Abstract

A generic feature of models of inflation obtained from string compactifications is correlations between the model parameters and the post-inflationary evolution of the universe. Thus, the post-inflationary evolution depends on the inflationary model parameters and accurate inflationary predictions require that this be incorporated in the evolution of the primordial spectrum. The fibre inflation model is a promising model of inflation constructed in type IIB string theory. This model has two interesting features in its post-inflationary evolution. The reheating temperature of the model is directly correlated with the model parameters. The model also necessarily predicts some dark radiation, which can be sizable for certain choices of discrete parameters in the model. We analyse this model in detail using publicly available codes - MODECHORD and COSMO-MC with the latest Planck+BICEP2/Keck array data to constrain the model parameters and $N_{\text{pivot}}$ (the number of e-foldings between horizon exit of CMB modes and the end of inflation). Our analysis sets up the basic methods necessary to extract precise inflationary prediction in string models incorporating correlations between model parameters and post-inflationary evolution.
1 Introduction

The inflationary paradigm provides an extremely attractive explanation for the observed spectrum and inhomogeneities in the Cosmic Microwave Background (CMB). Since observations in the future are likely to probe the CMB minutely [1], it is important to develop a systematic understanding of the methodology for extracting highly accurate predictions of inflationary models. The simplest method is to parametrize primordial perturbations with a set of empirical variables such as $A_s$ (the strength of the power spectrum), $n_s$ (the scalar tilt), $r$ (the tensor to scalar ratio), $f_{NL}$ (parametrizing the non-Gaussianity) etc. The best fit values of these are obtained by evolving the primordial fluctuations and comparing with observations of the CMB. Given a model of inflation, one can also compute the functional form of the primordial fluctuations in terms of the parameters of the model. One then requires that the predictions for the empirical parameters are in the best fit regions determined from evolution of the initial perturbations. Note that this is intrinsically a two step process where the empirical parameters characterising the primordial perturbations act as the matching points between observation and theory.

On the other hand, if one wants to confront a particular model of inflation with data, a more comprehensive method is to treat the model parameters as inputs for the cosmological evolution and directly determine the best fit regions for these parameters [2, 3] (see also [4]). This approach is particularly well suited if one is considering models which arise from a fundamental theory (such as string theory). In this case, one naturally expects various correlations between the model parameters and the post-inflationary evolution of the universe. Thus the post-inflationary evolution depends on the model parameters and accurate inflationary predictions require that this be incorporated in the evolution of the primordial spectrum.

Inflationary predictions of any model are sensitive to higher derivative corrections in the effective action. Hence, theories of quantum gravity are the appropriate setting to carry out inflationary model building. Fibre inflation [5] is a promising model of inflation set in IIB string theory. Phenomenologically, the model is interesting as it predicts a value of the tensor to scalar ratio ($r > 0.005$) which can be observationally verified with experiments planned in the near future. Thus, it is timely to carry out a detailed study of the model predictions. As we will discuss in detail in the next section, the model has two interesting features in its post-inflationary evolution. The reheating temperature of the model is directly correlated with the model parameters. The model also necessarily predicts some dark radiation, which can be sizable for certain choices of discrete parameters in the model. In this paper, we will use MODECHORD [3] and COSMO/MC [6], incorporate these features in the post-inflationary evolution and thereby perform a detailed analysis of the model predictions. These two features
are also expected to be generic in string constructions\textsuperscript{1}. For a discussion of dark radiation in string models see e.g. \cite{9}. Thus our analysis can serve as a template for the analysis of most string models. Effects of presence of dark radiation on cosmological observations is studied in \cite{10}.

Recently, the predictions of fibre inflation and their relationship to post inflationary dynamics have been analysed in \cite{2,7}. Our work develops on this, systematically incorporating the relationship between the model parameters and the post inflationary dynamics making use of the above mentioned publicly available packages. This allows us to obtain a detailed understanding of the model predictions.

