Random Access with Opportunity Detection in Wireless Networks
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Abstract—This letter proposes a novel random medium access control (MAC) based on a transmission opportunity prediction, which can be measured in a form of a conditional success probability given transmitter-side interference. A transmission probability depends on the opportunity prediction, preventing indiscriminate transmissions and reducing excessive interference causing collisions. Using stochastic geometry, we derive a fixed-point equation to provide the optimal transmission probability maximizing a proportionally fair throughput. Its approximated solution is given in closed form. The proposed MAC is applicable to full-duplex networks, leading to significant throughput improvement by allowing more nodes to transmit.

Index Terms—Random MAC, transmission opportunity, proportional fairness, stochastic geometry

I. INTRODUCTION
A distributed medium access control (MAC) plays a key role in efficient usage of wireless medium by allowing multiple users to share it without a central controller if they are temporally or spatially separated. Due to its simplicity and practicality, it is preferred over a centralized MAC to facilitate the simultaneous control of massive number of devices, while frequent collisions and unfair resource allocation can happen in the same vein. To reduce the collision, a spectrum sensing approach is widely used where a transmitter (TX) measures the interference before accessing the medium. The TX decides to access the medium if the interference is less than a predetermined threshold, e.g., Carrier Sensing Multiple Access (CSMA) [1]. However, this deterministic decision may lead to inaccurate estimation of the transmission opportunity because the interference levels of a TX and the paired receiver (RX) can be uncorrelated, resulting in hidden- or exposed-node problems when the interference at the RX is respectively less or more than that at the TX [2]. A straightforward approach to overcome the limitation is to add additional signaling protocols (see e.g., [3], [4]), which are inappropriate in dense networks because it rather brings about excessive interference [5].

This letter aims at developing a distributed MAC to maximize a network utility without additional signaling overhead. To this end, we propose a novel random access scheme based on the stochastic prediction of the transmission opportunity defined as a conditional success probability given a TX's measured interference. Contrary to the conventional random access without prediction (e.g., [6]), it helps each node to access the medium only when the transmission is likely to be successful, leading to reducing the hidden- and exposed-node problems, and collisions. Besides, this MAC can control each node’s transmission probability in a stochastic manner, which allows nodes to share the medium with achieving a proportional-fairness. Using a powerful tool of stochastic geometry (SG) [7], we derive a fixed-point equation to provide the optimal transmission probability in Proposition 1. The approximated closed form solution is also given in Proposition 2 giving useful insights for practical MAC design. It is worth noting that the proposed MAC is applicable to full-duplex (FD) networks [8]. Compared with a half-duplex (HD) based random MAC as in [9], it allows more nodes to access the medium, leading to significant throughput improvement.

II. SYSTEM MODEL
Consider a Poisson bipolar network comprising two Poisson point processes (PPPs). A set of nodes \( \{ \ell \} \) follows a homogeneous PPP \( \Phi_1 \) with density \( \lambda \) and another set of nodes \( \{ k \} \) also follows \( \Phi_2 \) where node \( k \) is randomly located at a distance of \( d \) from node \( \ell \). Based on Displacement Theorem [7], we regards \( \Phi = \Phi_1 \cup \Phi_2 \) as a superposition of two homogeneous PPPs with intensity \( \lambda \). All nodes in the network are assumed to be FD-capable, and they can transmit and receive simultaneously.

Each node in the network transmits with its transmission probability specified in the sequel. The signal between nodes \( \ell \) and \( k \) experiences Rayleigh fading with a unit mean \( ( h_{\ell,k} \sim \exp (1) ) \). A signal-to-interference ratio (SIR) of the transmitted signal from nodes \( \ell \) to \( k \) is expressed as:

\[
\text{SIR}_k = \delta_k P_k \beta + \sum_{m \in \Phi \setminus \{ \ell, k \}} \delta_m P_m h_{mk} \| m - k \|^{-\alpha},
\]

where \( P_m \) is the transmission power of node \( m \). The parameter \( \delta_m \) is an indicator that takes value 1 if node \( m \) transmits and 0 otherwise, and \( \alpha > 2 \) is a path-loss exponent. In the denominator, the first term represents the residual self-interference when both \( \ell \) and \( k \) are transmitting, i.e. FD communication, while the second one represents the aggregate interference from other TXs denoted by \( Z_k \). The parameter \( \beta \in [0, 1] \) denotes residual self-interference of a node depending on a cancellation algorithm. In this letter, we consider a unit transmission power \( P_m = 1 \) for all \( m \in \Phi \). The transmission is said to be successful when \( \text{SIR}_k \geq \theta \), where \( \theta \) is an SIR threshold. That is to say, the desired signal cannot be decoded if SIR value is smaller than \( \theta \).

