Massive MIMO Channel Estimation Based on Compressed Sensing

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Abstract. Obtaining accurate channel state information (CSI) is an important prerequisite for achieving good performance of the Massive MIMO system. The traditional pilot-based channel estimation algorithm has low spectrum utilization. The channel estimation algorithm based on compressed sensing can make full use of the sparsity of the channel and realize channel estimation with fewer pilots. In this paper, wavelet denoising is combined with a sparse channel estimation algorithm based on Orthogonal Matching Pursuit (OMP), and Cramer-Rao Lower Bound (CRLB) is derived. The simulation results show that this method has better performance.

1. Introduction

The Massive MIMO system is mainly based on the MIMO system, increasing the number of base station transmitting antennas to obtain better system performance, significantly improving the transmission rate, spectrum efficiency and energy utilization of the system[1-2], these good system performance depends on whether the base station can obtain accurate channel state information (CSI)[3]. The repeated use of pilots makes the pilot pollution problem serious. The increase in the number of antennas increases the matrix dimension of the channel and the pilot overhead, and the complexity of channel estimation also increases. Traditional channel estimation algorithms are mainly based on pilot channel estimation, such as Least Square (LS) algorithm, Minimum Mean Square Error (MMSE) algorithm [4], these algorithms require a large number of orthogonal pilot sequences to obtain better channel estimation results, and are not suitable for Massive MIMO system. Therefore, it is very important to study fast and accurate channel estimation.

Compressed sensing theory is a new theory proposed by Candes Randes, Romberg and Tao that can take advantage of signal sparsity [5]. The theory points out that if the signal is sparse in a certain transform domain, the signal can be projected into a low-dimensional space, and the original signal can be recovered with less signal. In recent years, compressed sensing theory has been widely used in various fields, especially in the field of communications. The literature [6] pointed out that the wireless channel is sparse, and the compressed sensing theory is applied to channel estimation. The compressed sensing reconstruction algorithm is used to recover the channel parameters, which can reduce the pilot overhead and computational complexity of the system. At present, channel estimation based on compressed sensing algorithms generally uses the orthogonal matching pursuit algorithm (OMP), which has low computational complexity and is easy to implement. This algorithm can obtain better estimation performance under the condition of high signal-to-noise ratio. Low signal-to-noise ratio conditions, poor performance. To solve this problem, this paper considers the influence of noise in the communication
system, uses wavelet denoising method to denoise the transmitted signal, uses OMP algorithm to reconstruct the signal, and derives the CRLB. The simulation results show that the mean square error after denoising is smaller than that before denoising.

2. System model
Considering the uplink model of the MU-MIMO system, in the Massive MIMO system, the base station side obtains good system performance by configuring a large number of antennas. Suppose there are L cells in the system, each cell has K single-antenna users, a base station is configured at the center of each cell, and M antennas (M>K) are configured at the base station [7].

The elements in the channel matrix can be expressed as follows:

\[ h_{jk} = g_{jk} (\beta_{jk})^{1/2} \] (1)

where \( h_{jk} \) represents the channel transmission coefficient between the mth antenna of the base station in cell j and the kth user in cell i, \( g_{jk} \) represents the small-scale fading coefficient, and \( \beta_{jk} \) represents the large-scale fading coefficient, which can be regarded as a constant in the experiment. Through formula (1), the channel matrix can also be expressed as:

\[
H_j = \begin{bmatrix}
    h_{j11} & \ldots & h_{j1k} \\
    \vdots & \ddots & \vdots \\
    h_{jk1} & \ldots & h_{jkk}
\end{bmatrix} = G_j B_j
\] (2)

where \( B_j = \begin{bmatrix}
    \beta_{j1} & \ldots & 0 \\
    \vdots & \ddots & \vdots \\
    0 & \ldots & \beta_{jk}
\end{bmatrix} \) is a Gaussian random matrix with a mean value of 0 and a variance of 1.

