Power, Rate, and Decoding Order Optimization for NOMA-Based Vehicular Communications

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Abstract—This paper considers a two-user non-orthogonal multiple access (NOMA) based infrastructure-to-vehicle (I2V) network, where one user requires reliable safety-critical data transmission and the other pursues high-capacity services. Leveraging only slow fading of channel state information, we aim to maximize the expected sum throughput of the capacity hungry user subject to a constraint on the payload delivery success probability of the reliability sensitive user, by jointly optimizing the transmit powers, target rates, and decoding order. We introduce a dual variable and formulate the optimization as an unconstrained single-objective sequential decision problem. Then, we design a dynamic programming based algorithm to derive the optimal policy that maximizes the Lagrangian. Afterwards, a bisection search based method is proposed to find the optimal dual variable. The proposed strategy is shown by numerical results to be superior to the baseline approaches from the perspectives of expected return, performance region, and objective value.

Index Terms—Markov decision process, power allocation, infrastructure-to-vehicle, reliability, dynamic programming.

I. INTRODUCTION

As a key enabler to intelligent transportation systems, vehicular communications take charge of information exchange among various types of entities on or near roads, including infrastructure-to-vehicle (I2V), vehicle-to-vehicle (V2V), and pedestrian-to-vehicle (P2V) communications. Depending on the applications of the carried data, different connections can be concerned with different quality of service (QoS), e.g., safety-critical data transmission usually requires high reliability and low latency while inforntainment data traffic often desires high capacity [1].

To embrace the challenge of fast channel variation in high-mobility vehicular environment, slowly varying large-scale channel fading information have been leveraged to develop smart wireless resource allocation to satisfy diverse QoS requirements of various links [2]–[5]. In [2], transmit power and resource block have been jointly allocated to maximize the throughput of cellular users subject to constraints on the rate outage probability of V2V users. In case of spectrum sharing between vehicle-to-infrastructure (V2I) and V2V links, the spectrum and power allocation proposed in [3] has optimized the sum capacity and minimum capacity of the V2I links while guaranteeing the signal-to-interference-plus-noise ratio (SINR) outage probability of the V2V connections. The average queueing latency and packet dropping probability of V2V links have been guaranteed by the resource allocation in [4]. The authors in [5] have jointly optimized channel, power, and blocklength allocation for finite-blocklength V2V and infinite-blocklength vehicle-to-network (V2N) communications, where the maximum latency of the V2V links is minimized subject to constraints on the rate outage probability of the V2V links and ergodic capacity of the V2N links.

While resource allocation decision is made only once for each realization of large-scale fading in [2–5], resource management can be further designed to adapt to the variation of fast fading [6]–[9]. The deep reinforcement learning based decentralized power level and subchannel allocation scheme proposed in [6] has provided lower latency for V2V links and higher capacity for V2I links. Leveraging multi-agent reinforcement learning approach, the spectrum sharing and power allocation have been designed in [7] to maximize the capacity of V2I links and improve data transmission reliability of V2V links. To promote the adaptability to fast environment variation, a meta-reinforcement learning based resource allocation scheme has been developed in [8] to enhance the QoS of V2I and V2V links. In [9], the sum throughput of V2I links have been maximized subject to constraints on the latency and reliability of V2V links by a multi-agent reinforcement learning based spectrum access strategy, where local channel state information (CSI) independent observations are exploited.

Although different links with different QoS requirements have been considered to share the same channel to improve spectrum utilization in the aforementioned works, the non-orthogonal multiple access (NOMA), as another spectrum-efficient technique, has received little attention for dynamic resource allocation in vehicular communications, especially from the optimization perspective. To this end, we consider power, rate, and decoding order optimization in a two-user downlink NOMA based vehicular system, where one user requires reliable payload delivery and the other pursues high capacity. Particularly, we maximize the expected data throughput of the capacity hungry user subject to a data delivery outage constraint of the reliability sensitive user. The main contributions are threefold. First, a finite Markov decision process (MDP) with appropriate reward design is developed such that the transmitter, acting as the agent, can optimize the Lagrangian of the original problem by equivalently maximizing its expected return. Second, a dynamic programming based algorithm is proposed to maximize the Lagrangian. Third, the dual variable is fast optimized by bisection search.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we will introduce the network scenario, signal transmission model, and problem formulation successfully.

