Modelling the He I triplet absorption at 10 830 Å in the atmosphere of HD 209458 b

M. Lampón1, M. López-Puertas1, L. M. Lara1, A. Sánchez-López1, M. Salz2, S. Czesla2, J. Sanz-Forcada3, K. Molaverdikhani4, F. J. Alonso-Floriano5, L. Nortmann6,7, J. A. Caballero3, F. F. Bauer1, E. Pallé6,7, D. Montes8, A. Quirrenbach9, E. Nagel2, I. Ribas10,11, A. Reiners12, and P. J. Amado1

1 Instituto de Astrofísica de Andalucía (IAA-CSIC), Glorieta de la Astronomía s/n, 18008 Granada, Spain
e-mail: m.lampon@iaa.es
2 Hamburger Sternwarte, Universität Hamburg, Gojenbergsweg 112, 21029 Hamburg, Germany
3 Centro de Astrobiología (CSIC-INTA), ESAC, Camino bajo del castillo s/n, 28692 Villanueva de la Cañada, Madrid, Spain
4 Max-Planck-Institut für Astronomie, Königstuhl 17, 69117 Heidelberg, Germany
5 Leiden Observatory, Leiden University, Postbus 9513, 2300 RA, Leiden, The Netherlands
6 Instituto de Astrofísica de Canarias, Calle Vía Láctea s/n, 38200 La Laguna, Tenerife, Spain
7 Departamento de Astrofísica, Universidad de La Laguna, 38026 La Laguna, Tenerife, Spain
8 Departamento de Física de la Tierra y Astrofísica & IPARCOS-UCM (Instituto de Física de Partículas y del Cosmos de la UCM), Facultad de Ciencias Físicas, Universidad Complutense de Madrid, 28040 Madrid, Spain
9 Landessternwarte, Zentrum für Astronomie der Universität Heidelberg, Königstuhl 12, 69117 Heidelberg, Germany
10 Instituto de Ciencias de l’Espai (CSIC-IEEC), Campus UAB, c/ de Can Magrans s/n, 08193 Bellaterra, Barcelona, Spain
11 Institut d’Estudis Espacials de Catalunya (IEEC), 08034 Barcelona, Spain
12 Institut für Astrophysik, Georg-August-Universität, Friedrich-Hund-Platz 1, 37077 Göttingen, Germany

Received 22 November 2019 / Accepted 21 February 2020

ABSTRACT

Context. HD 209458 b is an exoplanet with an upper atmosphere undergoing blow-off escape that has mainly been studied using measurements of the Lyα absorption. Recently, high-resolution measurements of absorption in the He I triplet line at 10 830 Å of several exoplanets (including HD 209458 b) have been reported, creating a new opportunity to probe escaping atmospheres.

Aims. We aim to better understand the atmospheric regions of HD 209458 b from where the escape originates.

Methods. We developed a 1D hydrodynamic model with spherical symmetry for the HD 209458 b thermosphere coupled with a non-local thermodynamic model for the population of the He I triplet state. In addition, we performed high-resolution radiative transfer calculations of synthetic spectra for the helium triplet lines and compared them with the measured absorption spectrum in order to retrieve information about the atmospheric parameters.

Results. We find that the measured spectrum constrains the [H]/[H'] transition altitude occurring in the range of 1.2–1.9 Rp. Hydrogen is almost fully ionised at altitudes above 2.9 Rp. We also find that the X-ray and extreme ultraviolet absorption takes place at effective radii from 1.16 to 1.30 Rp, and that the He I triplet peak density occurs at altitudes from 1.04 to 1.60 Rp. Additionally, the averaged mean molecular weight is confined to the 0.61–0.73 g mole−1 interval, and the thermospheric H/He ratio should be larger than 90/10, and most likely approximately 98/2. We also provide a one-to-one relationship between mass-loss rate and temperature.

Based on the energy-limited escape approach and assuming heating efficiencies of 0.1–0.2, we find a mass-loss rate in the range of (0.42–1.00) × 10−11 g s−1 and a corresponding temperature range of 7125–8125 K.

Conclusions. The analysis of the measured He I triplet absorption spectrum significantly constrains the thermospheric structure of HD 209458 b and advances our knowledge of its escaping atmosphere.

Key words. planets and satellites: atmospheres – planets and satellites: individual: HD 209458 b – planets and satellites: gaseous planets

1. Introduction

Close-in exoplanets receive considerable high-energy irradiation that triggers an outgoing bulk motion and expands their atmospheres. This in turn results in a very efficient mechanism for atmospheric mass loss. In some cases, the velocity reaches supersonic values that give rise to the so-called blow-off escape. This escape process is key for understanding the formation and evolution of observed (exo)planets. Lecavelier des Etangs et al. (2004) showed that such an efficient escape could lead to the loss of a significant fraction of the atmosphere in hot Jupiters, or even to a complete atmospheric loss in some hot Neptunes. In the same vein, Locci et al. (2019) showed that the occurrence of gas giants of less than 2 MJup in young stars is higher than in old ones. Furthermore, evaporation escape has also been invoked to explain the distribution with respect to the star–planet separation of super-Earths and mini-Neptunes (Jin & Mordasini 2018). Observations of extended atmospheres and their modelling are therefore essential for understanding the physical conditions that trigger evaporation escape and for obtaining a deeper understanding of planetary diversity, formation, and evolution.

The first detection of an evaporating atmosphere was reported by Vidal-Madjar et al. (2003), who observed the
Lya line in absorption in the atmosphere of HD 209458 b and concluded that the signal originated from escaping hydrogen. Subsequent studies, both theoretical (e.g., Yelle 2004; Tian et al. 2005; Penz et al. 2008; Sanz-Forcada et al. 2010, 2011; Koskinen et al. 2013; Salz et al. 2016) and observational (Vidal-Madjar et al. 2003, 2004, 2008, 2013; Ballester et al. 2007; Ballester & Ben-Jaffel 2015; Linsky et al. 2010; Ehrenreich et al. 2008; Ben-Jaffel & Hosseini 2010; Jensen et al. 2012), supported the prediction that the extended atmosphere of HD 209458 b can be explained by a highly effective escape. Following this thread, hydrodynamic escape models were used to explain the observations of the absorption excess in Lya and ultraviolet (UV) in other planets, such as in HD 189733 b, WASP-12 b, GJ 436 b, GJ 3470 b, and KELT-9 b (Vidal-Madjar et al. 2013; Ehrenreich et al. 2008, 2015; Lecavelier des Etangs et al. 2010, 2012; Fossati et al. 2008, 2013; Ben-Jaffel & Ballester 2013; Kulow et al. 2014; Lavie et al. 2017; Yan & Henning 2018).

Lya observations are restricted to space-based telescopes and are significantly affected by absorption in the interstellar medium, which leads to a scarcity of observational data. The He I 2 3S − 2 3P triplet, hereafter He(2S), composed of three lines at 10 830.33, 10 830.25, and 10 829.09 Å, is not strongly absorbed by the interstellar medium (Indriolo et al. 2009) and can be observed from the ground. Seager & Sasselov (2000) studied transmission spectra for close-in planets and estimated significant absorption of He(2S) that could be observable, particularly when the planet has an extended atmosphere. More recently, Oklopcić & Hirata (2018) developed a model suitable for estimating He(2S) absorption in extended atmospheres.

He(2S) absorption was detected for the first time in the atmosphere of WASP-107 b with the Wide Field Camera 3 (WFC3) onboard the Hubble Space Telescope (HST; Spake et al. 2018), and almost simultaneously with the high-resolution spectrograph CARMENES (Calar Alto high-Resolution search for M dwarfs with Exoearths with Near-infrared and optical Échelle Spectrographs; Quirrenbach et al. 2016, 2018) at the 3.5 m Calar Alto Telescope in the atmospheres of WASP-69 b (Nortmann et al. 2018), HAT-P–11 b (Allart et al. 2018), and HD 189733 b (Salz et al. 2018). A few more observations were also recently performed. Thus, HAT-P–11 b data were acquired with WFC3 (Mansfield et al. 2018), whereas CARMENES was also used to observe WASP-107 b (Allart et al. 2019) and HD 209458 b (Alonso-Floriano et al. 2019).

