DIRECT AND INDIRECT TRANSACTIONS AND REQUIREMENTS

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ABSTRACT. The indirect transactions between sectors of an economic system has been a long-standing open problem. There have been numerous attempts to define and mathematically formulate this concept in various other scientific fields in literature as well. The existing indirect effects formulations, however, can neither determine the direct and indirect transactions separately nor quantify these transactions between two individual sectors of interest in a multisectoral economic system. The novel concepts of the direct, indirect and transfer (total) transactions between any two sectors are introduced, and the corresponding requirements matrices are systematically formulated relative to both final demands and gross outputs, based on the system decomposition theory. It is demonstrated theoretically and through illustrative examples that the proposed requirements matrices accurately define and quantify the corresponding direct, indirect, and total interactions and relationships. The proposed requirements matrices for the US economy using aggregated input-output tables for multiple years are then presented and briefly analyzed.

1. INTRODUCTION

The analysis of observable direct transactions is relatively straightforward even in complex economic systems. The indirect transactions, however, is a complicated concept that has derived attention in many scientific fields, such as economics, ecology, graph theory, and network theory, for the last several decades. We define direct relationships as pairwise immediate interactions between two sectors in an economic system, and indirect relationships as pairwise interactions between two sectors through other sectors. With the directness and indirectness throughout the manuscript, we will constantly be referring to these fundamental definitions. The main focus of this manuscript is to introduce the novel direct, indirect, and transfer (total) transactions concepts defined pairwise between any two sectors of a multisectoral economic system and explicitly formulate the corresponding requirements matrices and coefficients for system analysis.

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Modeling interactions among industries and their interconnectedness goes back to the concept of the “circular flow” in an economy [24]. This idea is related to Petty’s concept of the interdependence of industries. François Quesnay created the Tableau Économique (economic table) in which he depicted the idea of the economy as a circular flow of income and output among economic sectors. The table is known for its diagrammatic representation of how transactions can systematically be traced through an economic system [20, 31]. Achille-Nicholas Isnard is known to be the first person to represent the circular flow of income and expenditure as an algebraic system of equations [15, 23].

The Tableau Économique is considered the first method for the explicit conceptualization of the nature of economic equilibrium. It is also hailed as a forerunner of general equilibrium theory pioneered by Léon Walras [37, 20, 31]. Walras used production coefficients that compared the required resources for a product and its total production [23]. Leontief’s empirical economic studies were based on Quesnay’s table and Walras’s formulations of general equilibrium, although his conclusion was that an economy is never in equilibrium. He made the circular flow transactions into a table which then led to the founding of the analytical tool called the input-output model [18]. The input-output analysis as we know today with contributions of many other economists analyzes intersectoral interactions in economic systems.

It is generally accepted that the input-output economics derive its significance largely from the fact that the total requirements coefficients measuring the combined effects of the direct and indirect repercussions of a change in final demands can easily be determined [30]. There have been numerous definitions and corresponding mathematical formulations of the indirect effects concept in the literature for about a century since the development of the input-output model [11, 23, 35, 27, 2, 16, 19, 1, 2, 13, 12, 14, 13, 29]. Each of these attempts, however meaningful, does not seem to be accurately describing and quantifying the indirect transactions. The existing approaches use the total—direct and indirect—effects formulation to define the indirect effects. The total effects, however, can neither determine the direct and indirect transactions separately nor quantify these transactions pairwise between any two sectors individually.

A mathematical theory, known as the system decomposition theory, and associated methodologies for the analysis of dynamic nonlinear compartmental systems was recently introduced by [3, 5, 4]. The static version of this theory has also been developed recently [6, 7]. This system decomposition partitions the system into subsystems, each of which separately represents all economic activities induced by an individual sector within the system. The system decomposition enables tracking the evolution of initial stocks, external inputs, and arbitrary intercompartmental flows of currency, goods, and services, as well as the associated storages derived from these stocks, inputs, and flows individually and separately within the system. The transient and the direct, indirect, cycling, acyclic, and transfer (total) flows along any given flow path or from one sector to another—along all paths—are also systematically formulated. In the present manuscript, we use these direct and indirect flows and distributions notions to conceptually redefine and mathematically
formulate the direct, indirect and transfer (total) transactions and requirements in the context of multisectoral economic systems.

The conceptualization and redefinition of directness and indirectness are inseparable. We first define the direct transaction between any two sectors as the total immediate pairwise flows between these sectors in an economic system. The total pairwise flows from one sector indirectly through other sectors to another will then be defined as the indirect transaction between these two sectors. For example, since steel is used in car production, each car purchase directly from the automotive sector includes an indirect purchase from the steel sector. The proposed methodology can separately quantify this indirect purchase specifically from the steel sector and, as a matter of fact, from any other individual sector of interest. The existing formulations, however, cannot determine and quantify such indirect purchases.

An immediate application of the proposed direct and indirect transactions and requirements concepts is the impact analysis. The direct and indirect requirements matrices can respectively be used to separately determine how a change in final demand for the output of one sector directly and indirectly affects every other individual sector in the system. Considering a hypothetical economic system, if the final demand for the output of the automotive industry is cut in half, the direct and indirect implications can separately be determined for the output of any individual industry in the car production chain through the proposed methodology. The existing formulations, however, can only quantify the cumulative—direct and indirect—effects of the decrease in the final demand on the system, but not the direct and indirect effects separately. Moreover, they cannot quantify how the change in the final demand for the cars affects the indirect purchases from the steel or any other individual sector of interest.

Following on the same hypothetical model, the alteration of the amount of steel to make the cars in the first step, coal to produce the steel in the second step, energy to extract the coal in the third step, and so on, can be calculated, based on the change in the final demand. In the existing formulations, the transaction between the steel and automotive industries in the first step is considered as the direct effect of the change in the final demand. On the other hand, when this production chain cycles back to steel at any later production step again, the current methodologies consider the value of steel used to make cars at that step as an indirect effect of the change (see Fig. 1). That is, the transactions between the same two sectors, steel and automotive industries, are inconsistently classified as the direct and indirect effects, solely based on the step number or order of propagation of the final demand within the system.

From a different perspective, it can be seen that the existing indirect effects are formulated without actually defining the indirect transactions between any two sectors in an economic system. The indirect effects are considered to be the total transaction carried by all subsequent steps after the first entrance of goods and services into each sector in the existing formulations. Therefore, even the immediate transactions between two sectors of interest after the first step in their interactions are considered as indirect in these approaches. The indirect effects are, therefore, implicitly defined microscopic quantities with limited practical use and cannot
The system decomposition theory defines the direct and indirect transactions as supplementary flows. The transfer (total) transactions is accordingly defined as the sum of the direct and indirect transactions. The corresponding direct, indirect, and transfer (total) flow distributions are defined relative to gross outputs in the context of the system decomposition theory [6]. In economic system analysis, however, the requirements matrices relative to final demands are more desirable for many cases. The direct, indirect, and total requirements matrices, as well as the corresponding transactions at both sectoral and subsectoral levels are systematically defined and explicitly formulated relative to both gross outputs and final demands in the present manuscript. These requirements matrices are also expressed in terms of the make-use framework as well. The requirements matrices relative to gross outputs are called composite requirements matrices, and the ones relative to final demands are called the simple requirements matrices. The elements of these requirements matrices will be called the direct, indirect, and transfer (total) requirements coefficients or simply direct, indirect, and total coefficients. The simple requirements matrices can be considered as the measures for the exogenous impacts on the sectors, while their composite counterparts as those for the intersectoral dynamics [6].

There are two requirements matrices explicitly formulated and widely used in economic system analysis: the direct and total requirements matrices. The simple transfer (total) requirements matrix is different from the existing total requirements matrix in a way that it covers the internal workings of the system by considering only the producing sectors and excluding final demands from the coefficients. Similar suggestions in regard to such exclusion are also made in the literature [8, 9]. Both the simple transfer (total) and existing total requirements matrices are formulated relative to final demands. The composite transfer (total) flow concept is equivalent to the total flow definition introduced by [34]. In the context of the total flows, the economic implications of the total requirements matrix relative to gross outputs instead of final demands is also discussed in the literature [22, 33].

The composite direct requirements matrix and the existing direct requirements or coefficient matrix are also the same. The difference between the simple direct requirements matrix and the coefficient matrix is that the simple direct requirements matrix yields the total direct transaction between any two sectors of the system, while the existing direct requirements matrix yields one step propagation within the system relative to final demands. The simple direct requirements matrix is defined relative to final demands, and its existing counterpart is defined relative to gross outputs. The simple direct and indirect requirements matrices are conceptually defined and explicitly formulated in the present manuscript for the first time in
the literature for a comprehensive analysis of economic systems by addressing the full complexity of their activities.

The requirements tables are mainly used for impact and policy analyses—the impact of a specific change in final demands on the sectors of an economic system, their production, and other economic activities. The economic repercussions of alterations in final demands within the system is a critical information for policy and business planning. The existing formulations provide only the total—direct and indirect—impacts of such changes on the system. The proposed simple and composite direct and indirect requirements matrices, however, separately determine the total direct and indirect responses by an individual sector of interest to an exogenous change in a specific sector’s final demand or gross output within an economy.

The simple direct and indirect coefficients can, for example, be used in emergency planning, such as separately estimating the direct and indirect effects that petroleum shortages would have on the production in each sector individually. A standard policy analysis problem is to investigate the implications of a new governmental policy change that impacts final demands for an economy in terms of interindustry production generated in response to the change. The simple direct and indirect requirements coefficients enable the analysis of the separate direct and indirect impacts of such programs or changes targeting a specific sector on any other individual sector of interest. For example, the direct and indirect impacts of the government tax policy aimed at decreasing consumer demand for a particular product individually on any other product within the system can separately be determined using the simple direct and indirect coefficients. Such thorough and in depth analyses are not possible through the state-of-the-art techniques.

The United Nations and most of the governments of the industrialized countries including the United States are currently using the input-output data to measure and analyze their national economic systems. The case studies at the end of the manuscript demonstrate that the proposed direct, indirect, and total transactions and requirements concepts capture the corresponding interactions and relationships between sectors of economic systems accurately and provide additional critical information that is not available through the existing formulations. The proposed concepts are applied to the aggregated US input-output data for multiple years to demonstrate their practicality and efficiency. The numerical results and their graphical representations for the simple direct, indirect, and total requirements matrices using these real data sets are also presented and briefly analyzed in the case studies.

2. METHODS

In this section, the fundamental relationships of input-output economics are summarized. The simple and composite direct, indirect, and transfer (total) transactions and requirements coefficients are systematically introduced through the system decomposition theory [6]. The conceptual development and methodological advancement brought by the theory are discussed and the differences between
Figure 1. Schematic representation of the indirect transactions and indirect effects. The sector $i$ is denoted by $s_i$ in the figure. The numbers next to arrows represent the step numbers or order, $n$, in the geometric series expansion of the total requirements matrix, $L$, given in Eq. 2.5. In the existing formulations, the flow segments labeled with the power of the first order term, $Af$, $n = 1$, in both colors are generally considered as the direct effects, and with the powers of all higher order terms, $A^n f$, $n > 1$, as the indirect effects. The unshaded flow segments represent the sum of the flow segments generated by all the remaining higher order terms of propagation, $n > 4$. In the context of the system decomposition theory, however, $z_{ik}$ represents the composite direct transactions from sector $i$ to $k$. Both the blue-shaded flow segments labeled with 1 and 4 (cycling flow at $s_3$) within $z_{43}$ represent the simple direct transactions from $s_4$ to $s_3$. A flow segment initiated at a sector and transmitted through other sectors to another is then defined as the indirect transaction. The red-shaded flow segments labeled with 2, 3, and 4 within $z_{24}$, $z_{43}$, and $z_{32}$, for example, are the portions of the indirect transactions from $s_1$ to $s_4$, $s_1$ to $s_3$, and $s_1$ to $s_2$, respectively.

The standard mathematical representation of the flow regime of a multi-sectoral economic system can be expressed as follows:

$$x = Z 1 + f$$

where $x$ is the vector of the gross outputs, $f$ is the vector of the final demands, $Z$ is the transactions matrix representing the intermediate flows of goods and services between the sectors, and $1$ is the vector whose entries are all one.

Let $\hat{x}$ be the diagonal matrix whose diagonal elements are the corresponding elements of vector $x$, and $\hat{L}$ be the diagonal matrix whose diagonal elements are the diagonal elements of matrix $L$. The direct requirements or (technical) coefficients
direct and indirect transactions and requirements

Matrix is then defined as

\[ A = Z \hat{x}^{-1} \]

where \( \hat{x} = \text{diag}(x) \). The direct requirements matrix shows the amount of inputs from industries in each row, an industry in a column needs in order to produce one dollar of its output. This matrix is called the composite direct distribution matrix in the context of the system decomposition theory [6].

The total requirements matrix can be formulated based on the direct requirements matrix as follows:

\[ L = (1 - A)^{-1} = I + A + A^2 + A^3 + \cdots + A^n + \cdots. \]

This matrix is sometimes called the Leontief’s inverse and its derivation can be found in [23]. It is also formulated with a completely different rationale and called the cumulative flow distribution matrix in the context of the system decomposition theory [6]. The relationship between the total gross outputs vector and the final demands vector \( f \) can be expressed as

\[ x = Lf. \]

The terms in the geometric series expansion of \( L \) defines step-by-step effects or propagation of final demands throughout the system:

\[ x = Lf = if + Af + A^2f + \cdots + A^n f + \cdots \]

The final demands, \( f = if \), generate a need for inputs from the productive sectors. These inputs are satisfied by the outputs of the first step that is represented by the direct requirements matrix \( A, Af \). These outputs themselves, however, generate a need for additional inputs for the functioning of the economic system. The additional inputs are satisfied by the outputs of the second step that is represented by the second order term of \( Lf, A^2f \), and so on. The steps are ordered by the power of the direct requirements matrix, \( n \). It is worth noting that, for static systems, all these steps are considered to be taking place simultaneously, as static systems are independent of time by definition. Each step in the propagation can be considered as a “discrete computational time,” not real time. Needless to say that there is no infinitely many steps of production in a single year.

