Flipped SU(5): Origins and Recent Developments*

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ABSTRACT

We present an account of the early developments that led to the present form of the flipped SU(5) string model. We focus on the method used to decide on this particular string model, as well as the basic steps followed in constructing generic models in the free fermionic formulation of superstrings in general and flipped SU(5) in particular. We then describe the basic calculable features of the model which are used to obtain its low-energy spectrum: doublet and triplet Higgs mass matrices, fermion Yukawa matrices, neutrino masses, and the top-quark mass. We also review the status of proton decay in the model, as well as the hidden sector bound states called cryptons. Finally, we comment on the subject of string threshold corrections and string unification.

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1. The Road to Superstring Models

It is generally believed that the Standard Model (SM) of the strong and electroweak interactions is to be viewed as an effective gauge theory valid at energies below the electroweak symmetry breaking scale. Besides the usual arguments in favor of more fundamental theories which encompass and potentially explain the SM, there is a consistency requirement that must be satisfied by any extension of the SM. This is related to the experimentally observed breaking of the electroweak symmetry. At least two classes of mechanisms come to mind to effect this breaking: the Higgs mechanism of ordinary point-field theories induced by radiative corrections in the presence of softly broken supersymmetry [1], and dynamical symmetry breaking schemes based on condensates of known \((e.g., \bar{t}t\) condensates [2,3]) or unknown \((e.g., \text{technicolor }[4])\) fermions which mimic the elementary Higgs boson. One of the clear advantages of the former mechanism is that the gauge hierarchy problem is automatically solved, while supersymmetry may still be needed in the latter since the condensation scales need to be rather large.\footnote{In fact, such a scenario has been explored recently in the literature [3,3].}

Experimentally speaking, there is mounting evidence against certain class of composite Higgs theories [3]. However, the main drawback of these theories is more general. Due to their reliance on unknown nonperturbative dynamics, these theories are not very well understood and of limited calculability, and therefore their status as physical theories is questionable. On the other hand, point-field supersymmetric theories are perfectly calculable and their grand unified extensions highly predictive [7]. Furthermore, increasingly more precise measurements of the low-energy gauge couplings and their extrapolation to very high energies shows a remarkable unification picture in the minimal unified model only when low-energy superpartners are present [8].

Once supersymmetry is acknowledged as a major building block of modern unified theories, the dynamical question of why the scale of supersymmetry breaking is \(\lesssim 1\) TeV arises. This question is addressed naturally in no-scale supergravity theories [4] where a flat classical potential insures the vanishing of the cosmological constant (even after supersymmetry breaking). The last piece of the puzzle is quantum gravity, and this problem has only one known solution, namely superstring theory [10]. With all this in mind we set out to construct a unified supersymmetric superstring model. This task is however non-trivial due to the immense classical vacuum degeneracy of string theory. We need to make some educated choices.
2. How to Pick a String Model

It turns out that there is one property of string models which is surprisingly restrictive: to gut or not to gut? By which we mean, do we choose the effective low-energy theory below the Planck mass to be a unified theory or a at most abelian extension of the SM? Unified in this context means that at least $SU(3)_C$ and $SU(2)_L$ are inside a bigger non-abelian gauge group. This distinction is necessary since in string theory all gauge couplings of non-unified gauge groups “unify” at the string unification scale $M_{SU}$, even though no new degrees of freedom get excited at this scale (besides string massive modes).

There are two types of symmetry breaking mechanisms in string models: (a) at the string scale through the use of Wilson lines (e.g., $E_8 \times E_8 \rightarrow SU(3)^3 \times E_8$; $SO(44) \rightarrow SU(5) \times U(1) \times \cdots$), and (b) at lower energies via the usual Higgs mechanism. Wilson lines are used in all string models to break the large primordial gauge symmetry down to the initial high-energy gauge group of the model at scales $\approx M_{Pl}$. From there on unified models with suitable Higgs sectors can break down to the SM at intermediate scales. All these initial high-energy gauge groups are reflections of two-dimensional symmetries called Kac-Moody algebras which are parametrized by a positive integer called the “level”. Level-one realizations have the property that no adjoint matter fields are present in the spectrum. Higher-level realizations allow adjoint matter representations as well as many more higher-dimensional representations. However, model-building using these higher-level algebras is technically a rather difficult enterprise.

The choices are then clear:

1. Construct Wilson-line breaking models with gauge group $SM \times U(1)$’s at the string scale. These models can be built using level-one or higher-level Kac-Moody algebras, although the latter are not really needed. Several examples of this class have been constructed in the literature.

2. Construct models with unified gauge symmetry at the string scale which need adjoint Higgs fields for symmetry breaking, using higher-level Kac-Moody algebras. Realistic models of this type are beset with constraints, although some examples exist.

3. Construct models with unified gauge symmetry at the string scale which do not need adjoint Higgs fields for symmetry breaking, using level-one Kac-Moody algebras. An archetypal example of this class of models is flipped $SU(5)$, although other examples exist.
Clearly the flipped $SU(5)$ model constructed under class (3) above is not the unique string model. However, it certainly is the most developed string model to date. It remains to be seen whether the proponents of any of the other string models could eventually overcome some of the calculational or phenomenological problems that their models may have, so that they too could be brought to a level of development comparable to flipped $SU(5)$.

3. Flipped SU(5): Introduction and Historical Remarks

Group theoretically speaking, flipped $SU(5)$ is just an alternative embedding of $SU(5) \times U(1)$ into $SO(10)$ and as such its basic predictions for $\sin^2 \theta_w$ and the proton decay lifetime have been known for a while in its non-supersymmetric version [25]. The main point is that the electric charge generator $Q$ is only partially embedded in $SU(5)$, i.e., $Q = T_3 - \frac{1}{5} Y' + \frac{2}{3} \tilde{Y}$, where $Y'$ is the $U(1)$ inside $SU(5)$ and $\tilde{Y}$ is the one outside. This property affects the usual unification picture as follows: the low-energy values of $\alpha_3$ and $\alpha_2 = \frac{\alpha}{\sin^2 \theta_w}$ unify at a scale $M_X$, i.e., $\alpha_3(M_X) = \alpha_2(M_X) = \alpha_5(M_X) \equiv \alpha_X$. The third coupling $\alpha_1 = \frac{\alpha}{\cos^2 \theta_w}$ gets related to $\alpha_X$ and the $U(1)_{\tilde{Y}}$ coupling at the scale $M_X$ by

$$\frac{25}{\alpha_1} = \frac{1}{\alpha_X} + \frac{24}{\alpha_{\tilde{Y}}}. \quad (3.1)$$

Both $\alpha_{\tilde{Y}}$ and $\alpha_5$ evolve further up to $M_{SU}$ where they finally unify into $SO(10)$. This fact is used to fix the normalization of the $\tilde{Y}$ charge in much the same way that the embedding of $U(1)_{Y'}$ in $SU(5)$ is used to obtain the well-known factor of $\frac{5}{3}$.

