Flavour Transport in Schrödinger Holography

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Abstract. We study using gauge-gravity duality the transport properties of charge carriers in a strongly-coupled theory with nonrelativistic symmetry, which were obtained via an irrelevant deformation that breaks the initial relativistic conformal group down to the Schrödinger group. We compute the associated DC and AC conductivities in the dual gravity description of probe D7-branes in an asymptotically Schrödinger spacetime. We find generically that the conductivities exhibit nonrelativistic scaling with dynamical exponent \( z = 2 \) in the ultraviolet, while in the infrared the scaling is relativistic.

1. Introduction

Gauge-gravity duality [1, 2, 3], or holography, provides new tools to study strongly-coupled theories at finite density, and may also provide novel insights into quantum critical condensed matter systems, which exhibit scale-invariance under “Lifshitz” scaling: \( t \rightarrow \lambda^z t, \, \vec{x} \rightarrow \lambda \vec{x} \), where \( \lambda \) is real and positive. The parameter \( z \) is the so-called dynamical exponent.

A prototype often used in discussions of strongly-coupled systems is the relativistic \( \mathcal{N} = 4 \) \( SU(N_c) \) supersymmetric Yang-Mills (SYM) theory at infinite \( 't \) Hooft coupling. There is, however, another prototype from the realm of nonrelativistic physics: fermions at unitarity [4, 5]. This is the system of fermions interacting via a short-ranged potential fine-tuned to support a zero-energy bound state, and its symmetry is given by the Schrödinger group: the symmetry group of the Schrödinger equation in free space. This is the nonrelativistic version of the conformal symmetry [6], and its algebra is generated by time translation \( H \), spatial translation \( P^i \), rotation \( M^{ij} \), Galilean boosts \( K^i \), dilation \( D \), a number operator \( N \), and for \( z = 2 \) a special conformal operator \( C \).

Unlike in the relativistic case, many of the strongly-coupled nonrelativistic systems are experimentally accessible. It is thus very interesting if weakly-coupled dual gravity descriptions can be found for these systems. Building towards such a dual, first the Schrödinger symmetry should be realised geometrically. This can be accomplished by deforming the anti-de Sitter spacetime in \( d + 3 \) dimensions, \( AdS_{d+3} \) [7, 8]. The resulting metric is

\[
    ds^2 = \frac{1}{r^2} \left( -2(dx^+)^2 \frac{r^2}{r^2} - 2dx^+dx^- + dx^idx^i + dr^2 \right),
\]

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where $x^\pm$ are the usual light-cone coordinates, and the isometry of the metric realises the Schrödinger symmetry $Sch(d)$ in $d$ spatial dimensions, which in particular includes the “Schrödinger” scaling: $x^+ \to \lambda x^+, \ x^- \to x^-, \ \{\vec{x},r\} \to \lambda\{\vec{x},r\}$. Here $x^+$ plays the role of time, and the nonrelativistic Hamiltonian $H$ and the number operator $N$ are identified with the relativistic light-cone momentum operators $P_+ \sim \partial_+$ and $P_- \sim \partial_-$ respectively. Note since the number spectrum is expected to be discrete in nonrelativistic theories, this implies that $x^-$ should be compactified, and a Discrete Light-Cone Quantisation (DLCQ) performed.

Having now the spacetime geometry which manifests the Schrödinger symmetry, the next step is to find gravity solutions for such “Schrödinger spacetime” ($Sch$). One direct method for doing so is to apply the so-called Null Melvin Twist (NMT) to known supergravity (SUGRA) solutions [9, 10, 11]. In particular, applying NMT to type IIB SUGRA on $AdS_5 \times S^5$, which is dual to $\mathcal{N} = 4$ $SU(N_c)$ SYM in the limits of large $N_c$ and ‘t Hooft coupling, one obtains $Sch_5 \times S^5$ dual to $\mathcal{N} = 4$ SYM deformed by a dimension-five operator that breaks the conformal symmetry down to the Schrödinger one.

