Probing the physics of newly born magnetars through observation of superluminous supernovae

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The central engines of some superluminous supernovae (SLSNe) are generally suggested to be newly born fast rotating magnetars, which spin down mainly through magnetic dipole radiation and gravitational wave emission. We calculate the magnetar-powered SLSNe light curves (LCs) with the tilt angle evolution of newly born magnetars involved. We show that, depending on the internal toroidal magnetic fields $B_i$, the initial spin periods $P_i$, and the radii $R_{DU}$ of direct Urca (DU) cores of newly born magnetars, as well as the critical temperature $T_c$ for $^{14}N$ neutron superfluidity, bumps could appear in the SLSNe LCs after the maximum lights when the tilt angles grow to $\pi/2$. The value of $T_c$ determines the arising time and the relative amplitude of a bump. The quantity $R_{DU}$ can affect the arising time and the luminosity of a bump, as well as the peak luminosity of a LC. For newly born magnetars with dipole magnetic fields $B_d = 5 \times 10^{13}$ G, $B_i = 4.6 \times 10^{16}$ G, and $P_i = 1$ ms, there are no bumps in the LCs if $T_c = 2 \times 10^8$ K, or $R_{DU} = 1.5 \times 10^3$ cm. Moreover, it is interesting that a stronger $B_i$ will lead to both a brighter peak and a brighter bump in a LC. While keeping other quantities unchanged, the bump in the LC disappears for the magnetar with smaller $P_i$. We suggest that, once the SLSNe LCs with such kinds of bumps are observed, by fitting these LCs with our model, not only $B_i$ and $P_i$ of newly born magnetars but also the crucial physical quantities $B_i$, $R_{DU}$, and $T_c$ could be determined. Nonobservation of SLSNe LCs with such kinds of bumps hitherto may already put some (though very rough) constraints on $B_i$, $P_i$, $R_{DU}$, and $T_c$. Therefore, observation of SLSNe LCs may provide a new approach to probe the physics of newly born magnetars.

I. INTRODUCTION

Newly born millisecond rotating highly magnetized neutron stars (NSs) (dubbed as millisecond magnetars) are generally suggested to be associated with a variety of astrophysical phenomena, such as long/short gamma-ray bursts [1,2], superluminous supernovae (SLSNe) [3,5], bright mergernovae emissions [6] that are possibly accompanied by an x-ray precursor [7], some rapidly evolving and luminous transients [8], and fast radio bursts [9]. Especially, as a subclass of SLSNe, the hydrogen-poor SLSNe [10] (classified as type-I SLSNe [11]) are generally suggested to be powered by millisecond magnetars because their light curves (LCs) can be well reproduced within the magnetar scenario (see, e.g., Refs. [4,5,12–14]). In the magnetar-powered model, huge rotational energy of the central newly born magnetar can be extracted and converted into heating energy to heat the supernova (SN) ejecta, making the SN quite brilliant [3,5]. Through fitting the LCs of SLSNe, the ejected masses and some important parameters of the newly born magnetars, such as the dipole magnetic fields $B_d$, initial spin periods $P_i$ can be determined. In most cases, the magnetars are required to possess $B_d \sim 5 \times 10^{13}$–$5 \times 10^{14}$ G, and $P_i \sim 1$–8 ms (see Refs. [14,15] and references therein).

Besides the strong surface dipole fields, magnetars are generally considered to possess even stronger interior toroidal magnetic fields (see, e.g., Refs. [16,17]). Observationally, the x-ray LCs of some short gamma-ray burst afterglows [17], the slow phase modulation in the x-ray emission of Magnetar 4U 0142+61 [18], and the bright giant flare from SGR 1806-20 [16] all indicate that the toroidal field could be a few to ~100 times higher than the dipole field of a magnetar, reaching ~$10^{16}$ G or higher. Various mechanisms have been proposed to explain the formation of strong magnetic fields of magnetars; for instance, magnetic flux conservation during the core collapse of a highly magnetized progenitor [19], an $\alpha-\omega$ dynamo in a differentially rotating millisecond protoneutron star (PNS) [20], Kelvin-Helmholtz instability [21], or magnetorotational instability (MRI) arises during the merger of two NSs [22] and the core collapse of massive stars [23], and the r-mode and Tayler instabilities act in a fast rotating NS [24]. In core collapse supernovae (CCSNe) associated with SLSNe of interest here, the magnetic fields of PNSs can be amplified through a series of ways, such as magnetic flux compression, linear winding, stationary accretion shock instability, MRI, and an $\alpha-\omega$ dynamo [25]. Specifically, numerical simulations of CCSNe showed that a PNS with toroidal field of ~$10^{15}$ G (or even ~$10^{16}$ G [23]) can be produced via MRI (and a MRI-driven turbulent dynamo) after core bounce if the
pre-collapse iron core is highly magnetized and rapidly rotating [25–28]. Meanwhile, the PNSs may probably have initial spin periods of the order of milliseconds [28–32], which could further trigger the turbulent dynamo that is driven by differential rotation and convection and amplify the interior toroidal fields to \( \sim 10^{16} \) G [21]. Hence, newly born rapidly rotating magnetars with toroidal fields of a few\( \times 10^{16} \) G can possibly be formed in CCSNe with highly magnetized and fast rotating pre-collapse cores. It also seems that the strong magnetic fields of newly born magnetars are tightly related to their fast rotations. However, an estimation of what portion of newborn NSs are magnetars with such strong toroidal fields is impossible at present because we still know little about the properties of the progenitor cores and the evolution process from PNSs to newborn NSs. Strong magnetic fields can induce nonaxisymmetric quadrupole deformation, which manifests the newly born magnetars as strong gravitational wave (GW) sources [16, 33–37]. However, initially, the tilt angle between the spin and magnetic axes of a magnetar may be very tiny, the gravitational wave emission would be strongly suppressed consequently [34].

