Higher orders and infrared renormalon phenomenology in deeply virtual Compton scattering.

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Abstract

We present results for higher order perturbative corrections to Compton scattering in the generalized Bjorken kinematics. The approach we have used is based on the combination of two techniques: conformal operator product expansion on the one side, and resummation of the fermion vacuum insertions with consequent restoration of the full QCD $\beta$-function via the naive non-abelianization assumption, on the other. These are terms which are lost in the former approach. Due to the presence of the infrared renormalon poles in the Borel transform of the resummed amplitude the latter suffers from ambiguities which reflect the asymptotic character of perturbation series. The residues of these IR renormalon poles give an estimate for the size of power corrections in deeply virtual Compton scattering.

Keywords: deeply virtual Compton scattering, conformal operator product expansion, renormalons, higher twists

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1 Introduction.

A measurement of deeply virtual Compton scattering (DVCS) \cite{1, 2, 3, 4} should yield important information on the leading-twist non-forward parton densities which contain new information on the strong interaction dynamics and could open a new window for the exploration of the internal structure of the nucleon. To attain this goal one should have theoretical control of higher orders of perturbation theory as well as higher-twist corrections to the amplitudes. While QCD perturbation theory is well established up to the multiloop (2,3,4-loop) level, in practice all-order results for particular quantities are inaccessible due to the essential complexity of many loop calculations. The theory of power corrections is not settled completely. Even if Operator Product Expansion (OPE) or its generalizations — factorization theorems — exist for a particular process such that the power suppressed contributions can be expressed in terms of some multiparton correlators, the latter cannot be evaluated quantitatively mainly due to lack of a feasible non-perturbative approach. Therefore, in fact already a rough order of magnitude estimate for the power corrections would be extremely valuable. Furthermore the planned experiments on DVCS will obtain primarily data at rather limited $Q^2$ (this is especially true for CEBAF experiments). Thus there is legitimate concern that large higher-twist corrections might completely obscure the interpretation of such experiments at least for some kinematic regions.

Recently, it has been realized that both of the above issues can be addressed by the study of the perturbative corrections in the large-$N_f$ limit of QCD \cite{6, 7} to the leading twist contributions. First of all, these calculations can be performed exactly due to the relatively easy algebra. Second, when supplemented by some assumptions, to be discussed below, it gives reasonable results when compared to exact perturbative quantities available. Last but not the least, by studying the ambiguities of the QCD perturbation series one can get some insight into the size of power suppressed corrections by the simple reason that only the sum of large orders of perturbation theory and higher twist contributions is free from ambiguities and thus physically relevant.

One remark should be added to all that has been said above. Since the resummation of the vacuum insertions roughly corresponds to taking into account only effects related to the running of the coupling, all other radiative corrections are discarded at the same time. This turns out sometimes to be an unreliable approximation. On the other hand, by limiting oneself to the case when the $\beta$-functions is zero, the so called hypothetical conformal limit, one has the advantage of conformal covariance of the theory and can make use of conformal operator product expansion.

\footnote{The naming “densities” which is used by us sometimes in the paper is not completely correct since it implies a probabilistic interpretation for the corresponding entries, however, the latter is lost for the non-forward functions. Therefore, it should be understood only as a synonym.}

\footnote{The proofs of factorization for DVCS and diffractive meson production were given in Refs. \cite{3, 5}.}

\footnote{Of course, even for $\beta = 0$ conformal invariance is broken by the renormalization of the field operators, however,}
that allows to get strong restrictions on the form of the off-forward part of the massless amplitudes in higher orders of QCD perturbation theory. Therefore, below, when studying the higher order perturbative corrections we will combine both approaches, i.e., renormalon chain resummation and conformal OPE, which will amplify each other.

The main objects of the present investigation are structure functions similar to those measured in deep inelastic scattering (DIS) but generalized to non-forward kinematics. They appear as the coefficients in the decomposition of the correlation function of two electromagnetic currents into independent Lorentz tensor structures which individually respect gauge invariance and are free of fictitious kinematical singularities

\[
T_{\mu\nu}(\omega, \zeta, Q^2) = i \int d^4z e^{iqz} \langle h' | T \{ J_\mu(0) J_\nu(z) \} | h \rangle = \left(-g_{\mu\nu} + \frac{q_\mu Q_\nu}{Q^2} \right) F^V(\omega, \zeta, Q^2) + \left(p_\mu - \frac{Q_\mu}{Q^2} q_\nu \right) \left(p_\nu - \frac{Q_\nu}{Q^2} q_\mu \right) \tilde{F}^V(\omega, \zeta, Q^2) + \frac{i}{Q^2} \epsilon_{\mu\nu\rho\sigma} F^A(\omega, \zeta, Q^2). \tag{1}
\]

The form factors $F^\Gamma(\omega, \zeta)$ introduced above differ only by an overall constant from the ones considered by us in Ref. [9], namely $F^\Gamma = -\frac{1}{2} F^\Gamma$. The latter can be written in leading twist approximation as a convolution of the non-perturbative non-forward distributions and the perturbatively calculable coefficient functions

\[
F^\Gamma(\omega, \zeta) = \int dx \sum_{i=Q,G} i T^\Gamma(\omega, x, \zeta, Q^2 | \alpha_s) i O^\Gamma(x, \zeta). \tag{2}
\]

The former are defined as light-cone Fourier transformations of non-local string operators sandwiched between appropriate hadronic states $i O^\Gamma(\lambda, \mu)$:

\[
i O^\Gamma(\lambda, \mu) = \langle h' | \phi^*_i(\mu n) \Phi [\mu n, \lambda n] \phi_i(\lambda n) | h \rangle \tag{3}
\]

\[
= \int dx e^{i\mu(x-\zeta)-i\lambda x} \left\{ O^\Gamma(x, \zeta) \theta(x) \theta(1-x) - i O^\Gamma(\zeta - x, \zeta) \theta(\zeta - x) \theta(x + 1 - \zeta) \right\}.
\]

The limits of integration on the right hand side of Eq. (3) can be deduced by studying the support properties of the function introduced above with Jaffe’s approach [10] (see also [11] for a recent discussion) which results in $-1 + \zeta \leq x \leq 1$ [3]. Here $i = Q, G$ runs over the parton species, $\Gamma$ corresponds to different Dirac or Lorentz structures, depending on the spin of the constituents involved and $\Phi$ is a path ordered exponential. The parton momentum fractions $x$ and $x - \zeta$ are the Fourier conjugated variables of the light-cone positions $\lambda$ and $\mu$. The parameter $\zeta$ is one can redefine the scale dimensions of the fields [8] and embed them into the original conformal representation so that conformal covariance is preserved.

5 In the original derivation the author of Ref. [3] has used the perturbative analyticity approach for studying support properties of (multi)parton distributions [12] (see also [13]).
the skewedness of the distribution defined as a $+\$-component of the $t$-channel momentum. The perturbative expansion of the coefficient function is given by

$$iT'(\omega, x, Q^2|\alpha_s) = iT_{(0)}(\omega, x) + iT_{(1)}(\omega, x, Q^2|\alpha_s) + O(\alpha_s^2),$$

with the leading order (LO) hand-bag contribution $iT_{(0)}(\omega, x) = \omega/(x\omega - 1) \pm (x \to \zeta - x)$ with “+”-sign corresponding to unpolarized scattering and “−” to the spin-dependent case (and $iG_{(0)}(\omega, x) = 0$). We have followed above the conventions introduced by Radyushkin for DVCS [3], namely, $\omega = -2(pQ)/Q^2$, and $\Delta_+ = q_+ - Q_+ = \zeta$, with $Q (p)$ and $q (p')$ being the incoming and outgoing momenta of the photon (proton). They are related to those used by us in Ref. [4], (which are closely connected to the variables adopted by the authors [1, 4]) according to

$$\bar{\omega} \equiv -(\bar{P}\bar{Q})/\bar{Q}^2 = \omega(2 - \zeta)/(2 - \omega\zeta), \quad \eta = \zeta/(2 - \zeta), \quad t = (2x - \zeta)/(2 - \zeta),$$

where the averaged momenta are introduced as follows $\bar{P} = p + p'$, $\bar{Q} = \frac{1}{2}(Q + q)$ and $\Delta_+ = \eta\bar{P}_+$. An advantage of the first conventions is that the variable $x$ acquires a simple partonic interpretation as the momentum fraction of the incoming constituent, while the variable $t$ cannot be interpreted in this way. However, its range does not depend on the longitudinal asymmetry parameter $\eta$, contrary to $x$.

Considerable theoretical efforts have been undertaken recently to explore the properties of DVCS in leading order: The evolution kernels which govern logarithmic scaling violation have been evaluated in Refs. [15, 16, 2, 17, 18, 19, 9]. The solution of the renormalization group equation for the non-forward distributions given above was found in [3, 14], while explicit numerical studies were performed in [17, 20]. However, the values of $Q^2$ for which the handbag approximation can be trusted and gives satisfactory results need not be the same as in for usual forward DIS [21]. Therefore, this forces us to study the effect of higher twists and large order perturbative corrections to the amplitudes.

The first step for any analysis beyond LO is the evaluation of the NLO coefficient functions. This issue was addressed first in [22] for the spin averaged singlet channel. In a previous paper [3] we evaluated all of them (recently these results were confirmed, see Ref. [23]) by taking advantage of conformal OPE for the process in question. The way they were obtained will be discussed in great detail in the next section.

