A combined model reference adaptive control law for multirotor UAVs

Alia Farhana Abdul Ghaffar\textsuperscript{1,2} | Thomas Richardson\textsuperscript{2} | Collin Greatwood\textsuperscript{2}

\textsuperscript{1} Department of Mechanical Engineering, International Islamic University Malaysia, Jalan Gombak, Selangor, Malaysia
\textsuperscript{2} Faculty of Aerospace Engineering, University of Bristol, University Walk, Bristol, UK

Abstract

Model reference adaptive control (MRAC) offers the potential to adapt in real-time to changes in the performance of small unmanned air vehicles. There are significant challenges with their use, however, primarily in the implementation and assurance of long-term system stability. This paper presents flight test results for a combined model reference adaptive control (CMRAC) law applied to the height control loop of a multirotor. Key features include the implementation of CMRAC with a baseline controller allowing for in-flight switching between the two; the use of an augmented state to improve the tracking performance and a CMRAC implementation that provides a shorter transient phase, faster parameter convergence, and closer tracking of the desired reference model response when compared with standard MRAC. With the current exponential growth of interest in unmanned air vehicles, the potential benefits of using CMRAC for control system development are significant, particularly for new vehicles with short development and testing phases and in cases where there are significant configuration changes in flight or prior to rapid deployment.

1 | INTRODUCTION

1.1 | Background of work

Unmanned air vehicle (UAV) use continues to grow rapidly due to advances in sensor, battery, and computing technologies. Small multi-rotors are now used commercially in many sectors such as in construction [1, 2], load handling [3, 4], and aerial delivery [5]. The ability of multirotor UAVs to fly at low speed and hover-in-place makes them suitable for indoor and outdoor tasks in urban areas. Mixed-mode vehicles, such as quad-wing UAVs offer the potential for vertical take-off and landing (VTOL) combined with long-range operations.

Multi-rotor and fixed-wing UAVs typically have a short development cycle. As such, there is often not enough time to build a comprehensive mathematical model of the vehicle and linear approximations are used to model the dynamics and design the control laws [6]. In this case, a fixed-gain proportional-integral-derivative (PID) controller may not offer the best possible performance, in particular for nonlinear systems [7] and where there are uncertainties in the dynamic model. The response of UAVs can be highly nonlinear, especially during rapid, agile manoeuvres [8], and with vehicles that have unconventional designs [9]. Such systems require more advanced controllers to cope with model uncertainties and inherent nonlinearities. The work presented in this paper explores the potential benefits of using an adaptive controller for a new system that lacks a comprehensive mathematical model. The aim is to develop a stable control algorithm that accommodates modelling uncertainties and adapts to system changes in real-time. To cater for rapid development cycle of new UAV systems, the intent is also to implement the controller with minimal modelling and tuning.

1.2 | Literature review

A model reference adaptive controller (MRAC) allows a closed-loop system to adjust to provide a desired nominal performance. This is done without the need for a comprehensive dynamic model of the plant because the response is achieved through control parameters that can adapt by means of continuous estimation and real-time update [10, 11]. Adaptive control...
has shown promising results when applied to models with parametric uncertainties [12–14], adapting to changes in the system dynamics or partial system failures [15]. In [16], an adaptive neural network controller was successfully used to find the level-flight trim conditions for a fixed-wing UAV in various operating conditions. In [17], it is shown that an adaptive control autopilot can be transferred between platforms with different flying qualities without the need to manually re-tune the control loops. This self-tuning capability can therefore reduce the time required to implement the control law on a new platform. However, often there is a compromise between performance and complexity of these controllers.

With great interest in the research community, the adaptive control algorithm presented in the literature has seen many variations and modifications with other nonlinear controllers such as adaptive sliding mode controller [18] where adaptive element is utilised to estimate the unknown parameters in the quadrotor model. In neural network controller [19], adaptive law is used to train the neural network on-line, and in adaptive fuzzy controller [20], adaptive control is used to achieve automatic tuning for quadrotor with variations in payload mass, and its centre of gravity. It is seen that the adaptive elements have the desired attributes to help enhance the system’s performance, especially when applied in the presence of model uncertainty, parameter variations, and disturbances. Interestingly, fixed gain controller such as PID added to an adaptive control can also help improve the system performance by reducing oscillations as reported in [21]. Recent works on quadrotor trajectory tracking and attitude stabilisation problems also show variations of other nonlinear and intelligent control such as model predictive control, and employing techniques such as backstepping and dynamic inversion [22–25]. However, unlike adaptive control, the design and implementation of these controllers require modelling of the system and some knowledge on the uncertainties in the model.

The stability of MRAC is established based on Lyapunov theory which ensures asymptotic error convergence. However, a Lyapunov-based adaptive law cannot guarantee parameter convergence to ideal values if there are non-parametric or unmatched uncertainties, such as noise or unmodelled dynamics [26]. Previous work has shown that sufficient parameter convergence can be achieved if persistency of excitation of the reference input is satisfied such that the input is sufficiently rich [27]. This condition essentially requires the plant states to span the complete spectrum of the state space over all time intervals. However, since the plant model is uncertain, analysing the complete state vector is often not feasible. In the absence of persistency of excitation, instability can be encountered due to parameter drift that ultimately leads to a sudden large amplitude oscillations called “bursting” [28]. However, in the presence of “bursting” a sufficiently rich signal can be acquired, thereby improving parameter estimates and the system regains stability [29]. To avoid this sudden instability, it is essential in the MRAC design to consider the accuracy of the control parameter estimates in addition to the tracking error convergence. There are typically two approaches currently used to achieve this: (1) Robust adaptive control and (2) composite adaptive control.

