The Longitudinal Structure Function $F_L$
as a Function of $F_2$ and $dF_2/d\ln Q^2$ at small $x$.
The Next-to-Leading Analysis

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Abstract

We present a set of formulae to extract the longitudinal deep inelastic structure function $F_L$ from the transverse structure function $F_2$ and its derivative $dF_2/d\ln Q^2$ at small $x$. Our expressions are valid for any value of $\delta$, being $x^{-\delta}$ the behavior of the parton densities at low $x$. Using $F_2$ HERA data we obtain $F_L$ in the range $10^{-4} \leq x \leq 10^{-2}$ at $Q^2 = 20$ GeV$^2$. Some other applications of the formulae are pointed out.

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For experimental studies of hadron-hadron processes on the new, powerful LHC collider, it is necessary to know in detail the values of the parton (quark and gluon) distributions (PD) of nucleons, especially at small values of $x$. The basic information on the quark structure of nucleons is extracted from the process of deep inelastic lepton-hadron scattering (DIS). Its differential cross-section has the form:

$$
\frac{d^2\sigma}{dxdy} = \frac{2\pi\alpha^2_{em}}{xQ^4} \left[ (1 - y + y^2/2) F_2(x, Q^2) - \left(\frac{y^2}{2}\right) F_L(x, Q^2) \right],
$$

where $F_2(x, Q^2)$ and $F_L(x, Q^2)$ are the transverse and longitudinal structure functions (SF), respectively.

The longitudinal SF $F_L(x, Q^2)$ is a very sensitive QCD characteristic because it is equal to zero in the parton model with spin $-1/2$ partons (it is very large with spin $-0$ partons). In addition, at small values of $x$, $F_L$ data are not yet available\(^3\), as they require a rather cumbersome procedure (see \cite{3}, for example).

In the present article we study the behaviour of $F_L(x, Q^2)$ at small values of $x$, using the HERA data \cite{4, 5} and the method \cite{6} of replacement of the Mellin convolution by ordinary products. By analogy with the case of the gluon distribution function (see \cite{7} and its references) it is possible to obtain the relation between $F_L(x, Q^2)$, $F_2(x, Q^2)$ and $dF(x, Q^2)/dlnQ^2$ at small $x$. Thus, the small $x$ behaviour of the SF $F_L(x, Q^2)$ can be extracted directly from the measured values of $F_2(x, Q^2)$ and its derivative without a cumbersome procedure (see \cite{3}). These extracted values of $F_L$ may be well considered as new small $x$ “experimental data” of $F_L$. When experimental data for $F_L$ at small $x$ become available with a good accuracy, a violation of the relation will be an indication of the importance of other effects as higher twist contribution and/or of non-perturbative QCD dynamics at small $x$.

We follow the notation of our previous work in ref. \cite{7}. The singlet quark $s(x, Q_0^2)$ and gluon $g(x, Q_0^2)$ parton distribution functions (PDF) at some $Q_0^2$ are parameterized by (see, for example, \cite{8}):

$$
p(x, Q_0^2) = A_p x^{-\delta}(1 - x)^{\nu_p}(1 + \epsilon_p \sqrt{x} + \gamma_p x) \quad (p = s, g)
$$

The value of $\delta$ is a matter of controversy. The “conventional” choice is $\delta = 0$, which leads to a non-singular behaviour of the PD (as for example the $D_0'$ fit in \cite{8}) when $x \to 0$. Another value, $\delta \sim \frac{1}{2}$, was obtained in the studies performed in ref. \cite{8} as the sum of the leading powers of $\ln(1/x)$ in all orders of perturbation theory (PT) ($D_-'$ fit in \cite{8}). Experimentally, recent NMC data \cite{10} favor small values of $\delta$. This result is also in agreement with present data for $pp$ and $\bar{p}p$ total cross-sections (see \cite{11}) and corresponds to the model of Landshoff and Nachtmann pomeron \cite{12} with the exchange of a pair of non-perturbative gluons, yielding $\delta = 0.086$. However, the new HERA data \cite{4, 5} prefer $\delta \geq 0.2$.

