Experimental evidence of strong electromagnetic shower enhancement induced by high-energy photons in a thick oriented tungsten crystal

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Abstract

We have observed a significant enhancement in the energy deposition and secondary particle generation by 25–100 GeV photons in a 1 cm thick tungsten crystal oriented along its (111) lattice axes. At 100 GeV, this enhancement, with respect to that observed without axial alignment, is more than twofold. This is ascribed to the acceleration of the electromagnetic shower development by the strong axial electric field. Previous studies of the enhancement of electromagnetic interactions by coherent effects in this energy range have generally made use of samples of thickness 1 mm or less. Our results, obtained with a 1 cm thick crystal, could be crucial in the development of high-performance photon absorbers, ultra-compact electromagnetic calorimeters and beam dumps for use at the energy and intensity frontiers.

Keywords: channeling, crystals, electromagnetic radiation, pair production, photon absorption, strong field

1. Introduction

High-Z metals are widely used in high-energy physics for applications requiring their strength, radiation hardness and short radiation and nuclear interaction lengths. In particular, tungsten (W) is extensively exploited in a wide variety of applications: compact beam collimators \cite{1, 2, 3}, radiation shielding \cite{1}, beam absorbers \cite{2, 4} and targets for beam-dump experiments \cite{5}. Moreover, tungsten foils with a thickness of one radiation length ($X_0 = 3.504$ mm) or smaller are easy to manufacture and prove ideal as the passive absorber layers in sampling electromagnetic calorimeters, which are ubiquitous in high-energy physics experiments.
for the measurement of electron and photon energies. Similarly, thin tungsten foils have been used in space-borne γ-ray telescopes, such as in the AGILE [6] and FERMI LAT [7] trackers, to convert incoming photons into $e^+e^-$ pairs.

Currently, tungsten is employed as an amorphous material in all the aforementioned applications, while its crystalline nature is completely ignored. However, it has been well known since the 1950s that the electromagnetic interactions between high-energy particles and crystalline matter can be strongly affected by the atomic lattice structure of the latter [5]. It was firstly predicted theoretically and then proved experimentally [9] that the probabilities for bremsstrahlung emission by high energy $e^\pm$ and conversion of high-energy photons to $e^+e^-$ pairs (pair production or PP) are enhanced when the particles are incident nearly parallel to crystallographic planes or axes [8], via the so-called coherent bremsstrahlung (CB) and coherent pair production (CPP) [10]. CB, for instance, consists in the enhancement of bremsstrahlung that occurs when the momentum transferred by the $e^\pm$ to the crystal matches a reciprocal lattice vector, in analogy with the Bragg-Laue diffraction.

The CB (CPP) theories work as long as the straight trajectory approximation is applicable to the motion of the incident (emitted) particle. However, the latter fails [11] when the charged particle trajectory is aligned with the crystal plane/axis within the so-called Lindhard critical angle [12], $\theta_L = \sqrt{2U_0/E}$, with $U_0$ the depth of the potential well associated with the plane/axis and $E$ the projectile initial energy. Indeed, in this condition, the particle interacts coherently with the atoms in the same row/plane and is subject to transverse oscillations in the effective (averaged) electric planar/axial field of $\varepsilon \sim 10^{10}-10^{12}$ V/cm, i.e., channeling occurs [13, 14, 15, 16].

