Abnormal Quantum Gravity Effect: Experimental Scheme with Superfluid Helium Sphere and Applications to Accelerating Universe

Hongwei Xiong

1Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, Wuhan 430071, China

(Dated: May 26, 2011)

Abstract

From the general assumption that gravity originates from the coupling and thermal equilibrium between matter and vacuum, after a derivation of Newton’s law of gravitation and an interpretation of the attractive gravity force between two classical objects, we consider the macroscopic quantum gravity effect for particles whose wave packets are delocalized at macroscopic scale. We predict an abnormal repulsive gravity effect in this work. For a sphere full of superfluid helium, it is shown that with a gravimeter placed in this sphere, the sensitivities of the gravity acceleration $\Delta g/g$ below $10^{-8}$ could be used to test the abnormal quantum gravity effect, which satisfies the present experimental technique of atom interferometer, free-fall absolute gravimeters and superconducting gravimeters. We further propose a self-consistent field equation including the quantum effect of gravity. As an application of this field equation, we give a simple interpretation of the accelerating universe due to dark energy. Based on the idea that the dark energy originates from the quantum gravity effect of vacuum excitations due to the coupling between matter and vacuum, without any fitting parameter, the ratio between dark energy density and matter density (including dark matter) is calculated as 2.2, which agrees quantitatively with the result $7/3$ obtained from various astronomical observations.

PACS numbers: 04.60.Bc, 95.36.+x, 98.80.-k

*Electronic address: xiong.quantum@gmail.com
I. INTRODUCTION

Although Newton’s gravity law and Einstein’s general relativity have given marvelous understanding about gravity, it is still the most mysterious problem in the whole field of science [1]. It is widely believed that the unification of gravity and quantum mechanics and the unification of gravity and other three fundamental forces are impossible in the foreseeable future. One of the main reason is that the abnormal effects observed by experiments on earth are highly scarce. To observe possible abnormal effects, most recently, Adler, Mueller and Perl [2] proposed a terrestrial search for dark contents of the vacuum using atom interferometry, somewhat similarly to the Michelson-Morley experiment.

Recently, Verlinde’s work [3] has renewed enthusiasm [4, 5] of the possibility that gravity is an entropy force, rather than a fundamental force [6]. Of course, if gravity is not a fundamental force, we should reconsider the unification of quantum mechanics and gravity. Inspired by the thermodynamic origin of gravity for classical objects, we will try to consider several fundamental problems: (i) Why the gravity between two classical objects is attractive? (ii) Does there exist new and observable quantum gravity effect? (iii) Whether there is new clue to solve the mechanism of accelerating universe?

In this paper, the quantum effect of gravity is studied based on the general principle that gravity originates from the coupling and thermal equilibrium between matter and vacuum background. For classical particles, this general principle gives the Newton’s gravity law. For particles described by quantum wave packets, we predict an abnormal quantum effect of gravity. Based on this abnormal quantum gravity effect, we consider the possible origin of the dark energy from the coupling and thermal equilibrium between matter and vacuum background. Quite surprising, the ratio of the dark energy in our simple calculations is $2^{7/3}$, which agrees quantitatively with the result $7/3$ obtained from various astronomical observations [7–11]. Our works also show that, with a sphere full of superfluid helium, there is a feasible experimental scheme to test our idea with a gravimeter placed in the sphere. The sensitivities of $\Delta g/g$ below $10^{-8}$ could be used to test our idea, which satisfy the present experimental technique of atom interferometer [12], free-fall absolute gravimeters [13] and superconducting gravimeters [14].

The paper is organized as follows. First, in Sec. II, we consider the change of entropy for a particle with a displacement in space. For a particle with an acceleration, we give
a derivation of the vacuum temperature due to the coupling between matter and vacuum. Based on the consideration of local thermal equilibrium, we give the acceleration of a particle in the presence of a finite vacuum temperature field distribution. In Sec. III, we give a derivation of Newton’s law of gravitation. In particular, we explain the physical mechanism of the attractive gravity between two classical objects. In Sec. IV, we give a brief discussion of the physical mechanism of the equivalence principle based on the thermodynamic origin of gravity. In Sec. V, we consider an abnormal quantum gravity effect for a particle described by the wave function in quantum mechanics. In Sec. VI, we consider an experimental scheme to test this abnormal quantum gravity effect with a superfluid helium sphere. The application of this abnormal quantum gravity effect to test many-world interpretation and de Broglie-Bohm theory is discussed. We also discuss the application of this abnormal quantum gravity effect to condensed matter physics. In Sec. VII, we give the field equation including the quantum gravity effect of vacuum excitations. This gives a possible interpretation of the repulsive gravity effect of dark energy. In Sec. VIII, we calculate the dark energy density from the general principle in this work. In Sec. IX, the general field equation including the classical and quantum gravity effect of matter and radiation is given. We give relevant summary and discussions in the last section.

II. ENTROPY, VACUUM TEMPERATURE AND INERTIA LAW

Compared with electromagnetism, weak interaction and strong interaction, the gravitation has several different features. (i) The gravitation is universal. (ii) The gravitation “charge” is in a sense the energy-momentum tensor. Hence, the gravitation “charge” is not quantized. (iii) The coupling between the energy-momentum tensor and spacetime leads to the gravity force for another particle. (iv) The gravity law closely resembles the laws of thermodynamics and hydrodynamics [15,20]. These features strongly suggest that the gravitation deserves studies with completely different idea, compared with other forces. It is well known that the force in classical and quantum gases can be understood in a natural way with statistical mechanics. Following the intensive pioneering works to eliminate the gravitation from a fundamental force, we will study the thermodynamic origin of gravitation, and in particular the quantum gravity effect when both quantum mechanics and thermodynamics are considered.
Although the theoretical predication about the abnormal quantum gravity effect addresses a lot of subtle problems, our starting points are the following two formulas about the change of entropy for a displacement of a particle, and the vacuum temperature because of the vacuum excitations induced by an accelerating object.

(1) The formula about the change of entropy $S$ after a displacement $x$ for a particle with mass $m$.

$$S = 2\pi k_B \frac{mc}{\hbar} x.$$ (1)

In the original work by Verlinde \cite{3}, this postulation motivated by Bekenstein’s work \cite{15} about black holes and entropy, plays a key role in deriving the Newton’s law of gravitation. The above formula means that after a displacement $x$ for a particle with mass $m$, there is an entropy increase of $S$ for the whole system. To understand this formula, we emphasize three aspects about this formula. (i) The whole system about the entropy includes the vacuum background. (ii) There is a strong coupling between the particle and vacuum background. This can be understood after little thought. Without a strong coupling, it is nonsense to define the location and time for a particle existing in the spacetime (i.e., the vacuum background). (iii) As a medium for various matters, the zero-point (or ground-state) energy density of the vacuum background is extremely large with the standard quantum field theory. The vacuum zero-point energy density is of the order of $10^{122} eV/cm^3$ if the energy cutoff is the Planck energy. This also gives the reason why there is a strong coupling between matter and vacuum.

In a sense, the motion of a particle in the vacuum background is a little similar to a speedboat moving in the sea. The speedboat left behind a navigation path in the sea. After a navigation of distance $x$, the speedboat stopped. After waiting sufficient time, we cannot know the navigation path in the sea. If the location resolution in the navigation path in the sea is $l_c$, about $x/l_c$ bits of information are lost. In this situation, there is an entropy increase of $S \sim k_B x/l_c$. For a matter (similar to the speedboat) moving in the vacuum background (similar to the sea), it is similar to understand the relation $S \sim k_B x/l_c$ with $l_c$ being the location resolution in the sight of the vacuum background.

Here we consider a method to calculate the location resolution $l_c$. From special relativity, the energy of the particle is $E = mc^2$. Together with quantum mechanics, the eigenfrequency is $\omega = E/\hbar = mc^2/\hbar$. It is understandable that various gauge fields in the vacuum have the propagation velocity of light velocity $c$. Hence, the relative velocity between matter and
the gauge field in the vacuum is \( c \). In this situation, we get the coherence length \( l_c \) of the particle in the sight of the vacuum background. With the standard quantum mechanics method, we have \( l_c = 2\pi c/\omega = 2\pi \hbar/mc \). This coherence length can also be regarded as the location resolution in the sight of the vacuum background. More detailed discussions about this coherence length and entropy are given in the Appendix.

(2) Vacuum temperature induced by a uniformly accelerating object in the vacuum background.

Considering a particle with acceleration \( \mathbf{a} \), we have

\[
x_j = \frac{a_j t^2}{2}.
\]

Here \( j = 1, 2, 3 \). In this paper, all bold symbols represent vectors. From Eq. (1), we have

\[
dS = \frac{2\pi k_B mc}{\hbar} \sqrt{\sum_j (a_j t)^2} dt.
\]

In addition, from \( E = \sqrt{m^2 c^4 + p^2 c^2} \), we have

\[
dE \approx m \sum_j a_j a_j t dt.
\]

Using the fundamental thermodynamic relation \( dE = T_V dS \) for the whole system including the vacuum background, we have

\[
k_B T_V \approx \frac{\hbar}{2\pi c} \frac{1}{\sqrt{\sum_j a_j^2}}.
\]

In this process, there is no entropy increase for the particle itself. The entropy increase comes from the vacuum. Hence, \( T_V \) refers to the vacuum temperature at the location of the particle.

Because temperature is a concept of statistical average, rigorously speaking, the right-hand side of the above expression should be written as the form of statistical average, \( i.e. \)

\[
k_B T_V \approx \frac{\hbar}{2\pi c} \frac{1}{\sqrt{\sum_j \langle a_j^2 \rangle}}.
\]

If the fluctuations of the acceleration can be omitted, we have

\[
T_V \approx \frac{\hbar |\mathbf{a}|}{2\pi k_B c}.
\]
Figure 1: (Color online) The red line shows a vacuum temperature field distribution. The red spheres show two classical particles at locations A and B. For an acceleration $a_B$ for the particle B, the acceleration induces a temperature field distribution shown by the blue dashed line, with the peak value given by Eq. (8). The local thermal equilibrium requests that this peak value is equal to the temperature of the vacuum temperature field (red line) at location B. This gives the physical mechanism why the particle B will accelerate in the presence of a finite vacuum temperature field.

