Modelling ultra-fine structure in dark matter halos

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ABSTRACT

Various laboratory-based experiments are underway attempting to detect dark matter directly. The event rates and detailed signals expected in these experiments depend on the dark matter phase space distribution on sub-milliparsec scales. These scales are many orders of magnitude smaller than those that can be resolved by conventional N-body simulations, so one cannot hope to use such tools to investigate the effect of mergers in the history of the Milky Way on the detailed phase-space structure probed by the current experiments. In this paper we present an alternative approach to investigating the results of such mergers, by studying a simplified model for a merger of a sub-halo with a larger parent halo. With an appropriate choice of parent halo potential, the evolution of material from the sub-halo can be expressed analytically in action-angle variables, so it is possible to obtain its entire orbit history very rapidly without numerical integration. Furthermore by evolving backwards in time, we can obtain arbitrarily-high spatial resolution for the current velocity distribution at a fixed point. Although this model cannot provide a detailed quantitative comparison with the Milky Way, its properties are sufficiently generic that it offers qualitative insight into the expected structure arising from a merger at a resolution that cannot be approached with full numerical simulations. Preliminary results indicate that the velocity-space distribution of dark matter particles remains characterized by discrete and well-defined peaks over an extended period of time, both for single and multi-merging systems, in contrast to the simple smooth velocity distributions sometimes assumed in predicting laboratory experiment detection rates. In principle, this structure contains a wealth of information about the formation history of the Milky Way’s dark halo.

Key words: methods: numerical - Galaxy: evolution - Galaxy: halo - Galaxy: dynamics and kinematics - Galaxy: solar neighbourhood - dark matter

1 INTRODUCTION

Dark matter (hereafter DM) appears to be the dominant mass component of galaxies and large-scale structures in the Universe. The first evidence came in the 1930s (Zwicky 1933, Smith 1936), but it was only in the 1970s that observations of the rotation curves of galaxies demonstrated that DM dominates the masses of galaxies (Rubin & Ford 1970, Rubin, Thonnard & Ford Jr 1980). These observations showed that many rotation curves are approximately flat, or even rising, in the outer region of galaxies, where there is little luminous matter and so a Keplerian decline is expected. Subsequently, work on the hierarchical structure formation paradigm showed that non-baryonic material known as “cold dark matter” (CDM) is required to match the observed large-scale structure of the Universe (Peebles 1982). The term “cold” derives from the fact that this material was non-relativistic at the epoch of matter-radiation equality. The density of this CDM has subsequently been indirectly measured by various experiments such as 2dFGRS (Percival et al., 2001), WMAP (Dunkley et al. 2008) and the Sloan Digital Sky Survey (Tegmark et al. 2006).

Particle physics provides us with various well-motivated candidates for the CDM, including weakly interacting massive particles (WIMPs). WIMPs can potentially be directly detected in the laboratory via their elastic scattering on target nuclei, and numerous experiments are currently underway to try to detect this phenomenon (e.g. Angle et al. 2007, Ahmed et al. 2008). The signals expected in these experiments, the number of recoil events per unit energy (and in some case its temporal and angular dependence), depend on the WIMP velocity distribution in the solar neighbour-
hood. A single stream of dark matter particles produces a step in the energy spectrum, detectable by a detector. The energy at which the step occurs is determined by the speed of the particles composing the stream (in the rest frame of the detector), while the height and position of the step vary annually, due to the Earth’s orbit. Multiple streams would lead to a more complicated picture, and a superposition of enough such streams would ultimately be indistinguishable from a smooth distribution, depending on the detector resolution. The question that we seek to address here is what form one might expect for this distribution in reality.

Predictions of the expected signals are often based on simplified models, which assume, for example, that the WIMP velocity distribution is Maxwellian (Freese, Frieman & Gould 1988) or a multivariate Gaussian (Evans, Carollo & de Zeeuw 2000, Helmi, White & Springel 2002). These models rely on the assumption that the Milky Way halo has reached a steady state so that the ultra-local DM phase-space distribution is smooth. However since structures form hierarchically, and the age of the Universe is not large compared with relevant dynamical timescales (such as the crossing time), this assumption is somewhat questionable.

Numerous N-body simulations have been performed studying the hierarchical formation and evolution of DM halos. Such simulations find that DM halos contain ubiquitous substructure, in particular in their outer regions (Moore et al. 1999, Klypin et al. 1999). However, the salient question here is whether or not the DM distribution is smooth on the scales probed by direct detection experiments. Unfortunately, the resolution of even the best N-body simulations is many orders of magnitude larger than the relevant scales. The Sun’s circular velocity around the centre of the Galaxy is \( v_0 \approx 200 \text{ km/sec} \), so that over a course of a year a terrestrial DM detector travels a distance

\[
r_{\text{det}} \approx v_0 \tau \exp \sim (200 \text{ km/sec})(1 \text{ yr}) \sim 0.1 \text{ mpc},
\]

whereas current N-body simulations cannot resolve scales smaller than \( \sim 100 \text{ pc} \) (e.g. Diemand et al. 2007).

