Magnetic charge of finite lifetime in SU(2) gluodynamics

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Abstract. A self-dual, localized solution to the classical SU(2) Yang-Mills equation in Euclidean spacetime, which formally possesses infinite action, is investigated in view of its U(1) charge content after Abelian projection. This is suggested by noting that the solution satisfies ’t Hooft’s differential projection condition away from the singularities. As a result the existence of dynamical, magnetic charge of finite lifetime is established. A covariant cutoff for the action is introduced by demanding the solution to be close to an instanton topologically. This is in analogy to the calculation of the mass of a point charge in classical electrodynamics or the subtraction of diverging self-energies of magnetic monopoles as discussed in the literature. The Wilson loop is evaluated in the background of a dilute gas. Assuming identical integrals over size distributions, the corresponding static quark/anti-quark potential at infinite spatial separation can be sizably higher than the potential in a dilute instanton gas.

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1 Introduction

Quantum Chromodynamics (QCD) is widely accepted as the theory of strong interactions due to its experimentally verifyable feature of asymptotic freedom in the UV momentum regime (see for example ref. and references therein). A perturbative treatment of the theory in the IR domain is excluded by a large coupling constant. Non-perturbative techniques are necessary and partial successes in explaining the low energy phenomenology of strong interactions, such as spontaneous chiral symmetry breaking and the axial U(1) problem, have been achieved this way (see ref. and references therein). However, the basic observation that no free color charges exist in nature is understood only poorly so far.

The dual superconductor picture of the non-perturbative QCD vacuum dates back to the mid seventies. It pro-
vides an appealing mechanism for color confinement due to the formation of color electric fluxtubes linking colored objects immersed in a condensate of magnetic charges. The resulting potential, linear in the (large) separation between the charges, is determined by an effective, universal scale — the string tension. The occurrence of magnetic charges in SU(n) Yang-Mills theory was made transparent in the process of the so-called maximal Abelian gauge fixing (MAGF) [6]. Due to this fixing gauge transformations belonging to the maximal Abelian subgroup U(1)_{n-1} of SU(n) are unconstrained and the resulting Abelian gauge theory has electric and magnetic charges. A rather successful model — the dual Ginzburg-Landau theory — has been employed to effectively mimic the low energy sector of SU(n) Yang-Mills theory after MAGF [7]. To classically describe the deconfinement phase transition induced by finite temperature a non-renormalizable version of the dual Ginzburg-Landau theory has been put forward recently [8].

Yet no analytical derivation of magnetic monopole condensation has been given although rather impressive results were obtained from the lattice [5]. However, the lattice detection of monopole trajectories has its inherent uncertainties (see below). Exact classical solutions to the Yang-Mills equations subjected to Abelian projection have been investigated more recently in view of their magnetic charge content [10,11,12]. Concerning charges of finite lifetime the results are not conclusive in the case of instantons since either the gauge fixing functional diverges for non-infinitesimal monopole trajectories [11] or the monopole loop is infinitesimal [10,12]. The investigation of field configurations, which are not exact solutions to the Yang-Mills equations (instanton/anti-instanton pair), are usually carried out on the lattice. Since the lattice provides a UV cutoff and thereby has a limited resolution and since only approximative solutions have been looked at the claimed detection of finite monopole loops is not well established.

The purpose of this work is to point out the presence of long-lived magnetic charge due to exact, self-dual solutions to the SU(2) Yang-Mills equations and to investigate their effect on the static quark potential at infinite separation. The fact that a solution possessing infinite action has a vanishing contribution to the partition function is dealt with by an appropriate subtraction in the spirit of ref. [5]. The divergence of the action is traced back to singularities of the action density on a hypersurface. In ref. [6] the appearance of magnetic monopoles in SU(n) gauge theory was made manifest after imposing a non-propagating gauge which leaves the maximal Abelian subgroup U(1)_{n-1} invariant. This gauge condition demands that a matrix quantity, say the field strength component $F_{12}$, transforming homogeneously under gauge transformations, be diagonal. In particular, for the case of SU(2) it is easy to shown that at points, where this condition is no constraint on the corresponding gauge transformations, the inhomogeneous part of the gauge transformation of $A_{\mu}$ is singular and contains an Abelian, magnetic monopole [13]. Due to the Coulomb-like behavior of the magnetic field strength in the vicinity of such a point the action
of the field configuration formally diverges, and hence it must be regularized phenomenologically. The solution discussed obeys the differential Abelian projection condition away from the singularities, and hence the restriction to its diagonal component is suggested.