There are several indirect effects formulations proposed in the literature [27]. Some of the corresponding indirect requirements matrices are listed below:

\[ E_1 = L - I = A + A^2 + A^3 + \cdots + A^n + \cdots \]

\[ E_2 = L - A = I + A^2 + A^3 + \cdots + A^n + \cdots \]

\[ E_3 = L - I - A = A^2 + A^3 + \cdots + A^n + \cdots \]

\[ E_4 = L - \hat{L} = A - \text{diag}(A) + A^2 - \text{diag}(A^2) + \cdots + A^n - \text{diag}(A^n) + \cdots \]

The left-multiplication of these indirect requirements matrices by \( f \) yields the corresponding indirect effects, \( E_if, i = 1, \ldots, 4 \). They represent the way in which final demands are transmitted as gross outputs through the productive sectors of an
economic system, generally after the first transactions. However, since the propagation takes place in computational time, the existing indirect effects notions are essentially computational concepts rather than physical measures.

The first indirect effect formulation, \( E_1 f \), represents the impact of the total effects, direct and indirect, less that of the final demands, \( f \) \cite{27, 2}. This formulation excludes only the final demands, and all intersectoral intermediate transactions are counted as indirect. It cannot distinguish the direct and indirect transactions and, consequently, cannot quantify indirect interactions. The second indirect effects formulation, \( E_2 f \), removes only the impact of the first order propagation, \( A f \), which is called the direct effects, from all production chains \cite{23, 27, 35, 16, 36}. It was used by the US Department of Commerce and a variation of it by UK input-output analysts \cite{27}. This formulation cannot quantify the indirect transactions either, as it includes direct transactions generated by the higher order terms of propagation due to cycling (see Fig. 1). Yet, the third formulation, \( E_3 \), removes the impacts of both the final demands and the direct effects simultaneously \cite{27, 1}. This indirect effects formulation is used by the Bureau of Economic Analysis (BEA) of the US and also widely used in ecological network analyses \cite{13, 11}.

The last indirect effects formulation, \( E_4 f \), is reported by \cite{27} referring to \cite{17}. This formulation seems to be an attempt to removing the final demands and the cycling effects from the total effects (see Fig. 1). The cycling effects, however, cannot be determined by only the diagonal entries of \( L \). This is because of the fact that each element of matrix \( L \) can be interpreted as the total effects of one sector directly or indirectly on another. Since the cycling effects—effects of a sector through other sectors reflexively back on itself—are a special case of the indirect effects, the cycling effects are included also in the off-diagonal entries of the total requirements matrix, \( L \). A detailed derivation of the cycling flows through the system decomposition theory is introduced recently by \cite{6}.

Unlike the current methodologies, the system decomposition theory explicitly formulates indirect transactions between any two sectors in the system. The indirect transaction from sector \( i \) to \( k \) is defined as the total pairwise intersectoral flows of goods and services from sector \( i \) indirectly through other sectors to \( k \). The direct transaction from sector \( i \) to \( k \) is then defined as the total pairwise immediate intersectoral flows from sector \( i \) to \( k \), regardless of the order of propagation of goods and services in their potentially circular interactions within the system—that is, whether the goods and services are cycling at sector \( i \) and reentering sector \( k \) multiple times through \( i \) (see Fig. 1).

The subthroughflow matrix, \( T = (\tau_{ik}) \), is defined as

\[
(2.7) \quad T = L \hat{f} \quad \text{with} \quad x = T 1 = L f
\]

through the system decomposition theory \cite{6}. Note that this subsectoral level system partitioning is not possible through the existing methodologies. The system decomposition also introduces the simple and composite direct, indirect, and transfer (total) flows relative to gross outputs. The composite indirect distribution and flow matrices, \( N^1 = (n^1_{ik}) \) and \( T^1 = (\tau^1_{ik}) \) relative to gross outputs, are formulated
as follows:

\[(2.8)\]
\[
N^\ddagger = (L - I) \hat{L}^{-1} - A \quad \text{and} \quad T^\ddagger = N^\ddagger \hat{\chi}.
\]

These composite indirect distribution and flow matrices will respectively be called the composite indirect requirements and transactions matrices relative to gross outputs in the present study. The \((i, k)\)-elements of \(N^\ddagger\) and \(T^\ddagger\) represent the total purchases from sector \(i\) indirectly by \(k\) to produce a dollar’s worth of its output and its gross output, respectively, to satisfy both the intermediate and final demands.

The composite direct and indirect flows are defined as supplementary flows as follows:

\[(2.9)\]
\[
T^e = T^d + T^\ddagger \Rightarrow T^\ddagger = T^e - T^d \quad \text{and} \quad N^\ddagger = N^e - N^d
\]

where the composite transfer (total) and direct distribution matrices relative to gross outputs, \(N^e = (n_{ik}^e)\) and \(N^d = (n_{ik}^d)\), and the corresponding flow matrices, \(T^e = (\tau_{ik}^e)\) and \(T^d = (\tau_{ik}^d)\), are given by

\[(2.10)\]
\[
N^e = (L - I) \hat{L}^{-1} \Rightarrow T^e = N^e \hat{\chi} \quad \text{and} \quad N^d = A \Rightarrow Z = T^d = N^d \hat{\chi}
\]

through the system decomposition theory [6]. These composite direct and total distribution and flow matrices will respectively be called the composite direct and total requirements and transactions matrices relative to gross outputs in the present study. The \((i, k)\)-elements of \(N^e\) and \(T^e\) represent the total purchases from sector \(i\) by \(k\) to produce a dollar’s worth of its output and its gross output, respectively, to satisfy both the intermediate and final demands. The composite direct requirements matrix is the same as the existing direct requirements or technical coefficients matrix, \(N^d = A\). The \((i, k)\)-elements of \(N^d\) and \(T^d = Z\) represent the total purchases from sector \(i\) directly by \(k\) to produce a dollar’s worth of its output and its gross output, respectively, to satisfy both the intermediate and final demands.

Although derived with completely different rationale, the composite transfer (total) flow matrix of [6] is equivalent to the total flow matrix of [34] and the total output-to-output multiplier of [23] after slight modifications. The total or net multipliers has been a topic of scholarly conversations for the last three decades [25, 9, 26, 21, 10]. In the context of the total flows, the economic implications of the total requirements matrix relative to gross outputs instead of final demands is also detailed in the literature [14, 22, 33].

The simple indirect flow matrix is formulated through the system decomposition theory as \(T^\ddagger = N^\ddagger T\) where the diagonal matrix \(T\) is defined to be \(T = \text{diag} (T)\) [6]. We reformulate the simple indirect transactions, \(T^\ddagger = (\tau_{ik}^\ddagger)\), and define the corresponding simple indirect requirements matrix, \(N^\ddagger = (n_{ik}^\ddagger)\), as follows:

\[(2.11)\]
\[
N^\ddagger = N^\ddagger \hat{L} = L - I - A \hat{L} \quad \text{and} \quad T^\ddagger = N^\ddagger \hat{\chi}
\]

using Eqs. 2.7 and 2.8, as well as the relationship \(T = \hat{L} \hat{\chi}\). The \((i, k)\)-elements of \(N^\ddagger\) and \(T^\ddagger\) represent the total purchases from sector \(i\) indirectly by \(k\) to satisfy a dollar’s worth of final demand and final demand, respectively, for its output.
The simple direct and indirect transactions are also supplementary. Using Eq. 2.11, this relationship can be expressed as follows:

\begin{equation}
T^t = T^d + T^i \Rightarrow T^i = T^t - T^d \quad \text{and} \quad N^i = N^t - N^d
\end{equation}

where the simple transfer (total) and direct distribution matrices relative to final demands, \(N^t = (n^t_{ik})\) and \(N^d = (n^d_{ik})\), and the corresponding simple flow matrices, \(T^t = (\tau^t_{ik})\) and \(T^d = (\tau^d_{ik})\), are

\begin{equation}
N^t = L - I \Rightarrow T^t = N^t \hat{f} \quad \text{and} \quad N^d = A L \Rightarrow T^d = N^d \hat{f}.
\end{equation}

The simple direct and total requirement matrices, \(N^d\) and \(N^t\), are different from the existing direct and total requirements matrices, \(A\) and \(L\), as detailed below.

The \((i,k)\)-elements of \(N^t\) and \(T^t\) represent the total purchases from sector \(i\) by \(k\) to satisfy a dollar’s worth of final demand and final demand, respectively, for its output. Therefore, the simple total requirements matrix represents internal workings of the system by excluding the final demands, while the existing formulation, \(L\), represents all transactions including sales to final demands. The simple transfer (total) requirements and the existing total requirements matrices are both defined relative to final demands. Similar suggestions in regard to excluding final demands from the total requirements coefficients are also proposed in the literature [8, 9].

The \((i,k)\)-elements of \(N^d\) and \(T^d\) represent the total purchases from sector \(i\) directly by \(k\) to satisfy a dollar’s worth of final demand and final demand, respectively, for its output. While the proposed simple direct requirements matrix yields the total immediate pairwise direct transactions regardless of the order of propagation, the existing direct requirements or coefficients matrix provides one step propagation, \(A f\), as given in Eq. 2.5, relative to final demands.

Let the superscript (*) represent any of the \(d\), \(i\), and \(t\) symbols. The critical difference between the simple and composite requirements matrices is that, although the simple versions, \(N^*\), are defined relative to final demands, \(\hat{f}\), their composite counterparts, \(NNN^*\), are formulated relative to gross outputs, \(\hat{x}\). This functionality allows for the impact analysis of the alterations in final demands and sectoral changes on economic systems. From a different perspective, the difference between the simple and composite direct, indirect, and transfer (total) transactions from sector \(i\) to \(k\), \(\tau^i_{ik}\) and \(\tau^*_ik\), is that the simple transactions quantify the corresponding flows originating only from sector \(i\) to \(k\), and the composite transactions quantify the corresponding flows from \(i\) to \(k\) regardless of their origin.

The simple direct, indirect, and transfer (total) gross outputs vectors, \(x^d\), \(x^i\), and \(x^t\) can then compactly be expressed as follows:

\begin{equation}
x^* = T^* 1 = N^* f
\end{equation}

similar to the linear relationship between \(x\), \(T\), and \(f\) in Eq. 2.7. The composite counterparts of these relationships can similarly be formulated [6].

Graph theoretically, the sign of a simple indirect transaction or requirement coefficient shows the existence of an indirect path between the corresponding two sectors in the system. That is, if the \((i,k)\)-element of \(T^i\) and \(N^i\) is positive, \(\tau^i_{ik} > 0\) and \(n^i_{ik} > 0\), then there is a production chain from sector \(i\) indirectly to \(k\). It is
also worth emphasizing that the diagonal elements of simple indirect transactions matrix, $T^s$, represent the cycling transactions from the corresponding sectors reflexively back into themselves indirectly through other sectors. The composite indirect transactions matrix has the same properties as well [6].

Although assuming that requirements matrices are constant while final demands are changing is unrealistic, the impact of a change in the final demands on processing sectors can be approximated. This approach is sometimes called the impact analysis. The linearity of the relationships between the factors given in Eq. 2.7 and 2.14 imply that

$$\Delta T = L \Delta \hat{f} \Rightarrow \Delta x = L \Delta f \quad \text{and} \quad \Delta T^* = N^* \Delta \hat{f} \Rightarrow \Delta x^* = N^* \Delta f.$$ 

In other words, a change in final demands, $\Delta f$, enforces the corresponding changes in the direct, indirect, transfer, and total transactions, $\Delta T^d, \Delta T^i, \Delta T^t, \text{and} \Delta T$, as well as the corresponding gross outputs, $\Delta x^d, \Delta x^i, \Delta x^t, \text{and} \Delta x$. Note that the composite counterparts of these relationships relative to gross outputs, $\hat{x}$, can similarly be formulated.

Lastly, we provide the simple and composite direct, indirect, and transfer (total) requirements matrices in terms of the make-use framework in Table 1. The $D$ and $B$ matrices used in the table are defined as follows:

$$D = V \text{diag}(V^T 1)^{-1} \quad \text{and} \quad B = U \text{diag}(V 1)^{-1} \Rightarrow A = DB$$

where $U$ and $V$ are the use and make matrices, respectively, and the prime represents the matrix transpose [23, 13, 28]. The aggregated real US input-output data for multiple years are briefly analyzed based on these formulations in Case study 3.2. The requirements matrices given in the table are in the industry-by-industry terms under the industry-based technology assumption. They can similarly be formulated in the commodity-by-commodity, industry-by-commodity, and commodity-by-industry terms as well.

3. Results

A hypothetical economic system is analyzed for illustrative purposes in this section to elucidate the uses and meanings of the proposed direct, indirect, and transfer
Figure 2. Schematic representation of the hypothetical economic model. The gross outputs at the first two steps are $Af = [2, 6, 3]'$ and $A^2f = [0.6, 0.6, 0.6]'$ (Case Study 3.1).

(totals) transactions and coefficients, as well as to communicate the differences between these coefficients and their existing counterparts. In the second case study, the real US input-output data aggregated to seven sectors are analyzed for 15 years. Since the main focus of the present manuscript is to introduce the proposed novel concepts and formulations, the numerical results and their graphical representations are briefly analyzed in this section essentially to demonstrate their effectiveness and wide applicability.

3.1. Case study. In this case study, a simple hypothetical model is analyzed as an application of the proposed methodology.

Let the sectors of a three-sector economic system model be agriculture (sector 1), manufacturing (sector 2), and services (sector 3). Let also the technical coefficients matrix be given as

\[ A = \begin{bmatrix}
0 & 0.1 & 0 \\
0 & 0 & 0.2 \\
0.3 & 0 & 0
\end{bmatrix}. \]

Using “row-to-column” convention, this indicates that the production of a dollar’s worth of goods in the agriculture sector requires a direct input of $0.30$ from the service sector. Similarly, the production of a dollar’s worth of manufacturing requires a direct input of $0.10$ from the agriculture sector, and that of the service sector requires $0.20$ direct input from the manufacturing sector.

