Five Dimensional Gravity and Big Bang Singularity

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Abstract

A 5 - dimensional gravity theory, motivated by the brane world picture, with factorisable metric and with the Kaluza scalar $G_{55}(r)$, is shown to give rise to a positive contribution to the Raychaudhuri equation. This inhibits the focusing of geodesics and possibly cause non - focusing of the geodesics. This feature is translated into the situation in which the universe has infinite age and hence no beginning, avoiding the big bang singularity.

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1. Introduction

One of the basic questions in cosmology is whether the universe had a beginning or has existed eternally. The Standard Cosmological Model based on Einstein’s theory of gravitation implies that the universe began with a big bang singularity. Within the framework of Einstein’s theory, the above singularity cannot be avoided. This can be understood, for instance, by the use Raychaudhuri equation [1] which in the absence of torsion exhibit focusing of geodesics converging to the big bang singularity [2]. In order to avoid this singularity, one has to go beyond Einstein’s theory of gravitation. This has been investigated in the brane world scenario and the recent studies indicate possible avoidance of the big bang singularity [3,4,5]. Studies in String Theory motivated non singular cosmology [6,7] avoid the big bang singularity as well. Ellis and Maartens [8] and Ellis, Murugan and Tsagas [9] proposed the emergent scenario in which the universe stays in a static past eternally and then evolves to a subsequent inflationary era, suggesting that the universe originates from Einstein static state rather than a big bang singularity. An emergent scenario has been made possible in the modified theories of gravity such as $f(R)$ gravity, loop quantum gravity [10] and in Einstein - Cartan theory [11]. These studies motivate the consideration of higher dimensional gravity as a possible candidate for avoiding the singularity. Further motivation is provided by the trace anomaly of Conformal Field Theory dual to a 5 - dimensional Schwarzschild AdS geometry [12] in which $H^4$ ($H$ being the Hubble parameter) terms are present in the equation for $\dot{H}$ and which leads to an infinite age of the universe, avoiding the singularity. Similar resolution of the singularity comes from the corrections to Raychaudhuri equation in the brane world scenario [13], in the approach using the ‘generalized uncertainty principle’ of quantum gravity [14] and in the quantum corrected Friedmann equations from loop quantum black hole’s entropy - area relation [15,16].

While a theory of quantum gravity is far from being realized, quantum corrected Raychaudhuri equation has been proposed by Das [17] and this was the basis to obtain the corrected Friedmann equation for $H$ by Ali and Das [18] which allow the avoidance of the big bang singularity and predicting infinite age for the universe. The said corrections to the Raychaudhuri
equation cause defocussing of the geodesics thereby avoiding the singularity.

From the above studies, it is clear that in order to avoid the singularity, one needs to modify gravity such that defocussing of the geodesics occurs. One way to modify Einstein’s theory of gravity is to consider five dimensional gravity without electromagnetic fields, a minimum modification. This modification is motivated by the brane world scenario.

A direct way to understand the possible avoidance of big bang singularity is to consider the Raychaudhuri equation. This equation in 4 - dimensional gravity is

$$\frac{d\Theta}{ds} = -\frac{\Theta^2}{3} - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}u^\mu u^\nu + (\dot{u}^\mu)_{\mu},$$

where $\Theta = u^\mu$, characterizing the volume of the collection of particles with 4-velocity $u^\mu$ as they fall under gravity, $\sigma_{\mu\nu}$ is the symmetric tensor representing the shear, $\omega_{\mu\nu}$ is the antisymmetric tensor representing the vorticity and the last term $(\dot{u}^\mu)_{\mu}$ vanishes on account of the geodesic equations. The vorticity causes expansion while the shear contraction. In the absence of vorticity, the geodesics contract or focus causing the universe to have a beginning a finite time ago, creating the big bang singularity [2]. One could obtain solutions with shear and no vorticity; but not with vorticity and no shear [19] in Einstein’s theory. The singularity theorems of Penrose and Hawking use this feature to state that there is an inevitable spacetime singularity [2, 19].

Thus, attempts to avoid the singularity require either use of complicated field theoretic models of matter or modified gravity. The quantum corrections to the Raychaudhuri equation in [13,14,17] exactly do this, preventing focusing of geodesics. This feature in cosmological considerations led to the avoiding of the big bang singularity with the universe without a beginning.

