An Overspeed Protection Mechanism for Virtual Coupling in Railway

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ABSTRACT As a new technology to improve transportation capacity by compressing train tracking interval, virtual coupling (VC) has attracted significant attention from both industry and academia. The traditional overspeed protection mechanism cannot guarantee the safe running for a train fleet. Therefore, in this article, we study two key problems of the overspeed protection mechanism: the limit speed calculation and the safe control for the protection of train in the formation when overspeed occurs. This article describes the calculation methods of limit speed difference (LSD) based on relative coordinates and a collision mitigation approach by minimizing the relative kinetic energy, respectively. Finally, the effectiveness of the proposed method is verified through numerical analysis. The proposed calculation method of limit speed can realize large headway at low speed and small headway at high speed, which can meet the time requirement of switch and route arrangement in station areas. The performance of the model predictive control (MPC) based approach is compared with other two control strategies, i.e. basic adaptive cruise control (ACC) and directly maximum braking control (DBC). The simulation results show that the MPC strategy has the best performance among these three strategies in reducing the total relative kinetic energy of virtually-coupled train formation, followed by the DBC control strategy, and the basic ACC control strategy needs to be improved.

INDEX TERMS High-speed railway, virtual coupling, model predictive control, limit speed difference, collision mitigation.

I. INTRODUCTION Rail transport demand has been growing around the world in recent years, because of its fast, convenient and comfortable travel experience. The passenger rail activity has increased significantly over the past twenty years, but is concentrated in a few regions (Fig. 1): China, India, Japan, European Union and Russia together account for more than 90% of passenger rail activity worldwide [1]. In order to meet this demand, one trend of future railway is to apply advanced train control system to reduce train separation and increase safety of railway operations.

Virtual coupling (VC) is a new concept related to reducing train separation. With the continuous development of Train-to-Train communication, Train-to-Ground communication technology, Automatic Train Operation (ATO) and related rail transportation technologies, virtually-coupled train formation or VC (see Fig. 2) have received considerable attention of researchers from railway industry and academic in recent years. The VC makes the connection between trains no longer an actual physical coupler, but trains maintain a short distance travel together and a consistent velocity by communicating with each other. By means of electronic data transmission, these trains would form a fleet after entering a trunk line. And then, they drive one behind the other with a short headway. The trains leave the formation automatically when they reach a junction or station.

One of the European Shift2Rail programme (i.e. IP2, Advanced Traffic Management and Control System) addresses research in the field of moving block signaling and goes a step further by introducing the concept of VC [2]. The VC is also part of the research project Next Generation Train (NGT) of the German Aerospace Center (DLR) and the research project Closer Running of the Rail Safety and Standards Board (RSSB) [3].
The overspeed protecting methods in current research follows the moving block principle with absolute braking distance (ABD) or relative braking distance (RBD) [4].

In this article, we propose a novel overspeed protection mechanism for VC to improve the safety of train formation operation, focusing on solving two major problems: How to calculate the limit speed for trains in a fleet, and how to brake to stand if train runs over the limit speed. The main contributions are as follows:

1) Proposes an algorithm for calculating LSD based on relative coordinates for trains in a fleet. It considers the braking performance relative to the leading train, and could remove the risk caused by which the braking force applied by the leading train is bigger than the following train.

2) Proposes a coordinated collision mitigation approach for VC trains by using MPC. The problem is modeled with the objective of minimizing the total relative kinetic energy for a virtually coupled train formation with considering the braking dynamics of train.

The rest of this article is organized as follows. In section II, we review the literature on VC, focusing on its scenarios and control method. In section III, we propose an overspeed protection mechanism for supervising train speed. It includes the calculation of limit speed based on LSD and an MPC based braking control strategy if overspeed occurs. In section IV, firstly, numerical analysis of limit speed based on relative coordinates is given. Then based on the data of CRH3, a kind of high-speed train in China, we present the experimental studies through simulation and verify the effectiveness of the proposed control strategy. At last, the conclusions are given in section V.

II. LITERATURE REVIEW

An initial idea of VC appeared in [5], [6], mainly for freight trains. The authors describe the methods to design and develop the concept. Because of the complexity of the project, this article only gives an overview. The details on communication, location and the engineering framework are not discussed presented. Quantities of recent studies have focused on two main aspects of VC: the scenarios of VC operation, and the control method of the train formation.