For the final demands $f = [10, 20, 30]'$, in million dollars, the total gross outputs from each sector required to satisfy this demand becomes $x = Lf$. That is,

\[ f = \begin{bmatrix}
10 \\
20 \\
30
\end{bmatrix} \quad \text{then} \quad x = Lf = \begin{bmatrix}
12.6761 \\
26.7606 \\
33.8028
\end{bmatrix}. \]
The composite direct transactions between the sectors of the system can then be expressed as follows:

\[
Z = A \hat{x} = \begin{bmatrix}
0 & 2.6761 & 0 \\
0 & 0 & 6.7606 \\
3.8028 & 0 & 0
\end{bmatrix}
\]

using Eq. 2.2. That is, $2.6761$ million worth of goods are transferred from the agriculture to the manufacturing sector, \(z_{12} = 2.6761\), $6.7606$ million from the manufacturing to the service sector, \(z_{23} = 6.7606\), and $3.8028$ million worth of service from the service to the agriculture sector, \(z_{31} = 3.8028\).

The total requirements matrix becomes

\[
L = (1 - A)^{-1} = I + A + \cdots + A^n + \cdots = \begin{bmatrix}
1.0060 & 0.1006 & 0.0201 \\
0.0604 & 1.0060 & 0.2012 \\
0.3018 & 0.0302 & 1.0060
\end{bmatrix}
\]

using Eq. 2.3. A step-by-step computation shows that the terms in the series expansion of \(L\) diminishes quickly:

\[
A^2 = \begin{bmatrix}
0 & 0 & 0.02 \\
0.06 & 0 & 0 \\
0 & 0.03 & 0
\end{bmatrix}, \quad A^3 = \begin{bmatrix}
0.006 & 0 & 0 \\
0 & 0.006 & 0 \\
0 & 0 & 0.006
\end{bmatrix},

A^4 = \begin{bmatrix}
0 & 0.0006 & 0 \\
0 & 0 & 0.0012 \\
0.0018 & 0 & 0
\end{bmatrix}, \quad A^5 = \begin{bmatrix}
0 & 0 & 1.2 \times 10^{-4} \\
3.6 \times 10^{-4} & 0 & 0 \\
0 & 1.8 \times 10^{-4} & 0
\end{bmatrix}.
\]

This is generally the case and \(L\) can be approximated accurately enough with just a few terms of the series. As an example, \((2, 3)\) entry of \(L\), \(\ell_{23} = 0.2012\), indicates that, for a dollar’s worth of service to meet its final demand, the service sector requires a total—direct and indirect—input of $0.2012 from the manufacturing sector. Similarly, \((2, 3)\) entries of \(A^n\) give the value of goods that, for a dollar’s worth of service, the service sector requires directly from the manufacturing sector at step \(n\). All the other entries of the matrices can be interpreted similarly.

The gross outputs for the first step then become:

\[
A \hat{f} = \begin{bmatrix}
0 & 2 & 0 \\
0 & 0 & 6 \\
3 & 0 & 0
\end{bmatrix} \Rightarrow Af = \begin{bmatrix}
2 \\
6 \\
3
\end{bmatrix}.
\]

This computation indicates that to satisfy the final demands of $10 million worth of goods from the agriculture sector, $20 million from the manufacturing sector, and $30 million worth of services from the service sector, the manufacturing sector needs $2 million worth of goods from the agriculture sector, the service sector needs $6 million from the manufacturing sector, and the agriculture sector needs $3 million worth of service from the service sector. In order to satisfy these intermediate demands from each sector in the first step, the gross outputs in the second
step should become:

\[
A^2 \hat{f} = \begin{bmatrix}
0 & 0 & 0.6 \\
0.6 & 0 & 0 \\
0 & 0.6 & 0 \\
\end{bmatrix} \Rightarrow A^2 f = \begin{bmatrix} 0.6 \\
0.6 \\
0.6 \end{bmatrix}.
\]

All the subsequent steps can be calculated and interpreted similarly (see Fig. 2).

The subthroughflow matrix that represents the total flow distribution between the sectors of the system to supply the final demands can then be calculated as follows:

\[
T = L \hat{f} = \begin{bmatrix} 10.0604 & 2.0121 & 0.6036 \\
0.6036 & 20.1207 & 6.0362 \\
3.0181 & 0.6036 & 30.1811 \end{bmatrix} \quad \text{with} \quad x = T 1 = \begin{bmatrix} 12.6761 \\
26.7606 \\
33.8028 \end{bmatrix}
\]

based on the formulation given in Eq. 2.7. As an example, \((2,3)\)-entry of \(T\), \(\tau_{23} = 6.0362\), indicates that $6.0362 million worth of goods in the manufacturing sector is destined to contribute to the final demand of the service sector. All the other entries can be interpreted similarly.

There are several indirect requirements matrices proposed in the literature as partially listed in Eq. 2.6. For this hypothetical economic system, they become

\[
E_1 = \begin{bmatrix} 0.0060 & 0.1006 & 0.0201 \\
0.0604 & 0.0060 & 0.2012 \\
0.3018 & 0.0302 & 0.0060 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1.0060 & 0.0006 & 0.0201 \\
0.0604 & 1.0060 & 0.0012 \\
0.0018 & 0.0302 & 1.0060 \end{bmatrix}
\]

\[
E_3 = \begin{bmatrix} 0.0060 & 0.0006 & 0.0201 \\
0.0604 & 0.0060 & 0.0012 \\
0.0018 & 0.0302 & 0.0060 \end{bmatrix}, \quad E_4 = \begin{bmatrix} 0 & 0.1006 & 0.0201 \\
0.0604 & 0 & 0.2012 \\
0.3018 & 0.0302 & 0 \end{bmatrix}.
\]

The simple indirect requirements matrix for this hypothetical system, however, is

\[
N^1 = \begin{bmatrix} 0.0060 & 0 & 0.0201 \\
0.0604 & 0.0060 & 0 \\
0 & 0.0302 & 0.0060 \end{bmatrix}
\]

as given in Eq. 2.11. As an example, \((1,3)\)-entry of \(N^1\), \(n_{13}^1 = 0.0201\), indicates that, for a dollar’s worth of service to meet its final demand, the service sector requires an indirect input of $0.0201 from the agriculture sector. All the other entries can be interpreted similarly.

The inaccuracy of the existing indirect effect formulations is theoretically demonstrated above, in the discussion after Eq. 2.6. This can be verified numerically for this hypothetical economic system as well. The diagonal entries of \(N^1\) represent reflexive indirect, that is cycling, transactions. Since there is one closed path in this system, \(s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow s_1\), the cycling flow is the same at each sector along the path (see Fig. 2). Consequently, the diagonal simple indirect coefficients are all equal to each other: \(n_{11}^1 = n_{22}^1 = n_{33}^1 = 0.0060\). As seen from Fig. 2, there are no indirect transactions from \(s_1\) to \(s_2\), from \(s_2\) to \(s_3\), and from \(s_3\) to \(s_1\). Therefore, the corresponding simple indirect coefficients are zero: \(n_{12}^1 = n_{21}^1 = n_{31}^1 = 0\). The relationship \(n_{12}^1 = 0\), for example, indicates that there are no goods in the agriculture sector that is destined to contribute indirectly through the service sector to the final
demand of the manufacturing sector. Graph theoretically, this implies that there is no indirect path from sector 1 to 2, which can easily be verified from Fig 2. Since the existing indirect effects formulations, $E_1$ to $E_4$, cannot exclude the cycling effects at the sectors along the path, they all have nonzero values in these entries (see Figs. 1 and 2).

The simple indirect transactions matrix and the indirect gross outputs vector then become

$$T^i = N^i f = \begin{bmatrix} 0.0604 & 0 & 0.6036 \\ 0.6036 & 0.1207 & 0 \\ 0 & 0.6036 & 0.1811 \end{bmatrix}, \quad x^i = T^i 1 = \begin{bmatrix} 0.6640 \\ 0.7243 \\ 0.7847 \end{bmatrix}$$

as formulated in Eq. 2.12. The nonzero entries of $T^i$ represent the monetary value of the indirect transactions between sectors that induces the corresponding final demands. For example, $\tau^i_{12} = 0.6036$ indicates that $0.6036$ million worth of services from the service sector are purchased by the manufacturing sector indirectly through the agriculture sector to satisfy the final demand for its product. Three indirect transactions, however, are zero: $\tau^i_{12} = \tau^i_{23} = \tau^i_{31} = 0$. The relationship $\tau^i_{12} = 0$ indicates that there are no goods in the agriculture sector that is destined to contribute indirectly through the service sector to the final demand of the manufacturing sector. The absence of indirect transactions in the given directions are consistent with the values of the corresponding indirect coefficients, as discussed above. The other indirect transactions can be interpreted similarly.

The existing direct and total requirements matrices are given in Eqs. 3.1 and 3.4. The novel simple direct and transfer (total) requirements matrices are also presented below for comparison:

$$N^d = \begin{bmatrix} 0 & 0.1006 & 0 \\ 0 & 0 & 0.2012 \\ 0.3018 & 0 & 0 \end{bmatrix}, \quad N^t = \begin{bmatrix} 0.0060 & 0.1006 & 0.0201 \\ 0.0604 & 0.0060 & 0.2012 \\ 0.3018 & 0.0302 & 0.0060 \end{bmatrix}$$

using Eq. 2.13. As seen from these results, the nonzero coefficients of $N^d$ and $A$ are different. The only difference between the simple transfer (total) and existing total requirements matrices, $N^t = L - I$ and $L$, is that the diagonal transfer coefficients are one less than the total coefficients. Since the simple total requirements matrix only covers internal workings of the producing sectors, it excludes the final demands from the coefficients on the main diagonal.

As formulated in Eq. 2.10, the composite direct requirements matrix is equivalent to the existing direct requirements or coefficient matrix, that is $N^d = A$. The composite indirect and total requirements matrices can be computed as

$$N^i = \begin{bmatrix} 0.0060 & 0 & 0.0200 \\ 0.0600 & 0.0060 & 0 \\ 0 & 0.0300 & 0.0060 \end{bmatrix}, \quad N^t = \begin{bmatrix} 0.0060 & 0.1000 & 0.0200 \\ 0.0600 & 0.0060 & 0.2000 \\ 0.3000 & 0.0300 & 0.0060 \end{bmatrix}$$

using Eqs. 2.8 and 2.10. As examples, the $(3, 2)$—element of $N^i$, that is the composite indirect coefficient $n^i_{13} = 0.03$, quantifies the total indirect purchases from the service sector indirectly through the agriculture sector by the manufacturing
sector to produce a dollar’s worth of its output to satisfy both intermediate and final demands. The corresponding simple and composite indirect coefficients are zero, due to the absence of the production chains in these directions. Similarly, the \((3,2)\)-element of \(N^t\), that is the transfer coefficient \(n_{32}^t = 0.03\), measures the total purchases from the service sector by the manufacturing sector to produce a dollar’s worth of its output to satisfy both intermediate and final demands. Since the direct and indirect flows are supplementary, the relationship \(n_{32}^d = n_{32}^t = 0.03\) indicates that there is no direct composite flow in the given direction. This observation agrees with the corresponding composite direct coefficient or transaction, that is \(z_{32} = a_{32} = n_{32}^d = 0\).

As seen from these illustrative results, the proposed requirements matrices provide critical statistics to analyze complex economic systems comprehensively at both sectoral and subsectoral levels. The results demonstrate that the novel concepts accurately capture the direct, indirect, and transfer (total) relationships and interactions between sectors. Such detailed analysis is not possible through the existing methodologies.

3.2. Case study. In this case study, the US input-output data for 15 years are briefly analyzed using the aggregated input-output use and make tables, \(U\) and \(V\), provided by [23] in Appendix B. These \(U\) and \(V\) matrices are expressed in millions of US current year dollars. The sectors in these aggregated data sets for the US economy are as follows: Agriculture (1), Mining (2), Construction (3), Manufacturing (4), Trade, Transport & Utilities (5), Services (6), and Other (7).

The requirements matrices are computed using the formulations listed in Table 1 for the make-use framework. The results are in the industry-by-industry terms under the industry technology assumption. The numerical results for the composite indirect and total requirements matrices, \(N^t\) and \(N^t\), have already been presented in the context of ecosystem analysis [6]. Therefore, no numerical tables and graphical representations for these matrices are provided in the present manuscript.

The numerical results for the simple direct, indirect, and transfer (total) requirements matrices, \(N^d\), \(N^t\), and \(N^t\), are presented in Tables 2-16 and their graphical representations are presented in Figs. 4-6. Since the graphs of \(N^t = L - I\) and \(L\) differ only along their main diagonals, the graphs for \(L\) can easily be predicted from those of \(N^t\), and so, are omitted. The graphical representation of the composite direct requirements matrix, \(N^d = A\), is also presented for a comparison in Figs. 3.

The requirements coefficients provide detailed information about the internal workings of and exogenous influence on economic systems. We use some coefficients from Table 2 for 2006 below to exemplify the interpretations of the numerical results provided in the tables. The simple indirect coefficient \(n_{31}^i(2006) = 0.0051\) represents the total value of goods purchased from sector 3 indirectly through other sectors by sector 4 in 2006 to satisfy a dollar’s worth of final demand for its product. There is only one zero simple indirect coefficient in all simple indirect requirements matrices presented in Tables 2-16: \(n_{31}^i(1919) = 0\). This indicates that there was no indirect flow from the construction sector to the other sector (sector 7) in 1919. The composite indirect coefficient \(n_{34}^i(2006) = 0.0032\) quantifies the total
value of goods purchased from sector 3 indirectly through other sectors by sector 4 in 2006 to produce a dollar’s worth of its product.

The simple direct coefficient \( n_{34}^d(2006) = 0.0030 \) gives the total value of goods purchased from sector 3 directly by 4 in 2006 to satisfy a dollar’s worth of final demand for its product. The composite direct coefficient \( n_{34}^d(2006) = 0.0019 \) then measures the total value of goods purchased from sector 3 directly by 4 in 2006 to produce a dollar’s worth of its product. Similarly, the simple total coefficient \( n_{34}^t(2006) = 0.0081 \) determines the total value of goods purchased from sector 3 by 4 in 2006 to satisfy a dollar’s worth of final demand for its product. The composite total coefficient \( n_{34}^t(2006) = 0.0051 \) ascertains the total value of goods purchased from sector 3 by 4 in 2006 to produce a dollar’s worth of its product.

