Effective Dual-Sourcing Through Inventory Projection

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We consider a single-echelon inventory system under periodic review with two suppliers facing stochastic demand, where excess demand is backlogged. The expedited supplier has a shorter lead time than the regular supplier but charges a higher unit price. We introduce the Projected Expedited Inventory Position (PEIP) policy, and we show that the relative difference between the long run average cost per period of this policy and the optimal policy converges to zero when both the shortage cost and the cost premium for expedited units become large, with their ratio held constant. A corollary of this result is that several existing heuristics are also asymptotically optimal in this non-trivial regime. We show through an extensive numerical investigation that the PEIP policy outperforms the current best performing heuristic policies in literature.

Key words: dual-sourcing; inventory; projected inventories; optimal policy; asymptotic optimality

1. Introduction

Dual-sourcing is a common and effective supply chain management practice employed by companies that operate both globally and domestically. It generally refers to situations where companies can replenish their inventories from a regular supplier as well as from an expedited supplier; the latter offering a shorter lead time than the former but at the expense of a cost premium. Even though the expedited supplier charges an additional cost, companies can leverage its responsiveness when inventories deplete precipitously.

Examples of companies that have utilized a dual-sourcing strategy are manifold. Allon and Van Mieghem (2010) describe a $10 billion high-tech U.S. company with suppliers in China and Mexico. The Chinese supplier charges lower costs than Mexico but shipping its products to the U.S. takes considerably longer. Rao et al. (2000) describe a similar situation at Caterpillar; they too use a more expensive expedited supplier whenever a shorter lead time is desired. While both examples concern two distinct suppliers, dual-sourcing may also refer to situations where companies can replenish their inventories...
from one supplier using two distinct transport modes. For example, Threatte and Graves (2002) describe a situation in which Polaroid Corporation relies on ocean transport to ship products from a supplier in Asia to the U.S., but occasionally uses expedited air transport when inventory levels are critically low.

In this paper, we treat a setting similar to the ones described above. In particular, we consider a single-item, single-echelon dual-sourcing inventory system under periodic review facing stochastic demand, where excess demand is backlogged. We assume that demands across periods are independently and identically distributed (i.i.d.). The expedited supplier has a shorter lead time than the regular supplier but charges a higher unit price. There are no fixed costs associated with ordering at either supplier. At the end of each period, linear holding and backorder costs are charged for leftover inventory and backlogged demand, respectively. We seek to minimize the long run average total cost per period, which consists of ordering, holding, and backorder costs.

The dual-sourcing system introduced above has been studied for almost sixty years now, starting with the contributions of Barankin (1961) and Neuts (1964). They show that the optimal policy exhibits a simple order-up-to structure when the lead times of the regular and expedited supplier are one and zero periods, respectively. Fukuda (1964) later extends this result to the case where the difference between the lead times of both suppliers is one period, irrespective of their individual lead times. For lead time differences larger than one period, Whittmore and Saunders (1977) and more recently Feng et al. (2006) show that the structure of the optimal policy is complex, and depends in general on the entire vector of outstanding orders. Although the optimal policy for general lead time differences can be computed in principle by stochastic dynamic programming, this is not practical as the state space of the dynamic program grows exponentially in the lead time difference; that is, it suffers from the curse of dimensionality. Most researchers have therefore focused on devising relatively simple but non-optimal replenishment policies (we will provide a thorough review later in Section 2).

Even though some heuristic policies have been shown to perform numerically close to optimal in at least certain regimes, theoretical justifications for such useful asymptotic optimality performance guarantees are mostly lacking; the only exception being the Tailored Base-Surge (TBS) policy. This policy orders a fixed amount at the regular supplier every period and uses an order-up-to rule for placing expedited orders. The TBS policy
was first proposed by Allon and Van Mieghem (2010), though Rosenshine and Obee (1976) and Janssen and de Kok (1999) have studied a closely related policy before. Numerical analysis by Klosterhalfen et al. (2011) and Janakiraman et al. (2015) indicates that the optimality gap of the TBS policy closes as the lead time difference grows large. Xin and Goldberg (2018) have recently provided theoretical justification for these numerical observations by proving that a simple TBS policy is asymptotically optimal as the lead time of the regular supplier grows large, with the lead time of the expedited supplier held fixed. The intuition behind this result is that as the lead time difference grows, the amount of randomness between when a regular order is placed and when this order arrives grows so large that a constant order, which ignores the entire vector of outstanding orders, performs nearly optimal. To our knowledge, this is the only heuristic policy for dual-sourcing inventory systems under periodic review with general lead time differences that has an asymptotic performance guarantee. Unfortunately, the TBS policy performs poorly outside this asymptotic regime, particularly when either the backorder cost becomes large or when the unit price charged by the expedited supplier becomes large. When both cost parameters become large simultaneously, the performance of the TBS drops even precipitously (Klosterhalfen et al. 2011). Yet, it is exactly this setting that is prevalent in practice, and it therefore remains an open research problem to determine control policies that are provably asymptotically optimal in this regime.

In this paper, we propose such a new policy, which we call the Projected Expedited Inventory Position (PEIP) policy. The PEIP policy keeps track of the expedited inventory position (net inventory level plus outstanding orders arriving within the expedited lead time) and places expedited orders such that this inventory position reaches a certain target level. Regular orders entering the information horizon for placing expedited orders may cause the expedited inventory position to exceed its corresponding target level. This implies that the expedited inventory position in expectation exceeds the target level. Under the PEIP policy, regular orders are therefore placed such that the expected expedited inventory position at the time of the regular order entering the expedited inventory position is kept at a target level. In doing so, the PEIP policy leverages all the information contained in the pipeline – as does the optimal policy. This is in contrast with existing heuristic policies that either aggregate all outstanding orders or ignore them entirely and place a constant order every period regardless. We further provide a separability result that enables efficient
optimization of the PEIP policy. Hence, the PEIP policy is vastly more advanced than existing heuristic policies, and yet can be optimized efficiently and retains an intuitive appeal for practitioners. It is also assuring to note that for the case of consecutive lead times, the PEIP policy reduces to two order-up-to levels, which is known to be the optimal policy for that specific case (Fukuda 1964).

We show that, under some mild conditions on the demand distribution, the relative difference between the long average cost per period of the best PEIP policy and the optimal policy converges to zero when both the shortage cost and the price for expedited units become large, with their ratio held constant. Our approach relies on establishing lower and upper bounds on the average cost rates of the optimal policy and the best PEIP policy, respectively, in terms of the average cost rates of canonical inventory systems with only one supplier. We then show that the ratio of these bounds converge to 1 in the non-trivial asymptotic regime where both the shortage cost and the price for expedited units grow large. Our research is useful for practice as it directly leads to an easily implementable heuristic that has asymptotic performance guarantees in high service regimes where utilizing an expedited source is expensive. In the capital goods industry, for example, it is very common that the shortage costs for critical parts as well as the costs associated with performing an expedited shipment are relatively high (see, e.g., Westerweel et al. 2021).

The main contribution of this paper is threefold:

1. We propose the PEIP policy for periodic review dual-sourcing inventory systems with general lead time differences, and we present an efficient optimization scheme to find its optimal policy parameters. This optimization scheme hinges on a separability result that allows us to decompose the optimization into two simple one-dimensional optimization problems.

