The approach to the development of a probabilistic model to determine the optimal mechanism for an unfinished oil well fund subsidising

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Abstract. In the context of the coronavirus pandemic and quarantine measures taken by the international community, energy demand declined significantly in the second quarter of 2020. In April, oil prices fell to a record low of $13-$14 per barrel. In order to restore oil prices, OPEC+ imposed strict restrictions on production in May. This has in turn decreased the number of drilling wells significantly and has had a negative impact on oilfield service companies. The decline of the oilfield services can lead to a loss of skills in the development of hard-to-recover reserves and to a loss of global market share. In order to reduce risks and ensure accelerated recovery of production volumes, a number of countries are developing and introducing methods of state subsidising to stimulate the construction of oil wells up to the moment of their completion. In Russia, the Ministry of Industry and Trade is also developing a plan to provide preferential loans to oilfield service organisations, and then oil companies will redeem finished wells. Given the recession in the global economy, the high volatility of macro parameters, the risk of the second wave of coronavirus and the new OPEC+ restrictions, makes it necessary to determine the best and most sufficient, but not excessive, mechanism for supporting oilfield services and stimulating the optimal pace of construction of oil wells depending on the type of oil fields and their remoteness from world markets’ sales. In view of the high uncertainty of model parameters, this paper proposes to use probabilistic methods and Bayesian methods to solve the above problems.

1. Introduction
On April 12, 2020, an OPEC+ agreement was reached [1], which meant that oil production would be significantly reduced (the agreement was adjusted on June 6, 2020):
- by 9.7 million barrels per day in May 2020 (for Russia – by 2.5 million barrels per day from the baseline value of 11 million barrels per day);
- by 9.6 million barrels per day in July 2020 (for Russia – by 2.5 million barrels per day);
- by 7.7 million barrels per day in August-December 2020 (for Russia – by 2 million barrels per day);
by 5.8 million barrels per day from January 2021 to April 2022 (for Russia – by 1.5 million barrels per day).

The head of the Russian Ministry of Energy, Alexander Novak, in an interview with Bloomberg [3], said that without support measures, the drop in orders for oil services in 2020, due to production restrictions, could be up to 30-40%, and in the future it might be difficult to restore production as the market crisis passes.

To prevent this, the Russian Ministry of Energy proposes to create a fund of uncompleted wells, modeled on the fund of uncompleted wells (DUC – drilled but uncompleted), which exists in the United States [3]. The wells will be drilled in the future without putting them into production as the demand recovers and the oil production increases.

State support will save the workload and solvency of oilfield service companies, safeguard jobs (the segment employs about 300 thousand people [3]), and continue development in the field of creating high-tech equipment, which is necessary for the domestic oil and gas industry for further development.

As of the end of July 2020, the main support tool was to provide loans for the construction of wells with full interest repayment or its part from the state budget.

In order to create the best uncompleted oil fund (whether for the national budget or for the profit of the oil company), it is recommended to use probability assessment methods.

2. Approaches to modelling

Under normal circumstances, the decision to build a well is made on the basis of the field development project, oil demand, current production restrictions, and the project’s profitability in current macro parameters. The drilling and completion time usually takes about six months. In the context of production restrictions in accordance with the OPEC+ agreement the decision to commission wells planned for construction in 2020-2021 may be postponed for 1-2 years.

When preferential loans are provided to create an uncompleted oil well fund (about 70% of the construction is completed), the national budget costs will be determined as:

\[
\sum_{t=1}^{t_n} \alpha \times C_1 \frac{1}{(1 + r)^t}
\]

where:
- \(\alpha\) is a loan rate (in % per month). It is based on the key rate level of the Central Bank of the Russian Federation and the agreement between the state and federal banks that provide borrowed funds for the oilfield services;
- \(C_1\) is the cost of a construction of an uncompleted well (depending on the complexity of production, the cost of a well is about 100-500 million rubles [4]);
- \(r\) is a discount rate (in % per month). It is based on the level of government bonds yield;
- \(t_n\) is the time period when the oil company buys and puts it into operation.

