In the minimal supersymmetric standard model (MSSM) properties of the third generation sparticles are important from the viewpoint of discriminating the SUSY breaking models and in the determination of the Higgs boson mass. If gluinos are copiously produced at CERN LHC, gluino decays into \( tt \) through stop and sbottom can be studied using hadronic decays of the top quark. The kinematical endpoint of the gluino decays can be evaluated using a \( W \) sideband method to estimate combinatorial backgrounds. This implies that fundamental parameters related to the third generation squarks can be reliably measured. The top-quark polarization dependence in the decay process may also be extracted by looking at the \( b \) jet distribution near the kinematical endpoint.

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The minimal supersymmetric standard model (MSSM) is one of the promising extensions of the standard model (SM). The model requires superpartners of ordinary particles (sparticles), and LHC at CERN might confirm the existence of these new particles [1]. Since the sparticle masses are related to the SUSY breaking mechanism, measurement of the masses provides a way to probe the origin of SUSY breaking in nature.

The masses and mixings of stops (\( \tilde{t}_{1,2} \) and sbottoms (\( \tilde{b}_{1,2} \)) are sensitive to the flavor structure of scalar masses. First, the diagonal masses in the stop and sbottom mass matrices, \( m_{\tilde{t}_{1}}, m_{\tilde{t}_{2}}, \) and \( m_{\tilde{b}_{1,2}} \), are predicted to be less than those of other squarks in the minimal supergravity (MSUGRA) model [2] because of the Yukawa RGE running effects. Some SUSY breaking models, such as the flavor U(2) model [3] or the decoupling solution [4], and the superconformal model [5] for the SUSY flavor problem, have also special imprints on these parameters. The \( t_{L}-t_{R} \)-Higgs trilinear coupling \( A_{t} \) at the weak scale has a large coefficient proportional to \( m_{t} \), resulting in a large left-right mixing of stops; \( m_{L_{R}}^{2} = m_{t}(A_{t} - \mu \cot \beta) \). In the MSUGRA \( A_{t} \) is proportional to the universal gaugino mass \( M \) at the GUT scale \( M_{GUT} \), and insensitive to the \( A \) parameter at \( M_{GUT} \) [6]. Indeed, this is one of the robust predictions of SUSY breaking at \( M_{GUT} \). These relations are not guaranteed if the SUSY breaking mediation scale is much smaller than \( M_{GUT} \). It should be stressed that the stop masses and the mixing are very important parameters to predict the light Higgs mass [7].

It is possible to study the stop and sbottom at LHC through the gluino (\( \tilde{g} \)) decay modes listed below.

\[ \begin{align*}
\text{I)} & \quad \tilde{g} \to \tilde{b}_{1} \to bb\chi_{1}^{0} \to bbl^{-}\chi_{1}^{0}, \\
\text{II)} & \quad \tilde{g} \to t_{L} \to t_{L}\chi_{\pm}, \\
\text{III)} & \quad \tilde{g} \to t_{L} \to tb\chi_{1}^{0}, \\
\text{IV)} & \quad \tilde{g} \to \tilde{b}_{1} \to tb\chi_{1}^{0}. \\
\end{align*} \]  

In previous literatures [1, 8], the third generation sfermions are often studied using the mode I) (\( bbl^{-}\) channels). This mode is important when \( \tilde{\chi}_{1}^{0} \) has substantial branching ratios into leptons. Measurement of the kinematical endpoints of the signal distributions tells us the sparticle masses. Unfortunately, the branching ratios could be very small, and this mode is insensitive to the stop. We tried to study the mode II) in Eqs. (1), but the result was unsuccessful because of the small branching ratio in the MSUGRA.

In this paper we try to measure the endpoints of the modes III) and IV) in Eqs. (1). The decay modes are expected to be dominant in the MSUGRA since \( \tilde{\chi}_{1}^{0} \) is likely to be \( SU(2)_{L} \) gaugino-like and \( Br(\tilde{b}_{1}, \tilde{t}_{1} \to \tilde{\chi}_{1}^{0}) \) could be as large as 60%. We focus on the reconstruction of hadronic decay of the top quark, because the distribution of the \( tb \) invariant mass \( M_{tb} \) makes a clear endpoint in this case.

