Considerations on failure mechanisms of rock slopes involving toppling

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Abstract. Probably the most relevant issue in stability analysis of rock slopes is the correct identification of the potentially occurring failure mechanism, which should be mechanically analyzed to assess stability, later on. Traditional rock slope stability approaches consider planar, wedge, rotational and toppling failure as potential instability mechanisms. Whereas the three first types involve sliding associated to different geometries of the unstable element or mass, toppling often involves also sliding and very complex geometries of multiple elements. In this sense, toppling should be contemplated more like a group of mechanisms than like a simple mechanism such as planar or wedge failure. Toppling could involve moreover one or many blocks. Initial studies classified toppling failure mechanisms in three groups: block, flexural and block-flexural toppling. The stability analysis of rock slopes prone to toppling involves the mechanical analysis of individual slab like blocks, which are considered to present perfect rectangular cross-section. However, the actual shape of these rock elements may not be so regular, so the influence of more realistic irregular shapes is usually not accounted for. In this article, the author will address how some geometry variations may be included in this analysis based on analytical considerations and physical models. Additionally, failure mechanisms observed in rock cuts and open pits often combine toppling with other sliding phenomena in different more or less complex manners. These combined mechanisms involving toppling will be reviewed and some case studies worked out by the author will be presented. Moreover, all along this document, considerations will be put forward regarding the nature of toppling related phenomena where small equilibrium variations may produce a release of a large mechanical energy, which can ultimately produce the destabilization of large slopes or groups of blocks. This suggests that it is wise in these cases to analyze not only the factor of safety, but also the evolution of the potential failure mechanism to understand what is happening and eventually provide sensible and reliable designs or appropriate remedial measures.

1. Introduction

A highly relevant issue in rock slope stability analysis is the identification of the potentially occurring failure mechanisms, which should then be mechanically analyzed. Unlike for the case of more homogeneous materials like soils, in rock masses, a panoply of different failure mechanisms can take place involving sliding and/or toppling according to the orientation, spacing and geomechanical features of occurring rock joints in relation to the slope geometry. The mechanical stability analysis is then carried out in the light of the failure mechanism identified, being this stage critical. Traditional rock slope stability approaches typically consider planar, wedge, circular or rotational and toppling failure as
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potential instability mechanisms [1-3]. For the last case, block, flexural and block-flexural toppling are contemplated (Figure 1).

Once having available a good geometrical and geomechanical characterization of the case under study and, in order to have an appreciation of the stability against any of these type of failure mechanisms, a factor of safety is computed typically based on analytical (Limit Equilibrium Method, LEM) techniques. Factors of safety (FoS) against sliding mechanisms are typically computed based on sliding and stabilizing forces, whereas for the case of toppling the equilibrium is expressed in terms of force moments around a potential rotation axis. Approaches associated to numerical modelling in terms of strength reduction [4-5] can also be used for safety computations. In this case, a strength reduction factor (SRF), equivalent, but different to a traditional factor of safety (FoS), is obtained.

To compute these factors of safety, a number of hypothesis or assumptions are typically needed to allow a correct definition of the problem, and eventually to find a rigorous solution. However, due to the natural complex and heterogeneous nature of rock masses, these assumptions may not strictly hold, which could affect the reliability of the solution found [6]. This is particularly relevant to the analysis of instability phenomena associated to toppling, since small changes in the geometry of the blocks and discontinuities at stake do relevantly affect the mechanical response of the system.

Figure 1. Typical toppling failures as observed in-situ by the author: a) Flexural toppling of schist rocks in a road near Nerva, Huelva (Spain), b) Flexural toppling in a roofing slate quarry in León (Spain), c) Flexural toppling failure in a river valley near Pont de Suert in Lleida (Spain), d) Secondary block toppling in a cliff in Bluff, Utah (USA), e) Toppling of basaltic rock blocks some of which have rolled down the hill near Salta, (Argentina), f) Structure prone to toppling in sandstones in Arches national Park, Utah (USA), g) Toppled granitic blocks in Abadín, Lugo (Spain), h) Sandstone blocks prone to toppling in Table Mountain, Cape Town, South Africa and i) Limestone toppled blocks near the ski resort of Candanchú in the province of Huesca in the Pyrenees (Spain).
Toppling involves rotation of columns or blocks of rocks about a fixed base [2]. There are two distinct cases of instability phenomena associated to toppling failure – block toppling, when the element is already detached from the rest of the rock mass, and flexural toppling, when a tensile strength needs to be attained to break the rock release the block from the rock mass. It is then necessary to know what is the situation for each study and to use the appropriate analysis approach (Figure 1).

Toppling phenomena did not start to be clearly recognized until the late sixties of the past century by Professor Leopold Müller [7]. However, a review of publications in the last fifty years [1-55] as well as the personal experience of the author suggest that toppling mechanisms seem to be much more common than initially thought and they are probably the ones to blame in relation to some relevant problems in open pit mines according to Sjöberg [8]. Figure 1 illustrates a number of toppling problems in rock slopes photographed by the author. They are rather common in natural and engineered slopes for civil or mining engineering purposes all over the world and they can be found in different geotechnical quality rock masses associated to every kind of rocks, including sedimentary, metamorphic and igneous ones.

Moreover, the author has witnessed that toppling phenomena do not only take place in the field of rock mechanics, but they could be relevant to other disciplines as illustrated in the pictures of Figure 2.

![Figure 2. Toppling problems outside rock mechanics. a) Flexural toppling of straw bales in wheat fields near Burgos (Spain). b) Toppling of containers in a cargo ship. c) Tower of Pisa, which would be toppled today due to foundation problems shall appropriate measures would not have been taken.](image)

The simplest toppling mechanisms affect a single block (Figure 3, left hand side). The analysis of such a case is relatively simple [1, 9-11], but it is necessary to a priori know whether the block is already detached from the rock mass (single block toppling) or not (flexural toppling).

The most common toppling failures involve several blocks. According to Goodman and Bray [11] the toppling failure mechanisms usually encountered in the field can be classified in block toppling, flexural toppling and block flexural toppling (Figure 3, right hand side). Block toppling takes place when
in hard rock, individual blocks or columns are formed by two normal joint sets, the main one dipping steeply into the face. The upper blocks tend to topple pushing forward the short columns in the slope toe. Goodman and Bray [11] proposed an approach to compute the stability against this type of failure mechanism. Flexural toppling occurs when continuous columns of rock dipping steeply towards the slope break in flexure and bend forward. Block flexural toppling is a rather complex mechanism characterized by pseudo-continuous flexure along long blocks that are divided by a number of cross-joints. Sketches are provided in Figure 3 and some illustrative examples in Figure 1.a, 1.b and 1.c. Aydan and Kawamoto [12] and Adikhary and co-workers [13-15] proposed limit-equilibrium-based approaches to estimate factors of safety against this type of failure. Block flexural toppling combines features of block toppling and flexural toppling.

