Problem and models of multicriteria decision making and risk assessment of the arctic offshore oil and gas field development

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Abstract. The article is devoted to the systematic solution of problems of multi-criteria assessment and the possibility of implementing projects for the development of hydrocarbon deposits on the Arctic shelf under conditions of uncertainty and risk. Risks are considered in a narrow and broad sense. It is shown that the solution of tasks, in this case, comes down: 1) to a multi-criteria assessment of the quality level of a field based on a hierarchy analysis method that takes into account uncertainties and risks of operating objects through the relevant necessary indicators (criteria); 2) determining, with the help of game theory adopted with the nature of the criteria for selecting the best Bayesian strategies, of development options under both partial and full uncertainty conditions; 3) selection using the rules of the Borda, Pareto and multi-criteria dispersive risk model for choosing the best development strategy.

1. Introduction

Oil and gas production in the fields of the Arctic shelf is now becoming one of the most pressing problems in the development of the world’s hydrocarbon resources. The projected oil and gas resources of the Arctic shelf of Russia are quite large and are estimated at 700-1020 billion barrels of oil equivalent (BBOE) \cite{1, 2}. However, they are concentrated mainly in hard-to-reach territories, far from industrialized regions covered by an integrated system for supplying the country with hydrocarbon resources.

These oil and gas basins are also distinguished by the fact that they have different and insufficient degree of knowledge, the presence of difficult climatic conditions, an incomplete and heterogeneous database, high economic, environmental and production risks, which ultimately determine large investments and low economic efficiency of the arctic shelf development. By taking this into account the following types of risks in the hydrocarbon production can be considered: geological, technological, organizational, economic and social. A detailed informative description of them can be found in \cite{1, 2}.

In the general case, risk is a measure of quantitative measurement of a hazard, which is a vector quantity that includes the following main indicators: the amount of damage from the impact of a
hazard; probability of occurrence of the considered hazard; uncertainty in terms of both damage and the likelihood of a hazard.

Based on IEC standards, as well as foreign standards (see, for example, [3,4]), risk in a broad sense means the average damage to a system, process, etc. (usually in monetary terms) from the manifestations of a specific hazardous impact factor for a set period of time [0, t] (usually t = 1 year). The risk of the i-th hazardous effect (risk factor) is estimated using the expression:

$$ R_i(t) = S_i \cdot \Phi_i \cdot t $$

If $t = 1$ year then

$$ R_i = S_i \cdot \Phi_i, $$

where $S_i$ is the average damage (loss) associated with the complete elimination of the i-th hazardous consequence (impact); $\Phi_i$ is the frequency (probability) of occurrence of the i-th hazardous effect (impact) or the average number of occurrences of this effect per year, the $\Phi_i$ dimension is 1/year.

In the classification adopted by the IEC standard, the following degrees of hazardous effects are distinguished: i = I - minor effects, II - boundary, III - critical, IV - catastrophic, which in this case correspond to the above states of nature $P_j$ (see below).

Often in the literature on industrial safety, the risk in the strict sense of the i-th hazardous consequence means the quantity $\Phi_i$ – the frequency (probability) of the occurrence of a hazardous effect, while $\Phi_i$ is referred to as the risk or probability of the occurrence of a hazardous event per year. Typically, the value of $\Phi_i$ is used as a risk in a situation where there is no information about the magnitude of the damage $S_i$.

However, there are other models for risk assessment and analysis, among them – models based on the hierarchy analysis method [5 -7], on the theory of games with nature [8 - 10], the dispersive model of risk [11], as well as wider approaches with an in-depth philosophy of misconception of risk management [12-14]), models based on ISO standards and the achievements of the Council of the Society for Risk Analysis (SRA) [15].

Based on the methods of system analysis, we only consider problems and the implementation of models based on the IEC standard, the hierarchy analysis method, game theory with nature and the variance risk model.

2. Geological risks evaluation with probabilistic model

We will treat the geological risk as a non-confirmation of resources. To assess the risk of not confirming the resources of the arctic shelf deposit, a probabilistic model of geological risk assessment is proposed, the essence of which is explained below.

The probability of non-confirmation of resources (industrial reserves) will be called the risk. Then, the geological risk can be defined as the probability of geological failure – the risk of non-confirmation of the availability of resources, that is, the probability of a complex event, namely: a) the studied deposit is not productive; b) not found as a result of prospecting and c) its reserves are less than the minimum required value $V$, i.e.:

$$ Pr = (1 - P_o) \cdot (1 - P_p) \cdot (1 - P_v), $$

where

$P_o$ – probability 0f the exploration target productivity;

$P_p$ – probability of the presence of reservoirs with net pay acceptable for commercial development;

$P_v$ – probability that identified reserves are not less than the minimum required value $V$.

