Time domain Einstein-Podolsky-Rosen correlation

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We experimentally demonstrate creation and characterization of Einstein-Podolsky-Rosen (EPR) correlation between optical beams in the time domain. The correlated beams are created with two independent continuous-wave optical parametric oscillators and a half beam splitter. We define temporal modes using a square temporal filter with duration $T$ and make time-resolved measurement on the generated state. We observe the correlations between the relevant conjugate variables in time domain which correspond to the EPR correlation. Our scheme is extendable to continuous variable quantum teleportation of a non-Gaussian state defined in the time domain such as a Schrödinger cat-like state.

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Quantum entanglement is one of the most fundamental properties of quantum mechanics and also the essential ingredient in quantum information processing\textsuperscript{1,2}. The nonclassical correlation was first argued by Einstein, Podolsky, and Rosen (EPR) relating to canonically conjugate continuous variables (CVs)\textsuperscript{3}. This originally devised EPR correlation has been experimentally realized by employing a two-mode squeezed vacuum state\textsuperscript{4,5}, where the relevant variables are quadrature-phase amplitudes of optical fields. Since this significant milestone, there have been further experimental observations of the EPR correlations\textsuperscript{6,7,8,9} and theoretical characterizations\textsuperscript{10,11}. Moreover the EPR correlation, or the two-mode squeezed vacuum has been used as a resource in implementation of quantum protocols. Thanks to well-established tools of manipulating the EPR correlation, CV quantum information processing (QIP) has been rapidly developing in recent years\textsuperscript{2} and has been successful in implementing quantum protocols such as quantum teleportation\textsuperscript{12,13,14}.

Quantum teleportation is one of the fundamental quantum protocols which exploit the EPR correlation or quantum entanglement\textsuperscript{12,15}. By using an optical field, it has been experimentally realized for Gaussian input states; a coherent state\textsuperscript{12,14}, a squeezed state\textsuperscript{16}, and an EPR state, i.e., entanglement swapping\textsuperscript{12}. These successes are based on well-developed techniques of optical Gaussian operations consisting of beam splitters, phase shifting, squeezing, phase-space displacement and homodyne detection. In addition to the teleportation, CV quantum protocols have so far been performed only with Gaussian states and operations. However non-Gaussian states or non-Gaussian operations are further required to extract versatile potential of CV QIP\textsuperscript{13}. Thus quantum teleportation of a non-Gaussian state would become the next important challenge toward universal CV QIP.

For the generation of a non-Gaussian state in an optical system, there is considerable interest in the techniques where photon counting is performed for a tapped-off beam from a Gaussian state light beam. When counting a photon, the tapped Gaussian state is converted off beam from a Gaussian state light beam. When counting a photon, the tapped Gaussian state is converted to a non-Gaussian state, a technique called "photon-subtraction". Recently there are some reports which show the generation of non-Gaussian states (Schrödinger cat-like states) with this technique\textsuperscript{18,19}. Since the photon counting event is defined in the time domain, conventional CV teleportation schemes with a sideband of an optical carrier\textsuperscript{12} cannot be used for the teleportation of such states. Therefore, in order to realize CV teleportation of such non-Gaussian states, one need to generate EPR correlation in the time domain.

In this Letter we demonstrate experimental generation and characterization of a two-mode squeezed vacuum state in the time domain, a step towards CV quantum teleportation of non-Gaussian states like a Schrödinger cat-like state. The main feature of our EPR resource is that we use two squeezed vacuum states of continuous-wave (CW) light beams from two independent subthreshold optical parametric oscillators (OPOs) to obtain well-defined frequency and spatial modes, and also that we utilize almost the whole frequency bandwidth of the OPO cavities to define a quantum state in the time domain. The use of the broad bandwidth allows us to make time-resolved measurements like photon counting as described below. The experiment presented here is quite different from the previous works in the frequency domain\textsuperscript{6,7,8,9} which have dealt with only frequency sidebands a few MHz apart from the carrier frequency, and therefore the previous works are not compatible with photon counting. Our scheme would have advantages even over the time-domain pulsed scheme\textsuperscript{8} with respect to spatial modes. The well-defined frequency and spatial modes are important for efficient scaling of a quantum circuit via interference of several beams.