Therefore, the received signal \( y_j \) at the base station in cell j can be expressed as:

\[ y_j = \sum_{i=1}^{L} X_i H_j + w_j \] (3)

where \( X_i \) represents the transmitted signal of users in cell i, and \( w_j \) represents a Gaussian white noise matrix with a mean value of 0 and a variance of \( \sigma \).

The matrix can be expressed as:

\[ Y = XH + w \] (4)

3. Compressed sensing
Most of the signals in nature are not sparse signals, and need to be projected to a sparse domain for sparse representation. Sparse representation is the basis of compressed sensing, which represents the information in a signal as a set of few real or complex numbers. Using mathematical language, a signal \( x \) can be expressed as [8]:

\[ x = \sum_{i=1}^{N} \psi_i s_i = \Psi s \] (5)

where \( x \) and \( s \) are vectors of \( N \times 1 \), and \( \Psi = [\psi_1, \psi_2, \ldots, \psi_N]^T \) is a sparse transform base of N columns orthogonal to each other. When there are only \( K \) non-zero elements in \( s \) and \( K \) is much smaller than \( N \), the signal \( x \) is \( K \) sparse in the \( \Psi \) domain, sparsity is \( K \).

For the \( K \) sparse signal \( x \), through the transformation matrix, \( M \) measurement values are taken from it, which can be expressed in matrix form as:
\[ y = \Phi x = \Phi \Psi s = As \]  

Among them, A is the measurement matrix. Obviously, the number of unknowns in formula (6) is greater than the number of equations, and the equations have countless solutions, and the original signal cannot be reconstructed accurately.

However, the signal x is K sparse, and the sparse optimization algorithm can be used to find the sparse variable s of formula (6), and then substituting formula (5) to find x. In addition, the measurement matrix need to satisfy the Restricted Isometry Property (RIP) [14] under the following condition:

\[ (1-\delta)\|s\|_2^2 \leq \|\Phi s\|_2^2 \leq (1+\delta)\|s\|_2^2 \]  

(7)

This feature actually ensures that the transformation matrix \( \Psi \) and the measurement matrix \( \Phi \) are not related to each other. The sparse solution can be obtained by solving the \( l_0 \)-norm, but the \( l_0 \)-norm solution is not a convex optimization problem. It is an NP-hard problem that is not easy to solve, especially when there is noise. In the case of, when the RIP coefficient \( \delta \) satisfies b, the \( l_0 \)-norm minimization problem is transformed into the \( l_1 \)-norm optimization solution, namely:

\[ \arg \min_s \|s\|_0, \quad s.t. y = As \]  

(8)

Sparse channel estimation based on compressed sensing actually transforms the formula (4) to construct a sparse matrix. The transformation process can be expressed as:

\[ Y = XFh + w = Ah + w \]  

(9)

Then, according to the sparsity of the channel and the known pilot information, the observation matrix is designed, and the sparse matrix is recovered through formula (8), and finally the CSI is obtained.

4. Sparse channel estimation

4.1. Channel estimation algorithm

Common compressed sensing reconstruction algorithms mainly include convex relaxation algorithm and greedy tracking algorithm. Greedy tracking algorithm has fast convergence speed and low computational complexity. When the receiving end performs compression measurement on the signal, the noise is also strengthened, which affects the signal reconstruction performance. The flow of the orthogonal matching pursuit algorithm based on wavelet denoising is as follows [9]:

(1) Initialization: the signal to be processed is \( A \), atomic support set \( \Lambda_0 = \emptyset \), measurement matrix A, sparsity k
(2) Use wavelet denoising method to denoise the signal y to obtain the denoised signal \( y_i \)
(3) Find the column B, which has the largest inner product with the measurement matrix \( b_k \),
\[ b_k = \arg \max_{j=1,2,\ldots,M} \langle r_{-1}, A_j \rangle, \quad \text{The column number is marked as } \lambda_i \]
(4) Add all the obtained column numbers \( \lambda_i \) to \( \Lambda_j \), update the support set \( \Lambda_t = \Lambda_{t-1} \cup \{\lambda_i\} \), and update the matrix \( A_t = A_{t-1} \cup b_{\lambda_i} \)
(5) Obtain the estimated value \( \tilde{h}_n = (A^T A)^{-1} A^T y_i \) through the least square algorithm
(6) Update the residual \( r = y_i - \tilde{h}_n A \), let \( t = t+1 \), if \( t \leq k \), jump to (3), otherwise, stop the iteration and output \( \tilde{h} \)
4.2. Cramer-Rao Lower Bound (CRLB)

Cramer-Rao Lower Bound (CRLB) is an indicator that measures the unbiasedness of the estimation results. In this part, we derive the CRLB of the channel estimation.

