Angular distributions in pion-nucleon Drell-Yan process

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Abstract. We study the Drell-Yan process in the polarized pion-nucleon scattering. We calculate the $U(1)_A$ gauge invariant hadron tensors. To this end, we have to take into account two diagrams—the standard diagram and an additional one—both diagrams are required to get the gauge invariant hadron tensor. We obtain a new single transverse spin asymmetry that probes gluon poles and chiral-odd distribution functions. The process of pion-nucleon scattering in the limit of large Feynman $x$ is described by a convolution of two non-collinear exclusive-channel distribution amplitudes, which demonstrates the duality between different factorization regimes.

1. Introduction

For decades the Drell-Yan (DY) process has been remaining one of the most convenient objects in the studies of hadron structure, both from the theoretical and experimental sides. Several facilities are currently carrying out or developing programs dedicated to measuring DY processes—SPS (COMPASS) [1, 2], RHIC [3], NICA [4, 5]. An important family of observables in these experiments is single transverse spin asymmetries ($S_{T,A}$), which are related to the three-dimensional hadron structure owing to the inextricable link between the transverse spin and the parton transverse momentum dependence [6, 7, 8, 9, 10, 11].

In this report based mostly on Ref. [12], we apply ideas developed in [13, 14, 15] (see also talk given by I. Anikin in this workshop) to DY process induced by the polarized pion-nucleon scattering at large Feynman $x$. It turns out that the inclusion of gluon poles in the spirit of [13, 14, 15] predicts a new $S_{T,A}$ non-trivially dependent on the azimuthal angle of nucleon spin vector (in the Collins–Soper frame). Moreover, as we will see in the following, the non-trivial angular dependence of $S_{T,A}$ can be considered as a signal of the parton transverse momentum dependence.

On the other hand, the limit of large Feynman $x$ allows us to describe pions by wave functions and distribution amplitudes rather than parton distributions. In a sense, this reveals a duality between semi-inclusive and exclusive processes or, put in other words, transverse momentum and generalized parton distributions. Besides, it gives a possibility to work with the gluon poles in the most explicit form—represented by a hard gluon propagator rather than inclusive function $B^V(x_1, x_2)$. 

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2. Kinematics. Sudakov decompositions

We study DY process in pion-nucleon scattering (Fig. 1) with nucleon $N$ being transversely polarized, $\pi(P_1) + N(P_2, S) \rightarrow \gamma^*(q) + \bar{q}(K) +$ spectators $\rightarrow \ell^-(l_1) + \ell^+(l_2) + \bar{q}(K) +$ spectators. The virtual photon produces a pair of unpolarized leptons with an invariant mass squared $q^2 = Q^2$ assumed to be large enough to make use of the factorization theorem for the process.

Figure 1. Pion-nucleon Drell-Yan process. In the case we consider, the nucleon is transversely polarized, leptons are unpolarized.

The Sudakov decompositions of momenta and nucleon spin 4-vector read

$$P_1 \approx \frac{Q}{x_B \sqrt{2}} n^* + P_{1\perp}, \quad P_2 \approx \frac{Q}{y_B \sqrt{2}} n + P_{2\perp}, \quad S \approx \frac{\lambda}{M_N} P_2 + S_\perp$$

$$q = \frac{Q}{\sqrt{2}} (n^* + n) + q_\perp, \quad q_\perp^2 \ll Q^2,$$

where $\lambda$ and $M_N$ are nucleon helicity and mass, $x_B$ and $y_B$ are usual dimensionless Bjorken variables, $n^*$ and $n$ are plus and minus light-cone vectors,

$$n^{*\mu} = (1/\sqrt{2}, 0, 0, 1/\sqrt{2}), \quad n^{\mu} = (1/\sqrt{2}, 0, 0, -1/\sqrt{2}).$$

In what follows, all Lorentz non-invariant quantities are defined in the Collins–Soper frame [11, 16], which is a rest frame of the dilepton pair with unit vectors of the spatial axes

$$\hat{x} \sim -\hat{P}_1 - \hat{P}_2, \quad \hat{z} \sim -\hat{P}_1 + \hat{P}_2, \quad \hat{y} = \hat{z} \times \hat{x}.$$

Here, a hatted vector is a vector of the same direction and unit magnitude, $\hat{A} = A/|A|$. All relevant angles are depicted in Fig. 2—the polar $\theta$ and azimuthal $\phi$ angles of the lepton $\ell^-(l_1)$, the angle $\varphi_S$ between scattering plane and nucleon-spin plane (azimuthal angle of the nucleon spin vector).

