Exploring the Gamma Ray Horizon with the next generation of Gamma Ray Telescopes. Part 3: Optimizing the observation schedule of γ-ray sources for the extraction of cosmological parameters

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Abstract

The optimization of the observation schedule of γ-ray emitters by the new generation of Cherenkov Telescopes to extract cosmological parameters from the measurement of the Gamma Ray Horizon at different redshifts is discussed. It is shown that improvements over 30% in the expected cosmological parameter uncertainties can be achieved if instead of equal-observation time, dedicated observation schedules are applied.
1 Introduction

Imaging Čerenkov Telescopes (IACT) have proven to be the most successful tool developed so far to explore the γ-ray sky at energies above a few hundred GeV. A pioneering generation of installations has been able to detect a handful of sources and to start a whole program of very exciting physics studies. Nowadays a second generation of more sophisticated Telescopes is starting to provide new observations. One of the main characteristics of some of the new Telescopes is the potential ability to reduce the gamma ray energy threshold below ∼ 30 GeV [1].

In the framework of the Standard Model of particle interactions, high energy gamma rays traversing cosmological distances are expected to be absorbed through their interaction with the diffuse background radiation fields, or Extragalactic Background Field (EBL), producing $e^+e^-$ pairs. Then the flux is attenuated as a function of the gamma energy $E$ and the redshift $z_q$ of the gamma ray source. This flux reduction can be parameterized by the optical depth $\tau(E, z_q)$, which is defined as the number of e-fold reductions of the observed flux as compared with the initial flux at $z_q$. This means that the optical depth introduces an attenuation factor $\exp[-\tau(E, z_q)]$ modifying the gamma ray source energy spectrum.

The optical depth can be written with its explicit redshift and energy dependence [2] as:

$$\tau(E, z) = \int_0^z dz' \frac{dl}{dz'} \int_0^2 dx \frac{x}{2} \int_{\frac{2l}{\sqrt{x(1+z')}}}^{\infty} d\epsilon \cdot n(\epsilon, z') \cdot \sigma[2xE\epsilon(1+z')^2]$$ (1)

where $x \equiv 1 - \cos \theta$ being $\theta$ the angle between the photon directions, $\epsilon$ is the energy of the EBL photon and $n(\epsilon, z')$ is the spectral density at the given $z'$.

For any given gamma ray energy, the Gamma Ray Horizon (GRH) is defined as the source redshift $z$ for which the optical depth is $\tau(E, z) = 1$.

In a previous work [3], we discussed different theoretical aspects of the calculation of the Gamma Ray Horizon, such as the effects of different EBL models and the sensitivity of the GRH to the assumed cosmological parameters.

Later, on [4] we estimated with a realistic simulation the accuracy in the determination of the GRH that can be expected from an equal-time obser-
vation of a selection of extragalactic sources. The results obtained in that previous study assumed an observation schedule of equal observing time per source which was set to a canonical value of 50 hours (rather standard assumption in IACTs). Although the actual observing time per source might have a lot of constraints (such as significance of the observation, physics interest of the source, competition in time for the observation of other sources, etc...), in this work we want to explore, taking into account just the unavoidable observability constraints, which time scheduling would optimize the power of this method to extract cosmological parameters and by how much the measurement of these parameters could improve.

One must also take into account that the determination of the cosmological parameters extensively discussed in [4] is based on the observations of Active Galactic Nuclei (AGN), which are intrinsically variable. Therefore one of the main parameters to decide which AGN is observed at any time will be their flaring state. For some AGN, it is possible to estimate its activity from observations in other wavelength, for instance using the X-ray data [5] provided by the All-Sky Monitor (ASM) [6] onboard the Rossi X-ray Timing Explorer. Unfortunately, there are a lot of AGNs for which there is not online data that would allow to infer the flaring state. Actually in the current catalogue of sources that are monitored by ASM only 3 of the 22 used on these studies appear. So that, here we’ll present an observational scheduling for the AGNs in table 1, which does not care about the activity of the source, and just optimizes the observation time to get the best precision on the cosmological parameter measurements.

The work is organized as follows: in section 2, the expected improvement in the precision of the GRH determination as a function of the observation time is discussed. Section 3 deals with the observational constraints in the optimization procedure. In section 4 we describe the optimization technique employed and describe the actual algorithm used. Section 5 presents the results of the optimization procedure in different scenarios considered and finally in section 6 we summarize the conclusions of this study.

2 Gamma Ray Horizon energy precision

To optimize the observation time, the first step is the study of the GRH precision as a function of the time that is dedicated for each source. The
estimated precision on the GRH comes from the extrapolation of the detected spectra of each source by MAGIC (see [4]). In there, the observation time enters as a multiplicative term to get the number of $\gamma$s.

