Research on Autofrettage Mechanism in Ultra-High Pressure Thick-walled Vessel

Guiqin Li 1,*, Xuechao Deng 1, Haoju Song 1, Baoqing Zhang 2

1 Shanghai Key Laboratory of Intelligent Manufacturing and Robotics, Shanghai University, Shanghai, China
2 Changchun University of Science and Technology, Changchun, China

*Corresponding author e-mail: leeching@shu.edu.cn

Abstract. The stress distribution of the Ultra-High Pressure (UHP) vessel is uneven, the inner layer stress value is the maximum and the outer layer stress value is the minimum, resulting in insufficient utilization of the outer layer material and reduced fatigue strength. In order to obtain the pressure when the UHP vessel bursts into failure, this paper proposes a new formula for calculating the burst pressure and uses 30 experimental sample data to verify the accuracy. The error of the calculation result is about 5% lower than the error of the traditional Faupel burst pressure formula by 15%, and is not affected by the tensile strength. The UHP vessel improves the stress distribution and improves the fatigue life using autofrettage technology, reverse yielding does not occur when the best autofrettage pressure is unloaded, and the yield strength is calculated using the burst failure criterion method. Simultaneously, the Shigley method is used to predict the fatigue life of the autofrettaged UHP vessel to prove that its fatigue life has been significantly improved.

1. Introduction
With the development of industrial technology, the quality requirements of UHP vessels have become higher in certain fields such as the chemical industry and the petrochemical industry [1]. The UHP vessel runs under high stress and high-pressure conditions. The following are the characteristics of its stress distribution. 1) Among the three principal stresses, the circumferential stress $\sigma$ is the largest. 2) The circumferential stress distribution is the largest in the inner wall stress, and the distribution along the wall thickness is very uneven. 3) As the value of diameter ratio $k$ increases, the degree of unevenness becomes more serious, and when the operating pressure is greater than or equal to $\sigma / \sqrt{3}$, the inner wall will inevitably yield[2]. Therefore, the stress distribution of the UHP vessel needs to be changed structurally, which is one of the main considerations in the design of the UHP vessel.

Autofrettage technology can be used to solve the problem of uneven stress distribution in UHP vessels. In 1906, Autofrettage technology was proposed. In World War II, autofrettage technology was widely used in the manufacture of gun barrels. In the 1950s, the 320MPa high-pressure polyethylene tubular reactor manufactured by the former Federal Germany used autofrettage technology. The Autofrettage technology exerts pressure on the cylinder to make the cylinder part yield. After the pressure is relieved, prestress is generated to improve the stress distribution structure [3-4]. The advantage of the Autofrettaged container is that the stress distribution on the inner wall of the container...
is uniform, the average stress is reduced, and the fatigue life is improved. At the same time, the Autofrettaged process is easy to operate and industrially apply.

2. Stress analysis and strength calculation of UHP vessel

2.1. Stress analysis of UHP vessel

The thick-walled cylindrical high-pressure vessel is subjected to stress analysis. The material is assumed to be uniform and ideal elastoplastic, and there is no internal stress before bearing pressure. There is no change in the geometry, load, and support along the cylinder axis. Any cross section of the cylinder remains flat after deformation, and the cylinder is regarded as a plane strain axisymmetric problem [5]. The inner radius of the thick-walled cylindrical high-pressure vessel is \( r_i \), the outer radius is \( r_o \), and it bears internal pressure \( p_i \) and external pressure \( p_o \), as shown in Figure 1. Under the combined action of internal pressure and external pressure of the vessel, circumferential stress \( \sigma_t \), radial stress \( \sigma_r \) and axial stress \( \sigma_z \) will be generated.

