Baryons with D5 Brane Vertex and k-Quarks States

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Abstract

We study baryons in $SU(N)$ gauge theories, according to the gauge/string correspondence based on IIB string theory. The D5 brane, in which $N$ fundamental strings are dissolved as a color singlet, is introduced as the baryon vertex, and its configurations are studied. We find point- and split-type of vertex. In the latter case, two cusps appears and they are connected by a flux composed of dissolved fundamental strings with a definite tension. In both cases, $N$ fundamental quarks are attached on the cusp(s) of the vertex to cancel the surface term. In the confining phase, we find that the quarks in the baryon feel the potential increasing linearly with the distance from the vertex. At finite temperature and in the deconfining phase, we find stable k-quarks "baryons", which are constructed of arbitrary number of $k(< N)$ quarks.

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1 Introduction

In the context of string/gauge theory correspondence [1, 2, 3], the large-$N$ dynamics of baryons can be related to the D5-branes embedded in $AdS$ space [4, 5]. This point, in the context of the non-confining $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, have been studied in [6, 7] using the Born-Infeld approach for constructing strings out of D-branes [8, 9]. The fundamental strings (F-string) are dissolved in the D5 brane in these approaches, and they could flow out as separated strings from the singular point(s) appeared on the surface of the D5 brane. So we can consider the system of the D5 brane and F-strings, which are out of the D5 brane, as baryon. The approach in this direction has been applied to the confining gauge theories [10, 11, 12]. In this case, the no force condition, a balance of the tensions of the D5 brane and the F-strings, is imposed at their connected point(s) [13, 14]. However, in [13, 14], the configuration of D5 branes with dissolved F-strings has not been considered. In [10], the D5 configuration has been taken into account of, but the no force condition has been used only in a restricted direction, then the structure of the F-strings is neglected. However, taking into account of both structure of D5 brane and F-strings is important to obtain possible baryon configurations as a system of the both objects.

Here, we study the plausible baryon configurations by embedding these objects in two simple bulk backgrounds which correspond to a confining and a non-confining $SU(N)$ gauge theories. In a confining theory, we can see two kinds of tensions in the baryon. One is the string tension of the F-strings, which works on the quarks in the baryon. And the other is observed as the tension of the bundle composed of many F-strings. The latter is found in a special configuration where the D5 brane is stretched.

In the deconfining and high temperature phase, which is expressed by the AdS Schwartzchshild background, we could find color non-singlet baryon configuration constructed of the quarks with the number $k < N$. This configuration could be generated due to the fact that $N - k$ F-strings of the baryon could disappear into the horizon. As a result, the baryon is separated to free $N - k$ quarks and the remaining $k$-quarks connected to the D5 baryon vertex as k-quarks baryon.

The AdS$_5$ background expresses also the deconfining phase, so the k-string state has been found also in this theory with a constraint for the number $k$ as $5N/8 < k < N$ in [13, 14]. In these approaches, any configuration of D5 vertex has not been considered as a solution of the brane action. In our case, however, such structures are taken into account of, then the lower bound for $k$ is removed as a result. Due to the geometrical freedom of the brane and the F-strings, any $k$ quarks baryon is possible.

In Section 2 we give our model and D5-brane action with non-trivial $U(1)$ gauge field which represents the dissolved F-strings. And the equations of motion for D5 branes are given. In section 3, we give a point vertex solution and study possible baryon configurations in the confining phase. We show the linear relation of the baryon mass and the distance of a quark in the baryon from the vertex. In section 4, the split vertex
solutions are studied. This solution is constructed by the bundle of F-strings flux with a definite tension between two cusps on $S^5$. And we estimate the tension of this flux. In section 5, we study the baryon in the deconfinement phase at finite temperature, and stable color non-singlet baryons are shown. And in a final section, we summarize our results and discuss future directions.

2 Set of the Model

We derive the equations for a D5-brane embedded in the appropriate 10d background. As a specific supergravity background, we consider the following 10d background in string frame given by a non-trivial dilaton $\Phi$ and axion $\chi$ [15, 16],

$$ds_{10}^2 = e^{\Phi/2} \left( \frac{r^2}{R^2} A^2(r) \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 \right).$$  

First, we consider the supersymmetric solution

$$A = 1, \quad e^{\Phi} = 1 + \frac{q}{r^4}, \quad \chi = -e^{-\Phi} + \chi_0,$$

with self-dual Ramond-Ramond field strength

$$G_{(5)} \equiv dC_{(4)} = 4R^4 \left( \text{vol}(S^5) d\theta_1 \wedge \ldots \wedge d\theta_5 - \frac{r^3}{R^8} dt \wedge \ldots \wedge dx_3 \wedge dr \right),$$

where $\text{vol}(S^5) \equiv \sin^4 \theta_1 \text{vol}(S^4) \equiv \sin^4 \theta_1 \sin^3 \theta_2 \sin^2 \theta_3 \sin \theta_4$. The solution (2) is dual to $N = 2$ super Yang-Mills theory with gauge condensate $q$ and is chirally symmetric.

Here the baryon is constructed from the vertex and $N$ fundamental strings. In the string theory, the vertex is considered as the D5 brane wrapped on an $S^5$ on which $N$ fundamental strings terminate and they are dissolved in it [4, 5]. Then the D5-brane action is written as by the Born-Infeld plus Chern-Simons term

$$S_{D5} = -T_5 \int d^6 \xi e^{-\Phi} \sqrt{-\det (g_{ab} + 2\pi \alpha' F_{ab})} + T_5 \int \left( 2\pi \alpha' F_{(2)} \wedge c_{(4)} \right)_{0...5} ,$$

where $T_5 = 1/(g_s(2\pi)^5 l_s^6)$ is the brane tension. The Born-Infeld term involves the induced metric $g$ and the $U(1)$ worldvolume field strength $F_{(2)} = dA_{(1)}$. The second term is the Wess-Zumino coupling of the worldvolume gauge field, and it is also written as

$$S = -T_5 \int d^6 \xi e^{-\Phi} \sqrt{-\det (g + F)} + T_5 \int A_{(1)} \wedge G_{(5)} ,$$

in terms of (the pullback of) the background five-form field strength $G_{(5)}$, which effectively endows the fivebrane with a $U(1)$ charge proportional to the $S^5$ solid angle that it spans. Namely,

$$\int_{S^4} d^4 \theta \frac{\partial L}{\partial F_{(2)}} = \int_{S^4} d^4 \theta \frac{\partial L}{\partial B_{(2)}} = kT_F.$$
where \( T_F = 1/(2\pi \alpha') \) and \( k \subset \mathbb{Z} \).

