Kolmogorov Algorithmic Complexity and its Probability Interpretation in Quantum Gravity

V.D.Dzhunushaliev *
Department of the Theoretical Physics
Kyrgyz State National University, Bishkek, 720024

Pacs 02.90.+p, 04.60.-m

*E-mail address: dzhun@freenet.bishkek.su
The quantum gravity has great difficulties with application of the probability notion. In given article this problem is analyzed according to algorithmic viewpoint. According to A.N. Kolmogorov, the probability notion can be connected with algorithmic complexity of given object. The paper proposes an interpretation of quantum gravity, according to which an appearance of something corresponds to its Kolmogorov's algorithmic complexity. By this viewpoint the following questions are considered: the quantum transition with supplementary coordinates splitting off, the algorithmic complexity of the Schwarzschild black hole is estimated, the redefinition of the Feynman path integral, the quantum birth of the Euclidean Universe with the following changing of the metric signature.
I. INTRODUCTION

The spacetime with gravity and matter fields contains the notions of topological space, geometrical structures (metric + connection), signature of metric and so on. Some above-mentioned structures (for example, a topology on some space) cannot be the solutions of any differential equations. The other structures are the solutions of corresponding differential equations, for example, the metric defines from gravity equations, the gauge fields (connection) from Yang-Mills equations. Hence, it is necessary to distinguish the two different regions in quantum gravity:

1. In the first region we have to do with questions connected with appearance of one or another nondifferentiable structures of spacetime until the appearance of some kind of differentiable structures on this topological space.

2. In the second region we quantize the differentiable structures: gravity (metric) and gauge fields (connection).

It is clearly that the quantization in the first case cannot be carried out as in the second case. Intuitively, it seems that the probability of appearance one or another nondifferentiable structures from the first region can be connected with its complexity. That is, the more complicated structures have to appear with the smaller probability and conversely.

In 60s A.N. Kolmogorov investigated the probability notion from the algorithmic viewpoint. The classical probability definition is connected with calculation of a ratio of the number of ways in which the trial can succeed to the total number of ways in which the trial can result. Kolmogorov had investigated the notion of probability from the algorithmic viewpoint. He wrote in [1]:

1. the basic notions of the information theory must and may be defined without the application of probability theory, so that the “entropy” and “number of information” notions are applicable to the individual objects;

2. the notions of the information theory are introduced by this manner can form the basis of a new conception of the chance, corresponding to natural idea that the chance is the absence of laws.

For our quantum gravitational purpose this Kolmogorov idea is very important and the probability notion is adaptable to single object (one or another nondifferentiable structures on Universe). From Kolmogorov’s viewpoint “chance” = “complexity”, and therefore the probability in quantum gravity (QG) can be connected with the complexity of a given structure. Now we shall give the exact definition of the algorithmic complexity (AC) as Kolmogorov did [1]:

The algorithmic complexity $K(x \mid y)$ of the object $x$ by given object $y$ is the minimal length of the “program” $P$ that is written as a sequence of the zeros and unities which allows as to construct $x$ having $y$:

$$K(x \mid y) = \min_{A(P,y)=x} l(P)$$

where $l(P)$ is length of the program $P$; $A(P,y)$ is the algorithm calculating object $x$, using the program $P$, when the object $y$ is given.

According to this definition, it can assume that some Universe structures are appeared (in QG) with the probability depending from its AC by the following manner:

$$P[g] \approx P_0 e^{-K[g]},$$
where \( P_0 \) is the normalized constant; \( K[g] \) is the \( AC \) of given object. For example, it can assume that the Universe birth probability from “Nothing” by Eq. (3) is defined, where \( K[g] \) is \( AC \) of Universe. Furthermore, we shall assume that the \( AC \) of given object can be spontaneously changed. In this case the situation is analogous to the spontaneous electron transition between two energy levels in atom: electron that was on the exited-state level \( E_1 \) (quantum gravitational object with the \( AC \) \( K_1 \)) falls in the result of the spontaneous quantum transition on the ground-state level \( E_2 \) (appear the quantum gravitational object with lesser \( AC \) \( K_2 < K_1 \)). It is highly probable that these processes of the quantum birth of Universe and the quantum transition with the \( AC \) changing happen in Plank region. The splitting off of the supplementary dimension; the changing of the metric signature, topology and so on can be such quantum transition with the \( AC \) changing.

