Particle interactions predicted from minimum information

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Abstract. A complete structure of particle interactions is predicted in the scope of minimum information quantum gravity (MIQG). This structure comprises a large but finite chain of interaction generations, one of which is already realized by the electroweak and the strong interaction. All further claimed interactions of the chain have not yet been observed, but should be observable in future high energy particle collision experiments. MIQG only assumes abstract quantum number conservation, the existence of general space-time thermodynamics and minimization of the input degrees of freedom. As shown in a previous publication, no explicite microscopic quantum structure is required in order to recover all well established physics as special cases and compute all measurable quantities in arbitrary non-singular gravitational fields.

Keywords: Theoretical Particle Physics, Standard Model, Quantum Gravity, General Relativity

1. Introduction

Despite considerable efforts, conventional attempts to quantize gravity are still facing difficulties at the most fundamental level [1]. In contrast, a thermodynamic concept can solve many problems which are due to incompatibilities between general relativity and quantum field theory. First indications of a thermodynamic nature of gravity have been found by Bekenstein [2] and Hawking [3]. Moreover, variation of the statistical and Wald’s entropy give an equivalent result [4]. Minimum information quantum gravity (MIQG) infers that space-time parameters are fundamentally macroscopic.

Unlike many theories of emergent space-time and gravity [5], MIQG is based on only three general assumptions [6] [7] [8]:

(i) quantum statistics,
(ii) macroscopic interpretation of space-time and
(iii) local maximization of the number of output per input degrees of freedom.
MIQG does not allow any ambiguities in the derivations. In contrast, approaches like loop quantum gravity or precanonical quantization differ from each other by the quantization method. Precanonical quantization is based on the De Donder-Weyl formulation and attempts an explicitly covariant quantization [9], while loop quantum gravity uses the canonical quantization for which the time dimension is distinguished [10]).

The new results of this article are related to particle interaction structures. The symmetries arise because the fundamental quanta can be assigned arbitrarily to any location of space-time, and the particles emerge from constraints due to the first law of thermodynamics.

2. Basic concept

According to MIQG, the world is made up of abstract primary quanta. One may arbitrarily order the quanta and assign to the ordering a macroscopic parametrization [8]. Physical observations are invariant under changes of the parametrization. It has been shown in [8] that the parametrization space must be a smooth manifold with a local Lorentzian structure, provided that small departures from the thermal equilibrium are mathematically well-behaved.

Applying the second law of thermodynamics yields a generalized form of Einstein’s field equations. The parametrization is identified as the space-time and the primary quanta generate the ADM mass in asymptotically flat spaces, while secondary quanta of angular momentum generate the ADM angular momentum [11] [12] ‡.

3. From quanta to particles

The arbitrarily ordered primary and secondary quanta can be assigned arbitrarily to space-time locations. This yields an extra term in the first law, describing all quantum number variations on \( \mathcal{M} \). Consider a thermally small space-time region \( \mathcal{M} \) (small enough to be approximately in equilibrium [8]), with boundary \( \partial \mathcal{M} = \Sigma \cup \mathcal{T} \), its space-like part \( \Sigma \) and time-like part \( \mathcal{T} \) being smooth and normal to each other, and possessing an approximate rotation isometry with translation vectors (2d-spatial index \( a \) and Minkowski index \( A \)) contained everywhere within the subspaces associated to \( \partial \Sigma = \Sigma \cap \mathcal{T} \) and \( \mathcal{T} \). Then, the first law reads:

\[
\delta U = T \delta S + \omega^a \delta J_a - P_A^a \delta V^A_a - \mu_R \delta n^R,
\]

‡ In addition to the rotation gauge, there is also the freedom to perform Lorentz boosts (with local Lorentz invariance and asymptotical invariance with respect to the BMS group [13] [14]), which, however, are not related to a coordinate gauge parameter.
where the first three terms are the pure gravity terms, the last term contains the quantum numbers of mass \(n^0\) and three angular momentum parameters \(n^1 \ldots n^3\), with coefficients \(\mu\). If \(\mathcal{M}\) is immersed in a bath of quanta, we instead consider the Legendre transform \(\mu = \mu \to n^R \delta \mu^R\). Eqn. (1) may be rewritten as a boundary integral, with extensive "quantum number densities" \(s^R_A\) and coefficients \(\kappa_{AR}\), together with the gravity term:

\[
\delta S\Big|_{\partial \mathcal{M}} = \sum_{A=\Sigma,T} \int_{\partial A} d^2 x \sqrt{|\gamma|} \left[ e^I_i \delta \tau^i_A + \kappa_{AR} \delta s^R_A \right],
\]

whith indices \(I\) (Minkowskian) and \(i\) (Lorentzian) refering to the projections onto the local vector subspaces associated to \(\Sigma\) and \(T\), \(\gamma_{ij}\) denotes the corresponding intrinsic 3d-metrics with determinant \(\gamma\), and \(\tau^i_A\) is the variable conjugate to the triads \(e^I_i\), and following the conventions in [6], [7], [15], [16] and [18]. The \(s^R_A\) are not completely determined.

