Discriminating Between Dagum Distribution and Burr-III Distribution

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Abstract - Test statistics based on Likelihood function, population Quantiles are suggested to discriminate between Dagum distribution and Burr-III distribution. Because of the non tractability of their exact sampling distributions, the percentiles of the proposed test statistics are tabulated with the help of simulated sampling distributions of the test statistics. The power of the test statistics are also tabulated and a comparative study w.r.t the powers for a given sample and the level of significance are worked out.

Key words and phrases- Dagum distribution(DD), Burr-III distribution(BD), Likelihood function, Quantiles, power of the test.

I. INTRODUCTION

Having experienced with the lack of analytical expression for the classical maximum likelihood estimation (MLE) of parameters in Dagum distribution, we propose to study whether any other standard model be an alternative to Dagum distribution with a reasonably admissible risk. Accordingly, we have chosen Burr type III model to test whether it can be an alternative to Dagum distribution. This aspect is viewed as the problem of discriminating between Dagum and Burr III models where Dagum is null population. We have considered the principle of Likelihood ratio criterion in a practically usable way. Hence we may call our procedure as likelihood ratio type procedure. The distinction is- the likelihood procedure to the classical MLE is used for both null and alternative populations. In the likelihood ratio type procedure we use any admissible estimators of concerned models as given by [1,2, 3].

The Probability density function (pdf) of Dagum distribution is given by

\[ f(x,b,a,p) = \frac{ap}{x} \left( \frac{x}{b} \right)^{ap-1} \left( 1 + \left( \frac{x}{b} \right)^{ap} \right)^{-ap}, \quad x > 0, a > 0, b > 0, p > 0. \] (1.1)

The cumulative distribution function (cdf) is

\[ F(x;b,a,p) = 1 + \left( \frac{x}{b} \right)^{-p}, x > 0, a > 0, b > 0, p > 0. \] (1.2)

The likelihood function of Dagum distribution is

\[ L(x,a,p,b) = \frac{(ap)^{n} \left( \prod_{i=1}^{n} x_i^{(ap-1)} \right)}{(b)^{nap} \left( \prod_{i=1}^{n} \left( 1 + \left( \frac{x_i}{b} \right)^{-a} \right)^{-ap} \right)}. \] (1.3)

The Dagum distribution is a skewed, uni-modal distribution on the positive real line.

The probability density function (pdf) of Burr-III distribution is given by

\[ f(x;c,k) = ck x^{-(c+1)} \left( 1 + x^{-c} \right)^{-k}, \quad x > 0, c, k > 0. \] (1.4)

The cumulative distribution function(cdf) is

\[ F(x;c,k) = \left( 1 + x^{-c} \right)^{-k}. \] (1.5)

The likelihood function of Burr-III distribution is

\[ L(x,c,k) = (ck)^{n} \left( \prod_{i=1}^{n} x_i^{-(c+1)} \left( 1 + x_i^{-c} \right)^{-k} \right). \] (1.6)
Many people have studied the importance of these two distributions as life testing models. The related works include [4&5]. In fact inferential properties of Dagum distribution are simple, efficient and mathematically tractable, where as it is not the same case with Burr-III distribution. We are therefore motivated to study whether Dagum distribution is a reasonable alternative to Bur-III distribution at least for the sake of adopting the analytical, powerful inferential characteristics of the data following Burr-III distribution. With this backdrop we suggested two different test statistics to discriminate Dagum distribution(with a=0.5,p=0.5 and b=2/3) and Burr-III distribution(with c=0.5 and k=0.5). The pdfs and cdfs plotted for the mentioned values of both the distributions are given below.

The basic distribution characteristics of Dagum distribution(DD) and its Properties are Presented in Section I. The rest of the paper is organized as follows. The proposed test statistics based on ratio of likelihood functions along with their percentiles and power values are explained in section II and Population quantiles explained in section III along with their percentiles and power values.

Summary and Conclusions are given in section IV.

II .TEST STATISTIC BASED ON LIKELIHOOD RATIO

Let us consider Dagum distribution as the null population say ($P_0$), the Bur-III distribution is considered as the alternative population say ($P_1$). If a random sample of $x_1,x_2,...x_n$ is drawn from $P_0$ evaluate the estimation of the parameters of $P_1$ using the given sample of $x_1,x_2,...x_n$.

Let $P_0$ denote the value of the likelihood function of the sample $x_1,x_2,...x_n$ with respect to null population , at the estimates of its parameters using $x_1,x_2,...x_n$. Let $P_1$ denote the value of likelihood function w.r.t alternative population , the estimates of $P_1$ using the sample $x_1,x_2,...x_n$. The ratio $\frac{L_1}{L_0}$ in a way represents the ratio of the likelihood of $P_1$ to that of $P_0$ with a sample drawn from $P_0$. Therefore $\frac{P_1}{P_0}$ is the ratio of a smallest probability to a larger probability and hence is expected to be small. The null hypothesis “H0: The sample is drawn from $P_0$ ” can be tested using the percentiles of the likelihood ratio $\frac{L_1}{L_0}$.

Therefore statistics $T_i = \frac{L_i}{L_0}$ can be taken as test statistics to test the above null hypothesis.

Since the distribution of $\frac{L_1}{L_0}$ is not analytical, we have computed the percentiles of empirical sampling distributions of $T_i = \frac{L_i}{L_0}$ with the help of 10,000 simulation runs of sample size n=2,...,10 are given in table 1.

