DIFFRACTION AS A CRITICAL INGREDIENT IN SOFT
SCATTERING

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Abstract: The roll of diffraction in the formulation of soft scattering is investigated aiming to assess the resent SD TOTEM results.

I. INTRODUCTION

s channel Unitarity screening considerations date back to the ISR epoch. 40 years latter, theoretical estimates of soft scatterings channels at the TeV-scale require a unified analysis of elastic and diffractive scatterings (Good-Walker (GW)[1]) coupled to s and t unitarity screenings. In the following I shall utilize the GLM model[2].

The lowest order of s-channel unitarity bound on $a_{el}(s,b)$ is obtained from a diagonal re-scattering matrix,

$$2Ima_{el}(s,b) = |a_{el}(s,b)|^2 + G^{in}(s,b).$$

Its general solution is: $a_{el}(s,b) = i \left(1 - e^{-\Omega(s,b)/2}\right)$ and $G^{in}(s,b) = 1 - e^{-\Omega(s,b)}$. Arbitrary $\Omega$ leads to a bound $|a_{el}(s,b)| \leq 2$. It considerably over estimates TOTEM’s $\sigma_{tot}$ and $\sigma_{el}$ preseted in this meeting. A much better output is provided by Glauber’s eikonal approximation leading to a consequent $|a_{el}(s,b)| \leq 1$, identical to the black disc bound. The screened cross sections are:

$$\sigma_{tot} = 2 \int d^2b, \left(1 - e^{-\Omega(s,b)/2}\right), \sigma_{el} = \int d^2b, \left(1 - e^{-\Omega(s,b)/2}\right)^2, \sigma_{inel} = \int d^2b, \left(1 - e^{-\Omega(s,b)}\right).$$

In a single channel model, the unitarity bound is initiated by the s-channel black bound and the $ln^2(s)$ expanding amplitude radius. The consequent Froissart-Martin bound[3] is:

$$\sigma_{tot} \leq Cln^2(s/s_0), s_0 = 1GeV^2 \text{ and } C \propto 1/2m_n^2 \simeq 30mb. \text{ C is far too large to be relevant.}$$

s-channel unitarity implies: $\sigma_{el} \leq \frac{1}{2}\sigma_{tot}$ and $\sigma_{inel} \geq \frac{1}{2}\sigma_{tot}$. At saturation, $\sigma_{el} = \sigma_{inel} = \frac{1}{2}\sigma_{tot}$. Introducing diffraction, significantly changes the features of s-unitarity. However, the saturation signatures remain valid.
II. GOOD-Walker DECOMPOSITION

Consider a p-p scattering in which we identify two orthonormal states, a hadron $\Psi_h$ and a diffractive state $\Psi_D$. $\Psi_D$ replaces the continuous diffractive Fock states. GW noted that $\Psi_h$ and $\Psi_D$ do not diagonalize the 2x2 interaction matrix $T$. Let $\Psi_1$ and $\Psi_2$ be eigenstates of $T$. $\Psi_h = \alpha \Psi_1 + \beta \Psi_2$, $\Psi_D = -\beta \Psi_1 + \alpha \Psi_2$, $\alpha^2 + \beta^2 = 1$. The eigenstates initiate 4 $A_{i,k}$ elastic GW amplitudes ($\psi_i + \psi_k \rightarrow \psi_i + \psi_k$). $i,k=1,2$. For initial $p(\bar{p}) - p$ we have $A_{1,2} = A_{2,1}$.

I shall follow the GLM definition, in which the mass distribution of $\Psi_D$ is not defined and requires an independent specification.

