Writing Spin in a Quantum Dot with Ferromagnetic and Superconducting Electrodes

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Abstract

We propose an efficient mechanism for the operation of writing spin in a quantum dot, which is an ideal candidate for qubit. The idea is based on the Andreev reflection induced spin polarization (ARISP) in a ferromagnetic / quantum-dot / superconductor system. We find that on the resonance of Andreev reflection, the spin polarization of quantum dot strongly depends on the magnetization of ferromagnetic electrode, and the sign of the spin polarization is controllable by bias voltage. In the presence of intradot Coulomb interaction, we show that ARISP effect can still survive as long as the charging energy $U$ is comparable to the superconducting gap $\Delta$. Detailed conditions and properties of ARISP are also discussed.

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Qubit is the basic unit in the achievement of quantum computing. Among many quantum two-level systems, the spin of quantum dot (QD) is an ideal candidate for such purpose [1]. Firstly, QD fabricated in semiconductor 2DEG can be well controlled by metallic gates, parameters such as resonant levels, tunnel barriers, and charging energy are experimentally tunable. Second, the spin coherent time is extremely long in semiconductors (exceeding 100ns at low temperature [2]), which is orders larger than charge coherent time. Moreover, due to 0D nature of QD, many spin-flip mechanisms are further suppressed, resulting in an even longer lifetime of spin in QD [3].

One of the challenge to exploit the spin of QD as qubit is to efficiently control the local spin polarization in QD. A natural idea is to apply a magnetic field and induce the spin polarization by Zeeman splitting [4]. But the scheme contains practical difficulties because the field is required to be of the order of Tesla but confined within the scale of QD. An alternative idea is to attach QD to ferromagnetic electrodes [4,5]. In principle, the asymmetry between two spin bands in the electrodes may induce spin-dependent renormalization and broadening to the resonant level of QD and lead to spin polarization. However, such mechanism requires relatively large magnetization in electrodes and relatively strong coupling between QD and electrodes. More important, the sign of spin polarization in QD can not be controlled electrically.

It is the purpose of this paper to present an efficient mechanism for writing spin in a QD with ferromagnetic (F) and superconducting (S) electrodes. The idea is motivated by the fact that F can provide spin-polarized electrons while S can only accept Cooper pair which is spin singlet. We shall show below that on the resonance of Andreev reflection (AR) [6–9], the spin polarization of QD strongly depends on the magnetization of F, and the sign of spin polarization is controllable by the bias voltage. The effect can survive even in the presence of strong Coulomb interaction, as long as the charging energy of QD is comparable to the superconducting gap in S.

non-interacting case- To see the physics clearly, we start with the non-interacting ferromagnetic / quantum-dot / superconductor (F-QD-S) system modelled by the following Hamiltonian, \( H = H_F + H_S + H_{dot} + H_T \), in which \( H_F = \sum_{k\sigma} (\varepsilon_k - \sigma h) a_{k\sigma}^\dagger a_{k\sigma} \) is for the F electrode, \( H_S = \sum_{p\sigma} \varepsilon_p b_{p\sigma}^\dagger b_{p\sigma} + \sum_{p}(\Delta b_{p\uparrow}^\dagger b_{p\downarrow} + \text{H.c.}) \) for the S electrode, \( H_{dot} = \sum_{\sigma} E_{\sigma} c_{\sigma}^\dagger c_{\sigma} \) is for the noninteracting QD, and \( H_T \) describes the tunnel couplings between QD and electrodes. We introduce the retarded, advanced, and lesser Green function in the Nambu representation:
\[ G^{r,a,<} \equiv \int dt \, e^{i\omega t} \left( \langle \langle c^\dagger_r(t)c_r(0) \rangle \rangle^{r,a,<} \langle \langle c^\dagger_a(t)c_a(0) \rangle \rangle^{r,a,<} \right). \] (1)

The occupation numbers \( \langle n_r \rangle \equiv \langle c^\dagger_r c_r \rangle \) and \( \langle n_a \rangle \equiv \langle c^\dagger_a c_a \rangle \) are related to the integral of \( G^< \).

