DETERMINATION OF PULSATION PERIODS AND OTHER PARAMETERS OF 2875 STARS CLASSIFIED AS MIRA IN THE ALL SKY AUTOMATED SURVEY (ASAS)

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ABSTRACT

We have developed an interactive PYTHON code and derived crucial ephemeris data of 99.4\% of all stars classified as “Mira” in the ASAS database, referring to pulsation periods, mean maximum magnitudes, and whenever possible, the amplitudes among others. We present a statistical comparison between our results and those given by the International Variable Star Index (VSX) of the American Association of Variable Star Observers, as well as those determined with the machine learning automatic procedure of Richards et al. Our periods are in good agreement with those of the VSX in more than 95\% of the stars. However, when comparing our periods with those of Richards et al., the coincidence rate is only 76\% and most of the remaining cases refer to aliases. We conclude that automatic codes still require more refinements in order to provide reliable period values. Period distributions of the target stars show three local maxima around 215, 275, and 330 days, apparently of universal validity; their relative strength seems to depend on galactic longitude. Our visual amplitude distribution turns out to be bimodal, however, 1/3 of the targets have rather small amplitudes ($A < 2.5^m$) and could refer to semiregular variables (SR). We estimate that about 20\% of our targets belong to the SR class. We also provide a list of 63 candidates for period variations and a sample of 35 multiperiodic stars that seem to confirm the universal validity of typical sequences in the double period and in the Petersen diagrams.

Key words: methods: data analysis – stars: late-type – stars: variable: general

Supporting material: machine-readable tables

1. INTRODUCTION

Mira variables are evolved pulsating late-type stars at the last evolutionary state before the planetary nebula phase. They belong to the Asymptotic Giant Branch and therefore they enrich the interstellar material with gas and dust that has undergone nuclear processing. This interaction with their surroundings, due to their strong winds, plays an important role in the Milky Way recycling processes. The mass-loss of Mira stars could be accompanied by period changes, which, in some cases, exceed 30\% of the original period value within a few decades (Smith 2013). This period variability is normally attributed to He shell flash pulses, but alternative explanations could also be considered (Templeton et al. 2005, and references therein). An analysis of light curves and the determination of pulsation periods and other observational parameters provides vital information about the interior processes in stars and their evolutions. However, systematic studies of observational parameters of Mira stars are still scarce.

One of the possibilities for improving this situation is offered by the ASAS database (All Sky Automated Survey, Pojmanski 2002), which is the outcome of a sky patrol program with CCD photometry, conducted at the Las Campanas Observatory, Chile, between 2000 and 2009 ($-90^\circ < \delta < +28^\circ$). The visual limiting magnitude is about 14.5\textsuperscript{m} and a total of up to 500 measures per star are available. An automatic procedure identified 50122 ASAS variable stars and classified part of them according to their type of variability. A total of 2895 variables are listed as Mira stars in this database. However, their pulsation periods, as suggested by ASAS, are not at all reliable, and therefore they cannot be used for a statistical study of Mira properties. The aim of this paper is to re-analyze the light curves of all stars classified as Mira in ASAS, in order to offer to the community reliable periods, amplitudes, and other observational parameters (including mean errors) in a way that they can easily be combined with future studies of these stars. For this purpose we used a simple interactive method that is not fully automatic but requires human control and interaction, as explained in Section 2. Section 3 corresponds to our data presentation, while in Section 4 we give a statistical analysis of our results, with emphasis on a comparison to the International Variable Star Index (VSX) of the American Association of Variable Star Observers (AAVSO: Watson 2006), as well as to the fully automated machine learning method of Richards et al. (2012). We also estimate the SR fraction in our sample, select candidates for period changes, and give some examples of multiperiodicity. Section 5 contains our conclusions.

2. A SIMPLE METHOD DETERMINING EPHEMERIS DATA

In order to extract crucial information from the ASAS database we developed a PYTHON code that determines the epochs and magnitudes of the maxima in their light curves, assigns correct cycle count numbers, and derives the most important ephemeris data of each star. This procedure is not fully automatic; it requires interactive human collaboration. In this way, we could analyze 99.4\% of all stars listed in ASAS as Miras.

