Non-minimally coupled cosmological models with the Higgs-like potentials and negative cosmological constant

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Abstract
We study dynamics of non-minimally coupled scalar field cosmological models with Higgs-like potentials and a negative cosmological constant. In these models, the inflationary stage of the Universe evolution changes into a quasi-cyclic stage of the Universe evolution with oscillation behaviour of the Hubble parameter from positive to negative values. Depending on the initial conditions, the Hubble parameter can perform either one or several cycles before becoming negative forever.

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(Some figures may appear in colour only in the online journal)

1. Introduction
Models with scalar fields play a central role in the current description of the evolution of the Universe at the early epoch [1–6]. However, predictions of simplest inflationary models are in disagreement with the Planck2013 results [7, 8]. At the same time, many of these inflationary scenarios can be improved by adding a tiny non-minimal coupling of the inflaton field to
gravity [9, 10]. The models with the Ricci scalar multiplied by a function of the scalar field are being intensively studied in the inflationary cosmology [11–21]. Over the last few years, the Higgs-driven inflation has attracted a lot of attention [15, 16, 18–21]. The models, which include both the Higgs boson and dilaton [22], have been proposed to describe not only an inflationary stage, but also a late period of dark energy domination. All these models deal with positive definite potentials.

The goal of this paper is to explore new mathematical features of cosmological models with non-minimally coupled scalar fields in the case of potentials with negative minima. At the present moment, we cannot say that there is any necessity to consider such types of cosmological models. However, this type of models can appear as a result of renormalization, so, our consideration can give restrictions to a choice of the realistic models. Also as it has been shown [23], non-positive definite potentials can arise as a consequence of nonlocality. Theories with nonlocal scalar fields [24], inspired by the string field theory [25] have been considered as early universe models, either inflationary models [26–28] or models with bounce solutions [29, 30]. Perturbations in such models have been studied in [31, 32]. In one of these models, the scalar field is the tachyon of the Neveu–Schwarz–Ramond (NSR) fermion string and the model has the form of a nonlocal Higgs-type model. Due to the effect of stretching, the potential [26, 27] nonlocality destroys the relation between the coupling constant in the potential, the mass term and the value of the vacuum energy (cosmological constant) and produces the effective double-well potential with the negative extra cosmological term [23] (see details in the appendix). Models with a non-minimal coupling dilaton are being [33–36] actively studied and are, in particular, motivated by the string field theory (SFT).

In many scalar-driven inflationary scenarios, the inflationary potential has a minimum and the scalar field rolls to this minimum and starts oscillating around it. Such behaviour of the scalar field allows us to avoid never-ending inflation. In realistic models of inflation, it is usually assumed that at the end of the period of accelerated expansion (during the oscillatory regime), the scalar field decays into an ordinary matter, in other words, reheating takes place. For a positive definite Higgs potential we get convergent oscillations, whereas if we assume that the minimal value of the potential is negative, then such a process is not possible, because the value of the Hubble parameter should be real and there exists a minimal non-zero velocity of the scalar field in the minimum. The characteristic property of models with non-positive definite potential is the existence of the unreachable domain on the phase plane, which corresponds to the non-real Hubble parameter. In the case of a minimal coupling, it turns out that non-positivity of a potential leads to an inevitable catastrophic change of the Universe evolution: from expansion to contraction [23]. This is just an opposite to what is going in the cyclic model [37], where the ekpyrotic phase is generated by a scalar field with a negative potential and one has a bounce from a phase of contraction to a phase of expansion, see [38] for discussion of the Planck2013 results support of this model.

In more details, the goal of the paper is to explore the behaviour of the scalar field in the neighbourhood of the negative minimum of a potential in modified gravity models. We consider the modified gravity models with action including the term proportional to the Ricci scalar multiplied by a function of the scalar field and with a negative minimal value of double-well potential. By numerical calculations we explore the induced gravity model and model with both the Hilbert–Einstein term and the squared scalar field multiplied by the scalar curvature. We also choose such values of the negative cosmological constant that the unreachable domain is separated into two parts. We show that the phase trajectories are being attracted to the boundary of the unreachable domain, touch it and go to infinity. During this evolution, the Hubble parameter oscillates from positive to negative values. This quasi-cyclic behaviour ends up on a point of the boundary of the unreachable
domain and after this, the evolution changes and completes by a monotonically contracting stage.

This paper is organized as the following. We start, section 2, from a short remind of the action and equations for the non-minimally coupled cosmological models. We consider dynamics in the models of induced gravity and of the Higgs-like inflation with a non-positive definite double-well potential. We show that there are solutions that wind up on the unreachable domain on the phase plane. In section 3, we discuss the model in the Jordan and Einstein frames. In section 4, we study the special case $\xi = -1/6$, in which some equations, considered in section 2, have to be modified. Finally, section 5 is devoted to the conclusion.

2. Classical dynamics in cosmological models with non-minimal coupling and non-positive definite potentials

2.1. Cosmological models with non-minimally coupled scalar fields

Cosmological models in framework of a non-minimally coupled theory are being actively studied [12–20, 39–45] (see also [46, 47] and references therein).

