A Multi-UAV Formation Maintaining Method Based on Formation Reference Point

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Abstract. In recent years, Unmanned Aerial Vehicles (UAV), have been increasingly utilized by scholars. Efficient control of UAV swarms opens a set of new challenges. There are several formation control methods, however, artificial potential field (APF) method has imperfections in formation maintaining and virtual structure is insufficient in collision avoidance. In this paper, we proposed a multi-UAV formation maintaining method combined artificial potential field method and virtual structure approach. The method uses the concepts of attraction and repulsion in the artificial potential field method, combined with formation reference points inspired by the virtual structure points in the virtual structure approach to control the formation. On the other hand, we express the leaders, the followers, and the formation reference points uniformly. When the formation of UAVs is determined, after the leader arrives at target point and hovers, the followers can track the formation reference points. The method can avoid collision between UAVs and achieve the purpose of formation maintaining. Finally, a UAV swarm consists of four UAVs is used for demonstration and verification, and the results of experiment prove the validity of the algorithm.

Keywords. Formation maintaining; formation reference point; artificial potential field method; virtual structure approach; UAV swarm.

1. Introduction
With the improvement of computing capabilities of embedded devices, people find that multi-agent system can complete complex tasks at a smaller cost. Multi-agent formation control refers to approaches to maintain a presupposed geometry, usually referred to as formation, and can also adapt to environmental constraints, such as obstacle avoidance, during the movement to a specific target. The formation control of quadrotor becomes a top issue due to the flexibility of quadrotor. There are several formation control approaches, such as artificial potential field method, leader-follower method, virtual structure approach and behavior-based method.

Artificial potential field is one of the most commonly methods to avoid obstacle. The main idea is to find a certain potential function so that the energy of the system is minimized and reaches its minimum value, preferably only, at the goal position. Furthermore, different tasks need different potential functions, for example, a function for attractive behavior and another function for repulsive behavior [1]. Leader-follower requires to specify a UAV as the leader, the rest as followers to follow leader’s movement. Position of follower usually determined by a leader $(l-y)$, and a two leaders control is optional($l-l$) [2]. The UAVs of virtual structure approach need to track points which defined by a presupposed virtual structure [3]. This method works excellent in formation maintaining but not so effectively in obstacle avoidance and collision avoidance. A UAV in behavior-based method only needs...
to achieve its own goal, defined by some simple behavior, so that the UAV swarm can complete a complex task. Behaviors of UAVs are uncertain, which results in a more difficult way to model, analyze and experimentalize [4]. Considering the pros and cons of given methods, scholars tend to control formation with multiple methods [5, 6].

In this document, a method combined virtual structure approach and artificial potential field method is proposed. It brings in the attraction and repulsion, which proposed in artificial potential field, for obstacle avoidance and collision avoidance. Also, formation reference point is proposed, which is derived by virtual structure point of virtual structure approach, in order to form a specific formation.

2. Formation Control

Considering a UAV swarm with \( n \) UAVs, \( \mathbf{P} = [\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \ldots, \mathbf{p}_n] \), given by GPS (Global Positioning System) or RTK (Real-time Kinematic) GPS in NED (North East Down) coordinate such as \( \mathbf{p}_i = (x_i, y_i, z_i) \), is the position vector of UAV in swarm. Define \( \mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \ldots, \mathbf{v}_n] \) and \( \mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \ldots, \mathbf{a}_n] \), usually given by IMU (Inertial Measurement Unit), as the velocity vector and acceleration vector. Obviously, \( \mathbf{v}_i \) and \( \mathbf{a}_i \) should meet the constraint:

\[
\begin{align*}
\mathbf{v}_i &= \begin{cases} 
\mathbf{v}_i & (\| \mathbf{v}_i \| \leq v_{\text{max}}) \\
\frac{v_{\text{max}}}{\| \mathbf{v}_i \|} \mathbf{v}_i & (\| \mathbf{v}_i \| > v_{\text{max}}) 
\end{cases} \\
\mathbf{a}_i &= \begin{cases} 
\mathbf{a}_i & (\| \mathbf{a}_i \| \leq a_{\text{max}}) \\
\frac{a_{\text{max}}}{\| \mathbf{a}_i \|} \mathbf{a}_i & (\| \mathbf{a}_i \| > a_{\text{max}}) 
\end{cases}
\end{align*}
\]

Assume that all UAVs are at the same altitude, which means \( z_1 = z_2 = z_3 = \ldots = z_n \). The distance between two UAVs can be expressed as:

\[
d = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}
\]

Each UAV in swarm can access all information (position, attitude, etc.) of other UAVs over WiFi or 4G/5G [7].

