Recent developments in heavy baryon chiral perturbation theory: Selected topics

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I review recent results in baryon chiral perturbation theory, in particular related to pion–nucleon scattering and first systematic attempts to go beyond next-to-leading order in the case of three flavors. New insight into the chiral expansion of the baryon masses and magnetic moments is presented.

1 The two–flavor case: LECs and $\pi N$ scattering

Chiral perturbation theory with nucleons is by now in a fairly mature status. It exploits the chiral symmetry properties of QCD and makes use of methods borrowed from heavy quark effective field theory to consistently deal with the spin–1/2 particles. A consistent power counting in a triple expansion of external momenta, quark masses and inverse powers of the nucleon mass (collectively denoted by the small parameter $p$) allows to investigate many single–nucleon processes as detailed in the recent review [1]. In case of two or more nucleons, there is still active debate about how to formulate the chiral expansion (of the potential, of the inverse amplitude, of the interaction kernel, ...). Although highly interesting, for the sake of brevity I will not further address these topics here. Instead, I will briefly highlight two novel calculations related to the determination of the dimension two low–energy constants and to pion–nucleon scattering. The effective pion–nucleon Lagrangian consists of a string of terms with increasing dimension. At second order, it contains seven a priori unknown coupling constants, the so–called low–energy constants (LECs). While these have been determined before [1], these determinations involved some quantities in which large cancellations appear inducing some sizeable uncertainty. In ref.[2], the four LECs related to pion–nucleon scattering and isoscalar–scalar external sources (as measured e.g. in the $\sigma$–term) were fixed from a set of nine observables which to one–loop order $p^3$ are given entirely by tree and loop diagrams with insertions from the dimension one and two parts of the effective Lagrangian. The fifth LEC is only non–vanishing in case of unequal light quark masses and can thus be estimated from the strong contribution to the neutron–proton mass difference. The other two LECs are given by the anomalous magnetic moments of the proton and the neutron. In that paper, it was also shown that the numerical values of these LECs can indeed be understood from resonance exchange, however, in some cases there is sizeable uncertainty related to certain $\Delta$ couplings. Most interesting is the finding that the LEC $c_1$ reveals the strong pionic correlations coupled to nucleons well known from phenomenological models of the nucleon–nucleon force.

| occurs in                                                                 | determined from                      | value         | res. exch. |
|--------------------------------------------------------------------------|---------------------------------------|---------------|------------|
| $c_1'$ $m_N, \sigma_{\pi N}, \gamma N \to \gamma N$                     | phen. + res.exch.                     | $-1.7 \pm 0.2$| $-1.7^*$   |
| $c_2'$ $\pi N \to \pi (\pi) N, \gamma N \to \gamma N$                  | phen. + res.exch.                     | $6.3 \pm 0.4$ | $3.8 \ldots 7.5$ |
| $c_3'$ $\pi N \to \pi (\pi) N, \gamma N \to \gamma N$                  | phen. + res.exch.                     | $-9.9 \pm 0.5$| $-8.4 \ldots -10.0$ |
| $c_4'$ $\pi N \to \pi (\pi) N$                                          | phen. + res.exch.                     | $6.8$         | $5.8 \ldots 6.9$ |
| $c_5'$ $(m_n-m_p)^{\text{strong}}, \pi^0 N \to \pi^0 N$                  | phenomenology                         | $-0.17$       | $-$        |
| $c_6$ $\kappa_p, \kappa_n$                                              | phen. + res.exch.                     | $5.8$         | $6.1$      |
| $c_7$ $\kappa_p, \kappa_n$                                              | phen. + res.exch.                     | $-3.0$        | $-3.1$    |

Table 1: Values of the dimension two LECs $c_i = 2mc_i$ ($i = 1, \ldots, 5$) as determined in [2]. Also given are the ranges based on resonance saturation. The uncertainties and ranges are discussed in detail in that reference. The * denotes an input quantity.
The one-loop contribution to the $\pi N$-scattering amplitude to order $p^3$ has first been worked out by Mojžiš [6]. Here, I follow ref.[2] in which certain aspects of pion–nucleon scattering have also been addressed. In the center-of-mass frame the $\pi N$-scattering amplitude $\pi^a(q) + N(p) \to \pi^b(q') + N(p')$ takes the following form:

$$T^{ba}_{\pi N} = \delta^{ba} \left[ g^+(\omega, t) + i\vec{\sigma} \cdot (\vec{q} \times \vec{q'}) h^+(\omega, t) \right] + i\epsilon^{bac} \tau^c \left[ g^-(\omega, t) + i\vec{\sigma} \cdot (\vec{q}' \times \vec{q}) h^-(\omega, t) \right]$$