A. Network Scenario

Consider a downlink I2V network consisting of one infrastructure access point (AP) and two vehicular users, where the first user, $U_1$, requires reliable safety-related data transmission while the second user, $U_2$, is throughput hungry but reliability and latency insensitive. In particular, $N$ data packets are required to be delivered from the AP to $U_1$ within $T$ slots with success probability no less than $1 - \delta$, where each slot with identical length, $\tau$, can be regarded as the channel coherence time on the order of, e.g., hundreds of microseconds in vehicular environment. In this regard, the small-scale fading of the I2V channels keeps constant within each slot but changes rapidly from one slot to
another. To avoid substantial signalling overhead caused by CSI feedback in every slot, the AP is assumed to know the statistical information rather than the realization of the fast fading in each slot. However, the large-scale fading is considered to be available at the AP since it varies on a slow scale and can be fed back to the AP less frequently. The channel power gain of the \( k \)th 12V connection \((k \in \{1, 2\})\) in the 7th slot is modeled as

\[
h_k[t] = \beta_k g_k[t],
\]

where \( \beta_k \) and \( g_k[t] \) account for the large-scale fading and small-scale fading, respectively. We consider Rayleigh fast fading in this article, i.e., \( g_k[t] \) is independent and identically distributed exponential random variables with unit mean for all \( k \) and \( t \).

### B. Signal Transmission Mechanism

To improve system spectrum efficiency, the AP transmits wireless signals to the two users over the same spectrum of bandwidth \( W \), using the NOMA scheme. With the total power consumption, \( P \), the AP constructs the superposition signal at the 3rd slot as

\[
x[t] = \sqrt{P} V_1[t] \cdot x_1[t] + \sqrt{P} V_2[t] \cdot x_2[t]
\]

where \( x_k[t] \) is the normalized baseband signal of \( U_k \) with unit power and \( V_k[t] \) denotes the proportion of power allocated to \( U_k \) in the 7th slot \((k \in \{1, 2\})\). We consider \( L \) discrete possible power allocation choices, forming the power set given by \( \mathcal{V} = \{V_1^l, V_2^l\}|l = 1, 2, \ldots, L\), where each power allocation can make full utilization of the total power, i.e., \( V_1^l + V_2^l = 1 \) for \( l = 1, 2, \ldots, L \). The received signal of \( U_{1,7} \) at the 7th slot is

\[
y_k[t] = h_k[t] x[t] + z_k[t] = \beta_k g_k[t] x[t] + z_k[t],
\]

where \( z_k[t] \) with variance \( \mathbb{E}[|z_k[t]|^2] = \sigma_k^2 \) represents the additive white Gaussian noise (AWGN) and cochannel interference.

In downlink NOMA transmission with successive interference cancellation, it is crucial to determine the decoding order, \( O[t] \), the target transmission rate of \( U_1 \), \( R_1[t] \), and the target transmission rate of \( U_2 \), \( R_2[t] \), in every slot. The two possible decoding orders form the set \( \mathcal{O} = \{O_{1 \rightarrow 2}, O_{2 \rightarrow 1}\} \), where \( O_{1 \rightarrow 2} \) represents the order of decoding \( U_1 \) and \( U_2 \) successfully and \( O_{2 \rightarrow 1} \) denotes the order of decoding \( U_2 \) and \( U_1 \) sequentially. The selectable target rates of \( U_k \), in the unit of the number of packets per slot, are discrete and form the set \( \mathcal{R}_k \) for \( k \in \{1, 2\} \). The number of bits contained in each packet of \( U_k \) is denoted by \( Y_k \) for \( k \in \{1, 2\} \). In the following, we discuss the data transmission processes for the two decoding orders in the 7th slot, where the power allocation is \((V_1[t], V_2[t])\), the transmission rate of \( U_1 \) is \( R_1[t] \), and the transmission rate of \( U_2 \) is \( R_2[t] \).

