A Complete Version of the Glauber Theory for Elementary Atom – Target Atom Scattering and Its Approximations

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Abstract. A general formalism of the Glauber theory for elementary atom (EA) – target atom (TA) scattering is developed. A second-order approximation of its complete version is considered in the framework of the optical-model perturbative approach. A ‘potential’ approximation of a second-order optical model is formulated neglecting the excitation effects of the TA. Its accuracy is evaluated within the second-order approximation for the complete version of the Glauber EA–TA scattering theory.

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1. Introduction

The experiment DIRAC, now under way at PS CERNs, aims to observe the relativistic hydrogenlike EA consisting of $\pi^\pm$ and/or $\pi^\mp/K^\mp$ mesons (‘dimesoatoms’/‘hadronic atoms’) in proton-nucleus interactions at 24 GeV and measure their lifetime with a high precision. The interaction of relativistic dimesoatoms with ordinary target atoms is of great importance for this experiment as the accuracy of the EA–TA interaction cross sections is essential for the extraction of the dimesoatoms lifetime. So it has been shown that for the success of the DIRAC experiment it is essential that the excitation and ionization cross sections of the pionium ($A_{2\pi}$) are known to 1% or better. It is found that only using the Glauber cross sections one will be able to reach the desired 1% level in accuracy for the target atom charge $Z > 60$.

Originally, applications of the Glauber theory were confined to high-energy nuclear and fundamental particle physics. At relatively low energies, Glauber’s model for studying the elastic scattering of nucleons has been modified in order to take into account the Coulomb field effect. The paper reviews the use of the conventional Glauber approximation in ‘atomic collisions’, i.e. in intermediate- and high-energy target-inelastic scattering of structureless charged particles by neutral atoms (H, He, and alkali metal target atoms) (see also [6]). Only one of the works reflected these papers is devoted to atom–atom collisions. The authors have attempted to infer the cross section for H(2s) quenching in the H(2s)–He(1s$^2$) interaction within the eikonal approximation using an effective potential. The general formalism has not been developed in this work.

In the series of papers, developed was an eikonal approach for the calculations of the total excitation cross sections of the relativistic hadronic atoms ($A_{2\pi}$, $A_{\pi K}$, $A_{KK}$) interacting with a screened Coulomb potential of ordinary target atom (Ti, Ta, etc.). These eikonal EA excitation cross sections for Coulomb EA–TA interaction takes into account all multiphoton EA–TA exchange processes. However, in this approximation all possible excitations of TA in intermediate and/or final states are completely neglected. In other words, this description is essentially based on the

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† Elementary atoms are the Coulomb bound states of two elementary particles. One can enumerate here $A_{2\pi}$, $A_{\pi\mu}$, $A_{2\mu}$, $A_{\mu e}$, $A_{\mu\pi}$, $A_{2\pi}$, $A_{eK}$, $A_{KK}$.
assumption that Coulomb potential of the TA does not change during the EA–TA interaction. As a result, the calculated cross sections of the coherent interaction $\sigma_{coh}^{tot}$ has been identified with the total cross sections $\sigma^{tot}$ within this approximation.

In the context of the DIRAC experiment, the incoherent part of the total cross sections $\sigma_{incoh}^{tot}$ corresponds to scattering with excitations of the TA electrons from the ground state to all possible exited states. Nuclear excitations of the TA will not considered here, because much larger excitation energy is required that exceeds the energy range relevant for the application to the dimesoatom–atom scattering [9]. Some estimations of the ratio $\sigma_{incoh}^{tot}/\sigma_{coh}^{tot}$ for the EA–TA scattering were performed in papers [11,12]. A detailed study of the target electrons influence on the EA scattering and the evaluation of the incoherent contributions to the coherent scattering in Born approximation is given in [9]. In ref. [11], announced were the simplest results concerning the role of multiphoton exchanges in the incoherent EA–TA interaction.

In this work, the eikonal approximation for target-elastic scattering of EA neglecting all possible excitations of TA has been extended valid for account these effects within the second-order optical model of Glauber theory for EA–TA scattering. The paper is organized as follows. First, we develop a general formalism of the Glauber theory [3] for EA–TA interactions in section 2. Next, we consider a second-order perturbation approximation of its complete version and establish a relationship between the developed formalism and the results of refs. [8] in section 3. In section 4, we formulate a ‘potential’ approximation for the second-order optical model and evaluate its accuracy. We also consider the results of our analysis in the context of the DIRAC experiment. In conclusion, we summarize in short our findings.

