Characteristics of the circular subsonic laminar jets flow with an initial Poiseuille profile

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Abstract. In this work, the experimental data are compared with the version of the “strong” jet (Re >> 1) of the exact Landau-Squire solution. The experiments were performed for a submerged air jet flowing out of a tube with a diameter of D = 3.2 mm and a length of more than 100D at a Reynolds number equal to Re = 436. The initial conditions in the jet are the Poiseuille velocity profile, the level of velocity pulsations is less than 1%. Measurements were carried out using a hot-wire anemometer. It is shown that satisfactory agreement with theory is achieved at distances from the tube starting from x/D = 5.6 and up to the zone of transition to turbulence (x/D > 35). Turbulence along the jet axis will increase from 1% to 2.5%, while in the mixing layers it increases to 4.7%.

1. Introduction

In the field of fluid dynamics, over the past 20 years, there has been a noticeable interest in studying motion at micro and nano scales. Flows in channels and jets are widely used in MEMS and NEMS (Micro and Nano Electro-Mechanical Systems) [1-2]. Microscales and low velocities lead to a decrease in Reynolds numbers and implementation of laminar and transient regimes of fluid and gas motion.

For the class of jet flows, an essential role is played by the initial conditions: Reynolds number, nozzle geometry, presence of co-current flow, initial degree of turbulence, etc. At present, the jets emanating from confuser-type nozzles at large Reynolds numbers (Re > 10000) are most thoroughly studied due to their great practical importance, especially for aviation [3-4]. The laminar-turbulent transition in such flows occurs within the initial section (x/D < 4-6) [5-6]. The outflow from jet sources of different geometry (holes, pipes and channels, diffusers) has been studied much less [7-10]. At the same time, such configurations are encountered in practice. For example, long tubes are used in jet burners for laminar and transient flow regimes [11].

The first work on the calculation of laminar jets was the Schlichting model [12]. In the Schlichting solution for a round jet, the Prandtl approximation for the boundary layer and the momentum conservation condition are used. This theory does not work well near the source of the jet, where empirical relationships are used [13]. The exact solution for a laminar circular jet within the framework of the Navier-Stokes equation was obtained by Landau-Squire [14-15]. It corresponds to the outflow of a liquid jet into a space filled with the same liquid. In the Landau-Squire approach, two limiting cases are distinguished: “weak” (Re ≤ 1) and “strong” jets (Re >> 1). Unfortunately, there are no enough experimental works devoted to comparison with this theory. For example, for the “weak” jet variant, work [16] was recently published, where experiments were carried out on a water jet.
flowing out of a nanometer-sized tube at Reynolds numbers (Re $<< 1$). We did not find such a comparison in the literature for the version of a “strong” jet.

The number of experimental works on the study of laminar and transitional round jets is limited [7, 10, 17-20]. From the pioneering work of Andrade et al. [17] and until now, the main focus of the authors is on changing the profile of the average velocity. At the same time, the experimental technique after HWA was supplemented by such means as LDA and PIV. Even in the works of Andrade et. al. [17] it turned out that to compare the experimental data with theoretical works, it is necessary to use the coordinates of the virtual source of the jet. This is due to the fact that both in the Schlichting solution and in the analytical formulas of Landau-Squire, the jet propagates from a point source [14-15]. At the same time, in a physical experiment, a jet source always has a finite transverse size. Thus, the problem of the influence of the initial velocity profile is still relevant. Another feature of the jet flow at low Reynolds numbers is an increase in velocity pulsations. So, in theoretical models on the hydrodynamic stability of a long pipe jet, it turned out that the option of a non-exponential growth of velocity pulsations works better [21]. The study of pulsations inside the jet flow in the published literature is presented very little, especially for the pipe jet. In the literature on microjets at low Re numbers, a comparison is often made with turbulent macrojets at large initial Reynolds numbers [22–23]. This paper presents two aspects in a physical experiment with a submerged laminar jet. Firstly, the distribution of the average velocity for the jet which has the initial Poiseuille profile is compared with the Landau-Squire solution. Secondly, for such an initial profile, velocity pulsations in the near zone of the jet are considered.

