Phenomenological constraints on low-scale gravity

Veniamin Berezinsky
INFN, Laboratori Nazionali del Gran Sasso, I-67010 Assergi (AQ), Italy
and Institute for Nuclear Research of the RAS, Moscow, Russia

Mohan Narayan
Mumbai University, Institute of Chemical Technology, Mumbai 400076, India
and INFN, Laboratori Nazionali del Gran Sasso, I-67010, Assergi (AQ), Italy

We study the constraints on gravity scale \( M_P \) in extra-dimension gravitational theory, obtained from gravity-induced processes. The obtained constraints are subdivided into strong (though not robust) and reliable (though less strong). The strong constraints can be in principle relaxed due to some broken gauge symmetries, e.g. family symmetry. The strongest constraint is given by neutrino oscillations. For different assumptions the lower bound on \( M_P \) is \( 10^{15} - 10^{18} \) GeV. However, it can be, in principle, reduced by broken family symmetry. More reliable bounds are due to flavor-conserved operators or those which change the flavors within one family. These bounds, obtained using the electron mass and width of \( \pi \rightarrow e\nu \) decay, are \( 1 \times 10^5 \) GeV and \( 5 \times 10^5 \) GeV, for these two cases, respectively.

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I. INTRODUCTION

Starting from works \([1]\), there has been intense activity in the recent past in constructing theories, in which the fundamental gravity scale \( M_P \) is much less than the usual Planck scale \( M_{Pl} \sim 1 \times 10^{19} \) GeV. These theories necessarily involve more than four space-time dimensions, where the extra dimensions are compactified. Gravitation propagates in all the dimensions, while the standard model fields are usually restricted to four dimensions or can propagate in the extra dimensions in more complicated versions of this scenario. One of the main motivations of these models is to provide an alternative solution to the hierarchy problem, which is usually taken care of by invoking supersymmetry. The fundamental scale of gravity \( M_P \) is related to the Planck scale \( M_{Pl} \) via the number of extra dimensions involved and their radii of compactification. There is an extensive literature on this subject. We indicate recent reviews \([2, 3, 4, 5]\), in which the reader can find references to most of the relevant papers in this area.

Many phenomenological restrictions on the fundamental scale \( M_P \) have been studied in the references cited above and it was demonstrated that low-scale gravity with \( M_P \sim \text{TeV} \) survives. In this paper we shall study restrictions imposed by quantum gravity operators, which have been already discussed in \([6]\).

The general approach to description of gravity-induced processes consists in following.

The unknown quantum gravity Lagrangian is assumed to be expanded at low energies in series of unrenormalizable operators, each being inversely proportional to the powers of the fundamental scale \( M_P \) \([7]\):

\[
\mathcal{L}(\psi, \phi) \sim \frac{O(1)}{M_P} (\psi \psi \phi \phi) + \frac{O(1)}{M_P^2} (\bar{\psi} \psi \psi \psi) + \ldots ,
\]

where \( \psi \) and \( \phi \) are fermion and boson fields, respectively. The inverse proportionality to \( M_P \) is a natural condition of vanishing of these operators when \( M_P \rightarrow \infty \), i.e. when gravity is switched off. Assuming the coefficients \( O(1) \) in expansion \([11]\) we follow argumentation of Hawking \([8]\). The Lagrangian \( \mathcal{L} \) and the operators of its expansion \([11]\) are expected to break the global symmetries \([2, 3, 4, 5]\). It could be understood, for example, as absorbing a global charge by virtual black hole with its consequent evaporation. The topological fluctuations (wormhole effects) break conservation of global charges, too \([10]\). It could be understood as transition of global charges to baby universes. On the other hand the discussed operators should respect the gauge symmetries and gauge discrete symmetries. In particular, the Lagrangian \([11]\) must have SU(2)×U(1) symmetry for the Standard Model (SM) fields, before this symmetry is spontaneously broken.