Manufacturing seems to be the backbone of the US economy in terms of both direct and indirect transactions, as can be verified from both the numerical results in Tables 2-16 and their graphical representations in Figs. 3-6. Interestingly, however, although the manufacturing sector directly contributes more to itself (4th rows in Fig. 4), it indirectly contributes more to the construction sector (3rd rows in Fig. 5). The diagonal indirect coefficients quantify the cycling transactions—the reflexive circular flows initiated at and ends in the same sector after possibly being transmitted throughout the system. The diagonal simple indirect coefficients in Figs. 3-6 show that the cycling transactions are generally not significant for most years in the US economy, except for the manufacturing sector.

The simple indirect coefficients indicate that the US industries indirectly rely increasingly more on the service sector (6th rows in Fig. 5). Intriguingly, the indirect contributions of the service sector to the US economy are even more than those of the manufacturing sector in recent years (4th rows in Fig. 5). Figure 4 indicates that the service sector also directly supplies the other industries increasingly more but at a lower scale relative to other direct transactions. This increasing tendency in the US economy towards being service-oriented is discussed in some BEA reports as well [32]. Contrary to this observation, the indirect contributions of the trade, transportation, and utilities sector (5th rows in Fig. 5) to the other industries are gradually decreasing, following an increase from 1919 to 1939.

The impact analysis formulated in Eq. 2.15 implies that a change in the final demands of industries at the amount of \([1, 0, 0, 0, 0, 2, 0, 0]^t\), in million dollars, in 2006 enforces the following changes in the indirect transactions throughout the system:

\[
\Delta T^i = \begin{bmatrix}
0.0154 & 0 & 0 & 0 & 0 & 0.0148 & 0 \\
0.0444 & 0 & 0 & 0 & 0 & 0.0311 & 0 \\
0.0045 & 0 & 0 & 0 & 0 & 0.0029 & 0 \\
0.1792 & 0 & 0 & 0 & 0 & 0.1238 & 0 \\
0.0693 & 0 & 0 & 0 & 0 & 0.0472 & 0 \\
0.1871 & 0 & 0 & 0 & 0 & 0.0850 & 0 \\
0.0210 & 0 & 0 & 0 & 0 & 0.0122 & 0
\end{bmatrix}
\]

and \( \Delta x^i = \begin{bmatrix}
0.0302 \\
0.0755 \\
0.0074 \\
0.3030 \\
0.1164 \\
0.2721 \\
0.0332
\end{bmatrix} \)

where the simple indirect transactions and gross outputs are formulated in Eqs. 2.11 and 2.15. These results indicate that a change in final demands of $1 million of the
agriculture sector and $2 million of the service sector yield the changes in the indirect transactions throughout the system as specified by $\Delta T^i$. For example, since $\Delta \tau_{31} = 0.0045$, $\Delta \tau_{36} = 0.0029$, and $\Delta x_3 = 0.0074$, the agriculture and service sectors end up buying $4500$ and $2900$ ($7400$ in total) worth of products, respectively, from the construction sector indirectly through the other sectors to meet the specified changes in final demands. The construction sector is the least affected sector from these changes in the final demands. The sector that is the most effected indirectly is the manufacturing sector with a total change in its gross output at the amount of $\$303,000$ ($\Delta x_{44} = 0.3030$).

The analysis of all the other coefficients would further elucidate various other aspects of the US economy. The unique perspectives the proposed requirements coefficients bring to the understanding of the internal workings of economic systems, intersectoral interactions, and exogenous impacts on the system and individual sectors, are not available through the state-of-the-art techniques. The accuracy of the interpretations of economic activities increases with the disaggregation of the industries.

4. DISCUSSION

There have been numerous attempts in the literature to define and formulate the indirect transactions between sectors of an economic system or species in an ecological system for about a century. None of these formulations can describe and quantify the indirect transactions accurately.

The existing indirect effects are formulated by modifications of the total requirements matrix, $L$, at different levels. The idea has essentially been to remove some terms in the geometric series expansion of $L$ to distinguish the direct and indirect effects. In a nutshell, the flow segments are classified as direct or indirect in reference to final demands, based on the order of propagation they contribute to the final demands. In general, the flow segments contributing to final demands in the first step are considered as direct effects, and all the subsequent steps as indirect in these formulations. In other words, the existing indirect effects notions are mainly computational concepts rather than a physical measure. The existing coefficients, consequently, cannot even quantify the direct and indirect transactions separately, let alone the direct or indirect transactions between any two sectors of interest.

In the context of the system decomposition theory the directness and indirectness are determined based on the nature of interactions and relationships between sectors. The direct and indirect flows between any two compartments within a compartmental system has recently been defined and mathematically formulated relative to gross outputs [6]. Based on this theory, the composite and simple direct, indirect, and transfer (total) transactions between any two sectors, as well as the associated requirements coefficients for multisectoral economic systems are conceptually redefined and explicitly formulated relative to both gross outputs and final demands in the present manuscript. The simple requirements coefficients can be considered as measures for exogenous impacts on each sector, while their composite counterparts for impacts of sectoral changes. Therefore, they provide
multiple measures for more rigorous and detailed analyses of economic systems to address their full complexity, exogenous influence, and intersectoral dynamics. These requirements matrices are expressed in terms of the make-use framework as well.

The existing direct and total requirements matrices are defined relative to gross outputs and final demands, respectively. Therefore, the proposed composite direct requirement matrix is equivalent to the existing direct requirements, or coefficient matrix. The difference between the simple direct requirements matrix and coefficient matrix is that while the former provides total direct transactions pairwise between any two sectors regardless of the order of propagation in their potentially circular interactions, the latter provides only one step propagation relative to final demands. The difference between the proposed and existing total requirements matrices is that while the former provides coefficients for total transactions between only producing sectors by excluding final demands, the latter provides coefficients for total transactions including sales to final demands.

The existing direct and total requirements matrices are mainly used for impact and policy analysis in national and regional economic systems. The BEA publishes these requirements tables together with the annual US input-output data. The proposed simple and composite direct, indirect, and total requirements matrices relative to both final demands and gross outputs provide different unique perspectives through novel requirements statistics about the system response to exogenous and sectoral changes on economic systems that are not available through the existing formulations.

The accuracy and efficiency of the proposed transactions and requirements matrices in capturing the corresponding interactions and relationships between sectors of an economic system is demonstrated through a hypothetical model in the first case study. In the second case study, we then presented the requirements coefficients for the US economy, using the aggregated input-output data for 15 years. The numerical results for these real data sets and their graphical representations are also presented.

**DECLARATIONS**

**Authors’ contributions.** Conceived the ideas and designed the methodology; Analyzed and interpreted the data; Wrote the manuscript.

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APPENDIX A. THE US REQUIREMENTS TABLES

The aggregated input-output tables in terms of the make-use framework ($U$ and $V$), as well as the direct and total requirements matrices ($A$ and $L$ in industry-by-industry terms) for the US economy are presented for 15 years (1919, 1929, 1939, 1947, 1958, 1963, 1967, 1972, 1977, 1982, 1987, 1992, 1997, 2002, 2006) in [23]. The sectors in these aggregated data sets are: Agriculture (1), Mining (2), Construction (3), Manufacturing (4), Trade, Transport & Utilities (5), Services (6), and Other (7).

The numerical results for the simple direct, indirect, and transfer (total) requirements tables, $N^d$, $N^i$, and $N^t$ are presented in Tables 2-16 in this Appendix, together with the coefficient matrix, $N^t = A$. These numerical results are also graphically presented in Fig. 3-6.
### Table 2. The US Requirements Tables for 2006

#### Technical Coefficients Matrix ($A$)

|   | 1   | 2         | 3        | 4        | 5        | 6    | 7   |
|---|-----|-----------|----------|----------|----------|------|-----|
| 1 | 0.2403 | 0.0014 | 0.0345 | 0.0001 | 0.0018 | 0.0007 |
| 2 | 0.0028 | 0.1307 | 0.0079 | 0.0756 | 0.031   | 0.0004 | 0.0066 |
| 3 | 0.0035 | 0.0002 | 0.001  | 0.0019 | 0.0039 | 0.0072 | 0.0242 |
| 4 | 0.1858 | 0.0959 | 0.2673 | 0.3311 | 0.0581 | 0.0558 | 0.1027 |
| 5 | 0.0774 | 0.0379 | 0.1063 | 0.1003 | 0.0698 | 0.0329 | 0.0439 |
| 6 | 0.0875 | 0.1298 | 0.1262 | 0.1239 | 0.1846 | 0.2889 | 0.2029 |
| 7 | 0.0102 | 0.0096 | 0.0095 | 0.0233 | 0.0223 | 0.0192 | 0.0225 |

#### Simple Direct Requirements Matrix ($Nd$)

|   | 1   | 2         | 3        | 4        | 5        | 6    | 7   |
|---|-----|-----------|----------|----------|----------|------|-----|
| 1 | 0.3212 | 0.0014 | 0.0551 | 0.0001 | 0.0026 | 0.0007 |
| 2 | 0.0037 | 0.1531 | 0.1207 | 0.0343 | 0.0006 | 0.0069 |
| 3 | 0.0047 | 0.0002 | 0.0010 | 0.0030 | 0.0106 | 0.0251 |
| 4 | 0.2483 | 0.1124 | 0.2689 | 0.5288 | 0.0643 | 0.0818 | 0.1066 |
| 5 | 0.1034 | 0.0444 | 0.1069 | 0.1602 | 0.0773 | 0.0482 | 0.0456 |
| 6 | 0.1169 | 0.1521 | 0.1269 | 0.1979 | 0.2045 | 0.4235 | 0.2107 |
| 7 | 0.0136 | 0.0112 | 0.0096 | 0.0372 | 0.0247 | 0.0281 | 0.0234 |

#### Simple Indirect Requirements Matrix ($Ni$)

|   | 1   | 2         | 3        | 4        | 5        | 6    | 7   |
|---|-----|-----------|----------|----------|----------|------|-----|
| 1 | 0.0154 | 0.0101 | 0.0224 | 0.0184 | 0.0073 | 0.0074 | 0.0110 |
| 2 | 0.0444 | 0.0185 | 0.0486 | 0.0262 | 0.0181 | 0.0155 | 0.0236 |
| 3 | 0.0045 | 0.0033 | 0.0048 | 0.0051 | 0.0036 | 0.0014 | 0.0035 |
| 4 | 0.1792 | 0.0940 | 0.1961 | 0.0684 | 0.0780 | 0.0619 | 0.1106 |
| 5 | 0.0693 | 0.0379 | 0.0756 | 0.0411 | 0.0303 | 0.0236 | 0.0454 |
| 6 | 0.1871 | 0.1278 | 0.2024 | 0.1849 | 0.1300 | 0.0425 | 0.1592 |
| 7 | 0.0210 | 0.0127 | 0.0227 | 0.0153 | 0.0112 | 0.0061 | 0.0149 |

#### Simple Transfer (Total) Requirements Matrix ($Nt$)

|   | 1   | 2         | 3        | 4        | 5        | 6    | 7   |
|---|-----|-----------|----------|----------|----------|------|-----|
| 1 | 0.3365 | 0.0101 | 0.0238 | 0.0735 | 0.0075 | 0.0101 | 0.0118 |
| 2 | 0.0481 | 0.1716 | 0.0566 | 0.1470 | 0.0524 | 0.0161 | 0.0305 |
| 3 | 0.0092 | 0.0036 | 0.0058 | 0.0081 | 0.0079 | 0.0120 | 0.0286 |
| 4 | 0.4275 | 0.2064 | 0.4650 | 0.5972 | 0.1424 | 0.1437 | 0.2172 |
| 5 | 0.1727 | 0.0823 | 0.1825 | 0.2013 | 0.1076 | 0.0718 | 0.0910 |
| 6 | 0.3041 | 0.2799 | 0.3294 | 0.3828 | 0.3345 | 0.4660 | 0.3698 |
| 7 | 0.0346 | 0.0239 | 0.0323 | 0.0525 | 0.0359 | 0.0342 | 0.0382 |
### Table 3. The US Requirements Tables for 2002

**Technical Coefficients Matrix \((A)\)**

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.2638 | 0.002 | 0.0027 | 0.0379 | 0.0004 | 0.0008 | 0.0008 |
| 2 | 0.0032 | 0.0468 | 0.0099 | 0.0381 | 0.0236 | 0.0004 | 0.0042 |
| 3 | 0.0043 | 0.0359 | 0.0007 | 0.0032 | 0.0058 | 0.0081 | 0.0204 |
| 4 | 0.1491 | 0.0934 | 0.245 | 0.351 | 0.05 | 0.0472 | 0.0959 |
| 5 | 0.0852 | 0.064 | 0.0968 | 0.0913 | 0.0794 | 0.0254 | 0.0452 |
| 6 | 0.1333 | 0.2457 | 0.144 | 0.1386 | 0.1844 | 0.2682 | 0.2026 |
| 7 | 0.0087 | 0.0138 | 0.0073 | 0.015 | 0.0267 | 0.0162 | 0.0193 |

**Simple Direct Requirements Matrix \((N^d)\)**

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.3635 | 0.0021 | 0.0027 | 0.0616 | 0.0004 | 0.0011 | 0.0008 |
| 2 | 0.0044 | 0.0497 | 0.0100 | 0.0619 | 0.0263 | 0.0006 | 0.0043 |
| 3 | 0.0059 | 0.0381 | 0.0007 | 0.0052 | 0.0065 | 0.0115 | 0.0210 |
| 4 | 0.2055 | 0.0991 | 0.2469 | 0.5705 | 0.0557 | 0.0669 | 0.0989 |
| 5 | 0.1174 | 0.0679 | 0.0975 | 0.1484 | 0.0885 | 0.0360 | 0.0466 |
| 6 | 0.1837 | 0.2608 | 0.1451 | 0.2253 | 0.2054 | 0.3799 | 0.2090 |
| 7 | 0.0120 | 0.0146 | 0.0074 | 0.0244 | 0.0297 | 0.0229 | 0.0199 |

