Double transverse spin asymmetries in vector boson production

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We investigate a helicity non-flip double transverse spin asymmetry in vector boson production in hadron-hadron scattering, which was first considered by Ralston and Soper at the tree level. It does not involve transversity functions and in principle also arises in W-boson production for which we present the expressions. The asymmetry requires observing the transverse momentum of the vector boson, but it is not suppressed by explicit inverse powers of a large energy scale. However, as we will show, inclusion of Sudakov factors causes suppression of the asymmetry, which increases with energy. Moreover, the asymmetry is shown to be approximately proportional to $x_1 g_1(x_1) x_2 g_1(x_2)$, which gives rise to additional suppression at small values of the light cone momentum fractions. This implies that it is negligible for $Z$ or $W$ production and is mainly of interest for $\gamma^*$ at low energies. We also compare the asymmetry with other types of double transverse spin asymmetries and discuss how to disentangle them.

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I. INTRODUCTION

Double transverse spin asymmetries in high energy hadron-hadron collisions have attracted much theoretical attention (starting with the early investigations [1,2]), but no experimental studies have been performed so far. At the BNL Relativistic Heavy Ion Collider (RHIC) transversely polarized protons will be collided for the first time. Therefore it is important to make an analysis of the double transverse spin asymmetries. The proposed DESY HERA-$\vec{N}$ experiment also prompts such a study (see e.g. [3]). For the transversity double spin asymmetry in the Drell-Yan process such studies have been performed in considerable detail [4]. In this paper we will mainly investigate another type of double transverse spin asymmetry, one which does not involve transversity functions.

In general, helicity non-flip quark and gluon states in transversely polarized hadron-hadron scattering will lead to power suppression [O$(1/Q^2)$], where $Q^2$ is the vector boson virtuality. In the present study we will exploit the fact that if the transverse momentum of the produced vector boson is observed, this no longer holds true. By observing the transverse momentum certain azimuthal asymmetries can occur in the cross section without explicit power suppression. The possibility of such an unsuppressed helicity non-flip double transverse spin asymmetry has been noted before in the literature and a tree level expression for the case of a virtual photon has been given [5,6]. Here we investigate specifically the case of weak vector boson production and the effect of inclusion of Sudakov form factors.

The helicity non-flip nature will allow for double transverse spin asymmetries even in $W$ production (for which the helicity flip contribution is absent [7]). Unfortunately, such transverse momentum dependent azimuthal asymmetries will turn out to suffer from suppression due to Sudakov factors, which increases with energy, as was forseen in Ref. [7]. We will explore this issue quantitatively in detail and we will show that for the azimuthal double transverse spin asymmetry of interest, the inclusion of Sudakov factors causes suppression by at least an order of magnitude compared to the tree level result and effectively produces a power behavior of $1/Q^\alpha$, with $\alpha \approx 0.6$. Moreover, the asymmetry will be shown to be approximately proportional to $x_1 g_1(x_1) x_2 g_1(x_2)$, which gives rise to additional suppression at small values of the light cone momentum fractions. The conclusion will be that this asymmetry is of interest mainly at lower energies, i.e. for $\gamma^*$ production. This also leaves the option of studying possible contributions to double transverse spin asymmetries in $W$ production from physics beyond the standard model.

The outline of this paper is as follows. In Sec. II we will repeat the essentials of the transversity double spin asymmetry in the Drell-Yan process in order to contrast it to the helicity non-flip asymmetry (Sec. III). We
study the latter asymmetry in the neutral (Sec. IIIA) and in the charged (Sec. IIIB) vector boson case. In order to obtain estimates we will assume Gaussian transverse momentum dependence of the quarks (discussed in Sec. IIIC). We will then include Sudakov form factors in the asymmetry (Sec. IIII) and estimate its quantitative effects (Sec. IIIIE). In Sec. IIIF we will comment on the possible study of physics beyond the standard model via double transverse spin asymmetries in $W$ production.

II. TRANSVERSITY DOUBLE SPIN ASYMMETRY

The main characteristic of the transversity double transverse spin asymmetry of hadron-hadron collisions is that the gluon distribution does not contribute. Hence, at leading order in an expansion in inverse powers of the hard scale(s) only the quark transversity distribution function \[1,8\] (denoted in hadron-hadron collisions is that the gluon distribution does not contribute. Hence, at leading order in an expansion in inverse powers of the hard scale(s) only the quark transversity distribution function \[1,8\] (denoted $h_1, \delta q$ or $\Delta_T q$) contributes. This leads to the well-known expression for the double transverse spin asymmetry in the Drell-Yan process:

$$A_{TT} = \frac{\sigma(p^+ p^- \rightarrow \ell^+ \ell^- X) - \sigma(p^+ p^- \rightarrow \ell^+ X)}{\sigma(p^+ p^- \rightarrow \ell^+ \ell^- X) + \sigma(p^+ p^- \rightarrow \ell^+ X)} = \frac{\sin^2 \theta \cos 2\phi_2}{1 + \cos^2 \theta} \sum_{a,\bar{a}} c_a^2 h_1^a(x_1) \overline{h}_1^{\bar{a}}(x_2).$$

(1)

We would like to note that the sum is over flavors including anti-flavors, otherwise one should add a term in both numerator and denominator for the exchange ($x_1 \leftrightarrow x_2$), since one can use that $h_1^a = \overline{h}_1^{\bar{a}}$. The above asymmetry comes from the following azimuthal dependence in the cross section

$$\frac{d\sigma(p^+ p^- \rightarrow \ell^+ \ell^- X)}{d\Omega dx_1 dx_2} = \frac{\alpha^2}{3Q^2} \sum_{a,\bar{a}} c_a^2 \left\{ y (1 - y) |S_{1T}| |S_{2T}| \cos(\phi_{S_1} + \phi_{S_2}) h_1^a(x_1) \overline{h}_1^{\bar{a}}(x_2) + \ldots \right\}.$$

(2)

