Aerodynamic performance and blockage investigation of a cambered multi-bladed windmill

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Abstract. Windmills for water pumping typically operate at low speed and high torque owing to their multi-bladed nature. This, however, complicates the rotor aerodynamic behavior due to the mutual interaction between adjacent blades and low Reynolds number, Re, operation. While studies on their aerodynamic performance indicate that increasing blade number, N, and airfoil type are critical analysis and design parameters, its low Re behavior with cambered airfoils appears complicated and is still poorly understood. Accordingly, the performance of a windmill model of diameter 0.68 m with 3 ≤ N ≤ 24 identical blades was investigated in two open jet wind tunnels, with different test section sizes: one with high blockage of 36.3 % and the other with a negligible blockage of 4.5 %, for comparison with Blade Element Theory (BET) predictions of thrust, torque, and power. It was found that BET is accurate except at low tip speed ratios, λ where it under-predicts the torque primarily because of the high solidity at high N. Furthermore, the study reveals that high blockage impacts significantly on rotor performance and is a function of N. Overall, the experiment gave a better performance, highlighting the importance of accounting for solidity in aerodynamic performance prediction.

1. Introduction

Current aerodynamic research focuses, understandably, on high-speed wind turbines generating electricity on a range of scales. On the other hand, small wind turbines for direct mechanical drive applications such as water pumping have been receiving less attention, even when they are most suitable and convenient to use in rural or off-grid areas. Water pumping windmills characteristically start under load, and this requires high torque, which is mostly realized by increasing blade number, N or solidity, σ (ratio of projected blade area to the swept area), which is in the region of 0.5 – 0.8 for the classical American windmill [1]. An example of a water pumping windmill is shown in figures 1 and 2; despite the fact that this has been around for several centuries, not much is known about its detailed aerodynamic performance. Windmill blades are generally cambered and made from thin rolled metal sheets. A spar consisting of a circular tube running along the blade is used to hold and connect it to the hub, as illustrated in figure 2. The simplicity in design, cost-effectiveness, and ease of manufacture of the blades make them attractive for low wind speed operation and suitable for developing countries where access to sustainable energy is still a huge challenge. Compared to the fast-running horizontal axis wind turbines (HAWTs), the multi-bladed rotor usually operates at tip speed ratio, \( \lambda < 3 \),
and has small values of $C_p$; nonetheless, they are for the most part designed to maximize torque for water pumping duties [1].

Understanding the performance characteristics of cambered blades is instructive and provides design guidelines for small wind turbines operating at $Re < 10^5$. Okamoto et al. [4] showed aerodynamically that cambered blades perform better than flat ones at $Re$ range of $10^3$ to $10^4$. The work by Kentfield [5] examined the performance characteristics of three small HAWTs for powering water pumps with different blade configurations. It concluded that based on the starting torque requirement, the multiblade one produced the best performance. The influence of blade curvature ranging from $0 - 14\%$ on the lift and drag characteristics of circular arc steel plates at $Re$ of $2.23 \times 10^5$ and incidence angle, $\alpha$ between $-20^\circ$ and $90^\circ$ were specifically studied by Pandey et al. [6]. They recommended the plate with $8\%$ curvature for high $C_l/C_d$ to maximize $C_p$ for the theoretically examined eight and twelve bladed windmills. A related study was by Bruining [7], though at a single camber of $10\%$. He conducted airfoil tests to determine the influence of tubular spars mounted at different chord positions around the blade profile from the leading edge at $Re$ of $0.6 \times 10^5$, $1 \times 10^5$ and $2 \times 10^5$. The best performance was obtained for the $0.5c$ position on the high-pressure side, a spar position adopted in the current investigation.

The effect of variable $N$ or equivalently $\sigma$ with regards to windmill performance was studied by Wegereef [8] who experimented on a scaled model of an Intermediate Technology Development Group (ITDG) windmill. He measured the performance of $N = 6$, 12, and 24 rotors with identical blades in a wind tunnel. While the work showed significant increments in starting torque with increased blade root (twist) angles, there was no report on thrust measurements and validation of the torque and power results, rendering the analysis incomplete and necessitating a thorough performance assessment of the model. Given their slow rotation and high solidity, windmill aerodynamics performance differs significantly from conventional wind turbines due to the blade to blade interaction and low $\lambda$ operation and consequently low $Re$ [1]. Intriguingly, not much has been reported on this critical aerodynamic interaction between the rotor blades with a cambered profile, particularly as $N$ increases. In an attempt to account for blade interaction, Cascade theory, a two-dimensional method of accounting for finite solidity and often used in turbomachinery, has been deployed to analyze wind turbine performance. Better results in comparison to BET were obtained for high solidity multi-bladed wind turbine rotors, though with a penalty in computational time [9]. Fagbenro et al. [10] studied high solidity effects on
a circular-arc bladed windmill rotor. Their computational study showed that solidity influences lift and drag coefficients, which agrees with the findings of Yan [11] even for low solidity rotors. Singh [12] examined the effect of solidity on the performance parameters of $N = 6, 12, \text{ and } 24$ that replicate that tested in Ref [8]. By comparing BET with wind tunnel measurement, it was concluded that the solidity effects are minimal. Part of the current study is a further investigation of Ref [12] using the same model to examine and understand the aerodynamics of multi-bladed windmills.