Below we outline the ideas of the second approach which allows to restore the effect of non-vanishing $\beta$-function and repeat some assumptions which form the basis of this method. Its main idea is that the radiative corrections related to the evolution of the coupling constant represent the main source of large effects. In abelian theory these can be determined by resummation of any number of fermion vacuum polarization insertions in the gluon lines. In QCD this is no

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In the following consideration we will repeatedly omit the superscript $Q$ in the LO amplitude.
longer true, since there is a number of other diagrams which contribute to the evolution of the coupling. However, this statement can be justified in the limit of an infinite number of fermion flavours $N_f \to \infty$. For moderate $N_f$ it is clear that fermion bubbles do not produce sizable effects. Therefore, in a second step, which has no strong theoretical foundation, we add the sub-leading corrections in $N_f$ and restore the full QCD $\beta$-function by hand from the corresponding $N_f$-dependent coefficient $[24]$:

$$\beta_0 = \frac{2}{3} N_f \rightarrow \frac{2}{3} N_f - \frac{11}{3} N_c,$$

which is the first term in the perturbative expansion of the QCD $\beta$-function

$$\beta_\epsilon = \mu \frac{\partial g}{\partial \mu} = -\epsilon g + \beta(g), \quad \text{with} \quad \frac{\beta(g)}{g} = \frac{\alpha_s}{4\pi} \beta_0 + \left(\frac{\alpha_s}{4\pi}\right)^2 \beta_1 + \ldots.$$  

Thus as a first approximation one can restrict oneself to the mere summation of the vacuum bubble chains substituted into the NLO Feynman graphs. As we have noted at the very beginning in the large-$N_f$ limit the Borel transformed perturbation series for the coefficient function $T(\tau) = T_n/n!(-\tau/\beta_0)^n$ can be obtained in closed form. From this the $n$-th order Wilson coefficient $T_n$ can be obtained by taking the $n$-th derivative ($T_n = (\beta_0/\alpha_s)^{2n}/d\tau^n|_{\tau=0} T(\tau)$). This technique gives good results for the quantities which are dominated by renormalons, although this fact can be traced only a posteriori by comparing the quantity in question with its exact value derived by some other technique. The error $\Delta T$ due to the asymptotic character of the perturbation series for Wilson coefficients can also be estimated from this Borel transformed series, i.e:

$$\Delta T = \pm \frac{1}{\pi} \text{Im} \int_0^\infty d\tau \exp\left(\frac{4\pi}{\beta_0 \alpha_s} \tau\right) T(\tau).$$  

In the non-singlet case and in $d = 4$ dimensions $T(\tau)$ results in a pole structure

$$T(\tau) = \frac{1}{(Q^2)^\tau} \left\{ \frac{R_0(\tau)}{\tau} + \frac{R_1(\tau)}{1 - \tau} + \frac{R_2(\tau)}{2 - \tau} \right\},$$

corresponding to power corrections of the form $1/Q^2, 1/Q^4$ resulting from the $1/(1 - \tau)$ and $1/(2 - \tau)$ infrared renormalon poles, respectively. Hence $R_1(1)/Q^2$ and $R_2(2)/Q^4$ yield the renormalon contributions to the off forward structure functions $\mathcal{F}^\tau$. Of course, they are completely unphysical and should cancel against UV renormalon ambiguities of corresponding twist-4 and twist-6 entries due to the mixing with low-twist operators. But if one assumes that the “genuine” non-perturbative contribution (which is essentially related to the specific hadron) to the latter is small it may be used as a first approximation to the magnitude of the non-leading twist corrections. This hypothesis is referred to as “ultraviolet dominance”. In some sense this is the opposite assumption as the one made in the QCD sum rules approach where one takes only the phenomenological condensates into account which are accepted to be saturated by non-perturbative
phenomena. Power divergences due to renormalons are completely disregarded. Thus at finite \( \tau \) we can probe the non-perturbative effects seen as IR renormalons. This approach to power suppressed contributions cannot be claimed to be rigorous since it does not become exact in any limit of QCD, but rather as a sophisticated guess checked in a number of cases. In principle, the \( 1/N_f \) approximation kills the asymptotic freedom of QCD and thus is inadequate to describe the real strong interaction dynamics. However, it is by now established empirically that it allows to define the position of renormalons provided one replaces the QED \( \beta \)-function by real \( \beta_{QCD} \) (it gives reliable prediction for the power \( \tau \) in \( 1/Q^\tau \) corrections). Of course, this consideration does not introduce any new information for the processes where OPE is well established and the \( 1/Q^\tau \)-behaviour is known. But although OPE predictions being available for about two decades for DIS they have been rarely used in practice due to the absence of any systematic approaches for the study of non-perturbative higher-dimensional operators. In the absence of any adequate approach to higher twist phenomena it is worthwhile to take advantage of the renormalon-based method and try to fix the dependence of the power corrections on the momentum fraction variables up to some numbers that fix the overall normalization. When naively applied for various QCD observables the absolute magnitudes \( R(1) \) were not unreasonable. All of this suggests that it makes sense to apply the renormalon estimate to our problem. We do know at least that for \( \zeta \to 0 \) the imaginary part of our amplitude is related to the usual structure functions for which the renormalon analyses gave sensible results.

According to the hypothesis of UV-dominance the shape and magnitude of the higher twist corrections is determined by the intrinsic ambiguity of the summation of the perturbative series and looks like

\[
F_{\Gamma}(\omega, \zeta) = \int dx \left\{ T_{\Gamma}(\omega, x, \zeta, Q^2|\alpha_s) + \theta_1 \frac{\Lambda^2}{Q^2}\Delta_{tw-4}(x, \omega, \zeta) + \theta_2 \frac{\Lambda^4}{Q^4}\Delta_{tw-6}(x, \omega, \zeta) \right\} \mathcal{O}_{\Gamma}(x, \zeta),
\]

where \( \Lambda^2 = \mu^2 e^C \) (see below) and \( \theta_i \) are adjustable parameters which have to be fitted to data. Of course, the power dependence in \( Q^2 \) is defined modulo logarithms of the hard scale \( Q \) which are governed by the renormalization group equation. Unfortunately, for multiparticle operators the evolution equations are extremely complicated since the rang of the anomalous dimension matrix grows with the moment of the correlator involved due to an increase of the number of local operators mixed by renormalization. Therefore, the exact evolution equation are of the Faddeev-type rather then of DGLAP form. The solution of the problem at least for the non-singlet twist-3 sector can be found by considering the integrated quantities which depend only on one

\footnote{It is well known that due to the higher dimensionality of the quantities involved even the state-of-the-art QCD sum rules cannot claim an accuracy better than \( 50\%-100\% \), although sometimes people quote smaller errors what is of course not legitimate. Moreover, only few lowest moments can be studied. Lattice QCD cannot help in this case for the time being due to e.g. unsolved questions of operator mixing on the lattice.}
argument, and going to the multicolour limit. In the present case the situation is complicated by
the exclusiveness of the kinematics which results in a mixing of the operators with total derivatives
so that even on leading twist level the anomalous dimensions turn out to be matrices not numbers
(to say nothing about higher twists).

In the present paper we present, apart from results on higher order corrections derived from the
application of conformal OPE, the theoretical predictions for all-orders coefficient functions within
the NNA approximation and a model for the momentum fraction dependence of the higher twist
corrections coming from the assumption of the UV dominance of the non-leading twist matrix
elements. A complete and thorough numerical analysis will be presented elsewhere together with
the implication of different models for the non-forward distribution functions. Since the relative
contributions of the non-perturbative and perturbative corrections we have calculated will depend
on the functional forms assumed so will the final results. For the time being the status of such
models is still very unsatisfactory.

2 Beyond leading order.

It is well known since for a long time that the conformal invariance of the theory puts severe
restrictions on the possible form of the amplitudes. In massless field theory this is true only
at tree level, but fails when the interaction is switched on due to renormalization effects (we
discard for the moment the effect of gauge fixing in gauge theories since it is not of relevance
for the physical sector). The latter can be divided in two classes: i) renormalization of the field
operators and ii) running of the coupling constant. While the first one is not too dangerous
since after the redefinition of the conformal representations by shifting the scale dimensions of the
fields, given originally in terms of the canonical dimensions, by the anomalous ones, the theory
respect conformal covariance. However, the second effect inevitably breaks conformal symmetry.
Therefore, supposing the existence of nontrivial zero $g^*$ of the $\beta$-function ($\beta(g^*) = 0$) conformaly
covariant OPE can be proven to exist even for the interacting theory. Below we will shortly outline
some of the points which are of relevance for our further discussion.

2.1 Conformal OPE and non-forward processes.

There exists a complete basis of twist-2 conformal operators $O$, which are labeled by the conformal
spin $j$ and the scale dimension $d_j = d_1 + d_2 + j$, constructed from products of the fields of dimension
d_i and spin $s_i$ ($\nu_i = d_i + s_i - \frac{1}{2}$):

$$O_{\mu_1 \ldots \mu_l j} = \mathcal{S} \prod_{\mu_1 \ldots \mu_l} i \partial_{\mu_1} \ldots i \partial_{\mu_l} \phi_{d_1} \sum_{n=0}^j C_n(\nu_1, \nu_2) i \partial_{\nu_1} \ldots i \partial_{\nu_j-n} i \partial_{\mu_{j-n+1}} \ldots i \partial_{\mu_j} \phi_{d_2} - \text{traces},$$  (9)
with \( \partial = \partial_+ + \partial_- \) and \( \partial_+ = \partial_+ - \partial_- \) and coefficients \( C_n(\nu_1, \nu_2) \). Contracting this expression with light-like vectors \( n_\mu \) (its dual \( n^* = p \) is a null vector\(^8\) along the opposite tangent to the light cone, defined such that \( p^2 = 0, np = 0 \) (\( \mathcal{O}_{jl} = \mathcal{O}_{+++} \)) we obtain immediately the well know expression for the conformal operators

\[
\mathcal{O}_{jl} = (i\partial_+) l \phi_{d_1} P_{j}^{\nu_1-\frac{1}{2}, \nu_2-\frac{1}{2}} \left( \frac{\nu_-}{\partial_- / \partial_+} \right) \phi_{d_2},
\]

where \( P \) are the usual Jacobi polynomials \(^{26, 28}\).

From dimensional counting alone we can easily write for the \( T \)-product of two local (scalar) currents \( J_A \) and \( J_B \)

\[
T \{ J_A(0) J_B(x) \} = \sum_{j=0}^{\infty} \frac{1}{2} \left( d_A + d_B - d_j + j \right) D_{jl} x_{\mu_1} \cdots \mu_{d_j} \mathcal{O}_{\mu_1 \cdots \mu_{d_j} j}(0).
\]

Since the operators \( \mathcal{O} \) transform covariantly with respect to the algebra of the conformal group there exists a relation between the coefficients. This relation can be found by applying, for instance, the generator of the special conformal transformation to both sides of Eq. (11). This leads to a recurrence relation with the following solution:

\[
C_{jl} = (-1)^{l-j} \frac{\Gamma(d_j + j) \Gamma \left( \frac{1}{2} (d_A + d_B - d_j + j + 2l) \right)}{\Gamma(l - j + 1) \Gamma(d_j + l) \Gamma \left( \frac{1}{2} (d_A - d_B + d_j - j) \right)} C_{jj}.
\]

Performing the sum over \( l \) we get finally

\[
T \{ J_A(0) J_B(x) \} = \sum_{j=0}^{\infty} C_{jj} \left( \frac{1}{2} \left( d_A + d_B - d_j + j \right) \right) x_{\mu_1} \cdots \mu_{d_j} \mathcal{O}_{\mu_1 \cdots \mu_{d_j} j}(0).
\]

Thus the advantage of the conformal OPE for the non-forward processes is that the Wilson coefficients are fixed entirely by symmetry up to \( C_{jj} \)-coefficients which can be fixed from the forward matrix elements known, for instance, from DIS.

