Lu Cao · You-Chang Yang · Hong Chen

Charmonium States in QCD-Inspired Quark Potential Model Using Gaussian Expansion Method

Received: 12 February 2012 / Accepted: 21 July 2012 / Published online: 10 August 2012 © Springer-Verlag 2012

Abstract We investigate the mass spectrum and electromagnetic processes of charmonium system with the nonperturbative treatment for the spin-dependent potentials, comparing the pure scalar and scalar-vector mixing linear confining potentials. It is revealed that the scalar-vector mixing confinement would be important for reproducing the mass spectrum and decay widths, and therein the vector component is predicted to be around 22%. With the state wave functions obtained via the full-potential Hamiltonian, the long-standing discrepancy in M1 radiative transitions of $J/\psi$ and $\psi'$ are alleviated. This work also provides an inspection and suggestion for the possible $c\bar{c}$ states among the copious higher charmonium-like states. Particularly, the newly observed $X(4160)$ and $X(4350)$ are found in the charmonium family mass spectrum as $M(2^1D_2) = 4164.9$ MeV and $M(3^3P_2) = 4352.4$ MeV, which strongly favor the $J^{PC} = 2^{++}$, $2^{−+}$ assignments respectively. The corresponding radiative transitions, leptonic and two-photon decay widths have been also predicted theoretically for the further experimental search.

1 Introduction

Due to the impressive increase of experimental results, charmonium ($c\bar{c}$) spectroscopy has renewed great interest recently, coming along with the striking disagreement with theoretical expectations [1–3]. The unexpected and still-fascinating $X(3872)$ has been joined by more than a dozen other charmonium-like states, while the series of vacancy have been left on the $c\bar{c}$ list. It is urgent to identify the possible new members of charmonium family from the abundant observations.

The QCD inspired potential models have been playing an important role in investigating heavy quarkonium, owning to the presence of large nonperturbative effects in this energy region. Most quark potential models [4–16] have common ingredients under the non-relativistic limit, despite some differences in the detailed corrections regarding relativistic and coupled channel effects, which typically are the Coulomb-like term induced by one-gluon exchange plus the long-range confining potential expected from QCD. Anyway, the nature of confining mechanism has been veiled so far. In the original Cornell model [17, 18], it was assumed as Lorentz scalar, which gives a vanishing long-range magnetic contribution and agrees with the flux tube picture of quark confinement [19]. Another possibility [9, 20] is that confinement may be a more complicated mixture of scalar...
and timelike vector, while the vector potential is anticonfining. In pure $c\bar{c}$ models, the Lorentz nature of confinement is tested by the multiplet splitting of orbitally excited charmonium states.

In addition, calculating treatment cannot be belittled. As several numerical methods fail in the potentials with $1/r^2$ and even higher negative power, the $O(v^2/c^2)$ corrections to the quark–antiquark potential has to be treated as mass shifts using leading-order perturbation theory [20–22]. However, the accuracy of perturbation expansion has been concerned and alerted recently [23], which indicates the most significant effect of the different treatments is on the wave functions. The nonperturbative treatment brings every state with its own wave function, while the perturbative treatment leads to the same angular momentum multiplets sharing the identical wave function. It is known that the radiative transitions, leptonic and double-photon decay widths are quite sensitive to the shape of wave function and its information at the origin. Each physical particle owning the different quantum numbers should behavior with a distinguishing state wave function. Thus, it is intriguing to investigate the still-puzzling confining mechanism, considering the treatment accuracy as well as the determination of model-related parameters.

In our calculation framework, the spin-dependent and independent interactions have been totally absorbed into the Hamiltonian holding kinetic term, where the different confining assumptions are compared from the mass spectrum and decay properties. The efficient state-labeled wave functions are inspected through the electromagnetic processes, i.e. radiative transitions and leptonic decays, which are considered to be a niche targeting test for the overlap radial integration and the subtle information at the origin. Since the relativistic determination of model-related parameters.

In the preceding section, we contribute to a discussion related to the latest experimental observations in Sect. 4. Finally, Sect. 5 summaries the remarks and conclusions.

