3D gravity and non-linear cosmology

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Abstract

By the inclusion of an additional term, non-linear in the scalar curvature $R$, it is tested if dark energy could rise as a geometrical effect in 3D gravitational formulations. We investigate a cosmological fluid obeying a non-polytropic equation of state (the van der Waals equation) that is used to construct the energy-momentum tensor of the sources, representing the hypothetical inflaton in gravitational interaction with a matter contribution. Following the evolution in time of the scale factor, its acceleration, and the energy densities of constituents it is possible to construct the description of an inflationary 3D universe, followed by a matter dominated era. For later times it is verified that, under certain conditions, the non-linear term in $R$ can generate the old 3D universe in accelerated expansion, where the ordinary matter is represented by the barotropic limit of the van der Waals constituent.

Theories of gravity in lower dimensions are useful instruments to investigate open questions in realistic theories[1]. Looking from another perspective, they can be analysed as alternative theories of gravity (and cosmology); in fact several interesting (classical and quantum) results were obtained in 2D and 3D formulations [1,2,3]. In the particular case of cosmological applications, the results include the description of a 3D Universe filled with ordinary matter or/and electromagnetic radiation[4,8], an inflationary 2D universe and a three-eras scenario, whose final accelerated period could be associated to the effects of a dark energy constituent [5].

As is well-known, the investigation about the true nature of dark energy is a fundamental topic in 4D cosmology. One approach to describe this final accelerated era in 4D universes is to modify Einstein’s theory of gravitation: the fundamental ingredient here is the inclusion of extra terms in the gravitational

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dynamics that depend non-linearly in the curvature scalar $R$; the cosmological implications of this idea have been discussed in several works (for recent investigations and references see [6]). These ideas come in substitution to the standard formulations that take the dark energy as a “usual” source [6]. This approach was tested in a 2D context recently by these authors [7], working with the Jackiw-Teitelboim (JT) model [1, 9].

If one wants to follow this path in 3D models, starting with the 3D version of the Einstein-Hilbert theory, special care must be taken due to the presence some peculiar features like the impossibility of gravitational propagation in free space and the absence of a Newtonian limit [1, 2, 4]. On the other hand, these problems can be solved by taking as substitute the 3D scalar gravitational model [1, 8]. This theory offers several interesting results in cosmological applications like radiation-filled 3D universes and transition from inflationary to matter dominated scenarios [1, 8]. Taking into account the arguments above the proposal of our work is to include a non-linear contribution in $R$ in the scalar 3D model, and to investigate in which cases this non-linear term is responsible for a final period of positive acceleration playing the role of a dark energy constituent in 3D universes.

An important point when discussing 3D gravitational models is the existence of a Newtonian limit [1, 2, 4]. If we start with the 3D Einstein equations what is obtained (in the small velocities and weak field limits) is a decoupling of the gravitational field from sources that leads the Newtonian potential to obey to the Laplace equation [2, 4].

A Newtonian limit can be implemented in 3D gravity if we substitute the Einstein 3D equations by the scalar model [1, 4], that was analysed in a cosmological context in [4, 8]. A way of showing that a Newtonian limit indeed exists in this case is to use the conformally-flat metric [1]:

$$g_{\mu\nu} = \Phi^2(r)\eta_{\mu\nu} \approx [1 + 2\epsilon \theta(r)]\eta_{\mu\nu},$$

(1)

where we invoke the weak field approximation, keeping terms up to first order in the parameter $\epsilon$. The curvature scalar is in this case

$$R = -4\epsilon \frac{d^2}{dr^2} \theta(r).$$

(2)

In the weak field limit, we have that the trace of the energy-momentum tensor of sources ($T$) obeys $T \approx \rho$; so if it is required that in the Newtonian limit the gravitational interaction be an attractive force, the scalar gravitational dynamics must be ruled by

$$R = -2\kappa T,$$

(3)

that leads to the well known non-relativistic expression $\nabla^2 \theta = \kappa \rho$. Several cosmological solutions emerging from this scalar gravitational dynamics were analysed in [4, 5, 8]. The results include (using the 3D Robertson-Walker metric) the description of periods dominated by matter and/or radiation [4], inflationary
young 3D universes and finally old universes dominated by dark energy in transition from a matter dominated period\[8\]; where the dark energy constituent is modeled using different equations of state \[4, 5, 8\]. As we mentioned before the central idea of this work is to include in the field equations a new term, non-linear in the curvature scalar \(R\). This discussion was done in \[6\] for 4D models and in \[7\] for 2D models. We start with the new field equations

\[
R^2 + 2\kappa TR - \omega^2 = 0 ,
\]

where \(\kappa = 2\pi G^2\) is the 3D gravitational coupling constant\[2\] (with \(G^2 = 1\) in natural units). \(\omega\) is a parameter that controls the non-linear term effects; in the 4D case some experimental data put restrictions on the values of this parameter, here it will be in principle free. Equation (4) has two roots in \(R\); we maintain the one that preserves the limit encoded in equation (3).