Our goal is to study the properties of “charges” transport in such holographic Schrödinger system, with the charged carriers (or “flavours”) introduced via probe D7-branes. In the next section, we birefly review the NMT and $Sch_5$ solution. In Sec. 3, we describe the holographic setting, and how flavours are introduced into the Schrödinger spacetime. We then compute the DC conductivity directly from the non-linear Dirac-Born-Infeld (DBI) action in Sec. 4, and the AC conductivity in an analytical regime in Sec. 5. We conclude with a summary and some comments in Sec. 6.

2. The Null Melvin Twist

The NMT is a species of TsT (T-duality, shift, T-duality) transformation that generates from an input of some SUGRA solution with two commuting $U(1)$ isometries a new solution with different asymptotics. We start from $AdS_5$-Schwarzschild $\times S^5$. The metric is

$$ds^2 = \frac{1}{r^2} \left[ \frac{dr^2}{f(r)} - f(r) dt^2 + dy^2 + dx^2 \right] + (d\chi + A)^2 + ds_{\mathbb{CP}^2}^2, \quad f(r) = 1 - \frac{r^4}{r_H^4}, \quad (2)$$

where $r_H$ is the position of the balck hole horizon, and we have written the $S^5$ metric as a Hopf fibration over $\mathbb{CP}^2$. Working in units where the $AdS$ radius is unity, the black hole temperature is given by $T = (\pi r_H)^{-1}$.

Using the Hopf fiber direction $\chi$ and the field theory spatal direction $y$ as the two commuting isometry directions, the steps of NMT are:

(i) Boost in $y$ by $\gamma$.
(ii) $TsT$: T-dualize in $y$, $d\chi \to d\chi + dy$, T-dualize in $y$.
(iii) Boost in $y$ by $-\gamma$.
(iv) Take limit $\gamma \to \infty$, $\alpha \to 0$ keeping $\beta = \frac{1}{2} \alpha e^\gamma$ fixed.

The result is asymptotically $Sch_5$ with the metric:

$$ds^2 = \frac{1}{r^2} \left[ \frac{dr^2}{f(r)} - \frac{f(r)}{r^2 K(r)} (dx^+)^2 + \frac{2}{K(r)} dx^+ dx^- + \frac{1 - f(r)}{2K(r)} \left( \frac{dx^+}{\sqrt{2}\beta} - \sqrt{2}\beta dx^- \right)^2 + dx^2 \right]$$

$$+ \frac{1}{K(r)} (d\chi + A)^2 + ds_{\mathbb{CP}^2}^2, \quad K(r) = 1 + \frac{\beta^2 r^2}{r_H^4}, \quad (3)$$

$\partial_\mu \equiv \partial/\partial x^\mu$. 

$^2$
where
\[ x^+ = \beta(t + x), \quad x^- = \frac{1}{2\beta}(-t + x). \] (4)

Note that we have defined the light-cone coordinates with factors of \( \beta \) so that they have the correct Schrödinger scaling once \( x^+ \) is identified as the time coordinate. The solution includes also an NS two-form field
\[ B = -\frac{1}{2\pi^2 K(r)} (d\chi + \mathcal{A}) \wedge [(1 + f(r))dx^+ + (1 - f(r))2\beta^2 dx^-], \] (5)
and a dilaton
\[ \Phi = -\frac{1}{2} \log K(r). \] (6)

The pre-NMT solution is recovered when taking \( \beta \to 0 \).

The metric in Eq. (3) has a horizon at \( r = r_H \), and so an associated Hawking temperature \( T \). Taking \( r_H \to \infty \) one obtains the \( T = 0 \) solution. The geometry in this case is precisely \( S^5 \), and not just asymptotically so. The \( T = 0 \) solution breaks all supersymmetry [10] and has a singularity at \( r = \infty \) [11]. Furthermore, at \( T = 0 \), DLCQ produces a null circle in the compactified \( x^- \) direction, which invalidates the SUGRA approximation because as any closed string wrapping the null circle will be massless [10]. However, as long as the spacetime has momentum in the \( x^- \) direction, the \( x^- \) circle is not null, and the problem is avoided [10]. Indeed, the finite-\( T \) solution always has \( x^- \) momentum since the NMT involve boosts, and it is clear from Eq. (3) that the \( x^- \) direction is not null for \( T \neq 0 \) (\( r_H < \infty \)). The dual field theory is then in a state with a finite number density, or equivalently a finite chemical potential (associated with the \( U(1) \) of the compact \( x^- \)). The field theory temperature \( T \) and chemical potential \( \mu \) are given by [9, 10]
\[ T = \frac{1}{\pi r_H \beta}, \quad \mu = -\frac{1}{2\beta^2}. \] (7)