It is obvious that $P_o$, $P_p$, $P_v$ in the lack of real data are subjective probabilities, i.e. probabilities assigned by the expert.

Thus, the problem is to expertly determine these probabilities with the least errors. As a rule, direct determination (assignment) of the considered subjective probabilities leads to erroneous results. However, today there is a method that allows to level out this shortcoming. This method includes the
principle of pairwise comparison of objects and the eigenvalue method, which is widely used in the hierarchy analysis method (AHP) [5 - 7].

The application of this approach is especially simple in case of subjective probabilities of two single unique and mutually complementary events (for example, events A and B, which determine the probabilities \( P_o \) and \( 1 - P_o \)) [5]. Obviously, the considered model can be applied to the definition of \( P_p \) and \( P_r \). Note that there is also the likelihood that the probabilistic estimates made for different sections of the analyzed areas are different then it becomes possible to map the distribution of risks over the area.

3. Risk assessment (technological, organizational, economic and social) of projects for the development of the arctic offshore oil and gas field based on the hierarchy analysis process (AHP)

Let us consider a project for the arctic offshore hydrocarbon field development. There are three development strategies (scenarios), namely, \( A_1, A_2, A_3 \). Please, note that each of the options considered may have its own uncertainties in evaluating performance indicators that can be associated with the degree (probability) of project implementation. In petroleum industry the levels of 90%, 50% and 10% probability \( (P_{90}, P_{50} \) and \( P_{10} \) are used [2]. The decision maker, according to the results of calculations and estimates, has to select the best scenario and identify the associated risk – probability (degree) of the project failure (the risk of investment in the project).

A. Probabilistic model for comparing feasibility of the arctic offshore field development

Here we use the same methodical approach as in the geological risk assessment. We assume that all the strategies \( A_i \), according to the risk factors presented above and the conditions in which they are implemented, generate their risk of project failure. Assuming (at first approximation) that these probabilities do not depend on each other, the overall risk of project failure to implement \( A_i \) can be defined as:

\[
P_{r}^{A_i} = (1 - P_{t}^{A_i}) \cdot (1 - P_{o}^{A_i}) \cdot (1 - P_{e}^{A_i}),
\]

where

\( P_{t}^{A_i}, P_{o}^{A_i}, P_{e}^{A_i} \) – probability of successful implementation of technical, organizational and economic solutions (respectively) of project scenario.

Obviously, in the conditions of uncertainty of occurrence of the considered risks – the lack of appropriate precedents and statistics, these probabilities should be considered and identified as subjective. Just as for geological risk probabilities, we will determine (evaluate) them on the basis of the hierarchy analysis method, which is based on the expert's compilation of matrices of paired comparisons of options (expert judgment matrices), asking the expert when comparing them to avoid errors [5] question: “How much more likely is the realization of event C (one scenario) than the realization of event D (another scenario) if these specific conditions (criteria, indicators, etc., describing this task, such as when assessing geological risk) are met. To quantify the fact that it is more likely – the implementation of C or D – it is necessary to use the fundamental point scale from 1 to 9 [5 - 7]. Using the matrix of pairwise comparison of options (strategies C and D) elements of the priority vector are calculated and interpreted as probabilities \( P_{r}^{A_i} \). Then, based on the meaning of the risk of non-realization of the project scenario \( r_i \), we get:

\[
P_{r}^{A_i} = 1 - r_i.
\]

In order to rely on the results provided by the matrix of pairwise comparisons, it is necessary to calculate its consistency index (CI) [5,6,7]:

\[
CI = (\lambda_{\max} - n)/(n - 1),
\]
where \( n \) is the dimension of the matrix of pairwise comparisons; \( \lambda_{\text{max}} \) – the maximum value of the normalized eigenvector of the matrix.

In practice, the judgment matrix is considered consistent if \( CI = 0 \div 0.15 \) (it is known from matrix theory that the inverse symmetric matrix is absolutely consistent when \( \lambda_{\text{max}} = n \) [5]. Inconsistency occurs when an expert compares objects. For example, when comparing three objects C, D and E an expert writes: \( C = 5D \), \( C = 6E \), \( D = 4E \), that is, \( C = 20E \), but it writes down \( C = 6E \) – which breaks transitivity of estimates.

Further, it is obvious that in order to obtain \( P_{\text{r}}^{Ai} \), it is necessary to compose two more matrices of pairwise comparisons with respect to obtaining \( P_{\text{o}}^{Ai} \) and \( P_{\text{e}}^{Ai} \). Multiplying the probabilities \( P_{\text{r}}^{Ai} \), \( P_{\text{o}}^{Ai} \) и \( P_{\text{e}}^{Ai} \), we obtain the probability of successful implementation of the project scenario \( A_i \).

