3D Hall MHD Modeling of Solar Wind Plasma Spectra

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Abstract. We present fully self consistent 3D simulations of compressible Hall MHD plasma that describe spectral features relevant to the solar wind plasma. We find that a $k^{-7/3}$ spectrum sets in for the fluctuations that are smaller than ion gyro radius. We further investigate scale dependent anisotropy led by nonlinear processes relevant to the solar wind plasma. Our work is important particularly in understanding the role of wave and nonlinear cascades in the evolution of the solar wind, structure formation at the largest scales.

Keywords: MHD Plasma, Whistler waves, Space Plasmas

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1. INTRODUCTION

Solar wind (SW) fluctuations comprise a multitude of length and time scales that collectively exhibit numerous nonlinear processes. Despite it’s complex evolutionary dynamics, solar wind provides the best laboratory for testing many nonlinear theories and simulation models. Spacecraft databases provide compelling observational evidences that the inertial range magnetic field fluctuations associated with characteristic frequencies, that are smaller than the ion gyro frequency ($\Omega_i$), can be described predominantly by a Kolmogorov-like 5/3 spectrum. Theoretic analysis describing the 5/3 SW spectrum regime relies largely on the usual magnetohydrodynamic (MHD) model of plasma. Intriguingly, the higher time resolution databases of solar wind fluctuations depict a spectral break near the end of the 5/3 spectrum that corresponds to a high frequency ($>\Omega_i$) regime where turbulent cascades are not explainable by the usual MHD models. This refers to a second inertial range where turbulent cascades follow a $k^{-7/3}$ spectrum (where $k$ is a wavenumber) spectrum in which the characteristic fluctuations evolve typically on kinetic Alfven time scales. The onset of the second or the kinetic Alfven inertial range is still elusive to our understanding of SW turbulence and the issue has been under a constant debate since many years. The mechanism leading to the spectral break has been thought to be either mediated by the kinetic Alfven waves (KAWs), or damping of ion cyclotron waves, or dispersive processes, Hall effects.

In this paper, we describe results from our three dimensional simulations that explains that the $k^{-7/3}$ spectrum observed above the spectral break in SW may be led by the Hall effects in the KAW regime. The underlying model, in our simulation model, is based on a two fluid Hall MHD description of plasma that consists of both electrons and ions. In the high frequency regime, $\omega > \Omega_{ci}$, the inertialess electrons contribute to the electric field which is dominated essentially by the Hall term corresponding to $J \times B$ force. The latter, upon substituting in the ion momentum equation, modifies ion momentum, magnetic field and total energy in a manner to introduce a high frequency ($\omega > \Omega_{ci}$) and small scale ($kL_i > 1$, where $L_i$ is ion Larmour radius) plasma motion. The characteristic length scales ($k^{-1}$) associated with the plasma motions are smaller than ion gyro radii ($l_i$). The quasi-neutral solar wind plasma density ($\rho$), velocity ($\mathbf{U}$), magnetic field ($\mathbf{B}$) and total pressure ($P = P_e + P_i$) fluctuations can then be cast into a set of Hall MHD equations as follows (see Ref [6] for detail description).

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0,$$

$$\rho \left( \frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) \mathbf{U} = -\nabla P + \frac{1}{c} \mathbf{J} \times \mathbf{B} + \mu \left( \nabla^2 \mathbf{U} + \frac{1}{3} \nabla (\nabla \cdot \mathbf{U}) \right),$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left( \mathbf{U} \times \mathbf{B} - d \frac{\mathbf{J} \times \mathbf{B}}{\rho} \right) + \eta \nabla^2 \mathbf{B},$$

$$\frac{\partial e}{\partial t} + \nabla \cdot \left( \frac{1}{2} \rho U^2 \mathbf{U} + \frac{P}{\gamma - 1} \mathbf{U} + \frac{c}{4\pi} \mathbf{E} \times \mathbf{B} \right) = 0,$$

where $e = 1/2 \rho U^2 + P / (\gamma - 1) + B^2 / 8\pi$ is a total energy of plasma that contains both electron and ion motions. All the dynamical variables are functions of three space and a time, i.e. $(x, y, z, t)$, co-ordinates. Equations (1) to (4) are normalized by typical length $\ell_0$ and time $t_0 = \ell_0 / v_0$ scales in our simulations, where $v_0 = B_0 / (4\pi \rho_0)^{1/2}$ is Alfvén velocity such that $\mathbf{V} =$. 

