A Simple Approximation of the One-Loop Corrected Cross Section for $e^+e^- \rightarrow W^+W^-$ at LEP 2

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Abstract

Using the $SU(2)$ gauge coupling, $g_{W^\pm}(M_W^2)$, at the high-energy scale of $M_W$, defined by the (theoretical value of the) leptonic $W$-width, rather than using the low-energy value, defined via the Fermi coupling, $G_\mu$, in the Born approximation, and supplementing with Coulomb corrections and initial state radiation, errors with respect to the exact one-loop results for the differential cross section of $e^+e^- \rightarrow W^+W^-$ are below 1% at LEP 2 energies at all $W^+W^-$ production angles. A similar procedure is suggested to incorporate leading bosonic loop effects into four-fermion production in the fermion-loop scheme. The resulting accuracy below 1% is sufficient for LEP 2 experiments.

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The process of $W$-pair production in $e^+e^-$ annihilation is presently studied experimentally at LEP 2. In the future, it will be one of the outstanding processes at a linear collider in the TeV energy range. It yields direct experimental information on the non-Abelian couplings characteristic for the $SU(2) \times U(1)$ electroweak theory, and it allows to put bounds on potential non-standard $Z_0W^+W^-$ and $\gamma W^+W^-$ couplings [1, 2]. Within the $SU(2) \times U(1)$ electroweak theory, the calculation of the radiative corrections to this process has received much attention [3]-[8].

The exact evaluation of the full one-loop electroweak corrections leads to complicated and lengthy expressions in terms of twelve $s$- and $t$-dependent form factors. Actually, only those (three) form factors which appear in amplitudes of the form of the Born approximation are numerically important [3, 9]. Unfortunately, however, no simple analytic form for these form factors, valid at arbitrary $e^+e^-$ energies, has been given so far. For the LEP 2 energy range, however, a simple approximation has indeed been suggested [5]. In this approximation, the Born approximation is evaluated in terms of the appropriately introduced Fermi coupling, $G_\mu$, and the high-energy electromagnetic coupling, $\alpha(s)$, and it is supplemented by Coulomb corrections and by initial state radiation employing the structure function method. By comparing the improved Born approximation (IBA) with the full one-loop results, accuracies of 1.5% to 2% were found [9, 10] in the angular distributions in the LEP 2 energy range. For energies above 500 GeV, a simplification of the exact $O(\alpha)$ corrections has been given in terms of an asymptotic high-energy expansion [7].

It is the purpose of the present note to point out that a slight modification of the previously suggested [5] improved Born approximation for the LEP 2 energy range improves its accuracy to values well below 1%. Accordingly, such an approximation will be sufficient for all practical purposes at LEP 2. Our results are obtained by replacing the low-energy value of the $SU(2)$ gauge coupling, $g_{W^\pm}(0) \equiv 4\sqrt{2}M_{W^\pm}G_\mu$ previously employed in [3, 9, 10], by its high-energy value, defined by the leptonic $W^\pm$ width, $g_{W^\pm}(M_{W^\pm}^2) \equiv 48\pi\Gamma_W^W/M_{W^\pm}$ [11] more appropriate for the LEP 2 energy scale. In essence, this approach amounts to employing a different renormalization scheme, defined [11] by using $\Gamma_W^W$ instead of $G_\mu$ as experimental input.

The Born amplitude for the process $e^+e^- \rightarrow W^+W^-$, in the notation of refs. [9, 10], takes the form

$$M_{\text{Born}}(\kappa, \lambda_+, \lambda_-, s, t) = g_{W^\pm}^2 \frac{1}{2} \delta_{\kappa-} M_I + e^2 M_Q,$$

\(^1\)We approximate $g_{W^\pm}^2(s \geq 4M_W^2)$ by $g_{W^\pm}^2(M_W^2)$.
where the dependence on energy and momentum transfer squared, $s$ and $t$, and on the electron and $W^{\pm}$ boson helicities, $\kappa = \pm 1$ and $\lambda = 0, \pm 1$, is contained in $M_I$ and $M_Q$. The $SU(2)$ gauge coupling and the electromagnetic coupling in (1) have been denoted by $g_{W^{\pm}}$ and $e$, respectively. Even though (1) is easily obtained by starting from the Feynman rules for $t$-channel neutrino and $s$-channel $\gamma$ and $Z_0$ exchange, we prefer to gain (1) directly from the electroweak theory in the $BW_3$ base, i.e. before diagonalization of $BW_3$ mixing.

