Sliding mode synchronization between uncertain Watts-Strogatz small-world spatiotemporal networks

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Abstract  Based on the topological characteristics of small-world networks, a nonlinear sliding mode controller is designed to minimize the effects of internal parameter uncertainties. To qualify the effects of uncertain parameters in the response networks, some effective recognition rates are designed so as to achieve a steady value in the extremely fast simulation time period. Meanwhile, the Fisher-Kolmogorov and Burgers spatiotemporal chaotic systems are selected as the network nodes for constructing a drive and a response network, respectively. The simulation results confirm that the developed sliding mode could realize the effective synchronization problem between the spatiotemporal networks, and the outer synchronization is still achieved timely even when the connection probability of the small-world networks changes.

Key words  synchronization, sliding mode control, small-world network, parameter identification

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1 Introduction

Synchronization has attracted significant attention in the last decade because of its extensive practical applications such as robot control, metabolic pathway, and aircraft formation. In a typical synchronous system, the data should be received and transmitted at the same time. It is crucial to ensure fast and reliable synchronous transmission, especially when a large number of data need to be transferred quickly. In 1990, the synchronization of coupled chaotic systems was
realized by using ordinary signals of negative Lyapunov exponents\cite{1}. Since then, synchronization has become a fundamental subfield of nonlinear dynamics, and many theoretical and trial methods for the optimized synchronization of two complex systems have been proposed\cite{2-5}.

With the enhanced research, people have realized that synchronization is an essential requirement for many types of coupled systems of complex networks, not just for chaotic systems\cite{6-8}. In the time of artificial intelligence advances, driverless cars are supposed to be a big challenge for current transportation systems. In addition to legal and ethical concerns, the change in the driving method puts forward higher requirements for the data synchronization of the real-time traffic management. It is worth pointing out that early synchronization studies mainly focus on complex network systems where the linking topologies are regarded to be either totally random or fully regular. However, actual network systems are more complicated. In terms of such problems, a small-world network model is proposed and developed\cite{9-10}. Compared with conventional networks only with regular characteristics, the small-world network model can significantly enhance the synchronization ability\cite{11-14}. With advances in science and technology, our daily life seems to be constructed by a multiplicity of overlapping networks, e.g., socio-political networks and commercial networks. At the early stage, the studies in network synchronization mainly focus on the dynamic characteristics, e.g., modularity structure, hierarchy architecture, and small world effect, but only within one network. However, synchronization phenomena exist even among two or more coupled complex dynamical networks\cite{15}. A typical example is the transmission bird flu viruses. The interspecies transmission of these viruses can be seen as outer synchronization. Influenza pandemics are unpredictable but recurring events, which can have health, economic, and social network consequences worldwide. How to solve outer synchronization problems for complex networks remains largely challenging\cite{16-17}. Tremendous efforts have been made to synchronize two coupled systems of real-world multiplex networks since the synchronization occurs frequently. Li et al.\cite{18} studied the synchronization phenomenon between two discrete complex networks. Zhou et al.\cite{19} studied the synchrony and controlling between small worlds in the erbium-doped dual-ring fiber laser system with hyperchaotic characteristics. Arellano-Delgado et al.\cite{20} started off with a small-world network model, comparatively analyzed the effects of the coupling strength and the dynamic characteristics of nodes on the synchronization capabilities, and enumerated some simulation results.

