Proton breakup in high-energy $pA$ collisions from perturbative QCD

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We argue that the distribution of hadrons near the longitudinal light-cone in central high-energy $pA$ collisions is computable in weak coupling QCD. This is because the density of gluons per unit transverse area in the dense target at saturation provides an intrinsic semi-hard momentum scale, $Q_s$. We predict that the longitudinal distribution of (anti-)baryons and mesons steepens with increasing energy and atomic number of the target, and that the transverse momentum distribution broadens. We show that the evolution of high moments of the longitudinal net baryon distribution with $Q_s$ is determined by the anomalous dimension $\gamma_{qq}$.

The computation of leading hadron (baryon) production in inelastic hadronic collisions from perturbative QCD (pQCD) usually suffers from the absence of a large momentum transfer scale. For example, in $pp$ or $pA$ collisions (at not too high energy) many times the incident baryon stays intact and emerges in the final state with a large fraction $z$ of the projectile light-cone momentum (the “leading particle” effect, see the discussion and further references in \cite{1,2}). Phenomenologically, one may picture this as the interaction of only one of the valence quarks of the proton with the target, which is then shifted to smaller $z$. The spectator diquark hadronizes and carries away roughly $z \approx 2/3$ of the incident momentum. Simple models like the “inside-outside” cascade assume dominance of such soft processes for leading baryon production up to high energies and so predict that baryon number is largely concentrated about the rapidity of the incident proton, and at small transverse momentum. Nonperturbative models based on the “gluon junction” and “diquark breaking” mechanisms have been proposed in the literature to describe the breakup of the projectile, see e.g. \cite{3}. In such models, also, baryons and anti-baryons are produced mainly at small transverse momentum.

In this note, we argue however that the inclusive distribution of leading hadrons in fact can be computed in weak coupling for very high energy collisions when the target approaches the “black body” limit, and that leading hadron production is strongly suppressed. This is because for a dense target all incident proton constituents scatter and experience a large momentum transfer (which is set by the “saturation” density scale of the target). Thus, the coherence of the projectile is destroyed completely, and the scattered quarks and gluons fragment independently \cite{4,2,5,6}. As a consequence, the proton decays predominantly into a beam of leading mesons, with the baryon number shifted to small light-cone momentum fraction $z \lesssim 0.1$. Also, we expect that the $p_t$ distributions of leading baryons and mesons closely reflect that of the scattered quarks, which is rather flat up to transverse momenta on the order of the square root of the density of gluons per unit area in the dense target.

The phenomenon we discuss here can be understood in two complementary languages. The limiting case of scattering off a black body is more transparent in the frame where the nucleus is at rest. The partons up to a resolution scale $p_t(s)$ (which is somewhat higher for gluons than for quarks) interact with the target with the geometric cross section of $2\pi R_A^2$. This implies the natural though pretty strong assumption that the parton - nucleus cross section is not tamed at a substantially smaller value. Hence in this limit the projectile wave function is resolved at a virtuality of $\sim p_t^2(s)$ which is much higher than for soft processes. In this frame, the process of leading hadron production corresponds to releasing the resolved partons from the projectile wave function. Correspondingly, the partons fragment with large transverse momenta $\sim p_t$ and essentially independently since their coherence was completely lost in the propagation through the black body. In the case of $\gamma^*A$ scattering one is able to make model independent predictions for the leading hadron spectrum \cite{7} which differ drastically from the DGLAP limit, providing an unambiguous signal for the violation of the leading twist approximation.

