Wormholes and Child Universes

E.I. Guendelman

Physics Department, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel

Evidence to the case that classical gravitation provides the clue to make sense out of quantum gravity is presented. The key observation is the existence in classical gravitation of child universe solutions or "almost" solutions, "almost" because of some singularity problems. The difficulties of these child universe solutions due to their generic singularity problems will be very likely be cured by quantum effects, just like for example "almost" instanton solutions are made relevant in gauge theories with breaking of conformal invariance. Some well motivated modifications of General Relativity where these singularity problems are absent even at the classical level are discussed. High energy density excitations, responsible for UV divergences in quantum field theories, including quantum gravity, are likely to be the source of child universes which carry them out of the original space time. This decoupling could prevent these high UV excitations from having any influence on physical amplitudes. Child universe production could therefore be responsible for UV regularization in quantum field theories which take into account semiclassically gravitational effects. Child universe production in the last stages of black hole evaporation, the prediction of absence of transplanckian primordial perturbations, connection to the minimum length hypothesis and in particular the connection to the maximal curvature hypothesis are discussed. Some discussion of superexcited states in the case these states are Kaluza Klein excitations is carried out. Finally, the possibility of obtaining "string like" effects from the wormholes associated with the child universes is discussed.

Keywords: Child Universes; Wormholes; Quantum Gravity.
1. Introduction

Quantum field theory and quantum gravity in particular suffer from UV divergences. While some quantum field theories are of the renormalizable type, quantum gravity is not and the UV divergences cannot be hidden into a finite number of ”counter-terms”. Perturbative renormalizability does not appear to be available for quantum gravity.

In an apparently unrelated development, the ”child universe” solutions have been studied\textsuperscript{[1, 2]}. These child universes are regions of space that evolve in such a way that they disconnect from the ambient space time. Inflationary bubbles of false vacuum correspond to this definition\textsuperscript{[1, 2]}. In this case an exponentially expanding inflationary bubble arises from an ambient space time with zero pressure which the false vacuum cannot displace. The inflationary bubbles disconnect from the ambient space generating a child universe.

There are difficulties with these child universe solutions due to their generic singularity problems\textsuperscript{[3]} which will very likely be cured by quantum effects, just in the way that for example ”almost” instanton solutions are made relevant in gauge theories even with breaking of conformal invariance (this breaking does not permit the existence of exact classical solutions)\textsuperscript{[2]}. Other avenues for the resolution of the initial singularity problem of these child universe solution include an initial semiclassical tunneling region that replaces the singularity\textsuperscript{[5]} the consideration of violation of energy condition\textsuperscript{[6]} which itself can originate from quantum effects or the non existence of a Cauchy surface\textsuperscript{[2]}.

There are also some well motivated modifications of General Relativity where singularity problems could be avoided. For example, in this conference we have heard in the talk by Walter Greiner on his work with Hess on Pseudo Complex General Relativity\textsuperscript{[8]} which could do this.

Here we want to explore the possibility that high energy density excitations, associated to the UV dangerous sector of quantum field theory could be the source of child universes, which will carry the UV excitations out of the original ambient space time. Child universe production could be therefore responsible for UV softening in quantum field theory that takes into account gravitational effects. It implies also the existence of a maximum energy density and curvature.

We will now show now, using a simple model, that very high UV excitations have appreciable tendency to disconnect from the ambient space time.

2. The Super High UV Bubble

We describe now the model\textsuperscript{[12]} which we will use to describe a high UV excitation which will be associated with the production of a child universe. This model for high UV excitation will consist of a bubble with very high surface tension and very high value of bulk energy density inside the bubble.

The entire space-time region consists of two regions and a boundary: 1) \textbf{Region I} de Sitter space 2) \textbf{Region II}, Schwarzschild space and the domain wall boundary.
separating regions I and II.

In **Region I**: The de Sitter space. The line element is given by

\[ ds^2 = -(1 - \chi^2 r^2)dt^2 + (1 - \chi^2 r^2)^{-1}dr^2 + r^2 dΩ^2 \]  

(1)

where \( \chi \) is the Hubble constant which is given by

\[ \chi^2 = \frac{8}{3} \pi G \rho_0 \]  

(2)

\( \rho_0 \) being the vacuum energy density of the child universe and \( G = \frac{1}{m_P} \) where \( m_P = 10^{19} \text{ GeV} \).

