The Globular Cluster Luminosity Function

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Abstract. The main aspects of the globular cluster luminosity function needing to be explained by a general theory of cluster formation are reviewed, and the importance of simultaneously understanding globular cluster systematics (the fundamental plane) within such a theory is pointed out.

1 Review

A clear understanding of the physics driving the basic form of the globular cluster luminosity function, or GCLF – the distribution of cluster magnitudes, luminosities, or masses in a galaxy – remains elusive. To be sure, substantial progress has been made in the theory of globular cluster mass loss and dynamical evolution over a Hubble time in galaxies; in our ideas about the assembly of large galaxies from multiple smaller fragments; in our understanding of how dense pockets of gas are converted in general to stars and star clusters; and in our appreciation of the origin and evolution of self-gravitating structure in turbulent gas. But the GCLF ultimately is shaped to some extent by every one of these processes and relies on a complex interplay between them.

Traditionally, the GCLF was constructed by plotting the number of globulars around a galaxy in equal-sized bins of integrated cluster magnitude. Since the globulars in our own Galaxy, at least, are known to share a common core mass-to-light ratio [19], the result is equivalent to the mass distribution $N(\log m) \equiv dN/d \log m$. This is the function with the familiar, Gaussian-like appearance, shown in the lower panels of Fig. 1 for the globular cluster systems of M87 and the Milky Way. The location of its peak, at $m_* \simeq 1.2 \times 10^5 M_\odot$, corresponds to the classic GCLF “turnover” magnitude $M^* V = -7.4$, which serves remarkably well as a standard candle (see, e.g., the review in [12]) and was interpreted in the first serious theories of globular cluster formation [25] as the imprint of a Jeans mass set by specific thermal conditions at pre- or protogalactic epochs. However, when clusters are counted instead in intervals of equal linear mass, the physical distribution obtained is $N(m) \equiv dN/dm = m^{-1} N(\log m)$, which is necessarily different in shape from the usual GCLF. This function, generally referred to as the globular cluster mass spectrum, is shown for M87 and the Milky Way in the top panels of Fig. 1. There it is clear that, while $m_*$ retains some physical significance as a point of sharp change in the physical dependence of $N(m)$, the number of globulars per unit mass continues to increase down to the lowest observed masses. It is then no longer obvious that there exists any “preferred” globular cluster mass scale [13].
Fig. 1. Observed mass spectra and GCLFs of M87 and the Milky Way. Absolute cluster magnitudes have been converted to masses using a constant $M/L_V = 1.45 \, M_{\odot} \, L_{\odot}^{-1}$ \cite{10}. Data for M87 are taken from \cite{23} (filled points, coming from $\simeq 0.8$–4.5 effective radii in the galaxy) and \cite{14} (open points, from 0.1–1.5 effective radii). Those for the Milky Way are taken from \cite{11}. Normalizations are chosen so as to put exactly one cluster at the “turnover” mass, $m_\ast \simeq 1.2 \times 10^5 \, M_{\odot}$, marked by arrows.

Disagreement persists among theoretical studies (e.g., compare \cite{10} with \cite{26}, \cite{27}, \cite{28}) as to whether the change of slope in $N(m)$ around $m = m_\ast$ had to have been established almost immediately at the time of cluster formation, or if instead $N(m)$ might initially have risen towards $m < m_\ast$ just as steeply as it falls towards $m > m_\ast$, with its current form resulting entirely from evaporation and tidal shocks removing globulars from the low-mass end of the distribution. But however this issue resolves itself, the same discussions support the idea that the shape of $N(m)$ above $m > m_\ast$ reflects reasonably well the initial distribution. Theories of globular cluster formation should therefore aim to address three main points taken from Fig. 1 (see also \cite{13}, \cite{24}):

