Bounding the electromagnetic and weak dipole moments of the tau-lepton in a simplest little Higgs model

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Abstract

From the total cross section for the reaction $e^+e^- \rightarrow \tau^+\tau^-\gamma$ at the $Z_1$ pole and in the framework of a simplest little Higgs model (SLHM), we get a limit on the characteristic energy scale of the model $f$, $f \geq 5.4$ TeV, which in turn induces bounds on the electromagnetic and weak dipole moments of the tau-lepton. Our bounds on the electromagnetic moments are consistent with the bounds obtained by the L3 and OPAL collaborations for the reaction $e^+e^- \rightarrow \tau^+\tau^-\gamma$. We also obtained bounds on the tau weak dipole moments which are consistent with the bounds obtained recently by the DELPHI and ALEPH collaborations from the reaction $e^+e^- \rightarrow \tau^+\tau^-$. 

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I. INTRODUCTION

The production of tau-lepton pairs in high energy $e^+e^-$ collisions has been used to set bounds on its electromagnetic and weak dipole moments \cite{1,2,3,4}. In the Standard Model (SM) \cite{5,6,7}, the $\tau$ anomalous magnetic moment (MM) $a_\tau = (g_\tau - 2)/2$ is predicted to be $(a_\tau)_{SM} = 0.0011773(3)$ \cite{8,9} and the respective electric dipole moment (EDM) $d_\tau$ is generated by the GIM mechanism only at very high order in the coupling constant \cite{10}. Similarly, the weak MM and EDM are induced in the SM at the loop level giving $a_W^\tau = -(2.10 + 0.61i) \times 10^{-6}$ \cite{11,12} and $d_W^\tau \leq 8 \times 10^{-34} \text{ecm}$ \cite{13,14}. Since the current bounds on these dipole moments \cite{1,2,3,4} are well above the SM predictions, it has been pointed out that these quantities are excellent candidates to look for physics beyond the SM \cite{11,12,13,14,15,16,17,18,19,20,21,22,23}. The couplings of the photon and $Z$ gauge boson to charged leptons may be parameterized in the following form:

$$
\Gamma_V^\alpha = eF_1(q^2)\gamma^\alpha + \frac{ie}{2m_l}F_2(q^2)\sigma^{\alpha\mu}q_\mu + eF_3(q^2)\gamma_5\sigma^{\alpha\mu}q_\mu, \quad (1)
$$

where $V = \gamma, Z$, $m_l$ is the lepton mass and $q = p' - p$ is the momentum transfer. The $q^2$-dependent form-factors $F_i(q^2)$ have familiar interpretations for $q^2 = 0$: $F_1(0) \equiv Q_l$ is the electric charge; $F_2(0) \equiv a_l$; and $F_3 \equiv d_l/Q_l$. The weak dipole moments are defined in a similar way: $F_2^Z(q^2 = m_Z^2) = a_W^\tau$ and $F_3^Z(q^2 = m_Z^2) = d_W^\tau/e$. The measurement of $a_W^\tau$ and $d_W^\tau$ has been done in the $Z_1 \rightarrow \tau^+\tau^-$ decay mode at LEP. The latest bounds obtained for the electromagnetic and weak dipole moments from the DELPHI and ALEPH collaborations at the 95% C.L. are: $-0.052 < a_\tau < 0.013$, $-0.22 < d_\tau(10^{-16} \text{ecm}) < 0.45$ and $a_W^\tau < 1.1 \times 10^{-3}$, $d_W^\tau < 0.50 \times 10^{-17} \text{ecm}$ \cite{3,4}.

