IRSA Transmission Optimization via Online Learning
Laura Toni and Pascal Frossard

Abstract
In this work, we propose a new learning framework for optimising transmission strategies when irregular repetition slotted ALOHA (IRSA) MAC protocol is considered. We cast the online optimisation of the MAC protocol design as a multi-arm bandit problem that exploits the IRSA structure in the learning framework. Our learning algorithm quickly learns the optimal transmission strategy, leading to higher rate of successfully received packets with respect to baseline transmission optimizations.

Index Terms
slotted ALOHA, successive interference cancellation, multi-arm bandit problem, online optimisation strategies.

I. INTRODUCTION

Random random slotted ALOHA (SA) with successive interference cancellation (SIC) \cite{1} has been widely considered as an effective MAC strategy, for its good performance despite the distributed protocol. In particular, let us consider a system where $L$ sources send information to a central base station (BS), through an irregular repetition slotted ALOHA (IRSA) algorithm \cite{2}, which is a SA protocol with SIC. The time axis is discretized in MAC frames, each of those composed of $M$ time slots. Visual sensor networks, in which each source sends periodic data to a central BS, represent one possible scenario\footnote{Random MAC strategies offer scalability, and adaptability to possibly varying $L$, like in the scenario considered in this paper.}. Each source sends $K$ source packets per MAC frame and each source packet is sent in multiple replicas within the MAC frame. Each replica is transmitted within one time slot and replicas sent from the same source are allocated to different slots, which are uniformly selected at random among the $M$ available slots in a MAC frame. The replication rate $l$ is selected for each source packet at random according to a transmission probability $\Lambda(x) = \sum_{l=0}^{l_{\text{max}}} \Lambda_l x^l$, where $\Lambda_l$ is the probability that a source transmits $l$ replicas of a given source packet within the MAC frame, and $l_{\text{max}}$ is the maximum replication rate. If a transmission slot is selected by only one source (singleton slot), the corresponding message is correctly received. When multiple sources select the same time slot for transmission, a packet collision happens. The BS implements the SIC algorithm to recover the collided messages. Finally, the BS defines the transmission strategy characterised by the source rate $K$ and the transmission probability $\Lambda(x)$ in IRSA\footnote{The model can also be extended to the case in which each source sends only one source packet ($K = 1$) and where the BS defines both $\Lambda(x)$ and $L$.}.

In the example provided in Fig. 1(a) the message sent by user 2 and user 3 are not decodable due to collisions in slot 3 and slot 5. However, thanks to SIC techniques, the collision might be resolved. A packet in a collision-free (singleton) slot is successfully received and decoded, revealing the slots containing the other replicas. Their removal via SIC may turn some of the collided slots into singletons, enabling the recovery of new packets. This can be seen as message-passing in the bipartite graph Fig. 1(b), opening the possibility of applying theory of rateless codes to IRSA schemes to analyze both asymptotic \cite{2} and finite-length performance \cite{3}–\cite{5}.

In this paper, we are interested in the transmission strategy optimization performed by the BS. Performance bounds in the asymptotic regime can be easily evaluated and can act as guidelines for a proper selection of the transmission strategy. The accuracy of such guidelines however reduces in non-asymptotic settings. Exact performance analysis has been recently derived for the finite block length case; unfortunately it is computationally too expensive for medium block sizes in practical implementations. Frameless ALOHA protocols can compensate for inaccurate

L. Toni is with the Electrical and Electronic Department, University College London (UCL), London WC1E 7JE, U.K. (e-mail: l.toni@ucl.ac.uk). P. Frossard is with École Polytechnique Fédérale de Lausanne (EPFL), Signal Processing Laboratory - LTS4, CH-1015 Lausanne, Switzerland. (e-mail: pascal.frossard@epfl.ch.)
performance evaluation, since the MAC frame is not set a priori. New slots are added until a sufficiently high fraction of packets has been decoded. However, this comes at the price of having changing MAC frame size over time. Rather than analytically deriving the IRSA system performance, we propose to estimate directly the optimal design of IRSA algorithms by online learning. We cast the network optimisation problem as a multi-arm bandit (MAB) problem \[6\], i.e., sequential decision strategy in which a decision maker needs to select the optimal “arm”. The BS is the decision maker who measures the overall performance (reward) of each arm (transmission strategy) and learns the optimal network design (optimal actions) by trial and error. To improve the learning performance, we design a specific IRSA-based MAB algorithm where we infer the IRSA structure into the learning problem in order to improve on the low convergence of classical MAB solutions. Simulation results show that our learning algorithm reaches higher mean reward with respect to transmission strategies designed based on i) classical bandit problems ii) analytical performance studies.

