Max-Min Fairness of Rate-Splitting Multiple Access With Finite Blocklength Communications

Yunnuo Xu, Yijie Mao, Member, IEEE, Onur Dizdar, Member, IEEE, and Bruno Clerckx, Fellow, IEEE

Abstract—Rate-Splitting Multiple Access (RSMA) has emerged as a flexible and powerful framework for wireless networks. In this paper, we investigate the user fairness of downlink multi-antenna RSMA in short-packet communications with/without cooperative (user-relaying) transmission. We design optimal time allocation and linear precoders that maximize the Max-Min Fairness (MMF) rate with Finite Blocklength (FBL) constraints. The relation between the MMF rate and blocklength, as well as the impact of cooperative transmission are investigated. Numerical results demonstrate that RSMA can achieve the same MMF rate as Non-Orthogonal Multiple Access (NOMA) and Space Division Multiple Access (SDMA) with smaller blocklengths (and therefore lower latency), especially in cooperative transmission deployment. Hence, we conclude that RSMA is a promising multiple access for guaranteeing user fairness in low-latency communications.

Index Terms—RSMA, NOMA, SDMA, FBL, max-min fairness, cooperative transmission.

I. INTRODUCTION

During the past few years, there has been a consensus that 5G will support three scenarios, Enhanced Mobile Broadband (eMBB), Massive Machine Type Communications (mMTC) and Ultra-Reliable and Low-Latency Communications (URLLC). According to the 3rd Generation Partnership Project (3GPP), a general URLLC reliability requirement is 99.999% with latency being less than 1 ms [1]. To support low-latency communications, short-packet with Finite Blocklength (FBL) codes are adopted to reduce the transmission delay [2].

In contrast to Shannon’s capacity with infinite blocklength assumption, Block Error Rate (BLER) cannot be neglected in the FBL regime [2]. Polanskis et al. [16] provided information-theoretic limits on the achievable rate for given FBL and BLER in Additive White Gaussian Noise (AWGN) fading channels [3]. The authors in [4] extended the achievable rate expression to Single-Input Single-Output (SISO) stationary channels with perfect Channel State Information (CSI). The maximum achievable transmission rate was investigated over Multiple-Input Multiple-Output (MIMO) fading channels under both perfect and imperfect CSI settings [5].

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Yunnuo Xu is with Imperial College London, London SW7 2AZ, U.K. (e-mail: yunnuo.xu19@imperial.ac.uk).

Yijie Mao is with the School of Information Science and Technology, ShanghaiTech University, Shanghai 201210, China (e-mail: maoyj@shanghaitech.edu.cn).

Onur Dizdar was with Imperial College London, London SW7 2AZ, U.K. He is now with Viavi Solutions Inc., Stevenage SG1 2AN, U.K. (e-mail: onur.dizdar@viavisolutions.com).

Bruno Clerckx is with the Department of Electrical and Electronic Engineering at Imperial College London, London SW7 2AZ, U.K., and also with Silicon Austria Labs (SAL), A-8010 Graz, Austria (e-mail: b.clerckx@imperial.ac.uk). Digital Object Identifier 10.1109/TVT.2022.3229822

It is known that Non-Orthogonal Multiple Access (NOMA) outperforms Orthogonal Multiple Access (OMA) by employing Superposition Coding (SC) at transmitter and Successive Interference Cancellation (SIC) at receiver. The superiority of NOMA over OMA was shown in terms of capacity region under infinite blocklength assumption [6], and Max-Min Fairness (MMF) throughput with FBL constraints [7]. With cooperative transmission, cooperative NOMA (C-NOMA) can achieve lower BLER than conventional NOMA and cooperative OMA [8]. However, multi-antenna NOMA is shown to be an inefficient strategy in terms of multiplexing gain and use of SIC receivers in downlink multi-user multi-antenna systems [9].

Recently, Rate-Splitting Multiple Access (RSMA) has emerged as a novel multiple access for downlink multi-antenna networks under infinite blocklength assumption. By splitting user messages, RSMA can softly bridge two extreme interference management strategies, namely, NOMA and Space Division Multiple Access (SDMA) [10], [11]. RSMA offers significant gains in terms of energy efficiency, CSI feedback overhead reduction and user fairness [12]. With FBL constraints, it was demonstrated that RSMA could guarantee the same transmission rate while allowing a reduction in blocklength and therefore reducing the transmission delay compared to SDMA and NOMA [13].