In **Region II**: The Schwarzschild line element is given by

\[ ds^2 = -(1 - \frac{2GM}{r})dt^2 + (1 - \frac{2GM}{r})^{-1}dr^2 + r^2 dΩ^2 \]  

(3)

The Einstein’s field equations,

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}. \]  

(4)

are satisfied in regions I and II and determine also the domain wall evolution\(^\text{1}\) using the methods developed by Israel\(^\text{13}\). Using gaussian normal coordinates, which assigns to any point in space three coordinates on the bubble and considers then a geodesic normal to the bubble which reaches any given point after a distance \( \eta \) (the sign of \( \eta \) depends on which side of the bubble the point is found). Then energy momentum tensor \( T_{\mu\nu} \) is given by

\[ T_{\mu\nu}(x) = \begin{cases} -\rho_0 g_{\mu\nu}, & (\eta < 0) \text{ for the child universe, Region I, (negative pressure)} \\ 0, & (\eta > 0) \text{ for the Schwarzschild region, Region II} \\ -\sigma h_{\mu\nu}\delta(\eta) & \text{for the domain wall boundary.} \end{cases} \]  

(5)

where \( \sigma \) is the surface tension and \( h_{\mu\nu} \) is the metric tensor of the wall, that is \( h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu \), \( n_\mu \) being the normal to the wall.

The eq. of motion of the wall give\(^\text{1}\)

\[ M_{ct} = \frac{1}{2G\chi} \frac{\gamma^3 z_m^6 (1 - \frac{1}{2}\gamma^2)^{\frac{3}{2}}}{3\sqrt{3}(z_m^6 - 1)^{\frac{3}{2}}} \]  

(6)

where the \( M_{ct} \) is the mass at (or above) which there is classically a bubble that expands to infinity into a disconnected space, the child universe. In the above equation

\[ \gamma = \frac{8\pi G\sigma}{\sqrt{\chi^2 + 16\pi^2 G^2 \sigma^2}} \]

\[ z_m^3 = \frac{1}{2} \sqrt[3]{8 + (1 - \frac{1}{2}\gamma^2)^2 - \frac{1}{2}(1 - \frac{1}{2}\gamma^2)} \]  

(7)
where \( z^3 = \frac{\chi_1^2 r_3^3}{2GM} \) and \( \chi_1^2 = \chi^2 + \kappa^2 \), \( \kappa = 4\pi G\sigma \). \( r_m \) is the location of the maximum of the potential barrier that prevents bubbles with mass less than \( M_{ct} \) to turn into child universes.

We expect this representation of a high UV excitation to be relevant even for a purely gravitational excitation, which can be associated, after an appropriate averaging procedure, to an effective energy momentum, a procedure that gets more and more accurate in the UV limit.

Let us now focus our attention on the limit where \( \sigma \to \infty \) (while \( \rho_0 \) is fixed), which we use as our first model of a super UV excitation. Then, we see that \( \gamma \to 2 \) and \( M_{ct} \to 0 \). Alternatively, we could obtain another model of super UV excitation, by considering the energy density inside the bubble, \( \rho_0 \to \infty \), while keeping \( \sigma \) fixed. This also leads to \( M_{ct} \to 0 \) as well. Finally, letting both \( \sigma \to \infty \) and \( \rho_0 \to \infty \) while keeping their ratio fixed, also leads to \( M_{ct} \to 0 \). In all these limits we also get the radius of the critical bubble \( r_m \to 0 \) as well.

In [4] the above expression for \( M_{ct} \) was explored for the case that energy densities scales (bulk and surface) were much smaller than the Planck scale, like the GUT scale. This gave a value for \( M_{ct} = 56 kg >> m_p \). Here we take the alternative view that the scale of the excitations are much higher than the Planck scale, giving now an arbitrarily small critical mass. Defining the "scale of the excitation" through by \( \rho_0 \equiv M_{exc}^4 \), then the pre-factor \( \frac{1}{2\pi\sigma} \) in eq (6), goes like \( \left( \frac{m_p}{M_{exc}} \right)^2 m_p \). We see that for transplanckian excitations, i.e. if \( M_{exc} >> m_p \), we obtain a very big reduction for \( M_{ct} \). This is a kind of "see saw mechanism", since the higher the \( M_{exc} \), the smaller \( M_{ct} \).

This means that in these models for high UV excitations there is no barrier for the high UV excitation to be carried out to a disconnected space by the creation of a child universe. Notice also the interesting "UV -IR mixing" that takes place here: although we go to very high UV limits in the sense that the energy density in the bulk or the surface energy density are very high, the overall critical mass goes to zero.

