Prospects of the local Hubble parameter measurement using gravitational waves from double neutron stars

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\textbf{ABSTRACT}
Following the detection of the GW170817 signal and its associated electromagnetic emissions, we discuss the prospects of the local Hubble parameter measurement using double neutron stars (DNSs). The kilonova emissions of GW170817 are genuinely unique in terms of the rapid evolution of color and magnitude and we expect that, for a good fraction $\geq 50\%$ of the DNS events within $\sim 200\text{Mpc}$, we could identify their host galaxies, using their kilonovae. At present, the estimated DNS merger rate $(1.5^{+3.2}_{-1.2}) \times 10^{-6}\text{Mpc}^{-3}\text{yr}^{-1}$ has a large uncertainty. But, if it is at the high end, we could measure the local Hubble parameter $H_L$ with the level of $\Delta H_L/H_L \sim 0.042$ ($1\sigma$ level), after the third observational run (O3). This accuracy is four times better than that obtained from GW170817 alone, and we will be able to examine the Hubble tension at $2.1\sigma$ level.

\textbf{Key words:}

1 INTRODUCTION
The gravitational wave (GW) signal GW170817 was detected at the signal-to-noise ratio (SNR) of 32.4 that is the largest value among the GW signals detected so far (Abbott et al. 2017a). From the estimated masses, the signal is considered to be generated by a DNS inspiral. After the GW detection, the associated electromagnetic (EM) emissions were discovered worldwide at various wavelengths (Abbott et al. 2017b). These sequential events brought profound impacts broadly on astronomical and physical communities. Here, in the face of the current torrent of research papers, we do not mention general aspects of GW170817, but rather concentrate on our main topic, observational cosmology.

It has been long known that, using GWs from binary inspirals (often called the standard sirens), we can estimate the distance to the source, solely based on the first principle of physics (Schutz 1986, Krolak & Schutz 1987). This shows a remarkable contrast to the traditional distance ladder that relies heavily on various empirical relations. Meanwhile, because of the simple scaling property of general relativity, it is not straightforward to estimate the redshift of the binary only from GWs (see also Chernoff & Finn 1993, MacLeod & Hogan 2008, Messenger & Read 2012). Therefore, in order to utilize the standard sirens efficiently, it would be crucially advantageous, if we can identify the transient EM signals associated with the GW events (see e.g. Holz & Hughes 2005, Nissanke et al. 2010). But, we had been far from confident whether such multi-messenger observations actually work.

Now, this concern is largely untangled by the followup observations of GW170817 and the resulting identification of its host galaxy NGC4993 at $z = 0.010$ (after the peculiar velocity correction, Abbott et al. 2017c, see also Hjorth et al. 2017). In fact, its kilonova (also called the macronova) emission turned out to be genuinely unique in terms of the rapid evolution of color and magnitude, also showing a characteristic time profile. It is true that we only have the single DNS event and additional ones are essential to understand the basic properties of the EM counterparts. But, now, we can expect long-term development of observational cosmology, by using DNSs as a powerful probe.

In the near future, around the entrance of this new avenue, our primary target would be the Hubble parameter, as already discussed in the pioneering work by Schutz (1986) more than 30 years ago. Indeed, using the distance $\sim 40\text{Mpc}$ estimated from the GW170817 signal and the redshift of its host galaxy, the LIGO-Virgo team reported the Hubble parameter $H_0 = 70^{+12}_{-8} \text{km sec}^{-1}\text{Mpc}^{-1}$ (Abbott et al. 2017c). Here the error bar represents 68.3% probability range.

The Hubble parameter is one of the most fundamental cosmological parameters, since the discovery of the cosmic expansion in 1929. But this parameter has attracted much attention quite recently. We have a 9% mismatch between the locally estimated value $73.24 \pm 1.74 \text{km sec}^{-1}\text{Mpc}^{-1}$ and that determined from the cosmic microwave background $66.93 \pm 0.62 \text{km sec}^{-1}\text{Mpc}^{-1}$ (Riess et al. 2016, Planck Collaboration 2016). This tension might be caused by an unidentified systematic error in the two types of measurements or might, in fact, imply a challenge to the standard cosmological model. In any case, the newly established method
based on the DNSs could make a notable contribution to the Hubble tension.

