Abstract

Possible analogies among the vacuum state and a quantum fluid provide an approach to study the induced vacuum energy density produced by thermal corrections, space-time curvature, boundary conditions and quantum back-reaction. We find that vacuum energy density is not naturally of the order of matter energy density. We show how higher-order corrections in quantum back-reaction can also contribute to the vacuum energy density, and how the cosmological expansion is a manifestation of a universe out of mechanical equilibrium. This last fact implies to know the underlying microscopic physics to be able to calculate the associated vacuum energy density, since from simple thermodynamic arguments are not sufficient.

1 Introduction

The idea of building analogue models among laboratory physics (e.g. quantum optics and condensed matter) and cosmological phenomena has recently attracted a great interest [1]. It has been also considered analogies among the particle physics and condensed matter systems [2]-[5]. Possible analogies among many-body quantum mechanics and Relativistic Quantum Field Theory (RQFT) allow to consider the gauge bosons and Dirac fermions as quasiparticle excitations of a quantum liquid. Moreover, for low-energy phenomena, it is possible to consider the Standard Model of particle physics (SM) and the general relativity as effective theories, which emerge from fermion zero modes of the quantum liquid vacuum state [3, 4]. In particular, the cosmological constant problem has been studied in this scheme [3, 4, 6, 7, 8, 9].

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The physics for a weakly interacting Bose gas can be modeled as the ground state of an interacting boson system plus a set of excitations (quasiparticles). A phenomenological description at low energy of the Bose quantum liquid can be performed by means of the Bogoliubov transformations. The ground state (|0⟩) is such that the annihilation operator of quasiparticles (\(\hat{\alpha}_p\)) annihilates the ground state \(\hat{\alpha}_p|0⟩ = 0\) \[4, 5\], in a similar way as the annihilation operator of particles annihilates the RQFT vacuum state. The effect of the Bogoliubov transformations in the Fermi liquid is similar as the one of the Bose quantum liquid, but in this case the ground state is the BCS state \[4, 10\]. The RQFT vacuum state is not analogous to the ground state of a quantum mechanical system because of the ground state of this last system can scatter particles while RQFT vacuum state can not \[11\]. Owing to quasiparticles do not feel the homogeneous vacuum state of a quantum liquid, quantum fluctuations of the vacuum can not scatter quasiparticles \[4\].

The structure over a microscopic (trans-Planckian) scale, for the quantum liquids, remain known. This fact is opposed to what happens in the SM where the structure is unknown. However, starting from topological properties of the SM one might suspect that the vacuum state of this model, above the electroweak phase transition, has the same universality class as the one of \(^3\)He-A \[4\]. Since the gauge symmetry of a Hamiltonian describing a relativistic chiral particles system is a property that can emerge from the Fermi point universality class \[3, 4\], it has been conjecture that all bosons and fermions of the SM can emerge in the vicinity of Fermi points \[3, 4\].

On the other hand, it there exists a deep connection among the vacuum energy density and the cosmological constant. Einstein introduced the cosmological constant in the field equation for gravity with the motivation that it could allow for a finite, closed, static universe in which the energy density of matter determines the geometry \[12\]-\[14\]. Recent observations of SNIa \[15\] combined with CMB anisotropies \[16\] suggest that the expansion of the universe is increasing in an accelerated way. The acceleration is driven by an unknown form of dark energy having a relative density of \(\Omega_A = 0.721 \pm 0.015\) \[16\]. For the dark energy density the observations imply a possible value for \(\omega\) in the range \(-0.11 < 1 + \omega < 0.14\) \[16\], being \(\omega\) the parameter that relates the pressure \(P\) and the dark energy density \(\rho\) in the equation of state \(P = \omega \rho\). Although the nature of dark energy is a complete mystery, the observations are in agreement with the idea that dark energy could arises from a pure cosmological constant term in the Einstein’s field equations. A positive cosmological constant \(\Lambda\) of magnitude \(\Lambda(G\hbar/c^3) \leq 10^{-123}\) \[14\] can be associated with the dark energy density by mean of the Einstein’s field equations. To recognize what is the origin of the dark energy is known as the cosmological constant problem.