It is easy to understand that the key to dynamic control is to achieve high-efficiency synchronization between given complex networks\cite{21-24}. Depending on the dynamics features of studied systems, researchers have proposed many control methods to obtain synchronization, e.g., adaptive synchronization, pinning synchronization, sliding mode synchronization\cite{25-28}. The sliding mode variable structure is a nonlinear control method\cite{29}. In the sliding mode control technique, the controller can drive the system into a pre-specified sliding hyperplane to obtain the desired dynamic performance. The method is insensitive to parameters, and has many advantages such as good robustness to external disturbance and fast dynamic response. Therefore, it has been widely applied in robot control\cite{30}. The sliding mode control shows obvious advantages over other methods, e.g., fast response speed, strong anti-interference, and simple physical implementation\cite{31-33}. However, in industrial processes, the systems have inertial related behaviors and measurement errors, which makes the variable structure control be accompanied by high frequency chattering under sliding mode dynamics\cite{34}. To overcome such chattering problem, the terminal sliding mode with circuits and systems was proposed\cite{35-36}. Furthermore, for more complex network systems, the control method is improved by designing a developed fixed-time stable sliding surface and implementing synchronization of networks\cite{37}. After that, the non-singular terminal sliding surface was investigated to study the outer synchronization with time variant topology and interference\cite{38}. In this paper, we will synchronize two small-world networks by an optimized sliding mode control.

Currently, most studies related to small-world network synchronization are based on only time-dependent networks. However, networks in practice are more complicated, e.g., networks
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with spatiotemporal chaotic behaviors. Spatiotemporal chaotic system has been widely used in practice such as modeling, control, and optimization. Addressing these concerning issues, in this paper, two Watts-Strogatz (WS) small-world spatiotemporal networks with varied topologies are built. Instead of the traditional linear method, a nonlinear sliding mode controller based on the Lyapunov principle is designed. The Fisher-Kolmogorov and Burgers systems are served as nodes to construct a drive network and a response network, respectively. The high efficiency of the proposed synchronization approach is verified by numerical examples, even for WS small-world spatiotemporal networks with different connection probabilities.

2 Model description

For a spatiotemporal network composed of N interacting nodes, the state equation of the kth node is described as follows:\(^\text{[39]}\):

\[
\frac{\partial x_k(r,t)}{\partial t} = f(x_k(r,t)) + \rho_k \sum_{j=1}^{N} c_{kj} x_j(r,t), \quad k = 1, 2, \cdots, N, \tag{1}
\]

where \(x_k(r,t) = (x_{k1}(r,t), x_{k2}(r,t), \cdots, x_{kn}(r,t))^T \in \mathbb{R}^n\) is the time-varying state variable. \(t\) and \(r\) are the time and space coordinates, respectively. \(f\) is the nonlinear vector function. \(\rho_k\) is the coupling strength between the network nodes. \(C = (c_{kj})_{N \times N}\) is the diffusive coupling matrix. \(c_{kj}\) is as follows: if \(k \neq j\) and there exists a connection from node \(j\) to node \(k\), \(c_{kj} \neq 0\); otherwise, \(c_{kj} = 0\). The diagonal elements of \(C\) are defined as \(c_{kk} = - \sum_{j=1, j \neq k}^{N} c_{kj}\). The WS small-world network models are constructed. Construct a ring-shaped network with \(N\) nodes, where any single node is linked to its 2N neighbors. For each pair of the connected nodes, the edge is rewired, i.e., one end of the edge is remained while the other one is disconnected with the probability \(p\), and reconnected to a node randomly selected from the network. The rewiring process is conducted edge by edge on the initial ring-shaped network clockwise.\(^{[9,11]}\)

Based on the mode \((1)\), considering the parameter uncertainties in practice, we construct the kth node for an N-node response network model with sliding mode controllers as follows:

\[
\frac{\partial y_k(r,t)}{\partial t} = g(y_k(r,t)) + \rho_k \sum_{j=1}^{N} d_{kj}(t) y_j(r,t) + u_k(r,t), \quad k = 1, 2, \cdots, N, \tag{2}
\]

where \(y_k(r,t) = (y_{k1}(r,t), y_{k2}(r,t), \cdots, y_{kn}(r,t))^T \in \mathbb{R}^n\) is the state variable. \(g\) is a nonlinear vector function. \(D = (d_{kj})_{N \times N}\) denotes the coupling matrix of the drive network, where \(d_{kj}(t)\) are the uncertain matrix elements of the response network. \(u_k(r,t)\) are designed controllers.

