Dynamic Modeling of Composite Boring Bars Considering Different Boundary Conditions

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Abstract. Boring bar for deep hole cutting is often more prone to flutter problems. The traditional boring bar is generally made of metal material. There is a close relationship between the dynamic characteristics of the boring bar and the tightness of the fixed end. In this paper, based on Euler-Bernoulli beam theory, the distributed parametric differential equations of composite boring bar are presented. The dynamic characteristics of composite boring bar with three different conditions including fixed-free, pinned-pinned-free and spring-spring-free are investigated. The corresponding frequency equations are given. The natural frequency and dynamic stiffness of the composite boring bar are obtained by numerical calculation. The influences of the supporting conditions and the ply angle are studied. The results show that the supporting conditions, layer angle, support spring stiffness and support spacing have an important effect on the dynamic characteristics of the composite boring bar.

1. Introduction
The internal surface operation of machine components is one of the most chatter-prone metal cutting operations. The reason is that the slender boring bars are usually required in cutting processes. A slender boring bar is generally the weakest link in a machine tool system. High levels of boring bar chatter result in poor surface finish of workpiece, excessive tool wear and tool break. Thus, boring bar chatter has a negative impact on productivity. The chatter is a self-excited vibration which involves low-order bending modes of the boring bar and is dominated by the bending mode in the cutting speed direction. High speed cutting operations using steel boring bars are often hindered by the chatter vibration of boring bars due to the low dynamic stiffness and low natural frequencies of steel boring bars.

Due to its high strength, stiffness, light weight and fatigue resistance, composite materials can be used to enhance the dynamic stiffness and fundamental natural frequency of boring bars. Nagano et al. [1] presented a carbon fiber/epoxy composite boring bar and investigated the effects of steel core shape on the bending stiffness and the natural frequency of the composite boring bars by FEM analysis.

Lee et al. [2-3] designed and manufactured a composite boring bar. The dynamic characteristics of the composite boring bar were experimentally determined with respect to material types and dimensions of the boring bar through vibration tests. A number of analysis models and methods concerning the cutting processes with boring bar and the behavior of the dynamic system has been developed.

Zhang et al. [4] derived analysis model from a two-degree-of-freedom model of a clamped boring bar and four cutting force components. Rao et al. [5] produced a continuous system model of boring dynamics based on a dynamic boring force model, including variation of chip cross-sectional area, and
a uniform Euler–Bernoulli cantilever beam. Andren et al. [6] used time series approach to investigate boring bar chatter and the results were compared with an analytical Euler–Bernoulli model. Scheuer et al. [7] investigated the dynamic properties of a boring bar with different clamping housings based on experimental modal analysis. Their study indicates that different clamping conditions using a clamping housing with clamp screws may affect the fundamental bar bending modes slightly. As the above review indicates, it appears like the most of the continuous system model of boring bar is aimed at conventional metal boring bar. When it comes to analytical investigations concerning the clamping properties influence on the dynamic properties of a clamped composite boring bar, it appears that little work has been done.

In this paper, the dynamical behaviors of the anisotropic composite boring bar with different clamping conditions are studied. Firstly, the Euler–Bernoulli beam theory for the modeling of a composite boring bar with clamped-free boundary conditions is used. To take into account the flexibility of clamping, two-span composite boring bar models with the boundary conditions of pinned-pinned-free and spring-spring-free are studied.

The natural frequency and dynamic stiffness of the composite boring bar are obtained by numerical calculation. The influences of the boundary conditions, the ply angle and support spacing are studied. The results show that the boundary conditions, ply angle, spring stiffness and support spacing have an important effect on the dynamic characteristics of the composite boring bar.

2. Dynamical model of composite boring bar

2.1. Motion equations and clamped conditions

The clamped composite boring bar can be modeled by an Euler-Bernoulli model with clamped-free boundary conditions(C-F), which consists of a homogeneous single span beam with constant cross-sectional area $A$ and constant cross-sectional moment of inertia $I_x$, see Figure 1.