Robust adaptive control approaches modifies the adaptive law such that the overall system has bounded solutions, ultimately halting the drift of the controller parameters. This improves the controller response and the system performance in the presence of parametric uncertainties, noise, and varying plant parameter [30]. An example is the L1 adaptive control, which is a filtered version of MRAC [31, 32] with the use of high controller gains to efficiently suppress uncertainties whilst maintaining an adequate stability margin. However, the benefits of adaptation in the L1 adaptive controller is questionable in certain cases [33]. Another example is the dead-zone method which avoids parameter drift by turning off the adaptation when the tracking error reaches a specified bound of non-parametric uncertainties [34, 35]. Similarly, the parameter projection method can be used to impose bounds to prevent control parameter growth [36]. For both these methods it can be difficult to determine meaningful bounds that contain the true parameter values. Alternative robust methods include the ε-modification [37], Q-modification [38], and μ-modification [39] which involve modifying the adaptive controller with a general damping term or forgetting factor to bound the error to a compact set. It is noted that most of the robustification methods do not improve the accuracy of parameter estimation, and cannot guarantee parameter convergence unless the PE condition is satisfied. In addition, many of these methods increase the number of gains required for tuning and increase the complexity of the algorithm.

The combined or composite model reference adaptive control (CMRAC) approach extends the MRAC algorithm by including an identification or prediction model in the algorithm to obtain more accurate plant parameter estimates [40, 41]. MRAC is broadly grouped into two categories: the direct method and the indirect method. The direct method involves adjusting the control parameters directly based on the tracking error between the outputs of the plant and the reference model [11]. Meanwhile, the indirect method involves adjusting the controller parameters “indirectly” based on the plant estimates. In this case the plant estimates are updated on-line based on the error between the outputs of the plant and the identification model [36]. Therefore, in the combined MRAC approach, the adjustment mechanism for the control parameters are driven by two error sources which are the reference model tracking error and the closed loop estimation error. This method has been shown to improve the transient response characteristics [42, 43]. In [44], estimated filtered system dynamics are derived in a predictor model, effectively removing high-frequency oscillation through the use of a low pass filter on the input–output measurements. CMRAC typically allows for the use of higher learning gains, thus increasing the speed of parameter convergence. It also gives smooth transient characteristics with less oscillation in the control input and smaller tracking errors [45]. It is also shown in [46] that the CMRAC can give better accommodation of delays and unmodelled dynamics. CMRAC does however require PE to guarantee exponential parameter convergence although extensions proposed by [47] and [48] can relax the requirement of persistency of excitation for parameter convergence. In [49], it is shown that parameter convergence can be
achieved by using selected on-line recorded data concurrently with instantaneous data. The condition on the richness of the recorded data is sufficient to guarantee parameter convergence, without requiring persistency of excitation of the plant states. It is worth noting that although there is strong empirical evidence for the performance of CMRAC, currently there is no theoretical proof of asymptotic stability of the controller in the literature. However, the conventional direct MRAC scheme, in which the CMRAC is originally derived from, has been proven through Lyapunov synthesis that it can lead to asymptotic convergence under parametric uncertainties [30].

1.3 Contributions

The strong empirical evidences on the capabilities and benefits of CMRAC controller has been the main motivation for using this controller. This paper presents the development of a combined MRAC controller that incorporates an identification model and uses plant estimates in the adaptive law formulation, as originally presented by [40]. However, in this work, the CMRAC is extended to broader class of systems including higher-order multiple inputs and multiple outputs (MIMO) systems. In addition, the model augmentative method proposed in [44] is also implemented, where the state vector is augmented with the trajectory tracking error because it has been shown that this helps enhances the tracking performance. Although the same CMRAC augmentation technique is employed, this work is different than [44] in the sense that the identification model and plant estimates are formulated differently without the use of a low pass filter in the input–output measurements. Moreover, the augmented technique in [44] serves the purpose to incorporate a baseline LQR-i controller in the CMRAC’s reference model design. In contrast, this work presented an augmented CMRAC structure that still allow the designer to design the reference model based on the desired response characteristics. The CMRAC and its augmented version are implemented as a quadrotor height controller and tested in an indoor flight arena. The results shown in this paper serve to evaluate the CMRAC performance and to show an improved performance by applying this augmentative method.

This paper also describes the approach taken to the flight tests with simple mathematical equations used to model the plant. In practice, adaptive control applied to an uncertain model will exhibit an oscillatory transient phase as it is adapting to the plant and this poses a problem in flight tests because it could potentially damage the platform. A relatively simple way to deal with it is to apply the adaptive controller with a baseline fixed-gain controller [15, 51], and it was shown that this controller scheme helps stabilise oscillations while the adaptive law serves to increase the overall system robustness to parametric uncertainties. In contrast to [15, 51], the CMRAC in this work is implemented fully on the control loop without fitting it with the fixed-gain controller. This is done so to allow the CMRAC parameters to estimate the quadrotor plant without the influence on the baseline controller. However, to ensure successful flight tests, a PID controller is used during take-off until the quadrotor hovers at a safe distance from the ground before the CMRAC is applied to the system. Thus a safe switching method for transferring between the PID controller and the adaptive controller in mid-flight is formulated. The implementation is simple, and it allows for expedited testing and validation of the CMRAC from simulation to the quadrotor experimental platform.

