Invariant Regularization of Supersymmetric Chiral Gauge Theory. II

Takuya Hayashi, Yoshihisa Ohshima, Kiyoshi Okuyama and Hiroshi Suzuki

Department of Physics, Ibaraki University, Mito 310, Japan

ABSTRACT

By undertaking additional analyses postponed in a previous paper, we complete our construction of a manifestly supersymmetric gauge-covariant regularization of supersymmetric chiral gauge theories. We present the following: An evaluation of the covariant gauge anomaly; a proof of the integrability of the covariant gauge current in anomaly-free cases; a calculation of a one-loop superconformal anomaly in the gauge supermultiplet sector. On the last point, we find that the ghost-anti-ghost supermultiplet and the Nakanishi-Lautrup supermultiplet give rise to BRST exact contributions which, due to “tree-level” Slavnov-Taylor identities in our regularization scheme, can safely be neglected, at least at the one-loop level.

* e-mail: hayashi@physun1.sci.ibaraki.ac.jp
† e-mail: ohshima@mito.ipc.ibaraki.ac.jp
‡ e-mail: okuyama@mito.ipc.ibaraki.ac.jp
§ e-mail: hsuzuki@mito.ipc.ibaraki.ac.jp
1. Introduction

In a recent paper [1], we proposed a manifestly supersymmetric gauge-covariant regularization of supersymmetric chiral gauge theories. In the sense of the background field method, it was shown that our scheme provides a supersymmetric gauge invariant regularization of the effective action above one-loop order. On the other hand, our scheme gives a gauge covariant definition of one-loop diagrams, and, when the representation of the chiral supermultiplet is free from the gauge anomaly, the definition restores the gauge invariance. Our scheme also defines composite operators in a supersymmetric gauge-covariant manner. However, several important issues concerning properties of our scheme were postponed in Ref. [1]. In the present paper, to complete the construction, we present detailed analyses on those issues.

We first recapitulate the essence of our regularization scheme. We consider a general renormalizable supersymmetric model:

\[
S = \frac{1}{2T(R)} \int d^6z \; \text{tr} \; W^\alpha W_\alpha + \int d^8z \; \Phi^\dagger e^V \Phi + \int d^6z \left( \frac{1}{2} \Phi^T m \Phi + \frac{1}{3} g \Phi^3 \right) + \text{h.c.}
\]  

(1.1)

To apply the notion of the superfield background field method [3], we split the gauge superfield and the chiral superfield as [1]

\[
e^V = e^{V_B} e^{V_Q}, \quad \Phi = \Phi_B + \Phi_Q.
\]  

(1.2)

Furthermore, to make the supersymmetry and the background gauge invariance (in the unregularized level) manifest, we adapt the following gauge fixing term and

\footnote{For conciseness, we do not repeat the explanation of our notation and conventions given in Ref. [1]; these basically follow the conventions of Ref. [2].}
the ghost-anti-ghost term

\[ S' = -\frac{\xi}{8T(R)} \int d^8z \text{tr}(\bar{D}^2 V_Q)(D^2 V_Q) \]
\[ + \frac{1}{T(R)} \int d^8z \text{tr}(e^{-V_B} c^a \dagger e^{V_B} + c^a) \]
\[ \times \mathcal{L}_{V_B/2} \cdot [(c + e^{-V_B} c^a \dagger e^{V_B}) + \coth(\mathcal{L}_{V_B/2}) \cdot (c - e^{-V_B} c^a \dagger e^{V_B})] \]
\[ - \frac{2\xi}{T(R)} \int d^8z \text{tr} e^{-V_B} b^a \dagger e^{V_B} b, \]  
\[(1.3)\]

where the normalization of the Nielsen-Kallosh (NK) ghost \( b \) has been changed from Ref. [1].

In calculating radiative corrections to the effective action in the background field method, i.e., the generating functional of 1PI Green’s functions with all the external lines forming the background field, \( V_B \) or \( \Phi_B \), we expand the total action \( S_T \equiv S + S' \) in powers of the quantum field, \( S_T = S_{T0} + S_{T1} + S_{T2} + S_{T3} + \cdots \). (Hereafter, a number appearing in the subscript indicates the power of the quantum fields.) The quadratic action is further decomposed as \( S_{T2} = S_{T2}^{\text{gauge}} + S_{T2}^{\text{ghost}} + S_{T2}^{\text{chiral}} + S_{T2}^{\text{mix}} \).

The first part, which is composed purely of the gauge superfields, is given by

\[ S_{T2}^{\text{gauge}} = \int d^8z V_Q^a \left[ \frac{1}{8} \tilde{\nabla}^\alpha \tilde{\nabla}^2 \tilde{\nabla}_\alpha + \frac{1}{2} W_B^\beta \tilde{\nabla}_\alpha - \frac{\xi}{16} (\tilde{\nabla}^2 \tilde{\nabla}^2 + D^2 \tilde{\nabla}^2) \right]^{ab} V_Q^b \]
\[ = \int d^8z V_Q^a \left[ -\tilde{\nabla}^m \tilde{\nabla}_m + \frac{1}{2} W_B^\beta \tilde{\nabla}_\alpha - \frac{1}{2} W_B \tilde{\nabla}^i + \frac{1}{16} (1 - \xi)(\tilde{\nabla}^2 \tilde{\nabla}^2 + D^2 \tilde{\nabla}^2) \right]^{ab} V_Q^b. \]  
\[(1.4)\]

The ghost-anti-ghost action, to second order in the quantum fields, is given by

\[ S_{T2}^{\text{ghost}} = \int d^8z \left[ c'^a (e^{V_B})^{ab} c^b + c'^a (e^{V_B})^{ab} c^b - 2\xi b'^a (e^{V_B})^{ab} b^b \right]. \]  
\[(1.5)\]

There are two kinds of action which contain the chiral multiplet. One is the part
that survives even for $\Phi_B = 0$,

$$S_{T2}^{\text{chiral}} = \int d^8 z \, \Phi_Q^\dagger e^{V_B} \Phi_Q + \int d^6 z \frac{1}{2} \Phi_Q^T m \Phi_Q + \text{h.c.}, \quad (1.6)$$

and the other is the part that disappears for $\Phi_B = 0$,

$$S_{T2}^{\text{mix}} = \int d^8 z \left( \Phi_B^\dagger e^{V_B} V_Q \Phi_Q + \Phi_Q^\dagger e^{V_B} V_Q \Phi_B + \frac{1}{2} \Phi_B^\dagger e^{V_B} V_Q^2 \Phi_B \right)$$

$$+ \int d^6 z \, g \Phi_B \Phi_Q^2 + \text{h.c.} \quad (1.7)$$

Our regularization is then implemented as follows: We take propagators of the quantum fields that are given by formally diagonalizing $S_{T2}^{\text{gauge}} + S_{T2}^{\text{ghost}} + S_{T2}^{\text{chiral}}$. Then, for a finite ultraviolet cutoff $\Lambda$, we modify the propagators so as to improve the ultraviolet behavior and simultaneously preserve the background gauge covariance. For example, for the quantum gauge superfield, we use

$$\langle T^* V_Q^a(z) V_Q^b(z') \rangle$$

$$\equiv i \left[ f \left( [-\tilde{\nabla}^m \tilde{\nabla}_m + W_B^a \tilde{\nabla}_a / 2 - \bar{W}_B \bar{D}^a / 2 + (1 - \xi)(\bar{\nabla}^2 \bar{D}^2 + \bar{D}^2 \bar{\nabla}^2) / 16] / (\xi \Lambda^2) \right) \right.$$

$$\times \left. \left[ -\tilde{\nabla}^m \tilde{\nabla}_m + W_B^a \tilde{\nabla}_a / 2 - \bar{W}_B \bar{D}^a / 2 + (1 - \xi)(\bar{\nabla}^2 \bar{D}^2 + \bar{D}^2 \bar{\nabla}^2) / 16 \right] \right]^{ab} \delta(z - z'), \quad (1.8)$$

and, for the ghost superfields,

$$\langle T^* c^a(z) c^{\dagger b}(z') \rangle = \langle T^* c^{\dagger a}(z) c^{\dagger b}(z') \rangle = -2\xi \langle T^* b^a(z) b^{\dagger b}(z') \rangle$$

$$\equiv i \left[ f(-\bar{D}^2 \bar{\nabla}^2 / 16 \Lambda^2) \bar{D}^2 \frac{1}{\bar{\nabla}^2 \bar{D}^2} \bar{\nabla}^2 e^{-V_B} \right]^{ab} \delta(z - z'), \quad (1.9)$$

* There is no deep reason for doing this. It merely simplifies the expressions, because we do not include $S_{T2}^{\text{mix}}$ in diagonalizing the quadratic action.

† Hereafter, the brackets $\langle \cdots \rangle$ represent an expectation value in the unconventional perturbative picture in which $S_{T2}^{\text{gauge}} + S_{T2}^{\text{ghost}} + S_{T2}^{\text{chiral}}$ is regarded as the “un-perturbative part”.
and, for the quantum chiral superfield,

$$\langle T^* \Phi_Q(z) \Phi_Q^\dagger(z') \rangle = \frac{i}{16} f(-D^2 \nabla^2/16 \Lambda^2) D^2 \frac{1}{\nabla^2 D^2/16 - m^\dagger m} \nabla^2 e^{-V_B} \delta(z - z'), \quad (1.10)$$

and

$$\langle T^* \Phi_Q(z) \Phi_Q^T(z') \rangle = \frac{i}{4} f(-D^2 \nabla^2/16 \Lambda^2) D^2 \frac{1}{\nabla^2 D^2/16 - m^\dagger m} m^\dagger \delta(z - z'). \quad (1.11)$$

In these expressions, $f(t)$ is the regularization factor, which decreases sufficiently rapidly, $f(\infty) = f'(\infty) = f''(\infty) = \cdots = 0$ in the ultraviolet, and $f(0) = 1$, to reproduce the original propagators in the infinite cutoff limit $\Lambda \to \infty$. The argument of the regularization factor has the same form as the denominator of each propagator; this prescription is suggested by the proper-time cutoff [1]. In this way, the propagators obey the same transformation law as the original ones under the background gauge transformation on the background gauge superfield $V_B$. This property is crucial for the gauge covariance of the scheme.

