Development of combined scaling models for liquid and gas densities at the saturation line: Structures and numerical data for SF$_6$

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Abstract. Some thermodynamic functions are considered in the work. They have a scaling form and connected with thermodynamic properties at the saturation line (the fluid density, the gas density, the order parameter, the mean diameter, etc). Scaling models have the area of applicability near the critical temperature. A Wegner model is valid at relative temperatures, $0 < \tau < 0.02$. Its structure includes scaling components with indices $\alpha$ and $\beta$, a linear component. A new Fisher model is suggested in 2003. Its structure includes an additional component with an index, $2\beta$. Another model is developed in the work. The model is referred to as a combined scaling model. At first, its structure contains scaling components including a term with $2\beta$ index; secondly, there are regular components, which allowed us to increase the area of applicability up to $\tau \approx 0.3$. We have made numerical estimates, which are connected with parameters of the combined scaling model and based on experimental data related to the fluid and gas densities of SF$_6$. This model is compared with an Anisimov model and with experimental data. Numerical data are got in some extrapolation region near the critical temperature. These results are analyzed.

1. Introduction

These are several sources, which describe the liquid density ($\rho_l$) and the gas density ($\rho_g$) related to the saturation line near the critical point of SF$_6$. In the analysis of the sources, we have considered some forms:

- a group of equations of state (EOS) among them EOS [1], which is recommended as an international standard EOS of SF$_6$;
- scaling equations ($\rho_l(\tau), \rho_g(\tau)$) among them Anisimov models [2], here $\tau = (T_c - T)/T_c$ is the relative temperature, $T_c$ is the critical temperature.

EOS [1] has an analytical form. At first, it means that this EOS can satisfactorily reproduce $\rho_g, \rho_l, T$ data related to a regular part of the thermodynamic surface. Secondly, this EOS does
not follow the scale theory of critical phenomena (ST) and has no opportunity to describe a singular behavior of row functions, for example, $d\rho_1/d\tau \to \infty$ when $\tau \to 0$, $d\rho_1/d\tau \to \infty$ when $\tau \to 0$, here $f_d = (\rho_l + \rho_g)/(2\rho_c)^{-1} - 1$ is the mean diameter, $\rho_c$ is the critical density. Thirdly, this EOS allowed us to determine $\rho_g, \rho_l, T$ data in the interval $10^{-5} < \tau < 10^{-1}$ or in the critical region related to the phase transition from the liquid to the vapor. Our analysis has shown: the accuracy of these numerical data is lower than the accuracy of the data, which are calculated in the regular region. This is an interesting problem: to improve the accuracy of EOS in a wide range of temperatures and pressures, including the critical region. The problem of EOS designing has been considered in some works including [1,3–10]. Authors of [6–10] have elaborated EOSs, which are related to some metals. In the case, the critical points are located at high temperatures and high pressures. Authors of [4,5] have considered a singular behavior, which is related to functions $(\rho_l(\tau), \rho_g(\tau), f_d(\tau), . . .)$ and connected with EOSs of some metals (Al, Cu, U, . . .) in the critical region.

It is the most urgent problem in ST, namely, to determine a structure of the model, which is connected with a property $(\rho_l, \rho_g, f_d, f_s)$, the order parameter, $f_s = (\rho_l - \rho_g)/(2\rho_c)^{-1}$ and related to the critical region; a traditional Wegner model [11] has a form

$$f_s = B_0\tau^{\beta_1} + B_1\tau^{\beta_1 + \Delta}, \quad f_d = B_d\tau^{1-\alpha_1} + B_d\tau^{1-\alpha_1 + \Delta} + B_d\tau,$$

(1) (2)

where $(\alpha_1 = 0.109, \beta_1 = 0.325)$ are the critical indices, $\Delta = 0.5$ is the exponent of the first non asymptotic member, $C = (B_{si}, B_{di})$ are the coefficients determined by a statistical treatment of experimental data.

