General D-branes Solutions From String Theory

ABSTRACT

We give the general solution for the elementary and solitonic D-brane configurations as a result of a reinterpretation of the already known p-branes. These solutions are found by means of a relevant conformal transformation on the string inspired action and its dual form. From this point of view, the nature of the electric and magnetic charge is clearer and the elementary and solitonic behaviour dependence on the initial lagrangian set. We give a complete characterisation of the spacetime defined by these solutions. The dual pair of instanton and 7-brane solution is presented as an example.
1 Introduction

Recent developments on string theory and duality conjectures suggest the existence of an underlying theory, so-called M-theory, such that the different string theories are just perturbative approximations of it. From the standpoint of a given string theory, the other formulations are seen as solitonic non-perturbative states (for a review see [1]). Apart from these 1-dimensional extended objects there are other p-brane configurations which have a very important role in the understanding of the new picture of M-theory. These solitonic states are especially interesting as they conserve half of the initial supersymmetry, where the remaining non-broken symmetries are related to the existence of a $-$symmetry, a generalisation of the well-known Green-Schwarz world-sheet $-$symmetry.

In Type II theories, the massless bosonic states include the so-called Ramond-Ramond (RxR) sector on the top of the more conventional Neveu-Schwarz-Neveu-Schwarz (NSxNS) sector, graviton and dilaton.

Recently, the p-branes carrying the Ramond-Ramond sector have been associated with superstring configurations known as D-branes [2]. From this point of view, there should be solitons for all the values of p from -1 to 9. This range is related to the expansion of the Ramond-Ramond bispinor in string theory as a Lorentz tensor. The allowed values for the dimension of the relevant RxR form are 0, 2, 4 for Type IIA theory and 1, 3, 5 for Type IIB theory. This formulation also admits the so-called dual RxR forms.

The following solutions are already known: the 0-brane or black hole, the 4-brane, the 5-brane and the 6-brane solutions of Type IIA, the 1-brane or string, the self-dual 3-brane, 5-brane, the (-1)-brane or instanton and its dual the 7-brane [3, 4, 5, 6, 7, 8, 9]. It is worth mentioning that in fact there are an infinite set of SL(2,Z) multiple of both \textquotedblleft dyonic\textquotedblright\ 1-branes [10] and 5-branes in the Type IIB theory (for a review see [11]).

In this paper we give an explicit form for the general solution of all the D-branes in Type IIA and Type IIB theories. Although the construction resembles very much the previous general solutions for p-branes [12], the D-p-branes have fascinating new features, like the non-singularity of some solutions in the relevant frame (either string or Einstein frame), real worm holes and connections with the cosmic string, to mention a few.

Some of the general forms for these solutions can be found in previous works [5, 8, 13], where the focus was more into either the relation between supermembrane theory and superstring Type IIA theory or the T-duality between the different D-p-branes solutions. Here the emphasis is given to a reinterpretation from the point of view of p-branes as solutions of effective lagrangians with RxR forms and its dual formulations. In particular, this framework shows the dependence of the elementary or solitonic characteristic on the initial lagrangian [13].

In section 2 we present the lagrangian corresponding to Type II theories. Then, by means of a reinterpretation and a conformal transformation inspired on the string nature of the lagrangian, we recover the general form for the lagrangians for p-branes. In section 3 we give a short review of the general solution for the p-branes, the dual relations between the elementary and solitonic solutions on the same theory and the relation between solutions on dual theories. In section 4 we give the general properties for the solitonic solutions
placing special emphasis on the solution with $d=8$ and its dual formulation.

2 The General $D$-$p$-Brane Solution

The effective lagrangian for the massless modes in Type II theories is given by the expression,

$$S_{10} = \int d^D x \bar{P} \frac{g(e^2 (R + 4(\partial)^2))}{2(d+1)!} \frac{1}{e^{a_0 d}} F_{d+1}^2$$  \quad (1)

where the NSxNS field is not considered and the values of $d$ depend on which version of the Type II theory has been used, basically we have $d+1 = 0;2;4$ for Type IIA and $d+1 = 1;3;5$ for Type IIB.

