Performance of selection combiner over Nakagami fading channel with Laplace cochannel interference

Vinay Kumar Pamula\(^{1a}\) and Anil Kumar Tippari\(^{2b}\)
\(^1\) Department of ECE, University College of Engineering Kakinada, JNTUK, Kakinada 533001, India
\(^2\) Department of ECE, CMR Institute of Technology, Hyderabad 501401, India
\(a\) pamulavk@ieee.org
\(b\) tvkumar2000@yahoo.co.in

Abstract: Wireless communications channel is multipath propagation channel and its main drawback is fading that degrades system performance. Multipath fading effects can be mitigated by using diversity combing scheme at the receiver. Many diversity combining schemes were considered in the literature to study their performance in different multipath fading environments. Recently, N. C. Beaulieu considered generalized framework for performance analysis of selection combining diversity in Rayleigh fading channel by deriving expressions for average error rate. However, experimental measures shown that the Nakagami fading channel model is a good fit to different multi-path fading environments. Hence the performance analysis of selection diversity combining scheme in Nakagami fading channels with Laplace cochannel interference is presented in this paper by deriving the average error rate of binary phase shift keying signals.

Keywords: cochannel interference, diversity combining, fading channel, Nakagami distribution

Classification: Wireless Communication Technologies

References

[1] G. Brennan, “Linear diversity combining techniques,” Proc. IEEE, vol. 91, no. 2, pp. 331–356, Feb. 2003. DOI:10.1109/JPROC.2002.808163
[2] N. C. Beaulieu, “A novel generalized framework for performance analysis of selection combining diversity,” IEEE Trans. Commun., vol. 61, no. 10, pp. 4196–4205, Oct. 2013. DOI:10.1109/TCOMM.2013.082813.110387
[3] M. K. Simon and M. S. Alouini, Digital Communication over Fading Channels, 2nd ed., Wiley, New York, 2005.
[4] N. C. Beaulieu and A. M. Rabiei, “Linear diversity combining on Nakagami-0.5 fading channels,” IEEE Trans. Commun., vol. 59, no. 10, pp. 2742–2752, Oct. 2011. DOI:10.1109/TCOMM.2011.080111.100373
[5] I. S. Gradshteyn and I. M. Ryzhik, Tables of Integrals, Series, and Products, 6th ed., Academic Press, 2000.
1 Introduction

Wireless communications finds a broad usage among all the communication techniques. This technique plays a vital role in many areas of science and technology viz., cellular mobile radio communication. But the major limitation with this technique is signal degradation due to multipath effects i.e., fading. When a radio frequency (RF) wave is transmitted over a wireless link, the amplitude and phase of the received signal fluctuate randomly with respect to time. This random phenomenon is called as multipath fading [1]. The channels in wireless communication are multipath propagation channels and the fading occur at the receiver side. This in turn degrades the performance of wireless communication system [2]. Henceforth, the diversity combining techniques are implemented at the receiver to overcome the effects of channel fading. Selection combining (SC), maximal ratio combining (MRC) and equal gain combining (EGC) are considered as basic linear diversity combining techniques [3, 4] and studied extensively in the literature. The SC scheme finds a widely spanned significance over all the combining schemes because of its less complexity [3].

Recently, N. C. Beaulieu [2] considered the generalized framework for performance analysis of the $D$-fold SC scheme in Rayleigh fading channel with cochannel interference by deriving expressions for average error rate of binary phase shift keying (BPSK) modulation scheme.

In this work, some results that give the error rate performance of $D$-fold SC scheme over Nakagami fading channel with Laplace cochannel interference are presented.

The remaining portion of this paper is organized as follows: Selection diversity combining scheme principle and its importance is stated in Section 2. Whereas section 3 reports the evaluation of error rate performance of selection diversity reception scheme for different Nakagami fading channels. Analytical results are shown in section 4.

2 Selection diversity combining

First, signals are received at the receiver end and selecting the best signal among all the received signals, this is the basic principle SC diversity is based. Because of this principle [3] SC is termed as least complicated of all the basic diversity schemes MRC, EGC and SC, to receive transmitted signal in multipath fading channels. Highest signal-to-noise ratio (SNR) branch is chosen by the combiner. As the output of SC is only one branch signal [3] individual branch signals coherent sum is not required. And hence, SC is used with coherent and non-coherent modulation techniques. Due to this specification SC scheme is simple and low cost.