In this paper, we discuss the prospects of gravitational-wave observational cosmology with the forthcoming third observation run (O3) of the LIGO-Virgo collaboration (Abbott et al. 2016) and its follow-on operations (see Zhao & Wen 2017 for the third generation detectors). Our results would be also useful to discuss related topics such as the efforts to suppress the amplitude calibration error of the GW measurement (see e.g. Vitale et al. 2012, Tuyenbayev et al. 2017, Cahillane et al. 2017) or the observational strategy for the EM counterpart search (Cowperthwaite et al. 2017).

This paper is organized as follows. In §2, we discuss the kilonova signals and contaminations at the host galaxy identification, taking into account the actual observational results of GW170817. In §3, we derive analytical expressions to evaluate the expected number of DNS detections and the averaged distance error at the GW data analysis. Then, in §4, we discuss the prospects of the Hubble parameter measurement in the near future. §5 is a brief summary of this paper. Following the standard convention, we assume the DNS masses at $1.4 M_\odot + 1.4 M_\odot$ whose chirp mass is only $\sim 2\%$ different from that of GW170817.

2 KILONOVA SIGNALS FOR THE HOST GALAXY IDENTIFICATION

The loudness of the optical sky always stands in the way to identify kilonovae in followup observations (Cowperthwaite & Berger 2015, Tanaka 2016, Cowperthwaite et al. 2017). Here, we argue that a good fraction $\gtrsim 50\%$ of kilonovae for DNS events within $\sim 200$ Mpc can be identified by followup observations in optical (and hopefully near-infrared) bands incorporating insights obtained from observations of the kilonova associated with GW170817. Once an electromagnetic counterpart such as the kilonova is successfully identified, the host galaxy will be determined relatively easily, because the expected cosmological redshift will at most be 0.05–1. Real-time identification is not necessary for our purpose, i.e., determining the host galaxy, and accordingly we do not worry about the lack of template images (Cowperthwaite & Berger 2015).

The key findings from the kilonova associated with GW170817 are summarized as follows (see, e.g., McCully et al. 2017, Shappee et al. 2017, Siebert et al. 2017, Utsumi et al. 2017). The emission becomes bright right after the merger, say $\sim 1$ day, peaking in blue optical bands (Shappee et al. 2017). While the magnitude in optical bands such as the $g$-band drops very rapidly (Siebert et al. 2017), near-infrared bands sustain bright emission for a few to several days, where longer emission is found in longer wavelengths (Utsumi et al. 2017). Even in the relatively red $z$-band, the emission becomes dim by 2.5 mag in only 6 days (Utsumi et al. 2017). The corresponding change in the peak wavelength makes the color evolution exceedingly rapid (McCully et al. 2017). The rapid decline of magnitudes and the rapid reddening are both distinctive features of the kilonova (see also Cowperthwaite & Berger 2015, Cowperthwaite et al. 2017). Furthermore, the spectrum is mostly featureless and becomes red as early as a few days after the merger (McCully et al. 2017, Siebert et al. 2017). Such spectra are not observed for other known transients, and thus make the kilonova very unique (Shappee et al. 2017). It should be worth noting that GW170817 appears to be observed from relatively polar directions (Abbott et al. 2017a).

Turning now to identification of future kilonovae. Fast optical transients as significant contaminants are summarized comprehensively in Cowperthwaite & Berger (2015). Among the fast transients, the type Ia supernova outnumbers the kilonova at given brightness. Fortunately, they will be easily distinguished due to their significantly slow time evolution compared to that of kilonovae (see also Scolnic et al. 2017). Elimination of type Ia supernovae could further benefit from the line structures in the spectrum if it is taken and the presumably high redshift. On the other extreme, stellar flares last less than a day and will be eliminated by requiring detections in multiple nights. Quiescent emission of the underlying star could be detected later for further secure elimination.

