Research Article

Dynamic Modeling and Cutting Stability of Rotating Tapered Composite Cutter Bar considering Material Damping

Yuhuan Zhang, Ren Yongsheng, Bole Ma, and Jinfeng Zhang

College of Mechanical and Electronic Engineering, Shandong University of Science and Technology, Qindao 266510, China

Correspondence should be addressed to Ren Yongsheng; renys@sdust.edu.cn

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Traditional milling cutter bars are generally made up of metals and exhibit poor capacity of chatter suppression. This study proposes an anisotropic composites tapered cutter bar for increasing natural frequency and damping and finally achieves the goal of enhancing chatter stability. Based on Hamilton principle and Euler–Bernoulli beam theory, the partial differential motion equations of the cutting system with a 3D rotating tapered composite cutter bar are established. Next, using the Galerkin method, the equations of motion are discretized so as to derive ordinary differential equations. In the model, damping modeling of the composite cutter bar is achieved theoretically by using damping dissipation constitutive relations for viscoelastic composites. Moreover, by introducing the rotating effect of the 3D cutter bar in the 2-DOF analytical model of stability analysis first proposed for a fixed-type cutter bar, an improved prediction model is developed and used to solve the stability lobes of the cutting system in the frequency domain analytically. Furthermore, the influences of the gyroscopic effect, material, ply angle, stacking sequence, and taper ratio on chatter stability are also discussed.

1. Introduction

Due to low dynamic stiffness and natural frequency, traditional metal cutter tools can hardly perform high-speed boring operations or end-milling operations under chatter. Chatter can be roughly classified into two types, regenerative chatter and mode coupled chatter. Regenerative chatter refers to self-excited vibration under the interaction between cutting force and undulation on the workpiece surface after the cutting of the previous cutter tooth, while mode coupled chatter is induced by the coupling between two natural mode shapes since the vibration system exhibits slight discrepancy in stiffness along two directions. Chatter can inevitably reduce the workpiece’s machining precision and surface processing quality and simultaneously shorten service lives of both the cutting tool and machine tool. This study focuses on regenerative chatter on account of its importance in the cutting process.

Previous research reveals that chatter suppression capability of a cutting system is tightly connected with its damping and static stiffness. The boring bar with higher damping and static stiffness also has a greater dynamic stiffness, and the boring bar with a greater specific stiffness also exhibits a larger natural frequency. Traditional materials cannot ensure great dynamic stiffness and natural frequency at the same time. Owing to a series of advantages including high modulus, great damping and low density, advanced fiber reinforced composites seem to be the best choice for replacing metal materials and developing the boring bars with excellent chatter suppression stability. Fiber reinforced composites now have been widely applied in the structural dynamic design of rotating structures [1–7], such as composite automotive drive shafts, high-speed spindles, and rollers. As a result, the modeling theory and numerical method of rotating composite shaft were developed [8–15]. Meantime, a composite is used in the design of leaf spring for the lightweight of automobiles [16]. Scholars also paid increasing attentions to structural design and dynamics of composites boring bar [17–20]. Lee and Suh [17] performed vibration tests and cutting tests and found that the fundamental frequency and damping ratio of graphite/epoxy composite boring bar are 80% and 50% higher than those of
steel boring bar, and the maximum cutting depth using the graphite/epoxy composite boring bar is 5 times greater than that using the steel boring bar. Nagano et al. [19] employed finite element analysis and cutting tests for investigating dynamic properties and chatter stability of the carbon/epoxy composite boring bar with different steel core shapes. Ghorbani et al. [20] utilized an experimental method to study the vibration suppression performance of boring bars filled with epoxy granite, with different cross-sections. The vibro-acoustic signals were measured during a machining process. Results showed that application of this new design of boring bars in the machining process can improve the surface quality up to 30% compared to the conventional boring bar.

Nevertheless, abovementioned studies only examined the simple cases when the boring bar is fixed and the workpiece is rotating. In practical applications, the boring bar is mostly rotates at a high speed while the workpiece is fixed. Lee et al. [18] performed the vibration test and cutting test and optimized the dynamic properties of the carbon/epoxy boring bar with different sizes of cores made up of different materials. Without chatter, the cutting depth can be enhanced by 30% compared with that of the tungsten carbide boring bar. It should be noted that abovementioned studies were performed using a finite element analysis or test. Based on Timoshenko beam theory, Kim et al. [21] established the model of composite cantilever end-milling bar, as well as proposed the chatter analysis model of composite end-milling bar by assuming that regenerative delay cutting force acted on the free end. The research also demonstrates that composite boring bar can greatly improve cutting stability.

It is well known that composite material has higher damping than conventional metallic material. However, it has been shown in previous research [22] that internal damping in a rotating system may lead to whirl instabilities in high-speed rotors. Therefore, accurate prediction of effects of internal damping in composite material rotors, particularly effects of composite damping on the stability of a rotating cutter bar, is essential.

