Network configurations of dynamic friction patterns

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Abstract – The complex configurations of dynamic friction patterns, regarding real-time contact areas, are transformed into appropriate networks. In this letter, we analyze the dynamics of static friction, i.e. nucleation processes, with respect to friction networks. We show that networks can successfully capture the crack-like shear ruptures and possible corresponding acoustic features. We found that the fraction of triangles scales remarkably with the detachment fronts. There is a universal power law between nodes’ degrees and motifs’ frequencies (for triangles, it reads $T(k) \propto k^\beta$ ($\beta \approx 2 \pm 0.4$)). In particular, the evolutions of loops are scaled with power law, indicating the aggregation of cycles around hub nodes. Furthermore, the motif distributions and modularity space of networks—in terms of within-module degrees and participation coefficients—show universal trends, indicating a common aspect of energy flow in shear ruptures. Moreover, we confirmed that slow ruptures generally hold small localization, while regular ruptures carry a high level of energy localization. We proposed that assortativity, as an index to correlation of a node’s degree, can uncover acoustic features of the interfaces.

Introduction. – The transition from static to dynamic friction with regard to rupture nucleation and precursors is the key feature in the sliding process of frictional interfaces. A basic process in the transition from slip to sliding state (stick-slip) includes the propagation of detachment fronts where reductions of contact areas yield fast energy flow in the rupture tip. Initiation of detachment fronts is followed by the emission of acoustic waves. Detachment fronts (front-like ruptures) are the wave-like fronts that are formed during the local and global fast change of contact areas, crossed or arrested through the interface [1–3]. By employing recent advancements in data acquisitions systems, laboratory experiments reveal three rupture modes [1–7]: slow ruptures, sub-Rayleigh ruptures (commonly known as regular earthquakes) and super-shear ruptures. The formation, transition and arresting of shear rupture modes—as well as detachment fronts which are mostly sub-Rayleigh forms—are the most elusive problems in terms of loading configurations and the geometrical complexity of frictional interfaces. Although different numerical and analytical methods investigated several aspects of regular and super-shear ruptures [8–10], the inherent complexities embedded in fault surfaces and the possible collective deformation of an interface’s elements necessitates further detailed analysis of shear ruptures. Recent experimental observations [1–5] show a clear pattern of slow ruptures, as well as the transition of regular ruptures to slow fronts; slow ruptures to sub-Rayleigh fronts; or the arresting of the rupture fronts.

Very recently, two approaches with respect to laboratory observations of slow ruptures have been proposed [11–13]. The first involves the continuous formulation of an extended spring-block model presented in [14]. It considers the plasticity of contact areas with proper Heaviside function. It also predicts smaller localizations of energy in slow ruptures. The second approach [13] uses rate and state friction laws (with Runia’s friction laws [15] for evolution of the state parameter). By employing the proper values for coefficient and numerical modelling, it was postulated that the emergence of slow ruptures results from slow creep fronts [13]. From another perspective, careful investigation of the correlation patterns of particles in sheared materials—numerical and experimental evidence [16,17]—showed the emergence of anisotropic long-range correlated patterns during the deformation of the sample. Furthermore, the characterization of contact patterns and possible correlation among elements of the sheared system are the

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key elements in the analysis of sheared systems as well as frictional interfaces. We notice non-linear and collective behaviours of contact areas in frictional interfaces indicating a complex system with intricate responses to environment stimuli. One of the recent theories to analyze complex systems is network theory. Network theory is a fundamental tool for the modern understanding of complex systems in which, by a simple graph representation, the elementary units of a system become nodes, and their mutual interactions become links. With this transformation of a system to a network space, many properties of the structure and dynamics of the system itself can be inferred. With respect to single fracture behaviour, the opening spaces (i.e. aperture patches in quasi-static loading condition) were mapped onto networks based on a Euclidean and correlation metric [18,19]. The results showed the clustering coefficient of obtained networks roughly to scale with the mechanical or hydraulic properties of the shear fracture.

