Possible Molecular States Composed of Doubly Charmed Baryons with Couple-channel Effect

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Abstract. We systematically investigate the possible molecular states composed of (1) two spin- ¹/₂ doubly charmed baryons, and (2) a pair of spin- ¹/₂ and spin- ¹/₂ doubly charmed baryons. The one-boson-exchange (OBE) model is used to describe the potential between two baryons. The channel mixing effect is considered for the systems with the same quantum number (J(P)) but different total spin (S) and orbital angular momenta (L). We also study the channel mixing effect among the systems composed of various doubly charmed baryons if they have the same quantum number. Many of the systems are good candidates of molecular states.

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1 Introduction

The researches on exotic states attract much attention since the first charmonium-like exotic state X(3872) was reported by the Belle Collaboration in 2003. After that, more and more exotic states have been observed by many major experimental collaborations, such as CLEO, BaBar, Belle, BESIII, CDF, D0, LHCb and CMS. Those states include charmonium-like states such as Zc(3900)², Zc(4020)², Y(4260) ⁵c, Y(4140)⁷, and bottomonium-like states, Zb(10610) and Zb(10650)⁸ and so on. In 2015, the LHCb Collaboration discovered two pentaquark states Pₓ(4380) and Pₓ(4450) in the J/Φ invariant mass spectrum of the Λₓ⁰ → J/ΦK⁻ p decay [⁹]. Recently, the LHCb Collaboration reported that Pₓ(4450) was resolved into two states Pₓ(4440) and Pₓ(4457) and observed a lower state Pₓ(4312)¹⁰. The experimental and theoretical progress about exotic states can be found in the recent reviews [¹¹,¹²,¹³,¹⁴,¹⁵,¹⁶,¹⁷,¹⁸].

Some multiquark exotic states are near the threshold of two hadrons. They might be molecular states. A hadronic molecular state is a loosely bound system composed of two color-singlet hadrons. The two hadrons are bound together by the residual force of the color interaction. One can use the One-boson-exchange (OBE) model to describe the dynamics between the two baryons in a molecular system. The OBE model is very successful to describe the deuteron as a hadronic molecular state composed of a neutron and a proton. The meson exchange force together with the couple-channel effect between S-wave and D-wave renders the deuteron a loosely bound state.

About forty years ago, Voloshin and Okun proposed the hadronic molecule composed of two heavy mesons [¹⁹]. De Rujula et al. interpreted the ψ(4040) as a D⁺D⁻ molecule in Ref. [²⁰]. Törnqvist used the one-pion-exchange (OPE) potential to calculate the possible charmed meson-antimeson molecular state [²¹,²²]. Besides, there are also many other calculations about possible hadronic molecular states, such as the combination of two mesons [²³,²⁴,²⁵,²⁶,²⁷,²⁸,²⁹], or two baryons [³⁰,³¹,³²,³³,³⁴,³⁵,³⁶]. Similarly, one can also explain the hidden charm pentaquark states as a molecular state formed by one heavy meson and one heavy baryon [³⁷,³⁸,³⁹,⁴⁰,⁴¹,⁴²,⁴³,⁴⁴,⁴⁵].

In 2017, the LHCb Collaboration reported the doubly charmed baryon Ξcc at 3621.40 ± 0.72(stat) ± 0.27(syst) MeV in the Λcc⁺K⁻π⁺π⁻ mass spectrum [⁴⁶]. As an important member in the baryon family, there are many theoretical works to calculate the mass of the doubly charmed baryon [⁴⁷,⁴⁸,⁴⁹,⁵⁰,⁵¹,⁵²,⁵³,⁵⁴]. It is also very interesting to investigate the possible molecular states containing doubly charmed baryons. The system composed of Ξcc and a charmed meson or baryon was calculated in Refs. [⁵⁵,⁵⁶]. The possible molecular system with Ξcc and a nucleon is studied in Refs. [⁵⁷,⁵⁸].

In Ref. [⁵⁹], the authors investigated the possible deuteron-like bound states composed of two spin- ¹/₂ doubly charmed baryons in the SU(3) flavor symmetry. In this work, we extend the same formalism to investigate the possible hadronic
molecular states composed of two spin-$\frac{3}{2}$ doubly charmed baryons (include $\Xi_{cc}$ and $\Omega_{cc}$), denoted as $B^*B^*$, as well as system one spin-$\frac{3}{2}$ and one spin-$\frac{1}{2}$ baryon, denoted as $B^*B$. We use the ŌBE model to describe the potential between two baryons. The couple-channel effect among $B^*B^*$, $B^*B$ and two spin-$\frac{1}{2}$ baryons, $BB$, are also included in this work. When we calculate the pure $B^*B^*$ and $B^*B$ systems, we consider the couple-channel effect from $D$-wave and $G$-wave.

We organize this work as follows. We give the theoretical formalism in Section 2, including the effective Lagrangian, coupling constants and the effective interaction potentials. We present the numerical results of the $B^*B^*$ systems and the $B^*B$ systems in Section 3. In the calculation, we also include the couple-channel effect among $BB$, $B^*B$ and $B^*B^*$. Then we discuss our results and conclude in Section 4. In Appendixes A and B, we collect some useful formulae and functions. We also calculate the systems composed of one baryon and one antibaryon, such as $B^*B^*$ and $B^*\bar{B}$. The results are collected in Appendix C.

## 2 Formalism

For a doubly charmed baryon, the two charm quarks can be treated as a static color source in the heavy quark limit. For the two charm quarks, their color wave function should be in the antisymmetric $\bar{3}$-representation. The spatial wave function of the two charm quarks is symmetric for the ground state. As a consequence, their spin wave function must be symmetric because of the Pauli principle. Therefore, the total spin of the two charm quark is 1, and the total spin of a ground doubly charmed baryon is $\frac{1}{2}$ or $\frac{3}{2}$. In the present work, we focus on the possible deuteron-like systems composed of two doubly charmed baryons with both spin-$\frac{3}{2}$, or one spin-$\frac{1}{2}$ baryon and one spin-$\frac{1}{2}$ baryon. The systems with the same quantum number but different components may couple with each other. We include the couple-channel effect in this work.

### 2.1 The Lagrangian

For convenience, we use column matrices to describe the doubly charmed baryons as follows,

$$ B = \left[ \xi_{cc}^+ \xi_{cc}^+ \Omega_{cc}^+ \right]^T, \quad B^{*\mu} = \left[ \xi_{cc}^+ \xi_{cc}^+ \Omega_{cc}^+ \right]^\mu T, $$

(1)

where $B$ and $B^{*\mu}$ denote the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ baryons, respectively. The exchanged mesons between two baryons are denoted by two matrices as follows,

$$ M = \begin{bmatrix} \frac{a^+}{\sqrt{2}} + \frac{a}{\sqrt{6}} & \frac{a^+}{\sqrt{2}} + \frac{a}{\sqrt{6}} & K^+ \\ \frac{a^+}{\sqrt{2}} - \frac{a}{\sqrt{6}} & \frac{a^+}{\sqrt{2}} - \frac{a}{\sqrt{6}} & K^0 \\ K^- & K^- & 0 \end{bmatrix}, $$

$$ V^\mu = \begin{bmatrix} \frac{\rho^+}{\sqrt{2}} + \frac{\rho}{\sqrt{6}} & K^{*+} & \gamma^\mu \\ \frac{\rho^+}{\sqrt{2}} - \frac{\rho}{\sqrt{6}} & K^{*0} & \gamma^\mu \\ K^{*-} & K^{*0} & \phi \end{bmatrix}, $$

(2)

where $M$ and $V^\mu$ denote the octet pseudoscalar mesons and the nonet vector mesons, respectively.

We construct the effective Lagrangian as follows,

$$ \mathcal{L} = \mathcal{L}_{\text{hh}} + \mathcal{L}_{\text{phh}} + \mathcal{L}_{\text{vhh}}, $$

(3)

for the scalar meson exchange

$$ \mathcal{L}_{\text{hh}} = g_{\sigma BB} B^\sigma B - g_{\sigma B^* B} \bar{B}^{*\sigma} \sigma B^\mu, $$

(4)

for the pseudoscalar meson exchange

$$ \mathcal{L}_{\text{phh}} = -\frac{g_{\sigma BB}}{2m_B} B^\sigma \gamma_5 \partial^\mu MB + \frac{g_{B^{*\sigma} B}}{2m_{B^*}} \bar{B}^{*\sigma} \gamma_5 \partial^\mu MB^\mu + \frac{g_{B^* B}}{m_B + m_{B^*}} B^{*\mu} \partial^\mu MB + h.c., $$

(5)

and for the vector meson exchange

$$ \mathcal{L}_{\text{vhh}} = g_{\nu B^*} \bar{B} \gamma_\nu \sigma B^\mu + \frac{f_{\nu B^*}}{2m_B} \bar{B} \sigma_{\mu \nu} \partial^\mu B + g_{\nu B^*} \bar{B} \gamma_\nu \sigma B^\mu - \frac{f_{\nu B^*}}{2m_B} \bar{B} \sigma_{\mu \nu} \partial^\mu B^\mu + \frac{i f_{\nu B^*}}{2\sqrt{m_B m_{B^*}}} \bar{B}^{\*\mu} (\partial^\mu \partial^\nu - \partial^\nu \partial^\mu) B^\nu, $$

(6)

The notations $g_{\sigma B^* B^*}$, $g_{\sigma B^* B}$, $g_{\sigma B^* B}$, $g_{\sigma B^* B}$, $g_{\sigma B^* B}$, and $f_{\nu B^*} B$ represent the coupling constants. $m_B$ and $m_{B^*}$ are the masses of the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ heavy baryons, respectively.

