Distributionally Robust Multi-Instance Learning with Stable Instances

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Abstract
Multi-instance learning (MIL) deals with tasks where data consist of set of bags and each bag is represented by a set of instances. Only the bag labels are observed but the label for each instance is not available. Previous MIL studies typically assume that the training and test samples follow the same distribution, which is often violated in real-world applications. Existing methods address distribution changes by re-weighting the training data with the density ratio between the training and test samples. However, models are often trained without prior knowledge of the test distribution which renders existing methods inapplicable. Inspired by a connection between MIL and causal inference, we propose a novel framework for addressing distribution change in MIL without relying on the test distribution. Experimental results validate the effectiveness of our approach.

1 Introduction
Multi-instance learning (MIL) [Dietterich et al., 1997] deals with tasks where the data consist of a set of bags and each bag contains a set of instances. Unlike traditional supervised learning, only the bag labels are known whereas instance labels are unknown. A bag is labeled as positive if it contains at least one positive instance, and labeled as negative if otherwise. The main goal of the majority of MIL algorithms is to predict the labels of unseen test bags. MIL has been applied to many applications, including text categorization [Andrews et al., 2002], web index page recommendation [Zhou et al., 2005], and image classification [Chen et al., 2006], etc.

Most multi-instance learning methods assume that the training and test data follow the same distribution, which is frequently violated in real-world tasks [Shimodaira, 2000]. Distribution change occurs when the training and test data are collected in different times or environments. Consider the example of an image classification task in Figure 1, we have training images for classifying dog collected during summer when the backgrounds often contain grass, whereas the test samples are collected during winter where the backgrounds are mostly snowy [Zhang and Zhou, 2014; Shen et al., 2018]. Standard MIL methods may predict images with only grass as positive and images with dogs in snow as negative because of the difference between the training and test distributions.

Existing methods for addressing distribution change in MIL [Zhang and Zhou, 2014; Zhang and Zhou, 2017] are not applicable in many real-world situations. They require the availability of the test data in order to estimate the density ratio [Sugiyama et al., 2009] between the test and training distributions, and use the weight to train a classifier using the re-weighted training samples. Unfortunately, classifiers often need to be trained without seeing the test samples.

Causal relationships describe the underlying mechanism of complex systems and thus are not affected by the distribution changes during the data collection procedure. Classifiers based on causal relationships will achieve more stable performance than correlation based methods when distribution change occurs. Several methods have been proposed to use causal relationships to design stable learners in single instance learning [Kuang et al., 2018; Shen et al., 2018]. However, none of them are designed for MIL.

It is possible to distinguish causation from correlation in multi-instance learning. Under the standard multi-instance assumption [Foulds and Frank, 2010], if an instance is causally related to the bag label, adding the instance to a negative bag would change the label of the resulting bag; however, the bag label would remain negative if the instance is not causally related to the label regardless of the correlation between the bag and the instance. Inspired by this motivation, we propose the StableMIL (Stable Multi-Instance Learning) framework to address the distribution change in MIL without using the test distribution. Since covariate shift [Zadrozny, 2004] has attracted the greatest number of studies, we focus on this setting where the marginal distribution of the bag changes but the conditional probabilities of the label given the bag and its instances remain unchanged. To the best of our knowledge, StableMIL is the first multi-instance distributionally robust method without relying on the test distribution.

Empirical evaluations indicate that, without using the test distribution, StableMIL significantly outperforms state-of-the-art algorithms and performs competitively to distribution change based methods that access the test data.

The rest of this paper is organized as follows. We review related work in Section 2, and present the framework in Section 3. Then we report the experimental results in Section 4 and we conclude the paper in Section 5.
We consider the widely accepted “standard multi-instance assumption” [Foulds and Frank, 2010] throughout our discussion. Formally, the assumption can be stated as:

**Assumption 1.** A multi-instance bag is negative if and only if all of its instances are negative, and a bag is positive if at least one of its instances are positive: i.e.,

\[
\phi(X_i) \triangleq f(h(x_{i1}), h(x_{i2}), \ldots, h(x_{im})),
\]

where \( \phi \) is the bag labeling hypothesis, \( f \) is the boolean OR function, and \( h \in \mathcal{H} \) is a hypothesis for instances in \( \mathcal{X} \).