The communication topology of swarm is described as figure 1.
2.1. Potential Functions

The method uses APF for obstacle avoidance and collision avoidance. Key point is to choose appropriate potential functions, which have a close relationship with the complexity of calculation and rate of convergence.

In APF, attractive force works between target point and UAV, aiming at moving toward the presupposed target point. For follower, the target point is formation reference point. Repulsive force works when detected other UAVs or obstacles, with the purpose of obstacle avoidance and collision avoidance. Attractive force and repulsive force are given:

\[
\begin{aligned}
F_{\text{att}} &= k_{\text{att}} d \\
F_{\text{rep}} &= \frac{1}{2} k_{\text{rep}} \left( \frac{1}{d} - \frac{1}{d_{\text{opt}}} \right) \frac{1}{d^2}
\end{aligned}
\] (4)

When distance is over \(d_{\text{opt}}\), there is no repulsive force works. So, the resultant force is:

\[
F = F_{\text{att}} + bF_{\text{rep}}
\] (5)

We use a variable \(b\) to control if repulsive force take effect, and \(b\) meets:

\[
b = \begin{cases} 
0 & d > d_{\text{opt}} \\
1 & d \leq d_{\text{opt}}
\end{cases}
\] (6)

Direction of attraction points to target from UAV, as for direction of repulsion points to UAV from obstacle or other UAV. When use APF alone to control the formation, the final formation will perform as figure 2.

As shown in figure 2, after leader move forward, followers tend to achieve a balance, where repulsive force and attractive force are equal and opposite. A solution is bringing in some reference points to control the formation, as shown in section B.

![Figure 2. Formation control only with APF.](image)

Note that \(d_{\text{opt}}\) is maximum collision avoidance range in order to not getting too close to other followers, related to the size of UAV, hover error, etc. In this work, let \(d_{\text{opt}}=3\)m for UAVs in experiment. For example, the size of UAV in the experiment is 0.5 m and GPS error is \(\pm 1.5\) m (without RTK) and error-tolerant rate is 0.5. So \(d_{\text{opt}}=(0.5+1.5)(1+0.5)=3\) m.
2.2. Formation Reference Point
Virtual structure approach describes formation as a virtual rigid structure, each UAV has a fixed position in the formation. Track the fixed virtual structure point and the swarm can maintain a presupposed formation. Based on virtual structure point, formation reference point is proposed. The point only has an attraction to the UAV and attraction can be given:

\[ F_{\text{att-ref}} = k_{\text{ref}} \mathbf{d}_{\text{ref}} \]  

(7)

\( \mathbf{d}_{\text{ref}} \) is the distance vector between formation reference point and UAV, given by:

\[ \mathbf{d}_{\text{ref}} = \mathbf{p}_{\text{ref}} - \mathbf{p}_{\text{uav}} \]  

(8)

Obviously, each follower has a one-to-one correspondence with the formation reference point. The point only generates an attraction to the UAV. For followers, formation reference points are replacements for the leader. Which means, leader doesn’t give an attractive force to followers, but repulsive force still takes effect when the distance is lower than safe distance.

2.3. Formation Control Strategy
Compared with fixed-wing drones, quadrotors can hover at specific positions, given in GPS. Quadrotor hover at mission targets, which means we can assume the start velocity and the end velocity is zero. So, the velocity in \( \Delta t \) should meet:

\[ \mathbf{v}_{\text{end}} = \mathbf{v}_{\text{start}} = 0 \]  

(9)

Under the assumption, next status of a UAV only related to current status, state transition equation is:

\[ \begin{align*}
\mathbf{v}_{n+1} &= \mathbf{v}_n + \Delta \mathbf{v} \\
\mathbf{p}_{n+1} &= \mathbf{p}_n + \Delta \mathbf{p}
\end{align*} \]  

(10)

\( \Delta \mathbf{v} \) and \( \Delta \mathbf{p} \) is the velocity increment and position increment of two adjacent status. These two parameters effected by the capability of UAV, usually effected by \( v_{\text{max}} \), maximum velocity, and \( \Delta t \), related to information exchange frequency. In this work, position information exchanges four times a second and \( \Delta t=0.25 \) s.