### 2.2 Coupling constants

The coupling constants in the effective Lagrangian (4-6) should be extracted from the experiment data. However, there are no experiment data for the doubly charmed baryon scattering with light mesons. Thus, we compare the coupling constants of doubly charmed baryons with the ones of the nucleons, which are known. With the help of the quark model, we get the relations between the coupling constants of doubly charmed baryons and nucleons.

The details of this method can be found in Ref. [60]. Here we directly show the relations between the two sets of coupling constants,

$$ g_{\sigma B^* B^*} = g_{\sigma B^* B^*} = \frac{1}{3} g_{\sigma NN}, $$

(7)
In the above formula, \( \rho \) doescalar meson exchange. For the vector meson exchange, \( \pi \) is exchanged. The superscripts \( s \), \( p \) and \( v \) are used to mark the scalar, pseudoscalar and vector mesons exchange potentials. The potentials can be divided into four terms. They are the central potential term, the spin-spin interaction term, the spin-orbit interaction term and the tensor term. The spin-spin, spin-orbit and tensor terms contain the spin-spin operator \( \Delta SS \), spin-orbit operator \( \Delta LS \) and tensor operator \( \Delta T \), respectively. The specific definition of those angular momentum dependent operators are different for the systems composed of baryons with different spins. We will discuss the operators in detail after we introduce the couple-channel effect. Here we use subscripts \( C \), \( SS \), \( LS \), \( T \) to mark them respectively. We derive the general potentials between two doubly charmed baryons in the specific formulae for different mesons exchanged as follows.
For the initial and the final baryons of one Fermion line the system composed of two baryons with mass differences between the initial and final baryons. For the pseudoscalar meson exchange, the value of the \( \sigma \) is 0 for the direct diagram. For \( \sigma \) can be easily expressed with \( \theta_\sigma^2 = -u_\sigma^2 \).

For the vector meson exchange, the potentials change into

\[
V_{SS}^v(r, \alpha) = C_\sigma^v \frac{g_\sigma g_\sigma^2}{4\pi} \frac{\theta_\sigma^2}{12m_a m_b} M_1 \Delta_{SS},
\]

\[
V_{T}^v(r, \alpha) = C_\sigma^v \frac{g_\sigma g_\sigma^2}{4\pi} \frac{\theta_\sigma^2}{12m_a m_b} M_3 \Delta_{T},
\]

where \( \theta_\sigma^2 = -u_\sigma^2 \).

The coupling constants for the doubly charmed baryons.

Table 2. The coupling constants for the doubly charged baryons.

| \( g_{BB} \) | \( g_{BB} \) | \( g_{BB} \) | \( f_{BB} \) | \( g_{BB} \) | \( g_{BB} \) | \( g_{BB} \) | \( f_{BB} \) | \( g_{BB} \) | \( g_{BB} \) | \( f_{BB} \) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( \Xi_e^* \Xi_e^* \) | 2.82 | -14.26 | 4.60 | -29.76 | \( \Xi_e^* \Xi_e^* \) | 2.82 | 43.55 | 4.60 | 72.25 | \( \Xi_e^* \Xi_e^* \) | 49.84 | -87.95 |
| \( \Xi_e^* \Omega_e^* \) | 2.82 | -14.57 | 4.60 | -30.30 | \( \Xi_e^* \Omega_e^* \) | 2.82 | 44.65 | 4.60 | 74.16 | \( \Xi_e^* \Omega_e^* \) | 50.91 | -89.83 |
| \( \Omega_e^* \Omega_e^* \) | 2.82 | -14.88 | 4.60 | -30.85 | \( \Omega_e^* \Omega_e^* \) | 2.82 | 45.75 | 4.60 | 76.12 | \( \Omega_e^* \Omega_e^* \) | 51.11 | -90.14 |
| \( \Omega_e^* \Omega_e^* \) | 52.18 | -92.07 |

In the above expressions, \( u_{\sigma/\alpha/\beta} \) are defined as \( u_{\sigma/\alpha/\beta} = m_{xx}^2 - Q_0^2 \), where \( m_{xx} \) is the mass of the exchanged meson. \( Q_0 \) is the zeroth component of the transition momentum.

In the heavy baryon limit, \( Q_0 \) can be easily expressed with mass differences between the initial and final baryons. For the system composed of two baryons with masses \( m_1 \) and \( m_2 \), the value of the \( Q_0 \) is 0 for the direct diagram. For the cross diagram, the value of the \( Q_0 \) is \( |m_1 - m_2| \). \( m_a \) and \( m_b \) are the masses of two doubly charmed baryons. For the initial and the final baryons of one Fermion line with the mass splitting, we approximately choose the geometric means of their masses, \( m_{ab} = \sqrt{m_a m_b} \). In Table 2 we give the masses of the baryons, as well as the masses of the exchanged mesons. For the multiple hadrons, their averaged masses are used.

\( g_s, g_p \) and \( g_v \) are the coupling constants in Eqs. (17-13). The subscript 1 and 2 of the coupling constants are used to mark different vertices. \( C_\sigma^v, C_\sigma^p \) and \( C_\sigma^v \) are the isospin factors. Their values are given in Table 3. For a system composed of different baryons, we should consider both direct and cross diagrams, as in Fig. 1. In Table 3 we use \( [ \ ] \) to denote the isospin factors of the cross diagrams. The functions \( H_{\alpha} = H_{\alpha}(A, m_{\alpha/\beta/\gamma}) \) and \( M_{\alpha} = M_{\alpha}(A, m_{\alpha/\beta/\gamma}) \) come from the Fourier transformation. We give their specific expressions in Appendix A.

For the system composed of one baryon and one antibaryon, the Lagrangians \( [6] \) still work. For the process exchanging a meson with certain G-parity, \( I_G \), the potentials are the Eqs. (17-20) with the extra G-parity factor \( I_G \). With the help of the G-parity rule, we can directly write
down the effect potentials of the baryon and antibaryon systems. We let the isospin factors of baryon-antibaryon system absorb the extra G-parity factor. The values are also given in Table 3. For some systems, some terms in the Lagrangian are lacking. Their potentials can be described with part of the formulae (4-6). In those cases we can also directly use the same potential but set the relevant coupling constant to zero.

We consider the couple-channel effect between states with different orbital angular momentum. The spin of a system composed of two spin-$\frac{3}{2}$ baryons can be 0, 1, 2 and 3. For a $J = 0$ system, we consider the couple-channel effect between $^1S_0$ and $^5D_0$. For a $J = 1$ system, we consider four channels, $^3S_1$, $^3D_1$, $^7D_1$ and $^7G_1$. For a system with total spin 2, there are also four channels, $^5S_2$, $^1D_2$, $^5D_2$ and $^5G_2$. And there are five channels for a $J = 3$ system, $^7S_3$, $^3D_3$, $^7D_3$, $^3G_3$ and $^7G_3$. We list all possible channels in Table 3. Their wave functions can be expressed as follows,

- For a $J = 0$ system,

$$\Psi(r, \theta, \phi)^T \chi_{ssz} = \begin{bmatrix} T_S(r) \\ 0 \end{bmatrix} |^1S_0 \rangle + \begin{bmatrix} 0 \\ T_D(r) \end{bmatrix} |^5D_0 \rangle .$$

- For $J = 1$ system,

$$\Psi(r, \theta, \phi)^T \chi_{ssz} = \begin{bmatrix} T_S(r) \\ 0 \\ 0 \end{bmatrix} |^3S_1 \rangle + \begin{bmatrix} 0 \\ T_D_1(r) \\ 0 \end{bmatrix} |^3D_1 \rangle + \begin{bmatrix} 0 \\ 0 \\ T_D_2(r) \end{bmatrix} |^7D_1 \rangle + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} |^7G_1 \rangle .$$

- For a $J = 2$ system,

$$\Psi(r, \theta, \phi)^T \chi_{ssz} = \begin{bmatrix} T_S(r) \\ 0 \\ 0 \end{bmatrix} |^5S_2 \rangle + \begin{bmatrix} 0 \\ T_D_1(r) \\ 0 \end{bmatrix} |^1D_2 \rangle + \begin{bmatrix} 0 \\ 0 \\ T_D_2(r) \end{bmatrix} |^5D_2 \rangle + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} |^5G_2 \rangle .$$

- For a $J = 3$ system,

$$\Psi(r, \theta, \phi)^T \chi_{ssz} = \begin{bmatrix} T_S(r) \\ 0 \\ 0 \end{bmatrix} |^7S_3 \rangle + \begin{bmatrix} 0 \\ 0 \\ T_D_1(r) \end{bmatrix} |^3D_3 \rangle + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} |^7D_3 \rangle + \begin{bmatrix} 0 \\ 0 \\ T_G_1(r) \end{bmatrix} |^3G_3 \rangle + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} |^7G_3 \rangle .$$

In the expression, $\Psi(r, \theta, \phi)$ and $\chi_{ssz}$ are the spatial and spin wave functions, respectively.

The spin of a system composed of one spin-$\frac{3}{2}$ baryon and one spin-$\frac{1}{2}$ baryon can be 1 and 2. For a $J = 1$ system, we consider the channels mixing effect between $^3S_1$, $^3D_1$ and $^5D_1$. For a system with total spin 2, there are four channels should be considered, $^5S_2$, $^3D_2$, $^5D_2$ and $^5G_2$. We also list them in Table 3. Their wave functions are the same.