In the present work magnetic charge is identified by means of the Coulomb-like behavior of the field strength of a fictitious charge moving toward (away from) a spatial point which turns into a singularity at a certain pair of time slices. The definition of the physical action is performed in analogy to defining the mass of a static point charge in classical electrodynamics. A covariant cutoff for the action functional is introduced which, in principle, can be determined phenomenologically.

Summarizing, the line of reasoning in this work is as follows: Classical, self-dual, infinite action solution, which is regular almost everywhere, possesses a converging action integral in the infinite, and is in Abelian gauge (Section 2) → presence of magnetic charge ⇒ infinite action (Section 3) → phenomenological action regularization (Section 4). In addition, the static quark/anti-quark potential in a dilute gas is calculated for infinite spatial separation in Section 5. Thereby, the integral over size distribution is assumed to be identical to that of the instanton case. The result is compared to the potential due to a dilute instanton gas.

2 Classical solution

A class of winding, BPS saturated solutions to the SU(2) classical Yang-Mills equations in Euclidean spacetime was found a long time ago in ref. [15]. These solutions have finite classical action $S = 1/(4g^2) \int d^4xF^a_{\mu
u}F^a_{\mu\nu} = 8\pi^2/g^2$ and spontaneously break eight of the symmetries of the classical theory. They are (anti)self-dual, that is

$$F^a_{\mu\nu} = \pm\varepsilon_{\mu\nu\kappa\lambda}F^a_{\kappa\lambda},$$

and are known as instantons. As first suggested by Gribov instantons can be interpreted as tunneling trajectories linking topologically distinct vacua [16]. In the last 20 years or so there has been an enormous industry relating the presence of these solutions in the non-perturbative QCD vacuum to low energy phenomenology, namely the $U(1)_A$ problem and spontaneous chiral symmetry breaking (see ref. [4] and references therein). However, the issue of color-confinement, which should be an important consequence of QCD, has not been satisfactorily settled by the instanton calculus [17,19].

For gauge group SU(2), in singular gauge, and for center $z = 0$ the instanton solution is given as

$$A^a_\mu = \bar{\eta}_{a\mu\nu}x_\nu \frac{x_\mu}{x^2(x^2 + \rho^2)},$$

with $\bar{\eta}_{a\mu\nu} \equiv -\frac{i}{2}\text{tr}(\tau_a\tau^-_\mu\tau^+_{\nu})$, $\tau^\pm_{\mu} = (\tau, \mp i)$, and $\tau_a$ the Pauli matrices. The solution (3) can be obtained as follows [20]. Making the ansatz

$$A^a_\mu = \bar{\eta}_{a\mu\nu}x_\nu \frac{1 + \phi(x^2)}{x^2},$$

the action takes the form

$$S = \frac{8\pi^2}{g^2} \frac{3}{2} \int d\tau \left\{ \frac{1}{2}(\phi')^2 + \frac{1}{8}(\phi^2 - 1)^2 \right\}, \quad \tau \equiv \ln \left( \frac{x^2}{\rho^2} \right),$$

in ref. [15].
where $\rho$ is some length scale. A BPS saturated vacua interpolation of the theory \([4]\) is
\[
\phi = -\tanh\left(\frac{\tau}{2}\right). \tag{5}
\]
Substituting this into eq. \((3)\), one obtains the solution of eq. \((2)\). Indeed, evaluating the action yields
\[
S = \frac{8\pi^2}{g^2} \frac{3}{8} \int d\tau \cosh^{-4}\left(\frac{\tau}{2}\right) = \frac{8\pi^2}{g^2}. \tag{6}
\]
Another BPS saturated solution to the equation of motion corresponding to \((4)\) is obtained by letting $\tau \rightarrow \tau + \tau_0$ in \((5)\). Thereby, $\tau_0$ may be complex as long as the new solution is real. This is necessary to have a real action. Admissible complex shifts are $\tau_0 = i n \pi$ with $n \in \mathbb{Z}$. Choosing $\tau_0 = i \pi$ yields
\[
\phi \rightarrow -\coth\left(\frac{\tau}{2}\right). \tag{7}
\]
Note that \((5)\) and \((7)\) behave identically in the infinite.

