Dynamic effectiveness evaluation of weapon system based on PFCSM method

Lei Hu¹, Guoxing Yi*, Yi Nan¹, Hui Zhao¹, Hao Wang¹ and Zhihui Cao¹

¹ School of Astronautics, Harbin Institute of Technology, Harbin, Heilongjiang, 150001, China
*Corresponding author’s e-mail: ygx@hit.edu.cn

Abstract. Aiming at the problem of weapon system dynamic effectiveness evaluation, this paper proposes the weapon system dynamic effectiveness evaluation method based on probability finite combination state machine. The method can formally describe the key nodes in the whole process of weapon system operation in the way that states, inputs and transfer functions were applied to characterize the dynamic evolution of weapon system operation effectiveness, consequently, the operation effectiveness of the whole life cycle of weapon system can be evaluated. Taking the sea-skimming missile attacking warship as an example, we dynamically evaluate the operational effectiveness of the anti-ship missile in the whole life cycle and find the key nodes which cause the change of operational effectiveness, consequently, the dynamic effectiveness evaluation can provide auxiliary decision-making support for the optimization of combat planning. The reliability, dynamics, applicability and expansibility of the proposed method were analysed in conclusion.

1. Introduction
The American weapon system effectiveness advisory committee gave the definition of operational effectiveness, which is the function of the availability, credibility and capability of the weapon system that can be utilized to perform a set of specific missions [1]. The effectiveness evaluation has important guiding significance for the research, development and application of weapon system [2], meanwhile, it can provide auxiliary decision-making support for the formulation of weapon system operation scheme [3]. Relevant research works on weapon system effectiveness evaluation abroad has been carried out early, at present, they have built the complete theoretical system. The research work on weapon system effectiveness evaluation in China began in the 1980s, although some research results have been obtained, there is still a certain gap compared with foreign countries. Therefore, it is necessary to further expand the research work related to weapon system effectiveness evaluation.

Weapon system effectiveness evaluation methods include comprehensive evaluation methods, combat simulation methods, analytic methods, test statistics methods and other emerging related methods. Comprehensive evaluation methods include expert scoring method, Delphi method [4], analytic hierarchy process [5], fuzzy analytic hierarchy process, fuzzy comprehensive evaluation method [6], etc. The comprehensive evaluation methods have the advantages of simple formula and easy to understand and calculate, but its evaluation results are greatly affected by subjective factors. Combat simulation methods include Monte Carlo method [7], system dynamics method [8], etc. The advantage of combat simulation method is that it consumes less resources, but it is difficult to build the accurate model of weapon system and combat environment. Analytical methods include Lanchester equation [9], ADC method [10], grey evaluation method [11], etc. Analytic method has the
characteristics of simple theory and easy implementation, but it is highly subjective and cannot point out the role of internal factors of weapon system in the battlefield. The test statistics method realizes the effectiveness evaluation of weapon system through the collection and sorting of actual combat data. The evaluation results are the most reliable, but each test requires a lot of human, material and financial resources. Neural network method [12-13] and support vector machine [14] belong to other emerging related methods whose applications are limited for that these methods are only applicable to a certain kind of problems.

At present, the static evaluation results for the degree of completing the pre-fixed mission can be acquired by effectiveness evaluation methods of weapon system [4-6, 9-11], the research work corresponding to the dynamic effectiveness evaluation is less. Dou [15] pointed out that the performance of weapon system in different stages would change with the change of the combat environment, however, the evaluation result acquired in Literature [15] was still a static value of the degree for mission completion, which cannot reflect the dynamic impact of four combat information spaces including weapon system, operational target, operational planning and combat environment on operational effectiveness. Finite state machine can intuitively represent the state changes of weapon system in the process of operation, in this case, it is easy to evaluate the operational effectiveness of weapon system in different stages [2]. Seok [16] gave the assignment scheme of the radar-missile resource assignment problem based on the finite state machine, however, they didn’t research the dynamic effectiveness evaluation problem. Liu [17] realized the operational effectiveness evaluation of naval aviation force by using probabilistic finite state machine, but did not conduct dynamic effectiveness evaluation for the whole combat life cycle of naval aviation force, and could not analyse the key nodes which affected the operational effectiveness.