In this study, we investigate the possible dependency of shear rupture transitions with correlation patterns of contact areas. To characterize the similarity patterns of real-time friction patterns, we map the interface’s elements (hereafter patches) into the proper networks. With this transformation, the networks’ parameters are related to the state variable in friction laws, such that frictional interfaces can be modeled in terms of networks’ evolution. Remarkably, analysis of different real-time contact measures showed rupture transitions strongly scaled with the motif evolution of networks as well as the fraction of triangles (clustering coefficient). Furthermore, we found emerged assortative networks show a unique power-law scaling in terms of global and local similarities. Interestingly, irrespective of rupture speed, similar trends in motifs’ ranks are observed. Our findings about the modularity of friction-networks with respect to within-module degree (Z) and participation coefficient (P) indicate that evolvable frictional interfaces occupy certain regions of the modularity space.

**Data.** – Our data set includes developments of real-time contact areas in recent slip-slip friction experiments on transparent interfaces [1,3,4]. Generally, the experiments involve shearing two transparent blocks on each other while a uniform normal loading is on the top block and a tangential force is used in the trailing edge. Recordings of the real-time relative contact areas mostly are based on a 1D assumption of interface dimension. The later assumption considers average-optical intensity with employing a laser ray through the interface [1,4]. However, there are limited cases of 2D measures of relative contact areas corresponding to intensity of laser-light. We use a correlation measure over 2D relative contact patterns. Furthermore, for 1D interface, we employ a truncated norm in the analysis of time-series to compare the similarity of the patterns.

**Methodology.** – To set up a non-directed network over 2D contact areas in a certain time step, we considered each patch of measured contact areas perpendicular to shear direction as a node —fig. 1(a). Each profile has N pixels where each pixel shows the relative contact area of that cell. Then, we define correlations in the profiles by using

\[ C_{ij} = \frac{\sum_{l=1}^{N} [A_i(l) - \langle A_i \rangle] \cdot [A_j(l) - \langle A_j \rangle]}{\sqrt{\sum_{l=1}^{N} [A_i(l) - \langle A_i \rangle]^2} \cdot \sqrt{\sum_{l=1}^{N} [A_j(l) - \langle A_j \rangle]^2}}, \]

where \( A_i(l) \) is the i-th profile with \( 1 \leq l \leq N \). We may see each profile as a separated cycle of a spatial series (i.e., collection of cycles in the x-direction). To map the obtained series, we define each patch as a node. To make an edge between two nodes, relative-high correlated profiles are connected \( (C_{ij} \geq r_c) \) with non-direct links. To choose \( r_c \), we use a nearly stable region in the betweenness centrality (BC)-\( r_c \) space which is in analogy with minimum value in the rate of edges density (for more information see [18-20]). The latter method has been used successfully in analyzing time-series patterns in network space [20]. We notice that finding a nearly stable region in the BC-\( r_c \) space satisfies dominant structures of contact patterns. Despite the aforementioned case, most of the recorded photos had insignificant dimension in the x-direction (\(< 0.2 \text{mm}–6 \text{mm}\)). Then, naturally with respect to the apparatus, the observed patterns were the longitudinal contact zones. In other words, 2D imaging drastically slows down monitoring in the propagation direction. As a result the measurements were generally 1D, while the integration over the z-direction was performed by optical means [2,4]. To build networks over 1D spatial contact areas, we used two different methods. The first method is comparing “closeness” of contact areas; i.e., if \( |A(x_i, t) - A(x_j, t)| < \xi \rightarrow a_{ij} = 1 \), where \( a_{ij} \) is the component of connectivity matrix. We use a similar procedure (BC-\( \xi \) space) to select the threshold level.