A structure of equations (1) and (2) includes scaling components with $(\alpha_1, \beta_1)$ indices and a linear component. The structure correlates with ST. Due to ST, $(\alpha_1, \beta_1)$ indices have an error estimated as $\approx 0.1\%$ [2]. Equations (1) and (2) are valid in the interval $0 < \tau < 0.02$. Equation (2) comprises a leading singular member, $B_d\tau^{1-\alpha_1}$; it means that the derivative, $d\rho/d\tau$, is singular $(d\rho/d\tau \approx -B_0\tau^{-\alpha_1} \to -\infty$ when $\tau \to 0$). The structure of a Wegner model [12] allows us to describe properties $(\rho_l, \rho_g, f_d, f_s)$ on a basis of six characteristics, $D = (T_c, \rho_c, \alpha_1, \beta_1, B_0, B_d)$, in an asymptotic region $(\tau_{low} \to \tau_{as})$, here $\tau_{low}$ is a low border of experimental data, $\tau_{as} \approx 1 \times 10^{-3}$.

A new Fisher model [13] is appeared in 2003. Its structure includes an additional component with $2\beta_1$ index in a comparison with (1) and (2). In the case, the derivative, $d\rho/d\tau$, has got another form $(d\rho/d\tau \approx -B_0\tau^{2\beta_1} - 1$ when $\tau \to 0$).

Our analysis shows that there is a set of values, $\alpha = (\alpha_1, \alpha_2, . . .)$ in the literature. It covers a wide interval. There are experimental $(\alpha_{exp}, \beta_{exp})$ values obtained by statistical treatment of measured $\rho_g, \rho_l, T$ data for a wide range of substances [1,3]. Values $\alpha_{exp}$ are spanning the whole interval $(\alpha_{exp} = 0.01–0.14)$. There are no works, which use a statistical treatment of experimental $\rho_g, \rho_l, T$ data and provide $\alpha_{exp} \approx 2\beta_1 = 0.65$. The rate $(2\beta_1 = 0.65)$ is significantly smaller than value $(1 - \alpha_1 = 0.89)$.

We try to elaborate another model in the work. This variant is referred to as a combined scaling model and follows some border conditions:

- the model is connected with properties $(\rho_l, \rho_g, f_d, f_s)$ and includes critical characteristics $D = (T_c, \rho_c, \alpha_1, \beta_1, . . .)$ and coefficients $C = (B_{si}, B_{di})$;
- its structure contains scaling components including a term with $2\beta$ index;
- there are regular components, which allowed us to increase the area of the model applicability up to $\tau \approx 0.3$;
- characteristics $D$ are calculated together with $C$ coefficients; a statistical treatment of experimental $\rho_g, \rho_l, T$ data is used in the case.
2. An analyses of some scaling models
A Landau model [2,14] is based on a phenomenological approach where fluctuation effects are not considered. This approach uses a number of dependencies; at first, the dependence of the free energy, $F$, on the density and the temperature [2,14]. The following form of $F$ is considered

$$
\tilde{F}(T, \rho, \Delta \rho) = F_0(T, \rho) + \frac{1}{2} A(\Delta \rho)^2 + \frac{1}{3} B(\Delta \rho)^3 + \frac{1}{4} C(\Delta \rho)^4, \ldots,
$$

(3)

where $\tilde{F} = F/(V P_c)$ is the relative free energy, $\Delta \rho = \rho/\rho_c - 1$ is the relative density, $F_0(T, \rho)$ is the regular function; $(A, B, C)$ are the functions of the temperature.

Secondly, the formula of a relative pressure, $p$, is determined as a derivative, $d\tilde{F}/d(\Delta \rho)$, and written in the form

$$
p = a_0 \tau_L + A(\Delta \rho) + B(\Delta \rho)^2 + C(\Delta \rho)^3,
$$

(4)

where $p = (P/P_c) - 1$, $\tau_L = (T - T_c)/T_c$ is the relative temperature [14].

