Dynamical generation of Majorana edge-correlations in a ramped Kitaev chain coupled to non-thermal dissipative channels

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Abstract: We quantitatively study the out-of-equilibrium edge Majorana correlation in a linearly ramped 1D Kitaev chain of finite length in a dissipative environment. The chemical potential is dynamically ramped to drive the chain from its topologically trivial to non-trivial phase in the presence of couplings to non-thermal Markovian baths. We consider two distinctive situations: In the first situation, the bath is quasi local in the site basis (local in quasi-particle basis) while in the other it is local. Following a Lindbladian approach, we compute the early time dynamics as well as the asymptotic behavior of the edge-Majorana correlation to probe the interplay between two competing time scales, one due to the coherent ramping while the other is due to the dissipative coupling. For the quasi-local bath, we establish that there is a steady generation of Majorana correlations in asymptotic time and the presence of an optimal ramping time which is expected to facilitate a quicker approach to the steady state. In the second scenario, we analyse the action of a local particle-loss type of bath in which we have established the existence of an optimal ramping time, till which the correlations closely follow the unitary coherent dynamics. Remarkably, at later times when the dynamics is dominated by the dissipation, we identify a linear scaling of the defect in correlation with the dissipative coupling strength.

I. INTRODUCTION

Topological properties of out-of-equilibrium quantum matter is an emerging field of both theoretical\textsuperscript{1–18} and experimental\textsuperscript{19–28} investigations. For review, we refer to the articles \cite{29–31}. Topological properties of matter are established to be extremely robust against sufficiently weak and local time-independent perturbations\textsuperscript{1,32,33}. These robustness arises in the topological phase (e.g., of a topological insulator) that is characterised by a non-trivial value of the topological invariant and is separated from the topologically trivial phase by a gapless quantum critical point (QCP). Further, in the topological phase of a chain with an open boundary condition, there exist a bulk-boundary correspondence (BBC) reflected in the existence of robust zero energy edge states in its topologically non-trivial phase. More recently, the fate of such topological phases of matter and the BBC in the presence of a dissipative environment has garnered considerable attention.

Very promising among such topological systems is the one-dimensional (1D) p-wave topological superconductor, described theoretically by the 1D Kitaev model\textsuperscript{1,25,32}. It has been established that the 1D Kitaev chain in its topological phase, hosts zero energy Majorana fermionic modes which are topologically protected. Interestingly, Majorana qubits being inherently non-local in character, are robust against local perturbations and are hence expected to support fault-tolerant quantum information processing operations. Non-local Majorana fermions and their statistics are theorized to find tremendous application in the implementation of fundamental quantum gates. The experimental implementation of quantum gates in such a system however requires unitary operations to be performed on the topological Majorana fermions in the presence of environmental couplings. In this direction, there have been an upsurge in both theoretical\textsuperscript{34–37} and experimental\textsuperscript{38–40} studies which probe the robustness, dynamical engineering and manipulation of these topological Majorana modes under a unitary drive in the presence of dissipative environmental couplings. In the light of recent experimental realisation of Majorana modes in quantum nano-wires\textsuperscript{41}, the robustness of these modes against coupling to a dissipative environment is fundamental to experimental quantum information processing.

The possibility of unitarily transporting and braiding of Majorana modes in the closed Kitaev chain has been extensively explored with results indicating that the dynamical transport of the Majorana edge-modes through unitary annealing across a QCP is not feasible\textsuperscript{42} (See also, \cite{43–46}). This is the consequence of passage through the gapless QCP where the zero-energy Majorana end states mix with the bulk bands and therefore get completely delocalised into the chain without recovery. The dynamical fate of topology has also been studied in the context of an 1D Kitaev chain in a specially engineered environment\textsuperscript{47}; it has been observed that a dissipation free subspace is dynamically generated which can indeed preserve the equilibrium topological Majoranas Kitaev chain in the asymptotic steady state. However, the early time dynamics of the topological Majoranas in dissipative systems and the possibility of protecting unitarily prepared Majorana modes against dissipation is a largely unexplored area where our work focusses on.

