Supersymmetric Action for FRW Model with Complex Matter Field

V.I. Tkach\textsuperscript{1} and J.J. Rosales\textsuperscript{1,2}

\textsuperscript{1} Instituto de Física, Universidad de Guanajuato
Apartado Postal E-143, León, Gto., México

and

\textsuperscript{2} Universidad Autónoma Metropolitana-Iztapalapa
Dept. de Física, Apdo. Postal 55-534
México, D.F. México

Abstract

On the basis of the local $n = 2$ supersymmetry we construct the supersymmetric action for a set of complex scalar supermultiplets in the FRW model. This action corresponds to the dilaton-axion and chiral components of supergravity theory.

PACS Number(s): 04.60. Kz, 04.65. + e, 12.60. Jv., 98.80. Hw.

\textsuperscript{*}e-mail vladimir@ifug1.ugto.mx

\textsuperscript{†}e-mail: juan@ifug3.ugto.mx
One of the possible models of unification of interactions is supergravity interacting with matter, because it admits the solutions of the problem of cosmological constant and comes to degeneration of masses for fields in any supermultiplet [1].

In the last years spatially homogeneous minisuperspaces models have to be indeed a very valuable tool in supergravity theories. The study of minisuperspace have led to important and interesting results, pointing out usefull lines of research. Since then, several publications have appeared on the subject of supersymmetric quantum cosmology, also including matter [2].

More recently we proposed a new formulation to investigate supersymmetric quantum cosmological models [3,4]. This formulation was performed by introducing a superfield formulation. This is because superfields defined on superspace allow all the component fields in a supermultiplet to be manipulated simultaneously in a manner, which automatically preserves supersymmetry. Our approach has the advantage of being more simple, than proposed models based on full supergravity [2] and gives, by means of this local symmetry procedure, in a direct maner the corresponding fermionic partners.

In the paper [5] was considered the FRW model interacting with a simplest real matter supermultiplet, and was shown, that in this model of Universe with local supersymmetry, when energy density of vacuum is equal to zero, allows to have breaking supersymmetry.

In this work we will consider the homogeneous and isotropic FRW cosmological model interacting with a set of homogeneous and isotropic complex scalar matter supermultiplet. As will be shown, the supersymmetric action obtained for this model corresponds to dilaton-axion and chiral components of supergravity theory. It is well known, that the action of cosmological FRW models is invariant under the time reparametrization \( t' \rightarrow t + a(t) \). Then, we may obtain the superfield description of these models, when we introduce the odd complex “time” parameters \( \eta, \bar{\eta} \), which are the superpartners of time parameter [3,4]. This procedure is well known from superparticles [6-8].

So, we have the following superfield action for the FRW model interacting with a set of complex scalar matter supermultiplet
\[
S = \int \left\{ -\frac{N^{-1}}{2\kappa^2} R \overline{D}_\eta R D_\eta R + \sqrt{k} R^2 + \frac{N^{-1} R^3}{4} \left( \overline{D}_\eta \overline{Z}^a D_\eta Z^a + \overline{D}_\eta Z^a D_\eta \overline{Z}^a \right) - R^3 |g(Z)| \right\} d\eta d\overline{\eta} dt,
\]

(1)

where \( \kappa = \sqrt{\frac{4\pi G}{3}} \) and \( G \) is the Newtonian constant of gravity. In the action (1) \( \mathcal{N}(t, \eta, \overline{\eta}) \) is a real gravity superfield, and it has the form

\[
\mathcal{N}(t, \eta, \overline{\eta}) = N(t) + i\eta \psi'(t) + i\overline{\eta} \overline{\psi}'(t) + \eta \overline{\eta} V'(t),
\]