**Simple Indirect Requirements Matrix \((N^i)\)**

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.0146 | 0.0127 | 0.0238 | 0.0229 | 0.0072 | 0.0067 | 0.0111 |
| 2 | 0.0199 | 0.0118 | 0.0222 | 0.0085 | 0.0068 | 0.0064 | 0.0108 |
| 3 | 0.0068 | 0.0057 | 0.0069 | 0.0081 | 0.0051 | 0.0015 | 0.0046 |
| 4 | 0.1657 | 0.1192 | 0.1856 | 0.0548 | 0.0707 | 0.0521 | 0.1006 |
| 5 | 0.0621 | 0.0472 | 0.0667 | 0.0400 | 0.0255 | 0.0188 | 0.0385 |
| 6 | 0.2017 | 0.1854 | 0.1990 | 0.1824 | 0.1246 | 0.0367 | 0.1481 |
| 7 | 0.0176 | 0.0146 | 0.0176 | 0.0142 | 0.0086 | 0.0040 | 0.0116 |

**Simple Transfer (Total) Requirements Matrix \((N^t)\)**

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.3781 | 0.0149 | 0.0265 | 0.0845 | 0.0076 | 0.0078 | 0.0120 |
| 2 | 0.0243 | 0.0615 | 0.0322 | 0.0704 | 0.0331 | 0.0070 | 0.0151 |
| 3 | 0.0128 | 0.0438 | 0.0076 | 0.0133 | 0.0115 | 0.0130 | 0.0257 |
| 4 | 0.3711 | 0.2184 | 0.4325 | 0.6252 | 0.1264 | 0.1189 | 0.1996 |
| 5 | 0.1795 | 0.1152 | 0.1642 | 0.1884 | 0.1140 | 0.0548 | 0.0851 |
| 6 | 0.3854 | 0.4462 | 0.3441 | 0.4076 | 0.3300 | 0.4166 | 0.3571 |
| 7 | 0.0296 | 0.0292 | 0.0250 | 0.0386 | 0.0383 | 0.0270 | 0.0315 |
TABLE 4. The US Requirements Tables for 1997

Technical Coefficients Matrix \((A)\)

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.2618 | 0.0001 | 0.0015 | 0.0401 | 0.0013 | 0.002 | 0.0008 |
| 2 | 0.0017 | 0.115 | 0.0062 | 0.0306 | 0.0236 | 0.0003 | 0.0036 |
| 3 | 0.0039 | 0.0002 | 0.011 | 0.002 | 0.0052 | 0.006 | 0.0101 |
| 4 | 0.174 | 0.1162 | 0.2372 | 0.3627 | 0.0758 | 0.0583 | 0.0424 |
| 5 | 0.0731 | 0.0643 | 0.0975 | 0.098 | 0.0847 | 0.0288 | 0.0267 |
| 6 | 0.111 | 0.257 | 0.1376 | 0.1232 | 0.2294 | 0.2146 | 0.0902 |
| 7 | 0.0063 | 0.0181 | 0.0086 | 0.0177 | 0.0212 | 0.0169 | 0.0167 |

Simple Direct Requirements Matrix \((N_d)\)

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.3612 | 0.0001 | 0.0015 | 0.0669 | 0.0015 | 0.0026 | 0.0008 |
| 2 | 0.0023 | 0.1315 | 0.0062 | 0.0511 | 0.0267 | 0.0004 | 0.0037 |
| 3 | 0.0054 | 0.0002 | 0.011 | 0.0033 | 0.0059 | 0.0079 | 0.0103 |
| 4 | 0.2400 | 0.1328 | 0.2384 | 0.6053 | 0.0856 | 0.0770 | 0.0434 |
| 5 | 0.1008 | 0.0735 | 0.0980 | 0.1635 | 0.0957 | 0.0380 | 0.0273 |
| 6 | 0.1531 | 0.2938 | 0.1383 | 0.2056 | 0.2592 | 0.2834 | 0.0922 |
| 7 | 0.0087 | 0.0207 | 0.0086 | 0.0295 | 0.0240 | 0.0223 | 0.0171 |

Simple Indirect Requirements Matrix \((N_i)\)

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.0184 | 0.0165 | 0.0254 | 0.0251 | 0.0117 | 0.0085 | 0.0060 |
| 2 | 0.0202 | 0.0117 | 0.0207 | 0.0126 | 0.0102 | 0.0066 | 0.0052 |
| 3 | 0.0040 | 0.0044 | 0.0040 | 0.0040 | 0.0031 | 0.0009 | 0.0014 |
| 4 | 0.1960 | 0.1417 | 0.1980 | 0.0635 | 0.0993 | 0.0603 | 0.0521 |
| 5 | 0.0696 | 0.0536 | 0.0703 | 0.0404 | 0.0342 | 0.0213 | 0.0199 |
| 6 | 0.1721 | 0.1679 | 0.1710 | 0.1557 | 0.1194 | 0.0372 | 0.0599 |
| 7 | 0.0178 | 0.0161 | 0.0176 | 0.0129 | 0.0111 | 0.0044 | 0.0056 |

Simple Transfer (Total) Requirements Matrix \((N_t)\)

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.3796 | 0.0166 | 0.0269 | 0.0921 | 0.0131 | 0.0112 | 0.0068 |
| 2 | 0.0226 | 0.1432 | 0.0269 | 0.0637 | 0.0369 | 0.0070 | 0.0089 |
| 3 | 0.0094 | 0.0047 | 0.0051 | 0.0074 | 0.0089 | 0.0088 | 0.0117 |
| 4 | 0.4360 | 0.2745 | 0.4364 | 0.6687 | 0.1850 | 0.1373 | 0.0954 |
| 5 | 0.1705 | 0.1271 | 0.1683 | 0.2039 | 0.1299 | 0.0594 | 0.0472 |
| 6 | 0.3252 | 0.4617 | 0.3093 | 0.3613 | 0.3785 | 0.3206 | 0.1521 |
| 7 | 0.0264 | 0.0368 | 0.0263 | 0.0425 | 0.0350 | 0.0267 | 0.0226 |
### Table 5. The US Requirements Tables for 1992

#### Technical Coefficients Matrix ($A$)

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.2339 | 0.0003 | 0.0061 | 0.0419 | 0.0005 | 0.0036 | 0.0004 |
| 2 | 0.0018 | 0.1654 | 0.009  | 0.0329 | 0.0274 | 0.0002 | 0.003  |
| 3 | 0.0122 | 0.0117 | 0.0009 | 0.0061 | 0.0208 | 0.0187 | 0.023  |
| 4 | 0.1667 | 0.0787 | 0.2992 | 0.3454 | 0.056  | 0.0673 | 0.0135 |
| 5 | 0.0914 | 0.081  | 0.1061 | 0.1057 | 0.1048 | 0.0427 | 0.016  |
| 6 | 0.09   | 0.1514 | 0.1139 | 0.0712 | 0.1555 | 0.2039 | 0.0134 |
| 7 | 0.0038 | 0.0105 | 0.0048 | 0.0119 | 0.0201 | 0.0112 | 0.0034 |

#### Simple Direct Requirements Matrix ($N^d$)

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.3107 | 0.0004 | 0.0062 | 0.0675 | 0.0006 | 0.0047 | 0.0004 |
| 2 | 0.0024 | 0.2004 | 0.0091 | 0.0530 | 0.0317 | 0.0003 | 0.003  |
| 3 | 0.0162 | 0.0206 | 0.0009 | 0.0098 | 0.0240 | 0.0242 | 0.0231 |
| 4 | 0.2214 | 0.0954 | 0.3035 | 0.5561 | 0.0647 | 0.0872 | 0.0136 |
| 5 | 0.1214 | 0.0982 | 0.1076 | 0.1702 | 0.1211 | 0.0553 | 0.0161 |
| 6 | 0.1196 | 0.1835 | 0.1155 | 0.1146 | 0.1797 | 0.2641 | 0.0135 |
| 7 | 0.0050 | 0.0127 | 0.0049 | 0.0192 | 0.0232 | 0.0145 | 0.0034 |

#### Simple Indirect Requirements Matrix ($N^i$)

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.0177 | 0.0131 | 0.0316 | 0.0220 | 0.0097 | 0.0104 | 0.0026 |
| 2 | 0.0230 | 0.0114 | 0.0291 | 0.0183 | 0.0126 | 0.0095 | 0.0033 |
| 3 | 0.0118 | 0.0105 | 0.0134 | 0.0115 | 0.0072 | 0.0036 | 0.0016 |
| 4 | 0.1741 | 0.1108 | 0.2169 | 0.0539 | 0.0830 | 0.0715 | 0.0255 |
| 5 | 0.0780 | 0.0549 | 0.0933 | 0.0487 | 0.0347 | 0.0313 | 0.0118 |
| 6 | 0.1141 | 0.1017 | 0.1272 | 0.0993 | 0.0738 | 0.0310 | 0.0175 |
| 7 | 0.0118 | 0.0090 | 0.0136 | 0.0081 | 0.0053 | 0.0040 | 0.0016 |

#### Simple Transfer (Total) Requirements Matrix ($N^t$)

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.3284 | 0.0134 | 0.0378 | 0.0894 | 0.0103 | 0.0151 | 0.0030 |
| 2 | 0.0254 | 0.2118 | 0.0383 | 0.0712 | 0.0443 | 0.0098 | 0.0063 |
| 3 | 0.0280 | 0.0311 | 0.0143 | 0.0213 | 0.0312 | 0.0278 | 0.0247 |
| 4 | 0.3956 | 0.2062 | 0.5204 | 0.6100 | 0.1478 | 0.1586 | 0.0391 |
| 5 | 0.1994 | 0.1530 | 0.2009 | 0.2189 | 0.1559 | 0.0866 | 0.0279 |
| 6 | 0.2356 | 0.2851 | 0.2428 | 0.2139 | 0.2555 | 0.2951 | 0.0309 |
| 7 | 0.0168 | 0.0217 | 0.0184 | 0.0272 | 0.0286 | 0.0185 | 0.0050 |
### Technical Coefficients Matrix (A)

|   | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|---|-----|-----|-----|-----|-----|-----|-----|
| 1 | 0.3016 | 0.0002 | 0.0062 | 0.04 | 0.0003 | 0.004 | 0.0003 |
| 2 | 0.0023 | 0.0541 | 0.0093 | 0.0396 | 0.0204 | 0.0006 | 0.004 |
| 3 | 0.0076 | 0.0173 | 0.0006 | 0.0058 | 0.0206 | 0.0217 | 0.0292 |
| 4 | 0.1376 | 0.0715 | 0.2945 | 0.3419 | 0.0533 | 0.0836 | 0.0184 |
| 5 | 0.0834 | 0.0602 | 0.1029 | 0.095 | 0.1144 | 0.0461 | 0.0256 |
| 6 | 0.0933 | 0.1486 | 0.1118 | 0.0558 | 0.1446 | 0.2158 | 0.0123 |
| 7 | 0.0042 | 0.0095 | 0.0045 | 0.0143 | 0.0165 | 0.0124 | 0.0036 |

### Simple Direct Requirements Matrix (Nd)

|   | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|---|-----|-----|-----|-----|-----|-----|-----|
| 1 | 0.4386 | 0.0002 | 0.0063 | 0.0638 | 0.0003 | 0.0053 | 0.0003 |
| 2 | 0.0033 | 0.0577 | 0.0094 | 0.0632 | 0.0238 | 0.0008 | 0.0040 |
| 3 | 0.0111 | 0.0185 | 0.0006 | 0.0093 | 0.0240 | 0.0286 | 0.0294 |
| 4 | 0.2001 | 0.0763 | 0.2986 | 0.5453 | 0.0621 | 0.1100 | 0.0185 |
| 5 | 0.1213 | 0.0642 | 0.1043 | 0.1515 | 0.1332 | 0.0607 | 0.0257 |
| 6 | 0.1357 | 0.1586 | 0.1134 | 0.0890 | 0.1684 | 0.2841 | 0.0124 |
| 7 | 0.0061 | 0.0101 | 0.0046 | 0.0228 | 0.0192 | 0.0163 | 0.0036 |

### Simple Indirect Requirements Matrix (Ni)

|   | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|---|-----|-----|-----|-----|-----|-----|-----|
| 1 | 0.0157 | 0.0115 | 0.0334 | 0.0289 | 0.0101 | 0.0138 | 0.0036 |
| 2 | 0.0201 | 0.0095 | 0.0264 | 0.0085 | 0.0080 | 0.0108 | 0.0036 |
| 3 | 0.0123 | 0.0090 | 0.0133 | 0.0107 | 0.0074 | 0.0041 | 0.0020 |
| 4 | 0.1640 | 0.0941 | 0.2123 | 0.0497 | 0.0827 | 0.0854 | 0.0328 |
| 5 | 0.0722 | 0.0439 | 0.0868 | 0.0456 | 0.0313 | 0.0358 | 0.0151 |
| 6 | 0.1070 | 0.0813 | 0.1146 | 0.0888 | 0.0688 | 0.0323 | 0.0210 |
| 7 | 0.0118 | 0.0074 | 0.0139 | 0.0067 | 0.0056 | 0.0048 | 0.0021 |

### Simple Transfer (Total) Requirements Matrix (Nt)

|   | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|---|-----|-----|-----|-----|-----|-----|-----|
| 1 | 0.4544 | 0.0117 | 0.0397 | 0.0927 | 0.0105 | 0.0191 | 0.0039 |
| 2 | 0.0234 | 0.0672 | 0.0358 | 0.0717 | 0.0318 | 0.0116 | 0.0076 |
| 3 | 0.0234 | 0.0275 | 0.0139 | 0.0200 | 0.0313 | 0.0327 | 0.0314 |
| 4 | 0.3641 | 0.1704 | 0.5109 | 0.5950 | 0.1448 | 0.1955 | 0.0514 |
| 5 | 0.1935 | 0.1081 | 0.1912 | 0.1971 | 0.1646 | 0.0965 | 0.0408 |
| 6 | 0.2427 | 0.2399 | 0.2279 | 0.1778 | 0.2372 | 0.3163 | 0.0333 |
| 7 | 0.0179 | 0.0176 | 0.0184 | 0.0295 | 0.0248 | 0.0211 | 0.0057 |
### Table 7. The US Requirements Tables for 1982