Models with non-minimally coupled scalar fields are described by the following action:

$$ S = \int d^4x\sqrt{-g} \left[ U(\phi)R - \frac{1}{2}g^{\mu\nu}\phi,_{\mu}\phi,_{\nu} - V(\phi) \right], $$

(1)

where $U(\phi)$ and $V(\phi)$ are differentiable functions of the scalar field $\phi$. We assume that $U(\phi) \geq 0$. We use the signature $(-, +, +, +)$, and $g$ is the determinant of the metric tensor $g_{\mu\nu}$.

In the spatially flat Friedmann–Lemaître–Robertson–Walker (FLRW) metric with the interval:

$$ ds^2 = -dt^2 + a^2(t) \left( dx_1^2 + dx_2^2 + dx_3^2 \right), $$

(2)

we obtain the following equations:

$$ 6UH^2 + 6\dot{U}H = \frac{1}{2}\dot{\phi}^2 + V, $$

(3)

$$ 2U(2\dot{H} + 3H^2) = -\frac{\dot{\phi}^2}{2} - 2\ddot{U} - 4HU + V. $$

(4)

The equation for the field got by the variation of the action over the field is

$$ \ddot{\phi} + 3H\dot{\phi} - 6U' (H + 2H^2) + V' = 0 $$

(5)

here and hereafter a dot indicates the derivation over $t$, a prime indicates the derivation over $\phi$. The Hubble parameter is $H \equiv \dot{a}/a$.

Subtracting equation (3) from (4), we obtain

$$ 4UH = -\dot{\phi}^2 - 2\dddot{U} + 2HU. $$

(6)

From equations (3)–(6), we obtain the following system of the first order equations:

$$ \dot{\phi} = \psi, $$

$$ \dot{\psi} = -3H\psi - \frac{(6U'' + 1)\psi^2 - 4VU' + 2UV'}{2(3U'^2 + U)} , $$

(7)

$$ \dot{H} = -\frac{2U'' + 1}{4(3U'^2 + U)} \psi + \frac{2U'}{3U'^2 + U} H \psi - \frac{6U'^2}{3U'^2 + U} H^2 + \frac{U'V'}{2(3U'^2 + U)}. $$
Note that equation (3) is not a consequence of system (7). On the other hand, if equation (3) is satisfied in the initial moment of time, then from system (7) it follows that equation (3) is satisfied at any moment of time. In other words, the system (7) is equivalent to the initial system of equations (3)–(5) if and only if we choose such initial data that equation (3) is satisfied.

Let us introduce a new variable:

\[ Q = H + \frac{\dot{U}}{2U}, \]

(8)

In terms of \( Q \), equations (3) and (6) have the following form

\[ 3Q^2 = \frac{\dot{\phi}^2}{4U} + \frac{3U^2}{4U^2} + \frac{V}{2U}, \]

(9)

\[ \dot{Q} - \frac{\dot{U}}{2U} Q = -\frac{U + 3U^2}{4U^2} \dot{\phi}^2. \]

(10)

Therefore,

\[ \frac{d}{dt} \left[ \frac{Q}{\sqrt{U}} \right] = -\frac{U + 3U^2}{4U^2} \frac{\dot{\phi}^2}{\sqrt{U}} \leq 0. \]

(11)

For an arbitrary positive function \( U(\phi) \), we get that \( Q/\sqrt{U} \) is a monotonically decreasing function. If for some moments of time \( t_1 \) and \( t_2 > t_1 \) we have \( \dot{\phi}(t_2) = \dot{\phi}(t_1) \) and \( \dot{\phi}(t) \) is not a constant at \( t_1 \leq t \leq t_2 \), then \( Q(t_2) < Q(t_1) \). The physical reasons of inequality (11) will be clear in the next section, where we consider this model in the Einstein frame.

All above-mentioned formulae are valid for an arbitrary potential \( V(\phi) \). Let us consider a non-positive definite potential. In this case, on the \( (\phi, \dot{\phi}) \) plane there is the boundary, at any point of which \( Q = 0 \), that is equivalent to

\[ \frac{\dot{\phi}^2}{2} + \frac{3(U' \dot{\phi})^2}{2U} + V = 0. \]

(12)

Note that this boundary divides the phase plane into two domains: one corresponds to real values of the Hubble parameter, defined by (3), the other one corresponds to non-real values of this function. So, if a trajectory starts from the real value of \( H \), then it never crosses the line \( Q = 0 \), but can touch this line.

We will call the domain on the \( (\phi, \dot{\phi}) \) plane, which corresponds to non-real values of the Hubble parameter, defined by (3), as ‘unreachable domain’, because this domain is unreachable for solutions with the initial conditions, which satisfy (3) and correspond to real values of the Hubble parameter \( H \). The boundary of this domain is defined by (12).

2.2. Induced gravity models

Let us consider the induced gravity models \([14, 43]\) with

\[ U(\phi) = \frac{1}{2} \xi \phi^2, \]

(13)

where \( \xi \) is the non-minimal coupling constant.

