Separation of proton polarizabilities with the beam asymmetry of Compton scattering

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We propose to determine the magnetic dipole polarizability of the proton from the beam asymmetry of low-energy Compton scattering based on the fact that the leading non-Born contribution to the asymmetry is given by the magnetic polarizability alone; the electric polarizability cancels out. The beam asymmetry thus provides a simple and clean separation of the magnetic polarizability from the electric one. Introducing polarizabilities in a Lorentz-invariant fashion, we compute the higher-order (recoil) effects of polarizabilities on beam asymmetry and show that these effects are suppressed in forward kinematics. With the prospects of precision Compton experiments at the MAMI and HGS facilities in mind, we argue why the beam asymmetry could be the best way to measure the elusive magnetic polarizability of the proton.

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The current Particle Data Group (PDG) [1] values of the electric- and magnetic-dipole polarizabilities of the proton [2, 3], i.e.,

\[
\alpha_{E1} = (12.0 \pm 0.6) \times 10^{-4} \text{ fm}^3, \quad (1a) \\
\beta_{M1} = (1.9 \pm 0.5) \times 10^{-4} \text{ fm}^3 \quad (1b)
\]

are in significant disagreement with the most recent predictions of chiral effective field theory (ChEFT) [4, 5], as can be seen in Fig 1. The state-of-the-art ChEFT calculations, based on either the baryon (B) or heavy-baryon (HB) chiral perturbation theory (ChPT) with octet and decuplet fields [11], are in excellent agreement with the experimental Compton-scattering cross sections, but not necessarily in agreement with the polarizabilities extracted from these data by the experimental groups, c.f. [12] for review. The situation is becoming more acute as the demand for precise knowledge of nucleon polarizabilities is growing; they are for instance the main source of uncertainty in the extraction of the proton charge radius from the muonic hydrogen Lamb shift (see [13] for a recent review).

A likely source of these discrepancies is an underestimation of model dependence in the extraction of polarizabilities from Compton-scattering data. In principle, one should opt for a model-independent extraction, based on the low-energy expansion (LEX) of Compton-scattering observables, where the leading-order terms, beyond the Born term, are expressed through polarizabilities. For example, the non-Born (NB) part of the unpolarized differential cross section for Compton scattering off a target with mass $M$ and charge $Ze$ is given by [2]

\[
\frac{d\sigma^{(NB)}}{d\Omega_L} = -\frac{Z^2\alpha_{em}}{M} \left(\frac{\nu'}{\nu}\right)^2 \nu' \left[\alpha_{E1} (1 + \cos^2 \theta_L) + 2\beta_{M1} \cos \theta_L \right] + O(\nu^4), \quad (2)
\]

where $\nu = (s - M^2)/2M$ and $\nu' = (-u + M^2)/2M$ are, respectively, the energies of the incident and scattered photon in the lab frame, $\theta_L$ ($d\Omega_L = 2\pi \sin \theta_L d\theta_L$) is the scattering (solid) angle; $s$, $u$, and $t = 2M(\nu' - \nu)$ are the Mandelstam variables; and $\alpha_{em} = e^2/4\pi$ is the fine-structure constant. Hence, given the exactly known Born contribution [14] and the experimental angular distribution at very low energy, one could in principle extract the polarizabilities with a negligible model dependence. In reality, however, in order to resolve the small polarizability effect in the tiny Compton cross sections, most of the measurements are done at energies exceeding 100 MeV, i.e., not small compared to the pion mass $m_\pi$. It

FIG. 1: (Color online). The scalar polarizabilities of the proton. Magenta blob represents the PDG summary [1]. Experimental results are from Federspiel et al. [6], Zieger et al. [7], MacGibbon et al. [8], and TAPS [9]. ‘Sum Rule’ indicates the Baldin sum rule evaluations of $\alpha_{E1} + \beta_{M1}$ [9] (broader band) and [10]. ChPT calculations are from [4] (BχPT—red blob) and the ‘unconstrained fit’ of [5] (HBχPT—blue ellipse).

