Measuring CP violation within Effective Field Theory of inflation from CMB

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In this work we propose an Effective Field Theory of inflation taking into account the spin density of matter which contributes to torsion. We first explicitly show that torsion mimics the role of a scalar field which is instrumental in causing inflation. We have obtained a strict bound on CP violating $\theta$ parameter, $\mathcal{O}(10^{-10}) < \theta < \mathcal{O}(10^{-9})$, using Planck+WMAP9 best fit cosmological parameters.

It is well known that torsion and curvature of any manifold are related to translation and rotation respectively. When both the symmetries are operative, the gravity sector should contain contributions from both. More specifically, when the energy momentum tensor is represented by the spin density of matter sources, the torsion contributes to gravity whereas mass density gives rise to curvature. In the context of cosmology, in Ref. [1] it was pointed out that the general features of inflationary scenario can be explained through torsion. Further, in [2] it was shown that torsion can be treated as an alternative to cosmic inflation which is generally believed to be caused by scalar field called ‘inflaton’. In the realm of quantum gravity when Ashtekar’s formalism of quantization of gravity is transcribed to Lagrangian formulation as proposed by Capovilla, Jacobson and Drell (CJD) [3] we can include torsion through the introduction of CP violating $\theta$ term in the Lagrangian. In this work our prime motivation is to explicitly show that there is a hidden scalar field in torsion caused by the spin density of matter. From the modified CJD Lagrangian including torsion we develop an effective potential in terms of this hidden scalar field which is instrumental in causing inflation. Using this methodology we give a strict bound on CP violation using Planck+WMAP9 best fit cosmological parameters [4].

At the classical level a generalized theory of gravity incorporating torsion is given by the celebrated Einsten-Cartan-Kibble-Sciama (ECKS) formalism [5] 1. In this, the affine connection has non-vanishing antisymmetric contribution leading to torsion which can be represented by spin-spin interaction. A dual current-current interaction picture can be developed by translating the SU(2) spin basis into the topological current. Within this prescription the spin-current duality can be explained in terms of a four-vector $n_\mu \in \text{SU}(2)$ in the Casimir operator basis as:

$$ n_\mu = \left( \frac{1}{\sqrt{2}} \right) \begin{pmatrix} \psi_1^* & \psi_2^* \end{pmatrix} \sigma_\mu \begin{pmatrix} \psi_1 & \psi_2 \end{pmatrix} $$

where $\psi_1 = (\cos \theta/2) e^{i\phi/2}$, $\psi_2 = (\sin \theta/2) e^{-i\phi/2}$. Using this one can construct an SU(2) group element $g = n_0 I + i \tilde{n} \cdot \sigma$, in terms of which the topological current can be expressed as [6]:

$$ J_\mu = \left( \frac{1}{24 \pi^2} \right) \epsilon_{\mu \nu \lambda \sigma} \text{Tr}[(g^{-1} \partial^\nu g)(g^{-1} \partial^\lambda g)(g^{-1} \partial^\sigma g)] $$

where $\epsilon_{\mu \nu \lambda \sigma}$ is the rank-4 Levi-Civita tensor. Now by demanding that in 4-dimensional Euclidean space the field strength $F_{\mu \nu}$ of a gauge potential vanishes on the boundary $S^3$ of a certain volume $V = 1$ inside of which $F_{\mu \nu} \neq 0$, we can write the gauge potential as $A_\mu = g^{-1} \partial_\mu g \in \text{SU}(2)$. Then from Eqn.(2) the Kac-Moody like current $J_\mu$ can be recast in terms of the Chern-Simons secondary characteristic class as [7]:

$$ J_\mu = \left( \frac{1}{16 \pi^2} \right) \epsilon^{\mu \nu \lambda \sigma} \text{Tr} \left( A_\nu F_{\lambda \sigma} + \frac{2}{3} A_\nu A_\lambda A_\sigma \right) $$

which allows us to define a topological invariant as:

