Minimally Supervised Feature Selection for Classification

Radu Alexandra Maria

Master Thesis
University Politehnica of Bucharest
Faculty of Automatic Control and Computers
Computer Science and Engineering Department

Supervisor: Associate Prof. Dr. Marius Leordeanu

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Abstract

In the context of the highly increasing number of features that are available nowadays we design a robust and fast method for feature selection. The method tries to select the most representative features that are independent from each other, but are strong together. We propose an algorithm that requires very limited labeled data (as few as one labeled frame per class) and can accommodate as many unlabeled samples. We also present here the supervised approach from which we started. We compare our two formulations with established methods like AdaBoost, SVM, Lasso, Elastic Net and FoBa and show that our method is much faster and it has constant training time. Moreover, the unsupervised approach outperforms all the methods with which we compared and the difference might be quite prominent. The supervised approach is in most cases better than the other methods, especially when the number of training shots is very limited. All that the algorithm needs is to choose from a pool of positively correlated features. The methods are evaluated on the Youtube-Objects dataset of videos and on MNIST digits dataset, while at training time we also used features obtained on CIFAR10 dataset and others pre-trained on ImageNet dataset. Thereby, we also proved that transfer learning is useful, even though the datasets differ very much: from low-resolution centered images from 10 classes, to high-resolution images with objects from 1000 classes occurring in different regions of the images or to very difficult videos with very high intraclass variance.
Nowadays people aim to enable computers and robotic systems to perform tasks at the level of human beings. Day by day, computers are becoming more powerful in terms of computational speed, storage capabilities and software intelligence. Computers have started to impact virtually all aspects of human life, from helping the development of other scientific fields to improving different industrial, agricultural, medical and consumer sectors. Ultimately they are helping us improve the quality of our lives.

While enjoying successes on all aspects of technology, automatic computer systems are still far from being able to see at the level of the human eyes and brain. More than 80% of the information that humans process in their brains comes from sight. Even though we are capable to see and interpret the information we gain and learn by vision from the very first months of our lives, the process that takes place behind is still difficult to explain. There is no general algorithm found yet that solves this problem, and the task has not yet been formalized.

In this thesis we tackle an important task in visual learning and recognition, which is that of feature selection and classifier learning with minimal supervision. We provide an efficient solution to this problem, and show through extensive experiments that our approach outperforms established algorithms from the feature selection and classification literature, such as AdaBoost, SVM, and Lasso with different types of regularization. Our method is efficient and general enough to be applied to various domains and here we focus on recognition especially in video, but we also provide encouraging experiments on image classification. Our approach is unique especially in the fact that it focuses on learning from limited data, while performing selection at the same time. This is different and complementary to the current trend in learning with Deep Neural Networks. As we will discuss in the future work chapter, we envision a natural combination of our approach to learning and selection from limited labeled training samples with the deep hierarchical classifier structure paradigms, which, as of now, require huge amounts of supervised training samples.

We start from the fact that we can take advantage of the increasing number of features that can be computed on images. These features range from manually designed ones, as HOG, SIFT and color histograms to automatically designed features generated by deep learning methods. As time complexity is still so important despite the evolution of processors and the multiprocessing capabilities available now, we cannot afford to use all these features without a prior selection, because the task would become prohibitively costly. There is also a memory (space) problem caused by the need to store so much data. To make a clearer idea about the highly increasing number of features, we mention that in 1997 there
Figure 1.1: What classes can trigger the idea of a “train”? Many classes have similar appearance but are semantically less related (blue box); others are semantically closer but visually less similar (green box). There is a continuum that relates appearance, context and semantics. Can we find a group of classifiers, which are together robust to outliers, over-fitting and missing features? Here, we show classifiers that are consistently selected by our method from limited training data as giving valuable input to the class “train”.

were few domains that used more than 40 features, while now hundreds to tens of thousands of features are explored as mentioned in [1]. Other interesting facts presented and demonstrated in the same paper are: unsupervised learning is desirable not only because in many situations the samples are not labeled, but it is less prone to overfitting, one feature that alone is useless, together with others may bring an important contribution. Therefore, we thought about designing a method for selecting those features that preserve as much as possible from the potential of the whole pool of features, but optimal feature selection is a NP-hard problem [1, 2]. Efficient feature selection algorithms must be created so that each class is triggered by a limited number of key input features (Fig. 1.1).

We approach feature selection from the task of discriminant linear classification [3] with novel constraints on the solution and the features. We put an upper bound on the solution weights and require the solution to be an affine combination of soft-categorical features, which should have stronger outputs on the positive class vs. the negative. We term these signed features. Recent work [4] has examined correlations between the firing of units in deep neural networks for learning; we concentrate on much simpler relations: the sign between a feature and the target class, i.e. the difference in the mean feature response over positive samples vs. negative ones.

We present both a supervised and an unsupervised approach. Our supervised method is a convex
constrained minimization problem. We extend this formulation to the unsupervised learning case, with a clustering formulation that can accommodate large amounts of unlabeled data; the only information required is the signs of individual features, as defined shortly. Interestingly, the latter learning formulation is a concave minimization problem different from the convex supervised one. Both formulations have important sparsity and optimality properties as well as strong generalization capabilities in practice. The proposed learning schemes also serve as feature selection mechanisms, such that the majority of features with zero weights can be safely ignored while the remaining few features work together as a powerful classifier ensemble. Consider Fig. 1.1: here we use image-level CNN classifiers [5], pre-trained on ImageNet, to recognize trains in video frames from the YouTube-Objects dataset as done in [6]. Our method discovers a feature ensemble from a pool of 6000 classifiers that are relevant to the concept. Since each classifier corresponds to one ImageNet concept, we show a few of the classifiers that have been consistently selected by our method over 30 trials from different small training sets of 8 video shots per class.

Ensemble learning proved to be a very effective method because it unites the capabilities of simple classifiers in a more complex and powerful one. To have many images is easy, but to have many labeled images is a more expensive scenario, because we need humans to label them. We try to overcome this problem by creating an almost unsupervised algorithm for image classification. More precisely, this algorithm requires only a very limited set of labeled data and as many unlabeled samples. We arrived to the idea that it would be much easier to work with labeled features than with labeled data because the number of features is usually much smaller than the number of examples. In this thesis we will refer by labeled/signed/flipped to those features that are positively correlated with the examples. Positively correlated means that the mean on the positive samples of the given feature is higher than the mean on the negative samples. We force the features to be positively correlated by changing the sign of the features that do not comply with this condition. We start from the simple least squares formulation, then we add some constraints that ensure the sparsity of our solution and then apply the integer projected fixed point method for optimization. All that our algorithm needs is the pool of positively correlated features and then it can use only unsupervised learning. The method is described in great detail in Chapter 3.

Despite the small quantity of labeled data, as few as one frame per class, our algorithm manages to surpass many well-known methods like SVM, AdaBoost, Lasso etc especially in the case of a limited number of labeled data. In Chapter 4 we present a series of various experiments executed. We prove that our method is more accurate than many other methods, the training time is smaller and we also analyze other aspects of the method. The difference in performance between our method and the others is more visible in the case of very limited training data.

The main contributions of our approach are:

1. An efficient method for joint linear classifier learning and feature selection. We show that, both in theory and practice, our solutions are sparse. The number of features selected can be set to $k$ and the non-zero weights are equal to $1/k$. The simple solution enables good generalization and learning in an almost unsupervised setting, with minimal supervision. This is very different from classical regularized approaches such as the Lasso.

2. Our formulation requires minimal supervision: namely only the signs of features with respect to the target class, as defined in Sec. 3.1. These signs can be estimated from a small set of labeled samples,
and once determined, our method can handle large quantities of unlabeled data with excellent accuracy and generalization in practice.

3. In our extensive experiments, both supervised and unsupervised variants of our method demonstrate superior performance in terms of learning time and accuracy when compared to established approaches such as AdaBoost, the Lasso, Elastic Net and SVM. In particular, the unsupervised variant significantly outperforms its competitors and is the least sensitive to the quantity of labeled data.

The outline of this work is as follows. In Chapter 2 we present other existent approaches to our problem, in Chapter 3 we explain our own approach for both supervised and unsupervised variants and make a theoretical analysis of the algorithms proposed. In Chapter 4 we talk about the features used and the experiment settings and present a number of experiments done with our two approaches and with the methods that we compare with. Chapter 5 concludes our work with a summary and some ideas about the research we did. An initial version of this work appeared in our paper [7], which presents the supervised learning scheme. Later we developed the almost unsupervised variant of our method, which will appear at the AAAI-16 International Conference on Artificial Intelligence [8] along with the initial supervised case. An arxiv version of this paper [9] also appeared. Our work also draws inspiration from ideas initially proposed in the Classifier Graph paper [10] and validates some of those concepts. For example, in our experiments we demonstrate that it is possible to learn powerful novel classifiers by efficiently selecting and combining previously learned independent classifiers from a very large heterogeneous pool, which is one of the ideas presented in [10].
Chapter 2

Related work

In this chapter we are going to make a presentation of the main approaches to the feature selection problem, and we will focus on its application in the domain of computer vision, more precisely object classification. We will talk about regularized approaches to feature selection like: Lasso and Elastic Net, to ensemble learning methods like: bagging, boosting and decision trees ensembles. We will not limit to feature selection, we will also refer to some of the descriptors used in computer vision: HOG and SIFT. We used the former in our implementation. We will also talk about object recognition, more precisely about part based models.

