Revealing the invariance of vectorial structured light in complex media

Isaac Nape<sup>1,6</sup>, Keshaan Singh<sup>1,6</sup>, Asher Klug<sup>1</sup>, Wagner Buono<sup>1</sup>, Carmelo Rosales-Guzman<sup>2,3</sup>, Amy McWilliam<sup>4</sup>, Sonja Franke-Arnold<sup>4</sup>, Ané Kritzinger<sup>1,5</sup>, Patricia Forbes<sup>5</sup>, Angela Dudley<sup>1</sup> and Andrew Forbes<sup>1,6</sup>

Optical aberrations place fundamental limits on the achievable resolution with focusing and imaging. In the context of structured light, optical imperfections and misalignments and perturbing media such as turbulent air, underwater and optical fibre distort the amplitude and phase of the light’s spatial pattern. Here we show that polarization inhomogeneity that defines vectorial structured light is immune to all such perturbations, provided they are unitary. As an example, we study the robustness of vector vortex beams propagating through highly aberrated systems, demonstrating that the inhomogeneous nature of polarization remains unaltered even as the medium itself changes. The unitary nature of the channel allows us to undo this change through a simple lossless operation. This approach paves the way to the versatile application of vectorial structured light, even through non-ideal optical systems, crucial in applications such as imaging and optical communication across noisy channels.

Non-paraxial light is vectorial in three dimensions and has given rise to exotic states of structured light such as optical skyrmions<sup>14,15</sup>, knotted strands of light<sup>16–18</sup>, flying donuts<sup>19</sup> and Möbius strips<sup>20</sup>. Paraxial light too is vectorial, in two dimensions, characterized by an inhomogeneous polarization structure across the transverse plane<sup>21</sup>. Vectorial structured light in two and three dimensions has been instrumental in a range of applications<sup>22–24</sup>, for example, to drive currents with a direction dictated by the vectorial nature of the optical field<sup>25–27</sup>, imprinting the spatial structure into matter<sup>28</sup>, enhanced metrological measurements<sup>29,30</sup>, probing single molecules<sup>31</sup> and to encode more information for larger bandwidths<sup>32–34</sup>. They are easy to create in the laboratory using simple glass cones<sup>35</sup>, stressed optics<sup>36</sup> and gradient-index lenses<sup>37</sup>, as well as from spatial light modulators and digital micromirror devices (DMDs)<sup>38,39</sup>, nonlinear crystals<sup>40,41</sup>, geometric phase elements<sup>42,43</sup>, metasurfaces<sup>44</sup> and directly from lasers<sup>45</sup>.

Given the importance of these structured light fields, much attention has been focused on their propagation through optical systems that are paraxial<sup>46–48</sup>, guided<sup>49</sup> and tight focusing<sup>50–52</sup>, as well as in perturbing media such as turbulent<sup>53–55</sup>, stressed optics<sup>56</sup> and gradient-index lenses<sup>57,58</sup>, as well as from spatial light modulators and digital micromirror devices (DMDs)<sup>38,39</sup>, nonlinear crystals<sup>40,41</sup>, geometric phase elements<sup>42,43</sup>, metasurfaces<sup>44</sup> and directly from lasers<sup>45</sup>.

Here we show that the inhomogeneity of a vectorial light field is impervious to complex channels as long as they are unitary (where an inverse process exists) and one-sided, a condition fulfilled for many aberrated optical systems. We illustrate this with two examples featuring strong aberrations: a tilted lens and atmospheric turbulence simulated on a spatial light modulator. We then demonstrate an inverse process exists) and one-sided, a condition fulfilled for complex channels as long as they are unitary (where an inverse process exists) and one-sided, a condition fulfilled for many aberrated optical systems. We illustrate this with two examples featuring strong aberrations: a tilted lens and atmospheric turbulence.

Results

Vectorial light and unitary channels. A vectorial structured light field can be compactly written in the quantum notation as the following unnormalized state:

\[ |\Psi\rangle = |e_1\rangle_A |u_1\rangle_B + |e_2\rangle_A |u_2\rangle_B, \]

highlighting the non-separable nature of the two degrees of freedom (DoFs), namely, \( A \) and \( B \), denoting the polarization and spatial DoFs, respectively. In this way, the vectorial field is treated as a quantum-like state (but not quantum and without non-local correlations), by virtue of its non-separable DoFs, akin to a locally entangled state\(^{44–48}\). The polarization DoF is expressed as any pair of orthonormal states, namely, \( |e_1\rangle \) and \( |e_2\rangle \), whereas the spatial DoF is given by the orthonormal basis states \( |u_1\rangle \) and \( |u_2\rangle \). In a quantum sense, this vectorial structured field would be called a pure state. The vectorial nature can be quantified through a measure of its non-separability\(^{59}\), a vector quality factor (VQF) (equivalently concurrence), which we will call its ‘vectoriness’ for brevity, ranging...
from 0 (homogeneous polarization structure of scalar light) to 1 (ideally inhomogeneous vectorial polarization structure).

Now imagine that our vectorial light field, $|\Psi_{\text{in}}\rangle$, passes through an arbitrary aberrated optical channel, for example, static imperfect optics or dynamic turbulent air. If the channel acts on only one DoF, then this can be considered as a one-sided channel (terminology borrowed from quantum communication theory), describing a situation where only one of the two entangled photons is affected. For our classical light, an example would be when the spatial mode (DoF $B$) is affected (distorted) whereas the polarization (DoF $A$) is not, or vice versa. In our analysis here, we will use DoF $A$ as unaffected, but this may be changed by a simple relabelling exercise. An open question is whether distortions acting only on the spatial DoF would keep the entire structure of the vectorial light field intact? It has been argued that the unaffected DoF acts as an anchor of sorts, keeping the entire field structure intact, and sometimes phrased as the ‘topological protection’ of the optical mode when discussing distortions that affect the spatial DoF but not the polarization DoF.

