**Abstract**

The strongest threat model for voting systems considers coercion resistance: protection against coercers that force voters to modify their votes, or to abstain. Existing remote voting systems either do not provide this property; require an expensive tallying phase; or burden users with the need to store cryptographic key material and with the responsibility to deceive their coercers. We propose VOTEAGAIN, a scalable voting scheme that relies on the revoting paradigm to provide coercion resistance. VOTEAGAIN uses a novel deterministic ballot padding mechanism to ensure that coercers cannot see whether a vote has been replaced. This mechanism ensures tallies take quasilinear time, making VOTEAGAIN the first revoting scheme that can handle elections with millions of voters. We prove that VOTEAGAIN provides ballot privacy, coercion resistance, and verifiability; and we demonstrate its scalability using a prototype implementation of its core cryptographic primitives.

1 Introduction

Remote electronic voting, i.e., voting outside a poll-booth environment, in which voters cast their ballot from their devices is susceptible to large-scale vote buying and coercion [31]. Yet, many deployed electronic voting systems [2,26,37] do not support coercion resistance. This might be suitable in Western democracies where freedom and privacy are well rooted in society. However, under more authoritarian regimes [29] or in younger democracies [33], coercion is a serious problem.

There are two kind of coercion-resistant electronic voting systems in the literature. The first kind provides users with fake voting credentials that voters use/produce when coerced, enabling deletion of coerced votes [13,31]. This approach has several downsides: (i) voters need to store their true voting credential on their devices, (ii) the system cannot give feedback on whether the correct credential was used, and thus voters cannot be sure if their vote has been recorded correctly at the time of voting, and (iii) voters need to convincingly lie while being coerced which may be a challenge. The second kind relies on the revoting paradigm [1,25,32,34]. These schemes avoid the drawbacks associated with the fake-credential approach by allowing voters to submit fully to coercers, and instead enabling them to supersede coerced votes by casting a new ballot. This approach requires that the coercer cannot detect whether a voter has cast new ballots. To achieve this, state-of-the-art schemes [1,32] require a quadratic number of operations, concretely a pair-wise comparison of all ballots, to privately filter superseded ballots. As an example, for the Iowa Democratic caucus with only 176,574 voters, Achenbach et al.’s solution [1] would require 1.8 core years to filter the ballots.

We propose VOTEAGAIN, a scalable coercion-resistant (re)voting scheme. VOTEAGAIN’s efficiency relies on two key insights: First, one can hide the number of ballots per user by inserting a deterministic number of dummy ballots which depends solely on the number of voters and the number of cast ballots. Thus, it reveals nothing about the number of ballots cast by individual voters, hiding any (re)voting patterns induced by voters or coercers. Second, because of the deterministic nature of the approach one can execute filtering in the clear, reducing the filtering time from quadratic to quasilinear: $O(n \log n)$ where $n$ is the number of ballots. As a result, for the Iowa caucus our construction requires under 14 core minutes. We estimate that VOTEAGAIN using 224 cores (less than $50$ on Amazon, or $75K$ on dedicated hardware) can filter hundreds of millions of ballots in hours.

We make the following contributions:

✓ We introduce VOTEAGAIN, a novel remote electronic revoting scheme based on well defined and widely used cryptographic constructions.

✓ We introduce a novel efficient deterministic padding scheme that hides revoting at a low cost. The complexity of the resulting filtering phase is $O(n \log n)$ where $n$ is the number of ballots. Our experiments show that in many practical scenarios the cost can be even lower.

✓ We show that previous definitions of coercion resistance in
We prove that VOT
words [12, 18]). When coerced, the voter lies to the coercer, (JCJ), are used in several voting schemes [3, 10, 12, 13]. In
To obtain public verifiability these schemes use a distributed
voters need to maintain cryptographic state.

We evaluate the scalability of VOTE
✓
coercion resistance using fake authentication credentials.

Coercion-resistant voting schemes fall under two categories: either they enable voters to generate fake authentication credentials or they allow the voter to revote. Coercion resistant schemes using fake credentials, introduced by Juels et al. [31] (JCJ), are used in several voting schemes [3, 10, 12, 13]. In these schemes, the voter has both real and fake authentication credentials (or pre-registered passwords and panic passwords [12, 18]). When coerced, the voter lies to the coercer, using a fake authentication credential (or handling it to the coercer), resulting in a non-counted ballot. Ballots cast with the real credential are counted. These schemes provide the real authentication credential to the voter during registration phase (in which the coercer must be absent). The voter must securely store this authentication credentials for later use, i.e., voters need to maintain cryptographic state.

Coercion resistant schemes based on revoting allow voters to cast multiple ballots and then filter these ballots, typically counting the last ballot per voter. For such a scheme to be coercion resistant, the filtering stage must be deniable [1], i.e., it must not expose which ballots are filtered, as this would expose revoting actions. Black box filtering where a trusted third party (TTP) performs the filtering privately is deniable [23], but not verifiable. To the best of our knowledge, there exist two publicly-verifiable deniable re-voting schemes [1, 32]. To obtain public verifiability these schemes use a distributed authority to compare each pair of ballots (i.e., \(O(n^3)\) operations) before shuffling to privately mark superseded ballots. After shuffling, these marks are decrypted and the tallying server verifiably filters superseded ballots. As literally specified in these papers, these schemes are ‘not efficient for large scale elections’. We confirm in Section 7 that Achenbach et al.’s scheme [1] cannot efficiently handle small elections of a hundred thousand users.

Both the JCJ based and the private revoting based schemes offer a solution with a \(k\)-out-of-\(t\) assumption for coercion resistance. However, on top of that, these schemes require the existence of Anonymous Channels (AC) to avoid coercion attacks such as forced abstention.

For authentication, most schemes require users to store cryptographic state [1, 10, 13, 23, 31, 32, 36], or remember special passwords [12, 18]. Helios [2] and Apollo [22] rely on regular username/password. To improve verifiability (by distributing the trust of the entity deciding which users are eligible voters), some schemes require that voters authenticate to \(k\) out of \(t\) parties [3, 10, 13, 31]. However, this results in a complex registration phase for the user where, additionally, the coercer is assumed to be absent. Revoting based schemes (including VOTEAGAIN) can be extended to this setting to reduce the trust assumptions required for authentication correctness (and hence verifiability). Table 1 summarizes the comparison between VOTEAGAIN and previous work.

### 2 Related Work

Coercion-resistant voting schemes fall under two categories: either they enable voters to generate fake authentication credentials or they allow the voter to revote. Coercion resistant schemes using fake credentials, introduced by Juels et al. [31] (JCJ), are used in several voting schemes [3, 10, 12, 13]. In these schemes, the voter has both real and fake authentication credentials (or pre-registered passwords and panic passwords [12, 18]). When coerced, the voter lies to the coercer, using a fake authentication credential (or handling it to the coercer), resulting in a non-counted ballot. Ballots cast with the real credential are counted. These schemes provide the real authentication credential to the voter during registration phase (in which the coercer must be absent). The voter must securely store this authentication credentials for later use, i.e., voters need to maintain cryptographic state.

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### 3 System and threat model

**Actors.** There are five actors in VOTEAGAIN: voters, a polling authority, a bulletin board, a tally server, and trustees.

- **Voters** Voters interact with the polling authority and the public bulletin board to cast their ballots. Each voter has the means to authenticate herself to the polling authority (e.g., an electronic identity card). There are \(n\) voters.

- **Polling Authority (PA)** The PA authenticates users and provides them with ephemeral voting tokens. Voters use these tokens to sign their ballots before posting them to the public bulletin board.

- **Public Bulletin Board (PBB)** The PBB is an append-only list

| Table 1: Comparison of different voting schemes. |
|-----------------------------------------------|
| **Reving** | **Security Properties** |
| **Durability** | **Verif. Filter** | **Complexity** | **Crypto State** | **Authentication** | **Ballot Privacy** | **Verifiability** | **Coercion Res.** |
| JCI [10, 13, 31] | No \(^1\) | Yes | \(n^2\) | Yes | \(k\)-out-of-\(t\) | \(k\)-out-of-\(t\) | \(k\)-out-of-\(t\) + AC |
| Black-box [23] | TTP | No | \(n\) | Yes | Unclear | TTP | TTP |
| Revote [1, 32] | \(k\)-out-of-\(t\) | Yes | \(n^2\) | Yes | TTP | \(k\)-out-of-\(t\) | \(k\)-out-of-\(t\) + AC |
| Helios [2] | revote is not possible | No | TTP | N/A |
| VOTEAGAIN | TTP | Yes | \(n\log n\) | No | TTP | TTP |

\(^1\)Revoting is possible, but revotes are not deniable. JCI instead achieves coercion resistance using fake authentication credentials.
of cast ballots. Ballots are posted during the election phase by the voters. During the tally phase, the tally server and trustees post their proofs and results to the bulletin board. Ad-hoc implementations [27] or blockchain-based implementations [20, 21] would be suitable for our PBB.

Tally Server (TS) The TS filters the ballots. It adds dummy ballots, shuffles the ballots, groups them by voter, and selects the last ballot for each voter.

Trustees The trustees mix and decrypt the selected ballots to reveal the outcome of the election. Each trustee has a partial decryption key for a $k$-out-of-$t$ encryption system.

Threat model. We assume an adversary $A$ whose goal it is to coerce voters into casting votes for a particular candidate or to abstain. This adversary, although computationally bounded, may coerce any voter – but not all voters. Under coercion, the coerced voter does exactly as instructed (without needing to lie). The coerced voter learns all information stored and received by the voter at the time of coercion. We assume that after coercion the coerced voter does not control a voter for some time before the end of the election, such that the voter can cast at least one more vote. We also assume that the user’s means of authentication is inalienable [1], that is, a coerced voter can neither eliminate nor duplicate a voter’s means of authentication.

While these assumptions are strong, we point out that so are the assumptions behind coercion resistant solutions that rely on fake credentials [10, 13, 31] (see Table 2). Fake-credential based solutions assume that users cannot be coerced during registration and hence need inalienable means of authentication during this phase; that users can store and hide cryptographic key material and hence are required to have access to where this material is stored during the voting phase; and that users can lie convincingly. These assumptions are not needed in VOTEAGAIN. Our construction allows users to vote from any device, preventing coercion attacks that rely on destroying or stealing the voting device.

In VOTEAGAIN, voters authenticate against the PA every time before voting to obtain an ephemeral voting token. Thus, the PA must be honest with respect to verifiability and coercion resistance. To enable quasilinear filtering we also require that the TS is honest with respect to coercion resistance. This assumption is stronger than Achenbach et al.’s $k$-out-of-$t$ assumption on the trustees [1], but their relaxation comes at a quadratic computational cost, see Table 1.

Finally, we require VOTEAGAIN to satisfy the following informal properties. We formalize them in Section 6. Table 3 summarizes the trust required in each party for achieving each of the properties.

**Definition 1** (Ballot privacy [7]). Assuming that at least $k$ trustees are honest, no coalition of malicious parties (including the PA and TS) can learn the vote of an honest user.

**Definition 2** (Coercion resistance). Assuming that the PA and TS are honest, no coercer can use the PBB to determine if coercion was successful or not, provided that the election outcome does not leak this information.

**Definition 3** (Verifiability). Assuming that the PA is honest, VOTEAGAIN guarantees that: (i) the last ballot per voter will be tallied, (ii) adversary $A$ cannot include more malicious votes in the tally than the number of voters it controls, and (iii) honest ballots cannot be replaced. If voters do not verify that their ballots are correctly appended to the PBB, ballots can be dropped or replaced by earlier ballots if those exist.

### 4 VOTEAGAIN: High-level overview

We sketch the key ideas of VOTEAGAIN. For simplicity, we omit, in this section, the zero-knowledge proofs that parties use to show that they performed operations correctly. We describe the protocols in detail in Section 5.1.

VOTEAGAIN proceeds in three phases: the pre-election phase, the election phase, and the tally phase. During the pre-election phase, the polling authority (PA) assigns to each voter

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**Table 2: Comparison of assumptions in pre-election phase and election phase required to mitigate coercion attacks fake credentials and revoting based systems.**

| Assumptions                      | Fake Credentials | Revoting |
|----------------------------------|-------------------|----------|
| **Pre-election phase**           |                   |          |
| No coercion                      | ✓                  | N/A      |
| Inalienable authentication       | ✓                  | N/A      |
| **Election phase**               |                   |          |
| Lie convincingly                 | ✓                  | X        |
| Coercer absent some point during election | ✓       | ✓        |
| Absence of coercer after coercion | X               | ✓        |
| Device holding voting secrets or need to remember special pwds | ✓     | X        |
| Inalienable authentication       | X                  | ✓        |

**Table 3: Trust assumptions on VOTEAGAIN entities to achieve each property.**

|                      | Ballot Privacy | Verifiability | Coercion resistance |
|----------------------|----------------|---------------|---------------------|
| PA                   | Untrusted      | Trusted       | Trusted             |
| TS                   | Untrusted      | Untrusted     | Trusted             |
| PBB                  | Untrusted      | Untrusted     | Untrusted           |
| Trustees             | $k$-out-of-$t$ | Untrusted     | Untrusted           |
i a random voter identifier \( \text{vid}_i \), and a random initial ballot index \( m_i \). These values are known only to the PA.

