Magnetic Catalysis in the Higher-Order Quark Sigma Model

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Abstract

The effect of the higher-order mesonic interactions is investigated on the chiral symmetry breaking in the presence of an external magnetic field. The effective of higher-order mesonic potential is employed and is numerically solved in the mean-field approximation. The chiral symmetry breaking increases with increasing magnetic field. Two sets of free parameterization are investigated on the magnetic catalysis. A comparison is discussed with the original sigma model and other studies. The obtained results are included that the higher-order mesonic interactions play an important role in the magnetic catalysis.

Keywords: Chiral symmetry, Magnetic catalysis, Mean-field approximation

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I. INTRODUCTION

Chiral symmetry breaking is an important phenomenon in hadron physics and is of fundamental importance for hadron properties. The difficulties involved in obtaining low-energy properties directly from QCD, the fundamental theory of strong interactions, have motivated the construction of effective models due to their simplicity and effectiveness in describing hadrons at low energies [1]. The linear sigma model has been proposed as a model for strong nuclear interactions [2]. The model was first proposed in the 1960s as a model for pion-nucleon interactions. Today it serves as an effective model for the low-energy phase (zero temperature) of quantum chromodynamics [3-6] and its modification is suggested as Refs. [7-10] to provide a good description of the baryon properties. In addition, the quark sigma model is successfully applied to the description of static and dynamic baryon properties at finite temperature and density as in Refs. [11-15].

The study of the influence of external magnetic fields on the fundamental properties of quantum chromodynamic (QCD) theory, confinement and dynamical chiral symmetry breaking is still a matter of great interest theoretical and experimental activities [16 – 19]. In Ref. [20], the chiral symmetry structure in the original sigma model in the presence of an external uniform magnetic field is investigated. Authors of Ref. [21] examined the chiral phase transition in the presence of electromagnetic field and they found the magnetic field enhances the chiral symmetry breaking in the NJL model. Shushpanov and Smilga [22, 23] studied the quark condensate in the presence of external magnetic field using the Schwinger-Dyson equation. In Refs. [24 – 26], the proper time method is applied to a four-fermion interaction model to study the influence of a magnetic field. It is shown that an external magnetic field has the effect of enhancing chiral symmetry breaking.

Recently, the higher-order mesonic interactions play an important role to test nonperturbative chiral dynamics [27]. On same lines, the hadron properties are improved in comparison with other models and are in good agreement with experimental data by including higher-order mesonic interactions as Refs. [10, 28]

In this paper, we investigate the effect of higher-order mesonic interactions on the chiral symmetry breaking in the present of an external magnetic field in the framework of quark sigma model. In addition, a new parametrization for sigma mass and coupling constant on the behavior of the phase transition is studied. So far no attempt has been made to
include higher-order mesonic interactions on the chiral symmetry breaking in the presence of external magnetic field.

This paper is organized as follows: The original sigma model is briefly presented in Sec. 2. Next, the effective higher-order mesonic potential in the presence of external magnetic field is presented in Sec. 3. The results are discussed and are compared with other models in Secs. 4 and 5, respectively. Finally, the summary and conclusion are presented in Sec. 6.

II. THE CHIRAL SIGMA MODEL WITH ORIGINAL EFFECTIVE POTENTIAL

The interactions of quarks via the exchange of $\sigma$- and $\pi$- meson fields are given by the Lagrangian density [3] as follows:

$$L(r) = i\bar{\Psi}\gamma^\mu\gamma^5\Psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\pi\partial^\mu\pi) + g\bar{\Psi}(\sigma + i\gamma_5\tau\pi)\Psi - U_1(\sigma, \pi),$$

with

$$U_1(\sigma, \pi) = \frac{\lambda^2}{4}(\sigma^2 + \pi^2 - \nu^2)^2 + m^2_\pi f_\pi \sigma.$$  (2)

