Circuit Modelling for Dielectric Layers in Electrowetting Devices

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Abstract. In this paper, we present an electromechanical model for the electrowetting based micro-droplet driving device. We developed an equivalent circuit for this microfluidic device by using the method of lumped parameter electromechanics. The voltage distributions across the device dielectric layer and the droplet were calculated based on the model. The actuation force for the parallel-plate device was derived according to the principle of virtual work. Based on the force calculation, we studied the effects of droplet conductivity, dielectric constant, and the electric field frequencies on the electrical force exerted on the liquid droplet.

1. Introduction
In digital microfluidics, many methods have been presented to manipulate microdroplet. These include the direct modulation of liquid surface tension through the application of the temperature distribution [1], surface acoustic wave method [2] and electrical actuation method [3,4]. Among methods to manipulate liquid microdroplets, electrical actuation has many advantages, primarily voltage-driven process control [5]. The principle of electrical actuation are electrowetting-on-dielectric (EWOD) for conductive droplets and dielectrophoresis (DEP) for dielectric droplets[6-8]. EWOD has been a widely used method due to the high driving speed and flexible operation.

When a conductive droplet is placed on top of dielectric surfaces, a voltage applied between the electrode patterned beneath the dielectric and the droplet will result in a reduction in the solid/liquid contact angle. This phenomenon exemplifies the electrowetting effect. A typical electrowetting phenomenon is shown in figure 1.

Electrowetting was first discovered by Gabriel Lippmann [9] in 1875 when he observed the rise of mercury under the action of an electric field. He derived an equation to describe this electro-capillary effect, and his equation is now called Lippmann’s law:

$$d\gamma_{SL}^{eff} = -\rho_{SL}dU$$

Where $\gamma_{SL}^{eff}$ is the effective solid/liquid interfacial tension, $\rho_{SL}$ is the surface charge density of the counter-ions, and U is the applied voltage potential.

In the early 1990s, Berge[10] introduced a dielectric layer between the droplet and the metal electrode to solve the problem of electrolysis. This scheme is the basis of the electrowetting-on-dielectric (EWOD) effect. Berge described the relationship between the change in contact angle and the applied electric voltage by the Lippmann-Young equation:
\[
\cos \theta(V) = \cos \theta(0) + \frac{\varepsilon_0 E}{2\gamma_{LG}} V^2
\]

Where \(\gamma_{LG}\) indicate the liquid-gas interfacial tension, \(t\) is the thickness of dielectric layer, \(\varepsilon_0\) is the permittivity of vacuum, \(\varepsilon_r\) is the dielectric constant of dielectric, \(V\) is the applied voltage, \(\theta(V)\) and \(\theta(0)\) indicate the contact angle with and without applied voltage.

EW-based microfluidic devices were first demonstrated in 2000 [11]. The EWOD actuator consisting of two parallel plates is presented in figure 2. Droplets are sandwiched between parallel plates. The bottom plate consists of an array of independently addressable electrodes, the electrodes are covered with the dielectric layer and the hydrophobic layer. The top plate consists of a continuous ground electrode covered by hydrophobic layer. When a potential is applied to the electrodes adjacent to the droplet, the droplet can be moved to the activated electrode. A wide variety of fundamental fluid operations have been realized on EWOD devices by selectively applying voltage to an array of electrodes, including droplet dispensing [12-15], transport [15], separation [12,14], coalescence [12], and others.

![Figure 1](image1.png)

**Figure 1.** The electrowetting effect. (a) A droplet resting on top of the surface; (b) The contact angle is reduced upon applied voltage.

