Study of CP Violation:
Electroweak Baryogenesis and Anomalous W-Boson Couplings

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Abstract

The contributions from the Ochanomizu CP Study Group to the KEK meetings on "CP violation and its origin" (1993 – 1997) are summarized on electroweak baryogenesis and anomalous W-boson couplings. We survey planned new experiments which could examine some aspects studied in our contributions. We also discuss several issues on baryogenesis. Ten problems are presented for further studies.

*To appear in the Proceedings of the KEK meetings on 'CP violation and its origin' (1993 – 1997).
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1 Introduction

From February 1993 to March 1997, we had five meetings at KEK on "CP violation and its origin" once a year. This series of meetings was especially fruitful to our group: We presented the works so far done, and got, from discussions and talks, critical or helpful advices and in particular new ideas for our studies the next year and the year after.

The talks of our group given at the meetings were as follows:

• The first meeting ('93)
  1) Azusa Yamaguchi: Electroweak baryogenesis and the phase transition dynamics [1].

• The 2nd meeting ('94)
  2) Akio Sugamoto: Electroweak phase transition and the baryogenesis [1, 2].

• The 3rd meeting ('95)
  3) Akio Sugamoto: A memory of Yoshiki Kizukuri [3].
  4) Tomomi Saito: CP-odd form factors of the $WWZ$ coupling with a singlet quark [4].

• The 4th meeting ('96)
  5) Noriyuki Oshimo: The electric dipole moments of the $W$ boson and the neutron in the supersymmetric model [5].
  6) Mohammad Ahmady: Rare $B$ decays [6].
  7) Tomoko Uesugi: Baryogenesis in the vector-like quark model [7].

• The last meeting ('97)
  8) Emi Kou: Combined $B \rightarrow X_s\bar{\psi}$ and $B \rightarrow X_s\eta_c$ as a test of factorization [8].
  9) Mayumi Aoki: Electroweak baryogenesis from chargino transport in the supersymmetric model [9].

Among the works presented in these talks, we summarize those on electroweak baryogenesis and anomalous $W$-boson couplings, together with their related works by us. A summary of the talks 6) and 8) is given separately by Ahmady and Kou [10]. Works on the electric dipole moments (EDMs) of the neutron and the electron presented in the talks 5) and 9) and in the talks by Kizukuri [11] are also separately summarized [12].

Some aspects discussed in our contributions could be examined in planned new experiments. From this point of view, we survey experiments for the neutron EDM using ultracold neutrons, experiments for $CP$ asymmetries at $B$ factory, and long baseline experiments for neutrino oscillations. We also take this occasion to discuss
several issues on electroweak baryogenesis which we think are important for further studies.

2 Electroweak Baryogenesis

We have studied baryogenesis so far in three models which are extended from the standard model (SM). Here we summarize our works in terms of Sakharov’s three criterions, namely, 1) $CP$ violation, 2) thermal non-equilibrium, and 3) baryon number violation, accompanied with 4) the results and 5) the characteristic points.

2.1 Model with heavy neutrinos [1,2]

1) Right-handed neutrinos $N_i$ and a Majoron field $\phi$ are introduced. The origin of $CP$ violation is the complex mass matrix for the neutrinos having both Dirac and Majorana masses:

$$
\left( \begin{array}{cccc}
\overline{\nu}_1 & \overline{\nu}_2 & \ldots & \overline{N}_1 & \overline{N}_2 & \ldots
\end{array} \right) \left( \begin{array}{cc}
0 & \lambda_D v(x) \\
\lambda_D^T v(x) & \lambda_M v(x)
\end{array} \right) \left( \begin{array}{c}
\nu_1 \\
\nu_2 \\
\vdots \\
N_1^c \\
N_2^c \\
\vdots
\end{array} \right),
$$

where we have assumed that the vacuum expectation value of $\phi$ and that of the ordinary Higgs field have the same position-dependent shape $v(x)$ in the bubble wall.

2) Thermal non-equilibrium is accomplished in the first order phase transition with the nucleation and expansion of bubbles, for which the order parameter is $\phi$. The bubble wall is an interface of the broken phase and the unbroken phase.

3) Lepton number $N_L$ is not conserved inside the bubble wall, which induces the reflection from the wall $\nu_i \rightarrow \nu_j$ and its $CP$ conjugate process $\overline{\nu}_i \rightarrow \nu_j$. Owing to $CP$ violation, these processes have probabilities different from each other. The produced non-vanishing value for $N_L$ subsequently induces a non-vanishing value for baryon number $N_B$ through fast $N_B$-violating transitions generated by electroweak anomaly.