Additional revenues of the state budget will amount to:

\[
\sum_{t=t_n+1}^{t_N} \frac{d(t) \times Taxes (pr(t); cs(t))}{(1 + r)^t} + Ml
\]

where:
- \(t_N\) is the time to put the oil well into operation under normal project conditions (that is, no unfinished oil well funds have been created). It is based on the field development project and the current parameters of the OPEC+ deal;
- \(d(t)\) is an increase in oil production due to earlier well commissioning. Let’s assume the well production is equal to one in the design profile;
- \(pr(t)\) is the price of URALS crude oil in USD / barrel;
- \(cs(t)\) is a dollar exchange rate in rubles / dollar;
- \(Taxes (pr(t); cs(t))\) are tax revenues (MET, export duty, profit tax) in rubles / ton of oil.
$M_l$ is a coefficient representing the multiplicative and integral intersectoral effects of wells construction due to the maintenance of the profitability of the oilfield service sector.

The coefficient $M_l$ includes the following indicators:

- shortfall in budget revenues (social taxes and contributions, unemployment payments, the cost of creating qualified personnel) due to the reduction of jobs in the oilfield services;
- shortfall in budget revenues (income tax, property tax) due to a decrease in intersectoral orders;
- increasing integral coefficients as a result of reduced incomes in the sales of goods and services due to the fall in real incomes of the population employed in oilfield services and related industries.

As a result, the construction of an unfinished well will be effective for the state budget if:

$$
\sum_{t=t_n+1}^{t_N} \frac{d(t) \ast Taxes(p(t); cs(t))}{(1 + r)^t} + M_l > \sum_{t=1}^{t_n} \frac{\alpha \times C_1}{(1 + r)^t}
$$

or

$$
\sum_{t=1}^{t_N-t_n} \frac{d(t) \ast Taxes(p(t + t_n); cs(t + t_n))}{(1 + r)^t} > \alpha \times C_1 \times \frac{(1 + r)^{t_n} - 1}{r} - M_l
$$

Let’s check the existence of a relation between the URALS price in dollars / barrel, in rubles / barrel, and the dollar exchange rate using the Pearson correlation coefficient, calculated on the basis of actual data from January 2015 to July 2020 [5] (figures 1-3).

**Figure 1.** Pearson correlation coefficient = -0.48.

**Figure 2.** Pearson correlation coefficient = -0.19.
Thus, it is possible to transform the previous inequality (4) as follows:

$$\sum_{t=t_n+1}^{\infty} \frac{d(t) \times \text{Taxes} (\text{pr}(t + t_n) \times \text{cs}(t + t_n))}{(1 + r)^t} > \alpha \times C_1 \times \frac{(1 + r)^{\frac{n}{r}}}{r} - M_l$$

(5)

Let’s also accept the assumption of the independence of the random variables $t_n, \text{pr}_t, \text{cs}_{t}$, $\text{cs}_{t}$.

To buy out a well by an oil company, it is necessary that the well is profitable, i.e.:

$$\sum_{t=t_n}^{\infty} \frac{d(t) \times (\text{pr}(t) \times \text{cs}(t) \times 7.3 - \text{Tr}(\text{cs}(t))) - \text{Taxes} (\text{pr}(t) \times \text{cs}(t); \text{cs}(t))}{(1 + r)^t} \frac{C_1}{(1 + r)^{\frac{n}{r}}} > C_1$$

(6)

or

$$\sum_{t=1}^{\infty} \frac{d(t) \times (\text{pr}(t + t_n) \times \text{cs}(t + t_n) \times 7.3 - \text{Tr}(\text{cs}(t + t_n)))}{(1 + r)^t} \frac{\text{Taxes} (\text{pr}(t + t_n) \times \text{cs}(t + t_n); \text{cs}(t + t_n)) + \text{op}(t) + C_2(t)}{(1 + r)^t} > C_1$$

(7)

where:

$\text{Tr}(\text{cs}(t))$ – the transportation costs of oil exported from the oil field where the oil well is being drilled;

$\text{op}(t)$ – operating costs;

$C_2(t)$ – capital costs of well completion.

Thus, in order to make a decision on the provision of state support for the creation of a fund of uncompleted wells, it is necessary to determine the probability of the effectiveness of the construction of such wells for the state budget, subject to their purchase by oil companies, i.e., their profitability. It is convenient to use the Bayesian approach for this:

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

(8)

Assume that the event $A$ is the efficiency of the well for the state budget, and $B$ is the profitability of the well purchasing for the oil company.

