On ve-Degree Irregularity Indices

Abdulgani Şahin

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Abstract: In this paper, vertex-edge degrees (or simply, ve-degrees) of vertices in a graph are considered. The ve-degree of a vertex \( v \) in a graph equals to the number of different edges which are incident to a vertex from the closed neighborhood of \( v \). The author introduces the ve-degree total irregularity index here and calculates this index for paths and double star graphs. Finally, the maximal trees are characterized with respect to the ve-degree total irregularity index.

Keywords: ve-degree, Albertson index, ve-degree irregularity, ve-degree total irregularity.

INTRODUCTION

The examination of molecular structures expressed through graphs is one of the important pillars of graph applications. In an undirected graph, the degree sequence is a uniform sequence of the degrees of its vertices that does not increase. Invariants belonging to graphs are most commonly referred to as topological indices and they are often stated using the degrees of vertices, distances between vertices, eigenvalues, symmetries, and many other properties of the graphs. The term topological index first appeared in a study by Wiener. Consider a simple and finite graph. Let \( G \) be this graph with the set of vertices \( V(G) \) and the set of edges \( E(G) \). The degree of a vertex \( u \) of the graph \( G \) is the number of adjacent vertices with \( u \) in \( G \) and it is indicated by \( \text{deg}(u) \).

A graph \( G \) is called regular if all its vertices have the same degree. A graph that is not regular is called irregular. Albertson stated the graph invariant as \( \text{irr}(G) = \sum_{v \in V(G)} |\text{deg}(u) - \text{deg}(v)| \) and named it as irregularity of the graph \( G \). In other words, the Albertson index and irregularity mean the same thing. He obtained some upper bounds for trees, bipartite graphs and triangle-free graphs in his study. Graphs with the maximal irregularity were characterized by Abdo et al. They took a different approach than Albertson and found a sharp upper bound for graphs with \( n \) vertices and some lower bounds on the maximal irregularity of graphs. Also, the total version of the Albertson index was recently defined by Abdo et al. They determined all graphs with maximal total irregularity.
irregularity was made in\textsuperscript{10} and some inequalities were obtained for connected graphs and trees. Moreover, some well-known irregularity measures were compared\textsuperscript{11} and it was shown that any two of these irregularity indices are mutually inconsistent. This means that it is difficult to decide definitively which is more and which is less irregular. Gutman demonstrated the calculations of the irregularity measures on molecular graphs and made comparisons between these results.\textsuperscript{12} The trees which were the most and least irregular were characterized according to the Albertson index.\textsuperscript{13} The irregularity measure based on eigenvalues of graphs described by Collatz and Sinogowitz\textsuperscript{14} has been the oldest known numerical irregularity measure. Bell introduced a second such measure based on the variance of the vertex degrees of a graph, another irregularity measure.\textsuperscript{15} He determined the most irregular graphs with respect to these two measures. More details about the irregularity of the graphs can be found in the book.\textsuperscript{16}

Domination is one of the most important graph invariants. A subset $D \subseteq V(G)$ is a dominating set, if every vertex in $G$ either is an element of $D$ or is adjacent to at least one member of $D$. The domination number is the number of vertices in a smallest dominating set for $G$.\textsuperscript{17} Domination has been shown to be a very sensitive graph theoretical invariant to even the slightest changes in a graph.\textsuperscript{18} Domination was studied for chemical materials in recent years.\textsuperscript{19} The lower and upper bounds on the domination numbers in different graphs were studied.\textsuperscript{20} Also, total edge-vertex domination was introduced recently.\textsuperscript{21}

Chellali et al. introduced two degree concepts: ve-degree and ev-degree of the graphs based on ve-domination and ev-domination.\textsuperscript{22} The ve-degree of a vertex $v \in V(G)$ equals the number of edges ve-dominated by $v$. The ev-degree of an edge $e = uv$ equals the number of vertices ev-dominated by $e$. The regularity and irregularity of graphs about ve-degree and ev-degree were studied by Horoldagva et al.\textsuperscript{23} A graph is ve-regular if all its vertices have the same ve-degree. A graph is ev-regular if all its edges have the same ev-degree. A graph $G$ is called ve-irregular if no two vertices in $V(G)$ have the same ve-degree. A graph $G$ is called ev-irregular if no two edges in $E(G)$ have the same ev-degree.

