Early thermalization and shear viscosity to entropy ratio in heavy-ion collisions at energies of BES, FAIR and NICA

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Abstract

Equilibration of highly excited baryon-rich matter is studied within the microscopic model calculations in A+A collisions at energies of BES, FAIR and NICA. It is shown that the system evolution from the very beginning of the collision can be approximated by relativistic hydrodynamics, although the hot and dense nuclear matter is not in local equilibrium yet. During the evolution of the fireball the extracted values of energy density, net baryon and net strangeness densities are used as an input to Statistical Model (SM) in order to calculate temperature $T$, chemical potentials $\mu_B$ and $\mu_S$, and entropy density $s$ of the system. Also, they are used as an input for the box with periodic boundary conditions to investigate the momentum correlators in the infinite nuclear matter. Shear viscosity $\eta$ is calculated according to the Green-Kubo formalism. At all energies, shear viscosity to entropy density ratio shows minimum at time corresponding to maximum baryon density. The ratio dependence on $T$, $\mu_B$, $\mu_S$ is investigated for both in- and out of equilibrium cases.

Keywords: transport models, heavy-ion collisions at BES/FAIR/NICA energies, Green-Kubo formalism, $\eta/s$ ratio

1. Introduction

One of the goals of experiments on heavy-ion collisions at intermediate energies below $\sqrt{s} = 20$ GeV is the search for the predicted tricritical point of the QCD phase diagram. At this point the first-order deconfinement phase transition between the quark-gluon plasma (QGP) and hadronic matter should become of the second-order. Various signals of such a phenomenon were predicted. The ratio of shear viscosity to entropy density, $\eta/s$, looks very prominent, because for all known substances this ratio reaches minimum value in the vicinity of critical point [1]. The absolute limit for $\eta/s$ estimated within the AdS/CFT correspondence is $1/(4\pi)$ [2]. Except of Ref. [3], this ratio was usually studied in microscopic models as function of temperature $T$ taken at fixed baryochemical potential and zero chemical potential of strangeness [4,5,6,7,8].

The standard procedure of $\eta$ determination by means of a transport model relies on the Green-Kubo formalism. The system of hadrons is inserted into a box with periodic boundary conditions. The shear
The viscosity is calculated in system of natural units $c = \hbar = k_B = 1$ as

$$\eta(t_0) = \frac{V}{T} \int_{t_0}^{\infty} dt \langle \pi^{ij}(t) \pi^{ij}(t_0) \rangle,$$

(1)

Here $V$ and $T$ is the box volume and temperature, and $t_0$ and $t$ denote moments of time, respectively. The correlator $\langle \ldots \rangle$ reads

$$\langle \pi^{ij}(t) \pi^{ij}(t_0) \rangle = \lim_{t_{\text{max}} \to \infty} \frac{1}{t_{\text{max}}} \int_{t_0}^{t_{\text{max}}} dt' \langle \pi^{ij}(t + t') \pi^{ij}(t') \rangle,$$

(2)

containing the nondiagonal parts $\pi^{ij}$ of the energy-momentum tensor

$$\pi^{ij}(t) = \frac{1}{V} \sum_{i \neq j} \frac{p^j(t) p^j(t)}{E(t)}.$$

(3)

The formalism requires that the initially out-of-equilibrium system is relaxed to the equilibrium state. The developed procedure and the results of our study are presented below.

2. The method

The problem appears to be manifold. We have to define the area in heavy-ion collision most appropriate for studying the relaxation process. Previous studies show that the central cell with volume $V = 125 \text{ fm}^3$ is most suitable for our research [9, 10]. To determine whether or not the equilibration takes place, one has to employ the statistical model (SM) of an ideal hadron gas with essentially the same number of degrees of freedom as in the transport model. In equilibrium, all characteristics of the system are determined via a set of distribution functions

$$f(p, m_i) = \left[ \exp \left( \frac{\epsilon_i - \mu_i}{T} \right) \pm 1 \right]^{-1},$$

(4)

where $p$ is momentum of a hadron specie $i$, $m_i$ is its mass, $\epsilon_i$ and $\mu_i$ is energy density and chemical potential, respectively. The last depends on the particle’s baryon charge $B_i$ and strange charge $S_i$: $\mu_i = B_i \mu_B + S_i \mu_S$. Plus and minus signs stand for Fermi-Dirac and Bose-Einstein statistics. The number density and the energy density can be found as the first and the second moments of $f(p, m_i)$, and the entropy density is

$$s_i = -\frac{g_i}{2 \pi^2} \int_0^{\infty} f(p, m_i) \left[ \ln f(p, m_i) - 1 \right] p^2 dp,$$

(5)

where $g_i$ is the degeneracy factor. In the vicinity of equilibrium the particle yields and energy spectra in the cell should be close to those provided by the SM. To find the shear viscosity the extracted cell parameters $\epsilon, \rho_B$, and $\rho_S$ should be used as an input to initialize the box with periodic boundary conditions [11, 12]. The UrQMD model [13, 14] is employed for both the cell and the box calculations. The cubic box with volume $V = 1000 \text{ fm}^3$ was initialized. At this stage the correlator given by Eq. (2) is calculated. Because of the ceaseless change of the energy density, net-baryon density, and net-strangeness density in the tested volume, one has to perform a series of snapshots of the system bulk conditions. We opted for 20 time slices, from $t = 1$ to $20 \text{ fm}/c$, with the time step $\Delta t = 1 \text{ fm}/c$.