Assume that $\Omega_A$ is the state space $\{t_n; \text{pr}_t \times \text{cs}_{t}; \text{cs}_{t}\}$ when the well construction is effective for the state budget.

Assume that $\Omega_B$ is the state space $\{t_n; \text{pr}_t \times \text{cs}_{t}; \text{cs}_{t}\}$ when the well purchasing is profitable for the oil company.

Assume that $\xi$ is the random state $\{t_n; \text{pr}_t \times \text{cs}_{t}; \text{cs}_{t}\}$.

Then the equation (8) can be represented as:
\[ P(\xi \in \Omega_A | \xi \in \Omega_B) = \frac{P(\xi \in \Omega_B | \xi \in \Omega_A) \cdot P(\xi \in \Omega_A)}{P(\xi \in \Omega_B)} \]  

(9)

Well purchasing time \( t_n \) is a discrete random variable that can take values in the range \([t_0; t_N + x]\), where:

- \( t_0 \) is the maximum value between the planned date of the well commissioning in the pre-crisis period and the completion date of the construction of an unfinished well when an appropriate decision is made;

- \( x \) is the technologically maximum possible shift in the well commissioning time when restrictions on oil production are tightened (for example, during the "second wave" of the coronavirus).

Then the right side of equation (9) can be represented as:

\[ \sum_{t_i = t_0}^{t_N + x} P(t_n = t_i) \cdot \frac{P(\xi' \in \Omega_B | \xi' \in \Omega_A) \cdot P(\xi' \in \Omega_A)}{P(\xi' \in \Omega_B)} \]  

(10)

Where \( \Omega_{Ai} \) is the state space \{ \( \bar{p}_{rt} c_{st} c_{st} \) \} when the well construction is effective for the state budget at \( t_n = t_i \):

\( \Omega_{Bi} \) is the state space \{ \( \bar{p}_{rt} c_{st} c_{st} \) \}, in which the well purchasing is profitable for the oil company when \( t_n = t_i; \)

\( \xi' \) is a random state \{ \( \bar{p}_{rt} c_{st} c_{st} \) \}.

For \( t_n = t_i \) equation (5) can be represented as follows:

\[ \sum_{t_i = 1}^{t_N - t_i} d(t) \cdot T axes \left( pr(t + t_i) \cdot cs(t + t_i); cs(t + t_i) \right) > \alpha \times C_1 \frac{(1 + r)^{t_i} - 1}{r} - M_l \]  

(11)

The tax burden can be represented as a linear function, then:

\[ \sum_{t_i = 1}^{t_N - t_i} d(t) \cdot \left( K_1 \cdot pr(t + t_i) \cdot cs(t + t_i) + K_2 \cdot cs(t + t_i) \right) > const_{ti} \]  

(12)

We accept the assumption of a normal distribution of quantities \( \bar{p}_{rt} \sim N(\bar{p}_{0r}; \sigma_{pr}^2) \) and \( c_{st} \sim N(c_{0s}; \sigma_{cs}^2) \). Where \( c_{0s} \) – average current course level, \( \bar{p}_{0r} \) – oil futures price, \( \sigma_{pr}^2 \) – average historical deviation of the spot oil price from the futures, \( \sigma_{cs}^2 \) – rate volatility deviation.

Then, the function of the left side of the equation (12) will have a normal distribution with mathematical expectation:

\[ \mu_{ti} = \sum_{t_i = 1}^{t_N - t_i} d(t) \cdot \left( K_1 \cdot pr_0(t + t_i) \cdot cs_0 + K_2 \cdot cs_0 \right) \frac{(1 + r)^t}{(1 + r)^t} \]  

(13)

And variance is:

\[ \sigma_{ti}^2 = \sum_{t_i = 1}^{t_N - t_i} d^2(t) \cdot \left( K_1^2 \cdot \sigma_{pr}^2 + K_2^2 \cdot \sigma_{cs}^2 \right) \frac{(1 + r)^{2t}}{(1 + r)^t} \]  