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\[ \ell_0 \nabla \cdot \partial_i \partial_t \bar{U} = U/v_0, \bar{B} = B/\sqrt{4\pi\rho_0}^{1/2}, \bar{P} = P/\rho_0 v_0^2, \bar{\rho} = \rho/\rho_0. \]

The parameters \( \mu \) and \( \eta \) correspond respectively to ion-electron viscous drag term and magnetic field diffusivity. While the viscous drag modifies the dissipation in plasma momentum in a nonlinear manner, the magnetic diffusion damps the small scale magnetic field fluctuations linearly. The magnetic field is measured in the unit of Alfvén velocity. The dimensionless parameter in magnetic field \( \ell_0 \) i.e. ion skin depth \( d_i = d_i/\ell_0, d_i = C/\omega_{pi} \) is associated with the Hall term. This means the ion inertial scale length \( (d_i) \) is a natural or an intrinsic length scale present in the Hall MHD model which accounts for finite Larmour radius effects corresponding to high frequency oscillations in \( kd_i > 1 \) regime. Clearly Hall forces dominate the magnetoplasma dynamics when \( 1/\rho(J \times B) > U \times B \) term in Eq. (3) which in turn introduces time scales corresponding to the high frequency plasma fluctuations in \( kd_i > 1 \) regime. Furthermore, our model includes a full energy equation [Eq. (4)] unlike an adiabatic relation between the pressure and density. The use of energy equation enables us to study a self-consistent evolution of turbulent heating resulting from nonlinear energy cascades in the solar wind plasma.

2. TURBULENT SPECTRA

To study the nonlinear evolution of turbulent cascades in a Hall MHD solar wind plasma, we have developed a fully three dimensional compressible Hall MHD code. Our code is massively parallelized using Message Passing Interface (MPI) to run on cluster like distributed-memory supercomputers. The code is scalable and transportable on different cluster machines. The spatial discretization employs a pseudospectral algorithm based on a Fourier harmonic expansion of the bases for physical variables (i.e. the density, magnetic field, velocity, temperature and energy) whereas the temporal integration uses a Runge Kutta (RK) 4th order method. The boundary conditions are periodic along the \( x, y \) and \( z \) directions in the local rectangular region of the solar wind plasma.

The turbulent fluctuations are initialized by using a uniform isotropic random spectral distribution of Fourier modes concentrated in a smaller band of lower wavenumbers \( (k < 0.1 \ k_{max}) \). While spectral amplitude of the fluctuations is random for each Fourier coefficient, it follows a certain initial spectral distribution proportional to \( k^{-\alpha} \), where \( \alpha \) is initial spectral index. The spectral distribution set up in this manner initializes random scale turbulent fluctuations. A constant background magnetic field is included along the \( z \) direction to deal primarily with the large scale or background solar wind magnetic field. Turbulent fluctuations in our simulations are driven either at the lowest Fourier modes or evolve freely under the influence of self-consistent dynamics described by the set of Eqs. (1) to (4). The inertial range spectral cascades in the either cases lead to the nearly identical turbulent spectra. We have further carried out simulations for a range of various parameters and spectral distributions to ensure the validity of our codes and the physical results. The simulation parameters are: spectral resolution is \( 128^3 \), \( \eta = \mu = 10^{-3}, \beta = 1.0, kd_i \sim 0.1 \sim 10, L_x = L_y = L_z = 2\pi \). The nonlinear coupling of velocity and magnetic field fluctuations, amidst density perturbations, excites high-frequency and short wavelength (by the \( \omega/\omega_{ci} \) effect) compressional dispersive KAW’s. The nonlinear spectral cascade in the modified KAW regime leads to...
a secondary inertial range in the vicinity of $kd_i \simeq 1$, where the turbulent magnetic and velocity fluctuations form spectra close to $k^{-7/3}$ [8, 9, 10]. This is displayed in Fig. (1). It is shown in Shaikh & Zank (2009) that the spectra described in Fig (1) is led by Hall effects.

