Fano interferences in the transport properties of triple quantum dot T-shaped systems

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Abstract. We consider the transport and the noise characteristic in the case of a triple quantum dots T-shaped system where two of the dots form a two-level system and the other works in a detector-like setup. Our theoretical results are obtained using the equation of motion method for the case of zero on-site Coulomb interaction in the detector dot. The transport trough the T-shaped system can be controlled by varying the coupling between the two-level system dots or the coupling between the detector dot and the exterior electrodes. The Fano dips in the system's conductance can be observe both for strong (fast detector) and weak coupling (slow detector) between the detector dot and the external electrodes. Due to stronger electronic correlations the noise in the case of a slow detector are much higher. This setup may be of interest for the practical realization of qubit states in quantum dots systems.

1. Introduction

Transport through a complex quantum dot (QD) system is of high interest both from the practical application and theoretical point of view [1]. Particularly, the Anderson single impurity model [2] was extensively used to understand the electronic correlations in QD systems. The model, in which QD’s are represented by impurities, was successfully applied to the study of single or multiple-dot systems. A triple QD’s T-shape system was recently considered by Tanamoto and Nishi [3]. In this configuration one dot (detector dot) is coupled to the external leads and the other two can be viewed as a two-level system coupled to the detector dot (See Fig. 1). The authors studied the Fano-Kondo effect and found that the Fano dip in the detector’s QD electronic density of states (DOS) is modulated by interactions with electrons in the two strongly coupled QD’s forming the two-level system. The internal coupling between the two QD’s in the two-level system plays an important role in the system’s transport properties. The analysis is done using the slave boson approximation in the absence of an on-site Coulomb interaction for the detector dot.

Here we propose an investigation of the triple QD’s T-shape system based on the equation of motion (EOM) method. We will consider quantities such as the main dot Green’s function, the electronic density of states, the system’s conductance, and the corresponding current noise characteristics.
creation and annihilation operators for electrons with momentum

where the first term describes the free electrons in the leads with

electrons in the lead

combination

\( (\alpha \equiv V, \sqrt{2} V) \)

\( V \)

\( V \).

\( H \) the system’s relative conductance as function of the ratio

\( \frac{E_d}{t_d} \)

for (a) a fast detector (\( \Delta/t_d = 2 \)) and (b) a slow detector

\( \Delta/t_d = 0.4 \)

in the case of a strong coupling

\( t_c/t_d = 5 \) (full line) and a weak coupling

\( t_c/t_d = 1 \) (dashed line).

2. Model

The Hamiltonian for the three QD’s T-shape system (see Fig. 1) can be written based on the

Anderson impurity as:

\[
H = \sum_{k,\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_{k,\sigma} V_{kd;\alpha}(c_{k\sigma}^\dagger d_{\sigma} + d_{\sigma}^\dagger c_{k\sigma;\alpha}) + H_m ,
\]

where the first term describes the free electrons in the leads with \( c_{k\sigma;\alpha} \) and \( c_{k\sigma;\alpha} \) being fermionic creation and annihilation operators for electrons with momentum \( k \) and spin \( \sigma \) in the lead \( \alpha \) (\( \alpha \equiv \text{left (L), right (R)} \)). In the second term, \( V_{kd;\alpha} \) describes the interaction between the free electrons in the lead \( \alpha \) and the localized \( d \)-electrons in the detector dot. For simplicity we will consider \( V_{kdL} = V_{kdR} \), case in which the detector couples to the leads only in the symmetric combination \( c_{k\sigma} = (c_{k\sigma:L} + c_{k\sigma:R})/\sqrt{2} \) and the dot connects effectively to a single lead, with \( V_{kd} = \sqrt{2} V_{kdL} \). The mesoscopic part of the total Hamiltonian, \( H_m \), describes the three QD’s of the system and can be written as

\[
H_m = \sum_{\sigma} E_d d_{\sigma}^\dagger d_{\sigma} + U_d n_{d\sigma} n_{d-\sigma} + \sum_{\sigma} E_b b_{\sigma}^\dagger b_{\sigma} + U_b n_{b\sigma} n_{b-\sigma} + \sum_{\sigma} E_a a_{\sigma}^\dagger a_{\sigma} + U_a n_{a\sigma} n_{a-\sigma} + t_d \sum_{\sigma} (d_{\sigma}^\dagger b_{\sigma} + b_{\sigma}^\dagger d_{\sigma}) + t_c \sum_{\sigma} (b_{\sigma} a_{\sigma} + a_{\sigma}^\dagger b_{\sigma}) .
\]

Here, the first two terms describe the detector QD, \( E_d \) is the energy of the localized electrons from the \( d \)-level, \( U_d \) is the on-site Coulomb interaction, \( d_{\sigma}^\dagger \) and \( d_{\sigma} \) are the creation and annihilation operators for electrons with spin \( \sigma \) in the detector dot, and \( n_{\sigma} = d_{\sigma}^\dagger d_{\sigma} \). The following four terms describe the electrons in the two QD in the two-level system; \( a_{\sigma}^\dagger \) and \( a_{\sigma} \) are the creation and annihilation operators for the electrons localized at the energy level \( E_a \) and \( b_{\sigma}^\dagger \) and \( b_{\sigma} \) are the creation and annihilation operators for the electrons localized at the energy level \( E_b \). The last two terms in the mesoscopic Hamiltonian describe interactions between the localized electrons in the component dots. The strength of these interactions are given by two coupling constants, \( t_d \) and \( t_c \).
The EOM method can be used to find the Green’s functions of the system and thereafter calculate the $d$-electrons DOS. In the case of a general Anderson impurity model, the EOM method leads to an infinite hierarchy of higher-order Green’s functions. A well known approximation rely on truncating this hierarchy, introducing certain averages in the equations chain leading to an approximate form of the system’s Green’s functions. Following Hewson [4] we find

$$[G_{dd}^a(\omega)]^{-1} = \frac{(\omega - E_d)(\omega - E_d - U_d)}{\omega - E_d - U_d(1 - <n_{d-\sigma}>)} - \frac{t^2_d}{\omega - E_a - U_a(1 - <n_{a-\sigma}>)} + i\Delta. \quad (3)$$

