Dynamic modeling of the Stewart platform for the NanShan Radio Telescope

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Abstract
The NanShan Radio Telescope is a 26-m fully steerable radio telescope, and it adopts a 6-UPU Stewart platform with electric motors to adjust and align the position of the subreflector. In order to analyze the actual dynamic performance and control the Stewart platform of the NanShan Radio Telescope, this article models the inverse dynamic of the Stewart platform using the virtual work approach. The model improves the accuracy of the dynamic equations and considered the pitching motion of the base platform in the practical application of the radio telescope. Dynamic simulations of the Stewart platform are implemented, the conditions of the passive rotation of the piston of actuators are considered, and the results show that the effect of the passive rotation of the pistons of the actuators is important to obtain more accurate result. The conditions of the system under the different elevation angles of the radio telescope are also considered, and the results show that the change of the elevation angles of the radio telescope has a great impact on the driving forces of the Stewart platform. It is known from the analysis that the passive rotation of pistons of actuators and the elevation movement of the primary reflector of the radio telescope are not ignorable for the precise analysis and control of the Stewart platform of the NanShan Radio Telescope.

Keywords
Radio telescope, Stewart platform, inverse dynamic, virtual work

Introduction
The NanShan Radio Telescope (NSRT) is a 26-m Cassegrain dual-reflector antenna, with a parabolic reflector as the main reflector and a hyperboloid as the subreflector. In the case of a telescope observation, a dual-reflector antenna must maintain stringent alignment of the subreflector with respect to the primary reflector.1 However, it is inevitable for a telescope to suffer from misalignment of the subreflector with the primary reflector resulting from structural deformation due to the effect of the environmental loading, such as the gravity, temperature, and so on. A Stewart platform is a manipulator with six degrees of freedom, which has high stiffness, low inertia, high positioning precision, and large load capacity. Recently, the Stewart platform has been widely used for telescopes to actively adjust the position of the subreflector to align the primary reflector. There are some well-known successful applications for the radio telescopes, such as LMT (Large Millimeter Telescope),2 Effelsberg,3 ALMA (Atacama Large Millimeter Array),4 FAST (Five-hundred-meter Aperture Spherical Telescope),5 and so on.

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Stewart platform was first proposed as a flight simulator by Stewart in 1965, which consists of a moving platform, a base, six extendable actuators, and 12 joints. The extendable actuator has a rotating cylinder and a moving piston. The six actuators connect the moving platform with the base by spherical joints or universal joints. There are three kinds of structures depending on the joints the Stewart platform used: 6-SPS, 6-UPS, and 6-UPU, where U stands for the universal joint, S for the spherical joint, and P for the prismatic joint. For the 6-UPU Stewart platform, an actuator is connected to the moving platform and the base by two universal joints.

For design and control of the Stewart platform, the accurate dynamic model is very essential. The dynamic modeling of parallel manipulators is quite complicated due to the closed-loop structure. Several methods for the inverse dynamics have been proposed, including the Newton–Euler formulation, the Lagrangian formulation, the Kane method, the screw theory, the Hamilton method, and the virtual work approach. In some cases, different methods may be combined to obtain better result, for example, Zanganeh et al. used both the Lagrangian and Newton–Euler approaches. Gallardo et al. combined the screw theory with the principle of virtual work. Although the derived equations for the dynamics of parallel manipulators present different levels of complexity and computational loads, the results of the actuated forces computed by different approaches are equivalent. The main goal of recent proposed approaches is to minimize the number of operations involved in the computation of the manipulator dynamics. Zhang and Song and Mahmoodi et al. have made a thorough analysis of the computational cost of the different formulations. Kalani et al. compared virtual work approach with the Newton–Euler method and the Lagrange formulation, and the virtual work approach is more straightforward and systematic resulting in more concise dynamic equations.