To answer this question, we employ a quantum framework (Supplementary Sections I–III provide the complete details). We show that a unitary one-sided channel—since it may be written as a positive trace-preserving map—transforms a vectorial (pure) input state $|\Psi_{\text{in}}\rangle$ into a vectorial (pure) output state $|\Psi_{\text{out}}\rangle$. Such a transformation may be expressed as

$$|\Psi_{\text{out}}\rangle = \left(\mathbb{I}_A \otimes T_B\right) |\Psi_{\text{in}}\rangle,$$

where $|\Psi_{\text{in}}\rangle$ is of the form of equation (1), identity operator $\mathbb{I}_A$ acts on polarization DoF $A$ and transmission matrix $T_B$ acts on spatial DoF $B$. Without loss of generality, the transmission matrix may be expressed as $T_B = \sum_j t_j |u_j\rangle \langle u_j|$ for a unitary channel, $t_j = t_j^*$. This allows us to write the output state as

$$|\Psi_{\text{out}}\rangle = |e_1\rangle |v_1\rangle + |e_2\rangle |v_2\rangle,$$

where $|v_1\rangle = T_B |u_1\rangle$ and $|v_2\rangle = T_B |u_2\rangle$. The fact that the channel is unitary, moreover, preserves the overlap between the spatial basis states $\langle u_1 | v_2\rangle = \langle u_1 | T_B^* T_B | u_2\rangle$ and hence the vectorness of the beams. Orthogonal input modes are transformed into another set of orthogonal output modes, and the vectorness remains invariant.

The answer to our question is then as follows: the output remains a non-separable vector beam with the same vectorness as the initial beam (Supplementary Information provides the full proof). Its spatial structure, however, including its amplitude, phase and polarization profile, is altered by the distorting medium, and our description of the vectorial structure changes from equation (1) to equation (3), expressed in an adjusted basis set $|v_j\rangle$. This is a direct manifestation of the Choi–Jamiołkowski isomorphism in quantum mechanics, which establishes a correspondence between a channel operator and a quantum state such that a measurement on one returns the other.

Applied to classical vectorial light (justified because of its non-separability), we find that the channel is fully described by its action on the vectorial beam; furthermore, measuring the beam fully characterizes the channel.

A property of a unitary channel is that the inverse process can be written as

$$|\Psi_{\text{in}}\rangle = \left(\mathbb{I}_A \otimes T_B\right)^{-1} |\Psi_{\text{out}}\rangle,$$

because the Hermitian adjoint of the channel operator is simply its inverse, that is, $\left(\mathbb{I}_A \otimes T_B\right)^{-1} = \left(\mathbb{I}_A \otimes T_B^*\right)$. In experimental terms, this means that the aberration introduced by transmission through the unitary channel can be undone, when sending the beam back through the channel. Equations (2) and (4) suggest that (1) the vectorial nature of the final field shares the same vectorness as the initial field; (2) the mapping results in a new adjusted basis that can be used for efficient characterization of the state; and (3) since the

Fig. 1 | Vectorial light through a tilted lens. a. The experiment has four stages: a generation stage to create the vectorial fields, a perturbation stage to pass it through a perturbing medium and two detection stages to perform Stokes projections and modal decomposition. The insets show the initial beam profiles and, with the laser at wavelength $\lambda = 633$ nm, the experiment has four stages: a generation stage to create the vectorial fields, a perturbation stage to pass it through a perturbing medium and two detection stages to perform Stokes projections and modal decomposition. The insets show the initial beam profiles and, with the laser at wavelength $\lambda = 633$ nm. All the beam profiles are shown as false-colour intensity and polarization maps. Methods provides the complete experimental details.

Supplementary Sections I–III provide the complete details. We show that an arbitrary aberrated optical channel, for example, static imperfect optics or dynamic turbulent air. If the channel acts on only one DoF, then this can be considered as a one-sided channel (terminology borrowed from quantum communication theory), describing a situation where only one of the two entangled photons is affected.

For our classical light, an example would be when the spatial mode (DoF $B$) is affected (distorted) whereas the polarization (DoF $A$) is not, or vice versa. In our analysis here, we will use DoF $A$ as unaffected, but this may be changed by a simple relabelling exercise. An open question is whether distortions acting only on the spatial DoF would keep the entire structure of the vectorial light field intact? It has been argued that the unaffected DoF acts as an anchor of sorts, keeping the entire field structure intact, and sometimes phrased as the ‘topological protection’ of the optical mode when discussing distortions that affect the spatial DoF but not the polarization DoF.

To answer this question, we employ a quantum framework (Supplementary Sections I–III provide the complete details). We show that a unitary one-sided channel—since it may be written as a positive trace-preserving map—transforms a vectorial (pure) input state $|\Psi_{\text{in}}\rangle$ into a vectorial (pure) output state $|\Psi_{\text{out}}\rangle$. Such a transformation may be expressed as

$$|\Psi_{\text{out}}\rangle = \left(\mathbb{I}_A \otimes T_B\right) |\Psi_{\text{in}}\rangle,$$

where $|\Psi_{\text{in}}\rangle$ is of the form of equation (1), identity operator $\mathbb{I}_A$ acts on polarization DoF $A$ and transmission matrix $T_B$ acts on spatial DoF $B$. Without loss of generality, the transmission matrix may be expressed as $T_B = \sum_j t_j |u_j\rangle \langle u_j|$ for a unitary channel, $t_j = t_j^*$. This allows us to write the output state as

$$|\Psi_{\text{out}}\rangle = |e_1\rangle |v_1\rangle + |e_2\rangle |v_2\rangle,$$

where $|v_1\rangle = T_B |u_1\rangle$ and $|v_2\rangle = T_B |u_2\rangle$. The fact that the channel is unitary, moreover, preserves the overlap between the spatial basis states $\langle u_1 | v_2\rangle = \langle u_1 | T_B^* T_B | u_2\rangle$ and hence the vectorness of the beams. Orthogonal input modes are transformed into another set of orthogonal output modes, and the vectorness remains invariant.