4) For thermally generated neutrino masses of $50 - 100$ GeV with the phase transition temperature of $100$ GeV, the ratio of baryon number to entropy is consistent with its observed value, if $CP$ violation is $10^{-5} - 10^{-7}$ measured by the Jarskog parameter $J$ for the neutrino masses.
5) In the unbroken phase, the Majorana mass terms for neutrinos vanish and $N_B - N_L$ are conserved. Thus the net density of $N_L$ leads to the net density of $N_B$. The temporal development of the phase transition was simulated, which shows that the temporal change of the expansion velocity $v_w$ and the fusion effect of nucleated bubbles are important. Depending them, the final result could be modified considerably.

2.2 Model with vector-like quarks [7]

1) A vector-like quark $U$ with the charge $2/3$ and a Higgs singlet $S$ are introduced. The vacuum expectation value $V(x)$ of $S$ has a position-dependent complex phase in the bubble wall, which gives a complex mass matrix for the up-type quarks:

$$
\begin{pmatrix}
\vdots \\
\bar{t}_R \\
U_R
\end{pmatrix}
\begin{pmatrix}
\vdots \\
fv(x) \\
FV(x) + F'V^*(x) \\
m
\mu
\end{pmatrix}
\begin{pmatrix}
\vdots \\
t_L \\
\bar{U}_L
\end{pmatrix}.
\tag{2}
$$

The complex value for $V(x)$ could be induced spontaneously from the $CP$-conserving Higgs potential with a Higgs doublet and $S$.

2) During the electroweak phase transition the bubble formation is expected, for which the order parameter is the ordinary Higgs field. The first order phase transition is the origin of thermal non-equilibrium.

3) Difference of the reflection and transmission probabilities between the processes $t, U \rightarrow t, U$ and their $CP$ conjugate processes $\bar{t}, \bar{U} \rightarrow \bar{t}, \bar{U}$ at the wall produces a net flux of hypercharge outside the bubble. Then a non-vanishing value of the chemical potential $\mu_B$ for baryon number, i.e. "thermal pressure" to increase baryon number, is generated, which triggers the $N_B$-violating transitions to produce baryon number.

4) For the $U$-quark mass of $300 - 500$ GeV with the wall velocity $v_w = 0.1 - 0.5$ and the wall width $\delta_w = (100$ GeV)$^{-1}$ the baryon number to entropy ratio of $10^{-10} - 10^{-11}$ can be reproduced.

5) The model is economical as an extension of the SM, and the mechanism of spontaneous $CP$ violation can be embedded. Spontaneous $CP$ violation, however, should be adopted with care because of the appearance of both $CP$-even and $CP$-odd bubbles. In order to keep one type of bubble and eliminate another, tiny but explicit $CP$ breaking of order $10^{-16}$ is necessary.

[Problem 1] Study how to measure directly various types of $CP$ violation originating from the Higgs potential. Is it possible to distinguish experimentally spontaneous $CP$ breaking mechanism from explicit one?
2.3 Supersymmetric standard model \[9,13\]

1) An extra $CP$-violating phase is introduced in the mass matrices of gauginos $\lambda$ and Higgsinos $\psi$, the spin 1/2 superpartners of gauge and Higgs bosons, respectively. For the charged sector the mass matrix is given by

\[
\begin{pmatrix}
X - (i\psi_2^+)^c \\
- gv_1^*(x)/\sqrt{2} \\
-m_H
\end{pmatrix}
\begin{pmatrix}
\tilde{m}_2 \\
-gv_2^*(x)/\sqrt{2} \\
m_H
\end{pmatrix}
\begin{pmatrix}
\lambda^- \\
(i\psi_1^-)
\end{pmatrix}, \tag{3}
\]

where $v_1(x)$ and $v_2(x)$ denote the vacuum expectation values for the two Higgs doublets; $\tilde{m}_2$ the soft supersymmetry-breaking mass for the SU(2) gauginos; and $m_H$ the Higgsino mass parameter in the mixing term of the two Higgs doublet superfields. Among the four parameters in the mass matrix, one complex phase $\theta$ may remain in $m_H$, escaping from the rephasing of field variables. This is the origin of $CP$ violation intrinsic in the supersymmetric standard model (SSM) useful for baryogenesis.