To proceed, we use several characteristics of networks. Each node is characterized by its degree \( k_i \) and the clustering coefficient. The clustering coefficient (as a fraction of triangles) is \( C_l \) defined as \( C_l = \frac{2T_l}{N(N-1)}, \) where \( T_l \) is the number of links among the neighbors of node \( i \) and \( k_i \) is the number of links. Then, a node with \( k \) links participates on \( T(k) \) triangles. Furthermore, based on the role of a node in the modules of network, each node is assigned to its within-module degree \( (Z) \) and its participation coefficient \( (P) \). High values of \( Z \) indicate how well-connected a node is to other nodes in the same module, and \( P \) is a measure of well-distribution of the node’s links among different modules [21]. To determine modularity and partition of the nodes into modules, the modularity \( M \) (i.e., objective function) is defined as [21]

\[ M = \sum_{s=1}^{N_M} \left[ \frac{l_s}{L} - \left( \frac{d_s}{2L} \right)^2 \right], \]

in which \( N_M \) is the number of modules (clusters), \( L = \frac{1}{2} \sum_i k_i l_s \) is the number of links in module and \( d_s = \sum_i k_i^s \)
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Fig. 1: (Colour on-line) (a) A typical example of transferring contact areas into networks, (b) mapping the dynamic relative contact areas in 2D (courtesy of real-time contacts of J. Fineberg) to networks space an plotting clustering coefficient as a fraction of triangles reveals the relatively precise rupture speed: three distinctive rupture speeds is compatible with mean contact area; 1, 3 corresponds with sub-Rayleigh rupture and 2 is slow rupture; (c) the scaling of triangles in obtained networks with number of similar profiles (node’s degree), expressed with power-law relation with a universal exponent, the distribution of motifs of networks over different time steps shows a non-uniform/universal distribution; the transition from rupture (1) to rupture (2) occurs suddenly which is related to regular acoustic emission (high-frequency waveform).

(the sum of nodes degrees in modules). Using an optimization algorithm (here we use Louvain algorithm [22]); the cluster with maximum modularity is detected. To describe the correlation of a node with the degree of neighbouring nodes, assortative mixing index is used [23]:

\[ r_k = \frac{(j_i k_i) - \langle k_i \rangle^2}{\langle k_i^2 \rangle - \langle k_i \rangle^2} \tag{3} \]

where it shows the Pearson correlation coefficient between degrees \( j_i, k_i \) and \( \langle \cdot \rangle \) denotes average over the number of links in the network.

Results and discussion. – We start with 2D interfaces, monitored at discrete time steps \([0, 0.4, 0.75, 1, 1.2, \text{ and } 1.4] \text{ ms} [1]\). Transferring X-patches (then perpendicular to shear direction) to networks and plotting clustering coefficient revealed three distinct patterns of rupture evolution (fig. 1(b)) which are comparable with the previous results [1,2]. The three patterns correspond with sub-Rayleigh (1 and 3 in fig. 1(b)) and slow rupture (2). In other words, the movement of the rupture tip is followed by fast variation of the clustering coefficient. Furthermore, by considering 3 points cycles (\(T\)-triangle loops) vs. node’s degree from 0.4 to 1.4 ms we show a power-law scaling (fig. 1(c)):

\[ T(k) \sim k^\beta \tag{4} \]

where the best fit for collapsed data set reads \( \beta \approx 2.1 \pm 0.4 \) (we call it the coupling coefficient of local and global structures). Thus, our analysis of aperture patterns in discrete slip measurements (over 20 mm shear displacements) in rock samples reveals the same scaling law with a very close coupling coefficient [18]. Now, we assume that the variation of the clustering coefficient is proportional with variation of shear stress (all equations are developed in a non-dimensional form), i.e., \( \frac{\partial f}{\partial t} \sim -\frac{\partial f}{\partial t} C(k) \). Consequently, we obtain \( \frac{\partial f}{\partial t} \sim k^{\beta-2}(2-\beta)T(k)^{(1-\frac{1}{2})} \). With \( \epsilon \equiv k^{\beta-2}(2-\beta) \) we obtain \( f_s \sim \epsilon \ln t(t) \). Then, we showed that the presented ansatz approximately results in a logarithmic dependency of the links to shear force. In addition, comparing the links’ variation with real-time contact areas indicates nearly identical evolutionary trends (for example, see fig. 3(d)) which provide a similar analogy to the evolution of the contact area \( A \) with the state variable \( \theta \): \( A(\theta) \sim A_0(1 + \log(1 + \theta)) \) [12]. Then the results hint that the developing the state variable in terms of functional friction networks parameters. To accomplish the friction-based networks formulation we need to establish the evolution of links, communities or motifs (such as loops). Here we follow the distribution of fig. 2. Amazingly, irrespective of the different type of rupture modes in the considered time interval, a power law satisfies the obtained distributions:

\[ P(T(k)) \sim T^{-\gamma}, \tag{5} \]

in which \( \gamma \approx 2.01 \). The power-law nature of loops shows aggregation of cycles around “hubs”. In other words, during rupture evolution, loops (and generally subgraphs with loops) are not distributed uniformly. The later observation is in analogy with the cellular networks [24].

Let us transfer (5) into a “hubness” model (i.e., Barabasi-Albert model [25]), where we assume hub nodes tend to absorb nodes with rich loops rather than poor nodes. Then, it leads to

\[ \frac{\partial T_i(k)}{\partial t} \sim \frac{m}{\sum T_j} T_i k_i \tag{6} \]

in which \( m \) is a coefficient of growth (or decay). Plugging (4) in (6) yields

\[ \frac{\partial k_i}{\partial t} \sim \frac{m}{\beta} \frac{k_i}{\sum k_j^\beta} \tag{7} \]

For \( \beta = 1 \) the model yields scale-free networks. To complete our analysis, we extend the variation state parameter of state and rate friction law [26] as a superposition

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of local and global evolutions of the network’s structures. We consider a simplified form of the standard equation for the friction with assumption of constant sliding velocity \((v = 1): \mu_s \sim \ln \frac{\theta}{v}\) in which \(\theta\) is the variable describing the interface state and \(D_c\) is the characteristic length for the evolution of \(\theta\). The state variable carries information about the whole population of asperities [27]. Commonly used empirical laws for evolution of state variable are Ruina’s laws for ageing and slipping states [15]. For slip law, it reads (constant sliding velocity: \(v \equiv 1\)): \(\frac{\partial \theta}{\partial t} \sim -\frac{\partial}{\partial x}(\ln \frac{\theta}{D_c})\). Let us transfer state variable in networks space as follows:

\[
\frac{\partial \theta_i(t)}{\partial t} = a \frac{\partial k_i}{\partial t} + b \frac{\partial T_i}{\partial t},
\]

where \(a\) and \(b\) are proper weights of the variations. From (4) and (8), we eventually obtain

\[
\frac{\partial \theta_i(t)}{\partial t} = \frac{\partial k_i}{\partial t}(a + b3k_i^{\beta-1}).
\]

Assuming \(\beta = 1, \frac{\partial k_i}{\partial t} > 0\) and \(a < 0\) indicates a decaying model for the state parameter in terms of attacking to hubs. Then the evolution of the state variable was expressed with only links evolution. Plugging (7) into (9) and assuming \(\beta = 2\) leads to

\[
\frac{\partial \theta_i(t)}{\partial t} = \frac{\partial k_i}{\partial t}(a + b3k_i^{\beta-1}) = \frac{\partial k_i}{\partial t}(a + b3k_i^{-1}).
\]

which shows a complex non-linear nature of state parameter. It may explain the role of the state parameter to cover variation of ruptures. Further developments of (9) and (10) can be done by using non-linear growth models as well as rate equations [28]. Notably, the first part at the right hand of eq. (10) shows friction networks follow a gel-like state, in which a condensation of nodes occurs [28,29]. In this case (or when \(\frac{x}{v} \to 0\)), a single node is connected to almost all nodes in the friction network. This behaviour is biased with the second term if the weight of global evolution is significant.

