Testing astrophysical models for the PAMELA positron excess with cosmic ray nuclei

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The excess in the positron fraction reported by the PAMELA collaboration has been interpreted as due to annihilation or decay of dark matter in the Galaxy. More prosaically, it has been ascribed to direct production of positrons by nearby pulsars, or due to pion production during stochastic acceleration of hadronic cosmic rays in nearby sources. We point out that measurements of secondary nuclei produced by cosmic ray spallation can discriminate between these possibilities. New data on the titanium-to-iron ratio from the ATIC-2 experiment support the hadronic source model above and enable a prediction to be made for the boron-to-carbon ratio at energies above 100 GeV. Presently, all cosmic ray data are consistent with the positron excess being astrophysical in origin.

The PAMELA collaboration [1] has reported an excess in the cosmic ray positron fraction, i.e. the ratio of the flux of positrons to the combined flux of positrons and electrons, \( \phi_{e^+}/(\phi_{e^+} + \phi_{e^-}) \), which is significantly above the background expected from production of positrons and electrons during propagation of cosmic ray protons and nuclei in the Galaxy [2]. It has been noted that the observed rise in the positron fraction between \( \sim 5 - 100 \text{ GeV} \) cannot be due to propagation effects [3], rather it requires a local primary source of cosmic ray electrons and positrons, e.g. nearby pulsars [4, 5, 6, 7]. More excitingly, this could be the long sought for signature of the annihilation or decay of dark matter particles in the Galaxy [8, 9, 10, 11, 12, 13].

Alternatively the observed rise in the positron fraction could be due to the acceleration of positrons produced by the decay of charged pions, which are created through hadronic interactions of cosmic ray protons undergoing acceleration in a nearby source [14]. That the secondary-to-primary ratio should increase with energy if secondaries are accelerated in the same spatial region as the primaries has been noted quite some time ago in the context of cosmic ray acceleration in the interstellar medium [13, 16, 17]. This model is conservative since it invokes only processes that are expected to occur in candidate cosmic ray sources, in particular supernova remnants (SNR). One way to distinguish it from the other models is to e.g. compute the expected anti-proton-to-proton ratio, which is experimentally observed to be consistent with the standard background [18]. This is in fact in accord with the above model which predicts a rise in the \( \bar{p} \) fraction only at energies above \( \sim 100 \text{ GeV} \) [19] (see also ref. [20]). This prediction cannot however be tested presently but must await data from the forthcoming AMS-2 mission [21] as well as PAMELA.

Although dark matter annihilation or decay as the explanation of the positron signal would appear to be disfavoured by the absence of a corresponding antiproton signal, this can in principle be accomodated in models with large dark matter particle masses or preferential leptonic annihilation/decay modes [22, 23, 24, 25]. Nearby pulsars as the source of the positrons are of course quite consistent with the absence of antiprotons. To differentiate between these possibilities and the model [14] in which secondary positrons from hadronic interactions are accelerated in the same region, we consider secondary nuclei in cosmic rays which are produced by the spallation of the primaries. An increasing secondary-to-primary ratio (e.g. boron-to-carbon or titanium-to-iron) in the same energy region would confirm that there is indeed a nearby cosmic ray source where nuclei are being accelerated stochastically along with protons.

An issue with this model [14] is that a crucial parameter is not known a priori but needs to be obtained from observations. This is the diffusion coefficient of relativistic particles near the accelerating SNR shock which determines the importance of a flatter spectral component over the usual Fermi spectrum and leads to the rise in secondary-to-primary ratios. Its absolute value cannot presently be reliably calculated. Observations of SNR indicate that the magnetic field is quite turbulent so that relativistic electrons diffuse close to the 'Bohm limit' with diffusion co-efficient: \( D_{\text{Bohm}} = r_{\ell}/3 \), where the Larmor radius \( r_{\ell} \) of the nucleus is proportional to the rigidity \( E/Z \) [26]. We need to determine the actual diffusion co-efficient of ions in SNR in ratio to the Bohm value by fitting to data. A measurement of one nuclear secondary-to-primary ratio therefore allows us to make predictions for other ratios in the framework of this model.