**Technical Coefficients Matrix (A)**

|   | 1    | 2   | 3    | 4    | 5    | 6    | 7    |
|---|------|-----|------|------|------|------|------|
| 1 | 0.2853 | 0.0002 | 0.0019 | 0.0455 | 0.0005 | 0.0045 | 0.0047 |
| 2 | 0.0025 | 0.0467 | 0.0078 | 0.078  | 0.0486 | 0.0006 | 0.0041 |
| 3 | 0.0093 | 0.0247 | 0.001  | 0.005  | 0.0194 | 0.0238 | 0.0221 |
| 4 | 0.1806 | 0.0637 | 0.3201 | 0.3505 | 0.0764 | 0.0877 | 0.0194 |
| 5 | 0.0683 | 0.0414 | 0.0973 | 0.1042 | 0.1255 | 0.0454 | 0.0351 |
| 6 | 0.0816 | 0.1561 | 0.098  | 0.0542 | 0.121  | 0.17   | 0.0071 |
| 7 | 0.0035 | 0.0046 | 0.0043 | 0.015  | 0.0164 | 0.011  | 0.0043 |

**Simple Direct Requirements Matrix (Nd)**

|   | 1    | 2   | 3    | 4    | 5    | 6    | 7    |
|---|------|-----|------|------|------|------|------|
| 1 | 0.4081 | 0.0002 | 0.0019 | 0.0749 | 0.0006 | 0.0056 | 0.0047 |
| 2 | 0.0036 | 0.0499 | 0.0079 | 0.1284 | 0.0577 | 0.0007 | 0.0041 |
| 3 | 0.0133 | 0.0264 | 0.0010 | 0.0082 | 0.0230 | 0.0296 | 0.0222 |
| 4 | 0.2584 | 0.0680 | 0.3249 | 0.5771 | 0.0907 | 0.1090 | 0.0195 |
| 5 | 0.0977 | 0.0442 | 0.0988 | 0.1716 | 0.1489 | 0.0564 | 0.0353 |
| 6 | 0.1167 | 0.1667 | 0.0995 | 0.0892 | 0.1436 | 0.2113 | 0.0071 |
| 7 | 0.0050 | 0.0049 | 0.0044 | 0.0247 | 0.0195 | 0.0137 | 0.0043 |

**Simple Indirect Requirements Matrix (Ni)**

|   | 1    | 2   | 3    | 4    | 5    | 6    | 7    |
|---|------|-----|------|------|------|------|------|
| 1 | 0.0224 | 0.0124 | 0.0386 | 0.0315 | 0.0142 | 0.0155 | 0.0057 |
| 2 | 0.0481 | 0.0178 | 0.0574 | 0.0185 | 0.0193 | 0.0221 | 0.0077 |
| 3 | 0.0126 | 0.0084 | 0.0139 | 0.0140 | 0.0085 | 0.0041 | 0.0023 |
| 4 | 0.2080 | 0.0991 | 0.2442 | 0.0694 | 0.1047 | 0.0954 | 0.0364 |
| 5 | 0.0866 | 0.0435 | 0.0995 | 0.0530 | 0.0378 | 0.0397 | 0.0172 |
| 6 | 0.0942 | 0.0633 | 0.1029 | 0.0920 | 0.0621 | 0.0314 | 0.0189 |
| 7 | 0.0128 | 0.0067 | 0.0145 | 0.0070 | 0.0058 | 0.0050 | 0.0022 |

**Simple Transfer (Total) Requirements Matrix (N t)**

|   | 1    | 2   | 3    | 4    | 5    | 6    | 7    |
|---|------|-----|------|------|------|------|------|
| 1 | 0.4305 | 0.0126 | 0.0405 | 0.1064 | 0.0148 | 0.0211 | 0.0105 |
| 2 | 0.0517 | 0.0676 | 0.0653 | 0.1469 | 0.0770 | 0.0228 | 0.0118 |
| 3 | 0.0259 | 0.0348 | 0.0149 | 0.0222 | 0.0315 | 0.0337 | 0.0246 |
| 4 | 0.4663 | 0.1671 | 0.5691 | 0.6464 | 0.1953 | 0.2044 | 0.0559 |
| 5 | 0.1843 | 0.0877 | 0.1982 | 0.2246 | 0.1868 | 0.0961 | 0.0525 |
| 6 | 0.2109 | 0.2299 | 0.2023 | 0.1812 | 0.2057 | 0.2427 | 0.0261 |
| 7 | 0.0178 | 0.0116 | 0.0189 | 0.0317 | 0.0253 | 0.0187 | 0.0065 |
### Technical Coefficients Matrix (A)

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.2463 | 0.0004 | 0.0035 | 0.047 | 0.0011 | 0.0052 | 0.0007 |
| 2 | 0.0022 | 0.0712 | 0.0093 | 0.0575 | 0.0288 | 0.0005 | 0.0048 |
| 3 | 0.0107 | 0.0375 | 0.0011 | 0.0064 | 0.019  | 0.0293 | 0.0193 |
| 4 | 0.2021 | 0.095  | 0.3722 | 0.3816 | 0.0645 | 0.0885 | 0.0126 |
| 5 | 0.0716 | 0.0478 | 0.1148 | 0.0855 | 0.1058 | 0.0498 | 0.0228 |
| 6 | 0.0864 | 0.0999 | 0.0724 | 0.0482 | 0.12   | 0.151  | 0.0083 |
| 7 | 0.0029 | 0.0049 | 0.0043 | 0.0141 | 0.0137 | 0.009  | 0.004  |

### Simple Direct Requirements Matrix \( (N^d) \)

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.3352 | 0.0004 | 0.0036 | 0.0812 | 0.0013 | 0.0063 | 0.0007 |
| 2 | 0.0030 | 0.0780 | 0.0095 | 0.0994 | 0.0332 | 0.0006 | 0.0048 |
| 3 | 0.0146 | 0.0411 | 0.0011 | 0.0111 | 0.0219 | 0.0355 | 0.0194 |
| 4 | 0.2750 | 0.1040 | 0.3787 | 0.6594 | 0.0744 | 0.1072 | 0.0127 |
| 5 | 0.0974 | 0.0523 | 0.1168 | 0.1477 | 0.1221 | 0.0603 | 0.0229 |
| 6 | 0.1176 | 0.1094 | 0.0737 | 0.0833 | 0.1384 | 0.1830 | 0.0083 |
| 7 | 0.0039 | 0.0054 | 0.0044 | 0.0244 | 0.0158 | 0.0109 | 0.0040 |

### Simple Indirect Requirements Matrix \( (N^i) \)

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.0256 | 0.0164 | 0.0453 | 0.0281 | 0.0129 | 0.0160 | 0.0032 |
| 2 | 0.0381 | 0.0171 | 0.0500 | 0.0143 | 0.0141 | 0.0171 | 0.0043 |
| 3 | 0.0144 | 0.0086 | 0.0165 | 0.0141 | 0.0088 | 0.0045 | 0.0019 |
| 4 | 0.2407 | 0.1333 | 0.3055 | 0.0685 | 0.1029 | 0.1106 | 0.0298 |
| 5 | 0.0783 | 0.0460 | 0.0962 | 0.0448 | 0.0315 | 0.0362 | 0.0114 |
| 6 | 0.0825 | 0.0529 | 0.0942 | 0.0690 | 0.0445 | 0.0288 | 0.0120 |
| 7 | 0.0119 | 0.0065 | 0.0146 | 0.0051 | 0.0046 | 0.0048 | 0.0014 |

### Simple Transfer (Total) Requirements Matrix \( (N^t) \)

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.3608 | 0.0169 | 0.0489 | 0.1093 | 0.0142 | 0.0223 | 0.0039 |
| 2 | 0.0411 | 0.0951 | 0.0595 | 0.1137 | 0.0473 | 0.0177 | 0.0091 |
| 3 | 0.0289 | 0.0497 | 0.0176 | 0.0252 | 0.0308 | 0.0400 | 0.0213 |
| 4 | 0.5157 | 0.2374 | 0.6842 | 0.7279 | 0.1773 | 0.2179 | 0.0425 |
| 5 | 0.1757 | 0.0983 | 0.2130 | 0.1925 | 0.1536 | 0.0966 | 0.0344 |
| 6 | 0.2001 | 0.1623 | 0.1679 | 0.1522 | 0.1830 | 0.2118 | 0.0204 |
| 7 | 0.0158 | 0.0118 | 0.0190 | 0.0295 | 0.0204 | 0.0157 | 0.0054 |
TABLE 9. The US Requirements Tables for 1972

Technical Coefficients Matrix (A)

|    | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|----|------|------|------|------|------|------|------|
| 1  | 0.3141 | 0.0003 | 0.0028 | 0.0542 | 0.001 | 0.0053 | 0.0012 |
| 2  | 0.0019 | 0.0542 | 0.0091 | 0.0296 | 0.016 | 0.0002 | 0.002  |
| 3  | 0.0069 | 0.0282 | 0.0003 | 0.0043 | 0.0156 | 0.0263 | 0.0166 |
| 4  | 0.1436 | 0.0943 | 0.3522 | 0.3771 | 0.0407 | 0.0892 | 0.0078 |
| 5  | 0.0616 | 0.0481 | 0.1043 | 0.0786 | 0.098 | 0.0442 | 0.0202 |
| 6  | 0.0865 | 0.1471 | 0.0686 | 0.0591 | 0.1157 | 0.1621 | 0.0105 |
| 7  | 0.0023 | 0.0063 | 0.0042 | 0.0117 | 0.0118 | 0.0096 | 0.0033 |

Simple Direct Requirements Matrix (Nd)

|    | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|----|------|------|------|------|------|------|------|
| 1  | 0.4684 | 0.0003 | 0.0028 | 0.0916 | 0.0011 | 0.0065 | 0.0012 |
| 2  | 0.0028 | 0.0578 | 0.0092 | 0.0500 | 0.0181 | 0.0002 | 0.0020 |
| 3  | 0.0103 | 0.0301 | 0.0003 | 0.0073 | 0.0177 | 0.0323 | 0.0167 |
| 4  | 0.2142 | 0.1006 | 0.3563 | 0.6375 | 0.0461 | 0.1095 | 0.0078 |
| 5  | 0.0919 | 0.0513 | 0.1055 | 0.1329 | 0.1110 | 0.0542 | 0.0203 |
| 6  | 0.1290 | 0.1569 | 0.0694 | 0.0999 | 0.1310 | 0.1989 | 0.0105 |
| 7  | 0.0034 | 0.0067 | 0.0042 | 0.0198 | 0.0134 | 0.0118 | 0.0033 |

Simple Indirect Requirements Matrix (Ni)

|    | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|----|------|------|------|------|------|------|------|
| 1  | 0.0229 | 0.0200 | 0.0523 | 0.0436 | 0.0114 | 0.0198 | 0.0031 |
| 2  | 0.0154 | 0.0086 | 0.0234 | 0.0063 | 0.0051 | 0.0084 | 0.0017 |
| 3  | 0.0102 | 0.0086 | 0.0115 | 0.0100 | 0.0061 | 0.0029 | 0.0013 |
| 4  | 0.1838 | 0.1255 | 0.2692 | 0.0530 | 0.0726 | 0.0993 | 0.0214 |
| 5  | 0.0585 | 0.0423 | 0.0799 | 0.0375 | 0.0216 | 0.0308 | 0.0084 |
| 6  | 0.0788 | 0.0646 | 0.0948 | 0.0684 | 0.0412 | 0.0283 | 0.0106 |
| 7  | 0.0087 | 0.0061 | 0.0115 | 0.0044 | 0.0034 | 0.0038 | 0.0010 |

Simple Transfer (Total) Requirements Matrix (Nt)

|    | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|----|------|------|------|------|------|------|------|
| 1  | 0.4913 | 0.0204 | 0.0551 | 0.1353 | 0.0125 | 0.0263 | 0.0043 |
| 2  | 0.0183 | 0.0664 | 0.0326 | 0.0563 | 0.0232 | 0.0087 | 0.0037 |
| 3  | 0.0205 | 0.0387 | 0.0118 | 0.0173 | 0.0237 | 0.0352 | 0.0179 |
| 4  | 0.3979 | 0.2260 | 0.6255 | 0.6904 | 0.1187 | 0.2088 | 0.0292 |
| 5  | 0.1503 | 0.0936 | 0.1854 | 0.1703 | 0.1326 | 0.0850 | 0.0286 |
| 6  | 0.2078 | 0.2215 | 0.1642 | 0.1683 | 0.1723 | 0.2272 | 0.0212 |
| 7  | 0.0121 | 0.0128 | 0.0157 | 0.0242 | 0.0167 | 0.0155 | 0.0043 |
### TABLE 10. The US Requirements Tables for 1967

#### Technical Coefficients Matrix (A)

|   | 1       | 2       | 3       | 4       | 5       | 6       | 7       |
|---|---------|---------|---------|---------|---------|---------|---------|
| 1 | 0.3016  | 0       | 0.0025  | 0.0508  | 0.0061  | 0.0022  | 0.0177  |
| 2 | 0.0022  | 0.0515  | 0.009   | 0.028   | 0.0085  | 0.0001  | 0.0044  |
| 3 | 0.0095  | 0.0229  | 0.0003  | 0.0041  | 0.0248  | 0.0088  | 0.0534  |
| 4 | 0.136   | 0.0935  | 0.3634  | 0.3894  | 0.0418  | 0.1577  | 0.2452  |
| 5 | 0.1225  | 0.1726  | 0.1221  | 0.0834  | 0.1432  | 0.1438  | 0.2661  |
| 6 | 0.0278  | 0.0228  | 0.0526  | 0.0325  | 0.0548  | 0.0694  | 0.0703  |
| 7 | 0.0183  | 0.0962  | 0.0088  | 0.0408  | 0.0444  | 0.0286  | 0.0455  |

#### Simple Direct Requirements Matrix (Nd)

|   | 1       | 2       | 3       | 4       | 5       | 6       | 7       |
|---|---------|---------|---------|---------|---------|---------|---------|
| 1 | 0.4423  | 0       | 0.0025  | 0.0899  | 0.0075  | 0.0024  | 0.0195  |
| 2 | 0.0032  | 0.0549  | 0.0091  | 0.0495  | 0.0105  | 0.0001  | 0.0048  |
| 3 | 0.0139  | 0.0244  | 0.0003  | 0.0073  | 0.0305  | 0.0098  | 0.0588  |
| 4 | 0.1994  | 0.0997  | 0.3688  | 0.6890  | 0.0515  | 0.1748  | 0.2702  |
| 5 | 0.1796  | 0.1841  | 0.1239  | 0.1476  | 0.1763  | 0.1594  | 0.2932  |
| 6 | 0.0408  | 0.0243  | 0.0534  | 0.0575  | 0.0675  | 0.0769  | 0.0775  |
| 7 | 0.0268  | 0.1026  | 0.0089  | 0.0722  | 0.0547  | 0.0317  | 0.0501  |

