A realistic formulation of approximate CP

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Abstract

CP violation in the SM is naturally implemented as a small imaginary perturbation to real Yukawa couplings. For example, a large CP asymmetry in $B_d$ decays can arise if the imaginary parts of quark mass matrices are of order $10^{-3} m_t, b$ or smaller. Applying the principle of “additive CP violation” to soft SUSY-breaking terms, the electric dipole moments of the neutron and mercury atom are predicted near current experimental limits; for nonuniversal $A$-terms, EDM bounds can be satisfied given certain flavour structures. The proposal may be formulated in a democratic basis, with Yukawas and soft terms of the form $(\text{const.}) \times (1 + \epsilon + i \zeta)$ where $|\epsilon| \ll 1$, $|\zeta| \lesssim 10^{-3}$, motivated by approximate permutation $\times CP$ symmetry.

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# 1 Introduction: Standard Model vs. SUSY

Recent measurements of a time-dependent CP asymmetry in $B_d$ decays [1, 2], in the context of the Standard Model, indicate a large unitarity triangle angle ($\sin2\beta \simeq 0.7$), a correspondingly large CKM phase $\delta_{\text{KM}}$, with a value of the Jarlskog invariant parameter $J_{\text{CP}} = (3 \pm 1) \cdot 10^{-5}$. In contrast, the continued null results of increasingly sensitive searches for fermion electric dipole moments (EDM's) [3, 4, 5], in the context of softly-broken supersymmetry, strongly suggest that the complex phases of soft SUSY-breaking terms, namely gaugino masses, the Higgs bilinear $B$-term, and scalar trilinear $A$-terms, are of order $10^{-2}$ or smaller [6]; such bounds apply even in the limit of exact universality of soft terms.
Thus supersymmetry appears to face a naturalness problem, since if, as usually imagined, $\delta_{\text{KM}}$ arises from Yukawa phases of order unity, one also expects large soft term phases. The alternative to small SUSY-breaking phases is soft terms with large CP-violating parts, but which cancel against each other in the expressions for EDM’s. However, since recent experimental improvements provide three linearly independent limits, with a complicated dependence on the parameter space ruling out most of the parameter space where cancellation was claimed, this possibility seems equally unnatural. The “SUSY CP problem” might also be circumvented by heavy (few TeV) scalar superpartners for the two light fermion generations \[7\], which remains a possible solution, unless or until light superpartners are detected\[8\] or by assuming that SUSY-breaking takes certain special forms, for example gauge- or anomaly-mediation (which are, however, not without their own problems).

An attractive and predictive alternative, requiring no assumptions about the SUSY-breaking sector, is approximate CP symmetry \[10\], usually formulated by requiring that all complex phases be small. Approximate CP is motivated by spontaneous breaking of exact CP symmetry \[11\], supposing that we live in a vacuum that happens to be close to a CP-conserving one in the space of v.e.v.’s. The concept of approximate CP thus relies on the existence of a measure of CP violation in the theory, which will be important in the discussion. The proposal can be consistent with measurements in the $K^0$ system, if there are supersymmetric contributions to CP-odd flavour-changing interactions (which are required if $\delta_{\text{KM}}$ is to be small). The prediction for the $B_d$ decay asymmetry is small, hence approximate CP formulated in terms of small phases is ruled out (see also \[12\]).

If for some reason there are flavour-changing squark mass terms with relatively large imaginary part, then $a_{J/\Psi K_S}$ can be generated by SUSY alone \[13\]; but it turns out that this possibility cannot be described as approximate CP, since some phases in the soft breaking sector are not small (section \[12\]). Besides, with the increasingly exact fit of the SM unitarity triangle to CP-violating observables, it becomes more difficult to see how supersymmetry can be the dominant contribution to such observables without some conspiracy between different soft terms.

We will argue that there is a viable alternative implementation of approximate CP, namely CP violation with the imaginary parts of all couplings restricted to be small: we call this proposal “additive CP violation”. It will be immediately objected that such imaginary parts are not invariant under field redefinition. However, the usual implementation via small phases is also subject to this ambiguity, which reflects the well-known fact that the same physics may result from apparently different actions.

The proposal of approximate CP — either through small phases or small

\[^1\text{Although, see }\[8, 9\] for recent challenges to the decoupling solution\]
imaginary parts — requires a rule to deal with such redefinitions. A particular set of coupling constants \( \{ \lambda \} \) is admissible as approximately CP-symmetric if and only if it can be brought to a form \( \{ \lambda' \} \) by field redefinition, for which either the phases (in the usual implementation) or the imaginary parts (in our proposal) are of magnitude less than some given small number, and where the theory with \( \text{Im} \lambda = 0 \) would conserve CP exactly. In other words, all phases or imaginary parts, respectively, larger than the given size must be removable (or reducible in size) by redefinition: non-removable phases or imaginary parts should be small.

This is logically distinct from the question whether the small phases or imaginary parts of a particular set of couplings \( \{ \lambda' \} \) can be completely removed by field redefinition, in other words whether CP is really violated. To obtain a reasonable phenomenology, small phases or imaginary parts must be non-removable. However it is usually somewhat laborious to find the field redefinition that reduces the size of such parameters to the absolute minimum, since flavour rotations must in general be considered. Thus we will not require always that small imaginary parts should be impossible to reduce by redefinition. Besides, the value of an experimentally observable measure such as \( J_{CP} \) will quickly alert us to cases in which small imaginary parts can be made significantly smaller by field redefinition.

We give examples in which both CP violation and quark flavour can be satisfactorily described by adding small perturbations to an initial Lagrangian with unbroken CP and flavour symmetry (i.e. rank one fermion mass matrices and universal soft terms). In these examples, “small” means of order \( 10^{-2} \) or less for dimensionless quantities, or for mass terms, of order \( 10^{-2} \) or less compared with \( v \approx 250 \text{ GeV} \), the natural mass scale of the SM and of softly-broken SUSY. Since the perturbation breaking CP will mostly be of order \( 10^{-3} \), small soft term phases follow quite naturally in this proposal, and for universal soft terms EDM’s are predicted below, but close to, current limits. We can also relax universality and allow the structure and size of CP- and flavour symmetry-violating terms in the squark mass matrices to be comparable (but with coefficients differing by factors of order 1) to that in the quark mass matrices. Thus, while the smallness of the symmetry-violating terms remains to be explained, in this type of proposal the SUSY-breaking terms do not appear unnatural in comparison to the (SUSY-preserving) Yukawa couplings.

1.1 “Additive” CP violation

There is currently no definite indication of the theoretical origin of CP violation in Yukawa couplings and soft terms. The assumption that it occurs by complex phase rotations cannot give a good account of experimental results without an apparently unnatural distribution of phases between the Yukawas and soft terms. Both “large” and “small” phases are unsatisfactory, unless, as we shall see, “small
“phases” are implemented within a democratic model of flavour.

We will explore the consequences if CP violation originates from an imaginary perturbation to the couplings of an initial Lagrangian with real Yukawa and soft terms (section 2); the perturbations required will turn out to be small, usually order $10^{-3}$ or less, hence the proposal is an (unconventional) form of approximate CP.

This formalism can also accommodate a theory of flavour: the total Lagrangian of (potentially) CP-violating and flavour-dependent operators is

$$\mathcal{L} = \mathcal{L}_0 + \epsilon_1 \mathcal{L}_{f_1} + \epsilon_2 \mathcal{L}_{f_2} + \zeta \mathcal{L}_{\text{Im}}$$

where $\mathcal{L}_0$ has unbroken flavour symmetry (by which we mean rank 1 Yukawa matrices) and $\epsilon_1$, $\epsilon_2$ are small parameters generating flavour structure, which are typically different in the up and down sectors, and which we parameterise by $\epsilon_1 \sim m_2/m_3$, $\epsilon_2 \sim \sqrt{m_1 m_2/m_3}$, where $m_{1,2,3}$ are quark masses in ascending order. $\mathcal{L}_{\text{Im}}$ then consists of Yukawas and soft terms with imaginary coefficients.

Alternatively, since $\zeta$ will turn out to be about the same size as $\epsilon_2$, the last two terms could be combined, with the interpretation that CP is violated by small complex parameters, which also generate the small quark masses and mixing angles, but may also enter into non-flavour-dependent couplings. In the case of spontaneously-broken CP and flavour symmetry, Eq. (1) gives the effective couplings after integrating out the symmetry-breaking scalars, and $\epsilon_{1,2}$ and $\zeta$ are simply (products of) scalar v.e.v.’s normalised to some UV cutoff.

To take an extreme example, the quark mass matrices

$$\mathbf{m}^u = \frac{m_t}{3} \begin{pmatrix} 1 + i\xi & 1 & 0.9895 - i\xi \\ 1 & 1 - i\xi & 0.9895 + i\xi \\ 0.9895 - i\xi & 0.9895 + i\xi & 0.9944 \end{pmatrix},$$

$$\mathbf{m}^d = \frac{m_b}{3} \begin{pmatrix} 1.0844 & 1.0736 & 0.9312 \\ 1.0736 & 1.0629 & 0.9527 \\ 0.9312 & 0.9527 & 0.9377 \end{pmatrix},$$

where $\xi = 0.00033$, result in the CKM matrix

$$\begin{pmatrix} 0.9748 & -0.2232 + 9 \cdot 10^{-5} i & 0.0008 - 0.0021 i \\ 0.2231 & 0.9740 & -0.0406 - 0.0005 i \\ 0.0083 - 0.0022 i & 0.03971 & 0.9992 \end{pmatrix}$$

(in the Wolfenstein phase convention) with a phase $\delta_{\text{KM}} = -1.68$ and Jarlskog invariant $J = 1.9 \times 10^{-5}$. The complex phases of $m_{ij}^{u,d}$ are smaller than $7 \times 10^{-4}$.