\(^3\)In the time of preparing this article, the H1 collaboration presented \cite{2} the first (preliminary) measurement of $F_L$ at small $x$.

\(^4\)We use PDF multiplied by $x$ and neglect the nonsinglet quark distribution at small $x$. 

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From the theoretical side, the type of evolution of the PD in Eq. (1) depends on the value and form of δ (δq = δg). For example, a Q^{2}\text{-independent} δ obeys the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation when x^{-δ} \gg 1 (see, for example, \cite{13} - \cite{18}). However, if δ(Q^{2}_{0}) = 0 in some point Q^{2}_{0} \geq 1GeV^{2} (see \cite{14}, \cite{16}, \cite{18}) , the behaviour p(x, Q^{2}) \sim Const (p = (s, g)) is not compatible with DGLAP equation and a more singular behaviour is generated.

If we restrict the analysis to a Regge-like form of structure functions, one obtains (see \cite{18})
\[ p(x, Q^{2}) \sim x^{-\delta_{p}(Q^{2})} \]
with next-to-leading order (NLO) δ_{q}(Q^{2}) \neq δ_{g}(Q^{2}) intercept trajectories.

Without any restriction the double-logarithmical behaviour, i.e.
\[ p(x, Q^{2}) \sim \exp \left( \frac{1}{2} \sqrt{\delta_{p}(Q^{2}) \ln \frac{1}{x}} \right) \]  \quad (2)
is generated.

At NLO and for f = 4 active quarks one has:
\[ \delta_{g}(Q^{2}) = \frac{36}{25}t - \frac{91096}{5625}l, \quad \delta_{q}(Q^{2}) = \delta_{g}(Q^{2}) - 20l \]
where \( t = \ln(\alpha(Q^{2}_{0})/\alpha(Q^{2})) \) and \( l = \alpha(Q^{2}_{0}) - \alpha(Q^{2}) \).

As our goal is the extraction of \( F_{L} \) without theoretical restrictions, we will consider both, the Regge-like behaviour \( p(x, Q^{2}) \sim x^{-\delta} \) (if \( x^{-\delta} \gg 1 \)) and the non-Regge dependence of Eq. (2) (if \( \delta(Q^{2}_{0}) = 0 \)).

1. Assuming the \textit{Regge-like behaviour} for the gluon distribution and \( F_{2}(x, Q^{2}) \) at \( x^{-\delta} \gg 1 \):
\[ g(x, Q^{2}) = x^{-\delta}\tilde{g}(x, Q^{2}), \quad F_{2}(x, Q^{2}) = x^{-\delta}\tilde{s}(x, Q^{2}), \]
we obtain the following equation for the \( Q^{2} \) derivative of the SF \( F_{2} \):
\[ \frac{dF_{2}(x, Q^{2})}{dl\ln Q^{2}} = -\frac{1}{2}x^{-\delta} \sum_{p=s,g} \left( r_{sp}^{1+\delta}(\alpha) \tilde{p}(0, Q^{2}) + r_{sp}^{\delta}(\alpha) x\tilde{p}'(0, Q^{2}) + O(x^{2}) \right) \]
\[ F_{L}(x, Q^{2}) = x^{-\delta} \sum_{p=s,g} \left( r_{Lp}^{1+\delta}(\alpha) \tilde{p}(0, Q^{2}) + r_{Lp}^{\delta}(\alpha) x\tilde{p}'(0, Q^{2}) + O(x^{2}) \right), \]  \quad (3)
where \( r_{sp}^{\delta}(\alpha) \) and \( r_{Lp}^{\delta}(\alpha) \) are the combinations of the anomalous dimensions (AD) of Wilson operators \( \gamma_{sp}^{\eta} = \alpha^{(0,\eta)} + \alpha^{2}\gamma_{sp}^{(1,\eta)} + O(\alpha^{3}) \) and Wilson coefficients \( \alpha B_{L}^{p,\eta} \) (1 + \( \alpha R_{L}^{p,\eta} \)) + O(\alpha^{3})\) and \( \alpha B_{L}^{p,\eta} + O(\alpha^{2}) \) of the \( \eta \) "moment" (i.e., the corresponding variables extended from integer values of argument to non-integer ones):