We are interested in finding out solutions that could be interpreted as D-branes solutions. The key idea is to generalize the above lagrangian to get a new formulation that will give us a better known theory. Therefore, we consider a conformal transformation which takes us to the so called Einstein frame.

$$S_{10} = \int d^D x \bar{P} \frac{g(e^2 (R + 4(\partial)^2))}{2(d+1)!} \frac{1}{e^{a d^0} F_{d+1}^2}$$  \quad (2)

We note that $a_{d0}$ depends on the dimension of the background space $D$ and on the dimension of the field strength. Explicitly we have,

$$a_{d0} = \frac{D}{4} \frac{d+1}{2}$$  \quad (3)

To obtain the well known lagrangian for the p-branes, $a_{d0}$ should be equal to the characteristic coupling constant $a(d)$, defined in terms of $d$ and $d$ (dual dimension) by the expression,

$$a = \frac{d}{4} \frac{2dd}{d+d}$$  \quad (4)

As a result, $a_{d0}$ must satisfy two different algebraic equations: the equation which takes us to p-brane lagrangians and the restriction that takes us to Type II theory. These two algebraic equations can be written simultaneously by means of the equation,

$$\frac{a_{d0}}{(d)} = \frac{1}{2} (4 - d)$$  \quad (5)

Hence, we have the allowed range $d = 4$, for the corresponding solutions of the Type II theory. This restriction in the possible values of $d$ can be overcome by considering as starting point a positive value for $a(d)$, so that the parameter $a_{d0}$ goes to $a_{d0}$, recovering the sector $d = 4$. We will see that this change in $a(d)$ corresponds to the dual formulation of the Lagrangian for the p-branes. Therefore the character of the solutions will be, of course, reversed in the sense that the solitonic and elementary solutions will exchange places.
3 The General p-brane Solution

From the Heterotic string effective action for the bosonic sector, we can generalise to

$$S_0 = \frac{1}{2k^2} \int d^p x \sqrt{g} R - \frac{1}{2} \left( \frac{\partial \phi}{\partial x} \right)^2 - \frac{1}{2(d+1)!} e^{2\phi} F_{d+1}^2$$  \hspace{1cm} (6)

where we consider a spacetime background of dimension D, in which we have a (p+2)-form
field strength $F_{p+2}$ interacting with gravity, $g_{MN}$ and a dilaton field, via the above
action. $a(d)$ is a yet undetermined negative constant.

The dual dimension $d$ is defined as $d = D - 2$ and the magnetic and electric charges as

$$e_d = \frac{Z}{g^{d+1}} e^{a(d)} F ; \quad g_d = \frac{Z}{g^{d+1}} F$$ \hspace{1cm} (7)

To solve the above equations we consider the most general split invariant under $P_d \times SO(D,d)$, where $P_d$ is the d-dimensional Poincare group. We split the coordinates as $x^M = (x^a ; x^m)$ where $a = 0;1;\ldots; (d-1)$ and $m = (d;\ldots; D-1)$. For the dilaton we consider
dependence solely on the transverse coordinates, $e(x) = e(r)$ where $r = \sqrt{\sum y^m y^m}$ and the line
element is given by,

$$ds^2 = e^{2A(r)} dx^a dx^b + e^{2B(r)} dy^m dy_m$$ \hspace{1cm} (8)

For the strength we define,

$$F_{d+1} = dA ; \quad A = e^C(r)$$ \hspace{1cm} (9)

For the elementary solution, or

$$F_1 \ldots d+1 = \frac{y^1}{\sqrt{y^1 y^1}}$$ \hspace{1cm} (10)

for the magnetic solution.