II. ONLINE IRSA OPTIMIZATION

A. Problem Formulation

The performance of the system is measured by an utility function \( U(r) \) that is a non-decreasing function of the number of per-source decoded packets \( r \leq K \) at the BS. In this work, we consider a generic utility function of the form

\[
U(r) = w \log(r + 1)
\]

where \( w \) is a scaling factor. The above utility function is typical in multimedia transmission, (e.g., visual sensor networks) and \( w \) reflects different priorities among different sources. However, the proposed learning strategy applies to any other increasing utility function.

In the above framework, optimising the system performance consists in finding the best transmission strategy (i.e., \( \Lambda(x)^*, K^* \)), such that the overall mean utility function per source is maximised. Formally, the optimisation problem is

\[
(\Lambda(x)^*, K^*) : \arg\max_{\{\Lambda(x), K\}} \sum_{r=0}^{K} w \log(r + 1) P(r|\Lambda, K; M, L) \\
\text{s.t.} \ LK \leq M
\]

where \( P(r|\Lambda, K; M, N) \) is the probability of decoding \( r \) packets in the scenario with \( L \) sources transmitting \( K \) packets over \( M \) time slots. The network traffic \( LK/M \) is finally constrained to be lower than 1, which corresponds to the stability limit of the SA MAC protocol \[2\].

B. MAB problem formulation

We now cast the above optimisation as a MAB problem. In our case, the BS is the decision maker, which can periodically adjust the transmission strategy. Let \( t \) denote the decision opportunity timestamp\[3\]. Between two

\[3\] We consider a decision every MAC frame but less frequent decisions can also be considered.
Algorithm 1 UCB

**Input:** $n$ (horizon), $A$ (number of arms)

**Initialize:** set $\hat{\mu}_{a,0}$ from the asymptotic theoretical analysis.

$t = 1$

while $t \leq n$ do

\[ \alpha_t = \arg \max_{a \in A} \left\{ \hat{\mu}_{a,t} + \beta \sqrt{\frac{2 \log t}{N_{a,t}}} \right\} \]

Select arm $\alpha_t$, receive reward $X_{\alpha_t,t}$

\[ \hat{\mu}_{\alpha_t,t} = (N_{\alpha_t} \hat{\mu}_{\alpha_t,t} + X_{\alpha_t,t})/(N_{\alpha_t} + 1) \]

$N_{\alpha_t} = N_{\alpha_t} + 1$

$t = t + 1$

end while

consecutive decision opportunities, the BS observes the overall system performance (i.e., the number of correctly decoded packets) that is the instantaneous reward. Based on this observation, it selects the next action, which is then communicated to all sources. The overall goal is to minimize the experienced regret, i.e., the loss due to the fact that the globally optimal policy is not achieved all the times.

Let $a \in A$ be one possible transmission strategy, with $A$ being the set of all possible strategies (possible arms). In our IRSA problem, an arm $a$ is associated to the transmission strategy defined by $\{\Lambda_a(x) = [\Lambda_{a,1}^{(a)}, \Lambda_{a,2}^{(a)}, \ldots, \Lambda_{a,K}^{(a)}], K_a\}$. Further $\mu_a = \sum_{r=0}^{K} \log(r + 1) P(r|\Lambda_a(x), K_a; M, L)$ be the mean reward of the arm $a$. At time $t$, the learner selects the action $a$ and experiences an instant payoff $X_{a,t}$, which is a realization of a random variable with unknown distribution $F_a$ and unknown mean value $\mu_a$. When the performance value is known, $P(r|\Lambda_a(x), K_a; M, L)$ could be derived and (2) could be solved numerically. Conversely, if there is no precise value (or no low-complexity evaluation) of the performance evaluation, we can learn $P(r|\Lambda_a(x), K_a; M, L)$ from experience, i.e., by trial-and-error. During the learning process suboptimal actions might be selected, leading to a cumulative regret after $t$ decisions that is $R(t) = t\mu^* - \sum_{i=1}^{t} X_{a_i,i}$ with $a_i$ being the arm selected at the $i^{th}$ decision opportunity and $\mu^*$ the mean reward of the optimal arm.

