Possible modification of the large-scale flow structures by vortical structural interactions

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Abstract. We explore the mechanisms behind vortical structural interactions modifying large-scale structures in wall turbulence. The evidence for this in terms of vortex interactions, such as merging and intense vortex strengthening, is found in [4] in ideal flow conditions. Here, these interactions are studied experimentally and numerically in turbulent boundary layer and channel flows respectively. This is done by extracting statistical information from conditional averaging of different events based on the spanwise swirling strength. Experimental results showed vortex merger leading to vortex intensification. This was in good agreement with the results of [4]. However, numerical results did not show complete agreement with experimental results. This may be due to the difference in spatial resolution of experimental and numerical data. Furthermore, the peak Reynolds shear stress did reveal a relative increase in magnitude when two vortices merged in the numerical data.

1. Introduction
Understanding energy transfer and cascade processes in wall turbulence helps in developing better large-eddy-simulation models (LES). In LES models, large scales are explicitly computed and small scales are modelled. As the small scales contain only a small part of the turbulent kinetic energy, their effect on the computed large scales is considered limited, and they are modelled in simple ways based on scale invariance and on the universality of the small scales. Understanding the phenomenon behind the small-scale effects on the large scales may help in building better models. There has been some evidence of the small scale interactions with the large scales and of a reverse cascade in [1, 5, 11], but the mechanisms behind these cascade processes and interactions between scales is not completely understood.

Cascade processes and self-sustaining mechanisms in wall-bounded turbulence are likely dynamical, time-dependent processes. However, the study of the associated flow features has often concentrated on the description of the instantaneous structures and their spatial relationships. This can be attributed to the unavailability of time series of the three-dimensional flow-velocity field. Thanks to advances in both numerical simulations and experimental capabilities, such data are now becoming available, which allows us to consider the dynamical aspects of turbulent flow in a quantitative way.

Previous studies using the newly available time series have mainly dealt with the convection velocity and lifetimes of the various flow structures. They have also highlighted examples of vortex structures merging or splitting [2, 3, 8]. When presenting structural interactions, only
the (small-scale) vortices were included in the discussion. But these interactions may also affect the larger scales responsible for most of the turbulent kinetic energy.

Cimarelli et al. [1] showed that the energy generated near the wall (buffer region) is transferred towards the outer region and the region very close to the wall. This suggests that the small scales in the near-wall region carry energy to, or interact with, the large scales in the outer regions. This would be an example of a so-called reverse cascade in which energy is transferred from small to large scales. The net average energy transfer is still from large to small scales, in the forward direction. Ishihara et al. [5] also observe a large transfer of kinetic energy down-scale (regular cascade) as well as a large up-scale transfer (reverse cascade) in homogeneous isotropic turbulence. A strong dynamic coupling between the scales was also noted in the simulations of [11].

If the energy is transferred from the small scales to the large ones, it implies that the small-scale interactions may modify the large scales. Evidence for this in an ideal flow was observed in [4]. They studied the dynamics of two vortex interactions similar to the approach followed in [12]. The initial velocity field whose dynamics were studied was a superposition of the perturbation velocity field corresponding to two vortices conditionally extracted from a fully developed turbulent channel flow and the mean turbulent velocity profile at the same Reynolds number. It was found that the two vortices (small scale structures) merge to form a stronger vortex and increase the strength of the low-speed streaks (large-scale structures). So the merging of two vortices and strengthening of low speed streaks can be considered as one of the interactions behind the reverse cascade. However, all the simulations in [4] were in an ideal flow environment at low friction Reynolds number. Observation of this interaction in real turbulent flows at high Reynolds numbers would further strengthen the hypothesis that the large scale structures can be modified by the vortical interactions.

The purpose of this paper is to study vortex interactions and their effect on the large scales in a high-Reynolds-number fully developed turbulent channel flow [9]. A new statistical tool is used to extract the information on vortical interaction. Section 2 describes the methodology. Results and conclusions are discussed in sections 3 and 4 respectively.

