Comment on “Resolving the 180° Ambiguity in Solar Vector Magnetic Field Data: Evaluating the Effects of Noise, Spatial Resolution, and Method Assumptions”

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Abstract In a recent paper, Leka et al. (Solar Phys. 260, 83, 2009) constructed a synthetic vector magnetogram representing a three-dimensional magnetic structure defined only within a fraction of an arcsec in height. They rebinned the magnetogram to simulate conditions of limited spatial resolution and then compared the results of various azimuth disambiguation methods on the resampled data. Methods relying on the physical calculation of potential and/or non-potential magnetic fields failed in nearly the same, extended parts of the field of view and [Leka et al. (2009)] attributed these failures to the limited spatial resolution. This study shows that the failure of these methods is not due to the limited spatial resolution but due to the narrowly defined test data. Such narrow magnetic structures are not realistic in the real Sun. Physics-based disambiguation methods, adapted for solar magnetic fields extending to infinity, are not designed to handle such data; hence, they could only fail this test. I demonstrate how an appropriate limited-resolution disambiguation test can be performed by constructing a synthetic vector magnetogram very similar to that of [Leka et al. (2009)] but representing a structure defined in the semi-infinite space above the solar photosphere. For this magnetogram I find that even a simple potential-field disambiguation method manages to resolve the ambiguity very successfully, regardless of limited spatial resolution. Therefore, despite the conclusions of [Leka et al. (2009)], a proper limited-spatial-resolution test of azimuth disambiguation methods is yet to be performed in order to identify the best ideas and algorithms.

Keywords: Active Regions, Magnetic Fields; Instrumental effects; Magnetic Fields, Photosphere; Polarization, Optical
1. Introduction

Properly resolving the azimuthal 180°-ambiguity in the transverse (perpendicular to the line-of-sight) component of solar vector magnetograms inferred by the Zeeman effect is a prerequisite for further exploiting these valuable data. Calculation of electric currents, magnetic energy and helicity budgets, flow velocities via physical models, and most techniques of coronal magnetic field extrapolation rely on disambiguated vector magnetograms. The azimuth ambiguity was realized at the dawn of Zeeman-based vector magnetography (Harvey, 1969) and continues to be an open research topic to this day. Decent-quality vector magnetograms used to be rare, even in the recent past. This situation is reversed nowadays with vector magnetograms routinely provided by the ground-based Vector SpectroMagnetograph (VSM; Henney et al., 2009) of the Synoptic Optical Long Term Investigations of the Sun (SOLIS) facility (Keller, Harvey, and Giampapa, 2003) and by the space-based SpectroPolarimeter (SP; Lites, Elmore, and Streander, 2001) of the Solar Optical Telescope (SOT; Tsuneta et al., 2008) onboard the Hinode spacecraft. Vast amounts of seeing-free full-disk vector magnetograms are also anticipated by the Helioseismic and Magnetic Imager (HMI; Scherrer and SDO/HMI Team, 2002) onboard the Solar Dynamics Observatory (SDO) mission. For single-height magnetograms acquired either at photospheric or at chromospheric heights (see, e.g., Leka and Metcalf, 2003) azimuth disambiguation is an ill-posed problem: the height derivatives (∂/∂z) of any parameter (other than the normal field component $B_z$ in case one uses the divergence-free condition $\nabla \cdot B = 0$) are unknown. To tackle this problem, an array of disambiguation techniques of various sophistication levels have been proposed. For a detailed description of most of these methods see Metcalf et al. (2006) and references therein; in addition, Li, Amari, and Fan (2007) and Crouch, Barnes, and Leka (2009) investigated disambiguation based on the divergence-free condition per se and concluded independently that information in multiple heights is required for the method to work. More generally, the assumptions adopted by each disambiguation method are the ones ultimately responsible for the quality of the disambiguation results. Precisely and self-consistently disambiguating a vector magnetogram is a formidable problem, especially when the studied magnetograms represent complex magnetic structures with a multipolar, stressed, or sheared photospheric or low chromospheric boundary.