2. Experimental setup

The experimental setup is shown in figure 1. Air was supplied from the compressor (4) to the supply line (5), then through the flow regulator it entered the tube (7) with diameter $D = 3.2$ mm and length of more than 100$D$. The tube formed a submerged jet, which flowed into the space bounded by a flow chamber made of Plexiglas with a size of 150×150×400 mm. A hot-wire anemometer sensor (8) was placed in the jet field; it could be moved by a coordinate device with a step of 100 $\mu$m along two axes in the range of 0 - 1400 mm. The air flow rate was precisely controlled using a Bronkhorst digital flow controller (6) in the range of 0 - 2 g/s. In this experiments, the Reynolds number corresponded to the value Re = 436 (Re = $U_0D/\nu$, where $U_0$ is the bulk velocity, $\nu$ is the kinematic viscosity). The initial conditions in the jet are the Poiseuille velocity profile, the level of velocity pulsations with one maximum on the axis $Tu = u/U_m*100 = 1$% where $u$ is velocity pulsations (rms), $U_m$ is average velocity of the jet on the axis at the initial cross-section. No artificial disturbances were introduced into the gas path. The thermodynamic parameters at the beginning of the jet corresponded to atmospheric pressure and room temperature. The dynamic characteristics of the jets were measured using a DISA 55M (3) constant temperature hot-wire anemometer. A miniature DISA 55P11 probe (tungsten wire, thread diameter of 5 $\mu$m, thread length of 0.6 mm) was used as a sensor. To collect hot-wire anemometric data and save them on the computer hard disk, a 14-bit E14-140 ADC/DAC (2) with a maximum sampling frequency of 10 kHz was used. The computer (1) was used to collect the hot-wire data and control the air flow regulator. The measurement uncertainty for average velocity measurements was 3-5% and for the absolute value of pulsations rms it was 4-7%.

3. Results

The exact solution of Landau-Squire [14, 15] within the Navier-Stokes equation corresponds to the outflow of a liquid jet from a semi-infinite thin tube into an open space filled with the same liquid. In the Landau approach, there are two limiting cases: “weak” and “strong” jets. The limiting case of a “weak” jet, apparently, is implemented at numbers Re $\leq 1$. In our experiments, the number Re was 436, which was more consistent with the case of a “strong” jet (Re $>> 1$). In this case, the angle $\Theta_b$ approximately corresponding to the jet boundary is defined as [14]:

\[ \Theta_b = \arctan \left( \frac{1}{4Re} \right) \]

\[ \Theta_b = \frac{1}{2} \arctan \left( \frac{1}{4Re} \right) \]
\[ \Theta_0^2 = \frac{64 \pi v^2 \rho}{3P} \quad (1), \]

where \( \nu \) is the kinematic viscosity, \( \rho \) is the density, \( P \) is the total impulse of the jet. The momentum \( P \) in (1) was calculated from experimental data using the integral of the velocity profile at the tube exit.

For small angles \( \Theta \sim \Theta_0 < 3^\circ \) the angular and radial components of the velocity in the spherical coordinate system applied Landau in their theory (in coordinates \( r - \Theta \), where \( r \) is the radius emanating from the jet exit point, \( \Theta \) is the angle between the jet axis and the radius) are as follows [14]:

\[ U_\theta = -\frac{4 \nu \Theta}{(\Theta_0^2 + \Theta^2)r}, \quad U_r = 8 \nu \frac{\Theta_0^3}{(\Theta_0^2 + \Theta^2)^2} r \]

Here, \( U_\theta \) is the velocity component perpendicular to the radius (angular) emanating from a point source of the jet, \( U_r \) is the velocity component directed along this radius (radial). For the velocity on the axis \( \Theta = 0 \), the values of the velocity components in accordance with (2) become \( U_\theta = 0 \), \( U_r = 8 \nu \Theta_0^3 r \). Figure 2 shows a comparison of experimental data on the velocity distribution on the jet axis with the Landau-Squire theory.

**Figure 1.** The experimental setup: 1 – computer; 2 – ADC/DAC; 3 – hot-wire anemometer; 4 – compressor; 5 – supply line; 6 – air flow regulator; 7 – tube; 8 – hot-wire anemometer probe.

**Figure 2.** The velocity distribution on the jet axis along the jet. Experiment - \( Re=436 \), theory – Landau-Squire.
The figure shows the data in dimensionless form. The abscissa is the ratio of distance from the end of the tube $x$ to its diameter $D$. The ordinate is the relation of velocity $U$ on the axis along the jet to the velocity on the axis in the initial section $U_m$. As it can be seen from the figure, the Landau theory satisfactorily agrees with the experimental data at distance $> 15D$ from the end of the tube. In the initial sections, the discrepancy between theory and experiment is noticeable, which can be explained by the restructuring from the tube velocity profile to jet profile.

