Theoretical study on a water muffler

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Abstract. Theoretical computation on a previously studied water muffler is carried out in this article. Structure of the water muffler is composed of two main parts, namely, the Kevlar-reinforced rubber tube and the inner-noise-reduction structure. Rubber wall of the rubber tube is assumed to function as rigid wall lined with sound absorption material and is described by a complex radial wave number. Comparison among the results obtained from theoretical computation, FEM (finite element method) simulation and experiment of the rubber tube and that of the water muffler has been made. The theoretical results show a good accordance in general tendency with the FEM simulated and the measured results. After that, parametric study on the diameter of the inner structure and that of the rubber tube is conducted. Results show that the diameter of the left inner structure has the most significant effect on the SPL of the water muffler due to its location and its effect on the diameter ratio D2/D1.

1. Introduction

Mufflers or silencers are widely used as passive noise reduction measures and has been studied thoroughly. The major research contents include the design and optimization method of muffler structure, computational method on the acoustic performance of mufflers.

Selamet [1] presented a quasi-one-dimensional approach to analyse three-pass mufflers with perforated elements using numerical decoupling. The approach was further developed to include mufflers with ducts extended into the end cavities. Dokumaci [2] applied the distributed parameter method to the analysis of mufflers employing multiple co-axial pipes. Wu [3] proposed a direct mixed-body boundary element method to obtain the acoustic performance of silencers lined with bulk-reacting sound absorbing materials. Selamet [4] investigated the acoustic behavior of a circular dual-chamber muffler in detail by a two-dimensional (2-D) axisymmetric analytical approach based on the mode-matching technique for the concentric configurations, the finite element method and experimental work. A number of effects were studied and some of these effects were shown to modify the acoustic behavior drastically, suggesting potential means to improve the acoustic performance. Denia [5] investigated the acoustic behavior of perforated dissipative circular mufflers with empty extended inlet/outlet in detail by means of a two-dimensional (2D) axisymmetric analytical approach that matches the acoustic pressure and velocity across the geometrical discontinuities, and the finite element method (FEM). Panigrahi [6] suggested a one-dimensional (1-D) scheme based on an algorithm that used user-friendly visual volume elements along with the theory of transfer matrix.

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based muffler analysis. Xu [7] established acoustic characteristics of a dual Helmholtz resonator which consists of a pair of cylindrical neck and cavity connected in series in terms of a lumped-parameter theory. Mimani [8] analysed the acoustical behavior of an elliptical chamber muffler having an end-inlet and side-outlet port with a 3-D impedance matrix approach. Vijayasree [9] proposed a transfer matrix based approach namely the Integrated Transfer Matrix (ITM) method to analyse a general commercial muffler that consists of both perforated pipes and baffles.

Most studies on mufflers/silencers focus on mufflers with gas medium. However, water mufflers have been generally ignored because the early application fields of mufflers have largely concentrated on the internal combustion engine and relevant components in the automotive industry. Noise from gas-medium-conveying equipment is larger than that of liquid-medium-conveying equipment. However, given that the acoustic performance of liquid-conveying system is gaining increasing attention, methods should be developed to effectively control the noise generated by such systems.

Water muffler presented in previous studies [10-14] are functional only in the problems with single frequencies or narrow frequency bands, and appear with assembly difficulties due to the oversized structure against the limited installation spaces. Moreover the steel tubes in the major parts of muffler are of high rigidity and more helpful to the vibration propagation. Based on these considerations, a water muffler has been proposed by the authors and studied both numerically and experimentally in a previous article [15].

Theoretical computation on the previously studied water muffler is carried out in this article. Structure of the water muffler is composed of two main parts, namely, the Kevlar-reinforced rubber tube and the inner-noise-reduction structure. Rubber wall of the rubber tube is assumed to function as rigid wall lined with sound absorption material and is described by a complex radius wave number. Comparison among the results obtained from theoretical computation, FEM simulation and experiment of the rubber tube and that of the water muffler has been made. The theoretical results show a good accordance in general tendency with the FEM simulated and the measured results which validates the efficiency of the theoretical computation. After that, parametric study on the diameter of the inner structure and that of the rubber tube is conducted. Conclusion has been made and discussed in later sections.

2. Basic equations

Waves in a circular duct with stationary medium are governed by [16]:

\[
\frac{\partial^2 p}{\partial t^2} - a_0^2 \left( \frac{\partial^2 p}{\partial r^2} + \frac{1}{r \partial r} \frac{\partial p}{\partial r} + \frac{1}{r^2 \partial \theta^2} + \frac{\partial^2 p}{\partial z^2} \right) = 0
\]

(1)

Where \( r \) is the radius of the duct, \( a_0 \) is the sound speed, \( z \) is the axial direction of the tube, \( \theta \) is the circular direction of the tube and \( p \) is the sound pressure.

