PHYSICS OF THE POWER CORRECTIONS IN QCD.

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We review the physics of the power corrections to the parton model. In the first part, we consider the power corrections which characterize the infrared sensitivity of Feynman graphs when the contribution of short distances dominates. The second part is devoted to the hypothetical power corrections associated with non-perturbative effects at small distances.

1 INTRODUCTION.

General remarks on the power corrections.
We consider QCD and processes determined by physics at short distances. Which means that there is a generic large mass scale, $Q \gg \Lambda_{QCD}$ where $\Lambda_{QCD}$ is the position of the Landau pole in the coupling:

$$\alpha_s(Q^2) \approx \frac{1}{b_0 \ln Q^2/\Lambda_{QCD}^2}.$$  

(1)

For example, $Q$ may stand for the total energy in the $e^+e^-$ annihilation into hadrons or the 4-momentum of the virtual photon in deep inelastic scattering (DIS).

Then one can use the perturbation theory and predictions for a physical observable $O$ are given by a perturbative series:

$$< O > = < O >_{\text{parton model}} \left(1 + \sum_{n=1}^{\infty} a_n \alpha_s(Q^2)^n\right).$$  

(2)

Now, we reserve for powers of $\Lambda_{QCD}/Q$ as well:

$$< O > = < O >_{\text{parton model}} \left(1 + \sum_{n=1}^{\infty} a_n \alpha_s(Q^2)^n + \sum_{n=k}^{\infty} b_n (\Lambda_{QCD}/Q)^n\right).$$  

(3)

These are the power corrections.

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\[b\] In fact, the perturbative corrections may modify the parton model by a powers of $\alpha_s(Q^2)$ as an overall factor as well. These are so called anomalous dimensions, best known from the example of moments from structure functions in DIS. For simplicity we consider the case of zero anomalous dimensions.
First of all, the power corrections appear to be a pure nonperturbative effect. Indeed, on one hand we have
\[
\left( \frac{\Lambda_{QCD}}{Q} \right)^k = \exp(-k/2b_0\alpha_s(Q^2)).
\] (4)

On the other hand, the function \( \exp(-\text{const}/\alpha) \) with a positive \( \text{const} \) is a classical example from the math
courses of a function which has a trivial Taylor expansion at \( \alpha = 0 \):
\[
\exp(-\text{const}/\alpha)|_{\alpha=0} = 0 + 0 \cdot \alpha + 0 \cdot \alpha^2 + \ldots
\] (5)
since the function itself and all its derivatives vanish at \( \alpha = 0 \). Thus, this function, being a non-zero, vanishes
identically as a perturbative expansion, which is the expansion at \( \alpha = 0 \).

The interest in power-like corrections originates from various sources:

\( (i) \) Nowadays, one may say that the perturbative QCD is trivially correct and, if it were possible, the
best thing would be just to subtract it out and proceed to non-perturbative pieces. In particular, one may
say that the physics of the confinement is encoded in the power corrections, not the perturbation theory.

\( (ii) \) In some cases, the accuracy of theoretical fits to experimental data require for an account of the
power corrections. For example, error bars on measured values of \( \alpha_s \) are affected by the power corrections.

\( (iii) \) More pragmatically, one could say that we are brought to consider the power corrections by the
logic of the development in the field. If it is at all possible to distinguish between these motivations, we will belong rather to the first line. Namely, we will assume, explicitly or implicitly, that the onset of the power corrections at some so to say
moderately large \( Q^2 \) signifies new physical phenomena. At the end, we shall see whether we are in fact justified to assume so.

There is another important aspect of the power corrections. While calculating the perturbative expansions is a well defined procedure in QCD, at least as a matter of principle, the definition of the non-perturbative terms is close to saying that these are unknown terms, the rest of the amplitudes upon subtraction of the perturbative contributions. In other words, working with the power corrections relies to a great extent on intuition and heuristic models. Nevertheless, we shall be mentioning sometimes the “standard picture” of the nonperturbative physics. What could this mean if we a priori know that no precise form of the nonperturbative fluctuations is assumed? Still, there is a content to the notion of the standard picture. Namely, the standard assumption is that the nonperturbative fields are soft. In other words, the typical size of the nonperturbative fluctuations is of order \( \Lambda_{QCD}^{-1} \). Later, we will challenge this picture to some extent.

Finally, let us mention that actually working with two infinite series as indicated in (3) would be awfully
difficult in practice. Thus, in reality one is always relying on a kind of a truncated series like:
\[
< O > = < O >_{\text{parton model}} \left( 1 + a_1\alpha_s(Q^2) + b_k(\Lambda_{QCD}/Q)^k \right)
\] (6)

where \( k \) characterizes the leading power correction. The assumption behind this truncation is that the power
corrections are somehow enhanced numerically. There is no proof of this assumption but it is an indispensable
ingredient of any phenomenology based on the power corrections. Historically, this assumption worked very
well in case of the so called QCD sum rules (Shifman et. al., 1979). It might well fail, however, in other cases.
If there exist high-precision data one may try to check this assumptions varying the number of terms in the
perturbative expansion kept explicitly and watching how the fitted value for the power corrections depends
on this. So far, this procedure was implemented in the most careful way in (Kataev et.al, 1998) in case of
DIS.

Outline of the lectures.

The power corrections are a hot subject in QCD. In particular at this School there will be another course,
given by Yu. L. Dokshitzer also devoted to a great extent to the power corrections. To avoid overlap, these lectures will consist of two parts. The first one is about power corrections within the approach based on
the operator product expansion (OPE) which goes back to QCD sum rules and is about 20 years old. The second part, to the contrary, describes an unconventional (and hypothetical) source of power corrections, that is short strings. This part is based mostly on the original work (Gubarev et. al., 1998).
It is worth adding that the unification of these two parts, separated by almost 20 years in terms of the original papers, under the same title is not artificial at all. The point is that the power corrections associated with the short strings hopefully solve some outstanding problems left over from the OPE-based approach.

2 POWER CORRECTIONS AND SOFT VACUUM FIELDS.

Correlation functions.

It is clear that if we take the limit \( Q^2 \to \infty \) literally we would be left with parton model, with no corrections whatsoever. Moreover, the power-like corrections would die fast. Thus, our general strategy will be to start with large \( Q^2 \) so that \( \alpha_s(Q^2) \) is small numerically. This is needed to have some control over theoretical calculation since QCD is simple only at short distances. However, then we will move down in \( Q^2 \) until reach so to say moderate \( Q^2 \) where the corrections become sizable. This is most interesting region for us. Indeed, we are still able to sort out various corrections since they are small compared to unity. On the other hand, we may hope to distinguish between various mechanisms of breaking the asymptotic freedom. In language of distances, we will start with \( r \to 0 \) and then proceed to “moderate” \( r \). Everywhere we understand that generically, \( r \sim 1/Q \).

For the sake of definiteness we will concentrate on correlation functions \( \Pi_j(Q^2) \) and quark-antiquark potential \( V(r) \). Let us introduce these quantities in more detail.

At first sight, it would be most natural for QCD studies to consider hadrons themselves. However, observing hadrons we would not find much quarks at short distances since they are predominantly at a characteristic distances of order \( \Lambda_{QCD}^{-1} \). And physics at such distances is governed by a large \( \alpha_s \) where we do not have reliable theory. To ensure that quarks do not fly away one has to resort therefore to an external source of quarks such as electromagnetic current and consider unphysical kinematics with space-like total momentum of quarks \( q, -q^2 = Q^2 > \Lambda_{QCD}^2 \). Then according to the uncertainty principle quarks can exist for time of order

\[
\tau \sim \frac{1}{\sqrt{Q^2}}
\]

which is small if \( Q^2 \) is large. For consistency, after such time the quarks are to be absorbed by another current, see Fig. 1.

![Figure 1: a) Correlator of currents in the parton model approximation. b) \( q^2 \) plane.](image)

In the field theoretical language, we are considering in fact a correlation function \( \Pi_j(Q^2) \):

\[
\Pi_j(Q^2) = i \int d^4x \ e^{iqx} \ \langle 0 | T\{j(x),j(0)\} | 0 \rangle, \quad Q^2 = -q^2
\]

where the current \( j \) may have various quantum numbers, like spin, isospin and for simplicity we do not indicate these quantum numbers, i.e. suppress the Lorenz indices and so on.

The basic theoretical ingredient is that \( \Pi(Q^2) \) at large \( Q^2 \) can be calculated in the parton model approximation:

\[
\lim_{Q^2 \to \infty} \Pi_j(Q^2) = \Pi_j(Q^2)_{\text{parton model}}
\]

On the other hand, by using dispersion relations \( \Pi_j(Q^2) \) can be expressed in terms of the absorptive part which is non-vanishing only for time-like total momentum, \( q^2 > 0 \):

\[
\Pi_j(Q^2) = \frac{1}{\pi} \int \frac{Im \Pi_j(s)}{s + Q^2} ds
\]
The imaginary part is directly observable, provided that the current \( j \) is a physical one. In particular, in case of the electromagnetic current, \( j = j_{el} \) the imaginary part in Eq. (10) is proportional to the total cross section of \( e^+e^- \)-annihilation into hadrons:

\[
Im\Pi_{j,i}(s) = \text{const} \frac{\sigma_{tot}(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}
\]  

(11)

Upon substitution of (11), the Eq. (10) becomes a sum rule. Indeed, \( \Pi_{j,i}(Q^2) \) is calculable and the same as for free particles plus small radiative corrections, while \( Im\Pi(s) \) is observable.

There is one more technical point to be mentioned. The dispersion relations (10) suffer in most cases from ultraviolet divergences which could be eliminated at a price of subtractions. But then there would appear an arbitrary polynomial. There is, however, another way to deal with this problem (Shifman et al., 1979). Namely introduce

\[
\Pi_j(M^2) = \frac{Q^{2n}}{(n-1)!} \left( \frac{-d}{dQ^2} \right)^n \Pi_j(Q^2)
\]  

(12)

in the limit where both \( Q^2 \) and \( n \) tend to infinity so that their ratio \( M^2 \equiv Q^2/n \) remains finite. Then it is easy to see that the weight function in the dispersion relation becomes:

\[
\frac{1}{s+Q^2} \rightarrow \frac{1}{M^2} \exp(-s/M^2),
\]  

(13)

while all the subtraction constants are removed by the differentiation (12).

Let us give a realistic example of the sum rule in the \( \rho \)-meson channel:

\[
\int R_{I=1}(s) \exp(-s/M^2) ds \approx \frac{3}{2} M^2 \left( 1 + \frac{\alpha_s(M^2)}{\pi} \right),
\]  

(14)

where \( M^2 \) is considered to be large enough so that \( \alpha_s(M^2)/\pi \) is small. Moreover \( R_{I=1}(s) \) is the total cross section of \( e^+e^- \) annihilation into hadrons with isotopic spin \( I=1 \) in units of the standard cross section \( \sigma(e^+e^- \to \mu^+\mu^-) \).

It is worth noting that to apply the technique considered it suffices to ensure that the time which quarks exist is small indeed. Apart from imposing the condition that \( Q^2 \) is large (see Eq. (7)) there exist other possibilities. In particular, as far as production of heavy quarks is concerned one can consider \( Q^2 = 0 \) since in that case (Shifman et al., 1976)

\[
\tau \sim 1/m_H.
\]

Therefore, in case of heavy quarks it is convenient to consider \( Q^2 = 0 \) and get rid of possible ultraviolet (UV) divergences by differentiating the dispersion relations (10) with respect to \( Q^2 \) at \( Q^2 = 0 \). In this way one comes to the sum rules (Novikov et al., 1977):

\[
\int \frac{R_c(s)}{s^{n+1}} ds \approx \frac{A_n}{(4m_c^2)^n} \left( 1 + B_n \frac{\alpha_s(m_c^2)}{\pi} \right),
\]  

(15)

where \( R_c \) is the contribution of the current of the charmed quarks into the ratio \( R(s) \) and the integer number \( n \) corresponds to the \( n \)-th derivative from (10) while \( m_c \) is the mass of the charmed quark normalized off-mass shell at \( p^2 = 0 \). Moreover, \( A_n, B_n \) are calculable numbers:

\[
A_n = \frac{3}{4\pi^2} \frac{2^n(n+1)(n-1)!}{(2n+3)!!},
\]

\[
B_n = \frac{4}{3\sqrt{\pi}} \frac{\Gamma(n+3/2)[1-1/(3n+3)]}{\Gamma(n+1)[1-1/(2n+3)]} - \frac{\pi}{2} + \frac{3}{4\pi} \left( \frac{\pi}{2} - 3 \right) \frac{\Gamma(n+3/2)[1-2/(3n+6)]}{\Gamma(n+2)[1-1/(2n+3)]} - \frac{4n}{\pi} \ln 2.
\]  

(16)
Note that the integral over $R(s)$ is contributed both by resonances, like $J/\Psi$ and by continuum cross section of production of particles with open charm. The bound states are to be included since their contribution is of zero order in the small $\alpha_s(M^2)$. Indeed, the properties of the resonances are governed by $\alpha_s \sim 1$. Moreover it turns out that there exist such $M^2$ (or $n$ in case of heavy quarks) for which the resonances dominate the integral over the physical cross section while the theoretical, partonic part is still calculable reliably since $\alpha_s(M^2)$ is small.