The above is expressed in the so-called Collins-Soper frame \[1\] (for details see e.g. \[10,11\]):

$$\hat{t} \equiv q/Q,$n

$$\hat{z} \equiv \frac{x_1}{Q} \hat{P}_1 - \frac{x_2}{Q} \hat{P}_2,$n

$$\hat{h} \equiv q_T/Q_T = (q - x_1 P_1 - x_2 P_2)/Q_T,$n

where $q$ is the vector boson momentum, $P_i$ are the hadron momenta and $\hat{P}_i \equiv P_i - q/(2x_i)$. The azimuthal angles lie inside the plane orthogonal to $t$ and $z$. In particular, $\phi_{S}^\ell$ gives the orientation of $\hat{P}_i$ ($i$ = $1, 2$) and $S_{1T}$ and $S_{2T}$. In the cross sections we also encounter a dependence on $y = l^-/q^-$, which in the lepton center of mass frame equals $y = (1 + \cos \theta)/2$, where $\theta$ is the angle of $\hat{z}$ with respect to the momentum of the outgoing lepton $l$.

The perturbative corrections to the double transverse spin asymmetry, Eq. (1), have been calculated in \[12\] and using the assumption that at low energies the transversity distribution function $h_1$ equals the helicity distribution function $g_1$ or by saturating the Soffer bound, it has been shown in Ref. \[13\] that $A_{TT}$ is expected to be of the order of a percent at RHIC energies.

In Ref. \[14\] it was discussed that in the process $p^+ p^- \rightarrow W X$ the transversity distribution cannot contribute and this is a general feature of chiral-odd functions and charged current exchange processes (see also \[15\]). This means that only the suppressed contributions from the twist-3 distribution function $g_T$ and its gluon analogue $\Delta_T g$ \[14\] contribute (these are chiral-even functions; they mix under evolution). Of course there are contributions of the transversity functions via quark mass terms or via production of other particles that can compensate for the helicity flip, but these are all of higher order in the strong and/or weak coupling constants (e.g. one can think of $p^+ p^- \rightarrow Z X \rightarrow W^+ W^- X$, but this is negligible at RHIC energies). Since neither quarks nor gluons contribute without suppression to the asymmetry $A_{TT}$ in the process $p^+ p^- \rightarrow W X$, it might make this asymmetry a good place to look for contributions from physics beyond the standard model. For instance, scalar or tensor couplings of the quarks to the $W$ could in principle produce an asymmetry. We will return to this issue at the end of this article. First one has to investigate and estimate another standard model mechanism, namely, there is the possibility that the quarks (and gluons) are not exactly collinear to the initial proton, leading to a helicity non-flip asymmetry without explicit suppression factors of $1/Q$. In other words, if one measures the cross section differential in the transverse momentum of the vector boson, either in its angle compared to the other particles or in its magnitude, the helicity non-flip double transverse spin asymmetry can receive contributions at leading order, even for $W$ production. If one averages over this transverse momentum,
then the asymmetry will vanish, but an (inadvertent) incomplete averaging, for instance due to imposed cuts, might still have observable consequences, cf. for instance Ref. [13]. Even though we will show that for \( W \) production this will not be a problem since the asymmetry turns out to be negligible, for \( \gamma^* \) at lower energies this is important to take into consideration.

### III. HELICITY NON-FLIP DOUBLE TRANSVERSE SPIN ASYMMETRY

If one can measure the cross section differential in the transverse momentum of the vector boson, either in its angle compared to the other particles or in its magnitude, then there is the possibility to have a double transverse spin asymmetry at leading order, in principle even for \( W \) production. To illustrate this we will make use of the formalism pioneered by Ralston and Soper [1], which will be applicable in the region where the observed transverse momentum is small compared to the hard scale(s). Since this is a tree level formalism, we will later on include the effects of resummation of soft gluons by combining it with the approach of Ref. [14]. We focus on the Drell-Yan process, first on neutral vector boson production and later on charged vector boson production. However, the expressions also apply to \( p \bar{p} \) [16]. We focus on the Drell-Yan process, first on neutral vector boson production and later on charged vector boson production. If one can measure the cross section differential in the transverse momentum of the vector boson, either in its angle compared to the other particles or in its magnitude, then there is the possibility to have a double transverse spin asymmetry at leading order, in principle even for \( W \) production. To illustrate this we will make use of the formalism pioneered by Ralston and Soper [1], which will be applicable in the region where the observed transverse momentum is small compared to the hard scale(s). Since this is a tree level formalism, we will later on include the effects of resummation of soft gluons by combining it with the approach of Ref. [14].

#### A. Neutral vector boson production

In Eq. (2) we have given the contribution to the cross section that depends on the sum of the two transverse spin angles with respect to the lepton pair production plane, i.e. \( \cos(\phi_{S_1} + \phi_{S_2}) \). This means that if one integrates over the lepton pair orientation, then this azimuthal dependence will average to zero. At order \( 1/Q^2 \) both quarks and gluons can contribute to \( A_{TT} \) via a term in the cross section which does not depend on the lepton scattering plane

\[
A_{TT} \propto \cos(\phi^f_{S_1} - \phi^f_{S_2}) \frac{M_1 M_2}{Q^2} g_T g_T,
\]

but this is expected to be negligible at \( Q^2 = M_Z^2 \). Moreover, it is not at all clear that such a factorized description of the asymmetry holds at this level of next-to-next-to-leading twist, since it is well known that for the unpolarized case this order \( O(1/Q^4) \) does not factorize.

On the other hand, if one were to observe the transverse momentum of the lepton pair compared to the protons, there will be a double transverse spin asymmetry as a function of this transverse momentum \( q_T \), which appears at the leading order in \( 1/Q \). It will involve one more angle \( (\phi^f_{S}) \), but even if one would integrate over this angle (keeping only the magnitude of \( q_T \)) and over the lepton pair orientation \( (\phi^f) \), then there will remain an azimuthal dependence in the cross section that depends on the orientations of the two transverse polarization vectors only.