The tilt angle evolution of a newly born magnetar with strong toroidal field was first investigated in Ref. [34] and then involved in the calculation of gravitational wave background from newly born magnetars [38]. The tilt angle trends to increase to \( \pi/2 \) in order to minimize the NS’s spin energy. Generally, the angle evolution can be divided into two stages [34]. The first stage is when the stellar temperature is so high (\( \gtrsim 10^9 \) K) that the whole star is in the liquid state without superfluidity. The tilt angle evolution is determined by the competition between the bulk viscosity (BV) of stellar matter and gravitational radiation reaction (GRR) of the magnetar. As the newly born magnetar cools down due to intense neutrino emission, a solid crust can form for stellar temperatures lower than \( \sim 10^9 \) K [39]. Moreover, the \(^3P_2\) neutron superfluidity will occur in the core when the stellar temperature drops below a critical value \( T_c \). After the formations of a solid crust and neutron superfluidity, the tilt angle evolution goes into the second stage, in which the angle evolution is driven by viscous dissipation of the free-body precession due to core-crust coupling [40]. Evolution of the tilt angle can lead to a change in the magnetic dipole luminosity that is ejected into a SN; thus, the SLSN LC may be changed accordingly. This represents the fundamental starting point of this paper, and the essential difference compared to previous work, in which a constant angle is generally assumed (e.g., Refs. [4, 5, 12, 13, 41]).

The most crucial quantities that determine the angle evolution in the first stage are toroidal field \( \bar{B}_i \) (volume-averaged strength), spin period \( P \), and stellar temperature \( T \). In the later stage, since the angle could increase to \( \pi/2 \) in a very short time if the fast rotating magnetar has a strong toroidal field [16, 40], the start time of this stage becomes crucial. The start time of the second stage strongly depends on the critical temperature \( T_c \) adopted\(^2\) and the cooling mechanism of the newly born magnetar. Consequently, the coupled evolutions of the tilt angle, spin, and stellar temperature of a magnetar should be taken into account while calculating the magnetar-powered SLSNe LCs. Furthermore, the effects of physical quantities \( \bar{B}_i \) and \( T_c \) should also be involved.

In fact, the critical temperature for \(^3P_2\) neutron superfluidity is a function of density, \( T_c(\rho) \), which has a parabolic shape and peaks at a certain density \( \rho \) between the core-crust boundary and the stellar center. The maximum value of \( T_c(\rho) \) is usually dubbed as the critical temperature \( T_c \) for \(^3P_2\) neutron superfluid transition. The specific value of \( T_c \) is still uncertain since the roles of medium effects and complicated interactions on the result of \( T_c \) are not clearly known [42, 43]. Previous result suggests that \( T_c \) may be within the range from a rather small value to \( \sim 10^{10} \) K (see Ref. [44] and references therein). However, observations of the cooling behavior of the NS in Cassiopeia A [45, 46] and surface temperatures of other isolated NSs [47] suggest \( T_c \sim 10^{6–7} \) K. Following Refs. [34, 45], in this paper, \( T_c \) is left to be a parameter within the range \( 5 \times 10^8–2 \times 10^9 \) K. We will show the value of \( T_c \) can obviously affect the shape of a SLSN LC, which suggests that observation of SLSNe may provide another approach to determine \( T_c \).

The effect of magnetically induced GW emission on the magnetar-powered SLSNe has been studied in Refs. [15, 41, 48]. The results show that strong toroidal fields can overall reduce the emitted luminosities of SLSNe because a large amount of rotational energy of the central magnetars is released in the GW channel [41, 48]. To require that the majority of rotational energy can be used to power the SLSNe, the GW emission must be weakened, and an upper limit for toroidal fields is derived as \( \bar{B}_i \lesssim \text{several} \times 10^{16} \) G [15]. However, all these results are obtained under the assumption of constant tilt angles as mentioned before. A more detailed investigation about the role of \( \bar{B}_i \) on the SLSNe LCs is still needed as the angles should indeed evolve with time.