For testing in small wind tunnels, the rotor swept area and chord length are maximized to obtain $Re$ for which there are available $C_l$ and $C_d$ data, which results in a high rotor projected area to tunnel area, called blockage ratio ($BR$). Thus, performance data from a scaled experimental rotor model in a wind tunnel are higher than that of the same turbine tested in real wind conditions. This is due to blockage, which alters the flow around the model in a finite wind tunnel. A wide range of blockage assessment and correction studies have been conducted for HAWTs and VAWTs in both closed and open tunnels, e.g., [13, 14, 15, 16]. Blockage has been shown to be significant, especially when the rotor area is $> 10\%$ of the working section cross-section [18]. It is typically more significant in a closed test section than an open one, where the unconstrained streamlines can mimic some features of an unbounded flow. Moreover, in a closed tunnel, the Betz limit or ideal maximum $C_p$, has been reported to be exceeded, similar to a shrouded wind turbine rotor [17]. Even though tunnel blockage as high as $48\%$ [14] has been investigated, its influence on rotor aerodynamic performance with a varying $N$ and high $\sigma$ typical of a water pumping wind turbine has not been dealt with in-depth.

This study is a combined investigation of the effect of increasing blade numbers, $N$ or otherwise high solidity and high blockage on a circular-arc-bladed windmill model operating at low $Re$ tested in two open jet wind tunnels with high and low (negligible) blockage. The influence of blockage and solidity on the aerodynamic performance are presented and compared with a BET using a new tip loss formulation with the overall intent of broadening the current knowledge and understanding of multi-bladed windmill aerodynamics.

2. Experimental setup

2.1. Wind tunnels

The tests for this study were performed at the University of Calgary Red Wind Tunnel (RWT) and the TU Delft Open Jet Facility (OJF). Both tunnels are open jet. The main characteristics of the tunnels are summarized in table 1. For the $15 \text{ m/s}$ nominal wind speed investigated, the combined unsteadiness and turbulence intensity, and non-flow uniformity of the RWT were measured to be $< 0.3\%$ and $\pm 2\%$ respectively. The turbulent intensity of the OJF was reported to be $< 0.25\%$ [19].

2.2. Windmill model

In order to investigate the aerodynamic performance of a water pumping windmill, a previously used turbine model that replicated the experimental rotor of Wegereef [8] and referred to in the work of Singh [12] was tested. The rotor is such that its hub is capable of carrying different $N$

| Name | Location | Type | Test section | Cross section | Max. speed |
|------|----------|------|--------------|---------------|------------|
| OJF  | TU Delft, the Netherlands | Closed circuit | $2.85 \text{ m} \times 2.85 \text{ m}$ | Octagonal | $35 \text{ m/s}$ |
| RWT  | University of Calgary, Canada | Open circuit | $1 \text{ m} \times 1 \text{ m}$ | Square | $18 \text{ m/s}$ |
(taken, for convenience, to be multiples of 3) and perhaps types of blades up to a maximum of 24, as depicted in figure 3. The rotor is of diameter 0.68 m, and the blades with 13% curvature were made from Acrylonitrile Butadiene Styrene (ABS) plastic via rapid prototyping to ensure precision and homogeneity in their profile. The blade model has a constant chord and thickness of 0.04 m and 0.002 m, respectively and a span of 0.2 m. Other geometrical characteristics are illustrated in figure 4. On the concave (high pressure) side of the blade, similar to the scaled rotor of Wegereef, spars of 1/4” diameter (15.88 % chord) extending from the rotor hub were mounted at the 0.5c position to maximize the lift-drag ratio ($C_l/C_d$) as suggested by Bruining [7], as well as for reinforcement and attachment to the rotor hub.