Even though the internal viscous damping constant was included in Kim et al. [21] to investigate effects of composite damping on the cutting stability and the damping ratio of the composite damping cutter bar is given empirically, no composite damping modeling has been performed yet. Ren et al. [22] presented a damping analysis model for the prediction of the variation of the natural frequency and damping with rotating speed. The critical rotating speeds and instability thresholds of composite shaft are evaluated. However, no cutting stability of the composite cutting bar was involved in this study.

Altintas and Budak [23] presented the 2-DOF frequency-domain solutions for the milling process with stationary cutter bars and rotating workpieces in which the machining force directions on the cutter bars in the inertial coordinates are fixed. The cutting stability of the process with the high-speed rotating cutter bar can be conducted in the cutter bar’s rotating coordinate frame, in which case the directional force coefficients are time invariant. However, the two orthogonal coordinates of the tool become dynamically coupled as a function of spindle speed.

A new cutting dynamic model with a rotating tapered composite cutter bar is developed in this paper. The partial differential motion equations of the cutting system with the 3D continuously distributed parameters cutter bar which incorporated internal damping and rotating effects are derived based on Hamilton principle and Euler–Bernoulli beam theory. After discretization using the Galerkin method, ordinary differential equations were obtained. Here, inclusion of viscous damping has been achieved theoretically by using the damping analysis model for composite shaft considering damping dissipation [22]. At the same time, an improving analytical prediction of stability lobes for the cutting system with a 3D cutter bar has been proposed by introducing the rotating effects of the cutter bar in the 2-DOF model of Comak et al. [24] and Eynian and Altintas [25] based on the rotating frame approach and used to investigate the process stability of the cutting system.

By means of Laplace transformation, a time-domain chatter equation was converted into frequency-domain characteristic equation and the stability lobes of the cutting system were plotted. Moreover, the influences of the gyroscopic effect, material, ply angle, and taper ratio on the chatter stability are investigated.

2. Theoretical Formula

2.1. Differential Equation of Motion. As shown in Figure 1, the radius of cross-section of the rotating tapered composite cutter bar with rotating speed of \( \Omega \) and length of \( L \) changes linearly along the axial length, i.e.,

\[
R(x) = \left[ 1 - (1 - \sigma)x/L \right] R_g, \quad \sigma = \frac{R_f}{R_g}
\]

where \( \sigma \) denotes the taper of the cutter bar, \( R_f \) and \( R_g \) denote the cross-section radius on the fixed and free ends of the cutter bar, respectively.

The motion differential equation of the cutter bar is established based on the following Hamilton principle:

\[
\int_{t_1}^{t_2} \delta(T - U) dt + \int_{t_1}^{t_2} \delta W_d dt = 0, \quad (1)
\]

where \( T \) and \( U \) denote the variations of kinetic energy and potential energy, respectively, and \( \int_{t_1}^{t_2} \delta W_d dt \) denotes the virtual work done by the internal damping force.

Based on the Euler–Bernoulli beam theory, the strain-displacement relations can be described as (Ren et al. [22])

\[
\psi_x = -\frac{\partial u_x}{\partial x}, \quad \psi_y = -\frac{\partial u_y}{\partial x}
\]

where \( u_x \) and \( u_z \) denote the displacement of the neutral axis along \( y \)-axis and \( z \)-axis of the points on the reference axis of the cutter bar, while \( \psi_x \) and \( \psi_y \) denote the rotations about \( y \)-axis and \( z \)-axis, respectively. Here, the \( x-y-z \) is a rotating coordinate system attached to the neutral axis of the cutter bar.
In view of the viscoelastic mechanical characteristics of the composite, the stress components can be split into elastic stress and dissipative [22]:

\[
\sigma_x = \sigma_x^e + \sigma_x^d, \\
\tau_{xa} = \tau_{xa}^e + \tau_{xa}^d,
\]

where \(\sigma\) and \(\tau\) denote normal stress and shear stress, respectively, and the superscripts \(e\) and \(d\) correspond to elastic and dissipative damping parts. Because the axial displacement \(u_x\) and torsion angle \(\varphi\) have no effect on the bending deformation, they can be removed from equation (2). In this case, the elastic stress-strain relations can be expressed as

\[
\sigma_x^e = (\tilde{Q}_1 \epsilon_x),
\]

where \(\tilde{Q}_1\) denotes reduced stiffness. The detailed definitions for the reduced stiffness are given in Appendix. The damping dissipation stress-strain relations can be written as

\[
\sigma_x^d = (c_1 \dot{\epsilon}_x),
\]

where \(c_1\) denotes the coefficient of the internal damping of the cutter bar linked to material damping (or call it off-axis stiffness linked to material damping).

The expression for kinetic energy is

\[
U = \frac{1}{2} \int_0^l \left[ D_{11}(x) \left( \frac{\partial \psi_x}{\partial x} \right)^2 + \left( \frac{\partial \psi_z}{\partial x} \right)^2 \right] dx.
\]

\[
\int_{t_1}^{t_2} \delta W_d \, dt = \int_0^l \left[ \int_{t_1}^{t_2} \frac{\partial}{\partial x} \left[ -c_1 I(x) \frac{\partial^2 \psi_x}{\partial x^2} + c_1 I(x) \Omega \frac{\partial^2 \psi_x}{\partial x^2} \right] \psi_x \, dt \right. \, dx
\]

The mass per unit length \(m(x)\) and the moment of mass inertia \(I_m(x)\) can be written as

\[
m(x) = \frac{\pi}{4} \sum_{k=1}^{M} \rho_k \left[ r_{k+1}^2(x) - r_k^2(x) \right],
\]

\[
I_m(x) = \frac{\pi}{4} \sum_{k=1}^{M} \rho_k \left[ r_{k+1}^4(x) - r_k^4(x) \right],
\]
in which \( \rho_k \) is the density of layer \( k \).

