A Low Complexity OFDM Receiver with Combined GAMP and MF Message Passing
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Abstract—With a unified belief propagation (BP) and mean field (MF) framework, we propose an iterative message passing receiver, which performs joint channel state and noise precision (the reciprocal of noise variance) estimation and decoding for OFDM systems. The recently developed generalized approximate message passing (GAMP) is incorporated to the BP-MF framework, where MF is used to handle observation factor nodes with unknown noise precision and GAMP is used for channel estimation in the time-frequency domain. Compared to state-of-the-art algorithms in the literature, the proposed algorithm either delivers similar performance with much lower complexity, or delivers much better performance with similar complexity. In addition, the proposed algorithm exhibits fastest convergence.

Index Terms—Iterative receiver, OFDM, BP-MF, GAMP.

I. INTRODUCTION

Due to the excellent performance, especially when applied to discrete probabilistic models, belief propagation (BP) [1] on factor graphs has attracted much attention in the design of iterative receivers for communication systems [2]–[4]. However, BP may suffer from high or even intractable computational complexity in certain applications [13]. An alternative to BP is the mean field (MF) approximation (also known as variational message passing) [12], which can efficiently deal with continuous probabilistic models involving probability density functions (pdfs) belonging to an exponential family. Another notable approximate inference technique is expectation propagation (EP) [11], which can be seen as an approximation of BP where some beliefs are approximated by pdfs in a specific exponential family. Recently, to take advantage of the merits of different message passing techniques, unified message passing frameworks have been investigated and applied to low complexity communication receiver design, e.g., the combined BP-EP receivers in [7].

This work is supported by the National Natural Science Foundation of China (NSFC 61571402, NSFC U1204607, NSFC 61172086).

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[10] and the combined BP-MF receivers in [5], [8], [9], [13], [14].

With the unified BP and MF framework in [13], a message passing OFDM receiver for joint channel estimation and decoding was proposed in [5], which involves high computational complexity due to the operation of a large matrix inversion required in each iteration. An alternative message passing receiver that allows flexible complexity-performance trade-off was proposed in [14], where groups of contiguous channel weights are assumed to obey a Markov model, leading to an algorithm whose complexity is adjustable by changing the size of each group. In addition, the noise precision is treated as a random variable and estimated by using MF. A similar method for noise precision estimation was also used in [8] and [9].

With combined BP and EP, an OFDM receiver performing joint channel estimation and decoding was designed in [7], where the recently developed generalized approximate message passing (GAMP) [15] is employed to reduce the complexity. GAMP was firstly used in [16] for sparse channel estimation (jointly performed with decoding) in OFDM systems. Compared to the algorithm in [7], the algorithm in [16] achieves better performance with lower complexity. However, the precision of the noise is assumed to be known at receivers in [7] and [16], and the extension of the receivers to the case of unknown noise precision is not straightforward.

This work concerns the design of message passing receiver for joint channel estimation and decoding with unknown noise precision. The unified MF and BP framework in [13] is used, and the GAMP algorithm is incorporated into the BP-MF framework to significantly reduce the complexity. With a stretched factor graph which is inspired by [7], modified GAMP is developed to handle a densely connected subgraph (functioning as channel estimation). In addition, MF is used to deal with observation factor nodes with unknown noise precision, while BP is used for the subgraph of demodulation and decoding. Compared to the BP-EP receiver in [7], the proposed receiver has the capability of noise precision estimation, and can achieve the same performance as the receiver in [7] with known noise precision. Compared to the state-of-the-art BP-MF receivers in [5], the proposed receiver delivers same performance while with much lower complexity. In addition, the proposed receiver can achieve much better performance than the receiver in [9] (for a fair comparison, the group size of the receiver in [14] is adjusted so that it has similar complexity to the proposed receiver). It is also shown that the proposed receiver exhibits fastest convergence compared to the receivers.
in [7], [5] and [14].

This paper is organized as follows. In Section II, the OFDM system model is described and a factor graph representation is presented. The new low complexity OFDM receiver is proposed in Section III. Performance comparisons and complexity analyses are provided in Section IV and conclusions are drawn in Section V.