Our method is illustrated with ASAS 174436-8629.2 (Z Oct) as a typical example. The first two steps of our procedure are...
shown in Figure 1. First, the complete ASAS light curve is presented and the code selects a horizontal cut line that by default is placed at one-third of the total amplitude (the magnitude difference between the brightest and the faintest measurements in the data sample) below the brightest point of the data sample. Only values brighter than this limit are used in the period determination process. The time is given in HJD-2450000. Lower panel: the histogram shows the distribution of the time differences between subsequent data points in the light curve; the red line refers to the limit for the cycle counting process (for details see the text).

In the next step (Figure 1, lower panel) the code displays a histogram that shows the distribution of the time differences between subsequent data points in the light curve. In this example all observations with time differences <50 days belong to the same maximum, while time differences around 270 days refer to jumps from one maximum to the next one. The PYTHON code identifies the first large gap in this
distribution, and suggests a limit for the cycle counting process, if the two following conditions are fulfilled: the ratio between two adjacent time differences is >3.5 and the latter time
3. DATA PRESENTATION

This procedure leads to a total of nine general parameters, listed for all stars in our first online table and reproduced here in Table 1 for arbitrary chosen examples. Each line begins with the ASAS identification and that of the General Catalogue of

| Table 1 | Parameters of 3 Examples of Mira Stars, as Determined with our Method (First Online Data Set) |
|---------|--------------------------------------------------------------------------------------------------|
| ASAS Name | Alternative Name | $T_0$(day) | $\sigma(T_0)$(day) | $P$(day) | $\sigma(P)$(day) | $\sigma(T)$(day) | $M_{\text{max}}$(mag) | $\sigma(M_{\text{max}})$(mag) | $A$(mag) | $N$ |
| 000006+2553.2 | Z Peg | 2657.4 | 15.1 | 316.35 | 3.65 | 24.1 | 8.63 | 0.67 | 3.67 | 7 |
| 000017+2636.4 | AH Peg | 2910.7 | 8.1 | 374.23 | 2.19 | 11.6 | 11.21 | 0.36 | 9.10 | 6 |
| 000837–3913.2 | V Scl | 1934.4 | 4.7 | 297.41 | 0.88 | 7.5 | 9.17 | 0.20 | 5.03 | 7 |

(The table is available in its entirety in machine-readable form.)

| Table 2 | Cycle Count Numbers $E$ vs. $T_{\text{max},i}$ (HJD-2450000) and $E$ vs. $M_{\text{max},i}$ (V Magnitude) at Each Individual Maximum of All Stars in online Table 1 (Second Online Data Set) |
|---------|--------------------------------------------------------------------------------------------------|
| ASAS Name | Other Name | $E$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 000006+2553.2 | Z Peg | $T_{\text{max},i}$ | 2629.5 | 2983.5 | 3295.6 | 3617.7 | 3912.9 | 4273.9 | ... | ... | 5164.6 | ... |
| 000017+2636.4 | AH Peg | $T_{\text{max},i}$ | 2917.6 | 3270.6 | 3662.6 | ... | 4409.5 | 4794.5 | 5145.6 | ... | ... | ... |
| 000837–3913.2 | V Scl | $T_{\text{max},i}$ | 1928.5 | 2235.5 | 2529.7 | ... | 3133.9 | ... | 3707.6 | 4316.7 | 4610.9 |

(The table is available in its entirety in machine-readable form.)

| Table 3 | Eliminated Stars Classified as Mira in ASAS (Not Listed in Online Tables 1 and 2) |
|---------|--------------------------------------------------------------------------------------------------|
| ASAS name | Other Name | Remarks: (Reason for Elimination) |
| 020751+2756.2 | Z Tri | (1) |
| 035003–5721.2 | RY Ret | (2) |
| 042757+1602.6 | W Tau | (2) |
| 054731+2708.3 | AW Tau | (1) |
| 054957–5252.1 | ... | (2) |
| 060401+2113.5 | V342 Ori | (2) |
| 070722+2818.0 | AM Gem | (1) |
| 145641+2730.4 | NSV 06861 | (1) |
| 153557–4930.5 | R Nor | (2) |
| 155042+1508.0 | R Ser | (4) |
| 162229–4835.7 | IO Nor | (3) |
| 172019–2309.7 | ... | (1) |
| 173808–2217.1 | ... | (1) |
| 174224–4344.9 | RU Sco | (4) |
| 180444–2959.4 | ... | (1) |
| 183536–2409.2 | ... | (1) |
| 184041–1401.2 | ... | (1) |
| 190948+2804.3 | TY Lyr | (1) |
| 193352+2819.6 | TY Cyg | (1) |
| 204222+2728.8 | EN Vul | (1) |

Notes.
1. Only one or two maxima covered by ASAS.
2. Irregular light curve due to multiperiodicity (see Table 8).
3. Misclassified as Mira; RCB star according VSX.
4. No maxima are covered by ASAS due to seasonal gaps.

