Synchronous characteristics of two excited motors in an anti-resonance system

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Abstract
The aim of this paper is to research the synchronization of two excited motors in an anti-resonance system. After establishing the dynamic equations of the system by Lagrange equation, the displacement responses of the system and the ability of vibration isolation are determined. And the synchronous condition and synchronous stability criterion of the system are derived by using the average small parameter method and Hamilton's principle, respectively. Moreover, the influences of the structural parameters on synchronous characteristics between the rotors are discussed by numerical analysis. Then, the correctness of theoretical analysis is verified by numerical simulations. The research results show that, the ability of vibration isolation is mainly influenced by frequency ratios of the system; the synchronous ability of the system is improved with the increase of the distance between two motors; meanwhile, the synchronous behavior of the system is closely related to the mounting position and rotation direction between the motors. The research result can provide theory guidance for designing the anti-resonance drilling fluid shakers.

Keywords: Synchronization, Stability, Anti-resonance, Dynamics, Numerical simulations

1. Introduction

The phenomenon of synchronization commonly exists in our daily life, such as synchronization of neural network, synchronization between coupled pendulum and synchronization of the rotors (Peña Ramirez et al., 2013; Fang et al., 2015). In the fact, the phenomenon of synchronization is taken regard as an adjustment of frequency owing to internally coupling motion of system. The history of synchronization dates from the Huygens discovery of synchronous motion between two pendulum clocks hung in a removable beam. In 1950, Signal and Swedish inventor, applied a patent for the simplest self-synchronous vibration machine. Later, professor Blekman studied the stability of two synchronously operated exciters by the Poincare-Lyapunov method, and gave the definition of synchronization from the view of kinematics and dynamics (Blekman et al., 1988, 1995). Sperling et al discussed a synchronous problem of two-plane automatic balancing of a rigid rotor by numerical method, and described ‘Sommerfeld effect’ of the system in special situation (Sperling et al., 2002). Balthazar et al gave comments on self-synchronization of two and four non-ideal exciters in the near-resonant region (Balthazar et al., 2004). Wen used the average method to deduce synchronous conditions and synchronous stability for rotor systems, and introduced synchronization theory to design many self-synchronous vibrating machines (Wen et al., 2009). Then, Zhao proposed a revised small parameter algorithm to determine the synchronous behavior of dual-exciters systems (Zhao et al., 2010). Based on Zhao’s method, Zhang studied synchronous mechanism of the vibrating system driven by double-motor and multi-motor in far resonance systems, and the research results show the better the symmetry of vibrating system, the easier the synchronization of the system (Zhang et al., 2013, 2015, 2017). Besides, Min studied self-synchronous characteristics of vibration system with dual-frequency and dual-motor excitation, and found that synchronization of the system will be greatly influenced with the distance from the rotating center of the motor to the centroid of supporting body (Min et al., 2018). Fang proposed a rotor-pendulum system, and discussed the synchronization of the system in the different resonances (Fang and Hou, 2018). In light of active control strategy, Kong realized synchronous control of three and four eccentric rotors by using a sliding mode algorithm (Kong et al., 2016, 2018). The above scholars mainly considered synchronous mechanism and synchronous control theory for self-synchronous vibrating machines, but the self-synchronous theory in an anti-resonance system are less reported.

At present, the awareness of safety and efficiency requirement of production is gradually improved in our daily life, how to decrease the negative influence produced by vibrating equipment is extremely urgent. For example, drilling fluid shaker in petroleum drilling engineering are high-speed vibrating equipment, and the exciting force formed by drilling
fluctuation would transmit to foundation, which not only accelerates the fatigue damage of equipment on the foundation, but also generates huge noise and endangers people’s physical and mental health. Therefore, vibration isolation becomes vitally important for the similar machines. Scholars proposed some methods for vibration isolation, such as the vibration isolation of multi-body and the anti-resonance to implement vibration isolation (Wang et al., 2014; Li et al., 2014; Xiong et al., 2018), and the method of anti-resonance has excellent ability of vibration isolation. Liu proposed an anti-resonance vibrating system, and discussed the effect of material fluctuation on the amplitude stability of the system. Finally, his research indicated that controlling excitation frequency is in favor of vibration absorption (Liu Jie et al., 2006). Later, Liu proved that the synchronization and synchronous transmission between the motors can be realized in the anti-resonance machines (Liu Jintao et al., 2015), but the influences of spring stiffness of the system, mounting position of two motors and rotation direction of the rotors on synchronous characteristics are ignored.

In this present work, an anti-resonance model excited by two motors is proposed, which has many advantages compared with traditional vibrating machines, such as excellent vibration isolation effect, decreasing mass of the whole system and energy saving. The aim of this paper is to research the synchronization of this model. This paper consists of the following parts: in section 2, the dynamics equation of the system is established. In section 3, the synchronous behavior of two rotors is described, and the ability of vibration isolation in the system is discussed. In section 4, synchronous state between two rotors is revealed with numerical computation. In section 5, computer simulations are implemented to demonstrate the availability of theoretical analysis. In section 6, some important conclusions are summarized.