In this work we analyse the new He(2S) observations of HD 209458 b reported by Alonso-Floriano et al. (2019). Based on the fact that this absorption probes the region where the escape originates, we aim at gaining a better understanding of the atmosphere of HD 209458 b and establishing tighter constraints on its mass-loss rate, temperature, and composition (e.g. the H/He ratio). To this end, we developed a 1D hydrodynamic and spherically symmetric model together with a non-local thermodynamic model for the He(2S) state similar to that reported by Oklopcic & Hirata (2018). The overall model computes the He(2S) radial distribution that it is then entered into a high-resolution line-by-line radiative transfer model in order to compute He(2S) absorption profiles. Synthetic spectra are then compared with the observed one in order to derive the properties of the escaping atmosphere.

This paper is organised as follows: Sect. 2 summarises the observations and Sect. 3 describes the modelling of the He(2S) radial densities and its absorption. The results, and a discussion on comparisons with previous works for the mass-loss rate, temperature, and H density, are presented in Sect. 4. The main conclusions are summarised in Sect. 5.

2. Observations of He I \( \lambda 10830 \) Å absorption

Here we briefly summarise the observations reported by Alonso-Floriano et al. (2019) of the helium excess absorption from the atmosphere of HD 209458 b using CARMENES (see Fig. 1). The He excess absorption peaks at a value of 0.91±0.10% at mid-transit. The core of the absorption shows a net blueshift of 1.8 ± 1.3 km s\(^{-1}\), suggesting that the helium envelope moves at that velocity, in the line of sight from the Earth with respect to the rest-frame of the planet. This velocity is rather low, similar to that reported by Snellen et al. (2010) from observations of the carbon monoxide band near 2 μm that occurs in the lower thermosphere, and suggests that the He triplet absorption takes place at relatively low altitudes (within a few planetary radii, see Sect. 4 below).

Figure 1 also shows excess absorption near 10 829.8 Å. Since the absorption is well above the estimated errors, it might be caused by an additional atmospheric absorption component. However, it is largely blue shifted, by around 13 km s\(^{-1}\), and therefore could be due to atmospheric escape occurring beyond the thermosphere. Alonso-Floriano et al. (2019) constructed the light curve of the He I signal (see their Fig. 5) and obtained an average in-transit absorption of \(-0.44\%\), about a factor two smaller than the peak absorption. However, there is no clear evidence for pre- or post-transit absorption signals.

We assumed a one-dimensional hydrodynamic and spherically symmetric model for analysing this absorption. This simplification is justified because: (1) the bulk of the absorption takes place at relatively low radii, well below the Roche lobe height, and with rather low wind velocities; (2) there are no clear signs of asymmetry in the absorption; and (3) the light curve does not exhibit signs of pre- or post-transit absorption – for
example there are no cometary-like tail features. This model also
has the advantage that it is computationally very efficient with
respect to 1D models that solve the energy balance equation, and is
about one order of magnitude faster than the model of García-
Muñoz (2007; priv. comm.) and about two orders of magnitude
faster than the model of Salz et al. (2016). It also allows us to
explore a wide range of atmospheric parameters, and yet gives
reasonable density distributions (see, e.g. the comparison with the
model of Salz et al. 2016 in Sect. 3.1 and Fig. 3 below).

The analysis of the velocity shifts in the He(2S) absorption is
potentially interesting because it would provide us information
about the 3D velocity distribution and maybe also break some of
the degeneracy between temperature and mass-loss rates. Our
model, as it is one dimensional and spherically symmetric, cannot
explain net blue or red shifts and therefore such an analysis is
beyond the scope of this paper. Nevertheless, in order to
better fit the measured spectrum we assume a velocity of
−1.8 km s−1 along the observational line of sight superimposed
on the radial velocities of our model. In the following sections
we describe the He triplet density model and the radiative
absorption calculations used to constrain the characteristics of
the escaping atmosphere of this planet.

### 3. Modelling of the helium triplet density and the absorption

We computed the He(2S) radial density by means of a 1D
hydrodynamic and spherically symmetric model (see, e.g. Yelle
2004; Tian et al. 2005; García-Muñoz 2007; Koskinen et al.
2013; Salz et al. 2016) and following the methods discussed in
Oklopčić & Hirata (2018). The hydrodynamic equations for
mass and momentum conservation, for a steady-state radial
atmospheric outflow are given by

\[
\frac{d}{dr} \left[ \rho(r) v(r) \right] = 0, \tag{1}
\]

and

\[
v(r) \frac{dv}{dr} + \frac{1}{\rho(r)} \frac{dp}{dr} + \frac{GM_p}{r^2} = 0, \tag{2}
\]

where \( r \) is the distance from the centre of the planet; \( \rho \) and \( v \)
are the mass density and bulk radial velocity of the gas, respec-
tively; \( p \) is the gas pressure; \( G \) the gravitational constant; and \( M_p \)
the planet mass (see Table 1). The momentum equation con-
tains only the forces induced by the gravity of the planet and the
gas pressure, neglecting other forces such as the stellar gravita-
tional pull and fluid viscosity. We also consider the atmosphere
as an ideal gas, \( p = \rho k T/\mu \), where \( k \) is Boltzmann’s constant,
\( T \) is temperature, and \( \mu \) is the gas mean molecular weight. The
pressure gradient can then be written as

\[
\frac{dp}{dr} = \frac{k T(r) \rho}{\mu(r)} \frac{d}{dr} + \rho(r) \frac{d}{dr} \left( \frac{k T(r)}{\mu(r)} \right). \tag{3}
\]

The derivative of Eq. (1) gives

\[
\frac{1}{\rho(r)} \frac{dp}{dr} = -\frac{1}{v(r)} \frac{dv}{dr} - \frac{2}{r}, \tag{4}
\]

and including Eqs. (3) and (4) into the momentum continuity
equation (Eq. (2)), this becomes

\[
v(r) \frac{dv}{dr} + v^2(r) \left( -\frac{1}{v} \frac{dv}{dr} - \frac{2}{r} \right) + \frac{d}{dr} \left( \frac{v^2(r)}{r^2} \right) + \frac{GM_p}{r^2} = 0,
\]

where \( v_s(r) = \left[ k T(r)/(\mu(r)) \right]^{1/2} \) is the gas speed of sound.

To solve Eq. (5) we need to incorporate the energy and species
continuity equations into the system, which requires the
use of computationally expensive numerical methods. Our aim
is to develop a very fast hydrodynamic model in order to explore
and constrain the mass-loss rate and temperatures imposed by
the He(2S) absorption measurements. To that end, we assume here
a constant speed of sound, \( v_s0 \), which allows us to decouple the
momentum equation from the energy budget equation and obtain
a simple expression for Eq. (5) that can be solved analytically.
This expression is known as the isothermal Parker wind approxi-
mation, previously studied by Parker (1958) for describing stellar
winds, and can be written as

\[
\frac{1}{v_s(r)} \frac{dv}{dr} \left[ \frac{v^2(r)}{v_s^2} - 1 \right] = \frac{2}{r} - \frac{GM_p}{r^2}. \tag{6}
\]

However, our approach does not require temperature to be
necessarily constant, but to have the same inverse altitude depen-
dence as \( \mu(r) \) so that the ratio \( T(r)/(\mu(r)) \) is constant, i.e. \( v_{s,0} = \left[ k T_0/(\mu) \right]^{1/2} \).

We obtain the constant speed of sound from

\[
v_{s,0} = \frac{1}{\mu} \left[ k T_0 \right]^{1/2}, \tag{7}
\]

where we introduce a constant temperature \( T_0 \) and the corre-
sponding average mean molecular weight \( \bar{\mu} \) of the gas. This
quantity is calculated using Eq. (A.3), which is constructed such
that it provides an accurate calculation of the integrals over
velocity and radius of the hydrodynamic equations.

