Outage Probability of Dual-Hop Multiple Antenna
AF Systems with Linear Processing in the Presence
of Co-Channel Interference

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Abstract—This paper considers a dual-hop amplify-and-forward (AF) relaying system where the relay is equipped with multiple antennas, while the source and the destination are equipped with a single antenna. Assuming that the relay is subjected to co-channel interference (CCI) and additive white Gaussian noise (AWGN) while the destination is corrupted by AWGN only, we propose three heuristic relay precoding schemes to combat the CCI, namely, 1) Maximum ratio combining/maximal ratio transmission (MRC/MRT), 2) Zero-forcing/MRT (ZF/MRT), 3) Minimum mean-square error/MRT (MMSE/MRT). We derive new exact outage expressions as well as simple high signal-to-noise ratio (SNR) outage approximations for all three schemes. Our findings suggest that both the MRC/MRT and the MMSE/MRT schemes achieve a full diversity of $N$, while the $ZF/MRT$ scheme achieves a diversity order of $N-M$, where $N$ is the number of relay antennas and $M$ is the number of interferers. In addition, we show that the MMSE/MRT scheme always achieves the best outage performance, and the $ZF/MRT$ scheme outperforms the MRC/MRT scheme in the low SNR regime, while becomes inferior to the MRC/MRT scheme in the high SNR regime. Finally, in the large $N$ regime, we show that both the $ZF/MRT$ and MMSE/MRT schemes are capable of completely eliminating the CCI, while perfect interference cancelation is not possible with the MRC/MRT scheme.

Index Terms—Dual-hop relaying, amplify-and-forward, co-channel interference, linear precoding, performance analysis

I. INTRODUCTION

The relay channel was first introduced by Van der Meulen in 1971 [1]. Later, in the seminal work of [2], Cover and El Gamal laid foundations to the information-theoretic understanding of the relay channel. The attention on relay channels was recently rekindled as a means to improve the coverage and link reliability in the context of cooperative wireless communications systems [3]. Various relaying methods have been proposed in the literature [3], among which, the amplify-and-forward (AF) protocol is the most popular one, due to its simplicity and low-cost implementation. In AF systems, the relay simply forwards a scaled version of the received noisy signal from the source to the destination.

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To improve the spectrum efficiency, future cellular systems are likely to adopt a more aggressive frequency reuse strategy, which will inevitably result in an interference-limited communication environment [4]. When the relay technology is adopted in cellular systems [5], the interference environment becomes increasingly complex. Motivated by the need to understand the performance limitations, a number of works investigating the impact of co-channel interference (CCI) on the performance of relay systems have appeared. For example, [6,7] studied the performance of relay selection for AF systems with CCI. Assuming Rayleigh fading channels, [8] examined the outage probability of dual-hop fixed-gain AF relaying systems with an interference-limited destination, and [9] studied the outage probability and the average bit error rate of dual-hop variable-gain AF relaying systems with an interference-limited relay. Later, a scenario considering the more general Nakagami-$m$ fading model was investigated in [10,11]. Moreover, different cases with CCI at both the relay and the destination nodes have been investigated in [12–14]. More recent works have also investigated the effect of CCI on single antenna two-way relaying systems for the decode-and-forward protocol [15] and the AF protocol [16]. However, it is worth noting that all these prior works deal with the case where all nodes are equipped with a single antenna.

It is well known that the multiple-input multiple-output (MIMO) technology provides extra spatial degrees of freedom which can be efficiently utilized for interference cancellation. To this end, MIMO has been identified as one of the key enabling physical layer technologies in wireless standards such as LTE-Advanced and IMT-Advanced [17]. Despite the importance, so far only a few papers have investigated the impact of CCI in MIMO relaying systems [18–20].

In this paper, we consider the scenario with multiple antennas at the relay, and single antenna at the source and destination. This particular system setup studied in the relay communication literature [18,21] is applicable in several practical scenarios where two nodes (e.g., machine-to-machine type low cost devices) exchange information with the assistance of an advanced terminal such as a cellular base-station/cluster-head sensor. At this point, it is important to highlight the major differences between the current paper and state-of-the-art in the literature. Unlike [18] which only considered a single interferer, the current paper allows for arbitrary number of interferers at the relay node. Compared with [19], which assumed an interference-limited single antenna relay,
the current work considers a multiple antenna relay. Compared with [20], which investigated the outage performance of the scenario with interference-limited multiple antenna relay, the current work considers the more general setup by taking into account of the effect of additive white Gaussian noise (AWGN) at the relay. More importantly, in contrast to [18–20], where the simple maximum ratio transmission (MRT) and maximum ratio combining (MRC) schemes were used, the current paper adopts more sophisticated linear combining schemes to suppress the CCI.

In the presence of CCI, linear diversity combining schemes have been widely adopted in the multiple antenna systems because of the low complexity and good performance [22]. In the same spirit, in this paper we propose a heuristic two-stage relay processing scheme, i.e., the relay first applies linear combining methods to suppress the CCI, and then forwards the transformed signal to the destination by using the MRT scheme. Three popular linear combining methods, i.e., MRC, zero-forcing (ZF) and minimum mean square error (MMSE), are investigated. To the best of the authors’ knowledge, the analysis of diversity combining schemes for the suppression of CCI in dual-hop AF relaying systems has not been presented in the existing literature.

We present a detailed performance analysis of the considered MRC/MRT, ZF/MRT and MMSE/MRT schemes in Rayleigh fading channels. Our main contributions are summarized as follows:

- For the MRC/MRT scheme, we derive a new exact expression involving a single integral for the outage probability of the system, and present a tight closed-form outage lower bound. In addition, we obtain a simple high signal-to-noise ratio (SNR) outage approximation, and prove that the MRC/MRT scheme achieves a diversity order of $N$, where $N$ is the number of antennas at the relay.
- For the ZF/MRT scheme, we first obtain the optimal combining vector maximizing the end-to-end signal-to-interference-and-noise ratio (SINR) outage approximation, and then derive a new exact closed-form expression for the outage probability. We also characterize the high SNR outage behavior and show that it achieves a diversity order of $N - M$, where $M$ is the number of interferers.
- For the MMSE/MRT scheme, we derive a new exact expression involving a single integral for the outage probability, and propose a tight closed-form outage lower bound. We also characterize the high SNR outage behavior of the MMSE/MRT scheme, and show that it achieves a diversity order of $N$.
- Our results suggest that the MMSE/MRT scheme always attains the best outage performance, and the ZF/MRT scheme outperforms the MRC/MRT scheme in the low SNR regime, while the MRC/MRT scheme achieves a superior outage performance than the ZF/MRC scheme in the high SNR regime.

- We also look into the large $N$ regime and demonstrate that in this case, both the ZF/MRT and MMSE/MRT schemes are capable of completely eliminating the CCI, while perfect interference cancellation is not possible with the MRC/MRT scheme.

The remainder of the paper is organized as follows: Section II introduces the system model. Section III presents a detailed investigation of the outage probability achieved by the three different schemes. Numerical results and discussions are provided in Section IV. Finally, Section V concludes the paper and summarizes the key findings.

Notation: We use bold upper case letters to denote matrices, bold lower case letters to denote vectors and lower case letters to denote scalars. $\| \mathbf{h} \|_F$ denotes the Frobenius norm, $E\{x\}$ stands for the expectation of the random variable $x$, $\ast$ denotes the conjugate operator, while $T$ denotes the transpose operator and $\dagger$ denotes the conjugate transpose operator. $CN(0,1)$ denotes a scalar complex circular Gaussian random variable with zero mean and unit variance. $I_k$ is the identity matrix of size $k$. $\Gamma(x)$ is the gamma function and $K_v(x)$ is the $v$-th order modified Bessel function of the second kind [25, Eq. (8.407.1)]. $\Gamma(a, x)$ is the upper incomplete gamma function [25, Eq. (8.350.2)] and $2F_1(a; b; c; z)$ is the Gauss Hypergeometric Function [25, Eq. (9.100)].

II. SYSTEM MODEL

Let us consider a dual-hop multiple antenna AF relaying system as illustrated in Fig. 1 where both the source and destination are equipped with a single antenna, while the relay is equipped with $N$ antennas. We consider the scenario where the relay is subjected to $M$ independently but not necessarily identically distributed co-channel interferers and AWGN, while the destination is corrupted by AWGN only. We also assume that the direct link between the source and the destination does not exist due to obstacles or path loss attenuation/severe fading.

