Research on the Behavior of the Shill Bidding with Affiliated Values

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Abstract—On-line auction provides convenience, as well as causes the frequent occurrence of many deceits, the passage mainly provides the concealed conditions for the seller’s shill bidding behavior based on the anonymity of on-line auction, it discusses the bidder’s optimal decision quotation when the shill bidding behavior happens or not in the affiliated value pattern. To establish the bidder and seller’s incomplete information dynamic game model when occurring the shill bidding and analyze the probability of shill bidding can provide the reference for bidder’s quotation. To consider the time, reserved price (price accepted by the seller) and the current highest auction price which affect the shill bidding and match the relevant non-homogeneous poisson process and exponent distribution can provide the reference for bidders to judge if the shill bidding behavior occurs by sellers.

Keywords—affiliated values; shill bidding; game model; equilibrium price

I. INTRODUCTION

With the rapid development of the Internet, all kinds of traditional industries seeking for to update the service concept, structure the new business model and building the new execution mode, and explore the new development path that is suitable for their own development and accords with the support of new big data era and bring fresh energy into traditional industries. On-line auction puts the traditional bidding into the overall environment of Internet, which makes the auction process more clear with higher participation degree. However, online auction not only promotes the development of auction, but also causes a series of problems, due to the unique feature of auction “anonymity”, it is difficult to trace the detailed bidder information, which causes the frequent occurrence of on-line auction deceits, the main deceit behaviors include: shill bidding, multiple bidding and collusion[7]. As for the possible deceits of sellers, shill bidding is the most common one.

Based on the dual features of Internet, “invisibility” and “transparency”, bidders and sellers can observe the bidding condition of auction[7]. In order to attract more auctioneers, some bidders do not set the reserved price, in the auction process, when bidders observe that the bidding condition is not optimistic, which means that when the auction price is far lower than the acceptable price, sellers may increase the auction price as a legal bidder, but the shill bidding behavior of seller is in order to increase the auction benefits, so sellers also face the risks of auctioning again if winning the auction. In addition to the sellers’ own factors, the number of bidders and the level of affiliation are also related to the occurrence of shill bidding[5]. So we consider the three factors which affect shill bidding: the time, the number of bidders and the reserve price to consider the probability of shill bidding. Therefore, based on it, it is considerable to analyze the seller’s shill bidding behavior.

Milgrom and Weber established the affiliated value model, the private value model and the shared value model can be regarded as the special affiliated value models. The “relevance” of affiliated value supposes that each bidder has his own private signal, but the private signal can become the comprehensive function of all the signal[4]. Shill bidding may take place in all-pay auction, ascending auction and other auction models[5]. Besides, not only 1 items, there are multi-item auction[6]. The passage mainly researches the possible shill bidding behavior existed in the affiliated value model.

II. MODEL SETTING BASED ON AFFILIATED VALUES

Supposing that a seller is eager to put a unit of item and that is indivisible over the Internet, which can be visible to every bidders who may join this auction. There will be n bidders in this auction, and this auction model setting is based on affiliated values.

We have the following assumptions:

1) There are N honest bidders, and those equilibrium strategy are related to their valuation.
2) There are N neutral risk bidders, to maximize the benefits.
3) There is no difference between the N bidders who participate in this bidding, and they adopt exactly the same bidding strategy.

Without considering the seller's shill bidding in the auction, the seller opens the auction without reserve price, and this auction will continue T days.

Suppose that the bidder’s evaluation variable to the item is a random variable \( x_i, i = 1, 2, ..., n \), and the bidder is rational, \( x_i \in [\underline{x}, \bar{x}] \), each bidder only knows their own evaluation
variable and does not know the other bidder’s evaluation variables, and each bidder’s evaluation variable who take of the item is mutually independent. Then the bidder’s perceived value of this item is \( v_i = U(x_i, x_{-i}, S) \), when \( S \) is an environmental variable, such as certificate equivalent and so on that seller provide to prove the value of this item, and that \( x_{-i} = x_1, x_2, ..., x_{i-1}, x_{i+1}, ..., x_n \).