In this work, we consider a linearly quenched BDI symmetric\textsuperscript{48} 1D finite Kitaev chain coupled to either of the two different non-thermal baths to address the
following questions: (i) How does the coupling with a quasi-local/local bath in the presence of a linear time dependent drive, affect the out of equilibrium behavior of the topological edge-Majorana correlations? (ii) Is it at all possible to engineer topological Majorana correlations in the presence of dissipation? (iii) How does coupling to the environment quantitatively affect the adiabatic preparation of correlated Majorana modes?

In the completely unitary scenario in the adiabatic limit, we show that the edge-Majorana correlation dynamically assumes its maximum value at the end of the quench. To address the issues raised in the previous paragraph, firstly we consider a finite 1D Kitaev chain, with an open boundary condition, coupled to a Markovian quasi-local non-thermal bath\(^{17}\) in the Lindbladian approach. In the presence of such a dissipator, the chemical potential is linearly ramped in time starting from a topologically trivial phase (of the closed system) to a non-trivial phase of the bare Hamiltonian. At the end of the quench, the time dependent driving is switched off and the system evolves with the time-independent final Hamiltonian in the same dissipative environment. It is noteworthy that the dissipator is chosen such that the Lindbladian steady state is the pure ground state of the topological Kitaev chain\(^{47}\). We show that although the system asymptotically reaches a steady topological state, the deviation of the out of equilibrium Majorana correlations from its topological value is suppressed for small but non-zero dissipative coupling. At the time after which the bath dominantly takes over the dynamics, determined by the dissipative coupling strength, the edge-Majorana correlations are expected to quickly saturate to its maximum topological value. We quantitatively identify the competing time scales in the out of equilibrium edge-Majorana correlation function during the early time dynamics of the chain.

In the second situation, we consider a dissipator which acts locally and independently on each site of a topologically trivial 1D Kitaev chain\(^{49,50}\). In the presence of the dissipative coupling, the chemical potential is ramped linearly in time to a topological phase across a QCP for a finite size system. We study the out of equilibrium edge-Majorana correlation function and to probe the effect of competing coherent and dissipative dynamics on the Majorana correlations. We exhibit the presence of an optimal time following which the dissipator dominantly takes over the dynamics and the correlation starts deviating from the unitary behavior. Remarkably, we also establish that the defect produced in the edge-Majorana correlation right at the end of the quench, scales linearly with the dissipative coupling strength in the weak coupling limit although the Majorana correlation is an expectation over a localised state.

The paper is organised as follows: In Sec. II we provide a brief introduction to the 1D Kitaev chain and its equilibrium topological properties. We also discuss the bulk boundary correspondence and define the defect in the edge-Majorana correlation function and along with its out of equilibrium behavior under an unitary quench across a QCP. Further in Sec. III, we introduce the complete dissipation coupled system and the Lindblad equation, which governs the dynamics of the system coupled to a Markovian bath. We also introduce the scheme of the time-dependent drive chosen in this work and discuss the possibility of unitary adiabatic generation of edge-Majorana correlation in a finite chain. In Sec. IV, we proceed with a linearly quenched Kitaev chain coupled to a quasi-local bath and probe the interplay between the coherent and dissipative dynamics. We study the non-equilibrium behavior of the Majorana edge-correlation in this setting and discuss the possibility of generation of correlated Majorana modes in the driven dissipative chain. Lastly, in Sec. V we consider the Kitaev chain coupled to a local bath and study the comparative time scales associated with the coherent and the dissipative dynamics and thereby identify an optimal time till which the correlation follow the unitary annealing before the bath takes over the dynamics dominantly. We conclude in Sec. VI with a brief summary of the work where we discuss relevant connections with recent experiments and the scope of further research. We have also incorporated two Appendices to complement the discussions presented in the main text.

