Investigation of Harmonic Effects in Locational Marginal Pricing and Developing a Framework for LMP Calculation

Mahmood Narimani1, Seyed Hossein Hosseinian2*

1,2Department of Electrical Engineering, Amirkabir University of Technology (Tehran Polytechnic), Iran

1 First Author, Hafez Ave, Tehran, Iran, Mobile: +98 912 737 4378, E-mail: m.narimani@aut.ac.ir
2*Corresponding Author; Second Author, Hafez Ave, Tehran, Iran, Tel: +98 21 6454 3343, Mobile: +98 912 679 7233, Fax: +98 21 6454 3301, E-mail: hosseinain@aut.ac.ir

Abstract: Locational Marginal Pricing (LMP) is arguably the most effective and commonly employed mechanism to provide the most reliable economic signal to market participants. Meanwhile, nodal prices depend on active power losses and transmission congestion which may be affected by harmonics pollution. In the conventional method, power system and loads are assumed linear and nodal prices are obtained by results of optimal power flow (OPF) at the power frequency. Harmonics lead to skin effect and increasing loss. Further, harmonic flowing in branches in a power network occupies transmission capacity. For providing more accurate signals to market participants and achieving more accurate nodal prices, harmonic effects on LMP are investigated and a framework is developed for LMP calculation in a harmonic polluted power system. In this framework, skin effect, losses, and congestion which can be arisen by harmonic pollution are modelled in optimal power flow (OPF) and are considered in LMP calculation. The proposed concept is implemented with 9-bus and 30-bus test systems while nodal price changes are also indicated.

Key Words: Locational Marginal Pricing, Harmonic, Optimal Power Flow, Nodal Price, Skin Effect

Nomenclature

| Symbol | Description |
|--------|-------------|
| λ<sub>ref</sub> | the price at the slack bus |
| λ<sub>cong</sub> | marginal congestion price |
| λ<sub>loss</sub> | marginal congestion price |
| N | set of buses |
| B | set of branches |
| G | controllable generators located at a subset of the system buses |
| C<sub>n</sub>(P<sub>n</sub>·G) | nth bus generation cost function |
| P<sub>n</sub>·G | injected active power at bus n |
| Q<sub>n</sub>·G | injected reactive power at bus n |
| V<sub>n</sub> | the nth bus voltage |
| R<sub>ac</sub> | resistance of conductor when current distribution is alternating sinusoidal |
| ber( . ) | real part of the Bessel function of the first kind and zero order |
| bei( . ) | imaginary part of the Bessel function of the first kind and zero order |
| ber'( . ) | derivative of ber( . ) |
| bei'( . ) | derivative of bei( . ) |
| r | conductor radius |
| σ | the conductivity of the conductor |
| ρ | the resistivity of the conductor |
| LF<sub>k</sub> | kth line transmission capacity |
| LF<sub>k</sub> | kth line transmission capacity at power |
| LF<sub>k</sub> | equivalent kth line transmission capacity in the presence of harmonic |

1*Corresponding Author; Second Author, Hafez Ave, Tehran, Iran, Tel: +98 21 6454 3343, Mobile: +98 912 679 7233, Fax: +98 21 6454 3301, E-mail: hosseinain@aut.ac.ir
1. Introduction

Locational Marginal Price (LMP) is one of the most commonly tools employed in the electricity market to determine nodal prices and provide economic signals for market participants. LMP is defined as the cost of supplying the next load increment at that bus. Many of the well-known running power markets such as PJM, NYISO, ISO-NE, CAISO, ERCOT, MISO, and NEMCO have utilized LMP in their systems [1,2].

Optimal power flow results are sensitive to the constraints of the branch flow and the way the transmission losses are spotted in calculations [3].

Accurate values of each component of the price are required to achieve the desired purpose of LMP signals, i.e., efficient dispatch and fruitful incentives. Accordingly, consideration of the variations in the system parameters for assessing changes in LMPs is advantageous for the bidding strategies of producers and consumers [4].

A market framework for the practical implementation of lossy financial transmission rights (FTRs) was proposed in [5]. The lossy FTRs can be settled directly according to LMPs without requiring any LMP decomposition.

The lossy FTR mechanism was improved and upgraded through the introduction of lossy option FTRs in [6].
In [7], a mathematical model was provided for adding cost components of loss and power factor to the transmission pricing.

In [8], current pricing was proposed as a method to achieve nodal prices without marginal losses for DC grids. The nodal prices without marginal losses were derived by linearizing the quadratic OPF problem only through fixing the voltage, instead of using Taylor approximation.

Authors in [9] discussed the pricing of marginal transmission network losses in the locational marginal pricing. In this paper, the traditional loss model was studied and a new model was proposed. The proposed model achieved more defendable and predictable market-clearing results by introducing loss distribution factors to explicitly balance the consumed losses in the lossless DC power system model.

In [4], a new model was presented to evaluate harmonic losses in optimal power flow and nodal pricing. The presence of harmonic losses in the OPF problem was expressed as particular resistive elements added to the line impedance in this model. Also, the reactance variations were assumed to be a multiplication of the harmonic order. Skin effect and resistance changes were not considered in that paper.