Then we give the embedded configuration of the D5 brane in the given 10d background. At first, we fix its world volume as \( \xi^a = (t, \theta, \theta_2, \ldots, \theta_5) \). For simplicity we restrict our attention to \( SO(5) \) symmetric configurations of the form \( r(\theta), x(\theta), \) and \( A_t(\theta) \) (with all other fields set to zero), where \( \theta \) is the polar angle in spherical coordinates. The action then simplifies to

\[
S = T_5 \Omega_4 R^4 \int dt d\theta \sin^4 \theta \{ -e^\Phi (r^2 + r'^2 + (r/R)^4 x'^2) - F_{\theta t}^2 + 4A_t \},
\]

where \( \Omega_4 = 8\pi^2/3 \) is the volume of the unit four-sphere.

The gauge field equation of motion following from this action reads

\[
\partial_\theta D = -4 \sin^4 \theta,
\]

where the dimensionless displacement is defined as the variation of the action with respect to \( E = F_{t\theta} \), namely \( D = \delta S/\delta F_{t\theta} \) and \( \tilde{S} = S/T_5 \Omega_4 R^4 \). The solution to this equation is

\[
D \equiv D(\nu, \theta) = \left[ \frac{3}{2} (\nu \pi - \theta) + \frac{3}{2} \sin \theta \cos \theta + \sin^3 \theta \cos \theta \right].
\]

Here, the integration constant \( \nu \) is expressed as \( 0 \leq \nu = k/N \leq 1 \), where \( k \) denotes the number of Born-Infeld strings emerging from one of the pole of the \( S^5 \). Next, it is convenient to eliminate the gauge field in favor of \( D \) and Legendre transform the original Lagrangian to obtain an energy functional of the embedding coordinate only:

\[
U = \frac{N}{3\pi^2 \alpha'} \int d\theta \ e^{\Phi/2} \sqrt{r^2 + r'^2 + (r/R)^4 x'^2} \sqrt{V_\nu(\theta)}.
\]

\[
V_\nu(\theta) = D(\nu, \theta)^2 + \sin^8 \theta
\]

where we used \( T_5 \Omega_4 R^4 = N/(3\pi^2 \alpha') \). Using this expression (8), we solve the D5 brane configuration in the followings.

### 3 The Point Vertex

In this section we study solutions which correspond to a baryon localized at a particular point in our 4d space-time. To localize the vertex in \( x \), we set as \( x' = 0 \), and the equation of motion for \( r(\theta) \) is obtained from (8) as

\[
\partial_\theta \left( \frac{r'}{\sqrt{r^2 + (r')^2}} \sqrt{V_\nu(\theta)} \right) - \left( 1 + \frac{r}{2} \partial_\theta \Phi \right) \frac{r}{\sqrt{r^2 + (r')^2}} \sqrt{V_\nu(\theta)} = 0.
\]
For our background solution (2), it is rewritten as

\[
\partial_\theta \left( \frac{r'}{\sqrt{r'^2 + (r')^2}} \sqrt{V_\nu(\theta)} \right) - \frac{1 - q/r^4}{1 + q/r^4} \frac{r}{\sqrt{r'^2 + (r')^2}} \sqrt{V_\nu(\theta)} = 0 .
\] (11)

In the case of \( q = 0 \), the background reduces to \( AdS_5 \times S^5 \), where the equation is analytically solved and well studied in [7]. In this case, however, the quarks are not confined. In order to see the baryon configuration, we should consider the case of non-BPS confinement phase, which is realized in our model for non-zero \( q > 0 \). The parameter \( q \) represents the gauge condensate in the 4d YM theory, and provide the tension of the meson states. So we could expect to be able to extract the characteristic properties of the baryon in the confinement phase.

However, for \( q \neq 0 \), it is difficult to find an analytical solution, so we try to solve the above equation numerically. The "potential" \( V(\theta) \) has three extremum points. In solving the equation, we impose the boundary condition at one of these point, \( \theta = \theta_c \), which is the minimum of \( V_\nu(\theta) \) and is given by the solution of

\[
\pi \nu = \theta_c - \sin \theta_c \cos \theta_c .
\] (12)

Then the boundary conditions are set as

\[
r(\theta_c) = r_0, \quad \frac{\partial r(\theta_c)}{\partial \theta} = 0 ,
\] (13)

where \( r_0 \) is a parameter which determine the configuration of D5 brane. The typical solutions are shown in the Fig. 1. These solutions have cusps at the pole points, \( \theta = \pi \) and \( \theta = 0 \) for \( \nu = 0.2 \) and only at \( \theta = \pi \) for \( \nu = 0 \).

For simplicity, we consider here the case of \( \nu = 0 \). In this case, \( \theta_c = 0 \) and the solution \( r(\theta) \) is smooth at \( \theta = 0 \) but it has a cusp at \( \theta = \pi \) on \( S^5 \). The solutions depend on \( q \) and \( r_0 \) through the ratio \( \zeta = r_0/q^{1/4} \), and they are classified to two types, (A) and (B), by \( \zeta \) as follows;

(A) \( \zeta \geq 1 \);

(i) For \( \zeta >> 1 \), the solution is near the BPS ‘tube’ solution [7], which arrives at \( \theta = \pi \) only at \( r = \infty \).

(ii) When \( \zeta \) decreases toward \( \zeta = 1 \), the solution begins to tilt and crosses the symmetry axis, \( \theta = \pi \), at a finite value of \( r = r_c \). And \( r'(\pi) \) is finite then it forms a cusp.

(iii) And we notice that \( r(\theta) = q^{1/4} \) at \( \zeta = 1 \), and this constant value of \( r \) is an exact solution to the equations of motion. Its shape is completely spherical. This is the only one allowable constant solution.
Figure 1: Family of solutions for \( q = 2.8 \) and \( r(0) = q^{1/4} + \epsilon \) with various \( \epsilon \), \( \epsilon = 0.2, 0.1, -0.2, -0.3 \) for the curves from the above. The right hand side shows the solutions for \( \nu = 0.2 \) and \( r(\theta_c) = q^{1/4} + 0.1 \) (the upper) and \( q^{1/4} - 1.1 \) (the lower).