For example Z Oct. The operator checks these diagrams, especially $E$ versus $O - C$, which will reveal non-zero slopes and sudden discontinuities whenever the cycle counting procedure was not correct. In this case, the operator has the option to repeat the whole process, varying some of the input parameters until the result is satisfactory. If there are at least two well-defined minima covered by the ASAS observations, the operator also estimates the magnitude at minimum light based on the light curve display (accuracy about 0.1 mag), and enters this information into the database. The amplitude $A$ is calculated by subtracting the value of $M_{\text{max}}$ from the magnitude at minimum light. However, since for most stars their minimum phases are below the threshold of ASAS, amplitudes could be determined only for 855 of our targets.

(See Table 2 for details.)
Variable Stars GCVS (Samus et al. 2007–2015), if available, and lists the parameters $T_0$, $\sigma(T_0)$, $P$, $\sigma(P)$, $\sigma (T)$, $\sigma (M_{\text{max}})$, $\sigma (M)$, and $N$, which refers to the total number of light maxima observed. All $T$ values refer to HJD-2450000. Our second online Table (here shown as examples in Table 2) gives $E$ versus $T_{\text{max},i}$ and $E$ versus $M_{\text{max},i}$, the most important parameters of each individual light maximum, in order to enable observers to easily combine our data with those of other sources. In this way, we could analyze the data of a total of 2875 stars classified as Mira in the ASAS catalog, leaving only 20 stars that we were not able to analyze with our method. They are listed in Table 3, and in each case the reason for elimination is given.

4. STATISTICAL ANALYSIS

4.1. Comparison to Richards et al. (2012) and the VSX

Our results for pulsation periods, $V$ magnitudes, and amplitudes can now be compared with those determined by Richards et al. (2012); these authors had obtained a variability type classification for all 50122 ASAS variables in a fully automated way, using a machine learning procedure, and deriving some of the abovementioned parameters. In addition, we have compared our results with the parameters given by the AAVSO International Variable Star Index VSX, which is the most comprehensive catalog of variable stars the Milky Way available, featuring more than 398,000 entries.

The period distributions of stars that these two databases have in common are shown in Figure 3 (upper panel). The mean value of our periods is $\overline{P} = 271.1$ days ± 1.4 (see. Table 6), while for the VSX $\overline{P} = 269.5$ days ± 1.6, based on 2868 common stars (excluding an extreme period of 5000 days in VSX). Therefore, these both have mean periods that agree within the expected margin. On the other hand, we found a considerably shorter mean period in Richards et al. (2012) $\overline{P} = 241.1$ days. This included 99 stars with ~1 day period values, which apparently refer to cases whose periods remained undetermined by the machine learning procedure of Richards et al. (2012), resulting in a value that corresponds to the day–night rhythm at Las Campanas Observatory. Excluding these cases, we derived $\overline{P} = 249.2$ days ± 2.1 for the remaining 2776 stars of Richards et al. (2012), which is still significantly lower than our value and that of the VSX mean periods.

In Figure 3 (lower panel) we compare our distribution with that of all 18656 Mira stars with known periods listed in the VSX. Three local maxima at periods around 215, 275, and 330 days are present in all histograms shown in Figure 3. A very similar period distribution was already determined by Vogt (1980), whose Figure 8 shows a histogram based on about 5000 Mira stars with known periods listed in the GCVS at that epoch. To our knowledge, there is no reference in the literature for this three-peak distribution of Mira periods.

Figure 4 (upper panel) shows a histogram of period ratios between Richards et al. and our results. Several whole-number period ratios can be identified and are marked with different colors; the corresponding numbers are given in Table 4. Considering the correlation between our periods and those of Richards et al. (Figure 4, lower panel) we found significant differences in our results. Most of the whole-number ratios should be aliases of the true period. Only about 76% of Richard’s et al.’s periods are in agreement with our values.