3 Average error rate

This section presents the average error rate of BPSK signals to study the performance of $D$-fold SC scheme.
The error rate of \( D \)-branch SC for Rayleigh fading is given by [2]

\[
P_f^{SC}(\tilde{\gamma}) = \sum_{k=0}^{D-1} (-1)^k \binom{D}{k+1} P_f(\frac{\tilde{\gamma}}{1+k})
\]

(1)

where \( \tilde{\gamma} \) is the average faded signal-to-interference ratio (SIR) and \( P_f(\cdot) \) is the average probability of error in Rayleigh fading channel. It is assumed that the cochannel interference has Laplace distribution which is a good model for the multiuser interference in wireless communication networks [2]. For a Laplace cochannel interference environment, the error rate is given by [2]

\[
P_{f}^{Laplace}(\rho) = \frac{e^{-\sqrt{2}\rho}}{2}
\]

(2)

where, \( \rho \) is the instantaneous unfaded signal-to-interference ratio (SIR). In this work, Eq. (1) and Eq. (2) are used to derive the expressions for the average error rate of BPSK signals over Nakagami fading channels.

3.1 Nakagami-\( m \) fading channel

The probability density function (PDF) of Nakagami-\( m \) random variable is given by [3]

\[
P_r(r) = \frac{2m^m r^{2m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{mr^2}{\Omega}\right)
\]

(3)

where, \( m \) is the Nakagami fading parameter and \( \Omega \) is the mean-square value of the channel amplitude, \( r \), given by \( \Omega = E[r^2] \), here \( E[\cdot] \) is the statistical expectation operator. The average error probability over Nakagami-\( m \) fading channel with Laplace cochannel interference can be derived by averaging (2) over the Nakagami-\( m \) fading distribution (3), by replacing \( \rho \) with instantaneous faded SIR, \( r^2 \rho \), as in [2], we get

\[
P_{f}^{Laplace}(\rho) = \left(\frac{m}{\Omega}\right)^m \frac{1}{\Gamma(m)} \int_0^{\infty} r^{2m-1} e^{-\frac{mr^2}{\Omega}} \sqrt{2\rho} dr
\]

(4)

Expressing (4) in terms of average faded SIR, \( \tilde{\gamma} = 2\sigma^2 \rho \), and by letting \( \sigma^2 = \Omega \), we get

\[
P_{f}^{Laplace}(\frac{\tilde{\gamma}}{2\Omega}) = \left(\frac{m}{\Omega}\right)^m \frac{1}{\Gamma(m)} \int_0^{\infty} r^{2m-1} e^{-\frac{mr^2}{\Omega}} \sqrt{\tilde{\gamma}} dr
\]

(5)

The average symbol error rate of \( D \)-fold SC scheme in Nakagami-\( m \) fading with Laplace cochannel interference, \( P_f^{Laplace}(\tilde{\gamma}) \), can be obtained by using (5) and (1) as

\[
P_f^{Laplace}(\tilde{\gamma}) = \sum_{k=0}^{D-1} (-1)^k \binom{D}{k+1} P_{f}^{Laplace}\left(\frac{\tilde{\gamma}}{2\Omega(k+1)}\right)
\]

(6)

Substituting (5) in (6) and then applying numerical integration to compute the integral of (5), the average symbol error rate can be derived and the corresponding analytical result is shown in Fig. 1(a) for different order of diversity.
3.2 Nakagami-\(n\) fading channel

The PDF of Nakagami-\(n\) random variable is given by [3]

\[
P_f(r) = \frac{2(1 + n^2)e^{-n^2r}}{\Omega} e^{-\frac{(1+n^2)r}{\Omega}} I_0\left(2n\sqrt{\frac{1 + n^2}{\Omega}}ight) \tag{7}
\]

And across the Nakagami-\(n\) fading PDF given in (7),

\[
P_f^{Laplace}(\rho) = \frac{1 + n^2}{\Omega} \int_0^\infty re^{-(1+n^2)r}\sqrt{2n^2 r^2} I_0\left(2n\sqrt{\frac{1 + n^2}{\Omega}}ight) dr \tag{8}
\]

Expressing (8) in terms of \(\gamma\), as

\[
P_f^{Laplace}\left(\frac{\gamma}{2\Omega}\right) = \frac{1 + n^2}{\Omega} \int_0^\infty r I_0\left(2n\sqrt{\frac{1 + n^2}{\Omega}}\right) \exp\left(-\frac{(1+n^2)r^2}{\Omega} - \frac{\gamma r^2 - n^2}{\Omega}\right) dr \tag{9}
\]

The average symbol error rate of \(D\)-fold SC scheme in Nakagami-\(n\) fading with Laplace cochannel interference, \(P_f^{Laplace}(\gamma)\), can be derived by using (9) in (1) as

\[
P_f^{Laplace}(\gamma) = \sum_{k=0}^{D-1} (-1)^k \left(\frac{D}{k+1}\right) P_f^{Laplace}\left(\frac{\gamma}{2\Omega(k+1)}\right) \tag{10}
\]

substituting (9) in (10) and then applying numerical integration then the analytical result is obtained which is shown in Fig. 1(b).

