NUMERICAL SIMULATIONS OF CONVERSION TO ALFVÉN WAVES IN SUNSPOTS

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ABSTRACT

We study the conversion of fast magnetoacoustic waves to Alfvén waves by means of 2.5D numerical simulations in a sunspot-like magnetic configuration. A fast, essentially acoustic, wave of a given frequency and wave number is generated below the surface and propagates upward through the Alfvén/acoustic equipartition layer where it splits into upgoing slow (acoustic) and fast (magnetic) waves. The fast wave quickly reflects off the steep Alfvén speed gradient, but around and above this reflection height it partially converts to Alfvén waves, depending on the local relative inclinations of the background magnetic field and the wavevector. To measure the efficiency of this conversion to Alfvén waves we calculate acoustic and magnetic energy fluxes. The particular amplitude and phase relations between the magnetic field and velocity oscillations help us to demonstrate that the waves produced are indeed Alfvén waves. We find that the conversion to Alfvén waves is particularly important for strongly inclined fields like those existing in sunspot penumbrae. Equally important is the magnetic field orientation with respect to the vertical plane of wave propagation, which we refer to as “field azimuth.” For a field azimuth less than 90° the generated Alfvén waves continue upward, but above 90° downgoing Alfvén waves are preferentially produced. This yields negative Alfvén energy flux for azimuths between 90° and 180°. Alfvén energy fluxes may be comparable to or exceed acoustic fluxes, depending upon geometry, though computational exigencies limit their magnitude in our simulations.

Key words: methods – numerical – Sun: oscillations – sunspots

Online-only material: color figures

1. INTRODUCTION

Observations using the Solar Optical Telescope (SOT) on board Hinode (De Pontieu et al. 2007) and the Coronal Multi-Channel Polarimeter (CoMP; Tomczyk et al. 2007) unambiguously reveal ubiquitous Alfvénic oscillations in the solar chromosphere and corona, with implications both for the Sun’s atmosphere and the solar wind. Tomczyk et al. find that coronal Alfvénic power is broadly spread in frequency, but with a distinct peak around 3–4 mHz, characteristic of the Sun’s internal p-mode wave field. Whether these transverse oscillations are strictly Alfvén waves or instead kink waves (Van Doorsselaere et al. 2008) depends on the magnetic and density structuring of the atmosphere and may vary with height as cross-field structuring becomes more or less important. We might expect though that sunspot atmospheres, with their presumed large-scale magnetic structures, present the most ideal site for more-or-less pure Alfvén waves to propagate.

The generation of Alfvén waves at the photosphere and their propagation through the various layers of the solar atmosphere has been extensively modeled (e.g., Cranmer & van Ballegoijen 2005). However, Cally & Hansen (2011) recently suggested that Alfvén waves must also be produced by mode conversion from fast magnetoacoustic waves beyond their reflection height in the low chromosphere. This coupling only occurs if wave propagation is not coplanar with gravity and magnetic field, and so the problem is necessarily three dimensional (3D).

Conversion from fast-mode high-β magnetoacoustic waves (manifesting as p modes in the subphotosphere) to slow-mode waves in solar active regions is relatively well studied both analytically and numerically (e.g., Zhugzhda & Dzhalilov 1982; Cally & Bogdan 1997; Cally 2001; Schunker & Cally 2006; Cally 2006, 2007; Hansen & Cally 2009; Khomenko et al. 2009; Felipe et al. 2010a); see Khomenko (2009) for a review. In a two-dimensional (2D) situation, the transformation from fast to slow magnetoacoustic modes is demonstrated to be particularly strong for a narrow range of magnetic field inclinations around 20°–30° to the vertical. For this reason, and because of the reduction in acoustic cutoff frequency afforded by strong inclined magnetic fields, magnetic field concentrations on the solar surface may truly be called magnetoacoustic portals (Jefferies et al. 2006), coupling the Sun’s interior oscillations to those of its atmosphere.

The remainder of the wave-energy flux, however, which is near-total away from these preferred inclinations, enters the low-β atmosphere as fast, predominantly magnetic waves. Due to the steep Alfvén speed gradient with height, these fast waves soon reflect back downward at the height $z_{\text{ref}}$ at which $\omega \approx v_A k_h$, where $\omega$ is the frequency, $v_A$ is the Alfvén speed, and $k_h$ is the horizontal wavenumber.