This shows that the acceleration of a particle will induce vacuum excitations, and thus lead to finite vacuum temperature. Although the above formula is the same as the Unruh temperature [19], the physical meaning is in a sense different from that of the Unruh effect. In the present work, the temperature $T_V$ denotes the vacuum temperature due to the vacuum excitations. Because the Unruh effect itself addresses a lot of subtle problems, in this paper, we will not discuss the detailed difference between $T_V$ and Unruh temperature. We will apply the above expression with our understanding of $T_V$ in this work.

The meaning of the above equation is further shown in Fig. 1. For a particle at location B with acceleration $a_B$ (shown by the dashed red arrow), the coupling between the particle and the vacuum establishes a temperature field distribution shown by the dashed blue line with peak value $T_V(a_B) = \hbar |a_B| / 2\pi k_B c$. In a sense, the strong coupling between the particle
and the vacuum leads to a “dressed” state which includes the local vacuum excitations and
the particle itself. If the particle has no size, the width of the local vacuum excitations is of
the order of the Planck length, with the same consideration of the derivation of the Newton’s
law of gravitation in the following section.

In Fig. 1, there is a temperature field distribution shown by the red line. At location
B, there is a particle denoted by red sphere. To establish local thermal equilibrium, the
red sphere will accelerate so that the peak temperature of the dressed state is equal to the
temperature of the vacuum temperature field at the location B, i.e. $T_V(a_B) = T_V(B)$. In
this situation, we have

$$|a_B| \approx \frac{2\pi k_B c T_V(B)}{\hbar}.$$  

(8)

However, the acceleration is a vector, while the temperature is a scalar. Therefore, the
above formula is not clear about the direction of the acceleration. We further consider the
free energy of the whole system which is defined as $F_{fe} = U - T_V S$. Here $U$ is the overall
energy of the system, which is a conserved quantity. Because the system evolution has
the tendency to decrease the free energy with the most effective way, we get the following
formula to determine the magnitude and direction of the acceleration in the presence of a
vacuum temperature field.

$$a(R) \approx \frac{2\pi k_B c T_V(R)}{\hbar} \frac{\nabla_R T_V(R)}{|\nabla_R T_V(R)|}.$$  

(9)

Here $R$ denotes a three-dimensional spatial vector. In Fig. 1, for the particle at location B,
the direction of the acceleration is determined based on the consideration of the free energy.
We will show in due course that the above equation will interpret why the gravity force is
attractive between two spatially separated objects.

Note that Eq. (9) is invalid for uniformly distributed vacuum temperature field. For
uniformly distributed vacuum temperature field such as location A in Fig. 1, from the con-
sideration of the free energy, the direction of the acceleration of a particle is completely
random. This means that the direction of the acceleration is highly fluctuating. Although
$\langle a \rangle = 0$ for this case, $\delta a \equiv \sqrt{\langle |a - \langle a \rangle|^2 \rangle} > 0$ if the uniformly distributed vacuum tempera-
ture is nonzero. By using Eq. (6), we have $\delta a \sim 2\pi k_B c T_V / \hbar$.

From $dE = F \cdot d\mathbf{x}$ and $dE = T_V dS$, we have $F \cdot d\mathbf{x} = T_V dS$. Using Eqs. (2)-(5), we have

$$\sum_j F_j a_j = m \sum_j a_j a_j.$$  

(10)
From the above equation, we get the following inertia law (Newton’s second law)

$$F = ma.$$ 

(11)

Of course, from $dE = F \cdot dx$ we can directly get this inertia law. The above derivation of the inertia law shows that it is self-consistent to consider the origin of classic force from statistical mechanics and the coupling between matter and vacuum background.

If $T_V = 0$, we have $|a| = 0$. This is Newton’s first law. The coupling between the particle and vacuum background, and the consistency with Newton’s first law suggest that the vacuum background is in a sense a superfluid. In this superfluid, the propagator velocity of various gauge fields is the light velocity $c$. If $c$ is regarded as the sound velocity of the vacuum background, the critical velocity to break the superfluidity is $c$.

III. A DERIVATION OF NEWTON’S LAW OF GRAVITATION

As discussed in previous section, the coupling between particle and vacuum background leads to an entropy increase for a displacement. The physical mechanism of this entropy increase is that there is a strong coupling between matter and vacuum. In this section, we give a derivation of Newton’s law of gravitation physically originating from these vacuum excitations.

When there are vacuum excitations due to the coupling between matter and space, we assume $l_G$ as the correlation length of the vacuum excitations. At least for the situations considered in the present work, the vacuum excitation energy density is much smaller than the vacuum zero-point energy density. We will show in due course that $l_G$ is of the order of the Planck length $l_p \equiv \sqrt{\hbar G/c^3}$ with $G$ being the gravitational constant, which is the length scale at which the structure of spacetime becomes dominated by quantum gravity effect. In Fig. 2, we consider the three-dimensional space with $l_G$ showing the structure of the space due to quantum gravity effect. Although the microscopic mechanism of $l_G$ is not completely clear, it is not unreasonable to assume the existence of $l_G$ in the structure of the space. These lowest space structures are a little similar to atoms in a solid. We assume the energy of a particle shown by red sphere in Fig. 2 is $\varepsilon$. We further assume the freedom of the lowest space structure is $i$. From the local thermal equilibrium, at the location of the
Figure 2: (Color online) The grid shows the correlation length (∼ \( l_p \)) for the vacuum fluctuations. The red sphere shows a particle which leads to a vacuum temperature field distribution, through the coupling with the vacuum.

The temperature of the vacuum is determined by

\[
\frac{i}{2} k_B T_V (R = 0) = \gamma \varepsilon. \tag{12}
\]

Here \( \gamma \) denotes a dimensionless coupling strength between matter and space. From the ordinary statistical mechanics, \( \gamma \) should be of the order of 1. We have

\[
T_V (R = 0) = \frac{2 \gamma \varepsilon}{i k_B}. \tag{13}
\]

When \( R \to \infty, T_V = 0 \). Hence, one expects the temperature field distribution shown in Fig. 3. For another particle with mass \( m \) in this temperature field distribution, from Eq. \( \text{(9)} \), the acceleration field distribution is then

\[
a = -\frac{2 \pi k_B c T_V}{\hbar} e_R. \tag{14}
\]

Here the radial unit vector \( e_R \equiv \frac{\mathbf{R}}{|\mathbf{R}|} \). To get the above expression, the spherical symmetry of the system for \( R \gg l_p \) is also used. This explains the attractive gravity force between
two classical objects. It is worthy to point out that both in Newton’s gravitation law and Einstein’s general relativity, this attractive gravity force is imposed from the observation results, rather than microscopic mechanism. One of the merit of thermodynamics lies in that even we do not know the exact collision property such as scattering length between atoms, the macroscopic force such as pressure can be derived. When the thermodynamic origin of gravity is adopted, there is a request that one gets the correct result of the direction of gravity force.

From the continuous property (Gauss’s flux theorem) of the force $\mathbf{F} = ma$, we have

$$T_V (R) = \frac{\eta}{R^2}. \quad (15)$$

Note that the above expression holds for $R \gg l_G$, so that the spherical symmetry approximation can be used.

Combined with Eq. (13), we have $\eta/l_G^2 = 2\gamma \beta \varepsilon/ik_B$. Although we do not know the exact value of $\beta$, it is understandable that $\beta$ is of the order of 1. In this situation, we have

$$T_V (R) = \frac{2\gamma \beta Mc^2 l_G^2}{ik_B} \frac{1}{R^2}. \quad (16)$$
To get the above expression, we have used \( \varepsilon = Mc^2 \). Using Eq. (14), we have

\[
a = -\frac{4\pi\gamma\beta c^3 l_G^2 M}{i\hbar} \frac{M}{R^2} e_R.
\]

(17)

Assuming

\[
l_G = \sqrt{\frac{i}{4\pi\gamma\beta l_p}},
\]

we get the standard result of Newton’s gravitation law

\[
a = -\frac{GM}{R^2} e_R.
\]

(18)

We see that the correlation length of the vacuum excitation is of the order of the Planck length. The temperature field distribution for \( R \gg l_p \) becomes

\[
T_V(R) = \frac{\hbar GM}{2\pi k_B c R^2}.
\]

(19)

We consider above the situation of a particle whose size is of the order of the Planck length. If the size of the particle is larger than the Planck length, by using the Gauss’s flux theorem, for \( R \) being much larger than the size of the particle, the above result still holds.

It is worthy to further consider the meaning of the vacuum excitations due to the coupling between matter and space. The above derivations give the temperature field distribution. Assuming the vacuum zero-point energy density is \( \rho_{VG} \), we consider the situation that a particle with mass \( M \) suddenly emerges at the location \( R = 0 \). This will lead to the establishment of the vacuum temperature field distribution (19). However, one should note that in the establishment process of the temperature field distribution, the whole energy should be conserved. Hence, assuming that \( \rho_{ex} \) is the vacuum energy density in the presence of \( M \), we have

\[
\int \rho_{ex} dV \simeq \int \rho_{VG} dV,
\]

(20)

\[
\sqrt{\langle (\rho_{ex}(R) - \rho_{VG})^2 \rangle} l_p^3 \sim \frac{i}{2} k_B T_V(R).
\]

(21)

This physical picture is further shown in Fig. 4.

For an assembly of classical fundamental particles (here “classical” means that the quantum wave packet effect is negligible) shown in Fig. 5, we assume the temperature field distribution due to the \( i \)th particle is \( T_{Vi}(R) \). Because there is no quantum interference
Figure 4: (Color online) For a particle at $R = 0$, the blue line shows the fluctuating vacuum energy distribution $\rho_{ex}$ around the zero-point vacuum energy density $\rho_{VG}$. We will show in due course that when the whole universe is considered, $\rho_{ex}$ will exhibit fluctuation behavior around $\rho_{VG} + \rho_V$ with $\rho_V$ being the dark energy density.

