Zeeman smearing of the Coulomb Blockade

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Charge fluctuations of a large quantum dot coupled to a two-dimensional lead via a single-mode good Quantum Point Contact (QPC) and capacitively coupled to a back-gate, are investigated in presence of a parallel magnetic field. The Zeeman term induces an asymmetry between transmission probabilities for the spin-up and spin-down channels at the QPC, producing noticeable effects on the quantization of the grain charge already at low magnetic fields. Performing a quantitative analysis, I show that the capacitance between the gate and the lead exhibits — instead of a logarithmic singularity — a reduced peak as a function of gate voltage. Experimental applicability is discussed.

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Recent research on mesoscopic quantum dots has led to a revival of Kondo physics. There is an extensive literature on the (one-channel) Kondo behavior of small dots with an odd number of electrons and a finite level spacing on the (one-channel) Kondo behavior of small dots coupled to a back-gate, is mathematically equivalent to the two-channel model in the Emery-Kivelson limit [4] (Fig. 1). The two channels of the Kondo problem correspond to the two spin channels for tunneling through the QPC. For recent reviews, see Refs. [3].

The non-Fermi-liquid nature of the ground state of the two-channel Kondo model is here reflected by the non-analyticity of the capacitance measured between the gate and the reservoir near the points where the dot charge Q is half-integer [2] \( (C = 
\partial Q/\partial V_G) \)

\[ C(N) = C_o - bC_{gd}|\lambda|^2 \cos(2\pi N) \ln \frac{1}{|\lambda|^2 \cos^2(\pi N)}. \]  

(1)

N is a parameter proportional to the gate voltage \( eN = V_GC_{gd}, \) \( C_{gd} \) denotes the gate-dot capacitance, \( C_o \) is the total dot capacitance and \( |\lambda|^2 \ll 1 \) is the small reflection probability at the QPC. Here, \( b > 0 \) is proportional to the Euler constant \( \gamma = e^C, \) with \( C \approx 0.5772. \) The second term describes the cross-over from the linear charge-voltage dependence to the “Coulomb staircase” behavior (inset in Fig. 3). This effect has been observed by Berman et al., in AlGaAs/GaAs heterostructures [5].

Below, I investigate how the capacitance \( C(N) \) evolves if a magnetic field B is applied parallel to the 2DEG [4]. I will show that the logarithmic divergence in the differential capacitance \( \delta C(N) = C(N) - C_o \) with \( N \to 1/2 \) gets already cutoff by a small magnetic field (Fig. 3)

\[ \frac{\delta C}{C_{gd}}(N) = b|\lambda|^2(1 - \delta^2) \cos(2\pi N) \times \]

\[ \ln \text{Max}[|\lambda|^2\delta^2\sin^2(\pi N); |\lambda|^2\cos^2(\pi N)], \]

\[ 0 < \delta \ll 1, \text{ being proportional to the Zeeman energy, } \Delta = g\mu_BB. \] This results from a Zeeman-like asymmetry between reflection probabilities for the spin-up and spin-down channels at the QPC. For a high field \( (\delta \approx 1), \) only the spin-channel will be transmitted producing a one-channel QPC model. The capacitance (or charge Q) only exhibits periodic oscillations as a function of N \[ 3, \]

\[ C(N) = C_o - bC_{gd}|\lambda|\cos(2\pi N). \]  

(3)

The Coulomb staircase behavior gets completely smeared out (inset in Fig. 3). An experiment using AlGaAs/GaAs heterostructures is presumably appropriate to probe this effect in the capacitance [10]. Indeed, Zeeman splitting at the QPC in an in-plane magnetic field has been confirmed by Thomas et al via conductance measurements [11], and for few conducting modes at the QPC the Landé factor is enhanced \( g \approx 1 \) (the bulk value is \( g = 0.4). \)

Let me emphasize that below a quantitative analysis of the smearing out of the logarithmic peak for the capacitance \( \delta C(N) \) is performed (which has not been previously done in Ref. [2]). The crossover from the two- to the one-channel QPC model is carefully investigated.

First, it is useful to compute reflection probabilities at the QPC in presence of a magnetic field parallel to the 2DEG, and to discuss the necessary magnetic-field dependent adjustment of the QPC potential \( V_G(B). \) In the close vicinity of the contact, the (smooth) confining potential will be approximated as a harmonic one [4].