\footnote{Hereafter we use \( \alpha(Q^{2}) = \alpha_{s}(Q^{2})/4\pi \).}

\footnote{Because we consider here \( F_{2}(x, Q^{2}) \) but not the singlet quark distribution.}
\[ r_{Ls}^n(\alpha) = \alpha B_L^{s,\eta} \left[ 1 + \alpha \left( R_L^{s,\eta} - B_2^{s,\eta} \right) \right] + O(\alpha^3) \]
\[ r_{Lg}^n(\alpha) = \frac{e}{f} \alpha B_L^{s,\eta} \left[ 1 + \alpha \left( R_L^{s,\eta} - B_2^{s,\eta} B_L^{s,\eta} / B_L^{g,\eta} \right) \right] + O(\alpha^3) \]
\[ r_{ss}^n(\alpha) = \alpha \gamma_{ss}^{(0),\eta} + \alpha^2 \left( \gamma_{ss}^{(1),\eta} + B_2^{s,\eta} \gamma_{ss}^{(0),\eta} + 2\beta_0 B_2^{s,\eta} \right) + O(\alpha^3) \]
\[ r_{sg}^n(\alpha) = \frac{e}{f} \left[ \alpha \gamma_{sg}^{(0),\eta} + \alpha^2 \left( \gamma_{sg}^{(1),\eta} + B_2^{s,\eta} \gamma_{sg}^{(0),\eta} + \gamma_{gg}^{(0),\eta} (2\beta_0 + \gamma_{gg}^{(0),\eta} - \gamma_{ss}^{(0),\eta}) \right) \right] + O(\alpha^3) \]

and
\[ \tilde{p}'(0, Q^2) \equiv \frac{d}{dx} \tilde{p}(x, Q^2) \text{ at } x = 0, \]

where \( e = \sum_i e_i^2 \) is the sum of squares of quark charges.

With accuracy of \( O(x^{2-\delta}) \), we have for Eq. (3)
\[ \frac{dF_2(x, Q^2)}{d\ln Q^2} = -\frac{1}{2} \left[ \frac{1}{r_{sg}^{1+\delta}} \xi_{sg}^{-\delta} g(x/\xi_{sg}, Q^2) + r_{ss}^{1+\delta} F_2(x, Q^2) + (r_{ss}^{1+\delta} - r_{ss}^{1+\delta}) x^{-1-\delta} s'(x, Q^2) \right] + O(x^{2-\delta}) \]
\[ F_L(x, Q^2) = r_{Lg}^{1+\delta} \xi_{Lg}^{-\delta} g(x/\xi_{Lg}, Q^2) + r_{Ls}^{1+\delta} F_2(x, Q^2) + (r_{Ls}^{1+\delta} - r_{Ls}^{1+\delta}) x^{-1-\delta} s'(x, Q^2) + O(x^{2-\delta}), \]

with \( \xi_{sg} = r_{sg}^{1+\delta} / x^\delta \) and \( \xi_{Lg} = r_{Lg}^{1+\delta} / x^\delta \).

From Eq. (3) and (4) one can obtain \( F_L \) as a function of \( F_2 \) and the derivative.
\[ F_L(x, Q^2) = -\xi \left[ 2 \frac{r_{Lg}^{1+\delta} dF_2(x, Q^2)}{r_{sg}^{1+\delta} r_{Lg}^{1+\delta}} + \left( r_{Ls}^{1+\delta} - r_{Ls}^{1+\delta} r_{ss}^{1+\delta} \right) F_2(x, Q^2) \right] + O(x^{2-\delta}, \alpha x^{1-\delta}) \]

where the result is restricted to \( O(x^{2-\delta}, \alpha x^{1-\delta}) \).

To arrive to the above equation we have performed the substitution
\[ \frac{\xi_{sg}}{\xi_{Lg}} \rightarrow \xi = \gamma_{sg}^{(0),1+\delta} B_L^{g,\delta} / \gamma_{sg}^{(0),\delta} B_L^{s,1+\delta} \]
and neglected the term \( \sim s'(x, Q^2) \).