The first limits on the MM and EDM of the $\tau$ lepton were obtained by Grifols and M´endez using L3 data \cite{19}: $a_\tau \leq 0.11$ and $d_\tau \leq 6 \times 10^{-16} \text{ecm}$. Escribano and Massó \cite{18} later used electroweak precision measurements to get $d_\tau \leq 1.1 \times 10^{-17} \text{ecm}$ and $-0.004 \leq a_\tau \leq 0.006$ at the 2$\sigma$ confidence level. There is extensive theoretical work done in models beyond the SM that contribute to EDM of charged leptons. In Ref. \cite{24}, the EDM of charged leptons are studied assuming that they have Gaussian profiles in extra dimensions. In \cite{25}, the lepton EDM has been analyzed in the framework of the seesaw model. The electric dipole moments of the leptons in the version III of the 2HDM are considered in \cite{26}. The work \cite{27} was related to the lepton EDM in the framework of the SM with the inclusion of non-
commutative geometry. Furthermore, the effects of non-universal extra dimensions on the EDM of fermions in the two Higgs doublet model have been estimated in Ref. [28]. In [29, 30], limits on the electromagnetic and weak dipole moments of the tau-lepton in the framework of a left-right symmetric model (LRSM) and a class of $E_6$ inspired models with an additional neutral vector boson $Z_θ$ have been analyzed.

Theoretically, numerous new physics models are proposed with different roles of Higgs in the models. The little Higgs model (LHM) [31, 32] has been proposed for solving the little hierarchy problem. In this scenario, the Higgs boson is regarded as a pseudo Nambu-Goldstone boson associated with a global symmetry at some higher scale. Though the symmetry is not exact, its breaking is specially arranged to cancel quadratically divergent corrections to the Higgs mass term at 1-loop level. This is called the little Higgs mechanism. As a result, the scale of new physics can be as high as 10 $TeV$ without a fine-tuning on the Higgs mass term. In these models, relatively light Higgs boson mass is due to its identity as a pseudo Goldstone boson of some enlarged global symmetries. Among various little Higgs models, the simplest little Higgs model (SLHM) [33, 34, 35] is attractive due to its relatively simple theory structure. Detailed discussions on SLHM can be found in the literature [33, 34, 35].

On the $Z_1$ peak, where a large number of $Z_1$ events are collected at $e^+e^-$ colliders, one may hope to constrain or eventually measure the electromagnetic and weak dipole moments of the $τ$ by selecting $τ^+τ^−$ events accompanied by a hard photon. The Feynman diagrams which give the most important contribution to the cross section from $e^+e^− → τ^+τ^−γ$ are shown in Fig. 1. The total cross section of $e^+e^− → τ^+τ^−γ$ will be evaluated at the $Z_1$-pole in the framework of a simplest little Higgs model. The numerical computation for the anomalous magnetic and the electric dipole moments of the tau is done using the data collected by the L3 and OPAL collaborations at LEP [36, 37]. We are interested in studying the effects induced by the effective couplings associated to the weak and electromagnetic moments of the tau lepton given in Eq. (1). For this purpose, we will take the respective anomalous vertices $ττγ$ and $ττZ_1$, one at the time, in diagrams (1) and (2) of Fig. 1. The numerical computation for the respective transition amplitudes will be done using the data collected by these collaborations.

Our aim in this paper is to analyze the reaction $e^+e^− → τ^+τ^−γ$ in the $Z_1$ boson resonance. The analysis is carried out in the context of a simplest little Higgs model [33, 34, 35] and we
attribute electromagnetic and weak dipole moments to the tau lepton. Processes measured in the resonance serve to set limits on the tau electromagnetic and weak dipole moments. First, using as an input the results obtained by the L3 and OPAL collaborations \[36, 37\] for the tau MM and EDM in the process \(e^+e^- \rightarrow \tau^+\tau^-\gamma\), we will set a limit on the SLHM energy scale \(f\) which is similar to that obtained through oblique corrections \[38\], as well as that obtained recently from the \(Z_1\) leptonic decay \[35\]. We then use this limit on \(f\) to get bounds on the weak dipole moments of the tau from the same L3/OPAL data. We have found that these limits are consistent with the new bounds obtained by the DELPHI and ALEPH collaborations from the process \(e^+e^- \rightarrow \tau^+\tau^-\) \[3, 4, 39\].

This paper is organized as follows: In Sect. II we present the calculation of the cross section for the process \(e^+e^- \rightarrow \tau^+\tau^-\gamma\) in a simplest little Higgs model. In Sect. III we present our results for the numerical computations and, finally, we present our conclusions in Sect. IV.