Below we discuss (see Sect. 3.1) the validity and scope of
this approach. In particular, the total density of our model closely
matches that obtained by more comprehensive models when the
constant speed of sound is equal to the maximum speed of sound
of those models. Under these conditions, the \( T_0 \) of our model is
very close to the maximum of the temperature profile obtained
by those models.

Equation (6) has infinite solutions but only one has a physi-
cal interpretation in terms of escape: the transonic solution that
describes a subsonic velocity of the gas at distances below the
sonic point (the altitude where wind velocity is equal to the
speed of sound) and supersonic beyond that point (see, e.g.
Table 2. Production and loss processes included in the He(2S) model.

| Name   | Process                                                                 | Rate (a) | References                        |
|--------|-------------------------------------------------------------------------|----------|-----------------------------------|
| J_{11} | H + hν → H^+ + e^-                                                   | See text | Osterbrock & Ferland (2006)       |
| α_{11} | H^+ + e^- → H                                                        | 2.59 × 10^{-13} (T / 10^4)^{-0.700} | Osterbrock & Ferland (2006)       |
| J_{He:S} | He(1S) + hν → He^+ + e^-                                           | See text | Brown (1971)                      |
| α_{1}  | He^+ + e^- → He(1S)                                                  | 1.54 × 10^{-13} (T / 10^4)^{-0.866} | Benjamin et al. (1999)            |
| J_{He:S} | He(2S) + hν → He^+ + e^-                                           | See text | Norcross (1971)                   |
| α_{3}  | He^+ + e^- → He(2S)                                                  | 2.10 × 10^{-13} (T / 10^4)^{-0.778} | Benjamin et al. (1999)            |
| q_{13} | He(1S) + e^- → He(2S) + e^-                                         | 2.10 × 10^{-8} √(13.6 / kT) exp\left( -19.81 / kT \right) \Upsilon_{13} | Bray et al. (2000); Oklopčić & Hirata (2018) |
| q_{31a} | He(2S) + e^- → He(2S) + e^-                                         | 2.10 × 10^{-8} √(13.6 / kT) exp\left( -10.80 / kT \right) \Upsilon_{22} | Bray et al. (2000); Oklopčić & Hirata (2018) |
| q_{31b} | He(2S) + e^- → He(2P) + e^-                                         | 2.10 × 10^{-8} √(13.6 / kT) exp\left( -10.40 / kT \right) \Upsilon_{22} | Bray et al. (2000); Oklopčić & Hirata (2018) |
| Q_{31} | He(2S) + H → He(1S) + H                                          | 5.00 × 10^{-10} | Roberge & Dalgarno (1982); Oklopčić & Hirata (2018) |
| Q_{He^+} | He^+ + H → He(1S) + H^+                                         | 1.25 × 10^{-15} \left( 300 / T \right)^{-0.25} | Glover & Jappsen (2007); Koskinen et al. (2013) |
| Q_{He}  | He(1S) + H^+ → He^+ + H                                       | 1.75 × 10^{-11} \left( 300 / T \right)^{0.75} exp\left( 128000 / T \right) | Glover & Jappsen (2007); Koskinen et al. (2013) |
| A_{31}  | He(2S) → He(2S) + hν                                                  | 1.272 × 10^{-4} | Drake (1971); Oklopčić & Hirata (2018) |

Notes. (a) Units are [cm^{-3} s^{-1}] for the recombination and collisional processes and [s^{-1}] for the other processes. \( \Upsilon_i \) are the effective collision strengths taken from Bray et al. (2000) at the corresponding temperatures.

### Parker 1958; Lamers & Cassinelli 1999.

Integrating Eq. (6) and selecting the solutions that cross the sonic point, we have

\[
\frac{v(r)}{v_{s,0}} = \frac{2 r_e^2}{v_{s,0}^2} \exp\left( \frac{v_{s,0}^2}{2} \right) = \left( \frac{r_e}{r} \right)^2 \exp\left( -2 \frac{r_e^2}{r} + \frac{3}{2} \right),
\]  

where \( r_e = G M_p / 2 \nu_{s,0}^2 \) is the radial distance of the sonic point. Of the two possible solutions of Eq. (8) the transonic one is the solution that provides the velocity profile of the hydrodynamic escape.

Integrating Eq. (1) and taking into account spherical symmetry, the mass-loss rate, \( M \), can be expressed by

\[
M = 4 \pi r^2 \rho(r) v(r),
\]

and including the velocity profile from Eq. (8), the density profile results in

\[
\rho(r) = \frac{2 r_e}{\frac{3}{2} - \frac{v_{s,0}^2}{2 r_e^2}} \exp\left( -2 \frac{r_e^2}{r} + \frac{3}{2} \right),
\]

where \( \rho_s \) is the gas density at the sonic point.

The radial distribution of the species H, H^+, helium singlet, hereafter He(1S), He^+, and He(2S), are obtained by solving their respective continuity equations,

\[
- v(r) \frac{\partial f_i}{\partial r} + P_i + L_i = 0,
\]

where the first term accounts for advection, \( f_i \) is the fraction of species \( i \) (i.e. with respect to the total – neutral, ionised, and excited atom concentrations – of species \( i \)), \( P_i \) is the production, and \( L_i \) is the loss term.

The production and loss rates and corresponding rates included for the different species are essentially those considered by Oklopčić & Hirata (2018), but two additional processes have been included: the charge exchange reactions, \( Q_{He} \) and \( Q_{He^+} \), which were taken from Koskinen et al. (2013); see Table 2.

Thus, production and loss incorporate photo-ionisation and recombination processes of atomic hydrogen (processes \( J_{11} \) and \( \alpha_{11} \) in Table 2). For helium, these processes include photo-ionisation (\( J_{He:S} \) and \( J_{He:S} \)), recombination (\( \alpha_{1} \) and \( \alpha_{3} \)), collisional excitation and de-excitation with electrons (\( q_{13} \) and \( q_{31} \)), collisional de-activation with H atoms (\( Q_{31} \)), charge exchange between atoms and ions (\( Q_{He^+} \) and \( Q_{He} \)), and the radiative relaxation of the helium triplet state (\( A_{31} \)). Double ionisation processes for helium were not considered, and we assume that the charge exchange reactions between helium and hydrogen do not significantly affect the concentration of the latter. Furthermore, we assume that the electrons produced from helium ionisation are negligible in comparison to the production from hydrogen ionisation.

The hydrogen photo-ionisation cross sections were taken from Osterbrock & Ferland (2006) and the photo-ionisation cross sections for the helium triplet and singlet states were taken from Norcross (1971) and Brown (1971), respectively (see Fig. 2). The photo-ionisation rates are calculated as usual,

\[
J_{i} = \int_{\lambda_{\min}}^{\lambda_{\max}} \sigma_{\lambda} F_\lambda \exp(-\tau_{\lambda}(r)) \, d\lambda,
\]

and

\[
\tau_{\lambda}(r) = \int_{r}^{\infty} \sigma_{\lambda}(\lambda) n_i(\lambda) \, d\lambda,
\]

