Study of Magnetic Field Influence on the Stationary Plasma Thruster Plume

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Abstract. The purpose of this paper is to study the magnetic field influence on the stationary plasma thruster (SPT) plume. In consideration of collisionless motion of ions, it is possible to write an expression for distribution function for ions leaving a ring opening. Further, the pattern of ion density calculation is built as of the corresponding integral of distribution function. Results of calculations made using the constructed pattern are given.

1. Introduction

Nowadays electrojets are used everywhere in space technique. The set of stationary plasma engines or in abbreviated form SPE is the most widely used of them. This type of engines is used almost on all modern space-crafts. Unlike liquid jets SPE have significantly bigger specific impulse. At present there is a constant development of SPE for the purpose of increasing in draft of the engine when maintaining great values of specific impulse. In that direction the task of effective thrust vectoring arises. As the draft if SPE is created by a stream of the plasma which is thrown out in environmental space, so it is natural to try to control thrust vector by means of a magnetic field. Early studies in this direction were made in work [1]. In this research a possibility of magnetic field traction control was shown. In the work [2] this problem was investigated in further detail. In this work it was concluded that the thrust vector is directed in the direction of the Lorentz force. On the basis of the conducted researches au-thors of work [2] issued and protected the paten [3] where it is offered to use a magnetic field to rotate the thrust vector. The conclusion which was drawn in work [2] is based on consideration of the motion of a charged particle in a magnetic field.

2. The model under study

SPT is presented schematically in Fig. 1. The plume consisting of ions enters the environmental space through a ring opening in the thruster. The thruster is represented by parallelepiped ABCDA\(_1\)B\(_1\)C\(_1\)D\(_1\). In the center of the side ABCD, which is a square as it is shown in Fig. 1, the coordinate system OXYZ is placed. The parallelepiped LKMN\(_1\)K\(_1\)M\(_1\)N\(_1\) shown in Fig. 1 by thin lines limits the countable domain.

The constant magnetic field H = 100 Gauss is directed along the axis X. Such magnetic field, while influencing ions, turns the plume, and this can result in the thrust vector rotation.
problem is to define such influence of magnetic field on the plume.

\textbf{Figure 1.} Stationary plasma engine geometry

With such magnetic field, electrons leaving the cathode shown in Fig. 1 are not magnetized yet and do not have strong influence on the plume leaving the thruster. Therefore, we assume that the flow of ions to the environment is collisionless. We neglect action of an electric field on ions. According to the result of work presented in [4], the originating electric field poorly influences the high-energy ions leaving a ring opening. The above assumptions are natural because the purpose of work is to define impact of magnetic field on the ion flow. In this case, \( f(t, \vec{x}, \vec{\xi}) \) is the cumulative ion distribution function that is defined from the following equation:

\[
\frac{\partial f}{\partial t} + \xi_i \frac{\partial f}{\partial x_i} + \frac{e}{mc} \varepsilon_{ikl} \xi_k H_l \frac{\partial f}{\partial \xi_i} = 0,
\]

where \( e \) is the ion charge, \( m \) is its mass, \( c \) is the light velocity, and \( \varepsilon_{ikl} \) is the Levi-Civita symbol. In the above formula, the summing over the repeated indices is supposed everywhere, at that the indices vary from one to three.

Transition to the dimensionless values was made in the same way as in [1, 2].

\[
x_i = L x'_i, \quad \xi_i = \xi_0 \xi'_i, \quad i = 1, 2, 3; \quad t = L \xi_0 \xi'_t, \quad \xi_0 = \sqrt{\frac{2eU_0}{m}}, \quad f(t, \vec{x}, \vec{\xi}) = n_0 \frac{\xi_0}{\xi_0} f'(t, \vec{x}', \vec{\xi}')
\]

Here \( U_0 \) is the discharge voltage (a potential difference between the sides ABCD and \( A_1B_1C_1D_1 \)), \( L \) is the half-length of the side AB; \( n_0 \) is the characteristic value of the ion density. This value is defined in [2] as the characteristic value of ion density for ions leaving a ring opening.