Notation- Boldface lower-case and upper-case letters denote vectors and matrices, respectively. Superscripts $(\cdot)^*$ and $(\cdot)^T$ represent conjugation and transposition, respectively. The expectation operator with respect to a density $g(x)$ is expressed by $\mathbb{E}[f(x)] = \int f(x)g(x)dx/\int g(x)dx$. The probability density function (pdf) of a complex Gaussian distribution with mean $\bar{x}$ and variance $\nu_x$ is represented by $\mathcal{CN}(...).$ The relation $f(x) = cg(x)$ for some positive constant $c$ is written as $f(x) \propto g(x).$ The notation $\odot$ represents the element-wise product between two vectors.

II. SYSTEM MODEL

Consider an OFDM system employing $N$ data and $P$ pilot subcarriers with disjoint sets of indices $D$ and $P$, respectively, where $D \cup P = \{1 : N + P\}$ and $D \cap P = \emptyset$. A sequence of $K$ information bits $b = \{b_k, k = 1, \ldots K\}$ is encoded and interleaved using a rate $R = K/(NQ)$ channel code and a random interleaver, yielding an interleaved codeword vector $\mathbf{c}$, where $Q$ denotes the order of modulation. $Q$ coded bits in each sub-vector $c_n$ are mapped to a data symbol $x_{in} \in S_D$, $n \in D$, where $S_D$ denotes modulation alphabet of size $2^Q$. The data symbols $\{x_{in}, n \in D\}$ are multiplied with pilot symbols $\{x_j, j \in P\}$ which are randomly selected from $S_P$, resulting in a vector of transmitted symbols $x = \{x_{in}, n \in D \cup P\}^T$. The transmitted symbols are modulated by IFFT and then a cyclic prefix (CP) is added before transmission through a wireless channel with $L$ taps, $\alpha = (\alpha_1, \ldots, \alpha_L)^T$.

After the removal of CP and the Fourier transform at the receiver side, the received signal in the frequency domain can be represented as

$$y = h \odot x + \omega \quad (1)$$

where $h = \Phi \alpha$ stands for the vector of frequency-domain channel weights, $\Phi$ represents the first $L$ columns of a $(N + P) \times (N + P)$ discrete Fourier transform matrix, and $\omega$ is an AWGN vector with zero mean and covariance matrix $\lambda^{-1}I$.

A. Probabilistic Formulation and Factor Graph Representation

The joint pdf of the collected observed and unknown variables in the OFDM system can be factorized as

$$p(y, h, x, c, b, \lambda) = f_M(x, c, b) f_\lambda(\lambda) \prod_{i \in D} f_{\alpha_i}(x_i, h_i, \lambda) \prod_{j \in P} f_{\delta_j}(h_j, \lambda) \prod_{i \in D \cup P} f_{\bar{\delta}_i}(h_i, \alpha) \prod_{l \in [1 : L]} f_{\alpha_l}(\alpha_l) \quad (2)$$

where $f_{\alpha_i}(x_i, h_i, \lambda) \triangleq p(y_i|x_i, h_i, \lambda) = \mathcal{CN}(h_i x_i; y_i, \lambda^{-1})$ for all $i \in D$, $f_{\delta_j}(h_j, \lambda) \triangleq p(y_j|h_j, \lambda) = \mathcal{CN}(h_j y_j; y_j, \lambda^{-1})$ for all $j \in P$, $f_{\bar{\delta}_i}(h_i, \alpha) \triangleq p(h_i|\alpha) = \delta(h_i - \Phi \alpha)$ with

![Factor graph representation for the factorization in (2)](image)

$\Phi_i$ denoting the $i$-th row of matrix $\Phi$. The local function $f_{\alpha_l}(\alpha_l) \triangleq p(\alpha_l)$ represents the prior pdf of the $l$-th channel tap, and $f_M(x, c, b)$ stands for the modulation, coding and interleaving constraints.

The factorization in (2) can be visually depicted by the factor graph shown in Fig. 1 where $f_M(x, c, b)$ is represented by the subgraph in the dashed box. More details about $f_M(x, c, b)$ can be found in [5]. It is worth mentioning that, the factor graph used in this paper is a stretched version of that in [7] where the extra variable nodes $\{h_i\}$ and the corresponding hard constraint factor nodes $\{f_\delta\}$ are added. This enables the use of combined BP and MF message passing framework. We use MF to handle the observation nodes where the noise precision is treated as a random variable, and use GAMP for message updating in the densely connected subgraph in Fig. 1.

