Robust and Least Complex Online Secondary Path Estimation in Broadband Feed-Forward Active Control of Impulsive Noise using FxLMAT Family

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ABSTRACT Active noise control (ANC) systems operating on adaptive algorithms endure instability in the presence of impulsive noise (IN). Therefore, this manuscript investigates the performance of such adaptive algorithms with online secondary path modeling (OSPM) in the presence of impulsive noise in non-stationary acoustic paths. Herein, we propose three new solutions based on the filtered x least mean absolute third (FxLMAT) algorithm for OSPM with IN. Our first solution utilizes the FxLMAT algorithm in the control filter of ANC, achieving improved modeling accuracy, good convergence, and better stability than the already existing variants of the filtered x least mean square (FxLMS) solutions. However, our proposed FxLMAT based solution exhibit slow convergence and is not robust in time-varying acoustic paths. Therefore, we propose a modified FxLMAT (MFxLMAT) algorithm to enhance the robustness of the FxLMAT solution. However, the proposed MFxLMAT still lacks good convergence. Henceforth, we introduce our third solution, i.e., a variable step size filtered x robust normalized LMAT (VSSFxRNLMAT) algorithm that almost acquires the same performance as that of the already existing filtered x recursive least square (FxRLS) family-based solutions. Simulation results are provided which show that our proposed VSSFxRNLMAT solution outperforms the already existing algorithms in terms of noise and vibration reduction, robustness, and convergence. Moreover, a comparative analysis based on the computational complexity of some well known existing as well as our proposed solutions is provided, which shows that our proposed VSSFxRNLMAT has almost the same complexity as that of the existing techniques.

INDEX TERMS Adaptive filters; Impulsive noise; LMS; LMAT; Noise Cancellation

I. INTRODUCTION

Noise has adverse effects on the information signal resulting in system performance degradation [1]. To mitigate the effect of noise, there has been lots of interest in developing ANC systems due to their numerous applications [2]–[8]. ANC systems work on the principle of superposition, an anti-noise signal having 180° phase shift to that of the primary noise are superimposed on each other resulting in a noise-free signal [4]. In the last decade, researchers have focused on high-order error power (HOEP) based adaptive algorithms [9]–[11], which have the ability to enhance the performance of the system in terms of fast convergence and can effectively minimize the noise interference. This improved performance is achieved by minimizing the higher-order power of the error signal. The least mean absolute third (LMAT) algorithm depends on reducing the mean of error third power [10], no local minimum for the LMAT algorithm exists, i.e, the error function is a perfect convex function of the filter tabs. In [10], [12]–[14], better convergence of the LMAT algorithm with respect to the LMS is achieved in various noise scenarios, but suffer stability problem due to its dependence on the input signal. Therefore, a normalized variant of LMAT (NLMAT) is proposed in [13], which depicts enhanced stability and can suppress non-Gaussian noise as compared to other investigated algorithms. Moreover, the least mean fourth (LMF) algorithms also belong to the HOEP adaptive filter family.
whose convergence also depends on the power of input signal like the LMAT algorithm [11], [15]–[18]. The most common adaptive algorithm in the ANC systems is FxLMS algorithm [19]–[22]. However, the FxLMS algorithm’s stability relies greatly on the reference signal, which is passed through an estimated secondary path. Inaccurate estimation of the secondary path results in the instability of the FxLMS algorithm, as well as slow convergence [4]. Therefore, recent works employ OSPM to improve the efficiency of the ANC systems.

In literature, two OSPM techniques exist; the first technique injects random noise into the ANC system and works on system identification for modeling of the secondary path [23]-[27]. The second technique estimates the secondary path directly from the ANC output, which, thus, avoids the addition of an extra noise [23]. The performance of the second technique is accurate as it is independent of the signal nature, whereas, the first technique depends on its output signal. The first technique is usually favored for real-time applications, covering all frequency ranges. In the following, we further elaborate on the first OSPM technique, i.e., injection of random noise into the ANC systems.

The concept of OSPM for ANC systems using random noise \( v_w(i) \) (as shown in Fig. 1) was first proposed by Eriksson et al. in [23]. Figure 1 shows block diagram of the system model introduced in [23], where \( x(i) \) is the reference input signal, \( s(i) \) is the impulse response of the secondary path, \( p(i) \) is the primary path signal, \( e(i) \) is the error signal, \( v_w(i) \) is the random noise, and \( w(i) \) represents filter weight of the filter. The ANC error signal \( e(i) \) comprises two components. The first component is \( d(i) - y'(i) \), which is used to update the taps of the control filter \( w(i) \). \( w(i) \) acts as a stumbling block for the OSPM filter \( \hat{s}(i) \). The second component, \( y'(i) \), linked with random noise \( v_w(i) \) updates the modeling filter \( \hat{s}(i) \) and behave as a distraction for the control filter \( w(i) \). Overall, the ANC system performance is affected due to the disturbance between the control filter and the modeling filter. Different techniques have been proposed for enhancing the performance of ANC systems with OSPM by reducing their mutual disturbance [23]. For instance, authors in [24]-[26] added a third adaptive filter to lower the disturbance caused by the two filters. However, these approaches did not perform well in reducing the impact of noise. Nevertheless, the technique proposed in [25] successfully removed the mutual interference between the two filters at the cost of increased design complexity. Therefore, to reduce the design complexity, Akhtar et al. proposed an incorporated variable step size FxLMS (VSSFxLMS) based modeling filter and modified FxLMS (MFxLMS) based control filter in [27]. In their proposed techniques, the authors set the gain, \( G(i) \), of the injected noise to 1, i.e., \( v_w(i) = v(i) \) [23]-[27]. However, for an accurate ANC system, the gain must be near zero, i.e., for achieving a better approximation of the secondary path, the OSPM filter must reach its steady state.