The flipped electric charge relation implies that the matter fields in each generation are assigned to the $SU(5)$ representations as follows

$$\bar{5} = \left\{ u^c, \left( \begin{array}{c} \nu_e \\ e \end{array} \right) \right\}, \quad (3.2a)$$

$$10 = \left\{ d^c, \left( \begin{array}{c} u \\ d \end{array} \right), \nu^c \right\}, \quad (3.2b)$$

$$1 = \left\{ e^c \right\}, \quad (3.2c)$$

Clearly “flipped” relative to the usual assignments. The neutral components of the $10$ and $\bar{10}$ representations of Higgs fields are then used to effect the unique symmetry breaking of $SU(5) \times U(1)$ down to $SU(3) \times SU(2) \times U(1)$ without the use of adjoint Higgs representations.
There are two very nice features of flipped $SU(5)$ model building which are seldom found in regular unified models: (i) a natural solution to the doublet ($H$)-triplet ($D$) splitting problem of the Higgs pentaplets $h$ through the trilinear coupling of Higgs fields:

$$10_H \cdot 10_H \cdot 5_h \rightarrow \langle \nu^c_H \rangle d^c_H D,$$

and (ii) an automatic see-saw mechanism to get heavy right-handed neutrino masses through coupling to singlet fields $\phi$,

$$10_f \cdot \overline{10}_H \cdot \phi \rightarrow \langle \nu^c_H \rangle \nu^c \phi.$$

The left-handed neutrino fields have the same Yukawa couplings as the up-type quarks.

The field theory blueprint that the string flipped $SU(5)$ model hoped to emulate was proposed in 1987. It took two years and three papers to produce the so-called revamped flipped $SU(5)$ model, in the summer of 1989. This string model was indeed close to its field theory analog but had the major advantage that its cubic superpotential could actually be calculated (as opposed to just being postulated). This model was derived in the free fermionic formulation of four-dimensional strings (see next section) and has the gauge group $SU(5) \times U(1) \times U(1) \times SO(10)_h \times SU(4)_h$ at the string scale, where $SO(10)_h$ and $SU(4)_h$ are (semi)hidden gauge groups. The superpotential of the model was promising, but there remained several unanswered questions which were wishfully ascribed to uncalculated higher-order terms in the superpotential. Among these were the full set of doublet-triplet couplings, the determination of the number of light Higgs doublets and the mixing between them, the hierarchy of quark and lepton masses, and the elimination of unwanted fields from the light spectrum.

A major advance in free fermionic string model building came with the elucidation of the techniques to calculate higher-order terms in the superpotential. This breakthrough allowed a thorough investigation of the structure of the model and gave answers to all the above lingering questions. Two low-energy variants of the model have since been constructed, differing in the assumed pattern of $SU(5) \times U(1)$ symmetry breaking, as discussed below. We now describe the calculational framework of the free fermionic formulation where the revamped flipped $SU(5)$ model has been constructed.

4. The Free Fermionic Formulation

A reflection of the plethora of classical string vacua are the many “formulations” one can use to construct these vacua (i.e., “string models”). Basically all possible ways of getting string models are now known. The two-dimensional (2d) world-sheet theory can only be conformal invariant at the quantum level if extra 2d degrees of freedom are
introduced besides the 4d coordinates and their 2d superpartners. These extra ingredients can be represented generally by appropriate conformal field theories with their own set of 2d fields. The initial choice was to introduce additional spacetime dimensions, as in the ten-dimensional Calabi-Yau models [10]. This choice has the advantage of yielding basically one 10d gauge group \((E_8 \times E_8)\). However, the compactification process to 4d has never been fully understood. The free fermionic formulation [36] chooses extra free 2d fermions as the supplementary degrees of freedom, thus no compactification is needed. The drawback is that the number of models that can be constructed this way is very large. Many other choices exist [37], but for calculational purposes the free fermionic formulation is the most convenient one.

Free fermionic models are determined by the boundary conditions of the 2d fermions as they loop around the 2d one-loop worldsheet (a torus). These boundary conditions are arranged in “vectors” of phases (e.g., periodic, antiperiodic, etc.). Strict consistency requirements (2d supersymmetry, 2d modular invariance, etc) constrain the set of allowed vectors, and also the possible states in the Hilbert space through generalized GSO projections.

The Hilbert space of physical states is completely calculable and the states are built using standard 2d field theory tools (i.e., creation and annihilation operators acting on a degenerate (Ramond sector) or non-degenerate (Neveu-Schwarz sector) Fock vacuum). Each vector in the model generates a sector of states (which may or may not be projected out by the chosen GSO projections). It also generates a set of GSO projections on the states already present in the model. Only after all desired vectors have been accounted for can one be sure of the final spectrum of the model. The mass formula finally selects the massless states. From this information all interactions in the model (e.g., cubic and higher-order superpotential, D-terms, etc.) can be calculated [29].

