Group consensus coordination control in networked nonholonomic multirobot systems

Tiehui Zhang¹,², Jun Liu³, Hengyu Li¹, Shaorong Xie¹ and Luo Jun¹

Abstract

In this article, the coordination control problem of group tracking consensus is considered for networked nonholonomic mobile multirobot systems (NNMMRSs). This problem framework generalizes the findings of complete consensus in NNMMRSs and group consensus in networked Lagrangian systems (NLSs), enjoying capacious application backgrounds. By leveraging a kinematic controller embedded in the adaptive torque control protocols, a new convergence criterion of group consensus is established. In contrast to the formulation under strict algebraic assumptions, it is found that group tracking consensus for NNMMRSs can be realized under a simple geometrical condition. The system stability analysis is dictated by the property of network topology with acyclic partition. Finally, the theoretical achievements are verified by illustrative numerical examples. The results show an interesting phenomenon that, for NNMMRSs, the state responses exhibit negative correlation with the algebraic connectivity and coupling strength.

Keywords

Group consensus, networked nonholonomic mobile multirobot systems, coordination control, acyclic partition

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Introduction

An intensive area of research about mechanical control of nonholonomic robot system (NRS) has been investigated in the past decade.¹⁻⁵ Since the necessary conditions of smooth feedback stabilization do not hold, it has been corroborated that controlling such system is a difficult problem.⁶ Besides, as the velocity constraints are not integrable, the position of the system cannot be obtained directly without the help of dynamic equation. Accordingly, NRS usually exhibits characteristics of underactuated system, that is, n degrees of freedom can only be controlled by m inputs, n > m.⁷⁻⁹ Among others, two-wheeled mobile robot, as a classical type of NRS, has been paid a significant attention in the literature¹⁰⁻¹² due to its wide variety of applications, including surveillance and monitoring missions. However, complex tasks cannot be completed by an individual robot, such as large-scale search and rescue missions, and assembly line production. To solve this problem, the research on the coordination control of networked nonholonomic mobile multirobot systems (NNMMRSs) has drawn a lot of attractions due to its powerful operation ability and broad potential

¹ School of Mechatronic Engineering and Automation, Shanghai University, Shanghai, China
² College of Elementary Education, Jining University, Qufu, China
³ Department of Mathematics, Jining University, Qufu, China

Corresponding author:
Jun Liu, Department of Mathematics, Jining University, Qufu, Xingtian Road 1, 273155, China.
Email: sdwsj@163.com
applications.\textsuperscript{12–15} In general, the problems of consensus tracking and coordination control go hand in hand. Consensus pertains to the collective behaviors that agents achieve commonality in some way by designing protocols with local or global information and defined state upfront.\textsuperscript{16,17} To realize various collective behaviors/patterns in multiagent systems (MASs), including but not limited to flocking,\textsuperscript{18–20} rendezvous\textsuperscript{21} and formation,\textsuperscript{22–24} a lot of research has been done on finite-time consensus,\textsuperscript{25} consensus under communication limits including time delays,\textsuperscript{26,27} uncertain nonlinear systems,\textsuperscript{28} and asynchronization.\textsuperscript{29}

It is noteworthy that the literature mentioned above concentrates on studying complete consensus, namely, all the agent behaviors evolve to the same state. Nevertheless, as the modern engineering systems tend toward large-scale, intelligentized and elaborate strategies, the tasks dealt with by multirobots are very complicated. Accordingly, to cope with any unpredicted circumstances and changes, the agent behaviors are likely to evolve multiple consistent states. For example, formation tracking control of MASs often has complex tasks inevitably lead to the complexity of multiobjective, which requires the systems to make interactive decisions (group communication algorithm) to reach a certain degree of consensus. In this case, modularizing the robot system into subgroups and achieving group consensus turn out to be an efficient way to reduce computational complexity. Taken this way, the robots in each subgroup track a common reference trajectory and asymptotic,\textsuperscript{30} and asynchronization.\textsuperscript{31} It is of practical and theoretical interest in the aforementioned studies, concerning either NNMMRSs complete consensus or holonomic MAS group consensus. However, it should be noted that the existing approaches cannot be applied to the group consensus problem of multiple nonholonomic systems in the context of dynamics.

Compared with the aforementioned work, the presentation in this article is for a promising framework to solve the group-based consensus problem of NNMMRSs. Resorting to a kinematic controller embedded in the adaptive torque control protocols, a new convergence criterion of group consensus is established. The key contributions from this study are summarized below: (i) Compared with the existing results,\textsuperscript{15,38–40} a more applicable scenario, the group tracking consensus problem of NNMMRSs, is proposed. This scenario can better adapt to the unforeseen changes brought by environments, situations, and tasks. (ii) The conclusions in this article extend the results of complete consensus in NNMMRSs and group consensus in NLSs, taking the existing results\textsuperscript{15,37} as special cases. (iii) This article shows that it is relatively easy to achieve group consensus under a simple geometrical condition. In contrast to the existing results,\textsuperscript{36} there is no need to make the predefined strict assumptions about algebraic eigenvalues. (iv) In comparison with complete consensus, the lower complexity of group consensus can be achieved by decreasing the intragroup interaction. The work in this research is conducive to the deployment and adaptation of networked robot architecture and distributed robotic applications.

