Engineering Superconducting Phase Qubits

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The superconducting phase qubit combines Josephson junctions into superconducting loops and defines one of the promising solid state device implementations for quantum computing. While conventional designs are based on magnetically frustrated superconducting loops, here we discuss the advantages offered by \( \pi \)-junctions in obtaining naturally degenerate two-level systems. Starting from a basic five-junction loop, we show how to construct degenerate two-level junctions and superconducting phase switches. These elements are then effectively engineered into a superconducting phase qubit which operates exclusively with switches, thus avoiding permanent contact with the environment through external biasing. The resulting superconducting phase qubits can be understood as the macroscopic analogue of the ‘quiet’ \( s \)-wave–\( d \)-wave–\( s \)-wave Josephson junction qubits introduced by Ioffe et al. [Nature \textbf{398}, 679 (1999)].

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In a quantum computer the information is stored in quantum two-level systems — these qubits replace the familiar bits of the classical computer as the basic computational unit. Calculations are performed through the usual quantum state evolution, modifying the superposition of basis states on individual qubits and the entanglement of states between different qubits. Maintaining the coherence of the quantum device throughout the calculation is the prime challenge in the quest for quantum computation. While the quantum states have to be manipulated from outside in order to carry out the specific computational task, the device should be maximally decoupled from the environment in order to avoid decoherence. Proposals for qubits based on trapped atoms, photons in QED cavities, or nuclear spins (NMR) are prime candidates for meeting these requirements, however, it seems that upscaling to a useful computer with order \( 10^4 \) qubits is difficult. Here is where solid state implementations of qubits enter the field, offering high manufacturing variability as well as scalability based on highly developed nanoscale technology. The large number of degrees of freedom associated with solid state devices challenges the maintainance of coherence. As of today, this problem has been met by either resorting to well isolated spins (on quantum dots), or through deliberate doping of semiconductors, or making use of the gapped quasi-particle spectrum in superconducting structures. The charge–phase (cooper-pair–vortex) duality in Josephson devices then admits two classes of qubit implementations: The “charge” device operates in the regime \( E_C \gg E_J \) (where \( E_C = e^2/2C \) is the charging- and \( E_J = I_c \Phi_0/2\pi e \) is the Josephson coupling energy; \( C \) = capacitance, \( I_c \) = critical current, \( \Phi_0 = h/e \) is the flux quantum), distinguishing states through their charge, while the superconducting phase qubits (SPQB) are based on strongly coupled junctions with \( E_J \gg E_C \). In “phase” qubits the states are usually distinguished by the direction of a circulating current in the Josephson loop, but recently a proposal for a ‘quiet’ qubit has been made which is effectively decoupled from the environment. Here, we introduce the \( \pi \)-junction, a Josephson junction with a ground-state characterized through a \( \pi \)-phase shift across the contact, as a new building-block of superconducting phase qubits and discuss how it can be used in the design of stable and switchable phase qubits.

In the following, we first develop the main ideas leading to the qubit design based on small-inductance multi-junction SQUID loops. The analysis of the robustness of these loops against static and dynamic fluctuations in electric- and magnetic fields will then guide us in the specific engineering of our five-junction loop, frustrated by a \( \pi \)-junction and defining a perfectly degenerate two-level system. We show how to construct a double-periodic \( \pm \pi/2 \)-junction from this five-junction loop and discuss its similarity with the SD-Josephson junction introduced by Ioffe et al. We then concentrate on operational aspects of the superconducting phase qubit: its efficient isolation from the environment motivates the design and use of superconducting phase switches and its trivial idle state is realized through a dynamically decoupled degenerate two-level system. We end with a discussion of possible implementations of \( \pi \)-junctions.

The classic phase qubit has been introduced in an early paper by Bocko et al. and involves a conventional two-junction SQUID loop frustrated by an external magnetic field. The two states used in the construction of the qubit are characterized by clock- and counterclockwise circulating currents or by the corresponding phase drops across the junctions. For quantum computing we are interested in qubits with a minimal coupling to the environment, hence we wish to have a situation where the

\[ \text{...} \]
phase is maximally decoupled from the flux, with the two quantum states mainly distinguished by the phase variable. Such a ‘quiet’ phase qubit producing no currents in the SQUID loop has been suggested recently and involves doubly periodic Josephson junctions with minima characterized by the phase variable $\phi = 0$ and $\phi = \pi$ (some unavoidable current flow remaining in the junction area itself is discussed below). However, these junctions are made from sandwiches combining $s$- and unconventional $d$-wave superconductors and are difficult to fabricate, hence alternative designs compromising with some residual coupling between the phase and the flux are still highly welcome.

