The influence of small-scale magnetic field on the heating of J0250+5854 polar cap

D P Barsukov¹, A A Matevosyan², I K Morozov¹, A N Popov¹ and M V Vorontsov²
¹ Ioffe Institute, Saint Petersburg, Russia
² Peter the Great St. Petersburg Polytechnic University, Saint Petersburg, Russia
E-mail: bars.astro@mail.ioffe.ru

Abstract. The influence of surface small-scale magnetic field on the heating of PSR J0250+5854 polar cap is considered. It is assumed that the polar cap is heated only by reverse positrons accelerated in pulsar diode. It is supposed that pulsar diode is located near the star surface (polar cap model) and operates in the steady state space charge-limited flow regime. The reverse positron current is calculated in the framework of two models: rapid and gradually screening. To calculate the production rate of electron-positron pairs we take into account only the curvature radiation of primary electrons and its absorption in magnetic field. It is assumed that some fraction of electron-positron pairs may be created in bound state that can later be photoionized by thermal photons from star surface.

1. Introduction
Radio pulsar J0250+5854 rotates with period \( P = 23.54 \) s [1]. It is the slowest pulsar among rotation powered pulsars [2]. It is an old pulsar with spin-down age \( \tau = 13.7 \cdot 10^6 \) years, its period time derivative is \( \dot{P} = 2.71 \cdot 10^{-14} \), its spin-down energy loss rate is equal to \( \dot{E} = 8.2 \cdot 10^{38} \) erg/s, the strength of dipolar magnetic field at pole estimated by pulsar spin-down rate is \( B_{\text{dip}} = 5.1 \cdot 10^{13} \) G, the distance to the pulsar estimated by dispersion measure is \( D_{\text{DM}} = 1.56 \) kpc [2]. Such pulsars lie beyond conventional pulsar "death line" (see, for example, [3, 4]). The existence of radio pulsar emission in such pulsars is usually explained by the presence of surface small-scale magnetic field [5, 6, 7, 8]. The other explanation is presented by [9]. In key feature of this explanation is that the pulsar is very close to orthogonal state, i.e. the inclination angle \( \chi \approx 90^\circ \). In this paper, as well as in our previous paper [10], we consider the influence of surface small-scale magnetic field on the J0250+5854 polar cap heating and the corresponding polar cap X-ray luminosity with taking into account positronium generation and its photoionization by thermal photons from star surface. In previous paper [10] we consider only \( \chi \sim 60^\circ \) and can not explain the presence of the pulsar above "death line" in the case of small inclinations \( \chi \lesssim 45^\circ \). Here we use a slightly modified configuration of small-scale magnetic field and show that in the used model the pulsar may be above "death line" in the case of small inclinations too.
Figure 1. A sketch of the vicinity of the inner gap. Neutron star is shown by gray area, pulsar tube boundaries are shown by black lines, the inner gap is shown by brown area.

Figure 2. The dependence of polar cap luminosity $L_{pc}$ due to the reverse positron heating on the small-scale field strength $B_{sc}$ in case of rapid screening model for different parameters is shown. The case of $\chi = 30^\circ$ is shown on the left panel and the case of $\chi = 0^\circ$ is shown on the right panel.

2. Model

Let the neutron star have a dipolar magnetic field $B_{dip}$ and a small-scale magnetic field $B_{sc}$. We suppose that the magnetic field near by the polar cap may be described by two-dipole model, see [12, 13, 14] for details:

$$\vec{B}(\vec{x}) = \frac{3\vec{x}(\vec{x} \cdot \vec{m}) - \vec{m} r^2}{r^5} + \frac{3\vec{\rho}_{sc} (\vec{\rho}_{sc} \cdot \vec{m}_{sc}) - \vec{m}_{sc} \rho_{sc}^2}{\rho_{sc}^3},$$

where $r = |\vec{x}|$ is the distance from star center, the plane $z = 0$ corresponds to the star surface (see figure 1), $\vec{m} = m \vec{e}_z$ is the magnetic dipole moment of the neutron star, $B_{dip} = 2m/r_{ns}^3$ is the strength of dipolar magnetic field on the star surface, $\rho_{sc} = \vec{x} + \delta \vec{e}_x - (r_{ns} - \ell) \vec{e}_z$, $\vec{m}_{sc} = -m_{sc} \vec{e}_x$, $r_{ns}$ is the neutron star radius, $B_{sc} = 2m_{sc}/\ell^3$ is the characteristic strength of small-scale magnetic field on the star surface, $\ell$ is the characteristic scale of small-scale magnetic field. In this paper, as well as in [10], we assume that $\ell = r_{ns}/20$. We also assume that the angular rotation velocity $\Omega$, $\Omega = 2\pi/P$, lies in the plane $Oxz$ (see figure 1). The pulse width at 10% intensity level at 350 MHz is equal to $w_{10} \approx 2.9^\circ \pm 0.4^\circ$ [1]. The pulsar beam radius at 400 MHz may be estimated as $\rho_{10} = 2.3^\circ P^{-0.36} \approx 0.7^\circ$ [15]. If we accept this estimation of the
Figure 3. The same as in figure 2, for the case of gradually screening model is shown.

