ON THE TOP QUARK’S CHIRAL WEAK-MOMENT

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Abstract

For the observed $t \to W^+b$ decay, an intensity-ratio equivalence-theorem for two Lorentz-invariant couplings is shown to be related to symmetries of $tWb$-transformations. Explicit $tWb$-transformations, $A_+ = M A_{SM}, P A_{SM}, B A_{SM}$ relate the four standard model’s helicity amplitudes, $A_{SM} (\lambda_W, \lambda_b)$, and the amplitudes $A_+ (\lambda_{W+}, \lambda_b)$ in the case of an additional $t_R \to b_L$ weak-moment of relative strength $\Lambda_+ = E_W/2 \sim 53$GeV. Two “commutator plus anti-commutator” symmetry algebras are generated from $M, P, B$. These transformations enable a simple and uniform characterization of the values of $\Lambda_+, m_W/m_t$, and $m_b/m_t$.

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1 Introduction:

In this paper, for the observed \( t \to W^+b \) decay [1], an intensity-ratio equivalence-theorem [2] for two Lorentz-invariant couplings is shown to be related to symmetries of tWb-transformations, \( A_+ = M A_{SM}, P A_{SM}, B A_{SM} \), where \( M, P, B \) are explicit 4x4 matrices. These tWb-transformations relate the standard model’s helicity amplitudes, \( A_{SM}(\lambda_{W^+}, \lambda_b) \), and the amplitudes \( A_+(\lambda_{W^+}, \lambda_b) \) in the case of an additional \( t_R \to b_L \) weak-moment of relative strength \( \Lambda_+ = E_W/2 \sim 53 GeV \). Versus the standard model’s pure \((V-A)\) coupling, the additional tensorial coupling can be physically interpreted as arising due to a large chiral weak-transition moment for the top quark. \( \Lambda_+ \) is defined by (1) below and the (+) amplitudes’ complete coupling is (2). \( E_W \) is the energy of the final W-boson in the decaying top-quark rest frame. The subscripts \( R \) and \( L \) respectively denote right and left chirality of the coupling, that is \((1 \pm \gamma_5)\). \( \lambda_{W^+}, \lambda_b \) are the helicities of the the emitted W-boson and b-quark in the top-quark rest frame. The Jacob-Wick phase-convention [3] is used in specifying the phases of the helicity amplitudes and so of these transformations.

Due to rotational invariance, there are four independent \( A(\lambda_{W^+}, \lambda_b) \) amplitudes for the most general Lorentz coupling [4,5]. Stage-two spin-correlation functions were derived and studied as a basis for complete measurements of the helicity parameters for \( t \to W^+b \) decay as tests with respect to the most general Lorentz coupling. Such tests are possible at the Tevatron [1], at the LHC [6], and at a NLC [7]. In this paper, a subset of the most general Lorentz coupling is considered in which the subscript “\( i \)” identifies the amplitude’s associated coupling: “\( i = SM \)” for the pure \((V-A)\) coupling, “\( i = (f_M + f_E) \)” for only the additional \( t_R \to b_L \) tensorial coupling, and “\( i = (+) \)” for \((V-A) + (f_M + f_E)\) with a top-quark chiral weak-transition moment of
relative strength $\Lambda_+ = E_W/2$ versus $g_L$. The Lorentz coupling involving both the SM’s \((V-A)\) coupling and an additional $t_R \rightarrow b_L$ weak-moment coupling of arbitrary relative strength $\Lambda_+$ is

$$W^*_\mu J^\mu_{bl} = W^*_\mu \bar{u}_b (p) \Gamma^\mu u_t (k)$$

where $k_t = q_W + p_b$, and

$$\frac{1}{2} \Gamma^\mu = g_L \gamma^\mu P_L + \frac{g_{f_M + f_E}}{2\Lambda_+} i\sigma^{\mu\nu} (k - p)_\nu P_R$$

(1)

where $P_{L,R} = \frac{1}{2} (1 \mp \gamma_5)$. In $g_L = g_{f_M + f_E} = 1$ units, when $\Lambda_+ = E_W/2$ which corresponds to the (+) amplitudes, the complete $t \rightarrow b$ coupling is very simple

$$\gamma^\mu P_L + i\sigma^{\mu\nu} v_\nu P_R = P_R (\gamma^\mu + i\sigma^{\mu\nu} v_\nu)$$

(2)

where $v_\nu$ is the W-boson’s relativistic four-velocity.

