Local heat transfer estimation in microchannels during convective boiling under microgravity conditions: 3D inverse heat conduction problem using BEM techniques

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Abstract. Two-phase and boiling flow instabilities are complex, due to phase change and the existence of several interfaces. To fully understand the high heat transfer potential of boiling flows in microscale’s geometry, it is vital to quantify these transfers. To perform this task, an experimental device has been designed to observe flow patterns. Analysis is made up by using an inverse method which allows us to estimate the local heat transfers while boiling occurs inside a microchannel. In our configuration, the direct measurement would impair the accuracy of the searched heat transfer coefficient because thermocouples implanted on the surface minichannels would disturb the established flow. In this communication, we are solving a 3D IHCP which consists in estimating using experimental data measurements the surface temperature and the surface heat flux in a minichannel during convective boiling under several gravity levels (g, 1g, 1.8g). The considered IHCP is formulated as a mathematical optimization problem and solved using the boundary element method (BEM).

1. Introduction

Nowadays, a huge number of high-tech exchange devices use the phenomenon of heat transfer related to convective boiling inside microchannels. One of flow boiling characteristics is the high value of heat transfer coefficient which offers possibility of transferring huge heat fluxes. It means that minichannels cooling elements and heat exchangers are widely applied in industry where they enable huge heat flux density. Indeed, convective boiling is a very effective heat transfer mode and has been considered correlated with several parameters [1]. As a result, the physical phenomena which occur during the phase change needs to be well known in order to have better understanding. In fact, there is an increasing need to improve the performances of the equipment by miniaturizing them. This explains the very significant number of work and works published up to date. On convective boiling in minichannels [2], studies under investigation are limited. Recent number of papers have appeared on experimental investigations and theoretical analysis of flow boiling inside minichannels for various geometry scales. Exhaustive reviews by Kandlikar [3] and Tadrist [4] are providing a state of the art of many aspects of boiling heat...
transfer. However, there are not many studies dedicated to the analysis of biphasic flows with phase shift in microgravity. Nevertheless, to improve these studies, new experimental data are necessary to clarify these points.

In this paper, we will present a steady 3D IHCP which consists in the unknown boundary condition estimation using data measurements. Here, numerical procedure is used to obtain local heat transfer coefficient along a minichannel while convective boiling occurs. On Fig. 1, we can see that the thermocouple temperature we inverse present variations while we pass from one gravity level to another. Thus, prediction of the heat transfer coefficient requires an approach that accommodates the transition from a nucleate-pool-boiling-like condition at low vapor qualities to a nearly pure film evaporation condition at higher vapor qualities. Nethertheless, during some period the temperature measurements show that the heat conduction is "quasi" steady (Fig. 1 between 1700 and 1800), that is why here we only present the steady approach.

![Figure 1. Instabilities due to transition gravity level. We can see that there are some instabilities due to the gravity transition level](image)

This problem belongs to the class of IHCP [5] related to unknown boundary estimation. The main variable are the heat flux and the wall temperature applied on a surface which are not accessible to direct measurement. Our problem is ill-posed in the sense of Hadamard [6] and the stability condition is violated. Here we deal with a steady IHCP. We will present the steady formulation and particularly the linear heat source formulation. To perform this, we used the Boundary Element Method (BEM) which permits a direct connection between the measurements and the unknown boundary condition. This method is described in details in by Brebbia and al. [7] and applied by some authors for IHCP resolution [8]. Actually, we find in the literature some examples of 3D inverse problems using BEM - Martin and Dulikravitch [9], also FEM, conjugate gradient [10] and adjoint method. But none of them use experiments. Experimental examples are most of the time 2D problems - for example Abou khachfe and Jarny [11]. Here, the main difficulties for our inverse problem are its ill-posed problem character. As a consequence, the solution might become unstable considering measurements errors. In order to obtain a stable solution, we use some regularization procedure [12] [13]; here the SVD (Singular Value Decomposition). The objective is to acquire a better knowledge of the boundary conditions that influence the two-phase flow and the local heat transfer in the minichannel.

2. Experimental procedure

The experimental activities are performed in the frame of the MAP (Microgravity Application Program) Boiling project founded by ESA (European Space Agency) and embarked on Airbus A300-ZeroG (Zero gravity) to perform three Parabolic Flights (PF) campaigns (March 2007).
To study the influence of gravity level on the fluid flow and to take data measurements, an experimental setup is designed with two identical channels; one for the visualization and one for the acquisition of data (Fig. 2).