It is worthwhile to examine whether the non - focusing of the geodesics could emerge classically in higher dimensional gravity with minimum modifications. The aim of this paper is to show non - focusing of geodesics by considering gravity in 5 - dimensional Kaluza theory with non - compact fifth dimension - a minimal modification of the gravity. In Section.2, we show this by considering static and spherically symmetric space time. The '55' part of the 5-dimensional metric, a 4-dimensional scalar, is responsible for the non-focusing feature. In Section.3, we consider the corrected Raychaudhuri equation in cosmological context and show that the universe is without be-
ginning, consistent with the non-focusing of the geodesics in Section 2.

2. Corrected Raychaudhuri Equation from 5-d Gravity

We consider gravity in 5-dimensional spacetime as in Kaluza-Klein theory with non-compact fifth dimension. There are no $U(1)$ gauge fields. The factorisable metric chosen corresponds to
\[
(ds)^2 = g_{\mu\nu} dx^\mu dx^\nu - G_{55} (dx^5)^2, \tag{1}
\]
where $g_{\mu\nu}$ is the 4-dimensional metric and $G_{55} = G_{55}(r) ; r^2 = x^2 + y^2 + z^2$ the Kaluza scalar. A remarkable consequence of this metric is that 4-dimensional geodesic equation has an acceleration term due to $G_{55}(r)$, namely
\[
\frac{d^2 x^\mu}{ds^2} + \Gamma^\mu_{\nu\lambda} \frac{dx^\nu}{ds} \frac{dx^\lambda}{ds} = \frac{1}{2} \frac{a^2}{G_{55}} g^{\mu\lambda} (\partial_\lambda G_{55}), \tag{2}
\]
where $a$ is a constant along the geodesic [20], consequence of the independence of the metric components in (1) on $x^5$. The occurrence of the acceleration term (right side in (2)) in 5-dimensional Kaluza-Klein theory has been realized by Schmutzer [21], Kovacs [22], Gegensberg and Kunstatter [23], Mashhoon, Liu and Wessen [24], Wessen, Mashhoon, Liu and Sajko [25] and in the brane world scenario by Youm [26]. The author studied the effect of this acceleration to restore causality in brane world [27] and to cause ‘expansion’ of the geodesics in the Raychaudhuri equation [28].

The Raychaudhuri equation coming from 5-dimensional gravity in 4-dimensional spacetime, describing the evolution of a collection of particles following their geodesics, is given by
\[
\dot{\Theta} = -\frac{\Theta^2}{4} - 2\sigma^2 + 2\omega^2 - \frac{4\pi G}{c^4} (\rho c^2 + 3p) + (\dot{u}^\mu)_{;\mu}, \tag{3}
\]
where $u^\mu = \frac{dx^\mu}{ds}$, $2\sigma^2 = \sigma_{\mu\nu} \sigma^{\mu\nu}$, $2\omega^2 = \omega_{\mu\nu} \omega^{\mu\nu}$, $\rho$ and $p$ are the density and pressure in the collection of particles. The subscript ; stands for covariant derivative. For simplicity the cosmological constant $\Lambda$ is set to zero. We have considered the 4-dimensional Raychaudhuri equation as the 5-dimensional
theory is written in terms of 4-dimensional quantities. This has the factor $\frac{1}{4}$ in front of $\Theta^2$ instead of $\frac{1}{3}$. In (3), the last term involves $\dot{u}^\mu = u^\mu_\nu u^\nu$, the possible acceleration (orthogonal to $u^\mu$) of the collection of particles. In view of (2), this term is present now. This term is:

$$\dot{u}^\mu = u^\mu_\nu u^\nu = \left(\frac{du^\mu}{ds} + \Gamma^\mu_\lambda\nu u^\lambda\right) u^\nu = \frac{du^\mu}{ds} + \Gamma^\mu_\lambda\nu u^\lambda u^\nu = \frac{du^\mu}{ds} + \Gamma^\mu_\lambda\nu \frac{du^\lambda}{ds} \frac{du^\nu}{ds} = \frac{d^2}{ds^2} \sqrt{g} g^{\mu\nu} \left(\partial_\rho G_{55}\right) = -\frac{a^2}{2} g^{\mu\nu} \left(\partial_\rho \frac{1}{G_{55}}\right)$$

using (2). So, the last term in (3) becomes

$$(\dot{u}^\mu)_\mu = -\frac{a^2}{2} g^{\mu\nu} D_\mu \left(\partial_\rho \frac{1}{G_{55}}\right),$$