![Figure 2](image2.png)

**Figure 2.** The electrowetting-based parallel-plate microfluidic device.

The actuation force is an important factor in the performance evaluation of the electrowetting-based chip. The main ways to calculate the actuation force are the contact angle method [16] and the method of lumped parameter electromechanics [7]. The contact angle method is based on the change of contact angle. To calculate the contact angle, Mugele [17] and Berthier [18] detailed two different approaches: the classical thermodynamic approach and the energy minimization approach. In the classical thermodynamic approach, the electrowetting effect can be viewed as a change of the value of the effective solid–liquid surface tension, and it results in a change of the contact angle. In the energy minimization approach, the change of the contact angle is explained by the consequence of a competition between dielectric energies. However, the contact angle method is limited by the contact angle saturation and cannot reflect the effect of the applied electric field frequency.
In this paper, we presented a circuit model that can be used to calculate the forces acting on conductive and dielectric liquids in the electrowetting-based parallel-plate microdroplet device. The actual parameters of device and liquid used in the experiment have been plugged in the model to calculate the electromechanical force on the droplet. The factors affecting the force e.g. droplet conductivity, dielectric constant, and the voltage frequency were discussed.

2. Circuit modelling
The electrowetting actuation force is calculated by using the method of lumped parameter electromechanics [7]. This method represents electromechanical systems by discrete circuit components, it can avoid the difficulty of analyzing the geometry and details of liquid profiles. The first step to calculate the total electromechanical force is to determine the resistance and capacitance of each circuit element.

![Figure 3. Equivalent circuit model for the parallel-plate device.](image)

Figure 3 shows a parallel-plate microfluidic device and its equivalent circuit. This RC circuit is similar to those of Jones [7] and Chatterjee [19]. In diagram, the left side represents the droplet filled region, and the right side is the medium region. In this paper, the ambient medium is air, but the model is applicable to any medium. The air is modeled as pure capacitance. The slightly conductive droplet is modeled as a resistor and capacitor in parallel. The hydrophobic layer (Teflon-AF) on the top plate, and the hydrophobic (Teflon-AF) / dielectric (Parylene) layers on the bottom plate are assumed to be perfect dielectrics (capacitors). For simplicity, the Teflon-AF layer on the top plate and the Teflon-AF / Parylene layers on the bottom plate are grouped together as capacitor \(C_{ptt}\) in the following calculation.

The capacitances and conductance of each element in figure 3 are:

\[
C_L = \frac{\varepsilon_0 k_L x_w}{D}\quad (3a)
\]

\[
g_L = \frac{\sigma_L x_w}{D}\quad (3b)
\]

\[
C_M = \frac{\varepsilon_0 k_{al} x_w}{D}\quad (3c)
\]

\[
C_{ptt} = \frac{1}{\frac{1}{C_t} + \frac{1}{C_t} + \frac{1}{C_d}}\quad (3d)
\]
\[ C'_{pt} = \frac{1}{C_t + \frac{1}{C_{pt}} + \frac{1}{C_d}} \]  

Where \( x \) is the length of liquid filled region of the activated electrode, \( \varepsilon_0 \) is the permittivity of free space, \( k_L \) is the dielectric constant of liquid, \( w \) is the width of electrode, \( \sigma_L \) is the conductivity of liquid, \( k_t \) is the dielectric constant of hydrophobic (Teflon-AF) layer, \( k_d \) is the dielectric constant of dielectric (Parylene) layer. We determine the impedance of each circuit element and evaluate the voltage across each element. Let \( Z_1, Z_2 \) be the impedances of the liquid and the combined dielectric/hydrophobic layers for the liquid filled side, and \( Z_3, Z_4 \) be the impedance of air and the combined layers for the medium side.

\[ Z_1 = \frac{1}{g_L + j\omega C_L}, \quad Z_2 = \frac{1}{j\omega C_{pt}}, \quad Z_3 = \frac{1}{j\omega C_M}, \quad Z_4 = \frac{1}{j\omega C_{pt}} \]  

Where \( \omega \) is the angular frequency of applied voltage, and \( \omega = 2\pi f \), where \( f \) is the frequency of applied voltage. The corresponding voltages defined in figure 3 are:

\[ V_1 = \frac{Z_1}{Z_1 + Z_2} V, \quad V_2 = \frac{Z_2}{Z_1 + Z_2} V, \quad V_3 = \frac{Z_3}{Z_3 + Z_4} V, \quad V_4 = \frac{Z_4}{Z_3 + Z_4} V \]  

The actuation force for the parallel-plate device was derived according to the principle of virtual work, which states that the total work done by an external force acting on a body initially in equilibrium plus electrical energy input is equal to the internal energy stored in the body at the new state. The electromechanical force is equal to the derivative of stored energy with respect to the displacement.