On the other hand, the cooling mechanism of newly born NSs is still an open issue. It is hard to address this issue both theoretically and observationally because of the poor knowledge of dense matter properties and obscuration of thermal emissions of NSs by the surrounding dense ejected materials. As generally considered, in the early period, the classical NSs\(^3\) composed purely of neutrons, protons, and electrons cool down mainly through the modified Urca (MU) process [49] if the proton fraction in stellar interiors is below a threshold of about 11% [50, 51]. However, some dense matter equations of state (EOSs) (e.g., Akmal-Pandharipande-Ravenhall (APR) [52] and Prakash-Ainsworth-Lattimer (PAL) [53]) predict that in the central region of some NSs with sufficient masses \( 5.4 \)–\( 5.5 \) the proton fraction can surpass the thresh-

\(^2\) This seems to be a direct consequence of the field amplification due to MRI and the \( \alpha \rightarrow \omega \) dynamo.

\(^3\) In this paper, we focus on this type of NSs only as in Ref. [34].
old, leading to the occurrence of the direct Urca (DU) process \[50, 51\]. The DU neutrino emission in the central region of NSs can greatly expedite the cooling of NSs. The size of the DU core depends on the mass of a NS and EOS (see Ref. \[56\] for a review), both of which are uncertain for NSs embedded in SNe. Since the stellar temperature evolution could affect the tilt angle evolution, and further the shape of SLSNe LCs, observation of SLSNe may give some clues on two crucial issues: (i) could the DU process occur in a NS, and (ii) if it occurred, how large is the DU core? These could help one to understand the NS interior structures and constrain the EOS of dense matter.

The paper is organized as follows. We show the evolution of newly born magnetars in Sec. \[2\] The model for magnetar-powered SLSNe is briefly introduced in Sec. \[3\]. Our results are presented in Sec. \[4\]

Finally, a conclusion and discussions are given in Sec. \[5\].

II. EVOLUTION OF NEWLY BORN MAGNETARS

The collapse of a massive progenitor core may give rise to a differential rotating PNS with strong convective motions in its interior. Initially, the ultrahot (with a temperature of a few tens MeV) PNS is opaque to neutrinos and may have a radius of several tens of kilometers. Subsequently, as the PNS becomes transparent to neutrinos, it will contract and become a newly born NS with a radius of \(\sim 10\) km at \(\sim 10\) s after the core bounce \[29, 31\]. Contraction of the PNS can lead to spin-up of the newly born NS because of angular momentum conservation; thus, the newly born NS possibly has a spin period of \(\gtrsim 1\) ms at birth \[29, 57\]. Simulations of the collapse of massive progenitor cores have demonstrated that the initial spins of newly born NSs could indeed reach of the order of 1 ms \[30, 58\]. It should be noted that fallback accretion probably exists at early periods, which may further spin up the central remnant and result in a newly born magnetar with an initial period very close to 1 ms. Theoretically, fast spin \((\lesssim 5\) ms) is suggested to be an indispensable condition so as to avoid the early unstable phase that newly born magnetars will undergo \[59\]. In this paper, we take 1 ms as the possibly minimum initial spin periods of newly born magnetars. We also note that newly born magnetars may have different initial spin periods due to different progenitor properties, and more detailed simulations are still needed in order to know how rapidly such magnetars can rotate.

A newly born magnetar is generally considered to be spun down via magnetic dipole radiation (MDR) and magnetically induced GW emission, which can be expressed as follows (e.g., Ref. \[58\]),

\[
\dot{\Omega} = -\frac{B_0^2 R^6 \Omega^3}{6 I c^3} \sin^2 \chi - \frac{2 G M^5}{5 c^5} \sin^2 \chi (15 \sin^2 \chi + 1),
\]

where \(\Omega\) is the angular frequency, \(B_0\) is the surface dipole magnetic field at the magnetic pole, and \(I = 0.35 M R^2\) is the moment of inertia of the NS with \(M\) and \(R\) representing the stellar mass and radius, respectively \[60\]. From now on, we take typical values \(M = 1.4 M_\odot\) and \(R = 12\) km for newly born magnetars. For the toroidal-dominated interior magnetic field configuration, the quadrupole ellipticity of magnetic deformation is \(\epsilon_B = -5 B_0 R^4 / (6 G M^2)\) \[33, 34\]. Following Dall’Oss et al. \[34\], at the first stage, the tilt angle \(\chi\) of a liquid NS evolves under the combined effect of GRR and BV, which can be written as

\[
\dot{\chi} = \frac{c^2 \sin \chi}{\tau_d} \left( \frac{2 G M^5}{5 c^5} I_B^2 \Omega^4 \sin \chi \cos \chi (15 \sin^2 \chi + 1) \right). \tag{2}
\]

The first term on the rhs of Eq. (2) is related to the effect of BV of stellar matter on damping of the free-body precession, which can essentially increase \(\chi\) of a prolate star \((\epsilon_B < 0)\). The corresponding damping timescale of free-body precession calculated for a classical NS with only the MU process involved is \[33\]

\[
\tau_d \approx 3.9 \text{s} \frac{c^2 \sin \chi}{1 + 3 \cos^2 \chi} \left( \frac{\tilde{B}_0}{10^{16} \text{G}} \right)^2 \left( \frac{T}{10^{10} \text{K}} \right)^{-6} \left( \frac{P}{1 \text{ms}} \right)^2 \left( \frac{M}{1.4 M_\odot} \right)^{-1} \left( \frac{R}{12 \text{km}} \right)^3. \tag{3}
\]

The second term on the rhs of Eq. (2) (taken from \[63\]) represents the damping of the free-body precession due to GRR, which can actually lead to alignment of the spin and magnetic axes even for a NS with \(\epsilon_B < 0\).