Fast-to-Alfvén conversion occurs around and above this fast wave reflection height $z_{\text{ref}}$ in 3D (see Figure 1), localized more closely to $z_{\text{ref}}$ as frequency increases (Cally & Hansen 2011). However, at the 3–5 mHz frequencies characteristic of p-modes, the process is typically spread over much of the chromosphere. Studies of fast-to-Alfvén conversion were initiated by Cally & Goossens (2008), who found that it is most efficient for preferred field inclinations from vertical between 30° and 40°, and azimuth angles (the angle between the vertical magnetic and wave propagation planes) between 60° and 80°, and Alfvénic fluxes transmitted to the upper atmosphere can exceed acoustic fluxes in some cases. Newington & Cally (2010) studied the conversion properties of low-frequency (∼1–2 mHz) gravity waves, showing that even larger magnetic field inclinations
can support gravity wave to Alfvén conversion and resulting Alfvénic wave propagation to the upper atmosphere. In nonuniform magnetic field, the relevant angles for mode conversion, either fast-to-slow or fast-to-Alfvén, are those pertaining in the respective conversion regions. Figure 1 summarizes the overall picture of conversion between the fast, slow, and Alfvén waves.

Motivated by these recent studies, we attack the problem by means of 2.5D numerical simulations. The purpose of our analysis is to calculate the efficiency of the conversion from fast-mode high-$\beta$ magnetoacoustic waves to Alfvén and slow waves in the upper atmosphere in a spreading sunspot-like magnetic field configuration. Our initial results on the conversion to Alfvén waves in simple field configurations were reported in Khomenko & Cally (2011). In the present work we extend our simulations to a more realistic case of a sunspot atmospheric models spanning the shallow subphotosphere (−5 Mm) to the high chromosphere (+1.9 Mm).

Above this layer, the transition region (TR) acts as a partial Alfvén wave reflector, affecting their distribution in the corona and solar wind and producing Alfvén turbulence via nonlinear coupling (Cramer & van Ballegooijen 2005), but this is beyond the scope of our present modeling.

2. SIMULATION SETUP

We numerically solve the nonlinear equations of ideal MHD using our code Hæncha (Khomenko & Collados 2006; Khomenko et al. 2008; Felipe et al. 2010a). The code solves nonlinear equations for perturbations, the equilibrium state being explicitly removed from the equations. In this study, for simplicity, and coherence with previous studies (see Cally & Hansen 2011), we use a 2.5D approximation. This approximation means that we allow all vectors in three spatial directions, but the derivatives are taken only in two (one vertical and one horizontal) directions, so that our perturbations are only allowed to propagate in the $X$–$Z$ plane. In addition, the initial perturbation is kept small to approximate the linear regime.

As a background magneto-static model atmosphere we use one sunspot-like model from Khomenko & Collados (2008). For simplicity, this model is azimuthally symmetric, with no twist of the magnetic field lines. While twist is often seen in sunspot magnetic fields, it is not a necessary feature of the processes we wish to explore here. It will be added along with other physical features in later studies. The computational region is magnetized in all its volume (distributed currents), but the strongest magnetic field is concentrated around the axis of the structure. The dimensions of the simulated domain are 78 Mm in the horizontal $X$ direction and 7.4 Mm in the vertical $Z$ direction. The axis of the sunspot is placed at the middle of the domain at $X = 39$ Mm. The bottom boundary of the domain is located at $−5$ Mm below the photospheric level, $Z = 0$. This zero level is taken to be the height where the optical depth at 500 nm is equal to unity in the quiet-Sun atmosphere 39 Mm away from the sunspot axis. The thermodynamic variables of the atmosphere at 39 Mm from the axis are taken from Model S of Christensen-Dalsgaard et al. (1996) in the deep sub-photosphere layers and continuing according to the VAL-C model (Vernazza et al. 1981) in the photospheric and chromospheric layers. The sunspot axis in the atmospheric layers is given by the semi-empirical model of Avrett (1981). The magnetic field at the axis is about 900 G at $Z = 0$ Mm. This is quite moderate for a sunspot, but is adopted for numerical reasons. The Alfvén speed at the top
of our computational domain becomes quite excessive if the field strength is too high, thereby necessitating impractically small time steps. Taking larger field strength would mostly result in a different scaling of the problem, without new physical phenomena being introduced (see, e.g., Khomenko et al. 2009).

As our modeling is 2.5D, we have made cuts through the sunspot model at several Y positions. Here we will describe three simulations situated in vertical planes at Y = 7.5, 11.25, and 15 Mm from the sunspot axis. A cut through the axis is uninteresting, as it is strictly 2D and can produce no Alfvén waves. Figure 2 depicts variations of some characteristic parameters of the Y = 7.5 Mm cut through the sunspot model.