Unfortunately, the conventional two-junction SQUID cannot offer a solution: The constraint $2\pi\Phi/\Phi_0 = \phi_1 + \phi_2$ relates the total flux $\Phi$ through the loop with the gauge invariant phase drops $\phi_1$ and $\phi_2$ across the junctions. For a large inductance loop with $L_L/c \gg \Phi_0$ the potential energy $V(\phi_1, \phi_2; \Phi_{ext} = \Phi_0/2) = E_J[2 - \cos \phi_1 - \cos \phi_2] + (\Phi - \Phi_{ext})^2/2L$ exhibits pronounced minima of order $E_J$ but the states involve unfavorably large fluxes of order $L_L/c \approx \Phi_0$, easily leading to magnetic crosstalk in an array of qubits. On the other hand, in a small inductance loop, where $L_L/c \ll \Phi_0$, the minima are shallow, of order $E_J/L_L/c \phi_0$. In order to effectively decouple the phase variables from the flux we have to resort to small inductance loops with three or more junctions. Here, both requirements of well defined minima (of order $E_J$) and small fluxes ($L_L/c \ll \Phi_0$) can be satisfied simultaneously. A first proposal for a superconducting phase qubit using a four-junction loop has been discussed by Feigelman et al. and a detailed analysis of the three-junction loop involving a tunable third junction has recently been given by Mooij et al. see Fig. 1. In a small inductance three-junction loop (with $L_L/c \sim 10^{-3}\Phi_0$) frustrated at $\Phi_{ext} = \Phi = \Phi_0/2$, the third junction is slaved to the other two, $\phi_3 = \pi - \phi_1 - \phi_2$, but the remaining two degrees of freedom are sufficient to define pronounced minima ($\phi_1 = -\phi_2 \equiv \phi_{min} = \pm \pi/3$) in the potential energy (see Fig. 2)

$$V(\phi_1, \phi_2) = E_J[3 - \cos \phi_1 - \cos \phi_2 - \cos(\pi - \phi_1 - \phi_2)],$$

while at the same time the fields produced by these states are small and hence do not perturb neighboring qubits. Quantum operation of the loop is introduced through the finite junction capacitance $C$: with $E_J/C \approx 25$ we have a plasma frequency $\omega_p \sim \sqrt{E_J/C} \sim E_J/5$ (placing the device into the semiclassical regime with a well defined phase basis) and a tunneling gap $\Delta \sim \hbar \omega_p \exp(-\sqrt{\alpha E_J/C}) \sim 10^{-3}E_J$, with $\alpha$ a numerical of order one ($\alpha = 8$ for a conventional small capacitance Josephson junction). Single qubit operations are performed through changes in the magnetic fluxes through the main (three-junction) loop, $\Phi$, and the auxiliary (two junction) loop of the tunable third junction, $\Phi_{aux}$, see Fig. 1. Typical qubit operations are carried out within the time scale $t_{op} \sim h/\Delta$ set by the tunneling gap $\Delta$. The quantum entanglement between two qubits can be modified through a tunable inductive coupling, where the coupling loop is opened and closed by a tunable junction as realized through a two-junction SQUID loop. First estimates of decoherence rates for this type of device give favorable values.

![FIG. 1.](image1.png)

FIG. 1. (a) The three junction SQUID studied by Mooij et al. [15] where the two degenerate ground states with clockwise and counterclockwise circulating currents are realized at maximal frustration with the penetrating flux $\Phi = \Phi_0/2$; changing the flux $\Phi_{aux}$ in the auxiliary SQUID loop produces an effective junction with varying coupling, i.e., a tunable junction. (b) Five-junction loop studied in the present paper. The frustration is achieved through a $\pi$-junction.

![FIG. 2.](image2.png)

FIG. 2. Contour plot of the potential energy $V(\phi_1, \phi_2; \Phi_0/2)$ for the symmetric frustrated three junction loop versus gauge invariant couplings $\phi_1$ and $\phi_2$. The apparent difference in distance between the two intracell- and intercell minima is due to the projection on the $\phi_1-\phi_2$ plane.