The differential equation describing the transversal vibration of the slender composite boring bar which ignores the effects of shear deformation and rotary inertia is given by

$$
\rho A \frac{\partial^2 w(z,t)}{\partial t^2} + \frac{\partial^2}{\partial z^2} 
\left[ \frac{EI_x}{12} \frac{\partial^2 w(z,t)}{\partial t^2} \right] = f(z,t)
$$

where $EI_x$ is flexural stiffness and $\rho A$ is mass per unit length of the composite boring bar and $f(z,t)$ is the external force per unit length.

For composite boring bar, $EI_x$ and $\rho A$ can be obtained as follows

$$
EI_x = \frac{\pi}{4} \bar{Q}_{11} \left( r_{N+1}^4 - r_1^4 \right)
$$

$$
\rho A = \pi \rho \left( r_{N+1}^2 - r_1^2 \right)
$$

where, $\bar{Q}_{11}$ is the off-axial stiffness coefficient of composite boring bar, $r_1$ and $r_N$ are the inner and outer radii of the layered composite boring bar.

In reality, the composite boring bar may be clamped with either two screws on the top and two on the underside. If the screw is assumed to be a deformable elastic body, and only its tension rigidity is considered, then elastic supports boundary conditions can be yielded. The model with the elastic support of two springs (S-S-F) is presented in Figure 2. If the clamping housing and the screw are seen to be rigid body, then the pinned boundary conditions can be yielded which assumes an infinitely stiff spring. In fact, if we let the spring coefficient $k_f$ go to infinity, we will obtain the model with the pinned-pinned-free boundary conditions (P-P-F), see Figure 3.

The boundary conditions model corresponding to the C-F, S-S-F and P-P-F can be expressed as

A. The boundary conditions of C-F
\[ w(z,t)_{|z=0} = 0, \frac{\partial w(z,t)}{\partial z}_{|z=0} = 0 \]  
\[ EI_s \frac{\partial^2 w(z,t)}{\partial z^2}_{|z=0} = 0, \frac{\partial}{\partial z} \left[ EI_s \frac{\partial^2 w(z,t)}{\partial z^2} \right]_{|z=0} = 0 \]  
\[ EI_s \frac{\partial^2 w(z,t)}{\partial z^2} = 0, EI_s \frac{\partial^2 w(z,t)}{\partial z^2} = 0 \]

B. The boundary conditions of P-P-F
\[ w(z,t)_{|z=0} = 0, EI_s \frac{\partial^2 w(z,t)}{\partial z^2} = 0 \]  
\[ w(z,t)_{|z=l_c} = 0, EI_s \frac{\partial^2 w(z,t)}{\partial z^2} = 0 \]

C. The boundary conditions of S-S-F
\[ EI_s \frac{\partial^3 w(z,t)}{\partial z^3} = -K_T w(z,t)_{|z=0} \]  
\[ EI_s \frac{\partial^3 w(z,t)}{\partial z^3} = \left[ EI_s \frac{\partial^3 w(z,t)}{\partial z^3} - K_T w(z,t) \right]_{|z=l_c} \]

where, \( w(z,t) \) is the transverse displacement for part of the boring bar with the length \( l_c \).
\[ EI_s \frac{\partial^2 w(z,t)}{\partial z^2} = 0, \frac{\partial}{\partial z} \left[ EI_s \frac{\partial^2 w(z,t)}{\partial z^2} \right]_{|z=l_c} = 0 \]

Figure 1. Composite boring bar with fixed-free boundary condition.

Figure 2. Composite boring bar with spring-spring-free boundary condition.
2.2. Solution method of natural frequency

2.2.1. C-F composite boring bar
In order to calculate the natural frequency of composite boring bar from Eq. (1), we let \( f(z,t) = 0 \). The solution of Eq. (1) can be obtained by using the separation-of-variables procedure.

