MODEL ORDER REDUCTION OF LINEAR TIME INTERVAL SYSTEM USING STABILITY EQUATION METHOD AND A SOFT COMPUTING TECHNIQUE

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DOI: 10.15598/aeee.v14i2.1432

Abstract. This paper deals with a new method for model order reduction of linear continuous time interval system. This new method is based on the Kharitonov’s theorem, the Stability equation method and the error minimization by Differential Evolution. The reduced order interval model is determined by using Kharitonov’s polynomials, which make use of the Kharitonov’s theorem and general form of the stability equation method for denominator, while the numerator is obtained by minimizing the integral square error between the transient responses of original and reduced order models using Differential Evolution algorithm. This method generates stable reduced order interval system if the original higher order system is stable and retains the steady-state value. The proposed method is illustrated with the help of typical numerical example considered from the literature.

Keywords

Differential evolution, integral square error, interval system, Kharitonov’s theorem, model order reduction.

1. Introduction

In general, the original system model is fairly complex and is of higher order. The understanding of the behavior of the system is difficult due to complexity. The analysis of a higher order is both tedious and costly. Therefore, the use of an order reduction makes it easier to implement analysis, simulations, and control system design. It has become necessary to use reduced order modeling techniques for the fundamental understanding of the systems characteristics. Model Order Reduction (MOR) is a branch of systems and control theory, for reducing their complexity, while preserving their input-output behavior. Order reduction methods are broadly classified into two types. Frequency domain order reduction methods are for the transfer function model. Time domain order reduction methods are for the state space model. Several methods are available in the literature for the order reduction of linear continuous systems in the time domain as well as the frequency domain. The reduced order model obtained in the frequency domain gives better matching of the impulse response with its higher order system.

Some of the most popularly used frequency domain order reduction methods are Padé approximation and continued fraction method. These are computationally fast and being able to match exactly the maximum number of system parameters to the reduced model. However, one of the disadvantages of these methods is that the stability of the reduced model is not guaranteed for stable higher order system. The effort has been devoted to developing stability preserving methods such as Routh stability criterion, Mihailov criterion, Hurwitz polynomial. The stability of these methods is achieved only by the loss of accuracy. Among these various model order reduction methods for stability preservation available in the literature, the stability equation method is one of the most popular techniques. The advantage of this method is that it preserves stability in the reduced model, if the original higher-order system is stable, and retains the first two time-moments of the system.

There are several methods available in the literature for order reduction, which are based on the minimization of the Integral Square Error (ISE) criterion. In [12], [13], the values of the denominator coefficients of the low order system are determined by some stability preservation methods and then the numerator co-
efficients of the low order systems are determined by
minimization of the ISE using optimization technique.

Recently one of the most popular research fields
has been “Evolutionary Techniques”, inspired by the
natural evolution of species. Evolutionary techniques
have been successfully applied to solve numerous op-
timization problems. Differential Evolution (DE) was
first proposed by Rainer Storn and Kenneth Price in
1996, it is a branch evolutionary algorithm. DE is a
stochastic population based direct search algorithm.

The advantages of DE are simplicity, accuracy, rea-
sonable speed and the fact that it is a robust opti-
mization method, which is, therefore, used to optimize
real parameter value function. The differential evolu-
tion (DE) algorithm can be used to find approximate
solution non-differentiable, nonlinear and multi-modal
objective functions. The main difference from other Evolutionary Algorithms (EA) is in the
mutation and recombination phases. Another difference
between DE and other EAs such as GA is that DE has
the ability to search with floating point representation
instead of binary representation that used in GA. DE
employs a greedy selection. Also it has a minimum
number of EA control parameters, which can be tuned
effectively. The above methods are available for fixed
systems only.

However, for many of systems the coefficients are
fixed but uncertain within a finite range. Such sys-
tems are classified as interval systems. In [3] γ − δ
Routh Approximation method for interval systems is
proposed. The reduced model of interval system is un-
stable even when the original higher order interval sys-
tem is stable. An improvement is proposed in [5] to the
γ − δ Routh approximation for interval systems using
the Kharitonov’s polynomials such that the resulting
interval Routh approximant is robustly stable. To im-
prove the effectiveness of model order reduction many
mixed methods have been proposed recently in [3], [8],
and [10] based on interval arithmetic. Thus, the sta-
\bility of the reduced order model is not guaranteed,
if the original interval system is stable. In [17] and
[18], the linear interval systems reduction techniques
are presented using the Kharitonov’s theorem to gen-
erate stable reduced order linear interval models. In
[19], a reduction technique for linear interval systems
using Kharitonov’s polynomials and Routh Approxima-
tion is presented to generate a stable reduced or-
der order interval model. In [20], the reduced order interval
model is obtained using Kharitonov’s polynomials to
retain stability and full impulse response energy of the
higher order interval system in its reduced order inter-
val model.