User fairness is an important criterion, however, to the best of our knowledge, user fairness of RSMA under FBL assumption has not been investigated. The user rate is composed of private rate and common rate, while the common rate is constrained by the rate of the worst-case user since the common stream must be decoded by all users. The common rate may decrease when users experience heterogeneous downlink channel strength. One possible solution to the problem is combining RSMA with cooperative transmission utilizing user relaying. We also consider a complex scenario involving simultaneous transmitting distinct messages to multiple multicast groups, which is likely to occur in the future wireless network due to the content-based services [14].

Compared to the previous work [15], we propose a novel system model that unifies the multigroup multicast and the cooperative transmission models, and we extend the proposed model to FBL scenarios. We study the MMF rate of RSMA with/without cooperative transmission under the constraints on blocklength and transmit power in both underloaded and overloaded deployments. Our results show that by utilizing RSMA, the user fairness can be guaranteed more efficiently compared to power-domain NOMA1 and SDMA with the same FBL. Alternatively, the blocklength can be reduced, and the latency is therefore decreased while achieving the same MMF rate. With cooperative transmission, the gain of RSMA can be enhanced further.

The rest of the paper is organized as follows. In Section II, we introduce the system model of multigroup multicast RSMA with/without user relay and we formulate MMF optimization problem. The proposed One-Dimensional Search Successive Convex Approximation (1D-SCA) joint optimization algorithm is specified in Section III. Numerical results illustrating the benefits of RSMA and Cooperative RSMA (C-RSMA) are discussed in Section IV, followed by the conclusions in Section V.

The superscript $(\cdot)^T$ denotes transpose and $(\cdot)^H$ denotes conjugate-transpose. $\mathcal{CN}(\zeta, \varphi)$ represents a complex Gaussian distribution with mean $\zeta$ and variance $\varphi^2$. The boldface upercases represent matrices and lowercase letters represent vectors. $\text{tr}(\cdot)$ is the trace. $|\cdot|$ is the

1In the sequel, power-domain NOMA will be referred simply by NOMA.
absolute value and $\| \cdot \|$ is the Euclidean norm. $\mathbb{C}$ denotes the complex space. $|A|$ is the cardinality of the set $A$.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We design a system model that unifies the multigroup multicast and the cooperative transmission models. We assume that a Base Station (BS) is equipped with $N_t$ transmit antennas communicating with $K$ single-antenna users that are indexed by $K = \{1, 2, \ldots, K\}$. The users are divided into $M \in [1, K]$ separated groups $\mathcal{G}_1, \ldots, \mathcal{G}_M$, where $\mathcal{G}_m \subseteq K$ is the set of users belonging to the $m$th group, $m \in M$, and $\mathcal{M} = \{1, \ldots, M\}$ is the index set of all groups. The user grouping is done such that the groups satisfy $\bigcup_{m \in M} \mathcal{G}_m = K$ and $\mathcal{G}_m \cap \mathcal{G}_j = \emptyset$, $\forall m, j \in \mathcal{M}$, $m \neq j$. The mapping function $\mu(k) = m$ maps a user-$k$ to group-$m$. The BS sends the distinct messages $W_1, \ldots, W_M$ to users in $\mathcal{G}_1, \ldots, \mathcal{G}_M$, respectively, and users belonging to the same group, $\mathcal{G}_m$, require a same message $W_m$. We denote the size of the group as $|\mathcal{G}_m| = G_m$.

Fig. 1 shows the system model with $M = 3$ user groups. The transmission consists of two phases, namely, the direct transmission (in the first time slot) and cooperative transmission phases (in the second time slot). Unlike [8] that allocates equal transmission time to two phases, we consider a dynamic time allocation strategy. $\theta$ is the fraction of time allocated to the direct transmission phase. In the first time slot, signals are transmitted from BS to all groups. In the second time slot, the users in $\mathcal{G}_1$ cooperatively decode and forward the signals to the users in $\mathcal{G}_2$ and $\mathcal{G}_3$.