To simplified calculation process, let velocity between two statuses as figure 3.

![Figure 3. Velocity graph between two statuses.](image)

So, the \( \Delta \mathbf{v} \) and \( \Delta \mathbf{p} \) is:

\[ \begin{align*}
\Delta \mathbf{v} &= \mathbf{v}_{\text{end}} - \mathbf{v}_{\text{start}} = 0 \\
\Delta \mathbf{p} &= \frac{1}{2} (\mathbf{v}_{\text{max}} t) = \frac{1}{4} \mathbf{a} t^2
\end{align*} \]  

(11)
Let mass of UAV to be unit mass, according to \(\mathbf{a} = \frac{\mathbf{F}}{m}, \mathbf{v}_{n+1}, \mathbf{p}_{n+1}\) is:

\[
\begin{align*}
\mathbf{v}_{n+1} &= \mathbf{v}_n + \mathbf{a} \Delta t = \mathbf{v}_n = 0 \\
\mathbf{p}_{n+1} &= \frac{1}{4} \mathbf{F}_n \Delta t^2 + \mathbf{p}_n
\end{align*}
\]

(12)

The \(\mathbf{F}_n\) represents the resultant force which will impact UAV’s flight path. It consists of attraction of target and repulsion of other UAVs. The resultant force is given:

\[
\begin{align*}
\mathbf{F} &= \mathbf{F}_{\text{att}} + \mathbf{F}_{\text{rep}} \\
\mathbf{F}_{\text{att}} &= \mathbf{F}_{\text{att-target}} \\
\mathbf{F}_{\text{rep}} &= \mathbf{F}_{\text{rep-leader}} + \sum_{i=1}^{n} \mathbf{F}_{\text{rep-follower}} (i)
\end{align*}
\]

(13)

For the leader, the \(\mathbf{F}_{\text{att-target}}\) is attraction of target point. But for a follower, \(\mathbf{F}_{\text{att-target}}\) represents the attraction of formation reference point. \(\mathbf{F}_{\text{rep-leader}}\) is repulsion of leader and \(\sum \mathbf{F}_{\text{rep-follower}}\) is resultant force of all other followers, for collision avoidance. Furthermore, if obstacle exists, add a \(\mathbf{F}_{\text{rep-obstacle}}\) is necessary.

2.4. Simulation Results

Simulations were written in MATLAB. To show the dynamic behavior of the UAV swarm, we set up a leader and three followers, let \(k_{\text{att}}=5, k_{\text{rep}}=50000\). Solid points show how follower behavior during the flight and squares demonstrate first and last position of follower when simulation finished. The simulation results, in figure 4, clearly show that the swarm can maintain the initial geometry shape during the flight.

![Figure 4. Simulation results of 1 leader an 3 followers.](image)

And velocity of followers is diminishing with getting close to formation reference point. To point out the relationship of \(k_{\text{rep}}\) and \(k_{\text{att}}\), let \(\lambda = k_{\text{rep}}/k_{\text{att}}\), then \(\lambda\) should more than 10000, through several simulations and experiments.

3. Experiment Verification

In this section, we validate the benchmark of the method with four quadrotors. The experiment platform constituted by quadrotors with Pixhawk Cube. UAVs in experiments communicate over WiFi and use a
Raspberry Pi 3B+ as companion computer. Information transfer via HTTP protocol, with a web server set up on companion computer.

**Figure 5.** Experiment platform with 4 UAVs.

Before the verification, we set the same parameters as used in simulation. Additionally, let $v_{\text{max}}=5$ m/s and $\Delta t=0.25$ s. For security, let $d_{\text{gap}}=3$ m, due to the size and hover error of UAV, which means UAVs will keep away from each other when the distance of two UAVs less than 3 m.

