Astrometric microlensing: a channel to detect multiple lens systems

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ABSTRACT

If a source star is gravitationally microlensed by a multiple lens system, the resulting light curve can have significant deviations from the standard form of a single lens event. The chance of producing significant deviations becomes important when the separations between the component lenses are equivalent to the combined angular Einstein ring radius of the system. For multiple lens systems composed of more than two lenses, however, this condition is difficult to meet because the orbits of such systems are unstable. Even if events are caused by a multiple lens system with stable orbits where a pair of lenses are closely located and the other component (a third body) has a wide separation from the pair, identifying the lens multiplicity photometrically will be difficult because the event will be identified by either a binary lens event caused by the close pair of lenses or a single lens event caused by the third body. In this paper, we show that if a seemingly binary lens event is followed up astrometrically using future high-precision interferometers, the existence of an additional third body can be identified via a repeating event. We show that the signatures of third bodies can be unambiguously identified from the characteristic distortions they make in the centroid shift trajectories. We also show that owing to the long-range astrometric effect of third bodies, the detection efficiency will be considerable even for third bodies with large separations from their close lens pairs.

Key words: gravitational lensing – binaries: general.

1 INTRODUCTION

If a source star is gravitationally microlensed by a multiple lens system, the resulting light curve can have significant deviations from the standard form of a single lens event. The chance of producing significant deviations induced by the lens multiplicity becomes important when the separations between the component lenses are comparable to the combined Einstein ring radius of the system, which is related to the physical parameters of the lens system by

\[ \theta_E = \left( \frac{4Gm_{\text{tot}}}{c^2} \left( \frac{1}{D_{\text{ds}}} - \frac{1}{D_{\text{os}}} \right) \right)^{1/2}, \]  

(1)

where \( m_{\text{tot}} = \sum_i m_i \) is the total mass of the lens system, \( N \) is the total number of component lenses, \( m_i \) are the masses of the individual lenses, and \( D_{\text{ds}} \) and \( D_{\text{os}} \) are the distances to the lens system and the source star, respectively. Some fraction of binary lens systems can meet this requirement, and dozens of candidate binary lens events have been reported by the lensing survey and follow-up observations (Udalski et al. 1994; Dominik & Hirshfeld 1994; Alard, Mao & Guibert 1995; Afonso et al. 2000; Alcock et al. 2000; Alcock et al. 2000).

However, for multiple lens systems that are composed of more than two lenses, it is difficult to identify the lens multiplicity photometrically. There are two reasons for this. First, multiple systems with all constituent lenses having separations between them equivalent to \( \theta_E \) are very rare because the orbits of such systems are unstable. Secondly, even if events are caused by a multiple lens system with stable orbits where a pair of lenses are closely located and the other component (a third body) has a wide separation from the pair, the close pair and the third body would behave as if they were two independent lens systems, causing the event to be identified either by a binary lens or a single lens event. One case where the lens multiplicity can be identified is when the source trajectory approaches closely to both the close lens pair and the third body, causing a repeating event. However, owing to the short-range photometric effect of the third body, the chance of producing repeating events is very low (see Section 3). Although there was a claim that a triple lens system was discovered by Bennett et al. (1999), it was subsequently shown to be better explained by a rotating binary (Albrow et al. 2000). As a result, despite the substantial fraction of multiple systems (Evans 1968; Batten 1973; Batten & Fletcher 1989; Fekel 1981; Mayor & Mazeh 1987), not a single multiple lens event has been reported to date.

Until now, lensing observations have only been carried out photometrically. However, using several planned high-precision optical interferometers, such as those to be mounted on space-based platforms, e.g. the Space Interferometry Mission (SIM) and the Global Astrometric Interferometer for Astrophysics (GAIA), and those to be...
mounted on very large ground-based telescopes, e.g. Keck and VLT, it will become possible to observe lensing events astrometrically. When an event is observed astrometrically using these instruments, one can measure the lensing-induced displacement of the source star image centroid position with respect to its unlensed position (centroid shift \( \delta \)). Astrometric lensing observation is important because the lens mass can be better constrained by the measured centroid shift trajectory (Miyamoto & Yoshii 1995; Walker 1995; Paczyński 1998; Boden, Shao & van Buren 1998).