The angular momentum dependent operators, $\Delta_{SS}$, $\Delta_{LS}$ and $\Delta_T$ have different forms for various combinations.
Table 3. The isospin factors for two baryon systems and baryon-antibaryon systems. The factors $I_{G}$ from G-parity rule have been absorbed by the isospin factors in the right panel.

| States | $C_{\alpha}^{d}$ | $C_{\beta}^{d}$ | $C_{\alpha}^{e}$ | $C_{\beta}^{e}$ | $C_{\alpha}^{p}$ | $C_{\beta}^{p}$ | $C_{\alpha}^{k}$ | $C_{\beta}^{k}$ | $C_{\alpha}^{cc}$ | $C_{\beta}^{cc}$ |
|--------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $[\Xi_{cc}^{-}\Xi_{cc}^{+}]^{J=0}$ | 1 | -3/2 | 1/6 | 0 | -3/2 | 1/2 | 0 | 0 | |
| $[\Xi_{cc}^{-}\Xi_{cc}^{+}]^{J=1}$ | 1 | 1/2 | 1/6 | 0 | 1/2 | 1/2 | 0 | 0 | |
| $[\Xi_{cc}^{-}\Xi_{cc}^{+}]^{J=3}$ | 1 | 0 | -1/3 | 0[1] | 0 | 0 | 0[1] | |
| $[\Omega_{cc}^{-}\Omega_{cc}^{+}]^{J=0}$ | 1 | 0 | 2/3 | 0 | 0 | 0 | 1 | 0 | |

Table 4. The channels considered in this work for two baryons systems.

| $B^* B^*$ | $B^* B$ |
|-----------|---------|
| $S$ | $D_1$ | $D_2$ | $G_1$ | $G_2$ | $S$ | $D_1$ | $D_2$ | $G$ |
| $J = 0$ | $[1S_0]$ | $[5D_0]$ | | | |
| $J = 1$ | $[1S_1]$ | $[3D_1]$ | $[7D_1]$ | $[7G_1]$ | $[3S_1]$ | $[3D_1]$ | $[5D_1]$ | |
| $J = 2$ | $[5S_2]$ | $[5D_2]$ | $[5G_2]$ | | $[5S_2]$ | $[5D_2]$ | $[5D_2]$ | $[5G_2]$ |
| $J = 3$ | $[7S_3]$ | $[7D_3]$ | $[7G_3]$ | | $[7S_3]$ | $[7D_3]$ | $[7G_3]$ | |

3.1 Two spin-$\frac{3}{2}$ baryon system

For the $\Xi_{cc}^{-}\Xi_{cc}^{+}$ and $\Omega_{cc}^{-}\Omega_{cc}^{+}$ systems, we should consider the Pauli Principle. For example, the total isospin for the $\Omega_{cc}^{-}\Omega_{cc}^{+}$ system is symmetric, the total spin for the system can only be 0 and 2. For the $\Xi_{cc}^{-}\Omega_{cc}^{+}$ systems, all combinations of spin and isospin are possible. We show the binding energies and the root-mean-square radii of possible molecular states in Table 6. Because of the tensor operator, the channels with the same total angular momentum but different spin and orbital angular momenta mix each other. We also show the percentage of the different channels in the Table.

For the $J = 0$ system, we consider the $[1S_0]$ and $[5D_0]$ wave mixing effect. The contribution of the $D$-wave is actually quite small, less than 2%. For the $[\Xi_{cc}^{-}\Xi_{cc}^{+}]^{J=0}$ system, we find a loosely bound state with the binding energy 2.37-15.93 MeV, while the cutoff parameter is 2.0-2.4 GeV. We present the potentials of the system when the cutoff parameter is 2.2 GeV in Fig. 2, where we use $V_{11}$, $V_{22}$ and $V_{12}$ to denote the potentials for the S-wave channel, the D-wave channel, and the off-diagonal term mixing the S-wave and D-wave, respectively. The only attractive potential between the baryons appears in the S-wave. For the $[\Xi_{cc}^{-}\Omega_{cc}^{+}]^{J=0}$ and $[\Omega_{cc}^{-}\Omega_{cc}^{+}]^{J=0}$ systems, we get binding solutions when we change the cutoff parameter from 1.8 GeV to 2.2 GeV. Both of them have small binding energies and reasonable root-mean-square radii.

For the $J = 1$ systems, we calculate four channel mixing effects among $[3S_1]$, $[3D_1]$, $[3G_1]$, $[3S_1]$, $[3D_1]$, $[5D_1]$ and $[3S_1]$, $[3D_1]$, $[5D_1]$. Actually the contribution of G-wave almost vanishes. For the $[\Xi_{cc}^{-}\Xi_{cc}^{+}]^{J=1}$ system, a bound state with the binding energy 25.28-39.72 MeV appears when the cutoff parameter is chosen between 0.82-0.84 GeV. Given that the binding energy is sensitive to the cutoff parameter, and the root-mean-square radius is less than 1 fm, the system may not be a perfect candidate of the molecular state. For the $[\Xi_{cc}^{-}\Xi_{cc}^{+}]^{J=1}$ system, we obtain a bound state with binding energy 1.07-20.89 MeV when the cutoff parameter is 1.8-2.2 GeV. The contributions of D-wave channels increase with the cutoff parameter. The contribution of the dominant S-wave channel is 88.6% when the cutoff is 2.2 GeV.

For the $\Xi_{cc}^{-}\Omega_{cc}^{+}$ and $\Omega_{cc}^{-}\Omega_{cc}^{+}$ systems with total spin 2, we obtain binding solutions with the cutoff parameter from 2.4 GeV to 2.6 GeV. The binding solution of the former system is 1.50-11.00 MeV, and 2.71-13.03 MeV for the latter one. There is no bound state for the $[\Xi_{cc}^{-}\Xi_{cc}^{+}]^{J=2}$ sys-
tem, even if we include the couple-channel effect and vary the cutoff parameter from 0.8 GeV to 3 GeV. The relevant interaction potentials with the cutoff parameter 1.0 GeV is shown in Fig. 3. The S-wave and D-wave potentials of the system, $V_{11}, V_{22}$ and $V_{33}$, are hardly attractive.

For the $J = 3$ case, there are only two possible systems, $|ξ_{cc}|^I_{J=3}$ and $|ξ_{cc}|^I_{J=3}$. We consider the mixing effect of five channels, $S^3, D^3, D^3, D^3$ and $G_3$. The binding energy of the $|ξ_{cc}|^I_{J=3}$ system is 2.65-8.87 MeV, when we change the cut off from 1.2 GeV to 1.4 GeV. The contribution of the channel $D^3$ is important, the percentage of which is over 10%. For the $|ξ_{cc}|^I_{J=3}$ system, the binding energy changes from 8.39 MeV to 25.78 MeV while the cutoff parameter changes from 1.12 GeV to 1.4 GeV. In the wave functions, the $S$-wave is dominant, whose contribution is over 99%. The root-mean-square radius of the bound state is 0.64-1.02 fm, which seems a little small for a loosely bound state composed of two doubly charmed baryons.

Among all possible systems composed of two spin-$\frac{3}{2}$ baryons, the systems $|ξ_{cc}|^I_{J=3}$, $|ξ_{cc}|^I_{J=3}$ and $|ξ_{cc}|^I_{J=3}$ are good candidates of molecular states. For the system $|ξ_{cc}|^I_{J=3}$, and $|ξ_{cc}|^I_{J=3}$, the existence of the bound solutions is very sensitive to cutoff parameter. Meanwhile, their root-mean-square radii are less than 1 fm. We do not find a binding solution for the $|ξ_{cc}|^I_{J=3}$ system.

### 3.2 One spin-$\frac{3}{2}$ baryon and one spin-$\frac{1}{2}$ baryon system

We investigate the possible molecular systems composed of one spin-$\frac{3}{2}$ baryon and one spin-$\frac{1}{2}$ baryon. Their total spin can be $\frac{1}{2}$ and $\frac{3}{2}$. For the $J = 1$ system, we consider the couple-channel effect among $3S_1, 3D_1$ and $5D_1$ channels. For the $J = 2$ system, the couple-channel effect is among $3S_2, 3S_2, 5D_2$ and $5G_2$ channels. We show the binding energies, root-mean-square radii and contributions of different channels of the $|ξ_{cc}|^I_{J=3}$ and $|ξ_{cc}|^I_{J=3}$ systems in Table 5.

For the $|ξ_{cc}|^I_{J=3}$ system, we obtain a loosely bound state with binding energy 4.73-12.57 MeV while the cutoff parameter is 1.2-1.6 GeV. The $D$-wave contribution is about 10% and decreases as the cutoff parameter increases. For the $|ξ_{cc}|^I_{J=3}$ system, the contribution of the $S$-wave is over 99%. When we choose the cutoff parameter at 1.05 GeV, the binding energy of the bound state is 8.34 MeV. For the $|ξ_{cc}|^I_{J=3}$ system, we find a bound state solution with binding energy 8.97 while the cutoff parameter is 1.25 GeV. For the $|ξ_{cc}|^I_{J=3}$ system, a bound solution with a dominant $S$-wave appears when the cutoff is 1.2 GeV-1.3 GeV. The result of $|ξ_{cc}|^I_{J=3}$ is very similar to that of $|ξ_{cc}|^I_{J=3}$. The two systems are related by the $U$-spin and the $V$-spin symmetry.

For the systems with total spin 2, the contributions of the $G$-wave channels are almost zero. For the $|ξ_{cc}|^I_{J=3}$ system, a loosely bound state with binding energy 2.33-10.15 MeV appears when the cutoff parameter is 1.2-1.6 GeV. The contribution of $D_2$ is about 8% when the cutoff is 1.4 GeV. For the $|ξ_{cc}|^I_{J=3}$ system, we obtain a binding solution with the binding energy 8.78 MeV while the cutoff parameter is 1.05 GeV. In the system, the $D$-waves contributions are less than 1%. For the $|ξ_{cc}|^I_{J=3}$ system, we obtain a binding solution with the binding energy 9.19 MeV when we choose the cutoff parameter as 1.25 GeV. The $D$-waves contributions are also very small. The results for $|ξ_{cc}|^I_{J=3}$ and $|ξ_{cc}|^I_{J=3}$ are almost the same. When we choose the cutoff from 1.0 GeV to 1.1 GeV, we find a bound state with binding energy 3.68-42.66 MeV for the former system, and a binding solution with binding energy 1.53-32.4 MeV for the latter one.