A new locational marginal price (LMP) policy for the distribution system was presented in [10] with significant penetration of distributed generation (DG). Despite acknowledging that significant losses play an essential role in the LMP calculation, the effect of losses due to network imbalances and harmonic were considered.

The dynamic tariff concept was introduced in [11], which is based on the distribution locational marginal prices (DLMP) for solving congestion problems, obtained by controlling the price values at different nodes. This analysis was extended in [12] to consider intertemporal characteristics of the flexible load. A new quadratic programming-based approach was presented in [13] to address the multiple solutions of the decentralized aggregator optimization algorithm, utilized in [12].

A simple methodology was described in [14] based on the analysis performed by the Hydro One in-house computer program (PROCOSE) to calculate transmission congestion cost (TCC) for a given period of time in dollars per unit time and locational marginal pricing (LMP) in dollars per megawatt-hour (MWh) at any selected bus in the transmission system. Sensitivity studies for determining the effect of changes in the system parameters and operating conditions on the LMPs can be carried out by this methodology.

A congestion management approach taking the demand elasticity into account was proposed in [15].

Meanwhile, the issue of harmonic losses is always disputed in a deregulated power system. The rapid development of the industries and energy supply technology improvements are the main contributing factors of
the harmonic distortion [16]. Many industrial consumers employing electronic devices such as reactive power compensation devices and UPSs are the significant factors of harmonic distortion on the distribution network [17]. For example, Korea will develop the infrastructure battery charging for electric vehicles which can increase harmonic pollution [18]. The utility companies usually install mitigation equipment to maintain the quality of power supply. The cost of this installed equipment must be recovered from the consumers who have caused the power quality problem, i.e., harmonic distortion [19].

In [20] a market-based framework was proposed for central management of harmonic compensation actions in micro grids (MGs). To this end, a distortion power expected payment function (DEFP) is proposed for each active power filter (APF) representing various imposed costs for participation in the harmonic power market (HPM).

With the development of power electronics technology, nonlinear loads acting as harmonic current sources are increasing in power distribution systems [21–25]. Harmonics cause reactance variations, skin effect, heightened resistance of lines, and increased line losses. Further, harmonics occupy the capacity of the lines, which can changes in the power market signals to inaccurate signals. Thus, these effects should be considered in pricing, which has remained neglected in the above-mentioned papers. In [4], only harmonic loss and reactance variations were considered in LMP, but skin effect and capacity occupation were not included. The main focus of our work is considering these effects in nodal pricing. To this end, we model these items in a way that can be used in pricing without altering the basic optimal power flow equations.

The outline of this paper is as follows. In section 2 the required formulations for locational marginal pricing, optimal power flow, and skin effect are presented. The proposed algorithm for nodal price calculation by considering harmonic is discussed in Section 3, which is followed by two case studies in Section 4. Section 5 concludes the paper.

2. Formulation

2.1. Locational Marginal Pricing (LMP)

The theoretical prices of electricity at each node on the network is a calculated shadow price; it is assumed that the next increment of electric energy at a specific bus and the hypothetical incremental cost to the system that would result from the optimized dispatch of available units establish the hypothetical production cost of the hypothetical incremental demand.
LMP is the summation of the marginal cost of generation, congestion cost, and cost of marginal losses [26]:

\[ \lambda_n = \lambda_{\text{ref}} + \lambda_{\text{cong}} + \lambda_{\text{loss}} \]  

Thus, LMP is a function of line congestion and losses. As mentioned above, intensified losses and line capacity occupation are the effects of harmonics.

The LMPs can be computed by either ac optimal power flow (ACOPF) or dc optimal power flow (DCOPF). The ACOPF model is more accurate than the DCOPF model, but it is prone to divergence. Albeit the ACOPF form is complex, it has been implemented by NYISO (New York international system organization) and CAISO (California independent system operator) [27–29].

2.2. Optimal Power Flow (OPF)

The combination of an objective function with the power flow equations results in Optimal Power Flow (OPF). Most OPF variants are based on the classical formulation of Carpentier [30] and Dommel and Tinney [31]. The classical formulation is an extension of the classic economic dispatch. The economic dispatch problem can be thought of as maximizing the economic welfare of a power network or minimizing the total cost of electricity generation while meeting the system constraints.