In a recent paper, Leka et al. (2009) (hereafter LE2009) aimed to continue the seminal work of Metcalf et al. (2006) who evaluated the different disambiguation methods by comparing their results on given (different between the two studies), synthetic vector magnetograms. The two studies resulted from a series of Azimuth Disambiguation Workshops held by the respective Working Group. Target vector magnetograms were synthetic because in this case there is an a priori known true solution or “answer”, against which one may compare different disambiguation solutions. “Answers” are unknown for real solar data. While Metcalf et al. (2006) focused on complex, flux-imbalanced, but noise-free and fully-resolved magnetic structures, LE2009 focused on the effects of photon noise and limited spatial resolution in the disambiguation process. I believe
that the effect of photon noise and its impact was treated fairly appropriately by LE2009. The chosen limited-resolution ("flowers") magnetograms, however, exhibited a feature that effectively disabled most disambiguation methods: the magnetic field vector was defined only within a narrow layer of 0.18″ above the perceived "photosphere", i.e., the plane on which disambiguation was tested. For any disambiguation method attempting physical calculations (i.e., potential and/or non-potential magnetic field) the underlying assumption is that magnetic structures extend to the semi-infinite space above the photosphere. This is driven by the lack of knowledge of the structures’ outer edges but it appears obvious that these structures extend well above the photosphere due to the decrease of the plasma density and the subsequent passing from the forced, possibly discontinuous, photospheric, to the force-free, space-filling, coronal magnetic fields (e.g., Longcope and Welsch 2000). Here I show that the assumption of a field defined only on and slightly above the photosphere makes it practically impossible to reproduce the “answer” field of LE2009 by any physics-based disambiguation method. Therefore, physics-based methods were subjected to a test that was not possible to handle, not due to the complexity of the synthetic data but, rather, due to the design of these data. Had the synthetic magnetogram been designed to extend well above the photosphere, the reported results of LE2009 would have been very useful and revealing. But with such an unrealistic magnetic structure one must determine what really needs reconsideration: the test data that overlook fundamental physics of solar magnetic fields, or the methods and models that are more adapted to the real Sun.

LE2009 did not discuss the narrow validity problem in detail. Instead, they attributed the failure of most disambiguation methods to correctly reproduce certain parts of the “answer” field to lost information due to unresolved structure on the disambiguation plane. Here I show in two different ways that the limited spatial resolution is not the reason of these methods’ failure: first, by disambiguating the original, fully-resolved and unbinned magnetogram of LE2009. This test was not undertaken in that paper. I find that physics-based methods fail at practically the same areas as in limited-resolution magnetograms, so failure cannot be attributed to the limited spatial resolution. Second, by constructing a semi-infinite magnetic structure and its corresponding horizontal field on the disambiguation plane using the normal field component of LE2009 as the boundary condition. In this case I find that even a conventional, potential-field disambiguation reproduces both the full- and the limited-resolution “answer” fields much better than what LE2009 reported.

In Section 2 I provide the theoretical background of finite- vs. semi-infinite volume magnetic structures. Section 3 describes the semi-infinite synthetic data on which I test some disambiguation methods, along with the metrics quantifying the performance of these methods. Section 4 describes my disambiguation results while in Section 5 I discuss crucial aspects of physics- and optimization-based disambiguation methods. Closing remarks are presented in Section 6.

1Redoing the analysis for a semi-infinite magnetic structures was proposed to K. D. Leka and colleagues prior to publishing the LE2009 paper. These authors declined, which led to this work that undertakes this task.
2. Theoretical Background

Assume a current-free, potential magnetic field $\mathbf{B}_P$ ($\nabla \times \mathbf{B}_P = 0$) in a given, finite and bounded, volume $V$. Aly (1987) showed that $\mathbf{B}_P$ has a unique solution in $V$, fully constrained by the nonzero normal field component $B_n$ on the boundary $\partial V$ of the volume. For a semi-infinite volume bounded only by one (bottom) boundary (plane, sphere) on which $B_n \neq 0$ the problem becomes identical to assuming that infinity corresponds to a flux, or magnetic, surface where $B_n = 0$. Therefore, $B_n$ at the bottom boundary can fully constrain $\mathbf{B}_P$ above it (Schmidt, 1964; Chiu and Hilton, 1977; Alissandrakis, 1981; Sakurai, 1982; Gary, 1989). Without loss of generality I assume a planar lower boundary so that $B_n$ is replaced by the opposite of the vertical magnetic field component $B_z$ on this plane (assumed isolated and hence infinite, surrounded by areas of zero magnetic field). Different potential-field solutions $\mathbf{B}_P'$ applying to this planar boundary and a finite volume above it differ from the unique $\mathbf{B}_P$ by a gauge $\nabla \psi$, where $\psi$ is a smooth scalar ($\nabla^2 \psi = 0$) constrained only by the normal-field condition on the finite $\partial V$:

$$\mathbf{B}'_P = \mathbf{B}_P + \nabla \psi \ .$$

(1)

Unless information to constrain $\psi$ is available, it is practically impossible to determine $\mathbf{B}'_P$ in the finite volume given the infinity of possible choices for $\nabla \psi$. As Sakurai (1989) puts it, one must have a physical reason for choosing a finite volume. This is the core of the problem with the finite-size magnetic structure of LE2009: other than computational convenience, there are no physical reasons dictating its selection. Physics-based methods, however, follow the semi-infinite volume approach backed up by the well-known fact that strong-field magnetic structures observed in the photosphere extend well above it. Hence, any physics-based method attempting to reproduce $\mathbf{B}'_P$ (say, a potential-field disambiguation method) will, at best, reproduce $\mathbf{B}_P$ which can be very different. In this case it is of little meaning to attempt disambiguation because neither of the two possible disambiguation solutions for $\mathbf{B}'_P$ on a given plane can be reproduced, or even guessed, by any calculation expecting a semi-infinite magnetic structure.