A. SCENARIOS OF VIRTUAL COUPLING OPERATION

The scenarios of VC operation need to be discussed with traditional train control system. Reference [7] introduces VC in the context of ERTMS/ETCS. They gave some preliminary hints, models and results and draw conclusions about required safety analyses and future developments. A concept ‘Closer Running’ is introduced [8]. It sometimes referred to as ‘relative block’, where trains are separated according to their current relative speed, braking capability and location. This reference explores what such a system might look like, how it might work, and describes the potential benefits and safety, technical, operational and cultural challenges to making it a reality. Reference [9] proposes three fundamental questions seriously which include ‘Is it safe?’ The answer to this question still needs to be explored. The mechanical and structural design of the trains needs to be taken into account as well as the mode of operation that is envisaged, along with the possibility of designing-in additional controls and/or mitigations.

Schumann [10] points out that, the VC are unrealizable with the current switch technology. For the VC at low speed, e.g. in the proximity of a station, a safe brake is possible in case of a switch failure. This is a significant challenge to be addressed. Zhang and Wang [11] propose a topological manifold-based monitoring method to guarantee the safety of the train-centric VC systems. The trains’ dynamic behavior under specific control commands can also be supervised by a safety theorem. Due to a dynamical running target instead of the traditional static end of authority information, the traditional speed supervision principle, i.e. overspeed protection mechanism, is not suitable for trains in the formation.

B. CONTROL METHOD OF THE TRAIN FORMATION

Except for scenario analysis, the scholars proposed some control method of the train formation.

Felez et al. [12] propose a decentralized MPC framework for VC in metro. The comparison with moving block train control concept is given. However, the overspeed protection principle is not presented. The risk of overspeed still needs to be discussed. Di Meo et al. [13] provide a proof of concept of VC by introducing a specific operating mode within the European rail traffic management system/European train control system (ERTMS/ETCS) standard specification, and by defining a coupling control algorithm accounting for time-varying delays affecting the communication links. The emergency braking scenario is discussed. The results are based on a decentralized control strategy and stability analysis guaranteed by Linear Matrix Inequality (LMI) criterion, where borrowed from the automotive field. Reference [14] focuses on
the simulation of the principle of VC, and the safety braking model required by VC technology is discussed. The improvement of capacity is evaluated by using VC. Liu et al. [15], [16] propose a coordinated control method based on multi-agent system for VC. A simple scenario of virtual coupled train is given, which does not include the emergency braking scenarios. Gao et al. [17] propose a control method for multiple high-speed trains without requiring prior information of the empirical parameters of the operational resistances and online adjusts by proper adaptation laws. The operation is protected by the moving block principle.

The cruise control problems for high-speed trains are investigated in [18], [19]. Distributed control laws based on graph theory are designed to avoid collision. The simulation is implemented in a moving block mode. Quaglietta et al. [20] and Quaglietta [21] introduce a detailed capacity analysis of the VC concept which upgrades moving-block train operations by imposing a relative braking distance separation rather than an absolute one. The proposed car-following based model considers non-linear train dynamics and relevant factors such as motion resistances due to track gradient and curvature as well as power limitations of the traction unit and safety constraints at junctions. It introduced a state flow-diagram and state transition conditions of the VC, however, without emergency braking. In [22], a synchronization control of a train platoon was developed under moving block system. This synchronization control scheme could keep the successive trains following with a certain distance to ensure the minimum train following headway. A self-triggered MPC based cooperative control model for multiple trains is proposed to adjust train following headway [23]. It uses moving block with relative braking distance to guaranteed the safe following distance.

The major of previous research follow the developments of platooning coming from the car industry [24], [25], [26]. The platooning of autonomous vehicles is a group of vehicles that can travel very closely together, safely at high speed. Many researchers have proved its potential to significantly benefit road transportation, including improving traffic efficiency, enhancing road safety, and reducing fuel consumption [27], [28], [29], [30]. However, it is not enough because of the nature that trains cannot avoid collisions by changing lane. The VC in railway still needs the speed supervision mechanism, i.e. overspeed protection. This article will introduce a overspeed protection mechanism for VC.

### III. TRAIN OVERSPEED PROTECTION MECHANISM IN THE FORMATION

Two key issues in the overspeed protection mechanism of trains in the formation will be studied in this section:1. The calculation method of limit speed;2. The safety protection control algorithm when overspeed occurs in the formation. We proposed a calculation method of limit speed difference (LSD) based on relative coordinates and a collision mitigation approach by minimizing the relative kinetic energy in this section respectively.

#### A. CALCULATION OF LIMIT SPEED DIFFERENCE BASED ON RELATIVE COORDINATES

For further discussion, the notations defined in TABLE 2 will be used. Here, train $j$ is the following train, and train $i$ is the preceding train.