Prior to the windmill testing, the aerodynamic characteristics of a scaled-up two-dimensional model of the blade in figure 4 were determined with and without a spar at the 0.5c position in the RWT for $Re= 0.6 \times 10^5$ and $1 \times 10^5$ at $-10^\circ \leq \alpha \leq 90^\circ$. Details of the experimental procedure can be found in Ref [21]. Figure 5 shows the 24-bladed rotor model in the RWT with the main measuring instruments labeled for easy identification, and mounted at a distance of half rotor diameter downwind of the tunnel outlet. Experimental testing used $N = 3, 6, 12$ and 24 at nominal wind speeds of 15 m/s in both the RWT and OJF at $0.5 < \lambda < 2.5$. For all $N$, the blade pitch angle was the same and equal to zero at the blade tip. The wind speed of RWT was measured using a pitot-static tube connected to an Ashcroft® CXLdp differential pressure transmitter having an accuracy of 0.4 % and installed in front of the rotor and at a larger radius. The forces and moments acting on the model in the RWT were measured using the six-axis ATI Delta SI-330-30 transducer mounted at the bottom of the sting (rotor tower) and fixed to the base plate with accuracy within 1.25 % per component. Measured data signals were acquired using the National Instruments: NI myDAQ and NI-DAQ USB-6212, respectively, for the speed and force at a sampling rate of 100 Hz with the aid of a specially-written MATLAB program. In the OJF, the rotor was positioned $\approx 3$ rotor diameters from the contraction outlet, and inbuilt pitot tubes measured the flow speed, with the required tunnel speed set and controlled remotely.
from a control room. The forces and moments were measured using the OJF external balance (see the base of figure 6), and the balance gives a $\pm 6\%$ error on the average. Signals from the balance were acquired using NI 9237 at a sampling frequency of 2 kHz with the aid of an in-house LabVIEW program. The rotor speed and torque were measured using the same equipment in both tunnels. For the rotational speed, a US Digital, Hollow Bore Optical Encoder model no HB5M-360-250-IE-D-H with a resolution of 360 cycles per revolution having an estimated accuracy of $\pm 0.056\%$ was employed and the output signals captured along with the ATI F/T. At the same time, the rotor torque was measured with a Precision Tork Magnetic Particle Brake (MPB), model MPB70, that replicates the mechanical load on a windmill. The choice of the MPB is primarily influenced by its ability to accurately fix the torque and hence the speed of the rotor through DC supply to the brake. The torque is determined based on calibration data measured by the authors, as described below. The duration of measurement for each test case was 30 s. Based on the free-stream wind speed, $U_\infty$, thrust, $T$, rotor speed, $\Omega$, and torque, $Q$ for the four rotor configurations, the performance coefficients were calculated using:

$$C_T = \frac{T_{\text{net}}}{\frac{1}{2}\rho U_\infty^2 \pi R^2}$$

$$C_Q = \frac{Q_{\text{net}}}{\frac{1}{2}\rho U_\infty^2 \pi R^3}$$

$$C_p = C_Q \lambda$$

in which $\lambda$ is

$$\lambda = \frac{\Omega R}{U_\infty}$$

and $T_{\text{net}}$ is the difference between the measured thrust, $T$ and that due to the spars connecting the blades to the hub, $T'$, $\rho$, is air density, $R$, is the rotor radius. $Q_{\text{net}}$ is the difference between the measured torque, $Q$ which is due to the blades and spars, and that by the spars not part of blades, $Q'$. Depending on $\lambda$ and $N$, the contribution of the spars to $T$ averages between $6 - 14\%$, while that for $Q$ can be as high as $16\%$, particularly, at runaway $\lambda$.

From the measurements in both tunnels and initial calculated performance results, it was found that, contrary to the manufacturer’s claim, the torque produced by the MPB is speed dependent. Therefore, the MPB had to be calibrated dynamically. The experimental set up of figure 5 was modified to include a rotary torque sensor, TQ513-062 manufactured by Omega Engineering, to directly measure the rotor torque as depicted in figure 7. The sensor is a full Wheatstone bridge strain gauge type having an accuracy of $\pm 0.1\%$ and $mV/V$ range electrical output, which is difficult for the NI DAQ equipment to acquire precisely. Thus, for low noise distortion and clear signal capture, a voltage amplifier, a FUTEK IAA100 analog amplifier with voltage output was used for in-line amplification. A MATLAB program was used to acquire voltage signals with the aid of the NI USB-6212 and the MPB was calibrated based on the torque sensor output, the input current to the MPB, and rotor speed.

