Abstract

There is lot of excitement in recent literature about the possibility of parity doubling in observed experimental spectrum, which, we point out, - is known for a long time - over 40 years. At the group theoretical level of classification of collective excitations one can fit the observed hadronic resonances with a simple three parameter mass relation of the SU(3) subgroup of the underlying U(15/30) graded Lie group to an accuracy of 5%. The baryonic excitations are the appropriate supersymmetric partners of the mesons. The diquark - antiquark similarity which has been shown to be important in phenomenology of pulsars can explain the supersymmetry. The parity doubling may indicate restoration of chiral symmetry. Significantly, the ground state baryons and mesons have no place in the fit, so that the parity doubling is indicated only when large excitation energy is available. We show the correspondence of parity doubling with the calculation of compactness of some pulsars. These pulsar properties can only be explained by strange quark matter with chiral symmetry restoration (CSR) for high density. Taking the example of the recently determined experimental mesonic resonances we see that parity doubling and clustering occurs at energy densities which are comparable to surface densities of strange stars - where we expect CSR.

Keywords: Parity doubling, Chiral symmetry restoration

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1 Introduction

Dynamical groups and spectrum generating algebras were much used in the literature for classifying states of a composite system in terms of underlying group symmetry of the Hamiltonian. The pioneers in this field were Barut [1], Barut and Bohm [2] and Dothan, Gell Mann and Ne’eman [3]. The subject has been reviewed in detail recently in two volumes edited by Bohm, Ne’eman and Barut [4].

The possibility that parity doubling is observed in baryon resonances was realized by Dey and Dey in 1993 [5]. This was inspired by the early work of Barut [1] in 1965 where he had looked at parity doubled states in the conformal O(4,2) model. The conformal model was revived in the version of string theory given by Kutasov and Seiberg [6] and leads to a rich phenomenology as shown by the Freund and Rosner [7], Dey, Dey and Tomio [8], Cudell and Dienes [9] and Mustafa et al. [10]. This is based on old wisdom of the Regge model giving same trajectory for mesons and baryons. According to Kutasov and Seiberg, the appearance of the destabilizing tachyons in a string theory severely constrains the difference of the densities of bosons and fermions in that theory. The results show that tachyon elimination does not require full-fledged supersymmetry. Cancellation between the boson and fermion states is all that is needed. It turns out that though the density of states of mesons and baryons each rises exponentially with energy, their difference rises only like a low power of energy.

The parity doubling in baryon resonances seem to have been rediscovered in 2000 by Glazman [11] and there are many follow up papers [12] - including [13]. In the paper mentioned last - the authors claim that chiral symmetry realized in the Nambu Goldstone mode does not predict the existence of degenerate multiplets of hadrons of opposite parity. However they assert that their arguments do not preclude the restoration of chiral symmetry at high temperature or high chemical potential.

But the energy densities in the high-lying mesonic resonances are estimated in this paper to be the same as that of strange quark matter (SQM) at the surface of a strange star. Hence it is not surprising that CSR is observed, namely the density of states for positive and negative parity is comparable. The strange star model is crucial in explaining many observations like

(a) super bursts [14]

(b) the minimum magnetic field for all observed pulsars [15]

(c) absorption and emission bands along with high redshift [16]

The next section briefly revives the simple compact group structure used by Dey and Dey [5], and we reproduce the baryons referring the reader to the original paper for the mesonic resonances.

For the mesons we list states from 2 GeV to 2.45 GeV, which show the so called clustering observed recently by Afonin [17]. Most of the resonances are recently found and need confirmation according to the Particle Data Group. But the energy density of each resonance in these states is high - suggesting chiral symmetry restoration. With respect of the objections to chiral symmetry restoration we agree with Afonin who observes: one should be careful with any

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1The authors are grateful to F. Iachello for pointing out the current relevance of these papers.
2 Baryon resonances

The common feature of the two very different calculations [1] and [18] is retained in the model - namely that the ground state does not show parity doubling but the excited states of the baryons do.

Looking at the baryon spectrum we see that the lowest states, i.e. the nucleon octet \((N, \Lambda, \Sigma, \Xi)\), have no odd parity partners. But when some excitation energy is available, we encounter parity doubling. It is no surprising that one can get this at finite \(T\), in lattice as we have already discussed. The Laplace transform of the finite temperature partition function in fact gives the excitation spectrum at zero temperature [8].