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2 This exception also highlights the fact that the physical result of complex phases of a given size depends significantly on the choice of flavour basis.

3 Note that $\mathcal{L}_{\text{Im}}$ may not in fact violate CP, if it can be eliminated by field redefinition. Nevertheless, since we are only setting an upper bound on $\zeta$, the proposal remains well-defined.
and the range of nonremovable imaginary parts (i.e. \(\max|\text{Im } m_{ij} - \text{Im } m_{ik}|\)), which is independent of the flavour basis up to small numerical factors, is smaller than \(2.5 \times 10^{-4} m_t\).

Much of the paper is devoted to showing that acceptable values of \(\delta_{\text{KM}}\) and \(J\) arise from equally small imaginary parts for Yukawa couplings and that “additive CP violation” can be consistent with correct masses and mixings (section 3). Then almost by inspection, the small imaginary parts of soft terms that one naturally expects in this proposal satisfy the EDM bounds, at least in the limit of universality. Independently of any specific model of SUSY-breaking, for any soft mass term \(\tilde{m}_i\), we have the expected form \(\tilde{m}_i \sim (\text{const.}) \times (1 + i\tilde{\zeta}_i)\), where \(|\tilde{\zeta}_i| \lesssim 10^{-3}\) and CP is conserved in the limit \(\tilde{\zeta}_i \to 0\).

We also present a preliminary study of the proposal in the case of nonuniversal soft terms. Without a concrete model of SUSY-breaking we cannot make “hard” statements; however, it is very reasonable for the Lagrangian of flavour-dependent soft terms also to take a form analogous to Eq. (1), and we proceed on this basis. With no restriction on the flavour structure of the imaginary perturbations \(\text{Im } \tilde{m}_i\), the bounds on EDM’s are exceeded by orders of magnitude. However, since we consider models of flavour structure in the Yukawas motivated by approximate symmetries, we have expectations for the flavour structure of nonuniversal \(A\)-terms, which give the main new contributions to EDM’s. If the \(A\)-term matrices have a structure parallel to the Yukawa couplings, but with different (possibly complex) coefficients, EDM’s are just at or slightly above experimental limits (section 6.2). Off-diagonal imaginary parts of squark mass insertions are then very small, hence one does not expect SUSY contributions to flavour-changing interactions to be measurable.

These expectations are of course subject to modification, depending on what concrete models of nonuniversal SUSY-breaking exist consistent with our proposal of “additive CP violation”. The main point of the paper is to give a rather general framework which, while being extremely simple to formulate, gives a realistic picture of CP violation in both the SUSY-preserving and SUSY-breaking couplings of the MSSM. The proposal may result from, for example, spontaneous breaking of \(P_L \times P_R \times CP\) symmetry by small v.e.v.’s, where \(P_{L,R}\) are permutation symmetries acting on weak doublet and singlet matter generations respectively, and CP symmetry enforces small imaginary parts for flavour singlet and flavour-dependent couplings alike.

### 1.2 Related work

Of course, quark mass matrices consistent with Eq. (1) have been proposed previously, but little attention has been paid to the manifest smallness of CP violation in such \(\text{ansätze}\) and the implications for supersymmetry.

An exception is [14], in which CP violation occurs through a small, complex parameter, that is constrained by a \(U(1)^2\) flavour symmetry to appear only in
certain off-diagonal elements in the quark masses and soft terms (to first order). A large $\delta_{KM}$ is produced through $\text{Im} m_{13}^{d}/m_{b} \sim 10^{-2}$, while quark-squark alignment, coupled with the smallness of the CP-violating parameter, sufficiently suppresses any SUSY contributions to the $K^0$ system and to EDM’s.

A solution to the SUSY CP problem, based on the fact that CP is only violated by flavour-changing interactions in the SM (neglecting the QCD vacuum angle), was proposed in [15]. If the Yukawas and soft breaking terms and Hermitian in flavour space, “flavour-diagonal” quantities are automatically real, while off-diagonal entries may be complex with a priori arbitrary magnitudes and phases. EDM’s are then suppressed, while allowing contributions to other CP-violating observables from superpartner loops. However, Hermiticity does not arise from a field theory symmetry, except in left-right models [16], since it requires the exchange of the weak doublet and singlet chiral fermions. Thus, the property is not preserved by radiative corrections (although the resulting effects on the EDM’s are small).

In [13] the authors studied the implications for supersymmetry of universal strength of Yukawa couplings (USY) [17], a model of the democratic type which automatically has small phases and imaginary parts: the flavour structure is generated entirely by Yukawa phases. As a result of improved experimental constraints since [18], it is very likely impossible to obtain large $\delta_{KM}$ and $J \gtrsim 10^{-5}$ with USY consistent with correct magnitudes of CKM elements [19]. The authors chose a typical USY ansatz with a much smaller value of $J$, thus all CP-violating observables, including $a_{J/\Psi K^0}$, must be of supersymmetric origin.

Since they do not appreciably affect EDM’s, one can take the off-diagonal squark mass terms in the super-CKM basis to have imaginary parts large enough to generate $a_{J/\Psi K^0}$. However, the real parts of the same mass terms are tightly constrained by $B_d^\ast B_d$ mixing [20], so in this basis the term generating $a_{J/\Psi K^0}$ must have a large phase. Even in the USY basis the authors chose some non-universal $A$-terms to have a phase $10^{-1}$; hence this proposal cannot be described as approximately CP-symmetric. Thus the smallness of the phases of $B$, gaugino masses, and other $A$-terms (which are more tightly constrained by EDM’s), still require some explanation. In fact, as we show below, it is unusual for small phases in a democratic basis to produce small $\delta_{KM}$ when the USY condition is relaxed: random small imaginary parts, of the size of the $\text{Im} y_{u,d}$ used in [13], in most cases generate large $\delta_{KM}$. The small values of $J$ and $\delta_{KM}$ allow us to diagnose that the small imaginary parts considered by [13] may be reduced by some field redefinition; such a redefinition would, of course, result in Yukawas for which the USY condition was not manifest.

4However, it may be possible to reproduce the experimental value of $a_{J/\Psi K^0}$, with all phases in a particular basis being $\leq 10^{-2}$, using a more general democratic Yukawa ansatz in the presence of nonuniversal soft terms [21].
2 Measures of CP violation and small imaginary parts

One cannot begin to solve the “SUSY CP problem” without considering CP violation in the SM, i.e. in the CKM matrix $V$, which arises in changing from the weak interaction basis of quarks to the mass eigenstate basis. Both CP violation and flavour originate from the quark mass matrices, and thus from Yukawa couplings $y_{ij}^{u,d}$. Our understanding of CP violation depends on what assumptions are made about the form of the mass matrices and where their phases or imaginary parts come from.

In the SM, one cannot construct a CP-violating observable without involving all three generations and bringing into play the small (13) and (23) elements of $V$. Any such quantity is highly suppressed, compared to, say, a charged current amplitude involving diagonal or (12) elements of $V$: thus any CP-violating effect in the SM involves small amplitudes, where “small” means suppressed by at least three orders of magnitude. The “smallness” of CP violation is manifested in the Jarlskog parameter, and to some extent in the $K^0$ system. In the $B_d$ system, a large CP asymmetry in a particular channel simply means that we are comparing with a CP-even quantity that also happens to be very small: the branching fraction into any channel with a large CP asymmetry is inevitably suppressed. The same comparison also leads to $\delta_{KM}$ being order 1: we will argue that $\delta_{KM}$ is not a good measure of CP violation in most circumstances.

The usual picture of CP violation is to start with Yukawas $y_{ij}^{u,d}$ in a “heavy” or hierarchical basis (for which the (33) elements are large and the rest small), and introduce large (order 1) phases for some or all elements. In this picture, which can be generalised to superpartner interactions defined in the same flavour basis, CP violation appears to be a large effect. However, such phases may not be a good measure of CP violation on the space of couplings, as we discuss later: for the moment, note that large phases can always be removed from the large Yukawa couplings by field redefinitions, and also can change by orders of magnitude under a (real orthogonal) change of flavour basis, even after the phases of the large Yukawas have been removed).

In any given flavour basis, fields can be redefined by phases to reduce the size of $\text{Im} y_{ij}$ as far as possible. The remaining imaginary parts $\text{Im} y_{ij}^{u,d}$ stay about the same size under change of basis: thus they are largely independent of which theory of flavour one considers. Hence we think they are a better candidate for a measure of CP violation. In the SM, such imaginary parts can be smaller than $10^{-3}$ and still be consistent with $\delta_{KM} \sim 1$. If one similarly redefines phases on $V$ to reduce imaginary parts, one finds the Wolfenstein form with $\text{Im} V = \text{few} \times 10^{-3}$. However, the relation between $\text{Im} y^{u,d}$ and $\text{Im} V$ is more subtle and depends to some extent on flavour structure, as we discuss later.
2.1 “Amplification” of $\delta_{\text{KM}}$

A large KM phase results rather generically, if all quark Yukawa couplings (normalised to the largest coupling in the up or down sector respectively) have imaginary parts less than or equal to $10^{-3}$; equivalently, $|\text{Im} \, m_{ij}^{u,d}| \leq 10^{-3} m_{t,b}$. In a democratic basis, all mass matrix elements are equal to $\frac{1}{3} m_{t,b}$ up to small perturbations, and we can have $|\text{Im} \, m_{ij}^{u,d}| / m_{t,b} \approx 3 \times 10^{-4}$, thus the (relative) Yukawa phases $\text{Arg} \, y_{ij}$ are $10^{-3}$ or smaller. The democratic basis has the obvious advantage that small phases and small imaginary parts mean the same thing, therefore “additive CP violation” can be formulated unambiguously in this basis. A similar type of mass matrix, except that the imaginary parts $|\text{Im} \, m_{ij}^{d}/m_b|$ were somewhat larger, was described in [22].

