Phase transitions and social distancing control measures for SARS-CoV-2 on small world networks

Benjamin Braun*1, Başak Taraktaş2, Brian Beckage3,4,5, Jane Molofsky3

1 Department of Mathematics, University of Kentucky, Lexington, Kentucky, USA
2 Department of Political Science, Boğaziçi University, Istanbul, Turkey
3 Department of Plant Biology, University of Vermont, Burlington, Vermont, USA
4 Department of Computer Science, University of Vermont, Burlington, Vermont, USA
5 Gund Institute for Environment, University of Vermont, Burlington, Vermont, USA

* These authors contributed equally to this work.

* benjamin.braun@uky.edu

Abstract

We investigate the efficacy of three social distancing controls on the spread of SARS-CoV-2 using an agent based SIR model on a small world network structure: 1) Global social distancing with a fixed probability of adherence. 2) Individually initiated social isolation when a threshold number of contacts are infected. 3) Use of personal protective equipment (PPE) to reduce viral shedding and resultant infectivity. The primary driver of total number of infections is the viral shedding rate, with probability of social distancing being the next critical factor. These results suggest that higher compliance with PPE usage and personal hygiene has the potential to decrease the number of infections and shorten epidemic duration. Individually initiated social isolation was effective when initiated in response to a single infected contact. The combination of social isolation and PPE resulted in very low levels of infection. Our model suggests that widespread application of social distancing through government control can drastically reduce viral spread; even in the absence of widely adopted social distancing protocols, high use of PPE can also dramatically reduce the viral spread while short-duration quarantine following exposure to an infected individual was less effective.

Introduction

The SARS-CoV-2 virus that has spread throughout the globe has created societal disruption and had a massive impact on global health [1]. With no known treatment, public policy and human behavior are currently the only tools that are available to mitigate the spread [2]. Different countries, and different states within the US, have implemented different approaches to social distancing [2,3]. While most government plans include some social distancing, questions have arisen as to the efficacy of social distancing, how long social distancing should last and to what extent it is needed [4]. Moreover, citizens who have been told to shelter in place have grown frustrated and want confirmation that social distancing is having the desired effect [5,7].

In addition to control imposed by governments to reduce human contact, there have also been social changes that have taken place that may work in concert to mitigate the spread of the virus [8,9]. Individuals who live with an infected individual...
are being asked or required to quarantine for 14 days prior to interacting in the larger society [10]. Removing individuals who have known exposure to the virus from social contacts will affect the viral spread. One question is whether these individual responses of behavioral modification will be enough to moderate the spread and whether there are additive or non-additive effects when implemented with top-down government policy on social distancing [8].

One key parameter of the viral spread is the contact rate between infected and non-infected individuals [3]. As the number of contacts increases, the probability of getting the virus increases [11, 12]. While modeling contacts can be done in mean-field, statistical, and metapopulation SIR models [1, 13–16], agent-based models provide a way to directly investigate how the connectedness of the social network influences viral spread [17, 18].

In this paper, we examine three types of social distancing and control measures that can influence the viral spread in a social network. The first type is social distancing imposed on the social network at a global scale, where the likelihood of social distancing is applied uniformly to all agents in the network. This form of social distancing serves as a proxy for government policies such as shutting down businesses and issuing shelter in place orders [19]. The second type of social distancing occurs when agents have social connections that are infected and symptomatic [2, 20]. In this case, agents isolate from their neighbors in the network if they are surrounded by enough symptomatic neighbors. This type of behavior is frequently recommended by government authorities, though enforcement is sometimes limited and so requires that agents choose the appropriate action [19]. The third type of control measure involves the reduction of the rate of viral spread, for example through personal protective equipment (PPE) such as wearing a mask in public to reduce viral transmission [20, 21]. We examine how each of these measures alone and in concert with each other influence the viral outbreak.

We ask the following three questions:

1. When social distancing is top-down and imposed globally, is there a phase transition, i.e. a sharp phase transition where the total number of infected individuals varies dramatically, with varying levels of compliance? If such a phase transition exists, then government policy may seek to maximize the efficacy of their social distancing policy while mitigating the costs.