3. The holographic set-up

The field theory dual to type IIB SUGRA on \( Sch_5 \) is that of \( \mathcal{N} = 4 \) \( SU(N_c) \) SYM deformed by a dimension-five irrelevant operator, \( O_\perp \), which breaks the relativistic conformal group of the SYM down to the Schrödinger group [9].

To study charge transport, we need to have charged degrees of freedom. This is done by introducing into the Schrödinger background \( N_f \) coincident probe D7-branes, which dual to \( N_f \) \( \mathcal{N} = 2 \) supersymmetric hypermultiplets in the fundamental representation of the \( SU(N_c) \) gauge groups [12]. We work in the probe limit where \( N_f \ll N_c \). The \( U(N_f) \) gauge invariance of the D7-branes is dual in the field theory to the global \( U(N_f) \) flavour symmetry, whose overall diagonal \( U(1) \) factor we identify as the “baryon” number in analogy with QCD. From gauge-gravity duality, the conserved \( U(1) \) current \( J^\perp \) is dual to the worldvolume gauge field \( A_\perp \) living on the D7-branes, and the mass operator is dual to the worldvolume scalar \( \theta \) that describes the embedding of the D7-branes in the background geometry.

To study transport properties, we need a finite charge density, \( \langle J^\perp \rangle \), a finite current density, \( \langle J^x \rangle \), as well as an electric field that drives the charge flow, \( F_{tx} \). We should thus have the worldvolume gauge fields \( A_t, A_x, \) and \( F_{tx} \) present. The dynamics of D7-branes is then described by the Abelian DBI action
\[ S_{D7} = -N_f T_{D7} \int d^8 \xi \, e^{-\Phi} \sqrt{-\text{det}(\mathcal{P}[g + B]_{ab} + (2\pi \alpha' F_{ab})}, \] (8)

\[ \text{3 That this is so can be seen from the Kaluza-Klein reduction of the } S^5 \text{ solution on } S^5 \text{ in which the NS } B\text{-field give rise to a massive vector in } Sch_5 \text{ whose dual operator is the vector } O_\parallel. \]
where \( T_{D^7} \) is the D7 tension, \( \xi_a \) the D7 worldvolume coordinates, \( \Phi \) the dilaton, \( P^{[g + B]}_{ab} \) the pull-back of the metric and the NS \( B \)-field to the D7 worldvolume, and \( F_{ab} \) the D7 \( U(1) \) field strength.

4. DC conductivity

We calculate the DC conductivity associated with the transport of baryon number charges directly from the DBI action using the method of Ref. [13], which capture effects beyond those of linear response. Below we describe the method and record the main results. The details of the calculation is found in Ref. [14].

Because of the DLCQ, we work in light-cone coordinates. Our ansatz for the worldvolume gauge field is thus:

\[
A_+(x, r) = E_\beta x + h_+(r), \quad A_-(x, r) = -2\beta^2 E_\beta x + h_-(r), \quad A_x(r),
\]

where we have redefined the electric field to be \( E_\beta = E/(2\beta) \) so that it has the correct nonrelativistic scaling (\( [E] = z + 1 = 3 \)), and \( h_\pm(r) \) are functions to be solved for. With this gauge field ansatz, we can compute directly from the DBI action in Eq. (8) the one-point nonrelativistic scaling (\( [E] = E/\beta T \) \( \to \beta T \)).

