Generalized entropic gravity from modified Unruh temperature

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In this study, the effects of the generalized uncertainty principle on the theory of gravity are analyzed. Inspired by Verlinde’s entropic gravity approach and using the modified Unruh temperature, the generalized Einstein field equations with cosmological constant are obtained and corresponding conservation law is investigated. Then, Newton’s law of gravity and the Poisson equation are derived with some correction. In a certain limit, our generalized equations reduce their standard forms.

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I. INTRODUCTION

In theoretical physics, one of the main study is to generalize the theory of gravity. Indeed, we have several good reasons. For example, the integration of the general theory relativity (GR) with the Standard Model is still unclear and the unification of the quantum mechanics and gravitation has not been done. For this aim, there are many methods to cope with these difficulties such as the supergravity, the noncommutative geometry, the quantum gravity, and string theory.

It is interesting to study the generalized theory of gravity from thermodynamical perspective. There are several theoretical studies which describe the thermodynamical origin of gravity. The first studies were done by Bekenstein and Hawking [1–3] which considered the formulation of the laws of black hole mechanics in thermodynamic point of view. According to these studies, the relation has occurred between black hole area and its entropy which is called the Bekenstein-Hawking (BH) entropy. This results opened a new window to describe the emergent nature of space-time and gravity. In 1995, Jacobson was the first who claimed that the gravity may not be a fundamental interaction but is originated from the first law of thermodynamics on a local Rindler horizon and he found the Einstein’s equation based on this idea [4]. Another important work was achieved by Verlinde [5] (see also [6]) who formulated a thermodynamical description of gravity by considering the holographic point of view and using BH entropy, and he obtained Newton’s law of gravity and the Einstein field equation. Verlinde also suggested that gravity emerges as an entropic force and then, this idea was named as the entropic gravity.

In addition to this, the works on entropic gravity proposal can be classified with two approaches [7]. The first one is the thermodynamic gravity (TG) which uses to obtain the Einstein’s equation from the BH entropy [4–6, 8–10], and the second one is the holographic gravity (HG) in which the Einstein’s equation can be obtained by keeping entropy stationery in equilibrium under variations of space-time geometry and quantum states within a small region [11–14].

After this theoretical background, one can say that the entropic gravity approach provides a powerful framework to generalize the theory of gravity. Thus, it can help us a deep understanding of the nature of space-time and gravity. For instance, when you use generalized thermodynamic quantities such as entropy, this process may cause to find the generalized theory of gravity [15–23].

These works motivate us to study the relation between the generalized uncertainty principle (GUP) and the entropic gravity. In this way, we consider the GUP corrected Unruh temperature, to obtain contributions to the theory of gravitation. For this purpose, using Verlinde’s entropic gravity approach [5] and the modified Unruh temperature given in [24], the generalized Einstein equations with the cosmological term is derived and its conservation law is discussed. In this connection, some corrections to Newton’s law of gravity and the Poisson equation are obtained.

The paper is organized as follows. In Sec. 2, the historical background of the Unruh effect and the modification of Unruh temperature based on GUP is shortly reviewed. In Sec. 3, the generalized Einstein field equation is obtained. In Sec. 4, we find generalized versions of Newton’s law of gravity and the Poisson equation. The last section concludes the paper and contains some discussion on about our results and possible future works.
II. GUP CORRECTED UNRUH TEMPERATURE

Discovery of the Hawking effect [2, 3] demonstrates that black holes should radiate with a temperature $T_H = \frac{\hbar g}{2\pi k_B c}$ where $g$ is the gravitational acceleration, $k_B$ is Boltzman’s constant and $c$ is the speed of light. In other words, the Hawking effect describes the black-body radiation which is produced by the vacuum fluctuations near the event horizon of the black hole.

In addition to this, there is another approach which is called the Unruh effect (also called Fulling-Davies-Unruh effect) [25–27]. The Unruh effect describes that a uniformly accelerated observer moving through in a flat space-time (Minkowski space-time) will observe black-body radiation released from the black hole. In this condition, the observer experiences following temperature,

$$T_U = \frac{\hbar a}{2\pi k_B c},$$

as though it were in a thermal field where an inertial observer would observe none. Here, $a$ is the acceleration of the observer. This temperature is called the Unruh temperature (or the Hawking-Unruh temperature). The Unruh effect, similarly with the Hawking effect, is originated from the vacuum fluctuations which cause a particle-antiparticle pair creation near the Rindler event horizon of the black hole. This effect is important to understand the notion of particle emission from cosmological horizons and black holes (for more detail see [28, 29]).