We aims to implement outer synchronization between two different spatiotemporal networks. To calculate the unknown parameters, the synchronization errors are given by

\[
e_k(r,t) = y_k(r,t) - x_k(r,t), \quad k = 1, 2, \cdots, N. \tag{3}
\]

Combining the dynamics characteristics of the spatiotemporal networks yields

\[
\frac{\partial e_k(r,t)}{\partial t} = \frac{\partial y_k(r,t)}{\partial t} - \frac{\partial x_k(r,t)}{\partial t}. \tag{4}
\]

Combining Eqs. \((1)\) and \((2)\) yields

\[
\frac{\partial e_k(r,t)}{\partial t} = g(y_k(r,t)) - f(x_k(r,t)) + \rho_k \sum_{j=1}^{N} (d_{kj}(t) - c_{kj}) y_j(r,t) + \rho_k \sum_{j=1}^{N} c_{kj} e_j(r,t) + u_k(r,t). \tag{5}
\]
3 Main Results

The sliding mode control shows obvious advantages over other methods, including fast response speed, strong anti-interference, and simple physical implementation. In this section, a neat sliding mode control approach is presented. To qualify the effect caused by the unknown parameters of the response network, some parameter recognition rates are calculated and found to achieve a steady value in an extremely fast simulation time period.

The sliding mode surface $S_k^{[32]}$ is defined as

$$ S_k = \psi_1 e_k(r, t) + \psi_2 S_{\text{ank}}, $$

where $\psi_1$ and $\psi_2$ are positive constants, $S_k = (S_{k1}(r, t), S_{k2}(r, t), \ldots, S_{kn}(r, t))^T \in \mathbb{R}^n$, and

$$ S_{\text{ank}} = \begin{cases} 
  e_k^T(r, t), & |e_k| \geq \zeta, \\
  l_1 e_k + l_2 \text{sgn}(e_k(r, t)) e_k^T(r, t), & S_k \neq 0, \quad |e_k| < \zeta. 
\end{cases} \quad (7) $$

In the above equation,

$$\begin{align*}
  l_1 &= \left(2 - \frac{\gamma_1}{\gamma_2}\right)\zeta^{\frac{2}{\gamma_2} - 1}, \\
  l_2 &= \left(\frac{\gamma_1}{\gamma_2} - 1\right)\zeta^{\frac{2}{\gamma_2} - 2}, \\
  S_k &= \psi_1 e_k(r, t) + \psi_2 e_k^T(r, t), \\n  0 &< \frac{\gamma_1}{\gamma_2} = \gamma < 1,
\end{align*}\quad (8)$$

where $\gamma_1$ and $\gamma_2$ are positive odd integers, and $\zeta$ is a small constant. Thus, the derivation of the time-dependent function (7) leads to

$$ \frac{\partial S_k(r, t)}{\partial t} = \psi_1 \frac{\partial e_k(r, t)}{\partial t} + \psi_2 E_k \frac{\partial e_k(r, t)}{\partial t}, $$

where

$$ E_k = \begin{cases} 
  \gamma \text{diag}(e_k^{-1}(r, t)), & |e_k| \geq \zeta, \\
  l_1 I + 2l_2 \text{sgn}(e_k(r, t)) \text{diag}(e_k(r, t)), & S_k \neq 0, \quad |e_k| < \zeta, 
\end{cases} \quad (9) $$

and $I$ is the identity matrix.

According to the sliding mode surface (7), the controller of spatiotemporal networks could be expressed as follows:

$$ u_k(r, t) = - (\psi_1 I + \psi_2 E_k)^{-1} S_k(r, t) - g(y_k(r, t)) + f(x_k(r, t)) $$

$$ - \rho_k \sum_{j=1}^{N} (c_{kj} e_j(r, t) + \tilde{\varepsilon} y_j(r, t)), $$

where $\tilde{\varepsilon}$ is the configuration coefficient.