In the $t$ rest frame, the helicity-amplitude matrix element for $t \rightarrow W^+ b$ is

$$\langle \phi_1^t, \phi_1^t, \lambda_{W^+}, \lambda_b | 1/2, \lambda_1 \rangle = D^{(1/2)*}_{\lambda_1, \mu} (\phi_1^t, \theta_1^t, 0) A_\mu (\lambda_{W^+}, \lambda_b)$$

where $\mu = \lambda_{W^+} - \lambda_b$ in terms of the $W^+$ and $b$-quark helicities. The asterisk denotes complex conjugation, the final $W^+$ momentum is in the $\theta_1^t, \phi_1^t$ direction, and $\lambda_1$ gives the $t$-quark’s spin component quantized along the $z$ axis. $\lambda_1$ is also the helicity of the $t$-quark if one has boosted, along the \textit{"{z}} direction, back to the $t$ rest frame from the \textit{(tt)cm} frame. It is this boost which defines the $z$ axis in the $t$-quark rest frame for angular analysis [4]. Explicit expressions for the helicity amplitudes associated with each \textit{“i” coupling are listed in Sec. 2. We denote by $\Gamma$ the partial-width for the $t \rightarrow W^+ b$ decay channel and by $\Gamma_{L,T}$ the partial-width’s for the sub-channels in which the $W^+$ is respectively longitudinally, transversely polarized; $\Gamma = \Gamma_L + \Gamma_T$. Similarly, $\Gamma_{L,T}\big|_{\lambda_b=-\frac{1}{2}}$ denotes the partial-width for the $W$-longitudinal sub-channel with $b$-quark helicity $\lambda_b = -\frac{1}{2}$.

The intensity-ratio equivalence-theorem states, \textit{“As consequence of Lorentz-invariance, for the $t \rightarrow W^+ b$ decay channel each of the four ratios $\Gamma_{L,T}\big|_{\lambda_b=-\frac{1}{2}}/\Gamma$, $\Gamma_{L,T}\big|_{\lambda_b=-\frac{1}{2}}/\Gamma$, $\Gamma_{L,T}\big|_{\lambda_b=-\frac{1}{2}}/\Gamma$, $\Gamma_{L,T}\big|_{\lambda_b=-\frac{1}{2}}/\Gamma$, is
identical for the pure \((V - A)\) coupling and for the \((V - A) + (f_M + f_E)\) coupling with \(\Lambda_+ = E_W/2\), and their respective partial-widths are related by \(\Gamma_+ = v^2 \Gamma_{SM}\).” \(v\) is the velocity of the W-boson in the t-quark rest frame. Note that this theorem does not require specific values of the mass ratios \(y \equiv m_W/m_t\), and \(x \equiv m_b/m_t\), but that the relative strength of the chiral weak-transition moment for the top quark has been fixed versus \(g_L\).

The three tWb-transformations, \(A_+ = M A_{SM}, P A_{SM}, B A_{SM}\), are related to this theorem. The \(M\) transformation implies the theorem, but as explained below, \(M\) also implies the sign and ratio differences of the (ii) and (iii) type amplitude ratio-relations which distinguish the (SM) and (+) couplings. The \(P\) and \(B\) transformations more completely exhibit the underlying symmetry relating these two Lorentz-invariant couplings. In particular, these three 4x4 matrices lead to two “commutator plus anti-commutator” symmetry algebras, and together enable a simple and uniform characterization of the values of \(\Lambda_+, y \equiv m_W/m_t\), and \(x \equiv m_b/m_t\). In Sec. 2, it is shown how these three tWb-transformations successively arise from consideration of different types of “helicity amplitude relations” for \(t \to W^+ b\) decay: The type (i) are ratio-relations which hold separately for the two cases, “\(i = (SM), (+)\)” The type (ii) are ratio-relations which relate the amplitudes in the two cases. By the type (iii) ratio-relations, the tWb-transformation \(A_+ = M A_{SM}\) where \(M = v \text{ diag}(1, -1, -1, 1)\) characterizes the mass scale \(\Lambda_+ = E_W/2\). Similarly, the amplitude condition (iv)

