UV Completed Composite Higgs model with heavy composite partners

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We study electroweak symmetry breaking in minimal composite Higgs models $SU(4)/Sp(4)$ with purely fermionic UV completions based on a confining Hypercolor gauge group and find that the extra Higgs potential from the underlying preon mass can destroy the correlation between the mass of Higgs and composite partners. Thus the composite partners can be very heavy for successful electroweak symmetry breaking without enhancing the separation between the new physical scale and Higgs VEV. So this kind of model can be easily realized by ordinary strong dynamics theories without artificial assumptions and more likely consistent with lattice simulations. The UV completion of partial compositeness predicts a light singlet Goldstone boson which interacts with QCD and electroweak gauge bosons through Wess-Zumino-Witten terms. It can be produced through gluon fusion at LHC and decay into gauge boson pairs. We briefly discuss its phenomenology and derive its bounds from LHC searches.

I. INTRODUCTION

The naturalness of Higgs potential is one of the most profound problems in particle physics. To solve this problem, new physics should be introduced to stabilize Higgs potential. Among these new physics theories, composite Higgs models (CHMs) [1–4] is currently the most popular one. In this model, the Higgs is a composite pseudo-Nambu-Goldstone boson (pNGB) so it is insensitive to other physical scales, such as Planck scale, and thus big hierarchy between electroweak symmetry breaking (EWSB) and the Planck scale can be achieved.

In ordinary CHMs, the Higgs potential is assumed to be only from top and gauge loop corrections. To regularise Higgs potential and achieve a light Higgs, some composite partners should be introduced to collectively break Higgs shift symmetry or realize maximal symmetry, such as warped extra dimensions [5,6], Little Higgs [4], and maximal symmetric CHMs [7,8], which results in strong correlation between the mass of Higgs and composite partners. So there always exists anomalously light top partners, around pNGB decay constant scale $f$, for light Higgs [10,12]. This special spectrum pattern of composite resonances, very different from QCD (the only observed strong dynamics in nature), requires some artificial ultraviolet (UV) completions. Moreover, the existing lattice simulations on some confining theories do not support this spectrum pattern [13–14], which makes constructing UV completions of ordinary CHMs very challenging.

There is the kind of CHMs that is supposed to have fermionic UV completions based on a confining Hypercolor gauge group $G_{HC}$ [15,21]. This UV completions contains two species of underlying Weyl fermions called preons, $Q$ (QCD neutral and electroweak charged) and $\chi$ (QCD colored). The confinement of the gauge group $G_{HC}$ will induce the spontaneously global symmetry breaking in the preon sector, generating pNGBs. The doublet pNGBs composed by $Q$ can be treated as Higgs bosons. The colored fermionic bound states with wavefunction $QQ\chi$ or $Q\chi\chi$ can play the role of the top partners, which serves as UV completion of the partial compositeness [22]. With this setup, there are three types of CHMs with symmetric coset space in EWSB sector: $SU(N_Q)/SO(N_Q)$, $SU(N_Q)/Sp(N_Q)$ and $SU(N_Q/2)^2/SU(N_Q/2)$ ($N_Q$ is the number of chiral preon $Q$), corresponding to $Q$ in the real, pseudo-real and complex representations of $G_{HC}$.

In this work, we study EWSB in the minimal CHMs with global symmetry breaking pattern $SU(4)/Sp(4) \cong SO(6)/SO(5)$ in $Q$ sector. If preons $Q$ are massive, Higgs potential will get extra contributions from $Q$ mass naturally. This extra potential can trigger EWSB in a different way together with Higgs potential from top and gauge sector. Especially, the correlation between the mass of Higgs and composite partners is lost (Higgs mass is only related to the scale difference between the partners of top and electroweak gauge bosons) and thus we can get heavy composite partners (can be as heavy as the confine scale $\sim 4\pi f$ at cost of more fine tuning) and light Higgs without enhancing the separation between Higgs VEV and scale $f$. So this kind of CHMs with heavy fermionic and vector resonances can be easily realized by ordinary strong dynamic theories, such as $G_{HC} = Sp(2N_{HC})$ with $2N_{HC} \leq 36$ [15], and consistent with lattice simulations, unlike ordinary CHMs.

Besides the extra single scalar $\eta$ in EWSB sector, which
is extensively discussed \cite{23,24}, this model predicts another singlet pNGB $\sigma$ associated with $U(1)_\sigma$ global symmetry \cite{25}, which is the subgroup of $U(1)_Q$ and $U(1)_\chi$ (overall phase of preon $Q$ and $\chi$). This $U(1)_\sigma$ is anomaly free under $G_{HC}$ so $\sigma$ can be light and crucial for testifying the partial compositeness. This singlet can interact with SM gauge fields (such as gluons) through Wess-Zumino-Witten (WZW) terms. So this singlet can be produced through gluon fusion at LHC and then decay into gauge boson pairs. We briefly discuss its phenomenology at LHC and derive its bounds for different UV completions.

The paper is organized as follows. In Sec. \[II\] we build the concrete UV completions for CHMs based on a confining hypercolor gauge group $G_{HC}$. In Sec. \[III\] we calculate the Higgs potential from preon masses, top and gauge boson loops in two cases: ordinary and minimal maximal symmetric CHMs. In Sec. \[V\] we study EWSB in Higgs potential and discuss the fine tuning. We find that heavy top partners can be achieved. In Sec. \[V\] we discuss the phenomenology of $\sigma$ at LHC and derive its bounds. We conclude in Sec. \[VI\]. The appendices contain the detailed expressions of top partner multiplets and the form factors in the effective Lagrangian, the descriptions of the gauge sector, as well as the details of $\eta$ mass from the hidden sector and $\sigma$ mass from $\chi$ sector.

\[II.\] THE MODEL

The consistent UV completions of CHMs with partial compositeness are limited \cite{15} if satisfy some consistent conditions, such as asymptotically freedom and free of anomalies. In this work, we study the CHM with global symmetry breaking pattern $SU(N_Q)/Sp(N_Q)$ in $Q$ sector and $SU(N_\chi)/SO(N_\chi)$ in $\chi$ sector. The global symmetry breaking pattern can thus determine that the hypercolor group in the UV completion can only be $G_{HC} = Sp(2N_{HC})$ with $2N_{HC} \leq 36$ or $G_{HC} = SO(N_{HC})$ with $N_{HC} = 11, 13$ \cite{26}. For simplicity, we focus on the minimal case where $N_Q = 4$ and $N_\chi = 6$, and the SM custodial symmetry $SU(2)_L \times SU(2)_R \subset SU(4)$ (Hypercharge is embedded in $SU(2)_R$) and QCD $SU(3)_c \subset SU(6)$ are embedded in the global symmetry as

\[
SU(4) \supset SU(2)_L \otimes SU(2)_R : \quad 4 = (2,2) \\
SU(6) \supset SU(3)_c : \quad 6 = 3 \oplus 3.
\]