To make this explicit we will look at Eq. (A1) of Ref. [11], which gives the cross section for the polarized Drell-Yan process \( p^1 p^1 \to \gamma (Z) X \) in the formalism of Ralston and Soper [11] using transverse momentum dependent distribution functions. From the expressions for the production of the \( Z \) boson it is easy to obtain the expressions for the production of the \( W \) boson. We will not repeat all the details of the calculation of the cross section expressions, rather we will focus on the expression as given in the Appendix in Ref. [11]. For the sake of argument it is unimportant to include contributions from the (formally difficult) \( T \)-odd distribution functions, hence we neglect them, but they can be easily included. This leaves

\[
\frac{d\sigma(h_1 h_2 \to \ell \ell X)}{d\Omega dx_1 dx_2 dq_T} = \frac{\alpha^2}{32} \sum_{a,b} \left\{ -K_1^a(y) |S_{1T}| |S_{2T}| \sin(\phi_h - \phi^f_{S_1}) \sin(\phi_h - \phi^f_{S_2}) \frac{F}{M_1 M_2} p_T \cdot k_T g_T g_T + \ldots \right\},
\]

where
\[ K_1(y) = \left( \frac{1}{2} - y + y^2 \right) \left[ c_1^2 + 2g_V e_a g_a^\gamma \chi_1 + c_1^a c_2^a \right] - \frac{1 - 2y}{2} \left[ 2g_V^a e_a g_a^\gamma \chi_1 + c_3^a c_2^a \right], \tag{8} \]

which contain the combinations of the couplings
\[
\begin{align*}
c_1^2 &= (g_V^1)^2 + (g_A^1)^2, \\
c_3^2 &= 2g_V^1 g_A^1.
\end{align*}
\tag{9}
\]

The Z-boson propagator factors are given by
\[
\begin{align*}
\chi_1 &= \frac{1}{\sin^2(2\theta_W)} \frac{Q^2(Q^2 - M_Z^2)}{(Q^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2}, \\
\chi_2 &= \frac{1}{\sin^2(2\theta_W)} \frac{Q^2}{Q^2 - M_Z^2} \chi_1,
\end{align*}
\tag{10}
\]

and \( g_V \) and \( g_A \) are the vector and axial-vector couplings to the Z boson. We have summed over the polarization of the outgoing leptons. Furthermore, we use the convolution notation (Ralston and Soper use \( I[...] \))
\[
\mathcal{F} [f_T] = \int d^2 p_T \, d^2 k_T \, \delta^2(p_T + k_T - q_T) f^a(x_1, p_T^2) f^a(x_2, k_T^2),
\tag{11}
\]

where \( a \) is the flavor index.

The function \( g_{1T} \) is the function \( h^{LT} \) of Ralston and Soper and has as interpretation the distribution of longitudinally polarized quarks (\( \gamma^+ \gamma_5 \) projection) inside a transversely polarized hadron. It enters into the calculation compared to the unpolarized distribution function as follows:
\[
\Phi(x_1, p_T) = \frac{M_1}{2P_1^-} \left\{ f_1(x_1, p_T^2) \frac{P_1}{M_1} - \frac{(p_T \cdot S_{1T})}{M_1} g_{1T}(x_1, p_T^2) \frac{P_1 \gamma_5}{M_1} + \ldots \right\}.
\tag{12}
\]

The details of the momenta are: the momenta of the quarks which annihilate into the photon with momentum \( q_+ \) are predominantly along the direction of the parent hadrons. One hadron momentum \( P_1 \) is chosen to be along the lightlike direction given by the vector \( n_+ \) (apart from mass corrections). The second hadron with momentum \( P_2 \) is predominantly in the \( n_- \) direction which satisfies \( n_+ \cdot n_- = 1 \), such that \( P_1 \cdot P_2 = \mathcal{O}(Q^2) \). We decompose the momenta in \( +, - \) and transverse components, defined through \( p^\pm = p \cdot n_\pm \), where we note that [cf. Eqs. (11-13)]
\[
\begin{align*}
n_+^\mu &= \frac{1}{\sqrt{2}} \left[ \not{\hat{\nu}} + \not{\hat{n}} - \frac{Q_T}{Q} \not{\hat{h}} \right], \\
n_-^\mu &= \frac{1}{\sqrt{2}} \left[ \not{\hat{\nu}} - \not{\hat{n}} - \frac{Q_T}{Q} \not{\hat{h}} \right],
\end{align*}
\tag{14}
\]

The four-momentum conservation delta function at the vector boson vertex is written as (neglecting \( 1/Q^2 \) contributions)
\[
\delta^4(q - k - p) = \delta(q^+ - p^+) \delta(q^- - k^-) \delta^2(p_T + k_T - q_T),
\tag{15}
\]

and allows for integration over \( p^- \) and \( k^+ \). However, the transverse momentum integrations cannot be separated, unless one integrates over the transverse momentum of the vector boson or —equivalently— of the lepton pair.

The Drell-Yan cross section is obtained by contracting the lepton tensor with the hadron tensor. At the tree level we find, for the hadron tensor,
\[
\mathcal{W}^{\mu \nu} = \frac{1}{3} \int d^2 p_T d^2 k_T \, \delta^2(p_T + k_T - q_T) \, \text{Tr} \left( \Phi(x_1, p_T) V_1^\mu V_2^\nu \right) \bigg|_{p^+ \rightarrow k^+} + \left( q \leftrightarrow -q \right).
\tag{16}
\]

The vertices \( V_1^\mu \) can be either the photon vertex \( V^\mu = e \gamma^\mu \) or the Z-boson vertex \( V^\mu = g_V \gamma^\mu + g_A \gamma_5 \gamma^\mu \).