For the case of two electromagnetic currents in order to avoid complications which arise due to the fact that the latter carry Lorentz indices it is enough to consider the trace and the antisymmetric part of the amplitude. In this way the off-diagonal part (in the conformal basis) of the amplitude is fixed unambiguously. (Remark: The difference between the non-diagonal analogues (\( F_V \) and \( \tilde{F}_V \)) of the forward structure functions \( F_1 \) and \( F_2 \) comes entirely from the forward Wilson coefficient function of DIS since as long as the former carry the same operator content the corrections to

\(^8\)In what follows the plus and minus indices in the place of the Lorentz indices refer to a convolution with the vectors \( n \) and \( n^* \), respectively.
eigenfunctions are the same in both cases). Thus, we have to leading twist accuracy

$$\mathcal{P}_{\mu\nu} J_\mu(0) J_\nu(x) = \sum_{j=0}^{\infty} C_{jj}^{\mu\nu} \left( \frac{1}{x^2} \right)^{\frac{1}{2}(2d_j-d_j+j+1)} x_\mu x_{\mu_1} \ldots x_{\mu_j} 2F_1 \left( \frac{1}{2}(d_j + j + 1), \frac{1}{2}(d_j + j + 1) \right| x_\sigma \partial_\sigma \right) \mathcal{O}_\mu^R_{(0)} \ldots \mathcal{O}_\nu^R_{(0)},$$

(14)

where $\Gamma$ labels the polarization of the amplitude in question, and $1F_1$ is a confluent hypergeometric function \cite{27,28}. The projectors are defined as follows $\mathcal{P}_{\mu\nu}^V = g_{\mu\nu}$ for the spin averaged case and $\mathcal{P}_{\mu\nu}^A = i\epsilon_{\mu\nu+\ldots}$ for the spin dependent one.

In the interacting theory the conformal operators will mix under renormalization in MS-type schemes so that covariance will be lost. Since the mixing matrix is triangular due to Lorentz invariance (only the operators with the same Lorentz spin can mix with each other) its eigenvalues are given by the diagonal matrix elements which coincide with the anomalous dimensions known for forward scattering $\gamma_j^\Gamma$. Thus, the renormalization group equation for the latter must be diagonalized first. This can be done by a finite transformation expressing conformal operators $\mathcal{O}_{jl}$ of the free theory through the multiplicatively renormalizable ones denoted by $\tilde{\mathcal{O}}_{jl}$:

$$\mathcal{O}_{jl} = \sum_{k=0}^{j} B_{jk} \tilde{\mathcal{O}}_{jl},$$

(15)

with the transformation matrix $\hat{B}$. It is expressed in terms of the off-diagonal $\hat{\gamma}^{ND}$ matrix elements of the anomalous dimension matrix $\hat{\gamma}$ of the conformal operators $\mathcal{O}_{jk}$. On the other hand $\hat{\gamma}^{ND}$ can be fixed entirely from the constraints coming from the algebra of dilatation and conformal generators, namely \cite{24,30} $[\hat{a}(l, \frac{3}{2}) + \hat{c}(l), \hat{\gamma}] = 0$ for $\beta = 0$ (all entries will be specified below). As was shown in Ref. \cite{30} this holds true in every order of perturbation theory for $\beta = 0$. Therefore, in the interacting theory conformal OPE is of the same form as Eq. (14) provided we replace the canonical scale dimensions $d_{j}^{\text{can}}$ of the operators by $d_{j} = d_{j}^{\text{can}} + 2i\gamma_{j}^\Gamma$ and rotate the tree level conformal operators to the covariant ones.

Taking into account all that has been said above we sandwich Eq. (14) between hadronic states and get after some simple algebra the following result for the Fourier transformed amplitude

$$\mathcal{F}^\Gamma(\omega, \zeta) = \sum_{i} \sum_{j=0}^{\infty} i C_{j}^{\mu\nu}(\alpha_s, Q^2/\mu^2) \omega^{j+1} 2F_1 \left( 1 + j + i\gamma_{j}^\Gamma, 2 + j + i\gamma_{j}^\Gamma \right| 4 + 2j + 2i\gamma_{j}^\Gamma \right) \omega^{j+1} \omega^{j+1} 2F_1 \left( 1 + j + i\gamma_{j}^\Gamma, 2 + j + i\gamma_{j}^\Gamma \right| 4 + 2j + 2i\gamma_{j}^\Gamma \right) \langle h|\tilde{\mathcal{O}}_{jj}^R|h \rangle,$$

(16)

where $2F_1$ is the hypergeometric function \cite{27,28}, and $iC_{j}^{\mu\nu}$ is the Wilson coefficient for forward scattering.

Since the product of the coefficient function and the eigenfunctions of the evolution kernels is a scheme independent quantity we can use any scheme we want. However, for other schemes the manifestly conformal covariant form of the OPE (14) will be hidden.
2.2 NLO corrections.

The main idea of the conformal approach for the evaluation of the coefficient functions consists in the combination of the information coming from conformal OPE and from ordinary factorization theorems. On the one hand we have the prediction for the amplitude from conformal OPE outlined in the previous section with an input gained from the forward scattering coefficient functions and anomalous dimensions. On the other hand NLO factorization tells us that the total $\alpha_s$-correction is generated by two sources: i) $\alpha_s$-corrections to the coefficient function and ii) $\alpha_s^2$-corrections to the evolution kernels which lead to a modification of the eigenfunctions of the evolution equation of order $\mathcal{O}(\alpha_s)$. Thus, since the latter can be fixed from conformal constraints we can get information about the former from the combined use of the first and second representation of the amplitude. Schematically, it reads

$$iT = iT_{\text{COPE}} - iT_{\text{kernel}},$$

(17)

where the conventions we have used are self-explanative. In this section we will only deal with the $s$-channel contribution. The crossed amplitude can be obtained at the end by a mere substitution, namely $x \rightarrow \zeta - x$.

The general conformal decomposition of the non-forward distributions looks like

$$i\mathcal{O}(x, \zeta) = \sum_{k=Q,G} \sum_{j=0}^{\infty} i^{k} \phi_j(x, \zeta | \alpha_s) \langle h' | k \mathcal{O}_{jj} | h \rangle,$$

(18)

where the partial conformal waves are generalized, beyond one-loop level, to non-polynomial functions which are the subject of the constraints.

The correction to an eigenfunction is defined completely in terms of the $\hat{B}$-matrix

$$\phi_j = \sum_{k=Q,G} \frac{x^k \bar{x}}{N_{k\nu} \langle \nu \rangle} C_k^2 (2x - 1) B_{kj}, \quad N_j(\nu) = 2^{-\nu+1} \frac{\Gamma^2(\nu) \Gamma(2\nu+j)}{(\nu+j)\Gamma^2(\nu)\Gamma(j+1)},$$

(19)

which in the conformal limit is defined completely in terms of the special conformal symmetry breaking matrix $\hat{\gamma}^c$ via the following relation

$$\hat{B} = \frac{1}{1 + \mathcal{J} \hat{\gamma}^c}, \quad \mathcal{J} \hat{M} = \frac{M_{jk}}{a(j, k, 3/2)} \theta_{j,k+1}, \quad \text{for } \forall \hat{M} - \text{matrix},$$

(20)

where $a(j, k, \nu) = 2[(j + 1)(j + 2) - (k + 1)(k + 2) + (2\nu - 3)(j - k)]$. To obtain $\phi_j$ in the simplest way one should derive the differential equation for the latter by using the eigenvalue equation for the Gegenbauer polynomials as well as the identity $a(j, k, 3/2) B_{jk} = -\{\hat{\gamma}^c \hat{B}\}_{jk}$ which follows from the definition of the $\hat{B}$-matrix.
For the singlet quark case, which is relevant for our present analysis, they have the form
\[ Q^k \delta_j(x, \zeta | \alpha_s) = \left\{ \delta_{Qk} \delta(x - y) + \frac{\alpha_s}{2\pi} Q^k \Phi(x, y, \zeta) \right\} \otimes \left[ \frac{1}{N_j(\nu)} \frac{1}{\zeta^{j+1}} \left[ \frac{y}{\zeta} \left( 1 - \frac{y}{\zeta} \right) \right]^{\nu - \frac{1}{2}} C_{j}^{\nu} \left( 2\frac{y}{\zeta} - 1 \right), \right\} \]
where \( \nu = \nu(k) \), \( k \) sums up the parton species \((Q, G)\), and we introduced the shorthand notation \( \otimes \equiv \int dy \).

To fix the normalization of the amplitude coming from conformal OPE we insert the conformal partial waves expansion into the LO result \(^{(2)}\) which followed from factorization and get as prediction for the DVCS function:
\[ F_{LO}(\omega, \zeta) = -2 \sum_{j=0}^{\infty} B(j + 1, j + 2) \omega^{j+1} _{\frac{1}{4} + 2j} F_1 \left( \frac{1 + j}{4 + 2j} \right) \langle h|QO|_{\nu j}|h \rangle. \]

Here the conformal matrix elements of the non-forward distributions are defined in terms of the conformal operators sandwiched between the physical states in question
\[ \left\{ \begin{array}{c}
QO^V \\ QO^A
\end{array} \right\}_{jl} = \bar{\psi} (i\partial_+) \gamma^\mu \left\{ \begin{array}{c}
\gamma_+ \\ \gamma_+ \gamma_5
\end{array} \right\} C_{j}^{\frac{5}{2}} \left( \frac{\partial_+}{\partial_+} \right) \psi. \]

The gluonic case differs by the replacements \( \frac{5}{2} \rightarrow \frac{7}{2} \) and \((j, l) \rightarrow (j, l) - 1\), and \( \Gamma \) stands now for the Lorentz rather than Dirac structures:
\[ \left\{ \begin{array}{c}
QG^V \\ QG^A
\end{array} \right\}_{jl} = G_\mu (i\partial_+)^{-1} \left\{ \begin{array}{c}
g_{\mu\nu} \\ i\epsilon_{\mu\nu-+}
\end{array} \right\} C_{j-1}^{\frac{7}{2}} \left( \frac{\partial_+}{\partial_+} \right) G_{\nu+}. \]