With fixed \( G(i) \), the ANC system performance is deteriorated as \( v(i) \) continuously appear in the control filter’s error signal, \( e(i) \). In [28], the authors proposed an auxiliary noise power (ANP) scheduling technique for tracking the modeling process and making changes adaptively to \( G(i) \). The ANP technique certainly helps to improve the ANC system’s performance by reducing the effect of \( v(i) \) in \( e(i) \) of the control filter after acquiring an accurate estimate of the secondary path. Therefore, various works merge FxLMS and ANP for accurate modeling, low steady-state, and fastest convergence [27]-[32]. For instance, Carini et al. propose a new ANP scheduling that employed normalized FxLMS (FxNLMS) both in control as well as in the modeling filter [29]. Their approach has a high gain at the expense of additional computational complexity.

Ahmed et al., in [31], suggested a new normalized variable step-size method for both the control and the modeling filters, respectively. Pu et al. introduced a low complexity technique by employing the VSSFxLMS algorithm and ANP scheduling strategy in [32]. Furthermore, the authors in [33] suggested an improved technique, where the gain \( G(i) \) is computed through the error of the modeling filter. In all the techniques (discussed earlier), the authors consider Gaussian noise for simplicity. Therefore, Fareeha et al., in [34] considers a more practical system with IN and proposed FxRLS family-based solutions [35] with better convergence and robustness at the cost of high computational complexity. The authors modeled IN using symmetric-\( \alpha \)-stable (S\( \alpha \)S) distribution, defined as

\[
\phi(x) = e^{-\gamma|x|^\alpha},
\]

where \( \alpha \) ranges from 0 to 2 and represents the characteristic exponent to control the distribution shape. When \( \alpha \) approach 2, the noise becomes Gaussian, whereas it gets impulsive as \( \alpha \) approaches zero. A stable S\( \alpha \)S distribution is achieved by fixing the scaling parameter \( \gamma \) to 1, i.e., \( \gamma = 1 \). Figure 2 illustrates the impact of varying \( \alpha \) on symmetric-\( \alpha \)-stable distribution.

To mitigate the effect of the impulsive noise, all of the existing techniques employ alternatives of FxLMS algorithms for the modeling and the control filters. Therefore, it is necessary to investigate new strategies for active noise reduction of impulsive noise with OSPM. The promising results presented in [12], [13], [36] prove that the filtered signal...
least mean absolute third (FxLMAT) algorithm gives good convergence, enhanced stability, and better robustness than FxLMS algorithms in an IN environment. However, to the best of our knowledge, none of the previously proposed LMAT algorithms available in the literature consider mitigation of IN with OSPM.

Herein, we propose three new solutions based on the FxLMAT family for impulsive noise in ANC with OSPM. The first solution incorporates the FxLMAT algorithm in the control filter and improves accuracy in non-stationary environments, however, it converges slowly and suffers instability. Therefore, to enhance the convergence of the FxLMAT algorithm, we propose a second solution by modifying the LMAT (MFxLMAT) algorithm in the control filter. The improved performance is achieved by normalizing the step size of the MFxLMAT algorithm, however, it still lacks robustness.

Therefore, we propose a third solution, i.e., a variable step size filtered x robust normalized LMAT (VSSFxRNLMAT) algorithm to improve the robustness and convergence of the FxLMAT family. The proposed VSSFxRNLMAT solution exhibited almost the same convergence, robustness, and steady-state when compared to the FxRLS family-based existing solution [34], [35] with very low complexity for IN control with OSPM.

The rest of the paper is structured as follows: Section II presents our proposed solutions. Analytical expressions for our system model are provided in this section as well. Section III provides the simulation results, as well as a complexity analysis of our proposed solutions to some well-known existing algorithms available in the literature. Section IV finally concludes our paper.

II. PROPOSED METHODS

All the existing methods for OSPM in ANC uses variants of the FxLMS and FxRLS family for the control and the modeling filters. Our proposed solution is based on the FxLMAT family. This section contains details of the three proposed solutions for OSPM with ANC in the presence of IN.