Given this ansatz, the imaginary parts of the diagonalisation matrices $U_{u,d}^{L}$ will be larger: for random distributions of small imaginary parts, we find $|\text{Im} \, U_{L}^{u}| \lesssim 5 \times 10^{-2}$ and $|\text{Im} \, U_{L}^{d}| \lesssim 5 \times 10^{-3}$. The amplification of $\text{Im} \, U_{u,d}^{L}$ relative to $\text{Im} \, m_{u,d}^{u,d}/m_{t,b}$ is an essential part of our argument, and is related to (the inverse of) the small parameters that generate quark masses and mixings.

The complex phases of $U_{ij}^{u,d}$ are also not necessarily large: for mass matrices near the democratic form these matrices have elements of order 1. It is straightforward to see how (small) imaginary parts of $U_{ij}^{u,d}$ feed into a realistic CKM matrix (section 4.1).

In order to obtain $J = (3 \pm 1) \times 10^{-5}$, we find that $\text{Im} \, U_{u,d}^{L} \simeq 3 \times 10^{-3}$ is the absolute minimum; but one requires very specific structures of $\text{Im} \, U_{u,d}^{L}$, and of the mass matrices, for $J$ to be generated so “efficiently”. In most cases we find $J/\text{Im} \, U < 8 \times 10^{-4}$, implying that $\text{Im} \, U^{u}$ or $\text{Im} \, U^{d}$ should be order $5 \times 10^{-2}$, as found above. However, the flavour structure of small imaginary parts cannot be random, since it is constrained by the requirement that the quark mass hierarchy and mixing angles be stable, particularly the up mass and $|V_{13}|$. Even after imposing this requirement, several possibilities remain: see section 3.

2.2 Phases vs. imaginary parts

Many CP-violating observables do not directly determine the complex phases of (physical or invariant combinations of) couplings, but only the imaginary parts. For the soft terms, one does not in general know either the real part or absolute magnitude, whereas for the KM phase or $\sin 2\beta$, both the magnitude and imaginary part of the relevant quantity can be determined, so one may convert to a complex phase. We argue that imaginary parts of (physical, rephasing-invariant) soft terms, and $J = \pm \text{Im} \, V_{ij} V_{kl} V_{il}^{*} V_{kj}^{*}$ for the CKM matrix, are suitable

5We normalise $\text{Im} \, m_{ij}^{d}$ to $m_b$ because the bottom Yukawa could be order 1 if $\tan \beta_{H} \equiv (H_{U})/(H_{D})$ is large; if $\tan \beta_{H} \simeq 2$ then we might in principle allow $\text{Im} \, m_{ij}^{d}/m_b \simeq 0.05$, since then we would have $\text{Im} \, m_{ij}^{d}/m_t \sim 10^{-3}$. We take the more conservative limit on $\text{Im} \, m_{d}^{d}$.

6We suppress the $L$ suffix unless there is ambiguity.
quantities to compare to experiment and to characterise the strength of CP violation theoretically. If we normalise symmetry-violating effects by comparison to symmetry-preserving ones, which is the hidden assumption behind using complex phases, we get very different answers depending on which CP-even quantity we choose. The resulting measures are somewhat arbitrary: the size of CP-even quantities does not tell us much about CP violation.

The quantity \( \tilde{\delta}^u \equiv (\delta_{11}^u)_{LR} M_3 / \tilde{m} = v A_{11}^u M_3 / \tilde{m}^3 \), the (11) mass matrix element mixing L and R squarks in the super-CKM basis, times the gluino mass, normalised to an average squark mass, is dimensionless and rephasing invariant, and directly enters diagrams generating an EDM for the neutron. From experimental bounds, \(|\text{Im} (\delta_{11}^u)_{LR} M_3 / \tilde{m}| \) should be less than about \(10^{-6}\). From this bound, almost nothing can be deduced about the phase \( \text{Arg} \tilde{\delta}^u \) without further assumptions. One might expect \( \text{Re} \tilde{\delta}^u \) to be of order \( m_u / \tilde{m} \sim 10^{-5} \), but this can be highly model-dependent. With non-universal \( A \)-terms the real part might be much larger, implying a very strict bound on the phase, or it might even be smaller, given some (rather bizarre) structure of soft terms: neither is as yet experimentally excluded [7]. The size of soft term phases is thus an ill-defined way to discuss the SUSY CP problem, unless one takes some model-dependent assumption such as minimal supergravity. For the \( B / \mu \) contributions, which lead to a limit on \( |\text{Im} \mu M_{1,2} / \tilde{m}^2 | \) (in the phase convention where \( B \mu \) is set real), limits on the phase are somewhat better-defined since \(|\mu| \) and \(|B \mu| \) are constrained by correct electroweak symmetry-breaking, but the experimental limit is still found more directly for the imaginary part.

Now we give some examples showing that \( \delta_{\text{KM}} \) and \((\sin 2) \beta \) are not sensible measures of CP violation over the space of possible values of the CKM matrix. In the Standard Model, consider the limit \(|V_{13}| \to 0, \delta_{\text{KM}} = \text{const.} \), achieved by taking \( \theta_{13} \to 0 \) with all other angles constant. Clearly \( \text{Im} V \) and \( J \) vanish in this limit, and all CP-violating effects become unmeasurably small. Nevertheless, by the standard lore, CP violation would remain “large”! Approaching this limit, the unitarity triangle would be the same shape, the time-dependent decay asymmetry \( a_J/\Psi_{\text{K}^0} \) might well remain order 1, but eventually these measures would become meaningless, since measurements could not be made.

Now consider the case of \(|V_{13}| \) becoming larger by a factor of 2, with \( \text{Im} V_{13} \) varying such that \( J \) is constant (and other elements adjusted to preserve unitarity). CP-violating signals such as the EDM’s and \( \epsilon_K \) would stay the same size, but the conventional measures \( \delta_{\text{KM}} \) and \( \sin 2 \beta \) would become smaller. In the case of a CP-odd rate asymmetry defined as \( (R - \bar{R})/(R + \bar{R}) \) for some rare decay channel, the difference \( R - \bar{R} \) which signals CP violation would not change for a given luminosity, but the total rate \( R + \bar{R} \) might be larger. Division by \( (R + \bar{R}) \) is convenient, but the resulting ratio does not tell us anything essential about the size of CP violation. One might equally well use \( (R - \bar{R})/R_{\text{tot}} \) as a mea-

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7Bounds on \( \text{Re} \tilde{\delta}^u \) exist but are much weaker than \(10^{-5} \). [2].
sure of symmetry violation (where \( R_{\text{tot}} \) is the total rate over all channels), which would likely stay constant (and small) with constant \( J \). In both these cases, \( \delta_{\text{KM}} \) and \( \sin 2\beta \) give a misleading answer to the question of how large the symmetry violation is.

In what follows we will evaluate both \( \delta_{\text{KM}} \) and \( J \), but one should keep in mind that \( \delta_{\text{KM}} \) is a somewhat meaningless measure unless the quark masses and mixing angles are sufficiently close to the observed values.

3 Small imaginary parts in democratic \textit{ansätze}

In this section we study the influence of adding small imaginary parts or complex phases to quark mass matrices which are initially real and close to the democratic form \( \Delta \) with all entries equal to 1. As noted before, small imaginary parts and small phases are equivalent in this basis. The flavour structures we use are consistent with successive breaking of a symmetry group permuting 3 generations, to the 2-element group, then to nothing.

The initial structure for the quark mass matrices is

\[
M_{0}^{u,d} \equiv \frac{3m_{0}^{u,d}}{m_{t,b}} = \Delta + \epsilon D_{0}, \quad (3)
\]

where \( m_{u,d} \) is the conventionally normalised quark mass matrix, \( \Delta \) is the “democratic” matrix with all entries equal to unity, \( \epsilon \) is a small parameter and \( D_{0} \) is a real matrix of order 1. When one adds small imaginary parts to \( M_{0} \) to obtain

\[
M_{\zeta} = \Delta + \epsilon D_{0} + i\zeta D/2, \quad (4)
\]

this will in general violate CP and also change the mass spectrum and mixing angles. Note that the imaginary part is normalised so that the largest relative phases are of order \( \zeta \), the entries of \( D \) taking both signs.

