ROLE OF UNIFICATION IN STRING THEORY

Microscopic phenomena up to sub-TeV scales are successfully described by the Standard Model (SM). Its overlooked but important feature might be being a gauge theory based on a particular group. Quantum consistency of anomaly cancellation requires very special matter contents, and it is best understood by spontaneous symmetry breaking of a unifying theory extending the gauge group \([1]\). The minimal anomaly free chiral representation \(16\) of \(SO(10)\) contains all the observed fermions of SM including right-handed neutrino. With the aid of supersymmetry (SUSY), Higgs bosons belong to a hypermultiplet and can be treated on equal footing as matter. Thus matter and Higgs pair (as well as colored Higgs pair) are unified to a single representation \(27\) of \(E_6\), predicting another kind of singlet related to \(\mu\)-parameter. Continuing, larger groups in higher dimension, we also have to turn on so-called \(G\)-flux or gauge bundle in extra dimensions, supplements of SM including right-handed neutrino. With the aid of anomaly cancellation requires very special matter content, and it is best understood by spontaneous symmetry breaking of a unifying theory extending the gauge group \([1]\). The minimal anomaly free chiral representation \(16\) of \(SO(10)\) contains all the observed fermions of SM including right-handed neutrino. With the aid of supersymmetry (SUSY), Higgs bosons belong to a hypermultiplet and can be treated on equal footing as matter. Thus matter and Higgs pair (as well as colored Higgs pair) are unified to a single representation \(27\) of \(E_6\), predicting another kind of singlet related to \(\mu\)-parameter of SUSY \([2]\). Continuing, larger groups in higher dimension can even unify vector multiplets. This is well-known chain of \(E_n\) unifications in which the position of the SM group is unique \(E_3 \times U(1)\).

However, it is difficult to realize the unification in field theory, yielding the observed vacuum configuration by the scalar potential using only scalar fields. Magnetic flux or gauge bundle in extra dimensions supplements the idea, but anomaly cancellation is only guaranteed if it is embedded in consistent string theory, free of one-loop divergence. So we construct the model from heterotic string or its dual F-theory, which explains the origin of \(E_8\) gauge theory where the \(E_n\) series ends \([3]\), and provides all the above ingredients. Further constraints, such as repetition of matter generations and gauge coupling unification, show this unification pattern itself is compelling in any UV completion \([4]\).

Our strategy here is to construct the SM gauge group from the beginning, nevertheless the matter generations and Higgses can follow unification relation of a larger group. This feature is clearly realized by spectral cover construction \([5]\). It enables us to explicitly construct (poly)stable vector bundle on elliptic fibered manifold, using two data. The first is spectral cover that specifies the structure group ‘broken part of the \(E_n\)’ hence determines unbroken gauge group, here to be \(SU(3) \times SU(2) \times U(1)\). To obtain chiral spectrum in four dimension, we also have to turn on so called \(G\)-flux. If we partly turn on the flux on \(S[U(3)_S \times U(1)]\) spectral cover, the resulting identities of chiral matter fall into \(27\) of \(E_6\), the commutant of \(SU(3)\) in \(E_6\), and the numbers of matter generations and Higgs pairs obey a unification relation of \(G = SO(10)\), the commutant to \(SU(4)\). Some things happen in heterotic dual theory. Those two data are translated into a vector bundle \(V\) of \(H\) in the heterotic side by Fourier–Mukai transformation \([6,7]\). So, first the vector bundle \(V\) of the same structure group yields the unbroken group \(G\). Then there is a Wilson line on the elliptic fiber, breaking \(G\) down to the SM group. However, the language of spectral cover is easy to distinguish matter from Higgs and to control their multiplicities separately.

Finally, the four dimensional interactions follow from gauge invariant terms of the higher dimensional effective Lagrangian by dimensional reduction \([8,9]\). The invariance under the mother group \(E_6\) plays the role of selection rule.

GAUGE GROUP

The model is obtained from F-theory compactification on elliptic Calabi–Yau fourfold with a section, admitting heterotic dual. We construct the spectral cover for the structure group \(S[U(5) \times U(1)]\) as follows \([10,11]\). First, embed a twofold \(B_2\) into a projectivized threefold \(Z = P(K_{B_2} \oplus O) \vee B_2\) using the canonical bundle \(K_{B_2}\) \([6,12]\) (Its global embedding using heterotic dual is easily done \([7]\).) The spectral cover in it is described by

\[
(b_0 s^5 + b_1 s^4 + b_2 s^3 + b_3 s^2 + b_4 s + b_5)(s + a_1) = 0 \quad (1)
\]

where \(a_1 = -b_1/b_0\) and \(s\) is an affine coordinate parameterizing the normal direction to \(B_2 = \{s = 0\}\). Each coefficient \(b_m\), parameterizing the positions of the covers, is related to the elementary symmetric polynomial of degree \(m\), out of weights of the fundamental representations \(5_1 + 1_{-5}\) of the \(S[U(5) \times U(1)]\). The surviving group on \(B_2\) is the commutant, the SM group \(SU(3) \times SU(2) \times U(1)\). This a sufficient specification, but it also provides the information on the unbroken gauge group \([12]\).