Using the above propagators of the quantum fields, 1PI Green’s functions are evaluated as follows. There are two kinds of contributions, because we have diagonalized $S_{T^2}^{\text{gauge}} + S_{T^2}^{\text{ghost}} + S_{T^2}^{\text{chiral}}$ in constructing the propagators. (I) Most of radiative corrections are evaluated (as usual) by simply connecting quantum fields in $S_{T^2}^{\text{mix}}, S_{T^3}, S_{T^4}$, etc., by the modified propagators. This defines the first part of the effective action, $\Gamma_1[V_B, \Phi_B]$, which is given by the 1PI part of

$$\langle \exp \left\{ i \left[ S_{T^2} - \left( S_{T^2}^{\text{gauge}} + S_{T^2}^{\text{ghost}} + S_{T^2}^{\text{chiral}} \right) \right] \right\} \rangle. \quad (1.12)$$

(II) However, since the quadratic action $S_{T^2}^{\text{gauge}} + S_{T^2}^{\text{ghost}} + S_{T^2}^{\text{chiral}}$ depends on the background gauge superfield $V_B$ (but not on $\Phi_B$) non-trivially, the one-loop Gaussian determinant arising from this action has to be taken into account. To define
this part of the effective action, \( \Gamma_{\text{II}}[V_B] \), we adopt the following prescription:

\[
\frac{\delta \Gamma_{\text{II}}[V_B]}{\delta V^a_B(z)} \equiv \langle J^a(z) \rangle, \quad J^a(z) \equiv \frac{\delta}{\delta V^a_B(z)}(S_{T2}^{\text{gauge}} + S_{T2}^{\text{ghost}} + S_{T2}^{\text{chiral}}). \tag{1.13}
\]

The quantum fields in \( J^a(z) \) are connected by the modified propagators. Note that since \( J^a(z) \) is quadratic in the quantum fields by definition, (1.13) consists of only one-loop diagrams. The total effective action is then given by the sum \( \Gamma[V_B, \Phi_B] = \Gamma_{\text{I}}[V_B, \Phi_B] + \Gamma_{\text{II}}[V_B] \). Although naively the relation (1.13) is just one of many equivalent ways to define the one-loop effective action, it has a great advantage at the regularized level; the prescription (1.13) respects the gauge covariance of the gauge current [1]. Our prescription (1.13) is a natural supersymmetric generalization of the covariant regularization of Refs. [4] and [5]. (III) When a certain composite operator \( O(z) \) is inserted into a Green’s function, it is computed as usual (by using the modified propagators):

\[
\langle O(z) \exp \{ i \left[ S_T - (S_{T2}^{\text{gauge}} + S_{T2}^{\text{ghost}} + S_{T2}^{\text{chiral}}) \right] \} \rangle. \tag{1.14}
\]

Following the above prescription, it was shown [1] that the first part of the effective action \( \Gamma_{\text{I}}[V_B, \Phi_B] \) (which contains all the higher loop diagrams!) is always supersymmetric and background gauge invariant. It was also shown that a composite operator which behaves classically as a gauge covariant superfield, such as the gauge current superfield \( \langle J^a(z) \rangle \) in (1.13), is regularized as a (background) gauge covariant superfield. This implies, in particular, that if there exists a functional \( \Gamma_{\text{II}}[V_B] \) in (1.13) that reproduces the gauge current superfield \( \langle J^a(z) \rangle \) as the variation, it is also supersymmetric and gauge invariant (because \( V^a_B \) is a superfield). One might expect that such a functional, an “effective action,” always exists. However, this is not the case in our scheme. In fact, if the whole effective action were always gauge invariant, there would be no possibility of the gauge anomaly that may arise from chiral multiplet’s loop.
Actually, it is easy to see that the anomaly cancellation is the necessary condition for the existence of $\Gamma_{\Pi}[V_B]$ in (1.13). As should be the case, our scheme provides gauge invariant regularization only for anomaly-free cases. In the analysis of Ref. [1], however, it was not clear if the anomaly cancellation is also sufficient for the existence of $\Gamma_{\Pi}[V_B]$. In § 2, we establish the existence of $\Gamma_{\Pi}[V_B]$ in anomaly-free cases. Therefore, our scheme actually provides for anomaly-free cases a gauge invariant regularization of the effective action.

The prescription (1.14), on the other hand, is especially useful when one wishes to evaluate the quantum anomaly while preserving the supersymmetry and the gauge covariance (or invariance). As was announced in Ref. [1], we apply it to the superconformal anomaly in the gauge supermultiplet sector and present the detailed one-loop calculation in § 4. In this calculation, the gauge-fixing term and the ghost-anti-ghost sector cause another complication. To treat this complication systematically, in § 3, we introduce the notion of BRST symmetry and the corresponding Slavnov-Taylor identity in our scheme.

2. Integrability of the covariant gauge current

As was noted in the Introduction, the most important issue that was left uninvestigated in Ref. [1] is whether the second part of the effective action $\Gamma_{\Pi}[V_B]$ in (1.13) exists; we call this a problem of the integrability of the gauge current. The gauge anomaly cancellation $\text{tr} T^a \{ T^b, T^c \} = 0$ is the necessary condition for the integrability [1] because the covariantly regularized gauge current $\langle J^a(z) \rangle$ gives rise to the covariant anomaly (we shall explicitly show this below), but, on the other hand, $\langle J^a(z) \rangle$ should produce a consistent anomaly [6–13] if such a functional $\Gamma_{\Pi}[V_B]$ exists. These two requirements are consistent only if the gauge anomaly vanishes. In this section, we shall establish the converse: When $\text{tr} T^a \{ T^b, T^c \} = 0$, $\Gamma_{\Pi}[V_B]$ which satisfies (1.13) always exists in $\Lambda \to \infty$. Namely, the anomaly cancellation is also a sufficient condition for the integrability. Therefore, when the gauge representation is anomaly-free, our prescription (1.13) provides a gauge invariant
regularization of the effective action. We shall give two proofs from quite different viewpoints. However, before going into the proof, let us show how the gauge anomaly is evaluated in our scheme.

2.1. Covariant gauge anomaly

In this subsection, we present a calculation of the covariant gauge anomaly which reads

\[ -\frac{1}{4} \partial^2 C^{ab} \langle J^b(z) \rangle_{\Lambda \to \infty} - \frac{1}{64\pi^2} \text{tr} T^a W_B^a W_B^a, \]  

(2.1)

where the combination \( C^{ab} \) has been defined by

\[ C^{ab} \equiv \left[ \frac{V_B}{2} \left( \coth \frac{V_B}{2} - 1 \right) \right]^{ab}, \quad V_B \equiv T^a V_B^a. \]  

(2.2)

The gauge covariance of the right-hand side of (2.1) is in accord with our gauge covariant definition of the gauge current [1]. To see how the left-hand side of (2.1) is related to the “gauge anomaly,” i.e., the non-invariance of the effective action \( \Gamma_{II}[V_B] \) under the background gauge transformation, we introduce the generator of the background transformation:

\[ G^a(z) \equiv -\frac{1}{4} \partial^2 C^{ab} \frac{\delta}{\delta V_B^b(z)}. \]  

(2.3)

In fact, it is easy to see that a variation of an arbitrary functional \( F[V_B] \) of \( V_B \) under the background transformation (see (2.6) of Ref. [1])

\[ \delta V_B = i \mathcal{L}_{V_B/2} \cdot \left[ (\Lambda + \Lambda^\dagger) + \coth(\mathcal{L}_{V_B/2}) \cdot (\Lambda - \Lambda^\dagger) \right], \]  

(2.4)

can be written as

\[ \delta F[V_B] = \int d^6 z i \Lambda^a(z) G^a(z) F[V_B] + \text{h.c.} \]  

(2.5)

Therefore, assuming the identification in (1.13), the gauge variation may be written

This result itself is not new [14]; we present the calculation in our scheme for later use.

Recall that the first part of the effective action \( \Gamma_I[V_B, \Phi_B] \) is always gauge invariant in our scheme.
\[
\delta I_{\Pi}[V_B] = \int d^6 z \, i \Lambda^a(z) G^a(z) I_{\Pi}[V_B] + \text{h.c.}
\]
\[
= \int d^6 z \, i \Lambda^a(z) \left( -\frac{1}{4} \right) D^2 C^{ab} \left\langle J^b(z) \right\rangle + \text{h.c.}
\]

This is the reason we regard (2.1) as the gauge anomaly.

