Holographic Baryons

— Static Properties and Form Factors from Gauge/String Duality —

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Abstract

In this paper, we study the properties of baryons by using a holographic dual of QCD on the basis of the D4/D8-brane configuration, where baryons are described by a soliton. We first determine the asymptotic behavior of the soliton solution, which allows us to evaluate well-defined currents associated with the $U(N_f)_L \times U(N_f)_R$ chiral symmetry. Using the currents, we compute the static quantities of baryons such as charge radii and magnetic moments, and perform a quantitative comparison with experiments. It is emphasized that not only the nucleon but also excited baryons, such as $\Delta$, $N(1440)$, $N(1535)$, etc., can be analyzed systematically in this model. We also investigate the form factors and find that our form factors agree well with the results that have been well established empirically. Using the form factors, the effective baryon-baryon-meson cubic coupling constants among their infinite towers in the model can be determined. Some physical implications following from these results are discussed.
§1. Introduction

The gauge/string duality has opened up new technology to analyze strongly coupled gauge theories. Although it has not been proven yet, there is a lot of highly nontrivial evidence suggesting the duality between four-dimensional gauge theories and string theory in higher-dimensional curved backgrounds, at least for the cases with supersymmetry and conformal symmetry (for a review, see Ref. [4]). It is obviously important to see if this idea can be applied to realistic QCD in many respects. One of the advantages of QCD is that we can use experimental data to check if the duality really works. In general, the analysis of non-supersymmetric strongly coupled gauge theories is very difficult, and hence it is almost impossible to establish such duality. However, for QCD, we can omit the complicated calculations on the gauge theory side and simply compare the results on the string theory side with the experimental data to obtain nontrivial evidence of the duality.

If the gauge/string duality is really applicable to QCD, it will provide us with new deep insight into the theory of strong interaction in both conceptual and practical terms. It will tell us that both QCD and string theory in a higher-dimensional curved background (holographic QCD) can be a fundamental theory of strong interaction at the same time. Of course, in the high-energy weakly coupled regime, QCD is a better description. However, at least in the large-$N_c$ strongly coupled regime, the string theory can be used as a powerful tool to calculate various physical quantities.

In Refs. [5] and [6], it was proposed that QCD with $N_f$ massless quarks is dual to type IIA string theory with $N_f$-probe D8-branes in the D4-brane background used in Ref. [7] at low energy. In this model, mesons appear as open strings on the D8-branes and baryons are expressed as D4-branes wrapped on the nontrivial four-cycle of the background, as was anticipated in Refs. [8] and [9] in the context of the AdS/CFT correspondence. The effective action for the open strings turned out to be a five-dimensional $U(N_f)$ Yang-Mills-Chern-Simons (YM-CS) theory in a curved background. Via the equivalence between the wrapped D4-branes and the instantons in the D8-brane world-volume gauge theory the baryons were obtained as instanton configurations in the Euclidean four-dimensional space in this five-dimensional gauge theory (see also Ref. [11]).

The idea of realizing baryons as solitons was pioneered by Skyrme in the early 1960s. He started with an effective action for a pion with a fourth-derivative term (Skyrme term) and constructed a soliton solution (Skyrmion) in which the $U(N_f)$-valued pion field carries a nontrivial winding number interpreted as the baryon number. In the model of holographic QCD this construction is naturally realized. It can be shown that the five-dimensional gauge theory is equivalent to a theory of mesons including pions as well as infinitely many
vector and axial-vector mesons. The pion sector is given by the Skyrme model and the instanton configuration reduces to a Skyrmion, realizing the old idea of Atiyah and Manton\textsuperscript{13} who proposed expressing a Skyrmion using a gauge configuration with a non-zero instanton number.

The Skyrme model was further developed by Adkins et al.,\textsuperscript{14} who analyzed various static properties of the nucleon and $\Delta$ by quantizing the Skyrmion. It is natural to expect that the analysis can be improved by applying their strategy to the holographic model of QCD, since the model automatically contains the contribution from the vector and axial-vector mesons such as $\rho$, $\omega$, $a_1$, etc.\textsuperscript{1*} In fact, the quantization of the instanton representing a baryon was studied in Refs. \textsuperscript{16}–\textsuperscript{18}. It was found in Ref. \textsuperscript{16} that the spectrum of the baryons obtained in the model, including nucleons, $\Delta$, $N(1440)$, $N(1535)$, etc, is qualitatively similar to that observed in nature, although the prediction of the mass differences tended to be larger than the observed differences. In Refs. \textsuperscript{17} and \textsuperscript{18}, a five-dimensional fermionic field was introduced into the five-dimensional gauge theory to incorporate the baryon degrees of freedom, where the interaction terms between the two fields were analyzed by quantizing the instanton. Using this five-dimensional system, the interaction of the baryons with the electromagnetic field and mesons was computed, which was found to be in good agreement with the experimental data.

In this paper, we analyze the static properties of baryons, such as the magnetic moments, charge radii, axial couplings, etc. of nucleons, $\Delta$, $N(1440)$, and $N(1535)$, generalizing Refs. \textsuperscript{14} and \textsuperscript{16}. These quantities are obtained by calculating the currents corresponding to the chiral symmetry that contain the information of the couplings to the external electromagnetic gauge field. As we will see, most of our results are closer to the experimental values than the results given in Ref. \textsuperscript{14} for the Skyrme model. For some quantities of the excited baryons, our results are predictions. We also consider the form factors of the spin $1/2$ baryons and study how they behave as functions of the momentum transfer.

Note that there are some closely related works that address the same subject. As mentioned above, Refs. \textsuperscript{17} and \textsuperscript{18} analyze the properties of baryons using the five-dimensional gauge theory with a five-dimensional fermion. On the other hand, our analysis is based on the quantum mechanics obtained through the usual procedure of quantizing solitons. These two approaches should in principle give the same results, and in fact we will find that some of our results are consistent with those in the studies, although the relation is not completely transparent. The work of Hata et al.\textsuperscript{19} is more directly related to ours. The main difference is the definition of the currents. As discussed in their paper, their currents are problematic,

\textsuperscript{*1} See Ref. \textsuperscript{15} for an attempt to include the effect of the $\rho$ meson in the Skyrmion based on the holographic QCD.
since they are not gauge-invariant and have some ambiguities in the definition.

The paper is organized as follows. In §2, after briefly reviewing the main points of the model and the construction of the baryon, we calculate the currents corresponding to the chiral symmetry. The applications are given in §3. We calculate various physical quantities including charge radii, magnetic moments, axial coupling, etc., and compare them with the experimental values for nucleons. We also present these values for excited baryons as our predictions. In §4, we compute the form factors of the spin 1/2 baryons and study their properties including cubic baryon-baryon-meson couplings. Section 5 is devoted to a summary and discussion with tables summarizing our results including our predictions for future experiments. Two appendices summarize some technical details.

§2. Currents

2.1. The model

In this subsection, we give a brief review of Refs. [5], [6], and [16] with an emphasis on the construction of baryons. Based on the idea of gauge/string duality, it was proposed that the meson effective theory is given by a five-dimensional $U(N_f)$ YM-CS theory in a curved background. The action of the model is

$$S = S_{YM} + S_{CS},$$
$$S_{YM} = -\kappa \int d^4x dz \text{ tr} \left[ \frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2 \right], \quad S_{CS} = \frac{N_c}{24\pi^2} \int_{M^4 \times \mathbb{R}} \omega_5(A).$$ (2.1)

Here, $\mu, \nu = 0, 1, 2, 3$ are four-dimensional Lorentz indices, and $z$ is the coordinate of the fifth dimension. The quantity $A = A_\alpha dx^\alpha = A_\mu dx^\mu + A_z dz$ ($\alpha = 0, 1, 2, 3, z$) is the five-dimensional $U(N_f)$ gauge field and $F = \frac{1}{2} F_{\alpha\beta} dx^\alpha \wedge dx^\beta = dA + iA \wedge A$ is its field strength. The constant $\kappa$ is related to the ’t Hooft coupling $\lambda$ and the number of colors $N_c$ as

$$\kappa = \frac{\lambda N_c}{216\pi^3} \equiv a\lambda N_c. \quad (2.2)$$

Although it is not explicitly written in (2.1), the mass scale of the model is given by a parameter $M_{KK}$, which is the only dimensionful parameter of the model. In Refs. [5] and [6], these two parameters are chosen as

$$M_{KK} = 949 \text{ MeV}, \quad \kappa = 0.00745, \quad (2.3)$$

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* Our notation is mostly consistent with Ref. [16] except that we do not use the rescaled variables defined in (3.9) of Ref. [16].
to fit the experimental values of the $\rho$ meson mass and the pion decay constant $f_\pi \simeq 92.4$ MeV. In this paper, we mainly work with $M_{\text{KK}} = 1$ unit. The functions $h(z)$ and $k(z)$ are given by

$$h(z) = (1 + z^2)^{-1/3}, \quad k(z) = 1 + z^2,$$

and $\omega_5(\mathcal{A})$ is the CS 5-form defined as

$$\omega_5(\mathcal{A}) = \text{tr} \left( \mathcal{A} \mathcal{F}^2 - \frac{i}{2} \mathcal{A}^3 \mathcal{F} - \frac{1}{10} \mathcal{A}^5 \right).$$

The action (2.1) is obtained in Ref. 5 as the effective action of $N_f$-probe D8-branes placed in the D4-brane background studied in Ref. 7 and is thought to be an effective theory of mesons in four-dimensional (large $N_c$) QCD with $N_f$ massless quarks.

In this paper, we consider the $N_f = 2$ case, and the $U(2)$ gauge field $\mathcal{A}$ is decomposed as

$$\mathcal{A} = A + \hat{A} \frac{1_2}{2} = A^a \tau^a \frac{1}{2} + \hat{A} \frac{1_2}{2} = \sum_{C=0}^3 A^C \tau^C \frac{1}{2},$$

where $\tau^a$ ($a = 1, 2, 3$) are Pauli matrices and $\tau^0 = 1_2$ is a unit matrix of size 2. Then, the equations of motion are

$$- \kappa \left( h(z) \partial_\nu \tilde{F}^{\mu\nu} + \partial_2 (k(z) \hat{F}^{\mu z}) \right) + \frac{N_c}{128 \pi^2} \epsilon^{\mu\alpha_2 \cdots \alpha_5} \left( F^a_{\alpha_2 \alpha_3} F^a_{\alpha_4 \alpha_5} + \hat{F}^a_{\alpha_2 \alpha_3} \hat{F}^a_{\alpha_4 \alpha_5} \right) = 0,$$

$$- \kappa \left( h(z) D_\nu F^{\mu\nu} + D_2 (k(z) F^{\mu z}) \right) \hat{\tau}^a + \frac{N_c}{64 \pi^2} \epsilon^{\mu\alpha_2 \cdots \alpha_5} F^a_{\alpha_2 \alpha_3} \hat{F}_{\alpha_4 \alpha_5} = 0,$$

$$- \kappa k(z) \partial_2 \tilde{F}^{z\nu} + \frac{N_c}{128 \pi^2} \epsilon^{\mu_2 \cdots \mu_5} \left( F^a_{\mu_2 \mu_3} F^a_{\mu_4 \mu_5} + \hat{F}^a_{\mu_2 \mu_3} \hat{F}^a_{\mu_4 \mu_5} \right) = 0,$$

$$- \kappa k(z) (D_\nu F^a) \hat{\tau}^a + \frac{N_c}{64 \pi^2} \epsilon^{\mu_2 \cdots \mu_5} F^a_{\mu_2 \mu_3} \hat{F}_{\mu_4 \mu_5} = 0,$$

where $D_\alpha = \partial_\alpha + i A_\alpha$ is the covariant derivative. The baryon in this model corresponds to a soliton with a nontrivial instanton number on the four-dimensional space parameterized by $x^M$ ($M = 1, 2, 3, z$). The instanton number is interpreted as the baryon number $N_B$, where

$$N_B = \frac{1}{64 \pi^2} \int d^3 x dz \epsilon_{M_1 M_2 M_3 M_4} F^a_{M_1 M_2} F^a_{M_3 M_4}. \quad (2.11)$$

Unfortunately, because the equations of motion are complicated nonlinear differential equations in a curved space-time, it is difficult to find an analytic solution corresponding to the baryons. However, as observed in Refs. 16 and 17, the center of the instanton solution is located at $z = 0$ and its size is of order $\lambda^{-1/2}$, and hence we can focus on a tiny region

\[\text{Here we omit the symbol ‘∧’ for the wedge products of } \mathcal{A} \text{ and } \mathcal{F} \text{ (e.g. } \mathcal{A} \mathcal{F}^2 = \mathcal{A} \wedge \mathcal{F} \wedge \mathcal{F} \text{) in the CS 5-form.} \]
around $z = 0$ for large $\lambda$, in which the warp factors $h(z)$ and $k(z)$ can be approximated by 1. Then it follows that the static baryon configuration is given by the BPST instanton solution with the $U(1)$ electric field of the form

$$A_M^{cl} = -if(\xi)g\partial_Mg^{-1}, \quad \hat{A}_0^{cl} = \frac{N_c}{8\pi^2\kappa}\frac{1}{\xi^2}\left[1 - \frac{\rho^4}{(\rho^2 + \xi^2)^2}\right], \quad A_0 = \hat{A}_M = 0.$$ (2.12)

Here

$$f(\xi) = \frac{\xi^2}{\xi^2 + \rho^2}, \quad g(x) = \frac{(z - Z) - i(\vec{x} - \vec{X}) \cdot \vec{\tau}}{\xi}, \quad \xi = \sqrt{(z - Z)^2 + |\vec{x} - \vec{X}|^2},$$ (2.13)

with $X^M = (X^1, X^2, X^3, Z) = (\vec{X}, Z)$ being the position of the soliton in the spatial $\mathbb{R}^4$ direction and the instanton size $\rho$. Substituting this configuration into the action and taking the nontrivial $z$ dependence of the background into account as a $1/\lambda$ correction, we find that $\rho$ and $Z$ have a potential of the form

$$U(\rho, Z) = 8\pi^2\kappa \left(1 + \frac{\rho^2}{6} + \frac{N_c^2}{5(8\pi^2\kappa)^2}\frac{1}{\rho^2} + \frac{Z^2}{3}\right),$$ (2.14)

which is minimized at

$$\rho_{cl}^2 = \frac{N_c}{8\pi^2\kappa}\sqrt{\frac{6}{5}}, \quad Z_{cl} = 0.$$ (2.15)

In order to quantize the soliton, we use the moduli space approximation method and then the system is reduced to the quantum mechanics on the instanton moduli space. The $SU(2)$ one-instanton moduli space is simply given by $\mathcal{M} \simeq \mathbb{R}^4 \times \mathbb{R}^4/\mathbb{Z}_2$, which is parameterized by $(\vec{X}, Z)$ and $y_I$ ($I = 1, 2, 3, 4$) with the $\mathbb{Z}_2$ action $y_I \rightarrow -y_I$. The size of the instanton $\rho$ is related to $y_I$ by $\rho = \sqrt{y_1^2 + \cdots + y_4^2}$, and $a_I \equiv y_I/\rho$ represent the $SU(2)$ orientations of the instanton. The Lagrangian of the collective motion of the soliton was obtained in Ref. [16] as

$$L = \frac{M_0}{2}(\dot{\vec{X}}^2 + \dot{Z}^2) + M_0\dot{y}_I^2 - U(\rho, Z),$$ (2.16)

where $M_0 = 8\pi^2\kappa$. The Hamiltonian is given by

$$H = \frac{1}{2M_0}(\dot{\vec{P}}^2 + \dot{P}_Z^2) + \frac{1}{4M_0}\Pi_I^2 + U(\rho, Z)$$ (2.17)

with the canonical momenta

$$\vec{P} = M_0\dot{\vec{X}} = -i\frac{\partial}{\partial \vec{X}}, \quad P_Z = M_0\dot{Z} = -i\frac{\partial}{\partial Z}, \quad \Pi_I = 2M_0\dot{y}_I = -i\frac{\partial}{\partial y_I}.$$ (2.18)
This system is manifestly invariant under $SO(4)$ rotation acting on $y^I$. As argued in Refs. [14] and [16] the generators of $SO(4) \simeq (SU(2)_I \times SU(2)_J)/\mathbb{Z}_2$ symmetry correspond to the isospin and spin operators given by

\[
I_a = \frac{i}{2} \left( y_4 \frac{\partial}{\partial y_a} - y_a \frac{\partial}{\partial y_4} - \epsilon_{abc} y_b \frac{\partial}{\partial y_c} \right), \quad J_a = \frac{i}{2} \left( -y_4 \frac{\partial}{\partial y_a} + y_a \frac{\partial}{\partial y_4} - \epsilon_{abc} y_b \frac{\partial}{\partial y_c} \right),
\]

(2.19)

respectively.

The explicit eigenfunctions of the Hamiltonian (2.17) are obtained in Ref. [16]. They are characterized by the quantum numbers $(l, I, s, n_{\rho}, n_z)$ as well as the momentum $\vec{p}$. Here $l = 1, 3, 5, \ldots$ are positive odd integers related to isospin $I$ and spin $J$ by $I = J = l/2$. Note that (2.19) implies $\vec{I}^2 = \vec{J}^2$, and hence only the states with $I = J$ appear in the spectrum as in Ref. [14] for the Skyrme model. $I_3$ and $s$ denote the eigenvalues of the third component of the isospin and spin, respectively. $n_{\rho}$ and $n_z$ are nonnegative integers corresponding to the excitations with respect to $\rho$ and $Z$, respectively. The species of the baryon are specified by $B \equiv (l, I_3, n_{\rho}, n_z)$ and the wavefunctions are of the form

\[
|\vec{p}, B, s\rangle = |\vec{p}\rangle |B, s\rangle,
\]

(2.20)

with $|\vec{p}\rangle = \frac{1}{(2\pi)^{3/2}} e^{i\vec{p} \cdot \vec{X}}$. For later use, we normalize the baryon state as

\[
\langle B, s | B', s' \rangle = \delta_{BB'} \delta_{ss'}.
\]

(2.21)

For example, the proton and neutron are interpreted as the particles with $B = (1, 1/2, 0, 0)$ and $B = (1, -1/2, 0, 0)$, respectively, and the corresponding wavefunctions with $s = 1/2$ are

\[
|p \uparrow\rangle \propto R(\rho) \psi_Z(Z)(a_1 + ia_2), \quad |n \uparrow\rangle \propto R(\rho) \psi_Z(Z)(a_4 + ia_3),
\]

(2.22)

respectively, where

\[
R(\rho) = \rho^{-1+2\sqrt{1+N_c^2/5}} e^{-\frac{M_0}{\sqrt{\rho}} \rho^2}, \quad \psi_Z(Z) = e^{-\frac{M_0}{\sqrt{\rho}} Z^2},
\]

(2.23)

up to normalization constants. See Appendix A.1 for more details.