This work is devoted to the memory of my friend, the husband, and co-author, a remarkable human being and scientist Alexander Tarasov, who untimely passed away on March 19th, 2011.

2. Complete version of the Glauber theory for EA–TA scattering

The Glauber approximation for the amplitude of the EA–TA interactions may be given by

$$A_{i\rightarrow f} = \frac{i}{2\pi} \int d^2 b \exp(iqb) \Gamma_{i\rightarrow f}(b),$$  \hspace{1cm} (1)

where $q$ is a three dimensional momentum transverse to the target; the integration is carried out over a plane perpendicular to the direction of incidence; $b$ is an impact-parameter vector in this plane; $\Gamma_{i\rightarrow f}(b)$ is the so-called profile function.

We may formulate the problem in a general way by considering the EA scattering on a system of $Z$ constituents with the coordinates $r_1, r_2, \ldots, r_Z$ and the projections on the plane of the impact parameter $s_1, s_2, \ldots, s_Z$. If we introduce the configuration spaces for the EA wave functions $\psi_i(r), \psi_f(r)$ and the wave functions $\Psi_i(\{r_k\}), \Psi_f(\{r_k\})$ of the TA constituents in the initial $i, I$ and the final $f, F$ states, we may write the profile function as

$$\Gamma_{i\rightarrow f}(b) = \int d^3 r \psi^*_f(r) \psi_i(r) \prod_{k=1}^Z d^3 r_k \Psi^*_k(\{r_k\}) \Psi_k(\{r_k\}) \times (1 - S(b, s, \{s_k\}),$$  \hspace{1cm} (2)

with the interaction operator

$$1 - S(b, s, \{s_k\}) = 1 - \exp[i\Phi(b, s, \{s_k\})]$$  \hspace{1cm} (3)

and the phase-shift function

$$\Phi(b, s, \{s_k\}) = Z\Delta\chi(b, s) - \sum_{k=1}^Z \Delta\chi(b - s_k, s),$$  \hspace{1cm} (4)

where the EA constituents phase-shift difference $\Delta\chi(b, s)$ one can represent as follows:

$$\Delta\chi(b, s) = \frac{\alpha}{\beta} \int_{-\infty}^{\infty} dz \left[ |R + r/2|^{-1} - |R - r/2|^{-1} \right],$$  \hspace{1cm} (5)

‡ For the energy range relevant to the dimesoatom–atom scattering, $r_k (k \in 1, Z)$ is the position vector of a TA electron.
\[ \mathbf{R} = (b, z), \quad \mathbf{r} = (s, z), \quad \mathbf{r}_k = (s_k, z_k). \]

Here, \( Z \) denotes the TA nuclear charge, \( \alpha \) is a fine structure constant, \( \beta = v/c = 1, v \) is the EA velocity of the in the laboratory frame, \( z \) is a direction of incidence, \( \mathbf{R} \) is a radius-vector from the center mass of the target atom to the EA center mass, \( \mathbf{r} \) is a radius-vector from one EA constituent to another.

The amplitude \( \langle \Phi \rangle \) is normalized by the relations:
\[
4\pi \text{Im} A_{i+I\to f+F}(0) = \sigma_{\text{tot}}(i, I), \quad |A_{i+I\to f+F}(q)|^2 = d\sigma_{i+I\to f+F}/dq_{\perp},
\]
where
\[
\sigma_{\text{tot}}(i, I) = \sigma_{\text{coh}}(i, I) + \sigma_{\text{incoh}}(i, I) = \sum_f \sum_F \sigma_{i+I\to f+F},
\]
\[
\sigma_{\text{coh}}(i, I) = \sum_f \sigma_{i+I\to f+I}, \quad \sigma_{\text{incoh}}(i, I) = \sum_{f \neq I} \sigma_{i+\perp\to f+I},
\]
\[
\sigma_{i+I\to f+F} = \int d^2q \, d\sigma_{i+I\to f+F}/dq_{\perp}.
\]