From the diagrams (a), (b) and (c) in Fig. 1, one immediately obtains

$$M_I = \frac{1}{s - M_Z^2}M_s + \frac{1}{t}M_t. \quad (2)$$

One recognizes the correspondence of this expression to diagrams (a) and (b) in Figure 1, diagram (c) supplying the substitution $M_W^2 \rightarrow M_Z^2 = (1 + (g'/g_{W^{\pm}})^2)M_W^2$ in the $s$-channel term. For the explicit forms of the $s$-channel and $t$-channel quantities, $M_s$ and $M_t$, we refer to [9].

The $B$-propagator, to all orders in $BW_3$ mixing, according to diagram (e) becomes

$$B \quad \Rightarrow \quad B = \frac{s - M_W^2}{s(s - M_Z^2)}, \quad (3)$$

where $M_B^2 \equiv (g'/g_{W^{\pm}})^2M_W^2$ for the square of the $B$-boson mass and $-(g'/g_{W^{\pm}})\cdot M_W^2$ for the mixing strength were used. For right-handed electrons, only the $B$-coupling (to the weak hypercharge current) of diagram (d) is relevant, implying, with (3) and $g'^2M_W^2 = e^2M_Z^2$, that

$$M_Q = -\frac{M_Z^2}{s(s - M_Z^2)}M_s. \quad (4)$$

This expression holds equally well for left-handed electrons, where contributions from the diagrams (d) and (a)+(c) make up one half of $M_Q$ each.

We feel that the above derivation of (1) illuminates in the most straightforward manner the decomposition of (1) into a weak $SU(2)$ and an electromagnetic piece, where the latter one for right-handed electrons is entirely induced by the $B$-boson coupling to the hypercharge current. Moreover, the origin of the double-pole structure in (4) as a result of $BW_3$ mixing becomes immediately obvious. The double pole leads to a suppression of the amplitude (4) relative to (2) at high energies, which in the case of longitudinal $W^{\pm}$ bosons is compensated, however, by the longitudinal polarization vectors.

\[\text{In addition, diagram (c) yields a contribution proportional to } e^2 \text{ (via the relation } (g')^2M_W^2 = e^2M_Z^2) \text{ which is recognized as a part of } M_Q.\]
Figure 1: Evaluating the electroweak Born approximation in the $BW_3$ basis.
The calculation of the cross section for $e^+e^- \rightarrow W^+W^-$ from (1) requires the specification of a scale at which the $SU(2)$ gauge coupling $g^2_{W\pm}$ and the electromagnetic coupling $e^2$ are to be defined. As $W$ pairs are produced at LEP 2 at energies of $2M_{W\pm} \lesssim \sqrt{s} \sim 200 GeV$, it is natural to choose a high-energy scale, such as $\sqrt{s}$. We expect that it is sufficiently accurate to use the scale $M_W \approx M_Z$ instead of $\sqrt{s}$. Accordingly, we choose (12)

$$
\left( \frac{e^2}{4\pi} \right)^{-1} = \alpha^{-1}(M_Z^2) = 128.89 \pm 0.09, \quad (5)
$$

for the electromagnetic coupling, and define $g^2_{W\pm}(M_W^2)$ by the leptonic width of the $W^\pm$.

$$
g^2_{W\pm}(M_W^2) = 4\pi \frac{\Gamma_l^W}{M_{W\pm}}. \quad (6)
$$

Expressing $\Gamma_l^W$ in terms of the Fermi coupling,

$$
\Gamma_l^W = \frac{G_\mu M_W^3}{6\sqrt{2}\pi(1 + \Delta y^{SC})}, \quad (7)
$$

including one-loop corrections, one finds that $g^2_{W\pm}(M_W^2)$ differs from the gauge coupling, $g^2_{W\pm}(0)$, defined from $\mu$-decay, by the correction factor $(1 + \Delta y^{SC})^{-1}$,

