I - THE EXPANDING UNIVERSE

from

the

HUGE VOID CENTER:

-

THEORY & MODELLING

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ABSTRACT

To add to and refine the model presented at the Grado3 meeting, of a radial Hubble expansion from the Bahcall & Soneira huge void center, it is here outlined a formal analytical formulation of the theory and modelling on which the previous check work based itself. The new Hubble Law is:

\[ \dot{r} = Hr + \Delta H \cdot \left( r - R \cos \gamma \right) + R \dot{w} \sin \gamma \]

The meaning and expression of the total \( \Delta H \) have been obtained and examined through a Galaxy Hubble law analysis based on derivatives with respect to light-space. This \( H \)'s variation, as predicted by the model, is due to a combination of two dominant effects, respectively time (TE) and space (SE). Structurally the scattering noise of the nearby Universe Hubble ratios seems to be partially caused by the perturbative term \( R \dot{w} \sin \gamma / r \). In conclusion a fundamental confirmation test of the model is presented; as a physical result one supports the density formula:

\[ \rho_0 > \frac{3H^2_0}{2\pi G} \]
1. INTRODUCTION

It's well known that so far a lot of time has been devoted to the problem of the Hubble constant; indeed this has become one of the most controversial and crucial scientific questions in the astronomical debate of the century. But in the last decades the investigations about value and meaning of \( H \) have greatly increased within different schools of thought, in particular after the enlightening observation of Rubin, Ford and Rubin (1973), the so-called RFR effect. This was the starting point of a long study, that through the fundamental contribution of a lot of works in observational cosmology, led the author (Lorenzi, 1989-91-93-94-95) to acknowledge an expansion center in the huge void of Bahcall & Soneira (1982) (Lipovetsky, 1987) who, following up the pioneering detection by Kirshner et al. (1981) of an apparent absence of relatively bright galaxies (\( z \approx 0.04-0.06 \)) over an enormous volume in Bootes, discovered and described a \( \sim 300 \) Mpc void of catalogued nearby rich Abell clusters of galaxies in the direction \( l^{II} \approx 140^0-240^0, b^{II} \approx 30^0-50^0 \), with \( z \approx 0.03-0.08 \). Here theory and modelling of all the research has been re-run, beginning from the original toy-model, proceeding through a few fundamental mathematical developments, and concluding with remarkable consequences for the Hubble ratio behaviour and the cosmography and evolution of the Universe.
2. THE ORIGINAL TOY-MODEL

2.1 Preliminary remarks

The father of the Big Bang, the Belgian abbot Georges Lemaître, originally imagined a different situation to the one worked out by Gamow (1948), Alpher, Herman (1948). He hypothesized the primordial explosion as being caused by a sort of fission cosmo-bomb, whose fragments started the journey producing the present dilution of the Universe.

Matching the idea of Lemaître (1933a,b) with the conception of Ambartsumian (1961), by which the protogalaxies (and all the observed matter) could have come out of extremely small concentrated bodies, it’s possible to give one intuitive explanation for the appearances of the most remote objects of the Universe, the Quasars, which have the peculiarity, as well known, of appearing extremely bright and small in the sky. Anyway, if we think plausible that a ”Hot Hard Big Bang” may have occurred, following some elementary spherical symmetry, the observation of the Universe from any splinter might show a few peculiarities, and possibly allow to locate a preferred relative point, that is the ”expansion center”.

Let us remember the contributions of Hubble (1929) and Penzias & Wilson (1965), who gave crucial observation evidence for the Big Bang; but also the question of isotropy or anisotropy, still rather dimmed by the authoritative Cosmological Principle (Milne, 1933). A very stimulating astronomical observation, though it was not equally meaningful in statistical terms (Sandage & Tamman, VI-1975b), was surely the one pointed out by Rubin, Ford and Rubin (1973); it raised many perplexities about the full validity of Hubble law (Nottale,Pecker,Vigier,Yourgrau, 1976). From that time the RFR effect has become the object of many important surveys, which have closely examined and partially interpreted a few observed anisotropies (see ref. in Lynden-Bell et al., 1988).

2.2 Introduction to an elementary Big Bang mechanics

In the full awareness that the greatest singularity of all, that referring to time ”0”, cannot be analysed otherwise than by conjectures, a simple toy-model of Big Bang mechanics was developed by the author (1989), the starting point of which is the Lemaître fragmentation hypothesis, out of the quantic and relativistic extremes. Such approach bases itself on the fundamental principle of
dynamics. Let us imagine a spherical cosmo-bomb, a relative explosion according to the spherical symmetry and the derived impelling bomb force as the only one acting force simply proportional to the mass of the inner globe. In other words we might hypothesize a coefficient \( \varepsilon \) which should just represent the impelling force developed by the unitary mass and representing the exclusive action force at the beginning. In consequence of a simultaneous explosion in all the globe points, assuming also the instantaneous propagation of the involved shock wave, we can reasonably consider as maximum the impulse received by the first shell on the outside surface. The impulses taken up by each internal shell, submitted to the burst impact of a minor globe, would become smaller and smaller. Therefore the impulse distribution should go from the zero value in the globe centre to a maximum value on the surface, while the impelling force acting on different shells should always be proportional to the mass of the involved inner globe. Now let us consider one spherical shell, of radius \( r_b \), and write Newton law as

\[ F = \frac{dp}{dt} \]  

where the impelling force \( F \) for the chosen shell is assumed constant as

\[ F = \frac{4}{3} \pi r_b^3 \rho \varepsilon \]  

Of course, in eq. (2) \( \rho \) would represent the cosmic matter density in its primordial state, \( \varepsilon \) the above mentioned speculative coefficient. Eq. (1), with \( p = mv \), becomes

\[ m \Delta v = F \Delta t \]  

\( \Delta t \) is the action time of \( F \); \( \Delta v = v_0 \) the acquired expansion velocity of the shell as a whole during the explosion time \( \Delta t \) in which, it is important to underline that, there is no other acting force outside the speculative \( F \) of eq. (2); \( m \) the mass of the shell itself \( \Delta r_b \) thick, that is

\[ m = 4\pi r_b^2 \Delta r_b \rho \]  

So eq. (3) gives automatically

\[ v_0 = C r_b \quad \text{with} \quad C = \frac{\varepsilon \Delta t}{3 \Delta r_b} \to \infty \]  

The previous equation refers to the starting radial velocity \( v_0 \) of any splinter with respect to the primordial globe centre, in a simple Big Bang hypothesis of a primordial shattering (according to the spherical symmetry) produced by an impelling force proportional to the mass of the inner globe, and assuming also the identity "splinters = protogalaxies". Such velocity \( v_0 \) is proportional
to the distance \( r_b \) from the centre and to a function \( C \) of constant value and \( \to \infty \) at least in the same way of the impelling coefficient \( \varepsilon \).