The frequency equation for the composite boring bar with C-F boundary conditions is

\[
\cos \beta l \cosh \beta l = -1
\]

(11)

There are solutions for \( \beta_n l \), \( n=1,2,3... \), to Eq. (11). The corresponding natural frequencies \( f_n \) are given by

\[
f_n = \left( \frac{2\pi f_n}{\rho A} \right)^{1/4} \frac{\rho A}{EI} \]

(12)

2.2.2. P-P-F composite boring bar
By using Eq. (1) and boundary conditions (6) and (7), the frequency equation for the composite boring bar with P-P-F boundary conditions is derived as follows \[9\]

\[
2\sin(\beta l_c)\sinh(\beta l_c) + 2\sin(\beta l_o)\sinh(\beta l_o)\cos(\beta l)\cosh(\beta l) - 2\sin(\beta l_l)\sinh(\beta l_l)\sin(\beta l)\cosh(\beta l) + 2\sin(\beta l_c)\sinh(\beta l_c)\cos(\beta l)\cosh(\beta l) + \sin(\beta l_l)\sinh(\beta l_l)\sin(\beta l)\cosh(\beta l) - \cos(\beta l_c)\sinh(\beta l_c)\cos(\beta l)\cosh(\beta l) = 0
\]

(13)

By solving the Eq. (13), \( \beta_n l_c \) and \( \beta_n l_l \), \( n=1,2,... \), can be obtained. Thus, the natural frequencies are yielded based on \( \beta_n l_c \) and \( \beta_n l_l \).

2.2.3. S-S-F model
Based on transfer matrix method \[10\], the natural frequencies for the composite boring bar with S-S-F boundary conditions can be solved.

By means of the separation-of-variables procedure and considering the compatibility requirement including boundary conditions (8), (9) and (10), the frequency equation for the composite boring bar with S-S-F boundary conditions can be expressed as

\[
\begin{bmatrix}
0 \\
0
\end{bmatrix} = \begin{bmatrix}
r_{11} + r_{13} & r_{12} + r_{14} & A_l \\
r_{21} + r_{23} & r_{22} + r_{24}
\end{bmatrix} \begin{bmatrix}
A_l \\
B_l
\end{bmatrix}
\]

(14)

where, \( r_{ij} \) is element of the following 2\( \times \)4 matrix

\[
T_{2\times4} U_{2\times4} \cdots U_{2\times4} = \begin{bmatrix}
r_{11} & r_{12} & r_{13} & r_{14} \\
r_{21} & r_{22} & r_{23} & r_{24}
\end{bmatrix}
\]

(15)
\( U'_{4:4} \) \((i=1,2,\ldots,k)\) are \(4 \times 4\) transfer matrix, its elements may be written as

\[
u_{11} = \left( \cos(\eta L, \beta) + \kappa_i \lambda_i^2 \cos(\eta L, \beta) \right) / 2
\]

\[
u_{12} = \left( \sin(\eta L, \beta) + \kappa_i \lambda_i^2 \sin(\eta L, \beta) \right) / 2
\]

\[
u_{13} = \left( \cosh(\eta L, \beta) - \kappa_i \lambda_i^2 \cosh(\eta L, \beta) \right) / 2
\]

\[
u_{14} = \left( \sinh(\eta L, \beta) - \kappa_i \lambda_i^2 \sinh(\eta L, \beta) \right) / 2
\]

\[
u_{21} = \left( \frac{S_i \cos(\eta L, \beta)}{(EI)_{i+1}(\eta_{i+1})} - (\lambda_i + \kappa_i \lambda_i) \sin(\eta L, \beta) \right) / 2
\]

\[
u_{22} = \left( \frac{S_i \sin(\eta L, \beta)}{(EI)_{i+1}(\eta_{i+1})} + (\lambda_i + \kappa_i \lambda_i) \cos(\eta L, \beta) \right) / 2
\]

