Enhancement of superluminal weak values under Lorentz boost

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We study the local group velocity defined as the weak value of the velocity operator in the \((1+1)\) dimensional Klein-Gordon as well as Dirac theory. It was shown by Berry [J. Phys. A 45, 185308(2012)] that when the pre- and post-selected states for evaluating the weak value are chosen at random from an ensemble of available states, the local group velocity has a universal probability distribution which can have both subluminal and superluminal components. In this work, we numerically explore the role of Lorentz boost and its impact on the superluminal fraction of the total probability distribution. We show that the dependence (enhancement) of the superluminal fraction on Lorentz boost of the total probability distribution differs both qualitatively and quantitatively for the Klein-Gordon waves and Dirac waves. For the Klein-Gordon waves, the asymmetry in the distribution of group velocities around the zero velocity point in the laboratory frame is entirely responsible for the observation of relative enhancement in the boosted frame. On the other hand, for the Dirac waves, we observe an enhancement irrespective of whether the laboratory frame velocity distribution is symmetric or not.

Introduction: Since the birth of Einstein’s special theory of relativity in 1905 [1] and the velocity addition formula obtained therein, it is understood that superluminal velocity for propagation of an optical signal in vacuum is an impossibility. The idea of signal (or more precisely, information) propagating superluminaly together with special theory of relativity would naturally lead to conclusions violating causality. By 1910, Sommerfeld[3] theoretically established the fact that the velocity of wavefront of a square wave propagating through any medium is always equal to velocity of light in vacuum. But, this fact does not rule out the possibility for physical disturbances in a medium to propagate at speeds greater than the speed of light in vacuum without violating causality. In principle, the group velocity of light \((v_g)\) can exceed velocity of light in vacuum \((c)\). Such a situation can arise in case of propagation of light in medium with anomalous dispersion[4] properties i.e. the refractive index of the medium decreases as a function of frequency in a particular frequency range. In optics literature, such a medium is referred to as fast light optical medium[5].

Superluminal tunnelling times also appear quite generally in the physics of quantum tunnelling where reshaping of the wave function is primarily responsible for such effects (see [6], [7], [8], [9], [10], [11]). These are referred to as “the phenomena of mode reshaping”. It was shown by Aharonov et. al. that the tunnelling times can be understood in terms of weak quantum measurements[12]. The associated “weak value” of the position of a clock weakly coupled to the particle was shown to be directly connected to the superluminal tunnelling times[13].

Motivated by the connection of superluminality to weak values and in the backdrop of the predictions of the OPERA experiment, Berry et.al. [14] discussed a scenario which could be related to the observation of apparent superluminal velocity of neutrinos. It was shown that such an observation may be related to weak values. Later in Ref. [15] Berry considered a general study of superluminal speeds for \((1+1)\) dimensional relativistic random waves where he defined a notion of local group velocity in terms of the weak value for the velocity operator. In the present work, we extend Berry’s analysis by studying the influence of Lorentz boost on the probability distribution of the weak values of the group velocity operator. In particular, we explore the possibility of observing an enhancement in the superluminal fraction of the probability distribution by applying a Lorentz boosting to a desired frame of reference with respect to the laboratory frame where the wave packet was prepared. We perform this analysis for both \((1+1)\) dimensional Klein-Gordon and Dirac theories.

This article is organised as follows. We begin with a brief introduction to weak values and local group velocity. This is followed by a numerical study of the boost-dependence of the superluminal fraction of the probability distribution due to Lorentz boosting of the Klein-Gordon waves using the averaging techniques introduced by Berry and Shukla[16]. We then study the boost-dependence for the superluminal fraction of the probability distribution for Dirac waves. Finally, we highlight crucial differences between the Klein-Gordon and the Dirac waves and conclude.