Now, classical kinematics furnishes radial run and velocity, \( R \) and \( \dot{R} \), of any splinter after a proper period "\( t \)" from the end of the Big Bang \( t = 0 \). In fact, immediately after the exclusive explosive action, it is possible to fix at \( t = 0 \) the beginning of the normal gravity deceleration. So, considering such deceleration \( \ddot{R} \) as \( \ddot{R} = \dot{R} \) dt, it follows from the mean theorem: \( \int_{v_0}^{\dot{R}(t)} \ddot{R} = \int_0^t \dot{R} dt = \langle \dot{R} \rangle t \), that is \( \dot{R}(t) = v_0 + \langle \dot{R} \rangle t \), whose further integration gives the radial run \( R \) as:

\[
R = r_b + v_0 t + \frac{1}{2} \langle \dot{R} \rangle t^2
\]

(6)

and then we can write in sequence

\[
R = r_b (1 + Ct) + \frac{1}{2} \langle \dot{R} \rangle t^2
\]

\[
r_b = \frac{1}{1 + Ct} (R - \frac{1}{2} \langle \dot{R} \rangle t^2)
\]

\[
\dot{R} = Cr_b + \langle \dot{R} \rangle t = \frac{C}{1 + Ct} R - \frac{C}{1 + Ct} \frac{1}{2} \langle \dot{R} \rangle t^2 + \langle \dot{R} \rangle t
\]

\[
\lim_{C \to \infty} \frac{C}{1 + Ct} = \frac{1}{t}
\]

\[
\dot{R} = \frac{R}{t} + \frac{1}{2} \langle \dot{R} \rangle t
\]

(7)

Above we have the splinter radial velocity as function of \( t, R \) and of the medium radial deceleration, \( \langle \dot{R} \rangle \), to which we need to apply a correct dynamic formula. According to classical mechanics for an orbiting splinter-galaxy of negligible mass \( m \) is

\[
\ddot{R} - R \dot{\vartheta}^2 = -\frac{GM}{R^2}
\]

(8)

where \( R \dot{\vartheta}^2 = a_c \) is representing the hypothetical centripetal acceleration due to an undefined angular velocity \( \dot{\vartheta} \), that could be roughly considered as belonging to a same thick orbital plane only for the nearby Galaxy environment, we can rewrite eq. ( 8) according to the Newtonian formulation of the first Einstein equation as below:

\[
\ddot{R} = -\frac{4}{3} \pi G \rho(t, R) R(t) + \dot{\vartheta}^2(t, R, ...) R(t)
\]

(9)

At present it is not easy to imagine the physical meaning of such \( a_c \); however it does not affect the procedure to obtain our aim, and its inclusion has to have only an explanatory and qualitative meaning. Therefore, after integrating eq. (9) according to the mean theorem, we can obtain the following

\[
\langle \dot{R} \rangle = -\frac{4}{3} \pi G R_0 \rho_\ast
\]

(10)
having fixed mathematically
\[
\rho_\ast = \left[ \langle \rho \rangle - \langle \dot{\vartheta}^2 \rangle \right] \frac{3}{4\pi G} \int_0^{t_0} \frac{R(t)dt}{R_0 \int_0^{t_0} dt}
\]  
(11)
and being at the same time
\[
\rho_\ast \leq \frac{\langle \rho \rangle \int_0^{t_0} R(t)dt}{R_0 \int_0^{t_0} dt}
\]  
(12)
where, of course, the sign = implies \( \langle \dot{\vartheta}^2 \rangle = 0 \).

The (10) formula represents an acceptable expression for the fixed average radial deceleration, \( \langle \ddot{R} \rangle \), of all the splinter-galaxies which should be at the same distance \( R_0 \), at our epoch \( t_0 \), from the centre of the Big Bang sphere; \( \rho_\ast \) is a density rotation function which refers to the average density \( \langle \rho \rangle \) of the sphere expanded to \( R_0 \) at \( t_0 \) and to the proper average square hypothetical rotational velocity \( \langle \dot{\vartheta}^2 \rangle \), hence we can reasonably write \( \rho_\ast = \rho_\ast(t_0, R_0, ...) \); \( G \) is the gravitation constant. Substituting (10) in (7), we finally obtain
\[
\dot{R} = R_0 \left[ \frac{1}{t_0} - \frac{2}{3} \pi G t_0 \rho_\ast(t_0, R_0) \right]
\]  
(13)
that is to say
\[
\dot{R} = H_{s-1} R_0
\]  
(14)
Such \( \dot{R} = H_{s-1} R \) is a true radial Hubble law, in c.g.s. units, centred on the expansion center, with the following Hubble constant formulation:
\[
H_{s-1} = \frac{1}{t} - \frac{2}{3} \pi G t \rho_\ast(t, R)
\]  
(15)

3. GALAXY HUBBLE CONSTANT VARIATION: \( \Delta H_{MW} \)

The Hubble law in (14) can be applied to our Galaxy, of course, being \( R_0 \) its distance from the origin at the epoch \( t_0 \), \( \dot{R} \) the involved recession velocity, and \( H_{0_{s-1}} = H_{0_{s-1}}(t_0, \rho_\ast(t_0, R_0(t_0))) = H_{0_{s-1}}(t_0, R_0(t_0)) \) a function which can be considered constant in our epoch \( t_0 \), all over the sphere having radius \( R_0 \).

The total derivative with respect to time of \( H_{s-1} \), whose units are \( s^{-1} \), is then
\[
\frac{dH_{s-1}(t_0, \rho_\ast)}{dt_0} = \frac{\partial H_{s-1}}{\partial t_0} + \frac{\partial H_{s-1}}{\partial \rho_\ast} \frac{d\rho_\ast}{dt_0}
\]  
(16)
being by (15)
\[
\frac{\partial H_{s-1}}{\partial t_0} = - \left\{ \frac{2}{t^2_0} - \frac{H_{0_{s-1}}}{t_0} \right\}
\]  
\[
\frac{\partial H_{s-1}}{\partial \rho_\ast} = - \frac{2}{3} \pi G t_0
\]  
(17)
So, applying the Taylor series, it results
\[
\Delta H_{s-1} = - \left\{ \frac{2}{t_0^2} - \frac{H_{0^{-1}}}{t_0} + \frac{2}{3} \pi G t_0 \frac{d\rho}{dt_0} \right\} \Delta t_0 + ...
\]  

(18)

Now we proceed to transform the previous relations in Hubble units. For this purpose let us begin to consider \( \Delta t_0 \) as the number of light-seconds of the distance \( r \). In other words any luminous signal, owing to the finite speed of light \( c \), reaching us at the epoch \( t'_0 \) with a delay with respect to the emission epoch \( t''_0 < t'_0 \), will have covered during that time the distance \( r = -c(t''_0 - t'_0) \), that is
\[
r = \frac{\delta r}{\delta t_0} \Delta t_0 = -c \Delta t_0
\]  

(19)

In (19) we have used \( \delta r \) to indicate the infinitesimal space run by light travelling towards us during an infinitesimal \( dt_0 = \delta t_0 \) of our past time. Such indication is important only to avoid confusion with the conventional \( dr \), which being included in \( \dot{r} \) represents the infinitesimal distance variation of any galaxy observed at the light distance \( r \). Consequently we define here, in place of the usual total derivative with respect to time, an alternative total derivative, as \( \delta/\delta r \), computed with respect to light-space.