Assume now a non-potential, current-carrying magnetic field $\mathbf{B}$ in the semi-infinite volume above the lower planar boundary. The current-carrying component $\mathbf{B}_c$ will be superimposed to $\mathbf{B}_P$ in this case, so that

$$\mathbf{B} = \mathbf{B}_P + \mathbf{B}_c \ .$$

(2)

Notice that since $\mathbf{B}_P$ is fully constrained by $B_z$ on the boundary, $\mathbf{B}_c$ has only horizontal components on the boundary, i.e. $B_{cz} = 0$ (Georgoulis, 2005; Georgoulis and LaBonte, 2007). The current-carrying component will be responsible for the electric current density $\mathbf{J}$ on and above the boundary via Ampere’s law

$$\mathbf{J} = \frac{c}{4\pi} \nabla \times \mathbf{B}_c \ .$$

(3)

Consider now the field $\mathbf{B}' = \mathbf{B}'_P + \mathbf{B}'_c = \mathbf{B}_P + \nabla \psi + \mathbf{B}_c'$ applying to a finite volume. This will respectively give rise to an electric current density $\mathbf{J}'$ that can
be very different from \( \mathbf{J} \). Both \( \mathbf{J} \) and \( \mathbf{J}' \) on the boundary have no dependence on \( B_z \) because \( B_{z\epsilon} = B'_{z\epsilon} = 0 \) on it. A disambiguation method aiming to, say, reproduce both \( B'_p \) and \( J'_z \) on the boundary by semi-infinite volume calculations will have an untenable task: at best, it will reproduce \( B_p \) and \( J_z \), respectively. In LE2009 the various disambiguation methods were asked to reproduce the finite-volume, non-unique \( B'_p \) and \( J'_z \) with only the information needed to reproduce the semi-infinite, unique \( B_p \) and \( J_z \).

3. Two Versions of the Synthetic “Flowers” Case and Comparison Metrics

3.1. “Flowers” Cases for Finite and a Semi-Infinite Volumes

The so-called “flowers” case was designed by LE2009 to represent a current-free magnetic structure in full resolution (Figure 4 of that paper). Therefore, the discussion about potential fields and Equation (1) of Section 2 applies here. The reader is referred to Section 3.2 of LE2009 for a detailed description of the “flowers” construction. I only reiterate here that the structure is semi-analytical and produced by constructing a potential field on two planes: a lower plane, where disambiguation is attempted, and a higher plane at distance equal to six pixel sizes, or 0.18” per the designed pixel size of 0.03”. The narrowly spaced planes apparently allow some control of the potential-field solution within the thin layer in between without knowledge of the field on its lateral boundaries.

To translate the original flowers case of LE2009 into a potential-field structure valid in the semi-infinite volume above the lower boundary, I have obtained the “answer” vertical field component \( B_z \) (K.D. Leka and colleagues have made this available online\(^1\)) and extrapolated from it. To achieve zero currents, hence a potential-field solution, at machine accuracy I choose the accurate, but characteristically slow\(^2\) Green’s functions method of Schmidt (1964): in particular, I use the classical definition of the potential field, \( \mathbf{B}_p = -\nabla \chi \), where \( \chi \) is a smooth scalar. Then, if \( \mathbf{r} \) is the vector position on the lower boundary (\( z = 0 \)) and \( (\mathbf{r}, z) \) is the resulting vector position in the semi-infinite space \( z \geq 0 \), Schmidt (1964) showed that

\[
\chi(\mathbf{r}, z) = \frac{1}{2\pi} \int \int \frac{B_z(\mathbf{r}')dx'dy'}{\sqrt{(\mathbf{r} - \mathbf{r}')^2 + z^2}},
\]

where \( \mathbf{r}' = (x', y') \) and \( \mathbf{r} \neq \mathbf{r}' \) for \( z = 0 \). Calculation of \( \mathbf{B}_p \) for \( z \geq 0 \) becomes then straightforward.

At the lower (disambiguation) boundary, \( z = 0 \), the original flowers case of LE2009 and my semi-infinite flowers solution are depicted in Figure 1\(^3\). For an identical vertical field component \( B_z \) on the boundary (Figures 1(a) and 1(d)) there are very significant differences in both the horizontal field (Figures 1(b) and 1(e)).

\(^1\)http://www.cora.nwra.com/AMBIGUITY_WORKSHOP/2006_workshop/FLOWERS/

\(^2\)This run took about 24 days and 17 hours in a 16-core computing cluster.
Figure 1. Visual comparison between the field components of the original, finite-volume flowers case of LE2009 (a-c) and my semi-infinite volume flowers case (d-f) in full resolution. Shown are the identical vertical field component saturated at $\pm 2.5$ kG (a,d), the horizontal field strength saturated at 2.5 kG (b,e), and the azimuth angle (c, f), ranging between 0 (black) and $2\pi$ (white). The green contour in all images indicates the location of the magnetic polarity inversion line.
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Figure 2. Azimuth difference $\Delta \phi$ between the finite flowers solution of LE2009 and my semi-infinite solution. Colored areas correspond to $\Delta \phi > 90^\circ$.

and (e)) and the azimuth angle (Figures (c) and (f)). Both solutions give $J_z = 0$ on the boundary, so they are potential-field solutions, but the flowers case of LE2009 cannot be reproduced unless the exact $B_z$-solution is known for the top boundary, as well (Equation (1) of LE2009). Obviously there are infinite potential-field solutions for the same $B_z$ on the lower boundary, each of them determined by the location and $B_z$-solution on the top and/or the lateral boundaries, but there is only one potential-field solution for the semi-infinite space. This solution is given in Figures (d)-(f).

