Reservoir Interwell Dynamic Connectivity Inversion Method with Considering External Fluid Influx

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Abstract. Interwell Dynamic Connectivity is an important basis for reservoir development and adjustment. A method for calculating interwell connectivity using production and injection data was established, which was based on capacitance model and material balance equation. And the interwell connectivity, time lag and the influence of fluid influx were considered in the method. Using numerical simulation verify that Established method can effectively deal with reservoirs with fluid influx.

1. Introduction
In China, most water drive oilfields are in high water cut or extra high water cut stage. The reason is that water channels cause injected water to be invalidated. So it is necessary to identify water channels. and interwell connectivity can be used to identify water channel. At present, the methods of searching connectivity can be divided into static analysis method and dynamic analysis method. [1] Static analysis method depends geological data such as earthquake and electric logging data to analyze reservoir continuity. [2] However, it is hard to get whole geological data and this data cannot reflect the true condition because of the technical reasons. As a result, the connectivity is not accurate. Dynamic analysis method includes geochemical method [3], PI decision [4] and tracer test [5]. Although these ways are effective, they are complex and expensive and they will interrupt produce. While using production dynamic data to understand the research relationship between wells has become a hot topic of research, and an important method because production data can be get easily. It includes correlation analysis model [6], multiple linear regression model [7], capacitance model [8], systematic analysis method [9], Extended Kalman Filter [10] and multilayer reservoirs method [11]. Few of these ways, however, consider external fluid influx. And some of them just use a water influx constant [12]. This paper depending on the material balance equation considers the influence of external fluid influx, and uses the dynamic data of reservoir production to calculate the dynamic connectivity between wells.
2. Interwell connectivity equation

During reservoir development, injection wells, production wells and formation media can be seen as a complete system. The injection data of the injection well is the input signal, and the production data of the production well is the output signal. In this process, the whole system must meet the material balance equation. Based on this, interwell connectivity equation can be obtained.

2.1. Multiple injectors and single producer system. According to the material balance equation:

\[
\sum_{i=1}^{Nw} \left( C_i V_p \frac{dp}{dt} \right)_{i,j} = \sum_{i=1}^{Nw} \lambda_{i,j} i_i(t) - q(t) + \sum_{i=1}^{Nw} q_{ei,j}(t)
\]  \(1\)

\(C_i\) is comprehensive compression factor, \(V_p\) is pore volume, \(\bar{p}\) is average formation pressure, \(\left( C_i V_p \frac{dp}{dt} \right)_{i,j}\) is fluid volume change in area controlled by injector I and producer j, \(\lambda_{i,j}\) is interwell connectivity coefficient, \(i_i(t)\), \(q(t)\) and \(q_{ei,j}(t)\) are injection rate, production rate of injector i and fluid influx volume in area controlled by injector I and producer j at t time respective, \(Nw\) is Number of injection wells corresponding to production wells. Among these, \(q_{ei,j}(t)\) is get from steady water influx formula:

\[
q_{ei,j} = K_{i,j} (p_i - \bar{p})
\]  \(2\)

\(K\) is water influx coefficient, \(p_i\) is initial formation pressure. [13]

According linear productivity model well bottom flow pressure can be calculated from production well production rate:

\[
\bar{p}_{i,j} = \frac{q_{ij}}{J} + p_{wfj}
\]  \(3\)

\(J\) is liquid production index, \(p_{wfj}\) is well bottom pressure.

Then defining time factor \(\tau_{i,j} = \left( C_i V_p \frac{dp}{dt} \right)_{i,j}\), and factor \(a_{i,j} = 1 + \frac{K_{i,j}}{J}\), we can translate (1) to:

\[
\sum_{i=1}^{Nw} \tau_{i,j} \frac{d q_{ij}}{dt} + \frac{d p_{wfj}}{dt} \sum_{i=1}^{Nw} \tau_{i,j} i_i(t) = \sum_{i=1}^{Nw} \lambda_{i,j} i_i(t) - \sum_{i=1}^{Nw} q_{ij}(t) + \sum_{i=1}^{Nw} \left( p_{oi,j} - \frac{q_{ij}}{f_{ij}} - p_{wfj} \right)
\]  \(4\)
Solve (4):

\[ q(t) = \sum_{i=1}^{Nw} q_{0i,j} e^{-\frac{a_{ij}(t-t_o)}{\tau_{ij}}} + \sum_{i=1}^{Nw} \lambda_{ij} e^{\frac{-a_{ij}\xi}{\tau_{ij}}} \int_{\xi=t_0}^{t} i(\xi)d\xi \]  

