Upper energy limit of heavy baryon chiral perturbation theory in neutral pion photoproduction

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A R T I C L E   I N F O

Keywords:
Chiral perturbation theory
Effective field theory
Pion photoproduction
Heavy baryon

A B S T R A C T

With the availability of the new neutral pion photoproduction from the proton data from the A2 and CB-TAPS Collaborations at Mainz it is mandatory to revisit Heavy Baryon Chiral Perturbation Theory (HBChPT) and address the extraction of the partial waves as well as other issues such as the value of the low-energy constants, the energy range where the calculation provides a good agreement with the data and the impact of unitarity. We find that, within the current experimental status, HBChPT with the fitted LECs gives a good agreement with the existing neutral pion photoproduction data up to ~170 MeV and that imposing unitarity does not improve this picture. Above this energy the data call for further improvement in the theory such as the explicit inclusion of the Δ(1232). We also find that data and multipoles can be well described up to ~185 MeV with Taylor expansions in the partial waves up to first order in pion energy.

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1. Introduction

Chiral Perturbation Theory (ChPT) is an effective field theory (EFT) of Quantum Chromodynamics (QCD) in the low-energy domain where quarks and gluons are confined into hadrons and conventional perturbation theory cannot be directly applied. Due to the spontaneous breaking of chiral symmetry in QCD the π meson appears as a pseudoscalar Nambu–Goldstone boson [1] becoming the carrier of the nucleon–nucleon interaction. However, when fully relativistic spin-1/2 matter fields (i.e. nucleon) are introduced in the theory the exact one-to-one correspondence between the loop expansion and the expansion in small momenta and quark masses is spoiled [2]. This is due to the fact that the nucleon mass M does not vanish in the chiral limit. A consistent power counting scheme known as Heavy Baryon Chiral Perturbation Theory (HBChPT) [3] overcomes this difficulty considering the baryons as heavy (static) sources. For πN scattering and pion photoproduction HBChPT has been successful at describing experimental data in the near threshold region [3,4]. In this Letter we address the question of how well it works for the latest and most accurate γp → π 0p reaction taken from threshold through the Δ(1232) region. This was performed with a tagged photon beam with energy bins of 2.4 MeV. We also determined the low-energy constants (LECs) to see if they are actually constant as the photon energy is increased. The quality of the HBChPT fits —χ2 per degree of freedom (χ2/dof) — are also compared to a simple empirical benchmark fit, a Taylor expansion of the partial waves. The data in [5] are more accurate than previous experiments and the first measurement of the energy dependence of Σ. This has allowed an extraction of the real parts of the four dominant multipoles for the first time — the S-wave E0+, and the three P-wave multipoles P1/2,3 (E1+, M1+, M1−). This is a much more significant test of the agreement of HBChPT with experiment. As the photon energy increases and the calculations gradually stop agreeing with experiment we have determined whether or not this is caused by one particular multipole. This information, in addition to the behavior of the low energy constants with photon energy provide clues about what improvements are needed to make the HBChPT calculations more accurate.

2. Theoretical framework

Due to the symmetry breaking, the S-wave amplitude for the γp → π 0p reaction is small in the threshold region — vanishing in the chiral limit [4]. Additionally, the P-wave amplitude is large and leads to the Δ(1232) resonance at intermediate energies [6]. Hence, for the γp → π 0p reaction the S- and P-wave contributions
are comparable even very close to threshold [7] and even D waves have an important early contribution due to the weakness of the S wave [8]. The differential cross section and photon asymmetry can be written in terms of electromagnetic responses

$$\frac{d\sigma}{d\Omega}(s, \theta) = \frac{q}{k_y} W_T(s, \theta)$$

$$\Sigma(s, \theta) = \frac{\sigma_\perp - \sigma_\parallel}{\sigma_\perp + \sigma_\parallel} = -\frac{W_S(s, \theta)}{W_T(s, \theta)} \sin^2 \theta$$