The experiment uses NED coordinate and let home position of leader as origin. A transformation between GPS and NED is given in (14) and (15). Let $\text{lat}$ and $\text{lon}$ as latitude and longitude of UAV’s current GPS position and $\text{lat}_{\text{leader-home}}$ and $\text{lon}_{\text{leader-home}}$ as latitude and longitude as GPS position of home point of leader. The home point can be read from flight control system. $d_{\text{lat}}$ and $d_{\text{lon}}$ is given:

$$\begin{cases} d_{\text{lat}} = \text{lat} - \text{lat}_{\text{leader-home}} \\ d_{\text{lon}} = \text{lon} - \text{lon}_{\text{leader-home}} \end{cases} \tag{14}$$

Note that in APM (an open source flight control system), home position of UAV can be read only after the UAV is armed.

The $(x, y)$ in NED can be given:

$$\begin{cases} x = d_{\text{lat}} R \\ y = d_{\text{lon}} R \cos \left(\frac{\text{lat} \pi}{180}\right) \end{cases} \tag{15}$$

Let leader take off at $(0, 0)$ and followers at $(-3, 3)$, $(-3, 0)$ and $(3, -3)$. Then the leader hover at $(10, 0)$, $(10, -10)$ and $(0, -10)$ for 0.5 s, waiting for followers come along, then land at $(0, 0)$. After analyzing experiment results, we can figure out that the method works fine for formation maintaining and collision avoidance.

In figure 6a, which is initial position, follower 3 went away from follower 2 for collision avoidance. Then, the distance of follower 3 and follower 2 kept around 3m until the leader hovered at target, when $t=2$ s. It takes about 2 s~2.5 s for leader went to around $(10, 0)$, but for followers, it took longer. Overdrive and underdrive happened very often due to communication delay between UAVs. The situation shows in figure 6a follower 3 around $(7, -3)$ and follower 2 around $(5, 0)$. To get a better performance, a more stable communication is necessary.

As experiment goes on, in figure 6b, swarm turned right, the overdrive and underdrive were corrected. Process of correcting can be seen at $(7, 2)$ and $(7, -2)$. In this state, leader and followers kept their relative position as expected.

However, follower 3 came around $(7, -13)$, when $t=7$ s, another overdrive happened. As shown in figure 6c follower 3 hovered at $(7, -13)$ for a longer time than other UAVs, caused by a short offline from swarm network, for 0.25 s~0.5 s. The short interval lead to follower 3’s falling behind from follower 1 and follower 2.
In figure 6d, swarm performed as initial formation within the allowable error range. As we can see around (-3, -7) follower 3 went away from the presupposed formation again, it hovered at (-3.5, -4) for some time and then went closer to follower 2, effected by attractive force.

![Diagram](image.png)

(a) Initial state \((t=2s)\)
(b) Around \((10, -10)\) \((t=4s)\)
(c) Around \((0, -10)\) \((t=7s)\)
(d) Final state \((t=10s)\)

**Figure 6.** States of experiment.

Whole experiment shows that the method proposed worked well with 4 quadrotors. The formation maintained along the flight, corrected immediately when some UAVs went away from expect position or get too close to each other.

### 4. Conclusions

In this work, a method based on APF and virtual structure approach is proposed. The method uses attractive force and repulsive force, proposed in APF, for obstacle avoidance. It maintains a presupposed formation by formation reference point, inspired by virtual structure approach. When the distance of two UAVs is lower than secure distance, repulsion will take effect, in order to avoid collision.

Finally, we built up a swarm experiment platform to test the method. Experiment results show that the method works fine with a leader and three followers. The swarm consists of four quadrotors can maintain initial formation through whole flight.

In the future, we plan to extend our work significantly. First, taking obstacle into consideration is necessary, we will implement an algorithm, which can avoid obstacle and collision at the same time, on the same experiment platform. Secondly, we will focus on increasing rate of convergence and stability.
of swarm algorithm. Thirdly, possibility of UAV’s failure, both leader and followers, should be considered.

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