A. Direct Transmission Phase

We consider an RSMA-assisted transmission model where the BS splits the message $W_m$ for users in group-$m$ into $\{W_{c,m}, W_{p,m}\}$, with $W_{c,m}$ and $W_{p,m}$ being the common and private parts, respectively. All common parts are combined into one common message and encoded into common stream $s_c$ to be decoded by all users. The private message $W_{p,m}$ is encoded into $s_{m}$ and is only decoded by users in group-$m$. The overall symbol streams to be transmitted is denoted by $s = [s_c, s_1, s_2, \ldots, s_M]^T$, and $\mathbb{E}[ss^H] = \mathbf{I}$. The streams are precoded via a precoding matrix $\mathbf{P} = [\mathbf{p}_c, \mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_M]$, where $\mathbf{p}_j \in \mathbb{C}^{N_t \times 1}$ represents the linear precoder for the stream $s_j$, $j \in \{c, 1, 2, \ldots, M\}$. This yields the transmit signal

$$x^{[1]} = \mathbf{p}_c s_c + \sum_{m \in \mathcal{M}} \mathbf{p}_m s_m,$$  

(1)

where the superscript $[1]$ represents the first transmission phase. The power constraint is $\mathbf{tr}(\mathbf{PP}^H) \leq \mathcal{P}_t$. Denote the signal received at the $k$th user in the $m$th group as $y_k$

$$y_k = \mathbf{h}_k^H x^{[1]} + n_k,$$  

(2)

where $\mathbf{h}_k \in \mathbb{C}^{N_t \times 1}$ is the channel vector between the transmitter and the $k$th user, and $n_k \sim \mathcal{CN}(0, 1)$ is the receiver AWGN. After each user receives the signal, the common stream is decoded and eliminated from the received signal, while the private parts are treated as noise. The Signal to Interference plus Noise Ratio (SINR) of the common stream is

$$\gamma_{c,k}^{[1]} = \frac{|\mathbf{h}_k^H \mathbf{p}_c|^2}{\sum_{j \in \mathcal{M}, j \neq k} |\mathbf{h}_k^H \mathbf{p}_j|^2 + 1}, \forall k \in \mathcal{K}.$$  

(3)

We assume that the transmitter and receivers perfectly know the channel vectors and perfect SIC can be performed at receivers. Once the common stream $s_c$ is successfully decoded, it is reconstructed and removed from the received signal. Subsequently, each user in $\mathcal{G}_m$ decodes the group private stream while treating other private streams as noise. The SINR of decoding the private stream $s_m$ at user-$k$ is

$$\gamma_{p,k}^{[1]} = \frac{|\mathbf{h}_k^H \mathbf{p}_m|^2}{\sum_{j \in \mathcal{M}, j \neq k} |\mathbf{h}_k^H \mathbf{p}_j|^2 + 1}, \forall k \in \mathcal{G}_m.$$  

(4)
Once the common and private messages are decoded, each user in $G_m$ reconstructs the original message by extracting the decoded common message and combining it with the decoded $W_{p,m}$. We include the effect of blocklength by using the rate expression proposed in [3]. Denote the total blocklength during the transmission as $l_m = l_d + l_c$, where $l_d$ is blocklength allocated to the direct transmission phase and $l_c$ is the blocklength of the codeword in the cooperative transmission phase. The rates $R^{[1]}_{c,k}, R^{[1]}_{p,k}$ of decoding the common and private streams (respectively denoted as common rate and private rate) in the first time slot are expressed as

$$R^{[1]}_{c,k} = \log_2 \left( 1 + \gamma^{[1]}_{c,k} \right) - \frac{V \left( \gamma^{[1]}_{c,k} \right)}{\theta}, \quad i \in \{c, p\},$$

where $B = Q^{-1}(\epsilon) \log_2 e$, $\epsilon$ represents BLER, and $Q^{-1}(\cdot)$ corresponds to the inverse of the Gaussian Q function, $Q(x) = \int_x^{\infty} \exp(-t^2) dt$. $\theta = l_d/l_c$, since the blocklength of the codeword is proportional to the latency and can be approximately expressed as $l_n \approx BT$, where $B$ and $T$ are the bandwidth and time duration of the signal (i.e., latency), respectively [2]. $V(\cdot)$ is the channel dispersion parameter

$$V \left( \gamma^{[1]}_{c,k} \right) = 1 - \frac{1}{(1 + \gamma^{[1]}_{c,k})^2}.$$  

