Leader-Following Bipartite Formation Control of Multiple Nonholonomic Robot Systems Over Signed Graph

Yupei Liu, Wenjing Xie, Yixin Zhao and Wei Huang*
School of Computer and Information Science, Southwest University, Chongqing, 400715, China
*Corresponding author’s e-mail: weihuang@swu.edu.cn

Abstract. This paper studies the bipartite formation control problem of multiple nonholonomic robot systems over signed graph. To overcome nonholonomic and underactuated control challenges, the hand position output is defined to obtain a linear hand position subsystem. A distributed control law is designed, which uses sliding mode control technology to eliminate the influence of dynamic leader, so that the global index of position formation error converges to an arbitrary small neighborhood of the origin. Then, the zero dynamic equation is computed, and based on it and Lyapunov method, the convergence conditions on controller parameters and on leader’s velocities are derived to guarantee the angle error is bounded and convergent. Finally, a simulation example containing eight robots and one leader is given to demonstrate the effectiveness of the proposed control strategy. In contrast to related results, this paper presents the complete stability analysis and corresponding stability conditions, in addition to the control law and stability of position subsystems.

1. Introduction

In the past few decades, there has been a widespread interest in the cooperative control of multi-robot systems (MRSs) [1]. Formation control problem [2] is a typical cooperative control, aiming to encourage multi-robots to form and maintain a predetermined geometric structure (and simultaneously track a leader).

In most of the literature, the formation control problem of multiple nonholonomic robots was investigated under unsigned graph [3-8], and recently, the problem was studied under signed graph in fewer references [9, 10]. The former is known as traditional formation control, and the latter is referred to as the new bipartite formation control, where all robots are divided into two groups with each group forming desired formation configuration and moving in an opposite direction with another group. For the case of signed graph, both cooperative and competitive relationships are taken into consideration to research the formation control, having a wider application and deserving a further investigation.

Related results on bipartite formation control of nonholonomic robots are based directly upon linear results. Without digon sign-symmetry property, the work [9] reported a distributed control algorithm for double-integrator systems, guaranteeing bipartite consensus and cluster consensus. Considering homogeneous generic linear systems, [10] proposed a linear saturated control law to achieve semi-global bipartite consensus. Both of the two papers [9, 10] finally applied their theoretic results to the hand position systems of nonholonomic robots with the purpose of achieving position bipartite formation, because hand position system can be transformed into linear form. However, the nonholonomic robot is not only nonlinear but also nonholonomic, so the control approach of directly extending linear theoretic
results to the control of nonholonomic robots employed in [9, 10] is not adequate. Furthermore, the boundedness and convergence of orientation of robots were not analysed and not ensured in [9, 10].

The above reviews show that the bipartite formation control problem of nonholonomic robots has not been well solved so far. Therefore, this paper investigates the bipartite formation control problem of nonholonomic robots over signed graph. Similar to the previous works concerning bipartite consensus [9, 10], we first transform the hand position system of nonholonomic robots into a linear form. Then, based on hand position system we designed a distributed control law to realize bipartite formation of position variables of robots, then studied the convergence conditions of internal dynamics of robots by using Lyapunov method. Finally, simulation examples are given to show the effectiveness of the proposed control law. The contribution of this paper is that the proposed control law not only achieves position bipartite formation control of nonholonomic robots but also guarantees the boundedness and convergence of orientations of robots.

2. Problem formulation

In this paper, the multi-robot systems under consideration consist of n followers labelled 1 through n and one leader indexed by 0. All the considered robots and the leader are the classic nonholonomic unicycle ones moving in a planar environment, and their kinematic models can be represented by [11][12]

\[ \dot{x}_i = v_i \cos \theta_i, \quad \dot{y}_i = v_i \sin \theta_i, \quad \dot{\theta}_i = \omega_i, \]

where \( i = \{0, 1, 2, \ldots, n\} \) is the index of follower robot, \( p_i = [x_i, y_i]^T \in \mathbb{R}^2 \) is the position of the robots \( i, \theta_i \in \mathbb{R} \) denotes the orientation of robots \( i \) with respect to the x-axis, and \( v_i, \omega_i \in \mathbb{R} \) represent the linear and angular velocity control inputs of vehicle \( i \), respectively.

