Performance of eddy-viscosity turbulence models for predicting swirling pipe-flow: Simulations and laser-Doppler velocimetry

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Abstract

We use laser-Doppler velocimetry (LDV) experiments and Reynolds-averaged Navier–Stokes (RANS) simulations to study the characteristic flow patterns downstream of a standardized clockwise swirl disturbance generator. After quantifying the impact of the mesh size, we evaluate the potential of various eddy-viscosity turbulence models in providing reasonable approximations with respect to the experimental reference. The choice of turbulent models reflects current industry practice. Our results suggest that models from the $k$-$\epsilon$ family are more accurate in predicting swirling flows than models from the $k$-$\omega$ family. For sufficiently resolved meshes, the realizable $k$-$\epsilon$ model provides the most accurate approximation of the velocity magnitudes, although it fails to capture small-scale flow structures which are accurately predicted by the standard $k$-$\epsilon$ model and the RNG $k$-$\epsilon$ model. Throughout the article, we highlight practical guidance for the choice of RANS turbulence models for swirling flow.

Keywords: swirl, pipe-flow, laser-Doppler velocimetry, OpenFOAM, RANS modeling

1. Introduction

Swirling flows are ubiquitous in many industrial applications including furnaces, cyclone separators, heat exchangers, and turbines. Yet, efficient numerical prediction with Reynolds-averaged Navier–Stokes (RANS) approximations is challenging and large-eddy simulations (LES) are still prohibitively expensive for practical applications. For strongly rotating flows, various standard assumptions used in RANS turbulence closures are not expected to hold, as discussed, for example, by Jakirić et al. \cite{jakiri2012}. Consequently, the performance of conventional RANS modeling approaches for predicting swirling flow remains elusive and requires further verification and validation with experiments.

Kitoh \cite{kitoh1997} performed experiments with turbulent swirling flow in a straight pipe where the swirl component is generated through guide vanes with variable vane angle and a bell-shaped cone at the center of the swirl generator. Based on the characteristic tangential velocity distribution, Kitoh \cite{kitoh1997} categorized the flow field into three regions: wall, annular, and core. Due to the streamline curvature, skewed shear directions in the annular region, and the resulting anisotropy, Kitoh \cite{kitoh1997} concludes that Reynolds stress models seem more promising than eddy-viscosity models in providing accurate predictions of swirling flows. This conclusion confirms earlier simulation results of Kobayashi and Yoda \cite{kobayashi1997} and is also supported by subsequent numerical studies. For example, simulations of Hirai et al. \cite{hirai2001} show a better performance of Reynolds stress models over the standard $k$-$\epsilon$ model and a modified $k$-$\epsilon$ model for the prediction of the laminarization phenomenon in swirling pipe-flow. Similarly, Čočić et al. \cite{cotic2001} report that two-equation turbulence models including RNG $k$-$\epsilon$, Launder Sharma $k$-$\epsilon$, and $k$-$\omega$ SST fail to predict experimental data, whereas the Launder–Gibson Reynolds stress model and the Speziale–Sarkar–Gatski model are found to provide better agreement with experiments.

In contrast, Chen et al. \cite{chen2003} find that the differential Reynolds stress model proposed by Launder et al. \cite{launder1975} provides quantitatively inaccurate results for predicting tangentially injected swirling pipe flows, reaching only qualitative agreement with ex-
experimental results. Further, several workers successfully applied turbulence models of the $k$-$\epsilon$ family to model swirling flow. For example, Parchen and Steenbergen [8] studied turbulent swirling pipe-flows for gas-metering applications and reported an acceptable behavior of the $k$-$\epsilon$ model, which provided a better approximation of the experimental values compared to an algebraic slip model. Escue and Cui [9] found a superior performance of the RNG $k$-$\epsilon$ model over a Reynolds stress model for moderate swirl intensities in 2D simulations when compared to experimental results. Petit et al. [10] successfully applied a $k$-$\epsilon$ model for unsteady simulations of a swirl generator, achieving good agreement in averaged velocity profiles. In view of the available results, a general consensus regarding the performance of eddy-viscosity closures for swirling flows appears to be lacking.