For this solution, we obtain from (8) the following

\[
U = \frac{N}{3\pi^2 \alpha'} 2^{1/2} q^{1/4} j(0),
\]

where

\[
j(\nu) = \int_0^{\pi} \sqrt{v(\theta)}
\]

and it is estimated as \( j(0) = 10.67 \) for \( \nu = 0 \). This represents the vertex energy for the constant solution of \( r \). A similar solution has been given in [13, 14] for AdS\(_5\) background. In our case, however, this solution corresponds to a special one in our non-conformal background. Actually, this solution disappears into the horizon in the AdS\(_5\) limit of \( q \to 0 \).

(B) \( 0 < \zeta < 1 \); In this case, the term \( 1 - q/r^4 \) in the second term of Eq. (11) turns to negative then \( r \) decreases with \( \theta \). This implies that the north and south poles on \( S^5 \) are opposite to the one of the solution for \( \zeta > 1 \), then the direction of the force coming from the D5 tension at the cusp is also reversed. Further, the minimum, \( r(\pi) \), approaches to zero for \( r_0 \to 0 \), but the brane could not arrive at \( r = 0 \) since the action diverges there even if how small \( r_0 \) is. In other words, an infinite energy is necessary to realize \( r(\pi) = 0 \). In other words, the D5 vertex is never absorbed in the horizon due to the confinement force, and \( \zeta \) is restricted as \( \zeta > 0 \).

3.1 Baryon configuration and no-force condition

The schematic configurations of the baryons formed of the two types of vertex (A) and (B) are shown in the Fig. 2. In the present case, the fundamental strings, which are dissolved in the D5 brane, should flow out from the cusp at \( \theta = \pi \). The D5 brane configuration is singular at this point. This singularity is cancelled out by the boundary term of the fundamental string stretching from this point. This cancellation
is equivalent to the so called no force condition, namely the cancellation of the tension forces among the fundamental strings and the D5 brane at the cusp point. It is possible to consider various configurations of D5 brane and strings which satisfy the no force condition. They are supposed as the various possible flowing of the fundamental strings from the D5 brane.

Then the fundamental strings coming out from the D5 brane could stretch separately to any direction when they are allowed dynamically. The dynamical conditions to be considered are separated to two parts, (a) the equations of motion of F-strings in the given background and (b) the no force conditions at the cusp point.

As for the condition (b), when the stretching direction of the F-strings is restricted to the \( r \) direction only as considered in [10], then the resultant baryon configuration is reduced to the BPS flux tube of D5 since the tension force of the F-strings is stronger than the one of the D5 vertex. However we could consider other configurations of F-strings which spread also to the \( x \) direction. As a result, the force in the \( r \)-direction is weakened and the configurations given in the Fig. 2 become possible.

Such a string configuration has been considered in [13], but the authors in [13] did not take into account of the structure of the D5 brane except for the constant \( r \) configuration as mentioned above. In the present model, more general configurations than the one considered in [13] are studied.

Here we suppose that the two end points of F-string are connected to the cusp \( r = r_c \) of the D5 vertex and the other probe brane, for example the D7 probe brane, which corresponds to the brane providing the flavor quarks and is put at an appropriate position, \( r = r_{\text{max}}(> r_c) \). Here \( r_{\text{max}} \) plays a role of the cut off of \( r \), and we do not give a D7 brane configuration with the attached strings. This problem is postponed to the future works. The quark mass in this case is given approximately by \( T_F(r_{\text{max}} - r_c) \) for the AdS5 background.

Now we turn to the no force condition at \( r = r_c \). In the \( r \) direction, the tensions of the fundamental strings and the one of the D5 brane should make a balance. As for the \( x \) direction, the balance should be realized by F-strings themselves. For D5 brane, the tension in the \( r \) direction is estimated in terms of \( U \) given by (8) under the variation \( r_c \to r_c + \delta r_c[10] \), it can be seen from (8), by using the Euler-Lagrange equation, that the energy of the brane changes only by a surface term at \( r_c \),

\[
\frac{\partial U}{\partial r_c} = N T_F e^{\Phi/2} \frac{r_c'}{\sqrt{r_c'^2 + r_c^2}},
\]  

(15)

where \( r_c' = \partial \theta r|_{\theta = \pi} \). From this equation, we can see that the direction of the force is reversed at the point \( \zeta = 1 \) when \( \zeta \) changes from (A) \( \zeta > 1 \) to (B) \( \zeta < 1 \) since the sign of \( r_c' \) changes. This behavior is important in our model as found below.

As for the fundamental string, which extends to the \( x \)-direction, its action is written
as
\[ S_F = -\frac{1}{2\pi\alpha'} \int dt dx \, e^{\Phi/2} \sqrt{r_x^2 + (r/R)^4}, \quad (16) \]
where \( r_x = \partial r/\partial x \) and the world sheet coordinates are set as \((t, x)\). Then, its energy and \( r \)-directed tension at the point \( r = r_c \) are obtained as follows
\[ U_F = T_F \int dx \, e^{\Phi/2} \sqrt{r_x^2 + (r/R)^4}, \quad (17) \]
\[ \frac{\partial U_F}{\partial r_c} = T_F e^{\Phi/2} \frac{r_x}{\sqrt{r_x^2 + (r_c/R)^4}}, \quad (18) \]

For the remaining \( N-1 \) strings, it is possible to take their world sheet coordinates in a different directions from the \( x \) direction. Here, for the simplicity, we assume that they extend in the same plane, for example the \( x-y \) plane, and their forces make a valence with the same tension in this plane. The quarks are put on a circle whose center is the D5 vertex. Then, the forces of the strings in the \( r \)-direction are added up, and its value is \( N \) times of (18) and it should keep a valence with the one of the D5 brane. We notice that the direction of this tension force is also reversed at \( \zeta = 1 \).