The up quark mass is most likely to receive a significant contribution from the imaginary part, being the smallest eigenvalue of the initial mass matrices \( M_{0}^{u,d} \). Similarly, \( V_{ub} \equiv V_{13} \), being the smallest accurately-measured CKM matrix element, is likely the most sensitive to imaginary perturbations (although \( V_{12} \) may also be sensitive since it usually depends on small (differences between) mass matrix elements). If we assume for example that \( M_{0}^{u} \) has a vanishing smallest eigenvalue and that \( \zeta < \epsilon \), then it is clear that, unless \( D \) has some special structure, \( \det M_{\zeta} \) will be proportional to \( \zeta \epsilon/2 \). A quick calculation also yields that \( \chi \), the second invariant of the Hermitian matrix \( H \equiv MM^{\dagger} \), receives contributions proportional to \( 4\epsilon^{2} \). The largest eigenvalue of \( M_{\zeta}^{u} \) is equal to 3 to a good approximation and the 2nd. eigenvalue is much smaller, thus the smallest eigenvalue is given at leading order by \( 3m_{u}/m_{t} \sim \det M_{\zeta}/\sqrt{\chi} \sim \zeta/4 \). Thus, for random \( D \) the largest value of \( \zeta \) that one can allow in the up sector is about
12m_u/m_t \sim 1.2 \times 10^{-4}$, a severe constraint which as we will see prevents a large value of $\delta_{\text{KM}}$. One can try to evade this by choosing an initial $\mathbf{M}_0^0$ which has a smallest eigenvalue different from zero, but if $\zeta$ becomes too large the conclusion is unavoidable. The same arguments apply to the down sector, the largest allowable relative phase being of order $1.4 \times 10^{-2}$.

### 3.1 Two democratic schemes

As an example, take the democratic ansatz

$$\mathbf{M}_0^{u,d} = \begin{pmatrix} 1 & 1 & 1+b \\ 1 & 1 & 1+b-c \\ 1+b & 1+b-c & 1+b-c \end{pmatrix}$$  

where $b = 9m_2/2m_3$, $c = \pm 3\sqrt{3m_1m_2}/m_3$. These mass matrices reproduce the observed quark masses and mixings reasonably well, with remarkably few free parameters ($c_u$ and $c_d$ taking (+) and (−) signs respectively).

We also consider the mass matrices

$$\mathbf{m}_0^{u,d} = m^{t,b} \begin{pmatrix} \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{B}{3\sqrt{2}} \begin{pmatrix} 2 & 2 & -1 \\ 2 & 2 & -1 \\ -1 & -1 & -4 \end{pmatrix} + \\ \frac{C}{6} \begin{pmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ -2 & -2 & 4 \end{pmatrix} + \frac{D}{\sqrt{3}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix} \end{pmatrix}$$  

where $B = 1(0.9)m_1/m_2$, $C = m_2/m_3$, $D = 1.1(1.3)\sqrt{m_1m_2}/m_3$ in the up (down) sector. With all entries real, and slightly different input parameters $m_i$, this also gives acceptable masses and mixings $[25]$.

To introduce CP violation, we keep the same magnitudes $|m_{ij}|$ but introduce phases $\zeta_{ij}^{u,d}$ where $|\zeta_{ij}^{u,d}| \leq 10^{-2}$: this ansatz is clearly of the form $[3]$.

### 3.2 Random small imaginary perturbations

In the absence of definite clues as to the origin of CP violation, one can imagine the phases $\zeta$ in the democratic basis to be random subject to the above constraint: $\zeta_{ij}^{u,d} = \zeta d_{ij}^{u,d}/2$ where $d_{ij}^{u,d}$ takes a uniform distribution on $(-1, 1)$. Then for each initial mass matrix and random set of $d_{ij}$, we calculated quark masses, mixings and CP violating parameters as functions of $\zeta$. For small enough $\zeta$ one expects a linear behaviour of $\delta_{\text{KM}}$ and $J$ and a quadratic variation of the masses away from their values at $\zeta = 0$. $[1]$  

---

$^8$Note that $m_{1,2,3}$ are input parameters and will be only approximately equal to the resulting mass eigenvalues.

$^9$A imaginary, Hermitian perturbation of the real mass-squared matrix $\mathbf{H}_0 = \mathbf{M}_0^0 \mathbf{M}_0^{0\dagger}$ leaves the eigenvalues unchanged to first order; the proof is simple.
Table 1: Dependence of CKM parameters and $m_u$ on $\zeta$ for the two-parameter mass matrices, for two sets of random coefficients $d_{ij}^{u,d}$.

| $\zeta$ | $\delta_{KM}$ | $F_{CP} \equiv \delta_{KM}/\zeta$ | $J \times 10^5$ | $m_u$(MeV) | $|V_{13}| \times 10^3$ |
|---------|----------------|----------------|-------------|---------|----------------|
| 0.0001  | -0.065, 0.033  | -650, 340      | 0.16, -0.079| 4.0, 4.1| 2.2, 2.2       |
| 0.0003  | -0.19, 0.100   | -645, 330      | 0.47, -0.24 | 6.0, 7.0| 2.3, 2.2       |
| 0.001   | -0.574, 0.32   | -570, 330      | 1.56, -0.79 | 17, 20  | 2.7, 2.3       |
| 0.003   | -1.06, 0.79    | -350, 260      | 4.6, -2.3   | 49, 61  | 5.0, 3.0       |
| 0.01    | -1.25, 1.32    | -125, 130      | 13.6, -6.6  | 177, 230| 14, 6.3        |

To emphasize the point that small imaginary parts, or small phases in the democratic basis, lead to large $\delta_{KM}$, we define an “amplification factor” $F_{CP} \equiv \delta_{KM}/\zeta$. is the largest relative Yukawa phase allowed. We expect $F_{CP}$ to tend to a constant in the limit of small $\zeta$, and it turns out that the asymptotic value is in most cases of order $m_{n+1}/m_n$ or $\bar{\theta}^{-1}$, and may be larger. (We give an analytic derivation of the related quantity $J/\zeta$ in section 4.1.) With many uncorrelated small imaginary perturbations $\zeta_{ij} \equiv \zeta_{d_{ij}}$, the contributions to $F_{CP}$ add (although accidental cancellations are possible) and the largest amplification factors, of order $10^3$, win.

We display two typical sets of results for each mass ansatz. For the two-parameter matrices Eq. (5), we find large amplification factors in the CKM phase, such that it reaches order 1, and the Jarlskog parameter is the correct order of magnitude, for random phases of order 0.003 (Table 1). The amplification becomes smaller when $\delta_{KM}$ is no longer small, and enters a nonlinear regime, although $J$ continues to grow in a more nearly linear fashion. The CKM mixing angles and masses do not vary greatly with $\zeta$, except for the up mass and $V_{13}$. Already for phases less than 0.0003 the up mass is affected the perturbation. Recall that the upper bound on the largest relative phase is $\zeta \leq 12m_u/m_t \simeq 6 \times 10^{-5}$ if the up mass is not to be affected. Also, having been constant at small phases, $|V_{13}|$ receives contributions which result essentially from adding an imaginary part of magnitude similar or greater to the initial real part $V_{13}$. Since for $\delta_{KM}$ to be order 1 we only require the imaginary part to be about the same size as $V_{13}$, the absolute value can increase by a factor of at most two in reaching the correct value of $J$, which is not problematic and can actually give a better fit to experiment than in the case of no CP violation.

For the three-parameter mass matrices, Table 2, the amplification may be still larger, such that one achieves acceptable values of $\delta_{KM}$ and $J$ for $\zeta = 10^{-3}$, but the value of $m_u$ is still unacceptably high for these parameter values. We also see that $|V_{13}|$ grows too large with larger values of $\zeta$. In fact, in both ansätze we found that $|V_{12}|$ and $|V_{21}|$ could also exceed the experimental bounds for larger values of $\zeta$, although not so drastically as $|V_{13}|$. This can be traced to the fact that the (1-2) mixing also originates from entries of order $\sqrt{m_1m_2/m_3}$ which are
Table 2: Dependence of CKM parameters and \(m_u\) on \(\zeta\) for the three-parameter mass matrices, for two sets of random coefficients \(d_{ij}^{u,d}\).

| \(\zeta\) | \(\delta_{KM}\) | \(F_{CP} \equiv \delta_{KM}/\zeta\) | \(J \times 10^5\) | \(m_u\) (MeV) | \(|V_{13}| \times 10^3\) |
|---|---|---|---|---|---|
| 0.0001 | -0.122, 0.047 | -1220, 470 | 0.27, -0.10 | 4.3, 5.7 | 2.1, 2.1 |
| 0.0003 | -0.35, 0.14 | -1170, 470 | 0.81, -0.31 | 4.8, 12.3 | 2.2, 2.1 |
| 0.001 | -0.86, 0.45 | -860, 450 | 2.7, -1.0 | 8.6, 38 | 3.4, 2.3 |
| 0.003 | -1.17, 0.98 | -390, 330 | 7.8, -3.1 | 23, 115 | 8.1, 3.6 |
| 0.01 | -1.01, 1.47 | -101, 148 | 19, -10 | 89, 376 | 23, 9.9 |

sensitive to small perturbations.

The instability of the up mass shows that as expected, a random structure of imaginary parts is strongly disfavoured to produce the observed CP violation. If one enforces \(\zeta^u = 0\) for the up-type phases, then the problem is reduced to keeping \(m_d\) stable; but in this case the amplification factor \(F_{CP}\) tends to be much smaller, of order 50 or less, such that much larger phases \(\zeta\) are required to produce \(\delta_{KM} \sim 1\). The down mass is then marginally unstable, and the size of the required phases or imaginary parts is only marginally consistent with CP violation being a small perturbation.

Thus, any small perturbation giving rise to realistic CP violation must have a more specific structure, which should in general be correlated with the structure of the original real mass matrices. In the next section we show how the amplification of small imaginary parts occurs, then in section 5 give some structures of quark mass matrices that preserve a correct mass hierarchy under imaginary perturbations consistent with \(\delta_{KM}\) being large and \(J\) of order \(\text{few} \times 10^{-5}\).