2. We prove that the PEIP policy is asymptotically optimal when both the shortage cost and the cost premium for expedited units become large, with their ratio held constant. Our proof technique can serve as a template that is readily applicable to other heuristic policies. We illustrate this by showing that several existing heuristic policies are also asymptotically optimal in this non-trivial regime of broad interest.

3. Through an extensive numerical investigation, we conclude that the PEIP policy outperforms the best performing heuristics in literature. Our numerical investigation further shows that, as the lead time difference between the regular and expedited supplier
grows, the PEIP policy converges to a TBS policy which is asymptotically optimal in this regime (Xin and Goldberg 2018).

The remainder of this paper is organized as follows. In the next section, we position our research with respect to other contributions that have been made. After that, in Section 3 we formally define the decision problem and introduce the notation that we use throughout this paper. Section 4 presents the PEIP policy, including analytical results that allow for an efficient optimization procedure to find the optimal policy parameters. We proceed with establishing our asymptotic optimality result in Section 5. After that, we report on a large numerical study in Section 6 and we provide concluding remarks in Section 7.

2. Positioning in the literature

Dual-sourcing systems operating in discrete time as well as their variants (e.g., continuous review models, multi-echelon systems, etc.) have been widely studied. For a broad overview of this rich field, we refer the interested reader to the surveys of Minner (2003) and Svoboda et al. (2021), as well as to the recent studies of, e.g., Gong et al. (2014), Boute and Van Mieghem (2015), Arts et al. (2016), Sapra (2017), Song et al. (2017), Drent and Arts (2021), and the references therein. Here we confine ourselves to contributions most relevant to our research, a large part of which revolves around heuristic policies.

Whittimore and Saunders show already in 1977 that optimal policies for the periodic review dual-sourcing inventory systems we typically face in practice (i.e. with lead time differences larger than one) are complex and difficult to compute. Yet it lasted until the beginning of this century before researchers began to devise various effective heuristic policies. Scheller-Wolf et al. (2007) propose the Single-Index (SI) policy that uses order-up-to rules for order placement at the regular and expedited supplier based on only the regular inventory position (inventory level plus all outstanding orders). By additionally tracking the expedited inventory position that includes only orders that will arrive within the expedited lead-time, Veeraraghavan and Scheller-Wolf (2008) introduce the Dual-Index (DI) policy. Regular and expedited orders are then placed according to order-up-to rules based on their respective inventory positions. They propose a simulation-based optimization and show that this is easily implementable and often results in near-optimal solutions in numerical examples.
Since then, the DI policy has been extended in various useful ways. Sheopuri et al. (2010) analyze the more general class of policies that use an order-up-to rule for placing expedited orders. Arts et al. (2011) incorporate stochastic lead times and provide an approximate evaluation method so that optimization can be done without simulation. Sun and Van Mieghem (2018) introduce the Capped Dual-Index (CDI) policy in which the DI policy is accompanied with a cap that smooths regular orders. Even though the CDI policy has been developed in a robust optimization setting under the worst-case cost performance criterion, numerical analysis indicates that it also performs well in terms of minimizing the long run average cost per period in a stochastic setting.

Sheopuri et al. (2010) show that the inventory system with lost sales is a special case of the dual-sourcing inventory system treated here given that the expedited supplier follows an order-up-to rule. The intuition behind this is that the orders placed at the expedited supplier can be thought of as demand lost from the regular supplier, with the cost premium of the expedited supplier being the lost sales penalty cost. Inspired by this relation, Sheopuri et al. (2010) propose the Vector Base-Stock (VBS) policy that uses an order-up-to rule for the expedited supplier, and a vector base-stock policy for the regular supplier. Hua et al. (2015) later generalize the VBS policy to the Best Weighted Bounds (BWB) policy. In addition, by using properties of $L^2$-convexity, they show that the optimal expedited orders are more sensitive to the older (i.e. soon-to-arrive) outstanding orders whereas the optimal regular orders are more sensitive to the younger outstanding orders. Extensive numerical investigations indicate that the BWB policy as well as the CDI policy described earlier are the best performing heuristic policies in the current literature, and can even outperform advanced deep reinforcement learning algorithms (Sun and Van Mieghem 2018, Gijsbrechts et al. 2022).

While the PEIP policy is similar to the aforementioned heuristic policies in that it also assumes an order-up-to rule for the expedited supplier, it differs significantly in how it operates the regular supplier. First, the PEIP policy projects the expedited inventory position in deciding upon regular orders. As such, it controls the expected expedited inventory position directly taking into account explicitly the excess by which the order-up-to level of the expedited supplier overshoots; existing heuristics do so only indirectly. Second, similar to the optimal policy, the PEIP policy leverages all available state information in deciding upon order placement at the regular supplier. Existing heuristics either aggregate all available information, or ignore it entirely and place a fixed amount every period regardless.
Our research also contributes to the growing body of literature that establishes asymptotic performance guarantees of heuristic policies for inventory systems whose optimal policies are difficult to analyze if not intractable. The relatively few studies that belong to this literature stream mostly focus on single-echelon inventory systems with lost sales. For such systems, Huh et al. (2009) and Bijvank et al. (2014) show that certain order-up-to policies are asymptotically optimal as the lost sales penalty cost approaches infinity while Goldberg et al. (2016) and Xin and Goldberg (2016) show that a simple constant order policy is asymptotically optimal as the lead time approaches infinity. Both heuristic policies perform however poorly outside their respective asymptotic regimes. This has recently led to the development of two heuristic policies that have asymptotic performance guarantees and additionally perform well in both regimes: the Capped Base-Stock (CBS) policy proposed by Xin (2022, 2021), and the Projected Inventory Level (PIL) policy proposed by Van Jaarsveld and Arts (2021).

The CBS policy can be interpreted as a hybrid between a constant order policy and an order-up-to policy, and it thus enjoys the superior performance of both policies in their respective regimes. The PIL policy places orders such that the expected inventory level at the time of arrival of an order is raised to a target level. Van Jaarsveld and Arts (2021) show that the PIL policy is asymptotically optimal for large lost sales penalty costs as well as for large lead times, though the latter only under the assumption that demand is exponentially distributed. The PEIP policy is related to the PIL policy: While a PIL policy places orders by projecting the expected inventory level, a PEIP policy places regular orders by projecting the expected expedited inventory position (while retaining an order-up-to rule for the expedited supplier).

Unlike inventory systems with lost sales, there currently exists only one heuristic policy for periodic review dual-sourcing inventory systems that has asymptotic performance guarantees. Indeed, a simple TBS policy is asymptotically optimal as the lead time difference between the regular supplier and expedited supplier grows large (Xin and Goldberg 2018). In this paper, we consider a different asymptotic regime and show that the best PEIP policy is asymptotically optimal as the unit cost of the expedited supplier and the shortage cost approach infinity simultaneously. Our main proof exploits the earlier described relation between lost sales inventory systems and dual-sourcing inventory systems; this allows us to draw on the asymptotic results for lost sales inventory systems of Huh et al.
We further argue that our proof technique in itself has merit too since it can serve as a template that is readily applicable to existing as well as future heuristic policies. We illustrate this by showing that the CDI, DI, and SI policy are also asymptotically optimal in the non-trivial asymptotic regime under consideration. As such, we extend the literature on asymptotic optimality results for dual-sourcing inventory systems considerably.