$$ Q_P = \left( \frac{1}{16 \pi^2} \right) \int d^4 x \partial_\mu J^\mu $$

which is commonly known as the Pontryagin index 2. Consideration of the Lagrangian $L = -\frac{1}{4} \text{Tr} (\epsilon_{\mu \nu \lambda \sigma} F^{\mu \nu} F^{\lambda \sigma})$ which effectively corresponds to the term $\partial_\mu J^\mu$ where $J^\mu$ is given by eqn.(3) leads to the construction of the current $J^\mu = \epsilon^{\mu \nu \lambda \sigma} a_\nu \otimes F_{\lambda \sigma} = \epsilon^{\mu \nu \lambda \sigma} \partial_\nu f_{\lambda \sigma}$ with $A_\mu = a_\mu \cdot \sigma$ and $F_{\mu \nu} = \partial_\mu A_\nu + [A_\mu, A_\nu]$ [8]. It can be shown that the axial vector current $J^5_\mu = \bar{\psi} \gamma_\mu \gamma_5 \psi$ is related

1 The ECKS theory [5] has two fold advantages over GR because-(1) torsion appears to prevent the formation of singularities from matter composed of particles with half-integer spin and averaged as a spin fluid, and (2) to introduce an UV cutoff in quantum field theory for fermionic degrees of freedom.

2 In fact the Pontryagin index is related to the Chern-Simons invariant through the relation:

$$ \int_{M_4} \text{Tr}(F \wedge F) = \int_{M_3} \text{Tr}(A \wedge dA + \frac{2}{3} A \wedge A \wedge A) $$

where $F$ is the two-form related to the field strength.
to the second component of the current $J_\mu$ through the relation $\partial^\mu J^{(2)}_\mu = -\frac{1}{2} \partial^\mu J_5^\mu \neq 0$. The consistency of the current conservation equations implies that: $J^{(1)}_\mu = -\frac{1}{2} J^{(2)}_\mu$, $J^{(3)}_\mu = +\frac{1}{2} J^{(3)}_\mu$ [9]. Consequently, the current-current interaction can be expressed in terms of $J^{(2)}_\mu$ only. Here we define a duality condition for the component $J^{(2)}_\mu$ as:

$$J^{(2)}_\mu = e^{\mu \nu \lambda \sigma} \partial_\nu j^{(2)}_\lambda = e^{\mu \nu \lambda \sigma} \phi(x)$$  \hspace{1cm} (5)

where $\phi(x)$ is the hidden scalar field and we will later see that it plays the role of inflaton, and $\epsilon^{\mu \nu \lambda \sigma}$ is the rank-3 Levi-Civita tensor. Therefore, the action turns out to be:

$$S_T = -\frac{\Sigma}{2} \int d^4x J^{(2)}_\mu J^{(2)}_\mu = \int d^4x \sqrt{-g(4)} \frac{m^2}{2} \phi^2$$  \hspace{1cm} (6)

which actually represents the CP conserving contribution from torsion. In Eqn. (6) $\Sigma$ represents the current-current interaction strength. Within EFT the effective mass of the scalar degrees of freedom is defined as:

$$m^2 = -\frac{\Sigma}{2} \epsilon^{\mu \nu \lambda \sigma} \epsilon^{\mu \nu \lambda \sigma} \epsilon_{\alpha \beta} = -\frac{144 \Sigma}{a^3(t)}$$  \hspace{1cm} (7)

where $a(t)$ is the scale factor in FRW space-time. Eqn. (6) suggests that the potential associated with torsion can be written as: $V_T(\phi) = -\frac{m^2}{2} \phi^2$. The negative sign of the time dependent coupling constant $m^2$ actually corresponds to the self interaction, when orientation of all the spin degrees of freedom are along the same direction.