The basic set up of our model is summarized in Table \[I\], where we list the SM quantum numbers of the two species of chiral preons (left-handed Weyl fermion): $Q_{1,4,..,4}$, $\chi_{1,3,..,6}$. Under this underlying strong dynamics, the global symmetry actually is $U(1)_\chi \times SU(4) \times U(1)_Q \times SU(6)$, $U(1)_\chi$ associated with the universal phase of preons $Q$ ($Q$), and is broken to $Sp(4) \times SO(6)$. One subgroup of the abelian group $U(1)_\chi \times U(1)_Q$ has anomaly with hypercolor symmetry and the corresponding pNGB mass is generally at cut-off scale. While the pNGB associated with the anomaly free subgroup $U(1)_\sigma$ of $U(1)_\chi \times U(1)_Q$ can be light, which is defined by the following $U(1)_\sigma$ charge assignment of the preons \cite{26},

\[
q_Q = N_\chi T_\chi, \quad q_\chi = -N_Q T_Q,
\]

where $N_{Q,\chi}$ is the number of Weyl fermions $Q/\chi (N_Q = 4$ and $N_\chi = 6$ in this model) and $T_Q, T_\chi$ is the Dynkin index of HC gauge group representation of $Q/\chi$. So in this model, the total number of light NGBs at a lower energy scale is

\[
26 = 1 + 5 + 20,
\]

where $1$ is from $U(1)_\sigma$ breaking, $5$ from $SU(4)/Sp(4)$, and $20$ from $SU(6)/SO(6)$. Before identifying the quantum number of these NGBs, we should choose consistent condensations of the underlying preons. Since the condensations in $Q$ ($\chi$) sector are in the anti-symmetric (symmetric) representation of global symmetry $SU(4)$ ($SU(6)$), we can choose the condensation of $Q$ and $\chi$ to be SM gauge invariant and in the form \cite{27}:

\[
\Sigma_Q = \begin{pmatrix}
 i\sigma_2 & 0 \\
 0 & -i\sigma_2
\end{pmatrix}, \quad \Sigma_\chi = \begin{pmatrix}
 0 & 1_{3\times3} \\
 1_{3\times3} & 0
\end{pmatrix},
\]

which will break the global $SU(4) \times U(1)_\sigma$ to $Sp(4)$ in the electroweak sector and $SU(6) \times U(1)_\sigma$ to $SO(6)$ in the $\chi$ sector. So the quantum number of the NGBs under $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_\chi$ is

\[
SU(4)/Sp(4) : \quad \pi_Q = (1,2,2)_0 + (1,1,1)_0, \\
SU(6)/SO(6) : \quad \pi_\chi = (8,1,1)_0 + (6,1,1)_{\pm 4}, \\
U(1)_\sigma : \quad \sigma = (1,1,1)_0,
\]

where the subscript represents $U(1)_\chi$ charge which is the subgroup of $SO(6)$ with embedding $X = \text{diag}[2/3,2/3,2/3,2/3,-2/3,-2/3, -2/3]$. Since $SU(4)/Sp(4)$ and $SU(6)/SO(6)$ are symmetric coset space, we can define its automorphism map

\[
T \rightarrow -VT^TV \Rightarrow U \rightarrow VU^TV,
\]

where $T$ is the broken generators in $SU(4)/Sp(4)$ or $SU(6)/SO(6)$ coset space and $V$ is the VEV of $SU(4)/Sp(4)$ or $SU(6)/SO(6)$. $U$ is Goldstone matrix fields for $SU(4)/Sp(4)$ or $SU(6)/SO(6)$. So the linearly

| $Q_{1,...,4}$ | $F/Spin$ | 1 | $(2,1) \oplus (1,2)$ | $q_Q$ |
| $\chi_{1,...,6}$ | $A/F$ | $3 \oplus 3$ | 1 | $q_\chi$ |

TABLE I: Quantum numbers of the Weyl preons under the gauge group $G_{HC} \times SU(3)_c \times SU(2)_L \times U(1)_\chi$ and global symmetry $U(1)_\sigma$. The hypercharge is $Y = T_R^c + X$ where $X$ is the VEV of $SU(6)$ with $X = \text{diag}[2/3,2/3,2/3,-2/3,-2/3, -2/3]$. The symbols F, A, Spin means fundamental, 2-index antisymmetric, and spinorial representation of $G_{HC}$ respectively.
realized sigma field $\Sigma$ and its transformation under global $SU(N)$ symmetry is

$$\Sigma \equiv U V U^T = U^2 V \Rightarrow \Sigma \rightarrow g \Sigma g^T, \quad g \in SU(N). \quad (7)$$

The linearly realized sigma in our model be parameterized as:

$$U_{Q,x} = e^{i \Pi_{Q,x}}, \quad \Sigma_{Q,x} = U_{Q,x}^2 \Sigma_{Q0,x0}$$

$$\Pi_Q = \cos \phi \sigma^{2}_{4Q} \mathbb{I}_4 + \frac{\sqrt{2} \pi_f T^a}{f} \Pi_f$$

$$\Pi_\chi = \sin \phi \sqrt{6} f_\chi + \frac{\sqrt{2} \pi_f T^a}{f_6} \Pi_f,$$

where $f_{Q,x}, f, f_6$ are the decay constants of the Goldstone bosons associated with $U(1)_{Q,x}$, $SU(4)/Sp(4)$, $SU(6)/SO(6)$, $T^a, \bar{A}$ are the $SU(4)/Sp(4)$ $(SU(6)/SO(6))$ broken generators with normalization $Tr[T^a T^b] = \delta^{ab}/2$, $\phi$ parametrizes the direction of anomaly free $U(1)_a$ subgroup of $U(1)_Q \times U(1)_\chi$, with value $\tan \phi \equiv \frac{f_\chi}{f_6}$.

In this section, we analyze the Higgs potential and EWSB based on UV completion. In CHMs the potential of pNGBs is generated by interaction terms that explicitly break global symmetry. In ordinary CHMs, it’s common that the SM gauge interaction and top Yukawa couplings are the main sources that contribute to pNGB potential. However, in this model, we will add another important contribution from the preon’s mass terms to pNGB potential. Which will bring significantly modifications to pNGB potential.

We want to emphasize again that since the pNGB $\eta$ and $\sigma$ are electroweak (EW) singlet, their VEV does not effect EWSB. Moreover, we will see that the quadratic terms of their potential can be easily kept positive without fine tuning. So without loss of generality, we always choose $\langle \eta \rangle = 0$ and $\langle \sigma \rangle = 0$ in the following discussions.

### A. pNGB potential from preon mass terms

In this model, the underlying preons can have masses, which will result in the pNGBs potential, like quark masses in QCD. In this subsection, we will discuss contributions of the mass of preon $Q$ to the Higgs potential. The most general gauge invariant mass terms of preon $Q$ that also preserve custodial symmetry are

$$\mathcal{L}_{mass} = Q_i^T \Sigma_{m_Q}^\dagger Q_j + h.c.,$$

where $\Sigma_{m_Q}$ is mass matrix.

This mass matrix transforms under global symmetry $SU(4)$ as

$$\Sigma_{m_Q} \to g_Q^T \Sigma_{m_Q} g_Q^\dagger.$$

where $g_Q$ is the $SU(4)$ element. According to global symmetry of the mass terms, the pNGBs potential generated

So we can find the relation between EWSB scale and global symmetry breaking scale $f$

$$\xi \equiv s_h^2 \equiv \frac{v_{SM}^2}{f^2}, \quad v_{SM} = 246 GeV,$$
by preon masses can be given by,
\[
V_m = -C_Q f^3 \text{Tr}[\Sigma_{m_Q} \Sigma_Q] + h.c.
\]
\[
= 8C_Q m_Q f^3 \cos \left( \frac{\sigma \cos \phi}{f} \right) \cos \left( \frac{\pi Q}{f} \right)
- 8C_Q \Delta_{m_Q} f^3 \frac{\eta}{\pi Q} \sin \left( \frac{\sigma \cos \phi}{f} \right) \sin \left( \frac{\pi Q}{f} \right),
\]
where we have defined
\[
m_Q = \frac{m_{Q_1} + m_{Q_2}}{2}, \quad \Delta_{m_Q} = \frac{m_{Q_1} - m_{Q_2}}{2}.
\]
Notice that \(C_Q \sim \langle QQ \rangle/(16\pi^2 f^3)\) is an unknown form factor determined by underlying Hypercolor dynamics \([10]\), which can be positive or negative. Generally, besides its potential from \(Q\) masses, the potential of singlet \(\sigma\) can be also from the mass of \(\chi\), more details can be found in App. [D].