where \( \lambda_{\min} \) and \( \lambda_{\max} \) correspond to the wavelength range of photo-ionisation, \( \tau_{\lambda}(r) \) is the optical depth, \( F_\lambda \) is the stellar flux density at the top of the atmosphere, and \( \sigma_{\lambda}(\lambda) \) and \( n_i(\lambda) \) are the absorption cross section and number density of species \( i \) (H and He), respectively.
The flux of HD 209458 in the 5–1230 Å range was calculated using a coronal model based on XMM-Newton observations of this star (Sanz-Forcada et al. 2010, 2011) that are combined to improve their statistical significance in the spectral fit, reaching a 3.2–σ detection. No substantial variability was registered in the X-rays observations, although the number of measurements is small. The coronal 1-T model, \( \log T(K) = 6.0 - 0.3 \log E M \) (cm\(^{-3}\)) = 49.52 ± 0.22, \( L_X = 6.9 \times 10^{30} \text{erg s}^{-1} \), is complemented with line fluxes from observations with the Cosmic Origin Spectrograph onboard HST published by France et al. (2010) in order to extend the model towards lower temperatures (\( \log T(K) = 4.0 - 6.5 \)). No substantial amount of emitting material is expected for higher temperatures. The XUV (5–920 Å) modelled luminosity is \( 1.5 \times 10^{31} \text{erg s}^{-1} \), which is about 2.6 times higher than in Sanz-Forcada et al. (2011). For the 1230–1700 Å range we used current observations with the Space Telescope Imaging Spectrograph taken from the HST archive. At \( \lambda > 1700 \) Å we used the stellar atmospheric model of Castelli & Kurucz (2004) scaled to the temperature, surface gravity, and metallicity of HD 209458 (see Table 1). The resulting stellar flux for the spectral range 5–2600 Å at the orbital separation of the planet (0.04707 au) is shown in Fig. 2. This flux shows several differences with respect to the solar flux (see, e.g. Claire et al. 2012; Linsky et al. 2014). Thus, it shows a weaker Ly\( \alpha \) continuum but more prominent emission lines in the 600–912 Å region. Also, the continuum in the 950–1200 Å range is significantly weaker, although this is expected to have a minor effect on the He(2\(^2\)S) population. In order to analyse the effects of these phenomena on the He(2\(^2\)S) density we performed some tests as shown in Sect. 3.2.

Equations (8), (10), (A.3), and (11) are solved iteratively until convergence is reached, ensuring a fully consistent model. As the abundances of neutral and ionised species and electrons vary with \( r \), so does the mean molecular weight. We assume pre-set neutral gas composition of H and He (we considered H/He mole-fraction ratios of 90/10, 95/5, and 98/2). To ease the numerical convergence of the model, we select the minimum value of \( \bar{\mu} \) (atmosphere fully ionised in the first iteration). The \( \rho(r) \) and \( \sigma(r) \) profiles derived from the solution of Eqs. (8) and (10) with

1 STIS/G140L combined spectra were acquired on the dates 9 Oct. 2003, 19 Oct. 2003, 6 Nov. 2003 and 24 Nov. 2003.

Fig. 2. Flux density for HD 209458 at 0.04707 au (left y-axis) as calculated here (black) and for the modern Sun (solar maximum) as reported by Claire et al. (2012) and scaled up to the size of HD 209458 (orange). We note that the resolution of the two spectra is different. The H, He singlet, and He triplet ionisation cross sections (right y-axis) are also shown.

In order to check the validity of our constant-speed-of-sound approach, we compared our results with those obtained by Salz et al. (2016), who solved the energy equation and did not include any assumption on temperature. For that comparison, we used the same HD 209458 b bulk parameters, mass-loss rate, H/He, and XUV stellar flux. We performed the calculations for two \( T_0 \) temperatures: (1) that obtained when considering the largest speed of sound computed by Salz et al. (2016) and the average mean molecular weight obtained in our model (Eq. (7)); and (2) the maximum of the temperature profile computed by Salz et al. (2016). Figure 3 shows these latter two temperature profiles and the one calculated by Salz et al. (2016), as well as the corresponding total density profiles. Except for altitudes very close to the lower boundary (\( r < 1.15 R_p \)), the density that we obtain with the temperature corresponding to the maximum speed of sound in Salz et al. (2016), 8750 K, agrees with that computed by these latter authors. This result should not be surprising because for the maximum of the speed of sound, its derivative is zero and then the equations of the full model reduce to those in our model, i.e. Eq. (5) reduces to Eq. (6).
The temperature $T_0 = 8750 \text{ K}$ is also close to the thermospheric maximum temperature calculated by Salz et al. (2016). Moreover, the density obtained for the maximum of the temperature profile of Salz et al. (2016), $T_0 = 9125 \text{ K}$, still gives very good agreement with that of Salz et al. (2016). Although at altitudes close to the lower boundary, where the He(2S) absorption is most important (see Fig. 9), the agreement is slightly poorer. Furthermore, the temperature profile obtained in our approach, dictated by $\nu_{\text{max}}(\text{Salz}) = [k T(r)/\mu(r)]^{1/2}$, is equal to $T_0 = 8750 \text{ K}$ at an altitude close to 1.6 $R_P$, which is where most of the stellar radiation is absorbed. We also performed this comparison with two other planets, HD 189733b and GJ 3470b, which show different parameters and maximum of their temperature profiles at significantly different altitudes (Salz et al. 2016). Since we obtain similar results, the input temperature to our model, $T_0$, can be considered as a good proxy (within $\sim 10\%$) for the maximum temperature obtained in the hydrodynamic models that also solve the energy budget equation.

### 3.2. He(2S) densities

Figure 4 shows several He(2S) density profiles and their dependence on the mass-loss rate (upper panel) and temperature (lower panel) for an H/He ratio of 90/10. As a general trend, for a given temperature, smaller mass-loss rates give rise to higher He(2S) peak densities and more compressed (narrower layers of He(2S) density profiles. Conversely, at larger mass-loss rates, the He(2S) density has a broader shape and its maximum occurs at larger planetary radii. Regarding the effect of temperature, for a given $M$, we can see that the peak of the He(2S) density occurs at smaller radii for larger temperatures, whereas colder thermospheres produce higher He(2S) abundances.

To understand this behaviour we analysed the production and loss terms of Eq. (11). As an example, Fig. 5 shows the production and loss terms of He(2S) for a temperature of 6000 K and a mass-loss rate of $4.2 \times 10^7 \text{ g s}^{-1}$. The production of He(2S) is dominated by the recombination of He$^+$ with electrons, $\alpha_3$, having its maximum contribution at about 1.2–1.4 $R_P$. The electron density is mainly driven by H photo-ionisation, $J_{\text{He}}$, whereas the electron productions from the photo-ionisation of He(1S) and He(2S) are negligible. In turn, the He$^+$ production is determined by the helium photo-ionisation, $J_{\text{He}^+}$, and the charge exchange process, $Q_{\text{He}^+}$, and its losses are mainly controlled by the recombination with electrons, $\alpha_1$ and $\alpha_3$, and the charge exchange process $Q_{\text{He}^+}$. At high altitudes, where He ionisation is very effective, the He$^+$ concentration is dominated by its photo-ionisation, $J_{\text{He}^+}$, but at lower altitudes the photo-ionisation production and the losses by recombination with electrons determine the resulting He$^+$ profile (see Fig. 5). The production of He(2S) by collisional excitation, $q_{13}$, is very small, i.e. below the lower limit of the scale of Fig. 5. We found that the inclusion of the charge-exchange processes $Q_{\text{He}}$ and $Q_{\text{He}^+}$ produces a net loss of He$^+$, which translates into a reduction of the He(2S) concentration peak (≈15% for this case).

Given the importance of H photo-ionisation in the production of He(2S), it can be seen that a thinner atmosphere (i.e. weaker $M$ or warmer thermosphere, see Fig. 6) produces ionisation of H at lower altitudes and hence effectively produces He(2S) at this region. In contrast, for larger $M$ or colder thermospheres, the density is higher and the absorption of the stellar flux takes place mainly at higher altitudes. Additionally we observe in Fig. 4 a broader extension of [He(2S)] (square brackets indicate concentration) at larger $M$, which is due to the weaker vertical gradient of the total density at higher altitudes (see Fig. 6). For these conditions, the attenuation of the stellar flux occurs more progressively with altitude than at low altitudes where the density abruptly increases (see Fig. 6). In addition to this, the hotter the gas is, the more expanded and thinner the thermosphere is.
Loses of He(\(^2\)S) are controlled by different processes. The major loss is due to photo-ionisation, \(J_{\text{He}^+}\), at higher altitudes and the collisional deactivation with electrons, \(Q_{31}\), at lower altitudes. Collisional deactivation with H, \(Q_{31}\), and advection are about one order of magnitude smaller than photo-ionisation and collisional deactivation with electrons. Overall, the helium triplet concentration is mainly controlled by the stellar flux: on the one hand, by the electron production from H photo-ionisation as well as by the density, which largely controls the atmospheric region where the stellar flux is fully absorbed.