Above, $<n_{i\sigma}>$ represents the average occupancy for the spin $\sigma$ electron in the dot $i$ and $\Delta = \pi N(E_F)[V_{kd}]^2$ with $N(E_F)$ the electronic DOS at the Fermi level.

3. Conductance and current noise characteristics

Transport properties in the triple QD’s T-shape system can be discussed in terms of system’s conductance and current noise characteristics. To calculate the system’s conductance we use the Meir and Weingreen formula:

$$G = G_0 \int_{-\infty}^{\infty} d\omega \frac{\Delta}{2} \left[ -\frac{\partial f(\omega)}{\partial \omega} \right] \rho_d(\omega), \quad (4)$$

where $G_0 = 2e^2/h$ ($e$ is the electron charge and $h$ the Plank constant), $\rho_d(\omega)$ is the DOS in the detector dot, and $f(x)$ represent the Fermi-Dirac distribution function. Quantum and thermal fluctuations are very important as they are one of the reasons why quantum correlations are difficult to observe. Such fluctuations can be estimated based on the current noise characteristics. The EOM formalism, which we considered at $T = 0$ K, allows us to estimate the quantum fluctuation for the triple QD’s T-shape system. The current shot noise, $S(V)$, when an external bias $V$ is applied to the detector dot is defined as a correlation function of current fluctuation. It can be proved that is related to the detector dot’s DOS via the transmission function $T(\omega) = \pi \Delta \rho_d(\omega)$ and can be obtain as:

$$S(V) = \frac{4e^2}{h} \int_{-eV/2}^{eV/2} d\omega T(\omega) \left[ 1 - T(\omega) \right]. \quad (5)$$

Also, we can consider the Fano factor $\gamma$ defined as the ratio of the shot noise and the current passing through the system, $\gamma = S(V)/(2eI)$. In the case of uncorrelated electrons $\gamma = 1$. The source current can be calculated as:

$$I_L = \frac{2e}{h} \int_{-\infty}^{\infty} d\omega T(\omega) [f_L(\omega) - f_R(\omega)], \quad (6)$$

where $f_\alpha(\omega) = [\exp \{ (\omega - \mu_\alpha)/k_B T \} + 1]^{-1}$ with $T$ being the temperature and $k_B$ the Boltzmann constant. For a symmetrical bias condition we set $\mu_L = E_F - eV/2$ and $\mu_R = E_F + eV/2$ with $E_F = 0$. Note that in our particular situation the current is conserved, $I_L = I_R$.

4. Discussion

For simplicity we will analyze the simple case with no on-site interaction in the component dots, i.e., $U_d = U_b = U_a = 0$. In Figure 2 we plotted the relative conductance, $G/G_0$, as function of the ratio $E_a/t_d$ for $E_F = 0$, $E_b = E_0$, and $E_d/t_d = -0.2$. One can clearly identify a double peak structure for both the slow and fast detector configurations. The strength of the coupling between the two-level system’s QD’s is responsible for the distance between the two
dip structures. Figure 3 presents the shot noise $S(V)$ (in units of $(e^2t_d/\hbar)$) as a function of the scaled applied bias $eV/t_d$. Figure 3a considers the case of the fast detector configuration for both weak and strong coupling in the two-level system at different values of the electron level in the detector dot. The value of the electron levels in the two QD’s of the two-level system is fixed at $E_a = E_b = 0$ and their value do not influence strongly the current shot noise. Figure 3b considers the case of a slow detector under the same conditions. One can clearly see that the shot noise is higher in the case of a slow detector as a result of the stronger coupling of the flowing electrons with the localized electrons in the triple QD’s T-shape system. In Figure 4 we plotted the Fano factor as a function of the applied external bias. Figure 4a considers the case of the fast detector configuration for both weak and strong coupling in the two-level system at different values of the electron level in the detector dot ($E_a = E_b = E_F = 0$). Figure 4b presents the case of a slow detector under the same conditions. There are major differences between the Fano factor for the cases of fast and slow detector, i.e., the factor is significantly larger in the second case. Such a behavior is expected for two reasons. First, we already seen that the shot noise is higher in the case of slow detectors. Second, the current in the case of a fast detector is higher due to smaller electronic correlations in the system. In general, one can conclude that a higher current noise is associated to stronger electronic correlations in the system.

In conclusion we presented an EOM analysis of the triple quantum dot T-shape system. Our analysis considered various coupling strengths in between the system’s component dots. We identify two main configurations based on the strength of the electronic coupling between the detector dot and the external electrodes, namely, the case of a fast detector characterized by a strong coupling, and the case of a slow detector characterized by a weak coupling.

References
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