ESTIMATION OF MEASURED DATA OF MICROCHANNEL GAS FLOWS

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ABSTRACT
The measured data of mass flow rates and streamwise pressure distributions at various experimental conditions of microchannels carried out by Pong et al (1994), Harley et al (1995), Shih et al (1996), Arkilic et al (1997, 2001), and Zohar et al (2002) are normalized by the kinetic factors $M_c$ and $p_k$, respectively. The normalized data are compared each other, and they are in excellent agreement, except the few with the small differences. This demonstrates that the measured data available are generally accurate.

NOMENCLATURE

- $h$: microchannel height
- $K_n$: Knudsen number
- $L$: microchannel length
- $M$: mass flow rate
- $M_c$: normalized factor of mass flow rate
- $p$: pressure
- $P$: normalized pressure, $P = p/p_o$
- $dp/dx$: streamwise pressure gradient
- $\lambda$: mean free path
- $\theta$: ratio of inlet to outlet pressure, $\theta = p_i/p_o$

Subscripts
- $i$: Inlet
- $o$: Outlet

INTRODUCTION

Many experimental studies [1-9] on gas flows through micro-channels were carried out to understand the microscale effects that are important for the design and optimization of MEMS devices. The mass flow rates and streamwise pressure distributions were measured at various conditions as shown in Table 1. The dimensions were about one micron high by several tens of microns wide and by several thousands microns long. The flow was driven by the pressure differences between the inlet and outlet, with a typical inlet velocity of about 0.2 m/s [10]. The flows are two dimensional because of the negligible spanwise effect for the large width-to-height ratio, while the isothermal assumption is valid under the low subsonic conditions without external heating.

Comparing these experimental data each other is helpful to assess their accuracy, and reveal the features of microchannel gas flows. Due to the differences between the experimental conditions, we have to normalize the measured data firstly. Let us image to slice up microchannels a cross section by cross section. Every cross section may be localized as the Poiseuille flow. The mass flow rate may be nicely related to the Knudsen number based on the channel height (see, e.g. Fig.6 in Ref. [11]) when a following normalization factor is used

$$M_c = \frac{2h^2 dp}{\nu_m dx},$$

where $h$ is the channel height, $\nu_m = \sqrt{2RT}$ is the most probable thermal speed, and $dp/dx$ is the pressure gradient.

Experiments [1,3,4,9] showed that the streamwise pressure distributions of gas flows through microchannel were nonlinear. This means that $dp/dx$ is not constant, which differs from the Poiseuille flow. Therefore, we have to obtain the solution of $dp/dx$, before the normalized factor $M_p$ may be extended to microchannels.
Table 1. Experimental conditions of microchannel gas flows

| Source     | Gas       | Height  | Width  | Length  |
|------------|-----------|---------|--------|---------|
| Pong et al [1] | N₂, He    | 1.2     | 40     | 3000    |
| Harley et al [2]  | N₂, He, Ar | 0.51 ~ 19.79 | 100 ~ 200 | 10000   |
| Shih et al [3,4]    | N₂, He    | 1.2     | 40     | 4000    |
| Arkilic et al [5-8] | He, Ar, N₂, CO₂ | 1.33 | 52.3   | 7490    |
| Zohar et al [9]     | He, Ar, N₂ | 0.53 ~ 0.97 | 40     | 4000    |

CONSERVATION OF MASS FLOW RATE THROUGH MICROCANCELS

Consider a cross section of microchannel. The mass flow rate through it may be written as

\[
M \cdot \frac{d}{dx} \ln \frac{M_{N-S}}{\phi(Kn)} = 0,  \tag{2}
\]

with

\[
M_{N-S} = \frac{2h^3}{3\mu RT} \frac{dp}{dx},  \tag{3}
\]

is the non-slip Navier-Stokes solution of mass flow rate for the Poiseuille flows, and with

\[
\phi(Kn) \equiv 1 + 6\alpha Kn + \frac{12}{\pi} Kn \ln(1 + \beta Kn),  \tag{4}
\]

where \(\alpha = 1.318889\), \(\beta = 0.387361\). \(\phi(Kn)\) reflects the local deviation from the N-S solution owing to the microscale effect. Eq. (4) is fitted based on the numerical solution of the linearized Boltzmann equation [12,13], under the tangential momentum accommodation coefficient \(\sigma = 1\).