(2)

where \( \psi'(t) = N^{1/2} \psi(t) \), \( \overline{\psi}'(t) = N^{1/2} \overline{\psi}(t) \) and \( V'(t) = N V - \overline{\psi} \psi \). The law transformation of the superfield \( \mathcal{N}(t, \eta, \overline{\eta}) \) may be written in the following way

\[
\delta \mathcal{N}(t, \eta, \overline{\eta}) = (\Lambda \mathcal{N}) \cdot + \frac{i}{2} \overline{D}_\eta \Lambda D_\eta \mathcal{N} + \frac{i}{2} D_\eta \Lambda \overline{D}_\eta \mathcal{N},
\]

(3)

where the superfunction \( \Lambda(t, \eta, \overline{\eta}) \) is written as

\[
\Lambda(t, \eta, \overline{\eta}) = a(t) + i\eta \beta'(t) + i\overline{\eta} \overline{\beta}'(t) + \eta \overline{\eta} b(t),
\]

(4)

and \( D_\eta = \frac{\partial}{\partial \eta} + i\eta \frac{\partial}{\partial t} \) and \( \overline{D}_\eta = -\frac{\partial}{\partial \overline{\eta}} - i\eta \frac{\partial}{\partial t} \) are the supercovariant derivatives, \( \beta'(t) = N^{-1/2} \beta(t) \) is the Grassmann complex parameter of the local “small” \( n = 2 \) susy transformations and \( b(t) \) is the parameter of local \( U(1) \) rotations of the complex \( \eta \).

So, the \( n = 2 \) local transformations of the supertime \((t, \eta, \overline{\eta})\) are

\[
\delta t = \Lambda(t, \eta, \overline{\eta}) + \frac{1}{2} \overline{\eta} \overline{D}_\eta \Lambda(t, \eta, \overline{\eta}) - \frac{1}{2} \eta D_\eta \Lambda(t, \eta, \overline{\eta}),
\]

\[
\delta \eta = \frac{i}{2} \overline{D}_\eta \Lambda(t, \eta, \overline{\eta}),
\]

\[
\delta \overline{\eta} = -\frac{i}{2} D_\eta \Lambda(t, \eta, \overline{\eta}),
\]

(5)

which are the generalization of the time reparametrization \( t' \to t + a(t) \) in the cosmological models.

The components of the superfield \( \mathcal{N}(t, \eta, \overline{\eta}) \) in (2) are gauge fields of the one-dimensional \( n = 2 \) extended supergravity, \( N(t) \) is einbein, \( \psi(t) \) and \( \overline{\psi}(t) \) are the time-like components of the Rarita-Schwinger fields \( \psi^\alpha_\mu \) and \( \overline{\psi}^\alpha_\mu \), which may be obtained by spatial reduction from
the four dimensional supergravity to the one-dimensional models and \( V(t) \), is \( U(1) \) gauge field.

The real “matter” superfield \( \mathcal{R}(t, \eta, \bar{\eta}) \) may be written as

\[
\mathcal{R}(t, \eta, \bar{\eta}) = R(t) + i\eta \lambda'(t) + i\bar{\eta}\bar{\lambda}'(t) + \eta\bar{\eta}B'(t), \tag{6}
\]

where \( \lambda'(t) = \kappa N^{1/2}\lambda(t) \), \( \bar{\lambda}'(t) = \kappa N^{1/2}\bar{\lambda}(t) \) and \( B'(t) = \kappa NB - \frac{\kappa}{2}(\bar{\psi}\lambda - \psi\bar{\lambda}) \). The law transformation for the superfield \( \mathcal{R}(t, \eta, \bar{\eta}) \) is

\[
\delta \mathcal{R} = \Lambda \dot{\mathcal{R}} + \frac{i}{2} \bar{D}_\eta \Lambda D_\eta \mathcal{R} + \frac{i}{2} D_\eta \Lambda \bar{D}_\eta \mathcal{R}. \tag{7}
\]

The component \( B(t) \) in (6) is an auxiliary degree of freedom, \( \lambda(t) \) and \( \bar{\lambda}(t) \) are dynamical degrees of freedom remaining from the spatial part of the Rarita-Schwinger field obtained from spatial reduction of pure supergravity theories to cosmological models and are the partners of the scale factor \( R(t) \).

