Radiative dark matter and neutrino masses from an alternative \( U(1)_{B-L} \) gauge symmetry

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Abstract. We propose a model where the masses of the active neutrinos and a dark matter candidate are generated radiatively through the \( U(1)_{B-L} \) gauge symmetry breaking. It is realized by a non-universal \( U(1)_{B-L} \) charge assignment on the right handed neutrinos and one of them becomes DM. The dark matter mass becomes generally small compared with the typical mass of the Weak Interacting Massive Particles and we have milder constraints on the dark matter. We consider the case where the dark matter is produced through the freeze-in mechanism and show that the observed dark matter relic density can be realized consistently with the current experimental constraints on the neutrino masses and the lepton flavor structure.

Keywords: dark matter theory, neutrino theory

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1 Introduction

The nature of neutrinos and dark matter is one of the most compelling puzzles in particle physics and cosmology. While we have hints of the scale of the neutrino masses, the dark matter (DM) mass is yet an arbitrary parameter ranging from $10^{-22}$ eV to $10^{19}$ eV. Huge efforts have been made by experimentalists especially on the weakly interacting massive particles (WIMP), e.g., direct searches such as XENON1T [1] and LUX [2], indirect searches such as Fermi-LAT [3], AMS-02 [4], and CALET [5], and collider searches at LHC [6]. Even though WIMPs have not been excluded, it would be better to consider lighter DM scenarios where the mass and the charges of DM are realized naturally. Some of the famous light DM scenarios are the axion DM [7] and hot/warm DM [8]. In this paper, we consider the possibility that the dark matter is one of the right handed neutrinos and obtain a small mass only radiatively in association with the radiative generation of the active neutrino masses. Since the active neutrino masses and the DM mass are both small but non-zero, it would be natural that they have the same origin. In addition, the DM is automatically neutral under the SM gauge symmetry because it is one of the right handed neutrinos.

We consider the radiative mass generation mechanism [9] for both of DM and the neutrinos as in [10–12], but with a different assignment of $U(1)_{B-L}$ charges on the right handed neutrinos. From the anomaly free condition, the three right-handed neutrinos can have non-universal charges of $(-4, -4, 5)$, whose applications are found in refs. [13–24]. Since it is different from the ordinary universal $U(1)_{B-L}$ charge assignment, we call it as the alternative $U(1)_{B-L}$ symmetry. Thanks to this unique charge assignment, only two right-handed neutrinos with charge $-4$ can contribute to the masses of neutrinos, while the remaining one with charge $5$ can become a DM candidate. Interestingly, one of the neutrino masses is vanishing in a minimal setup. To generate the masses of the active neutrinos and DM radiatively, we introduce several bosons that have different charges of the $U(1)_{B-L}$ symmetry. We also check that our model is consistent with the lepton flavor violations (LFVs), and the current DM relic abundance.

This paper is organized as follows. In section 2, we review our model and formulate the neutrino masses, LFVs, DM mass, and DM relic abundance. Then, we show two benchmark points with DM mass of 1 MeV and 100 MeV. In section 3, we conclude.

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1 The characteristic difference between our model and the others is that the DM mass can naturally be light, since its mass is generated at the one-loop level.
Table 1. Field contents of fermions and their charge assignment under $\text{SU}(2)_L \times \text{U}(1)_Y \times \text{U}(1)_{B-L}$, where the lower indices $a (= 1 - 3)$ and $i (= 1, 2)$ are the number of flavors.

| Bosons | $H$ | $\eta$ | $\chi$ | $\chi'$ | $\phi$ | $\phi'$ | $\phi''$ |
|--------|-----|--------|--------|--------|-------|-------|-------|
| $\text{SU}(2)_L$ | 2   | 2      | 1      | 1      | 1     | 1     | 1     |
| $\text{U}(1)_Y$   | 1/2  | 1/2    | 0      | 0      | 0     | 0     | 0     |
| $\text{U}(1)_{B-L}$ | 0   | -3     | 3      | 13     | 2     | 8     | 6     |

Table 2. Field contents of bosons and their charge assignment under $\text{SU}(2)_L \times \text{U}(1)_Y \times \text{U}(1)_{B-L}$.