2 Dynamic model

As shown in Fig.1, a new anti-resonance system of two excited motors is given, which consists of the rigid vibrating body, the isolation body, two excitation motors and some springs. The two identical motors are symmetrically mounted on the isolation body. The isolation body is connected by vibrating body and foundation through linear damping springs with different stiffness coefficients \( k_i \) and damping coefficients \( f_i \) in \( i \)-direction \( (i = x_1, x_2, y_1, y_2, \psi_1, \psi_2) \). The distance between the pivot of the motor and the centroid of the isolation body is expressed by \( l \) \( \beta_1 \) and \( \beta_2 \) represent installed angle of motor 1 and 2, respectively. The rotational centers of two rotors are denoted by \( \alpha_{1r} \) and \( \alpha_{2r} \), respectively, and their rotating radiuses are identical, denoted by \( r \). The centroid of the vibrating body and isolation body is described by \( o'_1 \) and \( o'_2 \), respectively. And the phases of rotors are denoted by \( \phi_i, i = 1, 2 \). \( \sigma \) represents the rotating direction of two rotors. The two rotors rotate in the same direction when \( \sigma = -1 \), and the two rotors rotate in the opposite direction when \( \sigma = 1 \). The transformation of reference coordinate is shown in Fig.1 (b), and rotation sequence of the reference coordinate is followed by \( (o'_1x'_1y'_1) \rightarrow (o'_1x'_2y'_2) \rightarrow (\alpha_1x_1y_1), i = 1, 2 \). \( \alpha_{1x_1y_1} \) is established by the centroid of the isolation body, defined as the global frame, and the auxiliary coordinate frame \( o'_1x'_1y'_1 \) is established by the centroid of the vibrating body.

Fig.1 Mechanical model of the anti-resonance system. (a) the simple physical model. (b)the reference frame of system.

In reference frame \( o''_1x''_1y''_1 \), the coordinates \( X'_1 \) and \( X'_2 \) of centroid of two unbalanced rotors can be separately expressed as:
\[
X_1^* = \begin{pmatrix}
-l \cos \beta_1 - r \cos \sigma \phi_i \\
l \sin \beta_1 + r \sin \sigma \phi_i
\end{pmatrix} \\
X_2^* = \begin{pmatrix}
-l \cos \beta_2 + r \cos \sigma \phi_i \\
l \sin \beta_2 + r \sin \sigma \phi_i
\end{pmatrix}
\] (1)

In reference frame \( \alpha_x, \alpha_y \), the coordinates \( X_j \) \( (j = 1, 2) \) of centroid of two rotors can be separately written as:

\[
X_j = X_0 + RX_j^* = \begin{pmatrix}
\cos \psi_j \\
-\sin \psi_j \\
\sin \psi_j \\
-\cos \psi_j
\end{pmatrix} \cdot X_0 = \begin{pmatrix}
x_2 \\
y_2
\end{pmatrix}, \quad j = 1, 2
\] (3)

In this case, the kinetic energy \( T \) of the vibration system can be obtained by:

\[
T = \frac{1}{2} \sum_{i=1}^2 M_i \left( \dot{x}_i^2 + \dot{y}_i^2 \right) + \frac{1}{2} \sum_{i=1}^2 J_i \dot{\phi}_i^2 + \sum_{i=1}^2 m_i \dot{X}_i \dot{X}_i
\] (4)

where, \( M_1 \) and \( M_2 \) are the mass of the vibrating body and the isolation body, respectively; \( m_j \) \( (j = 1, 2) \) is the mass of the two rotors; \( J_i \) \( (i = 1, 2) \) is rotational inertia of the vibration body and the isolation body, respectively; \( J_m \) \( (i = 1, 2) \) is rotational inertia of the two rotors; \( \psi_i \) \( (i = 1, 2) \) is angular displacement of the vibration body and isolation body on oscillating.

Meanwhile, the potential energy \( V \) of the vibration system can be obtained by:

\[
V = \frac{1}{2} k_{x_1} \dot{x}_1^2 + \frac{1}{2} k_{y_1} \dot{y}_1^2 + \frac{1}{2} k_{x_2} \dot{x}_2^2 + \frac{1}{2} k_{y_2} \dot{y}_2^2 + \frac{1}{2} k_{x_1} (x_1 - x_2)^2 + \frac{1}{2} k_{y_1} (y_1 - y_2)^2
\] (5)

Moreover, the dissipation energy \( D \) of the whole system can be described as:

\[
D = \frac{1}{2} f_{x_1} \dot{x}_1^2 + \frac{1}{2} f_{y_1} \dot{y}_1^2 + \frac{1}{2} f_{x_2} \dot{x}_2^2 + \frac{1}{2} f_{y_2} \dot{y}_2^2 + \frac{1}{2} f_{x_1} (x_1 - x_2) + \frac{1}{2} f_{y_1} (y_1 - y_2) + \frac{1}{2} f_{x_2} (\dot{x}_1 - \dot{x}_2) + \frac{1}{2} f_{y_2} (\dot{y}_1 - \dot{y}_2)
\] (6)

Finally, dynamics equation of the whole system can be deduced by Lagrange equation.

\[
\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = Q_i
\] (7)

The generalized force matrix \( Q \) of the system can be described as:

\[
\begin{bmatrix}
Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 & Q_8
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & T_1 & T_2
\end{bmatrix}
\] (8)

where, \( T_i \) \( (i = 1, 2) \) represents output torque of motor \( i \).

