Extending the Frontiers—Reconciling Accelerator and Cosmic Ray p-p Cross Sections

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Abstract

We simultaneously fit a QCD-inspired parameterization of all accelerator data on forward proton-proton and antiproton-proton scattering amplitudes, together with cosmic ray data (using Glauber theory), to predict proton-air and proton-proton cross sections at energies near $\sqrt{s} \approx 30$ TeV. The p-air cosmic ray measurements provide a strong constraint on the inclusive particle production cross section, as well as greatly reducing the errors on the fit parameters—in turn, greatly reducing the errors in the high energy proton-proton and proton-air cross section predictions.

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The energy range of cosmic ray experiments covers not only the energy of the Large Hadron Collider (LHC), but extends beyond it. Cosmic ray experiments can measure the penetration in the atmosphere of these very high energy protons—however, extracting proton-proton cross sections from cosmic ray observations is far from straightforward [1]. By a variety of experimental techniques, cosmic ray experiments map the atmospheric depth at which cosmic ray initiated showers develop. The measured quantity is the shower attenuation length ($\Lambda_m$), which is not only sensitive to the interaction length of the protons in the atmosphere ($\lambda_{p\text{-air}}$), with

$$\Lambda_m = k\lambda_{p\text{-air}} = k\frac{14.5m_p}{\sigma_{p\text{-air}}^{\text{inel}}}, \tag{1}$$

but also depends critically on the inelasticity, which determines the rate at which the energy of the primary proton is dissipated into electromagnetic shower energy observed in the experiment. The latter effect is taken into account in Eq. (1) by the parameter $k$; $m_p$ is the proton mass and $\sigma_{p\text{-air}}^{\text{inel}}$ the inelastic proton-air cross section.

The extraction of the pp cross section from the cosmic ray data is a two stage process. First, one calculates the $p$-air total cross section from the inelastic cross section inferred in Eq. (1), where

$$\sigma_{p\text{-air}}^{\text{inel}} = \sigma_{p\text{-air}} - \sigma_{p\text{-air}}^{\text{el}} - \sigma_{p\text{-air}}^{q\text{-el}}. \tag{2}$$

Next, the Glauber method [2] transforms the value of $\sigma_{p\text{-air}}^{\text{inel}}$ into a proton-proton total cross section $\sigma_{pp}$; all the necessary steps are calculable in the theory, but depend sensitively on a knowledge of $B$, the slope of $\frac{d\sigma_{el}}{dt}$, the $pp$ differential elastic scattering cross section, where

$$B = \left[ \frac{d}{dt} \left( \ln \frac{d\sigma_{el}}{dt} \right) \right]_{t=0}. \tag{3}$$

In Eq. (2) the cross section for particle production is supplemented with $\sigma_{p\text{-air}}^{\text{el}}$ and $\sigma_{p\text{-air}}^{q\text{-el}}$, the elastic and quasi-elastic cross section, respectively, as calculated by the Glauber theory, to obtain the total cross section $\sigma_{p\text{-air}}$. We show in Fig. 1 plots of $B$ as a function of $\sigma_{pp}$, for 5 curves of different values of $\sigma_{p\text{-air}}^{\text{inel}}$. This

![Figure 1: The $B$ dependence of the $pp$ total cross section $\sigma_{pp}$. The five curves are lines of constant $\sigma_{p\text{-air}}^{\text{inel}}$, of 440, 490, 540, 590 and 640 mb—the central value is the published Fly’s Eye value, and the others are $\pm 1\sigma$ and $\pm 2\sigma$. The dashed curve is a plot of our QCD-inspired fit of $B$ against $\sigma_{pp}$. The dot is our fitted value for $\sqrt{s} = 30$ TeV, the Fly’s Eye energy.](image)

summarizes the reduction procedure from the measured quantity $\Lambda_m$ (of Eq. 1) to $\sigma_{pp}$. Also plotted in Fig. 1 is a curve (dashed) of $B$ vs. $\sigma_{pp}$ which will be discussed later. Two significant drawbacks of this extraction method are that one needs:

1. a model of proton-air interactions to complete the loop between the measured attenuation length $\Lambda_m$ and the cross section $\sigma_{p\text{-air}}^{\text{inel}}$, i.e., the value of $k$ in Eq. (1).
a simultaneous relation between \( B \) and \( \sigma_{pp} \) at very high energies—well above the region currently accessed by accelerators.

