Theoretical investigations on the $Y(4260)$ being an hybrid meson.

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Abstract

The new recently experiments at the B-factories yield a renewed interest in the charm and charmonium spectroscopy. New intriguing states have been observed, which appear to be non-conventional mesons, such as $X(3862)$ and $Y(4260)$ and request more theoretical investigations.

We will explore the possibility for the $Y(4260)$ to be a $c\bar{c}g$ hybrid meson. Using the quark-gluon constituent model, we exclude its existence as QE-hybrid meson, and as mixing of $c\bar{c}g$ hybrid with conventional $c\bar{c}$ meson. We find the only interpretation as GE-hybrid, decaying in $D\bar{D}$ channels. Then more experimental studies are needed to confirm the existence of this resonance and to give its properties.

1 Introduction

In the last few years, an intense activity has been done at the B-factories, and has seen important progress of charm and charmonium spectroscopy. New charm mesons and baryons have been observed, and mainly charmonium-like mesons that need particular attention, since they do not classify in the standard hadronic spectroscopy (by the naive quark model). The more interesting observed new states are the $X(3872)$ and the $Y(4260)$.

• The $X(3872)$ was first discovered by BELLE\cite{1} in $B^\pm \rightarrow K^\pm X(3872)$, decaying in $\pi^+\pi^-J/\psi$, and confirmed by BABAR\cite{2}, CDF\cite{3} and DO\cite{4} collaborations. The mass of the $X(3872)$ is within errors of the $D^0\bar{D}^{*0}$ threshold $(3871.3\pm1.0\ MeV)$\cite{5}. Other decay modes have been observed:
  i) in $\gamma J/\psi$\cite{6} with a branching ratio of $(19\pm7)\%$ relative to the $X(3872)\rightarrow \pi^+\pi^-J/\psi$ mode,
ii) in $D^0\pi^0$ \cite{7}; this mode being dominant to have a branching ratio $(9.7 \pm 3.4)$ times higher than the $\pi^+\pi^- J/\psi$ mode.

Recent studies from BELLE, that combine angular and kinematic properties of the $\pi^+\pi^-$ mass strongly favor the assignment of $J^{PC} = 1^{++}$ \cite{6}, which is in agreement with expectations of models interpreting the $X(3872)$ as a molecule-like $D^0\bar{D}^{*0}$ bound state\cite{7}. Some other interpretations have been proposed for the $X(3872)$. Hence, the $X(3872)$ appears not to be a simple meson state.

- The $Y(4260)$.

The second interesting charmonium-like state is a new structure observed by BABAR Collaboration\cite{10} in ISR (Initial State Radiation) events at $4259 \pm 8_{-6}^{+2}$ $\text{MeV}$ with a width of $\Gamma = 88 \pm 23_{-4}^{+6}$ $\text{MeV}$, decaying in $\pi^+\pi^- J/\psi$, and having $J^{PC} = 1^{--}$. The $Y$ state has been confirmed by CLEO\cite{11} in $\pi^+\pi^- J/\psi$ and $\pi^0\pi^0 J/\psi$, which implies that it has Isospin $I = 0$.

Investigations have been done to observe other decay modes, like $D\bar{D}$ \cite{12}, $p\bar{p}$ \cite{13}, $J/\psi\pi^+\pi^-$ \cite{14}, without success, and limits for the decay rates are given; the upper limit for the $D\bar{D}$ mode is 7.6 times relative to the $Y \rightarrow \pi^+\pi^- J/\psi$ mode, which is much smaller than $BR[\Psi (3770) \rightarrow D\bar{D}] \approx 500 \times BR[\Psi (3770) \rightarrow \pi^+\pi^- J/\psi]$ \cite{15}; then the $Y(4260)$ cannot be a radially excited state such as $\Psi (4S)$. Furthermore, it should be noted that neither of $\Psi (4040)$, $\Psi (4160)$, $\Psi (4415)$, has been observed decaying in $\pi^+\pi^- J/\psi$ mode.

Several explanations have been proposed for the $Y(4260)$, the most favored being the $c\bar{c}g$ hybrid meson structure\cite{16,17}. We will in this work explore this hypothesis.

