Systematic effects in the determination of the 
$\pi NN$ Coupling from $\bar{p}p \rightarrow \bar{n}n$ differential 
cross section

Torleif Ericson

The Svedberg Laboratory, Uppsala University, Box 533, S-75121 Uppsala, Sweden

and

Benoît Loiseau

Division de Physique Théorique† Institut de Physique Nucléaire, F-91406 Orsay Cedex

and LPTPE Université Pierre et Marie Curie, F-75252 Paris Cedex 05, France

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Abstract

We show that the $\pi NN$ coupling constant extracted model- independently from $\bar{p}p$ charge exchange is subject to a systematic correction, and, more importantly, that the strong absorption in the critical region prevents a determination of the coupling constant to high precision using this process. This attenuates the possible conflict with the value determined from the $np$ charge exchange cross sections.

The value of the $\pi NN$ coupling constant has become a topic of hot debate in the last few years [1]. In addition from the 'classical' determinations from $\pi N$ scattering and forward NN dispersion relations with a 'high' value of $g^2$ near 14.3, the Nijmegen group has argued a lower value of about 13.6 on the

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*Also at Theory Division, CERN, CH-1211 Geneva 23, Switzerland
†Unité de Recherche des Universités Paris 11 et Paris 6 associée au CNRS
basis of their extensive analysis of NN data using the Nijmegen model \[2, 3\]. The issue of its precise value is an important question not only because it is a fundamental constant to nuclear physics \[4\], but also because it is of great importance also for the understanding of chiral symmetry breaking. Thus, the experimental error in the pion-nucleon coupling constant is the principal obstacle in the precise testing of the validity of the Goldberger-Treiman relation as predicted from chiral symmetry breaking \[3, 4\]. This relation would give $g^2 = 12.81$, if it were exact, which is not expected, however. To resolve the problem unambiguously we have undertaken a detailed study of the possibility of using methods of extrapolation to the pion pole in NN scattering for an accurate and model independent determination of the coupling constant directly from data. A first report on this work, based on high precision absolutely normalized data in the pole dominated region of np elastic charge exchange at 162 MeV, points to a high value of $g^2 = 14.60 \pm 0.30$ \[4\]. The dominant source of uncertainty is the experimental overall normalization. New, as yet unpublished, additional data give a slightly lower and more accurate value $g^2 = 14.4 \pm 0.2$ \[7, 8\]. The pole extrapolation method was originally suggested by Chew using a polynomial expansion \[7, 8\]. Its practical use is fraught with problems associated with systematic errors and instabilities which must be controlled and well understood. We have tentatively solved this problem on the one hand by stabilizing the extrapolation procedure using appropriate comparison functions, on the other hand by exploring systematic errors analyzing large numbers of 'pseudo experiments' generated from models with known coupling constants. This program is still being pursued on a wider body of np charge exchange data.

The corresponding information is in principle also available from the $\bar{p}p \rightarrow \bar{n}n$ charge exchange reaction, which has the same pole structure. Recently precise data on this reaction was obtained in the experiment PS206 by Birsa et al. \[10\] at 601 MeV/c (= 176 MeV kinetic energy) and the quality of these data are comparable and even superior to those of the best data on np charge exchange in a wide range of similar energies \[11\]. A preliminary analysis demonstrated strikingly that indeed both $p\bar{p}$ and $np$ charge exchange data extrapolate approximately to the same pole residue \[11\]. However, the further analysis \[12\] based on the classical Chew method gave the very low value $f_\pi^2 = 0.0708 \pm 0.0016 \pm 0.0011$ ($g^2 = 12.80 \pm 0.29 \pm 0.20$) (statistical and normalization errors, respectively), about 10% below our value above and also well below the Nijmegen value. This value is a cause of concern and raises
the question of consistency with \( np \) charge exchange and even more seriously of the overall validity of the approach as such. The analysis did not attempt to analyze and evaluate the systematic errors in the procedure. The present note aims to elucidate these questions as well as to clarify the structure of the contributions to antiproton charge exchange and their relations to the pion pole terms.