We end this review by remarking that recent advances on asymptotic optimality results for inventory systems that have complex optimal policies stretch beyond the lost sales and dual-sourcing inventory systems described above. See, for instance, Rong et al. (2017) for such results in the context of distribution systems consisting of one central warehouse and multiple local warehouses, Janakiraman et al. (2018) on inventory systems with multiple products that share production capacities, and Fu et al. (2019) on inventory systems for perishable products. We refer the interested reader to Goldberg et al. (2021) for a recent and comprehensive review of this growing field.

3. Notation and problem formulation

This section provides a formal problem description that mostly follows the notational conventions of Sheopuri et al. (2010). We consider a single-item, single-echelon inventory system under periodic review facing stochastic demand, where excess demand is backlogged. Products can be ordered each period both at a regular supplier and at an expedited supplier. Let $c^r (l^r)$ and $c^e (l^e)$ denote the unit prices (lead times) from the regular and expedited supplier, respectively. We assume that $c^e > c^r$ and $l^e < l^r$, that is, the expedited supplier offers a higher unit price, but its lead time is shorter. Notice that the present inventory problem reduces to a standard inventory problem with only one supplier if these assumptions do not hold. Both lead times are assumed to be deterministic as well as a non-negative integer multiple of the review period. For notational convenience, we also define the lead time difference $l = l^r - l^e \geq 1$.

Let $D_t$ denote the random demand in period $t$, with $t$ indexed forward in time starting from 0, i.e. $t \in \{0, 1, \ldots\}$. Demand across periods is a sequence of non-negative i.i.d. random variables. Whenever we drop the period index of $D_t$, we refer to the generic random variable with the same distribution as $D_t$. The cumulative demand over $n$ periods is denoted as $D^n$.

Let $I_t$ denote the net inventory level (on-hand inventory minus backlog) at the beginning of period $t$ after orders have arrived, but before demand has occurred. Let $q^r_t$ ($q^e_t$) denote
the order size placed with the expedited (regular) supplier in period \( t \). The orders that arrive in period \( t \) are thus \( q_{t-l}^e \) and \( q_{t-l}^r \), and we can consequently write the following recursion for the inventory level:

\[
I_t = I_{t-1} - D_{t-1} + q_{t-l}^e + q_{t-l}^r.
\]

At the end of each period \( t \), costs are levied as follows. Per unit in on-hand inventory \((I_t - D_t)^+\) carried over to the next period, a cost \( h > 0 \) is charged. Similarly, a cost \( p > 0 \) is charged for each unit in backlog \((D_t - I_t)^+\) carried over to the next period. Here we use the convention \( x^+ = \max(x, 0) \).

Observe that if the unit price charged by the expedited supplier \( c^e \) is higher than the total backlog cost incurred on a unit over \( l \) periods, i.e. \( l \cdot p \), then there is no cost benefit to be derived from ordering any units from the expedited supplier over the regular supplier. In fact, as shown in Sheopuri et al. (2010), the present inventory problem reduces to a standard single supplier inventory problem if \( c^e \geq l \cdot p \). We therefore assume throughout the remainder that \( c^e < l \cdot p \). We revisit this assumption in Section 5 when we expound on the particular asymptotic regime we are interested in.

Let the state of the system at the beginning of period \( t \) after orders have arrived be given by \( s_t = (I_t, q_{t-l}^r + 1, \ldots, q_{t-1}^r, q_{t-l}^e - 1, \ldots, q_{t-1}^e) \). The sequence of events in any period \( t \) is as follows:

1. Products that were ordered \( l^e (l^r) \) periods ago at the expedited (regular) supplier are received and added to the on-hand inventory, i.e. order \( q_{t-l}^e (q_{t-l}^r) \).
2. The size of the order placed with the expedited supplier \( q_{t}^e \) is determined based on the current state of the system \( s_t \).
3. The size of the order placed with the regular supplier \( q_{t}^r \) is determined based on the current state of the system \( s_t \) and the order just placed with the expedited supplier \( q_{t}^e \).
4. The demand \( D_t \) is realized, and holding or shortage costs are charged, in case that \((I_t - D_t)^+ > 0 \) or \((D_t - I_t)^+ > 0 \), respectively.

The total cost incurred in period \( t \) is denoted by \( C_t \), i.e. \( C_t = c^e \cdot q_{t}^e + c^r \cdot q_{t}^r + h \cdot (I_t - D_t)^+ + p \cdot (D_t - I_t)^+ \). An admissible policy \( \pi \) is as a rule that determines the size of the orders placed with the regular and expedited supplier based only on the historical information available in that period as outlined in the sequence of events above. All such admissible
policies $\pi$ are contained in the set $\Pi$. Since the total cost incurred in a period $t$ depends on the policy $\pi$, we henceforth write $C^\pi_t$ to denote this dependence explicitly. Our performance measure of interest is the long run average cost per period

$$C^\pi = \limsup_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} C^\pi_t.$$  

We assume throughout the remainder that $x_0 = 0$. That is, the initial inventory level equals zero, and there are initially no outstanding expedited or regular orders due to arrive within their corresponding lead times. This assumption simplifies our analysis later; it is however not strictly necessary as we are interested in the long run average cost per period, which is not affected by any specific choice for the initial state.

We end this section with two remarks related to the optimal policy $\pi^* = \arg\min_{\pi \in \Pi} C^\pi$. First, as shown by Sheopuri et al. (2010), $\pi^*$ depends on $c_e$ and $c_r$ only through the price premium of the expedited supplier, i.e. $c_e - c_r$. This result follows intuitively from the observation that any policy that has a finite long run average cost per period will incur at least a cost of $c^r \cdot E[D]$. We will therefore assume without loss of generality that $c_r = 0$. Second, even for small lead time differences, the structure of $\pi^*$ remains poorly understood and depends in general on the entire vector of outstanding orders. The policy we propose in the next section also takes into account this entire vector when deciding on the size of the orders placed with both suppliers. However, different from $\pi^*$, our policy does allow for tractable analysis and efficient optimization, even for large-sized problems often encountered in practice.

4. The Projected Expedited Inventory Position Policy

In this section, we present the Projected Expedited Inventory Position (PEIP) policy for operating dual-sourcing inventory systems. We first formally define the rules under which both suppliers operate, and we subsequently present analytical results that allow for an efficient solution procedure to find the optimal policy parameters.

4.1. Order placement rules

Under a PEIP policy, the ordering rule for the expedited supplier is assumed to be an order-up-to rule that operates as follows. At the beginning of each period $t$ after orders $q_{t-l'}^e$ and $q_{t-l'}^r$ have arrived, we review the expedited inventory position

$$IP^e_t = I_t + \sum_{i=t-l'+1}^{t-1} q^e_i + \sum_{i=t-l+1}^{t-l} q^r_i,$$  

(1)
and, if necessary, place an expedited order $q^e_t$ to raise $IP^e_t$ to its target level $S^e$, i.e. $q^e_t = (S^e - IP^e_t)^+$. Note that regular orders entering the information horizon for placing expedited orders may cause the expedited inventory position to exceed this target level. That is, immediately after expedited orders have been placed, there is an overshoot defined as $O_t = IP^e_t + q^e_t - S^e = (IP^e_t - S^e)^+$. This implies that the expedited inventory position following expedited ordering is in expectation greater than or equal to the target level. In fact, strictly greater unless products are sourced exclusively from the expedited supplier, which we have argued is sub-optimal under the assumptions we make.