**Casting ballots.** During the election phase, voters can cast as many votes as they want. To cast a vote, voter \( i \) first authenticates to the PA using her inalienable authentication means to obtain an ephemeral voting token. This voting token contains an encrypted voter identifier \( \gamma \), containing \( \text{vid}_i \), and an encrypted ballot index \( I \), containing \( m_i \). After each authentication, the PA increases \( m_i \) by one. Next, the voter encrypts her choice of candidate as \( v \). Finally, the voter sends the encrypted vote \( v \), the encrypted voter identifier \( \gamma \), the encrypted ballot number \( I \), and a signature using the ephemeral token to the bulletin board.

**Filtering ballots.** The encrypted voter identifiers and ballot indices enable the tally server (TS) to efficiently select the last ballot for each voter. The TS uses the simplest mechanism possible: It shuffles the ballots, and then decrypts the voter identifiers and ballot indices. The ballots can then publicly be grouped per voter, and the last ballot can be identified by inspection. Finally, the trustees tally the last ballot of each voter. See Figure 1.

**Hiding patterns using dummies.** By itself, shuffling and filtering is not a coercion-resistant mechanism: a coercer can perform the 1009 attack [38]. In this attack, the coercer forces a voter to cast a specific number of ballots and looks for a group of that size in the filtering step. If such group does not exist, the coerced voter has revoted. In \textsc{VoteAgain}, the TS inserts a deterministic number of dummy ballots and dummy voters before shuffling the ballots to hide such patterns while maintaining the simple public filtering procedure.

We illustrate our dummy mechanism in Figure 2, in a scenario with two voters (\( A \) and \( B \)) where, the coercer forces voter \( A \) to cast 2 ballots. At the end of the election phase the coercer observes 4 ballots and must determine whether \( A \) revoted (situation 2) or not (situation 1). Without dummies, distinguishing these situations is trivial: if \( A \) revoted there is a group of 3 ballots and one of 1 ballot, and there are two groups of 2 ballots otherwise. We add dummy ballots and voters to make both situations look identical. The idea is to find a \textit{cover} of ballots that could result from both situations. For instance, adding to either situation two dummy voters that cast four dummy ballots total yields groups of 1, 2, 2, and 3 ballots.

This observation makes both situations indistinguishable for the coercer (Figure 2, right).

To ensure that the cover is \textit{independent} from the voters’ real actions, its appearance must depend \textit{only} on the information available to the coercer: (1) the number of ballots \( n_B \) posted by users to the bulletin board; and (2) the number of voters \( v \) that cast a ballot. The goal of the dummy generation strategy is to allocate dummy ballots such that the adversary observes the \textit{same cover} regardless of the actual distributions of the \( n_B \) ballots over \( v \) voters.

Consider the case of two voters, i.e., \( v = 2 \), and 9 ballots, i.e., \( n_B = 9 \). As the filtering stage only reveals the sizes of the groupings and not their relation to voters the possible adversary’s observations are \( (1, 8), (2, 7), (3, 6), \) and \( (4, 5) \). To cover all these scenarios one needs 8 voters (6 of which are dummy) casting 1, 2, 3, 4, 5, 6, 7, and 8 ballots, for a total of \( 36 − 9 = 27 \) dummy ballots.

We add dummy ballots to real voters as well to reduce the number of group sizes that are possible. For example, in the previous scenario one can pad the cases \( (1, 8), (2, 7), (3, 6), (4, 5) \) to \( (1, 8), (2, 8), (4, 8), (4, 8) \). This can be covered with a cover containing voters with 1, 2, 4, 8 ballots each. Building this cover requires only 2 dummy voters and \( 15 − 9 = 6 \) dummy ballots. We stress that the \textit{number of added dummy ballots is independent of how the real ballots are actually distributed among the two voters}.

We refer to Section 5.2 for a generic and efficient algorithm for computing a cover.

**Filtering with dummies.** Before shuffling the ballots, the TS adds dummy ballots to achieve the desired grouping. We must ensure, however, that the TS cannot modify the election outcome. To this end, the TS tags real and dummy ballots with a different encrypted tag.

To determine how to add dummies, the TS inspects the decrypted voter identifiers and ballot indices; determines a cover; and then computes how many dummies to add to exist-
existing voter 531, and two dummy voters, 74 and 103, each with one dummy vote.

After adding the dummy ballots, the TS shuffles all ballots. Next, the TS decrypts the voter identifiers and ballot indices; groups ballots per voter, and selects the last ballot per voter. The tags enable the TS to prove that it did not omit real ballots cast by real voters, and it did not count dummy votes cast by dummy voters. In particular, the TS proves in zero-knowledge that the selected votes are either tagged as a real vote and therefore must correspond to the last ballot of a real voter; or the selected vote corresponds to a dummy voter (i.e., all the ballots in the group are tagged as dummies). Finally, the TS privately discards the selected votes corresponding to dummy voters. We refer to Section 5.1 for the full details.

**Design choices.** Obtaining coercion resistance require strong assumptions on some of the parties. In this section we discuss our design choices and motivate our trust assumptions (see Table 2 for a comparison with other protocols). First, we believe that revoting is an easy to understand solution to achieve coercion resistance. It requires no extra devices, no memorization, no interaction with several entities during registration, and no lying. For instance, Estonian elections have used a revoting model for years [28] with 44% of the electorate having used internet voting [17]. Second, it does not require voters to securely store cryptographic material, allowing a vote cast from any device. This further reduces the possibility of coercion attacks by confiscating the credential storage device.

Coercion resistance requires absence of the coercer at some point during the process. Fake-credential solutions assume that the coercer is absent during registration and at some point during the voting phase. Revoting, instead, assumes that a voter will have time after the coercion to cast the last vote. In the case of a remote registration process, a targeted attack will most likely succeed in both scenarios. However, attacks scale much better in the fake-credential setting: coercers have the entire registration period (e.g., 24 days in Spain) to coerce a voter. In contrast, coercers in the revoting setting must monitor all coerced voters after coercion to prevent them from revoting before the election closes.

We decide to trade-off trust with respect to coercion resistance on the PA and TS to obtain high gains in usability and efficiency: trust on the PA relieves users from keeping cryptographic state; and trust on the TS enables VOTEAGAIN’s quasilinear filtering of ballots.

### 5 The VOTEAGAIN voting scheme

**Preliminaries.** Let \( \ell \) be a security parameter. Let \( \mathbb{G} \) be a cyclic group of prime order \( p \) generated by generator \( g \). We write \( \mathbb{Z}_p \) for the integers modulo \( p \). We write \( a \in_R A \) to denote that \( a \) is chosen uniformly at random from the set \( A \).

VOTEAGAIN uses the ElGamal’s encryption scheme given by: A key generation algorithm \( \text{EC.KeyGen}(\mathbb{G}, g, p) \) which outputs a public-private key-pair \((pk=g^{a\sk}, \sk)\) for \( \sk \in_R \mathbb{Z}_p \); an encryption function \( \text{EC.Enc}(pk, m) \) which takes as input a public key \( pk \) and a message \( m \in \mathbb{G} \) and returns a ciphertext \( c=(c_1, c_2) = (g^r, m \cdot pk^r) \) for \( r \in_R \mathbb{Z}_p \); and an decryption algorithm \( \text{EC.Dec}(\sk, c) \) which returns the message \( m = c_2 \cdot c_1^{-\sk} \).

VOTEAGAIN uses deterministic encryption (with randomness zero) as a cheap verifiable ‘encoding’ for the ballot tags. Because the encryption is deterministic, verifiers can cheaply check that the encrypted tags have been correctly formed.

We use a traditional signature scheme given by: A key generation algorithm \( \text{Sig.KeyGen}(1^\ell) \) that generates a public-private key-pair \((pk_{\sigma}, sk_{\sigma})\); a signing algorithm \( \sigma = \text{Sig.Sign}(sk_{\sigma}, m) \) that signs messages \( m \in \{0, 1\}^* \); and a verification algorithm \( \text{Sig.Verify}(pk_{\sigma}, \sigma, m) \) that outputs \( \top \) if \( \sigma \) is a valid signature on \( m \) and \( \bot \) otherwise.

We use verifiable shuffles [4] to support coercion resistance in a private way. These enable an entity to verifiably shuffle
of a list of homomorphic ciphertexts in such a way that it is infeasible for a computationally bounded adversary to match input and output ciphertexts.

VoteAgain uses mixnets, a standard approach [9, 30, 35] to compute the election result given the filtered ballots output by the TS. The trustees jointly run the Vote.DKeyGen(1\(^{\ell}, k, t, n_c\)) protocol where \(\ell\) is the security parameter, \(k\), \(n_c\) the number of candidates, \(t\) the number of trustees, and \(k\) is the number of trustees needed to decrypt ciphertexts. This protocol outputs a public encryption key \(pk_T\) and each trustee \(i\) obtains a private decryption key \(sk_{T,i}\). To encrypt her vote for candidate \(c\), a voter calls \((v, \pi) = \text{Vote.Enc}(pk_T, c)\) to obtain an encrypted vote \(v\) and proof \(\pi\) that \(v\) encrypts a choice for a valid candidate. We denote the encryption of the zero candidate (i.e. no candidate) with explicit randomizer \(r \in_R \mathbb{Z}_p\) by \(\text{Vote.ZEnc}(pk_T; r)\). The algorithm Vote.Verify(\(pk_T, v, \pi\)) outputs \(1\) if the encrypted vote \(v\) is correct, and \(0\) otherwise. Given a list of selected votes \(\{V_1, \ldots, V_k\}\), the trustees jointly run the \((r, \Pi) \leftarrow \text{Vote.MixDecryptTally}(pk_T, \{V_1, \ldots, V_k\})\) protocol to compute the election result \(r\) and a proof of correctness \(\Pi\). Internally, Vote.MixDecryptTally uses a standard verifiable mix network and verifiable decryption to shuffle and decrypt the ballots, and then computes the final result in the clear. Any verifier can run Vote.VerifyTally(\(pk_T, \{V_1, \ldots, V_k\}, r, \Pi\)) to verify whether the result \(r\) is computed correctly.

The TS uses standard zero-knowledge proofs of knowledge [24] to prove that it operated correctly. We use the Fiat-Shamir heuristic [19] to convert them into non-interactive proofs of knowledge. We adopt the Camenisch-Stadler notation [11] to denote such proofs and write, for example,

\[
\text{SPK}\{(sk : pk = g_0^m \land m = \text{EC.Dec}(sk, I)}\}
\]

to denote the non-interactive signature proof of knowledge that the prover knows the private key \(sk\) corresponding to \(pk\) and that \(I\) decrypts to \(m\) under \(sk\).

### 5.1 VoteAgain description

VoteAgain proceeds in three phases: the pre-election phase, the election phase, and the tally phase. See Table 4 for a summary of frequently used symbols.

#### 5.1.1 Pre-election phase

In the pre-election phase, the PBB publishes the candidates, and the TS and the trustees prepare their cryptographic material. The PA assigns a unique, random voter identifier \(vid\) to each eligible voter. The correspondence between voters and their identifiers is private to the PA. The PA also generates a random token index \(m_i\) for each of the voters to enable the selection of the last ballot per voter. More formally:

**Procedure 1 (Setup).** To setup an election system with security parameter \(\ell\), electoral roll \(E\), candidate list \(C\), threshold \(k\), and \(t\) trustees, the different entities run the Setup(\(1^{\ell}, E, C, k, t\)) procedure. First, they pick a group \(G\) with generator \(g\) and prime order \(p\). They then proceed with the following steps:

1. The PBB initializes the bulletin board, and adds the list of candidates \(C\) to the bulletin board.
2. The PA stores the electoral roll \(E\). Let \(N\) be the number of eligible voters on the electoral roll. The PA generates a random and unique voter identifier \(vid\) to each eligible voter. The correspondence between voters and their identifiers is private to the PA. The PA also generates a random token index \(m_i\) for each of the voters to enable the selection of the last ballot per voter. They then proceed with the following steps:

   a. The PBB initializes the bulletin board, and adds the list of candidates \(C\) to the bulletin board.
   b. The PA stores the electoral roll \(E\). Let \(N\) be the number of eligible voters on the electoral roll. The PA generates a random and unique voter identifier \(vid\) to each eligible voter. The correspondence between voters and their identifiers is private to the PA. The PA also generates a random token index \(m_i\) for each of the voters to enable the selection of the last ballot per voter. They then proceed with the following steps:
   c. Procedure 1 (Setup). To setup an election system with security parameter \(\ell\), electoral roll \(E\), candidate list \(C\), threshold \(k\), and \(t\) trustees, the different entities run the Setup(\(1^{\ell}, E, C, k, t\)) procedure. First, they pick a group \(G\) with generator \(g\) and prime order \(p\). They then proceed with the following steps:

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Procedure 2 (GetToken(id, Auth)). On input her identity id and her inalienable means of authentication Auth:

1. The voter authenticates to the PA using Auth.
2. The PA looks up the corresponding voter identifier vid and ballot index $m_i$. Then, the PA encrypts the voter identifier $\gamma = \text{EC.Enc}(pk_{TS}, vid)$ and ballot number $I = \text{EC.Enc}(pk_{TS}, m_i)$ (it first encodes $m_i$ as an element of $G$), and increments the ballot index $m_i := m_i + 1$. The PA hides the index $m_i$ from the user to prevent coercers – who can see what users can see under coercion – from being able to detect whether the user revoted.
3. The PA creates an ephemeral signing key $(pk, sk) = \text{Sig.Keygen}()$ and signs this key together with the encrypted voter identifier and ballot number:

\[
\sigma^* = \text{Sig.Sign}(sk_{PA}, pk \| \gamma \| I)
\]

and returns the token $\tau = (pk, sk, \gamma, I, \sigma^*)$ to the user.
4. The user verifies the token $\tau = (pk, sk, \gamma, I, \sigma^*)$ by checking that $\text{Sig.Verify}(sk_{PA}, \sigma^*, pk \| \gamma \| I) = \top$.

