Preliminary investigation on a passive method for parametric instability control in advanced gravitational wave detectors

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Abstract. In this paper, we propose a passive method to suppress parametric instabilities. This method requires the design of a test mass mirror with appropriate loss distribution. We show that localized losses can significantly reduce the parametric gain without degrading the thermal noise for the advanced LIGO configuration. The method can be used individually or in conjunction with other active feedback methods. We present numerical analysis for both spherical and non-spherical mirrors.

1. Introduction
Significant improvement in the operation of interferometric gravitational wave detectors can be obtained by increasing the photon densities in the arm cavities. High energy density may however cause some undesired effects. Since the stability of the cavity is crucial for performance of an interferometer, any mechanical movement of the optical components would modulate the electromagnetic field [1] and effectively create higher order optical modes. Gravitational wave interferometers with long arms and test masses that have acoustic modes in the low ultrasonic range favour scattering between photons from the arm cavity and phonons in the mirror. Motion of the mirror surface associated with acoustic modes of the test mass modulates the electromagnetic field in the cavity. If such modulated field is spatially matched with test mass acoustic modes and if the frequency of vibrating surface together with optical modes satisfy resonance conditions, the positive feedback may occur [2,3]. This phenomenon is known as a high frequency parametric instability. Under such conditions mirror vibrations may grow in time and excite higher order optical modes. The exponential growth of ultrasonic acoustic modes can be characterised by the gain parameter $R$, as follow

$$R = \frac{2PQ}{McL\omega_m^2} \left( \frac{Q_o A}{1 + \frac{\Delta\omega^2}{\sigma^2}} \right), \quad \Delta\omega = \omega_0 - \omega_1 - \omega_m$$

where $Q$, $Q_o$ is the acoustic and the optical mode quality factor respectively, $\omega_m$ is the acoustic mode frequency, $\omega_0$ is the main mode frequency, $\omega_1$ is the Stokes mode frequency, and $A$ corre-
sponds to the overlapping factor. If $R$ is bigger than unity, the conditions for instability mode may be met.

Recently, it has been shown that such phenomena may be a serious issue in order to achieve the sensitivity of $h=10^{-22}$ for advance interferometers configuration. It has been pointed out that for such interferometers there may be as many as 10 unstable modes for sapphire and even up to 30 modes for fused silica test masses [4]. Therefore it becomes essential to investigate various methods in order to suppress parametric instabilities.

Several such methods have been proposed [4,5]. Thermal tuning is an example of an active method. It shifts an optical modes resonant frequency resulting in overlapping mismatch of acoustic modes. However, dynamically this process seems to be too slow in the case of fused silica optics. It will not enable actively damp the most unstable acoustic modes [5]. While it can actively detune the most unstable acoustic modes and possibly eliminate instability for room temperature interferometers with sapphire, it cannot be applied for cryogenic interferometers because the thermal expansion coefficient is much too small.

According to Eqn.1, sufficient suppression of the mechanical Q factor would lead to reduction of the gain parameter $R$. In order to achieve this effect it is necessary to increase the effective loss of the test mass. Loss modulation may be achieved by adding a damper in the form of strips, rings and coatings on the test mass. Unfortunately, the main issue related to this method is possible thermal noise (TN) degradation. Because TN strongly depends on loss localisation [6-8], the damper configuration must have small impact on the thermal noise level. Therefore it becomes essential to investigate a loss localization for which TN would not significantly increase. In the following study we present an analysis of Q factor suppression due to localised losses. Also an estimation of thermal noise for each loss configuration is shown. The simulations were carried out on a model of sapphire mirrors using the Advanced LIGO configuration.