- \( O[t] = O_{1 \rightarrow 2} \)

For this decoding order, \( U_1 \) decodes its signal by treating the signal from \( U_2 \) as interference, to channel capacity

\[
\tau W Y_1^{-1} \log_2 \left( 1 + \frac{P \beta_1 g_1[t] V_1[t]}{\sigma_1^2 + P \beta_1 g_1[t] V_2[t]} \right),
\]

which represents the number of packets that can be successfully carried in the 7th slot. Since the target rate \( R_1[t] \) can be supported only if the channel capacity can cover \( R_1[t] \), the number of packets that can be successfully received by \( U_1 \) in the 7th slot, denoted by \( D_1[t] \), is

\[
D_1[t] = \begin{cases} R_1[t], & \text{if } \tau W Y_1^{-1} \log_2 \left( 1 + \frac{P \beta_1 g_1[t] V_1[t]}{\sigma_1^2 + P \beta_1 g_1[t] V_2[t]} \right) \geq R_1[t] \\ 0, & \text{otherwise} \end{cases}
\]

At \( U_2 \), it first decodes the signal of \( U_1 \) by treating its own signal as interference, with channel capacity

\[
\tau W Y_1^{-1} \log_2 \left( 1 + \frac{P \beta_2 g_2[t] V_1[t]}{\sigma_2^2 + P \beta_2 g_2[t] V_2[t]} \right).
\]

If \( R_1[t] \) can be covered by this achievable rate, \( U_2 \) can decode its own signal suffering no interference from \( U_1 \) with capacity

\[
\tau W Y_2^{-1} \log_2 \left( 1 + \frac{P \beta_2 g_2[t] V_2[t]}{\sigma_2^2} \right).
\]

Otherwise, \( U_2 \) has to decode its information by treating the signal from \( U_1 \) as interference, giving rise to channel capacity

\[
\tau W Y_2^{-1} \log_2 \left( 1 + \frac{P \beta_2 g_2[t] V_2[t]}{\sigma_2^2 + P \beta_2 g_2[t] V_1[t]} \right).
\]

For each case, the data of \( U_2 \) can be successfully received at \( U_2 \) only if the capacity in \( \sigma_1^2 \) or \( \sigma_2^2 \) is no less than \( R_2[t] \). Overall, the number of packets that can be successfully received by \( U_2 \) in the 7th slot, denoted by \( D_2[t] \), can be figured out as

\[
D_2[t] = \begin{cases} R_2[t], & \text{if } \tau W Y_1^{-1} \log_2 \left( 1 + \frac{P \beta_2 g_2[t] V_1[t]}{\sigma_1^2 + P \beta_2 g_2[t] V_2[t]} \right) \geq R_1[t], \\ \tau W Y_1^{-1} \log_2 \left( 1 + \frac{P \beta_2 g_2[t] V_1[t]}{\sigma_1^2 + P \beta_2 g_2[t] V_2[t]} \right) \geq R_2 Y_2[t], \\ 0, & \text{otherwise} \end{cases}
\]

- \( O[t] = O_{2 \rightarrow 1} \)

By similar analysis as presented above, we derive the expressions of \( D_2[t] \) and \( D_1[t] \) for decoding order \( O_{2 \rightarrow 1} \) as

\[
D_2[t] = \begin{cases} R_2[t], & \text{if } \tau W Y_1^{-1} \log_2 \left( 1 + \frac{P \beta_2 g_2[t] V_2[t]}{\sigma_1^2 + P \beta_2 g_2[t] V_1[t]} \right) \geq R_1[t], \\ \tau W Y_1^{-1} \log_2 \left( 1 + \frac{P \beta_2 g_2[t] V_2[t]}{\sigma_1^2 + P \beta_2 g_2[t] V_1[t]} \right) \geq R_2[t], \\ 0, & \text{otherwise} \end{cases}
\]

and

\[
D_1[t] = \begin{cases} R_1[t], & \text{if } \tau W Y_1^{-1} \log_2 \left( 1 + \frac{P \beta_1 g_1[t] V_2[t]}{\sigma_1^2 + P \beta_1 g_1[t] V_1[t]} \right) \geq R_1[t], \\ \tau W Y_1^{-1} \log_2 \left( 1 + \frac{P \beta_1 g_1[t] V_2[t]}{\sigma_1^2 + P \beta_1 g_1[t] V_1[t]} \right) \geq R_2[t], \\ 0, & \text{otherwise} \end{cases}
\]

respectively.

