Mesons and Higgs branch in defect theories

Daniel Areán*1, Alfonso V. Ramallo*2 and Diego Rodríguez-Gómez†3

*Departamento de Física de Partículas, Universidade de Santiago de Compostela and Instituto Galego de Física de Altas Enerxías (IGFAE) E-15782 Santiago de Compostela, Spain

†Departamento de Física, Universidad de Oviedo Avda. Calvo Sotelo 18, 33007 Oviedo, Spain

ABSTRACT

We consider the defect theory obtained by intersecting D3- and D5-branes along two common spatial directions. We work in the approximation in which the D5-brane is a probe in the $\text{AdS}_5 \times S^5$ background. By adding worldvolume flux to the D5-brane and choosing an appropriate embedding of the probe in $\text{AdS}_5 \times S^5$, one gets a supersymmetric configuration in which some of the D3-branes recombine with the D5-brane. We check this fact by showing that the D5-brane can be regarded as a system of polarized D3-branes. On the field theory side this corresponds to the Higgs branch of the defect theory, where some of the fundamental hypermultiplet fields living on the intersection acquire a vacuum expectation value. We study the spectrum of mesonic bound states of the defect theory in this Higgs branch and show that it is continuous and gapless.
1 Introduction

One of the most exciting developments in the study of the gauge/gravity correspondence [1, 2] is the extension of this duality to include open string degrees of freedom, which corresponds, on the gauge theory side, to adding matter fields in the fundamental representation of the gauge group. The standard way to perform this extension is by including D-branes (flavor branes) in the supergravity background [3, 4]. If the number of extra D-branes is small, one can neglect their backreaction on the background and treat them as probes whose fluctuation modes are identified with the mesonic bound states of the theory with flavor (for a review see [5] and references therein).

The best studied flavor brane system is the one corresponding to the D3-D7 intersection which, in the decoupling limit, is equivalent to a D7-brane embedded in the $AdS_5 \times S^5$ background such that, in the UV, the induced worldvolume metric is of the form $AdS_5 \times S^3$. In the probe approximation the fluctuation modes of this system can be integrated analytically and the corresponding meson spectra can be obtained exactly [6]. In a more recent progress [7] the meson spectrum of this system in a mixed Coulomb-Higgs branch has been obtained. In this branch some fundamental hypermultiplet fields have non-vanishing vacuum expectation values. In the dual supergravity description the Higgs branch is described by instanton configurations of the worldvolume gauge field. This instantonic gauge field lives on the directions of the D7-brane worldvolume which are orthogonal to the gauge theory directions. The meson spectra for the case of two flavors has been computed in ref. [7] by using the non-abelian Dirac-Born-Infeld action with an $SU(2)$ instanton. The corresponding mass levels depend on the size of the instanton and display an spectral flow phenomenon.

In this paper we will perform a similar analysis for the supersymmetric intersection of D3- and D5-branes, according to the array:

\begin{equation}
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
D3 : & \times & \times & \times & \quad & \quad & \quad & \quad & \quad \\
D5 : & \times & \times & \quad & \times & \times & \times & \quad & \quad \\
\end{array}
\end{equation}

Notice that the D5-brane is of codimension one along the gauge theory directions of the D3-brane worldvolume. Actually, this D3-D5 system is dual [3] to a defect theory in which $N = 4, d = 4$ super Yang-Mills theory in the bulk is coupled to $N = 4, d = 3$ fundamental hypermultiplets localized at the defect [8, 9], which is located at a fixed value of the coordinate 3 in the array (1). These hypermultiplets arise from the 3-5 open strings and their mass is proportional to the separation of the two stacks of branes in the 789 directions of the array (1). When this distance is zero, the induced metric on the worldvolume of the D5-brane is $AdS_4 \times S^2$, while for non-zero distance we introduce a mass scale which breaks conformal invariance and, as a consequence, the induced metric is $AdS_4 \times S^2$ only asymptotically in the UV. The meson spectra in this latter case can be computed analytically [10] (see also [11]).

By switching on a worldvolume magnetic field along the $S^2$, one can still have a supersymmetric configuration if the D5-brane is appropriately bent along the 3 direction [12], which corresponds to a different $AdS_4 \times S^2 \subset AdS_5 \times S^5$ embedding. This worldvolume field induces D3-brane charge to the D5-brane probe. Actually, we will show that, in this case,
the D5-brane probe can be regarded as a bound state of polarized D3-branes. Moreover, by looking directly at the action of ref. [8] for the defect field theory, we will verify that our configuration corresponds to a situation in which some of the D3-branes end on a D5-brane and recombine with it. This recombination realizes the Higgs branch of the theory, in which some components of the fundamental hypermultiplets acquire a non-vanishing vacuum expectation value. Finally, we will look at the fluctuations of the probe around the static configuration. We will show that, contrary to what happens to the D3-D7 system, there is no discrete spectrum for the meson masses in this D3-D5 intersection with flux. The reason behind this result is the fact that our fluctuations are not localized at the defect and, instead, they spread over the direction 3 of the D3-brane worldvolume.

This paper is organized as follows. In section 2 we introduce the D3-D5 configuration with flux we are interested in. In section 3 we show that the D5-brane in this configuration admits a microscopic interpretation as bound state of D3-branes. The field theory analysis and the relation to the Higgs branch of the defect theory are the subjects of section 4. The fluctuations are studied in section 5. Finally, we end with some concluding remarks in section 6.

