The Nothing at the Beginning of the Universe
Made Precise

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ABSTRACT: We propose a new worldsheets approach to the McGreevy-Silverstein proposal: resolution of spacelike singularity via Scherk-Schwarz compactification and winding string condensation therein. Our proposal is built upon so-called three parameter sine-Liouville theory, which has useful features and could be solvable in conformal field theory method. Utilizing standard Wick rotation, we compute string pair production rate exactly in terms of renormalized worldsheets cosmological constant and find that the production rate is finite for six or less spacetime dimensions. We also find that the sine-Liouville potential excises string excitation in the asymptotic past, and that such "Nothing state" is realizable for a range of sine-Liouville coupling constants. We compute one loop vacuum-to-vacuum transition amplitude and again detect presence of the "Nothing state". We also survey various worldsheets approaches to the tachyon condensation based on timelike Liouville theory. We point out that string theory on a conifold provides the upper critical dimension for realizing the "Nothing state", thus making contact with the blackhole / string transition point.

KEYWORDS: big bang singularity, string theory, tachyon condensation.
1. Introduction

According to the standard 'big-bang' cosmology, the universe has started from a spacelike singularity. Within general relativity, it is established that such singularity is unavoidable. Near the singularity, curvature of the spacetime blows up, and classical treatment of the gravity breaks down. An outstanding question is whether effects beyond classical gravity can resolve the big-bang singularity. In the context of string theory, such effects may originate from string worldsheet and quantum fluctuations. As such, variety of possible string theoretic mechanisms for resolving spacelike singularity have been proposed in the past [1] – [7].

Recently, McGreevy and Silverstein (MS) proposed an extremely interesting mechanism for resolving cosmological singularity [8]. Their proposal relies on string theory; central to their proposal is utilization of "tachyon condensation" of winding string around cosmologically shrinking
$\mathbb{S}^1$, which in turn creates a mass gap to closed string worldsheet degrees of freedom \cite{3}. Take, for example, Type II superstring in an expanding universe of topology $\mathbb{R}_t \times \mathbb{S}^1_\theta \times \mathcal{M}_\bot$:

$$d s_{II}^2 = \ell_{st}^2 \left[ - d t^2 + a^2(t) d \theta^2 + d s_2^2 \right], \quad (1.1)$$

where the scale factor $a(t) = a_0 t^\nu$ is driven by a homogeneous and isotropic energy-momentum tensor. Around $\mathbb{S}^1$, one assigns spacetime fermions to obey ‘thermal’ boundary condition \cite{10}, thus cosmologically realizing the Scherk-Schwarz compactification \cite{11}. Accordingly, the spacetime supersymmetry is broken, causing Bose-Fermi mass splitting set by $a(t)/\ell_{st}$. In particular, winding string state around the Scherk-Schwarz circle $\mathbb{S}^1(\theta)$ survives the twisted Gliozzi-Scherk-Olive (GSO) projection \cite{12} and has mass spectrum $m^2_w = (a^2(t) - 1)/\ell_{st}^2$. Near the cosmological singularity at $t = 0$, proper size of the $\mathbb{S}^1$ shrinks smaller than the string scale $\ell_{st}$. The winding state becomes tachyonic, so it condenses over a region of the spacetime near the cosmological singularity. By taking adiabatic regime $0 < \nu \ll 1$, one can arrange the tachyon rolling to take place while string coupling and Hubble parameter remain small \cite{1}. Dynamics of a probe string is then describable by worldsheet theory involving Lorentzian signature ($\mathcal{N} = 1$ supersymmetric) sine-Liouville theory

$$Z_{t, \theta} \simeq \int [d t d \theta] \exp \left( \frac{i}{\ell_{st}^2} \int d^2 \sigma \left[ - |\partial t|^2 + |\partial \theta|^2 - \mu^2 e^{-2\kappa t} \cos^2 (\omega \bar{\theta}) + \cdots \right] \right) \quad (1.2)$$

where ellipses abbreviate terms involving worldsheet fermions. MS examined string dynamics in this region and concluded that, if $\mu$ is real (which plays the role of worldsheet cosmological constant), mass gap is generated for all worldsheet fields. In spacetime picture, this means that all closed string excitations are lifted up infinitely heavy. In effect, the tachyon condensation is seen to excise the epoch of big-bang singularity and replacing it by so-called ”Nothing state”. What makes MS proposal particularly attractive is that the ”Nothing state” is Lorentzian, string theoretic realization of the Hartle-Hawking no boundary proposal \cite{31}. MS also argued that their proposed mechanism is generic and can be extended to different types of spatial topology and boundary condition.

In drawing their claims from string worldsheet analysis, MS relied largely on saddle-point approximation. Such approximation is not likely to be self-consistent. Given that the singularity excision takes place within a string scale sized region of the spacetime \cite{11}, full-fledged worldsheet treatment of both the tachyon condensation and the string propagation therein are indispensable. Within the setup of MS proposal, however, such treatment is extremely difficult if not impossible \cite{3}.

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1Winding tachyon condensations was investigated e.g. in \cite{3} – \cite{21} in relation to various dynamical aspects of superstring theory, say, removing singularities, topology changes or space-time phase transitions, etc. Previous studies of cosmological backgrounds based on analytically continued solvable conformal field theories with linear dilatons are given e.g. in \cite{20} – \cite{25}. See also \cite{24} for a review and more complete reference list.

2We suppress decoupled free conformal field theory for transverse space in \cite{11} and Faddeev-Popov ghosts. We also suppress the fermionic degrees of freedom.

3At best, \cite{1} may be treated as a variant of $A_2$ Toda field theory.
This is because, for the Euclidean signature ($N = 1$ supersymmetric) sinh-Liouville theory

$$S_{\text{MS}} = \frac{1}{2\pi} \int d^2z \left[ \partial \varphi \bar{\partial} \varphi + \partial \theta \bar{\partial} \theta + 2\pi \mu e^{-2\kappa \varphi} \cosh^2(\omega \bar{\theta}) + \cdots \right]$$  \hspace{1cm} (1.3)

adopted by MS in prescribing the theory (1.2), systematic conformal field theory treatment is currently unavailable. The negative coupling of Wick-rotated tachyon background term ($\mu \rightarrow i\mu E$) causes the theory non-unitary and renders the difficulty even worse.

Given such difficulties, a host of physics questions arise. Is the tachyon condensation, especially dependence of physical processes to the cosmological constant, describable by exact conformal field theory approach? After tachyon condensation, is the spacetime geometry deformed only locally near the singularity or over an extended region? Is the rate of Bogoliubov pair production, which is caused by time-dependent background, finite and controllable? Is the "Nothing state" replacing the singularity unique or are there many possibilities? Related to it, given that the cosmological constant $\mu$ is subject to renormalization, is its phase determinable after string worldsheet effects are fully taken into account?

In this work, we propose a new worldsheet approach to winding tachyon condensation which bypasses all aforementioned difficulties. It is based on so-called the three-parameter model of the Euclidean signature sine-Liouville theory:

$$S_{\text{NRS}} = \frac{1}{2\pi} \int d^2z \left[ \partial \varphi \bar{\partial} \varphi + \partial \phi_1 \bar{\partial} \phi_1 + \partial \phi_2 \bar{\partial} \phi_2 + 2\pi \mu e^{\alpha \varphi} \cos(\beta \phi_1 + \delta \phi_2) + \cdots \right] .$$  \hspace{1cm} (1.4)

This theory is treatable by standard conformal field theory approach, in sharp contrast to the sinh-Liouville model (1.3). Consequently, we can examine full-fledged stringy effects to excision mechanism of the cosmological singularity and to onset and evolution of the tachyon condensation. Of particular interest is whether the 'Nothing state' is robust enough not to be affected by string worldsheet effects. We shall be able to answer these questions affirmatively by extracting renormalization effect to the worldsheet cosmological constant $\mu$ in a precise manner. The most significant point is that our results go beyond semiclassical approximations; we were able to do so since the proposed worldsheet theory (1.4) is treatable in full-fledged conformal field theory approach.