2. Simulation Methodology

We adopt the strain energy method in order to estimate the quality factor $Q$ and thermal noise. The quality factor is defined as

$$Q_n = \frac{\int E_n(\vec{r})dv}{\int E_n(\vec{r})\phi(\vec{r})dv},$$

(2)

where $E_n$ is the stored energy density for a given acoustic mode $n$, $\phi$ is the loss angle. For our thermal noise analysis we used the energy dissipation method of Levin [9,7].

$$S(f) = \frac{4k_BT}{\pi F_0^2} \int E_n(\vec{r})\phi(\vec{r})dv,$$

(3)

We adopt the strain energy method where $F_0$ is the transverse pressure profile corresponding to the laser beam profile produced by oscillatory force integrated over the mirror face, $k_B$ is Boltzmann’s constant, $f$ is frequency, and $T$ is temperature.

Due to the complexity of analytical energy calculations, simulation methods such as finite element modelling (FEM) were applied. This enables us to approximate the energy dissipation with good accuracy. We use the FEM software ANSYS for this study [10] in order to estimate the quality factor $Q$ and thermal noise. The quality factor is defined as

2.1. Test mass and loss definition

In order to construct the test mass mode we used tetrahedron brick and layered elements with total number of 146000 nodes. For the purpose of thermal noise analysis we created a full size 3D model. The meshing procedure was carried out in such way that all elements on the side wall had the same dimension. This was especially important for a lossy strip simulation. In case of
Table 1. ANSYS model properties.

| Material     | Young’s modulus \( [GPa] \) | Poisson ratio | Density \( [g/cm^3] \) | Number of nodes |
|--------------|-------------------------------|---------------|------------------------|-----------------|
| Sapphire \( Al_2O_3 \) | 400                           | 0.23          | 3.983                   | 146,000         |
| Tantala \( Ta_2O_5 \) | 140                           | 0.23          | 8.200                   |                 |
| Silica \( SiO_2 \)    | 70                            | 0.17          | 2.200                   |                 |

Figure 1. Simulated effective loss variation due to the coating loss frequency dependence. The Y axis corresponds to \( \phi_{eff} = \phi(f) - \phi_{const} \), the difference between frequency dependent and frequency independent effective loss. \( \phi_{const} \) was defined by the neglecting frequency part in Eqn. 4 and 5, respectively.

Table 2. Loss definition.

| Model | Loss Location                                      | Loss value               | Thickness [\( \mu m \)] |
|-------|----------------------------------------------------|--------------------------|--------------------------|
| A     | Loss strip and front face coating                   | \( 10^{-2} \)            | 20                       |
| B     | Front face coating only                             | [Eqn. 4 & 5]             | 15                       |
| C     | Back face coating and 50% cylinder wall coating     | \( 5 \times 10^{-4}, 5 \times 10^{-4} \) | 20                       |
| D     | Back face coating and 100% cylinder wall coating    | \( 5 \times 10^{-3}, 3 \times 10^{-3} \) | 20                       |
| E     | The same as D with high loss coatings               | \( 5 \times 10^{-3}, 5 \times 10^{-3} \) | 20                       |

modal analysis we used half size model with defined planar symmetry. We performed simulations for a proposed Advance LIGO sapphire mirror with a 157 mm radius and 130 mm thickness. Two different shapes of test mass were considered: a pure cylinder and a cylinder with flats. The flats lengths were 95 mm, see Fig.1. The mirror model properties are shown in Table 1. For optical coating simulations, additional layered version of the tetrahedral elements were used in the front face ot the test mass. We defined 60 layers in each element, what corresponding to a coating with 30 layers of tantala/silica \( Ta_2O_5/SiO_2 \). The thickness of each layer was \( \lambda/2 \), where \( \lambda = 1.064\mu m \). For each layer, the strain energy was calculated based on the parameters from Table 1. We have assumed a frequency dependent loss for the optical coating as follow [11].

\[
\phi_{SiO_2} = 0.4 \times 10^{-4} + 2.7 \times f \times 10^{-9}, \tag{4}
\]
Figure 2. An Example of test mass model with radius of $r=157$ mm, thick $d=130$ mm and with flats length of $f=95$ mm.