(4)

where $D_\mu$ stands for the covariant derivative, $D_\mu \left(\partial_\rho \frac{1}{G_{55}}\right) = \partial_\mu \partial_\rho \frac{1}{G_{55}} - \Gamma^\sigma_\mu\rho \left(\partial_\sigma \frac{1}{G_{55}}\right)$. Thus the Kaluza scalar corrected Raychaudhuri equation is

$$\dot{\Theta} = -\frac{\Theta^2}{4} - 2\sigma^2 + 2\omega^2 - \frac{4\pi G}{c^2} (\rho c^2 + 3p) - \frac{a^2}{2} g^{\mu\nu} D_\mu \left(\partial_\rho \frac{1}{G_{55}}\right).$$

(5)

This equation was obtained by the author in [28] and the presence of the last term was shown to induce expansion. When compared with the quantum corrected Raychaudhuri equation of Das [17], we see that the role of the quantum correction $\frac{\hbar^2}{m^2} h^{ab} R_{ab}$ of [17] (the other correction term in [17] has been set to zero in [17]) is played by $\frac{a^2}{2} g^{\mu\nu} D_\mu \left(\partial_\rho \frac{1}{G_{55}}\right)$, which is classical in origin. Further, usually the $\hbar$ terms in general will be much smaller compared to $\frac{\Theta^2}{4}$ or $R_{\mu\nu} u^\mu u^\nu$ which are classical. The quantum correction will be small, though crucial. In our case, the scalar correction term is classical and so robust as it is comparable to the remaining terms. It is known that vorticity $\omega$ induces expansion while the shear $\sigma$ induces contraction in the geodesics.

The acceleration term $\frac{a^2}{2} g^{\mu\nu} D_\mu \left(\partial_\rho \frac{1}{G_{55}}\right)$ can possibly induce expansion if this term turns out to be positive. We examine this for a 'static spherically symmetric' ansatz for the 5-dimensional metric

$$(ds)^2 = e^\mu e^\nu (dt)^2 - e^\nu (dr)^2 - r^2 \left((d\theta)^2 + \sin^2(\theta)(d\phi)^2\right) - \psi(r)(dx^5)^2,$$

(6)

where $\mu, \nu$ are functions of $r$ only and $\psi(r) = G_{55}(r)$. $\mu(r), \nu(r)$ and $\psi(r)$ are unknown functions of $r$ to be determined by the 5-dimensional Einstein equations, $\tilde{R}_{AB} = 0$, $A, B = 0, 1, 2, 3, 5$.

For the metric in (6), $\tilde{R}_{tt}, \tilde{R}_{rr}, \tilde{R}_{\theta\theta}, \tilde{R}_{\phi\phi}, \tilde{R}_{55}$ are evaluated and $\tilde{R}_{AB} = 0$ is solved for $\mu(r), \nu(r)$ and $\psi(r)$. Though this calculation is direct, it is involved. The details are given in [28]. We give three solutions for illustration.
**Solution.1:**

\[ \psi(r) = \frac{b}{r}, \]

\[ e^\mu = \left( C + \frac{2r}{3} \right) ; \quad e^\nu = \left( \frac{2}{3} + \frac{C}{r} \right)^{-1}, \]

(7)

where \( b, C \) are real constants and \( b > 0 \) (as \( b < 0 \) implies sign change in the metric (6) for the last term). For this solution, the last term in the Raychaudhuri equation (5) is found to be

\[ -\frac{a^2}{2} g^{\mu\rho} D_\mu \left( \frac{1}{G_{55}} \right) = \frac{a^2}{2br} \left( 2 + \frac{3C}{r} \right). \]

(8)

For \( C > 0 \), the right side is positive. For \( C < 0 \), we have from (7) \( e^\nu = \left( \frac{2}{3} - \frac{|C|}{r} \right)^{-1} \) and this must be positive. Therefore \( r > \frac{3|C|}{2} \). Setting \( r = \beta \frac{3|C|}{2} \) with \( \beta > 1 \), the right side of (8) becomes \( \frac{a^2}{2br}(2 - \frac{1}{\beta}) \). Since \( \beta > 1 \), this is clearly positive. Thus, the effect of the scalar in 5-dimensional gravity for the solution (7), on the Raychaudhuri equation, is to induce expansion.