(\text{a}) + \sum_{i=1}^{Nw} \int_{\xi=t_0}^{t} i(\xi)d\xi \]  

+ \sum_{i=1}^{Nw} a_{ij} \int_{\xi=t_0}^{t} p_{0i,j} e^{-\frac{a_{ij}(t-t_o)}{\tau_{ij}}} d\xi \]  

(\text{b}) + \sum_{i=1}^{Nw} \int_{\xi=t_0}^{t} \int_{\xi=0}^{t} e^{\frac{-a_{ij}\xi}{\tau_{ij}}} p_{wrfj}(\xi)d\xi \]  

(\text{c}) + \sum_{i=1}^{Nw} \int_{\xi=t_0}^{t} \int_{\xi=0}^{t} e^{\frac{-a_{ij}\xi}{\tau_{ij}}} p_{wrfj}(\xi)d\xi \]  

(\text{d}) (5)

(9) is divided into four parts: (a) represents decreasing in initial production; (b) is related water injection; (c) is related water influx; (d) is related well bottom pressure change. It is required attention that a part of injected water is counted into (c) instead of (d).

To simplify (5), (a) is simplified:

\[ \sum_{i=1}^{Nw} q_{0i,j} e^{-\frac{a_{ij}(t-t_o)}{\tau_{ij}}} = \beta_p e^{-\frac{t-t_o}{\tau_p} q_j_0} \]  

(6)

\[ \tau_p \]  

is initial item equivalent time constant, \( \beta_p \) is Initial capacity fit factor.

(d) is simplified as (a):

\[ v_j \left( p_{wrfj}(t_0) e^{\frac{z-t_o}{\tau_j}} - p_{wrfj}(t) \right) + e^{\frac{-t}{\tau_j}} \int_{\xi=t_0}^{t} \int_{\xi=0}^{t} e^{\frac{-t}{\tau_j}} p_{wrfj}(\xi)d\xi \]  

(7)

Discretize time, Step size is \( \Delta t \). The time is repressed as \{\( n_0, n_1, n_2, \ldots, n \} \). Equation (5) can be written as the following discrete form

\[ \hat{q}_j(n) = \beta_p q(n_0) e^{\left(\frac{n-n_0}{\tau_p}\right)} + \sum_{i=1}^{Nw} \lambda_{ij} I_i(n) + \sum_{i=1}^{Nw} a_{ij} \int_{\xi=t_0}^{t} \int_{\xi=0}^{t} e^{\frac{-t}{\tau_j}} p_{wrfj}(\xi)d\xi \]  

(8)

\[ \hat{q}_j(n) = \sum_{m=n_0}^{m=n} \left( 1 - e^{-\frac{\Delta t}{\tau_{ij}}} \right) a_{ij}^{(m-n)} e^{\frac{-\Delta t}{\tau_{ij}}} p_{wrfj}(m) \]  

When the production well bottom pressure does not change:
\[\hat{q}_j(n) = \beta_p q(n_0) e^{-\frac{(n-n_0)}{\tau_p}} + \sum_{i=1}^{Nw} \lambda_{i,j} I_{i,j}(n) + \sum_{i=1}^{Nw} \left[ \frac{1}{a_{i,j}} \left( 1 - e^{-\frac{a_{i,j}(n-n_0)}{\tau_{i,j}}} \right) \right] e^{-\frac{a_{i,j}(m-n)}{\tau_{i,j}}} K_{i,j}(p_{bi,j} - p_{wf,j}) \] (9)

Decreasing in initial production is caused by the injection water which is injected before the time to start calculation, in the middle and late stages of development. So (a) can be replaced by expansion (b):

\[\hat{q}_j(n) = \sum_{i=1}^{Nw} \lambda_{i,j} I_{i,j}(n) + \sum_{i=1}^{Nw} \left[ \frac{1}{a_{i,j}} \left( 1 - e^{-\frac{a_{i,j}(n-n_0)}{\tau_{i,j}}} \right) \right] e^{-\frac{a_{i,j}(m-n)}{\tau_{i,j}}} K_{i,j}(p_{bi,j} - p_{wf,j}) \] (10)

When use monthly production data, we found injected water over 6 months has very small impact, so we add the response of injectors to producer before the calculation 6 months.