2 The model

In this section, we review our scenario. In the fermionic sector, we introduce one Dirac fermion $S$ with 8 $B - L$ charge, one Majorana fermion $X_R$ with 5 $B - L$ charge, which is assumed to be a DM candidate, and two Majorana fermions $N_R$ with $-4$ $B - L$ charges, which are the source for the generation of the active neutrino masses and their oscillations. The fermion field contents and their assignments are summarized in table 1. In the bosonic sector, we introduce an inert isospin doublet boson $\eta$ with $-3$ $B - L$ charge, two inert isospin singlet boson $\chi(\chi')$ with 3(13) $B - L$ charges, and $\phi(\phi', \phi'')$ with 2(8, 6) $B - L$ charges having nonzero vacuum expectation values (VEVs), denoted by $\langle \phi(\phi', \phi'') \rangle \equiv v_{\phi(\phi', \phi'')}/\sqrt{2}$. Here $H$ is supposed to be the SM-like Higgs. The boson field contents and their assignments are summarized in table 2.

The relevant Lagrangian for Yukawa sector and nontrivial scalar potential under these assignments are given by

$$- \mathcal{L}_Y = y_{\alpha a} \bar{L}_a H e_{R \alpha} + y_{\alpha a} \bar{L}_a \tilde{\eta} N_{R \alpha} + y_{N i} \bar{N}_{R i} N_{R i} \phi' + y_{\chi} \bar{X}_R S_R \chi^* + y_{\chi} \bar{X}_R S_L \chi^* + M_S \bar{S}_L S_R + \text{h.c.}$$

$$+ \mu_0 H^\dagger \eta \chi + \mu_1 \phi \phi' \phi'' + \mu_2 \phi' \phi'' \chi^2 + \text{h.c.}$$

(2.1)

$$+ \left( \lambda_0 \chi^2 \phi \phi' + \lambda_1 \chi^2 \phi \phi'' + \lambda_2 H^\dagger \chi^2 \phi'' + \lambda_3 \phi^2 \phi' \phi'' + \lambda_4 \chi^2 \phi' \phi'' + \text{h.c.} \right) + \lambda_5 |H|^2 \eta^2,$$

where each of the index $a (= 1 - 3)$ and $i (= 1, 2)$ represents the number of families, and the first term of $\mathcal{L}_Y$ generates the masses of the SM charged-lepton fermions. Here we assume all the parameters above are positive real for simplicity.

Higgs sector. Here we formulate the Higgs sector. First of all, we decompose the fields as follows:

$$H = \begin{bmatrix} \frac{w^+}{\sqrt{2}} \\ \frac{\eta}{\sqrt{2}} + \frac{i z}{\sqrt{2}} \end{bmatrix}, \quad \eta = \begin{bmatrix} \frac{\eta^+}{\sqrt{2}} \\ \frac{\eta h + i m}{\sqrt{2}} \end{bmatrix}, \quad \phi(\phi', \phi'') \equiv \frac{v_{\phi(\phi', \phi'')}}{\sqrt{2}},$$

(2.3)

$$\chi(\chi') \equiv \frac{\chi_R(\chi')}{\sqrt{2}},$$

-2-
where $z$ and $w^+$ are eaten by the SM vector boson $Z$ and $W^+$, respectively. We have one massless state after diagonalizing the mass matrix of $(z_{\varphi^*}, z_{\varphi^*}, z_{\varphi^*})$ and it is eaten by the $B-L$ neutral vector boson $Z'$. Then, each of the mass matrices are denoted as follows: $M_R^2$ for CP-even mass matrix in basis of $[h, \rho, \rho', \rho'']^T$, $M_F^2$ for CP-odd mass matrix in basis of $[\tilde{z}_{\varphi^*}, z_{\varphi^*}, z_{\varphi^*}]^T$, $M_R^2$ for inert CP-even mass matrix in basis of $[\eta_R, \chi_R, \chi_R]^T$, $M_F^2$ for inert CP-odd mass matrix in basis of $[\eta_I, \chi_I, \chi_I]^T$. They are diagonalized as $A_M^\dagger O_A^\dagger$ and $V_A M_F^2 V_A^\dagger$, where $A \equiv R, I$. We denote the mass eigenstates as $[h, \rho, \rho', \rho'']^T \equiv O_R^\dagger [h_1, h_2, h_3, h_4]^T$, $[z_{\varphi^*}, z_{\varphi^*}, z_{\varphi^*}]^T \equiv O_I^\dagger [a_1, a_2, a_3]^T$, $[\eta_R, \chi_R, \chi_R]^T \equiv V_R^\dagger [H_1, H_2, H_3]^T$, $[\eta_I, \chi_I, \chi_I]^T \equiv V_I^\dagger [A_1, A_2, A_3]^T$, where $h_{\text{SM}} \equiv h_1$ is the SM-like Higgs boson, and $a_1$ is eaten by the $Z'$ boson. In the following, we take into account the constraints on the oblique parameters and simply impose $m_{\eta^\pm} \in m_{\eta^I} \pm 120 \text{ GeV}$ (from $\Delta T$) and $500 \text{ GeV} \lesssim m_{\eta_R} \approx m_{\eta_I}$ (from $\Delta S$) [25].