1.1. Swirling flows in flow metering

In this article, we study the swirling flow generated by a standardized clockwise swirl disturbance generator that is used for testing commercial flow meters (Figure 1 (a)). Commercial water, heat and cooling meters may be exposed to disturbed inflows when they are in operation, since the space available in realistic installations is often limited and prohibits the installation of a straight pipe long enough to create fully developed flow conditions upstream of the meter. Common flow disturbances induced by several standard installations such as bent pipes or valves may compromise the accuracy of meter readings. Consequently, it is necessary to assess the robustness of new products with respect to disturbed flow conditions. The standards EN ISO 4064-2:2014 [11] and OIML R 49-2:2013 [12] for the type-approval of commercial water meters and EN 1434-4:2007 [13] for the type-approval of commercial heat and cooling meters include standardized tests with artificially generated disturbed flows that are meant to emulate disturbed flow conditions in realistic installations. One of these artificially disturbed flows is generated by a standardized clockwise swirl disturbance generator (Figure 1 (a)), which emulates the flow structures induced by a double-bent pipe installation. (For a study of flow structures induced by double-bent pipes see, for example, Mattingly and Yeh [14].)

A detailed analysis of flow patterns generated by such swirl disturbance generators is not part of the standard type-approval procedure, but it is a key factor in enabling a meaningful interpretation of type-approval tests. Further, the detailed analysis of swirling flows has potential to assist the development process of robust meter designs and the associated testing and verification facilities. The swirling flow field generated with equipment according to EN ISO 4064-2:2014 [11] and OIML R 49-2:2013 [12] is expected to exhibit similar features as the flow fields in the studies discussed in Section 1. For example, Eichler and Lederer [15] conducted stereoscopic particle image velocimetry (SPIV) measurements downstream of a standardized DN80 swirl disturbance generator, showing that the swirling flow is maintained up to $87.0D$ downstream. Similarly, Tawackolian [16] provides experimental and numerical results using the $k$-$\omega$ model for a DN80 swirl generator and a comparison of the flow patterns produced by the swirl generator against those after bent pipes. While these studies provide valuable insight for large pipe diameters, no results appear to be available for smaller pipe diameters. Since the standardized parts for different diameters according to EN ISO 4064-2:2014 [11] and OIML R 49-2:2013 [12] are not exactly self-similar, the scalability of available results to smaller diameters remains unclear.

The present work aims to assess the potential of RANS simulations in providing accurate predictions of the flow patterns downstream of a standardized swirl disturbance generator. In view of the discussion in Section 1, available practical guidance for choosing turbulence models is ambiguous. To elucidate modeling choices for swirling pipe-flow, we study different numerical setups and compare them against experimental results obtained from laser-Doppler velocimetry (LDV) experiments. Our choice of turbulence models includes some of the most popular mainstream models currently used in industry. The aim is to identify best practices to achieve realistic and useful simulation results that can be used in practical applications such as flow meter testing and development. Additionally, we discuss our results in the context of available recommendations regarding the modeling of swirling flows. Within the study of different turbulence models, we also provide a detailed assessment of the impact of the mesh size and the length of the computational domain.
2. Materials and methods

We use a standardized clockwise swirl disturbance generator according to EN ISO 4064-2:2014 [11] and OIML R 49-2:2013 [12] with an inner diameter of $D = 15.0\,\text{mm}$. The working fluid is water and both simulations and experiments are realized with Reynolds number $Re = 4.0 \cdot 10^4$, where $Re$ is based on the volumetric velocity $u_{\text{vol}}$ and the pipe diameter $D$, such that

$$Re = \frac{u_{\text{vol}} D}{\nu},$$  \hspace{1cm} (1)

and

$$u_{\text{vol}} = \frac{Q}{A},$$  \hspace{1cm} (2)

with $\nu$ the kinematic viscosity of the water, $Q$ the volumetric flow rate, and $A = \frac{\pi}{4} D^2$ the area of the pipe cross-section.

2.1. Experiments

All experiments are performed on a calibration flow bench with an expanded uncertainty of 0.30% (coverage factor $k = 2.0$) for measurements against weight with test-volumes of 3 liter to 100 liter. The flow bench is equipped with three magneto-inductive (MID) master meters, each one responsible for controlling the flow in a certain flow-rate interval. The experiments are realized with water at temperature $T = 20.0\,\text{°C}$, corresponding to a kinematic viscosity of $\nu = 1004.79 \cdot 10^{-9}\,\text{m}^2/\text{s}$. During measurements, the volume flow-rate $Q$, the water temperature $T$, and the pressure $p$ are stabilized through PID controlled feedback loops. We verify the stability of $Q$, $T$, and $p$ by logging data from the master meters and the corresponding temperature and pressure sensors. Let $Q_M$ denote the time-averaged master flow-rate and let $\sigma_{Q_M}$ denote the associated standard deviation providing a measure for the stability of the flow-rate. We find $\sigma_{Q_M}/Q_M \approx 0.25\%$ and that the master meter signal has the characteristics of random white noise, which confirms that there is no preferred timescale and no low frequency disturbances that might bias the long-time accuracy of LDV measurements.