The ve-degree and ev-degree concepts of graphs were widely applied to Chemical Graph Theory.\textsuperscript{24} Many papers were written about the modified versions of the various topological indices with respect to ve-degree and ev-degree. Some chemical materials were investigated with these modified versions of the topological indices. For example, the ve-degree and ev-degree based topological properties of single walled titanium dioxide nanotube,\textsuperscript{25} $h$-naphtalenic nanotube,\textsuperscript{26} silicon carbide $SiC_{r}H[p,g]$,\textsuperscript{27} two carbon nanotubes,\textsuperscript{28} polycyclic graphite carbon nitride\textsuperscript{29} and crystallographic structure of cuprite $Cu_{2}O$\textsuperscript{30} were studied. It has been seen that ve-degree and ev-degree topological indices can be used as possible tools in QSPR researches.

The ve-degree irregularity index was defined recently.\textsuperscript{31} The definition of this concept is presented in the second section. Moreover, the maximal trees were characterized with respect to this index.\textsuperscript{32} In this paper, I define the ve-degree total irregularity index and compute this index for paths and double star graphs. Finally, I obtain the maximal trees with respect to the ve-degree total irregularity index.

**PRELIMINARIES**

Let $G$ be a simple graph with the vertex set $V(G)$ and the edge set $E(G)$ such that $|V(G)| = n$ and $|E(G)| = m$. For a vertex $u \in V(G)$, the open neighborhood of $u$ is defined as $N_{u}(u) = \{ v \mid uv \in E(G) \}$ and the closed neighborhood of $u$ is defined as $N_{u}(u) = \{ u \} \cup N_{u}(u)$.

The degree of a vertex $u$ is the cardinality of $N_{u}(u)$ and it is denoted by $\deg(u)$. A vertex which has degree one is called a leaf. The ve-degree of a vertex $v$ equals to the number of different edges which are incident to a vertex from the closed neighborhood of $v$ and it is denoted by $\deg_{v}(v)$. Moreover, the ev-degree of an edge $e = ab$ equals to the number of vertices of the union of the closed neighborhoods of $a$ and $b$, it is denoted by $\deg_{e}(e)$.

A graph $G$ is ve-regular if all its vertices have the same ve-degree. The paths, cycles, complete graphs and stars of order $n$ are denoted by $P_{n}$, $C_{n}$, $K_{n}$, and $S_{n,1,\ldots}$, respectively. The double star graphs $DS_{n,a}$ are consisted of the stars $S_{a}$ and $S_{b}$ such that $n = p + q + 2$. The subdivided star $S_{e}$ is obtained from a star $S_{e}$ by adding a vertex to every leaf of the star.
It is known that $S_{n,1}$ is ve-regular tree such that all its vertices have same ve-degree $n - 1$.\cite{21} The cycle $C_n$ $(n \geq 4)$ is the unique unicyclic graph which is ve-regular.\cite{22}

For simplicity, a ve-regular graph, each of whose vertices has ve-degree $r$, is called $r$-ve-regular.\cite{22} For example, the cycle graph is $4$-ve-regular for $n \geq 4$. Furthermore, the complete graph $K_n$ is $m$-ve-regular such that the size $m = n(n - 1) / 2$.