Based on fig. 1(b), we also estimate the speed of rupture fronts around the rupture zone with: \(\frac{\partial^2 C(z, t)}{\partial z^2} \sim -1/v_{\text{front}}\) which states the temporal-spatial gradient of the fraction of triangles correlates with the inverse of the speed of rupture fronts. Considering the spatial gradient of the state variable’s rate and recalling the relation of rupture speed and clustering coefficient requires (writing eq. (8) in terms of \(k\) and \(C\) and plugging \(\frac{\partial^2 C(z, t)}{\partial z^2} \sim -1/v_{\text{front}}\): \(1/v_{\text{front}} \sim -\chi(k)\frac{\partial^2 \theta_i(t)}{\partial k^2}\), where \(\chi(k)\) is a function of node’s degree and coupling coefficient. To confirm this relation, using Galilean transformation [12], we estimate front’s speed. Assuming \(\frac{\partial \theta}{\partial t} = -v_{\text{front}} \frac{\partial \theta}{\partial \tau}\) and recalling Ruina’s law with the assumption of unity rate of displacement, \(\frac{\partial \theta}{\partial t} = 1 - \frac{\theta}{D_c}\); then we obtain \(v_{\text{front}} = \frac{-\partial \theta_i(t)}{\partial t} = \frac{-\partial \theta}{\partial \tau} = 1\), which is comparable with our prediction of the front’s speed.

Next, we plot sub-graphs’ (i.e., motifs’) distributions (fig. 1(d)) which indicate a super-family phenomenon.

A similar trend in all the rupture speeds indicates the universality in energy flow in frictional interfaces, which is characterized by friction laws. We examined this universality by measuring contacts and apertures in terms of discrete displacement intervals and confirmed the achieved results [18]. Remarkably, the transition from sub-Rayleigh to slow rupture is correlated with a distinct spike in all types of the 4-points sub-graphs. Considering the variations of \(\langle k \rangle\) at the monitored interval \((10 < k < 30)\) and the abrupt change in loops which is of one order of magnitude, we conclude that the fluctuation of the coupling coefficient induces a remarkable growth of sub-growth. One can confirm that increasing sub-graphs is consistent with our formulation and observation (for 1D-friction patterns), and consequently increases \(\beta\) which leads to a rapid increase of loops.

Following 1D patterns of contact areas, we map 1D-net contact areas into friction networks by using closeness metric (fig. 3). According to [3] and fig. 3(b), the evolution of contact areas follows three remarkable evolutionary stages as follows: 1) detachment phase, 2) rapid slip, and 3) slow slip. We notice in this case, the first phase accompanies around 10% of contact area variation while phase 2 represents 2–3% of the changes in contacts. Notably, the evolution of pure contact areas does not reveal any more information on rupture speed or other possible mechanisms behind the fracture evolution.
However, the corresponding friction networks’ parameters, such as assortativity index and maximum modularity ($Q$) [30], indicate more features of the rupture speed. Each front passage is encoded as the remarkable spikes in assortativity index or maximum modularity (fig. 3(b), (f)). We also confirmed the coupling coefficient dramatically drops due to the front’s passage. Based on our formulation, the rapid drop of coupling coefficient induces fast variation of shear stress.

Moreover, the maximum variations of assortativity and modularity at the first phase are \( \sim 50\% \) and \( \sim 30\% \), respectively. A significant spike in transition to the phase (1) is followed by a nearly large stationary value of assortativity, where the transition time is comparable to the propagation of sub-Rayleigh’s front. Generally, our observation showed rapid sub-Rayleigh fronts encoded as sudden spikes in assortativity index, comparable with the radiated energy and acoustic emissions. Remarkably, the slow rupture stage is generally represented by monotonic growth of modularity while the variation of assortativity occurs in the two distinct categories: stable and non-stable slow rupture. Stability in assortativity occurs in two different stages; one is before sliding and another one takes place in the net-movement of the interface. However, it seems that the mechanisms of the similar stable-assortative stages are different. The first stationary interval accompanies the growth of modularity while the continuous decaying of links happens.