The excitation numbers $(n_{\rho}, n_z)$ are the quantum numbers which Skyrmions cannot have, and are thus peculiar to the instanton picture of baryons obtained in Ref. [16]. For example, $B = (1, \pm 1/2, 1, 0)$ corresponds to the Roper excitation $N(1440)$, and $B = (1, \pm 1/2, 0, 1)$ corresponds to $N(1535)$. Higher spin baryons are also included, such as $B = (3, I_3, 0, 0)$ with $I_3 = \pm 3/2, \pm 1/2$ giving $\Delta(1232)$.

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2.2. **Currents**

As in the usual effective field theory approach to the hadrons, it is useful to introduce the external gauge fields $A_{L\mu}$ and $A_{R\mu}$ associated with the chiral symmetry $U(N_f)_L \times U(N_f)_R$. The currents $J^\mu_L$ and $J^\mu_R$ associated with the chiral symmetry are then read off from the terms linear with respect to the external gauge fields in the effective action as

$$S \big|_{O(A_{L\mu}, A_{R\mu})} = -2 \int d^4x \, \text{tr} \left( A_{L\mu} J^\mu_L + A_{R\mu} J^\mu_R \right).$$

(2.24)

In terms of the vector and axial-vector current

$$J^\mu_V = J^\mu_L + J^\mu_R, \quad J^\mu_A = J^\mu_L - J^\mu_R,$$

(2.25)

(2.24) becomes

$$S \big|_{O(A_{L\mu}, A_{R\mu})} = -2 \int d^4x \, \text{tr} \left( V^{(+)}_\mu J^\mu_V + V^{(-)}_\mu J^\mu_A \right),$$

(2.26)

where

$$V^{(\pm)}_\mu = \frac{1}{2} (A_{L\mu} \pm A_{R\mu})$$

(2.27)

are external vector and axial-vector fields.

We can apply this idea to compute the currents in the present model, which leads us to a prescription analogous to that used in the AdS/CFT correspondence,\(^3\) as first explored in Ref.\(^1\) and discussed recently in Ref.\(^{19}\). As studied in Refs.\(^5\) and \(^6\), the chiral symmetry is identified with the constant gauge transformation at infinity $z = \pm \infty$, and the external gauge fields $A_{L\mu}$ and $A_{R\mu}$ can be introduced by considering the five-dimensional gauge field with the boundary conditions

$$A_\mu(x^\mu, z \to \pm \infty) = A_{L\mu}(x^\mu), \quad A_\mu(x^\mu, z \to \pm \infty) = A_{R\mu}(x^\mu),$$

(2.28)

respectively. To compute the currents, we substitute a solution of the equations of motion with this boundary condition into the action and keep only the linear terms in the external gauge fields $A_L$ and $A_R$ assuming that the external fields are infinitesimal. Consider the gauge configuration of the form

$$A_\alpha(x^\mu, z) = A_{\alpha_0}^{cl}(x^\mu, z) + \delta A_\alpha(x^\mu, z),$$

(2.29)

where $A_{\alpha_0}^{cl}$ is a classical solution of the equations of motion with $A_{\alpha_0}^{cl}(z = \pm \infty) = 0$ and $\delta A_\alpha(x, z)$ is an infinitesimal deviation from it satisfying

$$\delta A_\mu(x^\mu, z \to \pm \infty) = A_{L\mu}(x^\mu), \quad \delta A_\mu(x^\mu, z \to \pm \infty) = A_{R\mu}(x^\mu).$$

(2.30)
Substituting this configuration into the action (2.1), we obtain

\[ S \mid_{\mathcal{O}(A_L, A_R)} = \kappa \int d^4x \; 2 \; \text{tr} \left[ \text{tr} \left( \delta \mathcal{A}^\mu k(z) \mathcal{F}^\mu_{\mu z} \right) \right] \mid_{z = +\infty}^{z = -\infty}, \]  

which implies

\[ \mathcal{J}_{L\mu} = -\kappa \left( k(z) \mathcal{F}^\mu_{\mu z} \right) \mid_{z = +\infty}^{z = -\infty}, \quad \mathcal{J}_{R\mu} = +\kappa \left( k(z) \mathcal{F}^\mu_{\mu z} \right) \mid_{z = -\infty}^{z = +\infty}, \]  

\[ \mathcal{J}_{V\mu} = -\kappa \left[ k(z) \mathcal{F}^\mu_{\mu z} \right] \mid_{z = +\infty}^{z = -\infty}, \quad \mathcal{J}_{A\mu} = -\kappa \left[ \psi_0(z) k(z) \mathcal{F}^\mu_{\mu z} \right] \mid_{z = -\infty}^{z = +\infty}, \]  

where \( \psi_0(z) = \frac{2}{\pi} \arctan z \). Here only the surface terms at \( z = \pm \infty \) remain in (2.31) because of the equations of motion.

Note that the currents in (2.32) are consistent with the four-dimensional effective action obtained in Ref. [6]. It was shown in Ref. [6] that the four-dimensional effective action derived from our model has the following terms:

\[ S \mid_{\mathcal{O}(A_L, A_R)} = \int d^4x \; 2 \; \text{tr} \left( \mathcal{V}_i^{(+)} R_{\mu} \sum_{n=1}^{\infty} g_v^n \psi_n^\mu + \mathcal{V}_i^{(-)} R_{\mu} \sum_{n=1}^{\infty} g_a^n a_n^\mu \right), \]  

where \( \Pi(x), \psi_0^\mu(x) \), and \( a_n^\mu(x) \) are the pion, vector meson, and axial-vector meson fields, respectively, and \( f_\pi, g_v^n \), and \( g_a^n \) are the decay constants of these mesons, respectively. These mesons are related to the five-dimensional gauge field as

\[ \mathcal{A}_\mu(x, z) = \sum_{n=1}^{\infty} v_\mu^n(x) \psi_{2n-1}(z) + \sum_{n=1}^{\infty} a_\mu^n(x) \psi_{2n}(z), \quad \mathcal{A}_\mu(x, z) = \Pi(x) \phi_0(z), \]  

where \( \phi_0 = \frac{1}{\sqrt{2\pi k(z)}} \) and \( \{ \psi_n(z) \}_{n=1,2,...} \) is a complete set of the functions of \( z \) consisting of the eigenfunctions of the eigenvalue equation

\[ -h(z)^{-1} \partial_z \left( k(z) \partial_z \psi_n \right) = \lambda_n \psi_n, \]  

with the normalization condition

\[ \kappa \int dz \; h(z) \psi_n \psi_m = \delta_{mn}. \]  

We can show that \( \psi_{2n-1}(z) \) and \( \psi_{2n}(z) \) are even and odd functions of \( z \), respectively, if we arrange \( \psi_n(z) \) such that the eigenvalues satisfy \( \lambda_1 < \lambda_2 < \lambda_3 < \cdots \). The decay constants are given by

\[ f_\pi = 2 \sqrt{\frac{\kappa}{\pi}}, \quad g_v^n = -2\kappa \left( k(z) \partial_z \psi_{2n-1} \right) \mid_{z = +\infty}, \quad g_a^n = -2\kappa \left( k(z) \partial_z \psi_{2n} \right) \mid_{z = +\infty}. \]  

\[^{(1)}\text{See Eq. (5.12) in Ref. [6]}.\]
Note that the functions \( \psi_n(z) \) behave as \( O(z^{-1}) \) in the \( z \to \pm \infty \) limit and the decay constants \( g_v^n \) and \( g_a^n \) are determined as the coefficients in front of \( 1/z \) as
\[
\psi_{2n-1}(z \to \pm \infty) \simeq \pm \frac{g_v^n}{2\kappa z}, \quad \psi_{2n}(z \to \pm \infty) \simeq \frac{g_a^n}{2\kappa z}.
\] (2.39)
The following expressions are also useful:
\[
g_v^n = \lambda_2^{-1} \kappa \int dz h(z) \psi_{2n-1}, \quad g_a^n = \lambda_2 \kappa \int dz h(z) \psi_{2n} \psi_0,
\] (2.40)
which are obtained by using (2.38) and (2.36).

Comparing (2.26) and (2.34), the vector and axial-vector currents are obtained as
\[
\mathcal{J}_V^\mu = -\sum_{n=1}^{\infty} g_v^n v_n^\mu, \quad \mathcal{J}_A^\mu = -f_\pi \partial_\mu \Pi - \sum_{n=1}^{\infty} g_a^n a_n^\mu.
\] (2.41)
Using (2.35) and (2.38), we can easily check that (2.33) and (2.41) are equivalent. Note that the vector current is expressed by the vector mesons \( v_n^\mu \), which is a direct consequence of the complete vector meson dominance of the model found in Ref. 6. (See also Refs. 11 and 22.)

As in (2.3), we decompose the currents as
\[
\mathcal{J}_\mu = J_\mu + \hat{\mathcal{J}}_\mu = J_\mu + \hat{\mathcal{J}}_\mu = J_\mu + \hat{\mathcal{J}}_\mu = \sum_{C=0}^{3} \mathcal{J}_\mu^C \tau_C^C / 2.
\] (2.42)
Then the baryon number current is given by
\[
J_B^\mu = \frac{2}{N_c} \hat{J}_V^\mu = -\frac{2}{N_c} \kappa \left[ k(z) \hat{F}^{\mu z} \right]_{z=+\infty} - \frac{1}{N_c} \kappa \left[ k(z) \hat{F}^{\mu z} \right]_{z=-\infty}.
\] (2.43)

As a check, the baryon number density is computed as
\[
J_B^0 = -\frac{2}{N_c} \kappa \int dz \partial_z (k(z) \hat{F}^{0z}) = -\frac{1}{64\pi^2} \int dz \epsilon^{0M_1M_2M_3} F_{M_1M_2}^a F_{M_3M_4}^a + \text{(total derivative)},
\] (2.44)
where we have used the equation of motion (2.7). This is consistent with (2.11) as expected.

2.3. Asymptotic solution

In order to calculate the currents (2.32), we have to know how the field strength \( F_{\mu z} \) behaves at \( z = \pm \infty \). Unfortunately, we cannot directly use the solution (2.12) since it is only valid in the region \( \xi \ll 1 \)\(^\dagger\). In this subsection, we study how to extend the solution to the \( 1 \ll \xi \) region.

\(^\dagger\) Here we are assuming \( Z \sim Z_{cl} = 0 \), since the expectation value of \( f(Z) \) is approximated by its classical value \( f(Z_{cl}) \) for the large \( N_c \) and large \( \lambda \) limit.
Let us first summarize the gauge configuration in the $\xi \ll 1$ region. Here we include the time-dependent moduli parameters, which are treated as operators in quantum mechanics (2.17). As explained in Ref. [16], the $SU(2)$ gauge field takes the form

$$A_M = VA^0_M V^{-1} - i V \partial_M V^{-1},$$

(2.45)

where $A^0_M$ is the solution (2.12) and $V$ satisfies

$$- i V^{-1} \dot{V} = - \dot{X}^M A^0_M + \chi^a f(\xi) \frac{\tau^a}{2} g^{-1},$$

(2.46)

which follows from the Gauss law constraint. Here

$$\chi^a = -i \text{tr} (\tau^a a^{-1} \dot{a}) = -i \frac{1}{\rho^2} \text{tr} (\tau^a y^i \dot{y}^i) = \frac{1}{4\pi^2 \rho^2} J_a,$$

(2.47)

where $y = y_4 + iy_a \tau^a$, $a = a_4 + ia_a \tau^a = y/\rho$, and $J_a$ is the spin operator defined in (2.19).

It is convenient to perform the gauge transformation

$$A_\alpha \rightarrow A^G_\alpha = G A_\alpha G^{-1} - i G \partial_\alpha G^{-1},$$

(2.48)

with $G = a g^{-1} V^{-1}$. Then,

$$A^G_0 = -i(1 - f(\xi)) a \dot{a}^{-1} + i(1 - f(\xi)) \dot{X}^M a (g^{-1} \partial_M g) \dot{a}^{-1},$$

(2.49)

$$A^G_M = -i(1 - f(\xi)) a (g^{-1} \partial_M g) a^{-1}.$$

(2.50)

This choice of gauge is useful in considering the asymptotic behavior, as we will see in the following, while a singularity develops at $\xi = 0$.

The $U(1)$ part is treated as a perturbation in the background given by the $SU(2)$ gauge configuration obtained above. The leading contribution to the $U(1)$ part is obtained by solving the following linearized equations of motion (in the Lorenz gauge):

$$\partial_M \partial^M \hat{A}_0 = \frac{3}{\pi^2 a \lambda (\xi^2 + \rho^2)^2} \rho^4,$$

(2.51)

$$\partial_M \partial^M \hat{A}_i = \frac{3}{\pi^2 a \lambda (\xi^2 + \rho^2)^2} \left( \dot{X}^i + \frac{\chi^a}{2} (\epsilon^{iaj} x^j - \delta^{ia} z) + \frac{\dot{\rho} x^i}{\rho} \right),$$

(2.52)

$$\partial_M \partial^M \hat{A}_z = \frac{3}{\pi^2 a \lambda (\xi^2 + \rho^2)^2} \left( \dot{Z} + \frac{\chi^a x^a}{2} + \frac{\dot{\rho} z}{\rho} \right).$$

(2.53)

Here we substituted (2.49) and (2.50) into the equations of motion (2.7) and (2.9) with the warp factors $h(z)$ and $k(z)$ approximated by 1. We neglect the terms including $\partial_0^2$, because we are interested in slowly moving solitons.\(^1\)

\(^1\) The time derivative squared such as $\ddot{Z}$ and $\ddot{Z}^2$ can be traded by $Z$ or $Z^2$ via its Schrödinger equation or the relation $\dot{Z} = P_Z/M_0$; thus for large $N_c$ and $\lambda$, it is approximated by its classical value, which vanishes.
The regular solution is found to be

\[
\hat{A}_0 = \frac{1}{8\pi^2 a\lambda} \frac{1}{\xi^2} \left( 1 - \frac{\rho^4}{(\xi^2 + \rho^2)^2} \right) = \frac{1}{8\pi^2 a\lambda} \frac{\xi^2 + 2\rho^2}{(\xi^2 + \rho^2)^2}, \tag{2.54}
\]

\[
\hat{A}_i = -\frac{1}{8\pi^2 a\lambda} \left[ \frac{\xi^2 + 2\rho^2}{(\xi^2 + \rho^2)^2} \hat{x}^i + \frac{\rho^2}{(\xi^2 + \rho^2)^2} \left( \frac{\chi^a_i}{2} (\epsilon_{ija} \hat{x}^j - \delta^i_a z) + \frac{\dot{\rho} x^i}{\rho} \right) \right], \tag{2.55}
\]

\[
\hat{A}_z = -\frac{1}{8\pi^2 a\lambda} \left[ \frac{\xi^2 + 2\rho^2}{(\xi^2 + \rho^2)^2} \hat{z} + \frac{\rho^2}{(\xi^2 + \rho^2)^2} \left( \frac{\chi^a z}{2} + \frac{\dot{\rho} z}{\rho} \right) \right]. \tag{2.56}
\]

Note that (2.55) and (2.56) are neglected in Ref. 16, since the energy contributions from them are subleading in the 1/λ expansion given in Ref. 16. However, here we keep them because they give the leading contribution to the isoscalar current density. It can also be shown that the \( \hat{F}^2 \) terms in the equations of motion (2.7) and (2.9) are subleading in the 1/λ expansion, justifying the above perturbative treatment for the U(1) part.

So far, we have established a solution that is valid in the region \( \xi \ll 1 \). We now consider how to find the solution in the \( 1 \ll \xi \) region. The key observation is that all the components of the gauge field in (2.49), (2.50), (2.51), (2.55), and (2.56) are suppressed in the \( \rho \ll \xi \ll 1 \) region in the large \( \lambda \) limit. This implies that the nonlinear terms in the equations of motion can be neglected in this region for large \( \lambda \). Our strategy is to find a solution of the linearized equations of motion in the \( \rho \ll \xi \ll 1 \) region that smoothly connects the previous solution in the overlapping region \( \rho \ll \xi \ll 1 \).