3. ANISOTROPIC CASCADES

To measure the degree of anisotropic cascades (or spectral anisotropy) by employing the following diagnostics to monitor the evolution of the $k_{\perp}$ mode in time. The averaged $k_{\perp}$ mode is determined by averaging over the entire turbulent spectrum weighted by $k_{\perp}$, thus $\langle k_{\perp}(t) \rangle = \frac{\sum |k_{\perp}| Q(k, t)^2}{\sum |Q(k, t)^2|^{1/2}}$. Here $\langle \cdots \rangle$ represents an average over the entire Fourier spectrum, $k_{\perp} = (k_x^2 + k_y^2)^{1/2}$ and $Q$ represents any of $B, V, \rho, V \times B$ and $\nabla \times V$. Similarly, the evolution of the $k_{\parallel}$ mode is determined by the relation, $\langle k_{\parallel}(t) \rangle = \frac{\sum |k_{\parallel}| Q(k, t)^2}{\sum |Q(k, t)^2|^{1/2}}$. It is clear from these expressions that the $\langle k_{\perp}(t) \rangle$ and $\langle k_{\parallel}(t) \rangle$ modes exhibit isotropy when $\langle k_{\perp}(t) \rangle \simeq \langle k_{\parallel}(t) \rangle$. Any deviation from this equality corresponds to spectral anisotropy. We follow the evolution of the $\langle k_{\perp}(t) \rangle$ and $\langle k_{\parallel}(t) \rangle$ modes in our simulations. Our simulation results describing the evolution of $\langle k_{\perp}(t) \rangle$ and $\langle k_{\parallel}(t) \rangle$ modes are shown in Fig. (2). It is evident from Fig. (2) that the initially isotropic modes $\langle k_{\perp}(t) \rangle \simeq \langle k_{\parallel}(t) \rangle$ gradually evolve towards a highly anisotropic state in that spectral transfer preferentially occurs in the $\langle k_{\perp}(t) \rangle$ mode, and is suppressed in $\langle k_{\parallel}(t) \rangle$ mode. Consequently, spectral transfer in the $\langle k_{\perp}(t) \rangle$ mode dominates the nonlinear evolution of fluctuations in Hall MHD, and mode structures become elongated along the mean magnetic field or $z$-direction. Hence nonlinear interactions led by the nonlinear terms in the presence of background gradients lead to anisotropic turbulent cascades in the inertial range turbulent spectra.

Fig. (2) illustrates anisotropy corresponding to an averaged $k$ mode but the anisotropy exhibited by the small and large scale $B$ and $v$ fluctuations is not distinctly clear, nor is the degree of anisotropy in $B$ and $v$ fields clear from Fig. (2). The scale dependence of turbulent anisotropy is described in Fig. (3). Fig. (3) shows discrepancy in $k_{\perp}$ and $k_{\parallel}$ is prominent at the smaller $k$’s. This essentially means that the large scale turbulent fluctuations are more anisotropic than the smaller ones in a regime where characteristic length scales are smaller than $d_i$, i.e. $kd_i > 1$. It further appears from Fig. (3) that the smaller scales in the $kd_i > 1$ are virtually unaffected by anisotropic kinetic Alfvén waves that propagate along the externally imposed mean magnetic field $B_0$. Turbulent fluctuations with small characteristic scales in the $kd_i > 1$ regime of Hall MHD are not affected by the mean magnetic field or kinetic Alfvén waves. This leads us to conjecture that small scale turbulence in the $kd_i > 1$ regime behaves essentially hydrodynamically i.e. as eddies independent of the mean magnetic field or collisionless magnetized waves. Thus large and smaller turbulent length scales evolve differently in the $kd_i > 1$ regime of Hall MHD turbulence.

