BRST symmetry and Darboux transformations in Abelian 2-form gauge theory

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Abstract – We analyse the constraints of an Abelian 2-form gauge theory using the Faddeev-Jackiw symplectic formalism. Further, this theory is treated as a constrained system in the context of the Batalin-Fradkin-Vilkovisky formalism to retrieve the BRST symmetry. Using the fields decompositions the effective action for the Abelian 2-form gauge theory is written in terms of a diagonalized uncanonical part and the BRST exact one. The nilpotent BRST and contracting homotopy $\sigma$ closed transformations with field redefinitions are shown as the Darboux transformations used in the Faddeev-Jackiw formalism.

Introduction. – The quantization of nonsingular systems is in principle straightforward. On the other hand, the quantization of singular systems (i.e. systems with constraints) is nontrivial. The generalized Hamiltonian dynamics of singular systems was initiated by Dirac [1,2]. The dynamics of such systems are widely used in investigating theoretical models in contemporary elementary particle physics [3]. Dirac proposed a kind of bracket to quantize these (singular) systems. In the Dirac approach of dealing with singular systems, the dynamical equations involve the variables of the entire phase space, including also unphysical gauge degrees of freedom. However, a symplectic approach to quantize the singular systems has been used by Faddeev and Jackiw (FJ) [4], the so-called FJ approach, in which the systematic algorithm involves only the physical (unconstrained) degrees of freedom for arriving at a set of Hamilton equations of motion [5]. In ref. [6], this algorithm has been shown equivalent to the Dirac approach. In the FJ approach, the Lagrangian is treated in (symplectic) first order.

The Abelian antisymmetric rank-2 tensor field theory is an example of singular system where some of the constraints are not independent and said to be reducible. This theory is the subject of interests in various aspects [7–14]. For example, Kalb and Ramond have shown that Abelian rank-2 antisymmetric fields interact with classical strings [7], which was further applied to the dual description of the Abelian Higgs model [11,12]. The antisymmetric tensor field appears to couple the gravity or supergravity fields with a higher curvature term in four and ten dimensions [13] and a complete understanding of these couplings in superstring theories is crucial in order to have anomalies cancellation [14]. Abelian 2-form gauge fields have their relevance in the M-theory also. In particular, the action for multiple M2-branes was studied via BLG theory [15–19]. The gauge symmetry of this theory was generated by a Lie 3-algebra rather than a Lie algebra. However, this limited the scope of this theory to two M2-branes. So, this theory was generalized to the ABJM theory [20–24]. The gauge symmetry of this theory is generated by the gauge group $U(N) \times U(N)$. The BRST symmetry of the ABJM theory has also been studied [25–27]. The ABJM theory has been generalized to the theory of fractional M2-branes [28,29]. The gauge group of the ABJ theory is $U(N) \times U(M)$. Recently, the BRST symmetry of the ABJ theory has also been studied [30–32]. It is shown in [33] at the quadratic order of the Lagrangian that the M5-brane theory contains a self-dual two-form gauge field, in addition to the scalars corresponding to fluctuations of the M5-brane in the transverse directions, as well as their fermionic super-partners. The symplectic quantization for the Abelian rank-2 antisymmetric tensor field theory has been done in refs. [34,35]. However, the Darboux transformations are not studied for the Abelian...
rank-2 antisymmetric field in the FJ context. This provides a motivation for this present work.

The Batalin-Fradkin-Vilkovisky (BFV) formulation is a Hamiltonian path integral approach to quantize the constrained systems [36,37]. In this approach one extends the phase space of the theory by introducing a conjugate momentum for every Lagrange multiplier and a ghost field for every constraint. The induced effective action in the extended phase space exhibits a so-called BRST symmetry [38]. However, in the FJ approach the phase space is reduced by iteratively solving the constraints and performing the Darboux transformations, until we end up with an unconstrained and canonical Lagrangian. The relation between the BFV quantization scheme and the FJ approach for a gauged $SU(2)$ WZW model has been established in [39]. We explore it for the reducible gauge theory of the Abelian rank-2 tensor field.

In this work we start with the FJ constraint analysis for the 2-form gauge theory. The constraints which we found (primary and zero iterated) are exactly the same as those obtained from the Dirac analysis but in a more elegant manner. Then, we use the BFV approach by extending the phase space to analyse the BRST symmetry of the effective action. Further, the BFV action is written in two terms, the first one is an uncanonical term that we would obtain with the FJ method after solving the constraints and the second one is the BRST exact term. The BRST and contracting homotopy $\sigma$ closed transformations are calculated for the reducible 2-form gauge theory. Under the fields decompositions these transformations are shown as the Darboux transformations used in the FJ formalism.