**Definition 2.1.** For a connected graph $G$,

$$M_1(G) = \sum_{u \in V(G)} \deg^2(u).$$\cite{22}

**Definition 2.2.** For a connected graph $G$,

$$\sum_{u \in V(G)} \deg(u) = \sum_{e \in E(G)} \deg_e(e) = M_1(G) - 3n_0$$

such that $n_0$ is the total number of triangles contained in $G$.\cite{23}

It implies that for a triangle-free graph $G$,

$$\sum_{u \in V(G)} \deg(u) = \sum_{e \in E(G)} \deg_e(e) = M_1(G).$$

**Definition 2.3.** Let $G$ be a triangle-free graph. Then, for a vertex $u \in V(G)$

$$\deg_v(u) = \sum_{v \in N(u)} \deg(u).$$\cite{24}

**Definition 2.4.** Let $G$ be a triangle-free graph. Then, for an edge $e = ab \in E(G)$

$$\deg_e(e) = \deg(a) + \deg(b).$$\cite{24}

**Definition 2.5.** Let $G$ be a graph of order $n$. Then, the Albertson index of $G$ is computed by

$$irr(G) = \sum_{u \in V(G)} \deg(u) - \deg(v).$$\cite{7}

**Definition 2.6.** Let $G$ be a graph of order $n$. Then, the total irregularity index of $G$ is computed by

$$irr(G) = \frac{1}{2} \sum_{u \in V(G)} \left| \deg(u) - \deg(v) \right|.$$\cite{8}

If the degrees of vertices are ordered as $\deg(v_1) \geq \deg(v_2) \geq \ldots \geq \deg(v_n)$, the total irregularity index can be calculated by

$$irr(G) = \sum_{i=1}^{n-1} (\deg(v_i) - \deg(v_{i+1})).$$

**Lemma 2.7.** If $T$ is a tree of order $n$, then

i) $irr(T) \leq (n - 1)(n - 2)$ such that the equality holds if and only if $T$ is a star.\cite{13}

ii) $irr(T) \leq (n - 1)(n - 2)$ such that the equality holds if and only if $T$ is a star.\cite{9}

The ve-degree version of the Albertson index was expressed as in the following definition.

**Definition 2.8.** Let $G$ be a graph of order $n$. Then, the ve-degree irregularity of $G$ is computed by

$$irr_v(G) = \sum_{u \in V(G)} \left| \deg_v(u) - \deg_v(v) \right|.$$\cite{9}

It is clear that $irr_v(G) = 0$ for ve-regular graphs. The total ve-degree irregularity index can be defined as follows.

**Definition 2.9.** Let $G$ be a graph of order $n$. Then, the ve-degree total irregularity index of $G$ is computed by

$$irr_v(G) = \sum_{u \in V(G)} \left| \deg_v(u) - \deg_v(v) \right|.$$\cite{9}

If the ve-degrees of vertices are ordered as $\deg_{v_1}(v_i) \geq \deg_{v_2}(v_i) \geq \ldots \geq \deg_{v_n}(v_i)$, the ve-degree total irregularity index can be calculated by

$$irr_v(G) = \sum_{i=1}^{n-1} (\deg_v(v_i) - \deg_v(v_{i+1})).$$

It is denoted the ve-degree sequence by the notation $\deg_{v_1}(v_i), \deg_{v_2}(v_i), \ldots, \deg_{v_n}(v_i)$ for $\deg_{v_1}(v_i) \geq \deg_{v_2}(v_i) \geq \ldots \geq \deg_{v_n}(v_i)$. The repeated degrees can be shown by exponential numbers.

### MAIN RESULTS

**Theorem 3.1.** For paths,

$$irr_v(P_n) = \begin{cases} 0, & n = 2,3 \\ 6(n - 4) + 4, & n \geq 4 \end{cases}.$$\cite{9}

**Proof.** If a path has two or three vertices, their ve-degrees are equal. Then, the total ve-degree irregularity index equals to zero. Consider a path graph $P_n : v_1 \rightarrow v_2 \ldots \rightarrow v_n$ with $n \geq 4$. It is obtained that $\deg_{v_1}(v_i) = \deg_{v_2}(v_i) = 2$, $\deg_{v_3}(v_i) = \deg_{v_4}(v_i) = 3$ and $\deg_{v_5}(v_i) = 4$ for $3 \leq i \leq n - 2$. Therefore, I have the ve-degree sequence of paths as $[4,3^2,2^4]$. The ve-degree total irregularity index can be obtained by

$$irr_v(P_n) = 2(n - 4)(4 - 3) + 2(n - 4)(4 - 2) + 3(3 - 2) = 6(n - 4) + 4.$$\cite{9}

**Theorem 3.2.** Let $DS_{p,q}$ be a double star graph of order $n = p + q + 2$. Then,

i) $irr_v(DS_{p,q}) = 2pq$

ii) $irr_v(DS_{p,q}) = pq(p - q + 4)$.