By substituting equations (6), (9), and (10) into equation (1) and assuming that the cutting forces \( F_y \) and \( F_z \) are produced on the free end of the cutter bar along \( y \)-axis and \( z \)-axis directions, the bending-bending coupling forced vibration equation of the rotating tapered composite cutter bar can be written as

\[
m(x)\ddot{u}_y + C_\theta \dddot{u}_y - I_m(x) \frac{\partial^2 \ddot{u}_y}{\partial x^2} - 2I_m(x)\frac{\partial^2 \dddot{u}_y}{\partial x^2} - \frac{\partial^2 M_z}{\partial x^2} = F_y \delta_D(x - L),
\]

\[
m(x)\ddot{u}_z + C_\theta \dddot{u}_z - I_m(x) \frac{\partial^2 \ddot{u}_z}{\partial x^2} + 2I_m(x)\frac{\partial^2 \dddot{u}_z}{\partial x^2} - \frac{\partial^2 M_y}{\partial x^2} = L_\theta \delta_D(x - L),
\]

where \( \delta_D \) denotes the Dirac function:

\[
\delta_D(x - L) = \begin{cases} 
\infty, & x = L, \\
0, & \text{otherwise}.
\end{cases}
\]  

2.2. Galerkin Method. It is assumed that the differential equation (12) has the solutions:

\[
\begin{align*}
\ddot{u}_y &= \sum_j U_{yj} \Phi_j(x), \\
\ddot{u}_z &= \sum_j U_{zj} \Phi_j(x),
\end{align*}
\]

where

\[
\begin{align*}
\int_0^L m(x)\Phi_i \Phi_j dx &= M_{ij}, \\
\int_0^L I_m(x)\Phi_i \Phi'_j dx &= I_{mij}, \\
\int_0^L D_{ij} (x) \Phi_i \Phi'_j dx &= D_{ij}, \\
\int_0^L c_1 I (x) \Phi_i \Phi'_j dx &= D_{ij}, \\
\int_0^L F_y \Phi_i dx &= F_{yj}, \\
\int_0^L F_z \Phi_i dx &= F_{zj}.
\end{align*}
\]

Equation (15) is the dynamic equation of the cutting system. For analyzing free vibration characteristics in the bending of the cutter bar, it can be first assumed that \( F_y = F_z = 0 \).

In order to determine the internal damping coefficient, the following expression can be derived according to the

\[
\text{definition of the damping ratio of 1-DOF free vibration systems}
\]

\[
D_{11} = 2c_1 \sqrt{D_{11} (M_{11} - I_{m11})}.
\]

The internal damping ratio \( c_1 \) is related to the modal loss factor \( \eta_1 \):

\[
c_1 = \left( \frac{1}{2} \right) \eta_1,
\]

where \( \eta_1 \) denotes the modal loss factor, which can be defined as by using an energy approach [27]:

\[
\eta_1 = \left( \frac{1}{2\pi} \right) \left( \frac{|X|^T [\tilde{C}] [X]}{|X|^T [\tilde{K}] [X]} \right),
\]

where \( [X] \) is the mode deflection and [\( \tilde{K} \)] and [\( \tilde{C} \)] are the stiffness and damping matrices of a tapered nonrotating composite cutter bar, respectively, which have the forms
\[
[\tilde{C}] = \begin{bmatrix}
D^j_l & 0 \\
0 & D^j_l
\end{bmatrix},
\]
\[
[\tilde{K}] = \begin{bmatrix}
D & 0 \\
0 & D
\end{bmatrix}.
\] (20)

In the previous equations, the elements of damping matrix are given by [22]
\[
\int_0^l D^j_{11} (x) \psi_j''(x) \psi_j''(x) dx \equiv D^j_{ij}
\]
\[
D^j_{11} (x) = \left( \frac{\pi}{4} \right) \sum_{k=1}^{M_1} \bar{\eta}_u \Omega_{ij} \left[ r^j_{k+1}(x) - r^j_k(x) \right],
\]
\[i, j, k, s = 1, 2, 6,
\] (21)
in which \( \bar{\eta}_u \) denotes the off-axis damping dissipation factor of the composite layer. The detailed expressions can be found in Appendix.

2.3. Cutting Force and Stability Analysis. It is assumed that the milling cutter has N teeth that produce vibration displacements along y-axis and z-axis directions. The cutter bar rotates at a rotating speed of \( \Omega \) (rad/s) as shown in Figure 2.