It should be stressed that in the presence of the DU process the BV of stellar matter is several orders of magnitude higher than that with MU process involved \[64\]. Hence, if the DU process could occur in a NS core, the damping timescale \(\tau_d\) should be modified accordingly in principle. However, in the simple phenomenological NS structure models we consider hereinafter, the DU process, if it occurred in the core region, the radius of the DU core, \(R_{DU}\), is assumed to be much smaller than \(R\). This is reasonable because, depending on EOSs, the DU process can marginally occur or be quenched completely in a 1.4\(M_\odot\) NS (see, e.g., Refs. \[54, 56, 65\]). The DU core contains a rather small portion of the total precession energy due to the small \(R_{DU}\) and (thus) the small DU core mass \(M_{DU}\).

As a very rough estimation, taking the NS model derived in Ref. \[60\] for a 1.4\(M_\odot\) NS as an example, the precession energy of the DU core is \[33\]

\[
E_{pre,DU} = -\frac{1}{2} I_{DU} \Omega^2 \epsilon_B \cos^2 \chi
\]

\[
\approx 5.3 \times 10^{46} \text{erg} \left( \frac{M_{DU}}{0.023 M_\odot} \right) \left( \frac{R_{DU}}{2.4 \text{ km}} \right)^2 \left( \frac{\Omega}{10^4 \text{rad/s}} \right)^2 \left( \frac{16\tilde{B}_0}{10^{-10}} \right) \cos^2 \chi, \tag{4}
\]

where \(I_{DU} = 2M_{DU} R_{DU}^2 / 5\) is the moment of inertia of the DU core. While the precession energy of the remaining part of

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\[4\] Though some magneto-hydrodynamics simulations show that a twisted-torus magnetic configuration composed of both poloidal and toroidal fields may naturally form in NS interiors \[61\], the dominant one is still the toroidal component \[62\].
the star is $E_{\text{pre,DU}} \approx 8.1 \times 10^{49}$ erg $\Omega_1^2 [e_{B_{-1}} \cos^2 \chi] \gg E_{\text{pre,DU}}$, where we have adopted the notation $Q_1 = Q/10^4$ here and hereinafter. Thus, it can be seen that, though the damping rate of the precession energy $E_{\text{pre}}$ is much larger in the DU core due to its larger BV [34, 64], dissipation of most of the precession energy still occurs in the MU region. Consequently, the occurrence of the DU process in a small core region will not modify the form of $\tau_d$ given in Eq. (3) significantly.

If the orthogonal configuration between the two axes is not reached in the first stage, when the stellar temperature cools down to $T_c$, the second evolution stage will begin, in which the viscosity due to core-crust coupling plays a dominant role in damping of the free-body precession [40]. The tilt angle of a prolate star could increase to $\pi/2$ on a timescale $\tau = nP/|\kappa|$ with $n \sim 10^{-2} - 10^4$ representing the number of precession cycles [16, 33, 40, 66]. For a magnetar with $P \sim 30$ ms and $B_1 = 4.6 \times 10^{18}$ G as shown in Fig. 2 at the beginning of the second stage, the maximal orthogonal timescale is estimated to be $\tau_{\max} \approx 0.49 d (P/30 \text{ ms}) (B_1/10^{16, 66} \text{ G})^{-2}$ by taking $n = 10^4$, which is still much shorter than the evolution timescale (of the order of 100 d) of the magnetar. Hence, it is reasonable to assume that $\chi = \pi/2$ can be realized immediately when $T_c$ is reached (see, e.g., Ref. [38]).

Since the tilt angle evolution of a newly born magnetar is tightly related to the stellar temperature $T$, the thermal evolution is thus an important issue to be addressed. For the poor knowledge of dense matter EOS and the NS interior structures, here we adopt a phenomenological NS model, in which the NSs are comprised of a small DU core with radius $R_{\text{DU}}$ and a large MU shell of radius $R - R_{\text{DU}}$. Moreover, for simplicity, the isothermal assumption is adopted with temperature distributing uniformly in a NS. The “DU core+MU shell” model was used previously in Refs. [54, 65] while discussing the cooling of the 2.21$M_\odot$ NS with all calculations based on the realistic EOS APR. For a 1.4$M_\odot$ newly born magnetar, if the DU process could take place in the core region, the evolution of $T$ would follow the formula below,

$$C_v \frac{dT}{dt} = -L_{\text{DU}} - L_{\text{MU}}$$
$$= - \frac{4\pi}{3} Q_{\text{DU}} R_{\text{DU}}^3 - \frac{4\pi}{3} Q_{\text{MU}} (R^3 - R_{\text{DU}}^3),$$

