On Shapiro time delay in massive scalar-tensor theories

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Abstract. The problem of definition of the post-Newtonian parameter $\gamma$ in massive scalar-tensor theories is considered. We demonstrate an equivalent correspondence between the post-Newtonian parameter $\gamma$ and the parameter appearing in the equation of a null geodesic in massive scalar-tensor theories. We show that massive scalar-tensor theories can be distinguished from general relativity via the Shapiro time delay. All calculations are performed for hybrid metric-Palatini $f(R)$-gravity for the sake of illustration. The expression for Shapiro time delay in hybrid $f(R)$-gravity is obtained for the first time.

Keywords: Gauss-Bonnet-Lovelock-Horndeski-Palatini etc gravity theories, modified gravity

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1 Introduction

Originally, the parametrized post-Newtonian (PPN) formalism was invented as a universal way to compare theories of gravity with each other and with experiment [1–4]. This formalism is based on a premise that the metric of any metric theory of gravity can be parameterized in a certain form containing only post-Newtonian parameters and post-Newtonian potentials. The potentials do not change from theory to theory, while the set of 10 PPN parameters is unique for each theory of gravity. Moreover, all PPN parameters can be measured experimentally. Due to this fact, efficient tests of gravitational theories become possible. However, the PPN formalism was invented for theories that do not contain massive fields. If the PPN formalism is applied to massive scalar-tensor theories of gravity, there is still ambiguity.

The point is that in gravitational theories with a massive scalar field, in addition to the classical post-Newtonian potentials of the form $1/r$, Yukawa potentials are present [5]. Thus, additional terms appear in the PPN metric that cannot be included in the PPN metric in its original form. It becomes necessary to modify the PPN parameters and to make them distance-dependent (while in the classical PPN formalism PPN parameters are always constants), so that the PPN potentials remain universal for all theories [6]. And then the question arises, do these modified PPN parameters carry the same physical meaning as the previous ones? Are they determined by the same experiments? In this article we answer these questions for a single PPN parameter $\gamma$.

The parameter $\gamma$ has a certain physical meaning in the classical PPN formalism: it is responsible for the effect of light deflection in a field of a massive object [4, 7]. In the case of massless theories this parameter appears in the equation of a null geodesic. The main question is whether the parameter standing in the equation of a null geodesic in massive scalar-tensor theories of gravity coincides with the PPN parameter $\gamma$. Another question is whether such theories will predict the magnitude of the Shapiro delay different from general relativity.

This issue was partially investigated in the context of massive scalar-tensor theories. The most well-known scalar-tensor theory is Brans-Dicke theory [8]. Now a massive version of this theory is of great interest as it is considered as one of the ways to describe the accelerated expansion of the Universe. L. Perivolaropoulos in paper [9] considered the massive version of Brans-Dicke theory and applied PPN formalism to it. He found that PPN parameter $\gamma$ depends not only on parameters of the theory but also on the radial distance from the Sun.

On the other hand the Shapiro time delay was calculated in massive Brans-Dicke model in the work [5] by J. Alsing et al. The authors also found that the parameter in the null geodesic equation (which corresponds to parameter $\gamma$ for massless theories) depends on the distance.
In addition, in other work [10] X.-M. Deng and Y. Xie also calculated Shapiro time delay in massive Brans-Dicke theory and showed that light deflection and Cassini tracking [7] cannot distinguish massive scalar-tensor theory from general relativity (GR). In this work [10], authors obtained a result that $\gamma$ in massive scalar-tensor theories will be equal to $\gamma_{GR}$ in GR ($\gamma_{GR} = 1$). In all these works scalar-tensor theory with an arbitrary coupling function $\omega(\phi)$ and a generic potential $V(\phi)$ was considered.

In addition to the Brans-Dicke theory, in Horndeski gravity the light deflection was also calculated in two ways. In the work [11], S. Hou and Y. Gong found an explicit form of the PPN parameter $\gamma$, first from the post-Newtonian metric, and then from an expression for the Shapiro time delay. They obtained the same result in both cases, but since this was not the main goal of the article, so the authors did not pay much attention to the discussion of this result. Moreover, the PPN parameter $\gamma$ for Horndeski gravity was obtained earlier in the work [12].

