Geometric Design and Stability of Power Networks

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Abstract

From the perspective of the network theory, the present work illustrates how the parametric intrinsic geometric description exhibits an exact set of pair correction functions and global correlation volume with and without the inclusion of the imaginary power flow. The Gaussian fluctuations about the equilibrium basis accomplish a well-defined, non-degenerate, curved regular intrinsic Riemannian surfaces for the purely real and the purely imaginary power flows and their linear combinations. An explicit computation demonstrates that the underlying real and imaginary power correlations involve ordinary summations of the power factors, with and without their joint effects. Novel aspect of the intrinsic geometry constitutes a stable design for the power systems.

Keywords: Correlation; Geometry; Power Flow; Network; Stability.

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1 Introduction

Planning issues towards a stable power supply, viz., transmission and distribution systems have been in wide application of the power system since a decade [1]. In the transmission theory, this shows that the power flow varies as a function of the power factor of the network. In order to regulate the voltage of the network, the power factor $\phi$ is thus related to the impedance angle $a \geq \frac{\pi}{2} - \phi$. Interestingly, the determination of $a$ can be accomplished by tuning the network parameters, viz., resistance ($r$), inductance ($L$) and capacitance ($C$) [2, 3]. An optimal choice of $a$ improves the efficiency of power networks, and thus an appropriate design of the network parameters. For a given transmission line(s), we provide a proper network planning for the operations with a minimum reactive power requirement. From the outset of the intrinsic geometry, the present paper determines the required unit of the power flow under the fluctuations of the power factors.

Up to now, most of the network planning and design is based on the power flow equations [4, 5], and thus the network characterizations are linearly afforded by a set of chronological data analysis, heuristics methods, parametric estimations and optimization techniques [6, 7]. In this paper, this motivates us to define non-linear criteria for the electrical networks. Subsequently, we take an account of the fact that the set of voltages at all buses attains an equilibrium configuration, therefore the extremization of the power flow in the network can be determined in terms of $a$. The selection of the network parameters determined by the optimization techniques could be unreliable and thus the cause of bottle-necking. This shows an urgent need for compensator(s). For a safe operation and optimal power flow, our analysis provides stability criteria for the non-linearly reliable comportment of the electrical network. In this concern, our method provides a high degree of compensation strategy to reduce the fluctuation effects of the network parameters, viz., $r$, $L$ and $C$.

To be specific, we focus our attention on the power networks and determine the required set of voltage stability criteria, viz., the selection of power factors, network planning and compensation strategies. Further, the intrinsic geometry offers an effective determination of the power system characterization, power factor corrections and voltage regulation. It is worth mentioning that the voltage level at the buses could be tuned in the equilibrium. In the reverse engineering, the proposed solution keeps the power system in a stable voltage range, under the fluctuations of the network configuration. In this way, we provide an efficient power system characterization, which is non-linearly stable over the fluctuations of the network phases.

In this concern, the intrinsic geometry has been important in the configurations involving black holes in string theory [9, 15] and $M$-theory [16, 19], possessing a set of rich stability structures [20, 28]. Therby, there have been several investigations about the equilibrium perspective in black holes, explicating the nature of pair correlations and the associated stability of the solutions [34]. Besides several general notions analyzed in condensed matter physics [29, 33], we consider specific electrical network(s). As mentioned above, we analyze the parametric pair correlation functions and their correlation relations about the equilibrium configuration. We find that the intrinsic geometric consideration entails an intriguing feature of the underlying fluctuations, which are defined in terms of the network parameters.

Given a definite covariant intrinsic geometric description of the network configuration, we expose (i) the conditions for the stability, (ii) properties of the parametric correlation functions and (iii) scaling relations in terms of the parameters of the network. In this analysis, we enlist the complete set of non-trivial parametric correlation functions of the electrical networks. This anticipation follows from similar considerations in the black hole solutions in general relativity.
attractor black holes and Legendre transformed finite parameter chemical configurations, quantum field theory and associated Hot QCD backgrounds, as well in the stability of Quarkonia states. Thus, the intrinsic geometry plays an important role in the study of power networks and their stable design applications.

In the context of power flow equations, the usefulness of the present investigation may apparently seem to be limited, however, we illustrate that our preposition could in principle be generically applied to all electrical networks, in order to achieve the best understanding of the phenomenon and importance of the controlled power flow and network analysis. The intrinsic geometric consideration is capable to provide strategic planning criteria for the effective use of power system and voltage stability. We show that this notion follows from the standard laws of the electrical circuits. Subsequently, for an additional component, the criteria of the voltage stability can be used for an optimal selection of the network parameters.

2 Phases of the Electrical Network

In this section, we set up the formulation of the problem and outline the notion of the intrinsic Riemannian geometry. The exploration of the power flow equations gives the relation of the power flow with network phases, and thus the consideration for the analysis. Most efficiently, our intrinsic geometric model is designed to provide the critical values of the phases. It is worth mentioning that the present method is important towards the determination of the parameters, and thus the unstable mode of the network.

2.1 Hypothesis of the Power Flow

For the optimization of the power flow, we share the hypothesis of the prior solutions and use the load flow equations in order to solve the mentioned issues. The associated power conservation equations with the real (resistive) and imaginary (reactive) branch parameters are given by

\[ P_i = \sum |V_i||V_j||Y_{ij}|(G_{ij}\cos(a_{ij} + \delta_j - \delta_i) + B_{ij}\sin(a_{ij} + \delta_j - \delta_i)) \]

\[ Q_i = \sum |V_i||V_j||Y_{ij}|(G_{ij}\sin(a_{ij} + \delta_j - \delta_i) - B_{ij}\cos(a_{ij} + \delta_j - \delta_i)) \]

In the above equations, the phases are defined as

\[ \tan(a_{ij}) = \frac{X_{L_{ij}} - X_{C_{ij}}}{r_{ij}} \]

The inverse set of the impedances \( Z_{ij} \) and voltage angles \( \delta_j \) are

\[ Y_{ij} = \frac{1}{(r_{ij} + jX_{L_{ij}} - jX_{C_{ij}})} \]

\[ \delta_j = \frac{V_j}{|V_j|} \]

