MHD combined convection flow of non-Newtonian liquid with viscous dissipation and thermophoretic effects

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Abstract: The present article is concerned with combined convection MHD flow of Oldroyd-B liquid past over a stretchable sheet. Mathematical formulation of the problem is designed with the existence of viscous dissipation, radiation, thermophoresis and chemical reaction. With the help of suitable similarity transformation governing PDE’s are modified into nonlinear ODE’s. Graphs are portrayed to mention the variance on the velocity, concentration and temperature distributions. Moreover, Mass and heat transfer rates are illustrated through graphically.

1. Introduction

Studies of non-Newtonian liquids are growing fairly large because of the most number of industrial purposes, manufacturing processes and applications in other sub- fields of engineering. The applications of non-Newtonian liquids are oil pipeline friction reduction, cooling systems, flow tracers etc. The non-Newtonian liquids are different from the Newtonian liquids do not obey the Newton’s law of viscosity. Mainly, the non -Newtonian liquids have the behaviour of follow shear rate and shear stress. The relation between the shear rate and shear stress in non-Newtonian liquids are not a linear. The liquid which is opposes a strain and shear flow linearly with time when a pressure is applied is defined as a viscoelastic liquid. One of the essential models used to report the flow of viscoelastic liquid is Oldroyd-B liquid, it contains the retardation and relaxation time effects. Hayat et al. [1] explained the MHD convection of Oldroyd-B liquid passing through the stretchable surface with homogeneous and heterogeneous effects. Noor et al. [2] analyzed about the chemical reaction and radiation effects on MHD combined convective flow through a vertical plate. The study of chemical reaction and radiation on MHD has become more important industrially and the combined (mixed) convection is a combination of forced and free convection, which is used mostly in geophysics. Some other studies about combined convection are investigated by Ashraf et al. [3] and Naganthan et al. [4]. Shehzad et al. [5] dealt the presence of thermophoresis in MHD flow over a stretchable sheet under the mass and heat transfer. In addition, few more results based on mass and heat transfer in magneto hydrodynamic flow by using viscous dissipation and Joule heating was addressed by Baoku et al. [6] and Ganesh et al. [7].

The main motivation of this paper is to examine the effects of chemical reaction, viscous dissipation, radiation and thermophoresis in mixed convection of Oldroyd B flow through a stretchable sheet.

2. Flow formulation:

Consider a 2-D incompressible Oldroyd-B liquid flow influenced by a stretchable sheet. The surface velocity is taken into $u_1 = ax$, $a > 0$. Where $a > 0$ is described as a stretch rate. The temperatures divided into two divergent sheets are pointed by $T_w$ and $T_e$. $B_0$ is denoted the magnetic field strength and allocated to the stretchable surface. Variable concentration and Variable temperature and are noted by $C_w(x)$ and $T_w(x)$, while the liquid has a uniform and uniform ambient concentration $C_e$, ambient temperature $T_e$ with $C_w > C_e, T_w > T_e$. By using the above details the governing equations become as

$$\frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y} = \frac{1}{\rho}\left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} + 2u_1 v_1 \frac{\partial^2 u_1}{\partial x \partial y}\right) = \frac{\mu}{\rho} \frac{\partial^2 u_1}{\partial y^2}$$

(1)
\[ + \mu A_2 \left( u_1 \frac{\partial u_1}{\partial y} + v_1 \frac{\partial v_1}{\partial y} + \frac{\partial^2 u_1}{\partial x^2} - \frac{\partial u_1 \partial v_1}{\partial x \partial y} - \frac{\partial u_1 \partial^2 v_1}{\partial^2 x} \right) - \frac{\sigma T}{\rho} \left( u_1 + A_1 v_1 \frac{\partial u_1}{\partial y} \right) \]

\[ + g \left[ \beta_T (T - T_w) + \beta_C (C - C_w) \right] \]

\[ u_1 \frac{\partial T}{\partial x} + v_1 \frac{\partial T}{\partial y} \left[ \mu \frac{\partial u_1}{\partial y} \right]^2 + \frac{\sigma \rho m^2}{\rho c_p} u_1^2 \] \hspace{1cm} \left[ = \frac{\lambda_g}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \right] \hspace{1cm} \text{(3)}

\[ u_1 \frac{\partial C}{\partial x} + v_1 \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - \frac{\partial (y T C)}{\partial y} - K_m C \] \hspace{1cm} \text{(4)}

