Aeroelastic Stability Investigations for Large-scale Vertical Axis Wind Turbines

B C Owens
Senior Member of Technical Staff, Analytical Structural Dynamics
Sandia National Laboratories, P.O. Box 5800, Albuquerque, NM, 87185, U.S.A.
E-mail: bcowens@sandia.gov

D T Griffith
Principal Member of Technical Staff, Wind Energy Technologies
Sandia National Laboratories, P.O. Box 5800, Albuquerque, NM, 87185, U.S.A.
E-mail: dgriffi@sandia.gov

Abstract. The availability of offshore wind resources in coastal regions, along with a high concentration of load centers in these areas, makes offshore wind energy an attractive opportunity for clean renewable electricity production. High infrastructure costs such as the offshore support structure and operation and maintenance costs for offshore wind technology, however, are significant obstacles that need to be overcome to make offshore wind a more cost-effective option. A vertical-axis wind turbine (VAWT) rotor configuration offers a potential transformative technology solution that significantly lowers cost of energy for offshore wind due to its inherent advantages for the offshore market. However, several potential challenges exist for VAWTs and this paper addresses one of them with an initial investigation of dynamic aeroelastic stability for large-scale, multi-megawatt VAWTs. The aeroelastic formulation and solution method from the BLade Aeroelastic STability Tool (BLAST) for HAWT blades was employed to extend the analysis capability of a newly developed structural dynamics design tool for VAWTs. This investigation considers the effect of configuration geometry, material system choice, and number of blades on the aeroelastic stability of a VAWT, and provides an initial scoping for potential aeroelastic instabilities in large-scale VAWT designs.

1. Introduction
A vertical-axis wind turbine (VAWT) rotor configuration offers a potential transformative technology solution that significantly lowers COE for offshore wind due to its inherent advantages for the offshore market. For example, placement of the drive train components near the water level reduces support structure requirements (and costs) and improves accessibility for maintenance. The potential of scaling to much larger offshore VAWT rotors is an intriguing possibility as well. To remain a

1 To whom any correspondence should be addressed.
2 Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy’s National Nuclear Security Administration under contract DE-AC04-94AL85000.
viable option for offshore wind energy, however, VAWT technology will need to undergo significant
development in coming years. In particular, development of a better understanding of fundamental
dynamic behavior is needed for all of the VAWT configurations being considered.

Dynamic aeroelastic instability or flutter can be a concern for lift-generating structures under
aerodynamic loads. Coupling of aerodynamic forcing with a structure’s natural modes can lead to
large amplitude diverging motion. Recent studies have shown that flutter is a potential issue in very
large HAWT wind turbine blades [1-4] and may be a concern for very flexible multi-megawatt VAWT
structures under large aerodynamic loads as well. Aeroelastic instabilities have been observed in
smaller-scale VAWT designs [5], but this issue has not been explored for large-scale VAWT systems.
For an equivalent power rating, a VAWT design must have much longer (and likely more flexible)
blades than a HAWT design.

Analysis and design to avoid aeroelastic instabilities are the key issues addressed in initial design
studies. The design requirements and analysis techniques are well established for the conventional 3-
bladed HAWT, but these issues are not well-addressed for the range of VAWT configurations. This
paper presents an initial design study on the aeroelastic stability of large, multi-MW VAWT
configurations. Design parameters such as number of blades, rotor material selection, and fundamental
VAWT configuration are examined within the context of aeroelastic stability.

2. Previous Aeroelastic Design Tools and Investigations
Previous work by Lobitz [1] considered flutter of an isolated wind turbine blade rotating in still air.
The turbine blade was considered to be cantilevered at the root, and analysis was performed in a
rotating frame. This analysis tool accounted for rotational and stress-stiffening effects and employed
the NASTRAN finite element software [6] to account for the majority of structural dynamics
calculations. The use of DMAP [7] programming allowed the NASTRAN finite element matrices to
be modified to include aerodynamic effects in the form of aerodynamic mass, damping, and stiffness.
A key characteristic of Theodorsen's unsteady aerodynamic theory is the complex valued
"Theodorsen" function which accounts for the amplitude reduction and phase lag in aerodynamic
forcing on an oscillating structure as a result of shed vortices. Lobitz employed the use of the complex
eigensolver in NASTRAN to ease the DMAP implementation of the complex valued aerodynamic
terms. As shown in Reference 2, this aeroelastic representation requires special consideration beyond
conventional structural dynamics analysis methods. The tool, which was originally developed for
considering aeroelastic stability of vertical-axis wind turbines, was applied to utility scale horizontal-axis
wind turbine blades.