We drive waves by an imposed perturbation in a few grid points near the bottom boundary of the domain at Z = −5 Mm. The perturbation is calculated analytically as an acoustic-gravity wave of a given frequency and wavenumber, neglecting the magnetic field (dynamically unimportant at this depth), and neglecting the temperature gradient. We introduce self-consistent perturbations of the velocity vector, pressure, and density according to Mihalas & Mihalas (1984):

\[ \delta V_z = V_0 \exp \left( \frac{z}{2H} + k_{zr} z \right) \sin(\omega t - k_{zr} z - k_{xr} x) \]

\[ \frac{\delta P}{P_0} = V_0 |P| \exp \left( \frac{z}{2H} + k_{zr} z \right) \sin(\omega t - k_{zr} z - k_{xr} x + \phi_P) \]

\[ \frac{\delta \rho}{\rho_0} = V_0 |R| \exp \left( \frac{z}{2H} + k_{zr} z \right) \sin(\omega t - k_{zr} z - k_{xr} x + \phi_R) \]

\[ \delta V_x = V_0 |U| \exp \left( \frac{z}{2H} + k_{zr} z \right) \sin(\omega t - k_{zr} z - k_{xr} x + \phi_U) , \]

where the amplitudes and relative phase shifts between the perturbations are given by

\[ |P| = \frac{\gamma \omega}{\omega^2 - c_s^2 k_{zr}^2} \left( k_{zr} + \frac{1}{2H} \right)^2 (\gamma - 2) \]

\[ |R| = \frac{\omega}{\omega^2 - c_s^2 k_{zr}^2} \left( k_{zr} + \frac{\gamma - 1}{\gamma H k_{zr}} \right) \left( k_{zr} + \frac{1}{2H} \right)^2 \]

\[ |U| = \frac{k_{zr} c_s^2}{\gamma \omega} |P| \]

\[ \phi_P = \phi_U = \arctan \left( \frac{k_{zr}}{k_{zr} + \frac{1}{2H} \left( \gamma - 2 \right)} \right) \]

\[ \phi_R = \arctan \left( \frac{k_{zr} + \frac{\gamma - 1}{\gamma H k_{zr}} \left( k_{zr} + \frac{1}{2H} \right)}{1 - \frac{1}{2H k_{zr}^2}} \right) . \]

Given the wave frequency ω, and the horizontal wavenumber k_x, the vertical wavenumber k_z is found from the dispersion relation for acoustic-gravity waves in an isothermal atmosphere:

\[ k_z = k_{zr} + i k_{xi} = \sqrt{\left( \omega^2 - \omega_c^2 \right) / \left( c_s^2 - c_A^2 \left( \omega^2 - \omega_c^2 \right) / \omega^2 \right)} , \]

where \( \omega_c = \gamma g / 2c_s \) is the acoustic cutoff frequency and \( \omega_A = 2\omega_c \sqrt{\gamma - 1} / \gamma \).

In all simulations described in this work we set the perturbation frequency \( \nu = \omega/2\pi = 5 \text{ mHz} \) and horizontal wave number \( k_x = 1.37 \text{ Mm}^{-1} \). According to our sunspot model (see Figure 2), the driving frequency is just slightly below the maximum cutoff frequency reached at the temperature minimum. Over most of the spot these values result in the fast wave reflection level \( z_{\text{ref}} \) being higher than the Alfvén/acoustic equipartition surface \( z_{\text{eq}} \), i.e., \( \omega_c / k_x < c_s \) automatically satisfied for a propagating acoustic wave \( \omega^2 = c_s^2 \left( k_x^2 + k_z^2 \right) \). This is important, since it allows upcoming acoustic (fast) waves in \( v_A < c_s \) to convert to magnetic (fast) waves in \( v_A > c_s \) before reflecting at \( z_{\text{eq}} \).

To separate the Alfvén mode from the fast and slow magnetoacoustic modes in the magnetically dominated atmosphere \( \nu_A > c_s \) we use velocity projections onto three characteristic directions:

\[ \hat{e}_{\text{long}} = \{ \cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta \} ; \]

\[ \hat{e}_{\text{perp}} = \{- \cos \phi \sin^2 \theta \sin \phi, 1 - \sin^2 \theta \sin^2 \phi, \}

\[ - \cos \theta \sin \theta \sin \phi \} ; \]

\[ \hat{e}_{\text{trans}} = \{- \cos \theta, 0, \cos \phi \sin \theta \} . \]

These projections were shown to be rather efficient in separating the perturbations corresponding to all three modes both for idealized magnetic field configurations (Khomenko & Cally...
and vice versa. Note that projecting the velocities according to larger inclination of the ridges means lower propagation speeds.

The velocities are taken at \( \theta \) the magnetic field inclination propagating parallel to the field; the second one (\( \hat{F}_{\perp} \)) selects the Alfven wave, according to the asymptotic polarization direction derived by Cally & Goossens (2008); the remaining orthogonal direction (\( \hat{F}_{\text{trans}} \)) selects the fast magnetacoustic wave. In the rest of the paper we will address these projections as “acoustic,” “Alfven,” and “fast.”