There are four capacitive energy terms recognized in figure 3. The total energy \( U \) of the system is

\[ U = \frac{1}{2} \sum_{i=1}^{4} C_i V_i^2. \]  

Thus, the total electromechanical force \( F \) acting on the liquid is \( F = \frac{\partial U}{\partial x} \). Combining with equation (3) to (5), we obtain:

\[ |V_1| = \frac{\omega \varepsilon_0 k_{pt} D}{\left[ \frac{\omega^2 \varepsilon_0^2}{\omega^2 \varepsilon_0^2 (k_L (2d' + d) + k_{pt} D)^2 + \sigma_L^2 (2d' + d)^2} \right]^{1/2}} \]  

\[ |V_2| = \left[ \frac{\omega^2 \varepsilon_0^2 k_L^2 + \sigma_L^2}{\omega^2 \varepsilon_0^2 (k_L + k_{pt} D)^2 + \sigma_L^2} \right]^{1/2} V \]  

\[ F = \frac{1}{2} \varepsilon_0 w k_{pt} V^2 \left\{ \frac{\sigma_L^2 (2d' + d) + k_L \varepsilon_0 \omega^2 \left[ D k_{pt} + k_L (2d' + d) \right]}{\sigma_L^2 (2d' + d)^2 + \varepsilon_0 \omega^2 \left[ D k_{pt} + k_L (2d' + d) \right]^2} - \frac{k_{air}}{D k_{pt} + (2d' + d)k_{air}} \right\} \]
Where \(k_{\text{ptt}}\) is an equivalent dielectric constant for the combined dielectric/hydrophobic layers, \(k_{\text{ptt}} = \frac{k_{d}k_{d'}(2d'+d)}{k_{d}d + 2k_{d'}d}\), and \(k_{\text{air}}\) is the dielectric constant for air medium, \(k_{\text{air}} = 1\).

3. Results and discussion

3.1 Frequency-dependent nature

The electromechanical forces acting on the droplet in the parallel-plate device is calculated and plotted in the figure 4. The curve in the figure shows the correlation between the actuation force and the frequency of applied voltage. At low frequencies (EWOD region), it is primarily the EWOD force that causes droplet movement. At high frequencies (DEP region), the DEP force causes droplet movement. The transition between two regions can be accounted for by defining a critical frequency \(f_c\) [19] which a liquid transitions from conductive to dielectric behavior. The critical frequency was calculated by analysis of the liquid side circuit:

\[
    f_c = \frac{g_l}{2\pi(C_L + C_{\text{ptt}})}
\]  

Figure 4. The electromechanical forces acting on a 100 nL water droplet in parallel-plate device. All the applied AC voltages are set to be 100 Vrms.

When \(f << f_c\): \(V_1 \approx 0\), and \(V_2 \approx V\), calculated by (6.1) and (6.2). The liquid behaves as a pure conductor. Almost the voltage drops across the dielectric layers. This is the so-called EWOD region. Substituting \(V_1\) and \(V_2\) into equation (6.3), and because \(D >> (2d'+d)\), the electromechanical force is simplified to

\[
    F \approx \frac{1}{2} \varepsilon_0 \omega \frac{k_{\text{ptt}}}{(2d' + d)} V^2
\]

This equation can also be derived from the contact angle models[17,18].

