River mouth areas hydrodynamics mathematical modeling

A I Sukhinov¹, I Yu Kuznetsova², A E Chistyakov¹ and V N Litvinov¹,³

¹ Department of Mathematics and Informatics, Don State Technical University, 1, Gagarin Square, Rostov-on-Don, 344000, Russia
² Department of Intelligent and Multiprocessor Systems, Southern Federal University, 105/42, Bolshaya Sadovaya st., Rostov-on-Don, 344006, Russia
³ Department of Mathematics and Bioinformatics, Azov-Black Sea Engineering Institute of Don State Agrarian University, 21, Lenina st., Zernograd, 347740, Russia

E-mail: sukhinov@gmail.com, ikuznecova@sfedu.ru, cheese_05@mail.ru, litvinovVN@rambler.ru

Abstract. The article is devoted to the mathematical modeling results description of hydrodynamic processes in seas mouth areas. 3D hydrodynamics mathematical model description is given, which is used to calculate the 3D velocity vector fields of the aquatic environment movement in mouth areas. The approximation of the differential equations of the mathematical model is based on the balance method taking into account the fullness of the control areas. The use of this combination of methods made it possible to more accurately description the complex geometry of the coastline and seabed. Due to this, it was possible to reduce the calculation error at the boundaries of the computational domain and to increase the calculation accuracy. A software package for modeling hydrodynamic processes in mouth areas has been developed in C++. The description of the software package and the results of its operation are given.

1. Introduction

One of the most important factors in ensuring the safety of navigation is predictive modeling of the occurrence of man-made disasters and their consequences in coastal systems. The solution to this problem requires the provision of the possibility of prompt and accurate forecasting of siltation of shipping routes. Despite the constant scientific work of scientific schools and interested organizations, catastrophic phenomena of both natural and technogenic character periodically occur. For example, catastrophic storm in the Kerch Strait in November 2007 led to more than 20 ships wreck. Remains of pollution caused by oil spills on the coast of the Black and Azov Seas were found until 2011. At the same time, the area of contamination exceeded 200 km. In September 2014, a storm surge caused flooding of the coastal areas of the Azov Sea. The water level in the area of the port of Taganrog (Rostov Oblast) rose by more than 4 meters, which caused significant damage to the region's economy. In 2019 and 2020, in connection with low rainfall and strong moss, driving water away from the coast, in the Taganrog Bay and the basins of the rivers flowing into the Azov Sea, there was a restriction of navigation and grounding of tankers. The movement of bottom sediments from the mouth areas of the Don River to the west, observed over the past
decades, leads to an intensive bloom of the Taganrog Bay waters and the displacement of traditional species of flora and fauna. In the river systems of the South of Russia in the last decade, the frequency of occurrence of unfavorable and catastrophic events has increased, which include river floods associated with abnormal precipitation, which lead to human casualties and significant material losses, a drop in the water level, leading to the impossibility of navigation even to large rivers of the South of Russia, salinization of waters in the mouth area of the river, used after cleaning for the needs of the population, as a result of mixing river and sea waters, etc.