Now, indicating \( r \) in Mpc and writing \( H \) in Km s\(^{-1}\) Mpc\(^{-1}\) it is:
\[
\Delta t_0 = \frac{r_{em}}{-c} = -1.029 \times 10^{14} \cdot r_{Mpc}
\]
\[
H_{s-1} = 3.24 \times 10^{-20} H
\]  

(20)

The incremental variation (18), after the above transformation, becomes
\[
\Delta H_{MW} = 3.17 \times 10^{33} \left[ \frac{2}{t_0^2} - \frac{H_{0^{-1}}}{t_0} + \frac{2}{3} \pi G t_0 \frac{d\rho}{dt_0} \right] \cdot r + ....
\]  

(21)

Here the total derivative of the composed function \( \rho_* \) with respect to time concurs to define the Hubble constant variation \( \Delta H_{MW} \) of our Galaxy, being in particular \( \frac{d\rho_*}{dt_0} < 0 \) if the Universe is expanding with \( d\rho_/ dt_0 < 0 \) and \( dt_0 > 0 \). Consequently, after having assumed
\[
K_0 = \left( \frac{\delta H_{MW}}{\delta r} \right)_{r=0} = 3.17 \times 10^{33} \left[ \frac{2}{t_0^2} - \frac{H_{0^{-1}}}{t_0} + \frac{2}{3} \pi G t_0 \frac{d\rho}{dt_0} \right]
\]  

(22)

from (21) one obtains
\[
H_{MW}(r) \equiv H_0 + K_0 r + ...
\]  

(23)

that is the Milky Way Hubble constant trend as function of the light-space \( r \) in megaparsecs.
4. THE GALAXY HUBBLE LAW: $\dot{R}_{MW} = H_{MW} R_{MW}$

Here we are able to show the complete mathematically auto-consistency of the model of a radial Hubble expansion, based on the fundamental equation (24)

$$\left(\frac{dR_{(Km)}}{dt}\right)_{MW} = H_{MW} R_{MW}$$

(24)

that is on a pure Hubble law which, instead of being centred onto the Galaxy, has been here shifted into the center of the huge void of Bahcall & Soneira (Lorenzi, 1991-1996), at the distance $R_{MW}$ from the Milky Way ($MW$). So eq. (24) represents the radial velocity of our Galaxy, whose well known Hubble constant at our epoch $t_0$ takes the value $H_{MW} = H_0$. In the previous equation we can substitute $dt = dt_0$ with $dt_0 = -\delta r/c$ (see eq. 19), $dt_0$ being the negative number of light-seconds corresponding to the distance $\delta r$ covered in the past by the light emitted by any hypothetical observed source, and $c$ the speed of light, travelling towards the earth, in $Km\ s^{-1}$ units of course. Immediately it follows

$$\frac{\delta R_{MW}}{\delta r} = -\frac{H_{MW} R_{MW}}{c}$$

(25)

that, in first order of approximation for $r \rightarrow 0$, can be solved as

$$R_{MW}(r) \cong R_0 + q_0 r + ...$$

(26)

where

$$q_0 = -\frac{H_0 R_0}{c}$$

(27)

Let us note the dimensionless number $q_0$, and consequently the possibility to adopt directly the $Mpc$ units for $R, r$, as it is for the ratio $c/H_0$. Practically eq. (26), in which both $R_0$ and $q_0$ are here constant quantities, gives us the linear trend of $R_{MW}$ versus $r$, that is the distance $R$, of our galaxy from the assumed expansion center, corresponding to any covered light-distance $r$, therefore corresponding to the epoch of the light emission by the observed source; hence eq. (26) gives us the variation with time of the Milky Way radial distance $R_{MW}$ from the void center. Of course $R_0$ is our Galaxy $R_{MW}$ at our epoch $t_0$. Deriving (25) again, we have

$$\frac{\delta^2 R_{MW}}{\delta r^2} = -\frac{R_{MW}}{c} \frac{\delta H_{MW}}{\delta r} - \frac{H_{MW}}{c} \frac{\delta R_{MW}}{\delta r}$$

(28)

representing here a different way to indicate the deceleration $\ddot{R}$. So it follows:

$$\left(\frac{\delta H_{MW}}{\delta r}\right)_{r=0} = \frac{H_0^2}{c} - \frac{c}{R_0} \left(\frac{\delta^2 R_{MW}}{\delta r^2}\right)_{r=0} = K_0$$

(29)

from which, still in first order of approximation for $r \rightarrow 0$, we derive again the (23) equation.
4.1 BY A SIMULATION

At this point we have two theoretical relations, (23) and (26), that practically represent the values of $H_{MW}$ and $R_{MW}$ as functions of time. Consequently $\dot{R}_{MW} = H_{MW} R_{MW}$ can now be integrated within the limits of a simulation carried out by adopting as rigorously true the previous equations (23) and (26) (these being so for $r \to 0$), as was previously done in the contribution presented at the Sesto Pusteria International Workshop (Lorenzi, 1995b,c). Therefore:

$$\int_0^{R_0} dR_{(K_m)} = \int_0^{t_0} H_{MW} R_{MW} \cdot dt_0$$  \hspace{1cm} (30)

being

$$R_{MW} = R_0 \to r = 0 \to t_{R=R_0} = t_0 \text{ (our epoch)}$$ \hspace{1cm} (31)
$$R_{MW} = 0 \to r = -\frac{R_0}{q_0} \to t_{R=0} = 0 \text{ (adopted zero time)}$$ \hspace{1cm} (32)

So one obtains:

$$cR_0 = -\int_{-\frac{R_0}{q_0}}^{0} (H_0 + K_0 \cdot r)(R_0 + q_0 \cdot r) \cdot \delta r$$ \hspace{1cm} (33)

where, having imposed $q_0 = -\frac{H_0 R_0}{c}$, the $K_0$ of (29) assumes an appropriate simulation value. It follows

$$c = \int_0^{\frac{R_0}{q_0}} (H_0 + K_0 \cdot r)(1 - \frac{H_0}{c} r) \cdot \delta r$$ \hspace{1cm} (34)

whose solution, within the limits of the previous simulation, results to be finally

$$K_0 = \left( \frac{3H_0^2}{c} \right)_{r=0} = \left( \frac{\delta H}{\delta r} \right)_{r=0}$$ \hspace{1cm} (35)