#### Simple Indirect Requirements Matrix (Ni)

|   | 1       | 2       | 3       | 4       | 5       | 6       | 7       |
|---|---------|---------|---------|---------|---------|---------|---------|
| 1 | 0.0241  | 0.0274  | 0.0554  | 0.0439  | 0.0174  | 0.0304  | 0.0548  |
| 2 | 0.0155  | 0.0117  | 0.0236  | 0.0059  | 0.0061  | 0.0132  | 0.0221  |
| 3 | 0.0135  | 0.0170  | 0.0147  | 0.0148  | 0.0058  | 0.0116  | 0.0160  |
| 4 | 0.2132  | 0.1878  | 0.3215  | 0.0803  | 0.1128  | 0.1755  | 0.3035  |
| 5 | 0.1115  | 0.1225  | 0.1384  | 0.1039  | 0.0550  | 0.0904  | 0.1545  |
| 6 | 0.0414  | 0.0434  | 0.0505  | 0.0331  | 0.0192  | 0.0318  | 0.0593  |
| 7 | 0.0371  | 0.0344  | 0.0497  | 0.0262  | 0.0147  | 0.0303  | 0.0518  |

#### Simple Transfer (Total) Requirements Matrix (Nt)

|   | 1       | 2       | 3       | 4       | 5       | 6       | 7       |
|---|---------|---------|---------|---------|---------|---------|---------|
| 1 | 0.4664  | 0.0274  | 0.0579  | 0.1337  | 0.0249  | 0.0328  | 0.0743  |
| 2 | 0.0188  | 0.0666  | 0.0328  | 0.0555  | 0.0166  | 0.0133  | 0.0270  |
| 3 | 0.0274  | 0.0414  | 0.0150  | 0.0221  | 0.0363  | 0.0213  | 0.0748  |
| 4 | 0.4126  | 0.2876  | 0.6903  | 0.7693  | 0.1642  | 0.3504  | 0.5737  |
| 5 | 0.2911  | 0.3066  | 0.2624  | 0.2514  | 0.2313  | 0.2498  | 0.4477  |
| 6 | 0.0822  | 0.0677  | 0.1039  | 0.0906  | 0.0867  | 0.1087  | 0.1368  |
| 7 | 0.0639  | 0.1370  | 0.0586  | 0.0984  | 0.0694  | 0.0620  | 0.1019  |
TABLE 11. The US Requirements Tables for 1963

Technical Coefficients Matrix (A)

|   | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|---|-----|-----|-----|-----|-----|-----|-----|
| 1 | 0.31| 0.0038| 0.0574| 0.0087| 0.0006| 0.0322|
| 2 | 0.0022| 0.0553| 0.0086| 0.0314| 0.0085| 0.0002| 0.0077|
| 3 | 0.0099| 0.0202| 0.0003| 0.003| 0.0315| 0.0093| 0.055|
| 4 | 0.133| 0.0812| 0.37| 0.3983| 0.0401| 0.1496| 0.2574|
| 5 | 0.1054| 0.1935| 0.133| 0.0807| 0.1415| 0.1544| 0.257|
| 6 | 0.0246| 0.0143| 0.0429| 0.0267| 0.0473| 0.0604| 0.0662|
| 7 | 0.0198| 0.0982| 0.0075| 0.036| 0.0439| 0.034| 0.038|

Simple Direct Requirements Matrix (Nd)

|   | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|---|-----|-----|-----|-----|-----|-----|-----|
| 1 | 0.4619| 0.0039| 0.1029| 0.0107| 0.0007| 0.0352|
| 2 | 0.0033| 0.0593| 0.0087| 0.0563| 0.0104| 0.0002| 0.0084|
| 3 | 0.0148| 0.0217| 0.0003| 0.0054| 0.0387| 0.0102| 0.0601|
| 4 | 0.1982| 0.0871| 0.3760| 0.7140| 0.0493| 0.1637| 0.2813|
| 5 | 0.1570| 0.2074| 0.1352| 0.1447| 0.1739| 0.1689| 0.2808|
| 6 | 0.0367| 0.0153| 0.0436| 0.0479| 0.0581| 0.0661| 0.0723|
| 7 | 0.0295| 0.1053| 0.0076| 0.0645| 0.0540| 0.0372| 0.0415|

Simple Indirect Requirements Matrix (Ni)

|   | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|---|-----|-----|-----|-----|-----|-----|-----|
| 1 | 0.0281| 0.0334| 0.0668| 0.0538| 0.0221| 0.0352| 0.0721|
| 2 | 0.0170| 0.0128| 0.0272| 0.0068| 0.0072| 0.0146| 0.0255|
| 3 | 0.0140| 0.0196| 0.0160| 0.0163| 0.0056| 0.0135| 0.0186|
| 4 | 0.2133| 0.1842| 0.3326| 0.0786| 0.1170| 0.1783| 0.3228|
| 5 | 0.1050| 0.1209| 0.1377| 0.1015| 0.0551| 0.0911| 0.1582|
| 6 | 0.0333| 0.0377| 0.0430| 0.0279| 0.0164| 0.0279| 0.0508|
| 7 | 0.0333| 0.0322| 0.0474| 0.0263| 0.0138| 0.0286| 0.0513|

Simple Transfer (Total) Requirements Matrix (Nt)

|   | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|---|-----|-----|-----|-----|-----|-----|-----|
| 1 | 0.4900| 0.0334| 0.0706| 0.1566| 0.0328| 0.0359| 0.1073|
| 2 | 0.0203| 0.0721| 0.0359| 0.0631| 0.0176| 0.0148| 0.0339|
| 3 | 0.0288| 0.0412| 0.0163| 0.0217| 0.0443| 0.0237| 0.0787|
| 4 | 0.4115| 0.2713| 0.7087| 0.7925| 0.1663| 0.3419| 0.6040|
| 5 | 0.2620| 0.3283| 0.2729| 0.2461| 0.2290| 0.2600| 0.4391|
| 6 | 0.0699| 0.0530| 0.0866| 0.0758| 0.0745| 0.0940| 0.1232|
| 7 | 0.0628| 0.1375| 0.0551| 0.0908| 0.0678| 0.0658| 0.0928|
TABLE 12. The US Requirements Tables for 1958

Technical Coefficients Matrix \((A)\)

|   | 1  | 2  | 3  | 4  | 5  | 6  | 7  |
|---|----|----|----|----|----|----|----|
| 1 | 0.2954 | 0  | 0.0034 | 0.0703 | 0.0095 | 0.0003 | 0.0414 |
| 2 | 0.0019 | 0.0616 | 0.0109 | 0.0374 | 0.0077 | 0.0005 | 0.0084 |
| 3 | 0.0116 | 0.0006 | 0.0001 | 0.0021 | 0.0357 | 0.0124 | 0.068 |
| 4 | 0.1158 | 0.0794 | 0.3828 | 0.3802 | 0.0422 | 0.2247 | 0.2935 |
| 5 | 0.1122 | 0.1611 | 0.1368 | 0.0877 | 0.142 | 0.1387 | 0.2581 |
| 6 | 0.023 | 0.0232 | 0.0428 | 0.0245 | 0.0451 | 0.0662 | 0.0714 |
| 7 | 0.0207 | 0.1067 | 0.0055 | 0.0392 | 0.043 | 0.0292 | 0.0414 |

Simple Direct Requirements Matrix \((Nd)\)

|   | 1  | 2  | 3  | 4  | 5  | 6  | 7  |
|---|----|----|----|----|----|----|----|
| 1 | 0.4323 | 0  | 0.0035 | 0.1239 | 0.0117 | 0.0003 | 0.0457 |
| 2 | 0.0028 | 0.0666 | 0.0111 | 0.0659 | 0.0095 | 0.0006 | 0.0093 |
| 3 | 0.0170 | 0.0006 | 0.0001 | 0.0037 | 0.0440 | 0.0137 | 0.0751 |
| 4 | 0.1695 | 0.0858 | 0.3897 | 0.6703 | 0.0520 | 0.2483 | 0.3243 |
| 5 | 0.1642 | 0.1741 | 0.1393 | 0.1546 | 0.1751 | 0.1533 | 0.2852 |
| 6 | 0.0337 | 0.0251 | 0.0436 | 0.0432 | 0.0556 | 0.0732 | 0.0789 |
| 7 | 0.0303 | 0.1153 | 0.0056 | 0.0691 | 0.0530 | 0.0323 | 0.0457 |

Simple Indirect Requirements Matrix \((Ni)\)

|   | 1  | 2  | 3  | 4  | 5  | 6  | 7  |
|---|----|----|----|----|----|----|----|
| 1 | 0.0312 | 0.0399 | 0.0815 | 0.0615 | 0.0275 | 0.0561 | 0.0954 |
| 2 | 0.0180 | 0.0140 | 0.0329 | 0.0081 | 0.0093 | 0.0226 | 0.0336 |
| 3 | 0.0154 | 0.0223 | 0.0178 | 0.0192 | 0.0064 | 0.0159 | 0.0215 |
| 4 | 0.1985 | 0.1864 | 0.3392 | 0.0928 | 0.1318 | 0.2350 | 0.3710 |
| 5 | 0.1034 | 0.1198 | 0.1485 | 0.1090 | 0.0583 | 0.1120 | 0.1819 |
| 6 | 0.0318 | 0.0364 | 0.0437 | 0.0308 | 0.0177 | 0.0319 | 0.0554 |
| 7 | 0.0329 | 0.0322 | 0.0523 | 0.0294 | 0.0153 | 0.0370 | 0.0593 |

Total Requirements \((Nt)\)

|   | 1  | 2  | 3  | 4  | 5  | 6  | 7  |
|---|----|----|----|----|----|----|----|
| 1 | 0.4635 | 0.0399 | 0.0850 | 0.1854 | 0.0393 | 0.0565 | 0.1411 |
| 2 | 0.0208 | 0.0806 | 0.0440 | 0.0740 | 0.0188 | 0.0231 | 0.0429 |
| 3 | 0.0324 | 0.0230 | 0.0179 | 0.0229 | 0.0504 | 0.0296 | 0.0966 |
| 4 | 0.3680 | 0.2722 | 0.7289 | 0.7631 | 0.1838 | 0.4833 | 0.6953 |
| 5 | 0.2676 | 0.2939 | 0.2877 | 0.2636 | 0.2334 | 0.2653 | 0.4671 |
| 6 | 0.0655 | 0.0615 | 0.0873 | 0.0740 | 0.0734 | 0.1050 | 0.1343 |
| 7 | 0.0632 | 0.1475 | 0.0579 | 0.0986 | 0.0683 | 0.0693 | 0.1050 |
Table 13. The US Requirements Tables for 1947

|               | 1     | 2     | 3     | 4     | 5     | 6     | 7     |
|---------------|-------|-------|-------|-------|-------|-------|-------|
| **Technical Coefficients Matrix (A)** | 0.3272 | 0.0031 | 0.1212 | 0.0146 | 0.0053 | 0.0141 |
|               | 0.001 | 0.0835 | 0.0094 | 0.0334 | 0.0098 | 0.0013 | 0.0032 |
|               | 0.0122 | 0.0015 | 0.0002 | 0.0026 | 0.044  | 0.0081 | 0.0639 |
|               | 0.0949 | 0.098  | 0.3795 | 0.3733 | 0.0522 | 0.1709 | 0.2733 |
|               | 0.108  | 0.1091 | 0.1462 | 0.0735 | 0.1212 | 0.1254 | 0.3043 |
|               | 0.0081 | 0.0088 | 0.0436 | 0.0161 | 0.0387 | 0.066  | 0.0433 |
|               | 0.0011 | 0.0046 | 0.007  | 0.0232 | 0.0389 | 0.0338 | 0.0145 |
| **Simple Direct Requirements Matrix (Nd)** | 0.5054 | 0.0032 | 0.2085 | 0.0175 | 0.0058 | 0.0148 |
|               | 0.0015 | 0.0919 | 0.0096 | 0.0575 | 0.0118 | 0.0014 | 0.0034 |
|               | 0.0188 | 0.0017 | 0.0002 | 0.0045 | 0.0528 | 0.0088 | 0.0672 |
|               | 0.1466 | 0.1079 | 0.3866 | 0.6421 | 0.0627 | 0.1864 | 0.2872 |
|               | 0.1668 | 0.1201 | 0.1490 | 0.1264 | 0.1455 | 0.1368 | 0.3198 |
|               | 0.0125 | 0.0097 | 0.0444 | 0.0277 | 0.0465 | 0.0720 | 0.0455 |
|               | 0.0017 | 0.0051 | 0.0071 | 0.0399 | 0.0467 | 0.0369 | 0.0152 |
| **Simple Indirect Requirements Matrix (Ni)** | 0.0392 | 0.0429 | 0.1356 | 0.1077 | 0.0442 | 0.0755 | 0.1267 |
|               | 0.0137 | 0.0091 | 0.0298 | 0.0084 | 0.0088 | 0.0165 | 0.0284 |
|               | 0.0125 | 0.0099 | 0.0186 | 0.0172 | 0.0052 | 0.0153 | 0.0244 |
|               | 0.1447 | 0.1061 | 0.3169 | 0.0780 | 0.1249 | 0.1833 | 0.3144 |
|               | 0.0654 | 0.0511 | 0.1249 | 0.0924 | 0.0551 | 0.0851 | 0.1430 |
|               | 0.0180 | 0.0132 | 0.0299 | 0.0178 | 0.0125 | 0.0188 | 0.0386 |
|               | 0.0174 | 0.0128 | 0.0304 | 0.0116 | 0.0077 | 0.0184 | 0.0357 |
| **Simple Transfer (Total) Requirements Matrix (Nt)** | 0.5446 | 0.0429 | 0.1388 | 0.3161 | 0.0617 | 0.0813 | 0.1415 |
|               | 0.0152 | 0.1010 | 0.0394 | 0.0658 | 0.0206 | 0.0179 | 0.0318 |
|               | 0.0313 | 0.0116 | 0.0188 | 0.0217 | 0.0581 | 0.0241 | 0.0916 |
|               | 0.2912 | 0.2140 | 0.7036 | 0.7201 | 0.1875 | 0.3697 | 0.6016 |
|               | 0.2322 | 0.1712 | 0.2739 | 0.2188 | 0.2007 | 0.2219 | 0.4628 |
|               | 0.0305 | 0.0229 | 0.0743 | 0.0455 | 0.0589 | 0.0908 | 0.0841 |
|               | 0.0191 | 0.0179 | 0.0375 | 0.0515 | 0.0544 | 0.0552 | 0.0510 |
### Technical Coefficients Matrix \((A)\)