\[
u_{23} = \left( \frac{S_i \cosh(\eta L, \beta)}{(EI)_{i+1}(\eta_{i+1})} + (\lambda_i - \kappa_i \lambda_i) \sinh(\eta L, \beta) \right) / 2
\]

\[
u_{24} = \left( \frac{S_i \sinh(\eta L, \beta)}{(EI)_{i+1}(\eta_{i+1})} + (\lambda_i - \kappa_i \lambda_i) \cosh(\eta L, \beta) \right) / 2
\]

\[
u_{31} = \left( \cos(\eta L, \beta) - \kappa_i \lambda_i^2 \cos(\eta L, \beta) \right) / 2
\]

\[
u_{32} = \left( \sin(\eta L, \beta) - \kappa_i \lambda_i^2 \sin(\eta L, \beta) \right) / 2
\]

\[
u_{33} = \left( \cosh(\eta L, \beta) + \kappa_i \lambda_i^2 \cosh(\eta L, \beta) \right) / 2
\]

\[
u_{34} = \left( \sinh(\eta L, \beta) + \kappa_i \lambda_i^2 \sinh(\eta L, \beta) \right) / 2
\]

\[
u_{41} = \left( \frac{S_i \cos(\eta L, \beta)}{(EI)_{i+1}(\eta_{i+1})} - (\lambda_i - \kappa_i \lambda_i) \sin(\eta L, \beta) \right) / 2
\]

\[
u_{42} = \left( \frac{S_i \sin(\eta L, \beta)}{(EI)_{i+1}(\eta_{i+1})} + (\lambda_i - \kappa_i \lambda_i) \cos(\eta L, \beta) \right) / 2
\]

\[
u_{43} = \left( \frac{S_i \cosh(\eta L, \beta)}{(EI)_{i+1}(\eta_{i+1})} + (\lambda_i + \kappa_i \lambda_i) \sinh(\eta L, \beta) \right) / 2
\]

\[
u_{44} = \left( \frac{S_i \sinh(\eta L, \beta)}{(EI)_{i+1}(\eta_{i+1})} + (\lambda_i + \kappa_i \lambda_i) \cosh(\eta L, \beta) \right) / 2
\]

where \( \kappa_i = \frac{(EI)_{i+1}}{(EI)_{i+1}} \), \( \lambda_i = \frac{\eta_i}{\eta_{i+1}} \)

The matrix \( T_{2 \times 4} \) in Eq.(14) is given as

\[
T_{2 \times 4} = \begin{bmatrix}
-c_{k+1} & -s_{k+1} & c_{k+1} & s_{k+1} \\
-s_{k+1} & -c_{k+1} & s_{k+1} & c_{k+1}
\end{bmatrix}
\]

where
\[ c_{k+1} = \cos(\eta_{k+1} \lambda L_{k+1}), \quad s_{k+1} = \sin(\eta_{k+1} \lambda L_{k+1}), \]
\[ ch_{k+1} = \cosh(\eta_{k+1} \lambda L_{k+1}), \quad sh_{k+1} = \sinh(\eta_{k+1} \lambda L_{k+1}) \ quad (18) \]

By solving Eq. (14), \( \lambda_n \) is obtained, thus the natural frequencies for the composite boring bar with S-S-F boundary conditions can be found.

2.3. Dynamic stiffness

It is found that the maximum depth of cut of a boring bar is proportional to the dynamical stiffness of the boring bar\(^2\). In order to assess the stability of composite boring bar in boring operation and influence of clamped conditions, it is necessary to calculated the dynamical stiffness of the composite boring bar.

The dynamic stiffness \( D \) can be defined as the product of the static bending stiffness \( K \) and damping ratio \( \xi \)\(^3\), which is

\[ D = \xi K \quad (19) \]

The static bending stiffness \( K \) are defined by the quotient of a concentrated force at \( z = L \) and the resulting static tip displacement. For the problem at hand \( \xi \) is taken to be 0.015 for composite boring bar\(^1\).