Thus we find the following no force condition as discussed in [13, 14] with a slightly different setting,
\[ r_x^{(c)} = r_x' \frac{r_c}{R^2}, \quad (19) \]
at \( r = r_c \), where \( r_x^{(c)} \) denotes the value of \( r_x \) at \( r = r_c \). Here the sign of \( r_x^{(c)} \) and \( r_c' \) should be the same.

\[ \begin{array}{c}
\text{(A)}
\end{array} \]

\[ \begin{array}{c}
\text{(B)}
\end{array} \]

Figure 2: Point vertex baryon. For the solutions of (A) \( r_c > r_0 \) and (B) \( r_c < r_0 \).
3.2 Baryon mass and distance between quark and vertex

Thus we find the baryon configuration as depicted schematically in the Fig.2. For small baryon mass, its configuration is expressed by the figure (A), and the configuration changes to (B) when the mass increases. In the configuration (B), the fundamental string could become long with increasing baryon mass. As a result, any large baryon mass is realized by the configuration (B).

The energy (or the mass) of the baryon is therefore given as follows,

\[ E = NE_F + E_J^{(S)} \]  \hspace{1cm} (20)

where \( E_F \) is the F-string part and \( E_J^{(S)} \) represents the D5 brane part, i.e. the baryon vertex. The latter is obtained from (8) by setting as \( x' = 0 \),

\[ E_J^{(S)} = \frac{N}{3\pi^2\alpha'} \int_0^\pi d\theta \frac{e^{\Phi/2}(r^2 + r'^2)^{1/2}}{\sqrt{V_{\nu=0}(\theta)}} \]  \hspace{1cm} (21)

While the baryon vertex is seen as a point in our 4d space-time, it has a structure in inner space and has a finite value of \( E_J^{(S)} \).

Although \( r_0 \) is not equal to \( q^{1/4} \) in general, the rough estimation of this energy is given analytically in this limit of \( r_0 = q^{1/4} \) or \( \zeta = 1 \). The D5 brane is squashed, at this point, to a point in the \( r \)-direction. Then we obtain

\[ E_J^{(S)} |_{r_0=q^{1/4}} = \frac{N}{3\pi^2\alpha'} \frac{2^{1/2}q^{1/4}j(0)}{2} \]  \hspace{1cm} (22)

where \( j(0) = 10.67 \) as given above. Since \( q \) is written as \( q = \lambda \langle F_{\mu\nu}^2 \rangle \alpha'^4 \) [15], this vertex energy increases with the 'tHooft coupling like \( \lambda^{1/4} \) when \( \langle F_{\mu\nu}^2 \rangle \) is fixed. On the other hand, we find that the F-string tension is independent of \( \lambda \) as seen below. Then, the vertex energy is expected to become large and the main part of the baryon mass at large 'tHooft coupling.

Next, we turn to the energy of the F-string part. From (16), we can set the following constant \( h \),

\[ e^{\Phi/2} \frac{r^4}{R^4 \sqrt{r_x^2 + (r/R)^4}} = h \]  \hspace{1cm} (23)

Then, by eliminating \( r_x \) in terms of the above equation with a constant \( h \), we get the formula of the distance \( L \) between the quark and the vertex and the string energy \( E \) as

\[ L_{q-v} = R^2 \int_{r_e}^{r_{\text{max}}} dr \frac{1}{r^2 \sqrt{e^{\Phi/2} (h^2 R^4)/(e^{\Phi/2} R^4) - 1}} \]  \hspace{1cm} (24)

\[ E_F = T_\mathcal{F} \int_{r_e}^{r_{\text{max}}} dr \frac{e^{\Phi/2}}{\sqrt{1 - h^2 R^4/(e^{\Phi/2} R^4)}} \]  \hspace{1cm} (25)
These formula are equivalent to the one of the mesons made of quark and anti-quark except for the lower bound of the r-integration, which is given here as the D5 cusp point $r_c$. And the no force condition (19) is imposed at this point.

The equations (24) and (25) are evaluated for the solutions (A) and (B) separately since the string shapes are different in the two cases. First, we consider the solutions of (B). In this case, $r_x$ is negative at $r_c$ then $r$ decreases with increasing $|x - x(r_c)|$ after it departed from the cusp point $x(r_c)$, but the string can not never reach at the horizon $r = 0$ since the action diverges at this point. Thus $r$ reaches at the minimum $r(\equiv r_{\text{min}})$ at some $x = x_0$, where $r_x = 0$, then it begins to increase toward $r_{\text{max}}$ (see Fig. 2). The shape of this F-string is determined as the extremum of $U_F$, namely it is given as a solution of the equation of motion derived from (17). The total configuration of the baryon made of F-strings and the D5 vertex is controlled by the one parameter $r_0$. For a given $r_0$, all the value of $r'(\pi)$, $r_c$, $r_{\text{min}}$ and $r_x|_{r_c}$ are determined, so the energy of the baryon is also determined.

Therefore, when a value of $r_0(< q^{1/4})$ is fixed, it is convenient to separate the F-string to (i) the region of $r_{\text{min}} < r < r_{\text{max}}$ and of (ii) $r_{\text{min}} < r < r_c$. Then the energy can be written as

$$E_F^{(B)} = E_F^{(B_i)} + E_F^{(B_{ii})}. \tag{26}$$

The first term corresponds to the half of the meson configuration made of quark and anti-quark. When the energy becomes large or F-string grows long, $E_F^{(B_i)}$ is approximated by the following formula [16],

$$E_F^{(B_i)} = m_{q}^{\text{eff}} + \tau_M \frac{L_{q-\bar{q}}}{2} \tag{27}$$

where

$$\tau_M = T_F \sqrt{\frac{q}{R^4}}, \tag{28}$$

$$m_{q}^{\text{eff}} = T_F \int_{r_{\text{min}}}^{r_{\text{max}}} dr e^{\Phi/2} \tag{29}$$

In this calculation, the constant $h$ is taken as

$$h = e^{\Phi(r_{\text{min}})/2} \left( \frac{r_{\text{min}}}{R} \right)^2 e^{\Phi(r_c)/2} \left( \frac{r_c}{R} \right)^2 \frac{1}{\sqrt{\left(r'(r_c)/r_c\right)^2 + 1}} \tag{30}$$

Due to this boundary condition, $r_{\text{min}}$ is determined by using $r_c$, then by $r_0$ since $r_c$ is determined by $r_0$ as mentioned above. The remaining part of the F-string is obtained as

$$E_F^{(B_{ii})} = T_F \int_{r_{\text{min}}}^{r_c} dr \frac{e^{\Phi/2}}{\sqrt{1 - h^2 R^4/\left(e^{\Phi/4}ight)^4}}. \tag{31}$$