The PEIP policy aims to keep the expedited inventory position (and consequently the overshoot) at a fixed level. Although the expedited inventory position cannot actually be kept at a fixed level, it can be kept at a fixed level in expectation. More specifically, the PEIP places regular orders such that the expected expedited inventory position at the time those orders enter the information horizon for expedited order placement is raised to a target level. We henceforth refer to this target level as the projected expedited inventory position level, denoted $U$.

Before we provide a formal description of this ordering rule, we first state the following result related to the minimal required information for optimal regular order placement given that the expedited supplier follows an order-up-to rule. This information is sufficient to determine the expected expedited inventory position in the period that the regular order placed now enters the expedited inventory position.

**Lemma 1.** (Sheopuri et al. 2010, Lemma 4.1) Under the assumption that the expedited supplier follows an order-up-to rule, the optimal regular order in period $t$, $q^r_t$, depends on $x_t$ and $q^e_t$ only through the $l$-dimensional state description $x^r_t = (O_t, q^e_{t-1}, q^e_{t-2}, \ldots, q^e_{t-l+1})$.

Observe from the definition of the expedited inventory position in (1) that this inventory position in period $t + l$, i.e. $IP^e_{t+l}$, includes the regular order $q^r_t$ that we will place in the current period $t$. We write $\mathbb{E}[IP^e_{t+l}(q^r_t)|x^r_t]$ to denote the conditional expected expedited inventory position $l$ periods into the future in period $t$ given the current state of the system $x^r_t$ as a function of the regular order $q^r_t$ that we will place in period $t$. The ordering rule for the regular supplier can now formally be described as follows. In each period $t$ immediately after expedited ordering, we place a regular order $q^r_t$ such that $\mathbb{E}[IP^e_{t+l}|x^r_t]$
equals the projected expedited inventory position level $U$. That is, the period $t$ regular order $q^r_t$ solves
\[
\mathbb{E}[IP^e_{t+1}(q^r_t)|x^r_t] = U.
\] (2)

The solution to (2) is unique and can be computed efficiently via a simple bisection search.

Proposition 1 below shows that under the PEIP policy, it is always possible to place such a non-negative order (possibly zero) to attain the projected expedited inventory position level $U$. We note that Van Jaarsveld and Arts (2021) have established a similar result in the context of lost sales inventory systems. The proof of the proposition can be found in Appendix A.

**Proposition 1.** Under the PEIP policy, the conditional expected expedited inventory position in period $t+1$ before regular ordering in period $t$ is less than or equal to the projected expedited inventory position level $U$ for all periods $t \in \mathbb{N}$.

It is worth noting that the regular ordering rule under the PEIP policy is of the order-up-to type when the lead times of both suppliers are consecutive, i.e. when $l = 1$. To see this, recall that under such an order-up-to rule, regular orders are placed such that the usual inventory position is kept at a target level, say $S^r$. Now, observe that, following expedited ordering in some period $t$, we have $IP^e_{t+1} = S^e + O_t + q^r_t - D_t = S^r - D_t$, so that $\mathbb{E}[IP^e_{t+1}|x^r_t] = S^r - \mathbb{E}[D]$. Thus, when $l = 1$, the ordering rule for the regular supplier under a PEIP policy with projected expedited inventory position level $U = S^r - \mathbb{E}[D]$ is equivalent to an order-up-to rule with target level $S^r$. This observation implies that for the special case of consecutive lead times, the PEIP policy reduces to the optimal policy (Fukuda 1964). This feature is shared with the CDI, DI, SI, and BWB policy, but not with the TBS policy.

### 4.2. Optimization procedure

We now turn to the issue of finding the optimal PEIP policy parameters. Observe that the expedited inventory after ordering is bounded from below by the order-up-to level $S^e$. This implies that the projected expedited inventory position equals $S^e$ plus the projected overshoot. A given projected expedited inventory position level $U$ thus only scales $IP^e$ for a given $S^e$, which suggests that order placement depends on $S^e$ and $U$ only through their difference. We state this observation formally in the lemma below, which is a special case of Lemma 4.2 of Sheopuri et al. (2010):
Lemma 2. The distributions of $O$, $q^r$, and $q^e$ depend on $U$ and $S^e$ only through their difference $V = U - S^e$.

As our discussion preceding Lemma 2 already alludes to, the difference $V$ has an explicit meaning that allows for an alternative but equivalent specification and operation of the PEIP policy. Rather than keeping the projected expedited inventory position at a fixed level, this alternative specification places regular orders such that the expected overshoot is kept at a fixed level (while retaining an order-up-to level $S^e$ for the expedited supplier).

To state this equivalence relation formally, observe that $E[O_{t+l}|x^r_t]$ denotes the expected overshoot $l$ periods into the future at time $t$ given the system state at time $t$ following ordering, $x^r_t$. In any period $t$ after ordering, we have $E[IP^e_{t+l}|x^e_t] = S^e + E[O_{t+l}|x^e_t]$. Hence, placing regular orders such that $E[IP^e_{t+l}|x^e_t]$ equals the projected expedited inventory position level $U$ is equivalent to placing regular orders such that $E[O_{t+l}|x^e_t]$ equals what we henceforth refer to as the projected overshoot level $V = U - S^e$.

While the original specification of the PEIP policy has intuitive appeal, the alternative specification outlined in the previous paragraph allows us to decompose the optimization of the policy parameters into two simple uni-variate optimization problems. Indeed, letting $O^V$ denote the steady state overshoot random variable for a given $V$, the optimal order-up-to level $S^e^*$ for a given $V$ has a simple form:

Lemma 3. The optimal order-up-to level $S^e^*$ for a given $V$ equals

$$S^e^* = \inf \left\{ S^e \in \mathbb{R} : \mathbb{P} \left( D^{e+1} - O^V \leq S^e \right) \geq \frac{p}{p + h} \right\}.$$ 

The result above implies that a simple search procedure over the projected overshoot level $V$ suffices to find the globally optimal PEIP policy.

Lemma 3 is a special case of Lemma 4.3 of Sheopuri et al. (2010). Sheopuri et al. (2010) show that this result holds in general for all policies that have an order-up-to rule for the expedited supplier and operate some given regular ordering rule that uses only the information contained in $x^r_t$. For all such policies, the optimal order-up-to level for the expedited supplier can be interpreted as a standard Newsvendor model with one additional feature. The ordering rule for the regular supplier essentially dictates a return stream in the form of the overshoot which must therefore be subtracted from the lead time demand to arrive at the net lead time demand observed by the said Newsvendor. The PEIP policy has particular intuitive appeal in the context of this Newsvendor with returns interpretation; it keeps the expected amount to be returned $l$ periods into the future at a fixed level.
5. Asymptotic optimality

In this section, we prove that the best PEIP policy is asymptotically optimal when both the shortage cost and the cost premium for expedited units become large, with their ratio held constant. We first elaborate on why it is important and non-trivial to study this asymptotic regime. We then present the asymptotic optimality result itself, and we subsequently show that this result is robust in the sense that it holds true for a broad class of policies.

5.1. Regime of interest

We know from Xin and Goldberg (2018) that a simple TBS policy is asymptotically optimal as the lead time difference between the regular supplier and the expedited supplier approaches infinity. Unfortunately, the policy performs poorly outside this asymptotic regime, particularly when either the cost for a unit in backlog or the price charged by the expedited supplier is large. When these costs grow large simultaneously, the performance of the TBS drops precipitously (Klosterhalfen et al. 2011). It is however exactly this latter asymptotic regime that is prevalent in practice, and thus establishing asymptotic performance guarantees of the PEIP policy in this regime has important practical implications. In the capital goods industry, for instance, backordering a critical part comes at a high cost while expediting shipments from an emergency supplier are expensive too (see, e.g., Westerweel et al. 2021).