**B. pNGB potential from Fermion loops**

As ordinary CHMs, the pNGB Higgs can also get the contributions from the top loop. In this model, the UV completion constrains the top partners to be in the \(6\) or \(10\) or \(1\) representation of \(SU(4)\) and their wave function, and quantum number under \(SU(4) \times SU(6)\) is
\[
\psi_1 = \chi QQ \in (6, 6), \quad \psi_2 = \chi \bar{Q} \bar{Q} \in (6, 6),
\]
\[
\psi_3 = Q \bar{Q} \bar{Q} \in (1, 6), \quad \psi_4 = Q \chi \bar{Q} \in (15, 6).
\]
Notice that since composite partner \(\psi_3\) is a global \(SU(4)\) singlet, it can not mix with the top doublet. For the most general case, top fields can mix with these top partners at the same time. But in this work, since we just focus on Higgs potential and EWSB in the CHM with UV completion and this kind of mixings do not change the basic property of Higgs potential as in ordinary CHMs, we just work on the case where top quarks only mix with one multiplet of top partners through some specific dynamics so the shift symmetry of \(\sigma\) is always unbroken in case. \([22]\) In the following discussions we only focus on the simplest case that top partners are in \(6\) representation of \(SU(4)\). Actually, the top quark singlet \(t_R\) can mix with the top partners in two ways: one is that \(t_R\) is embedded in \(6\) representation of global \(SU(4)\) to mix with the operator \(\psi_1\); the other is that \(t_R\) is global \(SU(4)\) singlet and directly mix with the \(Sp(4)\) singlet component of \(\psi_1\). These two cases can result in different types of Higgs potential if maximal symmetry (MS) exists in the composite sector (corresponding to ordinary MS and minimal MS case respectively). \([13]\) In the rest of this subsection, we will discuss these two cases.

1. **ordinary maximal symmetry**

The left-handed fermionic operators \(\psi_1\) can be decomposed under unbroken subgroup \(Sp(4) \times SU(3)_c \times U(1)_X\) as
\[
(6, 6) = (5, 3, 2/3) + (5, 3, -2/3) + (1, 3, 2/3) + (1, 3, -2/3)
\equiv \Psi_{5L} + \Psi_{5R} + \Psi_{1L} + \Psi_{1R},
\]
where the superscript \(c\) represents charge conjugation. The contents of these multiplets can be found in Appendix \([A]\). In order to mix with these partners, top dou-blet and singlet, \(q_L\) and \(t_R\), should be embedded in \(6\) of \(SU(4)\) and the embeddings can be chosen as
\[
\Psi_{ql} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & Q_{ql} \\ -Q_{ql}^T & 0 \end{pmatrix}, \quad Q_{ql} = \begin{pmatrix} t_L \ 0 \\ b_L \ 0 \end{pmatrix},
\]
\[
\Psi_{t_R}^c = \frac{t_R^c}{2} \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix},
\]
where \(t_R^c\) is written in the left-handed form. Notice that the \(\eta\) shift symmetry is unbroken for these top embeddings. According to the transformation properties of the fields, the most general mixing terms between the SM fermions and the top partners invariant under \(SU(4)\) global symmetry can be obtained
\[
\mathcal{L}_{\text{mix}} = -\lambda_L f \text{Tr}[\Psi_{ql} U_Q (\Psi_{5R} + \epsilon_L \Psi_{1R}^c)] U_Q^T
- \lambda_R f \text{Tr}[\Psi_{t_R} U_Q (\Psi_{5L} + \epsilon_R \Psi_{1L}^c)] U_Q^T
- M_5 \text{Tr}[\Psi_{5L} \Sigma_{Q0} \Psi_{5R}^c \Sigma_{Q0}^c]
- M_1 \text{Tr}[\Psi_{1L} \Sigma_{Q0} \Psi_{1R}^c \Sigma_{Q0}^c] + h.c.,
\]
where \(\epsilon_{L,R}\) parameterizes the relative differences between the mixings of different top multiplets with elementary top fields. To reduce the fine tuning in Higgs potential for successful EWSB, the Higgs potential should be finite. To achieve this, we can assume there is a global symmetry \(SU(4)\) which different from Higgs shift symmetry in the composite sector, i.e. maximal symmetry, by following requirements,
\[
\epsilon_{L,R} = 1, \quad M \equiv M_1 = M_5.
\]
Under this condition, we can get the ordinary maximal symmetric model, similar to \([8]\). After integrating out the heavy top partners, we can get the top quark effective Lagrangian with the simplest form,
\[
\mathcal{L}_{\text{eff}} = \Pi_{0}^a(p) \text{Tr}[\Psi_{ql} p \Psi_{ql}] + \Pi_{1}^a(p) \text{Tr}[\Psi_{t_R}^c p \Psi_{t_R}^c]
+ M_1^a(p) \text{Tr}[\Psi_{1L} \Sigma_{Q0} \Psi_{1R}^c \Sigma_{Q0}^c] + h.c.,
\]
where \(\Pi_{0}^a\) and \(M_1^a\) are form factors and their expressions can be found in App. \([A]\). As discussed in \([8]\), we can find that the maximal symmetry can eliminate the Higgs dependent effective kinetic terms of top quarks in lower energy effective Lagrangian and only effective top Yukawa is dependent on Higgs. Since the effective top Yukawa is collectively generated, \(M_1^a \sim \lambda_L \lambda_R f^2 M\), and leading Higgs potential is proportional to top Yukawa coupling square, Higgs potential must be finite. The top mass is easily obtained,
\[
m_t = \frac{\lambda_L \lambda_R f^2 M}{\sqrt{2}M_{T_1}M_{T_2}} \sin \left( \frac{2\beta}{f} \right),
\]
where $M_{T_1}$ and $M_{T_2}$ are top partners mass and their full expressions are listed in Appendix A.

Now with this effective Lagrange, we can calculate the Coleman-Weinberg potential of Higgs at the full one-loop level with the form

$$V_i(h) = -2N_c \int \frac{d^4 p_E}{(2\pi)^4} \log \left( 1 + \frac{|M_i|^2 h^2}{2p_E^2 \Pi_0^2 \Pi_0^2 \pi^2} \sin^2 \frac{2\pi Q}{f} \right).$$

We can expand $V_i(h)$ in top Yukawa coupling $y_t$ up to $O(y_t^4)$

$$V_i(h) \simeq \gamma_f \left(-\sin^2 \frac{\pi Q}{f} + \sin \frac{\pi Q}{f}\right) \frac{h^2}{\pi^2},$$

where

$$\gamma_f = 4N_c \int \frac{d^4 p_E}{(2\pi)^4} \frac{|M_i|^2}{p_E^2 \Pi_0^2 \Pi_0^2}.$$ 

It is easy to find that the Higgs potential in the top sector is equivalent to the Higgs potential in ordinary maximal symmetry and the Higgs VEV naturally lies at $\xi = 1/2$.

### 2. minimal maximal symmetry

In this case, $t_R$ is a global $SU(4)$ singlet and thus can only mix with top partner singlet $\Psi_1$ directly without dressing nonlinear pNGB matrix $U_Q$ ($q$ shift symmetry is still unbroken). The interactions between top fields and top partners can be expressed as

$$\mathcal{L}_{mix} = -\lambda_L f \text{Tr}[\Psi_{qL} U_Q (\Psi_{SR} + \epsilon_L \Psi_{SR}^c) U_Q^T] + \lambda_R f \text{Tr}[\Psi_{1L} \Sigma_{Q0} - M_1 \text{Tr}[\Psi_{5L} \Sigma_{Q0} \Sigma_{SR}^c - M_1 \text{Tr}[\Psi_{1L} \Sigma_{Q0} \Sigma_{SR}^c] + h.c.].$$

To achieve finite Higgs potential from the above interactions, we impose MS in $\Psi_{SR}$ sector again by the conditions in Eq. (25). After integrate out these heavy partners, the lower energy effective Lagrangian can be obtained,

$$\mathcal{L}_{eff} = \Pi_{00}^4 (p) \text{Tr}[\Psi_{qL} \Psi_{qL}^c] + \Pi_{00}^4 (p) \bar{t}_R t_R^c + M_1^4 (p) \text{Tr}[\Psi_{1L} \Sigma_{Q0}^c t_R^c] + h.c.$$ 

