ASYMMENTRIC DIFFUSION OF MAGNETIC FIELD LINES

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ABSTRACT

Stochasticity of magnetic field lines is important for particle transport properties. Magnetic field lines separate faster than diffusively in turbulent plasma, which is called superdiffusion. We discovered that this superdiffusion is pronouncedly asymmetric, so that the separation of field lines along the magnetic field direction is different from the separation in the opposite direction. While the symmetry of the flow is broken by the so-called imbalance or cross-helicity, the difference between forward and backward diffusion is not directly due to imbalance, but a non-trivial consequence of both imbalance and non-reversibility of turbulence. The asymmetric diffusion perpendicular to the mean magnetic field entails a variety of new physical phenomena, such as the production of parallel particle streaming in the presence of perpendicular particle gradients. Such streaming and associated instabilities could be significant for particle transport in laboratory, space, and astrophysical plasmas.

Key words: astroparticle physics – diffusion – magnetic fields – magnetohydrodynamics (MHD) – turbulence

1. INTRODUCTION

Astrophysical plasmas feature a huge separation between the energy containing scale and the dissipation scale. Such a high-Reynolds number flows are necessarily turbulent (see, e.g., Armstrong et al. 1995). Conductive turbulent fluids generate their own magnetic fields by dynamo, as a result of which most of the astrophysical objects are magnetized to some degree, typically close to equipartition between kinetic and magnetic energies. One of the consequences of turbulence, whether in laboratory, astrophysical, or laboratory plasmas, is the magnetic field line stochasticity, which plays a crucial role in most key physical processes, such as thermal conduction, reconnection, particle transport, etc. Self-excited turbulence in tokamaks results in anomalous transport of particles perpendicular to the field and prevents reaching higher temperatures and densities. Recently, unusual features in the angular distribution of the arrival directions of cosmic rays (Abbasi et al. 2011) and spectral features for positrons (Adriani et al. 2009) revived studies of cosmic-ray propagation in stochastic magnetic fields (see, e.g., Beresnyak et al. 2011; Kistler et al. 2012). Indeed, carefully studying cosmic-ray diffusion could help discriminate between different models of positron excess, Fermi bubbles, and other currently unexplained cosmic-ray phenomena.

Turbulence is now understood as a multi-scale phenomenon, by and large owing to the pioneering paper of Richardson (1926), who studied turbulent diffusion and suggested that the diffusion coefficient depends on scale as $D \sim l^{1/3}$, known as Richardson’s law. Indeed, if two particles are separated by a distance $l$, the typical separation speed corresponds to the turbulent velocity on scale $l$, which is approximately $\delta v \sim l^{1/3}$ (Kolmogorov 1941). This suggests that separation between particles grow as $\delta l \sim t^{3/2}$. If turbulence is uniform and characterized only by the dissipation rate per unit mass, $\epsilon$, which has units of cm$^2$ s$^{-3}$, it is natural that the separation between two particles moving with the fluid, $\Delta r$, conforms to $(\Delta r)^2 = g_0 \epsilon t^3$, where $g_0$ is a dimensionless number known as Richardson’s constant. Richardson’s diffusion has been studied extensively by experimental, theoretical, and numerical means (see, e.g., Sawford et al. 2008 and references therein). The turbulent diffusion of particles embedded in the MHD fluid received relatively less attention, however, the same type of diffusion is expected in the perpendicular direction due to the same scaling $\delta v \sim l^{1/3}$ of strong MHD turbulence (see, e.g., Goldreich & Sridhar 1995; Beresnyak 2011, 2012a).

A different and very interesting question is how the magnetic field lines separate from each other in such an environment. This question is crucial because well-magnetized plasmas are often poorly collisional, with the ion Larmor radius being many orders of magnitude smaller that the mean free path from Coulomb collisions. In particular, the magnetized solar wind features mean free paths that are comparable to the distance from the Sun. In galaxy clusters the Coulomb mean free path is around 10–100 kpc. In most astrophysical environments there is also a high-energy component, called cosmic rays, for which Coulomb collisions are essentially negligible. Charged particles will therefore move along magnetic field lines for great distances and scatter mostly by magnetic perturbations. This will result in parallel diffusion being much larger than the perpendicular diffusion. In the absence of parallel momentum, particles will move along magnetic field lines and diffuse only due to the magnetic field line diffusion. The motion of the bulk of the plasma $\delta v$ that causes ordinary diffusion could be neglected if the ion speed $v_i$ is much larger than $\delta v$. For cosmic rays this condition is also very well satisfied because they move along field lines with the speed comparable to the speed of light $c$. In other words, at least for short timescales, the fluid is frozen from most particles’ perspective.