At present, the train overspeed protection mechanism is based on the calculation of limit speed. When calculating the limit speed, we consider train performance, line conditions, train driving status in the section, etc. The basic principle is to take the minimum value among various restrictions, i.e.:

$$V_L = \min(V_{L1}, V_{L2}, \ldots)$$

where $V_L$ is the limit speed, $V_{Li}(i = 1, 2, \ldots)$ includes such as the maximum speed of the vehicle structure, the limit speed of the line, the corresponding limit speed of each dangerous point in the authorized range of movement, etc. This traditional protection mechanism could be used between two
formations, which can be seen as two trains. When overspeed occurs, the following train/formation can brake to a stand safely.

For trains in the formation, the basic principle of overspeed protection mechanism needs to be discussed. With shortened headway, there is no enough space for braking to a stand. The ‘safe’ state for a formation, relatively, is that all trains in the formation could be applied braking. Specifically, suppose the braking rate of the first train in the formation could be $B_1$; then the braking rate of the second train should be $B_2 = B_1 + \Delta A_{21}(t)$; the braking rate of the third train should be $B_3 = B_2 + \Delta A_{32}(t)$; and so on for subsequent trains. The relative braking performance for each train is shown in TABLE 3.

The overspeed protection of trains in the formation is analyzed by using the speed-distance curve based on relative coordinates (as shown in Fig. 3). In Fig. 3, the horizontal axis represents the distance between the following and preceding trains, and the vertical axis represents the relative speed of the following and preceding trains. Corresponding to a certain distance, the basic relationship curve between relative speed and relative position in this relative coordinate system can be obtained based on the maximum braking rate $\Delta B_{ij}^{\max}(t)$ that the following train could apply, guaranteed in TABLE 3. The specific formula is as follows

$$\Delta V_{ij}(t) = \sqrt{2 \cdot \Delta B_{ij}^{\max}(t) \cdot \Delta S_{ij}(t)} \quad (1)$$

This relative speed cannot be exceeded at the certain distance $\Delta S_{ij}(t)$ in theory, otherwise the relative speed cannot be reduced to zeros within this distance, that is, a relatively safe state.

In addition, since it takes a certain amount of time for the traction system and the braking system to complete the conversion of working conditions, there are two typical transition states during this period: first, the relative acceleration working condition is maintained, and the reaction time is $T_{ij}^{\text{react}}(t)$, during which the relative acceleration of the train is $\Delta A_{ij}^{\text{react}}$; it can be taken as the maximum relative acceleration $\Delta A_{ij}^{\text{max}}(t)$; the coasting time after propulsion disable is $T_{ij}^{\text{coast}}(t)$, during which the relative acceleration of the train is $\Delta A_{ij}^{\text{coast}}$, it can be replaced by the coasting acceleration. Then the relative distance changes

$$d \Delta S_{ij} = T_{ij}^{\text{react}}(t) \cdot \Delta V_{ij}^{\text{safe}}(t) + \frac{1}{2} A_{ij}^{\text{react}} \cdot (T_{ij}^{\text{react}}(t))^2 + \left(\Delta V_{ij}^{\text{safe}}(t) + T_{ij}^{\text{react}}(t) \cdot \Delta A_{ij}^{\text{react}}\right) \cdot \Delta A_{ij}^{\text{coast}} + \frac{1}{2} \Delta A_{ij}^{\text{coast}} \cdot (T_{ij}^{\text{coast}}(t))^2. \quad (2)$$

According to the relative distance of train $i$ and $j$ at time $t$ and the change of the above-mentioned relative distance, the running distance for completing the conversion from traction to brake can be calculated:

$$d \Delta S_{ij}^{\text{limit}}(t) = \Delta S_{ij}(t) - d \Delta S_{ij} \quad (3)$$

The limit relative speed for $d \Delta S_{ij}^{\text{limit}}(t)$ can be calculated by the formula (4)

$$d \Delta V_{ij}^{\text{limit}}(t) = \sqrt{2 \cdot \Delta B_{ij}^{\max}(t) \cdot d \Delta S_{ij}^{\text{limit}}(t)} \quad (4)$$

Based on this limit relative speed, considering the speed change of the transition state, a safe relative speed can be calculated:

$$\Delta V_{ij}^{\text{safe}}(t) = d \Delta V_{ij}^{\text{limit}}(t) - T_{ij}^{\text{react}}(t) \cdot \Delta A_{ij}^{\text{react}} - T_{ij}^{\text{coast}}(t) \cdot \Delta A_{ij}^{\text{coast}} \quad (5)$$