Next, we turn to the solution (A), whose configuration is shown in the Fig. 2 (A). In this case, since $r'(\pi) > 0$, then the $r$ of the F-string increases from $r_c$ towards $r_{\text{max}}$
monotonically. There is no point of \( r_x = 0 \). So the configuration of F-string is obtained in terms of (24) and (25) by the setting of the lower bound of \( r \) integration as \( r_c \) and with the boundary condition at this point,

\[
h = e^{\Phi(r_c)/2} \left( \frac{r_c}{R} \right)^2 \frac{1}{\sqrt{(r'(r_c)/r_c)^2 + 1}}.
\]

(31)

In this case, the energy of the F-string is given as

\[
E_F^{(A)} = T_F \int_{r_c}^{r_{max}} dr \frac{e^{\Phi/2}}{\sqrt{1 - h^2 R^4/(e^{\Phi} r^4)}}.
\]

(32)

The above \( h \) could take its minimum, \( e^{\Phi(r_c)/2} (r_c/R)^2 \), at \( r'(r_c) = 0 \), which is realized at \( r_0 = q^{1/4} \), and the distance \( L_{q-v} \) becomes the maximum in the configuration (A). Actually, we can estimate the maximum of \( L_{q-v} \) as,

\[
L_{q-v}^{max} = R^2 \left( \frac{4\pi^2}{q} \right)^{1/4} \frac{\Gamma(3/4)}{\Gamma(1/4)}
\]

(33)

where we take \( r_{max} = \infty \). So the baryon is expressed by the solution (A) for \( 0 < L_{q-v} \leq L_{q-v}^{max} \) and by the (B) for \( L_{q-v}^{max} < L_{q-v} \).

Now we can calculate the total energy of the baryon \( E \) versus \( L_{q-v} \). The energy is given as

\[
E = NE_F + E_{D5}
\]

(34)

where \( E_F = E_F^{(A)} \) or \( E_F^{(B)} \) for short or long \( L_{q-v} \) respectively. Assuming that all fundamental quarks are put at the same distance from the vertex, \( E \) is numerically estimated as a function of \( L_{q-v} \). An example is shown in the Fig. 3 for \( r_{max} = 20 \).

In this figure, the value of \( E \) at \( L = 0 \) shows the value of \( E_{j(S)} \) with \( r_c = r_{max} \), namely the pure D5 brane energy. When \( L \) begins to increase, \( E_{j(S)} \) decreases rapidly with \( L \) and approaches to a constant, and \( E_F \) becomes dominant. It is expressed by \( E_F = E_F^{(A)} \) for small \( L \), in the region of \( L \leq 0.7 \), and by \( E_F = E_F^{(B)} \) for \( L \geq 0.7 \) in the present case. We can see for large \( L \) that \( E \) increases with \( L_{q-v} \) linearly with the tension,

\[
N\tau_M/2,
\]

(35)

where \( \tau_M \) is given in (27). This tension is equal to the sum of the one of the independent F-strings.

4 Split Vertices

The equations of motion for D5 embedding are solved here by adding the freedom of \( x(\theta) \) without the restriction \( x' = 0 \). In this case, it is convenient to use a parametric Hamiltonian formalism as in [10].
First we rewrite the energy (8) in terms of a general worldvolume parameter $s$ defined by functions $\theta = \theta(s), \ r = r(s), \ x = x(s)$ as:

$$U = \frac{N}{3\pi^2\alpha'} \int ds \ e^{\Phi/2} \sqrt{r^2 \dot{\theta}^2 + \dot{r}^2 + (r/R)^4 \dot{x}^2} \sqrt{V_\nu(\theta)},$$

(36)

where dots denote derivatives with respect to $s$. The momenta conjugate to $r, \ \theta$ and $x$ are

$$p_r = \dot{r} \Delta, \quad p_\theta = r^2 \dot{\theta} \Delta, \quad p_x = (r/R)^4 \dot{x} \Delta, \quad \Delta = e^{\Phi/2} \sqrt{V_\nu(\theta)} \sqrt{r^2 \dot{\theta}^2 + \dot{r}^2 + (r/R)^4 \dot{x}^2} \ .$$

(37)

Since the Hamiltonian that follows from the action (36) vanishes identically due to reparametrization invariance in $s$. Then we consider the following identity

$$2 \tilde{H} = p_r^2 + \frac{p_\theta^2}{r^2} + \frac{R^4}{r^4} p_x^2 - (V_\nu(\theta)) e^\Phi = 0 \ .$$

(38)

This constraint can be taken as the new Hamiltonian in order to get the following canonical equations of motion,

$$\dot{r} = p_r, \quad \dot{p}_r = \frac{2}{r^3} p_x^2 R^4 + \frac{p_\theta}{r^3} + \frac{1}{2} (V_\nu(\theta)) e^\Phi \partial_r \Phi,$$

(39)

$$\dot{\theta} = \frac{p_\theta}{r^2}, \quad \dot{p}_\theta = -6 \sin^4 \theta (\pi \nu - \theta + \sin \theta \cos \theta) e^\Phi,$$

(40)

$$\dot{x} = \frac{R^4}{r^4} p_x, \quad \dot{p}_x = 0 \ .$$

(41)

In solving these equations, the initial conditions should be chosen such that $\tilde{H} = 0$. 

---

Figure 3: $E-L_{q-v}$ plots for $q = 2.0, \ R = 1, \ r_{\text{max}} = 20$. The upper (lower) curve shows the baryon energy $E$ (the vertex energy $E_{\text{D5}}$).
Figure 4: The solution with cusps for $R = 1$, $q = 2$ and $\nu = 0.5$. The left figure is for $r(\theta = \pi/2) = 0.6$ and $r(\pi) = r(0) = 0.490507$. The right one is for $r(\pi/2) = 0.1$ and $r(\pi) = r(0) = 0.0336822$.

We now solve these equations numerically to obtain a configuration spreading in the $x$-direction with $p_x \neq 0$ (i.e. $x' \neq 0$). We find two types of configurations of D5 brane, the U shaped and the cap ($\bigcap$) shaped one. Their example solutions are shown in the Fig. 4. The U shaped one is obtained for large $r(0)$ or small mass baryon, and the cup shaped is obtained when $r(0)$ or baryon mass increases.