A proposal to minimize the impact of theory on these needs is the topic of this note. We have constructed a QCD-inspired parameterization of the forward proton-proton and proton-antiproton scattering amplitudes which is analytic, unitary and fits all accelerator data of \( \sigma_{tot} \), \( B \) and \( \rho \), the ratio of the real-to-imaginary part of the forward scattering amplitude; see Fig. 2. In addition, the high energy cosmic ray data of Fly’s Eye and AGASSA experiments are also simultaneously used, i.e., \( k \) from Eq. (1) is also a fitted quantity—we refer to this fit as a global fit. We emphasize that in the global fit, all 4 quantities, \( \sigma_{tot} \), \( B \), \( \rho \) and \( k \), are simultaneously fitted. Because our parameterization is both unitary and analytic, its high energy predictions are effectively model-independent, if you require that the proton is asymptotically a black disk. Using vector meson dominance and the additive quark models, we find further support for our QCD fit—it accommodates a wealth of data on photon-proton and photon-photon interactions without the introduction of new parameters. In particular, it also simultaneously fits \( \sigma_{pp} \) and \( B \), forcing a relationship between the two. Specifically, the \( B \) vs. \( \sigma_{pp} \) prediction of our fit completes the relation needed (using the Glauber model) between \( \sigma_{pp} \) and \( \sigma_{inel}^{p-air} \). The percentage error in the prediction of \( \sigma_{pp} \) at \( \sqrt{s} = 30 \) TeV is \( \approx 1.2\% \), due to the statistical error in the fitting parameters (see references [3, 8]). A major difference between the present result, in which we simultaneously fit the cosmic ray and accelerator data, and our earlier result [7], in which only accelerator data are used, is a significant reduction (about a factor of 2.5) in the errors of \( \sigma_{pp} \) at \( \sqrt{s} = 30 \).

In Fig. 3, we have plotted the values of \( \sigma_{pp} \) vs. \( \sigma_{p-air}^{inel} \) that are deduced from the intersections of our \( B-\sigma_{pp} \) curve with the \( \sigma_{p-air}^{inel} \) curves of Fig. 1. Figure 3 allows the conversion of measured \( \sigma_{p-air}^{inel} \) cross sections to \( \sigma_{pp} \) total cross sections. The percentage error in \( \sigma_{p-air}^{inel} \) is \( \approx 0.8\% \) near \( \sigma_{p-air}^{inel} = 450 \) mb, due to the errors in \( \sigma_{pp} \) and \( B \) resulting from the errors in the fitting parameters. Again, the global fit gives an error of a factor of about 2.5 smaller than our earlier result [7], a distinct improvement.

When we confront our predictions of the p-air cross sections (\( \sigma_{p-air}^{inel} \)) as a function of energy with published cross section measurements of the Fly’s Eye and AGASSA groups, we find that the predictions systematically are about one standard deviation below the published cosmic ray values. It is at this point important to recall Eq. (1) and remind ourselves that the measured experimental quantity is \( \Lambda_m \) and not \( \sigma_{p-air}^{inel} \). We emphasize that the extraction of \( \sigma_{p-air}^{inel} \) from the measurement of \( \Lambda_m \) requires knowledge of the parameter \( k \). The measured depth \( X_{\text{max}} \) at which a shower reaches maximum development in the atmo-
sphere, which is the basis of the cross section measurement in Ref. [5], is a combined measure of the depth of the first interaction, which is determined by the inelastic cross section, and of the subsequent shower development, which has to be corrected for. \(X_{\text{max}}\) increases logarithmically with energy with elongation rate (\(\Delta X_{\text{max}}\) per decade of Lab energy) of 50–60 g/cm\(^2\) in calculations with QCD-inspired hadronic interaction models. The position of \(X_{\text{max}}\) directly affects the rate of shower attenuation with atmospheric depth, which is the alternative procedure for extracting \(\sigma_{\text{inel}}\). The rate of shower development and its fluctuations are the origin of the deviation of \(k\) from unity in Eq. (1). Its predicted values range from 1.5 for a model where the inclusive cross section exhibits Feynman scaling, to 1.1 for models with large scaling violations [1]. The comparison between prediction and experiment is further confused by the fact that the AGASA [6] and Fly’s Eye [5] experiments used different values of \(k\) in the analysis of their data, i.e., AGASA used \(k = 1.349\) and Fly’s Eye used \(k = 1.6\).