In the non-interacting case considered here, \( G^r, G^< \), and \( G^> \) can be solved exactly in Keldysh formalism as \( G^r = (g_0^{-1} - \Sigma_0)^{-1} \), \( G^< = (G^r)^\dagger \), and \( G^> = G^r \Sigma_0 G^< \), where \( g_0 \) is the Green function of the isolated QD (see Eq. (7)) and \( \Sigma_0 \) the self energy caused by the tunnel coupling with electrodes. In the wide band limit, \( \Sigma_0 \) can be evaluated analytically as \( \Sigma_0 = \Sigma_{F0} + \Sigma_{S0} \) with

\[ \Sigma_{F0}^r = -\frac{i}{2} \begin{pmatrix} \Gamma_{F^\dagger} & 0 \\ 0 & \Gamma_{F^\dagger} \end{pmatrix}, \quad \Sigma_{F0}^< = i \begin{pmatrix} \Gamma_{F^\dagger} f(\omega - V) & 0 \\ 0 & \Gamma_{F^\dagger} f(\omega + V) \end{pmatrix}, \]

\[ \Sigma_{S0}^r = -\frac{i}{2} \Gamma_S \rho^r(\omega) \begin{pmatrix} 1 & -\frac{\Delta}{\omega} \\ -\frac{\Delta}{\omega} & 1 \end{pmatrix}, \quad \Sigma_{S0}^< = i \Gamma_S \rho^<(\omega) f(\omega) \begin{pmatrix} 1 & -\frac{\Delta}{\omega} \\ -\frac{\Delta}{\omega} & 1 \end{pmatrix}, \]

in which \( \Gamma_{F^\dagger}, \Gamma_{F^\dagger}, \) and \( \Gamma_S \) are the coupling strengths between QD and electrodes (notice that \( \Gamma_{F^\dagger} \neq \Gamma_{F^\dagger} \)), \( f(\omega) \equiv 1/(e^{\omega/k_B T} + 1) \) is the Fermi function, \( V \) is the bias voltage between F and S [10], \( \rho^r(\omega) \) and \( \rho^<(\omega) \) are the generalized BCS density of states defined as \( \rho^r(\omega) = \frac{|\omega|}{\sqrt{\omega^2 - \Delta^2}} \theta(|\omega| - \Delta) + \frac{\omega}{i\sqrt{\Delta^2 - \omega^2}} \theta(\Delta - |\omega|) \) and \( \rho^<(\omega) = \frac{|\omega|}{\sqrt{\omega^2 - \Delta^2}} \theta(|\omega| - \Delta).

Fig.1 shows the curves of the occupation number \( \langle n_{\sigma} \rangle \) vs the resonant level \( E_0 \equiv E_{\uparrow} = E_{\downarrow} \), with the ratio \( \Gamma_{F^\dagger}/\Gamma_{F^\dagger} = \frac{1}{3} \). (a), (b), and (c) are corresponding to the bias voltage \( V > 0, V = 0 \), and \( V < 0 \), respectively. For \( V = 0 \), there is a step from 0 to 1 in the occupation curve, indicating an electron filling when \( E_0 \) passes the chemical potential \( \mu_F = \mu_S \). Moreover, the curves of \( \langle n_{\uparrow} \rangle \) and \( \langle n_{\downarrow} \rangle \) are almost overlapped. For \( V > 0 \) (\( V < 0 \)), the step is shifted to \( E_0 = \mu_F = V \), and there emerges a resonant dip (peak) pinned at \( E_0 = \mu_S = 0 \). The results can be understood as follows (see also the schematic diagram in the bottom): Since no single particle states are available within the superconducting gap, electron filling to the QD is mainly determined by the F electrode, resulting in a step linked to the chemical potential of F. When the resonant level lines up with the chemical potential of S, however, two-particle process AR may occur, in which a spin\( \uparrow \) and a spin\( \downarrow \) electron in QD can leak into S by forming a Cooper pair and vice versa [11]. As a result, an Andreev dip (Andreev peak) is superposed on the step-like curve. The most remarkable features of AR dip (AR peak) are: (1) The spin polarization of QD, \( m \equiv \langle n_{\uparrow} \rangle - \langle n_{\downarrow} \rangle \), strongly depends on the magnetization of F. (2) The sign of \( m \) is controllable by the bias voltage, \( m > 0 \).
for $V > 0$ and $m < 0$ for $V < 0$. Hereafter, these nontrivial properties of AR resonance is referred to as Andreev reflection induced spin polarization (ARISP). Qualitatively, ARISP effect can be interpreted as follows: For $V > 0$ and $E_0 = \mu_S$, spin↑ and spin↓ electrons in QD tend to form Cooper pair and enter S. Since F can provide more spin↑ electron that spin↓ electron, the spin↓ electron will be depleted by spin↑ electron in the AR process, resulting in a strong spin polarization in QD. For $V < 0$ and $E_0 = \mu_S$, a Cooper pair is converted into a spin↑ and a spin↓ electron in QD. It is much easier for spin↑ electron than spin↓ electron to escape to the empty states of F, resulting in a reversed spin polarization.