This represents an insurmountable problem for the conventional simulation techniques, indicating that a completely different, specialized approach is necessary to describe in detail the ultra-fine DM distribution probed by direct detection experiments. A first attempt at such a specialized simulation was carried out by Stiff and Widrow (2003; hereafter SW). Their method used a reverse simulation process to calculate the DM speed distribution, \( f(v) \), at a single spatial point of the phase space, representing a detector. More specifically they ran a simulation of the formation of a DM halo and at the end of the simulation put down a uniform grid (in velocity space) of mass-less test particles at the point of interest. They then evolved the test and simulation particles back to the initial time, found where the phase-space sheet of the test particles intersects the initial DM phase-space distribution, and hence calculated the density of the test particles at the final detector position. In this way, they found that the DM distribution in the solar neighbourhood is characterized by a number of discrete peaks. This numerical approach allowed them to use initial conditions which reproduce a realistic hierarchical-formation model for the Milky Way. However, a drawback of this technique was that it proved numerically unstable, and, in order to stabilize the reverse integration, SW were obliged to introduce a softening length of some 20 kpc into the gravitational force law that they applied. This softening is worryingly large as it significantly exceeds the solar radius in the Milky Way, \( R_0 \approx 8.5 \text{ kpc} \), so might be expected to affect the inferred DM phase space distribution impinging on a terrestrial detector.

More recently Vogelsberger et al. (2008) have formulated a technique for calculating the evolution of the fine-grained DM density in both static potentials and N-body simulations. They argue that the small-scale DM distribution that a terrestrial experiment would observe can be described by a multivariate Gaussian, in apparent contradiction to the SW results, perhaps indicating that the SW analysis had been compromised by the modification that they had to make to the gravitational force.

In this paper we develop a complementary approach to the analysis of the ultra-fine DM distribution. Following SW, we calculate the DM distribution in the solar neighbourhood via a backward evolution method, but using a simplified model for the potential that allows the system to be expressed in action-angle (hereafter AA) variables. Although this simplified potential provides a less realistic representation of the Milky Way, its qualitative properties are similar, and it has the great benefit of being analytically soluble. Thus, the gravitational force does not have to be artificially softened (allowing one to test whether this effect did compromise the SW results), and one can very rapidly explore parameter space without the computational overhead of numerical integration.

The paper is organized as follows. In Section 2 we present the simplified model that we use to describe the interaction between a galaxy like the Milky Way and a merging sub-halo. Section 3 contains our initial results, and we conclude in Section 4 with a discussion.

## 2 THE MODEL

To study the evolution of the ultra-fine DM distribution contributed by a merging halo in a massive galaxy like the Milky Way, we adopt the isochrone potential,

\[
\Phi(r) = \frac{GM}{b + \sqrt{b^2 + r^2}},
\]

for the potential of the massive system. Although not intended as a realistic model for a complex system like the Milky Way, it can be tuned through its mass \( M \) and characteristic lengthscale \( b \) to approximate a range of systems. More importantly, orbits in such a potential can be calculated analytically by expressing the dynamics in AA variables, greatly simplifying the time evolution calculations (McGill & Binney 1990; Gerhard & Saha 1991).

Specifically, the Hamiltonian can then be expressed in terms of the actions, \( J_i \), as

\[
H(J) = -\frac{2}{(2J_r + L + \sqrt{4 + L^2})^2},
\]

where

\[
L \equiv |J_1 + 1_z|
\]

is the magnitude of the angular momentum vector. The actions remain constant with time, which we can express in spherical coordinates as
\[ J_i(t) = J_i(t_0) \]

where \( i = r, l, \varphi \). The corresponding angle variables, \( \theta_i \), can be determined from the solution to Hamilton’s equation,

\[ \dot{\theta}_i = \frac{\partial H}{\partial J_i} \equiv \Omega_i(J), \]

where the \( \Omega_i \) are the corresponding angular frequencies. These terms therefore evolve linearly with time, allowing the solution at any epoch (either forward or backward in time) to be expressed trivially in terms of the initial values,

\[ \theta_i(t) = \theta_i(t_0) - \Omega_i(t - t_0). \]

More details, including analytic expressions relating the AA variables \((J, \theta)\) and Cartesian coordinates \((x, v)\) can be found in McGill & Binney (1990) and Gerhard & Saha (1991).