Thus, the basic physical idea given here is:

\[ \text{In the Plank region can happen both the spontaneous quantum birth of the quantum gravitational object having the fixed } AC \text{ and the spontaneous quantum changing of it properties with corresponding changing the } AC. \]

Certainly it is necessary to sew all physical quantities by \( AC \) changing of this object. Let’s consider some questions of QG and attempt to connect the \( AC \) notion with the corresponding quantum probability.

II. THE QUANTUM TRANSITIONS FROM MULTIDIMENSIONAL TO 4D UNIVERSE WITH SUPPLEMENTARY COORDINATES SPLITTING OFF.

In this section the quantum transitions model is offered with splitting off of the supplementary dimensions (SD) in gravity during Universe evolution.

Thus, we suppose that the regions can exist in the spacetime in which SD turn on (off). In algorithmic viewpoint this means that we must describe the whole spacetime by not one algorithm (one field equations system) but more than one: each region has its nature laws (system of the field equation). Naturally, all field variables must be sewed on the boundaries of this regions. We consider the model example, in which the 4D Universe arises in the result of quantum spontaneous transition from the empty multidimensional Kazner’s Universe after splitting off SD [3].

A. 7-dimensional cosmological solution.

Let’s consider the empty 7D spacetime with the metrics in the following form:

\[
\begin{align*}
\text{ds}^2 &= dt^2 - a^2(t) dl_1^2 - b^2(t) dl_2^2, \\
\end{align*}
\]

(3)

where \( dl_{1,2}^2 \) is metrics on \( S^3 \) (3D sphere), \( L^3 \) (3D Lobachewsky space) or \( E^3 \) (3D Euclidean space). The Einstein’s vacuum equations have the following form:

\[
\begin{align*}
\frac{\ddot{a}}{a} &= \frac{\dot{b}^2}{b^2} - \frac{k_1}{a^2} + \frac{k_2}{b^2}, \\
\frac{\ddot{b}}{b} &= \frac{\dot{a}^2}{a^2} + \frac{\dot{b}^2}{b^2} - \frac{k_1}{a^2} + \frac{k_2}{b^2}, \\
3\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}^2}{a^2} + \frac{\dot{b}^2}{b^2} + \frac{k_1}{a^2} + \frac{k_2}{b^2} &= 0,
\end{align*}
\]

(4a, 4b, 4c)

where \( (\cdot) \) means the time derivative of \( t \); \( k_{1,2} = \pm 1, \quad 0 \) respectively for \( S^3 \), \( L^3 \) and \( E^3 \). We consider the simplest case for which it is possible to find analytical solution: \( k_{1,2} = 0 \). In this case system (4) has the following exact solution:
\[ a = a_0 \left( -\frac{t}{a_1} \right) ^\alpha; \quad b = b_0 \left( -\frac{t}{b_1} \right) ^\beta \]

where \( a_{0,1} \) and \( b_{0,1} \) are the constants that should be defined later by discussion of question of splitting off SD; \( t < 0 \).