As a result of the analysis of the existing hydrophysical models, such shortcomings as the lack of taking into account the complex geometry of the coastline and the bottom surface of the reservoir, evaporation, river runoff, and surge phenomena were revealed. Wind stresses, bottom friction, turbulent exchange and Coriolis force are often ignored. Some of the currently developed 3D hydrophysics models are implemented in software packages, for example, MARS 3D, POM (Princeton Ocean Models), CHTDM (Climatic Hydro Termo Dynamic Model), NEMO (Nucleus for European Modeling of the Ocean) [1]. Calculation of the current fields in shallow water in the case of a comparable water body depth with the wavelengths of surface waves requires taking into account the corresponding specifics and cannot be based on the use of oceanic models. In oceanographic models with sufficient accuracy of calculations at depths of more than 50 m, a hydrostatic approximation is usually used, which does not allow taking into account the significant effect of the bottom topography on wave motion, due to which turbulent exchange in the vertical direction occurs [2–4]. The sigma coordinate system used in oceanographic models does not allow to correctly describe the acceleration of the vertical movement of the water flow at significant depth differences in shallow water. Also, the process of drainage-flooding of coastal areas during surge phenomena, characteristic of shallow water bodies, is described incorrectly. The use of grids with a variable number of levels of the sigma coordinate system partially solves this problem, but significantly increases the complexity of calculations and the requirements for hardware resources of computing systems. Uniform rectangular grids are applicable and convenient for studying the motion of microparticles in the hydrosphere. It is known that their use can lead to the appearance of a solution error at the boundary of the computational domain. This disadvantage can be eliminated by using the method of partial filling of the calculation cells and the corresponding calculation algorithm. This will make it possible to more accurately simulate the behavior of the aquatic environment at the boundaries of the computational domain, presented in a stepwise form [5, 6].

The paper describes 3D model hydrophysical processes in coastal systems and mouth areas, taking into account friction on the bottom, complex topography of the bottom and coastline, nonlinear nature of microturbulent exchange in the vertical direction. An approach to the approximation of the model under consideration is described, taking into account the occupancy factors of the control areas. The proposed model formed the basis of the developed software package, which makes it possible to more accurately describe the hydrodynamic processes in mouth areas. The results of the software package operation are presented.

2. Problem setup
The developed model for calculating 3D velocity vector of the aquatic environment movement based on is hydrodynamics mathematical model of shallow water bodies [7]:

the equation of motion in three coordinate directions (system of Navier-Stokes equations)

\[
\begin{align*}
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} + v \frac{\partial u_i}{\partial y} + w \frac{\partial u_i}{\partial z} &= -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \left( \frac{\partial (\mu \nu_i)}{\partial x_j} \right)_{x_j} + \left( \frac{\partial (\mu \nu_j)}{\partial x_i} \right)_{x_j} + \left( \frac{\partial (\nu \nu_i)}{\partial z} \right)_{z_j},
\end{align*}
\]  
(1)
\[ v'_i + u v'_i + v v'_i + w v'_i = -\frac{1}{\rho} P'_i + \left( \mu v'_i \right)_i + \left( \mu v'_i \right)_i, \]
\[ w'_i + u w'_i + v w'_i + w w'_i = -\frac{1}{\rho} P'_i + \left( \mu w'_i \right)_i + \left( \mu w'_i \right)_i + g, \]

continuity equation (mass conservation law)
\[ \rho_i' + (\rho u)_i' + (\rho v)_i' + (\rho w)_i' = 0, \]

where \( \mathbf{V} = \{u, v, w\} \) is the water flow of shallow water body velocity vector; \( P \) is the hydrodynamic pressure; \( \rho \) is the aquatic environment density; \( \mu, \nu \) are turbulent exchange coefficients in the horizontal and vertical directions; \( t; g \) is the gravity acceleration.

The system of equations for the movement of the aquatic environment in mouth areas (1) - (4) is considered under the following initial condition
\[ \mathbf{V} = \mathbf{V}_0 \]
and boundary conditions:
– entrance (incoming streams from the sea and in the riverbed)
\[ \mathbf{V} = \mathbf{V}_0, P'_n = 0, \]
– lateral and lower water-bottom boundary
\[ \rho \mu \left( \mathbf{V}_n \right)'_n = -\mathbf{\tau}, \mathbf{V}_n = 0, P'_n = 0, \]
– lateral “water-water” boundary
\[ \left( \mathbf{V}_e \right)'_n = 0, \mathbf{V}_n' = 0, P'_n = 0, \]
– water surface (“water-air” boundary)
\[ \rho \mu \left( \mathbf{V}_e \right)'_n = -\mathbf{\tau}, w = -P'_n/\rho g, P'_n = 0, \]

where \( \mathbf{V}_n, \mathbf{V}_e \) are the velocity vector normal and tangential component; \( \mathbf{n} \) is the normal vector; \( \mathbf{\tau} = \{\tau_x, \tau_y, \tau_z\} \) is the tangential stress vector; \( \rho_s \) is suspension density.

