Game theoretic approach for public multi-mode transportation in smart cities

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Abstract
Here, a dynamic multi-mode transportation model is developed in which the passenger, for his trip, can use one or a combination of the transportation forms such as a car, a bus or a bicycle. This model is based on the game theory concept-based trip cost-optimization and it is called game theory multi-mode transportation. The proposed system is implemented through a realistic scenario in a specific city using the OMNET++ and the OpenStreetMap software tools. The results show that the average trip price and the average trip time are improved when using the proposed model.

1 | INTRODUCTION
Currently, transportation systems, which are the second highest source of pollution in cities [1], must deal with many issues to meet user expectations in terms of efficient, healthy, environmentally friendly, and convenient trips. For example, greenhouse gas (GHG) emissions [2], road congestion [3], and accidents [4] place a heavy burden on governments to provide an adequate environment for their population [5]. On the other hand, developments in computing technologies, vehicular communications, and sensing are expected to improve the transportation system to satisfy the user needs in terms of reducing the transportation cost and minimising the discomfort for users. Most previous studies, especially those studies conducted from 1999 to 2014 [6, 7], focussed on traditional ride-sharing schemes with a static itinerary in which the passenger shares his trip with other passengers who have the same origin and/or destination.

Nowadays, dynamic ride-share systems allow passengers to take another route with the possibility of reducing the trip’s time and cost. At the same time, road network developers are focussing on integrating multiple forms of transportation to provide the passenger with more dynamic transportation and a mobility-on-demand service. This study [8] proposed a model that combined carpooling with public transport to obtain multi-modal mobility planning. However, this system combines only two modes of transport and is designed for the sole purpose of finding the shortest route for the user to reach his destination without considering the cost of the trip.

One of the new trends in the modern transportation system planning models in smart cities is the game theory (GT) concept to solve congestion problems with multi-player objectives. The GT, with its different strategies, helps intelligent transport system developers to design a model that provides the shortest trip time and the lowest cost for transporting passengers [9].

The authors design a dynamic transport model that combines three modes of transport (bus, car ride-sharing, and bicycle) using the GT concept to provide more passenger-moving options with the lowest cost and shortest route. It should be noted that the car ride-sharing requires a group of persons using the same car and sharing the trip cost (time and price), while the bicycle requires a group of riders to rent the bike and share it for a specific time. According to [10], many residents prefer to use the bicycle for their short commutes. Cycling helps to reduce the incidence of many health problems, such as obesity, while helping to maintain a healthy heart. Here [11], a new dynamic mobility scheme that integrates car ride-sharing with the bus is modified.

The contributions of this work are summarised as follows:

- We are the first to propose a new mobility traffic model based on the GT concept to build a multiple-mode transportation scheme;

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• A dynamic mobility system based on GT is proposed and considered for the city of Ottawa; and
• A simulation is run to implement the proposed model. Through this system, passengers can ride buses, cars, or bicycles either exclusively or in combination during their trip for the fastest time, minimum waiting time, and the lowest cost.

The remainder of this article is organised as follows: Section 2 reviews previous works which are related to this study. Section 3 outlines the framework of the system. Section 4 explains the algorithm and mathematical formulation of the model. Section 5 describes the simulation and results; and finally, Section 6 concludes the article.

2 | RELATED WORK

Most transportation systems are interested in finding the route by which the passenger reaches their target as quickly as possible compared with other travel options. The reference [12] proposed to integrate dynamic ride-sharing into public transport. The authors suggested a system called XAR to provide the passenger with dynamic movement with the shortest possible time while considering limitations such as route change and walking preference. Some studies, such as [13], focussed on solutions for large-scale real-time ride-sharing. In this research, a kinetic tree algorithm is developed that deals with dynamic requests for scheduling. Compared with other models, the kinetic tree has proven to be more efficient and flexible in providing suitable route options with a faster arrival time for passengers. The article [14] designed a new route query for optimal multi-meeting-points in a real-time ride-sharing environment. This route query aims to find the shortest route from the starting point to the destination by minimising the ratio between the distance of the route and the shortest route to the destination. The model in [15] is based on the concept that the search for the shortest path of the public transport should be reduced to a specific area for each car after filtering the requests that would reduce the quality of service provided to the passenger. This system works by creating an efficient path-planning greedy algorithm strategy that helps to accelerate complex calculations. The algorithm in [16] helps to reduce the extra travel time resulting from a vehicle detour when a new sharing request occurs during the trip. In all these studies, the authors focus on only one mode of transportation and the need for a quick arrival.

Other studies have only focussed on finding solutions to reduce the cost of the trip. The authors of [17] proposed a price model that provides fair pricing to the riders based on each rider’s profile, so that the service provider makes a good profit without being circumvented. The dynamic ride-sharing pricing system in [18] is based on a double auction-based discounted trade reduction mechanism consisting of balanced budget, individual rationality, and incentive compatibility.

Recently, the trend in dynamic ride-sharing research is to create traffic systems that find the shortest route for the passenger while considering the passenger’s preferences, such as trip price. SHAREK in [19] is a model that allows the driver to provide his or her passenger with a sharing service by determining the vehicle’s current location and distance from the user. At the same time, the user specifies his current position, the distance to his last destination, and the maximum amount he is willing to pay for his trip. However, since SHAREK deals with only one group of passengers, this limits the use of the system’s ride-sharing capabilities. The taxi sharing system in [20] allows each user to determine the payment, the trip’s length of time, and the waiting time. The authors in [21] proposed a system to develop the current passenger sharing system to consider the current traffic situation in order to give passengers the preferred options for their journey. The article [22] designed utility-aware ride-sharing on road networks, a dynamic movement model based on maximising the vehicle-related utility and the rider-related utility without exceeding capacity-constrained and time-constrained riders.