**Theorem 1** If the elements of the unknown matrix in the response network $\hat{d}_{kj}(t)$ in the controller (10) satisfy the following adaptive laws:

$$ \frac{\partial \hat{d}_{kj}(t)}{\partial t} = -\check{\eta}_k \rho_k \psi_1 S_k^T(r, t)y_j(r, t) + \psi_2 S_k^T(r, t) E_k y_j(r, t) e^{\tilde{\mu} t}, $$

where $\rho_k$ and $\check{\eta}_k$ are both positive numbers, and $\tilde{\mu}$ is the exponential synchronization rate. Then, the synchronization between two small-world spatiotemporal networks with uncertainties can be implemented.
Proof A Lyapunov function of spatiotemporal networks is generally proposed as

\[ V(r, t) = \frac{1}{2} \sum_{k=1}^{N} S_k^T(r, t) S_k(r, t) e^{\hat{\sigma}t} + \sum_{k=1}^{N} \sum_{j=1}^{N} \frac{1}{2\eta_k} (\hat{d}_{kj}(t) - c_{kj} - \hat{\varepsilon})^2. \]  (12)

The corresponding differential equation is

\[ \frac{\partial V(r, t)}{\partial t} = \sum_{k=1}^{N} \left( (\psi_1 e_k(r, t) + \psi_2 S_{\text{ank}}) \left( \psi_1 \frac{\partial e_k(r, t)}{\partial t} + \psi_2 E_k \frac{\partial e_k(r, t)}{\partial t} \right) e^{\hat{\sigma}t} + \frac{\hat{\mu}}{2} \left( (\psi_1 e_k(r, t) + \psi_2 S_{\text{ank}}) \left( \psi_1 e_k(r, t) + \psi_2 S_{\text{ank}} \right) e^{\hat{\sigma}t} \right) + \sum_{k=1}^{N} \sum_{j=1}^{N} \frac{1}{\eta_k} (\hat{d}_{kj}(t) - c_{kj} - \hat{\varepsilon}) \frac{\partial \hat{d}_{kj}(t)}{\partial t} \right). \]  (13)

Substituting Eq. (6) into Eq. (13) yields

\[ \frac{\partial V(r, t)}{\partial t} = \sum_{k=1}^{N} (S_k(r, t))^T (\psi_1 I + \psi_2 E_k) \left( g(y_k(r, t)) - f(x_k(r, t)) + \rho_k \sum_{j=1}^{N} c_{kj} e_j(r, t) + \rho_k \sum_{j=1}^{N} (\hat{d}_{kj}(t) - c_{kj}) y_j(r, t) + u_k(r, t) \right) e^{\hat{\sigma}t} + \frac{\hat{\mu}}{2} \sum_{k=1}^{N} (\psi_1 e_k(r, t) + \psi_2 S_{\text{ank}})^T (\psi_1 e_k(r, t) + \psi_2 S_{\text{ank}}) e^{\hat{\sigma}t} + \sum_{k=1}^{N} \sum_{j=1}^{N} \frac{1}{\eta_k} (\hat{d}_{kj}(t) - c_{kj} - \hat{\varepsilon}) \frac{\partial \hat{d}_{kj}(t)}{\partial t}. \]  (14)

The further evolvement is done by introducing Eq. (10) as follows:

\[ \frac{\partial V(r, t)}{\partial t} = \sum_{k=1}^{N} S_k(r, t)^T (\psi_1 I + \psi_2 E_k) \left( g(y_k(r, t)) - f(x_k(r, t)) + \rho_k \sum_{j=1}^{N} c_{kj} e_j(r, t) + \rho_k \sum_{j=1}^{N} (\hat{d}_{kj}(t) - c_{kj}) y_j(r, t) - g(y_k(r, t)) \right) e^{\hat{\sigma}t} + \frac{\hat{\mu}}{2} \sum_{k=1}^{N} (S_k(r, t)^T S_k(r, t) e^{\hat{\sigma}t}) + \sum_{k=1}^{N} \sum_{j=1}^{N} \frac{1}{\eta_k} (\hat{d}_{kj}(t) - c_{kj} - \hat{\varepsilon}) \frac{\partial \hat{d}_{kj}(t)}{\partial t}. \]
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the node equations to build a drive network as follows:

$$\text{4 Numerical simulation and discussion}$$

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for traffic flow, atmosphere, heat transfer, and turbulence. This model can be expressed as

