Supersymmetric Quantization of Anisotropic Scalar-Tensor Cosmologies

James E. Lidsey*
Astronomy Unit, School of Mathematical Sciences,
Queen Mary and Westfield,
Mile End Road, London E1 4NS, United Kingdom

P. Vargas Moniz†‡§
Grupo de AsTrofisica e Cosmologia (GATC)
Departamento de Fisica, Universidade da Beira Interior (UBI)
Rua Marquês d’Avila e Bolama, 6200 Covilhã, Portugal

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Abstract

In this paper we show that the spatially homogeneous Bianchi type I and Kantowski–Sachs cosmologies derived from the Brans–Dicke theory of gravity admit a supersymmetric extension at the quantum level. Global symmetries in the effective one–dimensional actions characterize both classical and quantum solutions. A wide family of exact wavefunctions satisfying the supersymmetric constraints are found. A connection with quantum wormholes is briefly discussed.

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*E-mail: jel@maths.qmw.ac.uk
†E-mail: pmoniz@mercury.ubi.pt
‡URL: http://www.dfis.ubi.pt/~pmoniz
§Also at CENTRA, IST, Rua Rovisco Pais, 1049 Lisboa Codex, Portugal
1 Introduction

Quantum cosmology applies the fundamental principles of quantum physics to the entire universe. (For a review, see, e.g., Ref. [1]). The wavefunction of the universe is a functional on the configuration space (superspace) and obeys an infinite–dimensional partial differential equation – the Wheeler–DeWitt equation [2]. In view of the severe technical difficulties that arise in solving this equation, the normal procedure is to arbitrarily confine the fields to the neighbourhood of spatial homogeneity. Effectively, the infinite number of inhomogeneous modes and their interactions are truncated out and the configuration space (minisuperspace) is therefore finite–dimensional. The Wheeler–DeWitt equation then determines the evolution of the wavefunction on the minisuperspace and a given trajectory mapped out by the wavefunction may be interpreted as a cosmological space–time.

The validity of the minisuperspace approximation remains an open question to date. It is clearly inconsistent with the uncertainty principle since the amplitudes and momenta of the inhomogeneous modes are assumed to vanish simultaneously. Kuchař and Ryan addressed this question quantitatively within the context the Bianchi type IX cosmology and found that imposing additional symmetry on the model altered the nature of the physical predictions [3] (see also Refs. [3, 4]). On the other hand, Sinha and Hu employed the techniques of coarse–graining and interacting field theory to derive a condition that must be satisfied for the approximation to be justified [3]. Difficulties in coupling non–trivial spinor fields to highly symmetric spacetimes have also been highlighted by Henneaux [7].

Despite these uncertainties, however, the expectation is that the main features of the wavefunction should be preserved in a more general analysis [8]. In principle, the wavefunction of the universe yields the probability that a spatial hypersurface evolves from a given initial state. However, ambiguities arise when attempting to invoke such an interpretation due to the hyperbolic nature of the Wheeler–DeWitt equation: a conserved current with a positive–definite probability density is not possible.

One possible resolution of this and related difficulties is to extend the standard quantization of the universe in a *supersymmetric* fashion. (For a review, see, e.g., Refs. [8, 10]). Supersymmetry may help in the quantization of gravity for a number of reasons. Indeed, earlier work on supergravity theories [11] and recent developments in superstring theory [12] and M–theory [13] indicate that supersymmetry is a fundamental ingredient of any unified description of the fundamental interactions. Consequently, an analysis of the very early universe that includes supersymmetry is well motivated and a number of authors have developed models of the early universe where both quantum gravitational and supersymmetric effects are important [9, 10, 12, 13, 14, 15, 16, 17, 18]. The advantage of such an approach is that the quantum state of the universe, $\Psi$, is annihilated by supersymmetric constraints that are linear, first–order differential equations in the bosonic momenta variables. This is in contrast to nonsupersymmetric quantum cosmology, where a second–order Wheeler-DeWitt
equation has to be solved subject to suitable boundary conditions \[ [19, 20, 22] \]. The
supersymmetric algebra necessarily implies that \( \Psi \) also obeys the Hamiltonian con-
straint and it is therefore sufficient to solve the first–order constraint equations \[ 23 \].
In many cases, this resolves ambiguities in the choice of factor ordering. Furthermore, supersymmetric quantum cosmology places the results of standard quantum
 cosmology in a wider perspective \[ 1 \] and a study of quantum minisuperspaces with
supersymmetry may also provide (in spite of the obvious truncations) some helpful
insights concerning the set of states that represent a complete formulation of quantum
gravity with the other interactions.

In recent years, two attractive (and possibly related) approaches to supersymmet-
ric quantum cosmology have been developed. One approach is to begin with \( N = 1 \)
supergravity \[ [11, 14] \] in four dimensions and reduce the system to a one–dimensional
model by invoking a suitable homogeneous ansatz \[ 9, 10, 15, 16 \]. This leads to a
minisuperspace with \( N = 4 \) local supersymmetry. Alternatively, one may integrate
a purely bosonic action over the spatial variables to derive a \((1 + 0)\)–dimensional la-
grangian and then perform a supersymmetric extension of the corresponding Hamilton-
ian system by employing the quantization rules of the supersymmetric \( \sigma \)–model
\[ 24, 25, 26, 17, 18 \]. This results in an \( N = 2 \) supersymmetry. In particular, this
process could be related to the fact that any one-dimensional system is supersym-
metric provided its ground state is normalizable \[ 27 \]. Moreover, this technique can
be generalized to higher dimensions by employing Darboux transformations \[ 28 \].

In this paper we employ the latter approach to quantize spatially homoge-
neous cosmologies \[ 29 \] within the context of the Brans–Dicke theory of gravity \[ 30 \]. The
Brans–Dicke theory is relevant to the early universe and arises as the effective action
of higher–dimensional gravity theories and, in particular, superstring theory \[ 12 \].
Moreover, the spatially flat and isotropic Brans–Dicke cosmology exhibits a discrete
‘scale factor duality’ \[ 18, 31 \]. This symmetry forms the basis of the pre–big bang
inflationary scenario \[ 31, 32 \] (for a review see, e.g., Ref. \[ 33 \]) and its origin can
be traced to the \( T \)–duality of string theory \[ 34 \]. The consequences of scale factor
duality for string quantum cosmology have been explored by a number of authors
\[ 18, 35 \]. In particular, supersymmetric quantum states have been found that respect
the duality symmetry of the classical Hamiltonian \[ 18 \]. Our purpose in this paper is
to perform a supersymmetric extension of more general spatially homogeneous cos-
mologies. Specifically, we consider the locally rotationally symmetric (LRS) Bianchi
type I model and the Kantowski–Sachs universe \[ 29 \]. Generalizations of scale factor
duality have been shown to exist in these models \[ 36, 37 \] and we find supersymmetric
wavefunctions that respect these symmetries.