**Active neutrinos.** Here we formulate the active neutrino sector. We start with the Lagrangian written in terms of mass eigenstate:

$$-\mathcal{L}_\nu = \frac{y_{\nu_{\alpha}}(V_R^\dagger)_{1\alpha}}{\sqrt{2}} \bar{\nu}_{L\alpha} N_R H_\alpha - i \frac{y_{\nu_{\alpha}}(V_I^\dagger)_{1\alpha}}{\sqrt{2}} \bar{\nu}_{L\alpha} N_R A_\alpha,$$

(2.4)

where $\alpha (= 1 - 3)$ should be summed up. Then the active neutrino mass matrix, $m_{\nu}$, is given at the one-loop level via three inert bosons, and its formula is given by

$$m_{\nu}(m_{\nu})_{ab} = -\frac{1}{2(4\pi)^2} \sum_{\alpha=1}^3 \sum_{i=1}^2 y_{\nu_{\alpha}} M_i y_{\eta_{ab}}^T \left( (V_R)^2_{1\alpha} F_i [H_\alpha, i] - (V_I)^2_{1\alpha} F_i [A_\alpha, i] \right),$$

(2.5)

$$F_i [a, i] = \frac{m_\alpha^2}{M_i^2} \ln \left( \frac{m_\alpha^2}{M_i^2} \right),$$

(2.6)

where $M_i \equiv y_{\nu_{\alpha}} \sqrt{2} y_{\eta_{\alpha}}$. Here $m_{\nu}$ is diagonalized by the neutrino mixing matrix, $V_{\text{MNS}}$, as $m_{\nu} = V_{\text{MNS}} D_{\nu} V_{\text{MNS}}^\dagger$ ($D_{\nu} \equiv V_{\text{MNS}} m_{\nu} V_{\text{MNS}}^\dagger$) with $D_{\nu} \equiv (m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$ and $m_{\nu_1} (m_{\nu_3}) = 0$ for normal(inverted) ordering. Then we can parameterize the Yukawa coupling in terms of an arbitrary complex $3 \times 2$ rotation matrix with $O^T O = 1_{2 \times 2}$, as follows

$$y_\eta = V_{\text{MNS}}^\dagger D_{\nu} O R^{-1/2},$$

(2.7)

$$R = -\frac{1}{2(4\pi)^2} \sum_{\alpha=1}^3 M_i \left( (V_R)^2_{1\alpha} F_i [H_\alpha, i] - (V_I)^2_{1\alpha} F_i [A_\alpha, i] \right),$$

(2.8)

where $m_{\nu} \equiv y_\eta R y_\eta^T$. And $O$ for the normal hierarchy (NH) and the inverted hierarchy (IH) are given by

$$\begin{bmatrix}
0 & 0 \\
\cos \theta & -\sin \theta \\
\pm \sin \theta & \pm \cos \theta
\end{bmatrix}, \quad \begin{bmatrix}
\cos \theta & -\sin \theta \\
\pm \sin \theta & \pm \cos \theta \\
0 & 0
\end{bmatrix},$$