III. JOINT CHANNEL STATE AND NOISE PRECISION ESTIMATION AND DECODING

In this section, a joint channel state and noise procession estimation and decoding receiver is proposed by using the combined BP-MF message passing framework [13] on the factor graph shown in Fig. 1.

We denote the set of all factor nodes by $\mathcal{A}$ and divide it into two disjoint subsets, an MF set $\mathcal{A}_{MF} \triangleq \{f_\alpha, i \in D\} \cup \{f_P, j \in P\}$, and a BP set $\mathcal{A}_{BP} \triangleq \mathcal{A}/\mathcal{A}_{MF}$. For factor nodes in the BP part, the messages are updated using the BP rule, and extrinsic messages are passed to their neighbor nodes. For factor nodes in the MF part, messages are computed by the MF rule, and beliefs are used [13].

A. Message Passing for Channel Estimation

It can be seen from the graph shown in Fig. 1 that there is a densely connected part between variable nodes $\{\alpha_l, \forall l \in [1 : L]\}$ and factor nodes $\{f_\delta, h_i, \alpha\}, \forall j \in D \cup P$. As the relevant factor nodes are in the BP node set, we propose to apply the GAMP algorithm for this part to achieve low complexity message updating. Next, we detail the computations of incoming messages and outgoing messages for this part.

1) Incoming message (from the observation nodes) computations: We assume that the beliefs of noise precision $\lambda$ and data symbol $x_i$ are known, which are denoted by $b(\lambda)$ and $b(x_i)$ and given in (12) and (6) respectively. Then the
message \( m_{f_0} \to h_i(h_i) \), for \( j \in D \), from observation node \( f_0 \), to \( h_j \) is computed by the MF rule \[13\] as,

\[
m_{f_0} \to h_i(h_i) = \exp \{ (\log f_0(h_i, x_i, \lambda))h_i(x_i) \} \times \mathcal{N} \left( h_i; \hat{\theta}_i, \nu_{\theta_i} \right) \sim \mathcal{N} \left( h_i; \hat{\theta}_i, \nu_{\theta_i} \right)
\]

where \( \hat{\lambda} = (\lambda)_{h_\lambda(\lambda)} \).

Since for the observation nodes \( m_{f_j} \to h_j(h_j) \), for \( j \in P \) the value of \( x_j \) is known at the receiver, the message \( m_{f_j} \to h_j(h_j) \) is computed as

\[
m_{f_j} \to h_j(h_j) = \exp \{ (\log f_p(h_j, \lambda))h_j(x_j) \} \times \mathcal{N} \left( h_j; \hat{\theta}_j, \nu_{\theta_j} \right) \sim \mathcal{N} \left( h_j; \hat{\theta}_j, \nu_{\theta_j} \right)
\]

For the convenience of description, the Gaussian messages \( m_{f_j} \to h_j(h_j), \forall j \in P \) and \( m_{f_0} \to h_i(h_i), \forall i \in D \) are uniformly denoted as \( m_{f_k} \to h_{k_j}(h_k) \sim \mathcal{N}(h_k; \hat{\theta}_{k}, \nu_{\theta_k}), \forall k \in D \cup P \).

2) Outgoing message (to the observation nodes) computations:

For the first iteration, we initiate the messages \( f_{\delta_i} \to h_i(h_i), \forall i \in D \cup P \) as \( m_{f_\delta} \to h_i(h_i) = \mathcal{N}(h_i; \xi_i, \nu_{\xi_i}) \), which are later updated in \[10\]. The belief \( b(\alpha_i) \) for \( \alpha_i, \forall i \in [1 : L] \) is initiated as \( b(\alpha_i) = \mathcal{N}(\alpha_i; \hat{\alpha}_i, \nu_{\alpha_i}) \), which will be updated by \[9\].