where $W_T$ and $W_S$ are the electromagnetic responses, $\theta$ is the center of mass scattering angle, $k_y$ the center of mass photon energy, $q$ the pion momentum in the center of mass, and $s$ the squared invariant mass. The responses $W_T$ and $W_S$ are defined in term of the electromagnetic multipole responses:

$$W_T = T_0(s) + T_1(s)P_1(\theta) + T_2(s)P_2(\theta) + \cdots$$

$$W_S = S_0(s) + S_1(s)P_1(\theta) + \cdots$$

where $P_j(\theta)$ are the Legendre polynomials in terms of $\cos \theta$, the dots stand for negligible corrections, and

$$T_n(s) = \sum_{ij} \text{Re} \left[ \mathcal{M}_i^*(s) T_{ij} M_j(s) \right]$$

$$S_n(s) = \sum_{ij} \text{Re} \left[ \mathcal{M}_i^*(s) S_{ij} M_j(s) \right]$$

where $\mathcal{M}_i(s) = E_{0+}, E_{1+}, E_{2+}, E_{2-}, M_{1+}, M_{1-}, M_{2+}, M_{2-}$. The coefficients $T_{ij}^0$ and $S_{ij}$ can be found in Appendix A in [9].

The partial waves (electromagnetic multipoles) are not observables and have to be extracted from the experimental data within a theoretical framework (unless a complete experiment is possible [10]). In this Letter we employ three approaches to describe S and P waves that we present in forthcoming paragraphs: Section 2.1 HBChPT [11,12]; Section 2.2, Unitary HBChPT (U-HBChPT); and Section 2.3, Empirical. In all cases D waves are incorporated up to one loop and

$$E_{0+} = E_{0+L}(\gamma), E_{1+}, E_{2+}, E_{2-}, M_{1+}, M_{1-}, M_{2+}, M_{2-}$$

where $a_+$ is the leading order for the counter-term close to threshold ($\omega \approx m_{\pi^0}$) [8]. Hence, five LECs are fitted to the data under this approach: $a_+, a_-, \xi_1, \xi_2$, and $b_p$.

### 2.2. U-HBChPT

From general principles such as time reversal invariance and unitarity the S wave can be written as the combination of a smooth part and a cusp part [9,14,15]

$$E_{0+} = e^{i\delta_0} [A_0 + i\beta q_+/m_{\pi^+}], \quad s > 4m_{\pi^+}^2$$

$$E_{0+} = e^{i\delta_0} [A_0 - \beta |q_+|/m_{\pi^+}], \quad s < 4m_{\pi^+}^2$$

where $\delta_0$ is the $\pi^0\pi^0$ phase shift (which is very small), $\sqrt{s}$ is the invariant mass, $\sqrt{s_{\text{thr}}} = 2.08$ MeV, the invariant mass at the $\pi^0$ threshold, $q_+$ is the $\pi^+$ center-of-mass momentum, $A_0$ is $E_{0+}$ in the absence of the charge exchange re-scattering (smooth part), and

$$\beta = \text{Re}[E_{0+}(\gamma p \to \pi^+ n)] \times m_{\pi^-} a(\pi^+ n \to \pi^0 p)$$

parametrizes the magnitude of the unitary cusp and can be calculated [14] on the basis of unitarity. Eq. (8) takes the static isospin breaking (mass differences) as well as $\pi N$ scattering to all orders into account. In the electromagnetic sector it includes up to first order in the fine structure constant $\alpha$. The $\pi^+$ center-of-mass momentum, $q_+\pi^+$, is real above and imaginary below the $\pi^+$ threshold; this is a unitary cusp whose magnitude is parametrized by $\beta$ which can be calculated [14] on the basis of unitarity and taking into account a theoretical evaluation of isospin breaking [16], obtaining