**Figure 3.** Typical toppling failures as described in the classic literature [1, 2]. On the left-hand side, single block toppling including simple toppling and flexural toppling of one block. On the right-hand side, many blocks toppling including three type of mechanisms, block toppling, flexural toppling, and block-flexural toppling. Descriptions in the main text.

The stability assessment of rock slopes prone to toppling involves the mechanical analysis of individual slab-like blocks, which are typically considered to present a perfectly rectangular cross-section. However, the actual shape of these rock elements may not be so regular, so the influence of more realistic irregular shapes is usually not accounted for. In this article, the author will address how some geometry variations may be included in this analysis based on analytical considerations and physical models. Rounding of block corners, associated with spheroidal weathering processes, does affect stability of blocks against toppling. Other more complex geometries of blocks also control stability against toppling. So some guidelines will be provided to computing the stability against failure of irregular blocks. It is
relevant to remark that for this case of irregular rock elements a good knowledge of the geometry is needed to compute stability. This geometry can be today obtained in the field by means of contactless geometry surveying techniques (Photogrammetry or Laser Scanner). Once this geometry is well known, the projection of the centre of gravity (cog) of the rock element in relation with the contact base of the block and the potential rotation axis will be the key to reliably compute the FoS.

Additionally, failure mechanisms observed in rock cuts in roads, open pit and quarry walls and natural slopes sometimes combine toppling with other sliding phenomena. In particular, complex rock slope failure mechanisms including toppling phenomena in the lower part of the slope (slide-toe-toppling), in the upper part of the slope (slide-head-toppling) or at some intermediate part of the slope seem to be more common than initially thought [16-29].

In this paper, these types of mechanisms are also reviewed, the observed mechanisms classified and the triggering effects ultimately producing instability are addressed. Then, some case studies representative of these combined mechanisms involving toppling are presented, including a toppling – circular failure (slide-head-toppling) case in a quarry slope, a circular-toppling failure (slide-toe-toppling) representing typical failures in dry-masonry retaining walls and a circular-toppling-sliding case in a bench of a quarry. All these cases will be put into context, the occurring mechanism explained and some guidelines will be given in relation to FoS computation in these rock-slope unconventional instability cases.

2. Stability of a single block against toppling

In this section the author presents analytical, physical, and numerical studies of single block toppling stability in standard conditions (that is, located on a tilted plane, as in Figure 4.1.) and in cantilever or overhanging position (as in Figure 4.2). The cases where the block presents rounded corners (as it could be the case of eroded rock blocks, Figure 4.3), rough bases (Figure 4.4.) and when the regular block is not oriented in the direction of the strike of the resting plane (Figure 4.5) are also presented. Eventually, it will be briefly explained how to compute the stability against toppling of blocks of irregular shape resting on a plane, by means of two examples: a composite cylinder including a rock part and a steel part non-coaxially disposed and a natural irregular granite boulder.

In this section, the author puts forward the fact that, even if computing the stability against toppling of a simple regular block can be easy, computing or assessing the stability of a natural rock block, typically complex and with rough contacts in its base and with neighbors, is still a challenging task. So, when carrying out stability analyses involving many blocks results should be contemplated as only indicative [30].

2.1. Analytical, physical and numerical analysis of standard single block toppling at lab scale

The LEM factor of safety against toppling failure is generally computed as the relation of stabilizing moments against toppling or overturning moments computed according to the potential rotation axis (equation (1)):

$$FoS_{toppling} = \frac{\sum M_{stabilising}}{\sum M_{overturning}}$$

The stability of the block can also be studied by means of physical tilt test models, in which the block is placed on a horizontal surface or platform and being them progressively tilted until the block topples; so, the angle of actual toppling can be physically observed. To do this it is possible to use a tilting table [31-33], where tilt tests can be performed. In these tests, eventually sliding or toppling will be observed according to the case where the critical angle is smaller.

Toppling will be the typical instability mechanism for the case of slender blocks. Obviously, the block will slide when the dip of the table attains a tilt angle equal to the friction angle of the contact. We will try to avoid this situation to study toppling.

Distinct Element Methods and, particularly, those implemented in UDEC or 3DEC [34-35] can also be used to study the stability of blocks against toppling. To do that, models are set up with a large fixed base or platform on which the block is located. Tilting the platform progressively or trying various
models with different dips, the tilt angle at which the blocks topples or slides could be easily identified. Examples of such models for a simple block on a resting plane and for an overhanging block are shown in Figure 5.

![Figure 4](image-url)

Figure 4. Different cases of generally simple blocks whose toppling behaviour is under scrutiny. 1) Simple slab-like block resting on a tilting platform 2) Simple slab-like block overhanging on a tilting platform 3) Rounded corner block on a tilting platform 4) Block with a regular rough base and 5) Regular slab-like block oriented an angle in relation to the strike of the tilting platform.
One should be aware that UDEC considers a certain rounding of the block corners. For standard cases studied in this section, this rounding has been fixed in $10^{-6}$ m to be close to the ideal case of perfect corners. Also remark that the contacts of blocks are controlled by normal and shear stiffness, which do affect behavior of the system before sliding or toppling but, they generally scarcely affect critical stability angle outputs.

2.1.1. Stability of a single regular slab-like block.

As proposed by various authors for static and dynamic conditions [1-3; 9-11], for the case of single block toppling and considering a slab-like block with height $y_n$ and width $\Delta x$ with perfectly squared corners (Fig. 4.1), the factor of safety ($FoS$) against toppling can be calculated accordingly as (equation (2)):

$$FoS = \frac{M_{EST}}{M_{OVERT}} = \frac{W \cdot \cos \alpha \cdot \frac{\Delta x}{2}}{W \cdot \sin \alpha \cdot \frac{y_n}{2}} = \tan^{-1} \frac{\Delta x}{y_n}$$

(2)

Conversely, the critical dip angle at which toppling of a regular slab-like block takes place ($\alpha_{crit}$) can be obtained by solving equation (2) for $FoS = 1$, and therefore (equation (3)):

$$\alpha_{crit} = \arctan \left( \frac{\Delta x}{y_n} \right)$$

(3)

This provides a first analytical approach to estimate the stability of blocks. The instability condition fulfills when the weight of the block (a vertical vector with origin at its gravity center) falls out the base of the block. This condition can be generally applied to any kind of block.

As a brief example, Table 1 shows the physical (with a tilt testing machine), analytical (according to Eq. 3) and numerical (with code UDEC 5.20) results of the critical angle obtained for four white granite saw cut blocks tested. Additionally, the $FoS$ corresponding for the critical angle physically observed is also presented according to equation (2).