The regenerative dynamic milling force in the rotating coordinate system can be written as Comak et al. [24]
\[
[F] = b K_y \begin{bmatrix}
A(t) \end{bmatrix} [U] - [B(t)] [U]^T,
\] (22)
where
\[
[F] = \begin{bmatrix}
F_y & F_z
\end{bmatrix}^T
\]
\[U = \begin{bmatrix}
u_y & u_z
\end{bmatrix}^T
\]
\[U^T = \begin{bmatrix}
u_y (t - \tau) & u_z (t - \tau)
\end{bmatrix}^T,
\]
\[\begin{aligned}
A(t) &= \begin{bmatrix}
f_y & f_z
\end{bmatrix}.
\end{aligned}
\]
\[
M_y = \begin{bmatrix}
M_x & 0 \\
0 & M_z
\end{bmatrix}
\]
\[
C(\Omega) = \begin{bmatrix}
G^d_x & 2G^d \Omega \\
-2G^d \Omega & G^d
\end{bmatrix}
\]
\[
K(\Omega) = \begin{bmatrix}
G & G^d \Omega \\
-G^d \Omega & G
\end{bmatrix}
\]
\[
F = \int_0^l I_m(x) \Phi_1(x) \Phi_1'(x) dx,
\]
\[
M_y = M_z = \int_0^l m(x) \Phi_1^2(x) dx - \int_0^l I_m(x) \Phi_1(x) \Phi_1'(x) dx,
\]
\[
G = \int_0^l D_{11}(x) \Phi_1(x) \Phi_1'(x) dx,
\]
\[
G^d = \int_0^l C_1(x) \psi_1 \psi_1'(x) dx.
\] (27)

Average matrix \([A_0]\) has the form as follows [25]:
\[
[A_0] = \begin{bmatrix}
\phi_{ex} - \phi_{in} & \sin2\phi_{c} + 2K \cos^2\phi_{c} & -2\sin^2\phi_{c} - 2K \sin2\phi_{c} \\
2\cos^2\phi_{c} - 2K \sin2\phi_{c} & -\sin2\phi_{c} + 2K \sin^2\phi_{c}
\end{bmatrix}
\]
\[
\begin{bmatrix}
\alpha_{yy} & \alpha_{yz} \\
\alpha_{zy} & \alpha_{zz}
\end{bmatrix}
\] (25)
where \( \phi_{in} \) and \( \phi_{ex} \) are start and exit immersion angles of the cutter to and from the cut and \( K \) is the radial cutting constant.

2.4. Cutting Stability Analysis. It is assumed for simplicity that there is only one dominant vibration mode in each of the two principal axes of the cutter bar. Under the action of regenerative linear cutting force, the chatter equation of the rotating tapered composite cutter bar can be written as
\[
[M] [U] + [C(\Omega)] [U] + [K(\Omega)] [U] = \Phi_1' (L) b K_y [A_0] \begin{bmatrix}
U \\
U^T
\end{bmatrix},
\] (26)
where
\[
\begin{aligned}
\Phi_1 &= \Phi_1''(x) \Phi_1'(x) dx,
\end{aligned}
\]
\[
M_y = M_z = \int_0^l m(x) \Phi_1^2(x) dx - \int_0^l I_m(x) \Phi_1(x) \Phi_1'(x) dx,
\]
\[
G = \int_0^l D_{11}(x) \Phi_1(x) \Phi_1'(x) dx,
\]
\[
G^d = \int_0^l C_1(x) \psi_1 \psi_1'(x) dx.
\] (27)
where the symbol prime (’) denotes the first-order derivative with respect to \( x \).

By conducting Laplace transformation on equation (26) and assuming \( s = (\lambda + i\omega) \), it can be found that the value of \( s \) is connected with system stability.

If \( \lambda > 0 \), the system is in unstable state.

If \( \lambda < 0 \), the system is in stable state.

If \( \lambda = 0 \), the system is in the critical state between the stable and unstable state.

Let \( \lambda = 0 \), by substituting \( s = i\omega \) into the equation after Laplace transformation of equation (26), and the following expression can be written as

\[
(-\omega_s^2 [M] + i\omega_s [C(\Omega)] + [K(\Omega)])[U] = \Phi_2^\dagger (L) bK_c (1 + e^{-i\omega})[U],
\]

in which \( \omega_s \) denotes the chatter frequency.

The transfer function can be written as

\[
G(\omega_s, \Omega) = \left[ -\omega_s^2 [M] + i\omega_s [C(\Omega)] + [K(\Omega)] \right]^{-1} = \begin{bmatrix} G_{yy}(i\omega_s, \Omega) & G_{yz}(i\omega_s, \Omega) \\ G_{zy}(i\omega_s, \Omega) & G_{zz}(i\omega_s, \Omega) \end{bmatrix},
\]

(28)

Equation (28) can be rewritten as

\[
[I] - \Phi_2^\dagger (L) bK_c [A_0] (1 + e^{-i\omega}) [G(\omega_s, \Omega)] [U] = [0].
\]

(30)

The system of linear equation (30) has a nontrivial solution if its determinant is zero:

\[
det\left[[I] - \Phi_2^\dagger (L) bK_c [A_0] (1 + e^{-i\omega}) [G(\omega_s, \Omega)] \right] = 0,
\]

(31)

where \([I] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\) denotes an identity matrix.

