Influence of energetic ions on neoclassical tearing modes

Huishan Cai

Department of Modern Physics, CAS Key Laboratory of Geospace Environment, University of Science and Technology of China, Hefei 230026, People’s Republic of China

E-mail: hscai@mail.ustc.edu.cn

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Abstract
In addition to their effect on the linear stability of tearing modes, energetic particles can influence the nonlinear evolution of a magnetic island through an uncompensated cross field current due to the effect of charge separation when the orbit width of an energetic particle is much larger than the island width. The corresponding return parallel current may compensate the loss of bootstrap current in the magnetic island. This nonlinear effect depends on the island’s propagation frequency (the rotation frequency of the island relative to the plasma), the density gradient of energetic ions and magnetic shear. If the island’s propagation frequency is positive, the effect of the uncompensated current plays a stable role on neoclassical tearing modes. When the magnetic shear is sufficiently small, this effect becomes significant and can partially cancel or even overcome the destabilizing effect of the perturbed bootstrap current. In ITER this provides a possibility of using energetic ions to suppress the neoclassical tearing mode for the steady state and hybrid scenarios with weak magnetic shear.

Keywords: neoclassical tearing modes, energetic ions, tokamak

(Some figures may appear in colour only in the online journal)
beam. Hegna et al [17] showed that energetic ions can stabilize nonlinear tearing modes as a result of the perturbed parallel current of beam ions in the island region. This parallel current of beam ions is due to the deformation of the particle distribution function by the magnetic island. However, this stabilizing effect is expected to be small when the orbit width of energetic ions is much larger than the island width because the response of energetic ions to perturbation in the island region is weakened by orbital averaging. In this case, energetic ions can affect the linear stability of tearing modes through their interaction with tearing modes in the outer region [18–21, 23]. Furthermore, Mirnov et al [24] recently pointed out that the effects of energetic ions in the inner region of linear tearing modes cannot be neglected even when their orbit width is large for a reversed field pinch plasma. It was shown that an uncompensated cross field current is produced due to the effect of charge separation, and the effect of this uncompensated current is stabilizing for linear tearing modes. The uncompensated current comes from a net $\mathbf{E} \times \mathbf{B}$ current because the $\mathbf{E} \times \mathbf{B}$ current of beam ions is significantly reduced by the orbital averaging effect in the limit of large orbit width. In this work, we investigate the effect of the uncompensated current on the nonlinear evolution of NTMs. We will show that the effect is significant when magnetic shear is weak, and is stabilizing when the mode frequency is positive in the plasma frame.

In section 2, the evolution of NTMs including energetic ions effects is derived, and the effects of energetic ions on NTMs are analyzed and discussed. Finally, conclusions are given in section 3.

2. Influence of energetic ions on NTMs

The detailed calculation will be shown based on the above physics interpretation. The magnetic field is written as $\mathbf{B} = I \nabla \zeta + \nabla \zeta \times \nabla (\psi + \delta \psi)$, where the toroidal geometry is assumed to be axisymmetric, $\zeta$ is the toroidal angle, $\psi$ is the equilibrium poloidal magnetic flux, $\delta \psi = \partial \psi / \partial \zeta \cos \xi$ is the perturbed poloidal magnetic flux, where $\zeta = m(\theta - \zeta \omega_0) - \omega t$, and the familiar approximation of constant $\delta \psi$ is made, $m(\theta - \zeta \omega_0)$ is the helical angle in the direction perpendicular to the equilibrium magnetic field, $q = m/n$ is the value of the safety factor at the resonance surface, $\omega = \omega - n \omega_0$ is the rotation frequency of the island relative to the plasma and $\omega_0$ is the rotation frequency of the plasma. Here, $\omega$ is assumed to be constant with respect to $\zeta$, for simplicity. Then, based on the generalized Rutherford theory [25], by matching the solutions of outer and inner regions, the evolution of NTMs can be obtained:

$$\frac{4 \sqrt{2}}{s} q_i 4 \pi R_0 \frac{1}{c B_0} \int_{-1}^{\infty} d \Omega \left( J_4 \cos \xi \right) = \Delta, \quad (1)$$

where $w = 2 \sqrt{q_i \delta \psi / (\delta \psi)}$ is the island width (the prime denotes the derivative with respect to $r$), $s$ is the magnetic shear, $R_0$ is the major radius, $\Omega = 2(\psi - \psi_i)^2 h \psi - \cos \xi$ and $w^2 = (\psi_i^2 w)^2$.

