Numerical study of dimension effect on critical field of rectangular superconductor when The Ginzburg-Landau parameter is 3

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Abstract. Numerical studies have been done on the effect of size on critical field of superconductor with Ginzburg-Landau parameter ($\kappa$) is equal to 3. The object is a Type-II rectangular superconductor which was assumed to be in a vacuum and subjected to an external magnetic field as a function of time. The TDGL equation and boundary conditions are used to solve the problem numerically by using $\Phi_U$ method. The results show the data in the form of $\langle|\Psi|^2\rangle - H_{ext}$ graphic for each size variation. The $\langle|\Psi|^2\rangle - H_{ext}$ graphs are used to find the values of $H_{c1}$ and $H_{c3}$ against each size variation. From the results of the study, it can be seen that the larger size of superconductor will make the smaller prices of $H_{c1}$ and $H_{c3}$, until the value tend to be constant.

1. Introduction

The TDGL (Time Dependent Ginzburg-Landau) equation has been widely used to conserve the properties of superconductors [1 - 3]. However, this theory is not linear, so the solution is done by numerical study with $\Phi_U$ method [4]. The results that is obtained from this method can be said to be convergent even though the value of given external magnetic field is increased up to the value of the surface magnetic field ($H_{c3}$) [5]. Many researchers have used this solution to study the properties of superconductors with different geometric shapes and superconducting properties.

The TDGL equation can be written as below :

$$\frac{\hbar^2}{2m_0D} \left( \frac{\partial}{\partial t} + i \frac{es}{\hbar} \Phi(r, t) \right) \psi(r, t) = \frac{\hbar^2}{2m_0} \left( \nabla - i \frac{es}{\hbar} A(r, t) \right)^2 \psi(r, t) + |\alpha(T)| \psi(r, t) - \beta |\psi(r, t)|^2 \psi(r, t)$$

(1)

$$\frac{1}{\mu_0} \nabla \times \left( \nabla \times A(r, t) - \mu_0 H_{ext}(r, t) \right) = \frac{\hbar es}{2im_0} \left( \bar{\psi}(r, t) \bar{\psi}(r, t) - \psi(r, t) \bar{\psi}(r, t) - \frac{2i es}{\hbar} |\psi(r, t)|^2 A(r, t) \right)$$

$$- \sigma \left( -\nabla \Phi(r, t) - \frac{\partial A}{\partial t}(r, t) \right)$$

(2)

with the boundary condition equation is

$$\hat{n} \cdot \left( \nabla - \frac{is}{\hbar} A \right) \psi = 0,$$

(3)
where \( \mathbf{n} \) is the normal surface vector of the border field between the superconductor and the other materials, \( \psi \) is the order parameter, \( \mathbf{A} \) and \( \mathbf{H}_{\text{ext}} \) are the vector potential and the external magnetic field.

Several previous studies have successfully analyzed the properties of superconductor using this method. Including a study of the effect of the dimensions of the extent to the dynamics of Type-II mesoscopic superconductor in a square non-proximity condition with the Ginzburg-Landau parameter is \( \kappa = 2 \) [4]. Other study have analyzed the influence of Ginzburg-Landau parameter variation (\( \kappa \)) on the magnetic properties of a Type-II rectangular superconductor [6]. From some of these studies, a follow-up study was conducted to examine the effect of size on the critical fields \( H_{c1} \) and \( H_{c3} \) on a Type-II rectangular superconductor with \( \kappa = 3 \).

2. **Numerical Methods**

The superconductor that has been studied is a Type-II rectangular superconductor with \( \kappa = 3 \). Superconductor is assumed to be in a vacuum and subjected to a magnetic field as a function of time. The TDGL equation and the boundary condition equation are applied and solved by using \( \psi U \) method numerically [1,5,7 - 9]. Where for a rectangular superconductor with dimension \( N_x \times N_y \) it will consists of a set of cells with size \( \Delta x \times \Delta y \). Where for each cell is composed by three magnitudes, ie \( \psi', U^{x'} \) and \( U^{y'} \). As shown in Figure 1.

