CVC IN PARTICLE PHYSICS

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We review the hypothesis of the conserved vector current (CVC) within the Standard Model. In addition to the classic tests, such as pion beta decay and neutrino scattering, we mention recent tests involving LEP data. As well as providing a clear indication that the isovector current is not conserved, rho-omega mixing offers a fascinating opportunity to study CP violation at B-factories and we outline these ideas. Finally, we briefly touch on a new approach to mass generation in the Standard Model which, for example, leads to the up-down mass difference which breaks CVC.

1 Introduction

The CVC hypothesis arose at the time when only charged current weak interactions were known. The Hamiltonian for semi-leptonic, weak interactions was written as

\[ H_{CC}^{SL} = \frac{G}{\sqrt{2}} \left[ J_+^\lambda L^\lambda + J_-^\lambda L^\lambda \right], \]

with \( J_+^\lambda \) the charge changing hadronic current and \( L^\lambda \) the leptonic current. The hypothesis had two parts: firstly, if we break the hadronic weak current into vector and axial pieces, \( J_\lambda^\pm = V_\lambda^\pm - A_\lambda^\pm \), that the vector pieces \( V_\lambda^\pm / \cos \theta_C \) and the isovector piece of the electromagnetic current were the three components, \( j_{i\lambda} \), of a vector in isospace; secondly, that all 3 components of this current were conserved, \( \partial_\lambda j_{i\lambda} = 0 \). Viewed in hindsight, with the full Standard Model at hand, it is hard to appreciate the power and insight that it represented.

For the purposes of this brief review we begin with the Standard Model. The first part of the CVC hypothesis is then trivial because all interactions involving strongly interacting systems are built from the same vector (and axial vector) quark currents. On the other hand, the second part of the hypothesis is incorrect because the QCD Hamiltonian contains a piece proportional to \((m_u - m_d)(\bar{u}u - \bar{d}d)\). The fact that \( m_u \neq m_d \) implies that \( \partial_\lambda j_{i\lambda} \propto (m_u - m_d) \neq 0 \) and thus CVC can only be, at best, a good approximation. Nevertheless, it has proven such a successful approximation
that it is built into all phenomenological treatments. In the next section we review two of the classic applications of CVC as well as an important illustration of the fact that $m_u$ is not exactly equal to $m_d$.

## 2 Classic Tests

The most famous testing grounds for CVC are $\beta$-decay of the free neutron, bound nucleons and the pion. For the nucleon the most accurate measurements by far are made in finite nuclei and these are traditionally not considered part of particle physics. Therefore, we begin with the case of pion $\beta$-decay, which still gives the most precise, particle physics test. We then recall the potential importance of neutrino scattering where, as we shall see, the errors are still very large. To complete the section, we briefly review the most spectacular example of the effect of having $m_u \neq m_d$, namely $\rho - \omega$ mixing. This phenomenon is also very important in modern tests of CVC (e.g., at LEP) as well as for determinations of $CP$-violation at $B$-factories – as we shall discuss in the next section.

### 2.1 Pion Beta Decay

The decay

$$\pi^- \rightarrow \pi^0 + e^- + \bar{\nu}_e,$$

is severely depressed by the small mass difference between the charged and neutral pions. Only the vector part of the hadronic weak current can play a role in this decay. The CVC hypothesis then naturally relates the isovector, vector matrix element to the electromagnetic form factor of the pion ($F_{\pi}$):

$$< \pi^0(k')|J^\lambda|\pi^-(k) > = -\cos\theta_C \sqrt{2} \, < \pi^-(k')|j^\lambda_{C^-}\pi^-|\pi^-(k) > = -\cos\theta_C \sqrt{2} F_{\pi}(q^2)(k + k')^\lambda, \quad (3)$$

with $q = k' - k$.

The very small mass difference involved, $\Delta \sim 4$ MeV, means that the difference between $F_{\pi}(0) = 1$ and $F_{\pi}(q^2)$ is negligible. Hence the $\beta$-decay lifetime can be written in terms of essentially kinematic quantities:

$$\tau^{-1} = G^2 \cos\theta_C^2 \frac{\Delta}{30\pi^3} \left[1 - \frac{\Delta}{2m}\right] \Delta^5 F(1 + \delta_{\pi}). \quad (4)$$

The phase space factor, $\bar{F}$, is near one and the (electromagnetic) radiative correction ($\delta_{\pi}$) is of order $1\%$. This leads to a theoretical branching ratio $1.0482 \pm 0.0048 \times 10^{-8}$.