The operator $\langle \ldots \rangle = (1/2) \int [d \xi (\ldots)] / \Omega + \cos \xi$. $J_4$ is the parallel plasma current, which needs to be derived next.

Based on the quasi-neutrality equation, the parallel current satisfies

$$\mathbf{B} \cdot \nabla \left( \frac{J_{||}}{B} \right) + \nabla \times \left( \frac{B \times \nabla \mathbf{p}_h}{B^2} \right) + \nabla \cdot \mathbf{J}_s + \nabla \cdot \mathbf{J}_e + \nabla \cdot \mathbf{J}_p = 0, \quad (2)$$

where $\mathbf{J}_s$ and $\mathbf{J}_e$ are the polarization and neoclassical polarization currents [26], respectively. $\nabla \cdot \mathbf{p}_h$ is the pressure tensor of energetic ions. $\mathbf{J}_s \sim -n_i \mathbf{v}_E$ is the uncompensated cross field current, where $n_i$ is the density of energetic particles and $\mathbf{v}_E = c \mathbf{E} \times \mathbf{B}/B^2$ is the $\mathbf{E} \times \mathbf{B}$ drift. Here, because the island width is much smaller than the orbit width of energetic ions, the responses of energetic ions to the perturbations in the island are neglected. Then, the second term in equation (2) can be neglected, which was considered by Hegna et al [17]. The uncompensated cross field current is presented within the scope of fluid formalism, without solving the drift kinetic equation of energetic ions. Next, we will focus on the return parallel current $J_{||,u}$ induced by the uncompensated cross field current $J_u$, as

$$\mathbf{B} \cdot \nabla \left( \frac{J_{||,u}}{B} \right) = -\frac{c^2}{4 \pi v_d} \frac{m}{n_0} \frac{\mathbf{d} \mathbf{n}_{\delta \phi}}{\mathbf{d} \xi} \frac{\partial \delta \phi}{\partial \xi} \mathbf{w}_\psi \mathbf{w}_\xi, \quad (3)$$

where $v_d = B_l / \sqrt{4 \pi p}$ is the Alfvén velocity and $d = c \omega_d$ is the ion inertial length. $\delta \phi$ is the perturbed electrostatic potential, which can be determined by the quasi-neutrality condition.

It is convenient to transform the coordinates $(\psi, \theta, \xi)$ to the island coordinates $(\Omega, \theta, \xi)$. Then, based on the quasi-neutrality condition, $\delta \phi$ can be obtained from the ion continuity equation and the electron momentum balance equation, as [26]

$$\delta \phi = \omega' (\psi - \psi_i - h(\Omega)), \quad (4)$$

where the approximation is made that the ion Larmor radius is much smaller than the island width. $h(\Omega)$ is determined by the effect of the island on the radial transport [27], as

$$\frac{d h}{d \Omega} = \frac{\sqrt{2}}{4} \frac{w}{\Omega + 1/2} \frac{E(1/(\Omega + 1))}{H(\Omega - 1)}, \quad (5)$$

where $\sigma_v$ denotes the sign of $\psi - \psi_i$. $H(\Omega - 1)$ is the Heaviside function and $E(1/(\Omega + 1))$ is the elliptic function.

Thus, substituting equation (4) into equation (3), one can obtain

$$\frac{n_s q_i \mathbf{e} \cdot \mathbf{B}}{R_0 \mathbf{e} \cdot \mathbf{B}_0} \frac{\partial J_{||,u}}{\partial \xi} = \frac{c^2}{4 \pi v_d} \frac{m}{n_0} \frac{\mathbf{d} \mathbf{n}_{\delta \phi}}{\mathbf{d} \xi} \frac{\partial \delta \phi}{\partial \xi} \mathbf{w}_\psi \mathbf{w}_\xi \frac{d h}{d \Omega} \frac{\partial \psi}{\partial \xi}, \quad (6)$$

Then, by integrating equation (6), one can obtain

$$J_{||,u} = \frac{c^2}{4 \pi v_d} \frac{m}{n_0} \frac{\mathbf{d} \mathbf{n}_{\delta \phi}}{\mathbf{d} \xi} \frac{\partial \delta \phi}{\partial \xi} \mathbf{w}_\psi \mathbf{w}_\xi \frac{\sqrt{2}}{w} \left( x - \frac{\langle x \rangle}{\langle 1 \rangle} \right), \quad (7)$$