To measure the efficiency of conversion to Alfvén waves near and above the \( c_{S} = v_{A} \) equipartition layer, we calculate the time-averaged acoustic and magnetic energy fluxes as far as possible from the conversion layer in the magnetically dominated atmosphere (\( v_{A} \gg c_{S} \)):

\[
\hat{F}_{\text{ac}} = \langle \delta P \delta V \rangle, \\
\hat{F}_{\text{mag}} = \langle \delta \vec{B} \times (\delta \vec{V} \times \vec{B}_{0}) \rangle / \mu_{0}.
\]

The positive sign of the fluxes means energy propagating upward.

We also calculate a measure of the time-averaged energy contained in all three wave modes according to

\[
E_{\text{long}} = \rho_{0} c_{S} \langle \delta V_{\text{long}}^{2} \rangle, \\
E_{\text{perp}} = \rho_{0} c_{A} \langle \delta V_{\text{perp}}^{2} \rangle, \\
E_{\text{trans}} = \rho_{0} c_{A} \langle \delta V_{\text{trans}}^{2} \rangle,
\]

where in each case we use the corresponding velocity projections into characteristic directions (Equation (8)). For pure acoustic and Alfvén waves, where there is strict equipartition between kinetic and compressional or kinetic and magnetic energies, respectively, these would indeed be the true energies. Be that as it may, these forms are convenient for purposes of exposition and shall be used here.

3. VELOCITY PROJECTIONS

Figure 3 shows an example of the projected velocity components in our calculations as a function of height and time. The velocities are taken at \( X = 10.5 \) Mm, where the distinction between the three wave modes in the magnetically dominated atmosphere is clearly visible. In this representation the larger inclination of the ridges means lower propagation speeds and vice versa. Note that projecting the velocities according to Equation (8) allows us to separate the wave modes only in the magnetically dominated atmosphere, above the horizontal solid line in Figure 3. The figure shows how the incident fast-mode wave propagates to the equipartition layer \( c_{S} = v_{A} \) gradually changing its speed. After reaching the equipartition layer at about 5 minutes into the simulation, it splits into several components. The essentially magnetic low-\( \beta \) fast mode is produced above \( Z = 0.2 \) Mm (middle panel). This mode is reflected back down a few minutes after it has been produced. The reflection height, calculated as the height where the wave frequency \( \omega \) and wave horizontal wave number \( k_{h} \) are related by \( \omega = v_{A} k_{h} \) (ignoring the sound speed contribution to the fast wave speed), is well reproduced in the simulations, as the velocity variations associated with the fast mode decay rapidly above its reflection layer.

Unlike the purely 2D case, there is an Alfvén mode produced above the \( c_{S} = v_{A} \) level (right panel). This mode has a clearly distinct propagation speed compared to the acoustic mode (left panel). The ridges of the Alfvén mode are nearly vertical since the Alfven speed at these heights is large (above 100 km s\(^{-1}\)).

The low-\( \beta \) essentially acoustic slow mode escapes to the upper atmosphere tunneling through the cutoff layer due to the field inclination of \( \theta = 30^\circ \) that effectively reduces the cutoff frequency by the factor \( \cos \theta \).

The velocity amplitudes in Figure 3 are scaled with a factor of \( \sqrt{\rho_{0} v_{\text{ph}}} \), where \( v_{\text{ph}} = c_{S} \) for the \( V_{\text{long}} \) component and \( v_{\text{ph}} = v_{A} \) for the other two components. Although \( v_{A} \gg c_{S} \) above the solid line in Figure 3, the scaled amplitudes of the Alfvén mode are still smaller than of the slow acoustic mode at the selected \( X \) location.

Figure 3 also shows that the simulation enters into a stationary stage after about 10–15 minutes. A snapshot of the wave field developed in the stationary stage of the simulations is given in Figure 4. The movie of this simulation is available electronically.

The behavior of the three wave modes changes across the sunspot radial direction, as expected due to the change of the atmospheric properties. The magnetic field is more inclined at the periphery of the sunspot (much more so than is suggested by the figures, since the vertical scale has been stretched relative to the horizontal scale) and the azimuth varies from \( \phi = 0^\circ \) to \( 180^\circ \) from the right- to the left-hand side of the sunspot. The cutoff frequency reaches its maximum of 5.02 mHz at \( X = 0 \), \( Z = 0 \) Mm, decreasing steeply above this due to the rise of temperature in the chromosphere (Figure 2).