partial differential equation, the Burgers equation is one of the key research focuses in studies

build the response network due to its rich spatiotemporal dynamical behaviors. As a nonlinear

it is obtained that

$$\partial V(r, t) \over \partial t = \left( -S_k(r, t)^T S_k(r, t) e^{\hat{\mu} t} + \frac{\hat{\mu}}{2} S_k(r, t)^T S_k(r, t) \hat{\xi} t \right)$$

$$= -\sum_{k=1}^{N} \left( 1 - \frac{\hat{\mu}}{2} \right) S_k(r, t)^T S_k(r, t) e^{\hat{\mu} t}. \quad (16)$$

Ultimately, we come to conclude that, for any initial values, when \(1 - \hat{\mu}/2 \geq 0\) in Eq. (16),

it is obtained that \(\partial V(r, t) \over \partial t \leq 0\). In accordance with the Lyapunov stability principle, the

calculated deviation, which tends to be stable at zero, indicates the synchronization between

two spatiotemporal networks under Eq. (10). Thus, Theorem 1 is proven well.

4 Numerical simulation and discussion

Two numerical examples are given to illustrate the outer synchronization criteria obtained in

the preceding sections. It will show how to use the sliding mode controller (11) for synchronizing

two spatiotemporal networks (1) and (2) with different initial values according to Theorem 1.

As we all know, the Fisher-Kolmogorov system is well-established in life science. This spa-

tiotemporal chaotic system is intuitive for understanding the relationship between population

interactions and spread behaviors\[40\]. Therefore, the Fisher-Kolmogorov system is selected for

the node equations to build a drive network as follows:

$$\frac{\partial x(r, t)}{\partial t} = \varphi x(r, t) \left( 1 - \frac{x(r, t)}{\alpha} \right) + \xi \nabla^2 x(r, t), \quad (17)$$

where \(\varphi \) and \(\alpha \) are parameters, and \(\xi \) denotes the diffusion coefficient.

Meanwhile, the one-dimensional (1D) Burgers equation is selected as the node equations to

build the response network due to its rich spatiotemporal dynamical behaviors. As a nonlinear

partial differential equation, the Burgers equation is one of the key research focuses in studies

for traffic flow, atmosphere, heat transfer, and turbulence. This model can be expressed as

follows[41]:

$$\frac{\partial y(r, t)}{\partial t} = -\beta \frac{\partial y(r, t)}{\partial r} + \nabla^2 y(r, t), \quad (18)$$

where \(\beta \) is a system parameter.

The \(N\)-Fisher-Kolmogorov system (17) and 1D Burgers equation (18) are selected as the

nodes to construct the drive and response network, and the state equations of the \(k\)th node in

the network are described as follows:

$$\frac{\partial x_k(r, t)}{\partial t} = \varphi x_k(r, t) \left( 1 - \frac{x_k(r, t)}{\alpha} \right) + \xi \nabla^2 x_k(r, t) + \rho_k \sum_{j=1}^{N} c_{kj} x_j(r, t), \quad (19)$$

$$\frac{\partial y_k(r, t)}{\partial t} = -\beta \frac{\partial y_k(r, t)}{\partial r} + \nabla^2 y_k(r, t) + \rho_k \sum_{j=1}^{N} \hat{d}_{kj}(t) y_j(r, t) + u_k(r, t). \quad (20)$$
The drive and response network connection by the small-world spatiotemporal networks is considered, and the connection between the network nodes is rewired with the probability $p$. The sliding mode control input in the uncertain network node is confirmed as follows:

$$u_k(r, t) = -(\psi_1 I + \psi_2 E_k)^{-1} S_k(r, t) - \rho_k \sum_{j=1}^{N} (c_{kj} e_j(r, t) + \tilde{\varepsilon} y_j(r, t))$$

$$+ \beta \frac{\partial y_k(r, t)}{\partial r} - \nabla^2 y_k(r, t) + \varphi x(r, t) \left(1 - \frac{x(r, t)}{\alpha}\right) + \xi \nabla^2 x(r, t). \tag{21}$$

