A Unified Picture for Single Transverse-Spin Asymmetries in Hard Processes

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Using Drell-Yan pair production as an example, we explore the relation between two well-known mechanisms for single transverse-spin asymmetries in hard processes: twist-three quark-gluon correlations when the pair’s transverse momentum is large, \(q_\perp \gg Q_{\text{QCD}}\), and time-reversal-odd and transverse-momentum-dependent parton distributions when \(q_\perp\) is much less than the pair’s mass. We find that although the two mechanisms have their own domain of validity, they describe the same physics in the kinematic region where they overlap. This unifies the two mechanisms and imposes an important constraint on phenomenological studies of single spin asymmetries.

1. Single-transverse spin asymmetries (SSAs) have a long history, starting from the 1970s and 1980s when large SSAs were first observed in hadronic reactions at forward rapidities. The size of the asymmetries came as a surprise and has posed a challenge for researchers in Quantum Chromodynamics (QCD). SSAs have again attracted much interest in recent years, both experimentally and theoretically.\(^1\) Two mechanisms have been proposed in QCD for explaining the large SSAs: effects due to time-reversal-odd and transverse-momentum-dependent parton distributions (Sivers functions)\(^2\), or to spin-dependent twist-three quark-gluon correlations (Efremov-Teryaev-Qiu-Sterman (ETQS) mechanism)\(^3,4\). Much progress has been made in understanding theoretical issues involved in transverse-momentum-dependent (TMD) parton distributions. Their gauge properties have been clarified\(^5,6,11,12\), and the relevant QCD factorization formulas have been established\(^6,8,11,12\). Phenomenological studies have been made to explain the available data, using both mechanisms\(^4,13\). An interesting relation between the two types of distribution functions was discussed in\(^10\). However, a clear connection between the two seemingly different physical mechanisms, particularly at the level of physical observables, has so far not been established (see\(^14\)).

In this letter we explore the relation between the two mechanisms in Drell-Yan pair production at transverse momentum \(q_\perp\). If the pair’s mass is \(Q\), the quark-gluon correlation mechanism works when \(q_\perp, Q \gg Q_{\text{QCD}}\), the strong interaction scale. On the other hand, the TMD QCD factorization formalism applies when \(q_\perp \ll Q\). Therefore, one expects there to be a common kinematic region, \(Q_{\text{QCD}} \ll q_\perp \ll Q\), where both mechanisms should apply and describe the same physics. We perform calculations in this intermediate region in both formalisms and find that they indeed yield the same result. This conclusion imposes a rigorous constraint on phenomenological studies of the single-spin asymmetry data.

Consider scattering of a transversely polarized proton of spin \(S_\perp\) and momentum \(P\) on an unpolarized hadron (another proton, for example) of momentum \(P’\), producing a virtual photon that subsequently decays into a pair of leptons. The total center-of-mass energy is denoted by \(s = (P + P’)^2\). At the lowest order in QCD perturbation theory, there are two ways to produce such a pair: either a quark-antiquark pair annihilates into a virtual photon after emitting a gluon of transverse momentum \(q_\perp\), or a quark scatters with a gluon producing a quark and a virtual photon. If the lepton pair has a large positive rapidity, \(y\) (in the forward direction of the polarized proton), the dominant contribution comes from scattering involving a quark from the polarized proton. Our interest is to calculate the spin-dependent differential cross section \(d\Delta \sigma(S_\perp)/dQ^2 dy d^2q_\perp\) at large \(y\) with \(\Delta \sigma(S_\perp) = [\sigma(S_\perp) - \sigma(-S_\perp)]/2\).

2. When \(q_\perp, Q \gg Q_{\text{QCD}}\), the spin-dependent cross section can be calculated in terms of a twist-three quark-gluon correlation. The physics of this correlation can be seen as follows: when a transversely-polarized proton is traveling at nearly the speed of light, its internal color electric and magnetic fields have preferred orientations in the transverse plane. By parity invariance, the color electric field must be orthogonal to the spin of the proton. If averaged over the proton wave function, the field vanishes because the proton is color-neutral (also because of time-reversal symmetry). However, if one multiplies the color electric field with the quark color current, the average may be non-zero. This average defines a quark-gluon correlation function that characterizes a property of a polarized proton.