Figure 4. The dependence of the total number of produced unbound or photoionized pairs $n_{\text{pair}}$ in units $\Omega B_{\pi c e}$ for different parameters is shown. The case of $\chi = 30^\circ$ is shown on the left panel and the case of $\chi = 0^\circ$ is shown on the right panel. The colors mean the same as in figure 2.

pulsar beam radius then the value of an inclination angle $\chi$ (angle between vectors $\vec{\Omega}$ and $\vec{m}$, see figure 1) may be estimated as [16]

$$\chi \lesssim \beta_1 = \arcsin \left( \sin \left( \frac{\rho_{10}}{2} \right) / \sin \left( \frac{\psi_{10}}{4} \right) \right) \approx 31^\circ,$$

where $\beta_1$ is the upper limit of possible value of angle $\chi$. So we can expect that the pulsar has small inclination $\chi \lesssim 30^\circ$ or may even be close to aligned $\chi = 0^\circ$. It is worth to note that according to [17] the pulsar seems close to aligned $\chi \sim 3^\circ$ too and the ”maximal possible” value of inclination angle is $\chi \approx 36^\circ$. So in this paper we consider two cases: $\chi = 0^\circ$ and $\chi = 30^\circ$. We
consider only the case of inner gap [18] and assume that the inner gap occupies all the pulsar tube cross section and resides as low as possible. In most cases, the inner gap resides exactly on neutron star surface \( z = 0 \) (see [19] for details). We suppose that the inner gap operates in the steady-state space charge-limited flow regime [20]. For simplicity, we take into account the generation of electron-positron pairs only by curvature radiation emitted by primary electrons in magnetic field. The dependence of pair generation properties on the photon polarization is neglected in the calculations. The probability of positronium production is considered by the same way as in [10] (see also [11]). In order to take into account the positronium photoionization by thermal photons emitted from the star surface we use the following approximate formula for the photoionization rate [11]:

\[
\frac{dN}{dt} = W_0 \left( \frac{10^2}{\Gamma} \right)^3 \left( \frac{T}{10^6 K} \right)^2 (1 - \cos \theta_{\text{star}}),
\]

(3)

where \( \Gamma \) is positronium Lorentz factor, \( T \) is star surface temperature, \( \theta_{\text{star}} \) is star angular radius, \( W_0 = 6 \cdot 10^5 \text{s}^{-1} \). Due to small polar cap size, we neglect positronium photoionization by thermal photons from hot polar cap. In this paper, we do not take into account photon splitting and positronium decay. Reverse positron current is calculated by using two models: the model of rapid screening [21] according to which the electron-positron plasma screens parallel electric field \( E_\parallel = (\vec{E} \cdot \hat{B})/B \) almost immediately and the model of gradually screening [22, 23], which allows the parallel electric field to penetrate deep into the electron-positron plasma (see details of calculation in [24]).

3. Results

The dependence of the polar cap luminosity \( L_{\text{pc}} \) caused by the reverse positron current heating on strength of small-scale magnetic field \( B_{\text{sc}} \) in the case of rapid and gradually screening models is shown in figures 2 and 3, correspondingly. The dependence of the total number of produced unbound and photoionized pairs \( n_{\text{pair}} \) on strength of small-scale magnetic field \( B_{\text{sc}} \) is shown in figures 4 and 5. In the case of \( \chi = 0^\circ \), the pulsar lies below ”death line” at \( \delta = 0 \) and \( \delta = 0.1 r_{\text{ns}} \). In the case of \( \chi = 30^\circ \), the pulsar lies below ”death line” at \( \delta = 0 \) and \( T = 10^5 K \) and is very close to ”death line” \( n_{\text{pair}} \sim (0.1 - 0.2) \frac{\Omega_B}{2\pi c e} \) at \( \delta = 0 \) and \( T = 3 \cdot 10^5 K \).
4. Discussion

In the paper we use two assumptions about J0250+5854 pulsar: the presence of surface magnetic field with very small characteristic scale $\ell \sim 500$ m and neutron star surface temperature $T \sim (1 - 3) \cdot 10^5$ K. The main problem with using these assumptions is that the pulsar is very old $\tau = 13.7 \cdot 10^6$ years. Hence, it is difficult to explain why the field with so small scale has survived and why the star is so hot. But it is worth to note that radio pulsar B0950+08 has spin down age $\tau = 17.5 \cdot 10^6$ years and star surface temperature $T \sim (1 - 3) \cdot 10^5$ K [25]. Such temperature may be related to internal heating mechanisms like rotochemical heating and heating due to vortex friction [26]. We also may speculate that magnetic field decay event with Hall cascade has occur not so long ago in this pulsar [27]. Hence, small-scale magnetic field may be generated during Hall cascades and accompanying field decay may heat up the star.