We therefore decided to let \(k\) be a free parameter and to make a global fit to the accelerator and cosmic ray data, as emphasized earlier. This neglects the possibility that \(k\) may show a weak energy dependence over the range measured. Recently, Pryke [9] has made Monte Carlo model simulations that indicate that \(k\) is compatible with being energy-independent. Using an energy-independent \(k\), we find that \(k = 1.349 \pm 0.045\), where the error in \(k\) is the statistical error of the global fit. By combining the results of Fig. 2 (a) and Fig. 3, we can predict the variation of \(\sigma_{\text{inel}}^{p-\text{air}}\) with energy, \(\sqrt{s}\). In Fig. 4 we have rescaled the published high energy data for \(\sigma_{\text{inel}}^{p-\text{air}}\) (using the common value of \(k = 1.349\)), and plotted the revised data against our prediction.
sections are readily reconcilable using a value of $k$ respectively, at interactions. We predict high energy $\sigma$ and energy independent—this determination of $k$ severely constrains any model of high energy hadronic interactions. We predict high energy $\sigma_{pp}$ and $\sigma_{p-\text{air}}^{\text{incl}}$ cross sections that are accurate to $\approx 1.2\%$ and $0.8\%$, respectively, at $\sqrt{s} = 30$ TeV.

At the LHC ($\sqrt{s} = 14$ TeV), we predict $\sigma_{\text{tot}} = 107.9 \pm 1.2$ mb for the total cross section, $B = 19.59 \pm 0.11$ (GeV/c)$^{-2}$ for the nuclear slope and $\rho = 0.117 \pm 0.001$, where the quoted errors are due to the statistical errors of the fitting parameters.

In conclusion, the overall agreement between the accelerator and the cosmic ray $pp$ cross sections with our QCD-inspired fit, as shown in Fig. 3, is striking. We find that the accelerator and cosmic ray $pp$ cross sections are readily reconcilable using a value of $k = 1.349 \pm 0.045 \pm 0.028$, which is both model independent and energy independent—this determination of $k$ severely constrains any model of high energy hadronic interactions.

Recently, Pryke[9] has published a comparative study of high statistics simulated air showers for proton primaries, using four combinations of the MOCCA[10] and CORSIKA[11] program frameworks, and SIBYLL[12] and QGSJet[13] high energy hadronic interaction models. He finds $k = 1.30 \pm 0.04$ and $k = 1.32 \pm 0.03$ for the CORSIKA-QGSjet and MOCCA-Internal models, respectively, which are in excellent agreement with our measured result, $k = 1.349 \pm 0.045 \pm 0.028$.

Further, Pryke[9] obtains $k = 1.15 \pm 0.03$ and $k = 1.16 \pm 0.03$ for the CORSIKA-SIBYLL and MOCCA-SIBYLL models, respectively, whereas the SYBILL[12] group finds $k = 1.2$, which is not very different from the Pryke value. However, the SYBILL-based models, with $k = 1.15-1.20$, are significantly different from our measurement of $k = 1.349 \pm 0.045 \pm 0.028$. At first glance, this appears somewhat strange, since our model for forward scattering amplitudes and SIBYLL share the same underlying physics. The increase of the total cross section with energy to a black disk of soft partons is the shadow of increased particle production which is modeled in SYBILL by the production of (mini)-jets in QCD. The difference between the $k$ values of 1.15–1.20 and 1.349 results from the very rapid rise of the pp cross section in SIBYLL at the highest energies. This is an artifact of the fixed cutoff in transverse momentum used to compute the mini-jet production cross section, and is not a natural consequence of the physics in the model. There are ways to remedy this.

The plot of $\sigma_{pp}$ vs. $\sqrt{s}$, including the rescaled cosmic ray data is shown in Fig. 3. Clearly, we have an excellent fit, with good agreement between AGASA and Fly’s Eye. In order to extract the cross sections’ energy dependence from the cosmic ray data, the experimenters of course assigned energy values to their cross sections. Since the cosmic ray spectra vary so rapidly with energy, we must allow for systematic errors in energy dependence from the cosmic ray data, the experimenters of course assigned energy values to their cross sections. There are ways to remedy this.

In conclusion, the overall agreement between the accelerator and the cosmic ray $pp$ cross sections with our QCD-inspired fit, as shown in Fig. 3, is striking. We find that the accelerator and cosmic ray $pp$ cross sections are readily reconcilable using a value of $k = 1.349 \pm 0.045 \pm 0.028$, which is both model independent and energy independent—this determination of $k$ severely constrains any model of high energy hadronic interactions.
In the near term, we look forward to the possibility of repeating this analysis with the higher statistics of the HiRes[14] cosmic ray experiment that is currently in progress and the Auger [15] Observatory.

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