It is worth noting that direct experimental information on the correlation functions is available only in a limited number of cases since only very few currents, like the electromagnetic current is directly observable. However, the lattice simulations (Chen et. al., 1993) allow to measure $\Pi_j(x)$ for a much wider class of currents. These measurements refer to the Euclidean space directly since the lattice corresponds, of course, to Euclidean $x$.

To summarize: there are quite a few correlation functions known in Euclidean domain either from combined use of dispersion relations and experimental data or from the lattice simulations. The correlation functions are most suitable for studies of the power corrections.

The meaning of the OPE. Gluon Condensate.

In this section we will describe power corrections as they arise within the operator product expansion (OPE). The presentation goes back to the QCD sum rules (Shifman et. al., 1979). More detailed reviews can be found in (Reinders et. al., 1985), (Narison, 1989).

The central physical question which brought about the power corrections was as follows. Imagine that we study the sum rules (14) at very large $M^2$ and then go down with $M^2$. At some low $M^2$ the dispersive integral over $s$ would be dominated by the $\rho$-meson to such extent that it cannot be matched by the quark predictions. Then the question is: “Who stops the Asymptotic Freedom at some critical $M^2$?”

At first sight, the answer is almost trivial: the growth of the coupling at low momenta. It would be a common answer. However, if one addresses the problem in concrete, there arise doubts in the validity of this answer. For example, it is quite a common theoretical guess that the splitting between the vector mesons does not depend on flavor, say:

$$m(\rho') - m(\rho) \approx m(\psi') - m(J/\psi), \quad (17)$$

which is true experimentally. This simple-looking observation represents, however, a serious challenge to the wisdom that it is the growth of the effective coupling that stops the AF at moderate mass scales. Indeed, if expressed in terms of an invariant quantity, $s$, Eq. (17) implies that the $J/\psi$ is dual to a much larger interval of $s$ than the $\rho$ because the c-quark is heavy. In other words, the AF is violated in the $\rho$ channel much later than in the charmonium channel, if we start from very large $M^2$ downwards. A direct numerical analysis of the sum rules (14), (15) confirms this expectation. However, the coupling should run as a function of an invariant quantity and is flavor blind.

More generally, one feels that knowing the perturbation theory alone is not enough to plunge into the resonance physics. Therefore, let us assume for the moment that it is soft nonperturbative fields in the vacuum which are eventually responsible for the confinement. At first sight, having this idea does not help much since very little is known on the precise nature of these nonperturbative fields. The way out of this difficulty will be not to try to calculate the nonperturbative fields but describe them instead phenomenologically in terms of a few parameters.

One can understand the trick by considering the same Feynman graphs for the $\Pi_j(Q^2)$. Let us begin with the simplest graph of Fig. 2. As was explained above (see Eq. (6)), by taking $Q^2$ large we ensure that the correlation function is determined by short distances. Let us now estimate to which accuracy this assertion is correct. Consider to this end in more detail the route of the large $Q$, which is brought in by the current. The typical case is when the both quark lines in Fig. 1 carry momentum of order $Q$. “Typical” means favored by the phase space. But it is not forbidden that almost the whole momentum is carried by one line while the other quark line is soft. Then we get a problem since the soft line cannot be reliably described by the perturbative propagator because at large distances the perturbation theory is modified strongly.

Instead of making guesses, how the quark propagator looks like in the infrared (IR) we reserve for an unknown number for this propagator. The central point is of course that it is not an unknown function but
rather a number. To see that it is indeed true, prepare the graph as is shown in Fig. 2. Here the black

blob in the right-hand side denotes the hard quark line and currents. Along this line, since it is hard, all
the distances are small, \( x \sim 1/Q \) and everything is calculable. On the other hand, the soft line can travel
distances of order \( 1/k \sim \Lambda_{QCD}^{-1} \) which are large on the scale of \( 1/Q \). However, it is constrained to begin and
end up at the same point. We can substitute therefore the soft line by an unknown matrix element. This
matrix element is proportional to the so called quark condensate, \( \langle 0|\bar{q}q|0\rangle \neq 0 \), which is famous for the
fact that its non-vanishing value signals the spontaneous breaking of the chiral symmetry. If the current \( j \)
constructed on the light quarks, then it is easy to see that in fact the quark condensate is multiplied by the
quark mass \( m_q \) and the contribution of the IR region is additionally suppressed. Namely it is of the order

\[
\frac{\delta \Pi_j(Q^2)}{\Pi_j(Q^2)} \sim \frac{m_q \langle 0|\bar{q}q|0\rangle}{Q^4}.
\]  

(18)

Notice the factor \( Q^{-4} \) which we get on pure dimensional grounds. This suppression is the price we pay for
enforcing one of the lines to be soft.

The graph in Fig. 2 is very important, however, if one of the quarks is heavy. Then we replace \( m_q \) in
Eq. (18) by \( m_H \) while \( q \) still stands for a light quark. This is our first example of a nontrivial interplay of
perturbative and nonperturbative calculations. Indeed, on one hand, we consider perturbative graph. The
perturbation theory by itself respects the symmetries of the Lagrangian and \( \langle 0|\bar{q}q|0\rangle_{\text{pert}} \sim m_q \) and is
negligible. However, in the general treatment of the infrared sensitive contributions which we pursue now
the quark condensate \( \langle \bar{q}q \rangle \) is just a phenomenological number which is in fact does not go to zero for
\( m_q \to 0 \).

Coming back to consider currents constructed on the light quarks, the contribution (18) is small because
\( m_q \) is small and we have to go to the next order in perturbation theory to uncover IR sensitivity of the
Feynman graphs at large \( Q \). Thus, turn to the graph with a gluon exchange. Assuming that the momentum
flowing through the gluon line is in fact of order \( \Lambda_{QCD} \) we prepare the graph as indicated in Fig. 3. Again,
there is no reason any longer to use the perturbative expression for the gluon line since it is soft and modified strongly by the confinement. The quark lines propagating short distances become a receiver of long wave gluon fields in the vacuum. The receiver is well understood because of the asymptotic freedom. The intensity of the gluon fields is measured. It is characterized by the so called gluon condensate:

$$\langle 0| \alpha_s (G_{\mu\nu}^a)^2 |0 \rangle \equiv \langle 0| \alpha_s \left( (\tilde{H}^a)^2 - (\tilde{E}^a)^2 \right) |0 \rangle \neq 0$$

(19)

where $\tilde{H}^a, \tilde{E}^a$ are color magnetic and electric fields and $G_{\mu\nu}^a$ is the gluonic field strength tensor.

The question may arise, how do we know that it is the matrix element $\langle 0|\alpha_s (G_{\mu\nu}^a)^2|0 \rangle$ that arises in a straightforward evaluation of the IR sensitive part of the one-gluon exchange graph. The answer to this question is not difficult: this is the simplest expression compatible with the Lorentz and gauge invariance. In other words, the $(G_{\mu\nu}^a)^2$ is the simplest operator which is Lorentz and color singlet. It has dimension $d=4$ implying that the IR sensitive contributions are suppressed at large $Q^2$ as $Q^{-4}$.

What is described here in words, in its generality has an adequate formulation in terms of the Wilson operator expansion (Wilson, 1969). A systematic treatment of the power corrections within the Wilson OPE was given in (Shifman et al., 1979). In particular, it allows to evaluate the contribution of the gluon condensate $\langle 0|\alpha_s (G_{\mu\nu}^a)^2|0 \rangle$ to correlation functions corresponding to various currents $j$.

One may still wonder, how it could be possible to go beyond the perturbation theory through a simple preparation of perturbative graphs. To answer this question, let us try to evaluate the gluon condensate in perturbation theory. To lowest order it is given by a simple one-loop graph:

$$\langle 0|\alpha_s (G_{\mu\nu}^a)^2|0 \rangle \approx \frac{8 \cdot 3}{(2\pi)^2} \int d^4k \frac{k^2}{k^2}.$$  

(20)

where we did not cancel immediately $k^2$ in the denominator and numerator of the integrand to emphasize that the factor $k^{-2}$ is the gluon propagator while the factor $k^2$ upstairs is associated with the derivatives in the vertex. Also, we assumed that there are eight color gluons, as in the realistic case. From the first glance at the expression (20) it is clear that it diverges in the ultraviolet as $\Lambda^4_{\text{UV}}$. However, in all the applications we assume instead that

$$\langle 0|\alpha_s (G_{\mu\nu}^a)^2|0 \rangle \equiv c_G \Lambda^4_{QCD}$$

(21)

where the constant $c_G$ is to be found from the fitting procedure. And there is a rational behind this madness. The point is that the UV divergent part of the integral (21) corresponds the ordinary perturbative contribution. (Of course, there is no real UV divergence in the original Feynman graph (see Fig. 3) since at $k \sim Q$ the account for other lines in the graph provides with a kind of form factor so that the final result is finite.) Thus, we include only the IR contribution into the gluon condensate. Generally speaking, we should have subtracted then the perturbative IR contribution, included now into the matrix element of $(G_{\mu\nu}^a)^2$ from the ordinary perturbation theory. This is to avoid a double counting, as was emphasized by David (1984) and Mueller (1985). There are no difficulties of principle to deal with the problem (see, e.g., Shifman et al. (1979), Novikov et.al. (1985)). However the whole approach is becoming most interesting from the phenomenological point of view if the nonperturbative contribution is enhanced numerically. And this is the assumption which is commonly made at this point. Thus, the OPE allows to introduce in a consistent fashion the notion of the enhancement of the IR sensitive contributions to Feynman graphs.

Coming back to the problem of isolating soft lines in perturbative graphs, the same one-gluon exchange graph can be characterized by another route of the large $Q$ flow, see Fig. 4. Here we have two soft quark lines which means in turn that the corresponding correction to the right-hand side of the sum rules (14) is of the type:

$$\frac{< 0|\bar{q}Oq\bar{q}Oq|0 >}{Q^6}.$$  

(22)

where $O$ is constructed on the Dirac matrices $\gamma_\mu$ in the spinor space and on the Gell-Mann matrices $\lambda^a$ in the color space. Usually one assumes factorization of the 4-quark matrix element reducing it to the quark condensate mentioned above. This time, however, the condensate is not suppressed by power of the (practically vanishing) light quark masses. This is another example how the nonperturbative phenomenon

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of the spontaneous breaking of the chiral symmetry finds its way into the OPE based on analysis of the perturbative graphs.

With account of the quark and gluon condensates the sum rules in the $\rho$-channel become:

$$\int R_\rho(s) \exp(-s/M^2)ds \approx \frac{3}{2} M^2 \times (1 + \frac{\alpha_s(M^2)}{\pi}) + \frac{\pi}{3} \frac{(0|\alpha_s(G_{\mu\nu}^a)^2|0)}{M^4} - 32 \cdot 14\pi^3 \frac{|\langle 0|\alpha_s^{1/2}q\bar{q}|0\rangle|^2}{M^6} + \ldots$$

where the ellipses stand for perturbative and power corrections of higher order. In case of the charmonium sum rules (see (17)) there also appears a term proportional to the gluon condensate.

Eqs. (23) are the QCD sum rules (Shifman et. al., 1979). The first check was to see whether the gluon condensate explains the difference in the duality intervals in the $\rho$- and $J/\psi$- channels (see above). It does explain this difference naturally. Also in many other cases, the QCD sum rules turned to be a very straightforward and successful tool for orientation in the hadronic world. For our review it is most relevant that the sum rules provide with ample confirmation of the idea that it is the soft vacuum fields that are responsible for the breaking of the asymptotic freedom at moderate $M^2$ and signal in this way the formation of the resonances. One can characterize the power corrections by $M_{\text{crit}}^2$ which is defined as the value of $M^2$ where the power corrections become, say, 10% of the partonic contribution. Then the typical $M_{\text{crit}}^2$ associated with the quark and gluon condensates is

$$M_{\text{crit}}^2 \approx 0.6 \text{ GeV}^2$$

and its dependence on the channel for currents constructed on light quarks is not strong. The meaning of the $M_{\text{crit}}^2$ is that the power corrections blow up at smaller $M^2$. Clearly, the value (24) resembles the $\rho$-meson mass squared.

To summarize: the use of OPE allows to parameterize infrared sensitive contributions to the Feynman graphs in a systematic way in terms of vacuum expectation values. While the coefficients in front of the matrix elements are calculable as an expansion in $\alpha_s(Q^2)$, the matrix elements encode the information on nonperturbative phenomena.

**The heavy quark potential.**

Another useful quantity to consider in connection with the power corrections is the heavy quark potential at short distances, for a review and further references see (Akhoury and Zakharov, 1998).