The correlated boundary conditions are

\[ u_1 = u_{1w}(x) = ax, \quad v_1 = 0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0, \]

\[ u_1 \to 0, \quad v_1 \to 0, \quad T \to T_{\infty}, \quad C \to C_{\infty} \quad \text{as} \quad y \to \infty. \] \hspace{1cm} \text{(5, 6)}

Where, \((u_1, v_1, T, C, \mu, A_1, A_2, \sigma, \beta_T, \beta_C, \gamma, \rho, c_p, \kappa, \mu^*, D_m, V_T \) and \( k_m \) are the velocity components, kinematic viscosity, liquid density, relaxation time, retardation time, electrical conductivity, thermal expansion coefficients of temperature, thermal expansion coefficients of concentration, acceleration due to gravity, liquid thermal conductivity, specific heat at constant pressure, radiative heat flux, dynamic viscosity, molecular diffusivity of the species concentration, thermophoretic velocity and chemical reaction constant.

The radiative heat flux \( q_r \) is given below

\[ q_r = \frac{4 \sigma_1 \frac{\partial T}{\partial y}}{3 k^*} \] \hspace{1cm} \text{(7)}

Where, \( \sigma_1 \) is Stefan-Boltzmann constant and \( k^* \) is mean absorption coefficient.

The differences in temperature between the flows are concluded to sufficiently little. That is \( T^4 \) can be defined as a linear form of temperature. Developing \( T^4 \) with the help of Taylor series and disuse the higher order gives

\[ T^4 \cong 4 T_{\infty}^3 T - 3 T_{\infty}^4 \] \hspace{1cm} \text{(8)}

From equations (7) and (8), we get

\[ u_1 \frac{\partial T}{\partial x} + v_1 \frac{\partial T}{\partial y} + \frac{\lambda_g}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{4 \sigma_1 T_{\infty}^3 \frac{\partial^2 T}{\partial y^2}}{3 \rho c_p k^*} + \frac{\mu}{\rho c_p} \left( \frac{\partial u_1}{\partial y} \right)^2 \frac{\sigma B_0(x)}{\rho c_p} u_1^2 \] \hspace{1cm} \text{(9)}

The RHS terms of equation (9) represent the thermal radiation, viscous and magnetic terms.

The thermophoretic velocity \( V_T \) which view in the equation (4) can be defined as

\[ V_T = -k \left( \frac{\partial T}{\partial y} \right) \frac{\partial T}{\partial y} \] \hspace{1cm} \text{(10)}

A thermophoretic constant \( \tau \) can be explained as

\[ \tau = \frac{k (T_{\infty} - T_{\infty})}{T_{\infty}} \] \hspace{1cm} \text{(11)}

Above equations (2), (3) and (4) are modified into nonlinear ordinary differential equations by iterating the given below non-dimensional values

\[ \eta = \sqrt{\frac{\beta^*}{\rho}}, \quad u_1 = ax(\eta), \quad v_1 = -\sqrt{a} \mu f(\eta) \]
\[ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \Phi(\eta) = \frac{c - c_w}{c_w - c_w} \] \hspace{1cm} (12)

Using equation (12) in (2), (3) and (4) with the presence of boundary conditions are given by

\[ f'''' - f'' + \beta \left( f'' - 2ff' - f'^2 \right) + f'''' + \alpha_T \left( 2ff' - f'^2 f'' \right) - M \left( f'' - \alpha_T f'' \right) + \lambda (\theta + N \Phi) = 0 \] \hspace{1cm} (13)

\[ \left( 1 + \frac{4}{3} R_d \right) \theta'' + \text{Pr} C (f'' + f') + \text{Pr} \left( f'' + f' \right) = 0 \] \hspace{1cm} (14)

\[ \Phi'' + \text{Sc} \left[ f \Phi' + f' \Phi - \tau \left( \theta' \Phi' + \theta \Phi' \right) \right] - \text{Sc} \Phi = 0 \] \hspace{1cm} (15)

Subject to constraints

\[ f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \Phi(0) = 1 \quad \text{at} \quad \eta = 0 \] \hspace{1cm} (16)

\[ f' = 0, \quad f'' = 0, \quad \theta = 0, \quad \Phi = 0 \quad \text{as} \quad \eta \to \infty \] \hspace{1cm} (17)