In this section, we thus focus on the asymptotic regime where the unit expediting cost $c_e$ and the shortage cost $p$ grow large simultaneously. We do, however, need the assumption that their ratio is held fixed at a constant strictly smaller than $l$ (i.e. $c_e < p \cdot l$). Recall from Section 3 that if we would not impose this assumption, then ordering from the expedited supplier over the regular supplier would never be beneficial. That is, the dual-sourcing inventory problem would reduce to a trivial single supplier inventory problem if $c_e \geq p \cdot l$.

We remark that, following a reasoning similar to the above, the asymptotic regime where both $l$ and $c_e$ grow large with their ratio held fixed at a constant strictly smaller than $p$ is, at least theoretically, interesting as well. Indeed, this too is a non-trivial regime in which the optimal policy does not reduce to a trivial single supplier inventory problem. Xin (2022) has recently made advances on this latter regime by showing that for the continuous time variant of the problem, the CDI policy has a worst-case performance guarantee of 1.79.
5.2. Main results

To facilitate our analysis, we first introduce additional notation. Let $C^{PEIP^*}(p, h, c^e, l^r, l^e)$ and $C^\pi^*(p, h, c^e, l^r, l^e)$ denote the long run average cost per period of the dual-sourcing inventory system under the best PEIP policy and the optimal policy, respectively, as functions of the problem specific input parameters. We may occasionally omit certain input parameters when there is no ambiguity. We define the functions $p(n) = p \cdot n$ and $c^e(n) = c^e \cdot n$, and we are hence interested in the asymptotic regime where $n \to \infty$ with $\frac{c^e(n)}{p(n)}$ fixed at a constant strictly smaller than $l$. For a given $n > 0$, we further define $C^{PEIP^*}(n) = C^{PEIP^*}(p(n), c^e(n))$ and $C^\pi^*(n) = C^\pi^*(p(n), c^e(n))$ to denote the cost as we scale $p$ and $c^e$ proportionally with $n$.

Let $L(p, h, l)$ and $B(p, h, l)$ denote the canonical single supplier inventory systems with lost sales and backlogged demand, respectively, as defined in Janakiraman et al. (2007), Huh et al. (2009), and Bijvank et al. (2014). Following their notation, let $C^L^*(p, h, l)$ denote the long run average cost per period under an optimal policy in $L(p, h, l)$ as a function of the lost sales penalty cost $p$, the holding cost $h$, and the lead time $l$. Similarly, let $C^B^*(p, h, l)$ denote the long run average cost per period under the optimal policy in $B(p, h, l)$, with $p$ now denoting the penalty cost for backlogged demand. It is well-known that the optimal policy for $B(p, h, l)$ is a simple order-up-to rule. It is equally well-known that the optimal policy for $L(p, h, l)$ has a complex structure and depends in general on the entire vector of outstanding orders, very much akin to the present dual-sourcing inventory system.

We first establish a lower bound on the long run average cost per period of the optimal policy, i.e. $C^\pi^*(p, c^e, l^r, l^e)$, in terms of both the long run average cost per period of the optimal policy for $B(p, h, l)$, i.e. $C^B^*(p, h, l)$, and the optimal policy for $L(p, h, l)$, i.e. $C^L^*(p, h, l)$.

**Lemma 4.** $C^B^*(c^e/(l^r + 1), h, l^r) \leq C^L^*(c^e, l^r) \leq C^\pi^*(p, h, c^e, l^r, l^e)$

**Proof.** Observe that a lower bound on the long run average cost per period of the optimal policy is given by the long run average cost per period cost of an optimal policy for a system identical to the present inventory system but in which any backlogged demand at the end of a period is satisfied instantly by the expedited supplier at unit price $c^e$, where we assume without loss of generality that $c^e < p$. According to Theorem 3.1 of Sheopuri et al. (2010), this system is exactly $L$, where the lost sales penalty cost equals
Drent and Arts: Effective Dual-Sourcing Through Inventory Projection

c^e, i.e. \( C^L^*(c^e, l^r) \). Hence, we have \( C^{\pi^*}(p, c^e, l^r, l^e) \geq C^L^*(c^e, l^r) \). We further know from Theorem 5 of Janakiraman et al. (2007) that \( C^L^*(c^e, l^r) \geq C^B^*(c^e/(l^r + 1), l^r) \), and thus \( C^{\pi^*}(p, c^e, l^r, l^e) \geq C^B^*(c^e/(l^r + 1), l^r) \). \( \square \)

We note in passing that easily computable lower bounds for the optimal cost of dual-sourcing systems are relatively scarce. In fact, we are aware of only one such lower bound in terms of the optimal policy for an equivalent dual-sourcing system where the lead time difference is 1 (Sheopuri et al. 2010). For this equivalent system, the optimal policy is myopic in that both suppliers use an order-up-to level. In this respect, the lower bound in terms of \( B(c^e/(l^r + 1), l^r) \) in Lemma 4 is of independent interest as it is easy to compute and leads to an upper bound on the optimality gap for any given policy.

Our main result requires the following technical assumption regarding the cumulative demand over \( l^r + 1 \) periods:

**Assumption 1.** The random variable \( D_{l^r+1} \) has a positive and finite mean, i.e. \( 0 < E[D_{l^r+1}] < \infty \), and satisfies one of the following conditions:

1. \( D_{l^r+1} \) is bounded, or
2. \( D_{l^r+1} \) is unbounded and \( \lim_{d \to \infty} E[D_{l^r+1} - d | D_{l^r+1} > d]/d = 0 \).

This assumption first appeared in Huh et al. (2009). They show that many commonly used distributions in inventory models satisfy this assumption, including Poisson distributions, geometric distributions, negative binomial distributions with parameters \( r > 0 \) and \( 0 < p < 1 \), and (log-)normal distributions. A sufficient condition for this assumption to be satisfied is that the demand distribution has an increasing failure rate.

We are now in the position to state the main result of this section.

**Theorem 1.** Let the backorder penalty cost \( p \) and the expedited cost premium \( c^e \) be such that \( c^e/p < l \). Then, under Assumption 1, the best PEIP policy is asymptotically optimal as \( c^e \) and \( p \) approach infinity simultaneously, that is

\[
\lim_{n \to \infty} \frac{C^{PEIP^*}(n)}{C^{\pi^*}(n)} = 1.
\]

**Proof.** Observe that single sourcing from the regular supplier is a feasible PEIP policy by setting \( S^e \) equal to \( \infty \). This observation implies that the numerator is bounded from above by the long run average cost per period of an optimal policy in a single supplier inventory system with backlogged demand whose lead time is equal to the lead time of the
regular supplier, i.e. $C^B(p, h, l') \geq C^{PEIP^*}(p, h, c^e, l^r, l^e)$. Combining this observation with Lemma 4 we have

$$\frac{C^{PEIP^*}(n)}{C^\pi^*(n)} \leq \frac{C^B(p(n), h, l')}{C^B(c^e(n)/(l^r + 1), h, l')}.$$ 

By Theorem 2b of Huh et al. (2009), we know that under Assumption 1

$$\lim_{n \to \infty} \frac{C^B(p(n), h, l')}{C^B(c^e(n)/(l^r + 1), h, l')} = 1,$$

which gives the desired result. □

Define the subset $\tilde{\Pi} = \{\pi \in \Pi | C^\pi^*(p, h, c^e, l^r, l^e) \leq C^B(p, h, l')\}$. Note that it readily follows from the definition of $\tilde{\Pi}$ and the proof of Theorem 1 that any policy $\pi \in \tilde{\Pi}$ is asymptotically optimal in the regime under our consideration. This is an important observation that generalizes our result to a wide class of policies. We therefore state it as a theorem.