The elastic, SD and DD amplitudes in a 2 channel GW model are:

$$a_{el}(s,b) = i\{\alpha^4 A_{1,1} + 2\alpha^2\beta^2 A_{1,2} + \beta^4 A_{2,2}\}, a_{sd}(s,b) = i\alpha\beta\{-\alpha^2 A_{1,1} + (\alpha^2 - \beta^2) A_{1,2} + \beta^2 A_{2,2}\},$$

$$a_{dd}(s,b) = i\alpha^2\beta^2\{A_{1,1} - 2A_{1,2} + A_{2,2}\},$$

in which, $A_{i,k}(s,b) = \left(1 - e^{\frac{1}{2}\Omega_{i,k}(s,b)}\right) \leq 1$.

GW diffraction has distinct features:

1. The Pumplin bound: $\sigma_{el} + \sigma_{diff}^{GW} \leq \frac{1}{2}\sigma_{tot}$.
   $\sigma_{diff}^{GW}$ is the sum of the GW soft diffractive cross sections.

2. Below saturation, $\sigma_{el} \leq \frac{1}{2}\sigma_{tot} - \sigma_{diff}^{GW}$ and $\sigma_{inel} \geq \frac{1}{2}\sigma_{tot} + \sigma_{diff}^{GW}$.

3. $a_{el}(s,b) = 1$, when and only when, $A_{1,1}(s,b) = A_{1,2}(s,b) = A_{2,2}(s,b) = 1$.

4. When $a_{el}(s,b) = 1$, all diffractive amplitudes at the same $(s,b)$ vanish.

5. GW saturation signatures are valid also in the non GW sector.

6. The saturation signature, $\sigma_{el} = \sigma_{inel} = \frac{1}{2}\sigma_{tot}$, in a multi channel calculation, is coupled to $\sigma_{diff} = 0$. Consequently, prior to saturation the diffractive cross sections stop growing and start to decrease with energy. This may serve as a signature of approaching saturation.

III. CROSSED CHANNEL UNITARITY

Translating the concepts presented into a viable phenomenology requires a specification of $\Omega(s,b)$, for which Regge Pomeron ($IP$) theory is a powerful tool. Mueller applied 3 body unitarity to equate the cross section of $a + b \rightarrow M_{sd}^2 + b$ to the triple Regge diagram $a + b + \bar{b} \rightarrow a + b + \bar{b}$, with a leading $3IP$ vertex term. The $3IP$ approximation is valid when $\frac{m_s^2}{M_{sd}^2} << 1$ and $\frac{M_{sd}}{s} << 1$. The leading energy and mass dependences are
\[
\frac{d\sigma^{3P}}{dt \, dM_{sd}^2} \propto s^{2\Delta_{3P}} (\frac{1}{M_{sd}^2})^{1+\Delta_{3P}}.
\]
Mueller’s 3IP approximation for non GW diffraction is the lowest order of t-channel multi IP interactions, compatible with t-channel unitarity.

t-channel screening results in a distinction between GW and non GW diffraction. Recall that, unitarity screening of GW ("low mass") diffraction is controlled by eikonalization, while the screening of non GW ("high mass") diffraction is controlled by the survival probability. Note that the relationship between GW and Mueller’s diffraction modes needs further study.

IV. THE PARTONIC POMERON

Current IP models differ in details, but have in common a relatively large adjusted input \(\Delta_{IP}\) and a diminishing \(\alpha'_{IP}\). Traditionally, \(\Delta_{IP}\) determines the energy dependence of the total, elastic and diffractive cross sections while \(\alpha'_{IP}\) determines the forward slopes. This picture is modified in updated IP models in which s and t unitarity screenings induce a much smaller IP intercept at \(t=0\), denoted \(\Delta_{eff}^{\prime}\), which gets smaller with energy. The exceedingly small fitted \(\alpha'_{IP}\) implies a partonic description of the IP which leads to a pQCD interpretation.

Gribov’s partonic Regge theory[6] provides the microscopic sub structure of the IP where the slope of the IP trajectory is related to the mean transverse momentum of the partonic dipoles constructing the Pomeron. \(\alpha'_{IP} \propto 1/ <p_t^2>\).