The novelty and contribution of the work presented in this paper include the mathematical derivations that describe the implementation of the augmented CMRAC for both simulation and experimental test, especially the design of the augmented reference model and the augmented identification model which are slightly different than their counterparts in literature. In addition, the proposed techniques used to transfer the controller from PID to CMRAC while mitigating control input oscillations during the flight tests significantly help ensuring successful experimental flight tests. This allows the CMRAC controller to be quickly used on the quadrotor platform without the need to model the system comprehensively.

This paper is organised as follows. Section 2 shows the mathematical derivation and design of a combined MRAC (CMRAC) and the augmented version CMRAC; Section 3 describes the methods for implementing CMRAC on a quadrotor for flight experiments. Section 4 shows the experimental results and Section 5 discusses the results.

2. MATHEMATICAL DERIVATION AND CONTROLLER STRUCTURE

The combined MRAC (CMRAC) scheme applied in this work is based on the derivation from [40], and the algorithm is extended to include an additional state similar to the state augmentation proposed in [44]. The combined MRAC architecture is shown in Figure 1. In this CMRAC controller, two working models are designed; the reference model and the identification model. The reference model represents the desired response characteristics of the system, whereas the identification model represents the actual plant and contains estimates of the plant parameters.

2.1 Derivation of combined model reference adaptive control

2.1.1 Plant dynamics

A linear mathematical equation is used to describe the quadrotor plant including the main parameters that characterise the dynamics. Consider this linear system written in a state space form by:

\[
\dot{x}_p = A_p x_p + B_p \Delta u + d
\] (1)

and the output dynamics are given by:

\[
y_p = C_p x_p \in \mathbb{R}^n
\] (2)
where $x_p \in \mathbb{R}^n$ is the state vector of the system, $u \in \mathbb{R}^m$ is the control input to the system, $A_p \in \mathbb{R}^{n \times n}$ is the unknown and constant plant matrix, $B_p \in \mathbb{R}^{n \times m}$ is the known and constant input matrix, $C_p \in \mathbb{R}^{m \times n}$ is the output matrix, $\Delta \in \mathbb{R}^{m \times m}$ is an unknown control effectiveness matrix, and $d \in \mathbb{R}^n$ represents an unknown constant disturbance. $\Delta$ is a positive definite diagonal matrix, and takes the form of an identity matrix if there’s no uncertainties in the input matrix $B_p$.

The adaptive control law, $u$, is described as:

$$u = \hat{K}_{xp} x_p + \hat{K}_r r + \hat{K}_d$$

where $\hat{K}_{xp} \in \mathbb{R}^{m \times n}$, $\hat{K}_r \in \mathbb{R}^{m \times m}$, and $\hat{K}_d \in \mathbb{R}^n$ are the adaptive time varying control gain matrices that will compensate for uncertainties in the plant, the control effectiveness, and the disturbance respectively. In addition, $r \in \mathbb{R}^m$ is the trajectory demand.

### 2.1.2 Reference model dynamics

A linear reference model is designed such that it describes the desired behaviour of the system, written as:

$$\dot{x}_m = A_m x_m + B_m r$$

where $x_m \in \mathbb{R}^n$ is the desired reference trajectory, $A_m \in \mathbb{R}^{n \times n}$ is the desired reference system matrix, and $B_m \in \mathbb{R}^{n \times m}$ is the input matrix.

The control tracking error is defined by comparing the output from the reference model and the actual model output, written as:

$$e_t = x_m - x_p$$

The tracking error dynamics is then derived by taking the derivative of the tracking error (5) and applying the reference model and plant dynamics shown in (4) and (1) respectively:

$$\dot{e}_t = A_m e_t + B_p \Delta (\hat{K}_{xp} - \hat{K}_{xp}^*) x_p + (\hat{K}_d - \hat{K}_d^*) + (\hat{K}_r - \hat{K}_r^*) r$$

The superscript star (⋆) denotes the ideal control parameters that satisfy the following matching condition:

$$A_p + B_p \Delta \hat{K}_{xp}^* = A_m$$
$$B_p \hat{K}_r^* = B_m$$
$$B_p \hat{K}_d^* = -d$$

The adaptive controller is designed such that all the control parameters would eventually converge to these ideal values.

### 2.1.3 Identification model dynamics

An identification model is designed to contain estimates of plant parameters that will be continuously updated as the CMRAC adapts to the system. The identification model dynamics is derived as:

$$\dot{x}_p = \hat{A}_p x_p + (\hat{A}_p - A_m) x_p + B_p \hat{A} n + d$$

where $\hat{A}_p \in \mathbb{R}^{n \times n}$ is an estimate of the unknown plant matrix, $\hat{A} \in \mathbb{R}^{n \times n}$ is an estimate of the control effectiveness matrix, and $d \in \mathbb{R}^n$ is an estimate of the constant disturbance.

The identification error is defined by comparing the output from the identification model and the actual model,
written as:

$$e_i = \dot{x}_p - x_p$$  \hfill (9)

The identification error dynamics are then derived by taking the derivative of (9), and applying the identification model and the plant dynamics shown in (8) and (1) respectively.

$$\dot{e}_i = A_m (\dot{x}_p - x_p) + (A_p - A_p) x_p + \dot{d} - d + B_p (\Lambda - \Lambda)$$  \hfill (10)

If the matching condition stated in (7) is satisfied, the identification model dynamics should ideally converge to the reference model dynamics. This means that the parameters of the identification model in (8) must converge according to:

$$\dot{\Lambda}_p + B_p \dot{\Lambda}_d \to A_m$$

$$B_p \dot{\Lambda}_d \to B_m$$

Based on (11), the following additional closed loop estimation errors are defined:

$$\epsilon_x = \dot{\Lambda}_p + B_p \dot{\Lambda}_d - A_m$$

$$\epsilon_d = B_p \dot{\Lambda}_d + \dot{d}$$

$$\epsilon_r = B_p \dot{\Lambda}_r - B_m$$

These three equations can be solved simultaneously to find the three unknown parameters in the plant which are $A_p$, $\Lambda$ and $d$.