The rest part is briefly arranged as follows: In “Preliminaries” section, requisite mathematical backgrounds and problem formulation are introduced in order. The “Presentation” section solves the coordination control problem of group tracking consensus for NNMMRSs, “Simulations” section demonstrates numerical examples to verify the effectiveness of Theorem 1, and “Conclusions” section outlines the future work.

**Preliminaries**

**Graph theory**

Throughout this article, the notation \( \mathbf{G} = (\mathbf{V}, \mathbf{E}, \mathbf{A}) \) represents a weighted directed graph (diagraph), \( \mathbf{V} = \{1, 2, \ldots, d\} \) represents the node set, \( \mathbf{E} \) represents the edge set, and \( \mathbf{A} = [a_{ij}]^{d \times d} \) is the weighted adjacency matrix in which the entry is given by \( a_{ij} = 0 \) if \( (\chi_j, \chi_i) \notin \mathbf{E} \), otherwise, \( a_{ij} \neq 0 \) if \( (\chi_j, \chi_i) \in \mathbf{E} \). A directed path in the digraph is composed with a set of different edges from \( \chi_1 \) to \( \chi_j \), \( \{\chi_1, \chi_2, \ldots, \chi_{j-1}, \chi_j\} \), satisfying \( (\chi_n, \chi_{n+1}) \in \mathbf{E} \), \( n = 1, 2, \ldots, j - 1 \). A spanning tree in the digraph is defined as a directed path from \( \chi_j \) to any other nodes \( \chi_i \neq \chi_j \). Define \( L = [\ell_{ij}] \in \mathbb{R}^{d \times d} \) as the Laplacian matrix, where \( \ell_{ij} = \sum_{j=1}^{d} a_{ij} \), \( \chi_i = 1, 2, \ldots, w \), and \( \ell_{ij} = -a_{ij} \), \( \chi_i \neq \chi_j \).

**Dynamics of networked nonholonomic mobile multirobot systems**

Considering NNMMRSs of \( d \) robots subject to nonholonomic constraints, the \( i \)th system dynamics equation can be formulated as follows\textsuperscript{1,21}

\[
\begin{align*}
\dot{q}_i(t) &\in \mathbb{R}^n, \\
\dot{q}_i(t) &+ C_i(q_i, \dot{q}_i)q_i + G_i(q_i) = B_i(q_i)\tau_i + A_i^T(q_i)\lambda_i, \\
i &= 1, 2, \ldots, d
\end{align*}
\]

where \( q_i, \dot{q}_i \in \mathbb{R}^n \) are, respectively, the generalized coordinate vector and the generalized velocity vector. Denote
\[ M_i(q_i) \in R^{n \times n} \] as the inertial matrix, \( C_i(q_i, \dot{q}_i) \in R^{n \times n} \) as the Coriolis and centrifugal force matrix, and \( G_i(q_i) \in R^n \) as the generalized potential force, respectively. \( B_i(q_i) \in R^{p \times p} \) is the input transformation matrix, \( p = n - m \) and \( \tau_i \in R^n \) represent the input torque vector. The constraint matrix and the constraint force vectors are denoted as \( A_i(q_i) \in R^{m \times n} \) and \( \lambda_i \in R^m \), respectively.

The kinematic equation of the \( i \)'th system is expressed as

\[ A_i(q_i) \dot{q}_i = 0 \]

Assume that \( S_i(q_i) \in R^{n \times p} \) is the full-rank matrix,\(^{42} \) then

\[ S_i^T(q_i)A_i^T(q_i) = 0 \]

Thus, system (1) can be transformed into two formulas

\[ \dot{q}_i = S_i v_i \]  \hspace{1cm} (2)

\[ \overline{M}_i(q_i) \overline{v}_i + \overline{C}_i(q_i, \dot{q}_i) v_i = \overline{\tau}_i \]  \hspace{1cm} (3)

where \( v_i(t) \in R^p \) in equation (2) is the velocity vector applying for all \( t \), \( \overline{M}_i(q_i) = S_i^T M_i S_i \in R^{p \times p} \), \( \overline{C}_i(q_i, \dot{q}_i) = S_i^T (M_i \dot{S}_i + C_i S_i) \in R^{p \times p} \), \( \overline{\tau}_i = \overline{B}_i \tau_i \), \( \overline{B}_i = S_i^T B_i \). In addition, the properties of system (1) are available for those of system (3) after transformation. Furthermore, there are three significant dynamics properties.\(^{43} \)

**Property 1.** \( k_{mi}, k_{Mbi}, \) and \( k_{ei} \) are positive constants, satisfying that \( 0 \leq k_{mi} p_i \leq \overline{M}_i(q_i) \leq k_{Mbi} p_i \), \( ||\overline{C}_i(x, y, z)|| \leq k_{ei} ||v_i|| \), \( v_i, x, y, z \in R^p \).