4 How amplification works

4.1 From \(\text{Im} U^{u,d}\) to \(J\)

For diagonalisation matrices \(U^{u,d}\) for which large imaginary parts have been removed by field redefinition, leading contributions to \(J\) come from small imaginary parts \(\text{Im} U^{u,d}_{ij}\) multiplying large real parts of \(U^d\), and vice versa. Consider, in a “heavy” or hierarchical basis, small imaginary perturbations to the \(U^{u,d(h)}\) matrices, which correspond to infinitesimal \(U(3)\) transformations with symmetric generators.\(^{10}\) For the purpose of estimating \(J\) we write \(V = U^{u(h)T}_0 U_{\omega} U^{d(h)}_0\), where \(U^{u,d}_0\) are real. Then we find that imaginary parts in \(U^{u,d(h)}\) that can be expressed as diagonal \(U_{\omega}\) give negligibly small contributions to \(J\) (since off-diagonal elements of \(U^u\) are very small). With a small imaginary part \(\omega_{12}\) in the (1,2) and (2,1) elements of \(U_{\omega}\) we find \(J/\omega_{12} = \lambda^5 A^2 (1 - \rho) \simeq 3 \times 10^{-4}\) in terms

\(^{10}\)Written as \(U_{\omega} = 1 + i\omega S_1\) with real symmetric \(S.\)
of Wolfenstein parameters; for small $\omega_{23}$ and $\omega_{13}$ defined analogously we find $J/\omega_{23} = -A\lambda^4\rho \simeq -4 \times 10^{-4}$, $J/\omega_{13} = A\lambda^3 \simeq 9 \times 10^{-3}$ respectively. Thus we require either $\omega_{12} \sim 0.1$, $\omega_{23} \sim (-)1/13$ or $\omega_{13} \sim 1/300$ for $J$ to take the correct experimental value.

4.2 Diagonalisation

To see how $\text{Im} \ U$ can be much larger than $\text{Im} \ m/m_3$ and thus how the observed values of $J$, $\delta_{\text{KM}}$ can result from small imaginary parts, consider the mass matrix

$$m^{(h)} = m_0 \begin{pmatrix} 0 & w + i\zeta_{12}^{(h)} & i\zeta_{13}^{(h)} \\ w + i\zeta_{12}^{(h)} & v + i\zeta_{22}^{(h)} & u + i\zeta_{23}^{(h)} \\ i\zeta_{13}^{(h)} & u + i\zeta_{23}^{(h)} & 1 + i\zeta_{33}^{(h)} \end{pmatrix}$$  \hspace{1cm} (7)

where $u$, $v \sim m_2/m_3$, $w \sim \sqrt{m_1m_2/m_3} \sim (m_2/m_3)^{3/2}$ and the $\zeta_{ij}^{(h)}$ are of order $c$. Such a mass matrix in the “heavy” basis can always be related to a democratic one of the type we are considering via

$$m^{(h)} \equiv F m^{(d)} F^T$$  \hspace{1cm} (8)

where $F = \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & -2 \sqrt{2} \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{pmatrix}$ diagonalises $\Delta$; we do the analysis in the heavy basis since it is somewhat simpler. We consider symmetric matrices for simplicity and do not give a perturbation to $m_{11}$ since the smallest mass eigenstate would then be unacceptably unstable.

Then we construct $H \equiv MM^\dagger$ which is diagonalised as $U_L^\dagger H U_L = \text{diag}(m_i^2)$. The eigenvalues are given by $m_0(1 + \mathcal{O}(u^2), v^2 + \mathcal{O}(\zeta^{(h)2}), \mathcal{O}(w^4, \zeta^{(h)4})/v^2)$ and we find that $U_L$ is given by

$$U_L \simeq \begin{pmatrix} 1 & (w + i\zeta_{12}^{(h)})/v & uw + i\zeta_{13}^{(h)} \\ (-w + i\zeta_{12}^{(h)})/v & 1 & u + i\zeta_{23}^{(h)} \\ (uw + i(v\zeta_{13}^{(h)} - u\zeta_{12}^{(h)}))/v & -u + i\zeta_{23}^{(h)} & 1 \end{pmatrix}$$  \hspace{1cm} (9)

where we impose that the diagonal elements be real and keep only the leading real and imaginary parts (thus the matrix is only approximately unitary). Clearly the (12) imaginary part is amplified by the diagonalisation; the diagonal perturbations $\zeta_{ii}^{(h)}$ disappear (being unphysical as far as CP violation is concerned), while the (23) and (13) perturbations are unchanged in size. Identifying the imaginary parts of $U_L$ with the parameters $\omega_{12}$, etc., of the previous discussion, we have

$$\frac{J}{\zeta_{12}^{(h)}} \simeq \frac{\lambda^5 A^2 (1 - \rho)}{v} \simeq 0.07 \text{ (up)}, \ 0.006 \text{ (down)},$$
\[
\frac{J}{\zeta_{23}^{(h)}} \sim -A\lambda^4 \rho \simeq -4 \times 10^{-4}, \text{ or } \frac{J}{\zeta_{13}^{(h)}} \sim A\lambda^3 \simeq 9 \times 10^{-3}
\]

where each perturbation is considered separately. If we assume that the observed value \( J \simeq 3 \times 10^{-5} \) is correlated with \( \delta_{\text{KM}} = 0.5-1 \), we estimate the total amplification to be

\[
\frac{\delta_{\text{KM}}}{\zeta_{12}^{(h)}} \lesssim \frac{J}{\zeta_{12}^{(h)} J_{\text{exp}}} \sim 2300(\text{up}), \ 200(\text{down})
\]

since, other things being equal, the largest contribution evidently comes from \( \zeta_{12}^{(h)} \); subleading contributions are from \( \delta J_{\zeta_{23}^{(h)}} \sim 300 \), \( \delta' J_{\zeta_{23}^{(h)}} \sim 13 \).

Recall that the imaginary parts of \( m^u,d \) in the democratic basis are of order \( \zeta_{ij} m_3 / 3 = \zeta |d^u_{ij}| m_3 / 3 \), thus if \( d^u_{ij} \) are random in this basis one expects \( \delta_{\text{KM}} \) to be \( \sim 750 \zeta \) (or \( \sim 65 \zeta \), if only the down sector contributes), depending on the relative signs and magnitudes of the entries that feed into \( \zeta_{12}^{(h)} \). Thus the numerical results for the dependence of \( \delta_{\text{KM}} \) and \( J \) on \( \zeta \) can be understood analytically, at least in the linear regime.

## 5 Special mass matrices and phase structures

In order to preserve the small mass eigenvalues, we require \( \det M \) to remain of order \( m_1 m_2 / m_3^2 \) or \( (m_n / m_{n+1})^3 \sim \delta_0^3 \) under the CP-violating perturbation. As we argued previously, the expectation in the absence of any special structure is for \( \det M \sim \varepsilon \delta_0 \); for phases of order \( \zeta \), which are likely required to generate large \( \delta_{\text{KM}} \), the lightest mass eigenvalue would then approach the same magnitude as the 2nd. eigenvalue.

### 5.1 Decoupling of quark mass and CP

The most simple way to introduce CP violation in an almost democratic structure without spoiling the mass spectrum is of course one that leaves the masses exactly invariant. This is possible just by left multiplying \( M_0 \) with pure phase matrices of type \( K = \exp(i \zeta \cdot \text{diag}(\varphi_1, \varphi_2, \varphi_3)) \) with \( \zeta \) small. The mass hierarchy is then decoupled from CP violation, suggesting that the two effects have different origin. Given

\[
M^u = M_0^u, \ M^d = K M_0^d
\]

\footnote{Right multiplication would of course be unobservable and unphysical, corresponding to field redefinitions.}
it is easy to calculate the CKM matrix as $V = U_0^* K U_0^d$. Clearly, introducing another diagonal matrix of phases for the up-type quarks is redundant. The “amplification” occurs as large ($\sim 1/2$), approximately equal real entries in $U_{u,d}$, which cancel against each other in the small elements of $V$, are multiplied by small relative phases, leading to small imaginary contributions to $V$, of the right size for $\delta_{\text{KM}}$ to be large. The matrices $U_{u,d}$ are not required to contain large phases.

The larger mixing angles are not much affected by the introduction of $K$: they get a contribution of the order of $\zeta^2$. However, this contribution will be very significant for $V_{13}$ which now inherits a relatively large imaginary part; thus, the structure of $V$ is unavoidably coupled to CP violation. With the initial mass matrices of Eq. (5) and taking $K = \text{diag}(1, e^{0.008i}, 1)$, one obtains $\delta_{\text{KM}} = -0.719$ and $J = 2.9 \times 10^{-5}$, with $|V_{13}| = 0.0039$. Thus, realistic values of CP-violating parameters can originate from a small perturbation in conjunction with good values of masses and mixings.

However, it is difficult to imagine the origin of the phases $\zeta \varphi_i$, amounting to different, generation-dependent phase redefinitions for the up and down weak doublet quarks, within our proposal of “additive” CP violation. Also, the degree of amplification (order 100 or less) falls short of that achieved by random small imaginary parts. This is because the decoupling approach cannot take advantage of the amplification of imaginary parts that occurs in the diagonalisation of the mass matrices, as described in section 4.2.