Moreover our scheme is compatible with CV entan-
glement distillation with the photon subtraction operation [20]. Combining the entanglement distillation and teleportation would result in the improvement of teleportation fidelity [21]. Our entangled state could also be applicable to loophole-free tests of Bell’s inequalities [22, 23].

For an ideal two-mode squeezed vacuum state with infinite squeezing, the EPR correlations can be described by the Wigner function of the two modes $A$ and $B$:

$$W_{\text{EPR}}(x_A, p_A; x_B, p_B) = c \delta(x_A - x_B) \delta(p_A + p_B),$$

(1)

where $x$, $p$ are the quadrature amplitudes corresponding to the conjugate quadrature operators $\hat{x}$, $\hat{p}$ which obey the commutation relation $[\hat{x}, \hat{p}] = i/2$ ($\hbar = 1/2$) in analogy with the position and momentum operators. This Wigner function expresses that the quadrature amplitudes of the two modes are perfectly correlated ($x_A = x_B$) and anti-correlated ($p_A = -p_B$), respectively. In the real experiment where only finite squeezing is available, the state is no longer the ideal EPR state and shows finite variances in $\langle (\Delta(\hat{x}_A - \hat{x}_B))^2 \rangle$ and $\langle (\Delta(\hat{p}_A + \hat{p}_B))^2 \rangle$.

The two-mode squeezed vacuum generated by subthreshold OPOs has EPR correlations within the variance spectra $S_x(\Omega)$, $S_p(\Omega)$ as usually measured in the frequency domain experiments. Here, $\Omega$ is a sideband frequency around the fundamental wavelength. When measuring the CW beams in the time domain we need to specify a certain temporal mode within which the entangled quantum state is defined. Because of the broadband correlations of the CW beams, there is a large amount of freedom in choosing a mode which will suit an intended application and will show significant correlation below the vacuum level. For instance, in a teleportation experiment the temporal mode could be chosen to match that of the input state.

A simple choice of temporal mode is the square filter of duration $T$. Defining the filtered quadrature operators

$$\hat{x}^f = \frac{1}{\sqrt{T}} \int_0^T \hat{x}(t) dt,$$

(2)

and similarly for $\hat{p}^f$, these operators obey the same commutation relation, $[\hat{x}^f, \hat{p}^f] = i/2$. The EPR variances within this temporal mode will be given by [24]

$$\langle (\Delta(\hat{x}_A - \hat{x}_B))^2 \rangle = \frac{T}{2\pi} \int_{-\infty}^{+\infty} S_x(\Omega) \sin^2\left(\frac{\Omega T}{2}\right) d\Omega,$$

(3)

$$\langle (\Delta(\hat{p}_A + \hat{p}_B))^2 \rangle = \frac{T}{2\pi} \int_{-\infty}^{+\infty} S_p(\Omega) \sin^2\left(\frac{\Omega T}{2}\right) d\Omega.$$  

(4)

The variances are thus filtered by a sinc function, and by adjusting the integration time $T$ the frequency range which contribute to the integrated correlation can be selected. The square temporal filter is simple, but not necessarily the most appropriate for an application of the time domain entanglement. Theoretical investigations concerning the shape of the filter combined with photon counting experiments have been done in Refs. [24, 25].

The schematic diagram of our experiment is illustrated in Fig. 1. This is almost the same as the entanglement preparation part in our experiment of quantum teleportation [14]. The primary source of the experiment is a CW Ti:Sapphire laser at 860nm, most of whose output is frequency doubled in an external cavity. The output beam at 430nm is divided into two beams to pump two OPOs.

