An Error Compensation Method for Surgical Robot Based on RCM Mechanism

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ABSTRACT Remote center of motion (RCM) mechanisms, which are widely used in surgical robots, use mechanical constraints to realize movement around the RCM point. Errors in processing and assembly cause deviation of the RCM point and position error of the end-effector, these deviations cause the robot to exert excessive force on the incision during movement, leading to adverse effects that reduce the safety and accuracy of surgical robots based on the RCM. Based on kinematic calibration, this paper proposes a solution of forward and inverse kinematics considering the RCM point deviation, and this solution can simultaneously reduce the positioning error of the end-effector and the deviation of the RCM point. First, a calibration method based on the combination of the model and neural network is used to improve the kinematic accuracy of the robot. Second, a forward and inverse kinematics solution applied to RCM mechanisms is proposed, and this solution reduces the RCM point deviation during surgery. Finally, experiments were performed with an established surgery robot, verifying that the proposed method can simultaneously reduce the deviation of the RCM point and the position error of the end-effector. The proposed method is found to be superior to other methods and can improve the safety and accuracy of surgical robots.

INDEX TERMS Surgical robot, RCM mechanism, kinematics calibration, error compensation.

I. INTRODUCTION

In minimally invasive surgery (MIS), surgical instruments are operated through incision, and the remote center of motion (RCM) mechanism uses mechanical constraints to realize movement around the remote center of motion point (RCM point) [1]–[3], which provides a fixed point for the robot in the surgical incision so that the surgical instruments can reach the lesion area through an incision in the body surface to perform surgical operations [4], [5]. Therefore, this mechanism is widely used in surgical robots. In assisted or semiautonomous surgery based on the RCM mechanism surgical robot, the kinematic errors of the RCM mechanism cause two problems. 1) The position of the RCM point changes with the posture of the RCM mechanism; excessive force exerted by the robot on the incision during movement may lead to soft tissue damage or tearing at the incision [6]–[8] and may even cause movement of the soft tissue near the incision, leading to changes in the location of the lesion. 2) The end-effector of the RCM mechanism has positioning errors that lead to nonideal movements that may accidentally injure healthy tissues and cause other severe problems [3], [4]. Therefore, reducing the positioning error of the end-effector and the deviation of the RCM point is of great significance for improving the safety and accuracy of the surgery [5]–[12].

Kinematic calibration can significantly improve the accuracy of the robot without changing the hardware configuration [13]. Robot kinematics calibration methods are divided into two types: model-based calibration methods and model-free calibration methods. The model-based calibration method models the geometric error of the robot, and the model should fulfill the requirements of completeness, continuity and minimum [14]. The DH model proposed by Denavit et al. is widely used in robot calibration [15], but it violates the requirement of continuity when the adjacent joints are parallel. To fulfill the requirement of continuity, researchers proposed the CPC model [16], S model [17] and...
MDH model [18] based on the DH model. Other researchers used screw theory to describe the transformation of joint motion and proposed the POE model [19] and the FIS model [20], which are convenient for calculation and programming but unintuitive and difficult to understand [21]. Some researchers established error models, including geometric error and joint deflection error [22]–[24]. Model-based calibration methods focus mainly on modeling and compensating geometric errors, which have the advantages of accurately representing error source information and fast convergence speed. However, due to the complexity of nongeometric errors and the large number of error sources, it is difficult to establish a mathematical model including all error sources [13]. The model-free calibration method is an approximation of the robot kinematics relationship, which can realize the prediction of nongeometric errors. Some researchers have used Fourier polynomials or other polynomials to predict the end-effector positioning error of the robot [25], [26], but this method has high complexity and low accuracy. Guo et al. used the joint space grid method to predict nongeometric errors [27]. However, the spatial grid method is complicated to operate and is suitable only for situations where the motion space is small. Subsequently, researchers proposed using neural networks [13], [28], [29], fuzzy algorithms [30], and other methods to approximate robot positioning errors. Among them, neural networks have been widely used because of their learning ability and high flexibility. In contrast to the model-based calibration method, the model-free calibration method cannot analyze the error source of the robot, and the user cannot know the error source [13]. Although not all errors can be accurately and completely modeled, these errors can be compensated by the neural network calibration method. Therefore, the combination of the two methods can achieve better error compensation effects.

Generally, after the robot kinematics are calibrated, the correction of the kinematics model can improve the accuracy of the robot’s end-effector pose [31]–[33]. However, for surgical robots based on RCM mechanisms, RCM points are formed by mechanical constraints. Modifying the kinematics model cannot reduce the deviation of RCM points. In addition, changes in the end-effector pose of the RCM mechanism cause deviation of the RCM point, and compensation of the RCM point also causes changes in the end-effector position. Therefore, how to simultaneously reduce the deviation of the RCM point and the positioning error of the end-effector is worthy of attention because it directly affects the safety and accuracy of surgical robots based on the RCM mechanism.