The Unruh effect can be obtained by taking into account the Heisenberg uncertainty principle (HUP) when we consider a photon that has crossed the Rindler event horizon. In this case, the position uncertainty of the photon depends on the crossing point of the Rindler event horizon. A direct derivation of the Unruh effect by the help of HUP can be found in [30] (see also [31]).

One can say that the generalization of HUP may cause additional contributions to the Unruh effect. We know from early studies [32–35], HUP should be generalized when we consider gravitational effect in a micro scale where the gravitational effect is neglected. This process can be seen as a quantum gravitational correction to HUP and it is called the generalized uncertainty principle (GUP). This approximation was used various branches of physics such as the string theory, loop quantum gravity, and non-commutative quantum mechanics (for more detail see review [36]). For instance, GUP affects the Planck-scale black holes and their thermodynamics including the Unruh temperature [24] and the Hawking temperature [37]. Now we shall briefly review the paper [24] which includes the GUP corrected Unruh temperature. In order to obtain a generalized version of the Unruh temperature, the authors use the following form of GUP,

$$\Delta x \Delta p \geq \frac{\hbar}{2} \left[ 1 + \beta \left( \frac{\Delta p}{m_p} \right)^2 \right], \quad \left[ \hat{x}, \hat{p} \right] = i \hbar \left[ 1 + \beta \left( \frac{\hat{p}}{m_p} \right)^2 \right],$$

where $\beta$ is a constant dimensionless deformation parameter and the constants are chosen as $c = 1$, $k_B = 1$ throughout this paper. Considering this GUP background, using the quantum field theoretic calculations, the generalized Unruh temperature can be found as,

$$T \lesssim T_U \left[ 1 + \beta \pi \Omega \left( \frac{l_p a}{\pi} \right)^2 \right],$$

where $\Omega$ is the Rindler frequency, $l_p = \sqrt{\frac{G\hbar}{c^3}} \approx 10^{-35} m$ is the Planck length and $a$ is the acceleration of the observer. In a certain condition, we can take the Rindler frequency as $\Omega \approx 1/2\pi$ and thus, Eq.3 takes following form [24],

$$T \lesssim T_U \left[ 1 + \frac{\beta}{2} \left( \frac{l_p a}{\pi} \right)^2 \right] = T_U \left[ 1 + \frac{\beta}{2} \left( \frac{T_U}{m_p} \right)^2 \right].$$

Thus, we can write the last equations as,

$$T = T_U f(a) = \frac{\hbar a}{2\pi} f(a)$$

where the function $f(a)$ is,
\[ f(a) = \left[ 1 + \frac{\beta}{2} \left( \frac{l_p a}{\pi} \right)^2 \right] = \left[ 1 + \frac{\beta}{2} \left( \frac{T_U}{m_p} \right)^2 \right] \tag{6} \]

where \( m_p = \hbar/2l_p \simeq 10^{-8} \text{kg} \) is the Planck mass. Setting the deformation parameter \( \beta = 0 \), we can get the standard Unruh temperature.

### III. GENERALIZED EINSTEIN FIELD EQUATIONS

The thermodynamic origin of gravity, also known as entropic gravity, demonstrates the relation between gravity and thermodynamical quantities such as energy, temperature, and entropy. Thus, if one modifies the mentioned quantities, the gravity should be modified. In this section, we will follow Verlinde’s entropic gravity proposal \([5]\) and the method is given in \([20]\) to generalize the Einstein field equation by using the modified Unruh temperature Eq. \((6)\).

Verlinde considers the black hole horizon as a spherically symmetric holographic screen \( S \) based on Bekenstein’s idea \([1]\). In this case, we have a test particle with a mass \( m \) and an acceleration \( a \) moving along to the screen (see Fig. \(1\)). We start with the generalized Newton potential in general relativity \([38]\),

\[ \phi = \ln \left( -\xi^a \xi_a \right), \tag{7} \]

where \( \xi^a \) corresponds time-like killing vector in a static background. By the help of the Killing vector, the four velocity of the particle and its four acceleration can be written as follows,

\[ u^b = e^{-\phi} \xi, \quad a^b = e^{-2\phi} \xi^a \nabla_a \xi^b. \tag{8} \]