Our next step is to find the contribution in the action from curvature, for which we utilize the CJD model, where the action is given by [3, 10]:

$$S = \frac{1}{8} \int \eta(\Omega_{ij} \Omega_{ij} + a \Omega_{ii} \Omega_{jj})$$  \hspace{1cm} (8)

where $\Omega_{ij} = \epsilon^{\alpha \beta \gamma \delta} F_{\alpha \beta i} F_{\gamma \delta j}$ with $\alpha, \beta, \gamma, \delta$ as space time indices, $i, j$ the SU(2) group indices and $\eta$ is a scalar density. The canonical transformation of SU(2) gauge potential $(A_{ai})$ and the corresponding non-abelian fields $(E^a_i, B^a_i)$: $A_{ai} \rightarrow A_{ai}$, $E^a_i \rightarrow E^a_i - \theta B^a_i$ gives rise to a CP-violating $\theta$ term in the CJD Lagrangian so that for $a = -1/2$ [4] the action now reads [3, 10]:

$$S_C = \frac{1}{8} \int \left[ \phi \Omega_{ii} + \eta \left( \Omega_{ij} \Omega_{ij} - \frac{1}{2} \Omega_{ii} \Omega_{jj} \right) \right].$$  \hspace{1cm} (9)

In the first term the parameter $\theta$ essentially corresponds to the measure of CP violation which contributes to torsion and the rest is curvature contribution. The association of the second term in Eqn.(9) reveals that this term in 3+1 description corresponds to the contribution of the metric. Consequently Eqn.(9) can be recast as:

$$S_C = -\frac{\theta}{4} Q_P + \eta \int d^4x \epsilon^{\alpha \beta \gamma \delta} \epsilon^{\mu \nu \rho \sigma} \epsilon_{\alpha \beta \gamma \delta} \epsilon^{\mu \nu \rho \sigma} \epsilon^{\eta \tau \xi \eta} \phi \epsilon^{\xi \eta \tau \xi}$$

$$\times \int dx^\nu \phi \int dx^\nu' \phi \int dx^\xi \phi \int dx^\xi' \phi$$

$$- \frac{\eta}{2} \int d^4x \epsilon^{\mu \nu \lambda \sigma} \epsilon^{\alpha \beta \gamma \delta} \epsilon_{\nu \lambda \sigma} \epsilon_{\alpha \beta \gamma \delta} (\partial_\mu \phi)(\partial_\alpha \phi)(\partial_\delta \phi)(\partial_\delta \phi)$$  \hspace{1cm} (10)

where $\int dx^\nu \phi = \phi[\chi^\nu]$, and the symbol $[\cdots]$ signifies the boundary value of the coordinates in the affine parameter space. Now from Eqn.(10) we get:

$$S_C = -\frac{\theta}{4} Q_P + \int d^4x \sqrt{-g(4)} \left[ \frac{g^{\mu \nu}}{2} (\partial_\mu \phi)(\partial_\nu \phi) - \frac{\lambda}{4} \phi^4 \right]$$  \hspace{1cm} (11)

Noting that the asymptotic constancy of torsion compensates the bare cosmological constant [11] we can relate the contribution to torsion from the CP violating $\theta$ parameter in Eqn.(11) with the effective cosmological constant given by, $\Lambda_{eff} = \frac{\theta}{8 \Lambda_{UV}} Q_P$. We can define the vacuum energy $V_0$ through the relation, $V_0 = 3H^2 + \Lambda_{eff}^2 \Lambda_{UV}^4$. Here $\Lambda_{UV}$ signifies the UV cut-off scale of the proposed EFT theory [7]. Thus the expression for the

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[3] Though Eqn.(7) implies time dependence on the mass, but within the standard inflationary description from EFT the time dependence of the scale factor $a(t)$ follows the de-Sitter solution so that the time dependence of mass freezes. In more generalized EFT prescription this time dependence on mass can give rise to various interesting features and the non-trivial behavior can be studied via Renormalization Group Flow Equations.

[4] In 3+1 dimensional decomposition, $a = -1/2$ corresponds to Astekar’s action.