2 The D3-D5 intersection with flux

The near-horizon limit of the metric generated by a stack of N parallel D3-branes is $AdS_5 \times S^5$, which we will write as:

$$ds^2 = \frac{r^2}{R^2} \, dx_{1,3}^2 + \frac{R^2}{r^2} \, d\vec{y} \cdot d\vec{y},$$

(2)

where $\vec{y} = (y^1, \cdots, y^6)$ are the six coordinates orthogonal to the worldvolume of the D3’s, $r^2 = \vec{y} \cdot \vec{y}$ and the radius $R$ is given by:

$$R^4 = 4\pi g_s N (\alpha')^2.$$  

(3)

In eq. (2) $dx_{1,3}^2$ represents the $(3+1)$-dimensional Minkowski metric for the coordinates $x^0, \cdots, x^3$. Moreover, the D3-brane background is endowed with a Ramond-Ramond five-form $F^{(5)}$, whose potential will be denoted by $C^{(4)}$. The component of $C^{(4)}$ along the Minkowski coordinates is given by:

$$[C^{(4)}]_{x^0 \cdots x^3} = \left[ \frac{r^2}{R^2} \right]^2.$$  

(4)

For convenience, let us split the six $\vec{y}$ coordinates in two sets of three elements, according to the D3-D5 intersection represented by the array (1). The coordinates $(y^1, y^2, y^3)$ are those which are parallel to the D5-brane worldvolume in (1). We will write their contribution to the line element in (2) in spherical coordinates as $(dy^1)^2 + (dy^2)^2 + (dy^3)^2 = d\rho^2 + \rho^2 d\Omega_2^2$, where $d\Omega_2^2$ is the line element of a unit two-sphere. Moreover, let us denote by $\vec{z} = (z^1, z^2, z^3) = (y^4, y^5, y^6)$ the coordinates transverse to both the D3- and D5-branes. Clearly, $r^2 = \rho^2 + \vec{z}^2$ and the $AdS_5 \times S^5$ metric (2) can be written as:

$$ds^2 = \frac{\rho^2 + \vec{z}^2}{R^2} \, dx_{1,3}^2 + \frac{R^2}{\rho^2 + \vec{z}^2} \left( d\rho^2 + \rho^2 d\Omega_2^2 + d\vec{z} \cdot d\vec{z} \right).$$

(5)
The action of a D5-brane probe in the $AdS_5 \times S^5$ background is given by the sum of the Born-Infeld and Wess-Zumino terms, namely:

$$S_{D5} = -T_5 \int d^6 \xi \sqrt{-\det(g + F)} + T_5 \int d^6 \xi \ P[C^{(4)}] \wedge F,$$  

(6)

where $T_5$ is the tension of the D5-brane, given by $T_5^{-1} = (2\pi)^5 (\alpha')^3 g_s$, $g$ is the pullback of the metric (5), $F$ is the strength of the abelian worldvolume gauge field and $\xi^a (a = 0, \ldots, 5)$ are a set of worldvolume coordinates. In what follows we will use $x^0, x^1, x^2$ and the radial ($\rho$) and angular coordinates of eq. (5) as our set of worldvolume coordinates. The embedding of the D5-brane probe is then specified by the values of $x^3$ and $\vec{z}$ as functions of the $\xi^a$'s. We will consider static embeddings in which $|\vec{z}|$ is a fixed constant, namely $|\vec{z}| = L$. The simplest of such embeddings is the one in which the coordinate $x^3$ is also a constant and the worldvolume gauge field $F$ vanishes. This configuration was proposed in ref. [3], and studied extensively in ref. [8], as a prototype of a defect theory. In this case the defect is a flat wall determined by the condition $x^3 =$ constant and the induced worldvolume metric is, for large $\rho$, of the form $AdS_4 \times S^2$. The corresponding dual field theory is $\mathcal{N} = 4, d = 4$ super Yang-Mills theory coupled to $\mathcal{N} = 4, d = 3$ hypermultiplets localized at the defect which are in the fundamental representation. Let us generalize this configuration of the D5-brane probe by switching on a magnetic field $F$ along the two-sphere of its worldvolume. To be precise, let us assume that $F$ is given by:

$$F = q \text{Vol}(S^2) \equiv F,$$  

(7)

where $q$ is a constant and $\text{Vol}(S^2)$ is the volume form of the worldvolume two-sphere. To understand the implications of having a magnetic flux across the worldvolume $S^2$, let us look at the form of the Wess-Zumino term in the action (6), namely:

$$S_{WZ} \sim \int_{S^2} F \int P[C^{(4)}] \sim q x',$$  

(8)

where $x \equiv x^3$ and the prime denotes the derivative with respect to the radial coordinate $\rho$. It is clear by inspecting the right-hand side of eq. (8) that the worldvolume flux acts as a source of a non-trivial dependence of $x$ on the coordinate $\rho$. Actually, assuming that $x$ only depends on $\rho$, the action (6) of the probe takes the form:

$$S_{D5} = -4\pi T_5 \int d^3 x d\rho \left[ \rho^2 \left( 1 + \frac{(\rho^2 + L^2)^2}{R^4} x'^2 \right) \sqrt{1 + \frac{(\rho^2 + L^2)^2}{R^4} q^2 \rho^4} - \frac{(\rho^2 + L^2)^2}{R^4} q x' \right],$$  

(9)

where we have assumed that $\vec{z}$ is constant ($|\vec{z}| = L$) and we have integrated over the coordinates of the two-sphere. The Euler-Lagrange equation for $x(\rho)$ derived from (9) is quite involved. However, there is a simple first-order equation for $x(\rho)$ which solves this equation [12], namely:

$$x'(\rho) = \frac{q}{\rho^2}.$$  

(10)

Actually, the first-order equation (10) is a BPS equation required by supersymmetry, as can be verified by checking the kappa symmetry of the embedding [12]. The integration of eq.
(10) is straightforward:

\[ x(\rho) = x_0 - \frac{q}{\rho}, \tag{11} \]

where \( x_0 \) is a constant. The dependence on \( \rho \) of the right-hand side of eq. (11) represents the bending of the D5-brane profile required by supersymmetry when there is a non-vanishing flux of the worldvolume gauge field. Notice also that now the probe is located at a fixed value of \( x \) only at the asymptotic value \( \rho \to \infty \), whereas when \( \rho \) varies the D5-brane fills one-half on the worldvolume of the D3-brane (i.e. \( x^3 \leq x_0 \) for \( q > 0 \)).