On the free surface of a water body, the tangential stress is calculated as follows
\[ \mathbf{\tau} = \rho_s C_d |\mathbf{w}| \mathbf{w}, \]

where \( \mathbf{w} \) is the wind velocity relative to water; \( \rho_s \) is the atmosphere density; \( C_d = 0.0026 \) is the dimensionless surface resistance coefficient, which depends on wind speed [8].

At the water body’s bottom, the tangential stress has the form
\[ \mathbf{\tau} = \rho C_d h |\mathbf{V}| \mathbf{V}, \]

where \( C_d = g k^2 / h^{1/3}, k = 0.025 \) is the group roughness coefficient in Manning's formula; \( h \) is the distance from free surface to bottom.
3D computational grid is introduced

\[
\vec{w}_h = \{x^* = n\tau, x_1 = ih, y = jh, z_1 = kh; n = 0..N, i = 0..N, j = 0..N, k = 0..N; N, \tau = T, N, h = l, N, h = l, N, h = l,\}
\]

where \(\tau\) is the time step, \(h, h, h, h, h, h\) are space steps, \(N, \) is the time layers number, \(T\) is the upper bound on the time coordinate, \(N, N, N, N, \) are the nodes number by spatial coordinates, \(l, l, l, l, \) are space boundaries of a rectangular parallelepiped in which the computational domain is inscribed.

To discretize the model (1) - (10), we apply the pressure correction method, according to which the solution process is divided into three problems [9, 10]. The first task is to calculate the field of the fluid velocity vector without taking into account pressure on the basis of the system of diffusion - convection equations:

\[
\begin{align*}
\frac{\vec{u} - u}{\tau} + u\vec{u}_i + v\vec{v}_i + w\vec{w}_i &= \left(\mu\vec{u}_i\right)_x + \left(\mu\vec{v}_i\right)_y + \left(\mu\vec{w}_i\right)_z, \\
\frac{\vec{v} - v}{\tau} + u\vec{v}_i + v\vec{v}_i + w\vec{v}_i &= \left(\mu\vec{v}_i\right)_x + \left(\mu\vec{v}_i\right)_y + \left(\mu\vec{v}_i\right)_z, \\
\frac{\vec{w} - w}{\tau} + u\vec{w}_i + v\vec{w}_i + w\vec{w}_i &= \left(\mu\vec{w}_i\right)_x + \left(\mu\vec{w}_i\right)_y + \left(\mu\vec{w}_i\right)_z + g\left(\frac{\rho_0}{\rho} - 1\right)
\end{align*}
\]

where \(\{u, v, w\}\) is the velocity vector at the previous time level; \(\{\vec{u}, \vec{v}, \vec{w}\}\) is the velocity vector on the intermediate layer; \(\vec{u} = \sigma\vec{u} + (1 - \sigma)u, \sigma \in [0, 1]\) is the scheme weight.

The second task is based on the Poisson equation and allows to calculate the pressure values at each node of the computational domain

\[
P_{xx}^* + P_{yy}^* + P_{zz}^* = \frac{\rho - \rho}{\tau} + \left(\frac{\hat{\rho} \vec{u}}{\tau}\right)_x + \left(\frac{\hat{\rho} \vec{v}}{\tau}\right)_y + \left(\frac{\hat{\rho} \vec{w}}{\tau}\right)_z.
\]

Further, the distribution of water flow velocities is refined in the process of solving the third problem, built on the basis of an explicit scheme

\[
\begin{align*}
\frac{u - \vec{u}}{\tau} &= -\frac{1}{\rho} P'_{x}, \\
\frac{v - \vec{v}}{\tau} &= -\frac{1}{\rho} P'_{y}, \\
\frac{w - \vec{w}}{\tau} &= -\frac{1}{\rho} P'_{z},
\end{align*}
\]

where \(\{\vec{u}, \vec{v}, \vec{w}\}\) is the velocity vector at the current time level.