In figure 1, the expected $\sigma$ of the GRH ($\sigma_{grh}$) using several observation times is shown. In these plots only the statistical error from the fitting parameter is shown. That error comes from the error bars in the extrapolated spectra. On the one hand there is the uncertainty on the flux ($\Phi$) that is modeled as “n” times the square-root of $\Phi$, which is proportional to $\sqrt{N_\gamma}$ being $N_\gamma$ the number of detected $\gamma$s. On the other hand, the error on the determination of the energy improves also with $\sqrt{N_\gamma}$ if one assumes a gaussian statistic. Therefore, one expects that the $\sigma_{grh}$ decreases with the square-root of time. Actually, the extrapolated errors show a good agreement with the blue line that is a fit to:

$$\sigma_{grh} = k/\sqrt{\text{time}}$$  \hspace{1cm} (2)

This latter expression would mean that the $\sigma_{grh}$ can be as small as desired if enough observation time is used. But it does not represent the reality. One should also take into account the systematics, which become more and more important when reducing the statistical error. As it has already been discussed in [4] the main systematic errors in the GRH determination from the simulated experimental data are due to the uncertainty in the global energy scale and to some approximations used to fit the data. The former is a global systematic, which is absolutely independent and uncorrelated to the observation time, and hence it is not considered here. Instead, the latter should have an impact in the precision of the GRH determination as a function of the observation time that may be different for each source. The main effect of those approximations is that the value of the GRH differs slightly from the one that has been introduced. This difference is added quadratically to the statistical error to account for the systematic difference. Then the figure 1 has been repeated and the result for the same 4 sources are shown in figure 2. In that scenario the curve is fitted to:

$$\sigma_{grh} = a + k/\sqrt{\text{time}}$$ \hspace{1cm} (3)

where $a$ is the contribution coming from the systematic, which does not decrease with the amount of observation time and therefore becomes impor-
Figure 1: Evolution of the statistic precision of the GRH determination as a function of observation time for four of the used AGNs (3EG J1426+428, 3EG J1255-0549, 3EG J0340-0201 and 3EG J1635+3813). The blue line is the fit to one over square-root of time.

The parameterization that have been finally used is:

\[ \sigma_{grh} = a + GRH(50h) \times \sqrt{\frac{50 \text{ hours}}{\text{time(hour)}}} \]  

(4)

where \( GRH(50h) \) are the statistic errors for the GRH using 50 hours of observation time and \( a \) is the constant term of the above mentioned fit (table 1).
Figure 2: Evolution of the precision of the GRH determination, adding the systematics due to the approximation in the fit of the spectra, as a function of observation time for four of the used AGNs (3EG J1426+428, 3EG J1255-0549, 3EG J0340-0201 and 3EG J1635+3813). The blue line is the fit to equation 3.

3 Constraints

The aim of this work is the optimization of the time dedicated to each source, to understand which improvement can be obtained in the cosmological measurements. Nevertheless, that time should make sense in the frame of the possible observations performed by an IACT such as, for instance, MAGIC. Therefore some constraints should be set.

The first constraint is the total amount of time used for those observations. For that, we used 1000 hours to compare it with the “50 h per source” configuration. On that naive configuration 50 hours were chosen since it is a
reasonable time to spend in a single source and it was already a criteria to
do the list of the best MAGIC targets in [7]. Since 20 sources were used (see
reference [4]), it accounts for 1 000 hours. Moreover, taking into account that
Čerenkov Telescopes have typically observations times of about 1 200 hours
per year, the limit used could be reached even in one single year. And it is
more than acceptable for 2-3 years, since AGNs are one of the main targets
of the new generation of IACTs.

One of the singularities of the astrophysics field respect to other physics
disciplines is that it studies phenomena that cannot be generated by the
humans in a laboratory. Therefore one has to use what nature provides. In
this sense, IACTs cannot observe one given source for an infinite time during
a year, not even those 1200 hours of observation time, since each source is
only visible during some months every year. Based on that fact, we have
computed the amount of time that each source is visible below 45 degrees
zenith angle form the MAGIC location. In table 1, one can see that time for
each of the used AGNs, actually it holds for the year 2005 and it may change
a few percent from year to year due to the full moon periods. To compute the
optimal distribution of observation times, the constraint “MaxTime” used for
each source is:

\[
\text{MaxTime} < T(\text{1 year}) \ast \text{Years} \ast F
\]  

5

where \(T(\text{1 year})\) are the number of hours stated in table 1. “Years” is
the number of years during which data would be collected and it is set to 3.
And F is the fraction of the available time during which data would be taken.
It is set to 0.25 and it accounts for bad weather conditions, off data needed
for the standard “On-Off” analysis and time dedicated to other sources or
targets of opportunity.