2) The first order phase transition with the bubble formation is the origin of thermal non-equilibrium.

3) Difference of the probability between $CP$ conjugate processes for the reflections and transmissions of gauginos and Higgsinos at the bubble wall generates net hypercharge in the unbroken phase outside the bubble. The produced hypercharge density makes the chemical potential $\mu_B$ non-vanishing, which triggers the $N_B$-violating transitions for baryogenesis.

4) Having the new $CP$-violating phase not much small, $\theta > 0.1$, the observed ratio of baryon number to entropy can be excellently reproduced, if the mass parameters $\tilde{m}_2$ and $|m_H|$ are of order 100 GeV. The wall velocity and the wall width have been taken respectively for $v_w = 0.1 - 0.6$ and $\delta_w = 1/T - 5/T$, $T$ being the phase transition temperature.

5) The most interesting point of this scenario is the following: The value of $CP$-violating phase explaining the baryon number to entropy ratio of our universe is located in the range which is attainable in near future experiments for the EDM of the neutron ($D_n$). The value of $D_n$ is predicted to be $10^{-25} - 10^{-26}$ ecm. Top squarks, instead of gauginos and Higgsinos, could also assume the same role for baryogenesis.

3 Anomalous $W$-boson Couplings

Anomalous $W$-boson couplings for the $WWZ$ and $WW\gamma$ interactions have been studied in the models discussed also for baryogenesis in the preceding chapter. The
$WWZ$ or $WW\gamma$ vertex can generally be expressed as

$$
\Gamma^{\nu\lambda\mu}(q, \bar{q}, P) = f_1(q - \bar{q})^\mu g^{\nu\lambda} - f_2(q - \bar{q})^\mu P^\nu P^\lambda / m_W^2 + f_3(P^\nu g^{\mu\lambda} - P^\lambda g^{\mu\nu}) \\
+ i f_4(P^\nu g^{\mu\lambda} + P^\lambda g^{\mu\nu}) + i f_5 \varepsilon^{\mu\nu\lambda\rho}(q - \bar{q})_\rho - f_6 \varepsilon^{\mu\nu\lambda\rho} P_\rho \\
- f_7(q - \bar{q})^\mu \varepsilon^{\nu\lambda\rho\sigma} P_\rho (q - \bar{q})_\sigma / m_W^2.
$$

Among the seven form factors, $f_1$, $f_2$, $f_3$, and $f_5$ are $CP$-even and $f_4$, $f_6$, and $f_7$ are $CP$-odd. In the SM the form factors $f_1$ and $f_3$ alone do not vanish at the tree level, which holds also in the three models we discuss. Some of these form factors will be investigated through the process $e^+e^- \rightarrow W^+W^-$ at LEP2 or planned linear colliders. In particular, since the $CP$-odd form factors vanish at both the tree level and the one-loop level in the SM, new physics could be examined unambiguously through their effects.

### 3.1 Model with heavy neutrinos [14]

Radiative corrections to the $CP$-even form factors coming from the one-loop diagrams with the heavy neutrinos have been studied. Around the threshold for the pair production of neutrinos the behaviors of these form factors depend on whether the type of the neutrinos is Dirac or Majorana. The difference of cross section for the process $e^+e^- \rightarrow W^+W^-$ could be of order $10^{-4}$ between the process involving Dirac neutrinos and that involving Majorana neutrinos for their masses of $100-1000$ GeV. It will be possible to examine the parameters in the neutrino mass matrix in Eq. (1).

### 3.2 Model with vector-like quarks [4]

A vector-like quark $D$ with the charge $-1/3$ is introduced, and $CP$ invariance is violated either explicitly or spontaneously. The mass matrix for the down-type quarks is given by an equation similar to Eq. (2), which induces both $CP$ violation and flavor changing neutral current (FCNC). The SU(2) neutral current $J_\mu^3$ for the down-type quarks is given by

$$
J_\mu^3 = -\frac{1}{2} \sum_{\alpha,\beta=1}^{3} (\delta_{\alpha\beta} - V_{4\alpha}^* V_{4\beta}) \bar{d}_L^\alpha \gamma_\mu d_L^\beta,
$$

where $V$ denotes the unitary matrix which diagonalizes the mass matrix of the down-type quarks. This current gives non-vanishing values to $f_4$ and $f_6$ for the $WWZ$ vertex at the one-loop level, while $f_7$ remains vanishing. It turned out, however,
that the induced form factors are both much smaller than $10^{-4}$ for the $D$-quark mass of 1 TeV, which would be difficult to be detected in the near future.