A. FxLMAT BASED SOLUTION FOR FEED-FORWARD ANC WITH OSPM

Figure 3 shows a block diagram of our proposed model where FxLMAT algorithm is used in the control filter of an ANC system with OSPM. In Fig. 3, $P(z)$, $W(z)$, $S(z)$, $\hat{S}(z)$, and $H(z)$ are the $z$-transforms of the primary path $p(i)$, control filter $w(i)$, secondary path $s(i)$, modelling filter $\hat{s}(i)$ and the reference signal cancelling filter $h(i)$, respectively. The FxLMAT algorithm minimizes the third power of the absolute value of the error signal to find the filter coefficients [10]. The control filter’s weight is updated as

$$w(i + 1) = w(i) + \mu_w f(i)^2 \text{sgn}(f(i)) x'(i),$$

where $\text{sgn}$ is the signum function and $w(i + 1)$ is the weight of the control filter at $(i + 1)^{th}$ iteration, $\mu_w$ represents the step size, $f(i)$ is error of the control filter at the $i^{th}$ iteration, and $x'(i)$ is the filtered reference noise.

The error signal $e(i)$ can be expressed as

$$e(i) = [d(i) - y'(i)] + v'(i),$$

where $d(i) = p(i) * x(i)$ is the desired primary signal, $y'(i) = s(i) * y(i)$ is the cancelling signal and $v'(i) = s(i) * v(i)$ is the modeling signal. Note that $*$ represents the linear convolution. The modeling filter, $\hat{s}(i)$, is an FIR filter of length $M$ and its output $\hat{v}'(i)$ is given as

$$\hat{v}'(i) = \hat{s}(i)^T v(i),$$

where $\hat{s}(i) = [\hat{s}_0(i), \hat{s}_1(i), \cdots, \hat{s}_{M-1}(i)]$ is the impulse response of $\hat{s}(z)$ and $v(i) = [v(i), v(i-1), \cdots, v(i-M+1)]^T$.

The error signal $f(i)$ used to update the weight of the control filter $w(i)$ is expressed as

$$f(i) = e(i) - \hat{v}'(i) = [d(i) - y'(i)] + [v'(i) - \hat{v}'(i)],$$

where $y(i) = w^T(i) * x(i)$ is the output of the ANC filter. The modeling error signal $g(i)$ used to update the modeling filter $\hat{s}(i)$ and can be written as

$$g(i) = \varepsilon(i) + v(i) * [s(i) - \hat{s}(i - 1)],$$
where $\varepsilon(i)$ is defined as

$$
\varepsilon(i) = d(i) - s(i) \ast y(i) - x(i) \ast h(i).
$$

(7)

The modeling filter weights are updated using VSSLMS algorithm as

$$
\hat{s}(i + 1) = \hat{s}(i) + \mu_s(i)g(i)v(i).
$$

(8)

The modeling filter’s step size $\mu_s$ is calculated by

$$
\mu_s(i) = \frac{\beta}{\sqrt{T(i)v(i) + \delta_s(i))},
$$

(9)

where $v(i)$ is the white noise passed through gain, $\beta$ is a constant ranging from zero to one, i.e., $0 < \beta < 1$, and $\delta_s$ is the regularization parameter. To obtain $\delta_s$, first we need to find $\hat{g}(i)$ using Eq. 10, i.e.,

$$
\hat{g}(i) = \varepsilon(i) + v(i) \ast [s(i) - \hat{s}(i)].
$$

(10)

The component $\varepsilon(i)$ in the modeling error signal $g(i)$ can be discarded while updating the modeling filter $\hat{s}(i)$, as $\varepsilon(i)$ is a disturbance and ideally it should be equal to zero. However, in practical systems $\varepsilon(i)$ is present, thus, $v(i) \ast [s(i) - \hat{s}(i)] = 0$, assuming that $E(g^2(i)) = E(\varepsilon^2(i)).$ By putting Eq. (8) into Eq. (10) and using Eq. (6) to replace $\hat{s}(i - 1)$ we can get $\delta_s(i)$ as

$$
\delta_s(i) = \frac{P_v(i)[P_v(i) + \sqrt{P_g(i)P_e(i)}]}{P_g(i) - P_e(i)},
$$

(11)

where $P_v(i)$, $P_g(i)$, $P_e(i)$ are the powers of signal $v(i)$, $g(i)$, and $\varepsilon(i)$, respectively. The powers $P_v(i)$ and $P_g(i)$ can be approximated directly using the following two equations.

$$
P_v(i) = \lambda P_v(i - 1) + (1 - \lambda) v^2(i),
$$

(12)

$$
P_g(i) = \lambda P_g(i - 1) + (1 - \lambda) g^2(i).
$$

(13)

Now, we can obtain $P_e(i),$

$$
P_f(i) = P_d(i) - y^2(i) + P_{\varepsilon(i)} - v^2(i),
$$

(14)

and

$$
P_g(i) = P_d(i) - y^2(i) - x^2(i) + h(i) + P_{\varepsilon(i)} - v^2(i).
$$

(15)

From the above equations, we know that component in $g(i)$ related to the reference signal will approach 0 rapidly, hence, we can assume that $P_g(i) \approx P_{\varepsilon(i)} - v^2(i)$ and

$$
P_e(i) \approx P_f(i) - P_g(i) \approx P_f(i - g(i)) \ast (i),
$$

(16)

Then, the power of $P_{f(i - g(i))}$ is calculated as

$$
P_{f(i - g(i))}(i) = \lambda P_{f(i - g(i))}(i) + (1 - \lambda)[f(i) - g(i)]^2,
$$

(17)

where $0.9 \leq \lambda \leq 1.$ Now, we can find the regularization parameter by substituting Eqs. (15), (16), and (17) into Eq. (11). Finally, the gain $G(i)$ is computed by putting Eq. (15) in

$$
G(i) = c \sqrt{P_g(i)}.
$$

(18)

Where $c$ is a constant used to control the amplitude of the gain and $c > 1.$ The power calculation $P_g(i)$ employs the third filter, i.e., $h(i),$

$$
h(i + 1) = h(i) + \mu_h h(i) \ast x(i),
$$

(19)

where $\mu_h$ is the third filter’s step size, and its main purpose is to minimize the interference. Hence, it makes the secondary path’s online estimation faster.