The degree of filling \(o_{i,j,k}\) the cell \((i, j, k)\) is calculated through the pressure of the liquid column at the bottom of the cell [5]:

\[
o_{i,j,k} = \frac{P_{i,j,k} + P_{i+1,j,k} + P_{i,j+1,k} + P_{i,j+1,k}}{4\rho gh_z}.
\]
3. Discrete model

To construct a discrete model of the control area occupancy function \( q_m, m = 0, 6 \), which describe the ratio of the volumes of the filled part of the cell \( \Omega_m \) to the overall \( \Omega_D \) and is equal \( (q_m)_{i,j,k} = S_{\Omega_m} / S_{\Omega_D} \). The control area \( D_0 \) is the parallelepiped: \( x_i \leq x \leq x_{i+1}, y_j \leq y \leq y_{j+1}, z_k \leq z \leq z_{k+1} \). The rest of the control areas \( D_m, m = 1, 6 \) are half of the area \( D_0 \). For areas \( D_1 \) and \( D_2 \) the variable \( x \) takes on the values \( x_i \leq x \) and \( x \leq x_i \), respectively. Areas \( D_3 \) and \( D_4 \) are separated by a plane \( y = y_j \), and areas \( D_5 \) and \( D_6 \) a plane \( z = z_k \) [5].

An approximation of the problem of calculating the velocity field of the movement of an aqueous medium at an intermediate time layer (11) is written in the form:

\[
(q_0)_{i,j,k} \frac{v_{i,j,k} - v_{i,j,k-1}}{\tau} + (q_1)_{i,j,k} \frac{u_{i+1,j,k} - u_{i,j,k}}{2h_x} + (q_2)_{i,j,k} \frac{v_{i+1,j,k} - v_{i,j,k}}{2h_y} + (q_3)_{i,j,k} \frac{u_{i,j,k+1} - u_{i,j,k}}{2h_z} \\
\times v_{i,j,k} + (q_4)_{i,j,k} \frac{u_{i,j,k+1} - u_{i,j,k}}{2h_z} + (q_5)_{i,j,k} \frac{v_{i,j,k+1} - v_{i,j,k}}{2h_z} + (q_6)_{i,j,k} \frac{u_{i,j,k+1} - u_{i,j,k}}{2h_z}
\]

\[
+ (q_0)_{i,j,k} \frac{v_{i,j,k+\frac{1}{2}} - v_{i,j,k}}{h_y} + (q_1)_{i,j,k} \frac{u_{i+\frac{1}{2},j,k} - u_{i,j,k}}{h_x} + (q_2)_{i,j,k} \frac{v_{i,j,k+\frac{1}{2}} - v_{i,j,k}}{h_y} + (q_3)_{i,j,k} \frac{u_{i,j,k+\frac{1}{2}} - u_{i,j,k}}{h_x}
\]

\[
+ (q_4)_{i,j,k} \frac{v_{i,j,k+\frac{1}{2}} - v_{i,j,k}}{h_y} + (q_5)_{i,j,k} \frac{u_{i,j,k+\frac{1}{2}} - u_{i,j,k}}{h_x} + (q_6)_{i,j,k} \frac{v_{i,j,k+\frac{1}{2}} - v_{i,j,k}}{h_y}
\]

\[
\times \rho \left[ (\vec{V} - \vec{V}) \right] \frac{(u_{a} - u)}{\rho h_x} - \left[ (\vec{V} - \vec{V}) \right] \frac{u|\vec{V}|}{\rho h_x}
\]

where \( \vec{V} = \{u_a, v_a\} \) is the wind speed, \( H(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases} \) is the Heaviside function.