By omitting prime marks for the dimensionless variables, we obtain that the dimensionless cumulative ion distribution function \( f(t, \vec{x}, \vec{\xi}) \) is defined by the following equation:

\[
\frac{\partial f}{\partial t} + \xi_i \frac{\partial f}{\partial x_i} + \omega \varepsilon_{ikl} \xi_k H_l \frac{\partial f}{\partial \xi_i} = 0
\]

In [1]: \( \omega = L/r_L \), where \( r_L = mc\xi_0/eH \) is the Larmor radius and \( c \) is the light velocity.
Taking into account that the magnetic field component is \( (H, 0, 0) \), \( f \) can be written as:

\[
\frac{\partial f}{\partial t} + \xi \frac{\partial f}{\partial x} + \omega \xi \frac{\partial f}{\partial y} - \omega \xi_y \frac{\partial f}{\partial \xi} = 0
\]  

(2)

It follows from the above that for the cumulative ion distribution function the boundary condition can be given as in [1][2]. In the dimensionless form, it is the following:

\[
f = \begin{cases} 
\frac{\pi}{\sqrt{2}} \exp \left\{ \frac{-B_1(\xi - \bar{\eta})^2}{2} \right\}, & R_2 \leq r \leq R_1, \\
0, & (r < R_2) \cup (r > R_1),
\end{cases} 
\]  

(3)

Here: \( \bar{\eta}(r), \bar{\eta}(r) \) are the given functions. Their definition is presented in detail in [4]. \( B_1 = U_0/kT_i \gg 1 \) is the ion temperature at the thruster exit. It was given depending on thruster type; however the given inequality takes place always. The values \( R_1, R_2 \) are the dimensionless values for radiuses of the ring, through which the ions leave the thruster.

The characteristic system of the equation (2) is the following:

\[
\frac{d\tilde{x}}{d\tau} = \xi_x, \quad \frac{d\tilde{\xi}_x}{d\tau} = 0, \quad \frac{d\tilde{y}}{d\tau} = \xi_y, \quad \frac{d\tilde{\xi}_y}{d\tau} = \xi_z, \quad \frac{d\tilde{z}}{d\tau} = \omega \tilde{\xi}_z, \quad \frac{d\tilde{\xi}_z}{d\tau} = -\omega \tilde{\xi}_y.
\]

The solution of this system that at \( \tau = t \) satisfies the initial conditions \( \tilde{x}(t) = x, \ \tilde{y}(t) = y, \ \tilde{z}(t) = z, \ \tilde{\xi}_x(t) = \xi_x, \ \tilde{\xi}_y(t) = \xi_y, \ \tilde{\xi}_z(t) = \xi_z \) is:

\[
\begin{align*}
\tilde{\xi}_x(t) &= \xi_x, \quad \tilde{x}(\tau) = x - \xi_x(t - \tau), \\
\tilde{\xi}_y(t) &= \xi_y \cos \omega(t - \tau) - \xi_z \sin \omega(t - \tau), \quad \tilde{\xi}_z(t) = \xi_z \cos \omega(t - \tau) - \xi_y \sin \omega(t - \tau), \\
\tilde{z}(\tau) &= z + (\xi_y(\tau) - \xi_y)/\omega, \quad \tilde{y}(\tau) = y + (\xi_z(\tau) - \xi_z(\tau))/\omega.
\end{align*}
\]

It follows from above relations that \( \xi_x^2 + \xi_z^2 = \xi_y^2(\tau) + \xi_z^2(\tau) = \text{const} = V^2 \). It is obvious from the latter equation that it is convenient to introduce a polar coordinate system in the velocity space: \( \xi_x = \xi_x, \ \xi_y = V \cos \alpha, \ \xi_z = V \sin \alpha, \ 0 \leq V \leq +\infty, \ 0 \leq \alpha \leq 2\pi \). In such coordinate system, the equations for characteristics (2) will be written as:

\[
\begin{align*}
\tilde{\xi}_x &= \xi_x, \quad \tilde{\xi}_y = V \cos (\alpha + \omega(t - \tau)), \quad \tilde{\xi}_z = V \sin (\alpha + \omega(t - \tau)), \\
\tilde{z}(\tau) &= z - 2 \frac{V}{\omega} \sin \left( \frac{\omega(t - \tau)}{2} \right) \sin \left( \alpha + \frac{\omega(t - \tau)}{2} \right), \\
\tilde{y}(\tau) &= y + 2 \frac{V}{\omega} \sin \left( \frac{\omega(t - \tau)}{2} \right) \cos \left( \alpha + \frac{\omega(t - \tau)}{2} \right), \quad \tilde{x}(\tau) = x - \xi_x(t - \tau).
\end{align*}
\]

