The Holography Hypothesis and Pre-Big Bang Cosmology

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(Received)

Abstract

The consequences of holography hypothesis are investigated for the Pre-big-bang string cosmological models. The evolution equations are obtained from the tree level string effective action. It is shown that $S/A$ is bounded by a constant in each case, $S$ being the entropy within the volume bounded by the horizon of area $A$.

PACS number(s): 98.80.Cq, 04.30.Db
The study of the holographic principle has attracted increasing attention in the recent past. In essence, it states that if we consider a macroscopic theory of space and everything inside that region, we can represent it by a boundary theory living on the boundary of that region \([1,2]\). In the context of the entropy of the 4−dimensional black holes, it was argued that all phenomena inside a black hole of size \(V\) can be described by a set of degrees of freedom which live on the surface that bounds \(V\). Furthermore, each unit area, in Planck units, of the surface contains one bit of information if we imagine the surface to be a two-dimensional lattice. Thus the boundary theory, in the lattice picture, is discrete and consequently, the information density is bounded. It has been observed that there is a novel correspondence between theories that live in the bulk and their dual counterparts residing on the boundary through the holography hypothesis. It has been shown that type IIB string theory on the background \(AdS_5 \times S^5\) and \(N\) units of five-form flux on \(S^5\) is dual to 3 + 1 dimensional \(U(N)\) super Yang-Mills theory with 16 real supercharges which reside on the boundary of the AdS space \([3]\). There has been considerable activity \([4–6]\) to investigate various aspects of the aforementioned duality.

Recently, Fischler and Susskind (FS) \([7]\) explored the consequences of the holography principle in yet another unexplored and important direction i.e. the cosmological domain and derived very interesting results. In the cosmological context, the principle implies that the entropy contained within a volume of coordinate size \(R_H\) should not exceed the area of the horizon in Planck units. Therefore, holography principle imposes additional constraints on the cosmological models. First, the consequences of holography were examined \([8]\), where, the energy density of the Universe is dominated by a homogeneous minimally coupled scalar field and later they studied the Kasner’s Universe.

The purpose of this note is to examine stringy cosmological models and the compatibility of such models with holographic principle. For definiteness, we shall consider the pre-big-bang(PBB) scenario \([9]\) which is endowed with many attractive features. The basic ingredient of the PBB cosmology is that the Universe started initially from weak coupling regime with very small curvature. If one assumes homogeneity to start with, then the Universe
undergoes accelerated expansion due to the fact that the dilaton grows with time in the so-called (+)-branch \cite{9}. Thus, a cold, flat, weakly coupled Universe accelerates and expands towards a hot, curved and strongly coupled regime driven by the dilaton and the singularity lies in the future. This super-inflationary growth becomes evident when one works in the string frame metric, the metric appearing in the worldsheet action for a string in the curved target space. The inflationary solution is related to the expanding decelerating solution, in the post-big-bang era (the singularity is in the past), through scale factor duality and time reversal transformation. In the PBB scenario, we need an exit from the super-inflationary phase to the standard non-inflationary domain. Recently, in more general settings, the initial condition of homogeneity could be relaxed \cite{10,11} while the Universe evolves from weak coupling and low curvature regime. The Universe proceeds towards the PBB behavior in a suitable domain of space and it will fill almost whole space. The Universe appears very homogeneous, isotropic and spatially flat within that region. Furthermore, in the special case of homogeneous and isotropic cosmologies, which are not spatially flat, one can obtain explicit solutions \cite{12}. We refer the reader to recent review articles on the subject \cite{13} for the current status \cite{14} of PBB cosmology.

In view of the recent attentions on PBB cosmology, it is worthwhile to study the compatibility of the holographic principle with various solutions in the PBB scenarios.

We shall adopt the Einstein frame description throughout the course of this work. As is well known, the Einstein frame metric and the string frame metric are related through a conformal transformation involving the dilaton. The Einstein frame metric has been used to study several interesting aspects of PBB, especially the scenarios we intend to consider in this note. Furthermore, the Einstein frame metric is used in deriving the bounds as a consequence of the holography hypothesis.

We shall compute the entropy of the Universe within the horizon as follows: first, we determine the entropy per comoving volume, $S^c$ using thermodynamic arguments incorporating the effect of the fluid, namely, dilaton and/or axion; and then we obtain the total entropy, $S$, within the horizon as a product of $S^c$ and comoving volume, $V^c_H$. The horizon is
determined from the condition \( ds^2 = 0 \).