For the purpose of subsequent analysis, let us consider arbitrary \( i^{th} \)-bus such that the steady state condition is realized as \(|V_i| = 1\). This makes the underlying configuration reach an
equilibrium. Thus, the respective cases of the present interest reduce to the standard network considerations. For the purpose of the future analysis, we consider that a loss-less line is defined by $a_{ij} = 90^\circ$, where $+90^\circ$ represents the ideal case. In this case, it turns out that the network is purely inductive with $r = 0$. Notice that a realistic network would never reach the limit of the zero resistance. In the generic situations, the value of the $a_{ij}$ varies from $+90^\circ$ to $-90^\circ$. Further, the phase $a_{ij} = -90^\circ$ is also not feasible in the real situation, as the network possesses a finite capacitance. Thus, the phases for the inductor and capacitor circuits can be defined as

$$a_{(1)ij} = \tan^{-1}\left(\frac{X_{L_{ij}}}{r_{ij}}\right), a_{ij} = -90^\circ$$  \hspace{1cm} (5)$$

$$a_{(2)ij} = \tan^{-1}\left(\frac{X_{C_{ij}}}{r_{ij}}\right), a_{ij} = 90^\circ$$

For a general consideration of the network fluctuations, we have non-zero values for the network parameters, viz., $r$, $L$ and $C$, and thus the general phase angle $a_{(3)ij}$ is defined as

$$a_{(3)ij} = \tan^{-1}\left(\frac{X_{L_{ij}} - X_{C_{ij}}}{r_{ij}}\right)$$  \hspace{1cm} (6)$$

We take an account of the fact that the efficiency of the power flow on a transmission line is analyzed by the phases of the impedance pertaining to the transmission lines. As per the consideration of the next subsection, our method offers a non-linear characterization for the generic component of a realistic network, which we suppose neither a purely inductive nor capacitive component.

2.2 Hypothesis of the Intrinsic Geometry

In this subsection, we recall the motivation for intrinsic geometric analysis and set up the notations for the subsequent computations. Following the notations of the previous subsection, a given network can reach a local equilibrium, if we can fix one of the phases of the power network. The logic simply follows from the fact that the sum of the three angles of the tringle is a constant. To be specific, let us illustrate the consideration of the intrinsic geometry for the case of two parameter configurations. To be concrete, let the parameters be $\{a_1, a_2\}$ and let $S(a_1, a_2)$ be a smooth function of the network (real, imaginary) phases as defined in the Eqns.(1, 2) or any of their real combinations. For a given $S(a_1, a_2)$, the components of the correlation functions are described as the Hessian matrix $\text{Hess}(S(a_1, a_2))$ of the generalized power function under the flow of the parameters. Following this consideration, the components of the intrinsic metric tensor are given by

$$g_{a_1a_1} = \frac{\partial^2 S}{\partial a_1^2}, \hspace{0.5cm} g_{a_1a_2} = \frac{\partial^2 S}{\partial a_1 \partial a_2}, \hspace{0.5cm} g_{a_2a_2} = \frac{\partial^2 S}{\partial a_2^2}$$  \hspace{1cm} (7)$$

The components of the intrinsic metric tensor are associated to the respective pair correlation functions of the concerned power flow. It is worth mentioning that the co-ordinates of the underlying power factor lie on the surface of the parameters, which in the statistical sense, gives the origin of the fluctuations in the network. This is because the components of the metric tensor comprise the Gaussian fluctuations of the network power, which is a function of the parameters of the power configuration. For a given network, the local stability of the underlying system requires both the principle components to be positive. In this concern, the
diagonal components of the metric tensor, \( \{g_{a_i a_i} \mid i \in \{1, 2\} \} \) signify the heat capacities of the system, and thus they are required to remain positive definite quantities

\[
g_{a_i a_i} > 0, \; i = 1, 2
\]

From the perspective of intrinsic geometry, the stability properties of the network flows can thus be divulged from the positivity of the determinant of the metric tensor. For the Gaussian fluctuations of the two charge equilibrium power configurations, the existence of a positive definite volume form on the power surface imposes such a stability condition. Specifically, a power supplying configuration is said to be stable if the determinant of the tensor

\[
\|g\| = S_{a_1 a_1} S_{a_2 a_2} - S_{a_1 a_2}^2
\]

remains positive. For the two parameters networks, the geometric quantities corresponding to the chosen power elucidate the typical features of the Gaussian fluctuations about an ensemble of equilibrium states. Subsequently, the intrinsic scalar curvature, as a global invariant, accompanies the information of the correlation volume of the underlying power fluctuations. Explicitly, the scalar curvature \( R \) takes the following form:

\[
R = -\frac{1}{2} \left( S_{a_1 a_1} S_{a_2 a_2} - S_{a_1 a_2}^2 \right)^{-2} \left( S_{a_2 a_2} S_{a_1 a_1} S_{a_1 a_2} - S_{a_1 a_2}^2 \right) + S_{a_1 a_2} S_{a_1 a_2} S_{a_1 a_2} - S_{a_1 a_1} S_{a_1 a_2} S_{a_1 a_2} S_{a_1 a_2}
\]

Notice that the zero scalar curvature indicates that the power of the network fluctuates independently of the phases, while a divergent scalar curvature signifies a sort of phase transition, indicating an ensemble of highly correlated pixels of information on the power surface. In the case of black hole physics, Ruppeiner has interpreted the assumption “that all the statistical degrees of freedom of a black hole live on the black hole event horizon” as an indication that the state-space scalar curvature signifies the average number of correlated Planck areas on the event horizon of the black hole [29]. For the case of the two parameter systems, the above analysis of the surface shows that the scalar curvature and curvature tensor are related by

\[
R(a_1, a_2) = \frac{2}{\|g\|} R_{a_1 a_2 a_1 a_2}
\]

The scalar curvature thus defined offers the nature of the long range global correlation and underlying phase transitions originating from the power flow. In this sense, we anticipate that an ensemble of signals corresponding to the network are statistically interacting, if the underlying power configuration has a non-zero scalar curvature. Incrementally, we may notice further that the configurations under present consideration are allowed to be effectively attractive or repulsive, and weakly interacting, in general. The intrinsic geometric analysis further provides a set of physical indications encoded in the geometrically invariant quantities, e.g., scalar curvature and other geometrically non-trivial objects. For the electrical network, the underlying analysis would involve an ensemble or subensemble of the equilibrium configuration forming a statistical basis about the Gaussian distribution. With this brief introduction, we shall now proceed to systematically analyze the underlying stability structures of the network fluctuations in the real, imaginary power flows and their joint effects on the network.
3 Real Power Flow