The top mass can be extracted

$$m_t = \frac{\sqrt{2} \lambda_L \lambda_R f^2 M_{T1} M_{T2}}{M_{T1} M_{T2}} \sin \frac{\langle h \rangle}{f}.$$ 

The Higgs potential from full one loop is given by

$$V_f = -2N_c \int \frac{d^4 p_E}{(2\pi)^4} \log \left( 1 + \frac{2|M_i|^2 h^2}{2p_E^2 \Pi_0^2 \Pi_0^2 \pi^2} \sin^2 \frac{\pi Q}{f} \right).$$

In the limit of $\sin(\pi Q/f) \ll 1$, the Higgs potential can be expanded up to quartic order in $\sin(\pi Q/f)$,

$$V_f \simeq -\gamma_f \frac{h^2}{\pi^2} \sin^2 \frac{\pi Q}{f} + \beta_f \frac{h^4}{\pi^2} \sin^4 \frac{\pi Q}{f}.$$ 

with

$$\gamma_f = 2N_c \int \frac{d^4 p_E}{(2\pi)^4} \frac{2|M_i|^2}{p_E^2 \Pi_0^2 \Pi_0^2}.$$ 

### C. Higgs potential in gauge sector

As for other CHMs, the elementary EW gauge bosons interact with pNGBs through their mixing with composite vector mesons. According to the UV completion, the preons can confine to form vector mesons with wave function and quantum number under $Sp(4)$ as

$$\rho_p \sim Q^c T^a \sigma^0 Q : 10 \quad \sigma^0 \sim Q^c T^a \sigma^0 : 5, \quad (36)$$

where $T^a(T^a)$ is (un-)broken generators of $SU(4)$. These mesons’ interactions with EW gauge boson can be determined by hidden local symmetry (more details can be seen in App. B). The effective Lagrange of EW gauge boson can be obtained by integrating out these vector mesons,

$$\mathcal{L}^{eff} = \frac{P^{\mu\nu}}{2} \left[ g_2^2 \Pi_{00}^{\mu W} W^a_{\mu} W^{a}_{\nu} + g^2 \Pi_{00}^{B} B_{\mu} B_{\nu} \right]$$

$$(\gamma h^2) \left( W^a_{\mu} W^{a}_{\nu} + W^2_{\mu} W^{2}_{\nu} \right) + \Pi_{11}^4 \left( g' B_{\mu} - g W^3_{\mu} \right),$$

where $P^{\mu\nu} = g^{\mu\nu} - p^\mu p^\nu/p^2$ is the transverse projector and the explicit expression of form factors, $\Pi_{00}^{W,B}$ and $\Pi_{11}$, is shown in App. B. Using the full one-loop Higgs potential in Eq. (34), we can get the leading Higgs potential by expanding it up to $\sin^2(\pi Q/f)$ ($\sin^4(\pi Q/f)$) and higher power terms are suppressed by gauge coupling so can be neglected, comparing with Higgs potential in the top sector),

$$V_g \simeq \gamma_g \frac{h^2}{\pi^2} \sin^2 \frac{\pi Q}{f},$$

with

$$\gamma_g = \frac{3}{8(4\pi)^2} \int dp_E^2 \frac{2}{\left( \frac{3}{\Pi_{00}^4} + \frac{1}{\Pi_{11}^4} \right) \Pi_{00}^4}.$$ 

As QCD, the Higgs potential from gauge loop correct automatically satisfies Weinberg sum rules for CHMs based on fermionic UV completion. So Higgs potential is finite and the leading order of Higgs potential from electroweak gauge bosons loops after imposing Weinberg sum rules is $10 \ [20]$

$$V_g \simeq \frac{3f^2(3g^2 + g^2) m_p^2 \ln 2}{64\pi^2} \frac{h^2}{\pi^2} \sin^2 \frac{\pi Q}{f},$$

where for simplicity we require the scale $f_p$ associated with vector meson mixing with SM gauge boson to be equal to $f$, $f_p = f$. 

IV. ANALYSIS OF THE HIGGS POTENTIAL

In this section, we will discuss EWSB, the spectrum of new fields, and fine tuning in Higgs potential. We will find that the Higgs potential from preon $Q$ mass can weaken the correlation between Higgs mass and top partner mass. So in the CHMs with massive underlying preons, the composite partners can be as heavy as cut-off scale $\sim 4\pi f$ for successful EWSB.

A. EWSB in Higgs Potential

The total Higgs potential that determines the EWSB vacuum can be expressed as

$$V(h) = -\gamma_m^2 s_h^4 + \beta_f s_h^4 + \gamma_m c_h,$$  \hspace{1cm} (39)

where $c_h \equiv \cos((h)/f)$, $s_h \equiv \sin((h)/f)$, $\gamma \equiv \gamma_f - \gamma_g$ and $\gamma_m \equiv 8C_m m_Q f^3$ parametrizes Higgs potential from preon masses. In the ordinary MS case, $\beta_f = \gamma_f$. Notice that we always assume that the VEV of singlet $\sigma$ and $\eta$ is zero for simplicity so the terms in pNGB potential proportional to $\eta$ and $\sigma$ can not affect Higgs VEV and can be eliminated. We will find that this condition can be easily satisfied without fine tuning. The minimum of the potential that can be realized EWSB vacuum is one of the roots of the following equation

$$\gamma_m + 2c_h(\gamma - 2\beta_f)\xi = 0.$$  \hspace{1cm} (40)

If $\beta_f\xi < \gamma$, Higgs vacuum can be estimated as

$$c_h \approx -\frac{\gamma_m}{2\gamma} \Rightarrow \xi \approx \frac{4\gamma^2 - \gamma_m^2}{4\gamma^2}.$$  \hspace{1cm} (41)

From this expression, we can find that, different from ordinary CHMs, the EWSB can also be triggered by preon mass contributions. The mass of Higgs can be extracted from Higgs potential,

$$m_h^2 = \frac{2\xi[\gamma + 2(2 - 3\xi)\beta_f]}{f^2}.$$  \hspace{1cm} (42)

Comparing with ordinary CHMs, the Higgs mass contains extra factor $\gamma$ ($m_h^2 - 8\xi\beta_f f^2$ in ordinary CHMs). If impose some cancellation between $\gamma$ and $\beta_f$, we can thus easily get the light Higgs and heavy composite partners simultaneously (generally $\gamma$ and $\beta_f$ can have opposite sign and are independent). However, in ordinary CHMs, since Higgs mass is only proportional to $\beta_f$, Higgs mass is strongly correlated with top partner mass and there is no space to tune the parameters to achieve light Higgs and heavy partners simultaneously. So the extra Higgs potential from preon masses can weaken the correlation between Higgs mass and partner mass, which can be explicitly seen in the next subsection. In our model, the $\eta$ potential is only from the preon mass sector (the gauge and top sector preserve $\eta$ and $\sigma$ shift symmetry), and after EWSB, its mass can be expressed as (for $\xi \ll 1$)

$$m_h^2 \approx f^2 m_p^2 - 8\xi\beta_f f^2.$$  \hspace{1cm} (43)

Since $\beta_f$ is positive, to prevent $\eta$ from getting a VEV $\beta_f$ should satisfy the upper limit of $\beta_f < f^2 m_p^2/(8\xi)$, which will impose an upper bound on top partner mass. However, $\eta$ can obtain extra masses from some terms that only explicitly break $\eta$ shift symmetry so the top partners can be very heavy without violating $\eta$ mass bounds (more details can be seen in App. C). Notice that for simplicity we assume $\Delta m_\sigma = 0$ such that there is no mixing between $\eta$ and $\sigma$.