Equations (3)–(5) for such a choice of the function \( U(\phi) \) look as follows:

\[ H^2 = \frac{V}{3\xi \phi^2} + \frac{1}{6\xi} \left( \frac{\dot{\phi}}{\phi} \right)^2 - 2H \frac{\ddot{\phi}}{\phi}, \]

(14)

\[ 3H^2 + 2H = -2 \frac{\ddot{\phi}}{\phi} - 4H \frac{\dot{\phi}}{\phi} - \frac{4\xi + 1}{2\xi} \left( \frac{\dot{\phi}}{\phi} \right)^2 + \frac{V}{\xi \phi^2}, \]

(15)

\[ \ddot{\phi} + 3H \dot{\phi} + V' - 6\xi \phi (2H^2 + \dot{H}) = 0. \]

(16)
We assume that $\xi \neq -1/6$. The case $\xi = -1/6$ will be considered separately in section 4.

All calculations have been made for $\xi > 0$ that corresponds to $U(\phi) \geq 0$.

Equation (14) is the quadratic equation for $H$:

$$H^2 + 2H \frac{\psi}{\phi} - \frac{V}{3 \xi \phi^2} - \frac{1}{6 \xi} \left( \frac{\psi}{\phi} \right)^2 = 0,$$

and has the following solutions

$$H_{\pm} = -\frac{\psi}{\phi} \pm \sqrt{\left(1 + \frac{1}{6 \xi}\right) \left( \frac{\psi}{\phi} \right)^2 + \frac{V}{3 \xi \phi^2}}.$$

The function $H$ is a continuous function, so, if $V(\phi) > 0$ for all $\phi$, then evolution of the Universe in such a model is described either only $H_-$ or only $H_+$. It depends on initial conditions. If $V(\phi)$ is not a positive definite function, then it is possible that a part of evolution is described by $H_-$, whereas the other part by $H_+$. The simplest way to get a non-positive definite potential from the known positive definite one is to subtract a positive constant.

For the induced gravity model

$$Q \equiv H + \frac{\psi}{\phi}$$

and equation (10) has the following form:

$$\dot{Q} = \frac{\psi}{\phi} Q - \frac{6 \xi + 1}{2 \xi} \left( \frac{\psi}{\phi} \right)^2.$$

It is easy to see that for positive $\xi$ and $\phi$

$$\frac{d}{dt} \left[ \frac{Q}{\phi} \right] = -\frac{6 \xi + 1}{2 \xi} \left( \frac{\psi}{\phi} \right)^2 \leq 0.$$

Combining equations (14)–(16), we obtain the following system of the first order differential equations

$$\dot{\phi} = \psi,$$

$$\dot{\psi} = -3H\psi - \frac{\psi^2}{\phi} + \frac{1}{(1 + 6 \xi) \phi} [4V(\phi) - \phi V'(\phi)].$$

$$\dot{H} = -\frac{4H\psi}{(1 + 6 \xi) \phi} + \frac{V'(\phi)}{(1 + 6 \xi) \phi} - \frac{12 \xi}{1 + 6 \xi} H^2 - \frac{1}{2 \xi (1 + 6 \xi)} \left( \frac{\psi}{\phi} \right)^2.$$

Equation (17) is a consequence of system (21)–(23), under condition that the initial values of the functions $\phi$, $\psi$ and $H$ satisfy (17).

From equations (14) and (15) we get the following equation, whose form does not depend on the potential:

$$\dot{H} = -\frac{\dot{\psi}}{\phi} + H \frac{\psi}{\phi} - \frac{2 \xi + 1}{2 \xi} \left( \frac{\psi}{\phi} \right)^2.$$

Substituting the value of $H$ on the boundary of the unreachable domain, we obtain the following equation:

$$\frac{6 \xi + 1}{\xi} \left( \frac{\psi}{\phi} \right)^2 = 0.$$

It means that the boundary of the unreachable domain is not a solution for system (21)–(23) at any values of parameters. Only trivial solutions with $\psi = 0$ can belong to this boundary. The above-mentioned results are valid for any non-positive definite potential.
Figure 1. The solution of system (21)–(23) at $\Lambda = 0.05$. We choose $b = 1, \epsilon = 10, \xi = 10$. The initial conditions are $\phi_0 = 2, \psi_0 = 0, H_0$ is calculated by (18) with sign ‘+’.

On the left picture, the phase diagram is present. It is magenta at the corresponding value of $H = H_+ > 0$, blue at $H = H_+ < 0$, red at $H = H_- > 0$ and green at $H = H_- < 0$ brown. On this picture, brown dashed line corresponds to $H_+ = 0$, brown dashed line with long dashes corresponds to $H_- = 0$, black line is the boundary of the unreachable domain. On the right picture, the Hubble parameter as function of the cosmic time is present. Brown colour means that $H = H_+$, whereas $H = H_-$ is drown in dark green colour.

Models of this type in the case of the Higgs potential have been considered in a lot of works (see, for example, [15–19]). In this paper, dynamics in the model with the following non-positive definite Higgs-like potential

$$V_H(\phi) = \frac{\epsilon}{4}(\phi^2 - b^2)^2 - \Lambda, \quad \Lambda > 0,$$

(26)

where $\epsilon$, $b$ and $\Lambda$ are constants, is studied.

In the case of the $V_H(\phi)$ potential, the boundary of the unreachable domain ($Q = 0$) has the following form:

$$(1 + 6\xi)\dot{\phi}^2 = 2\Lambda - \frac{\epsilon}{2}(\phi^2 - b^2)^2.$$ (27)

Most attention will be paid to the case when $\Lambda < \epsilon b^2/4$. In this case, the unreachable domain on the $(\phi, \dot{\phi})$ plane consists of two separate parts.