**Property 2.** \( \frac{1}{2} \overline{M}_i - \overline{C}_i \) is skew symmetric.

**Property 3.** System (3) is linearly parameterizable with \( \rho_i \), which represents the constant dynamics parameter vector

\[ \overline{M}_i(q_i) \sigma + \overline{C}_i(q_i, \dot{q}_i) \varsigma = \Psi_i(q_i, \dot{q}_i, \sigma, \varsigma) \rho_i \]  \hspace{1cm} (4)

where \( \sigma, \varsigma \in R^p \) are differentiable vectors and \( \Psi_i(q_i, \dot{q}_i, \sigma, \varsigma) \) is regression matrix.

### Acyclic network

Here, the digraph \( G = (V, E, A) \) is employed to depict the network topology of \( d \) NNMMRS. For \( V = \{1, 2, \ldots, d\} \), assume that there is a partition \( \{V_1, V_2, \ldots, V_k\} \), satisfying (i) \( V_i \neq \emptyset \), (ii) \( \cup_{i=1}^{d} V_i = V \), and (iii) \( V_w \cap V_z = \emptyset \), \( w \neq z, w, z \in \{1, 2, \ldots, k\} \), and that the subgroup graph \( G_i \) of \( G \) is denoted as the network topology of \( V_i \).

Each subgroup node set can be described as \( V_i = \{ \sum_{j=0}^{i-1} n_j + 1, \ldots, \sum_{j=0}^{i} n_j \} \), where \( n_0 = 0, n_i > 0, \sum_{j=0}^{i} n_j = d, j = 1, 2, \ldots, k \). Denote \( h_0 = 0, h_i = \sum_{j=0}^{i} n_j \). Denote \( i = j \) if \( i \) and \( j \) come from the same subgroup node set.

Next, an acyclic partition of \( G \) is introduced as follows\(^{37,44} \)

**Problem formulation**

For NNMMRSs, the problem formulation of group tracking consensus is presented in this subsection. First, a sketch of the \( i \)'th two-wheeled nonholonomic robot is shown in Figure 1. Then, the configuration of the \( i \)'th robot can be described as follows

\[ q_i = [x_i, y_i, \zeta_i, \rho_{w, i}, \rho_{d, i}]^T \]

where \( P_0 = (x_i, y_i) \) represents the origin coordinate of the mobile robot body frame. \( \zeta_i \) is the steering angle of the mobile robot. \( \rho_{w, i} \) and \( \rho_{d, i} \) are, respectively, the angles of the right and the left actuated wheels.

Given that the wheels of each robot roll but not slip, the constraint matrix of \( A_i(q_i) \) in equation (1) is derived as\(^{41} \)

![Figure 1. Sketch of the \( i \)'th two-wheel actuated mobile robot.](image-url)
where $r_i$ is the radius of the wheel and $b_i$ is the half-width. Select $S_i(q_i)$ in equation (2) as

$$\begin{bmatrix}
\frac{r_i}{2} \cos \zeta_i & \frac{r_i}{2} \cos \zeta_i \\
\frac{r_i}{2} \sin \zeta_i & \frac{r_i}{2} \sin \zeta_i \\
\frac{r_i}{2b_i} - \frac{r_i}{2b_i} & 1 \\
0 & 1
\end{bmatrix}$$

(6)

For convenient discussion, let $q_i = [x_{i,t}, y_{i,t}, \zeta_{i,t}]^T$. The expression of the generalized velocity form is expressed as

$$\dot{q}_i = \begin{bmatrix}
x_{i,t} \\
y_{i,t} \\
\zeta_{i,t}
\end{bmatrix} = \begin{bmatrix}
\cos \zeta_i & 0 & 0 \\
\sin \zeta_i & 0 & 1 \\
0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
\mu_i \\
\eta_i \\
\zeta_i
\end{bmatrix}$$

where the vector $v_i = (\mu_i, \eta_i)^T$ is denoted as the velocity of $P_0$. Now, the concept of group consensus coordination control in NNMMRSs is defined below.