However, such an approach requires a very high qualification of the decision maker and does not explicitly take into account the multi-criteria assessment of the viability of development scenarios. Therefore it can be considered as preliminary. To solve this problem, a set of criteria should be selected, and the most significant ones should be highlighted.

**B. Model of ranking and criteria selection for the prospectivity assessment of the arctic offshore field**

For ranking and selection of criteria, the technique proposed in [6,7] can be applied. It is based on the calculation of the values and significance (weights) of the criteria according to the following expression

\[
K_i^* = \gamma_i F_i(K_i^D, K_i^S),
\]

where \( K_i^D \) – the value of the \( i \)-th criterion, assessed by the expert (or calculated) at the time of its assessment – state \( D \); \( K_i^S \) is is the value of the \( i \)-th criterion, estimated by the expert in points expressing his desire to improve the problem solution – the state \( S \); \( \gamma_i \) is the weight (importance) of the \( i \)-th criterion, determined on the basis of the expert’s experience and intuition, with \( \sum \gamma_i = 1 \) (100%).

The connection of the \( i \)-th criterion value in points with the physical parameters for \( S \) and \( D \) states can be expressed using basic scales, allowing the dimensional indicator (criterion) to be converted into points by mapping the dimensional criterion on the basic scale [6,7]. The ratio of the Base Scale and the Scale of Indicators usually makes sense “the more – the better”. For indicators “the less – the better” the scale is inverse.

The specific form of the function \( F_i \) can be the difference \( K_i^S \) and \( K_i^D \), showing how much the situation needs to be improved, or quotient of their division showing how many times it is necessary to improve the situation. The sequence \( K_i^* \) gives a series of decreasing importance of criteria from the point of view of the leader and shows where to focus attention. The initial sequence \( K_i \) is significantly different from the sequence \( K_i^* \).

The ranked sequence \( K_i^* \) can be used to reduce uncertainty while reducing the set of criteria in a decision-making process or, rather, when assigning objectives. It is performed on the basis of calculating the separation level \( \alpha(n) \), which determines the total relative weight of the group of the most important criteria:

\[
\alpha(n) = \frac{\sum_{i=1}^{n} K_i^*}{\sum_{i=1}^{N} K_i^*},
\]

where \( N \) – the number of criteria considered, and \( n \) – the highest criteria number in a reordered sequence that will be taken into account by the decision maker.

Let us apply this methodology to obtain the ranking and selection of criteria for assessing the feasibility of hydrocarbon production from the arctic offshore field (Table 1). These criteria are based on the analysis of information presented in [1, 2, 6, 7] and their (new) estimates that has been performed by translating the real (dimensional) values of the criteria into grades by the base scale method.
As follows from Table 1 the level of separation $\alpha_{14}$ covers 14 of 26 criteria considered. These are criteria 1, 2, 3, 5, 6, 7, 8, 11, 14, 15, 18, 19, 21, 25 and they contribute with 76% of the “weight” of all criteria. If you add four more criteria that have the value $K_i^* \geq 6$, their total relative weight will grow to 92%.

It is obvious that the decision maker may decide to choose any group of criteria for evaluating the effectiveness of field development. Usually a decision maker operates with 70% – 77% of the total weight. However, his decision may be guided by other considerations, for example, the Pareto principle 20/80, then (in our case) the number of criteria could be reduced to three ($\alpha_3=26\%$). In this case it will be criteria with ranks 1-3, namely, (1) Net Present Value, (2) Environmental Safety and (3) Recoverable resources. In any case, the choice of the level of separation always depends on the decision maker, his experience, intuition, and information content.

Table 1. Ranking and selection of criteria for the prospectivity assessment of production assets.