This situation is rather different if we compare our periods with those of the VSX (Figure 5). There are also aliases, but at a much lower extent, and only limited to half and double periods, while in more than 95% of common stars our periods coincide with those of the VSX. The few aliases can probably be explained by an insufficient amount of observations (either in the VSX, in ASAS, or in both). The most important values of this comparison, as well as those from the comparison with Richards et al. (2012), are listed in Table 4.

In Figure 6 (upper panel) we show the amplitude distributions. Richards et al. (2012) determined amplitudes for all variable stars in ASAS, using an automated procedure without considering that many targets could only be observed around their maxima by ASAS while the fainter phases of their light curves remained unobserved. This explains the large amount of
small amplitudes with a peak at \( A \approx 1.4 \) mag, an artifact caused by the large number of faint targets in the sample. In Figure 6 (lower panel) we restrict ourselves to the 855 stars in which we were able to determine the amplitude (i.e., whenever the ASAS light curve also contains the minimum magnitude level) and compare them with Richards et al. (2012), as shown in Figure 6 (lower panel). The slope, intercept, and correlation coefficient of the corresponding least-squares fit are given in Table 5.

Our amplitudes in Figure 7 show a slight tendency toward a bimodal distribution, with maxima at 2.5\( ^{m} \) and 4.5\( ^{m} \), and a deficit around 3.4\( ^{m} \). Apparently, the amplitudes are not correlated with the periods (Figure 7, lower panel). However, the amplitude distribution is severely affected by the presence of an SR population among the stars classified as Mira in ASAS. A total of 281 targets with known amplitude (33%) have \( A < 2.5^{m} \), smaller than expected for Mira stars. From the remaining 2020 stars, 915 have \( M_{\text{max}} > 12.0^{m} \), i.e., only 2.5\( ^{m} \) brighter than the ASAS limiting magnitude \( V = 14.5^{m} \). Among them there could be some SR variables, while all other targets will have larger amplitudes, typical for Mira stars. If we adopt the same fraction of 33\% (as found in the sample with known amplitude) to be of SR type among the stars \( M_{\text{max}} > 12.0^{m} \), we could expect 302 other SR variables in the “faint sample.” This estimation leads to a total of about 583 SR stars (20\%) of the total sample investigated here. As an example we could mention BI Car, which is the star with the largest period (817 days) in our sample, with an amplitude of only \( A = 1.42^{m} \); this star is classified as an SRB in the GCVS.

In Figure 8 we compare the mean maximum magnitudes, where we find a perfect agreement with Richards et al. (2012).
This is not surprising since $M_{\text{max}}$ should not be affected by any of the known selection effects.

4.2. Internal Correlations

In Table 6 we give the mean values of the parameters mentioned in Table 1, together with their standard deviations and extremes. The correlations of period $P$ versus the standard deviations of $P$, $T$, and $T_0$ are shown in Figure 9. Only $\sigma(P)$ seems to present a weak correlation with $P$, which, however, diminishes if we consider the ratio $P$ versus $\sigma(P)/P$. Figure 10 shows the relations of $M_{\text{max}}$ and $P$ versus $\sigma(M_{\text{max}})$. There seems to be a tendency of smaller standard deviations in $M_{\text{max}}$ for short periods, while longer periods show a large range of $\sigma(M_{\text{max}})$ values. Finally, in Figure 11 we show correlations of $P$, $\sigma(P)$, $\sigma(T)$, $\sigma(T_0)$, and $\sigma(M_{\text{max}})$ with the number $N$ of light maxima observed. All these diagrams show a tendency of small

**Table 4**

| Period Ratio | Richards et al. (2012) | VSX |
|-------------|------------------------|-----|
|             | $N_{\text{total}} = 2875$ | $N_{\text{total}} = 2868$ |
| 0:1         | 99 (3.4)                | 99 (3.4) |
| 1:3         | 38 (1.3)                | 38 (1.3) |
| 1:2         | 435 (15.1)              | 36 (1.3) |
| 2:3         | 23 (0.8)                | 23 (0.8) |
| 1:1         | 2181 (75.8)             | 2752 (95.9) |
| 2:1         | 34 (1.2)                | 17 (0.6) |
| 3:1         | 2 (0.1)                 | 2.1 |
| Other stars | 63 (2.2)                | 63 (2.2) |

**Note.** Other stars refer to the black dots in Figures 4 and 5. $N$ refers to star numbers.

Figure 5. Comparison of our periods with those of the VSX, following the same approach as that used for in Figure 4.
values for large \( N \), and a much higher spread of these parameters at low \( N \). This is mainly a selection effect, since small numbers of observations always are accompanied by larger uncertainties in the determined parameter values. The close correlation between \( P \) and \( N \) is an evident consequence of the fixed time interval, of about nine years, that is covered by the ASAS database. A comparison of our period distributions toward and away from the galactic center is shown in Figure 12, splitting our sample into those with galactic longitude \( l \in [270°,360°] \), \([0°,90°]\) and those with \( l \in [90°,270°] \); the peak at 330 days seems to be predominant around the galactic anticenter, but it disappears at the hemisphere around the central region of the Milky Way.