In this paper, we show that astrometric lensing can also be used for efficiently detecting third bodies in multiple lens systems. This is possible because owing to the long-range astrometric effect of third bodies, the lens multiplicity of a large fraction of events can be identified via repeating events. We investigate the properties of the deviations induced by third bodies on the centroid shift trajectories of events and estimate the efficiency of third-body detections expected from the future lensing observations. We note that this paper is an extension of the work of Han et al. (2002) who recently demonstrated the high efficiency of astrometric lensing observations in detecting very wide binary companions.

2 BASICS OF MULTIPLE LENSING

If a source star is lensed by a multiple lens system, the locations of the resulting images are obtained by solving the lens equation. When all lengths are normalized by the combined Einstein ring radius, the lens equation is expressed in complex notation as

\[
\xi = z + \sum_{i}^{N} \frac{m_i/m_{\text{int}}}{\xi_i - \hat{z}},
\]

where \( \xi_i \) are the positions of the lenses, \( \xi = \xi + i\eta \) and \( z = x + iy \) are the positions of the source and images, and \( \hat{z} \) denotes the complex conjugate of \( z \) (Witt 1990). The magnifications of the individual images are given by the inverse of the determinant of the Jacobian of the lens equation evaluated at the image position, i.e.

\[
A_j = \left( \frac{1}{\det J} \right); \quad \det J = 1 - \frac{\partial \xi}{\partial \xi} \frac{\partial \eta}{\partial \eta}.
\]

Then the total magnification and the source-star image centroid shift are obtained by

\[
A = \sum_{j} A_j,
\]

and

\[
\delta = \sum_{j}^{N} A_j \xi_j - \xi,
\]

where \( \xi = (\xi, \eta) \) and \( \xi_j = (x_j, y_j) \) are the source and image positions in vector notation and \( N \) represents the number of images. Since the lens equation describes a mapping from the lens plane on to the source plane, to find the image positions for given positions of the source and the lenses, it is required to invert the lens equation.

For a single lens system (\( N = 1 \)), the lens equation can simply be inverted. Solving the lens equation algebraically yields two solutions of the image positions, and the total magnification and the centroid shift are expressed in a simple form as

\[
A = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}
\]

and

\[
\delta = \frac{u}{u^2 + 2} \theta_h,
\]

where \( u \) is the dimensionless lens–source separation vector normalized by \( \theta_h \). For a single lens event, the light curve has a smooth symmetric shape (Paczynski 1986) and the centroid shift follows an elliptical trajectory (Walker 1995; Jeong, Han & Park 1997).

For a multiple lens system (\( N \geq 2 \)), on the other hand, the lens equation is non-linear and thus cannot be analytically inverted. However, since the lens equation can be expressed as a polynomial in \( z \), the image positions are obtained by numerically solving the polynomial. For an \( N \) point-mass lens system, the lens equation is equivalent to an \( (N^2 + 1) \)-order polynomial in \( z \) and there are a maximum of \( N^2 + 1 \) and a minimum of \( N + 1 \) images and the number of images changes by a multiple of two as the source crosses a caustic (Rhié 1997; Witt 1990). The caustic is the main new feature of the multiple lens system, which refers to the source position on which the magnification of a point source event becomes infinity. For a binary lens system (\( N = 2 \)), the sets of caustics form close curves. For a multiple lens system, the caustic structure becomes more complex and can exhibit self-intersecting and nested shapes. The order of the polynomial rapidly increases as the number of lenses increases, and thus it becomes difficult to solve the equation directly for systems composed of many lenses. One commonly used numerical method that allows one to study the lensing behaviours of multiple lens systems regardless of the number of lenses is the ‘inverse ray-shooting method’ (Schneider & Weiss 1986; Kayser, Refsdal & Stabell 1986; Wambsganss 1997). The disadvantage of using this method is that it requires a large computation time to study the detailed structures in the patterns of magnifications and centroid shifts.

In our analysis, we investigate the lensing properties of triple lens systems instead of testing systems with various numbers of lenses. We note, however, that for systems composed of more than three lenses, the individual sets of lenses composed of the close-pair lenses and each of the widely separated companion, in most cases, can be treated as independent triple lens systems. In addition, for triple lens systems, one can solve the lens equation directly instead of using the very time consuming inverse ray-shooting method.