Substituting Eqs. (4), (5), (6) and (8) into Eq. (7), the dynamic equations can be obtained as follows:
\[ M \ddot{x}_i + f_{x_i}(\dot{x}_i - \dot{x}_2) + k_{x_i}(x_i - x_2) = 0 \]
\[ M \ddot{y}_i + f_{y_i}(\dot{y}_i - \dot{y}_2) + k_{y_i}(y_i - y_2) = 0 \]
\[ J \ddot{\varphi}_i + f_{\varphi_i}(\dot{\varphi}_i - \dot{\varphi}_2) + k_{\varphi_i}(\varphi_i - \varphi_2) = 0 \]

\[ M \ddot{x}_i + (f_{x_i} + f_{x_2}) \dot{x}_i - f_{x_2} \dot{x}_2 + (k_{x_i} + k_{x_2}) \ddot{x}_i - k_{x_2} \ddot{x}_2 = -\sigma m_r (\ddot{\varphi}_i + \ddot{\varphi}_2) \sin \varphi_i + \ddot{\varphi}_i \cos \varphi_i + m_r \ddot{\varphi}_2 \sin \varphi_2 + \ddot{\varphi}_2 \cos \varphi_2 \]  
\[ M \ddot{y}_i + (f_{y_i} + f_{y_2}) \dot{y}_i - f_{y_2} \dot{y}_2 + (k_{y_i} + k_{y_2}) \ddot{y}_i - k_{y_2} \ddot{y}_2 = m_r \ddot{\varphi}_2 \sin (\varphi_2 - \varphi_1) + \ddot{\varphi}_2 \cos (\varphi_2 - \varphi_1) \]  
\[ J \ddot{\varphi}_i + (f_{\varphi_i} + f_{\varphi_2}) \ddot{\varphi}_i - f_{\varphi_2} \ddot{\varphi}_2 + (k_{\varphi_i} + k_{\varphi_2}) \dddot{\varphi}_i - k_{\varphi_2} \dddot{\varphi}_2 = m_r l_r \ddot{\varphi}_2 \sin (\varphi_2 - \varphi_1) + \ddot{\varphi}_2 \cos (\varphi_2 - \varphi_1) \]

where, \( M = M_2 + \sum_{i=1}^{2} m_i \), \( J = M_1 l_1^2 \), \( J_2 \approx M l_2^2 \), in which \( l_u \) and \( l_e \) are equivalent eccentric radius of the vibration body and the isolation body, respectively.

### 3 synchronous behavior of the rotors

#### 3.1 The steady state approximate solution of the system

Assuming that \( \varphi \) is average value of the phases of two rotors, and \( 2\alpha \) is the phase difference between rotor 1 and 2. Therefore, \( \varphi_1 \) and \( \varphi_2 \) can be described as

\[ \varphi_1 = \varphi + \alpha, \quad \varphi_2 = \varphi - \alpha \]  

Because of the periodic motion of the vibrating system, the change of average angular velocity of two motors is also periodic. The average value of angular velocity of motors must be a constant. At this moment, assuming that minimum positive period of vibration is \( T_{mn} \), the average integral value of angular velocity of motors in \( T_{mn} \) is a constant.

\[ \omega_{m0} = \frac{1}{T_{mn}} \int_0^{T_{mn}} \omega(t) dt = \text{constant} \]  

Assuming the coefficients of instantaneous variation of \( \dot{\varphi} \) and \( \ddot{\varphi} \) are \( \epsilon_1 \) and \( \epsilon_2 \) (\( \epsilon_1 \) and \( \epsilon_2 \) are functions of time \( t \)), respectively. We can obtain

\[ \dot{\varphi} = (1 + \epsilon_1) \omega_{m0}, \quad \ddot{\varphi} = \epsilon_2 \omega_{m0} \]  

Substituting Eq. (12) into Eq. (10), and assuming that \( \nu_1 = \epsilon_1 + \epsilon_2, \quad \nu_2 = \epsilon_1 - \epsilon_2 \), we obtain

\[ \dot{\varphi}_1 = (1 + \nu_1) \omega_{m0}, \quad \ddot{\varphi}_1 = (1 + \nu_1) \omega_{m0} \]
\[ \dot{\varphi}_2 = (1 + \nu_2) \omega_{m0}, \quad \ddot{\varphi}_2 = (1 + \nu_2) \omega_{m0} \]  

If the system can implement the synchronous operation, the period of steady operation is \( T_0 = 2\pi / \omega_{m0} \), and the average values of \( \epsilon_1 \) and \( \epsilon_2 \) over the single period are zero. Considering \( \dot{\varphi}_i, i = 1, 2 \) when the system operates in steady state, accelerated velocity (\( \ddot{\varphi}_i, i = 1, 2 \)) in the first six formulae of Eq. (9) can be ignored. In order to obtain the approximate solution of the system, the parameters are introduced as follows.
In order to reveal

\[ \omega_{n} = \sqrt{\frac{k_{i}}{M_{i}}} \], \( \xi_{n} = \frac{f_{n}}{2\sqrt{M_{i}k_{i}}} \), \( n_{n} = \omega_{n0}/\omega_{n} \)

\[ \omega_{y} = \sqrt{\frac{k_{i}}{M}} \], \( \xi_{y} = \frac{f_{y}}{2\sqrt{Mk_{y}}} \), \( n_{y} = \omega_{y0}/\omega_{y} \)

\[ \omega_{z} = \sqrt{\frac{k_{i}}{M_{i}}} \], \( \xi_{z} = \frac{f_{z}}{2\sqrt{M_{i}k_{z}}} \), \( n_{z} = \omega_{z0}/\omega_{z} \)

\[ \omega_{v} = \sqrt{\frac{k_{i}}{M_{i}}} \], \( \xi_{v} = \frac{f_{v}}{2\sqrt{M_{i}k_{v}}} \), \( n_{v} = \omega_{v0}/\omega_{v} \)

\[ \eta_{y} = \frac{m_{y}M}{M} \cdot \eta_{z} = \frac{m_{z}M}{M} \cdot \eta_{r} = 1/\rho \cdot \eta_{s} = J_{s}/M \]