For the Yang-Mills potential eq. \((7)\) implies
\[
A_\mu \rightarrow A_\mu = -2\eta_{\mu\nu}\rho^2 \frac{x_\nu}{x^2(x^2 - \rho^2)}. \tag{8}
\]
The corresponding anti-solution (negative topological charge density) is obtained by substituting $\eta_{\mu\nu} \rightarrow \eta_{\mu\nu}$. Formally, the action of solution \((8)\) reads
\[
S = \frac{8\pi^2}{g^2} \frac{3}{8} \int d\tau \sinh^{-4}\left(\frac{\tau}{2}\right). \tag{9}
\]
It diverges due to the quartic pole of the integrand at $\tau = 0$ and not due to a diverging integral in the infinite. An infinite action of a classical solution generally is sufficient cause to disregard this solution on grounds of its vanishing contribution to the partition function of the theory. As it will be explained below there is physical information in the solution \((8)\), and hence it will not be dismissed.

The surface of divergence of \((8)\) is the 3-sphere $x^2 = \rho^2$. For this reason it is referred to as spheron throughout the remainder of the paper. Writing the spheron as
\[
A_\mu^a = -\bar{\eta}_{\mu\nu}^{\ d} \partial_\nu \log \left(1 - \frac{\rho^2}{x^2}\right), \tag{10}
\]
it is obvious that away from the singularities the following two sets of conditions are fulfilled
\[
(\partial_\mu \pm i A_3^\pm) A_\mu^\pm = 0, \quad \partial_\mu A_\mu^a = 0, \tag{11}
\]
where $A_\mu^\pm \equiv A_\mu^1 \pm i A_\mu^2$. The first set expresses the fact that the spheron is 't Hooft's maximal Abelian gauge (MAG)\([6]\) which leaves a U(1) subgroup of SU(2) unfixed.

Globally, this is mirrored by a minimum of the functional
\[
G_M[A] = \frac{1}{4} \int d^4 x \{ A_\mu^1(x)A_\mu^1(x) + A_\mu^2(x)A_\mu^2(x) \} = 2 \int d^4 x \frac{\rho^4}{x^2(x^2 - \rho^2)^2} \tag{12}
\]
under gauge transformations \([11]\) if the divergence on $x^2 = \rho^2$ is cut out (see Section 4). Obeying of the second set of conditions in \((11)\) indicates that the spheron is Lorentz gauged. Restricting oneself to the diagonal component, which is suggested by MAG and Abelian dominance (see for example \([21]\)), the Lorentz gauge guarantees the minimization of the functional
\[
G_L[A] = \frac{1}{4} \int d^4 x A_\mu^3(x)A_\mu^3(x) = \int d^4 x \frac{\rho^4}{x^2(x^2 - \rho^2)^2} \tag{13}
\]
under U(1) gauge transformations \([14]\).

Back to the problem of a diverging action: It is argued in the following that our poor understanding of the nature of charged particles in classical electrodynamics must necessarily carry over to a classical, Abelian projected SU(2)
(and more general SU(n)) gauge theory in which the appearance of localized U(1) charges is expected [3,6]. The term ”poor understanding” refers to the fact that the classical self-energy of a static point charge is divergent. In classical electrodynamics the usual procedure is to constrain the corresponding energy functional to regions of space with a regular behavior of the integrand to produce the observed mass of the associated particle. In the quantum theory of electrodynamics (QED) the mere introduction of a mass parameter into the Lagrangian accounts in an effective fashion for the aforementioned process in the limit of zero coupling $1$. At nonzero coupling the mass starts to run (albeit slowly in QED due to an IR conformal point) with the momentum probing it. Going from the IR regime up to large momenta (yet below the Landau pole) the gross contribution to the mass is of a classical origin as the weak logarithmic dependence on momentum shows. From the viewpoint of Abelian projection it is thus not imperative to disregard a classical solution of infinite action due to divergences of the spheron field if these divergences can be attributed to U(1) charge in the Abelian projected theory (see ref. [5] where the subtraction of an infinite self energy was made explicit in the case of 2+1 dimensional QCD).