However, the same mechanism described above applies. The helium triplet abundance is largely determined by the stellar flux via H photo-ionisation as well as by the density, which largely controls the atmospheric region where the stellar flux is fully absorbed.

### 3.3. Spectral absorption

The spectral absorption of the He triplet lines was calculated computing the radiative transfer following the usual transit geometry (see Fig. 7). The transmission, \(T_\nu(r)\), along the line of sight (LOS) \(x\) at a radius \(r\) over the planet reference surface for an infinitesimal field of view of the planet’s atmosphere (see Fig. 7, top) at frequency \(\nu\) can be written as (see, e.g., López-Puertas & Taylor 2001, p. 64)

\[
T_\nu(r) = \exp\left[ -\int_{r_0}^{r_{\text{TOA}}} k_\nu(x) n(x) \, dx \right],
\]

where \(k_\nu\) is the absorption coefficient of the radiative transition and \(n(x)\) is the concentration of the absorbing gas, the helium triplet state in our case, and TOA stands for the top of the atmosphere.

Different He(\(^2\)S) density profiles are obtained when considering different H/He ratios, mass-loss rates, or temperatures. However, the same mechanism described above applies. The helium triplet abundance is largely determined by the stellar flux via H photo-ionisation as well as by the density, which largely controls the atmospheric region where the stellar flux is fully absorbed.
where \( R_\text{P} \) is the radius of the planet. We include the integration over the polar angle, \( \phi \), in order to account for potential atmospheric inhomogeneities in the density or velocity of the helium triplet state along this coordinate.

The absorption coefficients, \( K \), and the frequencies, \( \nu_0 \), for the three lines from meta-stable helium levels were taken from the NIST Atomic Spectra Database\(^2\). The He(\( 2^3S \)) density profiles were calculated as described above in this section. Temperature was kept constant, as given by the helium triplet density model. As most of the absorption is expected to take place at atmospheric regions below the Roche limit (see below), here we did not include the broadening of the lines due to turbulence (\( \nu_{\text{turb}} = 0 \) in Eq. (15)). The \( \nu_{\text{wind}} \) included in the nominal calculation corresponds to the line-of-sight component of the radial velocity outflow obtained in the density model of Sect. 3.

4. Results and discussion

To analyse the mid-transit absorption spectra of Fig. 1, the thermospheric model described in Sect. 3 has been run for a temperature range of 4000–11 500 K in steps of 125 K. The mass-loss rate interval is \( 10^8 \sim 10^{12} \text{ g s}^{-1} \) in eight steps per decade. This results in a total of 2013 simulations for each set of H/He abundances.

Subsequently, we computed the spectral transmission for each of the helium triplet profiles. In these radiative transfer calculations, we included the temperature and the escape velocity profile \( \nu(r) \) (see Eq. (8)) from the model. No turbulence term was included, as most of the absorption comes from radii below the Roche limit. Nevertheless, we performed a test for the case of Fig. 1 by including this term in the broadening of the line and found a less precise fitting; that is, the modelled line profile is wider than the measured profile.

A bulk Doppler blueshift of 1.8 \( \text{km s}^{-1} \) was included in the absorption calculation, as suggested by the observations. The additional absorption observed at around \( -13 \text{ km s}^{-1} \) (the magenta curve in Fig. 1) has not been included in these calculations (e.g. in the computation of \( \chi^2 \), see below). The reason is that this absorption seems to emerge from very high altitudes and possibly from material already ejected by the planet, but we are interested in determining the temperature and mass-loss rates of the bulk atmosphere.

The absorption spectra were computed at a very high resolution (wavelength step of \( 10^{-7} \text{ Å} \)) and then convolved with the CARMENES line spread function. We performed the integration over the radius, \( \nu \), in Fig. 7, from the lower boundary condition of the He triplet model (1.04 \( R_\text{P} \)) up to the Roche lobe boundary, located at 4.22 \( R_\text{P} \) (Salz et al. 2016).

In order to analyse the extent to which absorption contributes beyond that limit, we also computed the absorption including the atmosphere extended up to 10 \( R_\text{P} \). The results show that for \( M \) smaller than \( 10^{11} \text{ g s}^{-1} \) the additional absorption for radii beyond that boundary is very small; in particular, for \( M \approx 3.7 \times 10^8 \text{ g s}^{-1} \) at a temperature of 6000 K (Fig. 1) it is negligible (below 0.3%). This result is consistent with the light curve shown in Fig. 5 in Alonso-Floriano et al. (2019) which does not show signs of relevant ingress or egress absorption.

We compared the synthetic spectra to the measured one (see an example in Fig. 1) and calculated the \( \chi^2 \) in the spectral interval of 10 829.9–10 831.5 Å (avoiding the \( -13 \text{ km s}^{-1} \) component) for each of them. In this way, we obtained all possible pairs of \( M \) and \( T \) which are compatible with the measured absorption.

\(^2\) https://www.nist.gov/pml/atomic-spectra-database

![Fig. 8. Contour map of the \( \chi^2 \) of the model of the helium triplet absorption compared to that measured by CARMENES as reported by Alonso-Floriano et al. (2019). The \( \chi^2 \) has been scaled by \( 10^5 \) (or, equivalently, the reduced \( \chi^2 \) scaled by a factor of 3.8). An H/He ratio of 90/10 was assumed. The small black dots represent the grid of models. The white curve highlights the best fitting simulations.](image)

The results of the \( \chi^2 \) computations for H/He of 90/10, our nominal case, are shown in Fig. 8. The white curve highlights the best-fitting simulations. According to these results, the measured absorption is consistent with pairs of \( (T, M) \) ranging from \( (4625 \text{ K}, 10^{11} \text{ g s}^{-1}) \) to \( (11500 \text{ K}, 10^{11} \text{ g s}^{-1}) \). The relationship between temperature and \( M \) is not linear, being steeper at lower \( T \) and \( M \) values, and flatter at high temperatures.

In order to better understand the escaping atmosphere we plotted a selection of the He(\( 2^3S \)) densities in Fig. 9 (top panel for an H/He ratio of 90/10) that fit the observed absorption spectrum and cover the full range of temperatures and \( M \). This figure shows that the majority of the He(\( 2^3S \)) profiles peak at very short distances, in the range of 1.04–1.10 \( R_\text{P} \). The few profiles with peaks at distances greater than 1.30 \( R_\text{P} \) are more extended and generally correspond to higher temperatures and larger mass-loss rates. Another relevant feature of the atmospheres that best fit the measured absorption is that they all have a very similar averaged mean molecular weight, close to 0.69 g mole\(^{-1}\). This value corresponds to a partially ionised thermosphere, between the fully ionised (\( \mu = 0.65 \)) and the neutral (\( \mu = 1.3 \)) thermospheres.

As the atmospheric model provides \([\text{H}^+]\), we also analysed \([\text{H}/[\text{H}^+])\) for the different atmospheres that fit the observed He(\( 2^3S \)) absorption. In Fig. 10 we show the H mole fraction profiles for the same atmospheres as shown in Fig. 9. We see that for an H/He of 90/10 (top panel) most of the atmospheres reproducing the measured He(\( 2^3S \)) absorption have a rather sharp \([\text{H}/[\text{H}^+])\) transition region occurring at altitudes ranging from about 1.04 to 1.09 \( R_\text{P} \). That is, the measured He(\( 2^3S \)) spectrum suggests that the atmospheric hydrogen of HD 209458 b is fully ionised at altitudes above \( -1.6 R_\text{P} \), which is in agreement with the mean molecular weight of \( \sim 0.69 \text{ g mole}^{-1} \) mentioned above.