The stability of the cutter bar can be transformed into the characteristic value of the following second-order characteristic equation:

\[
det\left[I + \Lambda [G_0(\omega_s, \Omega)] \right] = 0,
\]

(32)

where

\[
\Lambda = -\frac{1}{2\pi} bK_c (1 + e^{-i\omega}).
\]

(33)

Substituting \( \Lambda = \Lambda_R + i\Lambda_I \) and \( e^{-i\omega} = (\cos \omega - \sin \omega i) \) into equation (33), one has

\[
b_{\text{lim}} = \frac{2\pi}{K_f} \frac{(\Lambda_R + j\Lambda_I)}{(1 + \cos \omega) - (j\sin \omega i)}.
\]

(35)

Since the limit cutting depth is a real number in the cutting process, \( b_{\text{lim}} \) is a real number, and then the imaginary part of equation (35) must vanish, which will lead to

\[
\kappa = \frac{\Lambda_I}{\Lambda_R} = \frac{\sin \omega i}{\cos^2 \omega i} = \frac{\sin \omega \tau/2}{\cos \omega \tau/2} = \tan \Psi.
\]

(36)

Finally, the critical axial depth \( b_{\text{lim}} \) and spindle speed \( \Omega \) (rpm) can be obtained as follows:

\[
b_{\text{lim}} = \frac{\pi}{K_f} (\Lambda_R + \Lambda_I \kappa),
\]

(37)

\[
\Omega = \left( \frac{60}{N} \right) \frac{\omega_s}{2(-\Psi + k\pi)}, \quad k = 0, 1, 2, \ldots.
\]

(38)

The stability lobes can be calculated as follows:

(1) Calculate the directional matrix in accordance with the milling cutter’s material and geometrical parameters

(2) Start a loop incrementing the spindle speed \( \Omega \)

(3) Scan the chatter frequency \( \omega_s \) by using the natural frequency as the reference, and calculate the transfer function

(4) Calculate real and imaginary parts according to equation (32), and solve the critical axial cutting depth \( b_{\text{lim}} \) and the corresponding rotating speed \( \Omega \)

(5) If the difference between the rotating speed obtained in the step 4 and input rotating speed in the step 2 satisfies the required precision, plot \( \Omega \) vs. \( b_{\text{lim}} \); otherwise, repeat the iterative procedures until the results converge

(6) Select the new \( j \) and calculate the adjacent lobes

3. Case Study and Discussion

3.1. Natural Frequency and Logarithmic Decrement. By neglecting the cutting forces in equation (15), the eigenvalue \( \lambda_n \) can be solved given a rotating speed \( \Omega \), whose imaginary part represents the natural frequency and the ratio of imaginary part to the real part defines the logarithmic decrement, respectively. Therefore, the variation curves of both natural frequency and damping with the rotating speed can be plotted.
In order to investigate the variations of the natural frequency and logarithmic decrement with the rotating speed, a case study is then performed on the tapered composite cutter bar with a cross-section thickness of $h = 0.01016$ m, $R_T = 0.176$ m. The length $L$ can be determined in accordance with the length-to-diameter ratio ($L/R_T$).

Figures 3 and 4 show the variation curves of the first natural frequency and logarithmic decrement of the cutter bar with the rotating speed when the ply angles are set as different values $0^\circ$, $30^\circ$, and $60^\circ$, respectively, with the taper ratio 0.5 and the length-to-diameter ratio 10. The composites have stacking sequence $[\pm \theta]_{16}$. Table 1 lists the material property parameters.

The numerical results of first natural frequency and logarithmic decrement are shown in Table 2 for an increasing number of mode shape functions. From these tables, it can be seen that to obtain the accurate results of the first natural frequency, no more than three mode shape functions are required.

As shown in Figures 3 and 4, as the rotating speed $\Omega$ increases from 0, both the variation curves of natural frequency and logarithmic decrement split into two curves. In the upper branch which is corresponding to forward whirling, the natural frequency increases with the rotating speed; in the latter branch which is corresponding to backward whirling, natural frequency drops with the rotating speed. The logarithmic decrement in the upper branch is always positive within the whole rotating speed range, suggesting that the forward whirling is stable. Within certain rotating speed range, the logarithmic decrement in the lower branch is positive; as the rotating speed increases to certain value, the logarithmic decrement in the lower branch changes from positive to negative, suggesting that the rotating cutter bar gradually lost motion stability. The value of $\Omega$ corresponding to zero logarithmic decrement is called as the instability threshold. It should be noted that, when the external damping equals to 0, the threshold of instability equals to the critical speed.

It can also be observed from Figure 3 that, decreasing of the ply angle increases the natural frequency of the composite cutter bar. This is because the closer the fiber is oriented to $0^\circ$, the greater the cutter bar rigidity and the higher the whirl frequencies are (see Table 1).

Figure 4 suggests that the instability thresholds decrease with ply angle. The reason is that the larger the ply angle, the greater the internal damping due to the composite materials is and the more likely the instability appears (see Table 1).