The paper is organized as follows. In the second section, we discuss the preliminaries of the FJ symplectic approach of a singular system. In the third section, we make an analysis to investigate the constraints structure of the Abelian rank-2 tensor field theory using a symplectic matrix. Then, we stress the BRST-BFV formulation to quantize such reducible gauge theory. In the fourth section, further, in the fifth section, we show that the BRST and contracting homotopy $\sigma$ transformations of the 2-form gauge theory are basically Darboux transformations used in FJ symplectic approach. The last section is kept for making concluding remarks.

**Faddeev-Jackiw approach: general formulation.**

In this section we discuss the methodology of the FJ approach to quantize the singular systems. In this formalism, we first write the Lagrangian of a singular system into the first-order form as follows:

$$L(\xi) = a_i(\xi)\dot{\xi}^i - V(\xi) \quad (i = 1, 2, 3, \ldots, n), \quad (1)$$

where $\xi^i$ is the symplectic variable and $V(\xi)$ is the symplectic potential. The first-order form can be implemented by introducing some auxiliary variables $(a_i)$ such as the canonical momentum [40]. The Euler-Lagrange equations of motion for Lagrangian (1) can be written as

$$f_{ij}(\xi)\dot{\xi}^j = \frac{\partial V(\xi)}{\partial \xi^i} \quad (i = 1, 2, 3, \ldots, n), \quad (2)$$

where $f_{ij}$ is the so-called symplectic matrix with the following explicit form:

$$f_{ij}(\xi) = \frac{\partial a_j}{\partial \xi^i} - \frac{\partial a_i}{\partial \xi^j} \quad (3)$$

If the matrix $f_{ij}$ is regular (invertible), all symplectic variables can be solved from (2)

$$\dot{\xi}^i = f^{-1}_{ij}\frac{\partial V(\xi)}{\partial \xi^j} \quad (i = 1, 2, 3, \ldots, n). \quad (4)$$

If the matrix $f_{ij}$ is singular, there are some constraints in this system. In order to quantize the system with constraints in the FJ method, Barcelos-Neto and Wotzasek [41,42] proposed the symplectic algorithm to extend the original FJ method [4]. We give a brief description of the symplectic algorithm here. The constraints arising from eq. (2) are

$$\Omega^{(0)}_a = (U_a)^T \frac{\partial V}{\partial \xi} = 0 \quad (\alpha = 1, 2, 3, \ldots, m), \quad (5)$$

where $U_a$ is the zero mode of the symplectic matrix $f$, $m = n - r$ ($r$ is the rank of $f$)

$$(U_a)^T f = 0 \quad (\alpha = 1, 2, 3, \ldots, m). \quad (6)$$

Now, we modify our original Lagrangian by introducing the constraint term multiplied with some Lagrange multipliers $(\nu^\alpha)$ as

$$L_{mod} = a_i(\xi)\dot{\xi}^i - V(\xi) - \nu^\alpha\Omega^{(0)}_a \quad (\alpha = 1, 2, 3, \ldots, m) \quad (7)$$

and calculate the symplectic matrix with modified Lagrangian density; if there is further constraint in the system then the matrix becomes singular otherwise it is nonsingular. But, doing iteratively, in the last we get a nonsingular matrix. This means there is no further constraint in the system. So according to the Darboux theorem [43] there exists a coordinate transformation

$$Q_1(\xi^{(0)}), \ldots, Q_m/2(\xi^{(0)}); \quad (8)$$

$$P_1(\xi^{(0)}), \ldots, P_m/2(\xi^{(0)}), \quad (9)$$

which transforms the first-order Lagrangian given in eq. (1) into

$$L^{(0)} = P_k\dot{Q}_k - V^{(0)}(P,Q) \quad (k = 1, 2, \ldots, m/2). \quad (10)$$

From the mathematical view, the key of the FJ method is just to construct such a Lagrangian that satisfies the Darboux theorem, and the FJ canonical quantization is established on such a form of the Lagrangian.