The most recent experiment to accurately determine this ratio was carried out at LAMPF more than 10 years ago. McFarlane et al. obtained $1224 \pm 36$ good events.
for $\pi^+\beta$-decay after observing more than $10^{11}$ pions. Their final branching ratio, $1.026 \pm 0.039 \times 10^{-8}$, dominates the current world average $(1.025 \pm 0.034 \times 10^{-8})$. Until this time the best measurement was from Depommier et al. in 1968. Clearly the theoretical and experimental values are completely consistent at the current experimental limit of about 3%.

2.2 Neutrino Nucleon Scattering

Neutrino nucleon scattering is the primary source of information on the weak axial vector current, notably $G_A(q^2)$. However, in order to extract information on the axial vector current it is more or less compulsory to assume that the vector matrix elements can be taken from electromagnetic interactions using CVC. Because of the difficulties of dealing with a neutrino beam it is preferable to leave a charged nucleon in the final state. Thus one is led to a deuteron target because of the lack of a free neutron target. However, by using the resolving power of a bubble chamber to tag a low momentum, spectator proton

$$\nu_\mu + d \rightarrow \mu^- + p + p_{\text{spectator}},$$

one can reduce the experimental uncertainties.

The data is not sufficiently accurate to determine the detailed shape of $G_A(q^2)$, which is usually parametrized as a dipole by analogy with the vector form factors. It is possible to relax the CVC constraint on the vector form factors and search simultaneously for the best-fit dipole masses $M_A$ and $M_V$. The values obtained in the early 80’s by Baker et al. ($M_V = 0.86 \pm 0.07, M_A = 1.04 \pm 0.14$) and Miller et al. ($M_V = 0.96 \pm 0.04, M_A = 0.80 \pm 0.10$) were consistent with CVC in that $M_V$ measured in electron scattering is 0.84 GeV. As there is now very good agreement on $M_A$, at about 1.02 GeV, the latter value is probably nothing to worry about, even though the apparent discrepancy in $M_V$ is $2 \frac{1}{2}$ standard deviations. The latest result from the Brookhaven group (Kitagaki et al.) is reassuring, yielding ($M_V = 0.89 + 0.04 - 0.07$ and $M_A = 0.97 + 0.14 - 0.11$).

The same data can also be used to search for evidence for the second-class vector current. The most recent limit comes from a measurement of the reaction $\bar{\nu}_\mu p \rightarrow \mu^+ n$ at Brookhaven. Unfortunately the sensitivity to the scalar form factor is reduced by a factor $(m_\mu/M_N)^2$. If it is parametrized as

$$F_S(q^2) = \rho \frac{F_V(0)}{1 - \frac{q^2}{M_S^2}},$$

then the limit on $\rho$ is $\rho < 1.8$, with $M_S = 1$ GeV and the axial tensor term set to zero. One can actually set a better limit on the axial tensor, second class current but that is not our concern.
2.3 Neutrino Deep Inelastic Scattering

Although it is not derived from CVC alone, the Adler sum-rule provides a fundamental test of the quark currents within the Standard Model. It relies on the equal-time commutation relation

\[ \delta(x_0) \left[ J_0^-(x), J_0^+ (0) \right] = -4 \delta^{(4)} (x) \left[ V^3_0 + A^3_0 \right] . \]  

(7)

Taking the matrix element of this relation between hadronic states and averaging over spin (so that \( < A^3_0 > = 0 \)) we find

\[ \int_0^\infty d\nu \left[ W_2^\nu (Q^2, \nu) - W_2^\nu (Q^2, \nu) \right] = -4 < I_3 > , \]  

(8)

and finally, changing the integration variable to Bjorken x, and using the fact that \( \nu W_2^\nu = F_2^\nu (x, Q^2) \) scales, we find

\[ \int_0^1 dx \frac{F_2^{\nu n} - F_2^{\nu p}}{2x} = 1. \]  

(9)

Note that this sum-rule is protected against \( 0(\alpha_s) \) corrections. Even now, the best experimental test of this fundamental sum-rule comes from the long extinct BEBC facility at CERN, with the result 1.01 \( \pm 0.08 \) (stat.) \( \pm 0.18 \) (syst.) 11. While the data is clearly consistent with expectations, an error of more than 20% is really unacceptable in such a fundamental quantity.