Considering the reasonable binding energies and root-mean-square radii of the above solutions, the spin-$\frac{1}{2}$ baryon and spin-$\frac{1}{2}$ baryon systems are all good candidates of molecular states.

### 3.3 The two doubly charmed systems with multi-channel mixing effect

We calculate the systems with channel mixing among $BB, B^*B$ and $B^*B^*$ in this subsection. $B$ and $B^*$ are the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ doubly charmed baryons, respectively. We present the possible systems with certain total spin and isospin in Tables 6 and 10. In the previous subsection, the couple-channel effect from the high angular momentum...
Table 6. The numerical results for the $B^*B^*$ systems. $\Lambda$ is the cutoff parameter. “B.E.” is the binding energy. $R_{rms}$ is the root-mean-square radius. $P_i$ is the percentage of the different channels.

| States | $\Lambda$(MeV) | E(MeV) | $R_{rms}$(fm) | $P_S$(%) | $P_{D1}$(%) | $P_{D2}$(%) | $P_{G1}$(%) | $P_{G2}$(%) |
|--------|----------------|--------|--------------|---------|-------------|-------------|-------------|-------------|
| $[\Xi^*_c\Xi^*_c]_{J=0}$ | 2000 | 2.37 | 2.25 | 99.5 | 0.5 |
| | 2200 | 7.46 | 1.45 | 98.9 | 1.1 |
| | 2400 | 15.93 | 1.17 | 97.9 | 2.1 |
| $[\Xi^*_c\Omega^*_c]_{J=0}$ | 1800 | 1.11 | 2.75 | 99.8 | 0.2 |
| | 2000 | 5.19 | 1.60 | 99.3 | 0.7 |
| | 2200 | 12.93 | 1.22 | 98.5 | 1.5 |

states are small. The components of $G$-wave in the total wave functions are usually negligible. Thus we only consider the $D$-wave in our calculation.

Because the masses of various systems are different, we define the binding energy relative to the $BB$ threshold. When we solve the coupled Schrödinger equations, we put the mass difference in the kinetic term. The effective potentials as well as the spin and orbital angular momentum dependent operators are consistent with what we defined before. However, we ignore the spin-orbital coupling effect in the off-diagonal elements of the interaction potentials. Because the effect is dependent on the external momentum, we can treat it as a high order correction compared with spin-spin and tensor interactions. Thus, it is reasonable to use only spin-spin and tensor interactions to describe the channel mixing effect. The numerical results including binding energies, root-mean-square radii and percentages of different channels are shown in Tables 11-13.

For the $\Omega^*_c\Omega^*_c$ system with $I(J^P) = 0(0^+)$, we find a binding solution when the cutoff is around 1.3 GeV. The dominant component of this solution is $\Omega_c\Omega^*_c|S_0\rangle$. The component of $\Omega^*_c\Omega^*_c|S_0\rangle$ increases with the cutoff parameter. The $D$-wave contributions to the system are insignificant. For the system with $I(J^P) = 0(2^+)$, we find a solution with the main component $\Omega_c\Omega^*_c|S_2\rangle$, which
agree with the single channel results in subsection 3.2. The mixing effect of the channel $\Omega_{cc}^*\Omega_{cc}^\ast |^3S_2\rangle$ is about 10% and should not be ignored.

For the $\Xi_{cc}^*\Xi_{cc}^\ast$ system with $I(J^P) = 0(1^+)$, we find a binding solution with a cutoff, 0.8 GeV. The channel mixing effect is very prominent among the $\Xi_{cc}\Xi_{cc}$, $\Xi_{cc}^*\Xi_{cc}^\ast$ and $\Xi_{cc}\Xi_{cc}^\ast$ with S-wave. Their contributions to the total wave function are about 50%, 35% and 15%, respectively. The D-wave channels mixing effect is also important. We obtain a binding solution with cutoff around 1.38 GeV. In the system, it is interesting that the D-wave contribution of $\Xi_{cc}\Xi_{cc}^\ast$ reaches up to 28%. For the $I(J^P) = 1(0^+)$ system, a binding solution with the main channel $\Xi_{cc}\Xi_{cc}^\ast |^1S_0\rangle$ appears when the cutoff is 1.06 GeV. For the $I(J^P) = 1(2^+)$ system, we obtain a binding solution based on $\Xi_{cc}\Xi_{cc}^\ast |^3S_1\rangle$. The result is consistent with the calculation considering only the couple-channel effect inside the $\Xi_{cc}\Xi_{cc}^\ast$ systems. The $\Xi_{cc}\Xi_{cc}^\ast$ channels contribute 7%-10% to the wave function, which makes the cutoff parameter a little larger when the binding energy is the same.

The isospin of the $\Xi_{cc}^*\Omega_{cc}^\ast$ systems is $\frac{1}{2}$. For the system with $J = 0$, we find a bound state with binding energy 1.21-28.64 MeV. The 97% component of the system is the $\Xi_{cc}\Omega_{cc}^* |^1S_0\rangle$ state when the cutoff is 1.17 GeV. The main part of the rest components is the S-wave of $\Xi_{cc}^*\Omega_{cc}$ for the $I(J^P) = \frac{1}{2}(1^+)$ system, we obtain a binding solution when the cutoff is around 1.2 GeV. The system has a main part of the $\Xi_{cc}\Omega_{cc}^* |^1S_0\rangle$ channel, which mixes with the channel $\Xi_{cc}^*\Omega_{cc}^\ast |^1S_0\rangle$. For the system with $J = 2$, a bound state solution appears when the cutoff is 1.1 GeV. The system is dominated by the S-waves of $\Xi_{cc}^*\Omega_{cc}$ and $\Xi_{cc}^\ast\Omega_{cc}^\ast$, and their contributions are 56.2% and 43.5%, respectively. For the system with $I(J^P) = \frac{1}{2}(3^+)$, we obtain a binding solution when the cutoff is around 1.3 GeV. In this system, the contribution of the $\Xi_{cc}^*\Omega_{cc}^\ast |^3S_1\rangle$ channel is over 93%, which is the only possible S-wave channel. The main component of the other channels are the $|^3D_3\rangle$ states of $\Xi_{cc}^*\Omega_{cc}$ and $\Xi_{cc}^\ast\Omega_{cc}^\ast$ each of which has the contribution about 2.7%.

In Ref. [34], the authors studied the systems composed of two spin-$\frac{1}{2}$ doubly charmed baryons. We show the results in Table 1 and make some comparison. After considering the channel mixing effect among the $BB$, $B^*B$ and $B^*B^*$ systems, we get some different conclusions for the $0(0^+)$, $1(0^+)$ and $\frac{1}{2}(0^+)$ systems. As an example, we show the potentials of the $0(0^+)$ systems in Fig. 4 where we give the potentials of the main channels. In the single channel calculation Ref. [33], the attraction of the $\Omega_{cc}^\ast\Omega_{cc}^\ast$ system is too weak to produce a bound state. But with the help of the channel mixing effects, especially the $\Omega_{cc}^\ast\Omega_{cc}^\ast |^1S_0\rangle$ channel, we can obtain a binding solution. The channel mixing effect plays the same role for the $1(0^+)$ and $\frac{1}{2}(0^+)$ systems.

Most of the $B^\ast(B^\ast)$ systems with the channel mixing effect are dominated by one channel with the percentage about 90%. However, the couple-channel effect from other channels may produce a bound state, which does not exist in the single channel case. Another interesting observation
Table 7. The numerical results for the $B^*B$ systems. $\Lambda$ is the cutoff parameter. “B.E.” is the binding energy. $R_{rms}$ is the root-mean-square radius. $P_i$ is the percentage of the different channels.

| States | $A$(MeV) | E(MeV) | $R_{rms}$(fm) | $P_S$(%) | $P_{D1}$(%) | $P_{D2}$(%) | $P_F$(%) |
|--------|----------|--------|---------------|---------|-----------|-----------|---------|
| $[\Xi^*_c \Xi^*_c]_{J=1}$ | 1200 | 4.73 | 2.56 | 87.3 | 6.1 | 6.6 |
| | 1400 | 8.29 | 2.15 | 88.0 | 5.4 | 6.6 |
| | 1600 | 12.57 | 1.86 | 90.2 | 4.3 | 5.5 |
| $[\Xi^*_c \Xi^*_c]_{J=1}$ | 1000 | 1.85 | 2.17 | 98.9 | 0.6 | 0.5 |
| | 1050 | 8.34 | 1.11 | 99.5 | 0.3 | 0.2 |
| | 1100 | 24.39 | 0.67 | 99.9 | 0.1 | 0.0 |
| $[\Omega^*_c \Omega^*_c]_{J=1}$ | 1200 | 3.01 | 1.64 | 99.5 | 0.3 | 0.2 |
| | 1250 | 9.68 | 1.06 | 99.5 | 0.3 | 0.1 |
| | 1300 | 24.39 | 0.67 | 99.9 | 0.1 | 0.0 |
| $[\Xi^*_c \Omega^*_c]_{J=1/2}$ | 1200 | 2.68 | 1.77 | 99.5 | 0.4 | 0.1 |
| | 1250 | 6.81 | 1.19 | 99.6 | 0.3 | 0.1 |
| | 1300 | 14.24 | 0.79 | 99.8 | 0.2 | 0.0 |
| $[\Xi^*_c \Omega^*_c]_{J=2}$ | 1200 | 2.33 | 2.06 | 99.2 | 0.0 | 0.8 |
| | 1050 | 8.78 | 1.05 | 99.6 | 0.0 | 0.4 |
| | 1100 | 25.32 | 0.64 | 99.9 | 0.1 | 0.0 |
| $[\Omega^*_c \Omega^*_c]_{J=2}$ | 1200 | 2.97 | 1.62 | 99.6 | 0.0 | 0.4 |
| | 1250 | 9.19 | 1.00 | 99.7 | 0.0 | 0.3 |
| | 1300 | 23.06 | 0.67 | 99.9 | 0.0 | 0.1 |

Table 8. Possible mixing channels for the $\Omega^*_c \Omega^*_c$ system with $J = 0$ and $J = 2$.

| $I(J^P)$ | $\Omega^*_c \Omega^*_c$ | $\Omega^*_c \Omega^*_c$ | $\Omega^*_c \Omega^*_c$ | $\Omega^*_c \Omega^*_c$ |
|----------|--------------------------|--------------------------|--------------------------|--------------------------|
| 0(0^+)  | $|1/2 S_0\rangle$ | $|1/2 S_0\rangle$ | $|5/2 D_0\rangle$ | $|5/2 D_0\rangle$ |
| 0(2^+)  | $|1/2 D_2\rangle$ | $|1/2 D_2\rangle$ | $|5/2 D_2\rangle$ | $|5/2 D_2\rangle$ |

is that the two or three main components of the systems are comparable, which may cause a large shift of the binding energy. Actually the numerical results are quite complicated with the couple-channel effect. The binding solutions become even more sensitive to the cutoff parameter.