$$\beta = (3.35 \pm 0.08) \times 10^{-3} / m_{\pi^-}$$

$$= (28.06 \pm 0.27 \pm 0.45) \times 10^{-3} / m_{\pi^-}$$

and $a(\pi^+ n \to \pi^0 p) = (0.1195 \pm 0.0016)/m_{\pi^-}$. In HBChPT up to one loop and $O(q^4)$, $\beta$ is fixed by the imaginary part of $E_{0+}$ — that is parameter-free— providing $\beta_{\text{HBChPT}} = 2.71 \times 10^{-3} / m_{\pi^-}$, which is far away from the unitary value. Because of the lack of unitarity of the S-wave amplitude [11] it is customary to substitute the S wave provided by HBChPT by a unitary prescription [9,11,12]. However, in this Letter instead of substituting the entire S wave for a prescription we prefer to substitute only the cusp part in $E_{0+}$ from HBChPT by the cusp part of $E_{0+}$ in Eq. (8), keeping the smooth part provided by HBChPT. In this way we keep the $E_{0+}$ counter-term and both HBChPT and U-HBChPT approaches have the same LECs to fit to the data.

### 2.3. Empirical fit

The empirical fit is parameterization of the S and P waves with a minimal physics input: unitarity in the S wave through the $\beta$ parameter and the angular momentum barrier. This is accomplished with a Taylor expansion in the pion energy in the center of mass $\omega$ up to first order on the smooth part of $E_{0+}$ and $P_1/q$ adding the cusp part in Eq. (8) to the S wave and keeping the imaginary part of the P waves equal to zero, in summary

$$E_{0+} = E_{0+L}(\gamma), E_{1+} + E_{2+} + E_{2-} + M_{1+} + M_{1-} + M_{2+} + M_{2-}$$

where $E_{0+L}, E_{1+}, E_{2+}, E_{2-}, M_{1+}, M_{1-}, M_{2+}, M_{2-}$ are free parameters that will be fitted to the experimental data. We note that

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1 The empirical parameterization in [5,9] expands on the photon energy in the center of mass frame $\omega$. We prefer to substitute only the cusp part in the data instead of substituting the entire S wave for a prescription we will be fitted to the experimental data. We note that
atic errors, starting the fits at 

 approaches we perform fits to the experimental data in [5] up to different maximum photon energies (fit).

 Each point represents a separate fit and the connecting lines are drawn to guide the eye. The points are plotted at the central energy of each bin, although the calculations take the energy variation inside of each bin into account. The value $\chi^2/$
dof = 1 is highlighted with a solid line.

this expansion goes to a lesser order in $\omega$ than HBChPT — i.e. $E^{\text{fit}}_{\text{max}}$ in Eq. (7) goes to order $\omega^3$ — but entails more parameters. We note that chiral symmetry is not imposed in this approach.

3. Results

Equipped with the HBChPT, U-HBChPT and empirical approaches we perform fits to the experimental data in [5] up to different maximum photon energies $E^{\text{fit}}_{\text{max}}$ within the range [158.72, 191.94] MeV and compute the $\chi^2$/dof as well as the corresponding error bars of the extracted parameters (see Appendix A). The energy bins of the data are approximately 2.4 MeV wide, which is taken into account in the fitting and calculations. We do not employ the first two energy bins from [5], 146.95 and 149.35 MeV, because they are less reliable due to systematic errors, starting the fits at $E^{\text{min}}_{\text{fit}}$ = 151.68 MeV. The amount of data employed in each fit depends on up to what energy we are fitting — i.e. for our lowest-energy fit ($E^{\text{max}}_{\text{fit}}$ = 158.72 MeV) we employ 100 experimental data (80 differential cross sections and 20 photon beam asymmetries) and for our highest-energy fit ($E^{\text{max}}_{\text{fit}}$ = 191.94 MeV) we employ 514 experimental data (360 differential cross sections and 154 photon beam asymmetries). The highest-energy fit has been chosen high enough to obtain a $\chi^2/$dof that ensures that the three approaches no-longer hold and the lowest-energy fit to ensure a reliable fit with enough experimental data. Systematics are not included in the $\chi^2$ and this uncertainty can amount up to 4% in the differential cross section and 5% in the photon asymmetry. The fits are performed employing a genetic algorithm whose details can be found in [19].