II. THE TOTAL CROSS SECTION

In this section we calculate the total cross section for the reaction \(e^+e^- \rightarrow \tau^+\tau^-\gamma\) using the neutral current lagrangian given in Eq. (22) of Ref. \[35\] for the SLHM for diagrams 1 and 2 of Fig. 1. The respective transition amplitudes are thus given by

\[
\mathcal{M}_1 = \frac{-g^2}{4\cos^2\theta_W(l^2 - m_l^2)}[(\bar{u}(p_3)\Gamma^\alpha(l + m_\tau)\gamma^\beta(g_\nu^\tau - g_A^\gamma_5)v(p_4)] \\
\quad \cdot \frac{(g_{\alpha\beta} - p_\alpha p_\beta/M_{Z_1}^2)}{[(p_1 + p_2)^2 - M_{Z_1}^2 - i\Gamma_{Z_1}^2]}[(\bar{u}(p_2)\gamma^\alpha(g_\nu^e - g_A^\gamma_5)v(p_1)]\epsilon_\lambda^\alpha, \tag{2}
\]

\[
\mathcal{M}_2 = \frac{-g^2}{4\cos^2\theta_W(k^2 - m_\tau^2)}[(\bar{u}(p_3)\gamma^\beta(g_\nu^\tau - g_A^\gamma_5)(k + m_\tau)\Gamma^\alpha v(p_4)] \\
\quad \cdot \frac{(g_{\alpha\beta} - p_\alpha p_\beta/M_{Z_1}^2)}{[(p_1 + p_2)^2 - M_{Z_1}^2 - i\Gamma_{Z_1}^2]}[(\bar{u}(p_2)\gamma^\alpha(g_\nu^e - g_A^\gamma_5)v(p_1)]\epsilon_\lambda^\alpha, \tag{3}
\]

where \(\Gamma^\alpha\) is the tau-lepton electromagnetic vertex which is defined in Eq. (1), while \(\epsilon_\lambda^\alpha\) is the polarization vector of the photon. \(l\) \((k)\) stands for the momentum of the virtual tau \((\text{antitau})\), and the coupling constants \(g_\nu^\tau\) and \(g_A^\gamma\) with \(l = e, \mu, \tau\) are given in Eq. (23) of Ref. \[35\].
The MM, EDM and the characteristic energy scale of the simplest little Higgs model give a contribution to the differential cross section for the process $e^+e^- \rightarrow \tau^+\tau^-\gamma$ of the form:

$$\sigma(e^+e^- \rightarrow \tau^+\tau^-\gamma) = \int \frac{\alpha^2}{48\pi} \frac{e^2a_\tau^2 + d_\tau^2}{4m_\tau^2} \frac{1}{4m_\tau^2} \frac{1}{x_W^2(1-x_W)^2} \left[ \frac{1 - 4x_W + 8x_W^2}{x_W^2(1-x_W)^2} \right] \frac{(1 - 4x_W + x_W^2)(s - 2\sqrt{s}E_\gamma) + \frac{1}{2}E_\gamma^2 \sin^2 \theta_\gamma}{(s - M_{Z_1}^2)^2 + M_{Z_1}^2 \Gamma_{Z_1}^2} \right] \left[ 1 + \frac{1}{8} \frac{(3 - 4x_W)}{(1 - x_W)^2} \left( \frac{v^2 f^2}{f^2} \right) \right] \frac{1}{E_\gamma} dE_\gamma d\cos \theta_\gamma, \quad (4)$$

where $x_W \equiv \sin^2 \theta_W$, $v$ is the vacuum expectation value and $E_\gamma$, $\cos \theta_\gamma$ are the energy and the opening angle of the emitted photon.

It is useful to consider the smallness of the factor $(\frac{v^2 f^2}{f^2})$, to approximate the cross section in Eq. (4) by its expansion in powers of $(\frac{v^2 f^2}{f^2})$ to the linear term: $\sigma = (\frac{e^2a_\tau^2 + d_\tau^2}{4m_\tau^2})[A + B(\frac{v^2 f^2}{f^2}) + O((\frac{v^2 f^2}{f^2})^2)]$, where $A$ and $B$ are constants which can be evaluated. Such an approximation for deriving the bounds of $a_\tau$ and $d_\tau$ is more illustrative and easier to manipulate.