$$
g^2_{W}(M_W^2) = \frac{g^2_{W\pm}(0)}{1 + \Delta y^{SC}}. \quad (8)
$$

The “scale-change (SC)” part, $\Delta y^{SC}$, of the coupling parameter $\Delta y$ of ref. [11], takes care of the change in scale between $\mu$-decay and $W^\pm$-decay. It is given by

$$
\Delta y^{SC} = \Delta y^{SC}_{\text{ferm}} + \Delta y^{SC}_{\text{bos}}, \quad (9)
$$

where the fermionic part, $\Delta y^{SC}_{\text{ferm}}$, is essentially due to contributions arising from light fermion loops in the $W^{\pm}$ propagator. For $m_t \rightarrow \infty$, it is given by

$$
\Delta y^{SC}_{\text{ferm}} |_{m_t \rightarrow \infty} = -\frac{3\alpha(M_Z^2)}{4\pi s_0^2} \approx -8.01 \cdot 10^{-3}, \quad (10)
$$

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Footnote:

3We note that $g^2_{W\pm}(M_W^2)$ as defined by (1) differs from $g^2_{W\pm}(M_W^2)$ as defined by (6) in ref. [11] by the factor $r \equiv 1 + c_\alpha^2 \cdot 3\alpha/4\pi$, where $c_\alpha^2 \cdot 3\alpha/4\pi \approx 1.34 \cdot 10^{-3}$. The factor $r$ corresponds to the factor $1 + 3\alpha/4\pi$ in the $Z^0$ width, where it is conventionally introduced in order to explicitly separate photon radiation from all other electroweak one-loop corrections. The introduction of the factor $r$ in the $W^{\pm}$ width allowed [11] to correctly define the magnitude of isospin breaking by one-loop weak interactions when passing from the charged boson coupling $g^2_{W\pm}(M_W^2)$ to the neutral boson coupling $g^2_{W^0}(M_Z^2)$, while keeping the usual convention of separating photon radiation from other loop corrections in the expression for the $Z^0$ width.

For the purposes of the present paper we have removed the factor $r$. This amounts to including all one-loop radiative corrections in $g^2_{W\pm}(M_W^2)$ as defined in (1). All qualitative conclusions of the present work remain the same if $\Delta y^{SC} \approx 3.3 \cdot 10^{-3}$ from (14), corresponding to (1), (6) and (8), is replaced by $\Delta y^{SC} \approx 4.6 \cdot 10^{-3}$ from ref. [11].
while for \( m_t = 180 \) GeV,
\[
\Delta y^{SC}_{\text{ferm}}|_{m_t=180\text{GeV}} = -7.79 \times 10^{-3}.
\]  

(11)

This negative contribution to \( \Delta y^{SC} \) is largely compensated by the bosonic one, \( \Delta y^{SC}_{\text{bos}} \), which is practically independent of the Higgs boson mass and is given by
\[
\Delta y^{SC}_{\text{bos}} = 11.1 \times 10^{-3}.
\]

(12)

Accordingly, the SU(2) coupling, \( g^2_{W^\pm}(M^2_{W^\pm}) \), to be used when evaluating the cross section for the process \( e^+e^- \rightarrow W^+W^- \) in the Born approximation is given by
\[
g^2_{W^\pm}(M^2_{W^\pm}) = \frac{4\sqrt{2}G_\mu M^2_{W^\pm}}{1 + \Delta y^{SC}}.
\]

(13)

with a correction term, due to scale change, whose magnitude,
\[
\Delta y^{SC} = 3.3 \times 10^{-3},
\]

(14)

is practically independent of the precise values of \( m_t \) and \( M_H \). Even though there is such a strong cancellation between fermions and bosons in \( \Delta y^{SC} \), thus implying the fairly small value of \( \Delta y^{SC} \) in (14), the correction induced by \( \Delta y^{SC} \) will be seen to be decisive for providing the announced accuracy, better than 1%, in the expression for the \( W^\pm \) pair-production cross section.