Upon making use of the method of separation of variables, and writing time dependence as \( e^{i\omega t} \) and \( \theta \) dependence as \( e^{im\theta} \), the general solution to equation (1) is given by:

\[
p(r, \theta, z, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} J_m(k_{r,m,n}r) e^{im\theta} e^{i\omega t} \left\{ C_{1,m,n} e^{-\beta_{r,m,n}z} + C_{2,m,n} e^{\beta_{r,m,n}z} \right\}
\]

(2)

Where \( k_{r,m,n} \) and \( k_{z,m,n} \) is the transmission wave number for the \((m, n)\) mode in radius direction and axial direction respectively.

Substituting in the equation of dynamical equilibrium for the axial direction:

\[
\rho_0 \frac{\partial u}{\partial t} + \frac{\partial p}{\partial r} = 0
\]

(3)

Yields the acoustic particle velocity:

\[
u(r, \theta, z, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} J_m(k_{r,m,n}r) e^{im\theta} \frac{k_{z,m,n}}{k_0 \rho_0} \frac{1}{k_{r,m,n}} \left\{ C_{1,m,n} e^{-\beta_{r,m,n}z} + C_{2,m,n} e^{\beta_{r,m,n}z} \right\}
\]

(4)

According to the cut off frequency relation

\[
f < \frac{1.84 a_0}{(\pi D)}
\]

(5)
one can assume that only plane waves could propagate in the main ducts under the considered frequency band (20 Hz-5000 Hz with $\omega_0=1500$ m/s and $D_{max}=0.125$ m). The sound pressure and acoustic particle velocity of the $(0, 0)$ mode is given by:

$$p(r, \theta, z, t) = e^{i\omega t} \left( C_0 e^{-jz \omega} + C_1 e^{jz \omega} \right)$$  \hspace{1cm} (6)

$$u(r, \theta, z, t) = \frac{\rho_0}{\omega} \left( C_0 e^{-jz \omega} + C_1 e^{jz \omega} \right)$$  \hspace{1cm} (7)

For conical tube with diameter being proportional to $z$ (distance from the hypothetical apex), the sound pressure and acoustic particle velocity of the $(0, 0)$ mode is denoted by:

$$p(z, t) = e^{i\omega t} \left( C_0 e^{-jz \omega} + C_1 e^{jz \omega} \right) e^{i\omega t} / z$$ \hspace{1cm} (8)

$$u(z, t) = e^{i\omega t} \frac{1}{\omega \rho_0 z} \left( -j k_0 - \frac{1}{z} \right) C_0 e^{-jz \omega} + (j k_0 - \frac{1}{z}) C_1 e^{jz \omega} \right)$$ \hspace{1cm} (9)

By assuming that the rubber wall functions as rigid wall lined with sound absorption material, the difference with duct of rigid wall is that $k_{r,m,n}$ is now determined from the boundary condition that the wall ($r=r_0$) has a finite impedance $Z_w$ while the rigid wall have infinite impedance. The momentum equation along the radial direction

$$\rho_0 \frac{\partial u_r}{\partial t} + \frac{\partial p}{\partial r} = 0$$ \hspace{1cm} (10)

Therefore

$$Z_w = \left( \frac{p}{u_r} \right)_{r=r_0} = -j \omega \rho_0 \frac{p}{\partial p/\partial r} = -j \omega \rho_0 \frac{J_m(k_{r,m,n} r_0)}{k_{r,m,n} J_m(k_{r,m,n} r_0)}$$ \hspace{1cm} (12)

Where

$$J_m(k_{r,m,n} r) = \frac{dJ_m(k_{r,m,n} r)}{dk_{r,m,n} r}$$ \hspace{1cm} (13)

Thus $k_{r,m,n}$, $n=0,1,2\ldots$ are the infinite roots of the transcendental eigenvalue:

$$\frac{J_m(k_{r,m,n} r_0)}{(k_{r,m,n} r_0) J_m(k_{r,m,n} r_0)} = j \frac{Z_w}{\rho_0 \omega_0} k_{r_0}$$ \hspace{1cm} (14)

For the acoustic particle velocity:

$$u(r, \theta, z, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} J_m(k_{r,m,n} r) e^{i\omega t} e^{im\theta} \frac{k_{r,m,n}}{\rho_0 \omega_0} \left( C_0 e^{-jz \omega} + C_1 e^{jz \omega} \right)$$ \hspace{1cm} (15)

It can be seen that ducts lined with acoustic absorptive material (that is, with complex wall impedance) would result in complex value of $k_{r,m,n}$ and hence $k_z$. The imaginary component of $k_z$ would introduce attenuation in the axial direction, and that is the basic principle of dissipative mufflers.