\section*{II. Kitaev chain and unitary dynamics across a QCP}

\subsection*{A. Topological properties}

The Kitaev chain is a one-dimensional system of spinless fermions on a lattice of linear dimension \(N\), and is represented by the Hamiltonian\(^{1}\)

\[ H = - \sum_{n=1}^{N-1} \left( J c_n^\dagger c_{n+1} + \Delta c_{n+1} c_n + h.c. \right) - \mu \sum_{n=1}^{N} \left( 2 c_n^\dagger c_n - 1 \right). \]  

(1)

In addition to the chemical potential \(\mu\) and the nearest-neighbor (NN) hopping interaction of amplitude \(J\), the Hamiltonian also incorporates an additional NN pairing interaction of amplitude \(\Delta\). Naturally, the total number of fermions is not conserved, even though parity is. The bulk of the chain located away from the edges can be modeled as a closed chain with periodic boundary conditions (PBC). Within the bulk, translation invariance allows a Fourier transformation of the bulk Hamiltonian into the (quasi) momentum basis where it assumes a particularly convenient form,

\[ H = \bigoplus_{k>0} H_k = \bigoplus_{k>0} \left( H_k^x \bigoplus H_k^y \right) - \left( 2J \cos k \right) \mathcal{I}_k, \]

(2)
\[ H = \sum_{i=1}^{N} E_i (d_i^\dagger d_i - d_i d_i^\dagger) = E_g + 2 \sum_{i=1}^{N} E_i d_i^\dagger d_i, \]  

where \( E_g \) is the ground state energy of the system and corresponds to the Bogoliubov vacuum \( |\text{GS}\rangle \) in which all the negative energy states are occupied, while \( E_i \) represents the energy of quasi-particle excitations generated by the Bogoliubov fermionic creation operators \( d_i^\dagger \) acting on the ground state. In other words, the operators \( d_i \) annihilates the Bogoliubov vacuum. The BBC is now explicitly identified as follows: for \( |\mu| < |J| \), the bulk winding number is quantized to unity; correspondingly, the two-point correlation function of the Majorana end modes in the ground state of the Hamiltonian, defined as \( \theta = i(a_1 a_2) \), also remains finite and approaches unity as \( \mu \to 0 \). The localization of the Majorana modes stem from the presence of a zero-energy quasi particle excitation \( (E_z = 0, E_i \in \{E_i\}) \) in the thermodynamic limit. On the other hand, for \( |\mu| > |J| \), \( \theta \) vanishes in the thermodynamic limit in the trivial phase. The two phases are demarcated by a quantum critical point (QCP) at \( |\mu| = |J| \): at this point, the bulk spectrum becomes gapless in the thermodynamic limit.

**B. Unitary dynamics across a quantum critical point**

The presence of a QCP or vanishing gap in the bulk spectrum presents a conundrum in the context of preparing a topological state — starting from a trivial phase of the system, it is impossible to drive the system into a non-trivial phase through a unitary dynamics \(^{42}\). Specifically, in the problem that we consider in this work, the system is initially in the ground state \( |\psi(0)\rangle = |\psi_0^0\rangle \) of an initial Hamiltonian \( H_i : |\mu_i| > |J| \), following which the Hamiltonian is ramped across the QCP at \( |\mu| = |J| \) to a final \( H_f : |\mu_f| < |J| \) using the protocol,

\[
\mu(t) = \left( \mu_i + (\mu_f - \mu_i) \frac{t}{\tau} \right) \Theta(\tau - t) + \mu_f \Theta(t - \tau), \tag{7}
\]

where \( \Theta(x) \) is the Heaviside step function. The protocol therefore linearly ramps the initial \( \mu_i \) to a final \( \mu_f \) during time \( \tau \) after which \( \mu \) remains frozen at the targeted \( \mu_f \). In the thermodynamic limit, the quantum adiabatic theorem breaks down; the system \( |\psi(t)\rangle \) therefore cannot be exclusively prepared in the ground state \( |\psi_0^f\rangle \) of \( H_f \). This results in generation of defects in the correlation of the Majorana end modes which we quantify as

\[
\chi(t) = \langle \psi_f^0 | \theta | \psi_f^0 \rangle - \langle \psi(t) | \theta | \psi(t) \rangle. \tag{8}
\]

Note that for the ideal situation of a perfect unitary adiabatic preparation, the quantity \( \chi(t) \) should vanish.