(2.9)

respectively. Notice that $\theta$ can be complex. We assume the perturbative bound; $y_\eta \lesssim \sqrt{4\pi}$. This parameterization allows us to use the neutrino oscillation data as input parameters in our numerical analysis. We use the values in NuFIT 5.0 [26].
Lepton flavor violations (LFVs). IFV processes $\ell_a \to \ell_b \gamma$ arise at the one-loop level from the same Yukawa couplings used for the generation of the neutrino masses, and its formula is given by \cite{27, 28}

$$BR(\ell_a \to \ell_b \gamma) \approx \frac{\alpha_{em} C_{\alpha\beta}}{3(4\pi)^4 G_F^2} \left| \sum_{i=1}^{2} (y_\eta)_{ai}(y_\eta^*)_{ib} F_{lfv}(i, \eta) \right|^2,$$

where $B_{lfv} \equiv 2m^6_a + 3m^4_a m^2_b - 6m^2_a m^4_b + m^8_b + 12m^4_a m^2_b \ln \frac{m_a}{m_a}$,\textsuperscript{(2.10)}

where we define $\alpha_{em} \approx 1/137$ is the fine-structure constant, $G_F \approx 1.17 \times 10^{-5}$ GeV$^{-2}$ is the Fermi constant, and $C_{21} \approx 1$, $C_{31} \approx 0.1784$, $C_{32} \approx 0.1736$. The experimental upper bounds are found in \cite{29, 30}:

$$BR(\mu \to e\gamma) \lesssim 4.2 \times 10^{-13}, \quad BR(\tau \to e\gamma) \lesssim 3.3 \times 10^{-8}, \quad BR(\tau \to \mu\gamma) \lesssim 4.4 \times 10^{-8},$$

(2.12)

where we define $\ell_1 \equiv e$, $\ell_2 \equiv \mu$, and $\ell_3 \equiv \tau$.

We estimate the size of LFVs in our model. We assume the scalar masses are the same order ($\sim m_S$) and the fermion masses are same order ($\sim M_f$). In the case of $r \equiv \frac{m_a}{M_f} \geq 1$, the Yukawa couplings and the branching ratios are written as

$$y_\eta \sim \frac{D_\nu}{M_f F_I[r, 1]}.$$

(2.13)

$$BR(\ell_a \to \ell_b \gamma) \sim \frac{\alpha_{em} C_{\alpha\beta}}{3(4\pi)^4 G_F^2} \left| \frac{D_\nu}{m^2_a F_I[1, 1/r]} F_{lfv}(1, r) \right|^2 \sim 10^{-36} \left( \frac{1 \text{ TeV}}{M_f} \right)^6 \left( \frac{F_{lfv}(1, r)}{F_I[1, 1/r]} \right)^2,$$

(2.14)

where $|F_{lfv}(1, r)/F_I[1, 1/r]| \leq \frac{1}{2}$ for $r \geq 1$. In the case of $r < 1$, we use the following expression:

$$y_\eta \sim \frac{D_\nu}{m^2_a F_I[1, 1/r]}.$$

(2.15)

$$BR(\ell_a \to \ell_b \gamma) \sim \frac{\alpha_{em} C_{\alpha\beta}}{3(4\pi)^4 G_F^2} \left| \frac{r D_\nu}{m^2_a F_I[1, 1/r]} F_{lfv}(1, r, 1) \right|^2 \sim 10^{-36} \left( \frac{1 \text{ TeV}}{m_a} \right)^6 \left( \frac{r F_{lfv}(1, r, 1)}{F_I[1, 1/r]} \right)^2,$$

(2.16)

where $|r F_{lfv}(1, r, 1)/F_I[1, 1/r]| < \frac{1}{2}$ for $r < 1$. In both cases, the branching ratios are much smaller than the experimental bounds. We can obtain $O(1)$ Yukawa couplings using the complex phases in the complex orthogonal matrix and a cancellation in eq. (2.8). However, since these situations need a fine tuning or a hierarchical structure in the scalar mass matrices, we do not consider the situation in this paper.

The warm dark matter candidate. Here, we derive the DM mass $M_X$ at one-loop level. We first write the relevant Lagrangian in terms of mass eigenstate as follows:

$$-\mathcal{L}_X = \frac{y_{\chi}(V_R^T)^{3\alpha}}{\sqrt{2}} \bar{S}_R X_R^{\alpha} H_\alpha + \frac{y_{\chi}(V_R^T)^{2\alpha}}{\sqrt{2}} \bar{X}_R S_L H_\alpha$$

$$- i \frac{y_{\chi}(V_R^T)^{3\alpha}}{\sqrt{2}} \bar{S}_R X_R^{\alpha} A_{\alpha'} - i \frac{y_{\chi}(V_R^T)^{2\alpha}}{\sqrt{2}} \bar{X}_R S_L A_{\alpha'},$$