3.3 Supersymmetric standard model [5,15]

The interactions for the chargino, neutralino, and $W$ boson violate $CP$ invariance, which induce the $CP$-odd form factors at the one-loop level. For the $WW\gamma$ vertex the magnitude of $f_6$ becomes of order $10^{-4}$, while $f_4$ and $f_7$ vanish. Although the induced form factor is not so large, it could be attainable experimentally in the near future. The $WWZ$ vertex has also been studied, obtaining the results $f_4, f_6 \sim 10^{-4} - 10^{-6}$, $f_7 = 0$.

[Problem 2] Study various methods to detect small $CP$ violation effects in the process $e^+e^- \rightarrow W^+W^-$. 

4 Prospects for New Experiments

For any theory in physics, experimental verification is indispensable. Even though a theoretical idea is beautiful and profound, it is not regarded as true before its experimental evidence comes out. We outline several interesting new experiments relevant to our contributions, hoping that some of the predictions could be examined.

4.1 Experiments of neutron EDM

In the SSM the baryon asymmetry of our universe implies that the EDM of the neutron is not much smaller than its present experimental upper bound. The improvement of its experimental measurement by one order of magnitude could provide an important test for the baryogenesis by the SSM. The neutron EDM gives constraints also on the model with vector-like quarks [16].

The technique of measuring the neutron EDM is progressing very rapidly. It was fortunate that this series of meetings was organized as a part of the big project on ”the ultracold neutron”. We learned various aspects for the neutron EDM experiments [17]. The experiment is now performed at the Institut Laue-Langevin in Grenoble: The cold neutron is guided by the Ni tube (totally reflecting ”fiber for neutron”) 20 m upwards, reducing the kinetic energy gravitationally. Then the neutron is scattered by the recessively rotating 690 (totally reflecting) Ni blades of the so-called ”turbines”, being further cooled down to the ultracold neutron (UCN) [18]. Let us remind ourselves of the high school physics that a non-relativistic particle
with velocity $V$ stops, after being scattered by a perfectly reflecting wall which recedes with velocity $V/2$. The electric field $E$ of 10 kV/cm is applied to the obtained UCNs, in parallel or anti-parallel with the magnetic field $B$ of 1 $\mu$T, measuring the change of the magnetic resonance frequency:

$$\nu \pm \Delta \nu = -2(\mu_n \cdot B \pm D_n \cdot E)/h \simeq 30 \text{ Hz} \pm \Delta \nu.$$  \hspace{1cm} (6)

The upper bound on $\Delta \nu$ has been obtained as $\Delta \nu < 1 \text{ $\mu$Hz}$ [19], which gives the bound on the neutron EDM $|D_n| < 10^{-25}$e cm. This result is consistent with the measurement at the Leningrad Nuclear Physics Institute [20].

The improvement of the measurement could be done, if a more effective source of UCNs is available. A hopeful candidate is the superthermal method [21]: The cold neutron with wavelength about 9 Å is guided into the liquid $^4\text{He}$ at 1 K. Then the neutron stops, by spending all its kinetic energy and momentum on a phonon emission, since the dispersions of neutron and phonon coincide at the wavelength. We hope that this technique will be successfully used in the neutron EDM measurement in the near future. The improvement is also possible by a more accurate monitoring system of the magnetic field which tends to fluctuate randomly and thus becomes the origin of a large systematic error.

[Problem 3] If there is a possibility of measuring $D_n$ very precisely to the order of $10^{-30}$e cm by e.g. some neutron interferometer, how much can one say about models beyond the standard model?

### 4.2 $B$ factory experiments

The model with vector-like quarks has FCNC interactions at the tree level as seen in Eq. (5) as well as the breaking of the unitarity triangles, which could be observed in experiments at $B$ factories. In these experiments the time dependent $CP$ asymmetry

$$A(t) = \frac{\Gamma(B^0 \to f) - \Gamma(\bar{B}^0 \to f)}{\Gamma(B^0 \to f) + \Gamma(\bar{B}^0 \to f)} \simeq \pm \sin(\Delta M_B t) \sin 2(\phi_M + \phi_D)$$  \hspace{1cm} (7)

can be measured for $B$-meson decay. The results may differ drastically from the SM predictions in various modes, such as $B_d \to \psi K_S, D^+D^-, \pi^+\pi^-$, and $B_s \to \rho K_S$ [22]. In the SSM, although effects of the new sources of $CP$ violation are small, new contributions to $B$-$\bar{B}$ mixing indirectly affect the $CP$ asymmetries [23].