The above given azimuthal dependence in the cross section, Eq. (15), means that if one observes the transverse momentum of the \( \gamma \) or Z boson, one can consider the cross section differential in the magnitude of the transverse
momentum only and integrate over the orientations of the leptons and of \( q_T \) itself. This results in the following double transverse spin asymmetry:

\[
A_{TT}(Q_T) = \frac{d\sigma \left[ p^+ \gamma \rightarrow \gamma (Z) X \right] - d\sigma \left[ p^+ \gamma \rightarrow \gamma (Z) X \right]}{d\sigma \left[ p^+ \gamma \rightarrow \gamma (Z) X \right] + d\sigma \left[ p^+ \gamma \rightarrow \gamma (Z) X \right]} = - \frac{\sum_{a,b} K^a_1(y) \mathcal{F} \left[ p_T \cdot k_T g_{1T} \gamma_{1T} \right]}{2M_1 M_2 \sum_{a,b} K^a_1(y) \mathcal{F} \left[ f_1 \bar{f}_1 \right]},
\]

where \( Q^2_T = -Q^2_T = q^2_T \ll Q^2 \). This can be seen from the following considerations. The angular dependence \( \sin(\phi^\ell_1 - \phi^\ell_2) \sin(\phi^d_1 - \phi^d_2) \) can be rewritten after integration over the angle \( \phi^d_1 \) as \( \cos(\phi^d_1 - \phi^d_2)/2 = \cos(\phi^d_2)/2 \). Since this does not depend on the orientation of \( \ell \) itself, one can integrate over it also. The angular dependence \( \cos(2\phi^d_1 - \phi^d_2 - \phi^d_3) \) averages out.

If on the other hand one only observes the angle of the transverse momentum and averages over its magnitude, one can also obtain a nonvanishing asymmetry (which can still be differentiated from the transversity asymmetry). The whole point is to prevent the averaging: \( \int d^2 q_T \mathcal{F} \left[ p_T \cdot k_T g_{1T} \gamma_{1T} \right] = 0 \). As said before, an incomplete averaging due to imposed experimental cuts might also result in a nonvanishing asymmetry, cf. for instance Ref. [4].

Before we continue we would like to point out that unlike for \( h_1 \) there is a leading twist gluon analogue of the function \( g_{1T} \). The function arises with \( ie^{-i\phi^{a,b}} \) in the correlation function \( \Phi^{a,b} \propto \langle P S \mathcal{F}^{a+b} F^{a+b} | P S \rangle \), which means that it is a \( \Delta g \) type of function with transverse momentum dependence. But since the transition \( gg \rightarrow \gamma(Z) \) is only possible via quarks, we implicitly include the gluon in the sum over flavors.

### B. Charged vector boson production

In order to arrive at the expressions for the cross sections of the charged current process (cf. Ref. [3]), one can take \( c_1 = 0 \) and replace

\[
\chi^Z_{a,b} \rightarrow \chi^W_{a,b} = \left( \frac{1}{8 \sin^2 \theta_W} \right) \frac{Q^4}{(Q^2 - M_W^2)^2 + \Gamma_W^2 M_W^2},
\]

in the above given coupling \( K^a_2 \). In addition, one replaces \( c_1 = \pm c_3 = 1 \), depending on the chirality of the quark or lepton, since \( c_1 = (g_R^2 + g_L^2)/2 \) and \( c_3 = (g_R^2 - g_L^2)/2 \). Hence, for a left-handed quark one finds \( c_1^q = c_3^q = 1 \) and for a right-handed quark one finds \( c_1^q = -c_3^q = 1 \); similarly for the leptons. We also note that a left- or right-handed quark or lepton has helicity \( \lambda_{q/e} = \mp 1 \). This results in

\[
K^{ab}_1(y) = 4 \chi^W_{a,b} |V_{ab}|^2 \left( \frac{1}{2} - y + y^2 - \lambda_q \lambda_e \frac{1 - 2y}{2} \right)
= 4 \chi^W_{a,b} |V_{ab}|^2 \begin{cases} y^2 & \text{for equal quark and lepton chiralities,} \\ (1 - y)^2 & \text{for opposite quark and lepton chiralities,} \end{cases}
\]

where \( a, b \) are the incoming quark and antiquark flavor indices, respectively, and \( V_{ab} \) stands for the appropriate Cabibbo-Kobayashi-Maskawa (CKM) matrix element. We illustrate the above by assuming that only \( u \) and \( d \) quark distribution functions contribute. This leaves two elementary subprocesses: \( ud \rightarrow W^+ \rightarrow e^+ \nu \) (ud and \( \nu \) have equal chiralities) and \( \bar{u}d \rightarrow W^- \rightarrow e^- \bar{\nu} \) (ud and \( \bar{\nu} \) have equal chiralities) for which one finds the couplings \( K^{ud}_1 = K^{u\bar{d}}_1 = 4 \chi^W_{u,d} |V_{ud}|^2 y^2 \). For the cross section one has to take into account that in the sum over final state polarizations there is now only one state that contributes, but for the asymmetry this is not relevant.

In case of \( p^+ p^+ \rightarrow W X \) and subsequent leptonic decay of the \( W \), we encounter the problem that the produced neutrino will prevent a determination of the transverse momentum of the \( W \) boson. Hence, in the case of a produced neutrino one cannot define a lepton scattering plane (one does not observe \( l' \)), hence azimuthal angles cannot be defined compared to the \( W \) boson direction. This holds unless one can reconstruct the direction of the neutrino by the momentum imbalance [17].

Another possible way of observing the transverse momentum of the \( W \) boson is looking at the \( W \) decay into 2 jets. The expressions for lepton pair production stay essentially the same for the 2 jets case after the obvious replacement of the coupling constants. By measuring the direction of the 2 jets, the transverse momentum of the \( W \) can be determined, but one problem is that it has \( \gamma^* / Z \rightarrow 2 \) jets as a background. Separation of \( \gamma^* / Z \) and \( W \) might only be possible with a very high transverse momentum cut [18], but then the given expressions are not applicable anymore. Another problem is that it also receives contributions from quark-quark scattering next to
the quark-antiquark scattering, but such contributions have rather large color factor suppression \cite{14,19}. In any case, the contribution coming from the transversity functions to the 2 jets asymmetry can always be eliminated by averaging over the orientation of the 2 jets, as explained above.

Here we will focus only on lepton pair production and assume that in case of W production the direction of the neutrino can be reconstructed to obtain the transverse momentum of the W.