It is also interesting to study the modifications of the induced metric introduced by the bending. Actually, when \( q \neq 0 \) this induced metric takes the form:

\[ G_{ab} d\xi^a d\xi^b = \frac{\rho^2 + L^2}{R^2} dx_{1,2}^2 + \frac{R^2}{\rho^2 + L^2} \left[ 1 + \frac{q^2}{R^4} \frac{(\rho^2 + L^2)^2}{\rho^4} \right] d\rho^2 + \rho^2 d\Omega_2^2. \tag{12} \]

It can be readily verified from (12) that the UV metric at \( \rho \to \infty \) (or, equivalently, when the D3- and D5-branes are at zero distance \( L \)) takes the form:

\[ AdS_4(R_{eff}) \times S^2(R), \tag{13} \]

where the radius of the \( AdS_4 \) changes from its fluxless value \( R \) to \( R_{eff} \), with the latter given by:

\[ R_{eff} = \left( 1 + \frac{q^2}{R^4} \right)^{\frac{1}{2}} R. \tag{14} \]

Notice that the radius of the \( S^2 \) is not affected by the flux, as is clear from (12).

One can understand the appearance of this UV metric as follows. Let us suppose that we have an \( AdS_5 \) metric of the form:

\[ ds_{AdS_5}^2 = \frac{\rho^2}{R^2} dx_{1,3}^2 + \frac{R^2}{\rho^2} d\rho^2. \tag{15} \]

Let us now change variables from \( (\rho, x^3) \) to new coordinates \( (\varrho, \eta) \), as follows:

\[ x^3 = \bar{x} - \frac{\tanh \eta}{\varrho}, \quad \rho = R^2 \varrho \cosh \eta, \tag{16} \]

where \( \bar{x} \) is a constant. It can be easily seen that the \( AdS_5 \) metric (15) in the new variables takes the form:

\[ ds_{AdS_5}^2 = R^2 (\cosh^2 \eta ds_{AdS_4}^2 + d\eta^2) , \tag{17} \]

where \( ds_{AdS_4}^2 \) is the metric of \( AdS_4 \) with unit radius, given by:

\[ ds_{AdS_4}^2 = \varrho^2 dx_{1,2}^2 + d\varrho^2. \tag{18} \]

Eq. (17) shows clearly the foliation of \( AdS_5 \) by \( AdS_4 \) slices with \( \eta = \) constant. The effective radius of the \( AdS_4 \) slice depends on the value of \( \eta \) as follows:

\[ R_{eff} = R \cosh \eta. \tag{19} \]
It can be straightforwardly checked by using the change of variables (16) with $\bar{x} = x_0$ that our embedding (11) corresponds to one of these $AdS_4$ slices with a constant value of $\eta$ given by:

$$\eta = \eta_q = \sinh^{-1}\left(\frac{q}{R^2}\right).$$  

(20)

Moreover, one can verify that the $AdS_4$ radius $R_{eff}$ of eq. (19) reduces to the expression given in (14) when $\eta = \eta_q$.

The worldvolume gauge field (7) is constrained by a flux quantization condition [13] which, with our notations, reads:

$$\int_{S^2} F = \frac{2\pi k}{T_f}, \quad k \in \mathbb{Z}, \quad T_f = \frac{1}{2\pi \alpha'}. $$

(21)

It is now immediate to conclude that the condition (21) restricts the constant $q$ to be of the form:

$$q = k\pi \alpha', $$

(22)

where $k$ is an integer.

3 Dielectric interpretation

The presence of a worldvolume flux as in (7) induces, through the Wess-Zumino term of the action (6), a D3-brane charge, proportional to $\int_{S^2} F$, on the D5-brane. For this reason it is not surprising that the D5-brane configuration of section 2 admits a microscopical description in terms of a bound state of coincident D3-branes. Actually, the integer $k$ of the quantization condition (21) has the interpretation of the number of D3-branes that build up the D5-brane. The dynamics of a stack of coincident D3-branes is determined by the Myers dielectric action [14], which is the sum of a Born-Infeld and a Wess-Zumino part:

$$S_{D3} = S_{BI}^{D3} + S_{WZ}^{D3}. $$

(23)

For the background we are considering the Born-Infeld action is:

$$S_{BI}^{D3} = -T_3 \int d^4\xi \text{ Str} \left[ \sqrt{-\det \left[ P[G + G(Q^{-1} - \delta)G]_{ab} \right]} \sqrt{\det Q} \right], $$

(24)

where we have set the worldvolume gauge field to zero. In eq. (24) $T_3$ is the tension of the D3-brane, given by $T_3^{-1} = (2\pi)^3 \alpha'^2 g_s$, and $G$ is the background metric (5). In this dielectric picture the D3-brane has non-commutative transverse scalars represented by matrices. In eq. (24) $\text{Str}(\cdots)$ represents the symmetrized trace and $Q$ is a matrix which depends on the commutator of the transverse scalars (see below). The Wess-Zumino term for the D3-brane in the $AdS_5 \times S^5$ background under consideration is:

$$S_{WZ}^{D3} = T_3 \int d^4\xi \text{ Str} \left[ P[C^{(4)}(\cdot)] \right]. $$

(25)
Let us now choose $x^0, x^1, x^2$ and $\rho$ as our set of worldvolume coordinates of the D3-branes. Moreover, we shall introduce new coordinates $Y^I (I = 1, 2, 3)$ for the two-sphere of the metric (5). These new coordinates satisfy $\sum_I Y^I Y^I = 1$ and the line element $d\Omega_2^2$ is given by:

$$d\Omega_2^2 = \sum_I dY^I dY^I, \quad \sum_I Y^I Y^I = 1 .$$

We will assume that the $Y^I$'s are the only non-commutative scalars. They will be represented by $k \times k$ matrices. In this case the matrix $Q$ appearing in (24) is given by:

$$Q^I_J = \delta^I_J + \frac{i}{2\pi\alpha'} [Y^I, Y^K] G_{KJ} .$$

Actually, we shall adopt the ansatz in which the $Y^I$'s are constant and given by:

