Dirac-Brueckner Approach to Hyperon Interactions and Hypernuclei

H. Lenske, C.M. Keil, F. Hofmann, S. Briganti
Institut für Theoretische Physik, Universität Gießen
Heinrich-Buff-Ring 16, D-35392 Gießen, Germany

The density dependent relativistic hadron (DDRH) theory is introduced as an effective field theory for nuclei and hypernuclei. A Dirac-Brueckner approach to in-medium nucleon-hyperon interactions is presented. Density dependent meson-baryon vertices are determined from DBHF self-energies in infinite matter. Scaling laws for hyperon vertices are derived from a diagrammatic analysis of self-energies. In a local density approximation the DB vertices are applied in relativistic DDRH Hartree calculations to finite hypernuclei. A single particle spectra and spin-orbit splittings are described reasonably well over the whole mass region.

II. DENSITY DEPENDENT HADRON FIELD THEORY WITH HYPERONS

A. The DDRH model Lagrangian

In hypernuclear models derived from a symmetry-broken SU(3)$_f$ Lagrangian neither of the $0^-$ $\pi$ and K meson fields contributes directly to a structure calculation, except in the u-channel through antisymmetrization. From the $1^-$ vector meson octet condensed isoscalar $\omega$ and isovector $\rho$ meson fields will evolve. In a system with a large fraction of hyperons also condensed octet $K^*$ and singlet $\Phi$ mesons fields can appear. A shortcoming of a pure SU(3)$_f$ approach is the missing of $0^+$ scalar mesons and, hence, the absence of a binding mean-field. A satisfactory description of the $0^+$ meson channels, e.g. in terms of dynamical two-meson correlations, is an unsolved question.

In view of these problems we use an effective Lagrangian including the degrees of freedom which are relevant for the nuclear structure problem. This is achieved by extending the original DDRH proton-neutron Lagrangian to the $1/2^+ S=-1$ ($\Lambda, \Sigma^+, \Xi^0$) and $S=-2$ ($\Xi^-$, $\Xi^0$) hyperon multiplets. In the meson sector the isoscalar $\sigma$, $\sigma_s$ ($\equiv$ scalar $s\bar{s}$ condensate), $\omega$ and $\phi$ meson, the isovector $\rho$ meson and the photon $\gamma$ are included. This leads to the isospin-symmetric Lagrangian.
\[ \mathcal{L} = \mathcal{L}_B + \mathcal{L}_M + \mathcal{L}_{\text{int}} \]
\[ \mathcal{L}_B = \overline{\Psi}_F \left[ i \gamma_\mu \partial^\mu - \hat{M} \right] \Psi_F \]
\[ \mathcal{L}_M = \frac{1}{2} \sum_{i=\sigma,\sigma_s} \left( \partial_\mu \Phi_i \partial^\mu \Phi_i - m_\Phi^2 \Phi_i^2 \right) - \frac{1}{2} \sum_{\kappa=\omega,\phi,\rho,\gamma} \left( \frac{1}{2} F^{(\kappa)}_{\alpha\beta} - m_\kappa^2 A^{(\kappa)}_{\alpha\beta} \right) \]
\[ \mathcal{L}_{\text{int}} = \overline{\Psi}_F \hat{\Gamma}_\sigma \Psi_F \Phi_\sigma - \overline{\Psi}_F \hat{T}_\omega \gamma_\mu \Psi_F A_\mu^\sigma - \frac{1}{2} \overline{\Psi}_F \hat{T}_\rho \gamma_\mu \Psi_F A_\mu^\sigma \]
\[ + \overline{\Psi}_F \hat{\Gamma}_\sigma \Psi_F \Phi_\sigma_s - \overline{\Psi}_F \hat{Q} \gamma_\mu \Psi_F A_\mu^\sigma - e \overline{\Psi}_F \tilde{Q} \gamma_\mu \Psi_F A_\mu^\sigma , \]

Here, \( \mathcal{L}_B \) and \( \mathcal{L}_M \) are the free baryonic and mesonic Lagrangians, respectively. Baryons are described by the flavor spinor \( \Psi_F \)
\[ \Psi_F = (\Psi_N, \Psi_{\Lambda}, \Psi_{\Sigma}, \Psi_{\Xi})^T . \]

The diagonal matrix \( \hat{M} \) contains the free-space baryon masses and \( \hat{Q} \) is the electric charge operator. The meson-baryon interactions are contained in \( \mathcal{L}_{\text{int}} \). The usual field strength tensor of either the vector mesons \( (\kappa = \omega, \phi, \rho) \) or the photon \( (\kappa = \gamma) \) is denoted by \( F^{(\kappa)}_{\mu\nu} \). Contractions of the field strength tensors are abbreviated as \( F^2 = F_{\mu\nu} F^{\mu\nu} \) etc.

