DOUBLE SPIN ASYMMETRY IN SEMI-INCLUSIVE DEEP INELASTIC SCATTERING

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We shortly review the various methods suggested for determining the transversity function. Among such methods, we consider especially those based on semi-inclusive deep inelastic scattering. In the framework of this kind of reactions, we propose to measure a double spin asymmetry, using a transversely polarized proton target and a longitudinally polarized lepton beam, and fixing the direction of the final pion. Under particular conditions, the asymmetry is sensitive to the transversity function.

1. Introduction

The transversity function, \( h_1 \), may yield nontrivial information on the nucleon structure. However, such a distribution is quite difficult to determine experimentally. In fact, in the last years, the problem has been debated at length, moreover, the data analysis of the recent HERMES experiment \(^1\) has met serious difficulties. In this situation any observable sensitive to \( h_1 \) should be taken into account. The aim of the present talk is to illustrate one such observable, consisting of a double spin asymmetry in Semi-Inclusive Deep Inelastic Scattering (SIDIS), using a transversely polarized proton and a longitudinally polarized charged lepton. We shall also review the various methods suggested in the literature for determining \( h_1 \), referring in particular to SIDIS, which presents some advantages over the other kinds of reactions.

In sect. 2 we recall the definition of \( h_1 \) and illustrate the kind of information we may extract from this function. In sect. 3 we show the difficulties concerning the measurement of the transversity function. Moreover we give a short review of the various methods suggested in the literature, referring, in particular, to the SIDIS single spin asymmetry measured in the HERMES experiment \(^2\). As an alternative, in sect. 4, we examine the possibility of a SIDIS double spin asymmetry. We consider two different cases, according as to whether the direction of the final hadron is fixed or not. We treat in detail the former case, suggesting an alternative experiment for extracting \( h_1 \). Lastly in sect. 5 we present numerical estimates of the asymmetry illustrated in sect. 4 and give a short summary.

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2. Definition of the transversity function

The transversity function is defined (see e. g., Jaffe and Ji) in terms of the tensor Dirac operator $\sigma_{\mu\nu}$. In order to understand the physical meaning of this distribution function, it is convenient to decompose the quark field in terms of given transversity. In a reference frame where the proton has a very large momentum directed perpendicularly to its polarization, it results that, for a given flavor $f$ and longitudinal fractional momentum $x$, $h_f^1(x)$ is the difference between the number density of quarks with spin aligned along the proton spin and the number density of quarks with opposite spin. Using different projection operators, we can establish two important properties of the transversity function, i. e., that

(i) $h_f^1(x)$ is a twist-two distribution function, which amounts to saying that it survives in the scaling limit;

(ii) $h_f^1(x)$ is chiral-odd, which makes it difficult to determine experimentally this function, as we shall see in the next section.

This distribution is different from the helicity distribution $\Delta q^f(x)$, for which one has to consider a proton travelling in the direction of its spin. The reason is that generally a Lorentz boost does not commute with a rotation. This would be the case, and the two distributions $h_1 = \sum_f e_f^2 h_f^1$ and $g_1 = \sum_f e_f^2 \Delta q^f$ (where $e_f$ is the fractional charge of the quark) would coincide, if the dynamics of the quarks inside the proton were non-relativistic. But we know that it is not so, because of the quark confinement and of the Heisenberg principle; furthermore some predictions of the non-relativistic quark model fail, like the value of the axial charge. Therefore we may really expect nontrivial information on the nucleon structure from the determination of $h_1$. Indeed, some authors have stressed the importance of transverse momentum in the difference between $h_1$ and $g_1$. They have shown, in the framework of the constituent quark model, that the quark transverse momentum induces nontrivial Melosh-Wigner rotations, owing to the boost from the proton rest frame to the infinite momentum frame. This causes a spin dilution both in $g_1$ and in $h_1$. This dilution, in turn, may explain, at least partially, the so-called spin crisis, consisting of a surprisingly small value, found by the EMC collaboration in 1987, of the first moment of $g_1$. But according to the model the dilution is less marked in $h_1$ than in $g_1$. Therefore the determination of $h_1$, compared with $g_1$, may shed indirectly some light on the spin crisis.