In section 2, we present the model used to represent the hybrid meson, as the quark-gluon constituent model, with the distinction between the two different modes (GE and QE).

In section 3, we give the selection rules for the decay of an hybrid $1^{--}$ meson in two mesons and we compute the decay widths.

In section 4, we explore the possibility for the $Y(4260)$ to be a mixing of an hybrid with a conventional charmed meson, as argued in \cite{16}. We conclude in section 5.

2 The hybrid state from the constituent model

In the framework of the strong-coupling regime, we can not yet be able to perform systematical calculations directly from the QCD Lagrangian, the process need alternative models, like Flux-Tube Model, Bag Model, QCD Sum Rules, Lattice QCD, ... or phenomenological potential models.

The quark model based on the instantaneous potential interaction between the hadron’s constituents is motivated by the experiment. Indeed, practically all our knowledge on hadron structure comes from phenomenological models, the constituent quark model particularly continues to have great success. We will use it to study the $J^{PC} = 1^{--} Y(4260)$.

In a recent work\cite{21}, we used a phenomenological potential which reproduces the QCD characteristics (asymptotic freedom and confinement), its expression
having the mathematical (Coulomb+linear) form. Relativistic corrections both in the kinetic and interaction parts of the Hamiltonian are considered. (taking into account spin-spin, spin-orbit and tensor interactions).

An important ingredient of the model is the mass of the constituent gluon \( m_g \). There is evidence for massive-like dispersion relation for the gluon, with mass ranging from \( 700 \sim 1000 \) MeV, both from lattice and Schwinger-Dyson equations. As for quarks, this represents a dynamical mass and is expected to add \( 0.7 \sim 1.0 \) GeV to the corresponding quarkonia.

We represent a state of the hybrid meson by the following quantum numbers:

- \( l_g \) is the relative orbital momentum of the gluon in the \( q\bar{q} \) center of mass;
- \( l_{q\bar{q}} \) is the relative orbital momentum between \( q \) and \( \bar{q} \);
- \( S_{q\bar{q}} \) is the total quark spin;
- \( J_g \) is the total gluon angular momentum;
- \( L = l_{q\bar{q}} + J_g \).

Parity and Charge Conjugation of the hybrid meson verify:

\[
P = (-)^l_{q\bar{q}} + l_g, \quad C = (-)^l_{q\bar{q}} + S_{q\bar{q}} + 1
\]  

For the lowest \( J^{PC} = 1^{--} \) hybrid states, equation (1) implies \( l_{q\bar{q}} = S_{q\bar{q}} \) and \( l_{q\bar{q}} + l_g \) odd, which gives two possibilities (see Table 1):

\[
l_{q\bar{q}} = S_{q\bar{q}} = 0, \quad l_g = 1 
\]  

and

\[
l_{q\bar{q}} = S_{q\bar{q}} = 1, \quad l_g = 0
\]  

We refer to the (2) as the gluon-excited mode (GE), and to the (3) as the quark-excited mode (QE).

| \( P \) | \( C \) | \( J \) | \( l_g \) | \( l_{q\bar{q}} \) | \( J_g \) | \( S_{q\bar{q}} \) | \( L \) |
|---|---|---|---|---|---|---|---|
| - | - | 1 | 0 | 1 | 1 | 1 | 0 |
| - | - | 1 | 0 | 1 | 1 | 1 | 1 |
| - | - | 1 | 0 | 1 | 1 | 1 | 2 |
| - | - | 1 | 1 | 0 | 1 | 0 | 1 |

Table 1: The lowest \( J^{PC} = 1^{--} \) hybrid states with their quantum numbers.

We found in [21] a significant mixing between the two QE and GE-modes, and we predict a \( 1^{--} q\bar{g} \) mass around 4.27 GeV (see Table 2):
Table 2: Mixed QE-GE hybrid mesons masses with spin corrections (in GeV)

|      | 1−− | 0−− | 0−+ | 1−+ |
|------|------|------|------|------|
| \(n\bar{g}\) | 1.40 | 1.58 | 1.73 | 1.93 |
| \(s\bar{g}\) | 1.66 | 1.87 | 2.02 | 2.21 |
| \(c\bar{g}\) | 4.27 | 4.35 | 4.38 | 4.48 |
| \(b\bar{g}\) | 10.50 | 10.66 | 10.68 | 10.80 |

The order of magnitude being in agreement with the masses obtained by the other models\[^{22}\].