C. Gaussian transverse momentum dependence

In order to obtain an estimate of the above asymmetry, we will consider a Gaussian transverse momentum dependence of the functions. Instead of using Gaussians, another way of obtaining an estimate of the asymmetry would be to use the spectator model for the function $g_{1T}(x, p_T^2)$ \cite{20}. But for simplicity we will assume a Gaussian transverse momentum dependence, e.g.

$$ g_{1T}(x, p_T^2) = g_{1T}(x) \frac{R^2}{\pi} \exp \left( -R^2 p_T^2 \right) \equiv g_{1T}(x) \mathcal{G}(p_T^2). \quad (22) $$

We would like to relate the function $g_{1T}(x)$ to a well-known function in order to be able to make some predictions in the end. This can be achieved by using $p_T^2$ weighted functions

$$ f^{(1)}(x) = \int d^2 p_T \frac{p_T^2}{2M^2} f(x, p_T^2). \quad (23) $$

For a Gaussian transverse momentum dependence we find that $g_{1T}^{(1)} = g_{1T}(x)/(2M^2 R^2)$. In the Wandzura-Wilczek approximation the function $g_{1T}^{(1)}$ is a well-known quantity: it equals (upon neglecting quark masses) $x g^{WW}_T(x)$, where $g^{WW}_T(x) = g_1 + g_2^{WW}$ is the Wandzura-Wilczek part of the function $g_T$ (see also Ref. \cite{21} for a discussion on this topic). This can be shown by using the equations of motion. The function $g_T$ has been studied by SLAC and the Spin Muon Collaboration (SMC) \cite{22} and the data are (still) consistent with $g_T = g^{WW}_T$. Also, the data show that $g_2$ is small compared to $g_1$, therefore, up to a few percent one can take $x g^{WW}_T(x) \approx x g_1(x)$. Thus, we find $g_{1T}(x) = g_{1T}^{(1)} \approx 2M^2 R^2 \approx x g_1(x) 2M^2 R^2$.

Furthermore, will assume that the Gaussians are the same for $f_1$ and $g_{1T}$ and for both protons, i.e., we take $R_1 = R_2 = R$ and $M_1 = M_2 = M$. In this way we find for example

$$ \mathcal{F} [p_T \cdot k_T g_{1T}] \approx -\frac{M^4 R^4}{\pi} \left( 1 - \frac{Q_T^2 R^2}{2} \right) \exp \left( -\frac{Q_T^2 R^2}{2} \right) x_1 g_1(x_1) x_2 g_1(x_2). \quad (24) $$

This results in the following tree level double transverse spin asymmetry at $Q_T = 0$

$$ A_{TT}(Q_T = 0) = M^2 R^2 \sum_{a, \bar{a}} K_1^a(y) x_1 g_1^a(x_1) x_2 g_1^\bar{a}(x_2) \sum_{a, \bar{a}} K_1^\bar{a}(y) f_1^a(x_1) f_1^\bar{a}(x_2). \quad (25) $$

We will also encounter the Fourier transforms of these functions. The function $\tilde{f}$ will denote the Fourier transform of $f$, and since we use the notation $f(x) = \int d^2 p_T f(x, p_T)$, we see that $f(x) = \tilde{f}(x, b = 0)$. Taking the Fourier transform of Eq. (22) yields

$$ \tilde{g}_{1T}(x, b^2) = g_{1T}(x) \exp \left( -\frac{b^2}{4R^2} \right). \quad (26) $$

D. Beyond the range of intrinsic transverse momentum

Monte Carlo studies including soft gluon resummation \cite{23} show that the largest contribution to the unpolarized cross section arises when the transverse momentum of the W is several GeV. This transverse momentum is too high to trust tree level expressions which involve only intrinsic transverse momenta. In order to go beyond this region, we will also include the Sudakov factor arising from resummed perturbative corrections to the transverse momentum distribution.

Resummation of soft gluons into Sudakov form factors \cite{24} results in a replacement in Eqs. (13) and (18) of
\[ \delta^2(p_T + k_T - q_T) \rightarrow \int \frac{d^2b}{(2\pi)^2} e^{-ib\cdot(p_T + k_T - q_T)} e^{-S(b)}, \]

where \( e^{-S(b)} \) is the Sudakov form factor and \( b^2 = b^2 \). This has been shown in Refs. \[23, 20\] for the leading twist and is discussed for the present context in more detail in Ref. \[26\]. The Sudakov form factor is found to be

\[ S(b, Q) = \int_{b_0^2/b^2}^{Q^2} \frac{db^2}{\mu^2} \left[ A(\alpha_s(\mu)) \ln \frac{Q^2}{\mu^2} + B(\alpha_s(\mu)) \right]. \]

One can expand the functions \( A \) and \( B \) in \( \alpha_s \) and the first few coefficients are known for unpolarized scattering \[27\] and for longitudinally polarized scattering \[28\]. The latter result is needed here since the function \( g_{1T} \) is a distribution of longitudinally polarized quarks; the asymmetry on the parton level is \( a_{LL} \). In order to obtain a first estimate of the effect of including the Sudakov factor we will take into account only the first term in the expansion of \( A \): \( A^{(1)} = C_F/\pi \). This leads to the expression \[29\]

\[ S(b, Q) = -\frac{16}{33 - 2n_f} \left[ \log \left( \frac{b^2 Q^2}{b_0^2} \right) + \log \left( \frac{Q^2}{\Lambda^2} \right) \log \left[ 1 - \log \left( \frac{b^2 Q^2/b_0^2}{\log (Q^2/\Lambda^2)} \right) \right] \right]. \]

with \( b_0 = 2 \exp(-\gamma_E) \approx 1.123 \). We will take for the number of flavors \( n_f = 5 \) and also \( \Lambda_{QCD} = 200 \text{ MeV} \).