Since $r$ and $\partial r/\partial \theta$ are finite at the end points of these configurations, the end points of both sides are the cusps. Then the baryon configurations given here are split into two distinct cusps, which are connected to $\nu N$ and $(1 - \nu) N$ quarks respectively for the case of a given value of $\nu$. We can see that the cusps of U shaped configuration are the type (A) which is given in the previous section, and type (B) cusps are seen for the cap shaped one. Then the quarks are attached as depicted in the Fig. 5 by considering the no force condition.

In both cases, the two cusps are connected by a confining flux tube of the gauge theory. We estimate the tension of this flux tube for a tuned configuration shown in the Fig. 6 as an example for the U shaped solution.

This tuned U-shaped D5 brane is very similar to the quark and anti-quark meson
Figure 6: The U-shaped solution of the D5-brane for $R = 1$, $r_0 = 0.32$, $q = 3.5$ and $\nu = 0.2$.

state configuration which is obtained by the fundamental string action. However the present case is for the D5 brane tube, so the tension of this tube is not equal to the one of the meson (27). From the Fig. 6 the tube sits almost at a constant $\theta \simeq \theta_c$ where $\dot{\theta} \simeq 0$. This behavior is understood from the fact that $\theta_c$, which is given above by the solution of (12), is the extremum of the potential $V_0(\theta)$.

Then we can estimate the tension ($\tau_v$) of the flux tube between the two cusp points as follows. From the above solutions for $r(x)$ and $x(\theta)$, we can approximate as $\theta = \theta_c$ and $x'(\theta_c) \gg 1$ in the flux region (see the right hand side of the Fig. 6). Then the D5 brane energy (8) can be approximated as follows

$$U_{\text{flux}} = \frac{N}{3\pi^2 \alpha'} \sin^3 \theta_c \int dx \ e^{\Phi/2} \sqrt{r_x^2 + (r/R)^4}$$

where $U_F$ is the energy of the fundamental string given by (17). This implies the linear relation of the energy of the flux and its length $L_{v-v}$. According to the method given for the meson case, we obtain

$$\tau_v = \frac{2N}{3\pi} \sin^3 \theta_c \tau_M$$

where $\tau_M = T_F \sqrt{\frac{\alpha'}{\pi \alpha'}}$ denotes the tension between the quark and anti-quark which is given above by (27). The dependence of the flux tube tension on $\nu$ is seen from the factor $\sin^3 \theta_c$. From equation (12), we find $\sin^3 \theta_c \simeq 3\pi \nu / 2$ for small $\nu$. This implies that $\tau_v(\nu = 1/N)$ reduces to the same tension with the one of meson state formed by quark and anti-quark [16]. This result has a natural gauge theory interpretation. When one quark is pulled out from the $SU(N)$ baryon (a color-singlet), the remaining $N - 1$ must be in the anti-fundamental representation of the gauge group. The flux tube extending between this bundle and a single quark should then have the same properties as the standard QCD string which connects a quark and an antiquark. On the other hand, for $\nu N \equiv k > 1$, the k-quarks are bounded by the confining force since $\tau_v < k \tau_M$.  

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Here we comment on a similar formula of the flux tension, which has been obtained in another confining 4d YM theory represented by AdS blackhole background \cite{10,12}. It is written as
\[
\sigma_B = \frac{8\pi}{27}N(g_{YM}^2 N)T^2 \nu(1-\nu) .
\] (45)
This has a similar \(\nu\) dependence to our \(\tau_v\) in (44), but \(\sigma_B\) differs from \(\tau_v\) in two points. (i) While \(\sigma_B/N\) increases with the 'tHooft coupling \(\lambda\) linearly, our \(\tau_v/N\) is independent of \(\lambda\). (ii) The scale of our \(\tau_v\) is determined by \(\langle F_{\mu\nu}^2 \rangle\) which is independent of the compact space scale, but \(\sigma_B\) is determined by the compact \(U(1)\) scale \(T\).

It would be useful to write the baryon energy or the mass formula for the split baryon by separating it to three parts as follows,
\[
E_{B}^{sp} = N (\nu E_{F}(r_{c_1}) + (1-\nu)E_{F}(r_{c_2})) + \tau_5 L_{v-v} + E_{J}^{(\nu)} + E_{J}^{(1-\nu)}
\] (46)
where \(L_{v-v}\) is the flux length and \(E_{F}(r_{c_i})\) are given by the same form with (25) for the two different cusp points \(r_{c_i}, i=1,2\). The last two terms describes the D5 brane parts between the cusps and the D5 flux, we call them as ”junction” here. They might be given as
\[
E_{J}^{(\nu)} \simeq \frac{N}{3\pi^2 \alpha'} \int_{\theta_{c}}^{\theta} d\theta \ e^{\Phi/2} (r^2 + r'^2)^{1/2} \sqrt{V_{\nu}(\theta)} ,
\] (47)
\[
E_{J}^{(1-\nu)} \simeq \frac{N}{3\pi^2 \alpha'} \int_{\theta_{c}}^{\pi} d\theta \ e^{\Phi/2} (r^2 + r'^2)^{1/2} \sqrt{V_{\nu}(\theta)} .
\] (48)
In the symmetric case of \(\nu=1/2\), we can set as \(r_{c_1} = r_{c_2} = r_{c}\), then the above formula are simplified as
\[
E = NE_{F}(r_{c}) + \tau_5 L + 2E_{J}^{(1/2)}
\] (49)
It is an interesting problem to compare the junction energy \(2E_{J}^{(1/2)}\) with the point vertex case at \(L=0\) and at the same \(r_c\). From the above formula, we obtain at \(r_c\)
\[
2E_{J}^{(1/2)} = \frac{2N}{3\pi^2 \alpha'} (4q)^{1/4} j^{1/2}(\pi/2) ,
\] (50)
where
\[
j^{1/2}(\pi/2) = \int_{0}^{\pi/2} \sqrt{V_{0.5}(\theta)} = 4.214 ,
\] (51)
Then the resultant junction energy is compared with \(E_{J}^{(S)}\) given by (22), and we find
\[
2E_{J}^{(1/2)} = \frac{2j^{1/2}(\pi/2)}{j(0)} E_{J}^{(S)} = 0.79 E_{J}^{(S)} .
\] (52)
This implies that the split vertex is energetically favourable rather than the point vertex when the flux length \(L_{v-v}\) is negligible.