For quantitative analysis, we evaluate $\langle n_\sigma \rangle$ near the AR resonance and obtain $\langle n_\uparrow \rangle = 1 - n(p)$ and $\langle n_\downarrow \rangle = 1 - n(-p)$ for $V > 0$, $\langle n_\uparrow \rangle = n(p)$ and $\langle n_\downarrow \rangle = n(-p)$ for $V < 0$, where

$$n(p) \equiv \frac{1}{2}(1-p) \frac{\Gamma_F^2}{4E_0^2(1-p^2) + \Gamma_F^2(1-p^2) + \Gamma_S^2},$$

in which $p \equiv (\Gamma_{F\uparrow} - \Gamma_{F\downarrow})/(\Gamma_{F\uparrow} + \Gamma_{F\downarrow})$ is the magnetization in F and $\Gamma_F \equiv (\Gamma_{F\uparrow} + \Gamma_{F\downarrow})/2$ is the spin averaged coupling strength. Fig.2a shows the AR dip for different magnetization $p$ of F electrode (also true for AR peak if upside down the plot). Notice that the resonance has the Lorentzian line shape with half-width $\sqrt[4]{(\Gamma_F^2(1-p^2) + \Gamma_S^2)/(1-p^2)}$, implying that AR resonance is broadened with the increase of $p$. Fig.2b shows the maximum spin polarization $m_0 = m(E_0 = 0)$ vs $p$ for different coupling strength ratio $r \equiv \Gamma_F/\Gamma_S$. Contrary to the intuition, ARISP effect is most pronounced when the ratio $r$ is small. Especially, in the limit $r \to 0$, $\langle n_\uparrow \rangle_0 = \frac{1}{2}(1 \pm p)$ and $\langle n_\downarrow \rangle_0 = \frac{1}{2}(1 \mp p)$ (upper sign for $V > 0$ and lower sign for $V < 0$). This means that both F and S electrodes are important in the ARISP effect: F provides the asymmetry between two spin categories, while S reinforces the asymmetry through AR process. Another noteworthy feature of ARISP is that the maximum spin polarization is determined by the ratio $r$ and $p$, rather than the magnitudes of $\Gamma_{F\uparrow}$, $\Gamma_{F\downarrow}$, and $\Gamma_S$. Below we shall take into account the intradot Coulomb interaction to investigate whether ARISP can still survive. Due to the symmetry between electron and hole, only the case of $V > 0$ is discussed.

**Interacting case**—To include the Coulomb interaction, we add the term $U n_\uparrow n_\downarrow$ to $H_{dot}$, which makes F-QD-S a strong correlated system. Notice that in the limit $U \to \infty$, double occupation is forbidden in QD, and electron number can not fluctuate by two. Hence AR is completely killed by Coulomb interaction in this limit. If, however, the charging energy $U$ is comparable to the superconducting gap $\Delta$, AR can still occur with the aid of bias voltage. Therefore, techniques for $U \to \infty$ limit (e.g., slave boson method) can not be applied to the
calculation of ARISP. Alternatively, we adopt the equation of motion (EOM) method, which is known to be reliable in the Coulomb blockade regime, and qualitatively correct for Kondo physics [12,13]. Moreover, EOM solution becomes exact in the $U \to 0$ limit. In the EOM approach, one can derive the equation for the retarded Green function $\langle (A(t_1)|B(t_2)) \rangle^r$ by differentiating with respect to $t_1$ or $t_2$, with new Green functions generated in the equation. To close the equation chain, we make the truncation in those Green functions containing two electrode operators in a mean field manner. After some algebra, the equation for $G^r$ can be obtained as