**Theorem 2.** Let the backorder penalty cost $p$ and the expedited cost premium $c^e$ be such that $c^e/p < l$. Then, under Assumption 1 any policy $\tilde{\pi} \in \tilde{\Pi}$ is asymptotically optimal as $c^e$ and $p$ approach infinity simultaneously, that is

$$\lim_{n \to \infty} \frac{C^\tilde{\pi}^*(n)}{C^\pi^*(n)} = 1,$$

where $C^\pi^*(n)$ is defined in a way similar as $C^\pi^*(n)$.

We conclude this section by noting that the CDI, DI, and SI policy are all contained in the set $\tilde{\Pi}$. They are thus asymptotically optimal when both $c^e$ and $p$ grow large with their ratio held fixed at a constant strictly smaller than $l$.

### 6. Numerical investigation

The theoretical results of the previous section are important as they provide performance guarantees for the PEIP policy in high service regimes where expedited orders are expensive. While this is particularly useful for practitioners operating in those regimes, it does not imply that the PEIP policy consistently performs well, especially outside those asymptotic regimes. In this section, we therefore undertake an extensive numerical investigation to examine the overall effectiveness of the PEIP policy, thereby complementing the theory of the previous section. This investigation mainly revolves around the comparison of the performance of the PEIP policy with the performance of several existing heuristic policies.

We first describe the benchmark heuristic policies in Section 6.1. We then continue with the numerical investigation itself, which consists of two parts. The first part, Section 6.2.
compares the performance of the PEIP policy with the performance of the benchmark heuristic policies based on a full factorial test bed of industrial size. The second part, Section 6.3, analyzes the performance of the PEIP policy as well as the benchmark heuristic policies as the lead time difference between both suppliers grows large. It is exactly this regime where a simple TBS policy is asymptotically optimal (Xin and Goldberg 2018).

6.1. Benchmark heuristic policies and computational aspects

The following five heuristic policies will serve as benchmarks: (i) The DI policy, (ii) the CDI policy, (iii) the VBS policy, (iv) the BWB policy, and (v) the TBS policy. Although we have discussed these five heuristic policies in our literature review, we revisit a few points here that are important to bear in mind while considering the results of the numerical investigation.

First, the CDI policy and the BWB policy are the current best performing heuristic policies. Their computational requirements are however significantly higher than the PEIP policy. Indeed, while the PEIP policy requires only a one-dimensional search to find the optimal policy parameters, both the CDI policy and the BWB policy require a search over two dimensions to arrive at their best policy parameters. By contrast, the DI policy, VBS policy, and the TBS policy have the same computational requirements as the PEIP policy. Second, the CDI policy is a generalization of the DI policy, and it is therefore to be expected that the former policy outperforms the latter. The same holds for the BWB policy and the VBS policy – the latter being a special case of the former. Third, all five benchmark heuristic policies use an order-up-to rule for the expedited supplier. As such, echoing our discussion after Lemma 3, the optimal order-up-to level under each policy for given regular ordering rule parameters follows a Newsvendor type equation. This is convenient numerically as it only requires us to simulate the steady state overshoot distribution for given regular ordering rule parameters to arrive at the corresponding optimal order-up-to level. We describe this simulation-based optimization scheme in detail next.

For all problem instances in this numerical investigation, we find the best policy parameters for the PEIP policy as well as for the benchmark heuristic policies via golden-section searches over their respective regular ordering rule parameters. In each iteration of this search procedure, we first compute the steady state overshoot distribution via simulation, we then determine the optimal order-up-to level via the Newsvendor equation, and we
finally compute the corresponding long run average cost per period. We use common ran-
dom numbers across all policies per problem instance, and we continue simulating the
steady state overshoot distribution until the width of the 95 percent confidence intervals
for the mean and for the standard deviation of the overshoot are less than 1 percent of
their respective point estimates. After we find the best control policy parameters for all
heuristic policies for a given problem instance, we compute the long run average cost per
period for each heuristic policy under its best policy parameters via simulation. As in
the simulation-based optimization scheme, this simulation uses common random numbers
across policies, and it continues until the width of the 95 percent confidence intervals for
the long run average cost per period is less than 1 percent of its point estimate.

6.2. Full factorial experiment

We start our numerical investigation by comparing the performance of the PEIP policy
with the performance of the heuristic policies described in the previous section. We do
so based on a large test bed of industrial size. Note that the optimal policy cannot be
computed for such large instances due to the complexity of the corresponding dynamic
program. We therefore exclude the optimal policy in this investigation.

In all instances of the test bed, the one period demand has a negative binomial dis-
tribution with mean 50 and the holding cost $h$ is fixed at 1. We further keep the price
of the regular supplier $c_r$ fixed at 0 since only the cost difference between the suppliers
affects the relative cost rate. The expedited lead time $l^e$ is kept fixed at 0 periods so
that the focus is on the lead time difference $l = l^r - l^e = l^r$. The remaining input param-
eters are varied over multiple levels. We vary the backorder penalty cost over five levels
($p \in \{4, 9, 19, 49, 99\}$), the coefficient of variation of the one period demand over five lev-
els ($\sigma_D/\mathbb{E}[D] \in \{0.15, 0.25, 0.5, 1, 1.5, 2\}$), and the regular lead time over four levels ($l^r \in \{2, 3, 4, 5\}$). We set the price of the expedited supplier $c^e$ equal to a fraction of the back-
order penalty cost multiplied by the lead time difference of the specific instance, that is,$c^e = \delta pl$ for $\delta \in \{0.1, 0.2, 0.4\}$. While this approach is different from most numerical studies
on dual-sourcing inventory systems, it does ensure that there is no instance in the test bed
for which exclusively sourcing from the regular supplier is optimal. Table 1 summarizes all
levels for each input parameter in our test bed that is not kept fixed. Permuting over all
these levels leads to a large test bed of 360 instances.
Table 1  Input parameter values for our test bed

| Input parameter | No. of choices | Values |
|-----------------|----------------|--------|
| 1 Backorder penalty cost, \( p \) | 5 | 4, 9, 19, 49, 99 |
| 2 Regular lead time, \( l^r \) | 4 | 2, 3, 4, 5 |
| 3 Coefficient of variation of demand, \( \sigma_{D} / \mu_{D} \) | 6 | 0.15, 0.25, 0.5, 1, 1.5, 2 |
| 4 Price of the expedited supplier, \( c^e \) | 3 | \( \delta pl \) for \( \delta = 0.1, 0.2, 0.4 \) |

We are interested in the performance of the PEIP policy in comparison with the performance of the benchmark heuristics. To that end, we compute for each instance of the test bed the percentage gap in the long run average cost per period of each benchmark heuristic policy \( \pi \in \{\text{DI, CDI, VBS, BWB, TBS}\} \) relative to the PEIP policy: \( 100 \cdot \frac{C_{\pi^*} - C_{\text{PEIP}^*}}{C_{\text{PEIP}^*}} \).

