Nonadiabatic Asteroseismology of GW Vir Stars

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Abstract. We present and explore a new method involving a nonadiabatic stability calculation to determine the position of the GW Vir band in the \( \log g - T_{\text{eff}} \) diagram. With this method, we calculate the position of two GW Vir stars, PG 1159-035, \( \log g = 6.80, T_{\text{eff}} = 144,500 \text{ K} \) and PG 1707+427, \( \log g = 7.38, T_{\text{eff}} = 81,500 \text{ K} \). The results are consistent with and complementary to the spectroscopic determination of atmospheric parameters and the chemical compositions of these stars.

1. Astrophysical Context

The GW Vir stars are extremely hot, compact objects exhibiting luminous variability due to gravity modes. They include two different hydrogen-deficient spectral types: the Wolf-Rayet Central Stars of Planetary Nebulae, or [WC] stars, and their descendants, the PG 1159 stars. The multiperiodic luminosity variations observed in GW Vir stars are due to a classic \( \kappa \)-mechanism associated with the partial ionization of the carbon and oxygen K-shells.

Quantitative spectroscopy, based on detailed NLTE model atmospheres and synthetic spectra, has revealed that GW Vir stars occupy a wide domain in the surface gravity–effective temperature plane, with objects in the ranges \( 5.5 \leq \log g \leq 7.5 \) and \( 80,000 \text{ K} \leq T_{\text{eff}} \leq 170,000 \text{ K} \). In addition, the inferred atmospheric abundances vary substantially from one star to another, the main atmospheric constituents being found in the intervals, \( 0.33 \leq X(\text{He}) \leq 0.76 \), \( 0.16 \leq X(\text{C}) \leq 0.55 \), and \( 0.06 \leq X(\text{O}) \leq 0.17 \). Of the 18 known GW Vir stars, 3 also exhibit hydrogen in their spectra, and are part of the hybrid-PG 1159 subclass. We refer the reader to the excellent review by K. Werner & F. Herwig \cite{1} for more details on the chemical compositions of these stars.

Unfortunately, at these very high temperatures and with these peculiar abundances, the uncertainties on the spectroscopic values of \( T_{\text{eff}} \) and \( \log g \) are quite large when compared to other compact stars. Typical uncertainties of 5–10\% in \( T_{\text{eff}} \), and \( \sim 0.5 \) dex in \( \log g \) are given for most GW Vir stars \cite{1}. We argue here that it is possible to infer the atmospheric parameters of GW Vir pulsators with a higher accuracy by combining spectroscopic results and nonadiabatic asteroseismology. The method used here involves the comparison of the modes observed in GW Vir with the ranges predicted by stability calculations.
2. Models and Results

It has already been shown [2, 3, 4] that the static stellar models with homogenous envelope composition, and the stellar pulsation code we are using here are suited for nonadiabatic studies. They have been used to reproduce the instability domain of GW Vir stars and to reproduce qualitatively individual instability ranges. The goal of the present analysis is to make the qualitative reproduction of instability bands presented in previous studies more quantitative. We are thus moving towards nonadiabatic asteroseismology.

The nonadiabatic effects responsible for modes excitation and damping are quasi-local, in the sense that they are concentrated in the envelope of the stars with only a small contribution from the stellar core. Since the structure of a GW Vir envelope is determined mainly by parameters obtained from spectroscopy, nonadiabatic asteroseismology can put new and different constraints on the atmospheric parameters and on the chemical composition of the envelope. However, it cannot put a direct constraint on either the core structure or the stellar mass.

Figure 1 shows that the instability band of GW Vir stars is strongly correlated with its location in the log $g$ - $T_{\text{eff}}$ diagram. We remark that the range of instability depends mainly on surface gravity for the “fluffy” stars of lower gravity, and that it is a more subtle function of temperature and gravity for the more compact/evolved objects of higher gravity. We also note that the excited quadrupole ($l = 2$) modes systematically have shorter periods than the dipoles ($l = 1$) modes. This feature can help to resolve ambiguous mode identification. For the two stars analyzed here, we rely on the identification made during observations, with the help of our model in case of ambiguity, to identified the degree $l$. In PG 1159-035 the shortest observed period, $P_{\text{min}}^o = 339$ s, is a dipole mode, whereas the long-period boundary $P_{\text{max}}^o = 982$ s is a quadrupole mode [5]. We did not try to fit individually the $l = 1$ and $l = 2$ instability bands because it was not always possible to determine the value of $l$ with certainty. For PG 1707+427, only dipole modes are observed [6]: $P_{\text{min}}^o = 224$ s, $P_{\text{max}}^o = 962$ s. We only compare them with the dipole instability band of our models. We emphasize that our approach only fits the range of GW Vir excited modes, without taking into account the distribution of the periods within this range. Both stars have been observed with the Whole Earth Telescope (WET).

Other than the surface gravity and the effective temperature, the most important parameter that influences the range of the instability band is the chemical composition of the envelope [4]. In the preliminary calculations presented here, we keep the composition or our models fixed, adopting the spectroscopic value. We notice that despite the uncertainties in spectroscopic values of log $g$ and $T_{\text{eff}}$, the chemical abundances are well constrained. The PG 1159-035 models calculated here adopt the latest spectroscopic determinations available for this star [7]. We also added traces of nickel to account for the “missing iron” of GW Vir stars. In that sense, we follow the idea that iron is transmuted into nickel and heavier elements by the s-process during the post-AGB phase. However, we have found that the heavy elements added to the models have only a small influence on the final results [4]. The chemical composition used for PG 1707+427 is less sophisticated since from the spectroscopic point of view this star has not been studied with the same level of detail as PG 1159-035 has. We used the ratio published in [1] for helium, carbon and oxygen; the heavier elements are in solar proportion.

