Robust tracking control for Robotic Manipulators based on Super-twisting Algorithm

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Abstract: Robotic manipulators are broadly used in industries for different kinds of specialized operational responsibilities and it is very complicated to design of a robust and stable controller for these systems. The performance of industrial robotic manipulators has augmented concerning stability and safety due to growths in robust controller design. However, designing a stable and reliable control method for a robot manipulator is important for real-world applications. This paper proposed a robust composite super-twisting sliding mode controller (STSMC) for robotic manipulators with uncertainties. To improve the robustness of robotic manipulators and reduce the chattering problem of sliding mode control, the Super-twisting algorithm is employed to design a continuous controller, and the stability is proved. Simulations on the 2-DOF robotic manipulator are shown to illustrate the effectiveness of the proposed method in which demonstrated that the proposed model has smoothness at 10⁻⁵.

1. Introduction

Robotic manipulators are widely used in different kinds of industries for different types of specialized operational responsibilities. These systems are enthusiastically coupled, time-varying and nonlinear. Therefore, it is very difficult to design a stable and robust controller for these kinds of systems. In recent few years, Research control methodologies for industrial robots have increased. The performance of industrial robotic manipulators has augmented regarding safety and stability due to developments in robust controller design. However, it is important to design a reliable and stable control method for a robot manipulator for real-world applications [1], [2]. It is essential to develop and apply control algorithms to complicated tasks. Several types of controllers have been recognized for robot manipulators. These controllers can be separated into two main classes: linear control systems and nonlinear control systems. Non-linear controllers are recommended in this research because in linear control some challenges are needed to overcome the limitation of the velocity and acceleration of the system. Nonlinear control algorithms are sub-divided into three main types: model-based control algorithms, soft computing (artificial intelligence)-based control theory and hybrid control actions. The main benefit of a nonlinear model-free controller is structure information, which has been improved by researchers over the years [1–6]. While it has several advantages, this method has challenges associated with system reliability, precision and robustness. Model reference control algorithms are suggested to improve the stability, robustness, and reliability. The feedback linearization method (FLC), back-stepping control procedure (BSC), passivity-based control (PBC), Lyapunov-based algorithm (PBC) and sliding mode control (SMC) are general methods for designing...
model-based controllers. However, selecting a suitable control technique is a foremost challenge for many researchers.

In recent years, different kinds of methodologies have been used to control the systems and the Sliding Mode Control (SMC) [7], [8] is one of the efficient techniques to control a complicated dynamic system. This controller reduces the sensitivity to disturbances and plant parameters differences. Another kind of control for robotics systems as the adaptive control, adaptive fuzzy sliding mode [9], [10], fuzzy logic, neural adaptive [11], neural networks [12], sliding mode neuronal network [13], and neuro-fuzzy.

SMC became the main working mode for the situation of robustness in the class of control systems [14], [15]. For the nature of this control, the principal disadvantage is the chattering phenomenon caused by the “sign” function while forcing the states to zero. The chattering problem is dangerous for the physical systems high-frequency oscillations. A method to reduce the chattering is the Super-twisting Algorithm (STA) [16], this algorithm was developed to avoid the chattering effect. The Super-twisting Control (STC) can be applied to any system where it can obtain the first derivative of the sliding variable. The main advantage of STC is that it compensates uncertainties and perturbations and guarantees the system of finite time convergence to the origin of the sliding variable and its derivate [17].

The main idea of this paper is to propose a controller for robotic manipulator based on a super twisting algorithm. To improve the trajectory tracking is the priority of this paper using a continuous controller. MATLAB/Simulink was used to validate the dynamic model and simulate the proposed controller for the realization of a particular task.

The rest of the paper is designated as a mathematical model of robot manipulator in section 2, the proposed design is discussed in section 3, and the stability will be described in section 4. In section 5 Results and analysis presented. In the last conclusion of this paper is described.

2. A mathematical model of Robot Manipulator
The primary challenge in this study is to control the robot manipulator using Super-twisting controller in the presence of uncertainty and disturbance. The dynamics of serial n-link is described in [18].

\[
M(q)\ddot{q} + V(q, \dot{q})\dot{q} + g(q) = \tau
\]  

(1)

Where \(M(q) \in \mathbb{R}^{n \times n}\) Inertial vector Matrix, \(V(q, \dot{q}) \in \mathbb{R}^{n \times 1}\) Centripetal and Coriolis vector Matrix, \(g(q) \in \mathbb{R}^{n \times 1}\) is the Gravitational vector Matrix, \(q \in \mathbb{R}^{n \times 1}\) is the Position Vector Matrix, \(\dot{q} \in \mathbb{R}^{n \times 1}\) is the Velocity vector Matrix and \(\tau \in \mathbb{R}^{n \times 1}\) is the applied input torque Matrix. The two-dimensional state vector \(x\) is defined as:

\[
x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}
\]  

(2)

Then the non-linear system in (1) can be represented as a state space model as follows:

\[
\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = M^{-1}(x_1)\left[\tau - V(x_1, x_2)x_2 - g(x_1)\right]
\end{cases}
\]  

(3)

Where \(y\) is the output of the system.