![System model](image1)

Fig. 1: System model: S, R and D denote the source, the relay and the destination, respectively.

In this paper, we consider half-duplex relaying, and hence a complete transmission occurs in two phases. During the first

1 The large $N$ regime analysis is of great interest due to the advent of large-MIMO (massive MIMO) technology [23, 24].
2 This scenario is also particularly relevant to frequency-division relay systems [26] where the relay and the destination experience different interference patterns.
phase, the source transmits the signal to the relay, and the signal received at the relay is given by
\[ y_r = h_1z + \sum_{i=1}^{M} h_is_i + n_1, \] (1)
where the \( N \times 1 \) vector \( h_1 \) denotes the channel for the source-relay link. The entries of \( h_1 \) follow identically and independently distributed (i.i.d.) \( CN(0,1) \). The \( N \times 1 \) vector \( h_is_i \) denotes the channel for the \( i \)-th interference-relay link, and its entries follow i.i.d. \( CN(0,1) \), and \( z \) is the source symbol satisfying \( E\{z^2\} = P \). The \( i \)-th interference symbol is \( s_i \) with \( E\{|s_i|^2\} = P_i \), \( n_1 \) is an \( N \times 1 \) vector and denotes the AWGN at the relay node with \( E\{n_1n_1^H\} = N_0I \).

In the second phase, the relay node transmits a transformed version of the received signal to the destination, and the signal at the destination can be expressed as
\[ y_d = h_2Wy_r + n_2, \] (2)
where \( h_2 \) is a \( 1 \times N \) vector and denotes the channel for the relay-destination link, and its entries follow i.i.d. \( CN(0,1) \), \( n_2 \) is the AWGN with \( E\{|n_2|^2\} = N_0 \), \( W \) is the transformation matrix at relay node with \( E\{|W|^2\} = P_r \).

Combining (1) and (2), the end-to-end SINR of the system can be computed as
\[ \gamma = \frac{\|h_2Wh_1\|^2P_r}{\sum_{i=1}^{M} \|h_2Wh_i\|^2P_i + \|h_2W\|^2P_rN_0 + N_0}. \] (3)
The optimal relay precoder matrix \( W \) maximizing the end-to-end SINR \( \gamma \) does not seem to be analytically tractable, due to the non-convex nature of the problem. Hence, in this paper, we propose a heuristic two-stage relay processing strategy, i.e., the relay first performs some linear combining method to suppress the CCI, and then forwards the transformed signal to the destination using the MRT scheme since it maximizes the SNR of the relay-destination link. Therefore, the heuristic relay precoder \( W \) is a rank-1 matrix, i.e., \( W = \omega \frac{h_1}{\|h_1\|_F} w_1 \), where \( \omega \) is the power constraint factor, \( \frac{h_1}{\|h_1\|_F} \) is used for matching the second hop channel and \( w_1 \) is a \( 1 \times N \) linear combining vector, which depends on the linear combining scheme employed by the relay and will be specified in the following section.

III. OUTAGE PROBABILITY ANALYSIS

In this section, we investigate the outage probability of the MRC/MRT, ZF/MRT and MMSE/MRT schemes. New exact analytical expressions are derived for the outage probability of all three schemes. In addition, simple high SINR approximations are presented, which provide a concise characterization of the achievable diversity order of the system, and enable a performance comparison of the three schemes.

The outage probability is an important performance metric, which is defined as the instantaneous SINR falls below a pre-defined threshold \( \gamma_{th} \). Mathematically, it can be expressed as
\[ P_{out} = \text{Prob}(\gamma < \gamma_{th}). \] (4)

A. MRC/MRT Scheme

For the MRC/MRT scheme, \( w_1 \) is set to match the first hop channel, hence, \( w_1 = \frac{h_1}{\|h_1\|_F} \). To meet the transmit power constraint at the relay, the constant \( \omega^2 \) can be computed as
\[ \omega^2 = \frac{P_r}{\|h_1\|^2P_1 + \frac{1}{\|h_1\|^2} + N_0}. \] (5)

Therefore, the end-to-end SNR for the MRC/MRT scheme, \( \gamma_{MRC} \) can be expressed as (6). Now, with the end-to-end SINR given in (6), we are ready to establish the outage probability of the MRC/MRT scheme. For notational convenience, we define \( \rho_1 = \frac{P_1}{N_0}, \rho_2 = \frac{P}{N_0}, \) and \( \rho_{1i} = \frac{P_i}{N_0}, i = 1, \ldots, M \). We have the following key result.

Theorem 1: In the presence of interferers at the relay, the outage probability of the dual-hop AF relaying system with the MRC/MRT scheme can be expressed as
\[ P_{out}^{MRC} = 1 - \frac{2e^{-\frac{\rho_1}{\rho_2}N}}{\Gamma(N)} \sum_{m=0}^{N-1} \left( \frac{\gamma_{th}}{\rho_1} \right)^m \frac{1}{m!} \sum_{j=0}^{m} \binom{m}{j} \frac{1}{\rho_2} \left( \frac{\gamma_{th}}{\rho_1} \right)^{m-j} \sum_{k=0}^{N+j-1} \left( \frac{\gamma_{th}}{\rho_1} \right)^k \frac{k!}{k!} \right) \cdot \mathcal{I}(\gamma_{th}) \] (7)
with \( \mathcal{I}(\gamma_{th}) \) given in (6), where \( \mathcal{D} = \text{diag}(\rho_1, \rho_1, \ldots, \rho_1) \), \( \rho(D) \) is the number of distinct diagonal elements of \( D \), \( \rho(D_1) > \rho(D_2) > \cdots > \rho(D_M) \) are the distinct diagonal elements in decreasing order, \( \tau(D) \) is the multiplicity of \( \rho(D_1) \) and \( \chi_{i,j}(D) \) is the \((i,j)-th\) characteristic coefficient of \( D \).

Proof: See Appendix [LB]

Theorem 1 presents the exact outage probability of the MRC/MRT scheme, which is quite general and valid for the system with arbitrary number of antennas and interferers. For the special case with a single interferer, Theorem 1 reduces to the result derived in [18, Theorem 13]. To the best of the authors’ knowledge, the integral \( \mathcal{I} \) does not admit a closed-form expression. However, this single integral expression can be efficiently evaluated numerically using software such as Matlab or MATHEMATICA, which still provides computational advantage over a Monte Carlo simulation method.

Alternatively, we can use the following closed-form lower bound of the outage probability, which is tight across the entire SNR range, and becomes exact in the high SNR regime.

Corollary 1: In the presence of interferers at the relay, the outage probability of the dual-hop AF relaying system with the MRC/MRT scheme is lower bounded by
\[ P_{out}^{MRC} \leq 1 - \frac{\Gamma(N, \frac{\rho_1}{\rho_2})}{\Gamma(N)} e^{-\frac{\rho_1}{\rho_2}N} \sum_{l=0}^{k} \frac{\gamma_{th}^l}{l!} \sum_{j=1}^{l} \chi_{i,j}(D) \frac{\Gamma(j+1)}{\Gamma(j)} \rho_1 \rho_{1(i)}^{-l} \gamma_{th}^{j-l}. \] (8)

Proof: See Appendix [LB]
dual-hop AF relaying system with the MRC/MRT scheme can achieve a diversity order of $\sum_{i=1}^M |h_i|^2 P_i$. Moreover, it implies that the number of interferers does not affect the achievable diversity order, as long as the number of the antennas equipped at the relay should be greater than the number of interferers. Hence, for the ZF/MRT scheme, it is assumed that $N > M$.

Theorem 2: In the high SNR regime, i.e., $\rho_2 = \mu \rho_1$, $\rho_1 \to \infty$, with $\mu$ being a finite constant, the outage probability of dual-hop AF relaying system with the MRC/MRT scheme can be approximated as

$$p_{\text{MRC}}^{\text{out}} \approx \frac{1}{\mu^N} + \sum_{k=0}^{N} \binom{N}{k} \sum_{i=0}^{\rho(D) \tau(D)} \chi_{i,j}(D) \frac{N}{k} \sum_{j=1}^{\rho(D) \tau(D)} \chi_{i,j}(D) \frac{N}{k} \frac{\mu^k}{\rho_{\text{MRC}}(i)}$$

$$\times \left( \frac{\alpha}{\rho_1} \right)^N + o \left( \left( \frac{\alpha}{\rho_1} \right)^{N+1} \right).$$

Proof: See Appendix LC.