This replacement is very useful. The NLO AD \( \gamma_{sp}^{(1),n} \) are singular in both points, \( n = 1 \) and \( n = 0 \), and their presence into the arguments of \( \tilde{p}(x, Q^2) \) makes the numerical agreement between this approximate formula and the exact calculation worse (we have checked this point using some MRS sets of parton distributions).

\[ \text{In the case of replacement Mellin convolution by ordinary product these singularities transform to logarithmically increasing terms (see [14] and [6]).} \]
Using NLO approximation of \( r_{sp}^{1+\delta} \) and \( r_{Lp}^{1+\delta} \) we easily obtain\(^8\) the final results for \( F_L(x, Q^2) \):

\[
F_L(x, Q^2) = \frac{B_L^{g,1+\delta}(1 + \alpha B_L^{g,1+\delta})}{\gamma_{sg}^{(0),1+\delta} + \gamma_{sg}^{(1),1+\delta}} \xi \left[ \frac{dF_2(x, Q^2)}{d\ln Q^2} + \frac{\alpha}{2} \left( \frac{B_{Lg}^{s,1+\delta}}{B_L^{g,1+\delta} \gamma_{sg}^{(0),1+\delta} - \gamma_{ss}^{(0),1+\delta}} \right) F_2(x, Q^2) \right] + O(\alpha^2, x^{2-\delta}, \alpha x^{1-\delta}) \tag{8}
\]

\[
F_L(x, Q^2) = -\frac{B_L^{g,1+\delta}(1 + \alpha B_L^{g,1+\delta})}{\gamma_{sg}^{(0),1+\delta} + \gamma_{sg}^{(1),1+\delta}} \xi \left[ \frac{dF_2(x, Q^2)}{d\ln Q^2} + \frac{\alpha}{2} \left( \frac{B_{Lg}^{s,1+\delta}}{B_L^{g,1+\delta} \gamma_{sg}^{(0),1+\delta} - \gamma_{ss}^{(0),1+\delta}} \right) F_2(x, Q^2) \right] + O(\alpha^2, x^{1-\delta}), \tag{9}
\]

where

\[
\gamma_{sg}^{(1),\eta} = \gamma_{sg}^{(1),\eta} + B_{Lg}^{g,\eta} + B_{Lg}^{g,\eta}(2\beta_0 + \gamma_{gg}^{(0),\eta} - \gamma_{ss}^{(0),\eta}), \quad \gamma_{ss}^{(1),\eta} = R_{Lg}^{g,\eta} - B_{Lg}^{g,\eta} \frac{B_L^{Lg,\eta}}{B_L^{Lg,\eta}}
\]

In principle any equation from above formulae (8), (9) may be used, because there is a strong cancelation between the shifts in the arguments of the function \( F_2 \) and its derivative, and the shifts in the coefficients in front of them. The difference lies in the degree of accuracy one can reach with them, which depends on the \( x \) and \( Q^2 \) region of interest.

For concrete values of \( \delta = 0.5 \) and \( \delta = 0.3 \) we obtain (for \( f=4 \) and \( \overline{\text{MS}} \) scheme):

if \( \delta = 0.5 \)

\[
F_L(x, Q^2) = \frac{0.87}{1 + 22.9\alpha} \left[ \frac{dF_2(0.70x, Q^2)}{d\ln Q^2} + 4.17\alpha F_2(0.70x, Q^2) \right] + O(\alpha^2, x^{2-\delta}, \alpha x^{1-\delta}) \tag{10}
\]

\[
F_L(x, Q^2) = \frac{1.04}{1 + 22.9\alpha} \left[ \frac{dF_2(x, Q^2)}{d\ln Q^2} + 4.17\alpha F_2(x, Q^2) \right] + O(\alpha^2, x^{1-\delta}) \tag{11}
\]

if \( \delta = 0.3 \)

\[
F_L(x, Q^2) = \frac{0.84}{1 + 59.3\alpha} \left[ \frac{dF_2(0.48x, Q^2)}{d\ln Q^2} + 3.59\alpha F_2(0.48x, Q^2) \right] + O(\alpha^2, x^{2-\delta}, \alpha x^{1-\delta}) \tag{12}
\]

\[
F_L(x, Q^2) = \frac{1.05}{1 + 59.3\alpha} \left[ \frac{dF_2(x, Q^2)}{d\ln Q^2} + 3.59\alpha F_2(x, Q^2) \right] + O(\alpha^2, x^{1-\delta}) \tag{13}
\]