Weak values and the local group velocity: Conventional theory of quantum measurement defines the act of measurement as a process that leads to collapse of a quantum state onto one of the eigenstates of the measurement operator and averaging over statistically large number of such measurements leads to the expectation value. This is referred to as the strong measurement. On the other hand, a lot of effort has gone into defining the notion of a weak measurement[17], where the process of measurement does not necessarily collapse the state and at the same time very little

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information regarding the state can be gleaned from such a measurement. These type of measurements are
typically referred to as unsharp or weak measurements.
By sandwicching such a weak measurement between two
strong measurements (pre- and post-selection of states),
Aharonov et. al. showed that the outcome of this
sequence of measurement (strong-weak-strong), led to
the “weak value” given below [13]

\[ A_{\text{weak}} = \Re \left[ \frac{\langle \text{post} | \hat{A} | \psi \rangle}{\langle \text{post} | \psi \rangle} \right] . \]  

(1)

Here, |ψ⟩ represents the pre-selected state in which the
system is initially prepared and |post⟩ represents the
post-selected state on which the system’s wavefunction
collapses due to a strong measurement which follows the
weak measurement. \( \hat{A} \) is an operator corresponding to
an observable whose weak value is determined. Unlike
the outcome of standard strong measurements, which
is bounded by the eigenvalue spectrum of the measure-
ment operator, this weak value is not bounded by the
range of eigenvalues of the operator and its real part has
the interpretation of conditional average of generalized
eigenvalues[18]. When the pre- and post-selected are
chosen to be the same state, the weak value reduces to
the standard expectation value. Further, one can generate a
probability distribution of the weak values if the pre-
and post-selected states are randomly chosen from an
ensemble of states. It was shown by Berry et. al. that if the
eigenvalue spectrum of the operator in a given range is
dense enough then the distribution function of the weak
values has a generalized Lorentzian form[18]. In this ar-
ticle, we will be using such an averaging scheme for weak
values following [16].

Next, we describe the notion of the local group ve-
locity which can be defined via the operator given by
\( \hat{v}(x) = 1/2(\delta(x - \hat{x}) \hat{v} + \delta(\hat{x} - x) \hat{v}) \) where \( \hat{v} = \partial_x \omega(\hat{k}) \) (k is the momentum operator). It was shown by Berry
that weak value of the group velocity operator, \( \hat{v} \) is the
same as the expectation value of the local group velocity
operator given above for appropriate choice of pre-
and post-selected states[15]. A distribution function for
the weak values can be obtained by choosing random
pre- and post-selected states. In what follows, we show
that for relativistic random waves in (1+1) dimensions,
if the frame of the observer is altered with respect to the
frame in which the wavepacket has been prepared (we
call this the laboratory frame), then the probability for
weak value of group velocity operator becoming su-
perluminal is also significantly altered. The superluminal
component of the probability distribution is defined as

\[ P_{\text{super}} = \int_{-c}^{-c} P(v) dv + \int_{c}^{\infty} P(v) dv , \]  

(2)

where the probability density of local group velocity is
given by \( P(v) \). In what follows, we shall use \( \hbar = 1, c = 1 \)
and \( m = 1 \).

**Klein-Gordon waves:** We begin with (1+1) dimen-
sional Klein-Gordon equation

\[ \frac{\partial^2 \psi}{\partial t^2} - \frac{\partial^2 \psi}{\partial x^2} = -\psi . \]  

(3)

An general initial state can be prepared as a super-
position of plane wave solutions given by
\[ \psi(x,t) = \sum_k c_k \exp(i\gamma_k(x,t)) \],
where the phases are given by \( \gamma_k = \mu_k + kx - t\sqrt{k^2 + 1} \). This state can be taken as the pre-
selected state for the weak measurement. In order to
define the wave packet \( \psi(x,t) \), we need to specify the set
of contributing wave vectors \( k \), their amplitudes \( c_k \) and
phases \( \mu_k \). (In the later sections, we will choose a certain
power spectrum for \( c_k^2 \), and \( \mu_k \) is taken to be a ran-
dom variable.) The angular frequency can be written as
\[ \omega = \sqrt{k^2 + 1} \]. The standard definition of group velocity
leads to

\[ \hat{v} = \frac{\hat{k}}{\sqrt{k^2 + 1}} . \]  

(4)

Now by choosing the pre-selected state to be the wave
packet \( \psi(x,t) \) and the post-selected state to be the
position eigenstate, the weak value for \( \hat{v} \) can be evaluated

\[ v(x,t) = \Re \left[ \frac{\sum_k c_k \frac{k}{\sqrt{k^2 + 1}} \exp(i\gamma_k(x,t))}{\sum_k c_k \exp(i\gamma_k(x,t))} \right] . \]  

(5)