3.3 Nakagami-\(q\) fading channel

The PDF of Nakagami-\(q\) fading channel is given by [3] as

\[
P_f(r) = \frac{(1 + q^2)r}{q\Omega} \exp\left(-\frac{(1 + q^2)^2}{4q^2\Omega} r^2\right) I_0\left(\frac{1 - q^4}{4q^2\Omega} r^2\right) \tag{11}
\]

The average error probability over Nakagami-\(q\) fading with Laplace cochannel interference can be derived by averaging (2) across the PDF of Nakagami-\(q\) fading given in (11),

\[
P_f^{Laplace}(\rho) = \frac{1}{2q\Omega} \int_0^\infty e^{-\frac{(1+q^2)^2}{4q^2\Omega} r^2} (1 + q^2) r I_0\left(\frac{1 - q^4}{4q^2\Omega} r^2\right) dr \tag{12}
\]

Expressing (12) in terms of \(\gamma\), we get

\[
P_f^{Laplace}\left(\frac{\gamma}{2\Omega}\right) = \frac{1}{2q\Omega} \int_0^\infty e^{-\frac{(1+q^2)^2}{4q^2\Omega} r^2} (1 + q^2) r I_0\left(\frac{1 - q^4}{4q^2\Omega} r^2\right) dr \tag{13}
\]

The average symbol error rate of \(D\)-fold SC scheme in Nakagami-\(n\) fading with Laplace cochannel interference, \(P_f^{Laplace}(\gamma)\), can be derived by using (13) in (1) as

\[
P_f^{Laplace}(\gamma) = \sum_{k=0}^{D-1} (-1)^k \left(\frac{D}{k+1}\right) P_f^{Laplace}\left(\frac{\gamma}{2\Omega(k+1)}\right) \tag{14}
\]

The analytical result is obtained by applying numerical integration to (14) which is shown in Fig. 2(a).
3.4 Nakagami-0.5 fading channel

The probability density function (PDF) of Nakagami-0.5 fading channel is given in [4] as

\[ f_\rho(r) = \frac{2}{\sqrt{2\pi\Omega}} e^{-\frac{r^2}{2\Omega}} u(r) \quad (15) \]

In a D-branch combining scheme, the PDF of D-branch selection combining is given in [4] as

\[ f_{SC}(r) = \frac{2D}{\sqrt{2\pi\Omega}} e^{-\frac{r^2}{2\Omega}} \left[ 1 - 2Q\left(\frac{r}{\sqrt{\Omega}}\right)\right]^{D-1} u(r) \quad (16) \]

where \( u(r) = \begin{cases} 1 & \text{for } r \geq 0 \\ 0 & \text{for } r < 0 \end{cases} \) and \( Q(\cdot) \) is the Gaussian Q-function [5]. The average probability of error is obtained by using the equation given in (2) as,

\[ P_f^{SC}(\tilde{\gamma}) = \int_0^\infty f_{SC}(r) P_e(r\sqrt{\text{SNR}}) dr \quad (17) \]

where \( P_e(r\sqrt{\text{SNR}}) \) is the instantaneous error rate that is a function of instantaneous SNR. Substituting (16) in (17) yields

\[ P_f^{SC}(\tilde{\gamma}) = \int_0^\infty D[1 - e^{-\frac{r^2}{2\Omega}}]^{D-1} \frac{2e^{-\frac{r^2}{2\Omega}}}{\sqrt{2\pi\Omega}} P_e(r\sqrt{\text{SNR}}) dr \quad (18) \]

Now, using the binomial expansion in (18) gives

\[ P_f^{SC}(\tilde{\gamma}) = \int_0^\infty D \sum_{k=0}^{D-1} (-1)^k \binom{D-1}{k} e^{-\frac{r^2}{2\Omega}} \frac{2}{\sqrt{2\pi\Omega}} P_e\left(\frac{\text{SNR}}{k+1}\right) d\tilde{\gamma} \quad (19) \]