In this article we demonstrate an equivalent correspondence between the PPN parameter $\gamma$ and the parameter appearing in the null geodesic equation for one more massive scalar-tensor theory. For this purpose, we consider hybrid metric-Palatini f(R)-gravity which can be represented as massive scalar-tensor theory [13]. Moreover, we show that massive scalar-tensor theories and general relativity predict different results regarding the magnitude of Shapiro delay in the gravitational field of a massive object.

The hybrid metric-Palatini f(R)-gravity belongs to a family of f(R)-theories [14, 15]. The action in f(R)-theories is constructed by a generalization of the gravitational part of the Einstein-Hilbert action as an arbitrary function of the curvature $R$. There are two possible approaches that can be used to obtain field equations from those modified actions: the metric one and the Palatini one. In the metric approach $g_{\mu\nu}$ is the only dynamical variable. Furthermore, the action is varied in respect to only $g_{\mu\nu}$. The Palatini method considers an idea of a connection which defines the Riemann curvature tensor to be a priori independent from the metric. Thus, variations with respect to the metric and to the connection are performed independently. Additionally, the Palatini method provides second order differential field equations, while in the metric approach these equations are of fourth order [16, 17]. Unfortunately, both methods lead to some unsolvable problems. The metric f(R)-theories, in general, cannot pass the standard Solar System tests without invoking any kind of screening mechanism [18–20]. All the Palatini f(R)-models aimed to explain the accelerated Universe expansion lead to microscopic matter instabilities and to unacceptable features in evolution patterns of cosmological perturbations [21, 22]. Hybrid f(R)-gravity was constructed as a mixture of Palatini and metric f(R)-theories. It unites all of the advantages of both approaches but lacks their shortcomings [23].

In a recent work [6] it was shown that a presence of a light scalar field in hybrid f(R)-gravity does not contradict the experimental data from the Solar system. This conclusion is based not only on the $\gamma$ parameter, but on all the other parameters of the post-Newtonian formalism. Thus, in contrast to metric f(R)-theory (excluding some metric f(R)-models [24–26]) hybrid f(R)-gravity can pass the full post-Newtonian test. Therefore, it will be interesting to show the equality of the $\gamma$ parameters obtained from the PPN metric directly and from the equation of a null geodesic for massive scalar-tensor theories.

The structure of the paper is the following. In section 2 we consider an action and field equations of the hybrid metric-Palatini theory in a general form and in a scalar-tensor representation. In section 3, we calculate the PPN parameter $\gamma$ and Shapiro time delay in detail. We conclude in section 4 with a summary and discussion.
Throughout this paper the Greek indices \((\mu, \nu, \ldots)\) run over \(0, 1, 2, 3\) and the signature is \((- , +, +, +)\). We will be working in units \(h = c = k_B = 1\) throughout the paper.

2 Hybrid f(R)-gravity

The action of hybrid f(R)-gravity has the form \([13, 23]\):

\[
S = \frac{1}{2k^2} \int d^4x \sqrt{-g} [R + f(\mathcal{R})] + S_m, \tag{2.1}
\]

where \(k^2 = 8\pi G\), \(G\) is the Newtonian gravitational constant, \(R\) and \(\mathcal{R} = g^{\mu\nu} R_{\mu\nu}\) are the metric and Palatini curvatures respectively, \(g\) is the metric determinant, \(S_m\) is the matter action. Here the Palatini curvature \(\mathcal{R}\) is defined as a function of \(g_{\mu\nu}\) and the independent connection \(\hat{\Gamma}^\alpha_{\mu\nu}\):

\[
\mathcal{R} = g^{\mu\nu} R_{\mu\nu} = g^{\mu\nu}(\hat{\Gamma}^\alpha_{\mu\alpha,\nu} - \hat{\Gamma}^\nu_{\alpha\lambda} \hat{\Gamma}^\lambda_{\mu\alpha} - \hat{\Gamma}^{\alpha\mu}_{\lambda\nu} \hat{\Gamma}^\lambda_{\mu\alpha}). \tag{2.2}
\]