In the rest frame of the incident particle, this effective field is enhanced by a Lorentz factor, $\gamma_{\text{eff}} = E/mc^2$ ($\gamma_{\text{eff}} = h\omega/mc^2$) for $e^\pm$ ($\gamma$), with $m$ the electron mass and $\omega$ the incident $\gamma$ frequency; at sufficiently high incident particle energy, it can become comparable to the QED critical field, $\varepsilon_0 = m^2c^3/e\hbar \sim 1.32 \times 10^{16}$ V/cm, introduced by Schwinger [17]. In the so-called strong field regime, in which $\gamma_{\text{eff}} \varepsilon \gtrsim \varepsilon_0$, a considerable enhancement in both radiation emission and PP is attained, outperforming both the standard Bethe-Heitler processes in amorphous media [18] and the CB/CPP processes in crystals. The strong crystalline field is characterized by the emission of quantum synchrotron radiation, for which the deflection angle in the oscillatory motion exceeds the typical radiation cone opening angle $1/\gamma_{\text{eff}}$, by high-energy $e^\pm$ and by intense PP by high-energy photons [19, 11, 20], which occur where the applicability of the uniform field approximation holds true [21, 19, 11, 20]. In practice, it comes into effect when the trajectory of the incident particle forms an angle with the lattice direction not much larger than $\Theta_0 = U_0/mc^2$ [22, 23, 24, 25].

The main features of the crystalline strong field can be described in terms of the parameter $\chi = \gamma_{\text{eff}}\varepsilon/\varepsilon_0$ [21, 20]. For $\chi \ll 1$ the effective field is subcritical, the synchrotron-like PP is strongly suppressed and the synchrotron-like radiation is soft (i.e., the emitted photon energy is a small fraction of the projectile one). On the other hand, for $\chi \sim 1$ the critical field is reached and quite radical changes occur: the synchrotron-like PP rapidly attains observable rates [25, 26] and the synchrotron-like radiation recoil becomes highly important, which results in an enhancement in the hard part of the photon energy spectrum. As a consequence, a strong acceleration of the electromagnetic shower development, or equivalently, a strong reduction of the radiation length, $X_0$, is attained. The cross section enhancement grows with photon energies ranging from a few tens of GeV up to a saturation value in the multi-ToV region [19, 27, 16, 26].

The enhancement of radiation and PP by high-energy $e^\pm$ and photons in strong and sub-strong crystalline fields was first experimentally investigated in the 1980s. The studies on PP were firstly focused on lighter elements such as silicon and germanium [28], and only at a later stage on high-$Z$ crystalline metals such as tungsten and iridium [29, 30, 31]. These studies were driven by the need to test the feasibility of an intense positron source [32] and to develop a compact photon converter, i.e., a device to separate the photon and neutral hadron beam components with minimal absorption or scattering of the latter [29, 30], for the NA48 experiment at CERN; eventually, an iridium crystal with a thickness of 0.98 $X_0$ was employed [33]. Indeed, with the aim of demonstrating the radiation or the PP enhancement, all these measurements featured samples with a thickness of $\lesssim 1X_0$ or, in case of a few studies limited to sub-strong field energies, slightly thicker ones, thus precluding a full investigation of the electromagnetic shower acceleration caused by the strong crystalline field.
Figure 1: Experimental setup at the CERN H2 beamline. The input electrons are incident on a copper radiator (TGT), producing bremsstrahlung photons; the latter are then separated from the charged beam by a bending magnet (BM). The S1–4 scintillating counters are used to generate the trigger signal ($S_1 \land S_2 \land S_3$) and to measure the downstream charged particle multiplicity ($S_4$). The silicon microstrip detectors SD1–3 track charged particles. An array of lead-glass blocks measures the energy of the interaction products of the photon ($\gamma$-CAL) and of the deflected electron (e-CAL). Bottom left insertion: tungsten $\langle 111 \rangle$ axial potential as a function of the transverse distance from the axis and BCC lattice, with the $\langle 111 \rangle$ axis highlighted in red.

and only allowing for either the radiative or the PP enhancement to be studied individually.

In this letter, we present an experimental investigation into the acceleration of the development of the electromagnetic shower occurring in interactions between 25–100 GeV photons with a thick ($2.85 X_0$) tungsten crystal oriented along the $\langle 111 \rangle$ axes, i.e. in the full strong field regime — $\chi \gtrsim 1$. The results were obtained at the CERN SPS, and demonstrate the strong enhancement of the electromagnetic shower development and, in particular, of the absorption power when the photon beam is aligned with the crystal axes. This effect might be advantageously exploited in future experiments at the energy and intensity frontiers, with applications in photon absorbers for fixed-target and beam dump experiments and in new generation detectors such as ultra-compact sampling calorimeters, as explained later in the text.