Let us first notice that although the static potential between heavy quarks is obviously a fundamental quantity, its definition in non-Abelian theories is not straightforward at all. The point is that we are interested in case when the quarks are in a color-singlet state. On the other hand, the gauge invariance allows to rotate quarks in the color space locally and, at first sight, it is just impossible to single out a color singlet state. The way out is to consider the so called Wilson loop, that is the average of an operator constructed on the
gauge field potential $A^a_\mu$ along a closed path $C$. Moreover, it is convenient to choose the contour $C$ as a stretched rectangle $C = r \times T$ with large $T$. Then the potential $V(r)$ is:

$$V(r) = - \lim_{T \to \infty} \frac{1}{iT} \ln \langle Tr P \exp \left( ig \oint_C dx^\mu A^a_\mu T^a \right) \rangle. \quad (25)$$

This definition is gauge invariant and refers to the Euclidean space. The main source of knowledge of $V(r)$ are the lattice simulations (see Bali et al, (1995); Bali, (1999)).

The most famous property of the potential $V(r)$ is that it grows linearly at large distances $r$:

$$\lim_{r \to \infty} V(r) = \sigma_\infty r. \quad (26)$$

Note that here we imply that the dynamical light quarks are not taken into account and consider the interaction of infinitely heavy external quarks embedded into the vacuum of pure gluodynamics.

We are mainly interested in the potential $V(r)$ at short distances. Let us begin our analysis with a trivial Yukawa attractive potential and expand it in powers of $(\lambda r)$ at small $r$:

$$V_{Y_u}(r) = - \frac{C}{r} e^{-\lambda r} = - \frac{C}{r} + C\lambda - \frac{C r \lambda^2}{2} + \frac{C r^2 \lambda^3}{6} + \ldots , \quad (27)$$

where $C$ is a positive constant. This simple equation carries an important message, namely, that the physics behind the odd and even powers of the mass $\lambda$ in the expansion (27) is different. Indeed, naively we should have had only powers of $\lambda^2$ since $\lambda^2$ is the only mass parameter entering the Lagrangian of a massive boson. Thus, our idea could be that we start with a Coulomb like potential at small $r$ and develop a perturbative expansion in $\lambda^2 r^2$. Eq. (27) implies that such an approach would fail because of infrared divergences. This can be readily checked explicitly of course. But it suffices to notice that the odd powers in (27) could arise only from infrared divergences which are cut off at distances of order $1/\lambda$.

Appearance of terms non-analytical in $\lambda^2$ in (24) allows to make important conclusions about the heavy quark potential in QCD. Generally, if a physical observable is sensitive to large distances at level of a certain power correction then it cannot be evaluated to such accuracy because the effective coupling is large. Rather, one should reserve for the strength of the correction as a phenomenological parameter. In our case, we can conclude from Eq. (27) that the quark-anti-quark potential in QCD can be parameterized as:

$$V(r) = - \frac{C_{-1}}{r} + C_0 A_{QCD} + C_2 r^2 A_{QCD}^3 + \ldots \quad (28)$$

where $C_{-1}$ is calculable perturbatively as an expansion in $\alpha_s(r)$ while $C_{0,2}$ account phenomenologically for the contribution of large distances. Moreover, $C_0$ can be included into definition of the (heavy) quark masses and the message is that the power corrections to the potential at short distances start with $r^2$ terms (Balitsky (1986). Dosch and Simonov(1988)). A salient feature of Eq. (28) is absence of a linear in $r$ term at short distances. The proof was in two steps. First, we learn from (27) that the linear correction to the potential is not contaminated by large-distance effects and, then, conclude that these corrections vanish since the only parameter of dimension $d=2$ is the gluon mass squared and it vanishes in QCD. Thus, presence of a linear term at short distances in the quark potential would signal new non-perturbative physics at short distances.

The crucial assumption behind Eq. (28) is that typical size of nonperturbative fluctuations is of order $A_{QCD}^{-1}$. This connection can readily be visualized in the abelian case. Namely, begin with an identity for the potential:

$$V(r) = \frac{1}{4\pi} \int d^3r' E_1(r') \cdot E_2(r + r'), \quad (29)$$

where $E_1, E_2$ are electric fields associated with each charge. Eq. (29) is convenient to account for the effect of the change in the electric fields at large distances. Consider an example of two charges of opposite signs in a cavity of size $R$. The electric field of the charges in empty space is that of a dipole at large distances, $E^2 \sim \alpha r^2 / r^6$. Now, because of the cavity the field is changed at $r' \sim R$ and the corresponding feedback to the potential $V(r)$ is of order:

$$\delta V(r) \sim \alpha \frac{r^2}{R^6} \int_R^\infty \frac{d^3r'}{(r')^6} \sim \frac{\alpha r^2}{R^3} \quad (30)$$
which is in agreement with the correction to the static potential discussed above.

After, hopefully, explaining thoroughly that the correction to the potential \( V(r) \) at short distances is of order \( r^2 \) in the standard picture we are going to introduce a new notion, that is retardation effects (Casimir and Polder, 1948). Discussion of this point is a kind of deviation from the course to our immediate goals. We are in fact interested in the static potential at short distances. And then there are no retardation effects, of course. However, historically atom-like systems of heavy quarks were discussed at most and in this case the retardation effects may be important. This is the only reason to include discussion of the retardation effects into this review.

Thus, Casimir and Polder did discuss the interaction of atoms with nonperturbative long-wave fields in QED about 25 years before the advent of QCD. What kind of nonperturbative fields did they consider? At first sight at least, there no nonperturbative fields in QED. They considered atom in a cavity. Then zero-point fluctuations are modified by the presence of the cavity at frequencies of order \( \omega \sim R^{-1} \) where \( R \) is the cavity size. If in the QCD case, we substitute the cavity of size \( R \) by a nonperturbative field of the size \( \Lambda_{QCD}^{-1} \), the problems turn similar!

Of course, here we present only a very sketchy view of the paper by Casimir and Polder. One starts with the dipole interaction (see also above):

\[
H_{int} = - e \mathbf{d} \cdot \mathbf{E}.
\]

The shift in energy levels of atoms arises then in second order in this interaction:

\[
\delta E_n \sim \sum_k V_{nk}(E_n - E_k + \omega_{\text{char}})^{-1}V_{kn},
\]

where the characteristic frequency of the nonperturbative fields is of order \( \omega_{\text{char}} \sim 1/R \). The result for the energy shift \( \delta E_n \) depends crucially on the relative magnitude of \( \omega_{\text{char}} \) and \( E_n \sim m\alpha^2 \sim \alpha/a_B \) where \( a_B \) is the Bohr radius. Namely, if \( \omega_{\text{char}} \gg E_n \), then

\[
\delta E_0 \sim \alpha d^2 E^2 R \sim \alpha \frac{a_B^2}{R^3}, \quad R \ll a_B/\alpha
\]

since \( d^2 \sim a_B^2 \) and \( E^2 \sim R^{-4} \).

The shift \( \delta E \) corresponds to our estimate given above in the language of the classical electrodynamics. In this case, the potential picture applies to the evaluation of the energy shifts.

On the other hand, if \( R \gg a_B/\alpha \) then \( \omega_{\text{char}} \) in the energy denominator in \( \delta E \) can be neglected and

\[
\delta E_0 \sim \alpha d^2 E^2 \frac{a_B}{\alpha} \sim \frac{a_B^2}{R^4}, \quad R \gg a_B/\alpha,
\]

where we used the estimates of \( E^2, d^2 \) given above and substituted \( E_n \sim \alpha/a_B \).

The estimate \( \delta E_0 \) in clear violation of the potential picture. Moreover, Eq. \( \delta E_0 \) could be interpreted by saying that if the distance \( a_B \) between the particles is much smaller than \( \alpha R \) then the \( r^2 \) potential is replaced by a \( r^3 \) potential \( \mathbf{d} \cdot \mathbf{E} \). Note also that the emergence of the scale \( R \sim a_B/\alpha \) in the equations above can be understood as an effect of retardation. Indeed, the time needed to communicate with the distances of order of the cavity size \( R \) can be called the retardation time, \( T_{\text{ret}} \sim R \). For the potential picture to be valid, \( T_{\text{ret}} \) is to be smaller than the revolution time which is of order, \( T_{\text{rev}} \sim a_B/\nu \sim a_B/\alpha \). The potential picture becomes distorted once \( T_{\text{ret}} \approx T_{\text{rev}} \).

In QCD case one considers (see Voloshin (1979), Leutwyler (1981)) atom-like systems of heavy quarks \( Q \) such that

\[
\alpha_s(M_H) \cdot M_H \gg \Lambda_{QCD}
\]

\(^{c}\)But this is true only as far as rough estimates are concerned. Rigorously speaking, there is no potential whatsoever corresponding to the shifts obtained in this way. This was emphasized much later, in connection with QCD, by Voloshin (1979) and Leutwyler (1981).
where $M_H$ is the mass of the heavy quark and the condition $\frac{\Lambda_{QCD}^2}{\Lambda_{QCD}^4}$ is that the Bohr radius is much smaller than $\Lambda_{QCD}^{-1}$. Again, the dipole interaction is relevant to describe interaction with soft gluon fields:

$$H_{int} = -\sqrt{\alpha_s(t_1^a - t_2^a)} \mathbf{d} \cdot \mathbf{E}^a,$$

where $t_i^a (i = 1, 2)$ are generators of the color group and refer to the quark $Q$ and anti-quark $\bar{Q}$ in the quarkonium while $\mathbf{E}^a$ denotes the soft gluonic field in the vacuum.

Moreover, one assumes that $\omega_{char} \sim \Lambda_{QCD}^{-1}$. As for the intensity of the gluonic fields, it is characterized again by the gluon condensate $<\alpha_s(G_{\mu\nu}^a)^2>$. The resulting shift of the energy levels of the $S$-states depends strongly on the principal quantum number $n$:

$$\frac{\delta E_{nl}}{M_H} = n_0 \pi <\alpha_s(G_{\mu\nu}^a)^2> (M_H C_F \alpha_s)^{3/2} \epsilon_{nl},$$

where the numbers $\epsilon_{nl}$ depend on the quantum numbers of the level, $n, l$, and are of order unity, e.g., $\epsilon_{10} \approx 1.5$. The growth with $n$ is due to the growth of the size of the atom-like state. However, to apply (37) one should assume that the retardation effects are already very important. For a concrete value of the heavy mass $M_H$ the balance between these requirements can be quite delicate and we refer the reader to a review by Yndurain (1998) for details and references.

To summarize: according to the standard QCD there is no linear correction to the Coulomb potential at short distances. This is in variance with naive unification of the linear potential (dominating at large distances) and Coulomb-like potential (dominating at small distances).

Moreover, because of the retardation effects the effect of the leading $r^2$ correction in $V(r)$ would be washed out in heavy quarkonium and the nonperturbative corrections to the Coulombic potential at short distances would start with even smaller terms of order $r^3$.

Other techniques: infrared renormalons, infinitesimal gluon mass.

Notice that while discussing $V(r)$ we did not resort to the OPE. It is not by chance of course. The point is that no matter how simple the notion of the potential $V(r)$ could appear, the OPE cannot be applied directly to evaluate $V(r)$. The reason is that we consider the static potential which means, potential energy averaged over large period of time. Thus, although we consider small spatial distances $r$, the potential $V(r)$ is sensitive to large intervals in the time directions.

It is not the only case when the OPE does not apply of course. Moreover, as was mentioned above, the OPE applies actually in the Euclidean space and only in some cases, when the analytical properties are simple, the results can be continued to the Minkowski space.

Thus, it is desirable to develop techniques to evaluate the power corrections directly in the Minkowski space. And there exist such techniques. In particular, we will discuss here infrared renormalons and infinitesimal gluon mass. We do not have time to cover the topics in detail. Instead we will illustrate the technique by two examples, rederiving the results which we already know. A more detailed exposition and further references can be found in reviews, see, e.g., Dokshitzer et. al. (1996), Akhoury and Zakharov (1997), Zakharov (1998), Beneke (1998).