Table 3 shows the first frequency and logarithmic decrement of the tapered cutter bars which are made of carbon fiber/epoxy composite material and steel. The taper ratios 0.75, 0.5, and the length-to-diameter ratio 10 are used. The composite cutter bar has stacking sequence $[0^\circ]_{16}$. Table 1 gives the carbon fiber/epoxy composite material property parameters. Table 4 gives the mechanical properties of steel.

It can be observed from Table 3 that the improvement of frequency and logarithmic decrement in using the anisotropic composite tapered cutter bar when compared with an isotropic tapered cutter bar is obvious. For $\sigma = 0.75$ and $\sigma = 0.5$, the differences in the first natural frequency are 168% and 143%, and the differences in the first mode logarithmic decrement are 256% and 263%.

Figure 5 shows the variation of the modal loss factor of the composite cutter bar with ply angle. It can be seen that the loss factor in the first mode increases with the value of ply angle increasing from $0^\circ$ to $81^\circ$. It is minimum at $0^\circ$ ply angle, about 0.45%, and has a maximum value, about 4.28%, at $81^\circ$ ply angle. An insignificant amount of reduction in the modal loss factor is observed in the range of $81^\circ$ to $90^\circ$ (about 4.22% for $90^\circ$ ply angle). Such behavior is explained by the fact that the closer the fiber is oriented to $90^\circ$, the greater the modal loss factor since the transversal specific damping capacity is greater than that along the longitudinal direction (see Table 2).
3.2. Stability Lobes. The stability analytical model is applied to a two-fluted composite end mill cutter for a half immersion down milling ($\phi_m = 0^\circ$ and $\phi_{ex} = 90^\circ$), $K_e = 732$ MPa and $K_s = 0.076$ as reported by Comak et al [24]. The composite cutter bar has the stacking sequence $[\pm \theta]_s$, with a mean radius 0.0352 m and a thickness 0.002 m. The length $L$ was determined according to the length-to-diameter ratio.

First, the convergence study of the stability lobes is given in Figure 6. It is seen that the convergence of the stability lobes is good with respect to $N$, and $N = 3$ is found to be adequate for convergence. Therefore, in this section, all results have been obtained by three mode shape functions.

Figure 7 shows the effect of rotation of the cutter bar on the cutting stability with the length-to-diameter ratio 10, the ply angle $0^\circ$, and the taper ratio 0.5, in which the stability lobes without and with the consideration of gyroscopic effect are marked in black solid lines and red dash lines, respectively. Apparently, as the rotating speed increases, the backward frequency drops and forward frequency increases under gyroscopic/rotating effect. The backward frequency is less than the forward one, and thus the position of the stability diagram associated with the backward mode is lower, thereby the backward whirling determines the stability of the cutting system.

It can also be observed from Figure 7 that the limit critical cutting depth of the cutter bar without gyroscopic effect is a constant, and the envelopes are horizontal lines. By taking the gyroscopic effect into account, the critical cutting depth is no longer a constant but relies on the rotating speed, whose envelopes decrease obliquely. Accordingly, unstable region of the cutter bar considering the gyroscopic effect is larger than those of the cutter bar without gyroscopic effect, suggesting that gyroscopic effect plays the role of negative damping in the cutting process. Therefore, the gyroscopic effect imposes nonnegligible effects on the system’s dynamic characteristics and stability limit during high-speed milling process.

Figure 8 shows the stability lobes of the cutter bars made up of carbon fiber/epoxy composite materials and steel. The taper ratio 0.5, the length-to-diameter ratio 10, and the ply angle $0^\circ$ are used, respectively. Table 4 gives the mechanical properties of steel.

As shown in Figure 8, the maximum value of the limit cutting depth for the carbon fiber/epoxy cutter bar is 0.412 mm, and the maximum value of the limit cutting depth for the steel cutter bar is 0.188 mm. This is due to the fact that, among three different types of cutter bars, the carbon/epoxy cutter bar has the greater bending stiffness and damping than the steel cutter bar. Also, the cutting speeds of the carbon fiber/epoxy composite cutter bar are larger than those of the steel cutter bar due to higher specific stiffness and in consequence higher fundamental natural frequency.

Figure 9 shows the effect of the ply angle on the stability lobes of the carbon/epoxy resin cutter bar with a taper ratio of 0.5 and a long-to-diameter ratio of 10.

It can be observed from Figure 9 that the stable region of the composite cutter bar increases with the decreasing ply angle. As the ply angle drops, the bending stiffness of the

### Table 1: Mechanical properties of carbon/epoxy composite material [13].

| $\rho$ (kg/m$^3$) | $E_{11}$ (GPa) | $E_{22}$ (GPa) | $G_{12}$ (GPa) | $G_{23}$ (GPa) | $\gamma_{12}$ | $\psi_1$ (%) | $\psi_2$ (%) | $\psi_4$ (%) | $\psi_5 = \psi_6$ (%) |
|-----------------|----------------|----------------|----------------|----------------|-------------|--------------|--------------|--------------|------------------|
| 1446.2          | 172.7          | 7.2            | 3.76           | 3.76           | 0.3         | 0.45         | 4.22         | 7.05         | 7.05             |