The result is

\[
\sigma = \sqrt{N^2 64 E_\beta^2 \beta^6 \cos^2 \theta(r_s)} + \frac{f(r_s)^2}{4\beta^4 E_\beta^3 r_s^4} (J^+)^2,
\]

where \( N = N_f T_{D^7} 2\pi^2 \), and \( r_s \) is determined from the real action condition by

\[
\frac{1}{E_\beta^2} = \frac{4r_s^2 \beta^2}{f(r_s)} [r_s^2 - \beta^2 f(r_s) \sin^2 \theta(r_s)].
\]

As in the relativistic \( AdS_5 \) case, there are two contributions to \( \sigma \) (see Ref. [13]). One comes from the charge carriers we explicitly introduced via the density \( \langle J^+ \rangle \) and is proportional to \( \langle J^+ \rangle^2 \). The other is proportional to \( \cos^6 \theta(r_s) \), and describes the contribution to charge-neutral pairs, which appears to arise from Schwinger and/or thermal pair production. This may seem strange in a nonrelativistic field theory, since the number of particles should not change. But given that the theory here is obtained by first deforming a relativistic one by an irrelevant operator and then perform a DLCQ, nothing is wrong with pair production.

To explore the scaling behaviour of the conductivity, we take two limits. First, in the “weak field” limit where \( E_\beta \ll \beta T^2 \), we have from Eq. (11) \( r_s \to r_H \), and

\[
\sigma \approx \sqrt{N^2 \pi^2 \frac{T^2}{\mu^2} \cos^6 \theta(r_s)} + \frac{16 \langle J^+ \rangle^2 \mu^2}{\pi^2 T^2 T^2}.
\]

Going further to the low temperature (\( T/\mu \to 0 \)) or large mass (\( \cos \theta(r_s) \to 0 \) [13]) regime, pair production is suppressed and we get

\[
\sigma \approx \frac{4 \langle J^+ \rangle}{\pi^2} \frac{|\mu|}{T}.
\]
In a scale-invariant theory with dynamical exponent $z$, the conductivity should behave as $\sigma \sim \langle J^+ \rangle T^{-2/z}$. Thus we see if we fix the chemical potential $\mu$, the scaling of $\sigma$ is relativistic with $z = 1$. On the other hand, if we fix the ratio $\mu/T$ instead while varying the temperature, the scaling is nonrelativistic with $z = 2$. We can understand this intuitively from the geometry. With the conductivity evaluated essentially at $r_H$, fixing $\mu/T$ is the same as fixing $r_H/\beta$. We are thus probing the geometry on the scale of the deformation $\beta$, and so a nonrelativistic behaviour is expected. If instead we fix $\mu$, taking $T/\mu \to 0$ is the same as having $r_H/\beta \to \infty$. The horizon then enters the region that is $AdS$-Schwarzschild-like, and relativistic scaling is then expected.

In the opposite “strong field” limit where $E_\beta \gg \beta T^2$, going to the small mass $(\theta(r_*) \approx 0$ [13]) regime, we have from Eq. (11) $r_* \sim (2E_\beta \beta)^{-1/2}$, and

$$\sigma \approx \sqrt{\frac{N^2}{8\frac{2}{3} \mu^2}} \frac{E_\beta}{\mu^{3/2}} \cos^6 \theta(r_*) + \frac{2|\mu|^2}{E_\beta^2} \frac{\langle J^+ \rangle}{\theta(r_*)} \to \frac{N}{8\frac{2}{3} \mu^{3/2}} \sqrt{\frac{E_\beta}{|\mu|^{3/2}}}.$$  

We see that the conductivity is non-vanishing after setting both mass and density to zero, and so must arise from the Schwinger process. Fixing $\mu$, we see $\sigma \propto \sqrt{E_\beta}$, which has the same scaling with the electric field as the $T = 0$ relativistic case [13]. On the other hand, fixing the ratio $E_\beta/|\mu|^{3/2}$ while varying $E_\beta$ leads to a constant conductivity, which is expected from a (2+1)-dimensional theory with nonrelativistic scaling if the only scale is the electric field.

5. AC conductivity

We compute the AC conductivity using linear response. To proceed, the external electric field $s$ now turned off, and we consider a small frequency-dependent perturbation of the subsequent background solution

$$A_x(x^+, x^-, r) = \Re \left[ e^{-i\omega(x^++2\beta^2x^-)}a_x(r, \omega) \right].$$  

For simplicity, we work with zero spatial momentum here.