The dynamic stiffness \( D \) for the composite boring bar with different boundary conditions are

C-F composite boring bar

\[ D = K \xi = \frac{3EI_s}{\ell^3} \xi \quad (20) \]

P-P-F composite boring bar

\[ D = \frac{3EI_s}{\ell^3 \left( l_e + 1 \right)} \xi \quad (21) \]

S-S-F composite boring bar

\[ D = K \xi \quad (22) \]

where

\[ K = \frac{k_{11}k_{22}}{k_{11} + k_{22}} \quad (23) \]

\( k_{11} \) is the equivalent static bending stiffness of an infinitely composite boring bar with two spring supports. \( k_{22} \) is the equivalent static bending stiffness of composite boring bar with P-P-F clamped conditions.

\( k_{11} \) and \( k_{22} \) can be derived as following, respectively

\[ k_{11} = \frac{l_e^2}{(l^2 + l_e)^2 - l_e^2} k_T \quad (24) \]
\[ k_{22} = \frac{3EI_s}{\ell^3 \left( l + l_e \right)} \quad (25) \]

It is obvious that if we let \( K_T = \infty \), then \( k_{11} = \infty \) and \( K = k_{22} \), the dynamic stiffness of the S-S-F composite boring bar will reduce the dynamic stiffness of the P-P-F composite boring bar.

3. Results

The mechanical properties of composite boring bar used in this work are shown in Table 1. The boring bar geometrical characteristics are the external radius \( R=10\text{mm} \), section thickness \( h=5\text{mm} \), lamination [\( \theta \)]\(^{30}\).
### Table.1 carbon/epoxy mechanical properties $^{[12]}$

| $\rho$ (kg/m$^3$) | $E_{11}$ (GPa) | $E_{22}$ (GPa) | $G_{23}$ (GPa) | $G_{12}$ (GPa) | $\nu_{12}$ | $\eta_1$ (%) | $\eta_2$ (%) | $\eta_4$ (%) | $\eta_5=\eta_6$ (%) |
|------------------|----------------|----------------|----------------|----------------|-----------|-------------|-------------|-------------|---------------------|
| 1446.2           | 172.7          | 7.2            | 3.76           | 3.76           | 0.3       | 0.45        | 4.22        | 7.05        | 7.05               |

#### 3.1. Natural frequency

Figure 4 presents the variation of the first three natural frequencies with ply angle of the composite boring bar with different boundary conditions. The related quantities used in the numerical simulation are given as $l=0.2$m, $l_c=0.05$m, $k_T=4.881\times10^5$ N/m.

Figure 4 shows that natural frequencies increase as the fiber ply angle decreases. The composite boring bar is stiffer when fibers are directed along the boring bar axis (longitudinal moduli is higher than the transversal moduli). From Figure 4, it is also observed that when the C-F composite boring bar model is changed to the P-P-F composite boring bar model, the first natural frequency for the ply angle $0^\circ$ drops by approximately 200 Hz, and approximately 600 Hz when C-F composite boring bar model is changed to the S-S-F composite boring bar model. This shows that the boundary conditions have significant influence on dynamic properties of the composite boring bar.

Figure 5 presents that the effect of the support spacing on the first three natural frequencies of the P-P-F composite boring bar ($l=0.2$m, $\theta=0^\circ$). It can be seen that natural frequencies of the P-P-F composite boring bar decrease as $l_c$ increases.

Figure 6 presents that the effect of support spacing on the first three natural frequencies of the S-S-F composite boring bar ($l=0.2$m, $k_T=4.881\times10^7$ N/m, $\theta=0^\circ$). As can be seen from Figure 6, the spacing of spring supports $l_c$ has same effect on the natural frequencies of the S-S-F composite boring bar as on those of P-P-F composite boring bar.
Figure 4. The first three natural frequencies vs. ply angle for the composite boring bar with different boundary conditions.