**Relation to feature selection through regularization:** Feature selection is such a general topic that it is used in many domains, from machine learning and bioinformatics to data mining and computer vision. Feature selection is a procedure used in machine learning for extracting a subset of relevant features in order to create a less complex model of learning. The motivation behind this technique is the fact that when the initial pool consists of a huge number of features, some of them are redundant (they do not bring any useful information). When we have a huge number of features and a small number of examples, we need to discard some of the features in order to avoid overfitting. Being such a vast topic, there are many approaches to feature selection: from ensemble learning methods, to $L_1$, $L_2$ regularization, greedy selection [11], genetic algorithms [12] (optimize feature selection in accordance with the resource limitations) and ant colony optimization [13]. Some benefits of feature selection techniques are mentioned in [14]: better understating of the data, curse of dimensionality reduction, improvement of predictor performance as noisy features can degrade the learning process. According to [14] the feature selection methods can be divided into two classes: filter and wrapper. The former refers to those methods that evaluate each feature individually, rank the features and give up those that are below a threshold. The later category refers to evaluating subsets of features and deciding which one is the best. Because the number of subsets is $2^N$, an exhaustive search is impossible, thus some search algorithms are used to maximize the objective function.

$L_0$ regularization is more suitable for feature selection than any other regularization type because $L_0$ norm is equal to the number of non-zero values. In feature selection we should minimize the number of features selected, namely the $L_0$-norm. As stated in [15, 16], $L_0$ regularization leads to very difficult minimization problems, therefore it is common to use other types of regularization as $L_1$ or $L_2$. In fact, $L_0$ optimization is an NP-hard problem. $L_1$ regularization is known to be the best relaxation of $L_0$ regularization, being solved by gradient descent.
L1 and L2 regularization are used for feature selection in [17, 18, 19]. L1 regularization, usually referred as Lasso [20], has three main benefits: it avoids overfitting by penalizing big weights, leads to sparse solutions and is a convex optimization problem, which has some disadvantages. If the number of features p is greater than the number of samples n, then Lasso will select at most n features and it fails to select grouped features - if a group of features are related, then it selects only one of them. Another regularized approach named ridge regression uses the L2-norm. Its drawbacks come from the fact that it only shrinks some parameters towards zero without setting them to exactly 0, thus all the features are kept, none of them being discarded. This makes the model difficult to interpret. Elastic Net [21] combines these two methods and benefits from their advantages. The Elastic Net regularization is expressed by: 

$$
\beta^* = \text{argmin}_{\beta} \|y - X\beta\|^2 + \lambda_2 \|eta\|^2 + \lambda_1 \|eta\|_1.
$$

The L1 regularization term enforces the sparsity of the solution, while L2 regularization term removes the limitation on the number of selected features and encourages grouping effect.

In [17] the authors use decision trees for feature selection with L1 regularization. Each tree uses some features in its construction and the total weight for each feature is given by the sum of the weights of the trees in which that feature appears. The optimization problem is non-convex and non-differentiable, but up to a fixed point it can still be optimized with gradient boosting. At each iteration only one dimension of the weights is optimized. In [18], the authors try to recover w given their input vectors and the output variables. They assumed that the weight matrix is sparse, and they focus on estimating the non-zero weights. The L1 regularized formulation of the problem can be solved with convex programming. It was observed that L1 regularization leads to sparse solutions. In [15], L2-L0-norm is used in the optimization problem and the authors try to solve it by DC programming (Difference of Convex functions) and approximate the L0-norm by a concave function. They want to select features for multiclass SVM. The objective function is decomposed as a difference of convex functions and the solution is obtained iteratively.

**Relation to ensemble learning:** During the last decades of research in machine learning, it has been observed that a combination of multiple individual classifiers is stronger than a single classifier. As stated in [22], the two necessary conditions for an ensemble to work are the following: 1) the base classifiers must be better than random and 2) the base classifiers must be as diverse as possible. If the first condition does not hold, then the ensemble will have a higher error than the base classifiers. If the second condition is not true and the classifiers are identical (or very similar), they will make the same errors and the ensemble will do the same. The result of the ensemble classifier is the outcome of a voting procedure of the composing classifiers. If all vote the same, then the ensemble will bring no improvement. The main approaches to ensemble learning [23, 24] are boosting [25], bagging [26] and decision trees ensembles [27, 28]. Selecting a subset of features from the whole pool presents two main advantages over the approach in which all the features are considered as stated in [29]. First, the computational costs are much more reduced. Second, the noisy features are removed and the algorithm can generalize better when it has to deal only with more representative features.

Bagging was the first efficient machine learning ensemble method. It is a meta-algorithm whose main idea is to train more classifiers on different subsets of the training set. The classifiers are trained independently, even in parallel. The subsets are obtained by sampling with replacement examples from the whole set. For each subset created, a new classifier is trained and finally the results of the classifiers are averaged in order to obtain a single result (regression) or they are subject to a voting
procedure (classification). The advantage of this method are the ability to reduce variation and to improve performance for unstable classifiers that vary significantly when small changes in the data appear. A particularity of bagging that is different in the case of our method is the fact that the former averages the whole set of classifiers, while the later uses only a subset of them. We believe that fewer classifiers could be better than all, this idea being also supported in [30], where an algorithm GASEN-b is proposed. In this algorithm a weight is assigned to each classifier. The weights are optimized using genetic algorithms and then, the classifiers whose weights were greater than a threshold are selected.

Boosting is a machine learning ensemble meta-algorithm. Through boosting a set of weak classifiers are combined in order to form a strong one. It is a sequential algorithm in which at each iteration the weak classifiers that have not been chosen yet are trained and the one that performs best in the given state is chosen. The goal is to choose the classifiers so as to avoid redundancy as much as possible. The examples in the training set are weighted, so that those that are more difficult will weight more, while the simpler ones will weight less; those examples that were misclassified at a given iteration will weight more in the next ones, so that the new classifiers focus more on them. There are many kinds of boosting algorithms: AdaBoost, LPBoost, LogitBoost etc. A weakness of the method is the fact that it performs well only when the base classifiers are weak, it cannot take advantage of stronger base classifiers. This is different in the case of our method, because it can work very well also with stronger classifiers. Other problems are the fact that initial classifiers tend to have a much greater importance than those that are chosen in later stages of the algorithm and the training phase is very slow. This method became famous through its particularization AdaBoost when it was successfully used for face detection by Viola and Jones in [31].

Decision trees ensembles represent a machine learning method that constructs a multitude of decision trees at training time and the result of the method is the average of the results of these trees (regression) or the mode of their results (classification). This method solves the problem of individual decision trees of overfitting. Unlike decision trees ensembles, our method averages only a subset of classifiers, not the whole pool, which makes a big difference.

**Image descriptors:** As stated before, feature selection is a very general problem applied in many domains. As in computer vision we do not usually work directly with the raw pixels, we should apply some descriptors on the image and then use their values as features. Given that the number of descriptors has increased very much during the last years, we should select only a part of them in order to apply then some machine learning algorithms. Image descriptors can be automatically or manually designed. Automatically designed image descriptors can be obtained using deep neural networks, such a network is described in [5]. Such descriptors are very numerous, for example with Caffe we can obtain 1000 descriptors. On the other hand, there are fewer manually designed descriptors, but stronger. We can mention Scale Invariant Feature Transform (SIFT) [32] - more suitable for object recognition, Histograms of Oriented Gradients (HOG) [33] - better for object classification and color histogram descriptors as: Dominant Color, CSD, CLD, all presented in [34]. In the following paragraphs we present the most important among them.

David Lowe proposes in [32] a new method for object recognition that consists in identifying a set of features in images that are invariant to scaling, rotation, translation, illumination, some affine transforms and also are sufficiently distinctive. The main difficulty in object recognition is the identification of such features. In each image a number of the order of 1000 of such features are identified in less than a second. The nearest-neighbour algorithm is used to identify the model that best matches the current object. If the
model and the object have at least 3 features in common, then the object is considered recognized. Given that the number of initial features for each object is about 20, this approach is effective also when the object is partially occluded because of the fact that only three key points are necessary to identify the object. The most frequent approaches at that time were based on template matching, but this solution was good only for the situations in which the illumination and the position of the object were invariant. Another solution was to identify only the features that are invariant to certain conditions and template match only on these: line segments, regions, grouping of edges. Another kind of approach was to detect the corners in an image and to focus the matching only on these areas, not on the whole image. The weakness of the last approach stands in the fact that it is not invariant to scaling, this means that the image has to be scanned at many scales in order to obtain a reliable result. In order to ensure scale and rotation invariance, an image pyramid is built to create the scale space. The scale space of the image is built and the difference-of-gaussian of the images obtained is computed. For each point, a 3 by 3 vicinity is considered from 3 neighbouring scales. If a point is a minima or a maxima in the three considered images, then it is an interest point. After the interest points are identified, the nearest neighbour algorithm is used to group the SIFT features and to detect the objects found in the image. Each key feature in the image has associated an orientation and based on these orientations, a histogram of orientations is built using 36 bins that cover the range of 360 degrees. Lowe’s approach has similarities with the mechanisms that primates use in order to recognize an object. Neuroscientists discovered neurons that are specialized in recognizing shapes like dark five-sided star shape, or a circle with a protruding element. These being intermediate features that help primates together with color and texture cues to recognize objects.