### 3.2 The framework

The heart of this work is the claim that there exists a connection between multi-instance learning and randomized experiments in the causal inference literature [Imbens and Rubin, 2015]. Generally speaking, an experiment can be used for determining whether a causal relationship exists between a binary treatment variable and an outcome variable. In other words, whether the action of changing the value of the treatment would affect the value of the outcome.

With the standard multi-instance assumption, let us consider adding instance \( x \) to a bag \( X_j \) as the action of treatment and the label \( Y \) of the bag as the outcome, then the causal relationship between instance \( x \) and label \( Y \) can be determined by whether the treatment changes the label from 0 to 1. In other words, if instance \( x \) has causal relationship with \( Y \), adding it to a negative bag would flip the label from negative to positive. On the other hand, if instance \( x \) is not causally related to \( Y \), adding \( x \) will not change the label. Formally we state the definition of causal instance as:

**Definition 2. (Causal instance)** An instance \( x_{ik} \) from a positive bag \( X_j^+ \) is a causal instance with regard to the bag label \( Y \) if for any negative bag \( X_j \in B^- \), it satisfies

\[
\phi^*(x_{ik} \cup X_j) - \phi^*(X_j) = 1,
\]

where \( \phi^* \) denotes an oracle bag classifier which always return the correct label of a multi-instance bag, and \( x_{ik} \cup X_j \) denotes a treated bag containing the instance of interest \( x_{ik} \) along with all the instances in the pre-treatment bag \( X_j \).

With Definition 2, we can now group the instances in a multi-instance bag into three categories: for the first category, the conditional expectation of the label has a non-zero dependence on the instances and the dependence does not change when other instances are added to the conditional set, we call these causal instances. For example, dogs are causal instances of a photo labeled as animal which will not change between the training and the test distributions (Figure 1). For the second category of instances, we term them as noisy instances. Noisy instances are correlated with either the causal instances, the bag label, or both, but do not themselves have causal relationships with the label. In other words, when conditioned on the full set of causal instances, they are independent of the bag label. For example, grasses and snows are noisy instances of an animal photo. Although they are highly correlated with the causal instances and the label, their correlations is vulnerable to changes during the collection of the training and test data. The third category of instances are termed similar as in existing literatures as negative instances,
which contains instances that are not significantly correlated to the label, i.e., random background objects without any significant correlation to the bag label.

The performances of existing MIL methods degenerate when distribution changes because they do not differentiate causal instances from noisy instances. When the training and test distributions differ, the correlations between noisy instances and the label will not be consistent. Therefore, models built by existing methods will be misleading by spurious correlations that only exist in the training data but are not valid in the test data. On the other hand, a multi-instance classifier based on causal instances will achieve a more stable performance because causal relationships are not affected by the distribution changes in the training and test distributions.

### Learning Causal Instances from Experiment

The key of building a distributional robust multi-instance classifier lies in differentiating causal instances from noisy ones. In this section, we will discuss the identification of causal instances from an ideal experiment setting.

By Definition 2, instances from negative bags cannot have causal relationships with the label. Therefore, we only need to consider the instances from the positive bags. Let \( \bigcup B^+ \) denote the candidate instance pool which contains all the instances from the positive bags and \( x_k \in \bigcup B^+ \) denote a candidate instance. For the sake of conciseness, subscript \( k \) will be dropped when the context is clear. To determine whether \( x \) is a causal instance, we need to estimate the causal effect of \( x \) on the bag label \( Y \) which can be defined as the difference between the expected label of a bag if it were treated minus the expected label of the bag if it were not treated:

\[
\tau(x) = E[Y(T = 1)] - E[Y(T = 0)].
\]

Here we use \( Y(T = 1) \) to denote the potential bag label if it were treated, i.e., the candidate instance \( x \) is present in the bag; and we use \( Y(T = 0) \) to denote the potential bag label if the bag were not treated, i.e., \( x \) is not present in the bag.