Figure 2. Collins–Soper frame. Cyan, olive and red planes are scattering, lepton and nucleon-spin planes, respectively. The colour of a vector corresponds to the colour of a plane the vector lies in.

3. Hadron tensor

The cross section of the DY process can be written [11, 16] as

$$\frac{d\sigma}{d^4q} = \frac{\alpha_{em}^2}{2Q^4} L_{\mu\nu} W^{\mu\nu},$$

where $L_{\mu\nu}$ and $W^{\mu\nu}$ are the hadron tensor and its conjugate, respectively.
where $s = (P_1 + P_2)^2$; $\Omega^{CS}_{k_1}$ is the solid angle of lepton in the Collins–Soper frame; $L_{\mu\nu}$ and $W_{\mu\nu}$ stand for the lepton and hadron tensor, respectively. Since the leptons are unpolarized in the case we consider, the lepton tensor is real. As a consequence, the hadron tensor has to be real as well.

Before the collinear factorization [13, 14, 15], the standard (Fig. 3a) and non-standard (Fig. 3b) diagrams give the following contributions to the hadron tensor:

\begin{align*}
W_{\mu\nu}^{\text{(stand.)}} & \sim \int d^4k_1 d^4k_2 \delta^{(4)}(k_1 + k_2 - q) \\
& \times \int d^4\ell \, D_{\alpha\beta}(\ell) \Tr [\gamma_{\nu} \Gamma_{\gamma} S(\ell - k_2) \gamma_{\mu} \Gamma_{1} \gamma_{3} \Gamma_{2}] \bar{\Phi}^{[\Gamma]}(k_2) \Phi^{[\Gamma]}_{(1)}(k_1; \ell) \Phi^{[\Gamma]}_{(2)}(k_1) \delta((P_1 - k_1)^2), \quad (6)
\end{align*}

\begin{align*}
W_{\mu\nu}^{\text{(nonstand.)}} & \sim \int d^4k_1 d^4k_2 \delta^{(4)}(k_1 + k_2 - q) \\
& \times \int d^4\ell \, D_{\alpha\beta}(\ell) \Tr [\gamma_{\nu} \Gamma_{\gamma} S(\ell - k_2) \gamma_{\mu} \Gamma_{1} \gamma_{3} \Gamma_{2}] \bar{\Phi}^{[\Gamma]}(k_2) \Phi^{[\Gamma]}_{(1)}(k_1; \ell) \Phi^{[\Gamma]}_{(2)}(k_1) \delta((P_1 - k_1)^2), \quad (7)
\end{align*}

where

\begin{align*}
\bar{\Phi}^{[\Gamma]}(k_2) & = \int \frac{d^4\eta_2}{(2\pi)^4} e^{-i k_2 \eta_2} \langle P_2, S | \bar{\psi}(\eta_2) \Gamma \psi(0) | S, P_2 \rangle, \quad (8) \\
\Phi^{[\Gamma]}_{(1)}(k_1; \ell) & = \int \frac{d^4\eta_1}{(2\pi)^4} e^{i (P_1 - \ell - k_1) \eta_1} \langle 0 | \bar{\psi}(\eta_1) \Gamma_1 \psi(0) | P_1 \rangle, \quad (9) \\
\Phi^{[\Gamma]}_{(2)}(k_1) & = \int \frac{d^4\xi}{(2\pi)^4} e^{-i k_1 \xi} \langle P_1 | \bar{\psi}(\xi) \Gamma_2 \psi(0) | 0 \rangle. \quad (10)
\end{align*}

In Eqs. (6) and (7), the $\delta$-functions $\delta((P_1 - k_1)^2)$ indicate that the quark with the momentum $P_1 - k_1$ is on its mass shell.