3 Magnetic charge

In this Section the consequences of Abelian projection applied to the spheron are investigated. For fixed times $|x_4| \leq \rho$ there is a 2-sphere of divergence with radius $ho(x_4) = \sqrt{\rho^2 - x_4^4}$. At $x_4 = -\rho$ a pointlike singularity in 3-space emerges which starts expanding with Minkowski-space velocity $v(x_4 = -\rho) = 1/\sqrt{2}$. The 2-sphere reaches its maximal radius $\rho = \rho$ at $x_4 = 0$ with velocity $v(x_4 = 0) = 0$, and shrinks to a point at $x_4 = +\rho$. At times $|x_4| > \rho$ there is no 2-sphere, and the field is regular in 3-space.

The underlying theory is matter-free and non-Abelian. The luxury of placing a static $\delta$-source on the right-hand side of Maxwell’s equation to obtain a Coulomb law for the corresponding field strength is absent. If attributed to the spatial divergences of the spheron field the life of a U(1) charge is highly dynamic and of limited duration. Hence, there are strong retardation and threshold effects which necessarily blur the Coulomb picture. Moreover, in the case of accelerated magnetic charge the conservation of magnetic flux can be realized by a smeared flux distribution in contrast to the static Dirac string. Thus, at $x_4 = \pm\rho$ there may be divergences of $B^2$ higher than quartic in $|x|$. It is then not justified to read off the magnetic charge from the ”residue” belonging to a fourth-order Coulomb pole $|x| = 0$ of $B^2$ at $x_4 = \pm\rho$. However, if there is charge a Euclidean space observer placed at $|x| = 0$ measures squares of field strengths that are quartically divergent as $x_4 \to -\rho$. Close to the singularity this behavior can be attributed to a fictitious charge moving uniformly at Euclidean ”speed-of-light” along one of the
spatial coordinate axis toward the observer\textsuperscript{2}. At $x_4 = -\rho$ the hypothetical charge materializes, that is, there is a genuine singularity at $x = 0$. After Abelian projection the squares of electric and magnetic charges can thus be read off as the "residues" of the fourth-order Coulomb pole at $x_4 = -\rho$ ($x = 0$) in $E^2$ and $B^2$, respectively. At $x_4 = \rho$ ($x = 0$) a similar argument applies. The situation is summarized in Fig. 2.

Using eq. (13) and the properties of the $\eta$-symbols, one easily obtains

$$E^2 \equiv (\partial_4 A_i^3 - \partial_i A_4^3)^2 = (\partial_4 A_i^3)^2 - 2\partial_4 A_i \partial_i A_4^3 + (\partial_i A_4^3)^2 = (\partial_4 h(x^2))^2 + (\partial_2 h(x^2))^2 + (\partial_3 h(x^2))^2 - 2\partial_2 h(x^2) \partial_3 h(x^2), \quad h(x^2) \equiv \log(1 + \frac{4x^2}{m^2}) \quad (15)$$

For $x_4 \to \pm \rho$ and $x = 0$ eq. (15) reduces asymptotically to

$$E^2_{x_4 \to \pm \rho, x=0} = \frac{1}{|x_4 \mp \rho|^4}. \quad (16)$$

Converting from the non-perturbative to the perturbative definition of $A_\mu$ by virtue of $A_\mu \to \frac{1}{g} A_\mu$ and comparing with the standard Coulomb expression, the magnitude of

\begin{equation*}
E^2 \equiv (\varepsilon_{ijk} [\partial_j A_k^3 - \partial_k A_j^3])^2 = 4 \left( (\partial_m A_n^3)^2 - \partial_m A_n^3 \partial_n A_m^3 \right), \quad (18)
\end{equation*}

where the expression in terms of derivatives of $h(x^2)$ has been omitted. The "residue" of the Coulomb pole $m$ turns out to vanish. Eqs. (17) signals that $e$ fulfills the Dirac condition with respect to the SU(2) coupling $g$. Thus it should be interpreted as a magnetic charge\textsuperscript{3}. This can be made manifest by exploiting the following charge rotation symmetry of Maxwell’s equations \textsuperscript{3}

$$m \to m \cos \alpha + e \sin \alpha, \quad e \to -m \sin \alpha + e \cos \alpha, \quad (19)$$

which allows to rotate $e$ into $m$ for $\alpha = \frac{\pi}{2}$. In Fig. 3 the angular distribution of $E^2$ at three different instants is depicted.