The answer to our question is then as follows: the output remains a non-separable vector beam with the same vectorness as the initial beam (Supplementary Information provides the full proof). Its spatial structure, however, including its amplitude, phase and polarization profile, is altered by the distorting medium, and our description of the vectorial structure changes from equation (1) to equation (3), expressed in an adjusted basis set $|v_j\rangle$. This is a direct manifestation of the Choi–Jamiołkowski isomorphism in quantum mechanics, which establishes a correspondence between a channel operator and a quantum state such that a measurement on one returns the other. Applied to classical vectorial light (justified because of its non-separability), we find that the channel is fully described by its action on the vectorial beam; furthermore, measuring the beam fully characterizes the channel.

A property of a unitary channel is that the inverse process can be written as

$$|\Psi_{\text{in}}\rangle = \left(\mathbb{I}_A \otimes T_B\right)^{-1} |\Psi_{\text{out}}\rangle,$$

because the Hermitian adjoint of the channel operator is simply its inverse, that is, $\left(\mathbb{I}_A \otimes T_B\right)^{-1} = \left(\mathbb{I}_A \otimes T_B^*\right)$. In experimental terms, this means that the aberration introduced by transmission through the unitary channel can be undone, when sending the beam back through the channel. Equations (2) and (4) suggest that (1) the vectorial nature of the final field shares the same vectorness as the initial field; (2) the mapping results in a new adjusted basis that can be used for efficient characterization of the state; and (3) since the
mapping is unitary, it can always be unravelled before or after the channel in a lossless manner. Note that in this analysis, we neither need to specify the details of the DoFs (what the modes or polarization might look like) nor the details of the channel itself (water, air and so on). We now showcase this generality through a series of exemplary test cases covering a wide range of input fields and channel conditions.

Experimental demonstration: tilted lens. To validate this perspective, we built the setup shown in Fig. 1a, first creating our test vectorial fields before passing them through some perturbing medium. Without any loss of generality, we chose the left- and right-circular basis, \( |\ell, \pm \rangle \equiv |R \rangle \) and \( |\ell, \mp \rangle \equiv |L \rangle \), for the polarization DoF and spatial modes imbued with orbital angular momentum (OAM) following \( |\ell \rangle \equiv |\ell \rangle \) and \( |\ell \rangle \equiv |\ell \rangle \), where \( \ell \) is the topological charge, forming the topical cylindrical vector vortex beams\(^{65,66}\). The initial cylindrical vector vortex beam was then analysed by both Stokes measurements and modal decomposition (Methods). The superimposed intensity and polarization profile of the initial beams (no perturbation) are shown in Fig. 1b, with both a radial mode of \( p = 0 \), with corresponding mode numbers \( N = 2p + |\ell| + 1 \) of 2 and 5, respectively.

Next, we pass these beams through a highly aberrated system—a tilted lens (Fig. 1b). This is known to severely distort the OAM modes\(^{65}\) and is routinely used as an OAM detector by breaking the beam into countable fringes\(^{66}\). The results at illustrative distances \( (Z_r - Z_o) \) after the lens are shown in Fig. 1c for typical tilt angles up to \( 15^\circ \) (adjusted to distort the beam). The superimposed intensity and polarization profiles reveal that although the vectorial structure distorts as one moves towards the far field, the inhomogeneity as measured by the vectorness does not, corroborated by the vectorness of each beam (Fig. 1c, insets), all remaining above 93%, that is, remaining fully vectorial as predicted by the unitary nature of the channel.

In contrast, we see that the intensity profiles change the morphology and concomitantly the polarization structure. This change can be explained by the coupling of modes outside the original subspace by virtue of the channel operator: the channel scatters the original OAM modes into new mode sets that maintain the same mode number as the input modes. We can visualize this by using the higher-order Poincaré (HOP) sphere, a geometric representation of vectorial structured light\(^{65,66}\). The case for \( \ell = \pm 4 \) is shown in Fig. 2 as an illustrative example. The initial cylindrical vector vortex beam is visualized as an equatorial vector of unit length (a pure state) (Fig. 2a). The channel (our tilted lens) maps this initial state to a new field (Fig. 2b) expressed across multiple HOP spheres (Fig. 2c).

All the new HOP spheres are made up of modal pairings that have non-zero modal powers (scattering probabilities). We quantify this by reporting the scattering probabilities and phases for every subspace (Fig. 2d,e). The initial modes \( (\ell, p) = (\pm 4, p) \) now only contain \( \sim 12\% \) of the total modal power, with new modal subspaces emerging to carry the rest. The HOP spheres are made of pairings of these modes, one on each pole, but not all contribute to the vectorness. To determine the contributing pairs, we measure the vectorness for every possible pairing (there are \( N(N - 1)/2 = 10 \) possibilities); the results are shown as a graph (Fig. 2f). In this graph, each vertex corresponds to a non-zero state (Fig. 2d) and the edges represent possible pairings to form an HOP sphere. The weight of each edge corresponds to the vectorness of that pairing. The initial subspace \( (\pm 4,0) \) is no longer a non-separable state, with a vectorness of 0, whereas some of the new subspaces (new HOP spheres) can be as high as 98%, that is, pure vectorial states. The graph can be rearranged with the zero-weighted edges removed to reveal a \( K_{11} \) bipartite graph structure with two independent vertex sets, namely,
The unitary channel (tilted lens; $1A \otimes TB$) maps the initial $\ell \pm 4$ vector field from the equator of its corresponding subspace onto a new HOP sphere spanned by an adjusted basis (shown as experimental images on the poles) where the vectorial structure is also a maximally non-separable pure state. a, b, Examples of $\ell = \pm 4$ (a) and $\ell = \pm 1$ (b). Because the unitary channel is a change of basis, the inverse $(1A \otimes TB)\dagger$ can be applied to map the field back to the original sphere. c, State $(1A \otimes TB)\dagger |\Psi_{in}\rangle$ is inserted as a pre-channel correction, resulting in the initial field $|\Psi_{in}\rangle$ as the output (shown for $\ell = \pm 4$). d, Post-channel correction is achieved on the output (shown for $\ell = \pm 1$), where the operation is simply a QWP, since $1A \otimes TB = TA \otimes 1B$ for this subspace.