| No. | Criteria                               | $K_i^D$ | $K_i^S$ | $K_i^{S-D}$ | $\gamma_i$ | $K_i^* = (K_i^{S-D}) \gamma_i$ | Rank $K_i^*$ |
|-----|----------------------------------------|---------|---------|-------------|------------|-------------------------------|-------------|
| 1   | Recoverable oil (gas) resources        | 3       | 6       | 3           | 6          | 18                            | III         |
| 2   | Reservoir porosity                     | 6       | 8       | 2           | 6          | 12                            | IV-VIII     |
| 3   | Social factors of development          | 1       | 3       | 2           | 6          | 12                            | IV-VIII     |
| 4   | Total number of production wells        | 4       | 5       | 1           | 5          | 5                             | XIX-XXI     |
| 5   | Net present value (NPV)                | 5       | 9       | 4           | 6          | 24                            | I           |
| 6   | Internal rate of return (IRR)          | 4       | 6       | 2           | 5          | 10                            | IX-XI       |
| 7   | Profitability index (PI)               | 4       | 7       | 3           | 4          | 12                            | IV-VIII     |
| 8   | Pay-off period                         | 5       | 7       | 2           | 5          | 10                            | IX-XI       |
| 9   | Capital expenditures                   | 3       | 4       | 1           | 5          | 5                             | XIX-XXI     |
| 10  | Operating costs                         | 2       | 4       | 2           | 3          | 6                             | XV-XVIII    |
| 11  | Government take                        | 3       | 5       | 2           | 5          | 10                            | IX-XI       |
| 12  | Justification of selection of technical solutions | 4       | 6       | 2           | 2          | 4                             | XXII-XXV    |
| 13  | Reliability of field development control | 5       | 6       | 1           | 2          | 2                             | XXVI        |
| 14  | Options for the sales products         | 4       | 7       | 3           | 3          | 9                             | XII-XIV     |
| 15  | Environmental safety                   | 3       | 7       | 4           | 5          | 20                            | II          |
| 16  | Preservation of flora and fauna        | 4       | 6       | 2           | 2          | 4                             | XVIII-XIX   |
| 17  | Noise                                  | 2       | 4       | 2           | 2          | 4                             | XVIII-XIX   |
| 18  | Reservoir permeability                 | 4       | 7       | 3           | 4          | 12                            | IV-VIII     |
| 19  | Reservoir depth                        | 4       | 7       | 3           | 4          | 12                            | IV-VIII     |
| 20  | Initial oil / gas saturation           | 5       | 8       | 3           | 2          | 6                             | XV-XVIII    |
| 21  | Reservoir temperature                  | 6       | 9       | 3           | 3          | 9                             | XII-XIV     |
| 22  | Reservoir thickness                    | 4       | 5       | 1           | 5          | 5                             | XIX-XXI     |
| 23  | Lithology of reservoir rock            | 6       | 8       | 2           | 3          | 6                             | XV-XVIII    |
| 24  | Rock metamorphism degree               | 4       | 6       | 2           | 2          | 4                             | XXII-XXV    |
| 25  | Reservoir area                         | 3       | 6       | 3           | 3          | 9                             | XII-XIV     |
| 26  | Tectonics of the area / field          | 4       | 7       | 3           | 2          | 6                             | XV-XVIII    |

C. Method of analyzing hierarchies for multi-criteria risk assessment for the arctic offshore field development project

For the further analysis 14 criteria selected in a previous section will be grouped as follows:

- B₁. Economic key performance indicators
- B₂. Risks
- B₃. Environmental protection
- C₁. Net present value
- C₂. Internal rate of return
- C₃. Pay-off period
- C₄. Profitability index
- C₅. Government take
- C₆. Environmental protection
- C₇. Geologic and technical conditions
- C₈. Environmental Safety
- C₉. Recoverable oil resources
- C₁₀. Reservoir porosity
- C₁₁. Reservoir permeability
- C₁₂. Reservoir depth
C6. Social factors of field development
C7. Options for the sales products
C8. Reservoir temperature
C11. Reservoir area

Based on this set of criteria, the decision maker needs to select the best development scenario from A1, A2 and A3 considered [1,2]. To assist in solving this task, the following graph can be built (Figure 1).

**Figure 1.** Hierarchical structure of selecting scenarios of the prospectivity assessment of the arctic offshore field development.
Legend: K – global criterion for the feasibility of field development strategies; B – indicators of field development efficiency; C – specific criteria from the above list; A1, A2, A3 – scenarios (strategies).

For the hierarchy on Figure 1 the expert compiles a matrix of judgments (pairwise comparisons)

Level 2.
Priority matrix of criteria in relation to the main criterion K (Figure 1). The vector of priorities has the elements $\Pi_j$: 0.21; 0.26; 0.29; 0.24. $\lambda_{\text{max}} = 4.2$; CI = 0.07.

Level 3.
This level considers three matrices of pairwise comparisons:
- Comparisons of economic criteria in relation to $B_1$ criterion. The priority vector has the elements: $\Pi_j$: 0.290; 0.195; 0.189; 0.162; 0.146. $\lambda_{\text{max}} = 4.7$; CI = 0.0.
- Comparisons of risk criteria in relation to $B_2$ criterion. The priority vector has the elements: $\Pi_j$: 0.33; 0.66. $\lambda_{\text{max}} = 1.88$; CI = 0.0.
- Comparisons of environmental criteria in relation to $B_3$ criterion. The priority vector has the elements: $\Pi_j$: 1.0. $\lambda_{\text{max}} = 1.0$; CI = 0.0.
- Comparisons of geo-technical criteria in relation to $B_4$ criterion. The vector of priorities has the elements: $\Pi_j$: 0.248; 0.192; 0.165; 0.203 0.109; 0.083. $\lambda_{\text{max}} = 6.786$; CI = 0.157.