Furthermore, this determination could allow an important test for a QCD prediction on the $Q^2$ dependence of the two distribution functions. Indeed, while the QCD evolution of the singlet part of $g_1$ is coupled to the gluon polarization, which may produce sensible scaling violations, such an effect should be absent in $h_1$.

3. Difficulties in determining $h_1$

The difficulties in determining the transversity function are connected with its chiral-odd character. Indeed a massless quark conserves its chirality under any type of interactions, either electroweak or strong. It follows that, if a given asymmetry
is sensitive to this function, it must depend on the product of $h_1$ by another chiral-odd function. Therefore we have to consider reactions in which two hadrons are involved, either in the initial or in the final state. Totally inclusive Deep Inelastic Scattering (DIS) is by no means suitable for determining $h_1$.

The double spin asymmetry in Drell-Yan (DY), with two transversely polarized proton beams, results to be proportional to \[ \sum_f e^2_f (x_a) h_1^f (x_b) \], where $x_a$ and $x_b$ are the longitudinal fractional momenta of, respectively, the active quark and antiquark that annihilate into a timelike photon. Here the drawback is that this asymmetry is quite small (1-2 % at most), moreover, presumably, $|h^f_\bar{f}| << |h^f_1|$, since the antiquark necessarily belongs to the sea. Something we could gain using a polarized antiproton beam instead of one of the two proton beams, but for the moment this kind of experiment looks quite unrealistic.

More promising look asymmetry experiments based on SIDIS, that is, on reactions of the type

\[ \ell^p \to \ell^h X, \]  

where $\ell$ is a charged lepton and $h$ a hadron. The proton target is polarized, which yields information on the polarization of the initial active quark. If also the lepton beam is polarized, or if the final hadron is a spinning, unstable particle, whose decay may be detected, we are faced with a double spin asymmetry. This allows to get information on the final quark polarization and, in principle, to extract $h_1$.

But even a single spin asymmetry, i.e., with an unpolarized lepton beam, may be sensitive to the the final quark polarization, provided we are able to exploit the so-called Collins effect in the angular distribution of $h$.

A single spin asymmetry is proportional to a mixed product of the type $S \times p_a \cdot p_b$, where $S$ is the proton spin and $p_a$ and $p_b$ are any two (non collinear) momenta of the particles involved in the reaction. This object is invariant under parity inversion, but changes sign under time reversal. In other words, the cross section contains a $T$-odd term. This can only come from the interference between two amplitudes with different phases. In reaction (1), this interference is produced by the final-state interaction between $h$ and other hadrons in the final state. Therefore in this case it is the fragmentation function of $h$ that has a $T$-odd part, sensitive to the polarization of the fragmenting quark, as follows from the above mixed product. Here we identify $p_a$ and $p_b$ with the momenta, respectively, of the final quark and of $h$, and approximate the quark momentum by the momentum $q$ of the virtual photon. Then the problem amounts to determining the $T$-odd part of the fragmentation function.

To this end, consider the transverse momentum dependent (t.m.d.) fragmentation function of the final hadron, $\varphi^f (z, P_\perp)$, where $z$ and $P_\perp$ are, respectively, the longitudinal fractional momentum and the transverse momentum of the pion with respect to the fragmenting quark. While the usual fragmentation function, $D^f (z)$, is obtained simply by integrating $\varphi^f$ over the tranverse momentum, the $T$-odd part
can be extracted by weighing $\phi^f$ with the above mixed product, that is, 

$$D_{\text{odd}}^f(z) = \int d^2P_\perp \phi^f(z, P_\perp) \sin \Phi,$$  

$$\sin \Phi = \frac{S \times q \cdot p_h}{|S \times q||p_h|}.$$  

(2)