We divide the computations of the messages into the following 5 steps:

1) Using \([15\) Eq. (35)], the belief \( b(h_i) \) of each frequency-domain channel weight \( h_i \) can be calculated as

\[
b(h_i) \sim m_{f_{\delta_i}} \to h_i(h_i) \sim \mathcal{N}(h_i; \hat{\theta}_{\delta_i}, \nu_{\theta_{\delta_i}})
\]

where

\[
\nu_{\delta_i} = \left( \frac{1}{\nu_{\theta_i}} + \frac{1}{\nu_{\xi_i}} \right)^{-1}; \quad \hat{\delta}_i = \nu_{\delta_i} \left( \frac{\hat{\theta}_i - \xi_i}{\nu_{\theta_i}} \right)
\]

2) Compute the two intermediate parameters \( \hat{s}_i \) and \( \tau_{s_i} \) for each \( i \) by using \([15\) Eqs. (6a), (6b), (36) and (37)]

\[
\hat{s}_i = g_{\text{out}}(\xi_i, \nu_{\xi_i}, \hat{\theta}_i, \nu_{\theta_i}) = \frac{\hat{h}_i - \xi_i}{\nu_{\xi_i}}
\]

\[
\tau_{s_i} = -\frac{\partial}{\partial \xi_i} g_{\text{out}}(\xi_i, \nu_{\xi_i}, \hat{\theta}_i, \nu_{\theta_i}) = \frac{1}{\nu_{\xi_i}} \left( 1 - \frac{\nu_{\delta_i}}{\nu_{\xi_i}} \right)
\]

3) Update the variance \( \nu_{\hat{r}_i} \) and mean \( \hat{r}_i \) of message \( n_{\alpha_i} \to f_{\alpha_i}(\alpha_i) \sim \mathcal{N}(\alpha_i; \hat{r}_i, \nu_{\hat{r}_i}) \) for each \( i \) by using \([15\)

\[
b_{\text{MF}} \sim \mathcal{N}(\alpha_i; \hat{r}_i, \nu_{\hat{r}_i})
\]

\[
eq \exp \left\{ -\lambda \left( |y_j - x_j h_j^2| b_{\text{MF}} \right) \right\}
\]

Eqs. (7a) and (7b),

\[
\nu_{\hat{r}_i} = \left( \sum_{i \in P \cup D} \tau_{s_i} \right)^{-1}; \quad \hat{r}_i = \nu_{\hat{r}_i} \sum_{i \in P \cup D} \hat{s}_i \Phi_{\alpha} + \hat{\alpha}_i.
\]

4) With the Gaussian priori distribution of the channel tap \( \alpha_i \) in time-domain \( p(\alpha_i) = \mathcal{N}(\alpha_i; \hat{\alpha}_i, \nu_{\alpha_i}) \), calculate the belief \( b(\alpha_i) \) of each \( \alpha_i \)

\[
b(\alpha_i) \sim p(\alpha_i) m_{f_{\delta_i}} \to f_{\alpha_i}(\alpha_i) \sim \mathcal{N}(\alpha_i; \hat{\alpha}_i, \nu_{\alpha_i})
\]

where

\[
\nu_{\alpha_i} = \left( \frac{1}{\nu_{\theta_i}} + \frac{1}{\nu_{\xi_i}} \right)^{-1}; \quad \hat{\alpha}_i = \nu_{\hat{\alpha}_i} \left( \frac{\hat{r}_i}{\nu_{\hat{r}_i}} + \hat{\xi}_i \right). \quad (9)
\]

The mean and variance coincide those computed by \[15\] Eqs. (8a), (8b), (31) and (32) in this Gaussian scenario.

5) The variance \( \nu_{\xi_i} \) and mean \( \xi_i \) of each message

\[
m_{f_{\delta_i}} \to h_i(h_i) = \mathcal{N}(h_i; \hat{\xi}_i, \nu_{\xi_i}) \sim \mathcal{N}(h_i; \hat{\xi}_i, \nu_{\xi_i})
\]

is updated by using \([15\) Eqs. (5a) and (5b)]

\[
\nu_{\xi_i} = \sum_i \nu_{\alpha_i}; \quad \hat{\xi}_i = \sum_i \Phi_{\alpha_i} \hat{\alpha}_i - \hat{s}_i \nu_{\xi_i} \xi_i.
\]

It is noted that the computations of \( \{ \hat{r}_i \} \) and \( \{ \hat{\xi}_i \} \) in Steps 3 and 5 can be implemented using the fast inverse Fourier transform (IFFT) and fast Fourier transform (FFT) respectively, leading to lower complexity.