For $(\frac{v^2 f^2}{f^2}) < 1$, the total cross section for the process $e^+e^- \rightarrow \tau^+\tau^-\gamma$ is given by

$$\sigma(e^+e^- \rightarrow \tau^+\tau^-\gamma) = (\frac{e^2a_\tau^2}{4m_\tau^2} + d_\tau^2)[A + B(\frac{v^2 f^2}{f^2}) + O((\frac{v^2 f^2}{f^2})^2)], \quad (5)$$

where $A$ explicitly is

$$A = \int \frac{\alpha^2}{48\pi} \frac{1 - 4x_W + 8x_W^2}{x_W^2(1-x_W)^2} \left[ \frac{1 - 4x_W + x_W^2}{x_W^2(1-x_W)^2} \right] \frac{(1 - 4x_W + x_W^2)(s - 2\sqrt{s}E_\gamma) + \frac{1}{2}E_\gamma^2 \sin^2 \theta_\gamma}{(s - M_{Z_1}^2)^2 + M_{Z_1}^2 \Gamma_{Z_1}^2} \right] E_\gamma dE_\gamma d\cos \theta_\gamma, \quad (6)$$

while $B$ is given by

$$B = \int \frac{\alpha^2}{96\pi} \frac{1 - 4x_W + 8x_W^2}{x_W^2(1-x_W)^2} \left[ \frac{1 - 4x_W + x_W^2}{x_W^2(1-x_W)^2} \right] \frac{(1 - 4x_W + x_W^2)(s - 2\sqrt{s}E_\gamma) + \frac{1}{2}E_\gamma^2 \sin^2 \theta_\gamma}{(s - M_{Z_1}^2)^2 + M_{Z_1}^2 \Gamma_{Z_1}^2} \right] \left[ (3 - 4x_W) \frac{v^2 f^2}{(1 - x_W)^2} \right] E_\gamma dE_\gamma d\cos \theta_\gamma. \quad (7)$$
The expression given for $A$ corresponds to the cross section previously reported by Grifols and Méndez for the SM [19], while $B$ comes from the contribution of the SLHM. Evaluating the limit when the characteristic energy scale $f \to \infty$, the second term in (5) is zero and Eq. (5) is reduced to the expression (4) given in Ref. [19].

In the case of the weak dipole moments, to get the expression for the differential cross section, we have to substitute the $Z_1$ SLHM couplings given in Eq. (23) of Ref. [35] with the respective weak dipole moments included in Eq. (1), that is to say $a^W_\tau = F^Z_2(q^2 = m^2_Z)$ and $d^W_\tau = eF^Z_3(q^2 = m^2_Z)$. We do not reproduce the analytical expressions here because they are rather similar to the term given in Eq. (4). In the following section we will present the bounds obtained for the tau dipole moments using the data published by the L3 and OPAL collaborations for the reaction $e^+e^- \to \tau^+\tau^-\gamma$ [36, 37].

III. RESULTS

In practice, detector geometry imposes a cut on the photon polar angle with respect to the electron direction, and further cuts must be applied on the photon energy and minimum opening angle between the photon and tau in order to suppress background from tau decay products. In order to evaluate the integral of the total cross section as a function of the parameters of the SLHM, that is to say, $f$, we require cuts on the photon angle and energy to avoid divergences when the integral is evaluated at the important intervals of each experiment. We integrate over $\cos \theta_\gamma$ from $-0.74$ to $0.74$ and $E_\gamma$ from $5$ GeV to $45.5$ GeV for various fixed values of the characteristic energy scale $f = (1.7, 5.2, 5.4, 5.6, 7, 10) \ TeV$ (as illustrated in Fig. 2) according to Refs. [35, 38]. Using the numerical values $\sin^2 \theta_W = 0.2314$, $M_{Z_1} = 91.18$ GeV, $\Gamma_{Z_1} = 2.49$ GeV and $m_\tau = 1.776$ GeV, we obtain the cross section $\sigma = \sigma(f, a_\tau, d_\tau)$.