Now, from the definition of the gauge current in (1.13), we have
\[
\int d^6 z \, i \Lambda^a(z) \left( -\frac{1}{4} \right) D^2 C^{ab} \left\langle J^b(z) \right\rangle = \int d^6 z \left\langle i \Lambda^a(z) G^a(z) (S_{T2}^{\text{gauge}} + S_{T2}^{\text{ghost}} + S_{T2}^{\text{chiral}}) \right\rangle.
\]

However, \( S_{T2}^{\text{gauge}} \), \( S_{T2}^{\text{ghost}} \) and \( S_{T2}^{\text{chiral}} \) are unchanged if we make the background gauge transformation (2.4) and simultaneously
\[
\delta V^a_Q = -i \Lambda^c(T^c)^{ab} V^b_Q, \quad \delta \Phi_Q = -i \Lambda^a T^a \Phi_Q,
\]

because \( S_{T2}^{\text{gauge}} \), \( S_{T2}^{\text{ghost}} \) and \( S_{T2}^{\text{chiral}} \) are background gauge invariant. Therefore, the right-hand side of (2.7) is equal to the opposite of the variation of \( S_{T2}^{\text{gauge}} \), \( S_{T2}^{\text{ghost}} \) and \( S_{T2}^{\text{chiral}} \) under (2.8). This yields (in the super-Fermi-Feynman gauge \( \xi = 1 \))
\[
\int d^6 z \left\langle i \Lambda^a(z) G^a(z) (S_{T2}^{\text{gauge}} + S_{T2}^{\text{ghost}} + S_{T2}^{\text{chiral}}) \right\rangle
\]
\[
= -2i \int d^8 z \left\langle \Lambda^a(z) (T^a)^{bc} V^b_Q \left( -\slashed{\nabla} m \slashed{\nabla}_m + \frac{1}{2} W_B^a \slashed{\nabla}_\alpha - \frac{1}{2} \nabla_B \slashed{D} \right)^{cd} V^d_Q(z) \right\rangle
\]
\[
+ i \int d^8 z \left\langle c^{ld} (e^{V_B})^{db} c^{c}(z) \right\rangle + \left\langle c^{id} (e^{V_B})^{db} c^{c}(z) \right\rangle
\]
\[
- 2 \left\langle b^{ld} (e^{V_B})^{db} b^{c}(z) \right\rangle
\]
\[
+ i \int d^8 z \left\langle \Phi^l_Q e^{V_B} T^a \Phi_Q(z) \right\rangle + i \int d^6 z \left\langle \Phi^T_Q m T^a \Phi_Q(z) \right\rangle,
\]

where we have used the explicit form of the action given by (1.4)–(1.6). In the above expression, use of the modified propagators (1.8)–(1.11) is assumed according to
our prescription (1.13). Then the kinetic operators in $S^\text{gauge}_{T^2}$, $S^\text{ghost}_{T^2}$ and $S^\text{chiral}_{T^2}$ cancel the denominators of the propagators. As a result, from (2.7), we have

$$-\frac{1}{4} D^2 C^{ab} \langle J^b(z) \rangle$$

$$= \frac{i}{4} D^2 \lim_{z' \to z} \left\{ \mathcal{T}^a f \left( -\tilde{\nabla}^m \tilde{\nabla}_m + \mathcal{W}^\alpha_{B} \tilde{\nabla}_{\alpha}/2 - \mathcal{W}_{B\dot{\alpha}} \mathcal{D}^{\dot{\alpha}}/2 \right)/\Lambda^2 \right\}^{bb} \delta(z-z')$$

$$+ 3 \cdot \frac{i}{4} \lim_{z' \to z} \left[ \mathcal{T}^a f(-D^2 \tilde{\nabla}^2/16\Lambda^2) \right]^{bb} \mathcal{D}^2 \delta(z-z')$$

$$- \frac{i}{4} \lim_{z' \to z} \text{tr} \mathcal{T}^a f(-D^2 \tilde{\nabla}^2/16\Lambda^2) \mathcal{D}^2 \delta(z-z').$$

(2.10)

The evaluation of this expression is straightforward. First of all, the first term on the right-hand side vanishes as $\Lambda \to \infty$ because the number of spinor derivatives is not sufficient; the gauge multiplet does not contribute. Next, in the second (the ghost-anti-ghost multiplet) and the third (the chiral multiplet) terms, we can repeat the calculation in the Appendix of Ref. [1]. In this way, we finally obtain (2.1). Note that the ghost-anti-ghost multiplet does not contribute, because $\text{tr} \mathcal{T}^a \{ T^b, T^c \} = 0$.

2.2. Integrability of the gauge current: the first proof

We prove that the second part of the effective action $\Gamma_{\Pi}[V_B]$ in (1.13) always exists when the representation of the chiral multiplet is free of the gauge anomaly. Our first proof is a natural supersymmetric generalization of the procedure in Ref. [5], which starts with the ansatz

$$\Gamma_{\Pi}[V_B] = \int_0^1 dg \int d^8 z \, V_B^a(z) \langle J^a(z) \rangle_g.$$  

(2.11)

In this expression, the subscript $g$ implies the expectation value is evaluated with the “coupling constant” introduced by taking $V_B \to gV_B$ in our definition of the covariant gauge current (1.13). We show below that, when $\text{tr} \mathcal{T}^a \{ T^b, T^c \} = 0$, this
functional reproduces the gauge current in the limit $\Lambda \to \infty$,

$$\frac{\delta \Gamma_{II}[V_B]}{\delta V_B^a(z)} = \langle J^a(z) \rangle. \quad (2.12)$$

Note that, since the composite operator $\langle J^a(z) \rangle_g$ in (2.11) does depend on the background gauge field $V_B$, the relation (2.12) is by no means trivial.

We now proceed with the proof. We first directly take the functional derivative of $\Gamma_{II}[V_B]$ (2.11):

$$\frac{\delta \Gamma_{II}[V_B]}{\delta V_B^a(z)} = \int_0^1 dg \langle J^a(z) \rangle_g + \int d^8 z' V_B^b(z') \frac{\delta \langle J^b(z') \rangle_g}{\delta V_B^a(z)}$$

$$= \langle J^a(z) \rangle - \int_0^1 dg \frac{d}{dg} \langle J^a(z) \rangle_g + \int d^8 z' V_B^b(z') \frac{\delta \langle J^b(z') \rangle_g}{\delta V_B^a(z)}.$$  

(2.13)

In going from the first line to the second line, we performed an integration by parts. Then we note $\langle J^a(z) \rangle_g$ depends on $g$ only through the combination $gV_B$. Therefore

$$\frac{d}{dg} \langle J^a(z) \rangle_g = \int d^8 z' V_B^b(z') \frac{\delta \langle J^a(z) \rangle_g}{g\delta V_B^b(z')}.$$  

(2.14)

Substituting (2.14) into (2.13), we have

$$\frac{\delta \Gamma_{II}[V_B]}{\delta V_B^a(z)} = \langle J^a(z) \rangle + \int_0^1 dg \int d^8 z' V_B^b(z') \left[ \frac{\delta \langle J^b(z') \rangle_g}{\delta V_B^a(z)} - \frac{\delta \langle J^a(z) \rangle_g}{\delta V_B^b(z')} \right].$$  

(2.15)

The relation (2.12) follows if the quantity in the square brackets (the functional rotation) vanishes. To consider this quantity, we note the identity

$$-\frac{1}{4} D^2 C^{ac} \frac{\delta \langle J^c(z) \rangle_g}{\delta V_B^b(z')} = \frac{\delta}{\delta V_B^b(z')} \left( -\frac{1}{4} D^2 C^{ac} \langle J^c(z) \rangle + \frac{1}{4} D^2 \frac{\delta C^{ac}}{\delta V_B^b(z')} \langle J^c(z) \rangle \right).$$  

(2.16)

Note that the first term on the right-hand side is nothing but the gauge anomaly (2.1). On the other hand, it is not difficult to see that the gauge covariance of our gauge
current [1],
\[ \langle J^a(z) \rangle ' = \frac{\partial V^a_B(z)}{\partial V^c_B(z)} \langle J^b(z) \rangle , \tag{2.17} \]
where \( V'_B = V_B + \delta V_B \) and \( \delta V_B \) is given by (2.4), implies
\begin{align*}
- \frac{1}{4} D^2 C^{ac} \frac{\delta \langle J^b(z') \rangle}{\delta V^c_B(z)} &= \frac{1}{4} D^2 \frac{\delta C^{ac}}{\delta V^b_B(z')} \langle J^c(z) \rangle \\
\text{as a coefficient of the chiral gauge parameter } \Lambda. \text{ Therefore, from (2.18), (2.16) and (2.1), we find}
\end{align*}
\[ - \frac{1}{4} D^2 C^{ac} \left[ \frac{\delta \langle J^b(z') \rangle}{\delta V^c_B(z)} - \frac{\delta \langle J^c(z) \rangle}{\delta V^b_B(z')} \right] \xrightarrow{\Lambda \to \infty} \frac{1}{64\pi^2} \delta \left( \frac{1}{2} \right) \text{tr } T^a W^a_B W_B \tag{2.19} \]
if \( \text{tr } T^a \{ T^b, T^c \} = 0 \). To see the functional rotation in the square brackets itself vanishes in (2.19) and (2.20), we expand the functional rotation in powers of \( V_B \), as \( c_0 + c_1 V_B + c_2 V_B^2 + \cdots \). Then (2.19) and (2.20) require \( D^2 c_0 = D^2 c_1 = 0 \), as the \( O(V_B^0) \) term (note that \( C^{ab} = \delta^{ab} + O(V_B) \)) and thus \( (\Box + iD^m \partial m/2)c_0 = 0 \).

Assuming the boundary condition that the functional rotation vanishes as \( x-x' \to \infty \), we have \( c_0 = 0 \). This procedure can clearly be repeated for \( c_1, c_2, \) etc., and we finally conclude that \( c_0 = c_1 = \cdots = 0 \); that is, the functional rotation itself vanishes. Therefore, from (2.15), we have (2.12).

Let us summarize what we have shown. The variation of the functional (2.11) reproduces the covariant gauge current as (2.12) if the “gauge anomaly,” the

\* Otherwise, the integral in (2.15), and thus (2.11), would be ill-defined.
left-hand side of (2.1), vanishes. In the $\Lambda \to \infty$ limit, the “gauge anomaly” is given by the right-hand side of (2.1) which vanishes when the anomaly cancellation condition, $\text{tr} \, T^a \{ T^b, T^c \} = 0$, holds. Therefore, if $\text{tr} \, T^a \{ T^b, T^c \} = 0$, the effective action (2.11) satisfies (2.12) in the $\Lambda \to \infty$ limit. Put differently, even if $\text{tr} \, T^a \{ T^b, T^c \} = 0$, when the cutoff $\Lambda$ is finite, there may exist pieces in the covariantly regularized gauge current $\langle J^a(z) \rangle$ which cannot be expressed as a variation of some functional. However, those non-integrable pieces disappear in the limit $\Lambda \to \infty$ in the same sense that the gauge anomaly is given by the right-hand side of (2.1) in this same limit.