The mass flow rate conservation through microchannels requires

\[
\frac{dM}{dx} = 0.  \tag{5}
\]

Substituting Eq. (2) into (5) and eliminating the constant term \(2h^3/(3\mu RT)\) give rise to a simple relation between \(p\) and \(Kn\)

\[
\frac{d}{dx} \left[ 1 + 6\alpha Kn + \frac{12}{\pi} Kn \ln(1 + \beta Kn) \right] \frac{dp}{dx} = 0,  \tag{6}
\]

or

\[
\left[ 1 + 6\alpha Kn + \frac{12}{\pi} Kn \ln(1 + \beta Kn) \right] \frac{dp}{dx} = C,  \tag{7}
\]

where \(C\) is a constant undetermined, \(P = p/p_0\), \(X = x/L\), \(p_0\) is the outlet pressure, and \(L\) is the microchannel length.

Eq. (6) may be regarded alternatively as a special case of the generalized Reynolds equation with the bearing number \(\Lambda = 0\). The generalized Reynolds equation was firstly derived by Fukui and Kaneko [14] from the linearized Boltzmann equation, and it works quite well for air slider bearings. Recently C. Shen [15] suggested to apply it to microchannels. Eq. (6) is valid over the entire flow regime from continuum to free molecular, because its kernel \(\phi(Kn)\) is obtained based on the linearized Boltzmann equation.

NORMALIZED FACTORS OF PRESSURE AND MASS FLOW RATE

For hard-sphere molecules, the mean free path \(\lambda = kT/\sqrt{2\pi T \sigma_T}\), where the collision cross section \(\sigma_T\) is constant. Consequently, the Knudsen number along a microchannel may be expressed as follows

\[
Kn = \frac{\lambda}{h} = \frac{h_\circ}{h} - \frac{\lambda_\circ}{P},  \tag{8}
\]

where the subscript \(\circ\) denotes the outlet.

Substitution of Eq. (8) into (7) yields

\[
\left[ P + 6\alpha Kn_\circ \frac{12}{\pi} Kn \ln(1 + \beta Kn_\circ) \right] \frac{dp}{dx} = Cdx,  \tag{9}
\]

Integrating Eq. (9) from the inlet \(X=0\) and \(P = \vartheta = p_1/p_0\), we have

\[
\int_0^1 \frac{1}{2} P^2 + 6\alpha Kn_\circ \left[ \ln(1 + \beta Kn_\circ) \right] + 12 \frac{Kn_\circ}{\pi} \alpha + \frac{1}{2} \frac{Kn_\circ}{\pi} \beta + \frac{1}{2} \frac{Kn_\circ}{\pi} \gamma + \frac{1}{2} \frac{Kn_\circ}{\pi} \delta = Cx, \tag{10}
\]

At the outlet \(X=1, P=1\), therefore

\[
C = \frac{1}{2} \left( (1 - \vartheta^2) + 6\alpha Kn_\circ (1 - \vartheta) \right) + 12 \frac{Kn_\circ}{\pi} \left[ \ln(1 + \beta Kn_\circ) \right] + \frac{1}{2} \frac{Kn_\circ}{\pi} \beta + \frac{1}{2} \frac{Kn_\circ}{\pi} \gamma + \frac{1}{2} \frac{Kn_\circ}{\pi} \delta. \tag{11}
\]

The normalized factor of mass flow rate \(M\) through microchannels may be obtained using Eqs. (1), (7) and (11)

\[
M = \frac{2h^2}{3\mu v_m} \frac{dp}{dx} = \frac{2h^2}{3\mu v_m} \frac{Cp_0^2}{\phi(Kn_1)p_1L},  \tag{12}
\]

where the subscript \(1\) denotes the inlet.

The kinetic solution of the streamwise pressure distribution \(p_1\) may be numerically solved from Eq. (10) that depends upon the parameters \(\vartheta\) and \(Kn_\circ\).

COMPARISON OF NORMALIZED MEASURED MASS FLOW RATES AND STREAMWISE PRESSURE DISTRIBUTIONS

Figure 1 compares the normalized mass flow rates at different conditions carried out by Harley et al [2], Shih et al
Generally they agree well each other, whereas the small differences between the helium cases of Shih et al [4] and Arkilic et al [6,7] are observed. A relation of the normalized mass flow rate to the inlet $Kn_1$ may be simply fitted as

$$M = M_e \left( a + bKn_1 + cKn_1^2 + dKn_1^3 + eKn_1^4 \right),$$

with $a=0.021998$, $b=12.48288$, $c=-87.95779$, $d=397.432$, and $e=-716.724$.