Based on Theorem 1, the network (19) and (20) under the controller (10) is output synchronized. To validate the theoretical analysis in Section 3, the space dimensions of the node systems (17) and (18) are both divided to 100 grid points. It is evident from Figs. 1 and 2 that the dynamics of the nodes in the drive network system is quite different from that in the response network. Moreover, assume $\varphi = 0.5$, $\alpha = 1$, $\xi = 5$, $\beta = 4$, $\tilde{\mu} = 0.6$, $\tilde{\eta}_k = 0.4$, $\psi_1 = 2$, $\psi_2 = 0.2$, $\zeta = 0.001$, $\gamma_1 = 3$, and $\gamma_2 = 5$.

**Fig. 1** Evolution of the dynamical variable of the Fisher-Kolmogorov system (color online)

**Fig. 2** Evolution of the dynamical variable of the 1D Burgers equation (color online)

We build the WS small-world network system with the node number $N$ of 20. Based on the sliding mode surface designed in Eq. (7), the controller of the spatiotemporal network and the unknown parameter recognition laws are proposed as Eqs. (10) and (11), respectively. In this situation, the tracking information, regarding the two spatiotemporal networks, is derived, as shown in Figs. 3–12. Note that only the simulation results at the representative nodes, i.e., 1, 5, 10, and 20, are exhibited to analyze the synchronous features, since listing all 20-nodes information seems to be a tedious and time-assuming process.

**Example 1** In this example, the rewiring probability is $p = 0.1$, the spatiotemporal network size is $N = 20$, and $K = 2$. The parameter $\hat{d}_{kj}(t)$ denotes the topological structure of the response networks (2), related to the uncertain factors in the practical applications, which is derived by means of Eq. (11).

As shown in Figs. 3 and 4, the values of the corresponding errors and the sliding modes fluctuate only at the first couple of tenths of a second, because the node equation and the topological characteristics of the drive response network are quite different. In this case, the coupling matrix of the response network is uncertain and random due to the complexity of the WS small-world spatiotemporal network. But for a longer time, the corresponding errors and the sliding modes in both space and time are close to zero after an extremely short-time evolution, which means that the present sliding mode control approach is effective and robust in the outer synchronization of spatiotemporal networks.

Figure 5 reveals that the uncertain parameters of networks are identified. When $t > 0.2 s$, $\hat{d}_{kj}(t)$ keeps constant. Combined with Fig. 4, it is found that with a relatively steady value of parameters, the small-world spatiotemporal networks implement outer synchronization. It is also
Fig. 3 Evolution of the spatiotemporal errors of nodes 1, 5, 10, and 20 at $K = 2$ and $p = 0.1$ (color online)
proven that the topological characteristics and parameters play critical roles in the processing of outer synchronization. Particularly, the total cumulative error of the small-world spatiotemporal network can be designed as \( e = \frac{1}{N} \sum_{k=1}^{N} \sqrt{e_k(r, t)^2} \). From Fig. 5, it is clear that the drive and response networks can achieve a zero cumulative error rapidly. Such result further confirms that the proposed synchronization method is efficient and reliable to manage small-world spatiotemporal networks with a variety of node equations and connection modes.

Example 2 The rewiring probability is set up to \( p = 0.2 \), the spatiotemporal network size is \( N = 20 \), and \( K = 2 \). The corresponding results are exhibited in Figs. 6–10.
Fig. 6 Evolution of the spatiotemporal errors of nodes 1, 5, 10, and 20 when $p = 0.2$ (color online)
Fig. 7  Evolution process of the total error when $p = 0.1$ (color online)

Fig. 8  Evolution of the sliding mode when $p = 0.2$ (color online)

Fig. 9  Recognition process of the coupling matrix $d_{ij}$ (color online)
Fig. 10 Evolution process of the total error when \( p = 0.2 \) (color online)

Obviously, the error of every single observed node and the total network error with time and space can quickly reach zero during the calculation processes. It only experiences a very short time period to identify the uncertain parameters. All these observations demonstrate that the designed control method is effective to fulfill the synchronization of spatiotemporal networks. Moreover, all observations reveal that the synchronization managed by the sliding mode controller (11) could be implemented effectively. It also confirms the significant importance of utilizing the topological characteristics and parameters during the process of outer synchronization. Obviously, our established complex network model with spatiotemporal chaotic behaviors is more realistic and universal for practical applications, since most reported works only focus on time-dependent synchronization.