Despite being collisionless, plasmas in many circumstances can be described as fluids on scales larger than the ion Larmor radius (Schekochihin et al. 2009). The inertial range of MHD turbulence features strongly anisotropic perturbations which are much smaller in amplitude than the mean magnetic field. The key component of this turbulence is Alfvénic mode, which is why it is often called Alfvénic turbulence. Due to the fact that the Alfvén mode is driven by magnetic tension, not pressure, it is relatively unaffected by the lack of collisions. The presence of the slow mode in such highly anisotropic turbulence neither affects dynamics (Goldreich & Sridhar 1995; Beresnyak 2012a) nor influences magnetic field lines, as the anisotropic slow
mode perturbation is mostly along the mean field. Therefore, the equations for the Alfvénic components, which are conventionally called reduced MHD (RMHD), are sufficient for studying field lines.

Perturbations in a strong mean magnetic field could be decomposed into forward and backward propagating components \( \mathbf{w}_\pm = \mathbf{v} \pm \mathbf{b}/\sqrt{4\pi \rho} \) called Elsässer variables. Since perturbation sources are not uniform, MHD turbulence is naturally imbalanced, i.e., the amplitudes of \( \mathbf{w}_+ \) and \( \mathbf{w}_- \) are not equal. This is verified by direct observations in the solar wind, where the dominant always propagates away from the Sun (see, e.g., Wicks et al. 2011). Other astrophysical sources are expected to have strong imbalance, for example, stellar winds and jets will emit predominantly outward-propagating component. Simulations of MHD turbulence with strong mean field. The balanced simulation B1 has been previously reported in Beresnyak (2011) and imbalanced simulations I1–I6 have been reported in Beresnyak & Lazarian (2010). More details concerning these simulations can be found in the above references. The parameters of the simulations are summarized in Table 1, with the defining feature of each imbalance simulation being the ratio of the dissipation rates \( \epsilon^\pm \) for Elsässer components \( \mathbf{w}_\pm \).

We were tracking the pairs of magnetic field lines started at random positions throughout the box and initially separated by distance \( r_0 \) by Equation (1). Figure 1 shows the tracking results for the B1 simulation. The transition to Richardson’s

| Run | Resolution | \( \epsilon^+ / \epsilon^- \) | \( \mathbf{w}_+ / \mathbf{w}_- \) | \( \ell_B^+ / \ell_B^- \) | \( \gamma_m^+ / \gamma_m^- \) |
|-----|------------|-----------------|-----------------|-----------------|-----------------|
| B1  | 1536^3     | 1               | 1               | 1               | 1               |
| I1  | 512 \times 1024^2 | 1.19 | 1.16 | 1.07 | 1.03 |
| I3  | 512 \times 1024^2 | 1.41 | 1.37 | 1.15 | 1.15 |
| I5  | 1024 \times 1536^2 | 2.00 | 2.36 | 1.36 | 1.31 |
| I6  | 1024 \times 1536^2 | 4.50 | 6.70 | 1.78 | 1.71 |

We used magnetic field snapshots obtained in simulations of Alfvénic turbulence. These simulations solved the RMHD equations with explicit dissipation and driving to achieve statistically stationary state. Further details behind the RMHD rationale, simulation setup, driving, numerical scheme, etc., can be found in Beresnyak (2012a). Each simulation represents stationary, strong MHD turbulence with strong mean field. The balanced simulation B1 has been previously reported in Beresnyak (2011) and imbalanced simulations I1–I6 have been reported in Beresnyak & Lazarian (2010). More details concerning these simulations can be found in the above references. The parameters of the simulations are summarized in Table 1, with the defining feature of each imbalance simulation being the ratio of the dissipation rates \( \epsilon^\pm \) for Elsässer components \( \mathbf{w}_\pm \).