In section 2, we first recapitulate the three-parameter sine-Liouville theory. We then propose a prescription of Lorentzian sine-Liouville theory relevant for the winding string tachyon condensation via the standard Wick rotation of the Euclidean theory. In section 3, via the Wick rotation, we propose a concrete realization of the "Nothing state" in a precise manner. Using conformal field theory approach, we extract bulk reflection amplitudes of the sine-Liouville theory and, from this, obtain the Bogoliubov pair production amplitudes by performing the Wick rotation carefully. We study renormalization of the tachyon mass parameter (worldsheet cosmological constant) and find that the 'Nothing state' indeed emerges over a range of worldsheet sine-Liouville coupling parameters. We also study the one-loop vacuum-to-vacuum transition amplitude, again via the standard Wick rotation, and find that it leads to the same conclusion as those based on the
Bogoliubov amplitudes. In section 4, we first discuss several possible Liouville theory models of winding tachyon condensation for various worldsheet supersymmetries. We then show that realistic tachyon condensation with $N = 1$ superconformal symmetry is mappable to the three-parameter sine-Liouville theory. We also point out that the upper critical dimension for realizing the "Nothing state" corresponds to string theory on a conifold and that this coincides with the black hole-string transition point. We finally present an intuitive spacetime picture that underlies emergence of the "Nothing state".

2. Proposal

Begin with our proposal for the cosmological winding tachyon condensation. A criterion we set out for a viable worldsheet approach is that the standard Wick rotation of cosmological string background is achievable within well-defined conformal field theories. Though the setup also adopts specifically cosmological Scherk-Schwarz compactification and tachyon condensation of winding string near the cosmological singularity, our proposal differs significantly from MS proposal. First, our proposal is based on well-defined conformal field theory at the outset: the cosmological background is described by first starting from Euclidean signature sine-Liouville (instead of sinh-Liouville) theory and then make the standard Wick rotation to Lorentzian signature sine-Liouville theory. Second, utilizing conformal field theory approach, our treatment is capable of capturing full-fledged worldsheet effects describing spacetime dynamics all the way down to string scale.

2.1 Three-parameter sine-Liouville theory

Our proposal is based on so-called three-parameter sine-Liouville field theory [30], which is a generalization of the models with smaller number of parameters [28, 29]. This is a theory defined in terms of three spacelike scalar fields: a Liouville field $\varphi$ and two compact scalar fields $\phi_1, \phi_2$, and is described by the action

$$S = \frac{1}{2\pi} \int d^2z \left( \partial \varphi \partial \varphi + \partial \phi_1 \partial \phi_1 + \partial \phi_2 \partial \phi_2 + 2\pi \mu e^{\alpha \varphi} \cos(\beta \phi_1 + \delta \phi_2) + \frac{q}{2} R(2, \varphi) \right)$$

(2.1)

where the coupling parameters $\alpha, \beta, \delta$ and the background charge $q$ are all real-valued. The last two terms describe sine-Liouville tachyon and linear dilaton backgrounds. For the moment, we view (2.1) as a model exhibiting the ‘Nothing state’ at the beginning of the universe. In this context, we view $\phi_1$ as the T-dual variable of $\tilde{\theta}$ in (1.2) and $\phi_2$ as other relevant $c = 1$ worldsheet degrees of freedom. In section 4, we shall show that the proposed model can be embedded into Type II string theory once we set $\delta = 1$ and fermionizing $\phi_2$.

The theory (2.1) is characterized by the coupling parameters $(\alpha, \beta, \delta)$. The background charge $q$ is related by the conformality condition to these parameters as

$$q = \frac{1}{2\alpha} (\alpha^2 - \beta^2 - \delta^2 + 2)$$

(2.2)

We adopt conventions that set $d^2z d\sigma^1 d\sigma^2$, $\hat{c}_{\alpha} = 2\pi$ so that $\langle \varphi(z, \bar{z}) \varphi(w, \bar{z}) \rangle = -\log |z - w|^2$, etc.
and sets total central charge of the theory to \( c = \tau = 3 + 12q^2 \). To ensure that the sine-Liouville potential cuts off the strong coupling region, we shall restrict to the range \( q\alpha > 0 \). The central charge is invariant under the exchange map

\[
\alpha \leftrightarrow \tilde{\alpha} \equiv \frac{1}{\alpha}(2 - \beta^2 - \delta^2),
\]

but, unlike the Liouville theory, this map does not yield self-duality of the theory. For example, the correlators are not self-dual but are paired up between \( \alpha \) and \( \tilde{\alpha} \) theories. A special subset of the theory is the two-parameter family \( 2q\alpha = 1 \) \([29]\) with \( \delta = 1 \) (i.e. \( \alpha = \beta = \frac{1}{2q} \)), the well-known \( \mathcal{N} = 2 \) Liouville theory, which is mirror to super-coset \( \text{SL}(2,\mathbb{R})_k/\text{U}(1) \) theory \([28, 33, 34, 35]\). In this case, the \( \phi_2 \) boson arises from bosonizing the \( \mathcal{N} = 2 \) superconformal fermions. Finally, the \( c = 3 \) sine-Liouville theory is obtainable by taking the limit \( q \to 0 \).

In the asymptotic region \( \varphi \to -\infty \), the tachyon condensate vanishes. The theory reveals \( \text{U}(1) \times \tilde{\text{U}}(1) \) worldsheet symmetries, whose holomorphic currents are

\[
J = i\left(\frac{1}{\beta} \partial \phi_1 - \frac{1}{\delta} \partial \phi_2 \right); \quad \tilde{J} = i\left(\frac{\beta \partial \phi_1 + \delta \partial \phi_2}{\beta^2 + \delta^2} \right),
\]

normalized so that the tachyon condensates carry integer-valued charges. For primary vertex operators, conformal dimension and global charges are given by

\[
V(a, b, c) = e^{a\varphi + ib\phi_1 + ic\phi_2} : \Delta_V = \frac{1}{2}(-a^2 + 2aq + b^2 + c^2) \quad \text{and} \quad Q_V = \frac{b}{\beta} - \frac{c}{\delta} \quad \text{and} \quad \tilde{Q}_V = \frac{b\beta + c\delta}{\beta^2 + \delta^2}.
\]

The tachyon background in (2.1) is neutral under \( \text{U}(1) \) but carries \( \pm 1 \) unit charges under \( \tilde{\text{U}}(1) \). As such, at finite \( \varphi \), the tachyon condensate breaks \( \tilde{\text{U}}(1) \) symmetry to \( \mathbb{Z} \). Hereafter, we shall refer \( \text{U}(1) \) charge the momentum quantum number and \( \tilde{\text{U}}(1) \) charge as the winding number of the string around the Scherk-Schwarz compactification circle.

For later considerations, we are interested in two-point correlation functions. By examining \( \text{U}(1), \tilde{\text{U}}(1) \) quantum numbers, we see that there are two possible classes of two-point correlation functions \(^5\):

\[
\mathcal{R} \equiv \left\langle V(a, b, c)V(a, -b, -c) \right\rangle \quad \text{and} \quad \mathcal{D} \equiv \left\langle V(a, b, c)V(a, b, c) \right\rangle.
\]

\(^5\)Our conventions for the \( \text{SL}(2, \mathbb{C}) \) states, charge conjugation and normalization are such that

\[
\left\langle V(2q - a, b, c)V(a, -b, -c) \right\rangle = 1.
\]
They define two possible types of reflection amplitudes: as the incoming wave hits the sine-Liouville potential, part of the reflection amplitude conserving the winding quantum number ($\Delta \tilde{Q} = 0$) is described by $R$, while the part shifting the winding number by integer unit ($\Delta \tilde{Q} = \pm 1, \pm 2, \ldots$) is described by $D$. Thus, compared to the Liouville theory, the correlators $D$ are new and highly nontrivial. Based on recursion relations for different $\alpha$, results for integer-valued $a, b, c$ and analytic continuation thereof, a closed form of $D$ was proposed in [30]. As the simplest situation, consider scattering a mode carrying vanishing momentum and winding quantum numbers off the sine-Liouville potential. The reflection amplitude is given by

$$D(a, 0, 0)|_{Q_v = 0} = R(a, 0, 0) = (\xi)^{\frac{\gamma - \alpha}{\alpha}} \frac{\Gamma(1 + \frac{\alpha - 2q}{\alpha})\Gamma\left(\frac{\alpha}{2} - \frac{a - q}{\alpha}\right)\Gamma(1 + \alpha(a - q))\Gamma(1 - \alpha(a - q) + \alpha q)}{\Gamma(1 - \frac{a - q}{\alpha})\Gamma\left(\frac{\alpha}{2} + \frac{a - q}{\alpha}\right)\Gamma(1 - \alpha(a - q))\Gamma(1 + \alpha(a - q) + \alpha q)} \times \exp \left[ \int_0^\infty \frac{ds}{s} \left( \frac{e^{-(1 + a^2 + \alpha a - 2q) s} - e^{-(1 + \alpha^2 - \alpha a)^s}}{(1 - e^{-s})(1 - e^{-\alpha^2 s})} \right) (1 - e^{-2\alpha q s}) + 4(a - q)qe^{-s} \right] .$$

(2.4)

Here, $\xi$ denotes the "renormalized" cosmological constant on the worldsheet:

$$\xi = (\pi \mu)^2 \frac{\lambda \left( -\frac{1}{2} \delta^2 \right) \gamma(\frac{1}{2})}{\gamma\left( \frac{1}{2} \right)} \frac{(1 + \alpha^2)\gamma(1 + \alpha^2)\gamma(1 + \frac{1}{2}(\alpha^2 - \beta^2 - \delta^2))}{(1 + \alpha^2)\gamma(1 + \alpha^2)\gamma(1 + \frac{1}{2}(\alpha^2 - \beta^2 - \delta^2))} ,$$

(2.5)

where $\gamma(x) \equiv \Gamma(x)/\Gamma(1 - x)$.