However, for a finite system of size \( L \), the bulk spectrum is not truly gapless; the gap \( \delta \) at the QCP scales as \( \delta \sim \)
In this section, we consider the dissipative dynamics of the Majorana edge modes with the Lindblad operators in Eq. (9) chosen as

\[ L_j = d_j^\dagger, \]  

where \( d_j^\dagger \) are the Bogoliubov annihilation operators defined in Eq. (6). Further, the operators \( d_j^\dagger \) annihilate the ground state of the final Hamiltonian \( H_f \), i.e., \( d_j^\dagger |\psi_0^f\rangle = 0 \), \( \forall j \) (hence the additional superscript \( f \)). We also assume that all the Lindblad operators act uniformly on the system (\( \kappa_j = \kappa, \forall j \)). Following the scheme outlined in Sec. III, we proceed to analyze the defect generated in the edge mode correlations \( \chi(t) \).

Let us first consider the asymptotic steady state of the system \( \rho_{ss} = \lim_{\tau \to \infty} \rho(t) \). As the system evolves under the action of the constant Hamiltonian \( H_f \) for \( t > \tau \), the asymptotic steady state can therefore be found by substituting \( H(t) = H_f \) in Eq. (9) and equating the r.h.s to zero,

\[ -i[H_f, \rho_{ss}] + \kappa \sum_j \left( 2d_j^\dagger \rho_{ss} d_j^\dagger - \{ d_j^\dagger d_j^\dagger, \rho_{ss} \} \right) = 0. \]  

\[ (12) \]
Solving the above equation, one obtains $\rho_{ss} = |\psi_f^0\rangle \langle \psi_f^0|$ (see Appendix. B). Hence the system asymptotically approaches the topological ground state of the final Hamiltonian $H_f$. Naturally, $\lim_{t \rightarrow \infty} \chi(t)$ vanishes as can be seen by substituting $\lim_{t \rightarrow \infty} \rho_t = \rho_{ss}$ in Eq. (10). The dissipative environment therefore induces the preparation of localized edge Majorana modes.

For a dissipative evolution governed by Eq. (9) with a time independent Hamiltonian, the time-scale of relaxation to steady state is of the order $\tau_B \sim 1/\kappa$. Although the bath asymptotically drives the system to its pure topological state, in the presence of a time-dependent driving the non-equilibrium state of the system is necessarily mixed. This is reflected in the increasing deviation of the edge-correlation function from its unitary value with increasing dissipation strength. This suggests that the fastest preparation of the edge modes should be possible in the sudden quench limit ($\tau \rightarrow 0$) of the ramp protocol, in which case the defects vanish asymptotically with a time scale $\tau_B$. Remarkably though, as shown in Fig. (2), a finite but small ramp duration ($0 < \tau < \tau_B$) results in generation of lesser defects in the edge mode correlations after time $\tau_B$ hence, the non-equilibrium state has a higher fidelity to the asymptotic steady state at $t = \tau_B$. This in turn is expected to facilitate a quicker stabilization of the edge correlation into its topological steady value.

The lowering of defects (for $\tau \lesssim \tau_B$) is a consequence of the following – (i) within the ramp duration, the bath induces negligible dissipation, the dynamics is therefore dominated by the unitary ramp. This results in lesser generation of defects at the end of the ramp with increasing ramp duration (see Fig. (1)). (ii) The short duration of the ramp (having $\tau \ll \tau_B$) does not significantly alter the dissipative relaxation time scale of the system. In other words, after the ramp is switched off, the non-equilibrium state develops a high fidelity with the topological state. This in turn is expected to result in a faster dissipative relaxation into the topological steady state. We therefore conclude that a finite but short ramp duration further speeds up the preparation of edge modes using this bath.