It is accepted [14] that functions $A$, $B$ and $C$ can be written as

$$
A = a \tau_L, \quad B = b \tau_L, \quad C > 0.
$$

(5)

Relationship (5) corresponds to the known conditions in the critical point

$$
(\partial P/\partial \rho)_T \approx A = 0, \quad (\partial^2 P/\partial \rho^2)_T \approx B = 0, \quad (\partial^3 P/\partial \rho^3)_T \approx C > 0.
$$

(6)

It is shown in [14] that condition (4)–(6) allowed us to obtain a quadratic equation of the relative densities at $\tau_L = -\tau < 0$ in the form

$$
\Delta \rho_{g,l} = \pm (a/(C)^{1/2}) \tau^{1-\alpha_2} = \pm f_s + f_d,
$$

(7)

$$
f_s = B_{d0} \tau^{\beta_2}, \quad f_d = B_{d0} \tau^{1-\alpha_2},
$$

(8)

where $\Delta \rho_g$ and the sign “−” refer to the gas branch, $\Delta \rho_l$ and the sign “+” refer to the liquid branch, $\alpha_2 = 0$ and $\beta_2 = 0.5$ are theoretical values obtained by Landau [14].

Equations (8) contain other values of the indices in a comparison with equations (1) and (2). It represents a Landau hypothesis (LH) that includes the following conditions:

- models (8) must contain two scaling component, six $D$ characteristics and universal ($\beta_2$, $\alpha_2$) indices;
- the mean diameter $f_d$ contains a scaling term with exponent $1 > \alpha > 0$ and does not include a linear component;
- models (8) have positive coefficients ($B_{d0} > 0, B_{d0} > 0$).

From the treatment of modern experimental data on the liquid and gas densities, it follows that $f_d$ can be presented in the interval ($\tau_{ow}$ to $\tau_{as}$) for many substances [2,15] as

$$
f_d = B_{d0} \tau^{1-\alpha}, \quad \alpha \approx 0.1, \quad B_{d0} > 0.
$$

(9)

We can conclude from (9) that $f_d$ (7) should contain a function, $C = f(\tau)$, which has a scaling ratio [15]

$$
C = c \tau^v, \quad v \approx 0.1, \quad C > 0.
$$

(10)

In the case, $\Delta \rho_{g,l}$ models can be presented in the form [16]

$$
\Delta \rho_{g,l} = \pm \left( (a/c)^{1/2} (\tau^{1-v})/2 \right) + (b/(2c)) \tau^{1-v} = \pm B_{d0} \tau^{\alpha_3} + B_{d0} \tau^{2\beta_3}.
$$

(11)

The order parameter $f_s$ and the mean diameter $f_d$ can be written as

$$
f_s = B_{d0} \tau^{\beta_3}, \quad f_d = B_{d0} \tau^{2\beta_3},
$$

(12)

where $2\beta_3 = 1 - v < 1$.

Model (12) corresponds to a modified Landau hypothesis (MLH), which includes the following conditions:
the model contains five characteristics $D = (B_{d0}, B_{s0}, \rho_c, T_c, \beta_2)$, while $B_{d0} > 0$ and $B_{s0} > 0$;

- the mean diameter $f_d$ involves a singular component, $B_{d0} \tau^{2\beta_3}$, and does not contain a linear member.

If we accept $v = 0.333$, then it follows from (12): $\beta_3 = 0.333$. We note that Novikov [17] also has used a phenomenological approach and accepted a condition ($\partial^3 P/\partial \rho^3)_T = 0$ for equation (4) in the critical point. He has received models ($f_s = B_{s0} \tau^{\beta_3}$, $f_d = B_{d0} \tau^{2\beta_3}$), where $\beta_3 = 1/3$.