**Role of measurement.** Given that the state vector after the channel lives on many HOP spheres in the original basis ($|u_\ell\rangle$ and $|u_\ell\rangle$) but only one HOP sphere in the adjusted basis ($|v_\ell\rangle$ and $|v_\ell\rangle$), it is pertinent to ask in which basis (or HOP sphere) should one make the vectorial measurement? In the quoted vectorness values thus far, we have circumvented this problem by using a Stokes measurement approach to extract the degree of non-separability, with the benefit of sampling in a basis-independent fashion (Methods). In contrast, many measurements of structured light are basis dependent, for example, in classical and quantum communication, where the basis elements form the communication alphabet. In Fig. 4a, we show the outcome of a basis-dependent vectorness measurement (Methods) in the original basis and in the adjusted basis, using the tilted lens as the channel. We see that for some symmetries, the choice of basis has no impact on the outcome, as in the case of $\ell = \pm 1$; however, for other vectorial fields, the impact is large ($\ell = \pm 4$). It should be noted that the $\ell = \pm 1$ beam through a tilted lens is a special case, because in modal space, the state vector has simply been rotated. In general, it is the adjusted basis that always reveals the invariance of the vectorness. The failure of the
The error bars are plotted as standard deviations from 50 instances of turbulence strengths (low, medium and strong). Each point on the spheres represents one instance from the experimental turbulence ensemble.

This result highlights the important role of measurement in determining the salient properties of vectorial light fields.

Figure 4b (spheres in the inset) shows the projections of the left (red) and right (blue) state vectors on the \( \ell = \pm 1 \) modal sphere\(^3\) for low, medium and strong turbulence strengths. Each instance of a turbulence strength is a red (blue) point (Fig. 4b), the scatter of which and deviation from the poles is indicative of modal crosstalk. This is a visual representation of why the vectorness decays when one considers only one sphere in the original basis: the original states are orthogonal (points on opposite poles with little scatter), but as the turbulence increases, they disperse across the sphere, resulting in superpositions of OAM, which become maximally mixed. For example, looking only at the \( \ell = \pm 1 \) subspace, the state may evolve as follows (ignoring normalization): \( | R \rangle | 1 \rangle \rightarrow | R \rangle | 1 \rangle + i| R \rangle | -1 \rangle \); similarly, \( | L \rangle | -1 \rangle \rightarrow | L \rangle | -1 \rangle - i| L \rangle | 1 \rangle \). The original vectorial state becomes \( | R \rangle | 1 \rangle + | L \rangle | -1 \rangle \rightarrow | L \rangle | -1 \rangle + i| L \rangle | 1 \rangle + | R \rangle | 1 \rangle + | R \rangle | -1 \rangle = (| R \rangle + i| L \rangle ) (| 1 \rangle + | -1 \rangle) \), which is a scalar, diagonally polarized, Hermite-Gaussian beam with a vectorness of 0. From this simple example, one can deduce that if only some modal spaces are considered in the beam analysis, then vectorial modes can reduce to scalar modes, but not vice versa.

In Fig. 5, we show typical crosstalk matrices with and without turbulence, where the input and output modes are both expressed in the original basis (Fig. 5a) and the adjusted basis (Fig. 5b). The crosstalk shown in Fig. 5b is deleterious for both classical and quantum communication. However, the unitary nature of the channel means that there is an adjusted basis, that is, \( | R \rangle | 1 \rangle, | R \rangle | 2 \rangle, | L \rangle | 1 \rangle \), and \( | L \rangle | 2 \rangle \), where the state vector is pure. Consequently, a post- or pre-channel unitary can undo the action of the channel, removing the crosstalk (Fig. 5c,d). The post-channel unitary is simply a measurement in the new adjusted basis, requiring nothing more than a change to the detection optics (holograms in our example) based on the channel under study. In Fig. 5d, the preparation optics are programmed to prepare the state in the adjusted basis, but measure it in the original basis, once again returning a crosstalk-free result. Although the action of the channel is to distort the initial beam (Fig. 5e), the channel action can be reversed (Fig. 5f), restoring the initial beam. This is a visualization of the low-crosstalk matrix (Fig. 5d), sending in the adjusted basis but measuring in the original basis. The scalar versions of the basis modes are shown in Fig. 5e,f (insets below each polarization profile), starting with the original modes that become perturbed due to turbulence. The adjusted basis, with their scalar versions (Fig. 5f, insets), maintains the orthogonality of the modes, and shows the key to the restoration of the initial field. When the adjusted basis is the input, the output is the corrected mode in the original basis. These results suggest that crosstalk-free communication is possible with a judiciously selected basis set for preparation or measurement, exploiting the fact that the vectorial state is intact in the adjusted basis. We use this fact in Fig. 5g to encode graphical information using our modal set, send it across the channel and decode it on the other side. Turbulence causes crosstalk, distorting the image, but this can be easily overcome by simply decoding (measuring) in the adjusted basis (Fig. 5g shows the results for medium and strong turbulence). A measurement in the adjusted basis reveals minimal modal decay, minimal crosstalk and high-fidelity information transfer across this noisy channel, with small deviations due to experimental imperfections.

**Demonstrating and exploiting invariance.** Finally, in Fig. 6, we show the results that highlight the power and versatility of this framework, and the importance of its implications (Supplementary Sections X–XII provide the complete experimental details). The central message is the invariance of the polarization inhomogeneity of vectorial light in a wide range of complex media. In Fig. 6a, we report the results on a 200 mm physical path through a heated channel where the induced temperature gradients in the air path measurement in the original basis is easily explained by the concatenation of the measurement subspace to just one of the many HOP spheres in which the state resides, as well as the consequent reduction in the state vector to a mixed and separable state because information is lost to other OAM subspaces.