The right part of the equation (2) is the function \( f(t, \tilde{x}, \tilde{\xi}) \) derivation owing to the characteristic system; so, the equation (2) can be written as: \( \frac{df}{d\tau} = 0 \). Taking the boundary condition (3) into account, the solution of the equation (2) can be written as:

\[
f(t, \tilde{x}, \tilde{\xi}) = \theta(\tilde{\xi}(\ell)) \theta((R_1 - \tau)(\tau - R_2)) \frac{\bar{\eta}(\tau)}{\pi^{3/2}} \exp \left\{ -B_1(\tilde{\xi}(\ell) - \bar{\eta}(\tau))^2 \right\} \]

(4)

where \( \ell \) is defined by:

\[
\tilde{z}(\ell) = z - 2 \sin \left( \frac{\omega(t - \ell)}{2} \right) \sin \left( \alpha + \frac{\omega(t - \ell)}{2} \right) \frac{V}{\omega}
\]

(5)
Let \( \tilde{\omega}_3 \). Computational scheme

Crucially new way.

In our case this is carried out in a domain to the integration over the SPT exit opening, but in our case this is solved by transition from integration over the velocity situation appeared to be more complicated comparing to that described in the above presented publications. As in [4] this problem is solved by transition from integration over the velocity domain to the integration over the SPT exit opening, but in our case this is carried out in a crucially new way.

3. Computational scheme

Let \( \tilde{x}(\tau) = r \cos \varphi, \tilde{y}(\tau) = r \sin \varphi \) and let us change to variables \((r, \varphi, \tau)\) by the formulas:

\[
\xi_x = \frac{(x - r \cos \alpha)}{t - \tau}, \quad ctg(\beta) = \frac{y - r \sin \varphi}{z}, \quad V = {\frac{\omega d}{2 \sin \left(\frac{\omega (t - \tau)}{2}\right)}},
\]

(7)

where \( \beta(\alpha, \tau) = \alpha + \omega \frac{(t - \tau)}{2} \), \( d(r, \varphi) = \sqrt{(y - r \sin \varphi)^2 + z^2} \). The Jacobian of transformation of replacement (7) will be:

\[
J = \begin{vmatrix}
- \frac{\cos \varphi}{t - \tau} & \frac{r \sin \varphi}{t - \tau} & \frac{x - r \cos \varphi}{(t - \tau)^2} \\
- \frac{\omega z \sin \varphi \cos \beta}{2 \sin \left(\frac{\omega (t - \tau)}{2}\right)} & - \frac{\omega r z \cos \varphi \cos \beta}{2 \sin \left(\frac{\omega (t - \tau)}{2}\right)} & \frac{\omega^2 \cos \left(\frac{\omega (t - \tau)}{2}\right)}{4 \sin^2 \left(\frac{\omega (t - \tau)}{2}\right)} \\
\frac{z}{d^2} \sin \varphi & \frac{z}{d^2} r \sin \varphi & \frac{\omega}{2} \\
\end{vmatrix}
\]

In the above formula it is considered that \( \cos \beta = \frac{y - r \sin \varphi}{d}, \quad \sin \beta = \frac{z}{d} \).

In new variables, we have:

\[
n(t, \tau) = \left( \frac{B_1}{\pi} \right)^{3/2} \int_{R_2}^{R_1} \int_0^{2\pi} \int_0^t n(\tau) \exp \left\{ - B_2 g^2 \frac{\omega^3 r \sin (\alpha + \omega (t - \tau))}{8(t - \tau) \sin^2 \omega (t - \tau)/2} \right\} dr d\varphi d\omega
\]

(8)
In (8): \( \sin (\alpha + \omega(t - \tilde{t})) \geq 0 \). This condition meets the following: \( \tilde{\xi}_z \geq 0 \).

\[
g^2 = \left( \frac{x - r \cos \varphi}{t - \tilde{t}} \right)^2 + \frac{\omega^2 d^2}{4 \sin^2(\omega(t - \tilde{t})/2)} - 2 \left( \frac{x - r \cos \varphi}{t - \tilde{t}} \right) \pi_r \cos \varphi - \omega \cos (\omega(t - \tilde{t})/2) U_1 + U_2,
\]

\( U_1 = \pi_r \sin \varphi (y - r \sin \varphi) + \pi_z z, \quad U_2 = \pi_r^2 + \pi_z^2 - \omega (y - r \sin \varphi) \pi_z - z \pi_r \sin \varphi \).

