Two-dineutron condensation in the excited state of $^8\text{He}$

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**Abstract.** One of the aims of this work is to investigate a two-dineutron condensate state in $^8\text{He}$ and to discuss the properties of two dineutrons in that state. We suggest that the excited $^8\text{He}(0^+)$ state that is the candidate for the two-dineutron condensation would exist above the $\alpha + 4n$ threshold energy by a few MeV and this state contains two compact dineutrons in the lowest $S$-wave around an $\alpha$ core. Different from $\alpha$ clusters, dineutron clusters in that state is soft in size changing, because two neutrons are not bound in free space and dineutron correlation becomes weak in the low-density structure of the dineutron condensation.

1. Introduction

Recently, neutron-rich nuclei have been investigated eagerly and many exotic phenomena have been suggested in them. Dineutron correlation is one of the most interesting topics of the physics of neutron-rich nuclei. A dineutron is a spatially compact spin-singlet pair of two neutrons, which can be regarded as a kind of cluster (quasi-boson). Although two neutrons are not bound in free space, it is theoretically suggested that a compact dineutron can be formed in the low-density region of nuclear matter [1] and in the neutron-halo or neutron-skin region of neutron-rich nuclei [2]. Such compact dineutrons whose size is comparable to that of an $\alpha$ is expected to behave as bosonic clusters individually and, therefore, various dineutron cluster excitation such as dineutron cluster condensation, in which some dineutrons behave as bosons and occupy the lowest-$S$ orbit simultaneously, may be realized. Cluster condensation, especially $\alpha$ condensation, attracts great interests as a novel and exotic structure in finite nuclei and has been investigated eagerly [3]. However, dineutron condensation has not been discussed so well, and the multi-dineutron dynamics and the systematics of formation of dineutron condensation are very interesting issues. Moreover, a dineutron is not naturally as rigid as an $\alpha$ cluster because of the unbound feature of two neutrons in free space, so the dineutron properties, that is, the degree of formation and the size, would change significantly depending on the circumstances such as the nuclear structure. We would like to clarify the diversity of the universal dineutron properties which has not been known at all.

In order to investigate dineutron correlation and condensation systematically, we have constructed a framework “a dineutron condensate (DC) wave function” [4]. This framework describes a nuclear system as a core plus one or a few dineutrons under explicitly considering the core structure such as deformation and excitation and the dineutron-size change. We apply
the DC wave function to $^8$He, which is well described with a system of an $\alpha$ core plus four valence neutrons [5]. One of the aims of this work is to investigate a two-dineutron condensate state in $^8$He and to discuss the properties of two dineutrons in that state. We suggest that the excited $^8$He($0^+$) state that is the candidate for the two-dineutron condensation would exist above the $\alpha + 4n$ threshold energy by a few MeV and would have a gaslike structure where one $\alpha$ and two dineutron clusters are weakly interacting with each other and expanded greatly, similar to an $\alpha$ condensate state. We show that two dineutrons in that state are surely rather compact and change their size readily. Dineutron correlation would become weak to some extent in a very low-density structure such as a gaslike one of two-dineutron condensation in $^8$He.

2. Framework

We superpose two kinds of wave functions to describe $^8$He($0^+$) states, the extended $^6$He+2$n$ cluster wave function and the $^8$He DC wave function. We project the those wave functions to $J^z = 0^+$, and we diagonalize the Hamiltonian by using those wave functions to obtain the $0^+$ states. In the following, we explain these frameworks briefly.

2.1. Extended $^6$He+2$n$ cluster wave function

The extended $^6$He+2$n$ cluster wave function describes $^8$He as a system of $^6$He plus extended 2$n$ (denoted as 2$n^*$ hereafter) clusters. The $^6$He cluster is composed of an $\alpha$ cluster and two valence neutrons in $(0p)^2$ orbits around the $\alpha$ and we describe these valence neutrons with a shifted Gaussian from the $\alpha$ particle. We locate the extended 2$n$ cluster around $^6$He described in such a way. By “extended”, we mean that we take into account the dissociation of the 2$n$ cluster due to the spin-orbit interaction. In the ordinary cluster wave function, the single-particle wave function is a Gaussian which have a real center parameter. A real part of the Gaussian center is the Gaussian width and common to all the nucleons in this wave function. In this work, we superpose the wave functions with different $\lambda$, $\psi(\lambda)$, and we describe these valence neutrons with a shifted Gaussian from the $\alpha$ particle. We locate the extended 2$n$ cluster around $^6$He described in such a way. By “extended”, we mean that we take into account the dissociation of the 2$n$ cluster due to the spin-orbit interaction. In the ordinary cluster wave function, the single-particle wave function is a Gaussian which have a real center parameter. A real part of the Gaussian center corresponds to the mean position of the nucleon. In the present one, we add an imaginary part, $\lambda$, to each Gaussian center as done in Ref. [6]. An imaginary part corresponds to the mean momentum of the nucleon. When $\lambda = 0$, the 2$n^*$ cluster is a spin-singlet cluster, that is, a dineutron. When $\lambda \neq 0$, the spin-up and spin-down neutrons in the 2$n^*$ cluster have the opposite momentum to gain the spin-orbit attraction. Then, the 2$n^*$ cluster is not a spin-singlet cluster any longer exactly. In this way, we take into account the dissociation of spin-singlet pairs and describe the shell-model configurations such as $(0p_{3/2})^4$ which is expected to be significant in the $N = 6$ nuclei such as $^8$He. The form of the $^6$He+2$n^*$ cluster wave function is as follows.