(1) Over about a decade in globular cluster mass above $m > m_\ast$, $N(m)$ can be approximated by power laws with exponents in the range $-1.7$ to $-2$ [$N(m) \propto m^{-1.65}$ is drawn as a solid line in the upper M87 panel of Fig. 1 and $N(m) \propto m^{-2}$ in the upper Milky Way panel], which are strikingly similar to those describing the mass distributions of giant molecular clouds (GMCs) in the Milky Way, of the dense star- and cluster-forming “clumps” inside GMCs, and of the super star clusters currently forming in mergers such as the Antennae (where $N_{SSC} \propto m_{SSC}^{-2}$ \cite{29}). Thus, any theory of the GCLF ought to be a special case of a more general theory of structure and star (cluster) formation in the interstellar medium.
(2) While $m_*$ is essentially identical in M87 and the Milky Way (as in many other galaxies), the overall shape of the globular cluster mass spectrum is not universal: The $m^{-2}$ power law that applies in the Milky Way and the Antennae (and M31: [21]) is also shown (as a dotted line) against the observed $N(m)$ in M87. It clearly fails to describe those data, and the same is true in several other giant ellipticals [13]. It is difficult to ascribe this difference to the effects of dynamical evolution, since much of the M87 data are taken from beyond one effective radius in the galaxy (where the GCLF should be closer to its original form than in the inner regions [27]), and since GCLF evolution is expected to have been less drastic in larger galaxies in the first place [24] [27]. Thus, any theory for the origin of the GCLF must allow for real differences in the shape of $N(m)$ from galaxy to galaxy but leave little room for variations in the mass $m_*$.  

(3) There is no detectable variation in either the value of $m_*$ or the shape of $N(m)$ above $m_*$ as a function of galactocentric position inside M87 [23] [14] [17]. Thus, a theory of globular cluster formation should couple with a realistic scenario of galaxy formation and evolution to produce a GCLF that is fairly insensitive to intragalactic environment.  

Recent studies of dynamical evolution of the GCLF [26] [27] [28] [10] appear (modulo some disagreements in detail, and only by assuming a host galaxy potential that is static over a Hubble time) to have made promising headway in understanding the third of these items. However, such calculations always assume the initial form of the GCLF a priori; they do not aim to address either of points (1) or (2). Two models that do have been developed in the literature.  

Elmegreen and collaborators construct an essentially geometric theory (no underlying dynamics are drawn upon or constrained) relating the fractal dimension of structure in any turbulent interstellar medium to a power-law mass spectrum for star- and cluster-forming gas clouds, thus exploiting similarities between our local interstellar medium and the GCLF [7] [8]. This approach suffered at first from an oversimplified view of the fractal geometry of both turbulence and the real interstellar medium (see, e.g., [4]), as well as an overly strict expectation of universality in the globular cluster mass spectrum [cf. point (2) above]. However, recent refinements [6] stand potentially to remove these concerns.  

Harris & Pudritz [13] and McLaughlin & Pudritz [21] instead build on older models of structure in the local interstellar medium and calculate the steady-state spectrum of gaseous protoglobular cluster (PGC) masses that develops through a competition between mass build-up by coalescent PGC-PGC collisions vs. the destruction of massive PGCs by feedback from their own star formation. The shape of $N(m)$ is then determined primarily by two fitting parameters: a mass dependence in the feedback destruction timescale ($\tau_d$) of the larger PGCs and the ratio ($\beta$) of a fiducial self-destruction time to a typical PGC collision timescale. Good fits to the observed $N(m)$ above $m_*$ in the globular cluster systems of both M87 and the Milky Way can be obtained [21] [12], but the required behaviour of $\tau_d$ and the values of $\beta$ are strictly ad hoc.  

Both of these models really concern themselves with a description of the mass spectrum of gaseous protoclusters and assume that the globulars themselves
will directly inherit the PGC $N(m)$. Thus, they require that the star-formation efficiency ($\epsilon \equiv m_{\text{GC}}/m_{\text{PGC}}$) in any PGC be independent of its initial mass. They also connect explicitly with descriptions of the local interstellar medium and current star formation by applying scalings between physical parameters (masses, radii, and velocity dispersions) for turbulent PGCs that are virialized and in pressure equilibrium with an ambient medium. Just how well justified these assumptions are, is a question that arises from the consideration of other properties of globular clusters.

