Radiation Damping of a Yang-Mills Particle Revisited

Sair Arquez
Departamento de Física, Escuela Superior de Física y Matemáticas del Instituto Politécnico Nacional
Unidad Adolfo López Mateos, Edificio 9, 07738, Ciudad de México, México

Departamento de Ciencias Naturales y Exactas, Universidad de la Costa,
Calle 58 num. 55-66, Barranquilla, Colombia.

Rubén Cordero
Departamento de Física, Escuela Superior de Física y Matemáticas del Instituto Politécnico Nacional
Unidad Adolfo López Mateos, Edificio 9, 07738, Ciudad de México, México

Hugo García-Compeán
Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN
P.O. Box 14-740, 07000, Ciudad de México, México

Abstract

The problem of a color-charged point particle interacting with a four dimensional Yang-Mills gauge theory is revisited. The radiation damping is obtained inspired in the Dirac’s computation. The difficulties in the non-abelian case were solved by using an ansatz for the Liénard-Wiechert potentials, already used in the literature [37] for finding solutions to the Yang-Mills equations. Three non-trivial examples of radiation damping for the non-abelian particle are discussed in detail.
1 Introduction

The problem of motion for a classical radiating particles has been studied for a long time in the non-relativistic and relativistic contexts [1, 2]. It is well known that this particle suffers a radiation damping due the self-force. In the Dirac’s correction there is a third-order time derivative of the position of the particle. Thus there are an unphysical phenomena such as runaway’s behavior and preaccelerating [3, 4, 5]. It is right now known that this bad behavior is a consequence of the pointlike nature of the particle. An exhaustive study of the mentioned phenomena was performed by many authors and the systematic description of some divergences require to renormalize the electron charge and mass. For some recent overviews, see [6, 7, 8, 9]. The solution of the mentioned problem has been of great importance to understand the relativistic dynamics of the particles interacting with radiation. Particularly in the Ref. [2], Dirac calculated the self-force of the emitted electromagnetic radiation (or radiation damping) from an electron. He considered expressions for retarded and advanced potentials, in terms of a parameter $\sigma$ that up on regularization is function of the electron size, which obviously must be taking to zero at the end of the computation.

The equations of motion for a particle carrying isotopic-spin in a non-abelian Yang-Mills field has been originally worked out in the 70s by S.K. Wong [10]. Moreover the study of non-abelian particles and their equations of motion have been of great importance along the years. Later important developments were given in Refs. [11, 12]. In these papers Lagrangian and Hamiltonian for the charged particle in a non-abelian gauge field are proposed such that Wong’s equations are obtained from them. The canonical quantization and some detailed cases were worked out in the second paper [12]. Further developments using Wong’s equations and incorporating spin were described in [13]. The non-abelian particle is also described by using the world-line formalism in Ref. [14]. The application to the transport properties of non-abelian fluids or plasmas and in general to non-abelian hydrodynamics were discussed in Refs. [15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]. In particular the importance of Wong’s equations in the problem of the quark-gluon plasma has been considered in Refs. [26, 27, 28, 29, 30]. More recently, the consideration of the Wong’s equations of motion has been discussed in the context of the double copy approach [31] and also as a possible candidate of dark matter [32].

As a next logical step, to extend the ideas in [2] to the case of non-abelian theories, however this simple idea has strong complications. The problem of motion for particles in a Yang-Mills fields have been discussed in a subsequent series of papers [33, 34, 35, 36]. In [33] the equations of motion were derived from the energy and momentum conservation. Also some solutions were obtained in terms of the first order abelian Liénard-Wiechert solution. In [34] it was studied the radiation damping of color in Yang-Mills theory. In Ref. [35] it was argued that the color charge can be changed for a suitable gauge group. If the gauge group is compact and semisimple then the color charge remains constant, however there is a transfer of energy described by the Liénard-Wiechert (LW) potentials. Moreover in Ref. [36] it was studied the variation of the color charges with respect to the choice of the non-abelian waves.

One of the most important problems in this context is to find suitable Liénard-
Wiechert’s potentials as solutions to the Yang-Mills equations. However, it is possible to construct these potentials satisfying physical conditions and being also compatible with the Yang-Mills equations. The simplest case is to consider an expansion in the form \( \frac{1}{R} \), where \( R \) gives the retarded distance, for a retarded potential such that it vanishes at infinity. However, it is possible to make an ansatz and write the LW potentials as superposition of terms with different negative powers of \( R \). The non-abelian dependence lies on certain \( \alpha_i \) functions that take values in the Lie algebra of the Lie group and also depend on the proper-time \([37]\). In the present paper we use this ansatz and the Dirac’s procedure \([2]\) in order to obtain the self-force and consequently the radiation damping of a Yang-Mills particle following Wong’s equations.

This paper is organized as follows. In Section 2 we introduce the notation and conventions we will follow along the article. In particular we give a brief review of the Liénard-Wiechert’s potentials and the ansatz \([37]\) that we will use to compute the radiation damping in subsequent sections. In this Section 2 we compute the retarded Yang-Mills field strength and the associated radiated field strength. Section 3 is devoted to obtain the self-force correction to Wong’s equations. We follow Dirac’s method of computing the flux in the volume of the energy-momentum tensor. In this same section we find the equations of motion for different cases. In the first case we recover the abelian Dirac’s modified equations of motion. Later we present three non-trivial cases of the radiation damping of the non-abelian particle. Finally in Section 4 we present our final comments. Three appendices were included in order to write down some important details of the very long computations relevant to Sections 2 and 3.

## 2 Liénard-Wiechert Potentials for a Non-Abelian Yang-Mills Charge

In the present section we will give some preliminaries and the set up of the next sections. We also introduce the notation and conventions we will follow along the article.

Consider the Yang-Mills potential \( A_\mu(x) = A_\mu^a(x)T_a \), where \( T_a \) \( a = 1, \ldots, \text{dim}(G) \) are the generators of the compact and simple gauge group \( G \), and \( x \) labels the points in the Minkowski space-time with signature \((+,-,-,-)\). These fields have a field strength given by

\[
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g [A_\mu, A_\nu],
\]

where \( g \) is the coupling constant. The action of the gauge field \( A_\mu \) in the presence of an external source \( J_\mu \) reads

\[
S = - \frac{1}{4} \int d^4x \text{Tr} [F_{\mu\nu}(x)F^{\mu\nu}(x) - J_\mu(x)A_\mu(x)].
\]

The corresponding Euler-Lagrange equations for \( A_\mu \) are the Yang-Mills equations

\[
D_\mu F^{\mu\nu} = -J^\nu.
\]
The classical interpretation of $J_\mu$ is known as the isotopic spin current \cite{10}, and expressed in the simple form

$$J_\mu = \int gI(s)\dot{z}_\mu(s)\delta^{(4)}(x_\mu - z_\mu(s))\,ds,$$  \hspace{1cm} (4)

which is the current associated to particles moving along the worldline $z_\mu(s)$. Here $I(s)$ is a generic element of the Lie algebra of $G$ and $z_\mu(s)$ stands for the position of the particle describing a smooth curve $\Gamma$ in spacetime and $x_\mu$ is any point in Minkowski spacetime.

Now let $(x_\mu - z_\mu(s))(x^\mu - z^\mu(s)) = 0$ be the light-cone condition, where $s$ denotes the proper time and $R = \dot{z}^\mu(x_\mu - z_\mu(s))$ is the retarded distance. In Ref. \cite{37}, the author proposed an ansatz in order to obtain some interesting classical solutions to the Yang-Mills equations in terms of the LW potentials. In the present article we will use this ansatz in order to study the radiation damping of an accelerated Yang-Mills particle.

Consider the ansatz for the LW potential worked out in \cite{37}

$$A^{(\text{ret})}_\mu = H(R)\dot{z}_\mu + G(R)\lambda_\mu,$$  \hspace{1cm} (5)

where $H$ and $G$ are smooth functions of $R$. $\lambda_\mu$ is given by $\lambda^\mu = \frac{(x^\mu - z^\mu(s))}{R}$, such that $\lambda^\mu\dot{z}_\mu = 1$ and $\lambda^\mu R_{\mu} = 1$ and $\lambda^\mu a_\mu = 0$ ($a_\mu \equiv \frac{\partial_\mu}{\partial s}$. In the latter condition for $\lambda_\mu$, $a$ represents the acceleration of the particle which is defined by $a = \frac{1}{R}\dot{z}^\mu(x_\mu - z_\mu(s))$. Moreover one can introduce additional scalars $a_k$ defined by $a_k \equiv \lambda^\mu a^k_{\mu}$, $k = 0, 1, 2, 3$ with $a_0 = a$. These scalars $a_k$ satisfy the following relation

$$\frac{(x^\mu - z^\mu(s))}{R}\partial_\mu a_k = 0.$$  \hspace{1cm} (6)

Taking account these considerations the ansatz \cite{3} is given by

$$A^{(\text{ret})}_\mu = \left(\frac{\alpha_1}{R} + \frac{\alpha_2}{R^2}\right)\dot{z}_\mu + \left(\frac{\alpha_3}{R} + \frac{\alpha_4}{R^2} + \frac{\alpha_5}{R^3}\right)(x_\mu - z_\mu(s)),$$  \hspace{1cm} (7)

where $\alpha_i$ with $i = 1, \ldots, 5$ are Lie algebra-valued functions of the proper time and they are $R$-independent functions.

For a smooth curve $\Gamma$ in space-time one has that the velocity and its respective derivatives in the proper time satisfies the following relations

$$v^2 = 1, \quad \ddot{v} = 0, \quad \dot{v} = v^2, \quad \dddot{v} = -3\ddot{v},$$

$$vv^{(4)} = -3\ddot{v}^2 - 4\dot{v}\dddot{v}, \quad vv^{(5)} = -5\dddot{v}(4) - 10\ddot{v}\dddot{v},$$

$$vv^{(6)} = -10\dddot{v}^2 - 6\dddot{v}(5) - 15\dddot{v}(4), \quad vv^{(7)} = -7\dddot{v}(6) - 21\dddot{v}(5) - 35\dddot{v}(4),$$

$$vv^{(8)} = -8\dddot{v}(7) - 28\dddot{v}(6) - 56\dddot{v}(5) - 35(\dddot{v}(4))^2,$$  \hspace{1cm} (8)

where we adopted Dirac’s notation \cite{2} for the scalar product: $vv \equiv v^2 \equiv v^\mu v_\mu$, and similarly for the other products, for instance $v\dot{v} \equiv v^\mu \dot{v}_\mu$ with $v_\mu = \dot{z}_\mu(s)$. Moreover the velocity of the particle $v^{(n)}$ in components reads as $v_\mu^{(n)}$ for $n \geq 4$ and they stand for
the $n$-th derivative of $v_\mu$ with respect to parameter $s$. Notice that in general $v^2 = \varepsilon$ with $\varepsilon = 0, \pm 1$. Within the adopted signature $\varepsilon = 0$ for null curves, $\varepsilon = 1$ for time-like curves, and $\varepsilon = -1$ for space-like curves.

We can use the properties of the Dirac delta functions to express Eq. (7) in the following form and to localize the LW potentials along the path of the particle. This procedure allow us to establish a relationship between the trajectory of the particle and the process of emission and absorption of radiation. Moreover it is easier to work in this representation given the possible divergences of the potential at $R = 0$. The expansion (7) in $1/R$ is finite for $R \neq 0$. This physical condition is also valid for non-abelian gauge theories, in which the potentials and the interaction is finite at long distances, at least within the classical description.

We follow the procedure introduced by Dirac in [2]. Thus we can express $A_\mu$ in the following form according to the ansatz [3] adapted for the retarded gauge field

$$A^{(ret)}_\mu = 2 \int \left[ \alpha_1 \dot{z}_\mu + \alpha_3 (x_\mu - z_\mu(s)) \right] \delta(\Omega) \, ds \!+\! 4 \int \left[ \alpha_2 \dot{z}_\mu + \alpha_4 (x_\mu - z_\mu(s)) \right] \delta(\Omega) \, \delta(\Omega) \, ds \, ds,$$

where $\Omega := (x_\mu - z_\mu(s))(x^\mu - z^\mu(s))$ vanishes as soon as $x^\mu$ coincides with the positions of the worldline of the particle $z^\mu(s)$.

In order to calculate the corresponding retarded field strength we compute (9) and plugging it into the Eq. (11), then we obtain

$$F^{(ret)}_{\mu\nu} = -\frac{\alpha_1}{R} \frac{d}{ds} \left[ \frac{1}{R} \dot{z}_\mu (x_\nu - z_\nu) - \frac{1}{R} \dot{z}_\nu (x_\mu - z_\mu) \right]$$

$$- 2\alpha_2 \frac{d}{ds} \left[ \frac{1}{R} \dot{z}_\mu (x_\nu - z_\nu) - \frac{1}{R} \dot{z}_\nu (x_\mu - z_\mu) \right]$$

$$+ \{ -2\alpha_2 + [\alpha_1, \alpha_4] + [\alpha_2, \alpha_3] \} \left[ \ddot{z}_\mu (x_\nu - z_\nu) - \ddot{z}_\nu (x_\mu - z_\mu) \right]$$

$$+ \left\{ [\alpha_1, \alpha_5] + [\alpha_2, \alpha_4] + [\alpha_2, \alpha_5] \right\} \left[ \ddot{z}_\mu (x_\nu - z_\nu) - \ddot{z}_\nu (x_\mu - z_\mu) \right].$$

(10)

In the process of getting Eq. (10) we used some relations between the $\alpha_i$’s found in [37].

To calculate the Yang-Mills field strength in the neighborhood of the particle along its trajectory, we need to perform a Taylor expansion for $x_\mu = z_\mu(s_0) + \gamma_\mu$, such that $\gamma_\mu$ is a small function of the parameter $\sigma$. Here $\sigma$ is the correction to the proper-time starting from its initial value at $s \to s_0 - \sigma$. This difference establishes to first order the retarded-time, which is delayed from the observer’s time $s$ and gives the time when the field begun to propagate, then we have the Taylor expansion in the form:

$$x_\mu - z_\mu(s_0 - \sigma) = \gamma_\mu + \sum_k \frac{(-1)^{k-1}}{k!} \sigma^k v^{(k-1)}_\mu$$

(11)
\[
\dot{z}_\mu (s_0 - \sigma) = - \sum_k \frac{(-1)^k}{(k - 1)!} \sigma^{k-1} v^{(k-1)}_\mu,
\]  

(12)

where \(k = 1, 2, \ldots, 9\). The possible values of \(k\) is due the last term in \(F_{\mu\nu}\) is of order \(\sigma^{-8}\), thus it is reasonable to take expansions up to order \(\sigma^9\) to have all terms independent of \(\sigma\), analogously to the process carried out by Dirac in Ref. [2]. Using equations (11) and (12) we can calculate the expansions for \(R\) and the products \(\dot{z}^\mu (x_\nu - z_\nu (s))\). This is done in appendix A.

Yang-Mills field is singular at \(R = 0\), thus in order to avoid these divergences it is convenient to introduce a regularization parameter \(\varepsilon\) which can be interpreted as the size of the particle. Equations of motion cannot depend on \(\varepsilon\), thus at the end of the computation we will have the finite and the divergent parts. By taking the limit \(\varepsilon\) tends to zero we will take zero all the positive powers of \(\varepsilon\) and the remaining few negative powers of \(\varepsilon\) will be renormalized.

To calculate the radiated field strength of the radiating particle \(F_{\mu\nu}^{(rad)}\), which can be determined by the equation:

\[
F_{\mu\nu}^{(rad)} = F_{\mu\nu}^{(ret)} - F_{\mu\nu}^{(adv)} = F_{\mu\nu}^{(out)} - F_{\mu\nu}^{(in)}.
\]

(13)

In analogy to the procedure performed by Dirac in Ref. [2] we can define the following relations

\[
f_{\mu\nu} = F_{\mu\nu}^{(act)} - \frac{1}{2} \left( F_{\mu\nu}^{(ret)} + F_{\mu\nu}^{(adv)} \right), \quad f_{\mu\nu} = F_{\mu\nu}^{(in)} + \frac{1}{2} F_{\mu\nu}^{(rad)}.
\]

(14)

The possibility to define the field in this way will allow to present the equations of motion in a compact and familiar form.

At this point it is worth highlighting the fact that in general these definitions were set out from first principles under the conditions of conservation of the moment and energy and in addition to the relativistic invariance of the Lagrangian. In our case all these conditions are satisfied classically, for this reason we have used these same tools. From Eq. (13) we can calculate \(F_{\mu\nu}^{(rad)}\) (for the details of the computation, see appendix B) which are expressed in the form

\[
F_{\mu\nu}^{(rad)} = \left( -\frac{3}{4} \ddot{\nu}^2 A_2 - \frac{7}{12} \ddot{\nu}^2 A_4 - \frac{11}{12} \dddot{\nu} A_5 \right) (v_\mu \dot{v}_\nu - v_\nu \dot{v}_\mu) \\
+ \frac{1}{3} \left( \frac{3}{2} \ddot{\nu}^2 A_5 + 2A_3 + 4A_1 \right) (\dddot{v}_\mu v_\nu - \dddot{v}_\nu v_\mu) \\
+ \frac{1}{4} (A_4 + 3A_2) \left( v_\mu \dddot{v}_\nu + \frac{2}{3} \dddot{v}_\mu \dddot{v}_\nu - \frac{2}{3} \dddot{v}_\nu \dddot{v}_\mu \right) \\
+ 2A_5 \left( -\frac{v_{\mu\nu}^{(4)}}{30} - \frac{\dddot{v}_\mu \dddot{v}_\nu}{24} + \frac{v_{\mu\nu}^{(4)}}{30} \right),
\]

(15)

where \(A_1 = -\alpha_1, A_2 = -2\alpha_2, A_3 = -2\dot{\alpha}_2 + [\alpha_1, \alpha_4] + [\alpha_2, \alpha_3], A_4 = [\alpha_1, \alpha_5] + [\alpha_2, \alpha_4], A_5 = [\alpha_2, \alpha_5].\)

To calculate the advance field strength \(F_{\mu\nu}^{(adv)}\) from \(F_{\mu\nu}^{(ret)}\) it is only required to change \(\varepsilon\) by \(-\varepsilon\), however unlike the Dirac case [2] in the computation of \(F_{\mu\nu}^{(rad)}\) there will
survive the even powers terms in $F_{\mu\nu}^{(rad)}$. This result is interesting because physically the radiated field has a subtle dependence on these terms which would involve divergences of the radiated field. However without these divergences the equations of motion would not be obtained correctly.