III. RANDOM ACCESS
Every node determines the transmission probability by detecting transmission opportunity defined as follows.

**Definition 1** (Opportunistic Probability [9]). The conditional success probability of the transmission from nodes \( \ell \) to \( k \) for a given TX-side interference \( I \) is defined as

\[
\text{OP}_I \equiv \Pr [ \text{SIR}_k > \theta | I ],
\]

where \( k \) denotes the dedicated RX of node \( \ell \).

A typical node \( \ell \in \Phi \) transmits with probability \( p_\ell = F (\text{OP}_I) \) in the next slot and remains silent otherwise, where \( F (\text{OP}) \) is a non-decreasing function. Although it is proved that a
linear increasing $F(\text{OP})$ gives a near-to-optimal performance to maximize an area spectral efficiency [9], it brings unfair resource allocation because the node with high OP monopolizes the radio resource. Hence, $F(\text{OP})$ needs to be redesigned for a fair resource allocation. Before finding the optimal transmission probability, we describe why random access is preferred and how the nodes predict their OP in the following.

A. Advantages of Random Access

We focus on two pairs in the network while the remaining interference from other nodes is constant. Figure 1 shows the SIR regions of two RXs where the mutual interference conditions are small and large, respectively. We assume the OP values are 0.8 (on X-axis) and 0.5 (on Y-axis), and we refer to the node as high- and low OP nodes, respectively. Three distinguishable points denote SIR values of three access modes: i) maximal power transmission (Max TX), ii) power-controlled transmission (PC TX), iii) random access transmission (Random TX). The node reduces its transmission power by OP in PC TX, $P_{\ell} = \text{OP}_{\ell}$, whereas the node sets its transmission probability, $p_{\ell} = \text{OP}_{\ell}$ in Random TX while fixing $P_{\ell}$ to the maximal power. In Figure 1, there is a small difference of 3% in the sum of SIRs of three schemes while PC TX and Random TX spend less radio resources: $(\text{Tx time}) \times (\text{Tx power})$. It implies an increase in the transmission success rate and a decrease in loss of transmission opportunity caused by the exposed node problem. In Figure 1, the SIR sums of Random TX and PC TX are even higher than Max TX as the mutual interference becomes large. The SIR sum as well as the SIR value of the low OP node are higher in Random TX than PC TX. This means that Random TX achieves more equitable and efficient resource allocation as the network becomes interference-limited. In PC TX, the low OP node experiences severe interference in every transmission, because the high OP node always transmits with a higher power. In Random TX, the low OP node has a chance to transmit with less interference when the high OP node is not transmitting. This not only improves the fairness by increasing the average SIR of the low OP node, but also increases the SIR sum because the SIR increase of the low OP node is greater than the decrease of the high OP node. Based on what we have so far, we use Random TX as the basis of our MAC.

B. Prediction of Opportunity

A node measures the aggregate TX-side interference, $I_k$, via the energy detection before the transmission. A direct derivation of OP is intractable because the isotropic property is violated due to the common interferers of TX and RX. For the analytical tractability, we introduce the following assumption:

**Assumption 1 (Empty Ball).** The interference $I_k$ can be decomposed into the strongest- and the residual interferences. In an average sense, the strongest interference comes from the nearest interferer located on a circle defined as an empty ball. In other words, there is no interferer inside the ball. Furthermore, the other interferers are assumed to follow PPP outside the ball.

It is derived in [9] that the expectation of the empty ball’s radius for a given $I$ should satisfy the following condition:

$$ IR^\alpha - 4\pi \lambda I R^2 - 1 = 0. $$

(3)

The above is solved in a closed form when $\alpha = 4$ as $R = I^{-\frac{1}{2}} \left[ \pi \lambda + \left( I + \pi^2 \lambda^2 \right)^{\frac{1}{2}} \right]^{\frac{1}{2}}$. With the radius, the OP value is computed as in [9].