2. Experimental setup

Crystalline tungsten has a body-centered cubic (BCC) lattice (Fig. 1 bottom left) with constant $a = 3.1652 \, \text{Å}$. The continuous potential associated with the $\langle 111 \rangle$ axis at room temperature is $U_0 = 887 \, \text{eV}$, which can be obtained from the curve plotted in Fig. 1 bottom left; this corresponds to $\Theta_0 \sim 1.75 \, \text{mrad}$. For this axis, $\chi = 1$ at about $13.6 \, \text{GeV}$ [19], at the maximum axial field $\varepsilon = \varepsilon_{\text{max}} \sim 5 \times 10^{11} \, \text{V/cm}$, and at $E_\gamma \gtrsim 22 \, \text{GeV}$ the coherent PP probability becomes larger than the Bethe-Heitler value.

The commercial sample under study was produced by Princeton Scientific; it was approximately cubic, 10 mm (i.e., $2.85 X_0$) thick. The $\langle 111 \rangle$ axes were normal to one of the faces. It was tested at the CERN H2 beamline with a tagged-photon beam obtained from a tertiary 120 GeV/c electron beam. The experimental setup is shown in Fig. 1. In the upstream stage, the electron beam was incident on a copper radiator in which photons were generated; the electrons were then deflected towards an array of lead-glass blocks [34], which measured the electron energy after bremsstrahlung emission and therefore served as a photon tagging system (e-CAL in Fig. 1). Furthermore, the trajectories of the input electrons were reconstructed via two $\sim 20 \times 20 \, \text{mm}^2$ silicon microstrip sensors.
(SD1–2), with an overall angular resolution of a few μrad [35]; this allowed the reconstruction of the impact point and trajectory of the photons incident on the crystal sample, given the small aperture of the bremsstrahlung emission cone at this energy (≈ 1/γ ≈ 4 μrad).

The sample was mounted on a high-precision goniometer, which allowed position and orientation in both the horizontal and vertical planes to be controlled remotely with a resolution of ≲ 5 μm and ≲ 5 μrad respectively [36, 37, 38]. Plastic scintillators were installed upstream and downstream with respect to the crystal position: the former (S3) served as a veto for the pairs produced by photon conversion in air (whose contribution was strongly suppressed by the use of vacuum pipes and a helium bag for transport of the deflected beam) whereas the latter (S4) allowed a measurement of the number of charged particles produced inside the crystal. Another lead-glass block (γ-CAL), placed along the photon beam trajectory, measured the energy transmitted in the forward direction. The input beam divergence of ≈ 85 μrad (≈ 60 μrad) in the horizontal (vertical) plane was considerably smaller than Θ₀, allowing the dependence of the strong field on the input angle to be studied with high precision. The crystal was tested at several different values of the angle of incidence of the photon beam with respect to the (111) axis, ranging from 0 to 12.5 mrad (≈ 7 Θ₀). Furthermore, data were collected at ≈ 45 mrad (≈ 2.6°) from the axes along a direction chosen to avoid proximity to other main axes and planes, where the lattice structure has no effect on the photon-matter interaction; this is equivalent to the case of random orientation.

3. Results and simulations

Fig. 2 shows the correlations between the transmitted energy directly measured by the γ-CAL and the initial photon one obtained via photon tagging, i.e., by subtracting the measured electron energy (by e-CAL) from the nominal value of the initial electron beam energy (120 GeV). The energy absorbed or dispersed by the crystal is strongly enhanced when the beam is incident along the crystal axis (blue points), compared to the random orientation (brown points). Indeed, for a 100 GeV photon incident at θ = 0 mrad, the transmitted energy is decreased by about 15 GeV relative to that for the random orientation, corresponding to a doubling of the energy absorbed inside the crystal. The maximum absorption power is maintained up to about 2.5 mrad from the crystal axes, while the size of the effect decreases as the angle grows. Nevertheless, even at an angle of incidence of 12.5 mrad (0.7°), the absorption is still ≈ 150% of that for the random orientation. As expected, since X₀ decreases as the input energy increases, the energy absorbed inside the crystal increases with the initial photon energy, whereas the fraction of input energy that is actually absorbed is nearly constant over the whole explored range. Surely, the broad angular range over which the macroscopic character of the enhancement in energy absorption is preserved is due to both the strength of the tungsten (111) axis potential and the high mosaicity of the sample, for which a value of ≈ 3 mrad was evaluated from simulations and X-ray diffraction data.