Further, in accordance with the hierarchy of Figure 1, a compiled priority matrix of criteria priority column-vectors in relation to criteria $B_1, B_2, B_3, B_4$ should be multiplied from the right by a vector of priorities in relation to the main criterion K. As a result, the following final vector of criteria weights is obtained: $\alpha_{c_1}=0.061$, $\alpha_{c_2}=0.041$, $\alpha_{c_3}=0.040$, $\alpha_{c_4}=0.034$, $\alpha_{c_5}=0.031$, $\alpha_{c_6}=0.086$, $\alpha_{c_7}=0.172$, $\alpha_{c_8}=0.290$, $\alpha_{c_9}=0.060$, $\alpha_{c_{10}}=0.046$, $\alpha_{c_{11}}=0.040$, $\alpha_{c_{12}}=0.049$, $\alpha_{c_{13}}=0.026$, $\alpha_{c_{14}}=0.020$
Level 4.

At this level of the hierarchy, 14 matrices of pairwise comparison of strategies $A_i$ are compiled with respect to each criterion $C_i$. After that, the corresponding priority matrix is compiled, which is then multiplied from the right by the vector generated in a previous step, the priority column of $\alpha_{C1} \ldots \alpha_{C14}$ criteria, and the priorities of the development scenarios are calculated. This priority matrix will eventually have the following form:

$$
\begin{pmatrix}
C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 & C_{10} & C_{11} & C_{12} & C_{13} & C_{14} \\
A_1 & 0.4 & 0.33 & 0.2 & 0.5 & 0.47 & 0.2 & 0.47 & 0.33 & 0.3 & 0.3 & 0.53 \\
A_2 & 0.4 & 0.4 & 0.2 & 0.3 & 0.3 & 0.3 & 0.3 & 0.2 & 0.33 & 0.5 & 0.29 \\
A_3 & 0.2 & 0.27 & 0.4 & 0.3 & 0.27 & 0.2 & 0.2 & 0.4 & 0.23 & 0.33 & 0.34 & 0.33 & 0.2 & 0.18
\end{pmatrix}
$$

and the priority vector of strategies $A_i$: $\alpha_{A_1}=0.325; \alpha_{A_2}=0.351; \alpha_{A_3}=0.324$.

Thus, the most preferable development strategy is $A_2$. Accordingly, the risk of failure of this development scenario is $P_r^{A_2}=1-0.351=0.649$.

In addition to the risk assessment models discussed above in the narrow sense, knowledge of risks in a broad sense is often required, that is, a possible loss of efficiency assessment. The theory that allows this is the theory of statistical solutions (theory of games with nature) \[8,10\].

4. Evaluation of the risk associated with the arctic offshore production asset based on the theory of statistical solutions (theory of games with nature)

It is known that in the theory of matrix games (games with nature) \[8, 10\] one player is active and “malicious”, the other player (the “nature”) is passive. Nature acts randomly, or rather, it is believed that its states are realized randomly.

Nature, for example, can be presented by oil and gas assets, natural conditions (land, sea, climate, etc.), geo-technical conditions (fluids and reservoir properties, etc.), and the uncertainty lies in the likelihood of possible outcomes of the states of nature. The task of the active player (decision maker) is to find a pure strategy that would secure the maximum benefit (or minimum loss), taking into account any state of nature. Note that the active player can’t change the states of nature.

In this paper an approach is proposed for determining the states of nature, which, based on the data in the Table 1, allows a multi-criteria assessment of the characteristics of the field quality by applying one of the compromise schemes, for example, convolution of criteria \[6, 7\].

Further, dividing the entire range of possible values of a complex field quality criterion into a certain number of intervals, for example, four. This enables to determine the following levels $\Pi_i$ of quality of the field: $\Pi_1$ – very high; $\Pi_2$ – high; $\Pi_3$ – medium; $\Pi_4$ – low quality level.

It is required to determine the best field development scenario (in our notation, the strategy $A_i$), using all possible criteria for evaluating the effectiveness and risks used in the theory of games with nature.

Table 2 contains the payment matrix of the games with nature. Parameters staying at the intersection of the columns and rows are complex indicators (criteria) of the field development options efficiency, generated by using the same methodical approach as the above-discussed states of nature. Note that these complex indicators are expressed in the base scale by a 20-point (French) grading system.

We will also assume that the greater the value of a complex indicator (regardless of whether it is a gain or a loss) – the better the solution. This follows from the properties of the inverse scale, which greatly simplifies calculations and their analysis.