\[
A_+(0, -1/2) = aA_{SM}(-1, -1/2),
\]

with \(a = 1 + O(v \neq y \sqrt{2}, x)\), determines the scale of the tWb-transformation matrix \(P\) and determines the value of the mass ratio \(y \equiv m_W/m_t\). \(O(v \neq y \sqrt{2}, x)\) denotes small corrections, see
below. The amplitude condition (v)

\[ A_v(0, -1/2) = -b A_{SM}(1 - 1/2), \quad (4) \]

with \( b = v^{-8} \), determines the scale of \( B \) and determines the value of \( x = m_b/m_t \). In Sec. 3, two symmetry algebras are obtained which involve the \( M, P, \) and \( B \) transformation matrices. Sec. 4 contains some remarks.

2 Helicity amplitude relations:

In the Jacob-Wick phase convention, the helicity amplitudes for the most general Lorentz coupling are given in [4]. In \( g_L = g_{fM+fE} = 1 \) units and suppressing a common overall factor of \( \sqrt{m_t(E_b + q_W)} \), for only the \((V - A)\) coupling the associated helicity amplitudes are:

\[
A_{SM} \left( 0, -\frac{1}{2} \right) = \frac{1}{y} \frac{E_W + q_W}{m_t}
\]
\[
A_{SM} \left( -1, -\frac{1}{2} \right) = \sqrt{2}
\]
\[
A_{SM} \left( 0, \frac{1}{2} \right) = -\frac{1}{y} \frac{E_W - q_W}{m_t} \left( \frac{m_b}{m_t - E_W + q_W} \right)
\]
\[
A_{SM} \left( 1, \frac{1}{2} \right) = -\sqrt{2} \left( \frac{m_b}{m_t - E_W + q_W} \right)
\]

For only the \((f_M + f_E)\) coupling, i.e. only the additional \( t_R \rightarrow b_L \) tensorial coupling:

\[
A_{fM+fE} \left( 0, -\frac{1}{2} \right) = -\left( \frac{m_t}{2\Lambda_+} \right) y
\]
\[
A_{fM+fE} \left( -1, -\frac{1}{2} \right) = -\left( \frac{m_t}{2\Lambda_+} \right) \sqrt{2} \frac{E_W + q_W}{m_t}
\]
\[
A_{fM+fE} \left( 0, \frac{1}{2} \right) = \left( \frac{m_t}{2\Lambda_+} \right) y \left( \frac{m_b}{m_t - E_W + q_W} \right)
\]
\[
A_{fM+fE} \left( 1, \frac{1}{2} \right) = \left( \frac{m_t}{2\Lambda_+} \right) \sqrt{2} \frac{E_W - q_W}{m_t} \left( \frac{m_b}{m_t - E_W + q_W} \right)
\]
From these, the amplitudes for the \((V - A) + (f_M + f_E)\) coupling of (1) are obtained by
\[ A_+ (\lambda_W, \lambda_b) = A_{SM}(\lambda_W, \lambda_b) + A_{f_M+f_E}(\lambda_W, \lambda_b). \]
For \(\Lambda_+ = E_W/2\), the \(A_+(\lambda_W, \lambda_b)\) amplitudes [8] corresponding to the complete \(t \to b\) coupling (2) are

\[
A_+ (0, -1/2) = \frac{1}{y} \frac{(q/E_W)}{m_t} \frac{E_W + q_W}{m_t} \\
A_+ (-1, -1/2) = -\sqrt{2} \frac{(q/E_W)}{m_t} \\
A_+ (0, 1/2) = \frac{1}{y} \frac{(q/E_W)}{m_t} \frac{E_W - q_W}{m_t} \left( \frac{m_b}{m_t - E_W + q_W} \right) \\
A_+ (1, 1/2) = -\sqrt{2} \frac{(q/E_W)}{m_t} \left( \frac{m_b}{m_t - E_W + q_W} \right)
\]