In this paper, model order reduction of interval sys-
tems is carried out by using the Kharitonov’s the-
orem, stability equation method and differential evo-
lution using ISE method. The denominator of the

2. Problem Formulation

Consider a higher order continuous time interval sys-
tem given by the transfer function:

\[ G_n(s) = \frac{N(s)}{D(s)} = \frac{[b_{n}^-, b_{n}^+]}{[a_{n}^-, a_{n}^+]} s^{n-1} + \cdots + \frac{[b_{1}^-, b_{1}^+]}{[a_{1}^-, a_{1}^+]} s + a_{0}^-, \tag{1} \]

where \([A_i^-, A_i^+]\) for \(i = 0, 1, \ldots, n\) are denominator coefficients of \(G_n(s)\) with \(A_i^-\) and \(A_i^+\) as lower and upper bounds of interval \([A_i^-, A_i^+]\) respectively, and \([B_i^-, B_i^+]\) for \(i = 0, 1, \ldots, n - 1\) are numerator coefficients of \(G_n(s)\) with \(B_i^-\) and \(B_i^+\) as lower and upper bounds of interval \([B_i^-, B_i^+]\) respectively.

It is proposed to obtain a reduced order interval model of the form:

\[ G_r(s) = \frac{[b_{r}^-, b_{r}^+]}{[a_{r}^-, a_{r}^+]} s^{r-1} + \cdots + \frac{[b_{1}^-, b_{1}^+]}{[a_{1}^-, a_{1}^+]} s + a_{0}^-, \tag{2} \]

where \([a_i^-, a_i^+]\) for \(i = 0, 1, \ldots, r\) are denominator coefficients of \(G_r(s)\) with \(a_i^-\) and \(a_i^+\) as lower and upper bounds of interval \([a_i^-, a_i^+]\) respectively, and \([b_i^-, b_i^+]\) for \(i = 0, 1, \ldots, r - 1\) are numerator coefficients of \(G_r(s)\) with \(b_i^-\) and \(b_i^+\) as lower and upper bounds of interval \([b_i^-, b_i^+]\) respectively.

3. Proposed Method

**Theorem 1** (Kharitonov theorem). An interval poly-
nomial family \(K(s) = \sum_{i=0}^{n} [a_i^- a_i^+] s^i\) with invariant
degree is robustly stable if its four Kharitonov polyno-
mials are stable.

According to the Thm. 1 every interval polynomial
\(K(s)\) is associated with the following four fixed para-
meter polynomials called Kharitonov polynomials. They are defined as:

\[
\begin{align*}
K_1(s) &= a_0^- + a_1^+ s + a_2^+ s^2 + \cdots + a_n^+ s^n, \\
K_2(s) &= a_0^- + a_1^+ s + a_2^+ s^2 + \cdots + a_n^+ s^n, \\
K_3(s) &= a_0^- + a_1^+ s + a_2^+ s^2 + \cdots + a_n^+ s^n, \\
K_4(s) &= a_0^- + a_1^+ s + a_2^+ s^2 + \cdots + a_n^+ s^n.
\end{align*}
\]
The interval system is stable if and only if its four Kharitonov polynomials satisfies Routh Hurwitz stability criterion.