With the help of the Killing equation \( \nabla_a \xi_b + \nabla_b \xi_a = 0 \), the four-acceleration can be expressed in terms of the Newton potential \( \phi \) as,

\[ a^b = -\nabla^b \phi. \tag{9} \]

From these definitions, the local temperature \( T \), given in Eq. \((5)\), on the holographic screen as an equipotential surface is now in analogue with non-relativistic case is defined as,

\[ T = \frac{\hbar}{2\pi} f(a) e^{\phi} N^a \nabla_a \phi, \tag{10} \]

where \( N^a \) a outward vector normal to the holographic surface \( S \) and \( \xi^a \). The exponential \( e^\phi \) represents the redshift factor that relates local time coordinate to that at a reference point with \( \phi = 0 \), because the temperature is measured with respect to the reference point at infinity \([5]\).

To find the relativistic analogue of Newton’s second law of gravity \( F = ma \), one can assume that the particle very close to the screen. In this case, the change in entropy at the screen is given by,

\[ \nabla_a S = -2\pi \frac{m}{\hbar} N_a. \tag{11} \]

So one can obtain the entropic force by using the last equation and Eq. \((10)\) as follows,

\[ F_a = T \nabla_a S = -m e^{\phi} f(a) \nabla_a \phi, \tag{12} \]

where \( f(a) \) corresponds GUP correction Eq. \((6)\). The last equation defines the force on a particle which is located very close the screen.

Now we would like to derive the generalized Einstein’s field equation originated from the entropic force by using generalized Unruh temperature which is given in Eq. \((10)\). According to Bekenstein, if there is a test particle near the black hole horizon which is distant from a Compton wavelength, the test particle increases black hole mass. This
Figure 1. Suppose that we have two masses, one is a test particle with mass $m$ close to spherical holographic screen $S$, and the other is the source with mass $M$ surrounded by the screen.

The process is defined as one bit of information. Considering the holographic screen on a closed surface with constant redshift, the bit density on the screen can be specified as,

$$N = \frac{A}{G\hbar},$$

where $A$ is the area of closed surface on the screen, and $G$ is a constant which will be identified as the Newton’s gravitational constant. Assuming that the energy is related with mass $M$ is distributed over all the bits. According to the equipartition law of energy $E = \frac{1}{2}TN$, and using the relation $E = M$, the total mass can be written as

$$M = \frac{1}{2} \int_S TdN. \quad (14)$$

Then substituting the Eq.(10) and Eq.(13) in Eq.(14), we get,

$$M = \frac{1}{4\pi G} \int_S f(a) e^\phi \nabla \phi dA, \quad (15)$$

where differential surface element is described by,

$$|dA| = dx^a \land dx^b \epsilon_{abcd} e^{-\phi} \xi^c N^d. \quad (16)$$

The Eq.(15) can be seen as the natural generalization of Gauss’s law to the General Relativity and the mass $M$ corresponds to the Komar mass. According to the papers [17, 19, 20] using the Stokes theorem, and the relations Eq.(17) and $\nabla_a \nabla^a \xi^b = -R^b_a \xi^a$, the Komar mass can be written as,

$$M = \frac{1}{4\pi G} \int_V \left\{ R_{ab} \xi^b n^a + [\nabla_b f(a)] [\nabla_a \xi^b] n^a \right\} dV, \quad (17)$$

where $R_{ab}$ is the Ricci curvature tensor, $V$ is the three dimensional volume bounded by the holographic screen and $n^a$ is its outward normal. Using the relation $\nabla^b \nabla_b f(a) = 0$ which is proved in [19], the second term in the parenthesis can be written by,

$$[\nabla_b f(a)] [\nabla_a \xi^b] n^a = -n^a \xi^b \nabla_a \nabla_b f(a). \quad (18)$$

Furthermore, the Komar mass can also be written as an integral over the enclosed volume of certain components of the energy-momentum tensor $T_{ab}$ [35] as,

$$M = 2 \int_V \left( T_{ab} - \frac{1}{2}g_{ab} T \right) \xi^b n^a dV; \quad (19)$$
where \( g_{ab} \) is space time metric tensor. When considering Eqs. (17) and (19), we find the generalized Einstein field equation as follows,

\[
f(a) R_{ab} - \nabla_a \nabla_b f(a) = 8\pi G \left( T_{ab} - \frac{1}{2} g_{ab} T \right).
\]