Charge stability: While the decoupling between phase and flux in a superconducting phase qubit can be achieved in small inductance $n \geq 3$-junction loops, the decoupling between phase and random offset charges in the environment can be realized by breaking the symmetry of the loop and choosing one junction weaker than the remaining ones. E.g., in a three junction loop the state $|+\rangle = |\phi_1 = \phi_2 = \phi_3 = \pi/3\rangle$ can decay into the state $|\rangle$ through three different paths $\gamma_i$.
where a flux $2\Phi_0/3$ enters through junction $i$ and leaves half-half through the other two, see Fig. 3 (due to the small inductance $L$, $\Phi \approx 0$ on the scale of $\Phi_0$). A tunneling process from $|+\rangle$ to $|-\rangle$ then must be calculated by adding the three amplitudes along $\gamma_i$ coherently, $\Gamma_{+\to-}^{i} = |\sum_{i=1}^{3} \gamma_{i}(-|+\rangle)^{2}$. The presence of a charge $Q_i$ on the $i$-th island, e.g., due to random offset charges, will then induce an additional Aharonov-Casher phase $\exp(2\pi i Q_i/2e)$ between the two flux paths enclosing this island, leading to unwanted changes in the tunneling amplitude $\Gamma_{+\to-}(Q_i)$. Breaking the loop’s symmetry with one weak junction channels the large flux through this junction and spoils this interference effect, rendering the qubit stable against random static offset charges. Note that dynamically fluctuating charges still can spoil the proper performance of the qubit through a time-like Aharonov-Casher interference effect. In order to prevent such dynamical modifications of the individual trajectory we have to protect the qubit from fast changes in the offset charges due to defects moving on a scale of the Rabi- or tunneling time $h/\Delta$.

**FIG. 3.** Reversing the current in the loop involves voltage pulses in the junctions, which can be viewed as arising from a flux $2\Phi_0/3$ entering through one junction and leaving half-half through the other two junctions. The choice for the flux to enter through any one of the three junctions leads to Aharonov-Casher type interference effects which can sensitively influence the performance of the qubit.

**Flux stability:** While the phase qubit can be constructed to resist charge fluctuations, it is very susceptible to any magnetic field variation, as this is the prime external signal used in setting up the two-level system and in manipulating the qubit. The requirement on the field stability is quite nontrivial: Assume an imprecision $\delta \Phi$ in the frustrating flux acting over $N_{\text{op}} \approx 10^{3} - 10^{6}$ operations (note that a device able to carry out $10^{4}$ operation can be run indefinitely using error correction techniques). In our current-phase decoupled loops such a change $\delta \Phi$ in the flux will produce a shift of order $(\delta \Phi/\Phi_0)E_J$ in the relative energy of the two states. The total accumulated phase $\delta \phi \sim E_J(\delta \Phi/\Phi_0)t/h \sim (E_J/\Delta)(\delta \Phi/\Phi_0)N_{\text{op}}$ should remain small (on the scale of $2\pi$) and hence requires a precision

$$\frac{\delta \Phi}{\Phi_0} \sim \frac{\Delta}{E_J N_{\text{op}}} \sim 10^{-6} - 10^{-7}.$$
SQUID-loop with one strong $\pi$-junction as our basic device, see Fig. 1(b). Since the loop cannot trap flux, the remaining four junctions are related via $\pi = \phi_1 + \phi_2 + \phi_3 + \phi_4$. The choice with four junctions in the loop is convenient (but not unique) as it allows for an easy construction of sub-units performing specific tasks. In particular, selecting the phase difference $\phi_1 + \phi_2$ through appropriate contacts we construct a $\pm \pi/2$-junction with minima at $\pm \pi/2$, see Fig. 4(a). Choosing all junction couplings $E_i, i = 1, \cdots, 4$ equally defines a $\pm \pi/2$-junction (due to the required charge stability we have to break the symmetry and thus have to use the somewhat more complicated couplings $E_1 = E_2 = E_J$, $E_3 = (1 - \gamma)E_J$, and $E_4 = (1 + \gamma')E_J$, $\gamma, \gamma' > 0$). It turns out, that the physics underlying the functionality of this $\pm \pi/2$-junction is identical to that of the SD-Josephson junction proposed earlier\[10\]. Consider the idealized five-junction loop shown in Fig. 4(b) where the four equal $s$-wave junctions ($E_i = E_J$) have been arranged in pairs. Let us fix the phase drop $\phi$ over the two first junctions and minimize the energy of this pair, $E_{p1}(\phi) = \min_\delta E_J[2 - \cos \delta - \cos(\phi - \delta)] \to \delta = \phi/2$. The resulting energy-phase relation $E_{p1}(\phi) = 2E_J[\cos(\phi/2)]$ is similar to that of a clean SNS-junction\[2\] with sharp cusps at $\phi = \pm \pi$, see Fig. 4(c). Treating the second pair of junctions in a similar way, a simple addition provides us with the optimized energy-phase relation of the $\pm \pi/2$-junction, $E_{\pm \pi/2}(\phi) = E_{p1}(\phi) + E_{p2}(\phi + \pi)$, where the phase of the second pair is shifted by $\pi$ due to the presence of the $\pi$-junction in the loop. For the symmetric setup with $E_{p1} = E_{p2}$ we obtain minima at $\pm \pi/2$, while breaking the symmetry will shift the minima away from $\pm \pi/2$ to values $\pm (\pi/2 - \alpha)$, see Fig. 4(c). It is then important to note that while breaking the symmetry does shift the minima it does not lift the degeneracy between the two minimal states, a crucial point in the design of a qubit with a trivial idle state (in fact, this degeneracy is a consequence of the time reversal symmetry in the Hamiltonian).

