CLEO Determinations of $|V_{cb}|$ from Inclusive Moments

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Moments of $\bar{B} \rightarrow X_c \ell \bar{\nu}$ and $B \rightarrow X_s \gamma$ decays can determine nonperturbative QCD parameters that relate the semileptonic decay width to $|V_{cb}|$. CLEO pioneered measurement of these moments, determined the relevant QCD parameters from the measured moments, and used these parameters to determine $|V_{cb}|$.

The width $\Gamma_{\text{SL}} \equiv \Gamma(\bar{B} \rightarrow X_c \ell \bar{\nu}) = B(\bar{B} \rightarrow X_c \ell \bar{\nu})/\tau_B$ for inclusive semileptonic decay to all charm states $X_c$ is related to the CKM matrix element $|V_{cb}|$ by $\Gamma_{\text{SL}} = \gamma_c |V_{cb}|^2$. Hence, $|V_{cb}|$ can be determined from measurements of the lifetime $\tau_B$ and the branching fraction for $\bar{B} \rightarrow X_c \ell \bar{\nu}$ decay, if the parameter $\gamma_c$ is known. Unfortunately $\gamma_c$ is a nonperturbative QCD parameter, and previously theoretical models had been the only means of estimating $\gamma_c$ [1, 2]. CLEO pioneered determination of $\gamma_c$ from measurements of energy moment of $B \rightarrow X_s \gamma$ decays [3] and measurements of hadronic mass moments in $B \rightarrow X_c \ell \bar{\nu}$ decays [4], and confirmed the results with measurements of lepton energy moments [5] in $\bar{B} \rightarrow X_c \ell \bar{\nu}$ decays. These and other moment analyses are reviewed in Ref. [2].

1 Theoretical Framework

The theoretical framework for these measurements is Heavy Quark Effective Theory, the Operator Product Expansion, and the assumption of parton-hadron duality in inclusive semileptonic $B$ decays. These lead to theoretical predictions that observables in $B \rightarrow X_s \gamma$ and $\bar{B} \rightarrow X_c \ell \bar{\nu}$ decays can be expanded in inverse powers of the $B$ meson mass $M_B$ [6]. To order $1/M_B^4$ the parameter $\gamma_c$ is

$$\gamma_c = \frac{G_F^2 M_B^5}{192 \pi^3} G(\bar{\Lambda}, \lambda_1, \lambda_2, \rho_1, \rho_2, T_1, T_2, T_3, T_4)$$

where $G$ is a polynomial in $1/M_B$ and $\bar{\Lambda}, \lambda_1, \lambda_2, \rho_1, \rho_2, T_1, T_2, T_3,$ and $T_4$ are nonperturbative QCD parameters. Of these parameters: $\bar{\Lambda}$ appears in all orders above $0$ in $1/M_B$; $\lambda_1$ and $\lambda_2$ first appear in second order; and the others first appear in third order. Some of the coefficients of the polynomial involve expansions in $\alpha_S$, which are carried out to order $\beta_0 \alpha_S^2$. The power series and the results depend on the renormalization scheme used in the theoretical calculations; the calculations used in these analyses were done in the $\overline{\text{MS}}$ scheme.

Physical interpretation of the parameters $\bar{\Lambda}, \lambda_1,$ and $\lambda_2,$ comes from the relationship between the $b$ quark mass $m_b$ and the $B$ and $B^*$ meson masses, $M_B$ and $M_{B^*}$: $M_B = m_b + \bar{\Lambda} - (\lambda_1 + 3\lambda_2)/(2m_b) + \ldots$ and $M_{B^*} = m_b + \bar{\Lambda} - (\lambda_1 - \lambda_2)/(2m_b) + \ldots$. Intuitively, $\bar{\Lambda}$ is the energy of the light quark and gluon degrees of freedom, $-\lambda_1$ is the average of the square of the $b$ quark momentum, and $\lambda_2/m_b$ is the hyperfine interaction of the $b$ quark and light degrees of freedom. Using these expressions, we determine $\lambda_2$ from $M_{B^*} - M_B \approx 46$ MeV/c².

There are similar expressions – involving the same nonperturbative QCD parameters – for the moments $(M_X^2 - M_{B^*}^2)/M_B^2$ of the hadronic mass $(M_X)$ spectrum in $\bar{B} \rightarrow X_c \ell \bar{\nu}$ decay and $\langle E_\gamma \rangle/M_B$ of the photon energy $(E_\gamma)$ spectrum in $B \rightarrow X_s \gamma$ decay. (Here $M_D = 0.25 M_D + 0.75 M_{D^*}$, the spin-averaged $D$ meson mass.) The coefficients $M_n$ and $E_n$ of the polynomials for these moments depend on the lepton momentum range measured in $\bar{B} \rightarrow X_c \ell \bar{\nu}$ decays and the energy range measured in $B \rightarrow X_s \gamma$ decays, respectively [7, 8]. In the $M_X^2$ moments $\bar{\Lambda}$ again appears in all orders of $1/M_B$, while $\lambda_1$ and $\lambda_2$ appear in second order. The latter two parameters do not appear up to third order in the expansion for $\langle E_\gamma \rangle/M_B$.