B. Noise Precision Estimation

The message \( m_{f_\delta} \to \lambda(\lambda) \) from pilot observation node \( f_\delta \) to \( \lambda, \forall \theta \in P \) is calculated by the MF rule,

\[
m_{f_\delta} \to \lambda(\lambda) = \exp \{ (\log f_\delta(h_j, \lambda))h_i \} \sim \mathcal{N}(\lambda; \hat{\lambda}, \nu_{\lambda})
\]

Analogously, the message \( m_{f_\delta} \to \lambda(\lambda) \) from data observation node \( f_{\delta_i} \) to \( \lambda, \forall i \in D \), can be represented as

\[
m_{f_{\delta_i}} \to \lambda(\lambda) = \exp \{ (\log f_{\delta_i}(h_i, \lambda))h_i \} \sim \mathcal{N}(\lambda; \hat{\lambda}, \nu_{\lambda})
\]

Supposing that the prior pdf \( p(\lambda) \) of \( \lambda \) is set to be \( 1/\lambda \), the belief \( b(\lambda) \) of noise precision \( \lambda \) is updated as

\[
b(\lambda) \sim p(\lambda) \prod_{j \in P} m_{f_j} \to \lambda(\lambda) \prod_{i \in D} m_{f_{\delta_i}} \to \lambda(\lambda)
\]

and its mean value is given by

\[
\hat{\lambda} = \sum_{j \in P} \left( \frac{P + N}{|y_j - x_j h_j|^2 b_{\text{MF}}} \right) + \sum_{i \in D} \left( \frac{P + N}{|y_i - x_i h_i|^2 b_{\text{MF}}} \right).
\]

C. Soft Demodulation and Decoding

The message \( m_{f_{\delta_i}} \to x_i(x_i) \) from data observation node \( f_{\delta_i} \) to variable node \( x_i, \forall i \in D \), is computed by using the MF

\[3\] The belief \( b(\alpha_i) \) in \( \[9\] \) is equivalent to the posterior function defined in \([15\) Eq. (33)].
rule,
\[ m_{f_{m} \rightarrow x_{i}}(x_{i}) = \exp \left\{ \langle \log f_{D}(h_{i}, x_{i}, \lambda) \rangle b(h_{i}) b(\lambda) \right\} \]
\[ \propto \mathcal{CN} \left( x_{i}; \frac{y_{i} h_{i}}{v_{h_{i}} + |h_{i}|^{2}}, \frac{1}{\lambda(v_{h_{i}} + |h_{i}|^{2})} \right). \]  
(14)
Massages \( \{n_{x_{i} \rightarrow f_{m}}(x_{i}) = m_{f_{m} \rightarrow x_{i}}(x_{i})\} \) for all \( i \in \mathcal{D} \) are passed to soft demodulation and decoding models, where demodulation is performed by the standard BP message update rule and decoding is implemented with the forward-backward (BCJR) algorithm [1]. Then the discrete extrinsic messages
\[ m_{f_{m} \rightarrow x_{i}}(x_{i}) = \sum_{s \in \mathcal{S}} \beta_{i}(s) \delta(x_{i} - s) \]
(15)
are passed back, where \( \mathcal{S} \) stands for modulation constellation, and \( \beta_{i}(s) \) represent extrinsic information on symbol \( x_{i} \). At last, the belief \( b(x_{i}) \) of data symbol \( x_{i} \), \( \forall i \in \mathcal{D} \) is updated by
\[ b(x_{i}) \propto n_{x_{i} \rightarrow f_{m}}(x_{i}) m_{f_{m} \rightarrow x_{i}}(x_{i}). \]  
(16)

D. Message passing schedule

From the factor graph in Fig. 1 we can find that there are multitudes of message passing schedules. In this paper, we firstly perform channel state and noise precision estimation with only pilots, and the number of iterations is denoted by \( T_{p} \). Secondly, the joint channel state and noise precision estimation and decoding is carried out iteratively using both the pilots and data, and the number of iterations is denoted by \( T_{d} \). The aforementioned schedule and the corresponding message updating are summarized in Algorithm 1, where lines 2-10 correspond to channel and noise precision estimation with only pilots and lines 12-22 correspond to joint channel and noise precision estimation and decoding with both pilots and data. Note that, the message computations in lines 6 and 9 are special forms of (8) and (13), since only pilots are used in lines 6 and 9.