Very recently, data on the titanium-to-iron ratio (Ti/Fe) from the ATIC-2 experiment have been announced [27] that indeed show a rise above \( \sim 100 \text{ GeV} \). We use this data as a calibration to determine the diffusion coefficient and then, extrapolating it according to its rigidity dependence, we predict the boron-to-carbon ratio (B/C) that should soon be measured by PAMELA (P. Picozza, private communication).

Galactic cosmic rays with energies up to the 'knee' in the spectrum at \( \sim 3 \times 10^{15} \text{ eV} \) are believed to be accelerated by SNRs. The strong shocks present in these environments allow for efficient diffusive shock acceleration (DSA) by the 1st-order Fermi process [28]. In the simple test-particle approximation, which is adequate for the level of accuracy of the present discussion [29], protons and nuclei that are injected upstream are accelerated to form a non-thermal power-law spectrum whose index de-
pends only on the parameters of the shock front, in particular the compression ratio $r$. For a supersonic shock with $r = 4$ the steady state energy spectrum for protons and nuclei is $N_d E \propto E^{-\gamma + \frac{2}{3}} dE$ where $\gamma = 3r/(r - 1)$. In the standard model of galactic cosmic ray origin, the accelerated primary nuclei produce secondaries by spallation on hydrogen and helium nuclei in the interstellar medium (ISM). In the simple ‘leaky box model’ an energy-dependent escape of the cosmic rays out of the galaxy is invoked to obtain a secondary-to-primary ratio that decreases with energy as observed to date in the region $\sim 1 \sim 100$ GeV. This is also obtained by using the GALPROP code which solves the full transport equation in 3 dimensions, and can yield both the time-independent as well as equilibrium solutions.

However, as the acceleration time for the highest energy particles is of the same order as the timescale for spallation, the production of secondaries inside the sources must be taken into account. In any stochastic acceleration process one then expects the secondary-to-primary ratio to increase with energy since particles with higher energy have spent more time in the acceleration region and have therefore produced more secondaries. This general argument can be quantified for the case of DSA by including the production of secondaries due to spallation and decay as a source term,

\[
Q_i(\varepsilon_k) d\varepsilon_k = \sum_j N_j(\varepsilon_k) \left[ \sigma^{\text{spall}}_{j-i} \beta c n_{\text{gas}} \frac{1}{\varepsilon_k \tau^{\text{dec}}_{j-i}} \right] d\varepsilon_k, \tag{1}
\]

where $\varepsilon_k$ is the K.E./nucleon (in GeV), and a loss term,

\[
\Gamma_i N_i(\varepsilon_k) d\varepsilon_k = N_i(\varepsilon_k) \left[ \sigma^{\text{spall}}_{i} \beta c n_{\text{gas}} \frac{1}{\varepsilon_k \tau^{\text{dec}}_{i}} \right] d\varepsilon_k, \tag{2}
\]

where $\sigma^{\text{spall}}_{j-i}$ and $\tau^{\text{dec}}_{j-i}$ are the partial (total) cross-sections and decay time, respectively. The transport equation for any nuclear species $i$ then reads

\[
u \frac{\partial f_i}{\partial x} = D_i \frac{\partial^2 f_i}{\partial x^2} + \frac{1}{3} \frac{\partial u}{\partial p} \frac{\partial f_i}{\partial p} - \Gamma_i f_i + q_i, \tag{3}
\]

where $f_i$ is the phase space density and the different terms from left to right describe convection, spatial diffusion, adiabatic energy losses as well as losses and injection of particles from spallation or decay. We consider the acceleration of all species in the usual setup: in the frame of the shock front the plasma upstream $(x < 0)$ and downstream $(x > 0)$ is moving with velocity $u_-$ and $u_+$ respectively. We solve eq. (3) analytically for relativistic energies $\varepsilon_k$ greater than a few GeV/nucleon such that $\rho \approx E$, $\beta \approx 1$ and $N_d E \approx 4\pi p^2 f_i d\rho$. At these energies ionization losses can be neglected and the spallation cross sections become energy independent.