**Simulated atmospheric turbulence.** To make clear that the tilted lens is not a special case, we alter the channel to atmospheric turbulence simulated on a spatial light modulator; this time, we measure the vectorness as a function of the turbulence strength (Fig. 6b). We find that a measurement in the original basis shows a decay in the vectorial nature of light as the turbulence strength increases, whereas the basis-independent approach reveals the unitary nature of turbulence: although the spatial structure is complex and altered, its vectorness remains intact and invariant to the turbulence strength. Here all the measurements were performed in the far field; therefore, the phase-only perturbations manifest as phase and amplitude effects. We clearly see the paradox: the vectorial structure can appear robust or not depending on how the measurement was done.
result in optical distortions (measured in the far field). Although the medium is dynamically changing, in principle, in some unknown manner, the vectorness remains intact—a fact tested across multiple input vectorial states (Supplementary Information) and shown for $\ell = \pm 10$ (Fig. 6a, middle). The data were collected at ~38 Hz (Fig. 6a, inset, shows a zoomed-in time window). Controlling the source temperature to alter the aberration strength does not alter the vectorness, as shown for $\ell = \pm 1$ and $\ell = \pm 5$ (Fig. 6a, right). We also show the results measured in the original modal basis: in this case, the invariance is lost, again confirming the important role of measurement in the context of vectorial light.

In Fig. 6b, we deliver a radially polarized vectorial beam through a setup for optical trapping and tweeze, and introduce aberrations into the path. We show that without any correction, the trap stiffness (a measure of how good the trap is) decreases by ten times; after applying a pre-channel unitary transformation, we can recover most (six times) of the original trap performance, as illustrated by the restricted motion of the particle in the trap. In our case, the adaptive optical routines were rudimentary (Methods and Supplementary Information), whereas state-of-the-art vectorial adaptive solutions promise a much enhanced performance and potential full recovery.

So far, all the channels have acted on the spatial mode and left the polarization as the unaffected DoF. In Fig. 6c, we show results after passing $\ell = \pm 1$ vectorial beams through a 1 m length of a few-mode optical fibre, stressed and bent to induce polarization coupling. The output beams were measured in both original basis and adjusted basis, with the crosstalk matrices shown for both (Fig. 6c, bottom). The results show high crosstalk when measured in the original basis (fidelity, 39%) and minimum crosstalk when measured in the adjusted basis (fidelity, 90%), confirming that there is a new HOP sphere where no correction is needed to recover the information carried by the state, which is important for optical communications.

Finally, in Fig. 6d, we show the results after passing vectorial beams through liquid media of 50 mm length, from water to the chiral media of d-limonene and sucrose. In chiral media, the polarization is the affected DoF, with a local polarization rotation of about 40° (Fig. 6d, left). Regardless of the medium and input beam type, the vectorness remains invariant. The graph (Fig. 6d, middle) shows the results for $\ell = \pm 2$ and $\ell = \pm 5$, with input VQFs of 0.5 and 1.0. Figure 6d, right, shows $\ell = \pm 20$ and $\ell = \pm 50$ output beam profiles from d-limonene, both with VQF above 0.99.

Discussion and conclusion
Our results show that vectorial structured light in complex media evolves from the near field to the far field, generally appearing spatially distorted in amplitude, phase and polarization structure,
although unaltered in vectorial inhomogeneity. This is explained by the unitary nature of such channels, mapping the state from an HOP sphere spanned by the original basis to a new HOP sphere spanned by an adjusted basis, as if only our perspective has altered. Any measurement in the original basis shows an apparent decay of the vectorness in strongly perturbing media even though the degree of polarization remains intact—a hidden invariance that can be observed through a judicious measurement. The role of measurement in quantum studies is well appreciated, and here, too, the vectorness of a classical beam can be found to be high and low, seemingly contradictory outcomes, yet both equally valid based on the choice of measurement. This is not only of fundamental importance but also of practical relevance: we have shown how to make a basis choice for preparation and/or measurement to negate modal crosstalk, with obvious benefits in classical and quantum communication across noisy channels, as well as in imaging through complex media.

The argument for robustness of vectorial light in channels where polarization is not directly affected is egregious: our quantum notation makes clear that the entire state is altered since its two DoFs are non-separable, in the same way that a true biphoton quantum state is altered if just one of the two photons is perturbed—both examples of one-sided channels. Our statement is corroborated here by theoretical examples and experimental proof, particularly illustrated by the example of operating on the ‘unaffected’ polarization DoF to correct the entire vectorial state.
Our analysis has considered unitary complex channels where one of the two DoFs is unaffected, covering a myriad of practical cases. These include channels with absolute losses (the same for each mode), which can be accounted for by a renormalization, whereas reduced amplitudes and adjusted phases due to modal or polarization crosstalk are fully accounted for in theory. We remark that not all the channels are one sided and unitary, for example, in strongly scattering biological systems.41 Although scattering can, in many cases, be reversible and thus handled by our theory, there are cases when this will not be true. Here it is the relative modal losses of the initial states, say $|L\rangle\langle\ell|$ and $|R\rangle\langle\ell|$, that break the reversibility, often due to detection that is either time averaged or inefficient: not all the light is collected, and the deficiency is mode dependent. Such cases may be handled by extending the applied quantum toolbox to cover a two-sided channel, as well as to introduce tools for mixed quantum states (for example, quantum state tomographies and purity measures) to cover the non-unitary channels. We also point out that the decay dynamics of the channel can be determined from a non-separable state irrespective of whether the channel is unitary41. Thus, although the invariance of the vectorness requires that the channel is unitary, pre- and post-channel correction do not, even though it may no longer be lossless. Our work has already revealed the benefit of a quantum framework applied to classical vectorial light and thus one can anticipate further benefits and insights as the scope is extended.

In conclusion, we have provided a general framework for understanding the impact of aberrations on vectorial light fields, thereby revealing the unitary nature of many complex media. Our work resolves a standing debate on the robustness of vectorial light in complex media, and will be invaluable to the exploding community working with vectorial structured light and its applications.