$$
\Phi_{^6\text{He}+2n^*} = A\Phi_{^6\text{He},b_{\alpha}}\phi_{n^+}(R_{\uparrow},b_\alpha) \phi_{n^\downarrow}(R_{\downarrow},b_\alpha).
$$

(1)

$\Phi_{^6\text{He}}$ and $\phi_{n^+\downarrow}$ are the wave functions of the $^6$He cluster and the spin-up/-down valence neutrons. $R$ is the Gaussian width, $R_{\alpha}$ is the Gaussian width, and $R_{\downarrow}$ is the Gaussian width. $b_\alpha = 1.46$ fm is the Gaussian width and common to all the nucleons in this wave function. In this work, we superpose the wave functions with $\lambda = 0, 0.4$ fm and $d_\lambda = 1, \ldots, 4$ fm.

2.2. $^8$He DC wave function

The DC wave function describes a system of a core plus one or a few dineutrons in the $S$ wave around the core. In the present work, the core is $\alpha+2n^*$ and one dineutron is distributed around the core. The form of the wave function is

$$
\psi_r \propto \exp \left[ -\frac{r^2}{4\beta^2} \right], \quad \psi_G \propto \exp \left[ -\frac{r^2}{\beta^2} \right],
$$

(3)

$$
\Phi_{\text{DC}} = A\Phi_{\alpha}(R_{\alpha},b_\alpha) \phi_{n^+}(R_{\uparrow},b_\alpha) \phi_{n^\downarrow}(R_{\downarrow},b_\alpha) \times \psi_r(b_\alpha) \psi_G(\beta).
$$

(2)
where $\Phi_\alpha \phi_{n_1} \phi_{n_2}$ is the core wave function of $\alpha + 2n^*$ and is described with the same manner as the $^{6}\text{He} + 2n^*$ cluster wave function. $\psi_r$ and $\psi_G$ are the relative and center-of-mass parts of the dineutron wave function composed of two neutrons coupled to a spin singlet. They have the relative coordinate between the two neutrons, $r$, and the center-of-mass coordinate of the two neutrons, $r_G$, respectively. In the relative part, the Gaussian width is $b_n$ and this parameter characterizes the relative distance between two neutrons, i.e., the dineutron size. In the center-of-mass part, the Gaussian width is $\beta$ and characterizes the distance between the center of mass of the two neutrons and that of the core, i.e., the spatial expansion of the dineutron from the core. In the DC wave function, we vary these parameters to describe a state containing a dineutron with various sizes and distributions. We superpose the DC wave functions with $\beta = 2, \ldots, 8$ fm and five $b_n$ values for each $\beta$. We prepare two kinds of parameter sets characterizing the $\alpha + 2n^*$ core, that is, $\lambda = 0.0$ fm and $d_\lambda = 1, \ldots, 8$ fm and $b_\lambda = b_n$, and $\lambda = 0.4$ fm and $d_\lambda = 1, \ldots, 4$ fm and $b_\lambda = b_n$. In the former set, the $2n^*$ cluster is also a dineutron and has the same size as the other dineutron and they can be expanded widely. In the latter, the $2n^*$ cluster is dissociated at the surface of the $\alpha$. Superposing these DC wave functions, we describe the configurations such as $\alpha + 2n[(0p_{3/2})^2]$ plus one developed dineutron and two-dineutron condensation where two dineutrons have the same compact size and both are in the lowest $S$ wave.

By superposing these wave functions, we describe the shell-model configurations and one- or two-dineutron configurations effectively.

3. Results
We discuss the $0^+$ states described by superposing the wave functions explained in Sec. 2. For the Hamiltonian in the present calculation, we use the effective two-body interactions, namely the Volkov No.2 force as the central force and the spin-orbit part of the G3RS force as the spin-orbit force. The parameters in the central force are chosen as $m = 0.55, b = h = 0.125$ and the strength of the spin-orbit force is $v_{LS} = 2000$ MeV, chosen to reproduce the $^8\text{He}$ ground state energy. In the present framework, we cannot exactly remove the total center-of-mass motion since all the Gaussian widths of the single-particle wave functions for the DC wave functions are not the same. Thus, we extract the expectation value of the total center-of-mass kinetic energy from the Hamiltonian to treat the center-of-mass motion in an approximated way.

