Shaping of Rack Cutter Original Profile for Fine-module Ratchet Teeth Cutting

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Abstract. The design models and the process of shaping the cutting edges of the rack cutter for cutting fine-module ratchet teeth are considered in the article. The use of fine-module ratchet teeth can reduce the noise and impact loads during operation of the freewheel mechanisms. Mathematical dependencies for calculating the coordinates determining the geometric position of the points of the front and back edges of the cutting profile of the rack cutter, the workpiece angle of rotation during cutting the ratchet teeth were obtained. When applying the developed method, the initial data are: the radii of the workpiece circumferences passing through the dedendum of the external and internal cut teeth; gradient angles of the front and back edges of the rail.

1. Introduction

The freewheel mechanisms in machine drives are used mainly for automatic connection and disconnection of kinematic chains. Their use makes it possible to simplify the kinematics and increase the functionality of the machines [1–9].

There are various constructive schemes of freewheel mechanisms, each of which has some disadvantages [4, 10–15]. The current trend of increasing the load capacity and durability of freewheel mechanisms while maintaining their small mass and dimensional parameters make the development of new structural schemes of mechanisms necessary, for example, eccentric of the non-friction type.

In eccentric freewheel mechanisms of the non-friction type, a fine-module ratchet engagement is used to transfer the load. A cutter as well as a rack cutter can be used for cutting fine-module ratchet teeth.

From the analysis of the design scheme of machine engagement of the cutting tool and workpiece it is established that in order to improve the quality of the fine-module ratchet engagement, it is necessary to increase significantly the cutter radius [16]. In this case, it is most advisable to use a rack cutter ($r_U = \infty$).

The existing calculation methods of the rack cutter intended for cutting the teeth of the most common profiles (evolvent, circular, cycloid) [17–29] cannot be directly applied when cutting fine-module ratchet teeth of the proposed profile.

The calculation method of the original profile of the rack cutter for cutting fine-module ratchet teeth of a rational profile is developed based on the general method of shaping cylindrical surfaces by the indexing method [28, 29].
2. Shaping of the front edge of the rack cutter teeth

Figure 1 shows the process of cutting the front edge of the external teeth with a conditionally stopped rack. In drawing up the design model, the following was set: \( r \) and \( r_1 \) – the radii of the workpiece circumferences passing through the dedendum of the external and internal cut teeth (\( r = r_{f1} \) and \( r_1 = r_{f2} \)); \( \gamma_1 \) and \( \gamma_2 \) – the gradient angles of front and back edges of rack cutter teeth.

![Figure 1. Design model for the profile shaping of the front edge of the rack cutter teeth.](image1)

![Figure 2. Design model for the profile shaping of the back edge of the rack cutter teeth.](image2)

The workpiece circumference of radius \( r_1 \) is rolled along the rack cutter without slipping. Shaping of the edge points of the cut teeth, for example point \( M \), occurs at the moment when the cross point of the normal to the tooth profile and a circumference of radius \( r_1 \) (point \( N \)) becomes the pitch point of the machine engagement, i.e. take the position of point \( N' \). Then point \( M \) will coincide with the conjugated point of the cutting edge of rack cutter \( M_U \).

The task of determining the profile of the tooth front edge of the rack cutter comes down to determining the coordinates of points \( M_U \), corresponding to all points \( M \) of segment \( AB \), which we solve in the following sequence.

The coordinates of point \( O_1 \) in system \( xOy \) are defined by the system of expressions [30]:

\[
\begin{align*}
x_{01} &= x_{02} + x_0^* \cos(-\Theta') - y_0^* \sin(-\Theta') ; \\
y_{01} &= y_{02} + x_0^* \sin(-\Theta') + y_0^* \cos(-\Theta') .
\end{align*}
\]  

(1)

where \( x_{02} \) and \( y_{02} \) are the coordinates of point \( O_2 \) in system \( XOy \); \( x_0^* \) and \( y_0^* \) – the coordinates of point \( O_1 \) in system \( x'O_2 y'^* \); \( (-\Theta') \) – the angle between the positive direction of axes \( Ox \) and \( O_2 x'^* \).

Coordinates: \( x_{02} = OC = n_1 \Theta'/180^\circ ; y_{02} = r_1 ; x_0^* = 0 ; y_0^* = -r_1 \).

After the substitution, the system of expressions (1) takes the form:
\[
x_{01} = \pi \Theta' /180' - r_1 \sin \Theta' ; \\
y_{01} = r_1 (1 - \cos \Theta') .
\]  
(2)

Coordinates of the point of the front edge of rack cutter tooth \( M_{U} \) in system \( xOy \):

\[
\begin{align*}
x_{51} &= x_{01} + x' \cos \psi_1 - y' \sin \psi_1; \\
y_{51} &= y_{01} + x' \sin \psi_1 + y' \cos \psi_1 .
\end{align*}
\]  
(3)

where \( x' \) and \( y' \) – the coordinates of point \( M_{U} \) in system \( x'O_1y' \), besides \( y' = 0; \psi_i \) – the angle between the positive direction of axes \( Ox \) and \( O_{1}x' \), \( \psi_i = 90 - \gamma_i + \Theta' - \Theta'_{U} \).