FIG. 1. The experimental setup: A large dot coupled to a 2DEG via a single mode QPC and capacitatively coupled to a back-gate, in a parallel magnetic field B (\( C_o = C_{gd} \)).
An electron with spin-projection \( \alpha = \uparrow, \downarrow \) (or \( \alpha = \pm 1 \)) along the magnetic field axis, is then subjected to the total potential \( V_\alpha(B) = m\omega_x^2 x^2/2 + m\omega_y^2 y^2/2 - \alpha\Delta/2 \). Here, \( \omega_x \) and \( \omega_y \) are the curvatures of the potential, and \( m \) the mass of an electron. The transverse part of the Hamiltonian produces “n transverse modes”. Then, the one-dimensional (1D) wave function \( \Psi_\alpha^n(x) \) for motion along \( x \) is determined by the effective potential

\[
V_\alpha^n(x) = V_\alpha(B) - \alpha \frac{\Delta}{2} + \hbar \omega_y(n + \frac{1}{2}) - \frac{1}{2} m \omega_x^2 x^2, \tag{4}
\]

the explicit form of which is of no interest here. The height of the barrier potential at the QPC for a spin-up channel of the transverse mode \( n = 0 \) remains at \( \hbar \omega_x > 0 \). Below, I will focus on low magnetic field effects. Using Eq. (5), similarly I adjust \( V_\alpha(B) = V_\alpha(B = 0) + \Delta/2 \), because I want field-independent reflection probabilities for spin-up channels.

Near the saddle-point \( (x \approx 0) \), the threshold energies of the mode \( n \) are then spin-splitted, as follows

\[
E_n^\uparrow = E_n = V_\alpha(B = 0) + \hbar \omega_y(n + \frac{1}{2}), \tag{5}
\]

\[
E_n^\downarrow = E_n + \Delta.
\]

Classically, modes with threshold energy below the Fermi energy \( E_F \) are perfectly open and the others remain closed. But, quantum mechanically transmission and reflection at the saddle are neither completely open nor completely closed \([13]\). Here, I fix the voltage \( V_\alpha(B = 0) \) such that \( E_0 \ll E_F < E_1 \). In Fig. 2, this corresponds to adjust \( \xi \) such that \( 0.5 \ll \xi < 1.5 \). Then, the spin-up channel of the transverse mode \( n = 0 \) remains at almost perfect transmission whatever the applied magnetic field. Moreover, I can disregard modes \( n \geq 1 \) which are almost perfectly reflected, because I am interested only in transport through the constriction.

The reflection/transmission amplitude of a 1D particle passing through an inverted parabolic barrier has been studied in detail by Connor \([14]\). Taking \( n = 0 \), small reflection probability for the spin-up channel reads

\[
R_0^\uparrow = |\lambda|^2 = \frac{1}{1 + \exp(2\pi i \xi_0^\uparrow)} \approx \exp(-2\pi i \xi_0^\uparrow), \tag{6}
\]

where \( \xi_0^\uparrow = (E_F - E_0)/\hbar \omega_x > 0 \). Below, I will focus on low magnetic field effects. Using Eq. (5), similarly I obtain \( (\delta = \Delta/2\hbar \omega_x \ll 0.4) \)

\[
R_0^\downarrow \approx \exp(-2\pi i \xi_0^\downarrow) \approx R_0^\uparrow (1 + 4\pi \delta). \tag{7}
\]

Both channels are transmitted but \( 1 \gg R_0^\downarrow > R_0^\uparrow \) (See Fig. 2). Applying an in-plane magnetic field, one gets what I call a two-channel anisotropic QPC model; This is defined, below. The limit of strong fields will be reached when the Zeeman energy approaches the curvature energies of the potential: \( E_0^\uparrow = E_1 \) and \( T_0^\uparrow = 1 - R_0^\uparrow \ll 0 \). A single channel will subsist in the constricton.

As I am interested in the dynamics of the system at energies much smaller than \( E_F \), I may linearize the spectrum of the 1D-fermions in state \( \Psi_\alpha(x) = \Psi_\alpha^n(x) \). One can always write \( \Psi_\alpha(x) = \exp(i k_F x) \Psi_{R_\alpha}(x) + \exp(-i k_F x) \Psi_{L_\alpha}(x) \); \( \Psi_{L_\alpha} \) and \( \Psi_{R_\alpha} \) respectively describe left- and right moving fermions, and \( E_F = k_F^2/2m \). Finite reflection in the channel \( \alpha = \uparrow, \downarrow \) then can be simply accounted for by adding a backscattering term \([5,6,15]\).

\[
H_{bs} = v_F \sqrt{R_0^\uparrow R_0^\downarrow} \Psi_{R_\alpha}(0) \Psi_{R_\alpha}(0). \tag{8}
\]
\( v_F \) denotes the Fermi velocity \([16]\). One must also include the kinetic energy through the constriction,

\[
H_{kin} = iv_F \int_{-\infty}^{+\infty} dx \{ \Psi_{R_\alpha}^\dagger \partial_x \Psi_{R_\alpha} - \Psi_{L_\alpha}^\dagger \partial_x \Psi_{L_\alpha} \}. \tag{9}
\]