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.1074 | 0.0788 | 0.0802 | 0.0209 | 0.0146 | 0.0232 |
| 2 | 0.0032 | 0.2228 | 0.1804 | 0.0294 | 0.0247 | 0.002 | 0.0203 |
| 3 | 0.0214 | 0.0085 | 0 | 0.0115 | 0.0304 | 0.0636 | 0.1379 |
| 4 | 0.1593 | 0.0561 | 0.2187 | 0.2319 | 0.1837 | 0.1724 | 0.3142 |
| 5 | 0.2352 | 0.2575 | 0.0304 | 0.2653 | 0.1129 | 0.0017 | 0.046 |
| 6 | 0.0386 | 0.0029 | 0.0003 | 0.0123 | 0.0217 | 0.0217 | 0.0419 |
| 7 | 0.0594 | 0.1764 | 0.0083 | 0.1762 | 0.2443 | 0.1732 | 0.0426 |

### Simple Direct Requirements Matrix \((N^d)\)

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.1293 | 0.0836 | 0.1433 | 0.0295 | 0.0154 | 0.0307 |
| 2 | 0.0039 | 0.3042 | 0.1915 | 0.0525 | 0.0349 | 0.0021 | 0.0268 |
| 3 | 0.0258 | 0.0116 | 0 | 0.0205 | 0.0429 | 0.0669 | 0.1824 |
| 4 | 0.1917 | 0.0766 | 0.2322 | 0.4144 | 0.2593 | 0.1814 | 0.4155 |
| 5 | 0.2831 | 0.3516 | 0.0323 | 0.4740 | 0.1594 | 0.0018 | 0.0608 |
| 6 | 0.0465 | 0.0040 | 0.0003 | 0.0220 | 0.0306 | 0.0228 | 0.0554 |
| 7 | 0.0715 | 0.2408 | 0.0088 | 0.3148 | 0.3449 | 0.1822 | 0.0563 |

### Simple Indirect Requirements Matrix \((N^i)\)

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.0743 | 0.0845 | 0.0753 | 0.0592 | 0.0864 | 0.0720 | 0.0990 |
| 2 | 0.0726 | 0.0611 | 0.0944 | 0.0806 | 0.0781 | 0.0660 | 0.0972 |
| 3 | 0.0763 | 0.1003 | 0.0615 | 0.1054 | 0.0864 | 0.0636 | 0.0286 |
| 4 | 0.3800 | 0.4388 | 0.3222 | 0.3724 | 0.3742 | 0.3118 | 0.3275 |
| 5 | 0.2508 | 0.2520 | 0.3090 | 0.1855 | 0.2523 | 0.2135 | 0.3078 |
| 6 | 0.0351 | 0.0454 | 0.0331 | 0.0470 | 0.0358 | 0.0292 | 0.0243 |
| 7 | 0.2744 | 0.2747 | 0.2581 | 0.2330 | 0.1731 | 0.1729 | 0.2661 |

### Simple Transfer (Total) Requirements Matrix \((N^t)\)

|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---|------|------|------|------|------|------|------|
| 1 | 0.2035 | 0.0845 | 0.1590 | 0.2025 | 0.1159 | 0.0873 | 0.1297 |
| 2 | 0.0765 | 0.3653 | 0.2859 | 0.1331 | 0.1130 | 0.0681 | 0.1241 |
| 3 | 0.1021 | 0.1119 | 0.0615 | 0.1260 | 0.1293 | 0.1305 | 0.2110 |
| 4 | 0.5717 | 0.5154 | 0.5544 | 0.7868 | 0.6335 | 0.4932 | 0.7431 |
| 5 | 0.5339 | 0.6035 | 0.3412 | 0.6595 | 0.4117 | 0.2153 | 0.3686 |
| 6 | 0.0816 | 0.0494 | 0.0334 | 0.0690 | 0.0664 | 0.0521 | 0.0797 |
| 7 | 0.3459 | 0.5156 | 0.2669 | 0.5478 | 0.5180 | 0.3551 | 0.3225 |
**Table 15. The US Requirements Tables for 1929**

**Technical Coefficients Matrix \( (A) \)**

|     | 1   | 2       | 3       | 4       | 5       | 6       | 7       |
|-----|-----|---------|---------|---------|---------|---------|---------|
| 1   | 0.344 | 0.0057 | 0.0439 | 0.0882 | 0.0168 | 0.0067 | 0.0178 |
| 2   | 0.0009 | 0.0794 | 0.1693 | 0.0516 | 0.0514 | 0.0098 | 0.0169 |
| 3   | 0.0006 | 0.0045 | 0     | 0.0077 | 0.025  | 0     | 0.0718 |
| 4   | 0.0949 | 0.0755 | 0.2443 | 0.259  | 0.2188 | 0.063  | 0.2553 |
| 5   | 0.06  | 0.1971 | 0     | 0.028  | 0.0194 | 0.0143 | 0.0679 |
| 6   | 0.0022 | 0     | 0.0146 | 0.0007 | 0     | 0.0228 | 0.014  |
| 7   | 0.0676 | 0.2903 | 0.1179 | 0.2708 | 0.1991 | 0.5214 | 0     |

**Simple Direct Requirements Matrix \( (N^d) \)**

|     | 1       | 2       | 3       | 4       | 5       | 6       | 7       |
|-----|---------|---------|---------|---------|---------|---------|---------|
| 1   | 0.5441 | 0.0065 | 0.0454 | 0.1437 | 0.0183 | 0.0069 | 0.0216 |
| 2   | 0.0014 | 0.0908 | 0.1752 | 0.0840 | 0.0561 | 0.0101 | 0.0205 |
| 3   | 0.0009 | 0.0051 | 0     | 0.0125 | 0.0273 | 0     | 0.0870 |
| 4   | 0.1501 | 0.0864 | 0.2529 | 0.4219 | 0.2387 | 0.0652 | 0.3095 |
| 5   | 0.0949 | 0.2255 | 0     | 0.0456 | 0.0212 | 0.0148 | 0.0823 |
| 6   | 0.0035 | 0     | 0.0151 | 0.0011 | 0     | 0.0236 | 0.0170 |
| 7   | 0.1069 | 0.3321 | 0.1220 | 0.4411 | 0.2172 | 0.5394 | 0     |

**Simple Indirect Requirements Matrix \( (N^i) \)**

|     | 1       | 2       | 3       | 4       | 5       | 6       | 7       |
|-----|---------|---------|---------|---------|---------|---------|---------|
| 1   | 0.0375 | 0.0830 | 0.1125 | 0.0975 | 0.0939 | 0.0815 | 0.0899 |
| 2   | 0.0353 | 0.0533 | 0.0587 | 0.0340 | 0.0521 | 0.0509 | 0.0546 |
| 3   | 0.0234 | 0.0473 | 0.0351 | 0.0416 | 0.0335 | 0.0554 | 0.0073 |
| 4   | 0.1867 | 0.3208 | 0.2943 | 0.2070 | 0.2689 | 0.3247 | 0.1985 |
| 5   | 0.0357 | 0.0574 | 0.0989 | 0.0761 | 0.0698 | 0.0771 | 0.0383 |
| 6   | 0.0041 | 0.0087 | 0.0065 | 0.0089 | 0.0073 | 0.0110 | 0.0024 |
| 7   | 0.1347 | 0.1833 | 0.2578 | 0.0864 | 0.1874 | 0.1541 | 0.2121 |

**Simple Transfer (Total) Requirements Matrix \( (N^t) \)**

|     | 1       | 2       | 3       | 4       | 5       | 6       | 7       |
|-----|---------|---------|---------|---------|---------|---------|---------|
| 1   | 0.5816 | 0.0895 | 0.1579 | 0.2412 | 0.1123 | 0.0884 | 0.1114 |
| 2   | 0.0367 | 0.1442 | 0.2339 | 0.1181 | 0.1082 | 0.0610 | 0.0751 |
| 3   | 0.0243 | 0.0524 | 0.0351 | 0.0541 | 0.0608 | 0.0554 | 0.0944 |
| 4   | 0.3368 | 0.4072 | 0.5472 | 0.6288 | 0.5076 | 0.3898 | 0.5079 |
| 5   | 0.1306 | 0.2829 | 0.0989 | 0.1217 | 0.0910 | 0.0919 | 0.1206 |
| 6   | 0.0076 | 0.0087 | 0.0217 | 0.0101 | 0.0073 | 0.0346 | 0.0194 |
| 7   | 0.2416 | 0.5155 | 0.3798 | 0.5275 | 0.4047 | 0.6935 | 0.2121 |
### Table 16. The US Requirements Tables for 1919

**Technical Coefficients Matrix \((A)\)**

|     | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|-----|------|------|------|------|------|------|------|
| 1   | 0.4009 | 0.0091 | 0.0802 | 0.1469 | 0.0129 | 0.0079 | 0.0397 |
| 2   | 0.0006 | 0.0716 | 0.198  | 0.038  | 0.0811 | 0.0124 | 0.017 |
| 3   | 0     | 0     | 0     | 0     | 0     | 0     | 0.0581 |
| 4   | 0.0746 | 0.0693 | 0.3189 | 0.2275 | 0.253  | 0.0034 | 0.3359 |
| 5   | 0.0441 | 0.1969 | 0     | 0.0158 | 0.0158 | 0.009  | 0.0622 |
| 6   | 0.0009 | 0     | 0.0274 | 0.0008 | 0     | 0     | 0.0108 |
| 7   | 0.035  | 0.401  | 0.105  | 0.2928 | 0.207  | 0.513  | 0     |

**Simple Direct Requirements Matrix \((Nd)\)**

|     | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|-----|------|------|------|------|------|------|------|
| 1   | 0.7039 | 0.0104 | 0.0824 | 0.2404 | 0.0141 | 0.0080 | 0.0505 |
| 2   | 0.0011 | 0.0821 | 0.2035 | 0.0622 | 0.0886 | 0.0125 | 0.0216 |
| 3   | 0     | 0     | 0     | 0     | 0     | 0     | 0.0739 |
| 4   | 0.1310 | 0.0794 | 0.3278 | 0.3722 | 0.2763 | 0.0034 | 0.4272 |
| 5   | 0.0774 | 0.2256 | 0     | 0.0259 | 0.0173 | 0.0091 | 0.0791 |
| 6   | 0.0016 | 0     | 0.0282 | 0.0013 | 0     | 0     | 0.0137 |
| 7   | 0.0614 | 0.4596 | 0.1079 | 0.4791 | 0.2260 | 0.5175 | 0     |

**Simple Indirect Requirements Matrix \((Ni)\)**

|     | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|-----|------|------|------|------|------|------|------|
| 1   | 0.0518 | 0.1961 | 0.2729 | 0.2061 | 0.1951 | 0.1445 | 0.2080 |
| 2   | 0.0262 | 0.0640 | 0.0649 | 0.0312 | 0.0463 | 0.0427 | 0.0549 |
| 3   | 0.0105 | 0.0403 | 0.0280 | 0.0326 | 0.0274 | 0.0388 | 0     |
| 4   | 0.1567 | 0.4566 | 0.4016 | 0.2640 | 0.3298 | 0.3520 | 0.2268 |
| 5   | 0.0228 | 0.0654 | 0.1121 | 0.0746 | 0.0747 | 0.0660 | 0.0388 |
| 6   | 0.0025 | 0.0092 | 0.0061 | 0.0074 | 0.0065 | 0.0087 | 0.0028 |
| 7   | 0.1191 | 0.2334 | 0.3744 | 0.0817 | 0.2451 | 0.1512 | 0.2719 |

**Simple Transfer (Total) Requirements Matrix \((Nt)\)**

|     | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|-----|------|------|------|------|------|------|------|
| 1   | 0.7557 | 0.2065 | 0.3554 | 0.4464 | 0.2092 | 0.1524 | 0.2585 |
| 2   | 0.0273 | 0.1460 | 0.2684 | 0.0934 | 0.1349 | 0.0552 | 0.0765 |
| 3   | 0.0105 | 0.0403 | 0.0280 | 0.0326 | 0.0274 | 0.0388 | 0.0739 |
| 4   | 0.2876 | 0.5360 | 0.7294 | 0.6362 | 0.6061 | 0.3555 | 0.6541 |
| 5   | 0.1002 | 0.2910 | 0.1121 | 0.1005 | 0.0920 | 0.0751 | 0.1179 |
| 6   | 0.0040 | 0.0092 | 0.0343 | 0.0087 | 0.0065 | 0.0087 | 0.0165 |
| 7   | 0.1805 | 0.6929 | 0.4824 | 0.5608 | 0.4711 | 0.6686 | 0.2719 |
Figure 3. The technical coefficients matrix (A) of the US economy for each year. The sectors are as follows: Agriculture (1), Mining (2), Construction (3), Manufacturing (4), Trade, Transport & Utilities (5), Services (6), and Other (7). (Case study 3.2).
Figure 4. The simple direct requirements matrices ($A^d$) of the US economy for each year. The sectors are as follows: Agriculture (1), Mining (2), Construction (3), Manufacturing (4), Trade, Transport & Utilities (5), Services (6), and Other (7). (Case study 3.2).
FIGURE 5. The simple indirect requirements matrices \(N^3\) of the US economy for each year. The sectors are as follows: Agriculture (1), Mining (2), Construction (3), Manufacturing (4), Trade, Transport & Utilities (5), Services (6), and Other (7). (Case study 3.2).
Figure 6. The simple transfer (total) requirements matrices ($N^T$) of the US economy for each year. The sectors are as follows: Agriculture (1), Mining (2), Construction (3), Manufacturing (4), Trade, Transport & Utilities (5), Services (6), and Other (7). (Case study 3.2).