3.1. Quality of the fits

Fig. 1 shows the $\chi^2$/dof for every fit performed versus the upper energy $E^{\text{max}}_{\text{fit}}$ of the fit as well as the number of data. It is shown that up to $\sim$170 MeV all the fits are equally good providing very low $\chi^2$/dof. Above 170 MeV the trend is different; while the empirical fit remains with a good and stable $\chi^2$/dof, both the HBChPT and the U-HBChPT with the fitted LECs start rising, a trend that shows clearly how the theory fails to reproduce the experimental data above that energy. Because we obtain very similar result for U-HBChPT and HBChPT, lack of unitarity cannot be blamed for the disagreement between theory and experiment. The HBChPT result contrasts with the empirical fit that up to 180 MeV provides a good description of the data. Above 185 MeV the $\chi^2$/dof of the empirical fit starts to rise showing the effects of higher orders in the partial waves and the appearance of a non-negligible contribution from the imaginary part of the P waves.

3.2. LECs as a function of $E^{\text{max}}_{\text{fit}}$

An important test of the accuracy of the HBChPT expansion is the stability of the empirical LECs versus $E^{\text{max}}_{\text{fit}}$. The empirical fit provides a solid benchmark because the parameters are the same (within errors) in the whole energy region [20]. Fig. 2 shows the $E^{\text{max}}_{\text{fit}}$ (fit) dependence of the LECs for both the HBChPT (with errors) and U-HBChPT approaches. This includes the S-wave LECs $a_1$ and $a_2$ — in Figs. 2(a) and 2(b) respectively and P-wave LECs $\xi_1$, $\xi_2$, and $b_p$ in Figs. 2(c), 2(d), and 2(e). Errors are larger for the fits with lowest $E^{\text{max}}_{\text{fit}}$ because of the smaller amount of data. The S-wave LECs are fairly stable in the whole energy range and both HBChPT and U-HBChPT are approximately constant within errors. On the contrary, P-wave LECs show a non-stable pattern with a positive slope for $\xi_1$ and $b_p$ and a negative slope for $\xi_2$. The large error bars make the extracted LECs compatible up to $E^{\text{max}}_{\text{fit}} \approx 175$ MeV except for $\xi_1$, whose value for the $E^{\text{max}}_{\text{fit}} = 170.53$ fit is already incompatible with the lower energy fit $E^{\text{max}}_{\text{fit}} = 161.08$, confirming that $\sim 170$ MeV above such energy the theory does not provide a good fit to the data. Besides, approximately at $\sim 170$ the U-HBChPT and HBChPT P-wave LECs start to be incompatible. The U-HBChPT LECs are systematically smaller in absolute value than the ones obtained through HBChPT, this is expected because the unitary $\beta$ is larger than $\beta_{\text{HBChPT}}$ giving a larger contribution by the Im $E_{\omega}$, which has to be compensated by the other multipoles. The slopes of the P-wave LECs show that higher order, relativistic and $\Delta$(1232) effects are absorbed into them, calling for improvement in the theory.

We have also looked into the correlations by computing the correlation coefficient $r(x,y) = \sigma_{xy}/(\sigma_x\sigma_y)$ for each pair of parameters and for every HBChPT and U-HBChPT fit. We find that the correlation remains or less stable for each pair throughout every fit. The S-wave LECs are highly correlated $r(a_1,a_2) \approx [0.78, 0.88]$, $\xi_2$ and $b_p$ provide $r(\xi_2,b_p) \approx [0.45, 0.6]$, which is not unexpected due to the photon asymmetry $W_{\text{response}}$ structure [5,21], and the rest are fairly uncorrelated lying within the range $[-0.2, 0.2]$. In the case of the S wave the correlation is responsible of the large error bars associated to $a_1$ and $a_2$ and indicates that energy dependence and threshold value of Re $E_{\omega}$ cannot be obtained separately without further experimental information. Regarding the magnitude of the empirical LECs, the empirical values are within a factor of two of the values estimated in Refs. [11,12] through resonance saturation.