In view of these shortcomings in the current works of effectiveness evaluation, this paper proposes a weapon system dynamic effectiveness evaluation method based on probabilistic finite combination state machine (PFCSM). The organizational structure of this paper is as follows, the first section analyses the research status of weapon system effectiveness evaluation. The second section introduces the PFCSM method proposed in this paper, the third section gives the detailed evaluation process of missile operational effectiveness. The fourth section depicts the dynamic evaluation results of missile operational effectiveness, and gives the dynamic evolution and optimization result of the whole life cycle of the missile operational effectiveness. The fifth section summarizes the proposed method from the aspects of reliability, dynamics, applicability and expansibility.

2. Probabilistic finite combination state machine
Finite state machine (FSM) can describe the development process of things through a finite number of states, these states can transfer between themselves triggered by input and behaviour. It is a calculation model abstracted from the study of the calculation process of finite states. The basic definition of FSM can be denoted as a ternary array $G$.

$$G = \{S, X, F\}$$

Where $S$ is a set of finite states, $X$ is a set of finite inputs, $F : S \times X \rightarrow S$ is state transition function. If the state transition function is regarded as a probability distribution function, then we can get a probabilistic finite state machine model, and the state transition function can be expressed as the following formula.

$$F(s, x, s') = p(x|s)p(s'|s, x)$$

Where $p(x|s)$ indicates the probability that the input is $x$ under the current state $s$. $p(s'|s, x)$ indicates the probability that the next state is $s'$ when the current state is $s$ and the input is $x$.

At current state $s$, $p(x|s)$ satisfies with the formula (3).

$$\sum_{x \in X_s} p(x|s) = 1$$

Where $X_s$ is a set of inputs in current state $s$ and $x \in X_s \subset X$. 


At current state $s$, $p(s'|s,x)$ satisfies with the formula (4).

$$\sum_{s'\in S} p(s'|s,x) = 1$$  \hspace{1cm} (4)

The combat process of weapon system includes multiple key stages, taking the missile attacking the warship as an example, the missile needs to go through three key stages: launch stage, cruise stage and attack stage. In different stages, both the flight altitude, flight path, flight speed of the missile and the interception and interference measures carried out by warship are changeable. Therefore, in order to clearly describe the state of weapon system in the combat process, this paper proposes a probabilistic finite state machine model based on combined state, which is called probabilistic finite combination state machine (PFCSM). The key idea of PFCSM method is to describe the specific state of weapon system during the combat process based on the combination of different symbols. The detailed definition is as follows.

$$s = S_{\langle q_1, q_2, \cdots, q_n \rangle}$$  \hspace{1cm} (5)

Where $q_i, i = 1, \cdots, n$ denotes the $i$-th description.

The current state is $s_{\langle q_1, q_2, \cdots, q_n \rangle}$ and the input is $x$, the next state is $s'_{\langle q_1, q_2, \cdots, q_n \rangle}$, we can get the formula (6) and formula (7).

$$\sum_{s \in S_{\langle q_1, q_2, \cdots, q_n \rangle}} p\left(x|s_{\langle q_1, q_2, \cdots, q_n \rangle}\right) = 1$$  \hspace{1cm} (6)

$$\sum_{s' \in S_{\langle q_1, q_2, \cdots, q_n \rangle}} p\left(s'|s_{\langle q_1, q_2, \cdots, q_n \rangle}, x\right) = 1$$  \hspace{1cm} (7)

Then, the dynamic effectiveness at any state $s_{\langle q_1, q_2, \cdots, q_n \rangle}$ can be denoted as follows.

$$E\left(s_{\langle q_1, q_2, \cdots, q_n \rangle}\right) = \sum_{s' \in S_{\langle q_1, q_2, \cdots, q_n \rangle}} \sum_{s \in S_{\langle q_1, q_2, \cdots, q_n \rangle}} p\left(x|s_{\langle q_1, q_2, \cdots, q_n \rangle}\right) p\left(s'|s_{\langle q_1, q_2, \cdots, q_n \rangle}, x\right)$$  \hspace{1cm} (8)