Dalal and Triggs in [33] have designed an algorithm for pedestrian detection. Despite the fact that they illustrated this method on pedestrians, it is a general object category detection algorithm that can be used very well for any category of objects. The idea of the authors was to find a representation that helps discriminating the human body over the cluttered background. HOG used to perform better than the existing methods. The authors of the study analysed every aspect of the algorithm in order to obtain the best possible results: the number of bins, the number of horizontal/vertical cells and the local normalization. They use a histogram of gradients in order to create an encoding of the image. For each training image the histogram of oriented gradients is computed. When an image should be tested, the same histogram is created and the image is classified, based on its histogram as containing a pedestrian, or not. The gradients of the image are computed, then the image is divided in equal cells. In each cell, each pixel will vote for a certain orientation of the edges. are cumulated in orientations bins. Figure 3.1 shows such a histogram computed for a pedestrian. Usually the range is between 0 and 180 degrees and the number of bins chosen by the authors was 9. It is a weighted vote in which each pixel contributes with the magnitude value. A local normalization step follows, when the cells are grouped in blocks, and they are normalized with regard to each block in which they appear. The blocks can or cannot overlap, in the former case the histogram will be bigger because some cells will be repeated, but the results proved to be better. The optimum parameters observed were: blocks of $3 \times 3$ cells, 9 bins and $6 \times 6$ pixels cells. They used linear SVM for classification. In order to detect pedestrians in each position and for each scale, Dalal and Triggs used a sliding window approach in which a filter is applied for all positions and for all scales. The results were very good, on the MIT database the accuracy was almost 100%, therefore they created a more challenging dataset with people in different position and different backgrounds. On INRIA set it obtains FPPW(false positives per window) score an order of magnitude better. HOG outperformed
the other algorithms: wavelet, PCA-SIFT and Shape Context. In this paper, Dalal and Triggs showed that histograms of oriented gradients, locally contrast normalized can give very good results compared to the methods that existed at that moment. They also showed that the simplest methods of computing the gradients are the most performant, but the performance is highly influenced also by the binning, the contrast normalization and by the overlapping of the blocks.

**Relation to object recognition:** During the last years, image classification evolved towards more complex higher-level models. Such an example is a method designed by Felzenszwalb et al. that learns *part-based models* and that we present in the following lines. In their study [35], Felzenszwalb et al. try to address the problem of the gap between the performance obtained using part-based models and other much simpler methods that outperform it on difficult datasets. This problem is caused by the fact that an object can be viewed from different orientations and thus, they propose to use mixture models to deal with this kind of situations. Complex part-based models being outperformed by simpler methods on more difficult datasets is due to the fact that they are more difficult to train, because learning methods like SVM cannot be used. A star model of the object is built using a coarse filter as root and finer filters to detect parts of the object. The model is represented as a tuple of the root part and the smaller parts. Each part is composed from its filter, the anchor (where it should be placed relative to the main part) and the penalties for changes in position. The score of an object is computed as the sum of the scores of each filter minus the deformation cost of each part relative to its expected position. The root location with highest scores represent detections and together with the placement of the parts constitute an object hypothesis. The system was tested on Pascal VOC dataset and also on INRIA, on 20 categories of objects.
Chapter 3

Problem description

In this chapter we will present our approach to feature selection, the mathematical formulation in which we start from the least squares formulation and add new constraints that lead to important theoretical properties. We also talk about our two methods: the supervised and the unsupervised one which is derived from the former. We present our algorithms and discuss the intuition behind our approach. We also make a theoretical analysis of the method.

3.1 Our method

Through our method we approach the case of binary classification, while for the multi-class scenario we apply the one vs. all strategy. Our training set is composed of \( N \) samples, each \( i \)-th sample being expressed as a column vector \( f_i \) of \( n \) features with values in \([0, 1]\); such features could themselves be outputs of classifiers. We want to find a vector \( w \), with elements in \([0, 1/k]\) and unit \( L_1 \)-norm, such that \( w^T f_i \approx \mu_P \) when the \( i \)-th sample is from the positive class and \( w^T f_i \approx \mu_N \) otherwise, with \( 0 \leq \mu_N < \mu_P \leq 1 \). For a positive labeled training sample \( i \), we fix the ground truth target \( t_i = \mu_P = 1 \) and for a negative one we fix it to \( t_i = \mu_N = 0 \). Our novel constraints on \( w \) limit the impact of each individual feature \( f_j \), encouraging the selection of features that are powerful in combination, with no single one strongly dominating. This produces solutions with good generalization power. In Sec. 3.2 we show that \( k \) is equal to the number of selected features, all with weights \( = 1/k \). The solution we look for is a weighted feature average with an ensemble response that is stronger on positives than on negatives. For that, we want any feature \( f_j \) to have expected value \( E_P(f_j) \) over positive samples greater than its expected value \( E_N(f_j) \) over negatives. From the labeled samples we estimate the sign of each feature \( \text{sign}(f_j) = E_P(f_j) - E_N(f_j) \) and if it is negative we simply flip the feature values: \( f_j \leftarrow 1 - f_j \).

3.1.1 Supervised learning

We begin with the supervised learning task, which we formulate as a least-squares constrained minimization problem. Given the \( N \times n \) feature matrix \( F \) with \( f_i^\top \) on its \( i \)-th row and the ground truth vector \( t \), we look for \( w^* \) that minimizes \( \|Fw - t\|^2 = w^\top (F^\top F)w - 2(F^\top t)^\top w + t^\top t \), and obeys the required
constraints. We drop the last constant term $t^\top t$ and obtain the following convex minimization problem:

$$w^* = \arg\min_w J(w)$$

$$= \arg\min_w w^\top (F^\top F)w - 2(F^\top t)^\top w$$

s.t. $\sum_j w_j = 1$, $w_j \in [0, 1/k]$.

Our least squares formulation is related to Lasso, Elastic Net and other regularized approaches, with the distinction that in our case individual elements of $w$ are restricted to $[0, 1/k]$, which leads to important theoretical properties regarding sparsity and directly impacts generalization power (Sec. 3.2). This also leads to our (almost) unsupervised approach, presented in the next section.

### 3.1.2 Unsupervised learning

Consider a pool of signed features correctly flipped according to their signs, which could be known a priori, or estimated from a small set of labeled data. We make the simplifying assumption that the signed features’ expected values for positive and negative samples, respectively, are close to the ground truth target values $(\mu_P, \mu_N)$. Then, for a given sample $i$, and any $w$ obeying the constraints, the expected value of the weighted average $w^\top f_i$ is also close to the ground truth target $t_i$: $E(w^\top f_i) = \sum_j w_j E(f_i(j)) \approx (\sum_j w_j)t_i = t_i$. Then, for all samples we have the expectation $E(Fw) \approx t$, such that any feasible solution will produce, on average, approximately correct answers. Thus, we can regard the supervised learning scheme as attempting to reduce the variance of the feature ensemble output, as their expected value is close to the ground truth target. If we now introduce the approximation $E(Fw) \approx t$ into the learning objective $J(w)$, we obtain our new ground-truth-free objective $J_u(w)$ with the following learning scheme, which is unsupervised once the feature signs are determined. Here $M = F^\top F$:

$$w^* = \arg\min_w J_u(w)$$

$$= \arg\min_w w^\top (F^\top F)w - 2(F^\top (Fw))^\top w$$

$$= \arg\min_w (-w^\top (F^\top Fw)) = \arg\max_w w^\top Mw$$

s.t. $\sum_j w_j = 1$, $w_j \in [0, 1/k]$.

Interestingly, while the supervised case is a convex minimization problem, the unsupervised learning scheme is a concave minimization problem, which is NP-hard. This is due to the change in sign of the matrix $M$. However, since $M$ in the unsupervised case could be created from larger quantities of unlabeled data, $J_u(w)$ could in fact be less noisy than $J(w)$ and produce significantly better local optimal solutions — a fact confirmed by our experiments.

### 3.1.3 Intuition

Let us take a closer look at the two terms involved in our objectives, the quadratic term: $w^\top Mw = w^\top (F^\top F)w$ and the linear term: $(F^\top t)^\top w$. If we assume that feature outputs have similar expected values, then minimizing the linear term in the supervised case will give more weight to features that are strongly correlated with the ground truth and are good for classification, even independently. However,
things become more interesting when looking at the role played by the quadratic term in the two cases of learning. The positive definite matrix $F^\top F$ contains the dot-products between pairs of feature responses over the samples. In the supervised case, minimizing $w^\top (F^\top F)w$ should find a group of features that are as uncorrelated as possible. Thus we seek group of features that are individually relevant due to the linear term, but not redundant with respect to each other due to the quadratic term. They should be conditionally independent given the class, an observation that is consistent with earlier research in machine learning (e.g., [22]) and neuroscience (e.g., [36]).

In the unsupervised case, the task seems reversed: maximize the same quadratic term $w^\top Mw$, with no linear term involved. We could interpret this as transforming the learning problem into a special case of clustering with pairwise constraints, related to methods such as spectral clustering with $L^2$-norm constraints [37] and robust hypergraph clustering with $L^1$-norm constraints [38, 39]. The problem is addressed by finding the group of features with strongest intra-cluster score — the largest amount of covariance. In the absence of ground truth labels, if we assume that features in the pool are, in general, correctly signed and not redundant, then the maximum covariance is attained by features whose collective average varies the most as the hidden class labels also vary. Thus, the unsupervised variant seeks features that respond in a united manner to the distributions of the two classes.

### 3.2 Algorithms

In our both approaches, we first need to determine the sign for each feature, as defined before. Once it is estimated, we can set up the optimization problems to find $w$. In Algorithms 1 and 2, we present our supervised and unsupervised learning methods. The supervised case is a convex minimization problem, with efficient global optimization possible in polynomial time. The unsupervised learning is a concave minimization problem, which is NP-hard and can only have local efficient optimization.