Procedure 3 (Vote(τ, c)). To cast a vote, the voter takes as private input the ephemeral voting token $\tau = (pk, sk, \gamma, I, \sigma^*)$ and a candidate $c \in \mathcal{C}$, and then proceeds as follows:

1. Encrypts her candidate $\beta = (v, \pi, pk_{TS}, \gamma, I, \sigma^*, \sigma)$ where $\sigma = \text{Sig.Sign}(sk, v \| \pi \parallel pk \| \gamma \parallel I \| \sigma^*)$. The voter posts the ballot $\beta$ to the bulletin board.
2. The public bulletin board runs Valid($\beta$), see below, to check that the ballot is valid, before appending it.
3. Finally, the voter verifies that the ballot $\beta$ has been appended to the bulletin board.

Procedure 4 (Valid(β)). The bulletin board verifies that the ballot $\beta = (v, \pi, pk_{TS}, \gamma, I, \sigma^*, \sigma)$ is valid with respect to the current state of the bulletin board as follows:

1. The PBB checks the correctness of the encrypted vote; of the user’s signature using the ephemeral key $pk$; and the PA’s signature on this ephemeral key $pk$, the encrypted voter identifier $\gamma$, and the encrypted ballot number $I$:

\[
\text{Vote.Verify}(pk_{TS}, v, \pi) = \top \quad \text{Sig.Verify}(pk_{TS}, \gamma, v \| \pi \| pk \| \gamma \| I \| \sigma^*) = \top \quad \text{Sig.Verify}(pk_{PA}, \sigma^*, pk \| \gamma \| I) = \top.
\]

2. The PBB checks that neither the encrypted vote $v$ nor the key $pk$ appear in any ballot $\beta'$ on the bulletin board. If any of these checks fails, the bulletin board returns $\bot$, otherwise, the PBB returns $\top$.

5.1.3 Tally phase

In the tally phase (see Figure 5), the TS takes the ballots from the PBB, adds dummy ballots, and shuffles them. Then, it selects the last vote per voter (see Figure 6). Then, to prevent dummy voters from making an overhead in the shuffle and decrypt phase, it shuffles the selected ballots and removes all ballots cast by dummy voters. Finally, the trustees shuffle and decrypt the selected ballots from real voters. Formally, we define two procedures, one to filter votes (Filter), and one to tally the selected ballots (Tally):

Procedure 5 (Filter). After the election closes, the TS selects the selected votes $V_i$ and produces the filter proof $\Phi$. If it aborts, it publishes the current $\Phi$ to the public bulletin board.

1. The tally server (TS) retrieves an ordered list of ballots $[\beta_1, \ldots, \beta_n]$ from the PBB, where $\beta_i = (v_i, \pi_i, pk_{TS}, \gamma_i, I_i, \sigma_i)$. The TS verifies the ballots by running step 1 of Valid and verifies that there are no duplicate votes $v_i$ or ephemeral public keys $pk_i$ on the bulletin board. If any of these checks fails, the TS sets $\Phi = \bot$, posts it to the bulletin board, and aborts.
2. The TS removes the proofs and signatures to obtain stripped ballots. It provably tags the ballots as ‘real’ ballots using a deterministic ElGamal encryption (with randomness zero) of the value $g^0 = 1_G$, $\theta_R = \text{EC.Enc}(pk_{TS}, g^0) = (g^0, g^0pk_{TS}) = (1_G, 1_G)$:

\[
\beta'_i = (v_i, \gamma_i, I_i, \theta_R).
\]
Next, the TS creates $n_D$ dummy ballots and provably tags them as such using a deterministic ElGamal encryption of the value $g$, $\theta_D = \text{EC.Enc}(pk_{TS}, g) = (1_g, g \cdot pk^7)$:

$$\beta'_i = (v_e, \gamma_i, I, \theta_D),$$

where $i < n_B$ and $v_e = \text{Vote.ZEnc}(pk_T, 0)$. We explain below how the TS determines the number of dummies $n_D$ as well as the values for $\gamma_i$ and $I$. The TS adds the stripped ballots $B' = [\beta'_1, \ldots, \beta'_{n_B+n_D}]$ to $\Phi$.

3. The TS shuffles the stripped ballots $B' = [\beta'_1, \ldots, \beta'_{n_B+n_D}]$ and randomizes the ciphertexts, to obtain a list of shuffled and randomized stripped ballots $B'' = [\beta''_1, \ldots, \beta''_{n_B+n_D}]$, which it adds, together with a proof $\pi_{\Phi}$ that this shuffle was performed correctly, to $\Phi$.

4. The TS now operates on each shuffled ballot $\beta''_i = (v'_i, \gamma'_i, I', \theta'_{I'})$. It decrypts $\gamma'_i$ to recover the shuffled and decrypted identifier, $\overline{vid}_i$. It also decrypts $I'$ to obtain the shuffled ballot index $\overline{m}_i$ and proves it did so correctly:

$$\overline{m}_i = \text{SPK}(\overline{vid}_i, \overline{m}_i, \gamma_i^{\text{dec}}),$$

$$\overline{vid}_i = \text{EC.Dec}(sk_{TS}, \gamma'_i) \land \overline{m}_i = \text{EC.Enc}(sk_{TS}, I'_i).$$

It then adds $C = [(\overline{vid}_1, \overline{m}_1, \gamma_1^{\text{dec}}), \ldots, (\overline{vid}_{n_B+n_D}, \gamma_{n_B+n_D}^{\text{dec}})]$ to $\Phi$. The TS aborts and adds $\perp$ to $\Phi$ if the decrypted ballot indices $\overline{m}_i$ are not unique for a given voter identifier. More precisely, it aborts if there exists indices $i, j, i \neq j$ such that $(\overline{vid}_i, \overline{m}_i) = (\overline{vid}_j, \overline{m}_j)$.

5. The TS groups the ballots with the same voter identifier, and selects the ballot with the highest ballot index from each group. Let $G_1, \ldots, G_k$ be the sets of ballot indices grouped by voter identifier. Consider group $G_j$ of size $\chi_j$. Let $j = \arg\max_{k \in G_j} \overline{m}_k$ be the index for which the ballot index $\overline{m}_j$ is maximal. Group $G_j$ either corresponds to a real voter, or to a fake voter. The TS produces a reencryption $\overline{v}_j$ of the encrypted votes as follows:

(a) If the group $G_j$ corresponds to a real voter, then the TS simply reencrypts the vote corresponding to the last ballot, i.e., it picks $r_j$ at random and sets $\overline{v}_j = v_{j_s} \cdot \text{Vote.ZEnc}(pk_T; r_j)$, to a randomized encryption of $v_{j_s}$.

(b) If the group $G_j$ corresponds to a fake voter, then picks $r_j$ at random and sets $\overline{v}_j$ to an empty vote: $\overline{v}_j = \text{Vote.ZEnc}(pk_T; r_j)$.

The TS proves that it computed the $\overline{v}_j$ correctly. If the corresponding voter is real, then the ballot $\beta''_j$ selected in (a) should be a real ballot, so EC.Dec$(sk_{TS}, \theta'_{I''_j})$ should equal $g^0$. If the voter is fake, then for all tags $\theta'_{I''_k}$ with $k \in G_j$, we have that $\text{EC.Enc}(sk_{TS}, \theta'_{I''_k}) = g^1$. Let $G_j = \{i_1, \ldots, i_{\chi_j}\}$ and $\theta = \prod_{k=1}^{\chi_j} \theta'_{I''_k}$, then the TS constructs the proof

$$\pi_{\sigma}^{\text{sel}} = \text{SPK}(r_j, \text{sk}_{TS}) : \text{PK}_{TS} = g^{\text{sk}_{TS}} \land ((g^0 = \text{EC.Dec}(sk_{TS}, \theta'_j)) \land \overline{v}_j = v_{j_s} \cdot \text{Vote.ZEnc}(pk_T; r_j)) \lor (g^1 = \text{EC.Enc}(sk_{TS}, \theta) \land \overline{v}_j = \text{Vote.ZEnc}(pk_T; r_j))) \to \Phi.$$

The TS adds the list of filtered encrypted votes $F = [(\text{vid}_1, \overline{v}_1, \pi_1^{\text{sel}}), \ldots, (\text{vid}_k, \overline{v}_k, \pi_k^{\text{sel}})]$ to $\Phi$.

6. The list $S_D = [\overline{V}_1, \ldots, \overline{V}_k]$ of selected votes contains ballots by dummy voters. In the next two steps, the TS removes these. First, the TS shuffles and randomizes the ciphertexts to obtain a new list $S'_D = [\overline{V}'_1, \ldots, \overline{V}'_k]$, which it adds, together with a proof $\pi_S$ of correct shuffle, to $\Phi$.

7. The TS knows the indices $D$ of votes in $S'_D$ that correspond to dummy voters and randomizers $r_i$ such that $\overline{v}_i = \text{Vote.ZEnc}(pk_T; r_i)$ for $i \in D$. The TS adds $D$ and $R = [r_i]_{i \in D}$ to $\Phi$.

8. Finally, the TS publishes the remaining votes $S = [V_1, \ldots, V_n]$ and the full proof $\Phi$ to the public bulletin board.
The filter procedure ensures that the TS cannot replace ballots by real voters: a selected vote must either correspond to a ballot by a real voter (condition a) or the selected vote is empty and the voter is a dummy voter (condition b). Moreover, the TS can only remove votes cast by dummy voters.

**Procedure 6 (Tally).** To compute the final tally, the trustees proceed as follows:

1. The trustees verify that the TS operated honestly by running the $\text{VerifyFilter}()$ algorithm (see below). If $\text{VerifyFilter}$ returns $\perp$ they return $(r, \Pi) = (\perp, \perp)$.

2. Let $\mathcal{S} = [V_1, \ldots, V_n]$. The trustees jointly run the $(r, \Pi) \leftarrow \text{Vote.MixDecryptTally}(pk_{\text{S}}, \mathcal{S})$. They publish the election result $r$ and the zero knowledge proof of correctness $\Pi$ to the public bulletin board.

### 5.1.4 Verification

Any external auditor can use the PBB to verify that all steps in the tally and filtering phases were performed correctly. We define the following verification procedures:

**Procedure 7 (VerifyFilter).** Any party can verify that the filtering processes was performed correctly by running $\text{VerifyFilter}()$. This algorithm examines the content of the bulletin board and performs the following checks:

1. First, check if all ballots are correct and that no duplicate votes or public keys are included in the ballots as per step 1 of Filter. If the checks fail, the bulletin board should contain $\Phi = \perp$; $\text{VerifyFilter}$ returns $\perp$ if that is not the case. Otherwise, it continues.

2. It next retrieves the selected votes $\mathcal{S}$ and the proof $\Phi$ from the bulletin board and continues as follows:

   (a) Verify that stripped real ballots are correctly formed. Consider ballots $[\beta_1, \ldots, \beta_{nb}]$, where $\beta_i = (v_i, \pi_i, pk_i, \gamma_i, I_i, \sigma_i^*, \sigma_i)$ and check that the stripped ballot $\beta'_i = (v_i, \gamma_i, I_i, \theta_R)$ has been added to $\Phi$ (where $\theta_R$ is as above).

   (b) Verify that the dummy ballots on the bulletin board are correctly formed. For ballots $[\beta_{nb+1}, \ldots, \beta_{nb+n_D}]$ where $\beta'_i = (v_i, \gamma_i, I_i, \theta_R)$, check that $v_i = v_e$ and $\theta_i = \theta_D$ (where $v_e$ and $\theta_D$ are as above).

   (c) Let $B' = [\beta'_1, \ldots, \beta'_{nb+n_D}]$ be all stripped ballots, and $B'' = [\beta''_1, \ldots, \beta''_{nb+n_D}]$ the shuffled and randomized ballots. Verify the shuffle proof $\pi_\sigma$ to check that $B''$ is a correct shuffle of $B'$.