Let us consider first the standard contribution, which exists even for real $B^V$-function. We constrain ourselves to the case with one of the pion distribution amplitudes being projected onto the matrix related to the chiral-odd distribution. Then, the pion chiral-odd distribution amplitudes “automatically” isolates the chiral-odd structure in the nucleon matrix element. So, the matrices $\Gamma$, $\Gamma_1$, $\Gamma_2$ and related distribution amplitudes in Eqn. (6) are as follows:

\begin{align*}
\Gamma \Phi^{[\Gamma]} = \sigma^{+-} \Phi^{[\sigma^{-}]} \quad \Gamma_1 \Phi^{[\Gamma]} = \gamma^{-}(\gamma_3) \Phi^{[\Gamma]}(\gamma_3) \quad \Gamma_2 \Phi^{[\Gamma]} = \sigma^{+}(\gamma_3) \Phi^{[\Gamma]}(\gamma_3). \quad (11)
\end{align*}
These are the twist-2 nucleon parton distribution and the pion distribution amplitudes of twist two and three. Following the factorization procedure (see, for example, [17, 18, 19, 20]), we have for the upper blob in Fig. 3a

\[ \bar{\Phi}^{[\sigma, \perp]}(y) \defeq \int (d^4k_2) \delta(y - k_2^2 / P_2^2) \bar{\Phi}^{[\sigma, \perp]}(k_2) = \varepsilon^{-\perp} S^{\perp} P^2 \bar{h}_1(y). \] (12)

For the lower blobs in Fig. 3a, we obtain a following convolution

\[ B_{\alpha\beta}^{[\Gamma_2, \Gamma_1]}(x_1, x_2) = \frac{g^{\perp}_{\alpha\beta}}{2P_1^+ (x_2 - x_1)} \Phi^{[\Gamma_2]}(x_1) k_1^{+} \otimes \Phi^{[\Gamma_1]}(x_2), \] (13)

where the integration over \( dk_1^- \) has been done with the help of \( \delta((P_1 - k_1)^2) \) and the convolution over the transverse momentum is defined as

\[ F \otimes G = \int (d^2k_1^+) F(k_1^+) \int (d\ell^- d\ell_\perp) \Delta(\ell^-, \ell_\perp) G(k_1^+; \ell^- \ell_\perp), \] (14)

\[ \Delta(\ell^-, \ell_\perp) = \frac{1}{\ell^- - \ell_\perp^2/(2(x_2 - x_1)P_1^+) + i \text{sign}(x_2 - x_1)0}. \] (15)

The convolution of the two non-collinear pion distribution amplitudes, the function \( B_{\alpha\beta}^{[\Gamma_2, \Gamma_1]}(x_1, x_2) \) of Eqn. (13), is an analogue of the function \( B^V(x_1, x_2) \) that is introduced for the description of the inclusive channel [13]. The representation through two non-collinear functions is physically sound, if the invariant mass of pion-induced spectators is not large. In the limit of \( |\ell_\perp| \gg |k_1^+| \), we can write a similar representation through two collinear distribution amplitudes. The resemblance between the functions \( B^V(x_1, x_2) \) and \( B_{\alpha\beta}^{[\Gamma_2, \Gamma_1]}(x_1, x_2) \) can be seen as a manifestation of duality between exclusive and inclusive channels.

In Eqn. (13), the gluon pole at \( x_1 = x_2 \) can be conveniently dealt with in the contour gauge (see Refs. [13, 14, 15] and the talk given by I. Anikin in this workshop). This leads to a modification

\[ \frac{1}{x_2 - x_1} \xrightarrow{c. g.} \frac{1}{x_2 - x_1 - i\epsilon}, \] (16)

which follows from the integral representation of the unit step function [14].

The non-standard term in the hadron tensor (Fig. 3b) can be considered along the same lines as the standard one [13]. Completing all proper decompositions and simplifications, we finally get the following expressions for the hadron tensor:

\[ W_{\mu\nu}^{(\text{stand.})} \sim i \int dx_1 dy \delta^{(4)}(x_1 P + y P_2 - q) \Phi^{[\Gamma]}(y) \]

\[ \times \int dx_2 \frac{1}{(x_2 - x_1)y_\perp + i0} \gamma_\mu \gamma_\alpha \gamma_\gamma \gamma_\beta \Gamma_2 \frac{1}{2} \frac{g^{\alpha\beta}_{\nu\mu} P_{1}^+}{x_2 - x_1 - i\epsilon} \Phi^{[\Gamma]}_1(x_1) \otimes \Phi^{[\Gamma]}_2(x_2), \] (17)

\[ W_{\mu\nu}^{(\text{nonstand.})} \sim i \int dx_1 dy \delta^{(4)}(x_1 P_1 + y P_2 - q) \Phi^{[\Gamma]}(y) \]