First, we will consider the power corrections to the potential $V(r)$ by using the IR renormalon technique (Aglietti and Ligeti, (1995); Akhoury and Zakharov, (1998)). Consider to this end the one gluon exchange potential

$$V(r) = -C_F \int \frac{d^3k}{(2\pi)^3} \pi \alpha_s(k^2) \exp(i\mathbf{k} \cdot \mathbf{r}) \frac{\exp(i\mathbf{k} \cdot \mathbf{r})}{k^2},$$

where $C_F$ is related to the color indices and $C_F = 4/3$ in the realistic case of the $SU(3)$ color group. Note also that running of the $\alpha_s(k^2)$ incorporates the leading logarithmic corrections. To find the renormalon contribution we rewrite (38) identically as

$$\alpha_s(k^2) = \int d\sigma \left( \frac{k}{\Lambda_{QCD}^2} \right)^{-2\sigma_b}.$$
Next, substitute (39) into (38) and perform integration over directions of \( k \) to get

\[
V(r) = \frac{2}{3\pi^2} \int_0^\infty dk \frac{\sin(kr)}{kr} \left( \frac{k}{\Lambda_{QCD}} \right)^{-2b_0}.
\]

Moreover, since we are interested in \( V(r) \) at small \( r \) we expand in \( kr \):

\[
V(r) = -\frac{2}{3\pi^2} \int_0^\infty d\sigma \Lambda_{QCD}^{2b_0} \int dk (k^{-2b_0} + k^2 r^2 + ...).
\]

Now, integrating over \( k \) introduces what is called renormalon poles at

\[
\sigma_{pole} = \frac{1}{2} b_0, \frac{3}{2} b_0, ...
\]

The next integration, that is over \( \sigma \), becomes undefined as a result of these poles. One can resort, say, to defining the integrals as their principal values. But this prescription is openly arbitrary and one is to reserve for an overall rescaling of the pole contributions. As a result, we get as corrections to the Coulombic potential at short distances:

\[
\delta V(r) = c_0 \Lambda_{QCD} + c_2 \Lambda_{QCD}^3 r^2 + ...
\]

reproducing the result speculated upon above. Since we expand in \( kr \) and \( k \sim \Lambda_{QCD}^{-1} \), Eq. (43) is valid at \( r \ll \Lambda_{QCD}^{-1} \)

Since we were able to determine the leading powers for the power corrections by means of the renormalons, one could think that the OPE is no better than the renormalons. It is, unfortunately, not true. Namely, renormalons do not allow, generally speaking, for a systematic expansion of the coefficients in front of the power corrections in small coupling, like \( \alpha_s(k^2) \). Generally speaking, all orders of perturbative expansion collapse to the same order contribution if projected onto the power corrections.

Let us illustrate the point by the same example of the quark potential. In fact three first orders of the perturbative expansion in front of \( 1/r \) were explicitly found, see Peter (1997):

\[
V(k^2) = -\frac{4C_F\pi\alpha_{MS}}{k^2} \left( 1 + \frac{\alpha_{MS}}{\pi} (2.583 - 0.278 n_f) + \frac{\alpha_{MS}^2}{\pi^2} (39.650 - 4.147 n_f + 0.077 n_f^2) \right)
\]

Moreover, in the approximation of the one-loop \( \beta \)-function,

\[
\alpha_s^2(k^2) = \frac{1}{2b_0} \Lambda_{QCD} \frac{d}{d\Lambda_{QCD}} \alpha_s(k^2).
\]

By differentiating \( n \) times \( \alpha_s \) with respect to \( \ln \Lambda_{QCD} \) we immediately find contribution of \( (n+1) \) renormalon chains, associated with the \( \alpha_s^{n+1}(k^2) \) in the perturbative expansion.

Thus, we have a unique possibility to compare the contributions of one-, two- and three renormalons:

\[
\delta V(r) \approx (const) \Lambda_{QCD}^3 r^2 (1 + 1.1 + 6.0 + ...).
\]

We see that there is no convergence in sight, even numerically.

The general rule is that if perturbatively

\[
V(k^2) = (4\pi \alpha_s/k^2)f(\alpha_s)
\]

then

\[
\delta V(r) = v_0 \Lambda_{QCD} f^B(\alpha_s = 1/2b_0) + v_2 \Lambda_{QCD}^3 f^B(\alpha_s = 3/2b_0)
\]

\[d\text{Strictly speaking, we should have kept further terms in the } \beta \text{-function since we consider a multi-loop effect. Thus, the example is still rather illustrative than rigorous.}\]
where \( v_{0.2} \) are constants and \( f^B \) is the Borel-improved expansion, i.e. the series with the expansion coefficients 
\[ a_n^B = a_n / n! . \]

Another technique which can be used directly in the Minkowski space is the introduction of a (fictitious) gluon mass \( \lambda, \lambda \rightarrow 0 \) and tracing terms non-analytical in \( \lambda^2 \). In fact, we used this technique when analyzed the Yukawa potential. Now we would like to emphasize its generality. Moreover, the idea is similar in fact to that underlying the introduction of the gluon condensate.

Indeed, let us consider again the gluon condensate perturbatively (see Eq. 20) but this time in case of a finite gluon mass (see Fig. 5):

\[
\langle \alpha_s (G^a_{\mu \nu} )^2 \rangle = \frac{6 \alpha_s \cdot 8}{(2 \pi)^4} \int \frac{d^4k}{k^2 + \lambda^2} \, .
\]

It diverges wildly in the ultraviolet, as discussed. Let us define the gluon condensate, however, as the non-analytical in \( \lambda^2 \) part of the perturbative answer (Chetyrkin and Spiridonov, 1988):

\[
\langle 0 | \alpha_s (G^a_{\mu \nu} )^2 | 0 \rangle = \frac{3 \alpha_s}{\pi^2} \int_0^\infty \frac{k^4 dk^2}{(k^2 + \lambda^2)} = - \frac{3 \alpha_s}{\pi^2} \lambda^4 \ln \lambda^2 .
\]

Note that there is no difficulty in practice to pick up the non-analytical in \( \lambda^2 \) terms. To this end, it is sufficient to differentiate (49) twice with respect to \( \lambda^2 \).

Moreover, to finally get rid of the gluon mass (which is not a pleasant sight for a theorist’s eye) we make a replacement:

\[
\alpha_s \lambda^4 \ln \lambda^2 \rightarrow c_4 \Lambda_{QCD}^4
\]

where \( c_4 \) is an unknown coefficient. The central point is that if we evaluate various correlation functions and isolate the terms \( \lambda^4 \ln \lambda^2 \) we would reproduce the sum rules (23). Indeed, as we explained above the gluon condensate in the sum rules (23) parameterizes the infrared sensitive part of the Feynman graphs associated with a soft gluon line. Picking up terms non-analytical in \( \lambda^2 \) is the same good for this purpose since the non-analyticity in \( \lambda^2 \) can obviously arise only from soft gluons, \( k \sim \lambda \).

So far we have not got any new result, though. The real advantage of introducing \( \lambda \neq 0 \) is that calculations can be performed now in Minkowski space as well and apply for this reason to a much wider class of observables than the OPE underlying the QCD sum rules (23). The link to QCD is again through the bald replacement of the non-analytical in \( \lambda^2 \) terms by corresponding powers of \( \Lambda_{QCD} \):

\[
\begin{align*}
\alpha_s \sqrt{\lambda^2} & \rightarrow c_1 \Lambda_{QCD} \\
\alpha_s \lambda^2 \ln \lambda^2 & \rightarrow c_2 \Lambda_{QCD}^2 \quad \ldots
\end{align*}
\]

where \( c_{1,2} \) are some coefficients treated as phenomenological parameters. The gluon condensate appears now merely as one of the terms in this sequence.

The phenomenology based on such rules turns successful (see, e.g., Webber (1999) and references therein). The problem is not so much a lack of success but rather too much of overlap (Akhoury and Zakharov, 1996) with old-fashioned hadronization models, like the tube model.

To summarize: there exist simple techniques which allow to parameterize the infrared sensitive parts of the Feynman graphs directly for observables in the Minkowski space. The success of this, most naive approach reveals that at least in the cases when the technique applies the nonperturbative effects reduce to a simple amplification of infrared sensitive contributions to the Feynman graphs.

\[\text{Footnote}: \text{The technique with } \lambda \neq 0 \text{ is in fact close to the technique based on the infrared renormalons, mentioned above. In the context of the gluon condensate, the infrared renormalons were considered first in (David, 1984) and (Mueller, 1985).}\]

\[\text{Footnote}: \text{There is a price to pay, however. Infrared renormalons if applied directly in the Minkowski space do not allow for model independent relations between power corrections to different observables. The reason is that the coupling } \alpha_s \text{ refers now to infrared region and is, therefore, of order unity. As a result, all orders in } \alpha_s \text{ are equally important, see (Zakharov, 1998). This is the same “collapse” of the perturbative expansion on the level of the power corrections illustrated above on the example of the potential } V(r).\]
Direct instantons

This simple picture is not universally true, however. First examples when it fails were found as early as in 1981 (Novikov et.al.) Namely, there exist channels where the infrared sensitive corrections described above cannot be the whole story. To give counter-examples, one can either rely on the analysis of experimental data or on theoretical methods which would allow to evaluate the power corrections without using the OPE. Both ways were exploited to demonstrate that there exist corrections which can be very important numerically and which go beyond the picture described above.

In particular, in the $0^+\rightarrow\rightarrow$-gluonium channel one can establish a low energy theorem relating the value of the correlation function at $Q^2 = 0$ to the same gluon condensate $<\alpha_s(G^a_{\mu\nu})>$. Moreover, one can utilize this knowledge to subtract the dispersion relations and convert the $\Pi(0)$ into a power correction at large momenta. As a result the following sum rules arise:

$$
\int ImG(s) \exp(-s/M^2)ds/s \approx G(M^2)_{\text{parton model}} \left[ 1 + \frac{<0|\alpha_s(G^a_{\mu\nu})^2|0>}{M^4} \left( \frac{2\pi^2}{\alpha_s(M^2)} + \frac{16\pi^2}{\beta_0\alpha_s^2(M^2)} \right) \right] \quad (52)
$$

where

$$G(Q^2) \equiv i \int d^4x \exp(iq\cdot x) \langle 0| T\{ (G^a_{\mu\nu}(x))^2 , (G^a_{\mu\nu}(0))^2 \} |0\rangle. \quad (53)$$

To reiterate, the power correction proportional to the gluon condensate originates here from two sources. First, there is a standard OPE correction (see (23)) and, second, the one evaluated via a low energy theorem specific for this particular channel. The correction which is not caught by the standard OPE is about (20-30) times larger!

Thus, if we characterize the scale where the asymptotic freedom gets violated by the power correction by the value $M^2_{\text{crit}}$ as is mentioned above, then $M^2_{\text{crit}}$ differs drastically in some channels:

$$
(M^2_{\text{crit}})_{\rho\text{-meson}} \approx 0.6 \text{ GeV}^2, \quad (M^2_{\text{crit}})_{0^+\text{-gluonium}} \approx 15 \text{ GeV}^2. \quad (54)
$$

We see that the proof of the low-energy theorem brought a proof of existence of qualitatively different scales in the hadron physics (Novikov et.al., 1981).

One may call the $0^+\rightarrow\rightarrow$-gluonium channel exceptional because of the appearance of this new correction. In case of the $\pi$-meson channel, it was also possible to show that the standard OPE corrections are not big enough to match the contribution of the pion into the dispersive part of the sum rules. Moreover, there arises a kind of hierarchy of exceptional channels, or of $M^2_{\text{crit}}$.

This hierarchy could be explained qualitatively in terms of transitions of the corresponding currents directly to instantons, see Fig. 5 (Novikov et. al., (1981), Geshkenbein and Ioffe (1980)). At large $Q$ the direct instanton contribution dies off fast, like $(Q^2)^{-4.5}$. However, at intermediate $Q^2$ these corrections may
become important first because of a big overall coefficient. Unfortunately, one cannot go far with clarification of
the situation by using only analytical means. Nowadays one relies on the model of instanton liquid which allows for a
much more quantitative treatment of the instanton effects (for a review and further references see (Shuryak and
Schafer, 1997). The model appears to be successful phenomenologically.

3 BEYOND THE OPE.

Elusive effects of confinement

Looking backward, it still remains a mystery whether any specific confinement effects are revealed through
the power corrections discussed so far. Indeed, consider the vacuum of pure gluodynamics. It is known
from lattice measurements that external heavy quarks are confined by this medium. On the other hand, the
effects included into the sum rules so far do not seem to encode the confinement. Indeed, the perturbative
QCD resembles ordinary bremsstrahlung in QED. The gluon condensate, as well as other newly found power
corrections can be detected by introducing a fictitious gluon mass which is not sensitive to the non Abelian
nature of gluons at all. Finally, instantons are known not to ensure the confinement either (Chen et.al.,
1998).

In an attempts to find power corrections related more directly to the physics of confinement one can
turn to the Abelian Higgs model (AHM) which underlies the dual superconductor model of the confinement
(Mandelstam (1974), Nambu, (1974); Polyakov (1975), ’t Hooft (1976)). The basic idea behind the model is
that the properties of the QCD vacuum are similar to the properties of an ordinary superconductor. Indeed,
if a pair of magnetic charges is introduced into a superconductor, the potential energy of the pair would
grow linearly with the distance \( r \) at large distances:

\[
\lim_{r \to \infty} V(r) \approx \sigma_\infty \cdot r \tag{55}
\]

where \( \sigma_\infty \) is the tension of the Abrikosov-Nielsen-Olesen string. In QCD, a similar phenomenon was postu-
lated to happen, with a change of magnetic charges to (color) electric, or dual charges.

If one introduces a pair of external magnetic charges into the vacuum of the AHM model in the Higgs
phase then the potential grows at large distances, see Eq. (55). The scale of distances is set up by the inverse
masses of the vector and scalar fields, \( m_{V,S}^{-1} \). This growth of the potential is due to the Abrikosov-Nielsen-
Olesen strings.

