Spin and Resonant States in QCD

M. Kirchbach

Instituto de Física, Universidad Autónoma de San Luis Potosí, Av. Manuel Nava 6, San Luis Potosí, S.L.P. 78240, México

Abstract. I make the case that the nucleon excitations do not exist as isolated higher spin states but are fully absorbed by \((\frac{3}{2}, \frac{3}{2}) \otimes (\frac{1}{2}, 0) \oplus (0, \frac{1}{2})\) multiplets taking their origin from the rotational and vibrational excitations of an underlying quark–diquark string. The \(\Delta(1232)\) spectrum presents itself as the exact replica (up to \(\Delta(1600)\)) of the nucleon spectrum with the \(K\)- clusters being shifted upward by about 200 MeV. QCD inspired arguments support legitimacy of the quark-diquark string. The above \(K\) multiplets can be mapped (up to form-factors) onto Lorentz group representation spaces of the type \(\psi_{\mu_1...\mu_K}\), thus guaranteeing covariant description of resonant states. The quantum \(\psi_{\mu_1...\mu_K}\) states are of multiple spins at rest, and of undetermined spins elsewhere.

SPECTRA OF LIGHT-QUARK BARYONS

Understanding the spectrum of the most simplest composite systems has always been a key point in the theories of the micro-world. Recall that quantum mechanics was established only after the successful description of the experimentally observed regularity patterns (such like the Balmer- series) in the excitations of the hydrogen atom. Also in solid state physics, the structure of the low–lying excitations, be them without or with a gap, has been decisive for unveiling the dynamical properties of the many-body system–ferromagnet versus superconductor, and the relevant degrees of freedom, magnons versus Cooper pairs. In a similar way, the regularity patterns of the nucleon excitations are decisive for uncovering the relevant subnucleonic degrees of freedom and the dynamical properties of the theory of strong interaction– the Quantum Chromo- Dynamics.

Despite its long history, amazingly, the structure of the nucleon spectrum is far from being settled. This is due to the fact that the first facility that measured nucleon levels, the Los Alamos Meson Physics Facility (LAMPF) failed to find all the states that were predicted by the excitations of three quarks. Later on, the Thomas Jefferson National Accelerator Facility (TJNAF) was designed to search (among others) for those “missing resonances”. At present, all data have been collected and are awaiting evaluation [1].

In a series of papers [2] a new and subversive look on the reported data in Ref. [3] was undertaken. There I drew attention to the “Come–Together” of resonances of different spins and parities to narrow mass bands in the nucleon spectrum and, its exact replica in the \(\Delta(1232)\) spectrum (see Fig. 1).

The first group of states consists of two spin-\(\frac{3}{2}\) states of opposite parities and a single spin-\(\frac{5}{2}^-\). The second group has three parity degenerate states with spins varying from \(\frac{1}{2}^{\pm}\) to \(\frac{5}{2}^{\pm}\), and a single spin-\(\frac{7}{2}^+\) state. Finally, the third group has five parity degenerate
states with spins ranging from $\frac{1}{2}^+ \text{ to } \frac{9}{2}^+$, and a single spin $\frac{11}{2}^+$ state (see Ref. [6] for the complete $N$ and $\Delta(1232)$ spectra). A comparison between the $N$ and $\Delta(1232)$ spectra shows that they are identical up to two “missing” resonances on the nucleon side (these are the counterparts of the $F_{37}$ and $H_{311}$ states of the $\Delta$ excitations) and up to three “missing” states on the $\Delta$ side (these are the counterparts of the nucleon $P_{11}$, $P_{13}$, and $D_{13}$ states from the third group). The $\Delta(1600)$ resonance which is most probably and independent hybrid state, is the only state that at present seems to drop out of our systematics.

The existence of identical nucleon- and $\Delta$ crops of resonances raises the question as to what extent are we facing here a new type of symmetry which was not anticipated by any model or theory before. The next section devotes itself to answering this question.

