Analysis of rainfall intensity-duration-frequency (IDF) curves of Baghdad city

Esraa S. Mahdi1,* and Mohammed Z. Mohamedmeki1
1 Highway and Transportation Engineering Department, College of Engineering, Mustansiriyah University, Baghdad, Iraq
*Corresponding author, email: esraasaadmahid@uomustansiriyah.edu.iq

Abstract. An appropriate formula of rainfall data is the Intensity Duration Frequency (IDF) relationship. The rainfall IDF relationship is one of the tools used considerably in water resources engineering, either in planning, designing and operating projects of water resources, or in flood control projects. The purpose of this paper is to update rainfall IDF curves of Baghdad city developed in previously. The frequency analysis of available rainfall data is perform by three statistical methods, namely, the Gumbel Distribution Theory, the Log Pearson Type III and Log Normal Distribution to attain rainfall intensities for various short durations (0.25, 0.5, 1, 2, 3, 6, 12 and 24) in hours and return periods (2, 5, 10, 25, 50 and 100) in years. IDF equation was derived based on Bernard equation and the results of three distributions were compared using Kolmogorov-Smirnov goodness of fit test with help of the Easy fit software 5.6. The results of three methods were close and accepted at 10% significant level. The maximum rainfall intensity of 118.052mm/hr was happened at the duration 0.25hr of the return period 100yr, whereas the minimum rainfall intensity of 1.257mm/hr was happened at the duration 24hr of the return period 2yr.

1. Introduction

It is well known that urban drainage systems treat with both wastewater and storm water. Storm water is the harvest of rainfall and other shapes of precipitation, for example snow, but in many of areas, rainfall is the most respectable. Therefore, representation and forecasting processes of rainfall are fateful in the design, analysis and operation of drainage systems. Representative rainfall data of one certain location on the catchment measured at an individual rainfall gauge is usually expressed either as depth in mm or intensity in mm/h. This type of rainfall data is more important if it can be related statistically to two other variables of rainfall, namely, duration and frequency [1]. “The duration indicates to the time period of t minutes of rainfall”. It should be noted that the duration does not necessarily mean the full time period of the event, where any event can be subdivided and analysed for a certain range of durations as needed. The rainfall frequency is commonly represented as a return period. An annual maximum rainfall event has a return period of T years when it is equalled in value once, averagely, each T years [2]. The rainfall IDF, relationships were established early by Sherman (1905) then Bernard (1932), as reported by [3]. Since, many of relationships have been formed for a number of the world parts [3].

The rainfall IDF curves give a graphical representation of the possibility that certain rainfall intensity, duration and return period will happen with similar characteristics. The rainfall IDF curves are an effective representation of the maximum event anticipated in an area, which reflects the average rainfall intensity for various durations of event at each return period [4].
2. Material and methods

2.1 Study area
Baghdad city is Iraq’s capital, located at 33° 20’ 19” N latitude and 44° 23’ 38” E longitude. The average monthly maximum air temperature is 44.4 C° in July, while the average monthly minimum air temperature is 4.2 C° in January. The city gains annual rainfall depth around 280 mm. The maximum relative humidity is around 71.7% in January, while the minimum relative humidity is around 25.3%. in July [1].

2.2 Available historical data of rainfall
For Baghdad city, available data are rainfall depths in mm for 24 hours during the last 16 years (2004-2019) obtained from Iraqi Meteorological Organization and Seismology. From these data, maximum daily rainfall depth \( P_{24} \) for each year as shown in figure 1 was used to update the rainfall IDF curves of the city developed by [1].

Figure 1. The maximum daily rainfall depth \( P_{24} \) in mm during 16 years (2004-2019)

2.3 Program of rainfall IDF curves derivation
The flowchart in figure 2 summarizes the work program followed to establish the rainfall IDF curves of Baghdad city.
2.3.1 Generation of shorter duration event. As mentioned above, the rainfall event can be analyzed according to various durations that are shorter than its full duration. Indian Meteorological Department (IMD) proposed empirical reduction formula in equation (1) to calculate rainfall depth for various durations less than 24hr [5].

\[ P_t = P_{24} \left( \frac{t}{24} \right)^{1/3} \]  

(1)

Where:
- \( P_t \) is the maximum hourly rainfall depth for each duration \( t \) in hour.
- \( P_{24} \) is the maximum daily rainfall depth.