Given a set of multi-instance bags, we can always obtain paired treated and untreated bags by adding the candidate instance \( x \) to a bag (if \( x \) is not in the pre-treatment bag) or removing \( x \) from a bag (if \( x \) is in the pre-treatment bag). Therefore, the causal effect can be estimated using the difference in the expectation of the realized outcomes provided by the data and the oracle classifier:

\[
\tau(x) = E[Y^*|T = 1] - E[Y^*|T = 0],
\]

where \( Y^* \) denotes the bag label after the treatment. Combining with the standard multi-instance assumption, we can obtain the following theorem:

**Theorem 3.** The causal effect of an instance \( x \) on the bag label \( Y \) obtained from an ideal experiment is

\[
\tau(x) = \frac{1}{2} E[Y^*|Y = 0, T = 1] + const,
\]

where const is the probability of a bag contains one and only one positive instance with the instance being \( x \).

**Proof.** For the simplicity of discussion, we assume that the classes are balanced, i.e. \( P(Y = i) = 0.5 \) for \( i \in \{0, 1\} \).

Utilizing the tower property of conditional expectation,

\[
\tau(x) = E[Y^*|T = 1] - E[Y^*|T = 0] = E[E[Y^*|T = 1, Y]] - E[E[Y^*|T = 0, Y]] = \frac{1}{2} \left( \sum_{Y=0}^{1} E[Y^*|T = 1, Y] - \sum_{Y=0}^{1} E[Y^*|T = 0, Y] \right).
\]

From the Standard MI assumption, adding any instance to a positive bag or removing any instance from a negative bag will not change the bag label. Therefore we have, \( E[Y^*|Y = 1, T = 1] = 1 \) and \( E[Y^*|T = 0, Y = 0] = 0 \). Accordingly,

\[
\tau(x) = \frac{1}{2} (E[Y^*|Y = 0, T = 1] - E[Y^*|T = 0, Y = 1] + 1).
\]

There exists two possibilities when a positive bag is under the control treatment: the pre-treatment bag contains positive instances other than \( x \) (may and may not contains \( x \) itself), the bag contains \( x \) and only \( x \) as its positive instance. Under the standard multi-instance assumption, the expectation is 1 and 0 for the two scenarios, respectively. Let \( p \) denote the probability of the second scenario, we can write \( E[Y^*|T = 0, Y = 1] = 1 - p \). Therefore, we have

\[
\tau(x) = \frac{1}{2} (E[Y^*|Y = 0, T = 1] + p).
\]

Theorem 3 indicates that the causal effect of an instance on the bag can be characterized by the expected treated bag label after adding the instance to a negative bag. It is safe to ignore the constant term during the estimation because \( p \) would be small for causal instances and \( p = 0 \) for non-causal instances.

### Learning Stable Instances from Data

Our proposed framework for distributionally robust multi-instance learning is inspired by the above discussion of learning the causal instances. However, since the oracle classifier does not exist, the instances identified using a surrogate empirical classifier may not be causal. We hence refer to them as stable instances to avoid misconceptions.

To identify the stable instances, first we train a multi-instance classification algorithm \( A \) with the training data and use \( A \) to denote the classifier that \( A \) returns. For each candidate instance \( x \), we then construct a set of bags which contains \( m \) number of “treated” bags from the original negative bags. The treated bags are constructed by adding the candidate instance \( x \) into the negative bags \( X_i^p \) as \( X_i^p = \bigcup X_i^+ \), for \( i = 1, \ldots, m \). For each treated bag, we use the previously trained classifier \( A \) to predict its label. Finally we use the average of predicted labels \( A(X_i^p) \) on the treated bags to estimate the expectation term in Equation 5 as

\[
\hat{\tau}(x) = \frac{1}{m} \sum_{i=1}^{m} A(X_i^p).
\]

After Equation 6 has been estimated for all the candidate instances, we choose those higher than the threshold \( \tau \) as stable
instances. We summarize the detailed procedure of learning the stable instances in Algorithm 1.