Online content
Any methods, additional references, Nature Research reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at https://doi.org/10.1038/s41566-022-01023-w.

Received: 31 August 2021; Accepted: 12 May 2022; Published online: 23 June 2022

References
1. Forbes, A., de Oliveira, M. & Dennis, M. R. Structured light. Nat. Photon. 15, 253–262 (2021).
2. Shen, Y., Hou, Y., Papasimakis, N. & Zheludev, N. I. Supertoroidal light pulses as electromagnetic skyrmions propagating in free space. Nat. Commun. 12, 5891 (2021).
3. Gao, S. et al. Paraxial skyrmionic beams. Phys. Rev. A 102, 053513 (2020).
4. Larocque, H. et al. Reconstructing the topology of optical polarization knots. Nat. Phys. 14, 1079–1082 (2018).
5. Galvez, E. J., Horne, B. L., Kumar, V. & Viswanathan, N. K. Generation of isolated asymmetric umbilics in light’s polarization. Phys. Rev. A 89, 031801 (2014).
6. Zdagkas, A. et al. Observation of toroidal pulsates of light. Preprint at https://arxiv.org/abs/2102.03636 (2021).
7. Keren-Zur, S., Tal, M., Fleischer, S., Mittleman, D. M. & Ellenbogen, T. Generation of spatiotemporally tailored terahertz wavepackets by nonlinear metasurfaces. Nat. Commun. 10, 1778 (2019).
8. Bauer, T. et al. Observation of optical polarization Möbius strips. Science 347, 964–966 (2015).
9. Brown, T. G. Unconventional polarization states: beam propagation, focusing, and imaging. Prog. Opt. 56, 81–129 (2011).
10. Wang, J., Castellucci, F. & Franke-Arnold, S. Vectorial light–matter interaction: exploring spatially structured complex light fields. AVS Quantum Sci. 2, 031702 (2020).
11. Otte, E., Alpmann, C. & Denz, C. Polarization singularity explosions in tailored light fields. Laser Photonics Rev. 12, 1700200 (2018).
12. Rosales-Guzmán, C., Ndagano, B. & Forbes, A. A review of complex vector light fields and their applications. J. Opt. 20, 123001 (2018).
13. Forbes, A. & Nape, I. Quantum mechanics with patterns of light: progress in high-dimensional and multidimensional entanglement with structured light. AVS Quantum Sci. 1, 011701 (2019).
14. Sederberg, S. et al. Vectorized optoelectronic control and metrology in a semiconductor. Nat. Photon. 14, 680–685 (2020).
15. Fang, Y. et al. Photoelectronic mapping of the spin–orbit interaction of intense light fields. Nat. Photon. 15, 115–120 (2021).
16. El Ketara, M., Kobayashi, H. & Brasselet, E. Sensitive vectorial optomechanical footprint of light in soft condensed matter. Nat. Photon. 15, 121–124 (2021).
17. Hawley, R. D., Cork, J., Radwell, N. & Franke-Arnold, S. Passive broadband full Stokes polarimeter using a Fresnel cone. Sci. Rep. 9, 2688 (2019).
18. Fang, L., Wan, Z., Forbes, A. & Wang, J. Vectorial Doppler metrology. Nat. Commun. 12, 4186 (2021).
19. Curcio, V., Alemán-Castañeda, L. A., Brown, T. G., Brasselet, S. & Alonso, M. A. Birefringent Fourier filtering for single molecule coordinate and height super-resolution imaging with dithering and orientation. Nat. Commun. 11, 5307 (2020).
20. Milione, G. et al. 4×20 Gbit/s mode division multiplexing over free space using vector modes and a q-plate mode (de)multiplexer. Opt. Lett. 40, 1980–1983 (2015).
21. Zhang, J. et al. Fiber vector eigenmode multiplexing based high capacity transmission over 5-km Fibre with Kramers-Kronig receiver. J. Lightw. Technol. 39, 4932–4938 (2021).
22. Zhu, Z. et al. Compensation-free high-dimensional free-space optical communication using turbulence-resistant vector beams. Nat. Commun. 12, 1666 (2021).
23. Zhao, Y. & Wang, J. High-speed vector beam encoding/decoding for visible-light communications. Opt. Lett. 40, 4843–4846 (2015).
24. Radwell, N., Hawley, R., Götte, J. & Franke-Arnold, S. Achromatic vector vortex beams from a glass cone. Nat. Commun. 7, 10564 (2016).
25. Beckley, A. M., Brown, T. G. & Alonso, M. A. Full Poincaré beams. Opt. Express 18, 10777–10785 (2010).
26. He, C. et al. Complex vectorial optics through gradient index lens cascades. Nat. Commun. 10, 4264 (2019).
27. Rosales-Guzmán, C. et al. Polarisation-insensitive generation of complex vector modes from a digital micromirror device. Sci. Rep. 10, 10434 (2020).
28. Chen, J. et al. Compact vectorial optical field generator based on a 10-megapixel resolution liquid crystal spatial light modulator. Opt. Commun. 495, 127112 (2021).
29. Wu, H.-J. et al. Vectorial nonlinear optics: type-II second-harmonic generation driven by spin-orbit-coupled fields. Phys. Rev. A 100, 053840 (2019).
30. Tang, Y. et al. Harmonic spin–orbit angular momentum cascade in nonlinear optical crystals. Nat. Photon. 14, 658–662 (2020).
31. Marrucci, L., Manzo, C. & Paparo, D. Optical spin-to-axial angular momentum conversion in inhomogeneous anisotropic media. Phys. Rev. Lett. 96, 163905 (2006).
32. Nassiri, M. G. & Brasselet, E. Multispectral management of the photon orbital angular momentum. Phys. Rev. Lett. 121, 213901 (2018).
33. Devlin, R. C., Ambrosio, A., Rubin, N. A., Mueller, J. B. & Capasso, F. Arbitrary spin–to-axial angular momentum conversion of light. Science 358, 896–901 (2017).
34. Forbes, A. Structured light from lasers. Laser Photonics Rev. 13, 1900140 (2019).
35. Beckley, A. M., Brown, T. G. & Alonso, M. A. Full Poincaré beams II: partial polarization. Opt. Express 20, 9357–9362 (2012).
36. Ma, Z. & Ramachandran, S. Propagation stability in focalized radially polarized vortex beams. Opt. Express 12, 384–393 (2004).
37. Youngworth, K. S. & Brown, T. G. Focusing of high numerical aperture cylindrical-vector beams. Opt. Express 7, 77–87 (2000).
38. Maman, S. et al. Transmission of classically entangled beams through mouse brain tissue. J. Biophotonics 11, e201800996 (2018).
39. Gianani, I. et al. Transmission of vector vortex beams in dispersive media. Adv. Photon. 2, 036003 (2020).
40. Biton, N., Kupferman, J. & Arnon, S. OAM light propagation through tissue. Sci. Rep. 11, 2407 (2021).
41. Suprano, A. et al. Propagation of structured light through tissue-mimicking phantoms. Opt. Express 28, 35427–35437 (2020).
42. Cox, M. A. et al. Structured light in turbulence. IEEE J. Sel. Topics Quantum Electron. 27, 1–21 (2020).
43. Gu, Y., Korotkova, O. & Gbur, G. Scintillation of nonuniformly polarized beams in atmospheric turbulence. Opt. Lett. 34, 2261–2263 (2009).
44. Cheng, W., Haas, J. W. & Zhan, Q. Propagation of vector vortex beams through a turbulent atmosphere. Opt. Express 17, 17829–17836 (2009).
46. Cai, Y., Lin, Q., Eyyuboğlu, H. T. & Baykal, Y. Average irradiance and polarization properties of a radially or azimuthally polarized beam in a turbulent atmosphere. Opt. Express 16, 7665–7673 (2008).