To find the total cross sections for all types of collisions in which EA and TA begins in the states \( i \) and \( I \), we must sum the partial cross sections in \( \langle 8 \rangle \) and \( \langle 9 \rangle \) over all states \( f \) and \( F \). The summation is easily carried out by using the completeness relations:
\[
\sum_f \psi_f(r)\psi_f^*(r') = \delta(r - r'),
\]
\[
\sum_F \Psi_F([\{r_k\}])\bar{\Psi}_F([\{r'_k\}]) = \prod_{k=1}^{Z} \delta(r_k - r'_k).
\]

Taking into account the expression
\[
\frac{1}{2\pi} \int d^2q_1 A_{i_1+i_2\to f+I}(q_1)A_{i_1+i_2\to f+I}(q_1 + q) = -i \left[ A_{i_1+i_2\to I}(q) - A_{i_1+i_2\to I}(q) \right]
\]
and entering the abbreviation \( S \equiv \exp[i\Phi] \), we find:
\[
\sigma_{\text{tot}}(i, I) = 2\text{Re} \int d^2b \left\langle 1 - \langle S \rangle \right\rangle,
\]
\[
\sigma_{\text{coh}}(i, I) = \int d^2b \left\langle 1 - 2\text{Re} \langle S \rangle + |\langle S \rangle|^2 \right\rangle,
\]
\[
\sigma_{\text{incoh}}(i, I) = \int d^2b \left\langle 1 - |\langle S \rangle|^2 \right\rangle,
\]
where the double brackets \( \langle \langle \rangle \rangle \) signify that an overage is taken over all the configurations of EA and TA in \( i \)-th and \( I \)-th states.

In doing so, the following expressions take place for the EA form factor
\[
\left\langle \tilde{f} \right\rangle = \int d^3r \, |\psi_i(r)|^2 \tilde{f}(r)
\]
and the correlated form factor of the TA
\[
\left\langle \langle \tilde{F} \rangle \right\rangle = \int \prod_{k=1}^{Z} d^3r_k \, |\Psi_F([r_k])|^2 \tilde{F}([r_k]).
\]

The relation defining the \( \langle S \rangle \) can be written in an abbreviated form as
\[
\langle S \rangle = \exp(i\Phi),
\]
where \( \Phi(b, s) \) is an effective (‘optical’) phase-shift function in the optical model of the full version of the Glauber theory.
3. Second-order approximation of the complete version

In the so-called optical-model perturbative approximation of the Glauber theory, the expansion, which we find for the 'optical' phase-shift function $\Phi(b)$, may be written as

$$\tilde{\Phi}(b, s) = \sum_{n=1}^{\infty} \frac{i^{n-1}}{n!} \Phi_n,$$

(20)

where

$$\Phi_1 = \langle (\Phi) \rangle, \quad \Phi_2 = \langle (\Phi - \Phi_1)^2 \rangle, \quad \Phi_3 = \langle (\Phi - \Phi_1)^3 \rangle, \quad \Phi_4 = \langle (\Phi - \Phi_1)^4 \rangle - 3\Phi_2^2, \ldots$$

(21)

$$\Phi_n \sim Z \left( \frac{\alpha}{\beta} \right)^n.$$

The first order of the expression for $\tilde{\Phi}(b, s)$ is the double average of the phase-shift function $\Phi(b, s, \{s_k\})$ over all configurations of EA and TA in $i$-th and $I$-th states. The second-order term of $\tilde{\Phi}(b, s)$ is purely absorptive in character and is equal in order of magnitude to the $Z\alpha^2$.

When the remainder term $R_3(b, s)$ in the series (20) is much smaller than unity

$$R_3(b, s) = \sum_{n=3}^{\infty} \frac{i^{n-1}}{n!} \Phi_n \ll 1,$$

(22)

it seems natural to neglect them and consider the following approximation:

$$\tilde{\Phi}(b, s) \approx \Phi_1(b, s) + \frac{i}{2} \Phi_2(b, s).$$

(23)

The last term in (23) corresponds to the incoherent scattering.

In order to consider the electron correlations in the TA ground state, it is useful to define inclusive densities. They may be defined by integrating over the remaining coordinates

$$\rho_{Z-1}(r_1, \ldots, r_{Z-1}) \equiv \int d^3r_Z \rho_Z(r_1, \ldots, r_Z),$$

(24)

with

$$\rho_Z(r_1, \ldots, r_Z) = |\Psi_0(r_1, \ldots, r_Z)|^2.$$

(25)

Each of these functions is symmetric and normalized to unity when integrated over all of its coordinates.

