A method of joint high-precision estimation of range and velocity in a radar using ultra-wideband frequency coded waveforms

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Abstract. The article describes a method of joint high-precision estimation of range and velocity in ultra-wideband short-range radar systems. The peculiarity of the method is the two-dimensional range-velocity compression function based on ambiguity function of ultra-wideband discrete frequency coded waveforms. Expressions of ambiguity function and compression function such as a block diagram of the radar transceiver for the proposed method are presented. Experimental data confirm the efficiency of the method.

1. Introduction
At the moment, in radar systems, there are various methods for estimating the velocity of small targets, such as small unmanned aerial vehicles (UAVs). The new methods are based on the use of the millimeter wave range and narrow-band step-frequency continuous-wave (SFCW) signals [1-3]. This article proposes a method for joint estimation of range and velocity in radars using ultra-wideband (UWB) signals with SFCW modulation and other waveforms of discrete frequency coded waveforms (DFCW) [4] based on the UWB ambiguity function (AF). A simplified expression for AF of UWB DFCW in range-velocity coordinates with corresponding simplification conditions is given. A distinctive feature of this expression from previously presented [5] is the ability to evaluate the compression characteristics for UWB DFCW with an unequal frequency step and unequal duration of the frequency components. A block diagram of the implementation of the radar for this method is presented, which allows one to work with signals in the band from 50 MHz to 6 GHz, as well as an expression for compressing a digital signal at the ADC output. The results of the velocity estimation for a target with an EPR of the order of 0.125 m² are presented by the proposed method compared to the convenience [6] method of estimating the velocity by two successively obtained High Resolution Range Profiles (HRRPs).

2. Materials and methods
2.1. DFCW ambiguity function
In the “classical” form proposed by Woodward in his monograph [7], the ambiguity function of a narrow-band signal is a convolution of the signal with its complex conjugate copy shifted in time and
frequency, where the time shift is caused by the delay of the signal to propagate and the frequency shift displays the Doppler effect due to the relative radial movement of the target and the radar:

\[
\left| \chi(\tau, \nu) \right| = \frac{1}{E} \left| \int_{-\infty}^{\infty} u(t) \overline{u(t - \tau)} \exp \{ j2\pi \nu t \} dt \right|,
\]

where

- \( u(t) \) – is the complex envelope of the received radio signal;
- \( \overline{u(t - \tau)} \) – a reference signal, which is a complex conjugate copy of the emitted signal, shifted in time by the value of \( \tau \);
- \( \tau = \frac{2R}{c} \) – signal delay due to the propagation of the signal to the target at a distance \( R \) from the radar;
- \( c \) – the speed of light;
- \( \nu = -\frac{2\nu}{c} f_c \) – the Doppler shift of the carrier frequency of the signal \( f_c \) when reflected from a target moving with velocity \( \nu \);
- \( E \) – the total energy of the signal necessary for normalizing the AF.

However, in the case of wideband and ultra-wideband signals (when the frequency bandwidth is higher than 25% of the center frequency \([8]\)), it is incorrect to describe the Doppler effect only by a carrier frequency shift (1). In the general case, the Doppler effect leads to scaling of the signal in time. \([9]\). In the case of radio signals, when the inequality \( \nu \ll c \) is true, we can write an expression for the received signal reflected from a moving target, taking into account the temporal scaling of the signal:

\[
s_R(t) = s[\alpha t - \tau],
\]

where

- \( s(t) \) – the transmitted signal;
- \( s_R(t) \) – the expected received signal used as a reference signal;
- \( \alpha = \frac{c + \nu}{c - \nu} \approx 1 - \frac{2\nu}{c} \) – the time scaling factor due to the Doppler effect;
- \( \tau = \frac{2R}{c - \nu} \approx \frac{2R}{c} \) – the signal delay due to the propagation of the signal to the target at a distance \( R \) from the radar.