We now analyze the different types of helicity amplitude relations involving both the SM’s amplitudes and those in the case of the \((V - A) + (f_M + f_E)\) coupling: The first type of ratio-relations holds separately for \(i = (SM), (+)\) and for all \(y = \frac{m_W}{m_t}, x = \frac{m_b}{m_t}; \Lambda_+\) values, (i):

\[
\frac{A_i(0, 1/2)}{A_i(-1, -1/2)} = \frac{1}{2} \frac{A_i(1, 1/2)}{A_i(0, -1/2)} \tag{5}
\]

The second type of ratio-relations relates the amplitudes in the two cases and also holds for all \(y, x, \Lambda_+\) values. The first two relations have numerators with opposite signs and denominators with opposite signs, c.f. Table 1; (ii): Two sign-flip relations

\[
\frac{A_+(0, 1/2)}{A_+(-1, -1/2)} = \frac{A_{SM}(0, 1/2)}{A_{SM}(-1, -1/2)} \tag{6} \\
\frac{A_+(0, 1/2)}{A_+(-1, -1/2)} = \frac{1}{2} \frac{A_{SM}(1, 1/2)}{A_{SM}(0, -1/2)} \tag{7}
\]

and two non-sign-flip relations

\[
\frac{A_+(1, 1/2)}{A_+(0, -1/2)} = \frac{A_{SM}(1, 1/2)}{A_{SM}(0, -1/2)} \tag{8} \\
\frac{A_+(1, 1/2)}{A_+(0, -1/2)} = 2 \frac{A_{SM}(0, 1/2)}{A_{SM}(-1, -1/2)} \tag{9}
\]
Eqs(7,9), which are not in [2], are essential for obtaining the \(P\) and \(B\) tWb-transformations and thereby the symmetry algebras of Sec. 3 below.

The third type of ratio-relations, holding for all \(y, x\) values, follows by determining the effective mass scale, \(\Lambda_+\), so that there is an exact equality for the ratio of left-handed amplitudes (iii):

\[
\frac{A_+(0, -1/2)}{A_+(-1, -1/2)} = -\frac{A_{SM}(0, -1/2)}{A_{SM}(-1, -1/2)}, \tag{10}
\]

Equivalently, \(\Lambda_+ = \frac{m_e}{4}[1 + (m_W/m_t)^2 - (m_b/m_t)^2] = E_W/2\) follows from each of:

\[
\frac{A_+(0, -1/2)}{A_+(-1, -1/2)} = -\frac{1}{2} \frac{A_{SM}(1, 1/2)}{A_{SM}(0, 1/2)}, \tag{11}
\]

\[
\frac{A_+(0, 1/2)}{A_+(1, 1/2)} = -\frac{A_{SM}(0, 1/2)}{A_{SM}(1, 1/2)}, \tag{12}
\]

\[
\frac{A_+(0, 1/2)}{A_+(1, 1/2)} = -\frac{1}{2} \frac{A_{SM}(-1, -1/2)}{A_{SM}(0, -1/2)}, \tag{13}
\]

From the amplitude expressions given above, the value of this scale \(\Lambda_+\) can be characterized by postulating the existence of a tWb-transformation \(A_+ = M A_{SM}\) where \(M = v \text{ diag}(1, -1, -1, 1)\), with \(A_{SM} = [A_{SM}(0, -1/2), A_{SM}(-1, -1/2), A_{SM}(0, 1/2), A_{SM}(1, 1/2)]\) and analogously for \(A_-\).

Assuming (iii), the fourth type of relation is the equality (iv):

\[
A_+(0, -1/2) = a A_{SM}(-1, -1/2), \tag{14}
\]

where \(a = 1 + O(v \neq y\sqrt{2}, x)\).