$$AG^r + G^r \tilde{A} = 2N + B + \tilde{B},$$

in which $B \equiv (g_1^{-1} - \Sigma^r_1)U^{-1}$, $A \equiv B(g_0^{-1} - \Sigma^r_0) + \Sigma^r_2$, $\tilde{A}$ or $\tilde{B}$ represents the transpose of $A$ or $B$, $U$ and $N$ are defined as

$$U \equiv \begin{pmatrix} +U & 0 \\ 0 & -U \end{pmatrix}, \quad N \equiv \begin{pmatrix} \langle n_\uparrow \rangle & 0 \\ 0 & \langle n_\downarrow \rangle \end{pmatrix}.$$ 

(6)

$g_0$ and $g_1$ are the bare Green function for the resonances $E_\sigma$ and $E_\sigma + U$,

$$g_0 = \begin{pmatrix} \frac{1}{\omega - E_\uparrow} & 0 \\ 0 & \frac{1}{\omega + E_\downarrow} \end{pmatrix}, \quad g_1 = \begin{pmatrix} \frac{1}{\omega - E_\uparrow - U} & 0 \\ 0 & \frac{1}{\omega + E_\downarrow + U} \end{pmatrix}.$$ 

(7)

$\Sigma^r_0$ and $\Sigma^r_1$ are the corresponding dressing self energies due to tunnel coupling with electrodes. $\Sigma^r_2$ is the self energy contributed by the spin flip in the cotunneling process, which is related to Kondo physics. In the wide band limit, these self energies can be evaluated analytically [14]. Let $\Sigma^r = \Sigma^r_{F1} + \Sigma^r_{S1}$, (find $\Sigma^r_{F0}$ and $\Sigma^r_{S0}$ in Eq.(2) and Eq.(3))

$$\Sigma^r_{F1} = -\frac{i}{2} \begin{pmatrix} \Gamma_{F\uparrow} + 2\Gamma_{F\downarrow} & 0 \\ 0 & \Gamma_{F\downarrow} + 2\Gamma_{F\uparrow} \end{pmatrix},$$

(8)

$$\Sigma^r_{F2} = \begin{pmatrix} \Gamma_{F\downarrow} Q \left( \frac{-\omega - V}{k_B T}, \frac{\omega - V}{k_B T} \right) & 0 \\ 0 & \Gamma_{F\uparrow} Q \left( \frac{-\omega - V}{k_B T}, \frac{\omega - V}{k_B T} \right) \end{pmatrix},$$

(9)

$$\Sigma^r_{S1} = \Gamma_S \begin{pmatrix} s_2(\omega_0) + s_2(\omega_1) + s_2(\omega_3) & s_1(\omega_3) \\ s_1(\omega_3) & s_2(\omega_0) + s_2(\omega_2) + s_2(\omega_3) \end{pmatrix},$$

(10)

$$\Sigma^r_{S2} = \frac{\Gamma_S}{2} \begin{pmatrix} s_2(\omega_1) + s_2(\omega_3) + s_4(\omega_1) - s_4(\omega_3) & s_1(\omega_0) + s_1(\omega_3) + s_3(\omega_0) + s_3(\omega_3) \\ s_1(\omega_0) + s_1(\omega_3) - s_3(\omega_0) - s_3(\omega_3) & s_2(\omega_2) + s_2(\omega_3) + s_4(\omega_3) - s_4(\omega_2) \end{pmatrix},$$