(5)

where $C_v \approx 10^{39} T_9$ erg K$^{-1}$ is the total heat capacity of the NS$^4$ and $Q_{\text{DU}} \approx 10^{47} T_9^6$ erg cm$^{-3}$ s$^{-1}$ and $Q_{\text{MU}} \approx 10^{21} T_9^6$ erg cm$^{-3}$ s$^{-1}$ are the DU and MU neutrino emissivities, respectively [51, 56]. From Eq. (5), one can see, by setting $R_{\text{DU}} = 0$, the thermal evolution returns to the pure MU cooling case, and the evolution equation is consistent with the analytical expression $T(t) = 10^9 K (t/T_c + 10^{-6})^{-1/6}$ with $T_c \approx 1$ yr [34, 51, 67]. It should be stressed that below the critical temperature $T_c$ the presence of neutron pairing can on one hand suppress the heat capacity, neutrino emissivities of the DU and MU processes of the stellar matter, and on the other hand provide new channels for neutrino emission that are associated with the pair breaking and formation processes [51]. For simplicity, below $T_c$, the thermal evolution of magnetars is ignored since it is trivial to our results.

### III. MODEL FOR MAGNETAR-POWERED SLSNE

Newly born magnetars lose rotational energy mainly via MDR and GW emission, of which only the energy in the MDR channel can be used to thermalize the SNe ejecta. The MDR luminosity emitted by a newly born magnetar is

$$L_m = \frac{B_1^2 R_6^6 \Omega_4^4}{6 c^3} \sin^2 \chi.$$  

(6)

During the expansion of SN ejecta, it loses internal energy due to adiabatic expansion and thermal radiation of the ejecta; meanwhile, the ejecta can also be heated by the energies that stem from MDR, $^{56}$Ni cascade decay, and ejecta-circumstellar medium interaction. Since in the magnetar-powered scenario the dominant energy source is MDR from magnetars, the evolution formula of the internal energy $E_{\text{int}}$ can thus be written as [4, 8, 41, 68]

$$\frac{dE_{\text{int}}}{dt} = -P_{\text{ej}} \frac{dV}{dt} - L_{\text{th}} + L_m,$$

(7)

where $P_{\text{ej}} = E_{\text{int}}/(3V)$ is the pressure dominated by radiation with $V$ denoting the volume of the SN ejecta and $L_{\text{th}}$ is the thermal radiation luminosity. At early times, the ejecta are optically thick, namely, the optical depth $\tau_0 = 3\kappa M_{\text{ej}}/(4\pi R_{\text{ej}}^2) \gg 1$ with $\kappa$, $M_{\text{ej}}$, and $R_{\text{ej}}$ representing the opacity, ejecta mass, and radius of the SN ejecta, respectively. In this case, $L_{\text{th}}$ has the following form [8]:

$$L_{\text{th}} = \frac{E_{\text{int}} c}{\tau_0 R_{\text{ej}}},$$

(8)

While at late times the ejecta will become optically thin ($\tau_0 \sim 1$), one then has [8]

$$L_{\text{th}} = \frac{E_{\text{int}} c}{R_{\text{ej}}},$$

(9)

Moreover, the dynamical evolution of the SN ejecta can generally be determined by the following equations [4, 8].

$$\frac{dR_{\text{ej}}}{dt} = v_{\text{ej}},$$

(10)

$$\frac{dv_{\text{ej}}}{dt} = 4\pi R_{\text{ej}}^2 P_{\text{ej}}/M_{\text{ej}},$$

(11)

where $v_{\text{ej}}$ is the expansion velocity of the SN ejecta.

### IV. RESULTS

Combining Eqs. (1), (2), (5), (7), (10), and (11) and taking into account the tilt angle evolution in the second stage, we

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\(^{5}\) For different values of $R_{\text{DU}}$, the expression for $C_v$ remains unchanged because the heat capacities of the stellar matter are the same for the DU and MU processes [4, 51, 65].
can determine the LCs (evolution of $L_{\text{th}}$) of magnetar-powered SLSNe. In all calculations below, typical values $M_\text{ej} = 5M_\odot$ and $\kappa = 0.2 \text{ cm}^2 \text{ g}^{-1}$ are taken for the ejecta mass and opacity, respectively [4, 11]. Moreover, the initial values for the SNe parameters, i.e., radius, velocity, and internal energy, are taken as $R_{\text{ej}} = 3 \times 10^8 \text{ cm}$, $v_{\text{ej}} = 10^9 \text{ cm s}^{-1}$, and $E_{\text{int}} = 10^{51} \text{ erg}$, respectively. We take $T_i = 10^{10} \text{ K}$ and $\chi_i = 1^\circ$ for the initial temperatures and tilt angles of newly born magnetars, respectively. The strength of $\dot{B_i}$ and the values of $R_{\text{DU}}$ and $T_c$ are set as free parameters in order to investigate their effects on the LCs. However, as discussed in Sec. [11] $\dot{B_i}$ are taken to be of the order of $10^{10} \text{ G}$, and no more than $100B_3$ as inferred from observations of magnetars.