The complex scalar matter supermultiplet \( Z^a \) consists of a set of spatially homogeneous scalar complex matter fields \( z^a(t), \bar{z}^a(t) \) \((a = 1, \ldots, n)\), four fermionic degrees of freedom \( \chi^a(t), \bar{\chi}^a(t), \phi^a(t) \) and \( \bar{\phi}^a(t) \), two bosonic auxiliary fields \( F^a(t) \) and \( \bar{F}^a(t) \) and a superpotential of the type \( |g(Z)| \). The components of the complex matter superfields may be written in the following way

\[
Z^a(t, \eta, \bar{\eta}) = z^a(t) + i\eta \chi^a(t) + i\bar{\eta}\bar{\chi}^a(t) + F^a(t)\eta\bar{\eta}, \tag{8}
\]

where \( \chi^a(t) = N^{1/2}\chi^a(t), \bar{\chi}^a(t) = N^{1/2}\bar{\chi}^a(t) \) and \( F^a(t) = NF^a - \frac{1}{2}(\bar{\psi}\chi^a - \psi\bar{\chi}^a) \). The law transformation for the complex matter superfield is written as

\[
\delta Z^a = \Lambda \dot{Z}^a + \frac{i}{2} \bar{D}_\eta \Lambda D_\eta Z^a + \frac{i}{2} D_\eta \Lambda \bar{D}_\eta Z^a. \tag{9}
\]

It is clear, that the superfield action (1) is invariant under the \( n = 2 \) local super time transformations (5) if the superfields transform as (3,7,9).

We will write the expression, which is found under the integral (1) by means of certain superfunction \( f(\mathcal{R}, \mathcal{N}, Z^a) \). Then, the infinitesimal small transformation of the action (1) under the superfield transformations (3,7,9) has the following form:
\[ \delta S = \frac{i}{2} \int \left\{ \bar{D}_\eta \left( \Lambda D_\eta f(\mathbb{R}, \mathbb{N}, Z^a) \right) + D_\eta \left( \Lambda \bar{D}_\eta f(\mathbb{R}, \mathbb{N}, Z^a) \right) \right\} \eta \bar{\eta} dt. \] \hspace{1cm} (10)

We can see, that under the integration it gives a total derivative. That is, the action (10) is invariant under the superfield transformations (3,7,9).

Making the corresponding operations from the action (1) one obtains the expression for the component action, where the auxiliary fields \( B(t), F^a(t) \) and \( \bar{F}^a(t) \) appear. Performing the variation with respect to these auxiliary fields we get three algebraical equations, which have the following solutions

\[ B(t) = \frac{\kappa}{2R} \bar{\lambda} \lambda + \frac{\sqrt{k}}{\kappa} + \frac{3\kappa}{4} R(\bar{\chi}^a \chi^a + \bar{\phi}^a \phi^a) - 3\kappa R |g|, \] \hspace{1cm} (11)

\[ F^a(t) = -\frac{3\kappa}{2R} (\lambda \bar{\phi}^a - \bar{\chi}^a) + 2 \frac{\partial |g|}{\partial \bar{z}_a}, \] \hspace{1cm} (12)

and

\[ \bar{F}^a(t) = -\frac{3\kappa}{2R} (\lambda \bar{\phi}^a - \bar{\chi}^a) + 2 \frac{\partial |g|}{\partial z_a}, \] \hspace{1cm} (13)

and after substituting them again into the component action obtained from (1) and making the following fields redefinitions \( \lambda \to R^{-1/2} \lambda, \ \bar{\lambda} \to R^{-1/2} \bar{\lambda}, \ \chi^a \to 2^{1/2} R^{-3/2} \chi^a, \ \bar{\chi}^a \to 2^{1/2} R^{-3/2} \bar{\chi}^a, \ \phi^a \to 2^{1/2} R^{-3/2} \phi^a \) and \( \bar{\phi}^a \to 2^{1/2} R^{-3/2} \bar{\phi}^a \). We get the following component action