For the special case where $\{\rho_{\text{MRC}}\}$ are equal, i.e., $\rho_{\text{MRC}} = \rho$ for any $i$, (10) reduces to

$$p_{\text{MRC}}^{\text{out}} = \frac{1}{\Gamma(N+1)} \left[ \frac{1}{\mu^N} + \sum_{k=0}^{N} \binom{N}{k} \frac{\Gamma(M+k)}{\Gamma(M)} \frac{\mu^k}{\rho_1} \right]$$

$$\times \left( \frac{\alpha}{\rho_1} \right)^N + o \left( \left( \frac{\alpha}{\rho_1} \right)^{N+1} \right).$$

Theorem 2 indicates that the MRC/MRT scheme achieves a diversity order of $N$. Moreover, it implies that the number of interferers does not affect the achievable diversity order, if it causes a detrimental effect on the array gain. This key observation suggests that, in the presence of strong CCI, the outage performance of the MRC/MRT scheme will be significantly affected. Hence, in such a scenario, the MRC/MRT scheme may not be suitable. Motivated by this, we now study the performance of more sophisticated linear combining techniques with superior interference suppression capability, namely, the ZF/MRT scheme and the MMSE/MRT scheme.

B. ZF/MRT Scheme

In the ZF/MRT scheme, the relay utilizes the available multiple antennas to completely eliminate the CCI. To ensure this is possible, the number of the antennas equipped at the relay should be greater than the number of interferers. Hence, for the ZF/MRT scheme, it is assumed that $N > M$.

Define an $N \times M$ matrix $H_1 = [h_{11}, h_{12} \cdots h_{1M}]$ as the interference channel matrix, the SINR expression in (3) can be alternatively expressed as

$$\gamma_{ZF} = \frac{\omega^2 \|h_2\|_F^2 \|w_1 h_1\|_F^2 \rho_1}{\omega^2 \|h_2\|_F^2 \|w_1 h_1\|_F^2 + \|h_2\|_F^2 \|w_1\|_F^2 + 1}.$$

Hence, the optimal combining vector $w_1$ should be the solution of the following maximization problem

$$w_1 = \arg \max_{w_1} \gamma_{ZF} \quad \text{s.t.} \quad w_1 H_1 = 0 \quad \|w_1\|_F = 1.$$ (13)

The problem in (13) can be solved as follows:

**Proposition 1:** The optimal combining vector $w_1$ is given by

$$w_1 = \frac{h_1 P_{h_1}}{\|h_1 P_{h_1}\|},$$ (14)

where $P = I_N - H_1 (H_1^H H_1)^{-1} H_1^H$.

Proof: See Appendix LC.

Having obtained the optimal combining vector $w_1$, the end-to-end SINR can be expressed as

$$\gamma_{ZF} = \frac{\|h_2\|_F^2 \|h_1 P_{h_1}\| \rho_1 \rho_2}{\|h_2\|_F^2 \rho_2 + \|h_1 P_{h_1}\| \rho_1 + 1}.$$ (15)

With the above SINR expression, we now study the outage probability of the ZF/MRT scheme.

**Theorem 3:** In the presence of interferers at the relay, the outage probability of the dual-hop AF relaying system with...
the ZF/MRT scheme can be expressed as

\[ P_{\text{out}}^{ZF} = 1 - \frac{2e^{-\frac{2\gamma_\text{th}}{\rho_2}}}{\Gamma(N)} \sum_{m=0}^{N-1} \frac{(\gamma_\text{th})^m}{m!} \sum_{j=0}^{m} \frac{(m)}{j!} \times \left( \frac{1}{\rho_2} \right)^{m-j} \frac{N+j-1}{k} \sum_{k=0}^{N+j-1} \frac{(N+j-1)}{k} \frac{(\gamma_\text{th})^k}{\rho_2^{k-j-1}} \times \frac{(1 + \gamma_\text{th})^{\gamma_\text{th}}}{\rho_1 \rho_2} K_{k-m+1} \left( 2 \frac{(1 + \gamma_\text{th})^{\gamma_\text{th}}}{\rho_1 \rho_2} \right). \]  

(16)

**Proof:** We start by expressing the end-to-end SINR given in (15) as

\[ \gamma_{ZF} = \frac{y_2 y_3 \rho_1 \rho_2}{y_2 \rho_2 + y_3 \rho_1 + 1}. \]  

(17)

where \( y_2 = \| h_2 \|^2_F \) and \( y_3 = \| h_1^\dagger P h_1 \| \). From [30], the probability density function (p.d.f.) of \( y_3 \) can be expressed as

\[ f_{y_3}(x) = \frac{x^{N-M-1}}{(N-M-1)!} e^{-x}. \]  

(18)

Hence, the outage probability of the ZF/MRT scheme can be written as

\[ P_{\text{out}}^{ZF} = \text{Pr} \left( \frac{y_2 y_3 \rho_1 \rho_2}{y_2 \rho_2 + y_3 \rho_1 + 1} \leq \gamma_\text{th} \right) \]

\[ = \text{Pr} \left( \frac{y_3}{y_2 + \frac{\rho_2}{\rho_1}} \leq \frac{\gamma_\text{th}}{\rho_1} \right). \]  

(19)

To this end, invoking the result of [18, Lemma 3] yields the desired result. \( \square \)

**Theorem 4** provides an exact closed-form expression for the outage probability of the ZF/MRT scheme. This expression only involves standard mathematical functions and hence, can be efficiently evaluated. To gain further insights, we now look into the high SNR regime, and present a simple and informative approximation for the outage probability.

**Theorem 4:** In the high SNR regime, i.e., \( \rho_2 = \mu \rho_1, \rho_1 \to \infty \), the outage probability of the dual-hop AF relaying system with the ZF/MRT scheme can be approximated as

\[ P_{\text{out}}^{ZF} = \frac{1}{\Gamma(N-M+1)} \left( \frac{\gamma_\text{th}}{\rho_1} \right)^{N-M} + O \left( \left( \frac{\gamma_\text{th}}{\rho_1} \right)^{N-M+1} \right). \]  

(20)

**Proof:** See Appendix II-B \( \square \)

As expected, we see that the interference power does not affect the outage probability of the ZF/MRT scheme. It is also interesting to observe that \( \mu \) does not affect the outage probability at high SNR. In addition, **Theorem 4** indicates that the achievable diversity order of the ZF/MRT scheme is \( N-M \). Compared with the MRC/MRT scheme, which attains a diversity order of \( N \), the ZF/MRT scheme incurs a diversity loss of \( M \). This important observation suggests that complete elimination of CCI may not be the best option in terms of the outage performance.

**C. MMSE/MRT Scheme**

The ZF scheme completely eliminates the CCI at the relay, which however may cause an elevated noise level. In contrast, the MMSE scheme does not fully eliminate the CCI, instead, it provides the optimum trade-off between interference suppression and noise enhancement. In the following, we study the outage performance of the MMSE/MRT scheme. To make the analysis tractable, we assume that \( \rho_i = \rho_1, \forall i = 1, 2, \ldots, M \). It is important to note that the equal interference power assumption adopted to simplify the ensuing analysis is of practical interest as well. For example, it applies when the interference sources are clustered together [28] or when the interference originates from a multiple antenna source implementing an uniform power allocation policy. In addition, we will later illustrate numerically that our analytical results provide very accurate approximations to the outage probability for scenarios with distinct interference power.