Taking into account (2) the ambiguity function for broadband and ultra-wideband signals can be written as follows:

\[
\left| \chi(\tau, \alpha) \right| = \frac{1}{E} \left| \int_{-\infty}^{\infty} s(t) \cdot s[\alpha t - \tau] dt \right|.
\]

DFCW equation:

\[
s(t) = \sum_{n=0}^{N-1} s_n(t) = \sum_{n=0}^{N-1} \text{rect} \left( \frac{t - nT - \frac{T_n}{2}}{T_n} \right) \cdot a_n \exp \{ j2\pi f_n t + j\varphi_n \},
\]

\( N \) – the number of signal pulses; \( s_n(t) \) – the \( n \)-th signal pulse; \( T_n \) – the duration of the \( n \)-th pulse; \( \sum_{m=0}^{n-1} T_m \) – the time elapsed before the emission of the \( n \)-th pulse;
The complex conjugate reference signal in (3) corresponding to (4) is written as follows:

\[
\begin{align*}
\bar{s}_n(t) &= \sum_{n=0}^{N-1} \text{rect} \left( \frac{t - \sum_{m=0}^{n-1} T_m - T_n}{2} \right) \cdot a_n \exp \left\{ -j 2\pi f_n \left[ \frac{\alpha t}{T_n} - j \varphi_n \right] \right\}. \\
&= a_n \cdot \left. \frac{\alpha (t - \sum_{m=0}^{n-1} T_m - T_n)}{\alpha T_n} \right\}_{0,1,0} \cdot \exp \left\{ -j 2\pi f_n \left[ \frac{\alpha t}{T_n} - j \varphi_n \right] \right\}. 
\end{align*}
\]

To simplify further conclusions, let us set the following conditions for the emission and reception of DFCW:

- the radar is monostatic;
- the maximum delay due to the initial range of the target, illustrated in figure 1, does not exceed the duration of the shortest discrete:

\[
\tau_{\text{max}} < \min_n(T_n), \quad R_{\text{max}} < \frac{\min_n(T_n) \cdot c}{2};
\]

\[T_n\]

\[\tau\]

\[a_{n-1}, f_{n-1}, \varphi_{n-1}\] \[a_n, f_n, \varphi_n\] \[a_{n-1}, f_{n-1}, \varphi_{n-1}\]

\[a_{n-1}, f_{n-1}, \varphi_{n-1}\] \[a_n, f_n, \varphi_n\] \[a_{n-1}, f_{n-1}, \varphi_{n-1}\]

**Figure 1.** Image of the reference (top) and received delayed (bottom) signals. Low range values lead to the fact that in time only adjacent pulses overlap.

- during the signal emission time, the target moves linearly at a constant velocity, while the time scaling of the reflected signal due to the maximum velocity of the target, shown in figure 2, is much shorter than the average pulse duration:

\[
\frac{2\tau_{\text{max}}}{c} \sum_{n=0}^{N-1} T_n < 0.1 \cdot \sum_{n=0}^{N-1} T_n / N, \quad \nu = \text{const}, \quad \tau \in \left[ 0; \sum_{n=0}^{N-1} T_n \right]
\]

\[
\nu_{\text{max}} < \frac{c}{20N}, \quad \nu = \text{const}, \quad \tau \in \left[ 0; \sum_{n=0}^{N-1} T_n \right].
\]
Figure 2. Image of the reference (top) and received delayed (bottom) signals in case of target movement with radial velocity.

These conditions make it possible to simplify expression (5) in such a way that for the reference signal the scaling effect is taken into account only in the phase of the signal and is not taken into account in its amplitude and duration.