The classical limit of the three-parameter sine-Liouville theory (2.1) should be well-defined. We see from (2.1) that $\varphi, \phi_1, \phi_2$ dynamics becomes semiclassical in the limits $\alpha \to 0, \beta \to 0, \gamma \to 0$, respectively. Because of the conformality condition (2.2), simultaneous classical limit is achieved if the dilaton slope $q$ is set to $1/\alpha \to \infty$. In these classical limits, the renormalized cosmological constant $\xi$ is replaced by a classical cosmological constant, which is always proportional to $\mu^2$.

The unitarity condition, $|R(a, 0, 0)| = 1$, restricts the coupling parameters further. For real-valued momentum $p$ condition $|p| = a - q$, it is straightforward to check that the ratios of gamma functions in (2.4) are pure phases. The third line in (2.4) is also a pure phase in so far as $\alpha^2 + \beta^2 + \delta^2 > 0$, which is always guaranteed in Euclidean theory (but will be nontrivial in Lorentzian theory).

\textsuperscript{6} We are using the different notation ($\alpha' = 2$) from that in [30] ($\alpha' = 1$). We have also corrected typos in the proposal [30], which was confirmed by the authors of [30].

\textsuperscript{7} Note that the validity of the minisuperspace analysis also forces us to take $\beta \to 0$ limit so as to treat $\phi_1$ as a weakly interacting field. Without taking the adiabatic limit for $\phi_1, \phi_2$, the minisuperspace analysis may not make sense due to the unboundedness of the sine-Liouville potential. We, nevertheless, believe that the theory is well-defined as a quantum theory. For example if one fermionizes $\phi_2$ at a special radius as we will do later, the potential term turns out to be a time-dependent mass term for worldsheet fermion. This is certainly a stable interaction classically. We would like to thank H. Ooguri for pointing out the unboundedness of the sine-Liouville potential.
$(\xi)^{(q-a)/a}$ is also a pure phase provided we restrict the coupling parameters in the neighborhood $\alpha, \beta, \gamma \to 0$ to range over the domain $\alpha^2 > \beta^2 + \delta^2$.

Once we set $2q\alpha = 1$, the proposed reflection amplitude is consistent with that for the two-parameter Sine-Liouville theory \[29\]. On the other hand, if one sets $\beta = \delta = 0$, the reflection amplitude reduces to that of the pure ($N = 0$) Liouville theory \[30\]. In this case, we verified that the reflection amplitude (2.4) satisfies not only the ordinary shift relation:

$$D(a + \alpha, 0, 0) = D(a, 0, 0) C^{-1}_{-1}(a; \alpha)$$

but also the dual shift relation,

$$D(a + \alpha^{-1}, 0, 0) = D(a, 0, 0) \tilde{C}^{-1}_{-1}(a; \alpha),$$

which is highly nontrivial given that \[30\] did not rely on any information of the dual recursion relations. Here, $C_{-1}(a; \alpha)$ and $\tilde{C}_{-1}(a; \alpha)$ refer to the structure constants \[32\].

A comment is in order. In \[30\], the reflection amplitude (2.4) was deduced from extrapolation of the amplitude at integer values of the Liouville momenta $a, b, c$. It is possible that the extrapolation misses a possible extra contribution, which vanishes at those discrete momenta. On the other hand, we have seen above that (2.4) passes consistency checks when reduced to Liouville theory. As such, we feel that the expression (2.5) of the renormalized cosmological constant is actually the correct result, valid for all values of the Liouville momenta. Throughout this work, we shall proceed with such tacit assumption.

### 2.2 Wick rotation to Lorentzian target space

String propagation in the Lorentzian cosmological background \[1.1\] can now be described via the Euclidean signature sine-Liouville theory \[2.1\]. This requires a suitable prescription for analytically continuation between the Euclidean and the Lorentzian target spaces. The prescription is expected to depend on physical situations, but should not be arbitrary. In particular, any physically sensible analytic continuation ought to be compatible with unitarity, causality and analyticity. Here, for the worldsheet theory \[2.1\], we argue that analytic continuation via the standard Wick rotation:

$$\begin{align*}
\phi &\rightarrow e^{i\epsilon}t \\
\alpha &\rightarrow e^{i\epsilon} \kappa \\
q &\rightarrow e^{i\epsilon} Q \\
\phi_{1,2} &\rightarrow \phi_{1,2} \\
\mu &\rightarrow \mu
\end{align*}$$

where $\epsilon \rightarrow +\pi/2$.

\[8\]At the same time, we need to rotate the worldsheet from Euclidean to Lorentzian one with the standard $+i\epsilon$ prescription for the Feynman boundary condition. Notice that the conformal invariance remains intact throughout these analytic continuations.
satisfies all these requirements. Range of the coupling constants $\beta, \delta$ is then fixed by Wick rotation of the conformality condition (2.2):

$$2\kappa Q = \kappa^2 + \beta^2 + \delta^2 - 2 .$$  
(2.9)

Analytic continuation for the parameters $\alpha, q$ is the same as that for the variable $\varphi$, and this is exactly the same as the standard Wick rotation for a canonically conjugate pair. The worldsheet action is now given as

$$S_L = \int \frac{d^2 \sigma}{2\pi} \left[ - \partial_\mu t \partial^\mu t + \partial_\mu \phi_1 \partial^\mu \phi_1 + \partial_\mu \phi_2 \partial^\mu \phi_2 + 2\pi \mu e^{-\kappa t} \cos(\beta \phi_1 + \delta \phi_2) - \frac{Q}{2} R_{(2)} t \right].$$  
(2.10)

Since the conformal invariance remains intact throughout the analytic continuation (2.8), the Lorentzian sine-Liouville theory (2.10) is a well-defined conformal field theory. The worldsheet action (2.10) now describes string propagation on the Lorentzian cosmological background (1.1) with the Scherk-Schwarz compactification. More precisely, as mentioned already, to obtain the superconformal model relevant for heterotic or Type II superstring theories, we need to set $\delta = 1$ and fermionize $\phi_2$ back to worldsheet fermions.

Notice that in our proposed analytic continuation (2.8) the worldsheet cosmological constant $\mu$ remains intact. Thus, the generating functional

$$Z[\mu] := \int_{\mathcal{C}} [dt d\phi_1 d\phi_2] \exp(iS_L)$$  
(2.11)

is an unambiguous and well-prescribed function of $\mu$ irrespective of the signature of the target spacetime, equivalently, the contour $\mathcal{C}$ prescribed in defining the worldsheet functional integral (2.11). As such, one can evaluate the generating functional utilizing the method of [39], viz. by first computing $\partial Z[\mu]/\partial \mu$ and then integrating back with respect to $\mu$ with the boundary condition that $Z[\mu = 0]$ yields the worldsheet partition function for $c = 3$ free field theory.

We again emphasize that the crucial advantage of our proposed worldsheet model is that the Euclidean theory (2.1) is a well-defined, unitary conformal field theory for any value of the linear dilaton and that the standard Wick rotation (2.8) manifestly carries over conformal properties to the Lorentzian theory.

3. The "Nothing state" made precise

In order for the proposed Lorentzian three-parameter sine-Liouville theory (2.10) to realize a technically natural "Nothing state" possessing predictability and calculability, it should be that

- back-reaction to the background is controllably small,

- no string excitation is present for $t \to -\infty$.