Numerical calculations give the following phase trajectory for system (21)–(23). As we see there are two stages of evolution on the phase plane. The first stage corresponds to some kind of the cyclic Universe, then the phase trajectory reaches the boundary of the unreachable domain and the second stage starts, during this stage the Hubble parameter rapidly decreases and the Universe contracts (see figure 1).

The fact which gives the change of the stages is that when the phase trajectory reaches the boundary of the unreachable domain, the square root in the equation for the Hubble parameter gets the ‘−’ sign.

We can describe this stage of the evolution in more details. The evolution of the system (21)–(23) is the following. Let us start with $\phi > 0$, some $\psi$ and the corresponding $H = H_+$. 

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Figure 2. Solutions at phase portraits of system (21)–(22) at $\Lambda = 0.05$. We choose $b = 1$, $\epsilon = 10$, $\xi = 1$. The function $H(t)$ is given by (17) as $H_+$ on the left picture and $H_-$ on the right picture. The initial conditions for the trajectories are $\phi_0 = 0.1$, $\psi_0 = 1$, $H = H_+$ (left) and $\phi_0 = 0.92$, $\psi_0 = 0$, $H = H_-$ (right). On these pictures, brown dashed line corresponds to $H = 0$, black line is the boundary of the unreachable domain, which corresponds to $Q = 0$.

The phase portrait in figure 2 (left picture) shows that the boundary $Q = 0$ is attractive. So, the trajectories are winding up this boundary and then, at some finite moment of time, touch it. From (19), we get that always $\dot{Q} \leq 0$ at $Q = 0$ and $\dot{Q} < 0$ at $Q = 0$ and $\psi \neq 0$. This means that if a solution for system (21)–(23) with $H_+$ touches this boundary at $\psi \neq 0$, then it changes to $H_-$, solutions with $H_-$ cannot touch this boundary at $\psi \neq 0$. After this, the solution tends to infinity (see figure 2, right picture) and $H$ tends to $-\infty$.

We can address the following question: is there a possibility that the process of reaching the boundary of the unreachable domain will take the infinite time and the evolution will not finish by the contraction? The answer is that the process of reaching the boundary of the unreachable domain in the case when the trajectory rotates around it will always take a finite time.$^4$

Indeed, if for some moments of time $t_1$ and $t_2 > t_1$ we have $\phi(t_2) = \phi(t_1)$, then we get from (20) the following:

$$\frac{Q(t_2)}{\phi(t_2)} - \frac{Q(t_1)}{\phi(t_1)} = \frac{1}{\phi(t_1)} (Q(t_2) - Q(t_1)) = -\frac{6\xi + 1}{2\xi} \int_{t_1}^{t_2} \frac{\psi^2}{\phi^3} \, dt \leq C_0 < 0$$

where $C_0$ is a negative number. So, for any circle value of $Q$ decreases on some positive value, which does not tend to zero, when number of circles tends to infinity, hence, only a finite number of circles is necessary to obtain the value $Q = 0$.

Note that, when $Q$ tends to zero,

$$\dot{Q} \to -\frac{6\xi + 1}{2\xi} \left(\frac{\psi}{\phi}\right)^2.$$

$^4$ This conclusion does not depend on a specific form of the potential $V(\phi)$. The important property of this potential is that its minimum cannot be reached with infinitely small velocity.
From (18) we get that $H$ is a real number at $\phi = b$, only if

$$(1 + 6\xi) \psi^2 > 2\Lambda.$$  

(28)

Therefore, at $\phi \approx b$ and $Q \rightarrow 0$ we obtain

$$\dot{Q} \approx -\frac{6\xi + 1}{2\xi} \left(\frac{\psi}{b}\right)^2 < -\frac{\Lambda}{\xi b^2}.$$  

(29)

We come to a conclusion that $\dot{Q}$ does not tend to zero at $Q \rightarrow 0$ if $\Lambda > 0$.

Let us analyse how the behaviour of the Hubble parameter depends on values of parameters $\xi$ and $\epsilon$. It is easy to see (figure 3) that the number of the Hubble parameter oscillations increases with the parameter $\epsilon$, whereas a period of the oscillations increases with the parameter $\xi$.

Note that the obtained behaviour of the solutions is essentially different from the behaviour of the solutions in the case $\Lambda = 0$. At $\Lambda = 0$, the unreachable domain is absent and the point ($H = 0, \phi = b, \psi = 0$) is an attractive fixed point and the Hubble parameter is positive at any point (see figure 4). We also can see the difference in the behaviour of the function $Q(t)$ (figure 5). This difference is a consequence of the unreachable domain existence.

