TRINIFICATION FROM SUPERSTRING TOWARD MSSM

JIHN E. KIM
School of Physics and Center for Theoretical Physics, Seoul National University, Seoul 151-747, Korea
E-mail: jekim@phys.snu.ac.kr

In this talk, I present a family unification in $Z_3$ orbifolded $E_8 \times E_8'$ heterotic strings. It is argued that trinification is a plausible candidate toward supersymmetric standard model at low energy.

1 Introduction

Even though the standard model (SM) is very successful, there exists the so-far unsolved family problem that there are 3 sets of fifteen chiral fields. "Is 3 a very fundamental number in the universe?" We try to investigate this problem.

A grand unification (GUT) in $SO(10)$ is very promising, but it has only one family. To have three families, one has to repeat the representations, which we want to avoid. In this respect a family symmetry such as $SO(3)$ or $SU(3)$ has been considered. But these extra family group will face the problem of Goldstone bosons or gauge unification. This is the reason to go beyond $SO(10)$, i.e. to $SO(4n + 2)$ with one spinor representation toward the unification of flavors. Most part of $SO(4n + 2)$ can be studied in $SU(2n + 1)$. For example, $SU(7)$ with the fermion representation, $\Psi \oplus \Psi^A \oplus \Psi_{ABC}$ contains 64 components of the $SO(14)$ spinor 64. But a naive breaking of $SO(14)$ down to $SO(10)$ leads no chiral fermions. One must twist the gauge group to obtain chiral fermions. This model, however, lacks the third quark families and is not phenomenologically successful. In addition, there are extra particles not present in the SM. In many models with twisting, extra particles are unavoidable. However, this road of attractive grand unification of flavor has not been considered any more since 1984, due to the possibility of understanding the family structure in the $E_8 \times E_8'$ heterotic string model.

One crucial thing needed for the flavor grand unification is that the gauge group should be big enough. Another thing is that the fermion representation should be anomaly free. It can be a reason to exclude the $SU(N)$ gauge groups since to cancel anomalies the $SU(N)$ representations should be matched miraculously. In this respect, $SO(4n + 2)$ allowing complex representations attracted a great deal of attention. Typically one assumes one spinor
representation in $SO(4n + 2)$. But, here introducing ‘one’ spinor does not have a strong rationale for the one. In this respect, we note that the gauge bosons has a fixed representation, i.e. the adjoint representation. So, it may be reasonable to relate fermions to the adjoint representation of the gauge group. To introduce fermions, it is better to have supersymmetry. Indeed, this road is exactly what the higher dimensional gauge theories take, and in particular in the superstring models.

2 Need for HESSNA

Superstring models are written in 10 dimensions(10D). A simple dimensional reduction down to 4D would not lead to chiral fermions. One has to twist the gauge group to obtain chiral families as in the previous example of $SO(14)$\footnote{1}. So, twisting the gauge group is necessary. Since we have to hide 6 internal spaces through compactification, there is a possibility for twisting the internal space. There are two well-known compactifications achieving these goals, the Calabi-Yau space compactification\footnote{2} and the orbifold compactification\footnote{3}. Among these the orbifold is simpler and easy to understand geometrically. So, the orbifold compactifications are most extensively studied.

The 4D string models were constructed in orbifold construction and in fermionic construction. The first standard-like models were constructed in orbifold compactifications\footnote{4,5}. On the other hand, the flipped $SU(5)$\footnote{6} was constructed in the fermionic construction\footnote{7}. These 4D string models can be considered as a 4D theory, not coming from 10D. But it is tempting to speculate that 4D models are the remnants of compactification of the 10D string, in which case the orbifold has the merit of geometrical interpretation, compared to the fermionic construction.

The simplest orbifold is to consider three two-dimensional tori $6D = 2D + 2D + 2D$ with a further identification of the point group. Among many $Z_N$ orbifold models, $Z_3$ orbifolds with $N = 1$ supersymmetry are especially fascinating because the multiplicity of matter fermions come in multiples of 3. This may be the reason that the family number is 3.