An important difference of the DDRH Lagrangian, eq.(3), to standard RMF approaches [4,5] is the description of medium effects and non-linearities in terms of density dependent meson-baryon vertex functionals \( \hat{\Gamma}_{\alpha B} = \hat{\Gamma}_{\alpha B}(\overline{\Psi}_F \Psi_F) \). The vertices are taken as Lorentz invariant functionals of the fermion field operators \( \Psi_F \) and since the baryon fields are treated as quantum fields, even in the mean-field limit a well defined class of quantum fluctuations with non-vanishing ground state expectation values is taken into account [17,18]. Dynamically, the vertices contribute to the Dirac equations as rearrangement self-energies describing the static polarization of the medium [17,22]. In bulk quantities, as for example total binding energies, the DDRH rearrangement self-energies are cancelled exactly but contribute to single particle quantities like separation energies, wave functions and density matrices [10,15].

A solvable model is obtained in the Hartree mean-field approximation. The operator-valued vertex functionals become c-number functions of the baryon densities [14,15]. The meson fields are obeying classical field equations, while the baryons are treated as quantum fields by solving the Dirac equation with static but density dependent self-energies, including rearrangement contributions [18]. For single \( \Lambda \) nuclei the hidden-strangeness \( \sigma_s \) and \( \phi \) fields are neglected because they are of order \( O(1/\Lambda) \).

**B. Dirac-Brueckner Approach to In-medium Hyperon Interactions**

It is obvious that the structure of the vertex functionals must be derived in a separate step. Provided that data are available the required information could be obtained from phenomenology, as in the approach of ref. [24] for isospin nuclei. A derivation on theoretical grounds has the advantage of providing - at least in principle - a deeper insight into the structure of interactions and, especially, the origin of medium-dependencies and inter-relations between the flavor sectors. Hence, we follow the original DDRH approach [17] and derive the vertex functionals from Dirac-Brueckner calculations. Full scale SU(3)_f DB calculations, however, are a considerable task whose solution is still pending. But the success of non-relativistic Brueckner calculations [45] clearly indicate the promising potential of such a microscopic approach.

The extension of Dirac-Brueckner calculations to the SU(3)_f multiplet have been outlined in [18]. Aiming at applications of the DB-results in relativistic mean-field calculations it is sufficient to have accurate knowledge on baryon self-energies rather than on the full momentum structure of in-medium interactions. A central result of [18] is the observation that Dirac-Brueckner interactions can be expressed in terms of medium-renormalized meson-exchange interactions with density dependent vertex factors,
\[ \Gamma_{\alpha B}(k_F^B) \equiv g_{\alpha B}^B s_{\alpha}^B(k_F^B) \]
given by the bare coupling constants \( g_{\alpha B}^B \) and the density dependent renormalization factor \( s_{\alpha}^B(k_F^B) \) for baryons of type B interacting through the exchange of mesons \( \alpha \).

With our choice of momentum independent, global vertices, the baryon self-energies are obtained as [18]
\[ \Sigma_{\alpha}^B(k|k_F^N, k_F^\Lambda) = \Gamma_{\alpha B}(k_F^B)\phi_{\alpha}(k|k_F^N, k_F^\Lambda, \Gamma) \]
where $\Phi_\alpha$ denotes a condensed meson field. From this equation the link to the DDRH Lagrangian and RMF theory is evident: Evaluating eq.(1) in DHF approximation self-energies of the same structure are obtained.