Including the Coulomb correction and the initial state radiation (ISR) in soft photon approximation, the improved Born approximation for the differential cross section takes the form
\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{IBA}} = \frac{\beta}{64\pi^2 s} \left| \frac{2\sqrt{2}G_\mu M^2_{W^\pm}}{1 + \Delta y^{SC}} \delta_{\kappa} + 4\pi\alpha(M^2_{Z})M^2_{\kappa} \right|^2 + \left( \frac{d\sigma}{d\Omega} \right)_{\text{Coul}} (1 - \beta^2) + \left( \frac{d\sigma}{d\Omega} \right)_{\text{ISR}}.
\]

(15)

The only difference of the present work with respect to refs. [3, 4, 10] consists in the inclusion of \( \Delta y^{SC} \) which introduces according to (13) the appropriate high-energy scale for the SU(2)-coupling strength. We note the dependence of \( (d\sigma/d\Omega)_{\text{ISR}} \) in (15) on the choice of the photon splitting scale, \( Q^2 \), inherently connected with the structure function method. This method consists of evaluating the leading logarithmic QED corrections thus all contributions proportional to \( (\alpha/\pi) \ln(m^2_e/Q^2) \).

The deviation of the differential cross section in the improved Born approximation (without the \( \Delta y^{SC} \) correction) from the full one-loop result normalized by the Born cross section, \( \Delta_{\text{IBA}} \), was worked out numerically in refs.

\footnote{The parameter \( \Delta y^{SC}_{\text{bos}} = 11.13 \times 10^{-3} \) for \( M_H = 100 \) GeV, while \( \Delta y^{SC}_{\text{bos}} = 11.08 \times 10^{-3} \) for \( M_H = 300 \) GeV and \( \Delta y^{SC}_{\text{bos}} = 11.07 \times 10^{-3} \) for \( M_H = 1000 \) GeV \[11\].}

\footnote{Note that \( \Delta y^{SC}_{\text{bos}} \) (this paper) = \( \Delta y^{SC}_{\text{bos}} \) (Table 1 in ref. [1]) \(-1.34 \times 10^{-3} \). Compare footnote 3.}
The introduction of $\Delta y^{SC}$ in (15) simply amounts to an additive correction to $\Delta_{IBA}$. This additive correction, $\delta \Delta_{IBA}$, is calculated by evaluating

$$\delta \Delta_{IBA} = \frac{(d\sigma/d\Omega)_{IBA}(\Delta y^{SC} \neq 0) - (d\sigma/d\Omega)_{IBA}(\Delta y^{SC} = 0)}{(d\sigma/d\Omega)_{Born}},$$

and the quality of the approximation (13) to the full one loop result is accordingly quantified by

$$\Delta_{IBA} + \delta \Delta_{IBA}.$$ (17)

We note that the magnitude of $\delta \Delta_{IBA}$ may easily be estimated due to the fact that the $M_I$ part dominates the cross section (13). Indeed, neglecting $M_Q$ in (15), one obtains from (16),

$$\delta \Delta_{IBA} \approx -2\Delta y^{SC} = -0.66\%$$

as a rough estimate. This value will be somewhat enhanced or diminished, depending on whether the interference term of the $M_I$ with the $M_Q$ term in (15) is negative (as in the forward region) or positive (as in the backward region).

The results for $\delta \Delta_{IBA}$ are given in Table 1 and the percentage quality of our improved Born approximation (13), $\Delta_{IBA} + \delta \Delta_{IBA}$, is compared with $\Delta_{IBA}$. The values for $\Delta_{IBA}$ are taken from Table 4 in ref. [9]. They are based on the choice of $Q^2 \equiv s$ for the photon splitting scale $Q^2$. One observes that indeed the deviation of the unpolarized cross section from the full one-loop results is improved to less than 1% as a consequence of introducing $\Delta y^{SC}$ in (13). As the scale $Q^2$ is by no means theoretically uniquely fixed, we also show, in Table 2, the results for the different choice of $Q^2 = M_W^2$. Even though the uncorrected quality of the approximation, $\Delta_{IBA}$, is better in this case than for $Q^2 = s$, the inclusion of $\delta \Delta_{IBA}$ again leads to an improvement of the quality of the approximation also in this case, and values below approximately 0.5% are reached. In other words, the qualitative improvement in the approximation (15), obtained by introducing $\Delta y^{SC} \neq 0$, is independent of the choice of $Q^2$.