According to the principle of MMSE [29], \( w_1 \) is given by

\[ w_1 = h_1^\dagger \left( h_1 h_1^\dagger + H_j H_j^\dagger + \frac{1}{\rho_1} I_{M-1} \right)^{-1} h_j. \]  

(21)

Also, in order to meet the power constraint at the relay, we have

\[ \omega^2 = \frac{\rho_2}{\| w_1 h_1 \|^2_{F} + \sum_{i=1}^{M} \| w_1 h_i \|^2_{F} + \| w_1 \|^2_{F}}. \]  

(22)

Therefore, the end-to-end SINR can be expressed as

\[ \gamma_{\text{MMSE}} = \frac{\| w_1 h_1 \|^2_{F}}{(1 + \frac{1}{\rho_2 \| h_2 \|^2_{F}}) \left( \sum_{i=1}^{M} \| w_1 h_i \|^2_{F} + \| w_1 \|^2_{F} \right) + \frac{\| w_1 h_1 \|^2_{F}}{\rho_2 \| h_2 \|^2_{F}}}. \]  

(23)

In order to study the outage probability, the remaining task is to characterize the distribution of \( \gamma_{\text{MMSE}} \). However, the involved SINR expression given in (23) is difficult to handle. Hence, we first express the SINR as

\[ \gamma_{\text{MMSE}} = \frac{\rho_2 \| h_2 \|^2_{F} Z}{(1 + \rho_2 \| h_2 \|^2_{F}) + Z}. \]  

(24)

where \( Z \) is defined as

\[ Z = \frac{\| w_1 h_1 \|^2_{F}}{(1 + \sum_{i=1}^{M} \| w_1 h_i \|^2_{F} + \| w_1 \|^2_{F})}. \]  

(25)

Now, let us focus on \( Z \) for the moment. The distribution of the random variable \( Z \) has been studied in [29], where an exact cumulative distribution function (c.d.f.) expression was presented. However, the final expression is a piecewise function, which is not amenable to further processing, and does not seem to be useful here. To circumvent this difficulty, we first derive an alternative unified expression for the c.d.f. of \( Z \).

**Proposition 2:** The c.d.f. of the random variable \( Z \) can be expressed as in (26) shown on the top of the next page, where
\[ F_Z(z) = 1 - \frac{\Gamma \left( \frac{N}{\rho_1} \right)}{\Gamma(N)} + \left( M + 1 \right) e^{-\frac{z}{\rho_1}} \left( \frac{z}{\rho_1} \right)^N N \sum_{m=m_1}^N \frac{\rho_i^{N-m+1}}{\Gamma(m) \Gamma(N-m+2)} \Gamma(m+1, N-m+1; N-m+2; \frac{z}{\rho_1}), \] (26)

where \( m_1 = \max(0, N - M) + 1 \).

Proof: See Appendix III-A \( \square \)

Having obtained the c.d.f. expression of \( Z \), we are now ready to study the outage probability of the MMSE/MRT scheme, and we have the following key result.

**Theorem 5:** In the presence of interferers at the relay, the outage probability of the dual-hop AF relaying system with the MMSE/MRT scheme can be approximated as in (27) shown on the top of the next page, where \( I_1(\gamma_{th}) \) can be written as in (28) also shown on the top of the next page.

Proof: See Appendix III-B \( \square \)

To the best of the authors’ knowledge, the integral \( I_1 \) does not admit a closed-form expression. Nevertheless, this single integral expression can be efficiently evaluated numerically. Alternatively, we can use the following closed-form lower bound, which is tight across the entire SNR range, and becomes exact in the high SNR regime.

**Corollary 2:** In the presence of interferers at the relay, the outage probability of the dual-hop AF relaying system with the MMSE/MRT scheme is lower bounded by

\[ P_{\text{out}}^{\text{MMSE}} = 1 - \frac{\Gamma \left( \frac{N}{\rho_1} \right)}{\Gamma(N)} + \left( M + 1 \right) e^{-\frac{z}{\rho_1}} \left( \frac{z}{\rho_1} \right)^N \sum_{m=m_1}^N \rho_i^{N-m+1} \times \]

\[ 2F_1 \left( M + 1, N - m + 1; N - m + 2; \frac{\rho_i}{\rho_1} \gamma_{th} \right) \frac{\Gamma(m) \Gamma(N - m + 2) \Gamma(m + N + M)}{\Gamma(N)(N + M + 1)} \]. (29)

Proof: See Appendix III-C \( \square \)

Having obtained the exact outage probability of the MMSE/MRC scheme, we now look into the high SNR regime, and derive simple analytical approximations for the outage probability of the system.

**Theorem 6:** In the high SNR regime, i.e., \( \rho_2 = \mu \rho_1, \rho_1 \to \infty \), the outage probability of the dual-hop AF relaying system with the MMSE/MRT scheme can be approximated as

\[ P_{\text{out}}^{\text{MMSE}} = \left[ \frac{\Gamma \left( \frac{N}{\rho_1} \right)}{\Gamma(N)} \right] + \left( M + 1 \right) e^{-\frac{z}{\rho_1}} \left( \frac{z}{\rho_1} \right)^N \sum_{m=m_1}^N \rho_i^{N-m+1} \times \]

\[ 2F_1 \left( M + 1, N - m + 1; N - m + 2; \frac{\rho_i}{\rho_1} \right) \frac{\Gamma(m) \Gamma(N - m + 2) \Gamma(m + N + M)}{\Gamma(N)(N + M + 1)} \]. (30)

D. Large \( N \) Analysis

In this section, we look into the large \( N \) regime with fixed \( M \), and examine the asymptotic behavior of the three proposed schemes. We have the following key result.

**Theorem 7:** When \( N \to \infty \), the end-to-end SINR of the ZF/MRT and the MMSE/MRT schemes converges to

\[ \gamma^\infty = \frac{\rho_2}{\rho_2} \frac{\left\| h_2 \right\|_F^2 \left\| h_1 \right\|_F^2}{ \left( \frac{\rho_2}{\rho_1} \right)^2 + 1 }, \] (31)

and the corresponding outage probability is given by

\[ P_{\text{out}}^{\infty} = 1 - \frac{2e^{-\frac{z}{\rho_1}} \sum_{m=0}^{N-1} \left( \frac{\gamma_{th}}{\rho_1} \right)^m \sum_{j=0}^{m} \left( \frac{m}{j} \right) \times \left( \frac{1}{\rho_2} \right)^{m-j} \sum_{k=0}^{N+j-1} \left( \frac{N+j-1}{k} \right) \left( \frac{\gamma_{th}}{\rho_2} \right)^{N+j-k-1} \times \left( \frac{1}{\rho_1} \right)^{k-m+1} \sum_{j=0}^{k-m+1} \frac{K_{k-m+1}}{2} \left( \frac{1}{\rho_1 \rho_2} \right)^{j} \right) \]. (32)

Proof: See Appendix IV \( \square \)

A close observation reveals that the asymptotic SINR \( \gamma^\infty \) presented in Theorem 7 is equivalent to the end-to-end SNR of the same dual-hop AF relaying system without CCI at the relay, which suggests that, when \( N \) is large, CCI at the relay has no impact on the ZF/MRT and the MMSE/MRT schemes. However, this is not the case for the MRC/MRT scheme. Let us make a careful scrutiny of the interference term

\[ U_1 = \sum_{i=1}^{K} \frac{\left\| h_i \right\|_F^2}{\left\| h_i \right\|_F^2} \frac{\left\| h_i \right\|_F^2}{\rho_i} \] in (6). It can be readily observed that \( U_1 \) is a hyper-exponential random variable which is independent of \( N \). Hence, for the MRC/MRT scheme, the effect of CCI persists even in the large \( N \) regime. This implies that in the presence of CCI, if the relay is equipped with a large number of antennas, adopting linear diversity combining schemes with superior interference suppression capability such as ZF/MRT and MMSE/MRT is preferred over the simple MRC/MRT scheme.
\[ p_{\text{out}}^{\text{MMSE}} = 1 - \frac{2e^{-\frac{\gamma_{\text{th}}}{\rho_1}} x_{\frac{\gamma_{\text{th}}}{\rho_1}} N - 1}{\Gamma(N)} \sum_{m=0}^{N-1} \left( \frac{\gamma_{\text{th}}}{\rho_1} \right) m \frac{1}{m!} \sum_{j=0}^{m} \binom{m}{j} \left( \frac{1}{\rho_2} \right)^{m-j} \sum_{k=0}^{N+j-1} \binom{N+j-1}{k} \left( \frac{\gamma_{\text{th}}}{\rho_2} \right)^{N+j-k-1} \times \left( \frac{1+\gamma_{\text{th}}}{\rho_1 \rho_2} \right)^{\frac{k-m+1}{2}} K_{k-m+1} \left( 2 \sqrt{\frac{(1+\gamma_{\text{th}}) \gamma_{\text{th}}}{\rho_1 \rho_2}} \right) + e^{-\frac{\gamma_{\text{th}}}{\rho_1}} \left( \frac{\gamma_{\text{th}}}{\rho_2} \right)^{N} \sum_{m=m_1}^{N} \frac{\rho_j^{-N-m+1}}{\Gamma(m) \Gamma(N-m+2) \Gamma(m-N+M)} \sum_{j=0}^{N} \binom{N}{j} \left( \frac{1}{\rho_2} \right)^{N-j} \sum_{k=0}^{N+j-1} \binom{N+j-1}{k} \left( \frac{\gamma_{\text{th}}}{\rho_2} \right)^{N+j-1-k} \mathcal{I}_1(\gamma_{\text{th}}), \right. \]