Even though there are no general rules, with some practice one can determine a minimal set of vectors that produce a model with N=1 spacetime supersymmetry and has three chiral families of quarks and leptons [38]. A particularly desired final spectrum may or may not be achievable and is mostly the result of a tedious trial-and-error process. Computer codes have been developed to this end [25]. As an example of these vectors and their role, we show in Table I the ones used in the revamped version of the flipped \(SU(5)\) model. Their roles are as follows: the vectors 1, S, and \(\zeta\) give an N=4 spacetime supersymmetric model with gauge group \(SO(28) \times E_8\). Adding \(b_1, b_2, b_3\) we achieve N=1 supersymmetry and \(SO(28) \rightarrow SO(10) \times SO(6)^3\). Adding \(b_4, b_5, 2\alpha\) breaks \(SO(6)^3 \rightarrow U(1)^6\) and \(E_8 \rightarrow SO(16)\). Finally adding \(\alpha\) breaks \(SO(10) \rightarrow SU(5) \times U(1), U(1)^6 \rightarrow U(1)^4\), and \(SO(16) \rightarrow SO(10) \times SO(6)\).
Table I: The set of basis vectors used in the construction of the revamped flipped $SU(5)$ model. The various entries correspond to the 22 left-moving (left of the colon) and 44 right-moving 2d fermions. A 1 (0) stands for periodic (antiperiodic) boundary conditions, and $1/2$ for a phase of $e^{i\pi/2}$. The symbol $A$ stands for $A = (\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} 1 1 1 1 0 0) \text{ and } 0_8 = 00000000$.

$$
S = (11 100 100 100 100 100 100 100 : 000000 000000 00000 0000 08) \\
b_1 = (11 100 100 010 010 010 010 : 001111 000000 11111 100 08) \\
b_2 = (11 010 010 100 100 001 001 : 110000 000111 11111 010 08) \\
b_3 = (11 001 001 001 001 100 100 : 000000 111100 11111 001 08) \\
b_4 = (11 100 100 010 001 001 010 : 001001 000110 11111 100 08) \\
b_5 = (11 001 010 100 100 001 010 : 010001 100010 11111 010 08) \\
\zeta = (00 000 000 000 000 000 000 : 000000 000000 00000 000 1_8) \\
\alpha = (00 000 000 000 011 000 011 : 000101 011101 1 1 1 1 1 1 1 \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} A)
$$

5. The Flipped SU(5) String Model

As explained in the previous section, to build a model in the free fermionic formulation one needs to specify a consistent set of vectors and generalized GSO projections. Well-defined rules then allow one to obtain the full massless spectrum of the model which is shown in Table II. Besides the usual gauge quantum numbers, string states possess internal 2d degrees of freedom which appear in 4d as continuous or discrete global symmetries. The latter are not shown in Table II but are obtained in the process and are essential in the calculation of the superpotential couplings [29]. All the cubic [23,30,31] and quartic [30] superpotential couplings have been calculated and the nonvanishing quintic ones are tabulated. For brevity here we just quote the cubic and quartic ones:

$$
W_3 = g\sqrt{2}(F_1F_1h_1 + F_2F_2h_2 + F_4F_4h_1 + F_4\bar{f}_5h_{45} + F_3\bar{f}_3h_3 + f_1\bar{f}_1h_1 + f_2\bar{f}_2h_2 \\
+ \bar{f}_5\bar{f}_5h_2 + \frac{1}{\sqrt{2}}(F_4\bar{F}_5\phi_3 + f_4\bar{f}_5\phi_2 + \bar{f}_4\bar{f}_5\phi_2) + \bar{F}_5\bar{F}_5h_2 + f_4f_4\bar{f}_1h_1 \\
+ (h_1h_2\Phi_{12} + h_2h_3\Phi_{23} + h_3h_1\Phi_{31} + h_3h_4\bar{h}_{45} + h.c.) \\
+ \frac{1}{2}(\phi_{45}\bar{\phi}_{45} + \phi^+\bar{\phi}^+ + \phi^-\bar{\phi}^- + \phi_1\bar{\phi}_1 + h_4h_5\bar{h}_{45})\Phi_3 + (\phi_1\bar{\phi}_2 + \bar{\phi}_1\phi_2)\Phi_4 \\
+ (\phi_3\bar{\phi}_4 + \bar{\phi}_3\phi_4)\Phi_5 + (\Phi_{12}\Phi_{23}\Phi_{31} + \Phi_{12}\phi^+\phi^- + \Phi_{12}\phi_1\phi_i + h.c.) \\
+ D_1^2\Phi_{23} + D_2^2\Phi_{31} + D_4^2\Phi_{23} + D_5^2\Phi_{31} + \frac{1}{\sqrt{2}}D_4D_5\Phi_3 \\
+ T_1^2\Phi_{23} + T_2^2\Phi_{31} + T_4^2\Phi_{23} + T_5^2\Phi_{31} + \frac{1}{\sqrt{2}}T_4T_5\phi_2
$$

6
Note that all couplings depend only on the unified string coupling $c$ where the coefficients $U$ is given by:

$$U = \text{gauge group, which is a linear combination of all the trace.}$$

It can be shown \[39\] that if there are properly normalized $U$ to Tr $U$ well as all the F-terms, that is want to preserve unbroken supergravity at $M$.

Indeed, the D-term of the anomalous $U$ really anomalous since there is a built-in mechanism \[40\] that cancels all the potential number of massive states. An effective low-energy theory of the massless modes is not arises because we are only considering the massless sector of a theory with an infinite orthogonal linear combinations are traceless and otherwise arbitrary. This phenomenon where the coefficients $c_i$ are given by:

$$c_1 = -3.07g^2, \quad c_2 = ic_1, \quad c_3 = -1.032ig^2, \quad c_4 = 2c_3,$$

$$c_5 = 0.487ig^2, \quad c_6 = -0.259g^2, \quad c_7 = -0.211ig^2, \quad c_8 = -0.829ig^2,$$

$$c_9 = 0.418ig^2, \quad c_{10} = 0.245ig^2, \quad c_{11} = 0.216g^2, \quad c_{12} = g^2/2. \quad (5.1c)$$

Note that all couplings depend only on the unified string coupling $g$.

A characteristic of this class of models is the presence of an anomalous $U_A(1)$ in the gauge group, which is a linear combination of all the $U(1)$’s in the model with nonvanishing trace. It can be shown \[39\] that if there are $n$ such $U(1)$’s, then the anomalous combination is given by $U_A = k \sum_i [\text{Tr} \, U_i] U_i$, where the coefficient $k$ is generally not important but for properly normalized $U(1)$’s it is given by $1/k^2 = \sum_i [\text{Tr} \, U_i]^2$. The remaining $n - 1$ orthogonal linear combinations are traceless and otherwise arbitrary. This phenomenon arises because we are only considering the massless sector of a theory with an infinite number of massive states. An effective low-energy theory of the massless modes is not really anomalous since there is a built-in mechanism \[41\] that cancels all the potential anomalies in triangle graphs. However, this state of affairs has observable consequences. Indeed, the D-term of the anomalous $U_A(1)$ receives a one-loop contribution proportional to $\text{Tr} \, U_A$ \[41\], and the “anomalous” gauge boson receives a two-loop order mass. Since we want to preserve unbroken supergravity at $M_{Pl}$, the D-terms of all $U(1)$’s must vanish, as well as all the F-terms, that is:

$$\langle W \rangle = \langle \frac{\partial W}{\partial \phi_i} \rangle = 0, \quad (5.2a)$$

$$\langle D_A \rangle = \sum_i q^i_A \langle |\phi_i| \rangle^2 + \epsilon = 0, \quad (5.2b)$$

$$\langle D_a \rangle = \sum_i q^i_a \langle |\phi_i| \rangle^2 = 0, \quad a = 1, 2, 3, \quad (5.2b)$$

\[\]
Table II: The massless spectrum of the revamped flipped \( SU(5) \) model. The transformation properties of the observable sector fields under \( SU(5) \times U(1) \) are as follows: \( F(10,1/2), \bar{f}(\bar{5},-3/2), l^c(1,5/2), h(5,-1) \). The hidden sector fields transform under \( SO(10)_h \times SU(4)_h \) as follows: \( T(10,1), D(1,6), \tilde{F}(1,4) \). The \( \tilde{F}_i, \tilde{F}_j \) fields carry \( \pm 1/2 \) electric charges.

| Observable Sector: | \( F_1 \bar{f}_1 l_1^c \) | \( h_1 \bar{h}_1 \) |
|-------------------|------------------|------------------|
| \( F_2 \bar{f}_2 l_2^c \) | \( h_2 \bar{h}_2 \) |
| \( F_3 \bar{f}_3 l_3^c \) | \( h_3 \bar{h}_3 \) |
| \( F_4 f_4 l_4 \) | \( h_{45} \bar{h}_{45} \) |
| \( \bar{F}_5 \bar{f}_5 l_5^c \) |

| Singlets: |
|-----------|
| \( \phi_{45} \bar{\phi}_{45} \) | \( \Phi_{12} \bar{\Phi}_{12} \) | \( \Phi_{1,2,3,4,5} \) |
| \( \phi^+ \bar{\phi}^+ \) | \( \Phi_{23} \bar{\Phi}_{23} \) |
| \( \phi^- \bar{\phi}^- \) | \( \Phi_{31} \bar{\Phi}_{31} \) |
| \( \phi_{1,2,3,4} \) | \( \bar{\phi}_{1,2,3,4} \) |

| Hidden Sector: | \( T_1 D_1 \bar{F}_1 \bar{\tilde{F}}_1 \) |
|---------------|------------------|
| \( T_2 D_2 \bar{F}_2 \bar{\tilde{F}}_2 \) |
| \( T_3 D_3 \bar{F}_3 \bar{\tilde{F}}_3 \) |
| \( T_4 D_4 \bar{F}_4 \bar{\tilde{F}}_4 \) |
| \( T_5 D_5 \bar{F}_5 \bar{\tilde{F}}_5 \) |
| \( \bar{F}_6 \bar{\tilde{F}}_6 \) |

where \( \epsilon = g^2 \text{Tr} U_A/192\pi^2 \). To restore the units one recalls that \( \kappa = \sqrt{8\pi}/M_{Pl} \) has been set to 1 in these equations. In the revamped model \( \text{Tr} U_A = 180 \) and thus \( \epsilon = (7.4 \times g \times 10^{17} \text{GeV})^2 \). Defining \( M \equiv M_{Pl}/2\sqrt{8\pi} \approx 1.2 \times 10^{18} \text{GeV} \), the “natural” scale of \( \langle \phi \rangle \) is then \( \langle \phi \rangle/M \sim 1/10 \). There are more unknowns than equations to be solved, so the flatness condition cannot determine the vevs, although some results actually follow [39,31], such as \( \langle \Phi_3, \Phi_{12}, \bar{\Phi}_{12} \rangle = 0 \). It can be shown [39] that at least three (the anomalous one and two anomaly-free ones) of the four \( U(1)'s \) are broken by any given solution to the flatness conditions, and most known solutions in fact break all four \( U(1)'s \). The gauge group is then reduced to \( SU(5) \times U(1) \times SO(10)_h \times SU(4)_h \) at scales \( \sim \langle \phi \rangle \sim 10^{17} \text{GeV} \).
There are three built-in mass scales in the model: (i) the singlet vevs $\langle \phi \rangle \sim 10^{17}$ GeV, (ii) the $SU(5) \times U(1)$ breaking vevs $V, \overline{V} \sim 10^{16}$ GeV (put in by hand), and (iii) the scale of hidden sector condensation $\Lambda_{10} \sim 10^{15}$ GeV [12] and $\Lambda_4 \sim 10^{10-12}$ GeV [12], determined dynamically. The ensuing hidden matter condensates $\langle TT \rangle, \langle DD, \tilde{F} \tilde{F} \rangle$ then provide another source of effective masses. With all these mass scales and the all-orders superpotential one can study the Higgs doublet and triplet mass matrices and the fermion Yukawa matrices to determine the low-energy content of the model. Before doing this, let us pause to appreciate the role of the higher-order terms in the superpotential. A generic cubic $SU(5) \times U(1)$ invariant $\phi_1 \phi_2 \phi_3$ can receive contributions from all orders of the form

$$c g^{N-2} \phi_1 \phi_2 \phi_3 \langle \phi \rangle^n (V \overline{V})^m \langle TT \rangle^p (\sqrt{2\alpha'})^{N-3},$$

(5.3)

where $N = n + 2m + 2p + 3$ and $c$ is an $O(1)$ calculable constant. Using [12,13] $\kappa = \frac{1}{2} g \sqrt{2\alpha'} = \sqrt{8\pi}/M_{Pl}$, that is $g \sqrt{2\alpha'} = 1/M$, we get

$$c g \phi_1 \phi_2 \phi_3 \left( \frac{\langle \phi \rangle}{M} \right)^n \left( \frac{V \overline{V}}{M^2} \right)^m \left( \frac{\langle TT \rangle}{M^2} \right)^p.$$

(5.4)

Clearly, higher-order terms get naturally suppressed by powers of $\langle \phi \rangle \sim 10^{-1}, V \overline{V}/M^2 \sim 10^{-4}$, $\langle TT/M^2 \rangle \lesssim (\Lambda_{10}/M)^2 \sim 10^{-6}$.

6. The low-energy spectrum

We now obtain the low-energy spectrum of the model by explicit examination of the Higgs doublet and triplet mass matrices, the fermion Yukawa matrices, and the neutrino see-saw matrix. We also run the third generation Yukawa couplings to low energies to obtain predictions for $m_t$ and $\tan \beta$.