An important point is that in the second term from (15) there is a term that corresponds exactly to Dirac's radiative field

$$F_{\mu\nu}^{(rad)} D = \frac{4}{3} A_1 \left( \bar{v}_\mu v_\nu - \bar{v}_\nu v_\mu \right), \quad (16)$$

which is the usual one for the radiation produced by an accelerated abelian charge.

3 The Equations of Motion

If a charged classical particle moves along its worldline it generates a flow on its trajectory. This flow can be calculated through the energy-momentum tensor of the field in the neighborhood of a point along the trajectory of the particle. The energy-momentum tensor is given by

$$T^F_{\mu\nu} = \text{Tr} \left( F^\rho_{\mu \rho \nu} - \frac{1}{4} g_{\mu \nu} F^{\alpha \beta} F_{\alpha \beta} \right). \quad (17)$$

We can consider a spherical hyper-surface of radius $\varepsilon$ covering the path. The integral on this surface is $4 \pi \varepsilon^2$, and the expression between the braces is precisely the equation of motion outlined in appendix C

$$\int \varepsilon^{-1} T_{\mu \nu} \gamma^\rho dx \left| dx \right| = \int \text{Tr} \left\{ A_5 f_{\mu \nu} \varepsilon^\nu + \frac{Q_{14} v_\mu + Q_{11} \bar{v}_\mu + Q_{17} \bar{v}_\mu + Q_{16} \bar{v}_\mu}{\varepsilon^2} \right. \right. \right.$$

$$+ \left. A_5 f_{\mu \nu} \varepsilon^\nu + Q_{36} f_{\mu \nu} v_\nu + Q_{39} v_\mu + Q_{33} \bar{v}_\mu + Q_{10} \bar{v}_\mu + Q_{15} \bar{v}_\mu \right.$$

$$+ \left. Q_{33} f_{\mu \nu} \varepsilon^\nu + Q_{38} v_\mu + Q_{6} \bar{v}_\mu + Q_{2} \bar{v}_\mu + Q_{12} \bar{v}_\mu - A_4 f_{\mu \nu} \varepsilon^\nu + Q_{35} f_{\mu \nu} v_\nu \right.$$

$$+ \frac{Q_{24} v_\mu}{\varepsilon^5} + \frac{Q_{23} v_\mu + Q_{22} v_\mu + Q_{20} \bar{v}_\mu}{\varepsilon^4} + \frac{Q_{13} v_\mu + Q_{21} \bar{v}_\mu + Q_{19} \bar{v}_\mu + Q_{18} \bar{v}_\mu}{\varepsilon^3} \right\} ds, \quad (18)$$

where the values of the functions $Q_i$'s are given in appendix C.

All terms depending on $\varepsilon$ must disappear when we take the limit $\varepsilon \rightarrow 0$. This is an impediment to obtain the equations of motion directly from the calculation, due to the terms that arise as negative powers of $\varepsilon$. These terms induce divergences in the equation of motion that should be removed. One strategy is to use the fact that the braces is an exact differential and match to a vector $\bar{B}_\mu$. However there is an obstruction to find the Dirac's constraint in which $v \bar{B} \equiv v_\mu \bar{B}^\mu = 0$. A useful procedure is to consider

$$\bar{B}_\mu = \left( \frac{Q_{23}}{\varepsilon^4} + \frac{Q_{13}}{\varepsilon^3} + \frac{Q_{14}}{\varepsilon^2} + \frac{Q_9}{\varepsilon} + Q_8 \right) v_\mu + \left( \frac{Q_{24}}{\varepsilon^5} + \frac{Q_{22}}{\varepsilon^4} + \frac{Q_{21}}{\varepsilon^3} + \frac{Q_{11}}{\varepsilon^2} + \frac{Q_3}{\varepsilon} + Q_6 \right) \bar{v}_\mu$$

$$+ \left( \frac{Q_{20}}{\varepsilon^4} + \frac{Q_{19}}{\varepsilon^3} + \frac{Q_{17}}{\varepsilon^2} + \frac{Q_{10}}{\varepsilon} + Q_2 \right) \bar{v}_\mu + \left( \frac{Q_{18}}{\varepsilon^3} + \frac{Q_{16}}{\varepsilon^2} + \frac{Q_{15}}{\varepsilon} + Q_{12} \right) \bar{v}_\mu$$

$$+ \left( A_5 + \frac{Q_{36}}{\varepsilon} + Q_{35} \right) f_{\mu \nu} \bar{v}_\nu + \left( A_5 - A_4 \right) f_{\mu \nu} v_\nu + Q_{33} f_{\mu \nu} \bar{v}_\nu. \quad (19)$$
We will determine a solution to the above expression in such a way that we can obtain the value of $B_\mu$. In the present case the product $vB \neq 0$ which complicates the calculation of the equations of motion. However a good strategy is to find, from our original set of vectors $\dot{B}_\mu, v_\mu, \dot{v}_\mu, \ddot{v}_\mu$, a new set of normalized vectors $\tilde{B}_\mu, \tilde{u}_1\mu, \tilde{u}_2\mu, \tilde{u}_3\mu, \tilde{u}_4\mu$ (see appendix C). It is now easy to prove that $\tilde{u}_i\dot{B} = 0, i = 1, \ldots, 4$. Here $\tilde{u}_i\mu = u_i\mu/|u_i|$ and we have $\tilde{u}_i\tilde{u}_i = 0$, due $\tilde{u}_i\tilde{u}_i = 1$. Thus a simple solution to the equation $\tilde{u}_i\dot{B} = 0$ is given by

$$B_\mu = k\tilde{u}_4\mu. \quad (20)$$

However, by replacing the value of $\tilde{u}_4\mu$ in Eq. (20), we have a non-homogeneous first-order differential equation, whose coefficients are function of the norm $|B| = \tilde{B}$, that involves the limit $\varepsilon \to 0$ and which does not exactly cancel the divergences. A solution to this problem is to choose $\dot{B}_\mu$ so that it respects the general form of $\dot{B}_\mu$, i.e. $\tilde{u}_i\dot{B} = 0$ and also eliminates the divergences. Therefore the best way to do this is to take the linear combination

$$\dot{B}_\mu = f_0(\varepsilon)v_\mu + f_1(\varepsilon)\dot{v}_\mu + f_2(\varepsilon)\ddot{v}_\mu, \quad (21)$$

for some suitable functions $f_i$. Moreover the choice of (21) is not arbitrary because $\tilde{u}_i\dot{B} = 0$ requires the additional condition: $Q_{18} = 0$ (since $\tilde{u}_i, v^{(j)} = 0$ except for $j = 3$). This implies that there is an abelian subgroup of the Lie’s algebra satisfying $[\alpha_2, \alpha_3]^2 = 0$. Consequently it reduces the equations to

$$\dot{B}_\mu = \left[\frac{-\frac{2}{3}(A_4^2 + A_5^2 + A_2A_4)}{\varepsilon^2} + \frac{(\frac{2}{3}A_4^2 + \frac{1}{2}A_2A_3 + \frac{5}{6}A_1A_4 + \frac{7}{6}A_3A_4)}{\varepsilon} \frac{\tilde{v}^2}{v} + \frac{(-\frac{1}{2}A_5^2 + \frac{5}{6}A_4^2 + \frac{5}{4}A_2A_4)}{\varepsilon} \tilde{v} \tilde{v} - \frac{1}{24}(8A_2^2 + 9A_3^2 + 13A_2A_4) \frac{v^4}{v}
\right.
\left. - \frac{1}{6}(2A_2^2 + A_4^2 + 3A_2A_4) \frac{\tilde{v}^2}{v} - \frac{1}{6}(2A_2^2 + A_4^2 + 3A_2A_4) \frac{\tilde{v}^2}{v}
\right.
\left. - \frac{1}{2}(A_3^2 + A_1A_3) \frac{\tilde{v}^2}{v} + \frac{1}{12}(9A_1A_2 + 17A_2A_3 + 9A_1A_4 + 13A_3A_4) \frac{\tilde{v}^2}{v}
\right]
\left. v_\mu + \left[\frac{-A_2^2 + A_4^2}{\varepsilon^3} + \frac{3A_1A_2 + A_1A_4 - A_2A_3 - 3A_3A_4}{2\varepsilon^2} + \frac{-A_3^2 \tilde{v}^2 - A_2A_1 \tilde{v}^2 + A_3^2 - A_1^2}{2\varepsilon}
\right.
\right.
\left. + \frac{1}{8}(5A_1A_2 + 3A_2A_3 + 3A_1A_4 + A_3A_4) \frac{\tilde{v}^2}{v} + \frac{1}{6}(A_4^2 - 5A_2^2 - 4A_2A_4) \frac{\tilde{v}^2}{v}
\right]
\left. \ddot{v}_\mu + \left[\frac{-2A_2^2 + A_4A_2 + A_4^2}{3\varepsilon^2} + \frac{2A_1A_2 + A_1A_4 - 2A_2A_3 - 3A_3A_4}{3\varepsilon} - \frac{1}{6}(2A_2^2 + A_2A_4) \frac{\tilde{v}^2}{v}
\right.
\right.
\left. + \left[\frac{A_2 - A_4}{\varepsilon} - A_1 + A_3\right] f_{\mu\nu}v^\nu - A_4 f_{\mu\nu}v^\nu.
\right. \quad (22)$$

We now can take $\dot{B}_\mu$ of the general form:

$$\dot{B}_\mu = f_0(\varepsilon)v_\mu + f_1(\varepsilon)\dot{v}_\mu + f_2(\varepsilon)\ddot{v}_\mu + k_3f_{\mu\nu}v^\nu, \quad (23)$$

where the last term in (23) is not consistent with (21). However, it can be proved that this term satisfies the condition $\dot{B}u_i = 0$ but this is possible only if $L_\nu \ddot{v}^\nu := 0$. However, we will not pursue this further.
where \( L_\nu := f_{\mu\nu} \tilde{A}_\mu \) is a null vector. The physical meaning of this is to have a privileged direction of the Yang-Mills field, see Eq. (105) in appendix C, for which it is allowed to add it to Eq. (23). Then the equations of motion can be expressed in the form

\[
\left[ \rho_1 \dot{v}^2 + \rho_2 \ddot{v} \dot{v} - \frac{1}{24} (8A_1^2 + 9A_1^2 + 13A_2A_4) \dot{v}^4 - \frac{1}{6} (2A_2^2 + A_1^2 + 3A_2A_4) \ddot{v} \dot{v} - \frac{1}{6} (2A_2^2 + A_1^2 + 3A_2A_4) \dot{v}^2 \right] v_\mu
\]

\[
+ \left[ m + \mu_1 \dot{v}^2 + \frac{1}{8} (5A_1A_2 + 3A_2A_3 + 3A_1A_4 + 4A_4) \dot{v} \right] \dot{v} + \frac{1}{6} \left( A_1^2 - 5A_2^2 - 4A_2A_4 \right) \dot{v} \dot{v} \right] \dot{v}_\mu
\]

\[
+ \left[ \mu_3 - \frac{1}{6} (2A_2^2 + A_2A_4) \dot{v} \right] \ddot{v}_\mu + (q - A_1 + A_3) f_{\mu\nu} v_\nu - A_4 f_{\mu\nu} v_\nu = 0, \tag{24}
\]

where \( \rho_1, \rho_2, \mu_1, \mu_3, m \) and \( q \) are suitable renormalization constants. Furthermore, the form of the functions \( f_i(\varepsilon) \) that cancels the divergences in Eq. (22) is of the form

\[
f_0 = \rho_1 \dot{v}^2 + \frac{\frac{2}{3} (A_1^2 + A_2^2 + A_2A_4) \dot{v}^2}{\varepsilon^2} + \frac{\left( \frac{2}{3} A_1A_2 + \frac{1}{3} A_2A_3 + \frac{5}{6} A_1A_4 + \frac{7}{6} A_3A_4 \right) \dot{v}^2}{\varepsilon}
\]

\[
+ \rho_2 \ddot{v} \dot{v} + \frac{\left( -\frac{1}{2} A_2^2 - \frac{5}{6} A_1^2 - \frac{5}{3} A_2A_4 \right) \ddot{v} \dot{v}}{\varepsilon}, \tag{25}
\]

\[
f_1 = m + \mu_1 \dot{v}^2 + \frac{A_3^2 + A_1^2}{\varepsilon^3} + \frac{3A_1A_2 + A_1A_4 - A_2A_3 - 3A_3A_4}{2\varepsilon^2}
\]

\[
+ \frac{A_2^2 \ddot{v}^2 - A_2A_4 \dot{v}^2 + A_3^2 - A_1^2}{2\varepsilon} \tag{26}
\]

\[
f_2 = \mu_3 + \frac{2A_2^2 + A_1A_2 + A_1^2}{3\varepsilon^2} + \frac{2A_1A_2 + A_1A_4 - A_2A_3 - A_3A_4}{3\varepsilon} \tag{27}
\]

Here \( k_3 \) is a correction to the charge of the non-abelian particle \([4, 5]\)

\[
k_3 = -q + \frac{A_2 - A_4}{\varepsilon}. \tag{28}
\]

### 3.1 The Dirac’s radiation damping effect

A particular case of Eq. (22) is when one considers \( \alpha_2, \alpha_3, \alpha_4, \alpha_5 = 0 \), which is justly the abelian case according to Eq. (10). For this case we have that Eq. (22) is of the form

\[
\dot{B}_\mu = \frac{1}{2} \alpha_1 \varepsilon^{-1} \dot{v}_\mu - \alpha_1 f_{\mu\nu} v_\nu, \tag{29}
\]

where \( f_{\mu\nu} \) is determined by Eq. (14).

In this case \( B_\mu = kv_\mu \) is solution to Eq. (29), where \( k = 1/2\alpha_1 \varepsilon^{-1} + \beta \). Thus we have the traditional equation of a charged particle in an electromagnetic field with corrections given the emission/absorption of radiation or the Abraham-Lorentz-Dirac equation \([2]\)

\[
m \dot{v}_\mu = -\text{Tr}(\alpha_1 f_{\mu\nu} v_\nu), \tag{30}
\]

where \( m = \text{Tr}(\beta) \).
3.2 The First non-abelian radiation damping effect

Now we consider the case in which \( \alpha_2 = \alpha_5 = 0 \). For this situation we find that the \( A_1 = -\alpha_1 \), and \( A_3 = [\alpha_1, \alpha_4] \). Plugging these expressions into Eq. (22) we have that the equations of motion take the simple form

\[
\dot{B}_\mu = \frac{1}{2} \left( A_3^2 - A_1^2 \right) \varepsilon^{-1} \dot{v}_\mu - \frac{1}{2} \left( A_3^2 + A_1 A_3 \right) \dot{v}^2 v_\mu + (A_3 - A_1) f_{\mu\nu} \dot{v}^\nu.
\]

(31)

In order to obtain the actual equations of motion we have to take the trace in both sides of the previous equation

\[
\text{Tr}(\dot{B}_\mu) = \text{Tr}\left( \frac{1}{2} \left( A_3^2 - A_1^2 \right) \varepsilon^{-1} \dot{v}_\mu - \frac{1}{2} \left( A_3^2 + A_1 A_3 \right) \dot{v}^2 v_\mu + (A_3 - A_1) f_{\mu\nu} \dot{v}^\nu \right).
\]

(32)

The first term on the right-hand side can be renormalized by taking \( B_\mu = k v_\mu \) and \( k = \frac{1}{2} ([\alpha_1, \alpha_4]^2 - \alpha_1^2) \varepsilon^{-1} + \beta \). Then we find the equation of motion

\[
m \dot{v}_\mu + \mu_1 v_\mu = \text{Tr}\left( \tilde{\mu} f_{\mu\nu} \dot{v}^\nu \right).
\]

(33)

Thus we have the simplest non-abelian equation with \( m = \text{Tr}(\beta) \) and \( \mu_1 = \text{Tr}(\tilde{\mu}_1) \), where \( \tilde{\mu}_1 = \frac{1}{2} ([\alpha_1, \alpha_4]^2 - \alpha_1 [\alpha_1, \alpha_4]) \) and \( \tilde{\mu} = [\alpha_1, \alpha_4] + \alpha_1 \), respectively. The field \( f_{\mu\nu} \) in (14) is modified only in the non-abelian charges of the radiation term which has the form

\[
F^{(rad)}_{\mu\nu} = \frac{2}{3} ([\alpha_1, \alpha_4] - 2 \alpha_1) (\dot{v}_\mu v_\nu - v_\mu \dot{v}_\nu).
\]

(34)

Let us now analyze the second non-abelian case of interest, such that \( k \) in \( B_\mu \) are both functions of velocity and acceleration.