Most of the previous studies have focused on the design of the RCM mechanism [34], [35] and have rarely paid attention to the compensation of the RCM point deviation, but this compensation in the high-precision surgical robot is more important to reduce potential damage associated with minimally invasive surgical incisions. The present work not only completes the kinematic calibration of the RCM mechanism-based surgical robot but also studies a method of simultaneously reducing the end-effector position error and RCM point deviation, which can reduce the deviation of the RCM point while fulfilling the needs of high-precision surgery so that the accuracy and safety of the surgical robot manipulator can be improved. In summary, the main content of this paper is as follows:

1. Robot kinematics calibration. A stepwise calibration strategy is adopted to improve the kinematic accuracy, and the manipulator’s driven error, geometric error and nongeometric error are compensated.

2. Compensation for RCM point deviation. Based on kinematic calibration, forward and inverse kinematics considering the deviation of the RCM point are proposed, which can reduce the positioning error of the end-effector and the deviation of the RCM point at the same time.

3. Experimental verification. Kinematic calibration of the surgery robot manipulator is carried out, and the deviation of the RCM point and the positioning error of the end-effector are evaluated. The proposed method is compared with other methods.

The content of this paper is as follows. Section II briefly introduces the surgery robot manipulator and establishes the error model of the robot. Section III uses a calibration method based on the combination of a model and neural network to calibrate the robot kinematics. Section IV derives the forward and inverse kinematics equations considering the RCM point deviation. Section V conducts experimental verification and program performance evaluation.

II. KINEMATICS MODEL OF SURGICAL ROBOT MANIPULATOR

A. INTRODUCTION OF SURGICAL ROBOT MANIPULATOR BASED ON THE RCM

This surgical manipulator based on an RCM mechanism is designed for MIS. The overall model is shown in Fig. 1. The surgical robot manipulator is mainly composed of prismatic joints and a dual-parallel six-bar RCM mechanism, which are used to achieve incision alignment and intraocular surgery, respectively. The surgical robot manipulator is referred to as the surgical robot. The laser tracker is used to evaluate the repeatability positioning accuracy of the robot, which is 50 µm.

For a typical robot-assisted surgery task, the surgical procedure is as follows. First, imaging equipment is used to determine the characteristics and location of the lesion area and the appropriate incision location, enabling formulation of a corresponding surgical plan. Subsequently, the doctor controls the robot’s RCM points to register with the surgical incision. Finally, the end-effector enters the body through the incision on the body surface to reach the lesion area for surgical operation.

B. FORWARD KINEMATICS OF THE SURGICAL ROBOT

A schematic diagram of the kinematic model of the surgical robot is shown in Fig. 1. The forward kinematics model from the base frame to the end-effector frame is as follows:

$$T_{tool}^b = T_1^b T_2^b T_3^b T_4^b T_5^b T_6^b T_{tool}$$  \hspace{1cm} (1)
The transformation matrix between adjacent joints is \( T_i^{-1} (i = 1, 2, 3, 4, 5) \), which is described by the following equation:

\[
T_i^{-1} = \text{rotz}(\theta_i) \ast \text{transl}(a_i, 0, d_i) \ast \text{rotx}(\alpha_i)
\]

(2)

where \( \theta_i, a_i, d_i \) and \( \alpha_i \) represent the joint angle, joint offset, link length and link twist angle, respectively. The transformation matrix \( T_{\text{tool}}^6 \) is described by the following equation:

\[
T_{\text{tool}}^6 = \text{transl}(x_{\text{tool}}, y_{\text{tool}}, z_{\text{tool}})
\]

(3)

The kinematic model from the measurement coordinate system to the robot end-effector coordinate system is as follows:

\[
T_{\text{tool}}^M = T_b^M \ast T_b^6 \ast T_{\text{tool}}^6
\]

(4)

where \( T_b^M \) represents the transformation matrix from the measurement coordinate system to the robot base coordinate system, expressed by the following equation:

\[
T_b^M = \text{transl}(x_0, y_0, z_0) \ast \text{rotx}(\alpha_0) \ast \text{roty}(\beta_0) \ast \text{rotz}(\gamma_0)
\]

(5)

Equation (4) can be abbreviated as \( f(\theta_1, g) \), where \( g \) represents the nominal parameter of the robot system. where

\[
\theta_1 = [d_1, d_2, d_3, \theta_1, \theta_5, d_6], \quad \text{where} \quad d_1, d_2, d_3 \quad \text{and} \quad d_6 \quad \text{are the joint positions of prismatic joints} \quad 1, 2, 3, \quad \text{and} \quad 6, \quad \text{respectively.}
\]

The parameters of the robot calibration system are shown in Tables 1 and 2. Here, \( a_1, a_2 \) and \( a_3 \) are the connection lengths of the corresponding prismatic joints, which have no effect on the motion accuracy; \( d_3 \) connects the prismatic axis and the rotation axis, which has no effect on the robot’s kinematic accuracy.