### 2.2.1 Polarized sector: gluons.

As we have seen above the correction to the eigenfunction \( Q^G \Phi \) can be found with the help of the special conformal anomaly matrix in the quark-gluon channel. For the \( QG \)-sector the derivation of the conformal Ward identities can be performed in abelian gauge theory following the same reasoning as for the non-singlet \( QQ \)-sector. This is true since in the conformal limit, the symmetry breaking parts of the kernels do not contain the Casimir operator \( C_A \) of the adjoint representation of \( SU(3) \). Moreover, for non-vanishing \( \beta \) the correct results can be reconstructed by substituting the QCD \( \beta \)-function for the QED one and, therefore, this treatment is sufficient to derive reliable NLO predictions in QCD. The special conformal anomaly matrix in LO looks like
\[ QG^{\tilde{\gamma}} = \frac{\alpha_s}{2\pi} [QG^{\tilde{\gamma}}, \tilde{b}(l)]_+ + QG^{\tilde{Z}}_1, \]

with \( b_{jk}(l) = \theta_{jk} \left\{ 2(l + k + 3) \delta_{jk} - [1 - (1)^{-j-k}][2k + 3] \right\} \) and \( QG^{\tilde{\gamma}} \) being the quark-gluon mixing anomalous dimension matrix. Thus, the only difference compared to the analysis of Ref. \(^{[29]}\)
Figure 1: One-loop Feynman diagrams for the $QG$ special conformal anomaly matrix. The empty vertex corresponds to the modified Feynman rules according to Eq. (26) while the full point denotes the usual quark-gluon vertex. The blob with cross stands for the quark conformal operator which mixes with the gluonic one due to renormalization.

is that now we have to evaluate the mixing renormalization matrix $QG\hat{Z}^* = \frac{1}{\epsilon}QG\hat{Z}^*[1] + \ldots$ of the conformal operator and the operator insertion

$$\Delta^g = g\frac{\partial}{\partial g} \int d^d x 2 x - L(x),$$

(26)

coming from the variation of the action with respect to the special conformal transformation $\delta_C S = -\frac{\Delta^g}{g}[\Delta^g] + \ldots$. This is the only source of symmetry breaking, in contrast to the situation in the $QQ$-channel where subtleties arise due to the quark equation of motion. Thus, only the renormalization problem of the above mentioned operator has to be solved\[10\]

$$i[Q\mathcal{O}_{jl}[\Delta^g] = i[Q\mathcal{O}_{jl}\Delta^g] + i \sum_{k=1}^j \{Q^Q\hat{Z}^*[k\mathcal{O}_{kl-1}] + i \sum_{k=1}^j \{QG\hat{Z}^*[j\mathcal{O}_{kl-1}] + GVC,$$

(27)

where GVC stands for the gauge-variant counterterms \[31\], which are not of relevance for the present discussion since they cannot affect the gauge invariant quantities we are interested in.

To lowest order in the coupling constant we have to calculate the diagram in Fig. 1 with one of the vertices being replaced by the operator insertion $i[\Delta^g]$. To $O(\alpha_s)$-accuracy we are limited to, it reduces to $i[\Delta^g] = ig\mu^e \int d^d x 2 x - \bar{\psi} \Delta \psi$ and thus in the Feynman rules it results in a mere differentiation with respect to the external gluon momenta of the last graph in Fig. 1 but with the familiar quark-gluon vertices. Since the differentiation of the Gegenbauer polynomial in momentum space is proportional to the $\hat{b}$-matrix acting on the latter

$$\left(\frac{\partial}{\partial p_+} + \frac{\partial}{\partial p'_+}\right)^l (p_+ + p'_+)C^\nu_j \left(\frac{p_+ - p'_+}{p_+ + p'_+}\right) = \sum_{k=0}^l b_{jk}(l)(p_+ + p'_+)^{l-1}C^\nu_j \left(\frac{p_+ - p'_+}{p_+ + p'_+}\right),$$

(28)

\[10\]Square brackets mean minimally (MS) subtracted operators \[31\].
we finally get
\[ QG\hat{Z}[l] = -\frac{\alpha_s}{2\pi} QG\hat{\gamma} \hat{b} (l). \] (29)

Thus, the special conformal anomaly can be obtained immediately and reads:
\[ QG\hat{\gamma}^c = -\frac{\alpha_s}{2\pi} \hat{b} (l) QG\hat{\gamma}. \] (30)

Following the discussion preceding this section we find from Eq. (30) that the corrections to
the eigenfunctions are completely expressed in terms of the shift operator:
\[ QG\Phi(x, y, \zeta) = (\mathcal{I} - \mathcal{D}) S(x, z) \otimes QGK^A(z, y, \zeta), \] (31)
where the generalized evolution kernel reads
\[ QGK^A(x, x', \zeta) = 2N_f T_F \Theta^{11}_{12}(x - \zeta, x - x'), \] (32)
and \( S \) generates the shift of the Gegenbauer polynomials index:
\[ S(x, y) \otimes [y\bar{y}]^{\nu - \frac{1}{2}} C^\nu_j (2y - 1) = \frac{d}{d\rho} \bigg|_{\rho=0} [x\bar{x}]^{\nu - \frac{1}{2} + \rho} C^\nu_j (2x - 1). \] (33)

In Eq. (31) \( \mathcal{I} \) is an identity operator and \( \mathcal{D} \) extracts the diagonal part of any test function \( \tau(x, y) \)
in its expansion with respect to a basis of Gegenbauer polynomials \( C^\nu_j \), i.e.
\[ \int_0^1 dx C^\nu_j (2x - 1) \left\{ \begin{array}{l} \mathcal{I} \\ \mathcal{D} \end{array} \right\} \tau(x, y) = \sum_{k=0}^j \tau_{jk} \left\{ \begin{array}{l} 1 \\ \delta_{jk} \end{array} \right\} C^\nu_k (2y - 1). \]

The diagonal matrix elements of the shift operator \( S \) which will be used below are for instance
\[ S_{jj} = 3\psi(2+j) - 2\psi(4+2j) - \psi(1) - \frac{2+2j}{2+j}. \] (34)

Here and below \( \psi^{(n)}(j) = \frac{d^{n+1}}{dx^{n+1}} \ln \Gamma(j) \) denotes polygamma functions. The evolution kernel introduced in Eq. (32) can be diagonalized with the following identity
\[ \zeta \int dy C^\frac{3}{2}_j \left( \frac{2y}{\zeta} - 1 \right) QGK^A(y, x, \zeta) = QG\gamma^A_{j-1} C^\frac{5}{2}_{j-1} \left( \frac{2x}{\zeta} - 1 \right) \text{ with } QG\gamma^A_j = -\frac{12 N_f T_F}{(j+1)(j+2)}. \] (35)

Technically, to get this equality one has to multiply both sides by the factor \((y\bar{y})^2\), differentiate three times with respect to \( y \) and use the relation
\[ [(y\bar{y})^2 C^{5/2}_{j-1} (2y - 1)]'' = \frac{\Gamma(j+4)}{6\Gamma(j)} C^{3/2}_j (2y - 1). \] (36)

\footnote{In this and subsequent formulae we followed our previous convention and introduced the generalized step functions \( \Theta_{i_1, i_2, ..., i_n}^{m}(x_1, x_2, ..., x_n) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} \Theta^{m}_k \prod_{k=1}^n (\alpha x_k - 1 + i0)^{-i_k}. \) \( \Theta^{m}_k \) denotes the generalized step function.}
The coefficient \( QG \gamma_j^A \) in front of the Gegenbauer polynomial coincides with the DGLAP moments of the kernel \( QG K^A(y, x, 0) \) up to the common factor 6/j, which arises as a result of the conventional definition of the Gegenbauer polynomials, namely, the coefficient of \( x^j \) in \( C_j^k(x) \) is 3/j times the coefficient of \( x^{j-1} \) in \( C_j^{k-1}(x) \); an additional factor of 2 comes from the argument of the polynomial.

Since now we know the \( \alpha_s \)-correction to the eigenfunctions of the two-loop \( QG \)-evolution kernel we can find coefficient function in the corresponding channel. On the one hand conformal OPE (16) to NLO gives the following prediction

\[
G_{\text{NLO}} = -2 \frac{\alpha_s}{2\pi} \sum_{j=0}^{\infty} B(j + 1, j + 2) \omega^{j+1} \left\{ G^A \! \! _j \, 2F_1 \left( \begin{array}{c} 1 + j, 2 + j \\ 4 + 2j \end{array} \bigg| \omega \zeta \right) \right. \\
+ QG \gamma_j^A \frac{d}{d\rho} \bigg|_{\rho=0} 2F_1 \left( \begin{array}{c} 1 + j + \rho, 2 + j + \rho \\ 4 + 2j + 2\rho \end{array} \bigg| \omega \zeta \right) \right\} \langle h'|G^A_{jj}|h \rangle, 
\]

with the forward coefficient function

\[
G^A_j(Q^2/\mu^2) = 2N_f T_j \frac{6}{j(j + 1)(j + 2)} \left\{ \ln \frac{-Q^2}{\mu^2} - \psi(j + 1) + \psi(1) - 1 \right\} 
\]

On the other hand factorization theorems tell us that the total \( \alpha_s \)-correction to the amplitude reads

\[
G_{\text{NLO}} = \frac{\alpha_s}{2\pi} Q T_0(\omega, y) \otimes QG \Phi(x, y, \zeta) \otimes G \mathcal{O}^A(x, \zeta) + G T^A(\omega, x, \zeta, Q^2) \alpha_s. 
\]

where \( G T^A_1 \) is the quantity in question. To proceed further we equate the right hand sides of Eqs. (37) and (39) and use the formulae

\[
T_0(\omega, y) \otimes S(y, z) \otimes QG K^A(z, x, \zeta) \otimes G \mathcal{O}^A(x, \zeta) = -2 \sum_{j=0}^{\infty} QG \gamma_j^A B(j + 1, j + 2) \omega^{j+1} \right. \\
\times \left\{ S_{jj} 2F_1 \left( \begin{array}{c} 1 + j, 2 + j \\ 4 + 2j \end{array} \bigg| \omega \zeta \right) + \frac{d}{d\rho} \bigg|_{\rho=0} 2F_1 \left( \begin{array}{c} 1 + j + \rho, 2 + j + \rho \\ 4 + 2j + 2\rho \end{array} \bigg| \omega \zeta \right) \right\} \langle h'|G^A_{jj}|h \rangle, 
\]

and

\[
T_0(\omega, y) \otimes \ln(1 - \omega y) QG K^A(y, x, \zeta) \otimes G \mathcal{O}^A(x, \zeta) = -2 \sum_{j=0}^{\infty} QG \gamma_j^A B(j + 1, j + 2) \omega^{j+1} \right. \\
\times \left\{ L_{jj} 2F_1 \left( \begin{array}{c} 1 + j, 2 + j \\ 4 + 2j \end{array} \bigg| \omega \zeta \right) - \frac{d}{d\rho} \bigg|_{\rho=0} 2F_1 \left( \begin{array}{c} 1 + j + \rho, 2 + j \\ 4 + 2j \end{array} \bigg| \omega \zeta \right) \right\} \langle h'|G^A_{jj}|h \rangle. 
\]