Taking the estimated rate of fast transients, we expect that the type Ia supernova and so-called Pan-STARRS fast transients can serve as significant contaminants (Cowperthwaite & Berger 2015). A remarkable point is that the peak brightness of the kilonova associated with GW170817 was found to be brighter by $\gtrsim 1.5$ mag (Utsumi et al. 2017), or equivalently by a factor of $\gtrsim 4$ than the model adopted in Cowperthwaite & Berger (2015). This means that the number of fast transients that can compete with kilonovae will be reduced by a factor of 8. Thus, the number of type Ia supernovae and Pan-STARRS fast transients will only be $\gtrsim 0.2$ and $\lesssim 1$, respectively, even for $100$ deg$^2$ sky localization. Furthermore, the rapid decline of the kilonova associated with GW170817 is hardly reproduced by Pan-STARRS fast transients. Thus, a large fraction of Pan-STARRS fast transients, say $\gtrsim 80\%$, would be removed by requiring a moderate decline rate that does not significantly remove blue kilonovae. Overall, we expect that more than $\gtrsim 50\%$ of blue kilonovae can be identified based on the brightness and decline rates.

The rapid reddening and/or red color, once detected, will serve as a powerful tool to distinguish kilonovae from other transients as previously thought (Cowperthwaite & Berger 2015, Tanaka 2016). Note that the blueness of the kilonova associated with GW170817 does not exclude existence of red kilonovae. Particularly, we still expect to observe kilonovae without blue components for edge-on binaries, for which lanthanide-enriched dynamical ejecta should be obscuring any blue emission (Kasen, Fernández & Metzger 2015). A substantial fraction of red kilonovae can be securely identified as electromagnetic counterparts to DNS events further aided by rapid magnitude evolution. Quantitatively, requiring $i - z \gtrsim 0.4$ mag will remove most of the contaminants, say $\gtrsim 90\%$, without discarding red kilonovae significantly (Cowperthwaite & Berger 2015). The requirement may be loosened to $\gtrsim 0$ mag when we additionally require the rise or decline time to be $\lesssim 4$ day, but about a half of kilonovae may be lost for the cut based on the decline (Cowperthwaite & Berger 2015).

To summarize, we expect that more than 50% of kilonovae may be identified successfully by first seeking rapid blue components, and next red components. In any case, properties of kilonovae discussed here rely heavily on a single event GW170817 combined with theoretical knowledge, and future detections of a variety of kilonovae in followup observations to DNS events are crucially important to refine the selection criteria. The fraction of identifiable kilonovae may be increased in the near future by understanding their characteristics with actual observations.

3 SIGNAL DETECTION AND ANALYSIS

In this section, we discuss analytical expressions for the detectable volume of DNSs and the appropriately averaged their distance estimation errors. We basically follow the formulation in Cutler & Flanagan (1994). In addition, using the method developed in Seto...
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(2015), we perturbatively include the geometrical information of the detector network. We also count the differences in sensitivities among detectors, and derive expressions convenient for statistical study of the local Hubble parameter measurement.

3.1 detectable volume

The SNR \( \rho_i \) of a detector \( i \) can be written as

\[
\rho_i^2 = 10 \left( \frac{d_{h,i}}{D} \right)^2 \left[ F_{+,(i)}(n, \psi)^2 \left( \frac{1 + v^2}{4} \right) + F_{\times,(i)}(n, \psi)^2 v^2 \right],
\]

where \( F_{+,(i)} \) and \( F_{\times,(i)} \) are the beam pattern functions that depend on the source direction \( n \) and the polarization angle \( \psi \) (Thorne 1987). The quantity \( d_{h,i} \) is the horizon distance for \( \rho_i = 10 \). We have the relation \( d_{h,i} = 2.26 d_{r,i} \) with the detection range \( d_{r,i} \) for the same SNR (see e.g. Chen et al. 2017). In eq. (1), \( D \) is the (luminosity) distance to the binary and \( v \) is the cosine of its inclination angle with \( |v| = 1 \) for face-on and \( v = 0 \) for edge-on.

If the detector noises are uncorrelated, the total SNR \( \rho \) of a detector network is given by

\[
\rho^2 = \sum_i \rho_i^2,
\]

Following Cutler & Flanagan (1994), we express \( \rho^2 \) in the form

\[
\rho^2 = \frac{\sigma(n)}{D^2} [c_0(v) + \epsilon(n)c_1(v) \cos 4\psi]
\]

with \( c_0(v) \equiv (1 + v^2)^2/4 + v^2 \) and \( c_1(v) \equiv (1 + v^2)^2/4 - v^2 \). Here the function \( \sigma(n) \) shows the total sensitivity of the network to GWs coming from the direction \( n \). Meanwhile, the function \( \epsilon(n) \) represents the asymmetry of the sensitivities to two (appropriately decomposed) orthogonal polarization modes. We generally have \( 0 \leq \epsilon(n) \leq 1 \). If the network is blind to one of the modes, we have \( \epsilon(n) = 1 \).