The singularity equation corresponding to the SM
we have the SM gauge group \([11]\). Referring to Tate’s part also special up to \(B\) extra \(\eta\) quantum numbers. All the indices take different value in \(S_1 = \{1, 2, 3\}\). The fields below middle line are decoupled.

The number of chiral \([4]\) vector bundle \(\pi\), respectively. The number of Higgs pairs turns out to be completely fixed to be twice the matter multiplicity \([10]\). Here we have one more parameter \(\zeta\), the trace part of \(U(3)\) in \(SU(4)\) vector bundle \([13]\), to relax the condition. So we turn on a universal spectrum.

The chiral spectrum in four dimension is completed by specifying G-flux. The field strengths along the Cartan direction come from the dimensional reduction of four-form field strength \(G\) of the dual M-theory and this induces vector bundle on the spectral cover \([5]\). Although the minimal \(SU(4)\) G-flux preserves unification relation of its commutant \(SO(10)\) in \(E_8\), the number of Higgs pairs turns out to be completely fixed to be twice the matter multiplicity \([10]\). Here we have one more parameter \(\zeta\), the trace part of \(U(3)\) in \(SU(4)\) vector bundle \([13]\), to relax the condition. So we turn on a universal flux

\[
\Gamma_3 = \lambda(3\sigma - \pi_3^*(\eta - 3c_1)) + \frac{1}{2} \pi_3^* \zeta, \quad \Gamma_Z = -\pi_2^* \zeta, \quad (5)
\]

where \(\sigma\) is the class for \(B_2\) inside \(\tilde{Z}\) and \(\pi_3, \pi_2\) are projections from \(U(3)_{-2}\) and \(U(1)_Z\) covers to \(B_2\), respectively. We do not turn on flux on \(X\) and \(Y\) cover thus the components \(t_5\) and \(t_6\) can be neglected in calculating the spectrum.

The number \(n_R\) of chiral \(R\) zero modes minus antichiral \(\bar{R}\) ones of the Dirac operator in \(\tilde{Z}\) is a topological number and counted by index theorem. It is simply given by the intersection between matter curve class and Poincaré dual of the G-flux, projected on \(B_2\) \([7, 15]\): \n
\[
n_R = P_R \cap \Gamma|_{B_2}. \quad (6)
\]
Because of identical geometry of spectral cover as in Ref. 16, 17, and we refer to it for the calculation of matter curves

\[ n_l = n_u^v = n_d^v = n_t^v = n_e^v = n_\nu^v = (3\sigma + \eta) \cap \sigma (\lambda(3\sigma_{\infty} - \eta) + \frac{1}{3}\zeta)|_{B_2}, \]

\[ n_{D_1} = n_h_u = (-2\sigma + \eta) \cap \sigma (\lambda(3\sigma_{\infty} - \eta) + \frac{1}{3}\zeta)|_{B_2} = (\lambda \eta - \frac{2}{3}\zeta) \cdot (\eta - 3c_1), \]

\[ n_{D_2} = n_h_d = (3\sigma + \eta) \cap \sigma (\lambda(3\sigma_{\infty} - \eta) + \frac{1}{3}\zeta)|_{B_2} = (\lambda \eta - \frac{2}{3}\zeta) \cdot (\eta - 3c_1), \]

\[ n_X = n_{N_Y} = n_{T_C} = n_{\Sigma} = 0, \]

\[ n_Q = n_{U^c} = n_{E^c} = n_{D^c} = n_L = n_{N_e} = c_1 \cdot \zeta, \]

\[ n_S = (3\sigma + \eta) \cap \sigma (\lambda(3\sigma_{\infty} - \eta) + \frac{1}{3}\zeta + \zeta)|_{B_2} = (-\lambda \eta + \frac{4}{3}\zeta) \cdot (\eta - 3c_1). \]

Here we omitted pullback and the product is for the divisors of \( B_2 \). We defined \( \sigma_{\infty} = \sigma + \pi^*c_1 \). The identities of matter are those in 27 of \( E_6 \), and their multiplicities manifest the \( SO(10) \) unification relation: it is a nontrivial check that \( h_u \) and \( h_d \) gives the same number in 89 and 19.