We first examine the contributions to the unpolarized differential cross section so as to illustrate the information that must be described by any extrapolation procedure. Here the total amplitudes and cross sections are defined in the usual fashion in terms of the five amplitudes \( a, b, c, d, e \) allowed by the invariance properties \([13, 14]\). Both the CM unpolarized cross section as well as the polarization transfer one are incoherent combinations of 5 amplitudes:

\[
\frac{d\sigma}{d\Omega} (t) = \frac{1}{2}(|a|^2 + |b|^2 + |c|^2 + |d|^2 + |e|^2);
\]

\[
\frac{d\sigma}{d\Omega} (t) (1 - K_{n00n}) = |c|^2 + |d|^2, \tag{1}
\]

where \( t \) is the squared 4-momentum transfer from the proton to the neutron. Taking into account the difference between the proton (\( M_p \)) and neutron (\( M_n \)) masses one has in the centre-of-mass system for the proton momentum \( k \) and scattering angle \( \theta \):

\[
t = +2k \sqrt{k^2 - M_n^2 + M_p^2} \cos \theta - 2k^2 + M_n^2 - M_p^2. \tag{2}
\]

There are five regularized pion Born amplitudes and in this case the \( r \)-space \( \delta \)-function has been subtracted \([13, 14]\):

\[
a_{\pi} + b_{\pi} = 0;
\]

\[
a_{\pi} - b_{\pi} = F(t);
\]

\[
c_{\pi} + d_{\pi} = F(t)(1 - 3\Pi(t));
\]

\[
c_{\pi} - d_{\pi} = F(t);
\]

\[
e_{\pi} = 0, \tag{3}
\]

where

\[
F(t) \equiv \frac{g^2}{\sqrt{s}} \left( \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - t} \right)^2; \quad \Pi(t) \equiv \frac{t}{t - m_\pi^2}. \tag{4}
\]
In Eq. (4) the charged pion mass is denoted by $m_{\pi}$ and the range of the form factor is chosen to be $\Lambda = 800$ MeV. This corresponds to an rms radius of .6 fm for the nucleonic pion source.

It is in practice convenient to chose a representation for the amplitudes such that the pion pole term appears in one single term, namely $(c + d)/\sqrt{2}$ for the $p\bar{p}$ case and $(b - d)/\sqrt{2}$ for the $np$ one. This regroups the pole term completely into the real part of these amplitudes, respectively, and they combine to an exact square in the cross section. These are therefore crucial to the extrapolation and they are also the dominant terms in the physical region. All the remainder is represented by terms which slowly and smoothly decrease with increasing $-t$. The differential cross section and polarization transfer then become

$$\frac{d\sigma}{d\Omega}(t) = 1/4[|a + b|^2 + |a - b|^2 + |c + d|^2 + |c - d|^2 + 2|e|^2];$$

$$\frac{d\sigma}{d\Omega}(t) (1 - K_{n10}) = 1/2[|c + d|^2 + |c - d|^2]$$

(5)

Let us now examine the behavior of the contributions to the cross section and their variation with $t$ in more detail. Such a study is most readily done using a model which represents the main physics even if it only qualitatively reproduces the data. The Paris antiproton model is sufficient for this purpose and readily reproducible [17]. The characteristic shape and magnitude of the terms in eq. (5) are illustrated for the pion Born terms of eqs. (3-4) and for the Paris model in Figs. 1a and 1b. Not unexpectedly, absorption is a prominent feature for antiproton charge exchange. Compared to $np$ charge exchange the cross section is quenched by a factor 4-5 for momentum transfers of a few pion masses, the region most sensitive to the pion pole. This feature is strikingly apparent also in the comparison of the $np$ and the $p\bar{p}$ pole extrapolation in fig. 2. There is an important difference in the physics between the two cases. In the $np$ case the amplitudes have imaginary parts only from unitarity. They are mostly nearly real and there are in practice only two important non-pole terms. In the antiproton charge exchange real and imaginary non-pole amplitudes are both important and four such terms contribute substantially. Fortunately our method permits us to ignore the detailed dynamics of these terms. In both case the term containing the pole amplitude has a zero at $t = -m_{\pi}^2/2$ in the Born approximation independent of form factors. The corresponding minimum survives with a slight shift in
the model cross sections as seen in Figs. 1a and 1b. Although the value is quenched in the antiproton case this suggests that similar extrapolation methods can, at least in principle, be used in both cases. Practice appears to confirm this. The minimum is accompanied by a steep rise of the differential cross section at small angles. It is this feature that is responsible for the overall minimum in the cross section, which is partly masked by slowly varying background terms. In the case of \( np \) scattering this minimum transforms into a shoulder in the cross section, but the corresponding structure is also present in that case.