$U_1(\sigma, \pi)$ is the meson-meson interaction potential where $\Psi$, $\sigma$ and $\pi$ are the quark, sigma, and pion fields, respectively. In the mean-field approximation, the meson fields are treated as time-independent classical fields. This means that we replace the power and products of the meson fields by corresponding powers and the products of their expectation values. The meson-meson interactions in Eq. (2) lead to hidden chiral $SU(2) \times SU(2)$ symmetry with $\sigma(r)$ taking on a vacuum expectation value

$$\langle \sigma \rangle = -f_\pi,$$  (3)

where $f_\pi = 93$ MeV is the pion decay constant. The final term in Eq. (2) included to break the chiral symmetry explicitly. It leads to the partial conservation of axial-vector isospin current (PCAC). The parameters $\lambda^2$ and $\nu^2$ can be expressed in terms of $f_\pi$, sigma and pion masses as,

$$\lambda^2 = \frac{m^2_\sigma - m^2_\pi}{2f^2_\pi},$$  (4)

$$\nu^2 = f^2_\pi - \frac{m^2_\pi}{\lambda^2}.$$  (5)
III. THE EFFECTIVE HIGHER-ORDER MESONIC POTENTIAL IN THE PRESENCE OF MAGNETIC FIELD

In this section, the higher-order mesonic potential $U_2(\sigma, \pi)$ is employed. In Eq. (6), the effective higher-order mesonic is included with the external magnetic field at zero temperature and density as follows,

$$U_{\text{eff}}(\sigma, \pi) = U_2(\sigma, \pi) + U_{\text{V accum}} + U_{\text{Matter}},$$

where

$$U_2(\sigma, \pi) = \frac{\lambda_1^2}{4} \left( \sigma^2 + \pi^2 - \nu_1^2 \right)^2 + \frac{\lambda_2^2}{4} \left( \left( \sigma^2 + \pi^2 \right)^2 - \nu_2^2 \right)^2$$

$$+ m_\pi^2 f_\pi \sigma.$$  

In Eq. 7, the higher-order mesonic potential satisfies the chiral symmetry when $m_\pi \to 0$ as well as in the standard potential in Eq. 2. Spontaneous chiral symmetry breaking gives a nonzero vacuum expectation for $\sigma$ and the explicit chiral symmetry breaking term in Eq. 7 gives the pion its mass.

$$\langle \sigma \rangle = -f_\pi.$$  

Where

$$\lambda_1^2 = \frac{1}{4f_\pi^2} (m_\pi^2 - m_\pi^2),$$

$$\nu_1^2 = f_\pi^2 - \frac{m_\pi^2}{2\lambda_1^2},$$

$$\lambda_2^2 = \frac{1}{16f_\pi^4} (m_\pi^2 - 3m_\pi^2),$$

$$\nu_2^2 = f_\pi^4 - \frac{m_\pi^2}{4\lambda_2^2 f_\pi^2}.$$  

For details, see Refs. [10,28]. To include the external magnetic field in the present model, we follow Ref. [19]. So, the vacuum energy potential is given by

$$U_{\text{V accum}} = \frac{N_c N_f g^4}{(2\pi)^2} (\sigma^2 + \pi^2)^2 \left( \frac{3}{2} - \ln \left( \frac{g^2(\sigma^2 + \pi^2)}{\Lambda^2} \right) \right),$$

where $N_c = 3$ and $N_f = 2$ are color and flavor degrees of freedom, respectively, and $\Lambda$ is mass scale.

$$U_{\text{Matter}} = \frac{N_c}{2\pi^2} \sum_{f=u}^{d} \left( |q_f| B \right)^2 \zeta^{(1,0)}(-1, x_f) - \frac{1}{2} (a_f^2 - x_f) \ln x_f + \frac{x_f^2}{4}.$$
In Eq. 14, we have used $x_f = \frac{g^2(\sigma^2 + \pi^2)}{2|q_f|B}$ and $\zeta^{(1,0)}(-1, x_f) = \frac{d \zeta(z, x_f)}{dz}\bigg|_{z=-1}$ that represents the Riemann-Hurwitz function, and also $|q_f|$ is the absolute value of quark electric charge in external magnetic field with intense $B$. In Eq. 6, the effect of the finite temperature and chemical potential is not included in the present model, in which the present model focuses on the study of magnetic catalysis at low energy.