The aim of this paper is to analyze some possible contributions of vacuum state to the cosmological constant in the scheme of considering the vacuum state as a quantum fluid. Several contributions to the cosmological constant have been studied
in this scheme \[6,7\]. One of these contributions is the presence of matter in the universe. But since this contribution depends only on the state equation of matter, it has been suggested that the coincidence problem\[3\] is partially solved \[6\]. However, a direct calculation, as the presented in the section \[3.1\], shows that the observed cosmological constant is four order of magnitude higher than the one induced by the matter contribution. We find in section \[3.3\] that the higher-order corrections to the vacuum energy density depend on the microscopic structure of the theory. Finally, we conjecture in section \[4\] that the expansion of the universe is a direct prove that the universe is not in mechanical equilibrium and this fact might to be a meaningful factor for the cosmological constant problem.

2 Vacuum state as a quantum fluid

The vacuum energy density in RQFT can be estimated from positive and negative contributions. Positive contribution comes from the zero-point energy of bosonic fields and negative contribution from occupied negative energy levels in the Dirac sea \[7\]. The energy density of the quantum vacuum \((\rho_\Lambda)\) can be expressed in terms of the number of bosonic \((\nu_b)\) and fermionic \((\nu_f)\) species \[6\] as

\[
\rho_\Lambda = \frac{1}{2V} \sum_{b,p} cp - \frac{1}{V} \sum_{f,p} cp \sim \frac{1}{c^3} \left( \frac{1}{2} \nu_b - \nu_f \right) E_{\text{Planck}}^4 = \sqrt{-g} \left( \frac{1}{2} \nu_b - \nu_f \right) E_{\text{Planck}}^4,
\]

since, for which the energy spectrum of particles is massless \(E \sim cp\), the largest contribution comes from the high momenta. The cut-off is provided by the Planck energy scale \(E_{\text{Planck}} \sim 10^{19}\) GeV. The vacuum energy density obtained is too large respect to the observed value.

On the other hand, the vacuum of a quantum liquid receives contributions from the trans-Planckian and sub-Planckian degrees of freedom. These degrees of freedom describe the interacting and correlated system of atoms in the real liquid. The calculation of an exact energy associated with the many-body wave function which describes the ground state of this real liquid is an extremely difficult exercise \[7\]. However, the calculations of the ground state energies for the weakly interacting Bose and Fermi liquids are known \[17\].

An appropriate model for the cosmological constant needs to satisfy the equation of state \(P_\Lambda = -\rho_\Lambda\), being \(P_\Lambda\) the pressure and \(\rho_\Lambda\) the energy density of the quantum vacuum. Owing to the ground state of the quantum liquids has associated thermo-
dynamical relations at temperature $T$ which lead to the necessary equation of state, vacuum state can be considered as a quantum fluid. This quantum fluid is constituted by weakly interacting Bose and Fermi liquids. The pressure $P$ for the quantum fluid, at $T = 0$, can be indistinctly determined through three different thermodynamic potentials: the Helmholtz free energy, internal energy and grand potential. Using the grand potential $\Omega(T,V,\mu)$ the pressure is \[ P = -\frac{1}{V} \langle 0 | \hat{H} - \mu \hat{N} | 0 \rangle \equiv -\bar{\epsilon}, \] being $|0\rangle$ the ground state of the quantum fluid, $\hat{H}$ the Hamiltonian operator, $\hat{N}$ the particle number, $\mu$ the chemical potential and $V$ the volume. We can identify $\bar{\epsilon}$ as the dark energy density due to the expression (2.2) is clearly the equation of state for the dark energy. Since the term $\hat{H} - \mu \hat{N}$ takes into account the ligature on the particle number, the grand potential is independent of the choice of the reference for the energy \[6\].

In this model it has been considered that if the universe is in thermodynamic equilibrium, in the absence of an external environment (it means a vanishing external pressure), the exact nullification of the vacuum energy $\bar{\epsilon}$ occurs without any special fine-tuning. This last fact can be obtained because of the thermodynamic relation allows to the whole equilibrium \[3, 4, 6, 7\]. The last is true because of in mechanical equilibrium the internal pressure of a quantum liquid is equal to external pressure.