### C. Problem Statement

From the above discussion, the data transmission processes of the two users are influenced by the action of jointly allocating the powers, rates, and decoding order, which needs to be determined for every possible system situation, also called state.

1) State: A state, \( s \), can be characterized by

\[
s = (E^s, Z^s),
\]

where \( E^s \) and \( Z^s \) denote the numbers of remaining slots and remaining packets of \( U_1 \), respectively. Clearly, there are \( T(N +
1) + 1 possible states, forming the state space $S$, among which the initial state is always $s_0 = (T, N)$. The states with nonzero remaining slots are referred to as the nonterminal states, forming the set $S^- = \{s|s \in S, E^s > 0\}$ with cardinality $(T-1)(N+1)+1$. Based on such definition, we can express the set of terminal states as $S - S^-$, having cardinality $N + 1$.

2) **Action**: An action, $a$, can be defined as

$$a = (O^a, V^a, R_1^a, R_2^a),$$

where $O^a \in O, V^a \in \mathcal{V}, R_1^a \in \mathcal{R}_1$, and $R_2^a \in \mathcal{R}_2$ represent the decoding order, power, target rate of $U_1$, and target rate of $U_2$, respectively. The set of actions, $\mathcal{A}$, can thus be expressed in the form of Cartesian product, i.e.,

$$\mathcal{A} = O \times \mathcal{V} \times \mathcal{R}_1 \times \mathcal{R}_2.$$ (14)

3) **Problem Formulation**: We model the power, rate, and decoding order allocation as a policy optimization issue, where a policy refers to a mapping, $a = \pi(s)$, from the set $S^-$ to the set $\mathcal{A}$. The policy optimization of maximizing the expected capacity of $U_2$ subject to a constraint on the payload delivery success probability of $U_1$ is formulated as

$$\max_{\pi \in \Pi} \mathbb{E}_\pi \left[ \sum_{t=1}^T D_2[t] \right]$$

s.t. $\mathbb{P}_\pi \left\{ \sum_{t=1}^T D_1[t] \geq N \right\} \geq 1 - \delta$, (15b)

where $\Pi$ is the policy space with cardinality $|\Pi| = |\mathcal{A}||S^-| = |\mathcal{A}|(T-1)(N+1)+1$ and $\delta$ is the maximum allowed payload delivery outage probability of $U_1$. Since the number of possible policies grows exponentially with $T$ and $N$, exhaustive search is unserviceable in most practical systems.

### III. Proposed Approach

In this section, we first derive the Lagrangian of the problem in (15) by introducing a Lagrange dual variable. Then, we construct a finite MDP and figure out the optimal policy to maximize the Lagrangian by dynamic programming. Finally, a bisection search based method is proposed to obtain the optimal dual variable.

#### A. Solution Structure

By associating a dual variable, $\lambda \in [0, +\infty)$, with the constraint, we have the Lagrangian of the primal problem given by

$$L(\pi, \lambda) = \mathbb{E}_\pi \left[ \sum_{t=1}^T D_2[t] \right] + \lambda \left[ \mathbb{P}_\pi \left\{ \sum_{t=1}^T D_1[t] \geq N \right\} - (1 - \delta) \right].$$ (16)

We can then derive the Lagrange dual function

$$f(\lambda) = \max_{\pi} L(\pi, \lambda),$$

where $\pi_\lambda = \arg \max \ L(\pi, \lambda)$ denotes the policy that maximizes the Lagrangian under fixed dual variable $\lambda$. Afterwards, we find the best dual variable, $\lambda^*$, by handling the Lagrange dual problem

$$\min_{\lambda} \ f(\lambda)$$

s.t. $\lambda \geq 0$. (18b)

Finally, policy $\pi_{\lambda^*}$ will be returned as the solution.