The usual hypothesis of homogeneity and isotropy, in the form of the 3D Robertson-Walker metric, \(ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2\right)\), leave the gravitational field equations in the following form (for \(k = 0\))

\[
2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = \pi T + \frac{\sqrt{4\pi^2 T^2 + \omega^2}}{2} ,
\]

(5)

When we substitute the perfect-fluid energy-momentum-tensor trace \((T = \rho - 2p)\) into the equation (5) and into the correspondent conservation law \((T^{\mu\nu}; \nu = 0)\) we obtain

\[
\ddot{a} = \frac{\dot{a}^2}{2a} - \frac{a\pi(\rho - 2p)}{2} + \frac{a\sqrt{4\pi^2(\rho - 2p)^2 + \omega^2}}{4} ,
\]

(6)

\[
\dot{\rho} + 2\dot{a}/a(p + \rho) = 0 .
\]

(7)

The early-times cosmic constituent is supposed to be ruled by the van der Waals equation of state \([10]\), \(p = b\rho^\alpha / (1 - \alpha\rho)\), \(b\) and \(\alpha\) are constants, proposed in the cosmological context \([10]\) to describe the behavior of a mixture of the inflaton with a matter constituent in a young universe. In fact, as the system evolves the vdW equation approaches a barotropic equation of state (modeling a period where matter starts to predominate over the inflaton contribution) \([10]\).

The coupled system of differential equations (6) and (7), is highly non-linear and we solve it numerically. The normalized boundary conditions are in this case : \(a(0) = 1, \dot{a}(0) = 1\) e \(\rho(0) = 1\). These conditions simulate a young 3D universe, in the beginning of an inflationary period\[5, 7, 8\].

In figure (1) is represented the evolution in time of the scale factor \(a(t)\) for several values of the parameters. What is observed is that the increasing of the barotropic parameter \(b\) makes the expansion faster, a result that is similar to the one found in the standard scalar case \([5]\); besides, the results show that the increase of parameter \(\alpha\) also implies into a faster expansion (not displayed in the figure). First, we see that the modifications in the non-linear term in \(R\) (controlled through the parameter \(\omega\)) give no sensible results for early times (in
Fact $1/R$ is negligible for early times) and therefore the standard 3D results are reproduced for a young 3D universe. On the other hand, for later times, the $\omega$ term promotes a faster expansion and, more importantly, a modification on the behavior of the acceleration, as we show below.

In figure (2) we show the behavior of the energy density. In agreement with the 3D universe expansion, the vdW energy density function is always decreasing. Accordingly, smaller values of $b$ make the fall of the curve smoother, and bigger values of $\alpha$ make the fall more abrupt.

Figure (3) shows the evolution in time of the acceleration. What is seen in the main picture is the first accelerated period and the first transition, from a 3D universe dominated by the inflaton to a period dominated by matter (decelerated period). It is confirmed that a second (decelerated) period exists, dominated by matter (as the vdW equation of state approaches a barotropic form). Till the end of this second period the non-linear term in $R$ gives a negligible contribution to the dynamics, following the results obtained in [5]. A different and interesting situation is found for later times, as it is shown in the small window: Increasing values of $b$ imply into a slower fall of the acceleration for early times and a slower transition to the third period for later times. More importantly, the link between dark energy and a geometrical origin can be done. In fact, we have that the non-linear contribution in $R$ (controlled by the free parameter $\omega$) is responsible for the transition to the final accelerated (third) period; increasing values of $\omega$ promote an earlier transition, making the matter-dominated era
shorter. As desired, when $\omega = 0$ there is no final transition at all. In summary, we have investigated, for 3D cosmological models, the possibility of giving to the contributions of dark energy a geometrical origin, adding a non-linear term in the scalar curvature $R$ to the dynamics. We have investigated and confirmed the presence of transitions to accelerated periods, using the 3D scalar formulation with the addition of a particular $1/R$ term, as a simulation of the beginning of a dark-energy dominated era; the modelling of a inflationary period was also possible. One constituent (ruled by the vdW equation) is enough to simulate the 3D young universe leaving the inflationary period. This is also the case to describe the transition to a final period (leaving the matter dominated era) when the non-linear term in $R$ starts to show its effects and promotes the desired accelerated regime, that would correspond to the dark energy dominated era.

The results have also shown that the role of the non-linear term is similar to the one found in 2D (when it is used the JT model); although this parallel can be done only qualitatively: different forms in the explicit expressions for the dynamics are found and also different values of parameters are necessary to select the more interesting scenarios. In both cases the analysis and results remain as theoretical explorations of hypothetical universes in lower dimensions.
Figure 3: Evolution in time of acceleration

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