Estimation of stability region for an interconnected AC/multi-terminal DC grid

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Abstract: This paper introduces a problem on large-signal stability of an interconnection of AC and MTDC (Multi-Terminal DC) grids. We formulate the problem as an estimation of stability region (or basin of attraction) of an asymptotically stable equilibrium point that is exhibited by coupled nonlinear differential-algebraic equations. A candidate of Lyapunov function is introduced for the interconnected AC/MTDC grid in terms of composite dynamical systems. We visualize several slices of the stability region of the interconnected grid under a practical setting of the grid’s parameters. The preliminary visualization shows that the stability region can be estimated with the candidate of composite Lyapunov function.

Key Words: power grid, multi-terminal DC grid, stability region, lyapunov function, composite system, voltage-source converter

1. Introduction

The stability of AC power grids has a long history of research with a close connection to nonlinear problems. In particular, the so-called transient stability analysis is governed by electro-mechanical motions of synchronous generators in an AC power grid with time-scale of typically seconds, called large-signal dynamics, and associated with the ability of the grid to maintain synchronism when subjected to a large disturbance [1]. The analysis is formulated as the estimation of a domain of attraction of an asymptotically stable equilibrium point, called stability region [2], for a nonlinear dynamical system that corresponds to a mathematical model of the electro-mechanical motions for a target power grid. Numerical and analytical methods such as Lyapunov/energy functions have been utilized for the estimation of stability regions for the transient stability analysis: see, e.g., [3, 4]. Complicated stability regions have been reported for a rudimentary configuration of the AC power grid [5, 6].

This paper addresses the estimation of stability region for a new configuration of power grids—an interconnection of AC and Multi-Terminal DC (MTDC) grids. The DC transmission has been
traditionally utilized in enhancement of stability and reliability of large-scale AC grids: see, e.g., [7, 8]. An MTDC grid implies that its possesses more than three terminals or buses connected via a network, which is different from the fact that most of existing DC transmissions possess two terminals only, i.e., point-to-point. For a rudimental AC grid with DC transmission based on Line-Commuted Converter (LCC), the authors of [9–11] formulated the problem of stability region and showed its complicated structures for the two different dynamical models. In the present paper, we focus on a MTDC grid based on Voltage-Source Converter (VSC). In comparison with LCC, it is pointed out in [12] that VSC is a promising technology for the future power technology in terms of modularity, controllability, and economy. The VSC-based MTDC grid is basically in the planning phase and mainly aims the transmission of electric energy from offshore wind farms to a loading center (see, e.g., [13, 14]) and the enhancement of stability and reliability of existing AC grids [15].

In [16, 17], the second author’s group reported a modularity-based modeling for large-signal dynamics in an interconnected system of AC and VSC-based MTDC grids. As mentioned above, this type of interconnection is mainly in the planning or development phase, and thus it is of basic requirement to establish a model-based method for design of the interconnection with specifications of stability and controllability. Because of the diversity of possible components, the modularity-based approach to the dynamic modeling is pursued in [16, 17].

Based on the preceding works [10, 16, 17], in this paper we estimate the stability region of an interconnected AC and VSC-based MTDC grid. The grid’s model is based on [16, 17] and introduced in Section 2. A nonlinear dynamical system to represent the large-signal dynamics of the grid is also derived in Section 2. We consider a candidate of Lyapunov function for the system in Section 3 and then investigate the dynamics of the system and associated stability region in Section 4. A preliminary version of this paper was reported as the non-reviewed proceeding [18] in Japan. The present paper contains novel formulations of the dynamical system and Lyapunov function, and numerical results on the stability region that are not published anywhere.

The contributions of this paper are three-fold. First, we formulate the estimation problem of stability region with a dynamical system described by coupled nonlinear differential-algebraic equations. This idea is originally utilized in [10] for the rudimental AC power grid with LCC-based DC transmission. The formulation in this paper for the interconnected AC/VSC-based MTDC grid is novel. Second, although still at a preliminary level, we visualize several slices of the stability region under a practical setting of the grid’s parameters. The visualization is parameterized by the amount of disturbance occurring in the MTDC grid. This will make it clear to see how the transient stability of the AC grid is affected by the MTDC grid. Third, we provide an analytical estimation of the stability region in terms of Lyapunov functions for composite dynamical systems. Several groups of researchers have constructed Lyapunov functions for stability analysis and stabilization of interconnected systems of VSC-based MTDC and AC grids [19, 20], and for design of a radial MTDC grid [21]. In this paper, we newly propose to use the idea of composite dynamical systems [22, 23] for exploring the estimation problem analytically. In particular, we focus on the approach in [2, 23] that is applied to dynamical systems with two different timescales. This is suitable to our problem because the VSC-based MTDC and AC grids exhibit multi-scale responses as numerically shown in Section 4. The idea of composite Lyapunov function is also direct to the modularity-based approach for the large-signal stability analysis.