**Definition 1.** For NNMMRSs consisting of $d$ robots, there is a partition $\{V_1, V_2, \ldots, V_k\}$ of the node set $V$, where $V_l$ is the subgroup of $V$, $l = 1, 2, \ldots, k$. The reference trajectory is given as

$$\begin{cases}
x_{r,l} = \mu_{r,l} \cos \zeta_{r,l} \\
y_{r,l} = \mu_{r,l} \sin \zeta_{r,l} \\
\zeta_{r,l} = \eta_{r,l}
\end{cases}$$

where $q_{r,l} = [x_{r,l}, y_{r,l}, \zeta_{r,l}]^T$ and $\mu_{r,l} \neq 0$. Denote $v_{r,l} = (\mu_{r,l}, \eta_{r,l})^T$. It refers to achieve group consensus tracking control in NNMMRSs if for robot $i$, $i = 1, 2, \ldots, d$, $i \in V_l$, utilizing the control input $\tau_i$, a smooth velocity control input vector, $v_{r,i} = [\mu_{r,i}, \eta_{r,i}]^T = P_i(e_i, v_{r,i}, \psi_{r,i})$ can be found, such that $v_i \rightarrow v_{r,i}, \lim_{t \rightarrow \infty} (q_i - q_{r,i}) = 0$, where $e_i$ is denoted as the position error with respect to $q_i$, and $\psi_{r,i}$ is denoted as the control gain vector. In addition, if $i = j$, one has $\lim_{t \rightarrow \infty} (q_i - q_j) = 0$, $\lim_{t \rightarrow \infty} (\dot{q}_i - \dot{q}_j) = 0$.

**Presentation**

Trajectory tracking is the core objective of basic navigation problems. Accordingly, the aim of this section is to design a unified group consensus tracking scheme for NNMMRSs. Assume that $V = \{1, 2, \ldots, d\}, V_l$ is the subgroup of $V$, as shown in “Acyclic network” subsection, $l = 1, 2, \ldots, k$. First, the position error of the $i$th robot is given below

$$e_i = \begin{bmatrix}
e_{i_1} \\
e_{i_2} \\
e_{i_3} \\
e_{i_4} \\
e_{i_5}
\end{bmatrix} = \begin{bmatrix}
\cos \zeta_i & \sin \zeta_i & 0 & x_{r,i} - x_i \\
-\sin \zeta_i & \cos \zeta_i & 0 & y_{r,i} - y_i \\
0 & 0 & 1 & \zeta_{r,i} - \zeta_i
\end{bmatrix}$$

(7)

where $i = 1, 2, \ldots, d, i \in V_l$. Next, take the derivative of equation (7)

$$\dot{e}_i = \begin{bmatrix}
\dot{e}_{i_1} \\
\dot{e}_{i_2} \\
\dot{e}_{i_3} \\
\dot{e}_{i_4} \\
\dot{e}_{i_5}
\end{bmatrix} = \begin{bmatrix}
\eta_i e_{i_2} - \mu_i + \mu_i \cos \zeta_i \\
-\eta_i e_{i_1} + \mu_i \sin \zeta_i \\
\eta_i - \eta_i \\
\eta_i - \eta_i \\
\eta_i - \eta_i
\end{bmatrix}$$

Second, introduce an auxiliary velocity control input of robot $i \in V_l$

$$\begin{bmatrix}
\hat{\mu}_{i \alpha} \\
\hat{\eta}_{i \alpha}
\end{bmatrix} = \begin{bmatrix}
\mu_i \cos \zeta_i + k_{i \alpha} e_{i_1} - \mu_i \sin \zeta_i \dot{e}_{i_1} \\
(\hat{\eta}_i + k_{i \alpha} \mu_i) e_{i_2} + k_{i \alpha} \mu_i \dot{e}_{i_2}
\end{bmatrix}$$

(8)

It can be obtained from the literature that $v_{i \alpha} = P_i(e_i, v_{r,i}, \psi_{r,i})$ guarantees the position tracking of NNMMRSs in $V_l$. Subsequently, for robot $i$, an auxiliary sliding reference velocity $v_{r,i} \in \mathbb{R}^p$ is introduced as

$$v_{r,i} = v_{i \alpha} - \alpha \sum_{j=1}^{d} a_{ij} \int_0^t (v_{i}(r) - v_{r,i}(r)) \, dr - \beta \int_0^t (v_i(r) - v_{i \alpha}(r)) \, dr$$

(9)

where $\alpha > 0$, and $\beta_i$ are the designed positive constants. In addition, one has

$$v_{r,i} = v_{i \alpha} - \alpha \sum_{j=1}^{d} a_{ij} (v_i(t) - v_{j \alpha}(t)) - \beta_i (v_{r,i}(t) - v_{i \alpha}(t))$$

(10)

Next, introduce the following sliding variable

$$s_i = v_i - v_{r,i}, s_i \in \mathbb{R}^p$$

(11)

then, the torque control protocol of the $i$th robot is given by

$$\tau_i = \mathbb{K}_i^{-1}(s_i, \dot{s}_i, v_r, v_{r,i}) \dot{\hat{\rho}}_i - \mathbb{K}_i s_i$$