4.3. Selection of Period Variability Candidates

In Figures 4 and 5, using black dots we have marked those stars that differ from our determinations but do not coincide with one of the other two databases or with one of the aliases. In the case of the VSX (Figure 5), they could be considered potential candidates for period changes because in many cases

| Table 5 | Linear Fit Parameters and Correlation Coefficients \( r \) |
|--------|---------------------|
|        | Slope   | Intercept | \( r \) | Reference Figure |
| \( A \) versus \( A(R) \) | 0.594   | 0.266     | 0.955 | 6             |
| \( M_{\text{max}} \) versus \( \sigma(M_{\text{max}}) \) | 0.950   | 0.603     | 0.963 | 8             |
| \( P \) versus \( \sigma(P) \) | 0.011   | -1.527    | 0.657 | 9             |
| \( P \) versus \( \sigma(P)/P \) | 0.00001664 | 0.000142 | 0.451 | 9             |
| \( P \) versus \( A \) | 0.000860 | 2.867     | 0.0668 | 7             |
| \( P \) versus \( \sigma(T) \) | 0.02662 | 5.254     | 0.365 | 9             |
| \( P \) versus \( \sigma(T_{21}) \) | 0.0253  | 0.968     | 0.485 | 9             |
| \( P \) versus \( \sigma(M_{\text{max}}) \) | -0.00252 | 12.021    | -0.165 | 10           |
| \( P \) versus \( \sigma(M_{\text{max}}) \) | 0.000526 | 0.199     | 0.301 | 10           |
| \( M_{\text{max}} \) versus \( \sigma(M_{\text{max}}) \) | -0.0238 | 0.611     | -0.208 | ...         |

Note. \( (R) \) refers to Richards et al. (2012).
VSX periods are based on observations from several decades, much longer than those in ASAS. For a comparison with Richards et al. (2012) we selected the limits according to the gaps visible in the histogram (Figure 4, upper panel). These limits are listed in the 4th column of Table 4. When comparing with the VSX we considered as variable candidates only those stars whose period ratios deviate more than 5% from the nominal ones (1:2, 1:1 or 2:1). The corresponding limits are listed in the last column of Table 4. Table 7 lists the 63 candidates for variability found according to these criteria. However, we should keep in mind that some of the stars with period ratio differences <5% could be real variables; this is also valid for some of the cases with half or double period values in the VSX, compared to ASAS. Finally, we suppose that there should be some erroneous period values in the VSX, because our ASAS periods listed in Table 7 have been rechecked and are rather certain. Therefore, the 63 candidates of Table 7 require confirmation using more extended light curve data.

4.4. A Sample of Target Stars with Multiperiodicity

In the course of our ephemeris determination, we could identify multiperiodic variations for a sample of 35 stars. For each star, the dominant period was detected with our PYTHON code as described in Section 2. In order to confirm the main period and to search for additional ones we used PERIOD04 (Lenz & Breger 2005), which is based on discrete Fourier transform. The frequencies were detected after the main frequency was subtracted. The appearance of harmonics of the main period and aliases has been controlled by adjusting the calculated combined light curve to the observations. Typical examples of this analysis are shown in Figure 13. The results are listed in Table 8. We found 22 stars with 2 periods and 13 with periods between 75 and 650 days. In order to find correlations between those multiperiods, we constructed the double period diagram (DPD), which compares contiguous periods, and the Petersen Diagram (PD), which presents period ratios (Figure 14). The solid lines A to F correspond to sequences found by Fuentes-Morales & Vogt (2014) for SR (semiregular) variables and dashed lines belong the sequences of Kiss et al. (1999). The multiperiodic stars adjust to these sequences for SR stars. Apparently sequences A and B are more populated by Mira stars compared to previously analyzed SR stars. Therefore, this sample is a valuable complement to previous investigations of sequences that correspond to similar periods.