It represents the safe relative speed between train $j$ and $i$ at time $t$. In order to further ensure safety and eliminate the influence of speed measurement errors, the LSD between train $j$ and train $i$ at time $t$ should be:

$$\Delta V_{ij}^{\text{limit}}(t) = \Delta V_{ij}^{\text{safe}}(t) - V_e \quad (6)$$

And the limit speed for train $j$ based on the relative coordinates can be calculated:

$$V_j(t) = \sqrt{2 \cdot \Delta B_{ij}^{\max}(t) \cdot \Delta S_{ij}(t) + V_i(t)} \quad (7)$$

\begin{table}[h]
\centering
\caption{Notations.}
\begin{tabular}{|c|c|}
\hline
Parameters & Description \\
\hline
$\Delta S_{ij}(t)$ & Distance between train j and train i at time t \\
$V_{ij}(t)$ & Speed of train j relative to train i at time t \\
$A_{ij}(t)$ & Acceleration of train j relative to train i at time t \\
$\Delta V_{ij}^{\text{limit}}(t)$ & Limit speed of train j relative to train i at time t \\
$\Delta V_{ij}^{\text{safe}}(t)$ & Safe speed of train j relative to train i at time t \\
$\Delta T_{ij}$ & Maximum error of speed measurement system \\
$d \Delta S_{ij}$ & Difference in the relative distance between train j and train i during the transition of the system operating conditions \\
$T_{ij}^{\text{react}}(t)$ & React time of train j relative to train i at time t \\
$T_{ij}^{\text{coast}}(t)$ & Coast time of train j relative to train i at time t \\
$\Delta A_{ij}^{\text{react}}$ & Acceleration of train j relative to train i during react time \\
$\Delta A_{ij}^{\text{coast}}$ & Acceleration of train j relative to train i during coast time \\
$\Delta A_{ij}^{\text{max}}(t)$ & Maximum deceleration of train j relative to train i at time t \\
\hline
\end{tabular}
\end{table}
The LSD based on the relative coordinates provides a new limit speed for the trains in the formation. This limit speed is set to meet the safety of trains driving in the formation. It is equal to other limit speed status. The basic principle of the train overspeed protection after adding the LSD based on relative coordinates is as follows:

\[ V_L = \min(V_{RL}, V_{L1}, V_{L2}, \ldots) \] (8)

Take the basic scenario of formation composed of two trains as an example. As shown in Fig. 4, the limit speed of train B2 is \( V_L = \min(V_{L1}, V_{L2}, V_{LR}, V_{LA}) = V_{LA} \). Where \( V_{L1} \) is the vehicle construction speed, \( V_{L2} \) is the line limit speed, \( V_{LR} \) is the limit speed based on relative coordinates, \( V_{LA} \) is the limit speed considering the dangerous point at the tail of the forward formation A, and \( V_{B1} \) is the current speed of train \( B_1 \).

The difference between the proposed method and moving block principle will be given in Section IV. A.

### B. A COLLISION MITIGATION APPROACH BY MINIMIZING THE RELATIVE KINETIC ENERGY

Most of the existing literatures studied proposing the control strategy to make the virtually-coupled train formation reach equilibrium. However, after reaching the equilibrium state, when the leader train implement emergency braking, the following train continue to use the previous control strategy which is used to form virtually-coupled train formation may lead to train collisions due to some random factors (limitation of braking ability, random perturbation of braking force, difference in deceleration performance, etc.). Therefore, how to stop the train fleet safely under these circumstances is a research point of interest in this article (Fig. 5).

The emergency braking process is introduced for virtually-coupled train fleet in Fig. 6. The leader train implements emergency braking after receiving the emergency braking instruction, at this time, the control goal of following trains is no longer to form a stable queue, but to evaluate the risk of collision and adjust the braking rate to minimize the risks of collision between the following trains, which is the main research content of this article.

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With reference to work in [31], we introduce the concept of relative kinetic energy which was used in the road traffic field to describe the impact of multi-vehicle collisions. The collision energy is also used to quantitatively measure the impact of accidents in the field of aviation. This article uses the relative kinetic energy to measure the potential impact of a collision between adjacent trains, which in turn is used to estimate the overall virtually-coupled train formation. The total relative kinetic energy $R(t)$ can be expressed as follows

$$ R(t) = \sum_{i=2}^{N} m_i (v_i(t) - v_{i-1}(t))^2 $$ (9)

where $v_i$ and $m_i$ are the velocity and mass of the i-th vehicle, respectively, and $N$ denotes the number of trains in the VC formations. In this way, the risk of train collision within the virtually-coupled train formation is reduced by reducing the total relative kinetic energy to achieve the purpose of safe stopping.