The concentration of field strength in the "northern" hemisphere at $x_4 = -\rho$ evolves to a mirror symmetric distribution at $x_4 = 0$. At $x_4 = \rho$ field strength is concentrated in the "southern" hemisphere.

At this point it is worth comparing the above results to the work of ref. \textsuperscript{11}. There, an infinitesimal magnetic monopole loop was obtained by looking at singular gauge transformations applied to an instanton in singular gauge.

These transformations were parametrized by a radius scale $\frac{1}{2}g$ \textsuperscript{3}. The corresponding quark doublet would have electric charge $\pm \frac{1}{2}g$.\textsuperscript{3}
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$R$ dividing spacetime into "outer" and "inner" regions, with the gauge transformed instanton in singular and regular gauge, respectively. The MAG functional turned out to be stabilized at $R = 0$, that is, back at the instanton. Now, the instanton has finite action. Does this contradict the things said before? It does not because the identified magnetic charge lives only infinitesimally short in such a way as to keep the action finite. In ref. [11] further investigations toward the detection of finite-size monopole loops were carried out on the lattice for an instanton/anti-instanton (I-A) pair. Since the lattice naturally provides a UV cutoff field strength divergences remain hidden. Furthermore, the I-A system is not an exact solution to the equation of motion and the results of an abelian projection may heavily depend on the chosen ansatz for a variational principle approach. Thus, the claimed detection of monopole loops in the I-A system using the lattice [11] remains questionable.

4 Regularized action

In this Section a covariant definition of the physical spheron action is suggested. For the spheron to be of relevance its contribution to the partition function (or tunneling amplitude in Minkowski space) must be seizable. Moreover, its topological charge is up to the factor $8\pi^2/g^2$ equal to the action (for the anti-spheron equal minus the action) and is strictly positive (negative) for finite cutoff. In classical electrodynamics the cutoff for the energy functional of the electron’s Coulomb field is determined from its measured mass. Similarly, estimating the mean topological susceptibility of the vacuum $\langle \varepsilon_{\mu\nu\lambda\kappa} F_{\mu\lambda}^a F_{\nu\kappa}^a \rangle$ in terms of the measured masses $m_{\eta'}, m_K, m_\pi, m_\eta$ by means of the Veneziano-Witten relation [24] or estimating the gluon condensate by means of QCD sum rules [25], one could, in principle, approximate the action cutoff for the spheron phenomenologically. For now the covariant cutoff $\tau_m$ is simply assumed to produce the classical action $S = 8\pi^2/g^2$ of the instanton. The cut off action integral for the spheron is evaluated as

$$S_s = \frac{8\pi^2}{g^2} \left( \frac{1}{2} - \frac{1}{8} [\cosh(3\tau_m/2) - 3 \cosh(\tau_m/2)] \sinh^{-3}(\tau_m/2) \right),$$

$$\tau_m \equiv \log(x_m^2/\rho^2).$$

(20)

Setting $S_s = 8\pi^2/g^2$, one obtains $\tau_m = \log 3$. In units of $\rho$ the minimal distance $\xi$ to the sphere consequently is

$$\xi \equiv \sqrt{\exp(\tau_m)} - 1 \sim 0.73.$$  

For this particular value the MAG functional of eq. (12) yields

$$G_M[A] = 4\pi^2\rho^2 \frac{1}{\exp(\tau_m) - 1} = 2\pi^2\rho^2$$

(21)

as opposed to $G_M[A] = 4\pi^2\rho^2$ for the instanton in singular gauge. In Section 5 the cutoff will be varied to test the sensitivity of the static quark potential.