5. DISCUSSION AND OUTLOOK

When we initiated this work, we selected all 2895 stars classified as “Mira” in the ACVS database of ASAS as our targets. Only during the analysis of the data did we notice that several stars did not fulfill the amplitude criterion for Mira classification, which should be A > 2.5σ according to the definitions given in the GCVS and VSX catalogs. Apparently, many stars classified as Mira in ASAS are in fact semiregular variables (type SR in the GCVS). However, since we could only determine amplitudes for 30% of our targets it was impossible to reclassify the rest of our stars, and we maintained the original sample, without any attempt of variability...
Statistically, we expect that about 20% of all targets could be of SR type.

One of the advantages of our method is the fact that we use only the observations around the light maxima in the light curves for our analysis; they tend to have smaller errors than the data with \( \sim 14 \) m, near the ASAS magnitude limit, as included by Richards et al. (2012) in their analysis. This inclusion of large-scattered data could be one of the reasons for the numerous period aliases, which appeared in a considerable fraction of stars when comparing our data with Richards et al. (2012). Our experience also shows that, in this type of work, human interaction is still necessary; apparently, machine learning methods need further refinement in order to achieve a quality and reliability comparable to that of generations of professional and amateur astronomers dedicated to variable star research. This latter statement is confirmed by our comparison to the VSX, which turned out to be much more reliable than that of the machine learning procedures by Richards et al. (2012). This, of course, refers only to the determinations of periods and amplitudes, not to the automatic variability type classification method developed by these authors and applied to ASAS. Any control or judgment of this method is outside the scope of this paper.

We also found indications of three peaks in the period distribution of targets, with maxima around 215, 275, and 330 days. According to Feast & Whitelock (2014) Mira periods are dependent on the star’s age or initial mass; the authors suggest mean ages of 12 Gyr at \( P = 200 \) days down to 2 Gyr at 500 days. The observed period distribution will reflect a combination of the star-forming rate and the lifetimes of Mira stars of different ages. The peaks in Figure 3 may indicate punctuations in the star-forming history in the volume sampled. In fact, we detected that the peak at 330 days is dominating if we look away from the galactic center, while it disappears in the distribution of stars toward the galactic center; here, the other two peaks at shorter periods are predominant (see Figure 12). In general, larger periods seem to be more frequent around the galactic anticenter, and shorter ones are more frequent toward the center. However, a stringent statistical analysis of this item should comprise a more complete sample, including stars from the northern hemisphere, which is beyond the scope of this paper. We also should bear in mind that there

Figure 9. Correlations between our periods and the parameters \( \sigma(P) \), \( \sigma(P)/P \), \( \sigma(T) \), and \( \sigma(T_0) \). The solid red lines correspond to the fit parameters given in Table 5.
is a “contamination” with about 20% semiregular variables in our sample, which could be relevant in the statistics of amplitude and period distributions, as well as the multi-periodicity occurrence.

We are aware that the ASAS time span of \( \sim 9 \) years is too short to determine significant period changes; however, we were able, based on comparative studies, to select candidates for period changes that are listed in Table 7. The 63 candidates in this Table represent about 2% of the entire sample of 2868 stars in common with the VSX. This fraction is of the same order as some literature values. According to Zijlstra & Bedding (2002) about 1% of Mira stars show evidence of period changes, but unstable periods may be more common among those with the longest periods. Sabin & Zijlstra (2006) also report instabilities for periods >450 days and discriminate between continuous, sudden, and meandering period changes. The most comprehensive study of period changes was given by Templeton et al. (2005) based on an analysis of 547 Mira stars. They found that about 10% of them are candidates for period changes, while 1.6% underwent strong period variability at a level of >6\( \sigma \) significance.

Figure 10. Correlations between our periods and the parameters \( M_{\text{max}} \) and \( \sigma (M_{\text{max}}) \). The solid red lines correspond to the fit parameters given in Table 5.