   (d) Next, let $\mathcal{C} = \{[\text{vid}_1, m_1, \pi_1^{\text{dec}}], \ldots, [\text{vid}_{nb+n_D}, m_{nb+n_D}, \pi_{nb+n_D}^{\text{dec}}]\}$ from the bulletin board, and verify the decryption proofs $\pi_i^{\text{dec}}$ for each of the shuffled ballots $\beta''_i$. Let $\text{vid}_i$ and $m'_i$ be the plaintexts verified in the previous step. Group the ballots by voter identifier into ballot groups $G_j$. For each group $G_j$, find ballot $\beta_{j,k}$ with the highest ballot index, recompute $\tau = \prod_{k=1}^{j} \tau_k$, and verify the reencryption proof $\pi_{e_{jk}}^{\text{dec}}$.

   (f) Let $\mathcal{S}_D$ be the selected votes $[\mathcal{V}_1, \ldots, \mathcal{V}_k]$ and $\mathcal{S}'_D = [\mathcal{V}'_1, \ldots, \mathcal{V}'_k]$ the shuffled and randomized votes. Verify the shuffle proof $\pi_\sigma$ for $\mathcal{S}_D$ and $\mathcal{S}'_D$.

   (g) Finally, for each $i \in \mathcal{D}$ verify that $\mathcal{V}'_i = \text{Vote.ZEnc}(pk_{\text{S}}, r_i)$ and that $\mathcal{S} = [\mathcal{S}_D[i] \mid i \notin \mathcal{D}]$.

If any of the checks fail, it returns $\perp$, and $\top$ otherwise.

**Procedure 8 (Verify).** Any party can verify the result $r$ and proof $\Pi$ against the public bulletin board. To do so, they proceed as follows:

1. Verify that the TS operated honestly by running the $\text{VerifyFilter}()$ algorithm. If $\text{VerifyFilter}$ returns $\perp$, then return $\top$ if $(r, \Pi) = (\perp, \perp)$, otherwise return $\perp$.

2. Given the selected votes $\mathcal{S}$, return the result of $\text{Vote.VerifyTally}(pk_{\text{S}}, \mathcal{S}, r, \Pi)$.

### 5.2 Hiding revoting patterns with dummies

In this section we provide a formal description of the dummy generation algorithm introduced in Section 4.

**Finding a cover.** Formally, a cover is a set $\mathcal{C} = \{(s_i, z_i)\}_i$ formed by groupings $(s_i, z_i) \in \mathbb{Z}^+ \times \mathbb{Z}^+$. Here, $s_i$ is the size of the ballot groups within that grouping, and $z_i$ is the upper bound on the number of times that such a ballot group can occur in any distribution of the $n_B$ real ballots among real voters. We aim to find a cover of minimal size $|\mathcal{C}| = \sum s_i \cdot z_i$ to minimize the number of dummies added.

**A sufficient cover.** We derive an upper bound on the amount of dummies required to build a cover. We do not use the number of real voters for this bound. Let $n_B$ be the number of real ballots on the PBB. For simplicity, assume padded group sizes are powers of two, i.e., $s_i = 2^i$ for $i \geq 0$. Given $n_B$ ballots, any distribution can have at most $z_0 = n_B$ groups of size $s_0 = 1$ (one ballot per voter). Similarly, any distribution can have at most $z_1 = \lfloor n_B / 2 \rfloor$ groups of size $s_1 = 2$. Recall we pad ballot groups to the next bigger size, so a ballot group of 3 would be padded to one of size $s_2 = 4$ ballots, therefore $z_2 = \lfloor n_B / 3 \rfloor$. More generally, there can be at most $z_i = \lfloor n_B / (2^i-1) + 1 \rfloor$ groups of $s_i = 2^i$ ballots. The biggest possible group (if all ballots were cast by the same voter), has size $2^{\lceil \log_2 n_B \rceil}$.

Therefore, the size of the cover $|\mathcal{C}|$ is bounded by:

$$|\mathcal{C}| \leq \sum_{i=0}^{\log_2 n_B} z_i \cdot s_i = n_B + \sum_{i=1}^{\log_2 n_B} 2^i \left\lfloor \frac{n_B}{2^{i-1} + 1} \right\rfloor \leq n_B + \sum_{i=1}^{\log_2 n_B} \frac{2^i}{2^{i-1} + 1} n_B \leq n_B + \sum_{i=1}^{\log_2 n_B} 2 n_B = (1 + 2^{\lceil \log_2 n_B \rceil}) n_B.$$

**An efficient cover.** Knowing the number of real voters $v$ enables to obtain a tighter cover. Consider the example of Sec-
tion 4 with \( v = 2 \) and \( n_B = 9 \). If we only consider \( n_B = 9 \), one of the possible distributions of votes would be having \( s_1 = \lceil 9/2 \rceil = 4 \) groups of size 2. However, knowing \( v = 2 \) rules out this possibility. There can be at most one group of size two: if there were 2 groups, each of the 2 voters could only cast 2 ballots, i.e., 4 ballots in total. However, we know there are 9 ballots so at least one voter has voted more than twice, implying that \( s_1 = 1 \).

When the number of ballots grows this reasoning becomes intractable. Consider ballot groups with group sizes, \( s = k' \) for \( i \in [0, \ldots, \lceil \log_B n_B \rceil] \) for a real number \( k > 1 \). We assume that \( n_B > v \), otherwise the cover would be trivial: \( C = \{(s_0 = 1, z_0 = v)\} \). We compute the cover as follows.

1. Consider groups of size \( s_0 = k^0 = 1 \). As \( n_B > v \), at least one voter must cast more than one ballot, resulting in \((s_0, z_0) = (1, v - 1)\).
2. Consider groups of size \( s_i = k^i \). We know that given \( n_B \), there can be at most \( \alpha_i = \lceil n_B/(k^{i+1} + 1) \rceil \) groups of size \( k^i \). The number of groups is also bound by the number of voters. If \( v \cdot s_i \geq n_B \) then all ballots can be assigned to the \( v \) voters given groups of maximum size \( s_i \), and we set \( v_i = v \), otherwise set \( v_i = v - 1 \) so that one voter is not in this grouping. Finally, we need at least \( z_i(k^{i+1} + 1) \) ballots to make \( z_i \) groups, but we must have enough ballots left over to make \( v \) groups in total, i.e., \( n_B \geq z_i(k^{i+1} + 1) + (v - z_i) \). Rewriting gives bound \( \beta_i = \lceil (n_B - v)/k^{i+1} \rceil \). We set \( z_i = \min(\alpha_i, v_i, \beta_i) \).

Assuming \( n_B > v \), the cover has \(|C| = \sum_{i=0}^{\lceil \log_B n_B \rceil} z_i s_i > n_B\) ballots, necessitating dummy ballots, and \( \sum_{i=1}^{\lceil \log_B n_B \rceil} z_i > v \) groups, necessitating dummy voters.

**Creating dummy voters and allocating dummy ballots.** The TS recovers all voter identifiers \( vid \) by decrypting the \( \gamma_s \), and the corresponding ballot indices by decrypting the \( \lambda_s \).

So far, we assumed that ballot index sequences are continuous. However, there can be gaps if some tokens were not used (e.g., the coercer does not use some tokens to identify index gaps in the filtering phase). The TS first requests the number of obtained tokens \( n'_B \) from the PA, and adds exactly \( n'_B - n_B \) dummy ballots to fill up any gaps, such that \( n'_B \) equals the number of obtained tokens. The TS can create a dummy ballot for voter \( vid \) by setting \( \gamma = EC.Enc(\text{pk}_{\text{TS}}, vid) \).

Given the current number of ballots \( n'_B \) and the number of real voters \( v \) the TS computes a cover \( C = \{(s_i, z_i)\}_1 \). To this end the TS performs a search to find the best \( k \), i.e., the one that gives the smaller cover. In our experiments in Section 7, \( k \) tends to be in the 2 to 4 range, and the search takes less than a second. The TS performs the following steps:

1. For every voter \( \text{vid}_j, j \in \{1, \ldots, v\} \) with \( t \) ballots, let \((s_i, z_i) \in C\) be the cover group with the smallest size \( s_i \) such that \( s_i \geq t \). To ensure that dummy ballots are never counted, the TS adds \( t - s_i \) dummy votes to \( \text{vid}_j \) with descending (and unused) ballot counters smaller than the last cast vote by this voter.
2. For each grouping \((s_i, z_i) \in C\) let \( z'_i \) be the number of real voters that were assigned to this group. The TS adds \( z_i - z'_i \) dummy voters. For each dummy voter, it picks a random \( \text{vid} \) and initial ballot index \( m \) and creates \( s_i \) dummy ballots with increasing ballot indices.

In total, the TS adds \( n_D = |C| - n_B \) dummies. Given \( n_B \) ballots on the bulletin board, \( n_B + n_D = O(n_B \log n_B) \), see the upper bound above. As the filtering phases is linear in \( n_B + n_D \) the time complexity of \( O(n_B \log n_B) \) follows.

### 6 Security Analysis

We analyze VOTEAĞAİN’S ballot privacy, verifiability, and coercion resistance. We follow Bernhard et al. [7] and model the trustees as a single trusted party with keys \((\text{pk}_T, \text{sk}_T)\), but we note that the result holds when trustees are distributed. We explicitly model the bulletin board \( \text{BB} \) as an append only string. BB. To ease modeling, we use the following redefinition of our voting scheme \( \mathcal{V}' = (\text{Setup}, \text{GetToken}, \text{Vote}, \text{Valid}, \text{Filter}, \text{VerifyFilter}, \text{Tally}, \text{Verify}) \) where the algorithms output changes to the bulletin board rather than posting to it directly. While Bernhard et al. model voter registration implicitly, we make the registration step explicit using the \text{GetToken} function because it forms an integral part of our voting scheme and may happen more than once. The redefined algorithms in \( \mathcal{V}' \) are as follows:

- **Setup**\((I', \mathcal{E}, \mathcal{C})\) as in Setup in procedure 1 but explicitly returns the public key \( \text{pk} = (\text{pk}_{\text{PA}}, \text{pk}_{\text{ST}}, \text{pk}_T) \) and the corresponding private keys \( \text{sk}_{\text{PA}}, \text{sk}_{\text{TS}}, \text{sk}_T \).
- **GetToken**\((i)\) returns a token \( \tau \) as in \text{GetToken}() in procedure 2.
- **Vote**\((\tau, c)\) returns \( \beta \) as in \text{Vote}() in procedure 3 but does not post the ballot to the bulletin board. Moreover, the voter first verifies the token \( \tau \) as in step 4 of procedure 2, and returns \( \bot \) if it does not validate.
- **Valid**\((\text{BB}, \beta)\) returns the result of \text{Valid}() in procedure 4 with respect to the bulletin board \( \text{BB} \).
- **Filter**\((\text{BB}, n'_B, \text{sk}_{\text{TS}})\) as in Filter in procedure 5, but takes the number of registrations \( n'_B \) as explicit input, and returns \( S \parallel \Phi \) instead of adding them to the board.
- **VerifyFilter**\((\text{BB}', S, \Phi)\) runs \text{VerifyFilter} from procedure 7 on \( \text{BB}' = \text{BB} \parallel S \parallel \Phi \) and returns the result.
- **Tally**\((\text{BB}, \text{sk}_T)\) returns \( (r, \Pi) \) as in \text{Tally} in procedure 6.
- **Verify**\((\text{BB}, r, \Pi)\) as in \text{Verify} in procedure 8 operating on the bulletin board \( \text{BB} \parallel r \parallel \Pi \).

### 6.1 Ballot privacy

We base our ballot privacy definition on the game-based definition by Bernhard et al. [7]. They model ballot privacy using
\[ \text{Exp}_{A, \psi}^{\text{bpriv}, b}(\ell, E, C): \]
\[ (pk, sk_{\text{ts}}, sk_{\text{ts}}) \leftarrow \text{Setup}(1^\ell, E, C) \]
\[ b \leftarrow A_{\psi}(pk, sk_{\text{ts}}, sk_{\text{ts}}) \]
\[ \text{Output } b' \]
\[ \text{OvoteLR}(\tau, c_0, c_1): \]
\[ \text{Let } \beta_0 = \text{Vote}(\tau, c_0) \text{ and } \beta_1 = \text{Vote}(\tau, c_1) \]
\[ \text{If Valid}(BB_0, \beta_0) = \perp \text{ return } \perp \]
\[ \text{Else } BB_0 \leftarrow BB_0 \parallel \beta_0 \text{ and } BB_1 \leftarrow BB_1 \parallel \beta_1 \]
\[ \text{Occast}(\beta): \]
\[ \text{If Valid}(BB_0, \beta) = \perp \text{ return } \perp \]
\[ \text{Else } BB_0 \leftarrow BB_0 \parallel \beta \text{ and } BB_1 \leftarrow BB_1 \parallel \beta \]
\[ \text{Oboard}(): \]
\[ \text{return } BB_0 \]
\[ \text{Otally}(S, \Phi) \]
\[ \text{If VerifyFilter}(BB_b, S, \Phi) = \perp \text{ return } \perp \]
\[ BB_b \leftarrow BB_b \parallel S \parallel \Phi \]
\[ BB_1 \parallel b \leftarrow BB_1 \parallel b \text{ Filter}(BB_1 \parallel b, [BB_1 \parallel b], sk_{\text{ts}}) \]
\[ (r, \tau_0) \leftarrow \text{Tally}(BB_0, sk_{\text{ts}}) \]
\[ \tau_1 = \text{SimTally}(BB_1, r) \]
\[ \text{return } (r, \tau_1) \]

Figure 7: In the ballot privacy experiment \( \text{Exp}_{A, \psi}^{\text{bpriv}, b} \), the adversary \( A \) has access to the oracles \( O = \{ \text{OvoteLR}, \text{Occast}, \text{Oboard}, \text{Otally} \} \). The adversary controls the TS and the PA. It can call Otally only once.