High-energy scattering probes the so-called light-cone correlations. For these, quark and/or gluon fields are separated along the light-cone direction \(\xi^-\) (if \(\xi^0\) denotes a space-time coordinate, the light-cone variables are defined as \(\xi^\pm = (\xi^0 \pm \xi^3)/\sqrt{2}, \xi_\perp = (\xi^1, \xi^2)\)). A typical diagram for scattering of an antiquark in the polarized gluon background is shown in Fig. 1. The blob in the
lower part of the diagram represents the spin-dependent quark-gluon correlation described above. One finds the following expression for the correlation \( \mathcal{I} \):

\[
\mathcal{I} = \frac{2}{2\pi} \int_{-\alpha}^{\alpha} d\zeta \int_{-\infty}^{\infty} dy \frac{dy}{2\pi} e^{i(k_1 - k_2 - k_3)^2} \epsilon_{\alpha \beta} S_{\perp, \beta}
\]

where the sums over color and spin indices are implicit, \( |PS\rangle \) denotes the proton state, \( \psi \) the quark field, and \( F_{\alpha}^{+} \) the gluon field tensor. In Eq. (1), \( x_1 = k_1^+/P^+ \) and \( x_2 = k_2^+/P^+ \) are the fractions of the polarized proton’s light-cone momentum carried by the quark in Fig. 1, while \( x_g = k_3^+/P^+ = x_2 - x_1 \) is the fractional momentum carried by the gluon; \( \mathcal{L} \) is the light-cone gauge link, \( \mathcal{L}(\zeta_2, \zeta_1) = \exp \left( -ig \int_{\zeta_2}^{\zeta_1} k^- A^-(\xi^-) \right) \) that makes the correlation operator gauge-invariant; and \( \epsilon_{\alpha \beta} \) is the 2-dimensional Levi-Civita tensor with \( \epsilon_{\perp 2} = 1 \).

When the antiquark of the unpolarized proton scatters off a quark of the polarized proton in the presence of a polarized color electromagnetic field (the gluon) in Fig. 1, the lepton-pair yield has a dependence on the transverse orientation of the spin. The strong interaction phase necessary for SSAs arises from the interference between an imaginary part of the partonic scattering amplitude with the extra gluon and a real part of the scattering amplitude without a gluon in Fig. 1. The imaginary part is from the pole of the parton propagator associated with the integration of the gluon momentum \( x_g \). Depending on which propagator’s pole contributes, \( \Delta \sigma(S_{\perp}) \) gets contributions from \( x_g = 0 \) (soft-pole) \( \mathcal{I} \) and \( x_g \neq 0 \) (hard-pole). \( \mathcal{I} \).

3. By considering all partonic diagrams in Fig. 1 and their complex conjugates, one finds the spin-dependent differential cross section:

\[
\frac{d^3 \Delta \sigma^{q\bar{q}}}{dQ^2 dy d^2 q_{\perp}}(S_{\perp}) = \int dx \int_{x_1}^{x_2} dx' q(x') \delta(\hat{t} + \hat{u} - Q^2) \times C \cdot \left[ H_q^a + H_q^b \right](x, x', \hat{t}, \hat{u})
\]

where the sum over all quark flavors, weighted with their electric charge squared, is implicit. We have defined the factor \( C = \sigma_0 e^{\alpha \beta} S_{\perp, \alpha} q_{\perp, \beta} \alpha_q/(2\pi^2) \) with \( \sigma_0 = 4\pi a_s^2/3N_c sQ^2, N_c = 3 \). \( x' \) is the momentum fraction carried by the parton from the unpolarized proton, and \( x \) is the total parton momentum fraction from the polarized proton while \( x_g \) is fixed using the pole (on-shell) condition. The partonic Mandelstam variables are defined as \( \hat{s} = (xP + x'P')^2, \hat{t} = (xP - q)^2, \hat{u} = (x'P' - q)^2 \). \( H_q^a \) and \( H_q^b \) are the soft- and hard-pole contributions, respectively.