Figure 11. Correlations between our cycle count number \( E \) and the parameters \( P, \sigma(P), \sigma(T), \sigma(T_0), \) and \( \sigma(M_{\text{max}}) \).
These results are consistent with theoretical expectations, in favor of the idea that large period changes are being caused by thermal helium flash pulses (Smith 2013 and references therein). But there are also alternative explanations, especially for minor oscillations around a mean period, whose timescales are consistent with the Kelvin–Helmholtz cooling time of the envelope. Period changes of this sort might be caused by thermal relaxation oscillations in the stellar envelope, perhaps in response to global changes caused by a thermal pulse (Templeton et al. 2005). But accurate period values and their errors are also important for other studies, for instance, in context with the mass outflow of red giants during the latest stage of their evolution. Uttenthaler (2013) found that a relation between dust mass-loss rate and Mira pulsation period exists, the largest mass-loss rate values correlating with long periods. On the other hand, the mass-loss rate is also related to infrared colors (Le Bertre & Winters 1998), underlining the need for simultaneous approaches at different fronts.

One of the most important characteristics of Mira stars is that they obey period–luminosity relations (PL); therefore, they are potentially important extragalactic distance indicators. Gaia will provide a distance calibration for all nearby Mira stars, and extensive and reliable information on their periods will be important (White et al. 2008; Whitelock 2012). PL relations of Mira stars could provide valuable information that is complementary to that given by classical Cepheids, in order to improve the distance calibration within the Milky Way, as well as to nearby galaxies.

Most past and present Mira star studies either refer to a single or a few well-studied representatives, or suffer from heterogeneous data, mostly without any indication of the detailed methods and of the errors involved. Here we hope to deliver a solid and homogeneous basis for future studies on periods and amplitudes of Mira stars. Those studies could extend the ASAS time interval of only nine years toward several decades in many cases. This should be possible with modern means, as the Virtual Observatory technology also involves scanned photographic patrol plates that are available.
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Figure 13. Left panel: light curve of R Cen showing two periods. The Fourier spectrum reveals two peak frequencies that correspond to the periods $P_0 = 499 \pm 2$ days and $P_1 = 250 \pm 1$ days. Right panel: light curve of ASAS 054957-5252.1, showing a typical behavior of resonance due to two similar periods.

