Temperature and Disorder Chaos in three-dimensional Ising Spin Glasses

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We study the effects of small temperature as well as disorder perturbations on the equilibrium state of three-dimensional Ising spin glasses via an alternate scaling ansatz. By using Monte Carlo simulations, we show that temperature and disorder perturbations yield chaotic changes in the equilibrium state and that temperature chaos is considerably harder to observe than disorder chaos.

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The fragility of the equilibrium state of random frustrated systems such as the Edwards-Anderson Ising spin glass\textsuperscript{1,2,3} has been predicted a long time ago\textsuperscript{4,5} and analyzed on the basis of scaling arguments\textsuperscript{6,7}. These scaling arguments predict that the configurations which dominate the partition function change drastically when the temperature or the disorder in the interactions between the spins are modified ever so slightly. The temperature chaos and disorder chaos effects have attracted considerable attention both from theory and experiment because of their potential relevance in explaining the spectacular rejuvenation and memory effects observed in hysteresis experiments in spin-glasses\textsuperscript{8,9} as well as other materials, such as random polymers and pinned elastic manifolds. Although there is evidence of disorder chaos in spin glasses, temperature chaos remains a controversial issue\textsuperscript{10,11,12,13,14,15} whereas for random polymers or pinned elastic objects\textsuperscript{16} chaos in general is well established\textsuperscript{17,18,19}.

Despite this lack of consensus, it has been surmised that temperature chaos would only be observable in spin glasses at very large system sizes and for large temperature changes\textsuperscript{20,21} thus making its presence unfathomable in simulations. These claims have recently been challenged. In particular, recent results point towards the existence of temperature chaos in four-dimensional Ising spin glasses\textsuperscript{22} where the free energy of a domain wall induced by a change in boundary conditions changes its sign chaotically with temperature in accordance with the droplet/scaling theories\textsuperscript{1,6,7}. In this work we study the overlap between states at different temperatures and disorder distributions directly for a physically relevant three-dimensional (3D) Ising spin glass. Our results show that the scaling laws that arise from the droplet theory are indeed well satisfied in 3D provided low enough temperatures are considered, although small corrections need to be applied. In addition, we show that temperature and disorder chaos have similar scaling functions. By rescaling the characteristic length scale in the problem, we show that disorder chaos appears at much shorter scales than temperature chaos.

The paper is organized as follows: We discuss first the model and Monte Carlo methods used, followed by the disorder- and temperature-chaos scaling approaches. We conclude with the results of our simulations of the 3D Ising spin glass and a general discussion.

\textbf{Model and numerical method} The Edwards-Anderson\textsuperscript{[4]} Ising spin glass is given by the Hamiltonian

\begin{equation}
\mathcal{H} = - \sum_{\langle ij \rangle} J_{ij} S_i S_j \tag{1}
\end{equation}

where the Ising spins $S_i \in \{\pm 1\}$ are on a cubic lattice with $N = L^3$ vertices and the interactions $J_{ij}$ are Gaussian distributed random numbers with zero mean and standard deviation unity. The sum is over nearest neighbor pairs. The model undergoes a spin-glass transition at $T_c = 0.95(9)$\textsuperscript{23,24,25}.

The order parameter of the system is defined via the overlap between two copies $\alpha$ and $\beta$, i.e.,

\begin{equation}
q_{\alpha,\beta} = \frac{1}{N} \sum_{i=1}^{N} S_i^\alpha S_i^\beta \tag{2}
\end{equation}

Following previous studies\textsuperscript{12,13}, we probe temperature chaos when the temperature between both replicas is shifted by an amount $\Delta T$. Disorder chaos is studied by introducing a perturbation $\Delta J$ in the disorder, i.e.,

\begin{equation}
J_{ij} \rightarrow \tilde{J}_{ij} = \frac{J_{ij} + x \Delta J}{\sqrt{1 + \Delta J^2}} \tag{3}
\end{equation}

which leaves the disorder distribution invariant. In Eq. (3) $x$ is a Gaussian distributed random number with zero mean and standard deviation unity. To monitor the changes induced by the perturbations of the system, we compute the \textit{chaoticity parameter}\textsuperscript{12,13} given by

\begin{equation}
Q_{T_1,T_2} = \frac{\langle q_{T_1,T_2}^2 \rangle}{\sqrt{\langle q_{T_1,T_1}^2 \rangle \langle q_{T_2,T_2}^2 \rangle}}_{av} \tag{4}
\end{equation}

for temperature chaos and by

\begin{equation}
Q_{\Delta J} = \frac{\langle \tilde{q}_{T_1,T_1}^2 \tilde{q}_{T_2,T_2}^2 \rangle}{\langle \tilde{q}_{T_1,T_1}^2 \tilde{q}_{T_2,T_2}^2 \rangle}_{av} \tag{5}
\end{equation}

for disorder chaos, respectively.
Because for typical droplets $F_T$ comes generally negative at temperature $T$, the free energy excitation of such droplets becomes generally negative at temperature $T_2$ (so that the droplet has to be flipped) for length scales larger than the chaotic length $\ell_c$ defined as

$$\ell_c = \left( \frac{\gamma(T_1)}{\sigma(T_2) - \sigma(T_1)} \right)^{1/\zeta}$$

with $\zeta = \frac{d_s}{2} - \theta$. (7)

Usually, small temperature changes are studied such that $S(T_1) \approx S(T_2)$. Here however, we do not use this approximation and since in the low temperature phase, when one can define droplets, the entropy is proportional to $\sqrt{T}$, we write

$$\ell_c(T_1, T_2) \propto \left( T_3^{3/2} - T_1^{3/2} \right)^{-1/\zeta}.$$ (8)