(11)
in which \( s_1(\omega) \equiv -\frac{1}{\pi} J \left( \frac{\omega}{\Delta} \right) \), \( s_2(\omega) \equiv -\frac{1}{\pi} J \left( \frac{\omega}{\Delta} \right) \), \( s_3(\omega) \equiv -\frac{1}{\pi} I \left( \frac{\omega}{\Delta} \right) \), \( s_4(\omega) \equiv -\frac{1}{\pi} J \left( \frac{\omega}{\Delta} \right) \), and \( \omega_0 \equiv \omega, \omega_1 \equiv \omega - E_{\uparrow} - E_{\downarrow} - U, \omega_2 \equiv \omega + E_{\uparrow} + E_{\downarrow} + U, \omega_3 \equiv \omega - E_{\uparrow} + E_{\downarrow} \). The dimensionless functions \( I, J, Q \) are defined as

\[
I(x) \equiv \begin{cases} \frac{\arcsin x}{x\sqrt{1-x^2}}, & |x| < 1 \\ \frac{x}{2\sqrt{x^2-1}} - \frac{1}{|x|\sqrt{x^2-1}} \ln \left( |x| + \sqrt{x^2-1} \right), & |x| > 1 \end{cases} \tag{12}
\]

\[
J(x) \equiv \begin{cases} \frac{\pi}{2} \frac{1}{\sqrt{1-x^2}}, & |x| < 1 \\ \frac{\pi}{2} \frac{-1}{\sqrt{1-x^2}} \text{sign}(x), & |x| > 1 \end{cases} \tag{13}
\]

\[
Q(x, y) \equiv -\frac{1}{2} \left( \frac{1}{e^{x+y}+1} + \frac{1}{e^y+1} \right) + \frac{1}{2\pi} \text{Re} \left[ \psi \left( \frac{1}{2} + \frac{ix}{2\pi} \right) - \psi \left( \frac{1}{2} + \frac{iy}{2\pi} \right) \right], \tag{14}
\]

with \( \psi \) being the digamma function.

As for \( \mathbf{G}^< \), we invoke the stationary condition

\[
\left( \frac{d}{dt} \left[ \sum_i c_i^\dagger c_i \right] \right) = \left( \frac{d}{dt} \left[ \sum_i c_i^\dagger c_i \right] \right) = 0 . \tag{15}
\]

Using Hensenburg equation, one can derive

\[
\int \frac{d\omega}{2\pi} \left[ \mathbf{G}^< \sum_0^\sigma + \mathbf{G}^> \sum_0^\sigma - \mathbf{G}^< \sum_0^\sigma - \mathbf{G}^> \sum_0^\sigma + \frac{E_{\uparrow\downarrow}}{2} \left( \mathbf{G}^< \sigma_3 - \sigma_3 \mathbf{G}^< \right) \right] = 0 , \tag{16}
\]

where \( E_{\uparrow\downarrow} \equiv E_{\uparrow} + E_{\downarrow} + U \) and \( \sigma_3 \) is the 3rd Pauli matrix. We remove the integral over energy and write down the equation for \( \mathbf{G}^< \) as

\[
\left( \sum_0^\sigma + \frac{1}{2} E_{\uparrow\downarrow} \sigma_3 \right) \mathbf{G}^< - \mathbf{G}^< \left( \sum_0^\sigma + \frac{1}{2} E_{\uparrow\downarrow} \sigma_3 \right) = \mathbf{G}^\sigma \sum_0^\sigma - \sum_0^\sigma \mathbf{G}^a , \tag{17}
\]