This gives us only a time-time component of the field equation \[5\]. The last equation can also be written as,

\[
f(a) \left( R_{ab} - \frac{1}{2} g_{ab} R \right) - \left( \nabla_a \nabla_b f(a) - \frac{1}{2} g_{ab} \nabla^2 f(a) \right) = 8\pi G T_{ab}.
\]

For more simplicity, Eq. (21) takes following form,

\[
R_{ab} - \frac{1}{2} g_{ab} R = \frac{8\pi G}{f(a)} \left\{ T_{ab} + T_{ab}(a) \right\}.
\]

Last result can be seen as the GUP corrected Einstein field equation with a new additional energy-momentum tensor as a function of acceleration which is defined,

\[
T_{ab}(a) = \frac{1}{8\pi G} \left( \nabla_a \nabla_b f(a) - \frac{1}{2} g_{ab} \nabla^2 f(a) \right).
\]

In this new condition, the laws of energy-momentum conservation should be reconsidered because we know that conservation laws are important in physics. From the definition of the Ricci tensor, we get the following expression,

\[
\nabla_a \left( R^a_b - \frac{1}{2} \delta^a_b R \right) = 0.
\]

Later, considering standard Einstein field equation, the covariant conservation law of the energy-momentum tensor can be written as \( \nabla_a T^a_b = 0 \). In our case, the covariant derivative of left-hand side of Eq. (21) should be zero. In order to satisfy this condition, we get the following equation,

\[
R \nabla_a f(a) + \nabla_a \nabla^2 f(a) = -16\pi G \nabla_a T^a_b.
\]

We know that \( \nabla_a T^a_b = 0 \), thus the left-hand side of the last equation should be zero, so we get following constraint,

\[
R \nabla_a f(a) = -\nabla_a \nabla^2 f(a).
\]

Under this constraint, the conservation of energy is satisfied. Moreover, if we set the constant \( \beta = 0 \) then the function takes \( f(a) = 1 \) thus the standard Einstein field equation is recovered,

\[
R_{ab} - \frac{1}{2} g_{ab} R = 8\pi G T_{ab}.
\]

Furthermore, considering an alternative expression of the Komar mass with cosmological constant \( \Lambda \)[16, 38],

\[
M = 2 \int_V \left( T_{ab} - \frac{1}{2} g_{ab} T + \frac{\Lambda}{8\pi G} g_{ab} \right) \alpha^a \xi^b dV,
\]

then comparing the last equation with Eq. (17), one can obtain the following field equation,

\[
R_{ab} - \frac{1}{2} g_{ab} R + \frac{\Lambda}{f(a)} g_{ab} = \frac{8\pi G}{f(a)} \left\{ T_{ab} + T_{ab}(a) \right\}.
\]

Thus, we obtained the Einstein field equation with cosmological constant plus an additional term and some corrections. In addition to this, according to Eq. (29), the constraint given in Eq. (26) leaves unchanged because the covariant derivative of cosmological constant is zero.
IV. CORRECTIONS TO THE NEWTON’S LAW OF GRAVITY AND THE POISSON EQUATION

Supposing that a particle of mass \(m\) moves a distance \(\Delta x\) orthogonal to the holographic screen as in Fig. 2. In this case, there is a relation between the change of entropy \(\Delta S\) and displacement of the particle from screen \(\Delta x\) \[5\],

\[
\Delta S = 2\pi m \frac{\hbar}{\Delta x}.
\] (30)

The last equation demonstrates that, the entropy is proportional to the information of the test particle and it can be seen as one of the main formula for the construction of the entropic force. Furthermore, if we consider the Compton wavelength \(\lambda_m = \hbar/m\), then the Eq. (30) can be written as \(\Delta S = 2\pi \Delta x/\lambda_m\). In the case of \(\Delta x \approx \lambda_m\), the particle is absorbed by the screen and it causes to increase the entropy of the system \[5\]. According to the Verlinde’s proposal, the entropic force can be written as follows,

\[
F = T \frac{\Delta S}{\Delta x}.
\] (31)

Substituting the modified Unruh temperature and Eq. (30) in Eq. (31) we get the gravitational force which acting on a particle of mass \(m\),

\[
F = ma f(a) = ma \left[ 1 + \frac{\beta}{2} \left( \frac{\ell_p a}{\pi} \right)^2 \right],
\] (32)

or it can alternatively be written in terms of the standard Unruh temperature,

\[
F = ma \left[ 1 + \frac{\beta}{2} \left( \frac{T_U}{m_p} \right)^2 \right].
\] (33)