At almost perfect transmission, the electronic wave function is shared between the reservoir and the dot. I can neglect finite size effects in a dot at the micron scale \( (\epsilon \to 0) \) \([17]\). Note that \( (H_{kin} + H_{bs}) \) with \( 1 \gg R_0^\dagger > R_0^\uparrow \) describes a two-channel anisotropic QPC model. Again, I ignore higher modes confined to the reservoir, and also neglect the Pauli contribution of the 2DEG. The charging process is described by the following usual term

\[
H_c = E_c \left( Q - N \right)^2, \tag{10}
\]

with \( E_c = e^2/(2C_{gd}) \ll E_F \) the (charging) energy that it costs to transfer a particle from the lead to the dot. The charge \( Q \) (of the dot) in \( H_c \) is now normalized to \( e \),

\[
Q = \int_0^{+\infty} dx \{ \Psi_{L_\alpha}^\dagger \partial_x \Psi_{L_\alpha} + \Psi_{R_\alpha}^\dagger \partial_x \Psi_{R_\alpha} \}. \tag{11}
\]

At low energies, I can proceed with this model by bosonizing the 1D Fermi fields \([5,6,15,18]\).

\[
\Psi_{ps} = \frac{1}{\sqrt{2\pi a}} \exp \left( i \sqrt{\frac{\pi}{2}} \left[ p(\phi_c + \alpha \phi_s) - (\theta_c + \alpha \theta_s) \right] \right). \tag{12}
\]
\( a \) is a short-distance cutoff, again \( a = \pm \) for spin up and spin down, and \( p = \pm \) for right and left movers. The spectrum of 1D free electrons yields separation of spin and charge. Resulting Hamiltonians are plasmon-like

\[
H_{\text{kin}} = \sum_{j=c,s} \frac{v_F}{2} \int_{-\infty}^{+\infty} dx \left[ (\partial_x \phi_j)^2 + \Pi_j^2 \right].
\]  

(13)

\( \partial_x \phi_j \) with \( j = (c, s) \) measures fluctuations of charge/spin density in the constriction and \( \Pi_j = \partial_x \theta_j \) being its conjugate momentum. In this representation, the charging Hamiltonian \( H_c \) reads

\[
H_c = E_c \left[ \sqrt{\frac{2}{\pi}} \phi_c(0) - N \right]^2.
\]  

(14)

To minimize energy, the charge in the dot is pinned at the classical value \( Q_c = \phi_c(0) \sqrt{2/\pi} \approx N \). \[ \[ \]

Now, one has to examine the quantum corrections to the charge entering the constriction, in presence of a small (parallel) magnetic field. For energies smaller than \( E_c \) and \( |\lambda| \ll 1 \), I can replace \( \cos[\sqrt{2\pi} \phi_c(0)] \) by the averaged value \( \sqrt{\gamma E_c a / \pi v_F \cos(\pi N)} \) (similarly for the sinuss).

The backscattering contribution to the ground state energy then takes the form

\[
\delta E(N) = -\frac{\Gamma(N)}{2} \ln(E_c / \Gamma(N)),
\]

where \( \Gamma(N) = J_{\perp x} / \pi v_F a \) is the Kondo resonance. The quantum correction \( \delta E \) to the charge in the dot, becoming equal to \( \delta Q = -b \delta E / \Gamma(N) \). This results explicitly in \( \delta Q \approx -b R_0^\dagger \sin(2\pi N \ln(E_c / \Gamma(N))) \).

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See e.g. Ref. [3], page 401. Another energy scale related to the magnetic field emerges: \( \Upsilon = J_{\perp y}^2 / \pi v_{F} a = \gamma E_{C} \mathcal{R}_{0}^{-1/2} \sin^2(\pi N)/\pi \). This is associated with the divergence of \( J_{\perp y} \). For high fields, adjusting correctly the voltage at the QPC one reproduces a single-channel QPC model; For \( |\lambda| \ll 1 \), the charge entering the dot exhibits only a weak quantum oscillation around its classical value. Experimentally, the prominent smearing of the Coulomb staircase — predicted by increasing the in-plane magnetic field — could be possibly observed for sufficiently low temperatures \( T \), probing (preferably) the capacitance line shapes of the large dot in a single-terminal geometry [10]. To account for nonzero temperature, one must rescale \( \max\{\Gamma; \Upsilon\} \) as \( k_{B}T + \max\{\Gamma; \Upsilon\} \). Experimentally, it would be crucial to minimize the broadening of the capacitance peaks due to thermal effects.

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