For completeness, in Tables 1 and 2, we also present the results for the cross section for left-handed electrons, which obviously do not differ much from the results for the unpolarized cross section, since the right-handed cross section is suppressed by about two orders of magnitude compared with the left-handed one. The right-handed cross section by itself is obviously unaffected by introducing $\Delta y^{SC}$.

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6We thank S. Dittmaier for providing us with the values of $\Delta_{IBA}$ for the photon splitting scale $Q^2 = M_W^2$. 
A final comment concerns the inclusion of the decay of the $W^\pm$'s into fermion pairs which has to be incorporated into a completely realistic description of the process of $W^\pm$ pair production. A gauge-invariant description of the process $e^+e^- \rightarrow 4$ fermions at one-loop order was recently given in the fermion-loop approximation [13]. In this connection it seems worth while to come back to the decomposition of $\Delta y^{SC}$ into fermion-loop and bosonic contributions in (9), (11) and (12). We note that taking into account fermion-loop contributions only, the estimate (18) becomes

\[ \delta \Delta_{IBA}|_{\text{ferm}} \simeq -2\Delta y^{SC}_{ferm} \mid_{m_t=180GeV} \simeq +1.56\%, \]  

and the total deviation $\Delta_{IBA} + \delta \Delta_{IBA}$ of (15) from the full one-loop results (using $\Delta_{IBA} \simeq 1.2\%$ from Table [1]) rises to values of the order of 2.8\% for the total cross section. Therefore, we expect that four-fermion production evaluated in the fermion-loop approximation [13] is also enhanced by as much as approximately 2.8\% relative to the (so far unknown) outcome of a full one-loop calculation incorporating bosonic loops as well. It is gratifying that a simple procedure of taking into account bosonic loops to improve the results of the fermion-loop calculations of four-fermion production immediately suggests itself. We suggest to approximate bosonic loop corrections by carrying out the substitution

\[ G_\mu \rightarrow G_\mu / (1 + \Delta y^{SC}_{bos}) \]  

with $\Delta y^{SC}_{bos} = 11.1 \cdot 10^{-3}$ in the four-fermion production amplitudes evaluated in the fermion-loop approximation. Substitution (20) practically amounts to using $g_{W^\pm}(M_{W^\pm}^2)$ in four-fermion production as well. With substitution (20) it is indeed to be expected that the deviation of four-fermion production in the fermion-loop scheme will be diminished from the above estimated value of $\simeq 2.8\%$ to a value below 1\%.

In summary, the simple procedure of introducing the $SU(2)$ gauge coupling $g_{W^\pm}(M_W^2)$ at the high-energy scale, approximated by $s \simeq M_W^2$, or in other words, by introducing a renormalization scheme, in which the $SU(2)$ coupling is defined by the (theoretical value of the radiatively corrected) leptonic width of the $W$-boson, allows one to incorporate most of the electroweak radiative corrections to the process of $e^+e^- \rightarrow W^+W^-$ in the LEP 2 energy range of $\sqrt{s} \lesssim 200GeV$. Adding the Coulomb corrections and the initial state radiation in the leading logarithmic approximation provides a scheme which approximates the full one-loop results with an accuracy better than 1\%. Moreover, we suggest a simple recipe to approximately incorporate bosonic corrections into four-fermion production calculations, which so far are available in the fermion-loop approximation only. The overall accuracy
thus obtained should be sufficient for the analysis of $W$ pair production at LEP 2.

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Table 1: The Table shows the quality of the improved Born approximation (IBA) for the total (defined by integrating over $10^0 < \vartheta < 170^0$) and the differential cross section (for $W^-$-production angles $\vartheta$ of $10^0$, $90^0$ and $170^0$) for $e^+e^- \rightarrow W^+W^-$ at various energies for unpolarized and left-handed electrons. The quantity $\Delta_{IBA}$ denotes the percentage deviation of the IBA from the full one-loop result, the numerical results being taken from ref. [9]. Our correction, $\delta \Delta_{IBA}$, as well as the final accuracy, $\Delta_{IBA} + \delta \Delta_{IBA}$, of our IBA are given in the second and third column, respectively. The photon splitting scale entering the cross section for initial state bremsstrahlung is chosen as $Q^2 = s$. 