We note in passing that polarization transfer in principle allows a clean separation of the terms containing the pion pole contributions from other terms as is apparent from eq. (5). However, the use of this possibility still requires the accurate knowledge of the unpolarized cross sections for a direct extrapolation to the pion pole. Since the eliminated terms are not dominant, even a perfect knowledge of the polarization is only a minor constraint on the pion coupling constant. The polarization information is of no particular use in the present context.

We have first followed a procedure identical to that of Bradamante et al., analyzing their data using the Chew method [11], [12]. In this case one defines the function \( y(x) \), which extrapolates the experimental data smoothly to the pion pole:

\[
y(x) = \frac{s x^2}{m^4 R^4} \frac{d\sigma}{d\Omega}(x) = \sum_{i=0}^{n-1} a_i x^i.
\]

(6)

Here \( s \) is the square of the total energy and \( x = -t + m_{\pi}^2 \). We will use the charged pion mass \( m_{\pi} \equiv 1 \) as the mass scale in the discussion. At the pion pole \( x = 0 \) the Chew function gives

\[
y(0) \equiv a_0 \equiv N g_f^4 / g_R^4,
\]

(7)

where \( N \) is the normalization of the experimental data, which may differ from the true value. The model independent determination of the coupling constant \( g^2 \) requires accurate single-energy data with absolute normalization \( N \) of the unpolarized differential cross section. The error in the coupling constant determined by such extrapolation methods is always proportional to \( \sqrt{N} \).

We accurately reproduce the Bradamante analysis for the \( \bar{p}p \) reaction using the Chew method and the results are given in Table 1. However, we
find minor discrepancies in their simultaneous analysis of the Uppsala 96 MeV np scattering data, which they use as a comparison. We traced this to their arbitrary omission of three data points. Therefore, at this first step our analysis reproduces their low value of the coupling constant.

The next problem is that of systematical errors in the extrapolation procedure and true extrapolation errors. Here the \( \bar{p}p \) case differs importantly from the np, since absorption is a major feature. Examination of the Chew function in fig. 2 shows that it has a pronounced minimum near 0°, the physical point closest to the value to be determined. The extrapolation point is about a factor of 4 larger than this value and lies opposite to the main trend of the function to be extrapolated. As a matter of principle this is a particularly unfavorable situation in any accurate extrapolation procedure. Another way of stating this problem is to note that the absorption produces an important reduction of the pion pole effects in the physical region, which is to say that it reduces the sensitivity to the coupling constant which we want to determine. Even so there remains a substantial sensitivity on the pion-nucleon coupling constant as already demonstrated by Bradamante et al. [11, 12]. The accuracy to which it can be determined is however a more delicate and quantitative question.

To investigate this matter we first use models with known \( g^2 \) for which we can generate randomly 'data points' at the exact angles and with the same statistical errors as in the actual experiment. For this we generate 10,000 equivalent pseudo experiments which we analyze in the same way as the actual experiment. As models we use a solution of a preliminary analysis of the Nijmegen group [18] and the Paris potential prediction [17]. However any other model with known pion exchange would do as well for the present purpose if it approximately reproduces the data. From Table 1 we find that the Chew extrapolation approach requires 6 polynomial terms to describe these 'experimental' data appropriately. From the results we conclude that the Chew extrapolation procedure systematically underpredicts the coupling constant by 2% (\( \delta g^2 \simeq 0.3 \)) for the case of 47 data points (n=6). This shift would appear even if the precision of the data were higher than at present. It is thus necessary to correct for such systematic shifts in the determination of the coupling constant using the Chew method.