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9Similar analytic continuation was advocated previously in the context of the Liouville theory [37, 38].
To ensure the first condition, which provides technical naturalness, string coupling parameter $g_{st} = e^{-Q/2}$ should be small outside the "Nothing state". To achieve this, we restrict $Q$ to the range $Q \geq 0$. In case $Q > 0$, we can then confine the strong string coupling region within the "Nothing state". In case $Q = 0$, we can keep the string coupling fixed and small by adjusting the coupling constants $\beta, \delta$ appropriately so that the conformality condition (2.9) is satisfied.

It now remains to ensure that string worldsheet effects are small enough. This is nontrivial since, even for free string theory, time-dependent background triggers Bogoliubov pair production. Thus, to ensure the first condition, we need to restrict the coupling constants $\kappa, \beta, \delta$ so that the pair production rate is sufficiently suppressed exceeding the Hagedorn growth of state density. This is a quite non-trivial issue in which the worldsheet quantum effect could play essential roles. In this section, we compute the rate and show that the condition is met for a non-trivial range of the coupling constants.

To ensure the second condition, which defines the "Nothing state", we need to set $\kappa > 0$ and to let the sine-Liouville potential provides an impenetrable barrier to closed string excitations in the asymptotic past $t \to -\infty$. That such a barrier is present, however, cannot be seen within the minisuperspace approximation. For one thing, the sine-Liouville potential is unbounded from below for a range of the compact boson fields, where $\cos(\beta \phi_1 + \delta \phi_2)$ takes a negative value. To probe the presence of potential barrier and hence the putative "Nothing state", we shall compute one-loop vacuum transition amplitude in section 3.3 and examine its extensivity with the temporal volume.

### 3.1 Bogoliubov pair production

Now, string dynamics (2.11) in the cosmological background (1.1) would involve Bogoliubov string pair production where the winding tachyon rolls rapidly during the early epoch of the big bang. The Bogoliubov production amplitude is determined by two-point correlators. MS computed the two-point correlator directly (with their own prescriptions of analytic continuation and in semi-classical approximation), and deduced that coherent Bogoliubov pair production effectively shows a thermal distribution once phase-correlations are averaged over.

Since the timelike theory is defined from the spacelike theory via the Wick rotation (2.8), physical observables are extractable accordingly. Indeed, taking the Wick rotation to the reflection amplitude (2.4) in Euclidean theory, we can extract 'reflection amplitude' in Lorentzian theory. This amplitude is interpretable as the Bogoliubov coefficient of particle pair production. We shall now show that the distribution functions extracted so behaves in a manner anticipated by MS, but with interesting and nontrivial twists due to full-fledged string worldsheet effects. Our prescription (2.8) involves Wick rotation of the Euclidean momentum to Lorentzian energy $a - q = ip \to -\omega$.

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$^{10}$The background (1.4) is also time-dependent but we expect that particle production induced by it is comparatively suppressed in the adiabatic regime $0 < \nu \ll 1$. 

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\( \alpha \to i\kappa \) and \( q \to iQ \). Taking also \( Q \to 0^+ \),\(^{11}\) we obtain the Lorentzian reflection amplitude \( R_L \) as
\[
R_L(\omega, 0, 0) = (\xi_L)^{-i\omega \kappa} \frac{\Gamma(1 + i\omega \kappa)\Gamma(-i\omega \kappa)}{\Gamma(1 - i\omega \kappa)\Gamma(+i\omega \kappa)} = -(\xi_L)^{-i\omega \kappa} . \tag{3.1}
\]
Here, \( \xi_L \) is the renormalized worldsheet cosmological constant in the Lorentzian sine-Liouville theory, obtained from (2.5) via the Wick rotation (2.8):
\[
\xi_L = (\pi\mu)^2 \frac{\frac{\gamma(-(-\kappa^2 + \beta^2 + \delta^2))}{\gamma(-\frac{1}{2}(-\kappa^2 + \beta^2 + \delta^2))} \gamma(1 - \kappa^2)\gamma(1 - \frac{1}{2}(\kappa^2 + \beta^2 + \delta^2))}{\gamma(-\frac{1}{2}(1 - \kappa^2))\gamma(1 - \frac{1}{2}(1 - \kappa^2))} . \tag{3.2}
\]
Here, the limit \( \kappa^2 + \beta^2 + \delta^2 \to 2 + \varepsilon \) (\( \varepsilon > 0 \)) is implicit (so that \( Q = 0 \) and the string coupling parameter is set to constant).

Throughout the analytic continuation, the Bogoliubov coefficient (3.1) remains well-defined. Moreover, the first line in (3.1) is a pure phase except for the first factor involving the renormalized cosmological constant \( \xi_L \). Now, suppose \( \xi_L \) takes a negative value. In this case, there are two possible vacuum branches, corresponding to the prescription \( \xi_L = |\xi_L|e^{\pm i\pi} \). We shall choose the lower branch and obtain the Boltzmann-like, convergent particle distribution function:
\[
\mathcal{P}(\omega) = |R_L|^2 = \left|\xi_L^{-i\omega \kappa}\right|^2 = e^{-\frac{2\omega}{T_{\text{eff}}}} . \tag{3.3}
\]
Here, the effective temperature \( T_{\text{eff}} \) is set entirely by the Liouville coupling constant \( \kappa \):
\[
T_{\text{eff}} \equiv \frac{\kappa}{\pi} . \tag{3.4}
\]
This is precisely the sort of the behavior anticipated in [8], except that the behavior here is deduced from entirely different definition of the time-dependent tachyon background and from different analytic continuations.

### 3.2 phases of renormalized tachyon condensate

According to (3.3), the "Nothing state" — state on which no finite energy string excitation is possible — arises in the regime wherever the renormalized cosmological constant \( \xi_L \) gets negative-valued. We are thus interested in whether \( \xi_L \) can change sign and, if it does, under what circumstances it does. We shall focus on \( c = 3 \) system obtained by taking \( Q \to 0^+ \) limit.\(^{12}\) Then, \( \xi_L \) in (3.2) takes the value:
\[
\xi_L = (\pi\mu)^2 \frac{\frac{\gamma(-2(1 - \kappa^2))}{\gamma(-1 - \kappa^2)} \gamma(1 - \kappa^2)\gamma(-0)}{\gamma(-\frac{1}{2}(1 - \kappa^2))\gamma(1 - \frac{1}{2}(1 - \kappa^2))} . \tag{3.5}
\]

\(^{11}\)This choice is automatically selected by requiring that the “Liouville wall” in the time-like theory is located in the “strongly coupled” region.

\(^{12}\)If we take \( Q \to 0^- \) limit, though we still obtain \( c = 3 \) system, the "Nothing state" obtained so have different characteristic. In particular, as we shall discuss later, \( Q \to 0^\pm \) theories exhibit quite opposite behavior in the worldsheet semiclassical limit. This indicates that there can be two distinct Lorentzian \( c = 3 \) system definable from the spacelike theory. We shall postpone such non-analyticity for the moment, and return back to it later.
As it stands, $\xi_L$ diverges because of the factor $\gamma(-0)$. This divergence is multiplicative, so it is absorbable by renormalizing the bare cosmological constant $\mu^{13}$. So, after the renormalization, the sign of $\xi_L$ depends solely on $\kappa$, the Lorentzian Liouville exponent.

It is important to recall that the three-parameter sine-Liouville theory has, in addition to the Liouville field $t$, compact boson fields $\phi_1, \phi_2$. The parameter $\kappa$ controls interaction of the Liouville field, while $\beta, \delta$ do so interaction of the compact bosons $\phi_1, \phi_2$. In the previous subsection, we took $Q \to 0^+$ so that we can set the string coupling parameter $g_{st}$ fixed to a small value.

We are eventually interested in superstring case, for which we need to fermionize $\phi_2$ by setting $\delta = 1$. The quantum conformality condition (2.9) then relates $\phi_1$ dynamics coupling parameter $\beta$ to the Liouville coupling parameter $\kappa$ as $\kappa^2 + \beta^2 = 1$. This implies that the worldsheet dynamics is always strongly coupled: the $\phi_2$ field is kept strongly coupled by $\delta = 1$, while $t$ and $\phi_1$ mutually interpolate between strong and weak coupling regime by $\kappa^2 + \beta^2 = 1$. In other words, at least two out of the three worldsheet fields $t, \phi_1, \phi_2$ are always strongly coupled. In Fig.(1), we plot the regions in the coupling parameter space, where the renormalized cosmological constant $\xi_L$ takes positive (blank) and negative (shaded) values, respectively. We also note that the condition $Q = 0$ is a line located on the verge of the sign change of $\xi_L$. Thus, the parameter range we are interested in indeed corresponds to strong coupling region. In fact, exact conformal field theory approach beyond mini- or midi-superspace approximation has been indispensable for us to determine the actual sign of $\xi_L$.