It is assumed that $\Delta t$ represents the sampling time for system discretization. The state space equation can be discretized into the following form

$$ \begin{align*}
x_i(k + 1) &= x_i(k) + v_i(k) \Delta t \\
v_i(k + 1) &= v_i(k) + (u_i(k) - w_i(k) - g_i(k)) \Delta t \\
u_i(k + 1) &= u_i(k) + \frac{1}{t_i} (u_{i,des} (k) - u_i(k)) \Delta t,
\end{align*} $$ (10)

where $k$ represents the time step.

The objective function over the predictive horizon is

$$ J(k) = \sum_{j=1}^{M} \sum_{i=2}^{N} m_i (v_{i-1}(k + j|k) - v_{i-1}(k + j|k))^2 $$ (11)

where $\ast (k + j|k)$ represents the predicted state obtained after predicting $j$ steps at the sampling time $k$, $m_i$ donates the mass of the train $i$, $N$ donates the number of trains in the virtually-coupled train formation, and $M$ donates the length of the predictive horizon (control horizon is assumed equal to the predictive horizon).

When the train is running on the line, the state of the system must be constrained by the inherent attributes of the train and the conditions of the line. The constraints we considered in the model are as follows

$$ \begin{align*}
u_i^{min} &\leq u_i(k + j|k) \leq u_i^{max} \\
0 &\leq v_i(k + j|k) \leq v_{lim}(x_i) \\
x_{i-1}(k + j|k) - x_i(k + j|k) &\geq s_m - \frac{v_{i-1}(k)^2}{2u_i^{min}} + \frac{v_i(k)^2}{2u_i^{min}} \\
i &= 1, 2, \ldots, N; j = 0, 1, \ldots, M - 1
\end{align*} $$ (12)

where $u_i^{min}$, $u_i^{max}$ are the maximum possible decelerate and accelerate force per unit mass, $u_i^{min}$ is less than zero. $v_{lim}(x_i)$ donates the maximum allowed speed depending on the position $x_i$ of the train on the track, and $s_m$ donates the minimum safe distance headway (i.e., safety margin). $s_m = \frac{v_{i-1}(k)^2}{2u_i^{min}} + \frac{v_i(k)^2}{2u_i^{min}}$ implies safe distance for adjacent trains.

Therefore, the optimization problem for the virtually-coupled train formation can be formulated as

$$ \begin{align*}
\min_{u_{des}(k)} &\quad J(k) = \sum_{j=1}^{M} \sum_{i=2}^{N} m_i (v_{i-1}(k + j|k) - v_i(k + j|k))^2
\end{align*} $$ (13)

subject to

$$ \begin{align*}
x_i(k + j + 1|k) &= A_i x_i(k + j|k) + B_i u_{i,des}(k + j|k) - W_i(k) \\
u_i^{min} &\leq u_i(k + j|k) \leq u_i^{max} \\
0 &\leq v_i(k + j|k) \leq v_{lim}(x_i) \\
x_{i-1}(k + j|k) - x_i(k + j|k) &\geq s_m - \frac{v_{i-1}(k)^2}{2u_i^{min}} + \frac{v_i(k)^2}{2u_i^{min}} \\
i &= 1, 2, \ldots, N; j = 0, 1, \ldots, M - 1
\end{align*} $$

where

$$ X(k + j|k) = [x_i(k + j|k), v_i(k + j|k), u_i(k + j|k)]^T $$

$$ A_i = \begin{bmatrix} 1 & \Delta t & 0 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 - \frac{\Delta t}{t_i} \end{bmatrix} $$

$$ B_i = \begin{bmatrix} 0 \\ \Delta t \\
0 \end{bmatrix}, \ W_i(k) = \begin{bmatrix} 0 \\ w_i(k + 1|k) \\ 0 \end{bmatrix} $$

$X(k + j|k)$ represents the predicted state obtained after predicting $j$ steps at the current time $k$, $W_i(k)$ donates the basic resistance and gradient resistance of the train $i$ during the prediction process. Note that, in order to facilitate the solution of the optimization problem (13), the corresponding linearization processing has been done here, that is at the current time instant $k$, the velocity $v_i(k)$ holds the same value during the predictive horizon from $k + 1$ to $k + M$ when used to calculate the basic resistance (i.e., $w_i(k + M|k) = \ldots = w_i(k + 1|k) = w_i(k)$). Gradient resistance is also assumed to be constant during the predictive horizon (i.e., $g_i(k + M|k) = \ldots = g_i(k + 1|k) = g_i(k)$). Because the changes in basic resistance and gradient resistance are negligible with short prediction time.