The singlet $\sigma$ both contains the freedoms of underlying preon $Q$ and $\chi$, its mass can be from both $Q$ and $\chi$ sector. In the $Q$ sector, its mass is only from the mass of preon $Q$ and can be easily extracted in the EWSB phase,

$$m_Q^2 = m_\eta \sqrt{(1 - \xi)} \cos \phi.$$  \hspace{1cm} (44)

Its mass from $\chi$ sector is also generated via the mass of preon $\chi$ (gauge interactions also preserve $\sigma$ shift symmetry in $\chi$ sector), more details can be seen in App. D.

In the rest of this section, we will numerically calculate the spectrum of the new fields and fine tuning of Higgs potential in two different models.

B. Ordinary Maximal Symmetry

In ordinary MS model, according to the analytical expressions of Higgs potential in Eq. (30) and Eq. (38), the Higgs potential from top and gauge sector are sensitive to the composite partners mass and can be generally parametrized as,

$$\gamma_f = \beta_f \simeq c_f \frac{N_2 y_t f^2 M_t^2}{8\pi^2}, \quad \gamma_g \simeq c_g \frac{3g^2 m_p^2 f^2}{16\pi^2},$$  \hspace{1cm} (45)

where $y_t$ is the top Yukawa coupling, $c_{f,g}$ is order one positive parameter, whose analytical expressions can be derived in Eq. (30) and Eq. (38), and $M_t$ is the top partner mass scale. As discussed above, the correlation between Higgs and top partner mass is weakened by the extra Higgs potential from preon mass. Substitute these expressions into Higgs mass in Eq. (42), we can see that Higgs mass is sensitive to the mass scale difference between top and gauge bosons partners,

$$m_h^2 \sim (5c_f M_t^2 - c_g m_p^2)\xi.$$  \hspace{1cm} (46)

Since $c_{f,g}$ are positive, the light Higgs only indicates that the scale difference between $M_f$ and $m_p$ is small while the masses of each composite partner can be very heavy without increasing the scale $f$, just only at the cost of increasing fine tuning. Different from ordinary CHMs, where the Higgs mass is proportional to the mass scale of top partners, $m_h^2 \sim M_t^2$, so the top partners always
be light for light Higgs, around $M_f \approx f$, no matter how to tune the parameters if $\xi$ is fixed. For example, in ordinary CHMs based on deconstruction, the maximal value of lightest top partner mass is around 1.5 TeV for $\xi = 0.1$ and $m_h = 125$ GeV \cite{11}. While, in our models, the mass of the lightest top partner can be as heavy as the cut-off ($\sim 4\pi f$) for the same benchmark point if the singlet $\eta$ can acquire extra mass through some hidden interactions that only explicitly break its shift symmetry, as shown in App. \cite{C} As discussed in Sec. IV.A if $\eta$ potential is only from preon mass, $\eta$ mass is correlated with top partners mass, which imposes the upper bound on top partners. For example if $\xi = 0.05$ and $m^2_\eta > 0$, using the expressions in Eq. \cite{15}, top partner mass $M_f$ should satisfy

$$M_f \lesssim \frac{\pi m_h}{y_t \sqrt{c_f N_c \xi}} \sim 1.6 \text{ TeV}. \quad (47)$$

Next, we will discuss the fine tuning in Higgs potential. Following the convention in \cite{23}, the fine tuning can be quantified as follows

$$\Delta = \max \{\Delta_i\}, \quad \text{with} \quad \Delta_i = \left| \frac{\partial \ln \xi}{\partial \ln x_i} \right|, \quad (48)$$

where $x_i$ is the free parameter of the model. Using the equation of Higgs vacuum in Eq. \cite{10}, we can get the analytical expression of $\Delta_i$,

$$\Delta_i = \frac{2x_i}{m^2_h f^2} \left[ \sqrt{1 - \xi} \frac{\partial \gamma_m}{\partial x_i} + 2(1 - \xi) \left( \frac{\partial \gamma}{\partial x_i} - 2 \xi \frac{\partial \beta_f}{\partial x_i} \right) \right]. \quad (49)$$

If $\xi \beta_f \ll \gamma$, according to the approximate expression of $\xi$ in Eq. \cite{11}, to get small $\xi$ the main tuning is from the cancellation between $\gamma_m$ and $\gamma$, which can be expressed as

$$\Delta_m = \frac{2 \xi m^2}{4 \gamma^2 - \gamma_m^2} = \frac{2}{\xi} (1 - \xi). \quad (50)$$

Under this condition, the tuning is always minimal even $\gamma_f \gg \beta_f$ which always results in double tuning $\Delta \approx \gamma_f / \xi \beta_f$ \cite{11} $\approx 1 / \xi$ in ordinary CHMs. So the preon mass can relax double tuning in ordinary CHMs. In general situation, the tuning is mainly from the cancellations among preon mass, gauge, and top sector. The tuning from these three sectors has the same behavior and almost is at the same order of magnitude. We can explicitly look at the tuning from $\rho$ meson mass through Eq. \cite{49},

$$\Delta_\rho = \left| \frac{\partial \ln \xi}{\partial \ln m_\rho} \right| = \frac{8(1 - \xi) \gamma_\rho}{m^2_\rho \sqrt{f^2}} \sim \frac{m_\rho^2}{m^2_h}. \quad (51)$$

If we choose $m_\rho = 3$ TeV and fix $\xi = 0.05$, we find $\Delta_\rho \sim 20$. Similar to other CHMs, the tuning increases as the mass of composite partners increases. This is because that the Higgs potential is sensitive to the partner mass scale. To get light Higgs, more precise cancellation among $\gamma_f, g$ and $\beta_f$ is needed if their masses increase.

Finally, we use the measurement of fine tuning in Eq. \cite{48} to do the numerical calculations for two cases.

One is the minimal case where $\eta$ mass is only from the preon mass sector. In this case, its mass is related to Higgs mass and top partners mass scale in the EWSB phase. The other one is that its mass can be also from a hidden sector as shown in App. \cite{C} so $\eta$ mass can decouple with physics in EWSB. In Fig. \cite{1} we show the fine tuning as the function of resonance mass for the minimal (left) and non-minimal (right) case with $\xi = 0.05$, $m_h = 125$ GeV, and $m_t \in [140, 160]$ GeV. In the minimal case, since $\eta$ suffers from stringent bounds from Higgs decay \cite{29}, we require $m_\eta > m_h / 2$ in the numerical scan for consistency. Comparing with the non-minimal case, we can find that, in the minimal case, the bounds of $\eta$ impose an upper limit on lightest top partner mass $M$ (see Eq. \cite{47}), which also imposes an upper limit on $m_\eta$ thorough Higgs mass. Since $M$ is around scale $f$, the tuning is minimal ($\sim 1 / \xi$). In the non-minimal case where $m_\eta$ is not related to $M$, as discussed above, these composite partners can be as heavy as possible for successful EWSB and the tuning increases as these partners become heavy. These numerical results confirm the above analysis.

C. Minimal Maximal Symmetry

In the minimal MS CHMs \cite{12}, the Higgs potential from the gauge sector is the same as ordinary MS CHMs. $\gamma_f$ is also sensitive to the top partner scale, whose parametrization is the same as in Eq. \cite{45}. While $\beta_f$ is suppressed at $O(y^2_t)$ and not sensitive to top partner mass, so Higgs mass is insensitive to $M_f$ in this kind of model. The factor $\beta_f$ can be generally parametrized as

$$\beta_f \simeq \frac{N_c y^2_t f^4}{16 \pi^2} \ln \frac{M^2_f}{m^2_t}. \quad (52)$$

where $b_f$ is just order one constant. In this model without preon mass contributions, since $\beta_f$ is suppressed, the Higgs mass is always too light, $m_h \approx 100$ GeV for $M_f \approx 10 f$. Meanwhile, since $\gamma_f$ is much bigger than $\beta_f$, this model suffer from double tuning, $\Delta \gtrsim 95 / \xi$ \cite{12} \cite{30}.
If the preon mass contribution to Higgs potential is included, the Higgs quartic can be enhanced so Higgs can be heavy enough and the fine tuning can be suppressed ($M_f$ can be reduced). On the other hand, according to the expression of $\eta$ mass in Eq. (13) in the minimal case, $m_\eta$ is not sensitive to top partner mass and is almost a constant for fixed $\xi$. For example, substituting the expression of $\beta_i$ into Eq. (13), we can get $m_\eta \approx 380$ GeV for $\xi = 0.1$. Unlike the first model, this model can contain heavy enough $\eta$ to escape the bounds without effecting top partners mass. So the non-minimal case is not necessary. The behavior of the tuning is the same as the first model. The tuning is minimal for $M_f \sim f$ while it increases as $M_f$ increases (see Eq. (51)). In Fig. 2, we numerically calculate the tuning as the function of resonance masses for $\xi = 0.1$ and $m_h = 125$ GeV, which confirms the above discussion.

