The Strange Magnetic Moment of the Proton in the Chiral Quark Model

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Abstract. The strange magnetic moment of the proton is small in the chiral quark model, because of a near cancellation between the quantum fluctuations that involve kaons and \( s \)-quarks and loops that involve radiative transitions between strange vector mesons and kaons.

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1 Introduction

The observation that the strangeness magnetic moment of the proton may be positive \( G_s^M(Q^2 = 0.1 \text{ GeV}^2) = 0.23 \pm 0.37 \) was unexpected as most predictions had given negative values [2, 3, 4, 6]. The expectation of a negative value for \( G_s^M(0) \) may be explained as follows: Consider a \( u \) quark with spin \( s_z \) projection +\( \frac{1}{2} \). If this fluctuates into a kaon and an \( s \) quark, the probability that \( s_z \) of the \( s \) quark be \( -\frac{1}{2} \) is twice that for the value +\( \frac{1}{2} \). Hence, as the charge of the \( s \) quark is \( -e/3 \), the \( s \) quark should contribute a positive amount to the magnetic moment of the \( u \) quark. On the other hand following spin-flip the \( \bar{s} \) quark in the kaon has orbital angular momentum \( l_z = +1 \), and thus as its charge is +\( e/3 \), it should also contribute a positive amount to the net magnetic moment of the \( u \) quark. As there are twice as many \( u \) quarks with \( s_z = +\frac{1}{2} \) as with \( s_z = -\frac{1}{2} \) in the proton it follows that this fluctuation will give a positive contribution to the magnetic moment of the proton. As \( G_s^M \) is defined as the matrix element of \( \bar{s} \gamma^\mu s \) this matrix element is obtained by multiplication of the “conventional” magnetic moment by \(-3\), and thus should be negative.

A positive value for \( G_s^M(0) \) has to arise from transition couplings between strange mesons. It has recently been noted that a fluctuation into a loop with a \( K^* \) meson, which decays into kaon that is reabsorbed with the \( s \) quark gives a positive value \( [2] \). That work based on the nonrelativistic oscillator quark

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model gave the result $G_M^s(0) = 0.035$. It is shown here that the chiral quark model leads to a small negative value for $G_M^s(0)$.

This calculation should be contrasted with those in refs. [3, 4], where the strangeness fluctuations were considered at the hadronic level. The consideration of the strangeness fluctuations of the constituent quarks is motivated by the fact that the magnetic moments of the nucleons are explained by the constituent quark model, but not by the hadronic constituent model [8]. The smaller meson-quark coupling constants imply loop corrections, which are sufficiently small so as not to perturb the overall quark model description of the magnetic moments. The quark model approach automatically takes into account all baryonic intermediate states.

2 Strangeness Loops with Transition Couplings

Consider the (hidden) strangeness fluctuations $q \rightarrow K^* s \rightarrow K \gamma s \rightarrow q$ of a light $u$ or $d$ quark $q$. The key vertex in this loop diagram is the $K^* \rightarrow K \gamma$ vertex, which is described by the transverse current matrix element:

$$< K^\alpha(k') | J_\mu | K^\beta(k) > = -i \frac{g_{K^*K\gamma}}{m_{K^*}} \epsilon_{\mu\lambda\nu\sigma} k_{\lambda} k'_{\nu} \delta^{ab}.$$  (1)

The coupling constant $g_{K^*K\gamma}$ is determined by the empirical decay widths for radiative decay of the $K^*$ as $g_{K^+K^+\gamma} = 0.75$ and $g_{K^-K^-\gamma} = 1.14$.

Given the current matrix element (1) the contributions to the anomalous magnetic moment of the $u$ and $d$ quarks from the $K^*K\gamma$ loop diagrams take the following form (when expressed in terms of nuclear magnetons and after assigning the kaon line a “strangeness charge” of $-1$):

$$G_M^s(0)_{u,d} = -\frac{g_{Kqs}g_{K^*qs}g_{K^*K\gamma}}{2\pi^2} \frac{m_p}{m_{K^*}} \int_0^1 dx (1 - x) \int_0^1 dy \{m_u(m_s - m_u x)$$

$$\{ \frac{1}{G_1} - \frac{1}{G_2} - \frac{1}{G_3} + \frac{1}{G_4} \} - \log\left(\frac{G_2 G_3}{G_1 G_4}\right) \}. \quad (2)$$

Here $m_p$ is the proton mass and $m_u$ and $m_s$ the constituent masses of the $u, d$ and $s$ quarks respectively, which use $m_u = 340$ MeV, $m_d = 460$ MeV [3]. The quantities $G_j$ above are defined as

$$G_1 = G(m_K, m_{K^*}), \quad G_2 = G(A, m_{K^*}), \quad G_3 = G(m_K, A^*), \quad G_4 = G(A, A^*), \quad (3)$$

where the function $G$ is defined as

$$G(m, m') = m^2 (1 - x) - m_u^2 x(1 - x) + m_s^2 x(1 - y) + m'^2 xy. \quad (4)$$

In the case of the $u$ quark the coupling $g_{K^*K\gamma}$ constant is that for the $K^+, K^{*+}$ mesons and in the case of the $d$ quark it is that for the $K^0, K^{*0}$ mesons.