\[ S = \int \left\{ -\frac{R}{2N \kappa^2} (D \eta)^2 + \frac{R^3}{2N} D \eta D \zeta \eta + \frac{NRk}{2\kappa^2} + \frac{9}{2} N \kappa^2 R^3 |g(z)|^2 - \frac{NR^3}{2} \frac{\partial |g|}{\partial \zeta} \right\} \left( \varphi^a - \bar{\phi}^a \right) \] 

\[ + \frac{3\sqrt{2k}}{4} iD \zeta (\bar{\lambda} \chi^a + \bar{\phi}^a) + \frac{3\sqrt{2k}}{4} iD \zeta (\lambda \bar{\chi}^a + \lambda \bar{\phi}^a) - \frac{\sqrt{k}}{2R} (\bar{\psi} \lambda - \psi \bar{\lambda}) - \frac{N \sqrt{k}}{2R} \lambda \bar{\lambda} + \frac{3N \sqrt{k}}{2R} (\bar{\phi}^a \phi^a + \bar{\chi}^a \chi^a) + \frac{9N \kappa^2}{4R^3} \bar{\chi}^a \chi^a \phi^a \hat{\phi}^a \phi^a 

\[ - \frac{3\sqrt{2}}{4R^{3/2}} \kappa (\bar{\psi} \lambda - \psi \bar{\lambda}) (\bar{\phi}^a \phi^a + \bar{\chi}^a \chi^a) - \frac{9}{2} N \kappa^2 |g| (\bar{\chi}^a \chi^a + \bar{\phi}^a \phi^a) \] 

\[ + \frac{9N \kappa^2}{2} \frac{\partial^2 |g|}{\partial \zeta \partial \psi} \bar{\phi}^a \phi^a \] 

\[ + \frac{3\sqrt{2}}{2} \kappa N \left[ \frac{\partial |g|}{\partial \zeta} (\bar{\phi}^a \lambda + \bar{\chi}^a \phi^a) + \frac{\partial |g|}{\partial \zeta} (\bar{\chi}^a \phi^a + \bar{\phi}^a \phi^a) \right] + 2N \frac{\partial^2 |g|}{\partial \zeta \partial \psi} (\bar{\phi}^a \phi^a + \bar{\chi}^a \phi^a) \]
\[ + \frac{\psi}{2} \left[ \sqrt{2} R^{3/2} \frac{\partial |g|}{\partial z_a} \chi^a + \sqrt{2} R^{3/2} \frac{\partial |g|}{\partial \bar{z}_a} \bar{\phi}^a + 3 \kappa R^{3/2} |g| \lambda \right] \\
- \frac{\psi}{2} \left[ \sqrt{2} R^{3/2} \frac{\partial |g|}{\partial z_a} \bar{\phi}^a + \sqrt{2} R^{3/2} \frac{\partial |g|}{\partial \bar{z}_a} \chi^a + 3 \kappa R^{3/2} |g| \bar{\lambda} \right] \right) \{ \bar{\eta}, \Pi_R \} = \frac{1}{2} \left( \frac{\partial^2}{\partial z_a^2} + \frac{\partial^2}{\partial \bar{z}_a^2} \right) - |g(Z)| d\eta \bar{\eta} dt. \tag{15} \]

From (1) we can see, that the action for a set of scalar complex supermatter has the form

\[ \int \left\{ \frac{1}{4} \left( \bar{D}_{\eta} \bar{Z}^a D_{\eta} Z^a + \bar{D}_{\eta} Z^a D_{\eta} \bar{Z}^a \right) - |g(Z)| \right\} d\eta d\bar{\eta} dt. \]

This action corresponds to the action obtained by spatial reduction from Wess-Zumino model in four dimensions with arbitrary superpotential \( g(Z) \). The action (15) gives two complex supercharges \( Q_1 \) and \( Q_2 \). Because of the action (15) is invariant under the change of \( Z^a \leftrightarrow \bar{Z}^a \), then the supercharges allow the invariance under the change of \( Q_1 \leftrightarrow \bar{Q}_2 \). Furthermore, we can join in a one complex supercharge \( \tilde{S} = Q_1 + Q_2 \) and \( \bar{\tilde{S}} = \bar{Q}_1 + \bar{Q}_2 \).