The Eq. (32) represents the generalized Newton’s law of inertia with the Planck scale correction based on GUP. Now we want to determine how the modified Unruh temperature affect the Poisson equation. For this purpose, we use a holographic screen \(S\) corresponding to an equipotential surface with fixed Newton potential \(\phi_0\). We assume that the volume which enclosed by the screen contains the mass distribution given by \(\rho(x)\), and all test particles are outside of this volume. Considering the Unruh temperature, we should define the acceleration. This acceleration can be found when we consider a test particle that is moving closer to the screen. The local acceleration of the test particle can be defined with the Newton potential as,

\[
\ddot{a} = -\nabla \phi.
\] (34)
Then, the modified Unruh temperature takes the following form,

$$T = \frac{\hbar}{2\pi f(a) \left| \vec{\nabla} \phi \right|}. \quad (35)$$

Substituting the last equation in Eq. (14) and using Eq. (13) we get,

$$M = \frac{1}{4\pi G} \int_S f(a) \vec{\nabla} \phi \cdot d\vec{A}. \quad (36)$$

The last equation can be written by using the divergence theorem as follows,

$$M = \frac{1}{4\pi G} \int_V \vec{\nabla} \cdot \left[ f(a) \vec{\nabla} \phi \right] dV, \quad (37)$$

where $V$ represents three dimensional volume element. Furthermore, the mass distribution of the volume in the closed surface $S$ can also be given as

$$M = \int_V \rho(\vec{x}) dV. \quad (38)$$

Comparing Eq. (37) with Eq. (38), we get the modified Poisson equation for gravity as follows,

$$\vec{\nabla} \cdot \left[ f(a) \vec{\nabla} \phi(\vec{r}) \right] = 4\pi G \rho(\vec{x}), \quad (39)$$

This result shows that there is an exact contribution to the Poisson equation comes from GUP. Consequently, one can say that changing the temperature leads to obtaining another form of gravity. When you take the constant $\beta = 0$ then the Eq. (39) reduces its standard form,

$$\nabla^2 \phi(\vec{x}) = 4\pi G \rho(\vec{x}). \quad (40)$$

V. CONCLUSION

In this work, we investigated the possible effects of generalized uncertainty principle (GUP) on Einstein’s theory of general relativity and Newton’s law of gravity. For this aim, we considered Verlinde’s entropic gravity proposal and the GUP corrected Unruh temperature. Using this theoretical background, we have derived generalized version of the Einstein field equations with cosmological constant and associated conservation law of energy-momentum tensor was analyzed. Considering Eq. (29), a new source term $T_{ab}(a)$ was occurred. Here, we can say that the modified temperature causes to a new energy source term to the Einstein equation. This kind of source term may be related to the dark energy [39, 40]. For this reason, the cosmological consequences of the Eq. (22) and Eq. (29) are open problems. We also investigated the conservation law of the generalized Einstein field equations and we found a constraint Eq. (26) which is required to satisfy corresponding conservation law.

Moreover, the relativistic and nonrelativistic forms of Newton’s law of gravity was found with GUP corrections. This modification has similar prediction with the Randall-Sundrum II model [41] which includes one uncompactified extra dimension. Parallel results can be found in [42] which was used a different model of GUP. We also found the GUP corrected Poisson equation by using the entropic gravity approximation.

The deformation parameter $\beta$ was studied in the framework of the gravitational and non-gravitational regimes in the literature (the corresponding results was revived in [43]). For instance, the deformation parameter should be $\beta < 10^{69}$ in the perihelion precession (Solar system data) in gravitational measurements. According to the paper [37], the parameter was also calculated as $\beta = 82\pi/5$ in the framework of the quantum field theory and the general theory of relativity. Taking the consideration of the non-gravitational regime, the parameter takes $\beta < 10^{20}$ and $\beta < 10^{46}$ for the Lamb shift and the Landau levels respectively. Our results provide a new background to study the mentioned fields [43] in the gravitational regime. Besides, it is easy to show that, in a certain limit ($\beta = 0$), all our results reduce their conventional forms.

Our results and the papers [15–23] show that the entropic gravity proposal provides a powerful framework to obtain the generalized theory of gravity by using thermodynamical quantities.
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