However, very little information on the pion pole is contained in the region of large momentum transfers. We therefore also truncated the data at \( x_{\text{max}} = 4.9m_\pi^2 \) (the 30 first points), so as to be able to use fewer parameters
in the extrapolation. One easily persuades oneself that the tensor amplitude makes it physically unreasonable to expect a description of the data with a polynomial of less than 5 terms in the original Chew polynomial extrapolation procedure and that data with large momentum transfer call for at least 6 terms.

In the case of 30 points, the analysis of the pseudodata for both the Paris and Nijmegen models in Table 2 demonstrates that 5 terms in the polynomial give a perfect fit to data such as these, although with a systematic downward shift of $g^2$ by 0.3 (Paris) to 0.5 (Nijmegen). The extrapolated value is very low (12.26) and it is not more than about $12.8 \pm 0.5$, even when corrected for the systematic shift. The shift disappears with one additional term in the polynomial, but at the cost of a much larger extrapolation error. For the corresponding experimental 30 data points the $\chi^2/DoF$ is only 0.46, which is an unexpectedly low value statistically. It reflects partly that the errors given by PS 206 consist of statistical and systematic errors added in quadrature [10]. Corrected for systematic shifts the Chew model in this case gives $12.84 \pm 0.46$ for $n = 5$ and $13.24 \pm 1.15$ for $n = 6$. The low $\chi^2$ raises the possibility of some correlations in the systematic errors, in particular at low momentum transfers. In every case the systematic error of 1.5%, i.e. $\pm 0.2$, due to the normalization uncertainty must be added to the overall uncertainty. These results are compatible with the Chew analysis in the larger data range with 47 data points. The latter appears to be more accurate with a value $g^2 = 13.1 \pm 0.3$ as seen in Table 1 for $n = 6$. This is somewhat disconcerting since the additional region is insensitive to the pion pole and the polynomial expansion is questionable over such a large region.

We now turn to the question of the precision that can be attained reliably in the analysis, since the errors given above are only the formal statistical uncertainty of the Chew method. To investigate this point it is convenient to use the Ashmore method as described in ref. [4]. We expect this method to work, since both the background terms and the interference term vary similarly to the $np$ case as already discussed above. It is thus the systematics of this method that will determine the precision which can be achieved. The Ashmore method parametrizes the differential cross section in terms of the regularized pion Born amplitudes, but has in addition a contribution simulating a $\rho$ meson pole with adjustable strength and shape, described with a form factor and polynomial terms in $x$ [4]. This method permits a phenomenological simulation of the absorption effects, but the dynamical
description as such is not realistic. The virtue of the method is its ease of
application and that it permits the explicit exploration of the influence of
changes in the pion-nucleon coupling constant. We note from Table 2 that
the Paris and Nijmegen model pseudodata are described in the Ashmore
model for $n=5$ with $\chi^2/DoF=1.00$ with a moderate systematic shift of 0.21
and 0.29, respectively, for 30 data points. An additional parameter as used
with $n=6$ will thus overparametrize the analysis.

In order to test the uncertainty of the coupling constant determined from
the full range of data in the Ashmore model we fixed the coupling constant for
$n=6$ to 13.21, 14.10 and 14.43 and determined the corresponding $\chi^2$ values
40.2, 51.0 and 58.6 for the 47 data points (41 degrees of freedom). This should
be compared to the corresponding Nijmegen description for $g^2=13.23$, which
is 48.6. Although the value of 13.23 gives a better $\chi^2$ in agreement with the
other methods all of these values give a good representation of the data as
is apparent from Fig. 3. We note in particular that the first few points near
t = 0 have a large contribution to the $\chi^2$. They play thus an essential role in
the discussion. For example, one half of the difference between the $\chi^2$ of the
Nijmegen model with a coupling constant of 13.23 and the Ashmore model
with 14.43 comes from the two points nearest $t = 0$. Since these points are
also the ones with the largest systematic errors [10], they are an important
potential source of systematic error in the extrapolation. If they are omitted,
the value of $g^2$ increases by .1 to .2. Interestingly, all of the Ashmore curves
have a generically similar shape which differs systematically from the one in
the Nijmegen description on the level of a few %.

These observations suggest
that the value of $g^2$ deduced from antiproton charge exchange cannot be
considered to fully model independent at a precision higher than 0.5 to 0.7
units (3 to 5%).