SD-Josephson junction versus five-junction loop: Consider then the SD-junction sketched in Fig. 5(a). We choose a contact with a clean normal metal layer (N) providing the coupling between the $s$-wave (S) and the $d$-wave (D) superconductor via the proximity effect: electrons incident on the $s$-wave superconductor from the left are reflected back as holes, a process known as Andreev reflection\[13\]. In turn, the hole is reflected from the $d$-wave superconductor as an electron and a phase-sensitive Andreev state is formed carrying supercurrent across the normal metal layer. Due to the $d$-wave symmetry, the Andreev levels naturally split into two families contributing the coupling energies $E_{f1}(\phi)$ and $E_{f2}(\phi + \pi)$, where the phase shift $\pi$ is due to the sign change in the $d$-wave condensate under a rotation by $\pi/2$, see Fig. 5(a). For a clean metal the coupling energies are given by a sequence of parabolas $\propto E_fo^2/2$ with sharp cusps at $\pm \pi$, see Ref.\[27\], and we obtain a similar energy-phase relation as for the five-junction loop. A symmetry breaking with $E_{f1} \neq E_{f2}$ corresponds to a misalignment of the SD boundary, producing a stronger coupling via one of the families — a misalignment in the $d$-wave crystal then leads to a shift of the minima away from $\pm \pi/2$ but does not destroy the degeneracy of the two-level system.

The obvious question to ask then is about the analogue of the circulating currents in the five-junction loop for the SND-junction: Indeed, as discussed by Huck et al.\[22\] the two degenerate ground states at $\pm \pi/2$ are linked to a current flowing up or down the intermediate metal layer and returning through the superconductors on the side, thereby producing local magnetic fields of size of the lower critical field $H_{c1} \approx \Phi_0/4\pi \lambda^2$ decorating the metal layer on either side, see Fig. 5(b) (here, $\lambda$ denotes the penetration depth; for a dirty metal of thickness $d$ and with a mean free path $l$ the current flow is suppressed by a factor $(l/d)^4$); hence the ‘quiet’ qubit advertised in Ref.\[13\] turns out to be less ideal then originally expected, with some residual magnetic coupling surviving on the level of the junction itself, a point which has been missed in Ref.\[13\]. In an idealized situation with a symmetric (back-) flow through the superconductors the fluxes trapped on the two sides of the normal-metal layer would be of opposite sign and thus would compensate one another, producing a magnetic far-field of quadrupolar nature. However, in a SND junction we cannot expect the backflow through the superconductors to be symmetric, hence the dipolar component of the magnetic field is only partly compensated. Still, the SND-junction teaches us a further trick in our struggle to minimize the coupling of the qubit to the environment: shaping the five-junction loop into a

\[\begin{align*}
D \quad (010) & \quad N \quad e \quad h \quad f_1 \\
\quad + & \quad \quad + \\
\quad - & \quad \quad - \\
\quad S & \quad \quad \quad f_2
\end{align*}\]
crossing double loop in the form of an ‘8’ produces a mutual compensation of the flux produced by the circulating current. Note that this idea can only be realized when the loop frustration is produced by a π-junction rather than an external magnetic field.

Qubits from five-junction loops and SD-Josephson junctions: Given the five-junction loops and SD-Josephson junctions above the qubit follows immediately from a simple shift-operation. Rather than having the minima at ±π/2, as is the case for the five-junction loop and for the SD-Josephson junction, we wish them placed at 0 and π. Such a shift in the minima is achieved by adding a second strong π/2-junction in series. This strong π/2-junction with large coupling energy and large capacitance acts as a classical shift-device and does not contribute to the quantum evolution of the qubit. For the five-junction loop we simply add a second, strongly coupled π/2-junction in series, see Fig. 6(a), while in the case of the SD-Josephson junction a simple termination with another DS'-junction defining a SDS'-Josephson junction will do. In order to distinguish these double periodic junctions with minima at 0 and π from the ±π/2-junctions we call them 2φ-junctions (alluding to the double periodicity) and use the special symbol shown in Fig. 6(a). The reason for adding this π/2 phase-shift is an operational one: combining the 2φ-junction together with a conventional s-wave junction into a SQUID loop we can lift the degeneracy between the two ground states as the s-wave junction is frustrated in the π-state — this will then allow us to define the phase-shift operation of the individual qubits.

