A Comparison of the LVDP and ΛCDM Cosmological Models

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Abstract

We compare the cosmological kinematics obtained via our law of linearly varying deceleration parameter (LVDP) with the kinematics obtained in the ΛCDM model. We show that the LVDP model is almost indistinguishable from the ΛCDM model up to the near future of our universe as far as the current observations are concerned, though their predictions differ tremendously into the far future.

Keywords: Variable deceleration parameter; Accelerating universe; Dark energy; Big rip

In a recent paper (Ref. [1]), we proposed a special law (LVDP) for the deceleration parameter $q = -kt + m - 1$ that is linear in cosmic time $t$, where $k > 0$ is a constant with the dimensions of inverse time and $m > 1$ is a dimensionless constant. This law allows us to generalize many exact cosmological solutions that one finds in the literature with a constant deceleration parameter ($q = m - 1$, see [2]), so as to obtain an expansion history of the universe that fits better with the observations. For instance, with the choice $k = 0.097$ and $m = 1.6$ in Ref. [1], we set $q = -0.73$ for the present universe (13.7 Gyr old) and predict that the transition from the decelerating to accelerating expansion should occur at $t_t \approx 6.2$ Gyr and at cosmic redshift value $z_t \approx 0.5$. Both of these values are consistent with current cosmological data.

The standard ΛCDM cosmological model is the simplest and arguably the one that most successfully describes the evolution of the observed universe. However, it suffers from important conceptual problems related with the presence of a cosmological constant. Besides that, the analyses of the cosmological data not only suggest an equation of state (EoS) parameter value $w \sim -1$ for the dark energy component of the universe but also do not exclude a time dependent EoS parameter that can pass below the phantom divide line ($w = -1$), i.e., the quintom dark energy and hence a Big Rip, in the future of the universe [4]. Nevertheless, ΛCDM is still considered a reference cosmological model such that any viable model should exhibit similar kinematics with the ΛCDM model up to the present age of the universe, independent of what it would predict for the future kinematics of the universe. Therefore, in this letter, we compare LVDP and ΛCDM models, and show that they cannot be distinguished observationally today, but would differ tremendously in the relatively far future. LVDP model predicts that the universe eventually enters into the super-exponential expansion phase and ends with a Big Rip while ΛCDM model predicts that the universe monotonically approaches the de Sitter universe. We compare the behavior of the scale factors, Hubble and deceleration parameters of the ΛCDM and LVDP models. We also compare the jerk and the snap parameters that involve, respectively, the third and the fourth derivatives of the scale factors. The jerk and snap parameters were not given before when the LVDP ansatz was first proposed in Ref. [1].

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We describe our observed universe by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric

\[ ds^2 = -dt^2 + a^2(t) \frac{dx^2 + dy^2 + dz^2}{1 + \frac{\kappa}{3}(x^2 + y^2 + z^2)} , \]  

where \(a(t)\) is the cosmic scale factor and the spatial curvature index \(\kappa = -1, 0, 1\) corresponds to spatially open, flat and closed universes, respectively. We introduce four cosmological parameters that describe the kinematics of the universe, namely the Hubble parameter and three (dimensionless) parameters; the deceleration, jerk and snap parameters given, respectively, as

\[ H = \frac{\dot{a}}{a}, \quad q = -\frac{\ddot{a}}{aH^2}, \quad j = \frac{\dddot{a}}{aH^3}, \quad s = \frac{\ddddot{a}}{aH^4}, \]  

where an overdot denotes \(d/dt\) (see \[5\] \[6\] \[7\]).

Einstein’s field equations are solved in the standard ΛCDM model for a mixture of pressure-less matter (including cold dark matter, CDM) and a cosmological constant \(\Lambda\) with positive sign for the spatially flat space-time (\(\kappa = 0\)). The kinematics for ΛCDM model follows \[8\] \[9\]:

\[ a_{\Lambda\text{CDM}} = a_1 \sinh^\frac{3}{2} \left( \frac{\Lambda}{3} t \right), \quad H_{\Lambda\text{CDM}} = \frac{1}{3} \coth \left( \frac{\Lambda}{3} t \right), \quad q_{\Lambda\text{CDM}} = 1, \quad j_{\Lambda\text{CDM}} = \frac{9}{2} + \frac{9}{2} \tanh^2 \left( \frac{\Lambda}{3} t \right). \]  

In the LVDP model on the other hand, we introduce the LVDP ansatz \(q = -kt + m - 1\) from the beginning and obtain the effective energy-momentum tensor by substituting the corresponding scale factor into the Einstein’s field equations, rather than introducing first the matter fields (see \[1\] for details). We find

\[ a_{\text{LVDP}} = a_2 e^{\frac{3}{2} \arctanh \left( \frac{1}{2} t^{-1} \right)}, \quad H_{\text{LVDP}} = -\frac{2}{3(kt-2m)}, \quad q_{\text{LVDP}} = -kt + m - 1, \quad j_{\text{LVDP}} = \frac{9}{2} k^2 t^2 - 3k (m - 1) t + (2m - 1)(m - 1) \]  

and

\[ s_{\text{LVDP}} = 3 k^2 t^3 - 9k^2 (m - 1) t^2 + 6k (2m - 1)(m - 1) t - 6m^3 + 11 m^2 - 6m + 1. \]

We would like to note here that the kinematics of the LVDP model can be generated from a constant such that \(k = -\dot{q}\), while the kinematics of the ΛCDM could be generated from the jerk parameter value \(j = 1\) \[8\] \[9\] \[10\].