Let us first describe the intrinsic stability of the electrical network with a given power factor. Following the Eqn. (1), the power defined with a set of desired corrections over the network power factors, chosen as the network variables $a_1$, $a_2$ for the present analysis, is given by

$$ P(a_1, a_2) := \frac{V^2}{R_0(1 + (\tan(a_1) - \tan(a_2))^2)} \tag{12} $$

The components of the correlation functions are described as the Hessian matrix $Hess(P(a_1, a_2))$ of the concerned power under the tuning response function. Following the Eqn. (12), the components of the metric tensor are

$$ g_{a_1a_1} = \frac{2c_1^2V^2}{R_0} \frac{n_{11}^R}{r_{11}^R} $$
$$ g_{a_1a_2} = \frac{2c_1^2c_2^2V^2}{R_0} \frac{n_{12}^R}{r_{12}^R} $$
$$ g_{a_2a_2} = \frac{2c_2^2V^2}{R_0} \frac{n_{22}^R}{r_{22}^R} \tag{13} $$

In this framework, we find that the geometric nature of the parametric pair correlations offers the notion of fluctuating networks. Thus, the fluctuating parameters may be easily divulged in terms of the intrinsic parameters of the underlying network configurations. For a given network, it is evident that the principle components of the metric tensor signify self pair correlations, which are positive definite functions over a range of the parameters. In order to simplify the subsequent notations, let us define the cosine and sine variables as

$$ \cos(a_i) := c_i, \quad i = 1, 2 $$
$$ \sin(a_i) := s_i, \quad i = 1, 2 \tag{14} $$

We have arrived at the conclusion that the numerator of the local pair correlation functions are expressed as the following trigonometric polynomials:

$$ n_{11}^R := -6c_1^2c_2 + 6s_1c_2s_2c_1^2 - c_1^2 + 6c_1^4c_2^2 $$
$$ -2s_1s_2c_1^2 + 3c_1^2c_2 - c_1^2c_3^2 $$
$$ n_{12}^R := 6s_1c_2s_2c_1 - 3c_1^2 + 7c_2^4c_3^2 - 3c_2^2 $$
$$ n_{22}^R := -c_1^4c_2^2 - c_1^2 - 2s_1c_2s_2c_1^2 + 6c_1^4s_2^2c_1^2 + 6c_1^2c_2^2 $$
$$ -6c_1^2c_2 + 3c_1c_2^2 \tag{15} $$

We further notice a similar conclusion for the denominator of the local pair correlation functions. In general, we find, for the real power flow pair correlations, that the denominator of the local pair correlation functions are

$$ r_{11}^R := -c_1^6 - 15c_1^4c_2^2 + 42c_1^2c_2^4 - 27c_1^4c_2^2 + 15c_1^6c_2^2 - 27c_1^2c_2^2 $$
$$ +15c_1^2c_2^4 + 6s_1c_2s_2c_1^2 + 20s_1^2c_2s_2c_1^2 - 15c_1^4c_2^2 $$
$$ +6s_1c_2s_2c_1 - c_1^2 + 13c_1^6c_2^2 - 20s_1c_2s_2c_1^2 $$
$$ -20s_1c_2s_2c_1^2 + 14c_1^2c_2^2s_2s_2 $$
$$ r_{12}^R := -c_1^6 - 15c_1^4c_2^2 + 42c_1^2c_2^4 - 27c_1^4c_2^2 + 15c_1^6c_2^2 - 27c_1^2c_2^2 $$
$$ +15c_1^2c_2^4 + 6s_1c_2s_2c_1^2 + 20s_1^2c_2s_2c_1^2 - 15c_1^4c_2^2 $$
$$ +6s_1c_2s_2c_1 - c_1^2 + 13c_1^6c_2^2 - 20s_1c_2s_2c_1^2 $$
$$ -20s_1c_2s_2c_1^2 + 14c_1^2c_2^2s_2s_2 $$
$$ r_{22}^R := -c_1^6 - 15c_1^4c_2^2 + 42c_1^2c_2^4 - 27c_1^4c_2^2 + 15c_1^6c_2^2 - 27c_1^2c_2^2 $$
$$ +15c_1^2c_2^4 + 6s_1c_2s_2c_1^2 + 20s_1^2c_2s_2c_1^2 - 15c_1^4c_2^2 $$
$$ +6s_1c_2s_2c_1 - c_1^2 + 13c_1^6c_2^2 - 20s_1c_2s_2c_1^2 $$
$$ -20s_1c_2s_2c_1^2 + 14c_1^2c_2^2s_2s_2 \tag{16} $$
It is worth mentioning that the real network is well-behaved for the generic values of the parameters. Over the domain of the \( \{a_1, a_2\} \), we observe that the Gaussian fluctuations form a set of stable correlations, if the determinant of the metric tensor
\[
g = \frac{8V^4c_1^2c_2^4 c_6^6}{R_0^4 r_R^4} (17)
\]
remains a positive function on the power factor surface \( (M_2(R), g) \). In terms of the trigonometric polynomial, we obtain that the numerator of the determinant of the metric tensor can be expressed as
\[
n_R^g : = 12s_1s_2c_1^4 - 4c_1^3c_2^2 + 4c_1c_2^4 - c_1c_2
+ 4c_1^3c_2^2 - c_1s_1s_2 - c_1s_2c_2^2 - c_1s_1s_2c_2^2
- c_1s_1s_2 - 7c_1^2 - 6c_1^2s_1s_2 - 7c_1c_2^2
+ 12c_1^2c_2^2 \quad (18)
\]
As per the expectation, the denominator of the determinant of the metric tensor turns out to be given by the following trigonometric polynomial:
\[
r_R^g : = -210c_1^6 - 210c_1^6c_2^4 + 910c_1^6c_2^6 - c_1^{10} - c_2^{10} + 121c_1^{10}c_2^{10}
+ 122c_1^6s_1s_2 + 252c_1^6s_2c_2^6 + 5848c_1^6s_2c_2^1 + 3328c_1s_2c_2^1
+ 808s_1s_2c_2^1 + 120s_1s_2c_2^1 - 120c_1^6s_1s_2 - 10s_1s_2c_2^1
+ 45c_1^{10}c_2^2 + 250c_1^6c_2^4 + 490c_1^6c_2^6 - 405c_1^6c_2^8 + 45c_1^{10}c_2^2
- 120s_1c_2^2c_2^1 + 10s_1c_2^2c_2^1 - 5848c_1c_2^2c_2^1 - 344s_1c_2^2c_2^1
- 344s_1c_2^2c_2^1 + 332c_1^6s_1s_2 + 120c_1^6s_2c_2^1 - 250c_2^{10}
+ 460c_2^8 - 45c_2^8s_1 + 490c_2^8c_2^4 - 1190c_2^8c_2^6 + 1180c_2^8c_2^1
- 405c_2^6c_2^0 - 1190c_2^6c_2^4 - 45c_2^6c_2^0 + 460c_2^6c_2^1 \quad (19)
\]
Herewith, the behavior of the determinant of the metric tensor shows that such a real power flow becomes unstable for the specific values of the parameters. For generic \( R_0 \) and \( V \), the nature of the determinant of the metric tensor is depicted in the Eqn. (17). It is worth mentioning further that the electric networks become unstable in the limit of vanishing \( \{a_1, a_2\} \).