The exact or spontaneously broken gauge symmetries can in principle suppress some gravity operators. Berezhiani and Dvali \([8]\) considered the family symmetry as mechanism of such suppression. An example can be given by the flavor-changing neutral currents (FCNC). If flavors are considered as global charges, FCNC, e.g. \( d \bar{s} \), appear as gravity induced operators being suppressed by powers of \( M_P \). FCNC effects should be small, restricted by \( K^0 - \bar{K}^0 \) mixing and \( \mu \rightarrow e\gamma \) decay. Treating this problem straightforwardly, one obtains the strong lower bound on \( M_P \). As was demonstrated by Berezhiani and Dvali \([8]\) the broken family symmetry may suppress additionally FCNC gravity operators. This interesting possibility reduces, however, the beauty of the original theory: the gauge family bosons...
have to propagate in the bulk, what was prerogative of graviton only. In addition, it also seems somewhat artificial that the gauge family bosons have this property, while the other standard model gauge fields do not.

There could be some other broken gauge symmetries which can suppress additionally the gravity operators. In this paper we shall consider the broken chiral symmetry as a mechanism of such suppression.

The plan of the paper is as follows.

We begin discussion with the family-flavor violating processes which in principle can be suppressed by family symmetry. Some of these processes have been already discussed, nevertheless we discuss them again, performing more accurate calculations. Not surprisingly, for this class of processes we obtain the strongest restrictions. Then we perform the calculations for processes which cannot be suppressed by family symmetry. In the section VI we discuss how these operators can be suppressed.

II. PROCESSES WITH FLAVOR VIOLATION

Quantum gravity interactions are supposed to violate the global symmetries, and thus the flavors of particles. However, they must respect gauge symmetries, and the broken gauge symmetries can suppress the flavor-changing process. In this section we shall obtain the bounds to the fundamental gravity scale $M_P$ from flavor-changing processes neglecting possible symmetry suppression, and keeping in mind that these bounds are not robust.

A. Constraint from $K^0 - \bar{K}^0$ mixing

$K^0 - \bar{K}^0$ mixing is the consequence of an $\Delta S = 2$ interaction, which is very well accounted for within the framework of the Standard Model. This mixing results in two mass eigenstates $K_1$ and $K_2$ with masses $m_1$ and $m_2$.

The mass difference between $K_1$ and $K_2$ is measured very precisely and is known to be $5 \times 10^9$ sec$^{-1}$ [11]. It has been realised since a long time that such a small value of this mass difference will be very sensitive to any new $\Delta S = 2$ contributions coming from beyond the standard model and hence will be able to strongly constrain such new contributions.

If one has an effective $\Delta S = 2$ four-fermion interaction, then the usual calculation for this mass difference is done using the "vacuum dominance approximation" for the hadronic matrix element, $\langle K^0|\bar{d}\gamma_{\mu}(1-\gamma_5)s|[\bar{d}\gamma_{\mu}(1-\gamma_5)s]|\bar{K}^0\rangle$, (see for example [12]). This gives, $\Delta m_{12} \approx G_2 f^K m_K$, where $f^K = 160$ MeV [11] is the constant appearing in the decay amplitude of $K^+ \rightarrow \mu^+\nu$ and $m_K = 494$ MeV, is the $K^+$ mass. $G_2$ parametrizes the strength of the new interaction and the approximation sign can be interpreted to imply that, there could be some dimensionless $\lambda_{\alpha\beta}$ coefficients in front of $G_2$, where $\alpha$ and $\beta$ are the quark flavors involved in the interaction. In our case, since low scale gravity is responsible for this contribution, the interaction is flavor blind and we set $\lambda_{\alpha\beta} \approx 1$.

We calculate effect of the hadronization using a scalar current in exact analogy with $\pi$ decay (see below):

$$\langle 0|\bar{d}\gamma_5 s|K^0\rangle = f_{sK}, \text{ with } f_{sK} = \frac{m_{K^+}}{(m_s + m_d)} f_K,$$

(2)

Hence we obtain

$$\frac{1}{M_P^2 m_K} f_{sK}^2 < 5 \times 10^9 \text{ s}^{-1},$$

(3)

resulting in

$$M_P > 10^7 \text{ GeV}.$$  

(4)

B. $\mu \rightarrow e\gamma$

The lepton number violating process $\mu \rightarrow e\gamma$ gives another bound on the fundamental gravity scale $M_P$. The decay rate of $\mu \rightarrow e\gamma$ in the Standard Model, extended with massive neutrinos is given by $\Gamma_{EW} = m_{\mu} F^3/8\pi$, where formfactor $F$ calculated [13] from the loop diagrams with exchange by neutrino, is very small

$$F = \frac{eG_F m_{\mu}}{32\pi^2} L,$$

(5)

with $L \sim (m_\nu/m_W)^2$. 