5.2 Weak coupling

A different approach is to weakly couple the mass spectrum to CP violation. This can be achieved, for example, by choosing the small parameters $b$ and $c$ of Eq. (3) to be complex [22], writing $b e^{i \beta}$ and $c e^{i \gamma}$ instead of $b$ and $c$. Although expressed in terms of complex phases, clearly this ansatz does not conflict with Eq. (1) as long as $\text{Im} b, c$ are sufficiently small. The invariants of $H$ then remain of the same order of magnitude but receive small corrections from the phases (corrections would also be small if $\text{Re} b, c$ were held constant and small imaginary parts were added). In [22] all parameters were taken real positive except $c^d$, which was assigned a phase $\pi/3$ (on top of its negative sign), generating $\delta_{\text{KM}} = 0.5$ while preserving acceptable values of quark masses. The amplification factor $F_{\text{CP}}$ is then found as $3\delta_{\text{KM}}/(\pi \text{Im} c_d) \simeq m_b/9\sqrt{m_d m_s} \simeq 19$, similar to the values resulting from random phases in the down sector and hence less favourable for the proposal of small phases, but with the advantage of stable $m_d$.

If we choose instead $c^u$ to have a phase $-\pi/3$ then one can obtain $\delta_{\text{KM}} \simeq -0.5$, with $J = 1.9 \times 10^{-5}$, for an amplification of $F_{\text{CP}} \sim m_t/9\sqrt{m_u m_c} \simeq 600$, with an up mass of 3.6 GeV (for the same input value $m_1^u$). We see as expected that the up sector dominates if one introduces imaginary parts of equal magnitude, since the amplification depends on mass ratios. One may also give $b^u$ an imaginary
part of order $|e^u|$, but this is less successful, resulting in $J \sim 10^{-6}$ and small $\delta_{KM} < 0.1$). Such an imaginary perturbation can be partially removed by phase redefinitions, due to the permutation symmetry: as we will verify by changing into the “heavy” basis, the contribution of $\text{Im} \, b$ to the perturbations $\zeta_{12}^{(h)}$ and $\zeta_{13}^{(h)}$ for which appreciable amplification occur, cancel. It was already shown in \cite{[23]} that one would require $b$ to have a large phase ($i.e.$ $\text{Im} \, b \sim \text{Re} \, b$) in order to generate large $\delta_{KM}$.

Similarly in the three-parameter ansatz Eq. (5), to preserve a small up mass one may give small imaginary parts to the parameters $B$, $C$, $D$, (keeping either $\text{Re} \, B$ or $|B|$, etc., constant) \cite{[24]} The most effective way is to give $D^u$ a phase $\pi/3$, which leads to $\delta_{KM} = -1.13$ and $J = 1.9 \times 10^{-5}$. The imaginary perturbation to $\Delta/3$ is $2 \text{Im} \, D^u / \sqrt{3} \approx \sqrt{m_u m_c / m_t}$, thus the amplification factor is $\delta_{KM} m_t / 3 \sqrt{m_u m_c} \approx 2000$. Recall that in the democratic basis we are adding phases $\pm \sqrt{3} \text{Im} \, D^u$ and it is relative phases, or equivalently nonremovable imaginary parts, that induce CP violation.

The effects of a correlated structure of phases on $\det \, M^{u,d}$, the up mass and $|V_{13}|$ can be understood intuitively if we change to the “heavy” basis where the initial real matrices are (neglecting $m_i/m_{i+1}$ next to 1)

$$
m_0^{(h)} = \begin{pmatrix}
0 & \mp \sqrt{m_1 m_2} & \pm \frac{\sqrt{m_1 m_2}}{\sqrt{2}} \\
\mp \sqrt{m_1 m_2} & -m_2 & -\sqrt{2}m_2 \pm \frac{3m_1 m_2}{2}
\end{pmatrix},
$$

for the two- and three-parameter matrices Eq. (5) and (3) respectively, and the imaginary perturbation is

$$
\zeta_D^{(h)} = \frac{m_3}{18}, \tag{15}
$$

where the notation $\sum \zeta_{i+j}$ in the (1,2), (1,3), (2,3) positions means copy the (2,1), (3,1), (3,2) entry respectively but transpose the labels on each of the $\zeta_{ij}$ \cite{[24]}

We set $\zeta_{11} = \zeta_{22} = \zeta_{33} = 0$ by a phase redefinition, without loss of generality. Then setting $\zeta_i = 0$, but allowing the input values of $m_2/m_3$ and $\sqrt{m_1 m_2}/m_3$ (or $b$ and $c$, or $B$, $C$ and $D$) to be complex in these expressions, will not have a large effect on the size of the small mass eigenvalues and mixing angles. Alternatively, one can also keep $m_0^{(h)}$ real but consider different correlated patterns of $\zeta_{ij}$.

\textsuperscript{12}In the down sector, the largest allowed phases of $B$, $C$ would be about 0.1, since the mass ratios (and resulting small mixing angles) are about 1/30.

\textsuperscript{13}We neglect $\zeta_{ij}$ next to 1, and in order to fit the matrix onto the page.
Clearly adding a small imaginary part to \( b \), equivalent to setting all \( \zeta_{i3}, \zeta_{3i} \) equal, cannot produce sufficient amplification since the crucial 12 and 13 elements in the heavy basis do not receive imaginary parts (due to cancellations). For complex \( c \), however, effectively written as \( \zeta_{23} = \zeta_{33} = \zeta_{32} \) (other \( \zeta_{ij} \) vanishing), the only cancellation is in the (2,2) element.

The “weak coupling” proposal, for complex \( c \) or \( D \) respectively, is successful in generating CP violation from very small (order \( 10^{-3} \)) imaginary parts, but from the point of view of a theory written in the democratic basis, the correlations required between different entries may be somewhat contrived. Finally in this section we consider the effect of small imaginary perturbations to individual mass matrix elements in the democratic basis (other elements being held real). Then for \( m_u \) to remain small, we enforce \( \zeta_{ij} = 0, i, j = 1, 2 \) for the imaginary perturbation in the democratic basis, and in order for the perturbation to affect the small (1,2), (1,3) \( \text{etc.} \) elements of \( m^{(h)} \) and hence potentially produce \( J \approx 10^{-5} \) we consider nonzero values of \( \zeta_{u13}, \zeta_{u23}, \zeta_{u31}, \zeta_{u32} \) in turn. In the three-parameter mass \( \text{ansatz} \), such a perturbation corresponds to a nonzero value of the (1-3) and (3-1) elements of \( m^{(h)} \), which is desirable since recent data on \( |V_{13}| \) disfavour the exact zero \[26\].

For \( \zeta_{u13} = -10^{-3} \) we find \( \delta_{\text{KM}} = -0.82, J = 2.0 \times 10^{-5} \) but the up mass is marginally affected: \( m_u \approx 6 \text{ MeV} \). For \( \zeta_{u23} = 10^{-3} \) the results are very similar. For \( \zeta_{u31} \) and \( \zeta_{u32} \) of the same size there is very little amplification and \( \delta_{\text{KM}} \) is about 0.0035, but the overlarge value of \( m_u \) persists. This difference simply reflects the fact that the matrix diagonalising \( m^u \) on the left, which will determine the observable CP violation, “feels” two perturbations related by transposition differently. The larger value of \( m_u \), compared to the results of taking complex \( c^u \) or \( D^u \), arises because adding imaginary parts to a fixed \( m^{(h)}_0 \) will increase the absolute value of the small parameters on which \( \det m \) and the up mass depend. When \( c^u \) or \( D^u \) receive a phase, the real parts of the small parameters of \( m^{(h)} \) decrease such that \( \det m \), and \( m_u \), remain of the same magnitude.

### 5.3 Magnification through \( V_{13} \)

If CP violation is to come from a small imaginary perturbation, which \( a \) priori is random, \( i.e. \) does not correspond to any particular pattern, then from our initial analysis it follows that it also has to be very small, so as not to affect the smallest quark mass eigenvalues. However, then \( \delta_{\text{KM}} \) will be too small, unless there is a mechanism that will magnify the influence of some tiny random imaginary perturbation, by a factor \( \text{parametrically larger} \) than the \( F_{\text{CP}} \gtrsim 500 \) (or \( \sim 50 \) for the down sector) which occurs generically in democratic scenarios. Clearly, this magnification process has to occur in \( V_{13} \) because the other off-diagonal elements are too large to be given non-negligible phases; \( V_{13} \) has to be already very small or even zero in an initial real quark mass matrix if the imaginary perturbation is
to play a rôle. In the “heavy” basis, we require the structure

\[ m_0^{(h)} = m_3 \begin{pmatrix} q & p & pq \\ p & r & -q \\ pq & -q & 1 \end{pmatrix} \]  

(16)

where \( q, r = \mathcal{O}(m_2/m_3) \) and \( p = \mathcal{O}(\sqrt{m_1 m_2}/m_3) \). The third eigenvalue of the matrix is approximately equal to \( m_3 \), hence we find that the (13) element of the diagonalisation matrix \( U \) is of order \( pqr \sim \sqrt{m_1 m_2}/m_3 = 1.5 \times 10^{-3} \), so one cannot possibly achieve an “amplification” greater than about 600 relative to a small imaginary perturbation in the heavy basis, thus random perturbations in the up sector are ruled out as the source of the observed effect. Since \(|V_{13}| \) has now been measured to be greater than or equal to 0.003, (see e.g. [26]) the smallest imaginary part consistent with large \( \delta_{\text{KM}} \) is order \( 2 \times 10^{-3} \), thus one expects an imaginary part of order \( \text{Im} m_{13}^{(h)} \sim 2 \times 10^{-3} m_b \) to be sufficient. But, on inspecting Eq. (15), it is clear that random imaginary parts of size \( m_b \zeta_{ij}/3 \) in the democratic basis will give rise to \( \text{Im} m_{13}^{(h)} / m_b \sim \zeta_{ij}/6 \), since one is adding four uncorrelated imaginary parts and the sum is likely to be twice the average size of each one. The average size of imaginary parts, in the notation of section \ref{sec:6}, is \( \zeta/2 \), thus \( \text{Im} m_{13}^{(h)} \), and \( \text{Im} V_{13} \), are expected to be of order \( \zeta/12 \) and the smallest viable value of \( \zeta \) is 0.024. Comparing this with the bound on random phases for \( m_d \) to be stable, \( \zeta \leq 12 m_d / m_b \sim 1.5 \times 10^{-2} \), the scenario is marginally ruled out, but remains the most attractive way of generating CP violation with small phases in the down sector. If phases smaller than 0.024 in the democratic basis happened to add constructively to give larger \( \text{Im} m_{13}^{(h)} \), the scenario could be successful, but the probability of such constructive interference is small given random perturbations \( \zeta_{ij} \). The main disadvantage of the proposal of this section is that it appears highly contrived to generate an initial mass matrix in the democratic basis which reproduces Eq. (16) with reasonable accuracy.