$$Y^I = \frac{J^I}{\sqrt{C_2(k)}},$$

where the $k \times k$ matrices $J^I$ correspond to the $k$-dimensional irreducible representation of the $SU(2)$ algebra:

$$[J^I, J^J] = 2i\epsilon_{IJK} J^K,$$

and $C_2(k)$ is the quadratic Casimir of the $k$-dimensional irreducible representation of $SU(2)$ ($C_2(k) = k^2 - 1$). Therefore, the $Y^I$ scalars parametrize a fuzzy two-sphere. Moreover, let us assume that we consider embeddings in which the scalars $\vec{z}$ and $x^3$ are commutative and such that $|\vec{z}| = L$ and $x^3 = x(\rho)$ (a unit $k \times k$ matrix is implicit). With these conditions, as the metric (5) does not mix the directions of the two-sphere with the other coordinates, the matrix $Q^{-1} - \delta$ does not contribute to the first square root on the right-hand side of (24) and we get:

$$\sqrt{-\det [P[G]]} = \frac{\rho^2 + L^2}{R^2} \sqrt{1 + \frac{(\rho^2 + L^2)^2}{R^4} x'^2} .$$

Moreover, by using the ansatz (28) and the commutation relations (29) we obtain that, for large $k$, the second square root appearing in (24) can be written as:

$$\text{Str} \left[ \sqrt{\det Q} \right] \approx \frac{R^2}{\pi\alpha'} \frac{\rho^2}{\rho^2 + L^2} \sqrt{1 + \frac{(\rho^2 + L^2)^2 (k\pi\alpha')^2}{R^4 \rho^4}} .$$

Using these results, the Born-Infeld part of the D3-brane action in this large $k$ limit takes the form:

$$S_{BI}^{D3} = -\frac{T_3}{\pi\alpha'} \int d^3 x d\rho \rho^2 \sqrt{1 + \frac{(\rho^2 + L^2)^2}{R^4} x'^2} \sqrt{1 + \frac{(\rho^2 + L^2)^2 q^2}{R^4 \rho^4}} ,$$

where we have already used (22) to write the result in terms of $q$. Due to the relation $T_3 = 4\pi^2 \alpha' T_5$ between the tensions of the D3- and D5-branes, one checks by inspection that the right-hand side of (32) coincides with the Born-Infeld term of the D5-brane action.
Notice also that the quantization integer \( k \) in (21) is identified with the number of D3-branes. Moreover, the Wess-Zumino term (25) becomes:

\[
S_{WZ}^{D3} = k T_3 \int d^3x \, d\rho \, \frac{(\rho^2 + L^2)^2}{R^4} x'.
\] (33)

The factor \( k \) in (33) comes from the trace of the unit \( k \times k \) matrix. By comparing (33) with the Wess-Zumino term of the macroscopical action (9) one readily concludes that they coincide because of the relation \( 4\pi q T_5 = k T_3 \), which can be easily proved.

4 Field theory analysis

In this section we will analyze the configuration described above from the point of view of the field theory at the defect which, from now on, we shall assume that it is located at \( x^3 = 0 \). Recall that the defect arises as a consequence of the impurity created on the D3-brane worldvolumes by the D5-brane which intersects with them according to the array (1). We are interested in analyzing, from the field theory point of view, the configurations in which some fraction of the D3-branes end on the D5-brane and recombine with it at the defect point \( x^3 = 0 \), realizing in this way a (mixed Coulomb-) Higgs branch of the defect theory.

The field theory dual to the D3-D5 intersection has been worked out by DeWolfe et al. in ref. [8]. The theory, which includes \( \mathcal{N} = 4 \) SU(\( N \)) SYM in 4d plus an \( \mathcal{N} = 4 \) hypermultiplet confined to the defect, has an SU(\( 2 \))\(_H \) \( \times \) SU(\( 2 \))\(_V \) R-symmetry. The SU(\( 2 \))\(_H \) (SU(\( 2 \))\(_V \)) symmetry corresponds to the rotations in the 456 (789) directions of the array (1). Written in terms of \( \mathcal{N} = 1 \) SUSY, this hypermultiplet gives rise to a chiral (\( Q \)) and an antichiral (\( \bar{Q} \)) supermultiplet, which are both doublets under SU(\( 2 \))\(_H \) while being in the fundamental representation of the gauge group. In addition, the 6 scalars of the bulk \( \mathcal{N} = 4 \), which are in the adjoint of the gauge group, naturally split in two sets, the first (which we will call \( \phi_I^H \)) forming a vector of SU(\( 2 \))\(_H \) and the second, which we denote by \( \phi^A_V \), a vector of SU(\( 2 \))\(_V \). Thus, the bosonic content of the theory is:

| Field | SU(\( N \)) | SU(\( 2 \))\(_H \) | SU(\( 2 \))\(_V \) |
|-------|-------------|--------------------|--------------------|
| \( A_\mu \) | adjoint     | singlet            | singlet            |
| \( \phi_I^H \) | adjoint     | vector             | singlet            |
| \( \phi^A_V \) | adjoint     | singlet            | vector             |
| \( q \) | fundamental | doublet            | singlet            |
| \( \bar{q} \) | fundamental | doublet            | singlet            |

We will assume that only the fields \( \phi_H, \phi_V, q \) and \( \bar{q} \) are non-vanishing. The defect action for this theory has a potential term which can be written as [8]:

\[
S_{\text{defect}} = -\frac{1}{g^2} \int d^3x \left[ \bar{q}^m (\phi_V^A)^2 q^m + \frac{i}{2} \epsilon_{IJK} \bar{q}^m \sigma_{mn} [\phi_J^I, \phi_K^H] q^n \right] - \frac{1}{g^2} \int d^3x \left[ \bar{q}^m \sigma_{mn} \partial_3 \phi_H^I q^n + \frac{1}{2} \delta(x_3) (\bar{q}^m \sigma_{mn} T^a q^n)^2 \right],
\] (34)
where the integration is performed over the $x^3 = 0$ three-dimensional submanifold and $g$ is the Yang-Mills coupling constant. In the supersymmetric configurations we are looking for the potential term must vanish. Let us cancel the contribution of $\phi_V$ to the right-hand side of (34) by requiring that:

$$\phi_V q = 0.$$  \hfill (35)