To estimate the period of the cyclic stage we can estimate a period of oscillations for a limit trajectory—a trajectory which coincides with a boundary of the unreachable domain $(1 + 6\xi)\psi^2 = -2V(\phi)$. We remind that this trajectory is not a solution for the system of equations under consideration. A solution to equation

$$\ddot{\phi} = -\frac{1}{1 + 6\xi} V'(\phi),$$

with the negative energy $E = \Lambda' - \varepsilon' b^4/4$, where $\Lambda' = \Lambda/(1 + 6\xi)$, $\varepsilon' = \varepsilon/(1 + 6\xi)$, has the form (see, for example, [48])

$$\phi(t) = A \text{dn}(\tilde{\Omega}t + c, k),$$  

(30)

where $\text{dn}(u, k)$ is the elliptic function of argument $u$ and modulus $k$. $A$ is the amplitude and $c$ is the phase. The frequency $\tilde{\Omega}$ and the modulus $k$ of the elliptic function are obtained from parameters $\varepsilon'$, $b$ of the equation and depend on the amplitude $A$:

$$\tilde{\Omega}^2 \equiv A^2 \frac{\kappa'}{2} \quad k^2 = 2 \left(1 - \frac{b^2}{A^2}\right),$$  

(31)

$$A^2 \equiv b^2 \left(1 + \left(1 + \frac{4E}{\varepsilon b^4}\right)^{1/2}\right).$$  

(32)

Parameter $c$ is defined by the relation

$$\phi(0) = A \text{dn}(c, k).$$  

(33)

Now we estimate a period of function $\phi(t)$ (it will be of the same order as the period for the function $H$). Then we will also have an estimation for a period of $H$. Let us suppose the period has the standard form

$$T = \frac{2\pi}{\tilde{\Omega}}$$  

(34)

and substitute $\tilde{\Omega}$ in the form (31). We obtain

$$T = 2\pi \sqrt{2} \sqrt{1 + 6\xi} \sqrt{2/\sqrt{\epsilon(b^4) + \Lambda}}.$$  

(35)
2.3. The model with non-minimal coupling of the type $U(\phi) = \frac{1}{2} \xi \phi^2 + K$

The model of the Higgs-driven inflation includes $R$, multiplied by $\frac{1}{2} \xi \phi^2 + K$, where $K$ is a nonzero constant. This model with $K = M_{Pl}^2/(16\pi)$, where $M_{Pl}$ is the Planck mass, and $\xi = 47000\sqrt{\epsilon}$, is being considered as a real candidate to describe the inflationary scenario [15, 16, 19–21], which is fully consistent with the Planck constraints [7].
Figure 4. The solution of system (21)–(23) at $\Lambda = 0$. We choose $b = 1$, $\varepsilon = 10$, $\xi = 10$. The initial conditions are $\phi_0 = 2$, $\psi_0 = 0$, $H_0$ is calculated by (18) with sign $`+`$ ($H_0 = \sqrt{3}/4$). The Hubble parameter is always $H_+$. 

Figure 5. The function $Q(t)$ at $\Lambda = 0.05$ (left) and $\Lambda = 0$ (right). We choose $b = 1$, $\varepsilon = 10$, $\xi = 10$. The initial conditions are $\phi_0 = 2$, $\psi_0 = 0$, $H_0$ is calculated by (18) with sign $`+`$.

Let us consider the model with

$$U(\phi) = \frac{\xi}{2} \phi^2 + \frac{M_{Pl}^2}{16\pi}.$$

Substituting (36) into action (1), we obtain:

$$S = \int d^4x \sqrt{-g} \left[ \left( \frac{M_{Pl}^2}{16\pi} + \frac{1}{2} \xi \phi^2 \right) R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right].$$

(37)
Equations (3)–(5) in this case are
\[ H^2 = \frac{8\pi}{3(M_{\text{Pl}}^2 + 8\pi \xi \phi^2)} \left( \frac{1}{2} \dot{\phi}^2 - 6\xi H \phi \dot{\phi} + V \right), \]  
\begin{equation} \tag{38} \end{equation}

\[ 3H^2 + 2\dot{H} = \frac{8\pi}{M_{\text{Pl}}^2 + 8\pi \xi \phi^2} \left[ \frac{1}{2} \dot{\phi}^2 + 2\xi \dot{\phi}^2 + 2\xi \ddot{\phi} \phi + 4\xi H \phi \dot{\phi} - V \right], \]  
\begin{equation} \tag{39} \end{equation}

\[ \ddot{\phi} + 3H \dot{\phi} + V'(\phi) + \frac{16\pi \xi \phi}{M_{\text{Pl}}^2 + 8\pi \xi \phi^2} \left( \frac{1}{2} \dot{\phi}^2 + 3\xi \dot{\phi}^2 + 3\xi \ddot{\phi} + 9\xi H \phi \dot{\phi} - 2V \right) = 0. \]  
\begin{equation} \tag{40} \end{equation}

Combining equations (38)–(40) we obtain the system of the first order differential equations describing dynamics of the model:
\[ \dot{\phi} = \psi, \]  
\begin{equation} \tag{41} \end{equation}

\[ \dot{\psi} = -3H \psi - \frac{(1 + 6\xi) \psi^2}{\phi(A + 6\xi)} + 4V - \phi AV', \]  
\begin{equation} \tag{42} \end{equation}

\[ \dot{H} = -\frac{\Lambda(1 + 2\xi) + 4\xi}{2\xi \phi^2 A(A + 6\xi)} \psi^2 + \frac{4H}{\Lambda \phi} \psi - \frac{4V - \phi AV'}{\phi^2 A(A + 6\xi)}, \]  
\begin{equation} \tag{43} \end{equation}

where \( \Lambda \equiv \frac{M_{\text{Pl}}^2}{8\pi \xi \phi^2} + 1. \)

Equation (38) is a quadratic equation for \( H \) and has the following solutions:
\[ H_\pm = -\frac{1}{\Lambda \phi} \pm \sqrt{\frac{1}{\Lambda \xi} \left( \frac{6\xi + \Lambda}{6\Lambda} \left( \frac{\psi}{\phi} \right)^2 + \frac{V}{3\phi^2} \right)} \].  
\begin{equation} \tag{44} \end{equation}

As in the previous case (the induced gravity model), if the potential is non-positive definite then there is a possibility of a transition between \( H_+ \) and \( H_- \).