The numbers of matter generations and Higgs pairs can be individually controlled, depending on the topological data on \( B_2 \). We require three generations of matter and one pair of Higgs doublet

\[ \lambda \eta \cdot (\eta - 3c_1) = -\frac{3}{2}, \quad \eta \cdot \zeta = 2, \quad c_1 \cdot \zeta = 0. \]

However this relation restricts the number of SM neutral field \( S \) be five. They are subject to quantization condition \( 3(\frac{1}{2} + \lambda) \in \mathbb{Z}, (\frac{1}{2} - \lambda) \eta + (3\lambda - \frac{3}{2})c_1 + \frac{1}{3}\zeta \in H_2(S, \mathbb{Z}) \).

We find a solution \( \lambda = \frac{1}{6} \) and the base as del Pezzo two surface with \( \eta = 2H, \zeta = H - 3E_1 \) do the job, where \( H \) is hyperplane divisor and \( E_1 \) is one of the exceptional divisor. In addition, because \( a_1 \) in \( 11 \) transforms as a section of \(-c_1 \), we have a scalar field \( O \) charged under \( SU(3) \) and another under \( SU(2) \), belonging to \( H^{2,0}(B_2) + H^{0,1}(B_2) \) 6. The other \( E_8 \) serves as hidden sector and is completely decoupled in smooth compactification and it can serve as supersymmetry breaking sector.

**Higgs Sector and Nucleon Decay**

The requirement of one pair of Higgs doublets fixed the factorization of spectral cover 4. It has the following phenomenological implications.

Firstly, it also distinguishes between up and down type Higgses. This is due to the structure of the \( U(3) \) monodromy \( S_3 \) 18, the permutation of the elements \( \{t_1, t_2, t_3\} \). It is the natural Weyl group, without a special monodromy further selected by hand. In terms of the \( S_3 \) representations, the fields having the same quantum number of lepton doublet under the SM group are

\[ l : \{t_1 + t_5 + t_6, t_2 + t_5 + t_6, t_3 + t_5 + t_6, t_4 + t_5 + t_6, t_4 + t_6 + t_5\}, \]

\[ h_u : \{t_1 + t_2 + t_6, t_2 + t_3 + t_6, t_3 + t_4 + t_6, t_4 + t_5 + t_6\}, \]

\[ h_d : \{t_1 + t_4 + t_6, t_2 + t_4 + t_6, t_3 + t_4 + t_6\}. \]

Effectively, the Higgs doublet is distinguished from the lepton doublet by an opposite matter parity or the \( U(1)_X \). It also forbids bare (super)renormalizable lepton and/or baryon number violating operators \( lh_u, l\nu^c, lq^c, w^c d^c \). Further factorization ruins this one Higgs pair structure but we obtain three pairs of Higgses, so our factorization seems the unique for the \( U(n) \) type spectral cover with universal flux.

Well-known is that the matter parity and \( U(1)_X \) alone cannot forbid dimension five proton decay operators such as \( qqq \) and \( w^c w^c d^c \). However, the above structure group forbids these operators. For instance, the former is not allowed because of nonvanishing sum of the weights \( (t_1) + (t_2) + (t_3) + (t_4) + (t_6) \) and \( (t_1) + (t_6) + (t_2) + (t_5) + (t_4) + (t_6) \), required by \( U(3)_L \) invariance, since one of \( S_3 \) index should appear twice 19. Once forbidden at the tree-level, it is also known that the induced operators are highly suppressed, probably explained by worldsheet instanton contribution 20.

Another prediction is the presence of an SM singlet field \( S \). Surveying the quantum number, it belongs to 27 representation of \( E_6 \), therefore, its interaction is restricted and we can calculate the corresponding terms. Because Higgs doublet and triplets are not simply vectorlike and up and down Higgses live on different matter curves, bare masses are forbidden by \( U(3)_L \) invariance. Instead we have Next-to-Minimal Supersymmetric SM (NMSSM) 21, 22. We can check that the only renormalizable superpotential for the surviving fields are (see also below)

\[ q_d u^c + q_h u^c + l h_d e^c + h_u \nu^c + S h_h \nu^c + S D_1 D_2^c + q d D_2^c + \nu^c d^c D_1 + u^c d^c D_2^c \]

omitting the flavor dependent coefficients. We expect the terms involving \( D_1 \) and \( D_2^c \) are all decoupled, yielding the \( \mu \)-term like \( S h_h \nu^c \). Bare quadratic or cubic terms in \( S \) are not allowed. Induced higher order terms include \( M_{\nu}^{-2}SS(Q D D L + D^c N D_1 + U^c E^c D_1) + M_{\nu}^{-4}SSS(U^c E^c L + Q D^c N^c L) \) but they are to be suppressed by a string scale \( M_s \). A Majorana mass for the \( \nu^c \) does not appear up to dimension five. There is an interesting room for this from Euclidean D3-brane or gauge instanton in F-theory 23, which might as well generate similar potential for \( S \). Remarkably, if one combination \( X - Z \) of \( U(1)_X \) and \( U(1)_Z \) is broken down, the orthog-
nal combination $X + Z$ inducing a Majorana neutrino mass also generates $S^3$ term $e^{-S_{int}(\nu^c\nu^c + SSS)}$.