In summary, the “weak coupling” proposal implemented by complex \( b, c \) or complex \( B, C, D \) in the two mass ansätze Eq. (5,6), can explain CP violation as a small perturbation consistent with the observed masses and mixings, and in the following we will take the “weak coupling” mass matrices as the main examples.

6 Soft terms

The result of applying the proposal of “additive CP violation” to the relevant soft breaking parameters, the gaugino masses \( M_i \), scalar bilinear \( B \)-term and trilinear

\[ \text{Im} m_{13}^{(h)} \]  

\footnote{In a basis where the large entries of \( V \) are real to good approximation.}
A-terms, is simply to set all potentially CP- and flavour-violating quantities to be of the form $\tilde{m}(1 + \epsilon_i + i\zeta_i)$, where $\tilde{m}$ is the magnitude predicted by one’s favourite mechanism of SUSY-breaking, $|\epsilon_i|$ are flavour-dependent parameters, and $|\zeta_i| \lesssim 10^{-3}$. In democratic models of flavour, this form may be determined by the transformation of the soft terms under $P_L \times P_R \times CP$. The soft scalar masses transform under either $P_L$ or $P_R$, hence in the limit of unbroken symmetry they take the form

$$m_{0ij}^2 = m_0^2(\delta_{ij} + \kappa \Delta_{ij}) = m_0^2(\delta_{ij} + \kappa),$$

where $i, j$ are flavour indices, i.e. both the unit and democratic matrices are allowed. However, we note that soft scalar masses generally turn out to be diagonal in the interaction basis, irrespective of the SUSY-breaking mechanism, thus for the time being we take $\kappa = 0$. Since flavour and CP symmetries are broken, we allow deviations from universality, which one might expect to be of order few $\times m_i/m_{i+1}$ or smaller. The flavour breaking parameter $m_d/m_s \simeq 0.05 \simeq \lambda^2$ is not particularly small, thus the SUSY flavour problem is unlikely to be solved without a more concrete model relating SUSY-breaking to the flavour symmetry, possibly analogously to alignment [27] for the case of continuous Abelian symmetry.

The $A$-terms are usually written as $L_{soft} \supset -A_{ij}^u Y_u^{ij} Q_i U^c_j H_u + (u \rightarrow d) + \cdots$ with flavour symmetry acting the same way on the scalar partners as on the fermions. Thus, in democratic models, the couplings $\tilde{A}_{ij}^u \equiv A_{ij}^u Y_u^{ij}$ (no sum) are expected to take the form $A_0^u(1 + \epsilon_{ij}^u + i\zeta_{ij}^u)/3$, where $A_0^u$ specifies the overall magnitude. The coefficients $\epsilon_{ij}^u, \zeta_{ij}^u$ may be different from those appearing in $M^u$, but are expected to be of the same order of magnitude as the coefficients $\epsilon, \zeta$ in the Yukawas. Then we may also write (with a similar expression in the down sector)

$$A_{ij}^u = A_0^u(1 + \tilde{\epsilon}_{ij}^u + i\tilde{\zeta}_{ij}^u)$$

(17)

with the coefficients $\tilde{\epsilon}, \tilde{\zeta}$ satisfying the same conditions as $\epsilon', \zeta'$ up to numerical factors of order 1. In an explicit model of flavour and SUSY-breaking there may be correlations between the parameters $\epsilon_{ij}, \zeta_{ij}$ in the Yukawa couplings and $\epsilon', \zeta'$ in the $A$-terms, but initially we consider a general structure of perturbations.

Gaugino masses and the $\mu$ and $B\mu$ terms are flavour singlets, hence only transform under CP: the small perturbation away from a CP invariant theory simply means that the (nonremovable) phases are order $10^{-3}$ or less, consistent with Eq. (1).

For example, some complex scalar v.e.v.’s may break flavour and CP, in which case there may be complex $F$-terms associated with the same multiplets, but both the scalar and $F$ components should break the symmetries by small amounts.
6.1 Supersymmetric CP: no longer a problem?

Predictions of EDM’s depend on the imaginary parts of the rephasing invariant combinations $\mathcal{M}_A \equiv \hat{A}_{(q,l)} M_i^* \mathcal{M}_B \equiv (B_\mu) \mu^* M_i^*$, $i = 1, 2, 3$, in the quark or lepton mass basis. In the presence of nonuniversal soft terms, the effects of CP- and flavour-violating interactions are most easily estimated by changing to the SCKM basis. The scalar (superpartner) mass matrices then receive off-diagonal or imaginary contributions which are treated perturbatively as mass insertions. For the EDM calculations one is only interested in the diagonal $A$-terms which are usually written $A_u$, etc., and enter into the observable phases as above. For a superpartner spectrum not too far above current experimental limits, and assuming no correlations between soft term phases, the tightest bounds apply to the “mu phase” $\text{Arg} \mathcal{M}_B$ which is bounded to be $< 10^{-2}$; the $A$-term phases are somewhat less restricted with bounds of order $< 10^{-1}$\[16\]This is to be compared with the amplification parameters $F_{CP}$ in the up and down sectors, of order $\gtrsim 500$ and $\sim 50$ respectively. We can easily allow all phases of soft terms to be of order $\delta_{KM}/F_{CP}^u < \text{few} \times 10^{-3}$, but phases of order $1/50 \sim \delta_{KM}/F_{CP}^d$ are potentially problematic. However, imaginary parts in the down sector need not be order $m_d/m_s$, and it is consistent with our realization of approximate CP by small imaginary parts, to set them also to $O(10^{-3})$. Thus as far as “flavour-diagonal” sources of CP violation are concerned, there is no SUSY CP problem. Assuming that phases of this size are present, EDM’s should be detected given a moderate improvement in experimental sensitivity.

6.2 Nonuniversal $A$-terms and EDM’s

If $A$-terms are nonuniversal, then there is a danger that large imaginary parts for the (11) entries $\hat{A}_{11}^{\text{SCKM}}$ in the super-CKM basis may be generated, even in the case of soft terms which are real in the interaction basis since these entries are no longer suppressed by the lightest quark or lepton mass (see \[22\]) but can get contributions proportional to $m_t, b$. We have

$$\hat{A}^{(d)} \rightarrow \hat{A}^{\text{SCKM}} = U_L^\dagger \hat{A}^{(d)} U_R.$$  \hfill (18)

Contributions to $\epsilon'$ and $\epsilon$ in the kaon system and $a_{J/ψK_S}$ would also be expected through off-diagonal $A$-terms in the SCKM basis.

If our proposal is implemented in the democratic basis, the left and right diagonalising matrices are close to the matrix $F$ which diagonalises $\Delta$. The phases of $U_L$ and $U_R$ need not be large, but they necessarily contain one or more entries with imaginary part $\ge 3 \times 10^{-3}$. For the most favoured “weak coupling”

\[16\]These order-of-magnitude bounds are taken to apply at the electroweak scale and are for $|A|$ comparable to soft scalar masses. As discussed earlier, bounds on the phases of $\mathcal{M}_A, \mathcal{M}_B$ only make sense if the absolute magnitudes are specified.
structures we find $|\text{Im } U^u_L|$ is order $5 \times 10^{-2}$ or less and $|\text{Im } U^d_L|$ is order $5 \times 10^{-3}$ or less.