In Fig. 2, we show the dependence of the total cross section for the process $e^+e^- \to \tau^+\tau^-\gamma$ with respect to the SLHM energy scale $f$. Using the data $\sigma = (1.472 \pm 0.006 \pm 0.020) \ \text{nb}$ Refs. [20, 37] for the cross section, where the first error is statistical and the second is systematic, we get the following limit for $f$:

$$f \geq 5.4 \ \text{TeV},$$

which is consistent with that obtained through oblique corrections [38], as well as that
obtained recently from an analysis on the decay width $\Gamma(Z_1 \rightarrow e^+e^-)$ [35].

As was discussed in Ref. [37], $N \approx \sigma(f, a_\tau, d_\tau)\mathcal{L}$, using Poisson statistic [37, 40], we require that $N \approx \sigma(f, a_\tau, d_\tau)\mathcal{L}$ be less than 1559, with $\mathcal{L} = 100 \text{ pb}^{-1}$, according to the data reported by the L3 collaboration Ref. [37] and references therein. Taking this into consideration, we can get a bound for the tau magnetic moment as a function of $f$ with $d_\tau = 0$. The values obtained for this bound for several values of $f$ are included in Table 1. The previous analysis and comments can readily be translated to the EDM of the tau with $a_\tau = 0$. The resulting bounds for the EDM as a function of $f$ are shown in Table 1. As expected, the limits obtained for the electromagnetic dipole moments of the tau lepton are consistent with those obtained by these collaborations from the data for the process $e^+e^- \rightarrow \tau^+\tau^-\gamma$ [36, 37].

| $f(\text{TeV})$ | $a_\tau$ | $d_\tau(10^{-16}\text{ecm})$ |
|-----------------|----------|-----------------------------|
| 1.7             | 0.0490   | 2.800                       |
| 5.2             | 0.0511   | 2.855                       |
| 5.4             | 0.0512   | 2.856                       |
| 5.6             | 0.0513   | 2.857                       |
| 7               | 0.0514   | 2.858                       |
| 10              | 0.0515   | 2.860                       |

Table 1. Bounds on the $a_\tau$ MM and $d_\tau$ EDM of the $\tau$-lepton for different values of the characteristic energy scale of the model $f$. We have applied the cuts used by L3 for the photon angle and energy.

The bounds for the weak dipole moments of the tau-lepton according to the data from the L3 and OPAL collaborations [36, 37] for the energy and the opening angle of the photon, as well as the luminosity and the event numbers, are given in the Table 2. As we can see, the use of the limit obtained for the $f$ characteristic energy scale of the model also induces bounds for the tau weak dipole moments, which are already consistent with those bounds recently obtained by the DELPHI and ALEPH collaborations in the process $e^+e^- \rightarrow \tau^+\tau^-\gamma$ [3, 4]. Our results in Table 2 for $f = (1.7, 5.2, 5.4, 5.6, 7, 10) \text{ TeV}$ [35] differ by about a factor of two of the bounds obtained by the DELPHI and ALEPH collaborations for the weak tau dipole moments [1, 3, 4]; our analysis is not sensitive to the real and imaginary parts of
these parameters separately. In order to improve these limits it might be necessary to study direct CP-violating effects \[41, 42\].

| \(f (\text{TeV})\) | \(a^W_\tau (10^{-3})\) | \(d^W_\tau (10^{-17} \text{em})\) |
|-----------------|-----------------|-----------------|
| 1.7             | 2.020           | 1.121           |
| 5.2             | 2.052           | 1.138           |
| 5.4             | 2.053           | 1.139           |
| 5.6             | 2.054           | 1.140           |
| 7               | 2.055           | 1.141           |
| 10              | 2.056           | 1.142           |

Table 2. Bounds on the \(a^W_\tau\) anomalous weak MM and \(d^W_\tau\) weak EDM of the \(\tau\)-lepton for different values of the characteristic energy scale of the model \(f\). We have applied the cuts used by L3 for the photon angle and energy.