For this purpose, we note that for \( \rho \ll \xi \ll 1 \), the gauge field (2.49), (2.50), (2.54), (2.55), and (2.56) are approximated as

\[
\hat{A}_0 \simeq -\frac{1}{2a\lambda} G^{\text{flat}}(\vec{x}, z; \vec{X}, Z), \tag{2.57}
\]

\[
\hat{A}_i \simeq \frac{1}{2a\lambda} \left[ \hat{x}^i + \frac{\rho^2}{2} \left( \frac{\chi^a_i}{2} (\epsilon_{ija} \frac{\partial}{\partial X^j} - \delta^i_a \frac{\partial}{\partial Z}) + \dot{\rho} \frac{\partial}{\rho} \frac{\partial}{\partial X^i} \right) \right] G^{\text{flat}}(\vec{x}, z; \vec{X}, Z), \tag{2.58}
\]

\[
\hat{A}_z \simeq \frac{1}{2a\lambda} \left[ \hat{z} + \frac{\rho^2}{2} \left( \frac{\chi^a z}{2} \frac{\partial}{\partial X^a} + \dot{\rho} \frac{\partial}{\partial Z} \right) \right] G^{\text{flat}}(\vec{x}, z; \vec{X}, Z), \tag{2.59}
\]

\[
A_0^G \simeq 4\pi^2 \rho^2 \hat{i} \hat{a} \hat{a}^{-1} G^{\text{flat}}(\vec{x}, z; \vec{X}, Z)
+ 2\pi^2 \rho^2 \hat{a}_i \hat{a}_a \hat{a}^{-1} \left( \hat{x}^i \left( \epsilon_{ija} \frac{\partial}{\partial X^j} - \delta^i_a \frac{\partial}{\partial Z} \right) + \dot{Z} \frac{\partial}{\partial X^a} \right) G^{\text{flat}}(\vec{x}, z; \vec{X}, Z), \tag{2.60}
\]

\[
A_i^G \simeq 2\pi^2 \rho^2 \left( \hat{a}_i \hat{a}_a \hat{a}^{-1} \frac{\partial}{\partial Z} + \epsilon_{ija} \frac{\partial}{\partial X^j} \right) G^{\text{flat}}(\vec{x}, z; \vec{X}, Z), \tag{2.61}
\]

\[
A_z^G \simeq -2\pi^2 \rho^2 \hat{a}_i \hat{a}_a \hat{a}^{-1} \frac{\partial}{\partial X^a} G^{\text{flat}}(\vec{x}, z; \vec{X}, Z), \tag{2.62}
\]

where

\[
G^{\text{flat}}(\vec{x}, z; \vec{X}, Z) = -\frac{1}{4\pi^2} \frac{1}{\xi^2}. \tag{2.63}
\]
is the Green’s function in the flat $\mathbb{R}^4$ that satisfies
\[
\partial_M \partial^M G_{\text{flat}}(\vec{x}, z; \vec{X}, Z) = \delta^3(\vec{x} - \vec{X})\delta(z - Z) .
\] (2.64)

We can easily check that the gauge configuration (2.57)–(2.62) satisfies the Maxwell and the linearized YM equations without sources:
\[
\partial_\beta \widehat{F}^{\alpha\beta} = \partial_\beta F^{\alpha\beta} \big|_{\text{linear}} = 0 ,
\] (2.65)
as well as the gauge condition
\[
\partial_\alpha \widehat{A}_\alpha = 0 , \quad \partial_\alpha A^G_\alpha = 0 .
\] (2.66)

In order to connect this solution to the large $\xi$ region, we have to take into account the effect of the curved background. The linearized equations of motion and the gauge condition that generalize (2.65) and (2.66) to the case with nontrivial warp factors $h(z)$ and $k(z)$ are
\[
h(z)\partial_\mu^2 \widehat{A}_i + \partial_\mu (h(z)\partial_\nu \widehat{A}_i) = 0 , \quad \partial_\mu \widehat{A}_z + \partial_\mu (h(z)^{-1}\partial_\nu (k(z)\widehat{A}_z)) = 0 ,
\] (2.67)
\[
h(z)\partial_\mu^2 A^G_i + \partial_\mu (k(z)\partial_\nu A^G_i) = 0 , \quad \partial_\mu A^G_z + \partial_\mu (h(z)^{-1}\partial_\nu (k(z)A^G_z)) = 0 ,
\] (2.68)
and
\[
h(z)\partial_\mu \widehat{A}_\mu + \partial_\nu (k(z)\widehat{A}_z) = 0 , \quad h(z)\partial_\mu A^G_\mu + \partial_\nu (k(z)A^G_z) = 0 ,
\] (2.69)
respectively.

To solve these equations, we define Green’s functions in the curved space as
\[
G(\vec{x}, z; \vec{X}, Z) = \kappa \sum_{n=1}^{\infty} \psi_n(z)\psi_n(Z)Y_n(|\vec{x} - \vec{X}|) ,
\] (2.70)
\[
H(\vec{x}, z; \vec{X}, Z) = \kappa \sum_{n=0}^{\infty} \phi_n(z)\phi_n(Z)Y_n(|\vec{x} - \vec{X}|) ,
\] (2.71)
where $\{\psi_n(z)\}_{n=1,2,...}$ is the complete set defined in (2.36) and (2.37), $\{\phi_n(z)\}_{n=0,1,...}$ is another complete set given by
\[
\phi_0(z) = \frac{1}{\sqrt{\kappa \pi k(z)}} , \quad \phi_n(z) = \frac{1}{\sqrt{\lambda_n}} \partial_\nu \psi_n(z) , \quad (n = 1, 2, \cdots)
\] (2.72)
and $Y_n(r)$ is the Yukawa potential with meson mass $m_n = \sqrt{\lambda_n}$,
\[
Y_n(r) = -\frac{1}{4\pi} e^{-\sqrt{\lambda_n} r} \frac{e^{-\sqrt{\lambda_n} r}}{r} ,
\] (2.73)
which satisfies
\[(\partial_i^2 - \lambda_n)Y_n(|\vec{x} - \vec{X}|) = \delta^i(\vec{x} - \vec{X}) . \quad (2.74)\]

Note that the normalization of the functions \(\{\phi_n\}_{n=0,1,2,\ldots}\) is fixed by
\[\kappa \int dz k(z)\phi_n\phi_m = \delta_{mn} . \quad (2.75)\]

Using (2.36), (2.72), (2.74), and the completeness conditions
\[\kappa h(z) \sum_{n=1}^{\infty} \phi_n(z)\phi_n(Z) = \delta(z - Z) , \quad \kappa k(z) \sum_{n=1}^{\infty} \phi_n(z)\phi_n(Z) = \delta(z - Z) , \quad (2.76)\]
it is easy to verify
\[h(z)\partial_z^2 G + \partial_z(k(z)\partial_z G) = \delta^i(\vec{x} - \vec{X})\delta(z - Z) , \quad (2.77)\]
\[\partial_z^2 H + \partial_z(h(z)^{-1}\partial_z(k(z)H)) = k(z)^{-1}\delta^i(\vec{x} - \vec{X})\delta(z - Z) , \quad (2.78)\]
\[\partial_z(k(z)H) + h(z)\partial_z G = 0 . \quad (2.79)\]

Then, the solution of equations (2.67)–(2.69) is obtained by replacing the Green’s function \(G^{\text{flat}}\) in (2.57)–(2.62) with \(G\) or \(H\) as follows:
\[\hat{A}_0 \simeq -\frac{1}{2a\lambda}G(\vec{x}, z; \vec{X}, Z) , \quad (2.80)\]
\[\hat{A}_i \simeq \frac{1}{2a\lambda} \left[\dot{X}^i + \frac{\rho^2}{2} \left(\frac{\chi^a}{2} \frac{\partial}{\partial X^a} - \delta^{ia} \frac{\partial}{\partial Z} + \frac{\hat{\rho}}{\rho} \frac{\partial}{\partial X^i}\right)\right] G(\vec{x}, z; \vec{X}, Z) , \quad (2.81)\]
\[\hat{A}_z \simeq \frac{1}{2a\lambda} \left[\dot{Z} + \frac{\rho^2}{2} \left(\frac{\chi^a}{2} \frac{\partial}{\partial X^a} + \frac{\hat{\rho}}{\rho} \frac{\partial}{\partial Z}\right)\right] H(\vec{x}, z; \vec{X}, Z) , \quad (2.82)\]
\[A_0^G \simeq 4\pi^2\rho^2ia\hat{a}^{-1}G(\vec{x}, z; \vec{X}, Z) + 2\pi^2\rho^2a^r\alpha^{-1} \left(\dot{X}^i \left(\epsilon^{iaj} \frac{\partial}{\partial X^j} - \delta^{ia} \frac{\partial}{\partial Z}\right) + \dot{Z} \frac{\partial}{\partial X^a}\right) G(\vec{x}, z; \vec{X}, Z) , \quad (2.83)\]
\[A_i^G \simeq -2\pi^2\rho^2a^r\alpha^{-1} \left(\epsilon^{iaj} \frac{\partial}{\partial X^j} - \delta^{ia} \frac{\partial}{\partial Z}\right) G(\vec{x}, z; \vec{X}, Z) , \quad (2.84)\]
\[A_z^G \simeq -2\pi^2\rho^2a^r\alpha^{-1} \frac{\partial}{\partial X^a} H(\vec{x}, z; \vec{X}, Z) . \quad (2.85)\]

Here, we neglected the terms including \(\partial_t^2\) as before.

Since the Green’s functions \(G\) and \(H\) approach \(G^{\text{flat}}\) for \(\rho \ll \xi \ll 1\), this solution is smoothly connected with the previous solution (2.57)–(2.62) in this region, as expected. Note also that the Green’s functions \(G\) and \(H\) vanish at \(\xi \to \infty\), and hence the linear approximation of the equations of motion does not break down all the way to infinity. Therefore, we can read off the behavior of the gauge field at \(z \to \pm\infty\), which is needed to calculate the currents, from (2.80)–(2.85).
2.4. Computation of the current

Since the wavefunctions of the low-lying baryon states, such as (2.22), are dominant for \( Z \sim \mathcal{O}(\lambda^{-1/2}N_c^{-1/2}) \ll 1 \), we can use the approximation \( h(Z) \simeq k(Z) \simeq 1 \) and the relation

\[
- \partial_z^2 \psi_n(Z) \simeq \lambda_n \psi_n(Z) ,
\]

which follows from (2.36). This implies

\[
\partial_z H + \partial_z G \simeq 0 , \quad (\partial_z^2 + \partial_z^2)G \simeq 0 , \quad (\partial_z^2 + \partial_z^2)H \simeq 0 .
\]

Using the asymptotic solution (2.80)–(2.85) and the relations (2.87), we obtain

\[
\hat{F}_{0z} \simeq \frac{1}{2a\lambda} \partial_z G ,
\]

\[
\hat{F}_{iz} \simeq \frac{1}{2a\lambda} \left[ \partial_z H - X^i \partial_z G - \frac{\rho^a}{4} \left( \left( \partial_i \partial_a - \delta^{ia} \partial^2 \right) H - \epsilon^{ija} \partial_j \partial_z G \right) \right] ,
\]

\[
F_{0z} \simeq 2\pi^2 \partial_0 (\rho^2 a \tau^a a^{-1}) \partial_0 H - 4\pi^2 \rho^2 i a \partial_0^{-1} \partial_z G - 2\pi^2 \rho^2 a \tau^a a^{-1} \partial_z \left( \left( \partial_i \partial_a - \delta^{ia} \partial^2 \right) H - \epsilon^{ija} \partial_j \partial_z G \right) ,
\]

\[
F_{iz} \simeq 2\pi^2 \rho^2 a \tau^a a^{-1} \left( \left( \partial_i \partial_a - \delta^{ia} \partial^2 \right) H - \epsilon^{ija} \partial_j \partial_z G \right)
\]

for \( Z \ll 1 \ll z \).

The following formulas are also useful:

\[
G^V(Z,r) \equiv \left[ k(z) \partial_z G \right]_{z=-\infty}^{z=+\infty} = - \sum_{n=1}^{\infty} g_{vn} \psi_{2n-1}(Z) Y_{2n-1}(r) ,
\]

\[
G^A(Z,r) \equiv \left[ \psi_0(z) k(z) \partial_z G \right]_{z=-\infty}^{z=+\infty} = - \sum_{n=1}^{\infty} g_{an} \psi_{2n}(Z) Y_{2n}(r) ,
\]

\[
H^V(Z,r) \equiv \left[ k(z) H \right]_{z=-\infty}^{z=+\infty} = - \sum_{n=1}^{\infty} \frac{g_{vn}}{\lambda_{2n-1}} \partial_Z \psi_{2n-1}(Z) Y_{2n-1}(r) ,
\]

\[
H^A(Z,r) \equiv \left[ \psi_0(z) k(z) H \right]_{z=-\infty}^{z=+\infty} = - \frac{1}{2\pi^2} \frac{1}{k(Z)} \frac{1}{r} - \sum_{n=1}^{\infty} \frac{g_{vn}}{\lambda_{2n}} \partial_Z \psi_{2n}(Z) Y_{2n}(r) ,
\]

where \( r = |\vec{x} - \vec{X}| \). Note that \( G^V \) and \( H^A \) are even functions with respect to \( Z \), while \( G^A \) and \( H^V \) are odd functions.

Now, we are ready to write down the currents from (2.32) and (2.33). The vector and axial-vector currents are obtained as

\[
\hat{J}_{0V,A}^0 = \frac{N_c}{2} G^{V,A} ,
\]

\[
\hat{J}_{0V,A}^i = -\frac{N_c}{2} \left[ \hat{Z} \partial_i H^{V,A} - \hat{X}^i G^{V,A} - \frac{\rho^a}{4} \left( \left( \partial_i \partial_a - \delta^{ia} \partial^2 \right) H^{V,A} - \epsilon^{ija} \partial_j G^{V,A} \right) \right] ,
\]

15
\[ J_{V,A}^0 = 2\pi^2 \kappa \left[ \partial_0 (\rho^2 \bar{a} e^{-1}) \partial_a H^{V,A} - 2\rho^2 i \dot{a} \bar{a}^{-1} G^{V,A} \right. \]

\[ \left. - \rho^2 \bar{a} e^{-1} \hat{X}^i ( (\partial^i \partial_\ell - \delta^{ia} \delta^i_j) H^{V,A} - e^{iaj} \partial_j G^{V,A} ) \right] , \] (2.98)

\[ J_{V,A}^i = -2\pi^2 \kappa \rho^2 \bar{a} e^{-1} \left( (\partial_i \partial_\ell - \delta^{ia} \delta^i_j) H^{V,A} - e^{iaj} \partial_j G^{V,A} \right) . \] (2.99)

Using the relation (2.87), it is easy to check that these currents are conserved (up to terms including \( \partial_0^2 \)).

\section{Static properties of baryons}

In this section, we study the static properties of baryons as applications of the currents obtained in the previous section.

### 3.1. Baryon number density, isoscalar mean square radius

The baryon number density is obtained from (2.43) and (2.96) as

\[ J_0^B = -\sum_{n=1}^{\infty} g_v \psi_{2n-1}(Z) \psi_{2n-1}(r) . \] (3.1)

As a check, the baryon number charge is calculated using this expression as

\[ N_B = \int_0^{\infty} dr 4\pi r^2 \left\langle J_0^B(r) \right\rangle \]

\[ = \sum_{n=1}^{\infty} g_v \lambda_{2n-1}^2 \langle \psi_{2n-1}(Z) \rangle \]

\[ = 1 , \] (3.3)

where \( \langle O \rangle = \langle B,s | O | B,s \rangle \) is the expectation value with respect to a baryon state \( | B,s \rangle \). Here we have used (2.40) and (2.76).

The baryon number density per unit \( r \) is given by the integrand of (3.2),

\[ \rho_B(r) \equiv 4\pi r^2 \left\langle J_0^B(r) \right\rangle = r \sum_{n=1}^{\infty} g_v \lambda_{2n-1}^2 \langle \psi_{2n-1}(Z) \rangle e^{-\sqrt{\lambda_{2n-1}^2} r} . \] (3.4)

Then the isoscalar mean square radius is

\[ \langle r^2 \rangle_{I=0} = \int_0^{\infty} dr r^2 \rho_B(r) = 6 \sum_{n=1}^{\infty} g_v \lambda_{2n-1}^2 \langle \psi_{2n-1}(Z) \rangle . \] (3.5)

In the large \( N_c \) and large \( \lambda \) limit, the baryon wavefunction is localized at \( Z = 0 \), as we can see from the wavefunction (2.22) for the nucleon states, and hence the expectation

[^1]: Here we neglect the effect of the \( U(1)_A \) anomaly, since it is subleading in the \( 1/N_c \) expansion.
value \( \langle \psi_{2n-1}(Z) \rangle \) can be approximated by its classical value
\( \psi_{2n-1}(Z_{cl}) = \psi_{2n-1}(0) \). In this approximation, it is possible to evaluate the isoscalar mean square radius \( (3.5) \) as follows. Note that the function
\[
F(z) \equiv 6 \sum_{n=1}^{\infty} \frac{g_{\nu n}}{\lambda_{2n-1}^2} \psi_{2n-1}(z)
\]
satisfies
\[
- \partial_z (k(z) \partial_z F(z)) = 6 h(z) \quad F(z) = F(-z)
\]
and the boundary condition \( F(z) \to 0 \) (\( z \to \pm \infty \)). The first relation is obtained from \( (2.36) \), \( (2.40) \), and \( (2.76) \). The solution of \( (3.7) \) is given by
\[
F(z) = F_0 - \int_0^z dz' k(z')^{-1} \int_0^{z'} dz'' 6 h(z'') .
\]
The constant \( F_0 \) is fixed by the boundary condition and we obtain
\[
F_0 = \int_0^\infty dz' k(z')^{-1} \int_0^{z'} dz'' 6 h(z'') \simeq 14.3 .
\]
Therefore \( (3.5) \) can be evaluated as
\[
\langle r^2 \rangle_{I=0} = \langle F(Z) \rangle \simeq F(0) = F_0 \simeq 14.3 / M_{KK}^2 ,
\]
and if we use the value \( (2.3) \) for \( M_{KK} \), we obtain
\[
\langle r^2 \rangle_{I=0}^{1/2} \simeq 0.785 \text{ fm} .
\]
The experimental value is \( \langle r^2 \rangle_{I=0}^{1/2} \mid_{\text{exp}} \simeq 0.806 \text{ fm} \) and the prediction of the Skyrme model in Ref. 14 is 0.59 fm.

It is interesting to note that this value is independent of \( \lambda \) and \( N_c \). The \( N_c \) independence is consistent with the analysis of baryons in large \( N_c \) QCD. The \( \lambda \) independence suggests that the size of the baryon number distribution is governed by the scale of the vector meson mass rather than the size of the soliton \( \rho_{cl} \sim O(\lambda^{-1/2}) \) for large \( \lambda \).