Approximations of equations (12) and (13) can be written in a similar way. The approximation of the equation for calculating pressure (14) is written in the following form:

\[
(q_1)_{i,j,k} \frac{\hat{P}_{i+1,j,k} - \hat{P}_{i,j,k}}{h_x^2} - (q_2)_{i,j,k} \frac{\hat{P}_{i,j,k} - \hat{P}_{i-1,j,k}}{h_x^2} + (q_3)_{i,j,k} \frac{\hat{P}_{i,j,k+1} - \hat{P}_{i,j,k}}{h_z^2}
\]

\[
- (q_4)_{i,j,k} \frac{\hat{P}_{i,j,k} - \hat{P}_{i,j,k-1}}{h_z^2} + (q_5)_{i,j,k} \frac{\hat{P}_{i,j,k+1} - \hat{P}_{i,j,k}}{h_z^2} - (q_6)_{i,j,k} \frac{\hat{P}_{i,j,k} - \hat{P}_{i,j,k-1}}{h_z^2}
\]
necessary to solve the operator equation

\begin{align*}
+ \left( q_1 \right)_{i,j,k} - \left( q_0 \right)_{i,j,k} \right) H \left( \left( q_1 \right)_{i,j,k} - \left( q_0 \right)_{i,j,k} \right) \left( \frac{\hat{p}_{i,j,k} - \hat{p}_{i-1,j,k}}{\tau^2 h_y} + \frac{\omega \hat{p}_{i,j,k}}{\tau h_z} \right)
= \left( q_0 \right)_{i,j,k} - \rho_{i,j,k} + \frac{\left( q_1 \right)_{i,j,k} \left( \hat{p}_{i,j,k} - \hat{p}_{i-1,j,k} \right) \left( \hat{p}_{i,j,k} - \hat{p}_{i-1,j,k} \right)}{\tau^2 h_y} + \frac{\left( q_1 \right)_{i,j,k} \left( \hat{p}_{i,j,k} - \hat{p}_{i-1,j,k} \right) \left( \hat{p}_{i,j,k} - \hat{p}_{i-1,j,k} \right)}{\tau h_z}
+ \left( q_2 \right)_{i,j,k} - \left( q_3 \right)_{i,j,k} \left( \frac{\hat{p}_{i,j,k} - \hat{p}_{i-1,j,k}}{\tau^2 h_y} + \frac{\left( q_1 \right)_{i,j,k} \left( \hat{p}_{i,j,k} - \hat{p}_{i-1,j,k} \right) \left( \hat{p}_{i,j,k} - \hat{p}_{i-1,j,k} \right)}{\tau h_y} \right) m_{i,j,k},
\end{align*}

where \( \left( \hat{p}_{i,j,k} \right)_{i+1/2,j,k} = \left( \hat{p}_{i+1,j,k} \hat{u}_{i+1,j,k} + \hat{p}_{i,j,k} \hat{u}_{i,j,k} \right)/2 \), \( m_{i,j,k} \) is the boundary condition mask. If the node \((i, j, k)\) belongs to the side or upper boundary, then \( m_{i,j,k} = 0 \), otherwise (at the entrance or exit) \(- m_{i,j,k} = 1 \).

A discrete analog of the system of equations (15) takes the following form:

\begin{align*}
\left( q_0 \right)_{i,j,k} \frac{\hat{u}_{i,j,k} - \hat{u}_{i,j,k}}{\tau} = \left( q_1 \right)_{i,j,k} \frac{\hat{p}_{i,j,k} - \hat{p}_{i-1,j,k}}{2 h_y \rho_{i,j,k}} + \left( q_2 \right)_{i,j,k} \frac{\hat{p}_{i,j,k} - \hat{p}_{i-1,j,k}}{2 h_y \rho_{i,j,k}},
\left( q_0 \right)_{i,j,k} \frac{\hat{v}_{i,j,k} - \hat{v}_{i,j,k}}{\tau} = \left( q_1 \right)_{i,j,k} \frac{\hat{p}_{i,j,k} - \hat{p}_{i-1,j,k}}{2 h_y \rho_{i,j,k}} + \left( q_2 \right)_{i,j,k} \frac{\hat{p}_{i,j,k} - \hat{p}_{i-1,j,k}}{2 h_y \rho_{i,j,k}},
\left( q_0 \right)_{i,j,k} \frac{\hat{w}_{i,j,k} - \hat{w}_{i,j,k}}{\tau} = \left( q_1 \right)_{i,j,k} \frac{\hat{p}_{i,j,k} - \hat{p}_{i-1,j,k}}{2 h_y \rho_{i,j,k}} + \left( q_2 \right)_{i,j,k} \frac{\hat{p}_{i,j,k} - \hat{p}_{i-1,j,k}}{2 h_y \rho_{i,j,k}}.
\end{align*}