As is well known, when one considers adiabatically expanding (contracting) Universe, the entropy per unit comoving volume remains constant. We recall [15] that the 0-th component of the conservation law, \( T^{\mu}_{;\mu} = 0 \) leads to

\[
\sqrt{g} \frac{dp}{dt} = \frac{d}{dt}(\sqrt{g}(\rho + p)),
\]

where, \( \rho \) and \( p \) are defined in terms \( T^{\mu}_{\mu} \) in the cosmological context and \( g \) is the determinant of the spatial part of the metric. Then it can be shown that the comoving entropy density remains constant in time throughout the PBB phase and can be expressed as

\[
S^c = \frac{(\rho + p)\sqrt{g}}{T},
\]

where, \( T \) is the temperature of the fluid. Moreover, for \( p = \rho \)

\[
\rho = \sigma_f T^2,
\]

where, \( \sigma_f \) can be identified to be “Stefan’s constant” of the fluid and we have set \( \hbar = k_B = 1 \) throughout. We can write \( S^c \) alternatively as,

\[
S^c = 2\frac{\sigma^{1/2}}{l_p}(\rho g)^{1/2},
\]

in terms of the dimensionless parameter \( \sigma \), where, \( \sigma_f = \frac{\sigma}{l_p} \).

Here, we have considered the Universe in PBB regime and assumed that there is no particle production during that era. Let us start with the simplest homogeneous PBB model, viz,

\[
ds^2 = -dt^2 + \sum (t/t_0 - 1)^{2\lambda_a} (dx^a)^2
\]

\[
\phi(x, t) = \phi_0 - \sqrt{2} \sqrt{1 - \sum \lambda_a^2 \ln\left(\frac{t}{t_0} - 1\right)}
\]

Here, \( \{\lambda_a\} \) are independent of \( x \), satisfying

\[
\sum \lambda_a = 1, \quad \sum \lambda_a^2 = \rho^2, \quad 1/3 \leq \rho^2 \leq 1
\]
Note that the first constraint on $\lambda_a$ is the well known Kasner condition. For this case, we get

$$V_H^c = \prod_a (X^a_H).$$  \hfill (8)

where, the horizon at any instant of time $t$, is at $X^a_H$ and is obtained using $X^a(t_0) = 0$ together with the relation $ds^2 = 0$ The relevant expression is

$$X^a_H = \frac{t_0(\frac{t}{t_0} - 1)^{1-\lambda_a}}{1 - \lambda_a}. \hfill (9)$$

The area of the surface bounding that volume is given by

$$A_H = \prod_a (X^a_H)(\frac{t}{t_0} - 1)^{\lambda_a}/3. \hfill (10)$$

Thus we arrive at

$$\frac{S}{A} = \frac{\sigma^{1/2} 1}{l^2_p \cdot 2\sqrt{\pi} \prod(1-\lambda_a)^{1/3}} \hfill (11)$$

. It is interesting to point out the similarity of the ratio $\frac{S}{A}$ with that of black holes. We note that if area is measured in $l^2_p$ units , $\frac{S}{A}$ becomes dimensionless. The Kasner condition and the constraint on $\lambda^2_a$ enable us to express the above ratio as a function of only one $\lambda_a$ and let us denote it as $Y$. Thus

$$\frac{S}{A} = \frac{\sigma^{1/2} 1}{l^2_p \cdot 2\sqrt{\pi} [(1 - Y)((Y)^2 + \frac{(1-\rho^2)}{2})]}^{1/3} \hfill (12)$$

. In order to derive an upper(a lower) bound on the above expression we need to maximize(minimize) the right hand side with respect to $Y$. The desired upper(lower) bound on $\frac{S}{A}$ is derived to be

$$\frac{S}{A} \leq \frac{\sigma^{1/2} 1}{l^2_p \cdot 2\sqrt{\pi} [(11/3 - 3\rho^2 - \frac{(3\rho^2-1)^{3/2}}{3\sqrt{2}})/9]^{1/3}}, \hfill (13)$$

$$\frac{S}{A} \geq \frac{\sigma^{1/2} 1}{l^2_p \cdot 2\sqrt{\pi} [(11/3 - 3\rho^2 + \frac{(3\rho^2-1)^{3/2}}{3\sqrt{2}})/9]^{1/3}} \hfill (14)$$