The paper is organized as follows. In Section II, the global symmetries of the ac-
tions and the supersymmetric quantization procedure are reviewed. The LRS Bianchi
type I model is quantized in Section III and the vacuum Kantowski–Sachs model is
quantized in Section IV. We conclude with a discussion in Section V, where we also
comment on the possible relationship between the supersymmetric minisuperspace ex-
tension \([17, 18, 27, 28]\) and minisuperspaces retrieved from more general supergravity theories \([9, 10, 16]\).

We assume throughout that \(\hbar = 1\).

# 2 Supersymmetric Quantum Bianchi Cosmology

## 2.1 Duality and the Wheeler–DeWitt Equation

We consider the four–dimensional Brans-Dicke action given by
\[
S = \int d^4x \sqrt{-g}e^{-\Phi} \left[ R - \omega(\nabla \Phi)^2 - 2\Lambda \right],
\]
(2.1)
where \(R\) is the Ricci curvature of the spacetime with metric \(g_{\mu\nu}\), \(g \equiv \det g_{\mu\nu}\) and \(\Phi\) represents the Brans–Dicke (dilaton) field. The coupling between the scalar and tensor fields is parametrized by the constant, \(\omega\), and \(\Lambda\) is the cosmological constant in the gravitational sector of the theory. A consistent truncation of the string effective action is given by Eq. (2.1) for \(\omega = -1\) and \(\Lambda < 0\) \([12]\). Dimensional reduction of higher–dimensional Einstein gravity on an isotropic, \(d\)–dimensional torus results in the above action, where \(\omega = -1 + 1/d\) and \(\Phi\) determines the volume of the internal space \([38]\).

The metric for the class of spatially homogeneous, LRS cosmological models with constant time hypersurfaces containing two–dimensional surfaces of constant curvature, \(k\), is given by \([29]\)
\[
ds^2 = -N^2dt^2 + e^{2\alpha - 4\beta} dr^2 + e^{2\alpha + 2\beta} d\Omega^2_{2,k}
\]
(2.2)
\[
ds^2 = -N^2 dt^2 + a_1^2 dr^2 + a_2^2 d\Omega^2_{2,k},
\]
(2.3)
where \(N\) is the non–dynamical lapse function, \(d\Omega^2_{2,k}\) is the unit metric on the constant curvature two–surfaces, \(e^{2\alpha(t)} \equiv a_1 a_2^2\) determines the effective spatial volume of the universe and \(\beta \equiv (1/3)\ln(a_2/a_1)\) determines the anisotropy of the model. The cases \(k = \{-1, 0, +1\}\) correspond to the Bianchi type III, I and Kantowski–Sachs universes, respectively. The geometry of the spatial sections of the Kantows–Sachs model is \(S^1 \times S^2\). The symmetry group of these surfaces is of the Bianchi type IX, but only acts transitively on two–dimensional surfaces that foliate the three–space.

Integrating over the spatial variables in Eq. (2.1) for the metric ansatz (2.2) yields the minisuperspace action:
\[
S = \int dt Ne^{3\alpha - \Phi} \left[ -6\frac{\dot{\alpha}^2}{N^2} + 6\frac{\dot{\alpha} \dot{\Phi}}{N^2} + \omega \frac{\dot{\Phi}^2}{N^2} + \frac{\beta^2}{N^2} + 2ke^{-2a_2 \beta} - 2\Lambda \right].
\]
(2.4)

Introducing the new variables\(^1\)
\[
\sigma = \sqrt{\frac{3 + 2\omega}{4 + 3\omega}} (\Phi - 3\alpha)
\]
(2.5)

\(^1\)We assume throughout this paper that \(\omega > -4/3\).
\[ u = \sqrt{\frac{8 + 6\omega}{2 + \omega}} \left( \frac{1}{4 + 3\omega} [\alpha + (1 + \omega)\Phi] + \beta \right) \] (2.6)
\[ v = \frac{1}{\sqrt{2 + \omega}} [\alpha + (1 + \omega)\Phi - 2\beta] \] (2.7)

implies that we may diagonalise the kinetic sector of the reduced action (2.4):

\[ S = \int dt \left\{ \frac{1}{N} e^{-\kappa \sigma} u^2 + \frac{1}{N} e^{-\kappa \sigma} v^2 - \frac{1}{N} e^{-\kappa \sigma} \dot{\sigma}^2 \right. \\
+ \left. 2Nke^{(C-\kappa)\sigma-Gu} - 2N\Lambda e^{-\kappa \sigma} \right\}, \] (2.8)

where

\[ \kappa \equiv \sqrt{\frac{4 + 3\omega}{3 + 2\omega}} \] (2.9)
\[ C \equiv \frac{2(1 + \omega)}{\sqrt{(3 + 2\omega)(4 + 3\omega)}} \] (2.10)
\[ G \equiv \sqrt{\frac{4 + 2\omega}{4 + 3\omega}}. \] (2.11)

Global symmetries in these models, corresponding to a generalization of scale factor duality, were uncovered in Ref. [36]. The action (2.8) is invariant under the discrete \( Z_2 \) ‘duality’ symmetry

\[ \bar{u} = u, \quad \bar{v} = -v, \quad \bar{\sigma} = \sigma \] (2.12)

and in terms of the original variables in Eq. (2.4), this is equivalent to

\[ \bar{\alpha} = \frac{4 + 3\omega}{3(2 + \omega)} \alpha - \frac{2(1 + \omega)}{3(2 + \omega)} \Phi + \frac{4}{3(2 + \omega)} \beta \] (2.13)
\[ \bar{\Phi} = -\frac{2}{2 + \omega} \alpha - \frac{\omega}{2 + \omega} \Phi + \frac{4}{2 + \omega} \beta \] (2.14)
\[ \bar{\beta} = \frac{2}{3(2 + \omega)} \alpha + \frac{2(1 + \omega)}{3(2 + \omega)} \Phi + \frac{2 + 3\omega}{3(2 + \omega)} \beta. \] (2.15)