Figure 5. Effect of support spacing on the first three natural frequencies of the composite boring bar with P-P-F boundary condition.
3.2. Dynamic stiffness

Table 2 presents the dynamical characteristics of the composite boring bar with various boundary conditions \( l=0.2m, l_c=0.1m, k_T=4.881\times10^6 N/m, \theta=0^\circ \). Table 2 shows that the dynamic stiffness of S-S-F composite boring bar is largest, followed by P-P-F and S-S-F composite boring bar.

Table 3 presents the effect of ply angle on the dynamical characteristics of the composite boring bar \( k_T=4.881\times10^6 N/m, l=0.2m, l_c=0.1m \). It shows that the larger ply angle, the higher dynamic stiffness is for the composite boring bar with same clamped conditions.

Table 4 presents the effect of spring stiffness \( k_T \) on the dynamical characteristics of the S-S-F composite boring bar \( l=0.2m, l_c=0.1m, \theta=0^\circ \). It was obvious that the dynamical stiffness of the S-S-F composite boring bar increases with \( k_T \).

Tables 5 and 6 present the effect of the supports spacing \( l_c \) on the dynamical characteristics of the composite boring bar with P-P-F and S-S-F boundary conditions, respectively. The results show that the dynamic stiffness increases as \( l_c \) increases.
## Table 2. Effect of boundary conditions on the dynamic characteristics of composite boring bar

| Ply angle | C-F | P-P-F | S-S-F |
|-----------|-----|-------|-------|
| $\theta = 0$ | $\omega_1$ (rad/s) | $\omega_2$ (rad/s) | $\omega_3$ (rad/s) | $D$ (N/m) |
| $\theta = 30$ | 856.122 | 660.843 | 507.302 | 558.194 | 4190.388 |
| $\theta = 60$ | 655.402 | 505.905 | 216.074 | 427.324 | 2862.845 |
| $\theta = 90$ | 279.925 | 216.074 | 134.932 | 182.512 | 646.257 |

## Table 3. Effect of ply angle on the dynamic characteristics of composite boring bar

| Ply angle | C-F | P-P-F | S-S-F |
|-----------|-----|-------|-------|
| $\omega_1$ (rad/s) | $D$ (N/m) | $\omega_2$ (rad/s) | $D$ (N/m) | $\omega_3$ (rad/s) | $D$ (N/m) |
| $\theta = 0$ | 856.122 | 9572.984 | 660.843 | 6381.989 | 3282.110 | 4190.388 |
| $\theta = 30$ | 655.402 | 5610.475 | 505.905 | 4284.782 | 216.074 | 2862.845 |
| $\theta = 60$ | 279.925 | 1023.441 | 216.074 | 682.292 | 134.932 | 646.257 |
| $\theta = 90$ | 174.805 | 399.115 | 134.932 | 4190.388 | 266.070 | 2862.845 |

## Table 4. Effect of spring stiffness on the dynamic characteristics of S-S-F composite boring bar

| $k_T$ | $\beta_1 l$ | $\beta_2 l$ | $\beta_3 l$ | $\omega_1$ (rad/s) | $\omega_2$ (rad/s) | $\omega_3$ (rad/s) | $D$ (N/m) |
|-------|-------------|-------------|-------------|-------------------|-------------------|-------------------|------------|
| $4.881 \times 10^5$ | 1.495 | 3.852 | 6.454 | 241.898 | 1605.918 | 4508.259 | 1024.385 |
| $4.881 \times 10^6$ | 2.165 | 5.488 | 9.015 | 507.302 | 3282.110 | 8795.942 | 4190.388 |
| $4.881 \times 10^7$ | 2.271 | 5.799 | 9.483 | 558.194 | 3696.292 | 9732.903 | 6064.780 |
| $4.881 \times 10^8$ | 2.365 | 5.941 | 9.885 | 605.359 | 3802.006 | 10595.582 | 6348.785 |
| $4.881 \times 10^9$ | 2.426 | 6.118 | 10.241 | 636.990 | 4051.071 | 11353.257 | 6380.936 |