#### Algorithm 1 Supervised learning

Learn feature signs from the labeled samples.
Create $F$ with flipped features from the labeled samples.
Set $M \leftarrow F^\top F$.
Find $w^* = \text{argmin}_{w} w^\top Mw - 2(F^\top t)^\top w$
\[\text{s.t. } \sum_j w_j = 1, \ w_j \in [0, 1/k].\]
\text{return } w^*$

#### Algorithm 2 Unsupervised learning from signed features

Learn feature signs from a small set of labeled samples.
Create $F$ with flipped features from unlabeled data.
Set $M \leftarrow F^\top F$.
Find $w^* = \text{argmax}_{w} w^\top Mw$
\[\text{s.t. } \sum_j w_j = 1, \ w_j \in [0, 1/k].\]
\text{return } w^*$

There are many possible fast methods for optimization. In our implementation we adapted the integer projected fixed point (IPFP) approach [40, 41], related to the Frank-Wolfe algorithm[42], which is efficient in practice (Fig. 3.1c) and is applicable to both supervised and unsupervised cases. The method converges to a stationary point — the global optimum in the supervised case. At each iteration IPFP approximates
Figure 3.1: Optimization and sensitivity analysis: a) Sensitivity to $k$. Performance improves as features are added, is stable around the peak $k = 60$ and falls for $k > 100$ as useful features are exhausted. b) Features ordered by weight for $k = 50$ confirming that our method selects equal weights up to the chosen $k$. c) Our method almost converges in 10–20 iterations. d) Runtime of interior point method divided by ours, both in Matlab and with 100 max iterations. All results are averages over 100 random runs.

The original objective with a linear, first-order Taylor approximation that can be optimized immediately in the feasible domain. That step is followed by a line search with rapid closed-form solution, and the process is repeated until convergence. In practice, 10–20 iterations bring us close to the stationary point; nonetheless, for thoroughness, we use 100 iterations in all our experiments. See, for example, comparisons to Matlab’s `quadprog` run-time for the convex supervised learning case in Fig. 3.1 and to other learning methods in Fig. 4.5. Note that once the linear and quadratic terms are set up, the learning problems are independent of the number of samples and only dependent on the number $n$ of features considered, since $M$ is $n \times n$ and $F^T t$ is $n \times 1$.

### 3.3 Theoretical analysis

First we show that the solutions are sparse with equal non-zero weights (P1), also observed in practice (Fig. 3.1b). This property makes our classifier learning also an excellent feature selection mechanism. Next, we show that simple equal weight solutions are likely to minimize the output variance over samples of a given class (P2) and minimize the error rate. This explains the good generalization power of our method. Then we show how the error rate is expected to go towards zero when the number of considered non-redundant features increases (P3), which explains why a large diverse pool of features is beneficial.
Figure 3.2: Sensitivity analysis for Lasso: Left: sensitivity to number of features with non-zero weights in the solution. Note the higher sensitivity when compared to our approach. Lasso’s best performance is achieved for fewer features, but the accuracy is generally worse than in our case. Right: sensitivity to lambda $\lambda$, which controls the L1-regularization penalty.

Let $J(w)$ be our objective for either the supervised or unsupervised case:

**Proposition 1:** Let $d(w)$ be the gradient of $J(w)$. The partial derivatives $d(w)_i$ corresponding to those elements $w^*_i$ of the stationary points with non-sparse, real values in $(0, 1/k)$ must be equal to each other.

**Proof:** The stationary points for the Lagrangian satisfy the Karush-Kuhn-Tucker (KKT) necessary optimality conditions. The Lagrangian is $L(w, \lambda, \mu, \beta) = J(w) - \lambda(\sum w_i - 1) + \sum \mu_i w_i + \sum \beta_i (1/k - w_i)$. From the KKT conditions at a point $w^*$ we have:

$$d(w^*)_i - \lambda + \mu_i - \beta_i = 0,$$

$$\sum_{i=1}^{n} \mu_i w^*_i = 0,$$

$$\sum_{i=1}^{n} \beta_i (1/k - w^*_i) = 0.$$

Here $w^*$ and the Lagrange multipliers have non-negative elements, so if $w_i > 0 \Rightarrow \mu_i = 0$ and $w_i < 1/k \Rightarrow \beta_i = 0$. Then there must exist a constant $\lambda$ such that:

$$d(w^*)_i = \begin{cases} 
\leq \lambda, & w^*_i = 0, \\
= \lambda, & w^*_i \in (0, 1/k), \\
\geq \lambda, & w^*_i = 1/k.
\end{cases}$$

This implies that all $w^*_i$ that are different from 0 or $1/k$ correspond to partial derivatives $d(w)_i$ that are equal to some constant $\lambda$, therefore those $d(w)_i$ must be equal to each other, which concludes our proof.

From Proposition 1 it follows that in the general case, when the partial derivatives of the objective error function at the Lagrangian stationary point are unique, the elements of the solution $w^*$ are either 0 or $1/k$. Since $\sum_j w^*_j = 1$ it follows that the number of nonzero weights is exactly $k$, in the general case. Thus, our solution is not just a simple linear separator (hyperplane), but also a sparse representation and a feature selection procedure that effectively averages the selected $k$ (or close to $k$) features. The method is robust to the choice of $k$ (Fig. 3.1.a) and seems to be less sensitive to the number of features selected than the Lasso (see Fig. 3.2). In terms of memory cost, compared to the solution with real weights for all features, whose storage requires $32n$ bits in floating point representation, our averaging of $k$ selected features needs only $k \log_2 n$ bits — select $k$ features out of $n$ possible and automatically set their weights to $1/k$. Next, for a better statistical interpretation we assume the somewhat idealized case.
when all features have equal means \((\mu_P, \mu_N)\) and equal standard deviations \((\sigma_P, \sigma_N)\) over positive (P) and negative (N) training sets, respectively.

**Proposition 2:** If we assume that the input soft classifiers are independent and better than random chance, the error rate converges towards 0 as their number \(n\) goes to infinity.

**Proof:** Given a classification threshold \(\theta\) for \(w^T f_i\), such that \(\mu_N < \theta < \mu_P\), then, as \(n\) goes to infinity, the probability that a negative sample will have an average response greater than \(\theta\) (a false positive) goes to 0. This follows from Chebyshev’s inequality. By a similar argument, the chance of a false negative also goes to 0 as \(n\) goes to infinity.

**Proposition 3:** The weighted average \(w^T f_i\) with smallest variance over positives (and negatives) has equal weights.

**Proof:** We consider the case when \(f_i\)'s are from positive samples, the same being true for the negatives. Then \(\text{Var}(\sum_j w_j f_i(j) / \sum_j w_j) = \sum w_j^2 / (\sum w_j)^2 \sigma_P^2\). We minimize \(\sum w_j^2 / (\sum w_j)^2\) by setting its partial derivatives to zero and get \(w_q (\sum w_j) = \sum w_j^2, \forall q\). Then \(w_q = w_j, \forall q, j\).
In this chapter we will present the features that we used, the ways in which we obtained them, the settings of our experiments and their corresponding results. In the first section we talk about the main datasets used in training and testing and the features created on them. Then we present the experiments that we did and we analyse the results obtained. We compare our supervised and unsupervised approaches with regularized methods like Lasso and Elastic Net and with other well known methods used in feature selection and classification: AdaBoost, SVM, FoBa. Our experiments refer to aspects like testing accuracy, training time, sensitivity to parameters and many others.

4.1 Youtube-Objects experiments

4.1.1 Features design

|                  | Training dataset | Testing dataset |
|------------------|------------------|-----------------|
| No. of frames    | 436970           | 134119          |
| No. of shots     | 4200             | 1284            |
| No. of classes   | 10               | 10              |

Table 4.1: Youtube-Objects dataset statistics.

Dataset description: To train and test our system we used Youtube-Objects video dataset [43] and features obtained on ImageNet and CIFAR10. Details about Youtube-Objects are found in Table 4.1. The 10 classes are aeroplane, bird, boat, car, cat, cow, dog, horse, motorbike, train. This information refers to the entire training dataset, but in our experimental design, we used only a part of it to train the two methods. Details about the actual training set can be found in the next sections. Each video in the dataset consists of a number of shots. The labeling is done per video; this means that some frames that appear in a video labeled as “dog” might contain only people and not dogs. This fact makes our task more difficult because we consider that some frames show a certain object even though they do not.
Figure 4.1: Three types of features are created to form our pool. They are classifiers trained on three different datasets: CIFAR10, Youtube-Objects and ImageNet. We encourage feature diversity and independence by looking either at different parts of the input space (type I) or at different locations in the image (types II and III). For type I we train separate classifiers for each cluster of each class. For types II and III we train different classifiers for each class at every sub-window. In total we get over 6160 features. As our experiments show the large variety of our features helps in improving classification.

**Features generation:** For the experiments we used a pool of 6160 features obtained on three different datasets, in different ways. The feature types are the following:

1. Features obtained by training binary classifiers on CIFAR10 dataset [44]. These classifiers are trained on the data obtained by clustering the images from each of the 10 classes into 5 clusters. The positive examples for each classifier are the examples from the corresponding class, while negative examples are those from other classes (8 times more negatives than positives). These make a total of 50 features. The classes from CIFAR10 coincide only partially (7 classes) with those from Youtube-Objects dataset. The classes from CIFAR10 are frog, truck, deer, automobile, bird, horse, ship, cat, dog, aeroplane.

2. Features obtained by training a multiclass SVM classifier on the HOG applied on different parts of the frames. We have applied PCA on the resulted HOG, thus we obtained smaller descriptors of length 46, so as to avoid as much as possible overfitting when using SVM. The training set that we used to obtain these features is a subset of Youtube-Objects (25000 frames, equally distributed...
between the 10 classes). Then, we applied these classifiers on each image and we considered as features the probabilities returned by SVM, this means 10 features for each part-classifier. The parts of the images that we considered are the whole image, the center of the image (length and height are half the initial size), the four corners of the image, the center of the center of the image and the corners of the center of the image. Finally, we have 11 classifiers, each with 10 probabilities, summing up to 110 features.