3.3. Comparison with experimental data

In order to compare with experimental data one has to choose a best fit (set of fitted parameters/LECs) for each approach. In our case we pick the fit up to 180.02 MeV for the empirical fit ($\chi^2$/dof = 1.21) and the fits up to 168.16 MeV for HBChPT ($\chi^2$/dof = 1.25) and U-HBChPT ($\chi^2$/dof = 1.21) approaches. Fig. 3 shows the single energy multipoles extracted from experimental data compared to the three approaches. The HBChPT fit is shown as an error band. The procedure to obtain the single energy multipoles from the data is explained in [5] and the error bars are computed as described in Appendix A. The data below the unitary cusp (146.95 and 149.35 MeV) are not reliable enough to accurately extract the single energy multipoles and, therefore, are not shown. Overall, the HBChPT and U-HBChPT do a reasonable job describing the
Fig. 2. (Color online.) Upper energy (fit) dependence of the LECs. Error band and full circles (green): HBChPT fits; Empty circles (blue): U-HBChPT fit. The high correlation between LECs $a_+$ and $a_-$ makes their error bars larger. Errorbars for the U-HBChPT LECs are not depicted but they are approximately of the same size as the HBChPT ones.

Fig. 3. (Color online.) Real part of the S and P waves. HBChPT fit up to 168.16 MeV: Error band and solid line (green); Empirical fit up to 180.02 MeV: Dashed. U-HBChPT fit up to 168.16 MeV: Dash-dotted; The data shown are the single energy multipoles extracted from the experimental differential cross sections and asymmetries in [5]. The gray area above the energy axis represents the systematic errors [5] and the red area at the top of (a) the uncertainty associated to our knowledge of the D waves.

Fig. 4 compares the empirical, HBChPT and U-HBChPT approaches to the differential cross section and photon beam asymmetry at two different energies, one within the HBChPT and U-HBChPT fitting region ($E_\gamma = 161.08$ MeV) and another outside it ($E_\gamma = 175.22$ MeV). Two results are noteworthy. First, the photon asymmetry is well reproduced for both energies by all the fits, Figs. 4(c) and 4(d); if we compare with other energies — higher, lower and intermediate — we find the same level of agreement between theory and data, obtaining that all the approaches are of the same quality and provide a good description of the photon asymmetry in the whole energy range considered in this Letter. Second, the HBChPT and U-HBChPT approaches underestimate the cross section for energies above the fitting limit ($E_{\gamma}^{\text{max}} = 168.16$ MeV), as can be seen in Fig. 4(b). This situation is clearer if we look into the component $T_0$ of the differential cross section response $W_{T}$ in Fig. 5(a) which above ~170 MeV is largely underestimated by the HBChPT approach. The $T_0$ component is essentially the total cross section and is dominated by $|M_{1+}|^2$ [9], which as seen in Fig. 3, is not so well described by the theory. The significant components of the $W_{T}$ response $T_0$, $T_1$, and $T_2$ are obtained fitting the
differential cross sections for each energy bin to Eq. (1). The same operation is done with the photon asymmetry, fitting the data to Eq. (2), extracting $S_0$ and $S_1$. The other two significant components of the $W_T$ response $T_1$ and $T_2$ are fairly well described up to $\sim 175$ MeV. In the case of $W_S$, $S_0$ is well determined and HBChPT with the fitted LECs describes it fairly well in the whole energy range.