2. Methodology
To retrieve statistical information about vortex merging and intense vortex strengthening, a method based on conditional averaging is used. Here the conditional average is approximated by a correlation analysis based on a particular event. We denote by \(x, y, z\) the streamwise, spanwise and wall-normal directions, and by \(u, v, w\) the corresponding instantaneous velocity components, unless mentioned otherwise. The conditionally averaged fluctuating flow field \(\bar{u}'_i(\Delta x, \Delta y, \Delta z)\) is estimated by

\[
\bar{u}'_i(\Delta x, \Delta y, \Delta z, \Delta t) = \frac{\langle u'_i(x + \Delta x, y + \Delta y, z, t + \Delta t)E(x, y, z_{ref}, t) \rangle}{\langle E(x, y, z_{ref}, t)^2 \rangle} \tag{1}
\]

where \(u'_i = (u', v', w')\) are velocity fluctuations, \(z_{ref}\) is a chosen height in the boundary layer (or equivalently a selected wall-normal distance in the channel) and \(E\) is the particular event function. The brackets \(\langle \cdot \rangle\) indicate averaging over the homogeneous flow directions (streamwise, \(x\) and spanwise, \(y\)) due to which the conditional average is only a function of the distances \(\Delta x\) and \(\Delta y\) between the reference and observation points.

Three different kinds of events are investigated in the present work. All of them are based on the two-dimensional spanwise swirling strength \(\lambda_y\). It highlights spanwise vortical elements which are correlated with the ejection events [4]. When \(\lambda_y\) increases, the ejection events are strengthened, and this strengthening is further correlated with intensification of the low-speed streaks or large-scale structures. The \(\lambda_y\) is the absolute value of the imaginary part of the
eigenvalue of the matrix

\[ J_{uw} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial z} \end{bmatrix}, \]  

and is given a sign to indicate the direction of swirl using the local spanwise vorticity \((\omega_y)\). The signed spanwise swirling strength (or signed swirling strength) is given by

\[ \lambda_y^* = \text{sign}(\omega_y) \lambda_y. \]  

The first type of event is defined in such a way that the extracted velocity field represents the evolution of a single vortex with the same sign as the mean flow velocity. It is also referred to as a type I event, and is defined by

\[ E(x, y, z_{\text{ref}}, t) = \begin{cases} \lambda_y^* & \text{if } \lambda_y^* > 0, \\ 0 & \text{otherwise.} \end{cases} \]  

The second type of event (type II) is based on the Lagrangian time development of \(\lambda_y^*\), i.e, the material derivative \(D\lambda_y^*/Dt\), and is specified as

\[ E(x, y, z_{\text{ref}}, t) = \begin{cases} \frac{D\lambda_y^*}{Dt} & \text{if } \frac{D\lambda_y^*}{Dt} > (a + b\lambda_y^*) \text{ and } \lambda_y^* > 0, \\ 0 & \text{otherwise.} \end{cases} \]  

where \(a\) and \(b\) are constants with the same dimensions as \(D\lambda_y^*/Dt\) and \(\lambda_y^*.\) The lines in the joint histogram (figure 1) of \(D\lambda_y^*/Dt, \lambda_y^*\) represent threshold levels for \(D\lambda_y^*/Dt\) in the above event. Constant \(a\) is the offset from the origin and \(b\) is the slope of the line. This event is defined in such a way that at a given point the signed swirling strength is very low, but has a large material derivative, i.e. growth rate.

The conditional average based on the third type of event (type III) gives the evolution of two vortical elements separated by a streamwise \((2\Delta x_s)\) and wall-normal \((2\Delta z_s)\) distance. Type III events are estimated by the product of the signed swirling strength at both locations as shown below:

\[ E(x, y, z_{\text{ref}}, t) = \begin{cases} \lambda_y^*(x - \Delta x_s, y, z_{\text{ref}} + \Delta z_s) \cdot \lambda_y^*(x + \Delta x_s, y, z_{\text{ref}} - \Delta z_s) & \text{if } \lambda_y^*(x - \Delta x_s, y, z_{\text{ref}} + \Delta z_s) > 0 \text{ and } \lambda_y^*(x + \Delta x_s, y, z_{\text{ref}} - \Delta z_s) > 0, \\ 0 & \text{otherwise.} \end{cases} \]  

Conditional averaging will next be carried out based on these events using experimental and numerical data of wall-bounded turbulent flows.

2.1. Numerical and Experimental data sets

The experimental dataset is a time resolved three-dimensional velocity field of a flat-plate zero-pressure-gradient turbulent boundary layer measured by tomographic-PIV [10]. The experiment was conducted in a water tunnel at TU Delft with free-stream velocity \(U_c = 0.53\) m/s. Measurements were performed at height \(0.1 < z/\delta < 0.3\), where the height of the boundary layer is \(\delta\). The streamwise location corresponds to a Reynolds number based on momentum thickness \(Re_\theta = 2460\), or to a friction Reynolds number \(Re_\tau = 800\). The spatial resolution was 55 wall units and the temporal resolution corresponds to the time taken to travel 10 wall units
in the convective direction, $\Delta t^+ = 0.47$ ('+' quantities represent wall units). The volume of each data set was $1.5\delta \times 1.5\delta \times 0.2\delta$ in streamwise, spanwise and wall-normal direction. The total number of data sets used to compute conditional average in terms of time series was equivalent to time taken to travel $100\delta/U_c$. More details on the experimental setup can be found in [10].