3. Features obtained by using a pretrained network from Caffe [5]. The convolutional neural network was trained on the ImageNet dataset [45] and it contains 1000 features. We applied these features on our own dataset so: on the whole image, on the center and on the 4 corners of each example, thus we obtained 6000 new features.

4.1.2 Experiments and results

We evaluate our method in the context of a limited training dataset and we intend to show that our method generalizes better than other well known methods. We combine features obtained from different image datasets and prove that knowledge transfer is useful by testing the system on videos. In all the experiments we consider the accuracy per frame. In order to compare the different methods that we took into consideration, we varied some dimensions of the problem: the number of shots for training, the number of frames from each shot and the number of features considered. To avoid overfitting, besides studying the testing accuracy, we also observe training accuracy vs testing accuracy.

The Youtube-Objects dataset is a difficult one because the movies are taken in the wild and there are more categories of objects that appear simultaneously in a frame. Moreover, in some frames of the videos the real object is even missing or other objects appear instead (e.g. a video is labeled as "dog”, but it contains only a car in some of its frames). The shots in each video may differ very much. The differences are caused by the orientation, size, luminosity, or by the presence of more object categories in the same frame. In some shots the object is occluded, it is in a corner or it is coming in and out. Due to these facts, learning a class of objects from a small number of frames becomes a very challenging task. The splitting of the videos in training and testing is done as in [6]. In Fig. 4.2 we show one of the random sets of training frames used in the case of one-shot learning. In this scenario we feed only one labeled frame from each class to the algorithm to learn the signs of the features. You can notice that some of the frames are not at all representative for their category even though we chose the frame found in the middle of the random shot. To ensure the accuracy of the results we have averaged the results of 30 or 100 random experiments for each method.

Regarding the type II of features we did an experiment to study whether there is a preference for features computed on a certain region. In Table 4.2 we show the distribution of the classifiers selected by our supervised method with respect to the position of the region on which they are obtained for each class. We can make some observations regarding this distribution. First, the whole image (W) is the most important for many classes, this means that apart from the object, the environment is also important. Secondly, for some categories like car, motorbike, aeroplane the center (C) of the image is important, while for others, regions off-center seem to be more representative than the center. Thirdly, for some classes that seem to be similar to humans, the classifiers chosen are rather different, as in the case of cats and dogs. For dogs the whole image is more representative, while for cats different off-center regions are
Figure 4.2: One of the 30 random training sets used for one-shot learning - we use only one labeled frame per class. Note that some frames do not contain the object, making the learning task really difficult.

Table 4.2: The distribution of sub-windows for the input classifiers selected for each category. The most selected one is whole, indicating that information from the whole image is relevant, including the background. For some classes the object itself is also important as indicated by the high percentage of center classifiers. The presence of classifiers at other locations indicates that the object of interest, or objects co-occurring with it might be present at different places. Note, for example the difference in distribution between cats and dogs. The results suggest that looking more carefully at location and moving towards detection, could benefit the overall classification.

| Locations | W   | C   | TL  | TR  | BL  | BR  |
|-----------|-----|-----|-----|-----|-----|-----|
| aeroplane | 65.6| 30.2| 0   | 0   | 2.1 | 2.1 |
| bird      | 78.1| 21.9| 0   | 0   | 0   | 0   |
| boat      | 45.8| 21.6| 0   | 0   | 12.3| 20.2|
| car       | 54.1| 40.2| 2.0 | 0   | 3.7 | 0   |
| cat       | 76.4| 17.3| 5.0 | 0   | 1.3 | 0   |
| cow       | 70.8| 22.2| 1.8 | 2.4 | 0   | 2.8 |
| dog       | 92.8| 6.2 | 1.0 | 0   | 0   | 0   |
| horse     | 75.9| 14.7| 0   | 0   | 8.3 | 1.2 |
| motorbike | 65.3| 33.7| 0   | 0   | 0   | 1.0 |
| train     | 56.5| 20.0| 2.4 | 12.8| 8.4 |
preferred. This might be due to the fact that cats can be found in more unconventional places than dogs that are bigger and usually found on the ground.

We evaluated and compared eight methods: ours, SVM, AdaBoost, ours + SVM (feed to SVM only the features selected by our method), Lasso, Elastic Net, forward-backward selection (FoBa) and averaging. For SVM we used the most recent version (at the moment of the experiments) of LIBSVM [46], while for Lasso and Elastic Net we used the implementation provided in MATLAB. All the features have values between 0 and 1 and are expected to be positively correlated with the positive class. In the case of our method, we select the features whose weights have a value greater than a threshold. After the features are selected, we can use them with any classifier. The features are used with most of the tested algorithms exactly as they are, while with AdaBoost they should be transformed into classifiers, by finding for each feature the threshold that minimizes the expected exponential loss at each iteration. This is the reason why AdaBoost proved to be much slower than the other methods.

We performed extensive experiments on both variants of our method (supervised and (almost) unsupervised) and also on the methods mentioned above. We evaluated: testing accuracy, training accuracy (to make sure the algorithm is not overfitting), training time, sensitivity to input parameters, accuracy of sign estimation, sparsity of the solutions and influence of the quantity of unlabeled data over the recognition accuracy. In the majority of our experiments we are going to use four subsets of features: 1) all features of type I (50 features), 2) all features of types I and II (160 features), 3) 2000 out of the 6000 features of type III - those computed on the whole image and on the central part of the image, 3) all features of types I and II and the 2000 features of type III also selected in the previous case.

**Signs estimation:** The (almost) unsupervised setting supposes very limited labeled data, only for computing the signs of the features. It leads to very good performance even when it uses only one labeled frame per class to flip the features. In Table 4.3 we show that the accuracy of signs estimation is usually high, it increases with the number of labeled shots and frames used. We can also notice an improvement in the sign estimation accuracy when the training sets contain stronger features like in the third and fourth case. The fact that the performance of our algorithm is good even for fewer labeled samples, as we will see in the next experiments, supports our affirmation that the algorithm is robust, not being sensitive to the signs of the features.

In Fig. 4.3 we show the accuracy of the sign prediction for each feature, which depends only on the ground truth and the value of the features. We considered the ground truth for feature signs as being the signs obtained by using the whole testing set as labeled data. In Table 4.4 we show how sign estimation accuracy varies for each class. In this case we computed the accuracy of the sign estimation particularly for the features selected with our unsupervised algorithm, not for all features. Both experiments were done on the four subsets of features described before. In the second experiment we studied only the case of 16 shots, each with 10 labeled frames per class, while in the first one we varied the number of shots and frames in order to see how it influences the sign accuracy. Notice (by comparing row 5 from Table 4.3 and last row from Table 4.4 which present same settings for the experiments) that our method chooses more features correctly flipped which means that the algorithm tends to select reliable features (the percent of the features selected by the algorithm that have correct signs is higher than the percent of the total number of features that have correct signs).
Table 4.3: Accuracy of feature sign estimation for different number of labeled shots: note that the signs are estimated mostly correctly even in the 1 labeled training frame per class case. While not perfectly correct, the estimation agreements between signs estimated from a single frame per class and signs estimated from the entire test set (containing more than 100K frames) show that estimating these feature signs from very limited labeled training data is feasible. Also, the experiments presented here indicate that our approach is robust to sign estimation errors.

| Number of shots | Features I | Features I+II | Features III | Features I+II+III |
|-----------------|------------|---------------|--------------|------------------|
| 1 (1 frame)     | 61.06%     | 64.19%        | 66.03%       | 65.89%           |
| 1 (10 frames)   | 62.61%     | 65.64%        | 67.53%       | 67.39%           |
| 3 (10 frames)   | 66.53%     | 69.31%        | 73.21%       | 72.92%           |
| 8 (10 frames)   | 72.17%     | 73.36%        | 78.33%       | 77.97%           |
| 16 (10 frames)  | 74.83%     | 75.44%        | 79.97%       | 79.63%           |
| 20 (10 frames)  | 75.51%     | 76.23%        | 80.54%       | 80.22%           |
| 30 (10 frames)  | 76.60%     | 77.15%        | 80.98%       | 80.70%           |
| 50 (10 frames)  | 77.41%     | 77.79%        | 81.52%       | 81.24%           |

Figure 4.3: Accuracy of features signs estimation when the signs computed on the testing set are considered to be the ground-truth (the testing set size is bigger than 100K and it is more probable to predict the signs correctly when the samples are more numerous). Note that the signs are better estimated when the features are stronger (as it happens in the case of type III and types I + II + III features), but the accuracy is quite high in all the four cases. The first value corresponds to 1-shot-1-frame case (please also see Table 4.3).
Table 4.4: Sign estimation accuracy for each class for the features selected by our algorithm when using 16 labeled shots with 10 labeled frames each. Results are averages of 30 different experiments.

| Locations | Feats. I | Feats. I+II | Feats. III | Feats. I+II+III |
|-----------|----------|-------------|------------|-----------------|
| Aeroplane | 99.00%   | 100.00%     | 100.00%    | 100.00%         |
| Bird      | 93.17%   | 85.33%      | 64.83%     | 63.87%          |
| Boat      | 92.67%   | 67.50%      | 77.67%     | 75.47%          |
| Car       | 90.67%   | 80.17%      | 60.67%     | 61.73%          |
| Cat       | 92.00%   | 93.83%      | 88.50%     | 86.00%          |
| Cow       | 90.50%   | 92.83%      | 70.17%     | 74.67%          |
| Dog       | 91.33%   | 96.50%      | 89.33%     | 88.47%          |
| Horse     | 92.67%   | 96.17%      | 88.67%     | 85.53%          |
| Motorbike | 90.17%   | 87.00%      | 71.83%     | 78.33%          |
| Train     | 96.00%   | 93.67%      | 81.50%     | 80.53%          |
| Mean      | 92.82%   | 89.30%      | 79.32%     | 79.46%          |

Methods comparison: In Fig. 4.4 we compare our supervised and unsupervised approaches with other methods used for feature selection. We notice that the results of ours-unsup$^2$ are better than those for all the other methods. Table 4.5 shows the results obtained with regularized methods like: Lasso and Elastic Net compared to the results obtained with our supervised method. Notice how our algorithm performs much better when the number of features is higher, while Elastic Net performs slightly better for the first set of features that are the least numerous. This suggests that our algorithm manages to better select from a higher number of features, which is quite encouraging because feature selection is more acutely needed when the number of features is very big. We tested Lasso and Elastic Net with different values of parameter $\lambda$ for each of the four subsets of features, and we chose the value for which we obtained the best results in the case of eight shots. The value of $\lambda$ is the same for Elastic Net and Lasso for the same subset of features.