Definition 4. Consider a voting scheme \( \psi = (\text{Setup}, \text{GetToken}, \text{Vote}, \text{Valid}, \text{Filter}, \text{VerifyFilter}, \text{Tally}, \text{Verify}) \) for an electoral roll \( E \) and candidate list \( C \). We say the scheme has ballot privacy if there exists an algorithm \( \text{SimTally} \) such that for all probabilistic polynomial time adversaries \( A \)

\[ \left| \Pr \left[ \text{Exp}_{A, \psi}^{\text{bpriv}, 0}(\ell, E, C) = 1 \right] - \Pr \left[ \text{Exp}_{A, \psi}^{\text{bpriv}, 1}(\ell, E, C) = 1 \right] \right| \]

is a negligible function in \( \ell \).

In Appendix A, we prove the following theorem.

**Theorem 1.** \( \text{VOTEAGAIN} \) provides ballot privacy under the DDH assumption in the random oracle model.

Bernhard et al. [7] also define strong consistency, to ensure that the result \( r \) does not leak information about individual ballots, and strong correctness to ensure that valid ballots are never refused by the bulletin board. We restate these notions and prove that \( \text{VOTEAGAIN} \) satisfies them in Appendix A.

### 6.2 Coercion resistance

Coercion resistance means that a coercer should not be able to determine whether a coerced user submitted to coercion – assuming it cannot learn this by seeing the result of the election (e.g., if there are zero votes for the selected candidate, the coercer knows the coerced user did not submit). In \( \text{VOTEAGAIN} \), this means that the coercer should not be able to determine whether a coerced user voted again, or not.

**Existing coercion resistant models are insufficient.** Juels, Catalano and Jakobsson (JCJ) model coercion resistance by comparing a real-world game with an ideal game [31]. In JCJ, voters evade coercion by providing the coercer with a fake credential. The real-world models normal execution. The adversary plays the role of the coercer and chooses a set of corrupted voters and identifies the coerced voter. Then, the honest voters cast their ballots (or abstain). If the coerced voter does not submit she also casts her true ballot. Thereafter, the adversary is given the credentials of all corrupt users, a credential for the coerced voter (which is fake if that voter resists), and the current bulletin board. The adversary can now cast more ballots. Upon seeing the result and the tally proof the adversary decides if the coerced voter submitted. In the ideal game, the adversary is not shown the content of the bulletin board, and she is given the true credential of the coerced voter and can therefore cast real ballots for the coerced voter. However, a modified tally function does not count ballots for the coerced voter cast by the adversary if the coerced voter resists. Once the election phase is over, the adversary is shown only the tally result, not the tally proof.

The JCJ model does not work for the revoting setting where the coerced voter casts another ballot after casting the ballot under coercion. Achenbach et al. [1] propose a variant in which the coerced voter acts after the adversary has cast his votes, revoting if she resists or doing nothing if she submits. Thereafter, the adversary is shown the new bulletin board and the resulting tally and proof. In the ideal model, the adversary is only provided the length of the bulletin board.
The model proposed by Achenbach et al. [1] does not capture coercion resistance. Following the real/ideal paradigm, in the ideal game it should hold with overwhelming probability that the adversary cannot distinguish between a submitting and a resisting coerced voter. Then, the proof would show that the adversary cannot learn more in the real world than it could in the ideal world. However, in the ideal game proposed by Achenbach et al., the coercion resistance property does not hold. The adversary can always distinguish between these two cases by simply observing the length of the bulletin board (which increases by one ballot if the coerced voter revotes). Therefore, any proofs in this model say nothing about whether the real scheme offers coercion resistance. While the Achenbach et al. scheme seems to be coercion resistant, coercion resistance does not follow from the proof in their model.

Finally, the model by Achenbach et al. does not capture the leakage resulting from the state kept by the voter, or as in our protocol, by the polling authority. Our protocol deliberately hides the ballot counter from the voter, so that when the coercer coerces the voter again, it cannot determine whether the coerced voter re-voted based on this counter. In Achenbach et al.’s model, the coercer cannot coerce a voter more than once.

A new coercion resistance definition. We propose a new game-based coercion resistance definition inspired by Bernard et al.’s ballot privacy definition. The game tracks two bulletin boards, BB₀ and BB₁, of which only one is accessible to the adversary (depending on the bit b). We ensure that regardless of the bit b, the same number of ballots are added to the bulletin board. The goal of the adversary is to determine b (see Figure 8). Recall that we assume that the PA, TS, and trustees are honest with respect to coercion resistance.

To model submits versus resists, we provide the adversary with an OvoteLR(ι₀,c₀,i₀,c₁) oracle to let voter ι₀, a “coerced” voter, cast a vote for candidate c₀ in BB₀, and voter i₁, any other voter, cast a vote for candidate c₁ in BB₁. The adversary is allowed to make this call multiple times. Regardless of the value of b, every call to OvoteLR results in a single ballot being added to each BB. This prevents the trivial win in the Achenbach et al. model. Since the polling authority keeps state, we work with two PAS: PA₀ and PA₁.

We model a coercion attack as follows. The adversary can cast votes using any user by calling Ogettoken(i) to obtain a voting token τ for voter i on the board that it can see, and a token τ’ for the other board. It can then run β = Vote(τ,c) and β’ = Vote(τ’,c) itself to create ballots for candidate c, on both boards and cast them using Ocast(β,β’). Note that per our assumptions, the adversary does not get access to the voter’s means of authentication. Moreover, we require that the adversary always casts valid ballots to both boards (but the encoded candidate need not be the same).

Finally, the adversary can make one call to Otally() which performs the filtering step and returns the result r (always computed on BB₀) and the tally proof. The result of Filter is accessible using Oboard. To correct for leakage stemming from the tally result, as in the ballot privacy game, we simulate the filter and tally proofs if the adversary sees BB₁.

This game models all the coercion attacks applicable to VoteAgain:

- **The 1009 attack.** The adversary casts a ballot as coerced voter ι₀ using τ, τ’ = Ogettoken(ι₀), β = Vote(τ,c), β’ = Vote(τ’,c) and then Ocast(β,β’) 1009 times. (Both boards now contain 1009 ballots by voter ι₀.) Then it calls OvoteLR(ι₀,c₀,i₁,c₁). If b = 0 the coerced voter revotes for candidate c on BB₀, otherwise it does not, and the alternative voter casts a ballot for candidate c on BB₁ visible to the adversary. Note that if the result of Filter Φ reveals the size of a group of ballots, the adversary can win this game (SimFilter does not model this leakage as it only gets n’ and r as input).

- **Returning coercer.** Let voter ι₀ be the coerced voter. First the coercer runs τ, τ’ = GetToken(ι₀), β = Vote(τ,c) and β’ = Vote(τ’,c), and Ocast(β,β’) to cast one vote as the
coercion user on both boards and to observe the token \( \tau \) corresponding to the board \( \mathbb{BB}_b \) it can see. Then it runs \( \text{VoteLR}(i_0, c_0, i_1, c_1) \), causing \( i_0 \) to cast a vote on the bulletin board \( \mathbb{BB}_b \) if \( b = 0 \), and \( i_1 \) to casts a vote on \( \mathbb{BB}_b \) if \( b = 1 \). Thereafter, it can examine the state by running \( \tau, \tau' = \text{GetToken}(i_0) \) again. If the new token \( \tau' \) leaks whether voter \( i_0 \) voted again (on board \( \mathbb{BB}_b \)), then the adversary wins the coercion resistance game.

**Definition 5.** Consider a voting scheme \( \mathcal{V}' = (\text{Setup}, \text{GetToken}, \text{Vote}, \text{Filter}, \text{VerifyFilter}, \text{Tally}, \text{Verify}) \) for an electoral roll \( \mathcal{E} \) and candidate list \( \mathcal{C} \). We say the scheme has **coercion resistance** if there exist algorithms \( \text{SimFilter} \) and \( \text{SimTally} \) such that for all probabilistic polynomial time adversaries \( \mathcal{A} \)

\[
\Pr\left[ \text{Exp}_{\mathcal{A}, \mathcal{V}}^\mathcal{V}(\ell, \mathcal{E}, \mathcal{C}) = 1 \right] - \Pr\left[ \text{Exp}_{\mathcal{A}, \mathcal{V}}^\mathcal{V}(\ell, \mathcal{E}, \mathcal{C}) = 1 \right] \text{ is a negligible function in } \ell.
\]

In Appendix B, we prove the following theorem.

**Theorem 2.** \texttt{VoteAgain} provides coercion resistance under the DDH assumption in the random oracle model.

### 6.3 Verifiability

In their analysis, Achenbach et al. [1] adapt the correctness definition of Juels et al. [31] to the revoting setting. However, Achenbach et al.’s model does not take into account that voters may not check that their ballots are cast correctly, nor that newer ballots should supersede older ballots even if voters have been coerced or corrupted. To address these cases, we adapt the qualitative game-based verifiability definition of Cortier et al. [14] – which accounts for a malicious bulletin board and voters not checking their ballots - to our setting by adding the GetToken function and explicitly modeling revoting. As in Cortier et al. [14], our game does not model voter’s intent, and assumes that the voting hardware, i.e., the device and software running Vote, is honest. We refer to Cortier et al. [15] for a formal process-based computational model that does model verifiability with voter intent. We note that the correctness definition by Juels et al. [31] was renamed to ‘verifiability’ by Cortier et al. [14], and therefore any model satisfying the latter satisfies the former.

In a nutshell, a voting scheme is verifiable [14] if for \( n_C \) corrupt voters, the result of the election always includes: (1) all votes by honest voters that verified whether their ballots were cast correctly, (2) at most \( n_C \) corrupted votes, and (3) a subset of the votes by honest voters that did not check if their ballots were cast correctly. These conditions ensure that while a malicious bulletin board can drop ballots of voters that do not check, it can insert at most \( n_C \) new votes.

**Extending the current verifiability definition.** We extend the definition presented by Cortier et al. [14] for the revoting setting to explicitly consider the number of votes cast by a voter, see Figure 9. The PA is honest, but the adversary controls the bulletin board, the TS, and the trustees. The system implicitly tracks the number of tokens \( \#\text{tokens}(i) \) that have been obtained by voter \( i \). The game tracks when each voter is corrupted in a (initially empty) list of corruption events \( \mathcal{C} \), and tracks the honest votes in \( \text{HVote} \). The adversary can call two oracles: \( \text{Vote}(i, c) \) to request that honest voter \( i \) outputs a ballot for candidate \( c \), and \( \text{GetToken}(i) \) to get a voting token for voter \( i \). Note that this models both corruption and coercion of voter \( i \). After a call to \( \text{GetToken}(i) \), voter \( i \) is considered corrupted until it casts an honest ballot using \( \text{Vote}(i, c) \). Eventually, the adversary outputs a bulletin board \( \mathbb{BB} \), the selected votes \( S \) and proof \( \Phi \), the election outcome \( r \in \mathbb{R} \), and a tally proof \( \Pi \) (line 3). The adversary loses if \( \Phi \) or \( \Pi \) do not verify (line 4). If it verifies, the adversary wins if the result does not satisfy the three intuitive conditions above.

The game computes the following groups of voters:

- **Corrupted** (line 6): voters considered corrupted, i.e., voters that were once corrupted (by calling \( \text{GetToken} \)) and thereafter never cast a checked honest vote.
- **Checked** (line 7): voters that verified a ballot and were not corrupted thereafter.
- **Unchecked** (line 8): voters that were never corrupted, but did not check their ballots either.

The game computes allowed candidates for honest voters:

- **AllowedVotes** \( [i] \) (line 9) A list of candidates that voter \( i \) honestly voted for in or after the last checked ballot.
- **Unchecked** \( [i] \) (line 10) A list of candidates that voter \( i \) never checked a ballot, this list includes all candidates this voter ever voted for.