2.3. Integrability of the gauge current: the second proof

In this subsection, we give another proof of the integrability. This proof, although being somewhat less rigorous and not applicable to the general form of the regularization factor $f(t)$, demonstrates an interesting relation between our prescription (1.13) and the generalized Pauli-Villars regularization [15–20]. We show below that, when the gauge anomaly vanishes, (1.13) can basically be realized by the generalized Pauli-Villars regularization. Since the Pauli-Villars regularization is implemented at the level of Feynman diagrams for which the Bose symmetry among gauge vertices is manifest, the corresponding effective action always exists; namely, the integrability is obvious. Since the gauge multiplet and the ghost-anti-ghost multiplet belong to the adjoint (i.e., real) representation, one may always apply the conventional Pauli-Villars prescription to those sectors; there is no subtlety associated with the gauge anomaly. Therefore, we shall present only analysis on the chiral multiplet.

To relate our prescription with the generalized Pauli-Villars regularization for the complex gauge representation, we introduce the “doubled” representation following Ref. [20] (see also Ref. [19]):

$$R^a \equiv \begin{pmatrix} T^a & 0 \\ 0 & -T^a \end{pmatrix}, \quad U_B \equiv R^a V^a_B, \quad (2.21)$$
where \( T^a \) is the original (complex) gauge representation. Then, with this notation, our regularized gauge current (1.13) for the chiral multiplet is expressed as

\[
\langle J^a_{\text{chiral}}(z) \rangle \equiv \left\langle \frac{\partial S_{\text{chiral}}}{\partial V_B^a(z)} \right\rangle
= \frac{i}{16} \lim_{z \to z'} \text{tr} \left( 1 + \sigma^3 \right) e^{-U_B} \frac{\partial e^{U_B}}{\partial V_B^a(z)} f(-D^2 \nabla^2/16\Lambda^2) \frac{1}{\nabla^2 D^2/16 - m^2} \nabla^2 \delta(z - z').
\]

(2.22)

Throughout this subsection, all the covariant derivatives are understood as the “doubled”, \( \nabla_{\alpha} \equiv e^{-U_B} D_{\alpha} e^{U_B} \). In (2.22), \( \sigma^3 \) is the Pauli matrix in the doubled space, and we have inserted the projection operator \( (1 + \sigma^3)/2 \) to extract the original representation \( T^a \) from the doubled representation. Since \( \sigma^3 R^a = R^a \sigma^3 \), the position of the projection operator does not matter.

We shall show that if \( \text{tr} \, T^a = 0 \) and \( \text{tr} \, T^a \{ T^b, T^c \} = 0 \), there exists a Lagrangian of Pauli-Villars regulators which basically reproduces our prescription (2.22). To write down this Lagrangian, we introduce a set of infinite regulators \([15]\) \( \phi_j (j = 0, 1, 2, \ldots) \). We assume \( \phi_j \) with \( j \) even is a Grassman-even chiral superfield and that with \( j \) odd is a Grassman-odd chiral superfield. Then the action is given by

\[
S_{PV} = \sum_{j=0,2,4,\ldots} \left( \int d^8 z \, \phi_j^\dagger e^{U_B} \phi_j + \int d^6 z \frac{1}{2} M_j \phi_j^T \sigma^1 \phi_j + \text{h.c.} \right)
+ \sum_{j=1,3,5,\ldots} \left( \int d^8 z \, \phi_j^\dagger \sigma^3 e^{U_B} \phi_j + \int d^6 z \frac{1}{2} M_j \phi_j^T i\sigma^2 \phi_j + \text{h.c.} \right),
\]

(2.23)

where \( M_0 \) is the original mass, \( M_0 = m \), and the mass of regulators is assumed to be of the order of the cutoff, \( M_j = n_j \Lambda \) for \( j \neq 0 \). Note that the use of the Pauli matrices \( \sigma^1 \) and \( i\sigma^2 \) makes possible the existence of mass that are consistent with the statistics of the fields. Noting the relations

\[
\sigma^1 R^a = -R^a T^1, \quad i\sigma^2 R^a = -R^a T^2 i\sigma^2,
\]

(2.24)

it is easy to verify that (2.23) is invariant under the background gauge transformation. Therefore, the generalized Pauli-Villars regularization cannot apply to the
evaluation of the gauge anomaly, and it is workable only for anomaly-free cases. The regularization proceeds as follows [15]: The zeroth field $\phi_0$ is introduced so as to simulate the one-loop contribution of the original field $\Phi_Q$. Therefore, the gauge representation $R^a$ must be projected on the original representation $T^a$ by inserting $(1 + \sigma^3)/2$ into a certain point of the loop diagram. The position and the number of the insertion do not matter, because the free propagator $\langle T^a \phi_0(z) \phi_0^\dagger(z') \rangle_0$ and the gauge generator $R^a$ commute with $\sigma^3$. On the other hand, contributions of regulators $\phi_j$ are simply summed over without the projection.*

It is quite convenient to re-express the above prescription in terms of the gauge current operator [18–20]. According to the above prescription, the gauge current of the zeroth field $\phi_0$ is defined by

$$\langle J^a_0(z) \rangle \equiv \frac{i}{16} \lim_{z \to z'} \text{tr} \, \frac{1}{2} e^{-U_B} \frac{\partial e^{U_B}}{\partial V^a_B(z)} \frac{D^2}{\nabla^2 D^2 / 16 - m^2} \nabla^2 \delta(z - z'), \quad (2.25)$$

where we have used the formal form of the propagator of $\phi_0$ in the background gauge field. The gauge current of the regulators is similarly given by

$$\langle J^a_{j \neq 0}(z) \rangle = \frac{i}{16} \lim_{z \to z'} e^{-U_B} \frac{\partial e^{U_B}}{\partial V^a_B(z)} \frac{(-1)^j}{\nabla^2 D^2 / 16 - n^2_j \Lambda^2} \nabla^2 \delta(z - z'), \quad (2.26)$$

where the statistics of each field have been taken into account. The sum of (2.25) and (2.26) gives the gauge current operator in the generalized Pauli-Villars regularization:

$$\sum_{j=0,1,2,\ldots} \langle J^a_j(z) \rangle$$

$$= \frac{i}{16} \lim_{z \to z'} \frac{1}{2} e^{-U_B} \frac{\partial e^{U_B}}{\partial V^a_B(z)} f(-D^2 \nabla^2 / 16 \Lambda^2) \frac{1}{\nabla^2 D^2 / 16 - m^2} \nabla^2 \delta(z - z')$$

$$+ \frac{i}{16} \lim_{z \to z'} \frac{\sigma^3}{2} e^{-U_B} \frac{\partial e^{U_B}}{\partial V^a_B(z)} D^2 \frac{1}{\nabla^2 D^2 / 16 - m^2} \nabla^2 \delta(z - z').$$

$$\quad (2.27)$$

* Of course, for the gauge invariance, the momentum assignment for all the fields must be taken the same.
In the above expression, we have introduced the function
\[ f(t) = 1 + 2 \sum_{j=1}^{\infty} \frac{(-1)^j t^{n_j^2}}{t + n_j^2} \] (2.28)
by omitting an irrelevant term for \( m \ll \Lambda \). The function \( f(t) \) can be regarded as a regularization factor; in fact, one can make \( f(t) \) sufficiently rapidly decreasing by choosing a suitable sequence of regulator masses. For example, when \( n_j = j \) \[15\], we have
\[ f(t) = \frac{\pi \sqrt{t}}{\sinh(\pi \sqrt{t})}. \] (2.29)

By comparing (2.27) with (2.22), we realize the following correspondence: The first part of (2.27) is identical to the 1/2 projected part of (2.22), although the regularization factor \( f(t) \) is limited to the form of (2.28). On the other hand, the second part of (2.27), the so-called “parity-odd” term, has no regularization factor. Therefore the second part of (2.27) is UV divergent and ill-defined in general. However, it is known in non-supersymmetric cases \[15–20\] that when \( \text{tr} T^a = \text{tr} T^a \{T^b, T^c\} = 0 \), coefficients of these divergent pieces in the “parity-odd” term vanish. To see this is also the case in (2.27), we first note that \( e^{-U_B} \partial e^{U_B} / \partial V_B^a \) is Lie algebra-valued (i.e., it is proportional to \( R^b \)). Then it is easy to see that, by examining the expansion in powers of \( V_B \), the quadratic, linear and logarithmic divergences are proportional to \( \text{tr} \sigma^3 R^a = 2 \text{tr} T^a \), \( \text{tr} \sigma^3 R^a R^b = 0 \), \( \text{tr} \sigma^3 R^a R^b R^c = \text{tr} T^a \{T^b, T^c\} \), respectively. Therefore, if \( \text{tr} T^a = \text{tr} T^a \{T^b, T^c\} = 0 \), the second part of (2.27) is also finite under the prescription \[15–20\] that the trace over the gauge indices is taken prior to the momentum integration.

Allow us to summarize the above results: The 1/2 part of our prescription (2.22) is basically equivalent to the 1/2 part of the generalized Pauli-Villars regularization, (2.27). The \( \sigma^3/2 \) part of (2.22) in general does not correspond to the \( \sigma^3/2 \) part of (2.27). However, if \( \text{tr} T^a = \text{tr} T^a \{T^b, T^c\} = 0 \), the \( \sigma^3/2 \) part of (2.27) (and thus also the \( \sigma^3/2 \) part of (2.22)) is finite. Therefore, we may take the \( \Lambda \to \infty \) limit in the \( \sigma^3/2 \) part of (2.22) which, since \( f(0) = 1 \), reduces to (2.27). This shows that,
if the gauge anomaly vanishes (and if tr $T^a = 0$), our prescription (1.13) in the $\Lambda \to \infty$ limit is basically equivalent to the generalized Pauli-Villars regularization. This proves the integrability. This conclusion is consistent with the result of the previous subsection that the integrability is ensured only for the $\Lambda \to \infty$ limit, even with tr $T^a \{T^b, T^c\} = 0$.

3. BRST symmetry and the Slavnov-Taylor identity

It is possible to introduce the notion of BRST symmetry in the superfield background field method. Since the $S$-matrix in the background field method is given by tree diagrams made from the effective action [21,22], which has $V_B$ and $\Phi_B$ as the argument, the relevance of the BRST symmetry for the unitarity of physical $S$-matrix is not necessarily clear in the present context. However, we see in the next section that the notion is in fact quite useful in systematically treating the gauge fixing part and the ghost-anti-ghost sector. In this section, with this application in mind, we present the method for (at least partially) incorporating the BRST symmetry in our scheme.