Now we will proceed with the hamiltonian analysis of the system. The momenta \( \Pi_R, \Pi_z^a \) and \( \Pi_{\bar{z}}^a \) conjugate to \( R(t), z^a(t) \) and \( \bar{z}^a(t) \) respectively, they are given by

\[ \Pi_R = - \frac{R}{N \kappa^2} \left[ \bar{R} - \frac{i}{2 \sqrt{R}} (\psi \bar{\lambda} + \bar{\psi} \lambda) \right], \tag{16} \]

\[ \Pi_z^a = \frac{R^3}{N} \left[ \bar{\dot{z}}^a - \frac{i}{\sqrt{2} R^{3/2}} (\psi \bar{\chi}^a + \bar{\psi} \phi^a) \right] + \frac{3i \sqrt{2} \kappa}{4 R^{3/2}} (\lambda \bar{\chi}^a + \bar{\lambda} \phi^a), \tag{17} \]

\[ \Pi_{\bar{z}}^a = \frac{R^3}{N} \left[ \dot{\bar{z}}^a - \frac{i}{\sqrt{2} R^{3/2}} (\bar{\psi} \chi^a + \psi \bar{\phi}^a) \right] + \frac{3i \sqrt{2} \kappa}{4 R^{3/2}} (\bar{\lambda} \chi^a + \lambda \bar{\phi}^a) \tag{18} \]

with respect to the canonical Poisson brackets

\[ \{ R, \Pi_R \} = 1, \{ z_a, \Pi_z^b \} = \delta_a^b, \{ \bar{z}_a, \Pi_{\bar{z}}^b \} = \delta_a^b. \tag{19} \]

As usual with fermionic systems the calculation of the momenta conjugate to the anticommuting dynamical variables introduces primary constraints, which we denote by
\[ a_i \equiv \Pi_{\lambda} + \frac{i}{2} \dot{\lambda} \approx 0, \quad a_{\bar{\lambda}} \equiv \Pi_{\bar{\lambda}} + \frac{i}{2} \dot{\bar{\lambda}} \approx 0, \]
\[ a_i^a \equiv \Pi_{\chi}^a - \frac{i}{2} \dot{\chi}^a \approx 0, \quad a_{\bar{\chi}}^a \equiv \Pi_{\bar{\chi}}^a + \frac{i}{2} \dot{\bar{\chi}}^a \approx 0, \]
\[ a_i^a \equiv \Pi_{\phi}^a - \frac{i}{2} \dot{\phi}^a \approx 0, \quad a_{\bar{\phi}}^a \equiv \Pi_{\bar{\phi}}^a + \frac{i}{2} \dot{\bar{\phi}}^a \approx 0, \]  
(20)

where \( \Pi_{\lambda} = \frac{\partial L}{\partial \dot{\lambda}} \) and \( \Pi_{\chi}^a = \frac{\partial L}{\partial \dot{\chi}^a} \) are the momenta conjugate to the anticommuting dynamical variables \( \lambda(t), \chi(t) \) and \( \phi(t) \) respectively. The momenta conjugate to \( B(t), \psi(t), \bar{\psi}(t) \) and \( V(t) \) are found equal to zero indicating, that these variables play the role of gauge fields, whose time derivative is arbitrary, so they are non-dynamical fields.