The earlier studies mainly focused on the dynamics of the 6-UPS or 6-SPS Stewart platform, and a few investigations were about the inverse dynamic of the 6-UPU Stewart platform. The 6-UPU Stewart platform has universal joints at both ends of legs, the platform’s carrying capacity is improved, and the piston of the actuator not only slides at the connection with cylinder but also has a passive rotation about the leg axis, so there is a force produced by the passive rotation of the piston relative to the cylinder. The passive rotation of the piston is often neglected in some dynamic models, and approximated as a 6-UPS Stewart platform, which brings errors. However, radio telescopes often require high accuracy, so it is important to improve the accuracy of the dynamic equation of the dynamic model. In this article, the principle of virtual work is employed to solve the inverse dynamic of the 6-UPU Stewart platform for the radio telescope. The force caused by the passive rotation of the piston is considered in our dynamic model, which improves the accuracy of the dynamic model. Since the Stewart platform is connected to the primary reflector of the radio telescope, the elevation movement of the primary reflector is taken into account in our model.

NSRT subreflector system description

As shown in Figure 1, the NSRT is a 26-m diameter, dual-reflector (Cassegrain) antenna. This antenna has a parabolic reflector and a hyperboloid. And the antenna focuses the radio waves by utilizing two reflectors, which means that the subreflector must keep high position and orientation accuracy. In this work, a 6-UPU Stewart platform is used to obtain desired position and orientation of the subreflector. The moving platform of the Stewart platform connects the subreflector, and the base platform is fixed to the primary reflector by four supporting legs. Therefore, while adjusting the pose of the subreflector, the Stewart platform follows the primary reflector in elevating motion. The elevation range of primary reflector during observations is $5^\circ - 88^\circ$, and the elevation rotatable range is $0^\circ - 90^\circ$.

As shown in Figure 2, the Stewart platform for the NSRT consists of a base platform, a moving platform, and six extendable actuators. Throughout this article, we assume that the base platform and the moving platform are both rigid and all six legs are identical. As shown in Figure 3, the piston of $i$th ($i = 1, 2, 3, 4, 5, 6$) leg is attached to the moving platform with an universal joint at point $p_i$, and the cylinder is attached to the base platform with an universal joint at point $b_i$. For the
convenience of expressions, the Cartesian coordinate frame $x_Py_Pz_P$ is fixed on the moving platform at origin $P$, and the Cartesian coordinate frame $x_By_Bz_B$ is fixed on the base platform at origin $B$. The pose of the moving platform is defined by a six-dimensional (6D) vector $\begin{bmatrix} x_P \\ y_P \\ z_P \\ a_P \\ b_P \\ g_P \end{bmatrix}$, where the first three components are the coordinates of frame $P$ in the reference frame $B$ and the last three components are parameters describing the orientation of the platform. And the rotation matrix $R_{BP}$ from the frame $P$ to $B$ consists of three Euler angles $\alpha, \beta, \gamma$ rotated about $x_B, y_B, \text{and } z_B$ axes, respectively, and can be defined as

$$R_{BP} = \begin{bmatrix} cac\beta & caa\beta + cac\gamma & cac\gamma + cac\gamma \\ sac\beta & ssa\beta + cac\gamma & ssa\beta + cac\gamma \\ -s\beta & csa\beta & cs\beta \end{bmatrix}$$

where $c$ and $s$ represent the cosine and the sine, respectively.

While the NSRT tracking extraterrestrial sources and satellites, the primary reflector of the antenna elevates $5^\circ - 88^\circ$. Since the base platform of the Stewart platform is fixed to the primary reflector, the base platform moves relative to the ground, which results a change of the gravity vector direction in frame $B$. In order to determine the influence of the base platform motion on the system dynamic, a geodetic reference coordinate frame $W$ is established at the antenna orientation center. As shown in Figure 4, the $x$-axis and the $z$-axis of frame $W$ coincide with the azimuth axis and the elevation axis of the antenna, respectively. As shown in Figure 5, the rotation matrix $R_{BW}$ from the frame $B$ to $W$ can be expressed as

$$R_{BW} = \begin{bmatrix} c\phi c\theta & -s\phi & c\phi s\theta \\ s\phi c\theta & c\phi & s\phi s\theta \\ -s\theta & 0 & c\theta \end{bmatrix}$$

where $\theta$ and $\phi$ are the elevation and the azimuth angle of the primary reflector, respectively.