The numerical data are from a direct numerical simulation of fully developed turbulent channel flow at the friction Reynolds number $Re_\tau = 950$, based on the channel half-height ($h$) [9]. The domain size normalized by $h$ is $[2\pi, \pi, 2\pi]$ in streamwise, spanwise and wall-normal directions, respectively. A total of 41 samples of the above volume separated by a time difference $\Delta t^+ = 8$ are used to calculate conditional averages. All the variables throughout the paper are normalized by inner wall units except for the reference wall normal height ($z_{ref}$) which is $0.2\delta$ in the case of the experiment data and $0.2h$ in numerical data.

2.2. Vortex Identification

Vortex identification is based on the local swirling strength suggested by [12]. It is defined as the imaginary part ($\tilde{\lambda}_ci$) of a complex eigenvalue of the velocity gradient tensor ($\partial \tilde{u}_i'/\partial x_j$). If all the eigenvalues are real, the local swirling strength is zero. Vortices are visualized by plotting the iso-surfaces of $\tilde{\lambda}_ci^2$ as shown in figure 5.

Vortices are also identified by the Q-criterion [6], as shown in figure 2, which is the second invariant of velocity gradient tensor ($\partial \tilde{u}_i'/\partial x_j$). Vortices visualized by both methods look very similar.

3. Results

In this section, statistical information extracted from experimental and numerical data is presented. This information is obtained by conditionally averaging to specific events, as mentioned in section 2. First, the analysis of experimental data is considered, and then the numerical results for different events are discussed. Mainly the observations connected with vortex merging and swirling strength intensification will be examined.

3.1. Experimental results

Velocity fields conditioned to type I events were extracted in time from the experimental data at $z_{ref} = 0.2\delta$. They represent a single vortex similar to the one shown in figure 5 for the numerical data. The vortex in figure 5 is visualized by iso-contours of the maximum swirling strength, and it is broadly similar to the vortices extracted by linear stochastic estimation in
The maximum spanwise swirling strength of the conditional average at time $\Delta t$ is normalized by its value at $\Delta t^+ = 0$ and is plotted in figure 3. It can be observed that the swirling strength increases until $\Delta t^+ = 0$, i.e., the time at which the event was specified, and then decreases steadily with increasing time. This decreasing strength corresponds to a loss of correlation with increasing time shifts, which can be attributed to variations in convection velocities of the structures considered in the averaging [3]. Similarly, figure 4 shows that the peak negative Reynolds shear stress, normalized by its value at $\Delta t^+ = 0$, increases until $\Delta t^+ = 0$ and then decreases.

When conditional averaging is conditioned to a type II event, the extracted velocity field represents the merging of two vortices. The evolution of two vortices in time is shown in figure 2, where $z_{ref} = 0.2\delta$ and the conditioning constants are $a^+ = 0.0005$, $b^+ = 0.21$. Those values of $a^+$ and $b^+$ in the experimental joint probability density function of $D\lambda^*_y/Dt$, $\lambda^*_y$ isolate extreme events similar to the lines representing the limit of an event in figure 1. The upstream vortex is located farther from the wall than the downstream one, and travels faster due to higher mean velocity at that position. This decreases the streamwise separation between the vortices, leading to merging. The evolution of the normalized maximum spanwise swirling strength ($\tilde{\lambda}^+_{y,max}$) and the peak Reynolds shear stress $\tilde{u}^+\tilde{\omega}^{+}_{min}$, conditioned on a type II event, are plotted in figure 3 and 4. It can be observed that both $\tilde{\lambda}^+_{y,max}$ and $\tilde{u}^+\tilde{\omega}^{+}_{min}$ decrease more slowly after time.
\[ \Delta t^+ = 0 \] than the single-vortex case. Figure 2 at the same time (\( \Delta t^+ = 0 \)) shows merging of two vortices. Based on these observations, it can be inferred that the spanwise swirling strength and Reynolds shear stress for merging case strengthen or decrease slower than the single-vortex case, due to the merging. This is in agreement with the ideal simulation results in [4].

