Simulation of cavitation phenomena near the wall

Yu Wan
Jiangsu Frontier Electric Technology CO.,Ltd, Nanjing 211102, China
Email: wanyu_ft@163.com

Abstract. The influence of cavitation phenomena near the wall was analyzed. When the cavitations occurred, the generation and annihilation of bubbles had the synchronous characteristic, which led to the periodic fluctuation of the medium density in the water supply pipe. The cavitation led to the fluctuation of pressure and caused the pipeline vibration. In addition, during cavitations, bubble collapse produced shock waves, impact damage to blades of pump.

1. Introduction
Cavitation is a common phenomenon that occurs in the flow of liquid. Cavitation bubble is the result of partial pressure of liquid is lower than the saturated vapor pressure, which lend to bubbles generation, expansion and contraction at the end. Cavitation phenomenon is usually accompanied by noise and jet (cavitation), especially at unsteady liquid flow, the pressure of the liquid violently changes and more prone to cavitation phenomenon.

The main reasons of pipeline vibration are medium pulse in tube, turbulence caused by vibration, equipment vibration, the water hammer, and pipe fatigue fracture [1]. The cavitation phenomenon is another reason of the pipeline vibration, which may appear gas-liquid two phase flow, medium physical properties and pressure change, resulting in a pressure shock, vibration. The harm of the vibration of the pipeline is increasing, and the vibration of the pipeline has become one of the hidden dangers affecting the safe of industrial production [2].

2. The cavitation theory
Cavitations is a common phenomenon, we must study the whole dynamic cycle process of bubbles from expansion, contraction and rebound to collapse.

2.1. Rayleigh equation
For a single static spherical bubble, the equilibrium condition can be obtained by analyzing its force condition:

$$p_R = p_v + p_g - \frac{2\sigma}{R}$$  \hspace{1cm} (1)

Where, $p_R$ is the water pressure near the bubble, Pa; $p_v$ is the saturated vapor pressure inside the bubble, Pa; $p_g$ is the partial pressure of gas inside bubble, Pa; $R$-the radius of the bubble, m; $\sigma$-coefficient of water surface tension, N/m.

Rayleigh [3] first gave the equation of bubble motion in an incompressible ideal fluid (Rayleigh equation) in 1917:

$$R\dot{R} + \frac{3}{2}\dot{R}^2 = \frac{p_R - p_\infty}{\rho}$$  \hspace{1cm} (2)
Where, \( p_{\infty} \) is the pressure at infinity away from the center of the bubble, Pa; \( \dot{R} \)-speed of the bubble wall, m/s; \( \ddot{R} \)-acceleration of the bubble wall, m/s\(^2\). The equation gives the distribution of transient pressure in the fluid around the bubble and the change in the radius of the bubble itself, but Rayleigh didn’t take into account other factors affecting the fluid.

In 1949, Plesset \[4\] derived the Rayleigh-Plesset equation of bubble motion in an incompressible ideal fluid through the continuous equation:

\[
\dddot{R} + \frac{3}{2} \ddot{R}^2 = \frac{1}{\rho} \left[ p_g(R, t) - p_0(t) - p_0 \right] + \frac{R}{\rho c} \frac{d}{dt} \left[ p_g(R, t) - p_a(t) \right] - 4\mu \frac{\dot{R}}{R} - \frac{2\sigma}{\rho R} \tag{3}
\]

\( p_g(R, t) \)-the gas pressure inside the bubble, Pa; \( p_a(t) \)-the external pressure acting on the bubble wall, Pa; \( p_0 \) is the fluid static pressure, Pa; \( c \)-the speed of sound in the fluid, m/s; \( \mu \)-the fluid dynamic viscosity coefficient, Pa\(\cdot\)s; \( \rho \)-the fluid density, kg/m\(^3\); \( \sigma \)-the liquid surface tension coefficient, N/m.

2.2. Bubble expansion process

After the bubble is formed, it will undergo an expansion process. It is generally assumed that the gas in the bubble is an adiabatic expansion process of an ideal gas in the expansion process.

\[
p_g = p_{g0} \gamma \left( \frac{R_0}{R} \right)^\gamma
\]

Where \( \gamma \) is the adiabatic index of gas.