2. Assuming the non-Regge-like behaviour for the gluon distribution and \( F_2(x, Q^2) \):

\[
g(x, Q^2) = \frac{\exp\left(\frac{1}{2}\sqrt{\delta g(Q^2)ln\frac{1}{x}}\right)}{(2\pi \delta g(Q^2)ln\frac{1}{x})^{1/4}} \tilde{g}(x, Q^2), \quad F_2(x, Q^2) = \frac{\exp\left(\frac{1}{2}\sqrt{\delta g(Q^2)ln\frac{1}{x}}\right)}{(2\pi \delta g(Q^2)ln\frac{1}{x})^{1/4}} \tilde{s}(x, Q^2),
\]

\(^8\)The LO analysis was given already in [\ref{ref}]
we obtain the following equation for the $Q^2$ derivative of the SF $F_2$:

$$
\frac{dF_2(x, Q^2)}{dlnQ^2} = -\frac{1}{2} \sum_{p=s,g} \exp\left(\frac{1}{2} \sqrt{\delta_p(Q^2)ln\frac{Q^2}{x}}\right) \left(\tilde{r}_{sp}^1(\alpha) \tilde{p}(0, Q^2) + O(x^1)\right),
$$

(14)

where $\tilde{r}_{sp}^1(\alpha)$ and $\tilde{r}_{lp}^1(\alpha)$ can be obtained from corresponding functions $r_{sp}^{1+\delta}(\alpha)$ and $r_{lp}^{1+\delta}(\alpha)$, respectively, replacing the singular term $1/\delta$ at $\delta \to 0$ by $1/\tilde{\delta}$:

$$
\frac{1}{\delta} \delta \to \frac{1}{\tilde{\delta}} = \sqrt{\frac{ln(1/x)}{\delta_p(Q^2)}} - \frac{1}{4\delta_p(Q^2)} \left[ 1 + \sum_{m=1}^{\infty} \frac{1 \times 3 \times ... \times (2m-1)}{4\delta_p(Q^2) ln(1/x)^m} \right]
$$

(16)

The singular term appears only in the NLO part of the AD $\gamma_{sp}^{(1),1+\delta}$ and the longitudinal Wilson coefficients $R_{lp}^{p,1+\delta}$ in Eq. (14). The replacement (16) corresponds to the following transformation:

$$
\gamma_{sp}^{(1),1+\delta} \equiv \tilde{\gamma}_{sp}^{(1),1+\delta} \overset{\delta \to 0}{\longrightarrow} \tilde{\gamma}_{sp}^{(1),1+\delta} = \gamma_{sp}^{(1),1+\delta} + \gamma_{sp}^{(1),1}
$$

$$
R_{lp}^{p,1+\delta} \equiv \tilde{R}_{lp}^{p,1+\delta} \overset{\delta \to 0}{\longrightarrow} \tilde{R}_{lp}^{p,1+\delta} = R_{lp}^{p,1+\delta} + \tilde{R}_{lp}^{p,1}
$$

(17)

where $\tilde{\gamma}_{sp}^{(1),1+\delta}$ ($\tilde{R}_{lp}^{p,1+\delta}$) and $\gamma_{sp}^{(1),1+\delta}$ ($R_{lp}^{p,1+\delta}$) are the coefficients corresponding to singular and regular parts of $\gamma_{sp}^{(1),1+\delta}$ ($R_{lp}^{p,1+\delta}$), respectively.

We restrict here our calculations to $O(x)$ because at $O(x^2)$ one obtains an additional factor in front of the function $F_2$ and its derivative, which complicates very much the final formulae.