We consider the non-positive definite Higgs-like potential (26). Numerical calculations give the following phase trajectory for this model (see figure 6, left pictures). As in the case of the induced gravity, there are two stages of evolution on the phase plane. The first stage corresponds to some kind of cyclic Universe, then the phase trajectory reaches the boundary of the unreachable domain (inside which, the values of the Hubble parameter are image) and the second stage starts. The significant difference from the induced gravity model is that on this stage of evolution the Hubble parameter does not decrease rapidly, but goes on oscillating evolution corresponding to the cyclic Universe. Moreover, first the phase trajectory rotates around one of the unreachable domains in such a way that the distance to the boundary of this domain increases with time, then the trajectory crosses the line \( \phi = 0 \) and starts rotating around both unreachable domains moving farther. Then at some point, the Hubble parameter rapidly decreases and the Universe contracts (see figure 6, right pictures).

We can use the same approach as in the previous subsection and show that if a trajectory rotates around one of the unreachable domains starting from \( H_+ \), then it will surely reach the boundary of the unreachable domain for a finite period of time. When the trajectory touches the boundary of the unreachable domain, the evolution which has been corresponding to \( H_+ \) changes to the evolution corresponding to \( H_- \).

### 3. Action and equation of motion in the Einstein frame

Once a modified gravity theory is recast into its scalar–tensor presentation, it immediately follows that both the Jordan frame (where the scalar fields non-minimally couples to gravity)
and the Einstein one (which has the same form as that of Einstein gravity with minimally coupled scalar fields) are available. These two frames are related by conformal transformation
\[ g_{\mu\nu} = \Omega^2 g_{\mu\nu}^{(E)} \rightarrow g = \Omega^8 g^{(E)}, \] (45)
where we denote the metric in the Jordan frame by \( g_{\mu\nu} \), while the one in the Einstein frame is labelled as \( g_{\mu\nu}^{(E)} \). In the following, we denote all quantities in the Einstein frame by adding a tag \( (E) \) or the index \( E \) to the corresponding ones in the other frame, in order to avoid confusion.

Let us consider the conformal transformation (45). Using \( g_{\mu\nu} = \Omega^{-2} g^{(E)}_{\mu\nu} \), one obtains the relationship between the Ricci scalars in the two frames [46, 47]:
\[ R = \Omega^{-2} \left[ R^{(E)} - 6 \left( \Box^{(E)} \ln \Omega + g^{(E)}_{\mu\nu} \nabla^{(E)}_{\mu} \ln \Omega \nabla^{(E)}_{\nu} \ln \Omega \right) \right]. \] (46)
Inserting this into action (37), one identifies the conformal factor as
\[ \Omega^{-2} = \frac{\kappa^2}{2} U \quad \rightarrow \quad \Omega = \frac{\sqrt{2}}{\kappa \sqrt{U}}, \] (47)
where \( \kappa^2 \equiv 8\pi / M_{Pl}^2 \).

The issue, which of the conformal frames, Jordan or Einstein, is the physical one, has been the subject of longstanding debate (see [49], and references therein). At the same time, it is known [50] that the spectrum of curvature perturbations and the amplitude of gravitational
waves obtained are invariant under conformal transformations. By this reason, it is possible to consider either frame as a physical one to study inflation and inflationary calculations are often performed in the Einstein frame. In this paper, we consider the conformal transformation from the mathematical point of view to connect the parameter $Q$ with the Hubble parameter in the Einstein frame and clarify the sense of inequality (20).

As is well known [46], conformally flat metrics are mapped into each other. The FLRW metric is conformally flat, so starting from the FLRW metric in the Jordan frame we obtain the corresponding FLRW metric in the Einstein one. This leads us to directly start from an FLRW metric with cosmic time in the Einstein frame

$$ds^2 = -dt^2 + a_E(t)^2 \delta_{ij} dx_i dx_j,$$

(48)

where in $dt_E$ and $a_E(t_E)$, the index $E$ denotes the corresponding quantities in the Einstein frame. We obtain

$$dt_E = \Omega^{-1} dr = \frac{\kappa \sqrt{U}}{\sqrt{2}} dr, \quad a_E = \frac{\kappa \sqrt{U}}{\sqrt{2}} a,$$

(49)

$$H_E = \frac{d \log a_E}{dt_E} = \Omega \left( H - \frac{\dot{\Omega}}{\Omega} \right) = \frac{\sqrt{2}}{\kappa \sqrt{U}} \left( H + \frac{\dot{U}}{2U} \right) = \frac{\sqrt{2}}{\kappa \sqrt{U}} Q.$$

(50)