A self-contained model is obtained by introducing an in-medium "renormalization" scheme. For that purpose it is of advantage to consider symmetric hyper-matter, i.e. $k_F^N = k_F^Y = k_F$ where one finds the relation

$$
\Gamma_\alpha^Y(k_F^Y) = \Gamma_\alpha^N(k_F^N) \frac{\Sigma_\alpha^Y(k|k_F)}{\Sigma_\alpha^N(k|k_F)} \bigg|_{k=k_F,k_F^N=k_F^Y}.
$$

which is exact in Hartree approximation. The significance of this result becomes apparent by expanding DB self-energies diagramatically with respect to the bare coupling constants $g_{\alpha}^{B}$ \[^{18}\]. Because the leading order contributions are given by tadpole diagrams one derives easily that the nucleon and hyperon self-energies are related by scaling laws

$$
R_\alpha^Y = \frac{\Sigma_\alpha^Y}{\Sigma_\alpha^N} \approx \frac{g_{\alpha}^{Y}}{g_{\alpha}^{N}}(1 + \mathcal{O}(1 - \frac{M_N}{M_Y})) + \cdots
$$

where the realistic case $g_{\alpha}^{Y} < g_{\alpha}^{N}$ is considered. The scaling factors $R_\alpha^Y$ are expected to be state-independent, universal constants whose values are close to the ratios of the bare coupling constants. For asymmetric hypermatter with a hyperon fraction $\zeta_Y = \frac{\rho_Y}{\rho} \ll 1$ a corresponding diagrammatic analysis shows that asymmetry terms are in fact suppressed because the asymmetry correction is of leading second order $\mathcal{O}(\frac{g_{\alpha}^{Y}}{g_{\alpha}^{N}} \zeta_Y)^2$). Thus, even in a finite nucleus where $\zeta_Y$ may vary over the nuclear volume, we expect $R_\alpha^Y = \text{const.}$ to a very good approximation.

These results agree remarkably well with the conclusions drawn from the analysis of single hypernuclei in purely phenomenological models. In the present context, eqs.(5) and (6) are of particular interest because they allow to extend the DDRH approach in a theoretically meaningful way to hypernuclei using the results available already from investigations of systems without strangeness. Below, the nucleon (Hartree) scalar and vector vertex functions $\Gamma_{\sigma,\omega N}(k_F)$ of \[^{17}\] will be used as reference values. The hyperon scaling factors $R_\alpha^Y$ are treated as phenomenological constants to be determined empirically. The $\chi^2$ distribution from fitting DDRH RMF-calculations to existing hypernuclear data deduced from $(\pi, K)$ \[^{34–38}\] experiments is shown in Fig. 1. The joined distribution of the scalar and vector scaling factors is characterized by a steep valley along the diagonal without a clearly pronounced minimum.

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**FIG. 1.** $\chi^2$ distribution for the scalar ($R_\sigma$) and vector ($R_\omega$) scaling factors. DDRH results for variations of ($R_\sigma, R_\omega$) are compared to $\Lambda$ single particle spectra obtained from $(\pi^+, K^+)$ reactions \[^{34–38}\]. The location of DDRH coupling constants and the values assumed in the naive SU(3)$_f$ quark counting model are indicated.
In order to stay as close as possible to the microscopic DDRH picture we use the theoretically derived value $R_\sigma = 0.490$ obtained from free space NA scattering with the Jülich potential [26]. Since theoretical values for the $\omega$ vertex are not available $R_\omega$ is taken from Fig. 1 leading to $R_\omega = 0.553$ with the above value of $R_\sigma$ [18]. The scaling factors are in surprisingly good agreement with RMF results [25] and are consistent with bounds on hyperon–nucleon couplings extracted from neutron star models [4, 11].

III. SPECTROSCOPY OF SINGLE $\Lambda$ HYPERNUCLEI

Relativistic DDRH Hartree theory and applications to isospin nuclei were discussed in great detail in ref. [17] and the references therein. Here, we present DDRH results only for single $\Lambda$ hypernuclei. The vertices are taken from DB calculations with the Bonn A NN potential [33]. The model parameters are compiled in Tab. I.

A. $\Lambda$ Single particle states and spin-orbit splitting

Hyperon single particle spectra for $S=-1$ hypernuclei can be seen as a very clean fingerprint of mean-field dynamics, since they are only weakly affected by many-body effects. The bulk structure contains information on the mean-field and, indirectly, the nucleonic density distributions. Of particular theoretical interest are spin-orbit splittings. At high energy resolution the fine structure of the spectra provides information also on dynamical correlations beyond static mean-field dynamics.