Now, we state the non-dimensional values corresponding to \( \alpha_T = A_1 a, \beta_\lambda = A_2 a \) are known as a relaxation time, retardation time constants, Hartmann number \( M = \frac{\sigma B_0^2}{\mu a} \), local buoyancy constant \( \lambda = \frac{Gr_\lambda}{Re_x^2} \), Grashof number \( Gr_\lambda = \frac{g \beta \left( T_w - T_\infty \right) a^3}{\mu^2} \), Reynolds number \( Re_x = \frac{u_w a}{\mu N} = \frac{\beta_\lambda (C_w - C_\infty)}{\beta_\lambda (T_w - T_\infty)} \), Radiation constant \( R_d = \frac{4 \epsilon_0 (T_w^4)}{k \lambda g} \), Prandtl number \( Pr = \frac{\rho c_p \mu}{k g} \), Eckert number \( E_c = \frac{u_w^2}{\left( c_p T_w - T_\infty \right)} \), Schmidt number \( Sc = \frac{\mu}{D_m} \) and chemical reaction constant \( K = \frac{k_m}{a} \).

The dimensionless forms of local skin friction, heat and mass transfer rates are represented below

\[ Re^{-3/2} Nu_x = -\left( 1 + \frac{4}{3} R_d \right) \theta'(0) \] \hspace{1cm} (18)

\[ Re^{-3/2} Sh_x = -\Phi'(0) \] \hspace{1cm} (19)

### 3. Results and discussion

The local similarity solution ODE’s, (13)–(15) related to the boundary condition (Eq. (16)) are solved for various values physical parameters. The impacts of different constants on temperature, velocity and concentration distributions for the stable values of \( M = 0.5, \alpha_T = 0.1, \beta_\lambda = 0.1, \lambda = 0.2, E_c = 0.5, Sc = 0.9, Pr = 0.9, N = 1.0, R_d = 0.3, K = 1.0 \) and \( \tau = 0.2 \). The influence of magnetic field (M) on the velocity distribution is sketched in Figure 1. Applied magnetic field has the trend to flow down diminishes the action of the liquid which reduces the velocity distribution. Action of relaxation time constant \( \alpha_T \) on velocity distribution is shown in Figure 2 it has inverse effect on the velocity distribution. Thus we conclude that the retardation time and relaxation time have inverse effects on velocity distribution. Figure 4 displays the changes in mixed convection constant (\( \lambda \)) on velocity distribution. Since \( \lambda \) is the ratio buoyancy forces to viscous forces. In fact higher range of \( \lambda \) reduces the viscous forces and shows an increment in the velocity. Figure 5 displays the result of Prandtl number (\( Pr \)) on the temperature distribution \( \theta(\eta) \). For a higher Prandtl number convection evolve into dominant in shifting energy from the plate compared to conduction. Thus rise in \( Pr \) corresponds to diminishing in the temperature and thermal boundary layer thickness. From Figure 6 we noted that an increasing in Eckert number (\( E_c \)) values give rise to the temperature variance that corresponds to higher temperature and thermal boundary layer thickness. Figure 7 shows higher values of chemical reaction (K) diminishes the concentration distribution. Figure 8 illustrated that concentration and its related boundary layer thickness is smaller for the higher thermophoretic constant \( \tau \). Figure 9 & 10 depicts the heat transfer rate increases while enhancing the values of radiation and mixed convection constants with the combination of magnetic field. Heat transfer rate increase while increasing the
values of \( R_d \) and \( \lambda \). Relaxation time constant \((\alpha_T)\) and retardation time constant \((\beta_T)\) shows the inverse effect on mass transfer rate (Figure 11 & 12).

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Figure 1 & 2. Influence of \( M \) and \( \alpha_T \) on velocity distribution \( f'(\eta) \)

Figure 3 & 4. Influence of \( \beta_T \) and \( \lambda \) on velocity distribution \( f'(\eta) \)

Figure 5 & 6. Influence of \( Pr \) and \( E_c \) on temperature distribution \( \theta(\eta) \)
Figure 7 & 8. Influence of $K$ and $\tau$ on concentration distribution $\Phi(\eta)$

Figure 9 & 10. Influence of $\lambda$ and $E_c$ on local Nusselt number $Re^{-\frac{1}{2}}Nu_x$

Figure 11 & 12. Influence of $\alpha_T$ and $\beta_T$ on local Sherwood number $Re^{-\frac{1}{2}}Sh_x$

4. Final remarks
The combined convection of an Oldroyd-B liquid flow upon a stretchable sheet with the appearance of viscous dissipation, thermophoresis and chemical reaction is inspected. The final observations are an enhance in magnetic field constant diminishes the velocity distribution. An enhance in magnetic field constant diminishes the velocity distribution, higher range of Eckert number rising the thermal boundary layer thickness, radiation and mixed convection constants boost up the heat transfer rate and solutal boundary layer thickness diminishes for larger thermophoretic constant.
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