In Eqn. (1), if we vary the "inner energy \(U\)" and the angular momentum \(J^a\), and fix all other variables to the physical values, we find that the mass and the angular momentum are not independent. Therefore, we impose constraint equations of the form

\[
\xi_J(x^i; s^R) = c_J(x^i)
\]

to the second law. We introduce Lagrange multipliers \(\lambda^J = \lambda^J(x^i)\) and write

\[
\delta S\Big|_{\partial \mathcal{M}} = \sum_{A=\Sigma,T} \int_{\partial A} d^2 x \sqrt{|\gamma|} \left[ e^I_i \delta \tau^i_A + \kappa_{AR} \delta s^R_A + \delta \left( \lambda^J_A \xi_{AJ} \right) \right] = 0,
\]

\[
\sum_{A=\Sigma,T} \int_{\partial A} d^2 x \sqrt{|\gamma|} \xi_{AJ} \delta \lambda^J_A = 0.
\]

After Legendre transformation, this yields the condition

\[
\delta \xi_{AJ} = -\frac{s^R_A \delta \kappa_{AR}}{\lambda^J_A} \approx -\frac{n^R \delta \mu^R}{\lambda^J_A \sum_{A} \int_{\partial} d^2 x \sqrt{\sigma}} \propto \frac{n^R}{\lambda^J_A}; \quad \lambda^J_A \propto n^R.
\]

\(\sigma\) is the determinant of the 2-metric of \(\partial \Sigma\). Conformly to Eqn. (6), define the particle as the smallest quantum number set. \(\xi_{AJ}\) is proportional to the numbers \(n^R\) of quanta per particle. \(\lambda^J\) is proportional to the particle number density. To obtain a quadratic form, we decompose the expression \(\lambda^J_A \xi_{AJ}\) to

\[
\lambda^J_A \xi_{AJ} = \psi^J_A \alpha^\mu \psi^J n_\mu = \pi^J_\mu \psi^J n_\mu = \pi^J \psi^J.
\]

The dagger in \(\psi^J_A\) has been incorporated because of the undetermined mathematical format required for \(\psi^J\) and \(\alpha^\mu\), the unit normal vector \(n_\mu\) on the boundary part \(A\) accounts for the flow of the particle field across the boundary, and \(\pi^J_\mu = \psi^J_\alpha \alpha^\mu\). Reinserting Eqn. (7) into Eqn. (4), applying Gauss’ theorem and finally using the
constraint Eqn. (5) and the procedure shown in [8] yields the (Legendre transformed) particle contribution to the entropy, with four degrees of freedom, $\Gamma = 0 \ldots 3$:

$$
\delta S_p|_{\partial \mathcal{M}} = \int_{\mathcal{M}} \mathrm{d}^4 x \sqrt{-g} \left[ (\nabla_\mu \delta \pi^\mu_\Gamma) \psi^\Gamma + (\delta \pi^\mu_\Gamma) \nabla_\mu \psi^\Gamma \right] 
$$

(8)

$$
S_p = \int_{\mathcal{M}} \mathrm{d}^4 x \sqrt{-g} \left[ (\nabla_\mu \psi^\dagger_\Gamma) \alpha^\mu \psi^\Gamma + \psi^\dagger_\Gamma \alpha^\mu \nabla_\mu \psi^\Gamma + \psi^\dagger_\Gamma (\nabla_\mu \alpha^\mu) \psi^\Gamma \right]. 
$$

(9)