Repeating the analysis of the previous section step by step using the replacement (17), we get (for $f=4$):

$$
F_L(x, Q^2) = \frac{1}{(1 + 30\alpha[1/\delta - \frac{116}{45}])} \left[ \frac{dF_2(x, Q^2)}{dlnQ^2} + \frac{8}{3} \alpha F_2(x, Q^2) \right] + O(\alpha^2, x)
$$

(18)

We have combined equations (9) and (18) in a more general formula valid for any value of $\delta$:

$$
F_L(x, Q^2) = -2 \frac{B_{L}^{g,1+\delta}}{\gamma_{sg}^{(0),1+\delta} + \tilde{\gamma}_{sg}^{(1),1+\delta}} \alpha \left[ \frac{dF_2(x, Q^2)}{dlnQ^2} + \alpha \left( \frac{B_{L}^{g,1+\delta} - \gamma_{ss}^{(0),1+\delta}}{B_{L}^{g,1+\delta}} F_2(x, Q^2) \right) \right] + O(\alpha^2, x^{-1-\delta}),
$$

(19)

9Using a lower approximation $O(x)$ is not very exact, because in this case $F_2$ and the gluon distribution can contain an additional factor in the form of a series $1 + \sum_{k} (1/\delta_p/ln(1/x))^k$, which is determined by boundary conditions (see discussion in Ref. [40]). We will not consider the appearance of this factor in our analysis.
where $\tilde{\gamma}^{(1),1+\delta}_{sg}$ and $\overline{R}_{L}^{(1),1+\delta}$ coincide with $\gamma^{(1),1+\delta}_g$ and $\overline{R}_{L}^{(1),1+\delta}$, respectively, with the replacement:

$$\frac{1}{\delta} \rightarrow \int_{x}^{1} \frac{dy}{y} g(y,Q^2) g(x,Q^2)$$

In the cases $x^{-\delta} \gg Const$ and $\delta \rightarrow 0$, the r.h.s. of (20) leads to $1/\delta$ and $1/\tilde{\delta}$, respectively.

3. In Fig. 1 it is shown the accuracy of Eqs. (10)- (13) and (18) in the reconstruction of $F_L$ at various $\delta$ values from MRS sets at $Q^2=20$ GeV$^2$. We have chosen for this test MRS(D$^0$) ($\delta=0$), MRS(D$_-$) ($\delta=0.5$) and MRS(G) ($\delta=0.3$) as three representative densities (see ref. [8] and references therein). It can be observed in Fig. 1a that using the formulae (10) and (11) one gets very good agreement with the input parameterization MRS(D$_-$) (less than 1 %) at low $x$.

Fig. 1b shows the degree of accuracy of the reconstruction formulae (12) and (13) with $\delta = 0$. Here one should expect the set MRS (G) to give also a very good ($\sim 1\%$ level) agreement, however this is not the case because set (G) distinguishes the exponents of the sea-quark part $\delta_s \sim 0$ from the gluon density ($\delta_g = 0.3$). Thus, Eq. (12) might be slightly modified to treat this case. Note that the agreement is improved when $x$ values decrease because the relative importance of the quark contribution becomes smaller.

Fig. 1c deal with the case $\delta = 0$. As in Fig. 1a, one can observe a very good accuracy in the reconstruction when $Q^2_0$ is closed to that of the test parameterization ($4 \text{ GeV}^2$ for MRS set). Notice also the lost of accuracy at high $x$ due to the importance of the $O(\alpha^2_s)$ terms neglected in Eq.(18).

With the help of Eqs. (10), (12) and (18) we have extracted the longitudinal SF $F_L(x,Q^2)$ from HERA data, using the slopes $dF_2/d\ln Q^2$ determined in ref. [19] and ref. [20]. When H1 data are used, the value of $F_2$ in Eq. (10) was directly taken from the parameterization given by H1 in ref. [4]. With ZEUS data we substitute directly the $F_2$ values presented in table 1 of ref. [20]. We have checked that the use of the H1 parameterization for $F_2$ when dealing with ZEUS data, does not change significantly the $F_L(x,Q^2)$ result. Another input ingredient in the extraction formulae is $\alpha_s(Q^2)$. We have used the NLO QCD approximation with $\Lambda_{\overline{MS}} = 225$ MeV, even though the results are no very sensitive to this value. For example, a variation in $\Lambda$ of around ±50 MeV changes the results less that a 1 %.