The constraints (20) are of second-class and can be eliminated by Dirac procedure. We define the matrix constraint \( C_{ik}(i, k = \lambda, \bar{\lambda}, \chi^a, \bar{\chi}^a, \phi^a, \bar{\phi}^a) \) as the Poisson bracket. We have the following non-zero matrix elements

\[ C_{\lambda \lambda} = C_{\bar{\lambda} \bar{\lambda}} = \{ \Pi_{\lambda a}, \Pi_{\bar{\lambda} b} \} = i \delta_a^b, \quad C_{\chi \chi} = C_{\bar{\chi} \bar{\chi}} = \{ \Pi_{\chi a}, \Pi_{\bar{\chi} b} \} = i \delta_a^b, \]
\[ C_{\phi \phi} = C_{\bar{\phi} \bar{\phi}} = \{ \Pi_{\phi a}, \Pi_{\bar{\phi} b} \} = i \delta_a^b \]

(21)

with their inverse matrices \( (C^{-1})_{\lambda \lambda} = -i, (C^{-1})_{\bar{\lambda} \bar{\lambda}} = i \) and \( (C^{-1})_{\phi \phi} = i, (C^{-1})_{\bar{\phi} \bar{\phi}} = i \). The Dirac brackets \( \{ , \}^* \) are then defined by

\[ \{ A, B \}^* = \{ A, B \} - \{ A, \Pi_i \}(C^{-1})^{ik}\{ \Pi_k, B \}. \]  
(22)

The result of this procedure is the elimination of the momenta conjugate to the fermionic variables, leaving as with the following non-zero Dirac brackets relations

\[ \{ R, \Pi_{\Pi} \}^* = \{ R, \Pi_{\Pi} \} = 1, \quad \{ z_a, \Pi_{\bar{z}}^b \}^* = \{ z_a, \Pi_{\bar{z}}^b \} = \delta_a^b, \]
\[ \{ \bar{z}_a, \Pi_{\bar{z}}^b \}^* = \{ \bar{z}_a, \Pi_{\bar{z}}^b \} = \delta_a^b, \]
\[ \{ \lambda, \bar{\lambda} \}^* = i, \quad \{ \chi_a, \chi_{\bar{b}} \}^* = -i \delta_a^b, \quad \{ \phi_a, \phi_{\bar{b}} \}^* = -i \delta_a^b. \]  
(23)

In a quantum theory Dirac brackets \( \{ , \}^* \) must be replaced by commutators [ , ] or anticommutators \( \{ , \} \). We get
\[ [R, \Pi_R] = i \{ R, \Pi_R \} = i, \quad [z_a, \Pi^b_z] = i \{ z_a, \Pi^b_z \} = i \delta^b_a, \]

\[ [\bar{z}_a, \Pi^b_z] = i \{ \bar{z}_a, \Pi^b_z \} = i \delta^b_a \quad (24) \]

\[ \{ \lambda, \bar{\lambda} \} = i \{ \lambda, \bar{\lambda} \} = -1, \quad \{ \chi_a, \bar{\chi}^b \} = i \{ \chi_a, \bar{\chi}^b \} = \delta^b_a, \]

\[ \{ \phi_a, \bar{\phi}^b \} = i \{ \phi_a \bar{\phi}^b \} = \delta^b_a, \]

where we choose the unity \( \hbar = c = 1 \). The first class constraints may be obtained from the action \((14)\), varying \( N(t), \psi(t), \bar{\psi}(t) \) and \( V(t) \) respectively. We obtain the following first-class constraints.