Popelka employed the original version of the NASTRAN based design tool to examine aeroelastic
instabilities of the smaller-scale Sandia 2-meter VAWT test-bed [5]. The model assumed two-
dimensional aerodynamic theory and did not consider inflow or wake in the derivation of aerodynamic
loads. Furthermore, other geometrical assumptions were imposed such as assuming the location of the
aerodynamic center at the quarter chord. Aeroelastic modeling efforts were experimentally validated
using the Sandia 2-meter VAWT test-bed by attempting to excite flutter for a range of operating
conditions. The study showed that flutter in the 2-meter VAWT resulted from a coupling of the first
flapwise bending mode and first torsional mode. The frequencies of the two blade modes, however,
did not have to coincide. It was also found that the tower and drive train torsional stiffness affect the
blade torsional frequencies which in-turn affects flutter speed. Finally, experimental observations of
the 2-meter VAWT revealed that wind velocity reduces the flutter speed, but this effect was
determined to be relatively small for operational wind speeds.

Hansen [4] also investigated aeroelastic of HAWTs, considering classical flutter as well as stall
induced vibration. Stall induced vibration is a fundamentally different phenomenon from classical
flutter. It should be noted that classical flutter tends to be a stronger, more rapidly occurring instability than stall induced vibrations. Furthermore, classical flutter is typically a concern for pitch regulated turbines while stall-induced vibrations tend to be a concern in stall regulated turbines. Hansen considered modeling of a complete turbine (tower and rotor) and the aeroelastic interaction of stall induced vibrations with the inflow/wake.

The recently developed BLade Aeroelastic STability Tool (BLAST) is a MATLAB based finite element tool capable of considering structural dynamics and aeroelasticity of a rotating HAWT blade [2]. The “in-house” custom finite element formulation implemented in BLAST avoids the need for commercial licensing, and provides a more accurate representation of blade geometry (flexural, center of mass, and aerodynamic center axes) than the Lobitz NASTRAN tool. Furthermore, BLAST also makes use of an aeroelastic formulation that is more consistent with conventional structural dynamics analysis techniques (see Reference 2). Finally, BLAST is highly automated allowing the analyst to efficiently conduct thorough design studies and provides a number of visualization options to aid in interpreting analysis results. The aeroelastic analysis capability employed in BLAST serves as the foundation for considering aeroelastic analysis of VAWTs in the current study.

3. Design Tool Overview
To facilitate the development of VAWT technology, robust design tools must be developed to assess innovative design concepts for off-shore wind energy technology. Therefore, an aeroelastic design tool is being developed for modeling large offshore VAWT configurations. The Offshore Wind ENergy Simulation (OWENS) toolkit [8] is able to explore a wide array of offshore VAWT configurations via modal and transient analysis. This tool has the capability to interface with aerodynamics, platform dynamics (hydrodynamics), and drivetrain/generator modules to predict the response of a VAWT of arbitrary configuration under a variety of conditions. The formulation also allows for stability analysis to identify potential resonance and aeroelastic stability issues. The core of the analysis tool is a robust and flexible finite element framework capable of considering the dynamics of large, flexible, rotating structures.

3.1 Finite Element Implementation
VAWTs are typically constructed of relatively slender structural components. Accordingly, Timoshenko beam theory has been utilized to characterize the motions of structural component. The equations of motion are developed for a beam element of arbitrary orientation in a rotating reference frame. This reference frame is allowed to translate in order to account for platform or foundation effects. Rotational effects of Coriolis and spin softening phenomenon are included in the formulation, and geometric nonlinearities are considered to include stress stiffening effects on the modal response of the structure under load. Technical details of the beam formulation and verification and validation studies are provided in References 8 and 9. The OWENS toolkit is capable of time-domain and frequency-domain analysis. Herein, frequency-domain analysis is considered to examine the modal aeroelastic response of VAWT configurations.