To quadratic order, the Lagrangian density for the perturbation is

$$\mathcal{L}_\epsilon = \alpha_+ f_{++}^2 + \alpha_- f_{--}^2 + 2\alpha_+ f_{+-} f_{-+} - \alpha_{\tau\tau} f_{\tau\tau}^2,$$

and the resulting equation of motion

$$a_x'' + \frac{\alpha_{\tau\tau}}{\alpha_{\tau\tau}} a_x' + \omega^2 \frac{\alpha_{++} + 4\beta^4 \alpha_{--} - 4\beta^2 \alpha_{+-}}{\alpha_{\tau\tau}} a_x = 0.$$  

The quantity $f_{mn} = \partial_m a_n - \partial_n a_m$ is the perturbation field strength, and the coefficients, $\alpha_{mn}(r)$, depends on the background solution for the worldvolume gauge fields present (through the charge density $\langle J^+ \rangle$) and the D7-brane embedding $\theta(r)$ (for their explicit form, see Re. [14]).

Following the now-standard recipe of computing transport coefficients holographically in linear response (for a review, see e.g. [15]), the AC conductivity is extracted, via the Kubo formula, from the retarded Green’s function computed from the functional derivative of the on-shell action. The result is

$$\sigma(\omega) \propto \lim_{r \to 0} \frac{\alpha_{\tau\tau} a_x'(r, \omega)}{\omega a_x(r, \omega)},$$

where we have omitted the overall factor, since we are primarily interested in scaling behaviour of $\sigma$ with frequency $\omega$. Here, the dependence on the background solution $\theta(r)$ is crucial. In

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4 The solution with the external electric field and the worldvolume gauge field $A_x$ turned off is obtained in the same way as described in the previous section. See Ref. [14] for more details.
In general, with finite temperature and density, $\theta(r)$ can only be solved for numerically. We will not do so here but instead we shall focus on a regime of physical interest where analytical results can be obtained with appropriate approximations.

The regime we are interested in is one that is dominated by the physics of a “quantum critical point” at $T = 0$. In particular, we work in the limit $T \ll \mu$, $\omega \ll m$, $\langle J^+ \rangle^{1/d}$, where temperature is the much smaller than all other scales (to suppress thermal effects), while mass and density are much larger (to avoid scale-invariance breaking effects from the probe D7 sector). Recall that the Schrödinger geometry interpolates between a UV critical point with $z = 2$ and an IR one with $z = 1$. By dialling the chemical potential $\mu$ with respect to $\omega$ (or equivalently, $\beta$ with respect to $r$), we can explore both the $\mu \ll \omega$ ($\beta \gg r$) UV region and the $\mu \gg \omega$ ($\beta \ll r$) IR region.

At finite temperature, if the density is zero, the D7-brane which wraps an equatorial $S^3 \subset S^5$ can “slip off” the $S^5$, collapse to a point at some $r = r_\Lambda$ and end there. But at finite density, the D7 must extend all the way to the horizon, and a spike forms when $r_\Lambda \ll r_H$ [16]. Along the spike $r_\Lambda < r \ll r_H$, $\theta(r)$ is approximately constant. Such regime corresponds to scales below the mass gap of the charge carriers, where scale invariance is approximately restored.

In the spike regime, the equations of motion in the IR and UV region becomes

\begin{align}
\text{IR} : & \quad \partial_r^2 \theta + \frac{\beta^2}{r^4} \sin \theta_0 \cos \theta_0 = 0, \\
\text{UV} : & \quad \partial_r \left( \frac{\partial_r \theta}{r} \right) + \frac{\cot \theta_0}{r^4} = 0,
\end{align}

where $\theta(r) \approx \theta_0$ along the spike. The respective solutions are

\begin{align}
\text{IR} : & \quad \theta(r) = \theta_0 + \frac{\beta^2}{6r^2} \sin \theta_0 \cos \theta_0, \\
\text{UV} : & \quad \theta(r) = \theta_0 + \frac{\cot \theta_0}{2} \log \frac{r}{r_0}, \quad |\cot \theta_0| \ll 1, \quad \left| \log \frac{r}{r_0} \right| \ll 1.
\end{align}

Following Ref. [17], we have introduced a reference scale $r_0$ where $r_\Lambda < r_0 < \beta$ to make the logarithm in the UV solution dimensionless.