Based on the assumption made (7), the (5) can be simplified to the following form:

$$s[\alpha t - \tau] = \sum_{n=0}^{N-1} s_n [\alpha t - \tau] \approx \sum_{n=0}^{N-1} \text{rect} \left( \frac{t - \sum_{m=0}^{n-1} T_m - \frac{T_n}{2} - \tau}{T_n} \right) \cdot a_n \exp \left\{ - j 2\pi f_n \left[ \alpha t - \tau - j \varphi_n \right] \right\}. \quad (8)$$

Thus, the convolved emitted (4) and reference (8) signals have the same pulses durations. Under condition (6) overlaps exist only between pulses with the current and previous indices, the remaining products are equal to zero. Then the convolution operation for the sum of the pulses can be replaced by the sum of the convolutions of each pulses with its shifted received copy and with a part of the previous pulse. The integration limits for such a convolution are determined by the duration of the convoluted pulses and the propagation delay from the target:

$$\left| \chi (\tau, \alpha) \right| = \frac{1}{E} \left| \int_{-\infty}^{\infty} s(t) \cdot s[\alpha t - \tau] \, dt \right| = \frac{1}{E} \int_{-\infty}^{\infty} \sum_{n=0}^{N-1} s_n(t) \sum_{n=0}^{N-1} a_n \exp \left\{ - j 2\pi f_n \left[ \alpha t - \tau - j \varphi_n \right] \right\} \, dt,$$

$$\left| \chi (\tau, \alpha) \right| = \frac{1}{E} \left| \sum_{n=0}^{N-1} \int_{-\sum_{m=0}^{n-1} T_m - \frac{T_n}{2} - \tau}^{\sum_{m=0}^{n} T_m - \frac{T_n}{2} - \tau} s_n(t) s_n[\alpha t - \tau] \, dt + \sum_{n=1}^{N-1} \int_{-\sum_{m=0}^{n-1} T_m - \frac{T_n}{2} - \tau}^{\sum_{m=0}^{n} T_m - \frac{T_n}{2} - \tau} s_n(t) s_{n-1}[\alpha t - \tau] \, dt \right|,$$

$$\left| \chi (\tau, \alpha) \right| = \frac{1}{E} \left| \sum_{n=0}^{N-1} \chi_{n,n} (\tau, \alpha) + \sum_{n=1}^{N-1} \chi_{n,n-1} (\tau, \alpha) \right|. \quad (9)$$

We substitute (4) and (8) into (9), calculate the integrals and obtain an expression for the ambiguity function of the UWB DFCW recorded for the variables of range and velocity, taking into account the assumptions made (6) and (7):

$$\left| \chi (R, \nu) \right| = \frac{1}{E} \left| \sum_{n=0}^{N-1} \chi_{n,n} (R, \nu) + \sum_{n=1}^{N-1} \chi_{n,n-1} (R, \nu) \right|, \quad (10)$$

where
This equation (10) allows us to analyze the compression characteristics of UWB DFCW with different sequences of frequencies $f_n$ and durations $T_n$ of each pulse. A distinctive feature of this expression from the existing ones [5] is the ability to analyze signals with a non-equidistant step in frequency and non-constant durations of pulses.

Some characteristics of the compressed signal depend of parameters of the DFCW:

- the range resolution: $\Delta R = \frac{c}{2F}$, $F = f_{\text{max}} - f_{\text{min}}$;
- the velocity resolution: $\Delta \nu = \frac{c}{N - 1}$, $f_c = \frac{f_{\text{max}} + f_{\text{min}}}{2}$;
- the unambiguous range interval: $R_{\text{max}} = \frac{c}{2\Delta f_{\text{min}}}$, $\Delta f_{\text{min}} = \min (f_n - f_{n-1})$, $n = 0,1,\ldots,N - 1$.

In the most cases, when $\frac{\min(T_n)}{\Delta f_{\text{min}}} > 1$ the second term $\chi_{n,n-1} (R, \nu)$ of (10) becomes negligible.

The implementation of the compression process of the received signal in the radar is proposed in two stages: analog and digital. The analog stage of compression is performed in the receiver circuit of the radar; the digital stage is performed using the expression of the two-dimensional compression function (11) for digital data at the output of the ADC receiver. The block diagram of the radar transceiver circuit and the compression function are proposed below.