#### B. Policy Optimization by Dynamic Programming

As the optimal solution of an unconstrained sequential decision problem for a given $\lambda$, $\pi_\lambda$ can be derived by constructing and solving a finite MDP, where an agent interacts with its environment during the MDP. Specifically, the agent, i.e., the AP, observes a state, $S_t$, at time $t$ from the state space, $S^-$, and on that basis chooses an action, $A_{t+1}$, from the action space, $\mathcal{A}$, based on a policy, $\pi$. One time slot later, the environment responds to the action taken in the previous slot by presenting a new state, $S_{t+1}$, and offering a reward, $R_{t+1}$, to the agent. It is worth noting that $S_t$ denotes the state at the end of the $t$th slot, $R_t$ is the reward received at the end of the $t$th slot, and $A_t$ represents the action taken in the $t$th slot, for $t = \{1, 2, 3, \cdots, T\}$. By defining $S_0 = s_0$ as the initial state observed at the beginning of the first slot, we have the episodic trajectory of the agent-environment interaction given as $S_0, A_1, R_1, S_1, A_2, R_2, S_2, \cdots, A_T, R_T, S_T$.

The reward $R_t$ is carefully designed as

$$R_t = D_2[t] + \lambda c_t,$$ (19)

where $c_t$ is given by

$$c_t = \begin{cases} 0, & \text{if } S_t \in S^- \\ \delta - 1, & \text{if } S_t \in S - S^-, Z^{S_t} > 0 \\ \delta, & \text{if } S_t \in S - S^-, Z^{S_t} = 0. \end{cases}$$ (20)

Since $R_t$ and $S_t$ have discrete probability distribution dependent only on the preceding state and action, the dynamics of the finite MDP can be characterized by

$$p(s', r|s, a) = \mathbb{P}\{S_{t+1} = s', R_{t+1} = r|S_t = s, A_t = a\},$$ (21)

denoting the probability of state $s'$ and reward $r$ at time $t$ given the preceding state $s$ at time $t - 1$ and action $a$ at time $t$, where $s' \in S, r \in R, s \in S^-$, and $a \in \mathcal{A}$.

Let $v_\pi(s)$ denote the value of state $s$ under policy $\pi$, standing for the expected return starting from $s$ and following $\pi$ thereafter, i.e.,

$$v_\pi(s) = \mathbb{E}_\pi \left[ \sum_{t=1}^{E^s} R_{T-E^s+j} \mid S_{T-E^s} = s \right]$$ (22)

for all $s \in S^-$ and $v_\pi(s) = 0$ for all $s \in S - S^-$. Since the reward design in (19) is related to $\lambda$, the optimal policy of this finite MDP is denoted by $\pi_\lambda^*(\lambda)$, which leads to the highest state values for all states, i.e., $v_{\pi_\lambda^*(\lambda)}(s) \geq v_\pi(s)$ holds for all $s \in S$ and $\pi' \in \Pi$. The following theorem provides an appealing property of $\pi_\lambda^*(\lambda)$.