This steep temperature increase produces a partial reflection of the slow acoustic mode in the magnetically dominated atmosphere. This reflection is visible as an interference pattern formed above the solid line in the \( V_{\text{long}} \) projection in the left part of the sunspot (upper panel). This additional reflection also produces stronger slow magnetic modes below the \( c_{S} = v_{A} \) cutoff frequency.
Figure 4. Snapshots of the three orthogonal components of the velocity taken at about 19 minutes after the start of the simulation at $Y = 7.5 \, \text{Mm}$ from the sunspot axis. Upper panel: $V_{\text{long}}$; middle panel: $V_{\text{trans}}$; bottom panel: $V_{\text{perp}}$. The black–white (blue–orange colors in the online journal) mean negative–positive velocity directions; the range of the gray (color) coding is the same in the three panels. The velocities are scaled with a factor of $\sqrt{\rho_0 c_S}$ (upper panel) and $\sqrt{\rho_0 v_A}$ (two lower panels). The horizontal solid line is the level where $c_S = v_A$; the horizontal dashed line is the fast-mode reflection level. Magnetic field lines are inclined black lines. The axes are not to scale. Note the presence of the Alfvén mode in $V_{\text{perp}}$ above $c_S = v_A$, where the fast mode ($V_{\text{trans}}$) is already reflected. Since the velocity polarizations "long," "trans," and "perp" are based on $v_A \gg c_S$ asymptotics, care must be taken not to overinterpret these figures well below the equipartition level. The apparent “discontinuity” seen in $V_{\text{perp}}$ near $Z = -1 \, \text{Mm}$ (seen also in Figure 3) is a node. (A color version of this figure is available in the online journal.)

Figure 5. Wave energies calculated according to Equation (10) at the top of the atmosphere (averages at heights from 0.4 Mm above the $c_S = v_A$ layer up to $z = 1.9 \, \text{Mm}$) for the three projected velocity components in turn, selecting respectively acoustic, fast, and Alfvén waves. The units of the gray (color in the online journal) coding are $10^6 \, \text{erg cm}^{-2} \, \text{s}^{-1}$. The three strips in each panel are the results of the three simulation runs at $Y = 7.5$ (lower), 11.25 (middle), and 15 Mm (upper) from the sunspot axis. (A color version of this figure is available in the online journal.)

4. ENERGY OF THREE WAVE MODES

We proceed by measuring the energies contained in each of the projected velocity components at the upper part of our simulation domain, above the Alfvén-acoustic equipartition layer $c_S = v_A$. As the properties of the atmosphere change in the horizontal direction across the sunspot, the height of the $c_S = v_A$ layer changes as well (see Figure 2). To be consistent, we take time-averaged energies at heights from 400 km above the $c_S = v_A$ layer up to the upper boundary of our simulation box in the stationary stage of the simulations.
Figure 5 illustrates the energies as a function of inclination and azimuth of the sunspot magnetic field lines at the corresponding horizontal locations. The orientation of the magnetic field is taken at heights where \( c_s = v_A \) in each of the simulations at three \( Y \) locations. The three strips in each panel of Figure 5 are for the three simulation runs. The format of the figure allows comparison with the previous studies, Cally & Goossens’s (2008) Figure 2 and Khomenko & Cally’s (2011) Figure 4.

This figure demonstrates that the maximum energy of slow acoustic waves is transmitted for inclinations around 30° (left panel). The dependence on the inclination is stronger than the dependence on the azimuth. This result is consistent with previous 2D theoretical models of fast-to-slow mode transformation in the homogeneous inclined magnetic field (Crouch & Cally 2003; Cally 2006; Schunker & Cally 2006). It is also consistent with the 3D analysis by Cally & Goossens (2008). The explanation of this effect is offered by the ray perspective. The fast-mode high-\( \beta \) waves launched from the sub-photospheric layers (with an angle about 90°, i.e., their lower turning point) reach the Alfvén-acoustic equipartition layer with an angle close to 20°–30° (Schunker & Cally 2006). Thus, for the moderately inclined magnetic field the attack angle between the wavevector and the magnetic field lines is small and the transformation is efficient. Note also that the frequency of waves is just at the cutoff frequency of the sunspot atmosphere, so due to tunneling effects the slow mode acoustic energy is also present over the wide range of inclinations.

The maximum energy of the Alfvén wave is present at inclinations about 35° and azimuth between 70° and 100° (right panel of Figure 5). The maximum power increases from the outer to the inner strip indicating a tendency toward larger transmission at larger inclinations. Thus, the behavior of waves in sunspot models is similar to the one found previously in the models with homogeneous field (Cally & Goossens 2008; Khomenko & Cally 2011; Cally & Hansen 2011). The transmission of Alfvén waves is more efficient for inclined fields, which is important in the sunspot penumbra.

Some transmitted energy is also present in the \( E_{\text{trans}} \) component (middle panel of Figure 5) near the sunspot periphery, though we believe this to be an artifact of our computational box not being tall enough to allow complete fast wave reflection there.