Having obtained all the cosmological parameters both for the LVDP and ΛCDM models, we are now able to compare them. We choose \(q_0 = -0.650\), in agreement with the more recent analyses of observational results (e.g., see \[11\] \[12\] \[13\]) of the deceleration parameter of the present day universe \(t_0 = 13.700\) (Gyr), and obtain \(\Lambda = 0.013\) for the ΛCDM model, which in return implies that the universe started to accelerate at \(t_c = 6.650\) (Gyr), i.e., \(t_0 - t_c = 7.050\) (Gyr) ago. Because these values are in good agreement with the observational studies and we want to compare ΛCDM and LVDP models, we simply use the above values to obtain the constants of LVDP model such that \(k = 0.092\) and \(m = 1.613\). We can also safely set \(a_{\Lambda\text{CDM}} = a_{\text{LVDP}} = 10\) at \(t = 0\) for simplicity.

Because our main goal in this letter is to compare the LVDP and ΛCDM models, for convenience, we regard the time parameter as dimensionless by taking \(t \rightarrow \frac{t}{10\text{Gyr}}\) and indicate Gyr in parantheses, i.e., (Gyr), to remind the reader of this.
$t_0 = 13.700$ (Gyr) by choosing $a_1 = 6.727$ and $a_2 = 13.144$. Determining all the constants we now know the time evolution of all the cosmological parameters both in $\Lambda$CDM and LVDP models. In Table 1 we calculate the values of all the cosmological parameters for four different ages of the universe that are cosmologically interesting and help us to compare the two models. The chosen ages are as follows: $t = t_t = 6.650$ (Gyr) when the universe starts accelerating in both models, $t = 13.700$ (Gyr) the present-day universe in both models, $t = 17.496$ (Gyr) when the universe reaches the exponential expansion and starts super-exponential expansion in LVDP model and $t = 34.992$ (Gyr) when the universe ends with a Big Rip in LVDP model. We note that the

| $t_t = 6.650$ (Gyr) | $t_0 = 13.700$ (Gyr) | $t = 17.496$ (Gyr) | $t = 34.992$ (Gyr) |
|------------------|------------------|------------------|------------------|
| $\Lambda$CDM | LVDP | $\Lambda$CDM | LVDP | $\Lambda$CDM | LVDP | $\Lambda$CDM | LVDP |
| $a$ | 5.339 | 5.351 | 10.000 | 10.00 | 13.168 | 13.144 | 42.660 | $\infty$ |
| $H$ | 0.114 | 0.115 | 0.075 | 0.074 | 0.070 | 0.071 | 0.066 | $\infty$ |
| $q$ | 0.000 | 0.000 | -0.650 | -0.650 | -0.823 | -1.000 | -0.994 | -2.613 |
| $j$ | 1.000 | 0.801 | 1.000 | 1.435 | 1.000 | 2.301 | 1.000 | 11.044 |
| $s$ | -2.000 | -1.602 | -0.050 | 2.346 | 0.471 | 6.204 | 0.982 | 64.488 |

Table 1: Values of the cosmological parameters for the $\Lambda$CDM and LVDP models at some crucial times.

deviations between the LVDP parameters and the $\Lambda$CDM parameters are negligibly small both during the time of transition and today and hence during all this time interval from the transition time to the present-day universe, since all the parameters vary monotonically during this time interval, as would be seen from the figures. The values of the jerk parameters are separated slightly, but remain in agreement with observations both for the LVDP and $\Lambda$CDM models. The snap parameters, on the other hand, get separated more compared to all the other cosmological parameters. However, the definition of the snap parameter involves the fourth derivative of the scale factor, and hence it is not an easy task to resolve observationally any deviation between the snap parameter values of the above models. Therefore, we conclude that the behavior of the LVDP and $\Lambda$CDM models are almost indistinguishable during the observed past of the universe. To substantiate this conclusion, we depict the scale factors in Fig.1, Hubble parameters in Fig.2, deceleration parameters in Fig.3, jerk parameters in Fig.4 and snap parameters in Fig.5 for $\Lambda$CDM (dashed lines) and for LVDP (solid line).

**Figure 1:** Scale factors $a$ versus cosmic time $t$ for the LVDP (solid) and $\Lambda$CDM (dashed) models. The vertical line represents the present time of the universe 13.7 (Gyr). The scale factor diverges at $t = 34.992$ (Gyr), i.e., Big Rip occurs, in the LVDP model.