2 The Globular Cluster Fundamental Plane

The multiple correlations between structural parameters of Milky Way globular clusters can be shown to reduce to just two independent relations that define a fundamental plane [5] and that physically signify [19] a constant mass-to-light ratio in the cluster cores ($M/L_V = 1.45 \, M_\odot \, L_\odot^{-1}$) and a tight scaling between cluster binding energy, total luminosity or mass, and Galactocentric position:

$$E_b(\text{GC}) = 3.4 \times 10^{39} \, \text{erg} \left( \frac{m_{\text{GC}}}{M_\odot} \right)^{2.05} \left( \frac{r_{gc}}{8 \, \text{kpc}} \right)^{-0.4},$$

which is drawn through the Galactic globular cluster data (normalized to a single Galactocentric radius of 8 kpc) in Fig. 2. Globular clusters in M31, M33, and NGC 5128 (Cen A) appear to fall along the binding energy-mass relation defined by the Galactic system, and in fact extend it to higher cluster masses [3] [18] [15]. But if gaseous protoglobulars were – like the star-forming clumps in Galactic GMCs today – in virial equilibrium within a pressurized ambient medium but

![Fig. 2. Binding energy vs. mass as observed for Milky Way globular clusters [19] and expected for virialized protoglobulars under high ambient pressure](image-url)
strongly self-gravitating (i.e., at the maximum equilibrium mass allowed under a given surface pressure \( P_s \)), then their binding energies should have shown quite a different dependence on total mass; and this ultimately suggests that the globular cluster \( N(m) \) might have differed significantly from the original protocluster distribution even before any dynamical evolution set in.

From the general relations given in [2], PGCs of the type just described would have obeyed the relation

\[
E_b \propto 7.2 \times 10^{42} \text{ erg} \left( \frac{m_{\text{PGC}}}{M_\odot} \right)^{3/2} \left( \frac{P_s}{10^8 k_B} \right)^{1/4} \tag{2}
\]

The pressure scale \( P_s = 10^8 k_B \) is significantly higher than typical pressures in our local interstellar medium, but this has been suggested as a necessary condition for the formation of bound, massive stellar clusters (e.g., [7] [2]) and so is worth explicit consideration. The dashed line in Fig. 2 therefore traces equation (2) with \( P_s \) fixed, as a representative PGC energy-mass relation. (Changing \( P_s \) even by as much as two orders of magnitude would move the line only slightly.)

If stars form in direct proportion to local gas density, then \( E_b \propto m^2 / R \) implies that \( E_b(GC)/E_b(PGC) \propto \epsilon^2 (R_{PGC}/R_{GC}) \) for \( \epsilon \) the star-formation efficiency in a PGC. The ratio of cluster to protocluster radii is further related to \( \epsilon \), in a way that depends on details of the massive-star feedback that ends star formation in a PGC and clears it of any remaining gas [16] [1]. Under the high ambient pressure \( P_s \) specified here, the freefall time of any PGC will be \( \sim 10^6 \) yr or less [2], suggesting that feedback might operate on a timescale longer than the dynamical time. In this case [16], \( R_{PGC}/R_{GC} = \epsilon \), and so \( E_b(GC)/E_b(PGC) \propto \epsilon^3 \). Then, noting that \( P_s \propto r_{gc}^{-2} \) if the ambient gas around the Galactic PGCs was distributed in an isothermal potential well, the ratio of equations [11] and [4] is nearly independent of Galactocentric position: \( \epsilon^3 \propto E_b(GC)/E_b(PGC) \propto m_{GC}^{2.05} / m_{PGC}^{1.5} \propto \epsilon^{2.05} m_{PGC}^{0.55} \), and thus \( \epsilon = (m_{GC}/m_{PGC})^{0.58} m_{PGC}^{0.58} \). This scaling obviously has to saturate at some protocluster mass for which star formation is 100% efficient. The value of this mass is not known from first principles (nor has any mass dependence in \( \epsilon \) been suggested from first principles), but in this highly simplified example it seems reasonable to assert that \( \epsilon = 1 \) where the energy-mass relations of the globulars and PGCs intersect, i.e., around \( m \sim 10^6 M_\odot \).