3. Operational effectiveness evaluation of missile based on PFCSM

In order to formally describe the combat process of anti-ship missile attacking warship, the operational effectiveness evaluation model of anti-ship missile attacking surface ships is constructed by using the proposed PFCSM method. The missile state can be denoted as follows.

$$s_{\langle q_1, q_2, q_3, q_4 \rangle} = \begin{cases} q_1 = \alpha, i \in \{1, 2, 3\} \\ q_2 = \beta, i \in \{1, 2, 3\} \\ q_3 = w, i \in \{1, \cdots, 7\} \\ q_4 = \{c, f\} \end{cases}$$  \hspace{1cm} (9)

Where $q_i$ depicts the combat stages of missile, and $\alpha, \alpha_1, \alpha_2, \alpha_3$ respectively depict the launch stage, cruise stage and attack stage. Further, $\alpha_{2,1}, \alpha_{2,2}, \alpha_{2,3}$ respectively depict the initial cruise stage, middle cruise stage and last cruise stage. $q_2$ depicts the flight altitude of missile, $\beta_1, \beta_2, \beta_3$ respectively depict the flight altitude: high, low and sea skimming. $q_3$ indicates the events that the missile maybe encounter during the combat process, including flight accuracy change ($w_1$), shipborne radar detection ($w_2$), bad sea state ($w_3$), dilution interference ($w_4$), ship-to-air missile interception ($w_5$), centroid jamming ($w_6$), phalanx artillery interception ($w_7$). $q_4$ denotes the mission execution results, symbol $c$ and symbol $f$ respectively depict success and failure.

The change of stage during the missile combat process is shown in formula (10).

$$\alpha_1 \rightarrow \alpha_{2,1} \rightarrow \alpha_{2,2} \rightarrow \alpha_{2,3} \rightarrow \alpha_3$$  \hspace{1cm} (10)

The change of flight altitude during the missile combat process is shown in formula (11).
\[ \beta_1 \rightarrow \beta_2 \rightarrow \beta_3 \rightarrow \beta_4 \rightarrow \beta_5, i, \ldots, i_n \in \{1, 2, 3\} \]  

(11)

All possible combination states of missile in launch stage can be denoted as follows.

\[
\begin{align*}
\alpha_1 - \beta_1 - w_i - c, f && \\
\alpha_1 - \beta_2 - w_i - c, f \\
\alpha_1 - \beta_3 - w_i - c, f
\end{align*}
\]  

(12)

After the missile is launched, the missile will fly to the specified altitude \((\beta_1, \beta_2, \beta_3)\), flight accuracy may change during the climbing process which may cause the mission failure, here the symbol \(c\) denotes the mission is in progress methodically, and we can omit it to simplify the expression. For instance, \(\alpha_1 - \beta_i - w_i - f\) denotes \(s_{(\alpha_i, \beta_i, w_i)}\), which means that the missile is in the launch stage and flying in high altitude, the flight accuracy changes during the climbing process which cause the mission failure. The advantage of the PFCSM method proposed in this paper is that it can express the rich and real-time battlefield dynamic information of the weapon system in each stage of the combat process.

All possible combination states of missile in initial, middle and last cruise stage can be depicted in formulas (13) – (15).

\[
\begin{align*}
\alpha_{2,1} - \beta_1 - w_2 - c, f \\
\alpha_{2,1} - \beta_2 - w_2 - c, f \\
\alpha_{2,1} - \beta_3 - w_2, w_3 - c, f
\end{align*}
\]  

(13)

\[
\begin{align*}
\alpha_{2,2} - \beta_1 - w_2, w_4, w_5 - c, f \\
\alpha_{2,2} - \beta_2 - w_2, w_4, w_5 - c, f \\
\alpha_{2,2} - \beta_3 - w_2, w_4, w_5 - c, f
\end{align*}
\]  

(14)

\[
\begin{align*}
\alpha_{2,3} - \beta_1 - w_5, w_6 \\
\alpha_{2,3} - \beta_2 - w_5, w_6 \\
\alpha_{2,3} - \beta_3 - w_5, w_6
\end{align*}
\]  

(15)

During the cruise stage, the missile can change the flight altitude, meanwhile, the warship can detect the coming missile based on radar system, and utilize shipborne weapon equipment to intercept or interfere the missile.