![Diagram](image)

(a) Combining a strong (classical; left circle with upward bar) π/2-junction and a (quantum) ±π/2-junction produces the 2φ-junction with minima shifted to 0 and π. (b) Switchable s-wave junction (phase switch): switching the ±π/2-junction between π/2 and −π/2 the phase drops across the two s-wave junctions either differ by π or 0. As a consequence, the total coupling across the loop changes from $E_1^\prime \pm E_2$ to $E_1^\prime \mp E_2$. (c) A sequential array of s-wave switches allows to enhance the dynamical range $E_1^\prime / E_2$ of the switch.

Qubit manipulation: Given the quantum state $|\Psi\rangle = |0\rangle + a \exp(\text{i} \chi)|1\rangle/\sqrt{1 + a^2}$ of the qubit, single qubit operations serve to manipulate the phase $\chi$ and the amplitude $a$ in the superposition (we call the corresponding operations a ‘phase-shifter’ and an ‘amplitude-shifter’, respectively). A further point of criticism and potential improvement then is in the manipulation of the qubit’s quantum state. Again, permanent connections to the qubit used to modify their relative position in energy or the coupling between the two states are potential carriers of noise. One might then come up with the following wireless scheme for qubit operation inspired by the usual NMR technique: A qubit with an energy separation $\Delta$ of its levels can be manipulated by means of resonant microwave pulses. A deliberate variation of the coupling energies $\Delta_k$, $k = 1, \cdots, N_{\text{qs}}$, during the construction phase then permits to address each qubit in an array individually by selecting the proper eigenfrequency $\Delta_k / \hbar$ for the microwave signal. With $N_{\text{qs}}$ qubits in the array, the typical distance between resonances is of order $\delta \Delta = \Delta / N_{\text{qs}}$. Assume we are manipulating the $k$-th qubit by tuning the microwave signal to its resonance frequency $\Delta_k / \hbar$. The transition time is related to the ac-signal via $t_{\text{op}} \sim h / \sqrt{(\delta \epsilon / 2)^2 + |V_k|^2}$, where $V_k = \langle 0 | V | 1 \rangle_k$ is the matrix element of the perturbing ac-field, and $\delta \epsilon$ denotes a possible deviation from resonance. What is the probability to excite other qubits closely in energy? Such erroneous transitions appear with a probability $|V_k|^2 / 2(\delta \Delta / 2)^2 + |V_k|^2$ and requiring them to be small implies the condition $|V_k|^2 \ll (\delta \Delta / 2)^2$. This translates into an operating time $t_{\text{op}} \gg N_{\text{qs}} h / \Delta$ scaling linearly with the number $N_{\text{qs}}$ of qubits in the system. Hence, while upsampling the number of operations $N_{\text{op}}$ puts restrictions on the variation of the applied field, upscaling of the number $N_{\text{qs}}$ of qubits puts unfavorable limits on the operation time $t_{\text{op}}$ and renders the problem with keeping the flux constant even more difficult.

Switches: An alternative scheme for a qubit manipulation with minimal coupling to the environment is provided through superconducting phase switches. The basic idea derives from the variable Josephson junction as realized through a small-inductance two-junction SQUID loop: Frustrating the loop with a flux $\Phi_0/2$ produces a flat potential and hence a small effective coupling across the loop. The problem with such a trivial implementation of the switch is again with the use of an external bias current producing the frustration in the ‘off’ state. Using our ±π/2-junction we can easily construct a ‘phase switch’ which operates without external bias field, see Fig. 6(b) and Ref. 13. We combine two (±)π/2-junctions and two conventional s-wave junctions into a small inductance SQUID loop. While the first π/2-junction is set at π/2, leads attached to the second ±π/2-junction allow to switch between the states ±π/2 via the application of voltage pulses. If the second junction is in the state $-\pi/2$, the two π/2-junctions compensate one another, the couplings of the two s-wave junctions add, and the
Unfortunately, we cannot hope to remove the coupling alltogether in the open state of the switch, as this would require very precise fabrication of identical junctions such that \( E_{sJ}^{1+} = E_{sJ,1} + E_{sJ,2} = 0 \). However, it is sufficient to achieve a cancellation \( E_{sJ}^{1+} \ll E_{sC} \), with \( E_{sC} \) the capacitive energy of the switch loop. Combining \( n \) switch-loops into an array (see Fig. 6(c)) will then further reduce the coupling by a factor \( (E_{sJ}^{1+}/E_{sC})^{n-1} \) (this follows trivially from the analogy to a 1D-Hubbard chain with hopping matrix element \( t = E_{sJ}^{1+} \) and interaction energy \( U = E_{sC} \)).