The above expression is based on the assumption of the linear superposition of gravity force $F = \sum_i F_i$. In the Newton's law of gravitation for an assembly of classical particles, this implicit assumption is also used. We stress here that, rigorously speaking, the summation in the above expression is about all fundamental particles.

One may still consider the following possibility of calculating the acceleration field

$$T_V = \sum_i T_{Vi},$$

$$a = \frac{2\pi k_B c T_V}{\hbar} \frac{\nabla_R T_V}{|\nabla_R T_V|}.$$
Obviously, the above expression contradicts with the Newton’s law of gravitation. Considering the astrodynamics addressing three bodies, this method to calculate the gravity force should be ruled out. The reason of the error in the above calculations lies in the method to get $T_V$. Even for the ordinary situation, the above method to get $T_V$ is also not correct.

We consider the presence of $N$ thermal sources at different location $\mathbf{x}_i$. The temperature increase for an observer due to these $N$ thermal sources is $\delta T$. If only one thermal source exists, we assume that the temperature increase for the observer is $\delta T_i$. It is obvious that when $N$ thermal sources coexist, the temperature increase for the observer is not $\delta T = \Sigma_{i=1}^{N} \delta T_i$.

IV. THE EQUIVALENCE PRINCIPLE

It is well known that the equivalence principle is based on the assumption that the gravitational mass $m_g$ is equal to the inertial mass $m_I$. It is still a mystery why $m_g = m_I$ because it seems that the gravitational mass and inertial mass are different physical concept, if the gravity force is regarded as a fundamental force, similarly to other fundamental forces.
For example, for electromagnetism, the inertial mass is completely different from the electric charge leading to electromagnetism. Previous studies clearly show that if the thermodynamic origin of gravity force is adopted, the gravitational mass and the inertial mass are the same mass in the mass-energy relation in special relativity. In all our derivations, we do not need especially introduce the gravitational mass at all. In this situation, the gravitational mass and inertial mass are in fact the same physical concept. Hence, it is not surprising that \( m_g = m_I \).

As discussed previously, the velocity of light is in a sense the sound velocity of the vacuum. If the vacuum is regarded as a superfluid, there is a breakdown of the superfluidity if the velocity of an object is larger than \( c \). This gives a possible physical mechanism why \( c \) is the speed limit for all objects. Hence, the validity of special relativity is in a sense determined by the vacuum background where there is a finite and extremely large zero-point energy density. In other words, it’s the vacuum background which leads to the theory of special relativity. This is the reason why there is an internal relation between \( c \) and vacuum zero-point energy density.

In a sufficiently small region of a freely falling system, the temperature due to the acceleration of this system is equal to the vacuum temperature field. In this small region, the object will not experience the finite vacuum temperature effect. This suggests the validity of the strong principle of equivalence. Hence, when the quantum effect is included in the field equation of gravitation, we will adopt the strong principle of equivalence, which is also adopted by Einstein in deriving his field equation for gravitation.

From \( S \sim k_B x/l_c \) and \( l_c \sim h/mc \), we see that there is another definition of the mass. For different fundamental particles, there are different correlation lengths in the coupling between particles and vacuum. The mass is then \( m \sim h/l_c \). Considering previous studies that Newton’s first law, second law and Newton’s gravitation law can be derived from \( S \sim k_B x/l_c \), this gives the definition of mass with the view of information. For the particle with inertial mass \( m \), we consider the possible change of \( m \) due to a large mass near us, such as the Milky Way galaxy. At the location of this particle, assuming the presence of the Milky Way galaxy leads to a change of the correlation length \( l_c \), we have \( \delta m/m = \delta l_c/l_c \).

Assuming at the location of the particle, the vacuum temperature field due to the Milky Way galaxy is \( T_V \), because \( l_c \) comes from the coupling between the vacuum and particle, to the first order, \( \delta l_c/l_c \) would be of the order of \( k_B T_V/E_{Pl} \) with \( E_{Pl} \) being the Planck energy.
In this situation, we have $\delta m/m \sim k_B T_V / E_{Pl}$. We see that, to observe the possible change of the inertial mass, extremely large $T_V$ would be needed. In most situations we can imagine, we cannot distinguish the gravity effect between Einstein’s general relativity and Mach’s principle. It is not clear, whether at the singular point of black hole or during the Planck epoch et al., there would be obvious effect predicted by Mach’s principle.

V. ABNORMAL QUANTUM EFFECT OF GRAVITY

In the thermodynamic origin of gravity, for an object with mass $M$, it establishes a temperature field of $T_V(R) \sim M / |R|^2$. Using the formula $a = 2\pi k_B c T_V \nabla_R T_V / \hbar |\nabla_R T_V|$ about the relation between acceleration and temperature, we get the Newton’s gravity law $F = -GMmR/R^3$ between two classical objects. In particular, the attractive gravity force between two classical objects is explained. In this section, we will consider the quantum gravity effect by including quantum mechanics in the thermodynamic origin of gravity.
To give a general study on the quantum gravity effect, we consider the following wave function for a fundamental particle (such as electron) with mass $m_q$,

$$\phi_q(x, t) \simeq \frac{1}{\sqrt{V}}, (R < R_0),$$

$$\phi_q(x, t) \simeq 0, (R > R_0).$$

The average density distribution $|\phi_q(x, t)|^2$ is shown by the black quantum sphere in Fig. 6(a).

For $R > R_0$, similarly to the consideration of a classical particle, it is easy to get

$$T_{Vq} = \frac{\hbar Gm_q}{2\pi k_B c R^2}. \quad (25)$$

At $R = 0$, from the consideration of the spherical symmetry, $a(R = 0) = 0$. Hence, we have $T_{Vq}(R = 0) = 0$. It is clear that the temperature field distribution is that shown in Fig. 7. For $R < R_0$, using again the spherical symmetry, we have

$$T_{Vq} = \frac{\hbar Gm_q R}{2\pi k_B c R_0^2}. \quad (26)$$
For the quantum wave packet shown in Fig. 6(a), from the relation between acceleration field and temperature field distribution given by Eq. (9), we have

\[ a = \frac{Gm_q R}{R_0^3}, (R < R_0), \]
\[ a = -\frac{Gm_q R}{R^3}, (R > R_0). \]  

(27)

This means a remarkable predication that in the interior of the quantum sphere, the gravity force is repulsive! This abnormal effect is further shown in Fig. 6(b). It is clear that this abnormal quantum gravity effect physically originates from the quantum wave packet effect for the particle in the black sphere. At \( R = R_0 \), the average acceleration field is zero. However, because of the finite vacuum temperature, the direction of the acceleration is highly fluctuating, \textit{i.e.} \( |a(R_0)| \sim Gm_q/R_0^2 \) and \( \langle a(R_0) \rangle = 0 \). It’s this highly fluctuating acceleration field leads to different value of \( \oint \mathbf{F} \cdot d\mathbf{S} \) in the interior and exterior of the quantum sphere.

If there are \( N \) particles in the same quantum state given by Eq. (24), we have

\[ a = \frac{2\pi k_B c}{\hbar} \sum_i T_{V_i}(R) \frac{\nabla R T_{V_i}(R)}{[\nabla R T_{V_i}(R)]} = \frac{G N m_q R}{R_0^3}, (R < R_0), \]
\[ a = \frac{2\pi k_B c}{\hbar} \sum_i T_{V_i}(R) \frac{\nabla R T_{V_i}(R)}{[\nabla R T_{V_i}(R)]} = -\frac{G N m_q R}{R^3}, (R > R_0). \]  

(28)

For this abnormal quantum gravity effect, one should be very careful. Without experimental verification, it is only a theoretical predication. Although we will show that it is possible that this abnormal quantum gravity effect just gives an interpretation to the accelerating universe due to dark energy, it is necessary to consider whether this theoretical predication is self-consistent.

To further understand the abnormal quantum gravity effect, we consider a classical sphere with the following density distribution

\[ n(x,t) \simeq \frac{N}{V}, (R < R_0), \]
\[ n(x,t) \simeq 0, (R > R_0). \]  

(29)

This density distribution for a classical sphere is shown in Fig. 6(c). At first sight, one may get the same acceleration field distribution, compared with the situation of the quantum sphere. This is not correct. For a classical sphere, assuming there are \( N \) particles, it is very clear that the wave packets of all particles are highly localized. Hence, for a particle at
location $x_j$, the temperature field distribution due to this particle is $T_{Vj}(R) \sim m_j/|R - x_j|^2$. From $a \sim \Sigma_j T_{Vj} \nabla T_{Vj} / |\nabla T_{Vj}|$, we get the result given by Newton’s law of gravitation, shown in Fig. 6(d). It is easy to show that the quantum states of the quantum sphere and classical sphere are completely different. In the quantum sphere, the many-body wave function is

$$\Psi_q(x_1, \cdots, x_N) = \phi_q(x_1) \cdots \phi_q(x_N). \tag{30}$$

Here $\phi_q$ is given by Eq. (24). For the classical sphere, the many-body wave function is

$$\Psi_c(x_1, \cdots, x_N) = c \Sigma_P P \left[ \phi_1(x_1) \cdots \phi_N(x_N) \right]. \tag{31}$$

Here $\phi_1, \cdots, \phi_N$ are highly localized wave functions. $P$ denotes all the permutations for the particles in different single-particle state. $c$ is a normalization factor. We consider here the situation of bosons. It is similar for Fermi system. It’s this essential difference of the many-body wave function that leads to different gravity effect.