3 PROPERTIES OF MULTIPLE LENSING BEHAVIOURS

If a lens system contains an additional third body, the lensing behaviour of the lens system is affected by the third body. The most important effect of the third body is that it makes the effective positions of other lens components shift towards it. The approximate amount of shift is

\[
\Delta z_i = \frac{q_T r_i}{d} |r_s|,
\]

where \( r_s \) is the position vector of the third body from the centre of mass of the two close-pair lenses (hereafter called the binary centre), \( d = |r_s|/\theta_h \) is the separation between the third body and the binary centre normalized by the combined Einstein ring of the two close-pair lenses, \( \theta_h = [4G(m_1 + m_2)/c^2] [(1/D_{\text{eff}}) - (1/D_{\text{int}})]^{1/2} \) and \( q_T = m_3/(m_1 + m_2) \) is the ratio of the third-body mass to the total mass of the close-pair lenses. If this effect is taken into consideration, however, the photometric lensing behaviour of events during the time when the source passes the region around the close lens pair is well approximated by that of the binary lens events without the third body. Similarly, besides the inverse effective positional shift of the third body towards the close lens pair, the lensing behaviour of events during the approach of the source to the third body is well approximated by that of the single lens event caused solely by the third body. That is, the close lens pair and the third body behave...
as if they are two independent lens system. In addition, since the photometric effect of both the close lens pair and the third body are confined to narrow regions around the individual lens systems, even if an event is caused by a triple lens systems, the event, in most cases, will be identified either as a binary lens or a single lens event. However, if a seemingly binary lens event is followed up astrometrically, the existence of an additional third body can be identified with a significantly increased efficiency. This is because, compared with the photometric effect, the astrometric effect of the third body endures to a large distance from it (Miralda-Escudé 1996), and thus it can cause noticeable deviations in the trajectories of the source star image centroid shifts even when the source approaches the third body with a considerable separations.

To show this, we construct maps of excess magnifications and centroid shifts of an example triple lens system. The excess magnification and the centroid shift are defined, respectively, by

\[ \epsilon = \frac{A_T - A_B}{A_T}, \]

and

\[ \Delta \delta = \delta_T - \delta_B, \]

where \( A_T \) and \( \delta_T \) represent the magnification and the centroid shift of the exact triple lens system, while \( A_B \) and \( \delta_B \) represent those of the binary lens event without the third body. The constructed maps are presented in Fig. 1. For the construction of the maps, we take

![Figure 1](https://academic.oup.com/mnras/article-abstract/335/1/189/1033090)

**Figure 1.** The maps of excess magnification (upper panel) and centroid shift (lower panel) of an example multiple lens system. The lens system is composed of three lenses where a pair of lenses are closely located and the other component (a third body) is widely separated from the pair. The two close-pair lenses have a mass ratio of \( q_B = m_2/m_1 = 1.0 \) and they are separated by \( a_B = 1.0 \) in units of \( \theta_E \). The third body has a mass ratio of \( q_T = m_3/(m_1 + m_2) = 0.6 \) and is separated from the centre of mass of the close-pair lenses by \( a_T = 15.0 \), also in units of \( \theta_E \). The filled dots represent the locations of the three lenses. Grey-scales are used to represent the regions of deviations with \( \epsilon \geq 5, 10 \) and 20 per cent for the excess magnification map and \( \Delta \delta \geq 5, 10 \) and 20 per cent of \( \theta_E \) for the excess centroid shift map. The coordinates of the maps are centred at the centre of the mass of the close-pair lenses and all lengths are normalized by \( \theta_E \). The straight lines with arrows represent the source trajectories of events whose light curves and centroid shifts are presented in Fig. 2. The solid curve near the close lens pair represents the caustics. The inset in each panel shows the enlarged view of the map in the region around the close-pair lenses. The dashed circle in each inset represents the combined Einstein ring of the close-pair lenses.
the effect of the positional shifts induced by the third body into consideration. The two close-pair lenses of the triple lens system have a mass ratio of \(q_B = m_2/m_1 = 1.0\) and the separation between them is \(a_B = 1.0\) in units of \(\theta_{E,B}\). The third body has a mass ratio of \(q_T = 0.6\) and is separated from the binary centre by \(a_T = 15.0\) (also in units of \(\theta_{E,B}\)) with an orientation angle of \(45^\circ\) with respect to the axis connecting the two close-pair lenses. The locations of the lenses are marked by filled dots on the maps. Grey-scales are used to represent the regions of deviations with \(\epsilon = 5, 10\) and 20 per cent for the excess magnification map and \(\Delta \delta = 5, 10\) and 20 per cent of \(\theta_{E,B}\) for the excess centroid shift map. For a typical Galactic lensing event, the angular Einstein ring radius is \(\sim (\mathcal{O}) 10^2\) \(\mu\)arcsec, and thus \(\Delta \delta = 0.1 \theta_{E,B}\) corresponds to \(\sim (\mathcal{O}) 10\) \(\mu\)arcsec. For comparison, we note that the SIM will be able to measure positional shifts of stars as small as several \(\mu\)arcsec (http://sim.jpl.nasa.gov).