B. Cooperative Transmission Phase

We assume that users in $G_1$ are cell-center users. They employ the Non-regenerative Decode-and-Forward (DF) protocol and act as Decode-and-Forward (DF) Half-Duplex (HD) relays. Users in $G_1$ re-encode the decoded common stream $s_c$ using a different codeblock generated independently from that of the BS, and forward $s_c$ to users in other groups (denoted as $G_{co} = \{G_2, \ldots, G_M\}$) with transmit power $P_c$. The transmit signal at user-$j$ of $G_1$ in the second time slot (denoted as superscript [2]) is given by $x^{[2]}_k = \sqrt{P_c s_c}$, $\forall j \in G_1$. The signal received at user-$k$ of $G_{co}$ is

$$y^{[2]}_k = \sum_{j \in G_1} h_{k,j} x^{[2]}_j + n_k, \forall k \in G_{co},$$

where $h_{k,j}$ is the SISO channel between user-$k$ and user-$j$. The SINR of decoding the common stream at user-$k$, $\forall k \in G_{co}$, is given by

$$\gamma^{[2]}_{c,k} = \sum_{j \in G_1} P_c |h_{k,j}|^2, \forall k \in G_{co}.$$  

The corresponding rate is

$$R^{[2]}_{c,k} \approx (1 - \theta) \left[ \log_2 \left( 1 + \gamma^{[2]}_{c,k} \right) - \frac{V \left( \gamma^{[2]}_{c,k} \right)}{\theta} \right], \forall k \in G_{co}.$$  

Users in $G_{co}$ combine the decoded common stream in both time slots, the achievable rate of the common stream is

$$R_c = \min \{ R^{[1]}_{c,k}, R^{[2]}_{c,k} \},$$

where $R^{[1]}_{c,k} = \min_{k \in G_1} \{ R^{[1]}_{c,k} \}$ and $R^{[2]}_{c,k} = \min_{k \in G_{co}} \{ R^{[1]}_{c,k} + R^{[2]}_{c,k} \}$ are the achievable common rate of users in $G_1$ and $G_{co}$, respectively. $R_c$ guarantees that each user is able to correctly decode the common stream [15, 16]. We define $R_c = \sum_{m \in M} C_m$, where $C_m$ is the rate at which $W_{c,m}$ is communicated, and $c = [C_1, C_2, \ldots, C_M]$. Once $s_c$ is decoded and removed from the received signal, user-$k$ decodes the intended private stream. Accordingly, the rate of group-$m$ is

$$R_m = C_m + \min_{k \in G_m} R^{[1]}_{p,k}, \quad m \in M.$$  

Since RSMA and NOMA employ interference cancellation which may cause error propagation, we set the BLER threshold of RSMA and NOMA to $\epsilon^{RSMA} = \epsilon^{NOMA} = 5 \times 10^{-5}$ so as to guarantee the approximated overall BLER is not larger than $10^{-5}$. The BLER of SDMA is $\epsilon^{SDMA} = 10^{-5}$.

Remark 1: If $\theta = 1$, the cooperative transmission phase is turned off and the system model becomes a conventional multigroup multicast case without user relaying. If $0 < \theta < 1$ and each user group has a single user, the system model reduces to a $K$-user cooperative transmission scenario.

C. Problem Formulation

The user fairness issue is the main focus of this work. In order to maximize the minimum group rate, the precoder $\mathbf{P}$, the common rate allocation $\alpha$, and the blocklengths, $l_d$ and $l_c$, are jointly optimized. The MMF problem is formulated as

$$\max_{\mathbf{P}, \alpha, l_d, l_c} \min_{m \in M} R_m$$

subject to

$$\sum_{m' \in M} C_{m'} \leq R_e$$

$$(\mathbf{PP}^H)^t \leq P_t$$

$$c \geq 0.$$  

The common stream decodability is guaranteed by constraint (12b), (12c) is the transmit power constraint.