The scale factors transform such that

\[ \bar{a}_1 = a_1 e^{\frac{\omega}{2 + \omega} \Phi} \] (2.16)
\[ \bar{a}_2 = a_2. \] (2.17)

Thus, the scale factor \( a_2 \) is invariant under the symmetry transformation, whereas \( a_1 \) undergoes a direct inversion for the string inspired case, \( \omega = -1. \)
The spatially flat Bianchi type I model \((k = 0)\) also exhibits a global \(SO(2)\) symmetry that acts non-trivially on the variables \(\{u, v\}\):

\[
\bar{u} = \cos \theta u - \sin \theta v \\
\bar{v} = \sin \theta u + \cos \theta v,
\]

(2.18)

where \(\theta\) is a constant. The equivalent transformations on the scale factors and dilaton field were presented in Ref. [36]. The variable, \(\sigma\), transforms as a singlet under Eq. (2.18).

The field equations for these models can be expressed in the form of an unconstrained Hamiltonian system, where the Hamiltonian vanishes. The momenta conjugate to the variables \(\{u, v, \sigma\}\) are given by

\[
\pi_u = 2\dot{u}e^{-\kappa \sigma} \\
\pi_v = 2\dot{v}e^{-\kappa \sigma} \\
\pi_\sigma = -2\dot{\sigma}e^{-\kappa \sigma},
\]

(2.19)

(2.20)

from which the classical Hamiltonian constraint follows:

\[
H = -\pi_u^2 - \pi_v^2 + \pi_\sigma^2 + 8ke^{(C-2\kappa)\sigma}e^{-Gu} - 8\Lambda e^{-2\kappa \sigma}.
\]

(2.21)

Eq. (2.21) may be written in the more compact form

\[
H = G^{ab}\pi_a\pi_b + W(q^a),
\]

(2.22)

\[
W(q^a) = -8ke^{(C-2\kappa)\sigma}e^{-Gu} + 8\Lambda e^{-2\kappa \sigma},
\]

(2.23)

where \(q^a = (\sigma, u, v)\) \((a = 0, 1, 2)\) and \(G^{ab} = \text{diag}(-1, 1, 1)\) is the minisuperspace metric. By identifying the conjugate momenta with the operators \(\pi_{q^a} = \pi_a = -i\partial/\partial q^a\) and neglecting ambiguities that arise in the factor ordering, we arrive at the Wheeler-DeWitt equation:

\[
\left[-\frac{\partial^2}{\partial \sigma^2} + \frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2} + 8ke^{(C-2\kappa)\sigma}e^{-Gu} - 8\Lambda e^{-2\kappa \sigma}\right] \Psi = 0.
\]

(2.24)

### 2.2 Supersymmetric Quantum Cosmology

In this subsection we summarize the procedure for attaining a supersymmetric extension of the models. In general, such an extension of the system is possible if a solution, \(I = I(q^a)\), can be found to the Euclidean Hamilton–Jacobi equation [24, 25, 17]:

\[
G^{ab}\frac{\partial I}{\partial q^a}\frac{\partial I}{\partial q^b} = W(q^a)
\]

(2.25)

A quantum Hamiltonian, \(\hat{H}\), may be defined by the conditions

\[
2\hat{H} = [Q, \bar{Q}]_+, \quad Q^2 = \bar{Q}^2 = 0
\]

(2.26)
and
\[ [\hat{H}, Q]_+ = [\hat{H}, \bar{Q}]_+ = 0, \quad (2.27) \]
where \( Q \) is a non–Hermitian supercharge and \( \bar{Q} \) is its adjoint. The functional forms of these supercharges are
\[ Q = \psi^a \left( \pi_a + i \frac{\partial I}{\partial q^a} \right), \quad (2.28) \]
and
\[ \bar{Q} = \bar{\psi}^a \left( \pi_a - i \frac{\partial I}{\partial q^a} \right), \quad (2.29) \]
respectively, where the corresponding fermionic (Grassmannian) variables are defined by
\[ \bar{\psi}^a = \theta^a, \quad \psi^b = G^{ab} \frac{\partial}{\partial \theta^a} \]
\[ \psi^a \psi^b + \psi^b \psi^a = 0, \quad \bar{\psi}^a \bar{\psi}^b + \bar{\psi}^b \bar{\psi}^a = 0. \quad (2.31) \]

Eqs. (2.26) and (2.27) represent the algebra for a \( N = 2 \) supersymmetry. For the three–dimensional minisuperspace that we are considering, the supersymmetric wavefunction can be expanded in terms of the Grassmann variables \( \theta^a \):
\[ \Psi = A_+ + B_0 \theta^a + \frac{1}{2} \epsilon_{abc} C^c \theta^a \theta^b + A_- \theta^0 \theta^1 \theta^2, \quad (2.32) \]
where the bosonic variables \( \{ A_+, B_0, C, A_- \} \) are functions of the minisuperspace variables, \( \epsilon_{abc} \) is totally antisymmetric on all its indices and \( \epsilon_{012} \equiv +1, \) etc. The supersymmetric wavefunction is then annihilated by the supercharges:
\[ Q \Psi = 0 \quad (2.33) \]
\[ \bar{Q} \Psi = 0 \quad (2.34) \]
and automatically satisfies the Hamiltonian constraint due to Eq. (2.26).

The Euclidean Hamilton–Jacobi equation for the LRS Bianchi models we are considering is given by
\[ -\left( \frac{\partial I}{\partial \sigma} \right)^2 + \left( \frac{\partial I}{\partial u} \right)^2 + \left( \frac{\partial I}{\partial v} \right)^2 = -8ke^{(C-2\kappa)\sigma} e^{-G_u} + 8\Lambda e^{-2\kappa\sigma}. \quad (2.35) \]

Thus, the problem of quantizing these models in a supersymmetric fashion involves finding a solution to Eq. (2.35) and then solving the simultaneous constraints (2.33) and (2.34) subject to the ansatz (2.32). Furthermore, since the models exhibit a classical, global symmetry, it is natural to consider those solutions to Eq. (2.26) that respect this symmetry.