Calculations of the field development effectiveness and risks can be carried out in two cases: (1) partial uncertainty situations, when the probabilities of the realization of the states of nature are given or can be calculated; (2) situations of complete uncertainty when probabilities are unknown or cannot be determined due to different conditions or when they are not important to the decision makers.
Table 2. Example of the payment matrix of the games with nature for various strategies for the arctic offshore field development.

| Development options | Field quality level | | | |
|---------------------|---------------------|---------------------|---------------------|---------------------|
|                     | Π₁                  | Π₂                  | Π₃                  | Π₄                  |
| A₁                  | 14                  | 10                  | 13                  | 15                  |
| A₂                  | 10                  | 12                  | 14                  | 16                  |
| A₃                  | 9                   | 12                  | 14                  | 16                  |

In the first approach several criteria described below are used.

**Bayes-Laplace criterion.** According to his approach, there is such a strategy $A_i$, which gives the maximum value to the expectation of gain for all possible outcomes of the states of nature $P(Π_j)$.

$$\bar{a}_i = \sum_{j=1}^{n} a_{ij} P_j \rightarrow \max,$$

where $\bar{a}_i$ – weighed arithmetic mean.

Minimum average risk criterion $\bar{r}_i$.

$$\bar{r}_i = \sum_{j=1}^{n} r_{ij} P_j \rightarrow \min,$$

Here the probabilities $P_j$ are set in the same way as in a previous case.

The risk of decision-making when using the strategy $A_i$ in the conditions of $Π_j$ is the difference:

$$r_{ij} = \beta_i - \alpha_{ij} = \max_i \alpha_{ij} - \alpha_{ij}$$

where $\beta_i = \max_i \alpha_{ij}, \ r_{ij} \geq 0$.

If a decision maker considers that none of the states of nature is a priority, then the probabilities $P(Π_j)$ are considered equal, ie:

$$P(Π_j) = P_j = \frac{1}{n}$$

– so called principle of insufficient justification (Laplace criterion);

In this case the best strategy securing the highest gain is:

$$\max(1/n) \sum_{j=1}^{n} a_{ij}$$

(11)

For minimizing the risk this strategy gives:

$$\min (1/n) \sum_{j=1}^{n} r_{ij} ,$$

(12)

where $r_{ij}$ – risk of choosing the strategy $A_i$ in the state of nature $Π_j$ (see above).

Another approach to assign probabilities $P(Π_i)$ is to arrange states of nature in the order of their plausibility. Then the probabilities $P_j$ are assigned to the proportional members of a decreasing arithmetic progression:

$$P(Π_1): P(Π_2): \cdots P(Π_n) = n: (n-1): \cdots: 1$$

Taking into account that $P(Π_1) + P(Π_2) + \cdots + P(Π_n) = 1$, probabilities are calculated as follows:

$$P_j = \frac{2(n+j-1)}{n(n+1)}$$

(Fishburne point estimate). Results are substituted into the Bayes-Laplace criterion.

In any of the cases considered, the optimal decision corresponds to strategy $A_i^*$, which gives the average maximum gain (or minimum total risk). Hodges-Lehmann criterion relies simultaneously on the maximin criterion and the Bayes-Laplace criterion. According to this criterion, the choice of the best strategy is determined by the expression:
\[ Z = \max_i \left[ \alpha \sum_{j=1}^n a_{ij} \cdot P_j + (1 - \alpha) \min_j a_{ij} \right], \tag{13} \]

The choice rule for this criterion is described below:

1. The payment matrix with elements \( a_{ij} \) is complemented by a column composed of the sum of weighted average (with a weight \( \alpha = \text{const} \)) of mathematical expectations and the smallest result of each row.

2. Solution with the largest value in the rows of this column are selected.

When \( \alpha = 1 \), the Hodges-Lehmann criterion converts into the Bayes-Laplace criterion, and with \( \alpha = 0 \) – into the maximin Wald criterion. The parameter \( \alpha \) expresses confidence in the probability distribution used.

Germeyer’s criterion is focused on the magnitude of the losses, that is, the negative values of all \( a_{ij} \) and is determined by the expression:

\[ Z = \max_i \min_j (a_{ij} P_j) \tag{14} \]

Selection rule according to the Germeyer’s criterion is formed as follows: the decision matrix with elements \( a_{ij} \) is supplemented with one more column containing in each row the smallest product of the result available in it by the probability of the corresponding state of nature \( \Pi_j \), and then among the obtained values of this column a strategy with the largest value is selected.

Germeyer’s criterion generalizes the Wald maximin criterion (see below). In the case of a uniform distribution of \( P_j = 1/n \), they become identical.

In the second approach, in a situation of complete uncertainty, that is, when \( P(\Pi_j) \) are unknown and they are not supposed to be calculated according to the above rules, the following criteria are used to make decisions: product criterion, Wald criterion, Savage test and Hurwitz criterion.

