An Optimization Algorithm of Robust Principal Component Analysis and Its Application

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Abstract. With the rapid development of robust principal component analysis (RPCA), it has been widely used in signal processing, pattern recognition, computer vision and other fields. The RPCA model has characteristics of complete reconstruction of the original signal from the noise pollution, high-dimensional and high-order complex signals. In order to solve the problems of slow iteration speed and low recovery accuracy in typical algorithms, an improved robust principal component analysis (RPCA) algorithm is studied. Firstly, the idea of smooth approximate zero norm is introduced to build the objective optimization function, then the inertial momentum is used to optimize each iteration of the matrix recovery process, finally, the model parameters are optimized by the grid method. Through simulation and comparative analysis, the results show that the improved algorithm has high accuracy, fast processing speed and remarkable application effect in field of logging data processing.

1. Introduction
The emergence of RPCA provides solutions for large-scale data processing problems. It has been successfully applied in data reconstruction, feature analysis, signal completion, signal recovery and other aspects [1-2]. It is applied in signal processing, pattern recognition and computer vision [3-6]. For the conventional RPCA algorithm, the basic idea of the commonly used algorithm is iterative threshold optimization with alternating updates. Similarly, for the $l_0$ norm optimization problem, the main methods are iterative hard-thresholding optimization algorithm and basis matching pursuit optimization algorithm [7]. Mohimani proposed an algorithm [8], that the smoothing function is used to approximate the minimized $l_0$ norm. So that the operation of solving the minimized norm is transformed into the extreme value of the smoothing function to solve the problem, which is called the reconstruction algorithm of approximate zero norm.

For the above problem, we analysis and research an improved algorithm that combining the principle of smooth functions with a kind of RPCA, and the RPCA optimization model using the approximate zero norm is constructed. The main improvement is in the smoothing optimization processing of the approximation method between the sparse matrix and the low-rank matrix in the RPCA algorithm, which ensure the noise robustness and improve the sparse recovery performance of the algorithm at the same time. In order to improve the efficiency of the low-rank matrix approximation problem with higher precision, a constraint method of high-efficiency inertial calculation is added in the improved algorithm, and the optimal parameter configuration is obtained by combining the grid search method. In the process of iteration in each layer, the incremental step is increased, which can keep the recovery precision and obviously shorten the convergence time. The improvement of this algorithm mainly includes three aspects: optimization model, algorithm iteration and parameter optimization. The result shows that the system has practical significance and remarkable effect in processing and analysing logging data.
2. The RPCA optimization model

Firstly, the idea of smooth approximate zero norm is introduced to build the objective optimization function, then the inertial momentum is used to optimize each iteration process of the matrix recovery, finally, the model parameters are optimized by the grid method.

2.1. The RPCA optimization model with Approximate zero norm

The key to the sparse matrix optimization is to find an approximate norm, the minimization problem of an approximate \(l_0\) norm can be replaced by the minimization of \(l_0\) norm, so that better sparse signal recovery effect can be obtained with a lower computation. In actual matrix recovery problem, the continuous smooth function is faster in data processing, and it is more difficult to use discontinuous functions such as \(l_0\) norm. There are many classical approximate norms such as Gaussian function[8], hyperbolic tangent function[9], arctangent function, etc. Therefore, the approximate zero norm function model in compressed sensing field is adopted to construct a new model in RPCA field, a smooth approximate \(l_0\) norm is used to approximate the discontinuous \(l_0\) norm in the RPCA algorithm.

Generally, the approximate zero norm function is selected to meet the characteristics of \(l_0\) norm. In this paper, the simplified fast fractional function \(\rho_\alpha(t)\) is studied to replace the discontinuous \(l_0\) norm.

The fractional function is shown in equation (1).

\[
\rho_\alpha(t) = \frac{bt}{bt+1}, \quad t \geq 0
\]

(1)

Where, \(b \in (0, +\infty)\).

According to equation (2), define \(F_\alpha(x) = \sum |x_i| \), then formula (1) can be optimized to be solved by a simplified smooth function, and its optimization model is as follows equation (2).

\[
\min_{A,E} \|A\|_1 + \lambda P_\alpha(E)
\]

(2)

2.2. Improved RPCA algorithm with inertial quantity constraint

In order to further improve the recovery speed and performance of the algorithm, we consider to add an inertial quantity in the model to further constrain the calculation compensation amount of the improved RPCA algorithm, so as to adapt the step length of iterative update. There are many applications in the field of computer vision. For example, it can be described as the constraint quantity in the form of \(f\)-norm matrix according to the actual noise form[10], so as to achieve better robustness of the results. The sparse noise can be effectively removed by adding the constraint term of total variation[11]. Based on the above analysis, we study a simple and efficient inertial quantity constraint term, and add this constraint term into the original main equation (2). The optimization model is shown in equation (3).

\[
\min_{A,E} \|A\|_1 + \lambda \|E\|_1 + \beta \|Z\|_1
\]

(3)

Among them, \(\beta\) is the equilibrium factor, \(\|Z\|_1\) is the constraint term, whose essence is the constraint inertia obtained by the convergence of the matrix \(A\) and \(E\) in each iteration.