From the gamma functions, we see that $\xi_L > 0$ for $0 < \kappa^2 < \frac{1}{2}$ and $\xi_L < 0$ for $\frac{1}{2} < \kappa^2 < 1$. Notice that, for superstring theories, $\kappa^2 = 1$ sets the upper limit of the parameter space since the limit corresponds to infinitely small compactification (or time-like Liouville theory in the T-dual

\[ \text{Figure 1: The sign of the renormalized cosmological constant } \xi_L \text{ as a function of coupling parameters, } \kappa^2 \text{ and } \beta^2. \text{ In the shaded/blank region, } \xi_L \text{ takes positive/negative value. The solid red line corresponds to the } Q = 0 \text{ condition.} \]

\[ \text{...} \]

\[ \text{...} \]
description). Accordingly, the particle production probability behaves as

\[ P(\omega) = |R_L(\omega)|^2 \propto \begin{cases} 
1 & \text{for } 0 < \kappa^2 < \frac{1}{2} \\
 e^{-2\omega/T_{\text{eff}}} & \text{for } \frac{1}{2} < \kappa^2 < 1
\end{cases} \] (3.6)

and the particle number distribution is determined to be \( N(\omega) = P(\omega)/(1 \mp P(\omega)) \) for boson and fermion particles, respectively.

Spectral moments of inclusive string production are then given by

\[ \langle E^n \rangle := \int_0^{\infty} d\omega \omega^n \rho(\omega)N(\omega) \quad (n = 0, 1, 2, \cdots) \] (3.7)

where \( \rho(\omega) \) refers to density of closed string states. We are eventually interested in our proposal in the superstring context. In this case, the density of state behaves for large \( \omega \) as

\[ \rho(\omega) \sim e^{2\pi \sqrt{\frac{c_{\text{eff}}}{3}} \omega}, \] (3.8)

where \( c_{\text{eff}} \) is the ‘effective central charge’ [40] of the transverse sector, which counts the net degrees of freedom. In simple cases when the transverse sector is a \( D-2 \) dimensional flat space with linear dilatons \( q_i \), the criticality condition is written as

\[ \frac{3}{2}(D-2) + 3 \sum_i q_i^2 = 12, \] (3.9)

and we just have

\[ c_{\text{eff}} = 12 - 3 \sum_i q_i^2 = \frac{3}{2}(D-2). \] (3.10)

For \( 0 < \kappa^2 < \frac{1}{2} \), the transition probability is flat, viz. the effective temperature of the pair produced string distribution is infinite. This may look counter-intuitive since worldsheet dynamics of the Liouville field \( t \) becomes classical in the limit \( \kappa \to 0 \). However, as discussed above, \( Q = 0 \) and conformality conditions imply that \( \frac{1}{2} < \beta^2 < 1 \) in this range. This means that the \( \phi_1 \)-field is strongly coupled. We thus interpret the divergent string pair production as being triggered by strongly coupled worldsheet dynamics of the compact boson fields, \( \phi_1, \phi_2 \).

For \( \frac{1}{2} < \kappa^2 < 1 \), the pair production probability is no less than \( e^{-2\sqrt{T_\pi}\omega} \). The maximum is at \( \kappa = 1 \), at which the probability is \( e^{-2\pi\omega} \). Now, the density of string states \( \rho(\omega) \) scales as \( \rho(\omega) \sim e^{2\pi \sqrt{c_{\text{eff}}/3} \omega} \). Therefore, if \( c_{\text{eff}} < 3 \), we always obtain ultraviolet finite particle production irrespective of the value of \( \kappa \). If \( 3 < c_{\text{eff}} < 6 \), we can achieve ultraviolet finite particle production if the Liouville coupling \( \kappa \) is chosen suitably. If \( c_{\text{eff}} > 6 \), we always face the ultraviolet divergence of the particle production for any value of \( \kappa \). In section 4, we will see that the threshold at \( c_{\text{eff}} = 6 \) corresponds to the string compactification on a conifold and also curiously coincides with the "black hole / string transition" [41, 42, 43] point \( k = 1 \) of \( SL_k(2)/U(1) \) coset conformal field.
theory. We note that, in this range of Liouville coupling \( \kappa \), worldsheet dynamics of the Liouville field \( t \) is strongly coupled, yet not strong enough to warrant unsuppressed pair production as in the \( 0 < \kappa^2 < \frac{1}{2} \) region. It implies that, in so far as the Bogoliubov pair production is concerned, the compact bosons \( \phi_1, \phi_2 \) play more significant role than the Liouville field \( t \).

We thus conclude that, in the range \( \frac{1}{2} < \kappa^2 < 1 \), the Lorentzian three-parameter sine-Liouville theory (2.10) yields an exactly soluble conformal field theory realization of the "Nothing state" at the beginning of the Universe. In particular, fermionic string theories with less than six dimensions (in the case \( D < 6 \) in (3.10)) can have finite and controllable Bogoliubov pair production and hence a viable model of nonsingular string cosmology. We emphasize that the pair production behaves in the present case quite differently from that in the Liouville theory. As elaborated above, this is attributed to the sine-Liouville potential involving \( \phi_1, \phi_2 \). We also emphasize that, because we set \( Q = 0 \) and the conformality condition, we cannot achieve worldsheet dynamics entirely semiclassical: if the Liouville field \( t \) is weakly coupled the fields \( \phi_1 \) is strongly coupled and vice versa.

### 3.3 "Nothing state" from vacuum amplitude

Alternative route for probing the "Nothing state" is to compute the vacuum-to-vacuum transition amplitude and examine whether the temporal volume of the vacuum energy is cut off at a scale set by the Liouville cosmological constant. Adopting the semiclassical approximation [39], MS estimated the one-loop transition amplitude \( Z_L \) as the product of zero mode \( Z_0 \) and nonzero mode \( \hat{Z} \) contributions:

\[
Z_L = Z_0 \cdot \hat{Z} \quad \text{where} \quad Z_0 = \left( -\frac{1}{\kappa} \ln \frac{|\mu|}{\Lambda} - \frac{i}{2T_{\text{eff}}} \right),
\]

(3.11)

Here, \( \ln \Lambda \) is the infrared cutoff for the Louville direction. The real part (proportional to \( \ln |\mu| \)) is interpreted as excising the temporal volume (out of the total volume \( \frac{1}{\kappa} \ln \Lambda \)) at the onset of "Nothing state". The imaginary part is interpreted as exhibiting effective thermal distribution of Bogoliubov pair production with temperature \( T_{\text{eff}} \). In getting these results, however, it was technically crucial for MS to rely on non-standard analytic continuation of the cosmological constant \( \mu \to i\mu \) they adopted.

From Lorentzian spacetime viewpoint, Bogoliubov pair production and vacuum energy are related each other. The one-loop vacuum transition amplitude can be understood as the time evolution under the tree-level Bogoliubov pair production. Let us examine if such relation can be checked directly by worldsheet analysis via the Wick rotation (2.8). It is well-known that the reflection amplitude \( \mathcal{R}(p) \) of the Euclidean Liouville theory is related to the density of states by

\[
\rho_{\text{finite}}(p) = \frac{1}{2\pi i} \frac{d\mathcal{R}(p)}{dp}.
\]

(3.12)
Assuming that this assumption holds even after the Wick rotation to the Lorentzian theory, we get

\[ \rho_{\text{finite}}(\omega) = \frac{1}{2\pi i} \frac{dR_L(\omega)}{d\omega}. \] (3.13)

We see that the Lorentzian relation (3.13) then leads to appearance of the imaginary part, reaching the same conclusion as MS. Let us now apply this general consideration to our proposal. Substituting the two-point function (3.1) to (3.13), we obtain

\[ \rho_{\text{finite}}(\omega) = -\frac{1}{2\pi \kappa} \ln \xi_L + \rho_0(\omega), \] (3.14)

where we separated explicitly the \( \omega \)-independent part from the \( \omega \)-dependent part. The full density of states contains an additional infrared cutoff factor \( \ln \Lambda \). We are primarily interested in the limit \( \Lambda \to \infty \) (removing the infrared cutoff). We then have

\[
Z_{\text{torus}} = \lim_{\Lambda \to \infty} \frac{1}{2\pi} \int d\omega \frac{e^{2\pi i r_1 P(\omega) - 2\pi r_2 H(\omega)}}{2\pi} \frac{1}{2\pi} \ln \left( \frac{\xi_L}{\Lambda^2} \right) \int d\omega \frac{e^{2\pi i r_1 P(\omega) - 2\pi r_2 H(\omega)}}{2\pi},
\]

where \( \text{Tr} \) refers to summing over all other quantum numbers than the energy.