We can insure this property by taking $q$ as:

$$q = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \alpha_1 \\ \vdots \\ \alpha_k \end{pmatrix},$$  \hfill (36)

and by demanding that $\phi_V$ is of the form:

$$\phi_V = \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix},$$  \hfill (37)

where $A$ is an $(N - k) \times (N - k)$ traceless matrix. Moreover, we shall take $\phi_V$, $q$ and $\bar{q}$ constant, which is enough to guarantee that their kinetic energy vanishes. Notice that the scalars $\phi_V$ correspond to the directions 789 in the array (1), which are orthogonal to both the D3- and D5-brane. Having $\phi_V \neq 0$ is equivalent to taking $|\vec{z}| = L \neq 0$ in the approach of sections 2 and 3, and it corresponds to a non-zero value of the mass of the hypermultiplets (see the first term in the defect action (34)).

Let us now consider the configurations of $\phi_H$ with vanishing energy. First of all we will impose that $\phi_H$ is a matrix whose only non-vanishing entries are in the lower $k \times k$ block. In this way the mixing terms of $\phi_V$ and $\phi_H$ cancel. Moreover, assuming that $\phi_H$ only depends on the coordinate $x^3$, the surviving terms in the bulk action are [8]:

$$S_{\text{bulk}} = -\frac{1}{g^2} \int d^4x \text{Tr} \left[ \frac{1}{2} (\partial_3 \phi_H^I)^2 - \frac{1}{4} [\phi_H^I, \phi_H^K]^2 \right],$$  \hfill (38)

where the trace is taken over the color indices. It turns out that the actions (34) and (38) can be combined in such a way that their sum can be written as an integral over the four-dimensional spacetime of the trace of a square. In order to write this expression, let us define the matrix $\alpha^I = \alpha^I_a T^a$, where the $T^a$'s are the generators of the gauge group and the $\alpha^I_a$'s are defined as the following expression bilinear in $q$ and $\bar{q}$:

$$\alpha^I_a \equiv \bar{q}^m \alpha^I_m T^a q^n.$$  \hfill (39)

It is now straightforward to check that the sum of (34) and (38) can be put as:

$$S_{\text{defect}} + S_{\text{bulk}} = -\frac{1}{2g^2} \int d^4x \text{Tr} \left[ \partial_3 \phi_H^I + \frac{i}{2} \epsilon_{IJK} [\phi_H^I, \phi_H^K] + \alpha^I \delta(x^3) \right]^2,$$  \hfill (40)
where we have used the fact that $\epsilon_{IJK} \text{Tr} \left( \partial_3 \phi^I_H [\phi^J_H, \phi^K_H] \right)$ is a total derivative with respect to $x^3$ and, thus, can be dropped if we assume that $\phi^H$ vanishes at $x^3 = \pm \infty$. It is now clear from (40) that we must require the Nahm equations [15]:

$$\partial_3 \phi^I_H + \frac{i}{2} \epsilon_{IJK} [\phi^J_H, \phi^K_H] + \alpha^I(x^3) = 0 .$$

(41)

(For a nice review of the Nahm construction in string theory see [16]).

Notice that when $\alpha^I$ vanishes, eq. (41) admits the trivial solution $\phi^H = 0$. On the contrary, if the fundamentals $q$ and $\bar{q}$ acquire a non-vanishing vacuum expectation value as in (36), $\alpha^I$ is generically non-zero and the solution of (41) must be non-trivial. Actually, it is clear from (41) that in this case $\phi^H$ must blow up at $x^3 = 0$, which shows how a non-vanishing vacuum expectation value of the fundamentals acts as a source for the brane recombination in the Higgs branch of the theory. Let us check these facts more explicitly by solving (41) for $x^3 \neq 0$, where the $\delta$-function term is zero. We shall adopt the ansatz:

$$\phi^I_H(x) = f(x) \phi^I_0 ,$$

(42)

where $x$ stands for $x^3$ and $\phi^I_0$ are constant matrices. The differential equation (41) reduces to:

$$\frac{f'}{f^2} \phi^I_0 + \frac{i}{2} \epsilon_{IJK} [\phi^J_0, \phi^K_0] = 0 ,$$

(43)

where the prime denotes derivative with respect to $x$. We shall solve this equation by first putting:

$$\phi^I_0 = \frac{1}{\sqrt{C_2(k)}} \begin{pmatrix} 0 & 0 \\ 0 & J^I \end{pmatrix} ,$$

(44)

where the $J^I$ are matrices in the $k$-dimensional irreducible representation of the $SU(2)$ algebra, which satisfy the commutation relations (29), and we have normalized the $\phi^I_0$’s such that $\phi^I_0 \phi^I_0$ is the unit matrix in the $k \times k$ block. By using this representation of the $\phi^I_0$’s, eq. (43) reduces to:

$$\frac{f'}{f^2} = \frac{2}{\sqrt{C_2(k)}} ,$$

(45)

which can be immediately integrated, namely:

$$f = -\sqrt{\frac{C_2(k)}{2x}} .$$

(46)

For large $k$, the quadratic Casimir $C_2(k)$ behaves as $k^2$ and this equation reduces to:

$$f = -\frac{k}{2x} .$$

(47)

Let us now take into account the standard relation between coordinates $X^I_H$ and scalar fields $\phi^I_H$, namely:

$$X^I_H = 2\pi \alpha' \phi^I_H ,$$

(48)
and the fact that \( \rho^2 \equiv X_{\mu}^I X_{\mu}^I \). Using these facts we immediately get the following relation between \( \rho \) and \( f \):

\[
\rho = 2\pi \alpha' f ,
\]

and the solution (47) of the Nahm equation can be written as:

\[
\rho = -\frac{\pi k \alpha'}{x} ,
\]

which, if we take into account the quantization condition (22), is just our embedding (11) for \( x_0 = 0 \). As expected, \( \rho \) blows up at \( x = 0 \), while its dependence for \( x \neq 0 \) gives rise to the same bending as in the brane approach. Notice also that, in this field theory perspective, the integer \( k \) is the rank of the gauge theory subgroup in which the Higgs branch of the theory is realized, which corresponds to the number of D3-branes that recombine into a D5-brane.