V. PHENOMENOLOGY AT THE LHC

If the top partners are very heavy, the first smoking gun of our model may be the presence of the extra neutral light pNGBs, especially their interactions with SM gauge bosons through WZW terms which can give rise to very typical signatures. We expect that the $\sigma$ field is the first signature of this class of CHMs besides the deviation of the Higgs property, which is the main prediction of partial compositeness, because it has the anomaly interactions with gluon fields, which can result in large production cross-section (the phenomenology of $\eta$ is extensively discussed in [24] [25]). Since its production cross-section is very small, its bounds are very weak, $m_\eta > m_h/2$. We will not discuss it in this work). We will sketch its phenomenology at the LHC in this section and give some bounds to the parameter space in our model according to the experimental data.

Generally, as discussed before, $\sigma$ can acquire mass from both $Q$ and $\chi$ mass sector,

$$m_\sigma = m_Q^Q + m_\chi^\chi,$$

(53)

where $m_Q^Q, m_\chi^\chi$ is the mass from $Q$ ($\chi$) sector in Eq. (44) (Eq. (D4)). If $\sigma$ mass is only from $Q$ sector, it is very light (always light than $m_\eta$ in minimal case) and is excluded by LHC detections. To have heavy $\sigma$, its mass should be dominated by $\chi$ mass contributions in Eq. (D4). Since the gauge and top sector preserve its shift symmetry, it does not interact with top or gauge bosons through Yukawa or gauge interactions. However, since $\sigma$ is composed by both $\chi$ and $Q$ freedoms, it mainly interacts with SM gauge fields $A^i_\mu$ through WZW terms, which can be parametrized as follows

$$L_{WZW} = \frac{\kappa_i}{32\pi^2} f_\sigma \epsilon^{\mu\nu\lambda\beta} A^i_\mu A^j_\nu A^k_\lambda,$$

(54)

where $f_\sigma \equiv \sqrt{(q_0^2 f_0^2 + 3q_0^2 f_\chi^2/2)/(q_0^2 + q_\chi^2)}$ is the decay constant associated with $\sigma$, $A^i_\mu$ generally denotes the gauge field strength of type $i = W, B, g$ (EW triplet, Hypercharge, gluon). Since $U(1)_\chi$ is the subgroup of $U(1)_Q$ and $U(1)_1$, the coefficients $\kappa_i$ can be obtained from corresponding coefficients of the $U(1)_q$ Goldstone bosons:

$$\kappa_i = \frac{q_0 \kappa_i^Q + q_\chi \kappa_i^\chi}{\sqrt{q_0^2 + q_\chi^2}},$$

(55)

where $\kappa_i^Q, \kappa_i^\chi$ only depends on the coset space of the $Q$ and $\chi$ condensates, and for our case we have

$$\kappa_i^Q = \kappa_i^B = d_Q, \; \kappa_i^\chi = 2d_\chi, \; \kappa_B^B = 12X^2d_\chi,$$

(56)

where $d_Q/d_\chi$ are the dimension of Hypercolor representation of $Q/\chi$ and $X$ is the $U(1)$ Hypercharge defined in TABLE. [1] The main production channel for the $\sigma$ field is through gluon-gluon fusion and the cross-section at proton-proton center-of-mass frame can be parametrized by the partial decay width and the parton luminosities:

$$\sigma(pp \rightarrow \sigma) = \frac{1}{m_\sigma s} C_{gg} \Gamma(\sigma \rightarrow gg),$$

(57)

where $\Gamma(\sigma \rightarrow gg)$ is its decay width to gluon pairs, $s$ is the center-of-mass energy square, the dimensionless partonic integral $C_{gg}$ is

$$C_{gg} = \frac{\pi^2}{8} \int_{m_Z/s}^{1} \frac{dx}{x} g(x) g(m_\sigma^2/8x),$$

(58)

The remarkable feature here is that $\sigma$ interactions with gauge fields are completely fixed by the representations of the preons under $G_{HC}$. So $\sigma$ decay widths can reflect the physics of UV completion. The analytical formulæ
of the partial decay widths to the SM gauge bosons are
\[ \Gamma(\sigma \to gg) = \frac{\alpha_W^2 \kappa_B^2 m_{A}^3}{8 \pi^3} \frac{m_A^3}{f_{A}^2} \]
\[ \Gamma(\sigma \to \gamma\gamma) = \frac{\alpha_W^2}{64 \pi^3} (\kappa_W + \kappa_B) \frac{m_{A}^3}{f_{A}^2} \]
\[ \Gamma(\sigma \to W^+W^-) = \frac{\alpha_W^2 \kappa_W^2 m_{A}^3}{32 \pi^3} \left( 1 - \frac{4 m_W^2}{m_A^2} \right)^{3/2} \]
\[ \Gamma(\sigma \to ZZ) = \frac{\alpha_W^2}{64 \pi^3} \frac{m_{A}^3}{f_{A}^2} \left( 1 - \frac{4 m_Z^2}{m_A^2} \right)^{3/2} \]
\[ \Gamma(\sigma \to Z\gamma) = \frac{\alpha_W^2}{32 \pi^3} \frac{m_{A}^3}{f_{A}^2} \left( 1 - \frac{m_Z^2}{m_A^2} \right)^3 \]
where \( \alpha_W = \alpha/s_W^2 \) and \( s_W(t_W) \) is the sine (tangent) function of Weinberg angle \( \theta_W \). Taking the \( \sigma \to \gamma\gamma \) decay channel as the reference channel, we can obtain the ratios of \( \sigma \) decay widths for \( G_{HC} = SO(11) \), which is only determined by the UV completion and independent on \( f_A \):
\[ \frac{\Gamma_{gg}}{\Gamma_{\gamma\gamma}} = 870, \quad \frac{\Gamma_{WW}}{\Gamma_{\gamma\gamma}} = 4.2, \quad \frac{\Gamma_{ZZ}}{\Gamma_{\gamma\gamma}} = 0.416, \quad \frac{\Gamma_{Z\gamma}}{\Gamma_{\gamma\gamma}} = 3.1, \]
and for \( G_{HC} = SO(13) \):
\[ \frac{\Gamma_{gg}}{\Gamma_{\gamma\gamma}} = 711, \quad \frac{\Gamma_{WW}}{\Gamma_{\gamma\gamma}} = 2.4, \quad \frac{\Gamma_{ZZ}}{\Gamma_{\gamma\gamma}} = 0.17, \quad \frac{\Gamma_{Z\gamma}}{\Gamma_{\gamma\gamma}} = 2.3. \]
For \( G_{HC} = Sp(2N_{HC}) \) with \( N_{HC} = 2 \), the ratios are
\[ \frac{\Gamma_{gg}}{\Gamma_{\gamma\gamma}} = 68399, \quad \frac{\Gamma_{WW}}{\Gamma_{\gamma\gamma}} = 713.6, \quad \frac{\Gamma_{ZZ}}{\Gamma_{\gamma\gamma}} = 144, \quad \frac{\Gamma_{Z\gamma}}{\Gamma_{\gamma\gamma}} = 206.2. \]
For the maximum situation, \( N_{HC} = 18 \), we obtain
\[ \frac{\Gamma_{gg}}{\Gamma_{\gamma\gamma}} = 76439.5, \quad \frac{\Gamma_{WW}}{\Gamma_{\gamma\gamma}} = 1179.7, \]
\[ \frac{\Gamma_{ZZ}}{\Gamma_{\gamma\gamma}} = 259.8, \quad \frac{\Gamma_{Z\gamma}}{\Gamma_{\gamma\gamma}} = 282.4. \]
From the above calculations, we can explicitly see that the decay channel into gluon pairs is dominant over the other channels.