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is $m_\pi$, the onset of the pion-production branch cut, that severely limits the applicability of a polynomial expansion in energy such as LEX. At the energies around the pion-production threshold one obtains a very substantial sensitivity to polarizabilities but needs to resort to a model-dependent approach in order to extract them (see [15, 16] for reviews).

The magnetic polarizability $\beta_{M1}$ seems to be affected the most: the central value of the BChPT calculation is a factor of 2 larger than the PDG value. This is attributed to the dominance of $\alpha_{E1}$ in the unpolarized cross section.\(^1\) It is desirable to find an observable sensitive to $\beta_{M1}$ alone, such that the latter could be determined independently of $\alpha_{E1}$. According to the leading-order (LO) LEX for cross sections involving linearly polarized photons [19], the difference of cross sections for photons polarized perpendicular or parallel to the scattering plane,

$$\frac{(d\sigma_\perp - d\sigma_\parallel)}{d\Omega}$$

depends only on $\alpha_{E1}$, while the combination

$$\frac{(\cos^2 \theta \, d\sigma_\perp - d\sigma_\parallel)}{d\Omega}$$

only on $\beta_{M1}$. New experiments at the Mainz Microtron (MAMI) and the High Intensity Gamma Source (HIGS) are planned to measure these two combinations in order to extract $\alpha_{E1}$ and $\beta_{M1}$ independently. This Letter aims to show that $\beta_{M1}$ can directly be extracted from the beam asymmetry,

$$\Sigma_3 \equiv \frac{d\sigma_\perp - d\sigma_\parallel}{d\sigma_\parallel + 2d\sigma_\perp},$$

and that such extraction is potentially more accurate than the one based on the observable given by Eq. (4).

Indeed, applying the LEX for the beam asymmetry of proton Compton scattering we arrive at the following result:

$$\Sigma_3 = \Sigma_3^{(B)} - \frac{4M\omega^2 \cos \theta \sin^2 \theta}{\alpha_{em}(1 + \cos^2 \theta)^2} \beta_{M1} + O(\omega^4),$$

(6)

where $\Sigma_3^{(B)}$ is the exact Born contribution, while

$$\omega = \frac{s - M^2 + \frac{t}{4}}{\sqrt{4M^2 - 4t}}, \quad \theta = \arccos \left( 1 + \frac{t}{2\omega^2} \right)$$

are the photon energy and scattering angle in the Breit (brick-wall) reference frame. In fact, to this order in the LEX the formula is valid for $\omega$ and $\theta$ being the energy and angle in the lab or center-of-mass frame.

Equation (6) shows that the leading (in LEX) effect of the electric polarizability cancels out, while the magnetic polarizability remains. Hence, our first claim is that a low-energy measurement of $\Sigma_3$ can in principle be used to extract $\beta_{M1}$ independently of $\alpha_{E1}$, just as it was proposed for the combination of polarized cross sections given in Eq. (4).

In reality the low-energy Compton experiments on the proton are difficult because of small cross sections and overwhelming QED backgrounds. Precision measurement only becomes feasible for photon-beam energies above 60 MeV and scattering angles greater than 40 degrees. The upcoming experiments at HIGS and MAMI are planned for the energies between 80 and 150 MeV. As mentioned above, at these energies the effect of higher-order terms may become substantial. One way to see it is to compare the LEX result with the dispersion-relation calculations or calculations based on chiral perturbation theory.

Figures 2 and 3 demonstrate such a comparison of the leading-LEX result to the next-next-to-leading order (NNLO) BChPT result of Ref. [4] for the two observables defined in Eqs. (4) and (5). The observables are plotted for the case of proton Compton scattering as a function of magnetic polarizability of the proton. From Fig. 2 one

\(^1\) The problem is quite similar to the case of proton form factors, where the angular (Rosenbluth) separation from unpolarized scattering did not yield the correct result for the electric form factor, due to the dominance of the magnetic contribution, and only separating the electric form factor from the magnetic one by using polarization has yielded a break through. See [17, 18] for reviews.
achieve. Still, very high-intensity photon beams would be required to achieve the statistics necessary to pin down the magnetic polarizability model-independently to the accuracy currently claimed by the PDG, c.f. Eq. (1b). The high-intensity electron facility MESA being constructed in Mainz is very promising in this respect.