(14)

Then the probability \( P(\xi' \in \Omega_{Ai}) \) is:

\[ 1 - \Phi\left( const_{ti} \right) = 0.5 - 0.5 \times \text{erf} \left( \frac{const_{ti} - \mu_{ti}}{\sqrt{2} \cdot \sigma_{ti}^2} \right) \]  

(15)

For \( t_n = t_i \) the equation (7) can be represented as follows:

\[ \sum_{t = 1}^{\infty} d(t) \cdot \left( pr(t + t_i) \cdot cs(t + t_i) + 7.3 - Tr(cs(t + t_i)) \right) \frac{(1 + r)^t}{(1 + r)^t} - \frac{\text{Taxes}}{(1 + r)^t} \]  

(16)

Or, taking into account the linearity of the functions of tax burden and transport costs, it is:
\[ \sum_{t=1}^{\infty} \frac{d(t) \cdot (K_3 \cdot pr(t + t_i) \cdot cs(t + t_i) + K_4 \cdot cs(t + t_i))}{(1 + r)^t} > \text{const}_2 \]  

(17)

Then, the function of the left side of equation (17) will have a normal distribution with mathematical expectation:

\[ \mu_{2i} = \sum_{t=1}^{\infty} \frac{d(t) \cdot (K_3 \cdot pr_0(t + t_i) \cdot cs_0 + K_4 \cdot cs_0)}{(1 + r)^t} \]  

(18)

And variance is:

\[ \sigma^2_{2i} = \sum_{t=1}^{\infty} \frac{d^2(t) \cdot (K_3^2 \cdot \sigma^2_{pr} + K_4^2 \cdot \sigma^2_{cs})}{(1 + r)^{2t}} \]  

(19)

Then the probability \( P(\xi' \in \Omega_{B1}) \) is equal to:

\[ 1 - \Phi(\text{const}_2) = 0.5 - 0.5 \cdot \text{erf} \left( \frac{\text{const}_2 - \mu_{2i}}{\sqrt{2 \cdot \sigma^2_{2i}}} \right) \]  

(20)

As \( K_3 = 7.3 - K_1 \), and \( K_4 = - Tr_5 - K_2 \), where \( Tr_5 \) are foreign exchange transport costs (for example, freight), then equation (17) can be represented as:

\[ \sum_{t=t_{N-t_i+1}}^{\infty} \frac{d(t) \cdot (K_3 \cdot pr(t + t_i) \cdot cs(t + t_i) + K_4 \cdot cs(t + t_i))}{(1 + r)^t} \]

\[ + \sum_{t=1}^{t_{N-t_i}} \frac{d(t) \cdot (7.3 \cdot pr(t + t_i) \cdot cs(t + t_i) - Tr_5 \cdot cs(t + t_i))}{(1 + r)^t} > \text{const}_{1i} + \text{const}_2 \]  

(21)

Then, the function of the left-hand side of equation (21) will have a normal distribution with the mathematical expectation:

\[ \mu_{3i} = \sum_{t=t_{N-t_i+1}}^{\infty} \frac{d(t) \cdot (K_3 \cdot pr_0(t + t_i) \cdot cs_0 + K_4 \cdot cs_0)}{(1 + r)^t} \]

\[ + \sum_{t=1}^{t_{N-t_i}} \frac{d(t) \cdot (7.3 \cdot pr_0(t + t_i) \cdot cs_0 - Tr_5 \cdot cs_0)}{(1 + r)^t} \]  

(22)

And variance is:

\[ \sigma^2_{3i} = \sum_{t=t_{N-t_i+1}}^{\infty} \frac{d^2(t) \cdot (K_3^2 \cdot \sigma^2_{pr} + K_4^2 \cdot \sigma^2_{cs})}{(1 + r)^{2t}} + \sum_{t=1}^{t_{N-t_i}} \frac{d^2(t) \cdot (7.3^2 \cdot \sigma^2_{pr} + Tr_5^2 \cdot \sigma^2_{cs})}{(1 + r)^{2t}} \]  

(23)