In the experiment, the profiles of average velocities and velocity pulsations were measured in nine sections along its flow. The measurements were carried out in sections, the closest of which was located at a distance of $x = 0.5$ mm from the tube exit, and the farthest was at $x = 100$ mm. This corresponded to the range of values of the dimensionless coordinates $x/D = 0.16 - 31$.

In the experiments, a coordinate device was used to move the hot-wire anemometer sensor in Cartesian coordinates. The Landau-Squire theory uses a spherical coordinate system. To compare the experimental data with theoretical dependences, it is required to construct the measured velocity profiles also in a spherical coordinate system. For this, the method proposed in [24] was used. The situation with coordinates at a separate point of the jet velocity profile in some cross-section is shown in figure 3.

![Figure 3. Relation between velocity components in spherical and Cartesian coordinate systems.](image-url)

In a cylindrical coordinate system, the velocity vector at point 1 (figure 3) is divided into two components $U_r$ and $U_\theta$ acting along the orthogonal axes of the Cartesian coordinate system (x, y). In a spherical system ($r, \theta$) point 2, (figure 3), the velocity components are divided into radial $U_r$ and angular $U_\theta$ acting, respectively, along the radius outgoing from the jet beginning and perpendicular to it. According to [24], the radial velocity component in a spherical coordinate system is related to the velocity components in a cylindrical coordinate system by the formula $U_r^2 + U_\theta^2 = U_r^2 + U_\theta^2$. Since at small angles ($\theta < 3 ^\circ$) $U_r << U_r$ and $U_\theta << U_r$, one can approximate $U_r \sim U_r$. Then, the radial velocity at point 2 in figure 3 is equal to $U_r = U_r^* (r^2/r)$, $U_\theta = U_\theta = \frac{8\pi \theta r}{\theta^2}$, $\theta_\theta$ is calculated by formula (1). The profiles of average velocities processed in this way are shown in figure 4a.

Comparison of experimental data and Landau-Squire theory is presented in dimensionless form. So, along the abscissa axis is the ratio of current angle $\theta$ and the angle corresponding to the jet boundary $\theta_0$. The ratio of average velocity $U$ and the maximum velocity in the local cross-section $U_{cl}$ is on the ordinate axis.

The profile of average velocities (figure 3a) in the initial section $x/D = 0.16$ differs sensibly from the theoretical profile of the Landau-Squire theory, which is explained by the proximity to the tube
exit. The velocity profile, most likely, does not have time to rebuild from pipe distribution to jet distribution. This correlates with the data in figure 2 and explains the discrepancy between the axial velocity in the initial sections of the jet and theory. In the downstream sections ($x/D = 12.5$ and $x/D = 31$), the experimental data are in satisfactory agreement with the theory.

Figure 4. Average velocity profiles (a) and profiles of turbulent pulsations (b) along the jet, theory - Landau-Squire.

Figure 4b shows the profiles of the turbulence degree ($Tu = u/U_{cl}*100\%$) which represent the ratio of velocity pulsations $u$ to the maximum velocity $U_{cl}$ in the local cross-section. The transverse coordinate $y$ is related to the tube radius ($R = D/2$). As it follows from the figure, in the section closest to the exit from the tube ($x/D = 0.16$), pulsations have one maximum on the jet axis ($Tu = 1\%$), which corresponds to a maximum in the distribution of velocity pulsations in a laminar flow in the tube. Turbulence on the jet axis tends to increase downstream slightly. More significant flow turbulization is observed in the mixing layer of the jet. Despite the fact that the jet, as it is shown by comparison with the theory in figure 4a, corresponds to the Landau-Squire laminar solution along the entire length, pulsations in the mixing layers reach a level of 4.7% at $x/D = 31$. Downstream, according to our previous works, a transition to a turbulent flow occurs [10].

4. Conclusions
A study of the characteristics of a submerged axisymmetric jet with $Re = 436$, flowing from a tube with a diameter of $D = 3.2$ mm and a length of more than $100D$, has been carried out. The initial conditions in the jet are the Poiseuille velocity profile, the level of velocity pulsations with one maximum on the axis $Tu = 1\%$. Profiles of average velocities and velocity pulsations were measured in several cross sections of the jet. A comparison of experimental data on the distribution of the average velocity with the Landau-Squire theory is carried out. It is shown that in the initial sections there is a discrepancy between the experiment and the theory, while in more distant sections ($x/D = 5.6 - 31$) the agreement with the theory is satisfactory. Measurements have shown that turbulence on the jet axis tends to grow slightly up to 2.5%, while in the mixing layers there is an intense increase in pulsations of up to 4.7%.

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