(2.17)
where $\alpha, \alpha' (= 1 - 3)$ should be summed up. Then the DM mass is given by
\begin{equation}
M_X = -\frac{y_X M_S y_{X'}}{(4\pi)^2} \sum_{a=1}^{3} \left[(V_R^T)_{3a}(V_R^T)_{2a}F_I[S,H_a] - (V_I^T)_{3a}(V_I^T)_{2a}F_I[S,A_a]\right].
\end{equation}

**Relic density.** We consider the case where the DM is $X$ and is generated via the freeze-in mechanism. In order to simplify our discussion, we consider a reheating temperature that is much lower than the $W/Z$ masses and the scalar masses but much higher than the $Z'$ mass so that the main production processes are through the $Z'$ boson.\(^2\) Notice that the $Z'$ mass becomes small due to the small gauge coupling required to realize the correct DM abundance. We assume that the initial abundance of $Z'$ and that of $X$ are initially zero and that $Z'$ and $X$ are in kinetic equilibrium with the SM particles. We calculate their current abundance by solving the Boltzmann equations given below.

Since we consider a rather light DM, which can also be produced after the QCD phase transition, we use different Boltzmann equations above and below the transition temperature, $T_{\text{QCD}}$. For $T > T_{\text{QCD}}$, they are given by\(^3\)
\begin{align}
\frac{dY_{Z'}}{dx} &= c(x) \left[\sum_f C(\ell \bar{\ell} \to Z') + C(X X \to Z')\right], \quad (2.19) \\
\frac{dY_X}{dx} &= 2c(x) \left[ C(Z' \to XX) + \sum_f C(\ell \bar{\ell} \to Z^* \to XX)\right], \quad (2.20)
\end{align}

where $Y_{Z'}$ and $Y_X$ are the yields of $Z'$ and $X$, respectively, and the sum is taken over all the SM leptons and quarks. The collision terms, $C$'s, are given in appendix A. We have defined
\begin{equation}
x = \frac{m_X}{T}, \quad (2.21)
\end{equation}
\begin{equation}
c(x) = x^4 \frac{135\sqrt{18}}{2\pi^4} \frac{m_{Pl}^2}{m_X^3} \frac{1}{g_{s s}^{1/2}} \left(1 + \frac{1}{3} \frac{d\ln g_{s s}}{dt}\right). \quad (2.22)
\end{equation}

Here, $g_s$ is the temperature dependence of the effective degrees of freedom for the energy density and $g_{s s}$ is those for the entropy. Their evolution is taken from [32].

For $T < T_{\text{QCD}}$, eq. (2.20) is modified as
\begin{equation}
\frac{dY_X}{dx} = 2c(x) \left[ C(Z' \to XX) + \sum_\ell C(\ell \bar{\ell} \to Z^* \to XX)\right]. \quad (2.23)
\end{equation}

Here, the sum of $\ell$ is taken over the SM leptons. For eq. (2.19), we modify if $m_{Z'} \lesssim 1$ GeV is satisfied since otherwise we cannot rely on the effective theory of hadrons. We use
\begin{equation}
\frac{dY_{Z'}}{dx} = c(x) \left[ \sum_\ell C(\ell \bar{\ell} \to Z') + C(XX \to Z') + C(\pi^0 \gamma \to Z') + C(\pi^0 \pi^+ \pi^- \to Z')\right]. \quad (2.24)
\end{equation}

We ignore $\pi^0 \gamma \to XX$ and $\pi^0 \pi^+ \pi^- \to XX$ processes via the off-shell $Z'$ since the pions disappear soon after the QCD phase transition and will not affect the abundance significantly. The other hadronic decay channels are known to be smaller than the above two processes [33].

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\(^2\) We have neglected the kinetic mixings between $U(1)_Y$ and $U(1)_{B-L}$ for simplicity, although they exist among gauged $U(1)$ symmetries in general.

\(^3\) Similar analysis has been done by, e.g., ref. [31].
We solve the Boltzmann equation from \( x = M_X/T_R \) with \( T_R \) being the reheating temperature, to a sufficiently large \( x = x_\infty \). The dark matter relic density should satisfy

\[
\Omega_X h^2 = \frac{Y_X(x_\infty)s_0 M_X}{3M_{Pl}^2 H_{100}^2} = 0.1193 \pm 0.0018, \tag{2.25}
\]

where \( s_0 \) is the current entropy density, \( H_{100} = 100 \text{ km/s/Mpc} \).