with $\omega = \sqrt{v \cdot q} = \sqrt{v \cdot q'}$ the pion cms energy and $t = (q - q')^2$ the invariant momentum transfer squared. $g^\pm(\omega, t)$ refers to the isoscalar/isovector non-spin-flip amplitude and $h^\pm(\omega, t)$ to the isoscalar/isovector spin-flip amplitude. After renormalization of the pion decay constant $F_\pi$ and the pion-nucleon coupling constant $g_{\pi N}$, one can give the one-loop contributions to the cms amplitudes $g^\pm(\omega, t)$ and $h^\pm(\omega, t)$ at order $p^3$ in closed form, see ref.[2]. The $t$-dependences of the loop-amplitudes $g^\pm(\omega, t)_\text{loop}$ and $h^\pm(\omega, t)_\text{loop}$ show an interesting structure, if one discards terms proportional to $g^a_A$. The $t$-dependence of $h^+(\omega, t)_\text{loop}$ is then given by $(2t - M_\pi^2)/(3M_\pi^2 F_\pi^2) \sigma(t)_\text{loop}$, with $\sigma(t)$ the nucleon scalar form factor. Furthermore, the $t$-dependence of $g^-(\omega, t)_\text{loop}$ becomes equal to $\omega/(2F_\pi^2) G^{V}_E(t)_\text{loop}$, with $G^{V}_E(t)$ the nucleon isovector electric form factor (normalized to unity). Finally, $h^-(\omega, t)_\text{loop}$ has the same $t$-dependence as $-1/(4m F_\pi^2) G^{V}_M(t)_\text{loop}$, with $G^{V}_M(t)$ the nucleon isovector magnetic form factor. The one-loop calculation of these nucleon form factors can be found in [7]. In table 2, I show the predictions for the remaining S, P, D and F-wave threshold parameters which were not used in the fit to determine the LECs. In some cases, contributions from the dimension three Lagrangian appear. The corresponding LECs have been estimated using resonance exchange. In particular, the 10% difference in the P–wave scattering volume $P^-_1$ and $P^+_2$ is a clear indication of chiral loops, because nucleon and $\Delta$ Born terms give the same contribution to these two observables. Note also that the eight D– and F–wave threshold parameters to this order are free of contributions from dimension three and thus uniquely predicted. The overall agreement of the predictions with the existing experimental values is rather satisfactory.

| Obs. | CHPT | Order | Ref. | Exp. value | Ref. | Units |
|------|------|-------|------|------------|------|-------|
| $a^{+}$ | 9.2 ± 0.4 | $p^4$ | [3] | 8.4 ± 10.4 | [4] | $10^{-2} M_\pi^{-1}$ |
| $b^{-}$ | 2.01 | $p^3$ | [2] | 1.32 ± 0.62 | [5] | $10^{-2} M_\pi^{-1}$ |
| $P^-_1$ | $-2.44 \pm 0.13$ | $p^3$ | [2] | $-2.52 \pm 0.03$ | [5] | $M_\pi^{-3}$ |
| $P^+_2$ | $-2.70 \pm 0.12$ | $p^3$ | [2] | $-2.74 \pm 0.03$ | [5] | $M_\pi^{-3}$ |
| $a_{2+}^{-}$ | $-1.83$ | $p^3$ | [2] | $-1.8 \pm 0.3$ | [5] | $10^{-3} M_\pi^{-5}$ |
| $a_{2+}^{+}$ | 2.38 | $p^3$ | [2] | 2.20 ± 0.33 | [5] | $10^{-3} M_\pi^{-5}$ |
| $a_{2+}^{-}$ | 3.21 | $p^3$ | [2] | 3.20 ± 0.13 | [5] | $10^{-3} M_\pi^{-5}$ |
| $a_{2-}^{+}$ | $-0.21$ | $p^3$ | [2] | 0.10 ± 0.15 | [5] | $10^{-3} M_\pi^{-5}$ |
| $a_{2+}^{+}$ | 0.29 | $p^3$ | [2] | 0.43 | [5] | $10^{-3} M_\pi^{-7}$ |
| $a_{2-}^{+}$ | 0.06 | $p^3$ | [2] | 0.15 ± 0.12 | [5] | $10^{-3} M_\pi^{-7}$ |
| $a_{3+}^{+}$ | $-0.20$ | $p^3$ | [2] | $-0.25 \pm 0.02$ | [5] | $10^{-3} M_\pi^{-7}$ |
| $a_{3-}^{+}$ | 0.06 | $p^3$ | [2] | 0.10 ± 0.02 | [5] | $10^{-3} M_\pi^{-7}$ |

Table 2: Threshold parameters predicted by CHPT. The order of the prediction is also given together with the experimental values.
2 Three flavors: General aspects

Heavy baryon CHPT was originally formulated for the three–flavor case [8], however, most of the numerous calculations performed only included the leading and sub–leading non-analytic corrections and some counter terms. Furthermore, it was argued that the spin–3/2 decuplet should be included in the effective field theory since it was found that in many cases the large kaon loop corrections were cancelled by diagrams with intermediate decuplet states. For an early review on these activities, see ref.[9]. To really assess the validity of the approach, it is mandatory to perform calculations that include all terms at the given order and should also be extended to order $p^4$ in the chiral expansion, guided by the experience from the two–flavor sector. Clearly, one can not expect calculations in SU(3) to be as precise as their SU(2) counterparts (for the same order in the chiral expansion), simply because the expansion parameter is significantly larger,

\[
\text{SU}(2): \quad \frac{M_\pi}{4\pi F_\pi} \simeq 0.1, \quad \text{SU}(3): \quad \frac{M_K}{4\pi F_K} \simeq 0.4.
\]

(2)

Similarly, the thresholds in scattering or production processes are much higher in energy. For example, producing neutral pions off protons by real photons has a threshold energy of about $E_\gamma = 145$ MeV, whereas the production of neutral kaons only starts at $E_\gamma \approx 900$ MeV. In what follows, I will present the results of some calculations which try to systematically explore the applicability and limitations of the three–flavor approach. Before discussing these specific examples, let me turn to some more theoretical aspects, i.e. the problem that to one loop in the chiral expansion divergences appear. The divergence structure of the one–loop generating functional to order $p^3$ has been worked out in ref.[10]. It extends previous works by Ecker and Mojžiš [11,12] for the pion–nucleon Lagrangian to the SU(3) case. While the method outlined in [11] can also be used in SU(3), the fact that the baryons are in the adjoint representation of SU(3) whereas the nucleons are in the fundamental representation of SU(2), complicates the calculations considerably. In fig. 1 the various contributions to the one–loop generating functional together with the tree level generating functional at order $\hbar$ are shown. The solid (dashed) double lines represent the baryon (meson) propagator in the presence of external fields. Only if one ensures that the field definitions underlying the mesonic and the baryon-meson Lagrangian match, the divergences are entirely given by the irreducible self–energy ($\Sigma_1$) and the tadpole ($\Sigma_2$) graphs.