It shall be shown next that the particle contribution to the entropy is $U(1) \times SU(2)$ invariant. For thermally small $\mathcal{M}$, choose $\alpha^\mu$ so that $|\psi^\dagger_J (\nabla_\mu \alpha^\mu) \psi^J| \ll |\psi^\dagger_J \alpha^\mu \psi^J n_\mu|$. One can modify $\alpha^\mu(x^\nu)$ smoothly towards $\alpha'^\mu(x^\nu) = \text{constant}$, while $\psi^J \rightarrow \psi'^J$ smoothly and while keeping the symmetries of $S_p$. Consider a local Minkowski frame ($g = -1$). The plane wave ansatz $\psi_J = \psi_{J0} e^{ip_\mu x^\mu}$ solves $\delta S_p = 0$. Insert the plane wave ansatz and the constrained factor $\delta (\psi^\dagger_J \zeta) = \nabla_\mu \delta \pi^\mu_J |_{\mu, \mu'=m^2}$ into (8):

$$
\delta S_p|_{\partial \mathcal{M}} = \int_{\mathcal{M}} \mathrm{d}^4 x \left[ \delta (\psi^\dagger_J \zeta) \psi^J + (\delta \pi^\mu_J) i p_\mu \psi^J \right]. 
$$

(10)

After replacing $ip_\mu \rightarrow \partial_\mu$ and writing $S_p$ conformly to [8], the Euler-Lagrange equation yields

$$
\psi^\dagger_J \zeta - \partial_\mu \psi^\dagger_J \alpha^\mu = \psi^\dagger_J \zeta - i p_\mu \psi^\dagger_J \alpha^\mu = 0, 
$$

(11)

with the plane wave inserted in the second expression. Solutions of $\delta S_p = 0$ also are solutions of

$$
\psi^\dagger_J (\zeta - i p_\mu \alpha^\mu)(\zeta - i p'^\nu \alpha^\nu) = 0. 
$$

(12)

The product of the terms containing $p_\mu$ yields the expression $\psi^\dagger_J \alpha^\mu \alpha^\nu p_\mu p'^\nu$. The definition of $\alpha^\mu$, Eqn. (7), fixes $\alpha^\mu \alpha_\mu = 1$. Also imposing $p_\mu p'^\mu = m^2$ leads to the anti-commutator relation for $\alpha^\mu$,

$$
\{\alpha^\mu, \alpha^\nu\} = 2 \delta^\mu_\nu. 
$$

(13)

This is the same structure as for the Dirac equation describing spin 1/2 particles and makes $S_p$ $SU(2)$-invariant which accounts for three degrees of freedom of the symmetry. An additional one-parameter symmetry makes $S_p$ invariant under phase transformations $\psi^J \rightarrow e^{i\alpha(x^\mu)} \psi^J$ ($U(1)$-invariance) and accounts for the fourth degree of freedom. In summary, we obtain $U(1) \times SU(2)$ invariance, as claimed above.

The boundary variation term $\sim \kappa_{\text{Ar}} \delta s^\text{N}_A$ also contributes to the dynamics of matter and can be brought to a similar form as Eqn. (7), although the "vector structure" of $\delta s^\text{N}_A$ is different. This yields a potential analogous to $\psi^J$, with the plane wave as a solution in the Minkowski frame in the limit of negligible spin from $\psi^J$. The three degrees of freedom for spin yield three "charges" which are assigned arbitrarily, so that the entropy is invariant under complex transformations with unit determinant ($SU(3)$-invariance), in analogy to the strong interaction structure for colour charges in QCD. There remains a fourth degree of freedom due to $s^0_A$, which yields a 1-parameter symmetry.
4. Higher level constraints

The structure developed in Section 3 can be extended. We expand $\delta S$ using triads (omitting the subscript $A$):

$$
\delta S_p \bigg|_A = \int_A d^3x \sqrt{|\gamma|} \pi_I \delta \psi^I = \int_A d^3x \sqrt{|\gamma|} \pi_i \delta \psi^i,
$$

where $\pi_i = \pi_I e_I^i$ and $\psi^i = \psi^I e_I^i$. In order for the temperature to be smooth, $\pi_i$ and $\psi^i$ must also be smooth and thus slowly varying across $\mathcal{M}$, and the diffeomorphism invariance of $S$ forces $\pi_i \psi^i$ to transform as a scalar. Correspondingly, $\pi^I$ and $\psi^I$ have the structure of Lorentz vectors.