III. PROPOSED ALGORITHM

We propose a 1D-SCA algorithm to solve problem (12). One can notice that $l_d = l_n - l_c$. Hence for a given $l_n$ and a fixed value of $l_c$, $l_d$ can be obtained. We first fix the value of $l_n$, and then we use Successive Convex Approximation (SCA) to solve the non-convex problem. Once the optimal precoder for the fixed $l_i$ is attained, we increase $l_i$ and solve the problem iteratively. The blocklength and precoder corresponding to the highest MMF are selected as optimal blocklength and precoder for the given blocklength $l_n$. For a given blocklength $l_n$ and fixed $l_i$ (hence, $l_d$ and $\theta$ are fixed, due to the relation $\theta = l_d/l_n$), the problem (12) can be transformed to

$$\max_{\mathbf{P}, \epsilon} \min_{m \in M} R_m$$

subject to

$$(12b), (12c), (12d).$$

Problem (13) is non-convex due to the non-convex rate expressions. We introduce the following variables, namely $t$, $\alpha_c = [\alpha_{c,1}, \alpha_{c,2}, \ldots, \alpha_{c,K}]$, $\alpha_p = [\alpha_{p,1}, \alpha_{p,2}, \ldots, \alpha_{p,K}]$, $\beta_c = [\beta_{c,1}, \beta_{c,2}, \ldots, \beta_{c,K}]$, $\beta_p = [\beta_{p,1}, \beta_{p,2}, \ldots, \beta_{p,K}]$, $t$ is the achievable rate lower bound for all user groups. $\theta_{\alpha_{p,m}}$ is the lower bound of the minimum private rate of users in group-$m$, i.e., $\min_{k \in G_m} R^{[1]}_{p,k} \geq \theta_{\alpha_{p,m}}$, $\theta_{\alpha_{c,k}}$ represents the lower bound of common rate $R^{[1]}_{c,k}$ in the direct transmission phase. $\beta_{c,k}$ and $\beta_{p,k}$ are the lower bounds of the SINR of the common and private streams in the first phase, respectively. $\nu(\rho) = 1 - (1 + \rho)^{-2}$. Problem (13) is equivalently written as

$$\max_{t, \alpha_c, \alpha_p, \beta_c, \beta_p} t$$

subject to

$$(13a), (13b).$$

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\begin{align}
&C_m + \alpha_{p,m} \geq t, \forall m \in \mathcal{M} \tag{14b} \\
&\theta_{c,k} \geq \sum_{j \in \mathcal{M}} C_j, \forall k \in \mathcal{G}_t \tag{14c} \\
&\theta_{c,k} + R_{c,k}^2 \geq \sum_{j \in \mathcal{M}} C_j, \forall k \in \mathcal{G}_{co} \tag{14d} \\
&\log_2(1 + \rho_{c,k}) - B \sqrt[4]{\frac{\nu(\rho_{c,k})}{l_d}} \geq \alpha_{c,k}, \forall k \in \mathcal{K} \tag{14e} \\
&\log_2(1 + \rho_{p,k}) - B \sqrt[4]{\frac{\nu(\rho_{p,k})}{l_d}} \geq \alpha_{p,\mu(k)}, \forall k \in \mathcal{K} \tag{14f} \\
&\sum_{j \in \mathcal{M}} |h_j^H p_j|^2 + 1 - \frac{|h_j^H p_j|^2}{\rho_{c,k}} \leq 0, \forall k \in \mathcal{K} \tag{14g} \\
&\sum_{j \in \mathcal{M}, j \neq \mu(k)} |h_j^H p_j|^2 + 1 - \frac{|h_j^H p_{\mu(k)}|^2}{\rho_{p,k}} \leq 0, \forall k \in \mathcal{K} \tag{14h} \\
&\text{s.t. (12c), (12d).} \tag{14i}
\end{align}