This paper is structured as follows. In section 2 we review some basic aspects of fibre inflation, in section 3 we discuss our methodology and perform our analysis, in section 4 we discuss our results and conclude.

\section{Review of Fibre Inflation}

The fibre inflation model is a model of inflation set in the Large Volume Scenario \cite{12} for moduli stabilisation of II B flux compactifications. Here, we briefly review aspects of the model that will be needed for our analysis and refer the reader to \cite{5,7,13} for further details.\textsuperscript{3}

The relevant dynamics during the inflationary epoch is that of the Kähler moduli fields\textsuperscript{4} of the Calabi Yau manifold associated with the compactification. The Kähler moduli are flat at tree level, but acquire a potential as a result of non-perturbative corrections to the superpotential, loop and $\alpha'$ corrections to the Kähler potential. The construction of fibre inflation models involves Calabi-Yaus with at least three Kähler moduli\textsuperscript{5}.

\begin{itemize}
  \item $T_1 = \tau_1 + i\theta_1$. For this field, the geometric modulus $\tau_1$ corresponds to the volume of a $T^4$ or $K3$ fibred over a $\mathbb{P}^1$ base. The field $\tau_1$ plays the role of the inflaton in the model.
  \item $T_2 = \tau_2 + i\theta_2$. Here the geometric modulus corresponds to the volume of the base.
  \item $T_3 = \tau_3 + i\theta_3$. Here, the geometric modulus corresponds to the blow up of a point like singularity. Non-perturbative effects on this cycle, play a important role in moduli stabilisation.
\end{itemize}

\textsuperscript{1}Another generic feature is epochs in the post-inflationary history in which the energy density is dominated by cold moduli particles. Its effect on inflationary predictions has been studied in detail in \cite{8}.

\textsuperscript{2}For a complimentary approach see \cite{11}.

\textsuperscript{3}We will follow the conventions and notation of \cite{7}.

\textsuperscript{4}The complex structure moduli are fixed by fluxes \cite{14}.

\textsuperscript{5}We will denote the Kähler moduli as $T_i = \tau_i + \theta_i$, with $\tau_i$ being a geometric modulus and $\theta_i$ its axionic partner.
The volume of the compactification can be expressed in terms of the volumes of the geometric moduli as
\[ V = \alpha \left( \sqrt{\tau_1 \tau_2} - \gamma \tau_3^{3/2} \right), \]  
(2.1)
where \( \alpha \) and \( \gamma \) are order one constants determined by the intersection numbers of the four cycles.

The potential developed as a result of the effects described above can be expanded in an inverse volume expansion. At order \( V^{-3} \), the geometric moduli \( \tau_2 \) and \( \tau_3 \) and the axion \( \theta_3 \) are stabilised. Loop effects at order \( V^{-10/3} \) provide a potential for the field \( \tau_1 \). This takes the form (in Planck units)
\[ V(\tau_1) = \left( g_s^2 \frac{A}{\tau_1} - \frac{B}{\sqrt{\tau_1}} + g_s^2 \frac{C \tau_1}{V^2} \right) \frac{W_0^2}{V^2}, \]  
(2.2)
where \( W_0 \) is the vacuum expectation value of the Gukov-Vafa-Witten superpotential and