5. ALFVÉN MODE POLARIZATION RELATIONS

To check if projecting the velocities in the directions given by Equation (8) gives an efficient way to separate the Alfvén mode from the fast and slow magnetoacoustic modes, we make use of the polarization relations for the Alfvén wave. In a classical Alfvén wave variations of the magnetic field and velocity should be related as (see Priest 1984)

\[
\delta \vec{V} = \mp \frac{\delta \vec{B}}{\sqrt{\mu_0 \rho_0}},
\]

(11)

where the upper sign is for Alfvén waves propagating parallel to the magnetic field and the lower sign is for those propagating in the opposite direction. The kinetic and magnetic energies for an Alfvén wave should be in equipartition, so the ratio \( R = \delta B / (\mu_0 \rho_0) / \delta V \) should be equal to one.

To confirm the Alfvén nature of the transformed waves, we checked the amplitude and phase relations for all three modes reaching the upper atmosphere. Figure 6 presents calculations of the amplitude ratio \( R \) for the three modes. In each of the cases \( \delta B \) and \( \delta V \) pairs are the projections in the corresponding characteristic direction for each mode (Equation (8)). The ratio is taken at the same heights as the energies from Figure 5 and is averaged in time in the stationary stage of the simulations. The ratio turns out to be different for the three modes. In the case of the slow acoustic mode (left panel), the magnetic field variations associated with the velocity variations are very small (ratio \( R \) about \( 10^{-2} \)). In contrast, in the case of the fast magnetic mode, magnetic field amplitudes are relatively strong, providing the ratio about \( 10^{2} \) (middle panel). For the Alfvén mode the ratio stays around \( 10^{0} \) (right panel), meaning that the velocity and magnetic field oscillations are in equipartition as expected for an Alfvén wave.

Figure 7 shows the phase shift between the \( \delta B_{\text{perp}} \) and \( \delta V_{\text{perp}} \) corresponding to the Alfvén mode. The situation here is very different from what one might naively expect. It is also different to the case of the simple atmosphere and homogeneous magnetic field considered by us previously in Khomenko & Cally (2011). In the right part of the sunspot, for the azimuth between 0° and 90° the Alfvén waves mostly behave as they should. The phase shift is about 180°, meaning an upward wave propagation. Nevertheless, in the left part of the sunspot (\( \phi \) between 90° and 180°) the value of the phase shift indicates a downward propagation. Taking into account that the amplitude ratio (Figure 6) gives clear evidence for the Alfvénic nature of the waves, we conclude that for \( \psi \) between 90° and 180°, downward-propagating Alfvén waves are generated by the mode transformation.

This accords perfectly with the uniform field modeling results of Cally & Hansen (2011; see their Figures 4 and 5) indicating that between 0° and 90° the upward propagating fast waves couple most efficiently to upward Alfvén waves. Since an alignment is needed between the direction of the wave propagation and magnetic field for efficient coupling, where the azimuth is between 90° and 180°, the strongest coupling happens between the downward propagating fast waves (after...
their reflection in the magnetically dominated atmosphere) and the Alfvén waves. In this case downward propagating Alfvén waves are produced. We believe that the same happens in the simulations, producing downward propagating Alfvén waves in the left part of the sunspot for $\psi$ above $90^\circ$.

All in these calculations confirm the Alfvén nature of waves selected by the $\hat{e}_{\text{perp}}$ projection in our simulations, including returning the predicted result regarding the coupling between refracted fast waves and downward propagating Alfvén waves in regions where the magnetic alignment is favorable.

6. ENERGY FLUXES

We proceed by calculating the acoustic and magnetic fluxes according to Equation (9). Figure 8 gives the time averages of the vertical magnetic and acoustic fluxes over the upper part of the simulation domain. In the case of the magnetic fluxes we only show them above the Alfvén-acoustic equipartition layer where they are produced and where the separation between the modes according to Equation (8) is meaningful.

All in these calculations confirm the Alfvén nature of waves selected by the $\hat{e}_{\text{perp}}$ projection in our simulations, including returning the predicted result regarding the coupling between refracted fast waves and downward propagating Alfvén waves in regions where the magnetic alignment is favorable.
of the sunspot. This appears to be in agreement with the visual impression from Figure 4 (upper panel) where the interference pattern of the slow acoustic modes is observed to the left of the axis. We believe that this partial reflection is due to strong temperature increase around the axis in the upper chromospheric part of the simulation domain. We have performed additional simulations placing the upper boundary of the domain at lower heights, below the chromospheric temperature increase. In this case we did not observe downgoing slow acoustic waves. This confirms that the reflection is a physical effect and not a numerical artifact from the upper boundary condition of the simulations.