A complementary way to discuss the limit of high densities is the infinite momentum frame treatment. Indeed, the wave function of a hadron (or nucleus) boosted to large rapidity exhibits a large number of gluons at small $x$. The density of gluons is expected to saturate when it becomes so large that gluon splitting is balanced by recombination \cite{8}. The density of gluons per unit of transverse area and of rapidity at saturation is denoted by $Q_s^2$, the so-called saturation momentum. This provides an intrinsic momentum scale \cite{9} which grows with atomic number (for nuclei) and with rapidity, due to continued gluon radiation as phase space grows. For sufficiently high energies and/or large nuclei, $Q_s$ can become much larger than $\Lambda_{QCD}$ and so weak coupling methods are applicable. Nevertheless, the well known leading-twist pQCD can not be used precisely because of the fact that the density of gluons is large; rather, scattering amplitudes have to be resummed to all orders in the density. When probed at a scale below $Q_s$, scattering cross
sections approach the geometrical size of the hadron (the “black body” limit), while far above \( Q_s \) the hadron is in the dilute regime where cross sections are approximately determined by the known leading-twist pQCD expressions. For practical applications we actually need a model which interpolates between the black body kinematics below the scale \( Q_s \) and large \( Q^2 \) where pQCD is applicable. For these purposes we shall employ the “Color Glass Condensate” model of [9].

We shall focus here on inclusive hadron production in the forward region of central \( pA \) collisions at high energies, such as at BNL-RHIC and CERN-LHC. The “forward region” is defined as the fragmentation region of the projectile, which may also be a small nucleus rather than a proton. (In the near future, deuteron-gold collisions will be studied experimentally at RHIC.) Such central, small impact parameter collisions can be selected using additional experimental triggers in the fragmentation region of the big nucleus.

The target nucleus, when seen from the projectile fragmentation region, is characterized by a relatively large saturation momentum. Its precise value can not be computed from first principles at present, but extrapolation of HERA fits to nuclei including the gluon shadowing effects yield values of \( Q_s^2 \sim 10 \text{ GeV}^2 \) at \( x \sim 10^{-4} \) reachable for BNL-RHIC energy and \( \sim 50 \text{ GeV}^2 \) at \( x \sim 10^{-6} \) for CERN-LHC energy (for \( A \sim 200 \) central nuclear targets) [10]. For LHC energies, it might even be interesting to consider head-on central \( pp \) collisions: seen from the fragmentation region of one of the protons, the other hadron might exhibit gluon densities as high as those for \( A \sim 200 \) nuclear targets at RHIC energy.

In the above-mentioned kinematic domain, the dominant process is scattering of quarks from the incident dilute projectile on the dense target. (In our numerical estimates we include scattering of gluons, using the factorization theorem proven in [11].) However, for leading hadron production, \( z > 0.1 \), this amounts to only a small correction since the gluon spectrum drops faster with \( z \), and because gluon \( \to \) hadron fragmentation is steeper.) At high energies (in the eikonal approximation), the transverse momentum distribution of quarks is essentially given by the correlation function of two Wilson lines \( V \) running along the light cone at transverse separation \( r_t \) (in the amplitude and its complex conjugate),

\[
\sigma^{qA} = \int \frac{d^2 q_t dq_q^+}{(2\pi)^2} \delta(q_+^+ - p_+^+) \left\langle \frac{1}{N_c} \text{tr} \left( \int d^2 z_t e^{i\vec{q}_t \cdot \vec{z}_t} [V(z_t) - 1]^2 \right) \right\rangle .
\]

(1)

Here, the convention is that the incident proton has positive rapidity, i.e. the large component of its light-cone momentum is \( P^+ \), and that of the incoming quark is \( p^+ = xP^+ \) (\( q^+ \) for the outgoing quark). The two-point function has to be evaluated in the background field of the target nucleus. A relatively simple closed expression can be obtained [5] in the “Color Glass Condensate” model of the small-\( x \) gluon distribution of the dense target [9]. In that model, the small-\( x \) gluons are described as a stochastic classical [12] non-abelian Yang-Mills field which is averaged over with a Gaussian distribution. The inelastic \( qA \) cross section is then given by [5]

\[
q^+ \frac{d\sigma^{qA\rightarrow qX}}{dq^+ dq_t dq_q^+ db} = \frac{q^+_+}{P_+^+} \delta \left( p^+_+ - q^+_+ \right) \frac{1}{(2\pi)^2} C(q_t),
\]

\[
C(q_t) = \int d^2 r_t e^{i\vec{q}_t \cdot \vec{r}_t} \exp \left( -2Q_s^2 \int \frac{d^2 l_t}{(2\pi)^2 l_t^2} \left( 1 - \exp(i\vec{l}_t \cdot \vec{r}_t) \right) \right) .
\]