Let us define \( u \equiv k/\sqrt{k^2 + 1} \) and \( c_k \equiv c_k \), \( \gamma_u(x,t) \equiv \gamma_k(x,t) \). Hence the expression for weak value of \( \hat{v} \) is found to be

\[ v(x,t) = \Re \left[ \frac{\sum_u c_k u \exp(i\gamma_u(x,t))}{\sum_u c_k \exp(i\gamma_u(x,t))} \right] . \]  

(6)

The probability density of the local group velocity is
given by

\[ P(v) = \langle \langle \delta(v - v(x,t)) \rangle \rangle_{\mu_k} . \]  

(7)

Here the notation \( \langle \langle . \rangle \rangle \) stands for statistical ensemble
average over random values of \( \mu_k \) distributed in the
interval \([0, 2\pi]\). It is important to note that ergodicity is
assumed which implies that the ensemble average value of
the velocity operator over the random values of \( \mu_k \) for
a fixed \( x, t \) will be the same as averaging over all possible
values of \( x, t \) while keeping \( \mu_k \) to be constant.

As mentioned in the introduction, the primary aim of
this study is to explore the influence of Lorentz boost
on \( P_{\text{super}} \). Before we go on, we make some remarks
on the conventions used. We call the frame in which
the wave packet is prepared as the “laboratory frame”
while the frame of reference after boosting will be called
the “boosted frame”. The laboratory frame variables
carry the subscript “lab” while the boosted frame variables carry no subscript. Under the action of the Lorentz boost on the two component wave-vector, it transforms as $k \rightarrow \gamma (k - \beta \omega)$ and $\omega \rightarrow \gamma (\omega - k \beta)$, where $v$ is the boost velocity, $k$ is the momentum and $\omega$ is the frequency and $\gamma = 1/\sqrt{1 - v^2}$. Hence the corresponding group velocity in the boosted frame is

$$u = \frac{k}{\omega} = \frac{k_{lab} - \omega_{lab} v}{\omega_{lab} - k_{lab} v} = \frac{u_{lab} - v}{1 - u_{lab} v}. \quad (8)$$

We first take a simple wavepacket consisting of a superposition of $N >> 1$ identical plane waves, with a group velocity spectrum given by

$$c^2_{u_{lab}} = \frac{1}{2} [\delta(u_{lab} - u_o) + \delta(u_{lab} + u_o)]. \quad (9)$$

This wave packet defined in the laboratory frame will be taken as a reference wave packet for studying the $P_{super}$ in the laboratory frame and the boosted frame. The numerous waves forming the wave packet move either left or right and on an average the wave packet has zero net velocity as the velocity distribution is completely symmetric about zero. Under Lorentz boost, this spectrum becomes $c^2_u = \frac{1}{2} (\delta(u - u_1) + \delta(u - u_2))$, where $u_1 = \frac{u_{o} - v}{1 - u_{o} v}$ and $u_2 = -\frac{u_{o} + v}{1 + u_{o} v}$. Now, in order to obtain the probability distribution numerically for superluminal velocity in the boosted frame, we follow the following steps:

(i) We discretize time and space coordinates in a given range. We have chosen the range of position coordinates to be $\in [-10, 10]$ and time to be $\in [0, 20]$ in the laboratory frame.

(ii) We consider 200 plane waves components whose superposition constitute the wave packet corresponding to the pre-selected state for evaluation of weak value of the velocity operator. The velocities of these 200 plane wave components are sampled from a given distribution in the laboratory frame. And the relative phases between these components are chosen from a flat random distribution.

(iii) Then we compute the group velocity at each space-time point on the grid and we add unity to the superluminal counter if it turns out to be greater than one. We repeat this step for the next grid point and continue till we have covered the entire region of space-time. We take the final count for the superluminal counter and divide it by the total number of iterations and this provides us with the superluminal fraction.

(iv) For any value of given boost velocity, the procedure in (iii) is repeated with the boosted distribution and boosted space-time coordinates. This superluminal fraction is then plotted against boost velocity.