\[ \alpha = 1 - \sqrt{5} \frac{6}{6}; \quad \beta = 1 + \sqrt{5} \frac{6}{6} \]

where \( G \) is determinant of the 7D metrical tensor, \( R_{4D} \) is 4D scalar curvature of the Einstein’s dimensions, the second term on the right part of Eq.(6) describes the metric dependence on SD from the Einstein’s coordinates, \( R_{SD} \) is the scalar curvature of the SD space. The metric on the supplementary coordinates \( \gamma_{ab} \) does not vary after the SD splitting off, that is \( \gamma_{ab} = \text{const} \). This leads to the fact that now (after SD splitting off) 4D Lagrangian has the following form:

\[ -L_{4D} = \sqrt{-g} (R_{4D} - \Lambda), \]

where \( g \) is determinant 4D metrics; \( \Lambda = R_{SD} = 6k_2/r_0^2 = \text{const} \) (it is suggested that the SD is \( S^3 \), \( L^3 \) or \( E^3 \)), \( r_0 \) is some characteristic length on the SD (later it will be showed that \( r_0 = b_0 \)). By this manner after the SD splitting off, we have \( \Lambda \) - term that has a physical meaning of scalar curvature of supplementary coordinates.

4D metric is being searched in the form:

\[ ds^2 = dt^2 - a^2(t)dl^2 \]

where element \( dl^2 \) is similar to \( dl^2 \) in Eq.(4).

The Einstein’s equations for this metrics are:

\[ 2 \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k_1}{a^2} + 3 \frac{k_2}{r_0^2} = 0, \]
\[ \frac{\dot{a}^2}{a^2} + \frac{k_1}{a^2} + \frac{k_2}{r_0^2} = 0, \]

where \( k_2/r_0^2 \) is the scalar curvature of the SD after its splitting off. For \( k_{1,2} = 0 \) we have the following solution:

\[ a = a'_0 = \text{const}. \]

This space is the Minkowski spacetime.

According to Kolmogorov’s definition both multidimensional and 4D Universes considered above have the various AC. Obviously, that the AC of the multidimensional Universe described by system (1a-1c) is more than the AC of 4D Universe, because the equations number in the system (1a-1c) is more than that in system (9a-9b).

Let’s consider the following model describing the 4D Universe birth: At the beginning the Universe is the empty multidimensional Kazner’s Universe evolving according to Eq. (5a,5b) to the singularity \( t = 0 \). Near the
singularity \(a(t) \to \infty\) and \(b(t) \to 0\). Therefore, the terms \((\dot{a}/a)^2, (\dot{b}/b)^2\) and \((\ddot{a}/a)^2, (\ddot{b}/b)^2\), are comparable with Plank curvature and make a key contribution to the scalar curvature \(R_{7D}\). It leads to that the quantities \(\dot{a}\) and \(\dot{b}\) have the essential quantum fluctuations.

When the quantities \(a/\dot{a}\) become comparable with the Plank length, the quantum transition happens from multidimensional equations \((\text{Eq.4a-4c})\) to the 4D equations \((\text{Eq.9a-9b})\). This process occurs due to the fact that the \(AC\) of the 4D Universe is less than the \(AC\) of the multidimensional Universe.

Let’s choose the next value for constants \(a_{0,1}\) and \(b_{0,1}\) in the \(\text{Eq.5a-5b})\): \(a_0 \gg b_0 \gg l_{pl}, a_1 = b_1 = l_{pl}\). Then for \(t \approx t_{pl}\):

\[
\begin{align*}
    a(t_{pl}) & \approx a_0; & b(t_{pl}) & \approx b_0, \\
    \frac{\dot{a}^2}{a^2} & \approx \frac{\dot{b}^2}{b^2} & \frac{\ddot{a}}{a} & \approx \frac{\ddot{b}}{b} \approx \frac{1}{l_{pl}^2}.
\end{align*}
\]

(11a)

(11b)

After this transition the following facts take place:

1. The linear 4D scale \(a_0\) and \(b_0\) (respectively in Einstein’s and supplementary dimension) remain in the classical region and they become equal to the respective multidimensional values before the \(SD\) splitting off;

2. 4D Universe evolves according to the Einstein’s equations \((\text{Eq.9a-9b})\);

3. The \(SD\) metrics becomes non-dynamical variable and doesn’t change with time.

Let’s estimate the probability \(P\) of given transition from multidimensional Universe to the 4D Universe as:

\[
P_{\text{multiD} \rightarrow 4D} = \frac{e^{-K_2}}{e^{-K_1} + e^{-K_2}},
\]