Problem (14) remains non-convex due to the non-convex constraints (14e)-(14h). Next, we approximate the non-convex parts \(\sqrt[4]{\nu(\rho_{c,k})}\), \(\sqrt[4]{\nu(\rho_{p,k})}\) in the constraints by the first-order Taylor series. Constraints (14e)-(14f) and (14g)-(14h) are approximated at \(\{p_{\mu(k)}^{[n]}, \rho_{c,k}^{[n]}\}\) and \(\{p_{p,k}^{[n]}, \rho_{c,k}^{[n]}\}\) at iteration \(n\) as (15) and (16), respectively.

\begin{align}
&\log_2(1 + \rho_{c,k}) - B \sqrt[4]{\frac{\nu(\rho_{c,k})}{l_d}} \left[ 1 - \left(1 + \rho_{c,k}^{[n]}\right)^2 \right]^{1/2} \left[ 1 + \rho_{c,k}^{[n]}\right]^{-3} \\
&\left(\rho_{c,k} - p_{\mu(k)}^{[n]} \right) - \left(1 + p_{\mu(k)}^{[n]} + 1\right) \geq \alpha_{c,k}, \tag{15a} \\
&\log_2(1 + \rho_{p,k}) - B \sqrt[4]{\frac{\nu(\rho_{p,k})}{l_d}} \left[ 1 - \left(1 + \rho_{p,k}^{[n]}\right)^2 \right]^{1/2} \left[ 1 + \rho_{p,k}^{[n]}\right]^{-3} \\
&\left(\rho_{p,k} - p_{p}^{[n]} \right) - \left(1 + p_{p}^{[n]} + 1\right) \geq \alpha_{p,\mu(k)}. \tag{15b} \\
&\sum_{j \in \mathcal{M}} |h_j^H p_j|^2 + 1 - \frac{2\Re\left\{\left|p_{\mu(k)}^{[n]}\right|^2 h_j^H p_{\mu(k)}\right\}}{\rho_{c,k}^{[n]}} \geq 0, \tag{16a} \\
&\sum_{j \in \mathcal{M}} |h_j^H p_j|^2 + 1 - \frac{2\Re\left\{\left|p_{p}^{[n]}\right|^2 h_j^H p_{p}^{[n]}\right\}}{\rho_{c,k}^{[n]}} \geq 0, \tag{16b}
\end{align}

Based on the approximation methods described above, the original non-convex problem is transformed to a convex problem and can be solved using the SCA approach. The main idea of SCA is to solve the non-convex problem by approximating it to a sequence of convex subproblems, which are solved successively. At iteration \(n\), based on the optimal solution \((p_{\mu(k)}^{[n]}, \rho_{c,k}^{[n]}, \rho_{p,k}^{[n]}))\) obtained from the previous iteration \(n - 1\), we solve the following subproblem

\begin{equation}
\begin{aligned}
&\max_{\theta_{c,k}, \rho_{c,k}, \rho_{p,k}} t \\
&\text{s.t. (15), (16), (12c), (12d).} \tag{17a}
\end{aligned}
\end{equation}

The proposed 1D-SCA algorithm is summarized in Algorithm 1, where \(\tau\) represents the tolerance of algorithm. Since the solution of Problem (17) at iteration \(n - 1\) is a feasible solution to the problem at iteration \(n\), the convergence of Algorithm 1 is ensured. The objective variable \(t\) is monotonically increasing and it is bounded above by the transmit power constraint. We propose a different algorithm (namely, 1D-SCA) to solve the formulated problem instead of using Weighted Minimum Mean Square Error (WMMSE) as in [15] due to the non-convex FBL rate expressions in (5) and (9). The proposed algorithm is more general than the one in [13] due to the optimization of the time allocation variable. By combining the one-dimensional search with SCA, less auxiliary variables and approximations are introduced and performance loss is hindered.

Algorithm 1 consists of two loops, one is due to the one-dimensional search, and the other is due to the SCA-based algorithm. The worst-case computational complexity of the one-dimensional search algorithm is \(O(\delta^{-1})\), where \(\delta \in (0, 1)\) is the increment between two adjacent candidates of \(\theta\). At each SCA iteration, the approximated problem is solved. Though additional variables \(\alpha_{c,k}, \alpha_{p,k}, \rho_{c,k}\) and \(\rho_{p,k}\) are introduced for convex relaxation, the main complexity still comes from the precoder design. The total number of SCA iterations required for the convergence is approximated as \(O(\log(\tau^{-1}))\). The worst-case computational complexity at each one-dimensional search iteration is \(O(\log(\tau^{-1}) \frac{K N_l}{\tau})\). Hence, the computational complexity of the proposed 1D-SCA is \(O(\delta^{-1} \log(\tau^{-1}) \frac{K N_l}{\tau})\).
Algorithm 1: 1D-SCA.