In the following Section we employ this method to quantize the LRS Bianchi I cosmology.
3 Supersymmetric LRS Bianchi I Quantum Cosmology

The Euclidean Hamilton–Jacobi equation for the LRS Bianchi type I cosmology is
\[
- \left( \frac{\partial I}{\partial \sigma} \right)^2 + \left( \frac{\partial I}{\partial u} \right)^2 + \left( \frac{\partial I}{\partial v} \right)^2 = 8\Lambda e^{-2\kappa\sigma}.
\] (3.1)

A solution to Eq. (3.1) that respects the global symmetry (2.18) and discrete $Z_2$ symmetry (2.12) of the reduced action (2.8) is given by
\[
\Lambda < 0 : \quad I = \pm \frac{1}{\kappa} \left[ \sqrt{A^2 - 8\Lambda x^2} - \text{Acotanh}^{-1} \left( \frac{\sqrt{A^2 - 8\Lambda x^2}}{A} \right) \right] + A\sqrt{u^2 + v^2},
\]
(3.2)
\[
\Lambda > 0 : \quad I = \pm \frac{1}{\kappa} \left[ \sqrt{A^2 - 8\Lambda x^2} - \text{Atanh}^{-1} \left( \frac{\sqrt{A^2 - 8\Lambda x^2}}{A} \right) \right] + A\sqrt{u^2 + v^2},
\]
(3.3)

where $A$ is an arbitrary constant and $x \equiv e^{-\kappa\sigma}$. In the $\Lambda > 0$ case one also requires $\sigma \leq \ln(2\sqrt{2}\Lambda^{1/2}/A)$ if $A > 0$ and $\sigma \geq \ln(-2\sqrt{2}\Lambda^{1/2}/A)$ for $A < 0$.

Given the solution (3.2), we could in principle quantise the system in a manifestly supersymmetric fashion. For simplicity, however, we consider the case where $A = 0$. The Euclidean action (3.2) then simplifies to
\[
I = \mp \frac{2\sqrt{-2\Lambda}}{\kappa} e^{-\kappa\sigma}
\] (3.4)

and it follows immediately from Eq. (3.4) that we require $\Lambda \equiv -\lambda < 0$ for consistency. The supercharges (2.28) and (2.29) are then given by
\[
Q = i \frac{\partial}{\partial \theta^0} \frac{\partial}{\partial \sigma} - i \frac{\partial}{\partial \theta^1} \frac{\partial}{\partial u} - i \frac{\partial}{\partial \theta^2} \frac{\partial}{\partial v} \mp i 2\sqrt{2}\lambda e^{-\kappa\sigma} \frac{\partial}{\partial \theta^0},
\]
(3.5)
\[
\bar{Q} = -i \theta^0 \frac{\partial}{\partial \sigma} - i \theta^1 \frac{\partial}{\partial u} - i \theta^2 \frac{\partial}{\partial v} \mp i 2\sqrt{2}\lambda e^{-\kappa\sigma} \theta^0,
\]
(3.6)

respectively.

The constraint (2.34) yields the set of coupled, first–order partial differential equations
\[
- i \frac{\partial A_+}{\partial \sigma} \mp i 2\sqrt{2}\lambda e^{-\kappa\sigma} A_+ = 0,
\]
(3.7)
\[-i \frac{\partial A_+}{\partial u} = 0, \quad (3.8)\]
\[-i \frac{\partial A_+}{\partial v} = 0, \quad (3.9)\]
\[-i \frac{\partial B_1}{\partial \sigma} + i \frac{\partial B_0}{\partial u} \mp 2\sqrt{2} e^{-\kappa \sigma} B_1 = 0, \quad (3.10)\]
\[-i \frac{\partial B_2}{\partial \sigma} + i \frac{\partial B_0}{\partial v} \mp 2\sqrt{2} e^{-\kappa \sigma} B_2 = 0, \quad (3.11)\]
\[-i \frac{\partial B_2}{\partial u} + i \frac{\partial B_1}{\partial v} = 0, \quad (3.12)\]
\[-\frac{1}{2} i \frac{\partial C_0}{\partial \sigma} - \frac{1}{2} i \frac{\partial C^1}{\partial u} - i \frac{1}{2} \frac{\partial C^2}{\partial v} \mp i \sqrt{2} e^{-\kappa \sigma} C^0 = 0. \quad (3.13)\]

From the corresponding constraint (2.33), it follows that
\[i \frac{\partial A_-}{\partial \sigma} \mp i 2\sqrt{2} \lambda e^{-\kappa \sigma} A_- = 0, \quad (3.14)\]
\[i \frac{\partial A_+}{\partial u} = 0, \quad (3.15)\]
\[-i \frac{\partial A_-}{\partial v} = 0, \quad (3.16)\]
\[-\frac{1}{2} i \frac{\partial C^1}{\partial \sigma} - \frac{1}{2} i \frac{\partial C^0}{\partial u} \pm i \sqrt{2} \lambda e^{-\kappa \sigma} C^1 = 0, \quad (3.17)\]
\[i \frac{\partial C^2}{\partial \sigma} + i \frac{\partial C^0}{\partial v} \mp i 2\sqrt{2} \lambda e^{-\kappa \sigma} C^2 = 0, \quad (3.18)\]
\[i \frac{\partial C^2}{\partial u} - i \frac{\partial C^1}{\partial v} = 0, \quad (3.19)\]
\[i \frac{\partial B^0}{\partial \sigma} - i \frac{\partial B^1}{\partial u} - i \frac{\partial B^2}{\partial v} \mp i 2\sqrt{2} \lambda e^{-\kappa \sigma} B^0 = 0. \quad (3.20)\]

Eqs. (3.7), (3.8) and (3.9) immediately imply that
\[A_+ = A_+^0 e^f \quad (3.21)\]
with \(A_+^0\) is an arbitrary constant and
\[f \equiv \pm \frac{2\sqrt{2} \lambda}{\kappa} e^{-\kappa \sigma}. \quad (3.22)\]