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Figure 2. Ratio of forward to backward diffusion in data cubes from simulation I5. The upper plot shows five curves from the same data cube with different initial separations, the same as in Figure 1. The lower plot shows time variability of the ratio.

Figure 3. Ratio of forward to backward diffusion as a function of imbalance.
difference is that the measurement of the diverging magnetic fields is quasi-Lagrangian, while the measurement of structure function that leads to anisotropy constant is Eulerian. This difference becomes even more pronounced in the imbalanced case, where each of the \( u^\pm \) components has its own anisotropy. The analogy between Richardson–Alfvén diffusion and critical balance would suggest that \( g^+ (\ell_p^+)^3 = g^- (\ell_p^-)^3 \). We presented the ratio of parallel scales in the middle of the inertial range in Table 1. As we see, the expression above is not satisfied and the ratio of magnetic diffusions cannot be explained by the anisotropy difference. So, despite the similar functional form, Richardson–Alfvén diffusion has no direct relationship to Goldreich–Sridhar anisotropy.

6. DISCUSSION AND IMPLICATIONS FOR PARTICLE TRANSPORT

Our measurement is the first clear demonstration of the \( \chi^3 \) superdiffusion of magnetic field lines in limited simulations of MHD turbulence. Earlier Maron et al. (2004) tried to obtain \( \chi^3 \) superdiffusion of magnetic field lines, but their results were inconclusive due to the limited size of the inertial range. This Letter is also the first observation of asymmetric superdiffusion. Superdiffusion of fast particles in the solar wind has been argued based on observational data (Perri & Zimbardo 2009). Superdiffusion of field lines has been discussed in Jokipii (1973). The reconnection model of Lazarian & Vishniac (1999) also uses perpendicular superdiffusion of field lines; see also Eyink et al. (2011). The superdiffusion of particles has been argued in Skilling et al. (1974), Narayan & Medvedev (2001), Lazarian (2006), and Yan & Lazarian (2008), however, the asymmetric superdiffusion has not been anticipated before. Our earlier measurement of perpendicular diffusion using MHD simulations (Beresnyak et al. 2011) has been made in the large separation limit and reproduced FLRW, which is symmetric. The measurements of cosmic-ray propagation in artificial random fields, such as Giacinti et al. (2012) can, in principle, reproduce superdiffusion, but since artificial fields lack the time-asymmetry of turbulent fields, they cannot reproduce asymmetric diffusion. Based on the similarity between Goldreich–Sridhar anisotropy and Richardson’s diffusion, Narayan & Medvedev (2001) suggested that magnetic field lines separate within the Goldreich–Sridhar cone; however, according to the section above, this analogy is misleading, especially in the imbalanced case. Time-asymmetry of turbulence, which we confirmed in this Letter, has consequences for small-scale dynamo as well (Beresnyak 2012b).

One of the consequences of asymmetric perpendicular diffusion is an induced streaming. Indeed, if we consider two close magnetic field tubes, one of which is filled with isotropically distributed particles and another empty, the asymmetric diffusion into the empty tube will result in an average streaming \( \sim (1 - g^+/g^-) \) of particle’s velocity. In particular, for relativistic particles, such as cosmic rays, this will result in a streaming velocity of \( 2c (1 - g^+/g^-) / \pi \), which could easily exceed the threshold for streaming instability, \( v_A \), as long as imbalance amplitude \( 1 - w^-/w^+ > (3\pi/2)u_A/c \), which is around \( 10^{-4} \) in the WISM. Therefore, the induced streaming will be counteracted by streaming instability (Kulsrud & Pearce 1969). Above the threshold for turbulent damping (Farmer & Goldreich 2004; Beresnyak & Lazarian 2008a) streaming instability will be suppressed and, according to the estimates in the above paper, the weak large-scale streaming should reappear at energies \( 10^{11} \) eV and strong streaming is expected above \( 3 \times 10^{13} \) eV, although such energies are already heavily influenced by pitch-angle scattering. The net effect of the streaming instability from particles with energies below \( 3 \times 10^{13} \) eV will be a flux of slab waves which will increase the rate of pitch-angle scattering for these particles. The modeling of this effect will be subject of a future publication.

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