Like in the pure metric and Palatini cases, the hybrid f(R)-gravity (2.1) can be rewritten in a scalar-tensor representation (for details see \([13, 23]\)):

\[
S = \frac{1}{2k^2} \int d^4x \sqrt{-g} \left[(1 + \phi)R + \frac{3}{2\phi} \partial_\mu \phi \partial^\mu \phi - V(\phi)\right] + S_m, \tag{2.3}
\]

where \(\phi\) is a scalar field and \(V(\phi)\) is a scalar field potential. This potential \(V(\phi)\) is related to the Palatini curvature \(\mathcal{R}\) by the following expression

\[V_{\phi} = \mathcal{R},\]

where \(V_{\phi}\) is the derivative of the potential \(V(\phi)\) with respect to the scalar field \(\phi\) (for details see \([13, 23]\)). Here and further we use the Jordan frame.

Then the metric and scalar field equations take the following forms \([13, 23]\):

\[
(1 + \phi)R_{\mu\nu} = k^2 \left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right) + \nabla_\mu \nabla_\nu \phi + \frac{1}{2} g_{\mu\nu} \left[V(\phi) + \nabla_\alpha \nabla^\alpha \phi\right] - \frac{3}{2\phi} \partial_\mu \phi \partial_\nu \phi, \tag{2.4}
\]

\[
\nabla_\mu \nabla^\mu \phi - \frac{1}{2\phi} \partial_\mu \phi \partial^\mu \phi - \frac{\phi[2V(\phi) - (1 + \phi)V_{\phi}]}{3} = -\frac{k^2}{3} \phi T, \tag{2.5}
\]

where \(T_{\mu\nu}\) and \(T\) are the energy-momentum tensor and its trace respectively.

3 Light deflection

3.1 PPN parameter \(\gamma\)

Firstly we show a calculation of PPN parameter \(\gamma\) in hybrid f(R)-gravity from PPN metric directly. To achieve this goal it is necessary to solve the field equations in the PPN approximation up to the order \(O(2)\) \([4, 27, 28]\). To obtain the linearized field equations in the weak-field limit we consider the following perturbations of a scalar field and metric tensor:

\[
\phi = \phi_0 + \varphi, \quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{3.1}
\]

where \(\phi_0\) is the asymptotic background value of the scalar field far away from a source, \(\eta_{\mu\nu}\) is the Minkowski background, \(h_{\mu\nu}\) and \(\varphi\) are small perturbations of tensor and scalar fields.
respectively. Here we consider \( \phi_0 \) as a constant. The scalar potential \( V(\phi) \) could be expanded in a Taylor series around the background value of the scalar field \( \phi_0 \) like

\[
V(\phi) = V_0 + V' \varphi + \frac{V'' \varphi^2}{2!} + \frac{V''' \varphi^3}{3!} \ldots,
\]

(3.2)

hence its derivative with respect to \( \varphi \) will be the following:

\[
V_\varphi = V' + \frac{V'' \varphi}{2!} + \frac{V''' \varphi^2}{3!} + \ldots.
\]

The field equation for the scalar field (2.5) in the leading perturbation order (\( O(2) \)) takes the form [6, 13, 23]:

\[
\left( \nabla^2 - m^2 \varphi \right) \varphi^{(2)} = -k^2 \phi_0 T,
\]

(3.3)

where we denote \( m^2 \varphi = \left[ 2V_0 - V' - (1 + \phi_0) \phi_0 V'' \right] / (2 \varphi) \) as a scalar field mass. The superscript \( (2) \) indicates the order of perturbation.