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The ratio between the contributions of uncompensated cross field current due to energetic ions and bootstrap current, $|\Delta_n^u|/|\Delta_n^b|$ against the fraction of energetic ion density $n_i/n_i$ with different values of magnetic shear $s = 0.05, 0.1, 0.5.$

where $\Omega = \sigma_s(\pi/2)(\Omega + 1)^{-1/2}E^{-1}H(\Omega - 1)$ and $x = r - r_c.$ Now, the parallel current can be written as

$$J_b = J_{b,x} + J_{b,u} + \langle J_b \rangle / (1),$$

(8)

where $J_{b,x}$ results from the neoclassical polarization current, while the contribution from the polarization current is neglected in comparison with the neoclassical polarization current. It can be seen that $(J_{b,x}) = (J_{b,u}) = 0.$ $(J_b)$ is determined by Ohm’s equation, as

$$\langle J_b \rangle = \frac{1}{\eta} \frac{d\delta \psi}{dt}(\cos \xi) + \langle \delta J_b \rangle,$$

(9)

where $\delta J_b$ is the perturbed bootstrap current. Then, substituting equations (7)–(9) into equation (1), one can obtain

$$\frac{8\pi}{\eta^2} \frac{h}{d} \frac{d\psi}{dt} = \Delta' + \Delta'' + \Delta'_x + \Delta'_n,$$

(10)

where $\Delta'_b$ [26], $\Delta'_i$ [27] and $\Delta'_u$ result from the contributions of bootstrap current, neoclassical polarization current and uncompensated cross field current, respectively, as

$$\Delta'_b = G_i \sqrt{r_c} \frac{\beta_{ih}}{s L_i} \frac{r_s}{w},$$

(11)

$$\Delta'_i = -1.64 \epsilon_s^3 G_2 \frac{r_s^2}{s^2 L_i} \frac{\beta_{ih}}{w^2} \frac{\omega' (\omega' - \omega_i)}{\omega_i^2},$$

(12)

$$\Delta'_u = -G_3 \frac{r_s^2}{s^2 L_i} \frac{\beta_{ih}}{w \omega_i} \frac{L_i}{L_i} \frac{n_i}{n_i}.$$  

(13)

Here the numerical coefficients are $L_i \approx 0.83,$ $G_1 \approx 2.31$ [28], $G_2 \approx 1.42,$ $G_3 \approx 1.58,$ $\omega_i = (cm/n_neq) d\psi/d\psi'$ is the ion diamagnetic current, $n_i$ is the density of thermal ions, $\beta_{ih} = 8 \pi p_i/B^2_0,$ and $L_i$ and $L_b$ are the scale lengths of ion density and energetic ion density, respectively. $\omega'$ is determined by the torque balance, which is still an open debate. Here, it is necessary to point out that the contributions of energetic ions are reflected in both $\Delta'_i$ and the stability criterion $\Delta$ (one can refer to [19] for details). From equation (13), it can be seen that the effect of uncompensated cross field current from energetic ions depends on the magnetic shear, the propagation frequency of the island and the density gradients of ions and energetic ions at the resonance surface. It plays a stable role for $\omega' > 0$ if the density gradients of ions and energetic ions at the resonance surface have the same sign. This is different from the effect of neoclassical polarization current, which is stabilizing for $\omega' < 0$ or $\omega' > \omega_i.$ Although the density of energetic ions is much smaller than the ion density, $\Delta'_i$ may become significant for weak magnetic shear, as in one of the scenarios of high $\beta$ and steady state, and hybrid operations in ITER [29] and some large tokamaks [7], where a lot of steady operational discharges have been realized with a configuration of zero or weak magnetic shear. For weak magnetic shear, the effect of uncompensated cross field current from energetic ions can be comparable to the contribution of the perturbed bootstrap current, and would enhance the onset threshold of NTMs or suppress them. For a typical tokamak like JT-60U, the main parameters $R_0 \approx 3.2$ m and $a \approx 0.8$ m, and the energetic ion density can be up to 0.02 $n_i$ during neutral beam injection [30]. Given $\omega' \sim \omega_i,$ $e_i \approx 1/8,$ $L_i = 0.16$ m, $L_b = 0.7$ m, the ratios $|\Delta'/\Delta_b|$ and $|\Delta'/\Delta_i|$ are plotted against $n_i/n_i$ in figures 1 and 2. From figure 1 it can be seen that $|\Delta'/|$$\Delta_b|$ increases as $n_i/n_i$ increases or $s$ decreases. For weak magnetic shear and a large fraction of energetic ion density, $|\Delta'| \sim |\Delta_b'|,$ or even $|\Delta'_u| > |\Delta'_b|.$ That is, the contribution of uncompensated cross field current from energetic ions becomes significant, and its stabilizing effect can partially cancel or overcome the destabilizing effect of the perturbed bootstrap current. Then, NTMs will be suppressed. From figure 2, it can be seen that $\Delta'_i \sim \Delta'_b$ with $w \sim p_i$ and $n_i/n_i \sim 1\%,$ which are typical values for a tokamak. $\Delta'_i$ and $\Delta'_b$ have the same dependence on magnetic