DDRH $\Lambda$ single particle levels for light to heavy nuclei are shown in Fig. 2. Two major differences between neutron and $\Lambda$ spectra are observed [18]:

1. $\Lambda$ and neutron single particle spectra are overall related by a constant shift and an additional compression because the $\Lambda$ central potential has a depth of only about -30 MeV, compared to -70 MeV for the neutrons.

2. The spin-orbit splitting of the $\Lambda$ states is reduced further being less than what is expected from the overall reduction of the potential strength.

![FIG. 2. DDRH $\Lambda$ single particle spectra for light to heavy nuclei.](image)

The DDRH calculations reproduce the experimentally observed very small spin-orbit splitting in $\Lambda$ hypernuclei, e.g. [39, 40], reasonably well even without an explicit dynamical suppression of spin-orbit interactions as e.g. a $\Lambda - \omega$ tensor coupling which, for example, is used in the QMC model [9]. $\Lambda$ and neutron spin-orbit potentials are compared in Fig. 3.
The reason for the small spin-orbit splitting is understood by considering the evolution with increasing mass number. From Fig. 4 it is seen that the splitting drops for higher masses. Such a behaviour is to be expected since the spin-orbit potential is a finite size effect and will vanish in the nuclear matter limit. A corresponding mass dependence is also found in pure isospin nuclei. The splitting also drops in the low mass region – now for the reason that the spin-orbit doublets approach the continuum threshold and get compressed before one of them or both become unbound. There is a remarkable similarity to the situation found in weakly bound neutron-rich exotic nuclei [49]. In both cases weak binding is an important reason for the reduction of spin-orbit effects. The wave functions of weakly bound states - either for the special dripline neutron states or Λ orbits in general - have a reduced overlap with the spin-orbit potential which remains well localized in the nuclear surface. As seen from Tab. II the binding energy effect leads to substantially larger spatial extensions of the Λ states.

FIG. 4. Spin-orbit splitting of single Λ levels. For our standard choice of coupling constants the $^{13}$C data point [39] is overestimated but reducing the vector coupling by about 2% ($R_\omega=0.542$) a better description is obtained (see text).

B. Comparison to data and phenomenological RMF calculations

In Fig. 5 the DDRH single particle spectra are compared to spectroscopic data from ($\pi^+, K^+$) reactions. States in intermediate to high mass nuclei are described fairly well by the model, while for masses below about $^{28}Si\Lambda$ deviations of up to 2.5 MeV arise, possibly indicating the limits of an approach using nuclear matter vertices in local density approximation. We seem to miss systematically a decrease of the repulsive vector interaction for low mass single Λ hypernuclei. This tendency already becomes apparent going from $^{51}V\Lambda$ to $^{28}Si\Lambda$. Actually, the dashed line for $R_\omega=0.542$ in Fig. 5 shows that a slight reduction of the vector repulsion improves considerably the description of the light mass data. This 2% variation of $R_\omega$ is largely within the uncertainties of the model parameters.
Especially for the heavier nuclei the microscopic DDRH results are at least of comparable quality as the phenomenological descriptions. This we consider as a remarkable success of the model because the coupling functionals were not especially adjusted to data except for the overall adjustment of the vector scaling factor $R_\omega$. In heavy nuclei the density dependence of the $\Lambda$ vertices act mainly as an overall reduction factor because the $\Lambda$ vector density is relatively small and varies only weakly across the nuclear volume. The essential density dependent effect is related to the dynamical redistribution of the surrounding nucleons by the rearrangement self-energies. Dynamically, it corresponds to a modification of the $\Lambda$ core potential due to static polarization in the nucleonic sector. Obviously, this effect is not accounted for by conventional RMF models.

Finally, DDRH and RMF results for $\Lambda$ particle-neutron hole configurations observed in $(K^-, \pi^-)$ reactions \cite{43,44} are given in Tab. III.

IV. SUMMARY, CONCLUSIONS AND OUTLOOK

The DDRH theory introduced previously for isospin nuclei was extended to hypernuclei by including the full set of SU(3)$_f$ octet baryons. Interactions were described by a model Lagrangian including strangeness-neutral scalar and vector meson fields of $q\bar{q}$ ($q=u,d$) and $s\bar{s}$ quark character. The medium dependence of interactions was described by meson-baryon vertices chosen as functionals of the baryon field operators. The DDRH vertices are chosen to cancel Dirac-Brueckner ground state correlations. Hence, the approach corresponds to a resummation of ladder diagrams into the vertices under the constraint that infinite matter ground state self-energies and total binding energies are reproduced. As the central theoretical result it was found that the structure of Dirac-Brueckner interactions strongly indicates that the ratio of nucleon and hyperon in-medium vertices should be determined already by the ratio of the corresponding free space coupling constants being affected only weakly by the background medium.

