QCD at work: from lepton to hadron colliders and back

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Abstract The astounding Physics results obtained with high-energy colliders in the last two decades owe much to an impressive progress in the understanding of the dynamics of strong interactions. I give here a personal overview of how the advance in QCD triggered by the Physics of hadronic final states at LEP has been exploited for New Physics searches at the LHC. Conversely, the need for precision calculations for LHC experiments has stimulated a huge progress in the understanding of the all-order structure of gauge theories. These results raise high expectations on the status of QCD at the start of a linear collider.

1 Introduction

With the start of the LHC Particle Physics has entered a new era. Not only will we probably have a final answer on the mechanism of spontaneous breaking of electroweak symmetry, but we could also observe novel phenomena like dark-matter or black-hole production. The LHC, being a hadron collider, can access a wide range of scales for Physics beyond the (known) Standard Model, from the LEP boundary of about 100 GeV up to the TeV scale. The price we have to pay is that events appear contaminated by the presence of a large number of hadrons. Among those, only a small fraction is related to the short-distance processes we are interested in, the rest comes either from secondary collisions of the remnants of the two broken protons (underlying event), or even from further soft collisions occurring within the same proton bunch, the so-called pile up. This is in sharp contrast with the situation at LEP, where only a few tens of hadrons were produced. A further important difference between $e^+e^-$ and hadronic colliders is the possibility of detection of individual hadrons. In the $e^+e^-$ case, the tracker has basically full solid-angle acceptance, so that information on charged hadrons is available everywhere in rapidity. At hadron colliders the tracker extends only inside a central region spanning a few units in rapidity, whilst outside the only available information comes from the calorimetric towers. Therefore, although both LHC experiments are able to combine detector information into objects like “topo-clusters” [1] or “particle flows” [2] which should be quite close to individual particles, at the moment the preferred objects for studies of hadronic final

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states are just jets. This experimental issue has also theoretical implications, as we will describe in the following.

QCD at LEP is a “theory of hadrons”, whose dynamics can be investigated from high momentum scales where quarks and gluons are produced, to the low momentum scales where the hadronisation mechanism is effective. This is done through final-state observables, like event-shape distributions or jet rates, which combine in various ways hadron momenta in numbers that provide an insight on the jet structure or the geometry of each event. The typical situation at LEP is that the hard scale of the process, the centre-of-mass energy $Q$, is much larger than the hadronisation scale $Q_0$, which is of the order of the mass of the proton. It is therefore possible to find values $Q_V$ for final-state variables such that $Q_0 \ll Q_V$, so that the corresponding distributions can be reliably computed in perturbative QCD. Furthermore, due to the fact that most $e^+e^-$ observables are global, i.e. sensitive to emissions everywhere in the phase space, and that in $e^+e^-$ annihilation one can safely rely on QCD coherence, it is always possible to approximate multiple soft-collinear parton matrix elements with a probabilistic angular ordered branching [3]. This feature is the key of the success in the description of QCD final states in $e^+e^-$ annihilation with both Monte Carlo event generators and analytical calculations (for a review see [4], and references therein). Just to recall the impressive accuracy reached by QCD calculations for hadronic final states in $e^+e^-$ annihilation, a fully differential code for $e^+e^-$ into three jets is available to order $\alpha_s^3$ (next-to-next-to leading order, NNLO) [5, 6], all-order resummation of large logarithms has been computed for selected event shapes (thrust, heavy-jet mass) at next-to-next-to leading logarithmic accuracy (NNLL) [7, 8, 9], and there exist also QCD inspired analytical models for (leading) hadronisation corrections [10, 11, 12, 13].

At the LHC QCD has to be the “theory of jets”, since resolving single hadrons is in general a difficult task. Although jet cross sections are generally within the domain of perturbative QCD, there are a number of features that make an all-order perturbative description of jet observables problematic. First of all, jets themselves are non-inclusive objects, there is no closed mathematical expression that relates the momentum of a jet to the momenta of final-state hadrons, not even approximately as happens for instance for the thrust in $e^+e^-$ in the two-jet limit. This makes it impossible to write jet cross sections in terms of operator matrix elements, as is done for many inclusive observables [14, 15, 16, 17], and sometimes also for event-shape distributions [7, 8, 18]. Another traditional worry expressed by all-order QCD practitioners is that jet cross sections are generally non-global observables [19, 20, 21]. Non-globalness, together with the fact that the presence of two hadrons in the initial state might spoil collinear factorisation [22, 23], cast serious doubts on the applicability of coherent branching to jet observables in hadronic collisions. At hadron colliders, especially at the LHC, there is also a major concern about the separation of scales between perturbative and non-perturbative Physics. Poorly understood phenomena like underlying event of pile-up can add several tens of