3. Results

About 50 000 central Au+Au collisions were generated at each of four energies, $E_{\text{lab}} = 10, 20, 30,$ and $40 \text{ AGeV}$. The calculations show that the matter in the cell expands with almost constant entropy-per-baryon ratio already after $t = 1 \text{ fm}/c$. Pressure in the cell also appears to be very close to the pressure calculated...
Fig. 1. Shear viscosity to SM entropy ratio $\eta/s_{\text{SM}}$ as function of (a) time $t$, (b) temperature $T$, (c) baryochemical potential $\mu_B$, and (d) strangeness chemical potential $\mu_S$ in the UrQMD calculations of central cell of central Au+Au collisions at $E_{\text{lab}} = 10$ AGeV (circles), 20 AGeV (triangles), 30 AGeV (squares), and 40 AGeV (diamonds). Lines are drawn to guide the eye.

within the SM for the hadronic gas in equilibrium. Both observations strongly support application of hydrodynamics to very early stages of nuclear collisions [15]. Relaxation to equilibrium in the box, however, takes much longer times compared to the cell case. The results of the calculations were averaged over the time period between 200 and 800 fm/c, where the correlator has a plateau. Figure 1 shows the dependencies of $\eta/s$ on (a) time, (b) temperature, (c) baryochemical potential, and (d) strangeness chemical potential. The statistical errors are smaller than the symbol sizes. The parts of the spectra related to nonequilibrium stages of the evolution are shown by the dashed lines. We see that the ratio $\eta/s$ decreases with decreasing bombarding energy from 40 to 10 AGeV. Also, it increases with the drop of temperature in the cell, accompanied by increasing $\mu_B$ and decreasing $\mu_S$. The smaller the bombarding energy, the lower the $\eta/s$ ratio. No distinct minima are observed. However, the entropy density and other macroscopic characteristics were calculated for the ideal hadron gas in equilibrium, whereas the system was out of equilibrium within the first few fm/c after beginning of the collision. The entropy density in the equilibrated system is larger than that in the non-equilibrated one. To account for this circumstance, we replace the distribution functions given by Eq. (1) to those provided by the momentum distributions of hadrons

$$f_i(p) = \frac{(2\pi)^3}{V g_i} \frac{dN_i}{d^3p}$$

In equilibrium, results obtained by both methods should coincide. Time evolution of $\eta/s$ in the cell and temperature dependence of this ratio, where the entropy density is calculated via Eq. (6), is shown in Fig. 2(a,b). Here all distributions reveal clear minima at $t = 5 \sim 6$ fm/c corresponding to maximum baryon density in the system. The minima become deeper with the decreasing energy of the collision. It would be important to study this effect at lower energies, say, up to $\sqrt{s} = 2 \sim 3$ GeV. If the dip in the ratio $\eta/s$ will stop to drop further, it could be taken as indication of change of the equation of state due to formation of non-hadronic objects, i.e., quark-gluon strings. These strings can be considered as a precursor of the QGP formation.

4. Conclusions

The following conclusions can be drawn from our study. We calculated the shear viscosity, the entropy density, and their ratio in the central cell with volume $V = 125$ fm$^3$ of central Au+Au collisions at energies
Fig. 2. Shear viscosity to nonequilibrium entropy ratio $\eta/s_{\text{noneq}}$ as function of (a) time $t$ and (b) temperature $T$ in the UrQMD calculations of central cell of central Au+Au collisions at $E_{\text{lab}} = 10$ AGeV (circles), 20 AGeV (triangles), 30 AGeV (squares), and 40 AGeV (diamonds). Lines are drawn to guide the eye.

from $E_{\text{lab}} = 10$ to 40 AGeV within the UrQMD model. First, the entropy density was estimated for an ideal hadron gas in equilibrium. Then, the entropy density of nonequilibrium state was calculated via the momentum distribution functions. For both cases shear viscosity and entropy density in the cell drop with time, whereas their ratio $\eta/s$ reaches minimum at $t \approx 5 - 6$ fm/c regardless of the collision energy. At later times this ratio increases. The lower the energy, the smaller the ratio. Further studies at lower energies are needed to check where $\eta/s$ will stop to decrease.

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