In particular, the two-particle and one-particle densities may be represented as

$$\rho_2(r_1, r_2) = \int d^3r_3 \rho_3(r_1, r_2, r_3), \quad \rho_1(r_1) = \int d^3r_2 \rho_2(r_1, r_2).$$

(26)

The two-particle density $\rho_2(r_1, r_2)$ describes the probability of finding any two of the properly antisymmetrized electrons at positions $r_1$ and $r_2$.

By taking a Fourier transform, we can discuss the one-particle $F_1(q)$ and two-particle $F_2(q_1, q_2)$ TA form factors, which are just expectation values of special one-particle and two-particle operators:

$$F_1(q) \equiv \int d^3r_1 e^{i qr_1} \rho_1(r_1),$$

(27)

$$F_2(q_1, q_2) \equiv \int d^3r_1 d^3r_2 e^{i q_1 r_1 - i q_2 r_2} \rho_2(r_1, r_2).$$

(28)

All the many-particle densities can be expressed in terms of one-particle static and transition densities. So, using the canonical anticommutation relations and a Fourier transform, we may immediately establish the following relations for the correlation term $W(q_1, q_2)$:

$$W(q_1, q_2) = F_1(q_1 - q_2) - F_1(q_1) F_1(q_2)$$

$$+ (Z - 1)[F_2(q_1, q_2) - F_1(q_1) F_1(q_2)]$$

(29)

$$W(q, q) \equiv F_{incoh}(q).$$

(30)
Finally, putting $b_\pm = b \pm s/2$, we express the quantities $\Phi_1(b, s)$ and $\Phi_2(b, s)$ as:

\[
\Phi_1 = \frac{2Z\alpha}{\beta^2} \int \frac{d^2q}{q^2} \left( e^{iqb_+} - e^{iqb_-} \right) \left[ 1 - \bar{F}_1(q) \right],
\]

\[
\Phi_2 = 4Z\alpha^2 \beta^2 \int \frac{d^2q_1}{q_1^2} \frac{d^2q_2}{q_2^2} \left( e^{iq_1b_+} - e^{iq_1b_-} \right) \left( e^{-iq_2b_+} - e^{-iq_2b_-} \right) \times W(q_1, q_2),
\]

with (20)–(30).

Accounting the relations

\[
\sigma_{\text{coh}}^{\text{tot}}(i, I) = \langle \sigma_{\text{coh}}^{\text{tot}}(s) \rangle, \quad \sigma_{\text{incoh}}^{\text{tot}}(i, I) = \langle \sigma_{\text{incoh}}^{\text{tot}}(s) \rangle,
\]

we may also find the following expressions for all type of the ‘dipole total cross sections’ $\sigma_{\text{coh(incoh)}}^{\text{tot}}(s)$, which depend only on the properties of the target material:

\[
\sigma_{\text{tot}}^{\text{coh}}(s) = 2 \int d^2b \left( 1 - \cos \Phi_1 e^{-\Phi_2/2} \right),
\]

\[
\sigma_{\text{tot}}^{\text{coh}}(s) = \int d^2b \left( 1 + 2 \cos \Phi_1 e^{-\Phi_2/2} + e^{-\Phi_2} \right),
\]

\[
\sigma_{\text{tot}}^{\text{incoh}}(s) = \int d^2b \left( 1 - e^{-\Phi_2/2} \right).
\]

In terms of the interaction operators $\Gamma_{\text{coh(incoh)}}(b, s)$, the simplest results concerning the total cross sections of the EA–TA interactions

\[
\sigma_{\text{coh}}^{\text{tot}} = \sigma_{\text{coh}}^{\text{tot}} + \sigma_{\text{incoh}}^{\text{tot}},
\]

are the following:

\[
\sigma_{\text{coh(incoh)}}^{\text{tot}} = \int d^3r |\Psi_{i}(r)|^2 d^2b \Gamma_{\text{coh(incoh)}}(b, s),
\]

where $\sigma_{\text{coh(incoh)}}^{\text{tot}}$ are the total cross sections of EA–TA interaction without or with excitation of the target atom. In [33], we applied the abbreviation $\int \prod_{k=1}^{Z} d^3r_k |\Psi_I(\{r_k\})|^2 \equiv \int d^3r |\Psi_I(r)|^2$

and operators who are looking like

\[
\Gamma_{\text{coh}}(b, s) = 1 - 2 \cos [\Phi_1(b, s)] \exp [-\Phi_2(b, s)/2] + \exp [-\Phi_2(b, s)],
\]

\[
\Gamma_{\text{incoh}}(b, s) = 1 - \exp [-\Phi_2(b, s)].
\]

