New results in fundamental theory of synchrotron radiation

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Abstract. Examining the characteristics of radiation from scalar particles in second excited state, which could be regarded at as a three-level system, we deliver (and solve) the problem on spectral properties in simplest form. Using the methods of quantum theory, we establish that partial contribution of fixed harmonic to the total power of radiation does not vanish in ultrarelativistic domain, and this fact contradicts the results known in classical approach. We also consider the evolution of radiation maximum. In particular, for a scalar particle we find the condition for radiation maximum to stay at highest possible harmonic.

1. Introduction
Classical electrodynamics is usually considered as an adequate tool for description of synchrotron radiation (SR) properties with respect to the parameters of contemporary facilities. Nevertheless, for the last few decades the main quantum parameter \( \zeta = \gamma H / H_0 \) increased from \( 10^{-7} \) to \( 10^{-5} \), i.e. by two orders. Since one knows that for the values \( \zeta \sim 10^{-3} - 10^{-2} \) quantum corrections have to be necessarily taken into account and assuming further development of SR sources techniques, important seems the consideration of basic SR properties in the framework of quantum theory.

It is known that according to classical theory the spectrum of SR is discrete and regardless of any values of parameters contains an infinite number of harmonics [1]. In quantum theory the concept of radiation spectrum corresponds to quantum transitions between particles' energy levels [2, 3].

To study the spectral properties of SR in the framework of quantum theory it is convenient to start with the simplest case where such properties may show up. Namely, if we consider quantum three-level system, we already deal with a spectrum that contains two harmonics, each related to a certain quantum transition. Comparing the characteristics of SR calculated in terms of quantum theory with their classical analogues one can find if and when (at which values of parameters) classical theory is representational.

Though from mathematical point of view it is easier to deal with weakly excited particles, the generalization to a case of arbitrary initial state appears more interesting concerning its possible applications, both theoretical and practical. For quantum transitions from an arbitrary initial state we focused on tracking the shift of radiation maximum - a problem thoroughly studied but so far only within classical approach.
2. Basic definitions and notations

The motion of a scalar particle (boson) in a constant uniform magnetic field \( \mathbf{H} = (0, 0, H) \) can be described by Klein-Gordon equation. Solving this equation one gets a discrete energy spectrum

\[
E = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - \beta^2}}, \quad \gamma = 1 + (2n + 1)b, \quad b = \frac{H}{H_0}, \quad H_0 = \frac{m_0^2 c^3}{|e_0| \hbar} = 4.41 \cdot 10^{13} \text{G} = 4.41 \cdot 10^9 \text{T}.
\]  
(1)

Here \( \gamma \) is the relativistic factor; \( m_0 \) is the rest mass of the particle; \( c \) is the speed of light; \( \beta = v/c \), where \( v \) is the speed of the particle in classical theory; \( n \) is the number of energy level; \( H_0 \) may be interpreted as the Schwinger field; \( \hbar \) is the Plank constant and \( e_0 \) is the charge of the particle.

Obviously, the process of photon emission corresponds to quantum transition to lower energy level. If the initial energy level number is \( n \), and one has a transition to level \( s < n \) then the number of radiated harmonic is \( n - s \).

3. Spectral Distributions and Polarization

The total radiation \( W_i \) emitted by a boson may be represented as the sum (parameter \( \theta \) describes the direction of photon emission with respect to external field) [3]

\[
W_i \sim \sum_{\nu=1}^{n} F_i(n, \nu, \beta), \quad F_i(n, \nu, \beta) = \int_{0}^{\pi} f_i(n, \nu, \beta, \theta) \sin \theta d\theta,
\]  
(2)

where each summand may be interpreted as the amount of radiation related to certain harmonic \( \nu \) of the spectrum. We choose standard labeling method for polarization components, \( i = \pm 1 \) stays for right and left circular polarization components (symmetry of the problem allows us to consider only right circular polarization \( i = 1 \)), \( i = 2, 3 \) is used for \( \sigma- \) and \( \pi-\)components of linear polarization respectively and \( i = 0 \) denotes total (summed over linear or circular polarization components) radiated power. To avoid cumbrous formulae we do not give exact definition of \( f_i \) and restrict ourselves to the mention of their connection with Laguerre functions.

In the case of three-level system, the initial state is characterized by \( n = 2 \) and there are two possible transitions \( n = 2 \rightarrow s = 1 \) (\( \nu = 1 \)) and \( n = 2 \rightarrow s = 0 \) (\( \nu = 2 \)). Therefore in this case \( W_i \) from (2) contains only two summands \( W_i \sim F_i(\beta) = F_i(2, 1, \beta) + F_i(2, 2, \beta) \). The evolution of \( F_i(\beta) \) is given in figure 1.

![Figure 1](image1.png)

**Figure 1.** The functions \( F_i(\beta) \). Curve numbers correspond with polarization components.

![Figure 2](image2.png)

**Figure 2.** Effective angles \( i = 0 \) for \( \nu = 2 \) (‘a’), \( \nu = 1 \) (‘b’), \( \nu = 1 \) or 2 (‘c’) and for total classical spectrum (‘d’).