Similarly, we deduce from Eqs. (3.14), (3.13) and (3.16) that
\[A_- = A_-^0 e^{-f}, \quad (3.23)\]
where \(A_-^0\) is a second, arbitrary constant.
To proceed in solving Eqs. (3.10), (3.11), (3.12) and (3.20), it proves convenient to redefine the functions $B_a$ as follows:

$$B_a = \hat{B}_a e^f, \quad a = (0,1,2).$$  \hspace{1cm} (3.24)

From the definition of $f(\sigma)$ given in Eq. (3.22), it follows from Eqs. (3.10), (3.11), (3.12), (3.20) and (3.24) that

$$\partial \hat{B}_1/\partial \sigma - \partial \hat{B}_0/\partial u = 0, \hspace{1cm} (3.25)$$

$$\partial \hat{B}_2/\partial \sigma - \partial \hat{B}_0/\partial v = 0, \hspace{1cm} (3.26)$$

$$\partial \hat{B}_2/\partial u - \partial \hat{B}_1/\partial v = 0, \hspace{1cm} (3.27)$$

$$\partial \hat{B}_0/\partial \sigma - \partial \hat{B}_1/\partial u - \partial \hat{B}_2/\partial v - 2\kappa f \hat{B}_0 = 0. \hspace{1cm} (3.28)$$

Similarly, by introducing the new set of variables $\hat{C}_b$ defined by

$$C_b = \hat{C}_b e^{-f}, \hspace{1cm} (3.29)$$

we derive a new set of equations that are equivalent to Eqs. (3.13), (3.17), (3.18) and (3.19):

$$\partial \hat{C}_1/\partial \sigma + \partial \hat{C}_0/\partial u = 0, \hspace{1cm} (3.30)$$

$$\partial \hat{C}_2/\partial \sigma + \partial \hat{C}_0/\partial v = 0, \hspace{1cm} (3.31)$$

$$\partial \hat{C}_2/\partial u - \partial \hat{C}_1/\partial v = 0, \hspace{1cm} (3.32)$$

$$\partial \hat{C}_0/\partial \sigma + \partial \hat{C}_1/\partial u + \partial \hat{C}_2/\partial v + 2\kappa f \hat{C}_0 = 0. \hspace{1cm} (3.33)$$

By manipulating Eqs. (3.25)–(3.28) we arrive at the following set of decoupled equations\footnote{For example, Eq. (3.34) is derived by applying the differential operator $\partial/\partial v$ on Eq. (3.28), then acting on Eq. (3.26) with $\partial/\partial \sigma$ and on Eq. (3.27) with $\partial/\partial u$. By employing Eq. (3.26), we then arrive at Eq. (3.34) above. A similar procedure leads to Eqs. (3.35) and (3.36).}:

$$\frac{\partial^2 \hat{B}_2}{\partial \sigma^2} - 2\kappa f \frac{\partial \hat{B}_2}{\partial \sigma} - \frac{\partial^2 \hat{B}_2}{\partial u^2} - \frac{\partial^2 \hat{B}_2}{\partial v^2} = 0, \hspace{1cm} (3.34)$$

$$\frac{\partial^2 \hat{B}_0}{\partial \sigma^2} - 2\kappa f \frac{\partial \hat{B}_0}{\partial \sigma} + 2\kappa^2 f \hat{B}_0 - \frac{\partial^2 \hat{B}_0}{\partial u^2} - \frac{\partial^2 \hat{B}_0}{\partial v^2} = 0, \hspace{1cm} (3.35)$$

$$\frac{\partial^2 \hat{B}_1}{\partial \sigma^2} - 2\kappa f \frac{\partial \hat{B}_1}{\partial \sigma} - \frac{\partial^2 \hat{B}_1}{\partial u^2} - \frac{\partial^2 \hat{B}_1}{\partial v^2} = 0. \hspace{1cm} (3.36)$$
Applying an equivalent technique to Eqs. (3.30)–(3.33) results in a set of decoupled equations for the amplitudes \( \hat{C}^c \):

\[
-\frac{\partial^2 \hat{C}^2}{\partial \sigma^2} - 2\kappa f \frac{\partial \hat{C}^2}{\partial u} + \frac{\partial^2 \hat{C}^2}{\partial v^2} = 0, \tag{3.37}
\]

\[
\frac{\partial^2 \hat{C}^0}{\partial \sigma^2} + 2\kappa f \frac{\partial \hat{C}^0}{\partial \sigma} - 2\kappa^2 f \hat{C}^0 - \frac{\partial^2 \hat{C}^0}{\partial u^2} - \frac{\partial^2 \hat{C}^0}{\partial v^2} = 0, \tag{3.38}
\]

\[
-\frac{\partial^2 \hat{C}^1}{\partial \sigma^2} - 2\kappa f \frac{\partial \hat{C}^1}{\partial \sigma} + \frac{\partial^2 \hat{C}^1}{\partial u^2} + \frac{\partial^2 \hat{C}^1}{\partial v^2} = 0. \tag{3.39}
\]

Eqs. (3.25)–(3.28) can be solved analytically if \( \hat{B}_{1,2} \) are independent of \( \sigma \). Eqs. (3.34) and (3.36) then imply that these variables satisfy the two–dimensional Laplace equation, subject to the integrability condition (3.27). Eqs. (3.25) and (3.26) further imply that \( \hat{B}_0 \) is independent of \( \{u, v\} \) and consistency between Eqs. (3.28) and (3.35) results in a further integrability constraint

\[
\frac{\partial \hat{B}_1}{\partial u} = -\frac{\partial \hat{B}_2}{\partial v}. \tag{3.40}
\]

The functional form of \( B_0 \) follows immediately up on integration of Eq. (3.28), \( B_0 = e^{-f} \). It is interesting that this is also the wavefunction (3.23) for the filled fermion sector. Similar conclusions follow for the functions \( \hat{C}^c \). If \( \hat{C}^{1,2} \) are independent of \( \sigma \), satisfy the two–dimensional Laplace equation, the integrability condition, \( \partial \hat{C}^1 / \partial u = -\partial \hat{C}^2 / \partial v \), and Eq. (3.32), then the function \( C^0 \) is given by the wavefunction (3.21) for the empty fermion sector, \( C^0 = e^f \).