Given the wavefunction of the baryon state, it is also possible to evaluate the expectation value \( \langle F(Z) \rangle \) numerically. For the nucleon wavefunction given by \( (2.22) \), we obtain
\[
\langle r^2 \rangle_{I=0}^{1/2} \simeq 0.742 \text{ fm} .
\]
\footnote{Note that in the previous section the chiral currents were obtained using the approximation \( Z \sim 0 \), and thus the classical value for \( Z \) was used. Here, we simply assume the same chiral current and evaluate it using the quantum states of the baryons.}
This value is the same for the other states with \( n_z = 0 \), such as \( \Delta(1232) \) and \( N(1440) \), since \( Z \) dependence of the wavefunction is the same for these states. For the states with \( n_z = 1 \), such as \( N(1535) \), we obtain

\[
\langle r^2 \rangle_{I=0}^{1/2} \simeq 0.699 \text{ fm}.
\]

See Appendix A.2 for more details.

3.2. Isoscalar magnetic moment

The isoscalar magnetic moment is defined as

\[
\mu^i_{I=0} = \frac{1}{2} \epsilon^{ijk} \int d^3x x^j J_k^B = \frac{1}{N_c} \epsilon^{ijk} \int d^3x x^j \hat{J}_V^k.
\]

It is easy to see that only the last term in (2.97) contributes to the integral. Then by evaluating the angular integral in (3.14) we obtain

\[
\mu^i_{I=0} = \frac{\rho^2 \chi^i}{12} \int_0^\infty dr 4\pi r^2 \partial_r J_B^0(r).
\]

Using (2.47), this can be evaluated as

\[
\mu^i_{I=0} = \frac{\rho^2 \chi^i}{4} = \frac{J^i}{2M_0},
\]

where \( J^i \) is the spin operator (2.19). For example, for nucleon states with up spin, this gives

\[
\langle p^\uparrow | \mu^i_{I=0} | p^\uparrow \rangle = \langle n^\uparrow | \mu^i_{I=0} | n^\uparrow \rangle = \frac{1}{4M_0} \delta^{3i}.
\]

The \( g \) factor is defined as

\[
\mu^i = \frac{g}{4M_N} \sigma^i,
\]

where \( \sigma^i \) is the Pauli matrix that acts on a spin doublet, and \( M_N \) is the nucleon mass. If we use (2.3) and the experimental value for the nucleon mass \( M_N \simeq 940 \text{ MeV} \), we have\(^*\)

\[
g_{I=0} = g_p + g_n = M_N/M_0 \simeq 1.68.
\]

The experimental value is \( g_{I=0}^{\exp} \simeq 1.76 \)\(^{23}\) and the prediction obtained from the Skyrme model in Ref. 14) is \( g_{I=0}^{\exp} \simeq 1.11 \)\(^{**}\).

\(^*\) Note that \( M_N \) is not calculated in Ref. 16) because the total contribution from the zero point energy of the fluctuations around the soliton solution is difficult to evaluate. Here the \( g \) factor is computed simply to express the magnetic moments in the unit of \( 1/(4M_N) \).

\(^{**}\) The same expressions as (3.17) and (3.19) were obtained using the current in Ref. 19). However, the numerical values were different, because the values of \( M_{KK} \) and \( \kappa \) used in Ref. 19) were different from ours.
3.3. **Isovector charge density and charge radii**

The isovector charge density is given by (2.98) and the isovector charge is evaluated using (3.3) as

\[ Q_V = \int d^3x \, J_V^0 = -4\pi^2 \kappa \rho^2 i a \hat{a}^{-1}. \]  

(3.20)

A relation similar to (2.47),

\[ -i \text{tr} (\tau^a a \hat{a}^{-1}) = -\frac{i}{\rho^2} \text{tr} (\tau^a y \hat{y}^\dagger) = \frac{1}{4\pi^2 \kappa \rho^2} I_a, \]  

(3.21)

where \( I_a \) is the isospin operator defined in (2.19), implies

\[ Q^a_V = I_a, \]  

(3.22)

as expected.

The isovector charge density per unit \( r \) is proportional to the angular integral of the isovector charge density with the normalization condition

\[ \int_0^\infty dr \rho_I = 1. \]  

This turns out to be identical to the baryon number density:

\[ \rho_{I=1} = \rho_B(r). \]  

(3.23)

Therefore, the isovector mean square charge radius \( \langle r^2 \rangle_I = 0 \) is the same as the isoscalar mean square radius \( \langle r^2 \rangle_{I=0} \) evaluated in §3.1. This result is somewhat puzzling since it is known that the isovector mean square radius is divergent in the chiral limit.\(^25\)

There is however no contradiction. This divergence is due to the IR divergence of pion loops.\(^26\) Our analysis only involves a string world-sheet with disk topology, and hence the pion loops are not included. Therefore it will be important to include the quark mass in the model\(^*\) and estimate the contribution from the annulus diagram to make a comparison with the experimental value, which is beyond the scope of the present paper.

The electric charge is defined by

\[ Q_{em} = I_3 + \frac{N_B}{2}, \]  

(3.24)

which gives \( Q_{em} = 1 \) for a proton \( (I_3 = 1/2) \) and \( Q_{em} = 0 \) for a neutron \( (I_3 = -1/2) \). Then, because of identity (3.23), the electric charge density is given by \( \rho_E = \rho_{I=0} \) for a proton and \( \rho_E = 0 \) for a neutron, and we obtain

\[ \langle r^2 \rangle_{E,p} = \langle r^2 \rangle_{I=0} \quad \text{(for a proton)} , \quad \langle r^2 \rangle_{E,n} = 0 \quad \text{(for a neutron)} . \]  

(3.25)

\(^*\) See Refs. 27)–34) for recent developments toward the incorporation of the quark mass in the model.
The experimental values in Ref. [23] are
\[
\langle r^2 \rangle_{E,p}^{\exp} \simeq (0.875 \text{ fm})^2, \quad \langle r^2 \rangle_{E,n}^{\exp} \simeq -0.116 \text{ fm}^2. \quad (3.26)
\]
Although our calculation does not reproduce the experimental value of the electric charge radius of the neutron \( \langle r^2 \rangle_{E,n} \), the vanishing of the neutron electric charge density seems to be a good approximation for reproducing the observed behavior of the electric form factor, as we will study in §4.

The charge radius for the excited baryons is similarly found to be the same as their isoscalar mean square radius \( \langle r^2 \rangle_{I=0} \) obtained in §3.1. In particular, our analysis predicts that the Roper excitation \( N(1440) \) has a charge radius equal to that of a proton \( (3.12) \), while that of \( N(1535) \), \( (3.13) \), is smaller.

3.4. Magnetic moment

The isovector magnetic moment is defined as
\[
\mu^i_{I=1} = \frac{1}{2} \epsilon_{ijk} \int d^3x \, x^j \text{tr}(J^k \tau^3) \times 2. \quad (3.27)
\]
Here the additional factor of 2 in the integrand is due to our normalization of the current. Substituting \((2.99)\) into this expression gives
\[
\mu^i_{I=1} = -4\pi^2 \kappa \rho^2 \text{tr}(a^i a^{-1} \tau^3). \quad (3.28)
\]
For the baryon states of \( I = J = 1/2 \), we can use the identity \((3.13)\)
\[
(B', s' | \text{tr}(a^i a^{-1} \tau^a) | B, s) = -\frac{2}{3}(\sigma^i)_{s's} (\tau^a)_{I_3' I_3}, \quad (3.29)
\]
with \( \sigma^i \) and \( \tau^a \) being the Pauli matrices corresponding to spin and isospin, respectively. Here we have used the notation
\[
(\sigma^i)_{s's} = \chi^\dagger(s') \sigma^i \chi(s), \quad (\tau^C)_{I'_3 I_3} = \psi^\dagger_I \tau^C \psi_I, \quad (3.30)
\]
where \( \chi(s) \) and \( \psi_I \) are defined as
\[
\chi(1/2) = \psi_{I_3=1/2} = \left( \begin{array}{c} 1 \\ 0 \end{array} \right), \quad \chi(-1/2) = \psi_{I_3=-1/2} = \left( \begin{array}{c} 0 \\ 1 \end{array} \right). \quad (3.31)
\]
Then, we obtain
\[
\langle p^\uparrow | \mu^i_{I=1} | p^\uparrow \rangle = -\langle n^\uparrow | \mu^i_{I=1} | n^\uparrow \rangle = \frac{8\pi^2 \kappa}{3} \langle \rho^2 \rangle \delta^{i3}, \quad (3.32)
\]
where $\langle \rho^2 \rangle$ is the expectation value of $\rho^2$ for the nucleon states. Using the wavefunction (2.22), $\langle \rho^2 \rangle$ is obtained as

$$
\langle \rho^2 \rangle = \int \frac{d\rho \rho^2 R(\rho)^2}{\int d\rho \rho^2 R(\rho)^2} = \frac{\sqrt{5} + 2\sqrt{5 + N_c^2}}{2N_c} \rho_{cl}^3.
$$

(3.33)

In the large $N_c$ limit, it can be approximated by its classical value $\rho_{cl}^2$ in (2.15). However, for $N_c = 3$, (3.33) implies $\langle \rho^2 \rangle_{\rho=0} \simeq 1.62 \rho_{cl}^2$, suggesting that the $1/N_c$ corrections are relatively large for this quantity.

Consequently, we obtain

$$
\langle p^\uparrow | \mu_{I=1}^I | p^\uparrow \rangle = -\langle n^\uparrow | \mu_{I=1}^I | n^\uparrow \rangle = \frac{1 + 2\sqrt{1 + N_c^2/5}}{\sqrt{6} M_{KK}} \delta^3.
$$

(3.34)

Here we have recovered the $M_{KK}$ dependence by dimensional analysis. The isovector $g$ factor, defined as in (3.18), is then

$$
g_{I=1} = g_p - g_n = \frac{4M_N}{M_{KK}} \cdot \frac{1 + 2\sqrt{1 + N_c^2/5}}{\sqrt{6}} \simeq 7.03
$$

(3.35)

for $N_c = 3$, $M_N \simeq 940$ MeV, and (2.3). The experimental value is $g_{I=1}|_{exp} \simeq 9.41^{22}$ and the prediction obtained from the Skyrme model in Ref. [14] is $g_{I=1}|_{ANW} \simeq 6.38$. If we approximate $\langle \rho^2 \rangle$ by its classical value $\rho_{cl}^2$, we have $g_{I=1} \simeq 4.34$, which is considerably smaller than the experimental value. It is interesting to note that going to $\langle \rho^2 \rangle$ from $\rho_{cl}^2$ by multiplying by the ratio $(N_c + \sqrt{5}/2)/N_c$ for large $N_c$, as can be seen from (3.33), has an effect similar to the “$N_c \to N_c + 2$” rule discussed in Ref. [17]. The value of the anomalous magnetic moment obtained using this rule from the five-dimensional effective spinor field theory approach in Ref. [17] is close to ours.

The magnetic moments for the proton and neutron (measured in units of the Bohr magneton $\mu_N = 1/(2M_N)$) are given as

$$
\mu_p = \frac{g_p}{2} = \frac{1}{4}(g_{I=0} + g_{I=1}) , \quad \mu_n = \frac{g_n}{2} = \frac{1}{4}(g_{I=0} - g_{I=1}) .
$$

(3.36)

If we insert the values (3.19) and (3.35), we get

$$
\mu_p \simeq 2.18 , \quad \mu_n \simeq -1.34 ,
$$

(3.37)

while the experimental values are $\mu_p|_{exp} \simeq 2.79$ and $\mu_n|_{exp} \simeq -1.91$. Note, however, that since the $N_c$ dependences of the $g$ factors and the magnetic moments are

$$
g_{I=0} \sim \mathcal{O}(1) , \quad g_{I=1} \sim \mathcal{O}(N_c^2) , \quad \mu_{I=0} \sim \mathcal{O}(1/N_c) , \quad \mu_{I=1} \sim \mathcal{O}(N_c) ,
$$

(3.38)

Again, the same expressions (3.32) and (3.35) can be found in Ref. [19] but with a different current. Eq. (3.32) also agrees with $\Delta\mu_{an}$ in Ref. [17], if we use the classical value (2.15) for $\langle \rho^2 \rangle$. 

\* \*
the contribution of the isoscalar component will be buried in the $1/N_c$ corrections of the isovector component in linear combinations such as (3.36). Therefore, it is more meaningful to consider $g_{I=0}$ and $g_{I=1}$ rather than $g_p$ and $g_n$ in our analysis.

Let us consider excited baryons. For spin 1/2 excitations of the baryons, the magnetic moment remains the same as that of the proton/neutron if $n_\rho = 0$ (for example, $N(1535)$), because it gives the same $\langle \rho^2 \rangle$. On the other hand, the Roper excitation $N(1440)$ has $n_\rho = 1$ for which, with $N_c = 3$,

$$\langle \rho^2 \rangle_{n_\rho=1} \simeq 2.37 \rho_{cl}^2 .$$  \hspace{1cm} (3.39)

See Appendix A.2 for the details. Substituting it into (3.32) and combining the result with the value of $\mu_{I=0}$ (which is the same value as that of the proton/neutron), we obtain, for the Roper excitation,

$$\mu_p \simeq 2.99 , \quad \mu_n \simeq -2.15 ,$$  \hspace{1cm} (3.40)

measured in units of the Bohr magneton of the nucleon $1/(2M_N)$. Our model predicts that the magnetic moment of the Roper is larger than that of the proton/neutron.

For spin 3/2 baryons such as $\Delta$, we need to reevaluate the matrix elements of the spin operator $J^i$, (3.29) and (3.33). We find (details are described in Appendix A.3)

$$\mu_{I=0}^i(B, J_3 = 3/2) = 3\mu_{I=0}^i(p \uparrow) , \quad (B = \Delta^{++}, \Delta^+, \Delta^0, \Delta^-)$$
$$\mu_{I=1}^i(\Delta^{++}, J_3 = 3/2) = \frac{9c}{5}\mu_{I=1}^i(p \uparrow) , \quad \mu_{I=1}^i(\Delta^+, J_3 = 3/2) = \frac{3c}{5}\mu_{I=1}^i(p \uparrow) ,$$
$$\mu_{I=1}^i(\Delta^0, J_3 = 3/2) = -\frac{3c}{5}\mu_{I=1}^i(p \uparrow) , \quad \mu_{I=1}^i(\Delta^-, J_3 = 3/2) = -\frac{9c}{5}\mu_{I=1}^i(p \uparrow) ,$$  \hspace{1cm} (3.41)

where $c \equiv \langle \rho^2 \rangle_{l=3} / \langle \rho^2 \rangle_{l=1} \simeq 1.34$. Therefore we obtain

$$\mu_{\Delta^{++}} \simeq 5.50 , \quad \mu_{\Delta^+} \simeq 2.67 , \quad \mu_{\Delta^0} \simeq -0.15 , \quad \mu_{\Delta^-} \simeq -2.97 ,$$  \hspace{1cm} (3.42)

using (3.19) and (3.35). The experimental values for $\Delta^{++}$ and $\Delta^+$ are\(^{23}\)

$$\mu_{\Delta^{++}}|_{\text{exp}} \simeq 3.7 - 7.5 , \quad \mu_{\Delta^+}|_{\text{exp}} \simeq 2.7^{+1.0}_{-1.3} \pm 1.5 \pm 3 ,$$  \hspace{1cm} (3.43)

with which our result (3.43) is found to be consistent. A recent lattice result gives\(^{35}\)

$$\mu_{\Delta^{++}}|_{\text{lattice}} \simeq 4.99 , \quad \mu_{\Delta^+}|_{\text{lattice}} \simeq 2.49 , \quad \mu_{\Delta^0}|_{\text{lattice}} \simeq 0.06 , \quad \mu_{\Delta^-}|_{\text{lattice}} \simeq -2.45 ,$$  \hspace{1cm} (3.44)

---

\(^{*)}\) This factor was missing in the earlier versions of the present paper. We thank T. Ishii for pointing out this error.
from which we see that the agreement with our result \((3.43)\) is not very good, particularly for \(\mu_{\Delta^0}\). This is because the contributions of \(\mu_{I=1} \sim \mathcal{O}(N_c)\) and \(\mu_{I=0} \sim \mathcal{O}(1/N_c)\) roughly cancel each other out for \(\Delta^0\) and hence the \(1/N_c\) corrections cannot be neglected. To make a more reasonable comparison, we should compare \(\mu_{I=0}\) and \(\mu_{I=1}\) with experimental or lattice results as explained above. Then, the average of the magnetic moments in (3.45),

\[
\frac{1}{4}(\mu_{\Delta^+} + \mu_{\Delta^0} + \mu_{\Delta^-})|_{\text{lattice}} \simeq 1.27 ,
\]

should be compared with our result for the isoscalar component \(\frac{3c}{5}\mu_{I=1}(p) \simeq 1.26\). The isovector components are extracted by considering the differences:

\[
\begin{align*}
\frac{1}{3}(\mu_{\Delta^+} - \mu_{\Delta^-})|_{\text{lattice}} & \simeq 2.48 , \\
\frac{1}{2}(\mu_{\Delta^0} - \mu_{\Delta^-})|_{\text{lattice}} & \simeq 2.47 , \\
(\mu_{\Delta^0} - \mu_{\Delta^-})|_{\text{lattice}} & \simeq 2.51 .
\end{align*}
\]

These values are compared with our result \(\frac{3c}{5}\mu_{I=1}(p) \simeq 2.82\).