IV. DISCUSSION OF RESULTS

In this section, we study the effect of external magnetic field on the symmetry breaking in the presence of higher-order mesonic interactions and also discuss the effect of coupling constant ($g$) and sigma mass ($m_\sigma$) on symmetry breaking. For this purpose, we numerically calculate the effective potential in Eq. (6). The parameters of the present model are the coupling constant $g$ and the sigma mass $m_\sigma$. The choice of free parameters of $g$ and $m_\sigma$ based on Ref. [4]. The parameters are usually chosen so that the chiral symmetry is spontaneously broken in the vacuum and the expectation values of the meson fields. In this work, we consider two different sets of parameters in order to get high and a low value for sigma mass. The first set is given by $\Lambda = 16.48$ MeV which yields $m_\pi = 138$ MeV and $m_\sigma = 600$ MeV. The second set as the first, yielding $m_\sigma = 400$ MeV.

In Fig. 1, the effect external magnetic field is plotted as a function of sigma field at $m_\pi = 0$. Three values of magnetic field ($B$) are taken. At $B = 0$, we note that the spontaneous chiral symmetry breaking is noted, in which chiral limit is satisfied ($m_\pi = 0$). By increasing magnetic field, we note that the curves shift to upper values and the symmetry breaking is clearly appeared in the present model. By increasing magnetic field up to $eB = 0.26 \text{ GeV}^2$. The potential has minima values. Two minima values of potential are larger from their values at zero magnetic field. This indicates that the generation fermion mass increases with strongly increasing magnetic field. Therefore, the catalysis feature is satisfied in the present model and agreement with NJL model [21] and Schwinger-Dyson equation [22, 23].
Fig. 1: The effective mesonic potential is plotted as a function of sigma field for different values of magnetic fields ($B$).

Fig. 2: The effective mesonic potential is plotted as a function of sigma field for the higher-order sigma model and original sigma mode at $eB = 0.128 \text{ GeV}^2$ and $m_\pi = 0$. 
In Fig. 2, effective potential of higher-order mesonic interactions is plotted as a function of sigma field at $eB = 0.128 \, GeV^2$. We note that qualitative agreement between the higher-order sigma model and the original sigma model. The curve of the original model shifts to lower values by including higher-order mesonic interactions, leading a small change in minima values of potential which gives generation fermion mass. The effective potential has two minima values of sigma field in the present model and the original sigma model. This means that the phase is remain unchanged by including higher-order mesonic interactions in the present of external magnetic field. This agreement with the logarithmic sigma model [28].

Fig. 3: The effective mesonic potential is plotted as a function of sigma field for the present model and the original sigma model in at $eB = 0.128 \, GeV^2$ and $m_\pi = 0.14 \, GeV$
Fig. 4: The effective mesonic potential is plotted as a function of sigma field for different values of sigma mass at $eB = 0.128$ GeV$^2$ and $m_\pi = 0$.

In Fig. 3, the effective potential is plotted as a function of sigma field where the explicit symmetry breaking is included ($m_\pi = 0.14$ GeV) at $eB = 0.128$ GeV$^2$. The explicit symmetry breaking is clearly appeared in the higher-order sigma model in comparison with the original sigma model. In addition, the phase transition is changed from first-order to crossover in the present model since the pionic mass with magnetic field are included.

It is important to investigate the effect of the role of the free parameters on the phase transition and symmetry breaking. In Fig. 4, the effective potential is plotted as a function of sigma field for two values of sigma mass. We note the curve shifts to lower values by decreasing sigma mass and the minima values of the potential are larger in comparison their values at $m_\sigma = 600$ MeV. Thus, the generated mass by symmetry breaking increases. In addition, the phase transition is remained unchanged by changing sigma mass as first-order. In Fig. 5, the effect of coupling constant ($g$) is drawn. By increasing coupling constant ($g$), the curve shifts to lower values with smaller two minima values of potential in comparison with their values at $g = 4.5$. Therefore, the generated mass is little changed by changing
coupling constant $g$ in the present of strong magnetic field.