2. Grid configuration and large-signal modeling

2.1 Grid configuration

The simple grid model which we study in this paper is based on [17] and shown in Fig. 1. The standard symmetric three-phase transmission is adopted to the AC grid. The AC grid is divided into the two areas: Area 1 as the left part in this figure and Area 2 as the right part. Area 1 includes one generation plant (equivalently, one synchronous generator), the infinite bus [24], and two VSCs connected to the MTDC grid. Area 2 includes the infinite bus and one VSC. The totally three VSCs are fed to the MTDC grid that is of bipolar-type transmission and forms a network of three DC lines.
Fig. 1. One-line diagram of interconnected AC/MTDC grid considered in this paper.

The bipolar-type transmission is normally used in practice [25].

Control scheme of VSC is crucial to dynamics of the interconnected grid. In this paper, following [17], we use the so-called Master-Slave control [26] for the three VSCs. The VSC labeled as VSC2 is “Master” in which it regulates the DC-side voltage by feedback, while the two VSCs as VSC1 and VSC3 are “Slave” in which they regulate the active power converted there. The control action in VSC is much faster than the electro-mechanical motions of the synchronous machines, as stated in [27]. Thus, we assume in this paper that the DC side-voltage at VSC2 and the active power at VCS1 and VSC3 are constant in time. In addition, it is assumed that the amount of reactive energy consumed in each VSC is constant in time regardless of the situation on the AC grid. This assumption would be relevant if the conversion dynamics, in particular, the voltage dynamics occurring the AC grid are well regulated.

2.2 Modeling of large-signal dynamics

Next, we introduce a mathematical model for large-signal dynamics of the interconnected AC/MTDC grid shown in Fig. 1. A set of variables and parameters used in the modeling is shown in Tables I and II. In what follows, the variables and parameters are normalized with the Per Unit (PU) system [10].
Table II. Variables and parameters of the MTDC grid.

| i<sub>kl</sub> | Current flowing from k-th bus to ℓ-th bus |
| i<sub>k</sub> | Current injected from VSC into k-th bus |
| v<sub>dc</sub><sup>k</sup> | DC voltage of k-th bus with slave-type VSC |
| P<sub>dc</sub><sup>k</sup> | Constant active power injected into k-th bus with slave-type VSC |
| v<sub>dc2</sub> | Constant DC voltage of the 2nd bus with master-type VSC |
| R<sub>kl</sub> | Line resistance between k-th and ℓ-th buses |
| L<sub>kl</sub> | Line inductance between k-th and ℓ-th buses |
| C<sub>k</sub> | Smoothing capacitance of VSC at k-th bus |
| C<sub>kl</sub> | Line capacitance between k-th and ℓ-th buses |

2.2.1 AC grid

We assume that the AC transmission network is loss-less, and all the AC loads are constant in time for the standard transient stability analysis. The large-signal dynamics of the AC grid are mainly governed by electro-mechanical motions of the (equivalent) synchronous generator. The motions are represented by a set of differential and algebraic equations [4] as follows:

\[
\begin{align}
\frac{d e'_q}{dt} &= -\left(\frac{T_{do}}{X_d - X'_d}\right)^{-1} \frac{\partial U_{ac}}{\partial e'_q}, \quad (1a) \\
\frac{d\delta}{dt} &= \omega, \quad (1b) \\
\frac{d\omega}{dt} &= \frac{1}{2H} \left(-D\omega - \frac{\partial U_{ac}}{\partial\delta}\right), \quad (1c) \\
0 &= \frac{\partial U_{ac}}{\partial\theta_k} + \begin{cases} 0 & k \in \{2, \ldots, 5, 7\} \\ p_{L6} & k = 6 \end{cases}, \quad (1d) \\
0 &= \frac{\partial U_{ac}}{\partial v_k}, \quad k \in \{2, \ldots, 7\} \quad (1e)
\end{align}
\]

where the scalar-valued function \( U_{ac} \) is important in this paper and defined as