It would be an interesting problem to consider a D5 brane configuration which splits to more number of fractions of string fluxes. An approach in this direction has been
given by Imamura [12, 11], and he has proposed the junction as the vertex part of
the splitted flux of the fundamental strings. Imamura has estimated this part by a
numerical calculation under an appropriate assumptions.

In the present model, however, it means to study the three or more number of split
D5 vertex. It would be a difficult task to obtain such a solution as a smooth numerical
solution. In this sense, this problem is the open one here.

5 Finite Temperature and k-quarks Baryon

Here, we consider the baryon configurations in the non-confining Yang-Mills theory.
Such a model is given by the AdS blackhole solution, which represents the high tem-
perature gauge theory. In our theory with dilaton, the corresponding background
solution is given as [17]

$$ds_{10}^2 = e^{\Phi/2} \left( \frac{r^2}{R^2} [-f^2 dt^2 + (dx^i)^2] + \frac{R^2}{r^2 f^2} dr^2 + R^2 d\Omega_5^2 \right).$$  (53)

$$f = \sqrt{1 - \left(\frac{r_T}{r}\right)^4}, \quad e^{\Phi} = 1 + \frac{q}{r_T} \log(\frac{1}{f^2}), \quad \chi = -e^{-\Phi} + \chi_0.$$  (54)

The temperature ($T$) is denoted by $T = r_T/(\pi R^2)$. The world volume action of D5
brane is rewritten by eliminating the $U(1)$ flux in terms of its equation of motion as
above, then we get its energy as

$$U = \frac{N}{3\pi^2 \alpha'} \int d\theta \ e^{\Phi/2} f \sqrt{r^2 + r'^2 / f^2 + (r/R)^4 x'^2} V_\nu(\theta).$$  (55)

where $V$ is the same form with (9).

For simplicity, we concentrate on the point vertex configuration. Then, we set $x' = 0$
as above, and the equation of motion of $r(\theta)$ is obtained as follows

$$\partial_\theta \left( \frac{r'}{\sqrt{r^2 f^2 + (r')^2}} \sqrt{V_\nu(\theta)} \right) - \frac{g}{\sqrt{r^2 f^2 + (r')^2}} \sqrt{V_\nu(\theta)} = 0.$$  (56)

$$g = \frac{1}{2e^\Phi} \partial_r \left( e^\Phi r^2 f^2 \right)$$  (57)

In this case also, we find the two types of solutions (A) and (B) which have been
given in the section 3 for $T = 0$ confinement phase. However, in the present finite $T$
case, the theory is in the quark deconfinement phase. So free F-strings could exist and
these F-strings touch on the horizon $r_T$. As a result, $N - k$ quarks disappear, and we
find $k$-quarks baryon. This implies that we could find the color non-singlet baryons as
depicted in the Fig.7. This baryon is therefore constructed from $k(< N_c)$ quarks and
the color singlet D5 vertex.
Figure 7: k-quarks baryons at finite temperature.

For fixed $k$, the solution (A) is obtained at small baryon mass or small $r_0$ where $g$ is positive. When the mass becomes large, then $g$ changes to negative value and we find the solution (B). From Eq.(57), we can estimate the temperature ($T_{c_1}$) where the solution changes from (A) to (B) as,

$$T < \frac{\gamma}{\pi R^2} q^{1/4} \equiv T_{c_1}, \quad \gamma = 0.579,$$

where $\gamma$ is obtained through a numerical analysis. In any case, we would find $k(< N_c)$ quarks baryon at finite temperature before it resolves to independent quarks completely at enough high temperature. This point is different from the meson, which is broken from the meson to the quark and antiquark at high temperature and there is no middle state as in the case of baryons.

Next, we consider the no force condition at the cusp of the k-quarks baryon for the configurations given in the Fig.7. The tension of D5 brane at $r_c$ is given as

$$\frac{\partial U}{\partial r_c} = NT_F e^{\Phi/2} \frac{r_c'}{\sqrt{r_c'^2 + r_c^2 f(r_c)^2}},$$

and for F-strings as

$$\frac{\partial U_F}{\partial r_c} = T_F e^{\Phi/2} \frac{r_c^{(c)}}{\sqrt{r_c^{(c)2} + (r_c/R)^4 f(r_c)^2}}.$$

Here, for the (N-k) strings ending on the horizon in the Fig.(A), we consider the limit of the vertical lines as the one in the Fig.(B). Then the no force condition is obtained
as
\[ N - \frac{r_c'}{\sqrt{r^2_c + r^2_c f(r_c)^2}} + (N - k) = k \frac{r^{(c)}_x}{\sqrt{r^{(c)}_x^2 + (r_c/R)^4 f(r_c)^2}} \] (61)

Notice that \( r^{(c)}_x \) and \( r'_c \) are positive for the solution (A) and negative for (B) respectively. Then, from \( 0 < k < N \), the no force condition Eq.(61) is rewritten as
\[ r^{(c)}_x > \frac{r_c}{R^2} r'_c. \] (62)

When this is compared with the no force condition (19) for the confinement phase, we can understand the above condition (62) as a reasonable one. The force of the remaining k-quarks must cover the lacked part of (N-k) quarks, so it must become larger.

Although the other complicated configurations are possible, these baryon configurations would be observed just after the deconfinement transition occurred at high temperature, \( T = T^{(dec)}_c \). The temperature increases above \( T^{(dec)}_c \) and nears \( q^{1/4} \), then the type (B) k-strings baryons will disappear firstly and the type (A) configurations are remained. When the temperature increases furthermore, the cusp point arrives at \( r_{\text{max}} \) at the temperature \( T_{\text{melt}} \). Since \( r_c < r_{\text{max}} \), all the k quarks (including N quarks) baryons should be collapsed to the free quarks in the medium of quark gluon plasma for \( T > T_{\text{melt}} \). In other words, k-quarks baryons are observed in a range of the temperature,
\[ T^{(dec)}_c < T < T_{\text{melt}} \]

We need some qualitative and phenomenological analyses to estimate this temperature range. This point is very interesting, but it is opened here.