The $B$ factory experiments will begin in the near future at KEK and SLAC. The KEK $B$ factory will operate the asymmetric $e^+e^-$ collider of the electron energy 8 GeV and the positron energy 3.5 GeV with luminosity $\mathcal{L} = 2 \times 10^{33}\text{cm}^{-2}\text{s}^{-1}$. 

7
4.3 Neutrino oscillation experiments

The introduction of masses for the neutrinos leads to neutrino oscillations as well as $CP$ violation. A test of $CP$ violation in the neutrino sector could be performed through neutrino oscillations in long baseline experiments [24], by measuring e.g. the difference between the transition probabilities

$$A(\nu) = P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e).$$

(8)

In order to have detectable neutrino oscillations, two neutrinos having different momenta should interfere effectively in the distance $L$. The phase difference is therefore required to satisfy the condition

$$\Delta \phi(L) = |p_1 - p_2|L \simeq \frac{L}{2E}|m_1^2 - m_2^2| \sim 1,$$

(9)

which is really the case for $L \simeq 100$ km, $E \simeq 1$ GeV, and $|m_1^2 - m_2^2| \simeq 10^{-2}$ eV$^2$. Then the magnitude of $A(\nu)$ could be of order $10^{-2}$ for the parameter values explaining the solar and the atmospheric neutrino anomalies.

The long baseline experiment is planned at KEK-Kamiokande, where neutrinos are produced by the upgraded PS at KEK and detected by the water Cherenkov counter at Kamiokande 250 km apart. The neutrinos coming from the proton machine are mainly $\nu_\mu$’s and $\bar{\nu}_\mu$’s produced by $\pi$-mesons or $K$-mesons, so that the asymmetry $A(\nu)$ of Eq. (8) could be measured. There is however one problem. The neutrino oscillations suffer the matter effects by the electrons in the earth, so that the results do not only reflect vacuum oscillations but also experimental circumstances. Since the latter effects contribute with different signs to the $\nu$ and the $\bar{\nu}$ oscillations, $A(\nu)$ contains twice as much the matter effects. For subtracting them, the difference of energy dependence between the vacuum oscillations and the matter effects may be useful.

[Problem 4] Invent a method to detect $CP$ violation related to heavy neutrinos of order 100 GeV in high energy experiments.

5 Discussions

The scenario of electroweak baryogenesis contains various issues for which further analyses are indispensable to obtain accurate results. We discuss some of them in an elementary and naive way, which we believe is useful for our studies.
5.1 Baryon number violation

In any model of electroweak baryogenesis, the main source of baryon number violation is the chiral gauge anomaly

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = -N_g \frac{1}{32\pi^2} F^{a\mu} F_{a\mu} = -N_g \partial_\mu K^\mu, \quad (10)$$

where $N_g$ represents the number of generations and $K^\mu$ is the Chern-Simons current. By this anomaly, baryon and lepton numbers can change while $N_B - N_g N_{CS}$ and $N_L - N_g N_{CS}$ being conserved, where $N_{CS}$ denotes the Chern-Simons number given by

$$N_{CS} = \int d^3x K^0. \quad (11)$$

Various configurations of the gauge fields give various values for $N_{CS}$, with different energies

$$E = \frac{1}{g^2} \int d^3x \{(F_{0i}^a)^2 + (F_{ij}^a)^2\}. \quad (12)$$

Since $N_{CS}$ becomes some integer for $E = 0$, the vacuum can be classified by $N_{CS}$ as $|\text{vac}, N_{CS} >$.

The baryon and lepton numbers shift through some configurations: $N_B \rightarrow N_B + N_g$, $N_L \rightarrow N_L + N_g$ with $|\text{vac}, N_{CS} > \rightarrow |\text{vac}, N_{CS} + 1 >$. One of such configurations is called "sphaleron" [25], which has $N_{CS} = 1/2$ and $E = 4\pi v/g^2 \simeq 10$ TeV. There are also a lot of other configurations to generate the changes of $N_B$ and $N_L$. Nambu’s electroweak string is one example [26]. These are, however, solutions in the broken phase after the electroweak phase transition ends and the transition rates between different baryon numbers are negligibly small for baryogenesis.