The $\pi -$quark coupling constant is determined by the $\pi NN$ coupling $g_{\pi NN}$ as [3]: $g_{Kqs} = g_{\pi qq} = 3/5(m_q/m_N)g_{\pi NN} = 2.9$. 


The corresponding vector meson coupling constant \( g_{K^*qs} \) is related to the nonstrange vector meson (\( \rho \)) quark coupling strength as

\[
  g_{K^*qs} \simeq g_{\rho qq} = \frac{3}{5}(m_q/m_N)g_{\rho NN}.
\]

With the usual value for the \( \rho NN \) vector coupling constant \( g_{\rho NN}^2/4\pi = 0.52 \) and the including the factor \((1 + \kappa)\), where \( \kappa = 6.6 \) (Table 10) is the \( \rho NN \) tensor coupling, the value \( g_{K^*qs} = g_{\rho qq} = 4.2 \) obtains.

As the \( K^*Ks \) loop diagrams are logarithmically divergent a cut-off is included in their evaluation by insertion of monopole form factors of the form \( (\Lambda^2 - m_{K^*}^2)/(\Lambda^2 + k^2) \) and \( (\Lambda^*^2 - m_{K^*}^2)/(\Lambda^2 + k^2) \) at the \( K \) and \( K^* \) vertices respectively. The cut-off masses \( \Lambda \) and \( \Lambda^* \) are free parameters. We shall here take \( \Lambda = \Lambda^* \) in the numerical calculations.

The contribution from these loops to the strange magnetic moment of the proton is obtained as

\[
  G_M^s(0)_p = \frac{4}{3} G_M^s(0)_u - \frac{1}{3} G_M^s(0)_d,
\]

and thus will have a value close to the corresponding constituent quark values. For \( \Lambda \) values in the range 1–1.5 GeV the contributions are positive, but small (Table 1).

These values are smaller than the corresponding value obtained in ref. [7] with the harmonic oscillator quark model. The difference is due to the non-relativistic nature of the latter which implies an overestimate of the radiative widths of the vector mesons. With the usual static fermion currents of the constituent quark model the coupling constant \( g_{K^*K^\gamma} \) should be 2, which is twice too large.

| \( \Lambda \) (MeV) | \( K^*K^\gamma \) | \( KK^\gamma \) | Sum |
|-----------------|-----------------|-----------------|-----|
| 800             | 0.0             | -0.016          | -0.016 |
| 1000            | 0.006           | -0.031          | -0.025 |
| 1200            | 0.026           | -0.046          | -0.020 |

3 Strange Fluctuations with Diagonal Couplings

The expressions for the contributions from the diagonal \( Ks \) loop contribution to the strange magnetic moments of the \( u \) and \( d \) quarks are similar to the corresponding expressions for the contribution of pionic fluctuation to the neutron magnetic moment [8]. These kaonic loops then give the same contribution to the anomalous strange magnetic moment of the \( u \) and \( d \) quarks. This may be written in the form

\[
  G_M^s(0) = \frac{g_{Kqs}^2}{4\pi^2}m_p \int_0^1 dx (1-x)(m_s - m_u x)(1-x)\left[ \frac{1}{H_1} - \frac{1}{H_4} - 2\frac{(A^2 - m_K^2)}{H_4^2} \right]
\]
Here the quantities $H_j$ are defined as

$$H_1 = H(m_K, m_K), \quad H_2 = H(m_K, \Lambda), \quad H_3 = H(\Lambda, m_K), \quad H_4 = H(\Lambda, \Lambda),$$

where $H(m, m') = (m^2 - m'^2)(1 - x) + m'^2 y + m'^2 x(1 - y)$. \hfill (7)

By (5) the kaonic loop contribution (6) to the constituent quark equals that of the proton. For $\Lambda$ below 1.2 GeV this negative contribution is larger in magnitude than the positive contribution of the $K^* K_s$ loop contributions considered above.

### 4 Discussion

The result obtained here is that the strange magnetic moment of the proton is small because of a strong cancellation between strangeness fluctuations, which involve radiative transition couplings and such which do not. The chiral quark model calculation leads to a stronger cancellation than the non-relativistic quark model calculation in ref. 8.

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