Very few current transportation systems include more than one means of transport as an option for the passenger during his journey. Reference [23] targets a dynamic multi-network mobility in urban areas. The research suggests a model to find solutions for traffic flow with the participation of carpooling, public buses, and trains within the framework of a single traffic system. The developers in [24] designed a mobile platform that offers suggestions for planning a trip that includes only two modes of transport, that is, carpooling and public buses. This solution ensures the shortest route for the passenger while considering delays caused by road accidents and traffic congestion. In [25], the purpose of the system is to combine two modes of transportation (ride-sharing and car-sharing) in one model so that the passenger can operate the multi-trip scheduling as one task. We note the shortcomings in this study. Most modern models focus on only one aspect, for example reducing time without taking into account lowering costs. Also, the article focuses on one or two modes of the transportation system without considering the diversity in terms of user needs. In [26], the proposed model uses public transportation to calculate the shortest route a passenger can take during the trip. To reduce the length of the searching process, the idea of the logarithm depends on calculating the new short path in each station. Therefore, it is determined whether the passenger will travel by car or will continue to travel by bus in real-time around the station and not for the entire journey. This work only focuses on reducing trip time, not on reducing both time and cost.

While many articles suggest multi-mode transportation systems to find solutions to some transportation problems, the time and the cost of the trip are not taken into account. Tripod’s optimization framework in [27] is a new rider order-management system that aims to reduce the energy consumed by the transportation system as a whole. The passenger here manages the online request as the model supports multiple means of transport and depends on giving the passenger the best option based on the current and predicate state.
of the network. However, this model has several limitations. First and most importantly, it does not take into account the passenger's desire to arrive at the lowest cost (time and price). Second, the simulation is applied in the morning period only. Reference [28] has proposed a framework for multiple-modal transportation networks to find a clearer picture of the different patterns and predictions of urban transport. The model consists of three layers of varying transportation networks, as a different coverage of these networks is studied for passengers in three different cities. However, mathematical details for this model do not calculate the time and cost required for the trip; as well, their effect on coverage is not studied.

As mentioned previously, in contrast to all these studies, a suitable scheme for the dynamic mobility multi-mode transportation based on the GT concept for passengers to plan their trip transit with the shortest time and the lowest fare is developed here.

3 METHODOLOGY AND PROBLEM FORMULATION

This part of the study explains the methodology for each transportation mode and discusses the mathematical optimisation equations which are used to implement the game theory multi-mode transportation (GT-MMT) model. Based on the GT concept, the GT-MMT model combines three different forms of transport to calculate the shortest periods and the lowest cost of the trip. Also, we assumed several relevant hypotheses that must be mentioned. First, we did not consider the person's walking time between the stations on his trip. Second, the stations of the different vehicles are located in the same position. Third, while the passenger's origin is the first station of the journey, the last station is the destination. Finally, security and trust have been assumed in this model.

Here, all mobile nodes are indicated by $V$. Public bus, car ride-sharing, bicycle, and passenger are symbolised by $b$, $c$, $r$, and $p$ respectively. Tables 1–4 indicate all assumed symbols in the study. Figure 1 shows the main components of the GT-MMT, which consists of car ride-sharing, public buses, bicycles, and passengers. The passenger, which is the element that is served in the proposed model, is denoted by $P = \{1, 2, \ldots p\}$.

In the next subsections, we show in detail the equations for calculating the travel time and fare of the three transportation modes.

3.1 Public bus transportation

Here the public bus set is represented as $B = \{1, 2, 3, \ldots b\}$, and the set of public bus stations as $SB = \{sb_{1,1}, sb_{2,1}, \ldots, sb_{b,1}\}$. Table 1 shows all parameters of the public bus.

| Notation | Definition |
|----------|------------|
| $b$      | Bus identification |
| $B$      | Bus set |
| $cab$    | Number of seats in bus $b$ |
| $cab(t)$ | Number of seats available on public bus $b$ at time $t$ |
| $sb_{i,b}$ | Bus station identification of bus $b$ |
| $sb_{i,pick}^{orig}$ | Origin station $i$ for bus $b$ in the trip |
| $sb_{i,drop}^{pick}$ | Destination station $i$ for bus $b$ in the trip |
| $sb_{i,b}^{pick}$ | Pickup public bus station |
| $sb_{i,b}^{drop}$ | Drop-off public bus station |
| $SB$     | Set of public bus station for all $b \in B$ |
| $SB_{b}$ | Set of public bus station for a specific public bus $b$ |
| $TPB_{p,b}$ | Total trip time for passenger $p$ when he takes public bus $b$ |
| $TB_{p}(x, sb_{i,b}^{pick})$ | Total bus time to reach the pickup station $sb_{i,b}^{pick}$ from its current location $x$ |
| $TW_{p,b}$ | Total passenger $p$ waiting time for bus $b$ |
| $\alpha$ | Whole trip flexible time |
| $PPB_{p,b}$ | Total bus trip fare for passenger $p$ when he takes the public bus $b$ |
| $z$      | Cost in dollar |
| $TB_{p}$ | Total time of used public bus for the passenger $p$ |
| $PB_{p}$ | Used public bus total fare for the passenger $p$ |
| $A_{b}$  | Strategy set of public bus |
| $I_{b}$  | Preference set of public bus |
| $U_{b}$  | Utility function of public bus $b$ |
| $B_{local}$ | Array containing all buses’ information that met the model conditions |
| $SB_{local}$ | Array containing all stations of buses that met the model conditions |
| $B_{final}$ | Array containing all buses’ information that gives the optimal solution |
| $SB_{final}$ | Array containing all stations of buses that gives the optimal solution |
| $B_{count}$ | Number of buses in $B_{local}$ |
| $y$      | Number of rows in $B_{local}$ in algorithm 2 |

\[
TPB_{p,b} = TV_{c,v}(x, sv_{v,m}^{drop}) + TB_{p,b}(x, sb_{i,b}^{pick})
+ TW_{p,b} + \sum_{sb_{i,b}^{drop}} TB_{p,b}(sb_{i,b}, sb_{i+1,b}) + \alpha
\]

\[
\forall p \in P, b \in B, v \in V, sv_{v,m}^{drop}
and sb_{i,b}^{drop}, sb_{i,b}^{pick} \in S_p, sb_{i,b} \in SB
\]

Here, all mobile nodes are indicated by $V$. Public bus, car ride-sharing, bicycle, and passenger are symbolised by $b$, $c$, $r$, and $p$ respectively. Tables 1–4 indicate all assumed symbols in the study. Figure 1 shows the main components of the GT-MMT, which consists of car ride-sharing, public buses, bicycles, and passengers. The passenger, which is the element that is served in the proposed model, is denoted by $P = \{1, 2, \ldots p\}$.