**Inverse kinematics analysis**

**Inverse position analysis**

Figure 6 represents vectors and coordinate frames used for the kinematic issue of the 6-UPU Stewart platform. For each kinematic chain, a closed vector-loop equation can be written as follows
where \( P^B_i \) is the position coordinate of the point \( P \) measured in the frame \( B \), \( P^P_i \) is the coordinate of the point \( P_i \) measured in the frame \( P \), \( b^B_i \) is the coordinate of the point \( b_i \) measured in the frame \( B \), \( l_1 \) is the distance from the point \( b_i \) to the lower plane of the cylinder, \( l_2 \) is the distance from the point \( P_i \) to the lower plane of the cylinder, \( n^B_i \) is the unit direction vector of leg \( i \) in the frame \( B \), from \( b_i \) to \( P_i \), and \( n^B_i \) can be obtained as

\[
n^B_i = \frac{P^B_i + R^B_P P^P_i - b^B_i}{l_i}
\]

For the \( i \)th leg, the leg length \( l_i \) is derived as

\[
l_i = l_1 + l_2
\]

where \( l_i \) is the length of the leg \( b_i P_i \), and \( l_i \) is derived as

\[
l_i = |P^B_i + R^B_P P^P_i - b^B_i|
\]

As shown in Figure 6, an actuator consists of two bodies: the rotating cylinder and the moving piston. The rotating body, with mass \( m_{1i} \) and a constant distance \( e_1 \), is connected with the universal joint to the base at \( b_i \). The moving actuator body with mass \( m_{2i} \) and a constant distance \( e_2 \) is connected with another universal joint to the platform at \( P_i \). The length of the actuator is adjustable with a sliding joint between these two bodies. The center of gravity positions of cylinder \( r^B_{1i} \) and piston \( r^B_{2i} \) are calculated as follows

\[
r^B_{1i} = b^B_i + e_1 n^B_i
\]

\[
r^B_{2i} = b^B_i + (l_i - e_2)n^B_i
\]

As shown in Figure 7, each leg connects to the base platform by a universal joint, and the rotation matrix which transfers leg frame \( L_i \) to frame \( B \) can be obtained as

\[
R^B_{L_i} = \begin{bmatrix}
c\phi_i & c\theta_i & s\phi_i & s\theta_i \\
c\phi_i & s\theta_i & -s\phi_i & c\theta_i \\
p & 0 & 0 & 1
\end{bmatrix}
\]

and the unit direction vector of leg \( i \) in the frame \( B \) is

\[
n^B_i = R^B_{L_i} n_i^L = \begin{bmatrix}c\phi_i & s\phi_i \\
c\theta_i & s\theta_i \\
0 & 0 & 1
\end{bmatrix}
\]

where \( n_i^L = [0 \ 0 \ 1]^T \). \( \phi_i \) and \( \theta_i \) can be obtained by solving equation (10).

**Inverse velocity analysis**

The generalized speed vector is defined as follows

\[
x_{mp} = [v^B_{mp} \ w^B_{mp}]^T
\]

where \( v^B_{mp} \) and \( w^B_{mp} \) are the linear and the angular velocity of the moving platform, and can be defined as

\[
v^B_{mp} = [\dot{x} \ \dot{y} \ \dot{z}]^T
\]

\[
w^B_{mp} = \begin{bmatrix}
c\alpha c\beta & -s\alpha & 0 \\
c\alpha s\beta & c\alpha & 0 \\
-s\beta & 0 & 1
\end{bmatrix}^T
\]
The linear velocity of the center of point $p_i$ expressed in the frame B can be derived by taking time derivative of the right-hand side of equation (3) as

$$v^B_{pi} = v^B_{mp} + w^B_{mp} \times R^B_{mp} p_i^B$$

(14)

where $p_i^B = R^B_{mp} p_i^B$. And transforming $v^B_{pi}$ to $i$th leg frame as

$$v^i_{pi} = R^{iT}_{pi} v^B_{pi}$$

(15)

the velocity of $i$th leg $l_i$ is derived as

$$l_i = v^i_{pi} n^i_i = n^i_i v^B_{mp} + (p_i^B \times n^i_i) w^B_{mp}$$

(16)