Product criterion. Here the selection rule is formed as follows: the decision matrix with elements \( a_{ij} \) is supplemented with a new column containing the products of all the results of each row, and then the option with the highest value is selected. The criterion makes sense when all the elements \( a_{ij} \) are positive. The product criterion is calculated as follows:

\[ Z = \max_i \prod_{j=1}^n a_{ij} \tag{15} \]

Wald maximin criterion (pessimism criterion – always count on the worst) – the worst result is declared the minimum gain, i.e.:

\[ \max_i \min_j (a_{ij}) \tag{16} \]

Savage's test (to avoid a big risk by all means) – the worst loss is declared not the minimum gain, but the maximum loss of gain compared to what could be achieved under given conditions:

\[ \min_i \max_j r_{ij} \tag{17} \]

Hurwitz criterion (pessimism-optimism criterion) – the degree of pessimism is estimated by experts with criterion \( \alpha \):

\[ \max_i \left\{ \alpha \min_j a_{ij} + (1 + \alpha) \max_j a_{ij} \right\} \tag{18} \]

where \( 0 \leq \alpha \leq 1 \) is the pessimism-optimism criterion. When \( \alpha = 1 \), it turns into the Wald criterion.

The obtained calculation results are summarized in Table 3. In this table, the best strategies by criteria are marked with asterisks (*). However, speculatively, it is very difficult to choose the best strategy.
In order to do this, let us apply a multi-criteria ranking according to the Borda rule. The essence of this rule is that for each criterion the facts of the advantages of this strategy over the other ones are recorded and summed up by all the criteria. The best strategy is the one that has the greatest total number of advantages. As a result of this ranking in the case of both partial uncertainty and complete uncertainty the best strategy was to develop scenario $A_2^*$ [6, 7].

The choice of the optimal strategy $A_i^*$ will be found in a similar way, by ranking them over several criteria according to Pareto [6, 7] and solving the problem in the conditions of partial and complete uncertainty. As a result, the optimal strategy is that strategy $A_i^*$, which is the best in the aggregate of all criteria. Here, in order to rank the strategies, the Pareto matrices of pairwise comparisons are compiled according to the rule:

$$a_{ij} = \begin{cases} 
1, & \text{if the } i\text{-th criterion has a strict preference over the } j\text{-th criterion} \\
0, & \text{otherwise}
\end{cases}$$

**Table 3. Evaluation of performance and risk criteria for strategy $A_i$.**

| №  | Criterion                                                      | Value of criterion | Strategy $A_1$ | Strategy $A_2$ | Strategy $A_3$ |
|----|---------------------------------------------------------------|--------------------|---------------|---------------|---------------|
| a) | Partial (incomplete) uncertainty                              |                    |               |               |               |
| 1  | Bayes-Laplace criterion, $P_j$ are set                        | 12.5               | 12.8*         | 12.6          |
| 2  | Criterion of minimum mean risk, $P_j$ are set                 | 1.0                | 0.8*          | 1.0           |
| 3  | Laplace uncertainty principle, $P_j$ are equal, minimum mean risk | 13.0*             | 13.0*         | 12.75         |
| 4  | Laplace uncertainty principle, $P_j$ are equal, minimum mean risk | 1.0*              | 1.0*          | 1.25          |
| 5  | Gemmeier’s criterion, $P_j$ are set                           | 1.5                | 1.6*          | 1.6*          |
| 6  | Hodges-Lehmann estimate, $P_j$ are set                        | 11.25              | 11.4*         | 10.8          |
| 7  | Fishburne point estimate, $P_j$ are set                       | 12.7*              | 12.0          | 11.6          |
| 8  | Fishburne point estimate $P_j$, minimum mean risk             | 0.6*               | 1.6           | 2.0           |
| b) | Complete uncertainty                                          |                    |               |               |               |
| 1  | Wald criterion (maximin)                                      | 10*                | 10*           | 9             |
| 2  | Savage test (minimax risk)                                    | 2*                 | 4             | 5             |
| 3  | Hurwitz criterion (pessimism-optimism)                        | 12.5               | 13*           | 12.5          |
| 4  | Criterion of product                                           | 27300              | 28880*        | 24192         |

As a result, two groups of strategies are obtained: the first group of strategies: $A_1, A_3$, and the second group – where the strategy $A_2^*$ is the best. However, it is easy to see that the set of criteria by which multicriteria ranking was carried out is meaningfully divided into two groups: criteria that have a sense of average gain, and criteria for average risk. We note here that it is not necessary to divide them into separate groups for multi-criteria ranking $A_i$, since it has been proven [8, 10] that, from the point of view of evaluating strategies, they give identical results. At the same time, to obtain a more systematic assessment of the effectiveness and risks of developing the deposits, it is necessary to consider another approach – the dispersion risk model.