Among the robots, an information interaction network containing both cooperative and competitive interactions is modelled by an undirected signed graph \( G(\mathcal{V}, \mathcal{E}, \mathcal{A}) \), where \( \mathcal{V} = \{1, 2, \ldots, n\} \) is the node of agents, \( \mathcal{E} = \{(v_i, v_j) | v_i, v_j \in \mathcal{V}\} \) is an edge set for communication links, and \( \mathcal{A} = [a_{ij}] \in \mathbb{R}^{n \times n} \) is the adjacency matrix of the signed graph. Note that \((v_i, v_j)\) indicates system \( j \) can receive information from system \( i \), and the index set of neighbourhood of system \( i \) is defined as \( \mathcal{N}_i = \{j \in \mathcal{V} | a_{ij} \neq 0\} \). The Laplacian matrix of \( G \) is denoted by \( L = [l_{ij}] \in \mathbb{R}^{n \times n} \) where \( l_{ii} = \sum_{j=1}^{n} |a_{ij}| \), and \( l_{ij} = -a_{ij}, \forall i \neq j \).

Definition 1: A signed graph \( G(\mathcal{V}, \mathcal{E}, \mathcal{A}) \) is said structurally balanced if the \( n \) systems can be divided into two groups: \( \mathcal{V}_1 \) and \( \mathcal{V}_2 \), with the property that \( \mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V} \) and \( \mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset \), such that \( a_{ij} \geq 0 \) if \( i, j \in \mathcal{V}_1 \) or \( i, j \in \mathcal{V}_2 \), \( a_{ij} \leq 0 \) if \( (i \in \mathcal{V}_1, j \in \mathcal{V}_2) \) or \( (i \in \mathcal{V}_2, j \in \mathcal{V}_1) \).

Assumption 2. The signed graph \( G(\mathcal{V}, \mathcal{E}, \mathcal{A}) \) is fixed, undirected, connected and structurally balanced.

If a signed graph is structurally balanced, then it can be transformed into unsigned graph \( G_u(\mathcal{V}, \mathcal{E}, \mathcal{A}_u) \) by letting the weight \( a_{u,ij} = |a_{ij}| \), where \( a_{u,ij} \) is the entry of \( \mathcal{A}_u \).

Lemma 1: Define a gauge transformation which is a change of orthant order in \( \mathbb{R}^n \) performed by the matrix \( D = [D \in \mathbb{R}^{n \times n} | D = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_n), \sigma_i \in \{1, -1\}, i = 1, 2, \ldots, n] \). A signed graph \( G \) is structurally balanced if and only if \( \exists D \in \mathbb{R}^{n \times n} \), such that \( DAD \) has all nonnegative entries.

Based on Assumption 2 and Lemma 1, \( DLD + H \) and hence \( L + H \) must be positive definite, where \( H = \text{diag}[1, 1, \ldots, 1] \in \mathbb{R}^{n \times n} \).

Assumption 1. The linear and angular velocities of leader are bounded and satisfying

\[ 0 \leq v_0 \leq v_\circ \leq v_\circ; \quad |\omega_0| \leq r_0, \]

where \((v_0, v_\circ, r_0)\) are known positive constants.

Control objective: design a distributed control law such that the bipartite formation errors

\[ p_i(t) - d_i - \sigma_i p_0(t), \quad \theta_i(t) - \theta_i, \quad i \in \mathcal{V} \]

converge to arbitrarily small neighbourhood of origin, where \( d_i = [d_{xi}, d_{yi}]^T \in \mathbb{R}^2 \) is formation information vector and \( \sigma_i = \{1, \quad i \in \mathcal{V}_1 \} \), and \( \theta_i = \{\theta_0 + 2k\pi, \quad i \in \mathcal{V}_1 \} \) \( k \in \mathbb{N} \).
3. Controller development

3.1. Model transformation
Define a hand position $\overline{p}_i$ as new output:

$$\overline{p}_i = [\overline{x}_i, \overline{y}_i]^T = p_i + l_i[\cos \theta_i, \sin \theta_i]^T,$$