The same statistical procedure is used to extract structures from the numerical data, and the results are discussed in the following section.

3.2. Numerical results

The velocity field is estimated by conditional averaging based on type I events, and is visualized in figure 5 by iso-contours of the swirling strength at \( \Delta t^+ = 0 \). The single vortex looks similar to the experimental results, and to the conditionally averaged vortices in [12]. The evolution of the single vortex in terms of the maximum spanwise swirling strength (\( \hat{\lambda}_{y,\text{max}}^+ \)), which is the maximum value of the spanwise swirling strength in the whole domain at time \( t \), is shown in figure 6 normalized by its value at \( \Delta t^+ = 0 \). The trend of increasing and decreasing normalized maximum spanwise swirling strength is similar to the observations found in experiments. The value of normalized maximum \( \hat{\lambda}_{y,\text{max}}^+ \) decreases due to the loss of correlation with time.

The extracted velocity based on a rapidly increasing swirl event, i.e. a type II event at time \( \Delta t^+ = 0 \), can be seen in figure 7. It is visualized by iso-contours of the swirling strength. The type II event was specified at \( z_{\text{ref}} = 0.2h \) with constants \( a^+ = 0.005 \) and \( b^+ = 2.4 \). Different values of \( z_{\text{ref}} \), \( a^+ \) and \( b^+ \) were also used to compute the conditional averages, and are listed in table 1. For all the values in table 1 and at all the time steps, the extracted velocity field only represented a single vortex, which is completely different from the merging-vortex scenario in experiments (figure 2). Figure 6 shows the evolution of the vortex in terms of its maximum spanwise swirling strength (\( \hat{\lambda}_{y,\text{max}}^+ \)) normalized by its value at \( \Delta t^+ = 0 \). The normalized \( \hat{\lambda}_{y,\text{max}}^+ \) increases and decreases faster than the extracted flow field based on the type I event.
Figure 7. Green contour represents 5% of maximum swirling strength of a vortex extracted using type II event at $z_{ref} = 0.2h$ and $\Delta t^+ = 0$ from numerical data. Left figure is side view ($xz$ plane) and right is front view ($yz$ plane).

Table 1. Overview of all evaluations based on numerical data for type II event.

| $z_{ref}$ | $a^+$ | $b^+$ |
|-----------|-------|-------|
| 0.1$h$, 0.2$h$, 0.4$h$ | 0.005 | 2.4 |
| 0.2$h$, 0.4$h$ | 0.005 | 4.7 |
| 0.2$h$ | 0.027 | 4.7 |
| 0.2$h$ | 0.005 | 1.17 |

These results are in complete disagreement with the experimental results, perhaps due to the difference in the spatial and temporal resolutions. The spatial resolution is 55 wall units in all directions in the experimental data, compared to 7.6 and 3.8 wall units in the streamwise and spanwise direction in the numerical data. Moreover, due to an unfortunate error at the time of selecting the dataset, the temporal resolution in the numerical data corresponds to the motion of a fluid particle by approximately 140 wall units at that height, compared to 10 wall units in experimental data. The higher spatial resolution in numerical data results in vortices being sharper and smaller compared to the experiments. As the result of the smaller vortex size (approx 25 wall units) in numerical data, the fluid particle will move out of the vortex, or the material derivative de-correlates quicker in one time step. Similarly, due to coarser spatial resolution in experiments, the vortex size is larger and material derivative de-correlates slower.

Hence, a different way of extracting two vortices was considered, to study whether merging leads to swirling strength intensification. One of the trivial ways was to define a type III event, which forces two initial vortices.

Velocity was extracted by conditional averaging based on a type III event at $z_{ref} = 0.2\delta$, with initial streamwise spacings $\Delta x_s^+ = 60$ and 120 and wall normal spacing $\Delta z_s^+ = 28$. The effect of streamwise spacing was also studied, along with merging and vortex intensification. Figures 8 and 9 show the evolution of the extracted velocity field visualized by iso-contours corresponding to 5% of the maximum swirling strength at time $\Delta t^+ = 0$, 8, 16 and 32. The two initial vortices were observed to merge for both the initial streamwise separation $\Delta x_s^+ = 60$ and 120 cases. The cause for merging was the higher velocity of the upstream vortex due to its higher wall-normal distance compared to the downstream vortex. This is similar to the idealized simulations of [4].

