Relative importance of entrained liquid fraction and mass transfer at the interface on pressure drop of annular flows

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Abstract. Fully developed adiabatic two-phase flows are commonly encountered in practice and their modeling is of primary importance for accurate, local calculations of pressure gradients and void fraction for the design of several industrial applications as heat exchangers and long fluid transfer piping. For large void fractions, the entrainment of liquid droplets into the vapor core can have a large effect on the pressure gradients. In this paper starting from theoretical considerations about the modeling of the pressure gradients in an annular two-phase flow, a numerical analysis as well as a discussion are proposed in order to quantify the relative importance of the entrainment on the calculations and to address further experimental studies and models on this topic.

Nomenclature

Symbols

A cross-sectional area [m²]
D channel diameter [m]
e entrainment ratio
f friction factor
g gravitational acceleration [m s⁻²]
G mass flux [kg m⁻² s⁻¹]
J superficial velocity [m s⁻¹]
m mass flow rate [kg s⁻¹]
p pressure [Pa]
R channel radius [m]
w velocity [m s⁻¹]
x vapor quality

γ spatial fraction
δ liquid film thickness [m]
ε void fraction
µ dynamic viscosity [Pa s]
ρ density [kg m⁻³]
τ shear stress [Pa]

Subscripts

i relative to liquid/vapor interface
l relative to liquid phase
le relative to entrained liquid
lf relative to liquid film
v relative to vapor phase
W relative to liquid/wall interface

Greek symbols

β channel inclination [°]

1. Introduction

Many industrial applications such as chemical processing, petrochemical transportation, evaporators and condensers or steam generators in nuclear plants involve two-phase flows. The structure of the
two-phase flow affects the performance of these applications: annular flow is one of the most important and frequent flow regimes in gas-liquid and vapor-liquid two-phase flows in tubes and channels. A thin film of liquid flows on the channel wall, surrounding a vapor core which flows in the center of the channel and may carry entrained liquid droplets in suspension. The vapor core, flowing faster than the liquid film, drags the liquid film and may atomize the tips of the waves which are formed at the liquid film surface. At the same time, entrained liquid droplets are continuously deposited back onto the liquid film after being accelerated in the gas or vapor core. Due to this continuous exchange of liquid between the liquid film and the gas or vapor core, mass and momentum exchange between the phases are strictly coupled, resulting in a complex physics of annular flows and in a particularly challenging analysis and modeling.

The optimal design and the energy efficient behavior of any two-phase flow system require accurate prediction of the pressure drop, needing the wall and the interface shear stresses as input for the modeling of mass, momentum and heat transfer in two-phase flows.

Revellin and Thome [1] developed a theoretical model for the prediction of the critical heat flux of refrigerants flowing in heated, round microchannels. In their model the basic idea was to integrate equations of mass, momentum and energy for each phase along the tube for a developing flow, in order to get a critical thickness determined by the Kelvin-Helmholtz theory, which causes the complete drying of the liquid film. The model was developed specifically for microchannels, when the relative effect of surface tension and of inertia forces guarantees both a symmetric annular flow and a marginal effect of the gravitational forces. In these conditions, the interface between vapor and liquid is smooth, the fraction of liquid entrained into the vapor core as droplets is marginal. The authors have proven that the model is able to predict experimental values with a good agreement, if the shear stresses between liquid and wall and between vapor and liquid phases are modeled with equations for smooth tubes.

Cioncolini, Thome and Lombardi [2] presented a model developed specifically for total pressure drops calculations including the actual value of the fraction of liquid entrained into the vapor core. In particular, the mass flow rate of liquid at the wall and the accelerational term reflect the effect of the liquid droplets entrained into the vapor core; consequently, the friction factor between the vapor and the liquid phases, based on the aerodynamic interaction between the phases through the Weber number of the vapor core, has been calibrated in such a way to include the effect of the entrainment.

Concerning the momentum balance for each phase, it is important to consider the momentum transfer between the phases due to the atomization of the liquid film, which goes into the vapor phase, and the corresponding deposition of liquid droplets moving from the vapor core into the liquid film.

The paper by Qu and Mudawar [3] includes this term. The model refers to a developing flow in a heated microchannel, assuming no atomization and a continuous deposition after a fully entrained boundary condition fixed at the transition between a churn and an annular flow. The friction factor for the interfacial shear stress is calculated based on the assumption of a laminar flow of the vapor core.