3.5. Axial coupling

As explained in Ref. \([14]\), the axial coupling \(g_A\) of the baryon states of \(I = J = 1/2\) is given by

\[
\int d^3x \langle B', s' | J_A^i | B, s \rangle \times 2 = \frac{2}{3} g_A (\sigma^i)_{s's}(\tau^a)_{I'_3I_3} .
\]

From \((2.99)\), the integral of the axial-vector current becomes

\[
\int d^3x J_A^i = 4 \frac{\pi^2 \kappa \rho^2}{3} \text{tr}(a \tau^i a^{-1} \tau^a) \int d^3x \partial^2 H^A .
\]

Although this is an integral of a total derivative, it does not vanish because of the term proportional to \(1/r\) in \((2.95)\). In fact, the integral can be performed using the Gauss’ divergence theorem as

\[
\int d^3x \partial^2 Y_{2n} = \int_S d\vec{S} \cdot \vec{\nabla} Y_{2n} = 4\pi \lim_{r \to \infty} r^2 \partial_r Y_{2n} = \delta^{0n} ,
\]

where \(Y_n(r)\) is defined in \((2.73)\) with \(\lambda_0 = 0\) for \(n = 0\). This implies that only the \(n = 0\) component of the mesons, that is the pion, contributes to the integral. Again using \((3.29)\), we obtain

\[
\int d^3x \langle B', s' | J_A^i | B, s \rangle = -\frac{16\pi \kappa}{9} \left( \frac{\rho^2}{k(Z)} \right) (\sigma^i)_{s's}(\tau^a)_{I'_3I_3} ,
\]

23
where \( \langle \rho^2/k(Z) \rangle \) is the expectation value with respect to the spin 1/2 baryon states. Comparing this with (3.48), we obtain

\[
g_A = \frac{16\pi\kappa}{3} \langle \rho^2 \rangle_{k(Z)} .
\]

(3.52)

If we approximate \( \rho \) and \( Z \) by their classical values, we obtain

\[
g_A \simeq \frac{2N_c}{3\pi} \sqrt{\frac{6}{5}} \simeq 0.697 .
\]

(3.53)

If we use the wavefunction (2.22) and (2.3) to numerically evaluate the expectation value for nucleons, we obtain

\[
\langle \rho^2 \rangle_{k(Z)} \simeq 1.05 \rho^2_{cl} ,
\]

(3.54)

and

\[
g_A \simeq 0.734 .
\]

(3.55)

The experimental value\(^{23}\) and the prediction obtained from the Skyrme model in Ref. 14 are

\[
g_A|_{exp} \simeq 1.27 , \quad g_A|_{ANW} \simeq 0.61 .
\]

(3.56)

For the excited baryons \( N(1440) \) (Roper) with \((n_\rho, n_z) = (1, 0)\) and \( N(1535) \) with \((n_\rho, n_z) = (0, 1)\), by evaluating \( \langle \rho^2/k(Z) \rangle \) using their wavefunctions, we obtain (see Appendix [A.2] for details)

\[
g_A^{(N(1440))} \simeq 1.07 , \quad g_A^{(N(1535))} \simeq 0.380 .
\]

(3.57)

We can see that the axial coupling for \( N(1440) \) is large while that for the negative parity baryon \( N(1535) \) is small compared with the proton/neutron.

There is another more direct way to perform the integral in (3.49), which will be useful in (3.51). Using (2.95) and (2.74), the integrand of (3.49) is

\[
\partial_j^2 H^A = \left( \frac{2}{\pi k(Z)} - \sum_{n=1}^{\infty} \frac{g_{2n}}{\lambda_{2n}} \partial_Z \psi_{2n}(Z) \right) \delta^3(\vec{x} - \vec{X}) - \sum_{n=1}^{\infty} g_{2n} \partial_Z \psi_{2n}(Z) Y_{2n}(r) .
\]

(3.58)

*) The sign of this equation is taken to be positive such that the axial coupling is defined to be positive. This sign can be flipped if one exchanges the definitions of “left” and “right” chiral sectors in the two asymptotes \( z \to \pm \infty \), i.e. the positive sign is a convention.

**) This expression, once the classical value \( \langle k(Z) \rangle = 1 \) is imposed, is equal to that obtained in Ref. 19. Furthermore, if we use \( \langle \rho^2 \rangle = \rho^2_{cl} \), it agrees with \( g_A, \text{mag} \) in Ref. 17.
Following the same logic as in (3.3), we can show that
\[ \sum_{n=1}^{\infty} \frac{g_{an}}{\lambda_{2n}} \psi_{2n}(Z) = \psi_0(Z), \]
where \( \psi_0(z) = \frac{2}{\pi} \arctan z \), using (2.30) and (2.76). This relation implies
\[ \sum_{n=1}^{\infty} \frac{g_{an}}{\lambda_{2n}} \partial_Z \psi_{2n}(Z) = \frac{2}{\pi} \frac{1}{k(Z)}, \]
and hence
\[ \partial^2_j H^A = -\sum_{n=1}^{\infty} g_{an} \partial_Z \psi_{2n}(Z) Y_{2n}(r). \]

Then, we can perform the integral as
\[ \int d^3x \partial^2_j H^A = \int_0^{\infty} dr \sum_{n=1}^{\infty} g_{an} \partial_Z \psi_{2n}(Z) e^{-\sqrt{\lambda_{2n}} r} = \frac{2}{\pi} \frac{1}{k(Z)}, \]
using (3.60). From this, it is easy to rederive (3.51).

3.6. Goldberger-Treiman relation

The axial coupling obtained in the previous subsection is related to the \( \pi NN \) coupling \( g_{\pi NN} \) by the Goldberger-Treiman relation
\[ g_A = \frac{f_\pi g_{\pi NN}}{M_N}. \]

We can derive this relation in our context following the argument given in Ref. [14] for the Skyrme model.

It is argued in Ref. [14] that the pion field behaves asymptotically as
\[ \langle \Pi^a(x) \rangle \simeq -\frac{g_{\pi NN}}{8\pi M_N} \frac{x^i}{r^3} \langle \sigma^i r^a \rangle \]
in the presence of a nucleon. Here, \( g_{\pi NN} \) is the \( \pi NN \) coupling and the expectation value is taken for the nucleon. In the present case, (2.85) provides us with information on the asymptotic pion field. The pion field can be read from the \( n = 0 \) component of (2.7) substituted in (2.85). We find
\[ A^G_z \simeq \Pi(x) \phi_0(z) + \cdots \]
with
\[ \Pi(x) = \frac{\sqrt{\kappa \pi}}{2} \alpha^i \alpha^{-1} \frac{p^2}{k(Z)} \frac{x^i}{r^3}, \]
which implies

\[ \langle \Pi^a(x) \rangle \simeq -\frac{\sqrt{\kappa \pi}}{3} \left\langle \frac{\rho^2}{k(Z)} \right\rangle \frac{x^i}{r^3} \langle \sigma^i x^a \rangle . \]  

(3.67)

Comparing (3.64) and (3.67), we obtain

\[ \frac{g_{\pi NN}}{M_N} = \frac{8\pi \sqrt{\kappa \pi}}{3} \left\langle \frac{\rho^2}{k(Z)} \right\rangle = \frac{1}{2} \sqrt{\frac{\pi}{\kappa}} g_A = \frac{g_A}{f_\pi} , \]

(3.68)

where we have used (2.38) and (3.52). This is nothing but the Goldberger-Treiman relation (3.63).

3.7. Axial radius

We define \( \rho_A(r) \) as a function proportional to the expectation value of the integrand of (3.62) with the normalization

\[ \int_0^\infty dr \rho_A(r) = 1 , \]

(3.69)

The axial radius is obtained as

\[ \langle r^2 \rangle_A \equiv \int_0^\infty dr r^2 \rho_A(r) = \langle F_A(Z) \rangle / \langle k(Z)^{-1} \rangle , \]

(3.70)

where

\[ F_A(z) \equiv 3\pi \sum_{n=1}^\infty \frac{g_{a_n} \partial_z \psi_{2n}(z)}{\lambda_{2n}^A} . \]

(3.71)

From (2.36) and (3.59), this function satisfies the differential equation

\[ \partial_z (k(z)F_A(z)) = -3\pi h(z)\psi_0(z) . \]

(3.72)

Integrating this equation, we obtain

\[ k(z)F_A(z) = f_0 - 3\pi \int_0^z dz' h(z')\psi_0(z') , \]

(3.73)

where \( f_0 \) is a constant. In order to fix \( f_0 \), we use the identity

\[ \int dz F_A(z) = 0 , \]

(3.74)

which follows from (3.71). Substituting (3.73) into (3.74), we obtain

\[ f_0 = 3 \int_{-\infty}^\infty dz - \frac{1}{k(z)} \int_0^z dz' h(z')\psi_0(z') \simeq 7.82 . \]

(3.75)
Therefore, if we approximate the expectation value by its classical value, we obtain

$$\langle r^2 \rangle_A = \langle F_A(Z) \rangle / \langle k(Z)^{-1} \rangle \simeq F_A(0) = f_0 \simeq 7.82/M_{KK}^2 \simeq (0.582 \text{ fm})^2.$$ (3.76)

If we numerically evaluate the expectation value of $F_A(Z)$ using the wavefunction (2.22), we obtain

$$\langle r^2 \rangle_A^{1/2} \simeq 0.537 \text{ fm}.$$ (3.77)

The experimental value is $\langle r^2 \rangle_A^{1/2} |_{\text{exp}} \simeq 0.674 \text{ fm}$.\(^*\)

We can compute the axial radius for excited baryons in the same manner as for the other quantities evaluated before. The computation of the axial radius is independent of the profile of $R(\rho)$ in the wavefunction; thus, for the $N(1440)$ (Roper) with $(n_\rho, n_z) = (1, 0)$, it gives the same result as that for the proton/neutron (3.77). For the $N(1535)$ with $(n_\rho, n_z) = (0, 1)$, we obtain (see Appendix A.2 for details)

$$\langle r^2 \rangle_A^{1/2} \simeq 0.435 \text{ fm}.$$ (3.78)

This is smaller than the axial radius of the proton/neutron.

§ 4. Form factors

In this section, we compute the form factors of spin 1/2 baryons associated with the currents $J_{V,A}^\mu$ obtained in the previous section. To this end, we first give a brief review of how to compute the form factors from the matrix elements of the currents. The present model enables us to compute the matrix elements easily with the tools formulated in §2.

4.1. Formalism

Let us first consider the matrix elements of a vector current for a baryon of spin 1/2,

$$\langle \vec{p}', B', s' | J_{V}^\mu (0) | \vec{p}, B, s \rangle. \quad (4.1)$$

Here $|\vec{p}, B, s\rangle$ and $|\vec{p}', B', s'\rangle$ denote the initial and final states of the baryon under consideration. In the present section, we focus on the case where $B$ and $B'$ have the same $n_\rho$ and $n_z$ while the isospins $I_3$ and $I'_3$ may be different. The states are normalized as in (2.21). The most general form of the matrix elements consistent with the symmetries and conservation of the current is

$$\langle \vec{p}', B', s' | J_{V}^\mu (0) | \vec{p}, B, s \rangle = i (2\pi)^{-3} \frac{g_{\rho} \mu I_3}{2} \bar{u}(\vec{p'}, s') \Gamma_{(C)}^\mu(p', p) u(\vec{p}, s).$$ (4.2)

\(^*\) This value is obtained by applying the formula $\langle r^2 \rangle_A = -6d/(dk^2) \log g_A(k^2)|_{k^2=0}$ to the axial form factor $g_A(k^2)$ in Ref. [36], which is fitted by a dipole.
where \( k = p - p' \) and \( m_B \) is the baryon mass. On the right-hand side of (4.2), no summation is taken for the index \( C \). \( F_1(k^2) \) and \( F_2(k^2) \) are the scalar functions of \( k^2 \) called the Dirac and Pauli form factors, respectively, whose dependence on \( n_s \) and \( n_z \) is indicated implicitly. They can be computed by evaluating the matrix elements of the current. \( u(\vec{p}, s) \) and \( u(\vec{p'}, s') \) are the Dirac spinors associated with the initial and final states of the baryon of mass \( m_B \), respectively. The normalization condition\(^4) \) is given by

\[
\overline{u}(\vec{p'}, s')u(\vec{p}, s) = \delta_{ss'} \frac{m_B}{\vec{p}'}.
\]

It is useful to write the matrix elements in the Breit frame with \( \vec{p} = -\vec{p}' = \vec{k}/2 \) and \( E = E' = \sqrt{m_B^2 + \vec{k}^2/4} \):

\[
\begin{align*}
\langle -\vec{k}/2, B', s'| \bar{J}_V^0(0) | \vec{k}/2, B, s \rangle &= (2\pi)^{-3} \frac{\delta_{\mu\nu}}{2} \frac{m_B}{E} \hat{G}_E(k^2) , \\
\langle -\vec{k}/2, B', s'| \bar{J}_V^1(0) | \vec{k}/2, B, s \rangle &= (2\pi)^{-3} \frac{\delta_{\mu\nu}}{2} \frac{im}{2E} \epsilon_{jla} k_i (\sigma^a)_{s's} \hat{G}_M(k^2) , \\
\langle -\vec{k}/2, B', s'| \bar{J}_V^2(0) | \vec{k}/2, B, s \rangle &= (2\pi)^{-3} \frac{(\tau^c)_{\mu\nu} \delta_{s's}}{2} \frac{m_B}{E} \hat{G}_E(k^2) , \\
\langle -\vec{k}/2, B', s'| \bar{J}_V^3(0) | \vec{k}/2, B, s \rangle &= (2\pi)^{-3} \frac{(\tau^c)_{\mu\nu} \delta_{s's}}{2} \frac{im}{2E} \epsilon_{jla} k_i (\sigma^a)_{s's} \hat{G}_M(k^2) .
\end{align*}
\]

Here \( G_{E,M}(k^2) \) are the Sachs form factors, related to the Dirac and Pauli form factors by

\[
\begin{align*}
\hat{G}_E(k^2) &= \hat{F}_1(k^2) - \frac{\vec{k}^2}{4m_B^2} \hat{F}_2(k^2) , & \hat{G}_M(k^2) &= \hat{F}_1(k^2) + \hat{F}_2(k^2) , \\
G_E(k^2) &= F_1(k^2) - \frac{\vec{k}^2}{4m_B^2} F_2(k^2) , & G_M(k^2) &= F_1(k^2) + F_2(k^2) .
\end{align*}
\]

The formulae of the Dirac spinor needed for this manipulation are summarized in Appendix B.2. The Sachs form factors can be obtained by evaluating the left-hand side of (4.6) using the baryon wavefunctions given in §2.1 and Appendix A.1.

\(^4)\) With this normalization, we assign \( \sqrt{2\vec{p}} u(\vec{p}, s) \) and \( \sqrt{2\vec{p}'} \overline{u}(\vec{p}, s) \) to an incoming and outgoing external line, respectively, in the computation of the Lorentz invariant matrix elements.
The electromagnetic form factors are defined by considering the matrix elements of the electromagnetic current,

\[ J_{em}^{\mu} = J_{V}^{a=3,\mu} + \frac{1}{N_c} \hat{J}^{\mu} . \]  

(4.8)

Then, for the states with \( I_3 = +1/2 \) (p) and \( I_3 = -1/2 \) (n), the Sachs form factors associated with the electromagnetic current are given by

\[ G_{E,M}^{p}(k^2) = \frac{1}{2} \left( +G_{E,M}(k^2) + \frac{1}{N_c} \hat{G}_{E,M}(k^2) \right) , \quad (\text{for } I_3 = +1/2) \]

\[ G_{E,M}^{n}(k^2) = \frac{1}{2} \left( -G_{E,M}(k^2) + \frac{1}{N_c} \hat{G}_{E,M}(k^2) \right) , \quad (\text{for } I_3 = -1/2) \]  

(4.9)

respectively. \( G_{E}^{p,n} \) and \( G_{M}^{p,n} \) are called the electric and magnetic Sachs form factors, and their Fourier transformation provides the distribution of the electric charge density and the magnetic current density, respectively. (See Refs. 37–39 for reviews.)

Next we study the axial form factor associated with the axial current \( J_{A}^{\mu}(x) \). As in the vector current case, we consider the matrix elements of the axial current for a spin 1/2 baryon,

\[ \langle \vec{p}', B', s'| J_{A}^{\mu}(0)| \vec{p}, B, s \rangle . \]  

(4.10)

The matrix elements consistent with the symmetries can be written in terms of the axial form factor \( g_{A}(k^2) \) and the induced pseudoscalar form factor \( g_{P}(k^2) \) as

\[ \langle \vec{p}', B', s'| J_{A}^{\mu}(0)| \vec{p}, B, s \rangle = (2\pi)^{-3} \left( \frac{\tau^{C}}{2} \right) \mu_{A}^{(0)}(p', p) \mu(p, s) \]  

(4.11)

with

\[ \mu(p', s') \mu_{A}^{(0)}(p', p) \mu(p, s) = \mu(p', s') \left[ i \gamma_{5} \gamma^{\mu} \hat{g}_{A}(k^2) + \frac{1}{2m_{B}} k^{\mu} \gamma_{5} \hat{g}_{P}(k^2) \right] u(p, s) , \]  

(4.12)

\[ \mu(p', s') \mu_{A}^{(1,2,3)}(p', p) \mu(p, s) = \mu(p', s') \left[ i \gamma_{5} \gamma^{\mu} g_{A}(k^2) + \frac{1}{2m_{B}} k^{\mu} \gamma_{5} g_{P}(k^2) \right] u(p, s) . \]  

(4.13)

On the right-hand side of (4.11), no summation is taken for the index \( C \). The current conservation law yields

\[ \hat{g}_{P}(k^2) = \frac{4m_{B}^{2}}{k^2} \hat{g}_{A}(k^2) , \quad g_{P}(k^2) = \frac{4m_{B}^{2}}{k^2} g_{A}(k^2) . \]  

(4.14)

In the nonrelativistic limit, the spatial component becomes

\[ \langle \vec{p}', B', s'| J_{A}^{(0)}(0)| \vec{p}, B, s \rangle \simeq - (2\pi)^{-3} \left( \sigma^{a} \right)_{s's} \delta_{I_{A}} \frac{\delta_{I_{A}}}{2} \left( \sigma_{ja} - \frac{k_{j} k_{a}}{k^2} \right) \hat{g}_{A}(k^2) , \]

\[ \langle \vec{p}', B', s'| J_{A}^{(0)}(0)| \vec{p}, B, s \rangle \simeq - (2\pi)^{-3} \left( \sigma^{a} \right)_{s's} \frac{\tau^{C}}{2} \left( \sigma_{ja} - \frac{k_{j} k_{a}}{k^2} \right) g_{A}(k^2) . \]  

(4.15)
Again, the axial form factor $g_A(k^2)$ can be computed by evaluating the matrix elements in (4.13) from the baryon wavefunctions given in [2.1] and Appendix A.1.