Unfortunately, the BRST symmetry is not manifest in our scheme, and it appears that the BRST symmetry or the associated Slavnov-Taylor (ST) identity has to be verified order by order. In the second half of this section, we show that there exists a natural extension of our prescription of § 1 which ensures ST identities at least at the tree level, under the classical equation of motion of the

* One might wonder why the condition tr $T^a = 0$, which did not appear in the previous subsection, is required here. The reason is that the finiteness of the “parity-odd” part is a stronger condition than the integrability. In fact, the $V_B$-independent piece of $\langle J^a(z) \rangle$ in our scheme acquires a quadratic divergence, $\int_0^{\infty} dt f(t) \operatorname{tr} T^a \Lambda^2/(16\pi^2)$ [1]. This does not spoil the integrability as this can be expressed as a variation of the Fayet-Iliopoulos $D$-term. However, this tadpole diagram is not regularized by the Pauli-Villars prescription in this subsection. This is the reason that tr $T^a = 0$ is required in the generalized Pauli-Villars regularization.

† On the other hand, the renormalizability of the effective action in the background field method is automatic in our scheme, because it is restricted by the background gauge invariance, instead of the BRST symmetry.
background field. This is sufficient for the application in the next section, where the superconformal anomaly in the gauge sector is evaluated to one-loop accuracy.

For later use, we summarize here the explicit form of the first three terms in the expansion of the classical action of the gauge sector,

\[
S_{\text{gauge}} = \frac{1}{2T(R)} \int d^6z \, \text{tr} \, W^\alpha W_\alpha, \quad (3.1)
\]

in powers of the quantum field \( V_Q \):

\[
S_{\text{gauge}}^0 = \frac{1}{2T(R)} \int d^6z \, \text{tr} \, W_B^a W_B^a, \quad (3.2)
\]

\[
S_{\text{gauge}}^1 = -\frac{1}{T(R)} \int d^8z \, \text{tr} \, V_Q \mathcal{D}^\alpha W_B^a = -\frac{1}{C_2(G)} \int d^8z \, (V_Q \mathcal{D}^\alpha W_B^a)^{a}, \quad (3.3)
\]

and

\[
S_{\text{gauge}}^2 = \int d^8z \, V_Q^a \left( \frac{1}{8} \nabla^a \nabla^2 \nabla_\alpha + \frac{1}{2} W_B^a \nabla_\alpha \right)^{ab} V_Q^b. \quad (3.4)
\]

### 3.1. BRST TRANSFORMATION

To make the BRST symmetry in the unregularized theory manifest, we adapt the following ghost-gauge fixing action:

\[
S' = i\delta_{\text{BRST}} \frac{1}{T(R)} \int d^8z \, \text{tr} \, (e^{-V_B c'^{\dagger}} e^{V_B} + c') V_Q \\
+ \frac{1}{T(R)} \int d^6z \, \text{tr} \, B f + \frac{1}{T(R)} \int d^6z \, \text{tr} \, B^{\dagger} f^{\dagger} \\
- \frac{2\xi}{T(R)} \int d^8z \, e^{-V_B} f^{\dagger} e^{V_B} f - \frac{2\xi}{T(R)} \int d^8z \, e^{-V_B} B^{\dagger} e^{V_B} b, \quad (3.5)
\]

where we have introduced the Nakanishi-Lautrup (NL) superfield \( B \), which is a Grassman-even chiral superfield, \( \overline{\mathcal{D}}_a B = 0 \). The BRST transformation of the gauge superfield is defined as usual by replacing the gauge parameter \( \Lambda \) of the
infinitesimal quantum field transformation by the ghost superfield, \( \Lambda \rightarrow c \) (see (2.5) of [1]):

\[
\begin{align*}
\delta_{\text{BRST}} V_B &= 0, \quad \delta_{\text{BRST}} e^{V_Q} = i (e^{V_Q} c - e^{-V_B} c^\dagger e^{V_B} e^{V_Q}), \\
\delta_{\text{BRST}} V_Q &= i \mathcal{L}_{V_Q/2} \cdot \left[ (c + e^{-V_B} c^\dagger e^{V_B}) + \coth(\mathcal{L}_{V_Q/2}) \cdot (c - e^{-V_B} c^\dagger e^{V_B}) \right], \\
\delta_{\text{BRST}} \Phi_B &= -ic \Phi_B, \quad \delta_{\text{BRST}} \Phi_Q = -ic \Phi_Q.
\end{align*}
\] (3.6)

Note that \( \delta_{\text{BRST}} \) is Grassman-odd in our convention. Then the nilpotency of the BRST transformation determines the transformation of the Faddeev-Popov (FP) ghost superfield:

\[
\begin{align*}
\delta_{\text{BRST}} c &= -icc, \quad \delta_{\text{BRST}} c^\dagger = -ic^\dagger c^\dagger.
\end{align*}
\] (3.7)

The BRST transformation of the anti-ghost is defined to be the NL superfield:

\[
\begin{align*}
\delta_{\text{BRST}} c' &= -iB, \quad \delta_{\text{BRST}} c'^\dagger = -iB'^\dagger, \\
\delta_{\text{BRST}} B &= 0, \quad \delta_{\text{BRST}} B'^\dagger = 0.
\end{align*}
\] (3.8)

We regard other fields, the gauge averaging function \( f \) and the NK ghost \( b \), as BRST scalars, i.e., \( \delta_{\text{BRST}} f = \delta_{\text{BRST}} b = 0 \).

The action \( S' \) (3.5) is equivalent to the conventional form of the ghost-gauge fixing action in the superfield background field method, (1.3). This can easily be verified by eliminating the NL field \( B \) from (3.5), using the equation of motion

\[
\begin{align*}
\overline{D}^2 V_Q &= 4f, \quad D^2 V_Q = 4e^{-V_B} f^\dagger e^{V_B}.
\end{align*}
\] (3.9)

Another equivalent form of \( S' \) is obtained by first integrating out \( f \) and \( f^\dagger \) and simultaneously \( b \) and \( b^\dagger \) in (3.5). This integration may be performed by shifting
the variables as

\[ f^a \rightarrow f^a - \frac{2}{\xi} \left( D^2 \frac{1}{\nabla^2 D^2} e^{-\nabla b} \right)^{ab} B^b, \quad f^{\dagger a} \rightarrow f^{\dagger a} - \frac{2}{\xi} \left( e^{\nabla b} \nabla^2 \frac{1}{D^2 \nabla^2} \right)^{ab} B^b. \]

Then the Gaussian integral over \( f \) is precisely cancelled by the integral of the NK ghost \( b \). After performing these integrations, the ghost-gauge fixing term \( S' \) (3.5) is effectively replaced by \( S' \rightarrow S_{FP} + S_{NL} \), where

\[ S_{FP} \equiv i \delta_{\text{BRST}} \int d^8 z V_Q^a \left[ (e^{-\nabla b})^{ab} c^{\dagger b} + c^a \right], \]

and

\[ S_{NL} \equiv \frac{8}{\xi} \int d^8 z B^a \left( \frac{1}{\nabla^2 D^2} e^{-\nabla b} \right)^{ab} B^{\dagger b}. \]

If one further integrates over the NL field \( B \), the conventional gauge fixing term, \( - (\xi/8) \int d^8 z (D^2 V_Q)^a (\nabla^2 V_Q)^a \), again results. Interestingly, the kinetic operator of \( B \), \( \nabla^2 D^2 / (\nabla^2 D^2) e^{-\nabla b} D^2 \), is the propagator of the NK ghost. The Gaussian integration of the bosonic NL field thus simulates the effect of the fermionic NK ghost; this should be the case because (3.11) plus (3.12) must be equivalent to (1.3). In what follows, we use \( S_{FP} + S_{NL} \) as the ghost-gauge fixing action because it exhibits the manifest BRST symmetry.

* As was noted in Ref. [1], the combination in the denominator of these expressions has to be interpreted as a representation of

\[ \tilde{\nabla}^2 D^2 \leftrightarrow 16 \tilde{\nabla}^m \tilde{\nabla}_m - 8 W_B^a \tilde{\nabla}_a - 4 (D^a W_B a), \]
\[ D^2 \tilde{\nabla}^2 \leftrightarrow 16 \tilde{\nabla}^m \tilde{\nabla}_m + 8 W_B^a D^{\dagger}_a + 4 (D_a W_B a), \]

respectively.
3.2. Slavnov-Taylor identity

As is well-known, the ST identities can be derived as expectation values of a BRST exact expression. Our first example, which is relevant to the discussion in the next section, is

$$\left\langle \left\langle T^* \delta_{\text{BRST}} [c'^a(z) B^{ib}(z')] \right\rangle \right\rangle = 0,$$

or

$$\left\langle \left\langle T^* B^a(z) B^{ib}(z') \right\rangle \right\rangle = 0.$$

In these expressions, the double bracket $\left\langle \left\langle \cdot \cdot \cdot \right\rangle \right\rangle$ implies the would-be “full” expectation value, in which the BRST symmetry is supposed to be exact. Similarly, we have

$$\left\langle \left\langle T^* \delta_{\text{BRST}} [c'^a(z) V^b_Q(z')] \right\rangle \right\rangle = 0,$$

or

$$\left\langle \left\langle T^* B^a(z) V^b_Q(z') \right\rangle \right\rangle = - \left\langle \left\langle T^* c'^a(z) \left[ \frac{V_Q}{2} \left( \coth \frac{V_Q}{2} + 1 \right) \right]^{bc} c^{\ell}(z') \right\rangle \right\rangle$$

$$+ \left\langle \left\langle T^* c'^a(z) \left[ \frac{V_Q}{2} \left( \coth \frac{V_Q}{2} - 1 \right) e^{-B} \right]^{bc} c^{\ell}(z') \right\rangle \right\rangle.$$

As mentioned above, the BRST symmetry is not manifest in our regularization scheme. Therefore, the validity of the above relations must be verified order by order. At present, we do not have a general comment on this point. However, at least at the tree level and under the classical equation of motion of the background field, there exists a natural extension of our prescription which ensures the above relations.