5 Fluctuations

Let us now analyze the small fluctuations around the static embedding of the D5-brane probe described in section 2. For simplicity we will restrict ourselves to study the fluctuations of the scalars transverse to both the D3- and D5-branes (i.e. those along the directions 789 in the array (1)). It can be shown that, at quadratic order, these fluctuations do not couple to those corresponding to the worldvolume gauge field and the scalar \( x^3 \). In the unperturbed configuration the distance \( |\vec{z}| \) between the two types of branes is constant and equal to \( L \) and, without loss of generality, we can assume that the branes are separated along the \( z^1 \) direction. Accordingly, let us consider a fluctuation of the type:

\[
z^1 = L + \chi^1 , \quad z^2 = \chi^2 , \quad z^3 = \chi^3 ,
\]

where the \( \chi^m \) are small. The induced metric for this perturbed configuration can be decomposed as:

\[
g = G + g^{(f)} ,
\]

where \( G \) is the metric written in (12) and \( g^{(f)} \) is the part of \( g \) that depends on the derivatives of the fluctuations, namely:

\[
\begin{align*}
g^{(f)}_{ab} &= \frac{R^2}{\rho^2 + L^2} \partial_a \chi^m \partial_b \chi^m .
\end{align*}
\]

The Born-Infeld determinant in the action (6) can be written as:

\[
\sqrt{-\det(g + F)} = \sqrt{-\det (G + F)} \sqrt{\det (1 + M)} ,
\]

where \( F \) is the worldvolume gauge field (7) and the matrix \( M \) is given by:

\[
M \equiv \left( G + F \right)^{-1} g^{(f)} .
\]
To evaluate the right-hand side of eq. (54), we shall use the expansion:

$$\sqrt{\det (1 + M)} = 1 + \frac{1}{2} \text{Tr}M + o(M^2).$$  \hspace{1cm} (56)

The prefactor multiplying this expansion in (54) is:

$$\sqrt{-\det (\mathcal{G} + \mathcal{F})} = \rho^2 \sqrt{\tilde{g}} \left( 1 + \frac{q^2 (\rho^2 + L^2)^2}{R^4} \right),$$  \hspace{1cm} (57)

where $\tilde{g}$ is the determinant of the round metric for the unit two-sphere. Moreover, let us separate the symmetric and antisymmetric part in the inverse matrix appearing in the expression of $M$ (eq. (55)):

$$\left( \mathcal{G} + \mathcal{F} \right)^{-1} = \hat{\mathcal{G}}^{-1} + \mathcal{J},$$  \hspace{1cm} (58)

where:

$$\hat{\mathcal{G}}^{-1} = \frac{1}{(\mathcal{G} + F)_S}, \quad \mathcal{J} = \frac{1}{(\mathcal{G} + F)_A}.$$  \hspace{1cm} (59)

Notice that $\hat{\mathcal{G}}$ is just the open string metric. After a straightforward calculation one can verify that $\hat{\mathcal{G}}$ can be written as:

$$\hat{G}_{ab} \xi^a \xi^b = \frac{\rho^2 + L^2}{R^2} dx^2_{1,2} + \frac{R^2}{\rho^2 + L^2} \left( 1 + \frac{q^2 (\rho^2 + L^2)^2}{R^4} \right) \left( d\rho^2 + \rho^2 d\Omega^2 \right).$$  \hspace{1cm} (60)

Moreover, the antisymmetric matrix $\mathcal{J}$ has only non-vanishing values when its two-indices are on the two-sphere. Actually, if $\theta$ and $\varphi$ are the standard polar coordinates on $S^2$, we have:

$$\mathcal{J}^{\theta\varphi} = -\mathcal{J}^{\varphi\theta} = -\frac{1}{\sqrt{\tilde{g}}} \frac{q}{q^2 + \frac{R^4 \rho^4}{\rho^2 + L^2} R^4}. $$  \hspace{1cm} (61)

Notice that the antisymmetric matrix $\mathcal{J}$ does not contribute to $\text{Tr}M$ since it is contracted with $g^{(f)}$, which is symmetric. The final result for $\text{Tr}M$ is:

$$\text{Tr}M = \frac{R^2}{\rho^2 + L^2} \hat{G}^{ab} \partial_a \chi^m \partial_b \chi^m.$$  \hspace{1cm} (62)

By using this result we get that the total lagrangian density for the $\chi$ fluctuations is given by:

$$\mathcal{L} = -\rho^2 \frac{\sqrt{\tilde{g}}}{2} \frac{R^2}{\rho^2 + L^2} \left( 1 + \frac{q^2 (\rho^2 + L^2)^2}{R^4} \right) \hat{G}^{ab} \partial_a \chi^m \partial_b \chi^m.$$  \hspace{1cm} (63)

It is clear from (63) that the open string metric $\hat{\mathcal{G}}$ governs the dynamics of the fluctuations. For this reason, it is interesting to look at $\hat{\mathcal{G}}$ closely and, in particular to compare it with the induced metric $\mathcal{G}$ of eq. (12). Notice that $\mathcal{G}$ and $\hat{\mathcal{G}}$ only differ in the term corresponding to the two-sphere. Actually, the metric $\hat{\mathcal{G}}$ in the UV ($\rho \to \infty$) becomes $AdS_4(R_{\text{eff}}) \times S^2(R_{\text{eff}})$, where $R_{\text{eff}}$ is the effective radius of eq. (14) (compare this result with (13)). If the separation distance $L$ is zero $\mathcal{G}$ and $\hat{\mathcal{G}}$ retain this $AdS \times S$ form for all values of $\rho$. However, when $L \neq 0$ the IR behaviour of these metrics changes drastically. Actually, it is clear from (12) that
the $S^2$ factor in $\mathcal{G}$ collapses at $\rho = 0$. On the contrary, when $q \neq 0$, the terms with $q$ in $\hat{\mathcal{G}}$ dominate over the others in the IR and the open string metric takes the form:

$$\hat{\mathcal{G}}_{ab} d\xi^a d\xi^b \approx \frac{L^2}{R^2} \left[ dx_{1,2}^2 + q^2 \left( \frac{d\rho^2}{\rho^4} + \frac{1}{\rho^2} d\Omega_2^2 \right) \right], \quad (\rho \approx 0).$$