This is equivalent to the velocity formula \(v = ay\sqrt{2}\left(\frac{1}{1-(E_b-q_W)/m_t}\right) = ay\sqrt{2}\) for \(m_b = 0\). In [2], for \(a = 1\) it was shown that (iv) leads to a mass relation with the solution \(y = \frac{m_W}{m_t} = 0.46006\) \((x = 0)\). The present empirical value is \(y = 0.461 \pm 0.014\), where the error is dominated by the 3% precision of \(m_t\). In [2], for \(a = 1\) it was also shown that (iv) leads to \(\sqrt{2} = v\gamma(1+v) = v\sqrt{\frac{1+v}{1-v}}\) so \(v = 0.6506\ldots\) without input of a specific value for \(m_b\). However, by Lorentz invariance \(v\) must
depend on $m_b$. Accepting (iii), we interpret this to mean that $a \neq 1$ and in the Appendix obtain the form of the $O(v \neq y\sqrt{2}, x)$ corrections in $a$ as required by Lorentz invariance. The small correction $O(v \neq y\sqrt{2}, x)$ depends on both $x \equiv m_b/m_t$ and the difference $v - y\sqrt{2}$.

Equivalently, by use of (i)-(iii) relations, (14) can be expressed postulating the existence of a second tWb-transformation $A_+ = P A_{SM}$ where

$$P \equiv v \begin{bmatrix} 0 & a/v & 0 & 0 \\ -v/a & 0 & 0 & 0 \\ 0 & 0 & 0 & -v/2a \\ 0 & 0 & 2a/v & 0 \end{bmatrix}$$

(15)

The value of the parameter $a$ of (iv) is not fixed by (15).

The above two tWb-transformations do not relate the $\lambda_b = -\frac{1}{2}$ amplitudes with the $\lambda_b = \frac{1}{2}$ amplitudes. From (i) thru (iv), in terms of a parameter $b$, the equality (v):

$$A_+(0, -1/2) = -b A_{SM}(1, 1/2),$$

(16)

is equivalent to $A_+ = B A_{SM}$

$$B \equiv \begin{bmatrix} 0 & 0 & 0 & -b \\ 0 & 0 & 2b & 0 \\ 0 & v^2/2b & 0 & 0 \\ -v^2/b & 0 & 0 & 0 \end{bmatrix}$$

(17)
The choice of \( b = v^{-8} = 31.152 \), gives

\[
B \equiv v \begin{bmatrix}
0 & 0 & 0 & -v^{-9} \\
0 & 0 & 2v^{-9} & 0 \\
0 & v^9/2 & 0 & 0 \\
-v^9 & 0 & 0 & 0
\end{bmatrix}
\]

and corresponds to the mass relation \( m_b = \frac{m_t}{v} \left[ 1 - \frac{v}{\sqrt{2}} \right] = 4.407 \text{GeV} \) for \( m_t = 174.3 \text{GeV} \).

3 Commutator plus anti-commutator symmetry algebras:

The anti-commuting matrices

\[
m \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},
p \equiv \begin{bmatrix} 0 & -a/v \\ v/a & 0 \end{bmatrix},
q \equiv \begin{bmatrix} 0 & a/v \\ v/a & 0 \end{bmatrix}
\]

satisfy \([m, p] = -2q, [m, q] = -2p, [p, q] = -2m\). Similarly, \( m \) and

\[
r \equiv \begin{bmatrix} 0 & -v/2a \\ 2a/v & 0 \end{bmatrix},
s \equiv \begin{bmatrix} 0 & v/2a \\ 2a/v & 0 \end{bmatrix}
\]

are anti-commuting and satisfy \([m, r] = -2s, [m, s] = -2r, [r, s] = -2m\). Note \( m^2 = q^2 = s^2 = 1, p^2 = r^2 = -1 \), and that \( a \) is arbitrary. Consequently, if one does not distinguish the (+) versus SM indices, respectively of the rows and columns, the tWb-transformation matrices have some simple properties:

The anticommuting 4x4 matrices

\[
M \equiv v \begin{bmatrix} m & 0 \\ 0 & -m \end{bmatrix},
P \equiv v \begin{bmatrix} -p & 0 \\ 0 & r \end{bmatrix},
Q \equiv v \begin{bmatrix} q & 0 \\ 0 & s \end{bmatrix}
\]
satisfy the closed algebra \([\overline{M}, \overline{P}] = 2\overline{Q}, [\overline{M}, \overline{Q}] = 2\overline{P}, [\overline{P}, \overline{Q}] = 2\overline{M}\). The bar denotes removal of the overall “\(v\)” factor, \(M = v\overline{M}\), ... Note that \(Q\) is not a tWb-transformation.