We obtained $\gamma_c$ from Equation (1), by determining $\bar{\Lambda}$ and $\lambda_1$ from measurements of $(M_X^2 - M_{B^*}^2)$ and $\langle E_\gamma \rangle$ after: determining $\lambda_2$ from $M_{B^*} - M_B$ and estimating $\rho_1, \rho_2, T_1, T_2, T_3, T_4$ to be at most $(0.5 \text{ GeV})^3$ from dimensional considerations. We also measured the second moments of these distributions, but do not use them to determine $\bar{\Lambda}$ and $\lambda_1$ due to the current state of theoretical uncertainties.

2 Measuring the $M_X^2$ and $E_\gamma$ Moments

We measured the $M_X^2$ moments using $3.2$ fb⁻¹ of $\Upsilon(4S)$ data and $1.6$ fb⁻¹ of continuum data of continuum data collected below the $B\bar{B}$ threshold. These data were accumulated using the CLEO II detector [4]. The continuum events were used to estimate backgrounds from continuum data in the $\Upsilon(4S)$ data sample. Calculation of the hadronic mass $M_X$ started with reconstruction of the neutrino in events with a single lepton by ascribing the missing energy and momentum to the neutrino. We then used
$M_X^2 \equiv M_B^2 + M_{\ell \nu}^2 - 2E_B E_{\ell \nu}$ where $M_{\ell \nu}$ and $E_{\ell \nu}$ are the invariant mass and the energy of the $\ell\nu$ system, respectively. (This expression is obtained by setting $\cos\theta_{B-\ell\nu} = 0$, where $\theta_{B-\ell\nu}$ is the unmeasurable angle between the momenta of the $B$ and the $\ell\nu$ system.) Neutrino energy and momentum resolution, and neglect of the modest term involving $\cos\theta_{B-\ell\nu}$ result in non-negligible width for the $M_X^2$ distributions of $B \to D\ell^-\bar{\nu}$ and $B \to D^*\ell^-\bar{\nu}$ decays. Figure 1 illustrates the $M_X^2$ distribution obtained in this analysis.

Moments were obtained from fits to the spectrum that include contributions from $D\ell\bar{\nu}$, $D^*\ell\bar{\nu}$, and $X_H\ell\nu$, where $X_H$ represents all higher mass resonant and non-resonant charm states. The moments determined in this manner are not very sensitive to the $M_X^2$ distributions assumed for the $X_H$ states and the modest sensitivity is included in the systematic error. The measured moments are:

$$\langle (M_X^2 - M_D^2) \rangle = 0.251 \pm 0.023 \pm 0.062 \text{ GeV}^2$$

$$\langle (M_X^2 - \langle M_X^2 \rangle)^2 \rangle = 0.639 \pm 0.056 \pm 0.178 \text{ GeV}^4$$

We measured $E_\gamma$ moments using 9.1 fb$^{-1}$ of $\Upsilon(4S)$ data and 4.4 fb$^{-1}$ of continuum data collected below the $B\bar{B}$ threshold. These data were accumulated using the CLEO II and CLEO II.V detector configurations. The analysis began with a search for an isolated $\gamma$ with $2.0 < E_\gamma < 2.7$ GeV. In this energy range, backgrounds are about a factor of 100 above the signal. Most of these backgrounds are $\gamma s$ from Initial State Radiation or photons from the decay of $\pi^0$’s in continuum events. Substantial background reduction is achieved with requirements on event shapes and energies in cones relative to $p_\gamma$, or with pseudoreconstruction of the $X_s$ state, or by requiring the presence of a lepton in the event. For each $\gamma$ candidate, all information was combined into a single weight that ranged between 0.0 for continuum events and 1.0 for $B \to X_s\gamma$ events. Using the continuum data to subtract backgrounds from continuum events in the $\Upsilon(4S)$ data was crucial for this analysis. The resulting weight distribution is illustrated in Figure 1. The $E_\gamma$ moments obtained from this weight distribution are:

$$\langle E_\gamma \rangle = 2.346 \pm 0.032 \pm 0.011 \text{ GeV}$$

$$\langle (E_\gamma - \langle E_\gamma \rangle)^2 \rangle = 0.0226 \pm 0.0066 \pm 0.0020 \text{ GeV}^2$$

Weights were obtained from fits to the spectrum and are plotted in Figure 2. The $E_\gamma$ and $M_X^2$ moments plotted in the $\lambda_1$–$\bar{\Lambda}$ plane.
\[ \Lambda = -0.350 \pm 0.070 \pm 0.10 \text{ GeV} \]  
(2)

\[ \lambda_1 = -0.238 \pm 0.071 \pm 0.078 \text{ GeV}^2, \]  
(3)

where the first errors are from the uncertainties in the moment measurements and the second errors are from the theoretical uncertainties, particularly the uncertainties in the values of the parameters \( \rho_1, \rho_2, T_1, T_2, T_3, \) and \( T_4, \) which are not measured.