5 Conclusions

In this work, a sliding mode control scheme is developed to achieve high-efficiency synchronization among WS small-world spatiotemporal networks with part uncertain parameters. The spatiotemporal chaotic system, which is highly sensitive to initialization and boundary conditions, has been widely applied in many fields. We start off with the dynamics behavior and system characteristics of the spatiotemporal chaotic system to build complex network models. Within network synchronization design environment, the expensive computations and time-consuming solutions of the high-dimensional differential dynamical system are the keys to network synchronization. In response to the outer synchronization research of spatiotemporal networks, a method for synchrony-and-controlling using high-order sliding mode observers is proposed. The nonlinear controller is found to be a powerful method for these small-world spatiotemporal networks. The simulation results indicate that the SW small-world spatiotemporal networks, even with different connection probabilities, reach synchronization in an extremely short time period, and all unknown parameters in the network are identified quickly. These findings demonstrate the obvious synchronization improvement by the sliding mode control pattern of the WS small-world spatiotemporal networks.

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References

[1] PECORA, L. M. and CARROLL, T. L. Synchronization in chaotic systems. Physical Review Letters, 64, 821–824 (1990)
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[2] ALBERT, R. and BARABASI, A. L. Statistical mechanics of complex networks. Reviews of Modern Physics, 74, 47–97 (2001)

[3] BOCCALETTI, S., LATORA, V., MORENO, Y., CHAVEZ, M., and HWANG, D. U. Complex networks: structure and dynamics. Physics Reports, 424, 175–308 (2006)

[4] TANG, L. J., LI, D., and WANG, H. X. Lag synchronization for fuzzy chaotic system based on fuzzy observer. Applied Mathematics and Mechanics (English Edition), 30(6), 803–810 (2009) https://doi.org/10.1007/s10483-009-0615-y

[5] MAO, X. C. and WANG, Z. H. Stability, bifurcation, and synchronization of delay-coupled ring neural networks. Nonlinear Dynamics, 84, 1063–1078 (2016)

[6] REN, H. R., XIONG, J. L., LU, R. Q., and WU, Y. Q. Synchronization analysis of network systems applying sampled-data controller with time-delay via the Bessel-Legendre inequality. Neurocomputing, 331, 346–355 (2019)

[7] LI, C. P., XU, C. X., SUN, W. G., XU, J., and KURTHS, J. Outer synchronization of coupled discrete-time networks. Chaos: An Interdisciplinary Journal of Nonlinear Science, 19, 013106 (2009)

[8] LI, C. P., SUN, W. G., and KURTHS, J. Outer synchronization of coupled complex networks. Physical Review E, 76, 046204 (2007)

[9] LIU, S. and WANG, Q. Y. Outer synchronization of small-world networks by a second-order sliding mode controller. Nonlinear Dynamics, 89, 1817–1826 (2017)

[10] ZHANG, H. H. and XIAO, P. C. Seizure dynamics of coupled oscillators with Epileptor field model. International Journal of Bifurcation and Chaos, 28, 1850041 (2018)

[11] ZHANG, C., WANG, X. Y., LUO, C., LI, J. Q., and WANG, C. P. Robust outer synchronization between two nonlinear complex networks with parametric disturbances and mixed time-varying delays. Physica A: Statistical Mechanics and Its Applications, 494, 251–264 (2018)

[12] LI, C. P., XU, C. X., SUN, W. G., and KURTHS, J. Outer synchronization of coupled discrete-time networks. Chaos: An Interdisciplinary Journal of Nonlinear Science, 19, 013106 (2009)