These scaling models ($f_s$, $f_d$, etc) are investigated in a wide range of substances and in a great number of papers; there are a row of numerical data on $D = (T_c, \rho_1, \alpha, \beta, B_{d0}, B_{s0}, \ldots)$. So, Anisimov [2] has developed a model for the H$_2$O, which includes $\alpha_1$ and $\beta_1$, has a satisfactory accuracy in the interval ($5 \times 10^{-4}$ to $5 \times 10^{-2}$) and written in the form

$$f_s = B_{s0} \tau^{\beta_1} + B_{s1} \tau^{\beta_1 + \Delta},$$

$$f_d = B_{d0} \tau^{1-\alpha_1} + B_{d1} \tau,$$

where $C = (B_{si}, B_{di})$ are the coefficients determined by a statistical treatment of experimental data on the densities of H$_2$O, $B_{d0} > 0$, $B_{s0} < 0$.

### 3. Combined scaling models of $f_d$ and $f_s$

A new hypothesis (AH) is developed from 2003 to 2017 in a few studies including [6, 7, 13, 17, 18]. It is based on ST conditions and considers a link, which exists between:

- the mean diameter $f_d$;

- the chemical potential, $\mu$, the entropy, $s$, the heat capacity, $C_v$ and the saturation pressure, $P$.

In accordance with AH, equations (2) and (7) include additional components; thus, Anisimov et al [18] has presented a numerical information of the following model

$$f_s = B_{s0} \tau^{\beta_1} + B_{s1} \tau^{\beta_1 + \Delta},$$

$$f_d = B_{d0} \tau^{1-\alpha_1} + B_{d1} \tau,$$ (15)

Form (15) meets the following conditions:

- the mean diameter $f_d$ includes the additional singular component in a comparison with a structure of (14);

- the critical indices $\alpha_1$ and $\beta_1$ follow the inequalities: $1 > 1 - \alpha_1 > 2\beta_1$.

These conditions lead to a conclusion that the second scaling member in $f_d$ is dominant over the other in some small region ($\tau_{\text{low}}$ to $\tau_{\text{as}}$). Equations (6) and (8) can be written in the interval ($\tau_{\text{low}}$ to $\tau_{\text{as}}$) in the form

$$f_s = B_{s0} \tau^{\beta_1},$$

$$f_d = B_{d2} \tau^{2\beta_1},$$ (16)

It is seen from (16) that derivative $df_d/d\tau$ is singular ($df_d/d\tau \approx -B_{d2} \tau^{2\beta_1 - 1} \rightarrow -\infty$ when $\tau \rightarrow 0$). Equation (16) is similar to equation (12): an index, $\beta_1$, is included in $f_d$ (16) in the doubled form.

There are numerical data on parameters ($B_{d0}, B_{d2}, B_{s0}, \ldots$) of (15) in [18] for multiple substances including SF$_6$ and N$_2$ in the range $\tau_{\text{low}} = 10^{-4}$ to $\tau_{\text{high}} = 6 \times 10^{-2}$. It is shown:

- the values of $B_{d2}$ depend on a substance and can be positive (SF$_6$, etc) and negative (N$_2$, etc);

- the authors [18] have attracted some information (experimental $\rho_g, \rho_l, T$ data, experimental data on $C_v$ and experimental data on the saturation pressure, $P$) to evaluate equations (15) and (16).
Fisher has introduced a term “complete scaling” in the frame of AH in his pioneer work [13]. Equation (15) is aimed to improve the traditional structure of \( f_d \) model and to increase an accuracy of \( f_d \) model. The equation reflects current trends of ST.

There is a model, which related to \( f_d \) and \( f_s \) and has the form discussed in [19, 20]

\[
f_d = B_{d0} \tau^{1-\alpha_4} + B_{d \text{exp}} \tau^{2\beta_4}, \quad f_s = B_{s0} \tau^{\beta_4} + B_{s1} \tau^{\beta_4+\Delta},
\]

where \( D = (T_c, \rho_l, \alpha_4, \beta_4, B_{d0}, B_{d \text{exp}}, \ldots) \) are the characteristics obtained from experimental \( \rho_g, \rho_l, T \) data [21] for SF_6.