4 Discussions and conclusions

In this work, with the help of the one-boson-exchange model, we systematically investigate the possible molecular systems composed of two spin-$\frac{3}{2}$ doubly charmed baryons,
Table 9. Possible mixing channels for the $\Xi_{cc}^{(*)} \Xi_{cc}^{(*)}$ system with total angular momentum 0, 1, 2 and 3.

| $I(J^P)$ | $\Xi_{cc} \Xi_{cc}$ | $\Xi_{cc}^{(*)} \Xi_{cc}$ | $\Xi_{cc} \Xi_{cc}^{(*)}$ |
|---------|-------------------|-------------------|-------------------|
| 0(1^+) | $|^3S_1\rangle$ | $|^3D_1\rangle$ | $|^3S_1\rangle$ |
| 0(3^+) | $|^3D_3\rangle$ | $|^5S_3\rangle$ | $|^3D_3\rangle$ |
| 1(0^+) | $|^5S_0\rangle$ | $|^5D_0\rangle$ | $|^5D_0\rangle$ |
| 1(2^+) | $|^5D_2\rangle$ | $|^5S_2\rangle$ | $|^5D_2\rangle$ |

Table 10. Possible mixing channels for the $\Xi_{cc}^{(*)} \Omega_{cc}^{(*)}$ system with total angular momentum 0, 1, 2 and 3.

| $I(J^P)$ | $\Xi_{cc} \Omega_{cc}$ | $\Xi_{cc}^{(*)} \Omega_{cc}$ | $\Xi_{cc} \Omega_{cc}^{(*)}$ |
|---------|-------------------|-------------------|-------------------|
| $\frac{1}{2}(0^+)$ | $|^1S_0\rangle$ | $|^1S_0\rangle$ | $|^1D_0\rangle$ |
| $\frac{1}{2}(1^+)$ | $|^3S_1\rangle$ | $|^3D_1\rangle$ | $|^3S_1\rangle$ |
| $\frac{1}{2}(2^+)$ | $|^5D_3\rangle$ | $|^5S_3\rangle$ | $|^5D_3\rangle$ |

Table 11. The numerical results for the $\Omega_{cc}^{(*)} \Omega_{cc}^{(*)}$ systems. $A$ is the cutoff parameter. “B.E.” is the binding energy. $R_{rms}$ is the root-mean-square radius.

| $I(J^P)$ | Result | $\Omega_{cc} \Omega_{cc}$ | $\Omega_{cc}^{(*)} \Omega_{cc}$ | $\Omega_{cc} \Omega_{cc}^{(*)}$ |
|---------|--------|-------------------|-------------------|-------------------|
| 0(0^+) | A (MeV) B.E. (MeV) $R_{rms}$ (fm) | $|^1S_0\rangle$ | $|^1S_0\rangle$ | $|^1D_0\rangle$ |
| 1300 | 5.64 | 1.34 | 94.8 | 4.1 | 0.1 | 1.0 |
| 1350 | 21.69 | 0.84 | 87.1 | 10.8 | 0.1 | 2.0 |
| 1400 | 57.81 | 0.61 | 78.9 | 18.5 | 0.0 | 2.6 |
| 0(2^+) | A (MeV) B.E. (MeV) $R_{rms}$ (fm) | $|^1D_2\rangle$ | $|^1S_2\rangle$ | $|^1D_2\rangle$ | $|^1S_2\rangle$ | $|^1D_2\rangle$ | $|^1S_2\rangle$ | $|^1D_2\rangle$ | $|^1S_2\rangle$ | $|^1D_2\rangle$ |
| 1360 | 10.24 | 0.50 | 0.6 | 6.5 | 0.1 | 0.4 | 92.4 | 0.0 | 0.0 |
| 1370 | 28.92 | 0.47 | 0.5 | 7.7 | 0.1 | 0.4 | 91.2 | 0.0 | 0.0 |
| 1380 | 49.90 | 0.45 | 0.5 | 8.9 | 0.1 | 0.4 | 90.1 | 0.0 | 0.0 |

$B^* B^*$, as well as the systems composed of one spin-$\frac{3}{2}$ and one spin-$\frac{1}{2}$ doubly charmed baryon, $B^* B$. We also study the couple-channel effect among various combinations of baryons. We consider the $S$-waves and $D$-waves of the $BB$, $B^* B^*$ and $BB^*$ systems together. The binding energies are defined relative to the $BB$ threshold, and the mass differences of different channels are put in the kinetic terms when calculating the coupled Schrödinger equations.

For the two spin-$\frac{3}{2}$ doubly charmed baryons systems, we consider the channels mixing among possible $S$-wave $D$-wave and $G$-wave. After considering the binding energies and root-mean-square radii, as well as the reasonable value of cutoff parameter, the following systems are good candidate of molecular states, such as $[\Xi_{cc} \Xi_{cc}]_{J=1,2}$, $[\Omega_{cc} \Omega_{cc}]_{J=1,2}$ and $[\Omega_{cc}^{(*)} \Omega_{cc}^{(*)}]_{J=1,2}$. The systems $[\Xi_{cc} \Xi_{cc}]_{J=1,2}$ are particularly interesting. Both of them have small binding energies around several MeVs, and large root-mean-square radii, 2-3 fm, when the cutoff is from 1.2 GeV to 1.6 GeV. We also consider the channel mixing between the $S$-wave and $D$-wave $BB$, $B^* B^*$ and $BB^*$ states. Most of the $B^* B^*$ states are dominated by one single channel, which has about 90% contribution. However, the channel mixing effect from the other channels may produce a bound state, which does not exist in the single channel case, such as the systems with $J = 0$. Moreover, the two or three main components of the systems are sometimes comparable, which may cause a large shift of the binding energy, such as the system with $I(J^P) = 0(1^+)$. As a byproduct, we consider the systems composed of one baryon and one antibaryon, $B^* B^*$ and $B^* B$. The results are collected in Appendix. The baryon-antibaryon systems may not be stable, because of the three meson decay modes through quark rearrangement when the threshold is open. Although molecular states composed of two doubly charmed baryons seem too hard to be produced in molecular states, such as $[\Xi_{cc} \Xi_{cc}]_{I=0}$, $[\Omega_{cc} \Omega_{cc}]_{I=0}$ and $[\Omega_{cc}^{(*)} \Omega_{cc}^{(*)}]_{I=0}$.
Table 12. The numerical results for the $\Xi(\ast)\Xi(\ast)$ systems. $\alpha$ is the cutoff parameter. “B.E.” is the binding energy. $R_{rms}$ is the root-mean-square radius.

| $I(J^P)$ | Result | $\Xi_{cc}\Xi_{cc}$ | $\Xi_{cc}^{\ast}\Xi_{cc}^{\ast}$ | $\Xi_{cc}^{\ast}\Xi_{cc}$ |
|---------|--------|-------------------|-------------------|-------------------|
| 0(1+)  | 0(1+)  | $|S_1\rangle\langle S_1|$ | $|D_2\rangle\langle D_2|$ | $|S_1\rangle\langle S_1|$ |
| 0(1+)  | 0(1+)  | 52.9 0.0          | 3.19 0.0          | 0.2              | 14.4 0.1 0.5 |
| 0(1+)  | 0(1+)  | 50.8 0.0          | 33.9 0.0          | 0.2              | 14.5 0.1 0.5 |
| 0(3+)  | 0(3+)  | 58.4 0.8          | 53.5 0.0          | 0.5              | 14.5 0.0 0.5 |
| 1(0+)  | 1(0+)  | 58.2 0.9          | 58.2 0.9          | 0.9              | 11.3 28.4 |
| 1(2+)  | 1(2+)  | 57.4 1.2          | 57.4 1.2          | 1.1              | 11.2 28.6 |

Fig. 4. The interactions potentials for the system with $I(J^P) = 0(0^+)$, $V_{11}$, $V_{22}$ and $V_{44}$ denote the diagonal terms in the potential matrix for the channels $\Omega_{cc}^0 \Omega_{cc}^0 |S_0\rangle$, $\Omega_{cc}^0 \Omega_{cc}^{\ast} |S_0\rangle$ and $\Omega_{cc}^0 \Omega_{cc}^{\ast} |D_0\rangle$ respectively.

the experiment, some possible structure composed of one baryon and one antibaryon may be discovered at LHCb in the future if they are not very broad.