3.3 The second non-abelian radiation damping effect

This case is more general than the previous one since it includes divergences of order \( \varepsilon^{-1} \), \( \varepsilon^{-2} \) and \( \varepsilon^{-3} \) in the mass and of order \( \varepsilon^{-1} \) in the color’s charge. If we take \( \alpha_2 = 0 \) and \( \alpha_4 = 0 \), then equations of motion read

\[
\dot{B}_\mu = \left( \frac{-2 A_4^2 \dot{v}^2}{3 \varepsilon^2} - \frac{3}{8} A_4^2 \dot{v}^4 - \frac{1}{6} A_4^2 \dot{v}^2 + \frac{3}{4} A_1 A_4 \ddot{v}^2 + \frac{1}{6} \dot{A}_4^2 \ddot{v}^2 + 5 A_1 A_4 \ddot{v}^2 - \frac{5 A_4^2 \dot{v}^2}{6 \varepsilon} \right) v_\mu
\]

\[
+ \left( \frac{3}{8} A_1 A_4 \dot{v}^2 + \frac{2}{3} A_4^2 \ddot{v}^2 + \frac{1}{6} A_4^2 \ddot{v}^2 + \frac{A_1^2}{\varepsilon^3} + \frac{A_1 A_4}{2 \varepsilon^2} - \frac{A_1^2}{2 \varepsilon} \right) \dot{v}_\mu + \left( \frac{A_1^2}{3 \varepsilon^2} + \frac{A_1 A_4}{3 \varepsilon} \right) \ddot{v}_\mu
\]

\[
- A_1 f_{\mu\nu} \dot{v}^\nu - \frac{A_4 f_{\mu\nu} v^\nu}{\varepsilon} - A_4 f_{\mu\nu} \dot{v}^\nu.
\]

(35)

The simplest expression for \( \dot{B}_\mu \) is given by

\[
\dot{B}_\mu = g_0 v_\mu + k_1 \dot{v}_\mu + k_2 \ddot{v}_\mu + k_3 f_{\mu\nu} \dot{v}^\nu,
\]

(36)

for suitable coefficients \( g_0 \), \( k_1 \), \( k_2 \) and \( k_3 \). In this case we can simplify the previous equation by defining the non-abelian relations

\[
k_1 := \varepsilon^{-3} A_4^2 + \frac{\varepsilon^{-2}}{2} A_1 A_4 - \frac{\varepsilon^{-1}}{2} A_1^2 - m, \quad k_2 := \frac{\varepsilon^{-2}}{3} A_4^2 + \frac{\varepsilon^{-1}}{3} A_1 A_4 - \mu_3, \quad k_3 := -\varepsilon^{-1} A_4 + q,
\]

(37)
where \( m, \mu_3 \) and \( q \) are some suitable renormalization parameters.

The first term in Eq. (36) depends on the function \( g_0 \) which is chosen as:

\[
g_0 = -\left( \frac{2}{3} \varepsilon^{-2} A_4^2 - \frac{5}{6} \varepsilon^{-1} A_1 A_4 + \rho_1 \right) \dot{v}^2 - \left( \frac{5}{6} \varepsilon^{-1} A_4^2 + \rho_2 \right) \ddot{v} v, \tag{38}
\]

where now \( \rho_1 \) and \( \rho_2 \) are the renormalization parameters.

The last term in Eq. (36) is not consistent with Eq. (21). However, it can be proved that this term satisfies the condition \( \dot{B}_\mu = 0 \), but this is possible only if \( L_\nu \dot{v}_\nu = 0 \), where \( L_\nu = f_{\mu\nu} \tilde{u}_4^\mu \) is null, (see the Eq. (105) in appendix C). Then it is allowed to add it in (36). Thus the equations of motion can be expressed in the form

\[
\left[ \rho_1 \ddot{v}^2 + \left( \rho_2 - \frac{3}{4} \alpha_1 [\alpha_1, \alpha_5] \right) \dddot{v}^2 - \frac{3}{8} \alpha_1 [\alpha_1, \alpha_5]^2 \dddot{v}^2 - \frac{1}{6} \alpha_1 [\alpha_1, \alpha_5]^2 [\dot{v}^2] \right] v_\mu \\
+ m \dot{v}_\mu + (\mu_1 \dddot{v}^2 + \mu_2 \ddot{v}) \dot{v}_\mu + \mu_3 \dddot{v}_\mu = (-\alpha_1 + q) f_{\mu\nu} \dot{v}_\nu - [\alpha_1, \alpha_5] f_{\mu\nu} \dot{v}_\nu. \tag{39}
\]

Of course, in order to find the equations of motion its is necessary to take the traces in both sides of the previous equation. This equation includes non-abelian corrections to the charge and a new coupling of the field strength with the velocity of the particle, where the tensor \( f_{\mu\nu} \) is expressed through the radiated field Eq. (14) that now has the form

\[
F^{(rad)}_{\mu\nu} = \frac{7}{12} \alpha_1 \alpha_5 [\dddot{v}^2 (v_\mu \ddot{v}_\nu - v_\nu \ddot{v}_\mu) - \frac{4}{3} \alpha_1 (\dddot{v}_\mu v_\nu - \dddot{v}_\nu v_\mu)] \\
+ \frac{1}{4} \alpha_1 \alpha_5 \left( \dddot{v}_\mu \dddot{v}_\nu + \frac{2}{3} \dddot{v}_\mu \dddot{v}_\nu - \frac{2}{3} \dddot{v}_\mu \dddot{v}_\nu - \dddot{v}_\mu v_\nu \right). \tag{40}
\]

We have defined the constants \( \mu_1 := -\frac{3}{8} \alpha_1 [\alpha_1, \alpha_5], \mu_2 := \frac{3}{4} [\alpha_1, \alpha_5]^2 \) in order to simplify the expression.

### 3.4 The third non-abelian radiation damping effect

In this case we choose the commutation relations to be \( [\alpha_1, \alpha_5] = [\alpha_2, \alpha_4] = [\alpha_2, \alpha_5] = 0 \). Then the equations of motion are

\[
\dot{B}_\mu = \left( -\frac{1}{3} A_2 A_4^2 - \frac{1}{2} A_3 v^2 + A_1 A_2 \dddot{v}^2 - \frac{1}{2} A_1 A_3 \dddot{v}^2 + \frac{A_2 A_3 \dddot{v}^2}{3 \varepsilon} - \frac{1}{3} A_2^2 \ddot{v}^2 - \frac{3}{4} A_1 A_2 \dddot{v}^2 + \frac{17}{12} A_2 A_3 \dddot{v} \right) v_\mu \\
+ \left( \frac{A_1^2}{2 \varepsilon} + \frac{5}{8} A_1 A_2 v^2 + \frac{3}{8} A_2 A_3 v^2 + \frac{3 A_1 A_2}{2 \varepsilon^2} + \frac{1}{6} A_1 A_3 \dddot{v}^2 + \frac{1}{3} A_2 \dddot{v} \right) v_\mu \\
- \frac{A_2 A_3}{2 \varepsilon^2} - \frac{A_2^2}{\varepsilon^3} \right) \dddot{v}_\mu \\
+ \left( -\frac{2 A_2 A_3}{3 \varepsilon} - \frac{1}{3} A_2 \dddot{v}^2 + \frac{2 A_1 A_2}{3 \varepsilon} - \frac{2 A_3^2}{3 \varepsilon^2} \right) \dddot{v}_\mu + \left( \frac{A_2}{\varepsilon} + A_3 - A_1 \right) f_{\mu\nu} \dot{v}_\nu. \tag{41}
\]
The vector $\dot{B}_\mu$ can be expressed in the form:

$$
\dot{B}_\mu = g_0 v_\mu + g_1 \dot{v}_\mu + k_2 \ddot{v}_\mu + k_3 f_{\mu\nu} v^\nu, 
$$

(42)

for suitable parameters coefficients $g_0$, $g_1$, $k_2$ and $k_3$ that eliminate the divergences as $\varepsilon$ goes to zero.

The renormalization of the equations of motion is carried out through the functions $g_0$ and $g_1$ depending on the velocities and accelerations, and the constants $k_2$ and $k_3$ that appear in the solution (12)

$$
g_0 := \frac{2A_1 A_2 \dot{v}^2}{3\varepsilon} + \frac{A_2 A_3 \dot{v}^2}{3\varepsilon} - \frac{2A_2^2 \dot{v}^2}{3\varepsilon} - \frac{A_2 \ddot{v} \dot{v}}{2\varepsilon} - \rho_1 \dddot{v} - \rho_2 \ddot{v}, 
$$

(43)

$$
g_1 := \frac{A_3^2}{2\varepsilon} + \frac{3A_1 A_2}{2\varepsilon^2} - \frac{A_1^2}{2\varepsilon} - \frac{A_2^2 \dot{v}^2}{2\varepsilon^2} - \frac{A_2 A_3}{2\varepsilon^2} - \frac{A_3^2}{\varepsilon^3} - \mu_1 - \mu_2 \dddot{v}^2, 
$$

(44)

$$
k_2 := -\frac{2A_2 A_3}{3\varepsilon} + \frac{2A_1 A_2}{3\varepsilon} - \frac{2A_2^2}{3\varepsilon^2} - \mu_3, \quad k_3 := \left( \frac{A_2}{\varepsilon} + q \right), 
$$

(45)

where $\rho_1$, $\rho_2$, $\mu_1$, $\mu_2$, $\mu_3$ and $q$ are the renormalization parameters.

The condition $\mathbf{B}\ddot{u}_\mu = 0$ and Eq. (21) to be fulfilled, the term $k_3 f_{\mu\nu} v^\nu$ in (12) must satisfy condition $f_{\mu\nu} v^\nu \ddot{u}_\mu = 0$. In order to fulfill the mentioned conditions we should have $A_1, A_5 = 0$ (see Eq. (105)). If we use the Eqs. (42)-(45), then the no-abelian equations of motion are

$$
\left( -\frac{4}{3} \alpha_2 \dot{v}^4 - \frac{1}{2} ( -2\dot{\alpha}_2 + [\alpha_1, \alpha_4] + [\alpha_2, \alpha_3]) \dot{v}^2 + \frac{1}{2} \alpha_1 ( -2\dot{\alpha}_2 + [\alpha_1, \alpha_4] + [\alpha_2, \alpha_3] ) \dddot{v}^2 + \rho_1 \dddot{v} - \frac{4}{3} \alpha_2 \dot{v}^2 + \frac{3}{2} \alpha_1 \alpha_2 \dddot{v} \dot{v} - \frac{17}{6} \alpha_2 ( -2\dot{\alpha}_2 + [\alpha_1, \alpha_4] + [\alpha_2, \alpha_3] ) \dddot{v} \dot{v} - \frac{4}{3} \alpha_2 \dddot{v} + \rho_2 \dddot{v} \right) v^\mu 
$$

$$
+ \left( \frac{5}{4} \alpha_1 \alpha_2 \dot{v}^2 - \frac{3}{4} \alpha_2 ( -2\dot{\alpha}_2 + [\alpha_1, \alpha_4] + [\alpha_2, \alpha_3] ) \dddot{v} - \frac{10}{3} \alpha_2 \dddot{v} + \mu_1 + \mu_2 \dddot{v} \right) \dddot{v} 
$$

$$
+ \left( -\frac{4}{3} \alpha_2 \dot{v}^2 + \mu_3 \right) \dddot{v} = (q - \alpha_1 + 2\dot{\alpha}_2 - [\alpha_1, \alpha_4] - [\alpha_2, \alpha_3]) f_{\mu\nu} v^\nu. 
$$

(46)

In this expression the interaction term $f_{\mu\nu} v^\nu$ contains non-abelian corrections to the charge of the Yang-Mills particle. Analogously to the previous cases we can determine the radiated field $F_{\mu\nu}^{(\text{rad})}$ through Eq. (17). In this case it becomes

$$
F_{\mu\nu}^{(\text{rad})} = \frac{11}{6} \alpha_2 \dot{v}^2 (v_\mu v_\nu - \ddot{v}_\mu v_\nu) + \frac{2}{3} (2\alpha_1 - 2\dot{\alpha}_2 + [\alpha_1, \alpha_4] + [\alpha_2, \alpha_3]) (\dddot{v}_\mu v_\nu - \ddot{v}_\mu v_\nu) 
$$

$$
- \frac{3}{2} \alpha_2 \left( v_\mu \dddot{v}_\nu + \frac{2}{3} \dddot{v}_\mu \dddot{v}_\nu - \frac{2}{3} \dddot{v}_\mu \dddot{v}_\nu \right). 
$$

(47)

4 Final Remarks

At the present time the classical problem of the dynamics of a radiating particle is still a subject of intense work. In the specific case of a color charged particle interacting with
a Yang-Mills field there is a huge interest in the problem of non-abelian hydrodynamics and the study of the quark-gluon plasma.

In the present article we continue exploring the equations of motion of a non-abelian charge in an external Yang-Mills field, where the dynamics is described by Wong’s equations [10]. There has been much work in this subject as we mentioned in the introduction section. However a self-consistent analysis of the classical theory, as was given by Dirac [2] for the abelian case, has not been worked out in the literature for the Yang-Mills case. This analysis has been carried out in the present article.

The strategy we followed here was to adopt the ansatz (7) for the retarded Liénard-Wiechert potentials, given in Ref. [37]. This allowed to compute the retarded, advanced and actual field strengths along the trajectory of the particle. A summary and the results of this derivation were collected in appendices A and B. Furthermore, using Dirac’s method of integration over the energy-momentum tensor, in appendix C is outlined the computation of the equations of motion and its corrections due the radiation damping. These results were used to give precise formulas in Section 3 of the corrections due the self-force to Wong’s equations. Moreover it is shown that under certain assumptions about the Lie algebra structure of the Dirac’s equation of motion [2] can be recovered. We also were able to find three non-trivial examples for the non-abelian equations of motion with the properly cancelation of divergences.

Finally, as a possible perspective of this work, we point out that a generalized Lorentz-Dirac equation was obtained in the context of the AdS/CFT correspondence [38]. This derivation In the near future we would like to address the problem of finding a close correspondence between gravity dual in the AdS space and some of the details that we found in the present article for the particle Yang-Mills theory.

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A Series expansions of fields near the trajectory of the particle

In the following appendices we present some computations and results we used in the body of the article. In this appendix we will give some preliminary results that will be necessary in the subsequent appendices.

In order to perform the calculation of the field strength $F_{\mu\nu}$ in Eq. (10) we need to compute first the terms with different negative powers of $R$. $R^{-1}$ is a function of $x_\mu$ and $z_\mu$ and therefore it can be expanded in powers of $\sigma$ as is shown in Eqs. (11) and (12). Neglecting the higher positive powers in $\sigma$ means that we are closer from the trajectory of the particle. Then the expansion reads

$$
R^{-1} = \left[ \dot{z}^\mu (x_\mu - z_\mu) \right]^{-1} \\
= \sigma^{-1} \left[ 1 + \gamma f(\sigma) \right]^{-1} \\
\times \left\{ 1 + \left[ -\frac{1}{6} \dot{\gamma}^2 \sigma^2 + \sum_{p=3}^{10} (-1)^p \sum_{n=0}^{p} \frac{\nu_n}{n! (p+1-n)!} \sigma^n \right] \left[ 1 + \gamma f(\sigma) \right]^{-1} \right\}^{-1},
$$

(48)

where $\gamma f(\sigma) = \sum_{k=1}^{10} (-1)^k \gamma \nu_k \sigma^k$ and $\gamma \nu_k = \gamma \nu_k^\mu$. The first factor can in turns be expanded as power series of $\sigma$ and it can be expressed as

$$
[1 + \gamma f(\sigma)]^{-1} = 1 - \gamma f(\sigma) + [\gamma f(\sigma)]^2 - \cdots - \left[ \gamma f(\sigma) \right]^{11} \\
= h_{10} \sigma^{10} + h_9 \sigma^9 + h_8 \sigma^8 + h_7 \sigma^7 + h_6 \sigma^6 \\
+ h_5 \sigma^5 + h_4 \sigma^4 + h_3 \sigma^3 + h_2 \sigma^2 + h_1 \sigma + h_0.
$$

(49)

In the process of expansion we have considered that the highest possible value of the powers of $\sigma$ relevant in our computation is up to 11. Moreover we have assumed that $\gamma$ is of the same order in $\sigma$, thus for instance we have $[\gamma f(\sigma)]^{10} = (\gamma \dot{\nu})^{10} - 5 \sigma (\gamma \ddot{\nu})^9 \gamma \dot{\nu}$ and $[\gamma f(\sigma)]^{11} = -(\gamma \dddot{\nu})^{11}$. The first coefficients $h_i$ of expansion in Eq. (19) are given by

$$
h_0 = \gamma \dddot{\nu}^{11} + \gamma \dddot{\nu}^{10} + \gamma \dot{\nu}^9 + \gamma \dot{\nu}^8 + \gamma \dot{\nu}^7 + \gamma \dot{\nu}^6 + \gamma \dot{\nu}^5 + \gamma \dot{\nu}^4 + \gamma \dddot{\nu}^3 + \gamma \ddot{\nu}^2 + \gamma \dddot{\nu} + 1,
$$

(50)

$$
h_1 = -5 \gamma \dddot{\nu} \gamma \dot{\nu}^9 - \frac{9}{2} \gamma \dddot{\nu} \gamma \dot{\nu}^8 - 4 \gamma \dddot{\nu} \gamma \dot{\nu}^7 - \frac{7}{2} \gamma \dddot{\nu} \gamma \dot{\nu}^6 - 3 \gamma \dddot{\nu} \gamma \dot{\nu}^5 - \frac{5}{2} \gamma \dddot{\nu} \gamma \dot{\nu}^4 \\
- 2 \gamma \dddot{\nu} \gamma \dot{\nu}^3 - \frac{3}{2} \gamma \dddot{\nu} \gamma \dot{\nu}^2 - \gamma \dddot{\nu} \gamma \dot{\nu} - \frac{\gamma \dddot{\nu}}{2},
$$

(51)

$$
h_2 = \frac{9}{6} \gamma \dddot{\nu} \gamma \dot{\nu}^8 + 9 \gamma \dddot{\nu} \gamma \dot{\nu}^7 + \frac{8}{6} \gamma \dddot{\nu} \gamma \dot{\nu}^6 + 7 \gamma \dddot{\nu} \gamma \dot{\nu}^5 + \frac{7}{6} \gamma \dddot{\nu} \gamma \dot{\nu}^4 + \frac{21}{4} \gamma \dddot{\nu} \gamma \dot{\nu}^3 \\
+ \frac{6}{6} \gamma \dddot{\nu} \gamma \dot{\nu}^5 + \frac{15}{4} \gamma \dddot{\nu} \gamma \dot{\nu}^4 + \frac{5}{6} \gamma \dddot{\nu} \gamma \dot{\nu}^3 + \frac{5}{2} \gamma \dddot{\nu} \gamma \dot{\nu}^2 + 4 \gamma \dddot{\nu} \gamma \dot{\nu}^3 + \frac{3}{2} \gamma \dddot{\nu} \gamma \dot{\nu}^2 \\
+ \frac{3}{6} \gamma \dddot{\nu} \gamma \dot{\nu}^2 + \frac{3}{4} \gamma \dddot{\nu} \gamma \dot{\nu}^3 + \frac{2 \gamma \dddot{\nu} \gamma \dot{\nu} \gamma \dot{\nu}^2}{6} + \frac{\gamma \dddot{\nu} \gamma \dot{\nu}^2}{4} + \frac{\gamma \dddot{\nu} \dot{\nu}^2}{6},
$$

(52)
where we adopted the notation $(\gamma v)^{\alpha} \equiv \gamma v^\alpha = (\gamma \mu v^\mu)^\alpha$. Furthermore we have written only the first coefficients of the expansion since that up on computation one can see that the higher order terms will not contribute to the equations of motion.