![Fig. 1: Contributions to the one–loop generating functional at order $\hbar$.](image)

The explicit calculations to extract the divergences from $\Sigma_{1,2}$ are given in [10]. The generating functional can be renormalized by introducing the following counterterm Lagrangian

\[
\mathcal{L}_{MB}^{(3)\text{ct}} = \frac{1}{(4\pi F_\pi)^2} \sum_{i=1}^{102} d_i \bar{H}_v^{ab}(x) O_i^{bc}(x) H_v^{ca}(x),
\]

with $'a, b, c' \in \text{SU(3)}$ indices, $H_v^{ab}(x)$ denotes the velocity eigenstate with eigenvalue +1 of the heavy baryon field and the field monomials $O_i^{bc}(x)$ are of order $p^3$. The dimensionless LECs...
$d_i$ are decomposed as

$$d_i = d^*_i(\mu) + (4\pi)^2 \beta_i L(\mu),$$

with

$$L(\mu) = \frac{\mu^{d-4}}{(4\pi)^2} \left\{ \frac{1}{d-4} - \frac{1}{2} \left[ \log(4\pi) + 1 - \gamma \right] \right\}. \quad (5)$$

Here, $\mu$ is the scale of dimensional regularization, $\gamma$ the Euler–Mascheroni constant and the $\beta_i$ are dimensionless functions of the axial couplings $F$ and $D$ that cancel the divergences of the one–loop functional. They are tabulated in [10] together with the $O_{bc}^i(x)$. These 102 terms constitute a complete set for the renormalization with off–shell baryons. As long as one is only interested in Greens functions with on–shell baryons, the number of terms can be reduced considerably making use of the baryon equations of motion. Also, many of these terms involve processes with three or more mesons. So for calculations of kaon–nucleon scattering or kaon photoproduction off nucleons, many of these terms do not contribute (or only start to contribute at higher orders). An example will be given below. At present, only few of the finite $d^*_i(\mu)$ have been determined. There are two main directions to extend these investigations. First, a systematic effort to pin down as many LECs as possible is needed and, second, the divergences at order $p^4$ should be extracted. Work along these lines is underway.

3 The scalar sector: Baryon masses and $\sigma$–terms

The scalar sector of baryon CHPT is particularly interesting since it is sensitive to scalar–isoscalar operators and thus directly to the symmetry breaking of QCD. This is most obvious for the pion– and kaon–nucleon $\sigma$–terms, which measure the strength of the scalar quark condensates $\bar{q}q$ in the proton. Here, $q$ is a generic symbol for any one of the light quarks $u, d$ and $s$. Furthermore, the quark mass expansion of the baryon masses allows to give bounds on the ratios of the light quark masses [13]. The most general effective Lagrangian to fourth order in the small parameter $p$ necessary to investigate the scalar sector consists of seven dimension two and seven dimension four terms with LECs plus some additional dimension two terms with fixed coefficients $\sim 1/m$. The dimension two terms with LECs fall into two classes, one related to symmetry breaking and the other are double–derivative meson-baryon vertices. The LECs related to the latter ones can be estimated with some confidence from resonance exchange. A method to estimate the symmetry breakers will be discussed below. The analysis of the octet baryon masses in the framework of chiral perturbation theory already has a long history, see e.g. [14]. However, only recently the results of a calculation including all terms of second order in the light quark masses, $O(m^2_q)$, were presented [15]. The calculations were performed in the isospin limit $m_u = m_d$ and the electromagnetic corrections were neglected. Previous investigations considered mostly the so–called computable corrections of order $m^3_q$ or included some of the finite terms at this order. This, however, contradicts the spirit of CHPT in that all terms at a given order have to be retained. The quark mass expansion of the octet baryon masses takes the form

$$m = \hat{m} + \sum_q B_q m_q + \sum_q C_q m_q^{3/2} + \sum_q D_q m_q^2 + \ldots$$

modulo logs. Here, $\hat{m}$ is the octet mass in the chiral limit of vanishing quark masses and the coefficients $B_q, C_q, D_q$ are state–dependent. Furthermore, they include contributions proportional to some LECs which appear beyond leading order in the effective Lagrangian. In contrast to the $O(p^3)$ calculation, which gives the leading non-analytic terms $\sim m^3_q$, the order $p^4$ one is no longer finite and thus needs renormalization. Intimately connected to the
baryon masses are the \( \sigma \)-terms, which are defined in a mass–independent renormalization scheme via,

\[
\sigma_{\pi N}(t) = \hat{m} \langle p' | \bar{u}u + \bar{d}d | p \rangle, \quad \sigma^{(1)}_{K\pi}(t) = \frac{1}{2} (\hat{m} + m_s) \langle p' | \bar{u}u + \bar{s}s | p \rangle, \\
\sigma^{(2)}_{K\pi}(t) = \frac{1}{2} (\hat{m} + m_s) \langle p' | - \bar{u}u + 2\bar{d}d + \bar{s}s | p \rangle, \tag{7}
\]