An attractive idea is to extend the above scheme to a switchable 2\( \phi \)-junction, combining two \((\pm)\pi/4\)-junctions and two \(\pm\pi/2\)-junctions into a SQUID loop (the operation follows the same scheme as that of the superconducting switch with the \((\pm)\pi/4\)-junctions replacing the \((\pm)\pi/2\)-junctions). Such a switchable 2\( \phi \)-junction then would allow for a change in the tunnelling gap through switching the barrier height separating the two ground states, producing an efficient amplitude-shifter. Unfortunately, small deviations in the precision of the junction parameters, producing an efficient amplitude-shifter. Unfortuntely, we cannot hope to remove the coupling alltogether in the open state of the switch, as this would require very precise fabrication of identical junctions such that \( E_{sJ}^{1+} = E_{sJ,1} + E_{sJ,2} = 0 \). However, it is sufficient to achieve a cancellation \( E_{sJ}^{1+} \ll E_{sC} \), with \( E_{sC} \) the capacitive energy of the switch loop. Combining \( n \) switch-loops into an array (see Fig. 6(c)) will then further reduce the coupling by a factor \( (E_{sJ}^{1+}/E_{sC})^{n-1} \) (this follows trivially from the analogy to a 1D-Hubbard chain with hopping matrix element \( t = E_{sJ}^{1+} \) and interaction energy \( U = E_{sC} \)).

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**Qubit functionality:** Let us finally combine the above elements into a qubit and discuss its functionality in terms of single- and two-qubit operations. We follow the scheme described previously in Ref. [13]. The qubit combines a 2\( \phi \)-junction, a switchable s-wave junction and a switchable large capacitance \( C_{ext} \) in a SQUID-loop, see Fig. 7(a). A switchable capacitance is easily realized by connecting a capacitance (\( C_{ext} \)) and a switchable s-wave junction (denoted by \( s' \) in Fig. 7(a) and in the following) in series: With a weak coupling \( E_{sJ}^{1+} < E_{ext} \) the phase \( \chi \) of the island fluctuates freely and the capacitor is effectively switched off. In order to switch on the capacitor \( C_{ext} \) we have to establish a sufficiently strong coupling across the \( s' \) junction, \( E_{sJ}^{1+} > E_{C,J} \), with \( E_{C,J} \) the charging energy of the \( s' \) junction, slaving the phase \( \chi \) of the island to the phase \( \phi \) of the 2\( \phi \)-junction. In the idle state the s-wave junction is in the off state (no coupling) and the capacitance is operating in parallel with the 2\( \phi \)-junction producing a large total capacitance in the loop. The resulting tunneling gap \( \Delta_{tot} = \sqrt{E_{J}E_{C,J}} \exp[-(\alpha E_{J}/E_{C,J})^{1/2}] \) is small and the two degenerate ground states are dynamically decoupled (the numerical \( \alpha \) is of order 1 to 10 and depends on the details of the potential barrier, \( \alpha \approx 2.6 \) for the four junction loop; a switchable 2\( \phi \)-junction would allow for a potential decoupling of the two ground states). Note that this idle state has a trivial state evolution with the relative amplitude and phase of the quantum superposition remaining unchanged. In order to shift the relative phase \( \chi \) in the superposition \( |\psi\rangle = (|0\rangle + a|\pi\rangle)|\pi\rangle/\sqrt{1 + a^2} \) the s-wave junction is switched into the strong coupling state (on). Its frustration in the |\pi\rangle state of the qubit removes the degeneracy between the states |0\rangle and |\pi\rangle. The time evolution of the two-level system is given by the unitary matrix \( u_{z}(\varphi) = \exp(-i\sigma_{z}\varphi/2) \) with \( \varphi = -2E_{sJ}^{1+}/\hbar \) and \( \sigma_{z} \) the usual Pauli matrix. Keeping the s-wave junction off during the time \( t \) will shift the phase by \( \chi = -2E_{sJ}^{1+}/\hbar \). Finally, the amplitude \( a \) is modified by switching off the large capacitance \( C_{ext} \) in the loop. With the capacitance now determined by the small capacitance \( C_{q} \) of the qubit (plus a residual small capacitance from the switch) the tunneling gap \( \Delta = \sqrt{E_{J}E_{C,J}} \exp[-(\alpha E_{J}/E_{C,J})^{1/2}] \) is large and the qubit exhibits Rabi oscillations with a frequency \( \omega = 2\Delta/\hbar \). The corresponding time evolution is given by the unitary matrix \( u_{z}(\varphi) = \exp(-i\sigma_{z}\varphi/2) \) with \( \varphi = -2E_{sJ}^{1+}/\hbar \). Starting from \( |\psi\rangle = |0\rangle \) and keeping the external capacitance off over a time \( t \) will shift the amplitude \( a = 0 \) to \( a = \tan^{2} \varphi \).