Here it is easy to recognize the first terms in the curly brackets in Eqs. (40) and (41) as the diagonal parts of the shift operator and the logarithm \( L_{jj} = -[\psi(j + 1) - \psi(1)] \), respectively. Extracting only the non-diagonal part from these identities, and adding the equation which comes from the forward coefficient function:

\[
2 \sum_{j=0}^{\infty} G^A_j(Q^2/\mu^2) B(j + 1, j + 2) \omega^{j+1} 2F_1 \left( \begin{array}{c} 1 + j, 2 + j \\ 4 + 2j \end{array} \bigg| \omega \zeta \right) \langle h'|G^A_{jj}|h \rangle 
\]

\[
= T_0(\omega, z) \otimes \left\{ QG K^A(z, x, \zeta) \ln \frac{-Q^2}{\mu^2} + [D \ln(1 - z \omega) - 1] QG K^A(z, x, \zeta) \right\} \otimes G \mathcal{O}^A(x, \zeta), 
\]
we can restore the form of the amplitude coming from OPE. Subtrac-
ting from the OPE result the correction to the eigenfunction coming from the usual factorization approach we get the coefficient function we are interested in
\[ G_{TA}(1) = -\frac{\alpha_s}{2\pi} T_{(0)}(\omega, y) \otimes \left\{ Q^G K^A(y, x, \zeta) \ln \frac{-Q^2}{\mu^2} + [\ln (1 - y\omega) - 1] Q^G K^A(y, x, \zeta) \right\}. \]

By subtracting the logarithm of the hard scale we obtain the net gluon coefficient function in the $\overline{\text{MS}}$ scheme.

2.2.2 Polarized sector: quarks.

In the quark sector there is no need to calculate the special conformal anomaly matrix for extended kinematics since it is known for the Efremov-Radyushkin-Brodsky-Lepage (ER-BL) case \[23\]. It was shown in Ref. \[34\] that the continuation to the whole $x/\zeta, y/\zeta$-plane is unique. To leading order this results in the substitution of the ordinary $\theta$-functions in the kernels by the generalized ones \[32\], namely
\[ k(x, y) \frac{\theta(x - y)}{1 - y} \rightarrow k \left( \frac{x}{\zeta}, \frac{y}{\zeta} \right) \Theta_{11}^0 (x - y, x - \zeta), \]
with a crossed contribution obtained by the substitutions $x \rightarrow \zeta - x, y \rightarrow \zeta - y$. The correspondence between extended kernels and ER-BL ones is the following $K(x, y, \zeta = 1) = -V_{ER-BL}(x, y)$. This means that, in principle, one could get the non-forward evolution kernels from their ER-BL analogues which are known for over fifteen years rather than calculate them once more. Coordinate space results could be reconstructed with a help of the Fourier transformation \[32\].

Taking into account the effect of non-vanishing $\beta$-function results in a modification of the eigenfunction of the NLO evolution kernel by an additional term $\sim -\beta_0 (\mathcal{I} - \mathcal{D}) S$, namely
\[ QQ\Phi(x, y, \zeta) = (\mathcal{I} - \mathcal{D}) \left\{ S(x, z) \otimes \left[ Q^G K^A(z, y, \zeta) - \frac{\beta_0}{2} \delta(z - y) \right] - Q^G(y, x, \zeta) \right\}, \]
where
\[ Q^G K^A(x, x', \zeta) = C_F \left[ \frac{x}{x - x'} \Theta_{11}^0 (x, x - x') + \frac{x - \zeta}{x - x'} \Theta_{11}^0 (x - \zeta, x - x') + \Theta_{111}^0 (x, x - \zeta, x - x') \right], \]
\[ Q^G(x, x', \zeta) = C_F \left[ \frac{x'}{x - x'} \ln \left( \frac{x'}{x - \zeta} \right) \Theta_{11}^0 (x - \zeta, x - x') + \frac{x'}{x - x'} \ln \left( \frac{x'}{x - x'} \right) \Theta_{11}^0 (x, x - x') \right]. \]

From conformal OPE it follows that
\[ -2 \sum_{j=0}^{\infty} Q^A C_j^j (Q^2 / \mu^2) B(j + 1, j + 2) \omega^{j+1} 2F_1 \left( 1 + j, \frac{2 + j}{4 + 2j} \left| \omega \zeta \right| \right) \langle h' | Q^A_{33} | h \rangle \]
\[ T_0(\omega, z) \otimes \left\{ \delta(z - x) - \frac{\alpha_s}{2\pi} \left[ QQK^A(z, x, \zeta) \ln \frac{Q^2}{\mu^2} + D \ln(1 - z\omega)QQK^A(z, x, \zeta) - \frac{3}{2}QQK^b(z, x, \zeta) - DQQG(z, x, \zeta) + \frac{3}{2}\delta(z - x) \right] \right\} \otimes QO^A(x, \zeta), \]

where the forward coefficient function is taken from Ref. [35]:

\[ QC_j^A(Q^2/\mu^2) = \left\{ 1 + \frac{\alpha_s}{2\pi} C_F \left[ 2S_{1,1}(j) - 2S_2(j) + S_1(j) \left( \frac{3}{2} + \frac{1}{j + 1} + \frac{1}{j + 2} \right) + \frac{3}{j + 1} - \frac{9}{2} + \left( \frac{3}{2} - S_1(j) - S_1(j + 2) \right) \ln \frac{-Q^2}{\mu^2} \right] \right\}. \]

Here the additional kernel reads

\[ QK^b(x, x', \zeta) = C_F \left[ \frac{x - \zeta}{x - x'} \Theta_0^0(x - \zeta, x - x') + \frac{x}{x - x'} \Theta_1^0(x, x - x') \right]. \]

In the derivation we have taken into account that the diagonal matrix elements of the matrices introduced above read

\[ \frac{1}{C_F} QQ_{ji} = 2[\psi(j + 2) - \psi(1)] - 2, \]

\[ \frac{1}{C_F} QQ_{jj} = \psi(j + 1) - \zeta(2) - [\psi(j + 1) - \psi(1)]^2, \]

\[ \frac{1}{C_F} QQ_{ji} = \frac{1}{C_F} QQ_{ji} = \psi(j + 3) + \psi(j + 1) - 2\psi(1) - \frac{3}{2}. \]

To recover the same form of the result as in Eq. (49) it is useful to modify these equalities by using the identity \( 2S_{1,1}(j) = S_1^2(j) + S_2(j) \) [36], where \( S_{i,m}(j) = \sum_{k=0}^i S_m(k)/k! \), \( S_m(k) = \sum_{i=0}^k 1/i^m \), \( \psi(1) = \zeta(2) - S_2(j) \) [28], and by noting that with these conventions \( QQ_{jj} = -2C_F S_{1,1}(j) \).

Thus, the same steps as before give

\[ QT_{i(1)}^A(\omega, x, \zeta, Q^2) = -\frac{\alpha_s}{2\pi} T_{0(1)}(\omega, y) \otimes \left\{ QQK^A(y, x, \zeta) \ln \frac{-Q^2}{\mu^2} - \frac{3}{2}QQK^b(y, x, \zeta) + \frac{3}{2}\delta(y - x) + \ln (1 - y\omega)QQK^A(y, x, \zeta) - QQG(y, x, \zeta) \right\}. \]

In the limit \( \zeta = 1 \) we obtain from Eq. (54) the results of Refs. [13] for the \( \pi \to \gamma\gamma \) transition form factor \( F_{\pi\gamma\gamma} \).

### 2.2.3 Unpolarized sector: quarks and gluons.

For the quark vector channel the only additional term comes from the forward DIS coefficient function [35] and reads \( C_F/(j + 1)(j + 2) \). It is easy to check that this eigenvalue corresponds to the evolution kernel \( \Theta_{011}^0 \), namely

\[ \int dx C_j^{3/2} \left( \frac{x}{\zeta} - 1 \right) \Theta_{111}(x, x - \zeta, x - y) = \frac{1}{(j + 1)(j + 2)} C_j^{3/2} \left( \frac{y}{\zeta} - 1 \right). \]
As we have noted in the previous subsection in order to verify this and similar eigenvalue problems there is no need to perform the integration rather one should differentiate $\nu + \frac{1}{2}$-times both sides of the equations multiplied by $(y\bar{y})^{\nu - \frac{1}{2}}$ with respect to $y$ and use the eigenvalue equation for the Gegenbauer polynomials of index $\nu$. To reduce the multi-argument $\Theta$-functions to the two-argument ones it is enough to use the following relations

$$
\Theta^n_{ijk}(x_1, x_2, x_3) = \frac{1}{x_1 - x_2} \left\{ \Theta^{n-1}_{i-1jk}(x_1, x_2, x_3) - \Theta^{n-1}_{ij-1k}(x_1, x_2, x_3) \right\} 
$$

$$
= \frac{1}{x_1 - x_2} \left\{ x_2 \Theta^n_{i-1jk}(x_1, x_2, x_3) - x_1 \Theta^n_{ij-1k}(x_1, x_2, x_3) \right\}.
$$

(56)

Thus, the problem to find the off-forward analogues to some forward functions is reduced to the solution of the inverse problem of determining a potential from the known eigenvalues

$$
\int dx C_{ij}^\nu \left( 2x - 1 \right) K(x, y, \zeta) = E_j(\nu) C_{ij}^\nu \left( 2y - 1 \right),
$$

(57)

which in general turns out to be quite nontrivial. If this problem would be solved in general we would be able to reconstruct the diagonal part of the kernel in the basis of Gegenbauer polynomials from the corresponding forward analogue and thus the whole NLO corrections to the ER-BL-type evolution kernel and not only its off-diagonal part.