1 Initialize: \( l_c = 100 \);

2 \textbf{repeat}

3 \textbf{Initialize: } \( n \leftarrow 0, \ell^{[n]} \leftarrow 0, P^{[n]}, \psi^{[n]}, \rho^{[n]} \);

4 \textbf{repeat}

5 \( n \leftarrow n + 1 \);

6 Solve problem (17) using \( P^{[n-1]}, \psi^{[n-1]}, \rho^{[n-1]} \) and denote the optimal value of the objective function as \( \ell^* \) and the optimal solutions as \( P^*, \psi^*, \rho^* \);

7 Update \( \ell^{[n]} \leftarrow \ell^*, P^{[n]} \leftarrow P^*, \psi^{[n]} \leftarrow \psi^*, \rho^{[n]} \leftarrow \rho^* \);

8 until \( |\ell^{[n]} - \ell^{[n-1]}| < \tau \);

9 \( l_c = l_c + 10 \);

10 until \( l_d \leq 100 \);

IV. RESULTS AND DISCUSSION

This section investigates the MMF performance of RSMA, NOMA, and SDMA for a variety of user deployments. Following the literature [10], [15], [16], the precoders of the proposed 1D-SCA algorithm are initialized by using Maximum Ratio Transmission (MRT) combined with Singular Value Decomposition (SVD). The tolerance of the algorithm is set to \( \tau = 10^{-3} \). We first show the performance of a multigroup multicast model when \( \theta \) is fixed to 1, then, we present the performance of a K-user cooperative model with optimized \( \theta \). The problem (17) is solved using the CVX toolbox in Matlab [17].

A. Multigroup Multicast Deployment

Under the non-cooperative multigroup multicast deployment, \( l_n = l_d \) and \( \theta = 1 \). Fig. 2 shows the MMF performance of RSMA, NOMA and SDMA with different number of transmit antennas and user deployments. \( P_t = 20 \text{ dB} \). The MMF performance is evaluated and averaged over 100 random channel generations, where the channel \( h_k \) has independent and identically distributed (i.i.d) complex Gaussian entries with a certain variance, i.e. \( \mathcal{CN}(0, \phi^2) \). In Fig. 2(a) and (b), we set the total number of users to \( K = 2 \) and \( K = 8 \) divided over \( M = 2 \) and \( M = 8 \) groups, respectively, such that each group contains one user. Fig. 2(c) shows the performances under 2 transmit antennas and 4 users deployment. The users are divided into \( M = 2 \) groups with 2 users per group. Due to the complexity of finding the optimum decoding order in multigroup multicast deployment, we assume that the NOMA SIC decoding order is performed in a descending order of channel gains in Fig. 2(a) and (b). In Fig. 2(c), we simplify NOMA by restricting all precoder directions to be the same, as the problem (27) in [14], and optimize the shared precoder by the SCA-based algorithm.

The notations “Inf” and “Fin” in figures represent the schemes when \( l_n = \infty \) and when \( l_n \) is finite, respectively. Non-cooperative RSMA is marked as “N-RSMA”. In Fig. 2, the MMF rates with FBL of three strategies increase with the blocklength as expected, and it is clear that with the same transmit power and blocklength, “Fin N-RSMA” achieves a higher MMF group rate. Compared with the underloaded deployment (in Fig. 2(a)), the relative gains of “Fin N-RSMA” over “Fin N-NOMA” and “Fin SDMA” in overloaded scenario (in Fig. 2(b)) are more pronounced as RSMA can manage the interference by partially decoding interference and partially treating interference as noise even in the overloaded deployment. “Fin SDMA” achieves certain MMF rate gain over “Fin N-NOMA” in underloaded scenario while “Fin N-NOMA” outperforms “Fin SDMA” in the overloaded deployments, since SDMA can manage interference in the underloaded scenario and it cannot manage multi-user interference efficiently in the overloaded deployment. From Fig. 2(c), the performance can be increased by 2.02 times by using RSMA compared with SDMA at the blocklength of 200, which is even higher than that in the infinite blocklength scenario. By splitting group messages, RSMA achieves MMF rate gain compared to NOMA and SDMA regardless of user deployments and blocklength, which guarantees the user fairness in both infinite and finite scenarios. Alternatively, RSMA can utilize smaller blocklength (and hence lower latency) for achieving the same MMF rate.