\[
U_{ac}(e'_q, \delta, \theta_2, \ldots, \theta_7, v_2, \ldots, v_7) := \frac{e'^2_q}{2(X_d - X'_d)} - \frac{E_f e'_q X'_d}{X_d - X'_d} + \sum_{k=2}^{7} P_{Lk}\theta_k + p_{L6}\theta_6 + \sum_{k=2}^{7} Q_{Lk} \ln v_k + \frac{1}{2X'_d}\{e'^2_q + v_2^2 - 2e'_qv_2\cos(\delta - \theta_2)\} - P_m\delta - \frac{1}{2}\sum_{k=2}^{7}\sum_{\ell=2}^{7} B_{kl}v_kv_\ell \cos(\theta_k - \theta_\ell) - \frac{1}{X_{inf}}v_3 \cos(\theta_{inf} - \theta_3) + \frac{v_3^2}{2X_{inf}} + \frac{X'_d - X_d}{4X'_d X_d} (v_2^2 - v_3^2 \cos(2(\delta - \theta_2))).
\]

Equation (1d) (or (1e)) shows the conservation law of active (or reactive) power. The constants \( P_{L6}, P_{L7}, \) and \( P_{L8} \) in (2) stand for the values of active power flowing into the three VSCs at a steady operating condition. The additional variable \( p_{L6} \) is introduced in (1d) for bus #6 with VSC2 to represent the transient of active power exchanged between the AC and MTDC grids. For the isolated Area 1 (with no connection via the two VSCs), \( p_{L6} \) is regarded as zero.

2.2.2 MTDC grid

The dynamic model for the MTDC grid in Fig. 1 is reported in [16,17]. The model is a set of differential equations and has the currents \( i_{kl} \) flowing through every line and the voltages \( v_{dc} \) at
the buses connected to slave-type VSCs. The direction of the positiveness of currents is shown with arrows in Fig. 1. The equations are

\[
\begin{align*}
\frac{di_{12}}{dt} &= \frac{1}{L_{12}} (-R_{12}i_{12} + v_{dc1} - V_{dc2}) \\
\frac{di_{13}}{dt} &= \frac{1}{L_{13}} (-R_{13}i_{13} + v_{dc1} - v_{dc3}) \\
\frac{di_{23}}{dt} &= \frac{1}{L_{23}} (-R_{23}i_{23} + V_{dc2} - v_{dc3}) \\
\frac{dv_{dc1}}{dt} &= \frac{1}{\bar{C}_1} \{i_1 - (i_{12} + i_{13})\} \\
\frac{dv_{dc3}}{dt} &= \frac{1}{\bar{C}_3} \{i_3 - (-i_{13} - i_{23})\}
\end{align*}
\]

where \(V_{dc2}\) is assumed to be constant as the master-type VSC, and

\[
\bar{C}_1 := C_1 + \frac{C_{12} + C_{13}}{2}, \quad \bar{C}_3 := C_3 + \frac{C_{13} + C_{23}}{2}.
\]

Here, based on the control scheme in Section 2.1, we have the following constraints of variables at every slave-type VSC:

\[
i_1 = \frac{P_{dc1}}{v_{dc1}}, \quad i_3 = \frac{P_{dc3}}{v_{dc3}} \quad (v_{dck} \neq 0)
\]

where \(P_{dc1}\) and \(P_{dc3}\) are constant. It should be noted that they are nonlinear constraints in terms of the variables (voltages). Thus, the differential Eq. (3) are nonlinear. This type of the nonlinear equations is investigated in [28, 29] for stability and bifurcation of DC-bus networks.

### 2.2.3 Interconnection

The modeling of interconnection is a key to the dynamic modeling of the AC/MTDC grid. In this paper, following [10], we introduce equations to represent the power conversion via VSC that is equivalent to the conservation law of power. For the master-type VSC (VSC2), the equation is given by

\[
p_{L6} + P_{L6} = V_{dc2}i_{2} = V_{dc2}(-i_{12} + i_{23})
\]

where \(P_{L6}\) is again the constant power fed from bus #6 linked to VSC2 at the steady operating condition. The value of \(P_{L6}\) can change in time when the states of the AC and MTDC grids are apart from the steady condition. Also, for the slave-type VSCs (VSC1 and VSC3), the equations are given by

\[
P_{L7} = P_{dc1}
\]

\[
P_{L8} = P_{dc3}
\]

where \(P_{L7}\) (or \(P_{L8}\) is the power fed from bus #7 (or #8) linked to VSC1 (or VSC3). These imply that the slave-type VSCs are regarded as constant loads from the AC grid, and that at the master-type VSC, the dynamics of the DC grid affect the AC grid in a unidirectional manner.