Here $p_h$ is the momentum of the hadron $h$. Moreover $\Phi$, the so-called Collins angle, is defined as the azimuthal angle between the $(q, S)$ plane and the $(p_h, S)$ plane. Notice that the $T$-odd fragmentation function is also chiral-odd. The Collins effect has been exploited in the recently realized HERMES experiment\textsuperscript{10}, where a longitudinally polarized proton has been used, and it is also invoked for the planned HERMES experiment\textsuperscript{19} with a transversely polarized target. The asymmetries, respectively twist-3 and twist-2, are both sensitive to the product $h_1^I(x)D_{\text{odd}}^f(z)$. Therefore the transversity function may be determined, provided we are able to extract the Collins fragmentation function from an independent experiment. In any case, a confirmation of the effect predicted by Collins comes both from HERMES data on the SIDIS single spin asymmetry\textsuperscript{10} and from $Z^0$ decay into two jets\textsuperscript{20}.

Variants of the Collins effect are the jet transversity determination\textsuperscript{8} in DIS and the two-pion interference method\textsuperscript{2}, applicable both to SIDIS and to proton-proton collisions.

4. Double spin asymmetry in SIDIS

Alternatively we can, in principle, extract $h_1$ from a SIDIS double spin asymmetry experiment, employing a transversely polarized proton target and a longitudinally polarized lepton beam, and detecting a pion in the final state. The asymmetry is defined as 

$$A = \frac{d\sigma_{\uparrow \rightarrow} - d\sigma_{\uparrow \leftarrow}}{d\sigma_{\uparrow \rightarrow} + d\sigma_{\uparrow \leftarrow}},$$  

(3)

where the arrows denote the the proton and lepton polarizations. Two kinds of experiments are possible, that is, fixing the final pion direction, or integrating the cross section over the pion transverse momentum with respect to the fragmenting quark. The latter possibility has been considered by Jaffe and Ji\textsuperscript{5} (JJ). The asymmetry they calculate contains twist 3 and twist 4 terms, the latter including the product $h_1^I(x)\tilde{e}_f^\pi(z)$; here $\tilde{e}_f^\pi(z)$ is the twist-3 fragmentation function of the pion, a chiral-odd function. Therefore, again, the determination of $h_1$ is subordinated to the knowledge, from an independent experiment, of another nonperturbative (and rather unusual) function.

If we keep the final pion direction fixed, the asymmetry contains one more twist 3 term, which disappears upon integration over the pion transverse momentum. This term survives for $S \cdot q = 0$, where the other terms - corresponding to the JJ asymmetry - vanish. Moreover, under this condition, such a term is sensitive to the t.m.d. transversity function, $\delta q^I(x, p_\perp)$. This can be intuitively seen by observing that, in this case, owing to the transverse momentum of the parton inside the hadron, the transverse polarization of the proton induces a longitudinal polarization
in the active quark, which may be related to $\delta q^f(x, \mathbf{p}_\perp)$. In principle there could be cancellations due to symmetries (e.g., under rotation, parity inversion, etc.), but under the conditions we shall impose this does not occur. The situation is quite analogous to the one described in ref.\[2]. There we considered a DY reaction of the type

$$p \, p^\uparrow \rightarrow \mu^+\mu^- \, X,$$

(4)

where we assumed to have one transversely polarized proton beam and to detect the longitudinal polarization of one final muon. In that case the asymmetry - which turns out to be the polarization of one of the muons - is non-zero, provided we consider nonvanishing, fixed values of the transverse momentum of the virtual photon with respect to the proton beams in the laboratory frame. Since a SIDIS reaction is kinematically isomorphic to DY, a similar effect occurs in the case considered, as we are going to show.