## Table 5. Effect of support spacing on the dynamic characteristics of P-P-F composite boring bar

| $l_s$ (m) | $\omega_1$ (rad/s) | $\omega_2$ (rad/s) | $\omega_3$ (rad/s) | $D$ (N/m) |
|-----------|-------------------|-------------------|-------------------|------------|
| 0.05      | 951.609           | 6170.081          | 17564.477         | 7658.387   |
| 0.1       | 660.843           | 4284.782          | 12197.554         | 6381.989   |
| 0.15      | 485.515           | 3148.001          | 8961.4682         | 6380.936   |

## Table 6. Effect of support spacing on the dynamic characteristics of S-S-F composite boring bar

| $l_s$ (m) | $\omega_1$ (rad/s) | $\omega_2$ (rad/s) | $\omega_3$ (rad/s) | $D$ (N/m) |
|-----------|-------------------|-------------------|-------------------|------------|
| 0.05      | 803.799           | 5241.066          | 14015.381         | 6444.922   |
| 0.1       | 558.194           | 3696.292          | 9732.903          | 6064.780   |
| 0.15      | 410.102           | 2674.013          | 7150.705          | 4717.106   |
3.3. Model validation

In order to validate the present model, the calculated results for the metal boring bar with S-S-F boundary conditions are compared with those obtained in Ref. [13]. This a three span boring bar with elastic support, its geometrical and material properties are \( l_1=0.035\text{m},\ l_2=0.005\text{m},\ l_3=0.215\text{m},\ \rho=7850\text{kg/m}^3,\ E=205\text{GPa} \). According to Ref.[13], the spring coefficients are \( k_T=4.881\times10^9\text{N/m} \), geometrical properties \( A=3.661\times10^{-5}\text{(m}^2)\), \( I_x=1.067\times10^{-5}\text{(m}^4)\), for the metal boring bar with four screws of size M8, and \( k_T=7.732\times10^9\text{N/m} \), geometrical properties \( A=5.799\times10^{-5}\text{(m}^2)\), \( I_x=2.676\times10^{-5}\text{(m}^4)\), for the metal boring bar with six screws of size M10. The results obtained using the present model are shown in Table 7 together with those of Ref. [13]. As can be seen from the table our results are well consistent with those in Ref. [13].

|                | Present | Ref. [13], analytical | Ref. [13], experimental |
|----------------|---------|-----------------------|-------------------------|
| C-F            | 698.309 | 698.331               | 601.791                 |
| S-S-F (M8)     | 520.412 | 519.432               | 555.357                 |
| S-S-F (M10)    | 526.883 | 525.241               | 555.351                 |

4. Conclusions

Based on Bernoulli-Euler beam theory, dynamical analysis has been carried out on the composite boring bar with different clamping conditions. The influences of the supporting conditions and the ply angle of composite boring bar are studied.

1) The clamped condition's model and its effect on the natural frequencies and dynamic stiffness of composite boring bar are significant. The clamped composite boring bar using clamped-free boundary conditions overestimates the natural frequencies and dynamic stiffness since it assumes rigid clamping which is not the case in reality. The dynamical model of the composite boring bar with pinned-pinned-free and spring-spring-free boundary conditions is an improvement to clamped-free composite boring bar model. These models can be used to predict the natural frequencies and dynamic stiffness with sufficient accuracy.

2) Ply angle, the supports spacing and spring stiffness affect the natural frequency and dynamic stiffness, thus affect the stability of composite boring bar during cutting. The natural frequency and dynamic stiffness decease as ply angle increases.

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