2.3. Grid search method optimization parameters

In the actual algorithm, we need accurate parameters to carry out reasonable weight matching for each matrix to achieve the optimal result. For the selection of parameters of equilibrium factors \(\rho\) and \(\beta\), we draw lessons from the ideas of grid search to improve the optimization of the parameters in the model, and grid search method is to try every possible \((\alpha, \beta)\) value. The method of grid search looks primitive but the expression is intuitive. In fact, there are many advanced algorithms, such as approximation algorithms or heuristic searches to reduce complexity. However, we tend to use the simple method of grid search for the following reasons: in the case of less parameter selection interval, the implementation of grid search idea is lower than the complexity of the advanced algorithm, and each pair \((\alpha, \beta)\) is
independent of the other and highly feasible. It will be realized automatically by using the exhaustive method to run all the parameters in the range used and find the most reasonable parameter matching.

3. The steps of improved optimization algorithm
The steps of improved optimization algorithm of smoothing inertial constraint RPCA (SICRPCA) is shown in table 1.

Table 1. Improved optimization algorithm of smoothing inertial constraint RPCA

| Algorithm 1. Improved optimization algorithm of smoothing inertial constraint RPCA |
|---------------------------------------------------------------|
| Input: The observation matrix $D$, $\lambda$, $b = 1.3$. |
| 1: Initialize $Y_0$, $E_0 = 0$, $\mu_0 > 0$, $\rho > 0$, $\beta > 0$, $k = 0$. |
| 2: While not converged do |
| 3: $(U, \Sigma, V) = svd(D - E_k + Y_k / \mu_k)$ |
| 4: $A_{k+1} = US_{i/k} \Sigma V^T$ |
| 5: $V_{k+1} = \mu_k^{-1}\lambda \text{sign}(E_k) - \mu_k^{-1}\lambda \text{sign}(E_k) + \frac{b}{|E_k| + 1} + \mu_k^{-1}\lambda \frac{b^2 E_k}{(b|E_k| + 1)^2}$ |
| 6: $E_{k+1} = S_{\rho \mu_k} (V_{k+1} + D - A_{k+1} + Y_k / \mu_k)$ |
| 7: $Z_{k+1} = \mu_k (D - A_{k+1} - E_{k+1}^*)$ |
| 8: $Y_{k+1} = Y_k + Z_{k+1} + Z_k \cdot \mu_{k+1} = \rho \mu_k$ |
| 9: $k = k + 1$ |
| 10: end while |

Output: $A_k$, $E_k$.

4. Experiment and analysis
The simulation data is used to simulate and verify the algorithm. The simulation data are generated according to randomly generated array $\{0, \ldots, m\}$ rank $\Lambda^*$. $E^*$ is a sparse matrix, and $m$ is the dimension of the generated matrix. The operating environment of the simulation experiment: Intel(R) Core(TM) i5 CPU, frequency parameter 2.30GHz, memory 8.00GB. The software platform: windows10 (64 bit). The simulation software: MATLAB R2016a.

4.1. Performance testing of the change with dimension
The experiment tests EALM, IALM, ST0RPCA (RPCA with approximate zero norm) and SICRPCA in the case of $r = 50$ and $r = 100$, where $\rho = 1.5$ in IALM and $\rho = 1.8$, $\beta = 0.22$ in the SICRPCA algorithm. In the case of 10% Gaussian noise, the matrix decomposition operation is carried out for the matrices with different dimensions of uniform increase of $m$. Figure 1 and figure 2 show the comparison between the iteration time and error rate of the basic model and the improved algorithm when the rank of the matrix is 50 and 100 respectively.

As can be seen from figure 1 and figure 2, the SICRPCA algorithm is slightly faster than the ST0RPCA algorithm in terms of iteration time, and its recovery accuracy correspondingly maintained the recovery level of ST0RPCA algorithm.
4.2. Performance testing of the change with rank

In order to further fully demonstrate the advantages of the improved algorithm, the experiment tests EALM, IALM, ST0RPCA and SICRPCA in the case of matrix dimension $m = 1000$ and the Gaussian noise is 10%, where $\rho = 1.5$ in IALM and $\rho = 1.8$, $\beta = 0.22$ in the SICRPCA algorithm. The testing results is shown in figure 3.