(3)

where $l_i > 0$. Then, the robot system (1) can be rewritten as

$$\dot{\overline{p}}_i = \begin{bmatrix} \cos \theta_i & -l_i \sin \theta_i \\ \sin \theta_i & l_i \cos \theta_i \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} \pm \begin{bmatrix} u_{x_i} \\ u_{y_i} \end{bmatrix} \pm u_i,$$

(4a)

$$\dot{\theta}_i = -\frac{\sin \theta_i}{l_i} \overline{x}_i + \frac{\cos \theta_i}{l_i} \overline{y}_i,$$

(4b)

where $(u_{x_i}, u_{y_i})$ are new control inputs. The inverse of input transformation is computed as

$$\begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} \cos \theta_i & \sin \theta_i \\ \sin \theta_i & -\cos \theta_i \end{bmatrix}^{-1} \begin{bmatrix} u_{x_i} \\ u_{y_i} \end{bmatrix} \pm u_i.$$

(5)

Define position formation tracking errors $\overline{e}_{p, i}$ and orientation errors $e_{\theta, i}$

$$\overline{e}_{p, i} = \overline{p}_i - d_i - \sigma_i \overline{p}_0, \quad e_{\theta, i} = \theta_i(t) - \overline{\theta}_i,$$

(6)

then the bipartite formation tracking control objective is converted as design a control law such that $\overline{e}_{p, i}$ converges to zero and $e_{\theta, i}$ enters an arbitrarily small neighbourhood of origin.

3.2. Controller design
In this paper, the following distributed bipartite tracking control law is proposed for each robot:

$$u_i = \sum_{j \in N_i} \left| a_{ij} \right| \left[ \begin{array}{c} \overline{p}_{i} - d_i - \text{sign}(a_{ij}) \left( \overline{p}_j - d_j \right) \\ -\overline{e}_{p, i} - \text{sign}(\overline{e}_{p, i}) \overline{v}_o \end{array} \right].$$

(7)

Here sign([\xi_1, \xi_2, \ldots, \xi_n]^T) = [\text{sign}(\xi_1), \text{sign}(\xi_2), \ldots, \text{sign}(\xi_n)]^T with $\xi_i \in \mathbb{R}$ for $i = 1, 2, \ldots, n$. Let

$$\overline{e}_p = [\overline{e}_{p, 1}, \overline{e}_{p, 2}, \ldots, \overline{e}_{p, n}]^T, \overline{p} = [\overline{p}_1, \overline{p}_2, \ldots, \overline{p}_n]^T, d = [d_1, d_2, \ldots, d_n]^T, H = \text{diag}(1, 1, \ldots, 1) \in \mathbb{R}^{n \times n}.$$

Then, under controller (7), system (4a) can be written into vectors form as follows

$$\dot{\overline{p}} = -(L \otimes I_2) (\overline{p} - d) - (H \otimes I_2) [\overline{e}_p] - \text{sign}(\overline{e}_p) \overline{v}_o \quad \text{(8)}$$

where $\otimes$ denotes the Kronecker product.

Based on the above equation and $\overline{e}_p = \overline{p} - d - (D_1 \otimes p_0), 1_n = [1, 1, \ldots, 1]^T \in \mathbb{R}^n$. then $\overline{e}_p$ system can be rewritten as

$$\dot{\overline{e}}_p = -(L_H \otimes I_2) \overline{e}_p - \text{sign}(\overline{e}_p) \overline{v}_o - (D_1 \otimes \overline{p}_0).$$

(9)

where $L_H = L + H$, the property $L D 1_n = 0$ is utilized for (9).

**Theorem 1.** Suppose $0 < l_i$, $0 < (l_i r_0 / \nu_0) < 1$, $(l_i r_0 / \nu_0)$ is a very small number, and Assumptions 1-2 hold, then the distributed control law (7) can globally exponentially regulate the position formation tracking errors $\overline{e}_{p, i}$ to zero, and $p_i(t) - d_i - \sigma_i p_0(t)$ and the orientation errors $e_{\theta, i}$ tend to an arbitrarily small neighbourhood of origin.