As the result of the above derivations, by integrating (1) for the AC grid, (3), (5), (6), (7), and (8) for the MTDC grid, it is possible to construct the mathematical model for large-signal dynamics of the AC/MTDC grid shown in Fig. 1. The model is a form of coupled nonlinear differential-algebraic equations as

\[
\frac{dx_{ac}}{dt} = f_{ac}(x_{ac}, y_{ac})
\]

\[
0 = g_{ac}(x_{ac}, y_{ac}) + g_{ac/mtdc}(x_{mtdc})
\]

\[
\frac{dx_{mtdc}}{dt} = f_{mtdc}(x_{mtdc})
\]
where \( x_{ac} := [\theta^*, \delta, \omega]^\top, \ y_{ac} := [\theta_2, \ldots, \theta_7, v_2, \ldots, v_7]^\top, \) and \( x_{mtdc} := [i_{d12}, i_{d13}, i_{qdc1}, i_{qdc3}]^\top \) (\( \top \) denotes the transpose operation of vectors). The vector-valued function \( f_{ac} \) represents the right-hand sides of the differential Eqs. (1a), (1b), and (1c), and \( f_{mtdc} \) represents the right-hand sides of (3). The function \( g_{ac} \) represents the right-hand sides of the algebraic Eqs. (1d) and (1e), and \( g_{ac/mtdc} \) represents the term \( pL_6 \) of (6).

We here provide several remarks on the DAE system. First, as mentioned before, the interconnection between the AC and MTDC grids appear in the algebraic Eq. (9b). The interconnection is unidirectional, precisely speaking, the MTDC grid can affect the AC grid in transient states, while the AC grid does not affect the MTDC grid. This originates from the choice of control scheme in Section 2.1 and the model simplification in this paper, for which we ignore detailed fast-timescale dynamics of power conversion. Second, related to the first one, it is indicated in [10] that the bidirectional interconnection appears between the AC and LCC-based DC grids. This difference is based on the fundamental characteristics of VSC and LCC. Regarding this, the model structure in this paper is relatively simpler than that in [10], implying that the idea of composite dynamical systems would work in this paper. Finally, both the interconnected grids with LCC and VSC are modeled as the coupled differential-algebraic equations. In this sense, the following statement in [30] can hold for the current analysis of VCS-based MTDC: “enhancement of transient stability via dc transmission.”

3. Candidate of composite lyapunov function

This section introduces a candidate of Lyapunov function for the estimation of stability region of the interconnected AC/MTDC grid described by the differential-algebraic Eq. (9). The notion of (local) Lyapunov function for the semi-explicit DAE system (9) with applications to power grids is presented in [31, 32].

In order to construct a candidate of Lyapunov function, we suppose that there exists an asymptotically stable Equilibrium Point (EP) of the DAE system (9), denoted by \( x_{ac}^0 := [\theta_q^0, \delta^0, 0]^\top, \ y_{ac}^0 := [\theta_2^0, \ldots, \theta_7^0, v_2^0, \ldots, v_7^0]^\top, \) and \( x_{mtdc}^0 := [i_{d12}^0, i_{d13}^0, i_{qdc1}^0, i_{qdc3}^0]^\top. \) See [10] for the definition of asymptotic stability of EP for semi-explicit DAE systems. As physical constraints, it is also supposed that \( v^{*}_{q}, v^{*}_{d} > 0, \) and \( v^{*}_{dck} > 0 \) hold. Here, in order to investigate trajectories of the DAE system (9), their existence and uniqueness in a neighborhood of the EP are also supposed, which will be mentioned in Section 4.