Even when inspected from a non-causal perspective, StableMIL is more robust to distribution change than standard MIL methods. Let us assume that the training bags in Figure 1 consist 40% of images with dog in grass background, 10% of images with dog in snow background, 10% of grass images, 10% of snow images, and 30% of other negative images. In the procedure of Algorithm 1, grass and snow instances will have a lower chance of flipping the label of a negative bag than the dog instances because dog instances exist in 100% of positive bags; however, grass instances only exist in 80% of the positive bags but also exist in 20% of the negative bags and snow instances only exist in 20% of the positive images but also exist in 20% of the negative ones. Therefore, stable instances can be differentiated from noisy ones.

Bag Embedding and Embedded Classification

After learning the stable instance set $C$, we used bag embedding [Chen et al., 2006; Fu et al., 2011; Zhang and Zhou, 2017] to map the bags into single instance representation using the stable instances, and then construct our classifier.

The multi-instance bag embedding is performed based on the similarity between the bags and each of the instances in $C$. The similarity between a bag and an instance can be measured using the following function:

$$d(X_i, x) = \max_{x_{ij} \in X_i} \exp(-\lambda \|x_{ij} - x\|^2)$$  

(7)

where $\lambda$ is the scaling parameter and can be chosen automatically by local scaling [Zelnik-Manor and Perona, 2004]. The intuition is that positive bags should have high similarities with at least one instance in $C$, and negative bags should have low similarities with all instances in $C$.

As a result, the embedded feature vector for bag $X_i$ is a $q$-dimensional vector which is consisted of the concatenated bag-to-instance similarities:

$$z_i = [d(X_i, x_1), \ldots, d(X_i, x_j), \ldots, d(X_i, x_q)],$$  

(8)

where $x_j \in C$ and $q$ is the cardinality of set $C$. After the bag embeddings are calculated, any standard machine learning algorithm can be used for training the model. In our experiments, we use SVM with RBF kernel for fairness of comparison since it is also used as the classifier for many multi-instance algorithms that employ bag embedding.

4 Empirical validation

In this section, we empirically validate the performance of the proposed framework by comparing it with state-of-the-art multi-instance algorithms. Firstly, we compare StableMIL with benchmark multi-instance algorithms including miSVM [Andrews et al., 2002], MILES [Chen et al., 2006], miGraph [Zhou et al., 2009], and miFV [Wei et al., 2017]. Secondly, we compare with distribution change MIL methods that utilize the test distribution, including MIILES with direct importance weighting [Sugiyama et al., 2007], MICS [Zhang and Zhou, 2014] and MIKI [Zhang and Zhou, 2017]. We use MIFV as the base multi-instance classifier of StableMIL and SVM [Chang and Lin, 2011] with RBF kernel for the modeling after embedding. Each experiment has been repeated for 30 times. The parameters for the compared methods are selected by grid search with cross-validation on the training data. The parameter $\tau$ of StableMIL is selected to ensure that the cardinality of stable instance pool $C$ is larger than the number of positive bags.

4.1 Results on Synthetic Data

We first perform empirical evaluation with synthetic data. The instances are generated from two positive and two negative concepts sampled from different Gaussian distributions. The goal is to simulate similar positive concepts but different negative concepts. For example, if the positive distributions describe “husky dog” and “shepherd dog”, then the negative distributions would describe “grass” and “snow”. Each bag contains around 10 to 20 instances. Each positive bag contains an average of 2 positive instances which comes from one of the two positive distributions, and each negative bag contains instances from one of the two negative distributions. A total number of 200 positive bags and 200 negative bags are generated with positive instances from each concept distributed evenly. We vary the dimensions of the distributions to generate low and high dimensional settings.