In (8), the replacement \( \omega(t - \tilde{t})/2 = u \) is made. Now:

\[
n(t, \bar{x}) = \left( \frac{B_1}{\pi} \right)^{3/2} R_1 \int_{R_2}^0 \int_{0}^{2\pi} \int_{n(\bar{x})} \exp \left\{ -B_2 g^2 \right\} \frac{\omega^3 (z \cos u + (y - r \sin u))}{8 \sin^4 u} d\theta d\phi.
\]

If \( \frac{\omega t}{2} \leq \pi \) then let’s assume: \( V = \frac{1}{\sin u} \). So:

\[
n(t, \bar{x}) = \left( \frac{B_1}{\pi} \right)^{3/2} R_1 \int_{R_2}^0 \int_{0}^{2\pi} F(r, \varphi) \int_{V^+}^{+\infty} \exp \left\{ -B_1 g_1^2 \right\} \frac{\omega^3 (z v_1 \cos u + (y - r \sin u))}{8 v_1 \arcsin \left( \frac{1}{v} \right)} d\theta d\phi d\theta d\phi,
\]

where \( F(r, \varphi) = r \pi(r) \exp \left\{ -B_1 U_2 \right\}, \quad v_1 = \sqrt{1 - \frac{1}{v^2}} \).

\[
g_1^2 = \omega^2 \left( \frac{x - r \cos \varphi}{\arcsin \left( \frac{1}{v} \right)} \right)^2 + \frac{\omega^2 d^2}{4} - \omega \left( \frac{x - r \cos \varphi}{\arcsin \left( \frac{1}{v} \right)} \right) \pi_r \cos \varphi - \omega \frac{v_1 v}{4} U_1,
\]

\( V^+ = \max \left\{ \frac{1}{\sin \omega t/2}, \frac{d}{z} \right\} \).

All attempts to calculate the improper integral in (9) failed numerically because not a single numerical scheme allowed us to consider the carrier of a delta-shaped integrand. It is easy to see that \( 0 \leq 1/v \leq \sin \omega t/2 < 1 \), so \( \arcsin \left( \frac{1}{v} \right) = v + O \left( \frac{1}{v^2} \right) \), \( v_1 = \sqrt{1 - \frac{1}{v^2}} = 1 + O \left( \frac{1}{v^2} \right) \). Therefore, the following computational scheme was offered.

The interval \( \left[ 0, \frac{\omega t_{\text{max}}}{2} \right] \cup \left[ \frac{\omega t_{\text{max}}}{2}, \frac{\omega t}{2} \right] \). Then \( n(t, \bar{x}) = I_1 + I_2 \), where for \( I_1 \) the integration is made from \( V^+ \) to \( V^* \). It is a proper integral. It was calculated by means of a rather narrow integration mesh. It is supposed in the improper integral \( I_2 \) that \( v \arcsin 1/v = 1, v_1 = 1 \). Then

\[
I_2 = \sum_{i=1}^{i_0} \sum_{j=1}^{j_0} F_{ij} \exp \left\{ -B_1 \left( U_{2ij} - \frac{U_{2ij}^2}{d_{ij}} \right) \right\} G_{ij} \Delta r \Delta \varphi,
\]

where any value with indexes \( i, j \) means that it is taken at \( r_{i+1/2} = R_2 + (i - 1) \Delta r + \Delta r/2, \varphi_{j+1/2} = (j - 1) \Delta \varphi + \Delta \varphi/2 \).