In the next section we will treat the Abelian 2-form gauge theory as a singular system and will analyse the constraints using the FJ symplectic approach.
Constraints analysis of the Abelian 2-form gauge theory: using the FJ approach. – We start with the Lagrangian density for the Abelian free Kalb-Ramond theory in \((1 + 3)\) dimensions (4D) \([7]\) given by

\[
\mathcal{L} = \frac{1}{12} F_{\mu \nu \rho} F^{\mu \nu \rho},
\]

where the antisymmetric field strength tensor in terms of the Kalb-Ramond field \((B_{\mu \nu})\) is defined as \(F_{\mu \nu \lambda} = \partial_{\mu} B_{\nu \lambda} + \partial_{\nu} B_{\lambda \mu} + \partial_{\lambda} B_{\mu \nu}\). This Lagrangian density is invariant under the following gauge transformation:

\[
\delta B_{\mu \nu} = \partial_{\mu} \Lambda_\nu - \partial_{\nu} \Lambda_\mu,
\]

where \(\Lambda_\mu\) is an arbitrary vector parameter.

This gauge transformation is reducible, since a particular choice of the vector parameter, \(i.e.\)

\[
\Lambda_\mu = \partial_\mu \xi,
\]

leads to \(\delta B_{\mu \nu} = 0\).

Now, the canonical momenta corresponding to the fields \(B_{0i}\) and \(B_{ij}\), respectively, are calculated as

\[
\Pi^{0i} = \frac{\partial \mathcal{L}}{\partial \dot{B}_{0i}} = 0,
\]

and

\[
\Pi^{ij} = \frac{\partial \mathcal{L}}{\partial \dot{B}_{ij}} = \frac{1}{2} F^{00ij}.
\]

The primary constraint of the theory thus obtained is

\[
\Pi^{0i} \approx 0.
\]

Now, in order to make the Lagrangian density for the Abelian 2-form gauge theory in the first-order symplectic form, we calculate the following:

\[
\Pi^{\mu \nu} \dot{B}_{\mu \nu} - \mathcal{L} = \Pi^{ij} \Pi_{ij} + \frac{1}{12} F_{ijkl} F^{ijkl} + 2 \Pi^{ij} \nabla_i B_{0j}.
\]

So, the first-order symplectic version of the Lagrangian density given in eq. (11) is given by

\[
\mathcal{L}^{(0)} = \Pi^{\mu \nu} \dot{B}_{\mu \nu} - \Pi^{ij} \Pi_{ij} - \frac{1}{12} F_{ijkl} F^{ijkl} - 2 \Pi^{ij} \nabla_i B_{0j},
\]

where \(V^{(0)}\) is the symplectic potential with the following expression:

\[
V^{(0)} = \Pi^{ij} \Pi_{ij} + \frac{1}{12} F_{ijkl} F^{ijkl} + 2 \Pi^{ij} \nabla_i B_{0j},
\]

The corresponding symplectic equations of motion can be calculated easily with the following relations:

\[
f^{(0)\xi k \lambda}_{ijk \lambda} = \frac{\partial V^{(0)}(\xi)}{\partial \xi^{ij}},
\]

where

\[
f^{(0)}_{ijk \lambda} = \frac{\partial a_{k \lambda} (y) - \partial a_{ij}(x)}{\partial \xi^{ij}} (21).
\]

The set of symplectic variables are

\[
\xi^{(0)}(x) = \{B_{ij}, \Pi_{ij}, B_{0i}\}.
\]

The components of the symplectic 1-form are calculated as follows:

\[
a^{(0)}_{B_{ij}} = \frac{\partial \mathcal{L}}{\partial \Pi_{ij}} = 0,
\]

\[
a^{(0)}_{\Pi_{ij}} = \frac{\partial \mathcal{L}}{\partial B_{ij}} = 0.
\]

Thus the matrix \(f^{(0)}\), whose general form reads

\[
f^{(0)} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & \frac{1}{2} (-\delta_{jk} \delta_{ji} + \delta_{ij} \delta_{jk}) \\
0 & \frac{1}{2} (\delta_{il} \delta_{jk} - \delta_{ij} \delta_{lk}) & 0
\end{pmatrix} \delta^3(\mathbf{x} - \mathbf{y}).
\]

It is a singular matrix. The zero mode of this matrix is \((0, 0, \nu^{B_{00}})\) where \(\nu^{B_{00}}\) is some arbitrary function. In terms of the FJ method \([44]\), using zero mode, we can obtain the constraint

\[
\Omega^{(0)} = \nu^{B_{00}} \frac{\partial V^{(0)}}{\partial \nu^{B_{00}}} \approx 0,
\]

\[
= \nu^{B_{00}} \frac{\partial V^{(0)}}{\partial B_{00}} \approx 0,
\]

\[
= \nabla_i \Pi^{ij} \approx 0.
\]

which is a zero-iterated constraint. This is not an independent constraint, since it satisfies the reducibility condition \(\nabla_i \Omega^{(0)} = 0\). However, it is easy to see that, even after calculating the symplectic matrix for the modified Lagrangian density with the above constraint, the zero modes do not lead to any new constraint. Hence, there is no further constraints in the theory.