3.2 VAWT Mesh Generator
A VAWT rotor consists of a tower, blades, and possibly support members (or struts). The blades may be affixed to the tower at their ends as in the Darrieus and V-VAWT configurations or via struts (H-VAWT). Struts may also provide a connection between the tower and blades at any position along the tower and blade spans. The VAWTGen mesh generator can consider VAWTs of arbitrary geometry, including H-type, V-type, and Darrieus configurations. The blades may be rotated into an arbitrary orientation at arbitrary locations about the tower. Therefore configurations with swept blades may be considered. The VAWT configuration will be discretized from continuous structural components into a finite number of beam elements. Figure 1 shows representative VAWT configurations generated with VAWTGen (from left to right: swept Darrieus, Darrieus with struts, V-VAWT, and H-VAWT).
The implementation also allows for concentrated structural components to be considered, and constraints of various joints may be imposed between structural components.

![Figure 1. Arbitrary VAWT configurations produced by VAWTGen.](image)

### 3.3 Aeroelastic Implementation

Theodorsen unsteady airfoil theory is employed to incorporate aerodynamic effects into the existing structural dynamics capability in OWENS. The equations of motion for a traditional second-order structural dynamics system are shown in Eq. (1). Here $\mathbf{q}$ is a generalized displacement vector (overdots denote explicit time derivatives), $\mathbf{F}_{np}$ is a generalized non-potential force vector and $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the mass, damping, and stiffness (potentially nonlinear) matrices respectively.

$$M\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F}_{np}$$  \hspace{1cm} (1)

For a flexible system considered in a co-rotating frame under a prescribed, constant angular velocity ($\Omega$) the equations of motion are shown in Eq. (2). Such that $\mathbf{G}$ and $\mathbf{S}$ are the Coriolis (skew symmetric) and “spin-softening” matrices respectively and $\mathbf{F}_{cent}$ is composed of body loads due to rotational effects (centrifugal force).

$$M\ddot{\mathbf{q}} + (\mathbf{C} + \mathbf{G}(\Omega))\dot{\mathbf{q}} + (\mathbf{K}(\mathbf{q}) - \mathbf{S}(\Omega))\mathbf{q} = \mathbf{F}_{cent}(\Omega) + \mathbf{F}_{sp}$$  \hspace{1cm} (2)

Considering Theodorsen’s unsteady airfoil theory allows aerodynamic effects to be introduced via the non-potential forcing term. The expressions for aerodynamic lift and moments are in terms of flapping and twisting motion of a cross-section as shown in Eqs. (3) and (4) below.

$$L = \pi \rho b^2 \left[ \dot{w} + V \dot{\theta} - b \alpha \dot{\theta} \right] + 2\pi \rho V b C(k) \left[ \dot{w} + V \theta + b \left( \frac{1}{2} - \alpha \right) \right]$$  \hspace{1cm} (3)

$$M = \pi \rho b^2 \left[ b a \dot{w} - V b \left( \frac{1}{2} - \alpha \right) \dot{\theta} - b^2 \left( \frac{1}{8} + c^2 \right) \dot{\theta} \right] + 2\pi \rho V b^2 \left( a + \frac{1}{2} \right) C(k) \left[ \dot{w} + V \theta + b \left( \frac{1}{2} - \alpha \right) \right]$$  \hspace{1cm} (4)

Here, $b$ is the semi-chord of an airfoil section, $a$ is the location of the elastic axis in semi chord fractions aft of the half chord, $V$ is the freestream velocity over the blade section, $\rho$ is air density, and $C(k)$ is the complex valued Theodorsen function. The flapwise motion of the blade section is represented by $w(t)$ and the torsional motion of the section is represented by $\theta(t)$. Furthermore, $k = b/V$ is a “reduced frequency” dependent on the oscillatory motion of the cross-section. The Theodorsen function $C(k)$ is complex in nature and models the amplitude reduction and phase lag in aerodynamic forcing as a result of unsteady effects due to shed vortices at the trailing edge of a blade.
section. While expressions for lift are traditionally expressed in terms of freestream velocity $V$, for a rotating turbine $V = r \Omega$ such that $r$ is the distance from the axis of rotation (tower axis) to blade section. Note that this expression for free-stream velocity over a blade section neglects the effects of inflow, and is valid for a turbine rotating in still air. Future development will seek to introduce inflow effects into the aeroelastic analysis capability. Under these assumptions, the aerodynamic loads are functions of generalized displacements, velocities, and accelerations as well as modal frequency $\omega$. Aerodynamic mass, damping, and stiffness matrices can be formulated in a finite element formulation, and the aeroelastic second order system with rotational effects is shown in Eq. (5).