Then the probability \( P(\xi' \in \Omega_{B1}|\xi' \in \Omega_{Ai}) \) is equal to:

\[ 1 - \Phi(\text{const}_{1i} + \text{const}_2) = 0.5 - 0.5 \cdot \text{erf} \left( \frac{\text{const}_{1i} + \text{const}_2 - \mu_{3i}}{\sqrt{2 \cdot \sigma^2_{3i}}} \right) \]  

(24)

Consequently, the probability of the effectiveness of an unfinished well construction for the state budget, subject to its subsequent purchase by an oil company, is:
\[ \sum_{t_i = t_0}^{t_N + x} P(t_n = t_i) \times 0.5 - 0.5 \times \text{erf} \left( \frac{\text{const}_{1i} + \text{const}_{2} - \mu_{3i}}{\sqrt{2 \times \sigma_{3i}^2}} \right) \times 0.5 - 0.5 \times \text{erf} \left( \frac{\text{const}_{1i} - \mu_{1i}}{\sqrt{2 \times \sigma_{1i}^2}} \right) \times 0.5 - 0.5 \times \text{erf} \left( \frac{\text{const}_{2} - \mu_{2i}}{\sqrt{2 \times \sigma_{2}^2}} \right) \] (25)

3. Conclusion

The development of information systems and tools for collecting and processing data has made it possible to use more accurate probabilistic evaluation methods instead of traditional scheme methods in decision-making.

The following algorithm is proposed to determine the scope of state support when creating uncompleted oil well funds:

1) Allocation of wells planned for commissioning in 2020-2021 from the design documentation, and the decision on the construction of which was postponed due to the coronavirus pandemic and decreased demand.

2) Request for current drilling plans and orders for oilfield services, which will allow excluding from the sample wells that have been planned for commissioning in the near future.

3) Combination of indicators of technologically connected wells (clusters). In the future, they will be counted as one well for evaluation.

4) Ranking and grouping of wells according to the order of commissioning according to the technological schemes of field development.

5) Determination of \( P(t_n = t_i) \) for \( t_i \in [t_0; t_N + x] \) for each group of wells. In the expert assessment of indicators, it is proposed to take into account the needful growth in oil production in Russia for the need to buy out wells at a certain moment and the likelihood of such growth, taking into account:
   - the current global balance of supply, demand and oil reserves;
   - growth rates of global oil demand;
   - forecasts of analytical agencies.

6) Determination of the current level of oil futures and variance \( \sigma_{PR}^2 \) based on historical data, taking into account the forecasts of analytical agencies.

7) Determination of the current level of the dollar exchange rate and variance \( \sigma_{CS}^2 \) based on historical data, taking into account the policy and forecasts of the Central Bank of Russia.

8) It is possible to use the following formula to determine the probability of an oil company buying an oil well:

\[ \sum_{t_i = t_0}^{t_N + x} P(t_n = t_i) \times 0.5 - 0.5 \times \text{erf} \left( \frac{\text{const}_{2} - \mu_{2i}}{\sqrt{2 \times \sigma_{2}^2}} \right) \] (26)

9) Exclusion from the sample of wells with a low probability of purchasing.

10) Calculation of coefficients characterising multiplicative and integral intersectoral effects, \( Ml \), taking into account the current economic indicators of oilfield service organisations.

11) Evaluation of the possibility of including a well in the unfinished well fund for the state budget, subject to its purchase by an oil company using the formula (25). Well ranking according to the probability of success.

12) Determination of the maximum amount of budget funds that can be used to support the oilfield services.
13) Creation of unfinished well funds, which will include objects with the highest probability of efficiency for the state budget, taking into account the limitation on the amount of funds provided by the state to support oilfield services.

The presented assessment model can be modified to take into account the risk of non-confirmation of the design level of production for various types of fields.

In the current environment of strong volatility in oil demand, to maintain its share of the global market for the Russian oil industry, it is essential to ensure sufficient production flexibility. For these purposes, the creation of an unfinished well fund can give the industry a strategic advantage. However, in order to avoid using this tool as an unprofitable reason for wasting national budget funds in the future, it is necessary to carefully evaluate all possible risks. Taking into account the development of software systems that make it possible to quickly process large amounts of data, to solve the problem, it is optimal to use probabilistic methods of assessment instead of the traditional scenario approach, which has a significant error.

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