2.2. Radar scheme for UWB discrete coded waveforms

Figure 3 presents the proposed analog circuit of the radar, which allows one to work with UWB signals in a wide frequency band from 50 MHz to 6 GHz.

2.2.1. Transmitter

Combining DDS and PLL in a transmitter synthesizer allows us to take advantage of both circuits. Two DDSS generate $N_{\text{DDS}}$ stable signals in bandwidth from 0 Hz to 300 MHz shifted 90° relative to each other with fast $T_{\text{DDS}}$ tuning times and a small frequency step $\Delta f_{\text{DDS}}$. PLL performs tuning of the signal frequency in the band from 50 MHz to 6 GHz in a slow time $T_{\text{PLL}} > T_{\text{DDS}}$ with a much higher frequency step $\Delta f_{\text{PLL}} \gg \Delta f_{\text{DDS}} (\Delta f_{\text{PLL}} = N_{\text{DDS}} \times \Delta f_{\text{DDS}})$. The signals from the DDS outputs are mixed with the PLL microwave signals of frequency $f_c$ in a quadrature modulator (IQM), at the output of which a frequency signal $f_n = f_{\text{PLL}} \pm f_{\text{DDS}}$ is generated. The signal is then amplified in a wideband amplifier.
(WBA). Low pass filters (LPF) in the transmitter are necessary to get rid of harmonics of the DDS clock frequency.

**Figure 3.** Block diagram of ultra-wideband radar.

2.2.2. Receiver

From the antenna output, the received signals are amplified in a low noise wideband amplifier (LNA). A quadrature detector (IQD) in the receiver channel performs the function of multiplying the received and reference signals. The receiver LPF time constant is consistent with the minimum sampling duration $T_{min} < T_{DDS}$. The LPF data has the function of integrating the signal at the output of the IQD. Thus, the joint use of a IQD and LPF acts as a convolution for one received discrete with its original copy $\chi_{n,a}(\tau,\alpha)$ in (9) and $\chi_{n,a}(R,\nu)$ in (10).

The narrowband quadrature signals at the output of the receiver are digitized in the ADC. The signals at the ADC output are quadrature digital samples for each frequency $f_n$ in the DFCW:

$$\chi_{n,a}(R_{\text{target}},\nu_{\text{target}}) = CS(n) = I(n) + i \cdot Q(n), \quad n = 0,1,...,N-1.$$ 

The next stage of compression is the convolution of the output quadrature digital samples in the coordinates of the range-velocity using the expression of the compression function.

2.3. Range-velocity compression function

The two-dimensional compression function of digital samples of the signal has the form:

$$|X(R,\nu)| = \left| \frac{1}{N} \sum_{n=0}^{N-1} [CS(n) - CS_0(n)] \cdot \exp \left\{ \frac{4\pi}{c} f_n \left[ R + u \left( \frac{\sum_{m=0}^{n-1} T_m}{2} + \frac{T_n}{2} + \frac{R}{c} \right) \right] \right\} \right|.$$ 

The result of expression (11) is a two-dimensional function with a maximum corresponding to the instantaneous range and velocity of the target. $CS_0(n)$ is the first signal from the accumulated data that is used to background compensation. The form of expression (11) depends on the time-frequency parameters of the signal and can be pre-modeled using the proposed expression of the ambiguity function (10). For example in figure 4 there is the 3D plot of compression function of ideal signal received from target at 3 meters with approach speed 3.8 m/s, signal parameters are the same as in experimental layout from below.
3. Experimental part and results

3.1. Experimental layout
For practical testing of the radar constructed according to the scheme in figure 3 an experimental prototype with an aluminum pendulum of radius 20 cm (RCS is about 0.125 m²) was built. Figure 5 shows the experimental layout diagram for studies of the proposed method for the joint estimation of range and velocity.