with \( |p\rangle \) a proton state with four–momentum \( p \) and \( t = (p' − p)^2 \) the invariant momentum transfer squared. A relation between \( \sigma_{\pi N}(0) \) and the nucleon mass is provided by the Feynman–Hellmann theorem, \( \hat{m}(\partial m_N/\partial \hat{m}) = \sigma_{\pi N}(0) \), with \( \hat{m} \) the average light quark mass. Furthermore, the strangeness fraction \( y \) and \( \hat{\sigma} \) are defined via

\[
y = \frac{2 \langle p|\bar{s}s|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle} = \frac{M_{\pi}^2}{\sigma_{\pi N}(0)} \left( M_{\pi}^2 - \frac{M_{\pi}^2}{2} \right)^{-1} m_s \partial m_N \partial m_s \equiv 1 - \frac{\hat{\sigma}}{\sigma_{\pi N}(0)}. \tag{8}
\]

Let me turn to the calculations presented in [15]. As stated before, there are ten LECs related to symmetry breaking. Since there do not exist enough data to fix all these, they were estimated by means of resonance exchange. To deal with such scalar-isoscalar operators, the standard resonance saturation picture based on tree graphs was extended to include loop diagrams. This is also done in calculations of the deviations from Dashen’s theorem, where one considers loops with photons and heavy (axial)vector mesons [16]. In contrast to the two-flavor case, the scalar mesons in SU(3) can not explain the strength of the symmetry breakers because these mesons are not effective degrees of freedom parametrizing strong pionic/kaonic correlations. To be precise, the dimension two symmetry breakers can be estimated by performing a best fit of the baryon masses based on a \( \mathcal{O}(p^3) \) calculation [17]. For scalar couplings of “natural” size, these values can not even be reproduced within one order of magnitude. One way to solve this problem, although it has its own conceptual difficulties, is to consider besides standard tree graphs with scalar meson exchange also Goldstone boson loops with intermediate baryon resonances (spin–3/2 decuplet and the spin–1/2 (Roper) octet) for the scalar–isoscalar LECs. In [15] a consistent scheme to implement resonance exchange under such circumstances was developed. In particular, it avoids double–counting and abides to the strictures from analyticity. Within the one–loop approximation and to leading order in the resonance masses, the analytic pieces of the pertinent graphs are still divergent, i.e. one is left with three a priori undetermined renormalization constants (\( \beta_\Delta \), \( \delta_\Delta \) and \( \beta_R \)). These have to be determined together with the finite scalar meson–baryon couplings \( F_S \) and \( D_S \) and the octet mass in the chiral limit. Using the baryon masses and the value of \( \sigma_{\pi N}(0) \) as input, one can determine all LECs in terms of one parameter, \( \beta_R \). This parameter can be shown to be bounded and the observables are insensitive to variations of it within its allowed range. Furthermore, it was also demonstrated that the effects of two (and higher) loop diagrams can almost entirely be absorbed in a redefinition of the one loop renormalization parameters. Within this scheme, one finds for the octet baryon mass in the chiral limit \( \hat{m} = 770 \pm 110 \) MeV. The quark mass expansion of the baryon masses, in the notation of Eq.(3), reads

\[
m_N = \hat{m} \left( 1 + 0.34 - 0.35 + 0.24 \right), \\
m_\Lambda = \hat{m} \left( 1 + 0.69 - 0.77 + 0.54 \right), \\
m_\Sigma = \hat{m} \left( 1 + 0.81 - 0.70 + 0.44 \right), \\
m_\Xi = \hat{m} \left( 1 + 1.10 - 1.16 + 0.78 \right). \tag{9}
\]

One observes that there are large cancellations between the second order and the leading non–analytic terms of order \( p^3 \), a well–known effect. The fourth order contribution to the
The nucleon mass is fairly small, whereas it is sizeable for the \( \Lambda \), the \( \Sigma \) and the \( \Xi \). This is partly due to the small value of \( \hat{m} \), e.g. for the \( \Xi \) the leading term in the quark mass expansion gives only about 55\% of the physical mass and the second and third order terms cancel almost completely. From the chiral expansions exhibited in Eq.\( (9) \) one can not yet draw a final conclusion about the rate of convergence in the three–flavor sector of baryon CHPT. Certainly, the breakdown of CHPT claimed in [13] is not observed. On the other hand, the conjecture [18] that only the leading non–analytic corrections (LNAC) \( \sim m_q^{3/2} \) are large and that further terms like the ones \( \sim m_q^2 \) are moderately small, of the order of 100 MeV, is not supported. The chiral expansion of the \( \pi N \) \( \sigma \)-term shows a moderate convergence, i.e. the terms of increasing order become successively smaller,

\[
\sigma_{\pi N}(0) = 58.3 \left( 1 - 0.56 + 0.33 \right) \text{ MeV} = 45 \text{ MeV} .
\]

(10)

Still, the \( p^4 \) contribution is important. For the strangeness fraction \( y \) and \( \hat{\sigma} \), one finds

\[
y = 0.21 \pm 0.20 , \quad \hat{\sigma} = 36 \pm 7 \text{ MeV} . \tag{11}
\]