In order to derive the form factors from the above formalism, it is useful to perform the Fourier transformation of the currents defined by

$$\tilde{J}_V^\mu(\vec{k}) = \int d^3x \ e^{-i\vec{k}\cdot\vec{x}} J_\mu(x).$$  \hspace{1cm} (4.16)

Using the explicit form of the current (2.96), (2.97), (2.98), and (2.99), it is not difficult to show that

$$\tilde{J}_V^0(\vec{k}) = e^{-i\vec{k}\cdot\vec{x}} \frac{N_c}{2} \sum_{n \geq 1} \frac{g_{\tau\nu} \psi_{2n-1}(Z)}{k^2 + \lambda_{2n-1}};$$  \hspace{1cm} (4.17)

$$\tilde{J}_V^\lambda(\vec{k}) = e^{-i\vec{k}\cdot\vec{x}} \frac{N_c}{2} \left( \frac{P^\lambda_X - k_\lambda / 2}{M_0} + \frac{i}{16\pi^2\kappa} \epsilon_{\lambda jla} k_l J_a \right) \sum_{n \geq 1} \frac{g_{\tau\nu} \psi_{2n-1}(Z)}{k^2 + \lambda_{2n-1}} + \cdots;$$  \hspace{1cm} (4.18)

$$\tilde{J}_A^0(\vec{k}) = e^{-i\vec{k}\cdot\vec{x}} \left[ I_\tau - i \frac{2\pi^2 \kappa \rho^2}{32\pi^2\kappa} \sum_{n \geq 1} \frac{g_{\tau\nu} \psi_{2n-1}(Z)}{k^2 + \lambda_{2n-1}} + \cdots \right];$$  \hspace{1cm} (4.19)

$$\tilde{J}_A^\lambda(\vec{k}) = e^{-i\vec{k}\cdot\vec{x}} \left[ -i \frac{2\pi^2 \kappa \rho^2}{32\pi^2\kappa} \epsilon_{\lambda jla} k_l \right] \sum_{n \geq 1} \frac{g_{\tau\nu} \psi_{2n-1}(Z)}{k^2 + \lambda_{2n-1}} + \cdots;$$  \hspace{1cm} (4.20)

where we have used (2.18), (2.17), and (3.21). Here ‘...’ denotes the terms that are odd with respect to $Z$, they do not contribute to the result for the matrix elements. Useful formulas here are

$$\int d^3x \ e^{-i\vec{k}\cdot\vec{x}} Y_n(|\vec{x} - \vec{X}|) = -e^{-i\vec{k}\cdot\vec{x}} \frac{1}{k^2 + \lambda};$$ \hspace{1cm} (4.23)

$$\int d^3x \ e^{-i\vec{k}\cdot\vec{x}} H^A(Z, |\vec{x} - \vec{X}|) = -e^{-i\vec{k}\cdot\vec{x}} \sum_{n=1}^{\infty} \frac{g_{\alpha\beta} \partial_Z \psi_{2n}(Z)}{k^2 + \lambda_{2n}}.$$  \hspace{1cm} (4.24)

The latter can be shown by using (3.60). In addition, it is important to note that the operator ordering in the Fourier transformation of the currents is fixed uniquely by the requirement

$$\left( \tilde{J}_{V,A}^\mu(\vec{k}) \right)^\dagger = \tilde{J}_{V,A}^\mu(-\vec{k}).$$  \hspace{1cm} (4.25)

* More precisely, $\tilde{J}_V^\lambda$ includes a term that is even in $Z$ and proportional to $\hat{Z}$. We discard this term, since it is negligible for large $N_c$ and $\lambda$. 

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which is equivalent to the Hermiticity of the current in the $x$-representation.

Using these expressions for the Fourier transformed currents together with the relation
\[ \langle \tilde{p}' e^{-i\tilde{k} \cdot \tilde{X}} | \tilde{p} \rangle = \delta^3 (\tilde{k} - \tilde{p} + \tilde{p}') , \]
the matrix elements can be calculated as
\[ \langle \tilde{p}', B', s' | \mathcal{J}^\mu (0) | \tilde{p}, B, s \rangle = \int \frac{d^3 k}{(2\pi)^3} \langle \tilde{p}', B', s' | \tilde{\mathcal{J}}^\mu (\tilde{k}) | \tilde{p}, B, s \rangle . \] (4·26)

4.2. Form factors

Comparing (4·17)–(4·20) with (4·6), we find
\[ \hat{G}_E (k^2) = N_c \sum_{n \geq 1} \frac{g_{I=0} \langle \psi_{2n-1}(Z) \rangle}{k^2 + \lambda_{2n-1}} , \quad \hat{G}_M (k^2) = N_c \frac{g_{I=1}}{2} \sum_{n \geq 1} \frac{g_{I=0} \langle \psi_{2n-1}(Z) \rangle}{k^2 + \lambda_{2n-1}} , \] (4·27)
\[ G_E (k^2) = \sum_{n \geq 1} \frac{g_{I=0} \langle \psi_{2n-1}(Z) \rangle}{k^2 + \lambda_{2n-1}} , \quad G_M (k^2) = \frac{g_{I=1}}{2} \sum_{n \geq 1} \frac{g_{I=0} \langle \psi_{2n-1}(Z) \rangle}{k^2 + \lambda_{2n-1}} , \] (4·28)

with
\[ g_{I=0} = \frac{m_B}{M_0} , \quad g_{I=1} = \frac{32\pi^2 \kappa m_B}{3} \langle \rho^2 \rangle \] (4·29)
being the isoscalar and isovector $g$ factors of the baryon $B$, as derived in §§3.2 and 3.4, respectively. Here we have used the formula (3·29). We note also
\[ \hat{G}_E (0) = N_c , \quad G_E (0) = 1 . \] (4·30)

Using (4·19), the electric and magnetic Sachs form factors for nucleons and their excited states with $I = J = 1/2$ are obtained as
\[ G_E^p (k^2) = \sum_{n \geq 1} \frac{g_{I=0} \langle \psi_{2n-1}(Z) \rangle}{k^2 + \lambda_{2n-1}} , \quad G_E^n (k^2) = 0 , \]
\[ G_M^p (k^2) = \frac{g_p}{2} \sum_{n \geq 1} \frac{g_{I=0} \langle \psi_{2n-1}(Z) \rangle}{k^2 + \lambda_{2n-1}} , \quad G_M^n (k^2) = \frac{g_n}{2} \sum_{n \geq 1} \frac{g_{I=0} \langle \psi_{2n-1}(Z) \rangle}{k^2 + \lambda_{2n-1}} , \] (4·31)

where
\[ g_p = \frac{1}{2} (g_{I=0} + g_{I=1}) , \quad g_n = \frac{1}{2} (g_{I=0} - g_{I=1}) \] (4·32)
are the $g$ factors of the nucleons (and their excitations). It follows that they satisfy the relation
\[ \frac{G_E^p (k^2)}{g_p} = \frac{2}{g_p} \frac{G_M^p (k^2)}{G_M^n (k^2)} = \frac{2}{g_n} \frac{G_M^n (k^2)}{G_M^p (k^2)} = \sum_{n \geq 1} \frac{g_{I=0} \langle \psi_{2n-1}(Z) \rangle}{k^2 + \lambda_{2n-1}} , \quad G_E^n (k^2) = 0 . \] (4·33)
Experimentally, the Sachs form factors for the proton and neutron are known to be well described as

\[ G_p^E(\vec{k}^2) = \frac{G_{pM}(\vec{k}^2)}{\mu_p} = \frac{G_{nM}(\vec{k}^2)}{\mu_n} = \left(1 + \frac{\vec{k}^2}{\Lambda^2}\right)^{-2}, \quad G_n^E(\vec{k}^2) = 0, \quad (4.34) \]

with \( \Lambda^2 = 0.71 \text{ GeV}^2 \). That is, the three form factors \( G_p^E, G_{pM}, \) and \( G_{nM} \) are proportional to each other and characterized by the dipole behavior. Furthermore, the electric charge density of the neutron can be well approximated to be flat. It turns out that our result (4.33) is in accord with these experimental results. In particular, the infinite sum in (4.33) can be approximated by a single dipole factor, showing the agreement as a function of \( \vec{k}^2 \).

To see this, we expand our result (4.33) as a Taylor series in \( \vec{k}^2 \),

\[ \sum_{n \geq 1} \frac{g_{\psi n}(\psi_{2n-1}(Z))}{\vec{k}^2 + \lambda_{2n-1}} = \langle f_0(Z) \rangle - \langle f_1(Z) \rangle \vec{k}^2 + \langle f_2(Z) \rangle (\vec{k}^2)^2 - \langle f_3(Z) \rangle (\vec{k}^2)^3 + \cdots, \quad (4.35) \]

where the coefficients are obtained as the expectation values of

\[ f_k(Z) \equiv \sum_{n \geq 1} \frac{g_{\psi n}\psi_{2n-1}(Z)}{(\lambda_{2n-1})^{k+1}}. \quad (4.36) \]

As seen in (3.3), we have \( \langle f_0(Z) \rangle = 1 \). Let us evaluate these coefficients using the classical approximation \( \langle \psi_{2n-1}(Z) \rangle \simeq \psi_{2n-1}(0) \). The first nontrivial coefficient, \( f_1(0) \), is in fact equal to \( F_0/6 \) given in (3.10). The higher coefficients can be obtained in the same way as \( F_0 \), as they satisfy the recursive relation

\[ -\partial_z(k(z)\partial_z f_k(z)) = h(z)f_{k-1}(z), \quad f_k(z) = f_k(-z), \quad f_k(\pm \infty) = 0. \quad (4.37) \]

We thus obtain

\[ f_1(0) = 2.38, \quad f_2(0) = 4.02, \quad f_3(0) = 6.20, \quad f_4(0) = 9.35, \quad f_5(0) = 14.0, \cdots. \quad (4.38) \]

On the other hand, the dipole expression (4.34) can be expanded using \( \Lambda^2 = 0.758 \text{ GeV}^2 \) as

\[ \left(1 + \frac{\vec{k}^2}{\Lambda^2}\right)^{-2} = 1 - 2.38\vec{k}^2 + 4.24(\vec{k}^2)^2 - 6.71(\vec{k}^2)^3 + 9.97(\vec{k}^2)^4 - 14.2(\vec{k}^2)^5 + \cdots, \quad (4.39) \]

where we have chosen the parameter \( \Lambda^2 \) in such a way that the first coefficient reproduces the \( f_1(0) \) of (4.38) with the unit \( M_{KK} = 949 \text{ MeV} = 1 \). We find a good agreement for the latter coefficients, suggesting that our form factors (4.33) exhibit the dipole behavior indicated by experiments. It is also useful to note that the relation (4.37) implies that the function

\[ F(\vec{k}^2, z) \equiv \sum_{n \geq 1} \frac{g_{\psi n}\psi_{2n-1}(z)}{\vec{k}^2 + \lambda_{2n-1}} \quad (4.40) \]
satisfies
\[ \partial_z (k(z) \partial_z F(\vec{k}^2, z)) = \vec{k}^2 h(z) F(\vec{k}^2, z) , \quad F(\vec{k}^2, z) = F(\vec{k}^2, -z) , \quad F(\vec{k}^2, \pm \infty) = 1 . \tag{4.41} \]

The solution of this equation evaluated at \( z = 0 \) is plotted in Fig. 1.

Fig. 1. A plot of \( F(\vec{k}^2, 0) \), which is equal to \( G_p^E(\vec{k}) \), with the classical value \( \psi_{2n-1}(0) \). Our result (solid line) reproduces the dipole behavior (dotted line) in (4.39) with \( \Lambda^2 = 0.758 \text{ GeV}^2 \).

The electric and magnetic charge radii can be computed from the first coefficient of the form factor expanded in powers of \( \vec{k}^2 \). Namely, they are given by
\[ \langle r^2 \rangle_{E,M} = -6 \frac{d}{d\vec{k}^2} \log G_{E,M}(\vec{k}^2) \bigg|_{\vec{k}^2=0} , \tag{4.42} \]
except for the neutron charge radius, which is defined by
\[ \langle r^2 \rangle_{E,n} = -6 \frac{d}{d\vec{k}^2} G_n^E(\vec{k}^2) \bigg|_{\vec{k}^2=0} . \tag{4.43} \]

Since all the form factors are proportional to each other (except for \( G_n^E \), which vanishes) as in (4.33), we conclude that
\[ \langle r^2 \rangle_{M,p} = \langle r^2 \rangle_{M,n} = \langle r^2 \rangle_{E,p} , \quad \langle r^2 \rangle_{E,n} = 0 . \tag{4.44} \]

It is easy to check that the definition of the charge radii in (4.42) is consistent with our previous calculation in (3.12) and using our result for the electric charge radius of the proton (3.12), we obtain
\[ \langle r^2 \rangle_{E,p}^{1/2} = \langle r^2 \rangle_{M,p}^{1/2} = \langle r^2 \rangle_{M,n}^{1/2} \approx 0.742 \text{ fm} . \tag{4.45} \]

The values observed in experiments are \( \langle r^2 \rangle_{E,p}^{1/2} |_{\text{exp}} \approx 0.875 \text{ fm} \) \( \langle r^2 \rangle_{M,p}^{1/2} |_{\text{exp}} \approx 0.855 \text{ fm} \) and \( \langle r^2 \rangle_{M,n}^{1/2} |_{\text{exp}} \approx 0.873 \text{ fm} \), which are reasonably close to our result.

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The axial form factors are given by
\[ \hat{g}_A(k^2) = \frac{N_c}{32\pi^2} \sum_{n \geq 1} g_{2n} \left( \frac{\partial Z \psi_{2n}(Z)}{k^2 + \lambda_2} \right), \quad g_A(k^2) = \frac{8\pi^2 \kappa}{3} \sum_{n \geq 1} g_{2n} \left( \frac{\partial Z \psi_{2n}(Z)}{k^2 + \lambda_2} \right). \] (4.46)

Note that the values at \( k^2 = 0 \) are obtained by using (3.60) as
\[ \hat{g}_A(0) = \frac{N_c}{16\pi^2} \left( \frac{1}{k(Z)} \right), \quad g_A(0) = \frac{16\pi \kappa}{3} \left( \frac{\rho^2}{k(Z)} \right). \] (4.47)

The latter reproduces (3.52).

It is empirically known that the axial form factor \( g_A(k^2) \) can also be well fitted by a dipole profile. Using the same technique as above, the Taylor expanded axial form factor with the classical approximation \( \langle \partial Z \psi_{2n}(Z) \rangle \approx \partial_z \psi_{2n}(0) \) is found to take the form
\[ \frac{g_A(k^2)}{g_A(0)} \approx 1 - 1.30k^2 + 1.09(k^2)^2 - 0.770(k^2)^3 + 0.511(k^2)^4 - 0.331(k^2)^5 + \cdots. \] (4.48)

These coefficients are close to those obtained from the dipole profile
\[ \left( 1 + \frac{k^2}{M_A^2} \right)^{-2} = 1 - 1.13k^2 + 0.958(k^2)^2 - 0.721(k^2)^3 + 0.510(k^2)^4 - 0.345(k^2)^5 + \cdots, \] (4.49)
with \( M_A \approx 1.26 \text{ GeV} \).

4.3. Cubic coupling

The form factors computed above are composed of an infinite tower of poles that correspond to the vector and axial-vector meson exchange. From the residues, we can extract information on cubic couplings among baryons and (axial-)vector mesons.

We assume that there exist cubic couplings of the form
\[ \mathcal{L}_\text{int}^v = \sum_{n \geq 1} \left( g_{v_{n}BB} \bar{v}_\mu^n \gamma_{\nu}^{\gamma} \frac{\tau^0}{2} B + g_{v_{n}BB} v_\mu^n \bar{v}_\mu^n B \right) + \frac{1}{4m_B} \sum_{n \geq 1} \left( h_{v_{n}BB} \left( \partial_\mu \bar{v}_\nu^n - \partial_\nu \bar{v}_\mu^n \right) \bar{B} \sigma^{\mu\nu} \frac{\tau^a}{2} B + h_{v_{n}BB} \left( \partial_\mu v_\nu^n - \partial_\nu v_\mu^n \right) \bar{B} \sigma^{\mu\nu} \frac{\tau^a}{2} B \right) \] (4.50)
in the four-dimensional baryon-meson effective action. Here \( \bar{v}_\mu^n(x) \) and \( v_\mu^n(x) \) are the \( U(1) \) and \( SU(2) \) parts of the \( n \)th vector meson \( v_\mu^n(x) \), respectively, and \( B(x) \) is the baryon field. Note that we do not include the direct interaction among baryons and the background gauge potential \( \mathcal{V}_\mu^{(+)} \), since the present model exhibits the complete vector meson dominance as argued in Ref. [6] and emphasized in Ref. [17] for the cases including baryons.
We recall that the vector and axial-vector mesons couple with the background gauge potentials as in (2.34). Using these interactions, the Dirac and Pauli form factors are computed to be

\[
\hat{F}_1(k^2) = \sum_{n \geq 1} \frac{g_{v^n} \hat{g}_{v^n BB}}{k^2 + \lambda_{2n-1}}, \quad \hat{F}_2(k^2) = \sum_{n \geq 1} \frac{g_{v^n} \hat{h}_{v^n BB}}{k^2 + \lambda_{2n-1}},
\]

and a similar relation holds for the \( SU(2) \) sector. On the other hand, in the present model, (4.27) and (4.28) together with (4.7) yield the Dirac and Pauli form factors of the form

\[
\hat{F}_1(k^2) = N_c \sum_{n \geq 1} \frac{g_{v^n} \langle \psi_{2n-1}(Z) \rangle}{k^2 + \lambda_{2n-1}}, \quad \hat{F}_2(k^2) = \frac{g_{l=0}}{2} \sum_{n \geq 1} \frac{g_{v^n} \langle \psi_{2n-1}(Z) \rangle}{k^2 + \lambda_{2n-1}},
\]

Here we have kept only the leading terms in the large \( N_c \) and large \( \lambda \) limit.