The first term of in the square brackets from the second factor of the right-hand side of Eq. (48) also can be expanded as powers of $\sigma$ with coefficients given by certain functions $p_i$:

$$h_3 = -\frac{1}{24} 8\gamma v^4 \gamma \ddot{v} - \frac{28}{6} \gamma \ddot{v} \gamma \dot{v} \gamma \ddot{v} - \frac{7}{24} \gamma v^4 \gamma \ddot{v} - 7\gamma v^3 \gamma \dddot{v} - \frac{21}{6} \gamma \ddot{v} \gamma \dddot{v} \gamma v^5 - \frac{6}{24} \gamma v^4 \gamma \dddot{v}$$

$$h_4 = \frac{7}{120} \gamma v^5 \gamma \dddot{v} + \frac{21}{36} \gamma \ddot{v}^2 \gamma \ddot{v} + \frac{21}{24} \gamma \ddot{v} \gamma v^4 \gamma \ddot{v} + \frac{6}{120} \gamma v^5 \gamma \ddot{v} + \frac{15}{36} \gamma \ddot{v}^2 \gamma \ddot{v}$$

$$+ \frac{105}{24} \gamma \ddot{v} \gamma \ddot{v} \gamma \dddot{v} + \frac{15}{24} \gamma \ddot{v} \gamma v^4 \gamma \ddot{v} + \frac{5}{120} \gamma v^5 \gamma \ddot{v} + \frac{35}{16} \gamma \ddot{v} \gamma v^4 \gamma \ddot{v} + \frac{10}{36} \gamma \ddot{v}^2 \gamma \ddot{v}$$

$$+ \frac{15}{12} \gamma \ddot{v} \gamma \ddot{v} \gamma \dddot{v} + \frac{6}{24} \gamma \ddot{v} \gamma v^4 \gamma \ddot{v} + \frac{3}{120} \gamma v^5 \gamma \ddot{v} + \frac{15}{16} \gamma \ddot{v} \gamma \ddot{v} + \frac{6}{36} \gamma \ddot{v}^2 \gamma \ddot{v}$$

$$+ \frac{3}{6} \gamma \ddot{v} \gamma \ddot{v} \gamma \dddot{v} + \frac{3}{24} \gamma \ddot{v} \gamma v^4 \gamma \ddot{v} + \frac{2 \gamma v^5 \gamma \ddot{v}}{120} + \gamma \ddot{v}^4$$

$$+ \frac{\gamma \ddot{v}^2}{36} + \frac{3}{24} \gamma \ddot{v}^2 \gamma \dddot{v} + \gamma \ddot{v} \gamma v^4 \gamma \ddot{v} + \frac{2 \gamma v^5 \gamma \ddot{v}}{120} + \gamma \ddot{v}^4$$

(54)

where the important coefficients for our further computations are

$$p_3 = -\frac{1}{6} \dot{v}^2, \quad p_4 = \frac{5}{24} \dddot{v}, \quad p_5 = -\frac{1}{5!} (8\ddot{v} + 9\dot{v} \dddot{v}), \quad p_6 = -\frac{1}{6!} (35\dddot{v} + 9\dot{v} \dddot{v}^2).$$

(56)
Moreover the expression \( \dot{z}_\mu(x_\nu - z_\nu(s)) \) is also expanded in powers of \( \sigma \) as follows

\[
\begin{align*}
\dot{z}_\mu(x_\nu - z_\nu(s)) &= \gamma_\nu v_\mu + (v_\nu v_\mu - \gamma_\nu \dot{v}_\mu) + \sigma^2 \left( k_{\mu\nu2} + \gamma_\nu \ddot{v}_\mu \right) + \sigma^3 \left( k_{\mu\nu3} - \frac{\gamma_\nu \dddot{v}_\mu}{3!} \right) \\
&\quad + \sigma^4 \left( \frac{\gamma_\nu v_\mu^{(4)}}{4!} + k_{\mu\nu4} \right) + \sigma^5 \left( k_{\mu\nu5} - \frac{\gamma_\nu v_\mu^{(5)}}{5!} \right) + \sigma^6 \left( k_{\mu\nu6} + \frac{\gamma_\nu v_\mu^{(6)}}{6!} \right) \\
&\quad + \sigma^7 \left( k_{\mu\nu7} - \frac{\gamma_\nu v_\mu^{(7)}}{7!} \right) + \sigma^8 \left( k_{\mu\nu8} + \frac{\gamma_\nu v_\mu^{(8)}}{8!} \right) + \sigma^9 \left( k_{\mu\nu9} - \frac{\gamma_\nu v_\mu^{(9)}}{9!} \right) \\
&\quad + \sigma^{10} \left( \frac{\gamma_\nu v_\mu^{(10)}}{10!} + k_{\mu\nu10} \right) + \sigma^{11} \left( k_{\mu\nu11} - \frac{\gamma_\nu v_\mu^{(11)}}{11!} \right),
\end{align*}
\]

(57)

where \( k_{\mu\nu i} \) are some functions depending on the velocities and their derivatives. Actually we need to compute the anti-symmetric version of the previous equation because this is the case that arises in the computation of \( F_{\mu\nu}^{(rad)} \), that is: \( \dot{z}_\mu(x_\nu - z_\nu(s)) - \dot{z}_\nu(x_\mu - z_\mu(s)) \). Thus, the relevant functions \( k_{\mu\nu i} \) are antisymmetric tensors and they are given by

\[
\begin{align*}
k_{\mu\nu2} &= \frac{v_\mu \ddot{v}_\nu}{2} - \frac{\ddot{v}_\mu v_\nu}{2}, & k_{\mu\nu3} &= \frac{\dddot{v}_\mu v_\nu}{3} - \frac{\ddot{v}_\mu \dddot{v}_\nu}{3}, \\
k_{\mu\nu4} &= \frac{v_\mu \dddot{v}_\nu}{8} + \frac{\ddot{v}_\mu \dddot{v}_\nu}{12} - \frac{\dddot{v}_\mu v_\nu}{12} - \frac{\ddot{v}_\mu \dddot{v}_\nu}{8} v_\mu v_\nu, \\
k_{\mu\nu5} &= \frac{v_\mu v_\nu^{(4)}}{30} - \frac{\ddot{v}_\mu \ddot{v}_\nu}{24} + \frac{\dddot{v}_\mu v_\nu}{24} + \frac{\dddot{v}_\mu \dddot{v}_\nu}{30}.
\end{align*}
\]

(58)

(59)

(60)

Now we give some definitions that will be useful later in order to simplify the relevant equations in the following appendices are

\[
\begin{align*}
g_1 &= h_2 - h_0^2 h_3, & g_2 &= -3 h_0^4 p_3 + 3 h_2 h_0^2 + 3 h_1^2 h_0, \\
g_3 &= 3 h_1 h_0 - 2 h_0^2 p_3 + 2 h_2 h_0^2 + 5 h_1^2 h_0, & g_4 &= 4 h_0^2 p_3 + 2 h_0^4 p_3 - 2 h_2 h_0^2 - h_1^2 h_0, \\
g_5 &= 2 h_1 h_0 + 2 h_0^3 p_3 - 2 h_2 h_0^2, & g_6 &= -4 h_0^4 p_3 + 4 h_0^2 h_3 + 6 h_1^2 h_2, \\
g_7 &= -4 h_0^5 p_3 + 4 h_0^2 h_3^2 + 6 h_1^2 h_0, & g_8 &= -4 h_0^4 p_4 - 20 h_1^2 h_0^4 + 4 h_3 h_0^4 + 12 h_1 h_2 h_0^2 + 4 h_3 h_0^2, \\
g_9 &= -5 h_0^6 p_4 - 30 h_1 h_0^6 p_3 + 5 h_3 h_0^4 + 20 h_1 h_2 h_0^3 + 10 h_1^3 h_0^2, & g_{10} &= 4 h_0^5 p_3 - 4 h_0^2 h_3^2 - 6 h_1^2 h_0^2, \\
g_{11} &= -5 h_0^6 p_3 + 5 h_2 h_0^4 + 10 h_1^2 h_3, & g_{12} &= -5 h_0^6 p_3 + 5 h_2 h_0^4 + 10 h_1^2 h_3, \\
g_{13} &= -5 h_0^6 p_4 - 30 h_1 h_0^6 p_3 + 5 h_3 h_0^4 + 20 h_1 h_2 h_0^3 + 10 h_1^3 h_0^2, & g_{14} &= 5 h_0^5 p_3 - 5 h_2 h_0^4 - 10 h_1^2 h_3, \\
g_{15} &= 15 h_0^7 p_3^2 - 5 h_0^6 p_5 - 30 h_0^5 p_3 - 30 h_1 h_3 p_5 - 75 h_1^2 h_0^4 - 30 h_0^3 p_5 - 5 h_4 h_0^4, \\
&\quad + 10 h_2 h_0^4 + 20 h_1 h_3 h_0^3 + 30 h_1^2 h_2 h_0^2 + 5 h_1 h_0, \\
g_{16} &= 5 h_0^6 p_4 + 30 h_1 h_0^6 p_3 - 5 h_3 h_0^4 - 20 h_1 h_2 h_0^3 - 10 h_1^3 h_0^2, & g_{17} &= -5 h_0^6 p_3 + 5 h_2 h_0^4 + 10 h_1^2 h_3, \\
g_{18} &= -\frac{1}{2} \frac{5 h_6 p_3}{2} + 5 h_2 h_0^4 + 5 h_1^2 h_3.
\end{align*}
\]

(61)

### B Computation of \( F_{\mu\nu}^{(rad)} \), \( F_{\mu\nu}^{(act)} \) and \( f_{\mu\nu} \)

In this appendix we will give some details of the computation of \( F_{\mu\nu}^{(ret)} \) and \( F_{\mu\nu}^{(adv)} \), through which the radiated field \( F_{\mu\nu}^{(rad)} \) is also obtained. To carry out the Taylor
expansion we use Eqs. (11) and (12) and the first term in the \( F_{\mu\nu}^{(ret)} \) from Eq. (10), then it is written as

\[
-R^{-1} \frac{d}{ds} \left[ R^{-1} \{ \dot{z}_\mu (x_\nu - z_\nu) - \dot{z}_\nu (x_\mu - z_\mu) \} \right]
= g_1 h_1 \gamma_\nu v_\mu - 2g_1 h_0 \sigma v_\mu + 2g_1 h_0 v_\mu v_\nu + 3h_1 h_0 k_{\mu\nu2} \\
+ 2h^2_0 k_{\mu3} + \frac{h^2_0 k_{\mu2} - h_1 h_0 \gamma_\nu \dot{v}_\mu + \frac{1}{3} h^2_0 \gamma_\nu \ddot{v}_\mu + h_1 h_0 v_\mu v_\nu}{\sigma} \\
+ h_1^3 (-p_4) \gamma_\nu v_\mu + 2h_1^2 p_3 \gamma_\nu v_\mu - \frac{2h^2_0 \gamma_\nu \ddot{v}_\mu}{6} - \frac{h^2_0 \gamma_\nu v_\mu}{\sigma^3} \\
- \frac{h_1 h_0 \gamma_\nu v_\mu}{\sigma^2} + h_3 h_0 \gamma_\nu v_\mu - h_1^2 \gamma_\nu \dot{v}_\mu + \frac{3}{2} h_1 h_0 \gamma_\nu \ddot{v}_\mu + h_1^2 v_\mu v_\nu, \quad (62)
\]

where the functions \( k_{\mu\nu i} \) and \( h_i \) were given explicitly in appendix A. Similarly we can calculate the Taylor expansion for the following terms in the retarded field \( F_{\mu\nu}^{(ret)} \). It is given by

\[
-R^{-2} \frac{d}{ds} \left[ R^{-1} \{ \dot{z}_\mu (x_\nu - z_\nu) - \dot{z}_\nu (x_\mu - z_\mu) \} \right]
= 3g_1 h^2_0 k_{\mu\nu2} + 5h_0 h^2_1 k_{\mu\nu2} + 2h^2_0 k_{\mu\nu2} + 7h^2_0 h_1 k_{\mu\nu3} + 3h^3_0 k_{\mu\nu4} - 2h^4_0 p_3 k_{\mu\nu2} + 4g_1 h_0 h_1 v_\mu v_\nu \\
- 3h^3_0 p_4 v_\nu v_\mu - 8h^3_0 p_3 v_\mu v_\nu - \frac{h^3_0 \gamma_\nu v_\mu}{\sigma^4} + 3h^3_0 h^4_0 v_\mu v_\nu + 2h_1 h_2 h_0 v_\mu v_\nu + h^4_1 v_\mu v_\nu \\
- \frac{2h^3_0 h_1 \gamma_\nu v_\mu}{\sigma^3} + g_1 h^2_0 \gamma_\nu v_\mu + 2g_1 h_0 h_2 \gamma_\nu v_\mu - h_0 h^2_2 \gamma_\nu v_\mu + 2h_0 h_1 h_3 \gamma_\nu v_\mu + h^2_0 h_4 \gamma_\nu v_\mu \\
- 2g_1 h^3_0 p_3 \gamma_\nu v_\mu - 5h^2_0 h^2_2 p_3 \gamma_\nu v_\mu - 4h^3_0 h_1 p_4 \gamma_\nu v_\mu - h^4_0 p_5 \gamma_\nu v_\mu - 4g_1 h_0 h_1 \gamma_\nu \dot{v}_\mu \\
- 3h^3_0 h_2 h_0 \gamma_\nu \dot{v}_\mu - 8h_1 h_3 h_0 \gamma_\nu \dot{v}_\mu + h^3_0 h_3 \gamma_\nu \dot{v}_\mu \\
+ \frac{3}{2} g_1 h^3_0 \gamma_\nu \ddot{v}_\mu + \frac{5}{2} h_0 h^2_1 \gamma_\nu \ddot{v}_\mu + h^2_0 h_2 \gamma_\nu \ddot{v}_\mu - h^2_0 p_3 \gamma_\nu \ddot{v}_\mu - \frac{7h^2_0 h_1 \gamma_\nu \ddot{v}_\mu}{6} + \frac{h^3_0 \gamma_\nu v_\mu^{(4)}}{24} \\
+ \frac{g_1 h^3_0 \gamma_\nu v_\mu + h^3_0 k_{\mu\nu2} + 2h^4_0 p_3 \gamma_\nu v_\mu - 2h_2 h^2_0 \gamma_\nu v_\mu - h^2_2 h_0 \gamma_\nu v_\mu + h_1 h^2_0 \gamma_\nu \dot{v}_\mu + \frac{1}{2} h^2_0 \gamma_\nu \ddot{v}_\mu + h_1 h^2_0 \gamma_\nu \ddot{v}_\mu}{\sigma^2} \\
+ \frac{1}{\sigma} \left[ 2g_1 h_1 h_0 \gamma_\nu v_\mu + 2g_1 h^2_0 v_\mu v_\nu + 4h_1 h^2_0 k_{\mu\nu2} + 2h^3_0 k_{\mu\nu3} - 2h_1 h_2 h_0 \gamma_\nu v_\mu + 2h^2_1 h_0 v_\mu v_\nu \\
- 2g_1 h^3_0 \gamma_\nu \dot{v}_\mu + 2h_1 h^3_0 p_3 \gamma_\nu v_\mu - \frac{2h^3_0 \gamma_\nu \ddot{v}_\mu}{6} - 2h_1 h_2 h_0 \gamma_\nu \ddot{v}_\mu + 2h_1 h^2_0 \gamma_\nu \ddot{v}_\mu \right], \quad (63)
\]

where \( h_i, p_i, k_{\mu\nu i} \) and \( g_i \) are given in appendix A.
Further terms of higher negative powers of $R$ are given by

$$R^{-3}\left[\ddot{z}_\mu(x_\nu - z_\nu) - \dot{z}_\nu(x_\mu - z_\mu)\right]$$

$$= 3h_1h_0^2k_{\mu\nu} + h_0^3k_{\mu\nu3}$$

$$+ \frac{1}{\sigma}\left[h_0^3k_{\mu\nu2} - 3h_0^4p_3\gamma_\nu v_\mu + 3h_2h_0^2\gamma_\nu v_\mu + 3h_1^2h_0\gamma_\nu v_\mu - 3h_1h_0^2\gamma_\nu \dot{v}_\mu\right]$$