The classical algorithm to solve MAB problems is the upper confidence bound (UCB) algorithm [6] (see Alg. 1). The UCB solves MABs using the optimism in face of uncertainty principles in order to find the best tradeoff between exploration and exploitation. Rather than selecting the action with the highest estimated reward $\hat{\mu}_{a,t}$, the UCB selects the reward with the highest bandit index $b_{a,t} = \hat{\mu}_{a,t} + \beta \sqrt{\frac{2 \log t}{N_{a,t}}}$, with $N_{a,t}$ being the number of times the arm $a$ has been selected up to $t$, and $\beta$ a multiplicative factor. The second term in the bandit index represents an upper confidence bound that reflects the uncertainty on the estimates of $\hat{\mu}_{a,t}$. Asymptotically, the UCB algorithm minimizes the regret $R(t)$ and therefore it maximizes the mean reward $\sum_{i=1}^{t} X_{a_i,i}$. Let define $X_{a,t} = (1/L) \sum_{i=1}^{L} \log(r_{a,t}^{(i)} + 1)$, with $r_{a,t}^{(k)}$ being the number of decoded packets for source $k$ with action $a$ taken at time $t$. This means that asymptotically the UCB optimizes the expected utility function for each source, i.e., $\sum_{r=0}^{K} w \log(r + 1) P(r|\Lambda, K; M, L)$. Defining the set of actions as $A : \{a = (\Lambda_a(x), K_a)|LK_a \leq M\}$ the UCB algorithm asymptotically optimises the problem in (2).

C. Proposed online learning algorithm

Classical UCB methods can be improved if prior information is inferred in the learning process. This is the reason why we propose an online learning solution based on a Bayesian UCB (or Bayes-UCB) algorithm [7] that allows us to infer information about the reward distribution. We represent the uncertainty about the system in terms of variance of the process rather than a confidence bound of the estimate as in classical UCB methods. Thus, the learning process consists in selecting at each time instant $t$ the arm $\alpha_t$ that maximises the following function $\alpha_t = \arg \max_{a} \{\hat{\mu}_{a,t} + \beta \sigma_{a,t}\}$, with $\hat{\mu}_{a,t}$ and $\sigma_{a,t}^2$ being respectively the mean and variance of the reward of action $a$ estimated at the decision opportunity $t$, and $\beta$ being a multiplicative factor. The Bayes-UCB algorithm thus automatically builds confidence intervals based on a Kullback-Leibler divergence, which best fits the geometry of the problem (see Appendix).

The Bayes-UCB algorithm initiates a mean and variance per arm. As there exists no exact information about the mean and the variance of the process in our IRSA system, we derive estimates from the asymptotic setting [2].
Algorithm 2 Bayesian UCB

**Input:** \( n \) (horizon), \( A \) (number of arms)

**Initialize:** set \( \hat{\mu}_{a,0} \) and \( \sigma^2_{a,0} \)

\[
t = 1
\]

while \( t \leq n \) do

\[
\alpha_t = \arg \max_a \{ \hat{\mu}_{a,t} + \beta \sigma_{a,t} \}
\]

Select arm \( \alpha_t \), receive reward \( X_{\alpha_t,t} \)

Perform Bayesian update to derive \( \hat{\mu}_a(t) \), and \( \sigma_a(t) \)

\[
t = t + 1
\]

end while

and under the assumption that each source independently assigns packets to transmission slots. In practice, the independency holds only among packets sent from different sources, therefore less collisions are actually experienced than the number predicted from the theory. The derived estimate is then refined by the Bayes-UCB algorithm at each decision opportunity.