The lowest order quantum gravity operator \( \mathcal{L} \) which induces \( \mu \to e\gamma \) decay is given by

\[
\mathcal{L} = \frac{1}{M_P^2} \phi \bar{e} \sigma_{\mu\nu} \mu F_{\mu\nu},
\]

where \( \phi \) is the SM scalar field and \( F_{\mu\nu} \) is e-m tensor field.

The width of \( \mu \to e\gamma \) decay calculated using operator \( \mathcal{L} \) after EW symmetry breaking is

\[
\Gamma = \frac{m_\mu^3 v_{EW}^2}{8\pi M_P^4},
\]

where vacuum expectation value is \( v_{EW} = \langle \phi \rangle = 174 \text{ GeV} \). The branching ratio \( B \) for this decay with respect to the dominant muon decay \( \mu \to e\bar{\nu}_e \nu_\mu \) is

\[
B = \frac{24\pi^2 v_{EW}^2}{M_P^2 G_F^2 m_\mu^4},
\]

results in \( M_P > 2 \times 10^7 \text{ GeV} \), if to use the present upper bound \( B < 1.2 \times 10^{-11} \) for \( \mu \to e\gamma \) decay.

C. Neutrino oscillations

Here we consider the gravitational interaction among the neutrinos as responsible for generating an additional term in the neutrino mass matrix, which is usually taken to be generated by some GUT dynamics. This additional term is taken as an perturbation to the GUT mass matrix. We follow the same approach and formalism developed in the work \[14\] (where the main aim was to consider the effect of this perturbation on the neutrino mixing angles). We refer the reader to it for further details.

Because of this perturbation, the mass splittings generated by the GUT mass matrix get an additional contribution due to the gravitational interaction. From Eq.(11) of \[14\], we get an expression for the modified mass splittings (\( M_i \) are the neutrino masses and \( \Delta M_{ij} = M_i^2 - M_j^2 \) are the mass splittings produced as a result of some GUT texture which is able to reproduce the observed masses and mixings).

\[
\Delta M_{ij}^2 = \Delta M_{ij}^2 + 2\mu (M_i \text{Re}[m_{ii}] - M_j \text{Re}[m_{jj}]),
\]

where \( m = U^\dagger U \), with \( U \) being the usual neutrino mixing matrix for three flavors, \( \lambda \) is a \( 3 \times 3 \) matrix with all elements equal to 1 and \( \mu = v_{EW}/M_P \).

The KamLAND \[15\] measurement of the lower mass splitting is a very robust result in neutrino physics confirming the earlier conclusions drawn from the solar and the atmospheric neutrino data. In the following analysis, we use this result. KamLAND gives the central value of \( \Delta M_{21}^2 \) as \( 7 \times 10^{-5} \text{ eV}^2 \). Let us estimate the correction to this scale coming from the second term in Eq.(9). Consider the case where the neutrino spectrum is hierarchial with \( |M_1| \ll |M_2| \ll |M_3| \), where \( M_3 \approx \Delta M_{31}^2 \) and \( M_2 \approx \Delta M_{21}^2 \). Also, unless the phases in the mixing matrix take very special values, the term \( \text{Re}[m_{22}] \) has a value \( O(1) \). Therefore the correction term can be written as \( 2\mu \sqrt{\Delta M_{21}^2} \).

The maximal correction due to the considered term can be parametrised with help of \( \alpha \equiv \delta(\Delta M_{31}^2)/\Delta M_{21}^2 \). For instance \( \alpha = 1 \) implies a 100 % correction. This can be translated into a bound on the scale \( M_P \):

\[
M_P > \frac{0.7}{\alpha} \times 10^{16} \text{ GeV}.
\]

The allowed values of \( \alpha \) may be obtained from analysis of the KamLAND data, respecting the general restriction \( \alpha < 1 \).