In order to explain the nature of transformation of the \( \{a_1, a_2\} \) forming the intrinsic surface, let us explore the functional behavior of the associated scalar curvature. Our computation shows that the scalar curvature reduces to the following form:
\[
R = \frac{1}{4R_0c_1^2c_2^2V^2} n_R^g \quad (20)
\]
The numerator and denominator of the scalar curvature take the following trigonometric polynomial expression:
\[
n_R^g : = -42c_1^6 - 42c_1^6c_2^4 + 444c_1^6c_2^6 + 3c_1^{10} + 3c_1^{10} + 702c_1^{10}c_2^{10}
+ 702c_1^6c_2^6s_1s_2 + 84c_1^6c_2^6s_1s_2 - 280s_1c_2^6c_2^6s_2c_2^1 + 356s_1c_2^6s_2c_2^1
+ 1080s_1c_2^6c_2^0c_2^1 - 24s_1c_2^6s_2c_2^1 + 24c_2^6c_2^0s_1s_2 - 18s_2c_2^6c_2^1
- 38c_1^{10}c_2^2 - 70c_1^{10}c_2^2 + 752c_1^{10}c_2^2 - 1349c_1^{10}c_2^2 - 38c_1^{10}
+ 24c_1^6c_2^6c_2^1 - 18s_1c_2^6c_2^6s_2c_2^1 - 280s_1c_2^6c_2^6s_2c_2^1 - 1008s_1c_2^6c_2^6c_2^1
- 1008s_1c_2^6c_2^6c_2^1 + 356c_1^6c_2^6s_1s_2 - 24s_1c_2^6c_2^6s_2c_2^1 - 70c_1^{10}c_2^2
+ 72c_1^6c_2^1 + 39c_2^6c_2^0 + 752c_2^6c_2^1 - 1082c_2^6c_2^1 + 2288c_2^6c_2^1
- 1349c_2^6c_2^1 - 1082c_2^6c_2^1 + 39c_2^6c_2^1 + 72c_2^6c_2^1 \quad (21)
\]
The denominator of the scalar curvature can be expressed as the trigonometric polynomial
\[
r_R^g : = -c_1^6 + 6s_1c_2^6c_2^1 + 20s_1c_2^6s_2c_2^1 + 6s_1c_2^6s_2c_2^1 + 8s_1c_2^6s_2c_2^1
+ 8s_1c_2^6s_2c_2^1 - 15c_1c_2^4 + 6s_1c_2^6c_2^1 + 53c_1c_2^4 + 4c_1c_2^4 + 53c_1c_2^4
+ 4c_1c_2^4 - 15c_1c_2^4 - 48c_1c_2^4 + c_2^4 + 2c_2^4 - 70c_1c_2^6s_1s_2 - 2c_2^4
- 12s_1c_2^6s_2c_2^1 + 72s_1c_2^6s_2c_2^1 - 12s_1c_2^6s_2c_2^1 - 12s_1c_2^6s_2c_2^1
- 12s_1c_2^6s_2c_2^1 - 11c_2^4c_2^1 + 2c_2^4c_2^1 - 48c_1c_2^4 + 72c_2^4c_2^1
- 48c_2^4c_2^1 + 2c_2^4c_2^1 - 11c_2^4c_2^1 \quad (22)
Figure 1: The determinant of the metric tensor plotted as the function of the power factors $a_1, a_2$, describing the real power fluctuations in electrical networks.

Figure 2: The determinant of the metric tensor plotted as the function of the equal power factor $a := a_1 = a_2$, describing the real power fluctuations in electrical networks.
Figure 3: The curvature scalar plotted as a function of the power factors $a_1, a_2$, describing the real power fluctuations in electrical networks.

Figure 4: The curvature scalar plotted as a function of the equal power factor $a := a_1 = a_2$, describing the real fluctuations in electrical networks.
We find that a typical real power network is globally correlated over all the generic Gaussian fluctuations of the parameters \( \{a_1, a_2\} \), unless \( n_{11}^R = 0 \). For the above real power networks, we observe that the scalar curvature diverges in the limit \( r_{11}^R = 0 \), showing a signature of the global instability on the \( \{a_1, a_2\} \) surface. Thus, the intrinsic geometric analysis shows that a real power network is interacting and locally stable over the surface of fluctuation, if the network parameters \( \{a_1, a_2\} \) are properly chosen.

For the choices \( V = 1 \) and \( R_0 = 1 \), the Fig. 1 shows the determinant of the metric tensor. These plots explicate the nature of the stability in the real power flow in the electrical network.

For the equal phases, viz., \( a_1 = a \) and \( a_2 = a \), the surface plots of the determinant of the metric tensor and scalar curvature are respectively shown in the Figs. 2 and 4. We observe that the instability is present only for a set of specific equal values of the parameters. For a limiting equal phases network, the limiting scalar curvature interestingly simplifies to the shown shape. In particular, the corresponding peaks in the Figs. 3 and 4 of the scalar curvature indicate the graphical nature of the global instability in the real power networks. Physically, the peaks in the curvature show the presence of non-trivial interactions in the network.