All possible combination states of missile in attack stage can be depicted in formulas (16).

\[
\begin{align*}
\alpha_3 - \beta_1 - w_3, w_6, w_i - c, f \\
\alpha_3 - \beta_2 - w_3, w_6, w_i - c, f \\
\alpha_3 - \beta_3 - w_3, w_6, w_i - c, f
\end{align*}
\]  

(16)

During the attack stage, the warship can utilize shipborne weapon equipment to intercept or interfere the missile.

4. Dynamic effectiveness evaluation

4.1. Combat scenario

The combat parameters of the anti-ship missile and warship is depicted in Table 1. Here two kinds of missile are given, the Missile B is twice as fast as A. Warship A can be regarded as a destroyer which equipped with abundant weapon system, Warship B is a supply ship who has little combat ability. About the combat environment, both good sea state and bad state are taken into consideration.
Table 1. Combat parameters of both sides

| Missile type and flight speed | Warship type and shipborne equipment | Sea state     |
|------------------------------|--------------------------------------|--------------|
| Missile A, 1Ma               | Warship I, detection, interference, interception equipment | Good         |
| Missile B, 2Ma               | Warship II, detection equipment      | Bad          |

The inputs of the system model based on PFCSM to formally describe the attack-defence confrontation process between anti-ship missile and warship are shown in Table 2.

Table 2. Inputs and description

| Input  | Description                  | Input  | Description                  |
|--------|------------------------------|--------|------------------------------|
| 0      | $x_0$ Launch missile         | 7      | $x_7$ Dilution interference  |
| 1      | $x_1$ Flight altitude climb  | 8      | $x_8$ Ship to air missile interception |
| 2      | $x_2$ Flight altitude descent| 9      | $x_9$ Centroid interference  |
| 3      | $x_3$ Flight altitude maintenance | 10      | $x_{10}$ Phalanx artillery interception |
| 4      | $x_4$ Flight accuracy change | 11     | $x_{11}$ Diving attack       |
| 5      | $x_5$ Shipborne radar detection | 12     | $x_{12}$ Jump attack         |
| 6      | $x_6$ Bad sea state          | 13     | $x_{13}$ Sea skimming attack |

In Table 2, $x_i, i \in \{0,1,\ldots,13\}$ denotes the possible inputs during the attack-defence confrontation process, $\overline{x}_i$ express the opposite meaning of $x_i$, taking $x_{10}$ as an example, $x_{10}$ means the warship takes phalanx system to intercept the coming missile, while $\overline{x}_{10}$ means the warship doesn’t takes phalanx system to intercept the coming missile.

The relevant combination states about the sea-skimming missile attacking the warship can be expressed in Table 3. The rest states indicate the mission failure $s_{\{a_1, f\}}$, $s_{\{a_1, \beta_1, f\}}$, $s_{\{a_1, \beta_1, \omega_1, f\}}$, $s_{\{a_2, \beta_1, \omega_1, f\}}$, and $s_{\{a_1, \beta_1, \omega_1, f\}}$ can be uniformly expressed as $s_f$.

Table 3. Combination states and abbreviation

| Combination state | Abbreviation | Combination state | Abbreviation |
|-------------------|--------------|-------------------|--------------|
| $s_0$             | $s_0$        | $s_{\{a_2, \beta_1, \omega_1\}}$ | $s_5$        |
| $s_{\{a_1\}}$    | $s_1$        | $s_{\{a_2, \beta_1, \omega_1\}}$ | $s_6$        |
| $s_{\{a_1, \beta_2\}}$ | $s_2$    | $s_{\{a_3, \beta_1, \omega_1\}}$ | $s_7$        |
| $s_{\{a_1, \beta_2, \omega_1\}}$ | $s_3$ | $s_{\{a_5, \beta_3, \omega_1\}}$ | $s_8$        |
| $s_{\{a_2, \beta_1\}}$ | $s_4$ | $s_{\{a_1, \alpha\}}$ | $s_e$        |