\[ \mathcal{I}_1(\gamma_{\text{th}}) = \int_0^{\infty} e^{-\frac{(1+\gamma_{\text{th}}) x_{\rho_1}}{\rho_2}} e^{-x} x^{k-N} F_1 \left( M+1, N-m+1; N-m+2; -\frac{\rho_j \gamma_{\text{th}}}{\rho_1} \left( 1 + \frac{\gamma_{\text{th}}}{\rho_2 x} \right) \right) dx. \] (28)

| Table I: Comparison of the MRC/MRT, ZF/MRT and MMSE/MRT Schemes |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| CSI requirement             | MRC/MRT                    | ZF/MRT                     | MMSE/MRT                   |
| Antenna number requirement  | h_1 and h_2                | h_1, h_2 and H_1           | h_1, h_2, H_1, and N_0     |
| Diversity order             | N                           | N - M                       | N                          |
| Impact of interference      | reduce the array gain       | no impact                   | reduce the array gain       |
| Decay rate of outage        | slow                        | fast                        | fast                       |
| probability vs. N           |                             |                             |                             |

E. Comparison of the Proposed Schemes

We now provide a more concrete comparison for the three different schemes studied. In the preceding analysis, the channel state information (CSI) requirement to perform relay precoding was not explicitly revealed. In practice, the acquisition of CSI involves additional feedback overhead, which must be considered in the design of wireless systems. On the other hand, if a large amount of CSI is available at the transmitting node, more sophisticated transmission schemes could be designed to improve the transmission efficiency and to achieve a better performance. Therefore, in order to make a fair comparison among the three different schemes, the CSI requirement of each individual scheme must be characterized. Table I gives a comparison of the MRC/MRT, ZF/MRT and MMSE/MRT schemes.

IV. Numerical Results and Discussion

In this section, we present numerical results to validate the analytical expressions derived in Section III. Note, the integral expressions presented in Theorem 1 and Theorem 5 are evaluated numerically with the build-in functions in Matlab, i.e., the “quad” command, and we choose the default absolute error tolerance value 1.0 \times 10^{-6} to control the accuracy of the numerical integration. In all simulations, we set \gamma_{\text{th}} = 0 dB, \rho_{\text{th}} = 0 dB, \forall i = 1, ..., M, \mu = 1, and all results are obtained with 10^8 runs.

Fig. 2 shows the outage probability of the dual-hop AF relaying system with the MRC/MRT scheme for different \( M \) and \( N \). As illustrated, the analytical results are in exact agreement with the Monte Carlo simulations, which demonstrates the correctness of the analytical expression given in (7). Also, the proposed lower bound is sufficiently tight across the entire SNR range of interest, and becomes almost exact in the high SNR regime (i.e., \( \rho_1 \geq 15 \text{ dB} \)), while the high SNR approximation works quite well even at moderate SNR values (i.e., \( \rho_1 = 15 \text{ dB} \)). In addition, we observe that increasing \( N \) reduces the outage probability by improving the diversity order of the system, while increasing \( M \) degrades the outage performance by reducing the array gain of the system.

Fig. 3 illustrates the outage probability of the dual-hop AF
relaying system with the ZF/MRT scheme for different $M$ and $N$. It is observed that, for fixed $M$, increasing the antenna number $N$ yields a significant outage improvement, since the achievable diversity order of the system is $N - M$. Moreover, comparing the curves associated with $N = 4$, $M = 3$ and $N = 3$, $M = 2$, we observe that, when $N - M$ is fixed, the outage probability difference between different $M$, $N$ pairs is almost negligible, and disappears in the high SNR regime, as shown in Theorem 4.

Fig. 4 examines the outage probability of the MMSE/MRT scheme for different $M$ and $N$. It can be readily observed that the analytical curves are in perfect agreement with the Monte Carlo simulation results, and the proposed lower bound and the high SNR approximation are sufficiently tight. In addition, we see that the curves associated with the equal interference power case have the worst outage performance. Hence, our analytical results could be used to serve as a tight outage upper bound in case of arbitrary interference power profiles.

Fig. 5 shows the outage probability of the MMSE/MRT scheme when the equal interference power assumption is no longer valid. As we can clearly observe, for a given total interference power, the gap between the analytical results and Monte Carlo simulations is sufficiently small, especially for the low interference power case where the difference is almost negligible. In addition, we see that the curves associated with the equal interference power case have the worst outage performance. Hence, our analytical results could be used to serve as a tight outage upper bound in case of arbitrary interference power profiles.

Fig. 6 compares the outage performance of the three relay precoding schemes under different cases of interference power, i.e., weak interference $\rho_I = 0$ dB and strong interference $\rho_I = 30$ dB. We observe the intuitive result that, the outage performance of the MRC/MRT and the MMSE/MRT schemes degrades when the interference power becomes stronger, while the outage performance of the ZF/MRT scheme remains the same regardless of the interference power levels. Comparing different curves, we see that the MMSE/MRT scheme always attains the best outage performance, and the ZF/MRT scheme outperforms the MRC/MRT scheme at the low SNR regime, especially when the interference power is large.
that of the MRC/MRT scheme. In other words, the minimum
MMSE/MRT schemes is almost the same, which is higher than
is large. Moreover, the outage decay rate of the ZF/MRT and
is small, and becomes inferior to the ZF/MRT scheme when
scheme always has the best outage performance, while the
ability is smaller for the ZF/MRT and MMSE/MRT schemes
compared with the MRC/MRT scheme.

V. CONCLUSION

We investigated the outage performance of a dual-hop AF
multiple antenna relaying system employing the MRC/MRT,
ZF/MRT and MMSE/MRT schemes in the presence of CCI.
Exact analytical expressions for the outage probability of all
three schemes were derived, which provide a fast and efficient
means of evaluating the performance of the system. Moreover,
simple and informative high SNR outage approximations
were presented, which enable the characterization of the impact
of key parameters, such as relay antenna number \(N\), number
of interferers \(M\) and interference power on the outage
performance of the system. Our findings suggest that both
the MRC/MRT and MMSE/MRT schemes achieve the full
diversity order of \(N\), while the ZF/MRT scheme achieves
a diversity order of \(N - M\). In addition, the MMSE/MRT scheme
always attains the best outage performance, and the ZF/MRT
scheme outperforms the MRC/MRT scheme in the low SNR
regime, while the opposite holds in the high SNR regime.
Finally, we have shown that, in the asymptotically large \(N\)
regime, perfect interference cancellation can be achieved by
using the ZF/MRT and the MMSE/MRT schemes.