Dynamical scaling of nucleon-hyperon vertices was tested in DDRH mean-field calculations for single $\Lambda$ hypernuclei. Calculations over the full range of known single $\Lambda$ nuclei led to a very satisfactory description of $\Lambda$ separation energies. The deviations from the overall agreement for masses below $A\approx16$ are probably related to the enhancement of surface effects in light nuclei which are not described properly by static RMF calculations with DB vertices obtained in the local density approximation. In a recent non-relativistic calculation indeed sizable contributions of hyperon polarization self-energies especially in light nuclei \cite{45} were found.

The results are encouraging and we conclude that DDRH theory is in fact an appropriate basis for a microscopic treatment of hypernuclei. The present formulation and applications are first steps on the way to a more general theory of in-medium SU(3)$_f$ flavor dynamics. Future progress on dynamical scaling and other theoretical aspects of the approach is depending on the availability of Dirac-Brueckner calculations for the full baryon octet including also the complete pseudoscalar $0^-$ and vector $1^+$ meson multiplets.

As work in progress the production of hypernuclei in hadronic reactions is presently investigated. Initial and final state interactions of the incident and outgoing mesons in $(\pi^+, K^+)$ and $(K^-, \pi^-)$ reactions are described in a relativistic eikonal approach. Good agreement with differential and total cross section data is obtained \cite{50}. A Lagrangian model, including s-channel production through nucleon resonances and t-channel production by e.g. $K^*$ exchange is under investigation. The nuclear structure results discussed here are entering into the (coherent) production amplitude. Electro-production of hypernuclei will be used as an independent and important test for the production vertex.
and the nuclear structure input.

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[1] H. Band, T. Motoba, J. Zofka, Int. Journ. Mod. Phys. A 5 4021 (1990).
[2] R.E. Chrien, C.B. Dover, Annu. Rev. Nucl. Part. Sci. 39 151 (1989).
[3] B.D. Serot, J.D. Walecka, Advances of Nuclear Physics, edited by J.W. Negele and E. Vogt (Plenum, New York, 1986), Vol. 16, p. 1; J.D. Walecka, Ann. Phys. 83 497 (1974).
[4] M. Rufa, J. Schaffner, J. Maruhn, H. Stöcker, W. Greiner, Phys. Rev. C 42 2469 (1990).
[5] N.K. Glendenning, D. Von Eiff, M. Haft, H. Lenske, M.K. Weigel, nucl-th/9211012 (1992).
[6] P. Papazoglou, D. Zschiesche, S. Schramm, J. Schaffner-Bielich, H. Stöcker, W. Greiner, Phys. Rev. C 59 411 (1999).
[7] P. Papazoglou, S. Schramm, J. Schaffner-Bielich, H. Stöcker, W. Greiner, Phys. Rev. C 50 2576 (1998).
[8] K. Tsushima, K. Saito, J. Haidenbauer, A.W. Thomas, Nucl. Phys. A 630 691 (1998); K. Tsushima, K. Saito, A.W. Thomas, Phys. Lett. B 411 9 (1997).
[9] A. Reuber, K. Holinde, H.-C. Kim, J. Speth, Nucl. Phys. A 608 243 (1996).
[10] V.G.J. Stoks, Th.A. Rijken, Phys. Rev. C 59 3009 (1999).
[11] Th.A. Rijken, V.G.J. Stoks, Y. Yamamoto, Phys. Rev. C 59 21 (1999).
[12] C. Keil, F. Hofmann, H. Lenske, Phys. Rev. C 61 064309 (2000).
[13] F. de Jong, H. Lenske, Phys. Rev. C 57 3099 (1998).
[14] F. de Jong, H. Lenske, Phys. Rev. C 58 890 (1998).
[15] H.F. Boersma, R. Malfliet, Phys. Rev. C 49 233 (1994).
[16] J.W. Negele, Rev. Mod. Phys. 54 913 (1982).
[17] Y.K. Gambhir, P. Ring, A. Thim, Ann. Phys. 198 132 (1990); P.-G. Reinhard, M. Rufa, J. Maruhn, W. Greiner, J. Friedrich, Z. Phys. A 323, 13 (1986).
[18] S. Haddad, M. Weigel, Phys. Rev. C 48 2740 (1993).
[19] R. Brockmann, R. Machleidt, Phys. Rev. C 42 1965 (1990).
[20] T. Hasegawa et al., Phys. Rev. C 53 1210 (1996).
[21] S. Ajimura et al., Nucl. Phys. A 585 173c (1995).
[22] P. H. Pile et al., Phys. Rev. Lett. 66 2585 (1991).
[23] B. ter Haar, R. Malfliet, Phys. Rep. 149 207 (1987).
[24] J.D. Bjorken, S.D. Drell, Relativistic Quantum Fields, McGraw-Hill Book Company (1965).
[25] H. Huber, K. Weigel, F. Weber, Z. Naturforsch. A 54 77 (1999); astro-ph/9811463.
TABLE I. Hadron masses and DDRH coupling constants at saturation density $\rho_0 = 0.16$ fm$^{-3}$ for $\Lambda$ particles and nucleons.