4 Imaginary Power Flow

In the present section, we analyze the nature of an ensemble of fluctuating electrical networks generated by a pair \( a_1, a_2 \). To focus on the most general case, we chose the variable \( a_1, a_2 \) as a function of \( L, C \) and \( r \) of the chosen network. Following Eqn. 2, when the imaginary power

\[
Q(a_1, a_2) := \frac{V^2}{R_0} \frac{(\tan(a_1) - \tan(a_2))}{(1 + (\tan(a_1) - \tan(a_2))^2)}
\]  

is allowed to fluctuate as a function of the \( a_1, a_2 \), we may again exploit the definition of the Hessian function \( Hess(Q(a_1, a_2)) \) of the imaginary power. Hereafter, the components of the metric tensor are given by

\[
\begin{align*}
g_{a_1 a_1} &= \frac{2V^2 n_{11}^I}{R_0 r_{11}^I} \\
g_{a_1 a_2} &= \frac{2V^2 n_{12}^I}{R_0 r_{12}^I} \\
g_{a_2 a_2} &= \frac{2V^2 n_{22}^I}{R_0 r_{22}^I}
\end{align*}
\]  

In this case, the numerators of the local pair correlation functions are expressed as the following trigonometric polynomials:

\[
\begin{align*}
n_{11}^I &= -5c_2^3 c_4^3 c_5^3 - s_1 c_1^3 + 8c_2^2 s_1 c_3^3 - 4c_2 s_2 c_4^3 + 3c_2 s_2 c_5^3 \\
&\quad - 3c_2 s_1 c_3^3 + 8c_2 s_2 c_3^3 - 3c_2 s_1 c_3^3 + 8c_2 s_2 c_3^3 + 3c_2 s_1 c_3^3 \\
n_{12}^I &= c_2^3 s_1 + 3c_2^3 s_1 c_2 - c_1^3 s_2 - 7c_2^3 c_3^3 s_1 + 7c_2^3 s_2 c_3^3 - 3c_2^2 s_2 c_3^3 \\
n_{22}^I &= -8c_2^3 c_3^3 s_1 - s_1 c_1^3 + 5c_2^3 s_1 c_1^3 - c_2 s_2 c_4^3 + 3c_2 s_2 c_5^3 \\
&\quad - 3c_2 s_1 c_3^3 + 7c_2^3 s_2 c_3^3 - 8c_2 s_1 c_3^3 + 8c_2 s_2 c_3^3 + 4c_2^3 s_1 c_3^3
\end{align*}
\]
While, the denominators of the local pair correlation functions take the following trigonometric expressions:

\[
\begin{align*}
{r}_{11}^f & := 13c_1^6c_2^6 + 14c_1^5c_2^7s_1s_2 + 6s_1c_1^2c_2^3c_1 + 15c_1^7c_2^6 \\
& + 6s_1c_2^3c_1c_2^3s_1s_2 - 20c_1^5c_2^3s_1s_2 - 20c_1^7c_2^3s_1s_2 - 15c_1^5c_2^4 \\
& + 20c_1^7c_2^3s_1s_2 - 27c_1^5c_2^4 + 15c_1^6c_2^4 - 27c_1^5c_2^3 + 42c_1^7c_2^4 \\
& - 15c_1^6c_2^4 - c_2^7 - c_1^6
\end{align*}
\]

\[
{r}_{12}^f := 13c_1^5c_2^6 + 14c_1^5c_2^5s_1s_2 + 6s_1c_1^2c_2^3c_1 + 15c_1^7c_2^6 \\
+ 6s_1c_2^3c_1c_2^3s_1s_2 - 20c_1^5c_2^3s_1s_2 - 20c_1^7c_2^3s_1s_2 - 15c_1^5c_2^4 \\
+ 20c_1^7c_2^3s_1s_2 - 27c_1^5c_2^4 + 15c_1^6c_2^4 - 27c_1^5c_2^3 + 42c_1^7c_2^4 \\
- 15c_1^6c_2^4 - c_2^7 - c_1^6
\]

\[
{r}_{22}^f := 13c_1^5c_2^6 + 14c_1^5c_2^5s_1s_2 + 6s_1c_1^2c_2^3c_1 + 15c_1^7c_2^6 \\
+ 6s_1c_2^3c_1c_2^3s_1s_2 - 20c_1^5c_2^3s_1s_2 - 20c_1^7c_2^3s_1s_2 - 15c_1^5c_2^4 \\
+ 20c_1^7c_2^3s_1s_2 - 27c_1^5c_2^4 + 15c_1^6c_2^4 - 27c_1^5c_2^3 + 42c_1^7c_2^4 \\
- 15c_1^6c_2^4 - c_2^7 - c_1^6
\]  

(26)

It follows that the pure pair correlations \( \{g_{11}, g_{22}\} \) between the parameters \( \{a_1, a_2\} \) remain positive, which is the same as for the flow of the real power. A straightforward computation further demonstrates the over-all nature of the parametric fluctuations. In fact, we find that the determinant of the metric tensor reduces to the following expression:

\[
g = \frac{V^4c_1^5c_2^6}{R_0} r^f \]  

(27)

In this case, the numerator of the determinant of the metric tensor is given by the following trigonometric polynomial:

\[
n^f_g := 10c_1^5c_2^6 + 37c_1^5c_2^7s_1s_2 - 3c_1^7c_2^2 - 3c_1^7c_2^3 - 39c_1^6c_2^2 + 28c_1^7c_2^4 \\
+ 10c_1^7c_2^2 - 39c_1^6c_2^3 + 3s_1c_2^3c_1s_2 - 20c_1^5c_2^3s_1s_2 - 20c_1^5c_2^3s_1s_2 \\
+ 3s_1c_2^3c_1s_2 + 2c_1^5c_2^3s_1s_2 + 38c_1^5c_2^3 - c_2^7 - c_1^6
\]  

(28)