The velocity of $p_i$ is also obtained by taking time derivative of the left-hand side of equation (3) as

$$v^i_{pi} = l_{1i} w^1_{1i} \times n^i_i + l_{2i} w^2_{2i} \times n^i_i + \dot{l}_i n^i_i$$

(17)

Since the 6-UPU Stewart platform has universal joints at both ends of legs, the piston of the actuator not only slides at the connection with cylinder but also has a passive rotation along the leg axis, so this part moves with cylinder as well as rotate about the leg axis. Previous literature often neglects the passive rotation of the piston, and assumes that the angular velocity of the cylinder $w^1_{1i}$ is equal to the angular velocity of the piston $w^2_{2i}$. Here, we take into account the passive rotation of the piston, and the angular velocity of piston can be obtained as

$$w^i_{2i} = w^i_{1i} + (R^{iT}_{1i} w^B_{mp} \cdot n^i_i) n^i_i$$

(18)

by substituting $w^i_{2i}$ in equation (17) with equation (18), we can get

$$v^i_{pi} = l_{1i} w^1_{1i} \times n^i_i + l_{2i} w^2_{2i} \times n^i_i + \dot{l}_i n^i_i$$

(19)

We assume that the cylinder does not spin about its own axis, which implies that $w^1_{1i}$ is perpendicular to $n^i_i$. This assumption will greatly simplify the kinematic equations for the angular velocity and acceleration of the strut. Therefore, $w^1_{1i} \cdot n^i_i = 0$, and the angular velocity of cylinder can be obtained by the cross product of equation (19) with $n^i_i$

$$w^i_{1i} = \frac{1}{l_i} (n^i_i \times v^i_{pi}) = \frac{1}{l_i} \begin{bmatrix} -v^i_{p, y} \\ v^i_{p, x} \\ 0 \end{bmatrix}$$

(20)

then $w^i_{2i}$ can be obtained by substituting $w^i_{1i}$ into equation (18)

$$w^i_{2i} = \frac{1}{l_i} \begin{bmatrix} -v^i_{p, y} \\ v^i_{p, x} \\ 0 \end{bmatrix} R^{iT}_{1i} (l_{1i} - l_{2i}) w^B_{mp}$$

(21)

where $R^{iT}_{1i} (l_{1i} - l_{2i})$ is the third row of the matrix $R^{iT}_{1i}$. The velocity of the center of mass of $i$th cylinder is found by differentiating equation (7)

$$v^i_{ci} = e_i w^i_{1i} \times n^i_i = \frac{1}{l_i} \begin{bmatrix} v^i_{p, y} \\ -v^i_{p, x} \\ 0 \end{bmatrix}$$

(22)

and the velocity of the center of mass of $i$th piston is found by differentiating equation (8)

$$v^i_{pi} = \dot{l}_i n^i_i (l_{1i} - l_{2i}) w^i_{2i} \times n^i_i = \frac{1}{l_i} \begin{bmatrix} (l_{1i} - l_{2i}) v^i_{p, y} \\ (l_{1i} - l_{2i}) v^i_{p, x} \\ l_i v^i_{p, z} \end{bmatrix}$$

(23)

**Inverse acceleration analysis**

The linear acceleration of moving platform is $\ddot{v}_{mp} = [\ddot{x}_p, \ddot{y}_p, \ddot{z}_p]^T$ and the angular acceleration is $\ddot{w}_{mp} = [\ddot{\alpha}, \ddot{\beta}, \ddot{\gamma}]$

$$\ddot{w}_{mp} = \begin{bmatrix} -\ddot{\beta} c \beta c \alpha - \ddot{\alpha} c \beta c \alpha - \ddot{\alpha} c \alpha \\ -\ddot{\beta} s \beta c \alpha c \beta - \ddot{\alpha} s \beta c \alpha c \beta - \ddot{\alpha} s \alpha \\ -\ddot{\beta} c \beta 0 0 0 \end{bmatrix} \begin{bmatrix} \ddot{\gamma} \\ \ddot{\alpha} \\ \ddot{\beta} \end{bmatrix}$$