| angle | unpolarized | left-handed |
|-------|-------------|-------------|
|       | $\Delta_{IBA}$ | $\delta \Delta_{IBA}$ | $\Delta_{IBA} + \delta \Delta_{IBA}$ | $\Delta_{IBA}$ | $\delta \Delta_{IBA}$ | $\Delta_{IBA} + \delta \Delta_{IBA}$ |
| $\sqrt{s} = 161$ GeV |
| total | 1.45 | -0.72 | 0.73 | 1.45 | -0.72 | 0.73 |
| 10 | 1.63 | -0.73 | 0.90 | 1.63 | -0.73 | 0.90 |
| 90 | 1.44 | -0.72 | 0.72 | 1.44 | -0.72 | 0.72 |
| 170 | 1.26 | -0.70 | 0.56 | 1.26 | -0.70 | 0.56 |
| $\sqrt{s} = 165$ GeV |
| total | 1.27 | -0.71 | 0.56 | 1.28 | -0.71 | 0.57 |
| 10 | 1.67 | -0.74 | 0.93 | 1.67 | -0.74 | 0.93 |
| 90 | 1.17 | -0.71 | 0.46 | 1.18 | -0.71 | 0.47 |
| 170 | 0.75 | -0.67 | 0.08 | 0.77 | -0.67 | 0.10 |
| $\sqrt{s} = 175$ GeV |
| total | 1.26 | -0.71 | 0.55 | 1.28 | -0.71 | 0.57 |
| 10 | 1.71 | -0.75 | 0.96 | 1.71 | -0.75 | 0.96 |
| 90 | 1.03 | -0.69 | 0.34 | 1.06 | -0.70 | 0.36 |
| 170 | 0.59 | -0.62 | -0.03 | 0.69 | -0.63 | 0.06 |
| $\sqrt{s} = 184$ GeV |
| total | 1.02 | -0.70 | 0.32 | 1.06 | -0.71 | 0.35 |
| 10 | 1.57 | -0.75 | 0.82 | 1.57 | -0.75 | 0.82 |
| 90 | 0.67 | -0.68 | -0.01 | 0.72 | -0.69 | 0.03 |
| 170 | 0.10 | -0.58 | -0.48 | 0.32 | -0.64 | -0.32 |
| $\sqrt{s} = 190$ GeV |
| total | 1.24 | -0.70 | 0.54 | 1.28 | -0.71 | 0.57 |
| 10 | 1.67 | -0.74 | 0.93 | 1.67 | -0.75 | 0.92 |
| 90 | 0.95 | -0.68 | 0.27 | 1.01 | -0.69 | 0.32 |
| 170 | 0.58 | -0.57 | 0.01 | 0.83 | -0.59 | 0.24 |
| $\sqrt{s} = 205$ GeV |
| total | 1.60 | -0.70 | 0.90 | 1.65 | -0.71 | 0.94 |
| 10 | 1.77 | -0.74 | 1.03 | 1.77 | -0.74 | 1.03 |
| 90 | 1.55 | -0.66 | 0.89 | 1.64 | -0.68 | 0.96 |
| 170 | 1.61 | -0.53 | 1.08 | 1.94 | -0.56 | 1.38 |
| angle | unpolarized | left-handed |
|-------|-------------|-------------|
|       | $\Delta_{IBA}$ | $\delta\Delta_{IBA}$ | $\Delta_{IBA} + \delta\Delta_{IBA}$ | $\Delta_{IBA}$ | $\delta\Delta_{IBA}$ | $\Delta_{IBA} + \delta\Delta_{IBA}$ |
| total | 0.97 | -0.72 | 0.25 | 0.97 | -0.72 | 0.25 |
| 10    | 1.14 | -0.73 | 0.41 | 1.14 | -0.73 | 0.41 |
| 90    | 0.95 | -0.72 | 0.23 | 0.96 | -0.72 | 0.24 |
| 170   | 0.78 | -0.70 | 0.08 | 0.78 | -0.70 | 0.08 |

$s = 161$ GeV

| total | 0.77 | -0.71 | 0.06 | 0.78 | -0.71 | 0.07 |
| 10    | 1.17 | -0.74 | 0.43 | 1.17 | -0.74 | 0.44 |
| 90    | 0.67 | -0.71 | -0.04 | 0.68 | -0.71 | -0.03 |
| 170   | 0.25 | -0.67 | -0.42 | 0.27 | -0.67 | -0.40 |