3. DYNAMICAL ALIGNMENT OF FLUCTUATIONS

To understand the strength of the nonlinear interactions in Hall MHD solar wind plasma, we determine the degree of alignment of the velocity and magnetic field fluctuations by defining the following alignment parameter $[11]$ that spans the entire $k$-spectrum in both the $kd_i > 1$ (Hall MHD) and $kd_i < 1$ (usual MHD) regimes. $\Theta(t) = \cos^{-1} \left( \frac{\sum k \cdot |B(k)| / \sum |V(k)| B(k)}{\sum |V(k)| B(k)} \right)$. The summation is determined from the modes by summing over the entire spectrum. In this sense, the alignment parameter depicts an average alignment of the velocity relative to the magnetic field fluctuations. Note carefully that this alignment can vary locally from smaller to larger scales, but the averaging (i.e. summing over the entire spectrum) rules out any such possibility in our simulations. Nonetheless, $\Theta$ defined as above enables us to quantitatively measure the average alignment of the magnetic and velocity field fluctuations while the nonlinear interactions evolve in a turbulent solar wind plasma.

The Alfvénic cascade regime of MHD turbulence $kd_i < 1$ in the solar wind plasma possesses relatively large scales ($kd_i < 1$) in which the velocity and magnetic field fluctuations are observed to be somewhat obliquely.
FIGURE 4. A progressive decrease in the angle of alignment from 90° indicates the eventual weakening of $\mathbf{V} \times \mathbf{B}$ nonlinear interactions. Evolution of the degree of alignment in the $kd_i > 1$ regime of Hall MHD. Small scale fluctuations possess orthogonal velocity and magnetic fields.

aligned. Hence our simulations show that the angle of alignment evolves towards $\Theta < 90^\circ$, as depicted in Fig. 4. Hence the strength of the nonlinear interactions corresponding to the $\mathbf{V} \times \mathbf{B}$ nonlinearity is relatively weak. This result is to be contrasted with characteristic turbulent length scales in the $kd_i > 1$ regime. The angle of alignment for the smaller scales corresponding to the $kd_i > 1$ regime is shown in Fig. 4. Significant differences are apparent in the angle of alignments associated with the large and small scales Fig. 4. It appears from our simulations that the small scale fluctuations ($kd_i > 1$) are nearly orthogonal as seen in Fig. 4. By contrast, the large scale fluctuations ($kd_i < 1$) in Fig. 4 show a significant departure from the orthogonality.

4. CONCLUSIONS

In summary, we have investigated the nonlinear and turbulent behavior of a two fluid, compressible, three dimensional Hall MHD model. In the presence of a large scale mean background magnetic field, small scale turbulent fluctuations exhibit anisotropic power spectra close to $k^{-7/3}$. We find that the long length scales in the $kd_i > 1$ KAW regime are more anisotropic compared to the shorter scales. Dynamical alignment and angular distribution of turbulence velocity and magnetic field fluctuations is found to play a critical role in determining the degree of nonlinear interactions. We find that characteristic turbulent fluctuations in the $kd_i > 1$ regime relax towards orthogonality, so that most of turbulent scale fluctuations have velocity and magnetic fields that are nearly orthogonal, i.e. $\mathbf{V} \perp \mathbf{B}$. For large scale fluctuations corresponding to the MHD regime, magnetic and velocity fields are not perfectly orthogonal, being instead on average nearly 70° to each other. By contrast, small scale fluctuations in the $kd_i > 1$ KAW regime exhibit nearly perfect orthogonality in that the average magnetic and velocity fields make an angle of nearly 90° with respect to each other.

The spectral properties of nonlinear Hall MHD are particularly relevant for understanding the observed solar wind and heliospheric turbulence. Hall MHD may also be useful for understanding multi-scale electromagnetic fluctuations and magnetic field reconnection in the Earth’s magnetosphere and in laboratory plasmas.

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