### 3.1. Reduction Procedure

Consider a family of real interval transfer Eq. (1). The four fixed Kharitonov’s transfer functions associated with \( G_n (s) \) are given as:

\[
G_n^1 (s) = \frac{N_n^1 (s)}{D_n^1 (s)} = \frac{B_0^1 + A_1^1 \cdot s + A_2^1 \cdot s^2 + \cdots + A_n^1 \cdot s^n}{A_0^1 + A_1^1 \cdot s + A_2^1 \cdot s^2 + \cdots + A_n^1 \cdot s^n}
\]

For first Kharitonov transfer function of the reduced order model:

\[
G^1_r (s) = \frac{N^1_r (s)}{D^1_r (s)} = \frac{b_{i0}^1 + b_{i1}^1 \cdot s + b_{i2}^1 \cdot s^2 + \cdots + b_{ir-1}^1 \cdot s^{r-1}}{a_{i0}^1 + a_{i1}^1 \cdot s + a_{i2}^1 \cdot s^2 + \cdots + a_{ir}^1 \cdot s^r}
\]

For stable first Kharitonov transfer function \( G^1_r (s) \), the denominator \( D^1_r (s) \) of the Higher Order System (HOS) is bifurcated into even and odd parts in the form of stability equations as:

\[
D^0_r (s) = A_{10} m_2 \left( 1 + \frac{s}{p_{10}^2} \right) \quad \text{and} \quad D^0_r (s) = A_{11} s n_4 \left( 1 + \frac{s}{p_{10}^2} \right)
\]

where \( m_1 \) and \( m_2 \) are the integer parts of \( \frac{z}{2} \) and \( \frac{n-1}{2} \) respectively and \( z^2 < p_{10}^2 < z^2 < p_{10}^2 \cdots \) Now by discarding the factors with large magnitudes of \( z^2 \) and \( p_{10}^2 \) in Eq. (11), the stability equations for \( r \)th order LOS are obtained as:

\[
D^0_r (s) = A_{10} m_3 \left( 1 + \frac{s}{p_{10}^2} \right) \quad \text{and} \quad D^0_r (s) = A_{11} n_4 \left( 1 + \frac{s}{p_{10}^2} \right)
\]

where \( m_3 \) and \( m_4 \) are the integer parts of \( \frac{z}{2} \) and \( \frac{r-1}{2} \) respectively. Combining these reduced stability equations and therefore proper normalizing it, the \( r \)th order denominator of LOS is obtained as:

\[
D^1_r (s) = D^0_r (s) + D^0_r (s) = \sum_{j=0}^{r} a_{1j} \cdot s^j
\]

Therefore, the denominator polynomial in Eq. (10) is now known, which is given by:

\[
D^1_r (s) = a_{10} + a_{11} \cdot s + a_{12} \cdot s^2 + \cdots + a_{1(r-1)} \cdot s^{r-1} + a_{1r} \cdot s^r
\]

### 2) Step 2

Determination of the numerator coefficients of the reduced model by Differential Evolution (DE). In this step, Differential Evolution (DE) is employed to minimize the objective function \( J \), which is the error between the original higher order system and the reduced order system. Therefore it is represented in the form:

\[
J = \int_0^\infty [y(t) - y_r(t)]^2 \, dt.
\]

Mathematically, the integral square error can be represented as:

\[
J = \sum_{i=0}^{M} [y(t) - y_r(t)]^2
\]
where, $y(t)$ is the unit step response of higher order and $y_*(t)$ is the unit step response lower order system at the $t^{th}$ instant in the time interval $0 \leq t \leq M$, where $M$ is to be chosen. The objective is to obtain a reduced order model, which is closely approximate original system. The objective function is to minimize ISE by using DE.

Differential evolution (DE) is a stochastic, population based direct search optimization algorithm introduced by Storn and Price in 1996 [15]. DE works with two populations; old generation and new generation of the same population. $NP$ is the size of the population and it is adjusted. The population consists of real valued vectors with a dimension $D$ that equals the number of design parameters/control variables. The population is randomly initialized within the initial parameter bounds. The three main operations carry optimization processes are: mutation, crossover and selection. In each generation, individuals of the current population become target vectors. For each target vector, the mutation operation produces a mutant vector. The mutation operation expands the search space. DE undergoes mutation operation after initialization. In mutation operation, it produce mutant vector $V_{i,G}$, with respective to each individual $X_{i,G}$, so called target vector, in the current population via mutation strategy:

$$V_{i,G} = X_{i,G} + F (X_{best,G} - X_{i,G}) + F (X_{r1,G} - X_{r2,G}).$$

For a given parameter vector $X_{i,G}$ two vectors $X_{r1,G}$ and $X_{r2,G}$ are selected randomly such that the indices $r_1, r_2$ are distinct. The mutation factor $F$ is a constant from $[0, 2]$. $V_{i,G}$ is called the donor vector.