V. OPTIMALITY IN THE PRESENCE OF LOCAL DISSIPATION

In this section, we model the dissipative effects through local Lindblad operators that act locally at each site on the Kitaev chain, barring the edge sites. Specifically, we choose

$$L_j = c_j, \quad j \in \{2, 3, ..., N - 1\},$$

where $c_j$ are the fermionic annihilation operators acting on site $j$ of the chain. In the presence of such local dissipative channels, the Majorana edge modes which were initially present in the system, are known to decay exponentially in time due to the finite overlap of the edge modes with the bulk for $\mu \neq 0$. However, for $\mu = 0$, the edge modes are essentially disconnected from the bulk and are therefore robust against any dissipation induced in the bulk. In our protocol, we therefore set $\mu f = 0$, so that the edge modes once created (with some defects) remain localized after the ramp, i.e. $\chi(t > \tau) = \chi(\tau)$. This allows us focus on the dissipative effects on the preparation stage of localized edge modes only and exclude any defects generated after the ramp is complete.

Unlike the previous case discussed in Sec. IV, the presence of local dissipative channels in the bulk is always detrimental to the preparation of edge states, as can be seen from Fig. 3a. Further, for sufficiently slow ramps ($\tau \gg \tau_B$), the defects generated monotonically increase with the coupling strength $\kappa$ (see Fig. 3b). The monotonic rise in defect can be explained as follows — an increase in coupling strength makes $\tau_B(\tau_B \sim 1/\kappa)$ even...
smaller than \( \tau \) which implies that the edge modes have ample time to decay (within \( 0 < t < \tau_B \)) before the chemical potential is eventually ramped to \( \mu_f = 0 \) at \( t = \tau \), following which the edge modes can no longer decay. Interestingly, for sufficiently low \( \kappa \), the dissipative defect in the Majorana end correlation scales with the ramp duration \( \tau \) as \( \lim_{\tau \to \infty} \chi(\tau) \sim \kappa \tau \). Although being an expectation over localised states, this resonates perfectly with the scaling of defect density in bulk residual energy under similar dissipation and driving protocols\(^{50}\).

In hindsight, we therefore conclude that in the limit of slow ramp \( \tau > \tau_B \), the defects generation is dominated by the dissipative dynamics of the environment.

On the other hand, for very fast quenches \( \tau_B \gg \tau \), the chemical potential \( \mu \) is quickly ramped to zero withing a short duration \( \tau \); within this duration the environment fails to induce any substantive decay in the Majorana edge correlations. The short duration of the quench however, itself results in generation of defects as discussed in Sec. II. The defect generation in the fast quench limit is therefore dominated by the unitary dynamics with the defects arising due to the fast (non-adiabatic) ramping.

It follows from the preceding discussion that, there exists an optimal ramp duration \( \tau_r \) at which the defect generation is minimized (see Fig. 3a) and the deviation from unitarity is small.

VI. DISCUSSIONS AND CONCLUSION

Choosing two fermionic dissipative environments of considerable difference in character, we quantitatively explore the pros and cons of dissipative annealing in a finite Kitaev chain. The issue of the protection of the correlated Majorana fermions against dissipation and enviromental interactions has been clearly addressed in this work with reference to the global nature of different environmental couplings.

Firstly, the chemical potential of a finite 1D Kitaev chain is slowly quenched across a QCP, to drive it from a topologically trivial to a non-trivial phase. In the thermodynamic limit, even under unitary dynamics, it is well established that annealing across a QCP necessarily induces excitations in the system which in turn annihilates the correlated edge-Majoranas and they delocalise into the bulk of the chain. On the other hand, for a finite chain, in the adiabatic regime, the system does not see a gapless point while crossing the QCP; exploiting this we have established that the adiabatic preparation of correlated Majorana fermions is possible in harmony with the topologically non-trivial final bulk Hamiltonian.