As can be seen from figure 3, the iterative time required for four kinds of algorithms are slowly increase with the increase of rank, and the corresponding error rate will also increase. In addition, the iteration time and error rate of the improved algorithm are small and are also stable, and the results obtained in different scenarios are superior.
4.3. Experiment of mixed noise

The experiment tests EALM, IALM, ST0RPCA and SICRPCA in the case of adding the zero-mean mixed noise close to the noise in the real environment. The results are shown in table 2.

As can be seen from table 2, the performance of several algorithms is reduced. The iteration of SICRPCA needs the shortest time and the error rate is relatively low. From this perspective, SICRPCA algorithm has the most advantages and the best effect in the experiment.

| Matrix dimension m | Algorithm | Iteration time (s) | Iteration times | rank (A) | ||(A) − A||_F / ||A||_F   |
|-------------------|-----------|-------------------|----------------|----------|-----------------------------|
| 500               | EALM      | 4.7932            | 7              | 25       | 0.03019                    |
|                   | IALM      | 3.4634            | 32             | 25       | 0.02984                    |
|                   | ST0       | 1.7236            | 33             | 25       | 0.03425                    |
|                   | SIC       | 1.6083            | 33             | 25       | 0.03458                    |
| 1000              | EALM      | 36.4677           | 7              | 50       | 0.01981                    |
|                   | IALM      | 17.5701           | 32             | 50       | 0.02046                    |
|                   | ST0       | 8.6101            | 34             | 50       | 0.02184                    |
|                   | SIC       | 7.3245            | 34             | 50       | 0.02176                    |
| 1500              | EALM      | 84.8424           | 7              | 75       | 0.01667                    |
|                   | IALM      | 46.3066           | 32             | 75       | 0.01835                    |
|                   | ST0       | 36.7190           | 34             | 75       | 0.01679                    |
|                   | SIC       | 36.0172           | 34             | 75       | 0.01641                    |
| 2000              | EALM      | 179.6730          | 8              | 100      | 0.01240                    |
|                   | IALM      | 118.7253          | 33             | 100      | 0.01454                    |
|                   | ST0       | 89.1948           | 34             | 100      | 0.01368                    |
|                   | SIC       | 81.0462           | 34             | 100      | 0.01373                    |
| 3000              | EALM      | 468.2457          | 8              | 150      | 0.01135                    |
|                   | IALM      | 192.0146          | 33             | 150      | 0.01248                    |
|                   | ST0       | 135.5349          | 35             | 150      | 0.01097                    |
|                   | SIC       | 119.1047          | 35             | 150      | 0.01091                    |

5. Log data processing

Before the use of logging data information, the process of cleaning and pre-processing the data information with efficiently stable method ensure the processed data can meet the minimum accuracy required for reservoir identification.

The process of logging data processing technology is as follows:

1. Input the initial sparse matrix.
2. Extract information from the original matrix to obtain background information with RPCA.
3. De-noising the background information with RPCA to get low-rank part and sparse part.
4. The clean logging data can be obtained by adding the target matrix and low-rank part.
5. Output the denoising recovery matrix.

The GR attribute of logging data is cleaned and analysed in figure 4, and the comparison of characteristics is shown in table 3.

![Figure 4. The processing of GR curve](image)
As can be seen from the Table 3, SICRPCA has the lowest processing error rate and the shortest processing time for logging data processing.

| Algorithm | 1st Iteration times | 2nd Iteration times | 1st Iteration time (s) | 2nd Iteration time (s) | Error rate |
|-----------|---------------------|---------------------|------------------------|------------------------|------------|
| EALM      | 8                   | 9                   | 2.1262                 | 1.1727                 | 0.07141    |
| IALM      | 32                  | 35                  | 0.5449                 | 0.5927                 | 0.07539    |
| ST0       | 23                  | 18                  | 0.3977                 | 0.2249                 | 0.01284    |
| SIC       | 27                  | 17                  | 0.3146                 | 0.2094                 | 0.01053    |

6. Conclusion
In this paper, a kind of improved robust principal component analysis (RPCA) algorithm is studied. The smoothing fraction structure is used as the approximate zero norm to approximate the $l_0$ matrix. The inertial quantity constraint term is introduced, and the compensation quantity can be dynamically distributed to increase the contraction range of the threshold value to achieve the constraint effect of the specified accuracy. The model parameters are optimized by the grid method. The matrix is recovered by Lagrange multiplier. It can be concluded through four groups of simulation experiments, that the improved optimization algorithm has the advantages of the shortest operation iteration time and relatively low recovery error rate. It is applied in the field of logging data processing and obtain the result of fast operation and low error rate, which has remarkable application effect.

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