(24)

the acceleration of $p_i$ can be obtained by differentiating equation (14)

$$v^i_{pi} = \ddot{v}_{mp} + w^B_{mp} \times R^B_{mp} p_i^B + w^B_{mp} \times (w^B_{mp} \times R^B_{mp} p_i^B) = \ddot{v}_{mp} + w^B_{mp} \times (w^B_{mp} \times R^B_{mp} p_i^B)$$

(25)
\[ \dot{v}_p^i = R_{li}^{BT} \dot{v}_p^B \]  

(26)

the acceleration of \( p_i \) also can be expressed in terms of the angular acceleration of the \( i \)th leg by differentiating equation (17)

\[ \dot{v}_p^i = \dot{i}_i n_i^i + l_{mi} w_{mi}^B \times n_i^i + l_{mi} w_{mi}^B \times (w_{mi}^B \times n_i^i) + \left( 2l_{mi} w_{mi}^B \times n_i^i + l_{mi} w_{mi}^B \times w_{mi}^B \times n_i^i + l_{mi} w_{mi}^B \times (w_{mi}^B \times n_i^i) \right) \]  

(27)

expression \( w_{mi}^B \) can be obtained by differentiating equation (18)

\[ \dot{w}_{mi}^B = \dot{w}_{mi}^B + n_i^i n_i^i \dot{w}_{mp}^B - n_i^i n_i^i \dot{w}_{mi}^B \times \dot{w}_{mp}^B \]  

(28)

since the cylinder does not spin about its own axis, cross product of equation (27) with \( n_i^i \) yields

\[ \dot{w}_{mi}^B = \frac{1}{i_i} n_i^i \times \dot{v}_p^i - \frac{2\dot{i}_i}{i_i} w_{mi}^B \]  

(29)

the accelerations of the mass centers of the cylinder and piston can be obtained by differentiating equations (22) and (23)

\[ \dot{v}_{li}^i = e_1 w_{li}^B \times n_i^i + e_1 w_{li}^B \times (w_{li}^B \times n_i^i) \]  

(30)

\[ \dot{v}_{2i}^i = \dot{i}_i n_i^i + 2\dot{i}_i w_{li}^B \times n_i^i + (l_i - e_2) w_{li}^B \times n_i^i + (l_i - e_2) w_{li}^B \times (w_{li}^B \times n_i^i) \]  

(31)

Jacobian

The Jacobian matrices are required in order to write the equation of motion of platform in compact form. Writing equation (14) in matrix form yields

\[ v_p^B = J_{p_i}^B \dot{x}_m^B \]  

(32)

where \( J_{p_i}^B \) is a 3 \( \times \) 6 matrix, defined as

\[ J_{p_i}^B = \begin{bmatrix} 1 & 0 & 0 & 0 & p_{i_z}^B & p_{i_y}^B \\ 0 & 1 & 0 & -p_{i_z}^B & 0 & p_{i_x}^B \\ 0 & 0 & 1 & p_{i_y}^B & -p_{i_x}^B & 0 \end{bmatrix} \]  

(33)

Substituting equation (33) into equation (15) yields

\[ \dot{v}_p^i = J_{p_i}^i \dot{x}_m^B \]  

(34)

where

\[ J_{p_i}^i = R_{li}^{BT} J_{p_i}^B \]  

(35)

and \( J_{p_i}^B, J_{p_i}^i, J_{p_i}^f \) and \( J_{p_i}^f \) are the rows of \( J_{p_i}^B \).