With the above described scheme for numerical evaluation we attempt to find appropriate distributions for $u_{lab}$ which lead to an enhancement in the superluminal probability distribution ($P_{super}$) under the action of Lorentz boost. Our analysis indeed leads to an affirmative answer. We note that from Eq. 9, the group velocity spectrum is symmetric about the zero velocity ($u_{lab} = 0$) which renders the laboratory frame to be the rest frame (zero average velocity for the wave packet). In this case, Fig. 1(a) shows that there is no enhancement in $P_{super}$ as a function of boost velocity. It turns out that if we take a asymmetric spectrum for $c_{u_{lab}}$ (the wave packet is not at rest in the laboratory frame), it opens up possibilities for enhancement of the $P_{super}$ as a function of boost velocity. This is shown numerically in Fig. 1(b). We take two sharply peaked wave packets (Gaussian with low variance) asymmetrically placed around $v = 0$, thus resembling Eq. 9. We see that the superluminal probability does increase with boost but not monotonically. And eventually, it falls off to zero in the ultra relativistic limit i.e., $v \rightarrow c$ (see Fig. 1(b)) after hitting a maximum value. This is interesting as it implies that given a group velocity distribution with a given degree of asymmetry in the laboratory frame, there is a corresponding value of boost speed for which $P_{super}$ attains a maximum value. In the specific case in Fig. 1(b), the maximum is reached at $v = 0.51c$ at which the superluminal fraction is 75% higher than the laboratory frame.

This particular observation of enhancement in the superluminal probability as a function of boost can be directly attributed to the asymmetry in the spectrum of $c_{u_{lab}}$. We also survey the impact of crucial parameters governing this asymmetry in the spectrum of $c_{u_{lab}}$. One such parameter is having different relative weights of the peaked Gaussian in the negative and positive velocity domain in $c_{u_{lab}}$ while keeping the width of the negative and positive velocity distribution to be the same. This is shown in Fig. 2(a). The effect of asymmetry in the relative weights of positive and negative velocity distri-
FIG. 2: (a) $P_{\text{super}}$ plotted as a function of boost velocity ($v$) for two different group velocity spectra prepared in the laboratory frame. Both the spectra are given by two Gaussians peaked at $u_1 = 0.1c$ and $u_2 = -0.9c$ and $\sigma = 0.01$. The red curve corresponds to 1.5 times higher weight factor for the positive velocity partial waves with respect to the negative velocity partial waves and the blue curve corresponds to having equal weights for both positive and negative velocity partial waves. (b) Comparison between two group velocity spectra, with two Gaussian packets peaked at $u_1 = 0.1c$ and $u_2 = -0.8c$ (the red curve corresponds to $\sigma = 0.01$ and the blue curve corresponds to $\sigma = 0.04$).

Distributions lead to large initial enhancement of the $P_{\text{super}}$ (red curve in Fig. 2a) over the symmetric case (blue curve in Fig. 2a) but eventually they reach similar maximum value and then tend to zero as $v \to c$. Next, we study the influence of width of the peaked Gaussian in the negative and positive velocity domain in $c_{\text{super}}$. This is demonstrated in Fig. 2b. The blue curve corresponding to wider spectrum does show an increase in superluminal fraction with boost but it is much lower than its narrower counterpart (red curve). Hence one can conclude that increase in width of the distribution could lead to large suppression in enhancement of the $P_{\text{super}}$ under influence of boost at low and intermediate values of $v$ though eventually both of them tend to zero as $v \to c$.

So far the numerical results presented above are for the case of wave packets comprising of positive energy (positive $\omega$) partial waves alone. Next, we will explore the influence of including partial waves with both positive and negative energy components in the composition of the wave packet. Here, we construct our pre-selected state $\psi$ as a equal superposition of positive and negative energy partial waves

$$\psi = \sum_u [c_{u+} \exp(i\gamma_{u+}) + c_{u-} \exp(i\gamma_{u-})] . \tag{10}$$

Also, $c_{u+} = c_{u-}$, $\gamma_{u+} = \mu_k + kx - t\sqrt{k^2 + 1}$ and $\gamma_{u-} = \mu_k + kx + t\sqrt{k^2 + 1}$. Following the same procedure with this modified wave function, we numerically investigate the difference with respect to the previous case. The result is shown in Fig. 3. A distribution with equal positive and negative energy components show a smaller superluminal fraction for smaller boost but for larger boost, one obtains an enhanced value (blue) with respect to the wave packet comprising of positive energy components alone (red). The relative increment is observed to be as large as 40–45% at around $v = 0.8c$. It is worth pointing out that in general we always observe a cross-over between two situations, one being the case where wave packets with combination of positive and negative components dominate the $P_{\text{super}}$ in the $v \to 1$ limit and the other being the lower $v$ limit where the opposite happens.