3.1. Energies and radii of $0^+_{1,2}$
We calculate the energy and the matter, proton and neutron radii of the $0^+_{1,2}$ states shown in Table 1. The $0^+_1$ energy agrees with the experimental value. Although the neutron radius of the $0^+_1$ state is underestimated compared with the experimental value, the large difference between the proton and neutron radii, which means a neutron skin, is well reproduced in the present calculation. We skip the details of the structure of the $0^+_1$ state, but the $(0p_{3/2})^4$ and one-dineutron components are important in this state. In the $0^+_2$ state, whose excitation energy

| Energy (MeV) | $r_m$ (fm) | $r_p$ (fm) | $r_n$ (fm) |
|--------------|------------|------------|------------|
| $0^+_1$      | −31.13     | 2.37       | 1.87       | 2.51       |
| $0^+_1$ (Expt.) | −31.40     | 2.49 − 2.52| 1.76 − 2.15| 2.64 − 2.69|
| $0^+_2$      | −23.24     | 4.85       | 2.26       | 5.44       |
Two dineutron component in $^8$He$(0^+_2)$

\[ N_{\text{dineutron}}(\beta, b_n) = |\langle \Phi_{2\text{DC}}(\beta, b_n) | \Psi_{\text{He}(0^+_2)} \rangle|^2, \]  

(4)

where $\Psi_{\text{He}(0^+_2)}$ is the wave function of the $0^+_2$ state described by superposing the $^6$He+$2n^*$ wave functions and $^8$He DC wave functions. $\Phi_{2\text{DC}}$ is the test DC wave function describing the system containing two dineutrons which have the same size, $b_n$, and expansion from the core, $\beta$. These two dineutrons are distributed around the core in the $S$ wave as Eq. (2), so this wave function with small $b_n$ and large $\beta$ corresponds to the component of two-dineutron condensation, which means that two compact dineutrons considered as a kind of cluster are in the lowest $S$ wave and expanded widely. We plot the overlap with $\Phi_{2\text{DC}}$ as a function of the dineutron size, $b_n$, and its spatial expansion from the core, $\beta$, in Fig. 1. A peak exists at $(\beta, b_n) \sim (5, 2)$ with a broad width in both the direction of $\beta$ and $b_n$. The peak value of $b_n \sim 2$ fm is much smaller than the size of the total system, and, therefore, the valence neutrons in this state form two compact dineutrons. However, this peak has the broad width in the $b_n$ direction (the peak spreads from $\sim 1.5$ fm to $\sim 3$ fm), which means that these dineutrons are certainly compact but their size changes readily. The broad width in the $\beta$ direction (the peak spreads from $\sim 3$ fm to $\sim 7$ fm) means that the interaction between the $\alpha$ and two dineutron clusters is weak and that these clusters are expanded widely. This corresponds to the gaslike structure of one $\alpha$ and two dineutrons. It should be noticed that two dineutrons in this state are certainly compact but very soft in size changing unlike an $\alpha$ cluster which is rigid and hardly changes its size. The strength of dineutron correlation can greatly depend on the circumstances such as the density, and the dineutron correlation in a dilute condensate state becomes somewhat weaker than, for
example, that in a ground state. Such a diverse property is an interesting point in dineutron correlation.

4. Summary
In this paper, we have mainly reported the possibility of the existence of the two-dineutron condensation in the excited state of $^8$He. The candidate has a much larger neutron radius than that of the ground state and the four valence neutrons are very spatially extended. They form two spin-singlet pairs and they become compact to be a dineutron. We have shown that, in that state, two dineutrons are rather compact but soft in size changing. These two dineutrons are expanded spatially from the core keeping a compact size, and one $\alpha$ and two dineutron clusters have little correlation with each other to form a gaslike structure of $\alpha+two$ dineutrons.

Two neutrons are not bound in free space, so a dineutron is naturally soft and changes its size readily, especially in such low-density structure as the gas-like one of the $0^+_2$ state. However, it is sure that a dineutron becomes so small that it can behave as a kind of cluster, and it is expected that a dineutron condensation can be realized in neutron-rich nuclei. It is challenging to investigate neutron-rich nuclei systematically in view of the dineutron condensation and to make clear the universal properties of the dineutron in such a system.

Acknowledgments
This work was supported by Grant-in-Aid for Scientific Research from Japan Society for the Promotion of Science (JSPS). It was also supported by the Grant-in-Aid for the Global COE Program “The Next Generation of Physics, Spun from Universality and Emergence” from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan. A part of the computational calculations of this work was performed by using the supercomputers at YITP.

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