**Theorem 1**: $\pi_\lambda^*(\lambda) = \pi_\lambda$ if the dynamics of the MDP are fixed.

**Proof**: We deploy policy $\pi$ for $M$ independent episodes with fixed dynamics, i.e., $p(s', r|s, a)$ keep fixed for all $s' \in S, r \in R, s \in S^-$, and $a \in \mathcal{A}$. Then, the expected return of the initial state, $s_0$, can be derived by averaging the return realizations of the $M$ episodes as $M$ approaches infinity. Let $I_m$ be the payload delivery success indicator of $U_1$ in the $m$th episode under policy $\pi$, such that $I_m = 1$ if the transmission is successful and $I_m = 0$ otherwise. In addition, $R_{t,m}$ refers to the reward received at time $t$ during the $m$th episode and $D_{k,m}(t)$ denotes the number of packets transmitted in the $t$th slot for $U_k$ during the $m$th episode under policy $\pi$. Finally, we can figure out the state value of $s_0$ as...
\[ P \{ D_2[t] = R_2[t] \} = \]
\[ \begin{cases} 
\exp(-\varphi_3), & \text{if } V_1[t] - V_2[t] \left( 2 \frac{Y_1 R_1[t]}{M} - 1 \right) > 0, V_2[t] - V_1[t] \left( 2 \frac{Y_2 R_2[t]}{M} - 1 \right) > 0, \varphi_2 > \varphi_1 \\
\exp(-\varphi_2) + \exp(-\varphi_3) - \exp(-\varphi_1), & \text{if } V_1[t] - V_2[t] \left( 2 \frac{Y_1 R_1[t]}{M} - 1 \right) > 0, V_2[t] - V_1[t] \left( 2 \frac{Y_2 R_2[t]}{M} - 1 \right) < 0, \varphi_2 \leq \varphi_1 \\
\exp(-\varphi_3), & \text{if } V_1[t] - V_2[t] \left( 2 \frac{Y_1 R_1[t]}{M} - 1 \right) \leq 0, V_2[t] - V_1[t] \left( 2 \frac{Y_2 R_2[t]}{M} - 1 \right) > 0 \\
\exp(-\varphi_2), & \text{if } V_1[t] - V_2[t] \left( 2 \frac{Y_1 R_1[t]}{M} - 1 \right) \leq 0, V_2[t] - V_1[t] \left( 2 \frac{Y_2 R_2[t]}{M} - 1 \right) \leq 0 \\
0, & \text{otherwise} 
\end{cases} \]

\[(29)\]

**TABLE I**

| Algorithm | Dynamic Programming for Obtaining \( \pi_\lambda \) [10] |
|-----------|--------------------------------------------------|
| 1: Initialization:  |
| • Initialize the dual variable, \( \lambda \), and the error tolerance, \( \xi \)  |
| • Set \( V(s) = 0 \) for all \( s \in S \)  |
| 2: repeat  |
| 3: \( \Delta = 0 \)  |
| 4: for \( s \in S \) do  |
| 5: \( v = V(s) \)  |
| 6: \( V(s) = \max_{a \in A} \left\{ \sum_{s' \in S} \sum_{r \in R} p(s', r | s, a) [r + V(s')] \right\} \)  |
| 7: \( \Delta = \max(\Delta, |V(s) - v|) \)  |
| 8: end for  |
| 9: until \( \Delta < \xi \)  |
| 10: \( \pi_\lambda(s) = V(s) \) for all \( s \in S \)  |
| 11: Return: policy \( \pi_\lambda \) with \( \pi_\lambda(s) \) given by  |
| \( \pi_\lambda(s) = \arg \max_a \sum_{s' \in S} \sum_{r \in R} p(s', r | s, a) [r + V_A(s')] \)  |

\[ v_\pi(s_0) = E_\pi \left[ \sum_{t=1}^{T} R_t \right] = \lim_{M \to \infty} \frac{1}{M} \sum_{m=1}^{M} \sum_{t=1}^{T} R_{t,m} \]
\[ = \lim_{M \to \infty} \left[ \frac{1}{M} \sum_{m=1}^{M} \sum_{t=1}^{T} D_{2,m}(t) \right] \]
\[ + \lambda \cdot \lim_{M \to \infty} \left[ \frac{1}{M} \sum_{m=1}^{M} \left[ \delta I_m + (\delta - 1)(1 - I_m) \right] \right] \]
\[ = \lim_{M \to \infty} \left\{ \left[ \frac{1}{M} \sum_{m=1}^{M} \sum_{t=1}^{T} D_{2,m}(t) \right] + \lambda \left[ \frac{\sum_{m=1}^{M} I_m}{M} - (1 - \delta) \right] \right\} \]
\[ = E_\pi \left[ \sum_{t=1}^{T} D_2[t] \right] + \lambda \left[ \mathbb{P}_\pi \left\{ \sum_{t=1}^{T} D_1[t] \geq N \right\} - (1 - \delta) \right] \]
\[ = L(\pi, \lambda). \]

(23)

Therefore, the maximizer of the left-hand-side expression, \( \pi^\dagger(\lambda) \), is the same as the maximizer of the right-hand-side expression, \( \pi_\lambda \), i.e., \( \pi^\dagger(\lambda) = \pi_\lambda \).