The results shown in Fig. 4.5 prove that our supervised method is much faster than the other methods, it has a constant training time and also it outperforms them in accuracy, even SVM in many cases. The difference is more prominent when the number of shots is smaller. In the unsupervised case, we added unlabeled training data. Here, we outperformed all other methods by a very large margin, up to over 20% (Table 4.6 and Fig. 4.4). We tested with different amounts of unlabeled data. While being almost insensitive to the number of labeled shots, (used only to estimate the feature signs), performance improved as more unlabeled data was added. Of particular note is when only a single labeled image per class was used to estimate the feature signs, with all the other data being unlabeled (Fig. 4.4).

Our method also exhibits good generalization as we can notice in Fig. 4.6, where training vs testing accuracy are plotted. This experiment is performed on the supervised variant of our algorithm. We analysed the evolution of the testing accuracy with respect to the training accuracy in order to avoid overfitting. The size of the pool of features from which we choose a subset is very important when it comes to accuracy as we can see in Table 4.7, this experiment is also done on the supervised method. In this experiment we wanted to emphasize that the performance of our algorithm increases with the number of features used.
Unsupervised learning: We also evaluate how testing accuracy is influenced by the quantity of unlabeled data used for learning. In order to assess this aspect, we trained our unsupervised algorithm on different quantities of unlabeled data and we realized that, as expected, the accuracy increases with this quantity, but the variation is not so high. Once more than 25% of the unsupervised data is provided, the accuracy reaches a plateau. These results are summarized in Table 4.10 and Fig. 4.7.

In Table 4.8 we show how testing accuracy varies among classes. For this experiment we used 16 shots with 10 labeled frames per class to compute the signs of the features; we used features of types I, II and III. Note that the accuracy is generally higher for classes designing human-made objects like: boat, aeroplane, car, while for natural classes as dog, cat, horse the recognition accuracy is smaller. The difference in accuracy might be explained by the fact that the videos for classes like boat, aeroplane and bird for which the accuracy is very high are rather static, the changes of the background and of the object itself are reduced, while dogs, cats, and horses are more dynamic and the recognition task becomes more difficult. Especially for videos in classes aeroplane and boat the foreground is very uniform.

For the unsupervised case we also performed experiments in which the unlabeled training set contained distinct examples than those used for testing, because we wanted to see what is the influence of the unlabeled set on the results. We considered four cases that we present in Table 4.9: 1) same frames for unsupervised training and testing, 2) different frames for unsupervised training and testing, 3) different shots for unsupervised training and testing, 4) different videos for unsupervised training and testing.
Figure 4.5: Accuracy and training time on YouTube-Objects, with varying training video shots (10 frames per shot and results averaged over 30 runs). Input feature pool, row 1: 50 type I features on CIFAR10; row 2: 110 type II features on YouTube-Parts + 50 type I features on CIFAR10; row 3: 2000 type III features in ImageNet; row 4: 2160 all features. Ours outperforms SVM, Lasso, AdaBoost and FoBa.
Table 4.5: Recognition accuracy for Lasso, Elastic Net and our supervised method.

(a) Features of type I

| Feats. I | Ours supervised | Lasso | Elastic Net (α = 0.5) |
|----------|-----------------|-------|-----------------------|
| 1 shot   | 17.83%          | 17.70%| 18.18%                |
| 3 shots  | 21.98%          | 22.73%| 22.76%                |
| 8 shots  | 29.12%          | 30.22%| 30.25%                |
| 16 shots | 29.75%          | 30.70%| 30.72%                |

(b) Features of types I + II

| Feats. I | Ours supervised | Lasso | Elastic Net (α = 0.5) |
|----------|-----------------|-------|-----------------------|
| 1 shot   | 36.71%          | 32.67%| 33.59%                |
| 3 shots  | 46.63%          | 42.89%| 44.00%                |
| 8 shots  | 51.06%          | 48.33%| 48.75%                |
| 16 shots | 51.94%          | 48.90%| 49.02%                |

(c) Features of type III

| Feats. I | Ours supervised | Lasso | Elastic Net (α = 0.5) |
|----------|-----------------|-------|-----------------------|
| 1 shot   | 49.58%          | 27.95%| 28.31%                |
| 3 shots  | 62.78%          | 45.83%| 46.37%                |
| 8 shots  | 69.16%          | 56.14%| 56.39%                |
| 16 shots | 70.23%          | 59.31%| 59.33%                |

(d) Features of type I + II + III

| Feats. I | Ours supervised | Lasso | Elastic Net (α = 0.5) |
|----------|-----------------|-------|-----------------------|
| 1 shot   | 52.39%          | 27.57%| 28.90%                |
| 3 shots  | 64.52%          | 45.81%| 46.66%                |
| 8 shots  | 71.21%          | 54.84%| 55.77%                |
| 16 shots | 72.66%          | 59.69%| 60.72%                |

Table 4.6: Experiments on our unsupervised learning. Improvement in recognition accuracy over our supervised method, by using unsupervised learning with unlabeled test data. Feature signs are learned using the same (1, 3, 8, 16) shots as in the supervised case. Note that the first column presents the one-shot learning case, when the unsupervised method uses a single labeled image per shot and also per class. Results are averages over 30 random runs.

| Training # shots | 1     | 3     | 8     | 16    |
|------------------|-------|-------|-------|-------|
| Feature I        | +15.1%| +15.2%| +12.6%| +12.6%|
| Feature I+II     | +16.7%| +10.4%| +6.4% | +6.2% |
| Feature III      | +23.6%| +11.3%| +5.1% | +3.7% |
| Feature I+II+III | +24.4%| +13.5%| +6.9% | +5.3% |

results are as expected because the accuracy is higher when the frames used for the unsupervised learning are also used for testing, and it decreases when the samples used for learning and testing are more distinct.
Figure 4.6: Our method shows a good generalization power. It quickly learns subsets of features that are more powerful on the testing set than when combined with SVM or SVM-alone. Note that, in our case, the training and testing errors are relatively closer to each other than for the competitors.

Table 4.7: Recognition accuracy of our supervised method on Youtube-Objects dataset, using 10 training shots (first row) and 20 training shots (second row), as we combine features from several datasets. The accuracy increases significantly (it doubles) as the pool of features grows and becomes more diverse. Features are: type I: CIFAR; types I+II: CIFAR + Youtube-Parts; types I+II+III: CIFAR + Youtube-Parts + Imagenet (6000).

| Accuracy   | Feats. I (50) | Feats. I+II (160) | Feats. I+II+III (6160) |
|------------|---------------|-------------------|------------------------|
| 10 train shots | 29.69%        | 51.57%            | 69.99%                 |
| 20 train shots | 31.97%        | 52.37%            | 71.31%                 |

Table 4.8: Mean accuracy per class, over 30 random runs of unsupervised learning with 16 labeled training shots.

| Mean accuracy per class (%) | Aero-plane | Bird | Boat | Car | Cat | Cow | Dog | Horse | Motor-bike | Train |
|-----------------------------|------------|------|------|-----|-----|-----|-----|-------|------------|-------|
|                             | 91.53      | 91.61| 99.11| 86.67| 70.02| 78.13| 61.63| 53.65  | 72.66      | 83.37 |

The frames in the same shot are very similar to each other, and the shots within the same video are more similar than the shots coming from different videos. However, the performance is still good even in the most difficult case, when the examples used for training are actually extremely different from those used for testing.
Table 4.9: Recognition accuracy for our unsupervised method for four cases: 1) same frames used for unsupervised learning and for testing, 2) different frames for unsupervised learning and for testing, 3) different shots used for unsupervised learning and for testing and 4) different videos used for unsupervised learning and for testing. In the first three cases we used the testing dataset for the unsupervised learning, while in the fourth case we used only the training set for the unsupervised learning, therefore the videos are different in training and testing.