In the case of $np$ charge exchange, which we previously investigated, it was
possible to improve the procedure considerably using a Difference Method
[4]. This method relies on an extrapolation of the difference of the Chew
function for data and a model with a known coupling constant. While this
method is effective in the case of $np$ charge exchange, it does not improve
significantly on the previous methods in the present case. The reason is the
scarcity of high precision information on the $p\bar{p}$ interaction other than charge
exchange. This has the consequence that either the model relies heavily on
the present data on charge exchange or lacks sufficient precision in the de-
scription of higher momentum transfer to be useful. In the former case the
The method would as expected accurately reproduce the $g^2$ of the model with considerable precision, but the argument would be circular. The method has the additional advantage of visually bringing out the details of the extrapolation in the low $t$ region at the level of precision under discussion here. It is therefore instructive to apply the method to the Paris model, which has qualitative agreement with data and incorporates constraints at large momentum transfer.

The results of the extrapolation of the difference function $y_M(x) - y(x)$ versus $x$ are given in Table 2 for 30 data points and are displayed in figure 4. We first calibrated the method using pseudodata from the Nijmegen model. These demonstrate that such data can be perfectly described by $n = 5$ with $\chi^2/DoF = 1.00$ and that the systematic extrapolation error is then negligible. However, applied to the actual data we find no improvement in the accuracy to which the coupling is determined. On the other hand Fig. 4 demonstrates that a major background has been removed. It is now easy to visualize the consequences of different values for the number of polynomial terms $n$ in the extrapolation. A good description is obtained already for $n = 4$ but with a substantial systematic correction. The values deduced for the coupling constant after the correction for the systematic shifts are in this case $13.10 \pm 0.16, 12.65 \pm 0.45$ and $13.24 \pm 1.15$ for $n = 4, 5$ and 6, respectively. These extrapolations give similar results to the previous ones and they are mutually consistent. We have also applied the method to the full set of data. In this case ($n = 6$) is needed, but the conclusions do not change.

We have already remarked that the absolute normalization of the cross section is a crucial number for the extraction of the coupling constant. Let us now discuss the uncertainties from this source, which is independent of the errors arising from the extrapolation uncertainty. The Nijmegen group has achieved a good description of the present data with $\chi^2/(data) = 1.035$ with an integrated charge exchange cross section $\sigma_{tot} = 12.14$ mb. The normalization error is most likely the experimental uncertainty in the experiment PS206 in view of the close fit to it. For the Paris model the corresponding $\sigma_{tot} = 13.45$ mb. This means automatically that if this normalization were used to normalize the same experimental data, then the deduced coupling constant would increase by 5.3%, that is $g^2$ would increase by 0.7. This emphasizes the importance of this question.

Some additional information on this point comes from other experiments. R.P. Hamilton et al. \[13\] have performed a dedicated absolute measurement
of the integrated charge exchange cross section with energy in small steps. By interpolation between 596 and 608 MeV/c this corresponds to a value of \( \sigma_{tot} = 11.80(11) \) at 601 MeV/c. The systematic error is stated to be less than 3 to 5 \%, depending on the energy region. Using 4\% as a reasonable estimate at the present energy this means a systematic uncertainty of about 0.47 mb, completely compatible with both the experiment PS 206 [10] and with the Nijmegen description. Experimental information can also be found in Nakamura et al. [20]. In this case the emphasis was on the angular shape and the forward interference dip and not on normalization. Their integrated cross sections are larger than in the other experiments by 15 to 20\%, but the systematic uncertainty was also larger (8.5\%). In view of the circumstances we, as the authors, do not believe these larger values to be significant. In conclusion, there appears to be no obvious reasons to question the presently quoted normalization in the experiment PS 206, but we emphasize that this issue must be kept in mind as a potential source of problems.