According to Theorem [1], \( \pi_\lambda \) can be derived by addressing the finite MDP. Generally, as long as the dynamics, \( p(s', r | s, a) \), are available, one can compute \( \pi_\lambda \) by dynamic programming with value iteration as presented in Table [1]. Upon the agent taking action \( a \) in the \( t \)th slot after observing state \( s \), the distribution of the next state \( s' \) and received reward \( r \) can be directly obtained based on the distribution of \( D_1[t] \) and \( D_2[t] \). In the following, we discuss the distribution of \( D_1[t] \) and \( D_2[t] \) for a general case, assuming the action taken in the \( t \)th slot is \((O_1,2, (V_1[t], V_2[t]), R_1[t], R_2[t]) \), where \( 0 < R_1[t] \leq Z^s \) and \( R_2[t] > 0 \).

According to (5), \( D_1[t] \) can take two values, i.e., \( R_1[t] \) and 0. Leveraging the exponential distribution of \( g_1[t] \) with unit mean, we can obtain
\[ P \{ D_1[t] = R_1[t] \} = \mathbb{P} \left\{ \frac{P \beta_1 g_1[t] V_1[t]}{\sigma^2 + P \beta_1 g_1[t] V_2[t]} \geq 2 \frac{Y_1 R_1[t]}{M} - 1 \right\} \]
\[ = \begin{cases} 
\exp(-\varphi_0), & \text{if } V_1[t] - V_2[t] \left( 2 \frac{Y_1 R_1[t]}{M} - 1 \right) > 0 \\
0, & \text{otherwise} 
\end{cases} \]

(24)

where \( \varphi_0 \) is given by
\[ \varphi_0 = \frac{\sigma_1^2 \left( 2 \frac{Y_1 R_1[t]}{M} - 1 \right)}{P \beta_1 \left[ V_1[t] - V_2[t] \left( 2 \frac{Y_1 R_1[t]}{M} - 1 \right) \right]} \]

(25)

Similarly, two values, \( R_2[t] \) and 0, can be taken by \( D_2[t] \) according to (6). Exploiting the exponential distribution of \( g_2[t] \) with unit mean and defining \( \varphi_1, \varphi_2, \) and \( \varphi_3 \) as
\[ \varphi_1 = \frac{P \beta_2 \left( V_1[t] - V_2[t] \left( 2 \frac{Y_1 R_1[t]}{M} - 1 \right) \right)}{\sigma_2^2 \left( 2 \frac{Y_2 R_2[t]}{M} - 1 \right)} \]
\[ \varphi_2 = \frac{P \beta_2 \left( V_2[t] - V_1[t] \left( 2 \frac{Y_2 R_2[t]}{M} - 1 \right) \right)}{\sigma_2^2 \left( 2 \frac{Y_2 R_2[t]}{M} - 1 \right)} \]
\[ \varphi_3 = \frac{P \beta_2 V_2[t]}{\sigma_2^2 \left( 2 \frac{Y_2 R_2[t]}{M} - 1 \right)} \]

(26)

(27)

(28)

it is not difficult to derive \( P \{ D_2[t] = R_2[t] \} \) shown in (29) on the top of this page, where the detailed deduction is omitted here due to space limitation. Then, we have \( P \{ D_1[t] = 0 \} = 1 - P \{ D_1[t] = R_1[t] \} \) and \( P \{ D_2[t] = 0 \} = 1 - P \{ D_2[t] = R_2[t] \} \).

From the above analysis, the dynamics, \( p(s', r | s, a) \), can be derived for decoding order \( O_1,2 \) in general cases. For the other decoding order, \( O_2,1 \), and special cases such as \( R_1[t] = 0 \) and/or \( R_2[t] = 0 \), \( p(s', r | s, a) \) can also be easily obtained using similar approaches. It is then straightforward to compute the expected capacity of \( U_2 \) during an episode and the payload delivery success probability of \( U_1 \) for any given policy, where the details are not provided here due to space limitation.

