Deposition of Charged Aerosol Particles Flowing through Parallel Plates†

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Abstract

Deposition of unipolarly charged aerosol particles flowing through parallel plates has been investigated both theoretically and experimentally. The equation of motion for the particles was solved numerically in consideration of the Coulomb force, image force, particle-inertia force, and fluid-velocity profile. Then the deposition efficiency was calculated based on the limiting trajectories of the particles.

It was found that the calculated deposition efficiency, assuming a laminar flow, was a little smaller than the corresponding analytical solution obtained for the plug flow distribution. It was also found that the effect of the image force and particle-inertia force was negligible in the experimental range. The experimental deposition efficiencies for charged fly-ash particles were well explained in so far as the actual velocity distributions were taken into consideration.

1. Introduction

The deposition of charged aerosol particles suspended in the air flow is enhanced by the effect of electric field caused by the particles themselves. This effect, called the space-charge effect, may be applicable to practical use such as dust collection. The reduction of the collection efficiency due to back discharge in an electrostatic precipitator has become a serious problem in recent years. This problem may be successfully solved and the collection efficiency will become predictable to considerable extent when the dust particles are precharged and then captured with the aid of the space-charge effect.

The deposition of monodispersed, charged particles on the circular tube due to the space-charge effect has been studied analytically by Wilson10) and experimentally by Kasper8), Adachi et al.11), and the author et al.7) These experimental results are found to have reasonable agreement with the analytical solution derived by Wilson. Adachi et al. have also studied the effect of image force. For the deposition of charged aerosol particles in parallel channel flow, the effects of space charge and thermal diffusion have been studied by Ingham5) and Chen2), the effect of image force by Yu et al.11,12), and the studies on the gravity effect has been reported elsewhere3,9). Unfortunately, however, there have been a very few reports which analyze these problems synthetically.

In this paper, the deposition efficiency of unipolarly charged aerosol particles flowing in a parallel channel was theoretically determined based on the critical particle trajectory, which is obtained through numerical integration of the equation of motion for the particles. In the numerical calculation, the effects of space charge, image force, particle-inertia force, and air-velocity profile were taken into considerations. Further, the deposition of unipolarly charged fly-ash particles was experimentally investigated, and the results were compared with the theoretical predictions.
2. Theory

Assuming a plug flow, a parallel plate model of \( L \) in length, \( 2Z_1 \) in width, and \( 2X_1 \) in space between the plates is considered with the coordinate system illustrated in Fig. 1. When unipolarly charged aerosol particles flow into the model channel, the \( X \)-component of the intensity of electric field at the position \( X_0 \) is given by the following equation:

\[
E = N_0 q X_0 / \varepsilon_0 \tag{1}
\]

where \( N_0 \) is the particle number per unit space volume, \( q \) is the electric charge on a particle and \( \varepsilon_0 \) is the air permittivity. The \( Y \)- and \( Z \)-components of the electrostatic field are negligible in the following analysis. Also, the electric force on a particle is given by

\[
F_e = q E = N_0 q^2 X_0 / \varepsilon_0 \tag{2}
\]

If the particles are assumed to be monodisperse, the Coulomb force \( F_e \) on a particle is unchanged throughout the movement. For this case, \( X \)-component of the particle velocity is determined by the force balance between the force \( F_e \) and the fluid resistance given by the Stokes law, as follows:

\[
v = \frac{F_e C}{3 \pi \mu D_p} = \frac{N_0 q^2 C X_0}{3 \pi \mu D_p \varepsilon_0} = \alpha X_0 \tag{3}
\]

The parameter \( \alpha \) in Eq. (3) is defined by the following equation:

\[
\alpha = \frac{N_0 q^2 C}{3 \pi \mu D_p \varepsilon_0} \tag{4}
\]

where \( \mu \) is the air viscosity, \( D_p \) is the particle diameter and \( C \) is the Cunningham’s slip correction factor. Thus the displacement of a particle in the \( X \)-direction is

\[
dX = v dT = \alpha X_0 dT \tag{5}
\]

By integrating Eq. (5),

\[
X_1 - X_0 = \alpha X_0 \tau \tag{5-a}
\]

where \( \tau \) is the average residence time. The deposition efficiency \( \eta \) is defined as

\[
\eta = (X_1 - X_0)/X_1 = \alpha \tau / (1 + \alpha \tau) \tag{6}
\]