**Proof.** It is known that a double star graph $DS_{p,q}$ consists of two stars $S_{p,q}$ and $S_{q,p}$. Assume that $p \geq q$. It means that there are $p + q$ vertices having degree one and two central vertices of stars having degree $p + 1$ and $q + 1$, respectively. Now the ve-degrees of the vertices in $DS_{p,q}$ are determined. The central vertices of stars ve-dominate all edges. Then, the ve-degrees of central vertices are $p + q + 1$. DOI: 10.5562/cca3813

Croat. Chem. Acta 2021, 94(2), 133–138
The ve-degree of the leaves of $S_{k,a}$ is $p + 1$ and the ve-degree of the leaves of $S_{k,a}$ is $q + 1$. Then, I obtain the ve-degree sequence of $DS_{a,x}$ as $\{(p + q + 1)^2, (p + q + 1)^2, (q + 1)^2\}$.

1. $\text{irr}_{ve}(DS_{a,x}) = p[p + q + 1 - (p + 1)] + q[p + q + 1 - (q + 1)] = 2pq$.

2. $\text{irr}_{ve}(DS_{a,x}) = 2[p[p + q + 1 - (p + 1)] + 2q[p + q + 1 - (q + 1)] + pq(p + q + 1 - q + 1)] = 2pq + 2pq + pq(p - q) = pq(p - q + 4)$.

Theorem 3.3. Let $T$ be a simple tree with order $n$, then

1. $\text{irr}_{ve}(T) \leq \frac{n^3 + n^2 - 17n + 15}{8}$

if $n = 2k + 1$ and the equality holds if and only if $T \cong S_k$.

2. $\text{irr}_{ve}(T) \leq \frac{n^3 + 5n^2 - 17n + 11}{8}$

if $n = 2k + 2$ and the equality holds if and only if $T \cong T_r$, which is indicated in Figure 1.

Proof. In order to prove the inequalities, I apply some operations to $S_{k,a}$ graphs. It is clear that the star graphs are ve-regular graphs. Then, the ve-degree irregularity index and ve-degree total irregularity index of stars equal to 0.

1. Assume that $n = 2k + 1$. If I remove a leaf from a star $S_{k,a}$ and attach it to another leaf, I obtain double star graph $T_1 = DS_{a,x}$, which is indicated in Figure 1. Then, $T_1$ is a double star graph which consists of a star graph $S_{k,s}$ with the central vertex $u$ and a path $P_2$ such that $u$ is joined to $x$. Thus, $\text{deg}_{ve}(y) = 2$, $\text{deg}_{ve}(x) = \text{deg}_{ve}(u) = n - 1$ and the remaining $(n - 3)$ vertices have ve-degree $(n - 2)$. Then, its ve-degree sequence is $\{(n - 3),(n - 2)^2\}$. So, $\text{irr}_{ve}(T_1) = 2(n - 3) + 2(n - 3) + 2(n - 3)(n - 3) = n^2 - 3n$.

If I remove a leaf and attach it (say that $z$) to the vertex $y$ on $T_r$, I obtain the tree $T_s$, which is indicated in Figure 1. It is obtained that $\text{deg}_{ve}(z) = 2$, $\text{deg}_{ve}(y) = 3$, $\text{deg}_{ve}(x) = n - 1$, $\text{deg}_{ve}(u) = n - 2$ and the remaining $(n - 4)$ vertices have ve-degree $(n - 3)$ for $T_s$. Then, the ve-degree sequence of $T_s$ is $\{n - 1, n - 2, (n - 3)^2, 3, 2\}$. So, $\text{irr}_{ve}(T_s) = 1 + 2(n - 4) + n - 4 + n - 4 + n - 4 + n - 4 + n - 5 + n - 4 + (n - 4)(n - 6) + (n - 4)(n - 5) + 1$.

Then, it can be seen that $\text{irr}_{ve}(T_r) - \text{irr}_{ve}(T_s) = n^2 - 9n + 18 > 0$ for $n \geq 7$. If the operation which is used in the transformation from $T_r$ to $T_s$ is used $(n - 3)$-times to a star, I obtain a path at the end.