**QUARK–DIQUARK STRING EXCITATIONS**

Baryons in the quark model are considered as constituted of three quarks in a color singlet state. It appears naturally, therefore, to undertake an attempt of describing the baryonic system by means of algebraic models developed for the purposes of triatomic molecules, a path already pursued by Refs. [7].

In the dynamical limit $U(7) \rightarrow U(3) \times U(4)$ of the three quark system, two of the quarks reveal a stronger pair correlation to a diquark (Dq), while the third quark (q) acts as a spectator. The diquark approximation [8] turned out to be rather convenient in particular in describing various properties of the ground state baryons [9], [10]. Within the context of the quark–diquark (q-Dq) model, the ideas of the rovibron model, known from the spectroscopy of diatomic molecules [11], can be applied to the description of the rotational-vibrational (rovibron) excitations of the q–Dq system.

Rovibron model for the quark–diquark system. In the rovibron model (RVM) the relative q–Dq motion is described by means of four types of boson creation operators $s^+, p_1^+, p_0^+, p_{-1}^+$. The operators $s^+$ and $p_m^+$ in turn transform as rank-0, and rank-1 spherical tensors, i.e. the magnetic quantum number $m$ takes in turn the values $m = 1$, 0, and $-1$. In order to construct boson-annihilation operators that also transform as spherical tensors, one introduces the four operators $\tilde{s} = s$, and $\tilde{p}_m = (-1)^m p_{-m}$. Constructing rank-$k$ tensor product of any rank-$k_1$ and rank-$k_2$ tensors, say, $A_{m_1}^{k_1}$ and $A_{m_2}^{k_2}$, is standard and given by

$$[A^{k_1} \otimes A^{k_2}]^k_{m} = \sum_{m_1,m_2} (k_1m_1k_2m_2|km)A^{k_1}_{m_1}A^{k_2}_{m_2}.$$  

Here, $(k_1m_1k_2m_2|km)$ are the standard $O(3)$ Clebsch-Gordan coefficients.

Now, the lowest states of the two-body system are identified with $N$ boson states and are characterized by the ket-vectors $|n_sn_plm\rangle$ (or, a linear combination of them) within a properly defined Fock space. The constant $N = n_s + n_p$ stands for the total number of $s$- and $p$-bosons and plays the rôle of a parameter of the theory. In molecular physics, the parameter $N$ is usually associated with the number of molecular bound states. The group symmetry of the rovibron model is well known to be $U(4)$. The fifteen generators
of the associated $su(4)$ algebra are determined as the following set of bilinears

$$A_{00} = s^+ \tilde{s}, \quad A_{0m} = s^+ \tilde{p}_m, \quad A_{m0} = p^+_m \tilde{s}, \quad A_{mm'} = p^+_m \tilde{p}_{m'}.$$  \hspace{1cm} (2)

The $u(4)$ algebra is then recovered by the following commutation relations

$$[A_{\alpha \beta}, A_{\gamma \delta}] = \delta_{\beta \gamma} A_{\alpha \delta} - \delta_{\alpha \delta} A_{\gamma \beta}.$$ \hspace{1cm} (3)

The operators associated with physical observables can then be expressed as combinations of the $u(4)$ generators. To be specific, the three-dimensional angular momentum takes the form

$$L_m = \sqrt{2} [p^+ \otimes \tilde{p}]_m^1.$$ \hspace{1cm} (4)

Further operators are $(D_m)$– and $(D'_m)$ defined as

$$D_m = [p^+ \otimes \tilde{s} + s^+ \otimes \tilde{p}]_m^1, \quad D'_m = i[p^+ \otimes \tilde{s} - s^+ \otimes \tilde{p}]_m^1,$$ \hspace{1cm} (5)

respectively. Here, $\vec{D}$ plays the rôle of the electric dipole operator.