Including the B matrix with \(b\) arbitrary, the algebra closes with 3 additional matrices

\[
\overline{B} \equiv \begin{bmatrix} 0 & d \\ f & 0 \end{bmatrix}, \overline{C} \equiv \begin{bmatrix} 0 & e \\ g & 0 \end{bmatrix}
\]

(22)

\[
\overline{G} \equiv \begin{bmatrix} 0 & h \\ k & 0 \end{bmatrix}, \overline{H} \equiv \begin{bmatrix} 0 & j \\ l & 0 \end{bmatrix}
\]

(23)

where

\[
d \equiv \begin{bmatrix} 0 & -b/v \\ 2b/v & 0 \end{bmatrix}, e \equiv \begin{bmatrix} 0 & b/v \\ 2b/v & 0 \end{bmatrix}, f \equiv \begin{bmatrix} 0 & v/2b \\ -v/b & 0 \end{bmatrix}, g \equiv \begin{bmatrix} 0 & v/2b \\ v/b & 0 \end{bmatrix}
\]

(24)

\[
h \equiv \begin{bmatrix} -2ab/v^2 & 0 \\ 0 & b/a \end{bmatrix}, j \equiv \begin{bmatrix} 2ab/v^2 & 0 \\ 0 & b/a \end{bmatrix}, k \equiv \begin{bmatrix} 1/2v^2ab & 0 \\ 0 & -a/b \end{bmatrix}, l \equiv \begin{bmatrix} 1/2v^2ab & 0 \\ 0 & a/b \end{bmatrix}
\]

(25)

The squares of the 2x2 matrices (24-25) do depend on \(a\), \(b\), and \(v\).

The associated closed algebra is: \([\overline{M}, \overline{B}] = 0, \{\overline{M}, \overline{B}\} = -2\overline{C}; [\overline{B}, \overline{C}] = 0, \{\overline{B}, \overline{C}\} = -2\overline{M}; [\overline{M}, \overline{C}] = 0, \{\overline{M}, \overline{C}\} = -2\overline{B};\) and \([\overline{P}, \overline{B}] = 2\overline{H}, \{\overline{P}, \overline{B}\} = 0; [\overline{H}, \overline{P}] = 2\overline{B}, \{\overline{H}, \overline{P}\} = 0; [\overline{H}, \overline{B}] = 2\overline{P}, \{\overline{H}, \overline{B}\} = 0 . Similarly, \([\overline{P}, \overline{C}] = 0, \{\overline{P}, \overline{C}\} = -2\overline{G}; [\overline{M}, \overline{H}] = -2\overline{C}, \{\overline{M}, \overline{H}\} = 0; [\overline{H}, \overline{C}] = 0, \{\overline{H}, \overline{C}\} = 2\overline{Q};\) and \([\overline{M}, \overline{G}] = -2\overline{H}, \{\overline{M}, \overline{G}\} = 0; [\overline{P}, \overline{G}] = 0, \{\overline{P}, \overline{G}\} = 2\overline{C}, \{\overline{P}, \overline{C}\} = 2\overline{G}; [\overline{G}, \overline{B}] = -2\overline{Q}, \{\overline{G}, \overline{B}\} = 0;\) and \([\overline{G}, \overline{C}] = 0, \{\overline{G}, \overline{C}\} = -2\overline{P}; [\overline{G}, \overline{H}] = 2\overline{M}, \{\overline{G}, \overline{H}\} = 0 . The part involving \(\overline{Q}\) is \([\overline{C}, \overline{Q}] = 2\overline{B}, \{\overline{C}, \overline{Q}\} = 0; [\overline{B}, \overline{Q}] = 2\overline{C}, \{\overline{B}, \overline{Q}\} = 0; [\overline{C}, \overline{Q}] = 0, \{\overline{C}, \overline{Q}\} = -2\overline{H}; [\overline{H}, \overline{Q}] = 0, \{\overline{H}, \overline{Q}\} = 2\overline{C}.\)
This has generated an additional tWb-transformation \( G \equiv v\overline{G} \); but \( C \equiv v\overline{C} \) and \( H \equiv v\overline{H} \) are not tWb-transformations.