But, as we discuss below, internal pressure is not only due to vacuum pressure.

Since the dark energy can be considered as a perfect fluid, it is interesting to analyze what is the role of the observer speed over the dark energy density. To do it, we remember that the stress-energy tensor for a perfect fluid is

\[ T^{\mu\nu} = -P g^{\mu\nu} + (\rho + P)u^\mu u^\nu, \]

being $g_{\mu\nu}$ the metric tensor and $u^\nu$ the 4-velocity. For simplicity, the universe is considered in special relativity, i.e., in the absence of gravity $G = 0$ \[19\]. In the general coordinate frame, the energy density $T^M_{00}$ and momentum density $T^M_{0i}$ are

\[ T^M_{00} = \gamma^2 \left( \rho^M + \frac{v^2}{c^2} P^M \right), \quad T^M_{0i} = \gamma^2 \left( \rho^M + P^M \right) v_i, \]

with $\gamma = 1/\sqrt{1 - v^2/c^2}$ and $v$ being the speed respect to the rest frame of the fluid \[19\]. Since the cosmological constant satisfy $P_\Lambda = -\rho_\Lambda$, the stress-energy tensor satisfies $T^\mu_\Lambda = P_\Lambda g^{\mu\nu}$ for any general coordinate frame. For a dark energy model in which the equation of state is $P \neq -\rho$, the energy density $T_{00}$ depends on the relative velocity among the rest frame of the fluid and the observer frame. This means that if dark energy is modeled by a fluid satisfying an equation of state given by $P \neq -\rho$, the dark energy density depends on the observer speed.
3 Contributions to the cosmological constant

We analyze in this section the possible contributions of vacuum state to the cosmological constant. A complete discussion about which are these possible contributions has been performed in the literature considering a set of scenarios for the quantum fluid [3, 4, 6, 7]. The vanishing value of vacuum energy obtained from the equilibrium condition of quantum liquids is perturbed by the different scenarios.

3.1 Vacuum energy from finite temperature

The external pressure ($P_E$) in a thermodynamical system vanishes if there is not external force. If this system is in equilibrium then the internal pressure ($P_I$) vanishes too. It is possible to think in a universe for which the pressure caused by quasiparticles (which play the role of matter) $P_M = 0$ is compensated by the negative vacuum pressure $P_\Lambda$, in such way that the internal pressure of the universe is $P_I = P_\Lambda + P_M = 0$. For the universe in equilibrium and assuming absence of external environment, the external pressure satisfies $P_E = P_I = 0$ [3, 4, 6, 7]. Considering that the universe is composed of species $i$ of matter and bearing in mind that the equation of state for a specie $i$ is $P_i = \omega_i \rho_i$, we obtain

$$P_E = P_\Lambda + \sum_i P_i = P_\Lambda + \sum_i \omega_i \rho_i = 0,$$

implying that $P_\Lambda = -\sum_i \omega_i \rho_i$. Using the equation of state for the cosmological constant, we obtain that the vacuum energy density is given by

$$\rho_\Lambda = \sum_i \omega_i \rho_i.$$  \hfill (3.2)

Species of matter contributing to vacuum energy density are baryonic matter, dark matter and hot relativistic matter (photons). For baryonic matter is known that $\omega_{bm} = 0$, meaning that it does not contribute to $\rho_\Lambda$. Since dark matter is cold, its equation of state is $P_{dm} = \omega_{dm} \rho_{dm}$. Cosmological bounds imply that $\omega_{dm}$ has a possible value in the range $-1.50 \times 10^{-6} < \omega_{dm} < 1.13 \times 10^{-5}$ [20]. This means that the dark matter contribution to $\rho_\Lambda$ can be depressed. The only contribution to $\rho_\Lambda$ is due to hot relativistic matter. This contribution can be seen as thermal quantum corrections to the ground state energy density (vacuum energy) of the weakly interacting Bose gas [6, 17]. For $T \ll 2\mu$, where the exponentially small evaporation can be neglected, the quantum liquid can be considered in equilibrium [7]. The ground state energy density $\tilde{\epsilon}_{rm}$, that include a thermal quantum correction coinciding with the Stephan-Boltzmann law, is given by