From eq. (3), for a given SNR threshold \( \rho_T \), the maximum detectable distance \( D_{\text{max}} \) is given by

\[
D_{\text{max}} = \frac{1}{\rho_T} \frac{\sigma(n)}{D^2} [c_0(v) + \epsilon(n)c_1(v) \cos 4\psi]^{1/2},
\]

and can be regarded as a function of the four angular parameters \((n, \psi, v)\).

To simplify expressions below, we introduced the averaging operation with the angular parameters;

\[
\int dA[\cdots] \equiv \frac{1}{4\pi} \int_0^{4\pi} dn \frac{1}{2\pi} \int_0^{2\pi} d\psi \frac{1}{2} \int_{-1}^1 dv [\cdots].
\]

Then the effective volume \( V \) for the detection threshold \( \rho_T \) is given as

\[
V = \int dA \int_0^{D_{\text{max}}} dD d4\pi D^2 = \rho_T^{-3} U,
\]

where we defined

\[
U \equiv \frac{4\pi}{3} \rho_T^{-3} \int dA D_{\text{max}}^3.
\]

Using eq. (4), we formally have

\[
U = \frac{4\pi}{3} \int dA \sigma(n)^3/2 [c_0(v) + \epsilon(n)c_1(v) \cos 4\psi]^{3/2}. \tag{8}
\]

We now evaluate this expression. Note that, because of the power 3/2, the four-dimensional integrals \( dA \) cannot be performed separately (Seto 2015). But we can overcome this difficulty by perturbatively expanding the term proportional to \( \epsilon(n) \) as follows

\[
U = \frac{4\pi}{3} \int dA \sigma(n)^{3/2} c_0(v) \left[ 1 + 3 \frac{\epsilon(n) c_1(v) \cos 4\psi}{c_0(v)} + \frac{3}{8} \left( \frac{\epsilon(n) c_1(v) \cos 4\psi}{c_0(v)} \right)^2 + \cdots \right]. \tag{9}
\]

After performing integrals separately, we obtain

\[
U \equiv \frac{4\pi}{3} g \times 0.82(1 + 0.01 s_2 + 2.1 \times 10^{-4} s_4 + \cdots).
\]

Here we used numerical results such as

\[
\frac{1}{2} \int_{-1}^1 dv c_0(v)^{3/2} = 0.821, \tag{11}
\]

and also defined

\[
g \equiv \frac{1}{4\pi} \int_{4\pi} dv \sigma(n)^{3/2}, \tag{12}
\]

\[
s_m \equiv \frac{1}{4\pi g} \int_{4\pi} dv \sigma(n)^{3/2} \epsilon(n)^m \tag{13}
\]

for even \( m \). For the quantity \( U \), all the geometrical information of the network are contained in \( g \) and \( s_m \).

Since \( 0 \leq \epsilon(n) \leq 1 \), we have the following inequalities for the integrals \( s_m \)

\[
0 \leq \cdots \leq s_4 \leq s_2 \leq 1. \tag{14}
\]

Therefore, after dropping the corrections \( \propto s_m \) in eq. (10), we get a good approximation

\[
U \simeq 3.44 g \tag{15}
\]

with relative error less than 1\% (Seto 2015, see also Schutz 2011).

For a network with a single detector \( i \), we identically have \( \epsilon(n) = 1 \) and \( s_m = 1 \). We also have

\[
V = \frac{4\pi}{3} \left( \frac{10}{\rho_i} \right)^3 d_{r,i}^3 \tag{16}
\]

because of the definition of the detection range \( d_{r,i} \).

3.2 distance error

We assume that the source direction \( n \) is accurately determined by the sky position of the EM counterparts such as the kilonova. Then from the information related to the extrinsic properties of GWs, we need to simultaneously estimate just the four extrinsic parameters, \( D, \psi, v \) and the initial phase of the wave. From the Fisher matrix of these parameters (Cutler & Flanagan 1994), the variance of the relative distance error for a binary is given by

\[
\left< \left( \frac{\Delta D}{D} \right)^2 \right> = 4D^2 \frac{(1 + v^2) - \epsilon(n) (1 - v^2) \cos 4\psi}{\sigma(n)(1 - \epsilon(n)^2)(1 - v^2)^2}. \tag{17}
\]

This expression depends on the four angular parameters \((n, \psi, v)\) as well as the distance \( D \). Due to the singularity of the Fisher matrix, this expression diverges at \(|v| \to 1 \) (face-on) and overestimates the variance, compared with a more elaborate nonlinear estimation (see e.g. Nissanka et al. 2013, Rodriguez et al. 2014). In contrast, for the edge-on binaries, eq. (19) would be an understimation, especially for low SNRs.