**TABLE I**
PARAMETERS SETTING OF THE OFDM SYSTEM

| Parameter                      | Value       |
|--------------------------------|-------------|
| Subcarrier spacing             | 15KHz       |
| Subcarrier number              | 512         |
| Number of evenly spacing pilots| 32          |
| Modulation for pilot symbols   | QPSK        |
| Modulation for data symbols    | 16QAM       |
| Channel interleaver            | Random      |
| Number of channel taps         | 32          |

IV. SIMULATION RESULT

We consider an OFDM system with parameters given in Table I and compare our proposed algorithm and the state-of-the-art algorithms in literatures in terms of BER performance. We use “BP-MF-GAMP” to denote our algorithm, and use “BP-MF-4”, “BP-MF-32” and “BP-MF-512” to denote the algorithm in [14] with group size (the state-space dimension of the Markov model) of 4, 32 and 512, respectively. Note that, when the group size \( G \) is selected to be 512, it is equivalent to the algorithm proposed in [5]. We also provide a comparison with the receiver [7] denoted by “EP-GAMP”, where the noise precision is assumed to be known. As a reference, the performance of the receiver with perfect channel state information \( h \) and noise precision \( \lambda \) is also included, denoted by “Perfect CSI”. The receivers, except “Perfect CSI”, first carry out \( T_{p} = 5 \) iterations for channel (and noise precision) estimation with only pilots. Then joint channel (and noise precision) estimation and decoding are performed with \( T_{d} = 15 \) iterations.

In Fig. 2 the BER performance of the receivers versus different SNRs is shown. It can be seen that “BP-MF-GAMP” and “BP-MF-512” perform similar to “EP-GAMP” with known \( \lambda \). But the performance of “BP-MF-G” (denoting the algorithm in [14] with group size \( G \)) deteriorates with the decrease of group size \( G \), and the performance degrades severely when \( G = 4 \). Note that, the complexity of “BP-MF-GAMP” is approximately the same as “BP-MF-4”.

Fig. 3 shows the performance of the receivers operating at an SNR of 10dB versus the iteration index. We can see that the proposed “BP-MF-GAMP” receiver converges faster than “BP-MF-G” receivers, and even faster than “EP-GAMP” with known \( \lambda \). It is also observed that, the convergence of “BP-MF-G” also becomes slower with the decrease of the group size \( G \).
Fig. 2. BER performance of the receivers versus SNR.

Fig. 3. BER performance of the receivers versus iteration index for an SNR of 10 dB.

A. Computational Complexity Comparison

The complexity of the proposed algorithm and those in [5] and [14] is dominated by the channel estimation part, so we only analyze the complexity of channel estimation. In [5], an inverse operation of a large matrix with dimension $(N + P) \times (N + P)$ is required in each iteration, so it has cubic complexity $O((N + P)^3)$. By assuming that the channel weight obey a Markov model, the large matrix inverse is converted into a number of small matrix inverses (with size $G$) in [14], and the complexity of the algorithm in [14] is reduced to $O(G^2(N + P))$.

Designed based on the factor graph in Fig. 1 where all variables are in scalar form, the proposed receiver avoids matrix inverses and its complexity is $O((N + P)L)$. Moreover, the computational complexity can be reduced to $O((N + P) \log(N + P))$ by using the IFFT and FFT for Steps S3 and S5.

V. Conclusion

By incorporating the GAMP algorithm into a unified BP-MF framework, we have designed a low complexity message passing receiver to perform joint channel state and noise precision estimation and decoding. The MF rule is used to tackle the observation factor nodes and GAMP is used to handle the message computations for the densely connected part of the factor graph. It has been shown that, the proposed algorithm outperforms the state-of-the-art algorithms in terms of computational complexity or performance.

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