In this paper, we get the transition probability distribution based on combat simulation and comprehensive evaluation methods, and we get the probability of $p(x|s)$ according to expert experience and knowledge. The specific information can be acquired in Table 4.
Table 4. Probability description

| $p(x|s)$ | $p(s'|s,x)$ | $p(x|s)$ | $p(s'|s,x)$ |
|---------|-------------|---------|-------------|
| $p(x_0|s_0) = 1$ | $p(s_1|x_0,x_0) = 0.99$ | $p(x_1|s_1) = 1$ | $p(s_2|x_1,x_1) = 0.95$ |
| $p(x_1|s_2) = 0.99$ | $p(s_3|x_2,x_4) = 0.9$ | $p(x_4|s_4) = 1$ | $p(s_5|x_5,x_2) = 1$ |
| $p(x_4|s_4) = 0.85$ | $p(s_5|x_4,x_6) = 0.8$ | $p(x_6|s_6) = 1$ | $p(s_7|x_6,x_5) = 1$ |
| $p(x_3|s_6) = 0.5$ | $p(s_8|x_6,x_8) = 0.5$ | $p(x_10|s_7) = 0.5$ | $p(s_9|x_7,x_10) = 0.35$ |
| $p(x_{11}|s_8) = 1$ | $p(s_{11}|s_8,x_{11}) = 1$ | - | - |

4.2. Dynamic effectiveness evaluation

Combat mission 1: missile A attack warship I under bad sea state. The combat process of missile A is shown in Figure 1. The combination state transition diagram of anti-ship missile during the combat process can be depicted in Figure 2. According to formula (8) and Table 4, the dynamic effectiveness evolution of missile A in the whole life cycle of attacking warship I can be calculated and depicted in Figure 3. In this case, we can get the clear and intuitive dynamic curve of missile operational effectiveness, meanwhile, it is easy to get the key factors that cause the operational effectiveness changeable.

One of key factors affecting missile operation effectiveness in combat mission 1 is the bad sea state, so the let’s carry out the combat mission 2: missile A attack warship I in good sea state. For combat mission 2, the combat process of missile A is shown in Figure 4, the combination state transition diagram of missile A is shown in Figure 5, the dynamic effectiveness evolution of missile A is shown in Figure 6.
According to Figure 2 and Figure 3, Figure 5 and Figure 6, it is obvious that missile A has suffered serious countermeasures including ship to air missile interception and phalanx artillery interception. The flight speed of missile A is 1 Ma, which gives the warship I much preparation time, so we apply missile B to attack warship I and carry out the combat mission 3: missile B attack warship I in good sea state. For combat mission 3, the combat process of missile B is shown in Figure 7, the combination state transition diagram of missile B is shown in Figure 8, the dynamic effectiveness evolution of missile B is shown in Figure 9.
At last, we carry out the combat mission 4: missile B attack warship II in good sea state. For combat mission 4, the combat process of missile B is shown in Figure 10, the combination state transition diagram of missile B is shown in Figure 11, the dynamic effectiveness evolution of missile B is shown in Figure 12.

According to the dynamic evaluation results about the combat mission 1, 2, 3, 4, the statistical results can be acquired and depicted in Table 5, it is easy to get that the operational effectiveness can be improved by adjusting the weapon system, operational target, operational planning.
5. Conclusions
In this paper, the dynamic effectiveness evaluation method of weapon system based on PFCSM is proposed, and according to the evaluation results of the specific example, we can get the following conclusions.

(1) Reliability
The reliability of the evaluation results depends on the data in Table 4, the PFCSM method is less affected by subjective factors. The operational effectiveness can be effectively improved by reasonably adjusting the operational planning, weapon system and operational target.

(2) Dynamics
As depicted in Section 4, the proposed method can describe the impact of four combat information spaces of weapon system, operational target, operational scheme and combat environment on the operational effectiveness of weapon system, and characterize the dynamic transfer of key nodes in the whole life cycle.

(3) Applicability
The dynamic effectiveness evaluation method of weapon system based on PFCSM can reflects the influence of key nodes on missile operation effectiveness at a certain level. In addition, the intelligent decision-making in weapon system countermeasure scenario must be a development hotspot. For the increasingly complex future battlefield, the method proposed in this paper can provide decision support for effectiveness optimization in the process of system countermeasure.