4.1. Sensitivity to the H/He ratio and comparison with previous H densities

We further explored whether or not we can constrain the H concentration with the measured He(\( 2^3S \)) spectrum. The H abundance in HD 209458 b has been derived from Ly\( \alpha \)
absorption measurements in several studies (see, e.g., García-Muñoz 2007; Koskinen et al. 2013; Salz et al. 2016). We then compared our derived H density from the He(^2S) fit with the ones reported in these latter studies. We found that, in general, our derived H densities for the usual H/He ratio of 90/10 (see Fig. 11, top panel) are significantly lower at all altitudes. This prompts us to consider other H/He densities in order to ascertain whether or not they can explain these discrepancies. Therefore, we repeated all the calculations above for the larger H/He ratios of 95/5, and 98/2, respectively.

Fig. 9. He(^2S) concentration profiles that best fit the measured absorption (see Figs. 8 and 12). Shown is a selection of the profiles along the black, cyan, and orange lines in Fig. 12, covering the whole range of temperatures and mass-loss rates of those figures. Panels from top to bottom are for H/He ratios of 90/10, 95/5, and 98/2, respectively.

Fig. 10. Range of the hydrogen molar fraction profiles resulting from the fit of the measured absorption (see Figs. 8 and 12) for three H/He ratios, covering the whole temperature and mass-loss rate ranges of those figures. The solid thicker lines are the mean profiles.

The most noticeable effect of increasing the H fraction (and thus decreasing the He abundance) is a global decrease of the thermospheric mass density. Therefore the thermosphere is lighter and, for given conditions of temperature and mass-loss rate, it is more expanded. Figure 13 (top panel) shows the change in the HD 209458 b density when assuming an H/He ratio of 90/10. The value at lower altitudes due to diffusive separation (see, Fig. 14 in Moses et al. 2005). Another possible process of depletion He in the upper atmosphere of a giant planet is the He sequestration in the interior of the planet due to the formation of an H–He immiscibility layer (Salpeter 1973; Stevenson 1975, 1980; Wilson & Militzer 2010).

The most noticeable effect of increasing the H fraction (and thus decreasing the He abundance) is a global decrease of the thermospheric mass density. Therefore the thermosphere is lighter and, for given conditions of temperature and mass-loss rate, it is more expanded. Figure 13 (top panel) shows the change in the HD 209458 b density when assuming an H/He ratio of 90/10.
95/5. Here we observe large changes in the density of close to two orders of magnitude for lower temperatures at low distances. The H density also shows large decreases (Fig. 13, middle panel), following the total density very closely. We also see the effects of absorption of the stellar radiation mainly for lower temperatures; that is, a change in the slope of the H density ratio at distances close to 1, 2, 3, and $4.5 \, R_p$ (for $M$ of $10^7$–$10^{12} \, g \, s^{-1}$) in response to the penetration of radiation to deeper altitudes for smaller H/He ratios.

As a consequence of the lower total density, and lower H density, for a larger H/He ratio, the helium triplet density also mainly follows the behaviour of H, and therefore we generally obtain smaller He($^3S$) densities (Fig. 13, bottom panel). It is also worth noting that the peak of the He($^2S$) density moves to slightly lower altitudes for larger H/He.

The results of fitting the measured He($^2S$) spectrum when using H/He of 95/5 and 98/2 in the model are shown in the corresponding figures: Fig. 12 (cyan and orange lines) for the constrained ($M$, $T$) pairs; Fig. 9 (middle and bottom panels) for the H($^2S$) density; and in Figs. 10 and 11 (middle and bottom panels) for the H mole fraction and H density, respectively.

Regarding the ($M$, $T$) relationship, this curve moves to colder temperatures and larger mass-loss rates for higher H/He (see Fig. 12), which is the consequence of a smaller He($^2S$) density. In order to fit the measured spectrum we need to lower $T$ or increase $M$ in order to increase the He($^2S$) concentration (see Fig. 4). For the relatively high H/He ratio of 98/2, the measured spectrum implies relatively low temperatures and high mass-loss rates.

Figure 9 (middle and bottom panels) shows the He($^2S$) densities for the higher H/He of 95/5 and 98/2. In comparison to the result obtained for the commonly used H/He ratio of 90/10, the concentrations peak at generally lower values. These peaks are located at larger radii, and the profile shapes are wider. The chosen H/He ratio used in the fitting of the He($^2S$) absorption also dictates the ionisation level of the atmosphere. Figure 10 shows that the atmosphere of HD 209458 b is ionised at higher altitudes for larger H/He.

By inspecting the results obtained for the H density for the three H/He ratios considered here (see Fig. 11), it seems that the derived H densities that agree better with those derived from previous Lyα measurements correspond to the 98/2 ratio (lower panel).

### 4.2. Comparisons of temperatures and mass-loss rates to those of previous works

Several models with different approaches and assumptions have been developed for studying the processes that drive the escape in HD 209458 b. A summary is listed in Table 3 for comparison.
Before discussing them, we should recall that our mass-loss rates refer to the total mass of hydrogen and helium, and that they were computed at the substellar location instead of the globally averaged to substellar values.

One of the approaches for escape modelling is to consider a particle model, which focuses on the exosphere or the unbound region. With the aim of interpreting HD 209458 b observations of H I Lyα, Vidal-Madjar et al. (2003) built a particle model where the atmosphere is composed of atomic hydrogen only. These latter authors considered the stellar radiation pressure and the planetary and stellar gravity. Thereby, they determined a lower limit for the substellar mass-loss rate of $0.4 \times 10^{11}$ g s$^{-1}$. This value, when compared to those derived here (we consider that our case of an H/He ratio of 98/2 gives the better overall agreement for the He(2S) and Lyα absorption; see Fig. 12, orange curve) would imply a temperature of about 7000 K, which seems plausible although generally lower than those calculated in different models (e.g. Salz et al. 2016). Higher temperatures lead to stronger mass-loss rates, which is consistent with Vidal-Madjar et al. (2003) since they provide a lower limit.

Similarly, Bourrier & Lecavelier des Etangs (2013) developed a 3D model for an atomic hydrogen exosphere including stellar radiation pressure. These latter authors suggested a substellar mass-loss rate range of $(0.04-4.0) \times 10^{11}$ g s$^{-1}$. This wide range of mass-loss rates is compatible with our results but does not significantly constrain the temperature range because, according to Fig. 12 (orange curve), the temperature varies in a wide range: 5125–11 500 K.

Other studies using hydrodynamic models have focused on thermospheric rather than exospheric simulations. In these approaches, several 1D models with spherical geometry have been developed. For example, Tian et al. (2005), in their case for an atmosphere of atomic hydrogen and a heating efficiency of $\eta = 0.6$, predicted a maximum substellar mass-loss rate of $2.4 \times 10^{11}$ g s$^{-1}$ for a temperature profile that grows monotonously with altitude from 800 K at 1 $R_p$ to $\approx 20 000$ K at 3 $R_p$. For this mass-loss rate, we derive from Fig. 12 a temperature close to 10500 K, which is significantly lower than most of the values predicted by Tian et al. (2005). Nevertheless, Shematovich et al. (2014) concluded (see discussion below) that heating efficiencies smaller than 0.2 probably yield more accurate mass-loss rates. Thus, the results from Tian et al. (2005) probably represent an overestimation of temperature.

Also, using atomic hydrogen thermospheric models and assuming heating efficiencies of 0.6 and 0.1, Penz et al. (2008) estimated substellar mass-loss rates of $1.4 \times 10^{11}$ and $0.52 \times 10^{11}$ g s$^{-1}$, and maximum temperatures of 9500 and 6300 K located near 1.5 $R_p$, respectively. For these mass-loss rates, we derive from Fig. 12 temperatures close to 9100 K (for their $\eta = 0.6$), and of 7200 K (for their $\eta = 0.1$), which is comparable for $\eta = 0.6$ but slightly higher than that of the latter authors for $\eta = 0.1$. We note, however, that they include only hydrogen. Thus, if we were to consider a lower He abundance in our analysis, our results would be in better agreement with theirs.