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Table 13. The numerical results for the $\Xi(0^{++}O^+(s))$ systems. $A$ is the cutoff parameter. “$B.E.$” is the binding energy. $R_{rms}$ is the root-mean-square radius.

| $I(J^P)$ | Result | $\Xi_{cc}^0 \Omega_{cc}$ | $\Xi_{c'}^0 \Omega_{cc}$ | $\Xi_{cc}^0 \Omega_{cc}$ | $\Xi_{c'}^0 \Omega_{cc}$ |
|----------|--------|----------------|----------------|----------------|----------------|
| $1/2(0^+)$ | A(MeV) B.E.(MeV) $R_{rms}$(fm) | $|S_0\rangle$ | $|S_0\rangle$ | $|^5D_0\rangle$ | $|^5D_0\rangle$ |
| 1170     | 1.21   | 2.50   | 97.0     | 2.3       | 0.0       | 0.3       | 0.3       |
| 1200     | 5.98   | 1.35   | 92.0     | 6.5       | 0.0       | 0.8       | 0.7       |
| 1250     | 28.64  | 0.81   | 81.7     | 15.7      | 0.0       | 1.3       | 1.3       |
| $1/2(1^+)$ | A(MeV) B.E.(MeV) $R_{rms}$(fm) | $|S_1\rangle$, $|D_1\rangle$ | $|S_1\rangle$, $|D_1\rangle$ | $|D_1\rangle$ | $|D_1\rangle$ |
| 1200     | 5.97   | 1.38   | 93.4     | 0.0       | 0.1       | 0.3       | 0.1       | 0.4       | 0.2       | 0.1       | 0.4       |
| 1220     | 12.05  | 1.10   | 90.2     | 0.0       | 0.0       | 0.4       | 0.2       | 0.5       | 0.2       | 0.2       | 0.5       |
| 1240     | 21.11  | 0.94   | 87.0     | 0.0       | 0.0       | 0.3       | 0.3       | 0.6       | 0.3       | 0.2       | 0.5       |
| $1/2(2^+)$ | A(MeV) B.E.(MeV) $R_{rms}$(fm) | $|S_2\rangle$, $|D_2\rangle$ | $|S_2\rangle$, $|D_2\rangle$ | $|D_2\rangle$, $|D_2\rangle$ | $|D_2\rangle$ | $|D_2\rangle$ | $|D_2\rangle$ |
| 1080     | 2.24   | 0.67   | 0.0      | 0.0       | 0.0       | 59.0      | 0.2       | 0.0       | 40.7      | 0.1       | 0.0       |
| 1100     | 29.0   | 0.59   | 0.0      | 0.0       | 0.0       | 56.2      | 0.2       | 0.0       | 43.5      | 0.1       | 0.0       |
| 1120     | 61.28  | 0.54   | 0.0      | 0.0       | 0.0       | 54.3      | 0.1       | 0.0       | 45.5      | 0.1       | 0.0       |
| $1/2(3^+)$ | A(MeV) B.E.(MeV) $R_{rms}$(fm) | $|S_3\rangle$, $|D_3\rangle$ | $|S_3\rangle$, $|D_3\rangle$ | $|D_3\rangle$, $|D_3\rangle$ | $|D_3\rangle$, $|D_3\rangle$ | $|D_3\rangle$, $|D_3\rangle$ |
| 1290     | 7.28   | 0.55   | 0.0      | 93.7      | 0.0       | 0.4       | 2.8       | 0.4       | 2.8       |
| 1300     | 23.69  | 0.53   | 0.0      | 93.9      | 0.0       | 0.4       | 2.7       | 0.4       | 2.7       |
| 1310     | 41.13  | 0.51   | 0.1      | 93.8      | 0.0       | 0.4       | 2.6       | 0.4       | 2.7       |

Table 14. The comparison of the $BB$ systems with and without channel mixing for the systems $B^+B^+$ and $B^+B$.

| $I(J^P)$ | This work | [34] |
|----------|-----------|------|
| $1/2(1^+)$ | 1200     | 5.97 | 1.38 | 1200    | 0.56  | 3.45 |
| 0(1^+)   | 800      | 36.47| 1.01 |         | 1100  | 0.68 | 3.32 |
| 0(0^+)   | 1300     | 5.64 | 1.34 |         | ×     |     |
| 1(0^+)   | 1060     | 0.87 | 2.91 |         | ×     |     |
| $1/2(0^+)$ | 1170     | 1.21 | 2.50 |         | ×     |     |

A Fourier transformation formulae

The specific expressions of the scalar functions $H_i = H_i(A, m_{\alpha/\beta}, r)$ and $M_i = M_i(A, m_{\alpha}, r)$ are as follows,

\[
\begin{align*}
H_0(A, m, r) &= Y(\alpha r) - \frac{3}{5}Y(\lambda r) - \frac{3}{2} \frac{\beta^2}{2x^2} Y(\lambda r), \\
H_1(A, m, r) &= Y(\alpha r) - \frac{4}{5}Y(\lambda r) - \frac{3}{2} \frac{\beta^2}{2x^2} Y(\lambda r), \\
H_2(A, m, r) &= Z_1(\alpha r) - \frac{3}{5}Z_1(\lambda r) - \frac{3}{2} \frac{\beta^2}{2x^2} Y(\lambda r), \\
H_3(A, m, r) &= Z(\alpha r) - \frac{x^\lambda}{\lambda^2} Z(\lambda r) - \frac{3}{2} \frac{\beta^2}{2x^2} Z_2(\lambda r), \\
M_0(A, m, r) &= -\frac{1}{\beta r} \left[ \cos(\beta r) - e^{-\lambda r} \right] + \frac{1}{\beta r^2} e^{-\lambda r}, \\
M_1(A, m, r) &= -\frac{1}{\beta r} \left[ \cos(\beta r) - e^{-\lambda r} \right] - \frac{1}{\beta r^2} e^{-\lambda r}, \\
M_2(A, m, r) &= -\left[ \cos(\beta r) - \frac{3}{\beta r} \sin(\beta r) \right] - \frac{3}{\beta r^2} \frac{\lambda^2}{\beta r} e^{-\lambda r}, \\
&\quad -\lambda^2 \frac{\beta^2}{2x^2} Z(\lambda r) - \frac{3}{2x^2} \frac{\lambda^2}{\beta r} Z_2(\lambda r). 
\end{align*}
\]
mulae.
\[
\begin{align*}
\frac{1}{u^2 + Q^2} F^2(Q) &\rightarrow \frac{\sigma}{4\pi} H_0(A, m, r), \\
\frac{Q^2}{u^2 + Q^2} F^2(Q) &\rightarrow -\frac{\sigma}{4\pi} H_1(A, m, r), \\
\frac{Q}{u^2 + Q^2} F^2(Q) &\rightarrow \frac{u^*}{4\pi} r H_2(A, m, r), \\
\frac{Q_i Q_j}{u^2 + Q^2} F^2(Q) &\rightarrow -\frac{u^*}{12\pi} [H_3(A, m, r) K_{ij} + H_1(A, m, r) \delta_{ij}].
\end{align*}
\]
(28)

If \( u^2 = m^2 - Q_0^2 < 0 \), the last formula above changes into
\[
\frac{Q_i Q_j}{u^2 + Q^2} F^2(Q) \rightarrow -\frac{\sigma}{12\pi} [M_j(A, m, r) K_{ij} + M_1(A, m, r) \delta_{ij}].
\]
(29)

If \( u^2 = m^2 - Q_0^2 < 0 \), the last formula above changes into

W. Operators

We extract some specific structures in the effective potentials and express them as some angular momentum dependent operators, \( \Delta_{SS}, \Delta_{LS} \) and \( \Delta T \). For the system with spin-3/2 initial and final states, their structures are as follows,
\[
\Delta_{SS} = \sigma \cdot \sigma, \quad \Delta_{LS} = L \cdot S, \quad \Delta T = \frac{3}{4} \sigma \cdot \sigma - \sigma \cdot \sigma,
\]
(30)

where \( L \) and \( S \) are the orbital angular momentum and total spin of two baryons. \( \sigma \) is the Pauli matrix.

For a spin-3/2 baryon, we introduce the Rarita-Schwinger field \( \Psi^\mu \). The field is defined through
\[
\Psi^\mu(\lambda) = \sum_{m_\lambda} \sum_{m_s} \langle 1m_\lambda, \frac{1}{2} m_s | \frac{3}{2} \rangle \epsilon^\mu(m_\lambda) \chi(m_s) = S^\mu_\lambda \Phi(\lambda),
\]
(31)

where \( \chi(m_s) \) is a two-component spinor with the third component of spin \( m_s, \epsilon^\mu(m_\lambda) \) is the polarization vector of a \( J = 1 \) field with the third component of spin \( m_\lambda, \epsilon^\mu(m_\lambda) \)

\[
\epsilon^\mu(+) = -\frac{1}{\sqrt{2}} [0, 1, i, 0]^T, \quad \epsilon^\mu(0) = [0, 0, 0, 1]^T,
\]
\[
\epsilon^\mu(-) = \frac{1}{\sqrt{2}} [0, 1, -i, 0]^T.
\]
(32)

\( \Phi(\lambda) \) is the eigenfunction of the spin operator of a spin-3/2 baryons.

\[
\begin{align*}
\Phi\left(\frac{3}{2}\right) &= [1, 0, 0, 0]^T, & \Phi\left(\frac{1}{2}\right) &= [0, 1, 0, 0]^T, \\
\Phi\left(-\frac{1}{2}\right) &= [0, 0, 1, 0]^T, & \Phi\left(-\frac{3}{2}\right) &= [0, 0, 0, 1]^T.
\end{align*}
\]
(33)

With the above specific form, we can obtain the transition operator \( S^\mu_\lambda \).

\[
S^\mu_0 = 0, \quad S^\mu_\lambda = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & -\frac{1}{\sqrt{3}} & 0 & 1 \end{bmatrix}, \quad S^\mu_\pm = \frac{i}{\sqrt{2}} \begin{bmatrix} 1 & 0 & \sqrt{2} & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 & 1 \end{bmatrix}, \quad S^\mu_{-\pm} = \frac{i}{\sqrt{2}} \begin{bmatrix} 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \end{bmatrix}.
\]
(34)

The spin operator for the spin-3/2 particles can be derived as \( S = \frac{3}{2} \sigma \), while \( \sigma \equiv -S^\mu_0 \sigma S^\mu_0 \).