The equation (13) is a quadratic programming problem (QP) with several linear constraints, and there are many efficient methods to solve such problem to realize real-time optimization. The solution to the problem given by equation (13) is the sequence of input signal

$$ u_{i,des} = [u_{i,des}(k|k), \ldots, u_{i,des}(k + M - 1|k)] $$

$$ i = 1, 2, \ldots, N. $$
Only the first input signal $u_{i,des}(k|k), i = 1, \ldots, N$ is applied to control the train operation. Next, we set $k = k + 1$, and repeat this procedure which named as moving horizon optimization.

IV. SIMULATION AND DISCUSSION

The numerical simulations of the calculation method of limit speed and collision mitigation approach proposed in Section III are conducted in IV.A and IV.B respectively.

A. NUMERICAL ANALYSIS OF LIMIT SPEED DIFFERENCE BASED ON RELATIVE COORDINATES

1) LIMITED SPEED CURVE BASED ON RELATIVE COORDINATES UNDER DIFFERENT CONDITIONS

In this section, considering different relative braking rates and relative distance conditions, calculate the limit speed based on LSD.

The parameters used in this analysis are set as follows: $T_{\text{react}}^{ij}(t)$ represents react time of train $j$ relative to train $i$ at time $t$, it is set to 2s considering the communication delay and the implementation cycle. $T_{\text{coast}}^{ij}(t)$ represents coast time of train $j$ relative to train $i$ at time $t$, taking into account the switching delay of the traction braking system, which is preset as 2s. $\Delta A_{\text{react}}^{ij}$ is acceleration of train $j$ relative to train $i$ during react time taking 1.5 m/s$^2$. $\Delta A_{\text{coast}}^{ij}$ is acceleration of train $j$ relative to train $i$ during coast time, which is 0.7 m/s$^2$ based on line conditions that the gradient of high-speed rail generally does not exceed 35 %.

The Fig. 7 shows the effect of relative speed on the LSD of the following train to preceding train. It can be concluded that the higher the speed relative to the preceding train, the greater the interval required for the following train at the same speed. The Fig. 8 shows the influence of relative braking performance on the LSD of the following train to preceding train. It can be observed that the better the relative braking performance, the higher LSD under the same interval.

In order to clearly observe the influence on the LSD, we draw the Fig. 9 (LSD vs the relative speed to preceding train and distance). It is assumed that the maximum relative braking rate $= 1.0 m/s^2$. Two pieces of useful information can be obtained from the Fig. 9. The first one is the minimum interval required for the following train when the relative speed to the preceding train is given; the other is that given the relative speed to preceding train and the distance between the preceding train. The LSD of the following train can be obtained, that is, the relative running speed to the preceding train that cannot be exceeded. The Fig. 10 combines all the information in Fig. 7, Fig. 8 and Fig. 9.

B. COMPARISON OF LIMIT SPEED BASED ON RELATIVE COORDINATES AND MOVING BLOCK PRINCIPLE

For comparison, we introduced a protection model that has been discussed by many scholars, that is, relative moving block principle. It is to consider that the train in front will not stop immediately, therefore, the position of the parking point after the preceding train applied emergency braking is taken as the end of the authorized movement of the following train.
The basic schematic diagram is shown in [4]. And without considering the conversion from traction to brake, the basic calculation formula for relative moving block principle is

$$\frac{V_j^2}{2 \cdot B_j^{\max}(t)} - \frac{V_i^2}{2 \cdot B_i^{\max}(t)} = \Delta S_j(t)$$  \hspace{1cm} (14)$$

Then, derived from it, the limit speed for moving block principle with RBD is

$$V'_j = \sqrt{2 \cdot B_j^{\max}(t) \cdot (\Delta S_j(t) + \frac{V_i^2}{2 \cdot B_i^{\max}(t)})}$$  \hspace{1cm} (15)$$

Meanwhile, without considering the conversion from traction to brake, the limit speed based on relative coordinates is as shown below, derived from (7)

$$V_j = \sqrt{2 \cdot (B_j^{\max}(t) - B_i^{\max}(t)) \cdot \Delta S_j(t)} + V_i$$  \hspace{1cm} (16)$$

According to the (15), (16), here we set $B_j^{\max}(t) = -1.5\text{m/s}^2$, $B_i^{\max}(t) = -1\text{m/s}^2$ (the braking performance of the following train is better than the preceding train), the distance between the two train ranges from 0 to 5000m, and the speed $V_i$ of the preceding train changes from 0∼100m/s. The numerical simulation result is shown in Fig. 11.