We perform measurements with a commercial LDV system (ILA/Optolution) in and industrial flow laboratory. The flow is seeded with neutrally buoyant silver coated hollow glass beads to improve the LDV signal. To ensure a fully developed flow profile upstream, the swirl generator is located after more than 100.0$D$ of straight pipe and the measurement section is located 12.0$D$ downstream from the swirl disturbance generator. For the present study, we focus on discussing the flow patterns in the near-field until 12.0$D$ downstream from the swirl disturbance generator, since this is the flow that will enter a flow meter in a type-approval test. The measurement grid comprises 241 measurement points as shown in Figure 1(b).

The amount of data acquired at each measurement point is determined by the choice of two experimental constraints: (I) the maximal number of single-point samples $n_{\text{max}}$ and (II) a timeout $t_{\text{max}}$ for each single-point measurement on the measurement grid. For the present measurements we choose $n_{\text{max}} = 10^3$ and $t_{\text{max}} = 60.0\,\text{s}$.

Axial velocity profiles are obtained through a collection of single-point measurements of the local axial mean velocity $\overline{w}$ over the measurement grid (Figure 1). To determine $\overline{w}$, we use the estimator

$$\overline{w} = \frac{1}{n} \sum_{i=1}^{n} w_i,$$

with $w_i$ single-point samples of velocities and $n$ the number of samples. The associated dimensionless turbulence intensity is

$$Tu = \frac{\sigma_w}{\overline{w}},$$

where

$$\sigma_w = \left( \frac{1}{n-1} \sum_{i=1}^{n} (w_i - \overline{w})^2 \right)^{1/2}.$$
is the standard deviation of samples \( w_i \). To estimate the reliability of the estimator \( \bar{u} \) at each spatial measurement point, we determine the associated standard error

\[
\sigma_{\bar{u}} = \frac{\sigma_w}{\sqrt{n}}.
\]

(7)

The standard error \( \sigma_{\bar{u}} \) provides an uncertainty estimate for the estimator of the mean velocity \( \bar{u} \). To obtain all velocity components, three consecutive LDV measurements are required. Notice that the velocity vector \( \bar{u} = (\bar{u}, \bar{u}, \bar{u})^\top \) is the experimental equivalent of the Reynolds-averaged velocity defined in Section 2.2.

### 2.2. Simulations

The numerical simulations are performed using a Reynolds-averaged Navier–Stokes (RANS) approach. Let a superposed bar denote Reynolds-averaging, so that (see, for example Pope [17])

\[
\bar{u} = \bar{u} + u',
\]

(8)

where \( \bar{u} \) is the RANS velocity and \( u' \) is the fluctuating velocity satisfying \( \bar{u}' = 0 \). Applying Reynolds-averaging to the incompressible Navier–Stokes (NS) equations yields the RANS equations

\[
\bar{u} = -\frac{1}{\rho} \text{grad} \bar{p} + \nu \nabla^2 \bar{u} - \text{div} \tau,
\]

\[
\text{div} \bar{u} = 0,
\]

(9)

where a superposed dot denotes material time-differentiation following \( \bar{u}, \bar{p} \) denotes the averaged pressure, \( \rho \) denotes the fluid density, and

\[
\tau = \overline{u' \otimes u'}
\]

(10)

is the Reynolds stress tensor. The closure problem in RANS modeling amounts to finding expressions for \( \bar{u}, \tau \) without using the fluctuating velocity \( u' \). The gradient-diffusion and the eddy-viscosity hypotheses assume that the deviatoric part of \( \overline{u'} \otimes \overline{u'} \) is proportional to the averaged stretching tensor \( \mathbf{D} = \frac{1}{2} (\text{grad} \bar{u} + (\text{grad} \bar{u})^\top) \) and yield the closure approximation

\[
-\tau + \frac{2}{3} \mathbf{I} \nu_T = \nu_T \mathbf{D},
\]

(11)

where \( \nu_T \) is the eddy-viscosity, \( \mathbf{I} \) is the identity tensor, and

\[
k = \frac{1}{2} |\overline{u'}|^2
\]

(12)

is the turbulent kinetic energy. In view of [9] and [12] the RANS equations with the gradient-diffusion hypothesis and the eddy-viscosity hypothesis are

\[
\dot{\bar{u}} = (\nu + \nu_T) \nabla^2 \bar{u} - \frac{1}{\rho} \text{grad} \bar{p} + \frac{2}{3} \rho \bar{k},
\]

\[
\text{div} \bar{u} = 0.
\]

(13)

Notice that \( k \) can be absorbed into a modified averaged pressure.