The linearized equations for the metric is given by [13, 23]

\[
- \frac{1}{2} \nabla^2 h_{\mu \nu} = - \frac{k^2}{(1 + \phi_0)} \left( T_{\mu \nu} - \frac{1}{2} T \eta_{\mu \nu} \right) + \frac{V_0 + \nabla^2 \varphi}{2(1 + \phi_0)} \eta_{\mu \nu}.
\]

(3.4)

To obtain this result we used Nutku gauge conditions [29]:

\[
h^\alpha_{\beta, \alpha} - \frac{1}{2} \delta^\alpha_{\beta} h^\mu_{\mu, \alpha} = \frac{\varphi, \beta}{1 + \phi_0}.
\]

(3.5)

Thus, the leading order of metric perturbations is defined as [6, 23]:

\[
h^{(2)}_{\mu \nu} = \frac{k^2}{4\pi(1 + \phi_0)} \frac{M}{r} \left( 1 - \frac{\phi_0}{3} e^{-m \varphi r} \right) + \frac{V_0}{1 + \phi_0} \frac{\varphi^2}{6},
\]

(3.6)

\[
h^{(2)}_{ij} = \frac{\delta_{ij} k^2}{4\pi(1 + \phi_0)} \frac{M}{r} \left( 1 + \frac{\phi_0}{3} e^{-m \varphi r} \right) - \delta_{ij} \frac{V_0}{1 + \phi_0} \frac{\varphi^2}{6},
\]

(3.7)

where \( M \) is the Solar mass and \( \delta_{ij} \) is the Kronecker delta. Here \( V_0/(\phi_0 + 1) \) is the cosmological constant term. It can be negligible at the Solar System scale in order not to affect the local dynamics [30]. That is why further this term is not considered [6].

Since \( h_{00} \) up to the order \( O(2) \) in the general post-Newtonian metric is defined as [4, 27, 28]

\[
h^{(2)}_{00} = 2 \frac{G^{\text{eff}} M}{r},
\]

(3.8)

it is possible to extract the effective gravitational constant from (3.6) [6, 13, 23]:

\[
G^{\text{eff}} = \frac{k^2}{8\pi(1 + \phi_0)} \left( 1 - \frac{\phi_0}{3} e^{-m \varphi r} \right).
\]

(3.9)

Throughout the article we add a superscript \( \text{eff} \) to the PPN parameters which are considered as spatially dependent functions.

After that we can extract PPN parameter \( \gamma^{\text{eff}} \) from the expression (3.7). The metric perturbation \( h_{ij} \) up to the order \( O(2) \) in the general post-Newtonian metric has the following form [4, 27, 28]:

\[
h^{(2)}_{ij} = 2 \gamma^{\text{eff}} \frac{G^{\text{eff}} M}{r} \delta_{ij}.
\]

(3.10)
Therefore, from the equation (3.7) we can express $\gamma_{\text{eff}}$ \cite{6, 13, 23}:

$$\gamma_{\text{eff}} = \frac{1 + \phi_0 e^{-m \varphi^2 / 3}}{1 - \phi_0 e^{-m \varphi^2 / 3}}. \quad (3.11)$$

As we can see, in hybrid f(R)-gravity, the PPN parameter $\gamma_{\text{eff}}$ depends on the distance as in the massive Brans-Dicke theory \cite{5}. This was an expected result since both theories are massive scalar-tensor models. In addition, the issue of applying the PPN formalism to massive scalar-tensor theories and the dependence of PPN parameters on a distance was considered in the recent paper \cite{6}.

### 3.2 Shapiro time delay

Secondly, we calculate the Shapiro time delay in the case of hybrid metric-Palatini f(R)-gravity. The equation of a photon motion along a null geodesic has the form:

$$g_{\mu \nu} u^\mu u^\nu = 0, \quad (3.12)$$

where $u^\mu = dx^\mu_a / d\tau_a$ is a four-velocity of a photon, $\tau_a$ is the proper time of particle $a$ measured along its worldline $x^\mu_a$.