![Figure 2: Energy measured by γ-CAL as a function of the tagged photon energy, at different angles between the beam and the (111) axis and in the random orientation (45 mrad). The points (dashed lines) represent the experimental data (simulated results).](image)

The experimental results presented in Fig. 2 demonstrate the faster electromagnetic shower development due to the strong crystalline field, with a resulting enhancement of secondary particle generation in the first layer of material. Each of these charged secondaries deposits energy inside the material during its passage, resulting in a significant increase of absorbed energy when the sample is oriented along its (111) axes.

We also directly measured the increase in the secondary production due to the acceleration of shower...
development with the S4 scintillating multiplicity counter, whose pulse height is proportional to the energy deposited inside the plastic layer and therefore to the number of incident charged particles (Fig. 3). As expected, the enhancement grows with the tagged photon energy, i.e., at higher initial $\chi$, and for decreasing angle of incidence with respect to the axis. The corresponding ratios between measured values at different angles of incidence and in the random orientation (i.e., 45 mrad off axis) range from 130–160% at $\sim 30$ GeV, depending on the incoming photon angle, to $\gtrsim 230\%$ at 100 GeV when on axis; indeed, even at an angle of 12.5 mrad with respect to the axis, the enhancement ratio is $\sim 170\%$ for 100 GeV photons.

![Figure 3: Energy deposited in the S4 plastic scintillator as a function of the tagged photon energy, at different angles between the beam and the (111) axis and in the random orientation (45 mrad). The points (dashed lines) represent the experimental data (simulated results).](image)

All of the experimental results presented here are validated by the agreement with the results of Monte Carlo simulations (see dashed lines in Figs. 2 and 3). The simulation of the full experimental setup was based on the Geant4 toolkit \cite{39}, in which the features of the radiation-matter interaction for amorphous media is implemented by default. The results for the axial configuration were obtained by rescaling the bremsstrahlung and PP cross sections according to the outcome of full Monte Carlo computations based on the Baier-Katkov quasi-classical operator method \cite{25, 26} to simulate the enhancement in oriented crystals — see, e.g., \cite{40, 41}. Indeed, since the crystal mosaicity exceeded the angle $\Theta_0$, our experimental conditions were far from those for which the uniform field approximation is applicable \cite{16}. For this reason, the full scale evaluation of Baier-Katkov formulae was used in simulations. It should be also noticed that the observed effects can be enhanced by reducing the mosaicity, i.e. by employing a better quality crystal.

4. Outlook on the applications

The observation of the enhancement in photon energy absorption and secondary pair production described in this work paves the way to manifold applications in high-energy physics and astrophysics. While the strong field effects result in a reduction of the effective $X_0$ that depends on the particle energy, the nuclear interaction length $\lambda_{\text{int}}$, i.e., the longitudinal scale of the inelastic hadronic processes, is unaffected by the lattice orientation. Therefore, axially oriented tungsten layers prove appealing to filter out the photon component in hadron beams with minimum effect on the hadronic component \cite{42}. As an example, this technique could be used to clean the prompt photons from a neutral hadron beam as proposed by the KLEVER (KExPeriment for VEry Rare events) experiment, which is planned to follow upon the NA62 experiment in the CERN North Area, extending the program of searches for rare kaon decays to include $K_L$ mesons.