Observing that

$$
\Theta^0_{111}(x, x - \zeta, x - y) = QQK^V(x, y, \zeta) - Q^b(x, y, \zeta) - \frac{1}{2} \delta(x - y),
$$

and that the corrections to the $QQ$-eigenfunctions in the spin-dependent and -averaged cases are the same we easily obtain

$$
Q_{(1)} T^V_{(1)}(\omega, x, \zeta, Q^2) = -\frac{\alpha_s}{2\pi} T_{(0)}(\omega, y) \otimes \left\{ QQK^V(y, x, \zeta) \ln \frac{-Q^2}{\mu^2} - \frac{5}{2} Q^b(y, x, \zeta) + \delta(y - x) \right. 
$$

$$
+ \left[ \ln (1 - y\omega) + 1 \right] QQK^V(y, x, \zeta) - QG(y, x, \zeta) \right\},
$$

(59)

$$
G_{(1)} T^V_{(1)}(\omega, x, \zeta, Q^2) = -\frac{\alpha_s}{2\pi} T_{(0)}(\omega, y) \otimes \left\{ QG^K(y, x, \zeta) \ln \frac{-Q^2}{\mu^2} - \frac{1}{2} \left[ QQK^V(y, x, \zeta) + QG^{KA}(y, x, \zeta) \right] \right. 
$$

$$
+ \ln (1 - y\omega) QG^K(y, x, \zeta) \right\}.
$$

(60)

The calculation of the gluon coefficient function quoted here does not introduce any new specific features and its derivation runs along the same line as before. The momentum space evolution kernels $Q(QG)K^V$ involved above read

$$
QQK^V(x, x', \zeta) = QQK^A(x, x', \zeta),
$$

(61)

$$
QGK^V(x, x', \zeta) = QG^{KA}(x, x', \zeta) - 4N_f T_F (x - x') \Theta^0_{111}(x, x - \zeta, x - x').
$$

12Note a misprint in Eq. (40) of Ref. 9 where the sign in the brackets of the fourth term should be changed.
2.2.4 Remark.

To get the corrections beyond NLO one should expand the amplitude coming from conformal OPE \([16]\) up to the required \(n\)-th order. However, these results would correspond to the special scheme where the conformal covariance is preserved. To be able to make the predictions in MS-type schemes we have to perform finite renormalization with \(\hat{B}\)-matrix which has to be known to \(n\)-th order. The latter requires evaluation of the special conformal anomaly \(\gamma^c\) in the corresponding channel. For instance, to NNLO this calculation goes along the same line as developed in Ref. \([37]\) for the two-loop non-singlet exclusive ER-BL evolution kernel, provided we replace the ordinary Feynman rules, by modified ones similar to those used by us in section 2.2.1. (Technically this analysis is of the same complexity as in the paper mentioned above). However, even then, this prediction will be only valid for the special case of vanishing \(\beta\)-function \(\beta = 0\) which, of course, may turn out to be far from reality. Therefore, it is worthwhile to study the importance of conformal symmetry breaking effects introduced by the running of the QCD coupling constant. This subject will be addressed in the next section.

3 Resummation of fermion vacuum insertions.

In this section we will present results for the resummation of the fermion bubble chains in the NLO coefficient functions of the DVCS amplitude. The idea of the NNA approximation is based on the observation that the effects related to the evolution of the coupling constant can be represented as a source of potentially large perturbative corrections. Its extraction can give important information on the higher order perturbative contributions.

The resummation of the fermion loops in the NLO coefficient function leads to a factorial growth of the series

\[
T - T_{(0)} = \sum_{n=0}^{\infty} d_n n! (-\alpha_s \beta_0)^n, \tag{62}
\]

which convergence radius is zero. Therefore, due to the asymptotic character of perturbation series the naive resummation is meaningless and to get reasonable predictions we should truncate the series at the order \(n_0\) at which the ratio of two successive terms is of the order 1. The first neglected term will give an estimate for the ambiguity of this approximation. This ambiguity is known to be power suppressed. This last point, discussed at length in the Introduction, will be used below for the construction of a model for the non-leading twist corrections.
3.1 Coefficient function.

In the following we adhere to the convention $b \equiv -\beta_0$ for the first coefficient of the $\beta$-function. After resummation of the fermion vacuum polarization blobs (the sum is defined up to infinity in the sense of a principal value (PV) prescription for the poles in the Borel integral, although in practice the truncation of the series at its minimal term is completely equivalent to this) in the NLO coefficient function (see Fig. 2) and with appropriate renormalization we get as result for the coefficient function for the $\Gamma$-amplitude (the details of this calculation can be found in Appendix A)

\[
T^\Gamma(\omega, x, \zeta, Q^2|\alpha_s) = -\frac{2C_F}{b} \operatorname{PV} \int_0^\infty \frac{d\tau}{\tau} e^{-\frac{4\pi}{(\alpha_s b)\tau}} \left\{ T^\Gamma(0, \tau|\omega, x, \zeta, Q^2) - \tilde{T}^\Gamma(-\tau, 0|\omega, x, \zeta, Q^2) \right\},
\]  

where $\tilde{T}^\Gamma(\tau, 0)$ is a Borel transform of $T^\Gamma$ with respect to the first argument. Note that the second term can be expressed in terms of the integral of the function $T^\Gamma(\tau, 0)$ itself

\[
\int_0^\infty \frac{d\tau}{\tau} e^{-\frac{4\pi}{(\alpha_s b)\tau}} \tilde{T}^\Gamma(-\tau, 0) = \int_0^{\frac{\alpha_s b}{4\pi}} d\tau \frac{\alpha_s b}{\tau} T^\Gamma(\tau, 0).
\]  

A straightforward calculation leads to the following representation of the Borel amplitude

\[
T^\Gamma(\epsilon, \tau) = F(\epsilon, \tau) D^\Gamma(\epsilon, \tau),
\]

with a common factor $F$

\[
F(\epsilon, \tau) = \left( \frac{\mu^2}{-Q^2} \right)^\frac{\epsilon}{2} \left[ \frac{\Gamma(1 - \epsilon) \Gamma^2(2 - \epsilon)}{\Gamma(4 - 2\epsilon)} \right]^{\frac{\epsilon}{2} - 1} \frac{\pi \tau}{\sin(\pi \tau)} \frac{\Gamma(2 - \epsilon)}{\Gamma(1 - \epsilon + \tau) \Gamma(3 - \epsilon - \tau)}.
\]  

In what follows we omit the dependence of the Borel transform $T^\Gamma(\epsilon, \tau)$ on the momentum fractions as well as other kinematical variables.
\((\mu^2 = 4\pi\mu^2)\) and a function \(D\) which is specific for the particular channel \((\Gamma = A, V)\). As can be seen from Eq. (53) in practice we need the function \(F\) only for vanishing first or second argument. We can easily get

\[
F(0, \tau) = \left(\frac{\mu^2 e^C}{-Q^2}\right)^\tau \frac{1}{(1 - \tau)(2 - \tau)}. \tag{67}
\]

where \(C\) differs for different subtraction schemes: \(C_{\text{MS}} = \frac{5}{3} + \ln 4\pi + \psi(1)\), \(C_{\text{MS}} = \frac{5}{3}\).

\[
F(\tau, 0) = \frac{1}{6} \frac{1 - \tau}{(2 - \tau)} \frac{\Gamma(4 - 2\tau)}{\Gamma(1 + \tau)\Gamma^3(2 - \tau)}. \tag{68}
\]

The contributions of individual diagrams to the function \(D\) in the polarized and unpolarized channels are given in Appendix A. Below we just quote these expressions for those particular values of its arguments which enter Eq. (63).

### 3.1.1 Polarized case.

\[
D^A(0, \tau) = T(0)_0(\omega, x) \left\{ \frac{3}{(1 - x\omega)^\tau} - \frac{2}{1 + \tau} - \frac{(2 - \tau)(1 - x\omega)}{\zeta \omega} \right\} 2F1 \left( \frac{1 + \tau}{2 + \tau} \left| x\omega \right. \right)
\]

\[
- \frac{1}{1 - \zeta \omega} \left[ \frac{2}{1 + \tau} + (2 - \tau) \frac{1 - x\omega}{\zeta \omega} \right] 2F1 \left( \frac{1 + \tau}{2 + \tau} \left| \frac{(x - \zeta)\omega}{1 - \zeta \omega} \right. \right). \tag{69}
\]

\[
D^A(\tau, 0) = T(0)_0(\omega, x) \left\{ \frac{3 - \tau^2}{1 - \tau} - \frac{2}{1 - \tau} \left[ 1 - (1 - \tau) \frac{1 - x\omega}{\zeta \omega} \right] 2F1 \left( \frac{1}{2 - \tau} \left| \frac{x\omega}{1 - \zeta \omega} \right. \right) \right.
\]

\[
- \frac{2 - \tau}{1 - \tau} \left[ 1 + (1 - \tau) \frac{1 - x\omega}{\zeta \omega} \right] 2F1 \left( \frac{1, 1 - \tau}{2 - \tau} \left| \frac{x - \zeta \omega}{1 - \zeta \omega} \right. \right). \tag{70}
\]

### 3.1.2 Unpolarized case.

\[
D^V(0, \tau) = T(0)_0(\omega, x) \left\{ \frac{3}{(1 - x\omega)^\tau} - \frac{1}{1 + \tau} \left[ 2 - (2 - \tau(1 - \tau)) \frac{1 - x\omega}{\zeta \omega} \right] 2F1 \left( \frac{1 + \tau}{2 + \tau} \left| x\omega \right. \right) \right.
\]

\[
- \frac{1}{1 + \tau} \left[ \frac{1}{1 - \zeta \omega} \left[ 2 + (2 - \tau(1 - \tau)) \frac{1 - x\omega}{\zeta \omega} \right] 2F1 \left( \frac{1 + \tau}{2 + \tau} \left| \frac{x - \zeta \omega}{1 - \zeta \omega} \right. \right) \right], \tag{71}
\]

\[
D^V(\tau, 0) = D^A(\tau, 0). \tag{72}
\]

The general feature manifested by the results we have just derived is the existence of a few IR renormalon poles in the Borel plane. This is a common point for all space-like processes \([25]\) which tells us what kind of power suppressed contributions should be added to make physical quantities free from ambiguities.