The value for \( y \) is within the band deduced in [19], \( y = 0.15 \pm 0.10 \) and the value for \( \hat{\sigma} \) compares favourably with Gasser’s estimate, \( \hat{\sigma} = 33 \pm 5 \text{ MeV} \) [13]. Further results concerning the kaon–nucleon \( \sigma \)-terms and some two–loop corrections to the nucleon mass can be found in [15]. Finally, two more comments concerning the difference of the pion–nucleon \( \sigma \)-term at \( t = 0 \) and at the Cheng–Dashen point are in order. First, in [20] it was shown that the remainder \( \Delta_R \) not fixed by chiral symmetry, i.e. the difference between the on–shell \( \pi N \) scattering amplitude \( D^+(0,2M^2_\pi) \) and the scalar form factor \( \sigma_{\pi N}(2M^2_\pi) \), contains no chiral logarithms and vanishes simply as \( M^4_\pi \) in the chiral limit. In addition, an upper limit was reported, \( \Delta_R \leq 2 \text{ MeV} \). Second, in [17] it was shown that a one–loop diagram with an intermediate \( \Delta(1232) \) allows to explain the numerical value of the scalar form factor \( \langle ff \rangle \). The leading \( p^3 \) graph with nucleon intermediate states gives only 7.4 MeV, i.e. half of the empirical value [19]. The \( \Delta \)-contribution, which formally starts at order \( p^3 \), adds another 7.5 MeV. However, it was already stressed in [17] that the spectral function \( \text{Im} \sigma_{\pi N}(t)/t^2 \) is much less peaked around \( t = 4M^2_\pi \) than the empirical one given in [19], see also fig. 2. The \( \Delta \)-contribution enhances the tail of the spectral function at larger \( t \), in contrast to the strong pionic correlations (higher loop effects), which tend to enhance the spectral function close to threshold. Furthermore, the SU(3) calculation of ref.[15] indicates fairly sizeable strangeness effects in this quantity. More detailed higher order calculations are necessary to clarify this issue.

![Fig. 2: Spectral function of the scalar ff. Solid line: \( N \), dashed line: \( N + \Delta \) contribution.](image-url)
4 The tale of the magnetic moments

The magnetic moments of the octet baryons have been measured with high precision over the last decade. On the theoretical side, SU(3) flavor symmetry was first used by Coleman and Glashow [21] to predict seven relations between the eight moments of the p, n, Λ, Σ±, Σ0, Ξ−, Ξ0 and the μΛΣ0 transition moment in terms of two parameters. One of these relations is in fact a consequence of isospin symmetry alone. In modern language, this was a tree level calculation with the lowest order effective chiral meson–baryon Lagrangian of dimension two, see fig. 3a,

\[ L^{(2)}_{MB} = -\frac{i}{4m} b_6^F \langle \bar{B}[S^\mu, S^{\nu}][F^{\mu}_{\nu}, B] \rangle - \frac{i}{4m} b_6^D \langle \bar{B}[S^\mu, S^{\nu}][F^{\mu}_{\nu}, B] \rangle \]  

(12)

with \( S_\mu \) the covariant spin operator, \( F^T_{\mu\nu} = -e(u^\dagger QF_{\mu\nu}u + uQF_{\mu\nu}u^\dagger) \) and \( \langle \ldots \rangle \) denotes the trace in flavor space. Here, \( Q = \text{diag}(2, -1, -1)/3 \) is the quark charge matrix, \( u = \sqrt{U} = \exp(i\phi/2F_\pi) \) and \( F_{\mu\nu} \) the conventional photon field strength tensor. Given the simplicity of this approach, these relations work remarkably well, truly a benchmark success of SU(3).

![Fig. 3: Chiral expansion of the magnetic moments to order \( p^2 \) (a), \( p^3 \) (b) and \( p^4 \) (c-i).](image)

The first loop corrections arise at order \( p^3 \) in the chiral counting [22], see fig 3b. They are given entirely in terms of the lowest order parameters from the dimension one (two) meson–baryon (meson) Lagrangian. It was found that these loop corrections are large for standard values of the two axial couplings \( F \) and \( D \). Caldi and Pagels [22] derived three relations independent of these coupling constants. These are, in fact, in good agreement with the data. However, the deviations from the Coleman–Glashow relations get considerably worse. This fact has some times been taken as an indication for the breakdown of SU(3) CHPT. To draw any such conclusion, a calculation of order \( p^4 \) is mandatory. This was attempted in [23], however, not all terms were accounted for. To be precise, in that calculation the contribution from the graphs corresponding to fig.3c-f were worked out. As pointed out in [24], there are additional one–loop graphs at \( O(p^4) \), namely tadpole graphs with double–derivative meson–baryon vertices (fig. 3g) and diagrams with fixed \( 1/m \) insertions from the dimension two Lagrangian, see fig. 3h,i (these could also be obtained by use of reparametrization invariance). In total, there are seven LECs related to symmetry breaking and three related to scattering processes (the once appearing in the graphs fig. 3g). These latter LECs can be estimated with some accuracy from resonance exchange. The strategy of [24] was to leave the others as free parameters and fit the magnetic moments. One is thus able to investigate the chiral
The Born terms subsume the leading electric and the subleading magnetic couplings of the photon to the nucleon/hyperon and $\gamma^*$ denotes a real ($k^2 = 0$) or virtual ($k^2 < 0$) transition moment. The chiral expansion of the various magnetic moments thus takes the form

$$\mu_B = \mu_B^{(2)} + \mu_B^{(3)} + \mu_B^{(4)} = \mu_B^{(2)} (1 + \varepsilon^{(3)} + \varepsilon^{(4)})$$

with the result (all numbers in nuclear magnetons)