C Numerical results of the baryon-antibaryon system

We calculate the possible molecular states formed by one baryon and one antibaryon. Although the baryon-antibaryon systems may be not stable, we give the possible molecular solutions for reference.

C.1 The \( B^+ \bar{B}^- \) system

We calculate the systems composed of one spin-3/2 baryon and one spin-3/2 antibaryon. For the \( \Xi_{cc}^- \Omega_{cc}^+ \) and \( \Xi_{cc}^+ \Omega_{cc}^- \) systems, they are antiparticles of each other and we only calculate the former system. We show the binding energies and the root-mean-square radii of possible molecular states in Tables 15 and 16. The mixing channels are the same as those for the two spin-3/2 baryons systems. Therefore, the angular momentum dependent operators are also the same.

For the \( \Xi_{cc}^- \Xi_{cc}^+ \) system, we find a binding solution with binding energy 1.96-8.78 MeV when we choose the cutoff parameter between 0.94 GeV and 0.98 GeV. The \( D^- \) wave contribution is significant and increases with the cutoff parameter. When we choose the cutoff around 1.0 GeV, the percentage of the \( D^- \) channel is over 30%. We give the interaction potentials of the system in Fig. [5] If only the \( S \)-wave is considered, the barely attractive potential could not produce a bound state. But the couple-channel effect from an adequately attractive \( D^- \) wave makes it possible. For the \( \Xi_{cc}^- \Xi_{cc}^+ \) system, the binding energy is 2.68-14.88 MeV while the cutoff is 1.0-1.2 GeV. For the \( \Xi_{cc}^- \Omega_{cc}^+ \) system, we obtain a bound state. The binding energy of the system is 12.54 MeV when the cutoff parameter is 1.5 GeV. Although the root-mean-square radius of the system is 0.76fm when we choose a large cutoff parameter, the system still seems to be a molecular states with the cutoff parameter less than 1.5 GeV. For the \( \Omega_{cc}^- \Omega_{cc}^+ \) system, a binding solution with the binding energy 1.48-27.87 MeV appears with the cutoff parameter from 1.25 GeV to 1.35 GeV.

For the \( \Xi_{cc}^- \Xi_{cc}^+ \) system, we obtain a bound state with the binding energy 1.63-27.92 MeV while the cutoff parameter is 0.9-1.0 GeV. The \( G \)-wave contribution of
Table 15. The numerical results for the $B^*\bar{B}^*$ systems with $J = 0, 1$. $A$ is the cutoff parameter. “B.E.” is the binding energy. $R_{rms}$ is the root-mean-square radius. $P_i$ is the percentage of the different channels.

| States | $A$(MeV) | B.E.(MeV) | $R_{rms}$(fm) | $P_{3}$(%) | $P_{D1}$(%) | $P_{D2}$(%) | $P_{G}$(%) |
|--------|----------|-----------|---------------|-----------|-------------|-------------|-----------|
| $[\Xi_{cc}^+ \Xi_{cc}^-]_{J=0}^{I=0}$ | 940 | 1.96 | 2.96 | 81.9 | 18.1 |
| | 960 | 4.45 | 2.28 | 75.7 | 24.3 |
| | 980 | 8.78 | 1.85 | 70.5 | 29.5 |
| $[\Xi_{cc}^+ \Xi_{cc}^-]_{J=0}^{I=1}$ | 1000 | 2.68 | 1.82 | 99.3 | 0.7 |
| | 1100 | 7.63 | 1.22 | 99.2 | 0.8 |
| | 1200 | 14.88 | 0.95 | 99.1 | 0.9 |
| $[\Xi_{cc}^+ \Xi_{cc}^-]_{J=0}^{I=\frac{1}{2}}$ | 1400 | 6.40 | 1.19 | 99.7 | 0.3 |
| | 1500 | 12.54 | 0.92 | 99.6 | 0.4 |
| | 1600 | 20.92 | 0.76 | 99.5 | 0.5 |
| $[\Omega_{cc}^+ \Omega_{cc}^-]_{J=0}^{I=0}$ | 1250 | 1.48 | 2.32 | 98.5 | 1.5 |
| | 1300 | 8.62 | 1.21 | 96.1 | 3.9 |
| | 1350 | 27.87 | 0.81 | 93.9 | 6.1 |
| $[\Xi_{cc}^+ \Xi_{cc}^-]_{J=1}^{I=0}$ | 900 | 1.63 | 3.26 | 85.9 | 8.4 | 5.7 | 0.0 |
| | 950 | 9.52 | 2.01 | 74.6 | 14.3 | 11.0 | 0.1 |
| | 1000 | 27.92 | 1.52 | 67.7 | 16.6 | 15.6 | 0.1 |
| $[\Xi_{cc}^+ \Xi_{cc}^-]_{J=1}^{I=\frac{1}{2}}$ | 1000 | 1.07 | 2.71 | 99.0 | 0.8 | 0.2 | 0.0 |
| | 1200 | 9.98 | 1.17 | 98.2 | 1.6 | 0.2 | 0.0 |
| | 1400 | 27.05 | 0.82 | 97.6 | 2.1 | 0.3 | 0.0 |
| $[\Xi_{cc}^+ \Xi_{cc}^-]_{J=1}^{I=\frac{3}{2}}$ | 1400 | 4.18 | 1.44 | 99.7 | 0.2 | 0.1 | 0.0 |
| | 1500 | 8.69 | 1.09 | 99.5 | 0.4 | 0.1 | 0.0 |
| | 1600 | 14.97 | 0.89 | 99.3 | 0.6 | 0.1 | 0.0 |
| $[\Omega_{cc}^+ \Omega_{cc}^-]_{J=1}^{I=0}$ | 1250 | 1.67 | 2.26 | 98.5 | 1.0 | 0.5 | 0.0 |
| | 1300 | 7.01 | 1.37 | 96.5 | 2.3 | 1.2 | 0.0 |
| | 1350 | 18.68 | 1.02 | 94.4 | 3.4 | 2.2 | 0.0 |

the system is almost zero. However, the total contribution of the $^3D_1$ and $^3D_2$ channel is about 30%. For the other three systems with total spin 1, $[\Xi_{cc}^+ \Xi_{cc}^-]_{J=1}^{I=0}$, $[\Xi_{cc}^+ \Omega_{cc}^+]_{J=\frac{1}{2}}^{I=\frac{1}{2}}$ and $[\Omega_{cc}^+ \Omega_{cc}^-]_{J=0}^{I=0}$, we all find binding solutions with reasonable binding energies and root-mean-square radii. The dominant parts of their wave functions are all S-wave.

For the systems $[\Xi_{cc}^+ \Xi_{cc}^-]_{J=0}^{I=0}$, a bound state appears with binding energy about 1.15 MeV, when the cutoff parameter is around 0.95 GeV. The D-waves are important for the system. The contribution of the $^3D_2$ channel is almost 10% when the cutoff parameter is 0.95 GeV. For the $[\Xi_{cc}^+ \Xi_{cc}^-]_{J=1}^{I=0}$ system, we obtain a loosely bound state solution, and dominant S-wave part in the total wave function. We show the binding energies, root-mean-square radii as well as the contributions of all the channels in the Table 16.

For the $[\Xi_{cc}^+ \Xi_{cc}^-]_{J=1}^{I=\frac{1}{2}}$ system, we obtain a bound state with binding energy 7.26-36.68 MeV when the cutoff varies from 0.8 GeV to 1.0 GeV. For the $[\Xi_{cc}^+ \Xi_{cc}^-]_{J=1}^{I=0}$ system, the loosely bound state solution appears until we change the cutoff parameter over 2.0 GeV. We change the cutoff parameter from 2.2 GeV to 2.6 GeV, and the binding energy increases from 2.16 MeV to 5.78 MeV. For the $[\Xi_{cc}^+ \Xi_{cc}^-]_{J=1}^{I=\frac{3}{2}}$ system, we find a binding solution with binding energy 1.69-5.53 MeV when the cutoff parameter is 1.8-2.2 GeV. The binding energies of the above two systems are stable for the cutoff parameter in the range over 2.0 GeV. For the $[\Omega_{cc}^+ \Omega_{cc}^-]_{J=2}^{I=0}$ system, a bound state with binding energy 1.15-14.06 MeV appears when the cutoff parameter changes from 1.2 GeV to 1.4 GeV.

For the possible systems composed of one spin-$\frac{1}{2}$ baryon and one spin-$\frac{3}{2}$ antibaryon, they are all good candidates of molecular states. A baryon-antibaryon system may be unstable especially when the threshold of three mesons is open. Some of these molecular systems may appear as an
enhancement in the baryon and antibaryon invariant mass spectrum, or a narrow resonance state etc.

C.2 The $B^+\bar{B}$ system

We also consider the systems composed of one spin-$\frac{3}{2}$ baryon and one spin-$\frac{1}{2}$ antibaryon, $B^+\bar{B}$. The mixing channels are the same as that for the two baryon systems with the same total angular momentum. The numerical results are shown in Table 11.