3 The decay of the hybrid meson

3.1 The decay process

In the model, at the leading order, the decay of the hybrid meson into two standard mesons may occur through two diagrams, the gluon annihilating into a \(q\bar{q}\) pair (namely \(q = u, d\)) (Fig.1, Fig.2).

The process of the Fig.1 dominates, the second (disconnected diagram) being suppressed by the OZI rule.

Fig.3 represents the process able to product directly \(\pi^+\pi^- J/\psi\), the mechanism being strongly suppressed.

This representation of the decay could not explain the experimental data, no signal of two-mesons decay mode being detected, especially as two-body decay usually dominates three-body decay.

On the other hand, a \(c\bar{g}\) 1−− hybrid meson in the mass range of \(\sim 4\) GeV can couple to \(D_1(2420)\overline{D}\) and \(D_1(2420)^\pm D^\mp\) (through the process of Fig.1) and a \(\pi^+\pi^- J/\psi\) signal may be produced by re-scattering effects, but the dominant final state should be in \(DD^*\pi\). This hypothesis suggests more experimental data to confirm the mass of the \(Y\) (the threshold of a \(D_1(2420)\overline{D}\) channel needs a mass lightly above) and to search for other decay channels.

3.2 The decay widths

The decay of an hybrid state \(A\) into two mesons \(B\) and \(C\) is represented by the matrix element of the Hamiltonian annihilating a gluon and creating a quark pair:

\[
(BC|H|A) = gf(A, B, C) (2\pi)^3 \delta_3 (p_A - p_B - p_C);
\]

where \(f(A, B, C)\) representing the decay amplitude by:

\[
f(A, B, C) = \sum_{(m), (\mu)} \Phi \Omega X \left( \mu_{\bar{q}q}, \mu_g; \mu_B, \mu_C \right) I(m_{\bar{q}q}, m_g; m_B, m_C, m) \times \left| \langle l_g m_g \mu_g | J_g M_g \rangle \langle l_{\bar{q}q} m_{\bar{q}q} | J_{\bar{q}q} M_{\bar{q}q} \rangle \langle L m' | S_{\bar{q}q} \mu_{\bar{q}q} | J M \rangle \times \langle L m_B S_{\bar{q}q} \mu_B | J_B M_B \rangle \langle L m_C S_{\bar{q}q} \mu_C | J_C M_C \rangle \right|.
\]
where \( \Phi, \Omega, X \) and \( I \) are the flavor, color, spin and spatial overlaps. \( \Omega \) is given by:

\[
\Omega = \frac{1}{24} \sum_a \text{tr} (\lambda^a)^2 = \frac{2}{3}
\]

(6)

From:

\[
\chi_{\mu_1}^+ \sigma^\lambda \chi_{\mu_2} = \sqrt{3} \langle \frac{1}{2} \mu_2 1 \lambda | \frac{1}{2} \mu_1 \rangle,
\]

(7)

we obtain the spin overlap:

\[
X (\mu \eta \mu g; \mu B, \mu C) = \sum_S \sqrt{2} \left[ \begin{array}{ccc} 1/2 & 1/2 & S_B \\ 1/2 & 1/2 & S_C \\ S_\eta \eta & 1 & S \end{array} \right] \times \langle S_\eta \mu g | S (\mu \eta + \mu g) \rangle \langle S_B \mu_B S_C \mu_C | S (\mu_B + \mu_C) \rangle;
\]

(8)

where

\[
\left[ \begin{array}{ccc} 1/2 & 1/2 & S_B \\ 1/2 & 1/2 & S_C \\ S_\eta \eta & 1 & S \end{array} \right] = \sqrt{3} (2S_B + 1) (2S_C + 1) (2S_\eta + 1) \left[ \begin{array}{ccc} 1/2 & 1/2 & S_B \\ 1/2 & 1/2 & S_C \\ S_\eta \eta & 1 & S \end{array} \right].
\]