Assuming different critical temperatures $T_c$, the evolutions of thermal radiation luminosities $L_{\text{th}}$ (upper panel) of SLSNe and tilt angles $\chi$ (lower panel) of the central magnetars are shown in Fig. [1]. We take dipole fields $B_d = 5 \times 10^{14} \text{ G}$, toroidal fields $B_t = 4.6 \times 10^{16} \text{ G}$, and initial spin periods $P_1 = 1 \text{ ms}$ for the magnetars, which are assumed to cool down via only the MU process ($R_{\text{DU}} = 0$) here. Since the growth of $\chi$ of the newly born magnetar can be suppressed by its strong $\dot{B_i}$ during the first evolution stage [see Eqs. (2) and (3)], $\chi$ will increase to $\pi/2$ only when the magnetar cools down to $T_c$ so that the viscosity due to core-crust coupling becomes effective. The higher $T_c$ is, the earlier $\chi = \pi/2$ can be achieved, as shown in the lower panel of Fig. [1]. For $T_c \leq 1.5 \times 10^9 \text{ K}$, the rapid growth of $\chi$ in the second evolution stage leads to an enhancement in the injected power $P_{\text{inj}}$, resulting in a bump in the LCs after the maxima. With the decrease of $T_c$, the bump arises at later times; however, its relative amplitude gradually grows. This can be understood as follows. The GW emission is more sensitive to $\chi$; thus, more rotational energy of a magnetar will be released in the GW channel if $\chi = \pi/2$ is achieved earlier. The energy in the MDR channel that can be used to energize the SN is therefore reduced, leading to a relatively small bump. In contrast, for $T_c = 2 \times 10^9 \text{ K}$, no apparent bumps appear in the resultant LC. The reason is the orthogonal configuration ($\chi = \pi/2$) can be achieved at the very beginning of the evolution if $T_c$ is high enough. In this case, the rapid increase in $L_{\text{inj}}$ cannot lead to a remarkable increase in $L_{\text{th}}$, the evolution at the very early period of which is essentially determined by the initial internal energy $E_{\text{int}}$ of a SN because very little energy of the central magnetar has been injected into the SN. The very short timescale needed for $\chi = \pi/2$ to be realized makes the resultant LC have little differences with that calculated by assuming a constant angle $\chi = \pi/2$ (see Fig. [2]).

The results above show that with the tilt angle evolution involved the arising of a bump after the maximum light in a SLSN LC can be related to the occurrence of $^3P_2$ neutron superfluidity in the magnetar core. Given a specific $T_c$, while other quantities remain unchanged, the arising time of the bumps in LCs is determined by the thermal evolution of central magnetars. This can be seen from Fig. [2] in which we show the results calculated by assuming that the magnetars cool down via both the DU and MU processes; however, the radii $R_{\text{DU}}$ of the DU cores are different. For a comparison, we also show the results obtained by assuming only MU cooling (black curves) and a constant angle $\chi = \pi/2$ [thick light-gray curves in panels (a), (b), and (c)]. The magnetar parameters $B_d$, $\dot{B_i}$, and $P_1$ are taken the same as in Fig. [1] while the critical temperature is fixed to be $T_c = 10^9 \text{ K}$.

The occurrence of the DU process in the core greatly accelerates the cooling of a magnetar [panel (d) of Fig. [2]]. The orthogonal configuration can thus be realized earlier in comparison with the result of only MU cooling [see panel (b)]. For $R_{\text{DU}} \leq 1.5 \times 10^4 \text{ cm}$, with the increase of $R_{\text{DU}}$, the arising time of the bumps in the LCs is gradually brought forward. However, the luminosities of the bumps as well as the peak luminosities are gradually enhanced. The reason is a larger $R_{\text{DU}}$ will result in a lower stellar temperature $T_c$ which can hinder the growth of $\chi$ of a magnetar in the first evolution stage. A small $\chi$ can, on one hand, reduce the rotational energy loss due to GW emission of the central magnetar and thus increase the energy in the MDR channel. On the other hand, in the case of MDR-dominated spin-down, the peak luminosity of a magnetar-powered SLSN can be approximately given

\[ L_{\text{th}} \propto \frac{\dot{E}}{R_{\text{DU}}} \]

For the values of $B_d$, $\dot{B_i}$, and $P_1$ taken in this paper, the braking effect of MDR overwhelms that of GW emission.
FIG. 2: Evolutions of the radiated luminosity $L_\text{th}$, tilt angle $\chi$, spin frequency $\nu$, and stellar temperature $T$, calculated by assuming different DU core radii $R_{\text{DU}}$ for the central magnetars as shown in the legends. The thick light-gray lines show the results derived for a constant angle $\chi = \pi/2$. The dotted line in panel (d) represents the critical temperature $T_\text{c}$.

different from the one obtained with a constant angle $\chi = \pi/2$ (see, e.g., Refs. [41, 48]), and (ii) the thermal evolution of a newly born magnetar, which is related to its interior structure, can obviously affect the shape of the SLSN LC that is powered by it.