Because $S_p$ is $SU(2)$-invariant, we can translate the ADM decomposition of $[8]$ to the Minkowskian vector space $Q$ with $\Phi^\Gamma(x^\mu) = x^\mu e^\Gamma = y^\Gamma \in Q$, as shown for the time-like boundary $T$:

$$
\delta S \bigg|_{\Phi^\Gamma(T)} = \int_{\Phi^\Gamma(T)} d^3y \left[ \tilde{N} \delta \tilde{e} + \tilde{N}_i \delta \tilde{e}^i + \tilde{N}_s \delta e + \ldots \right],
$$

where the tilde ($\sim$) refers to $Q$. In analogy to the angular momentum $[8]$, the decomposition of Eqn. (15) gives rise to a conserved quantity, the analogue "angular momentum" $J^A_\alpha$ in $Q$. Thus, a new set of quantum numbers associated to $J^A_\alpha$ emerges, with level 1 quantum density $s^{1}_{\alpha}. The interaction level is written in parentheses. In analogy to $s^0$ (or $s^N_0$ for clarity), a term with $s^1_0$ is added to the expression for $\delta S$,

$$
\delta S \bigg|_T = \int_T d^3x \sqrt{-\gamma} \left[ \tau^i_I \delta e^I_i + \pi_0 I \delta \psi^I_0 + \kappa_0 I \delta s^0 + \ldots \right].
$$

$s^0_1$ cannot be varied independently of the other quantum numbers because of the first law. We impose additional constraint equations on the second law,

$$
\xi_{(1)JK}(x^I; s^N_0; s^1_0) = c_{(1)JK}(x^I; s^N_0),
$$

where $J$ labels particles and $I$ labels degrees of freedom. We introduce level 1 Lagrange multipliers $\lambda^{JK}_{(1)}(x^I)$ and repeat the procedure of Section 3 to obtain:

$$
\delta S \bigg|_T = \int_T d^3x \sqrt{-\gamma} \left[ \tau^i_I \delta e^I_i + \pi_0 I \delta \psi^I_0 + \pi_{(1)JK} \delta \psi^{JK}_{(1)} + \ldots \right].
$$

This procedure also yields the level 1 particles, which are constrained by the level 1 quantum numbers. Thus, level 1 particles can only be added or removed from level 0 particles introduced in Section 3.
Due to spin conservation, the level 1 particles must have integer spin and thus the symmetry group on $\mathcal{Q}$ must be the Lorentz group. This may also be found by transforming the second index of $\psi^{IK}$ via $(\phi^I)^{-1}$ to a Lorentzian index (suppressing the level index), $\psi^{IK} \rightarrow \psi^I{}^\mu = \psi^{IK} e^K_\mu$. With $\omega_\nu = \delta \pi^{\mu \nu} \psi^I{}^\mu$ and applying Stokes’ Theorem yields

$$\int_{\partial \mathcal{M}} \omega = \int_{\mathcal{M}} d\omega = \int_{\mathcal{M}} d^4x \left[ (\nabla^\nu \pi_{I \mu \nu}) \psi^I{}^\mu - (\nabla^\mu \pi_{I \mu \nu}) \psi^I{}^\nu + \pi_{I \mu \nu} F^{I \mu \nu} \right]$$

with $F^{I \mu \nu} = \nabla^\nu \psi^I{}^\mu - \nabla^\mu \psi^I{}^\nu$. The spatial elements of $F^{I \mu \nu}$ are the generators of rotations in $SO(3)$. Combining this with the remaining one-parameter symmetry component yields the Lorentz invariance, as claimed.

Following the same procedure as for level 1, we obtain the level 2 quanta, $\psi^{s_1 s_2 s_3}$, one more variation term, the level 2 constraint, the level 2 particles, and so on, as long as higher level particles can be built. Increasing levels involve more and more quantum numbers entering the first law, letting some particle masses increase as well. However, the expansion of levels should stop before the Planck energy is reached. This yields, for levels running from 0 to $L$:

$$\delta S = \int_\mathcal{T} d^3x \sqrt{-g} \left[ \tau^I_{\mu} \delta e^I_{\mu} + \sum_{l=0}^L \pi_{(l)I_1...I_{l+1}} \delta \psi^{I_1...I_{l+1}+1} + \kappa_{(l)s_1...s_{l+1}} \delta s^{s_1...s_{l+1}} \right].$$