4. Materials and methods

Let \( A \) be a linear, positive definite operator \((A > 0)\) and in a finite-dimensional Hilbert space \( H \) it is necessary to solve the operator equation

\[ Ax = f, A : H \rightarrow H. \]

(17)

For the grid equation (17), iterative methods are used, which in canonical form can be represented by the equation [10]

\[ B^{\tau_{m+1} - \tau_{m}} + Ax = f, B : H \rightarrow H. \]

(18)

where \( m \) is the iteration number, \( \tau_{m+1} > 0 \) is the iteration parameter, \( B \) is the preconditioner. Operator \( B \) is constructed proceeding from the additive representation of the operator \( A_0 \) – the symmetric part of the operator \( A \)
The preconditioner is formed as follows

\[ B = (D + \omega R_1)D^{-1}(D + \omega R_2), \quad D = D^T > 0, \quad \omega > 0, \]

where \( D \) is the diagonal operator, \( R_1, R_2 \) are the lower- and upper-triangular operators respectively.

The algorithm for calculating the grid equations by the modified alternating-triangular method of the variational type is written in the form:

\[ r^m = Ax^m - f, \quad B(\omega_m)w^m = r^m, \quad \tilde{\omega}_m = \sqrt{\frac{(Dw^m, w^m)}{(D^{-1}R_1w^m, R_2w^m)}}, \]

\[ s_m = \frac{(A_0w^m, w^m)}{(B^{-1}A_0w^m, A_0w^m)} - \frac{(B^{-1}A_0w^m, A_0w^m)}{(B^{-1}A_0w^m, A_0w^m)}, \quad k_m = \frac{(B^{-1}A_0w^m, A_0w^m)}{(B^{-1}A_0w^m, A_0w^m)}, \]

\[ 1 - \sqrt{\frac{s_m^2k_m^2}{(1+k_m^2)(1-s_m^2)}}, \quad \tau_m = \theta_m \frac{(A_0w^m, w^m)}{(B^{-1}A_0w^m, A_0w^m)}, \quad \omega_m = \frac{x^m - \tau_m w^m, \omega_{m+1} = \tilde{\omega}_m}, \]

where \( r^m \) is the residual vector, \( w^m \) is the correction vector, the parameter \( s_m \) describes the rate of convergence of the method, \( k_m \) describes the ratio of the norm of the skew-symmetric part of the operator to the norm of the symmetric part.

The convergence rate of the method is:

\[ \rho \leq \frac{\nu' - 1}{\nu' + 1}, \]

where \( \nu' = \nu\left(\sqrt{1 + k^2} + k\right)^2 \), where \( \nu \) is the condition number of the matrix \( C_0 \), \( C_0 = B^{-1/2}A_0B^{-1/2} \).