In this article, we consider various two-equation eddy-viscosity models (Table 1) that determine \( \nu_T \) through solving two additional model transport equations. The first additional model transport equation is for the turbulent kinetic energy \( k \) and the second additional model transport equation depends on the model family. For models from the \( k-\varepsilon \) family, the second additional transport equation is for the turbulent dissipation

\[
\epsilon = 2 \nu_D \mathbf{D} : \mathbf{D},
\]

(14)

where \( D = \frac{1}{2} (\text{grad} \bar{u} + (\text{grad} \bar{u})^\top) \) is the fluctuation stretching tensor. Similarly, for models from the \( k-\omega \) family, the second additional transport equation is for the turbulent frequency \( \omega = \epsilon/k \). Despite the vast selection of available turbulence models, we focus on popular mainstream models that are commonly used in industrial application. More exotic models specifically developed for swirling applications might well yield improvements in performance with the downside of giving up universality, which can be problematic in industrial applications. For a discussion of model performance, also see Section 5.

#### 2.2.1. Numerical solution

We use the open-source CFD code OpenFOAM [18] and assess five different two-equation eddy-viscosity models, as summarized in Table 1. Both convective and diffusive terms are discretized with a second order Gaussian linear scheme. The near-wall region is modeled with enhanced wall functions available in the standard OpenFOAM distribution. A SIMPLE algorithm is used for the coupling between the pressure and the velocity equations. The pressure equation is solved with a geometric agglomerated algebraic multigrid (GAMG) method, and all other equations are solved with the smoothSolver solver of OpenFOAM using a Gauss–Seidel smoother.

The computational domain has a total length of 400.0 mm with the swirl disturbance generator.
placed at the center. Consequently, the distance from both the inlet and outlet to the swirl generator is approximately 13.0D. We use hex-dominant unstructured meshes generated with the OpenFOAM meshing tool *snappyHexMesh*. All meshes are generated with the mesh quality constraints summarized in Table 3.3 in [Appendix B]

### 3. Results

#### 3.1. Impact of the mesh size and domain length

To identify an optimal trade-off between simulation time and accuracy of results, we assess the impact of the mesh size and the impact of the length of the computational domain on the solution of different turbulence models. To that end, we run all turbulence models with five different mesh sizes. To quantify the effect of the domain length, we first perform simulations with a 20.0D inlet and outlet corresponding to a total domain length of 800.0 mm. In Figure 2, we compare results of both inlet lengths for three selected turbulence models. Panel (a) of Figure 2 shows the axial component $\mathbf{u}$ of the velocity 12.0D downstream from the swirl disturbance generator. Further we visualize the magnitude of the secondary flow

$$
\mathbf{v}_{xy} = \sqrt{v^2 + w^2},
$$

as shown in panel (b) of Figure 2. Visual comparison of both results shows that the characteristic flow patterns are maintained across different domain lengths although there are small-scale differences in the velocity field. To assess these differences in more detail, we compare velocity profiles over $r/R$ of each case upstream and downstream from the swirl generator. We compute averages of 10 individual linear profiles with equidistant angular spacing of 18° in analogy with the experimental measurement grid shown in Figure 1(b). The comparison of the averaged axial profiles upstream from the swirl generator shows that both RNG $k$-$\epsilon$ and $k$-$\omega$ SST models provide flow profiles closer to the fully developed reference with the longer inlet, while the realizable $k$-$\epsilon$ model appears to provide similar results for both inlet lengths (Figure 2(c)). In the cross-section 12.0D downstream from the disturbance generator, the RNG $k$-$\epsilon$ model shows a slight attenuation of the axial peak velocity for the long inlet and the $k$-$\omega$ SST model exhibits a smoother profile for the long inlet. The corresponding $\mathbf{v}_{xy}$ profiles are shown in panel (e) of Figure 2. Both the realizable $k$-$\epsilon$ model and the RNG $k$-$\epsilon$ model show almost identical $\mathbf{v}_{xy}$ for the long and the short computational domains. For different lengths of the computational domain, the $k$-$\omega$ SST model preserves main features including peaks and gradients close to the wall and the pipe center, but shows small differences in the $\mathbf{v}_{xy}$ profile for intermediate $r/R$. These results suggest that the impact of the domain length in the considered cases is minor. Further, this comparison illustrates, that the profile upstream from the swirl disturbance generator has a minor influence on the flow patterns downstream, suggesting that upstream disturbances do not influence the characteristic patterns of the swirling flow field downstream. Therefore, the study is performed using the shorter computational domain, which allows a reduction of the number of cells and the computational time.