We have here critically examined the accuracy to which the \( \pi NN \) coupling constant can be extracted from the recent precision data on \( p\bar{p} \) charge exchange and the importance of systematic theoretical corrections to pole extrapolation procedures. Our conclusion is that the situation is less favorable than for the corresponding \( np \) charge exchange reaction. This is due to the prominent role of absorption which reduces sensitivity to the pion in the most critical region of momentum transfers. In particular, it is not profitable to apply the Difference Method in the present context. Here, the accuracy does not increase and it was this method that was the key to high precision for \( np \) charge exchange. In spite of this limitation and the more important role of systematics in the extrapolation procedure it is possible to extract the coupling constant to a good degree of precision, though with an errors of about 4 to 5\%. Depending on the detailed procedure we find a range of plausible deduced values from 12.8 to 13.2 from the data. The statistical errors are typically of order .45 using only the range of data sensitive to the pion information. Formally, a higher statistical precision is achieved using a larger range of data. However, we found in model studies that a nearly indistinguishable description of the data is achieved with values for the coupling as large as 14.1. We therefore believe that caution should be used in quoting the formal errors above. We recommend a value \( g^2 = 13.0 \pm .7 \) with an additional systematic error of 0.3 from the overall cross section normalization. The value that is extracted in this way is a direct determination
the coupling constant for the $\pi N \bar{N}$ system. This value is low, but in view of the uncertainties it is compatible with the value for the coupling constant deduced from other sources. There is no obvious discrepancy with the value deduced from $np$ charge exchange.

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References

[1] For a review, see T.E.O. Ericson, Nucl. Phys. A543, 409c (1993).

[2] R. A. M. Klomp, V. G. J. Stoks, and J. J. de Swart, Phys. Rev. C44, R1258 (1991).

[3] V. Stoks, R. Timmermans, and J.J. de Swart, Phys. Rev. C47, 512 (1993).

[4] T. E. O. Ericson, B. Loiseau, J. Nilsson, N. Olsson, J. Blomgren et al., Phys. Rev. Letters 75, 1046 (1995).

[5] N. H. Fuchs, H. Sazdjian and J. Stern, Phys. Lett. B238, 380 (1990).

[6] C. A. Dominguez, Riv. Nuov. Cim. 8, 1 (1985).

[7] J. Blomgren, N. Olsson and et al., in preparation.

[8] T. E. O. Ericson, B. Loiseau, J. Blomgren and N. Olsson, II N Newsletter 12, in press.

[9] G.F. Chew, Phys. Rev. 112, 1380 (1958).

[10] R. Birsa, F. Bradamante, A. Bressan, S. Dalla Torre-Colautti, et al., Phys. Lett. B339, 325 (1994).

[11] F. Bradamante and A. Martin, Phys. Lett. B343, 427 (1995).
[12] F. Bradamante, A. Bressani, M., M. Lamanna, and A. Martin, Phys. Lett. **B343**, 431 (1995).

[13] J. Bystricky, F. Lehar, and P. Winternitz, Jour. Phys. **39**, 1 (1978).

[14] P. LaFrance, F. Lehar, B. Loiseau, and P. Winternitz, Helv. Phys. Acta **65**, 611 (1992).

[15] W.R. Gibbs and B. Loiseau, Phys. Rev. **C50**, 2742 (1994).

[16] The neutron-proton mass difference implies also a small correction to eqs. (3) and (4) (see ref. 14). This is neglected, since the effect on the coupling constant is minute.

[17] M. Pignone, M. Lacombe, B. Loiseau, and R. Vinh Mau, Phys. Rev. **C50**, 2710 (1994).

[18] R. Timmermans, private communication, see also R. Timmermans, Th. A. Rijken, and J.J. de Swart, Phys. Rev. **C50**, 48 (1994).

[19] R.P. Hamilton, T. P. Pun, R. D. Tripp, H. Nickolson, et al. Phys. Rev. Lett. **44**, 1179 (1980).

[20] K. Nakamura, T. Fujii, T. Kageyama, F. Sai, et al., Phys. Rev. Lett. **53**, 885 (1984).