The ultimate purpose is to optimize the total cost of generation, with the classical form of the formulation being:

\[ \min \sum_{n \in G} C_n(P_n^G) \]  

s.t.

\[ P_n(V, \delta) = P_n^G - P_n^L \quad \forall n \in N \]  

\[ Q_n(V, \delta) = Q_n^G - Q_n^L \quad \forall n \in N \]  

\[ P_{\min}^G \leq P_n^G \leq P_{\max}^G \quad \forall n \in G \]  

\[ Q_{\min}^G \leq Q_n^G \leq Q_{\max}^G \quad \forall n \in G \]  

\[ V_{\min} \leq V_n \leq V_{\max} \quad \forall n \in N \]  

\[ \delta_{\min} \leq \delta_n \leq \delta_{\max} \quad \forall n \in N \]  

\[ |LF_k| \leq LF_{\max}^k \quad \forall k \in B \]
The generation cost function of the $n$th bus can be illustrated as follows [32]:

$$C_n(P_n^{G}) = a_n + b_n P_n^{G} + c_n (P_n^{G})^2 / h$$  \hspace{1cm} (10)

### 2.3. Power System Harmonic Model

The method which is used in this paper is harmonic injections replacement with current sources with the frequency of the harmonic load [4].

Line impedance in $h$th harmonic is expressed as follows:

$$Z_h = R + jhX$$ \hspace{1cm} (11)

All loads in the system are represented by resistance paralleled with inductors or capacitors, given by the following equation:

$$G_n = \frac{P_n}{V_n^2} \hspace{1cm} (12)$$

$$Y_n = -\frac{Q_n}{V_n^2} \hspace{1cm} (13)$$

Basic load flow equations can be used to calculate the harmonic voltages of the buses.

$$I_h = Y_h \times V_h \hspace{1cm} (14)$$

### 2.4. Skin Effect

The skin effect causes the effective resistance of the conductor to increase at higher frequencies where the skin depth is smaller, thus reducing the effective cross-section of the conductor. The skin effect is due to opposing eddy currents induced by the varying magnetic field resulting from the alternating current.

The skin effect in a circular conductor is analysed by Ramo & Whinnery [33]. The skin effect dominates the resistance by increasing frequency. Also, the skin effect on the resistance of a circular conductor is expressed as follows [34]:

$$\alpha_R = \frac{R_{ac}}{R_{dc}} = \frac{m r}{2} \left[ \frac{\text{ber}(mr)\text{bei}’(mr) - \text{bei}(mr)\text{ber}’(mr)}{\left(\text{ber}’(mr)\right)^2 + \left(\text{bei}’(mr)\right)^2} \right] \hspace{1cm} (15)$$

$\text{ber}(mr)$, $\text{bei}(mr)$, $\text{ber}’(mr)$, $\text{bei}’(mr)$ are defined as following:

$$\text{ber}(mr) = 1 - \frac{(mr)^4}{2^4 \times 4^2} + \frac{(mr)^8}{2^8 \times 4^2 \times 6^2 \times 8^2} - \cdots \hspace{1cm} (16)$$
$$bei(mr) = \frac{(mr)^4}{2^2} - \frac{(mr)^6}{2^2 \times 4^2 \times 6^2} - \ldots$$  \hfill (17)

$$ber'(mr) = \frac{d}{d(mr)} ber(mr)$$ \hfill (18)

$$bei'(mr) = \frac{d}{d(mr)} bei(mr)$$ \hfill (19)

where $m$ is as below:

$$m = \sqrt{\sigma \mu_\epsilon \mu_\sigma}$$ \hfill (20)

$\sigma$ is the conductivity of the conductor and is calculated as follows:

$$\sigma = \frac{1}{\rho}$$ \hfill (21)

As an alternative to the above analysis, power companies often use approximations to the skin effect employing correction factors. Typical current corrections utilized by the NGC (UK) and EDF (France) are reported in Table 1 [35].

3. Proposed Algorithm

Energy pricing has been performed by OPF at the power frequency. Hence, harmonic effects are ignored in the pricing. Harmonics effects, including skin effect, resistance, and reactance elevation, as well as occupation of the capacity of the lines should be embedded in OPF calculation and pricing.

3.1. Lines’ Capacity Occupation

The occupation of transmission capacity is modelled by decrease transmission capacity via harmonic power flow. The division of the transmission capacity of the lines to fundamental power flow and harmonic power flow is illustrated in Fig. 1.

Thus, the transmission capacity of the line for nodal pricing in the presence of harmonics can be written as:

$$LF_{k,\text{max,new}}^h = LF_{k,\text{max}}^h - LF_{k,h}$$ \hfill (22)

Harmonic power flowing can be written as:

$$LF_{k,h} = \sum_{h=1}^{\infty} LF_{k,h}^h$$ \hfill (23)
3.2. **Harmonic Losses**

Harmonic losses are modelled as an additional resistance which is in series with the actual resistance.

Losses caused by total actual and additional resistances in each line in non-harmonic pollution are equal to losses arising from the actual resistance under the harmonic pollution conditions. Thus, when OPF is performed at the power frequency, harmonic effects have been considered in the calculation.

$$\sum_{h=1} R_{k,h} I_{k,h}^2 = (R_{k,1} + R_{k}^\text{'}) I_1^2$$  \hspace{1cm} (24)$$

$$\sum_{h=1} R_{k,h} I_{k,h}^2 = R_{k}^\text{new} I_1^2$$  \hspace{1cm} (25)$$

$$R_{k}^\text{new} = \frac{\sum_{h=1} R_{k,h} I_{k,h}^2}{I_1^2}$$  \hspace{1cm} (26)$$

Where, $R_{k,h}$ is the $k$th line resistance at $h$th harmonic affected skin effect obtained from Table 1.