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The power of the test statistic $T_1$ is also tabulated for 3 different levels of significance (1%, 2.5% & 5%) at the sample sizes $n=2, 3, \ldots, 10$ by simulating sampling from $P_i$ and using the values of $T_1$. The count of $T_1$ values that fall beyond the table values of table 1. These are given in the table 2.

Table 2: Power of $T_1$

| $n$ | 0.9900 | 0.9750 | 0.9500 |
|-----|--------|--------|--------|
| 2   | 9997   | 9987   | 9919   |
| 3   | 9846   | 9650   | 9328   |
| 4   | 9831   | 9627   | 9300   |
| 5   | 9885   | 9638   | 9282   |
| 6   | 9822   | 9752   | 9249   |
| 7   | 9789   | 9528   | 9142   |
| 8   | 9826   | 9610   | 9245   |
| 9   | 9788   | 9488   | 9107   |
| 10  | 9771   | 9528   | 9100   |

These tables indicate that even with the help of a small sample of size as small as 2 the power remains to be at more than 99%. It is therefore concluded that the $T_1$ statistic proposed in this section cannot discriminate between the null and alternative population with a high power values.

### III. TEST STATISTICS BASED ON QUANTILES

Let $x_i, x_2, \ldots, x_n$ denote sample from DD. The correlation type goodness of fit test as given by [6&7] in this case using percentiles can be formed as

$$T_2 = \frac{\sum_{i=1}^n X_i v_i}{\sqrt{\sum_{i=1}^n \sum_{j=1}^n v_i v_j}}$$

(3.1)

This statistic represents the correlation between $x_i$ & $v_i$, $i=1, 2, \ldots, n$ where $v_i$ is the $i^{th}$ quantile of the null population. The statistic $T_2$ is simulated through Monte Carlo method based on 10,000 simulations. Table 3 represents the percentiles of $T_2$ for sample sizes $n=2, 3, \ldots, 10$ and various levels of significance.

Table 3: Percentiles Of $T_2$ Based On Quantile

| $n$ | 0.99865 | 0.9900 | 0.9750 | 0.9500 | 0.0500 | 0.0250 | 0.01 | 0.005 | 0.00135 |
|-----|---------|--------|--------|--------|--------|--------|------|-------|---------|
| 2   | 1.0000  | 0.9999 | 0.9999 | 0.9999 | 0.0244 | 0.0244 | 0.0244 | 0.0244 | 0.0244 |
| 3   | 0.9999  | 0.9998 | 0.9992 | 0.9984 | 0.0032 | 0.0027 | 0.0026 | 0.0026 | 0.0026 |
| 4   | 0.9998  | 0.9991 | 0.9972 | 0.9952 | 0.0017 | 0.0007 | 0.0005 | 0.0005 | 0.0005 |
| 5   | 0.9996  | 0.9969 | 0.9936 | 0.9923 | 0.0020 | 0.0005 | 0.0002 | 0.0001 | 0.0001 |
| 6   | 0.9989  | 0.9937 | 0.9901 | 0.9896 | 0.0012 | 0.0007 | 0.0001 | 0.0001 | 0.0001 |
| 7   | 0.9987  | 0.9931 | 0.9883 | 0.9874 | 0.0025 | 0.0012 | 0.0005 | 0.0003 | 0.0001 |
| 8   | 0.9968  | 0.9874 | 0.9856 | 0.9839 | 0.0010 | 0.0003 | 0.0001 | 0.0001 | 0.0001 |
| 9   | 0.9961  | 0.9855 | 0.9838 | 0.9808 | 0.0007 | 0.0002 | 0.0001 | 0.0001 | 0.0001 |
| 10  | 0.9951  | 0.9831 | 0.9823 | 0.9713 | 0.0007 | 0.0002 | 0.0001 | 0.0001 | 0.0001 |
As we can see from table -3, the percentile points of $T_2$ increase as the sample size increases as well as the significance level increases. The power of the test statistic $T_1$ is also tabulate for 3 different levels of significance (1%,2.5%,5%) at the sample sizes $n=2,3 \ldots 10$ by simulating sampling from $P_1$ and using the values of $T_2$. The count of $T_2$ values that fall beyond the table values of table – 3. These are given in Table-4.

Table-4 Power of $T_2$

| n  | 0.99 | 0.975 | 0.95 |
|----|------|-------|------|
| 2  | 9991 | 9754  | 9500 |
| 3  | 9978 | 9877  | 9699 |
| 4  | 9980 | 9892  | 9774 |
| 5  | 9985 | 9897  | 9781 |
| 6  | 9972 | 9869  | 9723 |
| 7  | 9990 | 9926  | 9799 |
| 8  | 9987 | 9897  | 9742 |
| 9  | 9985 | 9902  | 9754 |
| 10 | 9991 | 9958  | 9938 |

IV. COMPARATIVE STUDY&CONCLUSION

The powers given in table 2 & 4 for the test statistics $T_1$ and $T_2$ respectively indicate that the two statistics do not discriminate between Dagum & Burr-III distributions significantly. i.e there is much similarity between these two distributions for larger values of n. If the sample size increases the Dagum distribution and Burr-3 distribution cannot be distinguished with respect to proposed test statistics. We may say that for data from Dagum distribution and the application of Dagum distribution is quite complicated, the methodology of Burr-III distribution can be used in large samples.

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