To study vortex intensification with the conditional eddies, the time evolution of maximum spanwise swirling strength ($\tilde{\lambda}_{y,max}$) and minimum of Reynolds shear stress ($\tilde{u}'w'_{min}$), normalized by their values at $\Delta t^+ = 0$, are plotted in figure 10 and 11 respectively. In figure 10, it can be observed that the normalized $\tilde{\lambda}_{y,max}$ for streamwise spacing $\Delta x_s^+ = 120$ decreases a little faster than for the spacing $\Delta x_s^+ = 60$. This is due to slower de-correlation with increasing time steps for $\Delta x_s^+ = 60$ compared to $\Delta x_s^+ = 120$ case. The drop in correlation is dominated by variations
Figure 8. Green contour represents 5% of maximum swirling strength of vortices extracted using type III event at $z_{ref} = 0.2h$, $\Delta x_s^+ = 60$, $\Delta z_s^+ = 28$ and $\Delta t^+ = 0$ from numerical data.

Figure 9. Green iso-contours represent vortices visualized by 5% of maximum swirling strength. Figure shows three stages of the evolution and merging of the two initial vortices shown in figure 8 at time $\Delta t^+ = 8$, 16 and 32 from left to right.

Figure 10. Temporal development of the normalized maximum spanwise swirling strength ($\lambda_{y,max}^+$) in the numerical data. Extracted velocity field is based on type III event. $\Delta x_s^+ = 60$, 120 represent initial streamwise separation between two vortices.

Figure 11. Temporal development of the minimum Reynolds shear stress ($\tilde{u}' \tilde{w}'_{min}$) in the numerical data. Extracted velocity field is based on type III event. $\Delta x_s^+ = 60$, 120 represent initial streamwise separation between two vortices.

in the convective velocities [3]. Therefore it is very difficult to interpret in terms of absolute or relative vortex strength.

Figure 11 shows a faster decrease in normalized $\tilde{u}' \tilde{w}'_{min}$ for $\Delta x_s^+ = 120$ before $\Delta t^+ = 32$ and slower decrease after $\Delta t^+ = 32$ compared to $\Delta x_s^+ = 60$. Merging occurs between $16 < \Delta t^+ < 32$ for $\Delta x_s^+ = 60$ case and between $24 < \Delta t^+ < 40$ for $\Delta x_s^+ = 120$. From these observations it can be inferred that merging does have a small effect on the normalized $\tilde{u}' \tilde{w}'_{min}$, but it is not sufficient to prove any vortex intensification or strengthening.
4. Conclusions
Small-scale vortical interactions were examined using a conditional averaging approach. The interactions were studied in terms of merging, leading to vortex intensification and possible changes in Reynolds shear stress $\tilde{u}'\tilde{w}'$, which is a large-scale property. The dynamics of vortex merging and vortical interactions were extracted from experimental and numerical turbulent flow data by conditional averaging based on events defined by the signed spanwise swirling strength ($\lambda_\theta^*$) and its material derivative ($D\lambda_\theta^*/Dt$). Statistics obtained by conditioning on the type I event represented the evolution of a single vortex for both experimental and numerical data. Both cases showed a qualitative agreement with the results obtained in [4] [12].

The velocity field extracted by conditional averaging using type II event showed different results for experimental and numerical data. Velocity fields extracted from experimental data showed the merging of two vortices. It also showed an increase in the vortex strength relative to the single vortex case, due to merging. This is in qualitative and quantitative agreement with the ideal simulation results by [4]. Conditional averages obtained from numerical data only showed a single vortex at all times, in contrast to merging-vortex scenario in the experimental data. Different values for $z_{ref}$ and the constants $a$ and $b$ were tested for numerical data. None of the extracted velocity fields showed a merging-case scenario. This may be due to the coarser spatial resolution in experiments, that do not resolve scales below 55 wall units, resulting in larger vortex sizes and slower de-correlation of $D\lambda_\theta^*/Dt$, or to the coarser temporal resolution of the numerical data, which do not resolve times shorter than 150 wall units. A re-analysis using more-closely spaced numerical data is currently under way.

To test merging and vortex amplification scenario in the numerical data, the type III event was used to extract the flow field. Even though merger between two vortices was observed, there was no clear vortex amplification or even increase in swirling strength. The peak Reynolds shear stress ($\tilde{u}'\tilde{w}'$) however did reveal a relative increase in magnitude near the point of vortex merging.

To conclude, statistics based on the numerical database showed weak signs of vortical interactions modifying the large scales. More elaborate approaches are needed to get more detailed insights into the dynamic interactions between vortices and the large scales.

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