$$ \begin{bmatrix} M + M_\Omega(\Omega) \end{bmatrix} \ddot{q} + \begin{bmatrix} C + G(\Omega) + C_\Omega(\Omega, \omega) \end{bmatrix} \dot{q} + \begin{bmatrix} K(q) - S(\Omega) + K_\Omega(\Omega, \omega) \end{bmatrix} q = F_{\text{cent}}(\Omega) + F_\Omega(\Omega) \quad (5) $$

Such that $M_\Omega(\Omega), C_\Omega(\Omega, \omega), K_\Omega(\Omega, \omega)$, are aerodynamic mass, damping, and stiffness matrices respectively. The vector $F_\Omega(\Omega)$ represents aerodynamic forces due to non-elastic effects (i.e. rigid angle of attack, manufactured blade twist, etc.). Details regarding the formulation of the aerodynamic system matrices are elaborated on in References 2 and 9. The method proposed by Wright and Cooper [10] is employed to maintain a real valued aeroelastic system. This is believed to be an improved formulation with analysis procedures more consistent with conventional structural dynamics analysis techniques than previous wind energy aeroelastic design tools [1]. For the interested reader, the iterative analysis procedure employed for the current aeroelastic design study is documented in References 2 and 9.

4. Configurations

A select number of VAWT configurations with a nominal rating of 5 MW were considered for aeroelastic analysis. These included Darrieus designs of either carbon or glass composite composition, with two or three blades and two “V-VAWT” designs of carbon composition with either two or three blades. All VAWTs employed a NACA-0021 airfoil with chord values described in Table 1.

Table 1 presents basic configuration information for the designs considered in this study. The Darrieus designs have a double tapered chord (6 meter maximum and 3 meters minimum) with two struts per blade at 10% and 90% blade height. The V-VAWT designs employ a power law for the blade shape profile shown in Eq. (6). Here, $H$ is the blade height/span and $R$ is the maximum radial distance of the blade from the axis of rotation. The V-VAWT designs have one strut per blade between the tower at 50% blade span and the blade at 70% blade span. Figure 2 shows wireframe visualizations of the VAWTs considered in this study.

$$ \frac{h}{H} = \left( \frac{r}{R} \right)^5 \quad (6) $$

**Figure 2.** Visualization of VAWT designs considered for aeroelastic analysis. (from left to right: DG2LCDT/DC2LCDT, DG3LCDT/DC3LCDT, VC2N5LC, and VC3N5LC)
Table 1. Overview of VAWT Configurations considered for aeroelastic stability analysis.

| Turbine ID | Type    | Material   | # of Blades | Rotor Speed (RPM) | Rotor Mass (mt) | Rotor Height (m) | Height To Diam. | Max/Min Chord (m) |
|------------|---------|------------|-------------|-------------------|-----------------|-----------------|-----------------|------------------|
| DG2LCDT    | Darrieus | Glass      | 2           | 7.2               | 1026.0          | 132.1           | 1.22            | 6.0 / 3.0        |
| DG3LCDT    | Darrieus | Glass      | 3           | 6.3               | 1385.0          | 132.1           | 1.22            | 6.0 / 3.0        |
| DC2LCDT    | Darrieus | Carbon     | 2           | 7.2               | 220.8           | 132.1           | 1.22            | 6.0 / 3.0        |
| DC3LCDT    | Darrieus | Carbon     | 3           | 6.3               | 243.8           | 132.1           | 1.22            | 6.0 / 3.0        |
| VC2N5LC    | V-VAWT   | Carbon     | 2           | 7.4               | 637.8           | 104.7           | 0.96            | 6.0 / 3.0        |
| VC3N5LC    | V-VAWT   | Carbon     | 3           | 6.5               | 924.0           | 104.7           | 0.96            | 6.0 / 3.0        |