Using WZW interaction in Eq. [4], we simulate different signatures of \( \sigma \) in LHC from the following channels,
\[ gg \to \sigma \to A_i A^i, \quad (59) \]
where \( A = \{ W^\pm, Z, \gamma, g \} \). By comparing with the experimental data from 8 TeV [31,35] and 13 TeV LHC [36,40], we derive the bounds of \( m_{\sigma} \) for different Hypercolor group with \( f_A = 800 \) GeV held fixed as shown in Fig. 3 (the color regions are excluded parameter spaces).

For \( G_{HC} = Sp(2N_{HC}) \) model, the bounds of \( \sigma \) increase as \( N_{HC} \) increases. For \( G_{HC} = SO(11/13) \) hypercolor model, the strongest constraints come from \( Z\gamma \) decay channel and we find \( m_{\sigma} < 2.6 \) TeV is excluded for \( SO(11) \) hypercolor group and \( m_{\sigma} < 2.8 \) TeV is excluded for \( SO(13) \).

VI. CONCLUSIONS

We studied the minimal composite Higgs model \( SU(4)/Sp(4) \) with purely fermionic UV completions based on a confining Hypercolor gauge group \( G_{HC} \). Under this strong dynamics, two species of underlying Weyl fermions in different representations of \( G_{HC}, Q_1, ... , Q_6 \) (QCD colorless) and \( \chi_1, ..., \chi_9 \) (QCD colored), should be introduced to generate the composite Higgs doublet, composed by \( Q \) alone, as well as top partners, composed by both \( Q \) and \( \chi \). Different from ordinary composite Higgs models, the Higgs potential is not only from top and gauge loop corrections but also from the masses of preon \( Q \). With this extra contribution, electroweak symmetry breaking can be realized differently and the correlation between the mass of Higgs and top partners is weakened.

To keep the Higgs potential from the top sector finite, we impose maximal symmetry in this model. Since the maximal symmetry can be realized in two different ways, we study its EWSB in two cases, the ordinary and minimal maximal symmetric CHMs. In the first case, even the Higgs potential from the top and gauge sector is sensitive to composite resonance mass scales, the Higgs mass is only sensitive to the mass scale difference between composite vector mesons and top partners. So the composite partners of top and gauge bosons can be as heavy as possible, even around the cut-off \( \sim 4 \pi f \), for successful EWSB just at the cost of high fine tuning. While the top partner mass is around scale \( f \) for light Higgs in ordinary CHMs without preon mass contributions no matter how to tune the parameters. However, since \( \eta \) mass is related with the top partner mass if it is only from preon masses, positive \( \eta \) mass square imposes the upper bounds on top partner mass, such as \( M < 1.6 \) TeV.
for $\xi = 0.05$ and $m_{\eta}^2 > 0$. But the mass of $\eta$ can also be generated from the hidden interactions which only break $\eta$ shift symmetry. If this contribution is dominant in $\eta$ mass, the correlation between the mass of top partners and $\eta$ is destructed and thus the composite top partners can be heavy arbitrarily (generally it should be smaller than cut-off scale) without any constraints. In the minimal maximal symmetric case, the Higgs quartic from the top sector is suppressed at quartic order in top Yukawa coupling and is not sensitive to top partner scale $M_f$. However, if without preon mass contributions, the Higgs mass is always too light even $M_f$ is around cut-off scale, which always results in double tuning $\sim 95/\xi$. While the extra Higgs potential from preon mass can enhance the Higgs quartic so the Higgs mass can be heavy enough for several TeV top partners with $\xi = 0.1$. Meanwhile, the tuning is significantly suppressed as low as minimal $\sim 1/\xi$ for $M > 1.5$ TeV. The $\eta$ mass is also insensitive to $M_f$ so it can be heavy enough to avoid the bounds $m_\eta > m_h/2$ and is almost fixed for fixed $\xi$ if its mass is only from preon mass sector (around 300 GeV for $\xi = 0.1$).

The partial compositeness predicts an extra $U(1)_\sigma$ pNGB $\sigma$ in this model. Since it contains both the freedoms of $Q$ and $\chi$, it can interact with SM EW and QCD gauge bosons through Wess-Zumino-Witten terms which is determined by UV completions. Especially its branch ratio into different gauge boson pairs can reveal the UV theory. This singlet can be resonance produced through gluon fusion and decay into gauge boson pairs. This can be the typical phenomenology of this kind of model at LHC and we derive the bounds of $\sigma$ mass for different $G_{HC}$ gauge group.

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Appendix A: Top partners and form factors in top effective Lagrangia

For the top partners $\Psi_{5,1}$ in the Lagrangian (24), their explicit embedding in representation 6 of $SU(4)$ are

$$
\Psi_{5L} = \left( \frac{1}{2} T_5 i \sigma^2 - \frac{1}{\sqrt{2}} Q T \right), \quad Q = \left( \begin{array}{c} T \ X_{5/3} \\ B \ X_{2/3} \end{array} \right) 
$$

$$
\Psi_{1L} = \left( \frac{1}{2} T_1 i \sigma^2 0 \right) \quad 0 - \frac{1}{2} T_1 i \sigma^2 
$$

$$
\Psi_{cR}^c = \left( \frac{1}{2} T_5 i \sigma^2 - \frac{1}{\sqrt{2}} Q T \right), \quad Q^c = \left( -X_{2/3} \ B^c \ X_{5/3} \ X_{5/3} \right) 
$$

$$
\Psi_{c}^c = \left( \frac{1}{2} T_1 i \sigma^2 0 \right) \quad 0 - \frac{1}{2} T_1 i \sigma^2 
$$

The form factors in Eq. (26) and masses of top partners in Eq. (27) in an ordinary maximal symmetric model can be expressed as

$$
\Pi_0^q = 1 - \frac{\lambda_2^2 f^2}{p^2 - M^2}, \quad \Pi_0^t = 1 - \frac{\lambda_2^2 f^2}{p^2 - M^2}, \quad M_{T_1} = \frac{\lambda_2^2 f^2 M^2}{M^2 - p^2} 
$$

$$
M_{T_2} = \sqrt{f^2 \lambda_2^2 + M^2}, \quad M_{T_3} = \sqrt{f^2 \lambda_2^2 + M^2}. \quad (A2)
$$