What expressed in Eq. (35) has indeed general validity in time; so it is possible correctly to carry out the integration

$$\int_{H_0}^{H_{MW}} \frac{\delta H}{H^2} = \frac{3}{c} \int_0^r \delta r$$ \hspace{1cm} (36)

whose solution gives:

$$H_{MW} = H_0 + \frac{3H_0^2}{c - 3H_0 r}$$ \hspace{1cm} (37)

that is the following formulas to $H_{MW}$ and $K_{MW}$.
\[ H_{MW} = \frac{H_0 c}{c - 3H_0 r} \quad \Delta H_{MW} = \frac{3H_0^2 r}{c - 3H_0 r} \quad K_{MW} = \frac{3H_0^2}{c - 3H_0 r} \] (38)

Analogously the eq. (25), where now the \( H_{MW} \) function is known, can be integrated to find the \( R_{MW} \) formula. The solution of the integral

\[ \int_{R_0}^{R_{MW}} \frac{\delta R_{MW}}{R_{MW}} = - \int_{0}^{r} \frac{H_0}{c - 3H_0 r} \delta r \] (39)

after the logarithmic reduction, gives finally:

\[ R_{MW} = R_0 \left(1 - \frac{3H_0 r}{c}\right)^{1/3} \] (40)

The last considerable result to be drawn is now that referring to the Galaxy radial deceleration formula (28), which at our epoch \( t_0 \), after the introduction of the appropriate derivatives, becomes:

\[ \left(\frac{\delta^2 R_{MW}}{\delta t^2}\right)_{r=0} = -2 \frac{H_0^2 R_0}{c^2} \] (41)

or, in c.g.s. units, the equivalent one:

\[ \ddot{R}_{MW=0} = \left(\frac{d^2 R_{cm}}{dt^2}\right)_{MW=0} = -2 \frac{H^2}{s-1} (t_0) \cdot R_{0,cm} \] (42)

Finally, by substituting the previous deceleration expression in eq. (9), as a physical result one obtains that at our epoch the inner Universe including the whole huge void of Bahcall & Soneira has a matter density \( \rho_0 \), with a lower limit, according to the following simple formula:

\[ \rho_0 > \frac{3H^2 (t_0, R_0)}{2 \pi G} \] (42b)

5. THE EXPERIMENTAL MODEL

In order to realize the operative purposes of the model, letting a side for the moment all the above developments, one may rather consider the only fundamental Eq. (14) as the general Hubble law

\[ \dot{R} = HR \] (43)

which can be applied to any place in the Universe, according to the Cosmological Principle, but taking into account the possibility that the assumed homogeneous and isotropic expansion is perturbed by local effects. In particular the choice could be that previously adopted (Lorenzi, 1991)
of applying Eq. (43) to the void center (VC) of the huge void of Bahcall & Soneira (1982), because the void itself is a deviation from the local homogeneity and isotropy, that is from the environmental standard conditions. This void (hereafter BSHV), extending 100° across the sky in the redshift range of $z \approx 0.03 - 0.08$, is centered approximately at $\alpha_{VC} \approx 9^\circ, \delta_{VC} \approx +30^\circ (l_{VC} \approx 195^\circ , b_{VC} \approx +40^\circ )$, and appears to extend, in projection, 300 $h^{-1}Mpc$ by $\geq 60 h^{-1}Mpc$ (Bahcall, 1988); consequently such a highly under-dense region dominates our sky. But there are other important scientific references, which seem to highlight the cosmological and cosmographic meaning of the BSHV and its center, VC. In particular we refer to the VC position belonging to the hemisphere with smaller Hubble ratios, inside Region 1 of Rubin, Ford and Rubin (RFR effect: 1973); to the detected Optical Dipole of Lahav (1987) (LOD: $l = 227^\circ \pm 23^\circ , b = +42^\circ \pm 8^\circ$), which follows from about 15000 optical galaxies at the low average depth of $50 h^{-1}Mpc$ practically in the same direction of VC; to the detection by Geller & Huchra (1989) of the "Great Wall" surrounding the BSHV with a minimum extent of $60 h^{-1}Mpc \times 170 h^{-1}Mpc$; and, finally, to the observed MBR dipole (COBE: Smoot et al., 1992).

That being stated, the basic hypothesis of research became, of course, that of a radial expansion, whose formulation (43) follows a pure Hubble law shifted into the center of the void (VC). The experimental model, here re-examined, is exclusively geometric, of the Euclidean type, within the limits of the present non relativistic observational cosmology ($cz \ll c$).

### 5.1 Analytical solution

Let us consider in Eq. (43) $R$ as representing the distance of a generic galaxy/group/cluster from VC, $\dot{R}$ the involved radial velocity, $H$ the corresponding Hubble flow parameter; and let us describe a generic perturbation by means of the following mathematical differential:

$$d\dot{R} = H \cdot dR + R \cdot dH + \delta$$  \hspace{1cm} (44)

Let us jointly consider the trigonometrical distance $r$

$$r^2 = R^2 + (R + dR)^2 - 2R(R + dR) \cos w$$  \hspace{1cm} (45)

between any two galaxies respectively $R$ and $R + dR$ distant from VC (see Fig. 1 of the vectorial solution). Such value of $r$ in (45) represents the distance covered from a galaxy to another by the luminous signal that, according to special relativity, travels all the time with a constant speed of light with respect to the receiver galaxy. Consequently the source distance $r$, registered by the
observer and computed in light-space, has to be considered to be the same as that of the source at the epoch of light emission, in which the galaxies find themselves at the distances $R$ and $R + dR$ from the void center VC, respectively.

If now we insert (43) and (44) into the derivative of (45) (as in the Appendix of the 1991 paper), with the second order differential $\delta$ fixed $= 0$ as first step of Eq. (44) applied to the very nearby Universe, and assume the basic hypothesis $\dot{\omega} = 0$ concerning the angle $\omega$ between two radial runs of the radial expansion from VC, that is with exclusion of any differential rotation, these mathematical steps follow:

$$2r\dot{r} = 4R\dot{R} + 2dR\dot{R} + 2R\dot{R} + 2dR - 4R\dot{R}\cos w - 2R\dot{dR}\cos w - 2R\dot{dR}\cos w$$

$$r\dot{r} = 2HR^2 + HdR^2 + RdHdR + H RdR + H RdR + dHR^2 +$$

$$-2HR^2 \cos w - H RdR \cos w - H RdR \cos w - dHR^2 \cos w$$

$$r\dot{r} = H(2R^2 + dR^2 + 2RdR - 2R^2 \cos w - 2RdR \cos w) + R^2 dH - R^2 dH \cos w + RdHdR$$

$$r\dot{r} = Hr^2 + RdHdR + R^2 dH(1 - \cos w)$$

$$\dot{r} = Hr + RdH \left[ \frac{dR + R(1 - \cos w)}{r} \right]$$

$$R = (R + dR) \cos w + r \cos \gamma$$

$$\cos w = \frac{R - r \cos \gamma}{R + dR}$$

$$r^2 = R^2 + (R + dR)^2 - 2R(R - r \cos \gamma) = dR^2 + 2RdR + 2Rr \cos \gamma$$

$$\frac{dR + R(1 - \cos w)}{r} = \frac{dR^2 + 2RdR + rR \cos \gamma}{r(R + dR)} = \frac{r^2 - rR \cos \gamma}{r(R + dR)}$$

$$\dot{r} = Hr + RdH \left[ \frac{r - R \cos \gamma}{R + dR} \right]$$

or

$$\frac{\dot{r}}{r} = H + \frac{dH}{dR} X = H + \frac{dH}{r} Y$$
with

\[ X = \frac{1 - (R/r) \cos \gamma}{1 + R/dR} \quad Y = \frac{r - R \cos \gamma}{R + dR} \quad dR = -R + \sqrt{R^2 + r^2 - 2rR \cos \gamma} \quad (48) \]