García-Muñoz (2007) studied the hydrodynamic escape of HD 209458 b for different atmospheric compositions. The most relevant case is that with a complete chemical scheme (H, He, C, O, N and D) and an H/He ratio of 91/9 (his DIV1 case), for which this latter author estimated a mass-loss rate of $5.0 \times 10^{11}$ g s$^{-1}$. Furthermore, the maximum temperature found by this latter author is 12 500 K at radii of 1.2 $R_p$. For his mass-loss rate we derive from Fig. 12 (orange curve) a temperature of 11 500 K or even larger, which is in good agreement with that computed by García-Muñoz.

Koskinen et al. (2013) carried out simulations for a thermosphere composed of H, He, and heavier elements (C, O, N, Si, Mg, Na, K and S). For their C2 case, which is the most representative, these latter authors obtained a mass-loss rate of $1.6 \times 10^{11}$ g s$^{-1}$ with a heating efficiency of 0.44, and a maximum temperature of 12 000 K at a distance of 1.5 $R_p$. For this mass-loss rate we obtain a temperature of about 9100 K, which is significantly lower than theirs.
3.0 2.5 1.5

Table 3. Comparison of temperature and mass-loss rates with other works.

| $F_{\text{XUV}}$ (erg cm$^{-2}$ s$^{-1}$) | H/He | $R_{\text{XUV}}$ (R$_p$) | $K$ | $\eta^{(a)}$ | $M^{(b)}$ ($\times 10^{11}$ g s$^{-1}$) | $T_{\text{max}}^{(c)}$ ($\times 10^3$ K) | $T_{\text{this work}}^{(d)}$ (K) | Model$^{(e)}$ | References |
|---|---|---|---|---|---|---|---|---|---|
| ... | 100/0 | >0.4 (f) | 3DP | (1) |
| (g) | 100/0 | 0.04–4.0 | See text | 10.5 HD | (3) |
| (b) | 100/0 | 0.6 | 2.4 | 9.5 | 9.1 HD | (4) |
| 1976 | 100/0 | 0.6 | 1.4 | 6.3 | 7.2 HD | (4) |
| 1976 | 100/0 | 0.1 | 0.52 | 12.5 | 11.5 HD | (5) |
| 2900 | 91/9 | See text | 5.0 | 12.0 | 9.1 HD | (6) |
| 1800 | ... | 0.44 | 1.6 | 9.1 | HD | (6) |
| 1148 | 90/10 | See text | 0.74 | 9.1 | 7.8 HD | (7) |
| (b) | 2400 | 98/2 | 1.16–1.30 | 0.76 | 0.1–0.2 | 10.5 HD | (3) |

Notes. $^{(a)}$Averaged heating efficiencies. Particle models (3DP) do not include this parameter. $^{(b)}$Substellar mass-loss rates. Globally averaged mass-loss rates were translated to substellar when necessary, multiplying by 4. $^{(c)}$Maximum temperature. Given only for hydrodynamic (HD) models. $^{(d)}$Temperature of our model for a 98/2 H/He abundance, corresponding to the $M$ of the compared model. $^{(e)}$Type of model: 3D particle estimation (3DP), Hydrodynamic model (HD) and Energy-limited (EL). $^{(f)}$Lower limit of mass-loss rate. $^{(g)}$Three to four times the current solar XUV flux. $^{(h)}$Current solar XUV flux. $^{(i)}$Upper limit of mass-loss rate. $^{(j)}$ $R_{\text{XUV}}$ range derived in this work for the assumed heating efficiency range (0.1–0.2).

References. (1) Vidal-Madjar et al. (2003); (2) Bourrier & Lecavelier des Etangs (2013); (3) Tian et al. (2005); (4) Penz et al. (2008); (5) García Muñoz (2007); (6) Koskinen et al. (2013); (7) Salz et al. (2015); (8) Lammer et al. (2003); (9) Sanz-Forcada et al. (2011); (10) Salz et al. (2016).

For an atomic hydrogen and helium thermosphere of 90/10, Salz et al. (2016) estimated a mass-loss rate of $0.74 \times 10^{-11}$ g s$^{-1}$ with a maximum temperature of 9100 K near 1.4 $R_p$. For that mass-loss rate our He(2$^3$S) absorption implies a temperature of about 7750 K (Fig. 12, orange curve, H/He of 98/2), which is significantly lower than the temperature of Salz et al. (2016). It is interesting to note that Salz et al. (2016) obtained an H density that underestimates the Ly$\alpha$ observations. For an H/He ratio of 90/10, our H density derived from the He(2$^3$S) absorption also underestimates the Ly$\alpha$ absorption (see top panel in Fig. 11). In order to fit the Ly$\alpha$ observations, Salz et al. (2016) supplied the required extra H using an H-only (no helium) model. In our case, to obtain a good fit to both the He triplet and Ly$\alpha$ absorption, we also need to decrease the H/He ratio; suggesting that a value of 98/2 could be enough.

Overall, we observe that although the ranges of mass-loss rates and temperatures in the literature are rather broad, they generally agree well with the constrained $M(T)$ curve found in this work (orange curve in Fig. 12). Only the values derived by Tian et al. (2005) and Koskinen et al. (2013), both larger than ours, fall relatively far from the derived constraints.

4.3. Energy-limited escape models

Estimates of the evaporation mass-loss rate of HD 209458 b have also been obtained by using the energy-limited approximation (Watson et al. 1981; Erkaev et al. 2007; Lammer et al. 2009). The main assumption of this model is that the escape is limited by the $F_{\text{XUV}}$, and it is useful for constraining the atmospheric mass-loss rate or the heating efficiency. With this approach, the substellar mass-loss rate, $M_{EL}$, can be written as

$$\dot{M}_{EL} = \frac{4\pi R_p^2 R_{\text{XUV}}^2 \eta F_{\text{XUV}}}{G K(\xi) M_p}, \quad (18)$$

For HD 209458 b, where $R_{\text{XUV}}$ is the atmospheric expansion radius, defined as the altitude where the optical depth is unity (Watson et al. 1981); $K(\xi) = 1 - 1.5 \xi + 0.5 \xi^3$ is the potential energy reduction factor, with $\xi = (M_p/M_\star)^{1/3} (a/R_p)$, this mass-loss rate is sometimes corrected by a factor that compensates for the underestimated kinetic and thermal energy gains. Here, however, we use uncorrected values.

We calculated the heating efficiency, $\eta_{EL}$, resulting from our He(2$^3$S) analysis using the aforementioned approach. To that end, we included in Eq. (18) the XUV stellar flux that we used, $F_{\text{XUV}} = 2.4 \times 10^2$ erg cm$^{-2}$ s$^{-1}$ (see Sect. 3), $K = 0.76$, and the $R_{\text{XUV}}$ values resulting from the model fit to the He(2$^3$S) absorption (see Fig. 14). The resulting heating efficiencies are shown in Fig. 15.

As discussed above, Shematovich et al. (2014) modelled the thermal-escape-related heating efficiency for HD 209458 b and...

Fig. 14. Ranges of the photo-ionisation coefficient of H, $J_H$, profiles that fit the measured absorption (Fig. 12) for the different H/He ratios.
concluded that \( \eta \) values smaller than 0.2 produce more realistic mass-loss rates. If we impose \( \eta = 0.1-0.2 \) and use the energy limited approach, our results of Fig. 15 limit the substellar mass-loss rate to the range of \((0.42-1.00) \times 10^{11} \) g s\(^{-1}\) (on the basis that the H/He ratio of 98/2 gives the best fit to the H density), which, together with the results of Fig. 12, leads to a temperature range of 7125–8125 K (see Table 3, last row).

Depending on the values used for the different parameters in Eq. (18), we find a wide range of mass-loss rate and heating efficiency estimates for HD 209458 b. For instance, Salz et al. (2016) estimated a heating efficiency of 0.21 using a mass-loss rate of 0.74 \( \times 10^{11} \) g s\(^{-1}\). These values are in good agreement with those obtained in this work.