Equation (16) can be written for each leg, and then assembled in matrix form

\[ \dot{l} = J_{mp}^B \dot{x}_m^B \]  

(36)

where \( l = [l_1, l_2, \ldots, l_6] \) represents the velocity of actuators. \( J_{mp}^B \) is a 6 \( \times \) 6 square matrix

\[ J_{mp}^B = \begin{bmatrix} n_{1z}^B & (p_{1z}^B \times n_{1y}^B)^T \\ n_{2z}^B & (p_{2z}^B \times n_{2y}^B)^T \\ \vdots & \vdots \\ n_{6z}^B & (p_{6z}^B \times n_{6y}^B)^T \end{bmatrix} \]  

(37)

And equations (20)–(23) in matrix form yield

\[ \dot{x}_{li}^i = \begin{bmatrix} \dot{v}_{li}^i \\ \dot{n}_{li}^i \\ \dot{w}_{li}^B \end{bmatrix} = J_{li}^i \dot{x}_m^B \]  

(38)

\[ \dot{x}_{2i}^i = \begin{bmatrix} \dot{v}_{2i}^i \\ \dot{n}_{2i}^i \\ \dot{w}_{2i}^B \end{bmatrix} = J_{2i}^i \dot{x}_m^B \]  

(39)

where

\[ J_{li}^i = \frac{1}{l_i} \begin{bmatrix} e_1 J_{p_i}^{ix} \\ e_1 J_{p_i}^{iy} \\ -J_{p_i}^{iz} \end{bmatrix} \]  

\[ J_{2i}^i = \frac{1}{l_i} \begin{bmatrix} (l_i - e_2) J_{p_i}^{ix} \\ (l_i - e_2) J_{p_i}^{iy} \\ J_{p_i}^{iz} \end{bmatrix} \]  

(40)

(41)

Dynamic model

Tsai\textsuperscript{16} proposed an explicit dynamic model for the Stewart platform used the virtual work approach, and we employed the virtual work approach to model the inverse dynamic of the 6-UPU Stewart platform for the NSRT. Because Tsai has given the detailed derivation of this part with result shown below

\[ J_{mp}^B F_{ac}^B + F_{mp}^B + \sum_{i=1}^{6} (J_{li}^i F_{p_i}^i + J_{2i}^i F_{p_i}^f) = 0 \]  

(42)
where \( \mathbf{F}_{ac} \) is the driving forces acting on the extendable actuators, and \( \mathbf{F}_{mp}^B \) is the resultant wrench due to the inertia of the moving platform, which is expressed as

\[
\mathbf{F}_{mp}^B = \begin{bmatrix} m_{mp} \mathbf{g}^B \\
0_{3 \times 1} \end{bmatrix} + \left[- \mathbf{F}_{mp}^B \mathbf{\omega}_{mp} - \mathbf{\omega}_{mp} \times \left( \mathbf{F}_{mp}^B \mathbf{\omega}_{mp} \right) \right]
\]

(43)

where \( m_{mp} \) and \( \mathbf{F}_{mp}^B \) are the mass and the inertia matrix of the moving platform, respectively, and \( \mathbf{I}_{mp}^B = \mathbf{R}_{mp}^{BT} \mathbf{I}_{mp}^B \mathbf{R}_{mp}^{BT} \). Since the base platform of the Stewart platform is fixed to the primary reflector, and the primary reflector elevates \( 5^\circ - 88^\circ \), so the base platform moves relative to the ground, which results a change of the gravity vector. And the gravitational acceleration \( \mathbf{g} \) defined in the B frame is derived as

\[
\mathbf{g}^B = \mathbf{R}_{p}^{BT} \mathbf{g}^W
\]

(44)

where \( \mathbf{g}^W = [g_{wx} = 0 \quad g_{wy} = 0 \quad g_{wz} = -9.8]_T \) in m/s\(^2\).