Notice that now $d\Omega_2^2$ is multiplied by a factor that diverges for $\rho \approx 0$ in (64). Actually, by changing variables from $\rho$ to $u = q/\rho$, this metric can be written as:

$$\frac{L^2}{R^2} \left[ dx_{1,2}^2 + du^2 + u^2 d\Omega_2^2 \right],$$

which is nothing but the six dimensional Minkowski space.

The equation of motion for the transverse scalars $\chi$ derived from (63) is:

$$\partial_a \left[ \sqrt{g} \frac{\rho^2}{\rho^2 + L^2} \left( 1 + \frac{q^2}{R^4} \frac{(\rho^2 + L^2)^2}{\rho^4} \right) \hat{\mathcal{G}}^{ab} \partial_b \right] \chi = 0.$$  

By using the explicit form of the effective metric $\hat{\mathcal{G}}^{ab}$ (eq. (60)), we can write this equation as:

$$\left[ \frac{R^4 \rho^2}{(\rho^2 + L^2)^2} + \frac{q^2}{\rho^2} \right] \partial^\mu \partial_\mu \chi + \partial_\rho \left( \rho^2 \partial_\rho \chi \right) + \nabla^i \nabla_i \chi = 0,$$

where the indices $\mu$ correspond to the $2+1$ Minkowski directions and the $i$'s are those of the two-sphere. Let us next separate variables and write the scalars $\chi$ in terms of the spherical harmonics on the two-sphere and plane waves in the Minkowski coordinates:

$$\chi = e^{ikx} Y^l(S^2) \xi(\rho),$$

where the product $kx$ is performed with the flat Minkowski metric and $l$ denotes the angular momentum on the $S^2$. The mass of the meson is defined as $M^2 = -k^2$. By using this ansatz, the equation of motion (67) reduces to:

$$\partial_\rho \left( \rho^2 \partial_\rho \xi \right) + \left\{ \left[ \frac{R^4 \rho^2}{(\rho^2 + L^2)^2} + \frac{q^2}{\rho^2} \right] M^2 - l(l+1) \right\} \xi = 0.$$  

Let us analyze the solutions of (69) when the distance $L \neq 0$. In general, the requirement of having a regular normalizable solution for the fluctuation $\xi$ determines the allowed values of the mass $M$. Let us now look at this regularity condition at the UV. For $\rho \to \infty$ one can easily show that there are two independent solutions of (69) which behave as $\rho^l$ or $\rho^{-l-1}$. Clearly, the admissible fluctuations are the ones decreasing as $\xi \sim \rho^{-l-1}$ for large $\rho$. For $q = 0$ (i.e. without flux) the fluctuations also behave as $\rho^\gamma$, with $\gamma = l, -l - 1$ when $\rho \approx 0$ and, thus, we should impose that $\xi \sim \rho^l$ at the IR. The matching of the $\rho \approx 0$ behaviour with that for $\rho \to \infty$ is only possible for a discrete set of values of the mass $M$. The corresponding discrete spectrum was found analytically in [10] and has a mass gap proportional to $L/R^2$. However, when $q \neq 0$ the $\rho \approx 0$ behaviour of the solutions of (69) changes drastically. Indeed, when $\rho$ is small and $q$ does not vanish eq. (69) reduces to:

$$\partial_\rho \left( \rho^2 \partial_\rho \xi \right) + \left[ \frac{q^2 M^2}{\rho^2} - l(l+1) \right] \xi = 0, \quad (\rho \approx 0).$$
Eq. (70) can be solved in terms of Bessel functions, namely:

\[ \xi = \frac{1}{\sqrt{\rho}} J_{\pm \left(\frac{\ell + \frac{1}{2}}{2}\right)} \left( \frac{qM}{\rho} \right), \quad (\rho \approx 0) . \] (71)

Near \( \rho \approx 0 \) the Bessel function (71) behaves as:

\[ \xi \approx e^{\pm i \frac{qM}{\rho}}, \quad (\rho \approx 0) , \] (72)

i.e. it oscillates infinitely as we approach \( \rho = 0 \). Notice that this gives rise to a continuous gapless spectrum for \( M \). Actually, one can understand this result by rewriting the function (71) in terms of the coordinate \( x^3 \) by using (11). Indeed, \( \rho \approx 0 \) corresponds to large \( |x^3| \) and \( \xi(x^3) \) can be written in this limit as a simple plane wave:

\[ \xi \approx e^{\pm i Mx^3}, \quad (|x^3| \to \infty) . \] (73)

Notice that this behaviour is consistent with the fact that the IR metric for \( q \neq 0 \) approaches the Minkowski metric (65) and the fluctuation spreads out of the defect locus \( x^3 = 0 \). We have checked that this fact is generic by analyzing the full set of fluctuations of the D3-D5 system. These fluctuations are coupled, but it turns out that they can be decoupled by using the same techniques as those employed in ref. [10] for the \( q = 0 \) case. One can show that these decoupled functions have the same qualitative behaviour as in (72), which implies that the spectrum is continuous for \( q \neq 0 \) [17].