Because of the displacement responses of the initial stage are all zero, i.e., \( x_{1}(0) = 0 \), \( x_{2}(0) = 0 \), \( y_{1}(0) = 0 \), \( y_{2}(0) = 0 \), \( \psi_{1}(0) = 0 \), \( \psi_{2}(0) = 0 \), the steady state responses of the system can be obtained by Laplace transform method.

\[ x_{i} = r_{i}\mu_{i}\eta_{i}\cos(\phi_{i} - \gamma_{i}) - \sigma_{i}\eta_{i}\cos(\phi_{i} - \gamma_{i}) \]

\[ x_{2} = r_{2}\mu_{2}\eta_{2}\cos(\phi_{2} - \gamma_{2}) - \sigma_{2}\eta_{2}\cos(\phi_{2} - \gamma_{2}) \]

\[ y_{1} = r_{1}\mu_{1}\eta_{1}\sin(\phi_{1} - \gamma_{1}) + \eta_{1}\sin(\phi_{1} - \gamma_{1}) \]

\[ y_{2} = r_{2}\mu_{2}\eta_{2}\sin(\phi_{2} - \gamma_{2}) + \eta_{2}\sin(\phi_{2} - \gamma_{2}) \]

\[ \eta_{1} = \frac{r_{1}\mu_{1}}{l_{y}}\eta_{1}\sin(\phi_{1} - \sigma\beta_{1} - \gamma_{1}) - \eta_{1}\sin(\phi_{1} - \beta_{1} - \gamma_{1}) \]

\[ \eta_{2} = \frac{r_{2}\mu_{2}}{l_{y}}\eta_{2}\sin(\phi_{2} - \sigma\beta_{1} - \gamma_{2}) - \eta_{2}\sin(\phi_{2} - \beta_{1} - \gamma_{2}) \]

where

\[ j = x, y, \psi \]

\[ a_{x} = n_{x}^{2}(1 - n_{x}^{2}) \], \( b_{x} = 2\xi_{x}n_{x}n_{x}^{2} \]

\[ a_{y} = n_{y}^{2} \], \( b_{y} = 2\xi_{y}n_{y}n_{y}^{2} \]

\[ c = (1 - n_{x}^{2})(1 - n_{y}^{2}) - n_{y}^{2}(4\xi_{y}n_{x}n_{y}^{2} + \eta_{x}n_{y}) \]

\[ d = 2\xi_{y}n_{y} + \xi_{y}n_{y}(1 - n_{y}^{2}) - (\eta_{x}\xi_{x} + \xi_{y}n_{y})n_{y}n_{y}^{2} \]

\[ \mu_{x} = \sqrt{\frac{a_{x}^{2} + b_{x}^{2}}{c^{2} + d^{2}}} \], \( \gamma_{x} = \arctan \frac{a_{x}d - b_{x}c}{a_{x}c + b_{x}d} \)

\[ \mu_{y} = \frac{a_{y}^{2} + b_{y}^{2}}{c^{2} + d^{2}} \], \( \gamma_{y} = \arctan \frac{a_{y}d - b_{y}c}{a_{y}c + b_{y}d} \)

where \( \gamma_{j} (i = x, y, \psi_{1}, x_{2}, y_{2}, \psi_{2}) \) is lagging phase of the system in \( i \) direction, and \( \mu_{i} (i = x, y, \psi_{1}, x_{2}, y_{2}, \psi_{2}) \) is magnification factor of amplitude of the system in \( i \) direction.

The magnitude of amplitude of the vibrating body and isolation body can be directly reflected the magnitude of excitation force acting on themselves. Therefore, the ability of vibration isolation is discussed by the ratio between the amplitude of the isolation body and vibrating body in this paper. In the light of Eq. (15), the amplitude of the system in \( y_{1} \) and \( y_{2} \) direction is a product of the magnification coefficient of amplitude and sine function. Considering weak damping of the system, lagging phase of the system can be ignored, i.e. \( \sin(\phi_{1} - \gamma_{1}) \approx \sin(\phi_{2} - \gamma_{2}) \). In order to reveal the relationship between vibration isolation ability and structural parameters of the system, the coefficient of vibration isolation ability is defined as

\[ \kappa = \frac{M_{x}\mu_{2}}{M_{x}\mu_{1}} \]

(17)
where, $\kappa$ is the coefficient of vibration isolation ability.

According to Eq. (17), the smaller the coefficient of vibration isolation ability, the better the ability of vibration isolation. Assuming that the coefficient of damping be ignored, the optimal ability of vibration isolation is appeared when $n_1 = 1$, i.e. $\omega_{\omega_0} = \sqrt{K_n / M_1}$. In the light of Eq. (17), the coefficient of vibration isolation is related to frequency ratio $n_1$, coefficient of damping $\xi_n$ and mass ratio $\eta_1$. Introducing parameter values of the system in Table 1, and the Fig. 2 is obtained by numerical calculation. As shown in Fig. 2(a), the ability of vibration isolation is firstly increased and then decreased with the increase of frequency ratio $n_1$, and the ability of vibration isolation is optimal when $n_1 = 1$. Moreover, the smaller the mass ratio $\eta_1$, the better the ability of vibration isolation. According to Fig. 2(b), the ability of vibration isolation cannot be influenced by coefficient of damping $\xi_n$.