### 3.3. Mutation

Mutation expands the search space. DE undergoes mutation operation after initialization. In mutation operation it produce mutant vector $V_{i,G}$, with respective to each individual $X_{i,G}$, so called target vector, in the current population via mutation strategy:

$$V_{i,G} = X_{i,G} + F (X_{best,G} - X_{i,G}) + F (X_{r1,G} - X_{r2,G}).$$

For a given parameter vector $X_{i,G}$ two vectors $X_{r1,G}$ and $X_{r2,G}$ are selected randomly such that the indices $r_1, r_2$ are distinct. The mutation factor $F$ is a constant from $[0, 2]$. $V_{i,G}$ is called the donor vector.

### 3.4. Crossover

Crossover incorporates successful solutions from the previous generation. After mutation, DE undergoes crossover. The trial vector $U_{i,G}$ is developed from the elements of the target vector, $X_{i,G}$, and the elements of the donor vector, $V_{i,G}$:

$$u^j_{i,G} = \begin{cases} v^j_{i,G} & \text{if } (\text{rand}_{1}[0, 1]) \leq \text{CR} \text{ or } (j = j_{\text{rand}}), \\ X^j_{i,G} & \text{otherwise.} \end{cases}$$

Elements of the donor vector enter the trial vector with probability CR (crossover rate) set to $[0, 1]$.

### 3.5. Selection

The newly generated values of trail vectors exceed the corresponding upper and lower bounds; we initialize them randomly and uniformly within the pre-specified range:

$$X^j_{i,G+1} = \begin{cases} U^j_{i,G} & \text{if } f(U_{i,G}) \leq f(X_{i,G}), \\ X^j_{i,G} & \text{otherwise.} \end{cases}$$

The trial vector $X_{i,G}$ is compared with trail vector $U_{i,G}$ and the one with lowest function value is admitted to the next generation.

Therefore the four $k^{th}$ order reduced Kharitonov’s transfer function denominators are obtained by using stability equation method and the numerators are obtained by minimizing integral square error using Differential Evolution Algorithm. These four $k^{th}$ order reduced Kharitonov’s transfer functions are represented as follows:

$$G^1_k(s) = \frac{b_{1k-1} \cdot s^{k-1} + b_{1k-2} \cdot s^{k-2} + \cdots + b_{10}}{a_{1k} \cdot s^k + a_{1k-1} \cdot s^{k-1} + \cdots + a_{10}},$$

$$G^2_k(s) = \frac{b_{2k-1} \cdot s^{k-1} + b_{2k-2} \cdot s^{k-2} + \cdots + b_{20}}{a_{2k} \cdot s^k + a_{2k-1} \cdot s^{k-1} + \cdots + a_{20}},$$

$$G^3_k(s) = \frac{b_{3k-1} \cdot s^{k-1} + b_{3k-2} \cdot s^{k-2} + \cdots + b_{30}}{a_{3k} \cdot s^k + a_{3k-1} \cdot s^{k-1} + \cdots + a_{30}},$$

$$G^4_k(s) = \frac{b_{4k-1} \cdot s^{k-1} + b_{4k-2} \cdot s^{k-2} + \cdots + b_{40}}{a_{4k} \cdot s^k + a_{4k-1} \cdot s^{k-1} + \cdots + a_{40}}.$$
Finally the reduced order interval model is obtained by the following equation:

\[ R_k(s) = \frac{\sum_{j=0}^{k-1} \left[ \min(b_{ij}), \max(b_{ij}) \right] \cdot s^j}{\sum_{j=0}^{k} \left[ \min(a_{ij}), \max(a_{ij}) \right] \cdot s^j}, \quad (26) \]

\[ I = 1, 2, 3, 4. \]

4. Numerical Example

Consider a higher order interval system from literature [4]:

\[ G(s) = \frac{[54.74]s^4+[90.166]}{(1.1)s^8+[2.8,4.6]s^6+[50.4,80.8]s^4+[80.1,133.9]s^2+[0.1,0.1]^2}. \]