The degree of polarization may be described by the functions \( q_i(\beta) = F_i(\beta)[F_0(\beta)]^{-1} \) and \( q_i(\nu, \beta) = F_i(2, \nu, \beta)[F_0(2, \nu, \beta)]^{-1} \). Here we consider the polarization in upper half-plane, i.e.
the integration limits in (2) are $[0, \pi/2]$. The behavior of $q_1(\beta)$ and $q_2(\beta)$ is demonstrated in figure 3, where dashed lines stay for classical theory predictions. It could be seen from figure 3 that, according to quantum theory, $\sigma$-linear polarization degree decreases with energy. On the contrary, the degree of circular polarization is increasing. Both these statements contradict classical results except for non-relativistic area where $\beta \to 0$.

It is convenient to introduce the functions $p_i(\nu, \beta) = F_i(2, \nu, \beta)|F_0(\beta)|^{-1}$ to describe partial contributions of separate harmonics to the total (summed over spectrum) radiation. Their evolution with $\beta$ is given in the figures 4 a and 4 b below.

For $p_i$ we have qualitative accordance with the results known in classical theory. Partial contribution of first harmonic decreases with energy and for the second harmonic it increases. So, at some energy value the second harmonic becomes dominating. However at $\beta = 1$ there is a non-zero value of contribution for each harmonic (classical theory says that for any harmonic its contribution tends to 0 at $\beta \to 1$).

4. Effective angles of radiation

We define the effective angle as an opening angle in which the main part of power is radiated. The dependence of effective angles on particles’ energy (on $\beta$) is shown in figure 2. Classical theory predicts the concentration of radiation near orbits’ plane in ultrarelativistic limit. It was shown [4, 5] that for first excited state particles (no matter spinor or scalar) quantum theory does not confirm this assumption. Here we demonstrated that for the case of shortest quantum spectrum no radiation concentration is observed. Thus, within ultrarelativistic region classical theory can not be considered as representational for weakly excited particles.
5. Spectral maximum

Classical theory predicts that increasing the energy of the particle, the number of harmonic corresponding to maximum of radiation $\nu_{\max}$ increases and turns proportional to $\gamma^3$ in ultrarelativistic limit. In quantum theory the number of any radiated harmonic may be less or equal to the number of energy level of a particle $\nu \leq n$.

Let us consider the functions $K(n, \beta) = F_0(n, n, \beta)[F_0(n, n - 1, \beta)]^{-1}$. Obviously, the condition $K(n, \beta) = 1$ for a fixed $n$ defines such $\beta = \beta_n$ and, therefore, such $\gamma = \gamma_n = (1 - \beta_n^2)^{-1/2}$ that the spectral maximum stays at highest harmonic. As one can see from figure 5, it turns out

![Figure 5](image1.png)

**Figure 5.** $K(n, \beta)$ (5 a) and the dependence of $\gamma_n^2$ on $n$ with its linear approximation (5 b).

that the dependence of $\gamma_n^2$ on $n$ is linear at $n \gg 1$, namely, $\gamma_n^2 = 2k_0n$, $k_0 \approx 1.146128792697$. The equality $\gamma_n^2 = 2k_0n$ can be correct only at certain value of $b_0$ (which characterizes the external field). Therefore, if $\gamma_n^2 = 2k_0n = 1 + (2n + 1)b_0$, i.e. $b_0 = k_0$ at $n \gg 1$ then it is possible to observe the shift of radiation maximum from $\nu = n - 1$ to $\nu = n$. Since $b$ does not depend on $n$, if $b < k_0$ then $\nu_{\max} \leq n - 1$, otherwise (when $b \geq k_0$) the maximum of radiation shifts to the highest harmonic $\nu_{\max} \leq n$. Thus, we assume that there exists an ordered set of numbers $k_0 > k_1 > k_2 > \ldots > k_s$ such that if $b < k_s$ then the spectral maximum lies on $\nu_{\max} \leq n - s$ and it shifts to higher harmonics with $n$. If $k_m < b < k_m-1$ ($1 \leq m \leq s$) then for any initial state $n > m$ the spectral maximum stays at $\nu_{\max} = n - s$. Finally, if $b > k_0$ then at any $n$ we have $\nu_{\max} = n$.

6. Conclusion

Concerning the radiation properties of weakly excited particles, classical and quantum results stay in good accordance only within non-relativistic area. The concentration of radiation predicted by classical theory for ultrarelativistic limit was not observed neither for two- nor for three-level systems. Still there is a certain agreement between classical and quantum results for separate harmonics of the spectrum. It means that for weakly excited particles classical theory stays representational only in some particular cases.

In the framework of quantum theory we also considered the evolution of SR spectral maximum. We found the condition for the maximum to lie at the highest harmonic of the spectrum, and, therefore, increasing the energy any other shift of the maximum for higher harmonics does not occur. The condition of its shift to the highest harmonic is that the external magnetic field intensity is higher than certain critical value. If the intensity of external field is less than this critical value, whatever other parameters are, the spectral maximum does not move to the highest harmonic. Thus, we found the restrictions for the rule concerning the evolution of spectral maximum in classical theory.

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