APPENDIX I

PROOF FOR THE MRC/MRT SCHEME

A. Proof of Theorem 7

We start by expressing the end-to-end SINR given in (6) as
\[
\gamma_{MRC} = \frac{y_1 y_2 \rho_1 \rho_2}{(y_2 \rho_2 + 1) (U_1 + 1) + y_1 \rho_1},
\]
where \(y_1 = \|h_1\|^2_E, y_2 = \|h_2\|^2_E, U_1 = \sum_{i=1}^{M} y_i \rho_i\) with \(y_i = \frac{|h_i|^2}{\|h_i\|^2_E}\). It is easy to observe that \(y_1\) and \(y_2\) are i.i.d. random variables with the p.d.f.
\[
f_{y_i}(x) = \frac{x^{N-1}}{(N-1)!} e^{-x}. \tag{34}
\]
Also, according to [19], \(y_i, i = 1, \ldots, M\), are i.i.d. exponential random variables with unit variance. Then, random variable \(U_1\) follows the hyper-exponential distribution with p.d.f.
\[
f_{U_1}(x) = \sum_{i=1}^{\rho(D)} \sum_{j=1}^{\tau_i(D)} \chi_{i,j}(D) \rho_{(i)}^{j-1} e^{-\frac{x}{\rho_I(i)}}. \tag{35}
\]

The outage probability of the system can be computed by
\[
P_{\text{out}}^{MRC} = \text{Prob} \left( \frac{y_1 y_2 \rho_2}{(y_2 \rho_2 + 1) (U_1 + 1) + y_1 \rho_1} \leq \gamma_{th} \right)
= \text{Prob} \left( \frac{y_2 - \frac{\gamma_{th}}{\rho_2}}{y_2 + \frac{\rho_1}{\rho_2}} \leq \frac{\gamma_{th}}{\rho_1} (U_1 + 1) \right). \tag{36}
\]

To this end, invoking the result presented in [18, Lemma 3], we can obtain the following outage probability expression conditioned on \(U_1\),
\[
P_{\text{out}}^{MRC} = 1 - \frac{2e^{-\frac{\gamma_{th}}{\rho_1} (U_1 + 1) - \frac{\rho_2}{\rho_1}}}{\Gamma(N)} \sum_{m=0}^{N-1} \left( \frac{\gamma_{th}}{\rho_1} (U_1 + 1) \right)^m \frac{1}{m!}
\times \sum_{j=0}^{m} \left( \begin{array}{c} m \\ j \end{array} \right) \left( \frac{1}{\rho_2} \right)^{m-j} N+j-1 \binom{N+j-1}{k}
\times \left( \frac{\gamma_{th}}{\rho_2} \right)^{N+j-1-k} B^{\frac{k-1}{N+j-1-k}} K_{k-1} \left( \frac{2\sqrt{B}}{\rho_1 \rho_2} \right), \tag{37}
\]
where
\[
B = \frac{\gamma_{th} + \rho_1}{\rho_1 \rho_2} (U_1 + 1)
\]
The desired result can be obtained by averaging over \(U_1\),
along with some simple basic algebraic manipulations.

B. Proof of Corollary 7

The end-to-end SINR can be upper bounded by
\[
\gamma_{MRC} = \frac{y_1 y_2 \rho_1 \rho_2}{(y_2 \rho_2 + 1) (U_1 + 1) + y_1 \rho_1} \leq \min \left( \frac{y_1 \rho_1}{U_1 + 1}, y_2 \rho_2 \right). \tag{38}
\]
Since \(y_1\), \(y_2\), \(U_1\) are independent random variables, the outage probability of the system can be lower bounded by
\[
P_{\text{out}}^{\text{MRC}} = 1 - \text{Prob} \left( \frac{y_1 \rho_1}{U_1 + 1} \geq \gamma_{th} \right) \text{Prob} \left( y_2 \rho_2 \geq \gamma_{th} \right). \tag{39}
\]
Conditioned on $U_1$, Pro(\(\frac{y_1 \rho_1}{U_1 + 1} \geq \gamma_{th}\)) can be computed as

$$\text{Pro}(\frac{y_1 \rho_1}{U_1 + 1} \geq \gamma_{th}) = 1 - \text{Pro}(y_1 < \frac{\gamma_{th}}{\rho_1} (U_1 + 1)) = \frac{\Gamma \left( N, \frac{U_1 + 1}{\rho_1} \gamma_{th} \right)}{\Gamma (N)}.$$  

(40)

The next step is to average over the distribution of $U_1$. After some algebraic manipulations, we arrive at

$$\text{Pro}(\frac{y_1 \rho_1}{U_1 + 1} \geq \gamma_{th}) = e^{-\frac{2\rho_1}{\rho_2}} \sum_{k=0}^{N-1} \frac{(\frac{\gamma_{th}}{\rho_1})^k}{k!} \sum_{l=0}^{k} \frac{\rho_1^l}{\rho_1 + \rho_{l(i)} \gamma_{th}}.$$  

(41)

Now, we look at the second part, Pro$(y_2 \rho_2 \geq \gamma_{th})$, which can be computed as

$$\text{Pro}(y_2 \rho_2 \geq \gamma_{th}) = \text{Pro}(y_2 \geq \frac{\gamma_{th}}{\rho_2}) = e^{-\frac{2\rho_2}{\rho_2}} \sum_{m=0}^{\infty} \frac{(\frac{\gamma_{th}}{\rho_2})^m}{m!} = \frac{\Gamma \left( N, \frac{2\rho_2}{\rho_2} \right)}{\Gamma (N)}.$$  

(42)

To this end, substituting (41) and (42) into (39) yields the desired result.

C. Proof of Theorem 2

Starting from (39), conditioned on $U_1$, the outage probability of the MRC/MRT scheme can be lower bounded by

$$P_{out}^{\text{MRC}} = 1 - \frac{\Gamma \left( N, \frac{2\rho_2}{\rho_2} \right)}{\Gamma (N)} \frac{\Gamma \left( N, \frac{U_1 + 1}{\rho_1} \gamma_{th} \right)}{\Gamma (N)}.$$  

(43)

Then, invoking the asymptotic expansion of incomplete gamma function [25, Eq. (8.354.2)], we have

$$\frac{\Gamma \left( N, \frac{2\rho_2}{\rho_2} \right)}{\Gamma (N)} = 1 - \frac{(\frac{2\rho_2}{\rho_2})^N}{\Gamma (N+1)} + o \left( \left( \frac{\gamma_{th}}{\rho_1} \right)^{N+1} \right).$$  

(44)

Similarly, we get

$$\frac{\Gamma \left( N, \frac{U_1 + 1}{\rho_1} \gamma_{th} \right)}{\Gamma (N)} = 1 - \frac{1}{\Gamma (N+1)} (U_1 + 1)^N \left( \frac{\gamma_{th}}{\rho_1} \right)^N + o \left( \left( \frac{\gamma_{th}}{\rho_1} \right)^{N+1} \right).$$  

(45)

Hence, the outage lower bound conditioned on $U_1$ can be approximated as

$$P_{out}^{\text{MRC}} = \frac{(\frac{2\rho_2}{\rho_2})^N}{\Gamma (N+1)} \left[ \frac{1}{\mu^N} + (U_1 + 1)^N \right] + o \left( \left( \frac{\gamma_{th}}{\rho_1} \right)^{N+1} \right).$$  

(46)

To this end, the remaining task is to compute the expectation of $(1 + U_1)^N$. Applying the binomial expansion,

$$(1 + U_1)^N = \sum_{k=0}^{N} \binom{N}{k} U_1^k,$$  

(47)

and averaging over $U_1$, we get

$$\text{E}_{U_1} \left\{ (1 + U_1)^N \right\} = \sum_{k=0}^{N} \binom{N}{k} \sum_{j=1}^{\infty} \sum_{i=1}^{\rho_{l(i)}} \chi_{ij}(D) \frac{\rho_{j(i)-1}}{(j-1)!} \int_{0}^{\infty} x^{j-1} e^{-\frac{x}{\rho_{l(i)}}} dx$$  

$$= \sum_{k=0}^{N} \binom{N}{k} \sum_{j=1}^{\infty} \sum_{i=1}^{\rho_{l(i)}} \chi_{ij}(D) \frac{\Gamma (j+k) \rho_{j(i)}^k}{\Gamma (j) \rho_{l(i)}}.$$  

(48)

Substituting (48) into (46) yields the desired result.

APPENDIX II

PROOF FOR THE ZF/MRT SCHEME

A. Proof of Proposition 2

Substituting $w_1 H_I = 0$ and $|w_1| = 1$ into (12), we have

$$\gamma_{ZF} = \frac{\omega^2 \|h_2\|^2_F |w_1 h_1|^2 \rho_1}{\omega^2 \|h_2\|^2_F + 1}.$$  

(49)

It should be noted that the power constraint constant $\omega$ is also dependent on the combining vector $w_1$ via the following relationship

$$\omega^2 = \frac{\rho_2}{|w_1 h_1|^2 \rho_1 + 1}.$$  

(50)

Hence, we have

$$\gamma_{ZF} = \frac{\|h_2\|^2_F \rho_2}{\|h_2\|^2_F + (|w_1 h_1|^2 \rho_1 + 1)}.$$  

(51)

Now, it is easy to show that $\gamma_{ZF}$ is an increasing function with respect to $|w_1 h_1|^2$. Therefore, the original optimization can be alternatively formulated as

$$w_1 = \arg \max_{w_1} |w_1 h_1|^2$$  

s.t. $w_1 H_I = 0$ & $\|w_1\|^2_F = 1.$  

(52)

To this end, the desired result can be obtained by following the similar lines as in the proof of [30, Proposition 1].