The second interval, which occurs with the same growth of assortativity (>20%), shows a decreasing trend in modularity. We concluded after a rapid-strong sub-Rayleigh front that a silent period is encoded in the system, followed by a fast transition to next stage. Indeed, transition to phase 3 occurs with \( \sim 25\% \) fast increment of assortativity index while the variation of the contact areas is only less than 4%. In other words, during this small time interval (\( \sim 60-100\mu s \)), what controls the behaviour of the interface is the pattern (shape) of contacts rather than pure contact areas where we guess the central rich-hubs drive the system. It immediately follows a relatively long stationary assortativity, indicating a stable mechanical characteristic of the interface (possible low sliding rate [1,3]). With respect to our observations, we hypothesize stable and unstable-slow ruptures carry high and small localization, respectively, while sub-Rayleigh ruptures hold high localization in terms of the evolution’s rate of the assortativity. Phase 3 clearly is characterized by a long term of near-stability in assortativity, net-contact areas, and nodes’ degrees (with a small \( \sim 3\% \) growth in modularity, clustering coefficient are observed). We have confirmed the theses’ results with three more data sets. It is worth mentioning that the maximum eigenvalues of the Laplacian matrix show approximately the same trend of degree correlation (fig. 3(g)). Mapping the nodes of friction networks in modularity space, defined by within-module index ($Z$) and the participation coefficient ($P$) revealed the collapsing of all nodes in a certain range of $P$ and $Z$ (fig. 3(C)). We confirmed the universality of observed patterns in $P-Z$ space across different case studies: either real-time contacts (1 or 2D) or aperture friction networks. Following [31], we divide the $P-Z$ space into 7 sub-regions: $R_1, \ldots, R_7$ which is based on the role of non-hubs and hubs nodes. The general categorization of $P-Z$ parameter space is as follows:

\[
\begin{align*}
\text{Non-hubs:} & \quad \{R_1 (P \leq 0.05), \quad R_2 (0.05 < P \leq 0.62), \\
& \quad R_3 (0.62 < P \leq 0.80), \quad R_4 (P > 0.80) \} \cap \{Z < 2.5\}, \\
\text{Hubs:} & \quad \{R_5 (P \leq 0.3), \quad R_6 (0.3 < P \leq 0.75), \\
& \quad R_7 (P > 0.75)\} \cap \{Z \geq 2.5\},
\end{align*}
\]

where each region nominates a unique characteristic of rich or poor nodes with respect to other modules. Our results for 8 case studies show that the probability of finding a profile in $R_4$ and $R_7$ is very unlikely, as most congestion of profiles happens in $R_1$, $R_2$ and $R_3$. A few nodes occupy $R_5$, indicating the role of hub nodes with the majority of connections within their module. We noticed a general negative fit of collapsed nodes in $P-Z$ space. Amazingly, plotting the evolution within-module index in $x-t-Z(x,t)$ shows a periodic nature (fig. 3(i)). We obtained the periodic property of $Z(x,t)$ in other real-time contact areas and aperture-friction networks. We believe that the periodicity of within-module degree and the participation coefficient —with respect to nearly the invariant nature of spatial-periodicity— are related somehow to the characteristic length ($D_c$) of the interface. Interestingly, in this parameter space, a clear super-shear front is revealed. Further studies will highlight the network-signatures of super-shear fronts (rupture). We conclude that fast fronts induce (or generally scale with) out-of-the-modules’ developments and that slow ruptures scale with in-modules’ evolution.

Conclusions. – As a conclusion to our study, we introduced friction networks over dynamics of different real-time contact areas. Based on our solid observations, we formulated a probabilistic frame for the evolution of the state variable in terms of friction networks. We used an ansatz to estimate the evolution of shear stress which helped us to estimate the rupture speed. The evolution of the state variable extended with the aggregation of loops and scaling of loops with the links. Moreover, we confirmed that slow ruptures generally hold small localization, while regular ruptures carry a high level of energy localization. We also introduced two new universalities with respect to the evolution of dry frictional interfaces: the scaling of local and global characteristics and the occupation of certain regions of modularity parameter space. Our results showed how the relatively highly correlated “elements” of an interface can reveal more features of the underlying dynamics. We proposed that assortativity as an index to correlation of node’s degree may uncover acoustic features of the interfaces. Our formulation can be coupled with elasto-dynamic equations to complete our understanding.
of the interface’s more realistic features. Further extension to our formulation with respect to the variation of particle velocity and loading conditions will be our future focus.

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