First of all, we introduce the candidate of Lyapunov function for the dynamics of the isolated AC grid, defined as

\[
W_{ac}(x_{ac}, y_{ac}) := \frac{1}{2}(2H)\omega^2 + U_{ac}(\theta^*, \delta, y_{ac}) - U_{ac}(\theta^*_{q}, \delta^*, y^*_{ac}). \tag{10}
\]

This function is assumed to be is positive-definite in a neighborhood of the EP. This assumption will be checked numerically in the next section. The time derivative of \( W_{ac} \) along feasible trajectories of the DAE system (9a,9b) without the term \( g_{ac/mtdc} \) is calculated as follows:

\[
\frac{dW_{ac}}{dt} = \left[ \frac{\partial U_{ac}}{\partial \theta^*_{q}}, \frac{\partial U_{ac}}{\partial \delta}, 2H\omega \right] \frac{dx_{ac}}{dt} + \left[ \frac{\partial U_{ac}}{\partial \theta_2}, \ldots, \frac{\partial U_{ac}}{\partial \theta_7}, \frac{\partial U_{ac}}{\partial v_2}, \ldots, \frac{\partial U_{ac}}{\partial v_7} \right] \frac{dy_{ac}}{dt}
\]

\[
= -D\omega^2 - \frac{T_{d0}^*}{X_d - X_d'} \left( \frac{dc^*_q}{dt} \right)^2 \leq 0. \tag{11}
\]

The last inequality holds because the parameters of synchronous machines generally satisfy \( T_{d0}^* > 0 \) and \( X_d > X_d' \) [24]. Also, the last equality holds if all the variables \( [x_{ac}, y_{ac}] \) are at the EP. Therefore, \( W_{ac} \) is a Lyapunov function for the EP in terms of the dynamics of the isolated AC grid. The Lyapunov function of the AC grid is composed of kinetic energy and potential energy. The kinetic energy is associated with the inertia and speed of the synchronous generator, and the potential energy
is associated with the electrical characteristics of transmission lines and loads, and the mechanical input power of generator [4].

Second, we introduce the following candidate of Lyapunov function for the dynamics of the isolated MTDC grid:

\[ W_{\text{mtdc}}(x_{\text{mtdc}}) := \frac{1}{2} \frac{1}{L_{12} L_{13} L_{23} C_1 C_3} \left\{ L_{12} (\Delta i_{12})^2 + L_{13} (\Delta i_{13})^2 + L_{23} (\Delta i_{23})^2 + C_1 (\Delta v_{dc1})^2 + C_3 (\Delta v_{dc3})^2 \right\} \]

(12)

where \( \Delta i_{12} := i_{12} - i_{12}^*, \Delta i_{13} := i_{13} - i_{13}^*, \Delta i_{23} := i_{23} - i_{23}^*, \Delta v_{dc1} := v_{dc1} - v_{dc1}^*, \) and \( \Delta v_{dc3} := v_{dc3} - v_{dc3}^* \). The function is clearly positive-definite. The time derivative of \( W_{\text{mtdc}} \) along trajectories of the system (9c) is derived as follows:

\[ \frac{dW_{\text{mtdc}}}{dt} = \frac{1}{L_{12} L_{13} L_{23} C_1 C_3} \left\{ -R_{12} (\Delta i_{12})^2 - R_{13} (\Delta i_{13})^2 - R_{23} (\Delta i_{23})^2 - \frac{P_{\text{dc1}} (\Delta v_{dc1})^2}{v_{dc1} v_{dc1}^*} \right\} \]

\[ \leq 0 \quad (P_{\text{dc1}}, P_{\text{dc3}} > 0). \]

(13)

The above equality holds only if all the variables are at the EP. Therefore, under the positiveness condition of loads \( P_{\text{dc1}} \) and \( P_{\text{dc3}} \), \( W_{\text{mtdc}} \) is a Lyapunov function for the EP in terms of the dynamics of the isolated MTDC grid.

Lastly, following [22, 23], we consider the candidate of composite Lyapunov function for the interconnected AC/MTDC grid, as a weighted sum of the two individual Lyapunov functions described by

\[ W(x_{\text{ac}}, y_{\text{ac}}, x_{\text{mtdc}}) := d_{\text{ac}} W_{\text{ac}}(x_{\text{ac}}, y_{\text{ac}}) + d_{\text{mtdc}} W_{\text{mtdc}}(x_{\text{mtdc}}) \]