### Table 1. Parameter value of coefficient equation (17) of vibration isolation capacity.

|                | Rotors | Vibrating body | Isolation body | Motors |
|----------------|--------|----------------|----------------|--------|
| $m_1$          | 4[kg]  | $M_1$=50[kg]   | $M_2$=40[kg]   | $m$=10[Kg] |
| $m_2$          | 4[kg]  |                | $k_n$=47326[N/m] | $L_m$=0.13[H] |
| $r$            | 0.04[m] |                | $k_n$=47326[N/m] | $L_m$=0.1[H] |
| $\omega_{\omega_0}$ | 157[rad/s] | $k_{v_1}$=19719[N/rad] | $k_{v_1}$=19719[N/rad] | $n_p$=2 |
| $f_{y_n}$      | 60[N•s/m] | $f_{y_n}$=60[N•s/m] | $R_y=0.54[\Omega]$ |
| $f_{y_n}$      | 60~300[N•s/m] | $f_{y_n}$=60[N•s/m] | $U_{50}$=220[V] |
| $f_{y_n}$      | 25[N•s/rad] | $f_{y_n}$=25[N•s/rad] | $\omega_y=60[Hz]$ |

![Fig.2 The coefficient of vibration isolation ability of the system](image)

#### 3.2 synchronous condition

In the light of Eq. (15), $\dot{x}_2, \dot{y}_2$ and $\psi_2$ can obtained. Then, substituting $\dot{x}_2, \dot{y}_2$ and $\psi_2$ into the last two formulas in Eq. (9), integrating them over $\phi = 0 \sim 2\pi$, neglecting higher order term, meanwhile, assume that the average values of $\alpha, \varepsilon_1, \varepsilon_2, \varepsilon_1$ and $\varepsilon_2$ represented by $\bar{\alpha}, \bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_1$ and $\bar{\varepsilon}_2$. In this case, the average differential equation of the system can be obtained.

\[
J_{01}\omega_{\omega_0}(\bar{\varepsilon}_1 + \bar{\varepsilon}_2) + f_{y_1}\omega_{\omega_0}(1 + \bar{\varepsilon}_1 + \bar{\varepsilon}_2) = T_{r1} - \bar{T}_{r1}
\]

\[
J_{02}\omega_{\omega_0}(\bar{\varepsilon}_1 - \bar{\varepsilon}_2) + f_{y_2}\omega_{\omega_0}(1 + \bar{\varepsilon}_1 - \bar{\varepsilon}_2) = T_{r2} - \bar{T}_{r2}
\]

(18)

where
\[
\bar{T}_{12} = \chi'_1\bar{\xi}_1 + \chi'_2\bar{\eta}_2 + \chi_{11}\bar{\xi}_1 + \chi_{12}\bar{\eta}_2 + \chi_4 + \chi_{13}
\]

where
\[
\begin{align*}
\chi'_{11} &= Mr^2\omega^2\eta_1^2W_{1i} + \eta_1\eta_2W_i\cos(2\alpha + \theta_i) - \eta_1\eta_2W_i\sin(2\alpha + \theta_i)/2, \\
\chi'_{12} &= Mr^2\omega^2\eta_1^2W_{1i} - \eta_1\eta_2W_i\cos(2\alpha + \theta_i) + \eta_1\eta_2W_i\sin(2\alpha + \theta_i)/2, \\
\chi_{11} &= Mr^2\omega^2\eta_1^2W_{1i} + \eta_1\eta_2W_i\cos(2\alpha + \theta_i) + \eta_1\eta_2W_i\sin(2\alpha + \theta_i), \\
\chi_{12} &= Mr^2\omega^2\eta_1^2W_{1i} - \eta_1\eta_2W_i\cos(2\alpha + \theta_i) - \eta_1\eta_2W_i\sin(2\alpha + \theta_i), \\
\chi_{21} &= Mr^2\omega^2\eta_2^2W_{2i} + \eta_1\eta_2W_i\cos(2\alpha + \theta_i) + \eta_1\eta_2W_i\sin(2\alpha + \theta_i)/2, \\
\chi_{22} &= -Mr^2\omega^2\eta_2^2W_{2i} + \eta_1\eta_2W_i\cos(2\alpha + \theta_i) + \eta_1\eta_2W_i\sin(2\alpha + \theta_i)/2, \\
\chi_n &= Mr^2\omega^2\eta_1^2W_{1i} - \eta_1\eta_2W_i\cos(2\alpha + \theta_i) + \eta_1\eta_2W_i\sin(2\alpha + \theta_i)/2, \\
\chi_k &= Mr^2\omega^2\eta_1^2W_{1i} - \eta_1\eta_2W_i\cos(2\alpha + \theta_i) + \eta_1\eta_2W_i\sin(2\alpha + \theta_i)/2, \\
\end{align*}
\]
\[
W_i = \mu_s\sin\gamma_{1i} + \mu_i\sin\gamma_{2i} + \mu_i^2\sin\gamma_{3i},
\]
\[
W_{oi} = \mu_s\cos\gamma_{1i} + \mu_i\cos\gamma_{3i} + \mu_i^2\cos\gamma_{3i},
\]
\[
a_i = -\sigma\mu_s\sin\gamma_{2i} + \mu_i\sin\gamma_{2i} - \mu_i^2\cos(\beta_i - \sigma\beta_i)\sin\gamma_{3i},
\]
\[
b_i = -\mu_i^2\sin(\beta_i - \sigma\beta_i)\sin\gamma_{3i},
\]
\[
a_e = -\sigma\mu_s\cos\gamma_{2i} + \mu_i\cos\gamma_{2i} - \mu_i^2\cos(\beta_i - \sigma\beta_i)\cos\gamma_{3i},
\]
\[
b_e = -\mu_i^2\cos(\beta_i - \sigma\beta_i)\cos\gamma_{3i},
\]
\[
\begin{align*}
\theta = \frac{\arctan(b_i/a_i)}{\pi + \arctan(b_i/a_i)}, & \quad a_i \geq 0, \\
\theta = \frac{\arctan(-b_i/a_i)}{\pi + \arctan(-b_i/a_i)}, & \quad a_i < 0,
\end{align*}
\]