Valuation function \( U(\cdot) \) satisfies the conditions of non-negative, continuous and finite expectation, random variable \( x \) is positively correlated with \( S \). The better the seller gives the value proof of the item, the more the bidder's evaluation will be, besides when the bidder observes the evaluation preference of other bidders, the greater the probability of their evaluation preference.

### III. THE FIRST-PRICE AUCTION WITH AFFILIATED VALUES

In the above assumptions, let \( Y_1 = \max(X_2, X_3, ..., X_n) \), obviously \( Y_1 \) and \( x \) are related. Let \( b(\cdot) \) be the equilibrium bidding strategy, and \( F_{n-1}(y|x) \) be the conditional probability distribution, then \( f_{n-1}(y|x) \) be the conditional density function. When \( X_1 = x, Y_1 = y \), the expected value of the item is \( v(x, y) = E(X_1 = x, Y_1 = y) \), auction website as agent usually charge a handling fee, which is \( K \) times as much as the bidder's bidding price, \( 0 < k < 1 \).

**Proposition 1.** The bidder finally obtains the auction in \( t \) time, and the equilibrium strategy \( b(x) \) is:

\[
b(x) = \frac{1}{1 + k} \frac{1}{F_{n-1}(x|y)} \int_\tilde{x}^x v(t, t) f_{n-1}(t|t) dt
\]

**Proof:** The expected return \( \pi \) of bidder 1 when the bidding price \( b(v_1) \) is:

\[
\pi = E[v(x, y) - (1 + k)b(v_1)] \int_{\tilde{x}}^x \frac{f_{n-1}(x|y)}{F_{n-1}(x|y)} dy
\]

By envelope theorem:

\[
(1 + k)b'(v_1) = [v(x, y) - (1 + k)b(v_1)] \frac{f_{n-1}(x|y)}{F_{n-1}(x|y)}
\]

By the first-order condition \( \frac{\partial \pi}{\partial v_1} |_{v_1=x} = 0 \):

\[
- \int_\tilde{x}^x (1 + k)b'(v_1)f_{n-1}(y|x)dy + [v(x, y) - (1 + k)b(v_1)]f_{n-1}(x|y) = 0
\]

As we know \( F_{n-1}(x|y) = 0 \), then:

\[
(1 + k)b'(v_1)f_{n-1}(x|y) + (1 + k)b'(v_1)f_{n-1}(x|y) = v(x, y) f_{n-1}(x|y)
\]

So the Equilibrium Bidding Strategy \( b(x) \) of bidder 1 is:

\[
b(x) = \frac{1}{1 + k} \frac{1}{F_{n-1}(x|y)} \int_\tilde{x}^x v(t, t) f_{n-1}(t|t) dt
\]

If the seller observes that the auction is not optimistic, he may join the auction in disguise as a bidder. Assuming that the seller has probability \( P, 0 < P < 1 \), if he joins the bidding, the shill bidding will occur. At this time, the number of bidder who participates in the auction becomes \( n + 1 \).

In the above assumption, let \( Y_1 = \max(X_2, X_3, ..., X_n) \), \( Y_1' = \max(X_2, X_3, ..., X_n, X_{n+1}) \), \( b(\cdot) \) the equilibrium strategies. When there is no shill bidding, \( F_{n-1}(y|x) \) is the conditional probability distribution, and \( f_{n-1}(y|x) \) is the conditional density function. When the shill bidding occurs, \( F_n(y'|x) \) is the conditional probability distribution, and \( f_n(y'|x) \) is the conditional density function.