It is revealing to compare the azimuth angles $\phi$ between the two flowers versions in full resolution (Figure 2). One notices that while the difference $\Delta \phi < 90^\circ$ for the majority of the boundary plane, there are extended areas where $\Delta \phi > 90^\circ$ (colored) or even $\Delta \phi \approx 180^\circ$ (dark red). The latter correspond mostly to the “ring of azimuth centers” that LE2009 opted to simulate at the upper-right part of the structure and to the simulated “plage” area that extends between $x \in (2500\,3000)$ and $y \in (900\,1800)$. The colored areas in Figure 2 are the ones for which at least an acute-angle potential-field disambiguation method is prone to fail. As I will show below, these are exactly the areas where much more sophisticated physics-based methods invariably fail, as well. The reason of failure is not the difficulty to reproduce such complex fields or the limited resolution but merely the fact that these methods expect a semi-infinite magnetic structure, distinctly different from what they actually find. In other words, these areas are the ones most heavily impacted by the finite-volume approach. LE2009 argued that the simulated ring of azimuth centers and the plage can be descriptive of some solar-like structures. I am not countering this argument, but I question the overall concept of the flowers case as a finite-volume magnetic structure.

Rebinning the full-resolution magnetogram to lower resolution, hence with a coarser pixel size, to simulate disambiguation in partially unresolved magnetic
Figure 3. Scatter-plot comparison between the magnetic field components of my semi-infinite flowers solution (ordinate) and the original flowers case of LE2009 (abscissa) at the lower boundary. Shown are comparisons for $B_z$ (a, d, g), $B_x$ (b, e, h), and $B_y$ (c, f, i) for the full resolution (a-c), the $300 \times 300$-rebinned (d-f), and the $100 \times 100$-rebinned (g-i) flowers cases. The solid lines demonstrate equality and the dashed lines show the least-squares best fit between any two compared components. Also shown are the linear (Pearson) and the non-parametric rank order (Spearman) correlation coefficients for each comparison.

Structures changes the “answer” field even more. LE2009 produced synthetic Stokes spectra and resulting images $I, Q, U, V$ and inverted them to obtain the “answer” field of Figures 1(a)-1(c). Then they rebinned the spectra and inverted them to obtain the rebinned “answer” fields. This author is not able to fully reproduce this inversion process. Therefore, the semi-infinite flowers solution of Figures 1(d)-1(f) has been spatially, rather than spectrally, rebinned. If anything, this rather simplistic spatial resampling might be expected to give more spurious structures when disambiguating the lower-resolution magnetograms. As we will see below, however, even simple rebinning of the data does not pose serious problems in the disambiguation.

Scatter-plot comparisons between the magnetic field components for the original, finite-volume flowers cases of LE2009 and the unique, semi-infinite volume flowers cases of this work are provided in Figure 3. Most of the differences correspond to $B_x$ and $B_y$, while the $B_z$-solutions in full-resolution (Figure 3(a)) are obviously identical. The small differences between the $B_z$-solution of LE2009 and my solution for the rebinned magnetograms (Figure 3(d) and 3(g)) refer primarily to the “plage” area - there, indeed, simple spatial rebinning incurs an impact on the lower-resolution $B_z$-maps.

The scatter plots of Figure 3 exhibit two notable features: first, correlation coefficients between any component $B_x$, $B_y$, $B_z$ for the original and the semi-infinite flowers solutions are nearly constant and insensitive to the rebinning.
process. This is strong indication that simple spatial rebinning does not introduce many more artifacts than the spectral rebinning and subsequent inversion of LE2009, which is encouraging for this test. Second, the flowers cases of LE2009 in both full and limited resolution invariably give stronger horizontal field components than the semi-infinite volume flowers case. These stronger fields cannot possibly be reproduced by a potential-field method seeking a minimum-energy field valid in the semi-infinite space.

An important point here (that LE2009 also point out) is that the rebinned flowers magnetograms (either in the finite or in the semi-infinite volume) do not correspond to a potential-field solution any more. The loss of information due to rebinning spoils the smoothness of the scalar potential $\chi$ ($\nabla^2 \chi$ now becomes nonzero - Section [5.1]), thus incurring spurious currents that any disambiguation method has to deal with. For the rebinned magnetograms the discussion about non-potential fields and Equations (2) and (3) of Section 2 apply. Overall, one should look critically on observed solar vector magnetograms as part of the inferred electric currents may be due to the incomplete recording of the fine structure of the actual fields. The reader is referred to Parker (1996) for an argument that all vertical current $J_z$ inferred from photospheric vector magnetograms is, in fact, fictional and caused by the limited resolution of the observing instruments. Further discussion exceeds the scope of this study.