[13] ZHOU, G. Y., LI, C. R., LI, T. T., WANG, C., HE, F. J., and SUN, J. C. Outer synchronization investigation between WS and NW small-world networks with different node numbers. Physica A: Statistical Mechanics and Its Applications, 457, 506–513 (2016)

[14] ARELLANO-DELGADO, A., LOPEZ-GUTIERREZ, R. M., MARTINEZ-CLARK, R., and CURZ-HERNANDEZ, C. Small-world outer synchronization of small-world chaotic networks. Journal of Computational and Nonlinear Dynamics, 13, 101008 (2018)

[15] CHEN, C., XIE, K., LEWIS, F. L., XIE, S., and FIERRO, R. Adaptive synchronization of multi-agent systems with resilience to communication link faults. Automatica, 111, 108636 (2020)

[16] DU, L., YANG, Y., and LEI, Y. M. Synchronization in a fractional-order dynamic network with uncertain parameters using an adaptive control strategy. Applied Mathematics and Mechanics (English Edition), 39(3), 353–364 (2018) https://doi.org/10.1007/s10483-018-2304-9
Pinning synchronization of complex switching networks with a leader of nonzero control inputs. IEEE Transactions on Circuits and Systems I: Regular Papers, **66**, 3100–3112 (2019)

Quasi-pinning synchronization and stabilization of fractional order BAM neural networks with delays and discontinuous neuron activations. Chaos, Solitons and Fractals, **131**, 109491 (2020)

Finite-time asynchronous sliding mode control for Markovian jump systems. Automatica, **109**, 108503 (2019)

Event-triggering dissipative control of switched stochastic systems via sliding mode. Automatica, **103**, 261–273 (2019)

Synchronization of networked multibody systems using fundamental equation of mechanics. Applied Mathematics and Mechanics (English Edition), **37**(5), 555–572 (2016) https://doi.org/10.1007/s10483-016-2071-8

Control chaos in transition system using sampled-data feedback. Applied Mathematics and Mechanics (English Edition), **24**(11), 1309–1315 (2003) https://doi.org/10.1007/BF02439654

Terminal sliding mode control for coordinated motion of a space rigid manipulator with external disturbance. Applied Mathematics and Mechanics (English Edition), **29**(5), 583–590 (2008) https://doi.org/10.1007/s10483-008-0503-1

Finite-time synchronization for chaotic gyros systems with terminal sliding mode control. IEEE Transactions on Systems, Man, and Cybernetics: Systems, **49**, 1131–1140 (2017)

Synchronization and tracking of multi-spacecraft formation attitude control using adaptive sliding mode. Asian Journal of Control, **21**, 832–846 (2019)

Variable structure systems with sliding modes. IEEE Transactions on Automatic Control, **22**, 212–222 (1977)

Second-order sliding mode controller design with output constraint. Automatica, **112**, 108704 (2020)

Discrete-time fast terminal sliding mode control for permanent magnet linear motor. IEEE Transactions on Industrial Electronics, **65**, 9916–9927 (2018)

Fast terminal sliding-mode control design for nonlinear dynamical systems. IEEE Transactions on Circuits and Systems I: Regular Papers, **49**, 261–264 (2002)

Brief non-singular terminal sliding mode control of rigid manipulators. Automatica, **38**, 2159–2167 (2002)

Fixed-time sliding mode controller design for synchronization of complex dynamical networks. Nonlinear Dynamics, **88**, 2637–2649 (2017)

Finite-time lag synchronization of uncertain complex dynamical networks with disturbances via sliding mode control. IEEE Access, **7**, 7082–7092 (2019)

Research on outer synchronization between uncertain time-varying networks with different node number. Physica A-Statistical Mechanics and Its Applications, **492**, 2301–2309 (2018)

Nonlinear waves in reaction-diffusion systems: the effect of transport memory. Physical Review E, **61**, 4177 (2000)

The Nonlinear Diffusion Equation: Asymptotic Solutions and Statistical Problems, Springer Science and Business Media, Boston (1977)