In the above equations, the functions $\Phi_1(b, s)$ and $\Phi_2(b, s)$ are given by (31) and (32). The phase-shift function $\Phi_2$ accounts TA excitations both in the intermediate and in the final states. If one puts $\Phi_2 = 0$, then the expressions (33)–(36) can be reduced to the corresponding relations of refs. [35]. In particular, $\sigma_{\text{incoh}}^{\text{tot}} = 0$ in this limit.

4. ‘Potential’ approximation of the second-order optical model and its accuracy

The eikonal approximation for EA–TA scattering neglecting effects of the intermediate excitations of TA (‘potential’ approximation) can be represented as follows:

\[
\left[ \sigma_{\text{incoh}}^{\text{tot}}(i, I) \right]_{\text{pot}} \approx 0, \quad \left[ \sigma_{\text{tot}}^{\text{tot}}(i, I) \right]_{\text{pot}} \approx \left[ \sigma_{\text{coh}}^{\text{tot}}(i, I) \right]_{\text{pot}}.
\]

Let us define the absolute accuracy of this approximation as

\[
\Delta \sigma_{\text{coh}}(i, I) \equiv \sigma_{\text{coh}}^{\text{tot}}(i, I) - \left[ \sigma_{\text{coh}}^{\text{tot}}(i, I) \right]_{\text{pot}} = \left( \Delta \sigma_{\text{coh}}^{\text{tot}}(s) \right),
\]

with

\[
\sigma_{\text{coh}}^{\text{tot}} = \sigma_{\text{coh}}^{\text{tot}} - \sigma_{\text{coh}}^{\text{tot}} = \sum_f \sum_{F\neq I} \sigma_{I+I \rightarrow F+F} - \sum_f \sum_{F\neq I} \sigma_{i+I \rightarrow f+F},
\]
Then, within the second-order perturbation theory, we get the following expression for its calculation:
\[
\Delta \sigma_{\text{coh}}^{\text{tot}}(s) = \sigma_{\text{coh}}^{\text{tot}}(s) - \left[ \sigma_{\text{coh}}^{\text{tot}}(s) \right]_{\text{pot}} = \int d^2 b \left[ e^{-\Phi_2} - 1 + 2(1 - \cos \Phi_1)e^{-\Phi_2/2} \right].
\] (43)

Here, the phase-shift functions \( \Phi_1 \) and \( \Phi_2 \) are defined by \((31)\) and \((32)\).

To estimate the other corrections, we will use the evaluation formulae given bellow:

\[
\int \Phi_1^2(b, s) \, d^2 b \sim (Z \alpha)^2 s^2 L , \quad \int \Phi_1^{2k}(b, s) \, d^2 b \sim (Z \alpha)^{2k} s^2 ;
\] (44)

\[
\int \Phi_2(b, s) \, d^2 b \sim (Z \alpha^2) s^2 L , \quad \int \Phi_2^2(b, s) \, d^2 b \sim (Z \alpha^2)^2 s^4 L^2 ;
\] (45)

\[
\int \Phi_3^2(b, s) \, \Phi_2(b, s) \, d^2 b \sim (Z \alpha^3)^2 s^4 L^2 ,
\]

\[
\int \Phi_3^{2k}(b, s) \, \Phi_2(b, s) \, d^2 b \sim (Z \alpha)^{2k} (Z \alpha^2) s^4 L^2 ;
\] (46)

with

\[
L = \ln \frac{R_+^2}{s^2} , \quad R_+ = R + \frac{r}{2} , \quad k \geq 1 .
\] (47)