3.2. Comparison Metrics

As in LE2009, the quality of the performed disambiguations will be judged by an array of different metrics, each highlighting a certain aspect of the disambiguation solution. In particular, I use the following metrics:

1. The area metric, $M_S$ (denoted by $M(a,s)_\text{area}$ in LE2009): take the ratio of the number of pixels $N_{\text{pixels}}^{\text{correct}}$, where ambiguity has been resolved correctly, over the total number of pixels $N_{\text{pixels}}^{\text{tot}}$:

$$M_S = \frac{N_{\text{pixels}}^{\text{correct}}}{N_{\text{pixels}}^{\text{tot}}}. \quad (5)$$

Achieving a $M_S$-value closer to 1 implies better disambiguation results with perfect disambiguation reflected in $M_S = 1$.

2. The transverse field metric, $M_{B_t>\tau}$ (denoted by $M(a,s)_{B_t>\tau}$ in LE2009): define a threshold $\tau$ in the transverse field strength and calculate the ratio between the sum of transverse field $B_t^{\text{correct},>\tau}$ above this threshold where ambiguity has been resolved correctly over the total transverse field $B_t^{>\tau}$ above the threshold $\tau$:

$$M_{B_t>\tau} = \frac{\sum (B_t^{\text{correct},>\tau})}{\sum (B_t^{>\tau})}. \quad (6)$$

Here, again, a value closer to 1 implies better disambiguation results with $M_{B_t>\tau} = 1$ implying perfect disambiguation.
The mean vector field difference metric, $M_{\Delta B}(a,s)$: take the magnitude of the difference between the disambiguation solution $B_{(s)}$ and the “answer” field $B_{(a)}$, sum it over the disambiguation plane, and normalize by the total number of pixels $N_{\text{pixels}}^{\text{tot}}$:

$$M_{\Delta B}(a,s) = \frac{\sum(|B_{(s)} - B_{(a)}|)}{N_{\text{pixels}}^{\text{tot}}}.$$  

(7)

This is a dimensional metric, providing the mean difference between the disambiguation solution and the “answer” field in magnetic field units. The smaller the value of $M_{\Delta B}(a,s)$ the better the disambiguation with $M_{\Delta B}(a,s) = 0$ implying perfect disambiguation.

The normalized electric current density metric, $M_{J_z}(a,s)$: take the vertical current density $J_z^{(s)}$ inferred by the disambiguation solution and the “answer” vertical current density $J_z^{(a)}$ to form the metric

$$M_{J_z}(a,s) = 1 - \frac{\sum(|J_z^{(s)} - J_z^{(a)}|)}{2\sum(|J_z^{(a)}|)},$$  

(8)

where $\sum(\ )$ corresponds to summation over all pixels on the disambiguation plane. Here also, better results are reached if $M_{J_z}(a,s)$ tends to 1 with perfect disambiguation reflected in $M_{J_z}(a,s) = 1$.

The total vertical current metric, $M_I$: this is another dimensional metric and corresponds to the absolute sum of the disambiguation-inferred $J_z^{(s)}$ over the disambiguation plane:

$$M_I = \sum(|J_z^{(s)}|).$$  

(9)

This metric is compared to the respective sum obtained for the “answer” vertical current density $J_z^{(a)}$.

4. Disambiguation Attempts

To disambiguate both the finite- and the semi-infinite versions of the flowers case I basically use the non-potential magnetic field calculation (NPFC) method of Georgoulis (2005) as revised in Metcalf et al. (2006). This is denoted as NPFC2 in the latter paper and in LE2009 but I call it NPFC hereafter. Briefly, the method uses Equation (2) for the potential and the non-potential fields, $B_P$ and $B_C$, respectively. It is iterative, producing interim $B_P$- and $B_C$-solutions, the former constrained by the interim $B_z$-solutions and the latter by the interim $J_z$-solutions. At the end of each iteration, each interim solution is replaced by the closest combination of the two possible disambiguation solutions for the heliographic field components on the image (observation) plane. The whole scheme converges to a stable, self-consistent disambiguation solution that is then translated to the line-of-sight reference system to determine the preferred orientation of the transverse field for each pixel. For another detailed description
of the NPFC method and its pipeline application to SOLIS/VSM data, see Georgoulis, Raouafi, and Henney (2008).

The NPFC method makes only one assumption: that the vertical component $B_c$ of the current-carrying field $B_C$ is constant with height ($\partial B_c/\partial z = 0$) on the disambiguation plane. This is a reasonable assumption given that $B_c = 0$ on the disambiguation plane (see discussion in Section 2) and that one generally expects $B_c$ to be small immediately above the plane unless the orientation of the magnetic field lines changes drastically.