**Solution.2:**

\[ \psi(r) = 1 + \frac{b}{r}, \]

\[ e^\mu = \text{constant} ; \quad e^\nu = \left( 1 + \frac{b}{r} \right)^{-1}, \]

(9)

where \( b \) is a real constant. For this solution, it is found that

\[ -\frac{a^2}{2} g^{\mu\rho} D_\mu \left( \frac{1}{G_{55}} \right) = \frac{3a^2b^2}{4r^2(r + b)^2}, \]

(10)

which is positive, thereby inducing expansion.

We have examined another static and spherically symmetric solution for 5-dimensional gravity theory by Chatterjee [29] and found that for this solution \(-\frac{a^2}{2} g^{\mu\rho} D_\mu \left( \frac{1}{G_{55}} \right)\) is positive [28]. Solutions 1 and 2 have \( \psi(r) \) becoming singular in \( r \to 0 \) limit. We give third solution with non-singular \( \psi(r) \).
Solution 3:

\[ \psi(r) = \frac{b}{r+C}; \quad b > 0; \quad C > 0, \]

\[ e^\nu = \frac{5r(r+C)}{(2r^2 + 7Cr + 8C^2)}, \]

\[ \mu(r) = \ell n \left( \frac{(r+C)^2(2r^2 + 7Cr + 8C^2)^{\frac{3}{2}}}{C^4 r} \right) - \sqrt{15} \arctan \left( \frac{4r+C}{\sqrt{15}C} \right). \]

(11)

For this solution, the last term in the Raychaudhuri equation (5) is found to be

\[ -\frac{a^2}{2} g^{\mu\rho} D_\mu (\partial_\rho \frac{1}{G_{55}}) = \frac{3a^2}{4b} \frac{(4r^2 + 9Cr + 6C^2)}{5r(r+C)^2}. \]

(12)

Since \( b > 0; \ C > 0 \), this term is positive. Thus, the effect of the scalar in 5-dimensional gravity for the solution (11), on the Raychaudhuri equation, is to induce expansion. Other solutions in (7), (9) were also found to be positive and hence induce expansion. The solution in (11) is non-singular for \( \psi(r) \).

In summary, we have shown that the term due to Kaluza scalar in the Raychaudhuri equation is positive for three different solutions. Now, consider a case of hyper surfaces orthogonal to geodesics (\( \omega = 0 \)), so that (5) is

\[ \dot{\Theta} = -\frac{\Theta^2}{4} - 2\sigma^2 - \frac{4\pi G}{c^4} (\rho c^2 + 3p) - \frac{a^2}{2} g^{\mu\rho} D_\mu (\partial_\rho \frac{1}{G_{55}}). \]

(13)

In the absence of the last term, the geodesics focus to a converging point in time pointing the existence of big bang singularity. With the presence of the last term whose contribution is found to be positive, the focusing is inhibited and could cause a non-focusing of geodesics. This could possibly avoid the big bang singularity. This is examined in the next Section.

3. Cosmological Implications
We now examine the 5-dimensional gravity for Friedmann equation satisfied by the scale factor \( R(t) \) in the 4-dimensional Friedmann-Robertson-Walker metric. This can be obtained from the Raychaudhuri equation (13) by replacing \( \Theta \) by \( \frac{4R(t)}{R(t)} \), as in [17, 30]. Then

\[
\dot{\Theta} = \frac{4\ddot{R}(t)}{R(t)} - \frac{4\dot{R}(t)^2}{R^2(t)},
\]

and (13) upon setting \( \sigma = 0 \), becomes

\[
\ddot{R}(t) = -\frac{4\pi G}{4c^4}(\rho c^2 + 3p) - \frac{a^2}{8}g^{\mu\rho}D_\mu \left( \partial_\rho \frac{1}{G_{55}} \right).
\]

In terms of \( H \),

\[
\dot{H} = \frac{\dot{R}(t)}{R(t)} ; \quad \ddot{H} = \frac{\ddot{R}(t)}{R(t)} - \frac{\dot{R}(t)^2}{R^2(t)} = \frac{\dot{R}(t)}{R(t)} - H^2,
\]

we get

\[
\dot{H} = -H^2 - \frac{4\pi G}{4c^4}(\rho c^2 + 3p) - \frac{a^2}{8}g^{\mu\rho}D_\mu \left( \partial_\rho \frac{1}{G_{55}} \right).
\]