In Fig. 3 we show the evolution curves derived by setting $T_\text{c} = 5 \times 10^8$ K, as inferred from the cooling behavior of the NS in Cassiopeia A [45]. One can see that for $T_\text{c}$ of this value bumps still exist in the LCs for the values of $\dot{B}_\text{th}$ and $R_{\text{DU}}$ taken (see the legends), while $B_\text{th}$ and $P_\text{th}$ are kept the same as in Fig. [1]. Again, the effect of the NS structure (the DU core radius $R_{\text{DU}}$) on the shape of a SLSN LC is clearly shown. Of the most important, in contrast to previous results [15, 41, 48], a higher $\dot{B}_\text{th}$ in a magnetar does not necessarily lead to a lower $L_{\text{th,p}}$ of the resultant LC. Instead, with the tilt angle evolution involved, a higher $\dot{B}_\text{th}$ can more obviously suppress the growth of $\chi$ [see panel (b) of Fig. 3]. Following the analysis above, the smaller $\chi$ will naturally lead to a higher peak luminosity and a brighter bump, as seen in panel (a). The close relation between the strength of the toroidal field in a magnetar and the shape of the magnetar-powered SLSN LC may enable us to probe the internal toroidal fields of newly born magnetars through observation of SLSNe LCs.

With $\chi = \pi/2$ assumed, by fitting the SLSNe LCs in the magnetar engine scenario, some central magnetars were found to have $B_d \lesssim 10^{14}$ G and $P_1 \gtrsim 1$ ms [14, 15]. Moreover, as suggested in Refs. [29, 30], newly born magnetars may have $P_1 \gtrsim 1$ ms. We therefore show in Fig. 4 the resultant LCs powered by magnetars with lower dipole magnetic fields of

\begin{equation}
L_{\text{th,p}} \sim \frac{E_{\text{rot},i} \tau_{\text{sd}}}{t_{\text{diff}}}.
\end{equation}

where $E_{\text{rot},i}$, $\tau_{\text{sd}}$, and $t_{\text{diff}}$ are the initial rotational energy, spin-down timescale, and the photon diffusion timescale, respectively. The spin-down timescale of a magnetar with tilt angle $\chi$ is $\tau_{\text{sd}} = 3c^3 P_i^2 / (2\pi^2 B_0^2 R^6 \sin^2 \chi)$. The peak luminosity is thus $L_{\text{th,p}} \sim 6L^2 c^3 / (B_0^2 R^6 \tau_{\text{diff}} \sin^2 \chi)$, which shows that a smaller $\chi$ will result in a higher $L_{\text{th,p}}$, as seen in Fig. 2. Furthermore, for $R_{\text{DU}} \lesssim 1.5 \times 10^4$ cm, at the points when the tilt angles grow to $\pi/2$, the magnetars have been considerably spun down, with spin frequencies $\nu(=\Omega/2\pi)$ in the range $\sim 24$–$50$ Hz [see panel (c)].

If $R_{\text{DU}}$ is large enough that the DU cooling becomes dominant, as is the case for $R_{\text{DU}} = 1.5 \times 10^5$ cm, the magnetar cools down to $T_\text{c}$ and has its tilt angle enlarged to $\pi/2$ almost immediately after its birth. The increase of $\chi$ does not lead to a bump in the LC, which is similar to the case of $T_\text{c} = 2 \times 10^9$ K, as shown in Fig. 1. The resultant LC for $R_{\text{DU}} = 1.5 \times 10^5$ cm differs little from that obtained with a constant angle $\chi = \pi/2$, except that the former has a higher $L_{\text{th,p}}$, which results from the suppression of GW emission and the promotion of $L_{\text{th,p}}$ before the orthogonal configuration is realized, as analyzed above. In short, from Fig. 2 two important conclusions can be drawn: (i) with the tilt angle evolution involved, under some conditions, the presence of strong $B_\text{th}$ in a newly born magnetar will result in a magnetar-powered SLSN LC that is quite
and, calculated for central magnetars with $B$ and $\chi$ show the results derived by assuming a constant angle $L_D = 2 < 2 \times 10^{14}$ G, while their toroidal magnetic fields $\tilde{B}_t$ are different. For comparison, we also show the results calculated by taking $P_i = 1$ ms and $\tilde{B}_t = 2 < 10^{16}$ G, but other quantities are kept unchanged (red solid lines). Obviously, for magnetars with lower $B_d$ and larger $P_i$, bumps still appear in the resultant LCs after the maxima if the magnetars have toroidal fields $\tilde{B}_t > 1.9 < 10^{16}$ G. The amplitudes of the bumps are very sensitive to the strengths of $\tilde{B}_t$, and a positive correlation exists between the former and the latter. Thus, bumps appear in the LCs only for magnetars with strong enough toroidal fields. The most interesting result is that compared to the no toroidal field case ($\tilde{B}_t = 0$), the strong $\tilde{B}_t$ in a magnetar could enhance the peak luminosity and the emission after the peak of a SLSN, rather than reduce them due to GW emission as formerly considered \[41, 48\], if the tilt angle evolution is involved. Furthermore, with other quantities kept the same, a smaller initial spin period of $P_i = 1$ ms does not lead to a bump in the LC, as seen in Fig. 3. This is because fast rotation of the magnetar can reduce the damping timescale $\tau_d$ [Eq. 3], so $\chi = \pi/2$ can be achieved very soon.