We explore the possibility of dissipative preparation of correlated Majorana edge modes through annealing in an open kitaev chain. It has been established that a quasi-local bath specifically engineered, dynamically isolates the edge Majoranas into a decoherence free subspace, thereby protecting the correlated Majoranas against dissipation. We study the non-equilibrium emergence of mutually correlated edge Majoranas in the steady state under the action of such a quasi-local dissipation. We establish that the dissipator takes the system into a steady topologically non-trivial dark state and the dynamical correlation between the edge-Majorana assumes a maximum value asymptotically. However, in the presence of the linear quenching, the non-equilibrium state deviates increasingly from the instantaneous ground state, unless the time of the ramping is much lower than the dissipative time scale, i.e., \( \tau < \tau_B \). We have observed that the bath starts playing significant role in the dynamics of the edge-correlation function at \( t = \tau_B \).

Interestingly, for \( \tau < \tau_B \), the defect produced at the time \( t = \tau_B \) is comparitively low and due to the bath, the system speedily achieves a high fidelity to the asymptotic topological steady state. Thereby, we find an optimal time of quenching for which the system relaxes quickly into the steady topological dark state.

In the second situation, we deal with a bath which is locally coupled individually and independently to each fermionic site of the chain. Furthermore, the bath is chosen to act as an infinite fermionic reservior accounting for local particle loss at every individual site of the chain. The end sites of the chain have been chosen not to couple with the bath explicitly, to study the indirect effect of dissipation through interaction of the end sites with the bulk chain. We explore the feasibility of adiabatic preparation of topologically protected edge-Majoranas in the presence of such a dissipator by quantitatively studying the interplay between the quenching and dissipative time scales in the dynamical edge-Majorana correlation function. At the end of the quench, the chemical potential of the quench vanishes and the bulk of the chain becomes completely decoupled from the end sites. This ensures that the bath does not induce any defects in the Majorana correlation after the quench is over. Remarkably, we establish the existence of a optimal region in which the bath induces considerably low excitations. \( \chi(\tau) \) deviates increasingly from the instantaneous unitary defects with increasing dissipation strength. Surprisingly, we establish that for sufficiently low dissipation strength, the dissipative deviation of \( \chi(\tau) \) at the end of the ramp follows an established scaling law with the dissipation strength and quenching time. The deviation of the defect produced in the dynamical Majorana correlation scales linearly in the dissipation strength for a fixed value of the quenching time, i.e. \( \chi(\tau) \sim \kappa \tau \).

At this point, it would be illuminating to draw a parallel with the behaviour of the residual energy at the end of the ramp \( (t = \tau) \), when plotted as a function of \( \tau \), in a driven dissipative Ising chain under action of a bath similar to that used in Sec. V\(^{50}\). The residual energy is defined as the excess energy of the time-dependent state over the instantaneous ground state and is quantitatively obtained by replacing \( \theta \) in Eq. (10) with \( H(t) \) so that \( H(\tau) = H_f \). It has been established in Ref. [50] that the
residual energy in such a situation exhibits a competition between the coherent and the dissipative dynamics. In the regime where the dissipative dynamics dominates over the unitary quench, the non-equilibrium residual energy of the system also scales linearly with dissipation strength while exhibiting a Kibble-Zurek (KZ) behaviour in the unitary dominated regime. This competition between the KZ mechanism and the dissipation results in the existence of an optimal $\tau$ at which the residual energy has a minimum. It is thus interesting to note that the Majorana correlation despite being an expectation over a localised state, displays a similar scaling law in the presence of dissipation in the small $\kappa$ limit. We also observe that similar to the residual energy, the edge-Majorana correlation exhibits a minima in its deviation from the unitary behavior at some optimal quenching $\tau_0$. However, unlike the residual energy the Majorana edge-correlation being non-extensive is not observed to follow a KZ scaling even in early time dynamics. Hence, the optimality discussed in this work is generically different from that observed in the residual energy.