![Figure 1](image)

**Figure 1.** A model of superconductor based on \( \psi U \) method.

3. **Results and Discussion**

Variations of the dimensions of superconductor that have studied in this study are 24. Therefore, to simplify the analysis, each base is divided into 4 quadrants. First and second quadrants for small sizes, third and fourth quadrant for large sizes. First quadrant consists of 8, 12, 16, 20, 24, and 28 bases. Second quadrant consists of 32, 36, 40, 44, 48, and 52 bases. Third quadrant consists of 56, 60, 64, 68, 72 , and 76 bases. Fourth quadrant consists of 80, 84, 88, 92, 96, and 100 bases.

Graphs of the root mean square modulus parameter as a function of the external magnetic field \( \langle |\psi|^2 \rangle - H'_{\text{ext}} \) are shown in figure 2 and figure 3. When the price of the external magnetic field \( (H'_{\text{ext}}) \) increases, the root mean square modulus parameter \( \langle |\psi|^2 \rangle \) will decrease to the first local minimum point then will rise in to its first local maximum point until going to be zero. The values of low critical field \( H_{c1} \) and high critical field \( H_{c3} \) can be determined from the first local maximum point and the zero point of the graph \( \langle |\psi|^2 \rangle - H'_{\text{ext}} \) [4].
In Figure 2 and Figure 3 show that the size can affect form of the graph $\langle |\psi'|^2 \rangle - H'_{ext}$. The larger size of the superconductor will generate more local maximum points and local minimum points. Since the values of $H_{c1}$ and $H_{c3}$ is also determined from the graph $\langle |\psi'|^2 \rangle - H'_{ext}$, so they are also affected by the size. The values of $H_{c1}$ and $H_{c3}$ for each size can be seen in Figure 4.

Figure 4 proves that size can affect the values of $H_{c1}$ and $H_{c3}$. It shows that the larger size of the superconductor then the values of $H_{c1}$ and $H_{c3}$ will be smaller and will tend to have a constant value. This is due to the application of TDGL equation and the boundary conditions equation. The TDGL
equation affects the value of $\psi$ at the center of the superconductor, whereas the equation of the boundary conditions affects the edge of the superconductor. As the size of the superconductor gets larger, the effect of the TDGL equation will be more dominant than the effect of the boundary conditions equation.

![Graph of $H_{c1}$ and $H_{c3}$ relation to size ratio.](image)

Figure 5. Graph of $H_{c1}$ and $H_{c3}$ relation to size ratio.

Figure 5 shows that the larger ratio of area / circumference, will get the smaller values of $H_{c1}$ and $H_{c3}$. This is happened because the area of the superconductor can be compared with the TDGL equation and the circumference of the superconductor can be compared with the boundary conditions equation.

4. Conclusion

From the results above, it can be concluded that the size of superconductor can affect its magnetic properties. The larger size of superconductor, the values of $H_{c1}$ and $H_{c3}$ are obtained will be smaller and in certain size will tend to be stable. As well as the larger size of the superconductor, the TDGL equation will be more influential than the equation of boundary conditions. Since the TDGL equation can be compared with the area and the equations of boundary conditions can be compared with the circumference.

5. References

[1] Barba-Ortega, J., Gonzalez, J. D., and Joya, M. R. 2013 *Journal of Physics* **410** 1
[2] Kato R, Enomoto Y and Maekawa S 1993 *Physical Review B* **47** 8016
[3] Barba-Ortega, J., Becerra, A. and Aguilar, J. A. 2010 *Physica C* **470** 225
[4] Anwar, F., Nurwantoro, P., dan Hermanto, A. 2013 *J. of Natural Sciences Research* Vol.3, No.15 pp 99-106
[5] Du, Q. 2005 *J. Math. Phys.* **46** 095109-1 – 095109-22
[6] Rosyida, R., Fuad, A., Darmanto. 2017 *Journal of Physics : Conf. Ser.* **909** 012009
[7] Bolech, C., Buscaglia, G. C. and Lopez, A. 1995 *Physical Review B* **52** R15719
[8] Gropp, W. D., Kaper, H. G, Leaf, G. K., Levine, D. M., Palumbo, M. and Vinokur V. M. 1996 *Journal of Computational Physics* **123** 254
[9] Winiecki, T. and Adams, C. S. 2002 *Journal of Computational Physics* **179** 127