6.1. Preliminaries

We must first make an educated guess as to the pattern of $SU(5) \times U(1)$ breaking. The 10 of Higgs is in general a linear combination of $F_1, F_2, F_3, F_4$. However, an analysis of the Higgs triplet mass matrix shows [30] that unless the vev is in $F_1$ or $F_2$ and/or $F_3$, there will not be enough light $d^c$ states. The choice $F_1$ and/or $F_3$ is preferred for the fermion mass spectrum. The two low-energy scenarios mentioned above arise when: (i) $\langle F_3 \rangle = 0$ [30] or (ii) $\langle F_3 \rangle \neq 0$ [34].
It so happens that these two choices were originally made together with a choice for the way in which the extra $f_4, l_4$ matter states got heavy. There are superpotential terms of the form: $f_4 \sum_i \alpha_i \bar{f}_i$ and similarly for $l_4$. In Model (i) one took $\alpha_{1,2,5} \gg \alpha_3$, whereas in Model (ii) the opposite was assumed. Below we will present results for Model (ii) only since these are more interesting. (Besides, Model (i) may suffer from too rapid proton decay \cite{44}.)

It has not become clear until a recent study of the hidden matter condensates in the model \cite{35} that the assumptions about $\langle F_3 \rangle$ and $\alpha_i$ which determine Models (i) and (ii), are actually compatible with each other. The point is that $\alpha_3 \propto \langle T_3 T_4 \rangle$ is non-vanishing and large only if $\langle F_3 \rangle \neq 0$, and in this case $\alpha_3 \gg \alpha_{1,2,5}$, as advocated in Model (ii). If $\langle F_3 \rangle = 0, \langle T_3 T_4 \rangle \sim 0$ and $\alpha_3 \ll \alpha_{1,2,5}$ as needed for Model (i). (However, the latter choice may lead to a pathological vacuum structure \cite{35}.)

6.2. Higgs doublet masses

The analysis is complicated because of the several sources of Higgs doublet masses, as follows

\begin{align*}
h_i \bar{h}_j & \rightarrow H_i \bar{H}_j, \quad (6.1a) \\
F \bar{f}_i \bar{h}_j & \rightarrow \langle \nu^c_{H_i} \rangle L_i \bar{H}_j, \quad (6.1b) \\
f_4 \bar{f}_j & \rightarrow \bar{L}_4 L_j, \quad (6.1c) \\
\bar{F}_5 f_4 h_i & \rightarrow \langle \nu^c_{H_i} \rangle \bar{L}_4 H_i. \quad (6.1d)
\end{align*}

Note the potential mixing between “Higgs” doublets $H_i$ and “lepton” doublets $L_j$. As remarked above, contributions of these types can arise at any order and one must be certain that the emerging structure of the matrix is indeed stable up to sufficiently high orders so that yet higher-order contributions are negligible. As it turns out, remarkable results hold which make the structure indeed stable \cite{33}, with many entries remaining uncorrected to all orders in nonrenormalizable terms. The reasons behind these results are the internal symmetries of the string states mentioned above. In passing, let us quote the following result \cite{33}

$$\phi^N \equiv 0, \quad N \geq 4, \quad (6.2)$$

where $\phi^N$ is an arbitrary product of $N$ singlets. This amazing result is very important since it implies that the F-flatness conditions only get contributions from the cubic $\phi^3$ couplings, that is, they are stable.
The analysis of the doublet matrix can be done in steps. First we consider only the all-orders contribution generated by singlet vevs. These leave \( H_1, H_{245} = \cos \theta H_2 - \sin \theta H_{45} \), and \( \overline{H}_{12} = \cos \theta \overline{H}_1 - \sin \theta \overline{H}_2, \overline{H}_{45} \) light, where \( \tan \theta = \langle \Phi_{23} \rangle / \langle \phi_{45} \rangle \) and \( \tan \tilde{\theta} = \langle \Phi_{31} \rangle / \langle \Phi_{23} \rangle \). It can be shown that \((TT)\)-generated effective mass terms give more structure to the remaining light Higgs doublet mass matrix, with \( H_1, H_{45} \) remaining light and a mixing term \( \mu H_1 H_{45} \), with 
\[ \mu = \langle \overline{\phi}_{45} \rangle \langle TT \rangle / M^2 \] . If \( \langle \overline{\phi}_{45} \rangle = 0 \) at \( M_{Pl} \) and it grows a vev \( \sim 10^{11} \text{ GeV} \) after supersymmetry breaking, then \( \mu \sim 10^3 \text{ GeV} \).

6.3. Higgs triplet masses

Analogously one can study the doublet-triplet splitting matrix and obtain the Higgs triplet masses [30,33]. The calculation is cumbersome but three important results follow: (i) the structure is stable as before, (ii) one must be careful with how the \( 10 \) vev is distributed among the decaplets, and (iii) there is a Higgs triplet with mass \( \sim 10^{10-11} \text{ GeV} \).

6.4. Fermion Yukawa couplings

After analyzing the doublet and triplet Higgs mass matrices we are ready to make the identification of the quarks and leptons with the specific string representations. This is a unique choice in Model (ii) [34]:

\[
\begin{align*}
    t b \tau \nu_\tau : & \quad Q_4 \, d_4^c \, u_5^c \, L_1 \, l_1^c, \\
    c s \mu \nu_\mu : & \quad Q_2 \, d_2^c \, u_2^c \, L_2 \, l_2^c, \\
    u d e \nu_e : & \quad Q_\beta \, d_\beta^c \, u_1^c \, L_5 \, l_5^c,
\end{align*}
\]

where the \( \beta \) subscript refers to \( F_\beta \propto -\langle F_3 \rangle F_1 + \langle F_1 \rangle F_3 \) which is the linear combination that does not get a vev. With this identification the Yukawa couplings can be written down immediately [34]:

\[
\begin{align*}
    \lambda_t = \lambda_b = \lambda_\tau = g \sqrt{2}, \\
    \lambda_c = 3.04g \langle \overline{\phi}_4 \rangle / M, \quad \lambda_s = \lambda_\mu = g \{ \sum_{i=1}^4 c_{si} \langle \overline{\phi}_i \rangle^2 + c_{s+} \langle \overline{\phi}^+ \overline{\phi}^- \rangle \} / M^2, \\
    \lambda_u = 0, \quad \lambda_d = \langle V_3 / V \rangle, \quad \lambda_e = g \{ c_{e1} \langle \overline{\phi}_1 \rangle^2 + c_{e4} \langle \overline{\phi}_4 \rangle^2 + c_{e+} \langle \overline{\phi}^+ \overline{\phi}^- \rangle \} / M^2,
\end{align*}
\]