Next, we study the influence of the mesh-size on the solution. Figure 3 shows the secondary flow predicted by different turbulence models and different meshes. The standard $k$-$\epsilon$ model and the RNG $k$-$\epsilon$ model show little dependency on the mesh size, maintaining similar flow patterns across different

| Model       | Main reference(s) |
|-------------|-------------------|
| standard $k$-$\epsilon$ | Launder and Spalding [19] |
| RNG $k$-$\epsilon$ | Yakhot et al. [20] |
| realizable $k$-$\epsilon$ | Shih et al. [21] |
| $k$-$\omega$ SST | Menter and Esch [23] & Hellsten [24] |

Figure 4: (a) Pressure along the centerline for the different turbulence models. (b) Darcy friction factor for the pipe section between $-3.0D$ and $-1.0D$. The inlet is located at $x/D \approx -13.0$, the swirl disturbance generator is located at $x/D \approx 0.0$ (shaded area), and the outlet is located at $x/D \approx 13.0$. |
mesh sizes. In contrast, the realizable $k$-$\varepsilon$ model exhibits a significant difference between solutions on coarser and finer meshes. While the coarser meshes show small-scale flow patterns, the finer meshes predict a homogeneous swirling flow without characteristic small-scale patterns. In general, the models from the $k$-$\varepsilon$ family tend to predict smaller magnitudes of $\tau_{xy}$ with increasing mesh sizes, whereas models from the $k$-$\omega$ family tend to predict higher magnitudes of $\tau_{xy}$ with increasing mesh size. Under the assumption of monotonic convergence, this suggests that swirling flows computed with models from the $k$-$\varepsilon$ family should be interpreted as a lower estimate for the secondary flow, whereas results computed with models from the $k$-$\omega$ family should be interpreted as an upper estimate for the secondary flow.

Further, we study the convergence of the pressure drop with increasing mesh size. The pressure along the center-line of the pipe is shown in Figure 2(a) for all considered turbulence models. As expected, there is a significant pressure drop across the swirl disturbance generator, while the pressure loss is approximately linear with respect to the $z$-coordinate for the pipe sections before and after the swirl disturbance generator. To compare the pressure loss in the straight pipe with theoretical references, we compute the Darcy–Weisbach friction factor

$$f = \frac{2D}{\rho_e L u_{vol}^2} \Delta \overline{p},$$

where $\rho$ is the density of water, $L$ is the associated length of the pipe-section (from $-3.0D$ to $-1.0D$, to consider the section where the flow is most developed), and $\Delta \overline{p}$ is the associated pressure loss. The friction factor is computed for all turbulence models and compared to the theoretical reference obtained from the Moody diagram [25]. Figure 2(b) shows that the pressure drop over the straight pipe downstream from the swirl disturbance generator is overpredicted by all considered turbulence models when compared to the theoretical reference.
from the Moody Diagram. The standard $k$-$\epsilon$ model and the RNG $k$-$\epsilon$ model appear to converge to a value that is higher than the theoretical reference as the mesh size increases. Moreover the realizable $k$-$\epsilon$ model shows a significant overprediction, where values for the finer meshes do not agree with the theory nor with the other models. Conversely, models from the $k$-$\omega$ family show a very close agreement with the theory and appear to converge towards the theoretical value for increasing mesh size. For the discussion in the remainder of the article, we use the results obtained with a mesh size of $20 \cdot 10^6$ cells.

### 3.2. Velocity fields

For the comparison of flow patterns at different cross-sections downstream of the swirl disturbance generator, we focus on three of the models since the structures predicted by the RNG $k$-$\epsilon$, the standard $k$-$\epsilon$ models, and both $k$-$\omega$ models are found to be qualitatively similar (see, for example, Figure 7). The flow patterns of the secondary flow predicted at four locations are shown in Figure 5 and iso-surfaces corresponding to $\overline{w} = w_{vol}$ of the domain downstream of the swirl disturbance generator are shown in Figure 6.

All considered models predict eight characteristic
flow patterns at the cross-section $2.0D$ downstream, which are associated with the eight blades of the swirl disturbance generator. At $6.0D$, the realizable $k$-$\varepsilon$ model does not show any small-scale flow patterns in $\overline{\tau}_{xy}$, but a radially homogeneous swirling flow. In contrast, the $k$-$\omega$ SST model and the $k$-$\varepsilon$ model still predict eight patterns with elevated $\overline{\tau}_{xy}$. The homogeneous swirling flow of the realizable $k$-$\varepsilon$ model is maintained further downstream with a continuous reduction of $\overline{\tau}_{xy}$. For the $k$-$\varepsilon$ model, the initial structures start dissipating and merging at around $10.0D$, resulting in only four visible flow structures at the $12.0D$ measurement cross-section. The $k$-$\omega$ SST model maintains the eight characteristic flow patterns throughout the entire simulation domain, although at the $12.0D$ measurement section the dissipation of four of them starts to become visible.