5 Wilson loop in a dilute spheron gas

In this Section the static quark/anti-quark potential $E(R)$ is estimated from the rectangular Wilson loop $\int_1$ in a dilute...
spheron gas background and compared to that of a dilute instanton gas for infinite spatial separation $R \to \infty$. All conventions and general results, which do only exploit the localization property of the instanton in singular gauge, can be taken from ref. [17]. There, $E(R)$ was obtained as

$$E(R) = -\int d^3x \frac{dp}{\rho^3} D_n(\rho) \frac{1}{n} \text{tr} (U^+(x)U^-(x+R) - 1) ,$$  

(22)

where

$$U^\pm \equiv \mathcal{P}\exp \left( \pm i \int dx_4 A_4(x) \right) ,$$  

(23)

$\mathcal{P}$ demands path-ordering, $D_n(\rho)$ denotes the spheron size distribution in a dilute gas for gauge group SU($n$), where quantum fluctuations are included in a semiclassical way. For $R \to \infty$ one has $U^-(x+R) \to +1$. In this limit eq. (22) reduces to

$$\lim_{R \to \infty} E_s(R) = -\int d^3x \frac{dp}{\rho^3} D_n(\rho) \frac{1}{n} \text{tr} (U^+(x) - 1)$$

$$\equiv -\int d^3x \frac{dp}{\rho^3} D_n(\rho) \frac{1}{n} f_s(u) ,$$  

(24)

where $u \equiv \sqrt{x^2/\rho^2}$. It is assumed here that the integral over spheron and instanton size distributions are identical.

Implementation of the covariant cutoff $\tau_m$ and comparing the one-loop contribution to tunnel-putation of the $x_4$ integral of eq. (24) yields

$$f_s(u) =$$

$$- \text{tr} \left( \exp \left( -2i\rho^2 \sigma \cdot x \int_{s}^{\infty} dx_4 \frac{1}{\sqrt{\rho^2 \varepsilon_m - x^2}} \left( \frac{x_4^2 + x^2}{x_4^2 + x^2 - \rho^2} \right) \right) \right) - 1$$

$$= -2 \left( \cos \left( \pi - 2 \arctan \left( \frac{\sqrt{s} - u^2}{u} \right) \right) + \theta(u - 1) \left( \pi + 2 \arctan \left( \frac{\sqrt{s} - u^2}{u^2 - 1} \right) \right) + \theta(1 - u) \left( \frac{u}{\sqrt{1 - u^2}} \log \left( \frac{\sqrt{s} - u^2}{\sqrt{s} - u^2 + \sqrt{1 - u^2}} \right) - 1 \right) \right) .$$  

(25)

Using $\varepsilon_m \equiv \exp(\tau_m) = 3$ from Section 4 and performing the remaining space integral of eq. (24), one has

$$\lim_{R \to \infty} E_s(R) = -\int \frac{dp}{\rho^3} \frac{1}{n} D_n(\rho) \int_0^{\sqrt{s}} du u^2 f_s(u)$$

$$\sim \int \frac{dp}{\rho^3} \frac{1}{n} D_n(\rho) \times 8\pi \times 1.52 .$$  

(26)

The result for the instanton in singular gauge is

$$f_s(u) \equiv - \text{tr} (U^+ - 1) = 2 \left( \cos \left( \pi \left[ 1 - \frac{u}{\sqrt{1+u^2}} \right] \right) - 1 \right) ,$$  

(27)

which implies

$$\lim_{R \to \infty} E_i(R) = -4\pi \int \frac{dp}{\rho^3} \frac{1}{n} D_n(\rho)$$

$$\int_0^{\infty} du u^2 f_s(u) \sim \int \frac{dp}{\rho^3} \frac{1}{n} D_n(\rho) \times 8\pi \times 1.10 .$$  

(28)