Assuming there are $N$ fundamental particles whose wave functions are $\phi_1(x, t), \cdots, \phi_j(x, t), \cdots, \phi_N(x, t)$, we give here the formulas to calculate the acceleration field due to these $N$ particles. Previous studies directly lead to the following two formulas.

$$a(R) = \frac{2\pi k_B c}{\hbar} \sum_{j=1}^{N} T_{Vj}(R) \frac{\nabla_R T_{Vj}(R)}{\nabla_R T_{Vj}(R)}, \tag{32}$$

and

$$T_{Vj}(R) = \frac{\hbar G m_j}{2\pi k_B c} \left| \int d^3x \phi_j^*(x, t) \frac{x - R}{|x - R|^3} \phi_j(x, t) \right|. \tag{33}$$

Here $m_j$ is the mass of the $j$th fundamental particle. The integral in the right-hand side of Eq. (33) is due to the quantum wave packet of the $j$th fundamental particle, while the norm of the vector after calculating this integral is due to the fact that $T_{Vj}$ is a scalar field larger than zero and $T_{Vj}$ is an observable quantity based on Eq. (32). It is easy to show that if all these $N$ particles are highly localized classical particles, we get the Newton’s gravity law $a(R) = -\sum_j Gm_j (R - x_j) / |x_j - R|^3$, with $x_j$ being the location of the $j$th particle.

We assume that a fundamental particle with mass $M$ is uniformly distributed in the sphere with radius $R_M$. For the vacuum temperature field due to this particle, the maximum temperature is

$$T_{V}^{\max} = \frac{1}{2\pi} \frac{hGM}{k_B R_M^2}. \tag{34}$$
From the mass-energy relation, even all the energy is transferred into temperature with only one freedom, we get a limit temperature
\[ T^0_V = \frac{2Mc^2}{k_B}. \] (35)

Obvious, there is a request of
\[ T_{V\text{max}} \leq T^0_V. \] (36)

This means that
\[ R_M \geq \sqrt{\frac{\hbar G}{4\pi c^3}} = \sqrt{\frac{1}{4\pi} l_p}. \] (37)

This result shows that any object having strong coupling with the vacuum background cannot be distributed within a sphere of radius \( l_p/\sqrt{4\pi} \), irrelevant to its mass. This provides a physical mechanism why in our calculations of \( T_V \), the volume of the smallest space unit is of the order of \( l_p^3 \).

VI. EXPERIMENTAL SCHEME AND POTENTIAL APPLICATIONS OF QUANTUM GRAVITY EFFECT

Now we turn to consider a feasible experimental scheme to test further the quantum gravity effect with superfluid \(^4\)He, shown in Fig. 8. For brevity’s sake, we consider a sphere full of superfluid \(^4\)He. There is a hole in this sphere. In this situation, from Eqs. (32) and (33), the gravity acceleration in the sphere due to superfluid \(^4\)He can be approximated as
\[ a = \frac{4\pi}{3} Gn_{He} R. \] (38)

Here the liquid helium density is \( n_{He} \approx 550 \text{ kg/m}^3 \). From this, the anomalous acceleration is \( a = 1.5 \times 10^{-7} \text{R/s}^2 \). The gradient of this anomalous acceleration is \( 1.5 \times 10^{-7} \text{/s}^2 \). Even only the condensate component of superfluid \(^4\)He is considered, for a superfluid \(^4\)He sphere whose radius is 1m, the maximum anomalous acceleration is about \( 10^{-8} \text{m/s}^2 \). Quite interesting, this value is well within the present experimental technique of atom interferometer [12] to measure the gravity acceleration. Nevertheless, this is a very weak observable effect. Thus, it is unlikely to find an evidence to verify or falsify this anomalous acceleration without future experiments. Apart from atom interferometer, the measurement of gravity acceleration with Bloch oscillation [21] for cold atoms in optical lattices, superfluid helium interferometry [22], free-fall absolute gravimeters [13] and superconducting gravimeters [14] provide other
methods to test the abnormal quantum gravity effect. In particular, the standard deviation of free-fall absolute gravimeters in the present technique is about $10^{-8} m/s^2$ [13], while the superconducting gravimeters have achieved the sensitivities of one thousandth of one billionth ($10^{-12}$) of the Earth surface gravity.

Because this abnormal quantum gravity effect could be tested by contrast experiment with superfluid helium and normal helium, and the prediction that this abnormal quantum gravity effect is location-dependent, the sensitivities of a gravimeter could be used to test the abnormal quantum gravity effect. This makes it very promising to test the abnormal quantum gravity effect in future experiments.

Because highly localized helium atoms will not lead to quantum effect of gravity, the measurement of acceleration will give us new chance to measure the fraction of highly lo-
calized helium atoms, which is still an important and challenging topic in condensed matter physics. In our theory, the quantum effect of gravity does not rely on superfluid behavior. It depends on whether the wave packet of a particle is localized. It is well known that whether there is wave packet localization is a central topic in condensed matter physics and material physics, such as the long-range order problem at a phase transition. Considering the remarkable advances in various gravimeters, it is promising that the quantum gravity effect would have potential applications in our understanding of condensed matter physics and material physics, etc.

A possible risk in the decisive test of the abnormal quantum gravity effect with superfluid helium sphere lies in our understanding of the superfluid behavior of liquid helium. In the ordinary understanding of superfluid helium, the superfluid fraction can achieve almost 100% while the condensate fraction is about 8% [23]. Because of the strong interaction between helium atoms, the liquid helium is a very complex strongly correlated system. Considering the fact that there are a lot of open questions in strongly correlated systems, we cannot absolutely exclude the possibility that the wave packets of all helium atoms are localized, although the whole system still exhibits superfluid behavior. This significantly differs from the Bose-Einstein condensate in dilute gases, where the wave function of the atoms in the condensate is certainly delocalized in the whole condensate.

The abnormal quantum gravity effect gives a possible experimental scheme to test many-world interpretation [24]. In many-world interpretation, there is no “true” wave packet collapse process. For a particle described by a wave packet, the measurement result of the particle at a location does not mean that the wave packet of this particle at other locations disappears. It suggests that the wave function of the whole universe evolves into a series of orthogonal many-body wave functions due to the interaction between the measurement apparatus and the particle. The observed result of the particle at a location corresponds to one of the orthogonal many-body wave functions. Considering again the superfluid helium sphere, if we increase the temperature so that it becomes normal liquid, based on the many-world interpretation, the wave packets of helium atoms (at least the fraction of the helium atoms initially in the condensate) are still delocalized in the whole sphere. In this situation, if the many-world interpretation is correct, it is possible that one may also get the abnormal quantum gravity effect. This would mean a gravity effect dependent on the history of a system. At least, it seems that all previous experiments or astronomical observations do
Figure 9: (Color online) Shown is the physical picture of $N$ particles with the same wave function $\phi$, based on the de Broglie-Bohm theory.

not overrule this possibility. The present work clearly shows that it’s time to consider more seriously the new view of gravity, in particularly by future experiments.

In the above studies of the quantum gravity effect, we adopt in a sense the ordinary understanding of quantum mechanics. It is worthy to consider the quantum gravity effect based on other understanding of quantum mechanics. Here we give a brief discussion with the de Broglie–Bohm theory. We consider $N$ particles with the same wave function $\phi (\mathbf{x})$ shown in Fig. 9. In this pilot-wave theory, the particle is still a highly localized particle with a well-defined trajectory guided by the wave function. We can not predict the exact location of a particle because the initial position of the particle is not controllable by the experimenter, and the motion of the particle guided by the wave function is determined by the initial position of the particle. If we assume further that the energy is mainly carried by the particle, rather than the wave, for the situation shown in Fig. 9, the gravity effect due to these $N$ particles would be the same as that of $N$ classical particles whose density is $N |\phi (\mathbf{x})|^2$. Hence, this experimental scheme also gives us chance to test the de Broglie-Bohm theory. If the abnormal quantum gravity effect is verified, the de Broglie-Bohm theory would
be excluded in a sense.

VII. FIELD EQUATION INCLUDING QUANTUM GRAVITY EFFECT OF VACUUM EXCITATIONS

As shown in Sec. IV, the gravitational mass and inertial mass are the same mass in Einstein’s mass-energy relation, when gravity force is regarded as a thermodynamic effect. This explains in a natural way the equivalence principle. For the existence of vacuum excitations at location $\mathbf{x}$, a local reference system with acceleration given by Eq. (32) will not experience these vacuum excitations. Therefore, in this reference system, the physical law of special relativity would hold. This strongly suggests that Einstein’s general relativity should be included to consider the quantum gravity effect. Because different locations can have different accelerations, different locations have different reference systems satisfying the physical law of special relativity. To construct the connection between the reference systems at different locations, Riemannian geometry is the most convenient mathematical tool. Therefore, although we try to argue here that thermodynamics and the coupling between matter and vacuum are the essential mechanisms of gravity force, Riemannian geometry is still needed to construct a systematic theory, because of the same reason in constructing Einstein’s general relativity.

To clearly introduce the new field equation including the quantum effect of gravity, we first give a brief introduction of Einstein’s field equation for classical object (see Ref. [1]). In weak field approximation, we have $g_{00} \simeq - (1 + 2 \phi_g)$ with $\phi_g$ being the gravitational potential. From $\nabla^2 \phi_g = 4\pi G n$ and $T_{00} \simeq n$, we have

$$\nabla^2 g_{00} = -8\pi G T_{00}.$$  

If $T_{00}$ is due to classical particles, from the attractive gravity force between classical objects originating from the consideration of the free energy, the negative sign in the right-hand side of the above equation is included. We stress again that, in Einstein’s derivation of his field equation, the negative sign in the right-hand side of the above equation is due to the observed phenomena that the gravity force between two classical objects is attractive, rather than a property derived from fundamental principle. The development of the above
equation to relativistic case gives

\[ G_{\mu \nu} = -8\pi GT_{\mu \nu}. \]  

(40)

With general considerations of Riemannian geometry and general covariance, we get the following Einstein’s field equation

\[ R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = -8\pi GT_{\mu \nu}. \]  

(41)

Note that we have adopted the unit with \( c = 1 \) and the following Minkowski space-time

\[ \eta_{\mu \nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \]  

(42)