(12)

where $\mathbb{K}_i$ is the symmetric positive-definite matrix, and the adaptive law $\dot{\hat{\rho}}_i$ is the estimation of $\rho_i$, which can be defined as

$$\dot{\hat{\rho}}_i = -\Xi_i v_{i \alpha}^T(q_i, \dot{q}_i, v_r, v_{r,i})s_i$$

(13)

where $\Xi_i$ is symmetric positive-definite.
and the derivative of equation (15) can be expressed as follows

\[ \dot{X} = X_T \]

where \( \dot{\rho}_i \) is \( \rho_i - \dot{\rho}_i \). Then, the compact form of equation (11) is written as

\[ \dot{W} = -\int_0^t ((\alpha L + \Theta) \otimes I_p) W \, dr + U + S \]  

(15)

the derivative of equation (15) can be expressed as follows

\[ \dot{W} = -((\alpha L + \Theta) \otimes I_p) W + \dot{U} + \dot{S} \]  

(16)

Define coordinate transformation \( \mathcal{X} = (\Delta \otimes I_p) W \). Then, \( \mathcal{X} \) can be expressed as \( \mathcal{X} = [X_I^T, X_T^T, \ldots, X_R^T]^T \), where

\[
\Delta = \begin{bmatrix}
\nu_1^T \\ \nu_2^T \\ \vdots \\ \nu_k^T \\
-1 & 1 & 0 & \cdots & 0 & 0 \\
0 & -1 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & -1 & 1 \\
\end{bmatrix}
\]

Since the matrix \( \Delta \) is invertible, reformulate equation (11) as

\[ \mathcal{X} = -\int_0^t ((\alpha L \Delta^{-1} + \Theta) \otimes I_p) \mathcal{X} \, dr + (\Delta \otimes I_p)(S + U) \]  

(20)

Then, equation (20) is decomposed into two formulas

\[ X_I = -\int_0^t (\Theta_I \otimes I_p) X_I \, dr + ((\nu_1, \nu_2, \ldots, \nu_k)^T \otimes I_p)(S + U) \]

where \( \Theta_I \) is the diagonal matrix with respect to \( \beta_i \int_0^t (v_i - v_{in}(r)) \, dr, \quad i = 1, 2, \ldots, k \) and

\[ X_R = -\int_0^t ((\alpha L + \Theta_R) \otimes I_p) X_R \, dr + S_R \]

(21)
where $\Theta_R$ is the diagonal matrix about $\beta_i^T \int_0^t (v_i(r) - v_{ia}(r)) \, dr$, $i = k + 1, k + 2, \ldots, d$. The structure of $S_R$ is written as

$$
\begin{bmatrix}
((s_2 - s_1) + (u_2 - u_1))^T, ((s_3 - s_2) + (u_3 - u_2))^T, \ldots, \\
((s_h - s_{h-1}) + (u_h - u_{h-1}))^T, \ldots, ((s_h - s_{h-1}) + (u_h - u_{h-1}))^T, \ldots, ((s_d - s_{d-1}) + (u_d - u_{d-1}))^T
\end{bmatrix}
$$

(22)

Based on the above analysis, Theorem 1 is readily obtained below.

Theorem 1. Given that $\{V_1, V_2, \ldots, V_k\}$ is an acyclic partition of $G$ and that Assumptions 1 and 2 hold, then coordination control of group tracking consensus for NNMMRSs (1) can be realized under the control protocol (12) and the adaptive law (13) if and only if Assumption 3 is satisfied.

Proof. ( Sufficiency ) The Lyapunov-like function is constructed as

$$
V_i = \frac{1}{2} s_i^T \tilde{M}_i(q_i) s_i + \frac{1}{2} \rho_i^T \tilde{\Xi}_i^{-1} \rho_i
$$

The derivative $\dot{V}_i$ is given by

$$
\dot{V}_i(t) = s_i^T \dot{\tilde{M}}_i \dot{s}_i + \frac{1}{2} s_i^T \tilde{M}_i \dot{s}_i + \rho_i^T \tilde{\Xi}_i^{-1} \dot{\rho}_i
$$

Combining Property 3 and closed-loop system (14) yields that

$$
\dot{V}_i(t) = s_i^T \left[ \frac{1}{2} \dot{\tilde{M}}_i(q_i) - \tilde{C}_i(q_i, \dot{q}_i) \right] s_i - s_i^T \tilde{K}_i s_i
$$

It can be seen from Property 2 that $\frac{1}{2} \dot{\tilde{M}}_i(q_i) - \tilde{C}_i(q_i, \dot{q}_i)$ is skew symmetric, therefore