Remark that analytical and numerical critical angle results tend to be equal with negligible errors of hundredths of degrees. However, the physical toppling angles observed tend to be somewhat smaller (between 0.5 and 3º) than the analytical ones, something the author attributes to the fact that due to cutting imperfections, some rounding of the edges does occur. This makes the block toppling slightly before than expected. The effect of this rounding effect will be further illustrated in next sections. Moreover, this rounding will be logically more relevant for slim blocks (block 2 in the table with 3º error) than for thick blocks (block 4 in the table with 0.3º error).

Figure 5. UDEC models of a simple toppling block and an overhung block.
Table 1. Results for simple slab-like granitic blocks tested against toppling in the tilt testing table including specimen, dimensions, tilting angle physically observed, analytically computed, and numerically estimated, and finally factor of safety against toppling in the moment of the observed toppling.

| Specimen | Material       | Average width \( \Delta \) (mm) | Average height \( y_n \) (mm) | \( \alpha \) \(_{\text{phys.}} \) (º) | \( \alpha \) \(_{\text{anal.}} \) (º) | \( \alpha \) \(_{\text{num.}} \) (º) | FoS for \( \alpha \) \(_{\text{phys.}} \) |
|----------|----------------|-------------------------------|------------------------------|----------------|----------------|----------------|----------------|
| 1        | White granite  | 15.93                         | 50.39                        | 16.10           | 17.55           | 17.54           | 1.10            |
| 2        | White granite  | 14.80                         | 48.73                        | 13.97           | 16.89           | 16.89           | 1.22            |
| 3        | White granite  | 21.40                         | 50.00                        | 22.33           | 23.17           | 23.17           | 1.04            |
| 4        | White granite  | 39.78                         | 100.25                       | 21.32           | 21.65           | 21.64           | 1.02            |

2.1.2. Stability of a single regular overhung slab-like block.

For the case of a single block able to topple located in a position overhanging a tilting plane a distance \( \delta \) (Fig. 4-2), the FoS against toppling can be analytically computed as the relationship between the stabilizing and overturning moments, which now reads (equation (4)):

$$
\text{FoS} = \frac{M_{\text{EST}}}{M_{\text{OVERT}}} = \frac{W \cdot \cos \alpha \left( \frac{\Delta x}{2} - \delta \right)}{W \cdot \sin \alpha \frac{y_n}{y_n}} = \tan^{-1} \alpha \left( \frac{\Delta x - 2\delta}{y_n} \right)
$$

Equating FoS to 1 in equation (4) as for limit equilibrium, we can deduce the corresponding critical dip angle at which toppling of a regular overhung slab-like block take place (\( \alpha \)\(_{\text{crit.}} \)):

$$
\alpha \text{_{crit.}} = \arctan \left( \frac{\Delta x - 2\delta}{y_n} \right)
$$

Table 2 illustrates physical, analytical and numerical (with code UDEC 5.20, as shown in Figure 5) results of the critical angles obtained for a white granite block overhung on surface some millimeters and then tilted until toppled. Again, the FoS corresponding for the critical angle observed is computed as per equation (5). Similar trends than those commented for the previous case are observed.

Table 2. Results for specimens tested against overhanging toppling in the tilt-testing table including specimen, dimensions, cantilever, tilting angle physically observed, analytically computed and numerically estimated, and FoS against toppling in the moment of the observed physical toppling.

| Specimen | Width \( \Delta \_k \) (mm) | Height \( y_n \) (mm) | Cantilever \( \delta \) (mm) | \( \alpha \) \(_{\text{phys.}} \) (º) | \( \alpha \) \(_{\text{anal.}} \) (º) | \( \alpha \) \(_{\text{num.}} \) (º) | FoS for \( \alpha \) \(_{\text{phys.}} \) |
|----------|-----------------------------|----------------------|-----------------------------|----------------|----------------|----------------|----------------|
| 4        | 49.8                        | 39.85                | 0                           | 21.63          | 21.65          | 21.64          | 1.00            |
| 4        | 49.8                        | 30.15                | 4                           | 17.30          | 17.59          | 17.57          | 1.02            |
| 4        | 39.85                       | 30.15                | 8                           | 12.95          | 13.35          | 13.33          | 1.03            |
| 4        | 39.78                       | 100.25               | 12                          | 8.20           | 8.95           | 8.93           | 1.09            |

2.1.3. Stability of a single regular round cornered slab-like block.

The case of a regular specimen with perfectly regular rounded corners (in line with eroded rock blocks observed in Nature, as shown in Fig. 4-3) behaves in a similar manner to a perfectly square block in an overhanging position [36]. Therefore, the stability of these blocks can be analytically computed considering the radius of the rounded corner (\( r \)), in such a way that the FoS against toppling could be calculated through equation (6):
\[ FoS = \frac{M_{est}}{M_{over}} = \frac{W \cdot \cos \alpha \left( \frac{\Delta x - r}{2} \right)}{W \cdot \sin \alpha \left( \frac{y_n}{2} \right)} = \tan^{-1} \left( \frac{\Delta x - 2r}{y_n} \right) \]  

(6)

And the critical angle can be computed for the equilibrium situation as (equation (7)):

\[ \alpha_{crit} = \arctan \left( \frac{\Delta x - 2r}{y_n} \right) \]  

(7)

In natural blocks, the radius \( r \) may not be constant all along the corresponding edge around which the block tends to rotate, so this value should be carefully considered. It is also important to have in mind that slab-like regular blocks considered sharp edged and perfectly regular may show some minor rounding in their corners due to the cutting process (for small blocks prepared in the lab) or of discontinuity formation in the field cases. This is why the analytical critical angle computed usually tends to overestimate that observed when tilt testing the specimens in the lab.

To check this approach, a slab-like 50 mm x 40 mm x 30 mm specimen with perfect 10-mm radius rounded corners was made with a 3D-printer. The specimen was named ABS. It was tilt tested in directions normal to their 3 principal axes. Results, in line with previous examples, are presented in Table 3, with 4 tilt tests averaged in every direction. Results show a rather good accuracy of the analytical and numerical methods with errors below 0.75º when compared to experimental results — an accuracy typically smaller than that expectable when measuring rock discontinuities in the field.

| Specimen | Width \( \Delta \) (mm) | Height \( Y_n \) (mm) | Radius \( r \) (mm) | \( \alpha_{phys} \) (º) | \( \alpha_{anal} \) (º) | \( \alpha_{num} \) (º) | FoS for \( \alpha_{phys} \) |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| ABS      | 39.85           | 49.83           | 10              | 22.12           | 21.73           | 21.73           | 0.98            |
| ABS      | 30.15           | 49.83           | 10              | 10.80           | 11.52           | 11.52           | 1.07            |
| ABS      | 30.15           | 39.80           | 10              | 14.27           | 14.30           | 14.29           | 1.00            |

2.1.4. Stability of a single regular slab-like block with concave, convex of rough base.

When the block base is not a perfect plane but it is a concave, convex or rough surface; this may affect the stability of the block. Let analyze the case of a block with a regular saw-tooth surface (Figure 4.4) or any other block with a convex, concave, or rough known-geometry base. It is possible to compute the effect of this roughness on the stability against toppling of the element by decomposing the block in simple forms (rectangles and triangles) and developing the formulation of the FoS in terms of stabilizing and overturning moments. For demonstrative purposes, Figure 6 shows the detail of the base of a block depicted in Figure 4.4 for the cases where the down dip side of the rough discontinuity, which is at the same time the potential rotation axis, coincides with a peak (Figure 6.a) and when this point coincides with a valley (Figure 6.b).