### C. Dual Variable Optimization

According to the optimization theory [11], the dual problem in (18) is always convex. In addition, as \( \lambda \) increases from zero to infinity, policy \( \pi_\lambda \) achieves nondecreasing payload delivery success probability of \( U_1 \) and nonincreasing expected capacity.
of $U_2$. Therefore, the dual problem in (13) is equivalent to minimizing $\lambda$ while keeping the reliability requirement of $U_1$ being satisfied, which can be efficiently addressed by bisection search. In particular, we initialize $\lambda_{\min}$ and $\lambda_{\max}$ to be the lower and upper bounds of the search range, respectively, where zero is assigned to $\lambda_{\min}$ and a large enough number is allocated to $\lambda_{\max}$ in general. Then, we look into the policy $\pi_{\lambda_0}$ with $\lambda_0 = (\lambda_{\min} + \lambda_{\max})/2$. In particular, we set $\lambda_{\min} = \lambda_0$ if the payload delivery success probability of $U_1$ is less than $1 - \delta$ under $\pi_{\lambda_0}$ and $\lambda_{\max} = \lambda_0$ otherwise. After continuously checking the middle point and narrowing down the search range, we have $\lambda^* = \lambda_{\max}$ when $|\lambda_{\max} - \lambda_{\min}| < \epsilon$, where $\epsilon$ is the error tolerance. It can be observed that such procedure has a low complexity as much as $O(\log(1/\epsilon))$.

IV. NUMERICAL RESULTS

Consider a specific experiment with $T = 4, N = 13, \tau = 1$ ms, $W = 1$ MHz, $P = 30$ dBm, $\beta_1 = \beta_2 = 10^{-6}$, $Y_1 = Y_2 = 1,500$ bits, $\sigma_1^2 = \sigma_2^2 = -70$ dBm, and $\delta = 0.1$. The power allocation and target rate spaces are set as $\mathcal{V} = \{(0, 0), (0, 1), (0.1, 0.9), (0.3, 0.7), (0.5, 0.5), (0.7, 0.3), (0.9, 0.1), (1, 0)\}$ and $\mathcal{R}_1 = \mathcal{R}_2 = \{0, 1, 2, 3, 4\}$. Fig. 1 demonstrates the superiority of the proposed strategy from the perspective of the expected return achieved by $\pi_2$. In particular, the joint optimization of power, rate, and decoding order achieves much higher return than the approaches with a fixed decoding order. Since the expected return of the optimal policy for any given $\lambda$ is the same as the $f(\lambda)$, it also validates the convexity of the dual function. To further look into the achieved performance, Fig. 2 shows the performance region of the strategies with and without decoding order optimization. From Fig. 2 the proposed scheme is more efficient in the sense that the interested performance metrics of the two users can be simultaneously enhanced when switching the policy from power and rate optimization to the proposed one with additional decoding order optimization.

Let $d_k$ denote the distances between the AP and $U_k$, which are uniformly distributed in the range from 10 m to 100 m. The large-scale fading is modeled as $\beta_k = 10^{-3}d_k^{-2}$ for $k \in \{1, 2\}$. By conducting 100 random realizations, Fig. 3 shows the average expected capacity of $U_2$ with different reliability requirement of $U_1$, where $T = 10, N = 16, Y_1 = Y_2 = 1,650$, $\mathcal{R}_1 = \mathcal{R}_2 = \{0, 1, 2\}$, and other parameters are set as before. It can be observed that the proposed algorithm leads to higher capacity comparing with the schemes optimizing power and rate but not decoding order. This is because the optimal decoding order may be different in different states and in different user distributions, making performance degradation under any fixed order. Moreover, the capacity of $U_2$ gets better as $\delta$ increases since the system could tilt in favor of $U_2$ as the QoS requirement of $U_1$ gets weaker.

V. CONCLUSION

A power, rate, and decoding order optimization algorithm has been developed for a two-user NOMA based 12V network with diverse QoS requirements. A finite MDP with appropriate reward design is constructed such that the agent can maximize the Lagrangian of the primal problem by optimizing its expected return in the agent-environment interaction. Then, a low-complexity bisection search based method is proposed to solve the dual problem. The superiority of the developed strategy to the baseline approaches has been validated by simulation results.

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