## 2 Potential Models and Calculational Approach

The confinement of quarks is assumed to be purely scalar linear type in NR model [21], and the scalar-vector mixed one in MNR [9]. Once the Lorentz structure of central part are decisive in the two cases,

\[
\begin{align*}
NR : \quad V_S &= br; \quad V_V = -\frac{4\alpha_s}{3r}, \\
MNR : \quad V_S &= b(1 - \epsilon)r; \quad V_V = -\frac{4\alpha_s}{3r} + \epsilon br,
\end{align*}
\]

the spin-orbit term and the tensor term can be directly derived from the standard Breit–Fermi expression to order $(v^2/c^2)$ with the charmed quark mass $m_c$. Summarily, the interaction potentials are

\[
\begin{align*}
V_{NR} &= -\frac{4\alpha_s}{3} + br + \frac{32\pi\alpha_s}{9m_c^2} \delta_\sigma(r) S_c \cdot S_\bar{c} + \left[ \frac{2\alpha_s}{m_c^2 r^3} - \frac{b}{2m_c^2 r} \right] L \cdot S + \frac{4\alpha_s}{m_c^2 r^3} T, \\
V_{MNR} &= -\frac{4\alpha_s}{3} + br + \frac{32\pi\alpha_s}{9m_c^2} \delta_\sigma(r) S_c \cdot S_\bar{c} + \left[ \frac{2\alpha_s}{m_c^2 r^3} + \frac{(4\epsilon - 1)b}{2m_c^2 r} \right] L \cdot S + \left[ \frac{\alpha_s}{3m_c^2 r^3} + \frac{\epsilon b}{12m_c^2 r} \right] T,
\end{align*}
\]

where $L$ is the orbital momentum and $S$ is the spin of charmonium. In the mixed-confining model, $\epsilon$ stands for the vector exchange scale. The singularity of contact hyperfine interaction within the spin-spin term has been smeared by Gaussian as in Ref. [21], $\delta_\sigma(r) = (\sigma/\sqrt{\pi})^3 e^{-\sigma^2 r^2}$. The involved operators are diagonal in a $| J, L, S \rangle$ basis with the matrix elements,

\[
\begin{align*}
\langle S_c \cdot S_\bar{c} \rangle &= \frac{1}{2} S(S + 1) - \frac{3}{4}, \\
\langle L \cdot S \rangle &= \frac{1}{2} [J(J + 1) - L(L + 1) - S(S + 1)], \\
\langle T \rangle &= \left\langle \frac{3}{r^2} (S_c \cdot r)(S_\bar{c} \cdot r) - (S_c \cdot S_\bar{c}) \right\rangle = \frac{-6 (\langle L \cdot S \rangle)^2 + 3 \langle L \cdot S \rangle - 2S(S + 1)L(L + 1)}{6(2L - 1)(2L + 3)}.
\end{align*}
\]

In addition, calculating treatment cannot be belittled. As several numerical methods fail in the potentials with $1/r^2$ and even higher negative power, the $O(v^2/c^2)$ corrections to the quark–antiquark potential has to be treated as mass shifts using leading-order perturbation theory [20–22]. However, the accuracy of perturbation expansion has been concerned and alerted recently [23], which indicates the most significant effect of the different treatments is on the wave functions. The nonperturbative treatment brings every state with its own wave function, while the perturbative treatment leads to the same angular momentum multiplets sharing the identical wave function. It is known that the radiative transitions, leptonic and double-photon decay widths are quite sensitive to the shape of wave function and its information at the origin. Each physical particle owning the different quantum numbers should behavior with a distinguishing state wave function. Thus, it is intriguing to investigate the still-puzzling confining mechanism, considering the treatment accuracy as well as the determination of model-related parameters.

In our calculation framework, the spin-dependent and independent interactions have been totally absorbed into the Hamiltonian holding kinetic term, where the different confining assumptions are compared from the mass spectrum and decay properties. The efficient state-labeled wave functions are inspected through the electromagnetic processes, i.e. radiative transitions and leptonic decays, which are considered to be a niche targeting test for the overlap radial integration and the subtle information at the origin. Since the relativistic reconstruction of the static confining potential is not unique, it complicates the account for the nature of confinement. Hence the special concerns are focused on the minimal but relatively well-understood model, with the aim of gleaning the actual influences from the potentials and treatments individually.

In the preceding section, we contribute to a discussion related to the latest experimental observations in Sect. 4. Finally, Sect. 5 summaries the remarks and conclusions.

\[
\begin{align*}
\langle S_c \cdot S_\bar{c} \rangle &= \frac{1}{2} S(S + 1) - \frac{3}{4}, \\
\langle L \cdot S \rangle &= \frac{1}{2} [J(J + 1) - L(L + 1) - S(S + 1)], \\
\langle T \rangle &= \left\langle \frac{3}{r^2} (S_c \cdot r)(S_\bar{c} \cdot r) - (S_c \cdot S_\bar{c}) \right\rangle = \frac{-6 (\langle L \cdot S \rangle)^2 + 3 \langle L \cdot S \rangle - 2S(S + 1)L(L + 1)}{6(2L - 1)(2L + 3)}.
\end{align*}
\]