The adversary wins if the result \( r \) verifies but violates any of the following conditions (lines 10–13): (1) For each honest voter that verified a ballot and was not thereafter corrupted (i.e., voters in Checked) the result should include either the candidate in that ballot, or a candidate in a later ballot. This corresponds to the candidates \( \{ c_i^{V_j} \}_{j=1}^{n_U} \) in the game. (2) Of the honest voters that did not check their ballots but were never corrupted (i.e., voters in Unchecked), at most one candidate that the honest voter voted for (in any ballot) can be included. This corresponds to the candidates \( \{ c_i^{U_j} \}_{j=1}^{n_U} \) in the game, where \( n_U \) is smaller than \( | \text{Unchecked} | \) or in fact 0. (3) At most \( n_C \) corrupted (or bad) votes were counted (i.e., the candidates \( \{ c_i^{B_j} \}_{j=1}^{n_B} \) )

In the game, the sum of these choices is modeled by the tallying function \( \hat{\rho} : \mathcal{C}^* \rightarrow \mathbb{R} \) that maps the voter’s choices in \( \mathcal{C} \) to an election result in \( \mathbb{R} \). This function should support partial tallying, i.e., for any two lists \( S_1 \) and \( S_2 \) we have that \( \hat{\rho}(S_1 \cup S_2) = \hat{\rho}(S_1) \ast_{\mathbb{R}} \hat{\rho}(S_2) \) for a commutative binary operator \( \ast_{\mathbb{R}} : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \). Note that a tally function that outputs the number of votes per candidate naturally admits partial tallying.
We evaluate the performance of VOTEAGAIN, which aims to support 8 million voters, has around 32 cores.

**Definition 6.** Consider a voting scheme \( \mathcal{N} \) = (Setup, GetToken, Vote, Filter, VerifyFilter, Tally, Verify) for an electoral roll \( \mathcal{E} \) and candidate list \( \mathcal{C} \). We say the scheme is **verifiable** if for all probabilistic polynomial time adversary \( \mathcal{A} \)

\[
\Pr[\text{Exp}^{\text{ver,b}}_{\mathcal{A},\mathcal{N}}(\ell, \mathcal{E}, \mathcal{C}) = 1] - \Pr[\text{Exp}^{\text{ver,l}}_{\mathcal{A},\mathcal{N}}(\ell, \mathcal{E}, \mathcal{C}) = 1] 
\]

is a negligible function in \( \ell \).

In Appendix C, we prove the following theorem.

**Theorem 3.** VOTEAGAIN is verifiable under the DDH assumption in the random oracle model.

### 7 Performance Evaluation

We evaluate the performance of VOTEAGAIN using a Python prototype implementation of its core cryptographic operations. We did not implement the GetToken protocol, but note that it can be implemented easily and cheaply using standard cryptography. We also did not implement the bulletin board as it is not core to our design. We use the petit [16] binding to OpenSSL for the group operations using the fast NIST P-256 curve. We ran all experiments in Linux on a single core of an Intel i3-8100 processor running at 3.60GHz. We expect nation-wide elections to have much more processing power available. For example, the Swiss CHVote system, which aims to support 8 million voters, has around 32 cores available per party in the system. We also include performance estimates of running the system on a large machine with 8 Intel Xeon Platinum 8280L processors, with 28 cores each, running at 2.7Ghz. As our scheme is almost completely parallelizable (only the hash functions for the non-interactive zero-knowledge proofs need to be computed sequentially), we estimate a 90\% parallelization gain: a speedup of 170 times when using the 8x28 cores with respect to the single core.

For all experiments we empirically select the best cover size \( k \) by sweeping over values from 1 to 64. In the majority of cases the optimal \( k \) is in the range [2, 4].

**Creating a ballot.** We use an ElGamal ciphertext to encrypt the voter’s choice, and a Bayer and Groth [5] zero-knowledge proof of membership to show that the selected candidate is eligible. Creating a ballot from 1000 eligible candidates costs 1.6 seconds, while verifying its correctness costs 0.24 seconds. The size of this proof is 1.5 kB.

**Impact of revoting.** Figure 10 shows the overhead, in terms of number of dummies per real ballot depending on the number of voters. This overhead influences the computation time of shuffling and filtering in the tally phase. We consider different revoting behavior. In the leftmost figure we model this behaviour as percentage of the number of voters: 50% models that half of the voters revoted once, and 200% models that all voters vote twice. We note that the overhead of 100% voters revoting once is equivalent to, for example, 25\% of the voters revoting 4 times. As expected, the overhead increases with both the number of voters and the number of revoted ballots. However, even for 100 million voters revoting twice (200\% revotes), the overhead is at most a factor of 32 (Figure 10 left).

However, casting a vote takes time. Thus, revoting patterns are constrained by the number of ballots that can be cast during an election. We consider an election period of 24h (larger than most countries), and bound how often a single voter can vote (1 ballot per second, per ten seconds, and per minute). As this limits the number of voters with a large amount of ballots, we do not need large covers, reducing the overhead (see Figure 10, center). Similarly, assuming that all voters will revote is very conservative. In a normal election one expects the vast majority of voters to vote once. In Figure 10, right, we show the overhead when the number of voters that cast more than one vote is limited. As fewer
voters revote, the total amount of votes is smaller and so are the covers.

**Filtering.** We implemented a non-optimized version of Bayer-Groths verifiable shuffle protocol [4] to implement steps 3 and 6 of Procedure 5. We measure the execution time of filtering and verifying, when varying the number of voters. Figure 11 left shows the times to run Filter and VerifyFilter on a single core machine. Figure 11 middle shows the estimated processing times on the big 8 processor Xeon machine. We estimate that the 8 processor machine can filter and tally the second round presidential election in Brazil (147 million registered voters) in 95 minutes if no voter revotes, and within a day assuming 50% extra ballots and at most one ballot per voter per ten seconds. We note that elections usually tally ballots per state, city, or smaller electoral district. Thus, in general we expect the number of ballots to be much smaller. All ballot groups in Figure 11 left and center have size one. Figure 11 right shows the effect of larger ballot groups resulting from revoting and dummy voters. As the average group size increases, the computation time goes down. Therefore, Figure 11 gives an upper bound on the processing time, given a known cover size.

For comparison we computed a lower bound on the filter cost of Achenbach et al.’s filter method by counting the number of group operations needed per ballot. We used this number to compute the estimate in Figure 11 left. A small-town election with 100,000 ballots takes 8 core months to filter in their scheme. Even on the large Xeon machine, an election with 1 million ballots takes over four months to complete. Our method needs respectively 10 core minutes and 30 seconds. The sizes of the tally proofs in VOTEAGAIN for these examples are 54 and 501 MB respectively.

**Smaller regions.** Many countries report election results per region, such as a province, a city, or a neighborhood. In those cases, results can be computed per region at lower computation cost. However, even in this setting, Achenbach et al.’s quadratic approach scales poorly. We note that the allowable size of reporting regions depend on local regulations, with the smallest regions likely being cities or neighborhoods, which can easily total 100,000s of voters. As Figure 11 (left) shows, even in this configuration, the quadratic approach requires 3 to 4 orders of magnitude more computation resources than VOTEAGAIN.

**Tallying.** We also measured the execution time of a single step of the mix network – a single shuffle and one verifiable decryption – using our verifiable shuffle implementation. Our results show that one step is a factor of three times faster than our filter protocol, e.g., mix-and-decrypting the 100,000 ballots takes around 3 core minutes and 1 million ballots takes 10 seconds on the Xeon machine.
8 Conclusion

Due to its complexity and cost, coercion resistance has been often overlooked in remote voting schemes. We introduced VOTEAGAIN, a revoting scheme that enables cleartext filtering thanks to efficient deterministic padding. VOTEAGAIN does not require users to store cryptographic material, and can efficiently handle millions of votes. We provided a new coercion resistance definition and updated existing definitions for ballot privacy and verifiability to the revoting setting. We have proven that VOTEAGAIN satisfies all of them.

References

[1] Dirk Achenbach, Carmen Kempka, Bernhard Löwe, and Jörn Müller-Quade. Improved Coercion-Resistant Electronic Elections through Deniable Re-Voting. USENIX Journal of Election Technology and Systems (JETS), (2), 2015.
[2] Ben Adida. Helios: Web-based Open-audit Voting. In USENIX Security Symposium, 2008.
[3] Roberto Araújo, Amira Barki, Solenn Brunet, and Jacques Traoré. Remote Electronic Voting Can Be Efficient, Verifiable and Coercion-Resistant. In Financial Cryptography Workshop VOTING, 2016.
[4] Stephanie Bayer and Jens Groth. Efficient zero-knowledge argument for correctness of a shuffle. In EUROCRYPT, 2012.
[5] Stephanie Bayer and Jens Groth. Zero-knowledge argument for polynomial evaluation with application to blacklists. In EUROCRYPT, 2013.
[6] David Bernhard, Véronique Cortier, David Galindo, Olivier Pereira, and Bogdan Warinschi. A comprehensive analysis of game-based ballot privacy definitions. Cryptology ePrint Archive, Report 2015/255, 2015.
[7] David Bernhard, Véronique Cortier, David Galindo, Olivier Pereira, and Bogdan Warinschi. Sok: A comprehensive analysis of game-based ballot privacy definitions. In S&P, 2015.
[8] David Bernhard, Olivier Pereira, and Bogdan Warinschi. How not to prove yourself: Pitfalls of the Fiat-Shamir heuristic and applications to Helios. In ASIACRYPT, 2012.
[9] Philippe Bulens, Damien Giry, and Olivier Pereira. Running mixnet-based elections with helios. In EVT/WOTE, 2011.
[10] Sergiu Bursuc, Gurchetan S Grewal, and Mark D Ryan. Trivitas: Voters Directly Verifying Votes. In VOTE-ID, 2012.
[11] Jan Camenisch and Markus Stadler. Efficient Group Signature Schemes for Large Groups (Extended Abstract). In CRYPTO, 1997.
[12] Jeremy Clark and Urs Hengartner. Selections: Internet Voting with Over-the-Shoulder Coercion-Resistance. In FC, 2012.
[13] Michael R. Clarkson, Stephen Chong, and Andrew C. Myers. Civitas: Toward a secure voting system. In S&P, 2008.
[14] Véronique Cortier, David Galindo, Stéphane Glondu, and Malika Izabachène. Election Verifiability for Helios under Weaker Trust Assumptions. In ESORICS, 2014.
[15] Véronique Cortier, David Galindo, Ralf Küsters, Johannes Müller, and Tomasz Truderung. SoK: Verifiability Notions for E-Voting Protocols. In S&P, 2016.
[16] George Danezis. Petlib: A python library that implements a number of privacy enhancing technologies. https://github.com/gdanezis/petlib. Accessed: June 2, 2020.
[17] Official e Estonia Website. e-governance / i-voting, accessed May 21, 2014. https://e-estonia.com/solutions/e-governance/i-voting/.
[18] Aleksander Essex, Jeremy Clark, and Urs Hengartner. Cobra: Toward concurrent ballot authorization for internet voting. In EVT/WOTE, 2012.
[19] Amos Fiat and Adi Shamir. How to Prove Yourself: Practical Solutions to Identification and Signature Problems. In CRYPTO, 1986.
[20] Conner Fromknecht, Dragos Velicanu, and Sophia Yakoubov. A Decentralized Public Key Infrastructure with Identity Retention. Cryptology ePrint Archive, Report 2014/803, 2014.
[21] Christina Garman, Matthew Green, and Ian Miers. Decentralized Anonymous Credentials. In NDSS, 2014.
[22] Dawid Gaweł, Maciej Kosarzecki, Poorvi L. Vora, Hua Wu, and Filip Zagórski. Apollo – End-to-end verifiable internet voting with recovery from vote manipulation. In E-VOTE-ID, 2016.
[23] Kristian Gjøsteen. Analysis of an Internet Voting Protocol. Cryptology ePrint Archive, Report 2010/380, 2010.
[24] S Goldwasser, S Micali, and C Rackoff. The Knowledge Complexity of Interactive Proof-systems. In STOC, 1985.
[25] Rüdiger Grimm and Melanie Volkamer. Multiple Cast in Online Voting – Analyzing Chances. In Electronic Voting, 2006.
A Proof of ballot privacy, strong correctness and strong consistency

Proof of theorem 1. This proof is very similar to the proof of ballot privacy of Helios in the full version of Bernhard et al. [6]. We start with the adversary playing the ballot privacy game with \( b = 0 \) and after a sequence of game steps transitions, the adversary finishes playing the ballot privacy game with \( b = 1 \). We argue that each of these steps are indistinguishable, and therefore the results follows.

The proof proceeds along the following sequence of games:

**Game \( G_0 \).** Let game \( G_0 \) be the \( \text{Exp}^{\text{bpriv},0}_{\mathcal{A},\mathcal{Y}} \) game (see Figure 7 and Definition 4).

**Game \( G_1 \).** Game \( G_1 \) is as in \( G_0 \) but we now compute

\[
\Pi_0 = \text{SimTally}(\mathcal{B}_b, r)
\]

by simulating the proof using the random oracle instead of using the real proof from \( \text{Tally}(\mathcal{B}_b, sk) \). Because of the simulation properties of the zero-knowledge proof system, \( \mathcal{A} \) cannot distinguish these two games.