Next, we compare the simulations with the experimental results at the section $12.0D$ downstream of the swirl generator. Figure 7 shows the secondary flow obtained from simulations along with the corresponding experimental results. Visual inspection of the contour plots suggests that the standard $k$-$\varepsilon$ model and the RNG $k$-$\varepsilon$ model provide the best approximations of the experimentally observed four characteristic patterns in the outer region of the pipe. Conversely, the realizable $k$-$\varepsilon$ model provides a more realistic approximation of the velocity magnitude close to the center of the pipe.

For the comparison of individual profile paths, we use 10 averaged profiles as described in Section 3.1. The individual measured velocity profiles are shown in Figure 8 along with the corresponding standard error. The standard errors associated with the averaged profiles are computed through error propagation calculus using the standard error $\sigma_{\overline{\tau}}$ of individual profiles. For the averaged axial velocity profile, we compute the associated standard error

$$\sigma_{\overline{\tau}} = \frac{1}{m} \left( \sum_{i=1}^{m} \sigma_{\overline{\tau}}^2 \right)^{1/2},$$

(17)

where $m = 10$ is the number of individual profiles. Similarly, for the secondary flow, assuming uncorrelated $\sigma_{\overline{\tau}}$ and $\sigma_{\overline{\tau}}$, we find

$$\sigma_{\overline{\tau}_{xy}} = \frac{1}{m} \left( \sum_{i=1}^{m} \sigma_{\overline{\tau}_{xy}}^2 \right)^{1/2},$$

(18)

The averaged profiles are shown in Figure 9. The shaded area indicates the associated standard error of the experimental results with a coverage factor $\kappa = 2.0$. The profiles of $\overline{\tau}$ indicate that the realizable $k$-$\varepsilon$ model is the only model that does not predict a spurious pronounced velocity peak at the pipe center, but an almost flat plateau-like profile that closely approximates the experimental data (Figure 9 (a)).

Similarly, the profiles of $\overline{\tau}_{xy}$ show that the near-field flow produced by the standardized swirl disturbance generator is similar to solid-body rotation (Figure 9 (b)). Importantly, the flow field generated by the standardized swirl disturbance generator according to EN ISO 4064-2:2014 [11] and OIML R 49-2:2013 [12] appears to lack a transition of the flow from solid body rotation to Rankine-vortex type flow as reported by Vaidya et al. [29] for a swirling flow generated by a rotating honeycomb. All turbulence models appear to overpredict $\overline{\tau}_{xy}$ and, as previously seen in Figure 7, the highest values are given by the models of the $k$-$\omega$ family. Overall, the best prediction of the experimental data is provided by the realizable $k$-$\varepsilon$ model which appears to capture the mean flow velocity more accurately than the other considered models.

### 3.3. Performance indicators

Performance indicators are useful integral metrics to quantify flow conditions. Following Yeh and Mattingly [27] and Müller and Dues [28], we compute the swirl angle $\phi$, the profile factor $K_p$, the asymmetry factor $K_a$, and the turbulence factor $K_{\tau_u}$. A detailed definition of the performance indicators is provided in Appendix A.
In Table 2, we compare performance indicators from computational and experimental results as well as available experimental results of Eichler and Lederer [15] with a DN80 pipe diameter and a flow rate of 96.221 m$^3$/h (Re ≈ 4.0 · 10$^5$). The values of performance indicators are reported as mean values of 10 profiles shown in Figure 1(b) along with the associated standard deviation. According to the definition A.4, the asymmetry factor $K_a$ is signed.

However, to report a meaningful non-zero mean, we average the magnitude of $K_a$. For the simulations, the turbulence factor $K_{Tu}$ is computed using the turbulent kinetic energy $k$ that is part of the solution of the corresponding turbulence model, such that

$$
Tu = \frac{\sqrt{2k/3}}{|\mathbf{U}|^2}.
$$

(19)
Notice that \(19\) corresponds to a three-dimensional turbulence factor, while the \(K_{Tu}\) value of the experiments corresponds to individual axial profiles.

The values in Table\(2\) indicate that the realizable \(k-\epsilon\) model provides low standard deviations in performance indicators. This is consistent with the circular symmetry of the predicted flow fields, which results in almost identical individual profiles, as discussed in Section \(3.2\). The profile factor \(K_p\) of the realizable \(k-\epsilon\) model shows good agreement with the LDV results, while all other models predict a higher value for this parameter. This substantiates the results discussed in Section \(3.2\) where the realizable \(k-\epsilon\) model is found to provide the best prediction of the experimental axial profile.