Eq. (46), when applied to human-made measurements, represents a modified formula for the local Hubble law. It holds when it is \( \dot{\omega} = 0 \) and \( \delta = 0 \). Otherwise, taking into account \( \dot{\omega} \neq 0 \), the further processing of (46) and the application of the sine theorem complete the previous formulation of radial velocity, through the addition of a new term. Finally it results:

\[ \dot{r} = Hr + dH \cdot (r - R \cos \gamma) + R \dot{\omega} \sin \gamma \quad (49) \]

being by definition \( dR \) the analytical differential, and so

\[ \lim_{dR \to 0} \left[ \frac{r - R \cos \gamma}{1 + dR/R} \right] = r - R \cos \gamma \quad (50) \]

Now we have to extend all the above differential process to the more realistic case of \( \delta \neq 0 \) in (44).

Indeed if \( \Delta H \) and \( \Delta R \) are considered as finite differences, rather than as the differentials \( dH \) and \( dR \), the (44) can be rewritten as follows

\[ \Delta \dot{R} = (H + \Delta H)(R + \Delta R) - HR = H \Delta R + R \Delta H + \Delta H \Delta R \quad (51) \]

Substituting such finite difference \( \Delta \dot{R} \) to \( d\dot{R} \) in the previous development (46), also including \( \dot{\omega} \neq 0 \), we’ll finally find the following finite difference equation

\[ \dot{r} = Hr + \Delta H \cdot \left[ \frac{r - R \cos \gamma}{R + \Delta R} \right] \cdot (R + \Delta R) + R \dot{\omega} \sin \gamma \quad (52) \]

that is

\[ \dot{r} = Hr + \Delta H \cdot (r - R \cos \gamma) + R \dot{\omega} \sin \gamma \quad (53) \]

Eq. (53), with the cancellation of \( R + \Delta R \) in (52), actually shows the possibility of existence of large \( \Delta R \) in the model characterized by small finite \( \Delta H \)'s; so it coincides formally with that obtained in (49) by adopting \( dR \to 0 \) as a consequence of \( \delta = 0 \).

In conclusion Eq. (53) results to be our searched new Hubble law. In it \( r = r_{ga} \) is the classical separation, in terms of light-space observed at our epoch and referring to the epoch of light emission, between the Milky Way (MW) and other galaxies (ga) ; \( \dot{r} = \frac{dr}{dt} \) is its variation in time as registered by us in our epoch and connected to the appropriate \( \Delta H \); \( \gamma \) is the observed angle at MW between the direction of the reference point VC and the direction referring to the
past galaxy/group/cluster position observed. \( \gamma \), being here \( \alpha, \delta \) known equatorial coordinates, can be calculated as follows:

\[
\cos \gamma = \sin \delta V C \sin \delta + \cos \delta V C \cos \delta \cos (\alpha - \alpha V C) \quad \sin \gamma = (1 - \cos^2 \gamma)^{1/2} \quad (54)
\]

Finally we shall note how \( \Delta H = 0 \) and \( \dot{w} = 0 \) reduce eq. (53) to the canonical Hubble law \( \ddot{r} = Hr \).

### 5.2 Vectorial verification

The previous analytical solution is clearly consistent with a finite difference scenario like that graphically represented in Figure 1, where the small increment \( \Delta H \) has to be referred to a galaxy/group/cluster that, at the epoch of emitted light we now receive, is respectively \( r \) distant from the Milky Way and \( R + \Delta R \) far from the expansion center VC.

Let us try a vectorial approach in order to verify the obtained solution more intuitively. In this case one can consider the radial velocity \( \dot{r} \) of the emission epoch as the difference of expansion velocity projected on the radial direction \( r \), between any galaxy/group/cluster and our Milky Way. So, starting as always from the basic hypothesis of radial expansion \( \ddot{R} = HR \) with the assumption \( \dot{w} = 0 \), it is easy to write

\[
\dot{r} = (H + \Delta H)(R + \Delta R) \cos \alpha - HR(- \cos \gamma) \quad (55)
\]

where the angle \( \alpha \) between \( r \) and \( R + \Delta R \) follows the simple equality

\[
\alpha = 180 - w - \gamma \quad (56)
\]

In (55) \( (R + \Delta R) \cos \alpha \) can be transformed trigonometrically as follows:

\[
(R + \Delta R) \cos \alpha = r - R \cos \gamma \quad (57)
\]

Consequently Eq. (55) becomes:

\[
\dot{r} = (H + \Delta H)(r - R \cos \gamma) + HR \cos \gamma \quad (58)
\]

which immediately gives the same solution

\[
\dot{r} = Hr + \Delta H \cdot (r - R \cos \gamma) \quad (59)
\]
6. INTERPRETATION OF THE TOTAL $\Delta H$

In order to clarify the meaning of the total $\Delta H$ of the finite difference equation (59), we must now return to the obtained results of the previous sections. Then, we must remember that $R = R_{MW}$ is the $MW$ distance from VC and $R + \Delta R$ the distance of a generic galaxy/group/cluster from VC, both referring to the epoch of the light emission, and consequently $\Delta R (\cong -r \cos \gamma$ in the nearby), still in light-space, represents the radial space separation of that past epoch; seemingly $\Delta H$ should be $H(R + \Delta R) - H(R) = \Delta H_{ga}(r)$, corresponding to the differential $\Delta R$, and $H(R)$ should be our Galaxy Hubble constant $H_{MW}$, both at the past epoch measured by the light-space $r$. Of course, in this context the $\dot{r}$ value of Eq. (59) refers to the instant of the emission epoch.

But the available observed $\dot{r}$ is different because it is registered by us at our epoch, and so it holds, owing to the light delay, the effects of the radial expansion variation occurred in the time elapsed during the light travel, that is after the light space $r$. Consequently our $\dot{r}$ in (59), when considered as observed $\dot{r}_{obs}$, needs to be implemented by a total $\Delta H$ representing as a whole the difference between the radial expansion of the observed galaxy/group/cluster at the emission epoch ($r = r_{ga}$) and the one of our Galaxy at the present time ($r = 0$).

In other words the true $\Delta H$ able to generate the observed velocity $\dot{r}_{obs}$ must be a combination of the Milky Way $\Delta H_{MW}$, which is tied to a finite difference of time, plus the above cited $\Delta H_{ga}$, which instead represents a finite difference of the density/rotation function $\rho_*$ at a precise moment of the past (cfr. Eq. (15)).