Similarly, beam-dump experiments for light dark matter searches could benefit from an oriented crystal dump target, which would allow for an increase in the experiment sensitivity; indeed, a crystalline target would allow the dump length to be reduced without any reduction in its electromagnetic shower containment capability and therefore the probability to be increased for dark photons to exit the dump before decaying, so that their decay products can be reconstructed in the apparatus downstream \cite{43}. Moreover, it is well known that, at high-energies, bremsstrahlung and PP are suppressed in amorphous media due to the Landau-Pomeranchuk-Migdal (LPM) effect \cite{44}. Since bremsstrahlung and PP are dramatically enhanced in crystals at high energy and at the same time are not reduced by the LPM effect, the use of aligned crystals is appealing for the design of beam dumps for use at the energy frontier.

The reduction in electromagnetic shower length is clearly visible in Fig. 4, which shows the simulated energy deposited in crystalline tungsten oriented
along the ⟨111⟩ axis by primary electrons/photons. The curves are obtained from simulation of highly collimated electron and photon beams incident on a 20 × 20 × 20 cm$^3$ tungsten crystal, i.e., wide enough to contain all the shower particles, oriented along the ⟨111⟩ axis: the fraction of the absorbed primary particle energy is plotted, at different input energies and for the random (top) and axial (bottom) cases, as a function of the depth inside the material. When on axis, the maxima of all of the curves are located approximately at the same depth, thus demonstrating that in case of an axially oriented crystal the position of the shower maximum is nearly independent of the initial energy between a few GeV and ~1 TeV. For a 100 GeV photon, the energy deposition at a depth of 1 cm is about three times higher in the axial orientation than in the random orientation. This enhancement is somewhat higher than our observed value, due to the limited transverse size of the sample tested and of the γ-CAL acceptance.

The acceleration of the shower development can be exploited to create sampling electromagnetic calorimeters with thinner passive layers. Sampling calorimeters are widely used in particle physics for the measurement of particle energies and consist of several high-Z, high-density passive layers, which force incoming electromagnetic particles to shower, interleaved with active layers (typically plastic scintillator) in which a signal proportional to the number of particles per stage is generated. The use of oriented tungsten absorber layers instead of amorphous ones would allow for a significant reduction of the calorimeter thickness, leading to the development of ultra-compact calorimeters with applications in fixed-target experiments, at the energy and intensity frontiers. Furthermore, as seen from Fig. 4 in a sampling calorimeter using oriented crystal absorbers, the depth profile of the electromagnetic shower is nearly independent of the incident particle energy, thus reducing or eliminating non-linearities due to shower leakage for high-energy particles. The results presented in this letter demonstrate that metallic crystals that are already commercially available are suitable for this purpose.

5. Conclusions

In summary, we have presented measurements of the acceleration of electromagnetic shower development for 25–100 GeV photons incident on a thick (multi-$X_0$), commercially available tungsten crystal at small angle relative to the ⟨111⟩ axis, demonstrating an enhancement in the secondary particle production and energy absorption for axially oriented crystals, compared to the case for random orientation. This behaviour is maintained over an angular acceptance of at least a few mrad. The magnitude of the observed effect is promising for the use of thick, commercial crystalline tungsten layers as compact photon or electron absorbers in current and future fixed-target experiments, which are intrinsically forward. These results also confirm that high-Z crystals such as tungsten prove interesting for the construction of next-generation, ultra-compact sampling calorimeters and targets for beam-dump experiments.