So the new scale must be even higher than a typical GUT scale. For a quasi degenerate neutrino spectrum with neutrino masses at \( \approx 1 \text{ eV} \), the constraint becomes stronger:

\[
M_P > \frac{0.9}{\alpha} \times 10^{18} \text{ GeV}.
\]

This result, however, comes with some caveats: (1) For the degenerate spectrum, for specific choices of the Majorana phases it is possible to arrange for \( \text{Re}[m_{ii}] \approx \text{Re}[m_{jj}] \) and hence to relax the constraint on \( M_P \). (2) For very specific choices of the \( O(1) \) coefficients, it is possible to use low scale gravity as the only contribution to the neutrino masses \[16\].
III. BOUND FROM PROTON DECAY

Now we discuss the bound on the gravity scale $M_P$ coming from proton decay. Even restricting our interest to $\Delta B = \Delta L$ operators, there are still many possible contributions:

$$L_{\text{eff}} \ni \frac{1}{M_P} \left[ \sum_{i=1}^{6} c_i^S O_i^S + \sum_{i=1}^{6} c_i^T O_i^T + \sum_{i=7}^{9} c_i^V O_i^V \right],$$

with $c_i^{S,T,V} = O(1)$.

In the above equation, the 6 operators of the scalar and of the tensorial type, $O_i^S$ and $O_i^T$ are $(q\bar{q})(q\bar{q})$, $(qq)(q\ell)$, $(uu)(de)$, $(ud)(ue)$, $(qq)(ue)$ and $(du)(q\ell)$. The 3 operators, $O_i^V$ of the vectorial type are $(qu)(q\ell)$, $(qu)(d\ell)$ and $(qd)(u\ell)$. These operators were first written down explicitly in [17]. It is simple to see that the weak hypercharge is conserved in all these. Regarding weak isospin, the first operator is built in the $I = 1$ channel, the last 3 in $I = 1/2$ and the remaining ones in $I = 0$. Finally the color of the three quark fields is contracted with $\epsilon^{abc}$. The flavor indices have been suppressed. If we restrict our attention solely to proton decay, i.e to $\Delta S = 0$ process, then the operators $O_1$ and $O_3$ do not contribute. To estimate the resulting bound on $M_P$, let us consider as an example the channel $p \to e^+\pi^0$, that has a strong bound of $\tau = 1.6 \cdot 10^{33}$ years from the Super-Kamiokande experiment. The typical hadronic matrix element e.g., $\langle \pi^0 | \epsilon^{abc} (q^a q^b) q^c | p \rangle$ can be presented as $\sqrt{m_p} |\psi(0)| u_A$, where $\psi$ is an quark overlap function with value at origin $|\psi(0)|^2 = \text{few} \cdot m_\pi^3$ and $u_A$ is the spinor describing the proton. In this way, we obtain the bound:

$$M_P \geq \left( \frac{m_p \cdot \tau}{32\pi} \cdot m_p |\psi(0)|^2 \xi^2 \right)^{1/4} \geq 5 \times 10^{15} \text{ GeV},$$

where $\xi$ is a renormalization factor from $M_P$ to the scale of proton decay. This bound is expected to be correct within a factor of the order of unity, however it is remarkably insensitive to the details of the hadronization.

IV. BOUND FROM THE MAJORANA NEUTRINO MASS

The quantum gravity dimension 5 operator can be constructed as SU(2) singlet from lepton and Higgs fields:

$$L = \frac{1}{M_P} \left( l_L^c \bar{t}_L \right) \left( \phi^c \bar{\psi} \right),$$

where $l_L$ is lepton SU(2) doublet, index $c$ denotes the charge conjugation and $\bar{\psi}$ the Pauli spin matrices. This operator generates the Majorana neutrino mass and as it was first suggested in Ref. [18]. After EW symmetry breaking, Eq.(13) gives the following Majorana neutrino mass term:

$$L_{\text{mass}} = \frac{v_{\text{EW}}^2}{M_P} \nu_l \nu^c_L.$$

From the cosmological limit on the neutrino masses, $m_\nu < 0.23$ eV [19, 20], valid for any flavor, we obtain using Eq.(14):

$$M_P > \frac{v_{\text{EW}}^2}{m_\nu} = 1.3 \times 10^{14} \text{ GeV}.$$  

Note that though Eqs. (13) and (14) violate lepton number, they do not allow transitions between flavors (lepton numbers) of different families, and therefore the bound (15) cannot be suppressed by family symmetry. For further discussion see section VI.