3. Blockage assessment

Based on the windmill model cross-sectional area and the test section size of the University of Calgary RWT and TU Delft OJF, the $BR$ correspond to 0.363 and 0.045, respectively, implying that blockage in the latter is negligible, which was confirmed from comparing measured $C_T$ with BET values. For the RWT, the blockage is significant. Preliminary investigation of the effect of blockage was conducted using solids disks and flat plates of varying sizes by measuring the drag force, $F_d$ experienced by them at increasing distances from the RWT exit. These geometries were chosen because the sharp edges should ensure fixed flow separation as $Re$ varies, and,
therefore, the drag should be independent of Re. It was observed that the drag coefficient, $C_d$, values are a function of the model size, $A_d$, and distance from the tunnel. The flow around the model differs with both the model position away from the tunnel and its size, increasing with $BR$ and proximity to the tunnel. Based on this knowledge and using standard $C_d$ values, the corrected upstream velocity at the disk was calculated using $U'_\infty = (2F_d/\rho C_d A_d)^{1/2}$. The velocity of the disk/plate at equivalent $BR$ as the windmill model was then used in calculating a blockage correction factor, $\gamma = U'_\infty/U_\infty$ as given in [20]. For $N = 3, 6, 12$ and 24, $\gamma$ ranges between 0.924 to 0.9733 and is less dependent on $N$, but varies with $\lambda$. Accordingly, Eqs. (1), (3) and (4) were then multiplied by $\gamma^2$, $\gamma^3$ and $\gamma$ to obtain the corrected thrust coefficients, $C_T'$, power coefficient, $C_p'$, and tip speed ratio, $\lambda'$ respectively. The corrected torque coefficient, $C_Q'$ was then determined using $C_p'/\lambda'$.

4. Blade element calculations

The blade of figure 4 was divided into 20 equispaced blade elements from the root to the tip. Two-dimensional airfoil lift and drag data for the airfoil sections were obtained from Ref [21]. The airfoil data, were, however, not corrected for stall delay effects or solidity. They were used as input into a MATLAB coded BET program `power_calc.m` detailed in Wood, [22] that incorporates a versatile and more generally accurate tip loss formulation given by Equation (19) of Wood et al. [24] and implemented with satisfactory outcome in [23], other than the commonly-used Prandtl tip loss. The blade element calculations were iterated for all elements to within a convergence tolerance of $10^{-4}$.

5. Results

Since the model used in this study is a scaled replica of Wegereef of Ref [8], its rotor performance for $N = 6, 12,$ and 24 denoted “WEG”, are compared to the OJF and BET in figures 8 and 9. As seen in figure 8, significant differences can be observed between the OJF and WEG results, however the closest match occurred for $N = 24$, where maximum power coefficient, $C_{p,max}$ is 0.252 occurring at $\lambda_{opt} = 1.008$ for the former compared to $C_{p,max} = 0.254$ at $\lambda_{opt}=1.148$ for the latter; a 0.71 % difference. Generally, the $C_p$ values for $N = 6, 12,$ and 24 are higher than
the OJF, but curiously, a decrease in N from 24 to 12 in the data of [8] increased the maximum $C_p$ by 9.34%. Wegereef’s rotor model had $BR = 0.465$, and since no blockage correction was reportedly made to its $C_p$ and $C_Q$, this possibly explains the observed discrepancies. Figure 8 also presents a performance comparison of the OJF experimental measurements and the BET prediction for $N = 3, 6, 12$ and 24 with $\sigma$ defined as $Nc/\pi R$ corresponding to 0.11, 0.22, 0.45 and 0.90 respectively. As expected, an increase in solidity by increasing $N$, shifts the $C_p$ curves toward lower $\lambda$, while also decreasing $\lambda_{opt}$ which is consistent with previous studies, e.g., [18]. Further, there is good agreement between the OJF measurements and the BET $C_p$ values. The BET predicted $C_{p,\text{max}}$ to within $\pm 4\%$ of measured OJF values, with no significant shift in $\lambda_{opt}$ for all four rotor test cases, see table 2. This is consistent with an analytical prediction of a high solidity, low-speed wind turbine reported by Duquette & Visser [25]; although they did not correct for $\sigma$, implying that high $\sigma$ effects are restricted to $\lambda < \lambda_{opt}$. It is important to note that runaway tip speed ratio, $\lambda_{run}$ (equivalent to maximum $\lambda$ at no load), reduces with increasing $N$ and shows a good correspondence between the OJF and BET, probably because of the low thrust at runaway. Wegereef’s rotor had comparable $Re$ to the present one, and since the OJF measurements match reasonably well with the BET, its results can then be considered more accurate.