(12)

where \(K_{1,2}\) in \((\text{Eq.12})\) are respectively the \(AC\) of multidimensional and 4D Universes described in Eq’s.\((\text{Eq.4a-4c})\) and \((\text{Eq.9a-9b})\). According to the \(AC\) definition \(K_{1,2}\) is calculated as a program length in bits for some universal machine (for example, for the Turing machine). It is known that the program describing even such simple arithmetical operation as addition in the Turing machine is very large. As a result, \(K_1 \gg K_2\). Then Eq.\((\text{Eq.12})\) can be approximately estimated as follows:

\[
P_{\text{multiD} \rightarrow 4D} \approx 1 - e^{-[K_1 - K_2]} \approx 1.
\]

(13)

III. THE ELECTRICAL CHARGE MODEL WITH SPLITTING OFF THE SUPPLEMENTARY COORDINATES

The aim of this section is the construction of a composite wormhole: it is the solution of 5D Einstein’s equations (Kaluza - Klein’s equation) under event horizon (\(EH\)), and the Reissner - Nordström’s solution outside \(EH\). The sewing the 5D Kaluza - Klein’s metric with 4D Einstein’s metric + electrical field is performed on \(EH\), that is, splitting off the supplementary coordinates happens.

A. 5D Wormhole in Kaluza - Klein’s theory

At first we remind the results achieved in Ref. \([7]\). 5D metric has the following wormhole-like view:
\[ ds^2 = e^{2\nu(r)} dt^2 - e^{2\psi(r)} (d\chi - \omega(r) dt)^2 - dr^2 - a^2(r) \left( d\theta^2 + \sin^2 \theta d\varphi^2 \right), \]  

where \( \chi \) is a supplementary coordinate; \( r, \theta, \varphi \) are 3D polar coordinates; \( t \) is time. Corresponding 5D Einstein’s equations have the following solution:

\[
\begin{align*}
a^2 &= r_0^2 + r^2, \quad (15a) \\
e^{-2\psi} &= e^{2\nu} = \frac{2r_0^2 + r^2}{r_0^2 - r^2}, \quad (15b) \\
\omega &= \frac{4r_0^2}{q} \frac{r}{r_0^2 - r^2}, \quad (15c)
\end{align*}
\]

where \( r_0 \) is a throat of this wormhole; \( q \) is a 5D “electrical” charge. It is easy to see that the time component of the metrical tensor \( G_{tt}(r = \pm r_0) = 0 \). This indicates that there is the null surface, as on its \( ds^2 = 0 \). The sewing of the 5D and 4D physical quantities happens in the following manner:

\[
\begin{align*}
e^{2\nu_0} - \omega_0^2 e^{-2\nu_0} &= G_{tt}(\pm r_0) = g_{tt}(r_+) = 0, \quad (16a) \\
r_0^2 &= G_{\theta\theta}(\pm r_0) = g_{\theta\theta}(r_+) = r_+^2, \quad (16b)
\end{align*}
\]

where \( G \) and \( g \) are 5D and 4D metrical tensors, respectively. \( r_+ = m + \sqrt{m^2 + Q^2} \) is the EH for Reissner-Nordström’s solution (\( m \) and \( Q \) are mass and charge of the Reissner-Nordström’s black hole). The quantities marked by \( (0) \) sign are taken by \( r = \pm r_0 \).

**B. The sewing Kaluza-Klein’s wormhole with Reissner-Nordström’s solution**

In this section we show that the Reissner-Nordström’s solution really can sew on \( EH \) with above received 5D wormhole. To do this, the 5D Kaluza-Klein’s metrical tensor is necessary to sew with (4D metric + Maxwell’s electrical field in Reissner-Nordström’s solution).

To sew \( G_{\chi t} \) and 4D electrical field we consider 5D \( R_{\chi t} = 0 \) and Maxwell’s equations:

\[
\begin{align*}
\left[ a^2 (\omega e^{-4\nu}) \right]' &= 0, \quad (17a) \\
(r^2 E_r)' &= 0, \quad (17b)
\end{align*}
\]

here \( E_r \) is 4D electrical field.