![Effective mesonic potential plot](image.png)

Fig. 5: The effective mesonic potential is plotted as a function of sigma field for different values of coupling constant $g$ at $eB = 0.128$ GeV$^2$ and $m_\pi = 0$.

V. SUMMARY AND CONCLUSION

In this work, we have employed the higher-order mesonic potential to study its effect on the chiral symmetry breaking in the presence of magnetic field. So, the novelty of this work, that the effect of the magnetic field is not investigated in the framework of higher-order sigma model. This model applied only to calculate the hadron properties at low energy without considering magnetic field as in Refs. [10, 28]. In addition, the effect of free parameters of the model is studied in the dense of magnetic field. A comparison with other models is presented, showing the obtained results are in the qualitative agreement with other models.

Therefore, the chiral higher-order sigma model successfully describes the magnetic catalysis. In addition, the present study shows that the parameters of the model play an important role in magnetic catalysis phenomenon. We hope to extend the present model to finite temperature and chemical potential which play an important role for studying properties of the universe and neutron star.
VI. REFERENCES

1. V. S. Timoteo and C. L. Lima, Physics Letters B 635, 168 (2006).
2. M. Gell-Mann and M. Levy, Nuovo Cinmento 16, 705 (1960).
3. M. Birse and M. Banerjee, Phys. Rev. D 31, 118 (1985).
4. B. Golli and M. Rosina, Phys. Lett. B 165, 347 (1985).
5. K. Goeke, M. Harvey, F. Grummer, and J. N. Urbano, Phys. Rev. D 37, 754 (1988).
6. T. S. T. Aly, J. A. McNeil, and S. Pruess, Phys. Rev. D 60, 1114022 (1999).
7. W. Broniowski and B. Golli, Nucl. Phys. A 714, 575 (2003).
8. M. Abu-Shady, Int. J. Mod. Phys. A 26, 235 (2011).
9. M. Abu-Shady, Int. J. Theor. Phys. 48, 1110 (2012).
10. M. Abu-Shady, Phys. Atom. Nuclei 73, 944 (2009).
11. N. Bilic and Nikolic, Eur. Phys. J. C 6, 515 (1999).
12. O. Scanvenius, A. Moscsy, I. N. Mishustin, and D. H. Rischke, Phys. Rev. C 64, 045202 (2001).
13. H. Mao, T.-Z. Wei and J.-S. Jin, Phys. Rev. C 88, 035201 (2013).
14. M. Abu-Shady, Int. J. Mod. Phys. E 21, 1250061 (2012).
15. M. Abu-Shady, Int. J. Theor. Phys. 49, 2425 (2010).
16. D. Kharzeev, K. Landsteiner, A. Schmitt, and H. -U Yee, "Strongly interacting matter in magnetic fields", Springer, 624, 117 (2013).
17. B. S. Kandemir and A. Mogulkoc, Phys. Lett. 379, 2120 (2015).
18. K. Kamikado and t. Kanazawa, JHEP 1, 129 (2015).
19. G. N. Ferrari, A. F. Garcia, and M. B. Pinto, Phys. Rev. D 86, 096005 (2012).
20. A. Goyal and M. Dahiya, Phys. Rev. D 62, 025022 (2011).

21. S. P. Klevansky and R. H. Lemmar, Phys. Rev. D 39, 3478 (1989).

22. I. A. Shushpanov and A. V. Smilga, Phys. Lett. B 16, 402 (1997).

23. I. A. Shushpanov and A. V. Smilga, Phys. Lett. B 16, 351 (1997).

24. H. Suganuma and T. Tastsumi, Annals. Phys. 208, 470 (1991).

25. K. G. Klimenko and T. Mat. Fiz. 89, 211 (1991).

26. V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, Phy. Rev. Lett. 73, 3499 (1994).

27. A. Feijoo, V. K. Magas, A. Ramos, and E. Oset, Phys. Rev. D 92, 076015 (2015).

28. M. Abu-Shady and M. Soleiman, Phys. Part. and Nucl. Lett. 10, 683 (2013).

29. M. Abu-Shady, Appl. Math. Inf. Sci. Lett. 4, 5 (2016).