The asymmetry factor \(K_a\) obtained from the simulations is significantly lower than the experimental value. Consequently, none of the models shows potential to provide an accurate estimation of this parameter. However, the experimental setup is always subject to small imperfections, which results in non-symmetric flow profiles, whereas the computational model is fully symmetric by definition. Further, previous investigations (see, for example, Eichler and Lederer [15]) reported that the swirling flow could be assessed computationally through small perturbations, for example in the boundary conditions. However, such an investigation goes beyond the scope of this article and should be investigated in future studies.

The turbulence factor \(K_{Tu}\) is approximated reasonably well by the considered turbulence models. The best prediction is provided by the RNG \(k-\epsilon\) model and the standard \(k-\epsilon\) model. The values obtained from the \(k-\omega\) model and the \(k-\omega\) SST model are lower than the experimental values. The realizable \(k-\epsilon\) model appears to fail in capturing the behaviour of \(K_{Tu}\), giving a considerable overprediction.

The swirl angle is well predicted by the three \(k-\)
The $k$-$\omega$ model by Wilcox [22] is designed to improve the performance of the standard $k$-$\epsilon$ model in the viscous layer of the near-wall regions and in adverse pressure gradients, promising more accurate results for free shear flows and separated flows. Additionally, the $k$-$\omega$ SST model combines the $k$-$\omega$ model and the standard $k$-$\epsilon$ model through a blending function that prompts the use of the $k$-$\omega$ model in the near wall regions and the $k$-$\epsilon$ model in regions far from the wall. The $k$-$\omega$ SST model was originally formulated by Menter [31] with the goal of modeling aerodynamics flows. Menter and Esch [25] presented a modification of the $k$-$\omega$ SST model which includes a new definition of the eddy viscosity using the strain rate instead of the vorticity. This modification is aimed at extending the applicability of the model to a wider range of flows. Yet, in the present scenario, we see no improvements on using the $k$-$\omega$ model or the $k$-$\omega$ SST model rather than the standard $k$-$\epsilon$ model. With respect to the experimental results, both models give the most unsatisfactory results for predicting realistic velocity profiles in the 12.0$D$ measurement plane. Conversely, we find that the $k$-$\omega$ model and the $k$-$\omega$ SST model provide realistic values of the pressure loss for sufficiently fine meshes. Further, the modeling assumptions suggest that the $k$-$\omega$ models are likely to provide a realistic prediction of the flow in the pipe section close the swirl disturbance generator and inside the swirl disturbance generator, were the influence of the wall region becomes more relevant. Yet, these modeling assumptions render the $k$-$\omega$ models deficient for predicting the evolution of the flow further downstream, where the influence of the wall is less dominant.
Table 2: Performance indicators at the cross-section $12.0D$ downstream from the swirl disturbance generator.

|                   | $K_p$ [−] | $K_a$ [%] | $K_m$ [−] | $\phi$ [deg] |
|-------------------|-----------|-----------|-----------|--------------|
| standard $k$-$\epsilon$ | 1.173 ± 0.131 | 0.062 ± 0.035 | 1.792 ± 0.004 | 12.269 ± 0.550 |
| RNG $k$-$\epsilon$      | 1.186 ± 0.174 | 0.051 ± 0.045 | 1.910 ± 0.007 | 12.514 ± 0.856 |
| realizable $k$-$\epsilon$ | 0.438 ± 0.005 | 0.002 ± 0.001 | 3.867 ± 0.000 | 11.292 ± 0.017 |
| $k$-$\omega$           | 1.163 ± 0.152 | 0.241 ± 0.325 | 1.176 ± 0.004 | 15.068 ± 1.062 |
| $k$-$\omega$ SST        | 1.360 ± 0.207 | 0.309 ± 0.356 | 1.340 ± 0.006 | 15.293 ± 1.521 |

Present experiments 0.470 ± 0.196 0.975 ± 1.120 2.071 ± 0.054 11.426 ± 0.561
Experiments 13.0D downstream with DN80 and $Re \approx 4 \times 10^5$ [13]