After the substitution of the system of expressions (2) into expressions (3), let us obtain a system of expressions for determining the coordinates of the front edge of the rack cutter teeth:

\[
\begin{align*}
x_{51} &= \pi \Theta' /180' - r_1 \sin \Theta' + x' \sin(\gamma_i + \Theta') ; \\
y_{51} &= r_1 (1 - \cos \Theta') + x' \cos(\gamma_i + \Theta') .
\end{align*}
\]  
(4)

To determine the coordinates of the profile points of the front edge of the rack cutter teeth, it is necessary to know the value of the workpiece angle of rotation \( \Theta' \) at the moment of their shaping.

3. Determination of the workpiece angle of rotation

The value of the workpiece angle of rotation according to figure 1 can be determined by \( \cos \Theta' = y_N / r \) or \( \sin \Theta' = x_N / r \).

This requires to define the coordinates of point \( N \) in system \( xOy \). Point \( N \) lies at the intersection of line \( MN \) and circumference of radius \( r_1 \), which are described by equations \( y = -xtg \gamma_i + |OK| \) and \( x^2 + (y - r_1)^2 = r_1^2 \). Besides \( |OK| = x'/\cos \gamma_i \).

Consequently, the equation system is valid for point \( N \):

\[
\begin{align*}
y_N &= -x_N t g \gamma_i + x'/\cos \gamma_i; \\
x_N^2 + (y_N - r_1)^2 &= r_1^2 .
\end{align*}
\]  
(5)

After solving equation system (5), let us obtain:

\[
\begin{align*}
x_N &= -\sin \gamma_i \cos \gamma_i (r_1 - |OK|) \pm [\sin^2 \gamma_i \cos^2 \gamma_i (r_1 - |OK|) - |OK|^2 \cos^2 \gamma_i + 2r_1 |OK| \cos^2 \gamma_i]^{1/2}; \\
y_N &= -\sin \gamma_i [r_1^2 \sin^2 \gamma_1 - 2r_1 x' \cos \gamma_1 - (x')^2]^{1/2} + r_1 \sin^2 \gamma_1 - |OK| \sin^2 \gamma_1 + |OK| .
\end{align*}
\]  
(6)

Figure 1 shows that that coordinate \( x_N \) of considered point \( N \) is larger than the coordinate for the second point of intersection of a straight line with a circumference. Therefore in the system of expressions (6) for definition \( x_N \) before a root, let us accept a "plus" sign \( (r > |OK|) \).

Finally:

\[
\begin{align*}
x_N &= \cos \gamma_i [r_1^2 \sin^2 \gamma_1 + 2r_1 x' \cos \gamma_1 - (x')^2]^{1/2} - \sin \gamma_i (r_1 \cos \gamma_2 - x') ; \\
y_N &= r_1 \sin^2 \gamma_i + x' \cos \gamma_i - \sin \gamma_i [r_1^2 \sin^2 \gamma_1 + 2r_1 x' \cos \gamma_1 - (x')^2]^{1/2}.
\end{align*}
\]  
(7)

4. Shaping of the back edge of the rack cutter teeth

The problem of determining the theoretical profile of back edge of rack cutter teeth is identical to the previous one.
The coordinates of point \( O_1 \) in system \( xOy \) (figure 2) are defined by the following system of expressions [30]:

\[
\begin{align*}
    x_{o1} &= x_{o2} + x'_{o1} \cos(\tau - \Theta^*) - y''_{o1} \sin(\tau - \Theta^*) ; \\
    y_{o1} &= y_{o2} + x'_{o1} \sin(\tau - \Theta^*) + y''_{o1} \cos(\tau - \Theta^*) .
\end{align*}
\]  

(8)

where \( x_{o2} \) and \( y_{o2} \) – the coordinates of point \( O_2 \) in system \( xOy \); \( x'_{o1} \) and \( y''_{o1} \) – the coordinates of point \( O_1 \) in system \( x'O_2y^* \); \( (\tau - \Theta^*) \) – the angle between the positive direction of axes \( O\!x \) and \( O_2x^* \).

Coordinates: \( x_{o2} = \pi \Theta'/180^\circ \); \( y_{o2} = r_1 \); \( x'_{o1} = 0 \); \( y''_{o1} = -r_1 \).