IV. OPTIMAL TRANSMISSION PROBABILITY

In this section, we derive the optimal transmission probability to maximize a proportionally fair throughput. The expected throughput of node $\ell$ is $p_{\ell} q_k$, where $q_k$ denotes the success probability at its RX $k$. Then, we define the normalized link throughput as follows:

**Definition 2 (Normalized link throughput).** The spatial average of the logarithmic link throughput is given by

$$ TP_{\ell} = \lim_{r \to \infty} \frac{1}{\lambda \pi r^2} \sum_{k \in \Phi \cap b(r)} \log(p_{\ell} q_k), $$

(4)

where $b(r)$ denotes the circle with radius $r$ centered at the origin.

Using Palm distribution of the point process $\Phi$ and the ergodicity of the PPP, we can treat the nodes $\ell$ and $k$ as a typical pair located at the origin where the corresponding throughput of the pair, $TP = TP_{\ell} + TP_k$. Then, we formulate a throughput-maximization problem of a typical FD pair to obtain the optimal transmission probability that achieves proportional fairness [10]:

$$ \max_{p_{\ell}} TP = E_k \left[ \log(p_{\ell} q_k) + \log(p_k q_k) \right] $$

(P)

s.t. 0 < $p_{\ell}$, $p_k$ 1, $i \in \Phi$.

We note that (P) is a general form of the network in which each node can transmit, i.e. FD network; we can also model a half-duplex network by setting $p_k = 0$ for $k \in \Phi_2$. By solving (P), we obtain the probability as follows:

**Proposition 1 (Optimal transmission probability).** The optimal transmission probability is given as $\min[p^*, 1]$ where $p^*$ is the solution of the following fixed-point equation:

$$ 1 = -\frac{e^{-\theta d_{\ell}} - 1}{(e^{-\theta d_{\ell}} - 1) p + 1} + \frac{1}{1 + (R^\alpha / \theta d_{\ell}) - p} + 4\pi \lambda d_{\ell} \int_{R/d}^\infty s^{-1} - p + s^\alpha / \theta \, ds. $$

(5)

Here, $R$ is the expected empty ball radius specified in [3]. When $\alpha = 4$, the above equation is reduced as

$$ 1 = -\frac{e^{-\theta d_{\ell}} - 1}{(e^{-\theta d_{\ell}} - 1) p + 1} + \frac{1}{1 + (R^\alpha / \theta d_{\ell}) - p} + \frac{2\pi \lambda d_{\ell}^2 \sqrt{\theta}}{\sqrt{1 - p}} \left( \arctan \left( \frac{R/d^2}{\sqrt{\theta (1 - p)}} \right) \right). $$

(6)
Proof. First of all, we derive the success probability \( q_k^{(H)} \) conditioned on \( \Phi \) for a HD communication.

\[
\mathbb{E}_{H_k} \left[ \Pr \left( SIR_k \geq \theta | H_k, \Phi, \delta_k = 0 \right) \right] = \prod_{i \in \Phi \setminus \{k\}} \left(1 - \frac{p_i}{1 + \|i - k\|^\alpha / \theta d^\alpha} \right) = q_k^{(H)}. \tag{7}
\]

We also have the success probability \( q_k^{(F)} \) as follows:

\[
q_k^{(F)} = \exp (-\theta d^\alpha) \prod_{i \in \Phi \setminus \{k\}} \left(1 - \frac{p_i}{1 + \|i - k\|^\alpha / \theta d^\alpha} \right). \tag{8}
\]

The expected SIR of the node \( k \), \( q_k = (1 - p_k) q_k^{(H)} + p_k q_k^{(F)} \). Then, TP is represented as follows:

\[
TP = \mathbb{E}_k \left[ \log \left( p_k (1 + p_k (\exp (-\theta d^\alpha) - 1)) q_k^{(H)} \right) + \log \left( p_k (1 + p_k (\exp (-\theta d^\alpha) - 1)) q_k^{(H)} \right) \right]. \tag{9}
\]

