Peculiarities of electron-ion collisions in strong magnetic field white dwarfs

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Abstract. We consider electron-ion collisions in strong magnetic field ($B > 10^7$ G) white dwarfs at a photospheric temperature $T \sim 10^4$ K. Under these unique conditions, the electron’s Larmor radius becomes less than the characteristic impact parameter of the close Coulomb collisions in a non-magnetized plasma. We analyze electron-ion collisions in the classical and in the strong quantum limits which both can be observed in the photospheres of the isolated magnetic white dwarfs. In both limits, the collision transport frequencies are determined by the close collisions in which an electron transits the region near an ion for a time of the order or less than the cyclotron period. In these effective collisions, an electron returns to an ion many times. The final state of an electron non-regularly (chaotically) depends on its initial state. The strong magnetic field significantly reduces the values of the collision transport frequencies in comparison with the analogous values in a non-magnetized plasma.

1. Physical conditions
In a completely ionized non-magnetized plasma, the collision transport frequencies are determined by the distant Coulomb collisions with the impact parameters varying from the distance

$$r_s = Z e^2/(2m_e E) = r_a (E/E_a)^{-1} \quad (1)$$

up to the Debye screening length $r_D \gg r_s$. A magnetic field qualitatively changes the Coulomb collisions when the electron’s Larmor radius

$$r_B = (2E/m_e)^{1/2}/\omega_B = r_a (E/E_a)^{1/2} (B/B_{cr})^{-1} \quad (2)$$

becomes smaller than the typical impact parameter of close collisions in a non-magnetized plasma given by Eq. (1) — see Zheleznyakov (1997), Zheleznyakov et al. (1999a), Koryagin (2000), Schmidt et al. (2000), and Hu et al. (2002a, b). Here, $E$ is the electron kinetic energy far from an ion, $m_e$ is the electron mass, $\omega_B = eB/(m_e c)$ is the cyclotron frequency, $e$ is the electron elementary charge, $Z > 0$ is the ion charge number, $B$ is the magnetic field strength, and $c$ is the speed of light. For the sake of comparison, the length scales given by Eqs. (1) and (2) are expressed using the Bohr radius $r_a = Z e^2/(2m_e E_a) = 0.53Z^{-1}$ Å. In this expression the ionization potential is given by $E_a = Z^2 e^4 m_e/(2\hbar^2) = 13.6Z^2$ eV. Furthermore, the magnetic field is expressed in terms of $B_{cr} = Z^2 e^3 m_e^2 c/\hbar^3 = 2.35 \cdot 10^9 Z^2$ G, for which the energy of the fundamental Landau level, $\hbar \omega_B/2$, equals $E_a$. The ratio of the length scales

$$r_B/r_s = (E/E_a)^{3/2} (B/B_{cr})^{-1} \ll 1 \quad (3)$$
is realized in the photospheres of magnetic white dwarfs whose temperatures and magnetic fields fall into the regions \( I_{\text{class}} \) and \( I_{\text{quant}} \) in Fig. 1.

In the region \( I_{\text{class}} \), the electron energy \( E \) exceeds the quantum one \( \hbar \omega_B \), therefore the electron motion can be described classically. In the quantum region \( I_{\text{quant}} \), the electron energy is smaller than \( \hbar \omega_B \). Nevertheless, the definitions of the parameter in Eq. (1) and of the Larmor radius of Eq. (2) keep valid in the region \( I_{\text{quant}} \). For this, one should use the energy of the fundamental Landau level \( \hbar \omega_B/2 = E_{n}(B/B_{\text{cr}}) \) as the kinetic energy \( E \) (Koryagin 2008a). Then, the electron energy \( E \) is not a free parameter in the quantum limit: \( E = \hbar \omega_B/2 \), the distance \( r_{s} \) of Eq. (1) starts decreasing for increasing magnetic field strengths: \( r_{s} \propto B^{-1} \), while the Larmor radius decreases slower: \( r_{B} \propto B^{-1/2} \). As a result, the ratio of Eq. (3) can be met only for magnetic fields \( (r_{B}/r_{s})_{E=\hbar \omega_B/2} = (B/B_{\text{cr}})^{1/2} \ll 1 \).