$$+ \frac{1}{2}h_0^3\gamma_\nu \ddot{v}_\mu + 3h_1h_0^2v_\mu v_\nu - 3h_0^4p_4\gamma_\nu v_\mu - 12h_1h_0^3\gamma_\nu v_\mu$$

$$+ 3h_0^4p_3\gamma_\nu \dot{v}_\mu - 3h_0^4p_3v_\mu v_\nu - \frac{h_0^3\gamma_\nu \dddot{v}_\mu}{6}$$

$$+ \frac{3h_1h_0^2\gamma_\nu v_\mu - h_0^3\gamma_\nu \dddot{v}_\mu + h_0^3\gamma_\nu v_\mu}{\sigma^2} + \frac{h_0^3\gamma_\nu v_\mu}{\sigma^3}$$

$$+ 3h_3h_0^2\gamma_\nu v_\mu + 6h_1h_0h_0\gamma_\nu v_\mu + h_1^3\gamma_\nu v_\mu - 3h_2h_0^2\gamma_\nu \dot{v}_\mu$$

$$- 3h_1^2h_0\gamma_\nu \ddot{v}_\mu + \frac{3}{2}h_1h_0^2\gamma_\nu \dddot{v}_\mu + 3h_2h_0^2v_\mu v_\nu + 3h_1^2h_0v_\mu v_\nu.$$  \hfill (64)
The following term is written as

\[
R^{-4} \left[ \dot{z}_\mu (x_\nu - z_\nu) - \ddot{z}_\nu (x_\mu - z_\mu) \right]
\]

\[
= 10 p_3^2 v_\mu \gamma_\nu h_0^6 - 4 k_{\mu 2} p_3 h_0^5 - 4 p_4 v_\mu \gamma_\nu h_0^5 - 4 p_5 v_\mu \gamma_\nu h_0^5
\]

\[
+ 4 p_4 \ddot{v}_\mu \gamma_\nu h_0^5 - 2 p_3 \dddot{v}_\mu \gamma_\nu h_0^5 + k_{\mu 4} h_0^4 - 20 h_1 p_3 v_\mu \gamma_\nu h_0^4
\]

\[
- 20 h_2 p_3 v_\mu \gamma_\nu h_0^4 - 20 h_1 p_4 v_\mu \gamma_\nu h_0^4 + \frac{v_\mu \gamma_\nu h_0^4}{\sigma^4} + 20 h_1 p_3 \dddot{v}_\mu \gamma_\nu h_0^4
\]

\[
+ \frac{v_\mu^{(4)} \gamma_\nu h_0^4}{24} + 4 h_2 k_{\mu 2} h_0^3 + 4 h_1 k_{\mu 3} h_0^3 + 4 h_3 v_\mu v_\nu h_0^3 + 4 h_4 v_\mu \gamma_\nu h_0^3
\]

\[
- 40 h_1^2 p_3 v_\mu \gamma_\nu h_0^3 - 4 h_3 \dddot{v}_\mu \gamma_\nu h_0^3 + 4 h_2 \dddot{v}_\mu \gamma_\nu h_0^3 - \frac{4 h_1 \dddot{v}_\mu \gamma_\nu h_0^3}{6}
\]

\[
+ 6 h_1 k_{\mu 2} h_0^2 + 12 h_1 h_2 v_\mu v_\nu h_0^2 + 6 h_2 v_\mu \gamma_\nu h_0^2 + 6 h_1 h_3 v_\mu \gamma_\nu h_0^2
\]

\[
- 12 h_1 h_2 \dddot{v}_\mu \gamma_\nu h_0^2 + 3 h_2^2 \dddot{v}_\mu \gamma_\nu h_0^2 + 4 h_3 v_\mu v_\nu h_0 + 12 h_1^2 h_2 v_\mu \gamma_\nu h_0
\]

\[
- 4 h_1 \dddot{v}_\mu \gamma_\nu h_0
\]

\[
+ \frac{1}{\sigma} \left[ - 4 p_3 v_\mu v_\nu h_0^5 - 4 p_4 v_\mu \gamma_\nu h_0^5 + 4 p_3 \dddot{v}_\mu \gamma_\nu h_0^5 + k_{\mu 3} h_0^4
\]

\[
- 20 h_1 p_3 v_\mu \gamma_\nu h_0^4 - \frac{\dddot{v}_\mu \gamma_\nu h_0^4}{6} + 4 h_1 k_{\mu 2} h_0^3 + 4 h_2 v_\mu v_\nu h_0^3
\]

\[
+ 4 h_3 v_\mu \gamma_\nu h_0^3 - 4 h_2 \dddot{v}_\mu \gamma_\nu h_0^3 + 2 h_1 \dddot{v}_\mu \gamma_\nu h_0^3
\]

\[
+ 6 h_1^2 v_\mu v_\nu h_0^2 + 12 h_1 h_2 v_\mu \gamma_\nu h_0^2 - 6 h_1^2 \dddot{v}_\mu \gamma_\nu h_0^2 + 4 h_3 v_\mu \gamma_\nu h_0
\]

\[
+ \frac{1}{\sigma^2} \left[ - 4 p_3 v_\mu \gamma_\nu h_0^5 + k_{\mu 2} h_0^4 + \frac{1}{2} \dddot{v}_\mu \gamma_\nu h_0^4 + 4 h_1 v_\mu v_\nu h_0^3
\]

\[
+ 4 h_2 v_\mu \gamma_\nu h_0^3 - 4 h_1 \dddot{v}_\mu \gamma_\nu h_0^3 + 6 h_1^2 v_\mu \gamma_\nu h_0^2
\]

\[
+ \frac{v_\mu v_\nu h_0^4 - v_\mu \gamma_\nu h_0^4 + 4 h_1 v_\mu \gamma_\nu h_0^3}{\sigma^3} + h_1^4 \dddot{v}_\mu \gamma_\nu.
\]
Finally the last expansion reads

\[
\begin{align*}
R^{-5} \left[ \dot{z}_\mu (x_\nu - z_\nu) - \dot{z}_\nu (x_\mu - z_\mu) \right] \\
= 15 p_3 v_\mu v_\nu h_0^7 + 30 p_3 p_4 v_\mu \gamma_\nu h_0^7 - 15 \frac{p_3}{2} \dot{v}_\mu \gamma_\nu h_0^7 - 5 k_{\mu\nu} p_3 h_0^6 - 5 k_{\mu\nu} p_4 h_0^6 - 5 p_5 v_\mu v_\nu h_0^6 \\
+ 105 h_{123} p_2 \gamma_\nu h_0^5 - 5 p_6 v_\mu \gamma_\nu h_0^5 + 5 p_5 v_\mu \gamma_\nu h_0^5 - \frac{5}{2} p_4 \dot{v}_\mu \gamma_\nu h_0^5 + \frac{5 p_3 \ddot{v}_\mu \gamma_\nu h_0^6}{6} + k_{\mu\nu} h_0^5 \\
- 30 h_1 k_{\mu\nu} p_3 h_0^5 - 30 h_{23}^2 v_\mu v_\nu h_0^5 - 30 h_1 p_4 v_\mu v_\nu h_0^5 - 30 h_3 p_3 v_\mu \gamma_\nu h_0^5 - 30 h_2 p_4 v_\mu \gamma_\nu h_0^5 \\
- 30 h_1 p_5 v_\mu \gamma_\nu h_0^5 + \frac{v_\mu \gamma_\nu h_0^5}{\sigma^5} + 30 h_2 p_3 v_\mu \gamma_\nu h_0^5 + 30 h_1 p_4 \dot{v}_\mu \gamma_\nu h_0^5 - 15 h_1 \dot{p}_3 v_\mu \gamma_\nu h_0^5 \\
- \frac{v_\mu (5) \gamma_\nu h_0^5}{120} + 5 h_3 k_{\mu\nu} h_0^4 + 5 h_2 k_{\mu\nu} h_0^4 + 5 h_1 k_{\mu\nu} h_0^4 + 5 h_4 v_\mu v_\nu h_0^4 - 75 h_1^2 p_3 v_\mu v_\nu h_0^4 \\
+ 5 h_5 v_\mu \gamma_\nu h_0^4 - 150 h_1 h_2 p_3 v_\mu \gamma_\nu h_0^4 - 75 h_2 p_4 v_\mu \gamma_\nu h_0^4 - 5 h_4 v_\mu \gamma_\nu h_0^4 + 75 h_1 p_3 v_\mu \gamma_\nu h_0^4 + \frac{5}{2} h_3 \ddot{v}_\mu \gamma_\nu h_0^4 \\
+ \frac{5 h_1 (4) v_\mu h_0^4}{24} - \frac{5 h_2 \ddot{v}_\mu \gamma_\nu h_0^4}{6} + 20 h_1 h_2 k_{\mu\nu} h_0^3 + 10 h_1^2 k_{\mu\nu} h_0^3 + 10 h_2 \dot{v}_\mu v_\nu h_0^3 + 20 h_1 h_3 v_\mu v_\nu h_0^3 \\
+ 20 h_2 h_3 v_\mu \gamma_\nu h_0^3 + 20 h_1 h_4 v_\mu \gamma_\nu h_0^3 - 100 h_1^2 p_3 v_\mu \gamma_\nu h_0^3 - 10 h_2 \dot{v}_\mu \gamma_\nu h_0^3 - 20 h_1 h_3 \ddot{v}_\mu \gamma_\nu h_0^3 \\
+ 10 h_1 h_2 \ddot{v}_\mu \gamma_\nu h_0^3 - \frac{10 h_1^2 \ddot{v}_\mu \gamma_\nu h_0^3}{6} + 10 h_1 k_{\mu\nu} h_0^3 + 30 h_2^2 h_\nu v_\mu h_0^3 + 30 h_1 h_2 v_\mu \gamma_\nu h_0^3 + 30 h_1^2 h_3 v_\mu \gamma_\nu h_0^3 \\
- 30 h_2^2 \dot{v}_\mu \gamma_\nu h_0^3 + 5 h_3 \ddot{v}_\mu \gamma_\nu h_0^3 + 5 h_1 \dot{v}_\mu \gamma_\nu h_0^3 - 10 h_1^2 v_\mu \gamma_\nu h_0^3 + 20 h_1 \ddot{v}_\mu \gamma_\nu h_0^3 \\
+ \frac{1}{\sigma^5} \left[ 15 p_3 v_\mu \gamma_\nu h_0^5 - 5 k_{\mu\nu} p_3 h_0^6 - 5 p_4 v_\mu v_\nu h_0^6 - 5 p_5 v_\mu \gamma_\nu h_0^6 + 5 p_4 \ddot{v}_\mu \gamma_\nu h_0^5 \right] \\
- \frac{5}{2} p_3 \dot{v}_\mu \gamma_\nu h_0^5 + k_{\mu\nu} h_0^5 - 30 h_1 p_3 v_\mu v_\nu h_0^5 - 30 h_2 p_3 v_\mu \gamma_\nu h_0^5 - 30 h_1 p_4 v_\mu \gamma_\nu h_0^5 \\
+ 30 h_1 p_3 \ddot{v}_\mu \gamma_\nu h_0^5 + \frac{v_\mu (4) \gamma_\nu h_0^5}{24} + 5 h_2 k_{\mu\nu} h_0^4 + 5 h_1 k_{\mu\nu} h_0^4 + 5 h_3 v_\mu v_\nu h_0^4 \\
+ 5 h_4 v_\mu \gamma_\nu h_0^4 - 75 h_2 p_3 v_\mu \gamma_\nu h_0^4 - 5 h_3 \ddot{v}_\mu \gamma_\nu h_0^4 + \frac{5}{2} h_2 \dot{v}_\mu \gamma_\nu h_0^4 - \frac{5 h_1 \ddot{v}_\mu \gamma_\nu h_0^4}{6} + 10 h_1 k_{\mu\nu} h_0^3 \\
+ 20 h_1 h_2 v_\mu v_\nu h_0^3 + 10 h_2 v_\mu \gamma_\nu h_0^3 + 20 h_1 h_3 v_\mu \gamma_\nu h_0^3 - 20 h_1 \ddot{v}_\mu \gamma_\nu h_0^3 \\
+ 5 h_1 \dot{v}_\mu \gamma_\nu h_0^3 + 10 h_3 v_\mu v_\nu h_0^3 + 30 h_1 \ddot{v}_\mu \gamma_\nu h_0^3 - 10 h_1^2 \ddot{v}_\mu \gamma_\nu h_0^3 + 5 h_4 v_\mu \gamma_\nu h_0^3 \\
+ \frac{1}{\sigma^5} \left[ - 5 p_3 v_\mu \gamma_\nu h_0^6 - 5 p_4 v_\mu \gamma_\nu h_0^6 + 5 p_3 \ddot{v}_\mu \gamma_\nu h_0^6 + k_{\mu\nu} h_0^5 - 30 h_1 p_3 v_\mu \gamma_\nu h_0^5 - \frac{\ddot{v}_\mu \gamma_\nu h_0^6}{6} \right] \\
+ 5 h_1 k_{\mu\nu} h_0^5 + 5 h_2 v_\mu v_\nu h_0^4 + 5 h_3 v_\mu \gamma_\nu h_0^4 - 5 h_2 \dot{v}_\mu \gamma_\nu h_0^4 + \frac{5}{2} h_1 \ddot{v}_\mu \gamma_\nu h_0^4 \\
+ 10 h_2^2 v_\mu v_\nu h_0^3 + 20 h_1 h_2 v_\mu \gamma_\nu h_0^3 - 10 h_2^2 \ddot{v}_\mu \gamma_\nu h_0^3 + 10 h_1 \ddot{v}_\mu \gamma_\nu h_0^3 \\
+ \frac{1}{\sigma^5} \left[ - 5 p_3 v_\mu \gamma_\nu h_0^6 + k_{\mu\nu} h_0^5 + \frac{1}{2} \ddot{v}_\mu \gamma_\nu h_0^5 + 5 h_1 v_\mu v_\nu h_0^4 + 5 h_2 v_\mu \gamma_\nu h_0^4 \\
- 5 h_1 \dot{v}_\mu \gamma_\nu h_0^4 + 10 h_2 v_\mu \gamma_\nu h_0^3 \right] \\
+ \frac{v_\mu v_\gamma h_0^5 - \dot{v}_\mu \gamma_\nu h_0^5 + 5 h_1 v_\mu \gamma_\nu h_0^4}{\sigma^4}.
\end{align*}
\]
Using these expansions obtained in this appendix we can calculate the field \( F^{(ret)}_{\mu \nu} \) in a neighborhood near the trajectory of the particle. To obtain this field in the points on the trajectories of the non-abelian particle

\[
F^{(ret)}_{\mu \nu} = \left(2k_{\mu \nu}h_0^2 - \frac{v_\mu \gamma_\nu h_0^2}{\sigma^3} + 3h_1k_{\mu \nu}h_0 - \frac{h_1v_\mu \gamma_\nu h_0^2}{\sigma^2} + \frac{k_{\mu \nu}h_0^2 + \frac{1}{2}v_\mu \gamma_\nu h_0^2 - h_1v_\mu \gamma_\nu h_0}{\sigma}\right)A_1
\]

\[
+ \left(k_{\mu \nu}h_0^3 + \frac{v_\mu \gamma_\nu h_0^3}{\sigma^3} + 3h_1k_{\mu \nu}h_0^2 + \frac{k_{\mu \nu}h_0^3 + \frac{1}{2}v_\mu \gamma_\nu h_0^3 - 3h_1v_\mu \gamma_\nu h_0^2 + g_2v_\mu \gamma_\nu}{\sigma}\right)A_3
\]

\[
+ \left(\frac{3h_0^3v_\mu \gamma_\nu - h_0^3v_\mu \gamma_\nu}{\sigma^2}\right)A_3
\]

\[
+ A_5 \left(k_{\mu \nu}h_0^5 + \frac{v_\mu \gamma_\nu h_0^5}{\sigma^5} + 5h_1k_{\mu \nu}h_0^4 + g_9k_{\mu \nu} + k_{\mu \nu}g_{11}
\]

\[
+ \frac{k_{\mu \nu}h_0^5 + \frac{1}{2}v_\mu \gamma_\nu h_0^5 - 5h_1v_\mu \gamma_\nu h_0^4 + g_12v_\mu \gamma_\nu + \frac{5h_0^4v_\mu \gamma_\nu - h_0^5v_\mu \gamma_\nu}{\sigma^4} + \frac{h_1 \dot{\gamma}_\nu + g_{16} \gamma_\nu + \dot{\gamma}_\nu \gamma_\nu g_{18}}{\sigma^2} + \frac{k_{\mu \nu}h_0^5 - \frac{1}{2}v_\mu \gamma_\nu h_0^5 + 5h_1k_{\mu \nu}h_0^4 + \frac{5}{2}h_1v_\mu \gamma_\nu h_0^4 + g_{12}v_\mu \gamma_\nu - g_{14}v_\mu \gamma_\nu}{\sigma^2}\right)
\]

\[
+ A_2 \left(3k_{\mu \nu}h_0^3 - \frac{v_\mu \gamma_\nu h_0^3}{\sigma^3} + 7h_1k_{\mu \nu}h_0^2 - \frac{2h_1v_\mu \gamma_\nu h_0^2}{\sigma^2}
\]

\[
+ g_3k_{\mu \nu} + \frac{k_{\mu \nu}h_0^3 + \frac{1}{2}v_\mu \gamma_\nu h_0^3 - h_1v_\mu \gamma_\nu h_0^2 + g_4v_\mu \gamma_\nu}{\sigma^2}
\]

\[
+ \frac{2k_{\mu \nu}h_0^3 - \frac{1}{2}v_\mu \gamma_\nu h_0^3 + 4h_1k_{\mu \nu}h_0^2 - 2g_1v_\mu \gamma_\nu h_0^2 + 2h_1v_\mu \gamma_\nu h_0^2 + 2h_1 \dot{\gamma}_\nu h_0^2 + g_5v_\mu \gamma_\nu}{\sigma^2}\right)
\]

\[
+ A_4 \left(k_{\mu \nu}h_0^4 + \frac{v_\mu \gamma_\nu h_0^4}{\sigma^4} + 4h_1k_{\mu \nu}h_0^3 + g_6k_{\mu \nu}
\]

\[
+ \frac{k_{\mu \nu}h_0^4 + \frac{1}{2}v_\mu \gamma_\nu h_0^4 - 4h_1v_\mu \gamma_\nu h_0^3 + g_7v_\mu \gamma_\nu}{\sigma^2} + \frac{4h_0^3v_\mu \gamma_\nu - h_0^4v_\mu \gamma_\nu}{\sigma^3}
\]

\[
+ k_{\mu \nu}h_0^4 + \frac{1}{2}v_\mu \gamma_\nu h_0^4 + 4h_1k_{\mu \nu}h_0^3 + 2h_1v_\mu \gamma_\nu h_0^3 + g_8v_\mu \gamma_\nu + \dot{\gamma}_\nu g_{10} \gamma_\nu}{\sigma}\right), 
\]

where \( A_1, A_2, A_3, A_4 \) and \( A_5 \) are those given after Eq. (15).