From Fig. 1, one finds the following facts. First, owing to the large separation between the third body and the close lens pair, both the photometric and astrometric effects of the third body are not important in the region around the close lens pair, implying that not only the photometric but also the astrometric lensing behaviours of triple lens events during the passage of the source through the region around the close lens pair are well described by the binary lensing approximation. Secondly, the region of significant astrometric deviation around the third body is much larger than the region of significant photometric deviation, implying that for an important fraction of multiple lens events the existence of third bodies can be identified from extended astrometric follow-up lensing observations.

To see the characteristics of the photometric and astrometric deviations induced by the third body, in Fig. 2, we present the light curves and the centroid shift trajectories of several events caused

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**Figure 2.** Example centroid shift trajectories (left-hand panels) and light curves (right-hand panels) of events caused by the triple lens system whose maps of excess magnification and centroid shift are presented in Fig. 1. The source trajectories responsible for the individual events are marked in Fig. 1, where the number in each panel corresponds to that of the source trajectory. In each panel, the dotted and solid curves represent the centroid shift trajectories and light curves produced with and without the third body. The insets inside the left-hand side panels show the detailed structures of the centroid shift distortions induced by the third body. The insets inside the right-hand side panels show the enlargement of part of the light curve during the passages of the source around the region of the close lens pair.
by the triple lens system. The source trajectories responsible for the individual events are marked in Fig. 1. In each panel, we also present the curves of the binary lens events computed without the third body taking the effective positional shifts of the close binary lens pair into consideration (dotted curves). On the centroid shift trajectories, we mark the centroid positions (‘x’ symbols) measured with a time interval of \( t_{E,B} / 4 \) to show the changing rate of the centroid position. For an event with \( t_{E,B} \sim 1 \) month, therefore, this interval corresponds to roughly a week. From the comparison of the centroid shift trajectories and the light curves of the corresponding events, one finds that the astrometric signatures of the third body can be clearly identified from the characteristic loop-shaped distortions in the centroid shift trajectories, while the photometric signatures are too weak to be noticed. One also finds that the astrometric deviations last for a long period of time.

4 Efficiency of Third Detections

In the previous section, we showed that astrometric lensing observations will enable one to identify the third body of a multiple lens system with little ambiguity from the characteristic deviations it makes in the centroid shift trajectories. In this section, we determine the efficiency of detecting third bodies expected from the future astrometric lensing observations. For this determination, we use the formalism introduced by Di Stefano & Scalzo (1999) and further developed by Han et al. (2002).

If we define \( b_1 \) and \( b_2 \) as the smallest separations from the source trajectory to the binary centre and the third body, respectively, the orientation angle of the source trajectory with respect to the line connecting the binary centre and the third body is represented by

\[
\alpha_\pm = \sin^{-1}\left(\frac{b_\pm \pm b_3}{|r_3|}\right),
\]

where the sign ‘\( \pm \)’ is for the case when the closest points on the source trajectory from the binary centre and the third body are on the same side with respect to the line connecting the binary centre and the third body, and the ‘\( + \)’ sign is when the closest points are on opposite sides. If all lengths are normalized in units of \( \theta_{E,B} \), equation (11) is expressed by

\[
\alpha_\pm = \sin^{-1}\left(\frac{b_\pm \pm \sqrt{q_T} \beta_3}{d}\right),
\]

where \( \beta_B \) and \( \beta_3 \) are the lensing impact parameters of the two independent events involved with the binary system composed of the close-pair lenses and the single third body, respectively.\(^2\) Let us additionally define \( \beta_{3,1} \), such that among events that were identified as being affected by the close lens pair by approaching the binary centre with an impact parameter \( \beta_B \), only events with source trajectories passing the third body closer than \( \beta_{3,1} \) can be identified as multiple lens events from centroid shift deviations greater than a threshold value, \( \Delta \delta_B \). Then, the fraction of multiple lens events for which third bodies can be identified astrometrically is computed by