The magnetic flux above the Alfvén-acoustic equipartition layer (middle panel of Figure 8) is mostly positive. Note that the absolute value of the magnetic flux is at all locations lower than the acoustic flux. The exception is the peripheral regions of the sunspot model where relatively large magnetic flux is observed. However, this artificially large flux is due to insufficient height of the simulation box above the equipartition layer, making it impossible to complete the reflection for the fast magnetic modes.

The magnetic flux in the middle panel of Figure 8 is due to a mixture of the fast and Alfvén modes. To separate the flux of the Alfvén mode we calculated the magnetic flux from Equation (9) using the projections $\delta B_{\text{perp}}$ and $\delta V_{\text{perp}}$. The result is given in the bottom panel of Figure 8. In agreement with previous considerations we observe that there is no significant flux in the peripheral part of the sunspot, confirming its origin due to incomplete reflection of the fast magnetic mode. As was already clear from the calculation of the phase relations (Figure 7) the Alfvén flux in the right part of the sunspot ($\psi$ between 0° and 90°) is positive and the flux in the left part of the sunspot ($\psi$ between 90° and 180°) is negative, indicating downward propagating Alfvén waves. These downward propagating waves are generated due to the coupling between the downgoing fast magnetic mode (after its reflection) and the Alfvén mode.

The Alfvén flux only represents a minor fraction of the acoustic flux in Figure 8. However, as explained at the end of Section 3, we believe this to be an artifact of the truncation of the broad Alfvén conversion region. To overcome this limitation, we performed an experiment with five times larger frequency ($\nu = 25$ mHz) and horizontal wavenumber ($k_x = 6.85$ Mm$^{-1}$) allowing us to significantly compact the conversion region. This high-frequency experiment is illustrated in Figure 9, where indeed the upward Alfvénic flux in the right half of the region now greatly exceeds the acoustic flux, and there is significant but very compact downward Alfvén flux in the left half. Thus, the conclusion is that Alfvén flux can actually exceed acoustic flux. Besides, Alfvén waves are more able to propagate to great heights than are acoustic or fast waves, which are limited, respectively, by shock formation and by reflection. This suggests that they do indeed represent a significant product of wave propagation and conversion in sunspots.

7. CONCLUSIONS

In this paper we have investigated the efficiency of conversion from fast to Alfvén waves by means of 2.5D numerical simulations in a complex sunspot-like magnetic field configuration. It is important to realize that quantitatively simulating mode transformation numerically is a challenge, as any numerical inaccuracies are amplified in such second-order quantities as wave energy fluxes. The tests presented in this paper prove the robustness of our numerical procedure and offer an effective way to separate the Alfvén from magnetoacoustic modes in numerical simulations.

In general, the conclusions from the previous models in simplified uniform magnetic field configurations (Cally & Goossens 2008; Khomenko & Cally 2011; Cally & Hansen 2011) apply also to our more complex sunspot-like magnetic field configuration, though there are some apparent differences. We note three points in particular.

First, the maximum of the magnetic energy of Alfvén waves transmitted to the upper atmosphere is shifted toward more inclined fields compared to the homogeneous field case.
Second, the amount of magnetic energy due to Alfvén waves transmitted to the upper atmosphere is about 10 times lower than for the acoustic waves. This differs from the conclusions reached by Cally & Goossens (2008) who find that at some inclination and azimuth angles the magnetic flux is larger. This discrepancy is entirely to be expected and is an artifact of the limitations of our present simulations. As seen in Figure 10(b) of Cally & Hansen (2011), the fast-to-Alfvén conversion region for frequencies comparable to our 5 mHz is spread more or less uniformly over some 20 scale heights from \( z_{\text{ref}} \) upward. Clearly, we do not have the luxury of such abundant space in our simulation box, and so only a small fraction of the total potential Alfvén flux is produced. In theory, this could be remedied in any of several ways.

1. The box could be made substantially taller. However, this would involve encompassing yet larger Alfvén speeds, especially if a TR is included, with unfortunate consequences for our numerical time step and therefore for the practical feasibility of the calculation.

2. The magnetic field could be increased in magnitude. This is an attractive course of action in any case, as the 900 G at \( Z = 0 \) adopted here on the axis is very conservative, being a factor of three less than might be expected in a mature large sunspot. This would lower both the \( c_s = v_A \) equipartition surface and the \( \omega = v_A k_x \) fast reflection level opening up more space above for mode conversion. Of course, the maximum Alfvén speed would again increase, with implications for the numerical time step.

3. Higher horizontal wavenumber waves could be modeled, increased by say a factor of five. As seen in Cally & Hansen (2011), Figure 10(b), conversion of waves with \( \kappa \approx k_x H \gtrsim 1 \) (where \( H \) is the density scale height) occurs in a considerably more compact region near \( z_{\text{ref}} \).