(14)

where \( d_{\text{ac}} \) and \( d_{\text{mtdc}} \) are non-negative constants. The candidate is clearly positive-definite in a neighborhood of the EP by the construction of the individual functions. Thus, it is necessary to check its time derivative along feasible trajectories of the DAE system (9). The derivative is calculated as follows:

\[ \frac{dW}{dt} = d_{\text{ac}} \frac{dW_{\text{ac}}}{dt} + d_{\text{mtdc}} \frac{dW_{\text{mtdc}}}{dt} \]

\[ = d_{\text{ac}} \left\{ -D \omega^2 - \frac{T_{\theta \phi}'}{X_d - X_d'} \left( \frac{de_{\theta}'}{dt} \right)^2 + d_{\text{ac}} P_{\text{L}, \theta} \frac{d\theta_\theta}{dt} + d_{\text{mtdc}} \frac{dW_{\text{mtdc}}}{dt} \right\} + d_{\text{ac}} V_{\text{dc2}} \left( -\Delta i_{12} + \Delta i_{23} \right) \frac{d\theta_\phi}{dt} + d_{\text{mtdc}} \frac{dW_{\text{mtdc}}}{dt}. \]

(15)

The term \( d_{\text{ac}} V_{\text{dc2}} \left( -\Delta i_{12} + \Delta i_{23} \right) \frac{d\theta_\theta}{dt} \) in (15) indicates the effect of interconnection between the AC and MTDC grids. Note that the first and third terms on the right-hand side of (15) are non-positive in a neighborhood of EP. Thus, to guarantee the condition of time derivative for Lyapunov functions, the following condition for the two parameters \( d_{\text{ac}} \) and \( d_{\text{mtdc}} \) is required:

\[ d_{\text{ac}} \left\{ -D \omega^2 - \frac{T_{\theta \phi}'}{X_d - X_d'} \left( \frac{de_{\theta}'}{dt} \right)^2 + d_{\text{mtdc}} \frac{dW_{\text{mtdc}}}{dt} \right\} \leq -d_{\text{ac}} V_{\text{dc2}} \left( -\Delta i_{12} + \Delta i_{23} \right) \frac{d\theta_\theta}{dt}. \]

(16)

We here provide two remarks on the above condition. The condition might hold by appropriately choosing \( d_{\text{ac}} \) and \( d_{\text{mtdc}} \). This is one of our future works in theoretical and numerical (optimization) studies. In the next section, we will show one choice of \( d_{\text{ac}} \) and \( d_{\text{mtdc}} \) in terms of the estimation of stability region. Also, the composite idea is related to the decomposition of energy function for hierarchical stability assessment in AC power grids [33], which might become applicable to general AC/MTDC grids.
Table III. Parameter setting of the AC grid.

| \( X_{d} \) | \( X_{q} \) | \( X_{d}' \) | \( X_{q}' \) | \( P_{m} \) | \( P_{l} \) |
|---------------|----------|----------|----------|----------|----------|
| 1.79          | 1.77     | 0.3      | 0.5      | 1.0      | 1000 MVA |
| \( T_{d0} \) | \( T_{d1} \) | \( D \) | \( H \) | \( E_{f,d} \) | 1.7026   |
| (12s) \( \times (120\pi s^{-1}) \) | \( 1.0 \) | \( 0.898 \) \( \times (120\pi s^{-1}) \) | 617     |
| Q_{1,6} \( \) | 0.1      | P_{L7}   | 0.2      | Q_{1,7}  | 0.1      |
| Q_{1,8} \( \) | 0.1      | V_{inf}  | 1.0      | \( \theta_{inf} \) | 0       |
| V_{g} \( \) | 1.0      | \( \delta \) | -0.0151  | \( \theta \) | 0       |
| B_{24} \( \) | 10       | B_{35}   | 20       | B_{16}  | 5.8824   |
| B_{22} \( \) | -15      | B_{33}   | -25      | B_{14}  | -15.8824 |
| B_{66} \( \) | -5.8824  | B_{77}   | -5.8824  | X_{inf} | 0.05     |

Table IV. Parameter setting of the MTDC grid.

| \( \theta \) | \( \delta \) | \( \delta \) | \( \delta \) | \( \delta \) |
|---------------|----------|----------|----------|----------|
| \( P_{dc1} \) | 0.2      | \( P_{dc3} \) | 0.3      | \( V_{dc2} \) | 1.0      |
| \( R_{13} \) | 0.10875  | \( R_{23} \) | 0.12     | \( L_{12} \) | 0.33134  |
| \( L_{23} \) | 0.47124  | C_{12}   | 2.20042  | C_{13}   | 2.83610  |
| C_{1}         | 4.93100  | C_{4}    | 5.39554  | 1.0      | 1.0      |

4. Numerical studies

This section provides numerical simulations of the DAE system (9) and the candidate of composite Lyapunov function in (14). In particular, we visualize several slices of the stability region of the interconnected AC/MTDC grid and their estimation using the candidate of composite Lyapunov function.