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References

[1] O. Brüning, L. Rossi, The High Luminosity Large Hadron Collider: The New Machine for Illuminating the Mysteries of Universe, World Scientific, 2015. doi:10.1142/9581
[2] H. Watanabe, et al., Nucl. Instr. Meth. Phys. Res. A 545 (2005) 542–553. doi:https://doi.org/10.1016/j.nima.2005.02.011
[3] T. Shimogawa, Nucl. Instr. Meth. Phys. Res. A 623 (2010) 585–587. doi:10.1016/j.nima.2010.03.078
[4] M. Quaresma, PoS DIS2016 (2016) 218. doi:10.22323/1.265.0218
[5] R. Moore, et al., Nucl. Instr. Meth. Phys. Res. A 936 (2019) 124–126. doi:10.1016/j.nima.2018.07.085
[6] A. Akhiezer, N. Shul’Ga, High Energy Electrodynamics in Matter, Taylor & Francis, 1996. doi:https://doi.org/10.1142/2216
[7] V. N. Baier, V. M. Katkov, V. M. Strakhovenko, Electromagnetic Processes at High Energies in Oriented Single Crystals, World Scientific, 1998. doi:10.1142/2216
[8] V. N. Baier, V. M. Katkov, V. M. Strakhovenko, Nucl. Instr. and Meth. in Phys. Res. Sect. A 90 (1997) 169–171. doi:10.1016/0168-9002(97)0088-2
[9] V. N. Baier, V. M. Katkov, V. M. Strakhovenko, Yadernaya fizika 36 (1982) 697–706.
[10] V. N. Baier, V. M. Katkov, V. M. Strakhovenko, Nucl. Instr. and Meth. in Phys. Res. Sect. B 16 (1996) 5–21. doi:10.1016/0168-583X(86)90221-1
[11] V. N. Baier, V. M. Katkov, V. M. Strakhovenko, Zh. Eksp. Teor. Fiz. 90 (1986) 801–815.
[12] J. Kimball, N. Cue, Physics Reports 125 (1985) 69–101. doi:10.1016/0370-1573(85)90221-3
[13] U. I. Uggerhøj, Rev. Mod. Phys. 77 (2005) 1131–1179. doi:10.1103/RevModPhys.77.1131
[14] J. C. Kimball, N. Cue, L. M. Roth, B. B. Marsh, Phys. Rev. Lett. 50 (1983) 950–953. doi:10.1103/PhysRevLett.50.950
[15] V. G. Baryshevskii, V. V. Tikhomirov, Sov. Phys. JETP 58 (1985) 135–140.
[16] V. G. Baryshevskii, V. V. Tikhomirov, Yadernaya fizika 1013–1032. doi:10.1016/j.nima.2005.02.011
[17] F. Sauter, Zeitschrift für Physik 69 (1931) 742–764. doi:https://doi.org/10.1007/BF01339461
[18] H. Bether, W. Heitler, Proc. R. Soc. Lond. A 146 (1934) 89–112. doi:https://doi.org/10.1016/BF01339461
[19] V. G. Baryshevskii, V. V. Tikhomirov, Phys. Usp. 32 (1989) 1013–1032. doi:10.1070/PU1989v032n11ABEH002778
[20] V. N. Baier, V. M. Katkov, V. M. Strakhovenko, Electromagnetic Processes at High Energies in Oriented Single Crystals, World Scientific, 1998. doi:10.1142/2216
[21] V. N. Baier, V. M. Katkov, V. M. Strakhovenko, Nucl. Instr. and Meth. in Phys. Res. Sect. A: Accelerators, Spectrometers, Detectors and Associated Equipment 234 (1985) 430–434. doi:https://doi.org/10.1016/0168-9002(85)90987-8
[22] V. G. Baryshevskii, V. V. Tikhomirov, Physics Letters A 133 (1988) 335–340. doi:https://doi.org/10.1016/0375-9601(88)90176-1
[23] V. G. Baryshevskii, V. V. Tikhomirov, Yadernaya fizika 36 (1982) 697–706.
[24] V. N. Baier, V. M. Katkov, V. M. Strakhovenko, Nucl. Instr. and Meth. in Phys. Res. Sect. B 16 (1996) 5–21. doi:10.1016/0168-583X(86)90221-1
[25] V. N. Baier, V. M. Katkov, V. M. Strakhovenko, Zh. Eksp. Teor. Fiz. 90 (1986) 801–815.
[26] J. Kimball, N. Cue, Physics Reports 125 (1985) 69–101. doi:10.1016/0370-1573(85)90221-3
[44] L. D. Landau, I. Pomeranchuk, Dokl. Akad. Nauk Ser. Fiz. 92 (1953) 535–536.