To show mathematically that above explained we have to calculate the finite difference $\Delta H$ as follows:

$$\Delta H = H_{ga}(r) - H_{MW}(0) = H_{ga}(r) - H_{MW}(r) + H_{MW}(r) - H_0 = \Delta H_{ga}(r) + \Delta H_{MW}(r) \ (60)$$

The previous (60) has general validity, but the value of $\Delta H_{ga}$ is crucial because its presence would mean the Universe being anisotropic.

First, let us try to carry out a differential analysis limited to the nearby Universe.

Taking into account the Hubble function $H_{ga}(r, R_{MW}(r) + \Delta R(r))$, relative to a galaxy observed as far as $r$ from us and $R_{MW}(r) + \Delta R(r)$ from VC, $R_{MW}(r)$ being the Milky Way distance from VC at the epoch of the light emission, and the $H_{MW}(r, R_{MW}(r))$, relative to the Milky Way always at the epoch $r$, and the contemporary $H_0 = H_0(0, R_0)$ of our Milky Way, we can apply
twice the Taylor series, in succession as follows:

\[ H_{ga}(r, R_{MW}(r) + \Delta R(r)) = H_{MW}(r, R_{MW}(r)) + \left( \frac{\partial H_{ga}}{\partial R} \right)_{R=R_{MW}(r)} \cdot \Delta R(r) + \ldots \]  

(61)

\[ H_{MW}(r, R_{MW}(r)) = H_{MW}(r) = H_0 + \left( \frac{\delta H_{MW}(r)}{\delta r} \right)_{r=0} \cdot r + \ldots \]  

(62)

from which it results the final one with two total derivatives, that is

\[ H_{ga}(r, R_{MW}(r) + \Delta R(r)) = H_0 + \left( \frac{\delta H_{ga}}{\delta r} \right)_{r=0} \cdot r + \left( \frac{\partial H_{ga}}{\partial R} \right)_{R=R_{MW}(r)} \cdot \Delta R(r) + \ldots \]  

(63)

Then our searched \( \Delta H = H_{ga}(r, R_{MW}(r) + \Delta R(r)) - H_0 \), after the introduction in (63) of the corresponding derivatives of the Hubble constant formulation (15), becomes

\[ \Delta H = \Delta H_{MW} + \Delta H_{ga} \cong K_0 r + Q \Delta R(r) \]  

(64)

where

\[ K_0 = 3.17 \times 10^{33} \left[ \frac{2}{t_0^2} - \frac{H_0}{t_0} - 1 + \frac{2}{3} \pi G t_0 \left( \frac{d\rho_{*MW}}{dt} \right)_{t=0} \right] \]

\[ Q = -9.51 \times 10^{43} \left[ \frac{2}{3} \pi G t_0 \left( \frac{\partial \rho_{*ga}}{\partial R(\text{cm})} \right)_{R_{MW}(r)} \right] \]

So the \( \Delta H \) of (64), referring to our local nearby Universe in the usual Hubble units, should not be zero; it should have a value depending on two components, the former of which, \( \Delta H_{MW} \cong K_0 r \), through the well defined \( K_0 \) coefficient in \( Km \ s^{-1} Mpc^{-2} \), represents a systematic time effect (TE, hereafter), while the latter, \( \Delta H_{ga} \cong Q \Delta R(r) \propto -\Delta \rho_{*ga} \), in \( Km \ s^{-1} Mpc^{-1} \), represents a space effect (SE, hereafter) depending both on the space position \( \Delta R(r) \) of the observed galaxy \( (ga) \) at the epoch \( t_s (= t_0 - t_{cm}) \) of its light emission and on the density rotation function variation in that epoch inside the hemispheres, having \( \Delta R > 0 \) or \( \Delta R < 0 \) respectively.

To conclude it is important to remark how \( Q \propto -\frac{\partial \rho_{*ga}}{\partial R} \), if present, may reasonably hold the same algebraic sign in the nearby environment. In fact, according to the model hypothesis of a spherical symmetry distribution around the void center VC, it is likely to have the sign change of \( \partial \rho_{*ga}(R) \) together with the sign change of \( \partial R \) when we change hemisphere. In other words the constant sign of \( Q \) should mean that our Galaxy does not find itself in a peculiar position like that of a density rotation peak in space. Let us remark how the quantities included in the square parentheses of the above (64) coefficients are all in c.g.s. units

7. HUBBLE FLOW IN THE NEARBY UNIVERSE

This section has the task to focus the equations of the previous experimental model, in order to make possible their easy interpretation in the most important samples of available data in
literature. The Hubble ratios to use for the check must be corrected by the motion of the Sun in the Local Group, in practice due to galactic rotation, with the standard vector of 300 Km/s towards $l = 90^\circ$, $b = 0^\circ$ (cf. Sandage & Tammann, 1975a). This means we consider Hubble ratios as seen from our Local Group, or from our Galaxy, the Milky Way, it being almost motionless within its Group. No correction, however, has to be applied for the motion of the Local Group in the cosmic microwave background (CMB)(see sub-section 9.2).

7.1 Experimental formulation

Indeed, eq. (53) represents the fundamental equation of all the present research. In the light of $\Delta H = H_{ga} - H_0 \cong K_0r + Q\Delta R$ and of the only linear relation $H = H_{MW} \cong H_0 + K_0r$, eq. (53), including the contribution due to $\dot{w}$, can be easily processed as Hubble ratio as follows

$$\frac{\dot{r}_{obs}}{r} \cong H_0 + K_0(2r - R \cdot \cos \gamma) + Q\Delta R \left(1 - \frac{R}{r} \cos \gamma \right) + \frac{R}{r} \dot{w} \sin \gamma$$

If now eq. (65) is referred to the very nearby Universe, a strong perturbative effect by the ratio $\dot{w}/r$ must be expected, being here the higher $\dot{w}$ the smaller $r$. The same effect should reasonably happen to the coefficient $Q = \left(\frac{\partial H_{ga}}{\partial R}\right)_{R=R_{MW}(r)}$, whose algebraic sign should be constant according to what is explained above, below eq. (64). In other words both $Q$ and $\dot{w}$ in the nearby environment seem able to produce much of the observed noise.

In order to better clarify the meaning of $Q$, it is useful to write a further approximated expression of (65), when $\Delta R \ll R$, that is

$$\Delta R \cong -r \cos \gamma$$

which, ignoring the noise of $\dot{w}$, transforms (65) in the following

$$\frac{\dot{r}_{obs}}{r_*} \cong H_0 + 2K_0r_* - (K_0R + Q \cdot r_*) \cos \gamma + QR \cos^2 \gamma + ...$$

The above (67) supplies a roughly quadratic equation in $x = -\cos \gamma$ to the Hubble ratio $y = \frac{\dot{r}_{obs}}{r_*}$ of a nearby sample of galaxies all at distance $r_*$, through the combination of a dipole and quadrupole type anisotropy.