Even for an adiabatic developed flow with negligible compressibility effect, when the net mass transfer is zero with the equilibrium between atomization and deposition, there still is a transfer of momentum between the liquid and vapor phases due to the liquid droplets acceleration/deceleration at the interface.

In the knowledge of the authors, the case of an annular two-phase flow in a macro-channel, including the effect of the entrainment into the vapor core and the effect of the momentum transfer at the interface, has never been considered in the literature, even if it is a very common situation in different fields of application.

An issue, concerning the modeling of such a case which limits the possibility to propose and validate a model, is the difficulty to find closure equations specifically developed for this situation.

As a first step, this paper presents a discussion trying to quantify the relative importance of each effect (correction of the entrainment effect onto the mass flow at the wall and, then, of the momentum transfer) for an adiabatic, annular developed two-phase flow. Consequently, different levels of complexity into the modeling as done by work [1], [2] and [3] are introduced and generalized without
any particular assumption. As a result, relative comparison of calculations with the different models are presented and discussed, in order to show their relative influence and to orient needs for further research.

2. Models description
Each of the following models assume a developed, adiabatic annular two-phase flow into a circular tube, with negligible compressibility effect (hence, no vapor quality and no pressure gradient variations).

Basic model (no entrainment) - CASE A
The conservation of momentum for the liquid and vapor phase could be expressed as follow.

\[-dp_y A_y -\tau_y dS_w + \tau_i dS_i + \rho_i A_i \, dz \, g \cos \beta = 0\]  \(1\)
\[-dp_i A_i -\tau_i dS_i + \rho_i A_i \, dz \, g \cos \beta = 0\]  \(2\)

Where \(A_y = (1-\epsilon)A_{nw}\) and \(A_i = \epsilon \cdot A_{nw}\) are the cross sections occupied by the liquid and the vapor, respectively, being \(\epsilon\) the void fraction. The term \(dS_w = 2\pi R \delta dz\) is the elementary liquid/wall interfacial area and \(dS_i = 2\pi (R-\delta) \delta dz\) is the elementary liquid/vapor interfacial area, being \(R\) the radius of the channel and \(\delta\) the liquid film thickness. \(\beta\) is the channel inclination angle in counterclockwise direction with respect to the gravitational acceleration vector (\(\beta=90^\circ\) for horizontal flow).

The shear stress can be calculated with equation (3), where the friction factor, \(f_w\), can be expressed according to the correlation proposed by Blasius for smooth tube, as done in [1] with no entrainment effect.

\[\tau_w = 0.5 f_w \rho_i w_y^2\]  \(3\)
\[f_w = 16\Re_y^{-1}\]  for \(\Re_y = \frac{2 \rho_i w_y \delta}{\mu_i} < 2300\)
\[f_w = 0.079 \Re_y^{-0.25}\]  for \(\Re_y > 2300\)

The shear stress at the liquid/vapor interface can be defined as follows, considering the correlation proposed by Blasius for smooth tube to evaluate the friction factor \(f_i\).

\[\tau_i = 0.5 f_i \rho_i \left( w_i - w_y \right)^2\]  \(4\)
\[f_i = 16\Re_i^{-1}\]  for \(\Re_i = \frac{\rho_i w_i D}{\mu_i} \left(1 - \frac{2\delta}{D}\right) < 2300\)
\[f_i = 0.079 \Re_i^{-0.25}\]  for \(\Re_i > 2300\)

From the momentum balance equations, the total pressure gradients for the liquid film and the vapor core can be expressed, in terms of void fraction and shear stresses as in equation (5).

\[-\frac{dp_y}{dz} = \frac{2 \tau_y - \tau_i \sqrt{\epsilon}}{R(1-\epsilon)} - \rho_i g \cos \beta; \quad -\frac{dp_i}{dz} = \frac{2 \tau_i}{R \sqrt{\epsilon}} - \rho_i g \cos \beta\]  \(5\)

The average velocity of the liquid film and the vapor phases to be used for shear stress calculations are:
\[ w_y = \frac{m_y}{\rho_y A_y} = G \left( \frac{1-x}{1-e} \right); \quad w_i = \frac{m_i}{\rho_i A_i} = Gx \] 

(6)

Equation (5) is valid when a symmetric annular flow exists.