In the present context, we are interested in the velocity distribution of particles passing through a fixed position (representing a DM detector) and its evolution with time. At this location, we can pick any velocity and analytically evolve these phase space coordinates backwards in time to determine their initial phase space location. We can then calculate the amount of material in a merging satellite that originated from these initial phase-space coordinates, and hence will end up at the selected velocity in a terrestrial detector today. By stepping through all such velocities, we can map out the present-day phase-space structure of this disrupted merging satellite on an arbitrarily fine spatial scale.

To complete this model, we need to specify the initial DM phase-space distribution of the merging halo. The simplest representation that provides enough freedom to explore the dependence on the properties of this merging halo is provided by a bivariate Gaussian,

\[ f(r, v) \propto e^{-[(r-r_0)^2/2\sigma_r^2]} e^{-[(v-v_0)^2/2\sigma_v^2]}, \]

where \( \sigma_r \) models the initial spatial extent of the merging halo while \( \sigma_v \) defines its velocity dispersion. We also impose the physically-motivated limit \( v < v_{\text{esc}} \) where \( v_{\text{esc}} \) is the escape velocity of the parent galaxy,

\[ v_{\text{esc}} = \sqrt{\frac{2k}{b + \sqrt{b^2 + r^2}}}. \]

In summary the principal steps in constructing this model for the present-day fine-scale phase-space distribution due to a merging satellite halo are:

(i) Select the spatial and velocity scale for the satellite and its initial position and velocity, and the time in the past, \( t_0 \), at which it was at that location.

(ii) Choose the present-day phase space coordinates of the detector location and a particular velocity \( v \).

(iii) Transform these phase-space co-ordinates into AA variables.

(iv) Analytically evolve these AA coordinates back to \( t_0 \).

(v) Transform them back into Cartesian co-ordinates.

(vi) Evaluate the initial phase space density due to the merging satellite for this phase-space location via equation (8), which is then also the phase space density in the present-day detector at velocity \( v \).

(vii) Repeat for a grid of velocities at this location to map out the full velocity distribution within the detector.

Figure 1. The distribution function of the component of the speed in the direction of the merger, denoted as \( x \), at the fixed position \( r = (1, 0, 0) \) at time \( t = 90 \) (when the satellite is on its second orbit, corresponding to \( \sim 1.20 \text{ Gyr} \) for a Milky-Way like galaxy).

Figure 2. Configuration as for Fig. 1 but with the evolution of the satellite calculated by evolving the co-ordinates of \( 10^5 \) particles forwards in time.

Clearly, this backwards-in-time approach allows us to pinpoint efficiently those DM particles from the initial merging satellite that are to be found passing through an arbitrarily-small detector today, and by choosing the grid of velocities appropriately we can map out the velocity structure with any resolution that we desire.

3 RESULTS

In this section we present some illustrative results obtained using the technique described in Sec. 2. This analysis is not exhaustive, nor is it intended to be used to describe quantitatively the merger history of the Milky Way. However, the control that we can impose on this simplified model, and the speed at which it can be computed at arbitrarily high resolution, means that it can be used to obtain unique qualitative insights into the likely signature of a halo merger event in a terrestrial DM detector.

As described above, this model simulates the fall of a
DM halo into a larger gravitational potential. The satellite is injected into the host galaxy and we calculate how the velocity distribution of particles passing through a fixed spatial position varies with time. The particles composing the satellite initially have similar (but not equal) energies. As a consequence, they follow slightly different orbits, have different orbital periods and with time they spread out in physical space.

As a concrete example we consider a satellite initially at position \( r_0 = (-5, 0, 0) \), with velocity \( v_0 = (0, 0.05, 0.1) \) and initial phase-space distribution function given by eq. (9). The spatial and velocity dispersions are respectively \( h = 0.5 \) and \( \sigma_v = 0.1 \) (scaled to Milky Way units corresponding to \( \sigma_v \sim 60 \text{ km s}^{-1} \)). We use the reverse AA evolution technique to calculate the velocity distribution at a position \( r = (1, 0, 0) \) at a range of times, which we can readily scale into physical “Milky Way” units corresponding to the detector’s location at \( R_0 \sim 8.5 \text{ kpc} \). This configuration has been chosen to describe a cold and concentrated satellite (as expected in reality), merging into the parent galaxy along \( x \), the axis defined by the direction of the merger, from a large distance (corresponding to \( r_0 \sim 40 \text{ Mpc} \)).

In Figure 1 we plot the distribution of the \( x \)-component of the velocity at the solar radius at time \( t = 90 \). For a Milky-Way-like parent galaxy, this corresponds to a physical time \( t = 1.2 \text{ Gyr} \). At this stage of the merger the satellite is on its second orbit. The satellite is still fairly coherent, but it is beginning to be disrupted by tidal forces. The large peak at positive speed corresponds to particles which are on their second orbit, while the smaller peak at negative speed is part of a tidal tail which has not yet finished its first orbit.