[5] Here we use the following spin-particle duality relations:

$$\eta e^{\mu \nu \lambda \sigma} \epsilon^{\alpha \beta \gamma \delta} \epsilon_{\nu \lambda \sigma} \epsilon_{\alpha \beta \gamma \delta} = -\sqrt{-g(4)} g^{\alpha \beta}$$

$$\eta e^{\alpha \beta \gamma \delta} \epsilon^{\mu \nu \rho \sigma} \epsilon_{\nu \mu \rho \sigma} \epsilon_{\alpha \beta \gamma \delta} \epsilon_{\eta \tau \xi \eta} [x^\nu x^\nu' x^\xi x^\xi'] = -\frac{\lambda}{4}.$$

[6] Here the effective cosmological constant $\Lambda_{eff}$ has mass dimension 2.

[7] Above the scale $\Lambda_{UV}$ it is necessarily required to introduce the higher order quantum corrections to the usual classical theory of gravity represented via Einstein-Hilbert term, as the role of these corrections are significant in trans-Planckian scale to make the theory UV complete [12]. However such quantum corrections are extremely hard to compute as it completely belongs to the hidden sector of the theory dominated by heavy fields [13]. String theory and Loop Quantum gravity are the two parallel approaches through which one can probe the various technical issues of such hidden sector. In the trans-Planckian regime the classical gravity sector is corrected by incorporating the effect of higher derivative interactions appearing through the local modifications to GR which plays significant role in this context [14]. In this case the appropriate choice of the co-efficients of the corrections would modify the UV behaviour of gravity. Such local higher derivatives can be renormalizable and only help to explain the UV features of gravity in 4D but they typically contain debris like massive ghosts. This problem can be addressed by incorporating the infinite higher derivatives appearing through non-local corrections to GR. In such a situation the non-local contributions, yielding a ghost-free condition for certain analytic
potential from CJD Lagrangian incorporating the CP violating $\theta$ term yields: $V_C(\phi) = V_0 + \frac{\lambda}{4} \phi^4$.

Now let us consider a situation where the superspace has Riemann structure. In such a case the contribution to the conserved current can be expressed as: $J^{\mu} g = \frac{1}{2} e^{\mu \nu \lambda \sigma \delta} R_{\lambda \sigma \delta} \phi^g$, where $\phi^g$ is an arbitrary vector and Riemann curvature tensor can be expressed as:

$$R_{\nu \lambda \sigma \delta} = \partial_{\nu} \omega_{\lambda \sigma} \delta + \omega_{\nu \mu}^{\rho} \omega_{\lambda \rho \sigma} - \omega_{\lambda \rho}^{\rho} \omega_{\nu \sigma} + e_{\nu \rho} e_{\delta \lambda}.$$  \hspace{1cm} (12)

As a result the gravitational part of the action can be written in terms of gravitational current-current interaction in the Riemann space as:

$$S_g = -\frac{\Lambda_{UV}^2}{2} \int d^4x \sqrt{-g} \left[ \frac{\Lambda_{UV}^2}{2} R + \frac{g_{\mu \nu}}{2} (\partial_{\mu} \phi)(\partial_{\nu} \phi) - V(\phi) \right].$$  \hspace{1cm} (13)

Now clubbing the contributions from Eqs.(6,11,13) the total action for the present field theoretic setup can finally be written as:

$$S = \int d^4x \sqrt{-g_4} \left[ \frac{\Lambda_{UV}^2}{2} R + \frac{g_{\mu \nu}}{2} (\partial_{\mu} \phi)(\partial_{\nu} \phi) - V(\phi) \right].$$  \hspace{1cm} (14)

such that the total effective potential is given by:

$$V(\phi) = V_T(\phi) + V_C(\phi) = V_0 - \frac{m^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4.$$  \hspace{1cm} (15)