We end this section by concluding that both the constraints primary and zero iterated are exactly the same as those obtained in the Dirac procedure.

The extended action and BRST symmetry. – In the above section, we obtained two constraints (primary and zero iterated) in the 2-form gauge theory, \(i.e.\) \(\Pi^{0i} = 0\) and \(\nabla_i \Pi^{ij} = 0\). In this section we discuss the nilpotent BRST symmetry for the Abelian rank-2 tensor field
theory. To do so, we introduce two pairs of canonically conjugate anticommuting ghosts \((\overline{C}_i, \overline{P}_i)\) and \((C_i, P_i)\) corresponding to the above constraints. Further, we need the following pairs of canonically conjugate commuting ghosts \((\beta, \Pi_\beta)\) and \((\overline{\beta}, \overline{\Pi}_\beta)\) which are ghosts of ghosts according to the property of reducibility. The ghosts numbers of these ghost fields are as follows:

\[
gh(C_i) = -gh(\overline{P}_i) = 1,
\]
\[
gh(\overline{C}_i) = -gh(P_i) = -1,
\]
\[
gh(\beta) = -gh(\Pi_\beta) = 2,
\]
\[
gh(\overline{\beta}) = -gh(\overline{\Pi}_\beta) = -2,
\]

and they satisfy the following (anti-)commutation relations:

\[
\{C_i(x), \overline{P}_j(y)\} = -i\delta_{ij} \delta^3(x - y),
\]
\[
\{\overline{C}_i(x), P_j(y)\} = -i\delta_{ij} \delta^3(x - y),
\]
\[
[\beta(x), \Pi_\beta(y)] = i\delta^3(x - y),
\]
\[
[\overline{\beta}(x), \overline{\Pi}_\beta(y)] = i\delta^3(x - y).
\]

The phase space is further extended by introducing the canonical conjugate pairs \((C_0, \overline{P}_0)\) and \((\overline{C}_0, P_0)\) as Lagrange multipliers to the pair \((C_i, P_i)\), \((\overline{C}_i, \overline{P}_i)\) and a canonical pair \((\varphi, \Pi_\varphi)\) as Lagrange multiplier to the gauge condition. Hence, the extended action is given by

\[
S_{\text{eff}} = \int d^4x \left[ \Pi_0 \dot{B}_0 + \Pi_i \dot{B}_i + P_i \dot{C}_i + \overline{P}_i \dot{\overline{C}}_i + \mathcal{L}_{\text{eff}} \right],
\]

which satisfies the following algebra:

\[
\{Q, Q\} = 0, \quad \{\mathcal{H}_e, Q\} = 0.
\]

The BRST symmetry transformation can be calculated with the following relation:

\[
s_b \phi = -i[\phi, Q]_s,
\]

where + is used for fermionic and − for the bosonic nature of generic fields \(\phi\). Using the above relation and the expression for the BRST charge given in eq. (32), we calculated the BRST symmetry transformations for the fields as follows:

\[
s_b B_{ij} = (\nabla_i C_j - \nabla_j C_i), \quad s_b B_{0i} = -\overline{P}_i,
\]
\[
s_b \Pi_{ij} = 0, \quad s_b C_i = 0, \quad s_b \overline{C}_i = \Pi_0, \quad s_b C_0 = \Pi_\beta,
\]
\[
s_b \overline{C}_0 = \overline{\Pi}_\varphi, \quad s_b \varphi = -\overline{P}_0, \quad s_b \beta = 0,
\]
\[
s_b \overline{\beta} = -\overline{\Pi}_0, \quad s_b \Pi_i = 0, \quad s_b \Pi_\beta = 0, \quad s_b \varphi = 0.
\]

These transformations are nilpotent (i.e. \(s_b^2 = 0\)) and the symmetry of the effective action is given in eq. (30).