The value \( \omega \) is optimal for

\[ \omega = \sqrt{\frac{(Dw^m, w^m)}{(D^{-1}R_1w^m, R_2w^m)}} \]

and the condition number of the matrix \( C_0 \) is estimated:

\[ \nu = \max_{y \neq 0} \left\{ \frac{1}{2} \left[ 1 + \sqrt{\frac{Dy, y}{(A_0y, y)}} \right] \right\} \leq \frac{1}{2} \left( 1 + \sqrt{\frac{\Delta}{\delta}} \right) = \frac{1 + \sqrt{\xi}}{2\sqrt{\xi}}, \]

where \( \xi = \frac{\delta}{\Delta}, \quad D \leq \frac{1}{\delta} A_0, \quad R_1D^{-1}R_2 \leq \frac{\Delta}{4} A_0 \).
5. Results and discussion
The described mathematical model of hydrodynamics formed the basis for a program complex (PC) developed in the C++ programming language [11], designed to calculate three-dimensional fields of the velocity vector of movement of an aqueous medium. Distinctive features of the PS are taking into account the complex geometry of the bottom and coastline, wind currents, friction against the bottom and turbulent exchange. The simulation of the movement of the calculation of the movement of the aquatic environment in the mouth areas was carried out in the area with linear dimensions of 50 m, 50 m horizontally and 4 m vertically. The lower boundary of the computational domain is at a depth of 2 m. The upper boundary rises 2 m above sea level. Computational grid parameters: horizontal steps - 0.5 m, vertical step - 0.1 m, time step - 0.25 s. Figure 1 shows the geometry of the computational domain.

The presented model of the hydrodynamics of shallow water bodies makes it possible to obtain three-dimensional fields of the vector of water flow velocities and pressure. At the same time, for shallow water bodies, to which in most cases river channels can be attributed, the geometry of the computational domain has a great influence on the flow fields; therefore, the use of the method of partial filling of the computational cells made it possible to take into account the complex geometry of the coastline and thereby increase the modeling accuracy.

![Figure 1](image)

**Figure 1.** Depth map of the computational domain.

The results of the work of the software complex when solving the problem of modeling the process of river runoff are presented. The developed software package can be used to predict overshoot phenomena in river deltas (figure 2). The speed of the water flow in the river bed is 0.2 m / s.
Figures 3, 4 show the results of sea tides numerical modeling in the mouth area of the river, carried out using the developed software package. The speed of the water flow oncoming from the sea is 0.4 m / s. The images were generated for four values of the model time, respectively equal to 10 and 20 seconds. Figure 3 shows the dynamics of the water rise level at the river mouth at the indicated time intervals.
Figure 4 shows the change in the water flow velocity values in the section along the center of the computational domain (XOZ section). Zero water level is indicated by a horizontal line. The effect of flooding is observed in the mouth area, caused by twice the speed of the incoming water flow from the sea, which indicates the qualitative adequacy of the proposed model.

![Figure 4. The water flow speed (m/s) in the center of the computational domain after 10 and 20 s.](image)

In 2017, members of the team of authors took part in an expeditionary study conducted in the water area of the Azov Sea [12, 13]. An array of field data was collected using hydrophysical probes ADCP, Sea Bird Electronics 19 Plus and other equipment. The use of algorithms for preliminary processing of experimental data, including those based on the Kalman filter, made it possible to eliminate the noisiness of expeditionary measurements of the water flow velocity vector and prepare them for further use in the processes of parameterization and verification of the mathematical model. Comparison of the values of the hydrophysical parameters obtained as a result of modeling with the data of field experiments showed their relative deviation in the range from 15 to 20%. The best agreement with the field data of the results obtained on the basis of the Smagorinsky subgrid turbulence model is revealed. Moreover, a further increase in the accuracy of the model is hampered by significant deviations of the experimental data from the mean.