$$
\dot{V}_i = -s_i^T \tilde{K}_i s_i \leq 0
$$

Consequently, $s_i$ belongs to $L_2$ space and $L_\infty$ space simultaneously, and $\rho_i \in L_\infty$. Because the auxiliary velocity control input $v_{ia}$ guarantees tracking steering stability of the $i$th robot, there always exists a positive constant $\alpha$, yielding that $-(\alpha \Delta \Delta^{-1} + \Theta) \otimes I_p$ is Hurwitz stable. Accordingly, system (20) is input-state stable with respect to input $S + U$ and state $\chi$. Since $S + U$ is bounded, $\chi$ is bounded, which implies that $X_i$ is bounded according to equation (17), $i = 1, 2, \ldots, k$. Consequently, $\mathcal{W}$ is bounded, which gives rise to the boundedness of $v_i$. Accordingly, $\ddot{q}_i$ is bounded as $\ddot{q}_i = \ddot{s}_i \ddot{v}_i$. For the sake of equation (11), $v_{ni}$ is bounded. Due to equation (10), $v_{ni}$ is bounded. According to Properties 1 and 3, $\mathcal{W}_3(q_i, \dot{q}_i, v_i, \nu_i)$ is bounded. Accordingly, $s_i$ is bounded because of equation (14). Thus, $\dot{V}_i$ is bounded. Then, $\dot{V}_i$ is uniformly continuous. In view of Barbalat’s Lemma, it is derived that $\dot{V}_i \to 0$, when $t \to \infty$, which results in $s_i \to 0_p$ as $t \to \infty$.

For another, $S_R \to 0_{p(d-k)}$ based on the structure of $S_R$ in equation (22). Since $-(\alpha L_R + \Theta_R) \otimes I_p$ is also Hurwitz stable, system (21) is input-state stable, that is, if $S_R \to 0_{p(d-k)}$, one has $X_R \to 0_{p(d-k)}$ as $t \to \infty$. From the structure of $X_R$ in equation (18), $X_{R_i} \to 0_{p(n_i-1)}$ as $t \to \infty$, $i = 1, 2, \ldots, k$. Then, from the structure of $X_{R_i}$ in equation (19), $\nu_{h_i+m+1} \to 0_{h_i+m}$ as $t \to \infty$, $l = 1, 2, \ldots, k$, $m = 1, 2, \ldots, n_i-1$. It follows that $v_i \to \gamma_j$ as $t \to \infty$, $i, j \in \{1, 2, \ldots, d\}$, where $i = j$. Since $\dot{q}_i = S_i \dot{v}_i$, $\lim_{t \to \infty} (\dot{q}_i - \ddot{q}_i) = 0$, $i = j$. Due to equation (15), one has $\mathcal{W} \to 0$ when $S + U \to 0$ as $t \to \infty$. Consequently, $v_i \to \nu_i$ as $t \to \infty$. For $\dot{q}_i = S_i \dot{v}_i$, it gives rise to $\lim_{t \to \infty} (\dot{q}_i - q_i) = 0$. Accordingly, $\lim_{t \to \infty} (q_i - \dot{q}_i) = 0$, $i = j$. Therefore, the group tracking consensus problem of NNMMRSs is solved. Sufficiency is proven.
(Necessity) Consider reductio ad absurdum. There exists at least one robot that cannot receive information from any other robots. In this case, group consensus cannot be realized, so contradiction arises. Necessity therefore applies. Theorem 1 is proven.

Remark 2. Compared with complete consensus, the dominant difficulty of group consensus scenario is to design feasible control algorithm to eliminate impacts between subgroups. Theorem 1 effectively expands the scopes of research interest of NNMMRSs. By the decomposition matrix $D$, it is clear to verify that the convergence of system (14) is equivalent to guaranteeing the stability of subsystems $X_i$ and $XR$. Thus, the case in the literature represents a special circumstance by specifying $k = 1$ in Theorem 1.

Remark 3. As NNMMRSs are intrinsically high-order nonlinear systems subject to underactuated characteristics, another difficult problem is that the component $z_i$ of the state $q_i$ usually has no corresponding input. By introducing a smooth kinematic reference vector encapsulated in $v_{ia}$, the designed algorithm guarantees the predefined transient state performance of NNMMRSs. When the constraint matrix $A_i(q_i)$ is equal to zero in system (1), the problem turns into the scenario of NLSs. Thus, our work extends the related results of NLSs.

Remark 4. Compared with the existing results, the network employed here satisfies one-way information transmission with a stable structure and multiple subgroups. Our problem framework better depicts the actual situation for complex tasks in engineering applications. Besides, stability analysis of Theorem 1 resorts to the propositions of network Laplacian matrix null space. Unlike the relatively strict algebraic assumptions, the realization of group consensus only relies on acyclic partition network topology.

Simulations

In view of the abovementioned discussion, it is necessary to verify the effectiveness and feasibility of our group consensus algorithm when the dynamics of multiple nonholonomic systems is considered. In addition, there has been no research regarding whether the existing convergence results of complete consensus are applicable to those of group consensus. These observations motivate us to carry out the following studies. In this section, three numerical simulation examples are illustrated under two network topologies.

Figure 4. Velocity responses of three subgroups in Figure 3 under $G_1 (\alpha = 1.0)$. t-axis represents time. V-axis and W-axis represent linear velocity and angular velocity, respectively.