5. Conclusions

We assessed the potential of five different turbulence models to predict flow patterns downstream of a standardized swirl disturbance generator with different meshes and mesh topologies. The numerical results were validated through a systematic comparison with LDV experiments. For sufficiently fine meshes, the realizable $k$-$\epsilon$ model gives the most reasonable prediction of both axial and in-plane velocity profiles, when compared to experimental results. However, the realizable $k$-$\epsilon$ model fails to predict the characteristic small-scale flow patterns determined experimentally, which are well-captured by the standard $k$-$\epsilon$ model and the RNG $k$-$\epsilon$ model. The pressure drop over the straight pipe-section before the swirl generator is overpredicted by models from the $k$-$\epsilon$ family when compared to a theoretical reference determined from the Moody diagram. In particular the realizable $k$-$\epsilon$ model appears to provide elevated values for the friction factors. On the other hand, the $k$-$\omega$ models predict values that are closest to the theoretical reference. We conclude that, in general, $k$-$\epsilon$ models appear to provide more accurate predictions of the velocity fields but fail to provide realistic predictions of the pressure drop. Conversely, $k$-$\omega$ models give a better prediction for the pressure drop. Overall, the performance of the investigated eddy-viscosity models appears to be limited, which is in agreement with expectations (see, for example, Jakirić et al. [1]). However, the usage of these models still reflects current industry practice and this detailed study including experimental validation facilitates a more targeted model selection and guidance for the interpretation of results. Additionally, a comparison of the present results with other references with different pipe diameters and flow rates shows that the performance indicators exhibit similar values. This suggests that, despite small differences in the swirl disturber geometries, the flow fields generated at different pipe diameters and flow rates are approximately self-similar. Consequently, small differences in the geometry of the swirl disturbance generators have only secondary effects on the flow field.

Appendix A. Definition of performance indicators

Appendix A.1. Profile factor

Following Yeh and Mattingly [27], the dimensionless profile factor $K_p$ is defined as

$$K_p = \frac{K_{p,m}}{K_{p,s}}, \quad (A.1)$$

with

$$K_{p,m} = \frac{1}{w_{vol}D} \int_{-R}^{R} (w_m - \overline{w}) \, dr \quad (A.2)$$

and

$$K_{p,s} = \frac{1}{w_{vol}D} \int_{-R}^{R} (w_{m,s} - w_s) \, dr, \quad (A.3)$$

where $w_m = \overline{w}(r = 0)$ is the velocity at the pipe center, $w_{m,s}$ is the velocity of the norm profile at the pipe center, and $w_s$ is the velocity of the norm profile. The profile factor is a measure for peakness ($K_p > 1$) or flatness ($K_p < 1$) of measured velocity profiles with respect to standard profiles such as Hagen–Pouiseille for laminar flow or Gersten and Herwig [32, 33] for turbulent flow.

Appendix A.2. Asymmetry factor

Following Yeh and Mattingly [27], the asymmetry factor

$$K_a = \frac{1}{D} \int_{-R}^{R} r \overline{w} \, dr \quad (A.4)$$

where $r$ is the radial coordinate from the axis of symmetry, $\overline{w}$ is the mean velocity, and $D$ is the pipe diameter.
quantifies the relative radial displacement of the center of gravity of the area under the flow profile with respect to the pipe center.

**Appendix A.3. Turbulence factor**

Each LDV point measurement is a collection of a large number of bursts resulting in a histogram (or probability density function) for the axial velocity component. The level of dispersion (i.e. the standard deviation) of this histogram quantifies the turbulence intensity \[\nu.\] As discussed by Durst et al. [34] and generalized by Pashtrapanask [35], the turbulence intensity \[\nu.\] in the core region \(-0.2 \leq r/R \leq 0.2\) can be estimated as

\[
Tu_s = 0.13 \left( \frac{Re}{w_{vol}} \right)^{-1/8}
\]

(A.5)

for \(Re/w_{vol} \geq 4500\). The turbulence factor \(K_{Tu}\) is defined as

\[
K_{Tu} = \frac{Tu_{\text{max}}}{Tu_s}, \quad \text{(A.6)}
\]

where \(Tu_{\text{max}}\) is the maximum of \[\nu.\] in the core region \(-0.2 \leq r/R \leq 0.2\).

**Appendix A.4. Swirl angle**

Following Yeh and Mattingly [36], the level of swirl can be measured quantitatively through the maximal swirl angle

\[
\phi = \arctan \left( \frac{|\tau_{xy}|_{\text{max}}}{w_{vol}} \right).
\]

(A.7)

However, the precise definition of the swirl angle may vary slightly depending on the author. Geometrically, the swirl angle \[\nu.\] is the angle between the ideal velocity vector and the actual velocity vector with swirl.

**Appendix B. Mesh quality parameters**

| OpenFoam parameter       | Value       |
|--------------------------|-------------|
| maxNonOrtho              | 65.0        |
| maxBoundarySkewness      | 20.0        |
| maxInternalSkewness      | 4.0         |
| maxConcave               | 80.0        |
| minFlatness              | 0.5         |
| minVol                   | 1.0 - 10^{-20} |
| minTetQuality            | 1.0 - 10^{-30} |
| minArea                  | -1          |
| minTwist                 | 0.05        |
| minDeterminant           | 0.001       |
| minFaceWeight            | 0.05        |
| minVolRatio              | 0.01        |
| minTriangleTwist         | -1          |
| nSmoothScale             | 4           |
| errorReduction           | 0.75        |

Table B.3: Mesh quality parameters used in the mesh generation with snappyHexMesh.

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