In the next subsections, we show in detail the equations for calculating the travel time and fare of the three transportation modes.

### 3.1 Public bus transportation

Here the public bus set is represented as $B = \{1, 2, 3, \ldots b\}$, and the set of public bus stations as $SB = \{sb_{1,1}, sb_{2,1}, \ldots, sb_{b,1}\}$. Table 1 shows all parameters of the public bus.

Equation (1) calculates the bus time period $TPB_{p,b}$ for a passenger $p$ when he takes a bus $b$ during his trip.
The time in Equation (1) is calculated by adding some values together. First, the period $TV_{v,p}(x,sv_{dr}^{\text{drop}})$ for the current vehicle to reach the passenger drop-off station $sv_{dr}^{\text{drop}}$ from its current location $x$. Second, the total bus time $TB_{b,p}$ to reach the pickup station $sb_{i,b}^{\text{pick}}$ from its current location $x$. Third, the passenger $p$ waiting time $TW_{p,b}$ for the bus $b$ in the pickup station $sb_{i,b}^{\text{pick}}$. Fourth, the summation of the time from the pickup station $sb_{i,b}^{\text{pick}}$ until the drop-off station $sb_{i,b}^{\text{drop}}$. Finally, the unexpected time $\alpha$ from the beginning to the end due to many possible factors on the route, such as accident.

### Table 2: Car ride-sharing notation and definition

| Notation | Definition |
|----------|------------|
| $c$ | Car ride-sharing identification |
| $C$ | Car ride-sharing set |
| $ca_c$ | Number of seats in car ride-sharing $c$ |
| $ca_c(t)$ | Number of seats available on car ride-sharing $c$ at time $t$ |
| $sc_{ij}$ | Car ride-sharing station identification $j$ of car $c$ |
| $sc_{ij}^{\text{orig}}$ | Origin station $j$ for car $c$ in the trip |
| $sc_{ij}^{\text{dest}}$ | Destination station $j$ for car $c$ in the trip |
| $sc_{ij}^{\text{pick}}$ | Pickup car ride-sharing station |
| $sc_{ij}^{\text{drop}}$ | Drop-off car ride-sharing station |
| $SC$ | Set of the car ride-sharing station for all $c \in C$ |
| $SC_c$ | Set of the car ride-sharing station for a specific car $c$ |
| $TPC_{p,c}$ | Total trip time for passenger $p$ when he takes car $c$ |
| $TPC_{c,p}(x,sc_{ij}^{\text{pick}})$ | Total car time to reach the pickup station $sc_{ij}^{\text{pick}}$ from its current location $x$ |
| $TW_{p,c}$ | Total passenger $p$ waiting time for car $c$ |
| $\eta$ | Whole trip flexible time |
| $PPC_{p,c}$ | Total car ride-sharing trip fare for passenger $p$ when he takes car $c$ |
| $z$ | Cost in dollar |
| $TC_c$ | The total trip time for car $c$ |
| $PC_c$ | The total trip cost for car $c$ |
| $TC_p$ | Total time of car ride-sharing for the passenger $p$ |
| $PC_p$ | Total fare of car ride-sharing for passenger $p$ |
| $A_c$ | Strategy set of the car ride-sharing |
| $I_c$ | Preference set of the car ride-sharing $c$ |
| $U_c$ | Utility function of car ride-sharing $c$ |
| $\sigma$ | Acceptable detour distance in the trip |
| $C_{\text{local}}$ | Array containing all cars’ information that met the model conditions |
| $SC_{\text{local}}$ | Array containing all stations of cars that met the model conditions |
| $C_{\text{final}}$ | Array containing all cars’ information that gives the optimal solution |
| $SC_{\text{final}}$ | Array containing all stations of cars that gives the optimal solution |
| $C_{\text{count}}$ | Number of cars in $C_{\text{local}}$ |
| $y$ | Number of rows in $C_{\text{local}}$ in algorithm 3 |
In the city of Ottawa, the bus trip fare is equal to 3.55 Canadian dollars for every 90 min. After 90 min, the passenger needs to pay for another bus ticket. It is essential to note that here we assume the price is the same for all ages and at any time or day.

### 3.2 Car ride-sharing transportation

Here, the car ride-sharing set is represented as $C = \{1, 2, 3, \ldots, c\}$, and the set of car ride-sharing satiations as $SC = \{sc_1, sc_2, \ldots, sc_c\}$. Table 2 shows all parameters of the car ride-sharing.

Equation (4) calculates the car ride-sharing time $TPC_{p,c}$ for a passenger $p$ when he takes a car ride-sharing $c$ in his trip.
TABLE 4  Trip notation and definition

| Notation | Definition |
|----------|------------|
| p        | Passenger id |
| P        | Passenger set |
| S        | Station set |
| S_p      | Set of stations that the passenger has passed through on it |
| n_p      | Number of the riders |
| O        | Origin set for all players |
| D        | Destination set for all players |
| o_p      | Origin for the passenger p |
| d_p      | Destination for the passenger p |
| V        | Set of all vehicles in the simulation |
| V_p      | Set of the vehicles ride by the passenger p |
| A_p      | Strategy set of the passenger p |
| I_p      | Preference set of passenger p |
| U_p      | Utility function of passenger p |
| w_max    | Maximum passenger waiting time |
| P_max    | Maximum price |
| T_max    | Maximum period |
| TV(x,sv,m) | Trip notation and definition |
| TV_c_p(x,sv,m) | Period for the current vehicle to reach the passenger drop-off station sv,m from its current location x |
| TP_c_p   | Total trip time for the passenger p |
| TP_p     | Total trip price for the passenger p |

\[
TPC_{p,c} = TV_{c,p}(x,sv_{c,m}) + TC_{c,p}(x,sc_{j,c}^{pick}) + TW_{p,c} + \sum_{j=1}^{d_{drop}} TC_{c,p}(sc_{j,c},sc_{j+1,c}) + \eta
\]