\footnote{For the special case of the thrust even NNNLL accuracy is claimed in Refs. [7, 10].}
GeV’s of extra transverse momentum to QCD jets, causing huge distortions in many commonly studied hadronic final-state observables (e.g. event-shape distributions) [24].

Given this situation, it might seem that the knowledge of QCD we have inherited from LEP is of little use for LHC Physics. While this consideration might be true for precision studies (e.g. measurements of $\alpha_s$), the insight on hadron dynamics we have at present can be largely exploited for LHC phenomenology. This will be the subject of the first part of my contribution (Section 2). In the second part (Section 3) I will shortly review the progress in QCD triggered by the quest for precision calculations in a multi-jet environment such as the LHC, and how these results have already influenced $e^+e^-$ phenomenology. I will conclude with my personal view on the challenges that we will have to face at the start of the Linear Collider (LC), and on what theoretical tools should be needed to tackle them.

## 2 LEP wisdom for LHC Physics

Before discussing how QCD results from LEP can be exploited at the LHC, it is worth asking ourselves whether at the LHC precision Physics has to be limited only to inclusive quantities like $Z$ or $W$ differential cross sections, or can also involve direct measurements of the hadronic energy-momentum flow. As already stated in the introduction, for precision purposes it is very difficult to exploit final-state observables, like event shapes, that are defined in terms of individual hadrons, since they get huge contributions from poorly understood phenomena like underlying event or pile-up. However, jets constructed with modern algorithms are less sensitive to these effects, and their cross sections can be computed in perturbative QCD and directly compared to data. In particular, for well separated jets, fixed order perturbation theory is enough to obtain a reliable description of data, allowing for measurements of the strong coupling constant (see for instance [25]). Furthermore, if the rapidity range in which jets are observed covers the full detector acceptance, observables like jet rates become global, and hence can be studied in the whole range of values of the jet resolution parameters with all-order resummation techniques [24]. Resummed jet rates are known from LEP to have small theoretical uncertainties, and therefore seem the best candidates for precision QCD studies. A close relative of jet rates is the jet-veto efficiency, for which one can obtain accurate QCD predictions, which can in turn be exploited in several New Physics contexts, for instance in Higgs or dark-matter searches.

Most observables at the LHC however are not suitable for precision studies, but, like jet masses, are relevant for New Physics searches. In this case LEP wisdom can be exploited in various ways, for instance one can try to answer the following questions:

- Can one reduce contamination from non-perturbative effects in jets?
- Is there an optimal procedure to filter jets originating from boosted object decays?
• Can we distinguish jets originating from colour singlet decays from pure QCD jets?

In the following I give examples on how the theoretical methods developed at LEP have been already exploited to gain some analytical insight on these issues.