Table 8
Double and Triple Periodicities of Selected Target Stars

| ASAS Name    | GCVS Name | Variability Class | $P_{\text{main}}$ (day) | $P_0$ (day) | $P_1$ (day) | $P_2$ (day) |
|--------------|-----------|-------------------|-------------------------|-------------|-------------|-------------|
| 024745−5903.1 | X Hor     | Mira              | 284.9                   | 283.2(0.2)  | 142.5(0.2)  | ...         |
| 035003−5721.2 | RY Ret    | Mira              | 128.1                   | 243.2(0.6)  | 226.8(0.6)  | 118.3(0.2)  |
| 042757+1602.6 | W Tau     | Semireg_PV        | 184.6                   | 243.7(0.4)  | 129.3(0.5)  | ...         |
| 054544+1838.2 | EG Tau    | Semireg_PV        | 219.3                   | 218.4(0.5)  | 119.7(0.3)  | ...         |
| 054657+1754.5 | El Tau    | Mira              | 368.2                   | 366.0(0.4)  | 123.2(0.9)  | ...         |
| 054906−2241.3 | ...       | Mira              | 200.6                   | 201.2(3.2)  | 99.8(0.2)   | ...         |
| 054957−5252.1 | ...       | Mira              | 131.3                   | 154.9(0.1)  | 144.7(0.2)  | 75.3(2.2)   |
| 060401+2113.5 | V0342 Ori | RSG               | 213.5                   | 282.7(2.5)  | 149.2(0.2)  | ...         |
| 062019+1121.1 | EO Ori    | Semireg_PV        | 277.1                   | 280.4(0.9)  | 138.7(0.3)  | ...         |
| 073753−3329.8 | KS Pup    | Mira              | 397.9                   | 399.8(0.6)  | 199.9(1.1)  | 132.7(0.6)  |
| 074752−2730.0 | NSV03743  | Mira              | 491.8                   | 484.0(1.5)  | 266.5(1.7)  | 244.7(1.05) |
| 075751−6517.8 | X Vol     | Mira              | 289.5                   | 288.2(0.2)  | 153.1(1.0)  | 143.2(1.5)  |
| 082113−1229.3 | AB Pup    | Mira              | 267.3                   | 264.9(0.2)  | 128.3(20.6) | ...         |
| 082508−4757.9 | ...       | Mira              | 486.4                   | 488.9(1.1)  | 249.1(12.6) | ...         |
| 083855−4656.6 | ...       | Mira              | 311.9                   | 333.0(1.0)  | 311.0(0.2)  | ...         |
| 084615−7717.4 | NSV18043  | Mira              | 268.7                   | 266.0(0.1)  | 244.3(4.3)  | ...         |
| 113359−7313.3 | NSV18797  | Mira              | 535.3                   | 545.0(1.5)  | 269.1(1.3)  | ...         |
| 113958−5346.9 | ...       | Semireg_PV        | 137.0                   | 298.9(1.7)  | 272.9(0.3)  | 138.2(0.4)  |
| 121617−5617.2 | BH Cru    | Mira              | 522.7                   | 530.3(0.4)  | 171.7(5.5)  | ...         |
| 124902−3645.8 | V0802 Cen | Semireg_PV        | 266.3                   | 266.7(0.4)  | 133.1(0.2)  | ...         |
| 130730−6853.5 | DW Mus    | Mira              | 368.5                   | 367.2(0.4)  | 177.9(0.7)  | ...         |
| 131930−6046.7 | TT Cen    | Mira              | 457.6                   | 458.5(0.6)  | 231.7(0.2)  | 153.6(1.5)  |
| 135102−7028.4 | Z Cir     | Mira              | 388.8                   | 389.6(0.3)  | 194.2(0.5)  | ...         |
| 141635−5954.8 | R Cen     | Mira              | 498.8                   | 499.5(2.2)  | 249.7(0.4)  | ...         |
| 142052−6730.9 | UZ Cir    | Mira              | 549.2                   | 546.1(0.6)  | 268.5(14.3) | 178.5(3.0)  |
| 151556−3935.6 | FL Lap    | Mira              | 265.0                   | 291.6(9.1)  | 268.3(0.4)  | 139.7(12.9) |
| 153557−4930.3 | R Nor     | Mira              | 391.6                   | 495.6(0.9)  | 248.8(1.1)  | 165.2(6.5)  |
| 161015+2504.3 | RU Her    | Mira              | 481.8                   | 485.2(1.4)  | 160.1(1.8)  | ...         |
| 162522−5827.8 | EQ Nor    | Mira              | 307.1                   | 299.7(0.2)  | 148.5(0.3)  | ...         |
| 173337−3615.6 | V1163 Sco | Mira              | 508.8                   | 519.0(1.1)  | 256.4(1.2)  | ...         |
| 175412−3420.5 | BN Sco    | Mira              | 615.1                   | 615.1(2.4)  | 306.6(0.7)  | 207.1(0.6)  |
| 190010−0134.9 | VX Aql    | Mira              | 634.7                   | 650.9(2.4)  | 325.4(4.3)  | 214.7(1.7)  |
| 202354+0056.8 | V0865 Aql | Mira              | 371.8                   | 371.8(19.5) | 184.6(0.3)  | ...         |
| 205300+2322.3 | RX Vul    | Mira              | 457.3                   | 459.7(1.3)  | 222.7(13.2) | ...         |
| 222313−2203.4 | RT Aqr    | Mira              | 246.8                   | 264.8(14.6) | 247.9(6.3)  | 121.4(8.3)  |

Note. ASAS and GCVS Names, variability class according to Richards et al. (2012), the dominant period $P_0$ obtained with the PYTHON code, and the longer and shorter periods $P_i$ ($i = 1, 2$).

for more than one century at different places, especially at the Harvard and Sonneberg observatories (Hudec 1999). Another task would be the extension toward fainter magnitudes, in order to get reliable amplitudes for a much larger number of Mira stars, for instance, with the Catalina Sky Survey (Drake et al. 2009, 2014), and the inclusion of other wavelengths,
especially those in the infrared, as done in VISTA (Dalton et al. 2006; Emerson & Sutherland 2010; Saito et al. 2012).

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Figure 14. Left panel: the DPD refers to longer periods log$(P_i)$ vs. shorter ones log$(P_{i+1})$, obtained for multi-periodic variables selected in this work (red symbols) compared with the DPD of semiregular variables found by Fuentes-Morales & Vogt (2014, black symbols). Right panel: the Petersen diagram (PD) gives log$(P_{i+1})$ vs. log$(P_i/P_{i+1})$. In both diagrams open circles refer to $i = 0$ for double periodic cases, filled squares to $i = 1$ and triangles to $i = 2$ for triple periodic cases. The denomination of sequences A–F is also identical to that of Fuentes-Morales & Vogt (2014). There are indications of an intermediate sequence between A and B for triple periodic stars.