There are three relevant timescales in the problem:

1. Acceleration time $\tau_{\text{acc}}$:

\[
\tau_{\text{acc}} = \frac{3}{u_- - u_+} \int_0^p \left( \frac{D^+_i}{u_+} + \frac{D^-_i}{u_-} \right) \frac{dp'}{p'} \approx 8.8 E_{\text{GeV}} Z^{-1} B_{\mu G} \text{ yr} \tag{4}
\]

for Bohm diffusion and the parameter values mentioned later.

2. Spallation and decay time $\tau_i$.

\[
\tau_i^{\text{spall}} \equiv \frac{1}{\sigma^{\text{spall}}_i} \sim 1.2 \times 10^7 \left( \frac{n_{\text{gas}}}{\text{cm}^{-3}} \right)^{-1} \text{ yr}, \tag{5}
\]

where an average $\sigma_i$ of $\sigma(100)$ nb has been assumed. The rest lifetime $\tau_i^{\text{dec}}$ of the isotopes considered ranges between $4 \times 10^{-2}$ yr and $10^{17}$ yr.

3. Age of the SNR under consideration.

\[
\tau_{\text{SNR}} = \frac{x_{\text{max}}}{u_+} \sim 2 \times 10^4 \text{ yr}. \tag{6}
\]

There are two essential requirements for SNR to efficiently accelerate nuclei by the DSA mechanism:

(a) $\tau_{\text{acc}} \ll \tau_i^{\text{spall}}$, which is equivalent to

\[
20 \frac{D_i}{u_+^2} \ll 1 \quad \Rightarrow \quad c_i \ll 6.4 \times 10^5 \frac{Z_i}{A_i} B_{\mu G} \text{ GeV}. \tag{7}
\]

(b) $\tau_{\text{SNR}} \ll \tau_i$ which implies,

\[
\frac{x_{\text{max}}}{u_+} \ll \frac{1}{\Gamma_i} \Rightarrow \frac{x}{u_+} \ll 1. \tag{8}
\]

The isotopes for which condition (b) is not satisfied at the lowest energy considered viz. $^{56}\text{Ni}$, $^{57}\text{Co}$, $^{55}\text{Fe}$, $^{54}\text{Mn}$, $^{51}\text{Cr}$, $^{49}\text{V}$, $^{44}\text{Ti}$ and $^{7}\text{Be}$ do not contribute significantly, so their decays in the source region are neglected.

We find that the general solution to eq. (3) for $x \neq 0$ is

\[
f_i^\pm = \sum_{j \leq i} \left( E_{ji}^\pm \lambda_{ji}^{\pm x/2} + F_{ji}^\pm \kappa_{ji}^{\pm x/2} \right) + G_i^\pm, \tag{9}
\]

where

\[
\lambda_{ji}^\pm = \frac{u_i}{D_i^\mp} (1 - \sqrt{1 + 4 D_i^\mp \Gamma_i^{\pm x/2} / u_+^2}),
\]

\[
\kappa_{ji}^\pm = \frac{u_i}{D_i^\mp} (1 + \sqrt{1 + 4 D_i^\mp \Gamma_i^{\pm x/2} / u_+^2}),
\]

where $\Gamma_i^\pm$ is the asymptotic value and $E_{ji}^\pm$ and $F_{ji}^\pm$ are determined by the recursive relations:

\[
E_{ji}^\pm = -4 \sum_{m > j} E_{mj}^\pm \Gamma_{j-i}^\pm / D_i^\mp \lambda_{ji}^{\pm x/2} - 2u_i \lambda_{ji}^\pm - 4 \Gamma_i^\pm, \tag{10}
\]

\[
F_{ji}^\pm = -4 \sum_{m < j} F_{mj}^\pm \Gamma_{j-i}^\pm / D_i^\mp \kappa_{ji}^{\pm x/2} - 2u_i \kappa_{ji}^\pm - 4 \Gamma_i^\pm. \tag{11}
\]