2.1 Non-perturbative effects in jets

One of the major theoretical achievements inherited from LEP is analytical models for hadronisation corrections. Within these approaches, leading hadronisation corrections to event-shape distributions and means are given as the product of a perturbatively calculable coefficient and a single universal non-perturbative parameter $\alpha_0$, which is extracted from experimental data [13]. The universality of $\alpha_0$ has been thoroughly tested at LEP, and is found to hold within 20% [4]. Since analytical hadronisation models rely basically on the universality of QCD soft radiation, they could be in principle equally applied to hadronisation corrections in hadronic collisions. This is what is done for instance in Ref. [26], where one finds the calculation of the transverse momentum loss of the leading jet due to hadronisation $\delta p_{t,\text{had}}$, which appears in a variety of jet studies at the LHC. This quantity is indeed related to the universal parameter $\alpha_0$, with a coefficient that scales as $1/R$, where $R$ is the jet radius. Furthermore, since $\delta p_{t,\text{UE}}$, the change in jet $p_t$ due to a hadron background approximately uniform in rapidity and azimuth (like underlying event or pile-up), is found to scale as $R^2$, one can compute the radius that minimises the two effects, which should be then used for precision studies, e.g. inclusive jet transverse momentum spectra. For New Physics searches however, where one wishes for instance to identify a peak in a jet-mass distribution, it is also important to minimise the amount of perturbative QCD radiation that escapes the jet $\delta p_{t,\text{pert}}$, which is found to scale as $\ln(1/R)$. The combined effect of the three sources of $p_t$ loss is illustrated in Fig. 1 (left), from which it is evident that there exists an optimal radius for which the total $\langle \delta p_t \rangle^2$ (computed neglecting interference among its different contributions) is minimised. Since both $\delta p_{t,\text{pert}}$ and $\delta p_{t,\text{had}}$ are triggered by QCD radiation, they depend on the total colour charge of the parton initiating the jet, whilst $\delta p_{t,\text{UE}}$ depends mainly on the centre of mass energy of the collider. Therefore we expect the optimal radius to change according to whether we consider quark or gluon jets, and whether we are at Tevatron or at LHC energies. This is confirmed by actual studies performed with parton shower event generators, and the resulting optimal radius as a function of the jet $p_t$ is shown in Fig. 1 (right).

2.2 Non-global observables and jet filtering

A relevant topic for New Physics searches at the LHC is the exploitation of boosted kinematics and jet substructure to detect high-$p_t$ heavy objects whose decay products fall inside the same jet (see [27] for a recent update). The basic search strategy consists in clustering each event...
into “fat” jets with a large radius, and then selecting a candidate jet which should contain the decay products of the heavy particle one is looking for. The best known example is a boosted Higgs decaying into a $b\bar{b}$ pair, where the candidate Higgs jet must contain at least two separated $b$-tagged subjets [28]. Once the candidate jet has been selected, the problem is how to clean it so that it contains as much as the Higgs decay products plus QCD radiation originated from them, and it is least contaminated by initial-state radiation or underlying event. This is the aim of the filtering procedure, which consists in reclustering the fat jet with a smaller radius $R_{\text{filt}}$ and reconstructing the candidate Higgs using only the hardest $n_{\text{filt}}$ subjets. The determination of the best $R_{\text{filt}}$ and $n_{\text{filt}}$ relies on the calculation of $\Sigma(\delta M)$, the fraction of events such that the difference between the Higgs mass and the jet mass is less than a given $\delta M$. Then one looks for the value of $\delta M$ for which $\Sigma(\delta M) = f$, with $f$ a given fraction of events, for instance 68%; clearly, the smaller $\delta M$, the better the mass resolution. The quantity $\Sigma(\delta M)$ is basically an event-shape fraction and, due to the fact that the Higgs is a colour singlet, can be computed with the theoretical tools developed for $e^+e^-$ (non-global) event shapes. It is then possible to determine analytically the values of $R_{\text{filt}}$ and $n_{\text{filt}}$ that minimise $\delta M$ (Fig.2 left), and the dedicated study of Ref. [29] indicates as optimal values $n_{\text{filt}} = 3$ and $R_{\text{filt}} = \min\{R_{b\bar{b}}/2, 0.3\}$ (where $R_{b\bar{b}}$ is the usual $\eta-\phi$ distance between the two $b$-subjets). These values give a good resolution for the Higgs mass peak also after a full event simulation with parton shower Monte Carlo’s (Fig.2 right) [28].
Figure 2: The width of the Higgs mass peak $\delta M$ \cite{29} for $n_{\text{fil}} = 3$ as a function of $\eta = R_{\text{fil}}/R$ (left), and the distribution in the invariant mass of the Higgs candidate jet corresponding to the selection cuts of Ref. \cite{28} (right).