Since the NPFC method reconstructs the magnetic field on the boundary by the superposition of $B_P$ and $B_C$, one can always enforce $B_c = 0$ on the boundary. This degenerates the NPFC method into a simple potential-field disambiguation method with $B_P$ calculated using fast Fourier transforms (FFT) for computational convenience.

Here I will use both the potential (FFT) and the NPFC disambiguation methods. Potential-field methods constitute the simplest possible disambiguation approach and are admittedly unrealistic for complicated magnetic structures. As such, their results are considered minimum standards. I use a potential-field method here (albeit somewhat more sophisticated than an acute-angle potential-field method) to show that even this works in some limited-resolution conditions if the test data are appropriately constructed, that is, they correspond to the semi-infinite magnetic field solution.

Figure 4 depicts the FFT and the NPFC disambiguation solutions on the original, finite-volume flowers case of LE2009. Contrary to that paper, here I also disambiguate the fully resolved flowers case (0.03′′ per pixel - Figures 4(a) and 4(d)) besides the two partially resolved cases: 0.3′′ per pixel (Figures 4(b) and 4(e)) and 0.9′′ per pixel (Figures 4(c) and 4(f)). The limited-resolution cases were disambiguated from the ambiguous magnetograms that LE2009 originally made available for the tests. For the fully resolved case I randomly scrambled the azimuth of the “answer’s” transverse field to create a 180°-ambiguous magnetogram.

Both the potential (Figures 4(b) and 4(c)) and the NPFC (Figures 4(e) and 4(f)) disambiguation solutions generally reproduce the results of LE2009 (the NPFC solution is present in LE2009 because one of the authors used the NPFC algorithm available online). Considering also the disambiguation of the fully resolved data reveals a key finding in Figure 4 regardless of spatial resolution (full or limited), both the potential and the NPFC methods fail (white areas) at nearly the same parts of the field of view, namely the areas where $\Delta \phi > 90^\circ$ in Figure 2, which are the ones most heavily impacted by the finite-volume approach. Therefore, failure in Figures 4(b), 4(e), 4(c), and 4(f) cannot be due to the limited spatial resolution, contrary to what LE2009 concluded.

Disambiguation tests on the semi-infinite-volume flowers cases are attempted in Figure 5. Here I scrambled the azimuth in both the full- and the limited-resolution “answers” to construct the ambiguous data. One now sees that the ambiguity is resolved correctly in the vast majority of pixels regardless of full
Figure 4. Comparison of disambiguation solutions with the “answer” field for the original flowers cases of LE2009. Shown are potential-field (FFT) disambiguations (a-c) and NPFC disambiguations (d-f). Comparisons refer to full resolution (a, d), the $300 \times 300$-rebinned field (b, e), and the $100 \times 100$-rebinned field (c, f). White (black) areas indicate where the ambiguity was incorrectly (correctly) resolved. The contours correspond to the line-of-sight field components and are taken at 0 and ±(100, 200, 600, 1000, 2000, 3000) G. Contours are blue (positive polarity), red (negative polarity), and green (magnetic polarity inversion line).
or limited spatial resolution and for both the potential and the NPFC methods. Minor inconsistencies (white areas) refer exclusively to the “plage” area in the case of full resolution. For limited resolution, problems in the “plage” area seem to be enhanced, at least for the 0.3″-per-pixel case (Figures 5b and 5e).
Table 1. Results of the first three comparison metrics ($M_S$, $M_{J_z}(a,s)$, $M_I$) for the original (finite-volume) flowers cases of LE2009 and for the semi-infinite-volume flowers cases introduced here. The metric values are rounded in their closest second decimal digit. I test a potential-field (FFT) and the NPFC disambiguation method. Notice that the $M_{J_z}(a,s)$-values in full resolution in the semi-infinite flowers case are not given because $J_{z(a)} = 0$ almost up to machine accuracy in this case (see also $M_I$ in full resolution), so the metric cannot be defined (Equation (8)).

|                         | Original (finite-volume) flowers cases of LE2009 | Semi-infinite-volume flowers cases introduced here |
|-------------------------|-----------------------------------------------|-----------------------------------------------|
|                         | $M_S$                                        | $M_{J_z}(a,s)$                                | $M_I$ ($\times 10^{13} A$)                      |
| “Answer”                | 0.03" 0.3" 0.9"                             | 0.03" 0.3" 0.9"                             | 0.03" 0.3" 0.9"                             |
| Potential (FFT)         | 0.84 0.85 0.85                               | -18.57 0.41 0.74                             | 0.84 0.90 0.98                               |
| NPFC                    | 0.84 0.85 0.86                               | -29.44 0.42 0.41                             | 1.30 0.92 1.16                               |
|                         |                                              |                                               |                                               |
| “Answer”                | 0.00 0.39 0.39                               | 0.00 0.39 0.39                               | 0.00 0.39 0.39                               |
| Potential (FFT)         | 1.00 1.00 1.00                               | N/A 0.88 1.00                                | 0.00 0.44 0.39                               |
| NPFC                    | 1.00 1.00 1.00                               | N/A 0.87 1.00                                | 0.00 0.44 0.39                               |

and, in addition, some minor problems occur in areas of strong gradients in the magnitude and orientation of the transverse field (Figure 1(e)) due to the lost structure. Problems in the plage area and elsewhere may be either due to the simple rebinning or, indeed, due to spurious effects caused by the loss of spatial resolution. This being said, Figure 5 undoubtedly shows that both the potential and the NPFC methods correctly resolve the ambiguity in most of the field of view, regardless of full or limited spatial resolution, when the magnetic structure corresponds to the semi-infinite volume above the boundary.