We note \( \eta = \frac{R}{R_0} \), \( \delta = \frac{p_{g0}}{p_0} \), \( \rho_0 = p_{g0} - p_{g0} \), \( \tau = \frac{t}{R_0} \), \( \psi = \frac{\sigma}{R_0 p_0} \).

Then formula (2) can be converted into a dimensionless Reynolds equation \[3\] :

\[
\eta \ddot{\eta} + \frac{3}{2} \dot{\eta}^2 = \frac{\sigma}{\rho_0} \eta^{-3\gamma} + 2\psi \eta^{-1} + 1 = 0
\]

For \( \ddot{\eta} = \frac{d\eta}{dt} \), \( \dot{\eta} = \frac{d^2\eta}{dt^2} \). We have

\[
\dddot{R} + \frac{3}{2} \ddot{R}^2 = \frac{1}{2\eta R} \frac{d}{dt} \left( R^3 \ddot{R} \right) \tag{6}
\]

In the initial conditions, when \( t = 0, R=R_0, \ddot{R} = \dot{R}_0 = 0 \). For a steam bubble, the pressure inside the bubble is constant \( p_v \), and when the liquid pressure is constant at infinity away from the bubble, set to \( p_0 \), then \( p_{\infty} = p_0 \). According to equations (2) and (6), we can get:

\[
\frac{1}{2\eta R} \frac{d}{dt} \left( R^3 \ddot{R} \right) = \frac{2p_v p_0}{\rho} \tag{7}
\]

Thus, it can be concluded that:

\[
\int_{R_0}^{R} d\left( R^3 \ddot{R} \right) = \frac{2p_v p_0}{\rho} \int_{R_0}^{R} R^2 dR \tag{8}
\]

According to condition \( R=R_0, \ddot{R} = \dot{R}_0 = 0 \), bubble expansion velocity and acceleration can be obtained:

\[
\dddot{R} = \frac{2}{3} \frac{p_v p_0}{\rho} \left( 1 - \frac{R_0^3}{R^3} \right) \tag{9}
\]

\[
\dot{R} = \frac{p_v p_0 R_0^3}{\rho R^4} \tag{10}
\]

It can be seen from the above equation that, at the early stage of bubble expansion, the bubble expansion speed rises sharply. When the multiple of \( \dot{R} \) is not larger than \( \dot{R}_0 \), the bubble expansion speed is close to the maximum.

2.3. Contraction process of bubble

The bubble shrinkage process can also be regarded as the adiabatic compression process of an ideal gas. Similarly, if the pressure of the fluid at infinite distance from the bubble center is constant \( p_{\infty} \), and the pressure inside the bubble is \( p_v \), and the \( p_v \) value is much less than \( p_0 \), it can be ignored, and then:
\[ R \ddot{R} + \frac{3}{2} \dot{R} = -\frac{P_0}{\rho} \]  

(11)

According to the initial conditions, the integral of the above formula:

\[ R^3 \dot{R}^2 = \frac{2}{3} \frac{P_0}{\rho} (R_0^3 R^3) \]  

(12)

Thus, the velocity and acceleration of bubble shrinkage can be respectively obtained as follows:

\[ \dot{R} = \sqrt{\frac{2}{3} \frac{P_0}{\rho} \left(\frac{R_0^3}{R^3}\right)} - 1 \]  

(13)

\[ \ddot{R} = -\frac{P_0 R_0^3}{\rho R^4} \]  

(14)

According to formula (13) and formula (14), when the bubble collapses, the speed and acceleration of the bubble are very high. When the bubble collapses in the water pump or near the wall surface, high-speed shock waves generate and harm the fixed wall surface, which is the main reason for cavitations [5-8].

3. Simulation results of bubble growth and collapse

3.1. Assumptions of numerical simulation

(1) the mass exchange between the inside of the bubble and the medium (water) is ignored;
(2) ignore the influence of gravity;
(3) the pressure outside the bubble wall is kept constant;
(4) the gas in the bubble is an ideal gas;

3.2. Simulation method

First of all, build the model and generate high quality structural grid, then import Fluent software, and use VOF model and unsteady state method to solve N-S equation. The dimensionless number \( \zeta \) \((\zeta = X/R_{max}, \text{where } X \text{ is the distance between the center point of the initial cavitation position and the wall, } R_{max} \text{ is the maximum radius of the cavitation bubble})\), which was changed to simulate a series of morphological changes of a single cavitation bubble. The results were visualized by importing the Fluent calculation results into CFD-POST, and a representative moment was found for comparative analysis within the collapse cycle of the cavity expansion and contraction.