\[ A = (c_1^{KK})^2 \quad B = 2\alpha c^W \quad C = 2(\alpha c_2^{KK})^2 \]
with \( c_1^{KK}, c_2^{KK} \) and \( c^W \) depending on the underlying compactification and fluxes. After incorporation of effects so that the minimum is a Minkowski one, canonical normalisation of \( \tau_1 \) and shifting the zero of the field to its minimum, the potential for the canonically normalised inflaton field (\( \hat{\phi} \)) is
\[ V = V_0 \left( 3 - 4e^{-k\hat{\phi}} + e^{-4k\hat{\phi}} + R \left( e^{2k\hat{\phi}} - 1 \right) \right), \]  
(2.3)
where
\[ k = \frac{1}{\sqrt{3}}, \quad V_0 = \frac{g_s^{1/3} W_0^2 A}{4\pi \lambda^2} \quad \text{with} \quad \lambda = \left( \frac{4A}{B} \right)^{2/3}, \quad \text{and} \quad R = 16g_s^4 AC B^2 \]  
(2.4)
The inflationary trajectory is \( \hat{\phi} \) rolling from positive values towards its minimum at zero. Note that \( R \propto g_s^4 \) and hence is naturally small. The potential has two inflection points: \( \hat{\phi}_{ip}^{(1)} \sim k \ln 4 \) and \( \hat{\phi}_{ip}^{(2)} \sim -k \ln R \). The second inflection point occurs as a result of competition between the positive exponential and the negative ones. Inflation occurs when the field lies between the two inflection points. If the value of \( R \) is small \( R < 2 \times 10^{-6} \), then horizon exit of the CMB modes takes place at a field value (\( \hat{\phi}_* \)) which is much less than the second inflection point \( \hat{\phi}_* \ll \hat{\phi}_{ip}^{(2)} \), the positive exponential term can be neglected. In this regime, a robust prediction of the model is a relationship between the spectral tilt (\( n_s \)) and the tensor to scalar ratio (\( r \))
\[ r = 6(n_s - 1)^2 \]  
(2.5)
On the other hand, for higher values of \( R \), the horizon exit of CMB modes takes place at a point which is closer to the second inflection point; the positive exponential term has to be incorporated in the analysis. With increase in the value of \( R \), the model predicts higher values of \( n_s \) and \( r \). Also the relationship \( (2.5) \) is broken.
The reheating epoch in fibre inflation models has been examined in detail in [7]. After the end of inflation the inflaton oscillates about its minimum and decays perturbatively, this is supported by full numerical analysis of the evolution of the scalar field after inflation [15] (a semi-analytic approach [16] has suggested the possibility of a preheating epoch, but the evidence from the fully numerical study is that the process is perturbative). The dominant decay channels are visible sector gauge bosons, visible sector Higgs, and ultra-light bulk hidden axionic fields (which act as dark radiation). The total visible sector and the hidden sector decay widths are given by

\[
\Gamma_{\phi}^{\text{vis}} = 12 \gamma^2 \Gamma_0 \quad \text{and} \quad \Gamma_{\phi}^{\text{hid}} = \frac{5}{2} \Gamma_0 ,
\]  

(2.6)

where \( \Gamma_0 = \frac{m_{\phi}^3}{48\pi^2 M_{\text{pl}}^2} \), and

\[
\gamma = 1 + \alpha_{\text{vis}} \frac{h(F_1)}{g_s} ,
\]  

(2.7)

where \( \alpha_{\text{vis}} \) is the high scale visible sector gauge coupling \( (\alpha_{\text{vis}}^{-1} \sim 25) \). \( h(F_1) \) depends on \( U(1) \) flux threading of the D7 brane on which matter fields are localised. It vanishes for zero flux, and is an order one quantity as the flux quanta is increased\(^6\).

Given the widths in (2.6) the prediction for dark radiation is easily computed. One finds

\[
\Delta N_{\text{eff}} = \frac{0.6}{\gamma^2} .
\]

Thus the model necessarily predicts some dark radiation. The prediction is high in the absence of any gauge flux, and can be sizable for small values of the flux quanta. Recall, that the analysis of Planck prefers higher values of \( n_s \) in the presence of dark radiation. As we have discussed earlier, this can be obtained with higher values of the parameter \( R \) in the inflationary potential (2.3).