We see that the function $Q$ is connected with the Hubble parameter in the Einstein frame. We get from (50) that $Q = 0$ is equivalent to $H_E = 0$. As known, for cosmological models with a minimally coupling scalar field, the Hubble parameter $H_E$ is a monotonically decreasing function, therefore, the trajectory cannot touch the boundary of unreachable domain twice. Also, it gives us the physical sense of inequality (11):

$$\frac{dH_E}{dt_E} = \frac{dH}{dt} = \frac{2}{\kappa^2 \sqrt{U}} \frac{d}{dt} \left[ \frac{Q \sqrt{U}}{U} \right] = -\frac{U + 3U_r^2}{2\kappa^2 U^3} \dot{\phi}^2 < 0,$$

(51)

at $U(\phi) > 0$. Note that the behaviour of the Hubble parameters $H$ and $H_E$ are essentially different in the model being considered. Formula (50) can be useful for numeric computations of the Hubble parameter $H_E$.

4. The special case $\xi = -1/6$

This case is of a special interest due to the fact that if $\xi = -1/6$ then equations (21)–(23) have no sense. To consider this case we put $\xi = -1/6$ into equations (14)–(16). These equations get the following form

$$H^2 + \left( \frac{\dot{\phi}}{\phi} \right)^2 + 2H \frac{\dot{\phi}}{\phi} = -\frac{2V(\phi)}{\dot{\phi}^2},$$

(52)

$$3H^2 + 2H = \left( \frac{\dot{\phi}}{\phi} \right)^2 - 2 \frac{\ddot{\phi}}{\phi} - 4H \frac{\dot{\phi}}{\phi} - \frac{6}{\phi^2} V(\phi),$$

(53)

$$\ddot{\phi} + 3H \dot{\phi} + V' + \phi(2H^2 + \dot{H}) = 0.$$ 

(54)

From equations (52) and (53) we obtain

$$\dot{H} = 2 \left( \frac{\dot{\phi}}{\phi} \right)^2 + H \frac{\dot{\phi}}{\phi} - \frac{\dot{\phi}}{\phi}.$$

(55)
Using (55), we eliminate $\dot{H}$ from equation (54) and obtain

$$V'(\phi) + 2\phi \left( H^2 + \frac{\dot{\phi}^2}{\phi} + 2H\phi \dot{\phi} \right) = V'(\phi) - 4\frac{V(\phi)}{\phi} = 0. \tag{56}$$

This means that a solution with a nontrivial $\phi(t)$ exists if and only if the potential $V(\phi) = V_0\phi^4$. The constant $V_0$ must be non-positive to satisfy the requirement that $H$ is real. Equation (55) is an identity in this case. We get that for any function $\phi$, there is function $H$ and they are solutions of equations.

5. Conclusion

We have considered the cosmological models with a non-minimally coupled scalar field and a non-positive definite potential, which is the Higgs-like potential plus a negative cosmological constant. This negative constant may appear as a result of the renormalization procedure (if there are no protection arguments) or a result of effective stretch of constants in string field theory inspired cosmological models.

The characteristic property of models with non-positive definite potential is the existence of unreachable domains in the phase plane which correspond to non-real values of the Hubble parameter. Boundaries of unreachable domains are given by equation (12). We have shown that behaviour of the Hubble parameter in these cosmological models with the double-well potential essentially depends on the sign of the minimum of the potential. For example, for the standard Higgs-like potential we have attractive fixed points $\phi = \pm b$, which correspond to the minima of the potential. If we subtract a positive constant from this potential, then these fixed points turn out into the unreachable domains in the phase plane.

We have explored the dynamics of cosmological models with such non-positive definite Higgs-like potentials on the examples of the induced gravity model and model with both the Hilbert–Einstein term and the induced gravity term. We choose such values of the cosmological constant and parameters of the potential that the boundary of the unreachable domain consists of two closed curves. We have shown numerically that the phase trajectories are being attracted to the boundary of the unreachable domain, touch it and go to infinity. It has been proved analytically that if a trajectory is attracted to the boundary of the unreachable domain, then this trajectory touches it at finite time.

The subtracting of the cosmological constant does not essentially change the behaviour of model during inflation, when the scalar field changes slowly, so, in the case of non-positive definite Higgs-like potential, the results should be qualitative the same: both ‘chaotic’ inflation and ‘new’ inflation can be obtained. In this context, note that the cosmological model (37) with the Higgs potential considered as a real candidate to describe the inflationary scenario [9, 15, 16, 19–21] is fully consistent with the Planck constraints [7]. The substraction of the negative constant may change crucially the reheating regime, even makes it non-realizable. The reheating for the Higgs-driven inflation has been considered in [16]. During the reheating stage, the energy stored in the scalar field is transferred to fields of the Standard Model and maybe other (hypothetical) fields. To take the process of the scalar field decay into account one should modify the initial equations (21)–(23), in particular, include the decay rate $\Gamma$ [17, 51]. Also, the quantum effects play important role (see [51] as a review). In this paper, we do not take into account these processes and obtain that the adding of even small negative cosmological constant changes the evolution of the Universe and the Hubble parameter tends to minus infinity instead of zero. To get the reheating for models with a non-positive definite potential, we have no choice as to provide a mechanism that compensates the negative cosmological constant at some stage of evolution, for example, by quantum effects or by switch on a new
type of matter. To study all stages of cosmological evolution, it is important to consider the models in presence of additional matter. This will be a subject of our future investigations.