According to formula (4), the conditional probability density of Y at a given H can be obtained [10]:

$$f(Y|H) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{(Y - XH)^2}{2\sigma^2} \right\}$$  \hspace{1cm} (10)

The Fisher information matrix of H is as follows:

$$I(H) = -E\left[ \frac{\partial^2 \ln f(Y|H)}{\partial H^2} \right]$$  \hspace{1cm} (11)

where,

$$\ln f(Y|H) = -\frac{1}{2} \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} \|Y - XH\|^2$$  \hspace{1cm} (12)

Find the partial derivative of Y to get

$$\frac{\partial^2 \ln f(Y|H)}{\partial H^2} = -\frac{1}{\sigma^2} A^T A$$  \hspace{1cm} (13)

Substituting formula (13) into formula (11), the Fisher information matrix is obtained as

$$I(H) = \frac{1}{\sigma^2} A^T A$$  \hspace{1cm} (14)

The CRLB of channel estimation can be expressed as

$$CRLB(H) = I^{-1}(H) = \sigma^2 (A^T A)^{-1}$$  \hspace{1cm} (15)

5. Simulation results

In order to verify the performance improvement of the improved algorithm, this paper will conduct simulation experiments to prove that the improved algorithm has better performance. The normalized mean square error (NMSE) is used to evaluate the performance of the sparse channel estimation algorithm. NMSE is defined as [11]:

$$NMSE = \frac{E\{|\hat{H} - H|^2\}}{E\{|H|^2\}}$$  \hspace{1cm} (16)

The smaller the value of NMSE, the better the estimated performance, and vice versa.
Figure 1. Reconstruction errors of different algorithms.  

Figure 2. Comparison of NMSE performance of algorithms under different signal-to-noise ratio Conditions.

Figure 1 compares the reconstruction errors of different sparse channel estimation algorithms. The measurement matrix selects the number of sampling points of the Gaussian white noise matrix, that is, the dimension of the measurement matrix. It is not difficult to see that as the sampling rate increases, the reconstruction error gradually decreases. In the case of a higher sampling rate, the reconstruction error of the StOMP algorithm is lower and the performance is better. While in the case of a low sampling rate, the reconstruction error of the OMP algorithm is lower, the algorithm performance is better, and the original signal can be restored well. Channel estimation based on compressed sensing can recover the signal at a lower sampling rate. Its estimation performance is related to the dimension of the measurement matrix. The more pilots the transmitter sends, the more channel information the receiver gets, the smaller reconstruction error and the higher the accuracy of channel estimation.

Figure 2 compares the OMP algorithm with its improved algorithm. It can be seen from Figure 2 that the improved OMP algorithm has better estimation performance than the OMP algorithm due to filtering out the influence of noise. Under the condition of low signal-to-noise ratio, when the NMSE value is the same, that is, the system has the same estimation performance, the improved OMP algorithm improves the estimation performance of the OMP algorithm by 2dB on average. It can also be seen from the figure that as the signal-to-noise ratio increases, the NMSE of the channel estimation algorithm is closer to the CRLB.

6. Conclusion
Based on the theory of compressed sensing and the sparsity of channels in Massive MIMO systems, this paper studies a sparse channel estimation algorithm based on compressed sensing. Taking into account the influence of noise on the estimation results under the condition of low signal-to-noise ratio, the received signal is processed by wavelet denoising, and then the signal is reconstructed by the OMP algorithm. Through simulation experiments, it is proved that the improved OMP algorithm has better performance.

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