(9)

The spatial overlap is given by:

\[
I (m \eta \eta; m_B, m_C, l, m) = \int \int \frac{d\vec{p}_1 \, d\vec{p}_2}{(2\pi)^6 \sqrt{2\omega}} \psi_{\eta \eta}^{l, m_B, m_C} (\vec{P}_B - \vec{p}_1 - \vec{p}_2) \psi_{\eta \eta}^{l, m_B, m_C} (\vec{P}_B - \vec{p}_2) \Omega_B d\Omega_B,
\]

(10)

where:

\[
\vec{p}_1 = - \frac{m_{\eta \eta}}{m_\eta + m_\eta} \vec{P}_B - \vec{p} - \frac{\vec{k}}{2}
\]

(11)

\[
\vec{p}_2 = - \frac{m_{\eta \eta}}{m_\eta + m_\eta} \vec{P}_B + \vec{p} - \frac{\vec{k}}{2}.
\]

(12)

\( l, m \) label the orbital momentum between the two final mesons.

Finally,

\[
\Phi = \left[ \begin{array}{ccc} i_1 & i_3 & I_B \\ i_2 & i_4 & I_C \\ S_\eta \eta & 1 & I_A \end{array} \right] \eta e;
\]

(13)

where \( I \)'s (i's) label the hadron (quark) isospins, \( \eta = 1 \) if the gluon goes into strange quarks and \( \eta = \sqrt{2} \) if it goes into non strange ones. \( e \) is the number of
diagrams contributing to the decay. Indeed one can check that since P and C are conserved, two diagrams contribute with the same sign and magnitude for allowed decays while they cancel for forbidden ones. In the case of two identical final particles, \( \epsilon = \sqrt{2} \).

The partial width is then given by:

\[
\Gamma (A \rightarrow BC) = 4 \alpha_s |f(A, B, C)|^2 \frac{P_B E_B E_C}{M_A}; \quad (14)
\]

with

\[
P_B^2 = \frac{\left[ M_A^2 - (m_B + m_C)^2 \right] \left[ M_A^2 - (m_B - m_C)^2 \right]}{4 M_A^2}; \quad (15)
\]

\[
E_B = \sqrt{P_B^2 + m_B^2}; \quad (16)
\]

\[
E_B = \sqrt{P_B^2 + m_C^2}.
\]

### 3.3 Selection rules for the decay

Computing the integral (10) in the two cases, for the decay respectively into two S-Wave mesons and into (S-wave + P-wave) mesons, leads to two selection rules.

Using the Harmonic oscillator potential, we find:

\[
I (m_q, 0; 0, 0, m) = 2^4 \sqrt{\frac{\pi}{3\omega}} \frac{R_{\sigma g}^{3/2} R_g^{3/2 + \delta} R_B^5}{(R_g^2 + R_B^2/2)^{3/2}} \frac{2m_q}{m_q + m} P_B \times \exp \left\{ -\frac{P_B^2}{2} \left[ R_{\sigma g}^2 + \frac{2m_q^2 R_B^2}{(m_q + m)^2} - \frac{2m_q R_B^2 + (m_q + m) R_{\sigma g}^2}{(R_{\sigma g}^2 + 2R_B^2) (m_q + m)^2} \right] \right\} \delta_{l_q, 0} \delta_{l_{\sigma g}} \delta_{m_q, 0} \quad (17)
\]

and

\[
I (0, m_q; m, 0, 0) = -\sqrt{\frac{\pi}{2\omega}} \frac{R_{\sigma g}^{3/2} R_g^{5/2} R_B^1}{(R_g^2/2 + R_B^2/4)^{5/2}} \left( R_{\sigma g}^2/2 + R_B^2 \right)^{3/2} \times \exp \left\{ -\frac{P_B^2}{2} \left[ R_{\sigma g}^2 + \frac{2m_q^2 R_B^2}{(m_q + m)^2} - \frac{2m_q R_B^2 + (m_q + m) R_{\sigma g}^2}{(R_{\sigma g}^2 + 2R_B^2) (m_q + m)^2} \right] \right\} \delta_{l_q, 1} \delta_{l_{\sigma g}} \delta_{m_q, 0} \delta_{m, 0} \quad (18)
\]
$q_iq_i$ is the quark created pair.