The replacement in Eq. \[(13)\] leads to (suppressing the flavor index)

\[ \mathcal{F} \left[ f\bar{f} \right] = \int \frac{d^2b}{(2\pi)^2} e^{ib\cdot q_T} e^{-S(b)} \tilde{f}(x_1, b) \tilde{\bar{f}}(x_2, b) \]

\[ = \frac{1}{2\pi} \int_0^\infty db J_0(b Q) e^{-S(b)} \tilde{f}(x_1, b) \tilde{\bar{f}}(x_2, b). \]

The functions also have a renormalization and factorization scale dependence, which we will choose to be equal \( \mu_R = \mu_F = \mu \). Hence, we have, for instance,

\[ f(x; \mu) = \int d^2p_T f(x, p_T; \mu) \equiv \tilde{f}(x, b = 0; \mu), \]

where also the boundary of the integration gives a \( \mu \) dependence. In Eq. \[(33)\] one usually takes \( \tilde{f}(x_1, b; \mu = b_0/b) \).

Of course, if one includes the effects of perturbative corrections, one should also include higher order corrections to the hard part not coming from soft gluons. But since the formalism, which means the factorized asymmetry will be the same also beyond the tree level. Here we will consider the transverse momentum structure of the tree level result and hence the transverse momentum weight in the asymmetry will be the same also beyond the tree level. Here we will consider \( H^{\mu\nu}(x_1, x_2, Q) \) to lowest order in \( \alpha_s \), therefore, only logarithmic \( Q^2 \) corrections to the results presented below are expected.

The numerator in Eq. \[(33)\] cannot be treated exactly like the denominator, so let us focus next on

\[ \mathcal{F} \left[ p_T \cdot k_T \right] \tilde{f} \tilde{\bar{f}} = \int \frac{d^2b}{(2\pi)^2} e^{ib\cdot q_T} e^{-S(b)} \int d^2p_T d^2k_T p_T \cdot k_T e^{-ib\cdot(p_T + k_T)} f(x_1, p_T^2) \tilde{f}(x_2, k_T^2). \]

As mentioned before, we assume that the distribution functions are Gaussians (as a function of transverse momentum), all of equal width: \( f(x_1, p_T^2) = \mathcal{G}(p_T^2) \) and \( \tilde{f}(x_2, k_T^2) = \mathcal{G}(k_T^2) \). One can then change variables to \( u = (p_T + k_T)/\sqrt{2} \) and \( v = (p_T - k_T)/\sqrt{2} \) and compute

\[ \int d^2p_T d^2k_T p_T \cdot k_T e^{-ib\cdot(p_T + k_T)} \mathcal{G}(p_T^2) \mathcal{G}(k_T^2) = -\frac{b^2}{4R^4} \exp \left( -\frac{b^2}{2R^2} \right), \]

which after application to Eq. \[(32)\] yields (see also Eq. \[(26)\)
\[ F(\mathbf{p}_T \cdot \mathbf{k}_T, \mathbf{f}) = - \int \frac{d^2b}{(2\pi)^2} e^{ib \cdot q_T} \frac{b^2}{4R^4} e^{-S(b)} f(x_1, b) f(x_2, b), \]  

(34)

which can be compared with Eq. (24). Both equations fulfill the property that the expression should vanish after \( q_T \) integration. One infers that the numerator of \( A_{TT}(Q_T) \) and hence \( A_{TT}(Q_T) \) itself oscillate, but in general \( \int dQ_T A_{TT}(Q_T) \neq 0 \), because of the \( Q_T \) dependence of the denominator.

For the asymmetry, Eq. (19), we then obtain

\[
A_{TT}(Q_T) = \frac{1}{8M^2 R^4} \frac{\sum a, \tilde{a} K_i(y) \int_0^\infty db b^3 J_0(bQ_T) e^{-S(b)} \tilde{g}_i^a(x_1, b) \tilde{f}_i^a(x_2, b)}{\sum a, \tilde{a} K_i(y) \int_0^\infty db b J_0(bQ_T) e^{-S(b)} \tilde{f}_i^a(x_2, b)} \approx \frac{M^2}{2} \frac{\sum a, \tilde{a} K_i(y) x_1 x_2 \tilde{g}_i^a(x_1, x_2) \int_0^\infty db b^3 J_0(bQ_T) \exp \left[-S(b) - \frac{1}{2} b^2 / R^2 \right]}{\sum a, \tilde{a} K_i(y) x_2 \tilde{f}_i^a(x_2) \int_0^\infty db b J_0(bQ_T) \exp \left[-S(b) - \frac{1}{2} b^2 / R^2 \right]},
\]  

(35)

where the approximation arises from taking \( g_{1T}(x) = g_{1T}^{(1)} \), \( 2M^2 R^2 \approx x g_1(x) 2M^2 R^2 \).

In order to extend the above equation to the nonperturbative region of large values of \( b \), one usually introduces \( b_s = b/\sqrt{1 + b^2 / b_{\text{max}}^2} \), and an additional term \( \exp \left[-S_{NP}(b)\right] \), needed to describe the low \( q_T \) region properly. In part, \( S_{NP}(b) \) is introduced to take care of the smearing due to the intrinsic transverse momentum, therefore, taking into account the term \( \exp \left(-\frac{1}{2} b^2 / R^2 \right) \) in addition will just produce a change in the coefficient of the \( b^2 \) term in \( S_{NP}(b) \). To keep the unpolarized cross section unaffected, we will therefore introduce as nonperturbative term \( \exp \left[-S_{NP}(b) + \frac{1}{b} b^2 / R^2 \right] \) and study the following final expression for the asymmetry:

\[
A_{TT}(Q_T) = \frac{M^2}{2} \frac{\int_0^\infty db b^3 J_0(bQ_T) \exp \left[-S(b) - S_{NP}(b)\right]}{\int_0^\infty db b J_0(bQ_T) \exp \left[-S(b) - S_{NP}(b)\right]},
\]  

(36)

where we define

\[
A(Q_T) = M^2 \frac{\int_0^\infty db b^3 J_0(bQ_T) \exp \left[-S(b) - S_{NP}(b)\right]}{\int_0^\infty db b J_0(bQ_T) \exp \left[-S(b) - S_{NP}(b)\right]}.
\]  

(37)

The denominator is then the conventional unpolarized expression. Also, we note that \( A(Q_T) \) is dimensionless and, for simplicity, we will take \( M = 1 \) GeV.