This approach has also been used to provide upper limits to the mass-loss rate, that is, when all radiation is thermalised (\( \eta = 1 \)). Thus, Lammer et al. (2003) estimated an upper limit for the mass-loss rate of \( 4 \times 10^{11} \) g s\(^{-1}\) using a calculated expansion radius of \( \approx 3 R_p \) and assuming \( K = 1.0 \). This value is, in effect, a very high upper limit as it is much larger than those derived in this work for reasonable temperatures (<11 500 K) of \( \approx 4 \times 10^{11} \) g s\(^{-1}\) (see Fig. 12).

Sanz-Forcada et al. (2011) estimated an upper limit of 1.8 \( \times 10^{11} \) g s\(^{-1}\) assuming \( K = 0.65 \), \( R_{XUV} \) equal to unity, and \( F_{XUV} = 910 \) erg cm\(^{-2}\) s\(^{-1}\). This upper limit agrees with our results. We note that these latter authors used a simplified value for \( R_{XUV} \) and an \( F_{XUV} \) flux that is 2.6 times lower than ours.

5. Conclusions

We present observational constraints on the thermosphere of HD 209458 b obtained from an analysis of the mid-transit He(2\(^3\)S) spectral absorption measurements reported in Alonso-Floriano et al. (2019). Under the assumption of an upper atmosphere in blow-off escape, we modelled its thermosphere using a 1D hydrodynamic model with spherical symmetry, coupled with a non-LTE model for the population of the He(2\(^3\)S) state. The model provides, among other quantities, profiles of the He(2\(^3\)S) concentrations, which were included into a radiative transfer model to calculate He(2\(^3\)S) absorption spectra. By comparing these spectra to the measured He(2\(^3\)S) spectrum of Alonso-Floriano et al. (2019), we constrained the mass-loss rate, temperature, He(2\(^3\)S) densities, and the degree of ionisation in the upper atmosphere of HD 209458 b. We also compared the derived H densities with previous profiles derived from the Ly\( \alpha \) absorption in order to further constrain the H density as well as the H/He ratio.

One of the main results we found is the close relationship between mass-loss rate and temperature for a wide range of these parameters (see Fig. 8). Additionally, we found that this relationship changes with the assumed H/He ratio (see Fig. 12). For a given temperature, the lighter the atmosphere, the stronger the mass-loss rate; and, for a fixed mass-loss rate, the lighter the atmosphere, the cooler the atmosphere. The H/He degeneracy was partially constrained by comparing with previous H density derived from Ly\( \alpha \) absorption measurements. Globally, we found that an H/He ratio of 98/2 gives a better overall fit to both the He(2\(^3\)S) and Ly\( \alpha \) measured absorption.

From the analysis performed for the considered ranges of \( M = 10^{-7} - 10^{-5} \) g s\(^{-1}\) and temperature of 4000 to 11 500 K, we obtain the following results: (i) we find that the He(2\(^3\)S) peak density is located in the altitude range of 1.04–1.60 \( R_P \); (ii) we find an [H]/[H\(^+\)] transition altitude ranging from about 1.2 to 1.9 \( R_P \); (iii) we obtain an effective radii of the XUV absorption in the range of 1.16–1.30 \( R_P \); (iv) the averaged molecular weight of the gas ranges from 0.61 to 0.73 g mole\(^{-1}\); and (v) we find that the thermospheric H/He ratio should be larger than 90/10, the most likely value being about 98/2.

Thus, the He(2\(^3\)S) absorption spectrum significantly constrains the thermospheric structure of HD 209458 b and advances our knowledge of its escaping atmosphere. In particular, we show that the H/He ratio should be larger than 90/10, that hydrogen is almost fully ionised ([H]/[H\(^+\)] < 0.1) at altitudes above 2.9 \( R_P \), and that He(2\(^3\)S) is accumulated at low thermospheric altitudes.

Comparing our results with previous works we find that, overall, they generally agree well with the constrained \( M(T) \) curve found in this work (orange curve in Fig. 12). Only the values derived by Tian et al. (2005) and Koskinen et al. (2013), both larger than ours, fall relatively far from the derived constraints.

Assuming the energy-limited approach, we derive the heating efficiency as a function of the mass-loss rate (see Fig. 15). If we additionally assume the heating efficiency range of 0.1–0.2 derived from the detailed study of Shematovich et al. (2014), we conclude that the most probable substellar total mass-loss rate is in the range of (0.42–1.00) \( \times 10^{11} \) g s\(^{-1}\), and that the temperature ranges from 7125 to 8125 K.

Our model, as it is inherently limited by its one dimension and spherical symmetry, cannot explain any net (either blue or red) shifts. Future work on the analysis of the observed net blueshift components and on possible cometary outflows with 3D modelling is encouraged.

Acknowledgements. We thank Prof. J. Linsky for his helpful refereed report. IAA authors acknowledge financial support from the State Agency for Research of the Spanish MICIU through the “Center of Excellence Severo Ochoa” award SEV-2017-0709. CARMENES is an instrument for the Centro Astronómico Hispano-Alemán de Calar Alto (CAHA, Almería, Spain). CARMENES is funded by the German Max-Planck-Gesellschaft (MPG), the Spanish Consejo Superior de Investigaciones Científicas (CSIC), the European Union through FEDER/ERF FICTS-2011-02 funds, and the members of the CARMENES Consortium (Max-Planck-Institut für Astronomie, Instituto de Astrofísica de Canarias, Hamburger Sternwarte, Centro de Astrobiología and Centro Astronómico Hispano-Alemán), with additional contributions by the Spanish Ministry of Economy, the German Science Foundation through the Major Research Instrumentation Programme and DFG Research Unit FOR2544.
The mean molecular weight, 

\[ \mu = \frac{\sum_{j} n_{j} \mu_{j}}{\sum_{j} n_{j}} \]  

where 

\[ \mu_{j} = m_{j}/(m_{j}+m_{\text{H}_{2}}) \]  

are the reduced masses of the molecules and atomic hydrogen, respectively. 

The mean molecular weight, \( \mu \), determines the gas contribution to the gravitational to escape ratio, \( \Pi \). 

\[ \Pi = \frac{M_{\text{gas}}}{M_{\text{star}}} \]  

The mean molecular weight, \( \mu \), enters in the first, second, and third terms of Eq. (A.2). Thus, we construct an expression for \( \Pi \): 

\[ G M_{P} \int_{r_{1}}^{r_{2}} \frac{\mu(r)}{r^{2}} \, dr + \int_{r_{1}}^{r_{2}} \mu(r) \, v(r) \, dv + \int_{r_{1}}^{r_{2}} \mu(r) \, d(1/\mu) = 0 \]  

By integrating Eq. (A.1) and re-arranging its terms, we obtain 

\[ G M_{P} \int_{r_{1}}^{r_{2}} \frac{\mu(r)}{r^{2}} \, dr + \int_{r_{1}}^{r_{2}} \mu(r) \, v(r) \, dv + \int_{1/\mu_{c}}^{1/\mu_{t}} \mu(r) \, d(1/\mu) = \int_{r_{1}}^{r_{2}} \frac{dv}{v} + 2 \int_{r_{1}}^{r_{2}} \frac{dr}{r} \]  

where \( r_{t} \) is the upper boundary distance of our model (10 \( R_{P} \)), and \( v_{t} \) and \( \mu_{t} \) are the bulk radial velocity and mean molecular weight, respectively.

The mean molecular weight, \( \mu(r) \), enters in the first, second, and third terms of Eq. (A.2). Thus, we construct an expression for \( \Pi \): 

\[ G M_{P} \int_{r_{1}}^{r_{2}} \frac{\mu(r)}{r^{2}} \, dr + \int_{r_{1}}^{r_{2}} \mu(r) \, v(r) \, dv + \int_{1/\mu_{c}}^{1/\mu_{t}} \mu(r) \, d(1/\mu) = \int_{r_{1}}^{r_{2}} \frac{dv}{v} + 2 \int_{r_{1}}^{r_{2}} \frac{dr}{r} \]  

where \( r_{t} \) is the upper boundary distance of our model (10 \( R_{P} \)), and \( v_{t} \) and \( \mu_{t} \) are the bulk radial velocity and mean molecular weight, respectively.