Assuming a homogeneous and isotropic binary distribution,
we can derive an averaged error in the relative distance $\sigma_{\text{inD}}$ for binaries with $\rho > \rho_T$ as

$$
\sigma_{\text{inD}}^2 = \frac{\int dA \int_0^{D_{\text{max}}} dD \left( \frac{\Delta D}{D} \right)^2 4\pi D^2}{\int dA \int_0^{D_{\text{max}}} dD 4\pi D^2}.
$$

(18)

After the $dD$ integral, we have

$$
\sigma_{\text{inD}}^2 = \rho_T^{-2} \frac{U}{X}.
$$

(19)

where we defined

$$
X = \pi \int dA \sigma(n)(1 - e^2)(1 - v^2)^2
\times \left[ \int (1 + v^2) - \epsilon(n)(1 - v^2) \cos 4\psi \right]^{-1} \int_0^{D_{\text{max}}} \text{d}D.
$$

(20)

$$
\epsilon(n) = \frac{1}{2} \left[ \frac{1}{2} c_0(v) + \epsilon(n)c_1(v) \cos 4\psi \right]^{1/2}
\times \left[ (1 - e^2)(1 + v^2) - \epsilon(n)(1 - v^2) \cos 4\psi \right]^{-1}.
$$

(21)

As shown in the $dD$ integral in eq. (20), $X$ is not dominated by small $D_{\text{max}}$ events. Therefore, for a sufficiently large number of DNS events, the statistical fluctuations of our estimation $\sigma_{\text{inD}}$ would be small.

In the same manner as $U$ in the previous subsection, after expanding the relevant factors in $X$ and averaging with the four angular parameters, we get

$$
X = g(0.966 - 0.574s_2 - 0.158s_4 - 0.068s_6 - 0.037s_8 \cdots).
$$

(22)

This expression is our new result and would be useful for statistical discussion on the local Hubble parameter measurement.

4 PROSPECTS OF O3 AND BEYOND

Based on the expressions derived in the previous section, we now discuss the prospects of the forthcoming observational runs.

4.1 observation plan

In Table 1, we summarize the actual results of the past two runs, O1 and O2, and the planned parameters for the future runs O3 and O4. Here, we denote the 2020+run (in Abbott et al. 2016) simply by O4. In Table 1, the duration $T_d$ for O1 and O2 are the total time for the simultaneous operation of the two LIGO detectors (based on Abbott et al. 2017a). For O3 and O4, the observational time relevant for the Hubble parameter estimation should be

$$
T_{\text{obs}} = f_d T_d.
$$

(23)

with the time fraction $f_d$ for the simultaneous operation of all the three detectors. The duration $T_d$ of O4 is not explicitly presented in Abbott et al. (2016).

In Table 1, the detection ranges $d_{r,i}$ are given for the threshold $\rho_T = 10$ (in contrast to the conventional $\rho_T = 8$). In the 6th column, we present the four-dimensional volume $V T_d$ using eq. (15) for $\rho_T = 10$. We also present $s_2, s_4, s_6, s_8$, and $\sqrt{U/X}$ required for the estimation of the relative distance error $\sigma_{\text{inD}}$.

At the stage 2024+ (Abbott et al. 2016), KAGRA is planned to join the detector network with $d_{r,i} = 112$ Mpc, in addition of two LIGOs ($d_{r,i} = 152$ Mpc) and Virgo ($d_{r,i} = 100$ Mpc). For these four detectors, we have $V = 68 \times 10^6$ Mpc$^3$ and $\sqrt{U/X} = 2.5$.