\[
H = -\frac{\kappa^2 \Pi_R^2}{2R} - \frac{kR}{2 \kappa^2} + \frac{2}{R^3} \Pi^a_z \Pi^a_z - \frac{9 \kappa^2}{2} R^3 |g(z)|^2 + \frac{R^3}{2} \frac{\partial g}{\partial z^a} \frac{\partial g}{\partial z^a} + \frac{\sqrt{k}}{2R} \bar{\lambda} \lambda \\
- \frac{3 \sqrt{2}}{2} i \kappa \frac{\Pi^a}{R^3} (\lambda \bar{\phi}^a + \bar{\lambda} \chi^a) - \frac{3 \sqrt{2}}{2} i \kappa \frac{\Pi^a}{R^3} (\bar{\lambda} \phi^a + \lambda \chi^a) - \frac{9 \kappa^2}{4R^4} (\bar{\phi}^a \phi^a + \bar{\chi}^a \chi^a) \bar{\lambda} \lambda \\
- \frac{3 \sqrt{2}}{2 R^2} (\bar{\phi}^a \phi^a + \bar{\chi}^a \chi^a) + \frac{9 \kappa^2}{2} |g| \bar{\lambda} \lambda - \frac{9 \kappa^2}{4R^3} \phi^a \bar{\phi}^a \chi^a \lambda + \frac{9 \kappa^2}{2} |g| (\bar{\phi}^a \phi^a + \bar{\chi}^a \chi^a) \quad (25) \\
- 2 \frac{\partial^2 |g|}{\partial z_a \partial z_b} \phi^a \chi^b + 2 \frac{\partial^2 |g|}{\partial z_a \partial z_b} \chi^a \phi^b - 3 \frac{\sqrt{2}}{2} \kappa \frac{\partial |g|}{\partial z_a} (\bar{\lambda} \phi^a + \bar{\chi}^a \lambda) \\
- \frac{3 \sqrt{2}}{2 \kappa} \frac{\partial |g|}{\partial z_a} (\bar{\lambda} \chi^a + \bar{\phi}^a \lambda) - 2 \frac{\partial^2 |g|}{\partial z_a \partial z_b} (\bar{\phi}^a \phi^b + \bar{\chi}^a \chi^b),
\]

\[
S = \left[ \frac{\kappa \Pi_R}{\sqrt{R}} - \frac{i \sqrt{2} \sqrt{R}}{\kappa} \frac{\partial |g|}{\partial z^a} \bar{\phi}^a \phi^a \right] + \left[ \frac{\sqrt{2}}{\sqrt{R^3}} \Pi^a_z + i \sqrt{2} \sqrt{R^3} \frac{\partial |g|}{\partial z_a} \right] \bar{\phi}^a \phi^a, \\
\bar{S} = \left[ \frac{\kappa \Pi_R}{\sqrt{R}} + \frac{i \sqrt{2} \sqrt{R}}{\kappa} \right] + \left[ \frac{\sqrt{2}}{\sqrt{R^3}} \Pi^a_z - i \sqrt{2} \sqrt{R^3} \frac{\partial |g|}{\partial z_a} \right] \bar{\phi}^a \phi^a, \quad (27)
\]

and

\[
\mathcal{F} = (-\bar{\lambda} \lambda + \bar{\phi}^a \phi^a + \bar{\chi}^a \chi^a). \quad (28)
\]

The constraints \((25, 28)\) follow from invariant action \((14)\) under the “small” local super-transformations \((5)\). The general hamiltonian is a sum of all the constraints, \(i.e.\)


\[ H_G = NH + \frac{i}{2} \bar{\psi} \bar{S} + \frac{i}{2} \bar{\psi} S + \mathcal{F}(\frac{1}{2} \nabla). \]  

(29)

In the quantum theory the first class constraints associated with the invariance of the action (14) become conditions on the wave function with the commutation rule (24), so that any physically allowed state must obey the following quantum constraints.

\[ H|\psi > = 0, \  S|\psi > 0, \  \bar{S}|\psi > = 0, \  \mathcal{F}|\psi > = 0, \]  

(30)

which are obtained when we change the classical dynamical variables by operators \( \Pi_R = -i \frac{\partial}{\partial R}, \Pi^a_Z = -i \frac{\partial}{\partial Z^a} \) and making the following matrix representation for fermionics variables \( \lambda, \bar{\lambda}, \chi, \bar{\chi}, \phi \) and \( \bar{\phi} \),

\[
\begin{align*}
\lambda &= -\sigma^- \otimes 1 \otimes 1, \quad \bar{\lambda} = \sigma^+ \otimes 1 \otimes 1, \\
\chi &= \sigma^3 \otimes \sigma^- \otimes 1, \quad \bar{\chi} = \sigma^3 \otimes \sigma^+ \otimes 1, \\
\phi &= \sigma^3 \otimes \sigma^3 \otimes \sigma^+, \quad \bar{\phi} = \sigma^3 \otimes \sigma^3 \otimes \sigma^-,
\end{align*}
\]  

