Depth-resolved birefringence and differential optical axis orientation measurements using fiber-based polarization-sensitive optical coherence tomography

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ABSTRACT

Conventional polarization-sensitive optical coherence tomography (PS-OCT) can provide depth-resolved Stokes parameter measurements of light reflected from turbid media. A new algorithm, which takes into account changes in optical axis, is introduced to give depth-resolved birefringence and differential optical axis orientation images using fiber-based PS-OCT. Quaternion, a convenient mathematical tool, is used to represent an optical element and simplify the algorithm. Experimental results with beef tendon and rabbit tendon and muscle show that this technique has promising potential for imaging the birefringent structure of multiple-layer samples with varying optical axes.

Key words: optical coherence tomography, medical and biological imaging, birefringence, polarization, turbid media

1. INTRODUCTION

Optical coherence tomography (OCT), a noninvasive imaging technique for turbid media, uses coherence gating of the light source to obtain two- or three-dimensional images [1]. Polarization-sensitive OCT (PS-OCT) can provide additional information on the changes in light polarization states caused by birefringence, diattenuation of the sample or both [2-10]. Conventional PS-OCT gives the overall phase retardation image, based on the assumption that the optical axis remains constant, as do the four Stokes parameter images of turbid media [4, 6-8]. However, these images represent accumulated birefringence effects and do not represent the local structure of the sample. Recently, Jiao et al. developed differential phase retardation imaging to determine birefringence structure by calculating the absolute value of the retardation difference between a given pixel and its adjacent pixel along the same longitudinal scan line [10]. This algorithm is restricted by the assumption of a constant optical axis in the sample under study. An algorithm which takes into account changes in the optical axis, must be used to determine the depth-resolved birefringent structure if the sample is modeled with multiple-layer optics. In this paper, we present a new algorithm with a fiber-based PS-OCT system, which can resolve the problem when the optical axis changes as a function of depth and the polarization state changes within the fiber. Quaternion, a mathematical tool, will be introduced to represent an optical element and simplify the algorithm. The birefringence and differential optical axis orientation images, which are calculated from PS-OCT measurements using this algorithm, will be used to provide additional contrast for tissue structure.

2. METHODS

2.1 Algorithm
To obtain the optical axis orientation and phase retardation, which varies with depth, the tissue and fiber are modeled with multiple-layer optics where each layer is considered as a linear phase retarder, assuming there is no polarizer in the system. Such an optical system can be completely described as the rotation of three normalized Stokes parameters \( S_1, S_2 \) or \( Q, U \) using a Poincaré sphere representation. Jones and Mueller matrices are the most common mathematical tools used to represent an optical element. If absorption and diattenuation are negligible, there are only three independent parameters in Jones and Mueller matrices. In such cases, quaternion, a much more convenient mathematical tool, can be used to represent the optical element. This is because Jones and Mueller matrices and quaternion are three different, but equivalent mathematical representations of rotations under the assumption mentioned above.
There are two independent parameters for each linear phase retarder. One is the optical axis orientation, \( \theta \), and the other is phase retardation, \( \Delta \). In the Poincaré sphere representation, the single linear phase retarder can be represented by rotation of angle \( \Delta \), about the axis \((\cos 2\theta, \sin 2\theta, 0)\). The quaternion representation \((q_n)\) of a single linear phase retarder can be described as

\[
q_n = \cos \frac{\Delta_n}{2} + \sin \frac{\Delta_n}{2} (\hat{i} \cos 2\theta_n + \hat{j} \sin 2\theta_n)
\]

where subscript, \( n \), represents the \( n \)th layer, and \( \hat{i} \), \( \hat{j} \) and \( \hat{k} \) are three imaginary units of a quaternion which satisfy \( \hat{i}^2 = \hat{j}^2 = \hat{k}^2 = \hat{i}\hat{j} = -\hat{1} \). The quaternion representation of multiple-layer linear phase retarders is the product of quaternions of individual retarders

\[
q_n \cdots q_2 q_1 = a_n + \hat{i} b_n + \hat{j} c_n + \hat{k} d_n
\]

where \( a_n^2 + b_n^2 + c_n^2 + d_n^2 = 1 \). We express the polarization states of both forward and backscattered light in the forward coordinate system. Using this convention, the quaternion representation of a single linear phase retarder for backscattered light is identical to that of forward light. Since the optical paths for forward and backscattered light are the same, but reversed in direction, if Eq. (2) is considered as the forward quaternion representation, \( Q_{n,1} \), the backscattered quaternion representation, \( Q_{n,b} \), can be described as

\[
Q_{n,b} = q_n q_2 \cdots q_1 = a_n + \hat{i} b_n + \hat{j} c_n - \hat{k} d_n
\]