**Entrainment effect on the frictional term of the total pressure drop - CASE B**

The entrainment ratio \( e \) is defined as the ratio between the liquid droplets mass flow rate inside the vapor core and the total liquid mass flow rate.

\[ e = \frac{\dot{m}_y}{\dot{m}_i} \] 

(7)

Assuming that the entrained liquid droplets flow at the same velocity of the vapor core, the spatial fraction of the total liquid occupied by the entrained liquid droplets, \( \gamma \), is related to the other parameters of the flow and thermodynamic properties by the following equation:

\[ \gamma = \frac{A_i}{A} = e \frac{1-x}{1-e} \rho_{li} \] 

(8)

The conservation of the momentum in equations (1) and (2) are formally the same when considering an annular flow with entrained liquid droplets into the vapor core. However, the entrained liquid droplets have an influence on the liquid film thickness meaning different results in the pressure drop. In this approach, named **CASE B**, the equations (1) and (2) can be reduced as a function of the spatial and mass fractions for the liquid entrained and vapor as the following.

\[ \frac{dP_y}{dz} = \frac{2\tau_y - \tau_i \sqrt{e + \gamma (1-e)}}{R(1-e)(1-\gamma)} - \rho_{li} g \cos \beta \] 

\[ \frac{dP_i}{dz} = \frac{2\tau_i - \rho_{li} \gamma (1-e) \cos \beta}{e + \gamma (1-e)} - \rho_{li} g \cos \beta \] 

(9)

The shear stresses are calculated with the same equations of the previous case (equations (3) and (4)), where the influence of the entrained liquid fraction is reflected into the velocities of phases through mass balances. In particular, the average velocity of the liquid film and the vapor phases to be used for shear stress calculations are:

\[ w_y = \frac{m_y}{\rho_i A_y} = G \left( \frac{1-x}{1-e} \right); \quad w_i = \frac{m_i}{\rho_i A_i} = Gx \] 

(10)

If the flow has no entrainment (\( \gamma = 0 \)), equation (9) is equivalent to equation (5); hence, **CASE B** includes also **CASE A**. If the flow has entrainment equal to one (\( \gamma = 1 \)), there is no more liquid at the wall and the flow is no more an annular but a mist flow. More details for the algorithm related to each case are presented in the next section.

**Entrainment effect both on the frictional term of the total pressure drop and on the momentum transfer between phases - CASE C**

In this case equation (5) is modified to account for the momentum transfer at the interface between vapor and liquid phases. Since for a developed flow the mass flow rate of liquid droplet atomized is in equilibrium with the one deposited (\( m_{atom} \)), equation (5) modifies in:

\[ -dP_y A_y \tau_x dS_v + \tau_i dS_i + \rho A_y dz \rho \cos \beta + m_{atom}(w_i - w_y) = 0 \] 

(11)

\[ -dP_i (A_i + A_y) \tau_x dS_v + (\rho A_i + \rho A_y) dz \rho \cos \beta - m_{atom}(w_i - w_y) = 0 \] 

(12)

The total pressure gradient for the liquid film and the vapor core can be reduced as a function of the spatial and mass fractions for the liquid entrained and vapor as the following.
The shear stresses are calculated with equations (3) and (4) using the velocity in equation (10). Closure equations are needed for the atomized mass flux at the interface and for the entrained liquid fraction into the vapor core.

2.1. Closure equations

2.1.1. Atomization/deposition rate. The work of Berna et al. [4] gives a review of the extensive literature existing on the annular two-phase flow, focusing specifically on the analysis of the main phenomena which are involved. Kataoka et al. [5] obtained separate equations for the atomization and the deposition rates, also in case of non-equilibrium conditions, when the atomization and the deposition rates are unbalanced. Furthermore, a correlation for the entrainment ratio in the fully developed region is also presented.

\[ m_{\text{atom}} = \frac{H_x dS}{D} \left[ 6.610^{-2} \left( \frac{\mu_r}{\mu_l} \right)^{0.26} (1-x)^{0.185} \text{Re}_l^{0.925} \text{We}_E^{0.925} \right] \]

where:

\[ \text{Re}_l = \frac{G(1-x)D}{\mu_l} \]

\[ \text{We}_E = \frac{G^2 x^2 D (\rho_l - \rho_g)^{0.5}}{\sigma \rho_v} \]

Ishii and Mishima [6] obtained one of the most known correlation to estimate the entrainment mass flux. The atomization rate is correlated to two dimensionless numbers, namely, a modified vapor Weber number and a liquid Reynolds number.

\[ G_{\text{atom}} = G(1-x) \tanh \left( 7.25 \cdot 10^{-7} \text{Re}_l^{0.25} \text{We}_E^{1.25} \right) \]