Harmonic power flow culminates in the calculation of the harmonic current flowing in $k$th line from bus $i$ to bus $j$:

$$I_{k,h} = \frac{V_{i,h} - V_{j,h}}{Z_{k,h}}$$  \hspace{1cm} (27)$$

3.3. **Reactance Changing**

Harmonic reactance ($X^h$) in $h$th harmonic is $X^h = hX$. Thus, reactances are increased by the harmonic order. Note that the equivalent reactance is calculated similar to calculating the equivalent resistance in the presence of harmonic.

The effect of all harmonics on $k$th line reactance between bus $i$ and $j$ can be calculated as follows:

$$Z_k = \frac{V_i - V_j}{I_k}$$  \hspace{1cm} (28)$$

$I_k$, $V_i$, and $V_j$ represent $k$th line current, $i$th and $j$th bus voltages, respectively and can be calculated as follows.

$$I_k = \sqrt{\sum_{h=1} I_{k,h}^2}$$  \hspace{1cm} (29)$$

$$V_i = \sqrt{\sum_{h=1} V_{i,h}^2}$$  \hspace{1cm} (30)$$
Then, the equivalent reactance of the \( k \)th line can be calculated as:

\[
X_{k}^{\text{new}} = \sqrt{Z_k^2 - R_k^{\text{new}}^2}
\]  

(31)

Fig. 2 indicates the flowchart of calculating nodal prices while considering the harmonic effects.

4. Case Study

4.1. 9-Bus System

A 9-bus system has been indicated in [36]. The characteristics of this system are shown in Table 2 and Table 3, where \( R \), \( X \), and \( B \) are the resistance, reactance, and susceptance, respectively.

The marginal costs of generators are given by the polynomial model as follows:

\[
MC_1 = 0.11P^2 + 5P + 150 \quad \$/MWh
\]

(32)

\[
MC_2 = 0.085P^2 + 1.2P + 600 \quad \$/MWh
\]

(33)

\[
MC_3 = 0.1225P^2 + P + 335 \quad \$/MWh
\]

(34)

Five cases have been considered for investigating the harmonic effect on local marginal pricing. These cases, including harmonic injections, are listed in Table 4.

Nodal prices by considering harmonic effects and LMP deviations are shown in Table 5. LMP deviations are also presented in Fig. 3. As can be seen, these deviations have significant values, which can result in inaccurate signals for power markets such as financial transmission rights (FTR). As can be seen in Table 6, these deviations have occurred while the THD index of currents and voltages have been less than 2.3% and 1.3%, respectively and below the limits specified in the traditional IEEE-519 standard [37].

4.2. 30-Bus System

The 30-bus system indicated in [38] has been tested by harmonic injections, as outlined in Table 7.

The deviation of LMPs due to harmonic effects has been illustrated in Fig. 4.

As can be seen in Fig. 4, the harmonics result in nodal price variations. Nodal prices in both demand and generation side are affected by harmonics. LMP deviations in some busses such as 8, 25, 26, and 28 have been significant, because of severe congestion occurring in the lines end to these busses. These significant deviations increase the financial transmission rights (FTR) and the gap between total generation revenue and total consumption cost. It suggests that consideration of the harmonic effects in pricing and power market enhances the transmission share value.
The contribution of harmonic effects is normally neglected in other OPF calculation methods, while Fig. 4 demonstrates that its effects may be considerable in the calculations.

5. Conclusion

This paper aimed to investigate the harmonic effects on nodal prices. The resistance, reactance, and transmission capacity for each line were replaced for calculating LMP with harmonics.

The results of simulations revealed the effects of harmonics on LMPs and changes in the nodal price values on the generation and demand side of the power market while harmonics were below the standard level. These deviations led to financial transmission rights (FTR) alterations. The elevation of the total cost is another effect of harmonics in the power systems.

Energy pricing was performed using optimal power flow at power frequency while harmonic effects were ignored in pricing. Numerical results showed difference between OPF results with and without consideration of harmonics. Consequently, capturing harmonics in the pricing process helps the power market in sending and receiving more accurate signals and enhancing the accuracy of power market signals, which can culminate in better decision-making.