For the $(0, 0)$ mode, the sound pressure and acoustic particle velocity are given by:

$$p(r, \theta, z, t) = J_0(k_z r) e^{i\omega t} \left( C_0 e^{-jz \omega} + C_1 e^{jz \omega} \right)$$ \hspace{1cm} (16)

$$u(r, \theta, z, t) = J_0(k_z r) e^{i\omega t} \frac{k_z}{\rho_0 \omega_0} \left( C_0 e^{-jz \omega} + C_1 e^{jz \omega} \right)$$ \hspace{1cm} (17)

3. Models and computation

3.1. Muffler structure

The water muffler (shown in figure 1) is composed of two main parts, namely, the Kevlar-reinforced rubber tube (simplified as the rubber tube in later discussions) and the inner-noise-reduction structure (simplified as the inner structure in later discussions). Principle of the noise reduction of the rubber tube is based on the dissipative muffler with compliant walls. As the rubber tube is a Kevlar-reinforced tube, the coupling effect between the wall of the rubber tube and the flow is assumed to be
weak and that can be neglected in the analysis. Then the rubber tube is functioned like a duct lined with acoustic absorptive material. The rubber wall is assumed to be locally reacting which means the impedance $Z_w$ is independent of $z$. The principle of the inner structure is the combination of conical tube and the extended-tube resonators, which are typical reactive muffler structures. These structures will reflect a substantial amount of the incident power back to the source by creating a mismatch of characteristic impedances [16]. The inner structure is symmetrically mounted on both sides of the muffler.

![Figure 1. Structure of the water muffler.](image1)

**Figure 1.** Structure of the water muffler.

### 3.2. Computational model

Computational model for the rubber tube and the water muffler is shown in figure 2 in which the small annular cavity in the muffler is omitted. The inner flow passage is divided into several elements according to the variation of the cross area and material (numbered in figure2). These elements are connected through the boundary conditions at the discontinuities. Two extended pipes are connected to these models to simulate the inlet and outlet pipe, the same as that modelled in the FEM simulation.

![Figure 2. Simplified inner flow passage of: (a) rubber tube, (b) water muffler.](image2)

**Figure 2.** Simplified inner flow passage of: (a) rubber tube, (b) water muffler.

### 3.3. Boundary conditions

At the discontinuities between two elements, the continuity of acoustic pressure and acoustic mass velocity should be fulfilled, that is:

\[
p_1 = p_2 \\
v_1 = v_2
\]

At the inlet, the sound source is set as the test results of the reference tube. At the outlet, outlet impedance obtained from the FEM simulation is applied.
4. Results and discussion

4.1. Verification of the theoretical computation
Comparisons among the results obtained from theoretical computation, FEM simulation and the experiment of the rubber tube and that of the water muffler are shown in figure 3 and figure 4 respectively. It can be seen from these figures that the theoretical results show a good accordance in general tendency with the FEM simulated and the measured results which validate the efficiency of the proposed computation method.

![Figure 3. Results of the rubber tube.](image)

![Figure 4. Results of the water muffler.](image)

4.2. Parametric study
The length and diameter of the rubber tube and the inner structure will significantly affect the acoustic performance of the water muffler. Because the length of the tube is normally determined due to the limited installation spaces, more attention is given to the diameter. Parametric study on the diameter of the inner structure (D2) and that of the rubber tube (D3) is conducted in this section. Comparisons on acoustic performance of the water muffler with symmetrically installed inner structures are shown in subfigure 5a and subfigure 5b. From these subfigures one can tell that the SPL increases with the increase of the diameter of the inner structure while shows little difference with the increase of the diameter of the rubber tube. Further comparisons on acoustic performance of the water muffler with asymmetrically installed inner structures in which the left inner structure has different radius with the right one (with \(D2_L\) and \(D2_R\) denotes the D2 of the left inner structure and that of the right one).
respectively) are shown in subfigure 5c and subfigure 5d. It can be seen from these subfigures that the SPL increases with the increase of the diameter of the left inner structure while shows little difference with the increase of the diameter of the right one. An overall view on the four subfigures in figure 5 shows that the radius of the left inner structure has the most significant effect on the SPL. This is because it is located near the sound source and the increase of D2 also results in the increase of the diameter ratio D2/D1 which will affect the sound reflection at the conic tube of the inner structure.

![Image of graphs showing acoustic performance](image)

**Figure 5.** Parametric study on acoustic performance of the water muffler: (a) Symmetric with inner structure in different radiiuses; (b) Symmetric with rubber tube in different radiiuses; (c) Asymmetric with the right inner structure in different radiiuses; (d) Asymmetric with the left inner structure in different radiiuses.

5. Conclusion

Theoretical computation on a previously studied water muffler is carried out in this article. Structure of the water muffler is composed of two main parts, namely, the Kevlar-reinforced rubber tube and the inner-noise-reduction structure. Rubber wall of the rubber tube is assumed to function as rigid wall lined with sound absorption material and is described by a complex radius wave number. Comparison among the results obtained from theoretical computation, FEM simulation and experiment of the rubber tube and that of the water muffler has been made. The theoretical results show a good accordance in general tendency with the FEM simulated and the measured results which confirm the efficiency of the computation method.

Parametric study on the diameter of the inner structure and that of the rubber tube is conducted. Results show that the radius of the left inner structure has the most significant effect on the SPL of the water muffler due to its location and effect on the diameter ratio D2/D1.

Acknowledgments

6
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