Finally, where the reflecting fast waves meet similarly inclined magnetic field in the conversion region as they propagate upward, they preferentially transfer their energy to upgoing Alfvén waves. However, if their upward path crosses field lines at large angles there is little transfer. But then this correspondence may be achieved on their way back down, in which case downward propagating Alfvén waves are the beneficiaries of their largess. This more efficient coupling to downward waves happens for azimuth angles above 90° and was predicted by Cally & Hansen (2011) based on their cold plasma (\( \beta = 0 \)) analysis.

In our future studies we will extend this analysis to fully 3D simulations and include both magnetic field twist and a chromosphere-corona TR. Of course twist contributes to the field orientation in the conversion regions, and therefore to the strength of mode conversion, both fast-to-slow and fast-to-Alfvén, but we do not anticipate significant novel effects beyond this. On the other hand, a TR is expected to be highly reflective to Alfvén waves, and therefore to greatly reduce coronal Alfvén fluxes from chromospheric mode conversion. Results in a zero-\( \beta \) model (Hansen & Cally 2011) confirm this, though they indicate that substantial coronal Alfvén fluxes are still possible if the fast wave reflection point is within a few chromospheric scale heights of the TR or indeed if the fast wave fails to reflect before it reaches the TR. It will be interesting to test this in realistic sunspot models.

Ultimately, our simulations will inform and be judged against observations of MHD waves in solar magnetic structures. The coronal observations of De Pontieu et al. (2007) and Tomczyk et al. (2007) are useful indicators of power reaching that height, though they are at the limits of our technology and subject to line-of-sight integration ambiguities. Wave observations at photospheric and chromospheric levels may be more reliable and precise. In particular, correlations between velocity and magnetic perturbation phases are crucial in disentangling the various MHD modes in the low atmosphere (Norton et al. 2001; Khomenko et al. 2003; Settele et al. 2002; Fujimura & Tsuneta 2009), though this is a very difficult task even in full 3D simulations.

Significant progress has recently been made in the identification of wave modes observed in sunspots and other solar magnetic features. On the one hand, simultaneous photospheric and chromospheric spectropolarimetric observations (such as those done with the TIP instrument on Tenerife, Canary Islands; Collados et al. 2007) introduce the possibility of “following” wave propagation with height (Centeno et al. 2009; Felipe et al. 2010b) and obtaining phase shifts between oscillations of magnetic field and velocity (Bellot Rubio et al. 2000). On the other hand, bidimensional spectrometers (such as, e.g., the IBIS instrument; Cavallini et al. 2000 or the Dutch Open Telescope on La Palma, Canary Islands) allow us to obtain high-resolution 2D velocity fields at different heights. Such simultaneous observations are crucial for our understanding of the physics of wave propagation in active regions. In sunspot umbrae, where the magnetic field is predominantly vertical, slow field-aligned magnetoacoustic waves are firmly detected and even reproduced in simulations including their particular observed wave pattern (Centeno et al. 2006; Felipe et al. 2011). In regions with an inclined magnetic field (sunspot penumbra and network canopies), a mixture of upward and downward propagating waves is often discovered (Braun & Lindsey 2000; Vecchio et al. 2007; Kontogiannis et al. 2010). At the photospheric level, these up- and downgoing waves are, possibly, fast magnetoacoustic waves undergoing a reflection process. In fact, from measured phase shifts between velocity and magnetic field oscillations, Khomenko et al. (2003) found that the contribution of slow mode waves is larger in umbral regions and the contribution of fast-mode waves becomes progressively more important toward the umbra–penumbra boundary. Alfvén waves are usually detected higher up in the corona (De Pontieu et al. 2007; Tomczyk et al. 2007). The simulations presented here suggest that the complete decoupling between the fast and Alfvén waves happens in the upper chromosphere or even above. Since the upcoming acoustic waves on the Sun are uncorrelated, their attack angle with respect to the sunspot magnetic field can be arbitrary, so a mixture of up- and downgoing Alfvén waves will be produced, mostly above regions with inclined magnetic field. Time variations of the chromospheric magnetic fields, together with velocity oscillations, are strongly desirable for the firm detection of Alfvén waves. However, measuring magnetic fields in the chromosphere may be challenging (see, e.g., Manso Sainz & Trujillo Bueno 2010) and actual measurements are scarce. As of today, very few measurements exist.
in active and quiet chromospheric regions (e.g., Socas-Navarro 2005, 2007; Trujillo Bueno et al. 2005; Centeno et al. 2010; Štepin & Trujillo Bueno 2010). But, as far as we are aware, no time variations have been reported. Further instrumental efforts, developments of chromospheric diagnostic techniques, as well as improved modeling, will be needed to constrain the results of the presented study.

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