On the other hand, similar k-quarks baryon configurations have been proposed with the restriction, \( k \geq 5N/8 \), for \( N = 4 \) SYM theory [13, 14]. In this theroy, the k-quarks baryon is possible since the quarks are not confined. But, the authors in [13, 14] have not considered the inner structure of the D5 brane with dissolved F-strings. In our high temperature model, insteadly, the k-quarks baryon is possible for any number of \( k \) with \( k < N \). This difference between the condition given in [13, 14] and ours could be reduced to the fact that the D5 brane structure is considered or not in deriving the no-force condition.

Actually, the no-force condition (61) is rewritten as
\[ N(1 + Q_5) = k(1 + Q_F), \] (63)
\[ Q_5 = \frac{r_c'}{\sqrt{r^2_c + r^2_c f(r_c)^2}}, \quad Q_F = \frac{r^{(c)}_x}{\sqrt{r^{(c)}_x^2 + (r_c/R)^4 f(r_c)^2}}. \] (64)
Since \(|Q_F| \leq 1\), we obtain

\[|Q_F| = \left| \frac{N(1 + Q_5) - k}{k} \right| \leq 1. \tag{65}\]

From this, we obtain the lower bound of \(k\) as

\[\frac{N}{2}(1 + Q_5) \leq k. \tag{66}\]

In the case of structureless D5 brane, we obtain \(Q_5 = 1/4 \) \([13, 14]\), then we find \(5N/8 \leq k\). However, in our model, \(Q_5\) could approach to -1 since \(r_c\) could approach to the horizon \(r_T\). This implies the lower bound of \(k\) is zero.

But we must notice that we need infinite energy to realize the limit of \(k=0\) since the vertex energy \(U\) approaches to infinity in this limit. When the energy of the baryon increases, its energy is used mainly to extend the length of the F-strings, namely the value of \(L_{q-v}\). However, at finite temperature and in the deconfinement phase, \(L_{q-v}\) has its maximum value due to the color screening. In this sense, the lower bound of \(k\) would be small but finite for \(T > T^{\text{dec}}\). So we expect \(k=0\) in the limit of the \(T \to T^{\text{dec}}\).

In our model, however, the deconfinement temperature is \(T = 0\), so we could find \(k=0\) in the limit of \(T \to 0\). In other words, small \(k\) state is seen just above \(T^{\text{dec}}\), and the lower bound of \(k\) increases with \(T\). So at enough high temperature, which would be below \(T^{\text{melt}}\) mentioned above, on the other hand, we can not see any \(k(< N)\)-quarks baryon and only the \(k=N\) baryon is allowed there. So the lower bound of \(k\) is dependent on the temperature. It is important to assure the details of this statement in a more realistic model with a finite \(T^{\text{dec}}\). This point is opened here.

The estimations of the mass of these states would be reported in the next work.

6 Summary and Discussion

The baryon configuration is studied based on the type IIB string theory by embedding D5-brane as a probe in the background corresponding to two kinds of large \(N\) Yang-Mills theories, the confining and the deconfining gauge theories. The D5 brane is needed as the baryon vertex of the quarks to make a color singlet, and the structure of the vertex is studied by solving the embedding equations for the fifth coordinate \((r)\) and a direction \((x)\) of our three dimensional space.

As for the \(x\) direction, two typical configurations, the point-like and the splitted vertex, are given. In the latter case, the quarks are separated into two clumps and a color flux tube is running between them. As found in other confining models, the energy of such a configuration is proportional to the separation between the two quark bundles. And its tension \((\tau_v)\), the energy per unit length, is given by a similar formula given before for the confining theory. Estimating \(\tau_v\), we find a natural dependence on
the color charges of the individual clumps and that this flux is interpreted as a bound state of a number of fundamental strings with a finite binding energy. The vertex energy is then given by the length of this flux times $\tau_v$.

As for the $r$-direction, we first show for the point vertex case. We find that the configurations are characterized by the relation of the position of the cusp(s) ($r_c$) and the extremum point of the D5 brane volume $r_0$, as (A) $r_c > r_0$ and (B) $r_c < r_0$. These configurations represent the same baryon at different energy (or mass) state. The configuration (A) corresponds to the low energy state and (B) does for high energy one. So the configuration of the fundamental strings in $r$-$x$ plane changes when the baryon energy increases. Considering of both configurations, the relation of the baryon energy ($E$), which is the sum of D5 brane and fundamental strings, and the distance ($L_{q-v}$) between a fundamental quark in the baryon and the vertex is examined. And we find a linear relation of $E$ and $L_{q-v}$ at large $L_{q-v}$. The tension in the case is given by the one of the meson times the quark number $N$ since the vertex energy is almost constant at large $L_{q-v}$. In obtaining this relation, the two vertex configurations (A) and (B) mentioned above appears. The configuration (A) is dominant at small $L_{q-v}$, where the linear relation of $E$ and $L_{q-v}$ is not still seen. At an appropriate $L_{q-v}$, the vertex configuration changes from (A) to (B) and the linear relation appears.

In the case of the split vertex, the configuration should be characterized in the $x$-$r$ plane as the U shaped and the cap ($\cap$) shaped. The U shaped one has two type (A) cusps, and the cap shaped one has two type (B) cusps. The problem to add the fundamental strings in this case could be solved by applying the results obtained in the point vertex case to each cusps of the split vertex. Then we would observe two kinds of the energy scale for the heavy baryon, $\tau_M$ and $\tau_v$, for the split vertex baryon.

As an example of deconfining Yang Mills theory, finite temperature theory is examined. At enough high temperature, the baryon collapses to quarks completely. Before arriving this temperature, we find that there is an intermediate temperature, where quarks are already not confined but they forms a "baryon" state, which is composed of $k$-quarks with $k < N$ and color singlet vertex. So this is a color non-singlet baryon. This baryon state would be made just after the phase transition of quark confinement and deconfinement. It is very interesting problem whether we could find this kind of baryon at high temperature.

As the next step, we should introduce the probe brane of the quarks and solve its embedding problem consistently with our baryon configurations obtained here. Another important problem is to quantize the baryon system to obtain their mass spectrum. Some approaches in this direction are seen [18, 19, 20].
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