(a) Features of type I

| No. of shots | Same frames | Diff. frames | Diff. shots | Diff. videos |
|--------------|-------------|--------------|-------------|--------------|
| 3 shots      | 37.24%      | 38.89%       | 27.18%      | 30.66%       |
| 8 shots      | 41.79%      | 43.11%       | 29.29%      | 34.21%       |
| 16 shots     | 42.41%      | 43.79%       | 27.65%      | 33.54%       |

(b) Features of types I + II

| No. of shots | Same frames | Diff. frames | Diff. shots | Diff. videos |
|--------------|-------------|--------------|-------------|--------------|
| 3 shots      | 57.01%      | 56.76%       | 52.96%      | 50.02%       |
| 8 shots      | 57.49%      | 57.29%       | 53.89%      | 50.07%       |
| 16 shots     | 58.11%      | 58.03%       | 54.01%      | 50.05%       |

(c) Features of type III

| No. of shots | Same frames | Diff. frames | Diff. shots | Diff. videos |
|--------------|-------------|--------------|-------------|--------------|
| 3 shots      | 74.04%      | 73.95%       | 56.66%      | 55.60%       |
| 8 shots      | 74.25%      | 74.05%       | 57.86%      | 55.33%       |
| 16 shots     | 73.87%      | 73.50%       | 58.29%      | 55.42%       |

(d) Features of types I + II + III

| No. of shots | Same frames | Diff. frames | Diff. shots | Diff. videos |
|--------------|-------------|--------------|-------------|--------------|
| 3 shots      | 78.02%      | 77.95%       | 61.24%      | 62.44%       |
| 8 shots      | 78.13%      | 77.96%       | 61.28%      | 62.11%       |
| 16 shots     | 77.93%      | 77.79%       | 61.24%      | 62.11%       |

For a better understanding of the performance of our unsupervised method, we present in Figs. 4.8 and 4.9 for each of the 10 categories of objects images that were classified correctly and incorrectly. For each class the proportion of correct and incorrect examples is consistent with the recognition accuracy per class. These results are obtained when testing our one-shot learning algorithm (only one labeled example is used per class). Note that we considered all frames in a video shot as belonging to a single category - even though sometimes a significant amount of frames did not contain any of the above categories. Therefore, often our results look qualitatively better than the quantitative evaluation.

**Signs transfer:** Another idea that we investigated during our experiments was the possibility to transfer the signs from a category to another. We computed the signs of the features for class *cat* and used them also for class *dog*. This would be a very useful idea if we have some classifiers already learnt and we want to learn a new category for which we do not have labeled images. Then we can take the signs from one of
Table 4.10: Testing accuracy when varying the quantity of unlabeled data used for unsupervised learning. We used 8 video shots per class with 10 frames each, for estimating the signs of the features. Results are averages over 30 random runs.

| Unsupervised data | Feats. I | Feats. I+II | Feats. III | Feats. I+II+III |
|-------------------|---------|------------|------------|-----------------|
| Train + 0% test $S_A$ | 30.86%  | 48.96%     | 49.03%     | 53.71%          |
| Train + 25% test $S_A$ | 41.26%  | 55.50%     | 66.90%     | 72.01%          |
| Train + 50% test $S_A$ | 42.72%  | 56.66%     | 71.31%     | 76.78%          |
| Train + 75% test $S_A$ | 42.88%  | 57.24%     | 73.65%     | 77.39%          |
| Train + 100% test $S_A$ | 43.00%  | 57.44%     | 74.30%     | 78.05%          |

Figure 4.7: Testing accuracy vs amount of unlabeled data used for unsupervised learning. Note that the performance almost reaches a plateau after a relatively small fraction of unlabeled frames are added.

...
Table 4.11: Mean binary accuracy per class, over 30 random runs of unsupervised learning with 8 labeled training shots. We tested three cases: 1) the signs are computed on each class, 2) the signs are borrowed from another very similar class, 3) the signs are borrowed from a very dissimilar class. Results are obtained on two subsets of features. See also Table 4.12a for the classes considered similar / dissimilar.

(a) Features of types \(I + II\)

| Class name | Its own sign | From sim. class | From dissim. class |
|------------|--------------|-----------------|-------------------|
| Aeroplane  | 96.85        | 84.93           | 87.77             |
| Bird       | 96.73        | 94.53           | 94.58             |
| Boat       | 98.34        | 88.34           | 87.50             |
| Car        | 97.53        | 96.90           | 96.81             |
| Cat        | 92.30        | 92.23           | 92.08             |
| Cow        | 94.16        | 94.14           | 94.13             |
| Dog        | 80.79        | 80.73           | 81.26             |
| Horse      | 88.95        | 88.99           | 80.72             |
| Motorbike  | 95.10        | 95.10           | 95.10             |
| Train      | 93.35        | 87.33           | 87.06             |
| Mean       | 93.41        | 90.32           | 89.70             |

(b) Features of types \(I + II + III\).

| Class name | Its own sign | From sim. class | From dissim. class |
|------------|--------------|-----------------|-------------------|
| Aeroplane  | 96.63        | 91.73           | 93.01             |
| Bird       | 99.01        | 97.68           | 94.59             |
| Boat       | 97.43        | 98.29           | 93.25             |
| Car        | 98.67        | 98.47           | 97.73             |
| Cat        | 94.48        | 94.66           | 93.12             |
| Cow        | 95.80        | 95.87           | 94.21             |
| Dog        | 84.87        | 82.13           | 80.88             |
| Horse      | 92.05        | 91.03           | 81.26             |
| Motorbike  | 98.09        | 97.67           | 97.19             |
| Train      | 94.28        | 92.26           | 87.82             |
| Mean       | 95.13        | 93.97           | 91.30             |

Table 4.12: The classes from which we borrowed the signs in the experiments with the sign transfer.

(a) The classes that we considered similar / dissimilar in order to borrow the feature signs from.

| Class name | Very sim. class | Very dissim. class |
|------------|-----------------|-------------------|
| Aeroplane  | Bird            | Train             |
| Bird       | Aeroplane       | Train             |
| Boat       | Aeroplane       | Cat               |
| Car        | Motorbike       | Bird              |
| Cat        | Dog             | Boat              |
| Cow        | Horse           | Cat               |
| Dog        | Cat             | Boat              |
| Horse      | Cow             | Aeroplane         |
| Motorbike  | Car             | Cat               |
| Train      | Car             | Cow               |

(b) Classes chosen to borrow the feature signs from for the experiments in which we kept 6 original signs and 4 original signs.

| Class name | 6 orig. signs | 4 orig. signs |
|------------|---------------|---------------|
| Aeroplane  | Aeroplane     | Aeroplane     |
| Bird       | Bird          | Aeroplane     |
| Boat       | Aeroplane     | Aeroplane     |
| Car        | Car           | Car           |
| Cat        | Cat           | Cat           |
| Cow        | Cow           | Horse         |
| Dog        | Cat           | Cat           |
| Horse      | Cow           | Horse         |
| Motorbike  | Car           | Horse         |
| Train      | Train         | Car           |
Figure 4.8: Lists of ten classified frames per category, for which the ratio of correct to incorrect samples matches the mean class recognition accuracy.

of the classes based on the percent of feature signs that coincide for each pair of classes. We try to find a more objective criterion (a numerical one) in order to decide which classes are similar and which are not. In Fig. 4.10 we show for each class how similar it is to all classes in the dataset. We can notice that the similarities computed in this way are quite intuitive and the more and stronger features we have, the more intuitive the similarities found are. The similarities computed with the fourth subset of features (types I +
II + III) are better than those found only with features of type I. Let us focus on the last case considered and look at the classes to which the similarity is $> 0.5$; for aeroplane: boat, motorbike, bird, train, for bird: cat, dog, motorbike, aeroplane, cow, for boat: train, car aeroplane, for car: train, boat, motorbike, for cat: dog, bird, cow, horse, for cow: horse, dog, cat, bird, for dog: cow, horse, cat, bird, for horse: cow, dog, cat, for motorbike: aeroplane, bird, car, for train: car, boat, aeroplane. We can remark the fact that generally the classes that designate animals are similar to each other, while classes related to transportation (which are also human-made) are more similar between them according to the signs of the features. This result is not at all surprising, we expected that the categories that are semantically related to be more similar than those that are not. It would have been counterintuitive to obtain that the train is similar to the cat, for example.
Table 4.13: Multiclass accuracy for 3 settings: 1) with the original signs, 2) with 6 original signs, 3) with 4 original signs. See also Table 4.12b for the classes from which the signs are borrowed.

(a) Features of type I

| No. of labeled shots | Multiclass accuracy |          |          |          |
|----------------------|---------------------|----------|----------|----------|
|                      | All orig. signs     | 6 orig. signs | 4 orig. signs |
| 1                    | 34.67%              | 35.14%    | 34.28%    |
| 3                    | 38.79%              | 37.37%    | 34.06%    |
| 8                    | 43.06%              | 40.05%    | 34.91%    |
| 16                   | 43.67%              | 40.45%    | 35.38%    |

(b) Features of types I + II

| No. of labeled shots | Multiclass accuracy |          |          |          |
|----------------------|---------------------|----------|----------|----------|
|                      | All orig. signs     | 6 orig. signs | 4 orig. signs |
| 1                    | 54.95%              | 54.14%    | 51.40%    |
| 3                    | 57.01%              | 55.67%    | 51.02%    |
| 8                    | 57.49%              | 55.25%    | 50.32%    |
| 16                   | 58.11%              | 55.77%    | 50.25%    |

(c) Features of type III

| No. of labeled shots | Multiclass accuracy |          |          |          |
|----------------------|---------------------|----------|----------|----------|
|                      | All orig. signs     | 6 orig. signs | 4 orig. signs |
| 1                    | 73.52%              | 72.46%    | 72.66%    |
| 3                    | 74.04%              | 72.46%    | 71.96%    |
| 8                    | 74.25%              | 72.12%    | 73.32%    |
| 16                   | 73.87%              | 72.38%    | 73.08%    |

(d) Features of types I + II + III

| No. of labeled shots | Multiclass accuracy |          |          |          |
|----------------------|---------------------|----------|----------|----------|
|                      | All orig. signs     | 6 orig. signs | 4 orig. signs |
| 1                    | 76.86%              | 76.08%    | 75.95%    |
| 3                    | 78.02%              | 76.01%    | 75.61%    |
| 8                    | 78.13%              | 76.34%    | 76.19%    |
| 16                   | 77.93%              | 76.29%    | 75.93%    |

4.2 MNIST experiments

In order to assess more accurately the performance of the algorithm that we developed we have tested it on MNIST dataset (containing images with digits). For these experiments we made a small change to the algorithm. The data are normalized so that they have the mean equal to 0 and the standard deviation equal to 1. Therefore, the values are not anymore between 0 and 1, they might also be negative and not subunitary. We noticed that when we flip the features it would be better to use $-f$ instead of $1 - f$ as we did before. This new way of flipping the features is used in the MNIST experiments. We show in Fig. 4.11 the classifiers chosen for each class. We represented in black the negatively correlated features (before flipping, because after flipping all features are positively correlated), in white the positively correlated features (before flipping) and in grey the features that were not chosen. The number of classifiers chosen was $k = 400$. The number of labeled images per class used for learning the signs was 2000. We can notice that the classifiers chosen are those from the center of the image. The positively correlated ones are precisely those that represent the shape of the digit, while the negatively correlated are around them and emphasize the shape of the digit.