3. Super-twisting Controller Design
3.1. Design of controller
For system (1) the desired homogenous linear time-invariant sliding surface can be written as follows in equation (4)
\[ s = cx_1 + x_2 \] (4)

where \( s, (c > 0), x_1, x_2 \) are sliding surface, sliding polynomial, position and velocity of the system respectively. In robotics, the main principle of designing super twisting control law is to move the error and derivative of error to zero. So, sliding surface can be written in the form of error as follows:

\[ s = ce + \dot{e} \] (5)

where \( e, \dot{e} \) are error and derivative of error and can be described in (6) and (7):

\[ e = x_1^d - x_1 = q_d - q = \begin{bmatrix} q_{d1} - q_1 \\ q_{d2} - q_2 \end{bmatrix} \] (6)

\[ \dot{e} = \dot{x}_1^d - \dot{x}_1 = x_2^d - x_2 = \begin{bmatrix} \dot{q}_{d1} - \dot{q}_1 \\ \dot{q}_{d2} - \dot{q}_2 \end{bmatrix} \] (7)

where, \( x_1^d \) and \( \dot{x}_1^d \) are the desired position and velocity of the system, respectively.

To designed super twisting control for this system, we need to take the derivative of the sliding surface. Equation (5) can be written in the following form:

\[ \ddot{s} = c\ddot{e} + \dot{\ddot{e}} \] (8)

where, \( \ddot{e} \) is the acceleration error. Now substituting the value of (6) and (7) in (8) we get

\[ \ddot{s} = c\ddot{e} + \dot{x}_2^d - \dot{x}_2 
\]

\[ = c\ddot{e} + \dot{x}_2^d - M^{-1}(x_1)[\tau - V(x_1, x_2)x_2 - g(x_1)] \] (9)

The total \( \tau \) of the proposed system can be designed as follows:

\[ \tau = M(x_1)\left[(c\ddot{e} + \dot{x}_2^d + u_{STA} + u_s) + V(x_1, x_2)x_2 + g(x_1) \right] \] (10)

where, \( u_s \) & \( u_{STA} \) are the sliding mode control and super twisting algorithm respectively, and can be defined in (11) & (12) respectively:

\[
\begin{aligned}
  u_{STA} &= k_2|s|^{1/2}\text{sign}(s) + v \\
  v &= -k_3\text{sign}(s)
\end{aligned}
\]  

where, \( k_2, k_3 > 0 \)

and

\[
\text{sign}(s) = \begin{cases} 
  -1, & s < 0 \\
  0, & s = 0 \\
  1, & s > 0 
\end{cases}
\]

\[ u_s = -k_1\text{sign}(s) \quad \text{where,} \quad k_1 > 0 \] (12)

Where, \( k_1, k_2 \) & \( k_3 \) are positive constants.

3.2. Stability Analysis

From (9) to (12) we obtain the following closed loop system

\[
\begin{aligned}
  \ddot{s} &= -k_1\text{sign}(s) - k_2|s|^{1/2}\text{sign}(s) + v \\
  \dot{v} &= -k_3\text{sign}(s)
\end{aligned}
\]  

(13)
Proof:

Let

\[\chi^T = [s\frac{1}{2}s\text{sign}(s), v] = [\chi_1, \chi_2]^T\]

Select a candidate Lyapunov function [19]:

\[V = \chi^T P \chi\]  \hspace{1cm} (14)

where \(P\) is the positive definite symmetric matrix and satisfies the following equation:

\[A^T P + P A = -Q\]  \hspace{1cm} (15)

where \(A = \begin{bmatrix} -\frac{1}{2}k_2 & \frac{1}{2} \\ -k_3 & 0 \end{bmatrix}\) is Hurwitz Matrix. \(Q\) is any positive definite symmetric Matrix.