The results for the beam energy of 135 MeV (Fig. 3) show that the leading LEX result does not apply at such energies. The differences between the LEX and BChPT curves at both 100 and 135 MeV are mainly due to $\alpha_{E1}$ (which here is equal to zero for the LEX and about 11 for the BChPT) and the $\pi^0$-anomaly contribution. While it is well known that the anomaly contribution is proportional to $t/(t-m^2_N)$ and hence is maximal at the backward angles, the $\alpha_{E1}$ contribution needs to be examined.

In order to uniquely define the 'recoil corrections' due to the polarizabilities, we ought to introduce them in a Lorentz-invariant fashion by writing down an effective Lagrangian that yields the right Hamiltonian in the static limit, i.e.,

$$\mathcal{L}_{NN\gamma\gamma} = \pi \beta_{M1} N N F^2 - \frac{2\pi (\alpha_{E1} + \beta_{M1})}{M^2} (\partial_\alpha N)(\partial_\beta N) F^{\alpha\beta} F^{\mu\nu} \eta_{\mu\nu}$$

where $F_{\mu\nu} = \partial_\mu A_\nu$ is the electromagnetic field-strength tensor, $N(x)$ is the nucleon Dirac-spinor field, $\eta_{\mu\nu} = \text{diag}(1,-1,-1,-1)$ is the Minkowski metric. Recalling that $F^2 = -2(F^{0\mu})^2 + (F^{0\mu})^2 = -2E^2 + 2B^2$, assuming the nucleon rest frame: $\partial_i N = 0$, $i\gamma^0 \partial_0 N = MN$, $i\partial_\alpha N \gamma^0 = -M N$, we obtain

$$\mathcal{L}_{NN\gamma\gamma} = 2\pi \{ \beta_{M1}(B^2 - E^2) + (\alpha_{E1} + \beta_{M1})E^2 \} NN,$$

which readily reproduces the well known nonrelativistic Hamiltonian: $4\pi(-\frac{1}{2}\alpha_{E1}E^2 - \frac{1}{2}\beta_{M1}B^2)$.

The Feynman amplitude corresponding to the Lagrangian in Eq. (8) can be written as

$$M_{NN\gamma\gamma}^{\mu\nu} = 4\pi \bar{u}(p') u(p) \left[ \beta_{M1}(q \cdot q' \varepsilon' \cdot \varepsilon - q \cdot \varepsilon' q' \cdot \varepsilon) + \frac{\alpha_{E1} + \beta_{M1}}{2M^2}(p'_\mu p_\beta + p_\alpha p'_\beta)(q'\varepsilon' \varepsilon' - q\varepsilon' \varepsilon)(q\varepsilon \varepsilon - q'\varepsilon' \varepsilon') \right]$$

where $p$ and $q$ ($p'$ and $q'$) are the four-momenta of incident (outgoing) nucleon and photon and the manifestly gauge-invariant polarization vectors are then

$$\varepsilon'_{\mu} = \varepsilon_{\mu} - \frac{(p' + p) \cdot \varepsilon}{(p' + p) \cdot q} q_{\mu}, \quad \varepsilon'_{\mu} = \varepsilon'_{\mu} - \frac{(p' + p) \cdot \varepsilon'}{(p' + p) \cdot q} q'_{\mu}. \quad (10)$$

The polarizability contribution to the invariant Compton amplitudes is thus given as follows:

$$A_{1}^{(NB)}(s,t) = 2\pi(\alpha_{E1} + \beta_{M1})(\nu^2 + \nu'^2) + 2\pi \beta_{M1} t, \quad (11a)$$

$$A_{2}^{(NB)}(s,t) = -4\pi \beta_{M1}. \quad (11b)$$

We note that the contribution of $\alpha_{E1} + \beta_{M1}$ to $A_1$ differs from conventional definitions by terms of higher order in the Mandelstam variable $t$, and, hence in energy. For instance, the difference of the present $A_1^{(NB)}$ with the one in
Ref. [12] is equal to $-\pi(\alpha E_1 + \beta M_1)t(\omega^2 - \frac{1}{4}t)$. It turns out, however, that this difference does not affect the NLO contribution of polarizabilities to the beam asymmetry, which together with the LO contribution of Eq. (6) reads