5. Results
The six VAWT configuration identified in the previous section were examined for aeroelastic instabilities. Pre-stressed modal analysis and iterative procedures were employed to conduct aeroelastic stability analysis at a number of specified rotor speeds. Frequency and damping trends were examined and potential aeroelastic stabilities were identified for each VAWT configuration. Figure 3 shows representative frequency and damping trends with respect to rotor speed for the DC2LCDT turbine. Frequency changes are apparent due to rotational, stress stiffening, and aeroelastic effects as rotor speed increases. Damping trends are dependent on aeroelastic damping. Some modes are minimally affected, while others have noticeable increases in damping. Finally, certain modes of the aeroelastic system cross from being positively to negatively damped. Herein, the cross-over point will be termed the “flutter speed” at which the onset of aeroelastic stability occurs, as negative damping brings about instability in the aeroelastic system. The modes of the system are labelled such that “tower” modes consist mainly of tower bending, “flatwise” modes consist mainly of blade deformation in the machine radial direction, and “edgewise” modes consist mainly of blade deformation towards or against the direction of blade travel.

Damping trends for the various configurations were observed and modes with negative aeroelastic damping were identified. Damping trends for these modes are shown in Figure 4. “Hard” instabilities (with a higher magnitude damping slope at the onset of instability) are observed for the Darrieus VAWTs while very “soft” instabilities (with a lower magnitude damping slope at the onset of instability) are observed for the V-VAWTs. No modal damping was assumed in the analysis, and these soft instabilities are likely to be mediated by small amounts of damping in the structure. Inspection of the mode shapes at the onset of aeroelastic instability are shown in Figure 5 and Figure 6 for the 2 and 3 bladed glass composite Darrieus designs. The mode shape reveals the unstable mode is primarily that of a first tower mode. Similar mode shapes were seen for the carbon composite Darrieus designs.

Examining results of aeroelastic analysis shows the VAWT designs considered in this study have sufficiently high flutter speeds outside the operating rotor speeds of these configurations. Interestingly, the carbon composite designs have much higher flutter speeds than a geometrically equivalent turbine composed of glass composite. Furthermore, it appears that the number of blades employed on a Darrieus design did not have a significant impact on the flutter speed. However, it can be noted that the magnitudes of damping in unstable modes of three-bladed designs tend to reach a higher magnitude than a two-bladed configuration employing the same blade design.
The mode shapes of the modes with soft instabilities for the 2 and 3 bladed carbon V-VAWT configurations are presented in Figure 7 and Figure 8 respectively. The unstable mode shape for the 2-bladed configuration in Figure 7 shows a coupled flap and edgewise mode with some twisting of the blades. The unstable mode shape for the 3-bladed configuration in Figure 8 is primarily a tower mode, with some blade deformation. The unstable mode shapes observed for the entire VAWT configuration can be hard to compare to classical flutter mode shapes of coupled pitch/plunging for aircraft wings or HAWT blades. Indeed, the VAWT configurations have fairly significant geometrical differences and the impact of this should be explored in future work.

![Figure 3](image1.png)

**Figure 3.** Frequency and damping vs. rotor speed for the DC2LCDT turbine.

![Figure 4](image2.png)

**Figure 4.** Damping trends of primary unstable modes for VAWT configurations.
Although Figure 4 identifies the flutter speed of the various turbines considered in this study it is important to view these within the context of the operating speed of the turbine. Thus, an aeroelastic stability margin (a unit-less metric) which normalizes the flutter rotor speed by the operating speed shown in Table 1 was computed as shown in Figure 9. An aeroelastic stability margin below or around 1 is undesirable, indicating instability could be excited during normal operating conditions. Although none of the configurations employed in this study had aeroelastic stability margins at or below unity, the glass Darrieus configurations had the lowest aeroelastic stability margins below 1.5. Additional design considerations may be desirable to further increase the aeroelastic stability margins of these designs. The carbon Darrieus designs performed well with aeroelastic stability margins above 3. Furthermore, the V-VAWT designs had relatively large aeroelastic stability margins, especially considering the very soft nature of damping in unstable modes which is likely to be mediated by the presence of modal damping.

Figure 5. Unstable mode shape for DG2LCDT.  
Figure 6. Unstable mode shape for DG3LCDT.