$s = 165$ GeV

| total | 0.70 | -0.71 | -0.01 | 0.73 | -0.71 | 0.02 |
| 10    | 1.17 | -0.75 | 0.42 | 1.17 | -0.75 | 0.42 |
| 90    | 0.48 | -0.69 | -0.21 | 0.51 | -0.70 | -0.19 |
| 170   | 0.05 | -0.62 | -0.57 | 0.15 | -0.63 | -0.48 |

$s = 175$ GeV

| total | 0.43 | -0.70 | -0.27 | 0.47 | -0.71 | -0.24 |
| 10    | 0.99 | -0.75 | 0.24 | 0.99 | -0.75 | 0.24 |
| 90    | 0.09 | -0.68 | -0.59 | 0.14 | -0.69 | -0.55 |
| 170   | -0.48 | -0.58 | -1.06 | -0.26 | -0.64 | -0.90 |

$s = 184$ GeV

| total | 0.63 | -0.70 | -0.07 | 0.67 | -0.71 | -0.04 |
| 10    | 1.07 | -0.74 | 0.33 | 1.07 | -0.75 | 0.32 |
| 90    | 0.35 | -0.68 | -0.33 | 0.41 | -0.69 | -0.28 |
| 170   | -0.02 | -0.57 | -0.59 | 0.23 | -0.59 | -0.36 |

$s = 190$ GeV

| total | 0.94 | -0.70 | 0.24 | 0.99 | -0.71 | 0.28 |
| 10    | 1.11 | -0.74 | 0.37 | 1.12 | -0.74 | 0.38 |
| 90    | 0.90 | -0.66 | 0.24 | 0.99 | -0.68 | 0.31 |
| 170   | 0.96 | -0.53 | 0.43 | 1.28 | -0.56 | 0.72 |

$s = 205$ GeV

Table 2: As Table 1, but for the photon splitting scale $Q^2 = M_W^2$. 

References

[1] M. Bilenky, J.L. Kneur, F.M. Renard and D. Schildknecht, Nucl. Phys. B409 (1993) 22; Nucl. Phys. B419 (1994) 240.

[2] I. Kuss and D. Schildknecht, Phys. Lett. B383 (1996) 470.

[3] M. Böhm, A. Denner, T. Sack, W. Beenakker, F. Berends and H. Kuijf, Nucl. Phys. B304 (1988) 463.

[4] J. Fleischer, F. Jegerlehner and M. Zrałek, Z. Phys. C42 (1989) 409.

[5] M. Böhm, A. Denner and S. Dittmaier, Nucl. Phys. B376 (1992) 29; err. ibid. B391 (1993) 483.

[6] J. Fleischer, J.L. Kneur, K. Kołodziej, M. Kuroda and D. Schildknecht, Nucl. Phys. B378 (1992) 443; err. ibid. B426 (1994) 246.

[7] W. Beenakker, A. Denner, S. Dittmaier and R. Mertig, Phys. Lett. B317 (1993) 622;
W. Beenakker, A. Denner, S. Dittmaier, R. Mertig and T. Sack, Nucl. Phys. B410 (1993) 245.

[8] W. Beenakker and A. Denner, Int. J. Mod. Phys. A9 (1994) 4837

[9] W. Beenakker et. al., in Physics at LEP2, eds. G. Altarelli, T. Sjöstrand, F. Zwirner, CERN 96-01 Vol. 1, p. 79, [hep-ph/9602351].

[10] S. Dittmaier,
Talk at the 3rd International Symposion on Radiative Corrections, Cracow, Poland, August 1996, [hep-ph/9610529].

[11] S. Dittmaier, D. Schildknecht and G. Weiglein, Nucl. Phys. B465 (1996) 3.

[12] H. Burkhardt and B. Pietrzyk, Phys. Lett. B356 (1995) 398;
S. Eidelman and F. Jegerlehner, Z. Phys. C67 (1995) 585.

[13] W. Beenakker et. al., [hep-ph/9612260], NIKHEF 96-031, PSI-PR-96-41