2.3.2 Hydrologic Frequency Analysis. Frequency analysis, with the help of statistical methods, is performed to predict rainfall events that may occur in the future from available historical data [6]. Statistical methods followed in this study to perform the frequency analysis: Gumbel Distribution Theory, Log Pearson Type III and Log Normal Distribution.
Gumbel Distribution Theory
In some references, this distribution is also called the Extreme Value Type I distribution. The Gumbel distribution is the most commonly used distribution in the frequency analysis, because of its appropriateness for modeling of extreme values. In this distribution, frequency factor $K$ is a noticeable parameter to be determined. As in equation (2), the frequency factor is a function of the return period $T$ [7].

$$K = -\frac{\sqrt{6}}{\pi} \left\{ 0.5772 + \ln \left[ \ln \left( \frac{T}{T - 1} \right) \right] \right\}$$  \hspace{1cm} (2)

In this distribution, the frequency rainfall depth $P_T$ for each duration $t$ at each certain return period $T$ is calculated using equation (3) [8].

$$P_T = (P_t)_{av} + KS$$  \hspace{1cm} (3)

Where:
- $(P_t)_{av}$ is the average of maximum hourly rainfall depth $P_t$ for each duration $t$.
- $S$ is the standard division of rainfall data in equation (4).

$$S = \left[ \frac{\sum_{i=1}^{n} ((P_t)_i - (P_t)_{av})^2}{n - 1} \right]^{1/2}$$  \hspace{1cm} (4)

Then, the conversion of the frequency rainfall depth $P_T$ to frequency rainfall intensity $I$ is performed by application of equation (5).

$$I = \frac{P_T}{t}$$  \hspace{1cm} (5)

Log Pearson Type III Distribution
Pearson Type III distribution is generally appropriate for skewed data [7]. Hydrologic data tends commonly to be positive skewed. To reduce this skewness, the natural logarithm of this data is taken. Thus, the hydrologic data is Log Pearson distributed [9].

The frequency rainfall depth for Log Pearson III distribution can be calculated by the same equation (3) followed in Gumbel distribution, but taking the natural logarithm of $P_t$ as shown in the equation (6) [10]. In addition, the frequency factor in this distribution is a function of both the return period and the skewness coefficient [7]. The skewness coefficient is calculated by equation (7) [10].

$$P_T^* = (P_t^*)_{av} + KS^*$$  \hspace{1cm} (6)

Where:
- $P_T^*$ is the frequency rainfall depth in the natural logarithm formula at each return period.
- $(P_t^*)_{av}$ is the average of maximum hourly rainfall depth in the natural logarithm formula for each duration $t$.
- $S^*$ is the standard division of rainfall data in the natural logarithm formula.

$$S^* = \frac{n \sum_{i=1}^{n} ((P_t^*)_i - (P_t^*)_{av})^3}{(n - 1)(n - 2)S^*3}$$  \hspace{1cm} (7)

Values of frequency factor $K$ can be obtained from tables found in many of Hydrology references; for example, reference [9]. Then, take values of antilog $10$ of $P_T^*$ calculated by equation (6) and convert it to $I$. 


Log Normal Distribution

“Normal distribution is a special case of the Pearson Type III distribution, describing a non-skewed variable” [9]. For the same purpose mentioned in the Log Pearson Type III distribution, the natural logarithm is taken for the data.

Log Normal distribution is performed with the same procedure as for the Log Pearson Type III distribution except for the frequency factor $K$ where it is extracted from table (12.13.1) in [9] based on skewness coefficient $C_s$ equal to zero. Where, the frequency factor is equal to the standard normal variable $Z$ [9].

2.3.3 Derivation of IDF equation. Bernard equation is one of the most commonly followed formulas to derive a relationship between the rainfall intensity and other effective rainfall variables, namely, the duration and the return period [11].

Bernard equation:

$$l = \frac{cT^m}{t^e}$$ (8)

Where: $c$, $m$ and $e$ are regional parameters required to determine.

In this study, the Bernard equation is converted to a linear equation by taking the natural logarithm of both its sides as in two equations (9) and (10), then performing a simple linear regression analysis to determine the parameters of $c$, $m$ and $e$.

$$\log_{10} l = K^* - e \log_{10} t$$ (9)

Where:

$$K^* = \log_{10} cT \Rightarrow K^* = \log_{10} c + m \log_{10} T$$ (10)

2.3.4 Goodness of fit test. The goodness of fit depicts how a statistical distribution fits well into a series of observed data. The goodness of fit determine the difference between the values of observed data and the values expected from a statistical distribution [12].

In this study, the goodness of fit was performed using Kolmogorov-Smirnov test with help of the Easy fit software 5.6 [1].