We require that the phase space distribution function converges to the adopted primary composition $Y_i$ (at the injection energy $p_0$) far upstream of the SNR shock:

\[
f_i(x, p) \xrightarrow{x \rightarrow -\infty} Y_i \delta(p - p_0) \quad , \frac{\partial f_i}{\partial p} (x, p) \xrightarrow{x \rightarrow -\infty} 0. \tag{12}
\]

We also require the solution to remain finite far downstream. As the phase space density is continuous at the
\[ f_i^- = f_i^0 e^{\gamma_i^- x^2/2} + \sum_{j<i} E_{ij}^- e^{\gamma_j^- x^2/2} + Y_i \delta(p - p_0) \left( 1 - e^{\gamma_i^- x^2/2} \right), \]

\[ f_i^+ = f_i^0 e^{\gamma_i^+ x^2/2} + \sum_{j<i} E_{ji}^+ e^{\gamma_j^+ x^2/2} + G_i^+ \left( 1 - e^{\gamma_i^+ x^2/2} \right). \]

Using eqs. (7-8), we can linearly expand \( \lambda_i^+ \) and \( \kappa_i^- \) in eq. (10) and the exponentials in eqs. (13-14) to obtain:

\[ f_i^+ = f_i^0 + q_i^+(x = 0) - \frac{\Gamma_i^+ f_i^0}{u_+} x, \]

where \( q_i^\pm \) denotes the downstream/upstream source term: \( q_i^\pm = \sum_{j<i} f_j^\pm v_i^j \).

Finally we integrate the transport equation over an infinitesimal interval around the shock, assuming that \( q_i^- / q_i^+ = \Gamma_i^- / \Gamma_i^+ = n_{gas}^- / n_{gas}^+ = r \) and that \( D_i^- \approx D_i^+ \):

\[ \frac{d f_i}{d p} = -\gamma_i f_i^0 - \gamma_i (1 + r^2) D_i^- f_i^0 - \nabla_i \delta(p - p_0) \]

\[ + \gamma \left[ (1 + r^2) q_i^-(x = 0) D_i^- + Y_i \delta(p - p_0) \right], \]

which is readily solved by

\[ f_i^0(p) = \int_0^p \frac{dp'}{p'} \left( \frac{p'}{p} \right)^\gamma e^{-\gamma (1 + r^2)(D_i^- (p) - D_i^- (p'))} u_+^2 \]

\[ \times \gamma \left[ (1 + r^2) q_i^-(x = 0) D_i^- (p') + Y_i \delta(p' - p_0) \right]. \]

Our eqs. (10-18) should be compared to eqs. (4-6) of ref. [14] where the loss terms \( \Gamma_i f_i \) were not taken into account. The exponential in our eq. (18) leads to a natural cut-off in both the primary and secondary spectra above the energy predicted by eq. (7). However, due to the approximations we have made, the secondary-to-primary ratios cannot be predicted reliably for \( 4\Gamma_i D_i / u_+^2 \gtrsim 0.1 \) i.e. much beyond \( \sim 1 \) TeV.

Starting from the heaviest isotope, eqs. (16) and (18) can be solved iteratively to obtain the injection spectrum after integrating over the SNR volume,

\[ N_i(E) = 4\pi \int_0^{u + T_{SN}} dx p^2 f_i(p) 4\pi x^2. \]

To account for the subsequent propagation of the nuclei through the ISM we solve the transport equation in the ‘leaky box model’ [30] which reproduces the observed decrease of secondary-to-primary ratios with energy in the range \( \sim 1 - 100 \) GeV by assuming an energy-dependent lifetime for escape from the Galaxy. The steady state cosmic ray densities \( N_i \) observed at Earth are then given by recursion, starting from the heaviest isotope,

\[ N_i = \frac{\sum_{j<i} \left( \Gamma_{i-j} \delta_{i-j} + 1/\epsilon_k \delta_{i-j} \right) N_j + \mathcal{R}_{SN} N_i}{1/\tau_{esc,i} + \Gamma_i}, \]

where \( \mathcal{R}_{SN} \sim 0.03 \text{ yr}^{-1} \) is the Galactic supernova rate.