Note the appearance of the constant prefactor $\frac{\sigma^{1/2} 1}{l^2_p \cdot 2\sqrt{\pi}}$ and therefore $S/A$ is bounded. We then obtain a bound on $\frac{S}{A}$ over a range of values of $\rho^2$. We have plotted the upper and lower bounds of $r = \frac{S}{A}$ against $k = \rho^2$ in Figure.1, where the bound is scaled in the unit of $\frac{\sigma^{1/2}}{l^2_p}$.
Next we consider a scenario proposed by Maharana, Onofri and Veneziano [16]. The starting point is the PBB classical epoch such that the coupling is weak and curvature is low; therefore, one can trust the tree level string effective action and consequently, the equations of motion are well known. Furthermore, the Universe is assumed to be spherically symmetric. It has been conjectured [10,11] that a Universe that gives rise to dilaton-driven inflation, converges in the past, to the Milne metric with constant dilaton. In [16], they studied how a small spherically symmetric lump of energy affects the evolution of the Universe due to classical instability of a perturbed Milne metric. As has been noted, for the spherical symmetric case, the problem gets simplified considerably when one looks at the Einstein equations and the matter field equation. It is possible to obtain analytic asymptotic solutions to the spherically symmetric field equations through the application of the gradient expansion technique. The dilaton, for this case, is given by

$$\phi(\xi, t) = \phi_0(\xi) - \frac{2}{\sqrt{3}}\sqrt{1 - \lambda^2}\ln\left(t/(t_0(\xi)) - 1\right),$$

(15)

and the line element is

$$ds^2 = -dt^2 + \left(\frac{t}{t_0} - 1\right)^{2(1-\lambda)/3}\left[\left(\frac{t}{t_0} - 1\right)^{2\lambda}e^{2\gamma}\,d\xi^2 + e^{2\delta}d\omega^2\right].$$

(16)

It looks as if the general solutions given by the above two equations depend on four functions of space: $\lambda, \gamma, \delta$ and $t_0$. We can absorb $\gamma$ in the redefinition of the coordinate $\xi$, $dr = e^\gamma d\xi$.
and by choosing equal time slices, the spatial dependence of $t_0$ can be removed. Therefore, one is left with two physically meaningful functions of $\xi$ as should be the case following the arguments of [16]. In fact, we set $\gamma = \delta = 0$ and assume $\lambda$ to be independent of $\xi$ in order to consider a simple scenario and then

$$
\text{ds}^2 = -dt^2 + \left(\frac{t}{t_0} - 1\right)^{2(1-\lambda)/3} \left[\left(\frac{t}{t_0} - 1\right)^{2\lambda} d\xi^2 + d\omega^2\right]. \tag{17}
$$

The position of the horizon $\xi_H$ at the time $t$ for this asymptotic metric of the Universe is

$$
\xi_H = \frac{3t_0}{2(1-\lambda)} \left(\frac{t}{t_0} - 1\right)^{2(1-\lambda)/3} \tag{18}
$$

At time $t = t_0$ the horizon is located at $\xi = 0$ or the Universe reaches the concentrated lump configuration [16]. Then, the comoving volume is given by $V_H^c = \int_0^{\xi_H} d\xi f d\omega = 4\pi \xi_H$. The area of the horizon turns out to be

$$
A_H = 4\pi \left(\frac{t}{t_0} - 1\right)^{2(1-\lambda)/3}. \tag{19}
$$

Therefore,

$$
\frac{S}{A} = \frac{\sigma^{1/2}}{l_p^2} \frac{3}{\sqrt{24\pi}} \sqrt{\frac{1 + \lambda}{1 - \lambda}}. \tag{20}
$$

So long as $\lambda$ is less than one $\frac{S}{A}$ is bounded. We may mention in passing that in the numerical simulations of [16] $\lambda$ turned out to be very small.

It is rather tempting to test whether the generalised holographic hypothesis is respected by the ”Dual Case”. It is easy to check, for our choice of the parameters, $\gamma = 0$ and $\delta = 0$, that the momentum and Hamiltonian constraints are satisfied together with the rest of the equations of motion under $\lambda \rightarrow -\lambda$ and $\phi \rightarrow -\phi$. Thus we can compute $\frac{S}{A}$ for the “dual” case and it is given by $\frac{S}{A} = \frac{2^{1/2}}{l_p^2} \frac{3}{\sqrt{24\pi}} \sqrt{\frac{1 + \lambda}{1 + \lambda}}$ satisfying an upper bound. This solution gives post-big-bang branch near singularity.