Consider now \textit{short} distances, \( r \ll m_{V,S}^{-1} \). Then the Coulomb like interaction dominates. However, there
is a stringy correction to the potential at any small distances (Gubarev et. al., 1998):

\[
\lim_{r \to 0} V(r) = - \frac{Q_M^2}{4\pi r} + \sigma_0 r \tag{56}
\]

where \( Q_M \) is the magnetic charge. The ANO string is a bulky object on this scale and is not responsible for
the linear correction. Instead, the stringy potential at short distances is due to an infinitely thin topological
string which connects the magnetic charges and which is defined through vanishing of the scalar field along the
string. Thus, it turns out that at least in this model the confined charges learn about confinement already
at small distances because of the short strings which are in fact seeds for future confining ANO strings.
Amusingly enough, it demonstrates that at short distances a dimension two quantity is not necessarily the
 gluon mass squared but could be a string tension as well. Eq. (56) is the main result which we are going to
substantiate in this part of the lectures. However, we need to make a few steps before we can explain (56).
Also, we shall spend some time later to discuss the validity of the OPE.

Field theoretical solenoid.

A key element in the conjectured mechanism of the quark confinement is the Abrikosov-Nielsen-Olesen string
(Abrikosov. 1957), (Nielsen and Olesen 1973). Although eventually it will \textit{not} be this string that captures
our attention, it is useful to begin with a brief review of the ANO string.
The strings are classical solutions to the Abelian Higgs model (AHM). The model describes a gauge field \( A_\mu \) interacting with a charged scalar field \( \Phi \) as well as self-interactions of the scalar field. The corresponding action is:

\[
S = \int d^4x \left\{ \frac{1}{4e^2} F_{\mu\nu}^2 + \frac{1}{2} \left[ (\partial - i A) \Phi \right]^2 + \frac{1}{4} \lambda (|\Phi|^2 - \eta^2)^2 \right\}
\]

(57)

where \( e \) is the electric charge, \( \lambda, \eta \) are constants and \( F_{\mu\nu} \) is the electromagnetic field-strength tensor, \( F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \). The scalar field condenses in the vacuum, \( \langle \Phi \rangle = \eta \), and the physical vector and scalar particles are massive, \( m_V^2 = e^2 \eta^2, m_H^2 = 2\lambda \eta^2 \). In the perturbative regime, interaction of the massive scalar and vector particles can be readily calculated order by order.

There exists, however, a topologically non-trivial stringy solution to the classical equations of motion which possesses a cylindrical symmetry and is characterized by a finite energy per unit of its length. In a way, it realizes an analog of a solenoid in field theory. The key element is to look for a solution with a non-vanishing electric current

\[
j_\mu = e (\Phi^* (\partial_\mu \Phi) - (\partial_\mu \Phi^*) \Phi).
\]

(58)

Moreover, if one chooses the scalar field of the form,

\[
\Phi(r) = e^{in\phi} f(\rho)
\]

(59)

then the electric current is circular and reminds, therefore, the solenoid current. The value of \( n \) in Eq. (59) is integer for the field \( \Phi \) to be a unique function of the coordinates. Since the current has only a nonvanishing \( j_\phi \) component, the functional form of the matching vector potential is

\[
A(\rho) = \frac{e}{\rho} A(\rho).
\]

(60)

The central question is whether it is possible to ensure finiteness of the energy (per unit length) with the ansatz (59), (60). The behavior of the fields at \( \rho \to 0 \) and \( \rho \to \infty \) is crucial at this point and it is easy to check that the conditions

\[
\begin{align*}
    f(0) &= A(0) = 0 \\
    f(\infty) &= \eta \\
    A(\infty) &= \frac{n}{e}
\end{align*}
\]

(61)

suffice to make the energy finite.

From these conditions alone, one can derive the magnetic flux carried by the ANO string. Indeed,

\[
\int \mathbf{H} \cdot d\mathbf{S} = \oint A_\mu dx_\mu = \frac{2\pi}{e} n,
\]

(62)

where to evaluate the latter integral we used the asymptotic value of \( A_\phi \), see Eq. (61). Thus, the flux is an integer of a minimal magnetic monopole charge. It is obvious then that the strings can end up with monopoles.

Explicit form of the functions \( f(\rho), A(\rho) \) introduced above can be found by solving the classical equations of motion:

\[
\begin{align*}
    \nabla^2 A_\mu &= \frac{i e}{2} (\Phi^* \partial_\mu \Phi - \Phi \partial_\mu \Phi^*) - e^2 A_\mu \\
    \nabla^2 \Phi + 2ieA_\mu \partial_\mu \Phi - e^2 A_\mu A^\mu &= \lambda \eta^2 \Phi - \lambda |\Phi|^2
\end{align*}
\]

(63)

In particular, one concludes from these equations that the function \( f(\rho) \) approaches its asymptotic value at \( \rho \to \infty \) exponentially, as \( \exp(-m_H \rho) \). The magnetic field falls off exponentially, as \( \exp(-m_V \rho) \). Notice, however, that the vector potential \( A_\phi \) falls off only as an inverse power of \( \rho \). There is no field strength tensor associated with the asymptotic value of \( A_\phi \). As for the full functions \( f(\rho), A(\rho) \) they can be obtained by numerical methods.
Details of the solution depend on the ratio of the masses, \( m_V \) and \( m_H \). Two limiting cases are of special interest. In the so called Bogomolny limit,

\[
m_V = m_H = m, \tag{64}
\]

the equations simplify greatly and can be solved in fact analytically, as a series in \( m \rho \). Another useful case is the so called London limit,

\[
m_H^2 \gg m_V^2 \tag{65}
\]

In this limit, the scalar field is frozen as \( \Phi = \eta \) and one can neglect in most cases the fluctuations of the field \( \Phi \) around its vacuum value. However, the string tension, or energy per unit length becomes logarithmically divergent in this limit:

\[
\sigma_\infty \approx \frac{\pi m_V^2}{2e^2} \ln \frac{m_H^2}{m_V^2} \tag{66}
\]

Indeed, without the Higgs field one cannot construct a solution with a finite string tension.

From the physical point of view, the most important manifestation of the ANO string is the linearly growing potential energy for an external monopole-antimonopole pair \( \Phi = \eta \), when the distance \( r \) is much larger than the inverse masses, \( m_V^{-1}, m_H^{-1} \). Thus, the magnetic charges are confined. This property of the Abelian Higgs model is the basis for the dual superconductor mechanism of the quark confinement. Namely, one assumes that the QCD vacuum is similar to the vacuum of the AHM in the sense that a scalar field with non-vanishing magnetic charge condenses in the QCD vacuum. Then the (color) electric charge is confined.

Note that once we allowed for point-like monopoles, we should have considered the energy of the Dirac string as well. Let us discuss this issue on another example, that is compact \( U(1) \) gauge theory.

**Compact Photodynamics.**

In this section we will outline the paper (Polyakov, 1975) which, from the point of view relevant to the present lectures, demonstrates that there can exist a very different source of nonperturbative effects than those we discussed so far.

The action of the theory we are going to consider is very simple:

\[
S = \frac{1}{4e^2} \int d^4x F_{\mu\nu}^2 \tag{67}
\]

where \( F_{\mu\nu} \) is the abelian field strength tensor, \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \). The action (67) is that of free photons and at first sight nothing interesting can come out from this theory. In particular, if we introduce external static electric charges as a probe, their potential energy would be given by one-photon exchange without any corrections.

However, we shall see in a moment that, in a particular formulation, the theory admits also magnetic monopoles. Hence, a few preliminary words on the monopoles. Monopoles have magnetic field similar to the electric field of a charge:

\[
H = qM \frac{r}{4\pi r^3} \tag{68}
\]

Then the flux of the magnetic field through a surface surrounding the monopole is:

\[
\Phi = qM \tag{69}
\]

On the other hand, because of the equation \( \text{div } H = 0 \) the magnetic flux is conserved. Thus, the magnetic monopole cannot exist by itself and one assumes that there is a string which is connected to the monopole and which brings in the flux. Moreover, to make the string invisible one assumes that the string is infinitely thin. Finally, to avoid the Bohm-Aharonov effect one imposes the Dirac quantization condition,

\[
e \oint A_\mu dx_\mu = e \int \mathbf{H} \cdot d\mathbf{s} = 2\pi \cdot n, \quad \text{or} \quad q_M = \frac{2\pi}{e} n \tag{70}
\]
Also, we ask for the energy (or action) associated with the Dirac string to vanish. Only then energies of the electric and magnetic charges are similar. We shall return to discuss the issue of the energy of the Dirac string later.

Now, the Dirac strings may end up with monopoles. The action associated with the monopoles is not zero at all but rather diverges in ultraviolet, since

$$\int \frac{d^3 r}{4\pi} H^2 \sim \frac{1}{e^2 a}$$

where $a$ is a (small) spatial cut off. If the length of a closed monopole trajectory is of order $L$, then the suppression of such configuration due to a non-vanishing action is of order

$$e^{-S} \sim \exp \left( -\text{const} \frac{L}{e^2} \right).$$

On the other hand, there are different ways to organize a loop of length $L$. This is the entropy factor. It is known to grow exponentially with $L$ as $\sim \exp \left( \text{const}' L \right)$.

Thus, one comes to the conclusion (Polyakov, 1975) that at some $e = e_{\text{crit}} \sim 1$ there is a phase transition corresponding to condensation of the monopole loops. As a result, if external electric charges are introduced as a probe, their potential energy grows with distance, $V(r) \sim r$ and they are confined.

To complete the presentation we should explain how one should understand the theory (67) that would imply a vanishing action for the Dirac string.

The crucial point is to define the theory by means of a lattice regularization. Then the action can be understood as a sum over plaquette actions:

$$S = \sum \frac{1}{2e^2} \left( 1 - \text{Re} \exp(i \oint A_\mu dx^\mu) \right) = \sum \frac{1}{2e^2} \left( 1 - \text{Re} \exp(i F_{\mu\nu} d\sigma_{\mu\nu}) \right) = \sum \frac{1}{2e^2} \left( 1 - \cos(F_{\mu\nu} d\sigma_{\mu\nu}) \right),$$

where the sum is taken over all plaquettes. In the continuum limit one reproduces of course the action (67). However, from the intermediate steps it is clear that the action admits for a large jump in $F_{\mu\nu}$:

$$F_{\mu\nu} \rightarrow F_{\mu\nu} + 2\pi \delta(\sigma_{\mu\nu})$$

where the $\delta$-function on the surface is defined as $\delta(\sigma_{\mu\nu}) d\sigma_{\mu\nu} = 1$ (no summation over $\mu, \nu$). The second term in Eq. (73) exactly corresponds to the Dirac string. Thus, the Dirac strings have no action in the compact version of the $U(1)$ gauge theory.

Note that the UV scale $1/a$ is the only scale at the model. Thus, it rather exists only as a lattice theory. The confining potential sets in for $e > e_{\text{crit}}$ at all the distances and in this sense the situation does not imitate QCD. There is a possibility that a truly continuum theory can be defined near $e = e_{\text{crit}}$ but we would not discuss this topic here (for a recent discussion and further references see, e.g., Jersak et.al (1999))

**Topology of gauge fixing**

The phenomenon of the monopole condensation is well established in QCD through numerical simulations on the lattice, for review see (Chernodub et. al., 1998), (Di Giacomo, 1998).

The path to understanding magnetic monopoles in gluodynamics necessarily goes through the paper by ’t Hooft (1980) where the monopoles were introduced as local objects and on general topological grounds. It is convenient to start, however, with the Abelian Higgs model, see Eq. (57), which contains rather strings than monopoles, to learn a new technique. The gauge transformation rotates the phase of the charged field $\Phi$:

$$\Phi \rightarrow e^{i\varphi} \Phi.$$  \hspace{1cm} (74)
In particular one can rotate Φ in such a way that

\[ \text{Im} \Phi = 0 \quad (75) \]

which defines what is called the unitary gauge. The reason for the name is that if \(<\Phi>\neq 0\) then the excitation spectrum in this gauge consists of physical massive vector and scalar particles, with no ghosts.

The crucial point is that, as remarked by 't Hooft, the condition (75) fixes the gauge uniquely unless the real part of Φ is also vanishing. Thus, there are exceptional points characterized by the equations

\[ \text{Im} \Phi = 0 \]
\[ \text{Re} \Phi = 0. \quad (76) \]

Viewed as two equations in four dimensional world, these conditions define a world sheet, or trajectory of a string. Note that the definition (76) is local in the sense that it defines an infinitely thin string.

The definition does not give any immediate clue as to whether the strings (76) are important dynamically. However, one may notice that Φ does vanish on the central axis of the ANO string, see (61). If one makes such an identification then it is easy to conclude that the world sheets (76) are either closed, corresponding to closed ANO strings, or end up with monopoles.