Let us take some examples, one can think of \(N(1535), \Lambda(1670)\) states \(N(1440), \Lambda(1600)\) \(1/2^+\); the \(N(1675), 5/2^-\) is almost degenerate with the \(N(1680), 5/2^+\) state etc. Chiral symmetry is realized at such high excitation within 5-10 %. In the non-relativistic and the semi-relativistic models, the occurrence of parity doubled states is contrived: the perturbative hyperfine interaction is adjusted to bring down the even parity to match with the odd levels, sometimes invoking multi-shell configurations [19] and sometimes deformation [20].

In an earlier paper [21], mesons and baryons were fitted in a simple model using the supersymmetric graded Lie group \(U(15/30)\). The odd parity baryons are added [5] and are important in view of the present interest. In the meson sector the interest is of a different nature.

Our model is based on excitations of bosons and fermions in the \(s, d\) and \(g\) shells of some effective potential, in terms of a \(U(15/30)\) graded Lie group. The reduction of this group into simpler structures, in particular to the \(SU(3)\) scheme, so well known in Nuclear Physics [22] have been worked out [23], [24]. In [21] a mass formula was given which fitted more than 60 mesons and baryons using the classification given by Yu. We now add more than 40 new states.

The scheme we follow is extremely simple with states belonging to a representation where the total number of particles is three. This implies that the mesons are different from other models. Here they are two fermion, one boson states: \(f \bar{f} b\) with the boson possibly representing some collective gluonic degree of freedom. The meson and baryon states are then classified according to the partition \([\nu]\) and the \(SU(3)\) representation \((\lambda, \mu)\). The \(SU(3)\) Casimir operators are \(L(L+1)\) and :

\[
C(\lambda, \mu) = \lambda^2 + \mu^2 + \lambda \mu + 3(\lambda + \mu)
\]  

Octet baryons (spin 1/2) belong to the partition [21]. This and the allowed values of \((\lambda, \mu)\) are given in Table (1). The isobars can be placed in the partition \(\nu = [111]\). The allowed \((\lambda, \mu)\) are also given in Table (1).

The mass \(M\) of the baryonic resonance is then given by

\[
M = 2700 - 9C(\lambda, \mu) + 8S(S + 1) + \alpha L(L + 1)
\]
We only list the baryonic resonances deferring discussion of the mesonic states till the next section. There are 54 resonances in the Tables (2, 3, 4, 5).

In an effective potential such a sdg-structure is natural. The SU(3) scheme diagonalizes the first two leading terms of the expansion of the effective potential about a local minimum, namely the oscillator term and the quadrupole term. To illustrate the procedure we list the baryon fits in Tables (4, 5). 8 positive and 6 negative parity baryons are fitted in these tables for which $S = 3/2$ in eq. (2) and $\alpha$ is taken to be $30 \, MeV$. The fit to experiment is 2.93%. The experimental states consist of 9 well established and 5 states with (**) status. All levels are fitted and the fit is generally similar.

Table 1: The partition $[\nu]$, SU(3) Young representation $(\lambda, \mu)$, the fermion number $N_F$ and spin $S$ given for $N \equiv N_F + N_B = 3$

| $[\nu]$ | $(\lambda, \mu)$ | $(N_F, S)$       |
|---------|------------------|-----------------|
| [3]     | (12,0) (8,2) (6,3) (6,0) (4,4) (3,3) .... (2,0) |                 |
| [21]    | (10,1) (8,2) (6,3) (7,1) (6,3) (6,0) .... (2,0) | (2,1) (3,1/2)   |
| [111]   | (9,0) (6,3) (6,2) (3,3) (3,0) (2,5) .... (2,1) | (3,3/2)         |

The Roper resonance N(1440) and the N(1710) are fitted very well in Table (2). The placement of 1710 in the second band is consistent with its very different properties: it has a very large two pion decay rate and very little $N\pi$ unlike the Roper. Altogether 18 positive parity baryons are fitted in Table (2). The quality of the fit is very good, the difference from experiment being 5.7%. All the well-established $\Lambda$-s and $\Sigma$-s are fitted along with three two-star $\Sigma$-s. It is interesting to note that in Table (2), and indeed also in the subsequent tables, the strange baryons fit along side nucleon excitations.

We include the odd parity baryons in Table(3) and suggest some parity doublets. In [25] the same is done, but there is a very important point of difference between Iachello’s work and ours. His model and the variant suggested by Robson [25] depend essentially on geometrically symmetric configurations. It is difficult to envisage why the ground states may be left out of such geometric symmetries. Our model, on the other hand uses chiral symmetry restoration, which happens only for excited configurations. The phenomenology may find justification only in an effective potential based on non-compact conformal group structure as indicated in Barut’s work [1]. Notice that the spectrum given by him has parity doubling, not in the ground state, but starting from the first excited state.