To compute the retarded Green’s function we need to solve the equation of motion for the gauge field fluctuation, Eq. (17), with in-going boundary condition. In the spike regime, the fluctuation equation of motion in the IR and UV region has simple forms and can be solved analytically:

\begin{align}
\text{IR} : & \quad a''_x + \frac{2}{r} a'_x + 4\beta^2 \omega^2 a_x = 0 \implies a_x(r, \omega) = a_x^0 e^{2\beta \omega r}, \\
\text{UV} : & \quad a''_x + \frac{1}{r} a'_x + 4\beta^2 \omega^2 a_x = 0 \implies a_x(r, \omega) = a_x^0 H_0^{(1)}(\omega r^2).
\end{align}

We now have all the ingredients to compute $\sigma(\omega)$. However, we cannot directly apply the Kubo formula in Eq. (18), since the $r \to 0$ limit is outside the regime where our approximations are valid. Instead, we will compute a “local conductivity” $\sigma(\omega, r_0)$ at the reference scale $r_0$, which is argued in Ref. [17] in the general Lifshitz case to have the same scaling behaviour with $\omega$ as $\sigma(\omega)$ in the limit $\omega r_0^2 \ll 1$. Specifically, we compute

\begin{align}
\sigma(\omega, r_0) = & \left. \frac{\alpha r a'_x(r, \omega)}{\omega a_x(r, \omega)} \right|_{r_0} \\
\propto & \begin{cases} \frac{(J^+)_0}{\omega} r_0^{-1}, & \beta \omega r_0 \ll 1, \quad \text{IR} \\
\frac{(J^+)_0}{16} (\omega \log(\omega r_0^2))^{-1}, & \omega r_0^2 \ll 1, \quad \text{UV} \end{cases}
\end{align}
Compare with the scaling found in Ref. [17]:

\[
\sigma(\omega, r_0) \propto \begin{cases} 
\langle J^t \rangle^{z/2} \omega^{-1}, & z < 2, \\
\langle J^t \rangle (\omega \log(\omega \Lambda^2))^{-1}, & z = 2, \\
\langle J^t \rangle \omega^{-2/z}, & z > 2,
\end{cases}
\]

(27)

where \( \langle J^t \rangle \) is the charge density and \( \Lambda \) a dimensionful scale, we see in our case the scaling with \( \omega \) is relativistic with \( z = 1 \) in the IR and nonrelativistic with \( z = 2 \) in the UV.

6. Conclusions

Using gauge-gravity, we computed both DC and AC conductivities associated with a finite density of charge carriers in a strongly-coupled theory with nonrelativistic symmetry. The dual field theory is that of \( \mathcal{N} = 4 \) SYM deformed by an irrelevant, dimension-five operator that breaks the conformal group down to the Schrödinger group, with dynamical exponent \( z = 2 \), and the charge carriers comprise of massive \( \mathcal{N} = 2 \) supersymmetric hypermultiplets. We found generically both DC and AC conductivities exhibit scaling with temperature or frequency as appropriate that is relativistic in the IR and nonrelativistic in the UV (with \( z = 2 \)). These results are in accord with our expectations, given that the nonrelativistic Schrödinger symmetry originates from the irrelevant deformation of an initially relativistic theory via the operations of the NMT.

In terms of model-building od the kind advocated in Ref. [17], the advantage of the set-up considered is the a straightforward way to engineer desired nonrelativistic scaling exponents from a known relativistic system using NMT, rather than come up with the relevant nonrelativistic system directly. The disadvantage is that the theory found via the NMT is not genuinely nonrelativistic. Indeed, there is no spontaneous breaking of the \( U(1)_N \) number symmetry generated by the number operator \( N \). Also, thermodynamic quantities such as the free energy density scale with negative powers of \( \mu/T \), and so diverges in the limit \( \mu/T \to 0 \), which appears to be a direct consequence of the DLCQ. Type IIB SUGRA in Schrödinger spacetime is thus apparently not a dual for fermions at unitarity.

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