Hence, for a cutoff chosen to give spheron and instanton identical topological charge the $\rho$-independent parts of the integrands are comparable. Setting $\xi = 1$ and thereby avoiding the problem of slightly different scales in the cut off solution, the number 1.52 in eq. (25) changes to 2.41.ing amplitude [18]. There is no easy estimate of these effects, and one would have to perform the one-loop calculation directly along the lines of ref. [18]. Furthermore, the cutoff for the size integrations may phenomenologically turn out to be different for the instanton as opposed to the spheron case.
For even higher cutoff, $\xi = \sqrt{5-1} \sim 1.24$, one obtains 3.52. Depending on the phenomenologically favorable cutoff this might have implications for color confinement would it qualitatively carry over to the spheron liquid. In ref. [19] the static quark/anti-quark potential in the instanton liquid was evaluated for two average instanton sizes $\bar{\rho}$. The values $\bar{\rho} = 600 \text{MeV}^{-1}$ and $\bar{\rho} = 400 \text{MeV}^{-1}$ yielded $\lim_{R \to \infty} E_i \sim 140 \text{MeV}$ and $\lim_{R \to \infty} E_i \sim 460 \text{MeV}$, respectively. This asymptotic behavior is approached rapidly (at $R \sim 2 \text{fm}$ one has $E_i \sim 110, \text{MeV}$ $\bar{\rho} = 400 \text{MeV}^{-1}$). Concentrating on the case $\xi = 1$, 3.2 times these values (instanton plus spheron contributions) are $\lim_{R \to \infty} (E_s + E_i) \sim 450 \text{MeV}$ and $\lim_{R \to \infty} (E_s + E_i) \sim 1.5 \text{GeV}$ which includes the range of spectral continuum thresholds $\sqrt{s_0} \sim 1.2 \ldots 1.4 \text{ GeV}$ successfully used in light-quark channel QCD sum rules [27]. This means that the out-of-the-vacuum creation of a light pair of current quark and anti-quark eventually turning into the massive constituents of hadrons (conceivably not without exciting more current quark/anti-quark pairs) would happen to a good approximation in the asymptotically free regime and thereby be supported by a large phase space. In this picture the notion of absolut quark confinement due to an infinite static quark potential for infinite separation seems too restrictive. It would be sufficient to demonstrate that the integrated probability of the aforementioned pair creation process is practically unity for separations comparable to or smaller than typical hadron sizes. At this point the somewhat speculative nature of the above considerations is stressed: Apart from the question of how physical the assumption is that the spheron’s topological charge takes values around unity it is necessary to calculate the numerical factor distinguishing instanton and spheron size distributions for decisive statements. Moreover, it is not quite clear how much the interaction between anti-instantons with sphersons (and vice versa) and the interaction between sphersons and antisphersons would alter what was said above. It is hoped that these uncertainties can be positively eliminated in the near future. At this stage the above results still should be taken as another serious indication that the physics of color confinement may be linked to the existence of long-lived magnetic charges in the vacuum of the corresponding SU(n) gauge theory.

6 Summary

In this work the identification of long-lived magnetic charges in the the vacuum of SU(2) Yang-Mills theory has been persued. The existence of a self-dual, infinite action solution to the classical Yang-Mills equation, which obeys the differential Abelian projection conditions away from its singularities, proved essential for this identification. Furthermore, the Abelian component of this solution is in Lorentz gauge which suggests an interesting connection to the recently discussed appearance of a mass dimension two condensate in QCD [14]. For the solution to contribute to the partition function in an essential way its physical action must be small. In analogy to the definition of the mass of a static point charge in classical electrodynamics a covariant cutoff for the action has been introduced. In this work it was chosen such as to give topological charge of
order one to the solution. The Wilson loop in a dilute gas was evaluated, and the static quark/anti-quark potential for infinite spatial separation was extracted. It turned out that this number can be sizably higher than the one obtained in a dilute instanton gas if identical integrals over size distributions are assumed.

The question whether one can describe low energy phenomena in QCD in a semi-classical way starting from its classical Lagrangian remains. It is well possible that the influence of quantum fluctuations generates a quantum effective action at low scales which is even qualitatively different from the classical theory. However, the hope is there that at least the relevant degrees of freedom in the low energy domain can be identified in the classical theory.

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Figure 1: The integration contours for the formal action functional.

Figure 2: The Euclidean world-volume of singular magnetic flux due to the spheron field after Abelian projection.

Figure 3: Angular distribution of $E^2$ at $x_4 = -\rho$ (a), $x_4 = 0$ (b), and $x_4 = \rho$ (c) for $\rho = 1$ and spatial distance from the singularity $\zeta = 0.5$. 
Fig. 1.

Fig. 2.

Fig. 3.