The time in equation \(4\) is calculated by adding some values together. First, the period \(TV_{c,p}(x,sv_{c,m})\) for the current vehicle to reach the passenger drop-off station \(sv_{c,m}\) from its current location \(x\). Second, the total car ride-sharing time \(TC_{c,p}\) to reach the pickup station \(sc_{j,c}^{pick}\) from its current location \(x\). Third, the passenger \(p\) waiting time \(TW_{p,c}\) for the car \(c\) in the pickup station \(sc_{j,c}^{pick}\). Fourth, the summation of the time from the pickup station \(sc_{j,c}^{pick}\) until the drop-off station \(sc_{j,c}^{drop}\). Finally, the unexpected time \(\eta\) from the beginning to the end due to many reasons may happen on the car way, such as accident. \(\forall p \in P, c \in C, v \in V, sv_{c,m}^{drop}\) and \(sc_{j,c}^{drop}, sc_{j,c}^{pick} \in S_p, sc_{j,c} \in SC\).

3.3 Bicycle transportation

Here, the bicycle set is represented as \(R = \{1, 2, 3, \ldots, r\}\) and the set of bicycle stations as \(SR = \{sr_1, sr_2, \ldots, sr_k\}\). Table 3 shows all the bicycle parameters.

Equation \(7\) calculates the bicycle time \(TPR_{p,r}\) for a passenger \(p\) when he takes a bus \(b\) in his trip.

\[
TPR_{p,r} = TV_{c,p}(x,sv_{c,m}) + TR_{r,p}(x,sr_{k,r}^{pick}) + TW_{p,r} + \sum_{k=1}^{d_{drop}} TR_{r,p}(sr_{k,r},sr_{k+1,r}) + \beta
\]

The time in Equation \(7\) is calculated by adding some values together. First, the period \(TV_{c,p}(x,sv_{c,m})\) for the current vehicle to reach the passenger drop-off station \(sv_{c,m}^{drop}\) from its current location \(x\). Second, the total bicycle time \(TR_{r,p}\) to reach the pickup station \(sr_{k,r}^{pick}\) from its current location \(x\). Third, the passenger \(p\) waiting time \(TW_{p,r}\) for the bicycle \(r\) in the pickup station \(sr_{k,r}^{pick}\). Fourth, the summation of the time from the pickup station \(sr_{k,r}^{pick}\) until the drop-off station \(sr_{k,r}^{drop}\).
Finally, the unexpected time $\beta$ from the beginning to the end due to many possible factors on the route, such as accident. $v \in V$ where $v$ is the current passenger vehicle, and $V$ represents the set of all vehicles in the model (car-ride-sharing, public bus, and bicycle). Also, the drop-off station of the current vehicle $s_{v,m}$, the drop-off station for the passenger $s_{k,r}^{drop}$, and the pickup station $s_{k,r}^{pick}$ for the passenger $p$ belong to $S_p$. Here, $s_{k,r}^{pick}$ represents the bicycle station for bicycle $r$ and belongs to the set of bicycle $SR$. If the bicycle is the first vehicle on the trip, then the period $TV_{v,p}(x,s_{v,m})$ is equal to zero. So, the Equation (7) becomes:

$$TPR_{p,r} = TR_{r,p}(x, s_{k,r}^{pick}) + TW_{p,r}$$

$$+ \sum_{s_{k,r}^{drop}} TR_{r,p}(s_{k,r}^{drop}, s_{k+1,r}^{drop}) + \beta$$

The trip cost, calculated by Equation (9), is equal to $\gamma$ dollars multiplied by the total time of the bicycle trip to reach the passenger’s last station ($T_{kp}$) per 1 min:

$$PPR_{kp} = \gamma \text{dollars} \times T_{kp}/\text{minute}$$

4 | Game Theory Multi-Mode Transportation (GT-MMT) Optimization Model

In this section, we discuss the game model type which is used in this work. In addition, the objective functions and constraints to be applied in the GT are explained. Finally, we illustrate the algorithms to calculate the utility function for all players. Figure 2 shows the main interaction between the players in the model. The car-ride-sharing, public bus, bicycle, and passengers have an account to save data in storage, and they make up the storage unit for the model. Each element has a set of parameters that contribute to the implementation of the logarithm of the proposed model. The parameter is mentioned in the third section and in Tables 1–4.

Here, Stackelberg Equilibrium GT model is used to apply the competition play between the players [29]. In this game, the passenger (the leader) has the priority to select his strategy before the car-ride-sharing, public bus, and bicycle (the followers).

We can express the game theoretic formulation for the GT-MMT as:

$$G = (V, \{A_v\}_{v \in V}, \{I_v\}_{v \in V}, \{U_v\}_{v \in V})$$

FIGURE 1 Game theory multi-mode transportation (GT-MMT) model overview

FIGURE 2 The main inputs and outputs parameters in game theory multi-mode transportation (GT-MMT) model
1. $V$ is a set of the players represented as $V = \{ B \cup C \cup R \cup P \}$

2. $A_v$ is a finite set of strategies for the player (a public bus, car ride-sharing, and bicycle) $v \in V$. Let $A_p$ be the strategy set for passenger and $A_b$, $A_c$ and $A_r$ be the strategy set for the public bus, car ride-sharing, and bicycle, respectively.