In the previous section, we found that the renormalized cosmological constant \( \xi_L \) is positive for \( 0 < \kappa^2 < \frac{1}{2} \) and becomes negative for \( \frac{1}{2} < \kappa^2 < 1 \). The one-loop vacuum transition amplitude then takes the form:

\[ Z_L = \begin{cases} 
-\frac{1}{2\pi} \ln |\xi_L|/\Lambda^2 - i0 \big) \tilde{Z} & \text{for } 0 < \kappa^2 < \frac{1}{2} \\
-\frac{1}{2\pi} \ln |\xi_L|/\Lambda^2 - i \frac{\pi}{2\pi} \big) \tilde{Z} & \text{for } \frac{1}{2} < \kappa^2 < 1.
\] (3.16)

We see that the result is in complete agreement with the tree-level Bogoliubov amplitude result. The real part exhibits excision of the time evolution of the universe at the beginning. Therefore, the dependence of the one-loop vacuum amplitude on \( -\frac{1}{2\pi} \ln |\xi_L| \) is a rigorous indication for the presence of the "Nothing state". The imaginary part agrees perfectly with (3.6) in the real-time thermal field theory formalism: for two-particle squeezed state, the time evolution runs over \( [-\infty, t] \) and \( [t, t - i\beta_{\text{eff}}/2] \). We see from the above result that the effective temperature \( T_{\text{eff}} \) is infinite for \( 0 < \kappa^2 < \frac{1}{2} \) and \( \kappa/\pi \) for \( \frac{1}{2} < \kappa^2 < 1 \), respectively.

3.4 Nothing state: from bare to renormalized

The most striking feature of the "Nothing state" based on the Lorentzian three-parameter sine-Liouville theory is the pattern of the Bogoliubov particle production rate as given in (3.6). Moreover, the rate (3.6) involves the renormalized cosmological constant \( \xi_L \) and is the exact conformal field theory result. Intuitively, we can interpret the result as follows. On general grounds, we expect two possible sources that the cosmological constant may change sign.
• In the semiclassical, mini-superspace approximation of the Liouville theory, Wick rotation provides a source of the sign flip: the mini-superspace Hamiltonian $H_E = +\partial_\varphi^2 + 4\pi\mu e^{\alpha\varphi}$ turns under Wick rotation into $H_L = -\partial_t^2 + 4\pi\mu e^{-\kappa t}$. The Hamiltonian constraint in Euclidean and Lorentzian cases are related each other by the flip of $\mu$ to $-\mu$. Hence, the phase shift of the reflection amplitude $\sim \mu^{ip}$ in the Euclidean theory becomes under Wick rotation the damping factor of the Bogoliubov coefficient $\sim e^{-\omega\pi}$ in the Lorentzian theory.

In sine-Liouville theory, the situation is drastically different. The Euclidean mini-superspace Hamiltonian

$$H_E = +\partial_\varphi^2 + \partial_{\phi_1}^2 + \partial_{\phi_2}^2 + 2\pi\mu e^{\alpha\varphi} \cos(\beta\phi_1 + \delta\phi_2) \quad (3.17)$$

becomes under the Wick rotation $\{2.8\}$ the Lorentzian mini-superspace Hamiltonian

$$H_L = -\partial_t^2 + \partial_{\phi_1}^2 + \partial_{\phi_2}^2 + 2\pi\mu e^{\alpha\varphi} \cos(\beta\phi_1 + \delta\phi_2) \quad (3.18)$$

In both cases, the sign of bare cosmological constant is irrelevant since it can be compensated by shifting $\beta\phi_1 + \delta\phi_2$ by $\pi$. This explains why both the reflection amplitude and the Bogoliubov coefficient depends on square of $\mu$. Thus, sign flip associated with the cosmological constant should arise not from the bare one but from some other combination of the coupling parameters.

• String worldsheet effects may provide another source of the sign flip. Take, for example, the $\mathcal{N} = 2$ SL$_k(2, \mathbb{R})/U(1)$ coset model describing the Euclidean two-dimensional black hole (cigar) geometry. The reflection amplitude behaves as

$$\mathcal{R}(p) \propto \left(\frac{M}{\Gamma(1 - \frac{k}{2})}\right)^{ip} = \xi^{ip}, \quad (3.19)$$

where $M$ is the black hole mass, which plays via the FZZ duality the role of bare cosmological constant in $\mathcal{N} = 2$ Liouville (sine-Liouville) theory. (See e.g. [46].) We see from (3.19) that what matters for the reflection amplitude is not the bare cosmological constant $M$ but the renormalized one $\xi$. For example, extrapolating the current algebra level $k$ naively across 1, we see that $\xi$ can change the sign.

Based on these intuitions and considerations in the previous subsection, we interpret the sign flip of $\xi_L$ arising from combination of both effects. For $0 < \kappa^2 < \frac{1}{2}$, we interpret $\xi_L > 0$ as a consequence of strong coupling dynamics of the compact boson fields $\phi_1, \phi_2$. For $\frac{1}{2} < \kappa^2 < 1$, we also interpret $\xi_L < 0$ as a consequence of strong coupling dynamics of the Liouville field $t$. Behavior of $\xi_L$ as a function of $\kappa^2$ is plotted in Fig.(2).
Figure 2: The renormalized cosmological constant $\xi_L$ as a function of $\kappa^2$. We set $Q = 0$ for an asymptotically flat universe. When $\xi_L$ becomes negative as in the region $\frac{1}{2} < \kappa^2 < 1$, the Bogoliubov coefficient $|R_L(\omega)|$ is exponentially suppressed.

4. Embedding into Superstring Theories

In the previous section, we proposed a viable string theory realization of the "Nothing state". The proposal facilitates exactly solvable conformal field theory approach, thus enabling us to define the "Nothing state" precise enough. An immediate question is whether our proposal can be made more realistic (i.e. free from bulk tachyon) by embedding the model into superstring theories. In this section, we shall argue that this can be done so provided we deform the sine-Liouville potential appropriately. In particular, we shall show that the deformed fermionic theory admits superconformal field theory description and features the same physics concerning emergence of the "Nothing state" as the bosonic theory did.

One possible route for constructing exactly solvable cosmological superstring background has been utilizing the Lorentzian Liouville theory: the $c = 1$ Liouville theory defined in terms of a timelike worldsheet boson, free of screening charge. The Lorentzian Liouville theory and tachyon condensation therein were much studied in recent years [37, 38]. In fact, the MS proposal belongs to a variant of the Lorentzian Liouville theory.

In fact, there are several possible Lorentzian Liouville theories, all classifiable according to the worldsheet supersymmetries. Furthermore, they are all related to various limits of the three-parameter sine-Liouville theory. Before presenting the relevant model, in this section, we shall begin with features in each of the models, with particular emphasis of advantage and shortcomings in utilizing them for concrete realization of the MS proposal.

4.1 models with $\mathcal{N} = 0$ conformal symmetry

Begin with the bosonic (no worldsheet supersymmetry) Lorentzian Liouville theory [37, 38]. Denoting the timelike worldsheet boson as $X^0$, the worldsheet action \textsuperscript{14} of the bosonic (no spacetime

\textsuperscript{14}Here, we set $\ell_{\text{st}}^2 = 4\pi$.}
supersymmetry) string theory is given by

\[
S_{\text{TLT}} = \frac{1}{4\pi} \int d^2 z \left( -\partial X^0 \overleftarrow{\partial} X^0 + 4\pi \mu e^{-2\beta X^0} \right)
\]  

(4.1)

plus the action for flat \( \mathbb{R}^{1,25} \) and for conformal ghosts. With vanishing background charge, which amounts to constant-valued dilaton, the worldsheet boson \( X^0 \) contributes central charge \( c = 1 \).