To examine the ST identities, we must first note that the two-point functions in (3.14) and (3.16) are not 1PI, and thus the tadpole vertex $S_1^{gauge}$ (3.3) also contributes through, say, $S_{T3}$. Without taking into account these tadpole contributions, the ST identities do not hold even in the unregularized theory. This is related
to the fact that each term in the expansion $S_{\text{gauge}} = S_{\text{gauge}}^0 + S_{\text{gauge}}^1 + S_{\text{gauge}}^2 + \ldots$ is not individually invariant under the BRST transformation (3.6). The tadpole contributions make a general analysis of the ST identities in our scheme complicated.

Fortunately, for the application in the next section, a great simplification occurs. We shall use the ST identities to conclude that BRST exact composite operators vanish at the one-loop level. These composite operators are defined by forming a one-loop diagram by the (modified) two-point functions in (3.14) and (3.16). Therefore even if the tadpole vertex $S_1$ is attached to the two-point functions, we can use the classical equations of motion for $V_B$ and $\Phi_B$, because we will work with the one-loop approximation. Moreover, we will assume $\Phi_B = 0$ in the next section. This implies $\mathcal{D}_a^a \mathcal{W}_{Ba} = 0$ under the classical equation of motion. Therefore the tadpole contributions arising from $S_{1\text{gauge}}^0$ (3.3) can be neglected because they are proportional to $\mathcal{D}_a^a \mathcal{W}_{Ba}$. When the tadpole contribution can be neglected, the two-point functions in (3.14) and (3.16) at the tree level are given by the propagators obtained by diagonalizing $S_{2\text{gauge}}^0 + S_{\text{FP}} + S_{\text{NL}}$. The Schwinger-Dyson equations corresponding to $S_{2\text{gauge}}^0 + S_{\text{FP}} + S_{\text{NL}}$ are given by

$\langle T^* B^a(z) V^b_Q(z') \rangle = -\frac{\xi}{8} (\mathcal{D}^2 \tilde{\nabla}^2)^{ac} \langle T^* V^c_Q(z) V^b_Q(z') \rangle$,

$\langle T^* B^a(z) V^b_{Q}(z') \rangle = -\frac{\xi}{8} (e^{V_B} \tilde{\nabla}^2 \mathcal{D}^2)^{ac} \langle T^* V^c_Q(z) V^b_Q(z') \rangle$,

$\langle T^* B^a(z) B^b_{\dagger}(z') \rangle = -\frac{\xi}{8} (e^{V_B} \tilde{\nabla}^2 \mathcal{D}^2)^{bc} \left[ \langle T^* B^a(z) V^c_Q(z') \rangle - i\delta^{ca} \delta(z - z') \right]$,

(3.17)

(the operator $e^{V_B} \tilde{\nabla}^2 \mathcal{D}^2$ acts on the $z'$-variable). In particular, these relations lead to $\langle T^* B^a(z) B^b_{\dagger}(z') \rangle = 0$ at the unregularized level.

In implementing the regularization, we must specify how to modify the original propagators. As a natural extension of the prescription of § 1, we regard the first two relations of (3.17) as the defining relations of $\langle T^* B^a(z) V^b_Q(z') \rangle$ and $\langle T^* B^a(z) V^b_{Q}(z') \rangle$. Namely, on the right-hand side of (3.17), the propagator of gauge superfield is given by the modified one (1.8). On the other hand, we
define \( \langle T^* B^a(z) B^{tb}(z') \rangle = 0 \) even in the regularized theory.

It is obvious that the prescription \( \langle T^* B^a(z) B^{tb}(z') \rangle = 0 \) is consistent with (3.14) at the tree level when \( S_1^{\text{gauge}} = 0 \). On the other hand, (3.16) at the tree level reads (by suppressing quantum fields \( V_Q \)),

\[
\langle T^* B^a(z) V^b_Q(z') \rangle = \left[ e^{-V_B(z')} \right]^{bc} \langle T^* c^a(z)c^c(z') \rangle
\]

\[
= \frac{i}{16} \left[ f \left( -\nabla^m \nabla_m + W_B^a \nabla_a/2 + (\mathcal{D}^a W_B^a)/4 \right) / \Lambda^2 \right] \delta(z - z'),
\]

where the last expression is the modified ghost propagator (1.9) in an un-abbreviated form [1]. At first glance, the left-hand side of (3.18), i.e., (3.17) with (1.8), does not seem to be identical to the right-hand side; in particular, the gauge parameter \( \xi \) must disappear in the expression. To see that they are in fact identical, when \( S_1^{\text{gauge}} = 0 \) or equivalently when \( \mathcal{D}^a W_B^a = 0 \), we first recall the original form of the quadratic action of the gauge sector, the first line of (1.4). Then, the first line of (3.17) gives

\[
\langle T^* B^a(z) V^b_Q(z') \rangle
\]

\[
= \frac{i}{16} \left[ \frac{D^2 \nabla^2 f( -D^2 \nabla^2/16\Lambda^2 + R/\xi \Lambda^2) / \Lambda^2}{D^2 \nabla^2/16 - R/\xi} \right] \delta(z - z'),
\]

(3.19)

where the combination \( R \) has been defined by

\[
R = \frac{1}{8} \nabla^\alpha D^2 \nabla_\alpha + \frac{1}{2} W_B^a \nabla_a + \frac{1}{4} \mathcal{D}^a W_B^a - \frac{\xi}{16} \nabla^2 \mathcal{D}^2
\]

\[
= \frac{1}{8} \nabla^\alpha D^2 \nabla_\alpha + \frac{1}{2} W_B^a \nabla_a - \frac{\xi}{16} \nabla^2 \mathcal{D}^2, \quad \text{when } \mathcal{D}^a W_B^a \equiv \{ \nabla^\alpha, W_B^a \} = 0.
\]

(3.20)

Since \( \nabla^2 R = 0 \), one can eliminate \( R \) in the regularization factor \( f(t) \) of (3.19), and then \( D^2 \nabla^2 \) can be exchanged with the regularization factor. Then, by noting the
identity

\[
\bar{D}^2 \bar{\nabla}^2 + \bar{\nabla}^2 \bar{D}^2 - 2 \bar{D}_\alpha \bar{\nabla}^2 \bar{D}^3 = 16 \bar{\nabla}^m \bar{\nabla}_m - 8 \mathcal{W}_{B}^\alpha \bar{\nabla}_{\alpha} - 4 (\mathcal{D}^\alpha \mathcal{W}_{B}) \alpha, \quad (3.21)
\]

we have

\[
\langle T^a B^a(z) V^b_{Q}(z') \rangle = \frac{i}{16} \left[ f \left( [-\bar{\nabla}^m \bar{\nabla}_m + W^{\alpha}_B \bar{\nabla}_{\alpha}/2 + (\mathcal{D}^\alpha \mathcal{W}_{B})/4]/\Lambda^2 \right) \right]^{ab} \times \frac{1}{\bar{D}^2 \bar{\nabla}^2 / 16 - R/\xi} \delta(z-z'). \quad (3.22)
\]

Hence, by noting the identity (3.8) of Ref. [1],

\[
\bar{D}^2 = \bar{D}^2 \frac{1}{\bar{\nabla}^m \bar{\nabla}_m - W^{\alpha}_B \bar{\nabla}_{\alpha}/2 - (\mathcal{D}^\alpha \mathcal{W}_{B})/4} \frac{\bar{\nabla}^2 \bar{D}^2}{16}, \quad (3.23)
\]

we realize that the last factor in (3.22) can be written as

\[
\frac{1}{\bar{D}^2 \bar{\nabla}^2 / 16 - R/\xi} = \frac{1}{\bar{D}^2 \bar{\nabla}^2 / 16 - R/\xi} \frac{\bar{D}^2 \bar{\nabla}^2}{16} \frac{1}{\bar{D}^2 \bar{\nabla}^2 / 16 - R/\xi} \quad (3.24)
\]

By substituting this into (3.22), we see that (3.19) is in fact identical to (3.18). Therefore, with the above prescription, the ST identity (3.16) holds to at least the tree level under the classical equations of motion, \( \mathcal{D}^\alpha \mathcal{W}_{B} = 0 \). With these understandings, we use the ST identities (3.14) at the tree level and (3.18) in the next section.
4. Superconformal anomaly in the gauge sector

In the remainder of this paper, we describe a one-loop evaluation of the superconformal anomaly in the gauge multiplet sector. This problem was also postponed in Ref. [1]. Although the one-loop result is well known [23–34], our formulation allows a direct calculation, relying neither on a supersymmetry–gauge symmetry argument nor on the connection to the β-function. (Our calculation is in spirit quite similar to that of Ref. [30].) We also clarify the underlying BRST structure; the conventional form of the superconformal anomaly accompanies BRST exact pieces that, due to the Slavnov-Taylor identities in the previous section, eventually vanish.

The superconformal anomaly is the spinor divergence of the superconformal current, \( \bar{D}^\alpha \langle \mathcal{R}_{\alpha\dot{\alpha}}(z) \rangle \). The classical form of the superconformal current is given by [35]

\[
R_{\alpha\dot{\alpha}} = -\frac{2}{T(R)} \text{tr} W_\alpha e^{-V} \overline{W}_{\dot{\alpha}} e^V. \tag{4.1}
\]

Basically, what we have to do here is to construct an expansion of this expression in powers of \( V_Q \) to \( O(V_Q^2) \) (the one-loop approximation) and a classification of various terms in \( \bar{D}^\alpha \langle \mathcal{R}_{\alpha\dot{\alpha}}(z) \rangle \) according to their nature: the explicit breaking of superconformal symmetry due to the gauge fixing, vanishing terms under the equation of motion (see below), and the intrinsic quantum anomaly. This approach was adopted in Ref. [1] for a computation of the superconformal anomaly arising from the chiral matter’s loop. However, the structure of the direct expansion of (4.1) is complicated, and it seems difficult to directly perform such a classification. Therefore, we will adopt a different strategy.