$$\phi_{Ta_2O_5} = 4.2 \times 10^{-4} + 0.4 \times f \times 10^{-9},$$ (5)

where $f$ is frequency. The influence of frequency dependence on effective loss is shown in the Figure 2. Additionally, we considered different dampers in the form of; side wall, strip, optical coating and back face. Five models with distinct mechanical loss configurations were chosen. The characteristic of each model is presented in Table 2. The reason for choosing such configurations is explained in the next section. All models include optical coating. The side wall, strip and back face were assumed to have the same material properties as the substrate. They are defined as layer of the substrate with specified loss angle. Such assumption however put some constrains on materials which could be used in practice. All materials with comparable acoustic impedance to the substrate meet requirements for such defined model. We assumed the constant substrate structural loss to be $10^{-8}$. To our knowledge, there is no evidence that the loss angle of sapphire material is frequency dependent. Relatively weak frequency dependence has been reported in the case of fused silica [12].

3. Effect of localised losses on thermal noise
A very important aspect of Q suppression analysis is the careful estimation of the TN level. In a perfect scenario we would expect substantial Q suppression with simultaneous preservation of the TN level. However, in reality any loss added up to the substrate’s intrinsic loss will cause degradation of thermal noise. The attempt of minimising this undesired effect required many trial methods in order to find the best configuration. Therefore, we analysed models with a damper defined as wall, back face, optical coating and strip. For this purpose we assumed the thickness of lossy surfaces to be 20 $\mu$m. For such defined dampers we ran thermal noise simulations for various values of loss angle. Our analysis was performed for a mesa beam size of $d=104$ mm in case of cylindrical mirror and Gaussian beam with radius of $r=60$ mm for both cylindrical and flat mirrors. The results are shown on Figures 3a and 3b. Figures 3c and 3d concern a case in which a damper is a single 5.2 mm loss strip. These figures show that the noise contribution from a strip has strong maxima and minima. It is clear that for a cylindrical mirror, Figure 3c, there is a location for which the effect on TN amplification is significantly
Figure 3. Effect of localised losses on thermal noise. Figures (a) and (b) correspond to simulation of thermal noise using various damper configurations for Gaussian and mesa beams, respectively. In the case of mesa beams, the analysis was carried out only for a cylindrical test mass. Figures (c) and (d) show thermal noise variation as function of a strip position on the test mass. The variation is with respect to model B’s thermal noise level. The test mass front face with a defined optical coating is located at the zero position.

small. However our preliminary investigation unveiled that for a strip loss value smaller than $10^{-2}$ such configuration itself cannot substantially suppress quality factors through out the entire frequencies range. Further studies with a various strip dimension and loss value must be performed. Based on these results and assuming that the TN should not exceed degradation more than 10%, we chose four of the most promising configurations, as shown in Table 2. For such configurations further investigation on the quality factor was performed.

4. Results

Our analysis was carried out on 380 acoustic modes in the frequency range from 9004 kHz to 106329 kHz. We perform FEM analysis for a sapphire mirror model with dimensions corresponding to the Advanced LIGO configuration. Quality factor for different acoustic modes were estimated using Eqn. 2. Figure 4a shows Q suppression for B,C,D, E models. For each model, the thermal noise level was estimated according to Eqn. 3. The results of thermal noise are shown in Fig. 4b. Our studies revealed that quality factor can be significantly reduced without significant noise degradation. The cost of only 14% degradation in thermal noise performance is achieved with model E for which the loss has the biggest value. Q may be suppressed by about 2 order of magnitude. Note that in a certain range of frequency we observe surprising linear features as shown in Fig. 4a. We have investigated this effect. We believe that
Figure 4. (a) Quality factor suppression for various acoustic modes. Each data point corresponds to a resonant mode. (b) Degeneration of thermal noise level due to the different loss localization. The results shown here concern only cylindrical model.