This higher order interval system can be represented as four fixed parameter Kharitonov transfer functions that are given as:

\[ G^1_4(s) = \frac{54 \cdot s + 90}{s^4 + 4.6 \cdot s^3 + 80.8 \cdot s^2 + 30.1 \cdot s + 0.1}, \quad (28) \]

\[ G^2_4(s) = \frac{74 \cdot s + 90}{s^4 + 2.8 \cdot s^3 + 80.8 \cdot s^2 + 33.9 \cdot s + 0.1}, \quad (29) \]

\[ G^3_4(s) = \frac{54 \cdot s + 166}{s^4 + 4.6 \cdot s^3 + 50.4 \cdot s^2 + 30.1 \cdot s + 0.1}, \quad (30) \]

\[ G^4_4(s) = \frac{74 \cdot s + 166}{s^4 + 2.8 \cdot s^3 + 50.4 \cdot s^2 + 33.9 \cdot s + 0.1}. \quad (31) \]

4.1. Step 1

Bifurcating the denominator of the above HOS in even and odd parts, we get the stability equations as:

\[ D_o^1(s) = s^4 + 80.8 \cdot s^2 + 0.1, \quad (32) \]

\[ D_o^2(s) = 4.6 \cdot s^3 + 30.1 \cdot s, \quad (33) \]

\[ D_o^3(s) = (s^2 + 0.00123764272) \cdot (s^2 + 80.7986236), \quad (34) \]

\[ D_o^4(s) = s (4.6 \cdot s^2 + 30.1). \quad (35) \]

Now by discarding the factors with large magnitude of \( z^2 \) and \( p^2 \) in \( D_o^2(s) \) and \( D_o^3(s) \) respectively, the stability equations for the second-order reduced model are given by:

\[ D_o^1(s) = 80.79876 \cdot (s^2 + 0.00123764272), \quad (36) \]

\[ D_o^2(s) = 80.9876 \cdot s^2 + 30.1 \cdot s + 0.1. \quad (37) \]

\[ D_o^3(s) = 80.79876 \cdot s^2 + 30.1 \cdot s + 0.1. \quad (38) \]

The reduced model is:

\[ G_r^1(s) = \frac{b_{11} \cdot s + b_{10}}{80.79876 \cdot s^2 + 30.1 \cdot s + 0.1}. \quad (39) \]

Same as remaining Kharitonov’s transfer function the reduced order transfer functions are:

\[ G_r^2(s) = \frac{b_{21} \cdot s + b_{20}}{80.79876 \cdot s^2 + 33.9 \cdot s + 0.1}, \quad (40) \]

\[ G_r^3(s) = \frac{b_{11} \cdot s + b_{30}}{50.39802 \cdot s^2 + 30.1 \cdot s + 0.1}, \quad (41) \]

\[ G_r^4(s) = \frac{b_{11} \cdot s + b_{40}}{50.39802 \cdot s^2 + 33.9 \cdot s + 0.1}. \quad (42) \]

4.2. Step 2

The numerator coefficients are obtained by minimizing integral square error using differential evolution.

The reduced order numerator coefficients obtained by minimizing integral square error by DE for 1st Kharitonov’s transfer function are (Tab. 1):

\[ N_1(s) = 50.01287 \cdot s + 90. \quad (43) \]

The reduced order numerator coefficients obtained by minimizing integral square error by DE for 2nd Kharitonov’s transfer function are (Tab. 2):

\[ N_2(s) = 74.01323 \cdot s + 90. \quad (44) \]

The reduced order numerator coefficients obtained by minimizing integral square error by DE for 3rd Kharitonov’s transfer function are (Tab. 3):

\[ N_3(s) = 50.008167 \cdot s + 166. \quad (45) \]

| Tab. 1: Typical parameter used by Differential Evolution for 1st Kharitonov’s transfer function. |
|-----------------------------------------------|
| Name            | Value |
| Population size | 50    |
| CR              | 0.8   |
| P               | 0.5   |
| Parameter 1: min, max | 50, 60 |
| Parameter 2: min, max | 80, 90 |
| Maximum generation | 10    |

| Tab. 2: Typical parameter used by Differential Evolution for 2nd Kharitonov’s transfer function. |
|-----------------------------------------------|
| Name            | Value |
| Population size | 20    |
| CR              | 0.8   |
| P               | 0.5   |
| Parameter 1: min, max | 70, 80 |
| Parameter 2: min, max | 80, 90 |
| Maximum generation | 10    |