We plot the total cross section in Fig. 3 as a function of the characteristic energy scale of the model \(f\) for the bounds of the magnetic moment given in Table 1. In Fig. 3, for \(f = 10 \text{ TeV}\) we reproduce the data previously reported in the literature. Our results for the dependence of the differential cross section on the photon energy versus the cosine of the opening angle between the photon and the beam direction \((\theta_\gamma)\) are presented in Fig. 4 for \(f = 5.4 \text{ TeV}\) and \(a_\tau = 0.0512\). Besides we plot the differential cross section in Fig. 5 as a function of the photon energy for the bounds of the magnetic moments given in Table 1. We observe in this figure that the energy distributions are consistent with those reported in the literature. Finally, we find that the effects induced by the tree level \(Z_1 e^+ e^-\) and \(Z_1 \tau^+ \tau^-\) couplings in the SLHM increase the cross section of the process \(e^+ e^- \rightarrow \tau^+ \tau^- \gamma\), and the predictions on the electromagnetic and weak dipole moments of the tau lepton are better estimated. However, it is necessary to make an analysis at loop level for the process \(e^+ e^- \rightarrow \tau^+ \tau^- \gamma\) in the context of a little Higgs model.

IV. CONCLUSIONS

We have determined limits on the electromagnetic and weak dipole moments of the tau-lepton using the data published by the L3 and OPAL collaborations for the \(e^+ e^- \rightarrow \tau^+ \tau^- \gamma\) at the \(Z_1\) pole. We were able to get limits on the weak dipole moments by constraining the
SLHM energy scale $f$ from the electromagnetic dipole moments obtained by these collaborations. We then used this limit to determined bounds on the electromagnetic and weak dipole moments of the tau lepton using the data published by the L3 and OPAL collaborations for the process $e^+e^- \rightarrow \tau^+\tau^-\gamma$. In addition, we obtained bounds for the weak dipole moments similar to those obtained recently by the DELPHI/ALEPH collaborations from the process $e^+e^- \rightarrow \tau^+\tau^-$. In particular, from the limit $f \rightarrow \infty$, our bound take the value previously reported in Ref. [19] for the SM. As far as the weak dipole moments are concerned, our limits given in Tables 1 and 2 are consistent with the experimental bounds obtained at LEP with the two-body decay mode $Z_1 \rightarrow \tau^+\tau^-$. In addition, the analytical and numerical results for the total cross section have never been reported in the literature before and could be of relevance for the scientific community.

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[1] W. Lohmann, *Nucl. Phys. Proc. Suppl.* **144**, 122 (2005).
[2] L3 Collab., P. Achard et al., *Phys. Lett.* **B585**, 53 (2004).
[3] DELPHI Collab., J. Abdallah et al., *Eur. Phys. J.* **C35**, 159 (2004), and references therein.
[4] ALEPH Collab., A. Heister et al., *Eur. Phys. J.* **C30**, 291 (2003), and references therein.
[5] S. L. Glashow, *Nucl. Phys.* **22**, 579 (1961).
[6] S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967).
[7] A. Salam, in *Elementary Particle Theory*, Ed. N. Svartholm (Almquist and Wiskell, Stockholm, 1968) 367.
[8] M. A. Samuel, G. Li and R. Mendel, *Phys. Rev. Lett.* **67**, 668 (1991); Erratum *ibid.* **69**, 995 (1992).
[9] F. Hamzeh and N. F. Nasrallah, *Phys. Lett.* **B373**, 211 (1996).
[10] S. M. Barr and W. Marciano in *CP Violation*, ed. C. Jarlskog (World Scientific, Singapore, 1990).
[11] J. Bernabeu et al., *Nucl. Phys.* **B436**, 474 (1995).
[12] J. Bernabeu et al., *Phys. Lett.* **B326**, 168 (1994).

[13] W. Bernreuther et al., *Z. Phys.* **C43**, 117 (1989).

[14] M. J. Booth, [hep-ph/9301293](https://arxiv.org/abs/hep-ph/9301293).

[15] M. C. González-García and S. F. Novaes, *Phys. Lett.* **B389**, 707 (1996).

[16] P. Poulose and S. D. Rindani, [hep-ph/9708332](https://arxiv.org/abs/hep-ph/9708332).