V. CONCLUSION AND DISCUSSIONS

Newly born magnetars are generally considered to possess small initial spin periods ($P_i > 1$ ms), strong internal toroidal magnetic fields, and initially small tilt angles. The last can
grow to $\pi/2$ due to damping of the free-body process of a magnetar by internal viscosities. In this paper, by involving the tilt angle evolution, we calculated the magnetar-powered SLSNe LCs. We find that, depending on $T_c$, $R_{DU}$, $B_i$, and $P_i$, at the point when the tilt angle of a magnetar grows to $\pi/2$, a bump could appear in the resultant LC after the maximum light. The arising of the bump can be associated with the occurrence of a $3P_2$ neutron superfluid in the magnetar interior; thus furthering the critical temperature $T_c$ and the cooling of the magnetar. We find that for newly born magnetars with $B_i = 5 \times 10^{14}$ G, $B_i = 4.6 \times 10^{16}$ G, and $P_i = 1$ ms there will be no bumps in the LCs if $T_c$ is as high as $2 \times 10^9$ K, or they have large DU cores with radii $R_{DU} = 1.5 \times 10^5$ cm. A lower $T_c$ can result in a bump with larger relative amplitude at later times. Similarly, a smaller $R_{DU}$ leads to a later but more dim bump, and a lower peak luminosity. The most interesting result is that the presence of strong toroidal magnetic field $B_t$ in a newly born magnetar does not lower the peak luminosity of the LC when the tilt angle evolution is involved. Instead, a stronger $B_t$ actually leads to both a brighter peak and a brighter bump. Moreover, for newly born magnetars with $B_i = 2 \times 10^{14}$ G and $P_i = 2$ ms, bumps still arise in the resultant LCs when certain values for $B_i$, $R_{DU}$, and $T_c$ are taken. While keeping other quantities unchanged, a smaller $P_i$ does not actually result in a bump in the LC because of a reduction in the damping timescale $\tau_d$. We therefore suggest that if the SLSNe LCs with such kinds of bumps could be observed in the future, by fitting these LCs with the model presented in this paper, one can determine not only $B_i$ and $P_i$ of the newly born magnetars but also $B_t$, $R_{DU}$, and $T_c$ that are not easy to probe in other ways. The latter three quantities are crucial for our understanding of internal magnetic fields of magnetars, EOSs, and neutron superfluidity at supranuclear densities.

Until now, none of the observed SLSNe shows such kinds of bumps in its LCs. This may be due to the following reasons: (i) A high critical temperature of $T_c > 1.5 \times 10^5$ K is favored. (ii) The central newly born magnetars have $B_i \leq 10^{16}$ G, which may be attributed to a less effective field amplification process related to MRI or an $\alpha - \omega$ dynamo. (iii) The DU process plays a dominant role in the cooling of the central magnetars ($R_{DU} \geq 10^5$ cm), and therefore EOS PAL is more favorable as compared to EOS APR. (iv) The magnetars have small $P_i$ (of 1 ms) but with relatively weak $B_i$ (of $2 \times 10^{16}$ G), and fast growths of the tilt angles in extremely early periods are thus unavoidable, as presented in Fig. 4. Consequently, in the current status, only very rough constraints may be set on these key physical quantities of magnetars. In order to obtain rigorous constraints, more observations of SLSNe with higher precision are needed so as to find such bumps in the LCs. On the other hand, further improvements of our model are also necessary (see the discussion in the last paragraph of this section). In a word, observation of SLSNe LCs may provide a new approach to probe the physics of newly born magnetars.

We note that, in addition to MDR and magnetically induced GW emission, newly born rapidly rotating magnetars may also spin down due to some instabilities that are driven by the emission of GWs [69, 70], for instance, secular instability [71], f-mode instability [72], and r-mode instability [73]. The first two are inclined to arise in more massive stars [52, 74], while the last one may arise in NSs with various masses. If a newly born magnetar can be considerably spun down through GW emissions associated with these instabilities in an extremely short time after birth, a bump may appear in the resultant LC due to a larger spin period of the magnetar, as inferred from Fig. 4 by increasing $P_i$ from 1 to 2 ms. Hence, whether bumps could appear depends on the braking effects of the GW emissions that related to these instabilities. Specifically, for the f-mode and r-mode, their rather uncertain saturation amplitudes are one of the key ingredients that determine the braking effects [74]. Detailed calculations with the effects of these instabilities involved are the aim of our future work.

In present paper, we adopted canonical values $M = 1.4M_\odot$ and $R = 12$ km for newly born magnetars. For more compact (massive) stars, larger DU cores may exist in their interiors, especially when a realistic EOS is considered [54]. This could expedite the cooling of the newly born magnetars, and possibly no bumps will appear in the LCs. Moreover, for more massive NSs with larger DU cores, the damping timescale $\tau_d$ should be recalculated, though we expect that $\tau_d$ may be considerably reduced due to stronger BV of stellar matter in the presence of DU process. In future work, by adopting some realistic EOS, we will calculate the structures and thermal evolutions of NSs with various masses. Based on these results, the magnetar-powered SLSNe LCs will be revisited, and by comparing them with the observed ones, we may get some information about the physical parameters of newly born magnetars as well as EOS of dense matter.

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