Baryogenesis is undertook not after the phase transition, but in the course of the phase transition, since baryon number violation is not suppressed in the unbroken phase. The transition rate between different baryon numbers has been estimated by computer simulations in lattice gauge theory: Observing the temporal change of $N_{CS}(t)$, the "lifetime" of having a nearly constant value (plateau) for $N_{CS}(t)$ is equated with $(\Gamma_{N_B-\text{violation}} V)^{-1}$ [27], $V$ being the space volume. The obtained transition rate $\Gamma_{N_B-\text{violation}}$ per unit volume and unit time is given by

$$\Gamma_{N_B-\text{violation}} = \kappa (\alpha_u T)^4, \quad (13)$$

with $\kappa = 0.1 - 1$. The reason why the rate is a function of the product $\alpha_u T$ is easily understood. The related Boltzmann weight of the pure gauge system is a function
of $\alpha_w T$, read from $\exp(-E/T) = \exp(-\text{constant}/g^2 T)$. Therefore, the rate per unit time and unit volume may be written as Eq. (13) from the dimensional analysis.

The $N_B$-violating transition rate implies that one transition occurs typically in the domain with the volume $(\alpha_w T)^{-3}$ and the time interval $(\alpha_w T)^{-1}$: Each domain has a configuration of the gauge fields with a value of $N_{CS}$. Each configuration appears with a proper Boltzmann weight. This is something like the bubble nucleations, although more complicate, since there are infinite types of bubble (infinite integers for the vacua and infinite real numbers for the excited configurations). If the quarks or leptons enter this medium, what really happens afterwards?

**[Problem 5]** What is the physical picture of the baryon number generating mechanism in the unbroken phase at finite temperature?

### 5.2 Phase transition

In the electroweak baryogenesis, thermal non-equilibrium is attributed to the first order phase transition of the universe accompanied by the electroweak symmetry breaking. Bubbles of the broken phase nucleate, expand, fuse with each other, and finally cover the whole space. Static properties of the phase transition can be obtained by analyzing the effective potential for the classical Higgs fields at finite temperature at least in the one-loop approximation. In the SM the one-loop effective potential is written in the large temperature approximation by

$$V_T(\phi) = \frac{\lambda_T}{4} \phi^4 - ET \phi^3 + D(T^2 - T_0^2) \phi^2,$$

where $T$ denotes temperature. The first order phase transition could be achieved, if the potential has the $\phi^3$ term. A bump then exists in the effective potential and separates the unbroken and the broken minima. The global minimum of the potential is at $\phi = 0$ for $T > T_c$ and at $\phi = \phi_{\text{min}} \neq 0$ for $T < T_c$, where the critical temperature $T_c$ is given by

$$T_c = \frac{T_0}{\sqrt{1 - E^2/\lambda_T D}}.$$

The $\phi^3$ term is induced exclusively by the interactions of bosons and the order parameter $\phi$ [28]. Therefore, the introduction of additional Higgs fields could increase the coefficient $E$, which raises the height of the bump and makes the first order phase transition stronger. This is one reason why multi-Higgs models are adopted in various attempts for baryogenesis. However, it necessitates elaborate analyses to
find in various models whether the electroweak phase transition is of the strongly first order or not.

[Problem 6] In multi-Higgs models, various kinds of bubbles may coexist having different vacuum expectation values for the Higgs fields. Discuss the complexity of the phase transition.

As the temperature $T$ cools down from the critical temperature $T_c$, the broken phase becomes more energetically favorable than the unbroken phase. When the temperature becomes lower than $T_c$ for a certain amount and the latent heat $\epsilon$ becomes nonnegligible, the phase transition begins by nucleations of bubbles with the broken phase. The latent heat is the difference of the effective potential between the broken and unbroken minima, being released during the transition. The shapes of the nucleated bubbles are determined by stationary configurations for the free energy $F$ of the Higgs fields roughly given by

$$F = \int d^3x \left[ \frac{1}{2} Z_T(\phi)(\nabla \phi)^2 + V_T(\phi) \right]. \quad (16)$$

The free energy of the bubble is essentially the sum of the surface energy $4\pi R^2 \sigma$ and the energy $-\left(\frac{4}{3}\pi R^3\epsilon\right)$ inside the bubble coming from the latent heat. Approximating that the width of the bubble wall is much smaller than the radius of the bubble, the width $\delta_w$ and the radius $R_c$ of the critical bubble are given by

$$\delta_w \approx \sqrt{\frac{\lambda_T}{ET}}, \quad R_c \approx \frac{(ET)^3}{\sqrt{\lambda_T^5 \epsilon}}, \quad \epsilon \approx D(T_c^2 - T^2)\phi_{\min}^2. \quad (17)$$

The critical bubble is the maximum point for the free energy, which is something like the sphaleron solution, connecting the smaller shrinking bubble and the larger expanding bubble. A problem is that we have essentially only one dimensional quantity, namely, the temperature $T$. Ignoring the details of the model, the wall width and the critical radius are of order $10^{-18}$ m. To have a clear interface of the bubble, i.e. $\delta_w \ll R_c$, the model has to give a large value to $E$ or a small value to $\epsilon$.