4  Time Optimization

In reference [4] the capability of the new IACTs to measure cosmological con-
stants has been discussed as well as the systematics on these measurements.
There, the main emphasis was put in the 68% contour in the \(\Omega_m - \Omega_\Lambda\) plane,
and it has been shown that it is competitive taking into account the system-
tatics (15% of energy scale, fit approximation and unknown Extragalactic
Background Light (EBL)) if a 15% external constraint on the Ultra Violet
Table 1: Parameters used for the optimization of the time observation dedicated to each source. The parameter $a$ is the time independent term contributing to the $\sigma_{GRH}$ (equation 4). And "$T(1\text{year})$" is the time that the source is below 45 degrees zenith angle during 2005 at the MAGIC location.

(UV) background is used. Under this scenario and scheduling 50 hours to each source, there is also the possibility to fit $\Omega_m$ and $\Omega_\Lambda$. Now, we would like to optimize the distribution of the observation time among the used sources to get the best precision on the measurement of the cosmological densities.

In order to optimize the distribution of the observation time by requiring a minimum error in some given parameter, a technique based upon a multi-dimensional constrained minimization using the Fisher Information Matrix has been used.

The Fisher Information Matrix, is defined as

$$F_{ij} = \left\langle \frac{-d^2 \log L}{d\theta_i d\theta_j} \right\rangle$$

where $L$ is the likelihood function of the measurements, $\theta_i$ and $\theta_j$ are fitting parameters and $\langle ... \rangle$ denotes expected value. In "normal" conditions,
it is the inverse of the error matrix for the parameters $i, j$. For large samples, a good estimator of $F$ is simply the function

$$F_{ij} = -\frac{d^2 \log L}{d\theta_id\theta_j} \quad (7)$$

evaluated at $\theta = \hat{\theta}$ namely, at the best fit parameter values.

In case $L$ could be simply approximated by a gaussian centered at the measured values, then

$$F_{ij} = \sum_{k,l} (df_k/d\theta_i)(V^{-1}kl)(df_l/d\theta_j) \quad (8)$$

evaluated at $\theta = \hat{\theta}$. The $i, j$ indices run over all the fitting parameters (the cosmological parameters in our case) and the $k, l$ run over all the measurements (the GRH measurements for different redshift in our case). $V$ is the error matrix of the measurements and $f_k(\theta)$ is the theoretical prediction for measurement $k$.

External constraints on the parameters are included by adding their corresponding Fisher Information Matrix. For instance, if $\Omega^\Lambda$ corresponds to parameter $i = 3$, and we want to include the constraint due to a measurement $\Omega^\Lambda_{\text{CMB}} \pm \Delta\Omega^\Lambda_{\text{CMB}}$ we just have to add to $F_{ij}$ a matrix $F'_{ij}$ with

$$F_{33}' = \left(\frac{\Omega^\Lambda_{\text{CMB}}}{\Delta\Omega^\Lambda_{\text{CMB}}}\right)^2 \quad (9)$$

and zero in all the other matrix elements.

This way, one can compute the expected fit parabolic error for any parameter without having actually to perform the fit. The expected error in $\Omega^\Lambda$ for instance would simply be $(F^{-1})_{33}$.

Now one must minimize this quantity (or any desired function of the fitting parameters) with respect to the observation time expended in each source (which enters in the evaluation of $V$ and hence, on $F$) with the relevant physical boundaries and constraints. For that we use the mathematical approach implemented in the code "DONLP2" developed by M.Spelucci [8]. The mathematical algorithm evaluates the function to be minimized only at points that are feasible with respect to the bounds. This allows to solve a smooth nonlinear multidimensional real function subject to a set of inequality and equality constraints. In our particular case:
• Problem function: It depends on the variable that we want to minimize but it is always a combination of the elements of the Fisher Information Matrix of the four dimensional fit in terms of $H_0, \Omega_\lambda, \Omega_M$ and the amount of UV background as described in [4].

• Equality constraints: The global amount of observation time, which is set to 1000 hours.

• Inequality constraints: The maximum available time for each source.

The result of this procedure is an array providing the optimal distribution of the observation times assuming parabolic errors, though we have explicitly checked that for the optimal time distribution the obtained precision on the fit parameters does not depend sizably on the assumption of parabolic errors.