Explicitly, we find that the denominator of the determinant of the metric tensor can be presented as

\[
r^f_g := 252c_1^5c_2^7s_1s_2 + 910c_1^5c_2^6 - 210c_1^5c_2^6 - 210c_1^5c_2^6 \\
- 12c_1^5c_2^6 + 122c_1^5c_2^7s_1s_2 + 121c_1^5c_2^6 + 10s_1c_2^3s_1s_2 \\
+ 120s_1c_2^3c_1s_2 - 120c_1^5c_2^6 + 332c_1^5c_2^7s_1s_2 + 332c_1^5c_2^6s_1s_2 \\
+ 344c_1^5c_2^7s_1s_2 - 344c_1^5c_2^6s_1s_2 + 490c_1^5c_2^6 - 250c_1^5c_2^6 \\
+ 420c_1^5c_2^6 - 20c_1^5c_2^6 + 405c_1^5c_2^6 - 250c_1^5c_2^6 \\
+ 420c_1^5c_2^6 + 405c_1^5c_2^6 + 45c_1^5c_2^6 + 45c_1^5c_2^6 \\
+ 120c_1^5c_2^7s_1s_2 - 584c_1^5c_2^6s_1s_2 + 120c_1^5c_2^7s_1s_2 - 584c_1^5c_2^6s_1s_2 \\
+ 10s_1c_2^3c_1s_2 + 808c_1^5c_2^6s_1s_2
\]  

(29)

It is not difficult to compute the exact expression for the scalar curvature describing the global parametric intrinsic correlations. In particular, we find that the scalar curvature reduces to the following form:

\[
R = \frac{R_0}{2c_1^5c_2^6V^2} \left( \frac{n^{(1)}_R}{r^f_R} + \frac{n^{(2)}_R}{r^f_R} \right) 
\]  

(30)

In the above equation, the numerator of the scalar curvature takes the trigonometric polynomial expression

\[
n^{(1)}_R := -297s_1c_1^6c_2^8 + 2235s_1c_1^7c_2^{10} + 4770s_1c_1^6c_2^8 - 15596s_1c_1^7c_2^{10} \\
- 19538s_1c_1^6c_2^8 + 45624s_1c_1^7c_2^{10} - 7857s_1c_1^6c_2^8 - 3954s_1c_1^7c_2^{12}
\]
Figure 5: The determinant of the metric tensor plotted as the function of the power factors $a_1, a_2$, describing the imaginary power fluctuations in electrical networks.

Figure 6: The curvature scalar plotted as a function of the power factors $a_1, a_2$, describing the imaginary power fluctuations in electrical networks.
The function \( r^I_R \) appearing in the denominator of the scalar curvature can be written as the following polynomial:

\[
\begin{align*}
\tau_1^I & := 42c_1^2 + c_2^2 + 5344c_1^2a_{12} + 10447c_1^{10}c_2^{10} + 4383c_1^{12}c_2^2 \\
& + 100c_1^2s_1s_2 - 708c_1^2s_1s_2 + 100c_1^2s_1s_2 + 2528c_1^2s_1s_2 \\
& - 4406c_1^2s_1s_2 + 2528c_1^2s_1s_2 - 4406c_1^2s_1s_2 + 2812c_1^2s_1s_2 \\
& - 6c_1^2s_1s_2 - 708c_1^2s_1s_2 - 6c_1^2s_1s_2 - 29c_1^2s_1s_2 \\
& + 4383c_1^2s_1s_2 + 518c_1^2s_1s_2 + 281c_1^2s_1s_2 - 1598c_1^2s_1s_2 \\
& - 518c_1^2s_1s_2 + 307c_1^2s_1s_2 + 307c_1^2s_1s_2 - 22s_1s_2c_1s_1s_2 - 22s_1s_2c_1s_1s_2 \\
& + 368c_1^2s_1s_2 + 368c_1^2s_1s_2 - 2132c_1^2s_1s_2 \\
& + 1826c_1^2 - 6442c_1^2 - 257c_1^2 + 1826c_1^2 + 2822c_1^2 \\
& - 482c_1^2 - 482c_1^2 - 257c_1^2 + 6442c_1^2 + 27c_1^2 + 27c_1^2 \\
& + 15c_1^2 + 3c_1^2 - 36c_1^2s_1s_2 - 36c_1^2s_1s_2 + 472c_1^2s_1s_2
\end{align*}
\]

Thus, we may easily analyze the underlying conclusions for the specific considerations of the variable power factor of the network. As in the case of real power flow, the global nature of scalar curvature and associated phase transitions can be thus determined over the range of power factors describing the important network of interest.

As in the previous subsection, we shall focus our attention on same electrical field and on the specific values \( V = 1 \) and \( R_0 = 1 \). The determinant of the metric tensor shown in the Fig.\([3]\) describes the phenomenological property of the Gaussian imaginary power fluctuations. The scalar curvature as shown in the Fig.\([7]\) depicts a couple of antisymmetrical fluctuations for the imaginary network power flow.

For the equal values \( a_1 = a \) and \( a_2 = a \), we notice, from the Fig.\([4]\), that the system acquires a couple of locally chaotic fluctuations. Herewith, we find the surprising fact that the imaginary power flow has a different geometric nature, and the scalar curvature turns out to be zero in the limit of the equal phases for the imaginary power flow. It is worth mentioning that such a power flow is globally non-interacting.

## 5 Complex Power Flow

In order to further understand the nature of generic electrical networks, we shall now consider the linear combination of the real and imaginary power flows in the network. The associated
joint power flow $F$ takes the following form:

$$F(a_1, a_2) := \frac{V^2}{R_0} \frac{(1 + \tan(a_1) - \tan(a_2))}{(1 + (\tan(a_1) - \tan(a_2))^2)}$$

(33)

In order to obtain the components of the metric tensor in the power space, we employ the definition of the Hessian matrix as indicated previously, and see that the metric tensor has the following expression for the components:

$$g_{a_1a_1} = \frac{2c_2 V^2}{R_0} \frac{n^C_{11}}{r^C_{11}}$$

$$g_{a_1a_2} = \frac{2c_2 V^2}{R_0} \frac{n^C_{12}}{r^C_{12}}$$

$$g_{a_2a_2} = \frac{2c_2 V^2}{R_0} \frac{n^C_{22}}{r^C_{22}}$$

(34)