This equation can be expressed up to the order $O(2)$ as:

$$-1 + h^{(2)}_{00} + \left( \delta_{ij} + h^{(2)}_{ij} \right) u^i u^j = 0. \quad (3.13)$$

Substituting (3.6) and (3.7) into (3.13), the equation (3.12) takes the following form:

$$-1 + \frac{k^2}{4\pi(1 + \phi_0)} \left( 1 - \frac{\phi_0}{3} e^{-m \varphi^2} \right) \frac{M}{r} + \left( 1 + \frac{k^2}{4\pi(1 + \phi_0)} \left( 1 + \frac{\phi_0}{3} e^{-m \varphi^2} \right) \frac{M}{r} \right) |u|^2 = 0. \quad (3.14)$$

If the photon was emitted at the point $x_e$ in the direction of $n$ at the time $t_e$, then its trajectory is described by the expression:

$$x^i(t) = x^i_e + n^i (t - t_e) + x^i_{PN}(t), \quad (3.15)$$

taking into account the post-Newtonian corrections $x^i_{PN}(t)$. Substituting this expression into (3.14), we obtain the following equation:

$$\mathbf{n} \cdot \frac{d\mathbf{x}_{PN}(t)}{dt} = \frac{dx^\parallel_{PN}(t)}{dt} = -\frac{k^2}{8\pi(1 + \phi_0)} \frac{M}{r}. \quad (3.16)$$

Then the time of a photon traveling from the $x_e$ to $x$ and back will be equal to

$$\Delta t = 2|x - x_e| - \frac{k^2}{8\pi(1 + \phi_0)} \int_{t_e}^t \frac{M}{r(t')} dt'. \quad (3.17)$$

The second term on the right-hand side of this equation is the correction $\delta t$ due to the Shapiro delay. It can be obtained after an integration:

$$\delta t = 4 \frac{k^2 M}{8\pi(1 + \phi_0)} \ln \left( \frac{(r_e + r_e \cdot \mathbf{n})(r_p - r_p \cdot \mathbf{n})}{r_b^2} \right), \quad (3.18)$$
where the photon is emitted from $r_e$ in the direction of $n$, traveling to $r_p$ and back again, $M$ is the mass of a body causing the time-delay and $r_b$ is the impact parameter of a photon with respect to the source. The most important point is that the mass appearing in (3.18) is not a measurable quantity [5]. The quantity that is actually measured is the Keplerian mass:

$$M_k = G_{\text{eff}} M = M \frac{k^2}{8\pi(1 + \phi_0)} \left(1 - \frac{\phi_0}{3} e^{-m_{\text{eff}} r} \right),$$

where $r$ should be thought of as a fixed quantity which depends on an exact way by which the Keplerian mass of the body was determined. For example, in the case of the Solar System $r$ should be set to 1 AU, since this is the typical scale used to determine the Keplerian mass of the Sun. In terms of $M_k$ we find

$$\delta t = \frac{4M_k}{1 - \frac{\phi_0}{3} e^{-m_{\text{eff}} r}} \ln \left[\frac{(r_e + \vec{r}_e \cdot \vec{n}) (r_p - \vec{r}_p \cdot \vec{n})}{r_b^2} \right],$$

This equation can be expressed as

$$\delta t = 2M_k (1 + \tilde{\gamma}) \ln \left[\frac{(r_e + \vec{r}_e \cdot \vec{n}) (r_p - \vec{r}_p \cdot \vec{n})}{r_b^2} \right],$$

where

$$\tilde{\gamma} = \frac{1 + \phi_0 e^{-m_{\text{eff}} r}/3}{1 - \phi_0 e^{-m_{\text{eff}} r}/3}. \quad (3.22)$$

Thus we obtain that

$$\tilde{\gamma} = \gamma_{\text{eff}} \quad (3.23)$$

in hybrid f(R)-gravity. It is worth noting that “$r$” is the distance at which the Keplerian mass of the Sun is measuring in both parameters $\tilde{\gamma}$ and $\gamma_{\text{eff}}$. Therefore, the deflection of a light ray by the gravitational field of the Sun depends not only on the mass of an object but also on the model parameters and the distance $r$. This is an important difference between massive scalar-tensor theories of gravity and general relativity. It should be said separately that the expression for Shapiro time delay in hybrid f(R)-gravity was obtained in this work for the first time. The restrictions imposed by $\gamma$ parameter on the parameters of the theory were obtained earlier in the works [6, 31].