| Mass   | Coupling Constants (at $\rho_0$) | $B = N$ | $B = \Lambda$ |
|--------|---------------------------------|---------|---------------|
| $m_N$  | 939.0 MeV                       | $g^2_{\sigma B}/4\pi$ | 6.781 | 1.628 |
| $m_\Lambda$ | 1115.0 MeV                   | $g^2_{\omega B}/4\pi$ | 9.899 | 3.022 |
| $m_\sigma$ | 550.0 MeV                      | $g^2_{\rho B}/4\pi$ | 1.298 | 0.000 |
| $m_\rho$  | 782.6 MeV                      |               |       |       |

TABLE II. r.m.s. radii for $\Lambda$, neutron and proton single particle states in $^{40}_{\Lambda}Ca$ and $^{208}_{\Lambda}Pb$.

| State | $\Lambda$ | $^{40}_{\Lambda}Ca$ | $^{208}_{\Lambda}Pb$ |
|-------|-----------|---------------------|---------------------|
| 1s$_{1/2}$ | 2.8 fm | 2.3 fm | 2.4 fm | 4.1 fm | 3.8 fm | 3.9 fm |
| 1p$_{3/2}$ | 3.5 fm | 3.0 fm | 3.0 fm | 4.8 fm | 4.5 fm | 4.6 fm |
| 1p$_{1/2}$ | 3.6 fm | 3.0 fm | 3.0 fm | 4.7 fm | 4.4 fm | 4.5 fm |
| 1d$_{5/2}$ | 4.7 fm | 3.5 fm | 3.6 fm | 5.3 fm | 5.0 fm | 5.1 fm |
| 1d$_{3/2}$ | 6.3 fm | 3.6 fm | 3.7 fm | 5.2 fm | 4.9 fm | 5.0 fm |

TABLE III. Transition energies of $\Lambda$ particle–neutron hole excitations in single $\Lambda$ hypernuclei observed in $(K^-, \pi^-)$ reactions. DDRH results and phenomenological RMF calculations [4] (phen. RMF), including nonlinear $\sigma$ self interactions, are compared to experimental values (exp.) [43,44].

| Hypernucleus | Neutron valence state | Configuration | exp [MeV] | DDRH [MeV] | phen.RMF [MeV] |
|--------------|-----------------------|---------------|-----------|------------|---------------|
| $^{12}_{\Lambda}C$ | 1p$_{3/2}$ | $(1s_{1/2}\Lambda, 1p_{3/2}n^{-1})$ | 6.72$\pm$2 | 6.69 | 5.02 |
| $^{16}_{\Lambda}O$ | 1p$_{1/2}$ | $(1s_{1/2}\Lambda, 1p_{1/2}n^{-1})$ | 3.35$\pm$2 | 5.76 | 3.53 |
| $^{40}_{\Lambda}Ca$ | 1d$_{5/2}$ | $(1p_{1/2}\Lambda, 1d_{5/2}n^{-1})$ | 19.20$\pm$2 | 18.40 | 20.07 |
|                   |           | $(1d_{5/2}\Lambda, 1d_{5/2}n^{-1})$ | 19.35$\pm$2 | 19.35 | 20.71 |