Now we consider the case where the effect of the antisymmetric tensor field is taken into account besides graviton and the dilaton in the four dimensional action [12]. Recall that in 4-dimensions the field strength $H_{\mu\nu\lambda}$, through Poincare duality is expressed as $H_{\mu\nu\lambda} =$
$e^{2\phi}e_{\mu\nu\lambda\rho}\partial^\rho h$, where $h$ is the axion. We would like to explore whether $\frac{S}{A}$ is bounded or not for the FRW type flat metric in the $(+)$ branch. Thus the metric has the form $ds^2 = -dt^2 + a(t)^2 dx^i dx^i$ in the Einstein frame and the equations of motion are

\begin{align}
(\dot{a})^2 + 2a\ddot{a} &= -\frac{a^2}{4}(((\dot{\phi})^2 + e^{2\phi}(\dot{h})^2), \quad (21) \\
a\ddot{\phi} + 3\dot{a}\dot{\phi} - e^{2\phi}a(\dot{h})^2 &= 0, \quad (22) \\
\frac{d}{dt}((a^3)e^{2\phi}\dot{h}) &= 0. \quad (23)
\end{align}

The Hamiltonian constraint equation is

\begin{equation}
(\dot{a})^2 = \frac{a^2}{12}((\dot{\phi})^2 + e^{2\phi}(\dot{h})^2). \quad (24)
\end{equation}

First two equations in the above describe time evolution of the scale factor and the dilaton. The equation of motion for the axion is the well known axion charge conservation law. We note that dots here denote time derivative. It is rather straightforward to get the first integral of motion,

\begin{equation}
H = \pm \frac{1}{\sqrt{12}}\sqrt{(\dot{\phi})^2 + e^{2\phi}(\dot{h})^2} \quad (25)
\end{equation}

where $H$ is the Hubble parameter and the conservation law yields $\dot{h} = La^{-3}e^{-2\phi}$, $L$ is chosen to be a positive number. Then the comoving volume of the Universe and the area of the horizon are given respectively by

\begin{equation}
V_H^c = \frac{4}{3}\pi(R_H)^3, \quad A = 4\pi(a^2)(R_H)^2 \quad (26)
\end{equation}

where, as usual, the horizon radius, $R_H$, is given by the $ds^2 = 0$ condition. It again turns out that

\begin{equation}
\frac{S}{A} = \frac{\sigma^{1/2}}{l_p^2} \frac{1}{\sqrt{24\pi}} \quad (27)
\end{equation}

where, in the above, $\sigma_d + \sigma_h = \frac{\sigma}{l_p}$. In summary, we have explored the implications of the holographic principle for several interesting cosmological scenarios in PBB cosmology and found that $\frac{S}{A}$ is bounded by constants. We mention that the solutions to the metric and
dilaton considered here follow from the field equations of the tree level effective action. Thus it is assumed that except the graviton and dilaton, all other fields which would arise as a consequence of dimensional reduction of the underlying ten dimensional theory to four dimensions are frozen i.e. carry no spacetime dependence. It is an interesting issue to envisage the scenario where moduli corresponding to internal (compact) dimensions also become time dependent [17]. As the dilaton takes large values (strong coupling domain), the higher derivative terms and higher order stringy correction effects play an important role and it will be essential to take into account these effects in string cosmology [18]. Again it will be nice derive the holographic bound for the case when stringy matter is present as was studied by Gasperini and Veneziano [8]. We hope to present our results in this direction in a future publication. Moreover, it is an interesting issue to investigate how $\frac{\mathcal{S}}{\mathcal{A}}$ is bounded at the end of the string phase (i.e. at the beginning of the FRW phase). Veneziano has obtained an interesting relation [19] for the ratio $\frac{\mathcal{S}}{\mathcal{A}}$ in a general $PBB$ scenario and with assumptions weaker than Fischler and Susskind [7]. It will be interesting to explore the consequences of holography hypothesis along these lines.

We would like to thank Gabriele Veneziano for very useful correspondence and encouragements. We have benefitted from interactions with S. Digal, J. Kamila, R. Roy and S. Sengupta during the course of this work.

Note added: After completion of this work we became aware of the preprint of Dongsu Bak and Soo-Jong-Rey, hep-th/9811008, which also discusses holography in string cosmology.
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