In the above equation, the numerators of the local pair correlation functions are expressed as the following trigonometric polynomials:

$$n^C_{11} := -c_1^2 s_2 + s_1 c_1^3 + 3c_1^2 c_2^2 - 2c_2 s_1 s_2 c_1^2 + 5c_1 c_2^2 s_2$$

$$+ 4c_1 c_2 s_2 - c_1 c_1^2 s_2 + 6c_1 c_1^3 s_1 s_2 - 3c_1^2 c_2 s_2$$

$$+ 7c_1^2 c_2 s_1 - 8c_1^2 c_2^2 s_2 + 3c_2 c_2^2 s_1 + 6c_1 c_2^4$$

$$- 6c_1^2 c_2^2 - 8c_1 c_2^3 s_2 + c_1 c_2^4 - c_2^4$$

$$n^C_{12} := 6c_1^2 c_2 s_1 s_2 - 3c_1^2 c_2 - 3c_1^2 s_1 c_2 + c_1^3 s_2 - 3c_1 c_2^3$$

$$+ 7c_1^2 c_2^3 + 7s_1 c_2^3 c_2^3 - 7s_2 c_2^3 c_2^3 - c_2^3 s_1 + 3c_2^3 c_2 s_1$$

$$n^C_{22} := -c_2^2 s_2 + s_1 c_1^3 + 3c_1^2 c_2^2 + 8c_1 c_2^3 s_2 + c_1 c_2 s_2$$

$$- 4c_1 c_2 s_1 + 6c_1^2 c_2 s_1 s_2 - 3c_1^2 c_2 s_2 + 8c_1^3 c_2 s_1$$

$$- 7c_1^2 c_2^3 s_2 + 3c_1 c_2^3 s_1 + 6c_1 c_2^4 - c_1 c_2^4$$

$$- 5c_1^2 c_2 s_1 - 6c_1^2 c_1^2 - 2c_3 s_1 s_2 c_1$$

(35)

The denominators of the local pair correlation functions reduce to the following trigonometric polynomials:

$$r^C_{11} := -20c_2^2 c_1^3 s_1 s_2 - c_1^3 - c_1^4 - 27c_2^4 c_1^3 + 14c_1 c_2^3 s_1 s_2$$

13
As before, it can be seen that the denominator of the metric is given by the following trigonometric polynomial:

\[ r_g := \frac{-20c_1^0c_2^0 - 26c_1^2c_2^2 + 15c_1^4c_2^4 - 27c_1^6c_2^6}{R_g^2} \]  

(36)

The determinant of the metric tensor turns out to be a rational polynomial function in the \( \{a_1, a_2\} \), which in turn is given, in compact notations, as

\[ g = \frac{4V^4c_2^2 n_g^C}{R_g^6} \]  

(37)

Herewith, the numerator of the determinant of the metric tensor is given by the following trigonometric polynomial:

\[ n_g^C := -25c_1^0c_2^0 - 18c_1^2c_2^2 + c_1^0 - 2c_1^4c_2^4 - 13c_1^6c_2^6 \]

(38)

As before, it can be seen that the denominator of the metric is given by the following trigonometric polynomial:

\[ r_g^C := -210c_1^0c_2^0 + 525c_1^2c_2^2 + 460c_1^4c_2^4 - 1190c_1^6c_2^6 + 460c_1^8c_2^8 \]

(39)

We see that the determinant of the metric tensor remains non-zero in the space of power factors and thus defines a non-degenerate intrinsic geometry on the surface of fluctuation of phases.

Finally, we may easily obtain the underlying scalar curvature which also has a rational polynomial form. It turns out that the scalar curvature can be reduced to the following form:

\[ R = \frac{R_g n_R^{(1)C} + n_R^{(2)C} + n_R^{(3)C} + n_R^{(4)C}}{r_g^{(1)C} + r_g^{(2)C} + r_g^{(3)C}} \]  

(40)

Our computation shows that the numerator of the scalar curvature takes the following trigonometric polynomial expressions:

\[ n_R^{(1)C} := -495c_1^0c_2^0 - 234c_1^2c_2^2 - 495c_1^4c_2^4 - 234c_1^6c_2^6 - 66c_1^8c_2^8 + 162c_1^{10}c_2^{10} \]

(41)

\[ +1356c_1^{12}c_2^{12} - 91736c_1^{14}c_2^{14} - 91736c_1^{16}c_2^{16} + 1356c_1^{18}c_2^{18} - 66c_1^{20}c_2^{20} \]

\[ +162c_1^{22}c_2^{22} + 392c_1^{24}c_2^{24} + 196c_1^{26}c_2^{26} - 21064c_1^{28}c_2^{28} + 220c_1^{30}c_2^{30} \]
Figure 8: The determinant of the metric tensor plotted as the function of the power factors $a_1, a_2$, describing the complex power fluctuations in electrical networks.

Figure 9: The determinant of the metric tensor plotted as the function of the equal power factor $a := a_1 = a_2$, describing the complex power fluctuations in electrical networks.
\[
\begin{align*}
n^{(2)C}_{R} &:= &90 s_c c^{10} &- 5796 s_c c^{12} &+ 594 s_c c^{12} &- 3878 s_c c^{10} c^2 \\
& &+ 14424 s_c c^{10} c^2 &- 6944 s_c c^{12} c^2 &+ 6996 s_c c^{12} c^2 &- 12292 s_c c^{12} c^2 \\
& &- 20348 s_c c^{12} c^2 &- 3600 s_c c^{12} c^2 &+ 2428 s_c c^{12} c^2 &+ 18194 s_c c^{12} c^2 \\
& &- 14436 s_c c^{12} c^2 &+ 6944 s_c c^{12} c^2 &+ 20348 s_c c^{12} c^2 &- 18194 s_c c^{12} c^2 \\
& &- 10386 s_c c^{12} c^2 &+ 9732 s_c c^{12} c^2 &+ 33034 s_c c^{12} c^2 &- 72538 s_c c^{12} c^2 \\
& &+ 38450 s_c c^{12} c^2 &- 5074 s_c c^{12} c^2 &- 902 s_c c^{12} c^2 &+ 694 s_c c^{12} c^2 \\
& &+ 5074 s_c c^{12} c^2 &+ 440 s_c c^{12} c^2 &- 30 s_c c^{12} c^2 &+ 2054 s_c c^{12} c^2 \\
& &- 5794 s_c c^{12} c^2 &- 14424 s_c c^{12} c^2 &- 2428 s_c c^{12} c^2 &+ 440 s_c c^{12} c^2 \\
& &- 660 s_c c^{12} c^2 &+ 12292 s_c c^{12} c^2 &+ 902 s_c c^{12} c^2 &- 6996 s_c c^{12} c^2 \\