The soft-pole contributions can be calculated in the same way as for direct-photon and inclusive-hadron production. A total of eight diagrams \( \mathcal{I} \) sum to

\[
H_q^a = \frac{1}{2N_c} \left[ \frac{2}{2\pi} \frac{d}{dx} T_F(x, x) \frac{D_{q\bar{q}}^\alpha}{\hat{u} + \hat{t}} + T_F(x, x) \frac{N_{q\bar{q}}^\alpha}{\hat{u} + \hat{t}} \right], \quad (3)
\]

where the hard coefficients \( D_{q\bar{q}}^\alpha \) and \( N_{q\bar{q}}^\alpha \) are

\[
D_{q\bar{q}}^\alpha = \frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + \frac{2Q^2}{\hat{u}} \hat{s}
\]

\[
N_{q\bar{q}}^\alpha = \frac{1}{\hat{t}^2 u} \left[ Q^2 (\hat{u}^2 - \hat{t}^2) + 2Q^2 \hat{s} (Q^2 - 2\hat{t}) \right]
\]

\[
- (\hat{u}^2 + \hat{t}^2) \hat{l}^2
\]

with \( 1/2N_c \) the color factor. In the real photon limit, \( Q^2 = 0 \), we obtain the direct-photon cross section.

Kinematics allows only a t-channel propagator to contribute to a hard pole, and there are a total of 12 such diagrams \( \mathcal{I} \). The sum of their contributions is

\[
H_q^b = T_F(x, x - x_g) \frac{N_{q\bar{q}}^b}{\hat{u}} \left[ \frac{1}{2N_c} + C_F \frac{\hat{s}}{\hat{s} + \hat{u}} \right], \quad (4)
\]

where

\[
N_{q\bar{q}}^b = \frac{(Q^2 - \hat{t})^3 + Q^2 \hat{s}^2}{-\hat{t}^2 \hat{u}}.
\]

As mentioned above, for the hard-pole contributions \( x_g = -x\hat{t}/(Q^2 - \hat{t}) \) differs from zero. We find that the hard-pole contribution has no “derivative-term”, \( \propto \partial T_F(x, x)/\partial x \), due to a cancellation among the diagrams. \( \mathcal{I} \). When \( q_{\perp} \ll Q \), single spin asymmetries can be generated from a spin-dependent TMD quark distribution introduced by Sivers \( \mathcal{I} \). The physics of the Sivers function can be understood as follows. Consider a transversely-polarized proton traveling with large momentum \( P \). The distribution of quarks with longitudinal and transverse momentum \( xP \) and \( \vec{k}_\perp \) can have a dependence on the orientation of \( \vec{k}_\perp \) relative to the polarization vector \( \vec{S}_{\perp} \). The TMD quark distributions are best defined through the following matrix:

\[
M^{\alpha \beta} = p^+ \int \frac{d^2 \xi}{(2\pi)^2} e^{-ix\xi \cdot \vec{P}^+} \int \frac{d^2 \vec{b}_{\perp}}{(2\pi)^2} e^{i\vec{b}_{\perp} \cdot \vec{k}_{\perp}} \times \left[ P^{0+}(\xi^-, 0, \vec{b}_{\perp}) \gamma^\alpha \Psi_{\perp}(0) \right] P^+ \right].
\]

Here the vector \( p = (p^+, 0^{-}, 0_{\perp}) \) is along the momentum direction of the proton, \( \Psi_{\perp}(\xi) = \)
\[ \mathcal{L}_v(-\infty; \xi) \psi(\xi), \] and the gauge link is \( \mathcal{L}_v(-\infty; \xi) = \exp \left( -ig \int_{0}^{\infty} d\lambda v \cdot A(\lambda v + \xi) \right) \). In a system where \( P^+ \gg P^- \), the vector \( v \) with \( v^- \gg v^+ \) is given a non-zero \( v^+ \) component in order to regulate the light-cone singularities. An expansion of the matrix in Dirac indices yields

\[ M = \frac{1}{2} \left[ q(x, k_{\perp}) \not p + q_T(x, k_{\perp}) \epsilon_{\mu\nu\alpha\beta} \gamma^\mu \not p^\nu \not k^\alpha S^\beta + \ldots \right] \]  

(6)

where \( q(x, k_{\perp}) \) is the spin-independent distribution and \( q_T(x, k_{\perp}) \) is the Sivers function. The dependence on \( v \) is of the form \( \zeta^2 = (2v \cdot P)^2/v^2 \) and is implicit.