However, the interesting aspect is that the enhancement is linked to the asymmetry of the distribution ($c_{\text{super}}$) and is a robust outcome. Even if we perturb the parameters of the wave packet, the feature of enhancement does not disappear.

**Dirac waves** : The Dirac equation in (1+1) dimension is given by

$$i\partial_t \psi = \hat{H}\psi = (\sigma_z \hat{\kappa} + \sigma_x)\psi , \tag{11}$$

where $|\psi\rangle$ is a two-component column vector and $\sigma_x, \sigma_z$ are the usual 2×2 Pauli matrices. We can express the above equation as

$$\begin{pmatrix}
  i\partial_t \eta \\
  i\partial_t \chi
\end{pmatrix}
= \begin{pmatrix}
  1 & 0 \\
  0 & -1
\end{pmatrix}
\begin{pmatrix}
  -i\partial_t \eta \\
  -i\partial_t \chi
\end{pmatrix}
+ \begin{pmatrix}
  0 & 1 \\
  1 & 0
\end{pmatrix}
\begin{pmatrix}
  \eta \\
  \chi
\end{pmatrix} . \tag{12}$$

The effect of Lorentz boost is incorporated as $(\eta, \chi) \to (\eta, \chi)\hat{U}^T$ where $\hat{U} = \text{diag}(\sqrt{1+v}, \sqrt{1-v})$ represents the action of boost on the two-component wave function and $v$ is the boost velocity. Thus, the boosted spinor is given by $(\sqrt{1+v}_\eta, \sqrt{1-v}_\chi)$.

Next, we note that the group velocity operator for the Dirac equation is given by $\hat{v} = \partial_k \hat{H} = \sigma_z$ whose eigenvalue spectrum is bounded within $\pm 1$ in units of velocity of light as expected. We wish to compute the weak values for the group velocity operator in this case by considering the pre-selected state to be a wave packet formed by superposition of plane wave solutions of the Dirac equation with random relative phase between them while the post-selected state is taken to be the position eigenstate chosen at random. To implement the random choice of states, we first note that the normalized wave functions of the Dirac equation can be mapped on to the Bloch sphere owing to two-component nature. Hence, we choose a random distribution for the two-component wave functions.
by choosing a uniform random distribution of these states on the Bloch sphere. Hence the post-selected state can be represented by \( \text{post} = (\cos(\theta/2), e^{-i\phi} \sin(\theta/2)) \) where \( \theta \) and \( \phi \) are taken to be uniform random variables.

Now, with \((\eta, \chi)\) as the pre-selected state, the weak value of the group velocity operator in the boosted frame of reference is given by

\[
V(x, t) = \Re \left[ \frac{\cos(\theta/2) \sqrt{\frac{1+u}{1+u}} - e^{-i\phi} \sin(\theta/2) \sqrt{\frac{1+u}{1+u}}}{\cos(\theta/2) \sqrt{\frac{1+u}{1+u}} + e^{-i\phi} \sin(\theta/2) \sqrt{\frac{1+u}{1+u}}} \right] 
\]

\[
= \frac{1 - \tan^2(\theta/2)[\frac{\pi}{\eta} - \frac{\eta^2}{\eta^2}]]^2 + 2 \tan(\theta/2)[\frac{\eta^2}{\eta^2} - \frac{1}{1+u} \cos \phi},
\]

where \( V(x, t) = \Re(\langle \text{post} | \hat{V} | \psi \rangle / \langle \psi | \psi \rangle) \) and \( \varphi = \phi - \arg(\chi/\eta) \). Next, we note the two-component wave function corresponding to the positive energy eigenstate of the Dirac equation (Eq. 11) in the laboratory frame can be expressed in terms of the group velocity for the partial plane waves \((u = k/\sqrt{1+k^2})\) defined in Eq. 4 as \((\sqrt{1+u}, \sqrt{1-u})\).

The wave packet representing the pre-selected state constructed from superposition of plane wave solutions of the Dirac equation with positive energy can be expressed as \((\eta, \chi) = (2\sum_{\mu_k} e^{i\eta\mu_k}\sqrt{\frac{1+u}{1+u}} + \sum_{\mu_k} e^{i\mu_k} \sqrt{\frac{1+u}{1+u}} \sin \phi)\) with an appropriate normalization imposed on it. On substituting this expression in Eq. 14 we can now obtain a distribution for the weak value for the group velocity operator by assigning a uniform random distribution on \( \mu_k \) and choosing a particular distribution for \( c_{\mu_k} \). Also, we pick the post-selected states from a uniform random distribution over the Bloch sphere.