   The strategies of each passenger are determined by the type of the transportation that should be taken on his trip to arrive at his last destination. Also, the strategy is dependent on whether the passenger needs to change the form of transportation at each station based on the updated information he receives from all the vehicles. As for car ride-sharing and public buses, they move from station to station and turn left and right in all directions. Also, the roads are designated for them to serve passengers. While bicycles do not differ much from cars and buses, they do not need to pass through stations to deliver the passenger to their destinations. Also, bicycles can travel down their own paths or paths that pedestrians usually take.

3. $I_i$ denotes the player preferences and information. Each player has many parameters and variables such as identification, stations, current positions etc. $I_p$, $I_b$, $I_c$, and $I_r$ represent the preferences of the passenger, the public bus, the car ride-sharing, and the bicycle, respectively.

4. $U_v$ is the utility function for each player where $v \in V$. The utility function $U_p$, $U_b$, $U_c$, and $U_r$ refers to the profit and loss when the strategies are applied for the passenger, the public bus, the car ride-sharing, and the bicycle, respectively. In other words, $U_v: S_1 \times \cdots \times S_n \rightarrow N$, where $N$ is a set of real numbers. In GT-MMT, the utility function for the passenger is to minimise the trip cost (the trip fee and time), the utility function of the car ride-sharing and the public bus is to maximise the number of passengers, and the utility function of the bicycle is to minimise its availability.

   The user has seven different options to select the transportation mode on his trip as we can see from the algorithm 1. Algorithms 2–8 are applied to determine the best combination of transportation methods for the optimal cost (price and time) based on the GT equations and the passenger's preference.

   According to the methodology of the car ride-sharing, the public bus, and the bicycle in the previous section, the cost (time and price fare) optimisation problem by applying the GT concept is expressed as:

$$U_p = \text{minimize}\{TP_p, TT_p\} \quad \text{subject to:}$$

$$U_b = \text{maximize}\{ca_b\} \quad \text{subject to:}$$

$$U_c = \text{maximize}\{ca_c\} \quad \text{subject to:}$$

$$U_r = \text{minimize}\{ca_r\} \quad \text{subject to:}$$

$$d_p \in SB_b \quad \text{(15)}$$

$$d_p \in SC_c \quad \text{(16)}$$

$$d_p \in SR_r \quad \text{(17)}$$

$$ca_b(t) \leq ca_b \quad \text{(18)}$$

$$ca_c(t) \leq ca_c \quad \text{(19)}$$

$$a_{\nu_k}(t) = \begin{cases} 1, & \text{if the bicycle is available} \\ 0, & \text{otherwise} \end{cases} \quad \text{(20)}$$
\[
\sigma \cdot D(o_p, d_p) \geq TD(o_p, d_p)
\]

\[
TV_{c,p}(x, sv_{o,m}^{drop}) \leq TB_{b,p}(x, s_{t,p}^{pick})
\]

\[
TV_{c,p}(x, sv_{o,m}^{drop}) \leq TC_{c,p}(x, sc_{j,p}^{pick})
\]

\[
TV_{c,p}(x, sv_{o,m}^{drop}) \leq TR_{c,p}(x, sr_{k,p}^{pick})
\]

\[
TB_{b,p}(x, s_{t,p}^{pick}) \leq w_{max}
\]

\[
TC_{c,p}(x, sc_{j,p}^{pick}) \leq w_{max}
\]

\[
TR_{c,p}(x, sr_{k,p}^{pick}) \leq w_{max}
\]

\[
TP_{b,p} \leq T_{max}
\]

\[
TPC_{c,p} \leq T_{max}
\]

\[
TPR_{p,r} \leq T_{max}
\]

\[
T_{P} \leq T_{max}
\]

\[
T_{P} \leq P_{max}
\]

Equations (11–35) are the objective function of the GT-MMiT. According to the GT concept, the lowest cost is chosen so that players can guarantee their minimum losses and maximum profit. The passenger \( p \) uses some strategies (preferences) to satisfy his goal. From Equation (11), the utility function of the passenger \( U_{p} \) is evaluated by minimising the total price \( TP_{p} \) and the total trip time \( TT_{p} \). From Equations (12) and (13), the utility function \( U_{b} \) of the public bus \( b \)
and the utility function \( U_c \) of the car ride-sharing c are evaluated by maximising the number of passengers in both. Finally, from Equation (14), the utility function \( U_r \) of the bicycle minimises the availability of the bicycle. Some constraints must be taken into consideration while evaluating this model. First (constraints 15–17), the destination \( d_p \) for the passenger \( p \) must be one of the stations in the vehicle station set. Second (constraints 18–20), the number of passengers (\( n_p \)) on the same trip must not exceed the capacity of the vehicle. \( c_a, c_b \) represent the car ride-sharing and the public bus capacity, respectively. As for the bicycle, it must be available at its station for the passenger to be able to use it on his journey. If it is available, the \( \alpha_{tr}(t) \) is equal to 1; otherwise, the value is zero. Third (constraint 21), the result of multiplying the time allowed to change the vehicle's path \( \sigma \) and the shortest distance between the passenger's departure point \( o_p \) and the point of his last arrival \( d_p \) must be greater than or equal to the total distance of the passenger's trip \( TD \). Fourth (constraints 22–24), the time the vehicle takes \( TV_{vp}(x, sv_{mp}^{drop}) \) or whether it is the public bus, the car ride-sharing, or the bicycle, to reach the next stop, must be equal to or less than the time taken for the other vehicle to travel from its current location to the passenger's pickup station \( TB_{vp}(x, sv_{mp}^{pick}) \) or \( TC_{vp}(x, sv_{mp}^{pick}) \) or \( TR_{vp}(x, sv_{mp}^{pick}) \). Fifth (constraints 25–27), the time that the vehicle takes to

---

**Algorithm 4: Bicycle trip cost**

Input: \( R, r, SR, SR, sr_{orig}^{ori}, sr_{dest}^{ori}, o_p, d_p, w_{max}, P_{max}, T_{max}, A_r, I_r \)  
Output: \( \langle R_{final}, SR_{final}, TT_p, TP_p \rangle \)  
Data: Finds the best set of bicycle \( r \) for each passenger \( p \in P \)