Some quintic couplings had to be calculated explicitly to obtain the result \( \lambda_s = \lambda_\mu \).
These Yukawa couplings at $M_{Pl}$ still need to be evolved down to low energies with a few uncertainties along the way, such as the effect of supersymmetry breaking at high scales, the details of the $SU(5) \times U(1)$ breaking, the decoupling of the various heavy modes along the way, and the usual low-energy questions about the values of the light quark masses. Nevertheless, let us point out some interesting features of this set of Yukawa couplings:

(a) The successful GUT relation $\lambda_b = \lambda_\tau$.
(b) The mass ratios $m_s/m_b \sim (\langle \phi \rangle/M)^2 \sim 1/30$ and $m_d/m_b \sim (V_3/V)^2 \sim 1/600$ could plausibly be accommodated by $\langle \phi \rangle/M \sim 1/6$ and $V_3/V \sim 1/25$.
(c) The successful (?) GUT relation $\lambda_s = \lambda_\mu$.
(d) The unexpected (successful (?)) relation $\lambda_c/\lambda_t \sim (\lambda_e/\lambda_\tau)^{1/2}$ (if $\langle \bar{\phi}_1 \rangle \sim \langle \bar{\phi}^+ \bar{\phi}^- \rangle \sim 0$).

6.5. Neutrino masses

The see-saw neutrino mass matrix in flipped $SU(5)$ involves $\nu, \nu^c, \text{ and singlet states } \phi$ [27,45], through the superpotential couplings: $\lambda_{ij} u_i F_j h_{45} \ni \lambda_{ij} u_i \nu_j \langle H_{45} \rangle$; $w_{ij} F_i F_5 \phi_j \ni w_{ij} V \nu_j \phi_j$; and $\mu_{ij} \phi_i \phi_j$. In the string model, the see-saw matrix turns out to be $14 \times 14 \{\nu_{1,2,5}, \nu^c_{2,2,4}, \phi_{1,2,3,4}, \bar{\phi}_{45}, \Phi_{3,4,5}\}$. An important point is that all the $\nu^c \nu^c$ entries vanish. However, there are two-loop radiatively induced contributions of the form [16] $m_{\nu \nu^c} \approx (\alpha_X/4\pi)^2(3/16)\lambda_{d,s,b} M_X \approx 5 \times 10^8 m_{d,s,b}$, which contribute decisively to the structure of the see-saw matrix. If we choose $\langle \bar{\phi}_2 \rangle = 0$, then the matrix breaks up into three blocks with only three light eigenvalues: $\nu_1, m_1 = 0; \sim \nu_2, m_2 = 0;\text{ and } \approx \nu_5, m_5 \approx m_t \sqrt{\lambda_d \langle TT \rangle}/M^2$. Hence (see Eqs. (5.3)) $\nu_\mu$ and $\nu_\tau$ are massless to this level of approximation, and $\nu_e$ has a mass in the eV range, within the current experimental limit $m_{\nu e} < 10 \text{ eV}$ [47].

6.6. Top-quark mass predictions

In principle one cannot determine any fermion mass since the ratio of vevs $\tan \beta = v_2/v_1$ has not been determined dynamically. The correct procedure would be to run all gauge and Yukawa couplings and demand adequate electroweak symmetry breaking. Without doing this one can only give an upper bound on $m_t$: $m_t = \lambda_t \sin \beta \times 174 < 174 \lambda_t \approx 174 \text{ GeV for } m_b = 5.0 \text{ GeV}$, since otherwise $\lambda_t$ would blow up before the unification scale. As an encouraging sign, an explicit calculation like the one outlined above for the $\lambda_t = \lambda_b = \lambda_\tau$ scenario in minimal supersymmetric GUTs gives [48] $m_t \approx 90 - 150 \text{ GeV}$ and $\tan \beta \approx 25 - 45$ for $m_b = 4.9 \pm 0.1 \text{ GeV}$. Some of these points have also been found to be compatible with adequate electroweak breaking.
7. Proton Decay

Baryon number violating operators of dimension four \((qqq/qql)\) and five \((qqql)\) are a generic menace to unified models. Basically \(d = 4\) operators must be forbidden by some extra symmetry, such as matter parity \([49]\), and acceptable Higgs-induced \(d = 5\) operators are allowed only in some regions of the parameter space of supersymmetric unified models \([50]\). Nonrenormalizable interactions at the Planck scale may also produce \(d = 4, 5\) baryon number violating operators. An analysis in a generic \(SU(5)\) supergravity model \([51]\) indicates that the nonrenormalizable point coupling \(g_b\) must be bounded by 
\[
g_b \lesssim 7 \times 10^{-25} \text{GeV}^{-1} \ll 1/M_{Pl}.\]
(In contrast, in the minimal \(SU(5)\) supersymmetric GUT model one finds 
\[
g_b \sim G_F m_c m_s \sin \theta_c / m_H,\]
which gives \(M_H \gtrsim 10^{17} \text{GeV}\).) Hence, if nonrenormalizable couplings are of order 
\[
g_b \sim 1/M_{Pl},\]
as one would naturally expect, then these new \(d = 5\) operators would be a disaster giving 
\[
\tau_p \sim 10^{20} \text{y}.
\]

The various operators contributing to proton decay in this model have to be studied carefully since some of them may be potentially disastrous. These operators are:

(a) Nonrenormalizable terms in the superpotential of the form \([32,44]\) \(FFF \bar{f}_M\) contain effective \(d = 5\) \(qqql\) operators which could be very large \([32,44]\) since the overall coefficient is expected \([30]\) to be \(\mathcal{O}(1)\). These terms appear first at fifth order \([30,44]\) (multiplied by \(\langle \phi \rangle / M\)) and always contain the \(\bar{f}_3\) field, and thus are harmless in Model (ii) since \(\bar{f}_3\) does not contain any light states.