(31)

where \( \sigma^\pm = \frac{\sigma_1 \pm i \sigma_2}{2} \) and \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) are the Pauli matrices. For the quantum generators \( H, S, \bar{S} \) and \( \mathcal{F} \) we obtain the following closed superalgebra

\[
\begin{align*}
\{S,\bar{S}\} &= 2H, & [S, H] &= 0, & [\mathcal{F}, S] &= -S, \\
S^2 &= \bar{S}^2 = 0, & [\bar{S}, H] &= 0, & [\mathcal{F}, \bar{S}] &= \bar{S},
\end{align*}
\]  

(32)

where \( H \) is the hamiltonian of the system, \( S \) is the single complex supersymmetric charge of the \( n = 2 \) supersymmetric quantum mechanics and \( \mathcal{F} \) is the fermion number operator.

**Conclusion**

Hence, on the base of the local “small” supersymmetry we considered the FRW cosmological model with the set of complex superfields and a superpotential \( g(Z^a) \). As effective supergravity theory coupled to “observable” sector with gauge group \( SU(3) \times SU(2) \times U(1) \) through a “hidden” sector [9] corresponding to four-dimensional superstrings, the next step
will be inclusion of Kähler function $K(Z^a, Z^b)$ to the scheme for chiral fields of observable supergravity sector, as well as, for dilaton-axion component of hidden sector of supergravity with their superpartners in “small” susy. We will also consider mechanism of spontaneous breaking of susy in the cosmological models and its influence on the Universe models.
Acknowledgments

We are grateful to J. Socorro, M.P. Ryan, O. Obregón and I.C. Lyanzuridi for their interest in this work. J.J. Rosales is also grateful for support by Universidad del Bajio, A.C., and CONACyT Graduate Fellowship. This work was also supported in part by CONACyT grant 3898P-E9607 and by ISF under grant U96000.
REFERENCES

[1] P. Van Nieuwenhuizen, Supergravity. Phys. Reports 68, (4), (1981). S. Weinberg, Phys. Rev. Lett. 50, 387, (1983); J. Ellis, A. D. Linde and D.V. Nanopoulos, Phys. Lett. B 118, 59, (1982).

[2] P.D. D’Eath, Supersymmetric Quantum Cosmology (Cambridge: Cambridge University Press, 1996); P.V. Moniz, Int. J. of Mod. Phys. A11, 4321, (1996).

[3] O. Obregón, J.J. Rosales and V.I. Tkach, Phys. Rev. D 53, 1750, (1986).

[4] V.I. Tkach, J.J. Rosales and O. Obregón, Class. Quant. Grav. 13, 2349, (1996).

[5] V.I. Tkach, O. Obregón and J.J. Rosales. To appear in Class. Quant. Grav.

[6] L. Brink, P. Divecchia and F.S. Howe, Nucl. Phys. B 118, 76, (1977); V.D. Gershun and V.I. Tkach, JETP Lett. 29, 320, (1979).

[7] D.P. Sorokin, V.I. Tkach and D.V. Volkov, Mod. Phys. Lett. A4, 901, (1989).

[8] D.V. Volkov, D.P. Sorokin and V.I. Tkach, Sov. J. Nucl. Phys. 49, 525, (1989); J. Kawalski-Glikman, J.W. Von Holten, S. Aoyama, and Lukierski, Phys. Lett. B201, 487, (1988).

[9] V.S. Kaplunovsky, And J. Louis, Phys. Lett. B, 306, 269 (1993).