2.4 Rho-Omega Mixing

The most spectacular evidence that the vector current is not conserved comes from electromagnetic interactions. In particular, the data for \( e^+ e^- \rightarrow \pi^+ \pi^- \), which is dominated by the isovector \( \rho \)-meson, shows a sharp interference pattern near the mass of the isoscalar \( \omega \)-meson. The natural explanation of this is that, because \( m_u \neq m_d \), the isospin pure \( \rho \) and \( \omega \) mesons are not eigenstates of the full QCD Hamiltonian. (N.B. One must, of course, include the mixing induced by coupling to the photon, namely \( \rho \rightarrow \gamma \rightarrow \omega \), but this is only 10% of the observed amplitude. Also, the \( \rho \) and \( \omega \) are resonances and therefore not eigenstates of any Hamiltonian, but one can give the statement some rigour within models, such as the cloudy bag 12, by turning off the coupling to decay channels.)

Suppose we make the standard simplification 3, which is that the direct decay of the isospin pure \( \omega \) to two pions cancels the imaginary piece of the two pion loop contribution to the mixing self-energy. This means that we can neglect the pure isospin state, \( \omega_f \), coupling to two pions (\( M_{\omega_f \rightarrow \pi \pi} = 0 \)) with the understanding that it is the real part of the mixing amplitude that is being extracted. To lowest order in
the mixing amplitude, the amplitude for the virtual $\gamma$ to decay to two pions can be written:

$$M_{\gamma \rightarrow \pi \pi} = M_{\rho \rightarrow \pi \pi} \frac{1}{s_\rho} M_{\gamma \rightarrow \rho I} + M_{\rho I \rightarrow \pi \pi} \frac{1}{s_\rho} \frac{1}{s_\omega} M_{\gamma \rightarrow \omega I},$$

(10)

where $1/s_V$ is the vector meson propagator.

The couplings that enter this expression, through $M_{\rho \rightarrow \pi \pi}$, $M_{\gamma \rightarrow \rho I}$ and $M_{\gamma \rightarrow \omega I}$, involve the pure isospin states $\rho_I$ and $\omega_I$. However, we can re-express it in terms of the physical states by first diagonalising the vector meson propagator. This leads to the result

$$M_{\gamma \rightarrow \pi \pi} = M_{\rho \rightarrow \pi \pi} \frac{1}{s_\rho} M_{\gamma \rightarrow \rho} + M_{\omega \rightarrow \pi \pi} \frac{1}{s_\omega} M_{\gamma \rightarrow \omega}$$

$$= M_{\rho \rightarrow \pi \pi} \frac{1}{s_\rho} M_{\gamma \rightarrow \rho} + M_{\rho \rightarrow \pi \pi} \frac{\Pi_{\rho \omega}}{s_\rho - s_\omega} \frac{1}{s_\omega} M_{\gamma \rightarrow \omega},$$

(11)

which is the form usually seen in older works. A recent analysis of the world data gave a value for the mixing amplitude of $\Pi_{\rho \omega} = -3800 \pm 370$ MeV$^2$.

3 Recent Tests and Applications

3.1 Testing CVC in $\tau^- $ Decay

All modern $e^+e^-$ colliders with sufficient energy (including LEP) have studied the decay

$$\tau^- \rightarrow \pi^- + \pi^0 + \nu_\tau.$$  

(12)

At present the world average for the branching ratio into this channel is $25.24\pm0.16\%$. Only the hadronic vector current is involved and according to CVC this will be only the $I=1$ piece of the vector current. Thus, unlike $e^+e^- \rightarrow \pi^+\pi^-$, there will be no $\rho - \omega$ interference.

The procedure is therefore to fit the $e^+e^- \rightarrow \pi^+\pi^-$ data, including $\rho - \omega$ mixing as well as the $\rho'(1450)$ and $\rho''(1700)$, and then use the purely $I=1$ piece of the amplitude to calculate the decay rate. The most recent estimate of the theoretical branching ratio comes from the analysis of Sobie, namely $24.6 \pm 1.4\%$. This differs from experiment by about $2\frac{1}{2}\%$ with a mainly theoretical error of $\sim 5\%$. Once again CVC works but the level of accuracy is relatively modest. In order to improve the accuracy one would need to resolve the differences between the existing data sets for $e^+e^- \rightarrow \pi^+\pi^-$. 

3.2 An Application to CP Violation

The fact that CVC is not exact, and in particular the mixing of $\rho$ and $\omega$, can be put to use in quite a spectacular fashion in the study of CP violation. We shall briefly report on the recent analysis of Enomoto and Tanabashi, following a suggestion of Lipkin. Rather than the traditional proposals to study $(B^0, \bar{B}^0)$ decays, these authors aim to use $\rho-\omega$ interference to generate a large CP violation signal in charged $B$ decays.