which can be interpreted as “detailed” stationary condition, i.e., the observables in the energy interval \( \omega \) and \( \omega + d\omega \) are time-invariant in the steady state. We note that Eq.(16) is exact while Eq.(17) is an approximation which neglects high order fluctuations in the presence of strong correlation (for details see [16]). Nevertheless, this approximation is sufficient for the study of ARISP which works in the Coulomb blockade regime. In addition, Eq.(17) becomes exact in the \( U \to 0 \) limit. For comparison, we also try the Keldysh equation \( \mathbf{G}^< = \mathbf{G}^\sigma \sum_0^\sigma \mathbf{G}^a \), and employ the commonly used Ng’s ansatz for \( \Sigma^< \) [15,13]. Although the calculated results are qualitatively agree with each other, the convergence is much poorer in the latter approximation scheme. To sum up, Eq.(5) for \( \mathbf{G}^\sigma \) and Eq.(17) for \( \mathbf{G}^< \) will be applied to the numerical study of ARISP.
Before presenting the numerical results, we qualitatively analyze the physics in the problem. As seen in the non-interacting case, spin polarization occurs on the AR resonance. The conditions for AR resonance are: (1) QD is occupied with even number electrons and (2) a pair of electrons can transfer freely between QD and S. In the noninteracting case, these conditions amount to $\mu_L > \mu_R$ and $E_0 = \mu_R$ for AR dip, or $\mu_L < \mu_R$ and $E_0 = \mu_R$ for AR peak. In the presence of Coulomb interaction, the occupation number of QD is quantized, and AR occurs in the even occupation region. Notice that the occupation of QD is mainly determined by F, let us cut off the S-QD coupling and consider the electron filling in F-QD system. QD has four occupation configurations: $|0\rangle$, $|\uparrow\rangle$, $|\downarrow\rangle$ and $|\uparrow\downarrow\rangle$, with energy $0$, $E_{\uparrow} = \mu_F - \mu_S$ and $E_{\downarrow} = \mu_F + U - 2\mu_F$, respectively. It is energy favorable that QD is empty when $E_0 - \mu_F > 0$, singly occupied when $-U < E_0 - \mu_F < 0$, and doubly occupied when $E_0 - \mu_F < -U$. On the other hand, AR resonance requires energy degeneracy when moving two electrons from QD to S, so that Coulomb blockade is lifted. It is straightforward to write down the equality, $E_{\uparrow} + E_{\downarrow} + U = 2\mu_S$, meaning that a spin $\uparrow$ and a spin $\downarrow$ electron of QD enter S and form a Cooper pair at the chemical potential $\mu_S$. Therefore the conditions for AR resonance are: $E_0 - \mu_S = -\frac{U}{2}$ and $E_0 - \mu_F < -U$ for AR dip, or $E_0 - \mu_S = -\frac{U}{2}$ and $E_0 - \mu_F > 0$ for AR peak. The key point is that electron filling is linked to $\mu_F$ while AR resonance is linked to $\mu_S$, and $\mu_F \neq \mu_S$ in nonequilibrium.

Fig.3a shows the curve of $\langle n_{\sigma}\rangle$ vs $E_0$ for $p = 0.5$. As expected, a spin polarized AR dip is superposed on a step-like pattern. The steps from 0 to $\frac{1}{2}$ and $\frac{1}{2}$ to 1 are located around $E_0 - \mu_F = 0$ and $E_0 - \mu_F = -U$, while AR dip located around $E_0 - \mu_S = -\frac{U}{2}$, which are in coincidence with the above discussion. $\langle n_{\uparrow}\rangle$ and $\langle n_{\downarrow}\rangle$ are nearly equal off the AR resonance. This is because the weak F-QD coupling can only induce small spin-dependent renormalization and broadening to $E_{\uparrow}$ and $E_{\downarrow}$. On the contrast, $\langle n_{\uparrow}\rangle$ and $\langle n_{\downarrow}\rangle$ are dramatically different on the AR resonance, which relies on the ratio of $\Gamma_{F\uparrow}$ to $\Gamma_{F\downarrow}$ rather than their magnitudes. The up-right inset shows the detailed lineshape of AR dip for several $p$. Comparing with Fig.2a, one can see that Eq.(4) derived in the non-interacting case can also describe the ARISP in the presence of Coulomb interaction. In fact, Coulomb interaction plays the role of quantizing the electron number of QD. Coulomb blockade is lifted for the conditions discussed above, under which AR occurs as if through a noninteracting QD. Next, we discuss the choice of charging energy $U$, which is determined by the size of QD. Fig.3b and Fig.3c shows the curves of $\langle n_{\sigma}\rangle$ vs $U$ on the AR resonance ($E_0 - \mu_S = -\frac{U}{2}$). Fig.3b is for the case of $V < \Delta$, in which single particle process is forbidden due to superconducting
gap and AR process dominates. One can see in the plot that the spin polarization is almost independent on $U$ when $U < 2V$, but gradually suppressed when $U > 2V$. The reason is as follows: on the AR resonance, QD favors double occupation when $U < 2V$ and single occupation when $U > 2V$. For double occupation, as shown in Eq.(4), ARISP is determined by the ratio $p$ and $r$, and independent on other parameters. For single occupation, the effective S-QD coupling is greatly suppressed, and therefore ARISP vanishes. One tends to think that increasing the bias voltage $V$ may help to overcome the charging energy $U$. This is true as long as $V < \Delta$. When $V > \Delta$, however, both single particle process and AR process are allowed, and the situation is more complicated. Fig.3c shows $\langle n_\sigma \rangle$ vs $U$ for $V > \Delta$. It turns out that the spin polarization is still independent on $U$ when $U < 2\Delta$, but has a maximum at $U = 2\Delta$, suppressed when $2\Delta < U < 2V$, and gradually vanishes when $U > 2V$. The suppression in the range of $2\Delta < U < 2V$ can be attributed to the onset of single particle process. We note that EOM solution is valid for relatively small $U$, and the suppression of ARISP is probably underestimated when $U > 2\Delta$. Nevertheless, the maximum at $U = 2\Delta$ is reliable, which is related to the singularity in the density of states of S electrode.