**BRST symmetry transformation as a Darboux transformation.** In this section, we study the BRST transformation and the contracting homotopy \(\sigma\) transformation for the Abelian 2-form gauge theory under the Darboux transformation. For this purpose we first decompose the field \(B_{ij}\) into transverse and longitudinal parts as follows:

\[
B_{ij} = B_{ij}^T + B_{ij}^L,
\]

\[
e_{ijk} \nabla_k B_{ij}^T + \nabla_i B_{j}^L - \nabla_j B_{i}^L,
\]

where \(B_{ij}^T = \epsilon_{ijk} \nabla_k B^T\) and \(B_{ij}^L = \mathcal{V}_i B_{j}^T - \nabla_j B_{i}^T\). Then we decompose corresponding momenta \(\Pi_{ij}\) into transverse and longitudinal parts as follows:

\[
\Pi_{ij} = \Pi_{ij}^T + \Pi_{ij}^L,
\]

\[
e_{ijk} \nabla_k \Pi^T + \frac{1}{\sqrt{2}} \left[ \nabla_i \Pi_j^T - \nabla_j \Pi_i^T \right],
\]

where \(\Pi_{ij}^T = \epsilon_{ijk} \nabla_k \Pi^T\) and \(\Pi_{ij}^L = \frac{1}{\sqrt{2}} \left[ \nabla_i \Pi_j^T - \nabla_j \Pi_i^T \right]\). Further, we exploit the relations (36) to solve the field variables \(C_i, \overline{C}_i, \Pi_{ij}^L\) and \(\Pi_{0i}\) in terms of BRST transformation as follows:

\[
C_i = s_b B_{0i}^L, \quad \overline{C}_i = s_b B_{0i}^T,
\]

\[
\Pi_{ij}^L = \frac{\nabla_j}{\sqrt{2}} s_b P_i,
\]

\[
\Pi_{0i} = s_b C_i.
\]

Using the fields decompositions the effective action given in eq. (30) is written as

\[
S_{\text{eff}} = \int d^4x \left[ \Pi_{0} \dot{B}_0^T + \Pi_{ij} \dot{B}_{ij}^T + \frac{1}{\sqrt{2}} \nabla_i \Pi^T \nabla^j \dot{B}_{ij}^T \right.
\]
\[
- 2 \frac{\nabla_i}{\sqrt{2}} \Pi_{ij}^T \nabla^j B_{ij}^T + \frac{1}{\sqrt{2}} \nabla_i \Pi^T \nabla^j \dot{B}_{ij}^T
\]
\[
+ \dot{\overline{C}}_i \dot{P}_0 + \dot{\overline{C}}_i \dot{P}_0 + \Pi_{ij} \dot{\overline{\beta}} + \Pi_{ij} \dot{\beta}
\]
\[
+ \frac{1}{\sqrt{2}} \nabla_i \Pi^T - \mathcal{H}_e - \{Q, \mathcal{H}_e\}.
\]

where the decomposed canonical Hamiltonian density is given by

\[
\mathcal{H}_e = \Pi_{ij} \dot{\Pi}_{ij}^T + 2 \frac{\nabla_i}{\sqrt{2}} \Pi_{ij} \nabla^j \Pi^T
\]
\[
- 2 \frac{\nabla_i}{\sqrt{2}} \Pi_{ij} \nabla^j \Pi_{ij}^T + \frac{1}{12} F_{ijjk} F^{ijjk}.
\]
We can easily see that using the symmetry transformations the effective action for the Abelian 2-form gauge theory can be recast as

\[ S_{\text{eff}} = \int d^4x \left[ \Pi^T_{ij} \dot{B}^{ijT} + \Pi_\beta \dot{\beta} - \mathcal{H} + s_b \left( \dot{c}^i B_{0i} - \mathcal{P}^i B^L_i \right) \right] + C_0 \dot{\beta} + \dot{c}_0 \dot{\varphi} + \frac{1}{4} s_b \mathcal{P}_i \left( \frac{1}{\sqrt{2}} \mathcal{P}^j \right) \right] = \{ Q, \Psi \}, \]  

(42)

where

\[ \mathcal{H} = \Pi^T_{ij} \Pi^{ijT} + \frac{1}{12} F_{ij}^k F^{ijk}. \]  

(43)

Hence, we can make the following choice for the gauge fermion:

\[ \Psi = i \left( \dot{c}^i B_{0i} - \mathcal{P}^i B^L_i + C_0 \dot{\beta} + \dot{c}_0 \dot{\varphi} + \frac{1}{4} s_b \mathcal{P}_i \left( \frac{1}{\sqrt{2}} \mathcal{P}^j \right) \right). \]  

(44)