![Figure 1. Thick-walled cylindrical UHP vessel subjected to internal and external pressure](image)

The stress distribution of the thick-walled cylinder is calculated using Lame's formula, where \( K \) is not the diameter ratio, and \( K>1 \).

\[
\sigma_t = -\frac{p_i K^2 + p_o}{K^2 - 1} \cdot \frac{(r_i - r_o) K^2 (K_i)}{(r_o - r_i) (K_o - 1)} \tag{1}
\]

\[
\sigma_r = -\frac{p_i K^2 + p_o}{K^2 - 1} \cdot \frac{(r_i - r_o) K^2 (K_i)}{(r_o - r_i) (K_o - 1)} \tag{2}
\]

The axial stress is related to the situation at both ends. For the closed end of the open head at both ends, the axial stress of the cylinder is \( \sigma_z = \frac{p_i - p_o K^2}{K^2 - 1} \).

The thick-walled cylindrical high-pressure vessel only bears internal pressure, and the three stresses can be written as the following formula.

\[
\sigma_t = \frac{p_i}{K^2 - 1} \left[ 1 - \left( \frac{K_i}{K_o} \right) \right] \tag{3}
\]

\[
\sigma_r = \frac{p_i}{K^2 - 1} \left[ 1 + \left( \frac{K_i}{K_o} \right) \right] \tag{4}
\]
It can be concluded from Equation 3 that the radial stress on the inner surface of the thick-walled cylindrical high-pressure vessel is the largest, and the value is \( \pi \). It is the smallest at the outer surface and has a value of 0. From equation 4, the circumferential stress on the inner surface of the thick-walled cylindrical UHP vessel is the largest, with a value of \( p_i (K^2 + 1/K^2 - 1) \), which is always greater than \( \pi \); it is the smallest on the outer surface, with a value of \( p_i (2/K^2 - 1) \). When \( k=3 \), the axial stress of the outer wall is only 20% of that of the inner wall, and the material properties of the outer wall are not fully utilized.

2.2. Blasting failure criterion and its strength calculation formula

As the internal pressure continues to rise, the plastic zone will gradually expand to the outer wall, after the inner wall of the thick-walled cylindrical high-pressure vessel enters a plastic state. Finally, the entire container becomes a plastic state. In this process, the strain hardening effect of the material is significant, and the load-bearing capacity of the container is continuously improved, but the wall thickness of the container is continuously reduced, causing the load-bearing capacity to continue to decrease. When the two effects of strain hardening and wall thickness reduction cancel each other out, the container will not be able to continue to increase the carrying capacity, and the container will soon burst. The pressure at this time is called the maximum bearing pressure of the cylinder, that is, the burst pressure \( P_b \). At present, it is most common to use Faupel formula to calculate burst pressure in engineering applications [6].

\[
\sigma_V = \frac{P_b}{K^2 - 1}
\]  

(5)

This Faupel formula has two shortcomings. On the one hand, there is an error in the calculation of the burst pressure; on the other hand, when the ratio of the yield strength of the container material to the tensile strength is low, the result of the Faupel formula is safer, and when the ratio is higher, The result is more dangerous. To solve the shortcomings of Faupel's formula, a new calculation formula for burst pressure is proposed.

\[
P_b = \sigma_c \left[ 2 + \left( \sqrt{3} - 2 \right) \eta + \sqrt{3} \eta^3 - \frac{2}{\sqrt{5}} \eta^4 \right] \frac{K^2 - 1}{K^2 + 1}
\]

(6)

(7)

The 30 sets of data in Table 1 are 15 kinds of materials and come from 4 experimental institutions. The yield ratio ranges from 0.4027 to 0.8852, and the vessel diameter ratio ranges from 1.33 to 4.71. The burst pressure of the 30 sets of samples is calculated using Formula 7.
Table 1. Measured value of blasting pressure of cylindrical vessel and calculated value and error of new improved formula