After imposing the necessary conditions to ensure supersymmetry conservation, we end
that the general solution for the elementary case is

$$ds^2 = H^{\frac{1}{2+d}} dx^a dx^b + H^{\frac{1}{2+d}} dy^m dy_m$$ \hspace{1cm} (11)

$$e = H^{\frac{1}{2+d}}$$ \hspace{1cm} (12)

$$H(r) = \begin{cases} 1 + \frac{K_o r}{a} & \text{if } d > 0 \\ C_o + K_o \ln(r) & \text{if } d = 0 \end{cases}$$ \hspace{1cm} (13)

where $a = \frac{q}{4 \frac{2\pi}{d+d}}$

To get the solitonic solutions, one replaces $d$ by $d$ and set $a(d) = a(d)$. This will give
a d-brane magnetically charged. In the above solutions, the relation between electric and
magnetic charge is given by $e_d g_d = 2n$, for $n$ integer.
The p-branes solutions presented before were regarded as solutions for the NSxNS sector. It is well known that these corresponding lagrangians can be rewritten in terms of dual objects where the relevant conformal factor must be taken into consideration. As a general rule we have,

\[ F_{d+1} = e^{a(d)} F_{d+1} \]  
\[ g_{MN}^d = e^{a(d)} g_{E}^d \] \( d \geq 0 \) \( M, N = 0, \ldots, d \) \( d > 0 \) \( d = 0 \) \( d + 1 \)

where \( g_{E}^d \) stands for the Einstein metric. Hence, the above conformal factors are the ones to be used if we want to consider change of frames from the Einstein picture to the d-brane picture.

From the analysis of section 2 and the specific form for the general p-branes, we get explicitly both types of D-p-brane solutions, elementary and solitonic.

For the elementary case, we have

\[ ds^2 = H^{1/2} dx \cdot dx + H^{1/2} dy_1 dy_m \]  
\[ e = H^{d-4} \]  
\[ H(r) = 1 + K d r^d \text{ if } d > 0 \]  
\[ C_0 + K d \ln (r) \text{ if } d = 0 \]  
\[ F_{m_1 \cdots m_d + 1} = \sum_{m = 1}^{d} \Theta_m H^1 \] \( m, n = 0, \ldots, d \) \( m > 0 \) \( m = 0 \) \( m \geq 1 \)

where \( = 0; \ldots; N \) \( N = d; \ldots; 9 \) \( N > 0 \) \( N = 0 \) \( N \geq 1 \)

For the solitonic case

\[ ds^2 = H^{1/2} dx \cdot dx + H^{1/2} dy_1 dy_m \]  
\[ e = H^{d-4} \]  
\[ H(r) = 1 + K d r^d \text{ if } d > 0 \]  
\[ C_0 + K d \ln (r) \text{ if } d = 0 \]  
\[ F_{m_1 \cdots m_d + 1} = \sum_{m = 1}^{d} \frac{y^1}{r^{d+2}} \] \( m, n = 0, \ldots, d \) \( m > 0 \) \( m = 0 \) \( m \geq 1 \)

where \( = 0; \ldots; N \) \( N = d; \ldots; 9 \) \( N > 0 \) \( N = 0 \) \( N \geq 1 \)

4 General behaviour and Singularities Issues

To consider the issue of singularity we must recalled that there are basically three different types of frames relevant in the general formulation of the theory: the Einstein frame, the
string frame and the so called source frame or sigma model frame. The last frame happens to be very important as the whole picture of duality between different p-branes actions becomes clearer [11]. Then, we can determine the relevant geometric invariant like the Ricci scalar on all three different frames. Obviously, on the sigma model frame is where, we should classify a given solution as elementary or solitonic. In our case the relation between the frame where the Type II solutions are given (string frame) and the sigma model frame is,

\[ g_{\alpha \beta}^{d} = e^{\frac{d}{2} + a(d)-d} g_{\alpha \beta} \]

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If we compute the curvature invariant for the solitonic case in string frame, some of the solutions will appear singular, depending on the dimension of the brane under consideration. On the other hand, if we calculate the curvature invariant on the sigma model frame, no singularities are found as expected. The different results are shown below.

| brane dimension d | Einstein frame | string frame | sigma model frame |
|-------------------|----------------|--------------|-------------------|
| d < 4             | R ! 1          | R ! 1        | R ! const         |
| d = 4             | R ! const      | R ! const    | R ! const         |
| d > 4             | R ! 1          | R ! 0        | R ! const         |