Using the definition

\[
\bar{L} = \ln \frac{R_+^2}{\langle s^2 \rangle}
\]

and the evaluation formulae \((45)\), we find the following relation between the total cross sections of incoherent scattering in Glauber and Born approximations:

\[
\sigma_{\text{incoh}}^{\text{tot}} = \left[ \sigma_{\text{incoh}}^{\text{tot}} \right]_{\text{Born}} \left[ 1 + O \left( Z \alpha^2 \frac{\langle s^2 \rangle}{R_+^2} \bar{L} \right) \right],
\] (48)

where

\[
\left[ \sigma_{\text{incoh}}^{\text{tot}} \right]_{\text{Born}} = \left\langle \int d^2 b \, \Phi_2(b, s) \right\rangle .
\] (49)

The difference between the first-order and second-order total cross sections of the incoherent scattering normalized to the first-order cross section reads:

\[
\frac{\Delta \sigma_{\text{incoh}}^{\text{tot}}}{\sigma_{\text{incoh}}^{\text{tot}} \left[ \sigma_{\text{incoh}}^{\text{tot}} \right]_{\text{Born}}} \approx 1 = \frac{\Delta \sigma_{\text{incoh}}^{\text{tot}}}{\sigma_{\text{incoh}}^{\text{tot}} \left[ \sigma_{\text{incoh}}^{\text{tot}} \right]_{\text{Born}}} = O \left( Z \alpha^2 \frac{\langle s^2 \rangle}{R_+^2} \bar{L} \right) .
\] (50)

It follows from \((40)\) that the incoherent interactions may be described by the Born approximation with the relative accuracy of order \(Z \alpha^2\). In terms of average radii of the interacting objects, they can be presented as

\[
Z \alpha^2 \frac{\langle r^2 \rangle_{\text{EA}}}{\langle r^2 \rangle_{\text{TA}}} \ln \left( \frac{\langle r^2 \rangle_{\text{TA}}}{\langle r^2 \rangle_{\text{EA}}} \right) .
\] (51)

From \((46)\) follows that the relative correction to the \(\sigma_{\text{coh}}^{\text{tot}}(i, I)\) caused the intermediate incoherent effects is of order

\[
Z^3 \alpha^4 \frac{\langle r^2 \rangle_{\text{EA}}}{\langle r^2 \rangle_{\text{TA}}} \ln \left( \frac{\langle r^2 \rangle_{\text{TA}}}{\langle r^2 \rangle_{\text{EA}}} \right) \ll 1
\] (52)

and can be successfully neglected. The same is true for all partial coherent cross sections. This result indicates that the theory of refs. \([5]\) provides quite an accurate description for the coherent sector of the EA–TA interactions.
For the cross sections of the elastic scattering
\[ \sigma_{i+1\rightarrow i+1}^{el}(i, I) = \int d^2q |A_{i+1\rightarrow i+1}(q)|^2, \]  
we can also obtain the relation to its ‘potential’ approximation:
\[ \sigma_{i+1\rightarrow i+1}^{el} = \sigma_{i+1\rightarrow i+1}^{el, pot} \left( 1 + \frac{1}{Z} \frac{\langle s^2 \rangle}{R^2_+} L \right). \]

The relative accuracy of this approximation can be then estimated by:
\[ \frac{\sigma^{el} - \sigma^{el, pot}}{\sigma^{el, pot}} = \frac{\Delta \sigma_{i+1\rightarrow i+1}^{el}}{\sigma_{i+1\rightarrow i+1}^{el, pot}} \approx \frac{1}{Z} \frac{\langle s^2 \rangle}{R^2_+} L. \]

For the purposes of the DIRAC experiment, the results of the performed analysis can be summarized as follows: (i) for the description of the coherent EA–TA interactions, it is enough to use the simplified version of the Glauber theory neglecting effects of the intermediate TA excitations; (ii) for the description of the incoherent EA–TA interactions, it is enough to use the Born approximation.

5. Conclusion

In this work, the complete version of the Glauber theory for EA–TA scattering accounting all possible excitations of EA and TA in intermediate and/or final states is formulated. The second-order optical model of its complete version is regarded. The accuracy of the ‘potential’ approximation is evaluated within the second-order optical-model. The work represents a natural generalization of the Glauber theory for describing the relativistic elementary atom – ordinary atom scattering. We would like to note that while the theory developed in this work is motivated by concrete experiment, it is also of general interest for high energy physics and atomic physics.

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