In both cases, the safe limit speed is positively correlated with distance or with the speed of the preceding train. That is to say, the greater the distance between the two train or the speed of the preceding train, and the safe limit speed of the following train will be higher. Under the relative coordinate system, the limit speed of the following train has a linear relationship with the speed of the preceding train, while under relative moving block system, its relationship is nonlinear.

From the simulation results (see Fig. 12), it can be observed that the limit speed in the relative coordinate system is always less than or equal to that in the relative moving block principle, with the same speed of the preceding train and headway. The theoretical proof of this conclusion is given in the appendix.

C. NUMERICAL ANALYSIS FOR COLLISION MITIGATION

In this section, numerical simulations are implemented to illustrate the effectiveness of the proposed control method. Typical scenario simulation results using different control strategies are discussed.
1) COMPUTATIONAL SETUP
Adaptive Cruise Control (ACC) is one of the earliest automated vehicle systems. The most widely used ACC controller is a linear state feedback controller, where the vehicle acceleration is proportional to the deviation of gap from a desired gap [26]. In general, the formulation of a linear control law in state feedback form can be written as

$$u_{i,d} = k_1 \varepsilon_i(t) + k_2 \dot{\varepsilon}_i(t)$$

where \( \varepsilon_i(t) = R_i(t) - d_i(t) \) is the deviation between the real spacing and the desired spacing, \( k_1 \) and \( k_2 \) are controller parameters, \( R_i(t) = x_{i-1}(t) - x_i(t) - l_i \) donates the actual distance between train \( i-1 \) and train \( i \), \( d_i(t) = hv_i(t) + s_m \) donates the desired distance between train \( i-1 \) and train \( i \) (constant time headway policy), \( l_i \) is the length of train \( i \).

In this chapter, the ACC control strategy [32] is selected and compared with coordinated collision avoidance by MPC control. Similarly, as a reference comparison, there is also a control method that directly adopted maximum braking (DBC).

It is assumed that the virtually-coupled train formation involves four trains and has already reached a stable state (i.e., the actual interval between trains is equal to the desired interval \( d_i(t) = hv_i(t) + s_m \)), and the constant time headway policy was used for tracking strategy of the formation. The initial speed of each train in the formation is \( v_0 = 60 \text{m/s} \). The braking force will fluctuate within a certain range due to environmental interference, therefore, the maximum deceleration will add a normal distributed interference \( D(0,0.2^2) \) on the basis of \( u_{i,\min} \). The main simulation parameters of the line and train are shown in TABLE 4.

D. NUMERICAL ANALYSIS
Case A - Emergency Braking for Homogeneous Fleet: Considering the simulation scene is that the leader train needs to perform emergency braking with the maximum deceleration to stop to standstill before the end of its movement authority (MA) because of overspeed for some reason. To verify the performance of the proposed MPC based coordinated collision mitigation approach, the other two control strategies (i.e., ACC and DBC) which are introduced in chapter computational setup also built for simulation studies.

Comparison of the total relative kinetic energies produced by the three control strategies over the entire braking process is illustrated in Fig. 13. The total relative kinetic energies under the DBC control strategy is not significantly different from the energy under the MPC control strategy under the simulation conditions described in TABLE 4. The reason is that there is very little difference in the deceleration ability of trains, which makes the following trains behavior basically consistent. This indicates that when the deceleration ability of trains is consistent and there is no communication delay, the DBC control strategy could achieve a good control effect, even if the emergency braking rate has a certain degree of randomness.

Case B - Emergency Braking for Heterogeneous Fleet: We considered the virtually-coupled train formation might including trains that have different deceleration ability, let \( u_{i,\min}^1 = -1.2, u_{i,\min}^2 = -1, u_{i,\min}^3 = -1.2, u_{i,\min}^4 = -1 \), which make the deceleration performance of the following trains

![FIGURE 13. Comparison of the Total Relative Kinetic Energies (case A, case B, case C from left to right).](image-url)
more different (see Fig. 5). It is obvious that the total relative kinetic energies under the MPC control strategy are smaller than that of the other two control strategy. The train formation achieved the lowest total relative kinetic energy when MPC based control algorithm was used. This has verified the effectiveness of the proposed algorithm.