The term \( \log \left( q_k^{(H)} \right) \) can be split into two terms depending on whether \( i = z_m \) or not, where \( z_k \) is the nearest interferer of node \( k \), and we have the following equation by Campbell’s Theorem.

\[
\mathbb{E}_k \left[ \sum_{i \in \Phi \setminus \{k, z_k\}} \log \left(1 - \frac{p_k}{1 + \|i - k\|^\alpha / \theta d^\alpha} \right) \right] = F(p_k, k). \tag{10}
\]

Finally, we can maximize the expectation when the following derivative expression holds:

\[
\frac{1}{p_m} + \frac{e^{-\theta d^\alpha} - 1}{(e^{-\theta d^\alpha} - 1) p_m + 1} - \frac{1}{1 + (R^\alpha / \theta d^\alpha) - p_m + \frac{\partial F(p_m, m)}{\partial p_m}} = 0,
\]

where \( m \in \{\ell, k\} \) and

\[
F(p_m, m) = 2\int_{i \in \mathbb{R}^2 \setminus \{\ell, k\} \setminus (\ell, m)} \log \left(1 - \frac{p_m}{1 + \|i - m\|^\alpha \theta d^\alpha} \right) \, di. \tag{12}
\]

For a given measured interference, the optimal \( p^* \) that maximizes the spatial average of the logarithmic throughput \( TP \) is determined by the fixed point equation \( [1] \). It is clear that the RHS of the equation monotonically increases continually with respect to \( p \) on \((0, 1]\) because its differential value is positive while LHS monotonically decreases, thereby the equation has a unique solution.

Remark 1 (Conservative Access). Noting that each pair distance \( d \) is different to each other in practice, our approach is said to be conservative such that the resultant access probability in Proposition 1 provides a lower bound of the practical case. Specifically, \([7] \) represents a lower bound for the random distance with mean \( \mu d \) by Jensen’s inequality. As a result, our approach is more likely to protect the transmissions of other devices, which is aligned with the mutual protection concept in spectrum-sharing networks \([11] \).

Remark 2 (Effect of Self-interference Factor). It is shown that as the residual self-interference factor \( \beta \) decreases, the optimal transmission probability \( p^* \) in Proposition \([6] \) increases. In other words, advanced self-interference cancellation technique makes it possible to reduce interference, allowing each node to access the medium more frequently.

The nodes can numerically compute the optimal probability by fixed-point iteration based on \([5] \), but the probability calculations are frequent and time-sensitive. To reduce the computational overhead, we approximate a closed-form expression of the optimal transmission probability as follows:

**Proposition 2 (Approximated Transmission Probability).** Given the path-loss exponent \( \alpha = 4 \), the optimal transmission probability is given as \( \min \left[ p^*, 1 \right] \) where \( p^* \) is approximated as

\[
p^* = \frac{-C_2 + \sqrt{C_2^2 - 4C_1C_3}}{2C_1}, \tag{14}
\]

where the coefficients \( C_1, C_2, \) and \( C_3 \) are given in \([13] \).

Proof. The arctan \( (x) \) rapidly increases when \( x \) is relatively small, and the value is saturated as \( x \) increases. We use two different functions, rational function in large interference and constant in small interference, for reflecting the distinct characteristics as follows:

\[
\arctan(x) \approx \begin{cases} 
\frac{1.632x - 0.1037}{x^2 + 0.3907} & \text{if } x \leq 10 \\
0 \text{ o.w.} & \end{cases} \tag{15}
\]

where \( x = (R/d)^2 \). The root mean square error between the function and \( \arctan(x) \) is 0.018 on \( x \in [0, 10] \).

Then, we can approximate \( p^* \) as a solution of the following quadratic equation by curve fitting,

\[
C_1 \cdot p^2 + C_2 \cdot p + C_3 = 0. \tag{16}
\]

The coefficients for each section are given by \([13] \). \( \square \)

Remark 3 (Effect of Parameters). Proposition \([2] \) shows that the approximated transmission probability decreases as node density \( \lambda \), pair distance \( d \), SIR threshold \( \theta \), and measured interference \( I \) increase. For large interference regime, two parameters \( \theta \) and \( d \) have large influence on the probability because the effect of the desired signal’s attenuation is large. On the other hand, the influence of \( \lambda \) becomes large in small interference regime because the growth of interference due to the increase of \( \lambda \) greatly reduces the success probability.