We have placed some of the odd parity baryons as $L = 0$ excitations of odd-parity quasi-particles in Table 3. This gives much better fit and fits in with the fact that $\Lambda(1405)$ and the $N(1535)$ are different from the 1520 $\Lambda$ or $N$. This is supported by the rather unusual 100 %
Table 2: Even Parity baryons $\alpha = 40$ MeV. All energies in MeV

| Band | $C(\lambda,\mu)$ | $L$ | IBFM | $J^P$ | Experiment |
|------|------------------|-----|------|-------|------------|
| 1    | 144              | 0   | 1410 | 1/2$^+$ | $N(1440) : (1430 - 1470)$ |
|      |                  |     |      |       | $\Lambda(1600) : (1560 - 1700)$ |
|      |                  |     |      |       | $N(1680) : (1675 - 1690)$ |
|      |                  |     |      |       | $\Lambda(1820) : (1815 - 1825)$ |
|      |                  | 2   | 1650 | 5/2$^+$ | $N(1720) : (1650 - 1750)$ |
|      |                  |     |      |       | $N(2210) : (2180 - 2310)$ |
|      |                  |     |      |       | $\Lambda(2350) : (2340 - 2370)$ |
|      |                  | 4   | 2210 | 9/2$^+$ | $\Sigma(2030) : (2025 - 2040)$ |
|      |                  |     |      |       | $N(1990) : (**)$ |
|      |                  | 6   | 3090 | 13/2$^+$ | $N(2700) : (**)$ |
| 2    | 114              | 0   | 1680 | 1/2$^+$ | $N(1710) : (1680 - 1740)$ |
|      |                  |     |      |       | $\Lambda(1810) : (1750 - 1850)$ |
|      |                  |     |      |       | $\Sigma(1660) : (1630 - 1690)$ |
|      |                  | 2   | 1920 | 3/2$^+$ | $\Lambda(1890) : (1850 - 1910)$ |
|      |                  |     |      |       | $\Sigma(2080) : (**)$ |
|      |                  | 2   | 1920 | 5/2$^+$ | $\Lambda(2110) : (2090 - 2140)$ |
|      |                  |     |      |       | $\Sigma(1915) : (1900 - 1935)$ |
|      |                  |     |      |       | $N(2000) : (**)$ |

$\Sigma \pi$ decay of the first one and the 45-55% $N\eta$ decay of the second one. The reader who does not like such exotic quasi-particles may group them together with the $(L = 1)$ 1520 $\Lambda$ and $N$.

The number of odd parity baryons fitted in Table (3) is 24, the fit is comparable to that of Table (2). In fact using all the 42 states the fit improves marginally to 5.03%.

3 Mesonic resonances.

For mesonic resonances we do not try to fit to any model in great detail. It is sufficient for us to note that there are 319 positive parity mesons and 306 negative parity ones in the range $\Delta E = 2$ to 2.45 GeV [26]. Specific examples of parity doubling are striking: for example $b_1 (I^G(J^{PC})1^+(1^+\mp))1960$, $\rho 1^+(1^-)1965$. But rather than specifics - the general fits appeal to us more for reasons which we will now discuss.

Most of these states are new. And in four pages of Particle Data Table one will find many states marked with X along with the states which we tabulate in Table (6). We hope these X-states will be found very soon with the improving experimental facilities. As such we have
Table 3: Odd Parity baryons $\alpha = 40$ MeV. All energies in MeV