1. for \( b = 1 \) to \( L \)  
2. if \( \alpha_{tr}(t) = 1 \) and \( TR_{vp}(x, sv_{mp}^{pick}) \leq w_{max} \) then  
3. for \( k = 1 \) to \( d_p \)  
4. \( TPR_{vp,r} = TV_{vp}(x, sv_{mp}^{drop}) + TR_{vp}(x, sv_{mp}^{pick}) + sr_{vp}^{drop} + \sum_{sr_{vp}^{drop}} TR_{vp}(sr_{jvp}, sc_{i+1}^{vp}) + \beta \)  
5. \( PPR_{vp} = \) y dollars \( \times T_{vp}/\text{minute} \)  
6. if \( \sigma \times D(o_p, d_p) \geq T_{max} \) and \( PPR_{vp,r} \leq T_{max} \) then  
7. \( R_{local}[y][0] \leftarrow k \)  
8. \( R_{local}[y][1] \leftarrow TPR_{vp,r} \)  
9. \( R_{local}[y][2] \leftarrow PPR_{vp,r} \)  
10. \( SR_{local}[k] \leftarrow s_{k,p} \)  
11. \( y + , \)  
12. \( R_{count} + + \)  
13. else  
14. GO to line 1  
15. else  
16. GO to line 1  
17. Find group of bicycles with the lowest cost (price and time) from \( R_{local} \)  
18. Result \( \langle R_{final}, SR_{final} \rangle \)  
19. for \( n = 0 \) to \( C_{count} (STEP 1) \) do  
20. \( TT_p = TT_p + C[n][1] \)  
21. for \( n = 0 \) to \( R_{count} \) (STEP 1) do  
22. \( TP_p = TP_p + B[n][2] \)  
23. return \( \langle R_{final}, SR_{final}, TT_p, TP_p \rangle \)

---

**Algorithm 5: Bus and car ride-sharing combination trip cost**

Input: \( B, b, SB, SB, ca_b, sh_{b}^{ori}, sh_{b}^{dest}, o_p, p, d_p, w_{max}, P_{max}, T_{max}, A_b, I_b, C_b, SC, SC_C, CA_c, SC_f, sc_{dest}^{ori}, sc_{dest}^{ori}, A_c, I_c \)  
Output: \( \langle B_{final}, C_{final}, SB_{final}, SC_{final}, TT_p, TP_p \rangle \)  
1. call algorithm 2 and algorithm 3  
2. Find the optimal combination of public buses and cars ride-sharing which satisfies the utility functions of the players.  
3. for \( j = 0 \) to \( B_{count} \) (STEP 1) do  
4. \[ TP_{B,b} = TP_{B,b} + B_{final}[j][1] \]  
5. for \( j = 0 \) to \( C_{count} \) (STEP 1) do  
6. \[ PP_{C,b} = PP_{C,b} + C_{final}[j][2] \]  
7. for \( j = 0 \) to \( C_{count} \) (STEP 1) do  
8. \[ TP_{C,C} = TP_{C,C} + C_{final}[j][1] \]  
9. for \( j = 0 \) to \( B_{count} + B_{count} \) (STEP 1) do  
10. \[ PP_{C,C} = PP_{C,C} + C_{final}[j][2] \]  
11. \[ TT_p = TP_{B,b} + TP_{C,b} \]  
12. \[ TP_p = PP_{B,b} + PP_{C,b} \]  
13. return \( \langle B_{final}, C_{final}, SB_{final}, SC_{final}, TT_p, TP_p \rangle \)

---

**Algorithm 6: Bus and bicycle combination trip cost**

Input: \( B, b, SB, SB, ca_b, sh_{b}^{ori}, sh_{b}^{dest}, o_p, d_p, w_{max}, P_{max}, T_{max}, A_b, I_b, R, r, SR, SR, sr_{b}^{ori}, sr_{b}^{dest}, A_r, I_r \)  
Output: \( \langle B_{final}, R_{final}, SB_{final}, SR_{final}, TT_p, TP_p \rangle \)  
1. call algorithm 2 and algorithm 4  
2. Find the optimal combination of public buses and bicycle which satisfies the utility functions of the players.  
3. for \( j = 0 \) to \( B_{b} \) (STEP 1) do  
4. \[ TP_{B,b} = TP_{B,b} + B_{final}[j][1] \]  
5. for \( j = 0 \) to \( C_{count} \) (STEP 1) do  
6. \[ PP_{B,b} = PP_{B,b} + B_{final}[j][2] \]  
7. for \( j = 0 \) to \( C_{count} \) (STEP 1) do  
8. \[ TP_{R,r} = TP_{R,r} + R_{final}[j][1] \]  
9. for \( j = 0 \) to \( R_{count} \) (STEP 1) do  
10. \[ PP_{R,r} = PP_{R,r} + R_{final}[j][2] \]  
11. \[ TT_p = TP_{B,b} + TP_{R,r} \]  
12. \[ TP_p = PP_{B,b} + PP_{R,r} \]  
13. return \( \langle B_{final}, R_{final}, SB_{final}, SR_{final}, TT_p, TP_p \rangle \)
move from its current location to the passenger’s location is equal to or less than the maximum waiting time \( w_{\text{max}} \). Here, \( T_{B_{b}}; \text{pick}_{i}; \text{p} \), \( T_{C_{c}}; \text{pick}_{j}; \text{p} \), and \( T_{R_{r}}; \text{pick}_{k}; \text{p} \) is the time from the current location of the bus, the car ride-sharing, and the bicycle, respectively, to the pickup station. Sixth (constraints 28–33), the total time or price for the passenger \( p \) when he takes any type of transportation must be less than or equal to the threshold value \( (T_{\text{max}} \text{ and } P_{\text{max}}) \). Finally, the total trip time \( T_{T_{p}} \) and the total trip price \( T_{P_{p}} \) must be less than or equal to the maximum total time \( T_{\text{max}} \) and the maximum total price \( P_{\text{max}} \), respectively.