We start with the observation made by Shizuya [36] that the superconformal current (4.1) may be regarded as the Noether current corresponding to a superfield variation,

\[
\Delta e^V = -\Omega^\alpha e^V W_\alpha - \overline{\Omega}_{\dot{\alpha}} \overline{W}^\dot{\alpha} e^V, \tag{4.2}
\]

where \( \Omega^\alpha \) is an infinitesimal Grassman-odd parameter. In fact, one can easily
verify that
\[
\Delta S^{\text{gauge}} = \int d^8z \left( -\frac{1}{2} \Omega^\alpha \bar{D}^\beta R_{\alpha \dot{\beta}} + \frac{1}{2} \Omega^\alpha D^\alpha R_{\alpha \dot{\beta}} \right),
\] (4.3)
by using the reality constraint on \( W_\alpha \).

To utilize the relation (4.3) in the background field method, we split \( \Delta e^V \) (4.2) into variations of \( V_B \) and of \( V_Q \). This splitting is of course not unique. Therefore, we can impose the condition that a variation of \( V_B \) depends only on \( V_B \) and a variation of \( V_Q \) is \( O(V_Q) \). By writing \( \Delta = \Delta_B + \Delta_Q \), this condition uniquely specifies
\[
\begin{align*}
\Delta_B e^{V_B} &\equiv -\Omega^\alpha e^{V_B} W_B \alpha, & \Delta_B V_Q &\equiv 0, \\
\Delta_Q V_B &\equiv 0, & \Delta_Q e^{V_Q} &\equiv \Omega^\alpha \left[ -e^{V_Q} (W_\alpha - W_B \alpha) + [W_B \alpha, e^{V_Q}] \right], \\
\Delta_Q \{ \text{(other fields)} \} &\equiv 0,
\end{align*}
\] (4.4)
and thus
\[
\Delta_Q V_Q^a = \Omega^\alpha \left( \frac{1}{4} \bar{D}^2 \bar{\nabla}_\alpha + \mathcal{W}_{B \alpha} \right)^{ab} V_Q^b + O(V_Q^2). \tag{4.5}
\]

Now, as is clear from (4.3), the calculation of the superconformal anomaly is equivalent to the evaluation of the 1PI part of
\[
\langle \Delta S^{\text{gauge}} \rangle = \langle \Delta_B S^{\text{gauge}} \rangle + \langle \Delta_Q S^{\text{gauge}} \rangle. \tag{4.6}
\]
At this stage, we should note the following fact. In classical theory, a conservation law follows from the classical equation of motion. In quantum theory, the classical equation of motion is replaced by the “quantum” equation of motion, \( \delta \Gamma[V_B]/\delta V_B = 0 \), where \( \Gamma[V_B] \) is the effective action in the background field

* In the following, we explicitly write only the \( \Omega^\alpha \)-parts.
method. Without using this equation to an accuracy consistent with the treatment, the conservation law does not follow even if there is no quantum anomaly nor explicit breaking. An example of such a “quantum” equation of motion is found in (5.18) of Ref. [1]. Therefore, we should consider the superconformal anomaly \( \mathcal{D}^{\dot{a}} \langle R_{a\dot{a}}(z) \rangle \) under the quantum equation of motion. In our present problem, \( \delta \Gamma[V_B]/\delta V_B = 0 \) implies, in particular, at the one-loop level,

\[
\langle \Delta_B S^{\text{gauge}} \rangle = - \langle \Delta_B S_{\text{FP}} \rangle - \langle \Delta_B S_{\text{NL}} \rangle,
\]

(4.7)

because of our prescription for the one-loop effective action (1.13). Recall that \( S^{\text{gauge}} + S_{\text{FP}} + S_{\text{NL}} \) is the total action (see (3.1), (3.11) and (3.12)).

On the other hand, we rewrite the quantum variation part of (4.6) as

\[
\langle \Delta_Q S^{\text{gauge}} \rangle = \langle \Delta_Q (S^{\text{gauge}} + S_{\text{FP}}) \rangle - \langle \Delta Q S_{\text{FP}} \rangle.
\]

(4.8)

Therefore we have, from (4.6)–(4.8),

\[
\langle \Delta S^{\text{gauge}} \rangle = \langle \Delta_Q (S^{\text{gauge}} + S_{\text{FP}}) \rangle - \langle (\Delta_B + \Delta_Q) S_{\text{FP}} \rangle - \langle \Delta_B S_{\text{NL}} \rangle,
\]

(4.9)

under the quantum equation of motion of \( V_B \).

In what follows, we show that the first piece \( \langle \Delta_Q (S^{\text{gauge}} + S_{\text{FP}}) \rangle \) in (4.9) gives rise to the quantum anomaly and that the remaining pieces \( (\Delta_B + \Delta_Q) S_{\text{FP}} \) and \( \Delta_B S_{\text{NL}} \) are BRST exact. This fact is obvious for the last term \( \Delta_B S_{\text{NL}} \) because the NL field \( B \) is the BRST transformation of the anti-ghost, (3.8) (note \( \Delta_B B = 0 \)). The BRST exactness of \( (\Delta_B + \Delta_Q) S_{\text{FP}} \) follows from the remarkable property of \( \Delta_B \) and \( \Delta_Q \) that their sum commutes with the BRST transformation \( \delta_{\text{BRST}} \). In

\[\dagger\] The equivalence of the effective action in the background field method and that of the conventional formalism is proven in Ref. [37] for non-supersymmetric gauge theories. It is argued in Ref. [38] for the supersymmetric case.
fact, one can confirm

\[
(\Delta_B + \Delta_Q)\delta_{\text{BRST}}e^{V_Q}
= i\Omega^\alpha \left[ (-e^{V_Q}W_\alpha + W_{B\alpha}e^{V_Q})c - e^{-V_B}c^\dagger e^{V_B}e^{V_Q}W_\alpha - W_{B\alpha}e^{-V_B}c^\dagger e^{V_B}e^{V_Q} \right]
= \delta_{\text{BRST}}(\Delta_B + \Delta_Q)e^{V_Q},
\]

and

\[
(\Delta_B + \Delta_Q)\delta_{\text{BRST}}(\text{other fields}) = 0 = \delta_{\text{BRST}}(\Delta_B + \Delta_Q)(\text{other fields}).
\]

Therefore these two operations commute \([\Delta_B + \Delta_Q, \delta_{\text{BRST}}] = 0\) on all the fields.

As a result of this property, we see

\[
\langle (\Delta_B + \Delta_Q)S_{\text{FP}} \rangle
= \left< i\delta_{\text{BRST}} \int d^8z (\Delta_B + \Delta_Q)V_Q^a \left[ (e^{-V_B})^{ab}c'^{l \dagger b} + c'^{b} \right] \right>
= \int d^8z \Omega^\alpha
\times \left< i\delta_{\text{BRST}} \left[ c'^{l \dagger a} \left( \frac{1}{4}e^{V_B}D^2\tilde{\nabla}_\alpha \right)^{ab} V_Q^b + c'^{a} \left( \frac{1}{4}D^2\tilde{\nabla}_\alpha + W_{B\alpha} \right)^{ab} V_Q^b + O(V_Q^2) \right] \right>.
\]

If the theory (including the regularization) is BRST invariant, the expectation value of a BRST exact piece must vanish in a BRST invariant state. Although this is not manifest in our regularization scheme, we have shown in the previous section that the tree level ST identities, \(\langle T^a B^a(z) B^b(z') \rangle = 0\) and (3.18), hold with an appropriate prescription under the classical equation of motion, \(D^a W_{B\alpha} = 0\) (we are assuming \(\Phi_B = 0\)). As a result, by defining the composite operators in \(\langle \Delta_B S_{\text{NL}} \rangle\) and (4.12) by connecting the quantum fields by modified propagators, according to (1.14), we can safely neglect the BRST exact pieces, at least at the one-loop level.

Having observed that the last two terms in (4.9) are BRST exact and can be neglected even in our present formulation, let us return to the first term on the
right-hand side of (4.9). To one-loop accuracy, there are three terms which may contribute to the 1PI part of $\langle \Delta Q(S^\text{gauge} + S_{FP}) \rangle$: (i) an $O(V_Q^2)$ term in $\Delta Q_{S^\text{gauge}}$, (ii) an $O(V_Q^2)$ term in $\Delta Q_{S^\text{gauge}2}$, and (iii) an $O(V_Q B)$ term in $\Delta Q_{S_{FP}}$. However, the first contribution (i) is proportional to $D_{\alpha} W_B^{\alpha}$ (see (3.3)), which is already a one-loop order quantity under the quantum equation of motion (recall that the classical equation of motion is $D_{\alpha} W_B^{\alpha} = 0$). As a result, (i) is a higher-order quantity and can be neglected. To evaluate (ii) and (iii), the expression (4.5) is sufficient. Therefore, from (3.4) and (3.11), we have

$$
\langle \Delta Q(S^\text{gauge} + S_{FP}) \rangle = 
\int d^8z 2\Omega^\alpha \left( \frac{1}{4} D^2 \tilde{\nabla}_\alpha + W_B^{\alpha} \right)^{ab} V_Q^b(z) \left( \frac{1}{8} \tilde{\nabla}^\beta D^2 \tilde{\nabla}_\beta + \frac{1}{2} W_B^{\beta} \tilde{\nabla}_\beta \right)^{ac} V_Q^c(z)
+ \int d^8z \Omega^\alpha \left( \frac{1}{4} D^2 \tilde{\nabla}_\alpha + W_B^{\alpha} \right)^{ab} V_Q^b(z) \left[ (e^{-V_B})^{ac} B^{|c}(z) + B^a(z) \right]
= -i \int d^8z \Omega^\alpha \lim_{z' \to z} \left[ f \left( -\tilde{\nabla}^m \tilde{\nabla}_m + W_B^{\beta} \tilde{\nabla}_\beta / 2 - \tilde{W}_B^{\beta} \tilde{D}^{\beta} / 2 \right) / \Lambda^2 \right]
\times (\tilde{D}^2 \tilde{\nabla}_\alpha - 4W_B^{\alpha}) \delta(z - z'),
$$

(4.13)

where, in the last expression, we have used the first two lines of (3.17) and (1.8) in the super-Fermi-Feynman gauge $\xi = 1$. Since (4.13) vanishes in the absence of the regularization factor $f(t)$, we realize that this is in fact the quantum anomaly (i.e., a naive interchange of the equal-point limit and $\Lambda \to \infty$ limit gives an incorrect result). The computation of the last expression proceeds in an almost identical way as that in the Appendix of Ref. [1] and we have $^*$ (the $-4W_B^{\alpha}$ part does not contribute).