Any separation between the $\Lambda$CDM and LVDP models do not grow continuously and hence the two models remain indistinguishable up to the present age of the universe. A continuous
separation between the two models just start after the present age of the universe. Hence both models would exhibit a similar behavior in the near future but evolve rather differently into the far future. The LVDP model reaches the exponential expansion phase \( q_{\text{LVDP}} = -1 \) at \( t = 17.496 \) (Gyr) and enters into a super-exponential expansion phase \( (q_{\text{LVDP}} < -1) \), while \( q_{\Lambda\text{CDM}} = -0.824 \). Moreover, in the LVDP model the size of the space and the Hubble parameter diverge at \( t = 34.992 \) (Gyr); while the size of the space remains finite and exhibits a power-law expansion with a deceleration parameter value \( q_{\Lambda\text{CDM}} = -0.994 \) for \( \Lambda\text{CDM} \) at \( t = 34.992 \) (Gyr). The divergence of the Hubble parameter in LVDP model, because the square of the Hubble parameter \( H^2 \) is proportional to the effective energy density of the universe in general relativity, also tells us that the energy-density of the universe diverges for \( t = 34.992 \) (Gyr). Hence, the universe ends with a Big Rip at \( t = 34.992 \) (Gyr) in the LVDP model, while it is still approaching the de Sitter phase in the \( \Lambda\text{CDM} \) model, where the universe is empty and only a vacuum energy (i.e., the cosmological constant) exists.

In conclusion, the cosmological kinematics we obtain via the LVDP ansatz are almost indistinguishable from those of the \( \Lambda\text{CDM} \) model up to the near future of our universe. Therefore, because the \( \Lambda\text{CDM} \) model fits the observational data quite well all the while, the LVDP would do so as well. The current observational data give us information concerning the past kinematics of the universe only and hence we do not have any reason to favor the \( \Lambda\text{CDM} \) model over the LVDP model on the observational basis alone. In the LVDP model, the effective energy-momentum tensor is not introduced as a source but rather is obtained using the LVDP kinematics in the gravitational field equations. The effective source thus we obtained [1] in Einstein’s theory
of general relativity exhibits a quintom DE-like behavior. Hence we say that the accelerated expansion in our LVDP model is driven by a quintom DE field. However, this has to be checked if one uses a generalized theory of gravity and/or includes a ‘DE component’ of the universe from the start. In ΛCDM model, on the other hand, the accelerated expansion is driven by the inclusion of a positive cosmological constant into the Friedmann equations in the presence of a dust source. However, the presence of a cosmological constant gives rise also to one of the most pressing conceptual problems in physics that one may avoid in the LVDP model.

Finally, we wish to re-assert that the LVDP ansatz can safely be used for generating exact cosmological models that generalize, in particular, many of the models in the literature already obtained via the constant deceleration parameter ansatz, which cannot be consistent with all observations since such models do not exhibit a transition from a decelerating expansion to an accelerating expansion, whereas LVDP ansatz does this consistently with observations. Furthermore, the LVDP ansatz is a good candidate for studying cosmological models with Big Rip futures and yet remain consistent with the present-day observations.

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References

[1] Akarsu, O., Dereli, T.: Cosmological models with linearly varying deceleration parameter. International Journal of Theoretical Physics 51, 621-621 (2012)

[2] Berman, M.S.: A special law of variation for Hubble’s parameter. Nuovo Cimento B 74, 182-186 (1983)

[3] Cunha, J.V.: Kinematic Constraints to the Transition Redshift from SNe Ia Union Data. Physical Review D 79, 047301 (2009)

[4] Cai, Y.F., Saridakis, E.N., Setare, M.R., Xia, J.Q.: Quintom cosmology: Theoretical implications and observations. Physics Reports 493, 1-60 (2010)

[5] Sahni, V., Saini, T.D., Starobinsky, A.A., Alam, U.: Statefinder-A new geometrical diagnostic of dark energy. JETP Letters 77, 201-206 (2003)

[6] Visser, M.: Jerk, snap and the cosmological equation of state. Classical and Quantum Gravity 21, 2603-2615 (2004)

[7] Dunajski, M., Gibbons, G.: Cosmic Jerk, Snap and Beyond. Classical and Quantum Gravity 25, 235012 (2008)

[8] Sahni, V., Saini, T.D., Starobinsky, A.A.: The Case for a Positive Cosmological Λ-Term. International Journal of Modern Physics D 9, 373-443 (2000)

[9] Grøn, Ø., Hervik, S.: Einstein’s General Theory of Relativity: With Modern Applications in Cosmology, Springer, 2007.

[10] Komatsu, E., et al.: Seven-year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation. The Astrophysical Journal Supplement 192, 18 (2011)

[11] Li, Z., Wu, P., Yu, H.: Examining the cosmic acceleration with the latest Union2 supernova data. Physics Letters B 695, 1-8 (2011)

[12] Yun, C., Bharat, R.: Hubble parameter data constraints on dark energy. Physics Letters B 703, 406-411 (2011)