\[
P = \frac{\alpha_{+, th} - \alpha_{-, th}}{\pi},
\]

where the threshold orientation angles have values of

\[
\alpha_{+, th} = \sin^{-1}\left(\frac{\beta_B + \sqrt{q_T} \beta_{1,3}}{d}\right),
\]

\[
\alpha_{-, th} = \sin^{-1}\left(\frac{\beta_B - \sqrt{q_T} \beta_{1,3}}{d}\right)
\]

for \( d > \beta_B + \sqrt{q_T} \beta_{1,3} \),

\[
\alpha_{+, th} = \pi/2
\]

\[
\alpha_{-, th} = \sin^{-1}\left(\frac{\beta_B - \sqrt{q_T} \beta_{1,3}}{d}\right)
\]

for \( |\beta_B - \sqrt{q_T} \beta_{1,3}| < d \leq \beta_B + \sqrt{q_T} \beta_{1,3} \), and

\[
\alpha_{+, th} = \pi/2
\]

\[
\alpha_{-, th} = -\pi/2.
\]

for \( d \leq |\beta_B - \sqrt{q_T} \beta_{1,3}| \). We note that the threshold orientation angles have different values depending on the relative size of the astrometrically effective region of the third body compared with the separation between the third body and the binary centre. For a given value of the threshold centroid shift deviation, \( \Delta \delta_B \), the corresponding threshold impact parameter to the third body is obtained by

\[
\beta_{3,1} = \left(1 + \frac{1}{(\Delta \delta_B / \theta_{E,3})^2 - 8}\right)^{1/2}.
\]

If one expresses the threshold astrometric deviation in terms of the fraction of the combined Einstein ring radius of the close-pair lenses, \( f = \Delta \delta_B / \theta_{E,B} \), equation (17) becomes

\[
\beta_{3,1} = \left(1 + \frac{\sqrt{q_T}}{f} + \frac{\sqrt{q_T}}{f}^2 - 8\right)^{1/2}.
\]

![Figure 3.](https://example.com/astrometric-microlensing.png)

Figure 3. The expected efficiency of detecting third bodies from astrometric follow-up lensing observations as functions of the separation, \( d \), and the mass ratio, \( q_T \), between the third body and the close lens pair. We assume that a third body is detected if the deviation induced by the third body is larger than 10 per cent of the combined angular Einstein ring radius of the monitored binary lens event, \( \theta_{E,B} \). The separation is expressed in units of \( \theta_{E,B} \).
In Fig. 3, we present the determined efficiency in the parameter space of the separation and the mass ratio between the third body and the close lens pair, $P(d, q_T)$. For this computation we set $f = 0.1$. For the impact parameter to the close lens pair we adopt $\beta_B = 0.0$, but we note that the dependence of the efficiency on the value of $\beta_B$ adopted is not important as long as $\beta_B \ll d$. We also note that $q_T$ can be larger than 1.0 because the third body can be heavier than the total mass of the close-pair lenses. From the figure one finds that the efficiency is substantial even for third bodies that are widely separated from their close lens pairs. It is worth noting that the scaling between $d$ and $q_T$ is quite linear. This can be understood as follows. The probability of detecting third bodies is proportional to the threshold orientation angle, i.e. $P \propto \alpha \propto \sin^{-1}(\sqrt{q_T}\beta_{3,th}/d)$ (see equations 12 and 13). In the limit of $d \gg \sqrt{q_T}\beta_{3,th}$, $\sin^{-1}(\sqrt{q_T}\beta_{3,th}/d) \sim \sqrt{q_T}\beta_{3,th}/d$, and thus $P \propto \sqrt{q_T}\beta_{3,th}/d$. Since $\beta_{3,th} \propto \sqrt{q_T}$ (see equation 18), one finds that $P \propto q_T/d$. Therefore, the scaling between $d$ and $q_T$ is linear.

5 CONCLUSION

In this paper, we show that future astrometric lensing observations will provide an efficient method of detecting third bodies of multiple lens systems, which could not have been detected from conventional photometric lensing observations. We showed that the deviation in the centroid shift trajectory induced by the third body of a multiple lens system has a characteristic loop that can be clearly distinguished from other types of deviations and thus can be identified unambiguously. In addition, since the deviations last for a long period of time, detecting third bodies will be possible even from sparse astrometric sampling.

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