To illustrate the power of the reverse AA evolution technique, we also carried out a simulation of the same configuration using the more conventional approach of evolving \( 10^5 \) DM particles initially in the satellite halo forward in time. Figure 2 shows a histogram of the speed in the \( x \)-direction of the particles at the same fixed point at the same time as for the backwards evolution calculation in Fig. 1. Clearly, the same peaks are reproduced, but since the vast majority of particles in this forward evolution ends up nowhere near the Earth, the sampling of these peaks is extremely poor. This problem becomes even more severe at later times as the particles populate a larger volume of phase-space. However, the forward evolution does provide complementary information by giving the big picture of how the satellite is being disrupted. Figure 3 shows the \( (x, v_x) \) phase space co-ordinates of \( 10^6 \) particles evolved forwards, showing how they have already spread out in phase space, with the peaks in the terrestrial detector arising from particles that have completed one or two orbits. The regions from which those particles come are labelled in the plot with circles.

A further indication of the flexibility of the backwards approach is provided by Figure 4. Each panel depicts the simulated phase-space distribution in a terrestrial detector after \( t \sim 14 \text{ Gyr} \) for a different set of initial conditions that vary the orbit of the satellite and its internal velocity dispersion. Because of the analytic nature of the orbit integration, this calculation was as simple as the shorter-evolution illustration, with no loss of precision in the results. As this figure illustrates, the velocity distribution does depend on the merging halo parameters: the higher velocity dispersion sub-halos, for example, wrap around the Milky Way more quickly, creating more peaks in the distribution. Nonetheless, the presence of persistent discrete fine phase space structure is a generic property of all the mergers.

This simplified model is not intended to predict quantitatively the experimental signal that terrestrial DM detectors should see, but we can take the qualitative analysis one step closer to the laboratory by considering the physical quantities that are most relevant for such experiments, particularly those with directional sensitivity. For such detectors, a useful diagnostic is provided by the speed of the DM particles as a function of the angle at which they impinge on the detector (measured relative to the direction of Solar motion in the Milky Way). Figure 5 shows this plot for the simulation described above at time \( t = 14 \text{ Gyr} \). Although the DM particles are quite well spread through the parameter space, it is apparent that even at this late time there is significant structure apparent in the plot, which would have an impact on the detectability of this particular merging sub-halo, and might even ultimately be used to reconstruct its origins.

In reality, of course, the Galactic halo is made up of multiple merging events, so the velocity distribution should...
be even more complex than that shown in Fig. 5. Since to a good approximation these multiple mergers do not interact with each other, we can model such a sequence by simply adding results like those obtained in Fig. 5. As Fig. 6 confirms, such superpositions further complicate the structure, but certainly do not produce a simple smooth distribution. A detector with even relatively crude angular resolution would be able to pick out the strong horizontal structures in this diagnostic figure, which arise from the constraint imposed by the cut-off at escape speed for DM particles from each individual merger event, suggesting that much information might be gleaned about the merger history of the Milky Way halo from future terrestrial DM detectors.

4 CONCLUSIONS
As a simple model for the signal that might be detected in a terrestrial DM detector due to a single sub-halo merger, we have studied the distribution of particles resulting from
the merger of a satellite into a parent galaxy described by an isochrone potential. Although not intended as a quantitative match to the Milky Way, this form of potential has the great benefit that the dynamics of the merger can be calculated remarkably simply, and the satellite can be evolved forwards or backwards in time analytically using action-angle variables. This simplicity allows us to quickly and accurately calculate the velocity distribution at any time at arbitrarily high spatial resolution, which is vital if we want to understand the sub-mpc-scale structure of the Milky Way’s halo probed by terrestrial DM detectors. We find that, even at late times (up to 14 Gyr) when the particles have spread out through phase-space, the velocity-space distribution function is characterized by discrete and distributed peaks. In agreement with Stiff & Widrow (2003), we find that the parameters that dictate the detectability of DM, particularly for experiments with directional sensitivity, contain persistent significant structure imprinted by the original merging sub-halo, which could well impact on the detectability of DM, as well as shedding light on the properties of the progenitor sub-halo.

Although the situation becomes more complex when one considers a halo built up from multiple mergers over the lifetime of the Galaxy, the evidence suggests that significant amounts of fine-grained features persist. Although such structures may have an impact on detectors’ ability to make an unequivocal detection of the DM halo, they also raise the fascinating possibility that it may be possible to use them to unravel the complete merger history of the Milky Way.

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