The $Z_3$ orbifold identifies the points related by 120° rotation, and hence there are three fixed points in the fundamental region as shown in Fig. 1. Since we must consider the direct product of three tori there are 27 fixed points. This geometrical twisting can be also accompanied in the twisting of the gauge group. The frequently discussed example is the shift vector $v = \frac{1}{3}(1 1 2 0 0 0 0 0 0 \cdots)$ which gives the gauge group $E_6 \times SU(3) \times E'_8$. Since $E_6$ contains the spinor representation of $SO(10)$ it attracted a great deal of attention. The reason that the fundamental representations of $E_{6,7,8}$
contain the $SO(10)$ spinor is the main phenomenological reason favoring the $E_8 \times E_8'$ heterotic string. In this talk also, we focus on the $E_8 \times E_8'$ heterotic string.

However, the symmetry breaking of $E_6$, $SO(10)$, and $SU(5)$ down to the standard model requires an adjoint representation for the GUT Higgs field(s). But in orbifold compactifications it is very difficult to obtain the adjoint representation\(^a\). This is the reason that the flipped $SU(5)$ GUT attracted so much attention.\(^8\) Because of this difficulty of GUT symmetry breaking, the direct derivation of the SM gauge group with reasonable fermionic spectrum attracted a great deal of attention, and are called standard-like models. These standard-like models pursue the following properties:

- The gauge group is $SU(3) \times SU(2) \times U(1)^n$.
- There are three families.
- In some cases, there are Higgs doublets but no color triplets.\(^5\) This is the doublet-triplet splitting.

In this talk, we try to delete like from standard-like.

There are two reasons that the standard-like models are not phenomenologically successful. One is the $\sin^2 \theta_W$ problem in that most of these standard-like models give the string value of $\sin^2 \theta_W$ too small compared to $\frac{3}{5}$.\(^10\) Another problem is that there are too many Higgs doublets appearing in the spectrum. Usually, the minimum number of the Higgs doublets is six, 3 from

\(^a\)For a high level Kac-Moody algebra, it is known that the adjoint representation is possible.\(^4\)
$Z_3$ and 2 from $H_{u,d}$ for the anomaly cancellation. If the SM is embedded in a simple GUT group, the bare value of the $\sin^2 \theta_W$ is

$$\sin^2 \theta_W^0 = \frac{\text{Tr} T_3^2}{\text{Tr} Q_{em}^2}$$  \hspace{1cm} (1)

where $T_3$ is the 3rd component of weak iso-spin and $Q_{em}$ is the electromagnetic charge. In general, there appear many charged electroweak singlet fields and hence $\sin^2 \theta_W$ can be much smaller than $\frac{3}{8}$. This is in gross contradiction with the LEP measurement of the seemingly unified gauge coupling constant at $2 \times 10^{16}$ GeV.$^b$ The basic reason of the $\sin^2 \theta_W$ problem is that the electroweak hypercharge $Y$ is leaked to uncontrollably many $U(1)$’s. A GUT is permissible in orbifold compactification, but it is difficult to obtain an adjoint representation. This has led to the consideration of the flipped $SU(5)$, i.e. $SU(5) \times U(1)$, but in the flipped $SU(5)$ there is also the problem of the leakage of the electroweak hypercharge to the extra $U(1)$. Therefore, we suggest GUT groups with the following property $^10$

- HESSNA = Hypercharge is Embedded in Semi-Simple group with No need for Adjoint representation.

Most probably, the QCD $SU(3)$ is already separated out, except in Pati-Salam type GUT $^{12}$. The simplest HESSNA is the trinification group $SU(3)_1 \times SU(3)_W \times SU(3)_c$ with the fermion representation

$$(\bar{3}, 3, 1) + (1, \bar{3}, 3) + (3, 1, \bar{3}).$$ \hspace{1cm} (2)

At non-string level the trinification has been extensively studied $^{13}$, but here the only requirement is the anomaly cancellation and phenomenological massage. But in the string trinification the theory is very restrictive. It is dictated from string theory. Breaking of $SU(3)_1 \times SU(3)_W \times SU(3)_c$ down to the SM is achieved by giving VEV’s to two fields in the spectrum $^2$. The electroweak hypercharge is represented as

$$Y = -\frac{1}{2}(-2I_1 + Y_1 + Y_W)$$ \hspace{1cm} (3)

with the subscripts denoting the respective $SU(3)$ groups.