One should notice that these limits where we take the surface tension or the energy density to very big values can be achieved as we go to early times (corresponding to the time of the creation of the child universes) in models where these quantities are dynamical variables. In this context [14] when considering for example models with dynamical tension one can show the existence of child universe production where the critical mass \( M_{ct} \) is indeed zero.

The difficulties related to the singularities for the solutions with \( M > M_{ct} \) should be solved as \( M_{ct} \) approaches zero, because then even a very small quantum fluctuation should be able to wash out this singularity if the mass is very, very small (the strength of the singularity is associated to the mass of the solution).
3. The String Gas Shell Example

Crucial parameters in the child universe formation models based on the description of vacuum bubbles in terms of thin relativistic shells are, apart from the total mass–energy of the asymptotically flat region, the false vacuum energy density and the shell surface tension. Let us then consider two spacetime domains \( M_{\pm} \) of two \((3+1)\)-dimensional spacetimes, separated by an infinitesimally thin layer of matter \( \Sigma \), a shell. We will also assume spherical symmetry: this simplifies the algebra, and is a non-restrictive assumption which almost always appears in the literature. Moreover, as a concrete case we will choose the one in which \( M_- \) is a part of Minkowski spacetime and \( M_+ \) is a part of Schwarzschild spacetime. Then, the equations of motion for the shell, i.e. Israel junction conditions, reduce to the single equation

\[
\epsilon_- \sqrt{\dot{R}^2 + 1 - \epsilon_+ \sqrt{\dot{R}^2 + 1 - 2GM/R}} = \frac{Gm(R)/R}{},
\]

where \( G \) is the gravitational constant and the only remaining degree of freedom is \( R(\tau) \), the radius of the spherical shell expressed as a function of the proper time \( \tau \) of an observer co-moving with the shell. \( \epsilon_{\pm} = \text{sgn}(n^\mu \partial_\mu r)|_{M_{\pm}} \) are signs, expressing the fact that the radial coordinate \( r \) can increase (\( \epsilon_{\pm} = +1 \)) or decrease (\( \epsilon_{\pm} = -1 \)) along the normal direction, defined by \( n^\mu|_{M_{\pm}} \) in \( M_{\pm} \), respectively (our convention is that the normal is pointing from \( M_- \) to \( M_+ \)). The function \( m(R) \) is related to the energy–matter content of the shell, and is what remains of the shell stress–energy tensor in spherical symmetry after relating the pressure \( p \) and the surface energy density \( \rho \) via an equation of state. Let us discuss this point in more detail, since our choice will be slightly unusual compared to the existing literature. We will, in fact, use \( p = -\rho/2 \), \( p \) being the uniform pressure and \( \rho \) the uniform energy density on \( \Sigma \). A string gas in \( n \) spatial dimensions satisfies \( p = -\rho/n \), therefore the two dimensional shell \( \Sigma \) we are dealing with is a sphere of strings. This, gives \( \rho = \rho_0/R \), where \( \rho_0 \) is a constant, and then, \( m(R) = cR \), with \( c = 4\pi\rho_0 \).

Making the above choice, we can then solve the junction condition to obtain the dynamics of the system. It can be seen that solving (8) is equivalent to solving the equivalent effective classical problem

\[
\dot{R}^2 + V(R) = 0, \quad V(R) = 1 - \frac{1}{4c^2} \left( \frac{2M}{R} + Gc^2 \right)^2,
\]

with the signs determined as \( \epsilon_- = +1 \) and \( \epsilon_+ = \text{sgn}(2M/R - Gc^2) \). The potential has the following additional properties:

\[
\lim_{R \to 0^+} V(R) = -\infty, \lim_{R \to \infty} V(R) = 1 - \frac{G^2c^2}{4}, \quad \frac{dV(R)}{dR} > 0.
\]

This shows that i) we can have unbounded trajectories only if \( c \geq 2/G \) and ii) this is independent from the choice of \( M > 0 \); moreover, from the result for \( \epsilon_+ \), we have that certainly iii) on all the unbounded trajectories \( \epsilon_+ \) changes sign (being positive for small enough \( R \) and negative for large enough \( R \); the general property that...
this change of sign must happen behind an horizon, is also obviously satisfied since 
\( c \geq 2/G \), so that at \( R = 2GM \) the sign \( \epsilon_+ \) is already negative although for small 

enough \( R \) it is positive). In view of the global spacetime structure associated with 
the above properties of all the unbounded solutions, it is clearly seen that they 
realize the formation of a baby universe. This happens for a large enough density 
of strings and for any positive value of \( M \). This simple model shows therefore very 
neatly the phenomena of “child universes out of almost empty space”. At this point 
we should mention some additional evidence that string matter has some peculiar 
features related to its capability of being responsible to produce universes out of 
nothing, see for example the interesting arguments of Trevisan presented in a poster 
in this conference, based on work by Berman and Trevisan. Another interesting 
fact in this respect is that a gas of string matter does not curve the spacetime in 
the context of the recently formulated theory with a dynamical time.