The DNS merger rate $R$ estimated after the GW170817 detection is (Abbott et al. 2017a)

$$
R = (1.5^{+3.2}_{-1.2}) \times 10^{-6}\text{Mpc}^{-3}\text{yr}^{-1}
$$

(90% probability range). We hereafter denote $R = f_R R_0$ with the median value $R_0 = 1.5 \times 10^{-6}\text{Mpc}^{-3}\text{yr}^{-1}$ and the scaling parameter $f_R$. Then the expected DNS events is given by

$$
N = RV(f_d T_d) = (f_d f_R R_0 T_d U_{\text{eq}})^{-3}.
$$

(25)

In reality, we will be able to identify the host galaxies for not all of the $N$ events. Therefore, we introduce the probability $f_E$ for the successful host galaxy identification, and use the total DNS events $N_E = f_E N$ for estimation of the local Hubble parameter. Here, for simplicity, we neglect the dependence of $f_E$ on the distance $D$ and the inclination $v$. Using Table 1, we explicitly have

$$
N_E = A(f_R f_d f_E) \left( \frac{T_d}{1\text{yr}} \right) \left( \frac{\rho_T}{10} \right)^{-3}
$$

(26)

with the numerical coefficients $A = 32$ for O3 and 69 for O4. In Table 2, we summarize the parameters that appear in this expression and are also useful for discussions below.

4.2 local Hubble parameter measurement

Next, we discuss the error in estimation of the local Hubble parameter $H_L$. For each DNS (label $j = 1, \cdots, N_E$) with identified host galaxies, we can estimate the Hubble parameter

$$
H_j = \frac{c z_j}{D_j}
$$

(27)

using the measured redshift $z_j$ of the host galaxy and the distance $D_j$ from GW data analysis. But both of them contain errors $\delta z_j$ and $\delta D_j$. The former would be dominated by the local peculiar velocity $v_j$ as

$$
\delta z_j \sim v_j/c,
$$

(28)

and the latter $\delta D_j$ would be the parameter estimation error at GW data analysis. Then we have

$$
\frac{\delta H_j}{H_L} \sim \frac{v_j}{c z_j} + \frac{\delta D_j}{D_j}.
$$

(29)

From eq. (19) and Table 1, the magnitude of the relative distance error is roughly estimated as

$$
\frac{\delta D_j}{D} \sim \frac{1}{\rho_T} \sqrt{\frac{U}{X}} \sim 0.3 \left( \frac{\rho_T}{10} \right)^{-1}.
$$

(30)

Meanwhile, given the typical one-dimensional velocity of galaxies $\sim 400 \text{km sec}^{-1}$ (Strauss & Willick 1995), we have $v_j/c z_j \sim 0.05$ for DNS distance $D_j \sim 100 \text{Mpc}$ ($c z_j \sim 7000 \text{km sec}^{-1}$). Therefore, for individual DNS events, the error for the Hubble parameter is approximately given by

$$
\frac{\delta H_j}{H_L} \sim \frac{\delta D_j}{D_j}.
$$

(31)

Statistically using totally $N_E$ DNSs, we have the estimation error for the local Hubble parameter

$$
\frac{\Delta H_L}{H_L} \sim f_E \frac{\sigma_{\text{inD}}}{\sqrt{N_E}} \sim \sqrt{\frac{U}{X}} \left( f_d f_R f_E R_0 T_d \right)^{1/2}.
$$

(32)

As mentioned earlier, the original expression (19) based on the Fisher matrix could be both over- and underestimate the variance, compared with a more elaborate nonlinear analysis. To include these mismatches, we introduced the correction factor $f_E$ in eq. (32). We should have $f_E \rightarrow 1$ in the limit $\rho_T \rightarrow \infty$. 

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we will have $\sim 1$ yr. Even for O3, there would be a strong motivation to suppress the amplitude calibration error much lower than 5% (Tuyenbayev et al. 2017).

### 4.3.1 optimistic case.

We assume the higher end of the DNS merger rate $f_R = 3.1$ and the high efficiencies $f_E = f_d = 0.8$ with the correction factor $f_F = 1.2$.

At the end of O3, we have $\Delta H_L/H_L \sim 0.042$ with $N_E \sim 63$. Therefore, if the current mismatch $\sim 9\%$ between the local and global Hubble parameters might be examined at 2.1$\sigma$ level. Then, after O4, we will have $\Delta H_L/H_L = 0.021$ with $N_E = 274$.