2.2. Final form of connectivity equation. In order to match the model with the actual, inversion calculation of the parameters in the model is required. It can be solved by optimization. Construct constrained nonlinear programming problems:

\[\text{Min} \sum_{j=1}^{J} \sum_{n=1}^{N} \left( q_j(n) - \hat{q}_j(n) \right)^2 \] (11)

\[\hat{q}_j(n) = \sum_{i=1}^{Nw} \lambda_{i,j} I_{i,j}(n) + \sum_{i=1}^{Nw} \left[ \frac{1}{a_{i,j}} \left( 1 - e^{-\frac{a_{i,j}(n-n_0)}{\tau_{i,j}}} \right) \right] K_{i,j}(\Delta p_{i,j}) \]

\[I_{i,j}(n) = \sum_{m=n-6}^{m=n} \left( 1 - e^{-\frac{\Delta t}{\tau_{i,j}}} \right) e^{-\frac{a_{i,j}(m-n)}{\tau_{i,j}}} i_i(m) \]

s. t. \[\sum_{j=1}^{J} \lambda_{i,j} \leq 1 \]

\[0 \leq \lambda_{i,j} \leq 1 \]

\[\tau_{i,j} > 0; a_{i,j} \geq 1 \]

If the injector i is inside the well network, the first constraint of I is 1.

3. Equation parameter solving

There are many parameters that need to be solved in equation (11), but the basic form is constrained nonlinear optimization. It is common to use Mixed penalty function method to solve. [14]

Such optimization problems can also be solved by an optimization toolbox in some software. For example, in MATLAB optimization toolbox has fmincon function which can easily solve the pending parameters in equation (11).
4. Dynamic inversion of typical reservoirs

To verify the accuracy of method, we use reservoir simulation to calculate the interwell connectivity of typical reservoirs models.

The building model is constituted by $61 \times 61 \times 4 = 14884$ cells which volume is $1 \times 10 \times 10 = 100$m$^3$. It is five-point well network (5 injectors and 4 producers). The pressure of producers is constant and the constant pressure water boundary is located in upper and left. The dynamic data of injectors is shown in Fig. 2.

4.1. Homogeneous reservoir.

Plane permeability of Homogeneous reservoir is 1000mD, and longitudinal permeability is 200mD. The method of this paper is compared with UCM model (Yousef) [8]. Correlation coefficient between the result calculated by this paper and Simulated value is over 0.99, while that of the UCM model is around 0.96.

The result of interwell connectivity is shown in Fig. 4. The triangles direction to injectors from producers. The bigger the triangle is the better the connectivity is. It is clear that the result of this paper are consistent with simulation. While UCM model has a great different.

![Figure 3. P1 production rate](image)

![Figure 2. Injection data](image)

![Figure 4. Connectivity of Homogeneous reservoir](image)
4.2. **Containing hypertonic strips.** A hypertonic strip which permeability is 10 times of other part is added into the model (Fig. 5). The result demonstrates exist of hypertonic strip and can show the position roughly.

![Figure 5. Reservoir with a high permeability belt.](image1)

![Figure 6. Reservoir with a fault](image2)

4.3. **Reservoir with closed fault.** A close fault is set in the reservoir and divide the model into two parts (Fig. 6). The result show that the injectors in the right part do not connect with P4. Although injectors in left part has connection with producers in right, their connectivity is very small. It is clear that there is a closed fault in the model.

5. **Conclusion**

(1) Based on material balance equation, considering External Fluid Influx, create a interwell connectivity inversion equation which can calculate connectivity correctly when has fluid influx.

(2) Comparing method of this paper and UCM model in homogeneous reservoir, correlation coefficient between the result calculated by this paper and Simulated value is over 0.99 which is better than UMC. What’s more the connectivity is more accuracy than UMC.

(3) In reservoir with hypertonic strips model and reservoir with closed fault model, the result shows the method can identify the hypertonic strips and fault model.

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