| Band | $C(\lambda, \mu)$ | $L$ | IBFM | $J^P$ | Experiment        |
|------|------------------|-----|-------|-------|-------------------|
| 1    | 144              | 0   | 1410  | 1/2−  | $N(1535) : (1520 - 1555)$ |
|      |                  |     |       |       | $\Lambda(1405) : (1407 \pm 4)$ |
|      |                  | 2   | 1650  | 5/2−  | $N(1675) : (1670 - 1685)$ |
|      |                  | 4   | 2210  | 9/2−  | $N(2250) : (2170 - 2310)$ |
|      |                  | 1   | 1490  | 3/2−  | $\Lambda(1520) : (1519.5 \pm 1)$ |
|      |                  |     |       |       | $N(1520) : (1510 - 1530)$ |
|      |                  |     |       |       | $\Sigma(1580) : (\ast \ast)$ |
| 1    |                  | 1   |       | 1/2−  | $\Sigma(1620) : (\ast \ast)$ |
| 3    | 1890            | 5/2− |       |       | $\Sigma(1775) : (1770 - 1780)$ |
|      |                  |     |       |       | $\Lambda(1830) : (1810 - 1830)$ |
| 3    |                  | 7/2− |       |       | $\Lambda(2100) : (2090 - 2110)$ |
| 5    | 2610            | 11/2− |       |       | $N(2600) : (2550 - 2750)$ |
| 2    | 114             | 1680 |       | 1/2−  | $N(1650) : (1640 - 1680)$ |
|      |                  |     |       |       | $\Sigma(1750) : (1730 - 1800)$ |
|      |                  |     |       |       | $\Lambda(1670) : (1660 - 1680)$ |
|      |                  | 1920 |       | 3/2−  | $\Sigma(1940) : (1900 - 1930)$ |
|      |                  |     |       |       | $N(2050) : (\ast \ast)$ |
|      |                  | 2   | 1920  | 5/2−  | $N(2200) : (\ast \ast)$ |
|      |                  | 1760 |       | 3/2−  | $\Sigma(1670) : (1665 - 1685)$ |
|      |                  |     |       |       | $N(1700) : (1650 - 1750)$ |
|      |                  |     |       |       | $\Lambda(1690) : (1685 - 1695)$ |
| 3    | 90              | 1896 |       | 1/2−  | $\Lambda(1800) : (1720 - 1850)$ |
|      |                  | 1976 |       | 3/2−  | $\Sigma(1940) : (1900 - 1950)$ |

a simple explanation for large number of nearly matching states of both parity which are not marked with X. In a recent paper aimed at finding the surface tension of strange stars [27] we found that

(1) the quarks, on the average, occupy a sphere of radius $r_n = 0.51$ fm and the mean interparticle distance $r_0 = 0.47$ at the surface of a strange star. Assuming $\sigma_{pp} = 3\sigma_{qq}$, following Heiselberg and Pethick [28], our estimate of $r_n$ is right [27].

(2) Thus at this density the size of a two particle cluster would be about 1 fm which will imply an energy density which is a quarter of the interval we have chosen $\Delta E$ of about 500 to 610 MeV/fm$^3$. This is the energy density at the surface of the star where chiral symmetry is restored. The chiral symmetry restoration was checked in a simple model by Ray, Dey and Dey [29].
Table 4: Even Parity isobars $\alpha = 30 \text{ MeV}$. All energies in MeV

| Band | $C(\lambda, \mu)$ | $L$ | IBFM | $J^P$ | Experiment          |
|------|------------------|-----|------|-------|---------------------|
| 1    | 108              | 0   | 1758 | $3/2^+$| $\Delta(1600) : (1550 - 1700)$ |
|      |                  | 2   | 1938 | $7/2^+$| $\Delta(1950) : (1940 - 1960)$ |
|      |                  |     |      | $5/2^+$| $\Delta(1905) : (1870 - 1920)$ |
|      |                  |     |      | $3/2^+$| $\Delta(1920) : (1900 - 1970)$ |
|      |                  |     |      | $1/2^+$| $\Delta(1910) : (1870 - 1920)$ |
| 2    |                  | 4   | 2358 | $11/2^+$| $\Delta(2420) : (2300 - 2400)$ |
|      |                  |     |      | $9/2^+$| $\Delta(2300) : (*)$ |
| 3    |                  | 6   | 3018 | $15/2^+$| $\Delta(2950) : (*)$ |

Table 5: Odd Parity isobars $\alpha = 30 \text{ MeV}$. All energies in MeV

| Band | $C(\lambda, \mu)$ | $L$ | IBFM | $J^P$ | Experiment          |
|------|------------------|-----|------|-------|---------------------|
| 1    | 108              | 0   | 1758 | $3/2^-$| $\Delta(1700) : (1670 - 1770)$ |
|      |                  | 2   | 1938 | $1/2^-$| $\Delta(1900) : (1850 - 1950)$ |
|      |                  |     |      | $5/2^-$| $\Delta(1930) : (1920 - 1970)$ |
|      |                  | 4   | 2358 | $9/2^-$| $\Delta(2400) : (*)$ |
|      |                  | 1   | 1818 | $1/2^-$| $\Delta(1620) : (1615 - 1675)$ |
| 4    |                  | 5   | 2718 | $13/2^-$| $\Delta(2750) : (*)$ |

4 Summary and conclusion.

In this paper we have revived the old models of Barut [1, 2], Dey and Dey [5, 21] and tried to establish that parity doubling and supersymmetry are high density phenomenon in excited state spectrum of the resonances which find parallel in the simple high density matter calculations rather than in complicated low energy models.