### 3.4. Probing D waves

The $S_1$ component of $W_S$ in Fig. 5(e) is due to the interference among P and D waves [8,9]. The empirical values of $S_1$ are consistent with the Born terms contribution of D waves but unfortunately the experiment is not accurate enough in the photon asymmetry to provide a quantitative measurement. The small non-zero effect between 175 and 185 MeV disappears if errors are computed at a 2$\sigma$ level. Hence, with the current experimental information only four quantities can be accurately obtained from each energy bin $T_0$, $T_1$, and $T_2$ from the differential cross section and $S_0$ from the photon asymmetry, that allows to extract the four multipoles in Fig. 3. If we intend to obtain information on the rest of the multipoles we need either more accuracy in the data to pin down $S_1$ (to obtain information on D waves), $T_3$ (P × D interference) or $T_4$ (D waves), or to measure other observables like the target asymmetry (to obtain Im $E_0$) [21,22], the $E$ asymmetry [9] (D waves) or the $F$ asymmetry [9,21,22] (Re $E_0$ and D waves). With our current knowledge of the P waves, the accurate extraction of Im $E_0$ and the $\beta$ parameter from the target asymmetry is feasible. This observable has been measured at Mainz together with the F asymmetry and data analysis is currently in progress [22].
Returning to D waves, they have been incorporated in our analysis as the Born terms and the $S_1$ component of $W_S$ is consistent with this approach. Up to order $O(q^3)$ in HBChPT this is the only contribution together with an $E_{2-}$ counter-term [23] which provides an additional LEC. However we have neglected it in our calculation because it has no impact in the $\chi^2$, and, therefore, it cannot be determined. Current experimental information does not allow to test our knowledge on D waves but we are hopeful about forthcoming experiments and we think that future more accurate data will provide a measure of the D-wave effects and allow to pin down the $E_{2-}$ counter-term if a deviation from Born terms is found.

4. Conclusions

Because of the high-quality experimental data gathered by the A2 and CB-TAPS Collaborations at Mainz we can assess the electromagnetic multipoles and their energy dependence to the best precision ever and we can accurately assess the energy range where HBChPT with the fitted LECs provides a good description of the data. Based on the accumulated evidence — LECs stability, $\chi^2$/dof and the empirical fit which works up to $\sim 180$ MeV — we find that HBChPT with the fitted LECs provides a good description of the experimental data up to 170 MeV. The lack of unitarity in the S wave is not responsible for the disagreement between HBChPT and the experimental data as we have proved through the U-HBChPT approach. The slopes of the P-wave LECs in Fig. 2 show how higher order, relativistic and $\Delta(1232)$ effects are absorbed into them, call for improvement in the theory. Some steps have been taken recently to improve the theory, i.e. Dispersive Chiral Effective Theory [24] which combines dispersion relations with ChPT, and relativistic Chiral Perturbation Theory [23] which does not provide better agreement with data than the HBChPT approach [5]. We have achieved an unprecedented accuracy in our empirical extraction of the multipoles from the data. This has provided a more sensitive test of the HBChPT calculations then has been previously been possible. What we have found is that there is a single multipole ($M_{1+}$) that is causing the gradual deviation from experiment (increasing $\chi^2$) with increasing energy so this disagreement is probably due to the fact that the $\Delta(1232)$ degree of freedom is not being taken into account in a dynamic way [25].

Acknowledgements

We thank the A2 and CB-TAPS Collaborations for making available the experimental data prior to publication. C.F.-R. is supported by “Juan de la Cierva” programme of Spanish Ministry of Economy and Competitiveness and his research has been conducted with support by Spanish Ministry of Economy and Competitiveness grant FIS2009-11621-C02-01, the Moncloa Campus of International Excellence (CEI Moncloa), and by CPAN, CSPD-2007-00042 Ingenio2010. A.M.B. research is supported in part by the US Department of Energy under contract No. DE-FC02-94ER40818.

Appendix A. Error bar calculation

Error bars have been computed through a Monte Carlo (MC) simulation. Once the minimum $\chi^2_{\text{min}}$ has been assessed the $\chi^2_1$ defined as $\chi^2_1 = \chi^2_{\text{min}} + 1$. Once we have the $\chi^2_1$ we run a MC varying the values of the parameters, we compute the corresponding $\chi^2$ for each set of parameter values, and we accept those sets which provide $\chi^2 < \chi^2_1$. If enough statistics are collected, the boundary of the simulation defines the confidence ellipse and the error bars for each parameter [26]. We also obtain correlation plots between parameters as well as the correlation matrix. Once the MC has been run we have a file with thousands of combinations of the parameters which are within the $\chi^2_1$ level. We use those sets to compute the bands in the partial waves and the observables which are shown in the figures. In this way the error bars in the partial waves and the observables take properly into account the correlations among parameters.

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