2.1.2. Entrainment ratio. Kataoka et al. [5] also proposed a correlation for the calculation of the liquid droplet fraction entrained in the vapor core at the equilibrium (fully developed flow), according to Equation (17).

\[ e(1-e)^{0.25} = 7.75 \cdot 10^{-7} \text{Re}_l^{0.25} \text{We}_E^{1.25} \]

Cioncolini and Thome [7] presented a correlation for the entrainment ratio. The value of the liquid fraction entrained into the vapor core depends on the core flow Weber number and the core flow density. Since the core flow density depends on the entrained liquid fraction, a procedure with two steps is required, as explained in [7].

\[ e = \left( 1 + 279.6 \text{We}_E^{-0.8395} \right)^{2.209} \]

Paleev and Filippovich [8] proposed a correlation including the effects of droplets concentration in the vapor core. The vapor density is replaced by an average mixture density as defined in [8].

\[ e = 0.015 + 0.44 \log \left[ \frac{\rho_v^*}{\rho_l} \left( \frac{\mu_{lv}}{\sigma} \right)^2 \cdot 10^d \right] \]
3. Algorithm
Given the input parameters (fluid, mass flux, saturation temperature, vapor quality and internal diameter), the entrainment fraction can be evaluated by the selected correlation. If the entrainment fraction is less than unity, the two momentum balances have to give the same total pressure gradient (unknown) for the same void fraction (unknown), and an iterative procedure is required, as shown in the flowchart (Figure 1).

It is worth noting that if the basic model (CASE A) is considered, to solve equations of CASE A or CASE B with $e=0$ ($\gamma=0$) are equivalent approaches. Meanwhile, if entrainment is equal to unity ($e=1$ and $\gamma=1$), the flow pattern changes to mist flow, when the liquid film at the wall no longer exists (all the liquid phase flows inside the vapor core as droplets). In this case, the model can be solved directly, assuming $\gamma=1$: void fraction can be calculated from equation (8) and the pressure gradient can be directly evaluated from equation (9) related to the vapor phase (being the vapor in contact with the wall, while mist flow occurs).

In Figure 2 the pressure gradient evaluated by the liquid momentum balance and by the vapor momentum balance as function of the void fraction are shown, for different values of the entrainment ratio from 0 to 0.9.

**Figure 1.** Iterative procedure for the calculation of void fraction and pressure gradient.
4. Results and discussion

4.1. Effect of the entrainment ratio on the phase velocities - CASE B vs CASE A

In this section the effects of the evaluation of the entrained liquid droplets on the pressure gradient will be studied. Figure 3 shows the entrainment ratio as function of vapor quality at given conditions. The fraction of liquid entrained as droplets increases with the vapor quality, with higher values for the correlation of Paleev and Filippovich. The model of Kataoka et al. assumes that the flow is at the thermodynamic equilibrium after \( x=1 \) (vapor single phase flow). On the contrary, the two other models (Cioncolini and Paleev) assumes a non-equilibrium flow for a vapor quality greater than unity meaning that liquid droplets at saturation coexist with the superheated vapor phase.

Figure 4 shows the entrainment ratio maps in terms of mass flux and diameter at saturation temperature of 25°C, vapor quality of 0.6 and R1234yf as working fluid. At high mass flux and internal diameter, Kataoka et al. gives entrainment ratio greater than one, which is physically impossible. As a consequence, the flow pattern is considered as mist flow: no liquid film occurs at the wall. In the investigated range of mass flux and diameter, Cioncolini and Thome calculates a maximum value of the entrainment ratio of 0.9. The correlation of Paleev and Filippovich is not sensitive to the diameter, so in the following analysis it will not be taken into consideration. The iso-lines reported in Figure 5 show the dimensionless pressure gradient evaluated in the CASE B with reference to the pressure gradient evaluated in CASE A, using the correlation by Kataoka et al. (Figure 5a) and Cioncolini and Thome correlation (Figure 5b).

Where low entrainment is expected, the pressure gradient is near to the reference pressure gradient. On the contrary, at high mass flux and diameter, the amount of the entrained liquid by the vapor core needs to be taken into account: error up to -60% in the pressure gradient prediction could occur if the entrainment ratio is neglected, in the investigated conditions.

Depending on the boundary conditions, the relative influence of the entrainment ratio on the pressure gradient prediction could be relevant. However, different correlations give different results in terms of both entrainment ratio and pressure gradient.
Figure 3. Entrainment ratio as function of vapor quality at the conditions on the top of the figure.