47. Ji-Xiong, P., Tao, W., Hui-Chuan, L. & Cheng-Liang, L. Propagation of cylindrical vector beams in a turbulent atmosphere. Chinese Phys. B 19, 089201 (2010).

48. Wang, T. & Pu, J. Propagation of non-uniformly polarized beams in a turbulent atmosphere. Opt. Commun. 281, 3617–3622 (2008).

49. Cox, M. A., Rosales-Guzmán, C., Lavery, M. P. J., Versfeld, D. J. & Forbes, A. Measuring the nonseparability of channel states. Nat. Photon. 13, 397–402 (2017).

50. Lochab, P., Senthilkumaran, P. & Khare, K. Designer vector beams containing orbital angular momentum. Phys. Rev. A 98, 023833 (2018).

51. McLaren, M., Konrad, T. & Forbes, A. Measuring the nonseparability of channel states. Sci. Rep. 6, 33306 (2016).

52. Spreeuw, R. J. C. A classical analogy of entanglement. Found. Phys. 28, 361–374 (1998).

53. Qian, X.-F. & Eberly, J. Entanglement and classical polarization states. Prog. Opt. 36, 281–364 (2008).

54. Kagalwala, K. H., Di Giuseppe, G., Abouraddy, A. F. & Saleh, B. E. J. Entanglement and classical polarization states. Phys. Rev. A 98, 023833 (2018).

55. Ren, Y. et al. Orbital angular momentum-based space division multiplexing for high-capacity underwater optical communications. Opt. Express 22, 093074 (2014).

56. Bouchard, F. et al. Quantum cryptography with twisted photons through a 30 meter flume tank using structured photons. New J. Phys. 22, 093074 (2020).

57. Hufnagel, F. et al. Investigation of underwater quantum channels in a turbulent atmosphere. Opt. Lett. 36, 4110–4112 (2011).

58. He, C., He, H., Chang, J., Chen, B., Ma, H. & Booth, M. J. Polarisation optics for biomedical and clinical applications: a review. Light Sci. Appl. 10, 194 (2021).

Publisher’s note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations. © The Author(s), under exclusive licence to Springer Nature Limited 2022, corrected publication 2022
Methods

Vector beam generation. To generate vector beams, a horizontally polarized Gaussian beam from a HeNe laser (wavelength $\lambda = 633$ nm) was expanded and collimated using a $x\times 10$ objective and a 250-mm-focal-length lens. The expanded beam was then passed through a half-wave plate before being separated into its horizontally and vertically polarized components using a Wollaston prism (WP). The plane at the WP was then imaged onto the screen of a DMD (DLPL6500) using a telescope system. The separation angle of the polarization components from the WP (−1°) resulted in the overlapping of the diffracted zeroth-order components. To create the desired scalar-component mode of the form $U(\tau) = |U(\tau)|\exp(\phi(\theta))$, where $\Phi$ is the phase of the field and $U$ has the maximum unit amplitude, the DMD was programmed with a hologram given by

$$H = \frac{1}{2} - \frac{1}{2} \text{sign} \left( \cos \left( \pi n + 2nG \right) - \cos \left( \pi A \right) \right),$$

(5)

where $G(\tau) = g(\tau) \cdot \mathbf{r}$ is a phase ramp function with grating frequencies $g(\tau) = (g_x, g_y)$, and $A(\tau) = \arcsin(\langle |U(\tau)| \rangle / \pi) / \pi$ and $\phi(\tau) = \Phi / \pi$ are the respective, appropriately enveloped, amplitude and phase of the desired complex fields, respectively, at pixel positions $x = (x, y)$. Holograms for each polarization component (denoted as $H_x$ and $H_y$) were multiplexed on the DMD, where the grating frequencies $(g_x, g_y)$ could be chosen to cause spatial overlap between the horizontally polarized first-order diffraction from $H_x$ and vertically polarized first-order diffraction from $H_y$. This combined first order contained our vector field, which was subsequently spatially filtered at the focal plane of a 4f imaging system and imaged onto a second DMD. The second DMD was addressed with a single hologram of the same form as $H_{\text{proj}}$ onto which the turbulence phase screens, along with any correlation filters, were encoded (a QWP was used to convert horizontal and vertical polarization to right-circular and left-circular polarization, respectively). Polarization projections were made using a linear polarizer and a QWP before the second DMD. The intensities at the Fourier plane were captured using a charge-coupled device (FLIR Grasshopper3) placed at the focal plane of a 2f imaging system.