References

1. Schweppe, F. C., Caramanis, M. C., Tabors, R. D., and Bohn, R. E., *Spot Pricing of Electricity*, Springer US (1988).
2. Yang, Z., Bose, A., Zhong, H., Zhang, N., Lin, J., Xia, Q., and Kang, C., “LMP Revisited: A Linear Model for the Loss-Embedded LMP”, *IEEE Trans. Power Syst.*, 32(5), pp. 4080–4090 (2017).
3. Leite da Silva, A. M. and de Carvalho Costa, J. G., “Transmission loss allocation: part I-single energy market”, *IEEE Trans. Power Syst.*, 18(4), pp. 1389–1394 (2003).
4. Norouzi, H., Abedi, S., Jamalzadeh, R., Rad, M. G., and Hosseinian, S. H., “Modeling and investigation of harmonic losses in optimal power flow and power system locational marginal pricing”, *Energy*, 68, pp. 140–147 (2014).
5. Vaishya, S. R. and Sarkar, V., “Implementation of lossy FTRs for perfect risk hedging under the marginal loss pricing”, *IET Gener. Transm. Distrib.*, 11(1), pp. 166–173 (2017).
6. Vaishya, S. R. and Sarkar, V., “Designing option FTRs for the lossy FTR system”, *IET Gener. Transm. Distrib.*, 12(9), pp. 2132–2139 (2018).
7. Avinash D. and Chalapathi, B., “MW-Mile method considering the cost of loss allocation for transmission pricing”, 2015 Conf. Power, Control. Commun. Comput. Technol. Sustain. Growth, IEEE, pp. 128–131 (2015).
8. Mackay, L., Ramirez-Elizondo, L., and Bauer, P., “Current pricing: Avoiding marginal losses in locational marginal prices for DC grids”, 2017 IEEE Manchester PowerTech, IEEE, pp. 1–6 (2017).
9. Litvinov, E., Zheng, T., Rosenwald, G., and Shamsollahi, P., “Marginal Loss Modeling in LMP Calculation”, *IEEE Trans. Power Syst.*, 19(2), pp. 880–888 (2004).
10. Shaloudegi, K., Madinehi, N., Hosseinian, S. H., and Abyaneh, H. A., “A Novel Policy for Locational Marginal Price Calculation in Distribution Systems Based on Loss Reduction Allocation Using Game
11. O’Connell, N., Wu, Q., Østergaard, J., Nielsen, A. H., Cha, S. T., and Ding, Y., “Day-ahead tariffs for the alleviation of distribution grid congestion from electric vehicles”, *Elec. Power Syst. Res.*, **92**, pp. 106–114 (2012).
12. Li, R., Wu, Q., and Oren, S. S., “Distribution Locational Marginal Pricing for Optimal Electric Vehicle Charging Management”, *IEEE Trans. Power Syst.*, **29**(1), pp. 203–211 (2014).
13. Huang, S., Wu, Q., Oren, S. S., Li, R., and Liu, Z., “Distribution Locational Marginal Pricing Through Quadratic Programming for Congestion Management in Distribution Networks”, *IEEE Trans. Power Syst.*, **30**(4), pp. 2170–2178 (2015).
14. Hamoud, G. and Bradley, I., “Assessment of Transmission Congestion Cost and Locational Marginal Pricing in a Competitive Electricity Market”, *IEEE Trans. Power Syst.*, **19**(2), pp. 769–775 (2004).
15. Sun, J. and Lo, K. L., “A congestion management method with demand elasticity and PTDF approach”, *2012 47th Int. Univ. Power Eng. Conf.*, IEEE, pp. 1–6 (2012).
16. Talacek, P. J. and Watson, N. R., “Marginal pricing of harmonic injections: an analysis of the resulting payments”, *IEEE Trans. Power Syst.*, **17**(3), pp. 640–645 (2002).
17. Aiello, M., Cataliotti, A., Favuzza, S., and Graditi, G., “Theoretical and Experimental Comparison of Total Harmonic Distortion Factors for the Evaluation of Harmonic and Interharmonic Pollution of Grid-Connected Photovoltaic Systems”, *IEEE Trans. Power Deliv.*, **21**(3), pp. 1390–1397 (2006).
18. Kim, K., Suk Song, C., Byeon, G., Jung, H., Kim, H., and Jang, G., “Power Demand and Total Harmonic Distortion Analysis for an EV Charging Station Concept Utilizing a Battery Energy Storage System”, *J. Electr. Eng. Technol.*, **8**(5), pp. 1234–1242 (2013).
19. Hojabri, M. and Toudeshki, A., “Power Quality Consideration for Off-Grid Renewable Energy Systems”, *Energy Power Eng.*, **5**(5), pp. 377–383 (2013).
20. Marini, A., Ghazizadeh, M. S., Mortazavi, S. S., and Piegari, L., “A harmonic power market framework for compensation management of DER based active power filters in microgrids”, *Int. J. Electr. Power Energy Syst.*, **113**, pp. 916–931 (2019).
21. Narimani, M., Hosseinian, S. H., and Vahidi, B., “A Method for Harmonic Power Tracing by Using Upstream and Downstream Distribution Matrices”, *Elec. Power Components Syst.*, **47**(13), pp. 1169–1179 (2019).
22. Chang-Song Li, Zhi-Xuan Bai, Xian-Yong Xiao, Ya-Mei Liu, and Yi Zhang, “Research of harmonic distortion power for harmonic source detection”, *2016 17th Int. Conf. Harmon. Qual. Power*, IEEE, pp. 126–129 (2016).
23. Wu, T., Ping Shum, P., Sun, Y., Huang, T., and Wei, L., “Third Harmonic Generation With the Effect of Nonlinear Loss”, *J. Light. Technol.*, **34**(4), pp. 1274–1280 (2016).
24. Kumar, D. and Zare, F., “Harmonic Analysis of Grid Connected Power Electronic Systems in Low Voltage Distribution Networks”, *IEEE J. Emerg. Sel. Top. Power Electron.*, **4**(1), pp. 70–79 (2016).
25. Mesas, J. J., Sainz, L., and Sala, P., “Statistical Study of Personal Computer Cluster Harmonic Currents from Experimental Measurements”, *Electr. Power Components Syst.*, **43**(1), pp. 56–68 (2015).
26. Bharatwaj, V. N. and Abhyankar, A. R., “Max-min fair financial transmission rights payment-based AC optimal power flow locational marginal price decomposition”, *IET Gener. Transm. Distrib.*, **8**(10), pp. 1724–1732 (2014).
27. Sarkar, V. and Khaparde, S. A., “Optimal LMP decomposition for the ACOPF calculation”, *IEEE Trans. Power Syst.*, **26**(3), pp. 1714–1723 (2011).
28. http://www.caiso.com/23cf/23cfe2c91d880.pdf.
29. Monica Gokul, C. H. and Sarada, K., “AC optimal power flow calculation for locational marginal pricing”, *Indian J. Sci. Technol.*, **8**(17) (2015).
30. Carpentie, J., “Contribution a l’étude du dispatching économique”, *Bull. la Société Française des Electr.*, **3**, pp. 431–447 (1962).
31. Dommel, H. and Timney, W., “Optimal Power Flow Solutions”, *IEEE Trans. Power Appar. Syst.*, **PAS-
32. Yang, Z., Zhong, H., Xia, Q., and Kang, C., “Solving OPF using linear approximations: Fundamental analysis and numerical demonstration”, *IET Gener. Transm. Distrib.*, **11**(17), pp. 4115–4125 (2017).