We also note that in a non-interacting disordered system, the defect density in connection with the post-quench residual energy at the best decreases logarithmically with increasing quenching time\textsuperscript{52}. This logarithmic behavior of the residual energy demands a much slower velocity of quenching for the adiabatic preparation protocol and we do expect a similar behavior in the Majorana correlation. The primary reason for choosing a disorder free situation in this work.

The Kitaev chain can be experimentally studied in optical lattices with trapped ultra-cold atomic systems\textsuperscript{53,54}. The dissipative baths employed in this work can also be engineered with dual interacting optical lattices coupled to a Bose Einstein condensate reservoir\textsuperscript{55}. The experimental study of the claims made in the work will further open up the possibility of dynamical preparation of topological Majorana fermionic modes even in contact with an environment.

Appendix A: Numerical scheme for calculating $\chi(t)$

In this appendix, we outline the numerical scheme for calculating the defect $\chi(t)$ generated in the edge Majorana correlations during the dissipative evolution of the system. In general, solving Eq. (9) tantamount to solving a differential matrix equation with dimensions $2^L \times 2^L$. The maximum system size, whose dynamics can be solved numerically is consequently limited. However, note that the Lindblad operators and the system Hamiltonian chosen in our work are linear and quadratic, respectively, in Majorana operators, i.e., they can be expressed as

$$L_j = l_j \cdot a = \sum_{k} l_{j,k} a_k, \quad (A1a)$$

$$H(t) = a \cdot \mathcal{H} \cdot a = \sum_{i,j} \mathcal{H}_{i,j} a_i a_j. \quad (A1b)$$

In such a scenario, one only needs to calculate a covariance matrix $C_{i,j}(t) = \text{Tr}(a_i a_j \rho(t)) - \delta_{i,j}$ of dimension $2L \times 2L$ that encodes all the pair-correlations in the non-equilibrium state of the system\textsuperscript{56–58}. The covariance matrix is obtained numerically by solving the equation\textsuperscript{56–58}

$$\dot{C}(t) = - X(t) C(t) - C(t) X^T(t) + i Y(t), \quad (A2)$$

where, $X(t) = 4 i H(t) + \text{Re}[M(t)]$, $Y(t) = 4 i \text{Im}[M(t)] - \text{Im}[M^T(t)]$ and $M(t) = \sum_l l_l \otimes l_l^T$. The defect in edge Majorana correlation is then calculated as

$$\chi(t) = \langle \psi_f^0 | \theta | \psi_f^0 \rangle - i C_{1,2L}(t). \quad (A3)$$

Appendix B: Asymptotic steady state of the system for Lindblad operators chosen as $L_j = d_j^f$

In this appendix, we illustrate that the asymptotic steady state of the system for the Lindblad operator $L_j = d_j^f$ (discussed in Sec. IV) corresponds to the topological ground state of the final Hamiltonian. On substituting $\rho_{ss} = |\psi_f^0\rangle \langle \psi_f^0|$ in the l.h.s of Eq. (12), the first term trivially vanishes as $|\psi_f^0\rangle$ is an eigen (ground) state of $H_f$. Further, by construction, the operators $d_j^f$ annihilates the state $\psi_f^0$.

$$d_j^f |\psi_f^0\rangle = \langle \psi_f^0 | d_j^f \rangle = 0. \quad (B1)$$

Consequently, the second term in the l.h.s of Eq. (12) also vanishes. Therefore, $\rho_{ss} = |\psi_f^0\rangle \langle \psi_f^0|$ is indeed an asymptotic steady state of the system when the Lindblad operators are chosen as $L_j = d_j^f$. In addition, we have verified numerically through an eigenvalue analysis of the Lindbladian super operator\textsuperscript{56} that the steady state is also unique.

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