Now to calculate the actual field we use the Eq. (14) and solve the equation for \( \sigma \) in terms of the parameter \( \varepsilon \). These arguments were already presented in [2], to calculate
\( F_{\mu\nu}^{(act)} \), it is necessary to determine first \( \sigma \) as a function of \( \varepsilon \) for which we have

\[
\sigma^2 = \frac{\varepsilon^2}{1 - \gamma \varepsilon} \left[ 1 + \frac{\varepsilon}{3} + \varepsilon^9 \left( \frac{\varepsilon^6}{19958400} + \frac{\varepsilon^4}{21600} + \frac{\varepsilon^2}{33600} + \frac{\varepsilon^2}{86400} + \frac{\varepsilon^2}{453600} \right) + \varepsilon^8 \left( \frac{\varepsilon^6}{1814400} + \frac{\varepsilon^4}{8100} + \frac{\varepsilon^2}{5184} + \frac{\varepsilon^2}{11340} + \frac{\varepsilon^2}{51840} \right) - \varepsilon^7 \left( \frac{\varepsilon^6}{1814400} + \frac{\varepsilon^4}{960} + \frac{\varepsilon^2}{1728} + \frac{\varepsilon^2}{6720} \right) + \varepsilon^6 \left( \frac{\varepsilon^6}{201600} + \frac{\varepsilon^4}{448} + \frac{\varepsilon^2}{315} + \frac{\varepsilon^2}{1008} \right) - \varepsilon^5 \left( \frac{\varepsilon^6}{2520} + \frac{\varepsilon^4}{72} + \frac{\varepsilon^2}{180} \right) + \varepsilon^4 \left( \frac{\varepsilon^6}{360} + \frac{\varepsilon^4}{45} + \frac{\varepsilon^2}{40} \right) - \varepsilon^3 \left( \frac{\varepsilon^6}{60} + \frac{\varepsilon^4}{12} \right) \right] .
\]

(68)

This expression is obtained from expansion (II) and the condition \((x_\mu - z_\mu(s))(x^\mu - z^\mu(s)) = 0\). The substitution of these equations, in \( F_{\mu\nu}^{(act)} \) gives the field strength very close to the trajectory of the Yang-Mills particle

\[
F_{\mu\nu}^{(act)} = f_{\mu\nu} - \frac{A_2 k_{\mu\nu}}{\varepsilon U_1} - \frac{A_3 h_0^5}{3\varepsilon U_1} \left[ v_\mu \gamma_\nu - v_\nu \gamma_\mu \right] + \frac{A_4 h_0^4}{6\varepsilon U_1} \left[ v_\mu \gamma_\nu - \bar{v}_\nu \gamma_\mu \right] 
\]

\[
- \frac{A_4 k_{\mu\nu}}{\varepsilon U_1} + \frac{A_2 h_0^4}{\varepsilon U_1} \left[ v_\mu \gamma_\nu - \bar{v}_\nu \gamma_\mu \right] 
\]

\[
- \frac{A_3 k_{\mu\nu}}{\varepsilon U_1} + \frac{A_3 h_0^3}{\varepsilon U_1} \left[ v_\mu \gamma_\nu - \bar{v}_\nu \gamma_\mu \right] - \frac{A_1 k_{\mu\nu}}{\varepsilon U_1} \left[ v_\mu \gamma_\nu - \bar{v}_\nu \gamma_\mu \right] 
\]

\[
+ \frac{A_2 k_{\mu\nu}}{\varepsilon U_1} \left( \frac{2a_1 h_0^3}{\varepsilon U_1^2} - \frac{4h_0^2 h_1}{\varepsilon U_1} \right) + \frac{A_4 k_{\mu\nu}}{\varepsilon U_1} \left( \frac{2a_1 h_0^2}{\varepsilon U_1^2} - \frac{4h_0^2 h_1}{\varepsilon U_1} \right) + \frac{A_5 k_{\mu\nu}}{\varepsilon U_1} \left( \frac{2a_1 h_0^1}{\varepsilon U_1^2} - \frac{5h_0^1 h_1}{\varepsilon U_1} \right) 
\]

\[
+ \frac{A_2}{\varepsilon U_1} \left( \frac{6a_1^2 h_0^3}{\varepsilon U_1^2} + \frac{2a_1 h_0^3}{\varepsilon U_1^3} - \frac{h_0^3}{\varepsilon U_1^3} \right) + \frac{A_4}{\varepsilon U_1} \left( \frac{4a_1 h_0^2}{\varepsilon U_1^3} + \frac{4a_1 h_0^2}{\varepsilon U_1^3} \right) + \frac{12a_1 h_0^2}{\varepsilon U_1^3} 
\]

\[
- \frac{6a_2 h_0^3}{\varepsilon U_1^3} + \frac{2h_0^3}{\varepsilon U_1^3} - \frac{g_5}{\varepsilon U_1^3} + \frac{2a_1 g_4}{\varepsilon U_1^3} \right] \left[ v_\mu \gamma_\nu - \bar{v}_\nu \gamma_\mu \right] 
\]

\[
+ \frac{A_3}{\varepsilon U_1} \left( \frac{6a_1^2 h_0^3}{\varepsilon U_1^2} + \frac{2a_1 h_0^3}{\varepsilon U_1^3} - \frac{h_0^3}{\varepsilon U_1^3} + \frac{6a_1 h_0^2}{\varepsilon U_1^3} - \frac{g_5}{\varepsilon U_1^3} \right) \left[ v_\mu \gamma_\nu - \bar{v}_\nu \gamma_\mu \right] 
\]

\[
+ \frac{A_4}{\varepsilon U_1} \left( \frac{20a_1^2 h_0^4}{\varepsilon U_1^4} - \frac{2a_1 h_0^4}{\varepsilon U_1^3} + \frac{4a_1 h_0^4}{\varepsilon U_1^3} + \frac{4a_1 h_0^4}{\varepsilon U_1^3} \right) + \frac{24a_1 h_0^1 h_1}{\varepsilon U_1^3} 
\]

\[
+ \frac{12a_2 h_0^3}{\varepsilon U_1^3} - \frac{4h_1^3}{\varepsilon U_1^3} - \frac{g_8}{\varepsilon U_1^3} + \frac{2a_1 g_7}{\varepsilon U_1^3} \right) \left[ v_\mu \gamma_\nu - \bar{v}_\nu \gamma_\mu \right] 
\]
$+ A_5 \left( \frac{-70a_1^4h_0^5}{\varepsilon U_1^3} - \frac{15a_2^3h_0^5}{\varepsilon U_1^4} + \frac{105a_2^2a_2h_0^5}{\varepsilon^2 U_1^5} - \frac{30a_1a_3h_0^5}{\varepsilon^3 U_1^5} + \frac{5a_4h_0^5}{\varepsilon^3 U_1^5} - \frac{15a_2^2h_0^5}{\varepsilon^3 U_1^5} + \frac{5a_2h_0^5}{\varepsilon^3 U_1^5} \right)
- \frac{h_0^5}{\varepsilon U_1^5} + \frac{100a_2^3h_1h_0^4}{\varepsilon U_1^4} - \frac{100a_1a_2h_1h_0^4}{\varepsilon U_1^4} + \frac{20a_3h_1h_0^4}{\varepsilon U_1^4} + \frac{20a_1h_1h_0^4}{\varepsilon U_1^4} - \frac{a_{15}}{\varepsilon U_1} + \frac{2a_{19}g_{12}}{\varepsilon U_1^2} - \frac{6a_2^2g_{12}}{\varepsilon U_1^3} + \frac{3a_2g_{12}}{\varepsilon U_1^3} - \frac{g_{12}}{\varepsilon^3 U_1^3} \right)[v_{\mu\nu} - v_{\nu\gamma}]$ 
$+ A_1 \left( \frac{6a_2^2h_0^2}{\varepsilon U_1^1} + \frac{3a_2^2h_0^2}{\varepsilon U_1^1} - \frac{2a_1h_1h_0}{\varepsilon^3 U_1^3} \right)[v_{\mu\nu} - v_{\nu\gamma}]$ 
$+ A_3 \left( \frac{3h_0^3h_1}{\varepsilon U_1} - \frac{2a_1h_3^2}{\varepsilon U_1^2} \right)[\dot{v}_{\mu\nu} - \dot{v}_{\nu\gamma}] + A_2 \left( \frac{2g_1h_0^2}{\varepsilon U_1} - \frac{2a_1h_1h_0}{\varepsilon U_1^2} + \frac{2h_7^2h_0}{\varepsilon U_1} \right)[\dot{v}_{\mu\nu} - \dot{v}_{\nu\gamma}]$ 
$+ A_4 \left( \frac{6a_2^2h_0^2}{\varepsilon U_1^1} - \frac{3a_2^2h_0^2}{\varepsilon U_1^1} + \frac{h_0^4}{\varepsilon^3 U_1^3} - \frac{8a_1h_1h_0^4}{\varepsilon U_1^3} - \frac{g_{19}}{\varepsilon U_1} \right)[\dot{v}_{\mu\nu} - \dot{v}_{\nu\gamma}]$ 
$+ A_5 \left( \frac{20a_3h_0^5}{\varepsilon U_1^3} + \frac{20a_1a_2h_0^5}{\varepsilon U_1^3} - \frac{a_3h_0^5}{\varepsilon U_1^3} + \frac{4a_1h_0^5}{\varepsilon U_1^4} + \frac{4a_1h_0^5}{\varepsilon^3 U_1^5} - \frac{30a_3h_1h_0^4}{\varepsilon U_1^3} - \frac{15a_2h_1h_0^4}{\varepsilon U_1^3} + \frac{5h_1h_0^4}{\varepsilon^3 U_1^3} \right)
- \frac{g_{16}}{\varepsilon U_1} - \frac{2a_{19}g_{14}}{\varepsilon U_1^1} \right)[\dot{v}_{\mu\nu} - \dot{v}_{\nu\gamma}]$ 
$+ A_2 \left( \frac{a_1h_3^3}{\varepsilon U_1^3} - \frac{2h_0^3h_1}{\varepsilon U_1} \right)[\ddot{v}_{\mu\nu} - \ddot{v}_{\nu\gamma}] + A_4 \left( \frac{a_1h_0^4}{\varepsilon U_1^2} - \frac{2h_0^3h_1}{\varepsilon U_1} \right)[\ddot{v}_{\mu\nu} - \ddot{v}_{\nu\gamma}]$ 
$+ A_5 \left( \frac{-3a_2h_0^5}{\varepsilon U_1^3} + \frac{3a_2h_0^5}{\varepsilon U_1^3} + \frac{h_0^5}{\varepsilon^3 U_1^3} - \frac{2a_1h_1h_0^4}{\varepsilon U_1^3} - \frac{g_{18}}{\varepsilon U_1} \right)[\dddot{v}_{\mu\nu} - \dddot{v}_{\nu\gamma}]$ 
$+ A_5 \left( \frac{5h_0^3h_1}{\varepsilon U_1} - \frac{a_1h_0^5}{\varepsilon U_1} \right)[\dot{v}_{\mu\nu} - \dot{v}_{\nu\gamma}],$ 

where $U_1 \equiv (1 - \gamma \dot{\nu})^{-1/2}$, and $a_1$, $a_2$, $a_3$ and $a_4$ are given by

$$a_1 = \frac{-\gamma \ddot{\nu}}{6}; \quad a_2 = \frac{1}{72} (-\gamma \dddot{\nu}^2 + 3 \gamma \dddot{\nu} + 3 \dot{\nu}^2)$$

$$a_3 = \frac{1}{432} (-\gamma \dddot{\nu}^2 + \frac{18\gamma \nu^{(4)}}{5} + 3 \gamma \nu \dddot{\nu} + \dddot{\nu}) - 18 \dddot{\nu}$$

$$a_4 = \frac{1}{128} \left( \frac{\gamma \dddot{\nu}^2}{3} + \frac{3}{16} \left( \frac{\gamma \dddot{\nu}^2}{12} + \frac{\dot{\nu}^2}{12} \right) \left( \frac{\gamma \dddot{\nu}^2}{3} \right) \right) - \frac{1}{4} \left( \frac{\gamma \nu^{(4)}}{60} + \frac{\dddot{\nu}}{12} \right) \left( \frac{\gamma \dddot{\nu}^2}{3} \right)$$

$$- \frac{1}{2} \left( \frac{\gamma \nu^{(5)}}{360} + \frac{\dddot{\nu}^2}{45} + \frac{\dddot{\nu} \dddot{\nu}}{40} \right) + \frac{1}{8} \left( \frac{\gamma \dddot{\nu}^2}{12} + \frac{\dot{\nu}^2}{12} \right) \left( \frac{\dddot{\nu}^2}{3} \right).$$

Now we can calculate the radiating field, in order to do that we calculate first the advanced field strength $F_{\mu\nu}^{(adv)}$. Similarly as in the abelian case, it can be done by changing $\varepsilon$ by $-\varepsilon$. Thus the radiated field strength $F_{\mu\nu}^{(rad)} = F_{\mu\nu}^{(ret)} - F_{\mu\nu}^{(adv)}$ is written
as

\[
F^{(rad)}_{\mu\nu} = A_5 k_{\mu\nu 2} \left( -\frac{20 a_1^2 h_0^5}{U_1^3} + \frac{24 a_1 a_2 h_0^5}{U_1^3} - \frac{6 a_3 h_0^5}{U_1^3} + \frac{30 a_2^3 h_1 h_0^4}{U_1^2} - \frac{20 a_2 h_1 h_0^4}{U_1^2} - \frac{2a_1 g_17}{U_1} + 2g_9 \right)
\]

\[
+ A_2 k_{\mu\nu 2} \left( \frac{6a_1^2 h_0^5}{U_1^2} - \frac{4a_2 h_0^5}{U_1^2} - \frac{8a_1 h_1 h_0^4}{U_1} + 2g_3 \right)
\]

\[
+ A_4 k_{\mu\nu 2} \left( \frac{6a_1^2 h_0^5}{U_1^2} - \frac{4a_2 h_0^5}{U_1^2} - \frac{8a_1 h_1 h_0^4}{U_1} + 2g_6 \right)
\]

\[
+ A_5 k_{\mu\nu 3} \left( \frac{6a_1^2 h_0^5}{U_1^2} - \frac{4a_2 h_0^5}{U_1^2} - \frac{10a_1 h_1 h_0^4}{U_1} + 2g_{11} \right)
\]

\[
+ A_1 k_{\mu\nu 2} \left( 6h_0 h_1 - \frac{2a_1 h_0^3}{U_1} \right) + A_3 k_{\mu\nu 2} \left( 6h_0^2 h_1 - \frac{2a_1 h_0^3}{U_1} \right) + A_2 k_{\mu\nu 3} \left( 14h_0^2 h_1 - \frac{4a_1 h_0^3}{U_1} \right)
\]

\[
+ A_4 k_{\mu\nu 3} \left( 8h_0^3 h_1 - \frac{2a_1 h_0^3}{U_1} \right) + A_5 k_{\mu\nu 4} \left( 10h_0^4 h_1 - \frac{2a_1 h_0^5}{U_1} \right)
\]

\[
+ 2A_3 h_0^3 k_{\mu\nu 3} + 4A_1 h_0^2 k_{\mu\nu 3} + 2A_4 h_0^2 k_{\mu\nu 4} + 6A_2 h_0^3 k_{\mu\nu 4} + 2A_5 h_0^5 k_{\mu\nu 5}.
\]

(71)

If we take all the \(A_i = 0\), \(i = 2, 3, 4, 5\), except \(A_1 \neq 0\), then we can calculate the radiated field by an abelian particle. Therefore the Yang-Mills field is reduced to the one of Dirac’s radiation damping \([2]\)

\[
F^{(rad)D}_{\mu\nu} = 4A_1 h_0^2 k_{\mu\nu 3} = 4A_1 h_0^2 \left( \frac{\dot{v}_\mu v_\nu - v_\mu \ddot{v}_\nu}{3} \right).
\]

(72)

Notice that in \(F^{(rad)}_{\mu\nu}\) there are two terms depending on \(A_1\). Moreover the terms depending on \(h_1\) and \(a_1\) will not contribute to the equations of motion, since they are linear in \(\gamma\) and they will be eliminated in the integral when the flow of the particle be computed.