\[ \times \text{Tr} \left[ \gamma_\mu \gamma_\alpha \gamma_\gamma \gamma_\beta \Gamma_2 \right] \frac{1}{2} \int dx_2 \frac{g^{\alpha\beta}_{\nu\mu} P_{1}^+}{x_2 - x_1 - i\epsilon} \Phi^{[\Gamma]}_1(x_1) \otimes \Phi^{[\Gamma]}_2(x_2). \] (18)
It can be readily seen that both terms (17) and (18) are not gauge invariant in QED sense, while the sum thereof is invariant. The sum reads

$$\mathcal{W}^{\mu\nu} = \int d^2q_{\perp} \mathcal{W}^{\mu\nu}$$

$$\sim i \int dx_1 dy \, \delta(x_1 P_1^+ - q^+ - q^-) \bar{h}_1(y) \int dx_2 \tilde{B}(x_1, x_2) \frac{\varepsilon_{\perp}^{\mu} P_1 P_2}{(P_1 P_2)^{\perp}} [\frac{1}{y} - \frac{P_{\perp}^\mu}{x_1}] ,$$

where

$$\tilde{B}(x_1, x_2) = \frac{1}{2} \frac{\Phi^{tw-3}_{(1)} (x_1) \otimes \Phi^{tw-2}_{(2)} (x_2)}{x_2 - x_1} .$$

The above expression coincides formally with the result obtained in [13] for the inclusive Drell-Yan process.

4. Single transverse spin asymmetry

The SS⊥A is written as

$$A = \frac{d\sigma^{(1)} - d\sigma^{(4)}}{d\sigma^{(1)} + d\sigma^{(4)}},$$

where $d\sigma^{(1)}$ and $d\sigma^{(4)}$ are cross sections (5) with opposite directions of the nucleon transverse spin. For the process under consideration, we can obtain the SS⊥A from Eqns. (19) and (20).

This asymmetry for the chiral-odd contribution can be put in terms of [2] as follows:

$$A = \left[ S_{\perp} Q \frac{\sin \phi S B_{UT}^{\sin \phi S}}{f_1(y_B) H_1(x_B)} \right], \quad D_{\sin 2\theta} = \frac{\sin 2\theta}{1 + \cos^2 \theta},$$

where $B_{UT}^{\sin \phi S} = 2\bar{h}_1(y_B) \Phi^{tw-3}_{(1)} (x_B) \otimes \Phi^{tw-2}_{(2)} (x_B); \tilde{f}_1(y_B)$ and $H_1(x_B)$ parametrize the following matrix elements contributing to the unpolarized cross section

$$\langle P_2 | \bar{\psi} \gamma \cdot q | P_2 \rangle \sim P_2 \tilde{f}_1(y), \quad \langle P_1 | \bar{\psi}_+ | q(K) \rangle \langle q(K) | \psi_+ | P_1 \rangle \sim P_1 H_1(x),$$

$$H_1(x) = \frac{1}{2 x_1 P_1^+} \int (d^2k_{\perp}) \phi_{(2)}^{\gamma + (\gamma \gamma)}(x_1, k_1^+) \phi_{(1)}^{[\gamma \gamma]}(x_1, k_1^+).$$

It should be stressed that the asymmetry (22) is different from both the leading twist Sivers asymmetry $A_{UT}^{\sin \phi S}$, which is accompanied by the depolarization factor $D_{[1+\cos^2 \theta]}$, and the higher twist asymmetries $A_{UT}^{\sin (\phi S \pm \phi)}$, which are related to a different tensor structure, although appearing with the same factor $D_{\sin 2\theta}$ as the asymmetry (22).

5. Conclusion

We derived the hadron tensor for pion-nucleon DY process with large Feynman $x$, transversely polarized nucleon and “primordial” transverse momenta. We took into account two diagrams (Fig. 3), standard diagram and non-standard [13] one—both diagrams are necessary to get manifestly $U(1)_{em}$ gauge invariant hadron tensor. The pion-related soft part of the diagrams is described by a convolution of two non-collinear distribution functions—a sort of decomposition of the inclusive function $B^V(x_1, x_2)$. This approximation implies that the invariant masses...
of undetected spectators in the pion sector are relatively small. We studied the case of the nucleon distribution and one of the pion amplitudes being chiral-odd. In this case, we obtained unexpected angular dependence of $S_S A$ that can be measured experimentally. The asymmetry can be seen as a signal for the transverse “primordial” momentum dependence probing both gluon poles and chiral-odd functions and related to the duality of different factorization regimes in exclusive and inclusive channels (see, [21]).

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