5 Results

After the optimization to get the minimum error on $\Omega_m$ or $\Omega_\lambda$, this precision is improved by about 35% (see table 2). It is worth to notice that the obtained uncertainties for $\sigma_{\Omega_m}$ and $\sigma_{\Omega_\lambda}$ do not significantly differ while optimizing for one or the other. Even optimizing for the area of 68% contour in $\Omega_m - \Omega_\lambda$ plane, which is done assuming that the contour is an ellipse, the precision obtained is at the same level. This effect is mainly due to the correlation between $\Omega_m$ and $\Omega_\lambda$. Therefore, we will refer as the optimum time the one that minimizes the area of the $\Omega_m - \Omega_\lambda$ contour. This optimum time for each source is shown in table 3. One should notice that in this table only some of the initial 20 extragalactic considered sources remain. For the others, the optimization suggests that it is less interesting to observe them in terms of cosmological measurements. Moreover, the remaining sources are the ones at lowest and highest redshifts as well as the ones with smaller errors. Both were expected to survive since the former give the capacity to disentangle the cosmological parameters (see reference [3]) and the latter give larger constraints with less dedicated time. The improvement for the 68% contour can be seen in figure 3.

It has already been mentioned in [4] that a different approach to extract information from the GRH can be done: one can use the present constraints of the cosmological parameters to get information on the EBL. In reference
Figure 3: Improvement on the 68% contour in the $\Omega_m - \Omega_\Lambda$ plane. The red solid line is the 68% contour, taking into account the systematics and imposing a 15% constraint on the UV background, when 50 hours for each of the 20 used sources are scheduled. The blue dashed line is the 68% contour under the same conditions but with the optimized time distribution.

[4], the complexity of such analysis is discussed but a simple first step can be done within the scenario of the 4 dimensional fit used for these studies. If one uses the current measurements of the cosmological parameters ($H_0 = 72 \pm 4$, $\Omega_m = 0.29 \pm 0.07$ and $\Omega_\Lambda = 0.72 \pm 0.09$ [9][10]) as external constraints and then optimizes the time distribution to get the minimum error on the fourth parameter which gives a scale factor for the UV background, one can reach a precision of 13.5%. It is worth to notice that, despite the distribution of time among them is different, the sources that are still used are roughly the same than the ones for the $\Omega_m - \Omega_\Lambda$ optimization.
Table 2: Error for the cosmological densities observing for a total of 1000 hours the considered 20 AGNs. In each column the distribution of these 1000 hours is done following different criteria. The first column is a distribution of 50 hours each source. Second and third are times optimized to minimize the uncertainty on $\Omega_m$ and $\Omega_\Lambda$. The last column optimizes the area of the 68% contour in the $\Omega_m - \Omega_\Lambda$ plane.

| Parameter | 50 h | $\Omega_m$ | $\Omega_\Lambda$ | 68% contour |
|-----------|------|------------|------------------|-------------|
| $\sigma_{\Omega_\Lambda}$ | 0.366 | 0.279 | 0.278 | 0.279 |
| $\sigma_{\Omega_m}$ | 0.417 | 0.241 | 0.245 | 0.246 |

Table 3: Time scheduled for each source. First column optimizes the area of the 68% contour and the second one the determination of the scale factor for the UV background.

| Source Name | $z$ | $T_{\text{area}}$ (hour) | $T_{\text{UV}}$ (hour) |
|-------------|-----|--------------------------|------------------------|
| W Comae, 3EG J1222+2841 | 0.102 | 60 | 278 |
| 3C 279, 3EG J1255-0549 | 0.538 | 78 | 21 |
| 3EG J0852-1216 | 0.566 | 7 | — |
| CTA026, 3EG J0340-0201 | 0.852 | 167 | 3 |
| 3C454.3, 3EG J2254+1601 | 0.859 | 14 | 7 |
| 3EG J0450+1105 | 1.207 | — | 13 |
| 3EG J1323+2200 | 1.400 | 10 | 278 |
| 3EG J1635+3813 | 1.814 | 351 | 351 |
| 1ES J1426+428 | 0.129 | 312 | 49 |

6 Conclusions

In our previous works on this subject [3, 4], it was shown that the precision reached to measure the GRH for 50 hours of observation time is not the same for each of the 20 considered extragalactic sources. Moreover, it was also clear that the sensitivity to $\Omega_m$ and $\Omega_\Lambda$ was larger at high redshift and that the capability to disentangle the cosmological parameters is based on having measurements at low and high redshift. Therefore, it is clear that a cleverer distribution of the observation time would lead to better results. The optimization of that time distribution pointed out the need of having low redshift measurements (3EG J1426+428 at $z = 0.129$) as well as others at
high redshift (3EG J1635+3813 at z = 1.814). Together with these extreme sources, the dedication of time at sources that reach the best precision of the GRH (3EG J0340-0201) would also help to improve the results.

In this work, the optimal distribution of the observation time, taking into account scheduling constraints, has been studied by applying a technique based upon a multidimensional constrained minimization using the Fisher Information Matrix. The results obtained show that a proper scheduling optimized for the determination of the cosmological parameters could allow to reduce by 35% the error on the determination of $\Omega_m$ and $\Omega_\Lambda$ and a notable reduction of the 68% contour in the $\Omega_m - \Omega_\Lambda$ plane.

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