The parton level distribution of the \( tb \) final state invariant mass is expressed as a function of \( m_{\tilde{g}}, m_{\tilde{t}_{1}}, m_{\tilde{b}_{1}}, \) and chargino mass \( m_{\tilde{\chi}_{1}^{+}} \): d\( \Gamma/dm_{tb} \propto m_{tb} \), and the endpoints \( M_{tb} \) for the modes III) and IV) are written as follows:

\( M_{tb}^{2}(\text{III}) = \frac{m_{t}^{2} - m_{\tilde{t}_{1}}^{2}}{2m_{\tilde{t}_{1}}} \left\{ (m_{\tilde{g}}^{2} - m_{\tilde{t}_{1}}^{2} - m_{\tilde{b}_{1}}^{2}) + \sqrt{(m_{\tilde{g}}^{2} - (m_{\tilde{t}_{1}} - m_{\tilde{b}_{1}})^{2})(m_{\tilde{g}}^{2} - (m_{\tilde{t}_{1}} + m_{\tilde{b}_{1}})^{2})} \right\}, \)

\( M_{tb}^{2}(\text{IV}) = \frac{m_{t}^{2} - m_{\tilde{t}_{1}}^{2}}{2m_{\tilde{b}_{1}}} \left\{ (m_{\tilde{b}_{1}}^{2} - m_{\tilde{t}_{1}}^{2} - m_{\tilde{\chi}_{1}^{0}}^{2}) + \sqrt{(m_{\tilde{b}_{1}}^{2} - (m_{\tilde{\chi}_{1}^{0}} - m_{\tilde{t}_{1}})^{2})(m_{\tilde{b}_{1}}^{2} - (m_{\tilde{\chi}_{1}^{0}} + m_{\tilde{t}_{1}})^{2})} \right\}. \)

Note that the endpoint of the final state \( tb\tilde{\chi}_{1}^{-} \) should be sensitive to both \( \tilde{t}_{1} \) and \( \tilde{b}_{1} \).

In order to demonstrate the endpoint reconstruction, we take an MSUGRA point with the scalar mass \( m = \)}
100GeV, the gaugino mass $M = 300$GeV, the $A$ parameter $-300$GeV at the GUT scale, $\tan \beta = 10$ and $\mu > 0$. This corresponds to the sample point A1 in Table 1. The masses and mixings of sparticles are calculated by ISASUSY/ISASUGRA [9]. The Monte Carlo SUSY events are generated by PYTHIA [10] using the masses and mixings, and then passed through a fast detector simulation program for the ATLAS experiment [11]. Jets are reconstructed by a cone-based algorithm with $\Delta R = 0.4$. The $b$ and $\tau$ tagging efficiencies are set to be 60% and 50%, respectively.

In our study we apply the following selection for the $tb$ signal: 1) $E^\text{miss}_T > 200$ GeV, 2) $m_{\text{eff}} > 1000$ GeV ($m_{\text{eff}} = E^\text{miss}_T + \sum_{\text{all jets}} p_T$), 3) two and only two $b$-jets with $p_T > 30$ GeV, 4) $4 \leq n_{\text{jet}} \leq 6$ ($n_{\text{jet}}$, number of additional jets with $p_T > 30$ GeV and $|\eta| < 3.0$). In addition, events with leptons are removed to reduce the background from $t\bar{t}$ production. At this stage the number of remaining $tt$ events is rather small, about 10% of the remaining SUSY events for the point A1.

To reconstruct the hadronic decay of the top quark, we first take a jet pair consistent with a hadronic $W$ boson decay with a cut on the jet pair invariant mass $m_{jj}$; $|m_{jj} - m_W| < 15$GeV. The invariant mass of the jet pair and one of the $b$ jets, $m_{bjj}$, is then calculated. All possible combinations are tried in an event, and the combination which minimizing $|m_{bjj} - m_t|$ is chosen. The jet combination is regarded as a top candidate if $|m_{bjj} - m_t| < 30$GeV. The energy and momentum of the jet pair are then rescaled so that $m_{jj} = m_W$.

The expected $tb$ endpoint is not clearly visible in the $m_{tb}$ distribution shown in Fig. 1(a). As there are 7 to 8 jets on average in a selected event, events with a fake $W$ boson (and a fake top quark) dominate the distribution. The distribution of the fake $W$ events is estimated from the events that contain jet pairs with the invariant mass in the ranges A) $|m_{jj} - (m_W - 15$GeV) $< 15$ GeV and B) $|m_{jj} - (m_W + 15$GeV$) < 15$ GeV; ‘the $W$ sidebands’. The energy and momentum of the jet pairs are then rescaled linearly to be in the range $|m_{jj} - m_W| < 15$GeV. The fake top candidates are reconstructed from the rescaled jet pairs and $b$ jets in the events.