Equation (17) corresponds to the QE-hybrid, which is the only mode allowed to decay only into two S-wave mesons\cite{20} and equation (18) shows that a GE-hybrid does decay only into a channel with one S-wave meson and one P-wave meson.

The same selection rules have been advocated for the light hybrid mesons\cite{19}.

In ref.\cite{17}, the selection rule for GE-hybrid is proved in any potential model.

### 3.4 The decay in two ground state mesons

The hybrid at $\sim 4.3 GeV$ can decay in $D^0\bar{D}^0$, $D^+D^-$, $D^+_sD^-_s$, $D^0\bar{D}^0$, $D^{*0}\bar{D}^{*0}$, $D^{*+}D^{*-}$.

Table 3 shows the results for the partial decay widths; the decays in $D^{*+}D^{*-}$ and $D^{*0}\bar{D}^{*0}$ for $S = 1$ are suppressed by the spin overlap.

| $L$ | 0     | 1     | 2     |
|-----|-------|-------|-------|
| $\Gamma_{D^0\bar{D}^0}$ | 129.5 | 388.5 | 647.5 |
| $\Gamma_{D^+D^-}$ | 135.1 | 406   | 676.2 |
| $\Gamma_{D^+_sD^-_s}$ | 142.8 | 428.4 | 714   |
| $\Gamma_{D^{*0}\bar{D}^{*0}} = \Gamma_{D^{*+}\bar{D}^{*-}}$ | 0.00  | 0.00  | 0.00  |

$\Gamma_{D^{*+}D^{*-}} = \Gamma_{D^{*0}\bar{D}^{*0}}$

$S = 0$

|   | 30.8 | 92.4 | 1.47  |

$S = 1$

|   | 0.00 | 0.00 | 0.00  |

$S = 2$

|   | 49.3 | 369.6 | 24.5  |

Table 3: Decay widths of the $(M=4.26)$ hybrid in $(S+S)$-standard mesons (in $\alpha_s MeV$).

The total decay width of a hybrid charmonium meson with mass $M = 4.26 GeV$ is very large; such state does not emerge from the continuum of the two mesons spectrum.

### 3.5 The decay in (S-meson+P-meson)

We give the results of the decay widths in Table 4.

\[
\Gamma_{D_s(2420)\bar{D}^0} = \Gamma_{D_s(2420)D^0} \approx \Gamma_{D_s^+(2420)D^-} = \Gamma_{D_s^+(2420)D^+}
\]

Table 4: Partial decay widths of the $(M=4.3)$ hybrid in $(L+S)$-standard mesons (in $\alpha_s MeV$).

We find each decay rate of $\sim \frac{10^7}{4} MeV$, which is enough small compared to the level spacing, to generate observable resonance $(\text{the total decay width } \sim 10^7 MeV)$. It should be noted that the decay width of the $Y(4260)$ is $\sim 90 MeV$.  

7
3.6 Is the $Y (4260)$ a mixed charmonium-hybrid meson?

In order to study this possibility, we can refer to [20]. The calculated transition Hamiltonian between a $c\bar{c}q 1^{--}$ hybrid and the conventional $c\bar{c}$ meson (such as $\Psi (3S)$) gives very small amplitudes, then very small resulting angles, which exclude the hypothesis to observe a mixed state.

4 Results and Conclusion

Therefore, according to the results, we may conclude that (in our model) a $J^{PC} = 1^{--}$ charmonium hybrid meson should have a mass around $\sim 4.3 GeV$ and decay preferably to $D_1 (2420)\bar{D}$. It is important to check the existence of such state; more experimental investigations are needed, searching for two-body decay channels (or $D^* D \pi$ channels), and confirmations on the mass and the decay width.

5 References

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Fig 1. Dominant decay process

Fig 2. OZI forbidden process

Fig 3. Direct production of $\pi^+ \pi^- J/\psi$