The above approach of including Sudakov factors in the tree level azimuthal asymmetry expressions can also be applied to expressions derived in \[38,11\] for electron-positron annihilation and in \[39\] for lepton-hadron scattering.

E. Estimating the asymmetry

For the case of \( W \) production the asymmetry becomes

\[
A_{TT}^W(Q_T) = \frac{1}{2} \frac{\sum a, \tilde{a}, b, \bar{b} |V_{ab}|^2 x_1 x_2 \tilde{g}_i^a(x_1, x_2) \tilde{f}_i^a(x_2)}{\sum a, \tilde{a}, b, \bar{b} |V_{ab}|^2 \tilde{f}_i^a(x_1, x_2) \tilde{f}_i^a(x_2)} A(Q_T),
\]  

(38)

If only the \( u \) and \( d \) quarks contribute, then also the CKM matrix elements drop out of the ratio.

For the nonperturbative Sudakov factor we use the parameterization of Ladinsky-Yuan, Ref. \[32\], which was fitted to relevant Fermilab data,

\[
S_{NP}(b) = g_1 b^2 + g_1 g_3 b \ln(100 x_1 x_2) + g_2 b^2 \ln \left( \frac{Q}{2Q_0} \right),
\]  

(39)

with \( g_1 = 0.11 \) GeV\(^2\), \( g_2 = 0.58 \) GeV\(^2\), \( g_3 = -1.5 \) GeV\(^{-1}\), \( Q_0 = 1.6 \) GeV and \( b_{\text{max}} = 0.5 \) GeV\(^{-1}\). We will take \( x_1 x_2 = 10^{-2} \), which is justified below. This leads to \( S_{NP}(b) = 1.98 b^2 \) at \( Q = 80 \) GeV and \( S_{NP}(b) = 0.77 b^2 \) at \( Q = 10 \) GeV.

The result for the asymmetry factor \( A(Q_T) \) at \( Q = 80 \) GeV is given in Fig. \[8\].
FIG. 1. The asymmetry factor $A(Q_T)$ at $Q = 80\text{GeV}$.

It is plotted up to $Q_T = Q/10$, since beyond that $Q_T$ range the magnitude only slowly decreases and also the approximation $Q_T^2 \ll Q^2$ is expected to become less valid. The asymmetry factor $A(Q_T)$ at $Q_T = 0$ is seen to be around 0.32 and at $Q_T$ values of a few GeV –relevant for the majority of produced $W$ bosons– the asymmetry factor has a sign change and consequently a smaller magnitude. On top of that the asymmetry is proportional to $|x_1g_1(x_1) x_2\bar{g}_1(x_2)| \leq x_1f_1(x_1) x_2\bar{f}_1(x_2)$. Therefore, the total asymmetry as a function of $Q_T$ is expected to be below the percent level, if one assumes that on average $x_1 = 0.4$ and $x_2 = 0.07$ for $W$ production at RHIC [18]. Since we have implicitly included the gluons in the sum over flavors, the latter argument is not valid if $\Delta g$ turns out to be extremely large at small $x$. Of course this will have even more serious implications for e.g. $A_{LL}$ in jet production, especially at low transverse momenta. We will just view this unlikely option as a proviso.

In Ref. [4] it is demonstrated that the transversity double spin asymmetry, which is a $q_T$ integrated asymmetry, is expected to be at most on the order of a few percent, which matches the level of sensitivity of RHIC. The present asymmetry is still a function of $Q_T$, requiring even more statistics. This will make the asymmetry $A_{TT}^{W(x)}(Q_T)$ invisible at RHIC. Moreover, since it oscillates as a function of $Q_T$, one expects that the asymmetry partly integrated over $Q_T$, will not lead to any significant result either.

A few remarks about the dependence of the result on the nonperturbative parameters. The asymmetry factor is seen to decrease with increasing Gaussian smearing width. Taking a higher value of $b_{max}$ and a lower value of $x_1x_2$ both increase this width. The above –optimistic– choices of $b_{max} = 0.5\text{GeV}^{-1}$ and $x_1x_2 = 10^{-2}$ are therefore expected to overestimate the asymmetry factor somewhat.

But at lower energies –where larger light cone momentum fractions can be achieved– this asymmetry for $\gamma^*$ production is still worth investigating.

FIG. 2. The asymmetry factor $A(Q_T)$ at $Q = 10\text{GeV}$. 
In Fig. 2 we have given the function $A(Q_T)$ at the scale $Q = 10$ GeV and find that at low values of $Q_T$ it is around 1. Measuring $A^W_{TT}(Q_T)$ at larger values of $x_1$ and $x_2$ might make this asymmetry observable.

If one studies the values of $\mathcal{A}(Q_T)$ at $Q_T = 0$ – where the asymmetry is largest – as a function of $Q$, one observes that inclusion of the Sudakov factor produces to good approximation a $1/Q^\alpha$ behavior, with $\alpha \approx 0.6$. Even though this suppression is not very strong as a function of $Q$, one actually needs to compare the resulting asymmetry Eq. (38) with the tree level asymmetry Eq. (25). This leads to a comparison of $\mathcal{A}(Q_T = 0)$ and $2M^2 R^2$. In a tree level analysis $R^2 = 1/(p_T^2)$ for a typical intrinsic transverse momentum squared value, i.e. $R^2 \approx 2 - 11$ GeV$^{-2}$, corresponding to the range of $\langle p_T^2 \rangle \approx (300 - 700$ MeV)$^2$. If we view $R^2 = 1$ as giving a lower bound for the tree level asymmetry factor, then $\mathcal{A}(Q_T = 0, Q = 80)$ is still an order of magnitude smaller.

We conclude that the Sudakov factors produce a strong suppression compared to the tree level result and the effect increases with energy. This will also have consequences for similar types of transverse momentum dependent azimuthal asymmetries appearing in for instance $e^+e^-$ annihilation at the $Z$ mass scale [13]31, where the same strong suppression due to Sudakov factors is expected.