2. How does behaviorally imposed social distancing both through self-isolation and the use of PPE interact with globally imposed social distancing? Are the effects additive or non-additive? If the effects of the different types of social distancing are synergistic then imposing all three social distancing measures may reduce the epidemic to levels that neither policy nor behavioral change could achieve alone.

3. Finally, how does the reduction of infectivity as through the mitigation of viral shedding using PPE, influence epidemic spread under different social distancing and control scenarios?

We examine these questions by developing an agent-based model implemented on a Watts-Strogatz small world network that reasonably approximate real-world social networks [22, 24].
Model and Parameters

Watts-Strogatz networks

We implement an agent-based model on Watts-Strogatz (WS) networks. The WS networks can simultaneously demonstrate both high clustering and short average path length, and thus serve as effective approximations of social networks that are neither completely random nor regular [22]. High clustering and short average path length allow for local interactions and more distant interactions to be incorporated [23–25], which are properties often found in real-world networks.

The WS small world network in our model is characterized by three parameters: number of nodes $N$, average node degree $K$ and rewiring probability. The rewiring parameter is used to determine the likelihood of rewiring each edge starting from a regular ring lattice. A rewiring parameter of 0 preserves the original ring lattice; a rewiring parameter of 1 simulates a random network. We fix the number of nodes $N = 500$ and the average degree $K = 20$, which allows $\ln(N) \ll K \ll N$. We then vary the rewiring probability among the values $\{0.0, 0.1, 0.25, 0.5\}$. For each of our four rewiring probabilities we construct 10 networks on which to run simulations.

Agent-Based Model

We develop an SIR, network-based, agent-based model where agents pass through various infection states (Fig. 1). Similarly to SARS-CoV-2, agents pass through an initial asymptomatic infection state followed by either a main asymptomatic infected period or a symptomatic infected period [3].

In our model, each agent carries an individual pathogen level that varies over time. Initially, this level is set to 0 pathogen units for susceptible agents. At each time step (conceived as a day) a susceptible agent increases their pathogen level by a fixed fraction of the pathogen levels of their infected neighbors. Model runs are initiated with a small number of infected agents, whose pathogen levels are initially set at 35 pathogen units, and the remainder of the agents are initially deemed susceptible. In our model, there is a global pathogen-level infection threshold that applies to all agents. We fixed that threshold to 25 pathogen units. If the pathogen level for an agent exceeds this threshold, then the agent become infected. These initial and threshold levels for the pathogen in the model are not based on real-world data, but rather were selected for simplicity to illustrate the mechanism of viral shedding.

Following the onset of infection, agents pass through an initial asymptomatic infection state, followed by a main infection state (either symptomatic or asymptomatic), before becoming resistant/removed (Fig. 1). The length of the initial asymptomatic state is the same for all agents, and can be set to last one or more days. Once the initial asymptomatic state passes, the agent enters one of two main states: infected and asymptomatic or infected and symptomatic. The lengths of the two main states are set independently from each other, but are the same for all agents. Following the main infection state, the agent is either resistant or removed.

In addition to the infection state parameters, each agent is in one of two behavior states: socially distanced or not socially distanced. The behavior state is reset each day. If an agent is not socially distanced on a given day, then that agent can interact with any neighboring agent. If an agent is socially distanced, the agent does not interact with any neighboring agents. Agents socially distance in a given day for one of two reasons. A global social distance probability is set, which determines the chance that an agent will socially distance on a given day. A local social distance threshold is set, and this value dictates individual responses to infected symptomatic neighbors. If the number of infected symptomatic neighbors of an agent equals or exceeds this
threshold, the agent will social distance independently of the global parameter.

**Fig 1. Agent Infection States.** Flow chart of infection pathways in the agent based model.

- **Susceptible (S)**
- **Initial Infected Asymptomatic ($I_{a}^{\text{init}}$)**
- **Main Infected Symptomatic ($I_{a}^{\text{main}}$)**
- **Main Infected Asymptomatic ($I_{a}^{\text{main}}$)**
- **Resistant/Removed (R)**

**Model Parameters and Simulations**

We ran two sets of simulations over different parameter spaces. Our primary simulation ran through ten networks for each set of parameters given in Table 1. Based on the results of this primary simulation, we ran a secondary set of simulations over a more refined parameter space given in Table 2 to analyze the phase transition behavior observed in the primary simulations. The parameters for the secondary simulation were selected based on our analysis of the primary data using regression trees to identify critical variables and on the observed ranges where phase transitions were observed.