By use of the following definitions:

\[
N_0 = c_o / m_p, \quad \dot{q} = q / m_p, \quad \tau = L / \bar{U}_y, \quad \text{and} \quad m_p = \frac{\pi}{6} D_p^3 \rho_p
\]

Hence Eq. (6) is rewritten as follows:

\[
\eta = \frac{\dot{q}^2 D_p^2 \rho_p c_o L C}{18 \mu \bar{U}_y \varepsilon_0} \tag{7}
\]

where \( c_o \) is the mass concentration of the particles, \( m_p \) is the particle mass, \( \dot{q} \) is the electric charge on a unit mass of the particles, \( \bar{U}_y \) is the average air velocity, and \( \rho_p \) is the particle density.

Although the particles are assumed to move along the plate (in the \( Y \)-direction) with the average air flow velocity, the effect of the actual velocity profile should be taken into consideration in a practical use. In addition, the inertia force and the image force should also be considered for relatively coarser parti-
Assuming that inlet air flow is fully developed so as to form a laminar velocity profile and the gravity effect is negligible, the equations of motion for particles are:

for X-component

\[
\frac{\pi}{6} D_p^2 \rho_p \frac{d^2 X}{dt^2} = F_e + F_i + 3\pi \mu D_p \left( U_x - \frac{dX}{dt} \right)
\]

(8)

for Y-component

\[
\frac{\pi}{6} D_p^2 \rho_p \frac{d^2 Y}{dt^2} = \frac{3\pi \mu D_p}{C} \left( U_y - \frac{dY}{dt} \right)
\]

(9)

where \( F_e \) is the Coulomb-force given by Eq. (2) and \( F_i \) is the image force given by the following equation:

\[
F_i = \frac{q^2 X_1 \sum_{n=1}^{\infty} \frac{(2n+1)X}{((2n+1)X_1^2 - X^2)^{3/2}}}{4\pi \varepsilon_o}
\]

(10)

If the air flow is assumed to be laminar, the air flow velocity is expressed as

for X-component

\( U_x = 0 \)

(11)

for Y-component

\[
U_y = \frac{3}{2} \bar{U}_y \left[ 1 - \left| \frac{X}{X_1} \right|^2 \right]
\]

(12)

By substituting Eqs. (11) and (12) into Eqs. (8) and (9) respectively, the following non-dimensional equations will be obtained:

\[
P \frac{d^2 x}{dt^2} = \frac{C(F_e + F_i)}{3\pi \mu D_p \bar{U}_y} - \frac{dx}{dt}
\]

(13)

\[
P \frac{d^2 y}{dt^2} = \frac{3}{2} (1 - x^2) - \frac{dy}{dt}
\]

(14)

where \( x = X/X_1 \), \( y = Y/X_1 \), and \( t = \bar{U}_y T/X_1 \).

The particle-inertia parameter \( P \) is defined as

\[
P = \frac{D_p^2 \rho_p \bar{U}_y C}{18 \mu X_1}
\]

(15)

The Coulomb-force parameter \( K_e \) and the image-force parameter \( K_i \) may be defined by the following non-dimensional forms;

\[
K_e = \frac{N_o q^2 X_1 C}{3\pi \mu D_p \bar{U}_y \varepsilon_o}
\]

(16)

Using the above parameters, Eq. (13) is rewritten in non-dimensional form as follows:

\[
P \frac{d^2 x}{dt^2} = K_e x + K_i \left[ \frac{x}{(1-x^2)} \right] - \frac{dx}{dt}
\]

(18)

The numerical integration of Eqs. (14) and (18) in combination yields the particle trajectories. Considering the particle which travels from the initial point \( X_0 \) to the boundary point \( (X = X_1) \) at the outlet of the parallel channel \( (Y = L) \), the deposition efficiency can be determined by the following equation;

\[
\eta = \frac{1}{x_0} \int_0^1 u_y dx - \frac{3}{2} x_o + \frac{1}{2} x_o^3
\]

(19)

where \( u_y = U_y/\bar{U}_y \).

In this study, the numerical integration based on the Runge-Kutta-Merson method was applied to Eqs. (14) and (18) for various initial conditions in order to obtain the deposition efficiency by use of Eq. (19).