From Figures 4 and 5 one notices that the finite-volume construction of the original flowers case, not the limited spatial resolution, is responsible for the problems in disambiguating these magnetograms. Methods that attempt to reproduce solar magnetic fields will fail by design to reproduce a field defined only within a narrow layer. On the other hand, such a structure is by no means expected in the solar atmosphere, where sunspot magnetic fields obviously extend well above the photosphere. Therefore, the original flowers case is not a proper test case for solar magnetic field disambiguation. Had LE2009 adopted a more “solar-like” semi-infinite structure they would have reached different results without having to sacrifice in fine detail: they could have produced their $B_z$ solution using the two-planes approach and then calculate the horizontal field pertaining to the semi-infinite volume above the lower boundary. LE2009 do make efforts to justify that their flowers magnetogram is “solar-like” (see their Figure 9 and subsequent discussion). I have attempted similar tests with my semi-infinite flowers case obtaining similar results. In this sense, the semi-infinite flowers case is also a “solar-like” structure. The fact, however, that the field of LE2009 is defined only within 0.18" above the photosphere could be much less justified in a solar context.
Comment on the Ambiguity Resolution Paper of Leka et al. (2009)

Table 2. Same as Table 1 but for the remaining metrics \( M_{B_t > T} \) and \( M_{\Delta B}(a, s) \).

| Original (finite-volume) flowers cases of LE2009 | Semi-infinite-volume flowers cases introduced here |
|-------------------------------------------------|--------------------------------------------------|
| \( M_{B_t > 100 \text{ G}} \) | \( M_{B_t > 500 \text{ G}} \) | \( M_{\Delta B}(a, s) \) (G) |
| 0.03" | 0.3" | 0.9" | 0.03" | 0.3" | 0.9" | 0.03" | 0.3" | 0.9" |
| Potential (FFT) | 0.94 | 0.94 | 0.94 | 0.98 | 0.99 | 0.98 | 72.66 | 65.87 | 61.70 |
| NPFC | 0.93 | 0.94 | 0.95 | 0.97 | 0.98 | 0.98 | 78.44 | 66.18 | 64.88 |

Tables 1 and 2 provide the results of the comparison metrics introduced in Section 3.2. While these results for the original flowers case of LE2009 are largely consistent with those included in that paper for the NPFC and the potential-field methods, the results for the semi-infinite flowers case introduced here demonstrate that the disambiguation has been tackled much more efficiently by both methods regardless of limited spatial resolution and even the simple rebinning.

A passing note should address the plausible question of why the simplistic potential-field disambiguation scores so highly in the semi-infinite flowers case, identically to (and, in a couple of cases, slightly better than) the more sophisticated NPFC method. The answer lies in the properties of the test case. Despite its structural complexity, flowers is a potential-field model that does not involve strong polarity inversion lines, significant magnetic stresses, or strong shear. If it did, as shown clearly in Metcalf et al. (2006), (acute-angle or not) potential-field disambiguation would largely fail in these areas.

5. Discussion

5.1. Optimization vs. Physics-Based Methods

Two of the tested disambiguation methods in LE2009 appeared to be less sensitive to the limited-resolution flowers cases than methods calculating potential / non-potential fields: the AZAM utility, implemented by B. W. Lites, and the “minimum-energy” (ME0) method, which is a revisit of Metcalf’s (1994) “minimum energy” method. The AZAM is a non-automatic disambiguation method and requires a human operator. It enforces smoothness on the disambiguation results along with an empirical compliance with the divergence-free condition. The results, therefore, are largely subject to the operator’s skill. The AZAM method has managed to reproduce some of the best disambiguation solutions in tests but since it is not automatic it cannot be part of massive disambiguation efforts. The ME0 method is an optimization and uses simulated annealing to minimize the functional

\[
\mathcal{F} = |\nabla \cdot \mathbf{B}| + \lambda |J_z|
\]
over the disambiguation plane. The choice for the weighting factor $\lambda$ changes the significance of the current-density term over the field’s divergence and can apparently lead to different disambiguation results, together with a number of other (mentioned but unspecified by LE2009) keywords in the code. LE2009 used $\lambda = 1$.