33. Ramo, S. and Whinnery, J. R., *Fields and Waves in Modern Radio*, Second, John Wiley & Sons, Ltd, New York (1962).

34. Mahmoud, A. and Shultz, R., “A Method for Analyzing Harmonic Distribution in A.C. Power Systems”, *IEEE Trans. Power Appar. Syst.*, **PAS-101**(6), pp. 1815–1824 (1982).

35. Arrillaga, J. and Watson, N. R., *Power System Harmonics*, John Wiley & Sons, Ltd, Chichester, UK (2003).

36. Chow, J. H. (Joe H.), Kokotović, P. V., and Thomas, R. J., *Systems and Control Theory for Power Systems*, Springer-Verlag (1995).

37. “519-2014 - IEEE Recommended Practice and Requirements for Harmonic Control in Electric Power Systems”, https://standards.ieee.org/standard/519-2014.html.

38. Alsac, O. and Stott, B., “Optimal Load Flow with Steady-State Security”, *IEEE Trans. Power Appar. Syst.*, **PAS-93**(3), pp. 745–751 (1974).

**Figures and Tables Captions:**

Table 1. Corrections for Skin Effect in Overhead Lines [35]

Fig. 1. Line’s Transmission Capacity Division to Fundamental Power Flow and Harmonic Power Flow

Fig. 2. Flowchart of Calculating Nodal Prices by Considering Harmonic Effects

Table 2. Characteristics of Lines for 9-Bus System

Table 3. Characteristics of Busses for 9-Bus System

Table 4. Harmonic Injections in Four Cases for 9-Bus System (%)

Table 5. New Nodal Prices by Considering Harmonic Effects

Table 6. THD Index of Currents and Voltages

Fig.3. LMP Deviation in 9-Bus System

Table 7. Harmonic Injections for 30-Bus System (%)

Fig. 4. LMP Deviation in 30-Bus System
| Company | Voltage (kV) | Harmonic order | Resistance |
|---------|-------------|----------------|------------|
| NGC     | 400, 275 (based on 0.4 sq.in. steel-core al. conductors) | $h \leq 4.21$ | $R_1 \left(1+\frac{3.45h^2}{192+2.77h^2}\right)$ |
|         | 132         | $4.21 < h \leq 7.76$ | $R_0 \left(0.806 + 0.105h \right)$ |
|         |             | $h > 7.76$ | $R_1 \left(1+\frac{0.646h^2}{192+0.518h^2}\right)$ |
| EDF     | 400, 225    | $h \leq 4$ | $R_1 \left(1+\frac{3.45h^2}{192+2.77h^2}\right)$ |
|         |             | $4 < h < 8$ | $R_1 \left(1.864 - 0.024\sqrt{h} + 0.105h\right)$ |
|         |             | $h > 8$ | $R_1 \left(0.267+0.485\sqrt{h}\right)$ |
|         | 150, 90     |             | $R_1 \left(1+\frac{0.646h^2}{192+0.518h^2}\right)$ |

$R_1$ is resistance of conductor at power frequency.