The estimated background distribution is shown in

![Fig. 1](image)

Fig. 1(b), which is obtained by averaging distributions from the sidebands A) and B). The estimation is based on an assumption that most of the jets in the events do not have significant correlation with the $b$ jets in the events. The estimated background distribution is subtracted from the signal distribution in Fig. 1(c). The estimated correct signal distribution (c) shows the better endpoint compared to (a). Fig. 1(d) is the same distribution as Fig. 1(c) but for the events which contain the mode III), the mode IV), or a decay mode irreducible to the mode III); $\tilde{g} \to \tilde{b}_1 \bar{b} \to \tilde{t}_1 (Wb) \to b\tilde{\chi}_1^-(bw)$. Note that if $(bw)$ has an invariant mass consistent to a top, the decay is kinematically equivalent to the mode III). Fig. 1(d) shows the expected clear edge at the right place ($M_{tb}(\text{III}) = 476$GeV and $M_{tb}(\text{IV}) = 420$ GeV), demonstrating that the sideband method works well. Here, the number of the generated SUSY events is $3 \times 10^6$, which corresponds to an integrated luminosity of 120fb$^{-1}$. The plots do not include the SM backgrounds.

Note that the signals from the modes III) and IV) in Eqs. (1) are significant in the total selected events. We fit the total distribution shown in Fig. 1(c) by a simple fitting function, which is described as a function of the endpoint $M_{tb}^\text{fit}$, the edge height $h$, and the smearing parameter $\sigma$ from the jet energy resolution. We assume that the signal distribution is sitting on a linearly-decreasing background. The $M_{tb}^\text{fit}$ is compared with the weighted endpoint $M_{tb}^\text{w}$ defined by

$$M_{tb}^\text{w} = \frac{Br(\text{III})M_{tb}(\text{III}) + Br(\text{IV})M_{tb}(\text{IV})}{Br(\text{III}) + Br(\text{IV})}.$$
where \( Br(\text{III}) \) and \( Br(\text{IV}) \) are branching ratios for the modes (III) and (IV), respectively. The fit is shown in Fig. 2 (a). We obtain \( M_{tb}^{\text{fit}} = 444.2 \pm 7.4 \) GeV, which is consistent with \( M_{tb}^{\text{ex}} = 459 \) GeV. The \( M_{tb}^{\text{fit}} \) changes moderately when one changes the \( m_{tb} \) range used for the fit. We choose the range so that the significance of the height \( S = h/\Delta h \) is at maximum. For the fit shown in Fig. 2(a), \( S(\text{max}) = 196.9/15.2 = 13.0 \) is obtained.

In order to check the availability of the endpoint measurement, we study twelve sample points in total, including the previous point, shown in Table 1, and compare \( M_{tb}^{\text{fit}} \) and \( M_{tb}^{\text{ex}} \). We choose two reference MSUGRA points as A1 and A2, where \( m = 100 \) GeV, \( M = 300 \) GeV, \( \tan \beta = 10, \mu > 0 \), and \( A = 300 \) GeV. We also study points with different mass spectrums from the MSUGRA predictions; two points with gluino masses heavier than the reference points, (G1, G2), and two points with modified stop masses (T1, T2). Furthermore, we include the MSUGRA points selected from [12] (B,C,G,I) and two non-SUGRA points E1 and E2 where the gluino decays exclusively into \( \tilde{t}_1 \). The result is summarized in Fig. 2 (b)[14]. The error bars represent the statistical errors for \( 3 \times 10^5 \) SUSY events at each point, and the systematic error of the jet energy scale (1%) is not included (see Table 1 for SUSY cross sections). This plot shows an impressive linearity between the expectation and the MC fits, although \( M_{tb}^{\text{fit}} \) is systematically lower than \( M_{tb}^{\text{ex}} \). This is reasonable since some of particles are always out of the cone to define jets. This effect may be corrected by comparing distributions with different jet definitions.

Another uncertainty may come from the jet fragmentation. If events are generated by ISAJET for the point A1, the reconstructed endpoint is smaller by 10%, and the number of events after the sideband subtraction is smaller by a factor of 1.5. The difference comes from the different jet fragmentation schemes. ISAJET radiates more soft jets for a parton, resulting in more background and smeared endpoint distribution. The event generators must be tuned carefully to extract the kinematical information from the signal distribution.

We now discuss the physics that might be studied with the tb endpoint measurement.