Figure 5. Surface energy ratio in respect to the total strain energy of the model. Each dot represents different acoustic mode.

this is a not a modelling artefact, but is a real effect associated with the fact that the ratio of modal strain surface energy to the volume energy is roughly constant for this set of modes. This idea may be supported by results shown in Fig. 5. We calculated the energy ratio. By making a comparison between the plots in Fig. 4a and Fig. 5 we find a good agreement.

In order to quantitatively investigate the dependence of Q suppression on parametric instability we used ten of the most unstable acoustic modes for which the parametric gain factor R is bigger than unity. The effect on the gain parameter R is show in Figure 6. Unstable modes were adopted from Ref. [5] which concerns a numerical investigation on parametric instability. The first observation shows that two of the four studied models meet the condition for which the parametric instability phenomenon would not occur. This fact indicates that the Q suppression method can reduce R by factor of the order of 100.

5. Discussion and Conclusion
The aim of this analysis was to determine the possibility of using dumpers as a passive method to control parametric instability. For this reason we have studied different loss configurations in the form of coating, strip, and side walls. Two types of sapphire test masses were considered;
pure cylinder and cylinder with flats. Dimensions of the test mass are agreeable with dimensions for proposed the Advanced LIGO sapphire mirrors. The analysis predicted here revealed some interesting aspects in favor of applicability of our method for future interferometers. Our analysis shows that for a single strip configuration there is a location on the test mass at which thermal noise has the least degradation. Both mesa beams and Gaussian beams show strong sensitivity of thermal noise to damper location. The mesa beam mirror has roughly double the noise degradation from a given loss strip. In both cases the loss strip should be located on the cylinder wall far from the mirror surface. A strip at this position with a defined loss below $10^{-2}$ does not effectively suppress modes in the full frequency range. However, if sapphire test masses are coated on the cylindrical wall and the back face with coatings comparable to typical optical coatings, the parametric gain $R$ may be reduced below unity for all previously unstable modes. Note that there is a set of modes between 75 kHz and 82 kHz that are difficult to damp. Fortunately we have not discovered parametric instability associated with these modes. This method can reduce $R$ by factor $\sim 100$ with a cost of 14% degeneration of the noise performance. We expect that for parametric gain bigger than 200 it may become difficult to suppress modes using only this method and keep thermal noise to an acceptable level. However, we think that Q suppression method may play crucial role in conjunction with other active feed-back methods.

In this work we have only investigated a small range of loss configurations. In future we will investigate thicker, narrower loss strips which may allow the minima in Fig. 3c and 3d to become deeper.

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**References**
[1] Savov P, Vyatchanin S et al 2004; available at http://www.ligo.org/pdf-public/savov.pdf
[2] Braginsky V B, Strigin S E, Vyatchanin S P et al. 2001 Phys. Lett. A 287 331
[3] Braginsky V B, Strigin S E, Vyatchanin S P et al. 2002 Phys. Lett. A 305 111
[4] Zhao C, Ju L, Degallaix J, Gras S, Blair D G et al. 2005 Phys. Rev. Lett. 94 121102
[5] Ju L, Gras S, Zhao C, Degallaix J and Blair D G submitted to JPCS
[6] Yamamoto K, Ando M, Kawabe K, Tsubono K et al. 2002 Phys. Let. A 305 18
[7] Gras S, Blair D G, Ju L et al. 2004 Phys. Lett. A 333 1
[8] K. Numata PhD thesis, University of Tokyo 2002.
[9] Levin Y et al. 1997 Phys. Rev. D 57 659
[10] ANSYS,Inc. http://www.ansys.com
[11] Harr G M, Armandula H, Black E, Crooks D R, Cagnoli G, Fejer M M, Hough J, Penn S D, Rowan S, Route R, Sneddon P et al. 2004 Proceedings of the SPIE 5527 33
[12] Penn S D, Ageev A, Busby D, Harry G M, Gretarsson A M, Numata K, Willems P (2005) 2004; available at http://www.ligo.org/pdf-public/penn.pdf