Equations (14) and (18) show that the deposition efficiency \( \eta \) is a function of \( P, K_e, K_i, L, \) and \( X_1 \). Figure 2 shows a typical set of correlations between \( \eta \) and \( K_e \) when \( K_i = 0 \) and \( L/X_1 = 100 \). All the curves using the parameter \( P \) indicate that \( \eta \) increases with an increase in \( K_e \), approaching the dotted line as \( P \) decreases. This dotted line represents the
analytical solution derived from Eq. (7) based on the assumption of plug flow. The influence of particle inertia becomes dominant when \( P > 0.1 \) and the slope decreases as \( P \) increases.

The effect of image force on deposition efficiencies for \( P = 0 \) is shown in Fig. 3 by use of the parameters of \( K_i \) and \( L/X_1 \). Figure 3 employs the same coordinate system as that of Fig. 2 and the dotted line also represents a solution for plug flow. Although \( \eta \) increases with increasing \( K_i \), the influence of image force is reduced as \( K_e \) increases; such an influence may be almost negligible when \( K_e > 0.1 \). The effect on deposition due to image force is enhanced as \( L/X_1 \) increases; in other words, when the plates are made longer or the clearance between the plates narrower.

By dividing Eq. (16) by Eq. (17), the ratio of the image force parameter \( K_i \) to Coulomb-force parameter \( K_e \) is expressed as

\[
\frac{K_i}{K_e} = \frac{1}{4 \pi N_o X_1^2} \tag{20}
\]

Figure 4 shows the ratio \( \eta/\eta_{K_i=0} \) as a function of \( K_e \) with the ratio \( K_i/K_e \) as a parameter under the conditions of \( P = 0 \) and \( L/X_1 = 100 \), where \( \eta_{K_i=0} \) represents the deposition efficiency for \( K_i = 0 \). The ratio \( \eta/\eta_{K_i=0} \) increases with increasing the ratio \( K_i/K_e \). This means that the influence of image force may be conspicuous with decreasing the particle number concentration and/or the space between the plates in consideration of Eq. (20).

3. Experiment

3-1 Experimental apparatus and method

The outline of the experimental arrangement is illustrated in Fig. 5. The powder material used for this work was JIS test dust No. 10 of fly ash with the density of 2.39 g/cm³ and the mass median diameter of 3.5 \( \mu \)m having the geometrical standard deviation of 2.18 (JIS Z-8901). After dispersed with a mixer-type disperser (made by Shimadzu Seisakusho Ltd.) at a constant feed rate with a table feeder (made by Sankyo Dengyo Co., Ltd.), the sample powder was unipolarly charged with the Boxer charger (made by Sankyo Dengyo Co., Ltd.), before being introduced into the test zone. Some of the sample particles were deposited on the test plates and the others which passed through them were captured with a glass fiber filter (made by Toyo Roshi Co., Ltd.). The deposition efficiency was given by

\[
\eta = (W_1 - W_2)/W_1 \tag{21}
\]

where \( W_1 \) is the weight of the sample discharged out of the table feeder and \( W_2 \) is the weight of it captured with the filter. It was observed that there was no adhesion of particles on the disperser wall and the deposition except for test plates was negligible (which was less than 3% of the total captured amount). The feeding rate of the table feeder fluctuated to some extent; the variation was about 5%
during 1 min operation with a range of 10 to 400 mg/s.

Figure 6 shows the outlook of the test plates with 39 cm height, 18 cm width, and 1 m length in which several steel plates were arranged in parallel and mounted with PVC holding spacers on top and bottom sides. The space between each test plate was to be varied with the spacer thickness. The flow after the charging zone was enlarged toward the deposition zone and the disk of 4 cm diameter with the attitude perpendicular to the air flow was mounted at the center of the enlarged duct.

Electric charges on aerosol particles were measured with a Faraday cage and an electric meter (made by Takeda Riken Co., Ltd.) as illustrated by the dotted line in Fig. 5. In addition, the size distribution of aerosol particles was determined with a cascade impactor (made by Nippon Kagaku Kogyo Co., Ltd.).