For the $[\Xi_{cc}\Xi_{cc}]_{J=\frac{3}{2}}$ system, we find a bound state with binding energy 1.67-18.21 MeV when the cutoff parameter is 0.9-1.1 GeV. For the $[\Xi_{cc}\Xi_{cc}]_{J=\frac{1}{2}}$ system, a very loosely bound state solution with binding energy 2.52-26.29 MeV when we choose the cutoff parameter from 1.0 GeV to 1.08 GeV. For the $[\Xi_{cc}\Xi_{cc}]_{J=1}$ system, we find a loosely bound

Table 16. The numerical results for the $B^+\bar{B}$ systems with $J = 2, 3$. $A$ is the cutoff parameter. “B.E.” is the binding energy. $R_{rms}$ is the root-mean-square radius. $P_i$ is the percentage of the different channels.

| States | $A$(MeV) | B.E.(MeV) | $R_{rms}$(fm) | $P_3$(%) | $P_{D2}$(%) | $P_{G1}$(%) | $P_{G2}$(%) |
|--------|----------|----------|--------------|----------|-------------|-------------|-------------|
| $[\Xi_{cc}\Xi_{cc}]_{J=2}$ $^{I=0}$ | 900 | 1.15 | 3.39 | 92.4 | 1.9 | 5.7 | 0.0 |
| 950 | 6.50 | 2.00 | 87.4 | 3.0 | 9.5 | 0.1 |
| 1000 | 18.13 | 1.51 | 83.9 | 3.8 | 12.1 | 0.2 |
| $[\Xi_{cc}\Xi_{cc}]_{J=2}$ $^{I=\frac{3}{2}}$ | 1400 | 3.80 | 1.69 | 98.3 | 0.2 | 1.5 | 0.0 |
| 1600 | 10.21 | 1.18 | 97.8 | 0.2 | 2.0 | 0.0 |
| 1800 | 19.13 | 0.94 | 97.2 | 0.3 | 2.5 | 0.0 |
| $[\Xi_{cc}\Xi_{cc}]_{J=2}$ $^{I=\frac{1}{2}}$ | 1600 | 4.31 | 1.46 | 99.6 | 0.1 | 0.3 | 0.0 |
| 1800 | 9.93 | 1.07 | 99.3 | 0.1 | 0.6 | 0.0 |
| 2000 | 17.62 | 0.87 | 98.9 | 0.1 | 1.0 | 0.0 |
| $[\Omega_{cc}\Omega_{cc}]_{J=2}$ $^{I=0}$ | 1300 | 4.64 | 1.55 | 97.9 | 0.4 | 1.7 | 0.0 |
| 1400 | 18.22 | 1.04 | 94.8 | 1.2 | 4.0 | 0.0 |
| 1500 | 45.21 | 0.84 | 90.9 | 2.3 | 6.7 | 0.1 |
| $[\Xi_{cc}\Xi_{cc}]_{J=3}$ $^{I=0}$ | 800 | 7.26 | 1.59 | 96.7 | 0.7 | 2.5 | 0.0 |
| 900 | 17.90 | 1.29 | 94.7 | 1.2 | 3.9 | 0.0 |
| 1000 | 36.68 | 1.19 | 92.4 | 2.4 | 8.0 | 0.1 |
| $[\Xi_{cc}\Xi_{cc}]_{J=3}$ $^{I=\frac{3}{2}}$ | 2200 | 2.16 | 2.27 | 96.9 | 0.2 | 2.9 | 0.0 |
| 2400 | 3.76 | 1.86 | 96.1 | 0.3 | 3.6 | 0.0 |
| 2600 | 5.78 | 1.59 | 95.3 | 0.3 | 4.4 | 0.0 |
| $[\Omega_{cc}\Omega_{cc}]_{J=3}$ $^{I=0}$ | 1800 | 1.69 | 2.21 | 99.4 | 0.1 | 0.5 | 0.0 |
| 2000 | 3.32 | 1.71 | 99.0 | 0.1 | 0.9 | 0.0 |
| 2200 | 5.33 | 1.45 | 98.5 | 0.1 | 1.4 | 0.0 |
| $[\Omega_{cc}\Omega_{cc}]_{J=3}$ $^{I=\frac{3}{2}}$ | 1200 | 1.15 | 2.56 | 99.3 | 0.1 | 0.6 | 0.0 |
| 1300 | 5.06 | 1.53 | 97.6 | 0.4 | 2.0 | 0.0 |
| 1400 | 14.06 | 1.17 | 94.5 | 0.9 | 4.6 | 0.0 |

"rms" is the root-mean-square radius.
state. When the cutoff parameter changes from 1.4 GeV to 1.8 GeV, the binding energy increases from 1.78 MeV to 11.66 MeV. For the $[\Xi^*_{cc}\bar{\Xi}_{cc}]_{J=0}^{1}$ system, we obtain binding solutions when the cutoff parameter changes from 1.5 GeV to 1.9 GeV. The binding energy of the $\Xi^*_{cc}\bar{\Xi}_{cc}$ system is 1.70-10.26 MeV, while that of the $\Xi_{cc}\bar{\Xi}_{cc}$ system is 1.74-10.38 MeV. For the $[\Xi^*_{cc}\bar{\Xi}_{cc}]_{J=0}^{1}$ system, a bound state with binding energy 2.55-30.91 MeV appears when the cutoff parameter is 1.3-1.5 GeV.

For the four $J=1$ systems, $[\Xi^*_{cc}\bar{\Xi}_{cc}]_{J=1}$, $[\Omega^*_{cc}\bar{\Omega}_{cc}]_{J=1}$, $[\Omega^*_{cc}\bar{\Omega}_{cc}]_{J=0}$, and $[\Omega^*_{cc}\bar{\Omega}_{cc}^{*}]_{J=0}$, their loosely bound solutions are very insensitive to the cutoff parameter.

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Table 17. The numerical results for the $B^*\bar{B}$ systems. $\Lambda$ is the cutoff parameter. “B.E.” is the binding energy. $R_{\text{rms}}$ is the root-mean-square radius. $P_i$ is the percentage of the different channels.

| States | $\Lambda$(MeV) | B.E.(MeV) | $R_{\text{rms}}$(fm) | $P_S$(%) | $P_D^{1}$(%) | $P_D^{2}$(%) | $P_G$(%) |
|--------|----------------|-----------|----------------------|---------|-------------|-------------|--------|
| $[^{1}\Sigma^*_{c\bar{c}}]^{I=0}_{J=1}$ | 900 | 1.67 | 2.21 | 99.8 | 0.1 | 0.1 | |
| | 1000 | 8.82 | 1.24 | 99.6 | 0.2 | 0.2 | |
| | 1100 | 18.21 | 1.03 | 99.3 | 0.4 | 0.3 | |
| $[^{1}\Sigma^*_{c\bar{c}}]^{I=1}_{J=1}$ | 2500 | 2.96 | 1.79 | 99.3 | 0.1 | 0.6 | |
| | 3000 | 5.68 | 1.39 | 99.0 | 0.1 | 0.9 | |
| | 3500 | 8.67 | 1.19 | 98.7 | 0.1 | 1.2 | |
| $[^{1}\Sigma^*_{c\bar{c}}]^{I=\frac{3}{2}}_{J=1}$ | 1800 | 1.82 | 2.06 | 99.9 | 0.0 | 0.1 | |
| | 2200 | 4.86 | 1.39 | 99.8 | 0.0 | 0.2 | |
| | 2600 | 8.06 | 1.15 | 99.7 | 0.0 | 0.3 | |
| $[^{1}\Omega^*_{c\bar{c}}]^{I=1}_{J=1}$ | 1300 | 1.78 | 2.09 | 99.8 | 0.0 | 0.2 | |
| | 1500 | 8.32 | 1.22 | 99.1 | 0.0 | 0.9 | |
| | 1700 | 19.23 | 0.97 | 97.6 | 0.1 | 2.3 | |
| $[^{1}\Omega^*_{c\bar{c}}]^{I=\frac{1}{2}}_{J=1}$ | 1000 | 2.52 | 2.26 | 96.3 | 0.9 | 2.8 | 0.0 |
| | 1040 | 10.77 | 1.39 | 94.6 | 1.3 | 4.1 | 0.0 |
| | 1080 | 26.29 | 1.04 | 93.7 | 1.4 | 4.9 | 0.0 |
| $[^{1}\Omega^*_{c\bar{c}}]^{I=0}_{J=1}$ | 1400 | 1.78 | 2.17 | 99.6 | 0.1 | 0.3 | 0.0 |
| | 1600 | 5.88 | 1.36 | 99.4 | 0.2 | 0.4 | 0.0 |
| | 1800 | 11.66 | 1.05 | 99.2 | 0.3 | 0.5 | 0.0 |
| $[^{1}\Omega^*_{c\bar{c}}]^{I=1}_{J=1}$ | 1500 | 1.70 | 2.10 | 99.9 | 0.0 | 0.1 | 0.0 |
| | 1700 | 5.26 | 1.32 | 99.8 | 0.1 | 0.1 | 0.0 |
| | 1900 | 10.26 | 1.02 | 99.7 | 0.1 | 0.2 | 0.0 |
| $[^{1}\Omega^*_{c\bar{c}}]^{I=\frac{3}{2}}_{J=1}$ | 1500 | 1.74 | 2.08 | 99.9 | 0.0 | 0.1 | 0.0 |
| | 1700 | 5.34 | 1.32 | 99.8 | 0.1 | 0.1 | 0.0 |
| | 1900 | 10.38 | 1.02 | 99.7 | 0.1 | 0.2 | 0.0 |
| $[^{1}\Omega^*_{c\bar{c}}]^{I=0}_{J=1}$ | 1300 | 2.55 | 1.83 | 99.6 | 0.1 | 0.3 | 0.0 |
| | 1400 | 11.71 | 1.05 | 98.8 | 0.3 | 0.9 | 0.0 |
| | 1500 | 30.91 | 0.77 | 97.8 | 0.5 | 1.7 | 0.0 |
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