By comparing these results, we obtain

\[
\hat{g}_{v^n BB} = N_c \langle \psi_{2n-1}(Z) \rangle, \quad \hat{h}_{v^n BB} = N_c \left( \frac{g_{l=0}}{2} - 1 \right) \langle \psi_{2n-1}(Z) \rangle, \]

\[
g_{v^n BB} = \langle \psi_{2n-1}(Z) \rangle, \quad h_{v^n BB} = \frac{g_{l=0}}{2} \langle \psi_{2n-1}(Z) \rangle.
\]

It is interesting to note the relation

\[
\hat{g}_{v^n BB} = N_c g_{v^n BB}.
\]

In particular, for the case with \( n = 1 \) and \( B \) being a nucleon, this means the \( g_{\omega NN} = N_c g_{\rho NN} \), which coincides with the constituent quark model prediction. This relation is found also in Ref. [17].

The axial form factors \( \hat{g}_A \) and \( g_A \) can be obtained from the effective cubic couplings with pion and axial-vector mesons of the form

\[
\mathcal{L}^{a}_{\text{int}} = \sum_{n \geq 1} \left( \hat{g}_{v^n BB} \hat{a}_\mu^n \overline{B} \gamma_5 \gamma^\mu \frac{\tau^0}{2} B + g_{v^n BB} \hat{a}_n^{aa} \overline{B} \gamma_5 \gamma^\mu \frac{\tau^0}{2} B \right) + 2i \left( \hat{g}_{v^n BB} \hat{\pi} \overline{B} \gamma_5 \frac{\tau^0}{2} B + g_{v^n BB} \hat{\pi}^a \overline{B} \gamma_5 \frac{\tau^a}{2} B \right),
\]

where \( \hat{a}_n^\mu(x) \) and \( a_n^{\mu a}(x) \) are the \( U(1) \) and \( SU(2) \) parts of the \( n \)th axial vector meson \( a_\mu^n(x) \), respectively, and \( \hat{\pi}(x) \) and \( \pi^a(x) \) are the \( U(1) \) and \( SU(2) \) parts of the pion field \( \Pi(x) \),

\(^{*)}\) Since the leading contribution to the baryon mass \( m_B \) is \( M_0 = 8\pi^2\kappa \), we consider \( m_B \) to be of order \( \lambda N_c \). However, we will not use the relation \( m_B \approx M_0 \), since we know that the subleading contributions in \( m_B \) are not small as discussed in Ref. [16].

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respective. As in (4.50), no direct coupling between the baryons and $V_\mu$ is assumed. It follows from this and (2.34) that the invariant amplitude with the incoming and outgoing baryons in the presence of the external gauge field $V_\mu$ is given by

$$\sqrt{2p^0} \sqrt{2p'^0} \pi(p', s') \frac{\delta_F(s)}{2} \left[ i\gamma_5 \gamma_\mu \sum_{n \geq 1} g_{an} \hat{g}_{an} BB \frac{k^2}{k^2 + \lambda_{2n}} \right. + k_\mu \gamma_5 \left( 2 f_\pi \hat{g}_{n\pi BB} \frac{1}{k^2} - 2 m_B \sum_{n \geq 1} g_{an} \hat{g}_{an} BB \frac{1}{k^2 + \lambda_{2n}} \right) ] u(p', s)$$

$$+ (SU(2) \text{ part}) . \quad (4.56)$$

By comparing this with the matrix elements (4.11)–(4.13), we obtain $\hat{g}_A, \hat{h}_A, g_A,$ and $h_A$ as functions of the cubic coupling constants in (4.55):

$$\hat{g}_A(k^2) = \sum_{n \geq 1} g_{an} \hat{g}_{an} BB \frac{k^2}{k^2 + \lambda_{2n}}, \quad g_A(k^2) = \sum_{n \geq 1} g_{an} \hat{g}_{an} BB \frac{k^2}{k^2 + \lambda_{2n}}, \quad (4.57)$$

$$\hat{g}_P(k^2) = 2 m_B \frac{2 f_\pi \hat{g}_{\pi BB}}{k^2} - 4 m_B^2 \sum_{n \geq 1} g_{an} \hat{g}_{an} BB \frac{1}{k^2 + \lambda_{2n}},$$

$$g_P(k^2) = 2 m_B \frac{2 f_\pi g_{\pi BB}}{k^2} - 4 m_B^2 \sum_{n \geq 1} g_{an} \hat{g}_{an} BB \frac{1}{k^2 + \lambda_{2n}} . \quad (4.58)$$

Equating these results with (4.46) then using (4.14) and (3.60) leads to the relations

$$\hat{g}_{\alpha \nu BB} = \frac{N_c}{32\pi^2 \kappa} \langle \partial Z \psi_{\alpha n}(Z) \rangle, \quad g_{\alpha \nu BB} = \frac{8\pi^2 \kappa}{3} \langle \rho^2 \rangle \langle \partial Z \psi_{\alpha n}(Z) \rangle \quad (4.59)$$

$$\hat{g}_{\pi BB} = \frac{m_B}{f_\pi} \frac{N_c}{16\pi^3 \kappa} \left( \frac{1}{k(Z)} \right), \quad g_{\pi BB} = \frac{m_B}{f_\pi} \frac{16\pi \kappa}{3} \left( \frac{\rho^2}{k(Z)} \right) . \quad (4.60)$$

Comparing (4.60) with (4.47), we note that the following relations hold:

$$\hat{g}_A(0) = \frac{f_\pi \hat{g}_{\pi BB}}{m_B}, \quad g_A(0) = \frac{f_\pi g_{\pi BB}}{m_B}, \quad (4.61)$$

i.e., the Goldberger-Treiman relation.

4.4. Numerical estimate

By solving (2.36) numerically using the shooting method, the expectation values of $\psi_{2n-1}(Z)$ and $\partial Z \psi_{2n}(Z)$ with respect to the wavefunction $\psi(Z)$ (see Appendix A.1 for

*) The propagator of $a_\mu^n$ is given by that of a Proca field: $\frac{1}{k^2 + \lambda_{2n}}(\eta_{\mu\nu} + k_\mu k_\nu/\lambda_{2n}).$
details) can be estimated as

\( n \) | \( \lambda_n \) | \( \langle \psi_n(Z) \rangle_{n_z=0} \) | \( \langle \psi_n(Z) \rangle_{n_z=1} \) | \( \langle \partial_Z \psi_n(Z) \rangle_{n_z=0} \) | \( \langle \partial_Z \psi_n(Z) \rangle_{n_z=1} \) 
---|---|---|---|---|---
1 | 0.669 | 5.80 | 4.51 | 0 | 0 
2 | 1.57 | 0 | 0 | 3.46 | 0.618 
3 | 2.87 | -2.70 | 0.766 | 0 | 0 
4 | 4.54 | 0 | 0 | -3.08 | 2.22

(4.62)

As an example, consider the nucleon with \( n_\rho = n_z = 0 \). Using this table, we obtain

\[
\begin{array}{c|c|c|c|c|c}
 n & \hat{g}_{v^*NN} & g_{v^*NN} & \hat{g}_{a^*NN} & g_{a^*NN} \\
---&---&---&---&---
1 & 17.4 & 5.80 & 4.42 & 6.14 \\
2 & -8.10 & -2.70 & -3.84 & -5.46 \\
\end{array}
\]

(4.63)

In particular, for the \( \rho \) meson we thus obtain

\[ g_{\rho NN} = g_{v^1NN} \simeq 5.80 \, . \] (4.64)

This is consistent with the experimental data \( g_{\rho NN|\text{exp}} = 4.2 - 6.5 \) (see also Ref. 17). Note that the functional form of \( (4.53) \) shows the universality of the \( \rho \) meson couplings among the spin 1/2 baryons with any \( n_\rho \) and \( n_z = 0 \). Moreover, for large \( N_c \) and large \( \lambda \), \( g_{v^1BB} \) tends to take a common value for any baryon state \( B \) with \( I = J = 1/2 \), because the corresponding wavefunction has a narrow support with a width of \( O(\lambda^{-1/2}N_c^{-1/2}) \). For reference, \( g_{\rho NN(1535)N(1535)} \), the cubic coupling of \( \rho \) with \( N(1535) \), can be numerically computed using the quantum wavefunction for \( n_z = 1 \) as

\[ g_{\rho N(1535)N(1535)} = \langle \psi_1(Z) \rangle_{n_z=1} \simeq 4.51 \, . \] (4.65)

To see if the \( \rho \) meson universality holds extensively in the meson sector, we list the meson cubic couplings \( g_{\rho \pi \pi} \), \( g_{\rho v^*v^*} \), and \( g_{\rho a^*a^*} \) computed in Ref. 6):

\[
\begin{array}{c|c|c}
 n & g_{\rho v^*v^*} & g_{\rho a^*a^*} \\
---&---&---
1 & 5.19 & 3.32 \\
2 & 3.12 & 2.98 \\
3 & 2.93 & 2.89 \\
4 & 2.87 & 2.85 \\
\end{array}
\]

(4.66)

It seems that the relation \( g_{\rho \pi \pi} \sim g_{\rho \rho \rho} \sim g_{\rho NN} \) roughly holds within 20% error, although \( g_{\rho v^*v^*} \) with \( n > 1 \) and \( g_{\rho a^*a^*} \) are not close enough to ensure the universality.

\* Here we show values only up to \( n = 4 \), as the quantum treatment of the values seems to break down for higher \( n \) because its deviation from the classical value becomes significant.
If we use \( m_B = 940 \text{ MeV} \) as an input, together with (2.3), the Yukawa couplings of the pion and the nucleons given in (4.60) are evaluated as

\[
\hat{g}_{\pi NN} \approx 5.37, \quad g_{\pi NN} \approx 7.46,
\]

while the experimental value is \( g_{\pi NN}|_{\text{exp}} \approx 13.2 \). The smallness of the predicted value is related to the similar observation for the axial coupling \( g_A \) analyzed in §3.5 through the Goldberger-Treiman relation (4.61). In the same manner, the Yukawa couplings involving \( N(1440) \) and \( N(1535) \) are easy to evaluate:

\[
\hat{g}_{\pi N(1440)N(1440)} \approx 8.23, \quad g_{\pi N(1440)N(1440)} \approx 16.7,
\]

\[
\hat{g}_{\pi N(1535)N(1535)} \approx 4.55, \quad g_{\pi N(1535)N(1535)} \approx 6.32.
\]

Here we have used (A-13), (A-14), (A-16), and (A-17)

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§5. Summary and discussion

In this paper, using the model of holographic QCD proposed in Refs. 5) and 6), we calculated the static quantities of the nucleon and excited baryons such as \( N(1440) \), \( N(1535) \), and \( \Delta \), and the form factors of the spin 1/2 baryons. The baryons are described as quantized instantons in five-dimensional YM-CS theory. By defining the chiral currents properly at the spatial infinity in the fifth dimension and by solving the YM-CS equations of motion using the instanton profile given in Ref. 16) and Green's functions, we obtain an explicit expression for the chiral currents depending on the baryon state. From the currents we computed various static quantities of the proton/neutron, such as the charge radii, magnetic moments, axial coupling and axial radius. See the summary table below. The Goldberger-Treiman relation is naturally derived. These quantities can be computed for excited baryons in the same manner, which are our theoretical prediction for the excited baryons (see the second table below). We also calculated the nucleon form factors (and also those for excited baryons). It was shown that the electric and magnetic form factors of the nucleon are roughly consistent with the dipole behavior observed in experiments. The electric as well as the magnetic charge radii of the baryons and their couplings to mesons are calculated from the form factors.

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\( \hat{g}_{\pi NN} \) is related to the Yukawa coupling of the isoscalar mesons \( g_{\eta NN} \) and \( g_{\eta' NN} \). It is, however, difficult to tell which component it corresponds to, since we are analyzing massless QCD with \( N_f = 2 \).

**\( \hat{g}_{\pi NN} \) is a computation of the Yukawa couplings involving the excited baryons is also performed in Ref. 44), where a five-dimensional spinor field is incorporated in the gravity side in the bottom-up approach in holographic QCD.
A table summarizing our results for the static properties for the proton and neutron is given below. For a comparison, the table includes the values obtained in experiments, and also the results obtained from the Skyrmion.

|                           | our model | Skyrmion \cite{14} | experiment |
|---------------------------|-----------|---------------------|------------|
| $\langle r^2 \rangle_{I=0}^{1/2}$ | 0.742 fm  | 0.59 fm            | 0.806 fm   |
| $\langle r^2 \rangle_{M,I=0}^{1/2}$ | 0.742 fm  | 0.92 fm            | 0.814 fm   |
| $\langle r^2 \rangle_{E,p}$        | (0.742 fm)$^2$ | $\infty$           | (0.875 fm)$^2$ |
| $\langle r^2 \rangle_{E,n}$        | 0         | $-\infty$          | $-0.116$ fm$^2$ |
| $\langle r^2 \rangle_{M,p}$        | (0.742 fm)$^2$ | $\infty$           | (0.855 fm)$^2$ |
| $\langle r^2 \rangle_{M,n}$        | (0.742 fm)$^2$ | $\infty$           | (0.873 fm)$^2$ |
| $\langle r^2 \rangle_{A}^{1/2}$    | 0.537 fm  | $-$                | 0.674 fm   |
| $\mu_p$                      | 2.18      | 1.87               | 2.79       |
| $\mu_n$                      | $-1.34$   | $-1.31$            | $-1.91$    |
| $\frac{|\mu_p|}{|\mu_n|}$     | 1.63      | 1.43               | 1.46       |
| $g_A$                        | 0.734     | 0.61               | 1.27       |
| $g_{\pi NN}$                 | 7.46      | 8.9                | 13.2       |
| $g_{\rho NN}$                | 5.80      | $-$                | $4.2 \sim 6.5$ |

For excited baryons, $N(1440)$ (Roper) and $N(1535)$, we provide our theoretical predictions in the following table.
As shown in the first table, we found good agreement with experiments for various quantities of baryons. Our numerical results presented here should be treated with caution since they were obtained for large values of the ’t Hooft coupling and $N_c$. In order to incorporate the difference among excited baryon states, we included subleading corrections only at the last stage of the calculations. This procedure is difficult to justify since we considered the leading order action (2.1) as our starting point, and there should be more corrections. Further investigation to improve the accuracy of the results would be interesting.

In this paper, baryons have been described as a soliton, and the baryon physics has been analyzed by a standard semiclassical quantization of the soliton. This approach appears to be completely different from the treatment of Ref. [17], in which a key step is to introduce a five-dimensional spinor field into the five-dimensional YM-CS system to represent the baryons. In order to relate the two approaches, consider the $\hat{v}BB$ coupling, for simplicity, in the four-dimensional meson-baryon effective Lagrangian (4.50):

$$\mathcal{L}_\text{int}^v = \sum_{n \geq 1} \left( \hat{g}_{\nu^+BB} \hat{v}_\mu^n(x) \overline{B}(x) i\gamma^\mu \frac{r_0}{2} B(x) + \cdots \right). \quad (5.1)$$

If we substitute $\hat{g}_{\nu^+BB}$ from (4.53), this can be written as

$$\mathcal{L}_\text{int}^v = \frac{N_c}{2} \int dz \left( \hat{A}_\mu(x,z) \overline{B}(x,z) i\gamma^\mu \frac{r_0}{2} B(x,z) + \cdots \right). \quad (5.2)$$

Here $\hat{A}_\mu$ is the $U(1)$ part of the five-dimensional gauge potential in (2.35), and $\overline{B}(x,z) \equiv B(x) \psi_Z(z)$ is a five-dimensional spinor field constructed by the four-dimensional baryon field.
$B(x)$ and the suitably normalized wavefunction $\psi_Z(Z)$ corresponding to the baryon $B$. This term can be summarized as a five-dimensional gauge interaction by regarding $\psi_Z(Z)$ as an eigenmode of a wave equation that reproduces the baryon spectrum found in Ref. [16]. Likewise, the interaction terms given in (4.50) and (4.55) are sufficient to reconstruct the five-dimensional spinor field action with the Pauli-type interaction in Ref. [17].

We have concentrated on a one-point function of mesons and the electromagnetic field in the presence of a single quantized soliton, mainly to extract the static properties of baryons. There are other interesting aspects of the force associated with the baryons, for example, interactions between baryons, in particular. When baryons are far from each other, we can use our results and compute the nuclear force based on a one-meson-exchange picture of the nuclear force. Resolving the issue of the nuclear force at short distance is interesting and will be reported in our forthcoming paper.[19]

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Appendix A

Excited Baryons

In this appendix, first we summarize quantum wavefunctions for the lowest/excited states of baryons obtained in Ref. [16], and present detailed calculations relevant to the static quantities of the baryons presented in §3.

A.1. Wavefunctions for excited baryons

In Ref. [16], fluctuations around the classical solution (2.12) of the five-dimensional YM-CS theory in the curved background (2.1) are quantized. The quantization of the soliton was done in the approximation of slowly moving (pseudo)moduli of the soliton solution. The
moduli degrees of freedom of the soliton are

\[ X^i(t), \quad Z(t), \quad \rho(t), \quad a^I(t). \]  \tag{A.1}

\( X^i \) and \( Z \) describe the center-of-mass motion of the soliton, while \( \rho \) and \( a^I \) (\( I = 1, 2, 3, 4 \)) describe the size of the soliton, which is instanton-like in the \( SU(2) \) sector, and the orientation of the instanton in the \( SU(2) \) group space, respectively, with \( (a^I)^2 = 1 \). The Hamiltonian for the moduli provides the quantized energy eigenstates of the baryon, specified by the quantum number \( B = (l, I_3, n_\rho, n_z) \) and its spin \( s \). The wavefunctions for the quantized states can be written explicitly for low-lying states. The nucleon wavefunctions, \( B = (1, \pm 1/2, 0, 0) \) and \( s = 1/2 \), are written as \[ |p \uparrow \rangle \propto R(\rho)\psi_Z(Z)(a_1 + ia_2), \quad |n \uparrow \rangle \propto R(\rho)\psi_Z(Z)(a_4 + ia_3), \] \tag{A.2}

with \[ R(\rho) = \rho^{-1+2\sqrt{1+N_z^2/5}}e^{-\frac{M_0}{\sqrt{6}}\rho^2}, \quad \psi_Z(Z) = e^{-\frac{M_0}{\sqrt{6}}Z^2}. \] \tag{A.3}

Here we present the first excited states for which static quantities are computed in §3.\textsuperscript{13} The excited state with \( B = (1, \pm 1/2, 1, 0) \) corresponds to \( N(1440) \), which is called the Roper excitation. The wavefunction is \[ R(\rho) = \left(\frac{2M_0}{\sqrt{6}}\rho^2 - 1 - 2\sqrt{1+N_z^2/5}\right)\rho^{-1+2\sqrt{1+N_z^2/5}}e^{-\frac{M_0}{\sqrt{6}}\rho^2}. \] \tag{A.4}

The excited state with \( B = (1, \pm 1/2, 0, 1) \) is \( N(1535) \). The wavefunction for the \( Z \) part is now given as \[ \psi_Z(Z) = Ze^{-\frac{M_0}{\sqrt{6}}Z^2}, \] \tag{A.5}

where, again, its normalization constant is not fixed yet. The other part of the wavefunction is the same as that of the proton/neutron. This excitation has a negative parity, which is reflected in the oddness of the wavefunction \( \psi_Z(Z) \).