Up to the insertion of an overall \( \iota = \sqrt{-1} \), each of these 4x4 barred matrices is a resolution of unity, i.e. \( \overline{P}^{-1} = -P, \overline{G}^{-1} = -G \), but \( \overline{Q}^{-1} = Q, \overline{B}^{-1} = B, \ldots \).

4 Remarks:

(1) **Summary:** The elements of the three logically-successive tWb-transformations are constrained by the helicity amplitude ratio-relations (i) and (ii). Thereby, the type (iii) ratio-relation fixes \( \Lambda_+ = E_W/2 \) and the overall scale of the tWb-transformation matrix \( M \). The amplitude condition (iv), \( A_+(0, -1/2) = aA_{SM}(-1, -1/2) \) with \( a = 1 + O(v \neq y\sqrt{2}, x) \), and the amplitude condition (v), \( A_+(0, -1/2) = -bA_{SM}(1 - 1/2) \) with \( b = v^{-8} \), determine respectively the scale of the tWb-transformation matrices \( P \) and \( B \) and characterize the values of \( m_W/m_t \) and \( m_b/m_t \).

The overall scale can be set here by \( m_t \) or \( m_W \). From an empirical “bottom-up” perspective of further “unification”, \( m_W \) is more appropriate to use to set the scale since its value is fixed in the SM.

(2) **Symmetries and Second Class Currents:** The additional \( t_R \rightarrow b_L \) weak-moment coupling violates the conventional gauge invariance transformations of the SM and traditionally in electroweak studies such anomalous couplings have been best considered as “induced” or “effective”. Nevertheless, in special “new physics” circumstances such a simple tensorial coupling as (2) might turn out to be fundamental. The “tensorial coupling” is of course a fundamental structure if considered from gravitation viewpoints. However, are the new symmetries associated
with the symmetry algebras of Sec. 3 sufficient to overcome the known difficulties [9] in constructing a renormalizable, unitary quantum field theory involving second class currents [10]? The $f_E$ component is second class. $f_E$ has a distinctively different reality structure, and time-reversal invariance property versus the first class $V, A, f_M$ [11].

(3) Form-factor Investigations: With respect to an “induced” or “effective” $t_R \to b_L$ tensorial coupling, the physics issue becomes one of investigating and excluding specific sources for producing such an additional weak-moment. One of the better motivated and more developed extensions of the SM is based on supersymmetry. The fundamental source of SUSY breaking and its details remains an important open problem, and the phenomenology associated with the SUSY spectra particles is largely unknown. Nevertheless, in the present context, SUSY provides a more general and useful off-shell theoretical framework in which to consider these theoretical patterns of the helicity amplitudes for $t \to W^+ b$ decay. Form factor effects naturally occur. In particular it might be possible to generate an additional effective $t_R \to b_L$ weak-moment coupling of relative strength $\Lambda_+ = E_W/2$. For what domains, if any, of parameters in R-violating SUSY models would this be possible? Does there exist dynamically salient phenomena and/or persistent features besides the (i)-(v) relations occurring in those parameter domains? In the case of the MSSM, a recent analysis through the one-loop level of SUSY QCD and SUSY electroweak corrections to $\Gamma_L, \Gamma_T$, and to the partial width $\Gamma$ shows that this is not possible [12]. However, the more general case of R-violating SUSY models remains to be investigated.

(4) Experimental Tests/Measurements: In on-going [1] and forth-coming [5,6] top-quark decay experiments, important information about the relationship of the tWb-transformation symmetry patterns of this paper to the observed top quark decays will come from:
(a) Measurement of the sign of the parameter $\eta_L \equiv \frac{1}{2} |A(-1,-\frac{1}{2})||A(0,-\frac{1}{2})| \cos \beta_L = \pm 0.46 (SM/+)$ so as to determine the sign of $\cos \beta_L$ where $\beta_L = \phi^L_{-1} - \phi^L_0$ is the relative phase of the two $\lambda_b = -\frac{1}{2}$ amplitudes, $A(\lambda_{W^+}, \lambda_b) = |A| \exp(i\phi^L_{\lambda_{W^+}}$).