The form factors in Eq. (32) and masses of top partners in Eq. (33) in the minimal maximal symmetric model can be expressed as

$$
\Pi_0^q = 1 - \frac{\lambda_2^2 f^2}{p^2 - M^2}, \quad \Pi_0^t = 1 - \frac{4 \lambda_2^2 f^2}{p^2 - M^2}, \quad M_{T_1} = \frac{\lambda_2^2 f^2 M^2}{p^2 - M^2} 
$$

$$
M_{T_2} = \sqrt{4 f^2 \lambda_2^2 + M^2}. \quad (A3)
$$

Appendix B: Gauge sector

According to the hidden local symmetry, the vector resonances $\rho_\mu$ transform non-linearly, while the axial resonances $a_\mu$ transform homogeneously, under a global $SU(4)$ transformation $g$,

$$
\rho_\mu = a^0_\mu T^a, \quad \rho_\mu \rightarrow h \rho_\mu h^\dagger + \frac{i}{g_\rho} h \partial_\mu h^\dagger 
$$

$$
a_\mu = a^0_\mu T^a, \quad a_\mu \rightarrow h a_\mu h^\dagger, \quad (B1)
$$

where $h = h(g, \pi^a)$ is the non-linearly realised $Sp(4)$ element. So at leading order in derivatives, the general Lagrangian allowed by Eq.(B1) is

$$
\mathcal{L}_\rho = -\frac{1}{2} \text{Tr} [\rho_\mu \rho_\nu^\dagger] + f_\rho^2 \text{Tr} [(g_\rho \rho_\mu - E_\mu^\rho T^a)^2] 
$$

$$
\mathcal{L}_a = -\frac{1}{2} \text{Tr} [a_\mu a_\nu^\dagger] + \frac{f_a^2}{\Delta^2} \text{Tr} [(a_\mu a_\mu - \Delta a_\mu^2 T^a)^2]. \quad (B2)
$$

where $iu^\dagger D_\mu U = d_\mu^a T^a + E_\mu^a T^a, \quad \rho_\mu = \partial_\mu \rho - \partial_\rho \rho_\mu - g_\rho [\rho_\mu, \rho_\nu], \quad a_\mu = \nabla_\mu a - \nabla_\nu a_\mu$ and $\nabla_\mu = \partial_\mu - i E_\mu^a T^a$. 

After integrating out the heavy resonances at tree level, the $SU(4)$ invariant Lagrangian, at quadratic order in the gauge fields and in momentum space, is
\[
\mathcal{L}^{\text{eff}} = \frac{P_{\mu\nu}}{2} \left( \Pi_0(p^2) \text{Tr}[A_\mu A_\nu] - p^2(W_\mu^a W_\mu^a + B_\mu B_\nu) \right) \\
+ \frac{\Pi_1(p^2)}{4} \left[ \text{Tr}[A_\mu \Sigma + \Sigma A_\mu^T] (A_\nu \Sigma + \Sigma A_\nu^T)^\dagger \right],
\]
where $A_\mu = g W_\mu^a T_L^a + g' B_\mu T_R^a$, $P_{\mu\nu} = g^{\mu\nu} - p^\mu p^\nu / p^2$ is the projector on transverse field configurations and $\Pi_{0,1}$ are form factors. From above Lagrangian, we get the most general effective Lagrangian for gauge bosons with explicit dependence on the Higgs field:
\[
\mathcal{L}^{\text{eff}} = \frac{P_{\mu\nu}}{2} \left( g^2 \Pi_0^W W^a_\mu W^a_\mu + g^2 \Pi_0^B B_\mu B_\nu \right) \\
+ \frac{g^2 \Pi_1}{h^2 + \eta^2} \left( W^3_\mu W^3_\mu + W^2_\mu W^2_\mu \right) \\
+ \frac{1}{h^2 + \eta^2} \left( g' B_\mu - g W^3_\mu / (g' B_\mu - g W^3_\mu) \right),
\]
where $s = \sin(\sqrt{h^2 + \eta^2} / f)$,
\[
\Pi_0^W = - \frac{p^2}{g^2} + p^2 - m^2_{\rho} / p^2, \quad \Pi_0^B = \Pi_0^W (g \rightarrow g'), \\
\Pi_1 = f^2 + 2p^2 \left( \frac{f_\rho^2}{p^2 - m^2_{\rho}} - \frac{f^2_\rho}{p^2} \right).
\]

**Appendix C: A mechanism to producing heavy $\eta$**

In gauge and top sector, these interactions are $U(1)_\eta$ invariant so the SM fields loops do not contribute to $\eta$ potential. To produce a heavy $\eta$ while preserving Higgs and $\sigma$ shift symmetry, we introduce an electroweak singlet complex scalar $\phi$. To break $\eta$ shift symmetry, we suppose it is embedded in $6$ representation of $SU(4)$ in the form
\[
\Phi = \frac{\phi}{2} \begin{pmatrix} i \sigma_2 & 0 \\ 0 & i \sigma_2 \end{pmatrix}.
\]
So its general couplings to the pNGBs are given by
\[
\mathcal{L}_\phi = \partial_\mu \phi_\dagger \partial^\mu \phi - m^2_\phi \phi_\dagger \phi - y_\phi f^2 \text{Tr}[\Phi \Sigma^\dagger] \text{Tr}[\Phi \Sigma]
\]
\[
= \partial_\mu \phi_\dagger \partial^\mu \phi - m^2_\phi \phi_\dagger \phi - 4y_\phi f^2 \frac{\eta^2}{\pi Q} \sin^2 \left( \frac{\pi Q}{f} \right) \phi_\dagger \phi.
\]
The pNGB potential at one $\phi$ loop level is
\[
V_\eta \sim \frac{y_\phi f^2 C_\phi}{(4\pi)^2} \frac{\eta^2}{\pi Q} \sin^2 \left( \frac{\pi Q}{f} \right) \Lambda_Q^2,
\]
where $\Lambda_Q \sim 4\pi f$ is the condensate scale of preon $Q$ and $C_\phi$ is order one constant. Generally, $y_\phi C_\phi$ can be positive so $\eta$ becomes massive from the scalar loop. Its mass is naturally at $O(f)$ so $\eta$ can be heavy enough to survive experimental bounds without any fine tuning.

**Appendix D: Mass of $\sigma$ from $\chi$ sector**

The $\chi$ preon mass can explicitly break the shift symmetry of $\sigma$ and thus can contribute to $\sigma$ mass. The gauge invariant mass term of preon $\chi$ can be aligned with condensation $\Sigma_{\chi 0}$.
\[
\mathcal{L}_{\text{mass}}^\chi = m_\chi \chi_\dagger \chi_\chi_0 + h.c.,
\]
where $m_\chi$ is the preon mass. Notice that we can formally keep the $SU(6)$ invariance by assigning the following transformation rules to the mass matrix
\[
\Sigma_{\chi 0} \rightarrow g_\chi^* \Sigma_{\chi 0} g_\chi^\dagger,
\]
where $g_\chi \in SU(6)$. Similarly, $\sigma$ potential from $\chi$ masses can be easily derived according to the global symmetry
\[
V_\sigma = - C_\chi f^3 m_\chi \text{Tr} \left[ \Sigma_{\chi 0} \Sigma_{\chi} \right] + h.c.
\]
\[
= - 12 C_\chi m_\chi f^3 \cos \left( 2 \sigma \sin \phi / \sqrt{6} f_6 \right),
\]
where $C_\chi \sim \langle \chi \chi \rangle / (4\pi)^2 f^3$ is the form factor related to strong dynamics [16]. We can also read the $\sigma$ mass from Eq. [D3],
\[
m_\sigma^2 = 2 \sin \phi \sqrt{2C_\chi m_\chi f_6}.
\]

[1] D. B. Kaplan and H. Georgi, Phys. Lett. 136B, 183 (1984).

[2] H. Georgi and D. B. Kaplan, Phys. Lett. 145B, 216 (1984).
The other orthogonal combination of $U(1)_{Q_X}$ corresponds to $U(1)_{\sigma}$, which has anomaly with $Sp(2N_{HC})$, and the associated scalar $\sigma$ can get heavy enough mass from anomaly and $Sp(2N_{HC})$ instanton effects thus decouple from the theory at lower energy. $
abla_1, \nabla_2$ and $\psi_4$ have different $U(1)_{\sigma}$ charge so if the top fields couple to two of them simultaneously, the $U(1)_{\sigma}$ symmetry is broken and the potential of $\sigma$ should be proportional to the product of their mixing couplings. If top quarks only mix with one multiplet of top partners, $U(1)_{\sigma}$ is always preserved and thus top loops do not con-
tribute to $\sigma$ potential.

[43] MS is the global symmetry in composite sector while the preon mass terms are in the representation of global symmetry $SU(4)$ and explicitly break it. So the mass terms do not influence maximal symmetry.