A variation of star-formation efficiency with protocluster gas mass was suggested by [20] as the origin of the steeper \( E_b = m \) slope for globulars relative to PGCs, and it was discussed in considerably more detail by [2], as the potential cause of the different mass-radius relations among globulars (roughly, \( R \propto m^{1/2} \) [19]) and virialized gas clouds (\( R \propto m^{1/2} \) [22]). These two approaches are clearly equivalent, and the results of [2] are identical to those found here. But to return to the main point, if any of this is even roughly correct, and \( \epsilon \) increases with PGC mass over most of the mass range of the observed GCLF, then the PGC mass spectrum, \( N(m_{PGC}) \), had to have been steeper than the globular cluster \( N(m_{GC}) \) obtained immediately after star formation and feedback were completed. The details of this are given in [2], from which it can be seen that if \( N(m_{GC}) \propto m_{GC}^{-\alpha} \) and \( \epsilon \propto m_{PGC}^{0.6} \), then \( N(m_{PGC}) \propto m_{PGC}^{-\beta} \) with \( \beta = 1.6\alpha - 0.6 \) (for \( m_{PGC} < 10^6 M_\odot \) or so). Thus, for \( \alpha = 1.65 \), as is found above \( m_{GC} > m_\ast \).
in the M87 globular cluster system, \( N(m_{\text{PGC}}) \propto m_{\text{PGC}}^{-2} \). But in the Milky Way (and M31, and the Antennae), the unevolved parts of the globular cluster spectrum already follow \( \alpha = 2 \), and \( N(m_{\text{PGC}}) \propto m_{\text{PGC}}^{-2.6} \) is implied. This slope is significantly steeper than any found or theorized for structures in the interstellar medium as we know it, and it presents a clear problem if we wish to hold on to the idea that globular cluster formation was simply a special case of generic star formation in a familiar interstellar medium.

It is possible that the removal of gas from a PGC by stellar feedback is not “slow” compared to the star-formation or dynamical times (e.g., if the pressure on protocluster surfaces were not as high as \( 10^8 \, k_B \), these internal timescales would not be so short). But even though \( \epsilon \) would not necessarily depend as strongly on \( m_{\text{PGC}} \) in such a case, the qualitative problem would remain of \( N(m_{\text{PGC}}) \) being steeper than \( N(m_{\text{GC}}) \), unless some other aspect of the analysis is also in error. Moreover, if the dynamical timescale of a protocluster grows to \( \sim 10^7 \) yr or more, there may be time for massive stars to explode as supernovae and dispel most of the gas before appreciable amounts of it can be converted to stars. This could make it difficult to produce tightly bound star clusters at all. (The energy injected by a single supernova into surrounding gas is \( \sim 10^{51} \) erg, and one Type II supernova is expected for every \( 135 \, M_\odot \) of stars formed with a Salpeter initial mass function. The combined energy exceeds the binding energy of any PGC in Fig. 2 by orders of magnitude.)

It may simply be that the inferred increase of \( \epsilon \) with \( m_{\text{PGC}} \) is the result of an incorrect assumption that the star formation inside a protocluster traces the local gas density. Perhaps gas clouds of all masses form stars in the same overall proportion, but more massive ones produce them in more centrally concentrated configurations. If so, then the GCLF above \( m_\star \) could indeed accurately reflect the PGC mass spectrum (as current models assume), which would in turn be explained within conventional theories of the interstellar medium. The binding energy–mass relation for globulars (not to mention the putative constancy of \( \epsilon \)) would still have to be produced from first principles in a theory of star formation with rigorous feedback calculations.

It could also be that the presumed scaling of PGC binding energy with mass, \( E_b(\text{PGC}) \propto m_{\text{PGC}}^{3/2} \), is incorrect because the dense gas clumps that produce bound stellar clusters do not originate in a state of virial or pressure equilibrium. There are a few interrelated points suggesting that this may be the case: the high pressures \( (P_s \sim 10^8 \, k_B) \) that are sometimes invoked \(^7\) \(^2\) to allow for various aspects of globular cluster formation are in excess of values calculated for any equilibrium setting; the problem of supernova energetics mentioned just above makes it almost inevitable that globulars (or any similarly tightly bound stellar cluster) had to have formed very rapidly from the collapse of the highest peaks in a field of density fluctuations, and these could plausibly have decoupled from pressure equilibrium with the ambient background; and in the most modern view of the interstellar medium as a large-scale turbulent flow or cascade (the sort of backdrop against which \(^3\) \(^7\) and \(^6\) work to compute mass spectra for protoclusters), hydrostatic equilibrium in transient substructures seems less
than guaranteed. But if PGCs were not in virial or pressure equilibrium, then it is not obvious that they couldn’t themselves have followed something like \( E_b(PGC) \propto m_{PGC}^2 \) or so – in which case the observed \( E_b - m \) scaling now seen in globulars presents no difficulties.

Whatever the true explanation of Fig. 2 it seems clear that further advances in our understanding of the globular cluster luminosity function will come only alongside a better appreciation of the process of star formation at a more detailed level. It is essential that attention be paid simultaneously to the multiple empirical clues provided by various properties of globular cluster systems.

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