The family of metrics presented here brings pathological behaviours at the limit \( r ! 0 \). To obtain the nature of the hypersurface \( r = 0 \), different change of coordinates should be done. The analysis on the sigma model frame [12], gives that the metric goes to \( (AdS)_{d+1} \times S^{10-d} \), as \( r ! 0 \), and goes to at Lorentzian space as \( r ! 1 \) interpolating two vacuum solutions. Also the hypersurface \( r = 0 \) is a degenerated event horizon, where a continuation through \( r < 0 \) can be done. The new region happens to be isomorphic to the original region. This behaviour was expected as it can be shown that these solutions resemble the extreme cases already discovered some time ago [13]. We should mention that the dilaton is singular at the horizon for \( d > 4 \), hence strictly speaking non of these solutions are solitonic, nevertheless in few cases the singularity can be overcome by interpreting these solutions as a dimensional reduction from 11D supergravity [14].

The above analysis was based on general characteristics of the solutions, in particular we can consider some features which depend on the dimension of the brane. First of all, the case \( d = 4 \) is selfdual and corresponds to the solution of the norm al p-branes, since the conformal factor gives the identity. From the elementary branch, the case \( d = 0 \) is very interesting, for we are in presence of the instanton found by Gibbons et. al [3], which is a worm hole. From the solitonic branch, we pay attention to the \( d = 8 \) (\( d = 0 \)). This is another worm hole solution such that the conserv ed charge is a topological charge. This worm hole corresponds to the previous worm hole but, in the dual theory. Both solutions are given by the equations
\[ ds^2 = H^{1-2} dy^m dy_m \]  
\[ e = H \]  
\[ H (r) = 1 + \frac{K_0}{r^8} \]  
\[ F_m = \theta_m H^{-1} \]

where \( m = 0; \cdots; 9 \) and

\[ ds^2 = H^{1-2} dy^m dy_m \]  
\[ e = H \]  
\[ H (y) = 1 + \frac{K_0}{y^8} \]  
\[ F_m = \theta_m \]  
\[ \frac{y^1}{r^{10}} \]

where \( m = 0; \cdots; 9 \).

Note that in the Einstein frame the elementary solution gives at space while the solitonic solution does not. On the top of these solutions we found their dual configurations, the elementary and solitonic 7-branes.

The elementary 7-brane is given by,

\[ ds^2 = H^{1-2} dx dx + H^{1-2} dy^m dy_m \]  
\[ e = H \]  
\[ H (r) = C_0 + K_0 \ln (r) \]  
\[ F_m = \theta_m H^{-1} \]

where \( m = 0; \cdots; 7, \ m = 8; 9 \).

In the Einstein frame the line element gives,

\[ ds^2 = dx dx + H^{1-2} dy^m dy_m \]

while the solitonic solution is given by,

\[ ds^2 = H^{1-2} dx dx + H^{1-2} dy^m dy_m \]  
\[ e = H \]  
\[ H (r) = C_0 + K_0 \ln (r) \]  
\[ F_m = \theta_m \]  
\[ \frac{y^1}{r^{10}} \]
where \( m = 0; \ldots; 9 \).

Finally in the Einstein frame the line element is

\[
\text{ds}^2 = H^{-1} \text{dx} \text{dx} + \text{dy}^m \text{dy}_m
\]  

(44)

5 Conclusions

We have seen how the general form for the solutions to Type II theory can be obtained from the already known conventional solutions of p-branes. The method is based on a reinterpretation of the conformal transformations which links dual formulations within the p-brane theories. This reinterpretation gives rise to a conformal transformation related to a unique string frame, which, after an algebraic imposition on the factor \( a(d) \), reproduces the solutions for Type II theory. The fact that this constraint in \( a(d) \) is compatible with its form from the p-brane solutions is what guarantees the consistency of the whole procedure.

The solutions presented here happen to be elementary or solitonic for new values of \( d \). This behavior is explained by the fact that these solutions come from different initial theories which are duals. This point of view follows from discussions in [11].

As an example, we have recovered explicitly some of the already known solutions (instanton and 7-brane) and we have also obtained their dual formulations as a gift from the generality of the method. All the possible solutions are contained here, together with the corresponding duals.

As a last remark, although the majority of these solutions were found already in the literature, the novelty of this method brings together mathematical structure of p-branes and D-branes.

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