Now I remove a leaf $s$ and attach it to a vertex $r$ which is incident to the central vertex $u$ on $T_r$. Thus, I obtain the tree $T_{s,t}$, which is indicated in Figure 1. For the tree $T_{s,t}$, $\text{deg}_{ve}(y) = \text{deg}_{ve}(s) = 2$, $\text{deg}_{ve}(x) = \text{deg}_{ve}(r) = n - 2$, $\text{deg}_{ve}(u) = n - 1$ and the remaining $(n - 5)$ vertices have ve-degree $(n - 3)$. Then, the ve-degree sequence of $T_{s,t}$ is $\{n - 1, (n - 2)^2, (n - 3)^2, 2\}$. So, $\text{irr}_{ve}(T_{s,t}) = 2 + 2(n - 5) + 2(n - 3) + 2(n - 5) + 4(n - 4) + 2(n - 5)^2$.

Consequently, $\text{irr}_{ve}(T_{s,t}) - \text{irr}_{ve}(T_r) = 2n - 8 \geq 0$ for $n \geq 4$. It is seen that the difference of the ve-degree total irregularity
index between $T_k$ and $T_k$ is greater than the difference of the ve-degree total irregularity index between $T_k$ and $T_k$. By this way, I obtain the subdivided star graph $S'_k$ which is the maximal tree with respect to the ve-degree total irregularity index with order $n = 2k + 1$. For $S'_k$, the ve-degree of the central vertex $u$ is $2k$, the ve-degree of the vertices at distance 1 from $u$ is $(k + 1)$ and the ve-degree of the vertices at distance 2 from $u$ is $2$. Therefore, the ve-degree sequence of $S'_k$ is $[2k, (k + 1)^2, 2^2]$. So,

\[
\text{irr}_k(S'_k) = k(k - 1) + k(2k - 2) + k^2(k - 1)
\]

\[
\text{irr}_k(S'_k) = k(k - 1) + 2k - 1 + k^2(k - 1) + k(k - 1)^2 + k
\]

\[
\text{irr}_k(S'_k) = \frac{(n - 1)^3}{2} + 3\left(\frac{n - 1}{2}\right)^2 + 4
\]

\[
\text{irr}_k(S'_k) = \frac{n^2 + 3n^2 - 9n + 37}{8}
\]

For the tree $T_k$, $\text{deg}_m(u) = 2$, $\text{deg}_m(v) = 2$, $\text{deg}_m(w) = 2$, the ve-degree of the $k$ vertices at distance 1 from $u$ is $(k + 1)$ and the ve-degree of the $k$ vertices at distance 2 from $u$ is $2$. Then, the ve-degree sequence of $T_k$ is $[2k + 1, (k + 2)^2, 2, 2]$. So,

\[
\text{irr}_k(T_k) = k(k - 1) + k(2k - 1) + k + k^2 + (k - 1)(k - 1)
\]

\[
\text{irr}_k(T_k) = k^3 + 4k^2 - k
\]

\[
\text{irr}_k(T_k) = \left(\frac{n - 1}{2}\right)^3 + 4\left(\frac{n - 1}{2}\right)^2 + \frac{n - 1}{2}
\]

\[
\text{irr}_k(T_k) = \frac{n^3 + 5n^2 - 17n + 11}{8}
\]

It implies that $T_k$ has the maximal ve-degree total irregularity index of even order in the trees.

**CONCLUSION**

After the introduction of the ve-degree irregularity index\(^{[31]}\) the ve-degree total irregularity index is defined in this paper. Moreover, the ve-degree total irregularity index of paths and double star graphs are obtained, and the maximal graphs with respect to this index are attained. Consequently, the present paper is a contribution to find the ve-degree based topological indices in different sciences. By means of the ve-degree based topological indices, the number of tools which are used in the computation of graph irregularity is increased.

As the paralleling of the rapid growing of science and technology, the importance of analysing in networks is increased. Then, the ve-degree irregularity indices may be used in the computation of the chemical, biological and other properties of chemical materials.

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**Croat. Chem. Acta** 2021, 94(2), 133–138
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