We cannot determine all of the relevant mass parameters from only the tb endpoint measurement, since the endpoint \( M_{tb} \) depends on \( m_{\tilde{g}}, m_{\tilde{b}} \), and \( m_{\tilde{\chi}} \). Thus, the study of \( bbl^+l^- \) final state would be important to single out the possible \( t \) and \( b \) contributions to the \( tb \) final state and to proceed to a model-independent study. For example, let us assume that errors of the endpoints \( M_{bbll} \) and \( M_{ull} \) are 10 GeV and that of the endpoint \( M_{ttll} \) is 30 GeV in the measurement. Here, \( b_1 \) is one of the two \( b \) jets for which the invariant mass \( m_{bbll} \) is larger. When we generate \( m_{\tilde{g}}, m_{\tilde{t}} \), \( \tilde{\chi}_i \), and \( \tilde{\chi}_i \) randomly around the reference point A1 fixing the endpoint \( M_{ttll} \) (which is expected to have a very small error), and require \( \Delta \chi^2 = \sum_i (M_i - M_i^{\text{fit}})^2 / \Delta M_i^2 < 1(9) \) (i runs over the possible endpoints), the deviation \( M_{tb}^{\text{fit}}(\text{IV}) - M_{tb}^{\text{fit}}(\text{III}) \) is always less than 15(45) GeV. For the point A1, \( M_{tb}^{\text{fit}}(\text{IV}) = 420 \) GeV and \( M_{tb}^{\text{ex}} \approx 460 \) GeV. The difference of \( M_{tb}^{\text{fit}}(\text{IV}) \) and \( M_{tb}^{\text{ex}}(\text{III}) \) therefore may be statistically distinguishable.

The measurement of the SUSY breaking parameters in different sectors might reveal an overall inconsistency of the SUSY breaking mediation models. The distribution of the invariant mass formed by combining the highest \( P_T \) jet and a same-flavor and opposite-sign lepton pair (jll channels) is sensitive to \( m_{\tilde{g}} \) and \( m_{\tilde{t}} \), and this may lead to the determination of \( m \) and \( M \) in the MSUGRA [1, 13]. Once \( M \) and \( m \) are fixed, they strongly constrain the endpoint \( M_{tb}^{\text{fit}}(\text{V}) \) in the MSUGRA — by comparing it with the measured value, one should be able to check if \( m_{\tilde{t}} \) or \( m_{\tilde{\chi}} \) are consistent to the MSUGRA predictions.

Note also that the formulae of \( M_{tb} \) or \( M_{bbll} \) involve \( m_{\tilde{\chi}} \), while the first generation squark mass is lower bounded by \( m_{\tilde{g}} \) in the model. If \( m_{\tilde{g}} \gg m_{\tilde{q}} \) is established by combining the squark mass scale determined through a \( jll \) analysis [1] and the analysis of the final state involving \( b \) jets, we can show that some new physics should occur below the scale where \( m_{\tilde{g}}^2 < 0 \). Note that, in the points G1 and G2 in Table 1, where the gluino mass is increased by 100 GeV from the MSUGRA predictions, \( m_{\tilde{\chi}}^2 \) becomes negative at the GUT scale. We study the jll distributions for the point G2 in a similar way as given in Ref. [13] and find that the jll endpoints are successfully reconstructed. Therefore the information on \( m_{\tilde{g}} \) should be obtained in this case.

In the framework of the MSUGRA, the measurement of \( M_{tb} \) is sensitive to the GUT scale \( A \) parameter. This effect is large when \( M \times A < 0 \) as can be seen in Fig. 3. Here we take \( M \sim m \) and \( M = 230 \) GeV, and vary \( A \) so that \( |A| < 3m \). The \( M_{tb}^{\text{ex}} \) and \( m_{\tilde{t}_i} \) vary by 50 GeV and 150 GeV, respectively, and the changes are again detectable. Note that the \( \tilde{\chi}_i \) decay into \( ll \) is closed in this case, therefore the information from the \( bbl^+l^- \) channels is not available. If \( m = 100 \) GeV and \( A \) is varied from \(-300 \) GeV to \( 300 \) GeV, \( M_{tb}^{\text{ex}} \) and \( m_{\tilde{t}_i} \) change by 30 GeV.

**Fig. 2:** (a) A fit to the \( m_{tb} \) distribution (point A1), and (b) comparison of \( M_{tb}^{\text{fit}} \) and \( M_{tb}^{\text{ex}} \) for the sample parameters given in this paper.
In this paper, we try to reconstruct final states which consisting of hadronic jets at LHC. This was considered difficult due to the large combinatorial background, but is overcome by a W sideband method developed to estimate the background. We reconstruct the \( t\bar{b} \) final state from the event containing two \( b \) jets. The reconstructed endpoint provides us an access to the gluino and the third generation sparticle masses without relying on the decay modes including leptons. The correct reconstruction of the events also allows us to consider the top-quark polarization dependence of the distribution. This information is important to determine the radiative correction to the Higgs mass, as well as the origin of SUSY breaking.

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[15] For events with \( m_{tt} < M_{th}(III) - 50 \text{ GeV} \) the gluino decay into \( \tilde{b}_1 \) starts to contribute.