When \( q_L \ll Q \), the Drell-Yan cross section in leading order \( q_L/Q \) can be factorized in terms of TMD quark distributions. A generic Feynman diagram can be decomposed into various contributions, each of which is represented by one of the reduced diagrams shown in Fig. 2 [4]. This factorization property can be seen from the diagram in Fig. 1 by considering the real gluon emitted either along the direction of \( P \) or \( P' \). The single-spin-dependent part of the Drell-Yan cross section has the following factorization [11].

\[ \frac{d^2 \Delta \sigma(S_{\perp})}{dQ^2 dy dq^2} = \frac{\sigma_0}{M_p} \int d^2 \vec{k}_{\perp 1} d^2 \vec{k}_{\perp 2} d^2 \vec{q}_{\perp} H \]  

(7)

\[ \times \left[ \vec{k}_{\perp 1} \cdot \vec{q}_{\perp} \right] \delta^{(2)}(\vec{k}_{\perp 1} + \vec{k}_{\perp 2} + \vec{q}_{\perp} - \vec{q}_{\perp}) \]  

\[ \times q_T(z_1, k_{\perp 1}, z_2) q_T(z_2, k_{\perp 2}, \zeta_1) S(\Lambda_\perp)^{-1}, \]

where \( z_1 = Q/\sqrt{\xi_1} \) and \( z_2 = Q/\sqrt{\xi_2} \) are the momentum fractions of the colliding hadrons associated with the observed Drell-Yan pair. \( H \) is a hard factor and is entirely perturbative. The soft-factor \( S \) is a vacuum matrix element of Wilson lines and captures the effect of soft gluon radiation. Since the soft-gluon contribution in the TMD distributions has not been subtracted, the soft-factor comes with inverse power. The soft-gluon rapidity cut-off \( \rho \) is defined as \( \rho = \sqrt{(2v_1 \cdot v_2)^2/v_1^2 v_2^2} \), where \( v_1 \) and \( v_2 \) are the directions of the gauge links in the Sivers and the unpolarized antiquark distributions. In a special coordinate frame, \( \xi_1^2 \xi_1^2 = \xi_2^2 \xi_2^2 = \rho Q^2 \) [11].

5. From what we have discussed so far, the two mechanisms for the single-spin asymmetries appear to be quite different. In the case of the twist-three correlation, the nonperturbative function itself is time-reversal even. On the other hand, in the TMD factorization, the Sivers function is time-reversal odd and inherently contains the final-state interaction through a Wilson line. There is, however, a common kinematic region, \( \Lambda_{QCD} \ll q_L \ll Q \), where both mechanisms should work. In this region, \( q_L \) is large enough so that the asymmetry is a twist-three effect. But at the same time, \( q_L \ll Q \), so the TMD factorization formalism also applies in this region.

In order to see the consistency of the two approaches, we need to calculate the explicit \( q_L \)-dependence in the TMD factorization. To do that, we let one of the transverse momenta \( \vec{k}_{\perp 1} \) and \( \vec{\lambda}_{\perp} \) be of the order of \( \vec{q}_{\perp} \) and the others small. When \( \vec{\lambda}_{\perp} \) is large, for example, we neglect \( \vec{k}_{\perp 1} \) in the delta function, and the integrations over these momenta yield either the ordinary antiquark distribution or a moment of the Sivers function. The latter is related to the twist-three correlation [10]:

\[ \int d^2 \vec{k}_{\perp} q(x, k_{\perp}) = \bar{q}(x), \int d^2 \vec{q}_{\perp} q_T(k_{\perp}, x) = T_F(x, x). \]

When one of the \( \vec{k}_{\perp 1} \) is taken to be of order \( \vec{q}_{\perp} \) and \( \vec{\lambda}_{\perp} \) is neglected in the delta function, one also needs

\[ \int d^2 \vec{\lambda}_{\perp} S(\Lambda_\perp) = 1. \]