The estimates of the plant parameters in the identification model are updated by taking account of the identification error (9) and the closed loop estimation errors (12). The update law for the estimate of the plant parameters is computed as:

$$\dot{A}_p = -\Gamma_A (\epsilon_x p e_T^i P + \gamma_c \epsilon_x T)$$

$$\dot{d} = -\gamma_d (e_T^i P + \gamma_c \epsilon_D T)$$

$$\dot{\Lambda} = -\Gamma_A (u e_T^i P + \gamma_c (\hat{A}_p e_x \epsilon_T^i + \dot{\Lambda}_d \epsilon_T^i + \dot{\Lambda}_d \epsilon_T^i))$$

where $\Gamma_A \in \mathbb{R}^{\infty \times \infty}$, $\gamma_d \in \mathbb{R}$, and $\Gamma_A \in \mathbb{R}^{\infty \times \infty}$, are the learning rate matrices that define the adaptation rate, and $\gamma_c \in \mathbb{R}$ is a weighting gain for the closed loop estimation errors, $\epsilon_x$, $\epsilon_d$, and $\epsilon_r$. The variable $P \in \mathbb{R}^{\infty \times \infty}$ is the unique, symmetric, and positive definite solution to the Lyapunov equation that exists for any arbitrary positive definite matrix: $Q \in \mathbb{R}^{\infty \times \infty}$

$$P A_m + A_m^T P = -Q$$  \hfill (14)

2.1.4 CMRAC update law

In the combined MRAC approach, a Lyapunov function candidate is formulated, based on both the tracking error dynamics (6) and the identification error dynamics (10), to define the CMRAC update law. The Lyapunov function is written as:

$$V = \frac{1}{2} e_P^T P + \frac{1}{2} \text{Tr} [\dot{\Lambda}_p \Gamma \dot{\Lambda}_p^T] + \frac{1}{2} \gamma_2^{-1} \dot{\Lambda}_d \dot{\Lambda}_d^T$$

$$+ \frac{1}{2} \text{Tr} [\dot{\Lambda}_p \Gamma \dot{\Lambda}_d^T] + \frac{1}{2} \gamma_2 \dot{\Lambda}_d \dot{\Lambda}_d^T$$

where $\text{Tr}(\cdot) = \sum \lambda_i$ i.e.; the trace of a matrix defined by the sum of its diagonal elements.

Based on the Lyapunov candidate function, the following update laws for the CMRAC parameters are derived:

$$\dot{\hat{\epsilon}}_y = -\gamma_x (\epsilon_x p e_T^i P + \gamma_c \epsilon_x T)$$

$$\dot{\hat{\epsilon}}_d = -\gamma_d (e_T^i P + \gamma_c \epsilon_D T)$$

$$\dot{\hat{\epsilon}}_r = -\gamma_r (e_T^i P + \gamma_c \epsilon_T T)$$

where $\epsilon_x \in \mathbb{R}^{\infty \times \infty}$, $\gamma_d \in \mathbb{R}$, and $\Gamma_r \in \mathbb{R}^{\infty \times \infty}$ are the gains that specify the learning rates of adaptation, and $P \in \mathbb{R}^{\infty \times \infty}$ is the positive definite solution to the Lyapunov equation as described in (14).

This update law renders the time derivative of the Lyapunov function, $\frac{dV}{dt}$, negative semi-definite, thus the system is globally stable and it guarantees that the tracking error, $\epsilon_x$, stays bounded in a compact neighbourhood of zero [36, 52]. The negative semi definite $\frac{dV}{dt}$ also implies that the controller parameters, $K_x$, $K_d$, and $K_r$ are bounded. Furthermore, the signal $x_p = \epsilon + x_m$ is also bounded. Since the signals $u$, $\epsilon$, and $x_p$ are bounded, then $\frac{dV}{dt}$ is bounded which means that that the Lyapunov function $\frac{dV}{dt}$ is uniformly continuous. Therefore, through Barbalat’s lemma, the error is guaranteed to converge to zero, thus the system is asymptotically stable [10, 53].

$$\lim_{t \to \infty} \epsilon_x(t) = 0$$  \hfill (17)

Moreover, the identification error, $e_i$ and closed loop estimation errors $\epsilon_x$, $\epsilon_d$ and $\epsilon_r$ converge to zero asymptotically, therefore the identification model dynamics eventually converge to the reference model dynamics.

$$\lim_{t \to \infty} (x_p(t) - x_m(t)) = 0$$  \hfill (18)

2.2 CMRAC with state augmentation

The MRAC algorithm is extended by incorporating the trajectory tracking error in the state vector, as shown in Figure 2. This augmentation is based on the work by [15, 44], who have shown empirical evidence that the augmentation leads to improved transient response and parameter convergence.
The output tracking error is defined as the difference between the response and the trajectory demand shown by:

\[ e_r = r - y_p \quad (19) \]

where \( r \in \mathbb{R}^m \) is a bounded demand input. Taking into account the integral of the output tracking error;

\[ e_{rI} = e_r s \quad (20) \]

The state vector can be extended such that:

\[ x = [xp \; erI]^T \in \mathbb{R}^{n+m} \]