Respectively:

\[
G_{ij} = \left( \frac{B_1}{\pi} \right)^{3/2} \int_{V^+}^{+\infty} \frac{\omega^3 (z + (y - r \sin \varphi) v_1)}{8 v^2} \exp \left\{ -B_1 \left( \frac{\omega d_{ij} v_1}{2} - \frac{U_{1ij} v_1}{d_{ij}} \right) \right\} dv,
\]

\( d = \sqrt{(x - r \cos \varphi)^2 + d^2}, \quad U_1 = (x - r \cos \varphi) \pi_r \cos \varphi + U_1 \).
Having made replacement \( \sqrt{B_1} \left( \frac{\omega d}{2} v - \frac{U_1}{d} \right)_{ij} = q \), we obtain:

\[
G_{ij} = \frac{B}{2\pi d^3} \left( z \left( \frac{1}{B_1} + \left( \frac{U_1}{d} \right)^2 + (y - r \sin \varphi) \frac{U_1}{d} \right) \left( 1 - \text{sign}(q^+) \text{Erf}(\sqrt{B_1} q^+) \right) + A \exp(-B_1 q^{+2}) \right)_{ij},
\]

\[
\sqrt{B_1} \left( \frac{\omega d}{2} v^+ - \frac{U_1}{d} \right)_{ij} = q^+.
\]

The value designated as \( A \) is not important, as the exponent following it nullifies it. We managed to find out by the described method that at \( \frac{\omega t}{2} = \frac{\pi}{4} \) the calculation of integral with partitioning and without it yields the same result.

**Figure 2.** Projections of the surface of the ion density level on the OYZ

Calculations showed that the contribution of the proper integral is small. This is seen in Fig. 2 where the patterns for the lines of level of a number-density distribution of plasma ions are presented in the plane \( Z0Y \). (a) \( \omega t/2 = \pi/4 \), (b) \( \omega t/2 = \pi/2 \), (c) \( \omega t/2 = 3\pi/4 \). It is obvious that there are no time variations at \( \omega t/2 > \pi/4 \).

The three-dimensional pattern for the ion density level surfaces is presented in Fig. 3. It is obvious that the plume rotates, twisting along the axis \( X \). The values on a color scale correspond to percents of the maximum density value that was equal to 4.

4. Conclusion

At \( \omega t/2 > \pi/2 \) the method of calculation remains the same, form for \( t - \tilde{t} \) through an arcsine will contain additional terms which will change an integrand, but the method will remain the same, having demanded addition of several more own integrals. From the analysis of Fig. 3 and formulas for calculation of components of a stress tensor, it is possible to draw a conclusion that the direction of a thrust vector at influence of a magnetic field is possible.

It is clear, that in this work the task about influence of a magnetic field on a stream of SPT is solved in rather approximate statement. To solve this problem it is completely necessary to consider influence of effects of a resonant recharge and self-consistent electric field. For this purpose, it is possible to use the splitting method of physical processes developed especially for the numerical solution of the kinetic equations. This method was for the first time offered in \[5\]. In \[6\] this method it was modified so that the numerical scheme had the second order of accuracy at a step on time. Now various variations of this method are widely used at the numerical solution of the kinetic equations. In \[7\] by the modification of this method, the system of the model kinetic equations was solved. All the above told implies that the method constructed in this article can be used for the solution of a task about definitions of influence of a magnetic field on SPT stream in the complete statement if to use it at a scattering stage.
Figure 3. Three-dimensional pattern for the ion density level surfaces

in constructed in [4] method of splitting. From other works devoted to this problem, it should be noted [8]. In this work functioning of a magnetic nozzle is investigated. For the description of the phenomenon authors use Vlasov’s equation, i.e. the equation (1). The difference of this work consists in absolutely different geometry. It allows them to reduce a task to one-dimensional. Boundary conditions in work [8] are such that in them there is no deltashaped cumulative distribution function therefore, it is possible to construct a numerical method by means of which the cumulative distribution function is defined and values of an ionic density are calculated. As the task is one-dimensional, schedules of density are provided in article. It is clear, that it is impossible to compare these results to the three-dimensional number-density distribution received in this work, and the sense of this comparison is unclear in view of essential differences in task geometry. It should be noted that carrying out comparisons of results of calculations of this work and results of calculations in work [4] encounters objective difficulties, as it isn’t possible to receive the solution of the task set above by method of statistical model operation yet. So in [9] by this method the problem about a stream is solved, but with other geometry. The numerical method is legibly not described and results – values of density aren’t given. The single work [9] known to authors where the method of statistical model operation solved a problem about SPT stream in the same statement, as well as in [4]. Results, which are received at the same time, don’t correspond to a number-density distribution in a stream at all. The reason of it was that the method of statistical model operation didn’t possible to simulate adequately a delta-shaped cumulative distribution function, and only the recharge is modelled. Actually, the main result of this article is creation of the numerical scheme, which would consider the specified deltoid.
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