The developed complex of interrelated algorithms and programs for solving spatial-three-dimensional non-stationary model problems of hydrophysics of channel systems, taking into account a number of major important factors affecting their course, can be used to assess hydrophysical changes in the geosystem monitoring of the ecological state of channel systems, to form scenarios for salinization of mouth areas, rivers, water purification, in the development of methods to eliminate possible negative consequences in case of emergency water pollution. The practical significance of numerical hydrodynamic algorithms and the complex of programs that implement them lies in the possibility of their application in the study of hydrophysical processes in channel water systems, as well as for constructing the field of velocities, pressure of the water medium and the function of elevation of the level, assessing the hydrodynamic impact on coastal protection structures and at the bottom and along the banks of rivers, as well as the calculation of the dynamics of flooding of coastal areas. The application of the research results is associated with the planning of rational use of natural resources: the construction of structures and the use of a specific section of the river to predict possible negative consequences.
6. Conclusion
The paper presents 3D model of the hydrodynamics of mouth areas, which makes it possible to describe with high accuracy the processes of waters mixing of interconnected reservoirs, as well as predict the penetration of sea waters into river floodplains. The use of the balance method taking into account the fullness of the calculated cells when approximating a continuous hydrodynamic model in spatial variables made it possible to take into account the complex geometry of the coastline and thereby increase the modeling accuracy. The resulting grid equations formed the basis of the developed software package, which includes the functions of calculating the water flow velocity field without taking into account pressure, hydrostatic and hydrodynamic pressure. On the example of modeling the dynamics of changes in the water flow currents in the coastal system mouth area, the results of its work are illustrated.

Acknowledgments
The study was carried out with the financial support of the Russian Foundation for Basic Research within the framework of scientific project № 19-07-00623.

References
[1] Bonaduce A, Staneva J, Grayek S, Bidlot J.-R. and Breivik O 2020 Sea-state contributions to sea-level variability in the European Seas Ocean Dynamics 70 pp 1547–1569
[2] Oliger J and Sundstorn A 1978 Theoretical and practical aspects of some initial boundary-value problems in fluid dynamica SIAM Journal on Applied Mathematics 35 pp 419–445
[3] Marchesiello P, Mc.Williams J C and Shchepetkin A 2001 Open boundary conditions for long-term integration of regional oceanic models Oceanic Modelling Journal 3 pp 1–20
[4] Androsov A A 2005 Straits of the World Ocean. General approach to modeling (St. Petersburg, Nauka) p 171 (in Russian)
[5] Sukhinov A I, Chistyakov A E, Sidoryakina V V and Protsenko E A 2019 Explicit-Implicit Schemes for Parallel Solving of the Suspension Transport Problems in Coastal Systems In: Voevodin V., Sobolev S. (eds) Supercomputing. RuSCDays 2019. Communications in Computer and Information Science 1129 pp 39–50
[6] Sukhinov A I, Chistyakov A E, Protsenko E A, Sidoryakina V V and Protosenko S V 2020 Accounting method of filling cells for the solution of hydrodynamics problems with a complex geometry of the computational domain Mathematical Models and Computer Simulations 12 pp 232–245
[7] Nieuwstadt F, Westerweel J and Boersma B J 2016 Turbulence. Introduction to Theory and Applications of Turbulent Flows (Springer, Cham)
[8] Alekseenko E, Roux B, Sukhinov A I, Kotarba R and Fougere D 2013 Coastal hydrodynamics in a windy lagoon Computers and Fluids 77 pp 24–35
[9] Samarsky A A and Vabishchevich P N 2009 Numerical methods for solving convection-diffusion problems (Moscow, URSS) p 248 (in Russian)
[10] Konovalov A N 2004 The steepest descent method with an adaptive alternating-triangular preconditioner Differential Equations 40 pp 1018–1028
[11] Browning J B and Sutherland B 2020 C++20 Recipes. A Problem-Solution Approach (Apress, Berkeley, CA) p 630
[12] Gushchin, V.A., Sukhinov, A.I., Nikitina, A.V., Chistyakov, A.E., Semenyakina, A.A. 2018 A Model of Transport and Transformation of Biogenic Elements in the Coastal System and Its Numerical Implementation Computational Mathematics and Mathematical Physics 58(8) pp 1316–1333
[13] Sukhinov, A.I., Chistyakov, A.E., Shishenya, A.V., Timofeeva, E.F. 2018 Predictive Modeling of Coastal Hydrophysical Processes in Multiple-Processor Systems Based on Explicit Schemes Mathematical Models and Computer Simulations 10(5) pp 648–658