To estimate the effects of nonuniversal soft terms we write the $A$-terms in the democratic basis as

$$\hat{A}^{u,d}_{ij} = A_0 \left( 1 + \epsilon^{u,d}(D_0^{u,d} + \tilde{D}_0^{u,d}) + i\zeta^{u,d}(D^{u,d} + \tilde{D}^{u,d}) \right) \frac{y_{tb}}{3},$$

where, on the expectation that the flavour structure in the soft terms will be parallel to that of the Yukawas, we take $\tilde{D}_0^{u,d}, \tilde{D}^{u,d}$ to be order 1 (where $\epsilon, \zeta, D$ are as defined in Eq. (4) and $\epsilon_u \sim 0.015, \epsilon_d \sim 0.1, \zeta_u,d \sim 10^{-3}$). It is convenient to first change to the heavy basis

$$\hat{A} \rightarrow \hat{A}^{(h)} = F \hat{A} F^\dagger$$

in which, in the absence of any correlated structure for the $\check{D}^{(0)ij}$, we expect the nonuniversal contributions also to be of order $A_0(\epsilon^{u,d} + i\zeta^{u,d})y_{tb}/3$. In this basis also, the imaginary parts of the diagonalisation matrices are order 5 $\times 10^{-2}$ $(5 \times 10^{-3})$ or smaller in the up (down) sector; we already know that the real parts are given by diagonalising Eq. (14). Then the mass insertions $\text{Im}(m^2_{11})_{LR}$ in the super-CKM basis arise from multiplying $\hat{A}$ by the appropriate Higgs v.e.v.:

$$\text{Im}(m^2_{11})_{LR} \simeq -m_3 A_0 \left( \zeta \text{Re } U_{iL}^{(h)} \frac{\check{D}^{(h)}_{ij}}{3} \text{Re } U_{jR}^{(h)} + \epsilon \left( \text{Im } U_{iL}^{(h)} \frac{\check{D}^{(h)}_{0ij}}{3} \text{Re } U_{jR}^{(h)} + \text{Re } U_{iL}^{(h)} \frac{\check{D}^{(h)}_{0ij}}{3} \text{Im } U_{jR}^{(h)} \right) \right)$$

in either the up or down sector, where the first term arises from complex $A$-terms in the theory basis, and the other terms come from nonuniversality of (real) $A$-terms.\footnote{Of course $\text{Im } A_0$ will also contribute, but this “universal” contribution was already considered in the previous section and does not dominate over the contributions considered here.}

Considering the first term only, one can take $i = j = 1$ to find a contribution giving rise to $\text{Im}(m^2_{11})_{LR}/\tilde{m}^2 \sim 10^{-3}A_0 m_t/3\tilde{m}^2 \sim 10^{-4}$, which is unlikely to be cancelled by any other term to good enough accuracy to respect the EDM bounds (of order $10^{-6}$–$10^{-7}$). Thus even without considering the “string CP” contributions from real nonuniversal $A$-terms, uncorrelated small imaginary parts for $A_{ij}$ are ruled out. This problem, arising from the (11) element in the heavy basis, is analogous to the up mass problem in the case of the Yukawas, which suggests that similar non-generic structures of imaginary parts in the $A$-terms may help to evade the EDM bounds while still allowing some nonuniversality. To take a simple example, if $A$-terms are of the form $\check{A} = A_L \cdot y + y \cdot A_R$, where $A_{L,R}$ are diagonal matrices,\footnote{\textsuperscript{23}}, the (11) elements in the SCKM basis are still proportional to $m_u$ or $m_d$, thus the EDM’s are as small as for universal $A$-terms (given small imaginary parts of $A_{L,R}$).
6.3 Nonuniversal benchmarks with additive CP violation

We now investigate whether some other restricted structures of nonuniversal A-terms, correlated to the Yukawa matrices, could also lead to suppressed EDM’s. For each of the quark mass matrices Eq. (5,6), we find the consequences if the A-terms have the same flavour structure but with different coefficients, which we will call \( \tilde{b}, \tilde{c} \), or \( \tilde{B}, \tilde{C}, \tilde{D} \), respectively. This form has not been derived from an underlying theory, but is imposed as a reasonable starting point or benchmark, based on Eq. (1) and on the expectation that the same operators generating quark flavour will also produce nonuniversality. If this ansatz turns out to be ruled out, even this restricted form of nonuniversality cannot be allowed; if it is successful, it motivates a search for theories in which such structures appear, and may suggest some characteristic signals of new physics. For example, if the quark mass matrix is as in Eq. (5) we take

\[
A_{u,d} = \frac{A_0 m_3}{3} \begin{pmatrix}
1 & 1 & 1 + \tilde{b} \\
1 & 1 & 1 + \tilde{b} - \tilde{c} \\
1 + \tilde{b} & 1 + \tilde{b} - \tilde{c} & 1 + \tilde{b} - \tilde{c}
\end{pmatrix}
\] (21)

with real parts of \( \tilde{b} \) and \( \tilde{c} \) randomly chosen on \( 9m_2/2m_3(-1,1) \) and \( 3\sqrt{3}m_1/m_3(-1,1) \) respectively. Note that while we used the running quark masses defined at a scale of 1 GeV in the analysis of section 3 (e.g. we used \( m_t(1 \text{ GeV}) \approx 400 \text{ GeV} \)), the appropriate RG scale is now the electroweak scale \( M_Z \), thus the prefactor \( m_3(M_Z) \) will be smaller, approximately by a factor 2. For the quark matrices of Eq. (6) one follows an exactly analogous procedure. Since these forms are merely a starting point for evaluating nonuniversal contributions, we do not consider RG running from high energies.

For each quark mass matrix ansatz we consider two scenarios: one “conservative”, in which only \( c_{u,d} \) or \( D_{u,d} \) are complex, and one “less conservative” in which \( b_{u,d} \) and \( C_{u,d} \) are complex. Imaginary parts are at most of order \( \text{Im} \), respectively.

Taking \( \tilde{m} = 400 \text{ GeV} \), we obtain for the mass matrices Eq. (5), in the “conservative” case, that the typical values of \( \text{Im} (\delta_{11})_{LR} \) and \( \text{Im} (\delta_{11})_{LR} \) are order \( 5 \times 10^{-6}, 5 \times 10^{-7} \) respectively. Thus the contribution of \( \tilde{B} \) to the EDM’s is marginally too large; however, one requires only mild cancellation (at the 10% level) to respect the bound. Accidental cancellations might occur between different contributions to \( \tilde{B} \): for example, we find that the first term involving \( \tilde{D} \) in Eq. (20), which arises from \( \text{Im} \tilde{c}^{u} \), is of the same order as the cross-term \( \text{Im} U_{Lij}^{u} \text{Re} \tilde{c}^{u} \), and may be of opposite sign. In the “less conservative” case, results are very similar, indicating that the \( \tilde{b} \) and \( \tilde{D} \) parameters have little influence on the EDM’s. Other imaginary parts of mass insertion parameters \( (\delta_{ij})_{LR} \) are of order \( 10^{-4} \) or smaller in the up sector and \( 10^{-6} \) in the down, hence contributions
to flavour-changing CP-odd observables appear to be negligible. Real parts of off-diagonal \((\delta_{ij})_{LR}\) are also below experimental FCNC bounds \([24]\), thus our approach is self-consistent, in that the form of nonuniversality that we have chosen is not ruled out by such bounds before even considering EDM’s.

For the mass matrices Eq. (\ref{eq:mass}), in the “conservative” case, we find that \(\text{Im} \,(\delta_{11}^u)_{LR}\) vanishes (or is order \(10^{-8}\) if \(A_0\) has a small overall phase) while \(\text{Im} \,(\delta_{11}^d)_{LR}\) is order \(10^{-6}\) or smaller; in the “less conservative” case, both \(\text{Im} \,(\delta_{11}^u)_{LR}\) and \(\text{Im} \,(\delta_{11}^d)_{LR}\) are order \(10^{-6}\) or smaller. Thus in this ansatz, which has slightly smaller coefficients of flavour symmetry-breaking operators, it appears easier to satisfy the EDM bounds with minimal fine-tuning or cancellations, consistent with a certain degree of nonuniversality.

7 Summary

If the world is approximately supersymmetric, it may be possible to test theories of the origin of CP violation and flavour, as well as the mechanism of SUSY-breaking, in the near future. However, first one must explain the absence, at the level of current experimental sensitivity, of EDM’s and flavour-changing processes resulting from superpartner loops. If the flavour problem is solved by universality or heavy scalars, and the CP problem by a mechanism giving automatically real soft terms, then we learn little about the origin of CP violation and flavour, which may lie at an arbitrarily high scale; conversely, if general soft terms are allowed then the parameter space is too large and the experimental constraints too complex for meaningful investigation.

We propose a guiding principle, additive CP violation, that provides a realisation of CP violation as a small perturbation consistent with experimental data, allowing a large CKM phase and nonzero phases of soft terms. In essence CP is broken by adding small imaginary parts to real couplings (rather than, for instance, by large phase rotations of real couplings). Applying this principle to universal soft terms, EDM’s are predicted just below the experimental bounds; for nonuniversal \(A\)-terms we require particular structures of deviations from universality to satisfy the bounds. Such structures are no more fine-tuned than the small perturbations to Yukawa couplings, which we know are required to generate quark masses and mixings, so one might reasonably expect whatever mechanism leads to Yukawa structure to also generate correlated patterns of soft terms.

An essential part of the proposal is the amplification which automatically generates the observed size of \(\delta_{KM}\) and \(J\) from small phases in the democratic basis, or equivalently small imaginary parts of Yukawa couplings in any flavour basis: one may have \(\text{Im} \,m_{ij} \leq 3 \times 10^{-4}m_t\) in the up sector and still obtain \(\delta_{KM} \sim 0.5\). Amplification of \(\text{Im} \,m_{u,d}\) occurs in the diagonalisation of quark mass matrices, and small imaginary parts in the \(U_{u,d}\) matrices can easily produce large \(\delta_{KM}\). There are several ways to implement CP violation as a small imaginary
perturbation while ensuring correct masses and mixings, the most attractive being to link the perturbation to a small parameter breaking flavour symmetry.

The Yukawas and soft terms that we use are not derived from a fundamental theory, nevertheless they can be seen as a consequence of flavour and CP symmetries broken by small parameters. The motivation for our phenomenological investigation is to draw attention to the possibility that complex and even non-universal soft terms are allowed, without requiring unnatural fine-tuning compared to the Yukawa couplings, and to provide guidelines for future model-building efforts which may give more definite predictions for new physics signals. Independently of the model, the parameter breaking CP cannot be smaller than $3 \times 10^{-4}$, thus nonzero EDM’s should be within one or two orders of magnitude of current bounds and the scenario may be testable within a few years.

If signals are found near the current level of sensitivity, the interpretation is either that the CP-violating parameter is somewhat larger, or that there is a mild degree of nonuniversality which enhances the $A$-term contributions.

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