When the OSPM coefficients are estimated accurately, the value of $G(i)$ starts to decrease, thus, ensuring better accuracy. However, a sudden change in the secondary path results in an increased $v(i)$ as gain $G(i)$ rises. The low value of $f(i)$ is achieved with faster convergence of OSPM filter, resulting in a small contribution of $v(i)$ in $e(i).$ However, for non-stationary acoustic paths, the proposed FxLMAT algorithm lacks robustness and stability, thus, adjustments are needed to improve the performance of this first proposed FxLMAT solution. Therefore, MFxLMAT based solution is devised in the next section.

### B. MFxLMAT BASED SOLUTION FOR FEED FORWARD ANC WITH OSPM

As mentioned earlier, the proposed FxLMAT performance in ANC with OSPM degrades in the presence of non-stationary acoustic paths. Therefore, we introduce another solution, i.e., a modified FxLMAT (MFxLMAT) algorithm in the control filter. The modeling filter employs VSSLMS algorithm where the modeling filter’s weights are updated as

$$
\hat{s}(i + 1) = \hat{s}(i) + \mu_s(i)g(i)v(i),
$$

(20)

where the modeling filter’s step size is $\mu_s,$ the error signal $g(i)$ is used for adaptation in the modeling filter, and $v(i)$ is the injected white noise passed through the gain and serves as an input to the modeling filter. The step size $\mu_s$ of the modeling filter can be calculated using Eq. (9). The regularization parameter $\lambda_s(i)$, as discussed in previous section, can be found by substituting Eq. (8) into Eq. (10) and using Eq. (6) to replace $\hat{s}(i - 1).$ The step size $\mu_{w_n}$ of the proposed MFxLMAT solution is normalized by the power of the reference and the error signal and is given as

$$
\mu_{w_n}(i) = \frac{\mu_w}{\delta_0 + |\hat{K}^2(i)| + E_n(i) + 1},
$$

(21)

where $\delta_0$ is a positive constant added to avoid division by zero in the denominator. The varying value of $\delta_0$ doesn’t affect the performance of the proposed algorithm. The energy of the error signal is computed using a low pass estimator, defined as

$$
E_n(i + 1) = (\lambda E_n(i) + (1 - \lambda)[f(i)]^2),
$$

(22)

where, the forgetting factor $\lambda$ has a range of $0.9 \leq \lambda \leq 1.$ The updated weight equation for control filter employing MFxLMAT is expressed as:

$$
w(i + 1) = w(i) + \mu_{w_n}(i)f(i)^2 sign(f(i)) x(i).
$$

(23)

With the occurrence of a large impulse at the $i^{th}$ iteration, amplitudes of the reference $x'(i)$ and the error signal $f(i)$
are increased, resulting in a decreased step size at the \(i^{th}\) iteration. This decrease in the step size momentarily halts the adaptation process of the weight update process, thus, ensuring enhanced stability of the proposed solution. Further, computation of the gain \(G(i)\) and the power calculation \(P_g(i)\) employing the third filter \(h(i)\) are done using Eq.(18), Eq. (13) and Eq. (19), as discussed in Section 2.1. MFxLMA has good stability but lacks convergence due to the freezing of the weight update process. This motivated us to present our third solution that enhances the overall system efficiency.

C. VSSFxRNLMAT BASED SOLUTION IN FEED-FORWARD ANC WITH OSPM

A new solution is suggested in this subsection, which incorporates VSSFxRNLMAT in the control filter and VSSLMS algorithm in the modeling filter. The RNLMAT algorithm reveals better convergence and robustness in the occurrence of the IN [13]. Motivated by the finding of RNLMAT in [13], we tested it in Feedforward ANC with OSPM and discovered that it has a slow convergence. Therefore, we modified normalized FxRNLMAT on the same lines as done in the previously proposed MFxLMA solution. The control filter’s weight update equation along with the modified step size are given as

\[
w(i + 1) = w(i) + \frac{\mu w(i)}{1 + \beta f(i)^3} f(i)^2 \text{sign}(f(i))x'(i),
\]

and

\[
\mu w(i) = \frac{\mu w_n}{||x'^2(i)|| + \delta_0 + E_n(i + 1)},
\]

respectively.