Data were accumulated for 332 signals with a repetition period of $T_{\text{frame}} = 100$ ms. Parameters of the used DFCW:
- Generally linear frequency code with equidistant frequency step;
- $T_{\text{DDS}} = 40$ us, $T_{\text{PLL}} = 300$ us, $N_{\text{DDS}} = 60$, $N_{\text{PLL}} = 18$, $N = N_{\text{DDS}} \times N_{\text{PLL}} = 1080$;
$T_{FULL} = N_{DDS}T_{DDS} + N_{PLL}T_{PLL} = 47.88 \text{ ms};$

$\Delta f_{DDS} = 5 \text{ MHz}, f_{DDS} = -153 \text{ MHz}, -148 \text{ MHz}, ..., +147 \text{ MHz};$

$\Delta f_{PLL} = 300 \text{ MHz}, f_{PLL} = 900 \text{ MHz}, 1200 \text{ MHz}, ..., 6000 \text{ MHz};$

$f_n = 752 \text{ MHz}, 757 \text{ MHz}, ..., 6147 \text{ MHz};$

$F = 5395 \text{ MHz}.$

3.2. Experimental results

The result of compression function (11) normalized to its maximum for 26-th of 332 accumulated signals received from pendulum (figure 5) is shown in figure 6. As can be seen from comprising figure 4 and figure 6 the practical implementation of the compression function from the real target has distortions. This distortion is due to imperfections in the frequency response of the transceiver path. In addition, in the vicinity of 0 meters, there is a certain peak associated with a change in the signal of the proliferating directly from the transmitting to the receiving antenna. However, as in the case of the theoretical function, the position of the maximum of the practical compression function shows the instantaneous values of the range and velocity of the pendulum.

![Figure 6. 3D plot of compression function of real signal received from pendulum at 3 meters with approach speed 3.8 m/s.](image)

According to the experimental data, the instantaneous values of the pendulum velocity were calculated in two ways. One way is the proposed method of fixing the maximum of the two-dimensional compression function. The second method is the convenience method for calculating the velocity using Fast Fourier Transform (FFT) from two sequentially received signals and calculating the shift of the maximum of RRRP for signal repetition interval:

$$V_k = \frac{R_s \{\max (|\text{FFT} \{CS_k\}|)\} - R_{s,k+1} \{\max (|\text{FFT} \{CS_{k+1}\}|)\}}{T_{SR}},$$

where $K$ – number of accumulated signals; $k$ – index of current signal; $|\text{FFT} \{CS_k\}|$ – RRRP of $k$-th signal; $T_{SR}$ – signal repetition interval.

The results of the experiment are shown in figure 7. The upper part of the graph shows the estimates of the pendulum velocity, calculated in two ways.

$V_{\text{shift}}$ – is the velocity estimated by the convenience method from two adjacent signals.
\( V_{AF} \) – is the velocity estimate obtained by calculating the coordinate of the maximum of the compression function (11).

\( V_{teor} \) – theoretical dependence of velocity on time, taking into account the attenuation of oscillations:

\[
V_{teor} = A \cdot \omega \cdot \cos(\omega \cdot t + \varphi_0) \cdot \exp\left(-t/\alpha\right), \quad \omega = 2\pi \left(\frac{g}{L}\right)^{1/2},
\]

where \( L \) – the length of pendulum [5.73m]; \( g \) – the gravitational acceleration of 9.8 [m / s]; \( A \) – swing amplitude 3 [m]; \( \omega \) – the cyclic oscillation frequency [Hz \times rad], depending on the length of the pendulum \( L = 5.73 \) [m]; \( \alpha \) – some empirical attenuation coefficient of the pendulum 60 [s] to comply with physical quantities.

The lower part of the graph shows the absolute estimation error for the two methods with respect to theoretical values.

The experiment repeated three more times with different numbers of accumulated signals.

**Figure 7.** Experimental results. The upper part of the graph shows the estimates of the pendulum velocity. The lower part of the graph shows the absolute estimation error between practical and theoretical values.

The table 1 shows the statistical characteristics of the absolute estimation errors.