Finally, it is interesting to compare the wavefunction (3.21) for the empty fermion sector with the general solution to the bosonic Wheeler–DeWitt equation (2.24). When the wavefunction depends only on the variable \( \sigma \), the general solution to Eq. (2.24) is given by

\[
\Psi = c_1 I_0(f) + c_2 K_0(f), \tag{3.41}
\]

where \( I_0 \) and \( K_0 \) are modified Bessel functions of the first and second kind with order zero, \( f \) is defined in Eq. (3.22) and \( c_i \) are arbitrary constants. In the large argument limit, the modified Bessel function of the first kind asymptotes to the form \( I_0 \propto f^{-1/2} \exp(f) \) and, consequently, there is a correlation, up to a negligible prefactor, with the fully bosonic component (2.21) of the supersymmetric wavefunction. Indeed, the solution \( \Lambda_+ = \exp(f) \) is an exact solution to the bosonic Wheeler–DeWitt equation if a suitable choice of factor ordering is made when identifying the momentum operator conjugate to the variable \( \sigma \). In general, the ambiguity in the factor ordering can be accounted for by identifying

\[
\pi^2 \propto -e^{p\sigma} \frac{\partial}{\partial \sigma} e^{p\sigma} \frac{\partial}{\partial \sigma} \tag{3.42}
\]

for some constant \( p \) in the classical Hamiltonian (2.21). In this case, the corresponding Wheeler–DeWitt equation is then solved by Eq. (3.21) for \( p = \kappa \).
4 Supersymmetric Kantowski–Sachs Quantum Cosmology

In this Section, we consider the supersymmetric quantization of the vacuum Kantowski–Sachs, Brans–Dicke cosmology where $\Lambda = 0$. The Wheeler–DeWitt and Euclidean Hamilton–Jacobi equations are given by

$$\left[-\frac{\partial^2}{\partial \sigma^2} + \frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2} + 8e^{A\sigma+Bu}\right] \Psi = 0 \quad (4.43)$$

and

$$\left(\frac{\partial I}{\partial \sigma}\right)^2 - \left(\frac{\partial I}{\partial u}\right)^2 - \left(\frac{\partial I}{\partial v}\right)^2 = 8e^{A\sigma+Bu}, \quad (4.44)$$

respectively, where $A \equiv C - 2\kappa$ and $B \equiv -G$.

We now assume the wavefunction does not depend on the variable $v$ and introduce ‘null’ variables over the reduced $(1 + 1)$–dimensional minisuperspace:

$$s \equiv \frac{8}{A^2 - B^2} \exp \left[\frac{1}{2}(A + B)(\sigma + u)\right]$$
$$\tau \equiv \exp \left[\frac{1}{2}(A - B)(\sigma - u)\right]. \quad (4.45)$$

The Wheeler–DeWitt equation (4.43) transforms into the unit–mass Klein–Gordon equation

$$\left[\frac{\partial^2}{\partial s\partial\tau} - 1\right] \Psi = 0 \quad (4.46)$$

and particular solutions to Eq. (4.46) are given by

$$\Psi_\mu = e^{-i\mu s + i\tau / \mu}, \quad (4.47)$$

where $\mu$ is an arbitrary, complex constant. If $\text{Im} \mu < 0$, the modulus of the wavefunction is square–integrable. The general solution to Eq. (4.46) may be expanded as a linear superposition of the family of solutions (4.47):

$$\Psi_{\text{gen}} = \int d^2 \mu \, F(\mu, \mu^*) \Psi_\mu \quad (4.48)$$

The function, $F$, represents a weighting function. If this is finite and only supported over a closed area of the $\text{Im} \mu < 0$ sector of the complex $\mu$–plane, Cauchy’s theorem implies that the integral in Eq. (4.48) may be reduced to a line integral over the real axis:

$$\Psi_{\text{gen}} = \int_{-\infty}^{+\infty} d\mu M(\mu) \Psi_\mu, \quad (4.49)$$

\footnotetext{We impose this restriction because it enables us to derive the 1– and 2–fermion states analytically. Solutions that depend on $v$ can also be considered, although in such cases it is not possible to proceed analytically.}
where $M(\mu)$ is an arbitrary function \[39\].

The Euclidean Hamilton–Jacobi equation (4.44) becomes

$$\frac{\partial I}{\partial s} \frac{\partial I}{\partial \tau} = 1$$

(4.50)

and admits the solutions

$$I = -bs - \frac{1}{b} \tau.$$  

(4.51)

where $b = i\mu$. Eq. (4.51) is invariant under the duality transformation (2.12). Moreover, we see the exact solution (4.47) to the Wheeler–DeWitt equation (4.43) is also a WKB solution, $\Psi = \exp(\pm I)$, to the Euclidean Hamilton–Jacobi equation (4.44).

In performing the supersymmetric quantization of this cosmology, it is convenient to diagonalise the minisuperspace metric. The reason is that the Grassmannian variables should satisfy the anticommuting relations $[\psi_a, \bar{\psi}^b]_+ = G^{ab}$. A non–diagonal minisuperspace metric would mean that fermionic states could not be clearly separated after the wavefunction has been annihilated by the supercharges. We therefore introduce the pair of variables

$$T \equiv \frac{1}{2}(s + \tau), \quad X \equiv \frac{1}{2}(s - \tau)$$

(4.52)

and this implies that the minisuperspace metric, defined in Eq. (2.22), has the non–trivial components $G^{00} = -G^{11} = -(A^2 - B^2)(T^2 - X^2)/2$ and $G^{22} = 1$.