2.3 Colour connections and the “pull”

Many of the heavy objects we wish to observe at the LHC are colour singlets. This raises the question on whether we can distinguish jets originating from hadronic decays of colour singlets from pure QCD jets. Although there is no definitive answer to this question so far, hints might be gained by studying the QCD radiation pattern in the interjet region, which is expected to be determined by the colour flow of each event. Reconstruction of colour connections between jets was extensively studied at LEP, for instance by counting the number of hadrons in the interjet region in three-jet events. There one observed that the hadron multiplicity was different in QCD three-jet events rather than in $q\bar{q}\gamma$ events, and the observed difference could be simply accounted for by considering the colour connections between the hard emitting partons, the so-called string/drag effect \cite{30,31,32,33}. An analogous analysis for hadron colliders has been recently proposed \cite{34}. It is based on the so-called “pull” vector of a jet, defined as

$$\vec{r} = \sum_{i \in \text{jet}} \frac{p_{t,i} |\vec{r}_i|}{p_{t,jet}} \vec{r}_i, \quad \vec{r}_i = (\Delta y_{i,jet}, \Delta \phi_{i,jet}).$$

The pull distribution “points” towards the jet to which the triggered jet is colour connected. For instance, following again Ref. \cite{34}, if one considers Higgs production in association with a $Z$ boson, the distribution in the pull angle $\Delta \theta_t$ of the higher $p_t$ is peaked around $\Delta \theta_t = 0$, corresponding to the “position” of the other jet, whilst that for the background $Zb\bar{b}$ is peaked around $\Delta \theta_t = \pm \pi$, corresponding to the beam (see Fig. 3 left). Experimental studies in $t\bar{t}$ events at the Tevatron confirm this difference \cite{35}. Indeed, the plot on the right-hand side of Fig. 3 shows the measured distribution in $\Delta \theta_t$ (labelled $\theta_{\text{rel}}$ pull in Ref. \cite{35}) for any of the

\footnote{Notice however that a definition of the pull vector as in eq. (1) raises a theoretical problem, since at tree level, when a jet consists of a single parton, the pull angle is undefined.}
two jets coming from the decay of a $W$ boson. The distribution is peaked around $\theta^{\text{rel pull}} = 0$, corresponding to the “position” of the other jet from $W$ decay. Ref. [35] shows also plots for the pull angle distribution for the two colour disconnected $b$-jets from top decay, which is instead peaked towards larger values of $\theta^{\text{rel pull}}$.

3 QCD predictions for multi-jet events

At LEP the majority of QCD precision studies has been performed for two-jet events. However, LEP produced many multi-jet events [36, 37, 38, 39], so that at present we have measurements that extend up to the inclusive six-jet rate [36]. However, these multi-jet events have not been fully exploited for QCD precision studies, the most notable exception being the three- and four-jet rates [40, 41, 42]. The main reason for this was the lack of fixed order calculations involving many legs in the final state. While at LEP one could restrict experimental analyses to low jet multiplicities, at the LHC many interesting phenomena, for instance production of top quarks or supersymmetric particles, involve a large number of jets in the final state. Notably, already now, there exists data for events $Z$ or $W$ boson production with six additional jets [43, 44, 45], whose $e^+e^-$ counterpart is the eight-jet rate! It is therefore clear that one of the main problem theorists had to face in view of the LHC was how to perform precision calculations (especially NLO) for multi-leg processes. The traditional approach based on Feynman diagram looks prohibitive due to the large number of diagrams (e.g. tens of thousands for processes like $t\bar{t}b\bar{b}$, involving four QCD hard emitters in the final state) which have to be computed. Although, as the calculation of Ref. [46] shows, it is still possible to perform multi-leg NLO calculations using Feynman diagrams, in recent years a number of revolutionary ideas changed our way of looking at one-loop diagrams. The main observation, based on the pioneering work of Ref. [47],
is that the coefficients of the one-loop master integrals into which any one-loop amplitude can be decomposed are actually tree-level matrix elements [48, 49, 50]. This was the starting point of the so-called “unitarity-cut” techniques, through which it is possible to compute one-loop amplitudes as a whole instead of the individual Feynman diagrams (for a review see [51], and references therein). With these methods NLO predictions are nowadays produced at an industrial rate by various collaborations, such as BLACKHAT [52, 53], HELAC-NLO [54], ROCKET [55, 56], GOSAM [57]. In the meantime many methods have been developed to efficiently compute tree-level matrix elements, like MADGRAPH [58] or COMIX [59]. There are also programs, like ALPGEN [60] or SHERPA [61], that implement algorithms to coherently combine tree-level matrix elements to parton showers. Last but not least, in recent years new methods have been developed to match even NLO calculations to parton showers [62, 63], nowadays automated in the aMC@NLO [64] and POWHEG-BOX [65] frameworks.