\( \mathbf{F}_{li}^i \) and \( \mathbf{F}_{2i}^i \) are the resultant wrench due to inertia of the cylinder and piston of \( i \)th actuators, which can be written as

\[
\mathbf{F}_{li}^i = \begin{bmatrix} m_{li} \mathbf{R}_{li}^{BT} \mathbf{g}^B \\
0_{3 \times 1} \end{bmatrix} + \left[- \mathbf{F}_{li}^i \mathbf{\omega}_{li} - \mathbf{\omega}_{li} \times \left( \mathbf{F}_{li}^i \mathbf{\omega}_{li} \right) \right]
\]

(45)

\[
\mathbf{F}_{2i}^i = \begin{bmatrix} m_{2i} \mathbf{R}_{2i}^{BT} \mathbf{g}^B \\
0_{3 \times 1} \end{bmatrix} + \left[- \mathbf{F}_{2i}^i \mathbf{\omega}_{2i} - \mathbf{\omega}_{2i} \times \left( \mathbf{F}_{2i}^i \mathbf{\omega}_{2i} \right) \right]
\]

(46)

where \( m_{li} \) and \( m_{2i} \) are the mass of cylinder and piston of the \( i \)th leg, respectively, and \( \mathbf{I}_{li}^i \) and \( \mathbf{I}_{2i}^i \) are the inertia matrix of cylinder and piston of the \( i \)th leg, respectively.

When \( \mathbf{J}_{mp}^B \) is not singular, the driving forces \( \mathbf{F}_{ac} \) acting on the extendable actuators can be obtained from equation (42)

\[
\mathbf{F}_{ac} = - \mathbf{J}_{mp}^{-BT} \left[ \mathbf{F}_{mp}^B + \sum_{i=1}^{6} (\mathbf{J}_{li}^i \mathbf{F}_{li}^i + \mathbf{J}_{2i}^i \mathbf{F}_{2i}^i) \right]
\]

(47)

\[p_1 = [0.3458 \quad 0.0927 \quad -0.2578]^T m\]
\[p_2 = [-0.0927 \quad 0.3458 \quad -0.2578]^T m\]
\[p_3 = [-0.2531 \quad 0.2531 \quad -0.2578]^T m\]
\[p_4 = [-0.2531 \quad -0.2531 \quad -0.2578]^T m\]
\[p_5 = [-0.0927 \quad -0.3458 \quad -0.2578]^T m\]
\[p_6 = [0.3458 \quad -0.0927 \quad -0.2578]^T m\]
\[b_1 = [0.4243 \quad 0.4243 \quad 0]^T m\]
\[b_2 = [0.1553 \quad 0.5796 \quad 0]^T m\]
\[b_3 = [-0.5796 \quad 0.1553 \quad 0]^T m\]
\[b_4 = [-0.5796 \quad -0.1553 \quad 0]^T m\]
\[b_5 = [0.1553 \quad -0.5796 \quad 0]^T m\]
\[b_6 = [0.4243 \quad -0.4243 \quad 0]^T m\]

The azimuth angle of the radio telescope

\[\alpha_0 = 30^\circ\]

Gravity centers of the cylinder and piston for all legs
\[e_1 = 0.35 m \quad e_2 = 0.3 m\]

Mass of the moving platform, cylinder, and piston of all legs
\[m_{mp} = 317.618 kg \quad m_{li} = 45.291 kg \quad m_{2i} = 38.424 kg\]

Moments of inertia of the cylinder and piston of all legs as well as the moving platform

\[
I_{li} = \begin{bmatrix} 1.505 & 0 & 0 \\
0 & 1.505 & 0 \\
0 & 0 & 7.352 \times 10^{-2} \end{bmatrix} \] \text{kg m}^2
\]
\[
I_{2i} = \begin{bmatrix} 2.759 & 0 & 0 \\
0 & 2.759 & 0 \\
0 & 0 & 1.251 \end{bmatrix} \] \text{kg m}^2
\]
\[
I_{mp} = \begin{bmatrix} 20.582 & 0 & 0 \\
0 & 10.556 & 0 \\
0 & 0 & 10.556 \times 10^{-2} \end{bmatrix} \] \text{kg m}^2