### Table 2: First frequency and modal logarithmic decrement of the tapered composite cutter bar ($\sigma = 0.5$, $\Omega = 0$).

| Ply angle $\theta$ (degree) | The first natural frequency (Hz) N = 1 | The first mode logarithmic decrement N = 3 N = 5 |
|-----------------------------|----------------------------------------|-----------------------------------------------|
| 0º                          | 441.52 438.27 438.27                  | 0.0095 0.0087 0.0087                          |
| 30º                         | 337.95 335.51 335.51                  | 0.0143 0.0128 0.0128                          |
| 60º                         | 146.05 143.24 143.24                  | 0.0585 0.0455 0.0455                          |

### Table 3: First frequency and modal logarithmic decrement for the tapered cutter bar made of different materials ($\Omega = 0$).

| Material                      | The first natural frequency (Hz) $\omega = 0.75$ | The first mode logarithmic decrement $\omega = 0.5$ |
|-------------------------------|-----------------------------------------------|-----------------------------------------------|
| Carbon fiber/epoxy            | 355.82 438.27                                | 0.0089 0.0087                                 |
| Steel                         | 132.56 180.38                                | 0.0025 0.0024                                 |

### Table 4: Mechanical properties of steel material.

| $\rho$ (kg/m$^3$) | $E$ (GPa) | $\gamma$ | $\zeta$ (%) |
|------------------|-----------|----------|-------------|
| 7850             | 210       | 0.3      | 0.01        |
composite cutter bar increases, which can lead to the increase of natural frequency (see Figure 3), thereby raising the rotating speed of machine tool spindle.

Figure 10 shows the effect of stacking sequence of the composite cutter bar on the stability lobes (\( l/d = 10, \sigma = 0.5, \theta = 00 \)). The greater the number of fibers oriented close to the longitudinal direction of the composite cutter bar (e.g., the stacking sequences unidirectional), the more they contribute to the cutter bar bending stiffness and, consequently, the larger the stable regions will be.

Figure 11 shows stability diagrams for the tapered composite cutter bar (\( \sigma = 0.3 \)) and uniform composite cutter bar (\( \sigma = 1 \)). Two cutter bars have same stacking sequence \([ \pm 30^\circ ]\) and same volume. The geometrical characteristics of the tapered carbon/epoxy cutter bar are \( R_T = 0.04028 \) m, \( R_R = 0.03012 \) m, \( h = 0.002032 \) m, and \( L = 0.704 \) m. The geometrical characteristics of the uniform carbon/epoxy cutter bar are \( R_T = R_R = 0.0352 \) m, \( h = 0.002032 \) m, and \( L = 0.704 \) m. As it can be seen in Figure 11 that stable regions of the tapered composite cutter bar are larger than those of the
uniform composite cutter bar, this may be due to the higher bending stiffness produced by cutter bar tapering.

4. Conclusions

An analytical model for predicting free vibration and chatter stability of a rotating tapered composite cutter bar, subject to dynamic regenerative cutting forces, is presented. Based on damping dissipation constitutive relations for viscoelastic composite, damping is introduced to the model. Moreover, the 2-DOF stability analysis model of Comak et al. [24] and Eynian and Altintas [25] for steel cutter bar based on the rotating frame approach is extended here to the case of the cutting process with rotating tapered composite cutter bar, which is used to predict stability lobes analytically. The continuum-based model of the cutter bar is developed by employing the Hamilton principle and Euler–Bernoulli beam theory, and the Galerkin method is used to discretize the partial differential motion equations with bending-bending coupling. The influences of some factors including the rotating gyroscopic effect, materials, ply angle, stacking sequence, and taper on the chatter stability are also examined. Results show that the gyroscopic effect of the composites acts as negative damping in the cutting process. By neglecting the rotating effect, the chatter stability is overestimated. Compared with the maximum limit cutting depth of the steel boring bar, the values of the carbon/epoxy cutter bar is enhanced by 263.8%. As the ply angle drops, the chatter stability increases. The chatter stability of the cutting process can also be improved by changing the taper ratio or stacking sequence of the composites cutter bar.