The excited states in the \( SU(2) \) group space have already been studied in the context of the Skyrmion\textsuperscript{14}. The lightest among these excited states is \( \Delta \) with \( I = J = 3/2 \). The isoquartet is composed of the four baryons \( \Delta^{++}, \Delta^+, \Delta^0, \) and \( \Delta^- \) with \( I_3 = 3/2, 1/2, -1/2, \) and \( -3/2 \), respectively. As shown in Ref.\textsuperscript{14}, \( \Delta^{++} \) with \( s = 3/2 \) is described by the wavefunction \[ (a_1 + ia_2)^3. \] \tag{A.6}
The rest of the $\Delta$ wavefunctions with $s = 3/2$ are obtained by letting the isospin lowering operator act on (A.6):

\[
(a_1 + ia_2)^2(a_4 + ia_3) , \quad (\text{for } \Delta^+ ) \\
(a_1 + ia_2)(a_4 + ia_3)^2 , \quad (\text{for } \Delta^0 ) \\
(a_4 + ia_3)^3 . \quad (\text{for } \Delta^- )
\] (A.7)

The wavefunction $R(\rho)$ for $\Delta$ (states with $l = 3$ and $n_\rho = 0$) is given as

\[
R(\rho) = \rho^{1+2\sqrt{1+N_c^2/5}} e^{-\frac{2\rho}{\sqrt{6}}}.
\] (A.8)

We use these wavefunctions to compute the static quantities of excited baryons.

A.2. **Expectation values relevant to $N(1440)$ and $N(1535)$**

First, we summarize what quantities are necessary for computing various static quantities presented in §3:

\[
\langle F(Z) \rangle , \quad \langle \rho^2 \rangle , \quad \langle \frac{\rho^2}{k(Z)} \rangle , \quad \langle F_A(Z) \rangle .
\] (A.9)

The first quantity $F(Z)$ is necessary for computing $\langle \rho^2 \rangle_{I=0}$, and thus $\langle \rho^2 \rangle_{E,M}$. The second, third, and fourth quantities, are for the magnetic moment $\mu$, the axial coupling $g_A$, and the axial radius $\langle \rho^2 \rangle_A^{1/2}$, respectively.

First, we compute $\langle \rho^2 \rangle$. The integral to be evaluated is

\[
\langle \rho^2 \rangle_{n_\rho=0} = \frac{\int \rho^5 R(\rho)^2 d\rho}{\int \rho^3 R(\rho)^2 d\rho} ,
\] (A.10)

and we substitute (A.4) into this to allow for the Roper excitation. By partial integration, we obtain

\[
\langle \rho^2 \rangle_{n_\rho=0} = \rho_{cl}^2 \frac{\sqrt{5} + 2\sqrt{5 + N_c^2}}{2N_c} ,
\] (A.11)

\[
\langle \rho^2 \rangle_{n_\rho=1} = \rho_{cl}^2 \frac{3\sqrt{5} + 2\sqrt{5 + N_c^2}}{2N_c} .
\] (A.12)

The former equation is for the proton/neutron, (3.33). For $N_c = 3$, these are numerically given by

\[
\langle \rho^2 \rangle_{n_\rho=0} \simeq 1.62 \times \rho_{cl}^2 ,
\] (A.13)

\[
\langle \rho^2 \rangle_{n_\rho=1} \simeq 2.37 \times \rho_{cl}^2 .
\] (A.14)
Next, we compute \( \left\langle \frac{1}{k(Z)} \right\rangle \), \( \langle F(Z) \rangle \), and \( \langle f_A(Z) \rangle \), that is, the quantities relevant to the \( Z \) directions. This is necessary for the \( N(1535) \) excitation. The classical values are given with \( Z = 0 \) as

\[
\left\langle \frac{1}{k(Z)} \right\rangle \simeq 1 , \quad \langle F(Z) \rangle \simeq F(0) , \quad \langle f_A(Z) \rangle \simeq F_A(0) .
\]  \( \text{(A.15)} \)

In the large \( \lambda \) expansion, the wavefunction for \( Z \) is localized at the origin \( Z = 0 \). Inclusion of the subleading terms causes differences in baryon states.

Numerical evaluation using the wavefunction (A.5) is performed as follows:

\[
\left\langle \frac{1}{k(Z)} \right\rangle_{n_z=0} = \int dZ \frac{1}{1+Z^2} e^{-\frac{2\sqrt{6} |Z^2|}{\kappa^2}} \simeq 0.649 ,
\]  \( \text{(A.16)} \)

\[
\left\langle \frac{1}{k(Z)} \right\rangle_{n_z=1} = \int dZ Z^2 \frac{1}{1+Z^2} e^{-\frac{2\sqrt{6} |Z^2|}{\kappa^2}} \simeq 0.337 ,
\]  \( \text{(A.17)} \)

where we have used \( \kappa = 0.00745 \).

For \( F(Z) \) and \( F_A(Z) \), we first solve the differential equations satisfied by them, (3.7) and (3.72), and then use the solutions to numerically evaluate the normalized integrals. We obtain

\[
\langle F(Z) \rangle_{n_z=0} \simeq 12.7 , \quad \langle F(Z) \rangle_{n_z=1} \simeq 10.9 ,
\]  \( \text{(A.18)} \)

\[
\langle F_A(Z) \rangle_{n_z=0} \simeq 6.67 , \quad \langle F_A(Z) \rangle_{n_z=1} \simeq 4.38 .
\]  \( \text{(A.19)} \)

Classical values are \( F(0) = 14.3 \) and \( F_A(0) = 7.82 \); thus, the ratios to the classical values are given as

\[
\frac{\langle F(Z) \rangle_{n_z=0}}{F(0)} \simeq 0.892 , \quad \frac{\langle F(Z) \rangle_{n_z=1}}{F(0)} \simeq 0.762 ,
\]  \( \text{(A.20)} \)

\[
\frac{\langle F_A(Z) \rangle_{n_z=0}}{F_A(0)} \simeq 0.852 , \quad \frac{\langle F_A(Z) \rangle_{n_z=1}}{F_A(0)} \simeq 0.560 .
\]  \( \text{(A.21)} \)

Thus, smaller values of the quantities are obtained for \( n_z = 1 \).

Using these expectation values, the static properties of \( N(1440) \) and \( N(1535) \) can be computed, as presented in §3.

A.3. Magnetic moment of \( \Delta \)

Evaluation of the isoscalar and isovector magnetic moments requires the matrix elements of

\[
\rho^2 \chi_i = -i\rho^2 \text{tr} \left( \tau^i a^{-1} \dot{a} \right) , \quad \text{tr} \left( a\tau^i a^{-1} \tau^a \right) , \quad \rho^2 .
\]  \( \text{(A.22)} \)
Here, we compute these for the wavefunctions of $\Delta$ given in (A.6), (A.7) and (A.8).

First of all, the third component of the spin of these states is chosen to be $s = +3/2$. Therefore, using (2.47), we can see that the value of $\rho^2 \chi^{i=3}$ is three times that of the proton/neutron for which $s = 1/2$.

Next, let us evaluate $\text{tr} (a^\tau a^{-1} \tau^a)$. From the symmetric nature of this quantity, it is enough to consider the index $i = a = 3$. For this choice of the index, we have

$$\text{tr} (a^\tau a^{-1} \tau^a) = 4a_4^2 + 4a_3^2 - 2 \ . \quad (A.23)$$

In order to calculate the expectation value of this quantity with the wavefunctions (A.6) and (A.7), we use the spherical coordinates

$$a_4 = \cos \theta_0 \ , \quad (A.24)
$$
$$a_3 = \sin \theta_0 \cos \theta_1 \ , \quad (A.25)
$$
$$a_2 = \sin \theta_0 \sin \theta_1 \cos \theta_2 \ , \quad (A.26)
$$
$$a_1 = \sin \theta_0 \sin \theta_1 \sin \theta_2 \ . \quad (A.27)$$

The Jacobian for this change of variables is $d\Omega_3 = \sin^2 \theta_0 \sin \theta_1 \sin \theta_2 \, d\theta_0 \, d\theta_1 \, d\theta_2$. Then we obtain, for instance,

$$\langle \Delta^{++}|(4a_4^2 + 4a_3^2 - 2)|\Delta^{++}\rangle = \frac{2\pi \int_0^\pi \int_0^\pi \sin^8 \theta_0 \sin^7 \theta_1 (4 \cos^2 \theta_0 + 4 \sin^2 \theta_0 \cos^2 \theta_1 - 2)}{\int d\Omega_3 (a_1^2 + a_2^2)^3} = -\frac{6}{5} \ . \quad (A.28)$$

$$\langle \Delta^+|(4a_4^2 + 4a_3^2 - 2)|\Delta^+\rangle = \frac{2\pi \int_0^\pi \int_0^\pi \sin^6 \theta_0 \sin^5 \theta_1 (\cos^2 \theta_0 + \sin^2 \theta_0 \cos^2 \theta_1)(4 \cos^2 \theta_0 + 4 \sin^2 \theta_0 \cos^2 \theta_1 - 2)}{\int d\Omega_3 (a_1^2 + a_2^2)^2 (a_4^2 + a_3^2)} = -\frac{2}{5} \ . \quad (A.29)$$

The denominators originate from the normalization of the states (A.6) and (A.7). It is also easy to see that

$$\langle \Delta^0|(4a_4^2 + 4a_3^2 - 2)|\Delta^0\rangle = \frac{2}{5} \ , \quad \langle \Delta^-|(4a_4^2 + 4a_3^2 - 2)|\Delta^-\rangle = \frac{6}{5} \ . \quad (A.30)$$

The expectation value of $\rho^2$ with respect to the wavefunction (A.8) is

$$\langle \rho^2 \rangle_{l=3} = \frac{\sqrt{5} + 2\sqrt{20 + N_c^2}}{2N_c} \rho_{cl}^2 \ . \quad (A.31)$$
\[ c \equiv \frac{\langle \rho^2 \rangle_{l=3}}{\langle \rho^2 \rangle_{l=1}} = \frac{\sqrt{5} + 2\sqrt{20 + N_c^2}}{\sqrt{5} + 2\sqrt{5 + N_c^2}} \approx 1.34 . \]  

(A.32)

for \( N_c = 3. \)

Comparing these with the proton state with up spin, we obtain (3.41) and (3.42). These ratios are used to compute the magnetic moments of \( \Delta \) as

\[ \mu_{\Delta^+} = 3 \times \frac{1}{2} (\mu_p + \mu_n) + \frac{9c}{5} \times \frac{1}{2} (\mu_p - \mu_n) \approx 5.50 , \]  

(A.33)

\[ \mu_{\Delta^0} = 3 \times \frac{1}{2} (\mu_p + \mu_n) - \frac{3c}{5} \times \frac{1}{2} (\mu_p - \mu_n) \approx -0.15 , \]  

(A.35)

\[ \mu_{\Delta^-} = 3 \times \frac{1}{2} (\mu_p + \mu_n) - \frac{9c}{5} \times \frac{1}{2} (\mu_p - \mu_n) \approx -2.97 , \]  

(A.36)

i.e., (3.43). Here we have substituted the magnetic moment for the proton and the neutron, (3.37).

**Appendix B**

**Useful Formulae and Notation**

**B.1. Useful formulae for currents**

Here we summarize useful formulae that are used in §2 for computing the currents. For \( g \) defined in (2.13), the following equations are obtained,

\[ g \partial_i g^{-1} = \frac{i}{\xi^2} (\frac{1}{2} (\mu_p + \mu_n) + \frac{9c}{5} \times \frac{1}{2} (\mu_p - \mu_n) \approx 5.50 , \]  

(B.1)

\[ g \partial_z g^{-1} = -\frac{i}{\xi^2} (\frac{1}{2} (\mu_p + \mu_n) + \frac{3c}{5} \times \frac{1}{2} (\mu_p - \mu_n) \approx 2.67 , \]  

(B.2)

\[ g^{-1} \partial_i g = -\frac{i}{\xi^2} (\frac{1}{2} (\mu_p + \mu_n) - \frac{9c}{5} \times \frac{1}{2} (\mu_p - \mu_n) \approx -0.15 , \]  

(B.3)

\[ g^{-1} \partial_z g = \frac{i}{\xi^2} (\frac{1}{2} (\mu_p + \mu_n) - \frac{3c}{5} \times \frac{1}{2} (\mu_p - \mu_n) \approx -2.97 , \]  

(B.4)

In particular, its relation to \( SU(2) \) generators is given as

\[ g \tau^a g^{-1} = \frac{1}{\xi^2} \left[ (z - Z)^2 - |\vec{x} - \vec{X}|^2 \right] \tau^a + 
\left( (z - Z)^2 - |\vec{x} - \vec{X}|^2 \right) \delta^{ab} - 
2 \left( \epsilon^{abc} (x^b - X^b)(z - Z) + (x^a - X^a)(x^c - X^c) \right) \tau^c , \]  

(B.5)

\[ \left[ g^{-1} \partial_i g , \tau^a \right] = \frac{2}{\xi^2} \left( (x^b - X^b)\delta^{ab} - (x^a - X^a)\delta^{ib} + (z - Z)\epsilon^{iab} \right) \tau^b , \]  

(B.6)
\[ [g^{-1}\partial_z g, \tau^a] = -\frac{2}{\xi^2} (x^k - X^k) \epsilon^{kab} \tau^b, \quad \text{(B.7)} \]

\[ g \tau^b + \tau^b g = \frac{2}{\xi} \left( (z - Z) \tau^b - i(x^b - X^b) \right), \quad \text{(B.8)} \]

\[ g \tau^b - \tau^b g = \frac{2}{\xi} (x^a - X^a) \epsilon^{abc} \tau^c. \quad \text{(B.9)} \]

A useful identity for the epsilon tensor is

\[ \epsilon_{ija} |\vec{x}|^2 = \epsilon_{abj} x^i x^b - \epsilon_{abi} x^j x^b + \epsilon_{ijb} x^a x^b. \quad \text{(B.10)} \]

### B.2. Useful formulae for Dirac spinors

We summarize the formulae of the gamma matrix and the Dirac spinor, which are used in the previous sections.

The Dirac representation of the gamma matrix is taken as

\[ \gamma^0 = -i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^j = -i \begin{pmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{pmatrix}, \quad \gamma_5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad \text{(B.11)} \]

We define

\[ \sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]. \quad \text{(B.12)} \]

It can be verified that

\[ \gamma^0 \gamma^\mu \gamma^0 = +\gamma^\mu, \quad \gamma^0 \gamma_5 \gamma^0 = +\gamma_5, \quad \gamma^0 \sigma^\mu_{\mu \nu} \gamma^0 = -\sigma_{\mu \nu}. \quad \text{(B.13)} \]

The on-shell Dirac spinor is given by

\[ u(\vec{p}, s) = \frac{1}{\sqrt{2E}} \begin{pmatrix} \sqrt{E + m_B} \chi(s) \\ \sqrt{E - m_B} \bar{n} \cdot \bar{\sigma} \chi(s) \end{pmatrix}, \quad \text{(B.14)} \]

with

\[ \bar{n} = \frac{\vec{p}}{|\vec{p}|}, \quad \text{(B.15)} \]

\[ \chi(1/2) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi(-1/2) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad \text{(B.16)} \]

This satisfies the Dirac equation

\[ (i \not{p} + m_B)u(\vec{p}, s) = 0, \quad \bar{u}(\vec{p}, s)(i \not{p} + m_B) = 0, \quad \text{(B.17)} \]
with
\[ \overline{\psi} = u^\dagger \beta . \quad (\beta = i \gamma^0) \]  
(B-18)

The Dirac spinor is normalized as
\[ \overline{\psi}(\vec{p}', s') u(\vec{p}, s) = \frac{m_B}{p^0} \delta_{ss'} . \]  
(B-19)

In the nonrelativistic limit, the Dirac spinor reduces to
\[ u(\vec{p}, s) = \left( \frac{1}{2m_B} \chi(s) \right) \]  
\[ + \mathcal{O}(m_B^{-2}) . \]  
(B-20)

It is easy to verify
\[ \overline{u}(\vec{p}', s') \gamma_5 u(\vec{p}, s) = \frac{1}{2m_B} k_a (\sigma^a)_{s's} + \mathcal{O}(m_B^{-2}) , \]  
(B-21)
\[ \overline{u}(\vec{p}', s') \gamma_5 \gamma^0 u(\vec{p}, s) = \frac{i}{2m_B} (p + p')_a (\sigma^a)_{s's} + \mathcal{O}(m_B^{-2}) , \]  
(B-22)
\[ \overline{u}(\vec{p}', s') \gamma_5 \gamma^j u(\vec{p}, s) = i(\sigma^j)_{s's} + \mathcal{O}(m_B^{-2}) . \]  
(B-23)

Here \( k = p - p' \) and we used
\[ \chi^\dagger_{(s')} \sigma^a \chi(s) = (\sigma^a)_{s's} . \]  
(B-24)

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