### C. GEOMETRIC ERROR MODELING

The actual model of the robot is inconsistent with the nominal model due to the influence of errors in processing, assembly and clearance. The relationship between the position error of the robot and the kinematic parameters can be expressed as:

\[
\Delta P_{3 \times 1} = J_B \ast \Delta B + J_R \ast \Delta q + J_T \ast \Delta T
\]

(6)

\[
\Delta P_{3 \times 1} = J_{3 \times 28} \ast \Delta \alpha_{28 \times 1}
\]

(7)

where \( \Delta P_{3 \times 1} \) is the position error vector of the end-effector, \( \Delta B \) is the error vector of the base frame, \( \Delta q \) is the geometric parameter error vector of the robot, and \( \Delta T \) is the parameter error vector of the tool coordinate system.

\[
\Delta B = [\Delta x_0, \Delta y_0, \Delta z_0, \Delta a_0, \Delta \beta_0, \Delta \gamma_0]^T, \quad \Delta T = [\Delta \theta_01, \Delta \theta_02, \Delta \theta_03, \Delta \theta_04, \Delta \theta_05, \Delta \theta_06]^T, \quad \Delta q = [\Delta \alpha^T, \Delta \beta^T, \Delta \gamma^T, \Delta \alpha^T, \Delta \beta^T, \Delta \gamma^T]^T, \quad \Delta \theta = [\Delta \alpha_1, \Delta \alpha_2, \Delta \alpha_3, \Delta \alpha_4, \Delta \alpha_5, \Delta \alpha_6]^T
\]

The Jacobian matrix corresponding to the geometric error. The Jacobian matrix corresponding to the error parameter of the base coordinate system is as follows [36]:

\[
J = [x_M, y_M, z_M, \theta_01, \theta_02, \theta_03, \theta_04, \theta_05, \theta_06]
\]

(8)

where \( x_M, y_M \) and \( z_M \) represents the direction vector of \{M\} (the measurement coordinate system). The coordinate system \{M’\} is formed after \{M\} rotates by \( \alpha_0 \) around axis \( x_M \). The vector \( P_{b, tool} \) connects the origin of the measurement

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**TABLE 1.** Nominal DH parameters of the surgical robot.

| Joint | \( \theta_i \) [rad] | \( d_i \) [mm] | \( a_i \) [mm] | \( \alpha_i \) [rad] |
|-------|-----------------|---------------|--------------|-----------------|
| 1     | 0               | \( d_1 \) | n/a           | \( \pi/2 \)      |
| 2     | \( -\pi/2 \)    | \( d_2 \) | n/a           | \( \pi/2 \)      |
| 3     | \( -\pi/2 \)    | \( d_3 \) | n/a           | \( 5 \times \pi/2 \) |
| 4     | \( \pi/2 \)      | \( d_4 \) | 0             | \( -\pi/2 \)    |
| 5     | \( -\pi/12+\theta_5 \) | 0 | 0 | \( -\pi/2 \) |
| 6     | 0               | \( d_6 \) | n/a           | n/a             |

**TABLE 2.** Parameters of the base coordinate system and tool coordinate system.

| Frames | X[mm] | Y[mm] | Z[mm] | Rot x [rad] | Rot y [rad] | Rot z [rad] |
|--------|-------|-------|-------|-------------|-------------|-------------|
| Base   | \( x_0 \) | \( y_0 \) | \( z_0 \) | \( \alpha_0 \) | \( \beta_0 \) | \( \gamma_0 \) |
| Tool   | \( x_{\text{tool}} \) | \( y_{\text{tool}} \) | \( z_{\text{tool}} \) | n/a | n/a | n/a |
coordinate system to the origin of the end-effector coordinate system. The Jacobian matrix corresponding to the error parameter of the robot is as follows [36]:

\[
\begin{align*}
   J_{dl} &= [x_{l-1} \times P_{l-1, tool}], \\
   J_{di} &= [z_{l-1}], \\
   J_{ai} &= [x'_{l-1}, J_{ai} = [x'_{l-1} \times P_{l, tool}]
\end{align*}
\]  

(9)

The coordinate system \(\{i'\}\) is formed when the link coordinate system \(\{i-1\}\) rotates \(\theta_i\) around the axis \(z_{i-1}\), and \(x'\) is the direction vector of the coordinate system \(\{i'\}\) along the axis \(x\). The vector \(P_{i, tool}\) connects the origin of the link coordinate system \(\{i\}\) to the origin of the end-effector coordinate system.

### D. IDENTIFICATION OF GEOMETRIC PARAMETER ERRORS

A complete error model with redundant parameters is established in equation (7). Redundant parameters will cause instability in the identification process. According to equation (2) and equation (5), \(\Delta \theta_1\) (the parameters of the base frame) and \(\Delta \theta_1\) (the parameters of joint 1) are coupled. \(\Delta \theta_1\) can be removed from the calibration model. Second, because the first three joints are prismatic joints, the parameters \(\Delta d_1\) and \(\Delta d_2\) are coupled with the parameters in the base coordinate system \(\Delta x_0, \Delta y_0, \Delta z_0, \Delta a_0, \Delta \beta_0\) and \(\Delta \theta_0, \Delta d_1, \Delta d_2\) and \(\Delta d_3\) are removed from the calibration model. According to equation (9), the parameters \(\Delta a_5\) and \(\Delta a_6\) are coupled, and the parameters \(\Delta a_0\) can be removed from the calibration model. Since the end joint is a prismatic joint, \(\Delta \theta_6\) has no effect on the positioning accuracy of the robot. In addition, the tool coordinate systems \(\Delta x_i, \Delta y_i\) and \(\Delta z_i\) are coupled with \(\Delta a_6, \Delta d_5\) and \(\Delta d_6\), respectively. In summary, the redundant parameters of the robot are removed from the calibration model, and a nonredundant calibration model is obtained as equation (10).

\[
\Delta P_{3 \times 1} = J^*_{3 \times 19} \Delta g^*_{19 \times 1}
\]  

(10)

where \(\Delta g^*_{19 \times 1}\) represents the identifiable parameter vector whose components are shown in Table 3. \(J^*_{3 \times 19}\) is the corresponding Jacobian matrix of \(\Delta g^*_{19 \times 1}\). For simplicity, the superscript “\(^*\)” is omitted in the following content.