In case of a nonabelian theory, one introduces first the so called \(U(1)\) projection which is nothing else but (partial) gauge fixing. Namely, one chooses a field in an adjoint representation. If we concentrate of \(SU(2)\), then this field is a triplet \(\Phi^a\). The field \(\Phi^a\) could be a component of the nonabelian field strength tensor \(F_{\mu \nu}^a\) or a composite operator. Then one chooses the gauge to rotate \(\Phi = \Phi^a \sigma_2\) to, say, the third direction in the color space:

\[ \Phi \rightarrow \Omega \Phi \Omega^{-1} = \Phi^a \frac{\sigma_3}{2} \quad (77) \]

where \(\Omega\) is the matrix of the gauge transformation. The condition (77) fixes the gauge only up to rotations around the third axis in the color space, that is up to a remaining \(U(1)\) symmetry.

What is crucial for introduction of the monopoles is that the condition (77) can be implemented everywhere except for the points

\[ \Phi^a \equiv 0. \quad (78) \]

Thus, the exceptional points are given by solutions to three equations in the four-dimensional space and specify therefore a trajectory. Actually, this is a monopole trajectory. To prove this, one follows exactly the same sequence of steps as in establishing the relation of the 't Hooft-Polyakov monopole to the Dirac magnetic monopole, see ('t Hooft, 1974). Indeed, in Georgi-Glashow model we diagonalize the higgs triplet \(\Phi^a\) as in (77) and \(|\Phi|^2\) vanishes at the center of a 't Hooft-Polyakov monopole. And the basic observation is that the algebra associated with the rotation of the 't Hooft-Polyakov monopole to the Dirac monopole is in fact uniquely determined by this circumstance. A detailed derivation can be found, e.g., in (Simonov, 1996) or in (Chernodub et.al., 1998).

Note that the definition of the monopole (78) is pure topological, the same as the definition of the string above, see (66). It does not at all imply existence of a classical monopole solution. However, it provides with a local definition of a magnetically charged field \(\Phi_M(x)\) and allows to study the monopole condensation. Note also that the definition of the monopoles depend on the choice of the auxiliary field (operator) \(\Phi^a\). The dual superconductor model of confinement is confirmed numerically most impressively within the so called maximal Abelian gauge. In this case, one fixes gauge, up to a \(U(1)\) subgroup by minimizing the value of \((A_1^1)^2 + (A_2^2)^2\), where \(A_1^1, A_2^2\) are components of the vector potential in the color space. The remaining \(U(1)\) symmetry is the phase rotation of the complex field \(A_1^1 + iA_2^2\). We note all this only in passing, and the reader is advised to consult (Chernodub et.al., 1998) for any detail.

To summarize: both strings in the Abelian Higgs model and monopoles in gluodynamics can be defined locally in topological terms. These definitions do not provide however, with any straightforward way to evaluate the dynamical significance of these objects.
Ultraviolet regularization and non-perturbative effects.

Now we would like to introduce a new idea on the connection of the topological strings and monopoles discussed in the preceding section and nonperturbative ultraviolet divergences. Indeed, intuitively it is appealing to guess that, if the string- or monopole-like solutions with finite energy exist only for theories with scalar fields, similar point-like, or topological excitations build up on the gauge fields alone have infinite action, generally speaking. Only if we accept a prescription of an ultraviolet regularization which eliminates these divergences, such excitations may play a dynamical role. Although intuitively the idea looks appealing, we shall not be able to prove it in its generality but rather illustrate it on examples.

Let us begin with the Dirac string. Naively, the energy \( \epsilon_D \) is equal to:

\[
\epsilon_D \sim \int \mathbf{H}^2 d^3r \sim \frac{(\text{Flux})^2}{A^2} \int dl,
\]

where \( A \) is the cross section of the Dirac string. Thus, \( \epsilon_D \) diverges as \( 1/A \) (quadratically) in ultraviolet once \( A \to 0 \).

Instead, we assumed \( \epsilon_D = 0 \). There are two ways to justify this assumption:

(i) Lattice regularization, as discussed in the preceding subsection.

(ii) Duality between electric and magnetic charges. Since the electric charges do not have strings attached, the action associated with the Dirac string is postulated to vanish.

Note that while in case of the compact \( U(1) \) we relied on point (i), while in case of the Abelian Higgs model we had to rely on (ii).

Turning to \( SU(2) \), there is of course a possibility to introduce a Dirac string associated with any \( U(1) \) subgroup. In this connection, let us mention another language for the strings, that is singular gauge transformations, for details see, e.g., Chernodub (1998). Gauge transformations can be specified in terms of a matrix \( \Omega \):

\[
\Omega(x) = \exp \left( i \alpha^a(x) \sigma^a \right)
\]

where \( \alpha^a(a = 1, 2, 3) \) are parameters of the transformation and \( \sigma^a \) are the Pauli matrices. Then

\[
\hat{A}_\mu \to \Omega^\dagger(x) \left( \hat{A}_\mu(x) - \frac{i}{g} \partial_\mu \right) \Omega(x)
\]

where \( \hat{A}_\mu = A_\mu^a \sigma^a/2 \). Moreover for the field strength one has

\[
\hat{F}_{\mu\nu} \to \Omega^\dagger \hat{F}_{\mu\nu} \Omega - \frac{i}{g} \Omega^\dagger (\partial_\mu \partial_\nu - \partial_\nu \partial_\mu) \Omega
\]

where we reserved for the possibility that the derivatives \( \partial_{\mu,\nu} \) do not commute when applied to \( \Omega(x) \). The noncommutativity is in fact the definition of a singular gauge transformation.

Consider furthermore

\[
\Omega(x) = \begin{pmatrix}
\cos \frac{\gamma}{2} & \sin \frac{\gamma}{2} e^{-i\alpha} \\
-\sin \frac{\gamma}{2} e^{i\alpha} & \cos \frac{\gamma}{2}
\end{pmatrix}
\]

where \( \alpha \) and \( \gamma \) are azimuthal and polar angles, respectively. Then it is straightforward to check that

\[
\hat{F}_{\mu\nu}(\hat{A}^{\Omega}) = \Omega^\dagger \hat{F}_{\mu\nu} \hat{A} \Omega - \sigma^3 \frac{2\pi}{g} (\delta_{\mu,1} \delta_{\nu,2} - \delta_{\mu,2} \delta_{\nu,1}) \delta(x_1) \delta(x_2) \Theta(-x_3)
\]

In other words, we generated a Dirac string directed along the \( x_3 \)-axis ending at \( x_3 = 0 \) and carrying the color index \( a = 3 \).

It is quite obvious that such Abelian-like strings are allowed by the lattice regularization of the theory. However, we cannot get too far using only Abelian-like field configurations. Indeed, imagine that the string (84) would go over into the same Abelian monopoles at its end points. Then we would have the same estimates as in the case of the compact \( U(1) \) but our coupling now tends to zero in the ultraviolet, \( \lim_{a \to 0} g^2(a) = 0 \) and these field configurations are strongly suppressed.
The best qualitative picture for QCD based on the analogy with the compact $U(1)$ seems to be as follows. Begin with a small lattice size $a$ but then go to a coarser lattice, with corresponding renormalization of the coupling $a la$ Wilson. Then once the effective coupling governing a $U(1)$ subgroup reaches the value of $e^2_{crit}$, then one may think that the monopoles are condensed, in the same way as in the $U(1)$ case. There is a semi-quantitative prediction based on this picture. Namely, it is natural to assume that the running of the coupling is changed drastically at this point as far as further advance to the infrared is concerned. Thus, we can speculate that the coupling is frozen at the value

$$g^2_{frozen} \sim 2e^2_{crit} \sim 2$$

which is not unreasonable phenomenologically.\(^h\)

To summarize, we touched upon a few issues in this subsection. First of all, we emphasized that introduction of the Dirac string implies, at least naively, a new quadratic ultraviolet divergence. We also argued that the Abelian-like monopoles cannot be important in the non-Abelian case since the bare coupling goes to zero. In a way, the non-Abelian monopoles are to be so to say empty at short distances in the sense that the action is determined by contribution of distances of order $\Lambda_{QCD}^{-1}$ where the coupling is not small. How this might happen, we shall explain in the next section.

**Magnetic monopoles in the limit $m_H \to \infty$.**

Magnetic monopoles of finite energy are found ('t Hooft (1974), Polyakov (1974)) as classical solutions to nonabelian field theories which include scalar fields. Thus, it is a common question, how one can understand monopoles in QCD when there are no scalar fields. We will provide a partial answer to this question, arguing that the scalar fields are replaced in some sense by singular gauge transformations as far as short distances are concerned.

Begin with the Georgi-Glashow model which includes a tripled $\Phi^a$ of scalar fields:

$$L = \frac{1}{4}(F^a_{\mu\nu})^2 + \frac{1}{2}(D_\mu \Phi^a)^2 + \frac{1}{8} \lambda ((\Phi^a)^2 - v^2)^2$$

Here,

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc}A^b_\mu A^c_\nu$$

$$D_\mu \Phi^a = \partial_\mu \Phi^a + g\epsilon^{abc}A^b_\mu \Phi^c,$$

and $\lambda, v$ are constants.

Furthermore, one introduces functions $A(r), \Phi(r)$:

$$A^a_\mu = \epsilon^{\mu ab}r^b A(r), \quad \Phi^a = r^a \Phi(r).$$

The monopole solution is characterized by its asymptotic at $r \to \infty$:

$$\lim A(r)_{r \to \infty} = -r^{-2}g^{-1}, \quad \lim \Phi(r)_{r \to \infty} = v \cdot r^{-1}.$$

The mass of the monopole is:

$$M_{\text{monopole}} = \frac{4\pi v}{g} C(\lambda/g^2),$$

where $C(\lambda/g^2)$ is in fact a very slow function of its argument, as emphasized already by 't Hooft (1974). Eq. (91) implies then that the mass of the monopole corresponds to the mass of a Dirac monopole with an ultraviolet cut off at distances of order $r \sim m^{-1}_v$.

What is a crucial point for us now, is that the mass of the monopole remains finite in the limit $\lambda \to \infty$ which is the same as the limit of infinite Higgs mass, $m^2_H = \lambda v$. Thus, we may take the scalar mass as an

\(^h\)Note that the lattice calculations show (Kato 1998, Chernodub 1999) that the abelian monopoles of finite physical size ("extended" monopoles (Ivanenko 1990)) are important for confinement.
ultraviolet cut off and hope to remove the scalar from the spectrum altogether. Now we are dealing with QCD still having a finite mass of monopole. Let us look closer (in the limit $m_H \gg m_V$) why we do need the scalar field.

Consider distances $m_V^{-1} \gg r \gg m_H^{-1}$. The point is that at such distances the field of the monopole can be nothing else but nearly a pure gauge,

$$F_{\mu\nu}^a \approx 0.$$  

(92)

Indeed, only in this case the mass of the monopole would be insensitive to distances much smaller than $m_V^{-1}$, as testified by (91). To find the solution more explicitly, let us look closer at the equations of motion. Since we consider $r \gg m_H^{-1}$ we choose $\Phi(r)$ in its asymptotical form, $r \to \infty$, see (90). Moreover, we make an ansatz:

$$A^a_{\mu} \approx c_{\mu} r^2$$  

(93)

where $c$ is a constant.

Then the equations of motion reduce to a simple algebraic equation (see, e.g., 't Hooft (1974)):

$$0 = 4cg^{-1} + 6g^{-1}c^2 + 2g^{-1}c^3 + 2v^2r^2g(1 + c).$$  

(94)

At the distances we are considering the product $gvr \ll 1$ and let us neglect for the moment the last term in (94). Then we have three solutions for the constant $c$:

$$c_1 = -1, \quad c_2 = -2, \quad c_3 = 0.$$  

(95)

The first solution is singled out by the fact that it nullifies also the $r^2$ terms in (94). This is crucial at large $r$ and that is why $c_1$ is valid asymptotically, see (90). However, this solution implies $(F_{\mu\nu}^a)^2 \sim r^{-4}$ and a large contribution to the mass from distances $r \ll m_V^{-1}$ which is not allowed, see (91). On the other hand, if we choose $c_2 = -2$ then Eq. (92) is fulfilled and the contribution to the action from the non-Abelian field vanishes. Thus, at small distances we should turn rather to the solutions $c_2, c_3$. To remove the contradiction with the Eq. (94) we include the next term in the expansion (93)

$$A^a_{\mu} \approx \frac{c}{gr^2} + dv^2 \ln r,$$  

(96)

where $d$ is a constant and the second term is a small corrections at the distances we are considering. The constant $d$ can be readily found: $d = \pm g/3$.

Since $F_{\mu\nu}^a = 0$, the potential

$$A^a_{\mu} \equiv -\frac{2}{r^2} \epsilon^{ab} r^b$$  

(97)

is obtainable from $A^a_{\mu} = 0$ by a gauge transformation:

$$-\frac{2}{r^2} \epsilon^{ab} r^b \sigma^a = i(\Omega^0)_{\mu} \partial_{\mu} \Omega^0.$$  

(98)

It is not difficult to find out the explicit form of the matrix $\Omega^0$:

$$\Omega^0 = i\vec{\sigma} \cdot \vec{n}.$$  

(99)

where $\vec{n}$ is the unit vector from the center of the monopole to the observation point.