(2)

(We define \( Q_s^2 \) as in [13], not [5].) The integrals over \( l_t \) are cut off in the infrared by a cutoff \( \Lambda \), which we assume is on the order of \( \Lambda_{QCD} \). At asymptotically large \( q_t \gg Q_s \), the correlator \( C(q_t) \) smoothly matches leading-twist perturbation theory: \( C(q_t) \rightarrow Q_s^2/q_t^4 \) [13]. For \( q_t \ll Q_s \), the cross section is much flatter than that from leading-twist perturbation theory [5,13]. Thus, the incident collinear quark pairs typically are scattered to large transverse momenta of order \( Q_s \gg \Lambda_{QCD} \). This is rather different from the behavior of leading-twist pQCD, where the momentum transfer peaks at \( q_t \sim \Lambda_{QCD} \). In eq. (2), \( Q_s^2 \) is in principle a function of \( x \), and of the impact parameter \( b \). In what follows, we shall consider only central collisions where \( Q_s^2 \) is largest. Also, we focus on a not very large rapidity interval in the forward region, and so assume that \( Q_s \) is essentially constant.

To compute inclusive production of hadrons, the quark distribution is convoluted with the (leading twist) quark distribution function of a proton and a fragmentation function:

\[
\int \frac{d^2 q_t dl_t}{(2\pi)^2} \frac{1}{2} \int d^2 q_t dl_t f_{q/p}(x, Q_s^2) D_{q/h}(\frac{z}{x}l_t, Q_s^2) \delta^2(xl_t/z + q_t - xk_t/z) C(q_t),
\]

(3)

Note that here we make the natural assumption, in line with the discussion above, that the projectile partons are resolved at the scale \( Q_s^2 = Q_s^2 \), i.e. that the factorization scale is given by the saturation momentum of the dense
In eq. (3), $k_t$ denotes the transverse momentum of the produced hadron, and $\log z$ is its rapidity relative to the proton beam; $l_t$ is the transverse momentum acquired in the fragmentation. We employ the CTEQ5L parameterization of the parton distribution functions in the proton [15].

Since the transverse spread in the fragmentation of the quark at $Q_s$ is much smaller than that originating from the scattering on the dense target, we can safely neglect this effect and use the fragmentation functions integrated over transverse momentum,

$$z \frac{d\sigma^{pA\rightarrow hX}}{dzdk_t^2} = \frac{1}{(2\pi)^2} \int_0^1 dz \frac{x}{z} f_{q/p}(x,Q_s^2) D_{h/q} \left( \frac{z}{x},Q_s^2 \right) C \left( \frac{xk_t}{z} \right).$$

Specifically, we employ the KKP parameterizations [16] for the fragmentation functions. The main contribution to the leading hadron yield comes from $x \sim z$. Thus, the transverse spectra of the leading hadrons (especially those of baryons) are tracking very closely the spectra of quarks.

We give our predictions for the $k_t$ spectra in Fig. 1. As expected, we find that the transverse momentum distribution at large $z$ flattens as the target density ($\sim Q_s$) increases. At the same time, the longitudinal ($z$-) distribution steepens, resulting in larger suppression of forward hadron production. If we integrate over $k_t$, we are left with a convolution of the quark distribution in the proton with the fragmentation function, $z(d\sigma^{pA\rightarrow hX}/dzdb) = \int_0^1 dx \frac{x}{z} f_{q/p}(x,Q_s^2) D_{h/q}(z/x,Q_s^2) d\sigma^{qA\rightarrow hX}/db$, similar to the expression given in [4,17].