For the arm \( a \) with transmission strategy \( (\Lambda_a(x), K_a) \), we evaluate \( P_e(\Lambda_a) \) as the probability for a packet to be lost after SIC. This is derived from the asymptotic analysis in [2], exploiting the SIC convergence analysis and the probability of a burst node edge being not resolved at an iteration, [2, Eq 2]. Due to the independency between sources, the probability for one source to correctly decode \( r \) source packets is

\[
P(r|\Lambda_a, K_a; M, L) = \left( \frac{K_a}{r} \right) P_e(\Lambda_a)^r [1 - P_e(\Lambda_a)]^{K_a - r}.
\]

We then note that the mean reward of the utility given in (1) is given by the logarithm of a binomial random variable. Therefore, by expanding the logarithm operator as a Taylor series in \( x = K_a P_e(\Lambda_a) \) and computing the binomial sum term by term, the mean reward and the reward variance of action \( a \) can be estimated with

\[
\mu_a = \log (K_a P_e(\Lambda_a) + 1), \quad \sigma^2_a = \frac{K_a P_e(\Lambda_a)(1 - P_e(\Lambda_a))}{(P_e(\Lambda_a)K_a + 1)^2}
\]

The initial estimates \( \hat{\mu}_{a,0} \) and \( \sigma^2_{a,0} \) are thus derived from the above equations, and the learning algorithms proceeds as described in Alg. [2].

**D. Computational Complexity**

Our proposed learning strategy is a low-complexity algorithm compared to transmission strategies designed based on finite-length performance analysis. At each decision opportunity, the only operations that need to be computed are \( i \) the selection of arms (which is a maximization of a known vector), \( ii \) the update of the mean reward only for the selected arm (a weighted sum). The initialization process requires the evaluation of (3) for all possible arms. The cardinality of the arm set depends on \( l_{max} \) and the maximum value of \( K \), but not on the MAC frame, neither on the number of sources. Moreover, this initialization step can be further simplified by approximating the packet error probability by a binary condition given by [2, Eq 7].

The exact finite length analysis [4], [5] has in contrary a complexity that scales exponentially. The combinatorial approach in [5] spans the all possible combinations of transmission realizations with a complexity \( O(|\mathcal{N}| l_{max}^L K) \), with \( |\mathcal{N}| \) being the cardinality of the set \( \mathcal{N} \), which is the set of possible edge-realizations in the bipartite graph. This is given by \( |\mathcal{N}| \leq \left( \frac{\hat{C} + M - 1}{2} \right) \), with \( \hat{C} = \sum_{n=0}^{L K - 2} \binom{L K}{M} \). A computational complexity that scales exponentially with the MAC frames and number of users is achieved also in [4], where the system performance evaluation is derived based on a finite Markov state machine and each state is characterized by the size of the cloud and ripples (with different degrees).

**III. Simulation results**

We now provide simulation results that show the performance of the proposed learning strategies with respect to \( (i) \) classical UCB algorithm, and \( (ii) \) MAC protocol optimisation based on asymptotic strategies. Results are
provided in terms of both regret \( R(t) = t \mu^* - \sum_{i=1}^t X_{\alpha,i} \) and experienced reward. Note that the finite-length analysis leads to the optimal decision strategy at each time step, meaning a null cumulative regret. Therefore, the regret shows the tradeoff between complexity (in evaluating the finite length analysis) and performance approximation (due to the learning process). The proposed algorithm (labeled “Proposed”) is compared with the classical UCB algorithm, both with \( \beta = 1 \). We also compare to a baseline solution (denoted “Asymptotic”) where the optimal transmission strategy is not learned but rather computed by assuming an asymptotic behavior of the system (i.e., very large MAC frame duration). Namely, \( P_e(\Lambda_a) \) in [3] is evaluated from [2] Eq 2 and the optimisation problem in (2) is solved numerically. Finally, we also consider a learning algorithm (labeled as “Proposed (\( \beta = 0 \))”) that does not take into account the confidence bound of the estimation. Namely, we consider our proposed algorithm with \( \beta = 0 \), which actually corresponds to the UCB algorithm with \( \beta = 0 \).