Now, it is clear that if we would introduce gauge rotations (97) ad hoc, without scalar fields, then we would conclude that the corresponding $A^{a}_{\mu}$ has a $\delta$-function singularity at the origin so that not only $F_{\mu\nu}^a$ but the total mass $\sim \int (F_{\mu\nu}^a)^2 d^4r$ is UV divergent. In this sense the only function of the scalar field is to smoothen the fields at the origin so that the contribution of the singularity is in fact normalized to zero. Now, the observation is that if we use the lattice regularization and ask the question whether the singular (at $r = 0$) potentials (98) are allowed or not, the answer would be in positive. Namely, the center of the singular monopole would fall into a center of a lattice cube and this would imply that the singularity does not contribute to the action in fact.

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Short Strings.

We proceed now to derivation of the stringy correction (56) to the potential $V(r)$ within the Abelian Higgs model (Gubarev et. al., 1998). Thus, we consider the following problem. Two external magnetic charges are put at a distance $r$ in the vacuum of the Abelian Higgs model. In particular, the vacuum expectation value of the scalar field $\Phi$ is non-vanishing. The problem is to find the potential energy $V(r)$ at distances much smaller than $m_H^{-1}, m_V^{-1}$.

The energy is determined in the classical approximation and, at first sight, the problem is not much of a challenge. The crucial point is that one has to impose an extra boundary condition, namely vanishing of the scalar field along a mathematically thin line connecting the magnetic charges. We have already mentioned this condition while discussing the topological strings (see Eq. (76 and the related discussion).

The language of the Dirac string can be useful to substantiate the point. Indeed, the infinitely thin line discussed above is nothing else but the Dirac string connecting the monopoles. The possibility of its challenge. The crucial point is that one has to impose an extra boundary condition, namely vanishing of the scalar field $\Phi$ along the string and it is just the condition mentioned above. In other words, Dirac strings always rest on the perturbative vacuum which is defined as the vacuum state obeying the duality principle. Therefore, even in the limit $r \to 0$ there is a deep well in the profile of the Higgs field $\Phi$. This might cost energy which is linear with $r$ even at small $r$.

The next question is whether this mathematically thin string realizes as a short physical string. Where by the physical string we understand a stringy piece in the potential, $\sigma_0 r$ at small $r$. In other words, we are going to see whether the stringy boundary condition implies a stringy potential. To get the answer we solve classical equations of motion. Let us note that the scheme of the calculation and some numerical results for the potential $V(r)$ can be found in a number of papers, see, e.g., (Ball and Caticha, 1988). However, prior to (Gubarev et. al., 1998) there were no measurements dedicated specifically to small corrections to the Coulombic potential at $r \to 0$.

We will consider the unitary gauge, $Im \Phi = 0$. Then the most general ansatz for the fields consistent with the symmetries of the problem is:

$$
\Phi = \eta f(\rho, z) \quad Im f = 0
$$

$$
A_a = \varepsilon_{ab}\hat{x}_b A(\rho, z) \quad A_0 = A_3 = 0
$$

$$
\rho = [x_a x_a]^{1/2} \quad z = x_3 \quad \hat{x}_a = x_a/\rho \quad a = 1, 2.
$$

In the limit $r \to 0$ the Coulombic contribution becomes singular. The easiest way to separate the singular piece is to change the variables $A = A_d + a$, where $A_d$ is the solution in the absence of the Higgs field:

$$
A_d = \frac{1}{2\rho} \left[ \frac{z_- - z_+}{r_-} \right] , \quad z = z \pm r/2 \quad r_\pm = [\rho^2 + z^2_\pm]^{1/2}.
$$

Let us introduce also a new variable $\kappa = \sqrt{2}\lambda/e = m_H/m_V$ and measure all dimensional quantities in terms of $m_V = \epsilon_\eta$. Upon the change of variables $f(\rho)$ the energy functional and the classical equation of motion take the form:

$$
E(r) = E_{sel} - \pi/r + \tilde{E}(r),
$$

$$
\tilde{E}(r) = \frac{z}{r} \int_0^{+\infty} d\rho \int_0^{+\infty} d\sigma \left\{ \left[ \frac{1}{\rho} \partial_\rho (\rho a) \right]^2 + \left[ \partial_\sigma a \right]^2 + \left[ \partial_\rho f \right]^2 + \left[ \partial_\sigma f \right]^2 + f^2(a + A_d)^2 + \frac{1}{4} \kappa^2 (f^2 - 1)^2 \right\}.
$$

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The energy functional has been minimized numerically. The numerical results (Gubarev et. al., 1998) for various \( \kappa \) values clearly demonstrate that there is a linear piece in the potential even in the limit \( r \ll 1 \). The slope \( \sigma_0 \) at \( r \to 0 \) was defined by the fitting the numerical data to:

\[
\tilde{E}_{\text{fit}}(r) = C_0 \left( \frac{1 - e^{-r}}{r} - 1 \right) + (\sigma_0 + \frac{1}{2}C_0)r = \sigma_0 r + O(r^2),
\]

(105)

The resulting slope \( \sigma_0 \) depends smoothly on the value of \( \kappa \). For the purpose of orientation let us note that for \( \kappa = 1 \) the slope of the potential at \( r \to 0 \) is the same as at \( r \to \infty \) That is, within error bars:

\[
\sigma_0 \approx \sigma_\infty
\]

(106)

where \( \sigma_\infty \) determines the value of the potential at large \( r \).

To summarize, existence of short strings has been proven in the classical approximation to the Abelian Higgs model. The linear piece in the potential at small distances reflects the boundary condition that \( \Phi = 0 \) along the straight line connecting the monopoles.

**Strings vs OPE.**

Let us emphasize that both in case of the compact \( U(1) \) and in case of the Abelian Higgs model the OPE does not apply to study the power corrections.

In case of the compact \( U(1) \) it is especially evident that the OPE does not work. Indeed, as we emphasized many times, the use of the OPE allows to parameterize infrared sensitive parts of the Feynman graphs. But there are no Feynman graphs at all now because we are considering the Lagrangian of a free gauge field. Nevertheless, the answer depends on the value of the coupling \( e^2 \). If \( e^2 \) is higher than the critical value \( e^2_{\text{crit}} \sim 1 \) then the system is in confinement phase (see above). As a result the physics is changed at all the distances. Indeed, the ultraviolet scale \( a \) (which may be thought of as the lattice size) is the only scale in the problem since the coupling does not run. As a result, the potential between external electric charges becomes linear at all the distances. One photon exchange is not seen at all. Clearly, this observation is equivalent to saying that the OPE does not work at any distance because the OPE is just the assumption that at short distances one sees particle exchange. Thus, we may say that the example of the compact \( U(1) \) is too strong. If \( e^2 > e^2_{\text{crit}} \) then there is no particle exchange at all. While in QCD we believe that perturbation theory applies to describe the leading effects at large \( Q^2 \).

Turn now to the AHM where we can check OPE against the direct calculation of the potential in the preceding subsection. At first sight, existence of the \( 1/Q^2 \) corrections in case of the AHM is not surprising since now there is an operator of dimension \( d = 2 \), that is \( |\Phi|^2 \). It is worth emphasizing therefore that a bit more careful analysis demonstrates that the power correction to the potential which we found in the AHM also contradicts the OPE.

Moreover, it is the existence of short strings that is manifested also through breaking of the standard operator expansion. Indeed, above we found the potential in the classical approximation. In this approximation the potential is usually directly related to the propagator \( D_{\mu \nu}(q^2) \) in the momentum space,

\[
V(r) = \int \frac{d^4q}{(2\pi)^4} e^{iq\cdot r} D_{00}(q^2)
\]

(107)

Moreover, as far as \( q^2 \) is in the Euclidean region and much larger than the mass parameters, the propagator \( D_{\mu \nu}(q^2) \) can be evaluated by using the OPE. Restriction to the classical approximation implies that loop contributions are not included. However, vacuum fields which are soft on the scale of \( q^2 \) can be consistently
accounted for in this way (for a review see, e.g., Novikov et al. (1985)). This standard logic can be illustrated by an example of the photon propagator connecting two electric currents. Modulus longitudinal terms, we have:

\[ D_{\mu\nu}(q^2) = \delta_{\mu\nu} \left( \frac{1}{q^2} + \frac{1}{q^2} e^2 \langle \Phi^2 \rangle \frac{1}{q^2} + \frac{1}{q^2} e^2 \langle \Phi^2 \rangle \frac{1}{q^2} e^2 \langle \Phi^2 \rangle \frac{1}{q^2} + \ldots \right) = \frac{\delta_{\mu\nu}}{q^2 - m_V^2}. \] (108)

Thus, one uses first the general OPE assuming \( |q^2| \gg e^2 \Phi^2 \) then substitutes the vacuum expectation of the Higgs field \( \Phi \) and upon summation of the whole series of the power corrections reproduces the propagator of a massive particle. The latter can also be obtained by solving directly the classical equations of motion.

This approach fails, however, if there are both magnetic and electric charges present. In this case, one can choose the Zwanziger formalism (Zwanziger, (1971), Brandt (1978)) and work out an expression for propagation of a photon coupled to magnetic currents following literally the same steps as in (108). In the gauge \( n_\mu D_{\mu\nu} = 0 \) the result is well known:

\[ \hat{D}_{\mu\nu}(q, n) = \frac{1}{q^2 - m_V^2} \left( \delta_{\mu\nu} - \frac{1}{(qn)} (q_\mu n_\nu + q_\nu n_\mu) + \frac{q_\mu q_\nu}{(qn)^2} + \frac{m_V^2}{(qn)^2} (\delta_{\mu\nu} n^2 - n_\mu n_\nu) \right). \] (109)

Here the vector \( n_\mu \) is directed along the Dirac strings attached to the magnetic charges and there are general arguments that there should be no dependence of physical effects on \( n_\mu \). On the other hand, if the potential energy is given by the Fourier transform of (109) then its dependence on \( n_\mu \) is explicit.

Note that Eq. (109) immediately implies that the standard OPE does not work any longer on the level of \( q^{-2} \) corrections. Indeed, choosing \( q^2 \) large and negative does not guarantee now that the \( m_V^2 \) correction is small since the factor \( (qn)^2 \) in the denominator may become zero. Of course, appearance of the poles in \( (qn) \) in the longitudinal terms is not dangerous since they drop due to the current conservation. However, the term proportional to \( m_V^2 \) in (109) cannot be disregarded and contribute, in particular, to (107).

The reason for the breaking of the standard OPE is that even at short distances the dynamics of the short strings should be accounted for explicitly. In particular, in the classical approximation the string lies along the straight line connecting the magnetic charges and affects the solution through the corresponding boundary condition, see above. More generally, the OPE allows to account for effects of vacuum fields, in our case for \( \langle \Phi \rangle \neq 0 \). The OPE is valid therefore as far as the probe particles do not change the vacuum fields drastically and the unperturbed vacuum fields are a reasonable zero-order approximation. In our case, however, the Higgs field is brought down to zero along the string and this is a nonperturbative effect. Thus, the stringy piece in the potential \( V(r) \) at \( r \to 0 \) is a nonperturbative correction which is associated with short distances and emerges already on the classical level.

4 REVISITING THE PHENOMENOLOGY.

Revisiting the quark potential at short distances.

We have shown above that in the AHM the quark potential contains a linear correction at short distances. Moreover, the reason for the breaking of the OPE seems to be a unified description of magnetic and electric charges. Since the monopole condensation is established within the abelian projection of QCD, we would speculate that a similar picture for the potential is true in QCD as well. Then we expect a linear piece to dominate over other non-perturbative corrections at short distances and this would make it unnecessary to account for the retardation effects discussed in part I.

Because the theoretical foundation for the new picture is not absolutely solid, phenomenological evidence would be especially important at this point.

Without going into detail, because of the space consideration, let us mention that there are a few areas where measurements, especially on the lattice, could clarify the situation:

(i) Direct measurements of the potential \( V(r) \) bring so far nonvanishing results for the effective string tension at short distances (Bali et al, 1995, Bali, 1999):

\[ \sigma_0 = (1/6) \sigma_\infty \] (110)
(ii) There exist lattice measurements of fine splitting of the $Q\bar{Q}$ levels as function of the heavy quark mass. The Voloshin-Leutwyler picture results in a particular pattern of the mass dependence of this splitting. Moreover, these predictions are very different from the predictions based on adding a linear potential to the Coulomb potential at short distances. Numerically, predictions based on short strings with $\sigma_0 \approx \sigma_\infty$ (see above) are close to the predictions obtained by Buchmuller and Tye (1980) within a phenomenological model for the potential.

The results of most advances measurements of this type are published in (Fingberg, 1998) and favor strongly the linear correction to the potential at short distances.

(iii) There is an interesting evidence that the nonperturbative fluctuations on the lattice responsible for the confinement can be identified as the so-called P vortices (Faber et al., 1999). If one measures the quark potential due only to these vortices, the numerical results seem to indicate that the slope of the potential is the same at large and small distances, see (L. Del Debbio et. al., 1997).