The presence of matter and radiation in the universe will establish various temperature field distributions. These temperature field distributions lead to the force for the matter and radiation in the universe. As shown previously, the physical mechanism of these temperature field distributions is due to the vacuum excitations. In higher-order calculations, the gravity effect of the vacuum excitations themselves should also be considered. Although this would be a complex nonlinear coupling process, general conditions (such as the principle of general covariance) could be imposed to attack this challenging problem. In a reasonable field equation including the gravitation effect of vacuum excitations themselves, one should introduce an extra energy-momentum tensor \( T_{\mu \nu}^{\text{vac}} \), apart from \( T_{\mu \nu} \) for ordinary matter and radiation. Because the vacuum excitations are virtual processes from the view of quantum field theory, these vacuum excitations are different from the ordinary matter. Because of this, these vacuum excitations cannot be described by the form of the energy-momentum tensor for the ordinary matter or radiation. In a sense, these vacuum excitations are integrated into the space time. Fortunately, in principle, one can add an extra term in Eq. (41) without the violation of Riemannian geometry and principle of general covariance. The sole form of the field equation is

\[ R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = -8\pi G \left( T_{\mu \nu} + T_{\mu \nu}^{\text{vac}} \right). \]  

(43)

Here

\[ T_{\mu \nu}^{\text{vac}} = \lambda g_{\mu \nu}. \]  

(44)
This general consideration leads to a request that the energy density in $T_{\mu\nu}^{\text{vac}}$ is uniform. Assuming $\rho_{VG}$ is the ground-state energy density of the vacuum, $\rho_{ex}(x,t)$ is the energy density of the vacuum in the presence of matter and radiation, we have

$$\delta \rho_{V}(x,t) \equiv \sqrt{\langle (\rho_{ex}(x,t) - \rho_{VG})^2 \rangle} \sim \frac{i}{2} k_B T_{\text{eff}}(x,t) / l_p^3.$$ 

Here $T_{\text{eff}}$ is determined by Eq. (7). The role of this spatially-dependent $\delta \rho_{V}(x,t)$ and $T_{\text{eff}}(x,t)$ has been included in Einstein field equation (41). The energy density of the vacuum excitations is $\rho_{V}(x,t) \equiv \langle \rho_{ex}(x,t) - \rho_{VG} \rangle$. There are two equivalent ways to understand the average $\langle \rangle$ considered here: the average about the time interval $\delta t_0$ which is much larger than the response time of the vacuum, and the average about the space scale $l_0$ which is much larger than the Planck length. We see that although $\delta \rho_{V}(x,t)$ is spatially dependent, in principle, $\rho_{V}(x,t)$ could be spatially independent. Of course, this is only a statement that there would be no contradiction between spatially dependent $\delta \rho_{V}(x,t)$ and spatially independent $\rho_{V}(x,t)$. The spatial independence of $\rho_{V}(x,t)$ is due to the general principle of covariance for spacetime itself. The role of $\rho_{V}(x,t)$ is included through $T_{\mu\nu}^{\text{vac}}$ in Eq. (43).

From $T_{66}^{\text{vac}} = \rho_{V}$, we have

$$T_{\mu\nu}^{\text{vac}} = \pm \begin{pmatrix} -\rho_{V} g_{00} & 0 & 0 & 0 \\ 0 & \rho_{V} g_{11} & 0 & 0 \\ 0 & 0 & \rho_{V} g_{22} & 0 \\ 0 & 0 & 0 & \rho_{V} g_{33} \end{pmatrix}. \quad (45)$$

For a flat universe, assuming the scale factor as 1 in the Fiedmann-Robertson-Walker (FRW) metric at the present time of our universe, we have

$$T_{\mu\nu}^{\text{vac}} = \pm \begin{pmatrix} -\rho_{V} & 0 & 0 & 0 \\ 0 & \rho_{V} & 0 & 0 \\ 0 & 0 & \rho_{V} & 0 \\ 0 & 0 & 0 & \rho_{V} \end{pmatrix}. \quad (46)$$

Considering the fact that various fields in the vacuum have the propagation velocity of $c$, it is not unreasonable to assume that various fields in the vacuum background have delocalized wave packets in the whole observable universe. Another reason is that when the big-bang model of the universe is adopted, various fields just have sufficient time to propagate in the whole observable universe. This could lead to delocalized wave packets for the vacuum excitation fields at least within the observable universe. The third reason is that when the
coupling between matter and vacuum is considered, the vacuum has the characteristic of superfluidity. These analyses suggest a repulsive gravity effect by nonzero and positive \( \rho_V \). This requests that one should take the negative sign in the above expression of \( T_{\mu\nu}^{\text{vac}} \). We finally get

\[
T_{\mu\nu}^{\text{vac}} = \begin{pmatrix} \rho_V & 0 & 0 & 0 \\ 0 & p_V & 0 & 0 \\ 0 & 0 & p_V & 0 \\ 0 & 0 & 0 & p_V \end{pmatrix}.
\] (47)

Here \( p_V = -\rho_V \). We see that vacuum excitations as a whole have positive energy and abnormal negative pressure. In our theory, it is clearly shown that the abnormal negative pressure physically originates from the quantum characteristic of vacuum excitations.

**VIII. DARK ENERGY DENSITY**

In this section, we will try to calculate \( \rho_V \) based on the whole thermal equilibrium in the coupling between matter and vacuum. If the gravity effect is regarded as the thermodynamics and the coupling between matters and vacuum, it is a natural request that the whole vacuum excitations (dark energy) should be determined based on the thermodynamic origin of gravity. Now we turn to consider the whole average effect of these vacuum excitations for large scale of the universe. We will try to calculate the whole average vacuum excitations, i.e. the dark energy density. In calculating the dark energy density, we will use the assumption of isotropy and homogeneity on the large scale of the universe. Another assumption is the spatially independent \( \rho_V \) discussed in previous section.

Using spherical polar coordinates for three-dimensional space, we have

\[
dr^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - Kr^2} + r^2 d\Omega \right].
\] (48)

Here \( d\Omega = d\theta^2 + \sin^2\theta d\phi^2 \). \( r, \theta, \phi \) are time-independent co-moving coordinates. \( a(t) \) is the scale factor. \( K = 1, -1, 0 \) for spherical, hyperspherical and Euclidean cases. In the present work, we consider the case of \( K = 0 \) verified by various astronomical observations.

If the evolution of the universe is considered, the radial coordinate \( r(z) \) of a source that is observed now with redshift \( z \) is

\[
r(z) = \frac{1}{a_0 H_0} \int_{1/(1+z)}^{1} \frac{dx}{x^2 \sqrt{\Omega_\Lambda + \Omega_M x^{-3} + \Omega_R x^{-4}}}. \] (49)
Here $\Omega_\Lambda \equiv 8\pi G \rho_{V0}/3H_0^2$, $\Omega_M \equiv 8\pi G \rho_{M0}/3H_0^2$, $\Omega_R \equiv 8\pi G \rho_{R0}/3H_0^2$. $\rho_{V0}$, $\rho_{M0}$ and $\rho_{R0}$ are the present values of the dark energy density, average cold matter (e.g. dust) energy density and hot matter (e.g. radiation) energy density. $a_0 \equiv 1$ and $H_0$ are the present values of the scale factor and Hubble constant. Because the gravity field (i.e. the vacuum excitations) has the propagation velocity of $c$, for the observer at $r = 0$, the above expression of $r$ about $z$ has the merit that, $r(z)$ is the radial coordinate of a source whose gravity field is observed now with redshift $z$. This gravity field at earlier time is that the observer could experience at the present time. This is the same reason why the above expression is very useful in calculating the luminosity of distant stars.

Various astronomical observations have given precision measurement of $\Omega_{M0}$ and $\Omega_{R0}$. Assuming $\alpha = \Omega_\Lambda / (\Omega_M + \Omega_R)$, we have

$$ r(\alpha, z) = \frac{1}{a_0 H_0 \sqrt{\Omega_M + \Omega_R}} \int_1^{1/(1+z)} \frac{dx}{x^2 \sqrt{\alpha + (\Omega_M / (\Omega_M + \Omega_R))^2} x^{-3} + (\Omega_R / (\Omega_M + \Omega_R))^2} \ . $$(50)

From this equation, we can also get $z \equiv z(\alpha, r)$. Note that the radial coordinate $r(\alpha, z)$ for co-moving sources is time-independent. We will use this sort of radial coordinate to calculate the dark energy density.

For an observer at $r = 0$ of the present time, the overall energy of cold matter and radiation the observer experiences is then

$$ E_{MR}(\alpha) = E_M(\alpha) + E_R(\alpha) \ . $$ (51)

Here $E_M$ for cold matter is given by

$$ E_M(\alpha) = \int_0^{\infty} \rho_{Mr}(r) \sqrt{\frac{1}{1 - v^2(\alpha, r)} \frac{1}{(1 + z(\alpha, r))^2}} r^2 dr d\Omega. $$ (52)

$\rho_{Mr}(r)$ is the cold matter density in the co-moving radial coordinate $r(\alpha, z)$, without considering the expansion of the universe. When this radial coordinate is adopted, the cold matter density in this coordinate system should not depend on the scale factor. Hence, when the present value of the scale factor $a$ is takes as 1, $\rho_{Mr}(r) = \rho_{M0}$. The existence of the average relative velocity $v(\alpha, r)$ between the observer and the matter at $r$ makes the cold matter energy density for the observer becomes $\rho_{M0} \sqrt{1/(1 - v^2(\alpha, r))}$. This is the physical origin of the factor $\sqrt{1/(1 - v^2(\alpha, r))}$ in the above expression. Because $z(\alpha, r)$ denotes the redshift, we have $v(\alpha, r) = ((1 + z(\alpha, r))^2 - 1) / ((1 + z(\alpha, r))^2 + 1)$. 

27
The factor $1/ (1 + z(\alpha, r))^2$ in the right-hand side of the above equation originates from two physical effects. When the thermodynamic origin of the gravity is considered, there are various sounds in the temperature field of the vacuum due to the presence of matter. In this situation, for the observer, the rate of arrival of the individual sounds in the gravitation field (temperature field) is reduced by the redshift factor $1/ (1 + z(\alpha, r))$. On the other hand, the energy of the individual sounds experienced by the observer is also reduced by the redshift factor $1/ (1 + z(\alpha, r))$. Hence, the effective energy density is reduced by the factor $1/ (1 + z(\alpha, r))^2$ in the above equation.