### TABLE 3. Identifiable parameters of the robot system.

| i  | Identified parameters | i  | Identified parameters |
|----|----------------------|----|----------------------|
| base | \( x_0, y_0, z_0, \alpha_0, \beta_0, \theta_0 \) | 4 | \( \theta_6, a_4, a_4 \) |
| 1 | \( \alpha_1 \) | 5 | \( \theta_6, d_4, a_4, a_4 \) |
| 2 | \( \theta_1, \alpha_2 \) | 6 | \( d_4 \) |
| 3 | \( \theta_1, \alpha_3 \) | tool | n/a |

We measure the end-effector position of robot \(n\) configuration and establish \(3 \times n\) equations to identify the geometric parameters of the robot. To minimize the linearization error of the system, the iterative least square method (ILS) is used to identify the parameters of the error model as follows:

\[
g_k = g_{k-1} + \Delta g_k
\]  

(11)

where \(g_k\) is the parameter updated in the \(k\)th iteration, and \(\Delta g_k\) is the parameter error identified in the \(k\)th iteration. \(\Delta g_k\) is expressed by the following equation:

\[
\Delta g_k = \left[J^T_k J_k\right]^{-1} J^T_k \Delta P_k
\]  

(12)

where \(\Delta P_k = \left[(\Delta P_{k}^1)^T (\Delta P_{k}^2)^T \cdots (\Delta P_{k}^n)^T\right]^T\), which is the position error vector of the end-effector at the \(k\)th iteration. \(J_k = \left[J^1_k J^2_k \cdots (J_k^n)^T\right]^T\), which is the corresponding identification Jacobian matrix. When the value \(\Delta g_k\) is less than \(\varepsilon\), the iterative process is terminated.

### E. PREDICTION OF NONGEOMETRIC ERRORS

After the geometric parameters are calibrated, the residual position error of the robot is still affected by nongeometric errors. The neural network is used to predict the residual position error after the model is calibrated. After the model is calibrated, the residual positioning error of the robot is as follows:

\[
\Delta P = P_0 - f(\theta, g')
\]  

(13)

where \(\Delta P\) represents the residual position error after calibration, \(P_0\) represents the actual measurement position, and \(f(\theta, g')\) represents the kinematic model of the robot after calibration. The input of the neural network is \(\theta_i = (d_1, d_2, d_3, \theta_4, \theta_5, \theta_6)\) (the angle value of each joint), while the output is \(\Delta P\) (the residual position error). The activation function of the hidden layer of the neural network is a tan-sigmoid function, and the activation function of the output layer is a linear function. For simplicity, the prediction equation of the trained neural network is abbreviated as \(\Delta P = f_{\text{nn}}(\theta_i)\).

After geometric parameter calibration and neural network training are completed, the position calculation equation of the robot end-effector is as follows:

\[
P_E = f(\theta, g') + f_{\text{nn}}(\theta_i)
\]  

(14)

where \(P_E\) represents the position of the end-effector after calibration.

### III. DERIVATION OF THE FORWARD AND INVERSE KINEMATICS CONSIDERING THE RCM POINT DEVIATION

Due to the influence of geometric and nongeometric errors, the three axes (joints 4, 5 and 6) of the RCM mechanism have difficulty intersecting difficult to intersect at one point, resulting in a deviation of the RCM point. The position of the RCM point changes with the posture of the RCM mechanism. A schematic diagram when the joint axes of the RCM mechanism do not intersect is shown in Fig. 2 (b).

In actual surgery, due to the limitation of the incision, we aim for the spatial position of the RCM point to be fixed when the end-effector reaches the desired joint position or spatial position. Since the RCM mechanism forms the RCM point through mechanical constraints, modifying the kinematics model cannot reduce the deviation of the RCM points. Therefore, it is necessary to compensate for the deviation
of the RCM point while ensuring that the robot reaches the desired joint position or the desired spatial position.

A. DERIVATION OF FORWARD KINEMATICS CONSIDERING RCM POINT DEVIATION

When performing in vivo surgical procedures, RCM point fixation is desired to minimize damage to the incision. However, due to the kinematic error of the robot, the position of the RCM point changes with the posture of the RCM mechanism, which reduces the safety of the operation. Therefore, it is necessary to compensate for the deviation of the RCM point.