These two equations are practically the Gauss law and they indicate that some quantity multiplied by area is conserved. In 4D case this quantity is 4D Maxwell’s electrical field and from this follows that the electrical charge is conserved. Thus, naturally we must join 4D electrical Reissner-Nordström’s field \( E_{RN} = Q/r_+^2 \) with “electrical” Kaluza-Klein’s field \( E_{KK} = \omega e^{-4\nu} \) on \( EH \):

\[
\omega_0^2 e^{-4\nu_0} = \frac{q}{2r_0^2} = E_{KK} = E_{RN} = \frac{Q}{r_+^2}. \quad (18)
\]

In this case the probability of splitting off the SD is calculated similar to Eq.(12) definition.

**IV. THE ALGORITHMIC COMPLEXITY OF THE SCHWARZSCHILD BLACK HOLE.**

Beckenstein and Hawking have showed that the black hole has entropy connected with the existence of event horizon. The entropy notion is usually employed to the statistical systems consisted of a great number of particles. But in given case this notion is used with the individual object as it was proposed by Kolmogorov.
According to the definition (1), the AC is defined as the smallest algorithm describing a given gravity field (GF). The metrical tensor is the solution of the gravity equations (Einstein’s equations, $R^2$-theory or the gauge gravity equations). Therefore, the algorithm describing given metrics is the corresponding gravity equations restoring the GF at the all spacetime from the initial and(or) boundary conditions. In this case the algorithm length describing GF is essentially reduced.

Now we shall calculate AC for the Schwarzschild black hole. For this purpose we shall write the metric in the following form:

$$ds^2 = dt^2 - e^{\lambda(t,R)} dR^2 - r^2(t,R) \left( d\theta^2 + \sin^2 \theta d\phi^2 \right),$$

where $t$ is time, $R$ is radius, $\theta$ and $\phi$ are polar angles. In this case Einstein’s equations are:

$$- e^{-\lambda} r'^2 + 2 r \ddot{r} + \dot{r}^2 + 1 = 0,$$

$$- \frac{e^{-\lambda}}{r} (2 r'' - r' \lambda') + \frac{\dot{\lambda} \dot{r}}{r} + \frac{\dot{\lambda}^2}{r} + \frac{2 \dot{r}}{r} = 0,$$

$$- \frac{e^{-\lambda}}{r^2} \left( 2 rr'' + r'^2 - rr' \lambda' \right) + \frac{1}{r^2} \left( r \ddot{\lambda} + \dot{\lambda} \dot{r}^2 + 1 \right) = 0,$$

$$2 \ddot{r} - \dot{\lambda} \dot{r}' = 0,$$

where (’) and (‘) mean respectively the derivations on $t$ and $r$. We give the wormhole $t = 0$ as a Caushy hypersurface. The assignment of initial data on this hypersurface define GF on the whole Schwarzschild spacetime. But “quantity” of the initial data can be essentially reduced. Actually, we examine the Eq.(20c) on the wormhole $t = 0$. Here, first time derivative of all components of the metrical tensor is equal to zero. Therefore the initial data must satisfy to equation:

$$2rr'' + r'^2 - rr' \lambda' - e^{\lambda} = 0.$$

To solve Eq. (21) on surface $t = 0$ it is necessary to define the boundary conditions. They may be taken on throat of wormhole in the following form:

$$r'(R = 0, t = 0) = 0, \quad r(R = 0, t = 0) = r_g,$$

where $r_g$ is radius at the event horizon. It means that the GF on the whole Schwarzschild spacetime is defined by the value of $G_{\theta \theta}$ component of metrical tensor at the origin. Therefore, the AC for the Schwarzschild metrics is defined by the following expression:

$$K \approx L \left[ \left( \frac{r_g}{r_{Pl}} \right)^2 \right] + L_{\text{Einstein}},$$

where $L \left[ (r^2/r_{Pl}^2) \right]$ is program length of the definition of dimensionless number $r_g^2/r_{Pl}^2$ made up for the some universal machine; $L_{\text{Einstein}}$ is program length of the solution of Einstein’s differential equations using some universal machine, for example, the Turing machine.