Figure 1. The ratio between the contributions of uncompensated cross field current due to energetic ions and bootstrap current, $|\Delta_n^u|/|\Delta_n^b|$ against the fraction of energetic ion density $n_i/n_i$ with $w/p_i = 0.5,$ $w/p_i = 1,$ $w/p_i = 2.$

Figure 2. The ratio between the contributions of uncompensated cross field current due to energetic ions and bootstrap current, $|\Delta_n^u|/|\Delta_n^b|$ against the fraction of energetic ion density $n_i/n_i$ with $w/p_i = 0.5,$ $w/p_i = 1,$ $w/p_i = 2.$
shear, but different dependences on the island’s propagation frequency. For \(0 < \omega' < \omega_\nu\), the effects of uncompensated current on NTMs and neoclassical polarization current are opposite. They would cancel each other. Then, the critical onset threshold of NTMs due to the neoclassical polarization current is enhanced. Actually, in some experiments, as in JT-60U [31], no NTM was observed in the discharge with NBI by optimization of the \(q\) profile with weak magnetic shear and small pressure gradient at the resonance surface, where the effects of energetic ions may be important. In DIII-D [14], it was also found that the onset threshold of NTMs increases with a co-injected beam.

As stated above, the effects of energetic ions on NTMs are reflected in both \(\Delta'_u\) and \(\Delta'.\) For a typical tokamak such as JT-60U, given the gyro-radius of energetic ions \(\rho_{\text{in}} \approx 0.018\) m, island width \(w \approx 0.01\) m, the temperature of thermal ions \(T_i \approx 1\) keV, 100 keV circulating energetic ions (CEI) and other main parameters as given in figures 1 and 2, and considering the mode \(m = 2, n = 1\), the comparison of the effects of energetic ions on NTMs is shown in figure 3 based on equation (13) and equations (19) and (20) in [19], where \(\Delta = 0\) without energetic ions is set. It can be seen that \(\Delta'_u\) and \(\Delta'\) both become more important as the magnetic shear decreases. For the given parameters, the effect of energetic ions on \(\Delta\) is more important than that on \(\Delta'_u\) for large magnetic shear, and they become comparable for weak magnetic shear. Furthermore, it can also be seen that \(\Delta\) depends on the direction of CEI as pointed out in [19], while \(\Delta'_u\) depends on the propagation frequency of the island. Hence, it is beneficial to enhance the onset threshold of NTMs with weak magnetic shear by co-CEI.

3. Conclusion

In conclusion, we have studied the effect of energetic ions on NTMs. It is found that energetic particles can significantly affect the nonlinear evolution of a magnetic island through an uncompensated cross field current from energetic particles. This uncompensated current is induced by the effect of charge separation when the orbit width of an energetic particle is much larger than the island width. The corresponding return parallel current may compensate the loss of bootstrap current in the magnetic island. This nonlinear effect depends on the island’s propagation frequency, the density gradient of energetic ions and magnetic shear. If the island’s propagation frequency is positive, the effect of the uncompensated current plays a stabilizing role on neoclassical tearing modes. When the magnetic shear is sufficiently small, this effect becomes significant and can partially cancel or even overcome the destabilizing effect of the perturbed bootstrap current. In ITER this provides a possibility of using energetic ions to suppress the neoclassical tearing mode for the steady state and hybrid scenarios with weak magnetic shear. Our analysis suggests that it is possible to enhance the onset threshold of NTMs or suppress them using energetic ions by optimization of the \(q\) profile, where the weak magnetic shear is beneficial in enhancing this suppressing effect. Furthermore co-NBI is advantageous since a co-passing energetic ion can reduce \(\Delta\) [19]. Here we have only considered the effect of energetic ions in the limit of large orbit width. When the orbit width of the energetic ion is comparable to or smaller than the island width, the effect of the uncompensated current will be smaller and the interaction of energetic ions with the inner region of tearing modes becomes important [17]. The different physics of the effects of energetic ions on NTMs also needs a quantitative comparison. The island’s propagation frequency is also not derived here, and needs to be determined from the momentum balance equation. These issues will be considered in the future.

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