Finally, let us come to the number of e-foldings before horizon exit. This is given by (see e.g. [18,19])

\[
N_e = 57 + \frac{1}{4} \ln r + \frac{1}{4} \left( \frac{\rho_s}{\rho_{\text{end}}} \right) + \frac{1 - 3w_{\text{rh}}}{12(1 + 3w_{\text{rh}})} \ln \left( \frac{\pi^2}{45} g_s(T_{\text{rh}}) \right) - \frac{1}{3} \frac{1 - 3w_{\text{rh}}}{(1 + 3w_{\text{rh}})} \ln \left( \frac{M_{\text{inf}}}{T_{\text{rh}}} \right) ,
\]  

(2.8)

where \( \rho_s \) and \( \rho_{\text{end}} \) are the energy densities of the universe at the time of horizon exit and the end of inflation. \( w_{\text{rh}} \) is the average equation of state during the reheating epoch, \( g_s(T_{\text{rh}}) \) the number of relativistic degrees of freedom at the end of reheating and \( T_{\text{rh}} \) the reheating temperature.

\(^6\)More precisely, \( h(F_1) = \frac{1}{2} k_{112} n_i^2 \), where \( k_{112} \) is a triple intersection number involving the two cycles dual to the three four cycles of the Calabi-Yau and \( n_i \) the integral coefficients of the expansion of the gauge flux in terms of these dual cycles [17].
The reheating temperature can be obtained from the Eq. (2.6). One finds
\[ T_{rh} = 0.12 \gamma m_{\hat{\phi}} \sqrt{\frac{m_{\hat{\phi}}}{M_{pl}}}, \] (2.9)
where \( m_{\hat{\phi}} \) is the mass of \( \hat{\phi} \) about the minimum at \( \hat{\phi} = 0 \) in (2.3). Note that this implies that the number of e-foldings before horizon exit in the model is correlated with parameters in the potential and the amount of dark radiation (although the dependence on the amount of dark radiation is very mild as \( \gamma \) is an order one quantity). Since the inflaton decays perturbatively and has a long lifetime \( w_{\text{rh}} = 0 \).

Before closing this section, we would like to emphasise that as in many string models, in fibre inflation there is a direct correlation between \( N_e \) and the parameters in the inflationary potential. The model has the interesting feature that for certain discrete choices in the parameter space a sizable amount of dark radiation is predicted. Furthermore, the model’s prediction for the tensor to scalar ratio is in the right ball park to be probed by upcoming observations. Given this, a detailed analysis of the model predictions which takes into account the above considerations is very well motivated. This is the primary goal of this paper.

3 Methodology and Results

In this section, we discuss our methodology for parameter estimations and report our results. First, we note that the potential in Eq. (2.3) has two parameters \( V_0 \) and \( R \), which themselves depend on some fundamental parameters (such as the volume of the compactification and \( W_0 \)). Thus, these two parameters broadly control the inflationary perturbations. But, these two parameters also control the post-inflationary history via Eq. (2.9). On the other hand, the parameter \( \gamma \) controls the amount of dark radiation \( \Delta N_{\text{eff}} \).

For given values the model parameters \( R \) and \( V_0 \), and \( N_{\text{pivot}} \), we have evaluated the cosmological perturbations by using ModeChord [3] (plugged together through Multinest [20]) without assuming slow-roll conditions. Along with these parameters, we have also varied \( \gamma \) which controls the amount of dark radiation produced. As usual, the Boltzmann solver CAMB [24] is used to evaluate the 2-point correlation functions for temperature and polarization, and then the model parameters are estimated and the goodness of fit is determined using CosmoMC [6]. The likelihoods used here are Planck’18 TT+TE+EE, Planck lowP, estimated using commander, Planck lensing and Planck+BICEP2/Keck array joint analysis likelihood [21]. The model parameters are then inferred from the chains using the code in [22].