Since the gauge coupling and the \( Z' \) mass are independent of the constraints discussed in the previous sections,\(^4\) we can always tune them to obtain the correct relic abundance. Thus, we only show two distinct parameter sets that give \( \Omega h^2 \simeq 0.12 \).

In figure 1, we plot the evolution of the yields of the DM and the mediator. In the left panel, we take \( M_X = 100 \text{ MeV}, \ m_{Z'} = 5 \text{ GeV}, \ g' = 2.3 \times 10^{-12}, \ T_R = 50 \text{ GeV} \).

With this parameter set, the mediator can decay into DM and the gauge coupling is required to be very small. The large part of the DM relic density is coming from the decay of the mediator at \( 1 \lesssim x \lesssim 100 \).

In the right panel, we take \( M_X = 3 \text{ GeV}, \ m_{Z'} = 5 \text{ GeV}, \ g' = 1 \times 10^{-6}, \ T_R = 50 \text{ GeV} \).

With this parameter set, DM is generated only through off-shell mediator and thus we need a rather large gauge coupling. Since the mediator cannot decay into a DM pair, the large yield of \( Z' \) does not affect the DM relic density.

A light \( Z' \) boson with a tiny gauge coupling can potentially be searched at the beam dump experiments\(^{35–39} \) or by the cooling of SN1987A\(^{40, 41} \). Currently, they constrain \( Z' \) mass is constrained roughly below 1 GeV; \( 10^{-8} \lesssim g' \lesssim (\text{GeV}/M_{Z'}) \times 10^{-7} \) is excluded by the beam dump experiments and \( 10^{-10} \lesssim g' \lesssim 10^{-7} \) is excluded by the SN1987A. Thus, the above parameter sets evade these constraints. The coming SHiP experiment\(^{42} \) can search for the \( Z' \) lighter than about a few tens of GeV, but will only constrain \( 10^{-9} \lesssim g' \lesssim 10^{-7} \) for the GeV mass regime. If we consider \( Z' \) boson lighter than 1 GeV, a part of the available parameter space will be covered at the SHiP experiment, but the detailed analysis is beyond the scope of this paper.

3 Conclusions

We have proposed a model where the active neutrino masses are generated radiatively and one of the right handed neutrinos becomes DM. We have naturally realized the tiny mass of DM as well as the neutrino mass matrix in the successful framework of the alternative gauged \( U(1)_{B-L} \) symmetry. We have shown that the LFV constraints are typically weak for this model. We have also investigated the relic abundance of DM through the freeze-in mechanism via the \( Z' \) gauge boson mediation. We have shown two successful benchmark points that realize the correct relic density. For simplicity, we have assumed all the scalar fields are heavy and respect the constraints from the oblique parameters. However, it may not be the case and some deviations from the SM may be detected in future.

\(^{4}\)There are many particles with large \( B-L \) charges, however, the \( B-L \) gauge coupling is very small and the \( B-L \) breaking scale is very large in the region of our interest. Therefore, the gauge coupling does not become non-perturbative below the Planck scale.
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A Collision terms

The collision terms for $1 \leftrightarrow 2$ processes are given by

$$
C(f \bar{f} \rightarrow Z'') = \frac{N_f Q_f^2 g^2 m_{Z'}^2 T}{8\pi^3} \left(1 - \frac{Y_{Z'}^2}{Y_{Z'}^{\text{eq}}}\right) \left(1 + \frac{2 m_f^2}{m_{Z'}^2}\right) \beta_f(m_{Z'}^2)K_1\left(\frac{m_{Z'}}{T}\right),
$$

(A.1)

and

$$
C(Z' \rightarrow XX) = -C(XX \rightarrow Z') = \frac{Q_X^2 Q_f^2 m_Z^2 T}{16\pi^3} \left(\frac{Y_{Z'}^{\text{eq}} - Y_X^2}{Y_{Z'}^{\text{eq}} Y_X^2}\right) \beta_X^2(m_{Z'}^2)K_1\left(\frac{m_{Z'}^2}{T}\right),
$$

(A.2)

where $N_f$ is 3 for the quarks, 1 for the charged leptons, and 1/2 for the neutrinos. Here,

$$\beta_f(s) = \Re\left[\sqrt{1 - \frac{4 m_f^2}{s}}\right], \quad \beta_X(s) = \Re\left[\sqrt{1 - \frac{4 m_X^2}{s}}\right].$$