In summary we have suggested that the high energy density in mesonic resonances should be explored experimentally since they may support the chiral symmetry restoration in models of strange stars.

Acknowledgements.

The authors wish to thank Professor Iachello for a useful discussion during a visit in 2005 by MD and JD to ECT, Trento and for pointing out that the papers [5, 21] is interesting and should be revived.
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Table 6: List of mesons at high energy

| Energy (in MeV) | $I^G(J^P_C)$ | State   | $I^G(J^P_C)$ | State   |
|-----------------|---------------|---------|---------------|---------|
| 2000            | $0^+(2^{++})$ | $f_2(2000)$ | $1^+(1^-)$   | $\rho(2000)$ |
|                 | $1^-(0^{++})$ | $a_0(2020)$ | $1^-(2^{--})$| $\pi(2005)$ |
|                 | $1^+(3^{+-})$ | $b_3(2025)$ | $0^+(0^{+-})$| $\eta(2010)$ |
|                 | $0^+(3^{++})$ | $f_3(2050)$ | $0^-(3^-)$   | $h_3(2025)$ |
|                 | $0^+(0^{++})$ | $f_0(2060)$ | $0^+(2^{++})$| $\eta_2(2030)$ |
|                 | $1^-(3^{++})$ | $a_3(2070)$ | $0^-(0^-)$   | $\pi(2070)$ |
|                 | $1^-(2^{++})$ | $a_2(2080)$ |               |         |
| 2100            | $0^+(2^{++})$ | $f_2(2140)$ | $0^+(0^{+-})$| $\eta(2100)$ |
|                 | $0^+(2^{++})$ | $a_2(2175)$ | $1^+(3^{-?})$| $X(2110)$  |
|                 | $0^+(2^{++})$ |            | $0^-(1^-)$   | $\omega(2145)$ |
|                 | $0^+(2^{++})$ |            | $0^+(0^-)$   | $\eta(2190)$ |
|                 | $0^+(2^{++})$ |            | $0^-(2^-)$   | $\omega_2(2195)$ |
| 2200            | $0^-(1^{++})$ | $\omega(2205)$ | $1^+(2^-)$ | $\rho_2(2240)$ |
|                 | $0^-(1^{++})$ | $h_1(2215)$ | $1^+(4^{-})$ | $\rho_4(2240)$ |
|                 | $0^-(1^{++})$ | $b_1(2240)$ | $0^-(2^-)$   | $\pi_2(2245)$ |
|                 | $1^-(1^{++})$ | $a_1(2270)$ | $0^+(2^{--})$| $\eta_2(2250)$ |
|                 | $1^-(2^{++})$ | $a_2(2270)$ | $1^-(4^{-})$ | $\pi_4(2250)$ |
|                 | $0^-(3^{++})$ | $h_3(2275)$ | $0^-(4^{-})$ | $\omega_4(2250)$ |
|                 | $1^-(4^{++})$ | $a_4(2280)$ | $0^-(3^-)$   | $\omega_4(2255)$ |
|                 |               |            | $0^+(4^{-?})$| $X(2260)$  |
|                 |               |            | $1^+(1^-)$   | $\rho(2265)$ |
|                 |               |            | $0^+(0^-)$   | $\eta(2280)$ |
|                 |               |            | $1^+(1^-)$   | $\rho(2280)$ |
|                 |               |            | $0^-(3^-)$   | $\omega_5(2285)$ |
| 2300            | $0^+(3^{++})$ | $f_3(2300)$ | $1^+(3^-)$   | $\rho_3(2300)$ |
|                 | $1^-(3^{++})$ | $a_3(2310)$ | $0^+(4^{+-})$| $\eta_4(2320)$ |
|                 | $0^+(1^{++})$ | $f_1(2310)$ | $0^+(1^{-})$ | $\omega(2330)$ |
|                 | $0^+(0^{++})$ | $f_0(2330)$ | $0^-(0^-)$   | $\pi(2360)$ |
|                 | $1^-(1^{++})$ | $a_1(2340)$ |               |         |