| Tab. 3: Typical parameter used by Differential Evolution for 3rd Kharitonov’s transfer function. |
|-----------------------------------------------|
| Name            | Value |
| Population size | 50    |
| CR              | 0.8   |
| P               | 0.5   |
| Parameter 1: min, max | 50, 60 |
| Parameter 2: min, max | 80, 90 |
| Maximum generation | 10    |

| Tab. 4: Typical parameter used by Differential Evolution for 4th Kharitonov’s transfer function. |
|-----------------------------------------------|
| Name            | Value |
| Population size | 50    |
| CR              | 0.8   |
| P               | 0.5   |
| Parameter 1: min, max | 50, 60 |
| Parameter 2: min, max | 80, 90 |
| Maximum generation | 10    |
The reduced order numerator coefficients obtained by minimizing integral square error by DE for 4th Kharitonov’s transfer function are (Tab. 4):

\[ N_2^4(s) = 74.00109 \cdot s + 166. \]  
(46)

The four reduced order Kharitonov’s transfer functions are:

\[ G_1^2(s) = \frac{54.01287 \cdot s + 90}{80.79876 \cdot s^2 + 30.1 \cdot s + 0.1}, \]  
(47)

\[ G_2^2(s) = \frac{74.01323 \cdot s + 90}{80.79876 \cdot s^2 + 33.9 \cdot s + 0.1}, \]  
(48)

\[ G_3^2(s) = \frac{54.00817 \cdot s + 166}{50.39802 \cdot s^2 + 30.1 \cdot s + 0.1}, \]  
(49)

\[ G_4^2(s) = \frac{74.00109 \cdot s + 166}{50.39801 \cdot s^2 + 33.9 \cdot s + 0.1}. \]  
(50)

Therefore the reduced order interval system obtained by Eq. (26) is:

\[ R_2(s) = \frac{[54.00817, 74.01323] + [90, 166]}{[50.39801, 80.79876] \cdot s^2 + [30.1, 33.9] \cdot s + [0.1, 0.1]}. \]  
(51)

Compare this with other method \( \gamma - \delta \) method [4].

\[ R_2(s) = \frac{[0.9893, 3.7103] \cdot s + [0.5269, 1.8628]}{[1.1] \cdot s^2 + [0.3308, 0.757] \cdot s + [0.0009, 0.0025, 0.1727]}. \]  
(52)

Therefore the step responses of the original and reduced order Kharitonov’s transfer functions are shown in Fig. 2, Fig. 4, Fig. 6 and Fig. 10 respectively.

The reduced order numerator coefficients obtained by minimizing integral square error by DE for 4th Kharitonov’s transfer function are (Tab. 4):

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(48)

\[ G_3^2(s) = \frac{54.00817 \cdot s + 166}{50.39802 \cdot s^2 + 30.1 \cdot s + 0.1}, \]  
(49)

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Therefore the reduced order interval system obtained by Eq. (26) is:

\[ R_2(s) = \frac{[54.00817, 74.01323] + [90, 166]}{[50.39801, 80.79876] \cdot s^2 + [30.1, 33.9] \cdot s + [0.1, 0.1]}. \]  
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(52)

Therefore the step responses of the original and reduced order Kharitonov’s transfer functions are shown in Fig. 2, Fig. 4, Fig. 6 and Fig. 10 respectively.
Fig. 4: Step Response (2nd Kharitonov’s TF).

Fig. 5: Convergence graph (3rd Kharitonov’s TF).

Fig. 6: Step Response (3rd Kharitonov’s TF).

Fig. 7: Convergence graph (4th Kharitonov’s TF).

Fig. 8: Step Response (4th Kharitonov’s TF).

Fig. 9: Step Response (4th Kharitonov’s TF).

Fig. 10: Step Response (4th Kharitonov’s TF).

Fig. 11: Step Response for lower bounds.
5. Conclusion

In this paper, a new method for order reduction is proposed by combining the advantages of conventional method and an optimization technique. The reduced order interval system is obtained by using the Kharitonov’s polynomial and the stability equation method for denominator coefficients, while numerator is obtained by minimising integral square error by using Differential Evolution. The use of interval arithmetic sometimes generates unstable reduced order model. Due to this, we use Kharitonov’s polynomial to make the reduced order interval models robustly stable.

The reduced interval system preserves stability when the original higher order interval system is stable, and also has better matching response. Therefore the error is minimised between the original higher interval system and reduced order interval system.

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