Carrying out the corresponding calculations for such a specimen formed by 3 glued 95-mm average high and 15-mm wide rock elements and a regular roughness of \( i = 20^\circ \), it turns out that for the peak axis case the toppling angle would be 3º higher than for the valley axis case (a difference also observed in practice). This difference is associated to the shorter moment arm of the component parallel to sliding of the main body of the element for the axis peak case and the stabilizing moment of the weight components parallel to sliding of the triangles close to the rough base (Figure 6).

Similar results can be obtained for concave and convex-based blocks (Figure 4.4) and results with variations in the range 0-5º are typically found. Even if these are not dramatic differences, in natural blocks this slight change could make the difference between toppling and stability.
2.1.5. Stability of a slab-like block oriented non-parallel to the strike of the resting plane.

In this case, the potential rotation axis does not coincide with plane strike line and therefore it is not a horizontal line, so the component of the weight parallel to the resting plane does not totally contribute to overturn the block. Only the component of this force acting perpendicular to the lower face of the slab-like block pushes the block (Figure 7).

Therefore, the FoS against toppling can be calculated in this case according to equation (8).

\[
FoS = \frac{M_{EST}}{M_{OVERT}} = \frac{\Delta x \cdot W \cdot \cos \alpha}{\frac{y}{2} \cdot W \cdot \sin \alpha \cdot \cos \gamma} = \tan^{-1} \alpha \cdot \frac{\Delta x}{y_n \cdot \cos \gamma}
\]  

where \( \gamma \) is the angle formed by the orientation of the potential rotation axis of the slab with the strike of the resting plane. The critical dip angle at which toppling of a regular slab-like block oriented non-parallel to the strike of the resting plane occurs can be obtained solving equation (8).

\[
\alpha_{crit.} = \arctan \left( \frac{\Delta x}{y_n \cdot \cos \gamma} \right)
\]  

As in previous cases, this condition fulfills when the weight of the block (a vertical vector passing through its center of gravity) falls out the base of the block. This base will be now a rectangle, but it will not be oriented parallel to the strike and dip of the resting plane. In case the block presents rounded corners, the corresponding equations can be extended to this situation, so the critical tilting angle will be now (equation (10)): 

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**Figure 6.** Potentially toppling block with irregular faces. Both depicted blocks are the same. In the case a) the block is tilted in such a way that the potential rotation axis coincides with a roughness peak and is more stable than in case b) where the block is positioned in such a way that the potential rotation axis coincides with a roughness valley.
\[ \alpha_{\text{crit.}} = \arctan \left( \frac{\Delta x - 2r}{y_n \cdot \cos \gamma} \right) \quad (10) \]

**Figure 7.** Simple slab-like block resting on a tilting plane oriented forming an angle gamma in relation to the plane strike. The resting plane strike and dip direction, the potential toppling rotation axis and the force decomposition are depicted.

**Table 4.** Results for a plastic printed specimen tested against toppling in different directions. Stars mark situation where the block topples in a different direction.

| Specimen | Width \( \Delta x \) (mm) | Height \( Y_n \) (mm) | Deviation \( \gamma \) | Radius \( r \) (mm) | \( \alpha_{\text{phys.}} \) (º) | \( \alpha_{\text{anal.}} \) (º) | FoS for \( \alpha_{\text{phys.}} \) |
|----------|--------------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| ABS      | 39.85                    | 49.83                | 10                   | 10                   | 22.03                | 22.37                | 0.98                 |
| ABS      | 39.85                    | 49.83                | 20                   | 10                   | 22.98                | 23.77                | 0.96                 |
| ABS      | 39.85                    | 49.83                | 30                   | 10                   | 24.71                | \( ^* \)              |                      |
| ABS      | 39.85                    | 49.83                | 40                   | 10                   | 27.49                | \( ^* \)              |                      |
| ABS      | 30.15                    | 49.83                | 10                   | 10                   | 10.85                | 11.70                | 1.08                 |
| ABS      | 30.15                    | 49.83                | 20                   | 10                   | 11.47                | 12.24                | 1.07                 |
| ABS      | 30.15                    | 49.83                | 30                   | 10                   | 11.05                | 13.25                | 1.21                 |
| ABS      | 30.15                    | 49.83                | 40                   | 10                   | 14.32                | 14.90                | 1.04                 |
| ABS      | 30.15                    | 39.80                | 10                   | 10                   | 15.18                | 14.51                | 0.95                 |
| ABS      | 30.15                    | 39.80                | 20                   | 10                   | 15.10                | 15.17                | 1.01                 |
| ABS      | 30.15                    | 39.80                | 30                   | 10                   | 16.33                | 16.40                | 1.00                 |
| ABS      | 30.15                    | 39.80                | 40                   | 10                   | 17.28                | 18.40                | 1.07                 |

\( ^* \) Block tilted in a different direction.
The sample named ABS presented in Figure 4.3 and tilted normal to its faces in section 2.1.3 has been now tested by orienting it in different directions in relation with the strike of the tilting platform. Results are again rather accurate with errors typically smaller than $2^\circ$. It is relevant to remark that when tilting this block with the 4 cm base and 5 cm height and rotated 30 and 40 $^\circ$ in relation with the platform strike, the specimen toppled in the lateral direction, so results were not computed. This observation could be relevant when analyzing the toppling of rock blocks resting in dihedral (wedge-like) surfaces.

### 2.2. Analytical, physical and numerical analysis of irregular blocks

Aiming at studying the stability against toppling of irregular blocks, as those typically found in Nature, the same approaches as those used for standard blocks (analytical, physical and numerical analyses) should be applied. However, in this case, some techniques should be used to properly handle geometry and to identify the position of the rotation axis. In particular, in irregular blocks, the normal projection of the center of gravity ($cog$) on the resting plane is typically not located in the center of the contact area, which tends to be moreover irregular. This makes the identification of the rotation axis not easy, which complicates all the stability calculation process.

In order to explain how to manage these problems, two illustrative examples will be presented in what follows regarding stability assessment of these irregular blocks, namely the stability of a composite cylinder and that of a natural large granite boulder.

#### 2.2.1. Stability of a composite cylinder.

This case is analyzed because the normal projection of the $cog$ of the element under scrutiny on the resting plane is not located at the centre of the cylinder base, so it is illustrative on how to manage this [32].