First, in the initial stage of surgery, it is necessary to determine \( P_{RCMP}^0 \) (the RCM point is aligned with the body surface incision), as shown follows:

\[
P_{RCMP}^0 = f(\theta_i^0, \mathbf{g}) + f_{\text{m}}(\theta_i^0)
\]

where \( \theta_i^0 = (d_1^0, d_2^0, d_3^0, \theta_4^0, \theta_5^0, 0) \), which represents the joint position of the robot when it coincides with the body surface incision. When \( T = T_n \), the desired joint positions are \( \theta_i^1, \theta_i^2 \) and \( d_i^6 \). We calculate the position of the RCM point as follows:

\[
P_{RCMP}^1 = f(\theta_i^1, \mathbf{g}) + f_{\text{m}}(\theta_i^1)
\]

where \( \theta_i^1 = (d_1^{n-1}, d_2^{n-1}, d_3^{n-1}, \theta_4^n, \theta_5^n, 0) \), which represents the joint position of the robot at the RCM point when \( T = T_n \), and the error between the position of the RCM point \( P_{RCMP}^1 \) and \( P_{RCMP}^0 \) is as follows:

\[
\Delta P_{RCMP}^n = P_{RCMP}^n - P_{RCMP}^0
\]

where \( \Delta P_{RCMP}^n \) represents the deviation of the RCM point. When \( T = T_n \), we calculate the joint position of the first three joint motion systems \( d_1^n, d_2^n \) and \( d_3^n \) to compensate for \( \Delta P_{RCMP}^n \) (the RCM point deviation) generated by the RCM mechanism motion, as shown follows:

\[
T_b^M T_1^M(d_1^n T_1^M(d_2^n T_3^n)) = T_b^M T_1^M(d_1^{n-1} T_2^{n-1} T_3^{n-1}) - \Delta P_{RCMP}^n
\]

We use equation (16) to calculate \( P_{RCMP}^n \) (the position of the RCM point) at this time. If \( P_{RCMP}^n \) is close to the initial position \( P_{RCMP}^0 \), the following conditions are fulfilled:

\[
\left\| \Delta P_{RCMP}^n \right\| \leq \varepsilon_1
\]

An iterative solution is completed until equation (19) is satisfied; otherwise, \( \theta_{iR}^n = (d_1^n, d_2^n, d_3^n, \theta_4^n, \theta_5^n, 0) \) and equations (16), (17) and (18) are executed in a loop until equation (19) is satisfied to realize the compensation of the RCM point deviation. The robot joint angle that satisfies equation (19) is \( \theta_i^n = (d_1^n, d_2^n, d_3^n, \theta_4^n, \theta_5^n, d_i^6) \). We send the joint angle to the robot controller for execution and use equation (14) to calculate the position of the end-effector.

In the stage of performing surgical operations, a forward motion solution scheme considering the deviation of RCM points is established, as shown in Fig. 3. Different from the standard forward kinematics solution method, the surgical robot based on the RCM mechanism needs to fulfill the requirements of the end-effector to reach the desired joint position and keep the RCM point position fixed. The specific operation steps are as follows. First, we calculate \( P_{RCMP}^0 \) (the position of the RCM point) at the initial time. Second, in the desired joint position, \( \Delta P_{RCMP} \) (the deviation of the RCM point) is calculated and compensated. The iteration process is terminated until \( \left\| \Delta P_{RCMP}^n \right\| \leq \varepsilon_1 \) is satisfied; otherwise, the state is updated and iterated again. Finally, the joint angles that fulfill the requirements are sent to the robot.

B. DERIVATION OF INVERSE KINEMATICS CONSIDERING RCM POINT DEVIATION

During the operation, due to the limitation of the incision, we aim to keep the position of the RCM point fixed when the end-effector reaches the desired spatial position. Based on the above ideas, the following work is carried out to determine \( P_{RCMP}^0 \), as shown in equation (15) and then control the end-effector to reach the desired spatial position while keeping the spatial position of the RCM point fixed. The specific workflow is as follows:

1. Solution of inverse kinematics without considering the RCM point deviation. When \( T = T_n \), the desired spatial position is \( P_{E}^n \). We use the iterative inverse kinematics method to calculate the joint position of the RCM mechanism [33], as shown follows:

\[
\Delta \theta_R^n = (J_R^n)^{-1}(P_{E}^n - P_{E}^{n-1})
\]

\[
\theta_R^n = \theta_R^{n-1} + \Delta \theta_R^n
\]

where \( J_R^n \) represents the Jacobian matrix corresponding to the RCM mechanism, which is shown in equation (9). Here, \( \theta_R^{n-1} \) and \( P_{E}^{n-1} \) represent the current joint position of the robot and the position of the end-effector, respectively. \( \theta_R^n = [d_1^{n-1}, d_2^{n-1}, d_3^{n-1}, \theta_4^{n-1}, \theta_5^{n-1}, d_i^6] \), and \( \Delta \theta_R^n = [0 \ 0 \ 0 \ \theta_4^n \ \theta_5^n \ d_i^6] \). Here, \( \theta_R^n = [d_1^{n-1}, d_2^{n-1}, d_3^{n-1}, \theta_4^n, \ \theta_5^n, d_i^6] \) represents the inverse kinematics solution of the RCM mechanism without considering RCM point deviation.
deviation. When the joint position is $\theta''$, we calculate whether $P_E''$ (the position of the end-effector) reaches $P_C''$ (the desired position), as shown follows:

$$P_E'' = f(\theta'', g''') + f_{nus}(\theta'')$$

$$\|P_C'' - P_E''\| \leq \epsilon_1$$

(22)  \hspace{1cm} (23)

The iterative process is terminated until the error is less than the termination criterion equation (23); otherwise, $\theta'' = \theta'',$ and equations (20), (21), and (22) are executed in a loop until equation (23) is satisfied. In addition, $\theta'' = [d_1'', d_2'', d_3'', \theta_4'', \theta_5'', d_6'']$, which represents the joint angle when the end-effector reaches the desired position without considering the RCM deviation.