Case C - Emergency Braking for Homogeneous Fleet with one Train Losing Part of Braking Deceleration: We also considered the scenario that there is a train in the virtually-coupled train formation could only achieve 75% of the maximum braking because of a failure of the braking system. Let \( u_1^{\min} = -1, u_2^{\min} = -1, u_3^{\min} = -0.75, u_4^{\min} = -1 \), the third train can only achieve part of deceleration ability. The Fig. 5 shows that the total relative kinetic energies are greatly increased with the DBC control strategy, and the total relative kinetic energies of MPC control strategy is still lowest compared to the other two strategies. From the above simulation results, we conclude that the proposed MPC based control algorithm could reduce the risk of collision between trains especially under the condition that the deceleration performance of following trains varies widely.

V. CONCLUSION

In the paper, considering the traditional overspeed protection mechanism cannot guarantee the safe running for trains in the formation, we study two key problems of the overspeed protection mechanism: the limit speed calculation and the safe control for the protection of train in the formation when overspeed occurs. This article describes the calculation methods of LSD based on relative coordinates and a collision mitigation approach by minimizing the relative kinetic energy, respectively. The new mechanism considers the risk caused by the relative braking performance of trains. It could alleviate the harm of rear-end collision when the formation brakes. Finally, the effectiveness of the proposed method is verified through numerical analysis. For these three typical scenarios: 1. Emergency braking for homogeneous fleet; 2. Emergency braking for heterogeneous fleet; 3. Emergency braking for homogeneous fleet with one train losing part of braking deceleration, the proposed MPC control method has better control effect than DBC and ACC, and can deal with the negative influence caused by changes in train braking performance.

For future work, the proposed mechanism limited the maximum braking rate for first train. In some extreme cases, a higher braking rate can be considered to reduce the loss. This situation remains to be studied. Further discussions will be needed about whether VC mode can be continued after emergency braking and how to continue if it can.

**APPENDIX**

Theorem 1: \( V_j \leq V'_j \), if \( \Delta S_{ij} > 0, B_{j}^{\max} > B_{i}^{\max} \)

where \( V_j \) is the limit speed based on relative coordinates, \( V'_j \) is the limit speed with relative moving block principle, \( \Delta S_{ij} \) is the distance between train \( j \) and train \( i \), \( B_{j}^{\max} \) and \( B_{i}^{\max} \) is the maximum braking deceleration for train \( i \) and \( j \) respectively.

**Proof:**

The formula for calculating the limit speed based on relative coordinates is as follow

\[
V_j = \sqrt{2 \cdot (B_{j}^{\max} - B_{i}^{\max}) \cdot \Delta S_{ij} + V_i^2} \quad (18)
\]

The formula for calculating the limit speed with relative block principle is

\[
V'_j = \sqrt{2 \cdot B_{j}^{\max} \cdot (\Delta S_{ij} + \frac{V_i^2}{2 \cdot B_{i}^{\max}})} \quad (19)
\]

Let \( f(V_i) = (17)^2 - (18)^2 \), it can be derived that

\[
f(V_i) = \left( 1 - \frac{B_{j}^{\max}}{B_{i}^{\max}} \right) V_i^2
+ 2V_i \sqrt{2(B_{j}^{\max} - B_{i}^{\max}) \Delta S_{ij} - 2B_{i}^{\max} \Delta S_{ij}}
\]

Here, \( f(V_i) \) is a quadratic function with respect to \( V_i \). When \( \Delta S_{ij} > 0, B_{j}^{\max} > B_{i}^{\max} \), The coefficient of quadratic term is negative and the coefficient of primary term is positive.

\[
A = \left( 1 - \frac{B_{j}^{\max}}{B_{i}^{\max}} \right) < 0
\]

\[
B = 2 \sqrt{2(B_{j}^{\max} - B_{i}^{\max}) \Delta S_{ij}} > 0
\]

So the symmetry axis of the image of \( f(V_i) \) is on the right side of the y-axis, and the direction of the parabola is downward. The discriminant of the quadratic polynomial is as follow

\[
\Delta = b^2 - 4ac = 8 \left( B_{j}^{\max} - B_{i}^{\max} \right) \Delta S_{ij} + 8 \left( 1 - \frac{B_{j}^{\max}}{B_{i}^{\max}} \right) B_{i}^{\max} \Delta S_{ij}
\]

It can be derived that \( 8 \Delta S_{ij} \neq 0 = 0 \). And, the graph of the quadratic function is tangent to the x-axis and \( f(V_i) \leq 0 \).

Therefore, the limit speed in the relative coordinate system is always less than or equal to that in the relative moving block principle, with the same speed of the preceding train and headway.

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