**Game \( G_2 \).** As in game \( G_1 \), but now \( \text{Tally}(\mathcal{S}, \Phi) \) ignores \( \mathcal{S} \) and \( \Phi \) provided by \( \mathcal{A} \) when computing the result \( r \). In particular, \( \text{Tally} \) now proceeds as follows:

\[
\text{VerifyFilter}(\mathcal{B}_b, \mathcal{S}, \Phi) = \perp \quad \downarrow \quad \perp
\]

\[
(r, \Pi_0) \leftarrow \text{Tally}(\mathcal{B}_b) \parallel \text{Filter}(\mathcal{B}_b, n_B, sk_T), \mathcal{S}, sk_T)
\]

\[
\mathcal{B}_b \leftarrow \mathcal{B}_b \parallel \mathcal{S} \parallel \mathcal{F}
\]

\[
\mathcal{B}_{1-b} \leftarrow \mathcal{B}_{1-b} \parallel \text{Filter}(\mathcal{B}_{1-b}, n_B', tsk)
\]

\[
\Pi_0 = \text{SimTally}(\mathcal{B}_0, r)
\]

\[
\Pi_1 = \text{SimTally}(\mathcal{B}_1, r)
\]

return \((r, \Pi_b)\)

The proofs included in \( \Phi \) ensure that \( \mathcal{A} \) honestly computed the filtering step. Therefore, the adversary’s view is indistinguishable from that in \( G_1 \).

**Game \( G_3 \).** As in game \( G_2 \), but in \( \mathcal{O} \) board we return \( \mathcal{B}_b \).

Note that in \( G_3 \) the adversary has the same view as in the \( \text{Exp}^{\text{bpriv},1}_{\mathcal{A},\mathcal{Y}} \) game. All that is left to show is that \( G_2 \) and \( G_3 \) are indistinguishable.

We now show that no adversary \( \mathcal{A} \) can distinguish \( G_2 \) from \( G_3 \). Let \( n_B \) be the number of \( \text{Vote}LR(\tau, c_B, \mathcal{C}_1) \) calls that the adversary \( \mathcal{A} \) made. In particular, for the \( i \)th call to \( \text{Vote}LR \), remember the tuple \((b_0, \beta_1, c_B, c_1)\) of candidates and resulting ballots. We now build a series of games \( H_0, \ldots, H_{n_B} \) and proceed by a hybrid argument.

In game \( H_i \) we show to the adversary a bulletin board where the first \( i \) ballots cast using \( \text{Vote}LR \) on \( BB_0 \) are replaced by those of \( BB_1 \). More precisely, in all games \( H_i \) we keep track of an additional bulletin board \( BB \) that is shown to the adversary, i.e., \( \mathcal{O} \) board now returns \( BB \). Whenever the adversary makes an \( \text{Ocast}(\beta) \) query, we also add \( \beta \) to \( BB \), i.e., \( BB \leftarrow BB \parallel \beta \).
Thereafter follows: Let $\Gamma$ do not use on Filter $sk$ directly compute the tally, as it does not know the decryption $j$.

Internally, show that $H$ is negligible in $\ell$.

Additionally set $BB$.

To show this, we create an adversary $A$ against NM-CPA. From Bernhard et al.’s work on Helios [8] we know that in the random oracle model under the DDH assumption the ballot encryption scheme based on ElGamal with a non-

the random oracle model under the DDH assumption the $H$ additionally set $BB$. It is then allowed to ask the encryption of the challenger. It is then allowed to ask the $BB$ in response to the first $BB$ as normal, but instead it sets $BB$.

For $BB$ = $BB$ in query $BB$, $BB$ = $BB$.

Finally, as in game $G_2$, $B$ simulates the tally proof. Note that if $BB$ = $BB$’s NM-CPA game, then $B$ perfectly simulates $H_{BB-1}$, and if $BB$ = $BB$ then it perfectly simulates $H_{BB}$. Therefore, any distinguisher between $H_{BB}$ and $H_{BB-1}$ breaks the NM-CPA security of the voting scheme.

A standard hybrid argument now shows that $H_{BB} = G_2$ is indistinguishable from $H_{BB-1} = G_3$. This completes the proof.

\[\Box\]

A.1 Strong Consistency

The ballot privacy definition ensures that ballots and the proof of correct tally $\Pi$ do not leak anything about how voters voted. However, maliciously crafted voting schemes might leak information about honest votes in the result $\Pi$ itself. To ensure that this is not possible, Bernhard et al. [7] introduced the notion of strong consistency. Intuitively, this notion ensures that the result $r$ is equal to the result function applied directly to the valid ballots (skipping the filter and tally phase). We follow the exposition of Bernhard et al., but make some changes to account for the fact that our scheme selects ballots with the highest corresponding ballot number $m$, rather than simply the last per voter.

Our voting scheme depends on a formal result function $\rho : ((G \times N) \times C)^* \rightarrow \mathbb{R}$, where $G$ is the space of voters identifiers, and $\mathbb{R}$ is the result space. Our result function selects, for every $\text{vid} \in G$, the ballot $((\text{vid}, m), c)$ where $m$ is the maximal counter for this voter. Then it counts the number of votes per candidate $c$ in the selected ballots and returns the result.

To model that the result $r$ output by Tally is consistent with the result function $\rho$, we require the existence of an extraction algorithm Extract that takes as input the TS’s key $sk_T$, the trustee key $sk_S$ and a ballot, and outputs a tuple $((\text{vid}, m), c) \in ((G \times N) \times C)$ with the corresponding voter identifier $\text{vid}$, ballot number $m$ and candidate $c$ in this ballot. If it fails to extract these values, it outputs $\perp$.

Moreover, we require a method ValidInd that validates ballots independent of the bulletin board. The function ValidInd takes as input the election public key $pk$ and a ballot, and outputs $\top$ if the ballot is valid, and $\perp$ otherwise.
Adapted from Bernhard et al. [7]

Definition 7 (Adapted from Bernhard et al. [7]). A voting scheme \( \mathcal{V}' = (\text{Setup}, \text{GetToken}, \text{Vote}, \text{Valid}, \text{Filter}, \text{VerifyFilter}, \text{Tally}, \text{Verify}) \) for an electoral roll \( \mathcal{E} \) and candidate list \( \mathcal{C} \) has strong consistency with respect to a result function \( \rho : ((\mathbb{G} \times \mathbb{N}) \times \mathcal{C}) \rightarrow \mathcal{E} \) if there exists algorithms Extract and ValidInd as above, such that the following three conditions hold:

1. For any \( (pk, sk_{PA}, sk_{TS}, sk_T) \) output by \( \text{Setup} \), for all voters \( i \in \mathcal{E} \) with voter identifier \( vid \), for all voter \( i \), \( vid \), \( c \) is a valid voter, and for any ballot \( \beta \) with \( \text{Vote}(\tau, c) \) with \( c \in \mathcal{C} \), we have that Extract\((sk_T, sk_{PA}, \beta)\) is valid. Then it decrypts \( \tau \) and \( I \) to get \( (vid, m) = (EC.\text{Dec}(sk_{PA}, \gamma), EC.\text{Dec}(sk_{PA}, I)) \). It returns \( ((vid, m), c) \).
2. For any \( (BB, \beta) \) we have that ValidInd\( (BB, \beta) = \top \) implies ValidInd\( (\beta) = \top \).
3. For all probabilistic polynomial time adversary \( A \) we have that
   \[
   \Pr\left[ \text{Exp}^{\text{cons}}_{A, \mathcal{V}'}(\ell, \mathcal{E}, \mathcal{C}) = 1 \right]
   \]
   is a negligible function in \( \ell \) (see Figure 13 for the game).

The first condition ensures that Extract can extract \( ((vid, m), c) \) correctly for honestly created ballots. The second condition ensures that ballots that are accepted by Valid with respect to the board \( BB \) must also be accepted by ValidInd. Finally, the third condition ensures that the adversary cannot produce bulletin boards where the result \( r \) does not correspond to the formal result function \( \rho \) executed on the individual ballots. (The adversary loses if Filter or Tally aborts because of an invalid bulletin board.)

A.2 Strong correctness

Finally, a malicious protocol designer might modify which ballots are accepted based on earlier ballots. To address this attack, Bernhard et al. [7] introduce the notion of strong correctness. Informally, a scheme has strong correctness if honestly generated ballots are accepted regardless of the content of the bulletin board.

Figure 13: In the strong-consistency experiment \( \text{Exp}^{\text{cons}}_{A, \mathcal{V}'}(\ell, \mathcal{E}, \mathcal{C}) \), adversary \( A \) must output a board \( BB \) with ballots that are not tallied correctly given Extract.

Definition 8 (Adapted from Bernhard et al. [7]). Consider a voting scheme \( \mathcal{V}' = (\text{Setup}, \text{GetToken}, \text{Vote}, \text{Valid}, \text{Filter}, \text{VerifyFilter}, \text{Tally}, \text{Verify}) \) for an electoral roll \( \mathcal{E} \) and candidate list \( \mathcal{C} \). We say the scheme has strong correctness if

\[
\Pr\left[ \text{Exp}^{\text{corr}}_{A, \mathcal{V}'}(\ell) = \bot \right]
\]

is a negligible function in \( \ell \) (see Figure 14 for the game).

Theorem 4. VOTEAGAIN provides strong-consistency and strong-correctness.

Proof of theorem 4. This proof roughly follows that of the strong consistency and strong correctness of Helios in the full version of Bernhard et al. [6]. To show that VOTEAGAIN is strongly consistent, we define the following Extract and ValidInd algorithms:

- Extract\((\beta, sk_{TS}, sk_T)\) operates on a ballot \( \beta = (v, \pi, pk, y, I, \sigma^2, \sigma) \). First, it verifies the proof \( \pi \) and the signatures \( \sigma^2 \) and \( \sigma \) as in step 1 of Valid in procedure 4. If any check fails, it returns \( \bot \). Otherwise, it recovers the candidate \( c = \text{Vote}.\text{Dec}(sk_T, v) \) (note that \( c \in \mathcal{C} \) because \( \pi \) is valid). Then it decrypts \( \gamma \) and \( I \) to get \( (vid, m) = (EC.\text{Dec}(sk_{PA}, \gamma), EC.\text{Dec}(sk_{PA}, I)) \). It returns \( ((vid, m), c) \).
- ValidInd\( (\beta) \) proceeds as in step 1 of Valid in procedure 4 to verify the ballot:

\[
\text{Vote}.\text{Verify}(pk_T, v, \pi)
\]
\[
\text{Sig}.\text{Verify}(pk, \sigma, v || \pi || pk || y || I || \sigma^2)
\]
\[
\text{Sig}.\text{Verify}(pk_{PA}, \sigma^2, pk || y || I)
\]

It returns \( \top \) if all are valid, and \( \bot \) otherwise.

First we show that the first condition of strong consistency is satisfied. By the correctness of the zero-knowledge proofs and decryption algorithms, Extract will indeed extract the required values for valid ballots. Since ValidInd executes a strict subset of the checks in Valid, it follows that the second condition is trivially satisfied.

For the third condition, we need to show that the adversary cannot create a valid bulletin board \( BB \) (i.e., one on which
Filter and Tally do not fail), but where the result is incorrect (respect to the output calculated with the extractor function).

Note that by the checks in steps 1 and 4 of procedure 5, we know that the identifier pairs \((vid, m)\) are unique. Consider the group \(G_j\) of ballots corresponding to \(vid_j\). The ideal result function \(p\) includes the vote where the ballot index is highest. In exactly the same way, Filter sets \(V_j\) to \(v_{j*}\) where the index \(j*\) maximizes the ballot index \(m_{j*}\). The equivalence of the ideal result and the result produced by tally now follows.

To see that \(\text{VOTEAGAIN}\) is strongly correct we need to prove that an adversary cannot create a ballot box \(BB\) such that an honest voter, when generating an honest ballot \(\beta\), that ballot will be rejected, i.e., \(\text{Valid}(BB, \beta) = \bot\). Note that the verification in \(\text{Valid}(\beta)\) is twofold. First, it verifies the validity of the ballot. It is trivial to see that this check passes for an honestly generated ballot. Second, it checks that the ephemeral public key \(pk\) and encrypted vote \(v\) do not yet appear on the bulletin board. Clearly, \(v\) does not appear because it was just generated honestly by the user. Moreover, neither does the public key \(pk\) appear before, because it was just freshly generated by the PA. Given that these two values contain a source of randomness when generated, it proceeds that \(A\) can only win with negligible probability.

\[\square\]

\section{Proof of Coercion Resistance}

\textbf{Proof of theorem 2.} We first specify how to construct \(\text{SimTally}\) and \(\text{SimFilter}\). As in the ballot privacy proof, \(\text{SimTally}(BB, r)\) simply simulates the proof of shuffle and the proof of correct decryption in Tally, so that regardless of the values in \(S\), \(r\) is the correct outcome.

