The most recent cosmological data point in the direction of a cosmological constant dominated universe. A suitable candidate for providing the required acceleration is a rolling scalar field named “quintessence”. We address the issue of building a particle physics model for quintessence in the context of SuSy QCD. We then go on to ask if the quintessence scalar can be identified with the inflaton field and give two examples in which this is indeed the case.

1 Introduction

The very last years have witnessed growing interest in cosmological models with $\Omega_m \sim 1/3$ and $\Omega_\Lambda \sim 2/3$, following the most recent observational data (see for example the discussion in the paper by Bahcall et al. and references therein). A very promising candidate for a dynamical cosmological constant is a rolling scalar field, named “quintessence”. The main motivation for constructing such dynamical schemes resides in the hope of weakening the fine tuning issue implied by the smallness of $\Lambda$. In this respect, a very suitable class of models is provided by inverse power scalar potentials which admit attractor solutions characterized by a negative equation of state.

Consider the cosmological evolution of a scalar field $Q$, with potential $V(Q) = M^4 + \alpha Q^{-\alpha}$, $\alpha > 0$, in a regime in which the scalar energy density is subdominant with respect to the background. Then it can be shown that the solution $Q \sim t^{1-n/m}$, with $n = 3(w_Q + 1)$ and $m = 3(w_B + 1)$, is an attractor in phase space. We have defined $w_Q$ to be the equation of state of the scalar field $Q$, and $w_B$ that of the background ($= 1/3$ for radiation and $= 0$ for matter). The equation of state of the scalar field on the attractor is found to be $w_Q = (\alpha w_B - 2)/((\alpha + 2)$, which is always negative during matter domination. As a consequence, the ratio of the scalar to background energy density is not constant but scales as $\rho_Q/\rho_B \sim a^{m-n}$, thus growing during the cosmological evolution, since $n < m$. The behaviour of these solutions is determined by the cosmological background and for this reason they have
been named “trackers” in the literature. A good feature of these models is that for a very wide range of the initial conditions the scalar field will reach the tracking attractor before the present epoch. Depending on the initial time, you can have several tens of orders of magnitude of allowed initial values for the scalar energy density. This fact, together with the negative equation of state, makes the trackers feasible candidates for explaining the cosmological observation of a presently accelerating universe. The point at which the scalar and matter energy densities are of the same order depends on the mass scale in the potential. This is fixed by requiring that $\Omega_Q = \mathcal{O}(1)$ today.

An interesting question, then, is whether the ‘quintessence’ scalar and the inflaton field, which dominate the expansion of the universe at very different times, could indeed be the same field. If this is the case, it should also be possible to uniquely fix the initial conditions for the ‘quintessential’ rolling from the end of inflation. Models in which one single scalar field drives both inflation and the late time cosmological accelerated expansion are named “quintessential inflation” models.

2 A particle physics model: Supersymmetric QCD

As first noted by Binètruy, supersymmetric QCD theories with $N_c$ colors and $N_f < N_c$ flavors may give an explicit realization of a quintessence model with an inverse power law scalar potential. The matter content of the theory is given by the chiral superfields $Q_i$ and $\overline{Q}_i$ ($i = 1 \ldots N_f$) transforming according to the $N_c$ and $\overline{N}_c$ representations of $SU(N_c)$, respectively. In the following, the same symbols will be used for the superfields $Q_i$, $\overline{Q}_i$, and their scalar components. Supersymmetry and anomaly-free global symmetries constrain the superpotential to the unique exact form

$$W = \left( N_c - N_f \right) \left[ \frac{\Lambda^{3N_c-N_f}}{\det T} \right]^{1/(N_c-N_f)}$$

where the gauge-invariant matrix superfield $T_{ij} = Q_i \cdot \overline{Q}_j$ appears. $\Lambda$ is the only mass scale of the theory.

We consider the general case in which different initial conditions are assigned to the different scalar VEV’s $\langle Q_i \rangle = \langle \overline{Q}_i \rangle \equiv q_i$, and the system is described by $N_f$ coupled differential equations. In analogy with the one-scalar case, we look for power-law solutions of the form

$$q_{tr,i} = C_i \cdot t^{p_i}, \quad i = 1, \ldots, N_f .$$

It is straightforward to verify that for fixed $N_f$ (and when $\rho_Q \ll \rho_B$), a solution exists with $p_i \equiv p = p(N_c)$ and $C_i \equiv C = C(N_c, \Lambda)$ and that it is the...
same for all the $N_f$ flavors. The equation of state of the tracker is given by
\[ w_Q = \frac{1 + r}{2} w_B - \frac{1 - r}{2}, \tag{3} \]
where we have defined $r \equiv N_f / N_c$. Then, even if the $q_i$‘s start with different initial conditions, there is a region in field configuration space such that the system evolves towards the equal fields solutions (3), and the late-time behavior is indistinguishable from the case considered by Binetrúy where equal initial conditions for the $N_f$ flavors were chosen. In spite of this, the multi-field dynamics introduces some new interesting features. For example, we have found that (in the two–field case) for any given initial energy density such that, for $q_1^n / q_2^n = 1$, the tracker is joined before today, there exists always a limiting value for the fields’ difference above which the attractor is not reached in time. A more detailed discussion and numerical results about the two-field dynamics can be found in Masiero et al.

3 Quintessential inflation

As already discussed, the range of initial conditions which allows $\rho_Q$ to join the tracker before the present epoch is very wide. Nevertheless, it should be noted that in principle we do not have any mechanism to prevent $\rho_Q$ from being outside the desired interval. In this respect, an early universe mechanism which could naturally set $\rho_Q$ in the allowed range for a late time tracking, would be highly welcome. Moreover, if we require the quintessence scalar to be identified with the inflaton, we would at the same time obtain a tool for handling the initial conditions and a simple unified picture of the early and late time universe dynamics, which we call “quintessential inflation”.

The basic idea is to consider inflaton potentials which, as it is typical in quintessence, go to zero at infinity like inverse powers. In this way it is possible to obtain a late time quintessential behaviour from the same scalar that in the early universe drives inflation. The key point resides in finding a potential which satisfies the condition that inflation and late time tracking both occur, and that they occur at the right times (see Peloso et al.).

Two models have been shown to fulfill these two requirements. One example is a first-order inflation model with potential going to zero at infinity like $\phi^{-\alpha}$. A bump at $\phi \ll M_p$ allows for an early stage of inflation while the scalar field gets “hung up” in the metastable vacuum of the theory. Nucleation of bubbles of true vacuum through the potential barrier sets the end of the accelerated expansion and starts the reheating phase. After the reheating process is completed, the quintessential rolling of the scalar $\phi$ starts and its initial
conditions (uniquely fixed by the end of inflation) can be shown to naturally be within the range which leads to a present day tracking. As an alternative, we considered the model of hybrid inflation proposed by Kinney et al. This is shown to naturally include a late-time quintessential behavior. As typical of hybrid schemes, the potential at early times (that is until the inflaton field is smaller than a critical value \( \phi_c \)) is dominated by a constant term and inflation takes place. Eventually the inflaton rolls above \( \phi_c \), rendering unstable the second scalar of the model, \( \chi \). This auxiliary field starts oscillating about its minimum and in this stage the universe is reheated. After \( \chi \) has settled down, the inflaton continues its slow roll down the residual potential, which goes to zero at infinity like \( \phi^{-2} \), thus allowing for a quintessential tracking solution. Also in this case the initial conditions for the quintessential part of the model are not set by hand, but depend uniquely on the value of the inflaton field at the end of reheating.

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