We first consider the case in which the transmission probability \( \Lambda(x) \) is fixed and the actions correspond to different number of packets per source \( K \). Fig. 2 provides the cumulative regret for the scenario with 300 time slots per MAC frame, 20 sources and \( \Lambda(x) = 0.75x^2 + 0.25x^3 \). The proposed learning solution (IRSA Bayes-UCB) generally outperforms all the other learning strategies. Finally, in this specific setting, the optimisation based on the asymptotic theoretical performance achieves good performance, even if it performs worse than our proposed learning strategy. Its performance however degrades in other settings as shown next.

In Fig. 3 the cumulative regret is provided when \( \Lambda(x) \) and \( K \) are jointly optimised in the scenario of 20 sources
and 300 time slots. The candidates $\Lambda(x)$ are defined as $\Lambda(x) = a_1x^2 + a_2x^3 + a_3x^8$, with the coefficients $a_i$ ranging from 0 to 1, i.e., $a_i \in [0:0.25:1], i = 1, 2, 3$. The candidate values of $K$ range from 1 to $\lfloor M/L \rfloor$. Moreover, note that the gain achieved by the proposed method (IRSA Bayes-UCB) with respect to the UCB algorithm has increased compared to the previous setting. This is due to the fact that the action space in this scenario has increased, leading to a slower learning curve and therefore to a higher gain in exploiting prior information while learning, as implemented in our algorithm. It is worth mentioning that the “Proposed ($\beta = 0$)” baseline performs very closely to the proposed learning strategy. This is due to the small randomness of the reward for all arms beyond the traffic threshold value $G^*$, as explained in the Appendix. However, the randomness in the remaining arms makes the “Proposed ($\beta = 0$)” worse than our IRSA Bayes-UCB algorithm. Finally, we provide the mean reward experienced over time in Fig. 4. The reward per instant (averaged over 100 runs) confirms the gain of the proposed learning strategy with respect to the baseline methods.

IV. CONCLUSIONS

We have proposed a learning framework for designing optimised IRSA transmission strategies. We have casted the optimal resource allocation and transmission rate optimisation as a multi-arm bandit (MAB) problem. We have then implemented a specific MAB algorithm that is able to exploit the initial knowledge about IRSA schemes in the form of asymptotic theoretical performance. This allows us to infer the structure of IRSA in MAB problems and improve the learning efficiency of the algorithm. Simulation results have validated our theoretical analysis and demonstrated the gain of the proposed learning strategy in all tested settings.

APPENDIX

SUBOPTIMALITY OF UCB

The key intuition of the UCB algorithm is the following. At time $t$, the estimated payoff for arm $a$ is $\hat{\mu}_{a,t}$ and it differs from the estimated one of at most $U$ (i.e., $|\hat{\mu}_{a,t} - \mu_a| \leq U$) with probability $p$ given by \[ p \leq e^{-2N_{t,a}U^2}. \]

Imposing a probability $p$ decreasing with time (e.g., $p = t^{-4}$) leads to $U_{\text{max}} = \sqrt{2 \log t / N_{t,a}}$. To ensure that the learner selects the optimal action as $t \rightarrow \infty$, the estimated reward $\hat{\mu}_{a,t}$ is added to the confidence bound $U_{\text{max}}$, which leads to Algorithm 1 in Sec. \[ \text{IV}. \]

However, arms with a traffic lower than the threshold $G^*$ will experience an almost deterministic payoff. Denoting the network traffic as $G = LK/M$, for $G < G^*$ packets are received with probability almost 1, while for $G > G^*$ the probability of correctly receiving the packets collapses to 0. This waterfall effect is typical for IRSA [2]. This means that $|\hat{\mu}_{a,t} - \mu_a| \leq \epsilon$ with probability almost 1, for small $\epsilon$ and $t$. Therefore, the confidence bound actually differs from different arms. For this reason, the UCB results in a suboptimal algorithm. Conversely, the Bayesian method allows us to infer this heterogeneity in the uncertainty of the arms, by imposing different variance values or the reward for different arms.

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