*Proof.* (I) Stability analysis of position closed-loop subsystem. Choose the Lyapunov function candidate $V_{\overline{e}_p} = 0.5 \overline{e}_p^T \overline{e}_p$. The derivative of the $V_{\overline{e}_p}$ is

$$\dot{V}_{\overline{e}_p} = -\overline{e}_p^T (L_H \otimes I_2) \overline{e}_p - \overline{e}_p^T [\text{sign}(\overline{e}_p)] \overline{v}_o - \overline{e}_p^T [(D_1 \otimes \overline{p}_0].$$

(10)

We can compute the third part of (10) as

$$-\overline{e}_p^T [(D_1 \otimes \overline{p}_0] = -\nu_0 \overline{e}_p (D \otimes I_2) (1_n \otimes [\cos \theta_0, \sin \theta_0]^T) \leq \nu_0 \|\overline{e}_p\|_1.$$

(11)

Here $\|\eta_1\|_1 = \eta_1^T \text{sign}(\eta_1) = \|\eta_1\|_2$ is used. Substituting (11) into (10), we get

$$\dot{V}_{\overline{e}_p} \leq -\overline{e}_p^T (L_H \otimes I_2) \overline{e}_p - \|\overline{e}_p\|_1 \overline{v}_o + \overline{v}_o \|\overline{e}_p\|_1 = -\overline{e}_p^T (L_H \otimes I_2) \overline{e}_p \leq 0,$$

(12)
where $L_H$ is positive definite under Assumption 2. Since $V_{\bar{p}}$ is positive definite, and $\dot{V}_{\bar{p}}$ is negative definite, we conclude that $\bar{r}_p$ globally exponentially converges to zero.

(II) Stability analysis of orientation subsystem. In order to compute the zero dynamics equation, on the manifold $(\bar{r}_p = 0, \bar{\sigma}_p = 0)$ then we get

$$\dot{\chi}_i, \dot{\psi}_i = \sigma_i [v_0 \cos \theta_0, v_0 \sin \theta_0]^T \pm \frac{1}{l_i} \left[ u_{x_i}, u_{y_i} \right]^T.$$  \hspace{1cm} (13)

Substituting the equation (13) into the $\theta_i$-equation in (4b) yields

$$\dot{\theta}_i = - \frac{1}{l_i} \left( u_{x_i} \sin \theta_i - u_{y_i} \cos \theta_i \right) = - \frac{\sigma_i v_0}{l_i} \sin (\theta_i - \theta_0).$$ \hspace{1cm} (14)

Case 1: $\sigma_i > 0$ for robot $i$ belongs to $\mathcal{V}_1$, and cooperative with leader.

In this case, let $\theta_{i,co} = \theta_i - \theta_0 - 2k\pi, k \in \mathbb{N}$, the zero dynamics equation can be written as

$$\dot{\theta}_{i,co} = - \frac{v_0}{l_i} \sin (\theta_{i,co}) - \omega_0.$$ \hspace{1cm} (15)

Case 2: $\sigma_i < 0$ for system $i$ belongs to $\mathcal{V}_2$, and antagonism with leader.

In this case, let $\theta_{i,an} = \theta_i - \theta_0 - (2k + 1)\pi, k \in \mathbb{N}$, the zero dynamics equation can be written as

$$\dot{\theta}_{i,an} = - \frac{v_0}{l_i} \sin (\theta_{i,an}) - \omega_0.$$ \hspace{1cm} (16)

Systems (15) and (16) are similar to the following system by viewing $(v_0)/l_i, \omega_0$ as $\kappa(t), \delta(t)$ respectively, and $\theta_{i,co}$ or $\theta_{i,an}$ as $\phi$,

$$\dot{\phi} = - \kappa(t) \sin \phi - \delta(t),$$ \hspace{1cm} (17)

where $\phi \in \mathbb{R}$ is a state variable. $0 < \kappa \leq \kappa_0$ and $\delta(t)$ is not identically equal to zero satisfying $|\delta(t)| < \delta_0 < \kappa_0$ with $2 \delta_0 / \kappa_0$ very small. We define the Lyapunov function for system

$$V_\phi = 1 - \cos \phi, \quad |\phi| < \pi.$$ \hspace{1cm} (18)

Tacking the derivative of $V_\phi$ yields

$$\dot{V}_\phi = - \kappa(t) \sin^2 \phi - \delta(t) \sin \phi \leq - \kappa_0 \sin^2 \phi + \delta_0 |\sin \phi| < 0$$ \hspace{1cm} (19)

on the condition of $|\phi| < \pi$ or $|\phi| < \arcsin(\delta_0 / \kappa_0)$.