$^*$ The expansion of the regulating factor $f(t)$ becomes considerably simpler if one notes that the whole expression is a superfield by construction, and thus explicit dependences on $\theta^\alpha$ and $\overline{\theta}_\alpha$ must vanish eventually.
\[
\lim_{z' \to z} \left[ f \left( -\Delta^m \Delta + W_\beta W_\beta / 2 - \overline{W}_\beta \overline{D}^\beta / 2 \right) \right]^{\alpha \alpha} \delta(z - z') \\
\overset{\Lambda \to \infty}{=} - \frac{i}{16\pi^2} (W_\beta D_\beta W_\alpha)^{\alpha \alpha}
\]
\[
= \frac{i}{32\pi^2} \left[ D_\alpha (W_\beta W_\beta)^{\alpha \alpha} + 2(W_\alpha D_\beta W_\beta)^{\alpha \alpha} \right]
\]
\[
= \frac{i}{32\pi^2} \frac{C_2(G)}{T(R)} D_\alpha \text{tr} W_\beta W_\beta, \quad \text{when } D_\beta W_\beta = 0.
\]  

Finally, by combining equations (4.14), (4.13), (4.9) and (4.3), we obtain, to one-loop accuracy,

\[
\overline{D}^\alpha \langle R_{\alpha \dot{\alpha}}(z) \rangle^{\Lambda \to \infty} = - \frac{1}{64\pi^2} \frac{C_2(G)}{T(R)} D_\alpha \text{tr} W_\beta W_\beta. \tag{4.15}
\]

We emphasize that we did not need the expansion in \(V_B\) in deriving this expression. Also, it is straightforward to include the effect of the chiral multiplet in the calculation, at least when the background chiral superfield is absent (i.e. when \(\Phi_B = 0\)). By utilizing the result of Ref. [1],† we have

\[
\overline{D}^\alpha \langle R_{\alpha \dot{\alpha}}(z) \rangle - 2 \left\langle \Phi_T^T m \nabla_\alpha \Phi_Q(z) \right\rangle + \frac{2}{3} D_\alpha \left\langle \Phi_T^T m \Phi_Q(z) \right\rangle
\]
\[
\overset{\Lambda \to \infty}{=} - \frac{1}{8\pi^2} \left[ \Lambda^2 \int_0^\infty dt f(t) + \frac{1}{6} \Omega \right] \text{tr} W_B(\alpha, \alpha) - \frac{3C_2(G) - T(R)}{192\pi^2} \frac{1}{T(R)} D_\alpha \text{tr} W_\beta W_\beta. \tag{4.16}
\]

One of the advantages of our scheme is that it provides a supersymmetric (background) gauge-invariant definition of the superconformal current operator [1]. Therefore, from the expression of the superconformal anomaly (4.16), we can immediately derive (quantum anomalous part of) the “central extension” of \(N = 1\) superconformal current.

† Or, one may use the fact [36] that the superconformal current of the chiral multiplet can be regarded as the Noether current associated with the variation \(\Delta \Phi = -\overline{D}^\alpha \Omega^\alpha e^{-V} D_\alpha e^V \Phi + (D^\alpha \Omega_\alpha) \Phi / 3) / 4\) and repeat the above procedure.
persymmetry algebra [39]; the procedure in Ref. [1] gives (for $m = 0$)

$$
\left\langle \{ \mathcal{Q}_\alpha, \mathcal{Q}_\beta \} \right\rangle = \frac{i}{2} \int d^3 x \sigma^{\alpha\beta\gamma} (\varepsilon_{\dot{\alpha}\dot{\beta}} D_{\dot{\gamma}} + 2 \varepsilon_{\dot{\beta}\dot{\gamma}} D_{\dot{\alpha}}) \mathcal{D}^{\delta} \left\langle R_{\gamma\delta}(z) \right\rangle \bigg|_{\theta = \bar{\theta} = 0}
$$

$$
\Lambda \rightarrow \infty \frac{1}{48 \pi^2} [3 C_2(G) - T(R)] \sigma_{\dot{\alpha}\dot{\beta}} \int d^3 x \partial_i (\lambda^{\alpha}_B \lambda^{\beta}_B).
$$

If we retained the BRST exact pieces in (4.9) in the superconformal anomaly, the central extension (4.17) also acquires BRST exact pieces. It is interesting that this structure is common to the result of Ref. [40], although we are using a supersymmetric invariant gauge fixing term.

5. Conclusion

In this paper, we have reported on the analyses of several issues postponed in Ref. [1], including the proof of the covariant gauge current in anomaly free cases. The basic properties of our scheme (except the BRST symmetry at higher orders) have now been clarified, and it has been established that our scheme actually provides a gauge invariant regularization of the effective action in the background field method. On the practical side, our scheme passed many one-loop tests performed in Ref. [1] and the present paper by reproducing the correct results in a transparent manner. Therefore, we are now ready to use our scheme in treating more realistic problems for which a manifestly supersymmetric gauge covariant treatment is crucial.

We are grateful to Professor K. Fujikawa and Dr. J. Hisano for helpful comments. The work of H.S. is supported in part by the Ministry of Education Grant-in-Aid Scientific Research, Nos. 09740187 and 10120201.
REFERENCES

1. T. Hayashi, Y. Ohshima, K. Okuyama and H. Suzuki, Prog. Theor. Phys. 100 (1998), 627.

2. J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton University Press, Princeton, 1992).

3. M. T. Grisaru, W. Siegel and M. Roček, Nucl. Phys. B159 (1979), 429.

4. K. Fujikawa, Phys. Rev. D29 (1984), 285.

5. H. Banerjee, R. Banerjee and P. Mitra, Z. Phys. C32 (1986), 445.

6. O. Piguet and K. Sibold, Nucl. Phys. B247 (1984), 484.

7. T. E. Clark and S. Love, Phys. Lett. 138B (1984), 289.

8. N. K. Nielsen, Nucl. Phys. B244 (1984), 499.

9. E. Guadagnini, K. Konishi and M. Mintchev, Phys. Lett. 157B (1985), 37.

10. K. Harada and K. Shizuya, Phys. Lett. 162B (1985), 322.

11. D. Nemeschansky and R. Rohm, Nucl. Phys. B249 (1985), 157.

12. G. Girardi, R. Grimm and R. Stora, Phys. Lett. 156B (1985), 203.

13. R. Garreis, M. Scholl and J. Wess, Z. Phys. C28 (1985), 628.

14. K. Konishi and K. Shizuya, Nuovo Cim. 90A (1985), 111.

15. S. A. Frolov and A. A. Slavnov, Phys. Lett. B309 (1993), 344.

16. R. Narayanan and H. Neuberger, Phys. Lett. B302 (1993), 62.

17. S. Aoki and Y. Kikukawa, Mod. Phys. Lett. A8 (1993), 3517.

18. K. Fujikawa, Nucl. Phys. B428 (1994), 169; Indian J. Phys. 70A (1996), 275.

19. K. Okuyama and H. Suzuki, Phys. Lett. B382 (1996), 117; Prog. Theor. Phys. 98 (1997), 463.
20. L. N. Chang and C. Soo, Phys. Rev. D55 (1997), 2410.

21. B. S. DeWitt, *Dynamical Theory of Group and Fields* (Gordon and Breach, New York, 1978).

22. L. F. Abbott, M. T. Grisaru and R. K. Schaefer, Nucl. Phys. B229 (1983), 372.

23. J. Lukierski, Phys. Lett. 70B (1977), 183.

24. T. Curtright, Phys. Lett. 71B (1977), 185.

25. H. Inagaki, Phys. Lett. 77B (1978), 56.

26. W. Lang, Nucl. Phys. B150 (1979), 201.

27. T. Hagiwara, S.-Y. Pi and H.-S. Tsao, Ann. of Phys. 130 (1980), 282.

28. H. Nicolai and P. K. Townsend, Phys. Lett. 93B (1980), 111.

29. O. Piguet and K. Sibold, Nucl. Phys. B196 (1982), 447.

30. S. Marculescu, Nucl. Phys. B213 (1983), 523.

31. M. T. Grisaru and P. T. West, Nucl. Phys. B254 (1985), 249.

32. M. T. Grisaru, B. Milewski and D. Zanon, Phys. Lett. 157B (1985), 174; Nucl. Phys. B266 (1986), 589.

33. M. Shifman and A. Vainshtein, Nucl. Phys. B277 (1986), 456.

34. M. R. Mehta, Phys. Rev. D44 (1991), 3303.

35. S. Ferrara and B. Zumino, Nucl. Phys. B87 (1975), 207.

36. K. Shizuya, Phys. Rev. D35 (1987), 1848.

37. L. F. Abbott, Nucl. Phys. B185 (1981), 189.

38. S. J. Gates, M. T. Grisaru, M. Roček and W. Siegel, *Superspace or One Thousand-and-One Lessons in Supersymmetry* (Benjamin/Cummings, Reading, Mass., 1983).
39. G. Dvali and M. Shifman, Phys. Lett. **B396** (1997), 64.
   A. Kovner, M. Shifman and A. Smilga, Phys. Rev. **D56** (1997), 7978.
   B. Chibisov and M. Shifman, Phys. Rev. **D56** (1997), 7990.

40. K. Fujikawa and K. Okuyama, Nucl. Phys. **B521** (1998), 401.