Following [17], we consider one species of fluid (collection of particles) with \( p = w\rho \) and set \( 8\pi G = c = 1 \) system of units. Then,

\[
\dot{H} = -H^2 - \frac{1}{8}\rho(1 + 3w) - \frac{a^2}{8}g^{\mu\rho}D_\mu \left( \partial_\rho \frac{1}{G_{55}} \right).
\]

We consider FRW flat metric \( k = 0 \) for simplicity. The Friedmann equation gives \( \rho = 3H^2 \) and so (18) becomes

\[
\dot{H} = -\frac{H^2}{8}(11 + 9w) - \frac{a^2}{8}g^{\mu\rho}D_\mu \left( \partial_\rho \frac{1}{G_{55}} \right) \equiv F(H).
\]

If the contribution from the scalar \( G_{55} \) is neglected, then \( \dot{H} = -\frac{H^2}{8}(11 + 9w) \) and the age of the universe is

\[
T = \int_0^\tau dt = \int_{H_0}^{H_F} \frac{dH}{H} = \frac{8}{(11 + 9w)}\left( \frac{1}{H_F} - \frac{1}{H_0} \right),
\]
showing finite $T$. The big bang singularity is not avoided.

If the contribution from the scalar $G_{55}$ is taken into account, then using (19),

$$ T = \int_{H_0}^{H_P} \frac{dH}{F(H)}, $$

where $F(H) = -\frac{H^2}{8}(11 + 9w) - \frac{a^2}{8} g^\rho_\mu D_\mu \left( \frac{\partial}{\partial \rho} G_{55}^{55} \right)$. In order to find $F(H)$, we need $G_{55}(r)$. For the sake of illustration, we take the non-singular solution (11) (that is $\psi(r) = G_{55}(r) = \frac{b}{r + c}$). Using FRW metric, we find

$$ F(H) = -\frac{H^2}{8}(11 + 9w) + \frac{a^2}{8br} e^{-2\int H ds}. $$

where $R$ in the second term of $F(H)$ is expressed as $e^{\int H ds}$. This expression is evaluated 'iteratively'. From (19) $\dot{H} = F(H)$. Neglecting the second term in $F(H)$, we write $\dot{H} = -\alpha H^2$ with $\alpha = \frac{1}{8}(11 + 9w)$. Then,

$$ -2 \int H ds = -2 \int H \frac{ds}{dH} dH = -2 \int \frac{H}{dH} dH, $$

$$ \approx \frac{2}{\alpha} \int_{H_P}^{H(s)} \frac{dH}{H} = \frac{2}{\alpha} \ell n \frac{H(s)}{H_P}, $$

$$ = \frac{2}{\alpha} \ell n \left( 1 + \frac{H(s) - H_P}{H_P} \right), $$

$$ \approx \frac{2}{\alpha} \left( \frac{H(s) - H_P}{H_P} - \frac{(H(s) - H_P)^2}{2H_P^2} + \cdots \right), \quad \text{(23)} $$

so that $F(H)$ is written as

$$ F(H) \approx -\alpha H^2 + \beta(r)e^{\frac{2}{\alpha} \left( \frac{H(s) - H_P}{H_P} - \frac{(H(s) - H_P)^2}{2H_P^2} + \cdots \right)}, \quad \text{(24)} $$

where $\beta(r) = \frac{a^2}{8br}$. The correction from the scalar $G_{55}$ changes the nature of $F(H)$. If $H_P$ is the nearest fixed point, then following the fixed point analysis of Awad [31], we approximate $F(H)$ as a polynomial in $H(s) - H_P$. Then, paralleling the steps of [17] (their equation (11)), we see that $T \to \infty$, showing a universe without a beginning. Thus, the conclusion of Section 1,
of possible non-focusing of geodesics, is consistent with the absence of singularity or the universe without a beginning. The above considerations hold good with $k = \pm 1$ as well, except for the change in $\beta(r)$.

In summary, we have shown that a simple 5-dimensional gravity theory (motivated by brane world picture) with the Kaluza scalar $G_{55}(r)$ leads classically to an inhibition of the geodesic focusing thereby avoiding big bang singularity (which requires focusing or converging geodesics) in Section 1. The use of FRW metric consistently yields infinite age of the universe or a universe without a beginning. These are in agreement with the conclusions of [17] and [18] which used quantum corrections. In the Raychaudhuri equation, the vorticity term could also give expansion or non-focusing of the geodesics. The presence of this term changes the spacetime to have torsion. We have presented a minimally modified higher dimensional gravity theory.

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