To this end we calculate the double spin asymmetry \[3\], taking $\mathbf{S} \cdot \mathbf{q} = 0$. In one-photon exchange approximation the differential cross section is of the type

$$d\sigma \propto L_{\mu\nu} H^{\mu\nu},$$

(5)

where $L_{\mu\nu}$ and $H_{\mu\nu}$ are the leptonic and the hadronic tensor respectively. The leptonic tensor reads, in the massless approximation,

$$L_{\mu\nu} = k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k \cdot k' + i\lambda_\ell \varepsilon_{\alpha\beta\gamma\delta} k^\alpha k'^\beta.$$  

(6)

Here $k$ and $\lambda_\ell$ are respectively the four-momentum and the helicity of the initial lepton, $k' = k - q$ the four-momentum of the final lepton and $q$ the four-momentum of the virtual photon.

As regards the hadronic tensor, we use a QCD-improved parton model\[21\]. The generalized factorization theorem\[22, 23\] in the covariant formalism\[24\] yields, at zero order in the QCD coupling constant,

$$H_{\mu\nu} \propto \sum_f e_f^2 \int d^2 p_\perp \sum_T q^f_T(x, p_\perp) \varphi^f(z, P^2_\perp) \text{Tr}(\rho^T \gamma_\mu \rho' \gamma_\nu).$$

(7)

Here $q^f_T$ is the probability density function of finding a quark or an antiquark in a pure spin state, whose third component along the proton spin is $T$. Moreover the $\rho$'s are the spin density matrices of the initial and final active parton, i.e.,

$$\rho^T = \frac{1}{2} \bar{\rho}[1 + 2T\gamma_5(\eta|| + \eta_\perp)], \quad \rho' = \frac{1}{2} \bar{\rho}'.$$  

(8)

$p$ and $p' = p + q$ are, respectively, the four-momenta of the initial and final parton; moreover $2T\eta||$ is component of the parton polarization along its momentum and $2T\eta_\perp$ the quark transverse Pauli-Lubanski four-vector; $\eta||$ is a Lorentz scalar, such that $|\eta|| \leq 1$. It is immediate to check that eqs. \[8\] are consistent with the Politzer theorem\[25\] in parton model approximation. Moreover we have

$$P_\perp = \Pi_\perp - 2p_\perp,$$

(9)
where \( \Pi \perp \) is the transverse momentum of the pion with respect to the photon momentum. We keep \( \Pi \perp \) fixed, therefore \( P \perp \) is a function of \( p \perp \). Furthermore we set \(|\Pi \perp| \leq 1 \text{ GeV}\), which is the condition for the factorization theorem to hold true \(^{18,23}\).

To calculate the asymmetry (3), we substitute eqs. (5) to (8) into that expression, resulting in

\[
A(Q, x; y, z, \Pi \perp) = \mathcal{F} \sum_{f=1}^{3} e_{f}^{2} \left( \delta Q_{f} + \delta \bar{Q}_{f} \right),
\]

\[
\mathcal{F} = \frac{y(2 - y)}{1 + (1 - y)^{2}}.
\]

Here we have set \( y = 1 - E'/E \), where \( E \) and \( E' \) are, respectively, the initial and final energy of the lepton. Moreover we have introduced the quantities

\[
Q_{f} = \int d^{2}p_{\perp} q_{f}(x, p_{\perp}^{2}) \varphi_{f}(z, P_{\perp}^{2}),
\]

\[
\delta Q_{f} = 2Q^{-1} \int d^{2}p_{\perp} \cdot S \delta q_{f}(x, p_{\perp}) \varphi_{f}(z, P_{\perp}^{2}),
\]

\[
q_{f} = \sum_{T=-1/2}^{1/2} q_{f}^{T}, \quad \delta q_{f} = \sum_{T=-1/2}^{1/2} 2Tq_{f}^{T}.
\]

\( q_{f} \) is the t.m.d. unpolarized quark distribution. We have slightly changed our notation, considering separately, for each flavor, the quark \((Q_{f}, \delta Q_{f})\) and antiquark \((\bar{Q}_{f}, \delta \bar{Q}_{f})\) contribution, the barred quantities being defined analogously to eqs. (11) and (12). Some remarks are in order.