One can show that the CP-violating difference in the decay rates $B^- \rightarrow \rho^0 \rho^-$ and $B^+ \rightarrow \rho^0 \rho^+$ is proportional to $\cos(\delta + \phi) - \cos(\delta - \phi)$, where $\phi$ is the CP violation phase and $\delta$ the known, strong phase arising from $\rho-\omega$ mixing. Notice that the signal vanishes if $\delta = 0$. Although the branching ratios for the decay modes $B^- \rightarrow \rho^0 h^-$ (with $h = K, K^*, \rho$, etc.) is very small ($\sim 10^{-8}$), the asymmetry can be as large as 90% ! This is clearly a very important suggestion to pursue further.

In concluding this brief summary we note one question which needs urgent study. At recent analysis of $\rho-\omega$ mixing by Maltman et al. has suggested that, contrary to earlier conclusions, the direct coupling of the $I = 0$ $\omega$ to two pions leads to a large uncertainty in $\Pi_{\rho\omega}$ and in the relative phase. In view of the need to know the strong phase $\delta$ very well, in order to extract $\phi$, this is a worrying conclusion. We simply note that this ambiguity vanishes identically at $q^2 = \hat{m}_\rho^2$, but that the residual ambiguity needs careful study.

4 Beyond the Standard Model

The Standard Model has proven very successful in every area of particle physics, including recent high-energy collider experiments. However, it has three features which are not well understood: the origin of mass, the three fermion generations and the phenomenon of CP violation. The question of mass is usually framed in terms of (fundamental) Higgs fields and why the corresponding Yukawa couplings take particular values. Instead, one might ask whether a formulation of the Standard Model with massless fermions makes sense. For example, it is well known that QED with massless electrons is not well defined at the quantum level.

In a recent paper, Bass and Thomas considered the pure Standard Model with gauge symmetry $SU(3) \otimes SU(2)_L \otimes U(1)$ and no additional interaction – i.e., with no grand unification. They examined the physical theory corresponding to the bare Standard Model Lagrangian with no elementary Higgs and just one generation of massless fermions and gauge bosons. At asymptotic scales, where the $U(1)$ coupling is significantly greater than the asymptotically free $SU(3)$ and $SU(2)_L$ couplings, the left and right handed states of any given charged fermion couple to the $U(1)$ gauge boson with different charges. At the Landau scale of this non-asymptotically free theory, it was suggested that there should be three separate phase transitions – corresponding
to each of the right-right, right-left and left-left interactions becoming supercritical. These transitions correspond to three generations of fermions. As one passes through each transition from a higher scale (shorter distance) the corresponding scalar condensate “melts”, releasing a dynamical fermion into the Dirac phase studied in the laboratory. In this picture the three generations emerge as quasi-particle states built on a “fundamental fermion” interacting self-consistently with the condensates.

Clearly this proposal differs in a fundamental manner from the conventional approaches to the Standard Model. While the conceptual framework is extremely simple and elegant, the techniques for dealing with non-perturbative physics at the Landau scale are not well developed. In particular, at the present stage it has not yet been possible to present a rigorous, quantitative derivation of all of the features of the Standard Model. Nevertheless, we believe that the potential for understanding so many phenomena, including mass, CP-violation and the generations, not to mention CVC, is so compelling that the ideas merit further study.

5 Conclusion

We have seen clearly that CVC is a very natural approximation within the Standard Model. Certainly the isovector, vector current (modulo the CKM matrix) is exactly the vector current involved in the charged current weak interactions. The hypothesis that the current is conserved is an approximation only because $m_u \neq m_d$ — and, of course, because of the electromagnetic interaction.

Within particle physics the best test of CVC is pion $\beta$-decay, with the present limit being about 3%. Using the decay of the heavier (1.777 GeV) $\tau^-$ to $\pi^0\pi^-\nu_{\tau}$ one can set a limit that is only slightly worse, around 5%. Tests involving neutrino-nucleon scattering are quite imprecise, with the fundamental Adler sum-rule (which, of course, tests current algebra not just CVC) being in desperate need of accurate data.

We also saw that $\rho - \omega$ mixing, which is a spectacular example of the non-conservation of the vector current, provides a very beautiful alternative way to study CP violation in $B$-decays. Finally, we briefly reviewed a rather ambitious framework for understanding the origin of many phenomenological features of the Standard Model, including the three generations and their masses. Such an approach may someday provide us with a real understanding of the origins of CVC.

6 Acknowledgements

I would like to thank a number of colleagues for helpful discussions, especially H. O’Connell and A.G. Williams concerning $\rho - \omega$ mixing, S.V. Gardner concerning the CP violation proposal and S.D. Bass concerning our generations proposal. It is a pleasure to thank Professor Minamisono and his colleagues for the opportunity
to participate in a most stimulating symposium. This work was supported by the Australian Research Council.

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