(4) Expansibility
The dynamic effectiveness evaluation of sea-skimming missile is given based on the proposed PFCSM method, in essence, we provide a theoretical framework for the dynamic evaluation of weapon system effectiveness, which is applicable to dynamic effectiveness evaluation of other weapon systems. Interested readers can try to use the PFCSM method to evaluate the operational effectiveness of other weapon systems.

References
[1] Yi C. (2009) Research on operational effectiveness evaluation and optimal control. Nanjing university of science & technology.
[2] Nan Y., Yi G.X., Wang C.H., et al. (2018) Application of probabilistic finite state machine in dynamic efficiency evaluation. Journal of Astronautics, 39(05):541-549.
[3] Hu L. (2018) Research on combat effectiveness algorithm of flying weapon under dynamic condition. Harbin institute of technology.
[4] Chao Z., Ma C., Liu Z., et al. (2006) A new multi-attribute optimal selecting method for weapon system through trapezoidal fuzzy analytic hierarchy process and Delphi. In: The Sixth World Congress on Intelligent Control and Automation. Dalian. pp. 7821-7825.
[5] Tang Z., Sun C., Liu Z.W., et al. (2012) Research on efficiency evaluation for underwater acoustic countermeasure system based on grey hierarchy analysis. Acta Armamentarii, 33(04): 432-436.
[6] Li Y.Q., Yan X.P., Hang H.P., et al. (2016) Evaluation of anti-jamming performance of radio fuze based on fuzzy comprehensive judgment. Acta Armamentarii, 37(05): 791-797.
[7] Zhang Y.Q., Xu Z.C., Sun H.B., et al. (2016) Optimization of carried sparse based on Monte Carlo simulation and parallel particle swarm optimization algorithm. Acta Armamentarii, 37(01): 122-130.

[8] Mu Z.L., Zhou Z.L., Yu L., et al. (2010) System dynamics method for effectiveness evaluation of formation attacking ground targets. System Engineering Theory and Practice. 30(03): 565-570.

[9] Chen X., Jing Y., Li C., et al. (2010) Effectiveness evaluation of warfare command systems with dissymmetrical warfare information. In: American Control Conference. Baltimore. pp. 5556-5560.

[10] Xia W., Liu X., Meng S., et al. (2016) Efficiency evaluation research of missile weapon system based on the ADC-model. In: the 6th International Conference on Machinery, Materials, Environment, Biotechnology and Computer. pp. 1227-1236.

[11] Xun Y.B., Huang C.Q., Wang Y., et al. (2011) Operational effectiveness evaluation of space weapons based on grey analytic hierarchy process. Journal of Air Force Engineering University. pp. 12(02): 32-37.

[12] Li H., Xing J. (2014) Combat effectiveness evaluation method of photoelectric defense system based on BP neural network optimized by bat algorithm. In: International Conference on Mechatronic Sciences, Electric Engineering and Computer. pp. 2481-2485.

[13] Han X.M., Chen J.J., Jiang K. (2012) Evaluating operational effectiveness of air and missile defense warhead based on LMBP neural network. Applied Mechanics & Materials. 192:301-305.

[14] Li H., Xing J. (2014) Combat effectiveness evaluation method of photoelectric defense system based on SVM optimized by particle swarm algorithm. In: International Conference on Mechatronic Sciences, Electric Engineering and Computer. Shenyang. pp. 2481-2485.

[15] Dou J. (2016) Ship-to-air missile system operational effectiveness evaluation credibility analysis. In: International Forum on Energy , Environment and Sustainable Development. Shenzhen. pp. 16-17.

[16] Seok J., Zhao J., Selvakumar J., et al. (2013) Radar resource management: dynamic programming and dynamic finite state machines. In: the 12th European Control Conference. Switzerland. pp. 17-19.

[17] Liu X.L., Pan J.H., Jin L., et al. (2011) Research on application of probabilistic finite-state automata to battle simulation of naval air force. Journal of System Simulation. 23(5): 853-856.