Nomenclature

| Symbol | Description |
|--------|-------------|
| \( \Omega \) | Rotating speed of the tapered composite cutter bar |
| \( R_T \) | The cross-section radii on the fixed end of the cutter bar |
| \( R_K \) | The cross-section radii on the free end of the cutter bar |
| \( \sigma \) | The taper of the cutter bar |
| \( T \) | Kinetic energy |
| \( U \) | Potential energy |
| \( W_d \) | Virtual work |
| \( u_y, u_z \) | The displacement of the neutral axis along \( y \)-axis and \( z \)-axis directions of the points on the reference axis of the cutter bar |
| \( \psi_x, \psi_y \) | The rotations about \( y \)-axis and \( z \)-axis |
| \( Q_{11} \) | Reduced stiffness |
| \( c_i \) | Off-axis stiffness linked to material damping |
| \( D_{11}(x) \) | The spatial varying flexural stiffness of the composite cutter bar |
| \( r_k(x), r_{k+1}(x) \) | The external and internal radius of layer \( k \) in section \( x \) |
| \( F_y, F_z \) | The cutting forces produced on the free end of the cutter bar along \( y \)-axis and \( z \)-axis directions |
| \( \Phi_j(x) \) | The mode function |
| \( \varsigma_i \) | Internal damping ratio |
| \( \{X\} \) | Mode deflection |
| \( [\tilde{K}] \) | The stiffness matrices of a tapered nonrotating composite cutter bar |
| \( [\tilde{C}] \) | The damping matrices of a tapered nonrotating composite cutter bar |
| \( b \) | Axial cutting depth |
| \( K \) | The tangential cutting constant |
| \( [A(t)], [B(t)] \) | Directional matrix whose coefficients depend on the instantaneous angular immersion of the cutter |
| \( \phi_b \) | The pitch angle of the cutter |
| \( \phi_c \) | The cutter pitch angle |
| \( [A_0] \) | Average matrix |
\[ \begin{align*}
\bar{Q}_{11} &= C_{11}\cos^4\theta + C_{22}\sin^4\theta + 2(C_{12} + 2C_{66})\sin^2\theta\cos^2\theta, \\
\bar{Q}_{16} &= (C_{11} - C_{12} - 2C_{66})\sin\theta\cos^4\theta + (C_{12} - C_{22} + 2C_{66})\sin^3\theta\cos\theta, \\
\bar{Q}_{66} &= C_{66}(\sin^4\theta + \cos^4\theta) + (C_{11} + C_{22} - 2C_{12} - 2C_{66})\sin^2\theta\cos^2\theta, \\
\bar{Q}_{12} &= C_{12}(\sin^4\theta + \cos^4\theta) + (C_{11} + C_{22} - 4C_{66})\sin^2\theta\cos^2\theta, \\
\bar{Q}_{26} &= (C_{11} - C_{12} - 2C_{66})\sin^3\theta\cos\theta + (C_{12} - C_{22} + 2C_{66})\sin\theta\cos^3\theta,
\end{align*} \]

in which \( C_{ij} \) are known functions of the layer material parameters and \( \theta \) represents ply angle.

\[ 
\begin{align*}
C_{11} &= \left( \frac{E_{11}}{1 - v_{12}v_{21}} \right), \\
C_{12} &= \left( \frac{v_{12}E_{22}}{1 - v_{12}v_{21}} \right), \\
C_{22} &= \left( \frac{E_{22}}{1 - v_{12}v_{21}} \right), \\
C_{66} &= G_{12}, \\
v_{21} &= v_{12} \cdot \left( \frac{E_{22}}{E_{11}} \right),
\end{align*} \]

in which \( E_{11} \) and \( E_{22} \) represent the elastic modulus, \( G_{12} \) represents the shear modulus, and \( v_{12} \) represents the Poisson ratio.

The formulas for calculating \( \bar{Q}_{ij} \) [22]:

\[ 
\begin{align*}
\bar{Q}_{11} &= \eta_{11}\bar{Q}_{11} + \eta_{12}\bar{Q}_{12} + \eta_{16}\bar{Q}_{16}, \\
\bar{Q}_{16} &= \eta_{11}\bar{Q}_{16} + \eta_{12}\bar{Q}_{26} + \eta_{16}\bar{Q}_{66}, \\
\bar{Q}_{66} &= \eta_{16}\bar{Q}_{16} + \eta_{26}\bar{Q}_{26} + \eta_{66}\bar{Q}_{66},
\end{align*} \]

where \( \eta_{ij} \) can be obtained as follows:

\[ \eta_{ij} = [T]^{-1} \begin{bmatrix} \eta_1 \\ \eta_2 \\ 0 \\ \eta_4 \\ \eta_5 \\ \eta_6 \end{bmatrix} [T], \quad (A.4) \]

where, \( \eta_j \) represent damping parameters, in which

**Appendix**

The detailed expressions for \( \bar{Q}_{ij} \) and \( \bar{Q}_{ij}^d \)

The formulas for calculating \( \bar{Q}_{ij} \) [22]:

\[ 
\begin{bmatrix} \cos^2\theta & \sin^2\theta & 0 & 0 & 0 & 2\cos\theta\sin\theta \\
\sin^2\theta & \cos^2\theta & 0 & 0 & 0 & -2\cos\theta\sin\theta \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \cos\theta & -\sin\theta & 0 \\
0 & 0 & 0 & \sin\theta & \cos\theta & 0 \\
-\cos\theta\sin\theta & \cos\theta\sin\theta & 0 & 0 & \cos^2\theta - \sin^2\theta & 0 \end{bmatrix}, \quad (A.5) \]

**Data Availability**

(1) The [1446.2 172.7 7.2 3.76 3.76 0.3 0.45 4.22 7.05 7.05] data used to support the findings of this study have been deposited in the website (https://doi.org/10.1016/j.compapt.2007.06.019). (2) The [0 90 732 0.076] data used to support the findings of this study have been deposited in the [DOI:10.1115/1.4032585]. (3) The [7850 210 0.3 0.01] data used to support the findings of this study were measured in the authors experiments.

**Conflicts of Interest**

The authors have declared that no conflicts of interest exist.

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