The second term denotes to spatially homogeneous condensation of the tachyon field: \( T(X^0) = 4\pi \mu \exp(-2\beta X^0) \), where the exponent \( \beta \) is set to \( \pm 1 \) by the tachyon on-shell condition. Choose the convention \( \beta = +1 \), viz. the tachyon condensation grows exponentially at early epoch \( X^0 \to -\infty \). At late epoch \( X^0 \to +\infty \), the tachyon condensation is turned off, and the spacetime becomes flat \( \mathbb{R}^{1,25} \). Implicit to the consideration is that, for real-valued \( X^0 \), \( \mu \) is restricted to positive definite value: in the mini-superspace approximation, the worldsheet reparametrization invariance puts the constraint \( (\dot{X}^0)^2 - 4\pi \mu e^{-2\beta X^0} = 0 \).

As it stands, path integral of the Lorentzian Liouville theory is ill-defined since the action is not positive definite. A prescription proposed first by [37, 38] involves analytic continuation \( (X^0, \beta) \to (\phi, b) = (-iX^0, +i\beta) \). This is the standard Wick rotation of the target spacetime. Taking the limit \( b \to \pm i \) along with the Wick rotation, the prescription leaves the central charge \( c = 1 \) intact [47]. In the classical limit, mini-superspace analysis shows that Bogoliubov pair production is exponentially suppressed. This agrees with full-fledged worldsheet analysis, in which the renormalized cosmological constant \( \xi_L = 2\pi \mu \gamma (b^2) \) is found to take negative value as \( b \to \pm i0 \).

Schomerus [48] criticized the proposal of Strominger-Takayanagi [38], and proposed another prescription with results on two- and three-point correlation functions. His proposal takes analytic continuation of the central charge \( c \): starting from the well-defined Liouville theory with \( c \geq 25 \), two different limiting theories were obtained in which conformal weights take real values. One is the Euclidean Liouville theory and another is the Lorentzian Liouville theory. The resulting two theories are not continuable by the Wick rotation proposed in [38], and this also explains nonanalytic factors in three-point correlation functions. In so far as two-point correlation functions are concerned, both prescriptions turn out to yield the same result. Moreover, within the mini-superspace approximation, [49] argued that the Bogoliubov amplitude is always unitary for a given choice of self-adjoint Liouville Hamiltonian. It was also observed that, if summed over all possible choice of self-adjointness, the Bogoliubov amplitude becomes exponentially damped and coincides with the result of [38].

The most serious difficulty of the bosonic Liouville theory is that, being bosonic string theory, the bulk tachyon is present in the spectrum: even at late epoch when the winding string tachyon background is turned off, the bulk tachyon would destabilize the flat spacetime \( \mathbb{R}^{1,25} \).

### 4.2 models with \( \mathcal{N} = 1 \) superconformal symmetry

The Lorentzian Liouville theory with \( \mathcal{N} = 1 \) worldsheet supersymmetry is the simplest generalization of the \( \mathcal{N} = 0 \) bosonic counterpart. With the \( \mathcal{N} = 1 \) worldsheet supersymmetry alone,
spacetime supercharges are not constructible. So, this class of models describes tachyon condensation of Type 0 string theories. The theory may be defined via the standard Wick rotation of the \( \mathcal{N} = 1 \) Euclidean Liouville theory, whose worldsheet action is given in terms of the \( \mathcal{N} = 1 \) supermultiplet \( (\varphi, \psi, \tilde{\psi}) \) by

\[
S_{\mathcal{N}=1} = \frac{1}{2\pi} \int d^2z \left( \partial \varphi \bar{\partial} \varphi + i \bar{\psi} \partial \psi + i \tilde{\psi} \tilde{\partial} \tilde{\psi} + \frac{1}{4} Q \varphi R(2) + 2\pi \mu b^2 \bar{\psi} \psi e^{b \varphi} \right),
\]  

(4.2)

where \( Q = (b + 1/b) \). In fact, the \( \mathcal{N} = 1 \) superconformal Liouville potential represents tachyon background with no winding mode excitations. In other words, the closed string tachyon is the bulk tachyon.

Furthermore, in this model, the Bogoliubov pair production is unsuppressed, as can be derived both from mini-superspace approximation and from exact two-point function. In fact, in manifestly \( \mathcal{N} = 1 \) supersymmetric formulation, the mini-superspace approximation is inert to the Wick rotation. This is because the \( \mathcal{N} = 1 \) supersymmetric Hamiltonian constraint is given as (in NS-NS sector)

\[
\left[ -\left( \frac{\partial}{\partial \varphi} \right)^2 + \left( \frac{\partial W}{\partial \varphi} \right)^2 \right] \Psi(\varphi) = 0,
\]

(4.3)

where \( W \) is the superpotential, and it changes only by an overall sign under the Wick rotation \( \varphi \to -iX^0 \). This also fits with the conformal field theory analysis, in which the renormalized cosmological constant is given by \( \xi_L = 2\pi \mu \gamma(bQ/2) \) \[50, 51\] and takes positive value as \( b \to \pm i0 \). Here, we are following the conventions of \[52, 53\].

4.3 models with \( \mathcal{N} = 2 \) superconformal symmetry

One can also start with Euclidean Liouville theory with \( \mathcal{N} = 2 \) worldsheet superconformal invariance and prescribe a Lorentzian counterpart by Wick rotation. Closely related model (‘T-dual version’) is constructed by the formal analytic continuation \( k \to -k \) in the SL\(_{(2, \mathbb{R})}/U(1) \) (super)coset, which is often called as the ‘cosmological’ SL\(_{(2, \mathbb{R})}/U(1) \) model \[23, 24, 25\]. These theories are easily embeddable into Type II superstring vacua with unbroken space-time supersymmetry. There is potentially a tachyonic infrared instability due to the negative mass gap along the timelike linear dilaton direction \[23\], but this is easily removable by adding spacelike linear dilaton (as in the light-like linear dilaton models \[34, 37\]).

However, these models also have a serious difficulty of ultraviolet divergence due to particle production. The particle production rate behaves \[23, 26\] for small radial momenta as \( \sim O(1) \) for all frequencies since there is no exponential damping. We thus face a uncontrollable ultraviolet divergence caused by the Hagedorn density of states. One may attempt to avoid this difficulty by taking the level \( k \) of SL\(_{(2, \mathbb{R})}/U(1) \) coset conformal field theory to \( |k| < 1 \) (see discussions around (3.19)). However, in the cosmological model, \( |k| < 1 \) means that the central charge is negative, thus it does not offer a sensible resolution.
A further difficulty has to do with ‘wrong’ sign Liouville wall: unlike $\mathcal{N} \leq 1$ counterparts, the sine-Liouville potential in this case is peaked at weak coupling region. This is inevitable because of the condition $2\alpha q = 1$, which is need if the $\mathcal{N} = 2$ worldsheet superconformal symmetry is to be retained \footnote{In \cite{25}, based on an interpretation of this peculiarity, a possibility was suggested for removing the cosmological singularity at string theory level. However, the MS (with ‘correct’ Liouville wall) seems more plausible.}. The resulting theory is known as the two-parameter sine-Liouville model \cite{29}. This is because the worldsheet action

$$S = \frac{1}{2\pi} \int d^2 z \left[ \partial \phi \bar{\partial} \phi + \partial \phi_1 \bar{\partial} \phi_1 + \partial \phi_2 \bar{\partial} \phi_2 + 2\pi \mu \cos(\beta \phi_1 + \delta \phi_2) + \frac{1}{4\alpha} \phi \mathcal{R}(2) \right]$$

is changed to

$$S = \frac{1}{2\pi} \int d^2 \sigma \left[ -\partial t \bar{\partial} t + \partial \phi_1 \bar{\partial} \phi_1 + \partial \phi_2 \bar{\partial} \phi_2 + 2\pi \mu e^{-\kappa t} \cos(\beta \phi_1 + \delta \phi_2) + \frac{1}{4\kappa} t \mathcal{R}(2) \right]$$

daft the Wick rotation of $\alpha \to i\kappa$ and $\phi \to it$. The resulting Lorentzian theory is pathological since the strong coupling region is not shielded by the Liouville potential. Although the conformal conditions are satisfied, the theory has a strong coupling singularity at $t \to +\infty$. The difficulty arises because of wrong Liouville potential term $\propto e^{-\kappa t}$, which is peaked at $t \to -\infty$ and decreases as $t \to +\infty$. (Compare this with the worldsheet action of three-parameter theory \footnote{This is essentially due to the fact that the background charge is given by $q = b + 1/b$ in bosonic Liouville theory and hence allows more choices of the sign after Lorentzian continuation $b \to i\beta$.}.) This difficulty, which does not exist in the bosonic Liouville theory \footnote{This is essentially due to the fact that the background charge is given by $q = b + 1/b$ in bosonic Liouville theory and hence allows more choices of the sign after Lorentzian continuation $b \to i\beta$.}, has led us to consider the three-parameter version in this work.