We search the best-fitted models in the log $g$ - $T_{\text{eff}}$ plane to PG 1159-035 and PG 1707+427. The goodness of the fit, $S^2$, is measured by comparing the maximum and minimum excited periods of the models $P_{\text{min}}^m$ and $P_{\text{min}}^m$ with those of the observations $P_{\text{max}}^o$ $P_{\text{min}}^o$.

$$S^2 = \frac{(P_{\text{max}}^m - P_{\text{max}}^o)^2}{\sigma_{\text{max}}^2} + \frac{(P_{\text{min}}^m - P_{\text{min}}^o)^2}{\sigma_{\text{min}}^2},$$ (1)

where the weight $\sigma_i = 1\% P_{\text{max}}^o$ is chosen to be proportional to the observed boundary periods.

The grid of putative PG 1159-035 stars covers and extends outside the spectroscopic error bars log $g = 6.4$ (0.025) 7.6 and $T_{\text{eff}} = 134,000$ (500) 146,000 K. Figure 2 shows that this
In stability bands calculated for models with homogeneous envelope composition, $X(\text{He}) = 0.333$, $X(\text{C}) = 0.49$, $X(\text{O}) = 0.17$ and solar ratios for heavier elements, along a $0.604M_\odot$ post-AGB track provided by F. Herwig. Black diamonds represent stable models and red dots represent unstable models. The red vertical strips map the excited period range common to both values of the dipole and quadrupole modes, and the dark (light) blue strips indicate the unstable periods for dipole (quadrupole) modes only. An obvious correlation between the position of the models in the $\log g - T_{\text{eff}}$ diagram and the range of their instability bands emerges.

resolution is already able to provide quantitative information about the surface gravity and effective temperature of the star. Since PG 1159-035 has a relatively low gravity, temperature has only a small effect on the instability band, and the minimum lies in the bottom of a valley carved around $\log g = 6.85 \pm 0.05$. At the minimum, we have $S^2 = .075$, $\log g = 6.80$ and $T_{\text{eff}} = 144,500$. While the nonadiabatic pulsation approach provides a good constraint on the surface gravity, one must rely on spectroscopy for the temperature. The measures are therefore complementary. They are also consistent.

We have applied the same procedure to PG 1707+427 on a more extended grid, $\log g = 6.9 \ (0.025)$ 8.1 and $T_{\text{eff}} = 70,000 \ (500) 115,000$ K. The results are presented in Figure 3. As expected from the analysis of Figure 1, the solution is affected by temperature more than in the fluffier case of PG 1159-035. At the minimum, we have $S^2 = .005$, $\log g = 7.38$ and $T_{\text{eff}} = 81,500$. For this star, the uncertainties in temperature are of the same order as those obtained from spectroscopy. However, the gravity is constrained more tightly than in the spectroscopic case. Again, complementarity and consistency is achieved.
Figure 2. Calculated $S^2$ function grid (in logarithmic units) for PG 1159-035 along the log $g$ – $T_{\text{eff}}$ diagram. The spectroscopy position of the star is shown with a red dot and its associate uncertainties. The arrow points at the best-fit model ($\log g = 6.8$ and $T_{\text{eff}} = 144,500$ K).

3. Conclusion

These results are quite promising. They open the door to further inquiry into nonadiabatic asteroseismology of GW Vir stars. However, they raise the following questions:

Q1 - What makes dipole and quadrupole modes visible in PG 1159-035 while only dipole modes are seen in PG 1707+427?

Q2 - Why is the instability band of GW Vir well reproduced by nonadiabatic modelling when that is not the case for most types of pulsating stars?

These questions are not easy, and we do not claim that the following potential answers are complete or even right. We only make these comments as starting point for further reflection.

A1 - The traditional answer is usually a geometrical argument stating that higher radial order modes are more difficult to see since their overall effect on the luminosity of the star tend to cancel when one integrates over the entire stellar disk. Nonetheless, this does not explain why the modes are seen in some GW Vir stars but not in the others. For PG 1159-035, in [5] the amplitude of the largest $l = 1$ mode is 6.9 mmag while the amplitude of the largest positively identified $l = 2$ mode is about a third of that at 2.0 mmag. For PG 1707+427, the highest amplitude $l = 1$ modes is 16.91 mmag [6] one third that value is well above the 1 mmag WET detection limit of that star. However, we notice that while the amplitude of the PG 1707+427 light curve is about twice as large as the one of PG 1159-035, the detection limit on PG 1159-035 $\sim 0.1$ mmag is about ten times better than the one of PG 1707+427 at $\sim 1.0$ mmag. Maybe a better limit would reveal quadrupole modes in PG 1707+427.

A2 - Carbon and oxygen, the two elements responsible for the driving, have well known opacity values and have a well determined abundance ratio. Uncertainties related to the stellar
metallicity are relatively small since the iron ionization bumps are far from the C/O partial ionization region [4]. Two other factors differentiate GW Vir stars from other stars. First, there is no convection associated with the driving. This removes all approximations that come with convection in the stellar modelling and in the application of the linear theory of stellar pulsation. Second, GW Vir stars are compact. We know that high gravity makes radial displacement in compact stars very low. One can state that nonlinear effects caused by large radial displacement in the driving layers of the stars, like the throttling effect [8], are negligible in compact stars. More thought and investigations are needed to establish the limits of the linear theory of pulsation on these two points. By their nature, the GW Vir stars are good candidates for this project.

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Figure 3. Calculated $S^2$ function grid (in logarithmic units) for PG 1707+427 along the log $g - T_{\text{eff}}$ diagram. The red dot and black rectangle are the spectroscopic position of the star with its associate uncertainties. The arrow points at the best-fit model (log = 7.38 and $T_{\text{eff}} = 81,500$ K).
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