The energy of the error signal is computed by low pass estimator and is given as

\[
E_n(i + 1) = (\lambda E_n(i)) + ((1 - \lambda))|f(i)^2|,
\]

where Eq.(26) is used to find energy of the modeling filter’s error signal, then, the weight equation of the modeling filter is updated according to [33] as

\[
\hat{s}(i + 1) = \hat{s}(i) + \mu s(i)g(i)v(i)
\]

where \(\mu_s\) is the step size for the modeling filter, \(g(i)\) is the error signal used for adaptation of the modeling filter, and \(v(i)\) is the white noise injected in the modeling filter. \(\mu_s\) is already explained in Section 2.1, i.e., Eq. (9). The regularization parameter \(\lambda_s(i)\) as discussed in the previous section can be found by putting Eq. (8) into Eq. (10) and using Eq. (6) to replace \(\hat{s}(i - 1)\). Further, the computation of gain \(G(i)\) and the power calculation \(P_g(i)\) employing the third filter \(h(i)\) are done using Eq.(18), Eq. (13) and Eq. (19), as discussed in Section 2.1. This VSSFxRNLMAT solution yields superior performance in terms of faster convergence, enhanced robustness, and better stability than the existing solutions of the FxLMS family.

III. RESULTS AND DISCUSSION

This section provides a detailed comparative analysis of our proposed and some already existing solutions via numerous simulations. The platform used for simulations is MATLAB 2019. Different parameters used for simulating the impulsive noise and ANC system are provided in Table 1. Moreover, we compare our proposed solutions to the following state-of-the-art algorithms

- Eriksson technique [23]
- Akhtar technique [27]
- Carini technique [29]
- Pu technique [32]
- Yang technique [33]
- Fareeha technique [34]

| Parameters | Values |
|------------|--------|
| Total Realization | Avg = 10 |
| Total Iteration | N = 100,000 |
| Characteristic Component | \(\alpha = 1.65 \text{ and } 1.85\) |
| Scale Parameter | \(\gamma = 1\) |
| Location Parameter | \(C = 0\) |
| Skewness Parameter | \(\delta = 0\) |

TABLE 1. Parameters for Impulsive Noise and ANC Systems used in Simulations

For the simulation results, we tested our proposed solutions to the already existing techniques both in stationary and non-stationary acoustic paths in the presence of IN. The performance metrics selected for simulations are mean noise reduction (MNR), relative modeling error \(\Delta S\), and the vibration reduction parameter (\(R\)).

MNR is calculated as

\[
\text{MNR}(i) = E \left( \frac{A_c(i)}{A_d(i)} \right),
\]

where \(A_c(i)\) and \(A_d(i)\) represents the absolute values of the disturbance and the residual error signal, respectively. \(A_c(i)\) and \(A_d(i)\) can be computed as

\[
A_c(i) = \lambda A_s(i - 1) + (1 - \lambda)|e(i)|,
\]

\[
A_d(i) = \lambda A_d(i - 1) + (1 - \lambda)|d(i)|.
\]

The relative modeling error \(\Delta S\) is computed as

\[
\Delta S(i) = 20 \log \frac{||\hat{s}(i) - \hat{s}(i)||^2}{||\hat{s}(i)||^2}.
\]

Finally, the vibration reduction parameter (\(R\)) is expressed as

\[
R(i) = -10 \log \left( \frac{\sum e(i)^2}{\sum d(i)^2} \right).
\]
first stage, the ANC filter was turned off for the first 50000 iterations and only the OSPM filter was used to estimate the secondary path. After 50000 iterations, both the ANC and OSPM filters were changed concurrently in the second stage. A white Gaussian noise $v_\text{w}(i)$ with zero mean and 0.05 variance is used as primary noise. Table 2 displays the values of the controlling parameters used in our simulations to get the best performance of all the existing and the proposed solutions.

Moreover, for simulations we consider the following three different cases.

- **Case 1:** Impulsive noise with $\alpha = 1.85$ in stationary acoustic path
- **Case 2:** Impulsive noise with $\alpha = 1.85$ in non-stationary acoustic path
- **Case 3:** Impulsive noise with $\alpha = 1.65$ in non-stationary acoustic path

In the following, we discuss each of these cases separately.

### A. CASE 1: IMPULSIVE NOISE WITH $\alpha = 1.85$ IN STATIONARY ACOUSTIC PATH

In case 1, we tested the performance of our proposed solutions as well as some existing solutions available in the literature [23], [27], [29], [32]–[34] of ANC for IN with OSPM. The comparison of $\Delta S$, vibration reduction, and MNR is illustrated in Fig. 5. Figure 5 (a) depicts the relative modelling error $\Delta S$ vs number of iterations. Figure 5 (a) clearly shows that Carini’s technique exhibits poor convergence and does not give a reliable estimation of the secondary path, whereas Yang’s and Fareeha’s techniques show fast convergence and have better precision by acquiring the lowest $\Delta S$ value, i.e., -40 dB at 12000 iterations. Figure 5 (b) shows the vibration reduction performance, where Carini’s and Fareeha’s techniques illustrate the fastest convergence, i.e., at 19dB and 16dB, respectively. Figure 5 (c) gives the MNR of all the techniques revealing that Carini’s technique exhibit the best convergence and approaches a steady-state value of -0.3 dB followed by Fareeha’s technique [34], which shows slow convergence and a steady-state value of -0.4 dB. As shown in Fig. 5, our proposedFxLMAT solution gives better noise reduction than Yang’s approach, however, it lacks robustness. Moreover, our proposed VSSFxRNLMAT solution reaches the same steady-state error and robustness of Fareeha’s technique [34] just after 50000 iterations along with the fastest convergence and with significant low complexity.