This yields an extended open loop dynamics described by:

\[ \dot{x} = Ax + B_\Delta u + d_+ + B_cr \]

\[ y = Cx \quad (21) \]

where the extended system matrices are given by:

\[
A = \begin{bmatrix}
A_p & 0_{m \times n+m} \\
-C_p & 0_{m \times m}
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
B_p \\
0_{m \times m}
\end{bmatrix}
\]

\[
d_+ = \begin{bmatrix}
d \\
0_{m \times m}
\end{bmatrix}
\]

\[
B_c = \begin{bmatrix}
0_{n \times m} \\
I_{m \times m}
\end{bmatrix}
\]

\[
C = [C_p \; 0_{m \times n+m}]
\]

Since the plant has been augmented such that the demand, \( r \), is described in the state vector, the control law for this system is slightly modified to exclude the feedforward term and can therefore written as:

\[ u = \hat{K}_e x + \hat{K}_d \quad (23) \]

The addition of the integral of the output error term in the control law signifies that the augmented CMRAC now tracks the trajectory demand in addition to the reference model response. With additional information on the demanded tracking signal in its adaptive algorithm, it may contribute to a better adaptation as shown in the experimental results.

2.2.1 Augmented reference model

As with the state augmentation of the plant, the reference model state is augmented with a third state, which is the integrated reference model output tracking error. The closed loop extended reference model is therefore rewritten as:

\[ \dot{x}_{maug} = (A_{maug} - B_{maug}K)x_{maug} + B_cr \]

\[ y_{maug} = C_{maug} x_{maug} + B_cr \quad (24) \]

where \( A_{maug}, B_{maug}, C_{maug} \in \mathbb{R}^{n+m \times n+m} \) is a Hurwitz matrix. \( K \in \mathbb{R}^{m \times n+m} \) is a pre-designed regulator that yields a stable closed loop reference model dynamics and where the extended system matrices are given by:

\[
A_{maug} = \begin{bmatrix}
A_m & 0_{m \times m} \\
-C_m & 0_{m \times m}
\end{bmatrix}
\]

\[
B_{maug} = \begin{bmatrix}
B_m \\
0_{m \times m}
\end{bmatrix}
\]

\[
B_c = \begin{bmatrix}
0_{n \times m} \\
I_{m \times m}
\end{bmatrix}
\]

The reference model is augmented in this way to ensure that the new closed-loop plant matrix of the reference model is Hurwitz. The augmented tracking error is defined as:

\[ e_c = x_{maug} - x \quad (26) \]

Comparing the augmented plant equation (21) and the augmented reference model equation (24), and given that the ideal control gains exist, the following matching condition can be
stated:

\[ A + BA\hat{K}_N^x = A_{m_{aug,CI}} \]
\[ BA\hat{K}_d^x = -d_- \]  

(27)

The superscript star (*) denotes the ideal control parameters that yield the desired system behaviour.

### 2.2.2 Augmented identification model

The identification model dynamics are also augmented to give:

\[ \dot{x} = A_{m_{aug,CI}}x + (\hat{A} - A_{m_{aug,CI}})x + B\hat{u} + \hat{d}_- + Br \]  

(28)

where the extended identification matrices are given by

\[ \hat{A} = \begin{bmatrix} \hat{A}_p \\ -C_p \end{bmatrix} \begin{bmatrix} 0_{n \times n} \\ 0_{n \times n} \end{bmatrix} \]
\[ B = \begin{bmatrix} B_p \\ 0_{n \times n} \end{bmatrix} \]
\[ \hat{d}_- = \begin{bmatrix} \hat{d} \\ 0_{n \times n} \end{bmatrix} \]
\[ B_c = \begin{bmatrix} 0_{n \times n} \\ I_{n \times n} \end{bmatrix} \]  

(29)

where \( \hat{A}_p \in \mathbb{R}^{n \times n} \) is an estimate of the originally unaugmented and unknown plant matrix, \( \hat{A} \in \mathbb{R}^{n \times n} \) is an estimate of the control effectiveness matrix, and \( \hat{d} \in \mathbb{R}^n \) is an estimate of the constant disturbance.

The augmented identification error is defined as:

\[ e_t = \hat{x} - x \]  

(30)

where \( \hat{x} \) is the estimated response of the augmented identification model and \( x \) is the actual response of the augmented closed loop system. The identification model dynamics should ideally converge to the reference model dynamics. Therefore, the parameters of the identification model should converge according to:

\[ \dot{\hat{K}}^x = A_{m_{aug,CI}} \]
\[ BA\hat{K}_d = -\hat{d}_- \]  

(31)

Note that in the augmented model, the matching condition is reduced to two equations. However, there are three uncertain parameters in the plant: \( A_{p}, \hat{A}, \) and \( d \). To ensure a correct estimate for each unknown parameter, the first equation in (31) is split into two conditional equations based on the components of the vector of \( \hat{K}_x \) and \( K \in \mathbb{R}^{n \times n} \), given by:

\[ A_{p} + B_{p}\hat{K}_x \rightarrow A_{m} - B_{m}K \]  

\[ B_{p}\hat{K}_d \rightarrow -B_{m}K \]  

(32)

This simply implies that the additional state of the integrated output tracking error would not be taken into account in the update of the identification system matrix, \( \hat{A} \), but it will instead be used to update the estimate of control effectiveness matrix, \( \hat{A} \).