The procedure of Kolmogorov-Smirnov test is summarized in the following points:

1) Observed data are arranged in descending order.
2) Weibull’s formula as in equation (11) is applied to find the cumulative probability $P_x$ for each value of observed data.
3) The theoretical cumulative probability $F_x$ for each value of observed data is calculated using a statistical distribution.
4) Calculating the difference $\triangle$ between $P_x$ and $F_x$ in absolute value, as in equation (12). And then taking the maximum value of calculated differences.
5) The value of Kolmogorov-Smirnov test $\triangle_0$ is extracted from the special tables found in the Easy Fit software 5.6 at 10%significant level.
6) If the maximum value of $\triangle$ less than $\triangle_0$, then the statistical distribution is good fit at 10%significant level.

$$P_x = \frac{m}{n + 1}$$ (11)
Where:

- \( n \) is the total number of the observed data.
- \( m \) is the order number of a value in a list arranged in descending order.

\[
\Delta = |P_x - F_x|
\]  
(12)

3. Results and discussion

Figures 3 to 5 show the rainfall IDF curves of Baghdad city obtained from the three followed distributions. All of them have been shown that the rainfall intensity was decreased with the increase of the rainfall duration at the same return period, and increased with the increase of the return period.

The values of rainfall IDF curves of Log Pearson Type III and Log Normal distribution were very close and slightly varying with the Gumbel Theory distribution, except for the values for two durations 0.25hr and 0.5hr at the return periods 50yr and 100yr where there was a rather greater difference.

Also the IDF equation parameters \((c, m, e)\) obtained from the three distributions can be considered somewhat close, with a note that the parameter \(c\) from the Gumbel Theory distribution was slightly greater than from the two remaining distributions. This convergence of the results enabled the average of these parameters to be taken to establish the final IDF equation for this study as shown in figure 6.

Kolmogorov-Smirnov goodness of fit test results is shown in figure 7. It has been shown that the Gumbel Theory distribution gave the lesser values of the maximum difference between the obtained data and the expected values for all durations. By and large, the values of the three distributions were close to a few, which could be accepted.

![Figure 3. IDF curves of Baghdad city according to Gamble Distribution Theory.](image-url)
Figure 4. IDF curves of Baghdad city according to Log Person Type III.

Figure 5. IDF curves of Baghdad city according to Log Normal Distribution.
4. Conclusions
It was difficult to adopt one distribution to establish the final IDF equation of this study. So, as explained above, the three distributions together have been based on to update the final IDF equation of Baghdad city. From this equation established, it can be seen that the maximum rainfall intensity of 118.052mm/hr was happened at the duration 0.25hr of the return period 100yr, whereas the minimum rainfall intensity of 1.257mm/hr was happened at the duration 24hr of the return period 2yr.

Taking into account the original IDF equation of the Baghdad city derived previously by [1] which was updated in this study, the results of the two studies were largely close.

5. References
[1] Al-Awadi A T Assessment of intensity duration frequency (IDF) models for Baghdad city, Iraq 2016 American-Eurasian Network for Scientific Information (AENSI Publication) 12 7-11
[2] Butler D and Davies J W 2011 Urban Drainage (London and New york: Spon Press)
[3] Rahman M M Development of rainfall intensity-duration-frequency relationships from daily rainfall data for the major cities in Bangladesh based on scaling properties 2015 *IJSRD International Journal for Scientific Research & Development* 3

[4] Basumatary V and Sil B S Generation of rainfall intensity-duration-frequency curves for the Barak River Basin 2018 *Meteorology Hydrology and Water Management. Research and Operational Applications* 6

[5] Nyamathi S J and Arelt A Modelling of short duration rainfall IDF equation for Bangalore city 2013 *Research & Reviews: Journal of Engineering and Technology* 2

[6] Baghel H, Mittal H, Singh P, Yadav K and Jain S Frequency analysis of rainfall data using probability distribution models 2019 *International Journal of Current Microbiology and Applied Sciences* 8 1390-6

[7] Dar A Q, Maqbool H and Raazia S 2016 *Proc. of The 4th Int. Con. on Advancements in Engineering and Technology (ICAET-2016) (Sangrur)* vol 57 (EDP Sciences) pp 274-80

[8] Hamaamin Y A Developing of rainfall intensity-duration-frequency model for Sulaimani city 2017 *Journal of Zankoy Sulaimani* 19

[9] Chow V T, Maidment D R and Mays L W 1988 *Applied Hydrology* (New York: McGraw-Hill)

[10] Rasel M M and Islam M M Generation of rainfall intensity-duration-frequency relationship for north-western region in Bangladesh 2015 *IOSR Journal of Environmental Science, Toxicology and Food Technology* 9 41-7

[11] Jaleel L A and Farawn M A Developing rainfall intensity-duration-frequency relationship for Basrah city 2013 *Kufa Journal of Engineering* 5 105-12

[12] Maydeu-Olivares A and Garcia-Forero C Goodness-of-fit testing 2010 *International encyclopedia of education* 7 190-6

**Acknowledgments**

Authors wishing to acknowledge the Iraqi Meteorological Organization and Seismology for providing the rainfall data for Baghdad city. The acknowledgments are extended to the staff of the College of Engineering in Mustansiriyah University for their assistance and encouragement during the preparation of this paper.