(b) Measurement of the closely associated parameter $\eta_L' \equiv \frac{1}{2} |A(-1,-\frac{1}{2})||A(0,-\frac{1}{2})| \sin \beta_L$ helicity parameter. This would provide useful complementary information, since in the absence of $T_{FS}$-violation, $\eta_L' = 0$.

(c) Measurement of the partial width for $t \to W^+ b$ such as in single-top production [13-15]. The $v^2$ factor which differs their associated partial widths corresponds to the SM’s $\Gamma_{SM} = 1.55 GeV$, versus $\Gamma_{+} = 0.66 GeV$ and a longer-lived $(+)$ top-quark if this mode is dominant.

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Appendix: The $O(v \neq y\sqrt{2}, x)$ corrections in $a$

In this appendix is listed the form of the $O(v \neq y\sqrt{2}, x)$ corrections in $a$ as required by Lorentz invariance:

For $a = 1 + \varepsilon(x, y)$, the (iv) relation is $v = (1 + \varepsilon)y\sqrt{2m_t/(E_W + q)}$ whereas from relativistic kinematics $v = q/E_W = [(1 - y^2 - x^2)^2 - 4y^2x^2]^{1/2}/[1 + y^2 - x^2]$. By equating these expressions and expanding in $x$, one obtains $\varepsilon = R + x^2 S$ where

$$R = \frac{1 - 4y^2 - 3y^4 - 2y^6}{4y^2(1 + y^2)^2}$$
$$S = \frac{-1 - 4y^2 + y^4}{2y^2(1 + y^2)^3}$$

and $v = y\sqrt{2}\left[1 + R + x^2(S + \frac{1+R}{1-y^2} + O(x^4)\right]$. From the latter equation, $R = (v - y\sqrt{2})/y\sqrt{2} + O(x^2)$. 

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For a massless b-quark \((x = 0)\) and \(a = 1\), the (iv) relation is equivalent to the mass relation \(y^3 \sqrt{2} + y^2 + y \sqrt{2} - 1 = 0\), and by relativistic kinematics to the W-boson velocity condition \(v^3 + v^2 + 2v - 2 = 0\) and the simple formula \(v = y \sqrt{2}\).

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Table Captions

Table 1: Numerical values of the helicity amplitudes for the standard model \((V - A)\) coupling and for the \((+\) coupling of Eq.(2). The latter consists of an additional \(t_R \to b_L\) weak-moment of relative strength \(\Lambda_+ \sim 53 GeV\) so as yield a relative-sign change in the \(\lambda_b = -\frac{1}{2}\) amplitudes. The values are listed first in \(g_L = g_{f_M} + f_E = 1\) units, and second as \(A_{new} = A_{g_L=1}/\sqrt{\Gamma}\). Table entries are for \(m_t = 175 GeV, \ m_W = 80.35 GeV, \ m_b = 4.5 GeV\).
|                | $A(0, -\frac{1}{2})$ | $A(-1, -\frac{1}{2})$ | $A(0, \frac{1}{2})$ | $A(1, \frac{1}{2})$ |
|----------------|-----------------------|------------------------|---------------------|---------------------|
| $A_{g_{L}=1}$ in $g_{L} = 1$ units |                       |                        |                     |                     |
| $V - A$       | 338                   | 220                    | -2.33               | -7.16               |
| $f_{M} + f_{E}$ | 220                   | -143                   | 1.52                | -4.67               |
| $A_{New} = A_{g_{L}=1}/\sqrt{T}$ |                       |                        |                     |                     |
| $V - A$       | 0.84                  | 0.54                   | -0.0058             | -0.018              |
| $f_{M} + f_{E}$ | 0.84                  | -0.54                  | 0.0058              | -0.018              |