The following equation calculates the total time trip for the passenger:

\[
T_{T_{p}} = \sum_{d_{b}^{\text{rop}}; i_{p}} T_{P_{B_{p},b}} + \sum_{d_{c}^{\text{rop}}; j_{p}} T_{P_{C_{c},c}} + \sum_{d_{r}^{\text{rop}}; k_{p}} T_{P_{R_{r},r}} \tag{36}
\]

Equation (37) below computes the total trip fare for the passenger:

\[
T_{P_{p}} = \sum_{d_{b}^{\text{rop}}; i_{p}} P_{P_{B_{p},b}} + \sum_{d_{c}^{\text{rop}}; j_{p}} P_{P_{C_{c},c}} + \sum_{d_{r}^{\text{rop}}; k_{p}} P_{P_{R_{r},r}} \tag{37}
\]

Algorithms 1 to 8 for the GT-MMT model are based on the following steps:

1. The passenger can get an information update of all players within the network range every 2 minutes.

2. The passenger can choose his preference as we can see from algorithm 1.

3. All algorithms from 2 to 8 work with the same idea, but the difference is in the type of transportation that the passenger prefers to take on his journey. The basic idea is that the model chooses a specific means of transportation or a group of vehicles that brings the most appropriate cost and most suitable period for the passenger. The time and cost are arranged from lowest to highest for all means of transportation within the scope of the passenger’s choice. Then, the optimization equations work on the outcome to choose the appropriate transportation mode for the passenger. The result is based on Equations (1)–(37) and the concept of the Stackelberg equilibrium model.

(a) Algorithm 2: The passenger prefers using the public bus only. The result is based on Equations (1)–(3), (10)–(12), (15), (18), (22), (25), (28), (31), (34)–(37).

(b) Algorithm 3: The passenger prefers using the car ride-sharing only. The result is based on Equations (4)–(6), (10), (11), (13), (16), (19), (21), (23), (26), (29), (32), (34)–(37).

(c) Algorithm 4: The passenger prefers using the bicycle only. The result is based on Equations (7)–(11), (14), (17), (20), (24), (27), (30), (33)–(37).
Bus, car ride-sharing and bicycle combination trip cost

(d) Algorithm 5: The passenger prefers using the public bus and the car ride-sharing. The result is based on Equations (1)–(3), (10)–(12), (15), (18), (22), (25), (28), (31), (34), (35), (4)–(6), (10), (11), (13), (16), (19), (21), (23), (26), (29), (32), (34)–(37).

Algorithm 6: The passenger prefers using the public bus and the bicycle. The result is based on Equations (1)–(3), (10)–(12), (15), (18), (22), (25), (28), (31), (34), (35), (7)–(10), (14), (17), (20), (24), (27), (30), (33)–(37).

Algorithm 7: The passenger prefers using the car ride-sharing and the bicycle. The result is based on Equations (4)–(6), (10), (11), (13), (16), (19), (21), (23), (26), (29), (32), (34), (35), (7)–(10), (11), (14), (17), (20), (24), (27), (30), (33)–(37).

Algorithm 8: The passenger prefers using the public bus, the car ride-sharing, and the bicycle. The result is based on Equations (1)–(37).

4. During the ride, the passenger continues to receive an update of the information so that if there is another vehicle that meets his requirement more than the current vehicle, he can disembark from the current vehicle and embark on the next vehicle from its station. This process can continue until the passenger reaches his final destination. From the description of our proposed GT-MMT, we can see that the complexity is in order of O (B, C, R, P), where B is the number of buses, C is the number of cars that are ride-sharing, R is the number of bicycles, and P is the number of passengers.

5 | SIMULATION AND RESULTS

Figure 3 presents a partial map of the city of Ottawa with three transportation forms (bus, car ride-sharing, and bicycle) to evaluate our GT-MMT model.

![Figure 3](image)

**FIGURE 3** Case study: overview of public bus, car ride-sharing and bicycle station in the city of Ottawa

5.1 | Simulation setup

Simulation setups are presented in Tables 5 and 6. To obtain part of the Ottawa map, we used Simulation of Urban Mobility (SUMO), and to set the route, we used OMNET++ programme. By using SUMO, the mobility traffic scenario is set-up randomly. This simulation measures the movement of car ride-sharing, public buses, and bicycles in the city of Ottawa from 12:00 AM to 11:59 PM. We compare GT-MMT performances with algorithm (without GT-MMT) in which we focus only on the availability of transportation forms without any update to the transportation system after the last passenger. For example, there is no update of the available seats in car ride-sharing, bus, or bicycle which could reduce the waiting time and the trip time.

To evaluate our model, we use OMNET++ to build the proposed model here. By extending Inet and Veins (SUMO and OMNET ++) modules, which are written in C++, the module interfaces and protocol messages have been implemented. Veins is used to realise the transportation mobility environment and Inet framework to build a Wifi interface. Moreover, we connect Veins with SUMO to create the random mobility scenario.

5.2 | Experiment evaluations and results discussion

By using the Monte Carlo technique to find the average value after 10,000 running times, our simulations are conducted for over 10,000 passengers. This simulation generates the random movement of passengers, car ride-sharing, public buses, and bicycles in the city of Ottawa for a duration of 24 h.

| TABLE 5 | SUMO parameters |
| --- | --- |
| Road network | NETCONVERT |
| Vehicle type | Car, public bus, bicycle |
| Time period to simulate | 12:00 AM–11:59 PM |
| Time set length | 1 min |
| Number of the stations | 10 |
| Simulation area dimension | 20 × 20 km |
| The integration method | Euler update |

| TABLE 6 | Simulation parameters |
| --- | --- |
| Number of cars | 4 |
| Number of buses | 50 |
| Number of bicycles | 10 |
| Number of persons | 10,000 |
| Number of stations | 10 |
| Simulation period | 12 AM–11:59 PM |
Figure 4 shows the average trip time and the average trip price variation for the GT-MMT model for the whole day. In this figure, the average trip price is constant and in the lowest value from midnight until 6:00 AM, from 10:00 to 11:00 AM, and from 2:00 PM until midnight. However, this value increases between 6:00 and 9:00 AM and also between 11:00 AM and 1:00 PM. For the average trip time, the highest values are recorded from 4:00 to 6:00 AM and from 7:00 to 8:00 PM. We note that the time is significantly decreased starting at 6:00 AM until it reaches the lowest value at 10:00 AM. However, the time substantially increases after midday before it becomes stable between 4:00 and 6:00 PM.