Two-qubit operations are carried out using a similar scheme. As two qubits are coupled into a SQUID loop by closing an s-wave switch, the two states |0\rangle and |\pi, 0\rangle are frustrated and are separated from the states |0\rangle and |\pi, \pi\rangle by the energy \( 2E_{sJ}^{1+} \). This allows us to shift the phases of the states |0\rangle and |\pi, 0\rangle relative to |0\rangle and |\pi, \pi\rangle and the two-qubit state evolves in time following the evolution

\[
U_{ps}(\chi) = \begin{pmatrix} u_{z}(\chi) & 0 \\ 0 & u_{z}(-\chi) \end{pmatrix}
\]

with \( \chi = -2E_{sJ}^{1+}/\hbar \). Combining this ‘phase shifter’ with several single-qubit operations \( U_{\mu}^{\pm}(\theta) \) rotating the qubit \( i \) by an angle \( \theta \) around the axis \( \mu \), \( \mu |u_{\mu}(\theta) = \exp(-i\sigma_{\mu}\theta/2) | \) then allows us to construct the nontrivial controlled NOT gate

\[
U_{CNOT} = \exp(-i\pi/4)U_{2y}(\pi/2)U_{1z}(-\pi/2)U_{2z}(-\pi/2)
\cdot U_{ps}(\pi/2)U_{2y}(-\pi/2).
\]

The proper operation of these qubits over \( N_{op} \) operations puts some constraints on the dynamical range of...
the switch: We require that the accumulated phase due to leaking in the open state is small,

\[ N_{\text{op}} t_{\text{op}} E_{s,J}^{\dagger}/\hbar < 1, \]

hence \( E_{s,J}^{\dagger} \leq \Delta/N_{\text{op}} \). On the other hand, we wish the phase shift operation to be equally fast as the amplitude shift, hence \( E_{s,J}^{\dagger} \gtrsim \Delta \). The two conditions then result in the dynamical range \( E_{s,J}^{\dagger}/E_{s,J}^{\dagger} \gtrsim N_{\text{op}} \sim 10^3 \), which can be achieved through the switch array described above, see Fig. 6(c).

![Diagram](image)

**FIG. 7.** (a) Qubit made from a 2φ-junction, a switchable large capacitance \( C_{\text{ext}} \), and a switchable s-wave junction. (b) Idle-state: the capacitance \( C_{\text{ext}} \) is switched in parallel to the 2φ-junction blocking Rabi oscillations — the s-wave-junction is switched off to keep the states degenerate. Phase-shift: The s-wave junction is on and its strong coupling \( 2E_{s,J}^{\dagger} \) removes the degeneracy of the two levels. (c) Amplitude-shift: The external capacitance is switched off allowing the 2φ-junction to execute Rabi-oscillations.