$$\begin{align*}
\mu_p &= 4.69 (1 - 0.57 + 0.16) = 2.79, \\
\mu_n &= -2.85 (1 - 0.36 + 0.03) = -1.91, \\
\mu_{\Sigma^+} &= 4.69 (1 - 0.72 + 0.24) = 2.46, \\
\mu_{\Sigma^-} &= -1.83 (1 - 0.41 + 0.04) = -1.16, \\
\mu_{\Sigma^0} &= 1.43 (1 - 0.93 + 0.38) = 0.65, \\
\mu_{\Lambda} &= -1.43 (1 - 0.93 + 0.35) = -0.61, \\
\mu_{\Xi^0} &= -2.85 (1 - 0.95 + 0.39) = -1.25, \\
\mu_{\Xi^-} &= -1.83 (1 - 0.86 + 0.22) = -0.65, \\
\mu_{\Lambda\Sigma^0} &= 2.47 (1 - 0.57 + 0.18) = 1.51.
\end{align*}$$

In all cases the $O(p^4)$ contribution is smaller than the one from order $p^3$ by at least a factor of two, in most cases even by a factor of three. Like in the case of the baryon masses [15], one finds sizeable cancellations between the leading and next-to-leading order terms making a precise calculation of the $O(p^4)$ terms absolutely necessary. In fact, in all (but one) cases the contribution from the double-derivative terms previously omitted is the largest at order $p^4$, one fings e.g. for the proton $\mu_p^{(4,e)} = 1.93$, $\mu_p^{(4,d+e+f)} = 2.87$, $\mu_p^{(4,g)} = -4.71$ and $\mu_p^{(4,h+i)} = 0.71$ leading to the total of $\mu_p^{(4)} = 0.79$. We predict the transition moment to be $\mu_{\Lambda\Sigma^0} = (1.51 \pm 0.01)\mu_N$ in good agreement with a recent lattice gauge theory result, $\mu_{\Lambda\Sigma^0} = (1.54 \pm 0.09)\mu_N$ [25]. Of course, this is not quite the end of the story. What is certainly missing is a deeper understanding of the numerical values of the symmetry breaking LECs which were used as fit parameters in [24].

## 5 Kaon photoproduction

Pion photo- and electroproduction in the threshold region has been studied intensively over the last few years by Bernard, Kaiser and myself [26] with high precision data coming from MAMI, SAL and NIKHEF. In addition, at the electron stretcher ring ELSA (Bonn) ample kaon photoproduction data have been taken over a wide energy range. Only a small fraction of these data is published [27], the larger fraction is still in the process of being analyzed. It therefore seems timely to study the reactions $\gamma p \rightarrow \Sigma^+ K^0$, $\Lambda K^0$ and $\Sigma^0 K^+$ in the framework of CHPT. This has been done in an exploratory study by Steininger [28], some of the results being published in [29]. Here, I will critically summarize the status of these calculations. The threshold energies for these three processes are 1046, 1048 and 911 MeV, in order. In the threshold region, for energies less than 100 MeV above the respective threshold, it is advantageous to perform a multipole decomposition. It suffices to work with $S$- and $P$-wave multipoles (the size of the $D$-waves has been estimated in [28]). The transition current for the process $\gamma^*(k) + p(p_1) \rightarrow M(q) + B(p_2)$ ($M = K^+, K^0$, $B = \Lambda, \Sigma^0, \Sigma^+$) calculated to $O(p^2)$ can be decomposed into Born, one-loop and counterterm contributions,

$$T = T^{\text{Born}} + T^{\text{1-loop}} + T^{\text{c.t.}},$$

where the Born terms subsume the leading electric and the subleading magnetic couplings of the photon to the nucleon/hyperon and $\gamma^*$ denotes a real ($k^2 = 0$) or virtual ($k^2 < 0$)
The calculation of the Borns term is standard, for charged kaon production the SU(3) generalization of the Kroll–Rudermann term gives the dominant contribution to the electric dipole amplitude. Of particular interest is the observation that the leading P–wave multipoles for $\Sigma^0K^+$ production are very sensitive to the yet unmeasured magnetic moment of the $\Sigma^0$ because it is enhanced by the coupling constant ratio $g_{pK\Lambda}/g_{pK\Sigma^0} = (D + 3F)/\sqrt{3}/(F - D) \simeq -5$. The one loop graphs are also easy to calculate. Two remarks concerning these are in order. First, the SU(3) calculation allows one to investigate the effect of kaon loops on the SU(2) predictions [26]. As expected, it is found that these effects are small, e.g. for neutral pion photoproduction off protons,

$$E_{0+}^{K} = \frac{eFM_\pi^3}{96\pi^2F_\pi^3M_K} = 0.14 \cdot 10^{-3}/M_{\pi^+},$$

which is just 1/10th of the empirical value and considerably smaller than the pion loop contribution. The result eq. (16) is in agreement with the famous decoupling theorem. In the chiral SU(2) limit, that is for a fixed strange quark mass, kaon loop effects must decouple, which means that they are suppressed by inverse powers or logs of the heavy mass, here $M_K$. Eq. (16) clearly shows this behaviour. Second, the loop graphs give rise to the imaginary part of the transition amplitude. Here, one encounters the standard problem of CHPT, namely that at a given order the imaginary parts are given to much less precision than the real ones due to the $O(p^3N)$ suppression for $N$–loop graphs. One finds that these imaginary parts come out much too big, which is caused by the pion loops. This can be understood by considering the recattering graph $\gamma p \to \pi^+ n \to nK^+$. By virtue of the Fermi–Watson theorem, one finds