After the substitution, the system of expressions (8) takes the form:

\[
\begin{align*}
    x_{o1} &= \pi \Theta'/180^\circ + r_1 \sin(\tau - \Theta^*) ; \\
    y_{o1} &= r_1[1 - \cos(\tau - \Theta^*)]^{1/2} .
\end{align*}
\]  

(9)

Coordinates of the point of back edge of rack cutter teeth \( M_U \) in system \( xOy \):

\[
\begin{align*}
    x_{s2} &= x_{o1} + x' \cos \psi \! - \! y' \sin \psi ; \\
    y_{s2} &= y_{o1} + x' \sin \psi + y' \cos \psi .
\end{align*}
\]  

(10)

where \( x, y \) – the coordinates of point \( M_U \) in system \( x'O_1y^* \); \( x' = 0 \); \( \psi \) – the angle between the positive direction of axes \( O\!x \) and \( O_1x^* \); \( \psi = \psi_2 - \psi_3 = \psi_2 + \tau - \Theta^* \); \( y' \) – set the position of point \( M \) to determine conjugate point \( M_U \).

After the substitution of the system of expressions (9), the expressions (10) take the form:

\[
\begin{align*}
    x_{s2} &= \pi \Theta'/180^\circ - r_1 \sin(\tau - \Theta^*) - y' \sin(\psi_2 + \tau - \Theta^*) ; \\
    y_{s2} &= r_1[1 - \cos(\Theta^*)] + y' \cos(\psi_2 + \tau - \Theta^*) .
\end{align*}
\]  

(11)

To determine the coordinates of the profile points of the back edge of rack cutter teeth, it is necessary to know the value of the workpiece angle of rotation \( \Theta^* \) at the moment of their shaping.

5. Determination of the workpiece angle of rotation

The value of the workpiece angle of rotation \( \Theta^* \) according to figure 3 can be determined by

\[
\begin{align*}
    \cos \Theta^* = (r_1 - y_N)/r_1 \quad \text{or} \quad \sin \Theta^* = x_N/r_1 .
\end{align*}
\]

To do this requires to define the coordinates of point \( N \) in system \( xO,y_1 \). Point \( N \) lies at the intersection of line \( KN \) and circumference of radius \( r_1 \), which are described by \( y = \operatorname{xtg} \gamma_2 + |OK| \) and \( x^2 + (y_1 - \gamma_1)^2 = r_1^2 \). Besides \( |OK| = y'/\operatorname{cos} \gamma_2 \).

Consequently, the equation system is valid for point \( N \):

\[
\begin{align*}
    y_{N1} &= x_{N1} \sin \gamma_2 + y'/\operatorname{cos} \gamma_2 ; \\
    x_{N1}^2 + (y_{N1} - \gamma_1)^2 &= r_1^2 .
\end{align*}
\]  

(12)

After solving the equation system (12), let us obtain:

\[
\begin{align*}
    x_{N1} &= -\sin \gamma_2(\gamma' - \gamma \cos \gamma_2) \pm \cos \gamma_2[r_1^2 \sin^2 \gamma_2 + 2r_1 \gamma' \cos \gamma_2 - (\gamma')^2]^{1/2} ; \\
    y_{N1} &= r_1 \sin^2 \gamma_2 + \gamma' \cos \gamma_2 \pm \sin \gamma_2[r_1^2 \sin^2 \gamma_2 + 2r_1 \gamma' \cos \gamma_2 - (\gamma')^2]^{1/2} .
\end{align*}
\]  

(13)
The coordinates of point \( N \) in system \( xOy \):

\[
\begin{align*}
  x_N &= x_{01} + x_{N1} \cos \tau - y_{N1} \sin \tau; \\
  y_N &= y_{01} + x_{N1} \sin \tau + y_{N1} \cos \tau. 
\end{align*}
\]  

(14)

After the substitution of the system of expressions (13) into the system of equations (14), one will finally obtain:

\[
\begin{align*}
  x_N &= \eta_1 \sin \tau - y' \sin(\gamma_2 + \tau) + \eta_2 \cos(\gamma_2 + \tau) \\
  &\pm \cos(\gamma_2 + \tau)[\eta_1^2 \sin^2 \gamma_2 + 2\eta_1 y' \cos \gamma_2 - (y')^2]^{1/2}; \\
  y_N &= \eta_1 (1 - \cos \tau) + y' \cos(\gamma_2 + \tau) + \eta_2 \sin \gamma_2 \sin(\gamma_2 + \tau) \\
  &\pm \sin(\gamma_2 + \tau)[\eta_2^2 \sin^2 \gamma_2 + 2\eta_2 y' \cos \gamma_2 - (y')^2]^{1/2}. 
\end{align*}
\]  

(15)

Signs before the root in the system of expressions (15) must be taken based on the results of calculations. The obtained dependences (4) and (11) make it possible to determine the coordinates of cutting edges of the rack cutter.

6. Conclusion

The use of the rack cutter when cutting fine-module ratchet teeth helps to improve quality of the ratchet engagement, as in this case the greatest possible length of the straight section of the front edge of the ratchet tooth is generated. The developed method for determining the original theoretical cutting profile of the rack cutter allows us to design a cutting tool for cutting ratchet teeth with module 0.3...1.0 mm having the necessary geometric parameters. It should be noted that using the rack cutter allows cutting only the external ratchet teeth.

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