Hence the construction for the probability distribution in Dirac case uses the same recipe as given in Eq. 7 for the pre-selected state (i.e., averaging over \( \mu_k \)) but additionally it also incorporates uniform averaging over \( \theta \) and \( \phi \) in the post-selected state which is what makes it different from the case of Klein-Gordon waves. The probability density function for Dirac waves is thus given by

\[
P(v) = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta \langle \delta(v - v(x, t)) \rangle_{\mu_k} \sin \phi. \tag{15}
\]

The resulting distribution of \( P_{\text{super}} \) (using Eq. 15) has been plotted in Fig. 4.

In this case, the numerical analysis has been carried out following steps outlined for the Klein-Gordon case along with an additional complication arising due to the Bloch sphere averaging of the wave functions. In this case, we also have a grid for \( \theta \) and \( \phi \) along with \( x \) and \( t \) which helps us to implement the random choice of Dirac spinors. For every point on the grid \((x, t, \theta, \phi)\), the group velocity is calculated and \( V > 1 \) is checked for adding unity to the superluminal counter. We move on to the next point on the grid and so on till we have covered the entire region of interest. We take the final value of the superluminal counter and divide it by the total number of iterations and this provides us the superluminal fraction.

Fig. 4 shows the evolution of superluminal probability corresponding to the above given distribution as a function of the boost velocity. Unlike the case of Klein-Gordon equation where the symmetric distributions never lead to increment of superluminality, the Dirac equation does lead to enhancement of superluminality irrespective of whether the initial spectrum for \( c_{\mu_k} \) is symmetric or asymmetric around the \( v = 0 \) point. This can be attributed to the two-component nature of the wave function in the Dirac case (as opposed to the Klein-Gordon case) which naturally leads to an averaging scheme over the Bloch sphere. Lastly, one should note that averaging over the full Bloch sphere automatically incorporates averaging over both negative and positive energy states.

**Klein-Gordon versus Dirac waves:** We next make a comparison of the observation of superluminality in the case of Klein-Gordon waves and the Dirac waves. We study the dependence of the \( P_{\text{super}} \) as a function of the boost velocity for a given specific post-selected state on the Bloch sphere and then contrast it with the results obtained for the Klein-Gordon case.

We choose the laboratory frame distribution for the group velocities to be given by

\[
c^2_{\mu_k} = \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(n_{\text{lab}} - n_1)^2}{2\sigma_1^2}} + \frac{1}{\sqrt{2\pi}\sigma_2} e^{-\frac{(n_{\text{lab}} - n_2)^2}{2\sigma_2^2}}, \tag{16}
\]

to carry out the simulations. The phase \( \mu_k \) is taken to be a random variable distributed uniformly between 0 and \( 2\pi \). Note that it is not sufficient to specify the random phases for the Dirac case, we also have to fix the post-selection angles \( \theta \) and \( \phi \). This additional input compared to Klein-Gordon leads to sharp distinction between the two cases. Depending on values of \( \theta \) and \( \phi \), the superluminality might show an increase, decrease or a non-monotonic behaviour as a function of the boost velocity. Fig. 5 shows two plots, (a) with \( \theta = \pi/2 \) shows a steady
It is a pleasure to thank the observation of superluminality can be attributed to probability distribution. For the Klein-Gordon waves, independent of variation of parameters influencing the two-component nature of the wave function for Dirac waves as opposed to the Klein-Gordon waves. The fact that superluminal probability for local group velocity can be incremented by changing the frame of the observer for relativistic waves is the key finding of the present work. Our results could motivate new avenues and possibilities for designing of experiments dealing with superluminal group velocity of light.

**Concluding remarks:** We show that Lorentz boost can lead to significant enhancement in superluminal component of the local group velocity for relativistic random waves both for (1 + 1) dimensional Klein-Gordon and Dirac cases. It is shown that this outcome is robust and is independent of variation of parameters influencing the total probability distribution. For the Klein-Gordon waves, the observation of superluminality can be attributed to the asymmetry in the velocity distribution function of the plane waves about the zero velocity point constituting the wave packet for the pre-selected state. In contrast to this, for the Dirac waves, this is no longer found to be true.

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