Table 2 summarizes the numerical results of the full factorial experiment. The table presents the average, maximum, and minimum gap percentages for each benchmark heuristic policy computed over all instances as well as the percentage of test instances in which the PEIP policy outperforms the specific benchmark heuristic policy. (A positive gap percentage implies that the PEIP policy outperforms the specific benchmark heuristic policy.) Detailed results are relegated to Appendix B. Based on this table, we conclude that the PEIP policy outperforms the TBS, DI, CDI, and VBS policy. The performance of the PEIP policy seems to be comparable with the BWB policy, though on average the PEIP policy outperforms this policy as well. It is also interesting to note that the PEIP policy can outperform the BWB and CDI policy by up to 2.67% and 4.67%, respectively. The opposite never holds. Indeed, when the PEIP policy is outperformed by the BWB or CDI policy, it only occurs by much smaller margins of up to 0.29% and 0.08%, respectively. Note that these gaps are smaller than the confidence interval around the estimated performance for any single instance. This is an interesting observation, especially in view of the fact that the BWB and CDI policy are the current best performing heuristic policies, and have considerably higher computational requirements than the PEIP policy. Furthermore, these policies have more parameters and the BWB policy is arguably difficult to communicate to practitioners.

Table 2  Summary of full factorial experiment

| Heuristic | Avg | Max | Min | %  |
|-----------|-----|-----|-----|----|
| VBS       | 0.15| 3.15| -0.05| 90 |
| BWB       | 0.03| 2.67| -0.29| 42 |
| DI        | 0.71| 4.97| 0.00 | 100|
| CDI       | 0.19| 4.67| -0.08| 94 |
| TBS       | 55.11| 373.88| 1.68 | 100|
6.3. **Long lead time difference regime**

We will now examine the performance of the PEIP policy as well as the benchmark heuristic policies for increasingly larger lead time differences \( l \). To that end, we will study a test instance in which we only vary \( l \). The cost parameters in this test instance are fixed as follows: \( h = 1 \), \( p = 19 \), and \( c^e = 5 \). As in the previous section, the one period demand has a negative binomial distribution with mean 50, with \( \sigma_D/E[D] \) now fixed at 0.25.

Since the TBS policy is asymptotically optimal as the lead time difference \( l \) grows large (Xin and Goldberg 2018), we will now use the relative difference between the long run average cost per period of each heuristic policy with the long run average cost per period of the TBS policy as the main performance measure. To that end, we compute the percentage improvement/reduction in the long run average cost per period of each heuristic policy \( \pi \in \{ \text{PEIP, BWB, VBS, CDI, DI} \} \) relative to the TBS policy: 
\[
\% \text{RED} = 100 \cdot \frac{C_{\text{TBS}}^* - C_{\pi}^*}{C_{\text{TBS}}^*}.
\]

Table 3 provides the gap percentages for increasingly larger values of \( l \), along with the variability of the orders placed with the regular supplier computed through simulation. Based on this table, we can draw three main conclusions. First, the PEIP policy has the best relative improvement over the TBS policy for each value of \( l \). Second, even for values of \( l \) that would be considered large in practice, the PEIP, BWB, CDI, and VBS policy outperform the TBS policy. Third, the variability of the orders placed with the regular supplier under the PEIP policy decreases in \( l \). This suggests that the PEIP policy will increasingly mimic the TBS policy as the lead time difference grows. (Note that the TBS policy has zero variability in its regular orders.) The same holds for the BWB, CDI, and VBS policy. Whether these four heuristic policies are asymptotically optimal when \( l \) grows large is an open question.

7. **Concluding remarks**

In this paper, we have proposed the PEIP policy for dual-sourcing inventory systems. This policy uses an order-up-to rule for the expedited supplier, and places regular orders such that the expected expedited inventory position at the time of the regular order entering the expedited inventory position is raised to a target level. In doing so, the PEIP policy
leverages all the information contained in the pipeline – as does the optimal policy. This is in contrast with existing heuristic policies that either aggregate all outstanding orders or ignore them entirely and place a constant order every period regardless. We have further established a separability result that reduces the optimization of the PEIP policy to two simple one-dimensional optimization problems. An extensive numerical investigation has indicated that the PEIP policy outperforms the current best performing heuristic policies, even the ones that employ more control parameters than the PEIP policy. The PEIP policy is thus vastly more advanced than existing heuristic policies, and yet retains an intuitive appeal for practitioners, has superior performance, and can be optimized efficiently.

We have shown that the PEIP policy is asymptotically optimal when the unit shortage cost and the unit price charged by the expedited supplier approach infinity simultaneously with their ratio held fixed at a constant. This result is useful for practice as it implies that the PEIP policy has asymptotic performance guarantees in high service regimes where utilizing an expedited source is expensive. We have further illustrated that this asymptotic optimality result is robust in the sense that it holds true for a wide class of dual-sourcing heuristic policies, including the CDI, DI, and SI policy. Our proof technique in itself has merit too since it can serve as a template that is readily applicable to future heuristic policies. As such, we have extended the literature on asymptotic optimality results for dual-sourcing inventory systems.

Our numerical investigation also poses an interesting question: Is the PEIP policy asymptotically optimal as the lead time difference between both suppliers grows large? Our numerical results provide strong evidence that this may be the case. An answer to this question for the PEIP and other policies is left for future research.

Acknowledgments

Table 3  Long lead times regime

| t   | PEIP %RED Var[Q'] | BWB %RED Var[Q'] | VBS %RED Var[Q'] | CDI %RED Var[Q'] | DI %RED Var[Q'] |
|-----|-------------------|-----------------|-----------------|-----------------|---------------|
| 2   | 21.81 104.02      | 21.80 100.03    | 21.75 113.03    | 21.76 110.57    | 21.53 132.25  |
| 4   | 12.03 59.90       | 11.98 48.97     | 11.78 73.87     | 11.95 46.98     | 10.30 127.67  |
| 6   | 7.50 33.64        | 7.37 34.61      | 6.86 51.23      | 7.32 24.62      | 4.06 125.51   |
| 8   | 5.18 21.27        | 4.97 24.97      | 4.23 36.49      | 5.09 15.72      | 0.17 125.45   |
| 10  | 3.81 14.42        | 3.52 18.24      | 3.06 19.86      | 3.66 9.03       | -2.53 124.25  |
| 12  | 3.05 9.23         | 2.72 14.85      | 2.55 11.38      | 2.84 8.81       | -4.60 124.05  |
| 14  | 2.43 6.97         | 2.04 12.08      | 2.00 9.89       | 2.09 2.55       | -6.26 122.81  |
| 16  | 2.06 5.29         | 1.78 10.47      | 1.78 3.63       | 1.83 2.74       | -7.63 123.37  |
| 18  | 1.68 3.84         | 1.61 4.26       | 1.56 2.95       | 1.57 2.51       | -8.57 123.48  |
| 20  | 1.45 3.01         | 1.36 2.79       | 1.34 2.50       | 0.09 0.02       | -9.25 122.61  |
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Appendix A: Proof of Proposition

Following our discussion after Lemma 2, it suffices to prove that in any period \( t \in \mathbb{N} \), we can place a non-negative regular order so that \( \mathbb{E}[O_{t+1}|X_t] \) equals the projected overshoot level \( V \). For ease of exposition, we will now denote the regular order placed in period \( t \) with \( q_{t+1}^r \) such that the order entering the expedited inventory position in period \( t \) is denoted \( q_{t+1}^e \).