$$\text{Im} \ E_{0+} = \text{Re} \ E_{0+}^{n+} \cdot a_{\pi K} \cdot \text{PS},$$

where PS denotes the phase space allowed for the virtual pion and $a_{\pi K}$ the $\pi K$ scattering length in the respective channel. Obviously, the initial charged pion photoproduction process is far away from its threshold, out of the range of applicability of CHPT. In [28,29] these multipoles were thus taken from the SAID data base. This is similar to the procedure adopted in the study of double neutral photopionproduction in [26]. Clearly, this needs refinement. At next order in the chiral expansion, one has e.g. additional contributions from $\pi^0$ and $\eta$ rescattering graphs. Note also that to this order, $p^3$, the loop graphs are not finite but need standard renormalization. This can either be done by direct Feynman diagram calculation [28,29] or by using the general method described above (this particular example is worked out in detail in [10]). Finally, there are the counter terms. Altogether, there are 13 various operators with unknown low–energy constants. One combination, $d_1 + d_2$, can be fixed from the nucleon axial radius. This also constrains the yet unmeasured $p \to \Lambda K^+$ transition axial radius,

$$\langle r_A^2 \rangle_{p \to \Lambda K^+} = \frac{3\sqrt{2}}{D + 3F} (d_1 + 3d_2) = 0.23 \ldots 0.70 \text{ fm}^2,$$

To fix the other LECs, $d_3, \ldots, d_{13}$, resonance saturation including the baryon decuplet and the vector meson nonet were used. A detailed account of this procedure can be found in [28]. I now summarize the results for the various final states (photoproduction case).

$K^{0\Sigma^+}$: All LECs are determined by resonance exchange. The total cross section has been calculated for the first 100 MeV above threshold. No data point exists in this range so far, but soon the new ELSA data should be available. The electric dipole amplitude is real at threshold, we have $E_{0+}^{\text{th}}(K^{0\Sigma^+}) = 1.07 \times 10^{-3}/M_\pi$. We also have given a prediction for the recoil polarization at $E_\gamma = 1.26$ GeV (which is the central energy of the lowest bin of the not yet published ELSA data).

$K^{+}\Lambda$: The total cross section from threshold up to 100 MeV above is shown in Fig.4a (left panel). The lowest bin from ELSA [27], $E_\gamma \in [0.96, 1.01]$ GeV, has $\sigma_{\text{tot}} = (1.43 \pm 0.14) \mu$b,
i.e. we slightly underestimate the total cross section. In Fig. 4b (left), we show the predicted recoil polarization $P$ at $E_\gamma = 1.21$ GeV (which is higher in energy than our approach is suited for). Amazingly, the shape and magnitude of the data [27] is well described for forward angles, but comes out on the small side for backward angles. Most isobar models, that give a descent description of the total and differential cross sections also at higher energies, fail to explain this angular dependence of the recoil polarization.

![Fig. 4: Left panel: (a) Total cross section for $\gamma p \rightarrow K^+\Lambda$ (solid line). The S-wave contribution is given by the dashed line. (b) Recoil polarization at $E_\gamma = 1.21$ GeV. Right panel: (a) Total cross section for $\gamma p \rightarrow K^+\Sigma^0$. (b) Recoil polarization at $E_\gamma = 1.26$ GeV. The data are from [27].](image)

$K^+\Sigma^0$: The total cross section is shown in Fig. 4a (right panel). It agrees with the two data points from ELSA [27]. The recoil polarization at $E_\gamma = 1.26$ GeV is shown in Fig. 4b (right). It has the right shape but comes out too small in magnitude. Nevertheless, we observe the important sign difference to the $K^+\Lambda$ case, which is commonly attributed to the different quark spin structure of the $\Lambda$ and the $\Sigma^0$. Notice that this argument is strictly correct for massless quarks only. Here, it stems from an intricate interference of the complex S- and P-wave multipoles. In any case, one would like to have data closer to threshold and with finer energy binning to really test the CHPT scheme.

Clearly, these results should only be considered indicative since we have to include (a) higher order effects (for both the S- and P-waves), (b) higher partial waves and (c) have to get a better handle on the ranges of the various coupling constants. In addition, one would also need more data closer to threshold, i.e. in a region where the method is applicable. However, the results presented are encouraging enough to pursue a more detailed study of these reactions (for real and virtual photons) in the framework of chiral perturbation theory.

6 Two other developments

Here, I will briefly draw the attention towards two other interesting developments, namely the consistent inclusion of the $\Delta(1232)$ resonance in the effective field theory and a coupled channel approach to deal with the three flavor sector.

