | Input factor | Torque |
|---------------|-----------|---------------------|---------------------|--------------|--------|
| 1             | 4975~4990 | 0.0130              | 0.0131              | 71.0000      |
| 2             | 4990~5005 | 0.0722              | 0.1065              | 71.0000      |
| 3             | 5005~5020 | 0.7231              | 0.7681              | 71.0000      |
| 4             | 5020~5035 | 0.3618              | 0.3939              | 71.0000      |
| 5             | 5035~5050 | 0.5223              | 0.5065              | 71.0000      |
| 6             | 5050~5065 | 0.4241              | 0.4148              | 71.0000      |
| 7             | 5065~5080 | 0.2783              | 0.3260              | 71.0000      |
| 8             | 5080~5095 | 0.2029              | 0.2230              | 71.0000      |
| 9             | 5095~5110 | 0.0858              | 0.0869              | 71.0000      |
| 10            | 5110~5125 | 0.0306              | 0.0303              | 71.0000      |
| 11            | 5125~5140 | 0.1011              | 0.1007              | 71.0000      |
| 12            | 5140~5155 | 0.1882              | 0.1832              | 71.0000      |
| 13            | 5155~5170 | 0.1994              | 0.2006              | 71.0000      |
| 14            | 5170~5185 | 0.2034              | 0.1928              | 71.0000      |
When the stepping motor runs 5 steps, the damping, elasticity and input torque coefficients of the system in different periods are identified [9], as shown in Table 2. It can be seen from Table 2 that the damping and elastic coefficients of the system change with time from 4975 to 5185.

In order to observe the speed change more clearly when the stepping motor starts, the strain and acceleration curves of the 1000 Hz filter are amplified from 4975 to 5200, as shown in Fig. 6. It is shown that the system runs up from 5001 to 5065 (the first peak of the acceleration curve), and after the point 5065, the acceleration has oscillation due to the sudden braking of the stepping motor [10]. The dynamics parameters identified by the sampling data of 5001~5065 points are: the damping coefficient 0.4287, the elastic coefficient 0.4284, and the input torque coefficient 68.0000. Therefore, the dynamic equation obtained from the identified coefficients is

\[ \dot{S} + 0.4287 \dot{S} + 0.4284S = 68.0000(F \cdot r \cdot R) \]  

(12)

Comparison of equation (3) and (12), the moment of inertia, the viscous damping coefficient and the elastic coefficient of the system may be obtained.

4. Conclusion

This paper studies the model technology of the M axis dynamic stepping motor driving system of Numerical Control Plate Printer. This technology is also suitable for rotating mechanical systems driven by other stepping motors; it's just that the identified parameters are different. The viscous damping and elastic coefficients of system obtained by the model method introduced in this paper provide a theoretical basis for the speed control of stepping motor and the optimization of system structure.

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