C Computation of equations of motion

In order to find the equations of motion we first should calculate the energy-momentum tensor \([17]\) in terms of the radiation field at points near the trajectory of the particle. For this we make use of \(F^{(act)}_{\mu\nu}\) on the world-line. When we use these relations in \([17]\), we can find the divergent terms in \(T_{\mu\nu}\). In the process of computing and simplifying the energy-momentum tensor \([17]\) it is useful to consider the following relations

\[
[\gamma_\mu v_\nu - \gamma_\nu v_\mu] [\gamma_\nu v_\rho - \gamma_\rho v_\nu] = -\gamma_\mu \gamma_\rho + \varepsilon^2 v_\mu v_\rho
\]

\[
[\gamma_\mu v_\nu - \gamma_\nu v_\mu] [\gamma_\nu v_\rho(i) - \gamma_\rho v_\nu(i)] = \gamma v(i)_\gamma v_\mu v_\mu + \varepsilon^2 v_\mu v_\rho(i) - vv(i)_\gamma v_\mu \gamma_\rho.
\]

\[
[\gamma_\mu v_\nu - \gamma_\nu v_\mu] [v_\nu v_\rho - v_\rho v_\nu] = \gamma v_\mu + (\gamma v)v_\mu v_\mu
\]

\[
[\gamma_\mu v_\nu - \gamma_\nu v_\mu] [v_\nu v_\rho - v_\rho v_\nu] = -\gamma^2 v_\gamma v_\mu v_\mu - (\gamma v)v_\mu v_\mu - \varepsilon^2 v_\mu v_\mu v_\rho
\]

\[
[\gamma_\mu v_\nu(i) - \gamma_\nu v_\mu(i)] [\gamma_\nu v_\rho(i) - \gamma_\rho v_\nu(i)] = \gamma v(i)_\gamma v_\mu v_\mu + \varepsilon^2 v_\mu v_\rho(i) - vv(i)_\gamma v_\mu \gamma_\rho
\]

\[
[\gamma_\mu v_\nu(i) - \gamma_\nu v_\mu(i)] [\gamma_\nu v_\rho(j) - \gamma_\rho v_\nu(j)] = \gamma v(i)_\gamma v_\mu v_\mu + \gamma v(j)_\gamma v_\mu v_\mu - vv(i)_\gamma v_\mu \gamma_\rho.
\]

(73)
Integrate out in a hyper-surface of radius $\varepsilon$ and using the Stokes theorem, with the normal vector to the surface being in the direction of $\gamma'$, which comes from the fact that the equation $\gamma dx = 0$ that is obtained from the variation of $(x_\mu - z_\mu)^2 = \varepsilon^2$ and $R = 0$ is satisfied in the same way as in [2] if we perform a splitting $dx \to dx_1 dx_2$ we get

$$- \int \varepsilon^{-1} T_{\mu\rho} \gamma^\rho ds \left| dx_2 \right| = - \int 4\pi \varepsilon T_{\mu\rho} \gamma^\rho U_1^{-2} ds$$

$$= \int \text{Tr} \left\{ \frac{A_5 f_{\mu\nu} v^{\nu} + Q_{14} v_\mu + Q_{11} \ddot{v}_\mu + Q_{17} v_\mu + Q_{16} \ddot{v}_\mu}{\varepsilon^2} 
+ \varepsilon (A_3 f_{\mu\nu} v^{\nu} + Q_{27} f_{\mu\nu} v^{\nu} + Q_{31} f_{\mu\nu} v^{\nu} + Q_{34} f_{\mu\nu} v^{\nu} + Q_{29} f_{\mu\nu} v^{\nu} + Q_{27} v_\mu + Q_1 \ddot{v}_\mu 
+ Q_5 \ddot{v}_\mu + Q_4 \ddot{v}_\mu) + \frac{A_5 f_{\mu\nu} v^{\nu} + Q_{36} f_{\mu\nu} v^{\nu} + Q_{9} v_\mu + Q_{3} \ddot{v}_\mu + Q_{10} \ddot{v}_\mu + \ddot{Q}_1 v_\mu}{\varepsilon} 
- A_4 f_{\mu\nu} v^{\nu} + Q_{35} f_{\mu\nu} v^{\nu} 
+ \varepsilon^2 (Q_{25} f_{\mu\nu} v^{\nu} + Q_{26} f_{\mu\nu} v^{\nu} + Q_{28} f_{\mu\nu} v^{\nu} + Q_{32} f_{\mu\nu} v^{\nu} + Q_{30} f_{\mu\nu} v^{\nu(4)}) 
+ Q_{33} f_{\mu\nu} v^{\nu} + Q_{8} v_\mu + Q_6 \ddot{v}_\mu + Q_2 \ddot{v}_\mu + Q_12 \ddot{v}_\mu + \frac{Q_{24} \ddot{v}_\mu}{\varepsilon^5} 
+ \frac{Q_{23} v_\mu + Q_{22} \ddot{v}_\mu + Q_{20} \ddot{v}_\mu}{\varepsilon^4} + \frac{Q_{13} v_\mu + Q_{21} \ddot{v}_\mu + Q_{19} \ddot{v}_\mu + Q_{18} \ddot{v}_\mu}{\varepsilon^3} \right\} ds, \tag{74}$$

where we have integrated on a hyper-sphere of area $4\pi \varepsilon^2$ and used the line element $\left| dx_2 \right| = (1 - \gamma \dot{v}) ds = U_1^{-2} ds$. If we remember that the linear terms in $\gamma$ do not contribute to the equations of motion and we neglect all terms that depend on positive powers of $\varepsilon$ (in the limit $\varepsilon \to 0$) we get Eq. (18). The functions $Q_i$ are given by

$$Q_1 = -\frac{35}{16} a_4 A_5^2 \dddot{v}^2 - \frac{5}{2} a_4 A_1 A_5 - \frac{5}{2} a_4 A_3 A_5 - \frac{17}{144} A_2 A_5 v^2 \dddot{v} - \frac{17}{288} A_4 A_5 \dddot{v}^2 \dddot{v}$$
$$- 17 A_5^2 \dddot{v}^2 v \dddot{v}^{(4)} + \frac{17}{1152} A_5^2 \dddot{v}^2 \dddot{v}^{(4)} + \frac{209}{9216} A_5^2 \dddot{v}^6 - \frac{25}{192} A_1 A_5 \dddot{v}^4 - \frac{1}{48} A_3 A_5 \dddot{v}^4 + \frac{21}{128} A_5^2 \dddot{v}^2 v^{(4)}$$
$$- \frac{13}{288} A_2 A_5 v \dddot{v}^2 v - \frac{17}{72} A_5 \dddot{v}^2 v^2 - \frac{3}{36} A_1 \dddot{v}^2 v^2 - \frac{1}{48} A_3 \dddot{v}^2 v^2 + \frac{17}{576} A_5^2 \dddot{v}^2 v^2$$
$$- \frac{1}{4} A_1 A_3 \dddot{v}^2 - \frac{1}{3} A_4 A_5 \dddot{v}^2 v - \frac{1}{12} A_1 A_2 \dddot{v} v - \frac{1}{12} A_2 A_3 V_2 \dddot{v} - \frac{1}{3} A_1 A_4 \dddot{v} v$$
$$- \frac{1}{3} A_3 A_5 \dddot{v} + \frac{3}{16} A_1 A_5 \dddot{v} + \frac{3}{16} A_3 A_5 \dddot{v} + \frac{1}{8} A_1 A_5 \dddot{v}^2 + \frac{1}{8} A_3 A_5 \dddot{v}^2$$
$$+ \frac{1}{72} A_2 A_5 (\dddot{v} v)^2 + \frac{1}{36} A_2 A_5 \dddot{v} v + \frac{1}{72} A_4 A_5 \dddot{v} v - \frac{1}{288} A_5 \dddot{v} v^{(4)} - \frac{1}{6} A_1 A_2 \dddot{v} v - \frac{1}{6} A_2 A_3 \dddot{v} v - \frac{1}{12} A_1 A_4 \dddot{v} v - \frac{1}{12} A_3 A_4 \dddot{v} v + \frac{1}{48} A_1 A_5 \dddot{v} v^{(4)} + \frac{1}{48} A_3 A_5 \dddot{v} v^{(4)}, \tag{75}$$

$$Q_2 = -\frac{10}{3} a_4 A_5^2 - \frac{3}{32} A_5^2 \dddot{v}^4 - \frac{1}{3} A_2 A_4 \dddot{v}^2 - \frac{1}{6} A_2 A_4 \dddot{v}^2 - \frac{1}{4} A_1 A_5 \dddot{v}^2 - \frac{1}{4} A_2 A_5 \dddot{v}^2 + \frac{17}{24} A_2 A_5 \dddot{v}$$
$$- \frac{55}{72} A_4 A_5 \dddot{v} v + \frac{1}{72} A_5^2 \dddot{v} v + \frac{2}{9} A_5^2 \dddot{v} v + \frac{1}{3} A_2 A_5 \dddot{v} v - \frac{1}{6} A_4 A_5 \dddot{v} v + \frac{1}{36} A_5^2 \dddot{v} v^{(4)}, \tag{76}$$
\[
Q_3 = -\frac{5}{2} a_4 A_5^2 + \frac{17}{192} A_5^2 \dot{v}^4 - \frac{1}{2} A_2 A_4 \dot{v}^2 - \frac{1}{2} A_2 A_4 \ddot{v}^2 - \frac{9}{16} A_1 A_5 \ddot{v}^2 \\
+ \frac{7}{16} A_3 A_5 \dot{v}^2 + \frac{3}{16} A_3 \dddot{v} \dot{v} - \frac{11}{12} A_2 A_5 \dddot{v} - \frac{13}{12} A_4 A_5 \dddot{v} + \frac{1}{8} A_3^2 \dddot{v}^2 \\
- \frac{1}{3} A_2 A_5 \dddot{v} - \frac{1}{4} A_4 A_5 \dddot{v} + \frac{1}{48} A_5^2 \dddot{v}^{(4)} + \frac{A_3^2}{2} - \frac{A_4}{2},
\]
(77)

\[
Q_4 = -\frac{5}{8} a_4 A_5^2 - \frac{1}{512} A_3^2 \dot{v}^2 - \frac{3}{64} A_1 A_5 \dot{v}^2 - \frac{1}{64} A_3 A_5 \dot{v}^2 + \frac{3}{64} A_5^2 \dddot{v} - \frac{1}{48} A_2 A_5 \dddot{v} \\
- \frac{1}{12} A_4 A_5 \dddot{v} + \frac{1}{24} A_5^2 \dddot{v} - \frac{1}{24} A_2 A_5 \dddot{v} - \frac{1}{48} A_4 A_5 \dddot{v} + \frac{1}{192} A_5^2 \dddot{v}^{(4)},
\]
(78)

\[
Q_5 = \frac{10}{3} a_4 A_2 A_5 + \frac{5}{3} a_4 A_4 A_5 - \frac{1}{288} 5 A_2 A_5 \dot{v}^4 - \frac{23}{576} A_4 A_5 \dot{v}^4 + \frac{59}{576} A_5^2 \dddot{v}^3 \\
+ \frac{1}{4} A_1 A_2 \dddot{v} + \frac{1}{12} A_2 A_3 \dddot{v} + \frac{1}{8} A_1 A_4 \dddot{v} + \frac{1}{24} A_3 A_4 \dddot{v} + \frac{1}{9} A_5^2 \dddot{v} \\
+ \frac{2}{9} A_1^2 \dddot{v} + \frac{1}{2} A_2 A_4 \dddot{v} + \frac{1}{24} A_1 A_5 \dddot{v} - \frac{1}{4} A_2 A_5 \dddot{v} + \frac{1}{24} A_3 A_5 \dddot{v} \\
- \frac{1}{8} A_4 A_5 \dddot{v} - \frac{1}{36} A_2 A_5 \dddot{v} - \frac{1}{72} A_4 A_5 \dddot{v} + \frac{1}{288} A_5^2 \dddot{v}^{(4)} - \frac{2}{9} A_2 A_5 \dddot{v} \\
- \frac{1}{9} A_4 A_5 \dddot{v} + \frac{2}{9} A_2^2 \dddot{v} + \frac{1}{18} A_4^2 \dddot{v} + \frac{2}{9} A_2 A_4 \dddot{v} - \frac{1}{36} A_2 A_5 \dddot{v}^{(4)} - \frac{1}{72} A_4 A_5 \dddot{v}^{(4)},
\]
(79)

\[
Q_6 = 5 a_4 A_2 A_5 + 5 a_4 A_4 A_5 + \frac{17}{288} A_5^2 \dddot{v}^2 \dddot{v} + \frac{37}{192} A_2 A_5 \dddot{v}^4 - \frac{5}{192} A_4 A_5 \dddot{v}^4 \\
+ \frac{125}{288} A_3^2 \dddot{v}^3 + \frac{5}{8} A_1 A_2 \dddot{v}^2 + \frac{3}{8} A_2 A_3 \dddot{v}^2 + \frac{3}{8} A_1 A_4 \dddot{v}^2 + \frac{1}{8} A_3 A_4 \dddot{v}^2 \\
+ \frac{1}{72} A_5^2 \dddot{v} \dddot{v}^2 + \frac{1}{6} A_2^2 \dddot{v} \dddot{v} + \frac{2}{3} A_4^2 \dddot{v} \dddot{v} + \frac{5}{6} A_2 A_4 \dddot{v} \dddot{v} + \frac{5}{12} A_1 A_5 \dddot{v} \dddot{v} \\
- \frac{3}{8} A_2 A_5 \dddot{v} + \frac{1}{3} A_3 A_5 \dddot{v} - \frac{3}{8} A_4 A_5 \dddot{v} - \frac{7}{24} A_2 A_5 \dddot{v}^2 - \frac{7}{24} A_4 A_5 \dddot{v}^2 \\
- \frac{1}{72} A_5^2 \dddot{v} \dddot{v} + \frac{1}{3} A_2^2 \dddot{v} \dddot{v} + \frac{1}{6} A_4^2 \dddot{v} \dddot{v} + \frac{1}{2} A_2 A_4 \dddot{v} \dddot{v} + \frac{1}{12} A_1 A_5 \dddot{v} \dddot{v} \\
+ \frac{1}{12} A_3 A_5 \dddot{v} \dddot{v} - \frac{1}{24} A_2 A_5 \dddot{v}^{(4)} - \frac{1}{24} A_4 A_5 \dddot{v}^{(4)},
\]
(80)
\[Q_7 = -\frac{1}{288} 31 A_2 A_5 \dot{v}^6 + \frac{41}{576} A_4 A_5 \dot{v}^6 + \frac{1}{12} A_1 A_2 \ddot{v}^4 - \frac{1}{12} A_2 A_3 \ddot{v}^4 + \frac{11}{48} A_1 A_4 \ddot{v}^4 + \frac{7}{48} A_3 A_4 \ddot{v}^4 + \frac{1}{128} A_1 A_5 \ddot{v}^4 - \frac{65 A_2^2 \dddot{v} \ddot{v}^4}{1536} - \frac{979 A_2^2 \dddot{v} \ddot{v}^4}{2304} - \frac{1}{9} A_2^2 \dddot{v} \ddot{v}^2 - \frac{1}{18} A_2 A_4 \dddot{v} \ddot{v}^2 - \frac{19}{24} A_1 A_5 \dddot{v} \ddot{v}^2 - \frac{1}{4} A_2 A_5 \dddot{v} \ddot{v}^2 - \frac{19}{24} A_3 A_5 \dddot{v} \ddot{v}^2 - \frac{1}{8} A_4 A_5 \dddot{v} \ddot{v}^2 + \frac{23}{144} A_2 A_5 \dddot{v} \ddot{v}^2 + \frac{41}{288} A_4 A_5 \dddot{v} \ddot{v}^2 - \frac{17 A_3^2 \dddot{v} \ddot{v}^4}{1152} + \frac{17}{24} A_2 A_5 \dddot{v} \ddot{v}^2 + \frac{17}{48} A_4 A_5 \dddot{v} \ddot{v}^2 - \frac{67}{384} A_3^2 \dddot{v} \ddot{v}^2 + \frac{1}{24} A_1 A_2 \dddot{v} \ddot{v}^2 + \frac{1}{6} A_2 A_3 \dddot{v} \ddot{v}^2 + \frac{1}{6} A_3 A_4 \dddot{v} \ddot{v}^2 + \frac{1}{12} A_1 A_4 \dddot{v} \ddot{v}^2 + \frac{1}{12} A_3 A_4 \dddot{v} \ddot{v}^2 - \frac{1}{48} A_1 A_5 \dddot{v} \ddot{v}^2 - \frac{1}{48} A_3 A_5 \dddot{v} \ddot{v}^2 + \frac{3}{64} A_2 A_5 \dddot{v} \ddot{v}^2 + \frac{1}{24} A_2 A_5 \dddot{v} \ddot{v}^2 + \frac{1}{6} A_1 A_3 \dddot{v} \ddot{v}^2 + \frac{1}{6} A_2 A_5 \dddot{v} \ddot{v}^2 + \frac{1}{48} A_4 A_5 \dddot{v} \ddot{v}^2 - \frac{1}{9} A_2 A_5 \dddot{v} \ddot{v}^2 - \frac{1}{18} A_4 A_5 \dddot{v} \ddot{v}^2 - \frac{1}{16} A_1 A_5 \dddot{v} \ddot{v}^2 + \frac{1}{48} A_2 A_5 \dddot{v} \ddot{v}^2 + \frac{5}{6} A_4 A_5 \dddot{v} \ddot{v}^2 + \frac{1}{6} A_3 A_4 \dddot{v} \ddot{v}^2 + \frac{1}{24} A_2 A_5 \dddot{v} \ddot{v}^2 + \frac{1}{24} A_2 A_5 \dddot{v} \ddot{v}^2 + \frac{1}{3} a_4 A_5 \dddot{v} \ddot{v}^2 + \frac{5}{3} a_4 A_5 \dddot{v} \ddot{v}^2 - \frac{1}{4} A_4 \dddot{v} \ddot{v}^2 - \frac{1}{4} A_5 \dddot{v} \ddot{v}^2 - \frac{1}{18} A_5 \dddot{v} \ddot{v}^2 - \frac{2}{9} A_2 A_4 V_{23} + \frac{1}{36} A_2 A_5 \dddot{v} \ddot{v}^2(4) + \frac{1}{72} A_4 A_5 \dddot{v} \ddot{v}^2(4) + \frac{1}{192} A_5 \dddot{v} \ddot{v}^2(4) + \frac{5}{8} a_4 A_5 \dddot{v} \ddot{v}^2 - \frac{1}{24} A_5^2 V_2 \dddot{v} \ddot{v} ^2, \quad (81) \]