**Table 1.** Statistical characteristics of the absolute estimation errors for 332 accumulated signals.

| Estimate value | Convenience method | Proposed method |
|----------------|--------------------|-----------------|
| Mean [m/s]     | 1.0698             | 0.6790          |
| Variance [$m^2/s^2$] | 32.487             | 1.8037          |
| Median [m/s]   | 0.3805             | 0.3855          |

The experiment repeated three more times with different numbers of accumulated signals.
Table 2. Statistical characteristics of the absolute estimation errors for 203 accumulated signals.

| Estimate value | Convenience method | Proposed method |
|----------------|--------------------|-----------------|
| Mean [m/s]     | 0.9188             | 1.1981          |
| Variance [m²/s²] | 17.9505           | 2.8118          |
| Median [m/s]   | 0.3844             | 0.4784          |

Table 3. Statistical characteristics of the absolute estimation errors for 198 accumulated signals.

| Estimate value | Convenience method | Proposed method |
|----------------|--------------------|-----------------|
| Mean [m/s]     | 0.7305             | 0.8673          |
| Variance [m²/s²] | 8.8321            | 2.0214          |
| Median [m/s]   | 0.2357             | 0.2646          |

Table 4. Statistical characteristics of the absolute estimation errors for 206 accumulated signals.

| Estimate value | Convenience method | Proposed method |
|----------------|--------------------|-----------------|
| Mean [m/s]     | 0.8864             | 1.0636          |
| Variance [m²/s²] | 15.3623           | 2.5811          |
| Median [m/s]   | 0.4147             | 0.3445          |

It can be seen from the tables 1-4 that the estimation carried out by the convenience method leads to a wider spread of values, despite the fact that the median error value of the convenience method is lower. This is explained by the duration of the necessary process of data accumulation for the convenience method. It is two or more times higher which ensures an improvement in velocity resolution. The proposed method allows one to obtain a similar estimate for the velocity using only one ultra-wideband signal. In addition, local emissions are observed in the graph of the convenience method, which led to a high dispersion of values. These outliers are explained by the fact that as the object's velocity increases, the usual FFT operation will lead to the signal spreading along its range and the jitter of the maximum position (described in [5] and [10]), as shown in figure 8. Since the FFT operation essentially displays the zero velocity cross section of the two-dimensional compression function, the quality of the speed estimation by the classical method using the FFT directly depends on the shape of the compression function. The shape of the compression function depends on both the signal parameters (bandwidth, duration, frequency step, frequency code) and the time-frequency parameters of the radar transmit-receive path. Thus, an accurate assessment of the applicability limits of the convenience and proposed methods requires complex parametric analysis and is not considered as part of this article.

With an increase in the velocity of the object the use of the usual FFT to estimate the velocity will lead to an increase in error.

4. Conclusions

The obtained expression (11) and the proposed radar block diagram allow realizing the ultra-wideband ambiguity function, which makes it possible to carry out a joint estimation of the range and velocity of the target for the time of the one probing. It allows one to reduce the time required to review a given sector of space and improve the quality of the radar. The experimental results obtained from the real radar prototype confirmed the correctness of the proposed method. According to expression (10), the result of this method depends on the time-frequency parameters of the signal. In the following works, it is supposed to study in more detail the influence of more complex forms of DFCW instead of the simplest SFCW signal.
Figure 8. The HRRP of 26-th signal in experimental data compressed by convenience FFT and by proposed range-velocity compression function. The velocity of target is about 3.8 m/s.

For example, signals with pseudo-random frequency coding, signals with variation of a frequency step or time step. As a comparison of the theoretical and experimental two-dimensional compression functions has shown, the frequency response of the transceiver path of the radar affects the result of compression. To improve the accuracy of estimating range and speed, it is required to use some path calibration algorithm, for example, signal predistortion in a DDS and PLL synthesizers, or post-distortion in a signal processor after ADC. A separate area of research is the optimization of finding the maximum in (11) to accelerate the decision on the instantaneous range and speed of the target, for example using the compressive sensing technique.

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