3 $Z_N$ embedding in $E_8$

Therefore, we looked for $SU(3)^3$ trinification groups from $E_8 \times E_8'$ heterotic string. We used the Dynkin diagram technique to find out the gauge groups,

---

$^b$To cure this problem, the so-called optical unification was suggested $^{11}$
by embedding $Z_N$ in $E_8$. This method was originally devised by Kac and Peterson [14], starting from the extended Dynkin diagram (Fig. 2) of $E_8$. The rank-8 $E_8$ has eight simple roots. The extended diagram has the ninth root $\alpha_0$, satisfying $\alpha_0 \cdot \alpha_i = 0$ if $i \neq 1$, and $\alpha_0 \cdot \alpha_1 = -1$. The Dynkin basis $\{\gamma_i; i = 1, 2, \cdots, 8\}$ is defined to satisfy $\gamma_i \cdot \alpha_j = \delta_{ij}$. In this Dynkin basis a shift vector $V$ is expanded. The embedding of $Z_N$ shift vector is $V = \sum s_i \gamma_i$ with

$$\sum_{i=1}^{8} n_i \gamma_i = 0 \mod N, \text{ for } s_i \geq 0. \quad (4)$$

Now for $s_i \neq 0$, remove that circle in the extended Dynkin diagram, which gives the surviving group with the $U(1)$’s added to make up rank 8. But the generalization with more than one shift vector is necessary to wrap the tori with Wilson lines [15]. Recently, we resolved this problem of adding more shift vectors [16]. This Dynkin diagram technique is a great help in finding out HESSNA, not counting the same model several times. Of course, it is better than trying every possible $V$ and $\alpha$’s, since we can figure out the gauge group from the beginning. Indeed, this method was the guiding idea finding out early string trinifications [18][19].

![Figure 2. Extended Dynkin diagram of $\hat{E}_8$ group. The numbers in the circle are the Coxeter label $n_i$ of the corresponding simple roots. $\alpha_0$ is the extended one.](image)

### 4 A mass matrix ansatz for MSSM

Now, let us show one example how one can obtain a MSSM. For $Z_3$ orbifolds, the bulk (untwisted) matter fields have multiplicity 3. The fields located at the fixed points have multiplicity 27. One Wilson line models give the multiplicity 9 at a fixed sector. Two Wilson line models give the multiplicity 3 at a fixed sector. Therefore, to construct a model with a reasonable spectrum it is plausible to consider two Wilson line models. It may be tempting to consider

\[\text{At field theory level, a similar study has been done}^{[17]}\]
three Wilson line models which distinguish 27 fixed points. However, with multiplicity=1 three families are not guaranteed at the outset. Therefore, two Wilson line models are the best at the moment. The Wilson lines (\(a_i\) \((i = 1, 3)\)) are shifts. In applying the Dynkin diagram technique, it is the same as the shift vector \(V\), but we must satisfy the modular invariance conditions:

\[ V^2 = \frac{1}{3} \cdot \text{(integer)} \]
\[ a_i^2 = \frac{1}{3} \cdot \text{(integer)} \]
\[ V \cdot a_i = \frac{1}{3} \cdot \text{(integer)} \]
\[ a_i \cdot a_j = \frac{1}{3} \cdot \text{(integer for } i \neq j) \]

(5)

The trinification spectrum comes in three different types of representations which we call the lepton humor \((\bar{3}, 3, 1)\) quark humor \((1, \bar{3}, 3)\), and anti-quark humor \((3, 1, \bar{3})\), respectively. The trinification spectrum \((2)\) is very similar to 27 of \(E_6\), as far as the spectrum is concerned. However, for the GUT symmetry breaking, the trinification is much better. The trinification spectrum contains two neutral components \((N_{10} \text{ and } N_5)\) in the lepton-humor sector) the VEV’s of which can break \(SU(3)_c\) down to the SM gauge group.

For a definite presentation of our argument, let us take a specific two Wilson-line \(Z_3\) trinification model \(20\),

\[ V = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0) \]
\[ a_1 = (\frac{1}{3} \ \frac{1}{3} \ 0 \ 0 \ \frac{1}{3} \ \frac{2}{3} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0) \]
\[ a_3 = (0 \ 0 \ 0 \ 0 \ 0 \ \frac{1}{3} \ 0 \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3}) \]

(6)

which gives the \(N = 1\) supersymmetry with the following gauge group

\[ SU(3)^3 \times U(1)^2 \times [SO(8) \times SU(3) \times U(1)^2]_h. \]

(7)