There is another way to understand the factor $1/ (1 + z(\alpha, r))^2$ in the above equation. For the matter with the radial coordinate $r(z)$, we consider the gravity field emitted at appropriate time $t(z)$. When this gravity field emitted at time $t(z)$ just arrives at the observer, the effective distance between the matter that we consider and the observer becomes $r(z)(1 + z)$ because of the expansion of the universe. For the observer, this is equivalent to the case that the energy the observer experiences for the matter at $r(z)$ is reduced by the multiplication of the factor $1/ (1 + z(\alpha, r))^2$.

As for $E_R$, because the radiation field propagates at a speed equal to $c$, we have

$$E_R(\alpha) = \int_0^\infty \rho_{R0} \frac{1}{(1 + z(\alpha, r))^2} r^2 dr d\Omega.$$  \hspace{1cm} (53)

It is understandable that the vacuum excitations leading to the gravity effect have also the propagation velocity of $c$. In this situation, it is similar to get $E_V$ for the vacuum excitations (dark energy), which is given by

$$E_V(\alpha) = \int_0^\infty \rho_{V0} \frac{1}{(1 + z(\alpha, r))^2} r^2 dr d\Omega.$$  \hspace{1cm} (54)

Assuming that there is a thermal equilibrium between matter and vacuum on the large scale of the universe, we have

$$E_{MR}(\alpha) = E_V(\alpha).$$  \hspace{1cm} (55)

The above equation is also due to the reason that the smallest space unit ($\sim l_p^3$) is the same for cold matter, hot matter and vacuum excitation, so that the overall freedom ($\sim V/l_p^3$ with $V$ being the volume of the universe) is the same for cold matter, hot matter and vacuum excitation.
It is straightforward to get
\[
\alpha = \frac{\int_0^\infty \left( \frac{\rho_{M0}}{\rho_{M0} + \rho_{R0}} \sqrt{1 - v^2(\alpha, r)} + \frac{\rho_{R0}}{\rho_{M0} + \rho_{R0}} \right) \frac{1}{(1 + z(\alpha, r))^2} r^2 dr d\Omega}{\int_0^\infty \frac{1}{(1 + z(\alpha, r))^2} r^2 dr d\Omega}. \tag{56}
\]
Combined with Eq. (50) to get \( z(\alpha, r) \), one can get the ratio \( \alpha \). The numerical result is \( \alpha \approx 2.2 \), agrees quantitatively with the result 7/3 in astronomical observations [7–11].

In the co-moving coordinate, the dark energy density is then
\[
\rho_{V0} = \alpha (\rho_{M0} + \rho_{R0}). \tag{57}
\]
Because the co-moving radial coordinate \( r(z) \) is time-independent, \( \rho_{M0} \) and \( \rho_{R0} \) can be also regarded as the cold matter density and radiation energy density in the co-moving coordinate. For an observer at other times, in the co-moving coordinate, \( \rho_{M0} \) and \( \rho_{R0} \) are also the cold matter density and radiation energy density for this observer. It is easy to show that the dark energy density is still given by the above equation. Together with this time-independent dark energy density in the co-moving coordinate, we see that \( \alpha \approx 2.2 \) is a universal value once the following conditions are satisfied.

(i) There is a big-bang origin of the universe.

(ii) The universe and its evolution are isotropic and homogeneous.

(iii) In the co-moving coordinate, the radiation energy density is much smaller than the cold matter energy density.

(iv) The evolution of the universe is a quasi-equilibrium process. This condition suggests that for cosmic inflation process, the calculation of \( \alpha \) deserves further studies.

One may consider the problem that why we do not introduce the similar concept of luminosity distance \( d_L = r(1 + z) \), so that the temperature field due to matter and radiation at the location of the observer becomes
\[
T_{MR}(\alpha) \propto \int_0^\infty \left( \rho_{M0} \sqrt{1 - v^2(\alpha, r)} + \rho_{R0} \right) \frac{1}{d_L^2} r^2 dr d\Omega. \tag{58}
\]
With the similar idea, the temperature field at the location of the observer due to vacuum excitations is
\[
T_V(\alpha) \propto \int_0^\infty \rho_{R0} \frac{1}{d_L^2} r^2 dr d\Omega. \tag{59}
\]
From \( T_{MR} = T_V \), we get
\[
\alpha = \frac{\int_0^\infty \left( \frac{\rho_{M0}}{\rho_{M0} + \rho_{R0}} \sqrt{1 - v^2(\alpha, r)} + \frac{\rho_{R0}}{\rho_{M0} + \rho_{R0}} \right) \frac{1}{(1 + z(\alpha, r))^2} dr d\Omega}{\int_0^\infty \frac{1}{(1 + z(\alpha, r))^2} dr d\Omega}. \tag{60}
\]
This method to calculate the dark energy density is wrong. In this method, at the location of the observer, the sum of the vacuum temperature due to the matter and radiation at different locations is adopted. As shown already previously, this sort of sum of the vacuum temperature due to the matter and radiation is nonsense. With the above equation, the numerical result of $\alpha$ is about 1.2. As expected, it does not agree with the astronomical observations.

In Eq. (57), the dark energy density is given in the time-independent co-moving coordinate. If the evolution of the scale factor $a(t)$ is considered, $\rho_V(t)$, $\rho_M(t)$ and $\rho_R(t)$ in the ordinary coordinate (where the proper distance $d(r, t) = a(t) r$ is adopted) need further studies. As shown previously, if the energy-momentum tensor given by Eq. (47) is adopted for the dark energy, the whole energy-momentum tensor can be written as

$$T_{\mu\nu} = T_{\mu\nu}^{\text{vac}} + T_{\mu\nu}^{\text{mat}} + T_{\mu\nu}^{\text{rad}}. \quad (61)$$

Here $T_{\mu\nu}^{\text{mat}}$ and $T_{\mu\nu}^{\text{rad}}$ are the energy-momentum tensors for cold matter and radiation. In the evolution of the universe, when the Friedmann-Robertson-Walker metric is used, the energy conservation law $T_{\nu}^{\mu;\nu} = 0$ leads to

$$\frac{d\rho}{dt} + \frac{3\dot{a}}{a} (p + \rho) = 0. \quad (62)$$

From this we have $\rho(t) \propto a(t)^{-3-3w}$. If $w = -1$ for dark energy as discussed in previous section, this requests that the dark energy density is constant in the evolution of the universe. It is similar to get the well-known results of $\rho_M(t) = \rho_{M0} (a(t))^{-3}$ and $\rho_R(t) = \rho_{R0} (a(t))^{-4}$.

For coexisting cold matter, radiation and dark energy, when there is no interchange of energy between different components, these evolutions for different components always hold. It is consistent with the result of the dark energy density in previous calculations. The above energy conservation law clearly shows a further condition in previous calculations of the dark energy density, i.e. the time-independent dark energy density relies on the condition that there is no energy interchange (or material conversion) between cold matter and hot matter including radiation. If this energy interchange happens, it is possible that the dark energy density becomes time-dependent. This deserves further theoretic studies for the early nonequilibrium universe, to find observable effect.

Most quantum field theories predict a huge value for the quantum vacuum. It is generally agreed that this huge value should be decreased $10^{120}$ times to satisfy the observation result.
Because there are no “true” wave functions for these “quantum zero-point states”, based on
our theory, even there are huge vacuum energy due to “quantum zero-point states”, the
gravity effect should be multiplied by zero! Another reason is that, the temperature of
the vacuum including only “quantum zero-point states” is zero. The finite temperature
characteristic of the vacuum is due to the excitations from the vacuum, which influence the
motion of the matters. Based on our theory, the cosmological constant problem is not a
“true” problem at all.

In this work, the vacuum energy calculated by us is due to the coupling and thermal
equilibrium between matter and vacuum background. The coupling and thermal equilibrium
lead to various “true” excitations from the vacuum. What we calculated in this work is in
fact aims at these excitations which can be described by sophisticated and delocalized wave
functions. In Fig. 10, we give a summary of the role of different forms of vacuum energy.

For the sake of completeness, now we consider the evolution of our universe in a brief
way. $g_{\mu\nu} (t)$ for flat universe is

$$g_{\mu\nu} (t) = \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & a^2(t) & 0 & 0 \\
0 & 0 & a^2(t) & 0 \\
0 & 0 & 0 & a^2(t)
\end{pmatrix}.$$  \hspace{1cm} (63)

We assume the present value of the scale factor $a$ as 1. If the matter (including dark matter)
and radiation are regarded as classical when the evolution of the whole universe is studied,
$T_{\mu\nu}$ in Eq. (43) takes the ordinary form. From Eqs. (43) and (63), we have \[26\]

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \left[\rho_{M0} (1 + z)^3 + \rho_{R0} (1 + z)^4 + \rho_V\right], \hspace{1cm} (64)$$

and

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3} \left[\rho_{M0} (1 + z)^3 /2 + \rho_{R0} (1 + z)^4 - \rho_V\right]. \hspace{1cm} (65)$$

**IX. FIELD EQUATION INCLUDING QUANTUM GRAVITY EFFECT OF MATTER**

The quantum gravity effect of vacuum excitations has been included in Einstein's field
equation, which explains in a simple way the remarkable astronomical observations about
Figure 10: (Color online) Shown is different role of the fluctuating vacuum energy $\rho_{ex}$. Because of the whole thermal equilibrium between matter and vacuum, and the request of general covariance on the gravity effect of the vacuum excitations, $\rho_{ex}$ has a fluctuation around $\rho_{VG} + \rho_{V}$. The fluctuations in $\rho_{ex}$ lead to the gravity effect in the general relativity for the ordinary matter and radiation, while $\rho_{V}$ (dark energy) leads to repulsive gravity effect which gives a physical mechanism of the accelerating universe.

dark energy. The basic clues of the combination of quantum gravity effect of matter and general relativity are:

(1) The direction of the acceleration field due to gravity force should be determined by the tendency of decreasing the free energy.

(2) The field equation should satisfy the principle of general covariance.

