Contact interaction of NEMS shell elements in a color noise field

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Abstract. A mathematical model of behavior a spherical size-dependent shell and plate taking into account contact interaction is constructed. The effect of the color noise field on the mechanical system is taken into account. The following hypotheses were taken as initial hypotheses: the shell and plate material is isotropic, the Cosserat elastic pseudo-continuum with constrained particle rotation, and the Kirchhoff–Love kinematic model. The equations of motion are obtained on the basis of the following theories: Theodore von Karman (geometric non-linearity), B.Ya.Cantor theory (accounting for contact interaction).

1. Introduction
NEMS devices have unique properties that determine their relevance for practical use. Among these properties, we can distinguish such as low mass, high dielectric strength, high frequencies of mechanical resonance, potentially large quantum mechanical effects, a large surface to volume ratio, which is important for sensitive elements of certain types of sensors, for example pressure sensors.

Nanosensors (cantilevers, nanowires, resonators, etc.) and nanoactuators (nanomotors, gears, etc.) are used in physics, biology, chemistry, medicine (diagnostics, cell nano- and microsurgery, drug delivery to the affected area of the body), electronic and the oil and gas industry. Modeling the behavior of NEMS elements involves the use of theories that take into account the effects of scale at the nanoscale. In particular, this is the micropolar theory, which lies in the plane of the scientific interests of many modern scientists [1-5].

2. Problem formulation
A mathematical model of the motion of a flexible shells element, taking into account the contact interaction between them, is obtained from the Hamilton-Ostrogradsky energy principle. Taking into account moment stresses, the potential energy of the mechanical system under consideration will take the form: 

\[ \Pi = \frac{1}{2} \int_\Omega \left( \sigma_{ij} \varepsilon_{ij} + m_{ij} \chi_{ij} \right) d\Omega, \]

where \( \varepsilon_{ij} \) - are the strain tensor components, \( \sigma_{ij} \) - are the stress tensor components, \( \chi_{ij} \) - are the components of the symmetric bending-torsion tensor, \( m_{ij} \) - are the components of the higher orders moments tensor, which are defined as follows:

\[ \varepsilon_{ij} = \frac{1}{2} \nabla u, \quad \chi_{ij} = \frac{1}{2} \nabla \theta, \quad \theta_i = \frac{1}{2} \left( \text{rot}(u) \right)_i, \quad m_{ij} = 2\mu \chi_{ij}, \]

where \( u \) – is the displacement vector with
components \( u_i \), \( i = \{x, y, z\} \), \( \theta_i \) - are the micro-rotate vector components.

\[
\lambda = \frac{E\nu}{(1 + \nu)(1 - 2\nu)}, \quad \mu = \frac{E}{2(1 + \nu)}
\]

- \( \lambda \) and \( \mu \) are the Lame parameters, \( \delta_{ij} \) - are the Kronecker symbols. In this work, the material is considered as the Cosserat pseudo-continuum with constrained particle rotation, which leads to the appearance of an additional independent length parameter in the model \( l \).

Figure 1. Design scheme

To model the contact interaction of elements of a mechanical system according to B.Ya.Kantor’s theory it is necessary to introduce the penalty function, as a consequence, the term appears in the equations

\[
\Psi = \frac{1}{2}\left[1 + \text{sign}(w_1 - h_k(x, y, t) - w_2)\right] [2], \quad K - \text{is the coefficient characterizing the rigidity of the transverse compression of the shell in the contact zone}, \quad h_k(x, y, t) - \text{is the gap between shell and plate}, \quad w_1, w_2 - \text{are the deflections of the shell and plate, respectively}.
\]

\[
\frac{\partial^2 N_{\nu_1}}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{1}{2} \frac{\partial^2 Y_{\nu_1}}{\partial y^2} + \frac{1}{2} \frac{\partial^2 Y_{\mu_1}}{\partial x\partial y} = \rho \frac{\partial^2 u_i}{\partial t^2},
\]