(2) Reduction in the deviation of the RCM point. The end-effector reaches the desired position, the change in the pose of the RCM mechanism causes the deviation of the RCM point. That is, when the desired joint position of the RCM mechanism $\theta_4'', \theta_5''$ and $d_6''$ are known, the deviation of the RCM point is calculated, and the deviation of the RCM point is compensated by the first three joints, relying on the forward kinematics derivation considering the RCM point deviation described in Section III part A.

(3) Evaluation of the position of the end-effector and RCM point deviation. Through (1) and (2), the joint position of the robot in the RCM point compensation state is obtained, $\theta'' = [d_1'', d_2'', d_3'', \theta_4'', \theta_5'', d_6'']$. However, the position of the end-effector changes due to the deviation of the compensation RCM point. It is necessary to check whether the position of the end-effector satisfies $\|P_C - P_E''\| \leq \epsilon_1$. If this condition is fulfilled, the iteration process is terminated; otherwise, $P_E^{n-1} = P_C$, $\theta^{n-1} = \theta''$, and processes (1), (2) and (3) are repeated until $\Delta P_{RCMP} \leq \epsilon_1$ and $\|P_C - P_E^{n}\| \leq \epsilon_1$ at the same time. The joint angle that satisfies the above requirements is $\theta'' = [d_1'', d_2'', d_3'', \theta_4'', \theta_5'', d_6'']$. The inverse kinematics workflow considering RCM point deviation is shown in Fig. 4.

In the surgical operation stage, an inverse kinematics solution considering the deviation of RCM points is established, as shown in Fig. 4. Unlike the standard inverse kinematics
solution, which guarantees only that the end-effector reaches the desired spatial position, the surgical robot needs to fulfill the requirements of the end-effector to reach the desired spatial position and keep the RCM point fixed. The specific operation steps are as follows: First, the joint angle of the RCM mechanism is calculated when the end-effector reaches the desired spatial position. Second, the deviation of the RCM point is calculated, and the joint positions of joints 1, 2, and 3 are calculated to compensate for the deviation of the RCM point. Then, we check whether $\|\Delta P_{RCMP}\| \leq \varepsilon_1$ and $\|P_E^* - P_E\| \leq \varepsilon_1$ are satisfied at the same time. If they are satisfied, the iterative process is terminated; otherwise, the state is updated and iterated again. Finally, the joint angles that fulfill the requirements are sent to the robot.

IV. EXPERIMENTAL VERIFICATION

In this section, experiments conducted to evaluate the validity of the calibration method and the proposed solution of forward and inverse kinematics considering RCM point deviation are described. First, we compensate for the driving error of the fifth joint. Subsequently, the robot is calibrated for geometric and nongeometric errors. Finally, the proposed solution of forward and inverse kinematics considering RCM point deviation is evaluated. Among the solutions, the effectiveness of the proposed forward and inverse kinematics solution considering the deviation of RCM points is examined from two perspectives: (1) the positioning error of the RCM point and the deviation of the RCM point during surgical operations; (2) the positioning error of the end-effector during surgical operations. The positions of the RCM point and the end-effector are measured separately, where the RCM point position is measured when joint 6 is at the zero position and the other joint positions are the same as the joint positions when the end-effector is measured. The following two methods are compared: the standard forward and inverse kinematics solution algorithm (method 1) and the forward and inverse kinematics solution algorithm, which considers only the geometric error to compensate for the deviation of the RCM point (method 2).

As shown in Fig. 5, the calibration system consists of a surgical robot, Leica AT960 laser tracker (with positioning accuracy 15 μm + 6 μm/m), magnetic base and SMR target. The axis fitting method is used to determine the initially identified $T_B^M$ [37]. The nominal parameters of the robot are shown in Table 1.

A. COMPENSATION OF THE FIFTH JOINT DRIVEN ERROR

The joint angle $\theta_b$ is transformed by the displacement of the push rod, as shown in Fig. 6. The transformation relationship between the displacement $c$ and angle of the push rod $\theta_b$ is shown in equation (24):

$$\theta_b = \arccos\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

where $c = c_I + x$, $c_I$ represents the initial position of the lead screw and $x$ represents the amount of displacement.

FIGURE 5. Experimental setup of the surgical robot calibration system.

FIGURE 6. Schematic diagram of the fifth joint drive structure.

The transformation relationship between the angle of the push rod $\theta_b$ and the displacement $c$ is shown in equation (25):

$$\theta_c = \pi - \theta_b - \arcsin\left(\frac{a}{b}\sin(\theta_b)\right)$$

$$c = \frac{b}{\sin(\theta_b)}\sin(\theta_b + \arcsin\left(\frac{a}{b}\sin(\theta_b)\right))$$

(25)

Here, $\theta_c$ represents the angle corresponding to the side length of triangle $c$. Equation (24) expresses how to calculate the angle value $\theta_b$ when value $c$ is known. Equation (25) represents how to calculate $c$ when angle $\theta_b$ is known.