F. Physics beyond the standard model

It is now clear that the standard model (SM) mechanisms seem to produce negligible double transverse spin asymmetries in $W$ production ($A^W_{TT}$). In summary the reasons are the following: the transversity distribution $h_1$ does not contribute [6]. At next-to-next-to-leading twist $[O(M_1M_2/Q^2)]$ the twist-3 distribution function $g_T$ (which is a helicity non-flip quark distribution) can contribute and its gluon analogue $\Delta_T g$ as well. At $Q^2 = M_W^2$ these contributions will be negligible. Furthermore, one can also neglect contributions which are of higher order in the strong and/or weak coupling constants. In the case of perturbative QCD corrections double helicity flip will be accompanied by quark mass terms and therefore will also be suppressed by at least two factors of $1/Q$. In the case of weak corrections one also expects negligible contributions, at least at RHIC energies, say around $Q^2 = M_W^2$. Any leftover asymmetry from incomplete averaging of the transverse momentum dependent asymmetry $A^W_{TT}(Q_T)$ is estimated to be negligible as well. So at RHIC energies $A^W_{TT}$ is expected to be negligible within the SM.

It is now fair to address the question: if a significant asymmetry would nevertheless be found in the polarized proton-proton collisions at RHIC, can one really conclude something about physics beyond the SM? For instance, there could be scalar or tensor couplings of the proton-proton collisions at RHIC, can one really conclude something about physics beyond the SM? For instance, for $Q^2 = M_W^2$ and $\Lambda = 1$ TeV the latter is a factor of 40 larger (although still quite small). But the problem of comparing to higher twist contributions disappears altogether if the new couplings violate symmetries.

This means that on top of the fact that the various SM mechanisms produce negligible asymmetries, one can also exploit the dependence on the orientation of the transverse spins compared to the lepton production plane to cancel out specific contributions exactly. We have seen that the transversity asymmetry $A^T_{TT}$ appears with an angular dependence $\cos(\phi_{S_1} + \phi_{S_2})$, whereas $A_{TT}(Q_T)$ and the $1/Q^2$ suppressed contribution from $g_T \mathbf{p}_T$ in Eq. (38) both appear with $\cos(\phi_{S_1} - \phi_{S_2})$. The latter does not depend on the lepton scattering plane, because the asymmetries are not double transverse spin asymmetries at the parton level. On the other hand, symmetry violation asymmetries can produce other angular dependences than any SM mechanism.

There might be $T$-odd asymmetries, for example the one of Ref. [34],

$$A^T_{TT} \propto \sin(\phi_{S_1} + \phi_{S_2}),$$

which would arise due to CP-violating vector couplings of the quarks to the $W$, which is assumed not to be $V - A$ anymore, but some complex linear combination of $V$ and $A$. It can clearly be distinguished from initial state interaction effects, which are $P$-even and only lead to asymmetries independent of the lepton scattering plane. To be more specific, if one assumes $T$-odd ($P$-even) distribution functions to be nonzero, then there will also be contributions proportional to [cf. Eq. (A1) of [34]]

$$\cos(\phi_{S_1}' - \phi_{S_2}') \mathcal{F} \left[ p_T \cdot k_T f^\perp_{TT} \mathbf{p}_T \right],$$

$$\sin(\phi_{S_1}' - \phi_{S_2}') \mathcal{F} \left[ p_T \cdot k_T f^\perp_{TT} \mathbf{p}_T \right].$$

The function $f^\perp_{TT}$ corresponds to the so-called Sivers effect [34]. Here one usually assumes that such a function might arise due to initial state interactions and its contributions indeed do not depend on the lepton pair orientation as can be seen from the above two angular dependences.
Therefore, these structures are distinguishable from the $T$-odd asymmetry $A_{T}^{+}$. However, it is important to note that $A_{T}^{+}$ can also effectively arise due to SM $CP$ violation, hence this contribution must first be calculated before any conclusion about physics beyond the SM can be made. Also, this specific asymmetry will be accompanied by the product $h_{1}^{u}(x_{1}){ar h}_{1}^{u}(x_{2})$, thus it will suffer from the same drawback as Eq. (4), namely that the transversity function for the antiquarks is presumably smaller than for the quarks, making this asymmetry hard (if not impossible) to detect at RHIC. But it illustrates how symmetry violation can be used in principle to disentangle SM asymmetries from new physics asymmetries.

IV. CONCLUSIONS

We have investigated a helicity non-flip double transverse spin asymmetry in vector boson production in hadron-hadron scattering, which was first considered by Ralston and Soper. It does not involve transversity functions and in principle also arises in $W$-boson production for which we have presented the expressions. The asymmetry requires observing the transverse momentum of the vector boson, but it is not suppressed by explicit inverse powers of the large energy scale $Q$. However, as we have shown, inclusion of Sudakov factors suppresses the asymmetry at least by an order of magnitude compared to the tree level result. This suppression increases with energy approximately as a fractional power, numerically found to be $\alpha \approx 0.6$. Moreover, the asymmetry is shown to be approximately proportional to $x_{1}g_{1}(x_{1})x_{2}{\bar g}_{1}(x_{2})$, which gives rise to additional suppression at small values of the light cone momentum fractions. This implies that the asymmetry is negligible for $Z$ and $W$ production at RHIC and is mainly of interest at low energies (for $\gamma^{*}$ production). The strong suppression with respect to the tree level result will also have consequences for similar types of transverse momentum dependent azimuthal asymmetries in for instance $e^{+}e^{-}$ annihilation at the $Z$ mass scale, where the same strong suppression due to Sudakov factors is expected.

We have also noted that unlike the transversity and $CP$-violating double transverse spin asymmetries, the helicity non-flip asymmetry $A_{T}(Q_{T})$ does not depend on the orientation of the transverse spin vectors compared to the lepton pair production plane orientation. This feature can be exploited to separate the different types of asymmetries.

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