The formulas (18) and (19) show that $\phi$ will converge to the neighborhood of origin with the invariant set $\{|\phi| < \pi | |\phi| < \arcsin(\delta_0 / \kappa_0)\}$. Since $2 \delta_0 / \kappa_0$ very small and $\phi$ can not stay at $\pm \pi$, we conclude that for any $|\phi(0)| < \pi$, $\phi$ enters $\{|\phi| < \pi | |\phi| < \arcsin(\delta_0 / \kappa_0)\}$. Apart from this, if $\delta(t) = 0$, then for any $|\phi(0)| < \pi$, $\phi$ converges to zero asymptotically.

Based on the above verification, we can conclude that $\theta_{i,co}$ and $\theta_{i,an}$ always converge to $\{\theta_{i,co} \in \mathbb{R} | |\theta_{i,co}| < \arcsin(\ell_i r_0 / \nu_0)\}, \{\theta_{i,an} \in \mathbb{R} | |\theta_{i,an}| < \arcsin(\ell_i r_0 / \nu_0)\}$. \hspace{1cm} (20)

where $0 < \ell_i, 0 < (l_i r_0 / \nu_0) < 1$ and $(l_i r_0 / \nu_0)$ very small.

This means that the followers in $\mathcal{V}_1$ will keep their directions similar to leader, and the followers in $\mathcal{V}_2$ will keep their directions against to leader. \hspace{1cm} $\square$

4. Simulation

Suppose that there nine agents (a leader labeled as 0 and eight followers labeled from 1 to 8) connected by the undirected communication topology with weights indicated on edges painted in Figure 1. All the considered robots are the classic unicycle ones show in Figure 2.

The initial positions and headings of the all robots, the desired formation information $d_i$ and parameters $l_i$ of followers are set as

$$(x_0, y_0, \theta_0)(0) = (5, 4, \pi/2),$$

$$(x_1, y_1, \theta_1)(0) = (1, 7, \pi/3),$$

$$(x_2, y_2, \theta_2)(0) = (6, 5, -\pi/4),$$

$$(x_3, y_3, \theta_3)(0) = (7, 1, 0),$$

$$(x_4, y_4, \theta_4)(0) = (10, 5, \pi/2),$$

$$(x_5, y_5, \theta_5)(0) = (-1, 1, -3/4\pi),$$

$$(x_6, y_6, \theta_6)(0) = (6, -1, -\pi),$$

$$(x_7, y_7, \theta_7)(0) = (-1, -1, -\pi/3),$$

$$(x_8, y_8, \theta_8)(0) = (-6, -7, -1/4\pi).$$
\[d_1 = d_2 = [-0.5, 0.5]^T,\]
\[d_3 = d_4 = [0.5, -0.5]^T,\]
\[l_1 = 0.12, \quad l_2 = 0.12,\]
\[l_5 = 0.14, \quad l_6 = 0.10,\]
\[d_5 = d_6 = [0.5, 0.5]^T,\]
\[d_7 = d_8 = [-0.5, -0.5]^T,\]
\[l_7 = 0.11, \quad l_8 = 0.09,\]
\[l_9 = 0.10, \quad l_0 = 0.08.\]

Figure 1. Communication topology of systems

Figure 2. The model of a classic unicycle robot.

Obviously, the communication topology is structurally balanced, and the followers can be divided into two clusters: \( V_1 = \{1, 2, 3, 4\}, V_2 = \{5, 6, 7, 8\}\). The matrix \( D = \text{diag}(1, 1, 1, -1, -1, -1, -1)\). Suppose that \( v_0 = 3, w_0 = 4\sin(t)\), one may thus get that \( v_0 = 4\).