The following additional closed loop estimation errors are defined based on the four equations of matching conditions:

\[ e_A = \hat{A}_p + B_p\hat{K}_x - A_m + B_mK \]  

\[ e_A = B_p\hat{K}_d + B_mK \]  

(33)

\[ e_d = B\hat{K}_d + \hat{d}_- \]  

To ensure consistency throughout this derivation, the closed loop estimation error for the augmented plant is defined as the combination of \( e_A \) and \( e_A \):

\[ e_x = \begin{bmatrix} e_A \\ e_A \end{bmatrix} \]  

(34)

By taking into account the identification error and the closed loop estimation errors, the update laws for the plant parameter estimate are then computed as:

\[ \dot{\hat{A}}^T = -\Gamma_A (x_t^T P + \gamma_e e_A^T) \]
\[ \dot{\hat{d}}_- = -\gamma_d (e_t^T P + \gamma_e e_d^T) \]
\[ \hat{A}^T = -\Gamma_A (u_t^T P + \gamma_e \left( \hat{K}_x e_x^T + \hat{K}_p e_p^T + \hat{K}_d e_d^T \right)) B \]  

(35)

where \( \Gamma_A \in \mathbb{R}^{n \times n} \), \( \gamma_d \in \mathbb{R} \), and \( \Gamma_A \in \mathbb{R}^{n \times n} \) are the learning rate matrices that defines the adaptation rate, and \( \gamma_e \in \mathbb{R} \) is a weighting gain for the closed loop estimation errors \( e \).

### 2.2.3 Control update law for augmented CMRAC

The augmented CMRAC control update law is written as:

\[ \dot{\hat{K}}_x^T = -\Gamma_x (x_t^T P + \gamma_e e_x^T) B \]
\[ \dot{\hat{K}}_d^T = -\gamma_d (e_t^T P + \gamma_e e_d^T) B \]  

(36)

where \( \Gamma_x \in \mathbb{R}^{n \times n} \) and \( \gamma_d \in \mathbb{R} \) are the gains that specify the learning rates of adaptation. The variable \( P \in \mathbb{R}^{n \times n} \) in both
Equations (35) and (36) is the unique, symmetric, and positive definite solution to the Lyapunov equation that exists for any arbitrary positive definite matrix \( Q \in \mathbb{R}^{n \times n} \).

### 3 CMRAC APPLIED TO THE HEIGHT CONTROL OF A QUADROTOR

#### 3.1 Controller setup

A linearized height dynamic equation is written to represent the quadrotor as:

\[
\begin{bmatrix}
\dot{\bar{x}} \\
\dot{\bar{y}} \\
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
0 & 0 \\
\end{bmatrix} \begin{bmatrix}
\bar{x} \\
\bar{y} \\
\end{bmatrix} + \begin{bmatrix}
0 \\
1 \\
\end{bmatrix} U + \begin{bmatrix}
0 \\
-\bar{g} \\
\end{bmatrix}
\]  

\( (37) \)

The plant matrix \( \mathbf{A} \) is not required to be known, but only the initial value and the sign of matrix \( \mathbf{B}_p \) must be known. Therefore, a simple and linearised model is sufficient to set up the CMRAC adaptive law. A second order reference model with a similar structure is used, written as:

\[
\begin{bmatrix}
\dot{\bar{x}}_m \\
\dot{\bar{y}}_m \\
\end{bmatrix} = \begin{bmatrix}
-\frac{1}{\tau_m} & -\zeta_m \\
0 & -\frac{1}{\tau_m} \\
\end{bmatrix} \begin{bmatrix}
\bar{x}_m \\
\bar{y}_m \\
\end{bmatrix} + \begin{bmatrix}
0 \\
1 \\
\end{bmatrix} r
\]  

\( (38) \)

where \( \tau_m = 1 \) s is the time constant of the reference model and \( \zeta_m = 0.7 \) is the damping ratio. The chosen reference model provides adequate signal smoothing. In the augmented CMRAC, the third order reference model is designed with a reduced second-order approximate characteristics. Referring to the augmented closed loop reference model as shown in Equation (24), pole placement method was used to compute \( \mathbf{K} \) based on the desired eigenvalues. Figure 3 shows the step response of both the second order and the third order reference model that has the characteristics equivalent to an ideally damped second order system.

#### 3.2 Benchmarking a baseline PID controller

The challenges in implementing CMRAC on a quadrotor experimental platform stem from the poor transient response that is inherent to adaptive control. For flight tests with CMRAC, the controller is implemented together with a well-tuned, fixed-gain PID controller that is used as a benchmark for the adaptive controller gains at the start of each test. The initial CMRAC parameters are initialised based on the PID values as shown in Table 1. The PID is also used in the take-off phase to ensure a stable take-off, however, once a stable hover is achieved, the control authority is passed over to the CMRAC controller. As a safety measure, the control authority can be reverted to the PID controller using a switch embedded in the real-time software interface at any time during the flight experiment.

#### 3.3 Dual controller in-flight

This section presents a method for transferring between two different controllers, namely a fixed-gain (PID) and an adaptive controller (CMRAC), for a single-closed loop system without causing a discontinuity in the control signal or inducing an unstable transient response. Figure 4 shows the dual controller structure with a simple switch to connect a CMRAC controller

| Parameter | Value |
|-----------|-------|
| \( K_p \) | 0.23  |
| \( K_i \) | 0.20  |
| \( K_d \) | 0.03  |

The identification model is written with a similar structure as the plant model as:

\[
\begin{bmatrix}
\dot{\bar{x}} \\
\dot{\bar{y}} \\
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
0 & 0 \\
\end{bmatrix} \begin{bmatrix}
\bar{x} \\
\bar{y} \\
\end{bmatrix} + \begin{bmatrix}
0 \\
\frac{1}{\tau_m} \\
\end{bmatrix} \dot{\bar{L}} + \begin{bmatrix}
0 \\
-\bar{g} \\
\end{bmatrix}
\]  