The effective potential is dominated by the vacuum energy correction term which determines the scale of inflation. To obtain the scale of inflation at $k_s \approx k_{\text{ cmb}}$, we express $V_0$ in terms of inflationary observables as:

$$V_s^{1/4} \approx V_0^{1/4} = 7.389 \times 10^{-3} \Lambda_{UV}^2 \times \left( \frac{r}{0.1} \right)^{1/4}. \hspace{1cm} (16)$$

where $r$ is the tensor-to-scalar ratio defined as: $r = A_T/A_S$ with $(A_T, A_S)$ being the amplitudes of the power spectra for scalar ($S$) and tensor ($T$) modes at $k = a H \approx k_s$. The effective cosmological constant or equivalently the CP violating parameter $\theta$ can be constrained as:

$$\Lambda_{UV} = \frac{\theta}{8 V_{ol4}} Q_P = 2.98 \times 10^{-9} \Lambda_{UV}^2 \times \left( \frac{r}{0.1} \right). \hspace{1cm} (17)$$

In order to compare the theoretical predictions with the latest observational data we use a numerical code CLASS [18]. In this code we can directly input the shape of the potential along with the model parameters. Then for a given cosmological background the code provides the estimates for different CMB observables. In the code we set the momentum pivot at, $k_s = 0.05 \text{ Mpc}^{-1}$ and used Planck + WMAP9 best fit cosmological parameters. Here Red and black colored points correspond to the upper and lower bound of the $\theta$ parameter for a given value of $Q_P$. All the parallel blue colored lines are drawn for different integer values of $Q_P$ which connects both the Red and black colored points. This plot suggests that as the value of $Q_P$ increases then the interval between the upper and lower bound of the $\theta$ parameter decrease and it will converge to very small value for large $Q_P$. Also the numerical value corresponding to the upper bound and lower bound of the $\theta$ parameter decreases once we increase the the value of $Q_P$.

#### FIG. 1: Constraint on CP violating topological $\theta$ parameter for discrete integer values of Pontryagin index $Q_P$ using Planck + WMAP9 best fit cosmological parameters. Here Red and black colored points correspond to the upper and lower bound of the $\theta$ parameter for a given value of $Q_P$. All the parallel blue colored lines are drawn for different integer values of $Q_P$ which connects both the Red and black colored points. This plot suggests that as the value of $Q_P$ increases then the interval between the upper and lower bound of the $\theta$ parameter decrease and it will converge to very small value for large $Q_P$. Also the numerical value corresponding to the upper bound and lower bound of the $\theta$ parameter decreases once we increase the the value of $Q_P$.

In trans-Planckian regime quantum corrections of matter fields and their interaction between various constituents modify the picture which are appearing through perturbative loop corrections [16]. However below $\Lambda_{UV}$ the effect of all such quantum corrections are highly suppressed and the heavy fields getting their VEV. Such VEV is one of the possible sources of vacuum energy correction in the spin-current dominated EFT picture which uplifts the scale of inflationary potential and the contributions of the VEV become significant upto a scale $\Lambda_c \lesssim \Lambda_{UV}$. But at very low scale, $\Lambda_{low} \ll \Lambda_C$, one can tune the vacuum energy correction, $V_0 \approx 0$ for which the contributions of the VEV can be neglected [17]. But such possibility is only significant when the contribution of the primordial gravity waves become negligibly small (see Eqn.(16)).

As a result, the CMB observables are constrained within the following range:

$$2.197 \times 10^{-9} \leq A_S \leq 2.202 \times 10^{-9},$$

$$0.957 \leq n_S \leq 0.962,$$

$$-1.08 \times 10^{-3} \leq \alpha_S \leq -0.99 \times 10^{-3},$$

$$0.055 \leq r \leq 0.057.$$  \hspace{1cm} (19)