**5. Dispersion model for evaluating the risk of hydrocarbon field development located at the arctic offshore**

There is another understanding of risk [11] – dispersion risk model. Let there be any operation, the income from which is a random variable, for example, the gain $a_{ij}$ in the payment matrix, defined in Table 2, depending on the adopted strategy $A_i$ of the field development and the state of nature $\Pi_j$. As mentioned above, according to the Bayes-Laplace and Hodges-Lehmann criterion, the average expected income is the expectation $M_i(a_{ij})$ of this random variable with development strategy $A_i$. 
The standard deviation \( \sigma(a_{ij}) = [D(a_{ij})]^{1/2} \) is a measure of the spread of possible values around the average expected income, where for a discrete random variable its variance that serves as a measure of risk is:

\[
D(a_{ij}) = M\left( a_{ij} - M_i(a_{ij}) \right)^2 = \sum_{j=1}^{n} (a_{ij} - a_{i,mean})^2 \cdot P_j
\]  

(19)

In order to choose best option, it is necessary to use weighting formulas in the approach to determining risk. For example, the weighting formula may be as follows [8]:

\[
\varphi(q, r) = q - r,
\]  

(20)

where \( q \) – income, \( r \) – risk.

The results of calculations by the ratio (20) are presented in Table 4. To choose the best strategy \( A_1^* \), in the rows of Table 4 we find the maximum value \( \varphi(q, r) \), which will determine the Pareto-optimal solution – the minimum possible risk and the maximum possible income.

**Table 4. Estimation of strategies \( A_i \) of the arctic offshore field development by two criteria: income (gain) and risk (standard deviation of income)**

| № | Criterion | The values of benefits, risks and their differences |
|---|-----------|---------------------------------------------------|
|   |           | Strategy \( A_1 \)                                |
|   |           | Benefit; Standard deviation \( \varphi(q, r) = q - r \) |
|   |           | Strategy \( A_2 \)                                |
|   |           | Benefit; Standard deviation \( \varphi(q, r) = q - r \) |
|   |           | Strategy \( A_3 \)                                |
|   |           | Benefit; Standard deviation \( \varphi(q, r) = q - r \) |
| 1 | Bayes-Laplace criterion, \( P_j \) set by expert | 12.5; 1.746 10.754 12.8; 1.833 10.967* 12.6; 2.154 10.446 |
| 2 | Bayes-Laplace criterion, Laplace uncertainty principle | 13.0; 1.871 11.129* 13.0; 2.236 10.764 12.75; 2.8 9.88 |
| 3 | Bayes-Laplace criterion, Fishburne point estimate | 12.7; 1.83 10.87* 12.0; 2.0 10.0 11.6; 2.491 9.109 |
| 4 | Hodges-Lehnmann criterion, \( P_j \) set by expert | 11.25; 2.15 9.1* 11.4; 2.31 9.09* 10.8; 2.8 7.2 |
| 5 | Hodges-Lehnmann criterion, Laplace uncertainty principle | 11.5; 2.40 9.1 11.5; 2.25 9.25* 10.86; 3.41 7.45 |
| 6 | Hodges-Lehnmann criterion, Fishburne point estimate | 11.35; 2.30 9.05* 11.0; 2.24 8.76 10.3; 2.65 7.65 |

Comparing further the best strategies for specific criteria and conditions for implementing states of nature, marked with asterisk (*) in the table, the best among them can be chosen – this is the \( A_1^* \) strategy, having \( \varphi(q, r) = q - r = 13.0 - 1.871 = 11.129 \) by Bayes-Laplace criterion and the Laplace uncertainty principle. However, the final decision is for decision makers.

**6. Conclusions**

The article proposed a systems approach for assessing risks and the effectiveness of possible strategies for the development and ranking of potential arctic offshore production assets, based on their multi-criteria assessment.
Several groups of criteria are considered, such as geological, economic, technological, climatic, social and environmental, which determine the possibility and significance of projects for the development of deposits of the arctic offshore. The selected criteria provide a framework for building up their ranked list and the model for evaluating the expected risks based on the hierarchy analysis method. Depending on the tasks to be solved, multi-criteria adequate interpretations and risk assessment models are proposed and substantiated.

The risks and effectiveness of development options for the arctic shelf deposit are assessed for partial uncertainty (the probabilities of different states of nature are known or can be estimated) and the total uncertainties (the probabilities of the states of nature are unknown).

To improve the quality of the choice of the best development option, it is proposed to apply the rule of multi-criteria ranking developed by Borda and Pareto, as well as the dispersion risk model.

The proposed models combine rigorous mathematical methods with the experience and intuition of experts who are an integral part of the relevant computer decision support system and machine learning, the essence of which in this case is the computer’s selection of the most significant criteria, the best compromise schemes and the corresponding optimization algorithms.

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