Eq. (3) presents the optical reversibility theorem using the quaternion representation when the same coordinate system is used. The overall roundtrip quaternion, \( Q_{T,n} \), can be obtained as

\[
Q_{T,n} = Q_{fiber,D} Q_{fiber,S,b} Q_{n,b} Q_{M,n} Q_{fiber,S,f} Q_f = A_n + \hat{i} B_n + \hat{j} C_n + \hat{k} D_n
\]

where \( Q_M = 1 \) is the quaternion of a mirror; \( Q_{fiber,S,b} \cdot Q_{fiber,S,f} \) and \( Q_{fiber,D} \) are quaternions of fibers in sampling (backscattered and forward) and detection arms, respectively. Analogously, the overall roundtrip quaternion of \( n+1 \) layers can be obtained as

\[
Q_{T,n+1} = Q_{fiber,D} Q_{fiber,S,b} Q_{n+1,b} Q_{M,n+1} Q_{fiber,S,f} Q_f = A_{n+1} + \hat{i} B_{n+1} + \hat{j} C_{n+1} + \hat{k} D_{n+1}
\]

Our goal is to obtain the phase retardation and optical axis orientation of the \((n+1)\)th layer. Based on Eqs. (1)-(5), the phase retardation of the \((n+1)\)th layer can be resolved as

\[
\cos \Delta_{n+1} = A_n A_{n+1} + B_n B_{n+1} + C_n C_{n+1} + D_n D_{n+1}
\]

Since birefringence is defined as the difference between ordinary and extraordinary refractive indices, the phase retardation between two neighboring pixels, \( \Delta \), will be used for the birefringence image.

For the optical axis orientation, \( \theta_{n+1} \), it is impossible to obtain a solution unless changes in the polarization state in the fiber and the optical axes orientations of all previous layers are taken into account. Fortunately, we find a parameter – the difference in optical axes orientations between two neighboring layers, which can present the optical axis information. Based on Eq. (1), the roundtrip quaternion of the \( n \)th and the \((n+1)\)th layers, \( q_n Q_{n+1} q_n \), can be obtained. \( q_n Q_{n+1} q_n \) represents an equivalent linear phase retarder, of which the real part is
\[
\text{Re}(q_n q_{n+1}^\dagger q_n) = \cos(\Delta_n + \Delta_{n+1}) + (1 - \cos 2\beta_n)\sin \Delta_n \sin \Delta_{n+1}
\]  
(7)

where \(\beta_n = \theta_{n+1} - \theta_n\) is the difference in optical axes orientations between the \(n\)th and \((n+1)\)th layers. The phase retardation of this equivalent retarder, \(\Delta_{n,n+1}\), can also be calculated with the roundtrip quaternions, \(Q_{T,n-1}\) and \(Q_{T,n+1}\):

\[
\cos \Delta_{n,n+1} = A_{n\rightarrow1}B_{n+1} + B_{n\rightarrow1}A_{n+1} + C_{n\rightarrow1}C_{n+1} + D_{n\rightarrow1}D_{n+1}
\]  
(8)

Since Eqs. (7) and (8) should be equal, we find

\[
\cos 2\beta_n = 1 - \frac{\cos \Delta_{n,n+1} - \cos(\Delta_n + \Delta_{n+1})}{\sin \Delta_n \sin \Delta_{n+1}}
\]  
(9)

We denote the image based on \(\beta_n\) value as the differential optical axis orientation image.

The quaternions are calculated from the measured Stokes parameters which have previously been described [4, 6]. Changes in the polarization state of light propagating through the sample can be described by rotation of the normalized Stokes vector \((S_1, S_2, S_3)\) in the Poincaré sphere representation. The rotation axis and angle can be determined by two pairs of initial and final Stokes vectors using a simple solid geometry analysis.

### 2.2 PS-OCT setup and data processing

Fig. 1 shows the single-mode fiber-based PS-OCT system. Light from the broadband light source (AFC Technologies, central wavelength \(\lambda = 1310\) nm, FWHM \(\Delta \lambda = 65\) nm, degree of polarization [DOP] less than 1%) enters a 2x2 fiber coupler. In the reference arm, a rapid canning optical delay line (RSOD) provides A-scans at 500 Hz without phase modulation. A phase modulator generates 500 kHz phase modulation for heterodyne detection. A four-step driving function is applied to the polarization modulator and each step introduces a \(\pi/4\) phase shift. Since only vertical linear polarized light can pass the phase modulator, four reference polarization states are selected, each separated by 45˚ angles over a great circle on a Poincaré sphere. The four corresponding polarization states scattered from the sample arm are obtained by phase-resolved processing of the interference fringe signals obtained from two perpendicular polarization detection channels. The rotation quaternions (i.e. both the rotation axis and angle) are calculated as follows: for each pixel, three pairs of polarization states, i.e. \((1&2, 2&3, 3&4)\), are selected to calculate the rotation angles, respectively. For each pair of polarization states, the relative position of the normalized Stokes vectors should be maintained in the Poincaré sphere after rotation in the absence of diattenuation. However, there is a small shift in the measured data. Suppose the angles between the two initial and final polarization states are \(\theta\) and \(\theta’\), respectively, \(\delta\) is the angle difference: \(\delta = \theta - \theta’\). Both \(\theta\) and \(\theta’\) are modified by \(\left(\theta - \frac{\delta}{2}\right)\) (i.e. \(\theta’ + \frac{\delta}{2}\)) by shifting all four Stokes vectors by the same angle \(\frac{\delta}{4}\) in different directions, as shown in Fig. 2. The relationships between these vectors are as follows:

\[
\begin{align*}
\hat{1}_{\text{new}} &= \frac{\sin(\theta - \delta/4)}{\sin \theta} \hat{1} + \frac{\sin(\delta/4)}{\sin \theta} \hat{2} \\
\hat{2}_{\text{new}} &= \frac{\sin(\delta/4)}{\sin \theta} \hat{1} + \frac{\sin(\theta - \delta/4)}{\sin \theta} \hat{2} \\
\hat{1}'_{\text{new}} &= \frac{\sin(\theta' + \delta/4)}{\sin \theta'} \hat{1}' + \frac{\sin(\delta/4)}{\sin \theta'} \hat{2}' \\
\hat{2}'_{\text{new}} &= -\frac{\sin(\delta/4)}{\sin \theta'} \hat{1}' + \frac{\sin(\theta' + \delta/4)}{\sin \theta'} \hat{2}'
\end{align*}
\]  
(10)

An exact rotation quaternion can be obtained with these shifted Stokes vectors. The average of the three rotation angles obtained from the three pairs of polarization states, respectively, is used as the final result.
Fig. 1. Fiber-based PS-OCT system. DOP, degree of polarization; Pol. Mod., polarization modulator; Pol. Control, polarization controllers; Phase Mod., phase modulator; PBS, polarization beam splitter; RSOD, rapid scanning optical delay line; and D1&D2, detectors.

Fig. 2. Schematic diagram for shifting the Stokes vectors. \( \mathbf{\hat{r}} \) and \( \mathbf{\hat{r}}' \) are two initial Stokes vectors. \( \mathbf{\hat{r}}'' \) and \( \mathbf{\hat{r}}''' \) are two final Stokes vectors. The vectors without subscript \( \text{new} \) represent Stokes vectors before shifting, those with subscript \( \text{new} \) represent Stokes vectors after shifting.

3. EXPERIMENTAL RESULTS

Fig. 3 shows the PS-OCT images of beef tendon (a-d) and rabbit muscle and tendon (a’-d’), respectively. Fig. 3b and 3b’ are roundtrip phase retardation images which are calculated based on the assumption of a constant optical axis and do not represent the local structure of the sample. Fig. 3c and 3c’ are birefringence images, in which a bright area represents a strongly birefringent region while a dark area represents a weakly birefringent region. Fig. 3d and 3d’ are differential optical axis orientation images, in which a dark area represents a region with constant optical axis while a bright line represents the boundary between two adjacent birefringent layers with different optical axes. Three special cases need to be explained as follows: (1) The boundary between two neighboring birefringent layers with different optical axes is represented by a dark line in the birefringence image. This is because the boundary region is composed of two layers and the equivalent birefringence is smaller than either layer. This is clearly shown in Fig. 3c where the dark line is the boundary region between two layers of tendon with different orientations of optical axes. (2) A region without birefringence is represented as a very dark area in the birefringence image. However, the same region will appear as a bright area in the differential optical axis orientation image. This is because the denominator in Eq. (9) is close to zero and thus very little noise will result in a large \( \beta_0 \) value. This is confirmed in Fig. 3d where the left side upper layer of the tissue represents a zero birefringence layer. (3) One of the limitations of our current analysis is that we only consider a linear phase retarder model. An elliptical phase retarder model that includes diattenuation will be considered in future work. A preliminary analysis indicates that the layer, which should be modeled with an elliptical phase retarder, will appear as a medium bright area in the differential optical axis orientation image due to this limitation. The muscle layer
in Fig. 3d' gives such an example. Histology of the beef tendon (Fig. 3e) is included for comparison. A clear boundary can be found in Figs. 3c, 3d and 3e, but not in the conventional OCT structural image (Fig. 3a). It is obvious that, in addition to the conventional OCT structural image, birefringence and differential optical axis orientation images can provide more detailed information, especially boundaries between different layers.

![Image](image_url)

**Fig. 3.** PS-OCT images of beef tendon (a-d), and rabbit tendon and muscle (a'-d'). (a) OCT structural images; (b) roundtrip phase retardation images; (c) birefringence images; (d) differential optical axis orientation images; and (e) histology image of beef tendon. Scale bar indicates 1mm.

### 4. CONCLUSION

In summary, previous PS-OCT research focused on roundtrip phase retardation and Stokes parameter imaging. Birefringence property measurements of skin have previously been reported [4]. However, it cannot provide specific birefringence information on layers with different optical axes. The novel depth-resolved birefringence and differential
optical axis orientation imaging technique reported herein can distinguish local birefringent structures in different layers, therefore providing better image contrast of multiple layered samples.

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