**4.4 three-parameter sine-Liouville theory:**

**deformation to $\mathcal{N} = 1$ superconformal model**

Finally, let us try to embed the three-parameter sine-Liouville model, studied in section 3, into superstring theory. This would lead to the most sensible cosmological model because of the following reasons:

1. We do not have the bulk closed string tachyon causing the infrared instability in the similar manner as the $\mathcal{N} = 2$ models (or the cosmological $\text{SL}(2, \mathbb{R})/U(1)$ models) as far as embedding into the GSO projected superstring backgrounds.

2. The particle production rate could be exponentially small exceeding the Hagedorn growth of state density similarly to the $\mathcal{N} = 0$ case, if the parameters are chosen suitably. This means that we are also free from the ultraviolet instability.

3. We have the parameter region in which the Liouville wall is located at the ‘correct’ side, which makes the quantum theory well-defined. This fact sharply contrasts with the $\mathcal{N} = 2$ models.
However, unfortunately the three-parameter model does not possess any worldsheet supersymmetry even if the scalar field $\phi_2$ is fermionized (except the $\mathcal{N} = 2$ enhancement for the two-parameter model). Instead, we may consider an alternative model with $\mathcal{N} = 1$ supersymmetric interaction

$$S_{\text{int}} = \int d^2z d^2\theta \mu e^{\alpha \Xi} \cos(\beta \Phi_1)$$

(4.4)

where $\Xi, \Phi_1$ are $\mathcal{N} = 1$ superfields with components $(\varphi, \psi, \bar{\psi})$ and $(\phi_1, \psi_1, \bar{\psi}_1)$, respectively. Fortuitously, this $\mathcal{N} = 1$ model bears exactly the same structure of the two-point correlators in the neutral sector as the three-parameter models discussed in section 3 except trivial rescaling of the bare cosmological constant:

$$\mu^2 \rightarrow (\alpha^2 + \beta^2)^2 \mu^2 .$$

(4.5)

This is because the only modification in the derivation of the reflection amplitude and the Bogoliubov amplitude is to replace the screening insertions

$$\mu^2 \int d^2z_1 d^2z_2 : e^{\alpha \varphi + i \beta \phi_1 + i \delta \phi_2} : (z_1) : e^{\alpha \bar{\varphi} - i \beta \phi_1 - i \delta \phi_2} : (z_2)$$

(4.6)

in (2.3) by

$$\mu^2 \int d^2z_1 d^2z_2 : (\alpha^2 \psi \bar{\psi} + i \alpha \beta \psi \bar{\psi}_1 + i \alpha \beta \psi_1 \bar{\psi} - \beta^2 \psi_1 \bar{\psi}_1) e^{\alpha \varphi + i \beta \phi_1} : (z_1)$$

$$\times : (\alpha^2 \bar{\psi} \bar{\psi} - i \alpha \beta \bar{\psi} \bar{\psi}_1 - i \alpha \beta \bar{\psi}_1 \bar{\psi} - \beta^2 \bar{\psi}_1 \bar{\psi}_1) e^{\alpha \bar{\varphi} - i \beta \phi_1} : (z_2) .$$

(4.7)

The two-point correlators in the neutral sector do not involve the fermions. So, contracting them among themselves, one effectively has screening operators exactly the same as those of the three-parameter model except rescaling the coupling parameters as in (4.5).

As such, the $\mathcal{N} = 1$ model with the worldsheet interaction (4.4) would provide a realistic ”Nothing state” in the context of fermionic string theories. Since the Bogoliubov particle production would be dominated by the neutral sector, we can adopt the reflection amplitudes of the three-parameter sine-Liouville theory extracted in section 3 to the present context.

In the previous section, we showed that a technically natural string theory setup of ”Nothing state” can be realized if the effective central charge $c_{\text{eff}}$ is less than or equal to 6. What kind of background does it correspond to? To answer this, let us consider a string theory background of the type

$$\mathbb{R}_Q^2 \times \mathbb{R} \times \mathcal{M} ,$$

(4.8)

where $\mathbb{R}_Q^2$ is identified as the transverse part of the four-dimensional spacetime, $\mathbb{R}^Q$ as $\mathcal{N} = 2$ Liouville theory with dilaton slope $Q$, and $\mathcal{M}$ as a unitary conformal field theory. Then, by the
conformality condition $c_{\text{tot}}=12$, we find that the critical situation $c_{\text{eff}} = 6$ is attained if $Q$ equals to $\sqrt{2}$ and $c_{\mathcal{M}} = 0$ (i.e. $\mathcal{M}$ sector should be trivial). This situation is quite interesting since it is interpretable as the four-dimensional superstring compactified on a conifold! The relation between $\mathcal{R}_Q$ and the conifold is well-known \cite{55, 56}. It is quite interesting that conifolds show up prominently in constructing nonsingular string cosmology. In this context, recall that most Calabi-Yau threefolds have conifold points in their moduli spaces and that the density of supersymmetric flux vacua is sharply peaked near the conifold points \cite{57, 58}. This may be an indication that nonsingular string cosmology model proposed in this work is abundantly realizable out of supersymmetric flux vacua.

The critical coupling $Q = \sqrt{2}$ coincides via FZZ duality with the critical level $k = 1$ of $\text{SL}_k(2, \mathbb{R})$ supercoset. Curiously, this is precisely where the ”black hole / string transition” is known to take place \cite{41, 42, 43} for string theory on three-dimensional anti-de Sitter (AdS$_3$) background with curvature $\sqrt{k\ell_{\text{st}}}$ and for string theory on linear dilaton background with dilaton slope $Q = \sqrt{2/k}$. The phase $c_{\text{eff}} < 6$ for realizing the ”Nothing state” coincides with the phase of long excited strings either at boundary of the AdS$_3$ or throat of the linear dilaton background.

Technically, the coincidence has to do with growth of Hagedorn density of states and ultraviolet behavior therein. For a D-brane rolling in five-brane \cite{44, 45} and related backgrounds, we found in \cite{41, 43} that the closed string emission is ultraviolet finite if $k \leq 1$ but divergent for $k > 1$. For the winding tachyon condensation, we found above that the Bogoliubov pair production is ultraviolet finite if $k \leq 1$ but divergent for $k > 1$. Whether the coincidence bears deeper connection between the two situations poses a very interesting question. We intend to report progress on this in a separate work.

### 4.5 Intuitive Spacetime Picture

We end the discussion with a heuristic remark that may explain how the ”Nothing state” can be understood intuitively. The crux of MS proposal is that winding string becomes tachyonic in an epoch near the cosmological singularity. Let us treat for simplicity the tachyon mass a constant-valued throughout the entire spacetime. Then, the effective field theory of the tachyon field $T$ coupled to the gravity $g_{mn}$ is given by

$$S_{10d} = \int d^{10}x \sqrt{-g} \left[ \mathcal{R} + \frac{1}{2} (\nabla_m T)^2 + \frac{\kappa^2}{2} T^2 + \cdots \right]. \quad (4.9)$$

Here, $-\kappa^2 < 0$ denotes the mass-squared of the winding string tachyon field. Taking the ansatz of the Einstein-de Sitter space for the metric:

$$ds^2 = -dt^2 + a^2(t)dx^2 \quad \text{and} \quad T(x) = T(t) \quad (4.10)$$

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the field equations read
\[
\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{6}(\dot{T}^2 - \kappa^2 T^2)
\]
\[
\ddot{T} + 9\frac{\dot{a}}{a}\dot{T} - \kappa^2 T = 0.
\]
(4.11)

An obvious solution is that \(T(t) = 0\) and \(a(t)\) takes a constant value. This describes a static universe in which the tachyon is not condensed. Another solution is that \(T(t) = \exp(-\kappa t)\) and \(a(t)\) takes again a constant value. This describes again a static and nonsingular universe in which the tachyon is coherently condensed. For both cases, the universe is static since the energy vanishes identically. Obviously, the latter solution approaches to the first as \(t \to +\infty\). In general, the tachyon may evolve differently and cause the spacetime to evolve cosmologically. Thus, starting from the solution \(T(t) = \exp(-\kappa t)\) as the asymptotic solution in the infinite past \(t \to -\infty\), by continuity, one can interpolate to the solution \(T(t) = 0\) via the cosmological solution. It then leads to the beginning of cosmological universe starting from a nonsingular, static universe.

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