The existence of such a “limiting fragmentation curve” [18] was confirmed recently by the PHOBOS collaboration at RHIC [19]. Note, however, that the “limiting fragmentation curve” keeps evolving with $Q_s$, even in the black body limit $Q_s \gg \Lambda$. This is due to QCD scaling violations, i.e. DGLAP evolution of the dilute projectile fields and of the fragmentation functions. The “limiting” distribution of net baryon number is given by

$$z \frac{d\sigma^{pA\rightarrow (B-\bar{B})X}}{dzdb} = \int_0^1 dx \frac{z}{x} [f_{q/p}(x,Q_s^2) - f_{\bar{q}/p}(x,Q_s^2)] \left[ D_{B/q}(\frac{z}{x},Q_s^2) - D_{\bar{B}/q}(\frac{z}{x},Q_s^2) \right] \frac{d\sigma^{qA}}{db}.$$

The evolution of the moments of this distribution with $Q_s^2$ in the black body limit is given by

$$\frac{d}{d\log Q_s^2} (z^n)_{B-\bar{B}} = \frac{\alpha_s(Q_s)}{\pi} \gamma_{qq}^{(0)} (n+1) (z^n)_{B-\bar{B}}, \tag{6}$$

with $\gamma_{qq}^{(0)}$ the LO anomalous dimension for the quark distribution and fragmentation functions.
In fig. 2 we show $z dN(p - p)/dz \equiv (zd\sigma^A/dzd^2b)/(d\sigma^A/d^2b)$. Lacking a parametrization of the net proton fragmentation function, we employ the ansatz $(D_B/q(z) - D_{B/q}(z))/D_{B/q}(z) \approx \sqrt{z}$, which appears consistent with the data of ref. [20] on the $p/p$ ratio. This is of course a rough qualitative ansatz which should not be used at small $z$. In any case, one does not expect that the assumption of independent fragmentation applies at small $z$, when the (longitudinal and transverse) momentum degradation in the fragmentation process is very large. This restricts the applicability of eq. (6) to sufficiently high $n$, say $\geq 5$.

Because of the softer fragmentation function of a quark to a baryon than to a meson we expect that in the "black-body" limit the leading particle spectrum will be dominated by production of mesons rather than nucleons, in qualitative difference to $pp$ scattering (see Fig. 1, right). Hence, though it would be possible to observe the $k_t$ broadening using a generic small angle neutron calorimeter, it will be necessary to do additional measurements to separate the $\pi^0, K_L, \eta, ..$ and neutron contributions. The current upgrade of STAR to extend the acceptance to larger rapidities and the BRAHMS detector will provide other opportunities at the BNL-RHIC accelerator.

Since the suppression of leading hadrons is so strong one may ask whether other mechanisms may compete. One such mechanism is the propagation of the nucleon through the target in a nearly point-like configuration. If its size is smaller than $1/Q_s$, it will not be resolved by the nucleus. However, this mechanism is power suppressed [4], and strongly peaked at small $k_t$. Also, the experiments with the deuteron beam at RHIC can measure directly the probability for a nucleon to go through the center of the target without interacting [21]. Hence, this contribution can be estimated, in principle.

In summary, we argue that for very high energy $pA$ collisions the production of leading hadrons, most notably that of forward baryons and anti-baryons, should be computable in weak coupling QCD. This is because the large gluon density of the "black body" target per unit transverse area at small $x$ provides an intrinsic semi-hard momentum scale. Resumming higher-twist contributions to the quark-nucleus cross section we predict a depletion of forward hadron production, which becomes stronger as the atomic number of the target nucleus and/or the energy increase. (This should be even more pronounced for $\Lambda$ hyperons than for nucleons because incident $u, d$ quarks are favored.) At the same time, the transverse momentum distribution of forward hadrons broadens.

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