Since the Higgs fields also obey $SO(10)$ unification relation, we have as many colored Higgs pairs $D_1, D_2$ as doublets. This doublet-triplet problem should be solved by an effect evading the unification structure, close but below the compactification scale. It is an interesting possibility to consider a vectorlike extra generation of matter fields, without changing the Dirac indices. Using aforementioned $SU(3)$ scalar $O$, there can be terms $(O) D_1 D_1 + (O) D_2 D_2 + M_O \text{tr} O^2 + \text{tr} O^3$, giving Dirac masses separately to $D_1, D_1$ and $D_2, D_2$ pairs. Conventional gauge coupling unification requires heavy triplets, so do a large VEV $\langle O \rangle$ and a large $M_O$. On the other hand, we expect a coupling $(\langle S \rangle + \mu_D) D_1 D_2$ is generated, with possible SUSY breaking $\mu_D$. The most strongly constrained nucleon decay operator is $qqql$, whose coefficient has upper bound $4 \times 10^{-7} M_{Pl}^{-1}$ [22]. At low energy scale, integrating out heavy fields, $qq D_1$ and $qf D_2$ may induce an operator $(\langle S \rangle + \mu_D)/M_D^2 \cdot qql ql D$ up to geometric suppression factor. If the numerator is naturally small or tuned, we can suppress this operator. A possible mixing from bare mass term $d^c d$ does not change this eigenvalue. The same argument goes to other induced operators for nucleon decay.

**GAUGE COUPLING UNIFICATION**

In type IIB language, Kaluza–Klein expansion of four-form field along the harmonic two-forms $w_i$ on $B_2, C_4 = C_2 \wedge w_2$, we have a term $\text{tr} Q^2 \int_M F_Q \wedge C_2 \int_B w_2 \wedge i F_Q$ from Chern–Simons interactions and here $F_Q$ is the field strength for $U(1)_Q$ flux and $i$ is immersion to $B$. We turn on is for $U(1)_Z$ thus the corresponding gauge boson acquire mass by Stückelberg mechanism and the symmetry is broken. We do not turn on flux along hypercharge or $X$ direction, so they are not broken and their gauge couplings receive no threshold correction from the flux [26, 27]. The existence of $U(1)$’s is clearly seen in the heterotic dual side. Four gauge couplings unify with the correct normalization in $SO(10)$

$$g_3 = g_2 = \sqrt{5/3} g_Y = \sqrt{40} g_X , \sin^2 \theta_W = 3/8,$$

at $M_s$. This is the consequence of the inheritance principle, for all the gauge group is embedded in $E_6$ and hence $SO(10)$. The $U(1)_X$ can survive at relatively low energy scale and be spontaneously broken down at relative low energy. Threshold corrections for the splitted Higgs triplets would modify the scale.

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[21] U. Ellwanger, C. Hugonie, A. M. Teixeira, Phys. Rept. 496, 1-77 (2010) and references therein; See also O. Lebedev, S. Ramos-Sanchez, Phys. Lett. B684, 48-51 (2010).
[22] M. Cvetic, J. Halverson, P. Langacker, JHEP 1009, 076 (2010).
[23] R. Blumenhagen, A. Collinucci, B. Jurke, JHEP 1008, 079 (2010); T. W. Grimm, M. Kerstan, E. Palti, T. Weigand, [arXiv:1105.3193 [hep-th]]; M. Cvetic, J. Halverson, I. Garcia-Exebarria, [arXiv:1107.2388 [hep-th]].
[24] G. F. Giudice, A. Masiero, Phys. Lett. B206, 480-484 (1988). J. E. Kim, H. P. Nilles, Phys. Lett. B138, 150 (1984).
[25] S. Raby, [hep-ph/0211024].
[26] R. Blumenhagen, Phys. Rev. Lett. 102 (2009) 071601.
[27] T. W. Grimm and T. Weigand, [arXiv:1006.0226 [hep-th]].
[28] K.-S. Choi, JHEP 1002, 004 (2010).