When measuring the driven error of $\theta_b$, first, we measure the direction vector of $L_1$. Subsequently, as the ball screw moves with equal increments, the spatial position of the target ball on the rod is measured $a$. We fit the spatial circle and the center of the measurement point, calculate the direction vector $L_1$ between the measurement point and the center of the circle, and calculate the angle $\theta_b$ between straight lines $L_1$ and $L_2$. When $\theta_b$ and $x$ are known, $a$, $b$ and $\theta_c$ are identified; the identification result is shown in Table 4.

B. KINEMATIC CALIBRATION

To perform geometric parameter identification, neural network training and kinematic accuracy evaluation. We generate 600 sets of joint configurations in the maximum measurable space of the robot and measure the corresponding end positions. The 100, 400, and 100 groups of samples covering the entire measurement space are selected as $S_1$, $S_2$ and $S_3$. 

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and the corresponding end-effector positions are \( P_{s1} \), \( P_{s2} \) and \( P_{s3} \), respectively. Among them, \( S_1 \) and \( P_{s1} \) are used for the identification of geometric parameters, \( S_2 \) and \( P_{s2} \) are used for the training of neural networks, and \( S_3 \) and \( P_{s3} \) are used for the evaluation and verification of the accuracy after calibration. The identification results of the robot DH parameters are shown in Tables 5 and 6.

To determine the structure of the ANN, ANNs with different numbers of neurons in each layer are trained to compare their performance. For each structure, the ANN is trained five times under random weights. Ninety percent of the samples are used for training, and 10% of the samples are used for verification. Table 7 shows the performance of the neural network with different hidden layer nodes. The experimental results show that the neural network with 30 hidden layers can predict the residuals better. Therefore, the ANN with 30 neurons was chosen in this paper.

The calibration results of the robot obtained by comparing the proposed calibration method with the model-based calibration method (calibration method 1) are shown in Fig. 8 and Table 8. Both method 1 and the proposed calibration method can effectively improve the end-effector positioning accuracy. At the same time, the selected calibration method reduces the impact of nongeometric errors on positioning accuracy on the basis of method 1. The results show that the calibration method based on the combination of the model and the ANN can effectively improve the kinematic accuracy of the robot by compensating for the geometric error and the residual position error.

### C. EVALUATION OF THE PERFORMANCE OF THE FORWARD KINEMATICS ALGORITHM

First, the RCM point of the control robot is aligned with the body surface incision, and the joint angle of the robot is \( \theta^I = [d_1^I, d_2^I, d_3^I, \theta_4^I, \theta_5^I, 0] \). Subsequently, the desired joint positions of joints 4, 5, and 6 are given. Finally, methods 1 and 2 and the proposed method are used to calculate the joint position of the robot and send their results to the robot for execution.
TABLE 8. Residual position error.

|                  | Mean (mm) | Maximum (mm) | Std. (mm) |
|------------------|-----------|--------------|-----------|
| Nominal model    | 4.7735    | 5.6756       | 0.4829    |
| Method 1         | 0.2310    | 0.3782       | 0.0620    |
| Proposed method  | 0.0866    | 0.1655       | 0.0290    |

FIGURE 9. Position error of RCM point.

FIGURE 10. Deviation of the RCM point.

The experimental results are shown in Figs. 9, 10, and 11 and Table 9. In contrast to the nominal model, the position error of the robot end-effector in method 1 is significantly reduced, but the deviation of the RCM is not reduced. This shows that although the robot kinematics accuracy can be improved through kinematic calibration, it cannot solve the RCM point deviation caused by hardware problems. Therefore, the standard forward and inverse kinematics solution algorithm cannot compensate for the RCM point deviation after calibration.

In method 2, the deviation of the RCM point is compensated, the deviation of the RCM point is reduced by 72.38%/70.62% compared with the average/maximum error of method 1, and the accuracy of the end-effector is almost the same as that of method 1. Therefore, the forward kinematics solution considering the deviation of the RCM point can effectively reduce the deviation of the RCM point.

In contrast to method 2, the proposed method effectively reduces the impact of nongeometric errors on the robot kinematics accuracy. By adopting the proposed error compensation method, the average/maximum deviation of the RCM point is reduced to 0.0516 mm/0.1082 mm. The average/maximum positioning error of the end-effector is reduced to 0.0735 mm/0.1717 mm.

The above experimental results show that the standard kinematics calibration algorithm can improve the kinematic accuracy of the end-effector but cannot reduce the deviation of the RCM point. On the basis of kinematics calibration, when the proposed forward kinematics solution considering the deviation of the RCM point is adopted, the effector can reach the desired joint position while effectively reducing the deviation of RCM points during the operation.

D. EVALUATION OF THE PERFORMANCE OF THE REVERSE KINEMATICS SCHEMES

To evaluate the correctness and effectiveness of the proposed inverse kinematics solution considering the RCM point deviation, the experimental steps are as follows. First, the RCM point of the control robot is aligned with the body surface...
incision. Subsequently, an accessible path of the RCM mechanism is defined, which is a space circle with a diameter of 14 mm. Forty-three target points are determined equidistantly along the path, and the distance between the points is 1 mm, so there are 43 expected positions on the path that need to be compensated. Finally, method 1, method 2 and the proposed method are used to compensate for the above 43 expected positions, and the compensated RCM point deviation and the positioning error of the end-effect are contrasted.