Figure 7. Unstable mode shape for VC2N5LC.  
Figure 8. Unstable mode shape for VC3N5LC.
5. Conclusions and Future Work
The availability of offshore wind resources in coastal regions, along with a high concentration of load centers in these areas, makes offshore wind energy an attractive opportunity for clean renewable electricity production. High infrastructure costs such as the offshore support structure and operation and maintenance costs for offshore wind technology, however, are significant obstacles that need to be overcome to make offshore wind a more cost-effective option. A VAWT rotor configuration offers a potential transformative technology solution that significantly lowers COE for offshore wind due to its inherent advantages for the offshore market. The large-scale of multi-megawatt VAWT configurations required for a significant reduction in COE are potentially prone to aeroelastic instabilities due to increased tip speed ratios and low system natural frequencies as a result of machine scale.

Aeroelastic analysis of a select few VAWT configurations has revealed varying degrees of sensitivity to aeroelastic instabilities. The aeroelastic analysis presented in this paper is considered to be an initial design study to scope preliminary VAWT configurations for aeroelastic instabilities. Overall, the multi-MW VAWT designs considered in this paper had sufficient aeroelastic stability margins with aeroelastic instabilities occurring outside of operational rotor speeds. Thus, other design drivers may be more critical than aeroelastic stability for the design considered. Predictions for carbon designs (Darrieus and V-VAWT) had larger stability margins than comparable glass designs. Indeed, the increased stiffness to mass ratio of carbon composite relative to glass composite may serve to elevate natural frequencies of the structure and alleviate flutter concerns (although this advantage comes at the expense of higher material costs).

The current aeroelastic implementation allowed for a convenient, straightforward extension of existing structural dynamics software to scope the aeroelastic stability of initial multi-MW VAWT designs and may serve as a foundation for the development of future analysis capabilities. Future work should seek to improve aeroelastic analysis for deepwater offshore VAWTs. This includes, considering inflow and wake effects which will bring about periodicity in the system. Thus, techniques for examining stability of linear time-varying periodic (LTP) systems should be employed. Furthermore, the aeroelastic formulation implemented into OWENS (and future enhancements) should be validated with available data sets such as that from Sandia 2-meter VAWT test-bed [5]. Finally, extensions should be made to consider effects of a floating platform support on the aeroelastic response of a VAWT.
Acknowledgements
The authors wish to thank Joshua Paquette of the Wind Energy Technologies Department at Sandia National Laboratories for his assistance in providing the turbine designs used in this study. This work was funded by the U.S. Department of Energy, Wind and Water Power Technologies Office.

References
[1] Lobitz D 2004 Aeroelastic Stability Predictions for a MW-sized Blade Wind Energy 7 211
[2] Owens B C, Griffith D T, Resor, B R, and Hurtado, J E 2013 Impact of Modeling Approach on Flutter Predictions for Very Large Wind Turbine Blade Designs Proceedings of the American Helicopter Society (AHS) 69th Annual Forum, (Phoenix, AZ, USA, 21-23, May 2013)
[3] Resor B R, Owens B C, and Griffith D T 2012 Aeroelastic Instability of Very Large Wind Turbine Blades Proceedings of the European Wind Energy Conference Annual Event (Copenhagen, Denmark, 16-19 April 2012).
[4] Hansen M 2007 Aeroelastic Instability Problems for Wind Turbines Wind Energy 10 551
[5] Popelka D 1982 Aeroelastic stability analysis of a Darrieus wind turbine, Sandia National Laboratories, SAND82-0672
[6] National Aeronautics and Space Administration 1986 The NASTRAN User’s Manual
[7] Douglas F 1980 The NASTRAN Programmer’s Manual, National Aeronautics and Space Administration.
[8] Owens B C, Hurtado, J E, Barone, M F, Griffith, D T, and Paquette, J A 2013 Aeroelastic Modeling of Large Offshore Vertical-Axis Wind Turbines: Development of the Offshore Wind Energy Simulation Toolkit Proceedings of the 54th AIAA Structures, Structural Dynamics and Materials Conference (Boston, MA, 8-11 April 2013)
[9] Owens B C 2013 Theoretical Developments and Practical Aspects of Dynamic Systems in Wind Energy Applications. Ph.D. Dissertation (College Station, Texas: Texas A&M University)
[10] Wright J and Cooper J 2008 Introduction to Aircraft Aeroelasticity and Loads, (Hoboken:Wiley)