### 3.2 Extended ER-BL evolution kernel.

The question of the NNA-corrections to the eigenfunctions of the generalized Efremov-Radyushkin-Brodsky-Lepage evolution equation was studied by us in Ref. \([9]\) (see also \([41]\)). They are given
by Eq. (13) but now the conformal anomaly $\gamma^c$ in the $\hat{B}$-matrix should be shifted by the term proportional to the $\beta$-function $\gamma^c \rightarrow \gamma^c + 2\hat{B}$. This follows from the conformal constraints extended to the case of a running coupling [29]. It was shown there that the eigenfunctions of the extended ER-BL evolution kernels with the fermion renormalon chains resummed to all orders leads to a shift of the index $\frac{3}{2}$ of the Gegenbauer polynomial by the amount $-\beta(g)/g$:

\[
\left[ \frac{x}{\zeta} \right]^{1-\beta/g} C^{3/2-\beta/g}_j \left( 2 \frac{x}{\zeta} - 1 \right).
\]

The NNA-anomalous dimension matrix is diagonalized in this basis

\[
\hat{B}^{-1} Q Q K_{\text{NNA}} \hat{B} = Q Q \gamma_{\text{NNA}},
\]

and its diagonal matrix elements coincide with the forward anomalous dimension of the DGLAP evolution equation with fermion-loop insertions resummed to all orders [38, 39, 40, 41]:

\[
QQ_{ij}^{\text{NNA}} \left( \tau = \frac{\alpha_s}{4\pi} \beta_0 \right) = C_F F(\tau, 0) \left\{ 1 - \tau - \frac{(1 - \tau)^2(2 - \tau)}{(1 - \tau + j)(2 - \tau + j)} \right. \\
\left. + \ (4 - 2\tau) \left[ \psi(2 - \tau + j) - \psi(2 - \tau) \right] \right\},
\]

Since the LO evolution kernels are the same in the polarized and unpolarized cases the eigenfunctions and eigenvalues are also the same. But, unfortunately, the eigenfunctions obtained by resumming only the fermion bubble chains with consequent restoration of the full $\beta$-function has nothing to do with reality since as has been observed in Ref. [42] there is significant cancelation in the evolution kernels between different conformal symmetry breaking parts for $N_f$ = light flavours, namely, between the special conformal anomaly and the $\beta$-function term. However, in the present approach while the latter is taken into account the former is discarded completely. The breaking of the NNA approximation for the anomalous dimension of forward deep inelastic scattering has been observed also in Ref. [39] which supports our conclusions.

The above result could be seen from the expressions for the eigenfunction found in the previous section 2.2.2. Adhering to the NNA we should neglect all terms in Eq. (45) except for those $\sim \beta$ and to $\mathcal{O}(\alpha_s)$ accuracy (for brevity we put $\zeta = 1$) we find

\[
\left\{ I - (I - D)\tau S \right\} \otimes [\bar{y}y] C^{3/2}_j (2y - 1) = N_j(\tau) [\bar{y}y]^{1-\tau} C^{3/2-\tau}_j (2y - 1) + \mathcal{O}(\tau^2),
\]

where $\tau = \frac{\alpha_s}{4\pi} \beta_0$, and $N_j(\tau) = 1 + \tau S_{jj}$ comes from the diagonal part of the shift operator which affects only the overall normalization. In this way we support the hypothesis of NNA since in Eq. (45) $\beta_0$ stands for the full QCD $\beta$-function rather than only for its fermionic piece up to limitation about its quantitative validity.
Note that the argument of the Gegenbauer polynomial $C^\nu_j(t)$ is defined on the segment $-1 \leq t \leq 1$, thus the above equalities are defined for a distribution with the support $0 \leq x \leq \zeta$. However, if one exploits the identity

$$\Theta_0^0(x, x - \zeta) \left[ \frac{x}{\zeta} \left( 1 - \frac{x}{\zeta} \right)^{-1/2} C^\nu_j \left( \frac{2x}{\zeta} - 1 \right) \right] = 2^{1-2\nu} \frac{\Gamma \left( \frac{1}{2} \right) \Gamma(j + 2\nu)}{\Gamma(\nu) \Gamma(j + \nu + \frac{1}{2}) \Gamma(j + 1)} \int_0^1 dt (tt^j)^{j+1/2} \delta^j(\zeta t - x),$$

and can understand the latter in the sense of a mathematical distribution, i.e. to get a meaningful result it should be convoluted first with some smooth function before integration over $t$, one can then restore the support properties to the whole range $0 \leq x \leq 1$.

### 4 Renormalon model for the higher twist corrections.

The model for higher twist contributions can be traced from the estimation of the ambiguity in the resummation of the perturbation series and is given by the imaginary part of the Borel integral which appears due to the necessity to deform the integration path in the complex plane of the $\tau$-parameter to escape from singularities (this leads to the undetermined overall sign owing to different possibilities to close the integration contour around the pole)

$$-\frac{2C_F}{b} \frac{1}{\pi} \text{Im} \int_0^\infty \frac{d\tau}{\tau} e^{-4\pi/(\alpha_s b) \tau} T^\gamma(0, \tau | \omega, x, \zeta, Q^2) = \pm \sum_{tw-n} \left( \frac{\mu^2 e^C}{Q^2} \right)^{(n-2)/2} \Delta^{\gamma}_{tw-n}(x, \omega, \zeta).$$

There are only two poles in the Borel plane which are located in the points $\tau = 1, 2$. Thus, the ambiguities generate only two types of power corrections corresponding to twist-4 and 6 operators. Of course, one can generate even higher power correction by considering the perturbative diagrams beyond one-loop order. However, in view of the success of the leading IR renormalon model we will limit ourselves to the one-loop approximation. From Eqs. (69), (71) of the preceding section we get for the spin-dependent sector

$$\Delta_{tw-4}^A(x, \omega, \zeta) = \frac{4C_F}{b} T_{(0)}(x, \omega) \left\{ \frac{(1 - \zeta \omega)(1 - x \omega + \zeta \omega)}{x \omega(x - \zeta \omega)} \ln(1 - \zeta \omega) + \frac{1}{x \omega} \left[ 1 - x \omega - \frac{(2x - \zeta \omega)(1 - \zeta \omega)}{x \omega(x - \zeta \omega)} \ln(1 - x \omega) + \frac{1}{1 - x \omega} \left[ \frac{1}{2} (x - \zeta \omega) - \frac{1 - \zeta \omega}{x \omega} \right] \right] \right\},$$

$$\Delta_{tw-6}^A(x, \omega, \zeta) = \frac{2C_F}{b} T_{(0)}(x, \omega) \left\{ \frac{1 - \zeta \omega}{(x - \zeta)^3 \omega^3} \ln(1 - \zeta \omega) - \frac{1}{(x \omega)^2} + \frac{1 - \zeta \omega}{(x - \zeta)^3 \omega^3} \ln(1 - x \omega) - \frac{1}{2(1 - x \omega)^2} \left[ \frac{2}{(x \omega)^2} \frac{1 - x \omega}{x \omega} \left( 3 - \frac{x \omega(1 - \zeta \omega)}{(x - \zeta)^2 \omega^2} \right) \right. \right. \right.$$

$$\left. \left. - \frac{1 - \zeta \omega}{(x - \zeta)^2 \omega^2} \left[ (1 - \zeta \omega)(2x - \zeta \omega) - \frac{x \omega}{(x - \zeta) \omega} \right] \right] \right\}.$$
and for the spin-averaged one

\[
\Delta_{\text{tw}-4}^V(x, \omega, \zeta) = \Delta_{\text{tw}-4}^A(x, \omega, \zeta),
\]

\[
\Delta_{\text{tw}-6}^V(x, \omega, \zeta) = \frac{2C_F}{b} T_{(0)}(\omega, x) \left\{ \left( 1 + 2 \frac{1 - x \omega}{\zeta \omega} \right) \frac{1 - \zeta \omega}{(x - \zeta)^3 \omega^3} \ln(1 - \zeta \omega) \right. \\
- \left. \left[ \frac{1}{(x \omega)^3} \left( 1 - 2 \frac{1 - x \omega}{\zeta \omega} \right) + \frac{1 - \zeta \omega}{(x - \zeta)^3 \omega^3} \left( 1 + 2 \frac{1 - x \omega}{\zeta \omega} \right) \right] \ln(1 - x \omega) \\
- \frac{1}{2(1 - x \omega)^2} \left[ \frac{2}{(x \omega)^2} - \frac{1 - x \omega}{x \omega} \left( 3 + 2 \frac{2 - 3x \omega}{x \omega \zeta \omega} \right) \\
- \frac{1 - \zeta \omega}{(x - \zeta)^2 \omega^2} \left( 1 + 2 \frac{1 - x \omega}{\zeta \omega} \right) \left( 1 - 3x \omega + \zeta \omega + \frac{x \omega}{(x - \zeta) \omega} \right) \right\} \right\}.
\] (81)

We should note that the twist-4 result can be considered as more reliable since the only contribution to it comes from the IR renormalons in the coefficient function for the twist-2 operators. For the twist-6 part there is additional input from IR ambiguities in the point \( \tau = 1 \) from the coefficient function of the twist-4 correlators. Thus the above presented results for tw-6 include only part of the total contribution. However, since the latter is suppressed by extra two powers of the momentum transfer its particular shape is not of relevance for us and we have cited them just for completeness.

There is another source of power suppressed contributions which comes from the non-convergent behaviour of the perturbation theory and is due to instantons [45]. However, they lead to strongly suppressed contributions due to high values of \( \tau \) and thus can safely be discarded.

5 Summary and outlook.

In this paper we have presented the theoretical framework for studying higher order and higher twist corrections based on the combination of two different formalisms. The first one is based on restrictions for the amplitudes of the massless theory coming from the algebra of the collinear conformal group in the hypothetical limit of vanishing \( \beta \)-function. We restore the dependence on the latter with the second approach which resums the fermion vacuum insertions to all orders in the coupling. Taken alone this approximation is insufficient since fermion loops do not dominate the radiative corrections to the amplitudes. However, supplied with the idea of NNA one may hope that they do and that thus all important perturbative corrections are taken into account. The validity of this approximation can be checked only by comparison with exact results. It turns out that not all quantities are approximated with good quality by this methods, but only those which are dominated by the renormalon poles. Due to the latter the resummed amplitude is plagued by the uncertainties that manifest the asymptotic character of the perturbative series. It was established that they are power suppressed and thus mimic power corrections to the amplitude.
This tells us that both higher orders and higher twist should be treated simultaneously to escape from ambiguities intrinsic to the perturbative series. By accepting that the non-perturbative higher dimensional operators are dominated by their UV renormalon poles it is possible to construct rough model for the momentum fraction dependence of the power suppressed contributions.