(A.3)

For $f \bar{f} \rightarrow XX$ process, we need to take care of the double counting. The sum of the on-shell and the off-shell $Z'$ contributions is calculated as

$$
C(f \bar{f} \rightarrow XX) = \frac{N_f Q_X^2 Q_f^2 g^4 T}{192\pi^5}\left(1 - \frac{Y_X^2}{Y_X^{\text{eq}}^2}\right) \int ds s^{\frac{s}{2}} \beta_X^2(s) \beta_f(s) \left(1 + \frac{2 m_f^2}{s}\right) K_1\left(\sqrt{s}/T\right).
$$

(A.4)
Since the on-shell part is already taken into account by the Boltzmann equation for \( Z' \), we eliminate it as
\[
\mathcal{C}(f \bar{f} \rightarrow Z' \rightarrow XX) = \mathcal{C}(f \bar{f} \rightarrow XX) - \mathcal{C}(f \bar{f} \rightarrow Z') \text{Br}(Z' \rightarrow XX) \left( 1 - \frac{Y_{\chi}^2}{Y_{\chi}^{eq}} \right) \left( 1 - \frac{Y_{Z'}^{eq}}{Y_{Z'}^2} \right)^{-1}.
\]  
(A.5)

Here, the decay width of the \( Z' \) boson is given by
\[
\Gamma_{Z'} = \sum_f \Gamma(Z' \rightarrow f \bar{f}) + \Gamma(Z' \rightarrow XX),
\]  
(A.6)

with
\[
\Gamma(Z' \rightarrow f \bar{f}) = \frac{N_f Q_f^2 g^2 m_{Z'}}{12 \pi} \left( 1 + \frac{2 m_f^2}{m_{Z'}^2} \right) \beta_f(m_{Z'}^2),
\]  
(A.7)

\[
\Gamma(Z' \rightarrow XX) = \frac{Q_{\chi}^2 g^2 m_{Z'}}{24 \pi} \beta_{\chi}(m_{Z'}^2).
\]  
(A.8)

The collision terms involving pions are [33]
\[
\mathcal{C}(\pi^0 \gamma \rightarrow Z') = \frac{\alpha_{EM} g^2 m_{Z'}^5}{256 \pi^6 f_{\pi}^2} \left( 1 - \frac{Y_{Z'}}{Y_{Z'}^{eq}} \right) \left( 1 - \frac{m_{Z'}^2}{m_{Z'}^2} \right)^3 K_1 \left( \frac{\sqrt{s}}{T} \right),
\]  
(A.9)

\[
\mathcal{C}(\pi^0 \pi^+ \pi^- \rightarrow Z') = \frac{g_{\rho \pi \pi} g_{B} m_{Z'}^3}{512 \pi^9 f_{\pi}^2} I(m_{Z'}) \left( 1 - \frac{Y_{Z'}}{Y_{Z'}^{eq}} \right) K_1 \left( \frac{\sqrt{s}}{T} \right).
\]  
(A.10)

Here, \( g_{\rho \pi \pi}^2 / (4 \pi) \simeq 3 \) and \( I(m_{Z'}) \) is the phase space integral given in [33]. We consider them only when they are kinematically allowed. Similarly, for the decay width of \( Z' \), we have [33]
\[
\Gamma_{Z'} = \sum_\ell \Gamma(Z' \rightarrow \ell \bar{\ell}) + \Gamma(Z' \rightarrow XX) + \Gamma(Z' \rightarrow \pi^0 \gamma) + \Gamma(Z' \rightarrow \pi^+ \pi^- \pi^0),
\]  
(A.11)

with
\[
\Gamma(Z' \rightarrow \pi^0 \gamma) = \frac{g^2 \alpha_{EM} m_{Z'}^3}{384 \pi^4 f_{\pi}^2} \left( 1 - \frac{m_{Z'}^2}{m_{Z'}^2} \right)^3,
\]  
(A.12)

\[
\Gamma(Z' \rightarrow \pi^+ \pi^- \pi^0) = \frac{g_{\rho \pi \pi}^2 g^2 m_{Z'}}{768 \pi^7 f_{\pi}^2} I(m_{Z'}).
\]  
(A.13)

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