As with $C_p$, a similar trend can be observed with $C_Q$, as shown in figure 9. The maximum $C_Q$ generally increases with $N$, and a high $C_Q$, particularly at $\lambda = 0$ as in the BET prediction, is advantageous for starting direct drive water pumping windmills. Unfortunately, measurements at lower and zero $\lambda$ could not be obtained, because while testing, the rotor stalls before reaching those values of $\lambda$. This precluded comparison with the BET at low $\lambda$. RWT measurements overestimated the model performance due to the effect of blockage, see figures 10 and 11. This causes the measured $C_p$ and $C_Q$ values to be greater than that of the low blockage tunnel and the prediction, and the discrepancy increases with increasing $N$ ($\sigma$). For example, compared to OJF, the RWT measured $C_p$ is 13.3 % higher for $N = 24$ at $\lambda = 1$. With reducing $\lambda$,
Table 2: Comparison of measured and predicted torque and power coefficients

| Rotor/Parameters | N=3 | N=6 | N=12 | N=24 |
|------------------|-----|-----|------|------|
|                  | OJF | BET | OJF  | BET  |
| $C_{p,max}$      | 0.097 | 0.095 | 0.165 | 0.161 | 0.219 | 0.228 | 0.252 | 0.247 |
| $\lambda_{opt}$  | 1.297 | 1.4  | 1.168 | 1.3  | 1.099 | 1.1  | 1.008 | 0.9  |
| $C_{Q,max}$      | 0.075 | 0.069 | 0.141 | 0.126 | 0.2196 | 0.207 | 0.350 | 0.299 |
| $\lambda_{run}$  | 2.317 | 2.2  | 2.244 | 2    | 2.197 | 1.9  | 2.058 | 1.9  |

The divergence becomes more pronounced, an indication that blockage and its correction are $\lambda$ dependent.

![Figure 8: BET power coefficient compared to the measurements by Wegereef and in the OJF](image1)

![Figure 9: BET torque coefficient compared to the measurements by Wegereef and in the OJF](image2)

The $C_T$ results are presented in figure 12. Similar to $C_p$ and $C_Q$ after maximum torque, the OJF corresponds reasonably well with BET. Except that for the cases of $N = 3$ and 6, the BET slightly over-predicted the thrust, while for $N = 24$, starting at $\lambda < 1$, a noticeable divergence is seen, similar to its corresponding $C_p$ and $C_q$ curves in figures 10 and 11 respectively. This low $\lambda$ behavior appears to be due to solidity effects not accounted for in BET calculations. However, the same cannot be said of the RWT. Disparities that are much larger than can be attributed to blockage or random measurement uncertainty are noticeable in the $C_T$ curves of figure 12. For example, at $\lambda=1$, the RWT overestimated $C_T$ by a factor of about 1.5 for $N = 24$. The reasons for this high value are currently under investigation.

The blockage correction previously explained was applied to the RWT measurements of $C_p$, and the results agree well with both the OJF and BET, as given in figure 13, although with a slight drop in $\lambda_{run}$. The evinced margin of difference is appreciably obvious as $N \rightarrow 24$. For all the studied rotor configurations, $\gamma$ drops with a rise in $\lambda$. At around $\lambda =1$ for $N = 24$, the correction of $C_p$ due to blockage was 12.75 % ($\gamma = 0.96$). While the RWT correction for $C_Q$ is expected to have the same effect as $C_P$ because of their direct relationship, the $\gamma$ required in the case of $C_T$ for $N = 3$, 6, 12 and 24 are surprisingly greater than that of its corresponding $C_p$ and $C_Q$ values. This is seen in the way the RWT and OJF results diverge at runaway. It appears, therefore, that the required blockage corrections for $C_p$ and $C_Q$ are complex, but far more complex for $C_T$ than suggested by the values of $\gamma$. 

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6. Conclusions
The aerodynamic performance of a 0.68 m windmill model with varying $N$ was investigated experimentally and computationally using BET. The experiments used two open jet wind tunnels, one with negligible blockage and one with a blockage ratio of 0.363. The main findings are summarized as follows:

(i) At small $\lambda$, the significantly higher measured torque for high $N$ implies that solidity effects have to be incorporated in BET for operation below the point of maximum torque. Otherwise, BET accurately predicted the $C_{p,\text{max}}$, the $\lambda$ at which this occurred, and the $\lambda_{\text{run}}$ for all blade numbers.

(ii) Blockage can be significant on rotor performance. The effect increases with $N$, as this increases the windmill torque and thrust as well as the blockage. Standard blockage corrections were applied and shown to be insufficient to explain the behavior at runaway and the large values of thrust measured in the RWT.

(iii) At higher $\lambda$ and lower angles of attack, solidity effects are small, and BET accurately predicts $\lambda_{\text{run}}$ with good accuracy.

(iv) Blockage factor, $\gamma$ increases with reducing $\lambda$. 
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