Our simulations are designed to determine the efficacy and interactions between several key parameters concerned with social interactions and control measures: 1) top-down imposed social distancing, 2) behaviorally-adjusted social distancing and 3) lowered viral shedding through increased PPE use and personal hygiene. We conceptualize our model parameters in the following way:

1. **Social distance probability:** the probability that an agent is socially distancing on any given day. This is conceived as representing government-imposed social distancing policies.

2. **Social distance threshold:** the number of infected symptomatic neighbors needed to cause an agent to social distance for that day. This is conceived as representing individual response to the presence of a symptomatic infected social connection.
| Parameter                                      | Description                                      | Value       |
|------------------------------------------------|--------------------------------------------------|-------------|
| **Infection**                                  |                                                  |             |
| Initial asymptomatic period                    | incubation period after exposure                | 1, 3, 5     |
| Pathogen infection threshold                   | amount of virus to become infected               | 25          |
| Main asymptomatic infection period             | period of infection without symptoms            | 5, 8, 10    |
| Main symptomatic infection period              | period of infection with symptoms               | 5, 8, 10    |
| Chance symptomatic                             | the probability an agent becomes symptomatic     | 0.25, 0.50, 0.75 |
| **Network**                                    |                                                  |             |
| Number of nodes (N)                            | number of agents                                 | 500         |
| Average node degree (K)                        | the average number of contact for agents         | 20          |
| Rewiring probability                           | probability each edge is rewired                 | 0.05, 0.10, 0.25, 0.50 |
| Initial outbreak size                          | number of infected agents at the start of the run| 5           |
| **Social Distance and Controls**               |                                                  |             |
| Social distance probability                    | the probability (out of 100) that an agent socially distances on a given day (reported as a percentage) | 0, 10, 20, 30, 40, 50, 60, 70, 80, 90 |
| Social distance threshold                      | number of infected neighbors needed for an agent to social distance | 1, 2, 4, 8 |
| Viral shedding                                 | the amount of pathogen level that the infected agents shed | 1%, 5%, 25% |
Table 2. Secondary Simulation Parameters

| Parameter                                | Value                  |
|------------------------------------------|------------------------|
| **Infection**                            |                        |
| Initial asymptomatic period              | 3                      |
| Pathogen infection threshold             | 25                     |
| Main asymptomatic infection period       | 8                      |
| Main symptomatic infection period        | 8                      |
| Chance symptomatic                       | 0.25, 0.50, 0.75       |
| **Network**                              |                        |
| Number of nodes (N)                      | 500                    |
| Average node degree (K)                  | 20                     |
| Rewiring probability                     | 0.10                   |
| Initial outbreak size                    | 5                      |
| **Social Distance and Controls**         |                        |
| Social distance probability (as percentage) | 60, 61, 62,…, 78, 79, 80 |
| Social distance threshold                | 1, 2, 3, 4, 5          |
| Viral shedding                           | 5%, 10%, 15%, 20%      |

3. Viral shedding: the fraction of viral load that an infected agent passes to each of its neighbors. This is conceived as representing general adherence to control measures such as mask-wearing and physical distancing.

**Results**

**Regression Tree**

We used a regression tree to partition the variation in final number of infected nodes across model parameters and runs in our primary simulation [29]. Reductions in viral shedding were associated with the primary partition in the regression tree in Fig. 2. Viral shedding rates below 15 relative to an unmitigated value of 25 were associated with a mean number of infections of 46 of 500 agents. Reduced viral shedding rates with social distancing over 25% led to overall infection of approximately 2% of the population. If the overall viral shedding is reduced dramatically to 1/5 of the unmitigated rate, even without additional social distancing, less than 1% of the population becomes infected.

Achieving low levels of infections in populations without reducing viral shedding requires significantly higher levels of top-down imposed social distancing (over 75%) which results in an approximately 1% infection rate in the population. If below 75% of the agents are social distancing, the infection rates for the populations are much higher; these range from a low of 21% (if individuals self-isolate in response to one infected social contact) all the way up to 97% with poor adherence to social distancing.