Table 2. The values of $p_{exp}/p_k$ at different conditions.

| Case | x (µm) | $p_i$ (psig) | $p_{exp}/p_k$ |
|------|--------|-------------|--------------|
| A1   | 400    | 25          | 1.013        |
|      |        | 1100        | 1.015        |
|      |        | 1800        | 0.994        |
|      |        | 2500        | 0.989        |
| A2   | 400    | 20          | 1.005        |
|      |        | 1100        | 1.018        |
|      |        | 1800        | 1.010        |
|      |        | 2500        | 1.013        |
| A3   | 400    | 15          | 1.005        |
|      |        | 1100        | 1.028        |
|      |        | 1800        | 1.004        |
|      |        | 2500        | 1.013        |
| A4   | 400    | 10          | 1.006        |
|      |        | 1100        | 1.012        |
|      |        | 1800        | 0.907        |
|      |        | 2500        | 1.030        |
| A5   | 400    | 5           | 1.017        |
|      |        | 1100        | 1.041        |
|      |        | 1800        | 1.024        |
|      |        | 2500        | 1.027        |
| B1   | 400    | 19          | 1.006        |
|      |        | 800         | 1.018        |
|      |        | 1200        | 1.029        |
|      |        | 1600        | 1.022        |
|      |        | 2000        | 1.038        |
|      |        | 2400        | 1.029        |
|      |        | 3200        | 1.054        |
|      |        | 3600        | 1.024        |
| B2   | 400    | 13.6        | 1.003        |
|      |        | 800         | 1.008        |
|      |        | 1200        | 1.011        |
|      |        | 1600        | 1.021        |
|      |        | 2000        | 1.038        |
|      |        | 2400        | 1.023        |
|      |        | 2800        | 1.054        |
|      |        | 3200        | 1.049        |
| B3   | 400    | 8.7         | 0.997        |
|      |        | 800         | 0.997        |
|      |        | 1200        | 0.999        |
|      |        | 1600        | 1.019        |
|      |        | 2000        | 1.018        |

Figure 1. Comparison of normalized measured mass flow rates through microchannels versus the inlet Knudsen number.

Figure 2. Comparison of normalized measured streamwise pressure distributions.
|    |    |    |    |
|----|----|----|----|
| 2400 | 8.7 | 1.007 |
| 2800 | 8.7 | 1.019 |
| 3200 | 8.7 | 1.004 |
| 3600 | 8.7 | 0.995 |
| C1 | 700 | 16.1 | 1.004 |
| 2000 | 16.1 | 1.045 |
| 3300 | 16.1 | 1.039 |
| C2 | 700 | 26.4 | 0.992 |
| 2000 | 26.4 | 1.044 |
| 3300 | 26.4 | 1.014 |
| C3 | 700 | 35.2 | 1.011 |
| 2000 | 35.2 | 1.032 |
| 3300 | 35.2 | 1.036 |
| D1 | 700 | 13.2 | 1.016 |
| 1400 | 13.2 | 1.005 |
| 2000 | 13.2 | 0.991 |
| 2600 | 13.2 | 0.993 |
| 3300 | 13.2 | 0.982 |
| D2 | 700 | 23.4 | 1.009 |
| 1400 | 23.4 | 0.979 |
| 2000 | 23.4 | 0.997 |
| 2600 | 23.4 | 1.015 |
| 3300 | 23.4 | 1.035 |
| D3 | 700 | 33.7 | 0.987 |
| 1400 | 33.7 | 0.987 |
| 2000 | 33.7 | 0.983 |
| 2600 | 33.7 | 1.008 |
| 3300 | 33.7 | 1.010 |
| D4 | 700 | 44.0 | 0.998 |
| 1400 | 44.0 | 0.983 |
| 2000 | 44.0 | 1.010 |
| 2600 | 44.0 | 1.012 |
| 3300 | 44.0 | 1.092 |

**CONCLUSIONS**

The measured data of mass flow rates and streamwise pressure distributions through microchannels at various experimental conditions are normalized by the kinetic factors $M_c$ and $p_k$, respectively. The normalized comparison is satisfactory, except the few that show the small differences. This demonstrates that the measured data available are generally accurate. Consequently, the fitting formula of the measured mass flow rates and the kinetic solution of streamwise pressure distribution may be reliably applied to the design and optimization of MEMS devices.

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