Finite-element simulation of cyclic compression of a cylinder with account for energy dissipation

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Abstract. The article considers the solution of the associated thermoelastic problem of dynamic compression of a cylindrical sample from hyperelastic material in an axisymmetric formulation using the Abaqus Simulia finite element modeling software package. The authors present the results of a numerical experiment, carry out its analysis, make a conclusion if it is necessary to take into account energy dissipation in the study of materials operating under cyclic loading.

1. Introduction

Researchers apply the laws of thermoelasticity to solve the problems of tire rolling simulation, taking into account the relationship between the processes of elastic deformation and thermal conductivity [1]. The field coupling has a significant effect on the distribution of deformations and temperature under non-stationary processes and pulsed loading, especially under resonance conditions [2]. Therefore, the dynamic problems of coupled thermoelasticity are of particular interest. As deformations and temperature are connected, dissipation occurs. This leads to a change in the nature of wave processes: in a thermoelastic medium. The waves decay and have dispersion. These features affect numerical models (for example, differential equations of thermoelastic motion are not self-adjoint) [3].

2. Materials and methods

We took a cyclic test for uniaxial compression of a rubber cylinder as a model for creating a numerical experiment scheme (figure 1). The sample was between two platforms (movable and fixed). The pre-stress of the sample was carried out by moving the upper platform by 4.5 mm (compression). To ensure constant amplitude of oscillations, we switched on the oscillator and applied a cyclic load. A cylinder 25 mm high and 17.8 mm in diameter was used as a sample. The compression rate was 100 mm min$^{-1}$.

At the next stage, we measured forces, displacements, and temperatures at the base of the sample. There were significant restrictions on the application of the methodology used for experimental investigation of energy dissipation arising from cyclic deformation due to the difficulty, and often the complete impossibility of measuring the temperature inside the test sample.
3. Numerical approaches to the problem solution

A solution to the associated thermoelasticity problem is necessary when the stress analysis depends on the temperature distribution, and the temperature distribution depends on the stress solution. For example, metalworking problems may include significant heating due to inelastic deformation of the material, which, in turn, changes the properties of the material. In addition, there are some problems in contact, when the heat between the surfaces can highly depend on the separation of the surfaces or the pressure transmitted through the surfaces. For such cases, thermal and mechanical solutions must be obtained simultaneously, and not sequentially. In the Abaqus / Standard software package, the temperature was taken into account by introducing the inverse difference scheme, and the nonlinearly coupled system was solved using the Newton method. Abaqus / Standard software package implements a technique for solving the related thermoelastic problem by the Newton method, both in an exact and an approximate way. The exact implementation of Newton’s method includes an asymmetric Jacobi matrix (1). It is shown in the following matrix representation of related equations:

$$
\begin{bmatrix}
K_{uu} & K_{u\theta} \\
K_{\theta u} & K_{\theta \theta}
\end{bmatrix}
\begin{bmatrix}
\Delta u \\
\Delta \theta
\end{bmatrix} =
\begin{bmatrix}
R_u \\
R_\theta
\end{bmatrix}
$$

(1)

where $\Delta u$ and $\Delta \theta$ are the corresponding corrections to the increment of the displacement and temperature; $K_{ij}$ are the submatrices of the fully connected Jacobi matrix; $R_u$ and $R_\theta$ are the mechanical and thermal residual vectors, respectively. When modeling material properties in the associated thermoelastic problem, it is necessary to take into account temperature effects of the material, inelastic fraction of heat, thermorheological temperature effects, in addition to standard mechanical properties.

4. Results and their discussion

The problem was solved in an axisymmetric formulation. When modeling the hyperelastic properties of the studied material, we used the Neo-Hookean model. The viscoelastic properties of the cylinder are specified by describing dimensionless shear moduli and volume relaxation for the Prony series in the time domain. Table 1 presents properties of a cylindrical rubber sample and metal bases.

Boundary conditions. The lower platform was rigidly fixed. The upper one was brought into cyclic uniaxial displacement. The supports and bases of the viscoelastic cylinder were rigidly fixed to each other. The oscillation amplitude of the upper support relative to the equilibrium position was 2.25 mm with an initial strain of 4.5 mm. The magnitude of the initial deformation was taken as the initial position. When we applied cyclic loading, deformation movement was recorded relative to the initial position. When simulating the cylinder heating process under cyclic loading in an axisymmetric setting, a 4-node axisymmetric thermally bonded element was selected that recorded the biaxial
displacement and temperature. The choice of this element was determined by the conditions of the formulated problem.

| Table 1. Material properties |
|-----------------------------|
| Value                       | Steel | Sample |
| Density (kg m⁻³)            | 7300  | 993    |
| Expansion coefficient (°C⁻¹) | 10.8x10⁻⁶ | 9.0x10⁻⁵ |
| Heat conductivity coefficient (W m⁻²°C⁻¹) | 70    | 0.6    |
| Heat capacity (J kg⁻²°C⁻¹)  | 320   | 1255   |
| Neo-Hookean Model Value     | C10   | D1     |
| Values                      | 1042.3| 9.6267E-006 |
| Viscoelastic Properties (Proni) | G₂₀₀₅₆₀₃₂₀    | Ti    |
| τ₀                           | 0     | 1.766  |
| Gₙ                          | 0     | 0.1536 |
| τₙ                          | 0     | 0.0127 |

We used numerical methods to obtain the temperature distribution field of the viscoelastic elastomeric material of the cylinder (figure 2).

Figure 2. Temperature distribution at the end of the experiment

Figure 3 is a graph of the temperature change of an arbitrary point inside the material of the test sample.

Figure 3. Graph of the temperature change of the point during the experiment
An analysis of the results has shown that the temperature inside the material increased by 1 degree due to energy dissipation during dynamic loading of the cylinder, even with a small number of cycles. It suggests a significant effect of energy dissipation at large values of the loading cycles (figure 4).

![Figure 4. Change in temperature field of an elastomeric sample over time](image)

5. Conclusion

The paper presents a developed algorithm for solving the associated thermoelastic problem of cyclic compression of a rubber cylinder by the finite element method using the Simulia Abaqus software package in an axisymmetric formulation. A temperature distribution field is obtained in the material of the cylindrical sample over time. It is shown that the temperature of the material increases by 1 degree with a small number of loading cycles. This change in
the temperature field indicates a significant effect of energy dissipation on the physical and mechanical characteristics of the material.

References
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