Such enormous progress had consequences also for $e^+e^-$ precision studies. For instance, for the first time it was possible to tackle the NLO calculation of the five-jet rate by crossing matrix elements used for $Z$ plus three jets at NLO [60]. The resulting theoretical analysis, in particular the extraction of a value of $\alpha_s(M_Z)$, took also advantage of the matching between tree-level five-jet matrix elements and parton shower implemented in the SHERPA Monte Carlo. Indeed, only using SHERPA was it possible to obtain a reliable estimate of hadronisation corrections, and hence a precise measurement of $\alpha_s(M_Z)$ (see Fig. 4). The limit on the jet multiplicity is nowadays being pushed further and further, and at the moment there exist (leading colour) NLO calculations for $e^+e^-$ up to seven jets [67]. It would be great to compare these predictions to LEP data, so as to have consistent extractions of $\alpha_s(M_Z)$ from all measured jet rates.

4 Outlook

The NLO revolution is just one example of the theoretical progress that has been triggered by LHC Physics in recent years. For processes like the production of colour singlets NNLO calculations are already available [68, 69, 70, 71, 72, 73] while considerable progress has been made towards NNLO predictions for top-antitop [74] or dijet production [75]. Given the complexity of two-loop calculations, many people have also tried to investigate whether the structure of QCD amplitudes could be deduced from general principles rather than obtained only through explicit calculations. This research stream involved on one hand the use of factorisation properties of gauge theories to arrive at a general formula for the infrared structure of gauge theories [76, 77]. On the other hand, also hard non singular contributions were investigated in theories with a high degree of symmetry, like $\mathcal{N} = 4$ Super Yang-Mills, hoping to be able to solve them at the quantum level (see for instance [78]). The latter studies have lead to the discovery of the simpler representation of multi-loop amplitudes in terms of mathematical objects called symbols [79]. The hope is to be able to associate to each amplitude its symbol content, so as to
Given the theoretical advances I have described so far, how can we imagine the state of the art of precision calculations at the start of the linear collider? Let us consider for instance the Higgsstrahlung process, the most widely used for Higgs searches at LEP, with both the Higgs and the recoiling vector boson decaying hadronically. The theoretical description of both signal and backgrounds (e.g. $e^+e^-$ to four jets) will be very different from that of LEP days. Definitely higher order corrections, both QCD and electro-weak, will be available at NNLO, and probably we will know the all-order structure of the dominant virtual corrections. Sophisticated methods based on jet substructure will be able to discriminate the signal from the backgrounds. Experimental analyses will also take advantage of the fact that all parton shower event generators will be matched to NLO matrix elements.

Concerning precision Physics, for two-jet event shapes hadronisation corrections will be very small, so that, just using the already available NNLO+NNLL predictions, we could have a measurement of $\alpha_s(M_Z)$ at the permille accuracy. Jet rates had already very small hadronisation corrections at LEP, at LC they will have basically none. In this case we already have NNLO predictions for three-jet production, and probably we will have them for four-jet production as well. Unfortunately no resummation beyond NLL accuracy is available for jet rates so far. An improvement in this direction would probably allow for the most precise determination of $\alpha_s(M_Z)$ ever.
I would like to conclude with a couple of remarks on non-perturbative effects, which are in fact the everlasting unknown in all collider experiments. The LC will not only be a precision machine, but also a means of investigation of those effects, especially in multi-jet events. For instance, when considering three-jet events, due to the extra radiation from a gluon, leading hadronisation corrections are expected to be roughly twice as large as in two-jet events. This feature, at LEP energies, made them too large to be allowed to neglect the contribution of subleading corrections, as was done for two-jet events. At the LC instead, non-perturbative corrections to three-jet event shapes like the $D$-parameter are of the same order of magnitude of the corresponding ones to two-jet event shapes at LEP. Therefore, more studies of the universality of the non-perturbative parameter $\alpha_0$ could be performed, opening for the first time the possibility of making quantitative statements about hadronisation from a gluon in a multi-jet environment. Last but not least, there might be experimental high-luminosity setups for the LC which imply contamination of signal events from pile-up. We hope that the LHC will teach us how to model this effect better and better, so as to be able to properly deal with it before the start of the LC.

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