Equation (8) shows that when temperature is changed, equilibrium configurations are changed on scales greater than $\ell_c$. Notice that by keeping the temperature dependence of the entropy, we obtain a slightly different scaling than usually considered $\ell_c(\Delta T)$, where a factor $\Delta T$ appears instead of $T_2^{3/2} - T_1^{3/2}$. While this makes no difference for small $\Delta T$ (which is the case in all simulations performed so far), this can be significant for temperature differences larger than the ones considered in this work. Similar arguments can also be applied to the case of a random perturbation in the disorder $\theta$ – see for instance Ref. [32] – where one obtains

$$\ell_c(\Delta J) \propto \Delta J^{-1/\zeta}. \quad (9)$$

Considering system-size excitations, these arguments thus suggest that the chaoticity parameters defined in Eqs. (4) and (5) have the following scaling behavior

$$Q_{T_1, T_2}(L, T_1, T_2) = F_T[L/\ell_c(T_1, T_2)],$$

$$Q_{\Delta J}(L, \Delta J) = F_J[L/\ell_c(\Delta J)],$$

where $F(x)$ is a function with $F(0) = 1$ that decays at large $x$. In what follows we test the aforementioned scaling relations via Monte Carlo simulations.

**Numerical results** The behavior of the chaoticity parameter for disorder chaos at zero temperature has recently been studied in Ref. [32] (see also Ref. [33]) and perfectly satisfies the predictions of the droplet model. We thus concentrate here on finite temperatures where the scaling relations were only tested in the simulations of Refs. [12], [13], and [22] in two and four space dimensions. We have performed low-temperature Monte Carlo simulations of the 3D Ising spin glass (see Table I). As can be observed in Fig. 1 scaling the data for disorder and temperature chaos according to Eqs. (10) works extremely well. The best scaling collapse determined by a nonlinear minimization routine [25] yields $\zeta \approx 1.04$ for temperature chaos and $\zeta \approx 1.16$ for disorder chaos, which is in rather good agreement with the accepted value $\zeta \approx 1.1$ from $d_s \approx 2.6$ [28, 34] and $\theta \approx 0.2$ [31, 35, 36]; see Eq. (7). Notice that we have a good scaling of the data even when $T_2$ is larger than $T_c$. We
have tested the temperature-dependence of the exponent ζ and find that for $T \leq 0.5$ its value is practically independent of temperature, i.e., at $T = 0.2$ we are probing the low-temperature regime.

Renormalization group arguments also suggest that the temperature and disorder chaos effects are deeply related and characterized by the same universal scaling function $\tilde{\Delta}(x) \propto x^{\xi(T)}$ with $A \approx 8.7$, i.e., disorder chaos appears at considerably shorter length scales than temperature chaos. Right: Log-log plot of the same data, plotted as $1 - Q$, to illustrate the quality of the scaling.

Finally, to better illustrate the mechanism of chaos, we study the distribution of the chaoticity parameter $Q$ as defined in Eq. (1) without the disorder average, and bin the data for different choices of the disorder to compute the distribution $P_L(Q_{T_1, T_2})$. According to the droplet model, in the weak chaos regime (where $\ell_c > L$), temperature chaos can manifest itself even on small length scales, but only for rare regions of space $[17]$.

The superposition of the scaling functions shows deviations at larger temperatures. This is not surprising as assumptions made when deriving the scaling function are not valid for high $T$. For example, the $\sqrt{T}$ dependence of the entropy or the very existence of droplets is only valid for $T \leq T_c$ [37]. We thus believe that to perform a definitive test of universality of the scaling functions, very large system sizes at low temperatures with small temperature changes should be used.

We also study the behavior of the scaling function. It decays as $(\ell_c/L)^{d/2}$ for strong chaos (when $\ell_c/L \leq 1$) [32]. However, in the limit $\ell_c/L \geq 1$, we obtain $1 - Q(x) \propto x^{\xi(T)}$ which differs from the $1 - Q(x) \propto x^d$ behavior found at zero temperature in Ref. [32]. This shows that the scaling function can have a more subtle behavior than what is naively expected from simple domain-wall arguments [22, 38]. Note that the results remain unchanged if one takes the disorder average of the different overlaps in Eq. (1) independently, as done in Eq. (11) of Ref. [13].

Conclusions We have studied numerically disorder and temperature chaos in 3D Ising spin glasses and show that both disorder as well as temperature chaos are well described within a scaling/droplet description. In particular, we find that the scaling variables have to be modified as done in Eq. (8) when the difference in temperature is large. In addition, we show that the weak chaos regime is dominated by rare events where system-size droplets are flipped. This has direct experimental implications be-

of the two scaling functions. From this $\ell$-scale renormalization we also conclude that the length scale at which temperature chaos appears is approximately ten times larger than the length scale needed for disorder chaos to appear, as has also been found in four space dimensions by Sasaki et al., see Ref. [22]. Therefore temperature chaos is harder to probe than disorder chaos, as has already discussed within a Migdal-Kadanoff approach on spin glasses [20].
cause the weak chaos regime has been argued to account in a quantitative way for the memory and rejuvenation effects [10]. Finally, we show that temperature and disorder effects could be described by similar scaling functions in the low-temperature regime, thus providing compelling evidence for the presence of a chaotic temperature dependence in spin glasses. This has also recently been proven for mean-field systems [11, 12]. This mechanism is also responsible for step-size responses that could in principle be observed experimentally in mesoscopic systems [11, 12]. Nevertheless, this behavior might change for larger system sizes and thus we propose to revisit the problem with better models [13]. Our findings will help interpret experiments on rejuvenation and memory effects in spin-glasses and other materials.

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