In conclusion, we have proposed an efficient mechanism for writing spin in a QD which is based on the ARISP effect in F-QD-S system. The scheme has the advantages that the magnetization of F is not required to be strong and the sign of spin polarization can be controlled in fully electric manner. Calculation shows that the optimal conditions for ARISP are: $\Gamma_F \ll \Gamma_S$, $U = 2\Delta$, $E_0 - \mu_S = -\frac{U}{2}$, and $|V| > U/2$. The properties of ARISP can be described by Eq.(4). In practice, the resonant level $E_0$ can be tuned by gate voltage, and AR resonance can be monitored by the small tunnel current between F and S electrodes.. In the context of intensive research and impressive progress in F/2DEG, S/2DEG, and F/S hybrid structures, the proposed F-QD-S system should be feasible with up-to-date nano-technology. We are looking forward to hearing the relevant experimental response.

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**FIGURE CAPTIONS**

Fig. 1 The occupation number $\langle n_\uparrow \rangle$ (solid) and $\langle n_\downarrow \rangle$ (dotted) vs the resonant level $E_0$ in a non-interacting F-QD-S system. The superconducting gap $\Delta = 1$ is set as energy unit, the bias voltage $V = 0.5, 0,$ and $−0.5$ for (a), (b), and (c), respectively. Other parameters are: $\Gamma_F = 0.01$, $\Gamma_S = 0.1$, $k_B T = 0.02$, and $p = 0.5$. The diagram in the bottom schematically shows the AR resonance in non-interacting F-QD-S system.

Fig. 2 Detailed analysis of the Andreev dip in Fig. 1a: (a) $\langle n_\uparrow \rangle$ (solid) and $\langle n_\downarrow \rangle$ (dotted) vs $E_0$ for different magnetization $p$ of F electrode. (b) $m_0 \equiv (\langle n_\uparrow \rangle - \langle n_\downarrow \rangle)_{E_0=0}$ vs $p$ for different coupling strength ratio $r$.

Fig. 3 (a) The occupation number $\langle n_\uparrow \rangle$ (solid) and $\langle n_\downarrow \rangle$ (dotted) vs the resonant level $E_0$ in an interacting F-QD-S system. The superconducting gap $\Delta = 1$ is set as energy unit, the bias voltage $V = 0.75$, and the charging energy $U = 1.2$. Other parameters are the same as Fig. 1. The up-right inset shows the details of the Andreev dip around $E_0 - \mu_S = -\frac{U}{2}$. The middle inset illustrates the possibility of using ARISP effect to manipulate the spin states of a QD array. (b) and (c) show the curves of $\langle n_\uparrow \rangle$ (solid) and $\langle n_\downarrow \rangle$ (dotted) at $E_0 - \mu_S = -\frac{U}{2}$ vs the charging energy $U$, with the bias voltage $V = 0.5$ and $V = 1.5$, respectively.
Fig.2
Fig. 3

(a) $N=2$

(b) $N=1$

(c) $N=0$

\[ \frac{\Lambda}{n_0} \]

\[ U_0 \]

\[ \frac{\Lambda}{n_0} \]

\[ U \]

\[ p=0 \]

\[ p=0.3 \]

\[ p=0.6 \]

\[ p=0.9 \]

\[ S \]

\[ \uparrow \]

\[ \downarrow \]