V. PROCESSES WITHOUT FAMILY-FLAVOR VIOLATION

In this section we shall study more reliable bounds on the gravity scale $M_P$. We restrict our consideration by flavor transitions within one family and by flavor-conserving operators.
A. Fermion masses

The Standard Model includes the Yukawa interaction

\[ \mathcal{L} = h(f_L \phi)f_R, \]

where \( f_L \) is fermion \( SU(2)_L \) doublet, \( \phi \) is the Higgs doublet and \( f_R \) is \( SU(2)_L \) singlet. The corresponding gravitational operator can be written, including \( SU(2)_L \) singlet (\( \phi \phi \)):

\[ \mathcal{L}_{\text{grav}} = \frac{1}{M_P^2}(f_L \phi)f_R(\phi \phi). \]

After EW symmetry breaking, the Eq.(17) gives the fermion mass term

\[ m_f = \frac{v^3_{\text{EW}}}{M_P^2} \]

Using \( m_f < m_e \), we obtain the lower bound

\[ M_P > 1 \times 10^5 \text{ GeV} \]

However, the Higgs sector of the Standard Model is most questionable part of the model. In particular, the differences of fermion masses in the first and third families requires the physics beyond SM.

The Lagrangian (17) may be forbidden imposing the chiral symmetry, and the lower bound on \( M_P \) may expected to become smaller. We shall demonstrate here that this is not the case.

Consider the chiral gauge symmetry within most natural \( SU(2)_L \times SU(2)_R \) model [21]. In this model for the first family \( q_L = (u, d)_L, l_L = (\nu, e)_L \) and \( \phi_L = (\phi^+, \phi^0)_L \) are transforming as \( (2, 1) \) and \( q_R = (u, d)_R, l_R = (\nu, e)_R \) and \( \phi'_R = (\phi^+, \phi^0)_R \) as \( (1, 2) \). For the other families \( q \) and \( l \) are defined in the identical way. The operator (17) is not \( SU(2)_L \times SU(2)_R \) singlet, and it does not conserve chiral charges. One can see it explicitly introducing the gauge interactions in the usual way:

\[ \mathcal{L} = g_L(\bar{q}_L \gamma_\mu \bar{q}_L + \bar{l}_L \gamma_\mu \bar{l}_L + \phi_L^* \partial_\mu \phi_L)\bar{W}_\mu L + g_R(\bar{q}_R \gamma_\mu \bar{q}_R + \bar{l}_R \gamma_\mu \bar{l}_R + \phi_R^* \partial_\mu \phi_R)\bar{W}_\mu R \]

Now we can write the gravity operator as \( SU(2)_L \times SU(2)_R \) singlet which conserves the chiral charges \( g_L \) and \( g_R \):

\[ \mathcal{L}_{\text{singl}} = \frac{1}{M_P^2}(f_L \phi L)(f_R \phi R). \]

After spontaneous symmetry breaking we obtain for the fermion mass

\[ m_f = \frac{1}{M_P} v_{\text{EW}} v_R, \]

which should be less than \( m_e \). The value of \( v_R \) is unknown, but it cannot be less than \( v_{\text{EW}} \). Then one obtains

\[ M_P > v_{\text{EW}}^2/m_e = 5.9 \times 10^7 \text{ GeV}. \]

Thus, submerging \( SU(2)_L \times U(1) \) into \( SU(2)_L \times SU(2)_R \) which respects chiral symmetry, we further increase the lower bound on \( M_P \). Further on we shall use only limit [19] as being more conservative.

B. Bound from \( \pi \rightarrow e\nu \)

Let us consider the operator

\[ \mathcal{L} = \frac{1}{M_P^2}(\bar{l}_L \gamma_\mu q_L)(\bar{q}_R \gamma_\mu l_R), \]

where \( l_L \) and \( q_L \) are \( SU(2)_L \) doublets and \( l_R \) and \( q_R \) are \( SU(2)_L \) singlets. The operator (24) is allowed also by \( U(1)_L \times U(1)_R \) chiral symmetry, if this symmetry is universal for all right/left fermions. See section [21] for further discussion of chiral-symmetry restrictions.
Therefore, all known symmetries and conservation laws are satisfied with (24), and we apply it to one family to avoid possible family-symmetry restrictions. In terms of the first generation fields it gives

\[ \mathcal{L} = \frac{1}{M_P^2} \bar{L} \gamma_{\mu} u_L (\bar{d}_R \gamma_{\mu} e_R). \]  