Due to our assumption on the initial state, i.e. \( x_0 = 0 \), we have that \( \exists q_{t}^e \geq 0 \) such that \( \mathbb{E}[O_t|x_0^e] = \mathbb{E}[(O_{t-1} + q_{t}^e - D_{t-1})^+ | x_0^e] = V \). We define \( Y_t = (O_t - D_t)^+ \). It suffices now to show that \( \mathbb{E}[Y_t|x_0^e, D_0 = 0] \leq \mathbb{E}[O_t|x_0^e] = V \) because \( \mathbb{E}[Y_t|x_0^e, D_0 = 0] \) is the minimum projected overshoot level as seen in period 1 before a regular order is placed. Observe that

\[
Y_t|(x_0^e, D_0 = 0) = (((...((O_0 + q_1^e)^+ + q_2^e - D_1)^+ \ldots)^+ + q_{t}^e - D_{t-1})^+ - D_t)^+,
\]

and

\[
O_t|x_0^e = (((...((O_0 + q_1^e - D_0)^+ + q_2^e - D_1)^+ \ldots)^+ + q_{t}^e - D_{t-1})^+.
\]

It is easy to verify through exhaustion and the fact that \( (x)^+ \) is monotone increasing that \( Y_t|(x_0^e, D_0 = 0) \leq O_t|x_0^e \) state-wise and thus \( \mathbb{E}[Y_t|x_0^e, D_0 = 0] \leq \mathbb{E}[O_t|x_0^e] = V \).

Appendix B: Detailed results numerical investigation

Table 4 presents the percentage gap in the long run average cost per period of each benchmark heuristic policy relative to the PEIP policy. In this table, we first distinguish between subsets of instances with the same value for a specific input parameter of Table 1 and then present the results for all instances.
## Table 4  Detailed result full factorial experiment

| Parameter Value | Avg | Max | Min | % | Avg | Max | Min | % | Avg | Max | Min | % | Avg | Max | Min | % | Avg | Max | Min | % |
|-----------------|-----|-----|-----|---|-----|-----|-----|---|-----|-----|-----|---|-----|-----|-----|---|-----|-----|-----|---|-----|
| p 4 | 0.32 | 0.65 | 0.08 | 100 | 0.09 | 0.45 | -0.11 | 78 | 1.74 | 4.97 | 0.23 | 100 | 0.36 | 4.19 | 0.06 | 100 | 10.73 | 30.99 | 1.68 | 100 |
| 9 | 0.29 | 0.62 | 0.02 | 100 | 0.01 | 0.27 | -0.21 | 47 | 1.01 | 3.27 | 0.11 | 100 | 0.28 | 4.32 | 0.02 | 100 | 21.74 | 58.51 | 4.48 | 100 |
| 19 | 0.08 | 0.33 | -0.05 | 96 | -0.04 | 0.66 | -0.29 | 22 | 0.50 | 1.73 | 0.04 | 100 | 0.15 | 3.10 | 0.01 | 100 | 38.35 | 90.34 | 9.34 | 100 |
| 49 | 0.12 | 3.15 | -0.03 | 83 | -0.03 | 0.46 | -0.18 | 26 | 0.17 | 0.61 | 0.00 | 100 | 0.04 | 0.43 | -0.07 | 92 | 76.82 | 199.62 | 23.71 | 100 |
| 99 | 0.02 | 0.09 | -0.02 | 72 | 0.12 | 2.67 | -0.13 | 35 | 0.11 | 2.81 | 0.00 | 100 | 0.12 | 4.67 | -0.08 | 79 | 127.90 | 373.88 | 32.34 | 100 |
| l 2 | 0.07 | 0.29 | 0.00 | 94 | 0.08 | 2.67 | -0.05 | 51 | 0.27 | 1.36 | 0.00 | 100 | 0.05 | 0.29 | -0.01 | 90 | 48.75 | 235.21 | 1.75 | 100 |
| 3 | 0.15 | 2.64 | -0.01 | 92 | 0.02 | 0.73 | -0.10 | 42 | 0.58 | 2.80 | 0.01 | 100 | 0.17 | 4.32 | -0.02 | 96 | 54.35 | 286.08 | 1.68 | 100 |
| 4 | 0.15 | 0.54 | -0.01 | 90 | 0.02 | 0.85 | -0.18 | 38 | 0.85 | 3.86 | 0.00 | 100 | 0.26 | 4.67 | -0.07 | 97 | 57.45 | 334.64 | 1.76 | 100 |
| 5 | 0.21 | 3.15 | -0.05 | 84 | 0.00 | 1.31 | -0.29 | 36 | 1.14 | 4.97 | 0.01 | 100 | 0.27 | 4.19 | -0.08 | 94 | 59.87 | 373.88 | 1.70 | 100 |
| σ_D 0.15 | 0.24 | 3.15 | -0.02 | 97 | 0.09 | 2.67 | -0.14 | 52 | 0.81 | 4.97 | 0.00 | 100 | 0.28 | 4.67 | -0.08 | 92 | 76.50 | 373.88 | 4.29 | 100 |
| 0.25 | 0.15 | 0.61 | 0.00 | 100 | 0.09 | 2.47 | -0.18 | 42 | 0.81 | 4.87 | 0.01 | 100 | 0.13 | 1.46 | -0.02 | 92 | 63.18 | 240.11 | 3.85 | 100 |
| 0.5 | 0.12 | 0.62 | 0.00 | 98 | -0.03 | 0.85 | -0.20 | 30 | 0.79 | 4.39 | 0.01 | 100 | 0.24 | 4.32 | -0.01 | 95 | 60.73 | 292.31 | 3.38 | 100 |
| 1.5 | 0.11 | 0.44 | -0.02 | 78 | 0.00 | 0.20 | -0.21 | 43 | 0.61 | 2.62 | 0.01 | 100 | 0.15 | 0.52 | 0.00 | 98 | 43.74 | 214.23 | 2.11 | 100 |
| 2.5 | 0.15 | 0.55 | -0.05 | 85 | 0.05 | 0.45 | -0.29 | 58 | 0.54 | 2.81 | 0.01 | 100 | 0.19 | 2.79 | 0.00 | 97 | 37.43 | 177.20 | 1.68 | 100 |
| c_D 0.1 | 0.20 | 0.65 | -0.02 | 93 | 0.02 | 0.45 | -0.18 | 51 | 1.15 | 4.97 | 0.03 | 100 | 0.27 | 4.67 | -0.08 | 90 | 24.68 | 88.41 | 1.70 | 100 |
| 0.2 | 0.15 | 0.62 | -0.03 | 89 | 0.02 | 0.27 | -0.20 | 37 | 0.78 | 3.39 | 0.02 | 100 | 0.21 | 4.19 | -0.01 | 94 | 48.60 | 182.03 | 4.48 | 100 |
| 0.4 | 0.11 | 3.15 | -0.05 | 89 | 0.06 | 2.47 | -0.29 | 44 | 0.43 | 2.81 | 0.00 | 100 | 0.15 | 2.79 | -0.01 | 94 | 89.73 | 373.88 | 9.82 | 100 |
| Total | 0.15 | 3.15 | -0.05 | 90 | 0.03 | 2.67 | -0.29 | 42 | 0.71 | 4.97 | 0.00 | 100 | 0.19 | 4.67 | -0.08 | 94 | 55.11 | 373.88 | 1.68 | 100 |