\[ \text{Realization of } \pi \text{-junctions:} \] Today, the most simple way to realize a \( \pi \)-junction makes use of a copper-oxide d-wave superconductor: connecting two s-wave superconductors (e.g., Pb) to the \((1,0,0)\) and \((0,1,0)\) surfaces of a d-wave material (e.g., YBa\(_2\)Cu\(_3\)O\(_7\)), the sign change in the wave function of the Cooper-pair produces the desired sign change or \( \pi \)-shift. However, long before the discovery of the high-\( T_c \) copper-oxides, suggestions have been made for the construction of \( \pi \)-junctions making use of magnetic impurities or bulk ferromagnetic interlayers: Bulaevskii, Kuzii, and Sobyanin[23] have been the first to note that introducing magnetic impurities into a tunneling junction leads to a sign change in the Josephson critical current if the tunneling channel through the magnetic impurities is strong enough to outplay the direct tunneling path. Later, critical-current oscillations have been predicted to occur in clean[24] and dirty[25] superconductor–ferromagnet–superconductor (SFS) junctions. Consider a Cooper-pair injected into a metallic layer under an angle \( \theta \) with respect to the interface normal \( n \). In a normal metal such a pair involves the states \([p \uparrow, \downarrow] \), \([−p \downarrow \uparrow] \). In a clean ferromagnet, the energies of the states with equal momenta \( p \) and opposite spins are split by the exchange field, \( E_{p,\pm} = \epsilon_p \pm E_{xc} \). Conservation of energy and momentum parallel to the SF interface implies that the perpendicular momenta of the two electrons are shifted in order to compensate for the different exchange energy, \( \delta p_n = \pm E_{xc} n/(\nu_v \cos \theta) \), the sign of the shift depending on the alignment of the spins with the exchange field. Thus in the ferromagnet the pair involves the states \([p + \delta p_n \uparrow, \downarrow] \), \([−p − \delta p_n \downarrow \uparrow] \) and carries a center of mass momentum \( P_{xc} = \pm E_{xc} n/(\nu_v \cos \theta) \), resulting in a spatially oscillating pair wave function \( \propto \cos(P_{xc} \cdot \mathbf{R}/\hbar) \) (here, \( \mathbf{R} \) denotes the center of mass coordinate of the pair). The proper integration over angles finally produces an oscillating dependence of the critical current in the parameter \( dE_{xc}/\hbar v_F \), where \( d \) is the thickness of the ferromagnetic layer. For a dirty ferromagnet the relevant parameter is \( \sqrt{d^2 E_{xc}/\hbar v_F} \), with \( l \) the mean free path of the dirty magnetic metal. Experimentally, \( T_c \) variations have been observed in Nb/Gd multilayers[27] with varying thickness of the ferromagnetic Gd layers, providing evidence for a \( \pi \)-coupling realized at a thickness \( d_{Gd} \approx 20 \) Å. Direct observation of a sign change in the critical current at a definite temperature and ferromagnetic-layer thickness has been recently reported by Veretennikov et al. in their study of Nb-Cu/Ni-Nb junctions.

\[ \text{2φ-junctions:} \] Another basic idea to be mentioned in the present context is the possibility to construct a ‘microscopic’ 2φ-junction (in contrast to the ‘macroscopic’ 2φ-junctions made from five-junction loops discussed above). Such junctions can be obtained by an accurate quenching of the first harmonic coupling, leaving the second harmonic with an energy \( E_1(\phi) \propto \cos(2\phi) \) as the leading term. A particular scheme for the implementation of such a junction has already been mentioned above, the SDS-‘junction introduced in Ref. [3] exploiting the node in the d-wave state, the elimination of the first harmonic is achieved by balancing the two lobes of opposite sign in wave function of the Cooper pair.

Another idea is to quench the lowest harmonic via disorder: Using a dirty SF–S junction, the averaging over the phase factors \( \exp(\mathbf{P}_{xc} \cdot \mathbf{R}) \) in the Cooper pair wave function produce an efficient (i.e., exponential) suppression in the Josephson coupling. Within a non-interacting electron description a similar suppression renders the second and higher harmonics small, too. However, includ-
ing effects of interaction in the calculation of the second harmonic coupling we can trade these exponential factors for an interaction vertex and arrive at a significant second order coupling $E_2^{(2)} \sim E_0(\lambda/k_Fd)$, with $E_0 = k^2_0 A h v_F / d$ the dimensionless interaction parameter ($\lambda = V N_0$, $V$ the interaction and $N_0$ the density of states; in obtaining these estimates we have mimicked the disorder by a rough interface and have applied estimates valid on short scales). In mesoscopic devices, care has to be taken regarding fluctuation effects; finite mesoscopic fluctuations in the first-order coupling can effectively compete with the second harmonic, see Ref. [31]. A specific discussion of a mesoscopic SF$_n$S junction exhibiting a double-periodic energy–phase relation (a mesoscopic dirty SF$_0$S $2\phi$-junction) has recently been given by Zyuzin and Spivak [34].

In conclusion, we have discussed the main basic principles involved in the construction of superconducting phase qubits. We have shown that this device family offers unique opportunities in the construction of a trivial idle state, in removing dangerous couplings to the environment, and the possibility to use phase switches rather than permanently attached bias leads in the manipulation of the qubit. The basic idea of the present work is that one can obtain a macroscopic doubly-periodic or $2\phi$-junction device using five-junction loops comprising a $\pi$-junction. Alternatively, a meso- or microscopic implementation of such $2\phi$-junctions involving SDS' or dirty SFS junctions opens up the possibility to design and develop a compact superconducting phase qubit.

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