Because the motors are identical and supplied power source at the same time, the output electromagnetic torque of two asynchronous motors can be described as
\[
T_{o1} = T_{o1} - k_{c1}(\bar{e}_1 + \bar{\eta}_2), \quad T_{o2} = T_{o2} - k_{c2}(\bar{e}_1 - \bar{\eta}_2)
\]

where, \(T_{o1}, T_{o2}\) and \(k_{c1}, k_{c2}\) are output electromagnetic torque and stiffness coefficient of angular velocity operating in steady state, respectively.

Substituting Eq. (19) into Eq. (18), the sum of two formulas is considered as the first row of the matrix, and the difference of two formulas is rearranged as the second row of the matrix. The coupled equation of two unbalance rotors can be obtained as follows
\[
A\hat{v} = Bv + u
\]

where
\[
\begin{align*}
\mathbf{v} &= \begin{bmatrix} \bar{\xi}_1 & \bar{\eta}_2 \end{bmatrix}^T, \quad \mathbf{u} = \begin{bmatrix} u_1 & u_2 \end{bmatrix}^T, \\
\mathbf{A} &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}
\end{align*}
\]
When the two unbalance rotors operate in the steady state, the coefficients of instantaneous variation of the system should be equal to zero in a single period, i.e. \( \ddot{\theta}_1 = 0 \), \( \ddot{\theta}_2 = 0 \), \( \ddot{\theta}_3 = 0 \), and \( \ddot{\theta}_4 = 0 \). Substituting them into Eq. (21), we have

\[
T_{e1} = [T_{e1} l / (M_r^2 \omega_{in}) - f_1 / (M_r^2)] + [T_{e2} l / (M_r^2 \omega_{in}) - f_2 / (M_r^2)]
\]

\[
T_{e2} = [T_{e2} l / (M_r^2 \omega_{in}) - f_2 / (M_r^2)] - [T_{e1} l / (M_r^2 \omega_{in}) - f_1 / (M_r^2)]
\]

\[
- \omega_{in}(\eta_1^2 + \eta_2^2)W_{so} / 2
\]

\[
+ \omega_{in}\eta_1\eta_2W_s (2\bar{\alpha} + \theta),
\]

\[
+ \omega_{in}\eta_1\eta_2W_s (2\bar{\alpha} + \theta) - \omega_{in}(\eta_1^2 - \eta_2^2)W_{so} / 2,
\]

According to Eq. (22), \( T_{e1} + T_{e2} \) is the sum of the output torque of two motors; \( (f_1 + f_2)\omega_{in} \) is the sum of the damping torque between two motors; \( \chi_{in} + \chi_{12} \) is the sum of load torque of the system. Therefore, the balance equation of torque of the vibrating system can be calculated with Eq. (22). The phase difference of the system can be obtained by Eq. (23).

In this case, the coupling torque and the difference of residual torque between the two motors can be described by

\[
T_{sys} = M_r^2 \omega_{in} \eta_1 \eta_2 W_s,
\]

\[
T_{def} = T_{m2} - T_{m1}
\]

where, \( T_{m1} = T_{e1} - f_1 \omega_{in} - M_r^2 \omega^2 \eta_1^2 W_{so} / 2 \) \( T_{m2} = T_{e2} - f_2 \omega_{in} - M_r^2 \omega^2 \eta_2^2 W_{so} / 2 \). \( T_{m1} \) and \( T_{m2} \) are the residual torques of motor 1 and 2, respectively. \( T_{sys} \) and \( T_{def} \) are the coupling torque and the difference of residual torque, respectively.