Since:

\[\dot{V} = \dot{\chi}^T P \chi + \dot{\chi}^T P \dot{\chi}
= \frac{1}{|\chi_1|} \left( A \chi - \begin{bmatrix} \frac{1}{2}k_1 \text{sign}(s) \\ 0 \end{bmatrix} \right)^T P \chi
+ \frac{1}{|\chi_1|} \left( A \chi - \begin{bmatrix} \frac{1}{2}k_1 \text{sign}(s) \\ 0 \end{bmatrix} \right)^T P \left( A \chi - \begin{bmatrix} \frac{1}{2}k_1 \text{sign}(s) \\ 0 \end{bmatrix} \right)
\]

\[= \frac{1}{|\chi_1|} \left( \chi^T A^T P \chi - \chi^T P A \chi - \frac{1}{4}k_1 \text{sign}(s) \right) P \chi - \chi^T P \left( \frac{1}{2}k_1 \text{sign}(s) \right) \]

\[\leq \frac{1}{|\chi_1|} \chi^T (A^T P + PA) \chi - \frac{1}{|\chi_1|} k_1 \text{sign}(s) |s|^{\frac{1}{2}} \text{sign}(s) \lambda_{\min}(P)
\]

\[\leq - \frac{1}{|\chi_1|} \chi^T Q \chi - k_1 \lambda_{\min}(P)
\]

\[\leq - \frac{1}{|\chi_1|} \lambda_{\min}(Q) \| \chi \|^2 \leq - \lambda_{\min}(Q) \| \chi \|^2 \] \hspace{1cm} (17)

\[\leq - \lambda_{\min}(Q) \| \chi \|^2 \] \hspace{1cm} (18)

For equation (14) we have

\[\lambda_{\min}(P) \| \chi \|^2 \leq \chi^T P \chi \leq \lambda_{\max}(P) \| \chi \|^2 \] \hspace{1cm} (19)

Then according to (19)
Combining (18) and (20), we obtain
\[ \dot{V} \leq -\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} \| \chi \| \]

Then the system (13) can converge to zero in finite time i.e. when \( t \to T, \ s \to 0 \). Since \( s = ce + \dot{e} \) we can conclude \( e \to 0, \ \dot{e} \to 0 \) when \( t \to \infty \). i.e. the robot system is asymptotically stable.

4. Simulations
The effectiveness of the proposed method a two-link robot manipulator described in [20] is considered, as shown in Figure 1. The dynamic model of the two-link robot manipulator and parameters selected from [17].

For STC the value of \( K2=0.8, \ K3=1.5 \) selected and constant reference is chosen as 0.2 Radian for simulations. Simulation Results of the considered robotic manipulator are shown below:

![Figure 1. Two Link Robotic Manipulator.](image)

Figure 2 and figure 3 shows the trajectory tracking of link 1 & 2 of proposed STC and [18] STC respectively. For the same parameters, the proposed STC shows the smooth trajectory tracking of the system while the trajectory tracking of the controller used in [18] is not smooth. The trajectory time for proposed system model is 0.04 seconds while the trajectory tracking time for the previously used controller is 0.02 seconds.

Figure 4 and figure 5 shows the Tau applied to link 1 & 2 of the proposed model and STC model used in [18] respectively. It is shown that the proposed controller is smoother and more robustness to the system while the [18] one is not smoother. From the zoomed view of figure 4 and figure 5 it is shown that the proposed controller has a minimal amount of changes rather than controller used in [18]. It is clearly shown that the proposed controller has more continuity and smoothness which proves that the system has more robustness while the STC used in [18] overshoot which can cause damage.

Tracking error of link 1 & 2 of proposed model and controller used in [18] is shown in figure 6 and figure 7, respectively. It is clear that the proposed model has less chattering effect than [18] model and error is smoothly going to zero. The Zoomed area of figure 7 and figure 7 shows the tracking error of the proposed model and it can be seen that the proposed model has very less error even in the zoomed...
section of figure 6 and figure 7 it can be seen that the controller used in [18] have an error in $10^{-4}$ while at this stage proposed controller has smoothness.

**Figure 2.** Trajectory tracking of link 1 using proposed STC vs. STC used in [18].

**Figure 3.** Trajectory tracking of link 2 using proposed STC vs. STC Used In [18].

**Figure 4.** Tau applied of link 1 using proposed STC vs STC used in [18].

**Figure 5.** Tau applied of link 2 using proposed STC vs. STC used in [18].

**Figure 6.** Error tracking of link 1 using proposed STC vs. STC used in [18].

**Figure 7.** Error tracking of link 2 using proposed STC vs. STC used in [18].

5. **Conclusions**

This study aim was to propose a Robust tracking control for Robotic Manipulators based on super-twisting Algorithm. According to simulations, the proposed super-twisting controller approach shows
the better performance in the term of reaching the object, in control sense and in the term of Error. The controller is smoother and continuous. The Result obtained from constant reference 0.2 radian for trajectory tracking shows that both controllers present good trajectory tracking for the stabilization of the position with the desired position, but the trajectory tracking error of the proposed STC is smaller than previously used STC. Additional types of controller and observer can be considered for Chattering attenuation In the future.

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