\( (39) \)

and the augmented identification model for the quadrotor height dynamics is written as:

\[
\begin{bmatrix}
\dot{\bar{x}} \\
\dot{\bar{y}} \\
\dot{\bar{z}} \\
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 0 \\
-1 & 0 & 0 \\
\end{bmatrix} \begin{bmatrix}
\bar{x} \\
\bar{y} \\
\bar{z} \\
\end{bmatrix} + \begin{bmatrix}
0 \\
\frac{1}{\tau_m} \\
0 \\
\end{bmatrix} \dot{\bar{L}} + \begin{bmatrix}
0 \\
-\bar{g} \\
0 \\
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
1 \\
\end{bmatrix} r
\]  

\( (40) \)

The initial values of the plant estimates \( \hat{\bar{A}}_p, \hat{\bar{A}}, \) and \( \hat{\bar{d}} \) are assigned based on the modelled plant equation. In addition, the initial condition given for \( \bar{L} \) is 1.
while disconnecting the PID controller. When disconnected from the plant, CMRAC enters a tracking mode \[54\]. In the tracking mode, the CMRAC tracks the output of the currently active PID controller. This signal is fed back into the adaptive control law block as a tracking signal, labelled as \( U_{\text{tr}} \), as shown in Figure 5. The control input, \( U \), is a product of the CMRAC input, \( U_{\text{ad}} \), and the control signal tracking input, \( U_{\text{tr}} \). The tracking signal \( U_{\text{tr}} \) ensures that \( U \) tracked the PID input when the CMRAC block is disconnected from the closed-loop system thus ensuring the continuity of the control signal. This means that the CMRAC controller continuously adjusts its internal state to track the active control signal when it is disconnected from the plant.

When CMRAC is being used, the residue term from the tracking signal, \( U_{\text{tr}} \), is removed in order to allow for a sufficient adaptation of the CMRAC gains. The residue is decayed exponentially with a specified time constant \( \tau \) as shown in (41) in order to avoid inducing a discontinuity in the control signal.

\[
U_{\text{tr residual}} = U_{\text{tr}} \exp \left( -\frac{t}{\tau} \right) \tag{41}
\]

Essentially, a smooth control signal is ensured as the control authority is gradually being transferred from the PID to CMRAC through an exponential decay of the tracking signal, \( U_{\text{tr}} \) and an asymptotic rise of the CMRAC parameters \( U_{\text{ad}} \). This is demonstrated in the simulation shown in Figure 6, where the control transfer is simulated at 40 s. The control input, \( U \) remains continuous throughout the simulation.

Additional techniques are also applied to mitigate large and undesirable transients when control authority switches to CMRAC; this includes initialising the adaptive control parameters, \( \hat{K}_{x} \), \( \hat{K}_{r} \), and \( \hat{K}_{d} \), at pre-specified stable values whenever the controller is transferred from PID to CMRAC. This is done by resetting the integral of the gains to their initial conditions when the adaptive controller is inactive. In addition, the learning rate gains of adaptation are minimized when CMRAC is activated to avoid oscillatory transients. However, to allow for sufficiently fast adaptation, the learning rate gains are then automatically tuned to their assigned values in an exponential growth pattern as shown in (42).

\[
\Gamma_{i}(t) = \Gamma_{i0} (1 - e^{-t/\tau}) \tag{42}
\]

where \( \Gamma_{i0} \) is the initial value of the learning rate gain, \( \tau \) is the time constant that defines the growth rate. In this work, the growth
rate time constant of 10 s was found to be reasonable, as shown in Figure 6(b).

4 EXPERIMENTAL RESULTS

The CMRAC controller was designed using Simulink and then tested on an off-the-shelf Lumenier QAV250 Mini RC Quadrotor [55] in an indoor flying arena at the Bristol Robotics Laboratory (BRL). The quadrotor weighs 0.5 kg and has a flight time of approximately 6 min with a 3-cell Lithium Polymer battery. Figure 7 shows the experiment setup in the flying arena. A closed-loop system between the hardware and software is constructed with the use of a Vicon motion capture system for providing the state feedback, and dSpace MicroAutoBox for real-time system communication via User Datagram Protocol (UDP) packets over a local area network (LAN) connection. An outer loop position controller runs on the ground station computer, and the control output is then translated to the stick command for throttle, pitch, roll, and yaw. The quadrotor is equipped with an on-board low-level attitude stabilising autopilot.

4.1 CMRAC height controller

In this experiment, the quadrotor is commanded to follow a staircase height demand while keeping $x$ and $y$ positions constant. The chosen height demand considers a minimum altitude of 1 m, and a maximum altitude of 2 m which to ensure it remains within the volume covered by the Vicon cameras.

The CMRAC initial parameters and learning rate gains are shown in Table 2. The experiment started with the use of the PID controller in the take-off phase. CMRAC is then activated when the quadrotor was at a safe height of 0.5 m from the ground.