Now making the substitution, \( \alpha = r\sqrt{k+1} \) in (19) to obtain

\[ P_f^{SC}(\tilde{\gamma}) = D \sum_{k=0}^{D-1} (-1)^k \binom{D-1}{k} e^{-\frac{\alpha^2}{2\Omega}} \frac{1}{\sqrt{k+1}} \int_0^\infty \frac{2}{\sqrt{2\pi\Omega}} e^{-\frac{\alpha^2}{2\Omega}} P_e\left(\frac{\text{SNR}}{k+1}\right) d\alpha \quad (20) \]

Note that

\[ P_f(\tilde{\gamma}) = \int_0^\infty f_{\rho}(\alpha) P_e(\alpha\sqrt{\text{SNR}}) d\alpha \quad (21) \]

By using (21), (20) can be written as

\[ P_f^{SC}(\tilde{\gamma}) = D \sum_{k=0}^{D-1} (-1)^k \binom{D-1}{k} \frac{1}{\sqrt{k+1}} P_f\left(\frac{\tilde{\gamma}}{k+1}\right) \quad (22) \]

The average error rate in Nakagami-0.5 fading channel with Laplace cochannel interference can be derived by averaging (2) across the Nakagami-0.5 PDF given in (15),

\[ P_f^{\text{Laplace}}(\rho) = \frac{1}{\sqrt{2\pi\Omega}} \int_0^\infty e^{-\frac{\rho^2}{2\Omega}} d\rho \quad (23) \]

After some algebraic manipulation (23) can be written as
\[ P_f^{Laplace}(\rho) = \frac{1}{2} \exp(\rho \Omega)[1 - \text{erf}(\sqrt{\rho \Omega})] \quad (24) \]

Substitute (24) in (22) yields
\[ P_f^D = D \sum_{k=0}^{D-1} (-1)^k \begin{pmatrix} D - 1 \\ k \end{pmatrix} \frac{1}{\sqrt{k + 1}} \frac{1}{2} e^{\frac{-\bar{\rho}}{2(k + 1)}} \left[ 1 - \text{erf}\left(\frac{\sqrt{\bar{\rho}}}{2(k + 1)}\right)\right] \quad (25) \]

4 Results and discussion

Consider a wireless transmission system having \( D \) (here \( D = 1, 2, 3, 4 \)) receiving antennas. In all the simulations, it assumed that \( \Omega = 10 \text{ dB} \). The average SER vs. average faded SIR in Nakagami-\( m \) fading for \( D \)-fold selection diversity is shown in Fig. 1(a) for \( m = 1 \). The fading severity of the Nakagami-\( m \) channel is given by the parameter \( m \). The fading parameter value \( m = 0.5 \) indicates severe fading condition and as it increases fading decreases, i.e., \( m \to \infty \) indicates no fading. As \( m \) increased to \( \infty \) the fading effect gets zero and Nakagami-\( m \) channel converges to non-fading. Both the Rayleigh (i.e., \( m = 1 \)) and Rician channels can be considered as the special cases of Nakagami-\( m \) channel. Rician distribution is described by a fading parameter \( k \), that is, the ratio of the power in the line-of-sight (LOS) component of the power in the multipath components. Furthermore, when \( m < 1 \) we obtain a one-to-one mapping between the \( m \)-parameter and the \( q \)-parameter allowing the Nakagami-\( m \) distribution to closely appears the Nakagami-\( q \). The average SER vs. average faded SIR for \( D \)-fold selection combiner in Nakagami-\( q \) channel is presented in Fig. 2(a).

The average SER vs. average faded SIR in Nakagami-\( n \) fading for \( D \)-fold selection diversity is shown in Fig. 1(b) for \( n = 0 \) (Rayleigh fading). In Nakagami-\( n \) fading, \( n \to \infty \) indicates no fading. The Nakagami-0.5 distribution is an important special case of the Nakagami distribution as it represents a worst case

![Fig. 1. The average SER vs. Average Faded SIR in (a) Nakagami-\( m \) fading with \( m = 1 \), and (b) Nakagami-\( n \) fading with \( n = 0 \), for \( D \)-fold selection diversity scheme.](image-url)
fading. The average SIR vs. SER in Nakagami-0.5 fading for $D$-fold selection diversity is shown in Fig. 2(b).

![Fig. 2. The average SER vs. Average Faded SIR in (a) Nakagami-$q$ fading with $q = 0$, and (b) Nakagami-0.5 fading for $D$-fold selection diversity.](image)

5 Conclusion

The average symbol error rate of SC scheme, operating over different Nakagami channels, has been derived. Analytical results are presented to show the performance of SC diversity scheme over Nakagami fading channels. Results show that the performance of selection combiner increases with increase in diversity order and SNR in the presence Laplace cochannel interference.