Thus, with a higher level of viral shedding, it becomes important to have agents self-isolate when a social contact becomes symptomatic. Even if this occurs, the infection rate in the population is an order of magnitude higher (10% vs. 1%) then if the viral shedding was reduced. Failure to achieve this strict social distancing in response to an infected social contact results in a wide-spread outbreak (approximately 62% of the population infected).
Fig 2. Regression Tree for Total Number of Infections. This regression tree identifies the input features with strongest influence on total number of infections. Each box contains the percentage of observations and associated mean number of infected agents.
Phase transitions

Based on the primary drivers of total number of infections found in the regression tree analysis, we conducted secondary simulations on a refined parameter space. In these simulations, we observed strong phase transitions in the total number of infections as a function of the social distancing and viral shed parameters. These transitions are shown in Fig. 3, Fig. 4, and Fig. 5. In all figures, the maximum number of infections is 500 (the number of nodes in the model).

In Fig. 3, a phase transition exists between viral shedding of 5% and 10%, across all levels of social distance thresholds and social distance probabilities. In Fig. 4, a clear phase transition exists at a social distance threshold of 1, across all levels of social distance thresholds and social distance probabilities. If the social distance threshold parameter is 2 or more, then it is possible to have epidemics that infect the entire population. In Fig. 5, a phase transition exists around a social distance probability of 73-74%, across the levels of social distance threshold and viral shedding. If the social distance probability is 74% or more, then our simulations end with a small number of infected agents.

Fig 3. Viral Shedding.

Viral shedding vs. final infection of agents for different levels of social distance thresholds (1 through 5) and for social distance probabilities varying from 60% to 80% at 1% intervals. A clear phase transition exists at viral shedding of 5-10% across the levels of social distance thresholds and social distance probabilities.

Number of Infections and Length of Epidemic

Given the regression tree analysis of our primary simulations, it is clear that viral shedding and social distance probability play key roles. In our secondary simulations over a refined parameter space, this becomes more clear. In Fig. 6 it is apparent that the primary driver of total number of infections is the viral shedding rate, with social
Fig 4. **Social Distance Threshold.** Social distance threshold vs. final infection of agents for different levels of viral shedding (5% to 20%) and for social distance probabilities varying from 60% to 80% at 1% intervals. A clear phase transition exists at a social distance threshold of 1, across all levels of social distance thresholds and social distance probabilities.

Distance probability being the next critical factor. Specifically, simulations with large total infections cluster to the upper left of the plot, where viral shedding rates are higher and social distancing is enacted by approximately 60% of agents.

There is also a clear interaction between the social distance probability and viral shedding parameters and the resulting number of infected agents and the length of the epidemic. These interactions are shown in Fig. 7 and Fig. 8. In Fig. 7, there is clustering of long epidemics when the chance is near 60% and the viral shedding rate is high. As the social distance probability increases to 80% and the viral shedding rate decreases, there is a phase transition where simulations result in outbreaks of short duration. In Fig. 8, most infections result in either a limited outbreak (less than 25% of population) or the vast majority of the population infected. As the social distance probability is increased from 60% to 80%, the length of the epidemics increase while remaining limited in total number of infections before jumping to high number of infections and then returning to short epidemic lengths.

**Discussion**

Mathematical modeling can provide tools to better understand epidemic dynamics and can vary from purely theoretical to more data driven and predictive [26]. While a simple model such as this one should not be used to make large-scale policy recommendations, it can provide a framework for specific hypothesis testing. Also, our model may be applicable to social networks of smaller size exhibiting small world characteristics, such as seen in college settings [32]. Here we use the model to ask
Fig 5. Social Distance Probability. Social distance probability vs. final infection of agents for different levels of viral shedding (5% to 20%) and for social distance threshold varying from 1 to 5. A phase transition exists around a social distance probability of 73-74%, across the levels of social distance threshold and viral shedding.

several specific questions about how different behaviors might impact population infection rates. We specifically examine three main actions that can be taken to curb the SARS-CoV-2 spread: 1) government imposed social distancing; 2) behaviorally induced social isolation following virus exposure and 3) infection rate reduction, e.g. through behavioral adaptation through PPE or increased hand-washing and disinfecting.