Inclusion of the $\Delta$: Among all the resonances, the $\Delta(1232)$ plays a particular role for essentially two reasons. First, the $N\Delta$ mass splitting is a small number on the chiral scale of
and second, the couplings of the \( N\Delta \) system to pions and photons are very strong, e.g.
\[
g_{N\Delta\pi} \simeq 2g_{NN\pi}. \tag{20} \]
So one could consider \( \Delta \) as a small parameter. It is, however, important to stress that in the chiral limit, \( \Delta \) stays finite (like \( F_\pi \) and unlike \( M_\pi \)). Inclusion of the spin–3/2 fields like the \( \Delta(1232) \) is therefore based on phenomenological grounds but also supported by large–\( N_c \) arguments since in that limit a mass degeneracy of the spin–1/2 and spin–3/2 ground state particles appears. Recently, Hemmert, Holstein and Kambor [30] proposed a systematic way of including the \( \Delta(1232) \) based on an effective Lagrangian of the type \( L_{\text{eff}}[U, N, \Delta] \) which has a systematic “small scale expansion” in terms of three small parameters (collectively denoted as \( \epsilon \)),
\[
\frac{E_\pi}{\Lambda}, \quad \frac{M_\pi}{\Lambda}, \quad \frac{\Delta}{\Lambda}, \tag{21} \]
with \( \Lambda \in [M_\rho, m_N, 4\pi F_\pi] \). Starting from the relativistic pion–nucleon–\( \Delta \) Lagrangian, one writes the nucleon (\( N \)) and the Rarita–Schwinger (\( \Psi_\mu \)) fields in terms of velocity eigenstates (the nucleon four–momentum is \( p_\mu = mv_\mu + l_\mu \), with \( l_\mu \) a small off–shell momentum, \( v \cdot l \ll m \) and similarly for the \( \Delta(1232) \) [8]),
\[
N = e^{-i\tau\nu \cdot x} (H_v + h_v), \quad \Psi_\mu = e^{-i\tau\nu \cdot x} (T_\mu v + t_\mu v), \tag{22} \]
and integrates out the “small” components \( h_v \) and \( t_\mu v \) by means of the path integral formalism developed in [7]. The corresponding heavy baryon effective field theory in this formalism does not only have a consistent power counting but also \( 1/m \) suppressed vertices with fixed coefficients that are generated correctly (which is much simpler than starting directly with the “large” components and fixing these coefficients via reparametrization invariance). Since the spin–3/2 field is heavier than the nucleon, the residual mass difference \( \Delta \) remains in the spin–3/2 propagator and one therefore has to expand in powers of it to achieve a consistent chiral power counting. The technical details how to do that, in particular how to separate the spin–1/2 components from the spin–3/2 field, are given in [30]. The method has been applied to the electric dipole amplitude in threshold \( \pi^0 \) photoproduction [31] and other applications will be published soon (like the \( E2/M1 \) ratio in the resonance region, the corrections to the \( P \)-wave LETs in neutral pion photoproduction and real as well as virtual Compton scattering).

**SU(3) with coupled channels:** It is well known that in certain reaction channels involving \( K \) and \( \eta \) mesons one encounters resonances very close to the respective thresholds. The most prominent example is the subthreshold \( \Lambda(1405) \) state seen in \( K^-p \) scattering. One might therefore question the applicability of three flavor CHPT as described before. The Munich group has set up a scheme to deal with such situations. It inputs the most general dimension two chiral meson–baryon Lagrangian into a multi–channel \( S \)-matrix [32]. For the case of \( K^-p \) scattering one has e.g. to take into account the pion–hyperon channels (\( \pi\Lambda, \pi\Sigma \)). The couplings between the various channels are determined by the chiral Lagrangian in terms of a few LECs. These build up the potential matrix \( V_{ij} \), which is then iterated to all orders by means of a Lippmann–Schwinger equation,
\[
T_{ij} = V_{ij} = \frac{2}{\pi} \sum_n \int_0^\infty dl \frac{l^2}{k_n^2 - l^2} \left( \frac{\alpha_n^2 + k_n^2}{\alpha_n^2 + l^2} \right)^2 V_{in} T_{nj}, \tag{23} \]
with \( k_n \) the relative meson–baryon momentum in the reaction channel \( n \). The range parameters \( \alpha_n \) are necessary to render the integrals finite, for physical reasons they are of a
size of a few hundred MeV. The resulting multi-channel S–matrix is then exactly unitary in the subspace of the open channels. Clearly, such a unitarization procedure induces some model–dependence and thus goes beyond the strict chiral counting. A best fit to the data in the various \( K^- p \) reaction channels \( (K^- p, K^0 n, \pi^+ \Sigma^-, \pi^0 \Sigma^0, \pi^- \Sigma^+, \pi^0 \Lambda) \) and to threshold branching fractions allows one to fix the various LECs. It is interesting to note that the resulting values are comparable to the ones found by means of resonance saturation [15]. In a similar fashion, one can deal with pion–induced \( \eta \) and \( K \) meson production as well as photopion production of the strange Goldstone bosons [33]. With a few parameters, one is then able to describe a wealth of data for a large energy range (at present, these calculations, however, only include the S–wave amplitudes). The most prominent result of this approach is the finding that the \( \Lambda(1405) \) as well as the \( S_{11}(1535) \) should be interpreted as (instable) meson–baryon boundstates. This is due to the fact that the chiral SU(3) dynamics predicts strongly attractive meson–baryon interaction in the antikaon–nucleon channel with total isospin \( I = 0 \) and in the kaon–\( \Sigma \) channel with \( I = 1/2 \).

7 Short summary and outlook

In my review talk about the status of baryon CHPT A.D. 1994 [34] I had addressed five open problems. Referring to that paper, let me briefly state what has happened in the mean time. (i) can be considered solved [35]. (ii) More precise data have been and are being produced and as discussed, a certain understanding of the LECs in terms of resonance exchange emerges. (iii) Similarly, first fully complete \( O(p^4) \) SU(3) calculations are becoming available supplemented by the coupled channel approach and we will soon be able to make more stringent statements about the precision of the approach. (iv) The \( \Delta \) can now be handled systematically and results of many calculations are expected soon. Finally, (v) is still in a very infant stage despite some impressive calculations like in [36] and some intriguing novel approaches like e.g. in [37]. So clear progress has been made but still lots of work remains to be done.

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