To simulate distribution change between the training and test data, we use a biased sampling procedure similar to those used in relevant literatures [Zadrozny, 2004; Zhang and Zhou, 2014; Zhang and Zhou, 2017]. Specifically, we define a selection variable $s_i$, where $s_i = 1$ indicates the $i$-th bag is selected into the training set and $s_i = 0$ indicates otherwise. Denote the positive and negative instances as $x^+$ and $x^-$, then for positive bags the sampling rules are $Pr(s_i = 1|x^+ \in P_1, x^- \in N_1) = a$ and $Pr(s_i = 1|x^+ \in P_2, x^- \in N_2) = b$; for negative bags the rules are $Pr(s_i = 1|x^+ \in N_1) = b$ and $Pr(s_i = 1|x^- \in N_2) = a$. Here $P_1, P_2, N_1$ and $N_2$ are the distributions where the positive and negative instances are sampled from. The values of sampling ratio $a$ are uniformly selected from $a \in [0.65, 0.95]$ and $b = 1 - a$. In other words, comparing to the test data, in the training samples there are more positive bags consists of instances from

Algorithm 1 Learning the stable instances

Input: Training bags $B_{tr} = \{(x_{i}, y_{i})\}_{i=1}^{m}$, multi-instance classification algorithm $A$, threshold $\tau$

Output: Stable Instance Pool $C$

1: Train $A$ using data in $B_{tr}$
2: $X^+ \leftarrow \emptyset$
3: for every $x \in \bigcup B^+$ do
4: \hspace{1em} $X^+ \leftarrow X^+ \cup x$
5: end for
6: for every $x \in X^+$ do
7: \hspace{1em} $s \leftarrow 0$
8: \hspace{1em} for every $x_i$ in $X^-$ do
9: \hspace{2em} $s = s + A(x \cup X_i^-)$
10: \hspace{1em} end for
11: \hspace{1em} $C = x \cup C$
12: end if
13: end for
14: return $C$
Table 1: Testing accuracy (\%, mean ± std) on synthetic datasets. The highest average accuracy is marked in bold.

| State-of-the-art multi-instance learning methods | Distribution change multi-instance methods | Proposed method |
|-----------------------------------------------|------------------------------------------|-----------------|
| miSVM                                        | MILES                                    | StableMIL       |
| Setting 1                                    | MILES+                                   |                 |
| 79.8 ± 2.0                                   | 83.1 ± 2.5                               | 93.9 ± 2.3      |
| 83.6 ± 1.6                                   | 91.4 ± 4.9                               |                 |
| 90.7 ± 3.9                                   | 87.3 ± 2.5                               |                 |
| 76.4 ± 3.8                                   |                                         |                 |
| Setting 2                                    | MICS                                     |                 |
| 72.7 ± 2.5                                   | 71.3 ± 3.6                               | 78.9 ± 3.5      |
| 71.0 ± 2.8                                   | 70.5 ± 5.9                               |                 |
| 70.7 ± 3.0                                   | 72.9 ± 2.3                               |                 |
| 68.3 ± 3.6                                   |                                         |                 |

Table 2: Testing accuracy (\%, mean ± std) on image classification tasks. The highest average accuracy is marked in bold. •/◦ indicates that StableMIL is significantly better/worse than the compared methods (paired t-tests at 95% significance level). The last row summarizes the Win/Tie/Lose counts of StableMIL versus other methods.

| State-of-the-art multi-instance learning methods | Distribution change multi-instance methods | Proposed method |
|-----------------------------------------------|------------------------------------------|-----------------|
| miSVM                                        | MILES                                    | StableMIL       |
| Setting 1                                    | MILES+                                   |                 |
| 6 and 9                                      | 75.5 ± 9.5•                              | 91.9 ± 7.1      |
| 0 and 6                                      | 75.2 ± 9.7•                              | 93.3 ± 4.6      |
| 0 and 8                                      | 76.3 ± 9.3•                              | 91.8 ± 8.8      |
| 0 and 9                                      | 70.5 ± 10.3•                             | 95.8 ± 3.3      |
| 4 and 7                                      | 70.0 ± 11.5•                             | 94.6 ± 3.3      |
| 1 and 7                                      | 90.5 ± 6.6•                              | 91.8 ± 8.8      |
| 2 and 7                                      | 70.4 ± 13.8•                             | 95.8 ± 3.3      |
| 3 and 6                                      | 68.8 ± 10.9•                             | 94.6 ± 3.3      |
| 6 and 8                                      | 76.1 ± 10.2•                             | 94.6 ± 3.3      |
| 2 and 4                                      | 71.7 ± 12.2•                             | 94.6 ± 3.3      |
| W/T/L                                       | 10/0/0                                   | 94.6 ± 3.3      |

$P_1$ and $N_1$ (i.e., “husky dog in grass”), but less bags containing instances from $P_2$ and $N_2$ (i.e., “shepherd dog in snow”); and there are more negative bags in the training samples consist of instances from $N_2$ (i.e., “snow”), but less negative bags which contain instances from $N_1$ (i.e., “grass”).