The supercharge (2.28) and its Hermitian conjugate (2.29) are then given by

$$Q = -iG^{00} \frac{\partial}{\partial \theta^0} \frac{\partial}{\partial T} - iG^{11} \frac{\partial}{\partial \theta^1} \frac{\partial}{\partial X} - iG^{22} \frac{\partial}{\partial \theta^2} \frac{\partial}{\partial v} + iG^{00} \frac{\partial}{\partial \theta^0} \frac{\partial I}{\partial T} + iG^{11} \frac{\partial}{\partial \theta^1} \frac{\partial I}{\partial X}$$

(4.53)

and

$$\bar{Q} = -i\theta^0 \frac{\partial}{\partial T} - i\theta^1 \frac{\partial}{\partial X} - i\theta^2 \frac{\partial}{\partial v} - i\theta^0 \frac{\partial I}{\partial T} - i\theta^1 \frac{\partial I}{\partial X},$$

(4.54)

respectively, where

$$I = -\left(b + \frac{1}{b}\right) T - \left(b - \frac{1}{b}\right) X.$$  

(4.55)

For the supersymmetric wavefunction, we consider the ansatz

$$\Psi = \alpha_+ + \beta_b \theta^b + \frac{1}{2} \epsilon^{abc} \gamma^c \theta^a \theta^b + \alpha_- \theta^0 \theta^1 \theta^2,$$  

(4.56)

where $\{\alpha_+, \beta_b, \gamma^c\}$ are bosonic functions of the minisuperspace variables. The annihilation of the wavefunction (4.54) by the supercharges (4.53) and (4.54) yields the set of coupled, first–order partial differential equations

$$G^{00} \frac{\partial \beta_0}{\partial T} + G^{11} \frac{\partial \beta_1}{\partial X} + G^{22} \frac{\partial \beta_2}{\partial v} + G^{00} \left(b + \frac{1}{b}\right) \beta_0 - G^{11} \left(-b + \frac{1}{b}\right) \beta_1 = 0$$

(4.57)
The wavefunctions for the empty and filled fermion sectors are readily deduced:

\[ \alpha_\pm = e^{\mp I}, \]  

where \( I \) is given by Eq. (4.55). To solve the remaining equations, we assume that the amplitudes \( \{ \beta_0, \gamma^c\} \) are independent of the variable, \( v \). In this case, Eqs. (4.68) and (4.69) yield the general solution, \( \beta_2 = \exp(-I) \), modulo a constant of proportionality, where \( I \) is given by the Euclidean action (4.51). Likewise, Eqs. (4.58) and (4.59) imply that \( \gamma^2 = \exp(I) \).

The wavefunctions for the one–fermion sector are completely determined by solving Eqs. (4.57) and (4.67). To proceed, it is convenient to transform back to the null coordinate pair \( (s, \tau) \) defined in Eq. (4.45). In terms of these variables, we find that

\[ \frac{\partial (\beta_0 - \beta_1)}{\partial s} + \frac{1}{b} (\beta_0 + \beta_1) = 0 \]  
\[ \frac{\partial (\beta_0 + \beta_1)}{\partial \tau} + b (\beta_0 - \beta_1) = 0. \]
Defining $Y \equiv \beta_0 - \beta_1$ and $Z \equiv b^{-1}(\beta_0 + \beta_1)$ implies that Eqs. (4.72) and (4.73) may be expressed in the more compact form:

$$\frac{\partial Y}{\partial s} = -Z, \quad \frac{\partial Z}{\partial \tau} = -Y. \quad (4.74)$$

Differentiating the first constraint in Eq. (4.74) with respect to $\tau$ and substituting in the second condition implies that both $\beta_{0,1}$ satisfy the unit–mass Klein–Gordon equation, i.e., the bosonic Wheeler–DeWitt equation (4.46). Thus, although these amplitudes satisfy the same equation as the wavefunction that arises in the standard quantum cosmological approach, the supersymmetry imposes strong constraints, as summarized in Eq. (4.74), on the functional form of the solutions that can arise. One class of allowed solution is given by $Y = \exp(-I)$ and $Z = -bY$, where $I$ is given by Eq. (4.51).

It now only remains to solve Eqs. (4.60) and (4.70) in order to determine the two–fermion sector of the supersymmetric wavefunction. By combining and subtracting these two equations, we find that

$$\frac{\partial (\gamma^0 + \gamma^1)}{\partial s} - \frac{1}{b} (\gamma^0 - \gamma^1) = 0 \quad (4.75)$$

$$\frac{\partial (\gamma^0 - \gamma^1)}{\partial \tau} - b (\gamma^0 + \gamma^1) = 0. \quad (4.76)$$

Defining $R \equiv \gamma^0 + \gamma^1$ and $W \equiv b^{-1}(\gamma^0 - \gamma^1)$ implies that Eqs. (4.73) and (4.76) are equivalent to:

$$\frac{\partial R}{\partial s} = W; \quad \frac{\partial W}{\partial \tau} = R. \quad (4.77)$$

Thus, the amplitudes $\gamma^{0,1}$ also satisfy the unit–mass Klein–Gordon equation. We find that one class of solution consistent with Eq. (4.77) is given by $R = \exp(I)$ and $W = -bR$. To summarize, therefore, the supersymmetric wavefunction that we have found for the vacuum Brans–Dicke, Kantowski–Sachs cosmology is given by

$$\Psi = e^{-I} + \beta_0 \theta^0 + \beta_1 \theta^1 + e^{-I} \theta^2 + \gamma^0 \theta^1 \theta^2 - \gamma^1 \theta^0 \theta^2 + e^I \theta^0 \theta^1 + e^I \theta^0 \theta^1 \theta^2, \quad (4.78)$$

where $\beta_{0,1}$ and $\gamma^{0,1}$ satisfy the unit–mass, Klein–Gordon equation (4.46) subject to the integrability conditions (4.74) and (4.77).

5 Discussion

In this paper, we have considered an $N = 2$ supersymmetric quantization of the LRS Bianchi type I and Kantowski–Sachs, Brans-Dicke cosmologies. In the former case, we found that a supersymmetric quantization is possible if a negative cosmological constant is introduced into the gravitational sector of the theory. For the
Kantowski–Sachs universe, the existence of such a term is not necessary, because this model has positive spatial curvature. In both models, supersymmetric quantum states were found for a given solution to the Euclidean Hamilton–Jacobi equation. Furthermore, these wavefunctions respect a global scale factor duality symmetry of the respective classical Hamiltonians.

Having found particular solutions to the supersymmetric quantum constraints, the immediate question that arises is the nature of the boundary conditions that such solutions satisfy. In general, the supersymmetric Hamiltonian has a spin term with a coefficient determined by the solution to the Euclidean Hamilton–Jacobi equation. This term implies that it is very difficult to complete a supersymmetric extension of the system with complex or imaginary solutions [17]. Thus, the boundary conditions that are typically most relevant in this quantization scheme are those due to Hartle and Hawking [19] and to Hawking and Page [21, 22].