4.1 Simulation setting

First of all, we summarize the setting for numerical simulations in this paper. Tables III and IV represent the values of constant parameters used here. The values in the PU system were set from [17]. The base quantities of the PU system were 1000 MVA and 500 kV for the AC grid and 1000 MW and 320 kV for the MTDC grid. The nominal frequency of the AC grid (for both left and right areas in Fig. 1) was set at 60 Hz (from Japan’s western transmission grid). All numerical integrations were performed in MATLAB with the implicit trapezoidal method [34]. The time step of the implicit trapezoidal method was 0.001 sec.

Here, to visualize the stability region, we compute trajectories of the DAE system (9) starting from multiple initial conditions of \((x_{ac}, y_{ac}, x_{mtdc})\). The initial conditions are taken from two-dimensional slices of the phase space that satisfy the algebraic Eq. (9b). For the visualization, it is required to check whether a trajectory \((x_{ac}(t; t_{0}), y_{ac}(t; t_{0}), x_{mtdc}(t; t_{0}))\) starting at initial time \(t_{0}\) sufficiently converges to the EP as time executes. The convergence is numerically judged when \((x_{ac}(t; t_{0}), y_{ac}(t; t_{0}), x_{mtdc}(t; t_{0}))\) satisfies the following equation of norm:

\[
\sqrt{\left\{ e_q^r(t; t_{0}) - e_q^r(t; t_{0}) \right\}^2 + \left\{ \delta(t; t_{0}) - \delta(t; t_{0}) \right\}^2 + \omega(t; t_{0})^2} \leq 0.01, \tag{17}
\]

where \(t - t_{0}\) is smaller than the pre-defined final execution time, 100 sec. If the trajectory does not satisfy (17) in 100 sec, then we judge that it does not converge to the EP. Also, we observe a case that the implicit trapezoidal integration stops because no trajectory (including initial condition) is obtained to satisfy the algebraic equations. This is related to the loss of existence or uniqueness of trajectories that originates from the singularity of the algebraic equations [10]. The values of the stable EP for the convergence condition are shown in Table V. At the located EP, all the components of partial derivatives of \(U_{ac}(e_q^r, \delta, y_{ac})\) are 0, and the associated Hessian matrix is positive-definite. Therefore, \(U_{ac}(e_q^r, \delta, y_{ac}) = U_{ac}(e_q^r, \delta, y_{ac})\) in (10) becomes positive in a neighborhood of the EP, implying that the assumption of positive-definiteness of the function \(W_{ac}(10)\) holds for the EP under Table V.
Table V. The values of stable equilibrium point of the DAE system.

|       | $\delta^*\rangle$ | $\omega^*\rangle$ | $e_{q}^*\rangle$ | $v_{dc1}^*$ | $v_{dc3}^*$ | $i_{12}^*$ | $i_{13}^*$ |
|-------|--------------------|--------------------|-------------------|-------------|-------------|------------|------------|
| $i_{11}^*$ | -0.0591            | -0.2328            | 1.0215            | 0           | 0           | 0.2549     | -0.0591    |
| $e_{1}^*$   | 0.9811             | 0.9869             | 0.9645            | 0.9424      | 0.9630      | 0.9645     | 0.9869     |
| $v_{0}^*$   | 0.9424             | 0.9630             | 0.2454            | 0.2969      | 0.0296      | 0.3882     | 0.0399     |
| $\theta_{1}^*$ | 0.2969          | 0.0296             | 0.3882            | 0.0064      | -0.0064     | 0.0399     | 0.0399     |

Fig. 2. Visualization of stability region of the interconnected AC/MTDC grid. The light blue point indicates located at the center the stable EP, the blue region the stability region, and the orange region the region from which trajectories do not converge to the EP. The black curves the level sets of the candidate Lyapunov function (14) under $d_{ac} = 1$ and $d_{mtdc} = 20$.  