| Number | Yield Strength (MPa) | Flexion Ratio | Diameter ratio K | Measured value | Formula 7 Calculated value | Error value | \( \tau_i \) |
|--------|----------------------|---------------|------------------|----------------|---------------------------|-------------|---------|
| 1      | 918.44               | 0.8852        | 2.28             | 935.00         | 913.13                    | -2.34       | 1.02    |
| 2      | 918.44               | 0.8852        | 2.28             | 942.00         | 913.13                    | -3.06       | 1.03    |
| 3      | 918.44               | 0.8852        | 2.05             | 774.00         | 805.54                    | 4.07        | 0.96    |
| 4      | 756.02               | 0.8793        | 1.75             | 503.00         | 527.99                    | 4.97        | 0.95    |
| 5      | 831.07               | 0.8750        | 2.75             | 985.27         | 996.84                    | 1.17        | 0.99    |
| 6      | 573.81               | 0.7894        | 3.69             | 1157.5         | 913.93                    | -21.04      | 2.17    |
| 7      | 556.52               | 0.7570        | 4.71             | 1326.3         | 1036.21                   | -3.17       | 1.28    |
| 8      | 525.71               | 0.7222        | 1.75             | 406.51         | 425.80                    | 4.75        | 0.95    |
| 9      | 508.19               | 0.7048        | 2.74             | 678.67         | 715.40                    | 5.41        | 0.95    |
| 10     | 425.41               | 0.6824        | 2.49             | 544.31         | 563.49                    | 3.52        | 0.97    |
| 11     | 429.48               | 0.6808        | 2.44             | 571.87         | 558.80                    | -2.29       | 1.02    |
| 12     | 284.46               | 0.5887        | 2.80             | 456.90         | 470.80                    | 3.04        | 0.97    |
| 13     | 284.46               | 0.5887        | 1.42             | 167.26         | 172.50                    | 3.13        | 0.97    |
| 14     | 245.35               | 0.5210        | 2.43             | 392.73         | 396.65                    | 1.00        | 0.99    |
| 15     | 263.29               | 0.5102        | 2.75             | 465.08         | 485.29                    | 4.35        | 0.96    |
| 16     | 227.39               | 0.4997        | 3.72             | 544.09         | 527.15                    | -3.11       | 1.03    |
| 17     | 227.39               | 0.4997        | 3.60             | 524.00         | 517.03                    | -1.33       | 1.01    |
| 18     | 227.39               | 0.4997        | 3.18             | 482.63         | 477.07                    | -1.51       | 1.01    |
| 19     | 227.39               | 0.4997        | 2.90             | 450.23         | 445.65                    | -1.02       | 1.01    |
| 20     | 227.39               | 0.4997        | 2.66             | 414.38         | 414.89                    | 0.12        | 1.00    |
| 21     | 227.39               | 0.4997        | 2.48             | 395.76         | 389.03                    | -1.70       | 1.02    |
| 22     | 227.39               | 0.4997        | 2.29             | 372.32         | 358.67                    | -3.67       | 1.04    |
| 23     | 227.39               | 0.4997        | 2.13             | 329.57         | 330.25                    | 0.21        | 1.00    |
| 24     | 227.39               | 0.4997        | 1.99             | 307.51         | 302.88                    | -1.51       | 1.02    |
| 25     | 227.39               | 0.4997        | 1.88             | 277.17         | 279.51                    | 0.84        | 0.99    |
| 26     | 227.39               | 0.4997        | 1.78             | 264.76         | 256.66                    | -3.06       | 1.03    |
| 27     | 227.39               | 0.4997        | 1.57             | 213.74         | 202.88                    | -5.08       | 1.05    |
| 28     | 227.39               | 0.4997        | 1.33             | 128.52         | 129.56                    | 0.81        | 0.99    |
| 29     | 235.33               | 0.4264        | 2.76             | 378.95         | 511.98                    | 35.10       | 0.74    |
| 30     | 248.88               | 0.4027        | 2.75             | 434.07         | 569.57                    | 31.22       | 0.76    |

Relative error range (%) = -21.87~35.1
Advantage rate (%) = 100.00
Special advantage rate (%) = 80.00
\( \tau_i \) variation range = 0.74~1.28
Accuracy = 1.00
Coefficient of Variation = 0.10