[21] T. Rönqvist, H. Condé, N. Olsson, R. Zorro, et al., Phys. Rev. **C45**, R496 (1992).
Table 1: Results applying the Chew method to the Birsa et al. data points at 176 MeV [10] and to the corresponding Nijmegen and Paris antinucleon pseudo-data (see text). The model coupling constants are $g_{\text{Nijmegen}}^2 = 13.23$ and $g_{\text{Paris}}^2 = 14.43$ with $\delta g^2$ the systematic shift from the true model value.

|       | PS206      |       | 'Nijmegen' |       | 'Paris'   |       |
|-------|------------|-------|------------|-------|-----------|-------|
| n     | $\chi^2/\text{DoF}$ | $g^2$ | $\chi^2/\text{DoF}$ | $g^2$ | $\delta g^2$ | $\chi^2/\text{DoF}$ | $g^2$ | $\delta g^2$ |
| 5     | 1.28       | 11.78 ± 0.15 | 1.77       | 11.53 ± 0.16 | 1.70   | 2.54       | 12.42 ± 0.14 | 2.01 |
| 6     | 0.90       | 12.76 ± 0.27 | 1.00       | 12.90 ± 0.27 | 0.33   | 1.01       | 14.17 ± 0.24 | 0.26 |
| 7     | 0.79       | 11.67 ± 0.58 | 1.00       | 13.10 ± 0.51 | 0.13   | 1.00       | 14.47 ± 0.45 | -0.03 |
Table 2: Results applying different extrapolation methods to the 30 first data points of PS 206 [10] as well as to the corresponding Nijmegen and Paris model pseudodata.

|                  | PS206       | 'Nijmegen'  | 'Paris'     |
|------------------|-------------|-------------|-------------|
|                  | \( n \)    | \( \chi^2/\text{DoF} \) | \( g^2 \) | \( \delta g^2 \) | \( \chi^2/\text{DoF} \) | \( g^2 \) | \( \delta g^2 \) |
| **Chew Method**  |             |              |             |               |               |              |              |
|                  | 5           | 0.47        | 12.26 ± 0.46| 1.00          | 12.85 ± 0.44  | 0.48          | 1.00          | 14.11 ± 0.39 | 0.32          |
|                  | 6           | 0.46        | 13.12 ± 1.15| 1.00          | 13.11 ± 1.17  | 0.12          | 1.00          | 14.38 ± 1.03 | 0.05          |
| **Ashmore Method**|             |              |             |               |               |              |              |
|                  | 4           | 0.44        | 12.40 ± 0.23| 1.04          | 12.70 ± 0.22  | 0.53          | 1.08          | 13.90 ± 0.20 | 0.53          |
|                  | 5           | 0.46        | 12.43 ± 0.34| 1.00          | 12.94 ± 0.33  | 0.29          | 1.00          | 14.22 ± 0.30 | 0.21          |
|                  | 6           | 0.46        | 13.43 ± 1.55| 1.00          | 13.14 ± 1.61  | 0.09          | 1.00          | 14.38 ± 1.43 | 0.04          |
| **Difference Method** |             |              |             |               |               |              |              |
|                  | 3           | 4.71        | 15.13 ± 0.06| 3.67          | 14.98 ± 0.06  | -1.75         |
|                  | 4           | 0.70        | 13.69 ± 0.16| 1.10          | 13.82 ± 0.16  | -0.59         |
|                  | 5           | 0.45        | 12.62 ± 0.45| 1.00          | 13.20 ± 0.43  | 0.03          |
|                  | 6           | 0.46        | 13.03 ± 1.15| 1.00          | 13.02 ± 1.17  | 0.21          |
Figure 1: The magnitude and structure of the different contributions in eq. 5 to the differential $\bar{p}p \to \bar{n}n$ charge exchange cross section at 176 MeV: a) using the pion Born amplitudes of eqs. 3 and 4; b) using the Paris antinucleon model. The pion pole contributions appear in $|a + d|^2$ only.
Figure 2: Comparison of the Chew pole extrapolation for \( np \) and \( \bar{p}p \) charge exchange at 162 MeV [4] and 176 MeV [10], respectively. The reference coupling constant is 14.1. For details, see text.
Figure 3: Fit to the data of Birsa et al. [10] by the Nijmegen group ($g_{\pi NN}^2 = 13.23$) [18] and using for the $n = 6$ Ashmore model with 3 different values for the $\pi NN$ coupling ($g^2 = 13.23, 14.10$ and 14.43, included in the definition of $n = 6$). Note the similar quality of the descriptions.
Figure 4: Extrapolation to the pole of the difference between the Chew function for the Paris model [17] and for PS206 [10] at 176 MeV for n=3, 4 and 5. Note that n = 4 or 5 is required by the data.