Finally, a quadrupole operator $Q_m$ can be constructed as

$$Q_m = [p^+ \otimes \tilde{p}]_m^2, \quad \text{with} \quad m = -2, \ldots, +2.$$ \hspace{1cm} (7)

The $u(4)$ algebra has the two algebras $su(3)$, and $so(4)$, as respective sub-algebras. The $so(4)$ sub-algebra of interest here, is constituted by the three components of the angular momentum operator $L_m$, on the one side, and the three components of the operator $D'_m$, on the other side. The chain of reducing $U(4)$ down to $O(3)$

$$U(4) \supset O(4) \supset O(3),$$ \hspace{1cm} (8)

corresponds to an exactly soluble RVM limit. The Hamiltonian of the RVM in this case is constructed as a properly chosen function of the Casimir operators of the algebras of the subgroups entering the chain. For example, in case one approaches $O(3)$ via $O(4)$, the Hamiltonian of a dynamical $SO(4)$ symmetry can be cast into the form [12]:

$$H_{RVM} = H_0 - f_1 (4\mathcal{C}_2 (so(4)) + 1)^{-1} + f_2 \mathcal{C}_2 (so(4)).$$ \hspace{1cm} (9)

The Casimir operator $\mathcal{C}_2 (so(4))$ is defined accordingly as

$$\mathcal{C}_2 (so(4)) = \frac{1}{4} \left( \vec{L}^2 + \vec{D}'^2 \right)$$ \hspace{1cm} (10)

and has an eigenvalue of $\frac{K}{2} (\frac{K}{2} + 1)$. Here, the parameter set has been chosen as

$$H_0 = M_{N/\Delta} + f_1, \quad f_1 = 600 \text{ MeV}, \quad f_2^N = 70 \text{ MeV}, \quad f_2^A = 40 \text{ MeV}.$$ \hspace{1cm} (11)

Thus, the $SO(4)$ dynamical symmetry limit of the RVM picture of baryon structure motivates existence of quasi-degenerate resonances gathering to crops in both the nucleon-
and Δ baryon spectra. The Hamiltonian that will fit masses of the reported cluster states is exactly the one in Eq. (9).

In order to demonstrate how the RVM applies to baryon spectroscopy, let us consider the case of q-Dq states associated with \( N = 5 \) and for the case of a \( SO(4) \) dynamical symmetry. It is of common knowledge that the totally symmetric irreps of the \( u(4) \) algebra with the Young scheme \([N]\) contain the \( SO(4) \) irreps \( (K, K) \) (here \( K \) plays the role of the four-dimensional angular momentum) with

\[ K = N, N-2, ..., 1 \quad \text{or} \quad 0. \quad (12) \]

Each one of the \( K \)-irreps contains \( SO(3) \) multiplets with three dimensional angular momentum

\[ l = K, K-1, K-2, ..., 1, 0. \quad (13) \]

In applying the branching rules in Eqs. (12), (13) to the case \( N = 5 \), one encounters the series of levels

\[ K = 1: \quad l = 0, 1; \]
\[ K = 3: \quad l = 0, 1, 2, 3; \]
\[ K = 5: \quad l = 0, 1, 2, 3, 4, 5. \quad (14) \]

The parity carried by these levels is \( \eta(-1)^l \) where \( \eta \) is the parity of the relevant vacuum. In coupling now the angular momentum in Eq. (14) to the spin-\( \frac{1}{2} \) of the three quarks in the nucleon, the following sequence of states is obtained:

\[ K = 1: \quad \eta J^\pi = \frac{1^+}{2}, \frac{1^-}{2}, \frac{3^-}{2}; \]
\[ K = 3: \quad \eta J^\pi = \frac{1^+}{2}, \frac{1^-}{2}, \frac{3^-}{2}, \frac{3^+}{2}, \frac{5^-}{2}, \frac{5^-}{2}, \frac{7^-}{2}; \]
\[ K = 5: \quad \eta J^\pi = \frac{1^+}{2}, \frac{1^-}{2}, \frac{3^-}{2}, \frac{3^+}{2}, \frac{5^-}{2}, \frac{5^-}{2}, \frac{7^-}{2}, \frac{7^+}{2}, \frac{9^-}{2}, \frac{11^-}{2}. \quad (15) \]