It was shown [19] that \( B_{d0} \) and \( B_{d0} \) correspond to inequalities \( B_{d0} > 0, B_{d0} > 0 \), and correlate with some conditions of MLH. Due to [19 ], estimates of parameters (17) are got as: \( \alpha_4 = 0.1099, \beta_4 = 0.3474, B_{d0} = 1.9575, B_{d0} = 0.4695, B_{d \text{exp}} = 0.0518 \). It can be seen [19] that \( B_{d \text{exp}} \) has to be positive. A comparison has allowed to conclude that \( (\rho_g, \rho_l, T) \)_calc obtained with the help of equations (17) are consistent satisfactory with the original points in the temperature range from \( \tau_{\text{low}} = 2 \times 10^{-4} \) to \( \tau < 0.01 \). Local deviations of experimental points are close to the error, \( \delta \rho_{\text{exp}} \approx 0.1\% \), related to data [21].

As the first step to a combined scaling model, we have considered a model, which is studied in [15, 16] and related to \( f_d \) and \( f_s \); it consists of scaling (\( F_{\text{scale}} \)) and regular (\( F_{\text{reg}} \)) parts

\[
f_s = B_{s0} \tau^{\beta_4} + B_{s1} \tau^{\beta_4+\Delta} + B_{s2} \tau^{\beta_4+2\Delta} + B_{s3} \tau^3 + B_{s4} \tau^2,
\]

\[
f_d = B_{d0} \tau^{1-\alpha_4} + B_{d1} \tau^{1-\alpha_4+\Delta} + B_{d2} \tau^{1-\alpha_4+2\Delta} + B_{d3} \tau^2 + B_{d4} \tau^3,
\]

where \( (B_{si}, B_{di}, i = 0,1,2) \) are the coefficients related to \( F_{\text{scale}} \); \( (B_{si}, B_{di}, i = 3,4) \) are the coefficients related to \( F_{\text{reg}} \).

Scaling part \( F_{\text{scale}} \) meets ST and is consistent with MLH, namely:

- characteristics \( B_{d0} > 0 \) and \( B_{d0} > 0 \);
- the mean diameter \( f_d \) does not contain a linear member.

There are characteristics \( D = (T_c, \rho_l, \alpha_4, \beta_4, B_{d0}, B_{d0}), \) which are included in equations (18), (19) and calculated together with the coefficients \( C = (B_{si}, B_{di}) \) on the basis of a statistical treatment of experimental \( \rho_g, \rho_l, T \) data, i.e. a nonlinear least squares method (NRMS) [15, 16]. It is given in [15] :

- characteristics \( D \) and \( C \) coefficients, which are belonged to equations (18), (19) and based on experimental \( \rho_g, \rho_l, T \) data [21] of SF_6;
- some results of comparisons.

In the analyses, equations (18) and (19) allowed us to estimate \( (\rho_g, \rho_l, T) \) _calc data, which are in a satisfactory agreement with the original points in the range from \( \tau_{\text{low}} \) to \( \tau = 0.3 \) while the local variances are close to the error, \( \delta \rho_{\text{exp}} \).

In the second stage, we have considered an equation, which describes \( f_d \) in the form

\[
f_d = B_{d0} \tau^{1-\alpha_4} + B_{d \text{exp}} \tau^{2\beta_4} + B_{d1} \tau^{1-\alpha_4+\Delta} + B_{d2} \tau^2 + B_{d3} \tau^3.
\]

Equation (20) includes \( F_{\text{scale}} \) and \( F_{\text{reg}} \), which are also presented in model (19). In our opinion, equations (18) and (20) are consistent with ST and should to follow MLH, namely:

- the mean diameter \( f_d \) should include the leading singular component, \( B_{d \text{exp}} \tau^{2\beta_4} \), and not contain a linear member \( B_{d \text{exp}} \);
- the coefficient, \( B_{d \text{exp}} \), confirms an inequality \( B_{d \text{exp}} > 0 \) (table 1).
Table 1. The parameters of the models (18) and (20).