Figure 5 and Figure 6 show the comparison of the average trip time and the average trip fare between the multi-mode transportation system based on GT and another model of multi-mode transportation that does not use the GT approach.

From Figure 5, we note that the price is the same from midnight until 6:00 AM in the GT-MMT model and in the model without GT-MMT. After six o’clock, the price for GT-MMT starts to increase slightly until eight o’clock in the
morning before decreasing. The price increases at 11 o'clock and then begins to decrease at 2:00 PM. The value remains almost constant for the rest of the day at the lowest cost compared with the model without GT-MMT. In Table 7, the average trip price-saving rate between the GT-MMT model and model without GT-MMT is zero at the beginning of the day and reaches $70 from 2:00 PM until midnight. However, the savings rate between the two models is almost constant and equal to 23.3.

Figure 6 represents the time trip variation value for the two models. We note that the average time value when the passengers use the GT-MMT and the model without GT-MMT is almost the same from midnight until 6:00 AM. After 6:00 AM, the time slightly drops for the GT-MMT model and then it returns to a slight ascent in the middle of the day before becoming constant at 4:00 PM and continuing at the 20 min to the end of the day. As for the model without GT-MMT, the time increases slightly after six o'clock in the morning and then increases significantly after one o'clock and continues to rise until 4:00 PM. In Table 8, the savings rate of the average trip time between GT-MMT and the model without GT-MMT is zero from midnight until 6:00 AM. After that, it starts to increase significantly until reaching 64.5 min at 4:00 PM.

According to the Stackelberg Equilibrium GT model, every player is trying to achieve the utility function within the model that best meets optimisation. The passengers want to reach their destination for the lowest price and at the least amount of time, but cars and buses want to increase the number of passengers to maximum capacity and bicycle reduces the availability. So:

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**Figure 6** Average trip time variation

**Table 7** Saving rate of average trip price variation with and without game theory multi-mode transportation model (GT-MMT)

| Day Time (hour)       | With GT-MMT ($) | Without GT-MMT ($) | Saving Rate (%) |
|-----------------------|-----------------|--------------------|-----------------|
| 12:00–6:00 AM         | 1.8             | 1.8                | 0               |
| 6:01–9:00 AM          | 2.3             | 3                  | 23.3            |
| 9:01 AM – 2:00 PM     | 2.3             | 3                  | 23.3            |
| 2:01–11:59 PM         | 1.8             | 6                  | 70              |

**Table 8** Saving rate of the average trip time variation with and without the game theory multi-mode transportation model (GT-MMT)

| Day Time (hour)       | With GT-MMT (min) | Without GT-MMT (min) | Saving Rate (%) |
|-----------------------|-------------------|----------------------|-----------------|
| 12:00 –6:00 AM        | 22                | 22                   | 0               |
| 6:01 AM –1:00 PM      | 10.8              | 23                   | 53              |
| 1:01–4:00 PM          | 10.9              | 50                   | 78.2            |
| 4:01–11:59 PM         | 22                | 62                   | 64.5            |
1. In general, the number of passengers and the number of available vehicles are higher during the peak period between 7:00 and 9:00 AM and between 2:00 and 6:00 PM. From the previous example, we can explain why time is at its lowest level as in Figure 4, where the abundance of vehicles and a large number of passengers with the freedom to choose the type or types of transportation leads to a reduction in the average trip time it takes for a passenger to reach his destination.

2. As shown in Figure 5, the total price in the two systems is almost the same at the beginning of the simulation when the traffic of the city is not heavy. However, during morning rush hour, while the average price in the multi-mode system that does not use the GT approach increases significantly, whereas the average price in the GT-MMT increases slightly. We observe that the rise in price is low in the GT-MMT scheme but high in the system without GT-MMT. The savings rate for the total trip price when using GT-MMT is higher than the savings rate when the GT-MMT system is not used, as shown in Table 7.

3. In Figure 6, similar to the trip price value, the trip period time without the GT-MMT model rapidly and continuously increases. In contrast, the trip period time is stable and short in the GT-MMT model. More time is saved when moving from the origin to the destination by using the GT-MMT scheme, as shown in Table 8.

6 | CONCLUSION

Here, we propose and develop a new system of transportation for citizens based on the GT-MMT model. We allow the traveller to enjoy his trip by using a combination of bus, car ride-sharing, and bicycle. Finding the best trip combination is based on the Stackelberg equilibrium GT model, where there is a leader who defines the rules of the game while the rest of the players follow him. Here, the passenger is the leader, and the rest of the vehicles follow the leader to define the strategies followed as well as the utility functions. The utility function varies from player to player within the scope of the same game theory. However, the passenger aims to reach the destination in the lowest possible time and for the lowest price depending on the type of vehicle he prefers to ride in, the bus and the car, and try to have the seats vacant as often as possible, as the bike is not available for this passenger.

The proposed model is simulated through a real scenario in a specific city, which is the city of Ottawa, using OMNET++ and OpenStreetMap tools, also Inet and Veins modules. Simulation results show that the GT-MMT gives riders an optimal trip at a lower cost and faster time, which increases the user satisfaction regarding the transportation service in the city.

For future work, we plan to improve the proposed algorithm by considering the user experience to improve the transportation service further and make mobility in the city more efficient, convenient, and enjoyable.

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