Accordingly: \(\alpha_s \propto \pi/ln (\frac{<p_t^2>}{\Lambda_{QCD}^2}) \ll 1\).

We obtain a IP with hardness changing continuously from hard (BFKL like) to soft (Regge like). This is a non trivial relation as the soft IP is a simple moving pole in J-plane, while the BFKL hard IP is a branch cut, approximated as a simple pole with \(\Delta_{IP} = 0.2 - 0.3\) and \(\alpha'_{IP} \simeq 0\).

GLM[2] and KMR[7] models are rooted in Gribov’s partonic IP theory with a hard pQCD IP input. It is softened by unitarity screening (GLM), or the decrease of its partons’ transverse momentua (KMR). The two definitions are correlated. GLM and KMR have a bound of validity at 60-100 TeV implied by their approximations.
V. UNITARITY SATURATION

Unitarity saturation is coupled to 3 experimental signatures:

\[
\frac{\sigma_{\text{inel}}}{\sigma_{\text{tot}}} = 0.5, \quad \frac{\sigma_{\text{diff}}}{B_{el}} = 9\pi, \quad \sigma_{\text{diff}} = 0.
\]

Following is p-p TeV-scale data relevant to the assessment of saturation:

CDF(1.8 TeV): \(\sigma_{\text{tot}} = 80.03 \pm 2.24 mb, \ \sigma_{el} = 19.70 \pm 0.85 mb, \ B_{el} = 16.98 \pm 0.25 GeV^{-2}\).

TOTEM(7 TeV): \(\sigma_{\text{tot}} = 98.3 \pm 0.2(stat) \pm 2.8(sys) mb, \ \sigma_{el} = 24.8 \pm 0.2(stat) \pm 2.8(sys) mb, \ B_{el} = 20.1 \pm 0.2(stat) \pm 0.3(sys) GeV^{-2}\).

AUGER(57 TeV): \(\sigma_{\text{tot}} = 133 \pm 13(stat) \pm 17(sys) \pm 16(Glauber) mb, \ \sigma_{\text{inel}} = 92 \pm 7(stat) \pm 9(sys) \pm 16(Glauber) mb\).

Note that AUGER output margin of error is large!

\(\sigma_{\text{inel}}/\sigma_{\text{tot}} = 0.754(\text{CDF}), \ 0.748(\text{TOTEM}), \ 0.692(\text{AUGER})\).

The numbers suggest a very slow approach toward saturation, well above the TeV-scale.

Since multi-channel models are bounded to the TeV-scale, I am limited to single channel models above 100 TeV. To this end I quote a calculation by Block and Halzen[8], who have checked the predictions of their model at the Planck-scale \((1.22 \cdot 10^{16} \text{TeV})\). They obtain \(\sigma_{\text{inel}}/\sigma_{\text{tot}} = 1131mb/2067mb = 0.547\).

It re-enforces the conclusion that saturation will be attained, if at all, at non realistic energies.

VI. TOTEM RECENT SD DATA

As noted, the predicted vanishing of the diffractive cross sections at saturation implies that \(\sigma_{sd}\), which up to TOTEM grows slowly with energy, will eventually start to reduce.

This may serve as an early signature that saturation is being approached.

The preliminary TOTEM measurement (reported in this meeting) of \(\sigma_{sd} = 6.5 \pm 1.3 mb\), corresponding to

\[3.4 < M_{sd} < 1100 GeV\] and \(2.4 \cdot 10^{-7} < \xi < 0.025\), suggests a radical change in the energy dependence of \(\sigma_{sd}\), which is smaller than its value at CDF.

This feature, if correct, suggests a much faster approach toward unitarity saturation than suggested by \(\frac{\sigma_{\text{inel}}}{\sigma_{\text{tot}}}\).
TOTEM diffractive data is very preliminary. Regardless, the compatibility between the information derived from different channels of soft scattering deserves a very careful study!

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