Exploiting the canonical fields decompositions given in eqs. (37) and (38), the nilpotent BRST symmetry transformations of eq. (36) have the following form:

\[ s_b B^T_i = c_i, \quad s_b B_{0i} = \mathcal{P}_i, \quad s_b \Pi_{ij} = 0, \quad s_b c_i = 0, \quad s_b \mathcal{C}_i = \Pi_{0i}, \quad s_b \mathcal{C}_0 = 0, \quad s_b \dot{c}_0 = 0, \quad s_b \beta = 0, \quad s_b \mathcal{P}_i = 0, \quad s_b \Pi_{ij} = 0, \quad s_b \Pi_{0i} = 0, \quad s_b \Pi_\beta = 0 = \]  

(45)

Here we notice that only transverse fields are BRST closed without being BRST exact. Therefore one can show that the functionals of these transverse fields are being used only in classical BRST cohomology. The contracting homotopy \( \sigma \) with respect to above BRST operator \( s_b \) is defined as

\[ \sigma(c_i) = B^T_i, \quad \sigma(B^L_i) = 0, \quad \sigma(\mathcal{P}_i) = B_{0i}, \quad \sigma(\Pi_{ij}) = 0, \quad \sigma(\mathcal{C}_i) = 0, \quad \sigma(\mathcal{C}_0) = 0, \quad \sigma(\beta) = 0, \quad \sigma(\varphi) = 0, \quad \sigma(\varphi_0) = 0, \quad \sigma(\beta_0) = 0, \]  

(46)

which is also nilpotent in nature. Further, the \( \sigma \) operator satisfies the following relation: \( \sigma s_b + s_b \sigma = N \), where \( N \) counts the degree in unphysical variables \( B^L_i, \mathcal{C}_i, \mathcal{P}_i, B_{0i}, \Pi_{ij}, \mathcal{C}_0, \Pi_{0i}, \varphi, \mathcal{P}_0, \varphi_0, \beta, \Pi_{0i}, \mathcal{P}_i, i.e. \)

\[ N = B^L_i \frac{\partial}{\partial B^L_i} + \mathcal{P}_i \frac{\partial}{\partial \mathcal{P}_i} + \mathcal{C}_i \frac{\partial}{\partial \mathcal{C}_i} + B_{0i} \frac{\partial}{\partial B_{0i}} + \]  

(47)

It follows that if the functionals \( \mathcal{G} \) of degree \( n \neq 0 \) is BRST closed in the unphysical variables,

\[ s_b \mathcal{G} = 0, \quad N \mathcal{G} = n \mathcal{G}, \]  

(48)

then it is BRST exact also, i.e. \( \mathcal{G} = s_b (1/n) \sigma \mathcal{G} \). However, only those BRST closed functionals, which are of degree \( n = 0 \) in the unphysical variables, are not BRST exact, i.e. the functionals of \( B^L_i, \Pi_{ij}, \beta, \Pi_\beta, \mathcal{P}_0, \mathcal{P}_i \) fields.

Therefore, the above BRST and \( \sigma \) closed transformations under which the fields transform are basically Darboux transformations used in FJ quantization.

**Conclusion.** – We have considered the Abelian rank-2 antisymmetric tensor field theory (which is a reducible gauge theory) as a singular system and have investigated the constraints involved in the theory using the FJ symplectic approach. Further, we have implemented the BFV formalism in which the scalar potential, \( B_{0i} \), is treated as a full dynamical variable with vanishing conjugate momentum, \( \Pi_{0i} \). According to the BFV formulation the phase space has been extended by introducing a canonical pair of ghost fields for each constraint in the theory. The conserved BRST charge as well as the BRST symmetry have been constructed for the Abelian 2-form gauge theory within the Hamiltonian framework. We have shown that using fields decompositions the effective action for the Abelian rank-2 tensor field theory can be written as a sum of an uncanonical term and the BRST exact one. Further, it has been shown that the field redefinitions under which the fields transform into nilpotent BRST and \( \sigma \) closed transformations are basically Darboux transformations used in the FJ approach.

A further use of a similar analysis in the quantum theory of gravity [45–50] and in higher derivative field theory [51] will be interesting. It is also important to mention that within the FJ framework the attempts to derive a non-Abelian version of this theory [52] will be exotic.

The path integral corresponding to the FJ quantization method has also been extensively studied under various aspects [53]. So far we have studied the Darboux transformations which appear in the FJ quantization as a symmetry of such path integral. However, it will be interesting to explore the Darboux transformations under which the path integral corresponding to the FJ quantization method is not invariant [54].

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