4. The Conjecture

This allows us to formulate the conjecture that the dangerous UV excitations that 
are the source of the infinities and the non renormalizability of quantum gravity 
are taken out of the original space by child universe production, that is, the con-
sideration of child universe production in the ultrahigh (trans planckian) sector of 
the theory could result in a finite quantum gravity, since the super high UV modes, 
after separating from the original space will not be able to contribute anymore to 
physical processes.

The hope is that in this way, child universes could be of interest not only in 
cosmology but could become also an essential element necessary for the consistency 
of quantum gravity. One situation where all the elements required (high energy 
densities, since the temperature is very big) necessary for obtaining a child universe 
appears to be the late stages of Black Hole evaporation. If the ideas explained here 
are correct, we should not get contributions to primordial density perturbations 
from the trans planckian sector, since these perturbations would have disconnected 
from our space time. Also, any attempt to measure distances smaller than the 
Planck length will be according to this also impossible since such a measurement 
will involve exciting a high UV excitation that will disconnect. This means that 
there must be a minimum length that we could measure, of the order of the Planck 

scale.

It appears there is a maximal energy density according to this, since now bub-
bles with high energy density will be quickly disconnected, being replaced in the 
observable universe by regions of Schwarzschild space, which has zero energy density, 
i.e., a very big energy density must decay in the observable universe. The "maxi-
mal curvature" hypothesis (here we focus on scalar curvature) is justified by this 
maximal energy density result, if we use eq. (11). An effective dynamics that takes 
into account the effect of child universe production (i.e. integrates out this effect) 
could resemble indeed that of obtained using the maximal curvature hypothesis.
5. Super Excited Kaluza Klein (KK) Modes

The gravitational effects on the spectrum of modes of particles with momentum in the direction of some periodic dimension have been analyzed. It was found there that the naive, uniformly spaced KK excitation spectrum gets drastically modified, since the size of the compact dimensions likes to grow near the region where the KK excitation lives. This leads to many orders of magnitude decrease of the energy of these KK excitations. The modification of the spectrum of KK excitations due to the growing of the extra dimensions was studied also in . Furthermore, once a configuration like that has been created, it is energetically possible for these modes to become superheavy by the recollapse of the extra dimension provided a child universe with an associated wormhole region is created . The superheavy modes must then decouple from the ambient universe for this to happen, in agreement with our general picture.

6. Child Universes, Wormholes and Strings

The creation of a child universe implies the creation of a wormhole region. Static wormhole configurations have been studied since the first construction by Einstein and Rosen , which consisted of simply joining two exterior Schwarzschild at the horizon, producing a doubling of the space for \( r > 2M \), but the elimination of the space \( r < 2M \). Although not noticed by Einstein and Rosen, the consistency of this construction (that is in order for it to be a solution of Einstein’s equations) requires the existence of light like matter at \( r = 2M \) . Generally wormholes are considered by joining exterior solutions outside the horizon \( r > 2M \), for a review see . Tranversable wormholes require in general exotic matter. Electric field lines can go from one universe to the other going through the wormhole and causing the appearence of charge, positive appearence on one universe and negative appearence on the other universe as has been pointed out by Misner and Wheeler .

The gauge fields that go from one side of the wormhole to the other can be used to construct wormhole throats which can be very, very long . Then it so happens that these very long wormhole throats have a dynamics that mimics that of string theory , which raises the interesting question of whether wormhole theory and in particular child universe theory could be origin of string theory, which could appear as the effective description of the dynamics of these long wormhole throats.

Acknowledgments

I want to thank the organizers of IWARA2009 conference for inviting me to this very interesting event and for support, to Vladimir Dzhunushaliev for reading the manuscript and for interesting comments, to Walter Greiner for interesting conversations concerning avoidance of singularities in his model and relevance to a child universe production, to Marcelo Samuel Berman and Luis Augusto Trevisan for interesting conversations concerning the possibility of creating a universe out of
nothing. This manuscript was prepared while I was visiting the Astrophysics and Cosmology Group at the Pontificia Universidad Catolica de Valparaiso, Chile.