Figure 8 shows the altitude tracking response of the quadrotor taken from an actual flight test. It is observed that there is an initial drop in quadrotor height at the onset of CMRAC. Subsequently, the tracking response improves, and within a short time, the response follows the reference model response. Figure 9 shows the CMRAC parameters. The parameters move smoothly to new values as the controller adapts to the system. It is seen that the tracking response improved as the adaptive controller parameters converged to their true values. In the flight test, it took approximately three manoeuvre cycles or 120 s for

| Parameter                  | Value |
|----------------------------|-------|
| $\Gamma_z$                 | 0.03  |
| $\Gamma_c$                 | 0.05  |
| $\Gamma_r$                 | 0.02  |
| $\Gamma_d$                 | 0.02  |
| Initial conditions         |       |
| $K_{\dot{z}}$              | -0.2  |
| $K_c$                      | 0.2   |
| $K_r$                      | 0.0   |
| $K_d$                      | 0.0   |

FIGURE 7 Flying arena hardware and software setup

FIGURE 8 Experiment of QAV250 quadrotor in altitude tracking with CMRAC showing a control transfer from PID to CMRAC at approximately $t \approx 50$ s

FIGURE 9 CMRAC parameters
the CMRAC parameters to reach the threshold of acceptable convergence values. Although the overall response is acceptable, there is a small error that persists in the tracking response. The tracking error and identification error associated with this flight test are shown in Figures 10 and 11 respectively. Both converge to the neighbourhood of zero relatively quickly and remained small for the remainder of the flight.

## 4.2 Augmented CMRAC height controller

A second flight test was conducted with the augmented version of the CMRAC. The augmented CMRAC controller was tuned using higher learning rate gains because it has been shown in preliminary simulation tests that higher learning rate gains can be used with the augmented CMRAC without exciting high frequencies or inducing unwanted oscillations in the response. Higher values of the learning rate gains will give faster adaptation and parameter convergence, which can be observed within the duration of the flight experiment. The controller tuning parameters are shown in Table 3. It should be noted that these same parameters when applied and tested on the nominal CMRAC flight experiment result in high-frequency content in the control input signal and the tracking response.

Figure 12 shows the experimental quadrotor height tracking response with the augmented CMRAC in the height control loop. A significant improvement is observed in the augmented CMRAC response, with a smaller transient at the controller onset and a faster improvement in tracking the reference model response compared to the previous results.

Figure 13 shows the progression of the CMRAC parameters as the controller adapts. These parameters were initialised arbitrarily, however it is observed that all control parameters
converged to new values smoothly and quickly when the adaptation was turned on. Comparing this to the CMRAC parameters shown in Figure 9, the augmented CMRAC parameters demonstrate reduced oscillation and faster convergence. Note that the converged values of the adaptive gains obtained from this experiment were slightly different than the values from the nominal CMRAC experiment, due to the augmented dynamics.

Figure 14 shows the tracking error and Figure 15 shows the identification errors associated with this flight test. It is seen that the errors converged to zero within 50 s of the application of CMRAC, and more importantly, the errors remained within a very small margin of zero for the remainder of the flight. The rapid convergence of the tracking error is due to the addition of the integral action in the augmented CMRAC control law together with the increased adaptation rates.

Several flights were conducted to test the performance of the augmented CMRAC. Similar results were observed, and the converged value of the adaptive controller gains obtained from five flight tests in similar settings are shown in Figure 16. It is seen that all the parameters from different flight tests converged to the same values, therefore, it can be concluded that the augmented CMRAC controller tested here results in control parameters close to the ideal values such that the closed-loop response tracks the reference model characteristics closely.

5 | DISCUSSION

The CMRAC controller is designed on a linearised model with uncertainties in the plant matrix, and a linear reference model is used to describe the desired response. Although linear model is used, it is worth noting that the adaptive control law is nonlinear, and when the controller is applied to real-time flight experiments, it is also subjected to uncertainties and nonlinearities in the quadrotor dynamics and environment that are not modelled in the simulation.

The experiments show good results for both the standard CMRAC and the augmented version. In both cases, the tracking error is bounded. However, the augmented CMRAC exhibits a better tracking response with reduced transient oscillations. In addition, augmented CMRAC gives smaller tracking and identification errors and faster error convergence as shown in the root mean square of the tracking error plot in Figure 17. The augmented CMRAC parameters adapt to the true values faster than the unaugmented version, leading to better tracking of the reference model dynamics. This significant improvement in the augmented CMRAC response is attributed to the use of an integral on the input signal tracking error in the control law. With a more robust response, the augmented CMRAC also allows for higher learning rate gains, $\Gamma_i$, which speed up the adaptation process.

It can be seen in the results that all the adaptive gains converged to their values and are bounded, except for the slight drift in the parameter $K_d$. This parameter compensates for unmodelled disturbances or uncertainties. It could be argued that in the flight experiments, the battery life degradation causes this
CONCLUSIONS

Combined model reference adaptive controller has been applied to a quadrotor altitude control loop in experimental flight tests. In addition, an extension called augmented CMRAC has also been designed and tested. The results show satisfactory tracking for both controllers, but the augmented CMRAC gives a significant improvement in the response and parameter adaptation characterised by smaller oscillations in the transient phase and faster convergence of adaptive parameters and the tracking errors. Furthermore, augmented CMRAC allows for higher adaptive learning rate gains without inducing high-frequency content in the control signal. This gives the advantage of achieving a faster adaptation. It can be concluded that the improved performance of the augmented CMRAC is attributed to the additional integrated output tracking error signal that was added to the state vector which helped mitigate tracking error quickly and effectively. These desirable characteristics of the augmented CMRAC open for further promising future developments such as utilising the controller for mitigating environmental disturbances or payload variations in quadrotor flights. Also, it is recommended to extend the application of the CMRAC controller for trajectory tracking in a 3D environment. Such challenging flight tests will better demonstrate the effectiveness of the adaptive mechanism. Furthermore, the effectiveness of this controller to learn a set of ideal control gains on-line and quantifiably the changes in the estimated plant parameters are worth to investigate further in future research. In terms of controller implementation for flight experiments, practical solutions have been proposed in this paper to achieve the intended safety and stability when testing the CMRAC controller.

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