When \(f >> f_c\), Neglecting the resistor in the circuit, the voltages in equation (6) are simply
\[ V_1 = \frac{k_{pl} D}{k_{pl} D + k_L (2d' + d)} V, \quad V_2 = \frac{k_L (2d' + d)}{k_{pl} D + k_L (2d' + d)} V \] (9)

Because \( D \gg (2d' + d) \), most of the voltage drop occurs across the liquid, the liquid is purely insulating and the DEP force causes the droplet movement. This is the DEP region. Using these results in the equation (6,3), we obtain

\[ F_{\text{DEP}} = \frac{\varepsilon_0 k_{pl} W^2}{2} V^2 \left( \frac{k_L}{k_L (2d' + d) + k_{pl} D} - \frac{k_{air}}{k_{air} (2d' + d) + k_{pl} D} \right) \] (10)

This equation expresses the DEP force acting on the droplets are exposed to the air. This equation is also usable for a liquid medium by replacing \( k_{air} \) with the dielectric constant of the medium.

### 3.2 Influence of liquid properties

Equation (6) reveals the influence of the conductivity \( (\sigma_L) \) and the dielectric constant \( (k_L) \) of the electromechanical force. See figure 5. When the \( \sigma_L \) increases with \( k_L \) fixed, the electromechanical force curve moves to high frequencies, but the force in low and high frequencies remain unchanged. This condition can be explained by the equations (8) and (10), which are independent of the conductivity. However, the critical frequency, \( f_c = \frac{g_{\text{water}}}{2\pi (C_{\text{water}} + C_{\text{total}})} = \frac{\sigma_L}{2\pi\varepsilon_0 (k_L + k_{pl} D)} \), is directly proportional to the liquid conductivity. Thus the main influence of liquid conductivity is on \( f_c \), the transition frequency between EWOD and DEP.

![Figure 5](image)

**Figure 5.** The influence of liquid conductivity on frequency-dependent behavior of force. The electromechanical force is calculated based on equations (6,3) for \( k_L = 78 \). The device parameters used in calculation are from the actual experimental equipment.

As shown in figure 6. When the liquid dielectric constant increases, the electromechanical force in EWOD region remains unchanged, but the force is increased in DEP region. This can also be explained by the equations (8) and (10). On the other hand, the liquid behaves like a conductor at EWOD frequency, which is independent of the dielectric constant. In DEP region, the liquid behaves like a dielectric, so the force is strongly influenced by \( k_L \).
Figure 6. The influence of liquid dielectric constant on frequency-dependent behavior of force. The electromechanical force is calculated based on equations (6) for $\sigma_L = 5.5 \times 10^{-6}$ S/m. The device parameters used in calculation are from the actual experimental equipment.

When the liquid conductivity increases with $f$ fixed, the electromechanical force curves in figure 7 are almost symmetric with the curves in figure 6. This is because the $f_c$ is decreased, when the liquid conductivity decreases. The voltage frequency, $f = 50$Hz, is a high frequency compared to the $f_c$, when the conductivity is small enough. So the force is strongly influenced by $k_L$ in low conductivity region. In high conductivity region, the $f_c$ is a low frequency compared to the $f_c$, and the dielectric property of liquid has no effect on the force.

Figure 7. The influence of liquid conductivity on frequency-dependent behavior of force. The electromechanical force is calculated based on equations (6) for $f=50$Hz. The device parameters used in calculation are from the actual experimental equipment.

4. Conclusions
In this paper, we presented an electromechanical model for the two-plate digital microfluidic device. We derived an equation consisting of chip parameters, conductivity and dielectric constant of liquid
and applied voltage to calculate the actuation force for the parallel-plate device according to this model. The transition between conducting and insulating behavior of droplet depends on the critical frequency $f_c$. Using this model, we studied the effects of conductivity, dielectric constant of fluids on electromechanical force. The liquid conductivity is directly proportional to the value of $f_c$, and the liquid dielectric constant has a great influence on the actuation force in high frequency DEP region. This model reveals the relationship between voltage frequency and actuation force, and provides a general and simple method to calculate the electromechanical force.

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