The chiral superfields are shown in Table 1. We can remove vectorlike representations as has been the practice in GUT’s. It is possible by giving VEV’s to the gauge group singlets. For a string calculation one should follow the Yukawa coupling derivation given in \(21\), but here we simply adopt the old GUT procedure since our presentation is just an idea toward MSSM. Then, we obtain the trinification spectrum from T0, and in addition more trinification-like fields,

\[ (3, 3, 1)(1, 1) + (1, \bar{3}, 3)(1, 1) + (3, 1, \bar{3})(1, 1) \]
\[ + (\bar{3}, 3, 1)(1, 1) + (3, 1, 1)(1, 3) + (1, \bar{3}, 1)(1, \bar{3}). \]

(8)

It is known that a vectorlike lepton-humor is needed toward neutrino masses and reasonable symmetry breaking pattern \(22\). We have a lepton humor in U, but does not have its antiparticles. So, the gauge group must be identified toward this purpose. We can identify \(SU(3)_1\) from \(E_8\) and \(SU(3)_h\) from \(E_8\) by the linkage field in T5, and obtain anti-lepton-humor from T8.
Table 1. The massless spectrum of the orbifold. The 3rd column denotes the multiplicity.

| sector | twist | mul. | fields |
|--------|-------|------|--------|
| U      | V     | 3    | (3, 3, 1)(1, 1) |
| T0     | V     | 9    | (1, 1, 1)(1, 1) |
| T1     | V + a1| 3    | (3, 3, 1)(1, 1) + (3, 3, 3)(1, 1) + (1, 3, 3)(1, 1) |
| T2     | V - a1| 3    | (1, 3, 1)(1, 1) + (3, 1, 1)(1, 1) + (1, 1, 3)(1, 1) |
| T3     | V + a3| 9    | (1, 1, 1)(1, 1) |
| T4     | V - a3| 9    | (1, 1, 1)(1, 1) + (1, 1, 1)(1, 1) |
| T5     | V + a1 + a3| 3    | (3, 1, 1)(1, 1) |
| T6     | V + a1 - a3| 3    | (3, 1, 1)(1, 1) + (1, 1, 3)(1, 1) |
| T7     | V - a1 + a3| 3    | (1, 3, 1)(1, 1) + (3, 1, 1)(1, 1) + (1, 1, 3)(1, 1) |
| T8     | V - a1 - a3| 3    | (1, 3, 1)(1, 1) |

Because there appear many Higgs doublets in the resulting spectrum, we take an ansatz that determinant of the Higgsino mass matrix vanish

\[
\text{Det. } M_{\tilde{R}} = 0. \tag{9}
\]

Then, we obtain only one pair of Higgs doublets at low energy, realizing the MSSM spectrum. This ansatz can be dictated from dynamics at high energy, such as the small instanton effects. The relevant instanton absorbs the vectorlike representations of Higgsinos as shown in Fig. 3. Then in this scheme it is possible to remove vectorlike color triplets.\(^{20}\) Even though the specific model we discuss supports our ansatz, it does not give a bare value of \(\sin^2 \theta_W^0 = \frac{3}{8} \). So, we may not take it as a fully satisfactory model. It will be seen whether a model realizing the Higgsino mass matrix ansatz with \(\sin^2 \theta_W^0 = \frac{3}{8} \) is present in \(Z_3\) orbifold models.

5 Conclusion

In this talk, I showed a road toward the construction of MSSM through the \(Z_3\) orbifold compactification of the \(E_8 \times E_8\) heterotic string. The \(Z_3\) is chosen to interpret three families. In standard-like models, there are too many Higgs doublets present. The problems of the extra \(U(1)\)'s and too many Higgs doublets are the obstacles for a bare weak mixing angle being \(\frac{3}{8} \). Therefore, gauge groups with no need of adjoint representation are proposed as GUT groups for an easy pattern of symmetry breaking. In particular, a trinification
Figure 3. A possible instanton interaction. All the Higgsino pairs should be included.

has been suggested toward a GUT group between the string scale and the electroweak scale. To obtain a MSSM from HESSNA, one should allow only one pair of Higgs doublets. We suggested a short distance dynamics toward realization of one pair of Higgs doublets through the ansatz, Det. $M_R = 0$. To obtain a bare $\sin^2 \theta_W = \frac{3}{8}$, it is better to obtain a trinification spectrum $6(27) \oplus 3(\overline{27})$, or $3(27) \oplus 3(\text{lepton-humor} + \text{anti-lepton-humor})$, where $27$ is the trinification spectrum. But, we have not obtained such a $Z_3$ model yet.