| $\rho_c$, kg/m$^3$ | $T_c$, K | $\alpha_4$ | $\beta_4$ | $B_{s0}$ | $B_{s1}$ | $B_{s2}$ |
|-------------------|---------|------------|-----------|----------|----------|----------|
| 741.61            | 318.7095| 0.1098     | 0.34745   | 1.9569   | 0.0112834| 0.0010612|
| $B_{s3}$          | $B_{s4}$| $B_{d0}$   | $B_{d\exp}$| $B_{d2}$| $B_{d3}$ | $B_{d4}$ |
|                   |         |           |           |          |          |          |
|                   | $-1.06553$| $1.344218$| 0.25491   | 0.08499 | 1.0729327| $-0.97233$| 0.769552 |

It is of interest to investigate a combine scale model consisted of equations (18) and (20). The parameters of this model (see table 1) have been calculated with the use of experimental $\rho_g, \rho_l, T$ data [21] and NRMS.

The initial approximation $C_0, D_0 = (T_{c0}, \rho_{c0}, \alpha_4, \beta_4, B_{d0}, B_{d\exp}, B_{s0}, \delta\rho)$ has been chosen in view of $C$ coefficients and $D$ characteristics, which are related to equations (17)–(19) and given in [15].

We note that $f_d$ (15) operates in the interval $(0 < \tau < 0.06)$; its parameters are obtained in [18] under the following restrictions:

- characteristics, $D = (T_c, \rho_c, \alpha_1, \beta_1)$, are selected as the data taken from the literature;
- other parameters ($B_{d0}, B_{d\exp}, B_{s1}, B_{s2}, B_{d1}, B_{d2}$) (15) are determined with an usage of a statistical treatment and experimental $\rho_g, \rho_l, T$ data [22].

4. Numerical characteristics of combined scaling equations on SF$_6$ example

Equations (18) and (20) have given a possibility to construct combined equations, $\rho_l(\tau, D, C)$, $\rho_g(\tau, D, C)$, on the basis of formulas (1) and (2). Our analysis shows that these models satisfactory reproduce $\rho_g, \rho_l, T$ data [21]; thus, deviations, $\delta\rho_l = 100(\rho_l - \rho_l) / \rho_l$, lie in the range from $-0.02$ to $0.09$% at temperatures $0 < \tau < 0.3$ and deviations, $\delta\rho_g$, are placed in the range from $-0.19$ to $0.16$% at temperatures $0 < \tau < 0.3$ (figure 1). Characteristics $D = (T_c, \rho_c)$ (see table 1) are in satisfactory agreement with those recommended in [21] (within the error of the latter).

A comparison shows that $\rho_g, \rho_l, T$ data [22] deviate systematically from corresponding values of $\rho_l(\tau, D, C)$ and $\rho_g(\tau, D, C)$ in the interval $0 < \tau < 0.06$ while the local deviations, $\delta\rho_{l,g}$, are positive and reach 1.6%.

Numerical data on the diameter $f_d$ (20) and some comparisons have shown, that values of $f_d$ (20) are in satisfactory agreement with values of $f_{d\exp}$, which are built on the basis of $\rho_g, \rho_l, T$ data [21] at temperatures $\tau_{low} < \tau < 0.3$ (figure 2). Local deviations ($\Delta f_{d_i} = |f_{d_i} - f_{dexp}|$, $i = 1, \ldots, N$) are determined as well as the maximum deviation, $\Delta f_{dmax}$, and the high boundary, $f_{dhigh} = f_d + |\Delta f_{dmax}|$ (see figure 2). The maximum deviation $\Delta f_{dmax}$ correlates well with $\delta\rho_{exp}$ of $\rho_g, \rho_l, T$ data [21].

We have analyzed components of $f_d$ (15). It is possible to see some of these results in figure 3 taken from [18]:

- this diameter satisfactorily coincides with $f_{d\exp}$ values calculated from the experimental data on the density of SF$_6$ [22];
- the component $B_{d0} \tau^{1-\alpha_1}$ of (15) is negative; we note that experimental values of $B_{d0}$ are positive for the overwhelming number of substances [2, 15, 16];
- the component $B_{d2} \tau^{2\beta_1}$ of (15) is positive and lies substantially higher than $f_{d\exp}$ values; the inequality $(B_{d\exp} \tau^{2\beta_1} > B_{d0} \tau^{1-\alpha_1})$ is satisfied at $0 < \tau < 0.06$. 
Figure 1. Relative deviations of the experimental data from the values obtained on the basis of models (18) and (20): 1—deviations of $\rho_g, T$ data [21]; 2—deviations of $\rho_l, T$ data [21].