Fig. 12 shows the end-effector positioning error of different methods and the deviation error of the RCM point (experimental results). The right column of Fig. 12 describes the deviation of the RCM point in the XYZ direction relative to the initial RCM point during the movement of different methods along the desired trajectory, and the color represents
In contrast to method 1, the proposed method establishes a forward and inverse kinematics solution considering the deviation of RCM points based on the compensation of geometric errors and residual positioning errors. This can simultaneously reduce the deviation of the RCM points and the end-effector position error. The proposed method can reduce the deviation of the RCM point, reduce the damage of the surgical instrument to the incision, improve the positioning accuracy of the end-effector, reduce the risk of tissue damage from nonideal motion, and improve the safety and accuracy of the operation.

### V. CONCLUSION

To address the problems of RCM point deviation and end-effector positioning error of surgical robots based on the RCM mechanism, this paper establishes a forward and inverse kinematics solution; this solution considers the RCM point deviation on the basis of kinematic calibration to achieve simultaneous reduction in the deviation of the RCM point and the positioning error of the end-effector. This method improves the positioning accuracy of the end-effector while reducing the deviation of the RCM point on the surgical incision. Experiments on an established surgery robot show that the proposed method effectively reduces the deviation of the RCM point and the positioning error of the end-effector, offering better performance than other methods. In addition, compensation experiments for a specific working path further verify the effectiveness of the proposed inverse kinematics solution. The proposed method is well suited for microsurgery with small body surface incisions and can improve the safety and accuracy of surgical robots based on RCM mechanisms.

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### TABLE 10. Compensation results of the test path with different methods.

| Project evaluation          | Method            | Mean (mm) | Maximum (mm) | Std. (mm) |
|-----------------------------|-------------------|-----------|--------------|-----------|
| RCM point position error    | Nominal model     | 5.2682    | 5.4716       | 0.1351    |
|                             | Method 1          | 0.2452    | 0.3391       | 0.0531    |
|                             | Method 2          | 0.2400    | 0.3172       | 0.0511    |
|                             | Proposed method   | 0.0792    | 0.1243       | 0.0421    |
| RCM point deviation         | Nominal model     | 0.7614    | 1.3225       | 0.3927    |
|                             | Method 1          | 0.4329    | 0.9101       | 0.325     |
|                             | Method 2          | 0.1259    | 0.2642       | 0.088     |
|                             | Proposed method   | 0.0689    | 0.1269       | 0.0397    |
| End-effector position error | Nominal model     | 5.312     | 6.011        | 0.4833    |
|                             | Method 1          | 0.2813    | 0.4428       | 0.1033    |
|                             | Method 2          | 0.2706    | 0.4337       | 0.103     |
|                             | Proposed method   | 0.0817    | 0.1770       | 0.0403    |

The size of the total deviation. The deviation of the RCM point is different under different methods, so the shape of the RCM point deviation is also different, and there is no correlation between the shape of the RCM point deviation under different methods. As shown in the right column of Figure 12, the proposed method significantly reduces the deviation of RCM points compared with other methods. Table 10 shows the results of the experiment. In contrast to the nominal model, the deviation of the RCM point in method 1 is smaller because the initial joint position of the robot in the nominal model is quite different from the joint position of the path starting point, resulting in a larger deviation error of the RCM point. In contrast to method 1, the average/maximum deviation of the RCM points in method 2 is reduced by 70.92%/70.97%, and the positioning accuracy of the end-effector is almost the same as that in method 1. The standard inverse kinematics solving algorithm cannot reduce the deviation of the RCM point.

In contrast to the nominal model, the maximum deviation error of the RCM point using the proposed method is reduced from 1.3225 mm to 0.1269 mm, which significantly reduces the deviation of the RCM point and significantly improves the accuracy of the desired path when the desired path is executed. In addition, because the proposed method considers the influence of nongeometric errors on the positioning accuracy of the robot, the performance of this method is better than that of method 2. The experimental results verify the accuracy and effectiveness of the proposed method and the superiority of the proposed method compared with other methods.

### E. DISCUSSION OF RESULTS

Compared with the nominal model, method 1 improves the kinematic accuracy of the robot by compensating for geometric errors. However, since the standard forward and inverse kinematics solution algorithm cannot compensate for the deviation of the RCM point, it is necessary to establish an algorithm that considers the deviation of the RCM point to fulfill the constraints of the body surface incision during the operation. In contrast to method 2, the proposed method takes the nongeometric impact on the robot positioning accuracy errors into account. The deviation of the RCM point and the positioning error of the end-effector are further reduced, as shown in Fig. 12 and Table 10. Therefore, the proposed method establishes a forward and inverse kinematics solution considering the deviation of RCM points based on the compensation of geometric errors and residual positioning errors. This can simultaneously reduce the deviation of the RCM points and the end-effector position error. The proposed method can reduce the deviation of the RCM point, reduce the damage of the surgical instrument to the incision, improve the positioning accuracy of the end-effector, reduce the risk of tissue damage from nonideal motion, and improve the safety and accuracy of the operation.
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