**Fig. 1**
Start

Run Harmonic Load Flow

Calculate Equivalent Transmission Capacity ($LF_{max,new}$) (Equation (22))

Calculate Equivalent Resistance ($R_{k,new}$) (Equation (26))

Calculate Equivalent Reactance ($X_{k,new}$) (Equation (31))

Replace Capacity, Resistance and Reactance with New Calculated Values in Previous Boxes for Each Line.

Run Optimal Power Flow

Calculate LMP

Finish

Fig. 2

| Line | From Bus | To Bus | R (p.u.) | X (p.u.) | B (p.u.) | MVA Rating |
|------|----------|--------|----------|----------|----------|------------|
| 1    | 1        | 4      | 0.02     | 0.0576   | 0        | 85         |
| 2    | 4        | 5      | 0.017    | 0.092    | 0.158    | 140        |
| 3    | 5        | 6      | 0.039    | 0.17     | 0.358    | 80         |
| 4    | 3        | 6      | 0.025    | 0.0586   | 0        | 120        |
| 5    | 6        | 7      | 0.0119   | 0.1008   | 0.209    | 150        |
| 6    | 7        | 8      | 0.0085   | 0.072    | 0.149    | 50         |
| 7    | 8        | 2      | 0.018    | 0.0625   | 0        | 180        |
| 8    | 8        | 9      | 0.032    | 0.161    | 0.306    | 95         |
| 9    | 9        | 4      | 0.01     | 0.085    | 0.176    | 100        |
Table 3

| Bus | Type | P<sub>0</sub> (MW) | Q<sub>0</sub> (MVar) | V (p.u.) |
|-----|------|-------------------|-------------------|--------|
| 1   | ref  | 0                 | 0                 | 1      |
| 2   | PV   | 0                 | 0                 | 1      |
| 3   | PV   | 0                 | 0                 | 1      |
| 4   | PQ   | 0                 | 0                 | -      |
| 5   | PQ   | 90                | 30                | -      |
| 6   | PQ   | 0                 | 0                 | -      |
| 7   | PQ   | 100               | 35                | -      |
| 8   | PQ   | 0                 | 0                 | -      |
| 9   | PQ   | 125               | 50                | -      |

Table 4

| Case | Case 1 | Case 2 | Case 3 | Case 4 | Case 5 |
|------|--------|--------|--------|--------|--------|
| Harmonic Order | 3 | 3 | 3 | 5 | 5 | 7 | 3 | 5 | 7 | 11 |
| Bus | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|     | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|     | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|     | 4 | 0.5 | 1 | 1 | 0 | 1 | 0.5 | 0 | 2 | 0.5 | 0.5 |
|     | 5 | 0.4 | 0.8 | 0 | 0.5 | 2 | 0.8 | 0.2 | 1 | 0.8 | 0.8 |
|     | 6 | 0.8 | 0.8 | 2 | 0.5 | 1.5 | 0 | 1 | 2 | 1.5 | 0.2 |
|     | 7 | 0.6 | 1 | 1.5 | 0.8 | 1.5 | 0 | 1 | 0 | 2 | 1 |
|     | 8 | 0.8 | 0.6 | 0.8 | 0 | 1 | 0.8 | 0.5 | 0.5 | 1.5 | 0.2 |
|     | 9 | 0 | 1 | 1 | 0.8 | 0 | 1 | 1 | 1.5 | 1 | 0.8 |