Non-separability measurements. We measured the non-separability of the vector beams in a basis-dependent and basis-independent approach using Stokes parameters and modal decomposition, respectively (Supplementary Information). First, to measure the Stokes parameters, we used the reduced set of four Stokes intensities, namely, $I_x$, $I_y$, $I_z$, and $I_t$, corresponding to the linearly polarized horizontal (H), diagonal (D) and circular-right (R) and circular-left (L) polarizations. From these measurements, we extracted the Stokes parameters as

$$S_x' = I_x + I_y,$$

(6)

$$S_y' = 2I_x - S_x,$$

(7)

$$S_z' = 2I_y - S_x,$$

(8)

$$S_t' = I_x - I_y.$$

(9)

The four intensity projections were acquired through the use of a linear polarizer (for $I_x$ and $I_y$) rotated by $\pi/4$ radians together with a QWP (for $I_z$ and $I_t$), oriented at $\pm \pi/4$ radians relative to the fast axis. Subsequently, the basis-independent VQF (equivalently non-separability) was determined from $V = \sqrt{1 - (S_x'^2 + S_y'^2 + S_z'^2)/S_i^2}$, where $S_i = \int S_i(r) dr$ are the global Stokes parameters integrated over the transverse plane.

For the basis-dependent approach, the overlap between orthogonally polarized projections of the field in question was used as a measure of non-separability, with unity overlap signifying that the field is completely scalar whereas a zero overlap indicated a maximally non-separable vector field. This overlap can be characterized by the magnitude of the Bloch vector, $s$, lying on the surface of a sphere spanned by superpositions of a chosen pair of basis states $|\Psi_{\pm}\rangle$. The magnitude can then be seen as a sum in quadrature of the Pauli-matrix expectation values $\langle \sigma_3 \rangle$, as their operation on the basis states gives the unit vectors of the sphere. We can determine these expectation values using projections $|\mathbf{P}|$ into superpositions of the spatial basis components described by

$$|\mathbf{P}| = \alpha_f |\Psi_f\rangle + \beta_f |\Psi_l\rangle.$$

(10)

Here $(\alpha_f, \beta_f) = (1, 0), (0, 1), \{\cos((1, 1), \frac{1}{\sqrt{2}}(1, -1), \frac{1}{\sqrt{2}}(1, 1), \frac{1}{\sqrt{2}}(1, -1))$ for both $|R|$ and $|L|$. These 12 on-axis intensity projections are used to calculate the length of the Bloch vector according to

$$|\Psi_{\pm}\rangle = (I_1 + I_2) - (I_1 + I_3),$$

(11)

$$|\Psi_{\pm}\rangle = (I_1 + I_2) - (I_3 + I_2),$$

(12)

$$|\Psi_{\pm}\rangle = (I_1 + I_2) - (I_1 + I_3),$$

(13)

where the index $i$ of $I_i$ corresponds to the $(R, L)$ polarization projections and the index $j$ represents the spatial-mode projections defined above. The non-separability is then given by $V = \sqrt{1 - p^2}$. In this work, the projections into the right- and left-circular polarization components was achieved using a linear polarizer and QWP. The subsequent spatial-mode projections were performed using a correlation filter encoded into a DMD, and a Fourier lens to produce on-axis intensities $I_{\text{proj}}$.

Adjusted basis measurement. To determine the adjusted basis, the complex amplitude of an aberrated probe field $|\Psi_{\text{probe}}\rangle$ needs to be measured (Supplementary Information). This field was approximated using a maximum likelihood estimation procedure, where far-field intensities of the right- and left-polarized components of the ideal vector beam through turbulence were captured:

$$|\Psi_{\text{probe}}\rangle = |R(L), k|\Psi_k\rangle^2,$$

(14)

where $|\Psi_{\text{probe}}\rangle$ denotes the Fourier transform of $|\Psi_{\text{probe}}\rangle$ and $k = (k_x, k_y)$. Simulated Fourier intensities, $|\Psi_{\text{ideal}}\rangle$ of ideal beams modulated by a phase (modelled by possible weighted combinations of Zernike polynomials $Z_j$), namely,

$$|\Psi_{\pm}\rangle_{Zern} = LG \frac{1}{\pi} \exp \left( i \sum_j c_j Z_j \right),$$

(15)

were generated (spatial dependence has been omitted). The set of coefficients $c_j$ that lead to the lowest square difference in intensity between the experimental and simulated cases, namely,

$$\chi^2 = (\bar{I}_{\text{probe}} - \bar{I}_{\text{Zern}})^2 + (\bar{I}_{\text{probe}} - \bar{I}_{\text{Zern}})^2,$$

(16)

was used to determine the required basis for recovery of the initial beam.

Data availability

The code used to reproduce the results is available at https://doi.org/10.5281/zenodo.6502858.

Code availability

The code used to reproduce the results is available at https://doi.org/10.5281/zenodo.6502858.

Acknowledgements

A.F. thanks the NRF-CSIR Rental Pool Programme.

Author contributions

L.N., W.B., A. Klug, A.M., K.S., C.R.-G. and A. Kritsinger performed the experiments. All the authors contributed to the data analysis and writing of the manuscript. A.F. supervised the project.

Competing interests

The authors declare no competing interests.

Additional information

Supplementary information The online version contains supplementary material available at https://doi.org/10.1038/s41566-022-01023-w.

Correspondence and requests for materials should be addressed to Andrew Forbes.

Peer review information Nature Photonics thanks Shawn Sederberg and the other, anonymous, reviewer(s) for their contribution to the peer review of this work.

Reprints and permissions information is available at www.nature.com/reprints.