Therefore, rovibron states of half-integer spin transform according to \( (\frac{K}{2}, \frac{K}{2}) \otimes [(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})] \) representations of \( SO(4) \). The isospin structure is accounted for pragmatically through attaching to the K–clusters an isospin spinor \( \chi^I \) with \( I \) taking the values \( I = \frac{1}{2} \) and \( I = \frac{3}{2} \) for the nucleon, and the Δ states, respectively. As illustrated by Fig. 1, the above quantum numbers cover both the nucleon and the Δ excitations. The states in Eq. (15) are degenerate and the dynamical symmetry is \( SO(4) \).

**Observed and “missing” resonance clusters within the rovibron model.** The comparison of the states in Eq. (15) with the reported ones in Fig. 1 shows that the predicted sets are in agreement with the characteristics of the non-strange baryon excitations with masses below \(~ 2500 \text{ MeV} \), provided, the parity \( \eta \) of the vacuum changes from scalar (\( \eta = 1 \)) for the \( K = 1 \), to pseudoscalar (\( \eta = -1 \)) for the \( K = 3, 5 \) clusters. A pseudoscalar “vacuum” can be modeled in terms of an excited composite diquark carrying an internal angular momentum \( L = 1^- \) and maximal spin \( S = 1 \). In one of the possibilities the total spin of such a system can be \( |L-S| = 0^- \). To explain the properties
of the ground state, one has to consider separately even \(N\) values, such as, say, \(N' = 4\). In that case another branch of excitations, with \(K = 4, 2, \) and 0 will emerge. The \(K = 0\) value characterizes the ground state. \(K = 2\) corresponds to \((1, 1) \otimes [(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})]\), while \(K = 4\) corresponds to \((2, 2) \otimes [(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})]\). These are the multiplets that we will associate with the “missing” resonances predicted by the rovibron model. In this manner, reported and “missing” resonances fall apart and populate distinct \(U(4)\)- and \(SO(4)\) representations. In making observed and “missing” resonances distinguishable, reasons for their absence or, presence in the spectra are easier to be searched for. In accordance with Ref. [13] we here will treat the \(N = 4\) states to be all of natural parities and identify them with the nucleon \((K = 0)\), the natural parity \(K = 2\), and the natural parity \(K = 4\)–clusters. We shall refer to the latter as ‘missing’ rovibron clusters. In Table I we list the masses of the \(K\)–clusters concluded from Eqs. (9), and (11).

**Spin and quark–diquark in QCD.** The necessity for having a quark–diquark configuration within the nucleon follows directly from QCD arguments. In Refs. [14], and [15] the notion of spin in QCD was re-visited in connection with the proton spin puzzle. As it is well known, the spins of the valence quarks are by themselves not sufficient to explain the spin-\(\frac{1}{2}\) of the nucleon. Rather, one needs to account for the orbital angular momentum of the quarks (here denoted by \(L_{QCD}\)) and the angular momentum carried by

**FIGURE 1.** Summary of the data on the nucleon and the \(\Delta\) resonances. The breaking of the mass degeneracy for each of the clusters at about 5\% may in fact be an artifact of the data analysis, as has been suggested by Höhler [4]. The filled circles represent known resonances, while the sole empty circle corresponds to a prediction. Figure taken from [5].
TABLE 1. Predicted mass distribution of observed (obs), and missing (miss) rovibron clusters (in MeV) according to Eqs. (9,11). The sign of $\eta$ in Eq. (15) determines natural- ($\eta = +1$), or, unnatural ( $\eta = -1$) parity states. The experimental mass averages of the resonances from a given K–cluster have been labeled by “exp”.