$$\tilde{\epsilon}_{rm} = \tilde{\epsilon}_{rm}(T = 0) + \rho_{rm} = \tilde{\epsilon}_{rm}(T = 0) + \sqrt{-\frac{\pi^2 k_B^4 T^4}{30 h^3}},$$  \hfill (3.3)
being $\bar{\epsilon}_{rm}(T = 0)$ the vacuum energy density at zero temperature, $k_B$ the Boltzmann constant and $\sqrt{-g} = c^{-3}$, with $c$ the sound velocity \[16\] 17. In this calculation we have considered the fact that the photon has two polarizations. Since hot relativistic matter satisfies the equation of state given by $P_{rm} = 1/3\rho_{rm}$, hot relativistic matter contributes to $\rho_{\Lambda}$ as

$$\rho_{\Lambda} = \frac{1}{3}\rho_{rm} = \frac{1}{3}\sigma T^4 = \frac{1}{3}\frac{\pi^2 k_B^4 T^4}{15 c^3 h^3}. \tag{3.4}$$

Then the energy density parameter associated with hot relativistic matter is $\Omega_{rm} \approx 4.77 \times 10^{-5} \[16\]$. Therefore the induced value for the vacuum energy density parameter is

$$\Omega_{\Lambda} \approx 1.59 \times 10^{-5}, \tag{3.5}$$

which is four orders of magnitude smaller than the observed. This value is not in agreement with the estimation given in \[3, 4, 6, 7\].

### 3.2 Vacuum energy due to a topological defect

A nonzero vacuum energy can be induced by the inhomogeneity of the vacuum in the quantum liquids. As an example of this phenomena, a topological defect called texture has been studied \[3, 4, 6\]. It was found that there is an equivalence among the energy gradient of twisted texture in $^3$He-A and the Riemann curvature of an effective space with time independent metric \[4\]. By this reason the vacuum energy density induced by the texture is proportional to the curvature $k$. As the universe is flat \[16\], i.e. $k \approx 0$, then the induced vacuum energy density vanishes. However, the analogy with quantum liquids suggests naturally that the universe is flat \[4, 7\].

### 3.3 Quantum back-reaction

As was previously mentioned it has been conjecture that all the bosons and fermions of the SM emerge in the vicinity of the Fermi points. Particularly, in the vicinity of a Fermi point, the quasiparticles (that by analogy correspond to Dirac fermions) are massless chiral fermions moving in gravitational and electromagnetic effective fields which are generated by the collective movement of the vacuum at low frequencies \[4\]. In this scheme the SM is equivalent to an effective theory of quasiparticles in the quantum liquid at low frequencies.

We can study the analogy among the SM particles and the quasiparticles at low frequency by considering the approach of quantum back-reaction of dilute Bose-Einstein condensates which has recently studied \[1, 21\]. This approach is based in to consider the classical quantities plus small quantum fluctuations $\hat{\psi} = \psi_c + \delta\hat{\psi}$. Implications of this approach at different levels have been studied in detail \[22\]. In particular,
the study of the time-dependent Gross-Pitaevskii equation reveals that the perturbation (linear quantum fluctuation) \( \delta \hat{\psi}(r, t) \) correspond to the quasiparticles (excitations). This perturbation is over the unperturbed wave function of the condensed state \( \psi(r, t) = \sqrt{n(r)}e^{-i\mu t/\hbar} \), being \( n(r) \) the equilibrium density of particles and \( \mu \) the chemical potential of the unperturbed system \([23]\). In this scheme of vacuum state as a quantum fluid, the linear quantum fluctuations (one-particle excitations) correspond with matter.