(25)

Performing the Fierz transformation [12] for operator (25) we obtain

\[ \mathcal{L} = \frac{2}{M_P^2} (\bar{u} (1 + \gamma_5) d) (\bar{e} (1 - \gamma_5) \nu_e). \]  

(26)

This operator results in two observational consequences: (i) it gives the matrix element for \( \pi \rightarrow e\nu \)-decay unsuppressed by factor \( m_e/m_\pi \), as it occurs in weak interaction, and (ii) it produces right-handed electrons in beta decays with the opposite helicity in comparison with weak interaction.

We shall concentrate here on much stronger effect (i).

For practical calculations it is enough to use the operator

\[ \mathcal{L}_{\text{eff}} = \frac{2}{M_P^2} (\bar{u} \gamma_5 d) (\bar{e} (1 - \gamma_5) \nu_e), \]  

(27)

where we omitted the scalar contribution in the \( \bar{u}d \) current since it vanishes between hadronic states \( <0|\pi|> \).

The standard calculations can be performed using the matrix element for pion decay \( \langle 0|\bar{u} \gamma_\mu \gamma_5 d|\pi \rangle = f_\pi p_\mu \), where \( p_\mu \) is pion momentum and \( f_\pi = 130 \text{ MeV} \) [11]. Multiplying this equation to \( p_\mu \) and using the Dirac equations for \( u \) and \( d \) quark fields, one obtains

\[ \langle 0|\bar{u} \gamma_5 d|\pi \rangle = \frac{m_\pi^2}{m_u + m_d} f_\pi, \]  

(28)

where the quark masses \( m_u \) and \( m_d \) are taken to be 4.5 and 8.5 MeV, respectively [11].

Using Eq. (28), we evaluate the width of \( \pi \rightarrow e\gamma \) decay, from the contribution coming from weak interaction and and from Eq. (27):

\[ \Gamma_{\text{tot}} = \frac{G_F^2}{8\pi} f_\pi^2 m_\pi m_e^2 \left[ 1 - \frac{1}{M_P^2 \sqrt{2}} \frac{m_\pi^2}{G_F m_e} \right]^2. \]  

(29)

However, the Standard Model prediction for the decay width \( \Gamma \) agrees with the experimental width within the error \( R = \delta \Gamma_X / \Gamma = 4 \times 10^{-3} \) [11], and this limits the new contribution. Thus:

\[ M_P > \left[ \frac{1}{R (m_u + m_d) G_F m_e} \frac{2\sqrt{2}}{G_F m_e} \right]^{1/2} \approx 5 \times 10^5 \text{ GeV}, \]  

(30)

VI. SUMMARY AND DISCUSSION

The status of the restrictions obtained for scale \( M_P \) varies when different processes are used.

The constraints from \( K^0 - K^0 \) transition and \( \mu \rightarrow e\gamma \) decay can be straightforwardly suppressed by the Berezhiani-Dvali mechanism [6]. Gravity-induced neutrino oscillations in principle is also suppressed by family symmetry, but this suppression would appear as well in the standard mechanism for neutrino oscillations, destroying its agreement with observational data. This problem can be probably solved in a model-dependent way (see section II C). In particular, \( M_P \) bound decreases when neutrino masses are produced due to gravitational effects only.

The bound due to Majorana neutrino mass is more robust than one due to oscillations. It is not suppressed by family symmetry and occurs if quantum gravity violates lepton numbers. The case of (almost) Dirac neutrino is given by smallness of this term, which implies even stronger bound on \( M_P \). A possible relaxation of this bound can be imposed in theory where lepton number is a gauge charge. Violation of lepton number is provided by breaking of this symmetry, and then operator (13) can include additional small factor, which reduces the bound (15). Notice, however, that if such mechanism allows transition between lepton numbers in different families, it affects the standard oscillation models, like in the case discussed above.

Since the proton decay is not discovered, the existence of some symmetry protecting the proton and respected by gravitational interaction cannot be excluded (see Antoniadis et al. in Ref. [1]). Note that the family symmetry [6]
does not work in this case. Moreover, it is always possible to find some ad hoc mechanism of suppression which makes proton “practically stable” (e.g. see [22]). Using all these arguments one may probably exclude bound due to proton decay from the list of robust restrictions.