7.2 Homogeneity & isotropy impose no quadrupole amplitude

Let us observe that the quadrupole amplitude $QR$ might be considered in terms of a true perturbative space density\rotation effect (SE), whose meaning may be connected to variations of the
matter density or of the space rotation of the cosmic sphere centred on the expansion center, when we consider objects having $\Delta R$ of our environment. In other words the eventual detection of this SE of the nearby Universe should allow us to identify the local density or rotation variation in the sphere.

It now becomes important to remark how the persistence of the matter density constancy in space could represent only homogeneity, but not isotropy, if we were in presence of a meaningful differential rotation, even if of local origin. **In this case all the quadrupole amplitude should be due to differential rotation.** Otherwise, if we find such perturbation to be rather undefined, one should assume the constancy of the density\rotation function with respect to space, according to the total homogeneity-isotropy condition which imposes the Universe to have no quadrupole component; this means assuming zero the value of $\Delta \rho_{\text{ren}}$, that is of $Q$ in eq. (64). However, even assuming rigorously true the exclusive homogeneity-isotropy condition, we must in any case consider the other important physical effect, the one entirely due to the light delay. This time effect (TE) is systematic and able to generate a true expansion dipole, having its amplitude measured by the value of $K_0 R$ in the formula

$$\frac{\dot{r}_{\text{obs}}}{r} \approx H_* - K_0 R \cos \gamma$$

(68)

### 7.3 Physical meaning of $H_*$ and $K_0 R \cos \gamma$

Eqs. (65)(66)(67) are indeed very important, as their application to the very nearby Universe permit accurate verification of the model. Furthermore they show now another important feature, that is the new Hubble parameter $H_*$, whose physical meaning immediately comes to light. In fact the $H_*$ of eq. (68), being $H_* = H_0 + 2K_0 r = H + K_0 r$ and $\dot{r}_{\text{obs}}$ $r$ observed quantities as radial velocity and light-space, is the Hubble ratio $\frac{\dot{r}_{\text{obs}}}{r}$ of observed sources located at $\gamma \approx 90^0$, in terms of the Hubble constant $H (= H_0 + K_0 r)$ at the light emission epoch, plus the same increment $K_0 r$ due to the observer deceleration and then to the slowing of the expansion, occurring during the time taken for the light to travel from the source, that produces a relative opposite effect of observed velocity increasing. In other words the measures from the earth on our Galaxy are affected both by seeing past epochs and by being referred to an observer having a decreased expansion velocity with respect to the time of the light emission. Such an effect is easier to understand and greatly amplified when we observe sources located at $\gamma \approx 0^0, 180^0$ directly along the radial expansion direction, becoming here $K_0 r - K_0 R \cos \gamma$ the resulting drop ($\gamma \approx 0^0$) or rise.
7.4 Differential rotation

The problem of rotation for the Universe is an open question, whose difficult analysis is particularly related to the physical interpretation given to it. Indeed a rigid rotation should not be detectable, while a differential one, as that due to angular momentum conserved or to local perturbative effects, and belonging to our nearby environment, should produce visible effects. Indeed the differential rotation, if present, may be described in the eqs. (53)(65) by the additional term \( R \dot{w} \sin \gamma \). Now, if there is differential rotation, one should have opposite algebraic values of \( \dot{w} \) corresponding to equal \( \gamma \) in the same hemisphere. And if this is the case, the global effect would be that of scattering of Hubble ratios data referring to the same angles \( \gamma \). Hence a simple way to remove such a systematic noise, as well as other random effects, may be the use of normal points when there is sufficient number of data in the numerical analysis. So the new basic assumption,

\[
\langle R \frac{\dot{w}}{r} \rangle = 0
\]  

adopted for a normal point corresponding to the same value of \( \gamma \) and to the average distance \( r_* \), is able to produce a normal Hubble ratio formulation of eq. (65) which might permit the cancellation of the eventual differential rotation of our nearby Universe.

8. HUBBLE FLOW ACCORDING TO \( K_0 = 3H_0^2/c \)

Lastly, according to the simulation solution of section 4, let us try to write the more general finite difference expansion equation, referred to a more distant Universe.

In this case we can substitute \( H, \Delta H, R \) in (53) the formulas (38)(40) derived by the Galaxy Hubble law, plus the finite difference \( \Delta H_{ga} \) of the (60) expression.

It results:

\[
\frac{\dot{r}_{obs}}{r} = H_0 + \frac{3H_0^2}{c - 3H_0r} \left[ 2r - R_0 \cos \gamma \left( 1 - \frac{3H_0r}{c} \right)^{\frac{1}{2}} \right] + \\
+ \Delta H_{ga} \left[ 1 - \frac{R_0 \cos \gamma}{r} \left( 1 - \frac{3H_0r}{c} \right)^{\frac{1}{2}} \right] + \frac{R_0 \dot{w} \sin \gamma}{r} \left( 1 - \frac{3H_0r}{c} \right)^{\frac{1}{2}}
\]  

\[ (70) \]}
9. CONCLUSIONS: A fundamental test

The conclusions and consequences, that may be drawn from the previous contents of theory and modelling of the expanding Universe from the huge void center, are indeed many and full of cosmological and astrophysical implications. Anyway, prior to enter into any details, it is fundamental to show experimentally the correctness of the model. On the ground of the successful check work carried out on a lot of available data samples, what has to be affirmed, at this advanced stage of the research, is the conclusive experimental confirmation of the expansion center presence predicted by the model and of the good formal accuracy shown by the obtained fundamental equations. In particular both the linear (dipole) and the quadratic structure (dipole + quadrupole) of the Hubble ratio trend in a Galaxy entourage of about constant \( r \) have had full experimental confirmation. However the negative constant value which results in the nearby Universe for the quadrupole amplitude \( QR \), or \( Q \), does not persist in the larger-scale environment of the nearby Aaronson (1986) clusters. Indeed it is even possible a sign change of \( Q \). This fact, together with the verified coincidence of the local expansion solution at different distances with \( Q = 0 \) assumed (see paper II), at present seems to support a local origin of the quadrupole amplitude \( Q \) of the very nearby Universe.

9.1 Expansion center check

Of course, the first fundamental check must give the uniqueness of the expansion center position.

About the coordinates of the Bahcall & Soneira void center as expansion center, both the linear and the quadratic formulation in \( \cos \gamma \), applied to our very nearby environment, seem to be able to allow their confirmation. From a series of (68)(67) least square fittings of data by the Aaronson et al. 308 nearby galaxy catalog (1982), processed according to the contents of their following 1986 paper, at the present time and limited to this Aaronson data set, a minimum standard deviation value has resulted, corresponding to a lightly shifted void center position, at \( \alpha_{VC} \approx 9^h.8, \delta_{VC} \approx +18^0 \) (dipole solution) and at \( \alpha_{VC} \approx 9^h.5, \delta_{VC} \approx +20^0 \) (dipole+quadrupole solution), that is at about \(+0.8^h\) and \(+0.5^h\) in right ascension and at about \(-12^0\) and \(-10^0\) in declination, respectively, with respect to the Bahcall & Soneira huge void center coordinates.