References

1. S. K. Blau, E. I. Guendelman and A. H. Guth, Phys. Rev. D 35, 1747 (1987).
2. For a review see S. Ansoldi , E. I. Guendelman, I. Shilon, e-Print: arXiv:0711.2198 [gr-qc].
3. E. Farhi, A. H. Guth, Phys.Lett.B183:149,1987.
4. G. ’t Hooft, Phys.Rev.D14:3432-3450,1976, Erratum-ibid.D18:2199,1978, I. Affleck, Nucl.Phys.B191:429,1981.
5. E. Farhi, A. H. Guth and J.Guven, Nucl.Phys.B339:417,1990.
6. E. I. Guendelman, N. Sakai, Phys.Rev.D77:125002,2008, Erratum-ibid.D80:049901,2009. (arXiv:0803.0286 [gr-qc]).
7. N. Sakai, K-ichi Nakao, H. Ishihara, M. Kobayashi, Phys.Rev.D74:024026,2006. (gr-qc/0602084) . A. Borde, M. Trodden, T. Vachaspati, Phys.Rev.D59:043513,1999. (gr-qc/9808060).
8. P. O. Hess, W. Greiner, Int.J.Mod.Phys.E18:51,2009 ( arXiv:0812.1735 [gr-qc]).
9. S. Ansoldi and E. I. Guendelman, Prog.Theor.Phys.120:985-993,2008 (arXiv:0706.1239 [gr-qc]).
10. M. S. Berman , L. A. Trevisan , E.I. Guendelman, arXiv:0911.0178 [gr-qc].
11. E. I. Guendelman, Int.J.Mod.Phys.D17:551,2008 (gr-qc/0703105).
12. W. Israel, Nuovo Cim.B44S10:1,1966, Erratum-ibid.B48:463,1967, Nuovo Cim.B44:1,1966.
13. S. Ansoldi and E.I. Guendelman, Prog.Theor.Phys.120:985-993,2008 (arXiv:0706.1239 [gr-qc]).
14. M. Markov, Pis'ma Zh. Eksp. Theor. Fiz. 36, 214 (1982); ibid 46, 342 (1987); V. P. Frolov, M.A. Markov, V. F. Mukhanov, Phys.Rev.D41:383,1990; V. F. Mukhanov and R. Brandenberger, Phys. Rev. Lett. 68, 1969 (1992), D. A. Easson, JHEP 0302:037,2003.
15. E.I. Guendelman, Int.J.Mod.Phys.A4:5119,1989, E.I.Guendelman, Gen.Rel.Grav.22:131,1990.
16. J. R. Morris, Phys.Rev.D67:025005,2003, (hep-th/0211175). E.I. Guendelman, J. R. Morris, Phys.Rev.D68:045008,2003, (hep-th/0307012) , J. R. Morris, Phys.Rev.D70:025007,2004. (hep-th/0405188). E.I. Guendelman, John R. Morris, Phys.Lett.B608:194,2005. (hep-ph/0501147).
17. E.I. Guendelman and D.A.Owen, Gen. Rel. and Grav. 21, 201, 1989.
18. A. Einstein, N. Rosen, Phys.Rev.48:73-77,1935.
19. E. I. Guendelman, A. Kaganovich, E. Nissimov, S. Pacheva, Phys.Lett.B681:457, 2009 (arXiv:0904.3198 [hep-th]).
20. Lorentzian wormholes: From Einstein to Hawking. Matt Visser, (Washington U., St. Louis) . 1995. Woodbury, USA: AIP (1995) 412 p.
21. C. W. Misner, J. A. Wheeler, Annals Phys.2:525,1957.
22. E.I. Guendelman, Gen.Rel.Grav.23:1415,1991.
23. V. Dzhunushaliev, Class.Quant.Grav.20:2407,2003 (gr-qc/0301118). V. Dzhunushaliev, Gen.Rel.Grav.35:1481,2003 (gr-qc/0301046). V. Dzhunushaliev, Class.Quant.Grav.19:4817,2002 (gr-qc/0205055). V. D. Dzhunushaliev , U. Kasper, D. Singleton, Phys.Lett.B479:249,2000 (gr-qc/9910092).