It becomes, then, a valid question to ask why the ME0 method scores better than AZAM and the physics-based methods for the limited-resolution flowers cases (Figure 11 of LE2009). From the methodology of simulated annealing one gathers that the method is guaranteed to asymptotically reach the global minimum of the functional $F$ at an infinite number of iterations (Metropolis et al., 1953; Press et al., 1992). Inspecting the functional, its minimum is determined by both terms, $|\nabla \cdot B|$ and $|J_z|$ ($\lambda = 1$). In case one term is much larger than the other, however, the global minimum of $F$ will be largely dictated by the dominant term. This author speculates that $|J_z| \gg |\nabla \cdot B|$ in the limited-resolution cases of LE2009, so the disambiguation naturally seeks the smoothest possible solution for the horizontal field, that is, the one giving rise to the smallest $|J_z|$ over the disambiguation plane, largely regardless of $|\nabla \cdot B|$. Physics-based methods, on the other hand, enforce $\nabla \cdot B = 0$ at each iteration. The smoothest possible solution of LE2009 is merely the one appropriate for this particular flowers case that is potential in full resolution, so ME0 manages to reproduce the solution better than the physics-based methods. It would be interesting to see how would ME0 score if the test structure included electric currents in full resolution and/or was placed far from disk center.

Per the above, two questions need to be addressed here:

1. Is it appropriate to work with magnetic field data in which $\nabla \cdot B \neq 0$ locally (see Figure 8 of LE2009)? At this point I agree with these authors in that the answer is yes. Solar vector magnetograms include unresolved structure so $\nabla \cdot B = 0$ may be locally violated - we must learn how to handle these data in order to provide disambiguation solutions as close to the divergence-free condition as possible.

2. Are optimization methods - that appear to work even in the finite-volume cases - preferable over physics-based methods? This remains to be determined and is subject to introducing test cases that are more likely to be encountered in the real Sun. This comment targets the narrow validity of the flowers model in LE2009, not the fine-scale structure it includes. This characteristic can hardly be considered realistic for solar magnetic fields and, as such, it disables all physics-based methods by design. Performing better in an unrealistic (finite-size) test case is not proof that optimization methods work better than others. Solar-like (semi-infinite) test cases of whatever complexity (electric currents, stresses, shear) including limited spatial resolution will unquestionably reveal the best-performing disambiguation techniques.

The computing time required to reach disambiguation results is another crucial aspect of this debate: albeit a viable concept ever since Metcalf (1994) devised it, the ME0 method, like every simulated annealing technique, is computationally intensive and hence inherently slow, significantly slower than other techniques. If disambiguation results between different methods are so similar
that no further benefit exists in additional computing, then one will naturally opt
to use the method reaching these results faster. The same will be even more true
in case slower optimization methods perform slightly worse than other, faster
methods. In case optimization methods are clearly the best performers then, of
course, they will be preferable despite their computational expense. Which of
the three is the case is yet to be seen despite the conclusions of LE2009; this
work clearly shows that further investigation is needed to determine the various
methods’ performance in limited-resolution conditions.

5.2. Conclusion

This work demonstrates that the disambiguation test for limited spatial res-
olution undertaken by LE2009 was problematic: without physical justification
it included a synthetic magnetic structure with such narrow validity in space
(only 0.18” above the photosphere) that it was unlikely for physics-based meth-
ods seeking a unique, semi-infinite solution for potential and/or non-potential
fields to work properly. Instead of the actual problem, LE2009 attributed these
methods’ failure to limited spatial resolution, which was misleading. I showed
that physics-based disambiguation - even a simplistic, potential-field method
- applied to a semi-infinite magnetic configuration with the same degree of
unresolved, fine-scale structure reproduces the correct disambiguation solutions
almost completely. Moreover, I showed that, regardless of fully resolved or un-
resolved structure, physics-based methods fail at precisely the areas where the
finite-volume approach influences transverse fields most heavily, forming angles
> π/2 compared to the transverse fields of the semi-infinite volume approach.

In conclusion, synthetic test data are often very useful research tools but they
come at a price: one must ensure that they fulfill to the best possible extent the
fundamental conditions of the physical system which they are designed to repro-
duce. Otherwise, problems that are not caused by the concepts or methodologies
employed to analyze the test data, but by the test data themselves, are likely to
appear.

6. Closing Remarks

In a series of private communications with K.D. Leka (2009) the author argued
that the original “flowers” case of LE2009 was a “perfect storm” for disambigua-
tion efforts and that if it could be used to “gently nudge” colleagues away from
potential-field acute-angle disambiguation, this would be a “major success”. In
principle the above are true (I also believe that potential-field disambiguation is
unrealistic for most photospheric conditions, especially those involving multipo-
lar, stressed, or sheared magnetic structures) but we have a responsibility to show
this using proper means. A way along these lines (i.e., the semi-infinite “flowers”
solution) was proposed to the authors of LE2009 but was not accepted, which
stimulated this work. Based on the results shown here, the problem of limited-
resolution tests and the performance of disambiguation methods in them remains
to be properly addressed by the community.
This author is open to further interaction and collaboration aiming to address the problems discussed here and to define the state-of-the-art in azimuth disambiguation. The semi-infinite “flowers” solution, in both its “answer” and its $180^\circ$-ambiguous versions in full and limited spatial resolution is available online for reproduction and validation by the interested researcher.

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