We first consider further the abnormal quantum gravity effect based on Eqs. (32) and (33). For a particle with mass $m$ and wave function $\phi$, Eqs. (32) and (33) have another equivalent form as follows.

$$a_c (\mathbf{R}) = Gm \int d^3x \phi^* (\mathbf{x}, t) \frac{\mathbf{x} - \mathbf{R}}{|\mathbf{x} - \mathbf{R}|^3} \phi (\mathbf{x}, t)$$
Here \( \mathbf{a}_c \) is the acceleration field without considering quantum effect of gravity. \( \mathbf{a}_q \) is the acceleration field when quantum effect of gravity is considered. For a classical particle, \( \mathbf{a}_q = \mathbf{a}_c \). Note that \( a_c = |a_c| \). From Eq. (66), we have \( \nabla \times \mathbf{a}_c = 0 \). In this situation, we get a simple relation between \( \mathbf{a}_c \) and \( \mathbf{a}_q \), which is given by

\[
\mathbf{a}_q (\mathbf{R}) = f_q (\mathbf{R}) \mathbf{a}_c (\mathbf{R}).
\]

Here \( f_q (\mathbf{R}) = \pm 1 \). The sign \( \pm \) at a location \( \mathbf{R} \) is determined by the rule that the direction of the acceleration \( \mathbf{a}_q (\mathbf{R}) \) should point to the increasing of \( T (\mathbf{R}) \sim |\mathbf{a}_c (\mathbf{R})| = |\mathbf{a}_q (\mathbf{R})| \) in the neighboring region including \( \mathbf{R} \). In classical case, this rule physically originating from the property of the free energy has explained why the gravity force between two classical objects is attractive. Obviously, it is natural to generalize this rule to quantum wave packet, because classical mechanics has been replaced by quantum mechanics, when fundamental physical law is addressed.

In reality, the wave function of a lot of particles may be very complex. For brevity’s sake, we consider a system of identical bosons which can be directly generalized to more complex case. The many-body wave function is assumed as

\[
\Psi_{\text{matter}} = \Psi_{\text{matter}} (x_1, x_2, \ldots, x_N, t).
\]

The single-particle density matrix is given by

\[
\rho_1 (x, x', t) = \int \Psi^*_{\text{matter}} (x, x_2, \ldots, x_N, t) \Psi_{\text{matter}} (x', x_2, \ldots, x_N, t) \, d^3x_2 \ldots d^3x_N \\
\equiv \left\langle \hat{\Psi}^\dagger (x, t) \hat{\Psi} (x', t) \right\rangle.
\]

This single-particle density matrix can be diagonalized, i.e. written in the form

\[
\rho_1 (x, x', t) = \sum_j N_j \phi_j^* (x, t) \phi_j (x', t).
\]

It is natural to consider the field equation of quantum gravity with this diagonalized single-particle density matrix.

To calculate the gravity field of the whole system when quantum effect is sufficiently considered, we first calculate the following field equation for \( N_1 \) bosons described by the wave function \( \phi_1 \)

\[
R_{\mu\nu} (1) - \frac{1}{2} g_{\mu\nu} (1) R (1) = -8\pi G T_{\mu\nu} (1).
\]
Here $T_{\mu\nu}(1) \equiv T_{\mu\nu}[N_1, \phi_1]$ can be calculated with the standard method.

The calculation based on the above equation does not consider the sign problem. From the following formula

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\lambda}(1) \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0,$$  \hspace{1cm} (73)

we can get the acceleration field $a(1)$. With the rule to solve the sign problem, we get a new field equation

$$R_{\mu\nu}(1') - \frac{1}{2} g_{\mu\nu}(1') R(1') = -8\pi G f_1(x^\tau) T_{\mu\nu}(1).$$  \hspace{1cm} (74)

Here, $f_1(x^\tau) = \pm 1$ is determined by the sign rule, and the acceleration field $a(1)$ obtained from Eqs. (72) and (73). It is clear that the above new field equation still satisfies general covariance. One should note that if at $x^\tau = \beta^\tau$, $f_1(\beta^\tau) = -1$ based on the sign rule. In calculating the above new field equation at $\beta^\tau$, $T_{\mu\nu}(1)$ in the whole space time is still the result based on the standard method. More specifically, Eq. (74) is not equivalent to the following field equation

$$R_{\mu\nu}(1') - \frac{1}{2} g_{\mu\nu}(1') R(1') = -8\pi G f_1(x^\tau) T_{\mu\nu}(1'),$$

$$T_{\mu\nu}(1') = \begin{cases} 
T_{\mu\nu}(x^\tau), x^\tau \in \Sigma_1 \\
-T_{\mu\nu}(x^\tau), x^\tau \in \Sigma_2 \\
T_{\mu\nu}(x^\tau), x^\tau \in \Sigma_3 \\
\ldots \ldots 
\end{cases} \hspace{1cm} (75)$$

It is very clear that this ‘wrong’ field equation is not self-consistent.

It is similar to consider $N_2$ bosons described by the wave function $\phi_2$. It is straightforward and natural to get the following field equation.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G \left( \sum_j f_j(x^\tau) T_{\mu\nu}(j) + T_{\mu\nu}^{\text{vac}} \right).$$  \hspace{1cm} (76)

Here $T_{\mu\nu}(j)$ is the ordinary energy-momentum tensor for $N_j$ identical particles described by the wave function $\phi_j$. $f_j(x^\tau)$ is determined by the sign rule for this mode. For the sake of completeness, $T_{\mu\nu}^{\text{vac}}$ is also included in the above equation.

X. SUMMARY AND DISCUSSION

In summary, Einstein’s field equation is extended to include the quantum effect of gravity, from the general assumption that gravity force originates from the coupling and thermal
equilibrium between matter and vacuum background. With this new field equation and physical mechanism of gravity force, without any fitting parameter, the accelerating universe of remarkable astronomical observations is quantitatively explained. We stress that the pioneering works on the connection between thermodynamics and gravity force, and the astronomical observations of accelerating universe play key role in the construction of the present theory. In fact, the present work is the result of the direct stimulation of these two works. The present paper is a significant extension of our previous works [31–34], with a lot of physical concepts being clarified and new method added to calculate the dark energy density by including general relativity.

The present theory has several significant features.

(1) It is a simple extension of Einstein’s field equation to include quantum effect of gravity.

(2) Different from Newton’s gravity law and Einstein’s general relativity, the present theory gives an interpretation why the gravity force between two classical objects is attractive. At least, the weak equivalence principle also becomes a derivable property.

(3) With the same reason to explain the attractive gravity force between two classical objects, the present theory predicts new physical effect, i.e. abnormal quantum gravity effect.

(4) The present theory quantitatively agrees with the astronomical observations of accelerating universe without any fitting parameter.

(5) If the present theory is verified, it has wide applications, even in condensed matter physics and experimental test of the many-world interpretation and de Broglie-Bohm theory, etc.

(6) Last but maybe most important, our theory can be falsified by experiment on earth by measuring gravity acceleration in the interior of a superfluid helium sphere.

Because of the above features, we believe the present theory should be taken seriously. In particular, these features mean great opportunity to relevant experiments on earth and astronomical observations.

One of the main results of the present work is the possible emergence of the quantum effect of gravity at the macroscopic scale. Nevertheless, it is an interesting issue to consider whether this macroscopic and abnormal quantum gravity effect could be derived from the microscopic mechanism about the quantum gravity, such as superstring with a positive cosmological constant [28], loop quantum gravity and twistor theory [29], etc. Recently,
the relation between gravity and thermodynamics has been studied in the frame of loop quantum gravity [30], which provides possible clue to this problem.

Although the microscopic interaction mechanism of quantum gravity at Planck scale is an unsolved problem, the thermodynamics of the macroscopic quantum gravity effect of this work is still meaningful. When statistical mechanics was initiated in 1870 by Boltzmann, the concept of atom is still an unrecognized hypothesis, not to mention the collision mechanism between atoms due to electromagnetic interaction. However, this does not influence the power of statistical mechanics in the description of gas dynamics. On the contrary, the theoretical and experimental advances of statistical mechanics greatly promoted further understanding of atoms. Once the thermodynamic origin of gravity is verified by future experiments and astronomical observations, the abnormal quantum gravity effect would also greatly promote the understanding of microscopic quantum gravity at Planck scale.

In this work, the repulsive gravity effect for superfluid helium sphere and dark energy physically originate from the same mechanism that the wave packet of energy is delocalized in the space scale we study. If the de Broglie-Bohm theory is correct, we would not observe the abnormal gravity effect for superfluid helium. However, for dark energy originating from various vacuum excitations, it is still possible that the energy is carried by various wave packets, rather than particles, because there is no stable particles in the vacuum excitations. Hence, even the abnormal gravity effect for superfluid helium is excluded in future experiments, it is still possible that the theory of the present work could be applied to accelerating universe.

One may consider the problem that whether there is abnormal quantum gravity effect in the interior of a neutron star. If the temperature (not the vacuum temperature) of the neutron star is zero and the wave packets of all neutrons are delocalized in the whole interior of the neutron star, we would expect an abnormal quantum gravity effect. However, the temperature of the neutron state is in fact extremely high. The typical temperature of a neutron star is about $10^6$ kelvins. In addition, the neutron star is not an ideal Fermi system without the consideration of the interaction between neutrons. The neutron star comprises very complex structures, such as the out core consisting of neutron-proton Fermi liquid and the inner core consisting of possible quark gluon plasma. In quantum statistical mechanics, the approximate model of the neutron star as an ideal Fermi system holds because of the validity of the local density approximation. The validity of this ideal Fermi system does
not mean that the wave packets of all neutrons are delocalized in the whole interior of the neutron star. Considering the extremely high temperature and complex structure of the neutron star, it is more reasonable to assume that the delocalized scale of the wave packet of the neutrons is much smaller than the size of the neutron star (the typical size of a neutron star is about 10 km). In this situation, we still expect the normal gravity effect in the interior of the neutron star.