V. ALGORITHMIC COMPLEXITY AND THE PATH INTEGRAL

In this section we will obtain well defined positive functional on the metric space that can be important for QG. On this basis we define the path integral in QG by the following manner [7]: The action functional in the path integral we replace with AC for a given metric, which is the positive defined functional of the metric:
\[
\int D[g]e^{-i(I[g] + \int g_{\mu\nu} J^{\mu\nu} dx)} \rightarrow \int D[g]e^{-i(K[g] + \int g_{\mu\nu} J^{\mu\nu} dx)} = e^{iZ[J^{\mu\nu}]} ,
\]

where \( g_{\mu\nu} \) is a metric; \( K[g] \) is the \( AC \) of given metric \( g \); \( Z[J] \) is a generating functional for QG.

It is sufficiently obvious that the most complicated gravitational fields are the metrics satisfying none field equations. According to Kolmogorov, they are random fields; in consequence of absence of the algorithm connecting the metric value in the neighbouring points. The metrics, which are the solutions of some gravity equations (Einstein’s, \( R^2 \) - theory, Euclidean, multidimensional and so on) have a much lesser \( AC \) in comparison with random metrics as they are calculated from initial and/or boundary conditions. The gravitational instantons are the simplest gravity objects: they are symmetrical spaces with the corresponding metrics possessing the some symmetry group. It is necessary to mark that the instantons can be defined not by field equations but by its topology charges that strongly simplifies its algorithmic definitions.

Thus, for a first approximation the path integral in QG is defined by sum on all gravity instantons. In the next approximation the contributions from the metrics which are the solutions of the Einstein’s equations, \( R^2 \) - theories, multidimensional theories and etc. are appeared. The contribution from complicated metrics (in above-mentioned Kolmogorov’s meaning) appears when the quantum field effects of gravitational field are takes into account. In the QG based on the integral \( (24) \) the Universe can contain a few regions with various gravity equations.

The effects connected with topology changing (spacetime foam) in the sum \( (24) \) are taken automatically into account, because it is quite evidently that the topological different spaces have the different \( AC \).

According to this idea, we could suppose that the changing of the Universe \( AC \) is possible during the process of its evolution. The Universe is born from “Nothing” with certain \( AC \) (it can be multidimensional and/or Euclidean) and then in own evolution process it can happen the processes with changing of its \( AC \): the supplementary coordinates splitting off; the metric signature changing; the simultaneous suppressing term like \( R^2 \) or \( R^a_{\,bde} R^{bij} \) (arising in the gauge theories of gravity) in Lagrangian and so on. J. Wheeler [8] used the similar idea discussing the question of appearance of our Universe from “Pregeometry”.

VI. THE QUANTUM BIRTH OF THE EUCLIDEAN UNIVERSE WITH THE FOLLOWING CHANGING OF THE METRIC SIGNATURE.

Every physical object in the nature may be described by some algorithm. For example, the Universe in Einstein’s theory is characterized by algorithm defining all notions laying at the base of the Universe. In this case the topological space, its geometrical structures (metrics, connection), Einstein’s equations and so on must be described. Such algorithm should be realized with some universal machine (for example, with Turing machine). As it was mentioned above, the length of such minimal algorithm is called the \( AC \) of given object and is defined by \( (1) \).

The Euclidean instantons have the simplest construction among all smooth manifolds with metrics and connection. If the algorithm is describing differential manifold and geometrical structure on spacetime to fix, then it is remaining to define the algorithm determining the concrete metric and connection only. In any geometrical gravity theory such algorithm will be field gravitational equations. In this case it is possible to build the following hierarchy in accordance with increasing complexity: Euclidean instantons, the spacetimes satisfying to any gravity equations (for example Einstein’s equations) and other random spaces. We remind that
the gravity equation is some algorithm computing the metric and connection from the initial and(or) boundary conditions.