3.2 Experimental results and discussion

The particle inertia parameter $P$ and the image force parameter $K_i$ were observed to be up to $10^{-2}$ and $10^{-6}$, respectively. Thus, the effects of such parameters would be negligible in the numerical calculation. When it is assumed that $P = 0$ and $K_i = 0$, $K_e$ multiplied by $L/X_1$ gives the following new parameter $\alpha \tau$ which is independent of the space between the plates.

$$K_e \frac{L}{X_1} = \frac{\dot{q}^2 D_p^3 \rho_p \varepsilon_0 LC}{18 \mu \bar{U}_y \varepsilon_o} = \alpha \tau$$  \hspace{1cm} (22)

The relationship between $\alpha \tau$ and $\eta$ is shown in Fig. 7 where the dotted line represents the
analytical solution for plug flow and the solid line represents a calculated result for laminar flow. The deposition efficiency increases with $\alpha r$ and the experimental results are somewhat smaller than the solid line as $\alpha r > 1$.

It has been recognized from the laboratory deposition experiment that the deposition efficiency may depend upon the plate material. In this work, we employed aluminum as well as PVC to examine the spacer effect. The result suggests the effect of the spacer material would be negligible as shown in Fig. 7 where the symbol $\bullet$ represents the measured value which was obtained by use of aluminum spacers.

It should be noticed that the actual velocity distribution of air flow would differ from the assumed one such as laminar or plug flow discussed above because of a lack of flow uniformity caused by the duct enlargement.

One of the velocity profiles at the inlet of the test plates measured by a wire anemometer is typically illustrated in Fig. 8. This figure, which is based on the non-dimensionalized coordinate system that corresponds to each inlet section in Figs. 1 and 6, implies that the flow velocity might be maximized at the center and the backward flow due to flow separation might take place near the wall. These phenomena are caused by the influence of duct enlargement, and they should be taken into consideration in a practical use.

The deposition efficiencies obtained from the modified flow distribution and the experimental flow distribution based on the actual profile indicated in Fig. 8 are also shown in Fig. 7. The latter curve provides a lower limit of the deposition efficiency because the flow development in the test plates may gradually form a laminar velocity profile. The experimental results are found within the range of the theoretical predictions, and thus the influence of the actual flow distribution may possibly justify the deviation of the experimental results from the theoretical solutions.

The experimental relationship between deposition efficiency and plate space $2X_1$ at $\alpha r = 4.2$ is shown in Fig. 9, where the solid line represents a theoretical solution based on the assumption of the laminar flow. The deposition efficiency was found to be independent of the plate space except that $2X_1 = 1.2$ cm. The deposition efficiency at $2X_1 = 1.2$ cm is close to the theoretical solution, and this agreement might result from the flow rectification effect of the parallel plates.

It is well-known that electric discharge takes place in atmosphere when the electric field of more than 30 kV/cm is applied. If this discharge occurs between the aerosol particles and the channel wall, the deposition efficiency may be reduced because of immediate disappearance of the electrostatic force on the particle. The intensity of electric field formed by charged particles in the channel is given by Eq. (1) when the space charge density is held constant.

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**Fig. 7** Comparison between calculated and experimental deposition efficiency (as a function of parameter $\alpha r$)

**Fig. 8** Experimental profile of flow velocities at the inlet of test plates
at the inlet. This intensity increases to the maximum value very near the wall, and thus its maximum value will be in proportion to the plate space. In this work, this intensity was found to be up to 10kV/cm when no steel plates were inserted (in other words, only the outer case of the test box shown in Fig. 6 was used). Although it was ascertained that this intensity was smaller than the critical value for initiating atmospheric discharge, experimental studies were carried out on the distributions of particle charge and particle concentration of charged aerosols at the inlet of the test plates by use of a Faraday cage.

The particle charge distributions at the inlet of the channel along the $X$ (width) and $Z$ (height) directions are shown in Figs. 10 (a) and (b) respectively. The symbols $\bigcirc$ and $\triangle$ in the figure denote the sampling in forward and in backward directions, respectively. With the sampling in each direction for the region where flow separation took place, it was found that the particle charge distribution in the chamber was nearly uniform. Consequently, it is reasonably considered that no electric discharge between particles and a wall of the equipment might take place.

Figures 11 (a) and (b) indicate the distributions of particle concentration in $X$ and $Z$ directions, respectively. The curve represents the velocity profile at the inlet of the parallel plate chamber. Although the particle concentration slightly increased near the wall, the correlation between particle concentration and velocity profile was not found.

It this work, the influences of deposition by gravity and by diffusion calculated from the equation derived by Marcus$^8$ were found to be up to 1% and up to 0.01%, respectively, showing that the influences might be reasonably neglected.

4. Conclusion

The deposition of unipolarly charged aerosol particles flowing through parallel plates were investigated both theoretically and experimentally.