\[
\frac{\partial^2 N_{\nu_1}}{\partial y^2} + \frac{\partial^2 T}{\partial x^2} - \frac{1}{2} \frac{\partial^2 Y_{\nu_1}}{\partial x^2} - \frac{1}{2} \frac{\partial^2 Y_{\mu_1}}{\partial x\partial y} = \rho h \frac{\partial^2 v_i}{\partial t^2},
\]
\[ \frac{\partial^2 M_{xy,1}}{\partial x^2} + \frac{\partial^2 M_{yx,1}}{\partial y^2} + 2 \frac{\partial^2 H_{1}}{\partial x \partial y} + k_{1,1}N_{xy,1} + k_{2,1}N_{yx,1} + \]
\[ + \frac{\partial}{\partial x} \left( N_{xy,1} \frac{\partial w_1}{\partial x} \right) + \frac{\partial}{\partial y} \left( N_{yx,1} \frac{\partial w_1}{\partial y} \right) + 2 \frac{\partial T_{1}}{\partial x} \frac{\partial w_1}{\partial y} + \]
\[ + 2 \frac{\partial T_{1}}{\partial y} \frac{\partial w_1}{\partial x} + 4T_{1} \frac{\partial^2 w_1}{\partial x \partial y} + \frac{\partial^2 Y_{xy,1}}{\partial x^2} + \frac{\partial^2 Y_{yx,1}}{\partial y^2} + \frac{\partial^2 Y_{xx,1}}{\partial x^2} + \frac{\partial^2 Y_{yy,1}}{\partial y^2} + \]
\[ + 2q_{1} + p_{x} - K(w_{1} - w_{2} - h_{1}(x,y,t)) \Psi = \rho_{1}h_{1} \frac{\partial^2 w_{1}}{\partial t^2} + \varepsilon_{1}\rho_{1}h_{1} \frac{\partial w_{1}}{\partial t}, \]
\[ \frac{\partial N_{xx,2}}{\partial x} + \frac{\partial T_{2}}{\partial y} + \frac{1}{2} \frac{\partial^2 Y_{xx,2}}{\partial y^2} + \frac{1}{2} \frac{\partial^2 Y_{xx,2}}{\partial x \partial y} = \rho_{2}h_{2} \frac{\partial^2 u_{2}}{\partial t^2}, \]
\[ \frac{\partial N_{yy,2}}{\partial y} + \frac{\partial T_{2}}{\partial x} - \frac{1}{2} \frac{\partial^2 Y_{yy,2}}{\partial x^2} - \frac{1}{2} \frac{\partial^2 Y_{yy,2}}{\partial x \partial y} = \rho_{2}h_{2} \frac{\partial^2 v_{2}}{\partial t^2}, \]
\[ \frac{\partial^2 M_{xx,2}}{\partial x^2} + \frac{\partial^2 M_{yy,2}}{\partial y^2} + 2 \frac{\partial^2 H_{2}}{\partial x \partial y} + k_{1,2}N_{xx,2} + k_{2,2}N_{yy,2} + \]
\[ + \frac{\partial}{\partial x} \left( N_{xx,2} \frac{\partial w_{2}}{\partial x} \right) + \frac{\partial}{\partial y} \left( N_{yy,2} \frac{\partial w_{2}}{\partial y} \right) + 2 \frac{\partial T_{2}}{\partial x} \frac{\partial w_{2}}{\partial y} + \]
\[ + 2 \frac{\partial T_{2}}{\partial y} \frac{\partial w_{2}}{\partial x} + 4T_{2} \frac{\partial^2 w_{2}}{\partial x \partial y} + \frac{\partial^2 Y_{xx,2}}{\partial x^2} + \frac{\partial^2 Y_{yy,2}}{\partial y^2} + \frac{\partial^2 Y_{xy,2}}{\partial x \partial y} + \frac{\partial^2 Y_{yx,2}}{\partial x \partial y} + \]
\[ + 2q_{2} + K(w_{1} - w_{2} - h_{2}(x,y,t)) \Psi = \rho_{2}h_{2} \frac{\partial^2 w_{2}}{\partial t^2} + \varepsilon_{2}\rho_{2}h_{2} \frac{\partial w_{2}}{\partial t}, \]

where \((N_{xx,2}, N_{yy,2}, T) = \int_{0}^{\frac{h}{2}} (\sigma_{xx}, \sigma_{yy}, \sigma_{xy}) dz, (M_{xx,2}, M_{yy,2}, H) = \int_{0}^{\frac{h}{2}} (\sigma_{xx}, \sigma_{yy}, \sigma_{xy}) dz, Y_{xx} = \int_{0}^{\frac{h}{2}} m_{xx} dz, \)
\(Y_{yy} = \int_{0}^{\frac{h}{2}} m_{yy} dz, Y_{xy} = \int_{0}^{\frac{h}{2}} m_{xy} dz. \)