We have also shown that in the case of the conformal coupling ($\xi = -1/6$), the induced gravity model has nontrivial solutions in the spatially flat Friedmann–Lemaître–Robertson–Walker metric if and only if $V(\phi) = V_0 \phi^4$.

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Appendix. The SFT motivation of the Higgs-like potential

Let us remind how the stretch appears and how we get\(^{[23]}\) a non-positive definite Higgs-like potential, in other words, the Higgs potential minus a positive constant. Following\(^{[24]}\), we consider the SFT inspired nonlocal action

$$ S_{nl} = \int \! d^4x \sqrt{-g} \left[ \frac{m_p^2}{2} R + \frac{1}{2} \phi (\Box + \mu^2) e^{-\lambda} \phi - V(\phi) - \Lambda \right], $$

where $V(\phi) = \varepsilon \phi^4/4$, the covariant d’Alembert operator for a scalar field

$$ \Box = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu \nu} \partial_\nu), $$

$\phi$ is a dimensionless scalar field and all constants ($m_p, \mu, \lambda, \Lambda,$ and $\varepsilon$) are dimensionless. A homogeneous scalar field on a spatially flat FLRW universe with interval (2) satisfies the following nonlocal equations:

$$ e^{\lambda (\Box^2 + 3H^2)} \left( \ddot{\phi} + 3H \dot{\phi} - \mu^2 \phi \right) = -V'_\phi, \quad 3H^2 = \frac{1}{m_p^2} \mathcal{E}. $$

The energy of the nonlocal field is:

$$ \mathcal{E} = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \sum_{l=0}^{n-1} \dot{\phi}_l \Box^{n-1-l} \phi + \sum_{n=2}^{\infty} \frac{\lambda^n}{n!} \sum_{l=1}^{n-1} \Box^l \phi \Box^{n-1-l} \phi - \frac{\mu^2}{2} \phi^2 + \frac{\varepsilon}{4} \phi^4 + \Lambda. $$

In the SFT inspired nonlocal cosmological models the value of the cosmological constant is fixed by the Sen conjecture\(^{[52]}\) (see also\(^{[25]}\)) that puts the potential energy in the nontrivial vacua equal to zero. In other words, we obtain the following potential:

$$ \tilde{V}(\phi) = \frac{1}{4} \varepsilon \phi^4 - \frac{1}{2} \mu^2 \phi^2 + \frac{1}{4} \left( \phi^2 - \frac{\mu^2}{\varepsilon} \right)^2, $$

that gives $\Lambda = \mu^4/(4\varepsilon)$.

It has been shown in\(^{[30, 31, 53–56]}\) that in the case of a quadratic potential, a cosmological model with one nonlocal scalar field can be presented as a model with finite or infinite number of local scalar fields and quadratic potentials. For the nonlocal model with a quadratic potential, it has been observed\(^{[26, 27]}\) that in the regime suitable for the early Universe, the effect of
the effective stretch of the kinetic terms takes place (see also [30]). One can generalize this result to the case of the quartic potential and assume that a nonlocal model can be described by an effective local theory with the following energy density:

\[ E \approx e^{\lambda \omega^2} \left( \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \mu^2 \phi^2 \right) + \frac{1}{4} \varepsilon \phi^4 + \frac{\mu^4}{4\varepsilon}, \] (A.4)

where \( \omega \) is a modified ‘frequency’ of the asymptotic expansion for equation (A.5) in the flat case. In the first approximation \( \omega = \mu \).

Equation (A.2) becomes

\[ \ddot{\phi} + 3H \dot{\phi} - \mu^2 \phi = -\varepsilon e^{-\lambda \omega^2} \phi^3, \] (A.5)

\[ H^2 = \frac{1}{3m^2_{\text{eff}}} \left( \frac{1}{2} \dot{\phi}^2 + \frac{\varepsilon_{\text{eff}}}{4} \phi^4 - \frac{1}{2} \mu^2 \phi^2 + \Lambda_{\text{eff}} \right), \] (A.6)

where

\[ m^2_{\text{eff}} = m^2_{p} e^{-\lambda \omega^2}, \quad \varepsilon_{\text{eff}} = \varepsilon e^{-\lambda \omega^2}, \quad \Lambda_{\text{eff}} = \Lambda e^{-\lambda \omega^2} = \frac{\mu^4}{4\varepsilon_{\text{eff}}} e^{-2\lambda \omega^2}. \]

As known [25], in the SFT inspired models \( \lambda > 0 \), so, we obtain the non-positive definite Higgs-like potential:

\[ V_L(\phi) = \frac{\varepsilon_{\text{eff}}}{4} \phi^4 - \frac{1}{2} \mu^2 \phi^2 + \Lambda_{\text{eff}} = \frac{\varepsilon_{\text{eff}}}{4} \left( \phi^2 - \frac{\mu^2}{\varepsilon_{\text{eff}}} \right)^2 + \frac{\mu^4}{4\varepsilon_{\text{eff}}} (e^{-2\lambda \omega^2} - 1). \] (A.7)

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