2. Close collisions
In the region for which Eq. (3) is valid, the collision transport frequencies are determined by the effective close collisions, in which an electron transits the region near an ion for a time of the order or less than the cyclotron period (Zheleznyakov 1997, Zheleznyakov et al. 1999a). The impact parameters of these collisions are of the order of or less than the cyclotron period (Zheleznyakov 1997, Zheleznyakov et al. 1999a). The classical motion near an ion unifies the regions \( I_{\text{class}} \) and \( I_{\text{quant}} \) in Fig. 1 and explains the identical values of the characteristic impact parameters of the effective close collisions in these regions (Koryagin 2008a).

In effective close collisions, the motion of an electron becomes quasi-bound — a particle returns to an ion many times before it goes away from a scattering center. Figures 2a and

![Figure 1](image-url)
Figure 2. The electron trajectories in the head-on (a), close (b), and distant (c) collisions.

Figure 3. The dependence of the backscattering probability $|T^{(b)}_{00}|^2$ on the electron energy $E$ in the close (a) and distant (b) collisions. The bars under the upper horizontal axis mark the energies and the energy widths of the resonant autoionizing states.

$2b$ show examples of the quasi-bound classic trajectories in a head-on collision and in a close collision with the impact parameter $p_h = L_u$: the initial angle between the electron velocity and the magnetic field (pitch-angle) $\theta = \pi/4$ and the energy $E = E_u/2$ (this corresponds to the equality $r_B = r_s$). The final pitch-angle non-regularly (chaotically) depends on the initial electron energy, the magnetic field strength, and the impact parameter (Hu et al. 2002a, b).

In the quantum-mechanical approach, the quasi-bound electron motion corresponds to the particle transition to the highly excited Landau levels at which the kinetic energy of the transverse to the magnetic field motion $(n + 1/2)\hbar\omega_B$ exceeds the electron kinetic energy far from an ion $E$. At these levels, an electron cannot go away from an ion and has to return back to a scattering center. In the quantum limit ($E \sim \hbar\omega_B$), the electron backscattering probability keeps non-regular (chaotic) dependence on the initial electron energy and the magnetic field strength (Koryagin 2008b). Figure 3a shows such a non-regular dependence on the energy of the electron motion along the magnetic field for a low-energy particle which can occupy only the fundamental Landau level before and after a collision; the impact parameter equals the double Larmor radius at the fundamental Landau level (this corresponds to the electron state with the magnetic azimuthal number $m = -1$), $B = 0.005B_{cr}$.

3. Distant collisions
In distant collisions with impact parameters $p_h \gg L_u$, an electron moves along the whole trajectory under the conditions of the drift approximation. Thus, in the classical description, the
final change of the electron pitch-angle exponentially decreases with increasing impact parameter as \( \Delta \theta \propto \exp(-0.62p_B^{3/2}/L_u^{3/2}) \) (Koryagin 2000). Figure 2c shows an example of a distant collision.

In the quantum limit \((E \sim \hbar \omega_B)\), an electron can completely scatter backwards even in a distant collision (Koryagin 2008b). For this, the electron energy must coincide with the energy of a so-called autoionizing state (Friedrich & Chu 1983, Chu & Friedrich 1984). The dependence of the backscattering probability on the electron energy acquires the form of a sequence of the resonant peaks (Fig. 3b). However, the energy width of the resonances exponentially decreases with increasing impact parameters. Thus, the contribution of distant collisions to the backscattering cross-section is small compared to that of close collisions.

4. Transport frequencies

The typical transport cross-sections are determined by close collisions. Therefore, their values are of the order of \( \pi L_u^2 \) which is much less than their typical values \( \pi r_s^2 \) without a magnetic field. For example, the deceleration of the non-zero hydrodynamic (mean) velocity of the thermal electrons along a magnetic field is described by the conventional transport frequency \( v_f = 2.2n_i v_T L_u^2 \ln^{4/3}(r_s/r_B) \) both in the classic (Koryagin 2000) and in the quantum (Koryagin 2008b) limits, where \( n_i \) is the ion space density and \( v_T = (k_B T/m_e)^{1/2} \) is the thermal velocity of electrons.

The decrease of the transport frequencies in a strong magnetic field conditions the increase of the required photospheric temperature (up to \( T = 4 \times 10^4 \) K) at which the cyclotron radiation pressure can exceed gravity, expel plasma from a photosphere, and, thus, inflate the extensive plasma envelope in a magnetosphere — the so-called radiation-driven diskon (Bespakov & Zheleznyakov 1990, Zheleznyakov et al. 1999b).

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