The boundary conditions should be added to the equations:
\[ \frac{\partial w_{1}}{\partial x} = w_{1} = \frac{\partial u_{1}}{\partial x} = u_{1} = \frac{\partial v_{1}}{\partial x} = v_{1} = \frac{\partial w_{2}}{\partial x} = w_{2} = \frac{\partial u_{2}}{\partial x} = u_{2} = \frac{\partial v_{2}}{\partial x} = v_{2} = 0 \text{ for } x = 0; \ a \]
\[ \frac{\partial w_{1}}{\partial y} = w_{1} = \frac{\partial u_{1}}{\partial y} = u_{1} = \frac{\partial v_{1}}{\partial y} = v_{1} = \frac{\partial w_{2}}{\partial y} = w_{2} = \frac{\partial u_{2}}{\partial y} = u_{2} = \frac{\partial v_{2}}{\partial y} = v_{2} = 0 \text{ for } y = 0; \ b \]

and initial conditions:
\[ \frac{\partial w_{1}}{\partial t} = w_{1} = \frac{\partial w_{2}}{\partial t} = w_{2} = \frac{\partial u_{1}}{\partial t} = u_{1} = \frac{\partial u_{2}}{\partial t} = u_{2} = \frac{\partial v_{1}}{\partial t} = v_{1} = \frac{\partial v_{2}}{\partial t} = v_{2} = 0 \text{ for } t = 0 . \]

Normal distributed load acts on the shell \( q_{1}(x,y,t) = q_{0} \sin(\omega_{r}t) \), where \( q_{0} \) - is amplitude, \( \omega_{r} \) - is frequency, \( q_{2}(x,y,t) = 0 \). Introduce the notation: \( h_{1}, h_{2} \) - are the shell and plate thickness, \( \varepsilon_{1}, \varepsilon_{2} \) -
are the damping factors, $\rho_1$, $\rho_2$ – are the density of the shell and plate material, respectively, $k_{x,1}$, $k_{x,2}$, - are the geometric curvature of the shell, $N_{x}, N_{y}, T$ - are the efforts, $M_{x}, M_{y}, H$ - are the moments, $Y_{xx}, Y_{yy}, Y_{xz}$ - are the higher order moments. Additive noise added to the system of equations in the form of a random term $p_n = p_{n0}(2.0 \text{ rand}()/(\text{RAND MAX} + 1.0) − 1.0)$ with constant intensity $p_{n0}$.

3. Numerical experiment
Consider a structure consisting of a nanoshell and a nanoplate. The nanoplate is located under the nanoshell at a distance $\delta_1$. A static transverse uniformly distributed load acts on the shell. In Figure 2 shows plots of the deflection versus load. Points are marked on the graph: A is the contact of the shell and plate, B is the subcritical load of the shell, if there is no contact with the plate, C is the critical load of the shell (without contact), D is the subcritical load during contact interaction of the shell and plate, E is the critical load in the contact interaction of the shell and plate, F is the supercritical load in the contact interaction of the shell and plate. As a numerical experiment shows, the loss of stability (“cotton”) at the shell (without contact with the plate) occurs under load $q_0 = 230$ and the deflections become larger than $8h$. Taking into account the contact interaction, the critical load almost doubles ($q_0 = 390$), and the deflection after the “clap” is equal to the deflection of the plate at the same load in the absence of contact interaction with the shell C.

![Figure 2. Deflection versus load.](image)

| Contact pressure | Shell deflection | Deflection of the plate | Shell moment | Plate moment | Sheath effort | Effort in the plate |
|------------------|------------------|-------------------------|--------------|--------------|--------------|-------------------|
| $q_0 = 180$      | No contact       | No contact              | No contact   | No contact   | No contact    | No contact        |
Consider the projection onto the x0y plane of contact pressure, deflections, moments, and forces for the shell and plate. Under load, contact has not yet occurred, the deflection of the shell is less than 0.5h. Upon contact, the zone of contact interaction between the shell and the plate is located around the central point; in the center, at $x = 0.5$, $y = 0.5$, there is no contact. Diagrams of moments and forces at the shell before contact and during contact do not change. With increasing load, the contact between the plate and the shell in the form of a ring, the moment at the shell in the center becomes positive. When “popping”, the shape of the contact pressure becomes a square, the moment in the corners of the shell changes sign. The maximum moment of the plate is in the center. The force diagrams in the shell and plate are dome-shaped.

4. Conclusion
A mathematical model of the nonlinear dynamics of the sensitive elements of nanoelectromechanical sensors in the form of a shell and a plate, taking into account their contact interaction under the influence of a transverse alternating load and additive white noise is constructed. The mathematical model is a system of differential equations in partial derivatives of high order and high non-linearity (geometric and constructive). The resulting system of nonlinear partial differential equations is reduced to the Cauchy problem by the finite difference method. The Cauchy problem is solved by the Newmark method. The stability analysis of the solution of the resulting system of differential equations is carried out.
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