The multiclass recognition accuracies obtained by our algorithm on the MNIST dataset are found in Table 4.14. We learnt the signs of the features on different numbers of images per class, ranging from one image per class, up to all (around 6000) images per class. We also present the results obtained when we use instead of the whole image, only its central part. Even though the number of features is halved in this case, the accuracy when the center is used nears the accuracy obtained with the whole image when the number of labeled examples increases, although for 1 labeled image the accuracy when only the center is
Figure 4.10: Youtube-Objects classes similarity based on the signs of the features. For each pair of features we present the percent of signs of features that coincide. The higher this percent is, the higher the similarity is.

In Fig. 4.12 we showed the testing accuracy of our unsupervised algorithm for two different settings: 1) the unsupervised learning was done on the testing set, 2) the unsupervised learning was done on the training set. We can notice that the difference in accuracy between the two is extremely small, this means that the algorithm can generalize very well and the power of the classification method does not necessarily come from the testing examples used during the unsupervised learning. We have also performed an experiment that assesses the similarity between the ten classes (digits) by evaluating the percent of signs that coincide between each pair of classes. In Fig. 4.13 we show the level of the sign coincidence for each pair of digits.

used is half the accuracy with the whole image.
Figure 4.11: We represented the classifiers chosen in white and black: white for the positively correlated, black for negatively correlated and grey for those not chosen.

Table 4.14: Mean multiclass accuracy for unsupervised learning on MNIST dataset for 30 random experiments. We varied the number of labeled images on which we learnt the signs, and we used the whole testing set for unsupervised learning.

| No. of labeled images | Whole image | Center |
|-----------------------|-------------|--------|
| 1                     | 64.36%      | 32.93% |
| 8                     | 74.75%      | 58.88% |
| 16                    | 75.70%      | 62.26% |
| 64                    | 76.57%      | 65.89% |
| 128                   | 76.63%      | 66.51% |
| 512                   | 76.63%      | 67.19% |
| 1024                  | 76.61%      | 66.90% |
| 2048                  | 76.68%      | 67.07% |
| 6000                  | 76.57%      | 67.80% |
Figure 4.12: Recognition accuracy for the unsupervised learning algorithm when: a) the testing set is used for unsupervised learning, b) the training set only is used for the unsupervised learning.

Figure 4.13: Digits similarity in MNIST dataset based on the percent of feature signs that coincide among them.
Chapter 5

Conclusions and future work

These last sections are reserved to the final discussions regarding the results presented in the previous chapter and the whole method. We will make a higher level analysis of the results. Then we draw some conclusions about our entire work, and finally, we present briefly how we intend to continue our research.

5.1 Discussion

**Discussion on (almost) unsupervised learning:** We demonstrated that our approach is able to learn superior classifiers in the case when no data labels are available, but only the signs of features are known. In our experiments, we only used minimal data to estimate these signs. Once they are known, any amount of unlabeled data can be incorporated. This aspect reveals a key insight: being able to label the features, and not the data, is sufficient for learning. For example, when learning to separate oranges from cucumbers, if we knew the features that are positively correlated with the “orange” class (roundness, redness, sweetness, temperature, latitude where image was taken) in the immense sea of potential cues, we could then employ huge amounts of unlabeled images of oranges and cucumbers, to find the best relatively small group of such features. Also note that since only a small number of images are used for estimating the feature signs (as few as one per class), some signs may be wrong. However, the very weak sensitivity of the method to the number of labeled training samples strongly indicates that it is robust to noise in sign estimation, as long as most of the features are correctly oriented.

**Discussion on the selected features:** We have noticed some surprising ways in which the class of a frame in Youtube-Objects is associated with a series of classes in ImageNet. There are different ways in which these associations are done:

1. similarity of the global appearance of the two objects, but no semantic relation: eg. train vs banister, tigershark vs. plane, Polaroid camera vs. car, scorpion vs. motorbike, remote control vs. cat’s face, space heater vs. cat’s head.

2. co-occurrence and similar context: helmet vs. motorbike

3. part-to-whole object relation: grille, mirror and wheel vs. car

4. combinations of the previous: dock vs. boat, steel bridge vs. train, albatross vs. plane.
Figure 5.1: For each training target class from Youtube-Objects videos (labels on the left), we present the most frequently selected ImageNet classifiers (input features), over 30 independent experiments, with 10 frames per shot and 10 random shots for training. In images we show the classes that were always selected by our method when \( k = 50 \). On the right we show the probability of selection for the most important 50 features together with other relevant classes and their frequency of selection presented as a list. Note how stable the selection process is and how related (but not identical) the selected classes are in terms of appearance, context or geometric part-whole relationships, to work robustly together as an ensemble. We find two aspects indeed surprising: 1) the high probability (perfect 1) of selection of the same classes, even for such small random training sets and 2) the fact that unrelated classes, in terms of meaning, could be so useful for classification, based on their surprising shape and appearance similarity.
Another observation would be the fact that some of the classes play a role of borders between the positive class and the others. This ensures the separation between the main class and the neighbouring classes. Another benefit is the fact that although there is no classifier for a certain class, it manages to learn how to distinguish this class from the others by using together other existent classes that are similar to it. For example, even though in ImageNet there is not a “cow” class, it learns the new concept from the ones that are available. In order to support our claims we show in Figure 5.1 for each class in Youtube-Objects the classes from ImageNet whose weights were the biggest, which means that they mattered more. We can notice that many selected classes are similar in appearance to the positive class, this is most visible in the case of the aeroplane class, while for other classes the resemblance is also at the semantic level, not only in appearance.

5.2 Conclusion

We present a fast feature selection and learning method that requires minimal supervision, with strong theoretical properties and excellent generalization and accuracy in practice. The crux of our approach is its ability to learn from unlabeled data once the feature signs are determined. Our contribution could open doors for new and exciting research in machine learning, with practical and theoretical impact. Both our supervised and unsupervised approaches can quickly learn from limited data and identify sparse combinations of features that outperform powerful methods such as SVM, AdaBoost, Lasso and greedy sequential selection — in both time and accuracy. With a formulation that permits very fast optimization and effective learning from large heterogeneous feature pools, our approach provides a useful tool for many recognition tasks, suited for real-time, dynamic environments. Our work complements much of the machine learning research on developing new, more powerful, classifiers. While this thesis has primarily demonstrated the effectiveness of our feature combinations in a specific context, our methods are general and could be used in conjunction with any machine learning algorithm.

We tested the method on a difficult video dataset and also showed that knowledge transfer is possible between datasets with very different characteristics: starting from different object classes, to different image quality and positioning of the target object. The method needs very limited labeled data for computing the signs of the features (whether they are positively or negatively correlated). It manages to compute the signs quite well even when only one frame per class is presented. And the method can handle successfully high quantities of unlabeled data. Moreover, after a percent of the unlabeled data are presented to the algorithm, the recognition accuracy reaches a plateau which means that even fewer examples are enough to learn. Either the supervised and the unsupervised approaches are better than most of the methods mentioned above.

The proposed method has strong theoretical properties; it guarantees the sparsity of the solution, all features have the same contribution and together, they sum to 1. The original supervised formulation was a convex optimization problem with a global minimum, while the unsupervised formulation is a concave problem, more difficult to solve, only with local minima.

Even though our algorithm is a standalone feature selection method, it can also be used in combination with other machine learning methods, an example could be to combine it with SVM: apply SVM only on the selected features by our method, as we did in our experiments.
5.3 Future work

In the next steps of this work we intend to apply our method on new datasets. We can also apply our feature selection method for different problems, because this is a general approach which is not designed especially for object recognition. Moreover we want to try to combine it with some neural networks to use features obtained on different levels of the networks and feed them to our feature selection algorithm. We also take into consideration using other features more video-oriented, like motion. Until now, we used only features that could have been applied also on images. We might also improve our prediction by taking into consideration the fact that some frames come from the same shot and take the class predicted in the majority of the frames as the class of the given shot.

Another idea would be to create an unsupervised hierarchy starting from our unsupervised variant of the algorithm. We want to add a new level to this algorithm by creating other features. We can consider regions of images that contain a pattern built from the pixels already chosen in the previous stage on which we can apply functions like max, min, mean and create new features. We can also make a local search around the centers of these regions because maybe the pattern contained in the current region responds better if it is shifted a few pixels. The values obtained by applying the functions mentioned on these regions might be considered higher level features that can be used either in parallel with the old ones, or separately. We need to optimize some parameters that characterize these features: the size of the region and the distance that we look around for a better position. We choose the centroids of these regions using our unsupervised algorithm, thus we create the new level of unsupervised learning. We are currently working on this idea, but it requires more investigation.
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