The importance of such result is especially due to the fact that the resulted expansion center is coinciding with a physical point of the sky. In fact Bahcall & Soneira detected the void center location by a different approach tied to a rich cluster distribution observed in the sky, entirely
independent of the expansion model. On the other hand the above obtained $\alpha \delta$ rectification could derive from the distortion produced by different effects like those listed below. Consequently a refinement of such coordinates is surely possible, but only through the convergence of different methods and contributors.

### 9.2 Dipole solution: $K_0 = 3H_0^2/c$ confirmed

The second fundamental check comes from the application of the dipole equation (68) to two separate important samples of individual galaxies. These are the above cited AA1 catalog of 308 individual nearby galaxies and the other Aaronson et al. (1986) sample (AA2) of 148 more distant individual galaxies. Both the samples may be considered homogeneous and rich enough, even if affected by large scattering in distance. Indeed, being the average individual distance of AA2 about 5 times greater than that of AA1, a solution can be attempted. To clarify the procedure, we can check two diagrams (see Fig. 5 and Fig. 6 of the optional section in Paper II: Mini check atlas of Hubble ratio dipoles), where the corresponding observed Hubble ratios $\dot{r}_{obs}/r$ of both the samples are plotted against the function $-\cos \gamma$ computed with respect to the original Bahcall & Soneira void center coordinates. The least square method applied to the algebraic system of the 308 and 148 dipole equations (68), respectively, gives the solutions listed in the small table below, where $s$ is the standard deviation of the fit and all the quantities are in Hubble units.

| Sample   | $\langle r \rangle_{sample}$ | $H_* = H_0 + 2K_0 \langle r \rangle$ | $K_0 R$ | $s$ |
|----------|------------------------------|-------------------------------------|---------|-----|
| 308AA1   | 16.13                        | 90.61 $\pm$ 1.80                   | 16.37 $\pm$ 2.99 | 28.67 |
| 148AA2   | 70.69                        | 98.79 $\pm$ 1.75                   | 15.42 $\pm$ 2.91 | 19.79 |

The previous table generates a simple algebraic system to solve, whose solution, with errors let aside, allows to find the following:

$$H_0 = 88.2 \quad K_0 = 0.075 \quad R_{AA1} = 218 \quad R_{AA2} = 206$$

This solution gives an accurate confirmation of the correlation between $K_0$ and $H_0$, as foreseen by the simulation result of Eq. (35) in section 4 ($3 \times 88.2^2 / 299800 = 0.078$). At the same time such solution is clearly affected by the Hubble ratio scattering, which is too high. Indeed the noise is necessarily great owing to a lot of combined effects, whose nature has been partially described in the previous sections, as connected to the presence of $\dot{\psi}$. Now, without entering here into
details, we can qualitatively conclude that the consistent registered deviations are due to at least
8 different effects. i.e. :

1) large scattering in distance inside the samples of galaxies;
2) random errors in the Hubble ratio measurements;
3) systematic errors in the Hubble ratios;
4) rough expansion center coordinates;
5) presence of a quadrupole perturbation;
6) perturbative actions inside groups and clusters;
7) possible limits in using the first order coefficient: $K_0$ ;
8) possible presence of differential rotation around the expansion center.

A few of these effects may be removed or reduced, applying a more precise equation than Eq.
(68) and, especially, passing to combined Hubble ratios of groups and clusters. In this case the
effects (1)(2)(6) tend to vanish, like (7) in the very nearby environment, while on the other hand
(5) and (8) should persist and become easier to study. Anyway it is important to remark that a
large part of the noise is probably due to a mixing of precise systematic physical effects, and not
to a generic chaotic thermic distribution.

In conclusion a few words have to be spent on an unconsidered effect among the 8 listed. We
refer to the motion of the Local Group (LG) in the cosmic microwave background (CMB). In fact
it must be said that the AA1 dipole here presented , after adding the correction of such a motion,
that is by means of the entire application of $M(Sun) = B(Sun) + C(LG) (= 300 \text{ Kms} \text{ to } l = 90^0,$
$b = 0^0 + 610 \text{ Kms} \text{ to } l \approx 268^0, b \approx +27^0)$ to the Sun’s velocity in the CMB, practically vanishes
together with the correlation between $K_0$ and $H_0$ , generating a standard deviation amplified more
than double ($s = 75.61$ ). Such exploratory check seems to find a reasonable explanation only
by assuming that the motion of the Galaxy or Local Group in the cosmic microwave background
might belong also to the nearby galaxies\groups, in a sort of large flow running almost along the
same direction. If this were the case, the Galilean relativity effect, while one is observing the
nearby Universe, results in the complete cancellation of the motion, while on the other hand its
consideration, exclusively in terms of applied correction to our Local Group, enormously increases
the noise, at the same time strongly involving the present solution.
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CAPTIONS OF THE FIGURES

Figure 1 (Paper I)
Local cosmographic section, as described in the vectorial verification of the text.

Figure 2 (Paper II)
Hubble ratios of 52 groups by de Vaucouleurs (1965) plotted against the $-\cos\gamma$ of each group.

Figure 3 (Paper II)
Hubble ratios of 20 groups by Sandage & Tammann (1975) plotted against the $-\cos\gamma$ of each group (cf. Table 1).

Figure 4 (Paper II)
The observed Hubble ratios of 83 individual galaxies by Sandage & Tammann (Tables 2-3-4:1975-V) plotted against the function $-\cos\gamma$ of each corresponding galaxy.

Figure 5 (Paper II)
The observed Hubble ratios of 308 nearby galaxies by Aaronson et al. (1982) plotted against $-\cos\gamma_{galaxy}$ of each corresponding galaxy.

Figure 6 (Paper II)
The observed Hubble ratios of 148 more distant individual galaxies by Aaronson et al. (1986) plotted against the function $-\cos\gamma_{cluster}$ of the corresponding cluster.

Figure 7 (Paper II)
Hubble ratios of 31 groups by Aaronson et al. (1982) plotted against the $-\cos\gamma$ of each group (cf. Table 2).

Figure 8 (Paper II)
Hubble ratios of 10 clusters by Aaronson et al. (1986) plotted against the $-\cos\gamma$ of each cluster (cf. Table 3).

Figures 2,3,7,8 (Paper II)
The sizes of the plotted points are in proportion to the member number, $n_{obs}$, of the group\cluster, according to the following:

$\Rightarrow n_{obs} \leq 4$; \hspace{1cm} $\Rightarrow 4 < n_{obs} \leq 8$; \hspace{1cm} $\Rightarrow 8 < n_{obs} \leq 16$; \hspace{1cm} $\Rightarrow 16 < n_{obs}$
This figure "fig_1.jpg" is available in "jpg" format from:

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