Making a better approximation, the full field operator \( \hat{\Psi} \) can be represented in terms of the particle-number-conserving mean-field ansatz \([1, 21, 24]\)

\[
\hat{\Psi} = (\psi_c + \hat{\chi} + \hat{\zeta})\hat{A}\hat{N}^{-1/2},
\]

being \( \psi_c \) the order parameter, \( \hat{\chi} \) the single-particle excitations and \( \hat{\zeta} \) the higher-order corrections originated in multi-particle excitations and correlations \([21, 24]\). Here single-particle means that Fourier components of \( \hat{\chi} \) are linear superpositions of annihilation and creation operators of quasiparticles and \( \hat{N} = \hat{A}^\dagger\hat{A} \) counts the total number of particles \([21, 24]\).

The vacuum pressure induced by the single-particle excitations only represent contributions of matter. At this respect we note that the higher-order corrections also contribute to the ground state energy, as can be seen in the equations of motion coupled for \( \psi_c, \hat{\chi} \) and \( \hat{\zeta} \) \([21]\). Owing to the higher-order corrections depend on the microscopic details of the interactions between the fundamental constituents of the quantum liquid, the microscopic physics of the system should be known to be able to obtain the full contributions to the vacuum pressure.

4 Vacuum energy in non-equilibrium

Additionally to the scenarios that might induce a cosmological constant for a universe in mechanical equilibrium, that we have analyzed in the last section, the surface tension is another one. This tension is provided by the boundaries of the system \([3, 4, 7]\), mineral that the universe is bounded by a surface in a 3-dimensional space. However this scenario is not considered here.

But a universe in mechanical equilibrium implies that there are no external forces acting over it, i.e. a universe in this state is not able to experiment a spontaneous change of state when it is subjected to certain boundary conditions. As was mentioned before the mechanical equilibrium of the universe occurs when the external pressure is equal to the internal one \( P_E = P_I \). Some implications of this mechanical equilibrium can be analyzed thinking in the following mechanical systems. First, in absence of gravity, let a gas into a cubic recipient of volume \( V \) having a piston over one of its walls. If the external pressure exerted by the piston is equal in magnitude, but in
opposite sense, to the pressure exerted by gas on the wall, then this system is in mechanical equilibrium. Second, let a drop of volume $V$ in a vacuum space. If there are not external forces acting over the drop, then the external pressure vanishes. In this case the drop will remain with its volume $V$ if the internal pressures are canceled, otherwise the drop will expand.

Owing to the universe is thinking in absence of a external environment, the external pressure vanishes $P_E = 0$. If the universe is in mechanical equilibrium, then the internal pressure vanishes $P_E = P_I = P_\Lambda + P_M = 0$. The expansion of the universe in this model is equivalent to the expansion of underlying quantum fluid, i.e. this is similar as if a drop expands in the absence of external forces. Owing to the universe is not in mechanical equilibrium, there is not a whole equilibrium. We conjecture that this state of the universe out of equilibrium, can be a meaningful factor which could contribute to the vacuum energy density. In that case the underlying microscopic physics should be known to be able to calculate the energy associated to the vacuum. In other words, owing to the simple thermodynamic arguments are not sufficient to calculate the vacuum energy density in a universe out mechanical equilibrium, it is necessary to know what is the underlying dynamics and structure of the quantum fluid which describes the vacuum state.

5 Summary

A thermodynamic analysis of the quantum vacuum, which is based in the scheme of considering the vacuum state as a quantum fluid, allows to analyze contributions to the cosmological constant. We have found that the vacuum energy density is not naturally of the order of the energy density of matter. The textures do not contribute to the vacuum energy density for a flat universe, while the higher-order corrections in quantum back-reaction contribute. Furthermore, we have conjectured that the expansion of the universe is a manifestation of a universe out of mechanical equilibrium. This fact makes necessary to know the underlying microscopic physics to be able to calculate the associated vacuum energy.

On the other hand, the Lorentz transformation for a general coordinate frame leads to that the stress-energy tensor for a perfect fluid in a model for the dark energy, with equation of state $P \neq -\rho$, depends on the relative velocity among the rest frame of the fluid and the observer frame.

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