The algorithm \(\text{SimFilter}(BB, n_B, r)\) proceeds similarly. It takes as input the bulletin board \(BB\), which it uses to determine the number of ballots \(n_B\), the number of registrations \(n'_B\), and the result \(r\). Moreover, it derives the number of real voters \(n\) using \(r\). It uses these data to compute the cover, and it adds the correct number of dummy ballots (for these, it sets \(\gamma\) and \(I\) to random ciphertexts) to obtain \(B'\). Then it computes a list of zero ciphertexts (encryptions of zero) of equal length, and simulates the shuffle proof \(\pi_{\sigma}\). It then generates fake voter identifiers \(\overline{vid}\) and \(m\) corresponding to the cover it computed earlier, associates these to shuffled ballot \(\beta_i\), and simulates the proofs \(\pi^\text{dec}\). Next, for each resulting group, it generates a random encryption of zero \(V_j = \text{Vote.ZEnc}(pk_i, r_j)\) and simulates the corresponding proof \(\pi^\text{sel}\). Then, it returns the randomness \(r_j\) and the indices of the dummy voters corresponding to the cover it computed early. Finally, for each remaining vote, it generates a random \(V_j\) and simulates the shuffle proof \(\pi_{\sigma}\).

In this proof, we will step by step replace all the ciphertexts that depend on the bit \(b\) by random ciphertexts. In particular, we first show that the adversary learns nothing about \(b\) during the election phase. We then show that it also learns nothing about \(b\) during the tally phase. The result follows.

**Game G1.** Game \(G1\) is as the \(\text{Exp}_{\gamma, \sigma}^{\text{cr}, \ell}(E, C)\) experiment. (Note that contrary to the proof of ballot privacy we do not fix the value for \(b\).)

**Game G2.** Game \(G2\) is as game \(G1\), but we compute the result directly based on the ballots on \(BB_0\). Let \([\beta_1, \ldots, \beta_{n_B}]\) be the list of ballots where

\[
\beta_i = (vid, \pi, pk_i, \gamma, I, \sigma^t_1, \sigma^t_2).
\]

Let \(c = \text{Vote.Dec}(sk, vid_i)\), \(vid_i = \text{EC.Dec}(sk_{TS}, \gamma_i)\), and \(m_i = \text{EC.Dec}(sk_{TS}, I_i)\). Then compute the result:

\[
r = \rho((vid_1, m_1), c_1), \ldots, (vid_{n_B}, m_{n_B}), (c_{n_B})
\]

As per strong consistency, games \(G2\) and \(G1\) are indistinguishable.

**Game G3.** Game \(G3\) is as game \(G2\), but with all the zero-knowledge proofs replaced by simulations. This includes the shuffle proof \(\pi_{\sigma}\), the decryption proofs of \(\pi^\text{dec}\) of the shuffled \(\gamma_i\) and \(I_i\), the reencryption proofs \(\pi^\text{sel}\), and the shuffle proof \(\pi'_{\sigma}\) produced in Filter; as well as the tally proof \(\Pi_i\) which we replace by the output of \(\text{SimTally}(BB_0, r)\). We use the random oracle to simulate this step, which is indistinguishable by the simulatability of the zero-knowledge proof system.

**Game G4.** Game \(G4\) is as game \(G3\) but we do not decrypt the \(\gamma_i\) and \(I_i\) anymore when running Filter. Instead, we proceed as follows. All ballots \(\beta_i = (vid, \pi, pk_i, \gamma, I, \sigma^t_1, \sigma^t_2)\) on the bulletin boards are valid. Hence, \(\sigma_i\) is a valid signature by \(PA_0\) resp. \(PA_1\) on \(\gamma_i\) and \(I_i\). Since the signature scheme is unforgeable, we know these ciphertexts were created by \(PA_0\) resp. \(PA_1\). Hence, we can associate to them the corresponding plaintexts \(vid_i\) and \(m_i\). Moreover, we know the permutation used by the TS during Filter, so we can also provide the correct plaintexts in step 4 of Filter on \(BB_0\) (recall the proofs of decryption \(\pi^\text{dec}\) are already simulated).

**Game G5.** Game \(G5\) is as game \(G4\), but we replace the ciphertexts \(\gamma_i\) and \(I_i\) in the token \(\tau_i\) by random ciphertexts for all tokens. Similarly, we replace the \(\gamma_i\) and \(I_i\) ciphertexts for the dummy ballots by random ciphertexts. Note that per the change in game \(G1\) we still associate the correct plaintexts \(vid_i\) and \(m_i\) in the Filter protocol. A hybrid argument with reductions to the CPA security of the ElGamal encryption scheme shows that games \(G5\) and \(G4\) are indistinguishable. This reduction is possible since we no longer need to decrypt these ciphertexts.

**Game G6.** Game \(G6\) is as game \(G5\), but we replace the encrypted votes \(v_i\) in the \(\text{Vote.ER}(\cdot)\) call by encryptions of the zero vector, i.e., \(v_i = \text{Vote.ZEnc}(pk_i, r)\) for a uniformly random randomizer \(r\). As in the ballot privacy proof, a hybrid argument with a reduction to the NM-CPA security of the ElGamal encryption scheme with zero-knowledge proof shows that games \(G6\) and \(G5\) are indistinguishable. Note that in this reduction we use the \(\text{Odec}\) of the NM-CPA challenger to decrypt votes in the adversary-determined ballots before computing the result \(r\).

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Note that as of game $G_6$, the adversary’s view of the bulletin board before calling $\text{Vote}()$ is independent of the value of $b$. (The ballots resulting from the $\text{Vote}LR$ call also contain a random ephemeral public key $pk$ and the signatures $\sigma^2$ and $\sigma$, but these are also independent of the actual voter selected.)

We now proceed to show that the adversary also cannot learn anything from the output of $\text{Filter}$. Notice that, regardless of the value of $b$, the filter step is computed with the same number of voters $v$, the same number of ballots $n_B$ and the same number of obtained tokens $n_B'$. Therefore, the output of $\text{Filter}$ applied to $BB_0$ and that of $\text{SimFilter}$ applied to $BB_1$ should be indistinguishable. In the following game steps we replace the ciphertexts after shuffling by zero-ciphertexts and show that these steps are indistinguishable for the adversary.

**Game $G_7$.** Game $G_7$ is the same as game $G_6$, but we replace the ciphertexts $c_i^j$, $I^j_1$, and $b_i^j$ after shuffling by random encryptions of zero. We proceed as if they still decrypt to the correct values. Note that we already simulate the shuffle proof and decryption proofs. Again, a hybrid argument with reductions to the CPA security of the ElGamal encryption scheme shows that the games $G_7$ and $G_6$ are indistinguishable. This reduction is possible since we no longer need to decrypt these ciphertexts.

**Game $G_8$.** Game $G_8$ is the same as game $G_7$, but we replace the shuffled encrypted votes $\mathbf{v}$ by random encryptions of zero. Similarly, we replace the randomizations, $R$, of the votes corresponding to dummy voters by the corresponding new randomization. This causes the pre-selected votes $\mathbf{V}_j$ per group to be incorrect, but this does not matter as we simulate the second shuffle proof, $\pi_\sigma'$, anyway. As before, the indistinguishability of this step follows from the NM-CPA security of the vote encryption scheme.

**Game $G_9$.** Game $G_9$ is the same as game $G_8$, but we replace the second shuffled votes $V_j$ by random encryption of zero. This causes the selected votes $V_j$ after the shuffle to be incorrect with respect to the result, but this does not matter as we simulate the proof of the tally. As before, the indistinguishability of this step follows from the NM-CPA security of the vote encryption scheme.

**Game $G_{10}$.** Game $G_{10}$ is as game $G_9$, but we replace the filter and tally proofs on $BB_0$ by simulations: we set $(S_0, \Phi_0) \leftarrow SimFilter(BB_0, n_B', r)$ and $\Pi_0 \leftarrow SimTally(BB_0, r)$. Note that this difference is purely syntactic, as per the changes we made before, we already computed exactly the output of $\text{SimFilter}$ on $BB_0$ and the result $r$.

Clearly the resulting view is independent of $b$. And coercion resistance follows. 

## C Proof of Verifiability

**Proof of theorem 3.** At the end of the filter procedure, the TS (or in our case, the adversary) outputs a list of selected votes $S = V_1, \ldots, V_n$, a proof $\Phi$, the result $r$ and the tally proof $\Pi$. Because $\Pi$ verifies, we know that the result $r$ is the addition of the votes contained in $V_1, \ldots, V_n$.

We first show that each encrypted vote $V_j$ contains a vote for a single candidate or an empty vote. Given that $\pi_\sigma$ and $\pi_\sigma'$ validate, we know that the vote $V_j$ corresponds to a group of ballot indices $G_j = i_1, \ldots, i_{\nu_j}$, corresponding to voter identifier $vid_j$. Moreover, the index $j^*$ is such that the decrypted index $m_j$ is maximal. Because $\pi_{\sigma'}^\text{sel}$ is valid, we know that either

1. $V_j$ is the reencryption of vote $v_{j^*}$ and $\text{EC.Dec}(\theta_{j^*}) = g^0$.
2. $V_j$ is the encryption of zero and $\text{EC.Dec}(\prod_{k=1}^{\chi_j} \theta_{i_k}) = g^{\chi_j}$. 

We now show that in the first case $v_{j^*}$ must be the encryption of a single candidate. Because $\theta_{j^*} = g^0$ and the correctness of the shuffle proofs $\pi_{\sigma}, \pi_{\sigma}'$, we know that $v_{j^*}$ originates from a valid ballot cast by a voter. This ballot included a proof that $v_{j^*}$ is the encryption of a single candidate.

Let $v$ be the number of voters that requested a voting token and for which at least one ballot is included on the bulletin board. We argue that the number of non-zero ballots that is included in the tally equals $v$. Let $vid_{i_1}, \ldots, vid_{i_{\nu_j}}$ be the corresponding voter identifiers. Because of the correctness of the shuffle, $\pi_\sigma$, there exists corresponding groups $G_{i_1}, \ldots, G_{i_{\nu_j}}$ to these voter identifiers after shuffling.

We show that any other group $G_{i'}$ contributes an empty vote to the tally. Let $vid_{i'}$ be the corresponding voter identity. The adversary cannot forge signatures by the PA, so any ballot with voter identity $vid_{i'}$ was added as a dummy ballot with $\theta_{j^*}$ as a tag. Therefore, in group $G_{i'}$, each tag encrypts $g^1$, so the only possible disjunct in $\pi_{\sigma'}^\text{sel}$ is therefore the second, and thus $\mathbf{V}_j$ is the encryption of zero.

We show that only such encrypted votes may be removed after the second shuffle. The adversary needs to find a random $r$ such that $V_j = \text{Vote.ZEnc}(pk_{\gamma_j}, r)$. Given that the DL-assumption holds, the adversary can only find such $r$ if the underlying plaintext is zero with very high probability. Given that the encryption of candidate zero is not a permitted option for real voters, and given the correctness of $\pi$, only votes corresponding to the above groups may be removed by the adversary after the second shuffle.

So, we now know that only the groups $G_{i_1}, \ldots, G_{i_{\nu_j}}$ each contribute exactly one candidate to the tally, and no more candidates are added by the other groups. We assign each group to one of the three groups in the game: the voters in Checked, the voters in Unchecked, or the voters in Corrupted. The result then follows.

We now show that the correct values are tallied for each of the voters in Checked that verified that their ballots were correctly cast. Consider a voter $i \in \text{Checked}$ with voter identifier $vid_i$. Let $ctr$ be the last ballot that it verified. We need to show that the tally includes either $i$’s ballot $ctr$, or a later ballot. We know ballot $ctr$ was added to the bulletin board.
Therefore, the corresponding group $G_i$ (matching voter identifier \( \text{vid}_i \)) containing $\chi_i$ ballots, must contain a shuffled ballot \((\nu'_j, \text{vid}_i, m_j, \theta'_j)\) corresponding to the original ballot \(\text{ctr}\) (because the shuffle proof and decryption proofs are valid). Note that $\theta'_j$ must be a decryption of $g^0$ by construction, therefore the tags in group $G_j$ (which must be encryptions of $g^0$ or $g^1$) can never sum to $g^0$ and therefore, we must take the first disjunct in the reencryption proof $\pi^\text{sel}_i$: $V_i$ must be the reencryption of an encrypted ballot $j^*$ where $\theta_j^*$ decrypts to $g^0$. Therefore, $\pi^\text{sel}_i$ must contain the encrypted vote corresponding to a real ballot cast by voter $i$. Finally, since $j^*$ maximizes $m_j$, in the group, we know in particular, that $m_{j^*} \geq m_j$ corresponding to the verified ballot. Therefore, we conclude that indeed $V_i$ reencrypts either ballot \text{ctr} by voter $i$, or a later ballot by voter $i$. Given the correctness of the second shuffled proved with $\pi^\sigma_i$, there will be a selected ballot $V_i$ encrypting the same value as $V_i$. Finally, given the correctness of the mixnet and decryption proofs in Tally, either \text{ctr} by voter $i$, or a later ballot by voter $i$, will be counted in the final tally.

Now suppose a group $G_i$ corresponds to a voter $i$ in Unchecked. Then, by the same argument as for voters in Checked, we know that the tally must either drop all ballots or include one of the ballots cast by voter $i$.

Finally, any remaining groups correspond to voters in Corrupted. Notice that any voter that is not in Checked or Unchecked must be in Corrupted. Since, each remaining group corresponds to an actual voter, and this voter is not in either of the former groups, it must indeed correspond to a voter in Corrupted. 

\[\square\]