Within the present context the field excursion [19–21] is defined as:

$$|\Delta \phi| = \Lambda_{UV} \int_0^{N_{cmb}} dN \sqrt{r(N)/8} \approx \sqrt{r/8} N_{cmb} \Lambda_{UV}.$$  \hspace{1cm} (20)
where $|\Delta \phi| = |\phi_s - \phi_f|$, in which $\phi_s$ and $\phi_f$ represent the field value corresponding to CMB scale and end of inflation respectively. Also $N_{\text{cmb}}$ is the number of e-foldings at CMB scale which is fixed at $N_{\text{cmb}} \approx 50 - 70$ to solve the horizon problem associated with inflation. Subsequently we get the following constraint on the field excursion: $|\Delta \phi| \sim \mathcal{O}(4.1-5.9) \times \Lambda_{\text{UV}}$, which implies to make the EFT of inflation validate within the prescribed setup for which we need to constrain the UV cut-off of the EFT within the following window: $\Lambda_{\text{UV}} \sim \mathcal{O}(0.16 - 0.24) \ M_p < M_p$, which is just below the scale of reduced Planck mass. Finally using Eqn.(17) we get the following bound on the CP violating parameter $\theta$:

$$3.48 \times 10^{-10} M_p^2 \leq \frac{\theta}{\Lambda_{\text{UV}}} Q_P \leq 7.62 \times 10^{-10} M_p^2.$$  \hspace{1cm} (21)

Thus once we fix $Q_P$, this will further give an estimate of $\theta$ according to the Eqn. (21). In Fig. (1) we have explicitly shown the constraint on $\theta$ from the proposed EFT picture which is obtained by using Planck + WMAP9 best fit cosmological parameters. To exemplify we have prescribed the bound on $\theta$ for different integer values of $Q_P$ lying within $1 \leq Q_P \leq 10$. From the plot it is easy to see that as the value of $Q_P$ increases the bound on the parameter $\theta$ converges to a very small value. This suggests that $\theta$ will converge to a constant value beyond a certain value of $Q_P$. It may be mentioned that the Pontryagin index can be taken to correspond to the fermion number \cite{[23]}. Indeed a fermion can be realized as a scalar particle encircling a vortex line which is topologically equivalent to a magnetic flux line and thus represents a skyrmion \cite{[23]}. The monopole charge $\mu = 1/2$ corresponding to a magnetic flux line is related to the Pontryagin index through the relation $Q_P = 2\mu$. In view of this, one may note that $Q_P$ represents the fermion number which is the topological index carried by a fermion. For an anti-fermion $Q_P$ takes the negative value. In any system the effective fermion number is given by the difference between the number of fermions and anti-fermions. Thus we can quantify the fermionic matter and hence the spin density through the total accumulated value of $Q_P$. As $Q_P$ increases we have the increase of fermions implying the increase in spin density. So from Eqn.(21) we note that for a fixed volume when $Q_P$ increases indicating the increase in spin density, the bound on the parameter $\theta$ converges to a small value representing the residual effect of torsion residing at the boundary. Thus the remnant of CP violation giving rise to torsion can be witnessed through the small value of $\theta$ which is operative at the boundary.

To summarize, we have proposed a methodology for generating hidden scalar field within EFT framework from the spin spin interaction picture. We have explicitly computed the vacuum energy corrected effective potential in sub-Planckian scale through which we give an estimate of inflationary CMB observables by constraining the model parameters- vacuum energy, mass and self-coupling from Planck + WMAP9 best fit values of the cosmological parameters. Finally, for the first time we constrain the CP violating topological $\theta$ parameter from the VEV of the heavy hidden sector fields appearing as vacuum energy correction within EFT.

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8 From experimental measurements of the neutron electric dipole moment, the experimental limit on the CP violating $\theta$ parameter is $\theta \leq 10^{-9}$ \cite{[22]}, which is consistent with our derived stringent bound on $\theta$.
9 In the context of canonical quantization of gravity it is observed that for small but non-vanishing value of the cosmological constant an exact solution to all the constraints of quantum gravity is given by the Chern-Simons state that describes the vacuum at the Planck scale which is chiral and implies an inherent CP-violation in quantum gravity \cite{[24]}.
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