| $K$ | sign $\eta$ | $N^{\text{obs}}$ | $N^{\text{exp}}$ | $\Delta^{\text{obs}}$ | $\Delta^{\text{exp}}$ | $N^{\text{miss}}$ | $\Delta^{\text{miss}}$ |
|-----|-------------|-----------------|-----------------|----------------|----------------|----------------|----------------|
| 0   | +           | 939             | 939             | 1232           | 1232           | 1612           | 1846           |
| 1   | +           | 1441            | 1498            | 1712           | 1690           | 1935           | 2048           |
| 2   | +           | 1612            | 1846            |                |                |                |                |
| 3   | -           | 1764            | 1689            | 1944           | 1922           | 2135           | 2102           |
| 4   | +           | 1935            | 2048            |                |                |                |                |
| 5   | -           | 2135            | 2102            | 2165           | 2276           |                |                |

the gluons (so called field angular momentum, $G_{\text{QCD}}$):

\[
\frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_{\text{QCD}} + G_{\text{QCD}} = \int d^3x \left[ \frac{1}{2} \bar{\psi} \gamma_5 \psi + \psi^i (\vec{x} \times (-\vec{D})) \psi + \bar{x} \times (\vec{E}^a \times \vec{B}^a) \right].
\]

In so doing one encounters the problem that neither $L_{\text{QCD}}$, nor $G_{\text{QCD}}$ satisfy the spin $su(2)$ algebra. If at least $(L_{\text{QCD}} + G_{\text{QCD}})$ is to do so,

\[
\left[ (L^i_{\text{QCD}} + G^i_{\text{QCD}}), (L^j_{\text{QCD}} + G^j_{\text{QCD}}) \right] = i \epsilon^{ijk} \left( L^k_{\text{QCD}} + G^k_{\text{QCD}} \right),
\]

then $\vec{E}^{ia}$ has to be restricted to a chromo-electric charge, while $\vec{B}^{ia}$ has to be a chromo-magnetic dipole according to,

\[
E^{ia} = \frac{g x^i}{\rho^3 T^a}, \quad B^{ia} = \left( \frac{3 x^i x^l m^l}{\rho^3} - \frac{m^i}{\rho^3} \right) T^a,
\]

where $x^i = x^i - R^i$. The above color fields are the perturbative one-gluon approximation typical for a diquark-quark structure. The diquark and the quark are in turn the sources of the color Coulomb field, and the color magnetic dipole field. In terms of color and flavor degrees of freedom, the nucleon wave function indeed has the required quark–diquark form $|p_\uparrow\rangle = \frac{\epsilon}{\sqrt{18}} \left[ u^+_i d^+_j \bar{u}^+_{i'} d^+_{j'} - u^+_i d^+_j \bar{u}^+_{j'} d^+_{i'} \right] u^+_k |0\rangle$. A similar situation appears when looking for covariant QCD solutions in form of a membrane with the three open ends being associated with the valence quarks. When such a membrane stretches to a string, so that a linear action (so called gonihedric string) can be used, one again encounters that very $K$-cluster degeneracies in the excitations spectra of the baryons, this time as a part of an infinite tower of states. The result was reported by Savvidy in Ref. [16]. Thus the covariant spin-description provides an independent argument in favor of a dominant quark-diquark configuration in the structure of the nucleon, while search for covariant resonant QCD solutions leads once again to infinite $K$-cluster towers. The
quark-diquark internal structure of the baryon’s ground states is just the configuration, the excited mode of which is described by the rovibron model and which is the source of the \( \left( \frac{K}{2}, \frac{K}{2} \right) \otimes \left[ \left( \frac{1}{2}, 0 \right) \oplus \left( 0, \frac{1}{2} \right) \right] \) patterns.