Figure 5. Trajectory tracking under network topology $G_2 (\alpha = 1.0)$ in X-Y plane.
Specify $A_i(q_i)$ and $S_i(q_i)$, respectively, as in equations (5) and (6) of “Preliminaries” section. $\mathcal{M}_i(q_i)$ and $\mathcal{C}_i(q_i, \dot{q}_i)$ are, respectively, expressed as

$$\mathcal{M}_i(q_i) = \begin{bmatrix}
\frac{r_i^2}{4b_i^2} (m_i b_i^2 + I_i) + I_{wi} & \frac{r_i^2}{4b_i^2} (m_i b_i^2 - I_i) \\
\frac{r_i^2}{4b_i^2} (m_i b_i^2 - I_i) & \frac{r_i^2}{4b_i^2} (m_i b_i^2 + I_i) + I_{wi}
\end{bmatrix}$$

and

$$\mathcal{C}_i(q_i, \dot{q}_i) = \begin{bmatrix}
0 & \frac{r_i^2}{2b_i} m_i I_i \dot{\xi}_i \\
-\frac{r_i^2}{2b_i} m_i I_i \dot{\xi}_i & 0
\end{bmatrix}$$

The regressor matrix can be described below

$$\mathcal{Y}_i(q_i, \dot{q}_i, \mathbf{v}_{ri}, \mathbf{v}_{rt}) = \begin{bmatrix}
\mathbf{v}_{ri1} & \mathbf{v}_{ri2} & \dot{\mathbf{v}}_{ri1} & \dot{\mathbf{v}}_{ri2} \\
\mathbf{v}_{rt1} & \mathbf{v}_{rt2} & \dot{\mathbf{v}}_{rt1} & \dot{\mathbf{v}}_{rt2}
\end{bmatrix}$$

Select the designed parameters below: $l_i = 0.3, a_i = 2, r_i = 0.15, b_i = 0.75, m_i = 30, m_{wi} = 1, m = 2m_{wi} + m_i, I_i = 15.625, I_{wi} = 0.0025, I_{wi} = 0.005, I_i = 2I_{wi} + 2m_{wi}b_i^2 + m_i I_i + I_{wi}, k_i = 5, m = 1, 2, 3, \Xi = 10$. For the convenience of comparison in the following examples, the initial values of linear velocities and angular velocities are all selected as $[\mu_i(0), \eta_i(0)]^T = [0, 0]$, $i = 1, 2, \ldots, 9$.

**Example 1.** The objective of this example is to verify the effectiveness of our group consensus algorithm. In other words, when the agents in one subgroup have impact on the agents in other subgroups, the control protocols ensure that the agents track multiple independent trajectories. For a team of NNMMRSs with nine robots, two directed graphs $G_1$ and $G_2$ are shown in Figure 2. Because each subgroup in $G_1$ and $G_2$ possesses directed spanning tree, Theorem 1 applies. Next, two composite tracking trajectories are given under $G_1$ and $G_2$, respectively.

For the first case, the tracking trajectories are provided under $G_1$

$$\mu_{r_1} = 1.5, \eta_{r_1} = 0.75$$

$$\mu_{r_2} = 1.0, \eta_{r_2} = 0.5, 0 \leq t < 5$$

$$\mu_{r_3} = 0.5, \eta_{r_3} = 0, 5 \leq t \leq 12$$

$$\mu_{r_4} = (t + 5)/15, \eta_{r_4} = 0$$

Select $\alpha = 1.0$, and the position initial values as $q_{r_1}(0) = [1.3, 0]^T, q_{r_2}(0) = [-1.16, -4.27, 4]^T, q_{r_3}(0) = 0.15, \ldots, q_{r_9}(0) = 0.15$.
It can be observed from Figure 3 that the circular trajectory and the line-circle trajectory can be well tracked at a constant speed. In addition, the accelerating straight-line motion tracking can be also realized. The velocity consistency of the three subgroups is shown in Figure 4. The simulation results indicate that nine robots belonging to three subgroups can work cooperatively to accomplish subtasks. Noting that any trajectory can be divided into straight line and circle arc segments between any two points, it is reasonable to demonstrate the abovementioned continuous trajectories.

For the second case, another composite tracking trajectories under $G_2$ are given by

$$
\begin{align*}
\mu_r & = 0.3t + 0.9, \eta_r = 0, \quad 0 \leq t < 5, \\
\mu_r & = 3.0, \eta_r = 0.75, \quad 5 \leq t \leq 8, \\
\mu_r & = 5, \eta_r = 1.25, \\
\mu_r & = \sqrt{2t}/2, \eta_r = 0
\end{align*}
$$
Select $\alpha = 1.0$, and the position initial values as $q_{1r}(0) = [2.95, 9.75, 0.21]^T$, $q_{2r}(0) = [18.0, 0.0]^T$, $q_{3r}(0) = [-12.15, 1.1]^T$, $q_{4r}(0) = [2.10, 0.0]^T$, $q_{5r}(0) = [2.8, 0.0]^T$, $q_{6r}(0) = [1, -2.0]^T$, $q_{7r}(0) = [1, -3.5, 0]^T$, $q_{8r}(0) = [1, -5.0]^T$, $q_{9r}(0) = [1, -10.0]^T$, $q_{10r}(0) = [1, -9.0]^T$, $q_{11r}(0) = [1, -21.3, 0]^T$, $q_{12r}(0) = [1, -20.0]^T$.