6 Concluding Remarks

In the field theory dual, the addition of a brane probe to a given background corresponds to the insertion of an impurity, which is generically located at a defect in the gauge theory directions. This defect hosts new open string degrees of freedom (hypermultiplets) which interact non-trivially with the original bulk fields. The AdS/CFT correspondence can be used to obtain the mass spectra of the mesonic operators, which are bilinear in the hypermultiplet fields, by studying the fluctuations of the flavor branes [4]. As reviewed in [5], when the hypermultiplets have a non-vanishing mass, one gets a well-defined discrete spectrum with a non-zero mass gap.

In this paper we have explored the effect of giving a vacuum expectation value to some components of the hypermultiplets. In string theory, such a Higgs branch is realized by recombining color and flavor branes in a non-trivial way. From the point of view of the worldvolume theory of the flavor brane, this recombination is realized by adding a suitable flux of the worldvolume gauge field, such that some units of charge of the color brane are dissolved in the worldvolume of the flavor brane. Then, supersymmetry requires that the flavor brane must be bent appropriately, as in eq. (11).

A natural question, which we addressed in this paper for the defect theory dual to the D3-D5 intersection, is how the meson spectrum is affected when one moves to the Higgs branch. We have shown above that the mesonic mass spectrum of the defect theory is drastically changed (it becomes continuous and gapless). The reason behind this behaviour
is the fact that the color and flavor branes are connected to each other, which delocalizes
the fluctuations in the direction orthogonal to the defect. Actually, this behaviour is generic
of any intersection dual to a defect theory with codimension greater than zero [17]. On
the contrary, this does not happen in the cases in which the flavor brane fills completely
the gauge theory directions, such as the D3-D7 intersection with a worldvolume instanton
studied in ref. [7]. We expect to report on these results in ref. [17].

Acknowledgments

We are grateful to J. D. Edelstein, M. Kruczenski, C. Núñez and to the participants of the
Ringberg Castle workshop “QCD and String Theory” (where a preliminary version of this work
was presented) for discussions. The work of DA and AVR was supported in part by MCyT,
FEDER and Xunta de Galicia under grant FPA2005-00188 and by the EC Commission under
grants HPRN-CT-2002-00325 and MRTN-CT-2004-005104. DRG is grateful to Universidad
Autonoma de Madrid, Universidad de Granada and Universidade de Santiago de Compostela
for their hospitality and support during various stages of this work. His work has been also
partially supported by CICYT grant BFM2003-00313 (Spain) and by the EC Commission FP6
program MRTN-CT-2004-005104 in which he is associated to UAM.

References

[1] J. M. Maldacena, “The large $N$ limit of superconformal field theories and supergravity”
Adv. Theor. Math. Phys. 2 (1998) 231, hep-th/9711200.

[2] For a review see, O. Aharony, S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, “Large $N$
field theories, string theory and gravity”, Phys. Rept. 323 (2000) 183, hep-th/9905111.

[3] A. Karch and L. Randall, “Locally localized gravity”, J. High Energy Phys. 0105 (2001)
008, hep-th/0011156; “Open and closed string interpretation of SUSY CFT’s on branes
with boundaries”, J. High Energy Phys. 0106 (2001) 063, hep-th/0105132.

[4] A. Karch and E. Katz, “Adding flavor to AdS/CFT”, J. High Energy Phys. 0206 (2002)
043, hep-th/0205236;
A. Karch, E. Katz and N. Weiner, “Hadron masses and screening from AdS Wilson
loops”, Phys. Rev. Lett. 90 (2003) 091601, hep-th/0211107.

[5] A. V. Ramallo, “Adding open string modes to the gauge/gravity correspondence”, Mod.
Phys. Lett. A21 (2006) 1, hep-th/0605261.

[6] M. Kruczenski, D. Mateos, R. Myers and D. Winters, “Meson spectroscopy in AdS/CFT
with flavour”, J. High Energy Phys. 0307 (2003) 049, hep-th/0304032

[7] J. Erdmenger, J. Grosse and Z. Guralnik, “Spectral flow on the Higgs branch and
AdS/CFT duality”, J. High Energy Phys. 0506 (2005) 052, hep-th/0502224;
R. Apreda, J. Erdmenger, N. Evans and Z. Guralnik, “Strong coupling effective Higgs
potential and a first order thermal phase transition from AdS/CFT duality”, Phys. Rev. D 71(2005) 126002, hep-th/0504151.

[8] O. DeWolfe, D. Z. Freedman and H. Ooguri, “Holography and defect conformal field theories”, Phys. Rev. D66 (2002) 025009, hep-th/0111135.

[9] J. Erdmenger, Z. Guralnik and I. Kirsch, “Four-dimensional superconformal theories with interacting boundaries or defects”, Phys. Rev. D66 (2002) 025020, hep-th/0203020.

[10] D. Arean and A. V. Ramallo, “Open string modes at brane intersections”, J. High Energy Phys. 0604 (2006) 037, hep-th/0602174.

[11] R. C. Myers and R. Thomson, “Holographic mesons in various dimensions”, hep-th/0605017.

[12] K. Skenderis and M. Taylor, “Branes in AdS and pp-wave spacetimes”, J. High Energy Phys. 0206 (2002) 025, hep-th/0204054.

[13] C. Bachas, M. Douglas and C. Schweigert, “Flux stabilization of D-branes”, J. High Energy Phys. 0005 (2000) 048, hep-th/0003037.

[14] R. C. Myers, “Dielectric branes”, J. High Energy Phys. 9912 (1999) 022, hep-th/9910053; N. R. Constable, R. C. Myers and O. Tafjord, “The non-commutative bion core”, Phys. Rev. D61 (2000) 106009, hep-th/9911136.

[15] W. Nahm, “The construction of all self-dual multi-monopoles by the ADHM method”, in “Monopoles in Quantum Field Theory”, eds. N. S. Craigie, P. Goddard and W. Nahm, World Scientific, Singapore, 1982.

[16] D. Tong, “TASI lectures on solitons: instantons, monopoles, vortices and kinks”, hep-th/0509216.

[17] D. Arean, A. V. Ramallo and D. Rodríguez-Gómez, work in progress, to appear.