We report the simulation results in Table 1. When compared to the state-of-the-art multi-instance learning methods, the proposed StableMIL method achieves significantly better performance. When compared with distribution change based multi-instance learning algorithms that utilized the test distribution, StableMIL also achieves better performance without the requiring the availability of the test datasets.

4.2 Results on Image Classification

We study the performance of StableMIL for image classification tasks with the MNIST dataset. Each sample in MNIST is an image of a hand-written digit of zero to nine. We generate 200 positive and 200 negative multi-instance bags where each bag contains an average of 20 images. A bag is labeled as positive if it contains one of the two visually similar digits (e.g., “1” and “7”), and negative if otherwise.

Biased sampling is generated similarly to the last section; however, we randomly separate the non-positive digits into two disjoint subgroups and sample the negative instances from each of them. For example, when compared to the test data, in the training data there are more positive bags with digit “1” as positive instances and “2”, “5”, “6”, “9” as negative instances but less positive bags with “7” and “3”, “4”, “8”, “0”, and there are more negative bags with “3”, “4”, “8”, “0” as instances but less negative bags with “2”, “5”, “6”, “9”.

We report the results Table 2. When compared with state-of-the-art MIL algorithms, the performance of StableMIL is significantly better on all datasets. When compared to distribution change methods, the performance of StableMIL is similar to MIKI; however, MIKI needs to access the test distribution whereas StableMIL achieves competitive accuracy using only the training data. MILES+ does not show significant improvement over MILES, which suggests that straightforward extension of single-instance distribution change method to MIL is not effective. MICS also does not show significant improvement, possibly due to the reason that MICS is not designed to address distribution changes in both the positive and negative instances.

4.3 Results on Text Categorization

To evaluate the performance of StableMIL for text categorization tasks, we utilize the widely used 20 News groups corpus [Zhou et al., 2009]. The corpus contains paragraphs belonging to 20 concepts. We use each paragraph as an instance and construct each multi-instance bag with an average of 20 instances. The bag construction and biased sampling procedures are performed similarly as previous subsections.

The results on text categorization task show similar trends to the results from image classification task. StableMIL performs significantly better than all compared state-of-the-art MIL algorithms. When compared to distribution change MIL algorithms, StableMIL performs competitive to MIKI and significantly outperforms MILES+ and MICS. Overall, our empirical evaluations have demonstrated that the proposed StableMIL framework is effective for addressing distribution change in multi-instance learning.
Table 3: Testing accuracy (%) on image classification tasks. The highest average accuracy is marked in bold. /o indicates that StableMIL is significantly better/worse than the compared methods (paired t-tests at 95% significance level). The last row summarizes the Win/Tie/Lose counts of StableMIL versus other methods.