Table 5

| Bus | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|---|---|---|---|---|---|---|---|---|
| LMP ($/MWh) | 23.6121 | 22.0659 | 29.9089 | 28.1364 | 30.0252 | 31.5721 | 33.7358 | 22.9207 | 26.9443 |
| LMP' ($/MWh) | 23.5640 | 22.1211 | 29.8733 | 28.6808 | 29.7986 | 31.4865 | 34.0925 | 22.9647 | 26.8707 |
| Deviation ($/MWh) | -0.0481 | 0.0552 | -0.0356 | 0.5443 | -0.2266 | -0.0855 | 0.3568 | 0.0440 | -0.0736 |
| LMP ($/MWh) | 23.5642 | 22.1171 | 29.8789 | 28.6813 | 29.8034 | 31.4928 | 34.1010 | 22.9604 | 26.8689 |
| Deviation ($/MWh) | -0.0479 | 0.0513 | -0.0299 | 0.5449 | -0.2218 | -0.0793 | 0.3653 | 0.0398 | -0.0754 |
| LMP ($/MWh) | 23.5642 | 22.1083 | 29.8921 | 28.6828 | 29.8167 | 31.5074 | 34.1216 | 22.9509 | 26.8647 |
| Deviation ($/MWh) | -0.0479 | 0.0424 | -0.0167 | 0.5464 | -0.2085 | -0.0646 | 0.3858 | 0.0302 | -0.0795 |
| LMP ($/MWh) | 23.5651 | 22.1099 | 29.8888 | 28.6701 | 29.8072 | 31.5037 | 34.1168 | 22.9527 | 26.8602 |
| Deviation ($/MWh) | -0.0470 | 0.0441 | -0.0201 | 0.5337 | -0.2180 | -0.0684 | 0.3810 | 0.0320 | -0.0841 |
| LMP ($/MWh) | 23.5648 | 22.0877 | 29.9220 | 28.6833 | 29.8396 | 31.5405 | 34.1627 | 22.9288 | 26.8564 |
| Deviation ($/MWh) | -0.0472 | 0.0219 | 0.0132 | 0.5469 | -0.1856 | -0.0316 | 0.4270 | 0.0081 | -0.0878 |
| Case | THD(i) | THD(v) | THD(i) | THD(v) | THD(i) | THD(v) | THD(i) | THD(v) | THD(i) | THD(v) |
|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Bus 1| 0.000  | 0.369  | 0.000  | 0.805  | 0.000  | 0.999  | 0.000  | 1.260  | 0.000  | 1.265  |
| 2    | 0.000  | 0.303  | 0.000  | 0.800  | 0.000  | 0.888  | 0.000  | 1.127  | 0.000  | 1.456  |
| 3    | 0.000  | 0.278  | 0.000  | 0.807  | 0.000  | 0.882  | 0.000  | 1.104  | 0.000  | 1.276  |
| 4    | 0.500  | 0.376  | 1.000  | 0.820  | 1.000  | 1.017  | 1.000  | 1.284  | 2.121  | 2.896  |
| 5    | 0.400  | 0.350  | 0.800  | 0.885  | 0.500  | 1.195  | 2.163  | 1.243  | 1.510  | 1.587  |
| 6    | 0.800  | 0.284  | 0.800  | 0.826  | 2.062  | 0.903  | 1.803  | 1.130  | 2.508  | 1.306  |
| 7    | 0.600  | 0.294  | 1.000  | 0.832  | 1.700  | 0.905  | 1.803  | 1.156  | 2.236  | 1.644  |
| 8    | 0.800  | 0.307  | 0.600  | 0.812  | 0.800  | 0.901  | 1.375  | 1.144  | 1.881  | 1.478  |
| 9    | 0.000  | 0.419  | 1.000  | 0.798  | 1.281  | 0.933  | 1.414  | 1.771  | 1.972  | 1.445  |

Fig.3
Table 7

| Bus No. | 3rd Harmonic | 5th Harmonic | 7th Harmonic | 11th Harmonic |
|---------|--------------|--------------|--------------|---------------|
|         | 0            | 0            | 0            | 0             |
|         | 0            | 0            | 0            | 0             |
| 3       | 1            | 0            | 0            | 0             |
| 4       | 1.5          | 2            | 0            | 0             |
| 5       | 1            | 0            | 0            | 0             |
| 6       | 0            | 0            | 0            | 0             |
| 7       | 0            | 0            | 2            | 0             |
| 8       | 0            | 1            | 0            | 0             |
| 9       | 0            | 1.5          | 0            | 1.7           |
| 10      | 0            | 0            | 0            | 0             |
| 11      | 0            | 0            | 1.5          | 0             |
| 12      | 2            | 0            | 0            | 0             |
| 13      | 0            | 0            | 0            | 0             |
| 14      | 0            | 1.7          | 0            | 1.2           |
| 15      | 0            | 0            | 0            | 0             |
| 16      | 0.8          | 0            | 0            | 0             |
| 17      | 1            | 0            | 1.5          | 0             |
| 18      | 0            | 0            | 0            | 1             |
| 19      | 0            | 0.8          | 0            | 0             |
| 20      | 0            | 0            | 0            | 0             |
| 21      | 0            | 0            | 0            | 0             |
| 22      | 0            | 0            | 2            | 0             |
| 23      | 1.7          | 0            | 0            | 1.5           |
| 24      | 0            | 0            | 0            | 0             |
| 25      | 0            | 2            | 0            | 0             |
| 26      | 0            | 0            | 0            | 0             |
| 27      | 0            | 0            | 1            | 0             |
| 28      | 2            | 0            | 0            | 2             |
| 29      | 0            | 1            | 0            | 0             |
| 30      | 0            | 0            | 0            | 0             |

Fig. 4

LMP Deviation ($/MWh)

Bus No.
Biographies

Mahmood Narimani was born in Iran. He received B.S. & M.S degrees in electrical engineering from Amirkabir University of Technology (AUT), Tehran, Iran in 2011 & 2013. Presently, he is a PhD. student at the department of electrical engineering of Amirkabir University of Technology, Tehran, Iran. His main fields of research are power quality, power market, restructuring, and deregulation in power systems.

Seyed Hossein Hosseinian was born in Iran. He received the Ph.D. degree from the Electrical Engineering Department, University of Newcastle, Newcastle upon Tyne, U.K., in 1995. At the present, he is a Professor in the Electrical Engineering Department at Amirkabir University of Technology (AUT), Tehran, Iran. He is the author of four books in the field of power systems. He is also the author and the co-author of over 200 technical papers. His special fields of interest include transient in power systems, power quality, restructuring, and deregulation in power systems.