B. Proof of Theorem 2

In the high SNR regime, the end-to-end SINR $\gamma_{ZF}$ can be tightly bounded by

$$\gamma_{ZF} = \frac{y_2 y_3 \rho_1 \rho_2}{y_2 \rho_2 + y_3 \rho_1 + 1} \leq \min (y_3 \rho_1, y_2 \rho_2).$$  

(53)

Hence, the outage probability of the system can be tightly lower bounded by

$$P_{out}^{\text{ZF}} \geq \text{Pro}(\min (y_3 \rho_1, y_2 \rho_2) \leq \gamma_{th}).$$  

(54)
Due to the independence of random variables $y_2$ and $y_3$, the outage lower bound can be written as

$$P_{out}^{ZF} = 1 - \text{Prob} \left( y_3 \geq \frac{\gamma \rho_1}{\rho_2} \right) \text{Prob} \left( y_2 \geq \frac{\gamma \rho_1}{\rho_2} \right)$$

$$= 1 - \frac{\Gamma \left( N - M, \frac{\gamma \rho_1}{\rho_2} \right)}{\Gamma \left( N - M + 1 \right)} \frac{\Gamma \left( N - M, \frac{\gamma \rho_1}{\rho_2} \right)}{\Gamma \left( N - M + 1 \right)}.$$  \hspace{1cm} (55)

Invoking the asymptotic expansion of incomplete gamma function [29, Eq. (8.354.2)], we have

$$P_{out}^{ZF} = \left( \frac{\rho_2}{\rho_1} \right)^N \frac{1}{\Gamma \left( N + 1 \right)} + o \left( \frac{\rho_2}{\rho_1} \right)^{N-M} \frac{1}{\Gamma \left( N - M + 1 \right)}. \hspace{1cm} (56)$$

It is obvious that the first item in (56) is negligible when compared with the second item in (56). Therefore, the desired result can be obtained after some simple algebraic manipulations.

**APPENDIX III**

**PROOF FOR THE MMSE/MRT SCHEME**

A. Proof of Proposition 2

The random variable $Z$ can be alternatively expressed as

$$Z = \rho_1 \rho_1 \left( \frac{1}{\rho_1} w_1 h_1^\dagger w_1^\dagger \right) = \rho_1 \rho_1 \left( \frac{1}{\rho_1} w_1 h_1^\dagger w_1^\dagger + w_1 h_1^\dagger w_1^\dagger \right). \hspace{1cm} (57)$$

where $I$ denotes the unit matrix.

Now define $R = H_1 H_1^\dagger + \frac{1}{\rho_1} I$, then the MMSE combining vector $w_1$ can be written by

$$w_1 = h_1^\dagger \left( h_1 h_1^\dagger + R \right)^{-1}. \hspace{1cm} (58)$$

Hence, we have

$$Z = \rho_1 \rho_1 \left( h_1 h_1^\dagger + R \right)^{-1} h_1 h_1 h_1^\dagger + \rho_1 \rho_1 \left( h_1 h_1^\dagger + R \right)^{-1} h_1. \hspace{1cm} (59)$$

Applying the well-known matrix inversion lemma, (59) can be simplified as

$$Z = \rho_1 \rho_1 \left( h_1 h_1^\dagger + R \right)^{-1} h_1. \hspace{1cm} (60)$$

To this end, invoking the results presented in [29, Eq.(11)], the c.d.f. of $Z$ can be expressed as

$$F_Z(z) = 1 - e^{-\frac{z}{\rho_1}} \sum_{m=1}^{N} \frac{A_m(z)}{(m-1)!} \left( \frac{z}{\rho_1} \right)^{m-1}, \hspace{1cm} (61)$$

where

$$A_m(z) = \left\{ \begin{array}{lr}
\frac{1}{1 + \sum_{i=1}^{M} \left( \frac{\mu^2}{\rho_1} \right)^i} & N \geq M + m, \\
\frac{1 + \sum_{i=1}^{M} \left( \frac{\mu^2}{\rho_1} \right)^i}{\left( 1 + \frac{\mu^2}{\rho_1} \right)^M} & N < M + m.
\end{array} \right. \hspace{1cm} (62)$$

To obtain a unified expression for the c.d.f. of $Z$, we find it convenient to give a separate treatment for the following two cases: 1) $N \geq M$, and 2) $N \leq M$.

1) $N \geq M$:

Noticing that

$$1 + \sum_{i=1}^{N-M+1} \left( \frac{\mu}{\rho_1} \right)^i = 1 - i \sum_{m=1}^{N-M} \left( \frac{\mu}{\rho_1} \right)^i,$$

the c.d.f. of $Z$ can be written as (64) shown on the top of the next page, which can be further simplified as

$$F_Z(z) = 1 - e^{-\frac{z}{\rho_1}} \sum_{m=1}^{N} \frac{1}{(m-1)!} \left( \frac{z}{\rho_1} \right)^{m-1} \frac{M}{1 + \frac{\mu^2}{\rho_1} \left( \frac{\mu}{\rho_1} \right)^M}. \hspace{1cm} (65)$$

Now, utilizing the following key observation,

$$\sum_{i=N}^{M} \left( \frac{\mu}{\rho_1} \right)^i = \frac{\Gamma(M + 1)}{\Gamma(M + 1) \Gamma(M + 1 - N)} \left( \frac{\rho_1}{\mu} \right)^N \times 2F_1 \left( M + 1, N - m + 1 ; -\frac{\rho_1}{\mu} \right). \hspace{1cm} (66)$$

$S_1$ can be alternatively expressed as

$$S_1 = \frac{\Gamma(M + 1)}{\Gamma(M + 1) \Gamma(M + 1 - N)} \left( \frac{\rho_1}{\mu} \right)^N.$$  \hspace{1cm} (67)

Finally, noticing that

$$e^{-\frac{z}{\rho_1}} \sum_{m=1}^{N} \frac{1}{(m-1)!} \left( \frac{z}{\rho_1} \right)^{m-1} = \frac{\Gamma(N, \frac{z}{\rho_1})}{\Gamma(N)}, \hspace{1cm} (68)$$

and after some algebraic manipulations, we obtain (69) shown on the top of the next page.

2) $N \leq M$:

Similarly, the c.d.f. of $Z$ can be alternatively expressed as

$$F_Z(z) = 1 - e^{-\frac{z}{\rho_1}} \sum_{m=1}^{N} \frac{1}{(m-1)!} \left( \frac{z}{\rho_1} \right)^{m-1} - \sum_{m=1}^{N} \sum_{i=N-m+1}^{M} \left( \frac{\mu}{\rho_1} \right)^i.$$  \hspace{1cm} (70)

Then, following the same lines as in the derivation of the $N \geq M$ case, we get (71) shown on the top of the next page.