**Proposition 2.** The bidder finally obtains the auction when shill bidding might occurs, and the equilibrium strategy \( b(x) \) is:

\[
b(x) = \frac{1}{1 + k} \frac{1}{F_{n-1}(x|y) + PF_n(y'|x)}
\]

\[
* \int_\tilde{x}^x [(1 - P)v(t, t) f_{n-1}(t|t) + P v(t, t) f_n(t|t)] dt
\]

**Proof:** The probability density function of winning this auction of the bidder 1 under the condition of \( P \) probability of shill bidding occurs is as follows:

\[
P_{n-1}(y'|x) + (1 - P)f_{n-1}(y|x)
\]

The expected return \( \pi \) of bidder 1 when the bidding price \( b(v_1) \) is:

\[
\pi = E[(v(x, y) - (1 + k)b(v_1)) 1_{b(v_1) > b(y)}] \int_{\tilde{x}}^x \frac{f_n(y'|x)}{F_n(y'|x)} dy + (1 - P) f_{n-1}(y|x)
\]

By the first-order condition \( \frac{\partial \pi}{\partial v_1} |_{v_1=x} = 0 \) and \( F_{n-1}(x|y) = 0 \), \( F_n(x|y) = 0 \):

\[
(1 + k)[P(b'(x)f_n(y'|x) + b(x)f_n(y'|x))] + (1 - P)(b'(x)f_{n-1}(y|x) + b(x)f_{n-1}(y|x)) = P v(x, y) f_n(y'|x)
\]

Solving the equation:

\[
b(x) = \frac{1}{1 + k} \frac{1}{F_{n-1}(x|y) + PF_n(y'|x)}
\]

\[
* \int_\tilde{x}^x [(1 - P)v(t, t) f_{n-1}(t|t) + P v(t, t) f_n(t|t)] dt
\]

### IV. GAME MODEL

Online auction can be regarded as an incomplete-information dynamic model, each bidder knows their own type of decisions (evaluative variables) while being ignorant of others. Meanwhile, by using the transparency and openness of Internet, bidders can detect the behaviors (quoted price) of their rival to speculate and estimate their types of decisions (evaluative variables), which could help bidders with their behaviors’ quoted price) amendments.

**Model Assumption:**

1) When sellers realized the poor condition of an auction, they might take shill bidding. Setting ‘\( p \)’ the probability that a seller chooses to take shill bidding; ‘\( t \)’ the probability that a bidder participates in the auction when seller have decided might take shill bidding.
2) ‘\(C_1\)’ stands for the cost that a seller has to pay for his behavior including paying the commission or account registration when disguised as a bidder to take part in the auction. ‘\(C_2\)’ is set as the commission that a bona-fide buyer needs to pay when taken part in the auction.

3) ‘\(L_1\)’, the earning of a seller does not take shill bidding. ‘\(L_2\)’, the illegal earning of a seller takes shill bidding.

4) ‘\(\pi_1\)’ the yield that a bidder earns winning the bidding without shill bidding. ‘\(\pi_2\)’, the yield that bidder earns winning the bidding under shill bidding.

**Proposition 3:** Nash equilibrium under mixed strategy is \([t^*, 1-t^*], (p^*, 1-p^*)\].

\[
t^* = \frac{C_1}{L_2-L_1}, \quad p^* = \frac{\pi_1-C_2}{\pi_1-\pi_2},
\]

**Proof:** From the figure above we can learn that:

- The expected revenue of bidder A: \(E_A(t, p) = t \cdot p \cdot (\pi_2 - C_2) + t \cdot (1-p) \cdot (\pi_1 - C_2)\)
- The expected revenue of bidder B: \(E_B(t, p) = t \cdot p \cdot (L_2 - C_1) + t \cdot (1-p) \cdot L_1 + (1-t) \cdot p \cdot -C_1\)

We can get the equilibrium strategy of shill bidding, by solving optimization problems:

\[
\max_{t \geq 0, p \in [0, 1]} E_A(t, p) = t \cdot p \cdot (\pi_2 - C_2) + t \cdot (1-p) \cdot (\pi_1 - C_2)
\]

\[
\max_{t \geq 0, p \in [0, 1]} E_B(t, p) = t \cdot p \cdot (L_2 - C_1) + t \cdot (1-p) \cdot L_1 + (1-t) \cdot p \cdot -C_1
\]

Let \(\frac{\partial E_A}{\partial t} = 0\) and \(\frac{\partial E_B}{\partial p} = 0\).

From the first order condition of optimization, we can get:

\[
t^* = \frac{C_1}{L_2-L_1}, \quad p^* = \frac{\pi_1-C_2}{\pi_1-\pi_2}
\]

After getting ‘\(p^*\)’ the probability that a seller chooses to take shill bidding, use it to be the guidance of optimal bid quotation.