Substituting Eqs. (24) and (25) into Eq. (23), the phase difference of two rotors can be deduced by

\[
2\bar{\alpha} = \arcsin \frac{T_{def}}{T_{sys}} - \theta_c
\]

According to Eq. (26), the phase difference between two rotors is a function of the coupling torque, the difference of residual torque and parameter \( \theta_c \). Due to \( |\sin(2\bar{\alpha} + \theta_c)| \leq 1 \) and \( T_{sys} > 0 \), the synchronous condition of the system can be described as

\[
T_{sys} \geq |T_{def}|
\]

Therefore, in order to satisfy the synchronous condition, the coupling torque of the system must be greater than or equal to the absolute value of difference of residual torque. In this condition, coefficient of synchronous ability \( \zeta \) is
defined as

\[ \zeta = \left| \frac{T_L}{T_{sys}} \right| \]  

(28)

where \( T_L = (f_1 + f_2) \omega_{m0} + Mr^2 \omega_{m0}^2 (\eta_1^2 + \eta_2^2)W_0 + Mr^2 \omega_{m0}^2 \eta_1 W_1 \cos(2\bar{\alpha} + \theta) \). The smaller the coefficient \( \zeta \), the better the synchronous ability. The effect of installation position of the motors on synchronous ability of the system is discussed, as shown in Fig. 3. In the light of Fig. 3 (a), when two motors rotate in opposite direction, the far installation distance between the motors is in favor of the synchronous implementation of the system; meanwhile, the synchronous ability is decreasing with the increase of installation angle of the motors. In Fig. 3 (b), the synchronous ability is firstly decreased and then increased with the increase of installation distance of two motors rotating in same direction; meanwhile it is also increased with the increase of installation angle of the motors. According to Fig. 3, it can be seen that, the effect of the installation angle of the motors on synchronous ability is decreasing with the increase of installation distance whatever the motors rotate in same or opposite direction.

3.3 Synchronous stability

According to Eq. (23), the phase difference can be obtained, and the Hamilton principle is used to estimate stability criterion of the system. In the light of the Hamilton principle, action quantity \( S \) can be described as

\[ S = \frac{1}{2\pi} \int_{0}^{2\pi} (T - V) d\varphi \]  

(29)

Substituting Eqs. (4), (5) and (15) into Eq. (29), the \( S \) can be determined by

\[ S = -\frac{1}{4} Mr^2 \omega_{m0}^2 \cos(2\bar{\alpha} + \theta) \]  

(30)

According to Ref (Kong et al., 2018), the value of phase difference is stable when Eq. (30) satisfy a minimum point of the Hamilton’s action quantity. In the other words, the second derivative of \( S \) about the phase difference is greater than zero.

\[ \frac{d^2S}{d(2\alpha)^2} \bigg|_{2\alpha = 2\alpha_0} > 0 \]  

(31)

Substituting Eq. (31) into (30), the stability criterion of the system can be described as
\[
\cos(2 \alpha + \theta_1) > 0
\] (32)

In the light of Eq. (32), if the phase difference is located in region \((-\pi/2 - \theta_1, \pi/2 - \theta_1)\), the phase difference is stability, and the two motors can operate in stable state. It shows that the synchronous stability of the system is related to the mounting position between two motors, the mass of rotors, the rotating direction of rotors and the frequency ratio of the system.

4 Numerical analysis

Through the theoretical analysis, the synchronous characteristics of the system are closely related to the mounting position, the mass of rotors, the rotating direction of the rotors and the frequency ratio of the system. Choosing the optimum values for the purpose of vibration isolation, the values of frequency ratio of the system are considered as \(n_y = 5, n_y = 5, n_y = 5, n_y = 5, n_y = 1\). Therefore, the influences of the mounting position, the mass and rotating direction of motors on synchronous characteristics are discussed by numerical analysis in the present section.

4.1 Synchronous state for \(\sigma = 1, n_1 = n_2 = n_3 = n_4 = n_5 = 5, n_5 = 1\)

The values of the dimensionless parameters are firstly considered as \(\sigma = 1, n_1 = n_2 = n_3 = n_4 = n_5 = 5, n_5 = 1\). As two motors of same type are used in present paper, the parameters of two motors are also alike. In this case, \((T_{r,01} - T_{r,02}) - (f_2 - f_1)\omega_{m0} = 0\) and \(\eta_1 = \eta_2\), and Eq. (23) can be simplified as \(\sin(2\alpha + \theta_1) = 0\). At this moment, the stable phase difference of the system is only related to \(\eta_1\). When \((T_{r,01} - T_{r,02}) - (f_2 - f_1)\omega_{m0} = 0\) and \(\eta_1 \neq \eta_2\), Eq. (23) can be simplified as \(\sin(2\alpha + \theta_1) = [(\eta^2_1 - \eta^2_2)W_{m1} / (2\eta_1\eta_2W_{r1})]\). Therefore, the stable phase difference of the system is related to \(W_{m1}, W_{r1}\) and \(\theta_1\). The influences of the mounting position and the mass of rotors on the synchronous state are as shown in Fig. 4. According to the Fig. 4 (a) and (c), the stable phase difference of the system is gradually decreased with the increase of installation angle \(\beta_1\) and coefficient of mounting distance \(r_1\) between the two motors. Meanwhile, the stable phase difference equal to zero when \(\beta_1 = \beta_2\). As shown in Fig. 4 (b) and (d), the stable phase difference is gradually increased with increase of coefficient of mounting distance \(r_1\) when \(\beta_1 > \beta_2\); the stable phase difference is gradually decreased with the increase of coefficient of mounting distance \(r_1\) when \(\beta_1 < \beta_2\); the stable phase difference equal to zero when \(\beta_1 = \beta_2\). Comparing Fig. 4 (a) with (b), the stable phase difference of the system is gradually increased with the increase of installation angle \(\beta_1\). Comparing Fig. 4 (a) and (c), the stable phase difference is increased with the decrease of the mass of rotors when \(\eta_1 \neq \eta_2\).
4.2 Synchronous State for $\sigma = -1, n_1 = n_2 = n_3 = n_4 = n_5 = 5, n_6 = 1$

When two rotors rotate in the same direction, i.e. $\sigma = -1$, the synchronous state is as shown in Fig. 5. In the light of Fig. 5, the stable phase difference of the system is gradually decreased with the increase of the installation angle; the stable phase difference is gradually increased with the increase of the coefficient of mounting distance. Comparing Fig. 5 (a, b) and (c, d), the change trend of the stable phase difference is not influenced by the mass of rotors, but the stable phase difference of the system is increased with the decrease of the mass of rotors.