Acknowledgments

We thank the discussions with Prof. Biao Wu, and his great encouragements. We also thank Prof. W. Vincent Liu’s great encouragements. This work was supported by National Key Basic Research and Development Program of China under Grant No. 2011CB921503 and NSFC 10875165.
Appendix

To get Eq. (1) about the entropy increase of a particle having a displacement, there is another request of the loss of information about the trajectory of the particle. In Fig. 11(a), we show the motion of a particle along the dashed line. Because of the location resolution from the view of the vacuum background, the dashed line is partitioned by the box with side length $l_c$. At time $t_1$, the wave packet of the particle is shown by the dashed line. At a later time $t_2 (= t_1 + l_c/v)$, the wave packet of the particle is shown by the solid line. At time $t_2$, the information of the location of the particle at time $t_1$ recorded by the vacuum background will be lost because of two reasons: (1) At time $t_2$, although the location of the particle with spatial resolution $l_c$ is recorded by the vacuum background, the velocity information is highly uncertain. From $\Delta x \Delta p \geq \hbar/2$, the velocity uncertainty of the particle is $\Delta v \sim c$. In this situation, at time $t_2$, after the position of the particle is recorded by the vacuum background, the history of the particle is lost from the view of the vacuum background. (2) At time $t_1$, although the location with spatial resolution $l_c$ is recorded by the vacuum background, it will be lost rapidly after a displacement at time $t_2$. At time $t_1$, the location record is due to the coupling between the particle and the vacuum background. In other words, due to the coupling between the particle and the vacuum background, at the location of the particle at time $t_1$, the vacuum has larger fluctuations. It’s these spatial-dependent vacuum fluctuations record the location of the particle with resolution $l_c$. Because the vacuum background is highly fluctuating, at time $t_2$, the location information at time $t_1$ will be lost because of the re-establishment of the thermal equilibrium in the vacuum background.

To justify further the above derivation of Eq. (1), we give here several discussions.

(i) In the above derivation, we assume a strong coupling between the particle and the vacuum background, so that the location of the particle can be recorded by the vacuum background. When quantum field theory is used, the vacuum background is in fact full of various gauge fields, virtual matter-antimatter pairs. This implies a strong coupling between the particle and the vacuum background.

(ii) One should distinguish two sorts of coherence lengths: the coherence length $l_c$ and the ordinary thermal de Broglie wave length. In the non-relativistic approximation, the thermal de Broglie wave length reflects the spatial coherence length which is $l_{de} \sim h/\Delta p$. We stress that $l_c$ and $l_{de}$ have different physical origins. $l_c$ originates from the coupling between the

38
Figure 11: (Color online) Fig. (a) and Fig. (b) illustrate the physical picture to get the starting point given by Eq. (1). Fig. (c) shows the essential difference between $l_c$ and $l_{de}$.

particle and the vacuum background. Because of the extremely large vacuum zero-point energy and strong coupling between matter and vacuum, the special relativity and quantum mechanics are used to calculate $l_c$. In contrast to $l_c$, $l_{de}$ originates from the coupling or interaction between the particle we study and other particles or environment. To clearly elaborate this, we take a hydrogen atom as an example, which consists of an electron and a proton. For a system consisting of a large number of hydrogen atoms, $l_{de} = \sqrt{2\pi \hbar / \sqrt{mk_B T_{sys}}}$ with $T_{sys}$ being the ordinary temperature in thermal equilibrium. $l_{de}$ physically originates from the interatomic interaction or the coupling with the environment. For Bose-Einstein condensation of atomic hydrogen [35], $l_{de}$ can arrive at 300 Å. In Fig. 11(c), we show $l_c$ and $l_{de}$ for an atom.

(iii) Although there is a velocity uncertainty $\Delta v \sim c$ for a particle due to the strong coupling with the vacuum, we stress that this does not mean that the particle will have highly random motion in the vacuum without other forces. The reason lies in that the coupling between a particle and the vacuum will lead a “dressed” state including the local vacuum excitations. As a whole, the velocity uncertainty could be much smaller than $c$. 

39
This is a little similar to the electron in a hydrogen atom. In the view of proton in the hydrogen atom, the velocity uncertainty of the electron is about $h/m_e l_{hy}$ with $m_e$ being the electron mass and $l_{hy}$ the size of the hydrogen atom. However, for the hydrogen atom as a whole, the velocity uncertainty is determined by the wave packet of the hydrogen atom, rather than the electron. In a sense, it is possible that there is a physical mechanism of spatially resolved quantum non-demolition detection \[36\] about the particle by the vacuum with spatial resolution $l_c$.

\[1\] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (New York, Wiley-VCH, 1972).

\[2\] R. J. Adler, H. Mueller and M. L. Perl, [arXiv:1101.5626](http://arxiv.org/abs/1101.5626) (2011).

\[3\] E. Verlinde, arXiv: 1001.0785 (2010).

\[4\] See, e.g., R. G. Cai, L. M. Cao and N. Ohta, Phys. Rev. D 81, 061501(R) (2010); T. Padmanabhan, Phys. Rev. D 81, 124040 (2010); F. W. Shu and Y. G. Gong, [arXiv:1001.3237](http://arxiv.org/abs/1001.3237) (2010); M. Li and Y. Wang, Phys. Lett. B 687, 243 (2010); T. W. Wang, Phys. Rev. D 81, 104045 (2010); P. Nicolini, Phys. Rev. D 82, 044030 (2010); Y. F. Cai and E. N. Saridakis, Phys. Lett. B 697, 280 (2011); R. Banerjee and B. R. Majhi, Phys. Rev. D 81, 124006 (2010).

\[5\] L. Modesto and A. Randono, [arXiv:1003.1998](http://arxiv.org/abs/1003.1998) (2010); J. W. Lee, [arXiv:1003.4464](http://arxiv.org/abs/1003.4464) (2010); I. V. Vancea and M. A. Santos, [arXiv:1002.2454](http://arxiv.org/abs/1002.2454) (2010); M. R. Setare and D. Momeni, [arXiv:1004.0589](http://arxiv.org/abs/1004.0589) (2010).

\[6\] T. Jacobson, Phys. Rev. Lett. 75, 1260 (1995).

\[7\] Adam G. Riess *et al*., Astronomical J. 116, 1009 (1998).

\[8\] S. Perlmutter *et al*., Astrophysical J. 517, 565 (1999).

\[9\] [http://lambda.gsfc.nasa.gov/product/map/current/map_bibliography.cfm](http://lambda.gsfc.nasa.gov/product/map/current/map_bibliography.cfm)

\[10\] M. Kowalski *et al*., Astrophys. J. 686, 749 (2008).

\[11\] [http://lambda.gsfc.nasa.gov/product/map/dr4/pub_papers/sevenyear/basic_results/wmap_7yr_basic_results.pdf](http://lambda.gsfc.nasa.gov/product/map/dr4/pub_papers/sevenyear/basic_results/wmap_7yr_basic_results.pdf).

\[12\] Alexander D. Cronin, Joerg Schmiedmayer, David E. Pritchard, Rev. Mod. Phys. 81, 1051 (2009).

\[13\] T. M. Niebauer, G. S. Sasagawa, J. E. Faller and F. Klopping, Metrologia 32, 159 (1995);
G. d’Agostino, S. Desogus, A. Germak, C. Origlia, D. Quagliotti, G. Berrino, G. Corrado, V. d’Errico and G. Ricciardi, Ann. Geophys. 51, 39 (2008); S. Svitlov, P. Maslyk, C. Rothleitner, H. Hu and L. J. Wang, Metrologia 47, 677 (2010).

[14] W. A. Prothero, J. Goodkind, Rev. Sci. Instrum. 39, 1257 (1968); J. Goodkind, Rev. Sci. Instrum. 70, 4131 (1999).

[15] J. D. Bekenstein, Phys. Rev. D 7, 2333 (1973).

[16] J. M. Bardeen, B. Carter, and S. W. Hawking, Commun. Math. Phys. 31, 161 (1973).

[17] S. W. Hawking, Commun Math. Phys. 43, 199 (1975).

[18] P. C. W. Davies, J. Phys. A 8, 609 (1975).

[19] W. G. Unruh, Phys. Rev. D 14, 870 (1976).

[20] T. Padmanabhan, Rep. Prog. Phys. 73, 046901 (2010).

[21] N. Poli, F. Y. Wang, M. G. Tarallo, A. Alberti, M. Prevedelli and G. M. Tino, Phys. Rev. Lett. 106, 038501 (2011); P. Clade et al., Europhys. Lett. 71, 730 (2005); G. Ferrari, N. Poli, F. Sorrentino and G. M. Tino, Phys. Rev. Lett. 97, 060402 (2006); F. Sorrentino et al., Phys. Rev. A 79, 013409 (2009).

[22] Emile Hoskinson, Yuki Sato, and Richard Packard, Phys. Rev. B 74, 100509(R) (2006).

[23] O. Penrose and O. Onsager, Phys. Rev. 104, 576 (1956).

[24] H. Everett, Rev. Mod. Phys. 29, 454 (1957).

[25] S. Weinberg, Cosmology (USA, Oxford University Press, 2008).

[26] P. J. E. Peebles and Bharat Ratra, Rev. Mod. Phys. 75, 559 (2003).

[27] A. J. Leggett, Quantum Liquids: Bose Condensation and Cooper Pairing in Condensed-Matter Systems (Oxford University Press, New York, 2006).

[28] Shamit Kachru, Renata Kallosh, Andrei Linde, and Sandip P. Trivedi, Phys. Rev. D 68, 046005 (2003).

[29] R. Penrose, J. Math. Phys. 8, 345 (1967).

[30] L. Smolin, arXiv:1001.3668 (2010).

[31] H. W. Xiong, arXiv:1012.5858 (2010).

[32] H. W. Xiong, arXiv:1101.0525 (2011).

[33] H. W. Xiong, arXiv:1101.1270 (2011).

[34] H. W. Xiong, arXiv:1101.4890 (2011).

[35] Dale G. Fried, Thomas C. Killian, Lorenz Willmann, David Landhuis, Stephen C. Moss, Daniel
Kleppner, and Thomas J. Greytak, Phys. Rev. Lett. 81, 3811 (1998).

[36] V. B. Braginsky and F. Khalili, *Quantum Measurements* (Cambridge Univ. Press, Cambridge, 1992).