A composite cylinder was prepared combining two cylindrical specimens. Both have 27-mm radius ($r$). The first one is 100 mm high and is made of hard rock with a density of 2650 kg/m$^3$ and the second one is 35-mm high and is made of steel with a density of 7800 kg/m$^3$. The second one is positioned on the top of the first one and is horizontally displaced outwards the coaxial position a distance $r/2 = 13.5$ mm with the aim of moving the $cog$ away of the symmetry axis of the regular element. Figure 8.a shows a photograph and projections of the specimen with the location of its $cog$, together with the equations for computing its position. The composite specimen is tilted until it topples.

Based on this test configuration (Figure 8b), it is possible to compute the FoS against toppling of the system:

$$
\text{FoS} = \frac{M_{STR.}}{M_{OVERT.}} = \frac{W \cdot x_{G-\text{crit.}} \cdot \cos \alpha}{W \cdot z_{G} \cdot \sin \alpha} = \frac{\sqrt{r^2 - y_G^2}}{z_G \cdot \tan \alpha} = \frac{\tan \alpha_{\text{crit.}}}{\tan \alpha}
$$

(11)

Where $y_G$ and $z_G$ refer to the coordinates of the $cog$ of the sample, which can be computed together with $x_{G-\text{crit.}}$ based on the formulae embedded in Figure 8. The toppling angle $\alpha_{\text{crit.}}$ at which the set topples can be computed solving equation (11) for limit equilibrium (FoS =1), in such a way that:

$$
\alpha_{\text{crit.}} = \tan^{-1} \frac{\sqrt{r^2 - y_G^2}}{z_G}
$$

(12)

Additionally, in this case, the element does not topple in the dip direction of the tilting plane, but in a direction forming an angle $\beta$ with this one (equation (13)).

$$
\beta = \tan^{-1} \frac{y_G}{\sqrt{r^2 - y_G^2}}
$$

(13)

The actual results of the calculations for the composite cylinder (Figure 8) are shown in Table 5, where a good matching of experimental and theoretical results is again found.
Figure 8. Photograph and projections of the composite cylinder used for this experiment and (b) Left view showing the evolution of the tilt test and the position of the cog while testing until toppling of the sample.

Table 5. Experimental, analytical and numerical results for tilt tests with an irregular specimen.

| Specimen           | $\alpha_{\text{phys.}}$ (°) | $\alpha_{\text{anal.}}$ (°) | $\alpha_{\text{num.}}$ (°) | FoS for $\alpha_{\text{phys.}}$ | $\beta_{\text{phys.}}$ (°) | $\beta_{\text{anal.}}$ (°) |
|--------------------|-----------------------------|-----------------------------|-----------------------------|--------------------------------|---------------------------|---------------------------|
| Composite cylinder | 16.1                        | 17.3                        | 17.3                        | 1.08                           | 15±1                      | 14.2                      |
| Granite boulder    | 30.1                        | 31.4                        | 30.5                        | 1.06                           | 10±2                      | 9.5                       |

From this experiment, two issues relevant for assessing the stability of real boulders derive. First, for a given tilting angle ($\alpha$) the FoS is reduced with respect to those configurations keeping the cog in a symmetry plane (e.g., tilt test with an equivalent rock specimen alone or with both specimens aligned). Second, the rotation point, for the misaligned set, becomes deviated from the line of maximum slope, as shown by $\beta$ angle in Figure 8b and as observed in the physical tilt test, which means in practice the orientation of the rotation axis should be painstakingly considered and not taken for granted.

2.2.2. Stability of a natural granite boulder.
The case briefly described here was reported in more detail elsewhere [32, 37], developed starting from previous studies on granite boulder stability [38, 39]. The stability of one of this natural granitic structures, a rocking stone not-surprisingly locally known as ‘Equilibrium Stone’ was studied. This is a large ellipsoidal boulder (about 8 m length and 3 m height) weighing 380 tons and resting on small contact area (less than 1 m$^2$) on a plane dipping 27°, so potential instability is naturally perceived (Figure 9.a). A very detailed knowledge of the geometry of the boulder was recovered by means of contactless survey methods, producing and accurate 3D point cloud (Figure 9.b), which allowed a rigorous knowledge of the boulder geometry including the location of the cog in its appropriate coordinates, the contact area extent and the projection of the cog in this area. It also allowed to 3D print a plastic replica of the boulder (Figure 9.c), which was later on used for a physical tilt testing model of the boulder.
Figure 9. a) Picture of the ‘Equilibrium Stone’ in NW Spain. b) Laser Scanner and 3D point cloud of the boulder and detail of the contact area within the boulder and projected on the resting plane c) 3D printed plastic replica of the boulder in a tilt test and d) detail of the contact area on the resting plane with normal and vertical projection of the cog for analytical stability computation.

Based on the coordinates of the cog and their projection on the contact area it was possible to compute the FoS of the boulder stability against toppling to be 1.20, following the procedure explained for the composite cylinder case. This FoS would go down to 1.09 and 0.99 if pseudo-static analysis were applied considering horizontal accelerations of 0.04·g (as recommended in seism-resistant legislation for the area) and 0.08·g (corresponding to the largest ever expected earthquake) [32].

Figure 10. Numerical 3DEC models of toppling of the non-standard rock blocks analyzed in this section as presented elsewhere [33, 37] and providing results consistent with analytical estimates (Table 5).
The critical dip value as computed and observed in the physical model with the 3D printed element are presented in Table 5, together with other relevant results. 3DEC models of this case [37] and the composite cylinder [33] are illustrated in Figure 10 and the corresponding FoS values reported in Table 5, where all the values and mechanisms indicate consistent trends.

The approaches presented in this section show that it is possible to compute the stability of any irregular rock block against toppling, provided a good knowledge on the nature and geometry of the block is available.

2.3. Stability of single block toppling in some quarry case studies

When extending the presented results to possible toppling mechanisms in rock slopes, other forces should be accounted for. In the case of quarries, water can play a relevant role. As an example, Figure 11 shows a bench in an aggregate quarry where periodically rockfall was observed associated to toppling failure. The analysis of the simple block flexural toppling is illustrated in Figure 11 including the computations of the FoS for the conditions applied. As it can be observed in the FoS results, while the block will be very stable for dry conditions (FoS = 6.42), it became totally unstable when the crack behind the block is saturated (FoS = 0.14). Indeed, the block would tend to topple (and produce rockfall) when its rear part is half-filled with water, a situation that can be expected to happen once every two or three years. Therefore, consideration of water flow tends to be extremely relevant when studying the field stability of blocks against toppling.

The author has been involved in a few stability studies [40, 41] of ornamental granite quarry overtilted benches (Figure 11) typically cut following the direction of discontinuities, namely rift and grain (parallel to the slope face) [42]. These stability studies generally show that there is a low probability of occurrence of individual block toppling instability phenomena in the studied quarries, associated to the large spacing and non-persistence of discontinuity planes. It is relevant to note that ranges of joint spacing and continuity as suggested by ISRM may be broad and accurate enough for this kind of applications, so a more detailed characterization is probably needed. Moreover, engineering geology issues, such as a good representation of conspicuous flow and occurrence of rock bridges, may turn standard studies conservative.