(b) Dimension-five Higgsino mediated operators \([52]\) are constructed via the usual tree-level diagrams involving three superpotential couplings: \(FFh, F \bar{f} \bar{h},\) and \(h \bar{h}\). Since we are interested in taking \(\bar{f}\) to be \(\bar{f}_{1,2,5}\), the associated \(\bar{h}\) is always \(\bar{h}_{45}\). The needed mixing term \(h \bar{h}_{45}\) is proportional to \(\mu\) for \(h_{1,2}\) and to \(\langle \phi_{45} \rangle\) for \(h_3\), and the \(F \bar{f} \bar{h}\) vertex coupling is in effect the up-quark Yukawa matrix. A careful analysis shows that the coefficients of the resulting effective \(d = 5\) operators are always suppressed enough, although the potential proton decay rates could be close to the current experimental limit.

(c) Dimension-six Higgs boson mediated operators could be important because of the existence of \(\mathcal{O}(10^{10-11}) \text{GeV}\) Higgs triplet states, as mentioned above. There are two classes of diagrams \([52]\) originating from superpotential couplings of the forms: \(F \bar{f} \bar{h} / F \bar{f} \bar{f} \bar{h}\) and \(FFh / f \bar{f} \bar{c} \bar{h}\). The first class yield effective operators suppressed by \(1/\tan^2 \beta\) relative to their counterpart in SUSY GUTs, while the second class give negligible contributions.
(d) Dimension-six gauge boson exchange operators are sufficiently suppressed in flipped $SU(5)$ models \cite{32}, where it is found that $\Gamma(p \to \nbar\nu\pi^+) = 2\Gamma(p \to e^+\pi^0) = \Gamma(n \to e^+\pi^-) = 2\Gamma(n \to \nbar\nu\pi^0)$, with $\tau(p \to e^+\pi^0) \approx 3 \times 10^{31} \left(\frac{M_{\text{Pl}}}{10^{15}\text{GeV}}\right)^4$. However, in the present model the particle assignment in (6) indicates that the dominant decay modes are actually $p \to \nbar\nu\tau\pi$ and $n \to \nbar\nu\tau\pi^0$. For comparison, the present experimental lower limit is $\tau(p \to e^+\pi^0) > 5.5 \times 10^{32}$ y \cite{53}.

8. Cryptons

The hidden sector of the model contains the 22 matter fields given in Table II. The electric charge generator $Q$ is given by $Q = T_3 - \frac{1}{5} Y' + \frac{2}{3} \tilde{Y}$, where $U(1)_{Y'}$ is the $U(1)$ inside $SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_{Y'}$. Since $Q$ is not completely embedded in a simple group, the possibility exists for electrically charged exotic states with no $SU(5)$ quantum numbers, such as the $\tilde{F}_i, \tilde{F}_j$ above which have $Q = \pm 1/2$ \cite{17,54}. Since light free fractionally charged particles are not observed \cite{53}, and their existence could have grave astrophysical consequences \cite{56}, there must exist a mechanism to either make them superheavy or somehow bind them into neutral bound states (as in QCD).

The solution to the charged quantization problem can be encoded in the following experimentally motivated charge quantization dogma: All (massless) fractionally charged particles must have nontrivial quantum numbers under unbroken nonabelian gauge groups, such that when confinement sets in and thus only gauge singlets are observable, the resulting physical states are integrally charged. In a string-derived model this condition can be conveniently implemented in the language of simple currents \cite{57} of the Kac-Moody algebra underlying the gauge group \cite{58,17}. Compatibility with the string rules places severe restrictions on the gauge groups and their Kac-Moody levels, which could enforce such a quantization condition. It can be shown \cite{17} that the following charge quantization rule can be consistently imposed on the spectrum of the flipped $SU(5)$ string model: $\alpha = \frac{1}{3} t_3 + Q + \frac{1}{2} t_4 + \frac{1}{2} c \in \mathbb{Z}$, where $t_3, t_4$, and $c$ are the triality, quadrality, and conjugacy classes of the respective $SU(3)_C, SU(4)_h$, and $SO(10)_h$ representations. One can readily verify that this condition is satisfied by all massless states in the model. Also, one can show \cite{17} that this holds at all massive levels as well.

Models which do not have a hidden sector sticky enough to confine the existing fractionaly charged states are likely to be in trouble. On the other hand, potentially realistic models that confine fractional charges will then generally contain integer-charge “hidden
hadrons” as their solution to the charge quantization problem. From the light spectrum of the model three kinds of $SU(4)_h \times SO(10)_h$ invariant bound states follow:

$$\text{cryptons} \begin{cases} \text{hidden mesons} : & T_i T_j, D_i D_j, \tilde{F}_i \tilde{F}_j, \ (0, \pm 1); \\ \text{hidden baryons} : & \tilde{F}_i \tilde{F}_j D_k, \tilde{F}_i \tilde{F}_j \tilde{D}_k, \ (0, \pm 1); \\ \text{tetrons} : & \tilde{\tilde{F}}_i \tilde{\tilde{F}}_j \tilde{\tilde{F}}_k \tilde{\tilde{F}}_l, \ (0, \pm 1, \pm 2); \end{cases} \tag{8.1}$$

where we have indicated in parenthesis the possible electromagnetic charges for each class of crypton. The superpotential for these hidden fields can be calculated, and it is seen that most of these fields become superheavy. However, there are four fields that remain as light as their respective confinement scales: $T_3, D_3, \tilde{F}_{3,5}, \tilde{\tilde{F}}_{3,5}$. Since a renormalization group analysis \[42\] indicates that $\Lambda_4 \approx 10^{11-12} \text{ GeV}$ and $\Lambda_{10} \approx 10^{14-15} \text{ GeV}$, we expect the $SU(4)_h$ bound states to be much lighter than $SO(10)_h$ bound states in general. The lightest $SU(4)_h$ bound states are expected to be the hidden mesons $\tilde{F}_{3,5} \tilde{F}_{3,5}$ and tetrons $\tilde{\tilde{F}}_{3,5}$ and $\tilde{\tilde{F}}_{3,5}$. Because of its larger crypto-charge, one would expect bound states of the $6$ field $D_3$ to be somewhat heavier. We expect the lightest hidden $SU(4)_h$ meson to be analogous to the $\pi^0 : \pi_4^0 \approx (\tilde{F}_3 \tilde{F}_3 - \tilde{F}_5 \tilde{F}_5)/\sqrt{2}, m_{\pi_4^0}^2 \approx \Lambda_4 \times m_{\tilde{\tilde{F}}_{3,5}, \tilde{\tilde{F}}_{3,5}}$, with charged $\pi_{4}^\pm \approx (\tilde{F}_3 \tilde{F}_5, \tilde{\tilde{F}}_3 \tilde{\tilde{F}}_5)$ states slightly heavier because of electromagnetic mass splitting analogous to that in QCD.