5 Simulations Results

In present section, the electromechanical coupling model is established by Runge-kutta method to verify the correctness of the theoretical analysis of the above.
5.1 The dynamic characteristics for \( n_{x_1} = n_{x_2} = n_{y_1} = n_{y_2} = 5, n_{x_i} = 1, \sigma = 1, r_i = 1.5 \)

The values of parameters in the simulation model are \( k_{x_1} = k_{x_2} = k_{y_1} = k_{y_2} = 47326, k_{x_i} = k_{x_i} = 19719, k_{x_i} = 1232450, f_{x_1} = f_{x_2} = f_{y_1} = f_{y_2} = 60, f_{x_i} = f_{x_i} = 25, f_{x_i} = 314, \sigma = 1, \beta_1 = \beta_2 = \pi / 6, l = 0.98, m_1 = m_2 = 4, M_1 = 50, M_2 = 40, J_1 = J_2 = 20, r = 0.04 \). The dynamic characteristics of the system can be obtained by computer simulations, as shown in Fig. 6. In the light of Fig. 6 (a), the velocities of the system can be obtained by computer simulations, as shown in Fig. 6 (a), the velocities of two motors started at the same time is gradually stabilized at 157 [rad/s].

As shown in Fig. 6 (b), the stable phase difference between the rotors is stable to zero, which is consistent with the numerical analysis in Fig. 4 (a). According to Fig. 6 (c) and (d), the amplitudes of the system in \( x_1, y_1, x_2, y_2 \) direction equal to zero. The amplitude of the isolation body in \( y_2 \) direction is approximately equal to zero, but the amplitude of vibrating body in \( y_1 \) direction is stable to 6 [mm]. Therefore, in this condition, vibration isolation of the system is realized.

5.2 The dynamic characteristics for \( n_{x_1} = n_{x_2} = n_{y_1} = n_{y_2} = 5, n_{x_i} = 1, \sigma = -1, r_i = 1.5 \)

The values of parameters in the simulation model are \( k_{x_1} = k_{x_2} = k_{y_1} = k_{y_2} = 47326, k_{x_i} = k_{x_i} = 19719, k_{x_i} = 1232450, f_{x_1} = f_{x_2} = f_{y_1} = f_{y_2} = 60, f_{x_i} = f_{x_i} = 25, f_{x_i} = 314, \sigma = -1, \beta_1 = \beta_2 = \pi / 6, \beta_3 = \pi / 3, l = 0.98, m_1 = 2, m_2 = 4, M_1 = 50, M_2 = 40, J_1 = J_2 = 20, r = 0.04 \). As shown in Fig. 7 (a), the velocities of two motors stimulated at the same time are different in initial phase but gradually stable to 157 [rad/s]. The fluctuation of the velocities between two motors is caused by the difference of the mass between two rotors. In the light of Fig. 7 (b), the value of the stable phase difference is stable to -1.956 [rad], closed to the value of numerical analysis -1.95 [rad] in Fig. 5 (c). According to Fig. 7 (c) and (d), the amplitudes of the system in \( x_1, y_1, x_2, y_2, y_2 \) direction are 0, 6, 0, 3.5, 0, 4 [mm], respectively. Therefore, when the
installation angles of the two motors are different, the vertical vibration of vibrating body can be still implemented. Comparing the amplitudes between the vibrating body and the isolation body, vibration isolation of the system is realized.

![Simulation Result](image)

**Fig.7** simulation result when $\beta_1 = \pi / 6, \beta_2 = \pi / 3, \sigma = -1, r = 1.5$

6 conclusions

The present work explores the ability of vibration isolation, synchronization and stability of the anti-resonance system excited dual-motor. The following conclusions should be stressed:

1. The system has the optimal ability of vibration isolation when $n = 1$, i.e., $\omega = \sqrt{k_1 / M_1}$; meanwhile, the smaller the mass ratio of the system $\eta$, the better the ability of vibration isolation.

2. The main influence factor of synchronous ability is the mounting distance. The greater the mounting distance of motors, the better the synchronous ability of the system.

3. The synchronous condition of the system is that the coupling torque of the system must greater than or equal to the absolute value of difference of residual torque. And the stability criterion of the system is $\cos(2\alpha + \theta) > 0$.

4. Considering two rotors rotating in the opposite direction, the stable phase difference of the system and the displacement of the system in $x, y$ direction are all equal to zero when $\beta_1 = \beta_2$; the stable phase difference of the system is increased with the increase of the mounting distance between two motors when $\beta_1 > \beta_2$; moreover, the stable phase difference of the system is decreased of the increase of the mounting distance between two motors when $\beta_1 < \beta_2$. Assuming two rotors rotating in the same direction, the stable phase difference of the system is gradually decreased with the increase of the installation angle $\beta_1, \beta_2$, and it is gradually increased with the increase of the mounting distance between two motors.
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