**Figure 11.** Bench in a quarry where rock table is prone to toppling. Simplified crosscut, photograph, formulation, parameters and results for various water conditions.
Figure 12. Photographs of various over-tilted benches in ornamental granite quarries in NW-Spain studied by the authors to check their stability.

3. Physical model examples of block toppling and comparison to LEM

The phenomenon of block toppling was studied in detail by Goodman and Bray [11], who proposed an analytical limit equilibrium solution, implemented in computer codes later on [43]. This has been the base of the analysis of toppling failure for the last 40 years. Numerical models have been able to reproduce the analytical results with a good accuracy [5, 29, 44, 45], but often the problem of fulfillment of assumptions needed to apply the approach is not found in practice [30, 46].

Goodman and Bray’s approach consisted on analyzing a regular system of blocks in which a slope at angle $\theta$ is excavated in a rock mass with layers dipping at $90-\alpha$ and the block base is stepped upward with overall inclination $\beta$ [11]. Based on these geometrical constrains and the height of the slope the number of blocks and the geometry of every block can be computed. In case the angle $\alpha$ is smaller than the friction angle of the contact base of blocks ($\phi$), block toppling is prone to take place.

To analyze this [11], the stability of every block is studied starting from the upper blocks and going downwards. Typically, upper blocks are plump, so they do not to topple. Below this upper zone, blocks are slender, so they tend to topple. Proceeding down the slope and in its lower part, blocks are again plump so they would not tend to topple by themselves, but due to forces transmitted by the upper blocks they usually tend to slide (Figure 13.a). Goodman & Bray [11] suggested computing equilibrium for all the blocks against toppling and sliding and going down the slope and iterate the friction angle until the limit equilibrium is found in the first block (accounting for the forces transmitted from the upper blocks). The FoS could then be estimated as the ratio between the actual friction coefficient ($\tan \phi$) and the friction coefficient that results in limit equilibrium of the whole array of blocks.

The core of the approach relies [11] on the equations computing the force ($P_{n-1}$) that should be applied in the lower face of every block ($n$) to ensure equilibrium against sliding (equation (14)) and against toppling (equation (15)), which are computed forcing the equilibrium (FoS = 1) for the equilibrium of every block of the system. The larger of these two forces (if positive) will be transmitted to block $n-I$, and this process will be repeated until achieving the first block. In this way, this force for sliding will be:

$$P_{n-1,s} = P_n - \frac{W_n \cdot (\tan \phi_n \cdot \cos \alpha - \sin \alpha)}{1 - \tan \phi_n \cdot \tan \phi_j}$$ (14)
And that against toppling:

\[
P_{n-1,t} = \frac{P_n \left[ M_n - \tan \phi_j \cdot \Delta x \right] + \frac{W}{2} \left[ y_n \cdot \sin \alpha - \Delta x \cdot \cos \alpha \right]}{L_n}
\]  

(15)

In case the blocks of the array present rounded corners, the equation to ensure equilibrium against sliding does not change, even if the block’s weights will be slightly smaller. However, the force against toppling should be recalculated to reflect the change in blocks’ geometry (Figure 13 b) [47]:

\[
P_{n-1,t} = \frac{P_n \left[ (M_n - r) - \tan \phi_j \cdot (\Delta x - r) \right] + \frac{W}{2} \left[ y_n \cdot \sin \alpha - (\Delta x - 2r) \cdot \cos \alpha \right]}{L_n + r \cdot (\tan \phi_j - 1)}
\]  

(16)

This rounding of the corners does affect stability so it could be convenient to account for it in calculations [47]. To check the validity of the Goodman & Bray [11] approach and analyze departure from the assumptions needed to apply it in practice, the author carried out physical scale models for two groups of 10 dedicatedly prepared blocks, one with sharp edges and the other one with rounded corners. These blocks were toppled in a planar and in a stepped base (Figure 14). Geometrical calculations were performed to compute the parameters needed to apply the Goodman and Bray’s approach [11], and the presented version of this approach for rounded corners [47].

The results of the tilt tests carried out with these models as shown in Figure 15 resulted in tilting angles as presented in Table 6. The reader can check how the theoretical approach sensibly coincides with the physical observations. It is also relevant to highlight the fact that rounding the corners of the block translates in instability for lower tilting angles. When equation accuracy was checked against the analytical approach, we found a standard deviation of 1°, which is quite an acceptable value for the rock mechanics’ discipline. This type of response contributes to explain some field reports, where blocks toppled at base angles lower than theoretically computed [47].

It could also be observed that the impact of edge rounding on stability largely depends on the overall slope geometry and on the friction angles, particularly for the contact base. When sliding was more relevant, the influence of rounded edges was limited. However, for a small number of not-so-slender blocks, the impact of rounded edges on stability tends to grow dramatically. Likewise, rounded corners do not play a significant role when there is a large number of slender blocks but they do for a small number of thicker blocks. It can be concluded that based on the developed approach [47], the role played by the rounding of block corners can be incorporated with a reasonable level of accuracy in both analytical approaches to single block and block toppling stability analyses.

![Figure 13. a) Typical stability trends of blocks in a block toppling array [43] and b) sketch of forces in a block of the array for the case of rounded cornered blocks [47].](image-url)
Figure 14. Geometrical approaches for computing geometrical parameters of block toppling on a planar and on a stepped base and groups of sharp edge and round corner blocks for physical modelling.

Figure 15. Four block toppling models at failure. a) Sharp edge model on flat wooden base, b) Rounded edge model on flat wooden base, c) Sharp edge model on stepped wooden base, d) Rounded edge model on stepped wooden base.

Table 6. Summary of the block toppling results (in degrees).

| Base            | Sharp Edges |         | Rounded Edges |         |
|-----------------|-------------|---------|---------------|---------|
|                 | Prediction  | Results | Prediction    | Results |
| Flat Wooden     | 20.9        | 21.4    | 20.5          | 19.4    | 19.6    | 17.1    |
| Stepped Wooden  | 18.7        | 18.8    | 18.8          | 16.6    | 16.6    | 15.8    |
4. Complex rock slope instability phenomena involving toppling
In this section an attempt to classify actual complex rock stability problems involving toppling is presented. Then, a brief review of triggering effects producing the instability phenomena in this type of mechanism is presented. Some real examples in rock slopes are presented in the next section.

4.1. Classification of complex rock slope instability failure involving toppling
A literature review carried out by the author has showed that a good number of rock slope stability problems available in the literature involve toppling phenomena, though other type of mechanisms takes place at the same time in different parts of the slope.