(iv) Analytical studies of the Bethe-Salpeter equation and comparison of the results with the data about the charmonium spectrum favor a nonvanishing linear correction to the potential at short distances (Badalian and Morgunov, 1999).

Thus, it appears that existing experimental data favor a linear correction to the heavy quark potential at short distances but much more work is needed to finalize the results.

To summarize: the stringy correction to the Coulombic potential at short distances found in the Abelian Higgs model violates the operator product expansion. The search for a linear piece in the potential at short distances in QCD is of great importance. We listed above some preliminary indications that such a correction is indeed present.

Revisiting correlation functions: tachyonic gluon mass at short distances.

The $1/Q^2$ corrections discussed in the previous section go beyond the standard OPE. Detection of the new type corrections through phenomenological studies would be of great interest. In this section we will discuss phenomenology in terms of a tachyonic gluon mass which is assumed to mimic the short-distance nonperturbative effects (Chetyrkin et al., 1998).

First, let us note that not all the $1/Q^2$ corrections in QCD are associated with short distances. For example, in case of DIS the $1/Q^2$ corrections are coming from the IR region and perfectly consistent with the OPE. Thus, the class of theoretical objects for which an observation of the $1/Q^2$ corrections would signify going beyond the OPE is limited. One example was the potential $V(r)$ discussed above. Other examples are the correlator functions where the IR-sensitive power corrections start with $Q^{-4}$ terms, as is explained in detail in the Part I of the lectures.

Thus, we concentrate on this set of variables and an interesting question is whether there is room for introduction of sizable non-standard $1/Q^2$ corrections to the correlator functions. The answer seems to be in positive.

Even if one accepts that the non-standard $1/Q^2$ correction to the potential has already been observed (see the preceding section), it is far from being trivial to evaluate the $1/Q^2$ corrections to other quantities. Qualitatively, however, one may hope that introduction of a tachyonic gluon mass at short distances would imitate the effect of the $\Lambda^2_{QCD}/Q^2$ corrections. Indeed, the linear term $\sigma_0 \cdot r$ in the potential at short distances can be imitated by the Yukawa potential with a gluon mass $\lambda$:

$$\frac{4\alpha_s}{6} \lambda^2 \sim - \sigma_0.$$  \(111\)

On the Born level, Eq. (111) is an identity. However, a tachyonic gluon mass can be consistently used at one-loop level as well. Of course Eq. (111) may serve only for a rough estimate.

Looking for other applications, let us remind the reader, that one of the basic quantities to be determined from the theory is the scale $M^2_{crit}$ at which the parton model for the correlators gets violated considerably via the power corrections. Now, a new term proportional to $\lambda^2$ is added to the theoretical side of $\Pi_j(M^2)$

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1 We are thankful to A. Leonidov for bringing this issue to our attention and for introducing to the literature.
which becomes:

$$\Pi(M^2) \approx (\text{parton model})(1 + \frac{a_j}{\ln M^2} + \frac{b_j}{M^4} + \frac{c_j}{M^2})$$  \hspace{1cm} (112)

where $c_j$ is calculable in terms of $\lambda^2$ (Chetyrkin et al., 1998).

$$c_\pi \approx 4c_\rho = \frac{4\alpha_s}{3\pi} c_{\text{gluonium}} = \frac{4\alpha_s}{\pi} \lambda^2.$$  \hspace{1cm} (113)

Phenomenologically, in the $\rho$-channel there are severe restrictions (Narison, 1992) on the new term $c_\rho/M^2$:

$$c_\rho \approx - (0.03 \div 0.07) \text{ GeV}^2.$$  \hspace{1cm} (114)

Remarkably enough, the sign of $c_\rho$ does correspond to a tachyonic gluon mass (if we interpret $c_\rho$ this way). Moreover, when interpreted in terms of $\lambda^2$ the constraint (114) does allow for a large $\lambda^2$, say, $\lambda^2 = -0.5 \text{ GeV}^2$.

As for the $\pi$-channel one finds now a new value of $M_{\text{crit}}^2$ associated with $\lambda^2 \neq 0$:

$$M_{\text{crit}}^2(\pi - \text{channel}) \approx 4 \cdot M_{\text{crit}}^2(\rho - \text{channel}).$$  \hspace{1cm} (115)

It is amusing that just this value of $M_{\text{crit}}^2$ was found in (Novikov et al., 1981) from an analysis of the pion contribution and it cannot be explained within the standard QCD sum rules. Moreover, the sign of the correction in the $\pi$-channel is what is needed for phenomenology (Novikov et al., 1981). Fixing the value of $c_\pi$ to bring the theoretical $\Pi_\pi(M^2)$ into agreement with the phenomenological input one gets

$$\lambda^2 \approx -0.5 \text{ GeV}^2.$$  \hspace{1cm} (116)

Finally, with this value of $\lambda^2$ in hand, we can determine the new value of $M_{\text{crit}}^2$ in the scalar-gluonium channel and, again, it turns to be what is needed for the phenomenology, see Eq. (115).

Further checks could be provided by measurements of various correlation functions on the lattice, similar to the measurements reported in (Chu et al., 1993). It would be most interesting to try to disentangle the $1/Q^2$ corrections discussed here from the effects of the direct instantons mentioned in Part 1. Let us give two particular examples:

(i) In case of instantons, the correlation function of the tensor currents, which essentially coincide with the energy-momentum tensor are protected against large corrections. Indeed, it is well known that the energy-momentum tensor vanishes on the instantons. If the tachyonic mass is responsible for large corrections, the tensor gluonic current is not singled out in any way.

(ii) It would be very important to measure the sum

$$\Pi_\sigma(x) + \Pi_\pi(x)$$

at critical values of $x$ where the violation of the asymptotical freedom becomes noticeable. The point is that the direct instantons drop off from this sum to the first order while the effect of the tachyonic mass would add up. The existing data (Chu et al., 1993), taken literally leave ample space for the tachyonic mass. Measurements with improved accuracy are highly desirable.

It is worth emphasizing also that the $\lambda^2$ terms represent nonperturbative physics and limit in this sense the range of applicability of pure perturbative calculations. This nonperturbative piece may well be much larger than some of the perturbative corrections which are calculable and calculated nowadays.

To summarize: at least qualitatively, the phenomenology with a tachyonic gluon mass which is quite large numerically stands well to a few highly nontrivial tests. Further crucial tests of the model with the tachyonic gluon mass could be furnished with measurements of various correlation functions $\Pi_j(M^2)$ on the lattice.

\footnote{Further discussion of the short-distance tachyonic gluon mass can be found in (Zakharov, 1998), (Simonov, 1999), (Huber et al., 1999).}
Revisiting vacuum condensates: condensates of dimension $d = 2$ in gauge theories.

As is explained in the first part of the lectures, introduction of various vacuum condensates turned a useful way to understand and characterize dynamics of QCD. The most famous example is the quark condensate:

$$\langle 0 | \bar{q} q | 0 \rangle \neq 0$$ (117)

where $q$ stands for light $u$- or $d$-quarks. In the realistic case of negligible small quark mass, a nonvanishing value of the quark condensate manifests spontaneous breaking of the chiral symmetry.

Within the framework of QCD sum rules there emerged also the gluon condensate

$$\langle 0 | \alpha_s (G_{\mu \nu}^a)^2 | 0 \rangle \neq 0$$

A non-vanishing value of this condensate does not signify breaking of any symmetry but rather provides with a measure of nonperturbative fields in the vacuum. The gluon condensate appears to be the simplest gauge invariant quantity characterizing nonperturbative vacuum fields. It has dimension $d = 4$ and this dimension is echoed in the observation that the leading non-perturbative corrections in the QCD sum rules at large $Q$ are of order $\langle 0 | \alpha_s (G_{\mu \nu}^a)^2 | 0 \rangle / Q^4$.

Now, we are driven to a rather astonishing conclusion that condensates of dimension $d = 2$ may also have dynamical significance. Indeed, the condensation of monopoles in QCD is described by $\langle \Phi_M^2 \rangle \neq 0$ which is a dimension $d = 2$ vacuum expectation value. Also, we saw that in the compact $U(1)$ there arises a nonvanishing string tension. The condensates are of nonperturbative nature and their appearance is related to the compactness of the gauge group. Clearly, such condensates cannot be detected by means of the standard OPE since they are not related to softness of propagators in the Feynman graphs.

There is no direct clash with the gauge invariance since $\Phi_M$ does not enter the Lagrangian but is constructed in a rather indirect way, see the preceding section. The situation when there emerge condensates not related at all to the original fields may appear, however, confusing. In terms of the original fields condensates of dimension $d = 2$ can be related only to the vacuum expectation value of $\langle A_\mu^2 \rangle$. And in this section we will argue, following (Gubarev et. al., 1999) that the condensate $\langle A_\mu^2 \rangle$ constructed directly on the vector-potential of the gauge field $A_\mu$ may also have dynamical significance, upon some clarifications Literally such a condensate is gauge non-invariant and devoid therefore of physical meaning. We will consider, however, the minimal value of it, $\langle A_\mu^2 \rangle_{\text{min}}$. Looking for the minimal value of $A_\mu^2$ implies, of course, a particular choice of the gauge. Also, we have to consider the Euclidean space-time so that the minimal value of $A_\mu^2$ could be meaningful.

The relevance of $\langle A_\mu^2 \rangle_{\text{min}}$ can be understood on a simple example of very thin cosmic strings borrowed from (Alford and Wilczek, 1989). Then the field strength tensor cannot be detected directly. However, because of the Aharonov-Bohm effect particles can be scattered off the strings. From our point of view, it is crucial that the system is characterized by a non-vanishing $A_\mu$. Indeed, because of the topological condition,

$$\int A_\mu dx^\mu = \int H \cdot ds \equiv \varphi$$ (118)

where $\varphi$ is the magnetic flux, the value of $A_\mu^2$ cannot be brought down to zero everywhere. Thus, the value of $\langle A_\mu^2 \rangle_{\text{min}} \neq 0$ distinguishes the system from the trivial vacuum for which, on the classical level, one can choose $A_\mu = 0$.

In case of the compact $U(1)$ gauge group, one can measure the $\langle A_\mu^2 \rangle_{\text{min}}$ numerically. Moreover, it is possible to substract explicitly the perturbative contribution to this quantity leaving only the dynamically relevant part $\langle A_\mu^2 \rangle_{\text{min, non-pert.}}$. The result for $\langle A_\mu^2 \rangle_{\text{min, non-pert.}}$ is presented in Fig. 6. We see that the behavior of this condensate clearly signals the phase transition to the confining phase near $e^2 = 1$. Although, it cannot be considered, strictly speaking, as an order parameter since even the nonperturbative part of $\langle A_\mu^2 \rangle_{\text{min}}$ does not vanish in any phase.

To summarize: dimension two vacuum expectation values, like $\langle A_\mu^2 \rangle_{\text{min}}$ may have a hidden topological meaning and be dynamically significant.
5 CONCLUSIONS.

In Part I we considered power corrections within the OPE approach, which culminates in the QCD sum rules (see, e.g., (23)). The sum rules allowed to tune the power corrections to resonances in some channels, like the ρ-meson channel. There are many further successful applications, see, e.g., Reinders et.al. (1985), Narison (1989), which we just did not have time at all to cover. More recently the technique was generalized to observables directly in the Minkowski space by using infrared renormalons or infinitesimal gluon mass.

On the other hand, in some so to say exceptional channels the sum rules certainly fail (Novikov et.al., 1981). Most conspicuous cases are the π-meson, 0⁺-gluonium channels. There are some explanations to the failures in terms of direct transitions of the currents to instantons (Novikov et.al. (1981), Shuryak and Schafer (1998)). However, the instanton corrections are lacking simple analytical structure. Also, neither the soft vacuum fields accounted for by the OPE nor the direct instantons seem to encode the effects of confinement.

Moreover, very recent studies of the power corrections in the Abelian Higgs model reveal novel \(1/Q^2\) corrections which are associated with small distances and are not accounted for in the operator product expansion. If such corrections exist in the QCD case as well, then there are quite a few places where the new corrections could be detected through measurements, mostly through measurements on the lattice. The existing data are not discouraging for the new corrections, not at all. Further checks are desirable.

To summarize, the nature may turn to be generous as far as power corrections are concerned. I am borrowing this term from a talk on dark matter. Indeed, first people assumed that there should be a single dominant source of the dark matter, and now it appears distributed among various equally important components. Similar picture may be true for the sum rules. Indeed, very first idea would be that theoretically the correlation functions \(\Pi_j(Q^2)\) could be found perturbatively at large \(Q^2\) and the growth of the effective coupling at smaller \(Q^2\) would signal the breaking of the asymptotic freedom. Then the picture got more involved and the effect of soft non-perturbative fields was included in terms of the quark and gluon condensates. It appears not suffice to explain the peculiarities of all the channels and the effect of direct instantons was invoked. As the latest development, hypothetical \(1/Q^2\) corrections associated with short strings are established within the Abelian Higgs model which is thought to mimic the QCD confinement.

Thus it appears now that practically all known “ingredients” of QCD have already found their way into the physics of the power corrections.
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