Next, let us come to our results. In Fig. 1 the dark radiation allowed from Planck’18 data with respect to the Hubble constant is plotted (the contours correspond to the 1-\( \sigma \) and 2-\( \sigma \) regions). \( \Delta N_{\text{eff}} \) represents the extra presence of radiation with respect to the theoretically
expected $N_{\text{eff}} \sim 3.046$ from the Standard Model (SM) of Particle Physics. We see that $\Delta N_{\text{eff}} = 0$ is fully consistent with the data. In Fig. 2, model parameters $R$ and $V_0$ are plotted against each other. The best fit value for the scale of inflation is around $2.3 \times 10^{10}$ GeV. In Fig. 3, the probability distribution of the number of e-foldings is plotted; the central value is around 53, which is quite close to the estimate in [7]. Finally, in Fig. 4, we have plotted the probability distribution for the reheating temperature and the most probable value is around $10^{11}$ GeV. A summary of the results is given in the Table 1. One of the interesting results is the central value of the $n_s$. The standard analysis by Planck gives this to be $n_s \sim 0.965$ [23], here we find a small shift, the central value is $n_s \sim 0.9691$. 

Figure 1: Favoured region in the $\Delta N_{\text{eff}}$ and the $H_0$ plane (the contours correspond to the $1 - \sigma$ and $2 - \sigma$ regions). $H_0$ is plotted in units of km s$^{-1}$Mpc$^{-1}$. 

Figure 2: Favoured region of the model parameter $R$ with respect to the scale of the inflation $V_0$ (in reduced Planck unit).
Figure 3: 1-D probability distribution of the number of e-foldings $N_{\text{pivot}}$.

Figure 4: 1-D probability distribution of the number of reheating temperature $T_{\text{reh}}$ (in reduced Planck Unit).

| Parameters           | Central value | 1σ    | 2σ    |
|----------------------|---------------|-------|-------|
| $R/(10^{-4})$        | 2.1451        | 0.1657| 0.3314|
| $V_0/(10^{-13})$     | 8.8911        | 1.1113| 6.1322|
| $\Delta N_{\text{eff}}$ | 0.00041      | 0.20721| 0.043101|
| $H_0$                | 68.011        | 2.553 | 3.979 |
| $n_s$                | 0.9691        | 0.0118| 0.0236|
| $r/(10^{-2})$        | 0.9321        | 1.0068| 1.2101|
| $N_{\text{pivot}}$  | 53.26         | 2.081 | 3.050 |
| $T_{\text{reh}}/(10^{-7})$ | 1.91     | 0.82  | 1.06  |

Table 1: Constraints on the model parameters and the cosmological parameters. Data combination used: *Planck'18 TT+TE+EE+ low P +lensing + BKPlanck15*. All dimensionful quantities are in reduced Planck units.

## 4 Discussion and Conclusions

The present work has focused on the phenomenology of fibre inflation. We would like to begin this section by noting some issues related to construction of the model in string compactifications. Firstly, there is the possibility of the presence of certain $\alpha'$ corrections [25] in the effective action (which are still not completely understood) that might contribute to the positive exponential term in (2.3). One consequence of this might be that the coefficient of the
positive exponential term can be pushed to higher values. In this case, our analysis would have to be redone taking into account the appropriate range for R. A similar issue is the geometric instability that can arise as a result of the ultralight field in the model \cite{27}. At this stage, it is unclear how relevant the instability is for fibre inflation, but it could have implications on the parameter space of the model. In any case, the methods in the paper are general enough so that they can be easily modified if there is improved understanding of the parameter space of the model.

Our results are interesting from the point of view of phenomenology. As reported in Table 1, the central value of the tensor to scalar ratio is $r \sim 0.00932$, which is in the observably verifiable range for the next generation of CMB-B mode surveys. In terms of future directions, it will be interesting to look for top down constructions of string models, with the model parameters in ranges obtained from our analysis. It will also be interesting to compare with the preferred ranges from the point of view of particle physics \cite{28}. Another phenomenologically exciting avenue is to carry out similar analysis for closely related models, including the $\alpha$-attractor class \cite{29}.

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