We can conclude that Carini’s technique is the most proficient reported solution for vibration reduction as well as the MNR among all the investigated techniques. Furthermore, as depicted in Fig. 5, Erikson’s, Akhtar’s, and PU’s methods diverged for IN. Henceforth, we therefore will only be considering Carini’s, Yang’s, and Fareeha’s solutions for the non-stationary acoustic path in the subsequent cases.

### B. CASE 2: IMPULSIVE NOISE WITH $\alpha = 1.85$ IN NON-STATIONARY ACOUSTIC PATH

In Case 2, non-stationary acoustic paths are used to validate the performance of our suggested solutions. By giving two sample right circular shifts at 50,000 iterations, a substantial perturbation is applied to both the primary and the secondary acoustic channels. The impulse response of the acoustic channels that have been disrupted is shown in Fig. 4. A comparison of $\Delta S$, vibration reduction, and MNR for our proposed solutions with the already existing techniques, for Case-2, are illustrated in Fig. 6.

Figure 6 shows that for all the performance indicators, Carini’s technique diverges when the acoustic paths are perturbed, whereas, Yang’s technique exhibit good accuracy,
i.e., $\Delta S = -31 dB$, however, it lags in convergence than our proposed solutions. Moreover, for the time-varying acoustic paths, our proposed VSSFxRNLMAT outperforms the already existing solutions in terms of faster convergence, increased noise reduction, high vibration reduction, and superior accuracy. Moreover, with low complexity, our proposed VSSFxRNLMAT solution achieves almost the same noise reduction, vibration reduction as well as have the fastest convergence as compared to Fareeha’s technique [34].

C. CASE 3: IMPULSIVE NOISE WITH $\alpha = 1.65$ IN NON STATIONARY ACOUSTIC PATHS

Case 3 explores the performance of all the investigated techniques for ANC with OSPM in the presence of perturbed acoustic paths with an impulsive environment, i.e., for $\alpha = 1.65$. Figure 7 (a-c) shows that our proposed solutions offer better performance as compared to the FxLMS family-based existing techniques in terms of greater accuracy, enhanced vibration and noise reduction, and improved robustness in the presence a highly impulsive environment.

Figure 7 depicts that our first proposed solution, i.e., FxL-MAT, lacks robustness and convergence in the case of high
impulses, whereas our proposed MFxLMAT shows faster convergence, improved accuracy as well as vibration reduction as compared to the FxLMAT solution. This improvement is achieved by the normalized step size of the proposed MFxLMAT by the power of the reference and the error signal. Moreover, our third solution, i.e., VSSFxRNLMAT, exhibits even better performance in terms of robustness, vibration reduction, and stability with almost the same computational complexity as that of different variants of the LMS family.

We can conclude from the simulation results that Carini’s technique exhibits the fastest convergence, best noise reduction, and high vibration reduction amongst the existing techniques for ANC with OSPM in the presence of a stationary IN environment. However, it fails to perform in the perturbed (non-stationary) acoustic paths scenarios. On the other hand, our proposed FxLMAT solution exhibit slow convergence and is not robust in the non-stationary acoustic paths. To improve the robustness of the proposed FxLMAT and achieve better accuracy and convergence, MFxLMAT is introduced, however, it still lacks fast convergence. To improve the convergence, we propose VSSFxRNLMAT which achieves the desired performances, i.e., fast convergence, excellent robustness with very low computational complexity.

D. COMPLEXITY COMPARISON

Besides robustness, computational complexity is essential in the analysis of algorithms. Based on the available resources, it is important to select algorithms according to their computational complexity. Table 3 shows complexity comparison of some well known existing solutions along with our proposed solutions, where \( L_w \), \( M \), \( K \), and \( L \) are the lengths of control filter \( w(i) \), modeling filter \( \hat{s}(i) \), reference signal cancelling filter \( h(i) \), and primary path, respectively. \( D \) is the length of the delay used for the injected random noise \( v_w(i) \) [29]. Table 3 clearly shows that the computational complexity of our techniques is almost the same as that of Carini’s, Pu’s, and Yang’s algorithms, however, it is least complex as compared to Fareeha’s algorithm.

IV. CONCLUSION

In this paper, we proposed three new solutions based on the filtered x least mean absolute third family for active control of impulsive noise with OSPM. The proposed solutions were evaluated through simulation using mean noise reduction, relative modeling error, and vibration reduction as performance metrics. Exhaustive simulations reveal that our first proposed solution, i.e., FxLMAT, has better stability and model precision as compared to the FxLMS solution, however, it suffers from slow convergence and poor robustness in the non-stationary acoustic paths. To improve the robustness of the FxLMAT solution in a time-varying IN environment, an MFxLMAT solution is presented, whose step size is updated using the energy of the error and the reference signal, however, it lacked robustness. Therefore, to cater for fast convergence and robustness, VSSFxRNLMAT solution was presented. It is shown through simulation results that the proposed VSSFxRNLMAT solution manifests the fastest convergence and good robustness as compared to the already existing algorithms as well as our proposed first two solutions. Moreover, the proposed VSSFxRNLMAT solution is computationally least complex and its overall performance in both stationary and non-stationary acoustic paths in the presence of IN is almost the same as that of the already existing FxRLS family-based OSPM solutions.