Figure 2. A mean diameter, $f_d$, and its components: 1—the diameter, $f_{d,\text{exp}}$, obtained on the basis of $\rho_g, \rho_l, T$ data [21]; 2—a border, $f_{\text{d,high}}$; 3—the diameter, $f_d$, obtained on the basis of model (15) [18]; 4—the diameter, $f_d$, obtained on the basis of model (20); 5—the component, $B_{d,\text{exp}} \tau^{2\beta_4}$; 6—the component, $B_{\alpha_4} \tau^{1-\alpha_4}$.

We have got $\rho_g, \rho_l, T$ data with a help of EOS [1] in an extrapolation region. Our comparison has shown:
Figure 3. A temperature dependents of $f_d$ (15): 1—$f_{d\exp}$ related to $\rho_g, \rho_l, T$ data [22]; 2—to $f_d$ related to (15); (1−$\alpha$)—the component, $B_{d0}\tau^{1-\alpha}$; 2$\beta$—the component, $B_{d\exp}\tau^{2\beta}$.

- deviations, $\delta\rho_l = 100(\rho_{l[1]} - \rho_l(\tau, D, C))/\rho_l(\tau, D, C)$, are increasing from 0.05 to 3.2% if $\tau$ decreases from $10^{-3}$ to $10^{-5}$;
- deviations, $\delta\rho_g = 100(\rho_{g[1]} - \rho_g(\tau, D, C))/\rho_g(\tau, D, C)$, are decreasing from −0.05 to −3.7% if $\tau$ decreases from $10^{-3}$ to $10^{-5}$.

5. Conclusions
Numerical data on the diameter $f_d$ have allowed us to draw some conclusions. First, it is shown:

- our values are located significantly higher than $f_d$ values recommended by Anisimov in the interval $0 < \tau < 0.06$;
- the component, $B_{d0}\tau^{1-\alpha}$, is positive and placed lower than $f_{d\exp}$;
- the component, $B_{d\exp}\tau^{2\beta}$, is positive and placed lower than $B_{d0}\tau^{1-\alpha}$.

Second, a comparison of $B_{d0}\tau^{1-\alpha}$ with $B_{d\exp}\tau^{2\beta}$ allows us to estimate: there is a relative temperature, $\tau_A \approx 1 \times 10^{-4}$, where an equality $B_{d0}\tau^{1-\alpha} = B_{d\exp}\tau^{2\beta}$ is valid. The inequality $B_{d\exp}\tau^{2\beta} > B_{d0}\tau^{1-\alpha}$ is satisfied at $\tau < \tau_A$. It is possible to write $f_s = B_{s0}\tau^{\beta}$, $f_d = B_{d\exp}\tau^{2\beta}$ at $\tau < \tau_A$; these equations contain five parameters ($B_{d\exp}$, $B_{s0}$, $\rho_c$, $T_c$, $\beta$). This form agrees satisfactorily with MLH.

Third, $(\alpha_4, \beta_4)$ indices coincide with $(\alpha_1, \beta_1)$ parameters within 1–3%.

Our analysis has shown that $\rho_l(\tau, D, C)$ and $\rho_g(\tau, D, C)$ represent experimental $\rho_g, \rho_l, T$ data with an acceptable accuracy in the interval $(2 \times 10^{-4} < \tau < 0.3)$. In our opinion, the component,
$B_{dexp} \tau^{2/3}$, can be determined more precisely on the basis of accurate experimental $\rho_g(\tau, D, C)$ data, which will be obtained in the interval $10^{-5} < \tau < 10^{-3}$.

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