We calculate the source densities \( N_i \) and ambient densities \( \mathcal{R}_{SN} \), taking into account all stable and metastable isotopes from \( ^{62}\text{Ni} \) down to \( ^{46}\text{Cr} / ^{46}\text{Ca} \) for the titanium-to-iron ratio, and from \( ^{18}\text{O} \) down to \( ^{10}\text{Be} \) for the boron-to-carbon ratio. Short lived isotopes that \( \beta^\pm \) decay immediately into (meta)stable elements are accounted for in the cross-sections. The primary source abundances are taken from ref. [31] and we have adopted an injection energy of 1 GeV independent of the species. The partial spallation cross-sections are from semi-analytical tabulations [32] and the total inelastic cross-sections is obtained from an empirical formula [23]. The escape time is modelled according to the usual relation:

\[ \tau_{esc,i} = \rho c x_{esc,i} = \rho c x_{esc,i} (E/Z_i)^{-\mu} \]

where \( x \) is the column density traversed in the ISM and \( \rho = 0.02 \text{ atom cm}^{-3} \) is the typical mass density of hydrogen in the ISM. We have neglected spallation on helium at this level of precision as its inclusion will have an effect \( < 10\% \). The parameters of the fit are sensitive to the adopted partial spallation cross-sections, for example \( \mu \simeq 0.7 \) for the Ti/Fe ratio but \( \sim 0.6 \) for the B/C ratio. For improved accuracy a current compilation of experimentally deduced cross-sections should be used.

FIG. 1: The titanium-to-iron ratio in cosmic rays along with model predictions — the ‘leaky box’ model with production of secondaries during propagation only (dashed line), and including production and acceleration of secondaries in a nearby source (solid line - dotted beyond the validity of our calculation). The data points are from ATIC-2 (triangles) [27] and HEAO-3-C3 (circles) [31].
The calculated titanium-to-iron ratio together with relevant experimental data is shown in Fig. 1. The dashed line corresponds to the leaky box model with production of secondaries during propagation only and is a good fit to the (reanalysed) HEAO-3-C3 data \cite{34}. The solid line includes production and acceleration of secondaries inside the source regions which results in an increasing ratio for energies above \(\sim 50\text{ GeV}/n\) and reproduces well the ATIC-2 data \cite{27} taking \(F^{-1} \approx 40\). This is similar to the value reported in refs. \cite{14,19}, thus ensuring consistency with the \(e^+\) as well as \(\bar{p}\) fraction measured by PAMELA.

Clearly the experimental situation is inconclusive so a new test is called for. Fig. 2 shows the corresponding expectation for the boron-to-carbon ratio with the diffusion coefficient scaled proportional to rigidity according to eq. (22). The CREAM data \cite{30} do show a downward trend as has been emphasized recently \cite{37}, but the uncertainties are still large so we await more precise measurements by PAMELA which has been directly calibrated in a test beam \cite{38}. Agreement with our prediction would confirm the astrophysical origin of the positron excess as proposed in ref. \cite{14} and thus establish the existence of an accelerator of hadronic cosmic rays within a few kpc.

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\[D_i(E) = 3 \times 10^{22} F^{-1} B_i^{-1} E_{\text{GeV}} Z_i^{-1} \text{cm}^2\text{s}^{-1} \quad (22)\]

where the fudge factor \(F^{-1}\) is the ratio of the diffusion coefficient to the Bohm value and is determined by fitting to the measured titanium-to-iron ratio.

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