Based on this, the author has attempted to classify these phenomena focusing on the part of the slope where toppling phenomena occur. Accordingly, it is proposed to classify complex rock slope instability phenomena involving toppling in the following three groups, which are schematically illustrated in Figure 16.

1. Combined rock instability where toppling occurs in the upper part of the slope and (planar or rotational) sliding occur in the lower part of the slope. This mechanism is also known as slide-head-toppling. This type of mechanism is illustrated in Figure 16.a. A typical case study of this type of mechanism worked out by the author is presented in section 5.1 corresponding to a slope in a clay and sandstone quarry [21]. Similar mechanisms have been reported by others [24, 48, 49].

2. Combined rock instability where (planar or rotational) sliding occur in the upper part of the slope and toppling occur in the lower part of the slope. This mechanism is also known as slide-toe-toppling. Fig. 16.b. illustrates this kind of phenomena. A typical case study of this type of mechanism and an approach to estimate its stability is proposed by Mohtarami et al. [50] corresponding to a mountain slope in Iran. Other authors also presented cases fitting this model [26, 29, 51]. Alejano et al. [52, 53] present the study of dry masonry retaining wall stability where the observed failure mechanism responds to this mechanism in that failure of a granular material behind the wall induces toppling of the masonry wall that behaves as an engineered rock mass. This case will be briefly introduced in this document.

3. Combined rock instability where toppling occurs in the middle of the slope accompanied in the upper and lower parts by sliding phenomena comprehending planar or rotational failure or wedge moving (sinking in the upper part or extruding in the lower part). An illustrative view of this type of mechanism is presented in Figure 16.c. A failure that took place in a quarry bench and worked out by the author is presented in section 5.3 [27]. Böhme et al. [22] described a complex landslide of this type, where, based on field observations, kinematic analyses and numerical modelling, the authors identified that the failure mechanism was composed by subsiding bilinear wedge failure, a toppling component and planar sliding along the foliation at the toe of the slope. Additionally, León-Buendía et al [23] described a complex slope failure in a quartzite rock cut excavated in highly fissured quartzite rock mass. In the instability phenomenon observed, three parts or mechanisms could be distinguished: an upper part in which sliding predominates over toppling, the intermediate part of the slope where toppling is more frequent and the lower part of the slope in which block or wedge extrusion took place. Gu and Guang [25] reported a similar case in a river valley in China and Hutchinson et al. [19] described a similar problem in an open pit mine wall in Australia.
Figure 16. Illustrative classification of complex rock failure mechanisms involving toppling including: a) complex failure involving toppling in the upper part of the slope, b) complex failure involving toppling in the lower part of the slope and c) complex failure involving toppling in the middle part of the slope.
4.2. Triggering effects producing instability phenomena associated to toppling

When a block or set of blocks prone to topple occur in the middle of a potentially unstable slope in a rock mass, small changes in forces or moments taking place in the block or set of blocks may trigger instability phenomena. These conclusions were reached in the light of observations on physical models, on the actual behavior of tilting blocks and on some failure mechanisms observed in practice.

Consider a slender rock block in the middle of a rock mass prone to sliding, with a geometry and applied forces such that it is stable, although close to limit equilibrium against toppling (the applied forces are projected within its base). If an additional small force causes the block to topple, then the whole weight of the block is applied to the back of the potential sliding mass in the lower part of the slope. Thus, when such a block becomes unstable, the significant and sudden force of its weight pushing the blocks below can contribute to the general instability of a rock zone. The additional toppling force referred above can be associated with a rise of the groundwater level, which increases the water pressure applied in the block. It may also be associated with deepening of the slope, seismic events, blasts in the slope zone or erosion causing block corners to round.

To illustrate this, consider the following simple analysis of block toppling. A typical case of block toppling using RocTopple [53] (Figure 17) was modelled. The parameters used were slope height 13 m, slope angle 79°, block width (Δx) 1.6 m, block base angle 9° (perpendicular to the counter slope dip of 81°), overall base inclination 33°, upper slope angle (20°) and base and block contact friction angle 40°. Whereas for the dry case, a FoS of 5.23 is obtained, which would satisfy any design engineer; a mild rise of the water level expectable after a few rainy days will downscale the FoS to 0.7, meaning failure.

![Figure 17. RocTopple [54] analysis of slender block toppling in a) dry conditions and b) wet conditions associated to a mild rise of the water level.](image-url)
5. Three case studies of complex rock slope instability phenomena involving toppling
In this section, three illustrative case studies are presented, reflecting different types of complex rock-slope failures involving toppling phenomena.

5.1. A block toppling – circular failure in a quarry slope
In the process of developing a study to the enlargement of a clay and sandstone quarry in E-Spain, an instability mechanism took place that was difficult to understand in the early stages of investigation [21]. The geology of the quarry comprehended a sedimentary formation, which appeared in the field with a minimum thickness of 100 m and had ten alternating sandstone and claystone beds, 4- to 12-m thick. This formation was overlaid with thin beds of conglomerates and marls and a very thick limestone formation more than 100 m thick. The geological structure of the mine zone was such that the sedimentary package to be mined was located in a block (between two sunken blocks) limited by two sub-vertical normal faults, which also marked the mine limits (Figure 18).

A geomechanical characterization was carried out to estimate the strength of the claystones and sandstones in the range 50-60 kPa of cohesion and 30-35º friction. Discontinuities were measured in the limestone to find out that, apart from rather horizontal bedding, a highly continuous (more than 20 m) and closely spaced (0.6-2 m) discontinuity set existed dipping around 75º towards the NW slope. With this information in mind, it was possible to identify a failure mechanism (Figure 19.a) in the NW slope as a complex one including toppling in the upper part and circular failure in the lower part in the weak sedimentary rocks. This clearly falls within the first group of classification of section 4.1.

Figure 18. Southeasterly-northwesterly geological cross-cut of the quarry.

Figure 19. a) Photograph of the unstable northwestern wall of the quarry showing the complex failure mechanism described in the text b) Numerical results with UDEC.
The factor of safety against this type of failure was computed, first, analytically and then, numerically (Figure 19.b). The analytical calculation was carried out by first computing the force needed to stabilize the upper toppling blocks up to the contact point, and transmitting this force in the form of stress, triangularly distributed, in the back of the weak formation to compute the $FoS$ against circular failure. A value of 1.00 was computed for the existing conditions, justifying the instability observed.

This factor of safety was also numerically computed with the help of DEM code UDEC [34] using the shear strength reduction technique. A factor of safety of 1.05 was obtained, together with a good representation of the observed phenomena (toppling and rotational failure were clearly observed, as shown in Figure 19.b). A more detailed and duly explained revision of this case study can be consulted elsewhere [21].