The bounds from fermion masses and $\pi \to e\nu$ are practically robust. The former does not include even global symmetry violation and flavor-changing currents. The operator (21) is one-family operator, it respects chiral symmetry $SU(2)_L \times SU(2)_R$, being a singlet of this group. If no $SU(2)_R$ sector exists, and the symmetry is reduced to $SU(2)_L \times U(1)$ of the SM, the electron mass is given by operator (17).

The operator (24) for $\pi \to e\nu$ is one-family operator, conserving hypercharge, lepton and baryon numbers. Being $SU(2)_L \times SU(2)_R$ singlet it respects the chiral symmetry. This decay is described by the flavor-changing current allowed in CC weak interaction, and it must be allowed in quantum gravity. No global symmetry is violated by this operator.

Therefore, all theoretically known symmetries allow the operators (21) and (24). In principle, these operators can be suppressed by a new chiral symmetry, given for example by the following chiral transformations:

$$l_L \to e^{i\theta_L^i} l_L, \quad q_L \to e^{i\theta_R^i} q_L, \quad f_R \to e^{i\theta_R f_R},$$

where the rotation angles $\theta_L^i, \theta_R^i, and \theta_R$ are not equal. It results in the different coupling constants $g_R^i, g_R^j, and g_R$ in interaction of fermions with gauge bosons $A_{L,R}^i, A_{L,R}^j, and A_{R}^i$.

Operator (24) is explicitly forbidden by this “ad hoc chiral symmetry” (AHCS), and the modified AHCS-conserving operators are suppressed when this symmetry is broken. Operator (21) can be suppressed also, but transformation of the Higgs fields must be specified.

This particular chiral symmetry meets two problems: the model is not anomaly-free because $g_L^i \neq g_L^j$, and for the simplest Higgs sector the EW symmetry breaking occurs at the AHCS scale. Indeed, for the massless fields one should introduce two scalars $\phi_l$ and $\phi_q$ to provide masses for $A_{L,L}$ and $A_{L,R}$ after spontaneous symmetry breaking. However, vev’s $\langle \phi_l \rangle$ and $\langle \phi_q \rangle$ break also EW symmetry giving too large masses to W- and Z-bosons.

Finally, AHCS symmetry does not fit the GUT models, while $SU(2)_L \times SU(2)_R$ symmetry is typical for most GUT models, most notably for SO(10).

Probably one can construct more complicated AHCS model with the problems indicated above being solved. However, in absence of any motivation for symmetry (31) we qualify this possibility as exotic.

Obtained bounds are summarised in Table I.

### Table I: Bounds on the gravity scale $M_P$ from various processes.

| Process                  | Lower bound on $M_P$ |
|--------------------------|----------------------|
| **Within single family:**|                      |
| Electron mass            | $10^5$ GeV           |
| $\pi \to ee$ decay       | $5 \times 10^5$ GeV  |
| Majorana neutrino mass   | $10^{14}$ GeV        |
| Proton decay             | $10^{15}$ GeV        |
| **Transitions between families:** |               |
| $K^0 - \bar{K}^0$ oscillations | $1 \times 10^7$ GeV |
| $\mu \to e\gamma$       | $1 \times 10^7$ GeV  |
| Neutrino oscillations    | $10^{15} - 10^{18}$ GeV |

In conclusion, the supergravity operators (11) impose the lower limits on the fundamental gravity scale $M_P \gg$ TeV. These limits are valid even if supergravity operators (11) break only fermion flavors, like in case of $\pi \to ee$ decay, or conserves all flavors like in case of a fermion mass. The obtained lower limits must be considered as a problem for TeV-scale gravity, though we do not interpret these results as exclusion of TeV-scale gravity. The simplest possibility is given by some symmetry which forbids or suppresses the dangerous operators like the AHCS considered above. However, TeV-scale gravity is only a possibility, and the lower limits obtained above, could be an argument in favor of extra-dimension gravitational theory with the larger scale, like e.g. $M_P \sim 10^{15}$ GeV in the Horava-Witten scenario [23]. These models have the interesting phenomenological applications, like for example the gravity-induced neutrino masses (see section [14] and [16]).
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