Also, as in QCD we expect the $\eta^0_4 \approx (\tilde{F}_3 \tilde{F}_3 + \tilde{F}_5 \tilde{F}_5)/\sqrt{2}$ state to be significantly heavier because of a $U_A(1)$ anomaly. By analogy with the nucleon and $\Delta$ states in QCD, the lightest tetrons are expected to be the neutral $\tilde{\tilde{F}}_3^2 \tilde{\tilde{F}}_3^2$ and $\tilde{\tilde{F}}_5^2 \tilde{\tilde{F}}_5^2$ states, with singly-charged $\tilde{\tilde{F}}_3^3 \tilde{\tilde{F}}_3^3, \tilde{\tilde{F}}_5^3 \tilde{\tilde{F}}_5^3$, and $\tilde{\tilde{F}}_3 \tilde{\tilde{F}}_5^3$ states somewhat heavier, and doubly-charged $\tilde{\tilde{F}}_3^4, \tilde{\tilde{F}}_5^4, \tilde{\tilde{F}}_3^4$, and $\tilde{\tilde{F}}_5^4$ states even heavier.

By considering the likely crypton decays one can show that the lightest neutral tetrons have lifetimes in excess of $10^{16}$ years, and hence are possible candidates for the dark matter of the Universe. There are also cosmological \[54\] constraints on metastable cryptons. A naive estimate of the crypton relic abundance $\Omega_C \approx (\Lambda_4/100 \text{ TeV})^2 \gg 1$ indicates the need for entropy releasing mechanisms. There are several mechanisms which dilute the crypton relic density to levels $\Omega_C \ll 1$. However, $\Omega_C \approx 0.1 - 1$ is not ruled out. Long-lived cryptons also satisfy all known constraints on massive unstable neutral relic particles \[59\].

9. String Unification

String theory predicts the scale at which all gauge couplings should unify to be $M_K \approx 7.3 \times g \times 10^{17} \text{ GeV}$ \[13\]. However, this scale must be corrected to include the so-called
string threshold corrections. These arise in much the same way as in regular gauge theories when thresholds of massive particles are crossed. The novelties in string theory are that the threshold is only approached from below and that the massive states are infinite in number. The one-loop renormalization group equation for the gauge couplings can be generally written as follows [13]

\[
\frac{16\pi^2}{g^2_a(\mu)} = \frac{16\pi^2}{g^2} + b_a \ln \frac{M_K^2}{\mu^2} + \Delta_a,
\]

(9.1)

where \(\Delta_a\) are the string threshold corrections and \(b_a\) is the coefficient of the one-loop beta function. In generic orbifold (and free fermionic) models these have been calculated to be [60, 61]

\[
\Delta_a = -\sum_\alpha \frac{1}{2} b_\alpha^a \ln \left[|\eta(iT_\alpha)|^4 \Re T_\alpha\right] + c_a + Y,
\]

(9.2)

where \(\eta\) is the Dedekind function and \(b_\alpha^a\) are the beta function coefficients of the three N=2 supersymmetry sectors into which the massless spectrum can be split. Equivalently [62] one can express \(\Delta_a\) in terms of the charges of the states under the modular transformations of the moduli fields \(T_\alpha\). These fields parametrize the size of the compatification manifold. The constant \(c_a\) is model-dependent, but in a large class of models \(c_a = b_a \cdot c\). This contribution is small in all known cases [13, 14, 15] and will be neglected in what follows. The quantity \(Y\) is irrelevant in practical applications but it is of some theoretical interest [62, 63, 64].

A nice simplification occurs when one takes the values of the three moduli to be the same, since then \(\frac{1}{2} \sum_\alpha b_\alpha^a = b_a\) [65] implies that [15]

\[
\Delta_a \approx b_a \left[\frac{\pi}{3} \Re T - \ln \Re T + \text{small corrections}\right].
\]

(9.3)

In the case of flipped \(SU(5)\), the free fermionic formulation has \(T = 1\) and the string unification scale becomes

\[
M_U \approx M_K e^{\pi/6} \approx 1.24 \times g \times 10^{18} \text{ GeV}.
\]

(9.4)

The significance of this result cannot be over-emphasized. At this scale the string couplings unify and start their running to low energies. Furthermore, the scale is determined by the string unified coupling, which becomes the only unknown in the theory. This unification scale is two orders of magnitude larger than the one expected from a straightforward evolution of the low-energy couplings in the minimal supersymmetric \(SU(5)\) model [8].
Put it differently, if one starts at $M_U$ with a value of $g$ such that $\alpha_{em}$ at low energies comes out right, then at low energies one obtains $\sin^2 \theta_W = 0.218$ and $\alpha_3 = 0.20$, which are in gross disagreement with their very precise experimental counterparts.

Two alternatives have been suggested to reconcile these results. One can throw in extra vector-like matter representations at suitable intermediate scales to delay unification \cite{66}, or one can exploit the $T_\alpha$ dependence of $\Delta_a$ to push $M_U$ down \cite{67}. A combination of both possibilities is in principle possible, although the latter cannot happen in free fermionic models \cite{67}. In the case of flipped $SU(5)$, things are more complicated due to the non-minimal matter content at only approximately known intermediate mass scales. However, it appears that the first alternative will need to be pursued to obtain a satisfactory model. This modification of the model is currently under investigation; we believe that the main features of the model outlined above will remain for the most part intact.

10. Conclusions

We have described the early developments leading up to the flipped $SU(5)$ string model, focusing on the rationale behind this particular string model and basic model-building in the free fermionic formulation of superstrings, which was used to construct the flipped $SU(5)$ model. A study of the all-orders superpotential allows us to obtain the low-energy spectrum of the model, which entails an analysis of the doublet and triplet Higgs mass matrices, the fermion Yukawa matrices, and the see-saw neutrino matrix. We also showed that in spite of several operators contributing to the proton decay rate, it is not a problem in the latest version of the model. We also reviewed one of the possible “smoking guns” of string, namely the hidden sector bound states called cryptons. Finally, we commented on the subject of string threshold corrections and string unification.

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