(i) Invariance of strong interactions under parity, time reversal and rotations (in particular rotations of \( \pi \) around the proton momentum) implies

\[
\delta q_{f}(x, p_{\perp}) = \delta q_{f}(x, -p_{\perp}).
\]

This relation has two important consequences. First of all the integral at the r. h. s. of eq. (12) vanishes for \( \Pi \perp = 0 \), therefore \( \delta Q_{f} \) is proportional to the the scalar product \( \Pi_{\perp} \cdot S \). Secondly, if we consider totally inclusive DIS - which amounts to replacing \( \varphi_{f} \rightarrow 1 \)-, the integral (12) is washed out by integration over transverse momentum.

(ii) It is worth observing that, owing to the non-collinearity of the quark with respect to the proton, the t.m.d. transversity function includes, unlike \( h_{1} \), a chiral-even term, which can be calculated by changing the quantization axis from the proton momentum to the quark momentum. It is just such a chiral-even function that appears in formula (12); this is why our asymmetry formula (14), unlike the other asymmetries considered in the literature, does not contain any chiral-odd distribution or fragmentation functions.

(iii) Gauge invariance implies that QCD first order corrections, in particular graphs with one gluon exchange, contribute to the above mentioned asymmetry. However a calculation in the light cone gauge assures that such contributions are about 10% of the zero order terms.
(iv) Lastly the twist-3 character of the asymmetry (10) - which can be immediately checked from eq. (12) - forces us to pick up not too large values of $Q^2$ ($\leq 10 \text{ GeV}^2$). However this is not a serious limitation with respect to the twist-2 azimuthal asymmetries, which are plagued by a strong Sudakov suppression at large $Q^2$.

5. Numerical results and summary

Here we calculate the order of magnitude of the asymmetry (10). To this end we assume $q^f$, $\delta q^f$ and $\varphi^f$ to have a gaussian transverse momentum dependence, with the same width parameter. Then eq. (10) results in

$$A(Q, x; y; z, \Pi_\perp) = S \cdot \frac{2zF}{1 + z^2} \sum_f e_f^2 \left[ h_1^f(x)D^f(z) + \bar{h}_1^f(x)\bar{D}^f(z) \right] .$$

(15)

Taking into account eq. (15) and the second eq. (10), we see that the optimal conditions for measuring the asymmetry are (i) $y$ and $z$ as close to 1 as possible and (ii) the pion transverse momentum relative to the photon parallel to the proton polarization. Under such conditions, and taking $|\Pi_\perp| \sim 1 \text{ GeV}$ and $Q = 2.5 \text{ GeV}$, we have $A \sim 0.4R$, where $R = h_1^f(x)/q^f(x)$ has been determined by HERMES, $|R| = (50 \pm 30)\%$.

To summarize, first of all, we have shortly reviewed the methods proposed in the literature for determining $h_1$. Then we have suggested a SIDIS experiment, with a longitudinally polarized lepton and a transversely polarized proton, detecting a pion in the final state. We demand to pick up events such that the lepton scattering plane is orthogonal to the proton polarization; moreover we select pions produced in a fixed direction and at not too large angles with respect to the virtual photon momentum. The relative asymmetry is sensitive to the t.m.d. transversity function, but, unlike the other methods proposed in the literature, it does not involve any other unknown functions. The t.m.d. functions $q^f$ and $\varphi^f$ involved in asymmetry (10) can be parametrized in a well determined way. This asymmetry is estimated to be, for not too large values of $Q^2$ and under favourable kinematic conditions, at least $\sim 10\%$. The experiment could be performed at some facilities, like CERN (COMPASS coll.), DESY (HERMES coll.) or Jefferson Laboratory, where similar asymmetry measurements have been realized or planned.

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