Acknowledgments

This work is supported in part by the KOSEF ABRL Grant No. R14-2003-012-01001-0, the BK21 program of Ministry of Education, and Korea Research Foundation Grant No. KRF-PBRG-2002-070-C00022.

References

1. J. E. Kim, Phys. Rev. Lett. 45, 1916 (1980); Phys. Rev. D23, 2706 (1981).
2. D. J. Gross, J. A. Harvey, E. J. Martinec, and R. Rohm, Phys. Rev. Lett. 54, 502 (1985); Nucl. Phys. B256, 253 (1985).
3. P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, Nucl. Phys. B258, 46 (1985).
4. L. J. Dixon, J. A. Harvey, C. Vafa, and E. Witten, Nucl. Phys. B261, 678 (1985).
5. L. Ibañez, J. E. Kim, H. P. Nilles, and F. Quevedo, Phys. Lett. B191, 282 (1987).
6. L. E. Ibanez, J. Mas, H. P. Nilles and F. Quevedo, Nucl. Phys. B301, 157 (1988); C. A. Casas, E. K. Katehou, and C. Munoz, Nucl. Phys. B317, 171 (1989); A. Font, L. E. Ibanez, F. Quevedo and A. Sierra, Nucl. Phys. B331, 421 (1990); D. Bailin, A. Love and S. Thomas, Phys. Lett. B194, 385 (1987).
7. S. M. Barr, Phys. Lett. B112, 219 (1982); J.-P. Derendinger, J. E. Kim, and D. V. Nanopoulos, Phys. Lett. B139, 170 (1984).
8. I. Antoniadis, J. Ellis, J. S. Hagelin, and D. V. Nanopoulos, Phys. Lett. B194, 231 (1987).
9. Z. Kakushadze and S.-H. H. Tye, Phys. Rev. Lett. 77, 2612 (1996).
10. J. E. Kim, Phys. Lett. B564, 35 (2003).
11. J. Giedt, Mod. Phys. Lett. A18, 1625 (2003); G. Cleaver, V. Desai. H. Hanson, J. Perkins, D. Robbins, and S. Shields, Phys. Rev. D67, 026009 (2003).
12. H. D. Kim and S. Raby, JHEP 0301, 056 (2003).
13. S. L. Glashow, in The 5th Workshop on Grand Unification, ed. K. Kang, H. Fried, and P. Frampton (World Scientific, Singapore, 1984) p.538.
14. V. G. Kac and D. H. Peterson, in Anomalies, Geometry, and Topology [1985 Argonne-Chicago Conference]. p. 276.
15. L. Ibanez, H. P. Nilles, and F. Quevedo, Phys. Lett. B187, 25 (1987).
16. K.-S. Choi, K. Hwang, and J. E. Kim, Nucl. Phys. B662, 476 (2003).
17. A. Hebecker and M. Ratz, Nucl. Phys. B670, 3 (2003).
18. J. E. Kim, JHEP 0308, 010 (2003).
19. K.-S. Choi and J. E. Kim, Phys. Lett. B567, 87 (2003).
20. K.-S. Choi, K.-Y. Choi, K. Hwang, and J. E. Kim, hep-ph/0308160.
21. S. Hamidi and C. Vafa, Nucl. Phys. B279, 465 (1987); L. J. Dixon, D. Friedan, E. J. Martinec, and H. Shenker, Nucl. Phys. B282, 13 (1987); A. Font, L. E. Ibanez, H. P. Nilles, and F. Quevedo, Nucl. Phys. B307, 109 (1988) and Nucl. Phys. B310, 764(E) (1988); A. Font, L. E. Ibanez, H. P. Nilles, and F. Quevedo, Phys. Lett. B210, 101 (1988) and Phys. Lett. B213, 564(E) (1988); A. Font, L. E. Ibanez, F. Quevedo, and A. Sierra, Nucl. Phys. B331, 421 (1990).
22. See, for example, B. Campbell, J. Ellis, M. K. Gaillard, D. V. Nanopoulos, and K. Olive, Phys. Lett. B180, 77 (1986); B. R. Greene, K. H. Kirklin, P. J. Miron, and G. G. Ross, Nucl. Phys. B292, 606 (1987).