The cosmic variance due to the coherence of the peculiar velocity field is estimated to be $\sim 1\%$ at the survey depth of $D \sim 200$Mpc (Shi & Turner 1998) and could become a potential concern at O4. Even for O3, there would be a strong motivation to suppress the amplitude calibration error much lower than 5% (Tuyenbayev et al. 2017).

### 4.3.2 standard case

We assume the median value $f_R = 1.0$ for the merger rate, and the efficiencies $f_E = f_d = 0.6$ with the factor $f_F = 1.3$. As easily understood from Eqs. (26) and (33), the factor $f_R$ is the major cause of the difference from the optimistic case above.

With O3, we have $\Delta H_L/H_L \sim 0.11$ with $N_E \sim 12$. Among the expected twelve events, the maximum SNR is roughly estimated to be $10 \times 10^{1.3} \sim 23$, and smaller than that of GW170817. The error $\Delta H_L/H_L \sim 0.11$ is not so different form 0.15 obtained from GW170817. If the time fraction $f_d$ is less $0.5$, the error becomes even larger. We expect that GW observation is not likely to play a critical role to examine the Hubble tension in the next five years for this case.

### 5 SUMMARY

The GW170817 event clearly demonstrated that the DNSs could become ideal standard sirens accompanied by characteristic EM signals for host galaxy identification (Abbott et al. 2017a). The kilonova of GW170817 is genuinely unique in terms of the rapid evolution of color and magnitude and we expect that, for a good fraction $\gtrsim 50\%$ of the DNS events within $\sim 200$Mpc, we can identify their host galaxies, using their kilonovae. Therefore, the DNSs will become a powerful and reliable tool for observational cosmology. Our immediate target would be the locally measured Hubble parameter that currently has a 9% tension with the value obtained from the cosmic microwave background (Riess et al. 2016, Planck Collaboration 2016). Considering these circumstances, we discussed the prospects of the Hubble parameter measurement using DNSs observed during the forthcoming LIGO-Virgo observational runs (Abbott et al. 2016).

In order to evaluate the measurement error of the Hubble parameter estimated form multiple DNSs, we derived convenient expressions, and applied them for the three observational scenarios. If the DNS merger rate is at the high end of the current constraint $\sim 3.7 \times 10^{-4}$Mpc$^{-3}$yr$^{-1}$ and the planned sensitivities are realized for LIGO and Virgo, we could attain the accuracy $\Delta H_L/H_L \sim 0.042$ with O3. Then, the Hubble tension might be verified at 2$\sigma$ or we might indicate a potential systematic error for the traditional cosmological distance probes. Also, this precision would give a strong motivation to suppress the amplitude calibration errors of the ground-based detectors. On the other hand, if the DNS merger rate is at the low end $\sim 0.2 \times 10^{-6}$Mpc$^{-3}$yr$^{-1}$, even with the 2020+ observation, it would be unlikely to go significantly beyond the level $\Delta H_L/H_L \sim 0.15$ already obtained by GW170817 whose SNR = 32.4 is contrastingly at the high end tail.

In any case, additional DNS events with O3 would be indispensable to further constrain the DNS rate and also better under-

| $d_{r,i}$ | $d_E$ | $d_F$ | $V T_d [10^3$Mpc$^3$yr$] | $s_2$ | $s_4$ | $\sqrt{U/X}$ |
|----------|-------|-------|------------------------|-------|-------|--------------|
| O1       | 0.14yr| 56Mpc | 56Mpc                  | 2.9   | 0.859 | 0.769 3.1    |
| O2       | 0.3yr*| 38Mpc | 77Mpc                  | 0.8*  | 0.756 | 0.626 2.8    |
| O3       | 1yr   | 116Mpc| 116Mpc                 | 21    | 0.756 | 0.626 2.8    |
| O4 (2020+) | 1yr  | 152Mpc| 152Mpc                 | 72Mpc | 0.776 | 0.651 2.9    |

Table 1. Parameters for each observational run. We denote 2020+ observation (Abbott et al. 2016) by O4. Detection ranges $d_{r,i}$ for each detector is given for $1.4M_\odot + 1.4M_\odot$ DNS and the threshold SNR=10 (from Abbott et al. 2016, but O2 from Abbott et al. 2017a). The four-dimensional volume $V T_d$ for O4 is given for $T_d = 1$ yr. The numbers with the asterisk are calculated without Virgo.
stand the EM counterparts, especially the anisotropies of kilonova emissions.

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