In Ref. [12] we presented the four dimensional Racah algebra that allows to calculate transition probabilities for electromagnetic de-excitation of the rovibron levels. The interested reader is invited to consult the quoted article for details. Here I restrict myself to reporting the following two results: (i) All resonances from a \( K \)-mode have same widths. (ii) As compared to the natural parity \( K = 1 \) states, the electromagnetic de-excitation of the unnatural parity \( K = 3 \) and \( K = 5 \) rovibron states appear strongly suppressed. To illustrate our predictions I compiled in Table 2 below data on experimentally observed total widths of resonances belonging to \( K = 3 \) and \( K = 5 \). The suppression of the electromagnetic de-excitation modes of unnatural parity states to the nucleon (of natural parity) is shown in Table 3. It is due to the vanishing overlap between the scalar diquark in the latter case, and the pseudo-scalar one, in the former. Non-vanishing widths can signal small admixtures from natural parity states of same spins belonging to even \( K \) number states from the “missing” resonances. For example, the significant value of \( A^{p}_{\frac{5}{2}^{+}} \) for \( N \left( \frac{5}{2}^{+}; 1680 \right) \) from \( K = 3 \) may appear as an effect of mixing with the \( N \left( \frac{5}{2}^{+}; 1612 \right) \) state from the natural parity “missing” cluster with \( K = 2 \). This gives one the idea to use helicity amplitudes to extract “missing” states.

The above considerations show that a \( K \)-mode of an excited quark-diquark string (be the diquark scalar, or, pseudoscalar) represents an independent entity (particle?) in its own rights which deserves its own name. To me the different spin facets of the \( K \)-cluster pointing into different “parity directions” as displayed in Fig. 2 look like barbs. That’s why I suggest to refer to the \( K \)-clusters as barbed states to emphasize the aspect of alternating parity. Barbs could also be associated with thorns (Spanish, espino), and espinons could be another sound name for \( K \)-clusters.

| \( K \) | Resonance | width [in GeV] |
|------|----------|--------------|
| 3    | \( N \left( \frac{1}{2}^{-}; 1650 \right) \) | 0.15 |
| 3    | \( N \left( \frac{1}{2}^{+}; 1710 \right) \) | 0.10 |
| 3    | \( N \left( \frac{3}{2}^{+}; 1720 \right) \) | 0.15 |
| 3    | \( N \left( \frac{3}{2}^{-}; 1700 \right) \) | 0.15 |
| 3    | \( N \left( \frac{5}{2}^{+}; 1675 \right) \) | 0.15 |
| 5    | \( N \left( \frac{3}{2}^{+}; 1900 \right) \) | 0.50 |
| 5    | \( N \left( \frac{5}{2}^{+}; 2000 \right) \) | 0.49 |
TABLE 3. Reported helicity amplitudes of resonances

| K parity of the spin-0 diquark | Resonance | $A_0^p$ | $A_2^3$ [in $10^{-3}$GeV$^{-\frac{1}{2}}$] |
|-------------------------------|------------|---------|-------------------------------------|
| 3 -                           | $N\left(\frac{1}{2}^+;1710\right)$ | 9 ±22   |                                     |
| 3 -                           | $N\left(\frac{3}{2}^+;1720\right)$ | 18±30   | -19±20                              |
| 3 -                           | $N\left(\frac{1}{2}^-;1700\right)$ | -18±30  | -2±24                               |
| 3 -                           | $N\left(\frac{3}{2}^-;1675\right)$ | 19 ±8   | 15±9                                |
| 3 -                           | $N\left(\frac{3}{2}^-;1680\right)$ | -15±6   | 133±12                              |
| 1 +                           | $N\left(\frac{3}{2}^-;1520\right)$ | -24±9   | 166±5                               |

"barbed" states
(espinons)

FIGURE 2. $K$-excitation mode of a quark-diquark string: barbed states (espinons).