Figure 2 shows the extracted values of the longitudinal SF and the prediction in QCD using MRS sets (G), (D$_-$) and (D$^0$) and the $O(\alpha^2_s)$ coefficients calculated in ref. [21]. For comparison we have included in the same figure the results from different formulae. In Fig. 2a the points extracted with $\delta = 0$ and $\delta = 0.5$ are spread over a band which could be considered as an indication of the theoretical uncertainty of the method, if $\delta$ were completely unknown. However, in a realistic situation, the uncertainty should be smaller if one could restrict in advance the value of $\delta$, as it is discussed below.

On the other hand, the deviation between the data points using $\delta = 0$ and the prediction of $F_L$ from MRS(D$^0$) parton distributions, is a signal that the formula is inadequate for the extraction of $F_L$. In this case the origin of the discrepancy is not clear. It could be
due to the importance of other contributions, not considered in the formula, or perhaps simply that $\delta$ is large.

In general it can be observed that the agreement, within the errors, with the calculation from sets MRS(G) and MRS(D$^-$) is excellent. There is also a relative good agreement with a preliminary experimental H1 point for $F_L$ ref. \cite{2}, if one takes into account the systematic error, not shown in Fig. 2a.

4. In summary, we have presented Eqs. (8)-(12) for the extraction of the longitudinal SF $F_L$ at small $x$ from the SF $F_2$ and its $Q^2$ derivative. These equations provide the possibility of the non-direct determination of $F_L$. This is important since the direct extraction of $F_L$ from experimental data is a cumbersome procedure (see \cite{3}). Moreover, the fulfillment of Eqs. (8)-(13) in DIS experimental data is a cross-check of perturbative QCD at small values of $x$.

We have found, as in the case of the gluon extraction formulas \cite{4}, that for singular type of partonic densities the results do not depend practically on the concrete value of the slope $\delta$, due to a cancelation of that dependence between certain coefficients. However, when $\delta \to 0$, the coefficients in front of $dF_2(x, Q^2)/dlnQ^2$ and $F_2(x, Q^2)$ have singularities leading to terms $\sim \sqrt{ln(1/x)}$. In this case there is a strong correlation between the results and the concrete form of small $x$ asymptotics of $F_2(x, Q^2)$.

Consequently, before to apply these formulae, some study of the experimental data is necessary in order to verify the type of $F_2(x, Q^2)$ behavior at $x \to 0$ (i.e. the value of $\delta$). Note that this study should be done at a fixed value of $Q^2$ and it does not require the knowledge of the quark and gluon content. For example, in Ref. \cite{15} it was suggested the determination of the slope from the observable $dF_2(x, Q^2)/dlnx$, which was measured in \cite{19}.

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\footnote{This happens in the framework of the double-logarithmic asymptotic. The singularities lead to terms $\sim ln(1/x)$ in the case of Regge-like asymptotic (see \cite{8}, \cite{14}, \cite{4}).}
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Figure captions

Figure 1: Relative difference between the reconstructed longitudinal SF using formulae in text and different input parameterizations at $Q^2 = 20 \text{ GeV}^2$.

Figure 2: The longitudinal SF $F_L$. The points were extracted from Eqs. (10) and (18) using H1 [19] data (Fig. 2a) and from Eqs. (10) and (12) using ZEUS [20] data (Fig. 2b). Solid, dashed and dotted lines are the calculation from sets MRS(G), MRS(D−) and MRS(D0) [8] using $O(\alpha_s^2)$ corrections. It is also shown a BCDMS data point at $x = 0.1$ and a preliminary H1 data point.
Fig. 1

(a) $\delta=0.5$, set (D$_{-}$)

Eq. (11)  
Eq. (10)

(b) $\delta=0.3$, set (G)

Eq. (12)  
Eq. (13)

(c) $\delta=0$, set (D$_{0}$)

$Q_{0}^{2}=3$ GeV$^{2}$  
$Q_{0}^{2}=4$ GeV$^{2}$  
$Q_{0}^{2}=5$ GeV$^{2}$

$X$