$$
\Sigma_3 = \Sigma_3^{(0)} - \frac{4 \cos \theta \sin^2 \theta}{(1 + \cos^2 \theta)^2} \frac{M^3 \beta_{M_1}}{\alpha_{em}} \left( \frac{\omega}{M} \right)^2 \left\{ 1 + \left( \frac{\omega}{M} \right)^2 \left[ a_1(\cos \theta) + \frac{M^3 \alpha E_1}{\alpha_{em}} \right] \right\} 
+ \frac{\sin^2 \theta}{(1 + \cos^2 \theta)^2} \left( \frac{\omega}{M} \right)^4 \left[ a_2(\cos \theta) \frac{M^3 \alpha E_1}{\alpha_{em}} + a_3(\cos \theta) \left( \frac{M^3 \beta_{M_1}}{\alpha_{em}} \right)^2 \right] + O(\omega^6),
$$

(12)

where the dimensionless coefficient functions are given by (with $\kappa$ denoting the anomalous magnetic moment)

$$
a_1(z) = -\frac{1}{4(1 + z^2)} \left[ 7 - 8z + z^4 + 2(8 - 16z + 7z^2 - z^4) \kappa + (32 - 40z - z^2 - z^4) \kappa^2 \right] 
+ 4\left(5 - 4z - 3z^2\right) \kappa^3 \left(5 - 3z^2\right) \kappa^4, \quad (13)
a_2(z) = 2(1 - z) + 2(2 - 4z + z^2) \kappa + (7 - 10z - 2z^2) \kappa^2 + 4(1 - z - z^2) \kappa^3 + (1 - z^2) \kappa^4,
a_3(z) = \frac{2(1 - 6z + z^2)}{1 + z^2}.$$

At order $\omega^4$ there are also contributions from higher polarizabilities, such as the following four spin polarizabilities [15]: $\gamma_{E1E1}, \gamma_{E1M2}, \gamma_{M1E2}, \gamma_{M1M1}$. Spin polarizabilities are dominated by the $n^d$ anomaly and, hence, their contribution is only appreciable in the backward angles. There is also a contribution from the fourth-order scalar polarizabilities $\alpha_{E2}, \alpha_{M2},$ and $\beta_{M\nu}$, which can be obtained by the following replacement in the $O(\omega^2)$ term:

$$
\beta_{M1} \rightarrow \beta_{M1} + \omega^2 \left( \beta_{M\nu} - \frac{1}{12} \alpha_{E2} + \frac{1}{2} \beta_{M2} \cos \theta \right). \quad (14)
$$

Examining the $O(\omega^4)$ contribution of scalar polarizabilities shown above, we note that the coefficient functions containing $\kappa$ (i.e., $a_1$ and $a_2$) can become large in the backward angles. To limit the impact of these terms, as well of the spin polarizabilities, on the extraction of $\beta_{M1}$, the scattering angle should be chosen in the vicinity of 60 degrees. As is illustrated in Fig. 2, in this case the leading LEX result is very close to the BChPT result, thus confirming the near-perfect cancellation of higher-order terms in this case.

To conclude, the beam asymmetry $\Sigma_3$ is instrumental in isolating the contribution of the magnetic polarizability $\beta_{M1}$ to low-energy Compton scattering. While the cross sections receive contributions from both the electric and magnetic polarizability, the effect of $\alpha_{E1}$ cancels out from the asymmetry at leading order in the low-energy expansion. We have shown that the next-to-leading corrections are suppressed at the forward scattering angles. A precise and model-independent determination of the proton $\beta_{M1}$ is feasible through a precision measurement of $\Sigma_3$ at beam energies below 100 MeV and forward scattering angles. Furthermore, when multiplied with the unpolarized cross section, $\Sigma_3$ yields the polarized cross section difference, which provides an exclusive access to the electric polarizability. A measurement of the beam asymmetry at these low energies has not been done yet, but is being planned at the MAMI and HIGS facilities and could be done in the very near future.

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