It is important to emphasize that the expression of the Shapiro delay (3.18) contains the mass of a gravitating object. For a calculating of the value of the Shapiro delay, it is necessary to use the experimental values of the quantities included in the theoretical expression. The mass of a gravitating object can be obtained from Kepler’s third law, which is modified in massive theories of gravity [32]. Instead of the Newtonian gravitational constant Kepler’s third law contains an effective gravitational constant. We cannot use the “true” mass of the object, since it is unknown from the experiment. Then it is necessary to change the mass in the expression (3.18) for the mass recovered from experiments and in accordance with Kepler’s third law it is $G_{\text{eff}} M$. It is the key difference between the our work and the article [10], since the authors of paper [10] did not take into account the necessity to use the Keplerian mass for calculating Shapiro delay.
4 Conclusions and discussion

In this investigation, we posed two questions: whether the parameter standing in the equation of a null geodesic in massive scalar-tensor theories of gravity coincides with the PPN parameter $\gamma$ and whether such theories will predict the magnitude of the Shapiro delay different from general relativity. We discussed these issues using the hybrid metric-Palatini f(R)-gravity which can be represented as massive scalar-tensor theory [13, 23]. In the framework of the considered theory we showed that the post-Newtonian parameter $\gamma^{\text{eff}}$ calculated from the post-Newtonian metric directly is equal to the parameter $\tilde{\gamma}$ obtained from the expression for the Shapiro time delay of a light ray passing near a massive body. Therefore, in the case of hybrid f(R)-gravity the parameters are identical. In addition the Shapiro time delay in hybrid f(R)-gravity was obtained in this work for the first time.

Previously, similar studies were conducted in a framework of the massive Brans-Dicke theory. In paper [9] the post-Newtonian parameter $\gamma$ was calculated from the post-Newtonian metric and later in [5] the Shapiro time delay was calculated. Similar to the case of hybrid f(R)-theory, both methods had led to the same result. Later on this problem was considered in [10]. Authors had also calculated the Shapiro time delay in the massive Brans-Dicke theory. However, they found that the light deflection and Cassini tracking [7] cannot distinguish a massive scalar-tensor theory from GR. This conclusion is based on the result obtained by the authors of paper [10] that the $\gamma$ parameter in massive Brans-Dicke theory is equal to the $\gamma_{GR} = 1$. They also demonstrate that the Shapiro delay is determined only by the mass of an object and does not depend on the parameters of a massive scalar-tensor theory. These conclusions are not consistent with result obtained earlier in paper [5] and in our work. The difference in results is that in the latter work [10] it was not taken into account an inequality of the observed Keplerian mass and mass of a body causing the time-delay.

In the paper [11] authors had obtained expressions for PPN parameter $\gamma$ using the two methods that were previously discussed in Horndeski theory. Both expressions were identical (see eq. (38) and eq. (50)-(51) in the paper [11]). Since Horndeski gravity is the most general scalar-tensor theory providing second-order field equations which evades Ostrogradski instabilities, this equality can be applied for all special cases of the Horndeski theory. This fact was demonstrated on the examples of Brans-Dicke theory and hybrid f(R)-gravity.

Based on the foregoing, we can conclude that both massless and massive scalar-tensor theories can be distinguished from GR based on the data of light deflection by the gravitational field of the Sun. In addition, within a massive scalar-tensor theory the light deflection is determined by the PPN parameter $\gamma$ as in the case of a massless theory. However, the difference between these models is that the parameter $\gamma$ in massive scalar-tensor theories depends on the distance at which the Keplerian mass was measured. Therefore, it can be argued that the PPN formalism gives the same predictions as a direct calculation of the Shapiro time delay and it is applicable to massive scalar-tensor theories.

This work is the first step on a way to obtain a universal apparatus for testing gravitational theories with massive fields in the weak-field limit which would be similar to the original PPN formalism for massless theories. The construction of such a formalism will be the aim of further extensive research in the future.

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– 7 –
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