The nucleation rate $I$ of the critical bubble per unit volume and unit time is given in terms of the free energy $F_c$ of the critical bubble as

$$I = T^4 \left( \frac{F_c}{2\pi T} \right)^{3/2} \exp \left( -\frac{F_c}{T} \right), \quad (18)$$

$$\frac{F_c}{T} \propto \frac{1}{(T_c^2/T^2 - 1)^2}. \quad (19)$$
This equation shows that the nucleation rate is almost vanishing during some interval of cooling down from $T_c$, but rapidly increases to become nonnegligible at some temperature $T = T_c - \Delta$. The phase transition occurs when the nucleation rate becomes larger than the expansion rate of the universe, i.e. $F_c/T < 140$. However, how can the exact value of $\Delta$ be determined?

The phase transition, if it is weakly first order, may proceed even at the temperature above $T_c$ through so-called "subcritical bubbles". The contributions of the subcritical bubbles to the phase transition have been studied, including both quantum and thermal fluctuations at finite temperature [29]. If the subcritical bubbles live long, the phase transition dynamics could drastically be changed from its conventional picture. However, a recent study suggests that the subcritical bubbles are easily deformed from spherical shape and non-spherical bubbles are generally short-lived [30].

The electroweak phase transition has also been studied by computer simulation in lattice gauge theory. In a recent work [31], where the Monte Carlo time history of the Higgs action $L_s(t)$ coupled to the gauge fields is simulated, we can see that appearances and disappearances of bubbles do occur in the course of the phase transition. Could we combine this analysis and that analysis on the time history of $N_{CS}(t)$ to see the local structure of the universe?

The nucleated bubble expands in the medium. The bubble wall is accelerated by the pressure inside the bubble, while decelerated by the scattering of thermal particles in the outside medium. The velocity of the wall $v_w(t)$ first grows exponentially and approaches gradually to a constant value $v_w(\infty)$ determined by the balance of the different forces. However, non-equilibrium interactions of the thermal plasma with the wall make the calculation of the wall velocity rather complicated.

**[Problem 7]** Study the electroweak phase transition and estimate rigorously the various obscure quantities, such as $T = T_c - \Delta$, $v_w(t)$, $\delta_w$, and $R_c$.

After expanding for some time, bubbles fuse with each other, which is a very complicate process. There exists an exactly solvable model for this dynamics, called Kolmogorov-Avrami theory [32]. This is a stochastic theory based on the Poisson distribution of the bubble formation. However, it is assumed that the wall velocity is independent of time, $v_w(t) = \text{constant}$, and the critical radius is zero, $R_c = 0$. A model without such specific assumptions is welcomed.

**[Problem 8]** By modifying the Kolmogorov-Avrami theory, construct a realistic theory of the phase transition in field theoretical manner, in which the bubbles of
three-dimensional domain are created and annihilated.

5.3 Other issues

Various time scales enter the discussion of baryogenesis. The phase transition temperature is roughly given by $T \approx 100$ GeV, which is the same order of magnitude as the weak boson masses. The length scale and the time scale are respectively given by $\ell_0 = 2 \times 10^{-18}$ m and $\tau_0 = 10^{-26}$ s. The time scales of the weak and strong interactions may be estimated by second order processes:

$$
\tau_{\text{weak}} = \frac{1}{\sigma_{\text{weak}} n} \approx \frac{1}{\alpha_w^2 T} \approx \frac{10^3}{T},
$$

$$
\tau_{\text{strong}} = \frac{1}{\sigma_{\text{strong}} n} \approx \frac{1}{\alpha_s^2 T} \approx \frac{10^2}{T},
$$

where $n$ denotes the number density of the relevant particles approximated by $n \approx T^3$. The time scale of the baryon number violation is estimated by

$$
\tau_{N_B-\text{violation}} = \frac{n}{\Gamma_{N_B-\text{violation}}} \approx \frac{1}{\alpha_w^4 T} \approx \frac{10^6}{T}.
$$