B. K-User Cooperative Deployment

In this subsection, \( \theta \in (0, 1) \), effects of cooperative transmission phase will be investigated. Fig. 3 shows the average MMF rates of different strategies with varied number of transmit antennas and channel strength disparities among users. The MMF values for blocklength 500 are given in the figure with arrows. We assume that \( M = 3, G_1 = G_2 = G_3 = 1 \), \( \phi^2_1 = 1 \), \( \phi^2_2 = 0.09 \), \( \phi^2_3 = 0.01 \). The variances of user channels are \( \phi^2_1 = 1, \phi^2_2 = 0.09, \phi^2_3 = 0.01 \). \( N_t = 4 \) in subfigure 3(a), while \( N_t = 2 \) in subfigure 3(b).

From Fig. 3(a) and (b), the performance of “Fin C-RSMA” outperforms “Fin N-RSMA” and “Fin SDMA”. Furthermore, in the underloaded scenario, the “Fin N-RSMA” is likely to turn off the common message and boil down to SDMA, especially when blocklength is small, resulting the relative gain (N-RSMA over SDMA) close to 0. But the trend is reversed through implementing cooperative transmission, and the MMF rate of RSMA attains nearly twice as much as that of SDMA. The system is overloaded when \( N_t = 2 \). Compared to the underloaded deployment, when blocklength is 500 bits the relative gains of “Fin C-RSMA” over “Fin SDMA” and “Fin N-RSMA” over “Fin SDMA” are enhanced from 0.55 to 1 and from 0.14 to 0.55, respectively. This is because each user experiences more severe multi-user interference in the overloaded scenario than in the underloaded case. In comparison to underloaded scenarios, a large portion of users’ messages are split and encoded into the common stream for each user to decode, and more power is allocated to the common stream, as shown in Fig. 4(a). The amount of interference that will be decoded at each user is further increased as the relaying user in C-RSMA retransmits the common stream.

Fig. 4(b) shows how the time allocated to direct transmission phase, \( \theta \), changes with blocklength for C-RSMA strategy. As depicted in Fig. 4(b), less time is allocated to the direct transmission phase when the blocklength is small, and as blocklength increases, the gap between “Inf \( \theta \)” and “Fin \( \theta \)” becomes smaller. Since FBL results in rate penalty, in
order to improve user fairness, more time is allocated to the cooperative transmission phase to increase the MMF rate of the cell edge users. C-RSMA with FBL is better suited to situations where users experience stronger multi-user interference as it has a higher capability to manage interference. Compared with N-RSMA and SDMA, C-RSMA achieves a higher MMF rate with the same blocklength or attains the same MMF rate with smaller blocklength (and therefore lower latency).

We analyze the MMF rate achieved by applying the precoder obtained under the infinite blocklength assumption to the scenario with FBL, i.e., “Inf-Fin C-RSMA”. From Fig. 5, a clear gap between “Fin RSMA” and “Inf-Fin RSMA” is observed, which justifies the effectiveness of our optimization.

V. CONCLUSION

This paper investigates the performance of RSMA with FBL in terms of user fairness in both non-cooperative and cooperative deployments. A novel system model that unifies non-cooperative/cooperative multigroup multicast deployments is proposed and an optimization problem is formulated to maximize the minimum group rate. A 1D-SCA algorithm is adopted to solve the problem. The results show that RSMA can better maintain user fairness with the same FBL or achieve the same MMF rate with smaller blocklength (and hence lower latency) in comparison with NOMA and SDMA. With cooperative transmission, the gain of RSMA is enhanced further. Consequently, we conclude that RSMA is a promising strategy for enhancing user fairness in FBL communications.

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