\[Q_8 = -\frac{10}{3} a_4 A_5^2 \dddot{v}^2 + \frac{1}{16} A_4 A_5 \dddot{v} \ddot{v}^2 V - \frac{5}{48} A_2^2 \dddot{v} \ddot{v} \ddot{v}^2 + \frac{85}{144} A_2^2 \dddot{v} \ddot{v}^6 - \frac{1}{3} A_2^2 \dddot{v}^4 - \frac{3}{8} A_4 \dddot{v}^4 + \frac{7}{24} A_2 A_4 \dddot{v}^4 + \frac{19}{48} A_3 A_5 \dddot{v}^4 - \frac{1}{96} A_2^2 \dddot{v}^2 \dddot{v}^2 + \frac{1}{4} A_5^2 \dddot{v}^3 - \frac{13}{32} a_2 A_5 \dddot{v}^3 + \frac{245}{256} A_4 A_5 \dddot{v} \dddot{v} \ddot{v}^2 + \frac{13}{288} A_2^2 \dddot{v} \ddot{v} \ddot{v}^2 - \frac{17}{24} A_5^2 V_2^2 \ddot{v}^2 + \frac{1}{9} A_2 A_5 \dddot{v} \ddot{v}^2 + \frac{1}{48} A_4 \dddot{v} \ddot{v}^2 + \frac{1}{3} A_2 \dddot{v} \ddot{v} - \frac{1}{6} A_2 \dddot{v} \ddot{v} - \frac{1}{12} A_2 A_5 \dddot{v} \ddot{v} + \frac{1}{24} A_2 A_5 \dddot{v} \ddot{v} + \frac{1}{24} A_2 A_5 \dddot{v} \ddot{v} + \frac{1}{3} A_2 A_5 \dddot{v} \ddot{v} - \frac{1}{3} A_2 A_5 \dddot{v} \ddot{v} - \frac{1}{3} A_2 \dddot{v} \ddot{v} - \frac{1}{3} A_2 \dddot{v} \ddot{v} - \frac{1}{48} A_5^2 \dddot{v} \ddot{v} + \frac{1}{9} A_5^2 \dddot{v} \ddot{v} \ddot{v} \dddot{v} + \frac{1}{16} A_2 A_5 \dddot{v} \ddot{v} + \frac{1}{16} A_4 A_5 \dddot{v} \ddot{v} - \frac{1}{2} A_2 A_5 \dddot{v} \ddot{v} - \frac{1}{2} A_2 A_5 \dddot{v} \ddot{v} - \frac{1}{2} A_1 A_3 \dddot{v} \ddot{v} + \frac{3}{4} A_1 A_2 \dddot{v} \ddot{v} + \frac{17}{12} A_2 A_3 \dddot{v} \ddot{v} + \frac{3}{4} A_1 A_3 \dddot{v} \ddot{v} + \frac{13}{12} A_3 A_4 \dddot{v} \ddot{v} + \frac{1}{3} A_2 A_5 \dddot{v} \ddot{v} + \frac{1}{6} A_4 A_5 \dddot{v} \ddot{v} - \frac{1}{36} A_5^2 \dddot{v} \ddot{v} \dddot{v} \dddot{v} \dddot{v} \ddot{v} + \frac{13}{12} A_3 A_4 \dddot{v} \ddot{v} + \frac{1}{3} A_2 A_5 \dddot{v} \ddot{v} + \frac{1}{6} A_4 A_5 \dddot{v} \ddot{v} - \frac{1}{36} A_5^2 \dddot{v} \ddot{v} \dddot{v} \dddot{v} \ddot{v} \ddot{v}, \quad (82) \]
\[ Q_9 = -\frac{5}{64}A_5^2\dddot{v}\dddot{v} + \frac{1}{4}(-5)A_2A_5\dddot{v} + \frac{11}{12}A_4A_5\dddot{v} + \frac{23}{72}A_5^3\dddot{v} + \frac{1}{9}A_5^3\dddot{v} \]

\[ + \frac{1}{3}A_2A_5\dddot{v} + \frac{3}{8}A_4A_5\dddot{v} - \frac{1}{48}A_5^{(4)}\dddot{v} + \frac{2}{3}A_2A_5\dddot{v} + \frac{1}{2}A_4A_5\dddot{v} - \frac{1}{16}A_5^2\dddot{v} + \frac{2}{3}A_1A_4\dddot{v} + \frac{5}{6}A_3A_4\dddot{v} + \frac{7}{6}A_3A_4\dddot{v} - \frac{1}{2}A_2^2\dddot{v} - \frac{5}{6}A_5^2\dddot{v} - \frac{5}{3}A_2A_4\dddot{v} - \frac{1}{2}A_1A_5\dddot{v} - \frac{7}{6}A_3A_5\dddot{v} - \frac{1}{9}A_5^2\dddot{v} + \frac{1}{8}A_1A_5\dddot{v} - \frac{1}{8}A_3A_5\dddot{v}, \quad (83) \]

\[ Q_{10} = \frac{43}{72}A_5^2\dddot{v} + \frac{1}{4}A_2A_5\dddot{v}^2 - \frac{1}{8}A_4A_5\dddot{v}^2 + \frac{1}{9}A_5^2\dddot{v} + \frac{2A_1A_2}{3} + \frac{A_1A_4}{3} - \frac{2A_2A_3}{3} - \frac{A_3A_4}{3}, \quad (84) \]

\[ Q_{11} = \frac{1}{3}A_2A_4\dddot{v} + \frac{9}{16}A_2A_5\dddot{v}^2 - \frac{9}{16}A_4A_5\dddot{v}^2 + \frac{1}{12}A_5^2\dddot{v} + \frac{3A_1A_2}{2} + \frac{A_1A_4}{2} - \frac{A_2A_3}{2} - \frac{3A_3A_4}{2}, \quad (85) \]

\[ Q_{12} = \frac{5}{48}A_5^2\dddot{v} + \frac{1}{16}A_2A_5\dddot{v}^2 + \frac{1}{48}A_5^2\dddot{v}, \quad (86) \]

\[ Q_{13} = -\frac{1}{12}A_2A_5\dddot{v} + \frac{1}{2}A_4A_5\dddot{v}^2 + A_2A_5\dddot{v}^2 - \frac{1}{8}A_5^2\dddot{v}, \quad (87) \]

\[ Q_{14} = \frac{9}{16}A_5^2\dddot{v}^4 - \frac{2}{3}A_2A_5\dddot{v}^2 - \frac{5}{24}A_5^2\dddot{v}\dddot{v} - \frac{1}{3}A_5^2\dddot{v}^2 - \frac{2}{3}A_5^2\dddot{v}^2 - \frac{2}{3}A_2A_4\dddot{v}^2 - \frac{7}{6}A_1A_5\dddot{v}^2 - \frac{1}{3}A_3A_5\dddot{v}^2 + \frac{17}{12}A_2A_5\dddot{v} + \frac{7}{4}A_4A_5\dddot{v} - \frac{1}{8}A_2A_5\dddot{v} - \frac{1}{8}A_4A_5\dddot{v}, \quad (88) \]

\[ Q_{15} = \frac{1}{64}A_5^2\dddot{v}^2 - \frac{A_1A_5}{8} + \frac{A_3A_5}{8}, \quad Q_{16} = \frac{A_2A_5}{8} - \frac{A_1A_5}{8}, \quad (89) \]

\[ Q_{17} = \frac{1}{6}A_5^2\dddot{v}^2 - \frac{1}{3}A_2A_5 + \frac{A_1A_2}{3} + \frac{A_3A_5}{3} + \frac{2A_2A_3}{3} - \frac{2A_1A_5}{3}, \quad (90) \]

\[ Q_{18} = \frac{A_2^2}{8}, \quad Q_{19} = -A_4A_5, \quad Q_{20} = \frac{2A_2^2}{3}, \quad (91) \]

\[ Q_{21} = \frac{1}{2}A_5^2\dddot{v}^2 - A_2^2 + A_4^2 + A_3A_5, \quad Q_{22} = -\frac{1}{2}A_2A_5 - \frac{3A_4A_5}{2}, \quad (92) \]

\[ Q_{23} = \frac{1}{6}A_5^2\dddot{v}^2, \quad Q_{24} = \frac{A_2^2}{2}, \quad (93) \]
\[ Q_{25} = -5a_4A_5 + \frac{65}{192}A_5\dot{v}^4 + \frac{1}{8}A_1\dot{v}^2 + \frac{3}{8}A_3\ddot{v}^2 + \frac{3}{8}A_5\dot{v}\ddot{v} + \frac{A_5\ddot{v}^2}{3} - \frac{2A_4\dot{v}\ddot{v}}{3} - \frac{A_2\dot{v}\dddot{v}}{6}, \]  

(94)

\[ Q_{26} = -\frac{1}{3}A_2\dot{v}^2 - \frac{13A_4\dot{v}^2}{24} + \frac{7A_5\dot{v}\dddot{v}}{8}, \quad Q_{27} = \frac{5A_5\dot{v}\dddot{v}}{6} - \frac{1}{2}A_4\dot{v}^2, \quad Q_{28} = \frac{17A_5\dot{v}\dddot{v}}{48} + \frac{A_1}{2} + \frac{A_3}{2}, \]  

(95)

\[ Q_{29} = \frac{A_5}{6}, \quad Q_{30} = \frac{A_5}{24}, \quad Q_{31} = -A_5\dot{v}^2, \quad Q_{32} = -\frac{A_2}{3} - \frac{A_4}{6}, \quad Q_{33} = \frac{A_5}{2}, \]  

(96)

\[ Q_{34} = -\frac{A_2}{2} - \frac{A_4}{2}, \quad Q_{35} = \frac{5}{8}A_5\dot{v}^2 - A_1 + A_3, \quad Q_{36} = A_2 - A_4. \]  

(97)

Now the equations of motion (22) need to be renormalized, thus as we will need in Section 3, a set of four-vectors can be built in an orthonormal basis from the set \( \{ \hat{B}_\mu, v_\mu, \dot{v}_\mu, \ddot{v}_\mu \} \), where these vectors have the following form

\[ \hat{B}_\mu = \hat{B}_\mu. \]  

(98)

Here \( \hat{B} \) is the norm of \( \hat{B}_\mu \). The first orthonormal vector is given by

\[ \hat{u}_{1\mu} = \frac{v_\mu - (\hat{v}\hat{B})\hat{B}_\mu}{u_1}, \]  

(99)

with contraction of the form \( |u_1|^2 = u_1^2 = 1 - \frac{\hat{v}\hat{B}}{B^2} \). For the second normalized vector in terms of \( \dot{v}_\mu \) we find

\[ \hat{u}_{2\mu} = \frac{1}{u_2} \left[ \dot{v}_\mu - (\hat{v}\hat{B})\hat{B}_\mu + \frac{(\hat{v}\hat{B})(\hat{\dot{v}}\hat{B})}{u_1^2} \left( v_\mu - (\hat{v}\hat{B})\hat{B}_\mu \right) \right], \]  

(100)

whose magnitude can be expressed in the simple form

\[ u_2^2 = \dot{v}^2 - (\hat{v}\hat{B})^2 - \frac{(\hat{v}\hat{B})^2(\hat{\dot{v}}\hat{B})^2}{u_1^2}. \]  

(101)

The vector \( u_{3\mu} \) is expressed in terms of \( \ddot{v}_\mu \) and it is given as follows

\[
\begin{align*}
u_{3\mu} = & \ddot{v}_\mu - \frac{\hat{v}\space\hat{v}B_\mu}{B^2} - \frac{v\dddot{v} - (\hat{v}\hat{B})(\hat{\dot{v}}\hat{B})}{u_1^2} \left( v_\mu - (\hat{v}\hat{B})\hat{B}_\mu \right) \\
& - \frac{1}{u_2^2} \left[ v\dddot{v} - (\hat{v}\hat{B})(\hat{\dot{v}}\hat{B}) + \frac{(\hat{v}\hat{B})(\hat{\dddot{v}}\hat{B})}{u_1^2} \left( \dddot{v} - (\hat{v}\hat{B})\ddot{v}\hat{B} \right) \right] \\
& \times \left[ \ddot{v}_\mu - (\hat{v}\hat{B})\hat{B}_\mu + \frac{(\hat{v}\hat{B})(\hat{\dddot{v}}\hat{B})}{u_1^2} \left( v_\mu - (\hat{v}\hat{B})\hat{B}_\mu \right) \right].
\end{align*}
\]  

(102)
Consequently we have

\[ u_3^2 = \mathbf{v}^2 - (\dot{\mathbf{v}}\mathbf{B})^2 - \frac{1}{u_1^2} \left[ \mathbf{v}\mathbf{v} - (\mathbf{v}\mathbf{B})\mathbf{v}\mathbf{B} \right]^2 \]

\[ -\frac{1}{u_2^2} \left[ \mathbf{v}\mathbf{v} - (\dot{\mathbf{v}}\mathbf{B})(\mathbf{v}\mathbf{B}) + \frac{(\dot{\mathbf{v}}\mathbf{B})(\mathbf{v}\mathbf{B})}{u_1^2} \left( \mathbf{v}\mathbf{v} - (\mathbf{v}\mathbf{B})(\mathbf{v}\mathbf{B}) \right) \right]^2. \] (103)

All these vectors are constructed in such a way that the conditions of orthonormalization are fulfilled in all the expressions of vectors \( u_i \). Here the index \( i = 1, 2, 3, 4 \) stands for a label of the basis. The index \( j = 1, 2, 3, 4 \) in \( \epsilon^{ij} \) stands to the derivative with respect to the proper-time of the particle. Analogously the last more general orthonormal vector that we build in this basis is

\[ u_{4\mu} = \dot{v}_\mu - (\dot{v}\mathbf{B})\dot{B}_\mu - \frac{\left[ \mathbf{v}\mathbf{v} - (\mathbf{v}\mathbf{B})(\mathbf{v}\mathbf{B}) \right]}{u_1^2} \left[ v_\mu - (\mathbf{v}\mathbf{B})\dot{B}_\mu \right] \]

\[ -\frac{1}{u_2^2} \left[ \mathbf{v}\mathbf{v} - (\dot{v}\mathbf{B})(\mathbf{v}\mathbf{B}) + \frac{(\dot{v}\mathbf{B})(\mathbf{v}\mathbf{B})}{u_1^2} \left( \mathbf{v}\mathbf{v} - (\mathbf{v}\mathbf{B})(\mathbf{v}\mathbf{B}) \right) \right] \times \left[ v_\mu - (\mathbf{v}\mathbf{B})\dot{B}_\mu \right] \]

\[ -\frac{1}{u_3^2} \left\{ \mathbf{v}\mathbf{v} - (\dot{v}\mathbf{B})(\mathbf{v}\mathbf{B}) - \mathbf{v}\mathbf{v} - (\dot{v}\mathbf{B})(\mathbf{v}\mathbf{B}) \right\} \left( \mathbf{v}\mathbf{v} - (\mathbf{v}\mathbf{B})(\mathbf{v}\mathbf{B}) \right) \]

\[ -\frac{1}{u_2^2} \left[ \mathbf{v}\mathbf{v} - (\dot{v}\mathbf{B})(\mathbf{v}\mathbf{B}) + \frac{(\dot{v}\mathbf{B})(\mathbf{v}\mathbf{B})}{u_1^2} \left( \mathbf{v}\mathbf{v} - (\mathbf{v}\mathbf{B})(\mathbf{v}\mathbf{B}) \right) \right] \times \left[ \mathbf{v}\mathbf{v} - (\mathbf{v}\mathbf{B})(\mathbf{v}\mathbf{B}) + \frac{(\dot{v}\mathbf{B})(\mathbf{v}\mathbf{B})}{u_1^2} \left( \mathbf{v}\mathbf{v} - (\mathbf{v}\mathbf{B})(\mathbf{v}\mathbf{B}) \right) \right] \right\} \times \left( \mathbf{v}\mathbf{v} - (\dot{v}\mathbf{B})(\mathbf{v}\mathbf{B}) - \frac{(\dot{v}\mathbf{B})(\mathbf{v}\mathbf{B})}{u_1^2} \left( \mathbf{v}\mathbf{v} - (\mathbf{v}\mathbf{B})(\mathbf{v}\mathbf{B}) \right) \right) \] (104)

Now with Eqs. (99), (100), (102), (104) and the conditions of orthonormalization, it can be proved that \( v_\mu^{(i)\nu_4} = v_\mu^{(i)\nu_4} = 0 \) with \( i = 0, 1, 2, 3 \) and \( v_\mu^{(0)} = v_\mu \). With Eqs. (104) it is easy to show that the product \( \mathbf{B}\mathbf{u}_4 \), where \( \dot{B}_\mu \) is taken from Eq. (22) and this give rises to

\[ (\varepsilon^{-1}Q_{36} + Q_{35}) L_\nu v^\nu - A_1 L_\nu \dot{v}^\nu = 0, \] (105)
where $L_{\nu} := f_{\mu\nu} \tilde{n}^{\mu}_{4}$. This expression is used in the subsections (3.3) and (3.4) to obtain the equations of motion.

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