[17] T. Huang, W. Lu and Z. Tao, *Phys. Rev.* **D55**, 1643 (1997).

[18] R. Escribano and E. Massó, *Phys. Lett.* **B395**, 369 (1997).

[19] J. A. Grifols and A. Méndez, *Phys. Lett.* **B255**, 611 (1991); Erratum *ibid.* **B259**, 512 (1991).

[20] L. Taylor, *Nucl. Phys. Proc. Suppl.* **B76**, 237 (1999).

[21] G. A. González-Sprinberg, A. Santamaria, J. Vidal, *Int. Jour. Mod. Phys.* **A16** (Suppl.1B), 545 (2001).

[22] Gabriel A. González-Sprinberg, Arcadi Santamaria, Jorge Vidal, *Nucl. Phys. Proc. Suppl.* **98**, 133 (2001).

[23] Gabriel A. González-Sprinberg, A. Santamaria, J. Vidal, *Nucl. Phys.* **B582**, 3 (2000).

[24] E. O. Iltan, *Eur. Phys. J.* **C44**, 411 (2005).

[25] B. Dutta, R. N. Mohapatra, *Phys. Rev.* **D68**, 113008 (2003).

[26] E. Iltan, *Phys. Rev.* **D64**, 013013 (2001).

[27] E. Iltan, *JHEP* **065**, 0305 (2003).

[28] E. Iltan, *JHEP* **0404**, 018 (2004).

[29] A. Gutiérrez-Rodríguez, M. Hernández-Ruiz and M. A. Pérez, *Int. Jour. Mod. Phys.* **A22**, 3493 (2007).

[30] A. Gutiérrez-Rodríguez, M. A. Hernández-Ruiz and L. N. Luis-Noriega, *Mod. Phys. Lett.* **A19**, 2227 (2004).

[31] N. Arkani-Hamed et al., *Phys. Lett.* **B513**, 232 (2001).

[32] N. Arkani-Hamed et al., *JHEP* **0208**, 021 (2002).

[33] D. E. Kaplan, M. Schmaltz, *JHEP* **0310**, 039 (2003).

[34] M. Schmaltz, *JHEP* **0408**, 056 (2004).

[35] Alex G. Dias, C. A. de Pires and P. S. Rodrigues da Silva, *Phys. Rev.* **D77**, 055001 (2008), and references therein.

[36] OPAL Collab., K. Ackerstaff et al., *Phys. Lett.* **B431**, 188 (1998), and references therein.

[37] L3 Collab., M. Acciarri et al., *Phys. Lett.* **B434**, 169 (1998), and references therein.
[38] G. Marandella, C. Schappacher, A. Strumia, *Phys. Rev.* **D72**, 035014 (2005), and references therein.

[39] L3 Collab., M. Acciarri et al., *Phys. Lett.* **B412**, 201 (1997).

[40] R. M. Barnett et al. *Phys. Rev.* **D54**, 166 (1996).

[41] M. A. Pérez, F. Ramírez-Zavaleta, *Phys. Lett.* **B609**, 68 (2005).

[42] F. Larios, et al., *Phys. Rev.* **D63**, 113014 (2001).
FIG. 1: The Feynman diagrams contributing to the process $e^+e^- \rightarrow \tau^+\tau^-\gamma$ in a simplest little Higgs model.

FIG. 2: The curves show the shape for $\sigma(e^+e^- \rightarrow \tau^+\tau^-\gamma)$ as a function of the energy scale $f$. $a_\tau = -0.052$
FIG. 3: The total cross-section for $e^+e^- \rightarrow \tau^+\tau^-\gamma$ as a function of the characteristic energy scale $f$ and $a_\tau$ (Table 1).

FIG. 4: The differential cross section for $e^+e^- \rightarrow \tau^+\tau^-\gamma$ as a function of $E_\gamma$ and $\cos\theta_\gamma$ for $f = 5.4$ TeV and $a_\tau = 0.0512$. 
FIG. 5: The differential cross section for $e^+e^- \rightarrow \tau^+\tau^-\gamma$ as a function of $E_\gamma$ and $a_\tau$ with $f = 5.4$ TeV.