In conclusion we have resummed fermion vacuum polarization bubbles in the coefficient function of the DVCS amplitude. It is known that NNA approximation overestimates radiative corrections. A more realistic estimate is given by the semi-sum of the exact NLO (section 2) and NNA (section 3.1) results. By using the UV dominance hypothesis we give an estimate of the shape and magnitude of the higher twist contributions. The numerical analysis will be performed in a separate publication. For that we have to accept one of the models for leading twist non-forward distribution. Several of them are already on a market [46, 47, 48].

Note added: After the present study was finished we learned about Refs. [49, 50] where the leading twist factorization for DVCS was proved to all orders of perturbation theory.

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A Fermion bubbles in the DVCS coefficient function.

A.1 Spin-dependent scattering.

In the treatment of the polarized sector we resolved the $\gamma_5$-ambiguity following Braaten’s recipe [43], namely, we used the anticommutativity property of $\gamma_5$ only in the box-type diagram and contract the string of $\gamma$-matrices through $\gamma_5$, while in all other cases we did it in the other direction such that no commutation with the chiral matrix occurred.

We have calculated the one-loop Feynman graphs represented in Fig. 2 in a $d = 4 - 2\epsilon$ dimensional space with the following effective gluon propagator in the Landau gauge ($\mu^2 = 4\pi\mu^2$):

\[(-i) D_{\mu\nu}^{ab}(k) = \frac{-i\delta^{ab}}{k^2 + i0} \left( \frac{\mu^2}{-k^2} \right)^{\sigma} \left( g_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2} \right). \tag{A.1} \]

A simple calculation leads to the results (since the sum of all diagrams forms a gauge invariant set, only the $g_{\mu\nu}$-part of propagator has been used in the real calculation)

\[ T_{(1, a)}^A = f(\sigma, \epsilon) \frac{1 - \epsilon}{(1 - x\omega)^{\sigma + \epsilon}}, \]
\[ T_{(1,b)}^A = f(\sigma, \epsilon) \frac{1}{(1 - \zeta \omega)^{\sigma + \epsilon}} \left( 1 + \epsilon(1 - \epsilon) - \epsilon(\sigma + \epsilon) \frac{1 - \zeta \omega}{1 - x \omega} \right) \]

\[ - \frac{1}{1 + \sigma} \left[ 2 - \epsilon + 2 \epsilon(\sigma + \epsilon) - \frac{\sigma \epsilon(\sigma + \epsilon)}{1 - \epsilon} \right] 2F_1 \left( 1 + \sigma + \epsilon, 1 + \sigma \left| \frac{(x - \zeta \omega)}{1 - \zeta \omega} \right. \right), \]

\[ T_{(1,c)}^A = T_{(1,b)}^A(\zeta = 0), \]

\[ T_{(1,d)}^A = f(\sigma, \epsilon)(1 - x \omega)(1 - \epsilon) \frac{\sigma(1 - \sigma) + 2 - 2 \epsilon(1 - \epsilon)}{(1 + \sigma)(2 + \sigma)} F_1 \left( 1 + \sigma + \epsilon, 1 + \sigma, 1 \left| x \omega, \zeta \omega \right. \right), \]

with the common overall factor \( f(\sigma, \epsilon) \):

\[ f(\sigma, \epsilon) = -\frac{\alpha_s}{2 \pi} C_F \left( \frac{\mu^2}{-Q^2} \right)^{\sigma + \epsilon} \frac{\Gamma(\sigma + \epsilon)\Gamma(2 - \epsilon)\Gamma(1 - \sigma - \epsilon)}{\Gamma(1 + \sigma)\Gamma(3 - \sigma - 2\epsilon)} T_{(0)}(\omega, x). \quad \text{(A.2)} \]

While the calculation of the self-energy and vertex-type corrections is straightforward we should mention some technical details about the box-type graph. As quoted above a straightforward calculation of the latter leads to a result in terms of the Appel function \( F_1 \) \([27, 28]\). However, the latter can be reduced to the difference of two hypergeometric functions \( 2F_1 \). The derivation proceeds in a very simple manner. One evaluates the momentum integral in \( d \)-dimensions with the integrand given by the product of the denominators of the box diagram and the factor \((pk)\). There are two possibilities to compute this integral: on the one hand we can join all four denominators via Feynman parameters and get at the end the result in terms of \( F_1 \). The second way is to represent the factor \((kp)\) as a difference of two propagators, namely

\[ (kp) = \frac{1}{2\zeta} \left\{ [(x - \zeta)p - k]^2 - [xp - k]^2 \right\}. \quad \text{(A.3)} \]

Then, the final result looks like a difference of the vertex-type graphs. Thus, we end up with the following relation between the Appel function and hypergeometric ones which we have failed to find in any textbook on special functions \([27, 28]\):

\[ F_1 \left( 1 + a + b, 1 + a, 1 \left| x, y \right. \right) \quad \text{(A.4)} \]

\[ \frac{2(2 + a)}{y(a + b)} \left\{ (1 - y)^{-(a+b)} 2F_1 \left( a + b, 1 + a \left| \frac{x - y}{1 - y} \right. \right) - 2F_1 \left( a + b, 1 + a \left| \frac{x}{2 + a} \right. \right) \right\} \]

\[ = \frac{1}{y(1 - b)} \left\{ 2F_1 \left( 1 + a + b, 1 + a \left| x \right. \right) - (1 - y)^{-(a+b)} 2F_1 \left( 1 + a + b, 1 + a \left| \frac{x - y}{1 - y} \right. \right) \right\}. \]

We put \( \sigma = n\epsilon \), with \( n \) being the number of the fermion bubbles inserted in the gluon line, and multiply the above expressions with a factor corresponding to the product of the fermion vacuum polarization blobs

\[ \pi_n = \frac{1}{\epsilon^n} \left( -\frac{\alpha_s}{4\pi} \beta_0 \right)^n \left[ \frac{\Gamma(1 + \epsilon)\Gamma^2(2 - \epsilon)}{\Gamma(4 - 2\epsilon)} \right]^n. \quad \text{(A.5)} \]
Performing trivial subtraction of sub- and overall divergences and resummation, which is particularly easy as only the end terms in the sum survive [4], along the line of Ref. [44] we obtain the result given in the main text in Eq. (63) with the functions $D_i$ corresponding to the contributions of particular diagrams

\[
D^A_a(\epsilon, \tau) = T_{(0)}(\omega, x) \frac{1 - \epsilon}{(1 - x\omega)^\tau} \quad (A.6)
\]

\[
D^A_b(\epsilon, \tau) = T_{(0)}(\omega, x) \frac{1}{(1 - \zeta \omega)^\tau} \left\{ 1 + \epsilon(1 - \epsilon) - \epsilon \tau \right\} \frac{(1 - \zeta \omega)}{1 - \epsilon} \left[ 2 - \epsilon + 2\epsilon \tau - \frac{\epsilon \tau (\tau - \epsilon)}{1 - \epsilon} \right] 2F_1 \left( \frac{1 + \tau, 1 - \epsilon + \tau}{2 - \epsilon + \tau}, \frac{(x - \zeta)\omega}{1 - \zeta \omega} \right), \quad (A.7)
\]

\[
D^A_c(\epsilon, \tau) = D^A_b(\epsilon, \tau|\zeta = 0), \quad (A.8)
\]

\[
D^A_d(\epsilon, \tau) = T_{(0)}(\omega, x) \frac{1 - x\omega}{\zeta \omega} \left[ 2 - \epsilon - \tau + \frac{2\epsilon \tau}{1 - \epsilon + \tau} \right] \times \left\{ 2F_1 \left( \frac{1 + \epsilon, 1 - \epsilon + \tau}{2 - \epsilon + \tau}, \frac{\epsilon \tau}{1 - \epsilon + \tau} \right) - (1 - \zeta \omega)^{-\tau} 2F_1 \left( \frac{1 + \epsilon, 1 - \epsilon + \tau}{2 - \epsilon + \tau}, \frac{(x - \zeta)\omega}{1 - \zeta \omega} \right) \right\}. \quad (A.9)
\]

Although our expression for the box-type diagram differs from the one calculated (for $\zeta = 1$) in Ref. [41] by the factor $\sim \epsilon \tau$ obviously this contribution has no impact, neither on the final answer (63) since it vanishes in both limits $\epsilon = \tau = 0$, nor on the one-loop results derived in section 2.2.2 as it proportional to $\epsilon^2$.

### A.2 Spin-averaged scattering.

For the spin-averaged amplitude there is no difficulty due to $\gamma_5$ and the calculation is straightforward

\[
T^A_{(1,a)} = T^V_{(1,a)}, \quad T^A_{(1,b)} = T^V_{(1,b)}, \quad T^A_{(1,c)} = T^V_{(1,c)},
\]

\[
T^A_{(1,d)} = f(\sigma, \epsilon)(1 - x\omega)(1 - \epsilon) \frac{(1 - \epsilon)(2 - \epsilon) - (\sigma + \epsilon)(1 - \sigma - 3\epsilon)}{(1 + \sigma)(2 + \sigma)} \times F_1 \left( \frac{1 + \sigma + \epsilon, 1 + \sigma, 1}{3 + \sigma}, x\omega, \zeta \omega \right),
\]

where the function $f(\sigma, \epsilon)$ is defined by Eq. (A.2). Some simple manipulations give finally

\[
D^A_a(\epsilon, \tau) = D^V_a(\epsilon, \tau), \quad D^A_b(\epsilon, \tau) = D^V_b(\epsilon, \tau), \quad D^A_c(\epsilon, \tau) = D^V_c(\epsilon, \tau), \quad (A.10)
\]

\[
D^V_d(\epsilon, \tau) = T_{(0)}(\omega, x) \frac{1 - x\omega}{\zeta \omega} \left[ 2 - \epsilon + \tau - \frac{4\tau(1 - \epsilon)}{1 - \epsilon + \tau} \right] \times \left\{ 2F_1 \left( \frac{1 + \epsilon, 1 - \epsilon + \tau}{2 - \epsilon + \tau}, x\omega \right) - (1 - \zeta \omega)^{-\tau} 2F_1 \left( \frac{1 + \epsilon, 1 - \epsilon + \tau}{2 - \epsilon + \tau}, \frac{(x - \zeta)\omega}{1 - \zeta \omega} \right) \right\}. \quad (A.11)
\]
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