CONCLUSIONS

We argued that the Come-Together of several parity degenerate states of increasing spins to $\psi_{\mu_1...\mu_k}$ multiplets can be explained through rotational-vibrational modes of an excited quark-diquark string, be the diquark scalar (in the respective observed $\psi_\mu$, and the “missing” $\psi_{\mu_1\mu_2}$, and $\psi_{\mu_1...\mu_4}$), or pseudoscalar (in the observed $\psi_{\mu_1...\mu_3}$, and $\psi_{\mu_1...\mu_5}$, respectively). Each $K$ state consists of $K$ parity couples and a single unpaired “has–been” spin-$J = K + \frac{1}{2}$ at rest. The parity couples should not be confused with parity doublets. The latter refer to states of equal spins, residing in distinct Fock spaces built on top of opposite parity (scalar, and pseudoscalar) vacua with $\Delta l = 0$. The parity degeneracy observed in the baryon spectra is an artifact of the belonging of
resonances to \( \left( \frac{K}{2}, \frac{K}{2} \right) \otimes \left[ \left( \frac{1}{2}, 0 \right) \oplus \left( 0, \frac{1}{2} \right) \right] \), in which case the opposite parities of equal spins originate from underlying angular momenta differing by one unit, i.e. from \( \Delta l = 1 \). Chiral symmetry realization within the \( K \)-cluster scenario means having coexisting scalar and pseudoscalar diquarks ("vacua"), and consequently coexisting \( K \)-clusters of both natural and unnatural parities. Stated differently, if TJNAF is to supplement the unnatural parity LAMPF "espinons" with \( K=3 \), and 5 by the natural parity \( K=2,4 \) ones, then we will have manifest mode of chiral symmetry in the baryonic spectra. The total number of ordinary-spin states in our scenario needs not be multiple of two, as it should be in case of parity doubling.

**ACKNOWLEDGMENTS**

Marcos Moshinsky and Yuri Smirnov brought their unique expertise into uncovering the quark-diquark dynamics behind the resonance clusters. Work partly supported by Consejo Nacional de Ciencia y Tecnología (CONACyT, Mexico) under grant number 32067-E.

Special thanks to the organizers for having managed such a memorable event. Congratulations to Arnulfo Zepeda and Augusto Garcia to their anniversaries and exemplary careers.

**REFERENCES**

1. V. Burkert, hep-ph/0210321.
2. M. Kirchbach, Mod. Phys. Lett. **A12**, 2373-2386 (1997); Few Body Syst. Suppl. **11**, 47-52 (1999).
3. Particle Data Group, Eur. Phys. J. **C15**, 1 (2000).
4. G. Höhler, in *Pion-Nucleon Scattering* (Springer Publishers, Heidelberg, 1983), Landolt-Börnstein Vol. I/9b2, Ed. H. Schopper.
5. M. Kirchbach, D. V. Ahluwalia, Phys. Lett. **B529**, 124-131 (2002).
6. M. Kirchbach, Nucl. Phys. **A689**, 157c-166c (2001).
7. R. Bijker, F. Iachello, and A. Leviatan, Phys. Rev. **C54**, 1935-1953 (1996); R. Bijker, F. Iachello, and A. Leviatan, Ann. of Phys. **236**, 69-116 (1994).
8. Proc. Int. Conf. *Diquarks 3*, Torino, Oct. 28-30 (1996), eds. M. Anselmino and E. Predazzi, (World Scientific).
9. M. Oettel, R. Alkofer, and L. von Smekal Eur. Phys. J. **A8**, 553-566 (2000).
10. K. Kusaka, G. Piller, A. W. Thomas, and A. G. Williams, Phys. Rev. **D55**, 5299-5308 (1997).
11. F. Iachello, and R. D. Levin, *Algebraic Theory of Molecules* (Oxford Univ. Press, N.Y.) 1992.
12. M. Kirchbach, M. Moshinsky, and Yu. F. Smirnov, Phys. Rev. **D64**, 114005 (2001).
13. M. Kirchbach, Int. J. Mod. Phys. **A15**, 1435-1451 (2000).
14. D. Singleton, Phys. Lett. **B427**, 155-160 (1998).
15. X. Ji, Phys. Rev. Lett. **78**, 610-613 (1997); X. Ji, Phys. Rev. Lett. **79**, 1255-1228 (1997).
16. G. Savvidy, Phys. Lett. **B438**, 69-79 (1998).