In particular, it is natural to consider whether the Kantowski–Sachs wavefunction derived above satisfies the Hawking–Page boundary conditions relevant to a wormhole configuration [21, 22]. Classically, a wormhole represents an instanton solution of the Euclidean field equations [11, 21]. At the quantum level, such a state may be interpreted as a solution to the Wheeler–DeWitt equation. The appropriate boundary conditions that must be satisfied are that the wavefunction should be regular, in the sense that it does not oscillate an infinite number of times, when the three–metric degenerates and that it should be exponentially damped when the three–geometry tends to infinity [21, 22].

The anisotropic geometry, $S^1 \times S^2$, of the Kantowski–Sachs model implies that there are different types of possible wormholes [11, 12]. These have been studied by Campbell and Garay within the context of Einstein gravity minimally coupled to a massless scalar field [12]. The geometry of the spacetime asymptotes to $\mathbb{R}^3 \times S^1$ if the radius of the circle, $\tilde{a}_1$, tends to a constant as the radius of the two–sphere diverges. Alternatively, if the volume of the two–sphere tends to a constant as $\tilde{a}_1 \to \infty$, the geometry is $\mathbb{R}^2 \times S^2$. The wavefunction representing the ground state of each of these wormholes is the path integral over all metrics that asymptotically have these geometries and over all matter configurations that vanish at infinity. For the $\mathbb{R}^3 \times S^1$ wormhole, the wavefunction is given by $\Psi \propto e^{-(4\tilde{a}_1\tilde{a}_2)}$ in the asymptotic limit. The corresponding limit for the $\mathbb{R}^2 \times S^2$ wormhole is $\Psi \propto e^{-\tilde{a}_1^2}$.

After transforming back to the original variables of Section II, we find that the bosonic component of the supersymmetric Kantowski–Sachs wavefunction (4.78) does not asymptote to either of these forms. Its interpretation as a quantum wormhole is therefore not clear. However, a further solution to the Euclidean Hamilton–Jacobi equation (4.44) that respects the scale factor duality (2.12) of the classical action is given by

$$I = \left(\frac{32}{A^2 - B^2}\right)^{1/2} e^{(A\sigma + Bu)/2}.$$  \hspace{1cm} (5.79)

Consequently, a supersymmetric quantization may be performed with this solution.
Due to the non-trivial functional form of Eq. (5.79), however, it has not been possible to find analytical solutions for the intermediate fermionic sectors. On the other hand, the empty fermion sector is given by $\Psi \propto e^{-I}$ and it is of interest to compare this wavefunction with the above ground state wormhole wavefunctions. For example, in the superstring inspired model, where $\omega = -1$, we find that $I = 4a_1a_2e^{-\Phi}$. Performing a conformal transformation on the four-metric, $\bar{g}_{\mu\nu} = \Theta^2 g_{\mu\nu}$, where $\Theta^2 \equiv e^{-\Phi}$, implies that the dilaton field is minimally coupled to gravity in the ‘Einstein-frame’, $\bar{g}_{\mu\nu}$. In terms of variables defined in this frame, the wavefunction is given by $\Psi \propto \exp(-4a_1\bar{a}_2)$ and this is precisely the wavefunction for the $R^3 \times S^1$ quantum wormhole that arises in the standard Wheeler–DeWitt quantization.

This is important because the ground state of the $R^3 \times S^1$ quantum wormhole has been selected by the supersymmetric quantization procedure. We emphasize that the bosonic component of the supersymmetric wavefunction is unique once a solution to the Euclidean Hamilton-Jacobi equation has been specified. In this sense, therefore, any ambiguities that arise in the operator ordering are eliminated.

Moreover, the interior of a Euclidean Schwarzschild black hole has the form of a Kantowski–Sachs metric [11] and it is possible, therefore, that supersymmetric quantum cosmology may relate a black hole interior to a quantum wormhole. It would be interesting to consider this possibility further. For example, such a relationship would have implications for the graceful exit problem of the pre–big bang inflationary scenario [32]. This problem arises because the classical, dilaton–driven inflationary solution becomes singular in a finite proper time. At present, no generally accepted mechanism has been proposed to avoid such a singularity and ensure a smooth transition to the standard, post–big bang expansion. However, an epoch of pre–big bang inflation may be formally interpreted in the Einstein–frame in terms of gravitational collapse [13]. If the final state of such a collapse were a non–singular supersymmetric wormhole configuration, such a problem could in principle be avoided. It is intriguing that whereas the pre– and post–big bang branches are related to one another by the scale factor duality of the classical action, the empty fermion component of the wavefunction is invariant under such a duality transformation.

In principle, the supersymmetric quantization of other homogeneous, scalar–tensor cosmologies can also be considered following the method outlined in this paper. The Bianchi type II, VIo and VIIo cosmologies also exhibit global symmetries at the classical level [36] and, in particular, the Wheeler–DeWitt equation for the Bianchi type II model reduces to Eq. (4.46) after appropriate field redefinitions [45]. Thus, a similar analysis to that presented in Section IV may also be performed for this model. Similarly, the effective potential arising in the Wheeler–DeWitt equation of the LRS type III model has an opposite sign to that given in Eq. (4.43). However, the Wheeler–DeWitt equation can be transformed into the unit–mass Klein–Gordon equation (4.46) after a suitable choice of null variables.

Finally, there remains the open question of the possible relationship between the different approaches to supersymmetric quantum cosmology. As we discussed in the
introduction, a supersymmetric minisuperspace may be obtained directly from a full four-dimensional $N = 1$ supergravity action with the assistance of a suitable dimensional reduction for both the bosonic and fermionic variables (see, e.g., Refs. [9, 10, 16]). Alternatively, a bosonic minisuperspace may be extracted from a $(1 + 0)$–dimensional lagrangian and a supersymmetric extension established along the lines of Refs. [24, 17, 18] or [27, 28]. Determining the fundamental similarities and differences between these two methods is a challenging problem. The one attempt to investigate this was made in Ref. [44], but unfortunately it was based on an incomplete ansatz for the supersymmetric Bianchi type IX model. This particular problem of the ansatz was eventually corrected [9, 10, 16] but no further studies have been made. Such a complex investigation is beyond the objectives and scope of this paper, but it would be interesting to consider this topic further.

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