Figure 4. Entrainment ratio evaluated by Kataoka et al., Cioncolini and Thome and Paleev and Filippovich as function of mass flux and internal diameter at: $T_{\text{sat}}=25^\circ\text{C}$, $x=0.6$. 
4.2. Effect of the entrainment/atomization both on the phase velocities and on the momentum transfer at the interface CASE C vs CASE A

Figure 6 shows the trend of the atomization rate per unit of length as function of the vapor quality for different entrainment ratio correlations. When using Kataoka et al. correlation for the atomization rate (Figure 6a), a slight difference between the correlations of the entrainment occurs. In the investigated conditions, at vapor quality of 0.6 the atomization rate reaches the maximum value. The correlation of Ishii and Mishima (Figure 6b) gives lower values of the atomization rate with respect to Kataoka et al. Furthermore, no relevant differences between the entrainment correlations occur.

The iso-lines reported in Figure 7 shows the dimensionless pressure gradient evaluated in CASE C with reference to the pressure gradient evaluated in CASE A, using the combinations of entrainment ratio and atomization rate correlations reported on the top of the figure, at saturation temperature of 25°C, vapor quality of 0.6 and R1234yf as working fluid.
When Kataoka et al. is used to estimate the entrained liquid droplets (Figure 7a and Figure 7b), the results in terms of dimensionless pressure gradient are similar at high values of mass flux and diameter. Meanwhile, if the entrainment ratio is low, Kataoka et al. for the atomization rate over-predicts the pressure gradient respect to the CASE A and Ishii and Mishima under-predicts it.

Using Cioncolini and Thome (Figure 7c and Figure 7d), different results can be obtained depending on the correlation for the atomization rate. In the major of the investigated conditions, using Kataoka et al. correlation for the atomization, the model over-predicts the pressure gradient with respect to the CASE A (up to 50%). Instead, the prediction is under-estimated when Ishii and Mishima correlation is implemented in the investigated conditions.

![Figure 7](image_url)

**Figure 7.** Dimensionless pressure gradient (CASE C) as function of mass flux and internal diameter at T\(_{\text{sat}}\) = 25°C, x=0.6.

### 4.3. Effect of the operating conditions

To take into account the influence of the vapor quality, an analysis varying the mass flux and vapor quality for two values of the internal diameter, 1 and 12 mm, is studied in this section. A saturation temperature of 25°C is considered. At small value of the diameter, the effect on the pressure gradient is not significant, so the results are not showed.

The iso-line at constant entrainment ratio is shown in Figure 8 as a function of the mass flux and vapor quality at saturation temperature of 25 °C and internal diameter of 12 mm. High entrainment ratio is observed for high mass flux and vapor quality. Similarly to Figure 4, Kataoka et al. gives entrainment ratio greater than one meaning mist flow occurs in these conditions.

Figure 9 shows the dimensionless pressure gradient evaluated in the CASE C (including entrainment ratio and atomization rate) as function of the mass flux and internal diameter. Using Kataoka et al. for the entrainment ratio (Figure 9a and Figure 9b), at high mass flux and vapor quality the model calculates lower pressure gradient with respect to the reference state.
When Cioncolini and Thome and Kataoka et al. are used for the entrainment ratio and the atomization rate respectively, the ratio between the pressure gradient evaluated in the \textit{CASE C} and the one evaluated in the \textit{CASE A} assumes values up to 1.6 in the area characterized by high diameters and high vapor quality.

**Figure 8.** Entrainment ratio as function of mass flux and vapor quality at: $T_{\text{sat}}=25^\circ\text{C}$, $D=12.0$ mm.

**Figure 9.** Dimensionless pressure gradient (CASE C) as function of mass flux and vapor quality at: $T_{\text{sat}}=25^\circ\text{C}$, $D=12.0$ mm.
5. Conclusion
Starting from a basic modeling of a two-phase symmetric, fully developed annular flow with no entrainment, other two models are presented to account for the effects of the entrainment ratio on the phase velocities and on the momentum transfer at the interface between phases. The analysis was run for different correlation both for the entrainment ratio and the atomization rate, in a wide range of mass flux, channel diameter and vapor quality.
The main conclusion can be summarized as follow:
- The liquid fraction entrained into the vapor core and the momentum transfer at the interface have a large influence on pressure drop calculations and should not be neglected especially for macro-channels and large mass flux.
- The closure equations chosen for the calculations have a remarkable influence on the results: specific correlations for friction factors using equations (13) and (14) should be used to obtain more general correlations, including consistently the effects of the entrainment.

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