REFERENCES

[1] S. M. Kuo and D. R. Morgan, Active Noise Control Systems-Algorithms and DSP Implementation. New York: Wiley, 1996.
[2] G. Panda and D. P. Das, “Adaptive filter based active noise controller under acoustic feedback”, *IEE Journal of Research*, Vol. 49, no. 6, pp. 439-444, 2003.
[3] P. Palanisamy and N. Kalyanasundaram, “Sonar target detection by modified adaptive noise cancellation using correlating filter”, *International Journal of Electronics*, Vol. 98, no.1, pp. 41-60, 2011.
[4] L. Lu, K. L. Yin, R. C. de Lamare, Z. Zheng, Y. Yu, X. Yang, B. Chen, “A survey on active noise control techniques–Part I: Linear systems”, arXiv preprint arXiv, pp.2110.00531, 2021.
[5] S. Fan, Y. Xiao, S. Fang, Y. Zhao and X. Zhou, “Clipping noise cancellation for signal detection of GSTFIM systems”, *IEEE Access*, Vol. 8, pp. 33830-33837, 2020.
[6] F. B. Félix, M. Magalhães and G. Papini,” An improved Anc algorithm for the attenuation of industrial fan noise”, *Journal of Vibration Engineering and Technologies*, Vol.9, no. 2, pp. 279-289, 2021.
[7] S. S. Haykin, *Adaptive Filter Theory*. Pearson Education India, 2008.
[8] P. L. Feintuch, N. J. Bershad and A. K. Lo, “A frequency-domain model for filtered LMS algorithms-stability analysis, design, and elimination of the training mode”, *IEEE Transactions on Signal Processing*, Vol. 41, no. 4, pp. 1518-1531, 1993.
[9] S. H. Cho and S. D. Kim, “Adaptive filters based on the high order error
### TABLE 3. Complexity Comparison of Investigated Techniques

| Techniques          | $\times$, $\div$  | $+$, $-$ | Total          |
|---------------------|-------------------|---------|----------------|
| Carini’s Techniques [29] | $7L_w + 6M + 4D + 26$ | $6L_w + 6M + 4D - 1$ | $13L_w + 12M + 8D + 25$ |
| Pu’s Techniques [32]  | $2L_w + 3M + 18$   | $2L_w + 3M + 3$  | $4L_w + 4M + 21$ |
| Yang’s Techniques [33] | $2L_w + 3M + 2K + 19$ | $2L_w + 3M + 2K + 5$ | $4L_w + 6M + 4K + 24$ |
| Fareeha’s Techniques [34] | $3L_w^2 + 4L_w + 3M^2 + 7M + 2K + 10$ | $2L_w^2 + 2L_w + 2M^2 + 5M + 2K - 2$ | $5L_w^2 + 6L_w + 5M^2 + 12M + 4K + 8$ |
| Proposed FxLMAT   | $L_w + 6M + 3K + 17$ | $2L + L_w + 6M + 3K + 5$ | $2L + 2L_w + 12M + 6K + 22$ |
| Proposed MFxLMAT  | $L + L_w + 6M + 3K + 20$ | $3L + L_w + 6M + 3K + 8$ | $4L + 2L_w + 12M + 6K + 31$ |
| Proposed VSSFxRNLMAT | $2L + L_w + 6M + 3K + 20$ | $4L + L_w + 6M + 3K + 9$ | $6L + 2L_w + 12M + 6K + 33$ |

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statistic", in In Proceedings of IEEE APCCAS’96-Asia Pacific Conference on Circuits and Systems, pp. 109-112, 1996.

[10] S. H. Cho, S. D. Kim, H. P. Moon and J. Y. Na, “Least mean absolute third (LMA3) adaptive algorithm: Mean and mean squared convergence properties,” In Proceedings of Sixth Western Pacific Reg. Acoustic Conference, Hong Kong, Vol. 22, no. 10, pp. 2303-2309, 1997.

[11] E. Walach and B. Widrow, “The least mean fourth (LMF) adaptive algorithm and its family”, IEEE Transactions on Information Theory, Vol.30, no. 2, pp. 275-283, 1984.

[12] Y. H. Lee, J. D. Mok, S. D. Kim and S. H. Cho, “Performance of least mean absolute third (LMA3) adaptive algorithm in various noise environments,” Electronic Letters, Vol. 34, no. 3, pp. 241-242, Jan. 1998.

[13] K. Xiong, S. Wang and B. Chen, “Robust normalized least mean absolute third algorithms”, IEEE Access, Vol. 7, pp. 10318-10330, 2019.

[14] S. Guan and L. Zhi, “Nonparametric variable step-size LMA algorithm,” Circuits, Systems, and Signal Processing, Vol. 36, no.3, pp. 1322-1339, 2017.

[15] A. M. Al Omour, A. Zidouri, N. Iqbal and A. Zerguine, “Filtered-X least mean fourth (FXLMAF) and leaky FXLMAF adaptive algorithms;” EURASIP Journal on Advances in Signal Processing, Vol. 2016, no. 1, pp. 1-20, 2016.