In the paper only curvature radiation of primary electrons is taken into account. It is worth to note that resonant compton scattering may give a similar quantity of pairs [28] that leads to decreasing anode altitude and hence decreasing polar cap heating $L_{pc}$ by the reverse positron current. In the paper the cases $\chi = 0^\circ$ and $\chi = 30^\circ$ are considered. The case $\chi = 60^\circ$ and $\delta = 0$ has been considered in [10]. It is necessary to note that if we would accept a estimation of pulsar beam radius $\rho_{10} = 2.87^o \cdot P^{-0.27}$ [16] we obtain a restriction $\chi \lesssim 48^\circ$. And if we would accept an estimation of pulsar beam radius at 400 MHz $\rho_{10} = 9.3^o \cdot P^{-0.36}$ [29] or $\rho_{10} \sim 3^o$ [1] we do not obtain any restriction on the value of inclination angle $\chi$ at all. Also it is worth to note that the observation of polarization of the pulsar emission together with the estimations by pulse width gives $\chi \sim 20^\circ - 50^\circ$ [30].

Acknowledgments

We sincerely thank A.I. Tsygan, O.A. Goglichidze, K.Yu. Kraav, V.M. Kontorovich, D.A. Rumyantsev, D.N. Sobyanin, I.F. Malov, V.A. Urpin and V.S. Beskin for help, comments and useful discussions.

References

[1] Tan C M and et al 2018 ApJ 866 54
[2] Manchester R N, Hobbs G B, Teoh A and Hobbs M 2005 Astron. J. 129 1993-2006
[3] Harding A K and Muslimov A G 2002 ApJ 568 862-77
[4] Kantor E M and Tsygan A I 2004 Astronomy Reports 48 1029-36
[5] Hbischman J A and Arons J 2001 ApJ 554 624-35
[6] Gil J and Mitra D 2001 ApJ 550 383-91
[7] Harding A K, Muslimov A G and Zhang B 2002 ApJ 576 366-75
[8] Harding A K and Muslimov A G 2011 ApJ 726 L10
[9] Novoselov E M, Beskin V S, Galishnikova A K, Rashkovetskyi M M and Biryukov A V 2020 MNRAS 494 3899-911
[10] Barsukov D P, Vorontsov M V and Morozov I K 2020 J. Phys.: Conf. Series 1697 012021
[11] Usos V V and Melrose D B 1995 Australian Journal of Physics 48 571-612
[12] Gil J A, Melikidze G I and Mitra D 2002 A&A 388 235-45
[13] Kantor E M and Tsygan A I 2003 Astronomy Reports 47 615-22
[14] Szary A 2013 Non-dipolar magnetic field at the polar cap of neutron stars and the physics of pulsar radiation Preprint astro-ph/1304.4203
[15] Malov I F, Malov O I and Nosov S L 1993 Astronomy Reports 37 26-34
[16] Malov I F and Nikitina E B 2011 Astronomy Reports 55 19-30
[17] Kon F F, Tong H, Xu R X and Zhou X 2019 ApJ 876 131
[18] Shibata S 1991 ApJ 378 220-54
[19] Barsukov D P, Poljakova P I and Tsygan A I 2009 Astronomy Reports 53 1146-54
[20] Fawley W M, Arons J and Scharlemann E T 1977 ApJ 227-43
[21] Arons J and Scharlemann E T 1979 ApJ 231 854-79
[22] Harding A K and Muslimov A G 2001 ApJ 556 987-1001
[23] Lyubarskii Yu E 1992 A&A 261 544-50
[24] Barsukov D P, Goglichidze O A and Tsygan A I 2016 Astronomy Reports 60 586-97


[25] Pavlov G G, Rangelov B, Kargaltsev O, Reisenegger A, Guillot S and Reyes C 2017 ApJ 850 79
[26] Guillot S, Pavlov G G, Reyes C, Reisenegger A, Rodriguez L E, Rangelov B and Kargaltsev O 2019 ApJ 874 175
[27] Igoshev A P and Popov S B 2018 Research Notes of the American Astronomical Society 2 171
[28] Zhang B, Harding A K and Muslimov A G 2001 ApJ 531 L135-38
[29] Malov I F 1986 Astrophysics 24 289-98
[30] Agar C H and et al 2021 MNRAS 2021 1102-14