| State-of-the-art multi-instance learning methods | Distribution change multi-instance methods | Proposed method |
|-------------------------------------------------|------------------------------------------|-----------------|
| miSVM                                           | MILES+                                   | StableMIL       |
| gra.os                                          | 56.2 ± 4.9•                             | 64.2 ± 4.2•     | 71.4 ± 3.5 |
| gra.ibm                                         | 57.8 ± 3.7•                             | 64.0 ± 3.3•     | 68.6 ± 4.0 |
| mac.win                                         | 56.7 ± 5.0•                             | 58.5 ± 5.6•     | 77.2 ± 2.3 |
| os.mac                                          | 54.7 ± 4.9•                             | 60.8 ± 3.2•     | 64.8 ± 3.7 |
| os.win                                          | 63.8 ± 6.0•                             | 62.8 ± 3.2•     | 72.0 ± 3.2 |
| auto.baseball                                   | 54.8 ± 4.6•                             | 59.6 ± 4.0•     | 62.8 ± 4.0 |
| auto.moto                                       | 54.5 ± 5.0•                             | 61.7 ± 5.2•     | 68.3 ± 3.6 |
| baseball.hockey                                 | 69.2 ± 8.0•                             | 68.6 ± 4.5•     | 73.6 ± 6.0 |
| moto.baseball                                   | 54.2 ± 3.8•                             | 55.3 ± 3.5•     | 61.2 ± 2.9 |
| moto.hockey                                     | 54.4 ± 4.5•                             | 58.4 ± 3.2•     | 60.0 ± 3.0 |
| crypt.med                                       | 52.0 ± 3.9•                             | 56.2 ± 4.2•     | 74.5 ± 5.7 |
| crypt.space                                     | 54.8 ± 6.5•                             | 59.4 ± 2.5•     | 69.4 ± 3.4 |
| elec.space                                      | 51.6 ± 2.9•                             | 57.9 ± 4.2•     | 59.8 ± 4.0 |
| med.space                                       | 52.1 ± 4.0•                             | 58.5 ± 5.2•     | 58.8 ± 6.5 |
| guns.mideast                                    | 58.3 ± 7.8•                             | 61.2 ± 4.2•     | 62.1 ± 3.3 |
| guns.misc                                       | 55.6 ± 6.8•                             | 58.9 ± 3.6•     | 64.4 ± 2.2 |
| mideast.misc                                    | 55.7 ± 3.2•                             | 58.2 ± 3.6•     | 65.7 ± 2.2 |
| religion.guns                                   | 58.0 ± 5.8•                             | 56.8 ± 4.8•     | 61.5 ± 3.8 |
| WTTL                                           | 20/0/0                                  | 19/1/0          | 11/4/5   |

Table 4: Testing accuracy (%) on benchmark datasets. The highest average accuracy is marked in bold.

| Dataset    | miSVM | MILES | miGraph | miFV | StableMIL |
|------------|-------|-------|---------|------|-----------|
| Musk1      | 87.4  | 84.2  | **88.9**| 87.5 | 87.6      |
| Musk2      | 83.6  | 83.8  | **90.3**| 86.1 | 90.0      |
| Elephant   | 82.0  | **89.1**| 86.8   | 82.4 | 84.2      |
| Fox        | 58.2  | **76.0**| 61.1   | 58.9 | 63.8      |
| Tiger      | 78.9  | **86.0**| 85.9   | 79.0 | 78.6      |

4.4 Results on Non-Distribution Shift Data

StableMIL can be used without prior knowledge of whether distribution change has occurred. We evaluate StableMIL with benchmark MIL datasets without distribution change using cross validation. As shown in Table 4, the performance of StableMIL is competitive with the compared state-of-the-art methods. The results indicate that although StableMIL is designed to produce robust predictions across different training and test distributions, it performs competitively to the state-of-the-art when there is no distribution change. This is reasonable, since the causal instances are useful for prediction regardless of the existence of distribution changes.

4.5 Parameter Influence

We perform experiments to study the influence of parameter \( \tau \) on the performance of StableMIL by varying the values of \( \tau \) and show the results in Figure 2. \( \tau \) controls the size of the instance set \( C \); decreasing the threshold \( \tau \) will include noisy instances and increasing the threshold will exclude stable instances. The figure shows that the accuracy on the unseen test data is affected by the size of \( C \), which indicates that our framework is effective in identifying the stable instances. We find that choosing a \( \tau \) value which ensures the cardinality of \( C \) is larger than the number of positive bags in the training data is a good strategy for all our experiments.

5 Conclusion

In this paper, we have identified a connection between MIL and causal inference. Inspired by the connection, we have proposed a novel framework for addressing distribution change in MIL without accessing the test distribution. Experiments have shown that the proposed framework is robust to the distribution changes between the training and test data. We have focused on the standard multi-instance assumption and the covariate shift setting in this work. Future work includes investigating the extension of StableMIL to more relaxed distribution change settings, and exploring the link between other MIL assumptions and causal inference.
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