[16] E. Eweda, “Dependence of the stability of the least fourth algorithm on target weights nonstationarity”, IEEE Transac tion of Signal Processing, Vol. 62, no. 7, pp. 1634-1643, 2014.

[17] P. Song and H. Zhao, “Filtered-x least mean square/fourth (FXLMS/F) algorithm for active noise control”, Mechanical Systems and Signal Processing, Vol. 120, pp. 69-82, 2019.

[18] S. Pradhan, X. Qiu and J. Ji, “Affine combination of the filtered-x LMS/F algorithms for active control”, In Vibration Engineering for a Sustainable Future, pp. 313-319, 2015. Springer, Cham.

[19] L. Lu and H. Zhao, “Improved filtered-x least mean kurtosis algorithm for active noise control,” Circuits, Systems, and Signal Processing, Vol. 36, no. 4, pp. 1586–1603, 2016.

[20] F. B. Félix, M. Magalhães, G. Papini, “Improved active noise control algorithm based on the convex combination method”, Journal of the Brazilian Society of Mechanical Sciences and Engineering, Vol.43, no. 3, pp. 1-12, 2021.

[21] A. Mirza, A. Zeb, M. U. Yasir, D. Ilyas and S. A. Sheikh, “Less complex solutions for active noise control of impulsive noise”, Analog Integrated Circuits and Signal Processing, Vol. 102, no. 3, pp. 507-521, 2020.

[22] C. Bao, P. Sas and H. Van Brussel, “Comparison of two-on-line identification algorithms for active noise control”, In Proceedings of the 2nd Conference on Recent Advances in Active Control of Sound and Vibration, pp. 38-54, 1993.

[23] L. J. Eriksson and M.C. Allie, "Use of random noise for online transducer modeling in an adaptive active attenuation system", The Journal of the Acoustical Society of America, Vol. 85, no. 2, pp. 797-802, 1999.

[24] S. M. Kuo and D. Vijayan, “A secondary path modeling technique for active noise control systems,IEEE Transactions on Speech and Audio Processing, Vol. 5, no. 4, pp. 374-377, 1997.

[25] M. Zhang, H. Lan and W. Ser, “Cross-updated active noise control system with online secondary path modeling”,IEEE Transactions on Speech and Audio Processing, Vol. 9, no. 5, pp. 598-602, 2001.

[26] M. Zhang, H. Lan and W. Ser, “A robust online secondary path modeling method with auxiliary noise power scheduling strategy and norm constraint manipulation”, IEEE Transactions on Speech and Audio Processing, Vol. 11, no. 1, pp. 45-53, 2003.

[27] M. T. Akhtar, M. Abe and M. Kawamata, “A new variable step-size LMS algorithm-based method for improved online secondary path modeling in active noise control systems”,IEEE Transactions on Audio, Speech, and Language Processing, Vol. 14, no. 2, pp. 720-726, 2006.

[28] M. Abe and M. Kawamata, "Noise power scheduling in active noise control systems with online secondary path modeling".IEEE Electronic Express, Vol. 4, no. 2, pp. 66-71, 2007.

[29] A. Carini and S. Malatini, "Optimal variable step-size NLMS algorithms with auxiliary noise power scheduling for feedforward active noise control",IEEE Transactions on Audio, Speech, and Language Processing, Vol. 16, no. 8, pp. 1383-1395, 2008.

[30] P. Davari and H. Hassanpour,"A new online secondary path modeling method for feed-forward active noise control systems", In 2008 IEEE International Conference on Industrial Technology, pp. 1-6, 2008.

[31] S. Ahmed, M. T. Akhtar and X. Zhang, "Robust auxiliary-noise-power scheduling in active noise control systems with online secondary path modeling",IEEE transactions on Audio, Speech, and Language Processing, Vol. 21, no. 4, pp. 749-761, 2012.

[32] Y. Pu, F. Zhang and J. Jiang, "A new online secondary path modeling method for adaptive active structure vibration control",Smart Materials and Structures Vol. 23, no. 6, 2014.

[33] T. Yang, L. Zhu, X. Li and L. Pang, "An online secondary path modeling method with regularized step size and self-tuning power scheduling",The Journal of the Acoustical Society of America, Vol. 143, no. 2, pp. 1076-1084, 2018.

[34] F. Jabeen, A. Mirza, A. Zeb, M. Imran, F. Afzal and A. Maqbool, “FxRLS
algorithms based active control of impulsive noise with online secondary path modeling”, IEEE Access, Vol. 9, pp. 117471-117485, 2021.

[35] A. Zeb, A. Mirza, Q. U. Khan and S. A. Sheikh, “Improving the performance of FxRIS algorithm for active noise control of impulsive noise”, Applied Acoustics, Vol. 116, pp. 364-374, 2017.

[36] A. Hussain, A. Mirza, A. Zeb and M. Y. Umair, “A modified filtered-x LMAT algorithm for active noise control of impulsive noise”, IEEE International Symposium on Signal Processing and Information Technology (ISSPIT), Dubai, UAE, Dec. 2019.