What remains is to calculate the large-\( k_{\perp} \) behavior of the TMD distributions and the soft factor. The latter has been determined in Ref. [11] and reads

\[ S(\Lambda_\perp) = \delta(\vec{\lambda}_{\perp}) + \frac{\alpha_s}{2\pi^2} \frac{1}{\lambda_{\perp}^2} C_F \ln \rho^2 - 2, \]

which is infra-red finite. To the same order, the unpolarized antiquark TMD distribution is given by

\[ \bar{q}(z_2, k_{\perp}) = \bar{q}(z_2) \delta(\vec{k}_{\perp}) + \frac{\alpha_s}{2\pi^2} \frac{1}{k_{\perp}^2} C_F \]

\[ \times \int \frac{dx'}{x'} \bar{q}(x') \left[ \frac{1 + \xi_2^2}{(1 - \xi_2)^+} + \delta(\xi_2 - 1) \left( \ln \frac{z_2^2 k_{\perp}^2}{\xi_2^2} - 1 \right) \right], \]

where \( \xi_2 = z_2/x' \) and an additional contribution involving the gluon distribution has been neglected [11].
As discussed above, the Sivers function at large $k_\perp$ can be calculated in terms of the twist-three quark-gluon correlation. There are also soft and hard pole contributions in this case. We show one example of the relevant Feynman diagrams in Fig. 3. There are a total of eight diagrams for the soft-pole contribution, and twelve for the hard-pole one. Adding all contributions, we find:

$$q_T^{(1)}(z_1, k_\perp) = \frac{\alpha_s}{4\pi^2} \frac{2M_F}{(k_\perp^2)^2} \int \frac{dx}{x} \left\{ A + C_F T_F(x, x) \times \delta(\xi_1 - 1) \left( \ln \frac{\xi_1^2}{k_\perp^2} - 1 \right) \right\}, \quad (10)$$

where the $1/(k_\perp^2)^2$ behavior follows from a dimensional analysis, and where

$$A = \frac{1}{2N_c} \left[ x \frac{\partial}{\partial x} T_F(x, x)(1 + \xi_1^2) + T_F(x, x - \bar{x}_g) \frac{1 + \xi_1}{(1 - \xi_1)^+} + T_F(x, x) \frac{(1 - \xi_1)^2 (2\xi_1^2 + 1) - 2}{(1 - \xi_1)^+} + C_F T_F(x, x - \bar{x}_g) \frac{1 + \xi_1}{(1 - \xi_1)^+} \right], \quad (11)$$

with $\bar{x}_g = (1 - \xi_1)x$ and $\xi_1 = z_1/x$.

Plugging the above results into the TMD factorization formula [1], we obtain the spin-dependent cross section for the $q + \bar{q}$ channel at large $q_\perp$:

$$\frac{d^3 \hat{q} \gamma^\gamma \gamma^\gamma}{dq^2 dy d^2 q_\perp} = C \frac{g^2}{q_\perp^2} \int \frac{dx}{x} \left\{ \delta(x_2 - 1)A + \delta(\xi_1 - 1)B \right\}, \quad (12)$$

where $A$ and $C$ have been given above and

$$B = C_F T_F(x, x) \left[ \frac{1 + \xi_2^2}{(1 - \xi_2)^+} + 2\delta(\xi_2 - 1) \ln \frac{Q^2}{q_\perp^2} \right]$$

where $\xi_2 = z_2/x'$. To compare this to the twist-three correlation calculation, we expand Eq. (2) for small $q_\perp/Q$. At leading order in $q_\perp/Q$, the partonic Mandelstam variables become $\hat{s} = q_\perp^2/(1 - \xi_1)(1 - \xi_2)$, $\hat{t} = -q_\perp^2/(1 - \xi_2)$, and $\hat{u} = -q_\perp^2/(1 - \xi_1)$. Using the above, we find that the spin-dependent cross section to leading order in $q_\perp/Q$ is exactly the same as the one shown in (12). The same conclusion applies for the contribution to the single-spin asymmetry generated by gluon-quark scattering.

To summarize, we have studied the single transverse-spin asymmetry in Drell-Yan pair production at both large and small transverse momenta $q_\perp$ of the lepton pair. At large $q_\perp$, one has a result in terms of a twist-three quark-gluon correlation function. At small $q_\perp$, a factorization formula in terms of TMD parton distributions is valid. We have demonstrated that in the intermediate region, both approaches give the same answer. This establishes an in explicit calculation for a physical process the connection between these two mechanisms for generating single-spin asymmetries. It is this connection that unifies the physical pictures for the underlying dynamics of single transverse-spin asymmetries and imposes an important constraint on phenomenological studies of the existing and future data. Extension to the gluon initiated subprocess and other physical processes, such as the semi-inclusive deep inelastic scattering, will be presented elsewhere, along with the details of the calculations described above.

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