In Figure 3, (a) shows the represent trajectory of the leader, (b) shows the trajectories of the leader and followers under control law (7). The diamond and star pattern in Figure 3(a) indicate the starting point and the end point of the trajectory respectively. The diamond shape and the circle shape in Figure 3(b) are the initial position of the leader and the followers respectively, the star pattern represents the end position of all robots. Figure 3 (b) show that robots 1,2,3,4 belong to \( V_1 \) make into desired formation shape and track the leader, others belong to \( V_2 \) tracking the mirror trajectory of leader, and they keep expected formation always. The bipartite position formation tracking errors trajectories are shown in Figure 4, where (a)(b) show that the position formation tracking errors \( p_i(t) - d_i - \sigma p_0(t)\) tend to an arbitrarily mall neighbourhood of origin. The trajectories of \( e_\theta\) are shown in figure 4.(c), indicating that \( e_\theta\) converge to an arbitrarily small neighborhood of origin.
5. Conclusion

In this paper, the leader-follower bipartite formation tracking control problem of unicycle vehicles is solved by proposing a distributed control law. By use of hand position and input-output feedback linearization, we obtain a linear hand position subsystem. Based on this model transformation, the position bipartite formation tracking control is achieved by deducing a distributed control law. Under this control law, the robots come into desired geometric position shape, moreover for the followers cooperating with the leader, their moving trajectories and attitude angles are the same as those of leader, and for the followers fighting against the leader, their moving trajectories and attitude angles have the same value as leader but with opposite signs. Future work will focus on solving the leader-follower bipartite formation tracking problems of multiple nonholonomic robot systems with directed network.

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References

[1] Xie, W. and Ma, B. (2017). Smooth time-invariant control for leaderless consensus of networked nonholonomic systems. International Journal of Advanced Robotic Systems, 14(6): 1729881417748442.
[2] Xie, W., Ma, B., et al. (2018). A new formation control of multiple underactuated surface vessels. International Journal of Control, 91(5): 1011-1022.
[3] Zhao, Y., Kim, D., et al. (2017) Consensus formation control of multiple wheeled mobile robots. In: Conference. Name.Conference. Location. 1081-1086.
[4] Cheng, Y., Jia, R., et al. (2018). Robust finite-time consensus formation control for multiple nonholonomic wheeled mobile robots via output feedback. International Journal of Robust and Nonlinear Control, 28(6): 2082-2096.
[5] Zhang, X., Peng, Z., et al. (2019). Distributed fixed-time consensus-based formation tracking for multiple nonholonomic wheeled mobile robots under directed topology. International Journal of Control: 1-10.
[6] Maghenem, M., Bautista-Castillo, A., et al. (2017). Consensus-based Formation Control of Nonholonomic Robots using a Strict Lyapunov Function. IFAC-PapersOnLine, 50(1): 2439-2444.
[7] Chen, N., Wang, Y., et al. (2020) Multi - Mobile Robot Leader -Follower Formation Control Under Switching Topology. In: Chinese Control And Decision Conference. 2673-2678.
[8] Peng, Z., Wen, G., et al. (2015). Distributed consensus-based formation control for multiple nonholonomic mobile robots with a specified reference trajectory. International Journal of Systems Science, 46(8): 1447-1457.
[9] Zhang, W., Zuo, Z., et al. (2020). Double-Integrator Dynamics for Multiagent Systems With Antagonistic Reciprocity. IEEE Transactions on Cybernetics, 50(9): 4110-4120.
[10] Qin, J., Fu, W., et al. (2017). On the Bipartite Consensus for Generic Linear Multiagent Systems With Input Saturation. IEEE Transactions on Cybernetics, 47(8): 1948-1958.
[11] Zhao, Y., Xing, B., et al. (2020) Global Exponential Rendezvous Control of Nonholonomic Unicycle Vehicles with Directed Communication Topology. In: Chinese Intelligent Systems Conference. 552-561.
[12] Zhao, Y., Huang, W., et al. (2020) Global Formation Control of Nonholonomic Unicycle Vehicles With Guaranteed Rate of Convergence, Asian Journal of Control, 1-10.