Finally, the total entropy is obtained from the above expressions and from [7]:

$$S_{\text{total}} = \int_{\mathcal{M}} d^4x \sqrt{-g} \left[ \epsilon^I_{\mu} \epsilon^J_{\nu} \Phi_{IJ}^{\mu \nu} + \omega_{\mu I J} \Omega_{IJ} \right]$$

$$+ \sum_{l=0}^L \sum_{b=\psi,s} \int_{(l)} b^{I_1...I_{l+1}} l^{I_1...I_{l+1}+1} + F^{I_1...I_{l+1}}_{b(l)j} F^{I_1...I_{l+1}}_{b(l)j},$$

where $\omega_{\mu I J}$ is the connection 1-form, $\Phi_{IJ}^{\mu \nu}$ is the generalized curvature 2-form, $\Omega_{IJ}$ is the gravitational variable conjugate to the connection 1-form, $F^{I_1...I_{l+1}}_{b(l)j}$ is the antisymmetrized covariant derivative of $b^{I_1...I_{l+1}}(l)$, $F^{I_1...I_{l+1}}_{b(l)j}$ is the generalized current density (the antisymmetrized covariant derivatives of $\pi_{(l)I_1...I_{l+1}}$ and $\kappa_{(l)s_1...s_{l+1}}$ for $b = \psi$ and $b = s$, respectively), $F^{I_1...I_{l+1}}_{b(l)j}$ is the generalized field corresponding to the conjugate potential, and the index notation has been relaxed ($\mathcal{N} \rightarrow I$).

5. Identification and prediction of interactions

5.1. Level 0

The level 0 potential $\psi^I_{(0)}$ has the same rank as the 4 formal Dirac functions $(\psi_Y^{I_3W}) = (\psi^0, \psi_{1/2}, -1/2, \psi_{-1/2}, 1/2, \psi_{-1})$, where $Y$ and $I_3W$ denote the weak hypercharge and the weak isospin, respectively. The spinor structure arises only once the variation with respect to $\psi^{I_3W}$ is carried out and the Euler-Lagrange Equation (Dirac Equation) is being solved. The level 0 provides the required $U(1) \times SU(2)$ invariance.
5.2. Level 1

The relevant level 1 potential \( \psi^{I\mu}_{(1)} \) has the same rank and index structure as the set of electroweak interaction potentials \( (B^\mu, W^\mu_K) \) associated to the photon, the \( Z^0, W^+ \) and \( W^- \) bosons. To lowest order, these bosons are exchanged between fermions (level 0 particles), as it should be. The gauge space associated to these bosons has Lorentz-symmetry with subgroup \( SO(3) \), as required for spin 1 bosons.

The quantum number variation leads to one more potential with 3 particle charges and 3 anti-particle charges with \( SU(3) \) symmetry. This interaction corresponds precisely to the strong interaction, while the charges are identified to be the colour charges and the exchanged ”particles” are the gluons. To lowest order, the gluons are exchanged between fermions, as it should be. Quarks have non-zero colour charge.

The mass quantum number also must be taken into account. The mass quantum potential must

(i) be a scalar potential and
(ii) be exchanged between particles that can be members of any level of interaction.

The only known potential which fulfills both of these requirements is the Higgs potential.

5.3. Higher level interactions

All other interactions predicted from MIQG according to our expansion, Eqn. (21), have not yet been observed. However, the particle collision energy might be required to be higher than for the lower level interactions, in order for the higher level particle resonances to be observed. In particular, consider the level 2 potential \( \psi^{IJK}_{(2)} \) and its associated particles which are exchanged between the bosons \( W^* (\gamma, Z^0, W^+, W^-) \) of the electroweak interaction. Conformly to Eqn. (21), there should be 16 level 2 bosons \( X^{IJ} \) which can interact with \( W^* : W^* \rightarrow W^{*I} + X^{IJ} \).

Finally, consider the level 2 quantum number interaction between bosons of the electroweak interaction, involving \( 4 \times 3 \) colour-like charges plus anti-charges. Thus, not all level 1 bosons are subject to the same level 2 interactions, and there must be families of different vector boson generations. Hence, there must be several families of level 0 fermions (including possibly the flavours).

6. Conclusions

Starting from essentially three assumptions, the most general form of quantum gravity plus matter has been derived and formally constructed. By construction, the model reproduces general relativity and quantum field theory as special cases. Moreover, all known structures of the standard model and additional structures of interactions and
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particles have been deduced from the model. In particular, the model predicts a chain of several levels of interactions. On each interaction level, there is an interaction involving the exchange of new particles, and there is an interaction involving the exchange of a new elementary set of quanta, also observable as "elementary particles". It is hoped that the existence of some of the higher level interactions might be confirmed in high energy collision experiments, via the observation of new particle resonances and the associated decay channels.

Acknowledgements

I would like to thank Philippe Jetzer for hospitality at University of Zurich.

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