(a) $i_{12}(0) = i_{12}^*$  

(b) $i_{12}(0) = i_{12}^* + 0.5$  

(c) $i_{12}(0) = i_{12}^* - 0.5$
4.2 Results and discussion

Figure 2 shows three slices of stability region of the EP for the interconnected AC/MTDC grid. For each figure, we used totally the 308481 initial conditions for the visualization (961 conditions for \( \delta \) and 321 for \( \omega \)). The initial conditions \( \delta(0) \) and \( \omega(0) \) apart from the EP represent disturbances to the left AC grid in Fig. 1. In addition to \( \delta(0) \) and \( \omega(0) \), the initial condition of the DC current \( i_{12} \) is also considered to represent disturbances to the MTDC grid: \( i_{12}(0) = i_{12}^* \) for Fig. 2(a), \( i_{12}(0) = i_{12}^* + 0.5 \) for Fig. 2(b), and \( i_{12}(0) = i_{12}^* - 0.5 \) for Fig. 2(c). In these figures, the light blue point located at the center indicates the stable EP, the blue region the stability region, and the orange region the region from which trajectories do not converge to the EP.

We discuss the obtained visualization of stability region in Fig. 2. On the one hand, the outer boundaries (smooth curves) between the blue and orange regions are similar in each of the three slices. This would originate from the unidirectional interconnection between the AC and MTDC grids as shown in the DAE system (9). An example of trajectories of converging to the EP is shown in Fig. 3(a). The three trajectories starting from the different values of \( i_{12}(0) \) behave in almost similar manner. This is because the dynamics of the MTDC grid is faster than those of the AC grid and converges to the steady condition for the set of initial conditions. Thus, the stability of the AC grid is less affected by the MTDC grid in this setting. This also holds for the non-convergence case as shown in Fig. 3(b) (note that the trajectories are terminated at about 0.4 sec). These suggest a reason why the outer boundaries are similar. On the other hand, the sharp erosion of the stability region appears in their right sides. This type of sharp erosion is reported in Figs. 2 and 3 of [10] for the coupled differential-algebraic equations. This would originate from the singularity of the algebraic equations mentioned above and can be interpreted physically in what follows. In these figures, we changed the initial values of the DC current \( i_{12} \) and thereby changed the loading condition at bus #6 in the left.
The large deviation of $i_{12}$ makes it difficult to locate a solution of the algebraic equations that are functions of the loading condition at bus #6. This becomes significant in the right sides of the figures because the rotor angle $\delta$ around them is larger than the value at EP, and hence the power-flow condition becomes heavy and close to the critical point [35]. This is why the sharp erosion of the stability region appears in the right sides and the area of the stability region becomes small in Figs. 2(b) and 2(c).

Also, we discuss the estimation of stability region based on the candidate Lyapunov function (14). In Fig. 2 the black curves show the level sets of the candidate Lyapunov function (14) under $d_{ac} = 1$ and $d_{mtdc} = 20$. The values of the level sets are 0.2258, 0.4516, 0.6773, 0.9031 in Fig. 2(a), 0.2712, 0.3433, 0.4155, 0.4876 in Fig. 2(b), and 0.4238, 0.6485, 0.8732, 1.0979 in Fig. 2(c). In each interior of the level set, we confirm that the condition (16) holds for the computed convergent trajectories. An example of the time evolutions of the values of candidate Lyapunov function along trajectories is shown in Fig. 4. This implies that they decrease monotonically as time executes. The short-term responses in Fig. 4(b) are affected by the initial conditions $i_{12}$ because the current $d_{mtdc}$ is larger than $d_{ac}$ and hence the function $W_{mtdc}$ controls dominantly the value of candidate Lyapunov function. Thus, we suggest that the level sets provide analytical estimations of the stability region. In each of the figures, the level set reaching the boundary of stability region (between the blue and orange regions) shows the optimal estimation under the current setting $d_{ac} = 1$ and $d_{mtdc} = 20$ in the sense that the area of the estimation is largest on the slice. The current choice of $d_{ac}$ and $d_{mtdc}$ is one example, and their better choice for the estimation is in our future work.

5. Concluding remarks

This paper formulated a nonlinear problem on the large-signal stability for the interconnected AC/MTDC grid. The coupled nonlinear differential-algebraic equations were derived to the problem. Numerical simulations of the equations were performed for visualizing several slices of the stability region. Also,
the idea of composite Lyapunov function was introduced to analytically estimate the stability region. These show that the idea works effectively for exploring the stability region of the interconnected AC/MTDC grid.

Many follow-up studies to the present paper are possible. One is to determine \( d_{ac} \) and \( d_{mtdc} \) with an optimization technique to derive the optimal estimation of stability region. Another is to construct a composite Lyapunov function for a model with general and practical control schemes of VSC. The other is to conduct the same modeling and analysis for a grid with modular multilevel converter [36] that has attracted a lot of interest in power electronics field.

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