### 3 Determining \(|V_{cb}|\) from \( M_X^2 \) and \( E_\gamma \) Moments

To compute \( \Gamma_{SL}^e \), we used: \( B(\bar{B} \rightarrow X_e \ell \bar{\nu}) = (10.39 \pm 0.46)\% \) \( \text{[10]} \), \( \tau_{B\pi} = (1.548 \pm 0.032) \text{ ps} \) \( \text{[10]} \), \( \tau_{B\rho} = (1.653 \pm 0.028) \text{ ps} \) \( \text{[10]} \), \( f_+ - f_0 = 1.04 \pm 0.08 \) \( \text{[11]} \), giving \( \Gamma_{SL}^e = (0.427 \pm 0.020) \times 10^{-10} \text{ MeV}. \) Then \( \Gamma_{SL}^e = |V_{cb}|^2 \) and Equation (4) for \( \gamma_e \) then yielded,

\[ |V_{cb}| = (40.4 \pm 0.9 \pm 0.5 \pm 0.8) \times 10^{-3}, \]  
(4)

where the first error is from the experimental determination of \( \Gamma_{SL}^e \), the second from the measurement of \( \bar{\Lambda} \) and \( \lambda_1 \), and the third from theoretical uncertainties, i.e., from \( \alpha_s \) scale uncertainties and ignoring the \( O(1/M_B^2) \) terms which contain the estimated parameters \( \rho_1, \rho_2, T_1, T_2, T_3, \) and \( T_4. \)

Note that – even with direct measurement of these nonperturbative QCD parameters – the residual theoretical uncertainty is comparable to the experimental errors!

### 4 Determining \(|V_{cb}|\) from Moments of the Lepton Spectra in \( \bar{B} \rightarrow X e \ell \bar{\nu} \)

Moments of the \( E_\ell \) spectrum in inclusive \( \bar{B} \rightarrow X e \ell \bar{\nu} \) decay can also be used to determine \( \bar{\Lambda} \) and \( \lambda_1. \) We define:

\[ R_0 = \frac{\int_{1.5 \text{ GeV}}^{1.7 \text{ GeV}} \frac{d\Gamma_{SL}^e}{dE_\ell} dE_\ell}{\int_{1.5 \text{ GeV}}^{1.7 \text{ GeV}} \frac{d\Gamma_{SL}^e}{dE_\ell} dE_\ell}, \]

\[ R_1 = \frac{\int_{1.5 \text{ GeV}}^{1.7 \text{ GeV}} \frac{E_\ell d\Gamma_{SL}^e}{dE_\ell} dE_\ell}{\int_{1.5 \text{ GeV}}^{1.7 \text{ GeV}} \frac{E_\ell d\Gamma_{SL}^e}{dE_\ell} dE_\ell}. \]

In order to avoid the necessity of removing \( \bar{B} \rightarrow X_u \ell \bar{\nu} \) decays from our data, we include both \( \Gamma_{SL}^e \) and \( \Gamma_{SL}^\mu \) in \( \Gamma_{SL} \). The moments \( R_0 \) and \( R_1 \) for \( \bar{B} \rightarrow X e \ell \bar{\nu} \) decay can be expressed in terms of expansions in \( \alpha_S \) and \( M_B \) (spin-averaged \( B \) mass) involving \( \bar{\Lambda} \) and \( \lambda_1 \). These expansions have been calculated to \( O(1/M_B^3) \) \( \text{[12]} \). Determining \( \bar{\Lambda} \) and \( \lambda_1 \) from these moments provides an important check of theory, particularly the importance of neglected higher order terms in the theoretical \( E_\ell, M_X^2, \) and \( E_\gamma \) moments.