The applicable range of formula 7 is determined. Diameter ratio: 1.33~3.72; Yield-strength ratio: 0.4497~0.8852. If both the diameter ratio and the yield-strength ratio are high, the calculated value of the formula will have a certain deviation. Formula 7 has an error of about 5% within a certain range, and the advantage rate can reach 100%. The formula has the characteristics of accurate result, simple form and convenient calculation. This formula has certain applicable value. The blasting failure criterion is most in line with the actual situation of the material, so it is most suitable as a design standard for UHP vessels.
3. Stress analysis and calculation of autofrettaged UHP vessel

The pressure on the inner wall of the container is greater than the initial yield pressure. The inner wall surface begins to undergo plastic deformation and expands outwards until the radius required by the design is reached, and the applied pressure is the autofrettaged pressure $p_a$. The thick-walled cylindrical container assumes that the geometric conditions are determined, the internal pressure is uniform and symmetrical, the radius of the interface is $r_c$, and the pressure on the interface is $p_c$. Part of the container under unloading autofrettaged pressure yields. The self-reinforcing pressure is calculated as formula 8.

$$p_s = \frac{\sigma_s}{\sqrt{3}} \left( 1 - \frac{r_c^3}{r_s^3} + 2 \ln \frac{r_c}{r_s} \right)$$  

(8)

The container can be divided into elastic state cylinder and plastic state cylinder. The elastic state cylinder is acted by the internal pressure $r_c$, and the plastic state is acted by the external pressure $r_c$ and the internal pressure $p_a$, as shown in Figure 2.

![Elastic cylinder](a) Elastic cylinder  ![Plastic cylinder](b) Plastic cylinder

Figure 2. Autofrettaged partially yielded thick-walled cylindrical container

The best degree of self-reinforcement should meet the two requirements, through stress analysis and calculation. 1) For a given circle diameter ratio $K$ value, when auto-reinforcement is performed according to the degree of auto-reinforcement, reverse yield will not occur after pressure relief; 2) In normal operating conditions of the cylinder, the equivalent stress of the superimposed stress is elastic. Plastic interface. According to these two principles, find the best auto-reinforcement degree. When the working pressure is $p$, the best auto-reinforcement pressure is formula 9.

$$p_a = \frac{\sigma_p}{\sqrt{3}} \left[ \frac{\sqrt{3} p}{\sigma_p} \exp \left( \frac{\sqrt{3} p}{\sigma_p} \right) \right]$$

(9)

4. Fatigue analysis of Autofrettaged UHP vessel

Fatigue life prediction is predicted using Shigley approximation. This method belongs to the stress-life method. It is operated by summarizing the distribution law of the fatigue strength of the specimen through a large number of rotary fatigue tests. The test involves different materials, different heat treatments and manufacturing processes, making the summarized law more universally applicable. The method takes the minimum fatigue strength line as the fatigue design curve, and the minimum fatigue strength is obtained through the rotary test. It has the characteristics of few design parameters, high calculation accuracy, and can be used for the fatigue strength design of UHP vessels and their
components. According to the Mises theory, the alternating stress amplitude and average stress of the inner wall of the vessel are formula 10.

\[ \sigma_{alt} = \sigma_m - \frac{\sqrt{3} p K}{2(K^2 - 1)} \]  

(10)

The mathematical expression of the fatigue design curve is formula 11, where \( m = \frac{1}{3} \log \frac{0.9}{\sigma_s} \) and \( b = \log \left( \frac{0.9 \sigma_s}{\sigma_s} \right) \).

\[ \log \sigma_{alt} = -m \log N + b \]  

(11)

5. Conclusion

Aiming at the deficiencies of the Faupel formula for burst pressure calculation, the burst pressure calculation formula is proposed. According to the application experiment data, it can get better results in a certain range. The reinforced UHP vessel is divided into elastic part and plastic part for analysis, and the best autofrettage pressure under certain working pressure is obtained. The Shigley approximation method is used for fatigue life prediction. The fatigue life of the autofrettaged UHP vessel can be accurately predicted. After comparing with ordinary containers and autofrettaged containers, the fatigue life of autofrettaged containers can be significantly improved.

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