Thus, we concluded that some Euclidean instanton from the gravity vacuum with the greatest probability in consequence of quantum fluctuations is appeared. We suppose that it is born from the vacuum in the cut form. For example, de’Sitters instanton $S^4$ (4Dsphere) can appear without neighbourhood of some point. This cut instanton has a boundary. Let’s suppose that the spontaneous changing of the metrics signature takes place on the boundary and the pseudo-Riemannian spacetime is appeared that begins its evolution from boundary according to one or another gravity equations. This is well known Hawking’s idea on the Universe birth from Euclidean region from the algorithmical viewpoint.

Now let’s define the AC of the instanton. The instanton is described by the self-duality equations which are differential equations of the first order, while the field gravity equations describing spacetime are the differential equations of the second order. Consequently, the instantons are simpler objects than another spacetimes. Let’s notice that the instanton can be described in still simpler manner. Indeed, the instanton is completely defined by its topological number - Pontragin’s index. Therefore the AC of instanton is equal to the length of the algorithm describing the integer $n = \text{Pontragin’s index}$. Thus:

$$K[\text{instanton}] = (\text{the record length of integer } n) + K_0$$

where $K_0$ is the length of the algorithm describing all other notions: the topological space, the geometrical structure on it and so on.

The algorithm length describing integer $n$ can be estimated in the following way: The integer $n$ can be written as $n$ “stick-unity”. Then, the considered algorithm consists of elementary steps, where each of them calculates one “stick-unity” from $n$. Then the length of this algorithm is approximately equal to the examined integer $n$.

Let’s consider the quantity $\exp(-\text{instanton action})$ connected with the probability of corresponding tunnel transition. It is well known that the instanton action is proportional to the integer $n$. Thus two integers (AC and the instanton action) coincide up to a constant. This is a supplementary argument for support of the expression.

If our algorithm could be broken into several steps describing metrics $g$ step by step, then the complete probability is equal to the product of the applicable probability for each separate step. For example, the whole algorithm describing the Universe includes consistently the algorithms describing the topological space, the differentiable structure on it, the geometrical structure (metrics and connections), the field gravity equations and so on. For our purposes we single out the separate factor, describing the algorithm computating the metrics and connection according to the field equations. The remained factor is one for all Universes and it can be inserted into the normalization constant.

Let’s propose that the changing of the metrics signature happens with the following probability:

$$P_{\text{change}} = \frac{e^{-K[\text{Mink}]}}{e^{-K[\text{Eucl}]} + e^{-K[\text{Mink}]}}$$

where $K[\text{Eucl}]$ and $K[\text{Mink}]$ are accordingly AC of the gravity equations in the Euclidean and pseudo-Riemannian spaces. It is necessary to note that this process is essentially quantum phenomenon rather than classical.
VII. ACKNOWLEDGMENTS

This research was supported by ISF Grant MYT000.

[1] A.N. Kolmogorov, Information theory and algorithm theory: Logical foundation of the information theory. Combinatorial foundation of the information theory and the probability calculus. (M.: Nauka, 1987).

[2] V.D.Dzhunushaliev, Izv. vuzov, ser. Fizika, N9, 55(1994) (In Russian).

[3] V.D.Dzhunushaliev, Izv. vuzov, ser. Fizika, N6, 78(1993) (In Russian).

[4] J.D.Bekenstein, Phys.Rev. D7, 2333(1973).

[5] J.D.Bekenstein, Phys.Rev. D9, 3292(1974).

[6] S.W.Hawking, Nature, 238, 30(1974).

[7] V.D.Dzhunushaliev, Izv. vuzov, ser. Fizika, N3, 108(1995) (In Russian).

[8] C.Misner, K.Thorne, J.Wheeler, Gravitation (W.H.Freeman and Company, San Francisco, 1973).