Figure 5 shows that each trajectory can be tracked at constant speed and accelerating speed effectively. Moreover, Figure 6 displays that the velocity states in each subgroup converge to a smooth line in a short time, implying the realization of group tracking consensus. It should be noted that the smoothness of the intersection point has a significant impact on the states of the line-circle trajectory. If the arc is tangent to the straight line, it can be seen from the magnified subgraph of Figure 4 that the change is smooth. If the arc intersects but is not tangent to the straight line, then the change is drastic, as shown in the magnified subgraph of Figure 6. This conclusion can provide certain reference for the path planning problems in engineering practice.

Example 2. The impact on the performance (or negotiation speed) with different algebraic connectivity is considered in this example. A well-known observation is that the algebraic connectivity of dense graphs is relatively large.\textsuperscript{26} It is clear that the algebraic connectivity of $G_2$ is larger than...
that of $G_1$. Select the same tracking trajectories and initial values as in Example 1. Figure 7 shows the tracking renderings under network topology $G_2$ ($\alpha = 1.0$). Compared with the simulation results under different network topologies, especially at the bottom right corner of Figures 8 and 9, the angular velocities (vertical axis) show that the negotiation speed in $G_2$ is not higher than that in $G_1$, that is, enhancing the algebraic connectivity of each subgroup, the manifestation of convergence rate is not significantly improved. This conclusion suggests that a simple network topology may be better since negative correlation arises between the performance and the algebraic connectivity in the evolution of group consensus for NNMMRss.

**Example 3:** This example focuses on the impact of the coupling strength on group tracking consensus. For convenience, three circular tracking trajectories are considered under network topologies $G_1$ and $G_2$. The tracking trajectories are given as follows

$$\mu_{r_1} = 3.0, \eta_{r_1} = 3.0, \mu_{r_2} = 1.5, \eta_{r_2} = 1.5, \mu_{r_3} = 0.5, \eta_{r_3} = 0.5$$

Select $\alpha = 3$, $\alpha = 0.75$, and the position initial values as $q_{r_1}(0) = [1, 3, 0]^T$, $q_{r_2}(0) = [1, 0, 0]^T$, $q_{r_3}(0) = [1, -3, 1.5]^T$.  

![Figure 12. Velocity responses of three subgroups in Figure 10(b) ($\alpha = 0.75$).](image)

![Figure 13. Tracking of three circular trajectories under network topology $G_2$ in X-Y plane when $\alpha = 3.0$ (c) and $\alpha = 0.75$ (d).](image)
The simulations exhibited in Figures 10 and 11 reveal that, for group consensus, the coupling strength has a remarkable impact on the final consistent state, yet an interesting phenomenon is that the performance is negatively related to the coupling strength. No matter which network topology is considered, it can be seen from Figures 10 and 11 that, when $\alpha = 3.0$, the position tracking of the third subgroup is not as effective as that when $\alpha = 0.75$. In addition, the third subgroup negotiation speed in Figure 12 ($G_1, \alpha = 3.0$) is not higher than that in Figure 13 ($G_1, \alpha = 0.75$), which can be clearly seen from the magnified figures. This conclusion can be also obtained from the comparison of Figures 14 and 15 under $G_2$. In sharp contrast to the complete consensus results of NNMMRSs, this distinction indicates that, for group consensus, weaker coupling strength may be better in practical engineering applications. Rigorous theoretical analysis will be pursued for further study.

To put it simply, for NNMMRSs, on the premise of the given geometrical assumptions in Theorem 1, the coordination control problem of group tracking consensus is realized with satisfactory expected effects. Notably, due to the influence of the former subgroup on the latter subgroup, it takes a longer time for the latter subgroup to reach agreement regardless of whether the acyclic partition structure is complicated or simple.
Conclusions
This article has investigated the group tracking consensus problem of NNMMRSs. To guarantee the transient state performance of NNMMRSs, the designed algorithm embedded a kinematic and torque controller in the scheme. A coordinate transformation matrix, composed of a special arrangement of left eigenvectors that were associated with the Laplacian matrix zero eigenvalue, was constructed to facilitate the equivalent subsystem stability analysis. In the future, how to quantify the amount of communication that affects group consensus will be a challenging topic. In addition, it is hoped to realize group consensus for NNMMRSs in more general network topologies for further study. 47–49

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ORCID iDs
Jun Liu https://orcid.org/0000-0002-2154-4037
Hengyu Li https://orcid.org/0000-0002-2243-5908

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