To this end, the desired result can be obtained by appropriately choosing certain parameters.
\[ F_Z(z) = 1 - e^{-\frac{z}{\rho_1}} \sum_{m=1}^{N-M} \frac{1}{(m-1)!} (\frac{z}{\rho_1})^{m-1} e^{-\frac{z}{\rho_1}} \sum_{m=M-N+1}^{N} \frac{1}{(m-1)!} (\frac{z}{\rho_1})^{m-1} \left( 1 - \frac{\sum_{i=N-m+1}^{M} (\frac{\rho_1}{\rho_1-z})^i}{1+\frac{\rho_1}{\rho_1-z}} \right), \]

(64)

\[ F_Z(z) = 1 - \frac{\Gamma\left(\frac{N}{\rho_1}, \frac{z}{\rho_1}\right)}{\Gamma(N)} + \Gamma(M+1) e^{-\frac{z}{\rho_1}} \sum_{m=M-N+1}^{N} \frac{\rho_1^{N-m+1} e_{\frac{\rho_1}{\rho_2}} z}{\Gamma(m) \Gamma(N-m+2) \Gamma(m-N+M)}. \]

(69)

\[ F_Z(z) = 1 - \frac{\Gamma\left(\frac{N}{\rho_1}, \frac{z}{\rho_1}\right)}{\Gamma(N)} + \Gamma(M+1) e^{-\frac{z}{\rho_1}} \sum_{m=M-N+1}^{N} \frac{\rho_1^{N-m+1} e_{\frac{\rho_1}{\rho_2}} z}{\Gamma(m) \Gamma(N-m+2) \Gamma(m-N+M)}. \]

(71)

**B. Proof of Theorem 3**

Starting from the end-to-end SINR presented in (24), the outage probability of the system can be expressed as

\[ P_{\text{out}}^{\text{MMSE}} = \text{Prob}\left( \frac{\rho_2 y_2 Z}{(\rho_2 y_2 + 1) + Z} \leq \gamma_{\text{th}} \right) \]

= \text{Prob}\left( Z \frac{y_2 - \frac{2y_2}{\rho_2}}{y_2 + \frac{1}{\rho_2}} \leq \gamma_{\text{th}} \right). \quad (72)

Then, applying the method used in [18, Lemma 3], and utilizing the c.d.f. expression of Z given in Proposition 2 and the p.d.f of y_2, the outage probability of the system is given by

\[ P_{\text{out}}^{\text{MMSE}} = 1 - \mathcal{I}_2 + \left( \frac{\gamma_{\text{th}}}{\rho_1} \right)^N \frac{\Gamma(M+1)}{\Gamma(N)} \sum_{m=m_1}^{N} \frac{\rho_1^{N-m+1} \mathcal{I}_3}{\Gamma(m) \Gamma(N-m+2) \Gamma(m-N+M)}, \]

(73)

where

\[ \mathcal{I}_2 = \int_{N}^{\infty} \frac{\Gamma\left(N, C\frac{\rho_1}{\rho_2}\right)}{\Gamma(N)} t^{N-1} e^{-t} dt, \]

(74)

and

\[ \mathcal{I}_3 = \int_{\frac{N}{\rho_2}}^{\infty} e^{-C\frac{\rho_1}{\rho_2}} e^{-t} t^{N-1} C^N \]

\[ 2F_1\left( M + 1, N - m + 1; N - m + 2; -\frac{\rho_1}{\rho_1} \gamma_{\text{th}} C \right) dt. \]

(75)

Hence, the outage probability can be lower bounded by

\[ P_{\text{out}}^{\text{MMSE}} \geq \text{Prob}\left( \min(Z, y_2 \rho_2) \leq \gamma_{\text{th}} \right). \]

(77)

Due to the independence of y_2 and Z, the outage probability lower bound can be computed as

\[ P_{\text{out}}^{\text{MMSE}} \left( \gamma_{\text{th}} \right) = 1 - \text{Prob}\left( Z \geq \gamma_{\text{th}} \right) \text{Prob}\left( y_2 \geq \frac{\gamma_{\text{th}}}{\rho_2} \right). \]

(78)

To this end, invoking the c.d.f. of Z given in Proposition 2 and the p.d.f of y_2, the desired result can be obtained after some simple algebraic manipulations.

**D. Proof of Corollary 3**

To prove the statement, we only need to show that \( A_{\text{MRC}} > A_{\text{MMSE}} \), where

\[ A_{\text{MRC}} = \frac{1}{(N+1) \left[ 1 + \sum_{k=0}^{N} \left( N \left( \frac{k+M}{\mu} \right) \right) \right]^{\mu} \rho_1^k}, \]

(79)

and

\[ A_{\text{MMSE}} = \sum_{m=m_1}^{N} \frac{\Gamma(M+1) \rho_1^{N-m+1} \left( 1 + \left( \frac{1}{\mu} \right)^N \right)^{\frac{1}{\gamma_{\text{th}}}}} {\Gamma(N+1)} \Gamma\left(N-m+2; \frac{M}{\rho_1} \right), \]

(80)

A close observation shows that both \( A_{\text{MRC}} \) and \( A_{\text{MMSE}} \) have the a common item \( \left( 1 + \left( \frac{1}{\mu} \right)^N \right)^{\frac{1}{\gamma_{\text{th}}}}} {\Gamma(N+1)} \). Hence, to proof \( A_{\text{MRC}} > A_{\text{MMSE}} \), we only need to show \( A_1 > A_2 \), where

\[ A_1 = \sum_{k=1}^{N} \frac{\Gamma(M+k) \rho_1^k}{\Gamma(N-k+1) \Gamma(k+1) \Gamma(M)}, \]

(81)

and

\[ A_2 = \sum_{m=m_1}^{N} \frac{\Gamma(M+1) \rho_1^{N-m+1}} {\Gamma(m) \Gamma(N-m+2) \Gamma(m-N+M)}. \]

(82)

**C. Proof of Corollary 2**

The end-to-end SINR can be upper bounded by

\[ \gamma_{\text{MMSE}} = \frac{\rho_2 y_2 Z}{(\rho_2 y_2 + 1) + Z} \leq \min(Z, \rho_2 y_2). \]

(76)
Define \( t = \min(N, M) \), after some tedious algebraic manipulations, \( A_2 \) can be alternatively expressed as

\[
A_2 = \sum_{k=1}^{t} \frac{\Gamma(M + 1) \rho_k^t}{(N - k + 1) \Gamma(k + 1) \Gamma(M - k + 1)}.
\]  

(83)

Comparing \( 83 \) and \( 81 \), it is easy to show that \( A_1 > A_2 \), which completes the proof.

**APPENDIX IV PROOF OF THEOREM 7**

In the asymptotic large \( N \) regime, the law of large number holds, and we have

\[
\frac{1}{N} \mathbf{h}_k^\dagger \mathbf{h}_i = 0, \quad \text{and} \quad \frac{1}{N} \mathbf{H}_i^\dagger \mathbf{H}_i = \mathbf{I}_M.
\]  

(84)

For the ZF/MRT scheme, starting from \( 15 \), and with the help of \( 83 \), we have

\[
\frac{1}{N} \mathbf{h}_i^* \mathbf{P}_1 \mathbf{h}_i = \frac{1}{N} \mathbf{h}_i \mathbf{h}_i^\dagger \left( \frac{1}{N} \mathbf{H}_i^\dagger \mathbf{H}_i \right)^{-1} \mathbf{H}_i \mathbf{h}_i = \mathbf{h}_i \mathbf{h}_i^\dagger \left( \frac{1}{N} \sum_{i=1}^{M} \mathbf{h}_i \mathbf{h}_i^\dagger \right) = \mathbf{h}_i \mathbf{h}_i^\dagger. 
\]  

(85)

Substituting \( 88 \) into \( 15 \), we have

\[
\gamma_{ZF} = \frac{\|\mathbf{h}_1\|_{P_r}^2}{\mathbf{P}_r \|\mathbf{h}_1\|_{P_r}^2 + N_0 \left( \|\mathbf{h}_1\|_{P_r}^2 P_s + N_0 \right)} = \gamma^\infty. 
\]  

(89)

For the MMSE/MRT scheme, with the help of \( 83 \), and applying the well-known Woodbury matrix identity, we have

\[
\mathbf{Z}^{-1} = \frac{1}{\rho_1} \mathbf{I} - \frac{1}{\rho_1} \mathbf{H}_i \mathbf{H}_i^\dagger. \quad \text{Hence, } \mathbf{Z} \text{ can be expressed as}
\]

\[
\mathbf{Z} = \frac{\rho_1}{\rho_1} \left( \frac{1}{N} \mathbf{h}_i \mathbf{h}_i^\dagger - \frac{1}{N} \mathbf{h}_i \mathbf{H}_i \mathbf{H}_i^\dagger \mathbf{h}_i \right) = \rho_1 \|\mathbf{h}_1\|_{P_r}^2. 
\]  

(90)

Substituting \( 90 \) into \( 24 \), we have

\[
\gamma_{MMSE} = \frac{\rho_2 \|\mathbf{h}_2\|_{P_r}^2 \rho_1 \|\mathbf{h}_1\|_{P_r}^2}{1 + \rho_2 \|\mathbf{h}_2\|_{P_r}^2 + \rho_1 \|\mathbf{h}_1\|_{P_r}^2} = \gamma^\infty. 
\]  

(91)

Given the asymptotic SINR, the desired outage probability can be computed by following the similar lines as in the proof of Theorem 3.

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