To see whether the system placed in the experimental apparatus of the universe is in thermal equilibrium or not, the time scales of interactions have to be compared with that of the expansion of the universe

$$
\tau_{\text{expansion}} \approx \frac{1}{H} \approx \frac{1}{\sqrt{\frac{M_{\text{Planck}}^2}{T^4}}} \approx \frac{10^{17}}{T},
$$

where $H$ denotes the Hubble constant. Comparing these results, we have the ratio

$$
\tau_{\text{strong}} : \tau_{\text{weak}} : \tau_{N_B-\text{violation}} : \tau_{\text{expansion}} = 10^{-1} : 1 : 10^3 : 10^{14}.
$$

Therefore, the effect of the expansion of the universe is completely negligible for the weak interaction, strong interaction, and $N_B$-violating transition. The $N_B$-violating transition is rather slow behaving as if the fourth order process of the weak interaction.

The amount of baryon asymmetry generated in the phase transition depends on the transport time $\tau_T$ of a net hypercharge emitted from the wall. This represents a time within which the particles carrying the net hypercharge can travel in the medium without suffering so much from thermal interactions with the medium. For the leptons and the quarks, $\tau_T$ may be given by $\tau_{\text{weak}}$ and $\tau_{\text{strong}}$, respectively. After moving approximately at the speed of light during the transport time, the particles
get thermalized to bias equilibrium conditions for favoring a non-vanishing value of baryon number, which is then generated by electroweak anomaly. The produced baryon number is swallowed from behind by the macroscopic bubble wall moving roughly at a constant speed $v_w$. The time for the wall to catch up with the baryon number is thus given by $\tau_{\text{catchup}} \simeq \tau_T/v_w$. This period is not so large, but should be sufficiently large for the system to establish thermal equilibrium and the $N_B$-violating transition to occur.

The net flux of hypercharge is emitted from the bubble wall to the symmetric phase, owing to $CP$ violation. In the processes that the particles are reflected or transmitted at the wall, $CP$ violation makes differences in their probabilities between $CP$-conjugate states, which leads to the non-vanishing density of hypercharge. For obtaining quantitatively the hypercharge flux, it is necessary to solve Dirac equations or Klein-Gordon equations. The methods of solving these equations numerically have been established and work very well now. On the other hand, analytical methods have been applied only for limited cases.

**[Problem 9]** Obtain analytically the $CP$ asymmetry in the reflection and transmission probabilities for the particles at the wall. Can we use the exact solvability in the soliton physics for this purpose? The scattering data and the potential are strongly related in the soliton theory.

**[Problem 10]** Discuss the conservation of angular momentum and the mixing of left-handed and right-handed particles, when the particles enter the bubble wall obliquely.

The electroweak baryogenesis necessitates new sources of $CP$ violation other than the Kobayashi-Maskawa phase of the SM. Up to now several models have been proposed. If the new $CP$-violating source of a model affects phenomena observable in experiments, the model could be constrained nontrivially. In these models detailed analyses for consistency between the baryon asymmetry and the experimental observables are indispensable. On the other hand, in a recently proposed model the new $CP$-violating source could be only viable within the bubble wall, which does not affect phenomena in the broken phase.

The microscopic picture for various transitions of the particles outside the bubble wall has not been well established. The particles reflected or transmitted from the bubble wall interact with the medium and then $N_B$-violating transitions occur. These processes depend on the details of the reactions for the particles. Although analyzing such a whole system is very complicated, the microscopic picture is nec-
necessary for discussing the electroweak baryogenesis more quantitatively.

**Acknowledgments**

We give our sincere thanks to Kaoru Hagiwara without whom this series of KEK meetings had not been organized so splendidly and to whom all the participants are indebted. We are grateful to Hajime Aoki, Jiro Arafune, Koichi Funakubo, Jun-ichi Kamoshita, Yasuhiro Katsuki, Minako Kitahara, Takao Ohta, Isamu Watanabe, Yoshio Yamaguchi, and Katsuji Yamamoto for fruitful and valuable discussions. The work of M.A. is supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Sciences and Culture, in Japan. This work is supported in part by the Grant-in-Aid for Scientific Research (No. 08640357) and by the Grant-in-Aid for Joint Scientific Research (No. 08044089) from the Ministry of Education, Sciences and Culture, in Japan.

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