A two-mode squeezed vacuum is produced by combining two squeezed vacua at a half beam splitter (HBS). Each squeezed vacuum is generated from a subthreshold OPO with a 10-mm-long KNbO$_3$ crystal. The crystal is temperature-tuned for type-I noncritical phase matching. Each OPO cavity is a bow-tie-type ring cavity consisting of two spherical mirrors (radius of curvature 50mm) and two flat mirrors. The round trip length is about 500mm and the waist size in the crystal is 20$\mu$m. An output coupler has transmissivity of 12.7%, while the other mirrors are highly reflective coated at 860nm. They have a high transmission for 430nm so that the pump beam passes the crystal only once. The pump power is about 70mW for each OPO. The total intracavity losses are around 2%, giving a cavity bandwidth of 7MHz HWHM. The resonant frequency of the OPO is locked via the FM sideband locking method [20], by introducing a lock beam which is propagating against the squeezed vacuum beam to avoid the interference between the two beams. However a small fraction of the lock beam reflecting from surfaces of the crystal circulates backward and contaminates the squeezed state. This problem is resolved by changing the transverse mode and the frequency of the lock beam, as described in detail in Ref. [25].
The output beams from the HBS are sent to Alice and Bob, and then measured using homodyne detectors with a bandwidth of 8.4 MHz. We lock relative phases between EPR beams and local oscillators (LOs) in the detection, i.e., \( x \) or \( p \) quadrature, in the following way. First, weak coherent beams which pass through the same paths as squeezed states are injected into the OPOs from one of the flat mirrors and used for the conventional dither and lock method. The relative phases between the weak coherent beams and the squeezed vacua and between the weak coherent beams at the half beam splitter (HBS) are actively controlled by applying feedback voltages to piezos (PZTs). The quadratures to be detected in the homodyne detectors are also determined by locking the phases between the LOs and the injected coherent beams. However these weak beams and the modulation on them contaminate EPR correlation in some frequency ranges. Therefore we need to remove such beams and lock the phases without the beams. To do so, we introduce an electronic circuit that holds the feedback voltage and keeps the phase relation for a while. Within 2\( \mu \)sec, the beams are blocked with mechanical shutters before the OPOs, and the EPR beams are measured with the homodyne detectors. Each output from the detectors is sampled with an analogue-to-digital converter (ADC) at a rate of 50M samples per second and then filtered on a PC to yield a number of measured quadrature values. We then estimate the degree of EPR correlation using the correlation diagrams with 10000 points as shown in Fig. 4. These results show clear correlations in

FIG. 2: Calculated Fourier analysis of the 50M-sampled raw data without average. (a) \( S_x(\Omega) \) for \( \langle (\Delta \hat{x}_A - \hat{x}_B)^2 \rangle \). (b) \( S_p(\Omega) \) for \( \langle (\Delta \hat{p}_A + \hat{p}_B)^2 \rangle \). Each trace is normalized to the corresponding vacuum level.

FIG. 3: Typical measured correlations. Only 50 points are picked up. (a) and (b) are measured quadrature values within time interval \( T = 0.2 \mu \)sec for \( x \) and \( p \) quadratures, respectively. In each figure, trace (i) is for Alice, while (ii) for Bob.
the relevant quadratures. By repeating the same measurement ten times, we obtained the correlations of \(\langle (\Delta (\hat{x}_A - \hat{x}_B))^2 \rangle = -3.30 \pm 0.28dB\) and \(\langle (\Delta (\hat{p}_A + \hat{p}_B))^2 \rangle = -3.74 \pm 0.32dB\) compared to the corresponding vacuum variances. Accordingly the sufficient criteria [10, 11] for quantum entanglement holds: \(\langle (\Delta (\hat{x}_A - \hat{x}_B))^2 \rangle \) \(+\langle (\Delta (\hat{p}_A + \hat{p}_B))^2 \rangle = 0.45 \pm 0.02 < 1\). Therefore our generated state is entangled in the temporal mode defined within \(T = 0.2\mu sec\). Note that we can decrease the integration time \(T\) for fast data acquisition and QIP, while the EPR correlation will degrade due to the contribution of higher frequency ranges. One would be able to obtain correlation over a broader band by use of small-size OPOs with shorter round-trip length or waveguide crystals, e.g., periodically poled Lithium Niobate waveguides [29].

In summary we have experimentally demonstrated generation and characterization of a two-mode squeezed vacuum state in the time domain. We have observed the nonclassical correlation between the temporally measured quadrature values. Our setup is almost the same as an entanglement preparation part of our quantum teleportation setup [14]. With slight modification of classical channels, we will be able to perform the broadband teleportation whereby it is possible to transfer quantum states defined in the time domain like a Schrödinger cat-like state, a single photon state and other non-Gaussian states. Moreover our scheme is compatible with CV entanglement distillation with photon subtraction [24]. Our system has the well-defined frequency, spatial and temporal modes. Therefore we could efficiently scale up quantum circuits by interfering several beams toward the universal CV QIP.

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