V. ANALYSIS OF SHILL BIDDING’S INDUCED FACTORS

Online auction usually sets a time \(t\), and \(n\) number of bidders would arrived and taken part in the auction during time \(t\). Friedman’s research shows that in traditional auction, the number of bidders follows the Poisson distribution, while due to the particularities of online auction, we considered the number of bidders follows non-homogeneous Poisson process.

**Theorem 1:** The counting process \(N(t), t \geq 0\) is said to be a non—homogeneous Poisson process with time-varying intensity function \(\lambda(t), t \geq 0\), if:

1) \(N(0) = 0\)
2) \(N\) has independent increments
3) \(P(N(t+\Delta t) - N(t) = 1) = \lambda(t)\Delta t + o(\Delta t)\)
4) \(P(N(t+\Delta t) - N(t) \geq 2) = o(\Delta t)\)

The actual number of bidders is not a fixed value, so it does not conform to the Poisson distribution. However, the bidding process can be approximated as the arrival process, so it can be regarded as the non-homogeneous Poisson process.

**Proposition 4:** The number of bidders participated in online auction follows nonhomogeneous Poisson process, the interval between two auctions’ occurrences follows Exponential distribution.

**Proof:** The number of bidders ‘\(n\)’ during time ‘\(t\)’ follows nonhomogeneous Poisson process:

\[
P(N(t) - N(0) = n) = \frac{[m(t) - m(0)]^n}{n!} e^{-[m(t) - m(0)]} (6)
\]

Then \(m(t) = \int_0^t \lambda(s)ds\).

Set \(T\) as the interval between two auctions’ occurrences, so that:

1) \(t \leq 0, F(t) = P(T \leq t) = 0\)
2) \(t > 0, F(t) = P(T \leq t) = 1 - P(T > t) = 1 - P[N(t) = 0] = 1 - e^{-[m(t) - m(0)]})\)

\[
f(t) = \{[m(t) - m(0)]e^{-[m(t) - m(0)]}, t > 0\}
\]

The occurrence of shill bidding is related to three factors: Reserve price \(r^\ast\) (seller’s price), current time \(t_0\), current highest price \(v_1\), the probability leading shill bidding happens \(P = g(r^\ast, t_0, v_1)\).

When the higher the seller’s evaluation of the auction item, it has been a while since the auction start, the current highest price is still far from seller’s expectation, sellers might take shill biddings. In another case, when there still has huge difference in prices but closes to the end time of auction, seller usually does not take chances to disguise as a bidder since bidder will win this auction in all probability at that time and it will be void.

We think, in practice, the number of bidders follows nonhomogeneous Poisson process, the interval between auctions follows corresponding Exponential distribution. If there is an interval between two auctions deviates greatly from Exponential distribution, it implies that there might have shill bidding behavior among these two auctions.

VI. CONCLUSION

The passage mainly researches the possible shill bidding behavior existed in the affiliated value model, and provides the reference of optima decision for bidders when the shill bidding behavior is considered to occur or not, so that bidders can estimate their own benefits. The process between bidders and sellers can be regarded as an incomplete information dynamic
game process, on this account, the game model is established, it analyzes the possible probability of shill bidding of sellers and adjusts the bidder’s optimal quotation dynamically when obtaining the relevant auction information. Finally, considering the auction time, reserved price (the acceptable price of seller) and the current auction quotation when the shill bidding behavior occurs, it matches the non-homogeneous Poisson process by the bidders, as well as matches the exponent distribution by the adjacent auction time interval, which observes that if the shill bidding behavior occurs by sellers and guarantees the bidder’s rights further.

The passage limitation and the research direction in the future are that the factors of shill bidding are limited without the interactions between factors and by the seller’s emotion[9], as for the non-homogeneous Poisson process data matching, there is insufficient theory support, so it is difficult to solve the exponent distribution issue, it seeks for matching the non-homogeneous distribution process model in the future researches to predict the period of shill bidding behavior more precisely.

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