Specific simulation Settings: because of the cavity expansion and contraction and the breaking process is a very quick process, so choose transient and pressure-based option in the Fluent software, select 2D axisymmetric, which can reduce the computation and computing time. Select VOF model and tick implicit to improve the convergence of solutions. Turn on the energy formula. The Liquid phase is set as water-liquid, the gas phase as water-vapor. Open the operating conditions option in cell zone and set the calculation field size to 40×40mm. Boundary conditions: select axial symmetry, interior type int-fluid, pressure-outlet and pressure on the boundary is 101325Pa, temperature is 298K. Select PISO which is the velocity and pressure coupling solution method, and select PRESTO method to solve the pressure. Define the monitor and set the absolute criteria value to 1e-06 in the residual option.

3.3. Result discussion

Figure 1 and Figure 2 are the figures of numerical simulation and visualization experiments [9], respectively. It is similar by comparing the numerical simulation results with the experimental values, in the 130 \( \mu \)s bubble is largest, near the 200 \( \mu \)s bubble begin to change flat, then from bubble sag, sag of top-down consistent with jet direction, dent and conform to the actual phenomenon. The sag eventually tears the bubble, which ruptures in the middle and creates a jet flow down the wall.
Figure 1. Diagram of steam bubble growth cycle when $\zeta=1.2$.

Figure 2. Visualization experiment results [9].

Figure 3. Diagram of steam bubble growth cycle when $\zeta=1.5$.

Figure 4. Diagram of steam bubble growth cycle when $\zeta=1.8$. 
Figure 3, Figure 4 are $\zeta=1.5$ and $\zeta=1.8$ of the simulation results, the comparison of Figure 1, Figure 3, Figure 4 shows that bubble expansion shrinkage speed almost unanimously, it will not be affected by the number $\zeta$ and bubble growth rate tends to be consistent, so once produce bubbles, the bubbles grow and breaking are almost synchronous, this is bound to cause water cyclical changes in physical properties of medium in the pipeline, resulting in a cyclical fluctuation pressure and vibration.

4. Conclusions
In this paper, the cavitation phenomenon was studied through the numerical simulation of a single bubble growing to collapse near the wall. It is found that the process of bubble growth and collapse is similar, and the change of flow rate will intensify cavitation phenomenon. The cavitations process is consistent with the change of the working medium parameters. Therefore, special attention should be paid to the cavitation and vibration of the pipeline at variable operating conditions.

References
[1] Wenjun Lv 2017 Reasons and prevention of vibration of power plant feed pump [J] Shandong Industrial Technology 18 234-235 (in Chinese)
[2] Xin Hu, Ming Liu 2016 Diagnosis and treatment of high pressure water supply pipeline vibration in thermal power plant [J] Huadian Technology 38(10) 37-38 (in Chinese with English abstract)
[3] Rayleigh J W 1917 On the pressure developed in a liquid during the collapse of a spherical cavity [J] Philosophical Magazine 34 94-98
[4] Plesset M.S. 1949 The Dynamics of Cavitation Bubbles [J] Journal of Applied Mechanics 16(03) 227-282
[5] Jitang Huang 1991 Principle and application of cavitation and cavitation [M] Beijing: Tsinghua University Press (in Chinese)
[6] Jing Luo, Jian Li, Guangneng Dong 2007 Computational simulation of bubble outburst in static flow field and high-speed water flow at wall surface [J] Journal of Tribology 27(6) 562-566 (in Chinese with English abstract)
[7] T. Hiroyuki, T. Matsuno, K. Shuto 2008 Numerical investigations of shock-bubble interactions in mercury [J]. Fluid Dynamics Research 40 510-520
[8] Q.Z, K. Bremhorst, H. Alehossein, et al. 2007 Simulation of cavitation bubbles in a convergent-divergent nozzle water jet [J] Journal of Fluid Mechanics 573 1-25
[9] Claus-Dieter Ohl, Manish Arora, Rory Dijjkink, Valihav Janvew and Detlef Loshse 2006 Surface cleaning from laser-induced cavitation bubbles Applied Physics Letters 89(7) 074102-074102-3