5.2. Failure of granite dry masonry retaining walls
A limit-equilibrium-based technique was proposed for the stability analysis and design of drystone masonry retaining walls, focusing granite walls as typically built in NW Spain [52]. Traditionally, walls were designed based on experience. The method was able to study and estimate stability of the wall for different wall heights associated with block rows, against sliding and two overturning mechanisms. Some calibration was done based on simple physical models in the lab. It was observed that, for the conditions encountered, the toppling failure mechanism for the wall was predominant. The method can also be applied to estimates of the stability of existing walls with known dimensions and properties.

The analytical method was applied to the back analysis of an 8 m high wall that fell after a high rainfall episode [53], paying special attention to estimating relevant parameters for the materials used. The friction angle between the stone blocks was studied by means of full-scale tests, and an empirical method was used to estimate the backfill strength properties.

The technique was tested by back analyzing the 8 m high wall failure shown in Figure 20.a, permitting to justify the failure of the wall for an increase of 3 m in the height of the water level behind the wall. Results were compared with numerical results from the UDEC code (Figure 20.b). The failure mechanism identified was sliding of the granular material behind the wall and toppling of the dry masonry retaining wall, as shown in the fact the clearer blocks initially located in the upper part of the wall appeared in the front line of the debris. This kind of mechanism already detected in old studies [55] implied sliding behind and toppling in the front of the slope so it clearly matches with the so called slide-toe-toppling, corresponding to the second type of complex toppling failures defined in Section 4.1.

Figure 20. a) Photograph of the unstable 8-m high dry granite masonry retaining wall toppled after a relevant rainfall episode. Remark the whiter blocks of the upper part of the wall in the front of the debris, a clear sign of toppling b) UDEC numerical model of the retaining wall showing the sliding-toppling failure mechanism that took place.
5.3. A complex circular-subsiding wedge-toppling-sliding case in a bench of a quarry

During the process of the geotechnical characterization of a quarry slope, in relation to a geotechnical study to understand some movements detected in the slope, a complex failure mechanism took place affecting a 30-m high bench in a granodiorite quarry. This happened at the end of the rainiest day in the last ten years in the area according to rainfall records [27].

After care inspection of the state of the area after the failure took place, the geology of the zone based on different degrees of weathering of the granodiorite was interpreted as follows (Figure 21.a and b). In the rear of the slope, a completely weathered granodiorite (CDG) associated to a fault existed. In front of this material, a block dipping toward the slope of moderately weathered granodiorite (FG-III) appeared followed by a wedge of highly decomposed material (HDG). In front of this materials, the rock became fresher and it was only slightly weathered (FG-II), however two discontinuities dipping against the slope and some sub-horizontal joints dipping towards the slope were present following joint sets.

The bench where the failure took place is illustrated in Figure 21.a, after this happened. More or less clearly observed and marked in this figure is the geological section previously described. Additionally, it can be seen how the wedge of HDG crumbled to produce some disaggregated sand. Some topple blocks coming from the block below the wedge (FG-II) can be also clearly identified. This permitted to interpret the observed failure mechanism based on the crosscut in Figure 21.b and described as follows.

Figure 21. a) View of the quarry bench after failure. The unmoved materials in the slope can be seen in the left lateral surface marked in colors. This includes a wedge of HDG (purple), a block of reddish granodiorite dipping towards the slope (red) and a block of fresh granodiorite (blue). The rear of the fallen area is a surface in slightly weathered granite dipping around 85° outwards. In the fallen material, the HDG turned into disaggregated material, rotated blocks apparently coming from original FG-II red block were observed and some blocks of grey fresh granodiorite appeared in the sides. b) Illustrative geometric and geological crosscut WE section of the slope where the failure took place.

The rockslide interpretation includes circular failure in completely decomposed granodiorite at the rear of an unstable area pushing on a block of granodiorite. This first block induced subsidence and failure of a wedge of highly decomposed granodiorite, which pushed at a slender block below it, causing it to topple. This toppling block eventually pushed at a lower block that slid out of the slope. A sketch with a crosscut view including the initial and final state of the bench is depicted in Figure 22 in order to illustrate the instability that took place.

In this rather complex mechanism, completely weathered rock interacted with rock blocks and ultimately produced a small landslide of around 1,500 m³ of material. This failure has been analyzed by means of analytical and numerical techniques. The obtained $FoS$ justified the observations carried out in the quarry, with values over one for the typical conditions, and below one for the hydro-geologic conditions associated to a very heavy rainfall day (around 140 l/m²) in which the failure phenomena took place.
Alejano et al. [27] presented a more detailed report of this case study. This failure mechanism can be classified within the third type of complex failure mechanisms involving toppling, where toppling is observed in the middle of the movement area. This type of mechanisms (classified in the group 3 of section 4.1) are difficult to identify in practice, particularly before they have taken place. However, they seem to be not as uncommon as initially thought and could be helpful to understand some failure mechanisms described as deep-seated toppling and referred by Sjöberg [8] and other authors.

6. Conclusions
In this study, the author has reviewed the present understanding on toppling phenomena in rock masses. To do that, first, analytical, physical and numerical approaches to analyse the stability of more or less simple or complex geometry blocks against toppling is reviewed, introducing some aspects not considered in the standard literature, permitting to analyse the toppling stability of irregular blocks.

Then, the author has introduced the basics of toppling phenomena in rock slopes formally presenting and describing the most typical toppling mechanisms observed in nature and comprehending some blocks: block, flexural and block-flexural toppling. Additionally, a preliminary classification of complex failure mechanism partially involving toppling has been presented, with the aim of having it available to be able to identify the possibility of occurrence of these phenomena when carrying out rock slope stability studies. This classification includes mechanisms where toppling phenomena take place in the upper part of the slope, in the lower part of the slope or only in its middle part, always in association with (rotational or planar) sliding mechanisms and/or subsiding or extruding wedges.

Moreover, this study puts forward the need to consider the possibility of complex failure mechanisms when analyzing rock slope instability. It also highlights the role of blocks prone to toppling that are located in some part of a rock slope. Particularly, when a slender block or group of blocks located in the middle of an initially stable slope experiences an increase in an external force — for instance, due to water pressure — that causes it to topple, all the weight is transmitted to the block immediately below the slender block, inducing a greater force that can render the entire area unstable. This type of phenomena may lay behind the not so uncommon occurrence of these complex failure mechanisms.
where toppling occurs in the middle of the slope, which is reflected in at least six of the references provided in this article.

But indeed, one of the main aims of this manuscript is to recall the paramount relevant thinking reflected in quote from the preface of the Rock Slope Engineering book by Hoek and Bray [1]: “Because the rock mass behind each slope is unique, there are no standard recipes or routine solutions which are guaranteed to produce the right answer each time they are applied”. In rock slope engineering — and in geology in general — failure mechanisms are often complex and sometimes very difficult, if not impossible, to predict. A good recognition of geology is extremely relevant [56]. To paraphrase the geologist Richard Fortey [57] in his Earth’s biography: “Nature has the habit of never being quite as simple as initially thought”. All in all, there is still a lot to learn in what concerns toppling of rock slopes.

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