We measured these moments with the same data sample that we use in the measurement of the hadronic mass moments. The principal experimental challenges are identifying leptons and eliminating leptons from sources other than \( \bar{B} \rightarrow X e \ell \bar{\nu} \) decay. Other sources include \( J/\psi \) decay, \( e^\pm \) from \( \pi^0 \) Dalitz decay, \( \gamma \) conversions, secondary leptons from \( b \rightarrow c \) decay followed by \( c \rightarrow s \ell \bar{\nu} \) decay, leptons from continuum events, and hadrons misidentified as leptons (this background is much more significant for muons than for electrons). After cuts to reduce leptons from these sources and subtraction of estimated residual yields, we subtract the yield from our continuum data. The resulting electron and muon momentum spectra are illustrated in Figure 3. Table 1 gives the \( R_0 \) and \( R_1 \) values obtained from electron, muon, and combined lepton data samples. The results for electrons and muons are obviously very consistent. The dominant systematic errors are from the secondary lepton contribution, lepton identification, electroweak radiative corrections, and the uncertainty in the absolute momentum scale.

![Figure 3](1630802-003)

**Figure 3.** Momentum spectra in the \( B \) meson rest frame for electrons (triangles) and muons (squares). The quantity \( d\Gamma/d\ell \) represents the differential semileptonic branching fraction in the bin \( \Delta p, \) divided by the number of \( B \) mesons in the sample and \( \Delta p. \)

### Table 1. Measured values of \( R_0 \) and \( R_1 \) from \( e \) and \( \mu \) data and for the weighted average of the two \( (\ell). \) The errors are statistical and systematic in that order.

| \( \ell \) | \( R_0 \) | \( R_1 \) |
|---|---|---|
| \( e \) | 0.6184 \( \pm 0.0016 \) \( \pm 0.0017 \) | 1.7817 \( \pm 0.0008 \) \( \pm 0.0010 \) GeV |
| \( \mu \) | 0.6189 \( \pm 0.0023 \) \( \pm 0.0020 \) | 1.7802 \( \pm 0.0011 \) \( \pm 0.0011 \) GeV |
| \( \ell \) | 0.6187 \( \pm 0.0014 \) \( \pm 0.0016 \) | 1.7810 \( \pm 0.0007 \) \( \pm 0.0009 \) GeV |

The values of \( \bar{\Lambda} \) and \( \lambda_1 \) obtained from the lepton energy moments are,
\[ \bar{\Lambda} = +0.39 \pm 0.03 \pm 0.06 \pm 0.12 \text{ GeV} \] and
\[ \lambda_1 = -0.25 \pm 0.02 \pm 0.05 \pm 0.14 \text{ GeV}^2, \]
where the errors are statistical, systematic, and theory. The uncertainties in the \( 1/M_B^2 \) terms dominate the theoretical errors. (Note that the theoretical uncertainties are larger than the experimental errors in this analysis!) These results are in excellent agreement with the values from the \( E_\gamma - M_X^2 \) moments given in Equations (2) and (3).

The agreement between the \( R_0 - R_1 \) and \( E_\gamma - M_X^2 \) moment analyses is illustrated in Figure 4, where all four measured moments with their total experimental errors are plotted in the \( \bar{\Lambda} - \lambda_1 \) plane. Due to the theoretical uncertainties in the relationships between moments and \( \bar{\Lambda} \) and \( \lambda_1 \) we do not make an overall fit to the two analyses. There is little correlation among these measurements, so the consistency of the \( \bar{\Lambda} \) and \( \lambda_1 \) values from \( E_\gamma - M_X^2 \) moments with those from from \( E_\ell \) moments increases confidence in the theories.

The value of \( |V_{cb}| \) obtained from these \( E_\ell \) moments is
\[ |V_{cb}| = (40.8 \pm 0.5 \pm 0.4 \pm 0.9) \times 10^{-3}, \]
where the first error is from the experimental determination of \( \Gamma_{S_L}^\gamma \), the second from the measurement of \( \bar{\Lambda} \) and \( \lambda_1 \), and the third from the theoretical uncertainties described following Equation (4). Of course, since the values of \( \bar{\Lambda} \) and \( \lambda_1 \) from this analysis are in excellent agreement with the corresponding results from the \( E_\gamma - M_X^2 \) analysis, this value of \( |V_{cb}| \) agrees very well with the value given in Equation (4).

5 Summary

CLEO determined \( |V_{cb}| \) from two different moment analyses. The result of the \( E_\gamma - M_X^2 \) analysis is \( |V_{cb}| = (40.4 \pm 0.9 \pm 0.5 \pm 0.8) \times 10^{-3} \) while the result of the \( R_0 - R_1 \) analysis is \( |V_{cb}| = (40.8 \pm 0.5 \pm 0.4 \pm 0.9) \times 10^{-3} \). In each case, the first error is the uncertainty due to the uncertainty in the measured semileptonic decay width \( \Gamma(B \rightarrow X_c \ell \nu) \), the second is from the determination of \( \bar{\Lambda} \) and \( \lambda_1 \) from moments, and the third is an estimate of the theoretical uncertainty due to the estimated range of values of additional nonperturbative QCD parameters that are not determined in the analyses. The two results are clearly in excellent agreement.

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