In order to explore the impact of the passive rotation of the piston to the inverse dynamic of the 6-UPU Stewart platform, dynamic simulation of the system both with and without considering the passive rotation of the piston is presented. The model that considers the passive rotation of the piston is called the complete dynamic model, and the model that does not consider the passive rotation of the piston is called the simplified dynamic model. The elevation angle of the primary reflector of the radio telescope is \( \theta = 0^\circ \). And the trajectory of the moving platform is...
\( \alpha = \beta = \gamma = 0.15\sin(3t) \) and \( P_{mp} = \begin{bmatrix} 0.05\sin(3t) \\ 0.05\sin(3t) \\ 0.05\sin(3t) \end{bmatrix} \)

Figure 8 shows the comparison of the driving forces of six extendable actuators obtained by the complete dynamic model and the simplified dynamic model. Each figure shows visible difference between two cases, especially in Figure 8(b) the actuator 2 has significant difference between the driving forces obtained by the complete dynamic model and the simplified dynamic model, which means that the passive rotation of the piston which is not considered in the simplified dynamic model has an influence on the dynamic model of the 6-UPU Stewart platform.

To explore the impact of the change of the elevation angle of the primary reflector for the dynamic model, dynamic simulations that compare the inverse dynamic of the system under different elevation angles are presented. And the given elevation angles of primary reflector of the radio telescope are \( \theta = 0^\circ \), \( \theta = 30^\circ \), and \( \theta = 90^\circ \), respectively. The trajectory of the moving platform is the same as the first simulation.
Driving forces of six extendable actuators while $u = 0^\circ$, $u = 30^\circ$, and $u = 90^\circ$ are plotted in Figures 9–11, respectively. When $u = 0^\circ$, the driving forces of the actuators are in the range of 200–2800 N. When $u = 30^\circ$, the driving forces of the actuators are in the range of 2200–4800 N. And when $u = 90^\circ$, the driving forces of the actuators are in the range of 2600–6500 N. Comparing to the two other situations when $u = 90^\circ$, the range of driving forces is the largest. And the result indicates that the elevation movement of the primary reflector of the radio telescope has a great impact on the dynamics of the Stewart platform. And for the given trajectory, the larger elevation angle of the radio telescope, the greater driving forces are required for the extendable actuators to drive the Stewart platform.

Figure 9. Driving forces of six extendable actuators when $\theta = 0^\circ$.

Figure 10. Driving forces of six extendable actuators when $\theta = 30^\circ$.

Figure 11. Driving forces of six extendable actuators when $\theta = 90^\circ$.

Driving forces of six extendable actuators while $\theta = 0^\circ$, $\theta = 30^\circ$, and $\theta = 90^\circ$ are plotted in Figures 9–11, respectively. When $\theta = 0^\circ$, the driving forces of the actuators are in the range of 200–2800 N. When $\theta = 30^\circ$, the driving forces of the actuators are in the range of 2200–4800 N. And when $\theta = 90^\circ$, the driving forces of the actuators are in the range of 2600–6500 N. Comparing to the two other situations when $\theta = 90^\circ$, the range of driving forces is the largest. And the result indicates that the elevation movement of the primary reflector of the radio telescope has a great impact on the dynamics of the Stewart platform. And for the given trajectory, the larger elevation angle of the radio telescope, the greater driving forces are required for the extendable actuators to drive the Stewart platform.

**Conclusion**

This study models the inverse dynamic of the 6-UPU Stewart platform for the NSRT using the virtual work approach. The dynamic model improves the accuracy of the dynamic equations and considered the pitching motion of the base platform in the practical application of the radio telescope. Simulation results indicate that the effect of the passive rotation of the pistons of the actuators is important to obtain more accurate result. And the driving forces required for the Stewart platform to move along same trajectory with different elevation angles are compared. The results show that the elevation motion of the radio telescope has a great impact on the dynamic of the Stewart platform, and the larger elevation angle of the radio telescope, the greater driving forces are required for the extendable actuators to drive the Stewart platform. The model can be used in the precise analysis and control of the Stewart platform of the NSRT.

**Declaration of conflicting interests**

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