**V. Numerical Results**

Figure \([2] \) shows the optimal transmission probability obtained by Proposition \([4] \) as a function of OP. Based on \([8] \), we set the residual self-interference factor \( \beta = -110dB \). The effect of \( \beta \) cannot be shown due to the lack of space, but the result remains unchanged if \( \beta \) is less than \(-50dB \). We figure out that the probability increases with a quasi-convex shape as OP increases. A transmission should be restricted if

\[
\begin{align*}
C_1 &= (10.4 \times 0.66 + 1.15) (3.5 \times 0.5 + 1.1) (0.85 - 0.2) (4.5 \times 10^{-4} - 1.75 - 0.12) (-28.84 \times \exp (-\theta d^4) + 29.85) \\
C_2 &= 3.8 \times 10^{-4} \times 1^{-1.75} \times (1.11 \times 10^3 \times 2^3 + 0.306 + 2.3) (0.14 \times 10^2 + 6) (-0.12 \times \exp (-\theta d^4) + 1.129) \quad \text{if } \frac{R^\alpha}{\theta d^\alpha} \leq 10 \\
C_3 &= - (\lambda^{1.63} + 0.014) (0.7 \times 0.7 + 0.5) (10 \times d^{-4} + 0.4) (0.9 \times I^{-1.763} + 49) (0.02 \times \exp (-\theta d^4) + 0.98) \quad \text{o.w.} \\
C_4 &= (273 \times 0.66)(1.11 - 2 - 1) (2.5 \times d^{-4} - 0.7) (1.2 \times 10^{-5} - 1.75 - 0.4) (7.96 \times \exp (-\theta d^4) - 6.958) \\
C_5 &= 10^{-4} \times 1^{-1.75} \times (3.8 \times 10^3 \times 2^3 + 1.4) (2.7 \times 0.1 + 3.5) (34 \times d^{-4} + 3) (0.526 \times \exp (-\theta d^4) + 1.526) \\
C_6 &= - (0.9 \times 0.63 + 0.013) (0.7 \times 0.7 + 0.5) (17 \times d^{-4} + 0.7) (0.5 \times I^{-1.763} + 30) (-0.046 \times \exp (-\theta d^4) + 1.051)
\end{align*}
\]
its success probability is low, since the communication failure causes unnecessary interference to the network. As density and threshold increase, the node transmits more conservatively with the same OP in order to generate less interference. In a similar vein, the smaller the parameter $\alpha$, the lower the probability of transmission even with the same OP, because the transmission of a node is more interfering with the others. Nevertheless, the nodes have non-zero probability even if OP is zero as a result of proportionally fair resource allocation.

Figure 3 shows the link throughput according to the access schemes: Max TX, PC TX, Random TX and CSMA/CA. The carrier sensing threshold in CSMA/CA and the transmission power of nodes are set as $-30$ dBm and $23$ dBm, respectively. The throughput difference between Random TX and CSMA/CA becomes larger as the node density increases, because applying OP enables nodes to determine a suitable probability even in the interference-limited situation while they wastes radio resources for the contention in CSMA/CA. Although PC TX also controls the transmission power by OP, the gain is insignificant because the weakened desired signal cancels out the effect of reduced interference. Random TX prevents excessive interference and exploits spectrum reusability in FD networks as well, resulting in an average throughput gain of 12%.

Figure 4 shows the optimal transmission probabilities and its approximation as $\theta$ varies when $\alpha = 4$, $d = 2m$, $\lambda = 0.001$ nodes/$m^2$; (b) Optimal transmission probabilities and its approximation as $\lambda$ varies when $\alpha = 4$, $d = 2$, $\theta = 1$.

VI. CONCLUSION

We proposed a random access scheme that prevents indiscriminate transmissions based on the prediction of transmission its approximation for a verification of the approximation used in Proposition 2. The approximation is accurate in the fact that the averaged and maximum errors are 0.5% and 1.4% in small interference environment (Figure 4), 1.76% and 1.99% in large interference region (Figure 4b).

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