n^{(3)C}_{R} &:= &+ 3878 s_c c^{12} c^2 &+ 3600 s_c c^{12} c^2 &- 6944 s_c c^{12} c^2 &+ 2428 s_c c^{12} c^2 \\
& &- 594 s_c c^{12} c^2 &+ 660 s_c c^{12} c^2 &- 2428 s_c c^{12} c^2 &- 20 s_c c^{12} c^2 \\
& &+ 30 s_c c^{12} c^2 &+ 20 s_c c^{12} c^2 &- 90 s_c c^{12} c^2 &+ 9048 s_c c^{12} c^2 \\
& &+ 31758 s_c c^{12} c^2 &+ 2160 s_c c^{12} c^2 &- 3214 s_c c^{12} c^2 &- 5512 s_c c^{12} c^2 \\
& &- 1862 s_c c^{12} c^2 &- 32 s_c c^{12} c^2 &+ 288 s_c c^{12} c^2 &- 5702 s_c c^{12} c^2 \\
& &+ 5288 s_c c^{12} c^2 &+ 2540 s_c c^{12} c^2 &- 8117 s_c c^{12} c^2 &- 31758 s_c c^{12} c^2 \\
& &- 123516 s_c c^{12} c^2 &+ 3114 s_c c^{12} c^2 &- 123516 s_c c^{12} c^2 \\
& &- 84592 s_c c^{12} c^2 &+ 2540 s_c c^{12} c^2 &- 84592 s_c c^{12} c^2 \\
& &- 28816 s_c c^{12} c^2 &- 32 s_c c^{12} c^2 &- 288 s_c c^{12} c^2 \\

n^{(4)C}_{R} &:= &- 5792 s_c c^{12} c^2 &+ 44762 s_c c^{12} c^2 &+ 5288 s_c c^{12} c^2 \\
& &+ 124 s_c c^{12} c^2 &- 12 s_c c^{12} c^2 &- 12 s_c c^{12} c^2 &+ 12 s_c c^{12} c^2 \\
& &+ 14504 s_c c^{12} c^2 &- 1552 s_c c^{12} c^2 &- 26392 s_c c^{12} c^2 &+ 122669 s_c c^{12} c^2 \\
& &- 26482 s_c c^{12} c^2 &+ 269177 s_c c^{12} c^2 &- 102868 s_c c^{12} c^2 &- 26392 s_c c^{12} c^2 \\
& &+ 122669 s_c c^{12} c^2 &- 26482 s_c c^{12} c^2 &- 1552 s_c c^{12} c^2 &+ 14504 s_c c^{12} c^2 \\
& &- 56702 s_c c^{12} c^2 &+ 11005 s_c c^{12} c^2 &- 281 s_c c^{12} c^2 &+ 64 s_c c^{12} c^2 \\
& &+ 70 s_c c^{12} c^2 &+ 1771 s_c c^{12} c^2 &+ 70 s_c c^{12} c^2 &+ 1771 s_c c^{12} c^2 \\
& &+ 122669 s_c c^{12} c^2 &+ 56702 s_c c^{12} c^2 &+ 59163 s_c c^{12} c^2 &+ 36560 s_c c^{12} c^2 \\
& &- 21276 s_c c^{12} c^2 &+ 21276 s_c c^{12} c^2 &+ 36560 s_c c^{12} c^2 \\
\end{align*}
\]
complex power fluctuations in electrical networks.

\[
\mathcal{R}^{(3)C} = \begin{pmatrix}
+220s_1c_1^2c_2^9 - 6560s_1c_1^7c_2^10 + 618s_1c_1^6c_2^7 + 2080s_1c_1^5c_2^9 \\
+4784s_1c_1^7c_2^3c_2^2 - 1116s_1c_1^7c_2^1c_2^1 + 120s_1c_1^7c_2^2c_2^2 \\

\end{pmatrix}
\]

For the choices \( V = 1 \) and \( R_0 = 1 \), the Fig. (8) shows that the determinant of the metric tensor has a commutative effect of the real and complex power flow fluctuations. This plot explicates the nature of the stability of a joint power flow in a realistic electrical network. The corresponding plot for the scalar curvature is depicted in the Fig. (10). This plot shows the global nature of a combined real and imaginary power flows in the electrical network. This analysis remains under the effect of Gaussian fluctuations of the realistic network of the resistance, reactance and impedance.

For the equal network parameters \( a_1 = a \) and \( a_2 = a \), the surface plots of the determinant of the metric tensor and scalar curvature are respectively shown in the Figs. (9) and (11). We observe herewith that the stability of the electrical network power flow exists in certain distorted bands. These distortions are strong enough, so that they are capable to modulate the global properties of the fluctuation of the power flow. Specifically, we notice, for equal values of the network power factors, that the global instabilities exit for three specific values.
Figure 11: The curvature scalar plotted as a function of the equal power factor $a := a_1 = a_2$, describing the complex fluctuations in electrical networks.

6 Conclusion and Remarks

We have studied a power system planning, where the voltage stability is the main concern. In this setup, advancements in the electricity market, thus introduced, involve radical changes in the structure of the power systems. In the electrical environment of the power industry, we have considered an intrinsic geometric model to make a power system non-linearly efficient. Our model gives promising optimization criteria to select the optimal network parameters. The robustness of the model has been illustrated by variation(s) of the impedance angles, viz., phases. For a set of impedance, resistance and capacitance, our construction describes a definite stability character, with respect to the power fluctuations of the network(s). As a function of the trigonometric polynomials, we have demonstrated that the canonical fluctuations can precisely be depicted without any approximation.

In the present paper, we have analyzed the statistical fluctuations in the real and imaginary power flows and thus characterized the network configurations. Furthermore, for the joint effect of the real and imaginary power equations, we have shown that the intrinsic geometric notion offers a clear picture of fluctuating network parameters. Such a configuration, as the limit of an ensemble of the power factor fluctuations, reduces to the specific electrical network. The present analysis does not stop here, but it invades the nature of the underlying imaginary power flow and explicates the global stability for the identical power factors. Our study thus offers an appropriate network design towards the stability of the existing linear optimization techniques.

Finally, our proposition offers suitable tests towards the planning of the network parameters of finite component power systems. In our analysis, the intrinsic geometric model takes an account of the non-linear effects, arising in the stochastic power systems. A novel approach is herewith made possible in the history of network designs, and thus the present investigation provides appropriate network designs. The criteria deduced from our method can be used for determining optimization constraints both in the economic analysis of power systems, as
well as for setting up the operating point(s) of the power system, viz., network planning and compensation techniques. This task is left for the future investigation.

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