Summarizing the results of this work:
(1) The deposition efficiency increased with
increasing the Coumb-force parameter and the theoretical solution based on the assumption of laminar flow was slightly smaller than that for plug flow. When the image force and particle-inertia force were negligibly small, the deposition efficiency was expressed as a function of the unique parameter $\alpha$ which did not depend on the space between the parallel plates.

(2) The deposition efficiency decreased with an increase in particle-inertia parameter $P$. The influence of the inertia was, however, reasonably negligible if $P$ was less than 0.1.

(3) The image force affects the deposition efficiency when the image-force parameter is larger than $10^{-4}$ and the Coulomb-force parameter was smaller than $10^{-1}$. The effect becomes large with increasing $L/X_1$. It should be noted that the image force could be dominant when the particle number concentration and the plate space were very small.

(4) The experimental deposition efficiency was slightly smaller than the theoretical one obtained by the assumption of laminar flow. This fact could be explained to some extent by considering the actual flow distribution.

(5) The distributions of particle concentration and particle charge were found to be nearly uniform, and the electric discharge was not observed.

**Nomenclature**

- $C$: Cunningham’s slip correction factor [-]
- $c_d$: particle mass concentration [kg/m$^3$]
- $D_p$: particle diameter [m]
- $E$: intensity of electric field [V/m]
- $F_e$: Coulomb-force on a particle [kg·m/s$^2$]
- $F_i$: image force on a particle [kg·m/s$^2$]
- $K_e$: Coulomb-force parameter [-]
- $K_i$: image-force parameter [-]
- $L$: plate length [m]
- $m_p$: particle mass [kg]
- $N_0$: particle number concentration [1/m$^3$]
- $P$: particle inertia parameter [-]
- $q$: electric charge on a particle [C]
\( q \) : electric charge on unit mass of particles \( [\text{C/kg}] \)

\( T \) : time \( [s] \)

\( \tau \) : non-dimensional time \( (= \frac{U_y T}{X_1}) \) \([-\text{}\)]

\( U_x \) : air flow velocity in \( X \) direction \( [\text{m/s}] \)

\( U_y \) : air flow velocity in \( Y \) direction \( [\text{m/s}] \)

\( U'_y \) : average air flow velocity in \( Y \) direction \( [\text{m/s}] \)

\( \bar{U}_y \) : non-dimensional air flow velocity in \( Y \) direction \([-\text{}\)]

\( \bar{v} \) : particle velocity in \( X \) direction \( [\text{m/s}] \)

\( \bar{w}_1 \) : total mass of sample \( [\text{kg}] \)

\( \bar{W}_2 \) : particle mass captured by filter \( [\text{kg}] \)

\( X \) : coordinate perpendicular to plate \( [\text{m}] \)

\( X_0 \) : initial position of particle in \( X \) coordinate \( [\text{m}] \)

\( X_1 \) : half distance of space between plates \( [\text{m}] \)

\( x \) : non-dimensional \( X \) coordinate \( (= \frac{X}{X_1}) \) \([-\text{}\)]

\( x_0 \) : initial position of particle in \( X \) coordinate (non-dimensional) \([-\text{}\)]

\( Y \) : coordinate in flow direction \( [\text{m}] \)

\( y \) : non-dimensional \( Y \) coordinate \( (= \frac{Y}{X_1}) \) \([-\text{}\)]

\( Z \) : coordinate perpendicular to \( X \) and \( Y \) directions \( [\text{m}] \)

\( Z_1 \) : half of plate width \( [\text{m}] \)

\( \varepsilon_0 \) : air permittivity \( [\text{F/m}] \)

\( \eta \) : deposition efficiency \([-\text{}\)]

\( \mu \) : air viscosity \( [\text{kg/m-s}] \)

\( \rho_p \) : particle density \( [\text{kg/m}^3] \)

\( \tau \) : average residence time \( [\text{s}] \)

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Explanation of the cover photograph

The cover photograph (by TEM) shows very fine particles of magnesium oxide (MgO) with the magnitude of 10000X. When ground down to sub-micron size, this material becomes highly active on the particle surfaces, although it is relatively stable before grinding from the chemical point of view. Brought into contact with water in the air, it is rapidly turned into magnesium hydroxide (Mg(OH)\(_2\)) with an acicular shape, as shown in the photograph. The X-ray diffraction chart on the right verifies such a reaction.