Early in the pandemic, government imposed social distancing was the primary tool used to slow infection rates or as commonly described “flatten the curve” [1]. Actions taken are quite varied and there is generally no uniform agreement about how much social distancing is necessary to significantly slow SARS-CoV-2 transmission. To examine the full potential for social distancing we examine a wide range of possible scenarios varying from no official social distancing to extreme social distancing (90%). While the two extreme scenarios are easy to envision (zero social distancing is business as usual and 90% is all but non-essential businesses closed), more moderate social distancing scenarios are harder to translate into direct societal actions. Nevertheless, we find a clear phase transition with social distancing in our small-world models. In general, social distancing compliance below 65% results in a wide-spread outbreak, while social distancing compliance above 75% contains the virus to low levels. For our secondary simulations over a refined parameter space, in the absence of stringent individual social distancing responses we observe that there is a phase transition that occurs as the percentage of agents socially distancing changes from 73% to 74%.

While it is difficult to relate these transitions to specific policies that may be put in place, it is apparent that aggressive social distancing policies are required to contain an outbreak. Moreover, social distancing measures that allow for more inter-agent contact will likely allow rampant viral spread.
In addition to government actions, individual behavior taken during the pandemic can greatly affect the dynamics of the pandemic. Most governments have issued guidelines for social isolation if in contact with a diseased individual. The most commonly recommended guideline is 14 days of self-isolation to avoid exposing other individuals [6, 8, 10]. However, despite these official guidelines, self-isolation following exposure requires that infected individuals inform their contacts and that exposed individuals voluntary comply. Given these caveats, we wanted to understand how self-isolation following an infected symptomatic social contact would influence final disease progression. In this model, self-imposed social distancing occurs only on the days when the agent has sufficiently many symptomatic contacts in the network. Thus, our model incorporates self-imposed social distancing as highly responsive to an agent’s short-term perceptions regarding infection risks within their community. Interestingly, for self-isolation to negatively impact the number of infections, an extreme level of responsiveness was needed by the agents involved. In our model, it was necessary for self-isolation to occur following exposure to only one infected agent. If self-isolation occurred following the symptomatic infection of two or more contacts, the effect on viral spread was minimal. Given the high number of asymptomatic infected people with SARS-CoV-2 [15, 27], it is likely that many individuals who have come in contact with an infected individual are not aware of their exposure. Our findings also highlight the well-known fact that contact tracing following an individual’s positive test is critical for limiting the spread of the infection [4].

Individuals responsiveness to recommended government actions are highly variable [6]. One control measure to decrease the virus spread is wearing a mask in public [21]. While this behavior is widely adopted in Asian countries that had experience with prior epidemics, it was much slower to catch on in Western
Mask wearing in our model is captured by viral shedding. In our model, if the viral shedding rate is very small, then the epidemic does not spread. With moderate rate of viral shedding, the social distance threshold at which someone decides to self-isolate after coming into contact with an infected individual becomes much more important. In our model, if the social distance threshold is set to 1 (agents self-isolate after coming into contact with at least one infected agent), then the final total infection rate in the population is approximately 12%. However, if the behaviorally induced social distancing does not take place or takes place at a higher threshold, then the final infection is much larger with the overall infection rate in the population approximately 62%. Finally, when the viral shedding in our model was set at a high level of 25% (simulating a comparatively more infectious disease), the government imposed social distancing was required to be greater than 80% to control the outbreak, resulting in an approximately 1% infection rate in the population. Other less stringent social distancing conditions result in a viral infection rate between 25% and 97.5%.

**Conclusion**

Although not prescriptive, the analysis of our simple agent-based model on synthetic networks provides strong evidence for generalizations. Social distancing controls in this model exhibit a sharp phase transition regarding total number of infections, either
when imposed at a large scale by government mandates or when based on individual response to infected contacts. Our models suggest that in order to be effective, mandatory social distancing such as stay-at-home orders need to be broadly applied across all segments of society with a high degree of adherence by individuals. Alternatively, self-isolation must be immediately enacted if a social contact is known to be infected. Finally, if it is possible to reduce viral shedding through adherence to mask wearing, then the size of the final infected population can be significantly reduced.

Supporting information

S1 File. NetLogo code and experimental data. https://github.com/braunmath/social-distance-effects-covid19

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