**Figure 2.** Coupling of the two rods during parabolic flights

The two devices enable us to study the influence of gravity on the temperature and pressure measurements. The two minichannels (Fig. 2) are modelled as a rectangular rod made up of three materials; a layer of polycarbonate ($\lambda = 0.2 \text{ W.m}^{-1}.\text{K}^{-1}$), a cement rod ($\lambda = 0.83 \text{ W.m}^{-1}.\text{K}^{-1}$) instrumented with 21 K-type thermocouples and in the middle a layer of inconel ($\lambda = 10.8 \text{ W.m}^{-1}.\text{K}^{-1}$) in which the minichannel is engraved (in Fig. 3). Inside the cement rods, 5 heating wires are providing a power of 11 W. The thermocouples sensors enable us to acquire the temperature in various locations (x, y and z) of the device. The heating wires are used to provide the power necessary to obtain a biphasic flow. Two pre-heaters are used to warm up the fluid to 2 °C below its saturation temperature. Thus it is possible to assume a constant saturated fluid at the inlet of the minichannels. Pressure and temperature measurements are acquired at 133 Hz to allow observation of non-steady flow. Flow visualization is performed using a Photron FastCam®. The experiments are conducted with HFE-7100, which presents many advantages: it is compatible with almost all materials, transparent, odorless, colorless, non-flammable, non-explosive and it has a low boiling temperature (61 °C at 1013.15 hPa compared with 100°C for water) and a low heat of vaporization (20 times less than water).

3. The Inverse solution Problem

In the case of steady IHCP, considering homogeneous domain $\Omega$ (Fig. 4), delimited by its boundary $\Gamma$, the linear steady state heat conduction equation is a Laplace equation which can be written:

$$\int_{\Omega} \Delta \theta T^*(M) d\Omega + \int_{\Omega} \frac{g}{\lambda} T^*(M) d\Omega = I + I_g$$

(1)

Where $\theta$ is the temperature, $g$ the heat source and $k$ the thermal conductivity. Let us consider the outer normal of the boundaries denoted by $n$. We choose a function $T^*$ which satisfies the fundamental heat transfer equation but without any consideration to the original boundary conditions imposed on $\Gamma$:

$$\Delta T^* = \delta(M, M_k) = \begin{cases} 0 & \text{if } M \neq M_k \\ \infty & \text{if } M = M_k \end{cases} \text{ and } \quad T^* = \left( \frac{1}{4\pi r} \right)$$

(2)
Function $T^*$ is known as the fundamental solution to the Laplace equation. The notation $\delta(M, M_k)$, $T^*(M, M_k)$ will be used now since the solution depends both on location of points $M_k$ and $M$. The difference between the fundamental solution $T^*$ and the Green’s function is that the former is any general solution to the differential operator equation with the Dirac delta forcing function, while the latter is a particular solution which satisfies a set of specified boundary conditions at the end points of the interval. Using the properties of function $T^*$ we can write the following statement:

$$\int_{\Omega} \theta \Delta T^* d\omega = -\theta_M \quad \text{which leads to} \quad I = \int_{\Gamma} \frac{\partial \theta}{\partial n} T^* d\gamma - \theta_M - \int_{\Gamma} \frac{\partial T^*}{\partial n} d\gamma$$  \hspace{1cm} (3)$$

which is known to be the fundamental Boundary Integral Equation (BIE), in this last equation $\theta_M$ is the temperature at the interior node $M$ situated at a distance $r$ from the boundary. If we consider the heat source part (Fig. 4), let us consider a set of $K$ line heat sources (here $K=5$) in domain $\Omega$ acting at line $L_k$ with the strength $g_k$, as a result we have in BEM:

$$I_g = \int_{\Omega} \frac{g}{\lambda} T^* d\omega \quad \text{with} \quad g = g(x, y, z) = \sum_{k=1}^{K} g_k f(L_k)$$ \hspace{1cm} (4)$$

We can explicit the final term where $L_k$ is the source length:

$$f(L_k) = H(M, L_k) = \begin{cases} 
0 & \text{if} \quad M \notin L_k \\
1 & \text{if} \quad M \in L_k
\end{cases}$$ \hspace{1cm} (5)$$

Here $g_k$ is the algebraic strength of the heat line source $k$ (W.m$^{-1}$) and $H(t)$ the heaviside function. As a result our domain $\Omega$ is subdivided into $J$ regions $\Omega_j$ ($\Omega = \bigcup_{j=1}^{J} \Omega_k$), in which the heat source term $g_k$ is supposed to be constant along line $L_k$. In this case the numerical integration can be written:

$$I_g = \int_{\Omega} \frac{g}{\lambda} T^* d\omega = \sum_{k=1}^{K} \frac{g_k}{\lambda} \int_{L_k} T^* dl$$ \hspace{1cm} (6)$$

Concerning the measurements, $N'=21$ is the number of domain (Fig. 5) interior points $\Omega$ provided here by the thermocouples and $N=235$ the number of boundary elements (triangular 3 nodes or quadrilaterals 4 nodes) son our rod, the system let appear $(N+N')=256$ equations.

Figure 4. Scheme of the line heat source scheme using BEM.

Figure 5. Coupe 2D of the problem of the unknown boundary conditions.
The number of unknowns, noted \((M=270)\), is a function of the boundary conditions applied on the different elements of \(\Gamma\). We deal with two domains one gathering the unknowns and one with the line heat source and internal points. Indeed, the polycarbonate presents an adiabatic boundary \(\Gamma_{30}\). Namely for each element \(\Gamma_i\), we have at least one unknown per element for the following boundary conditions; first kind of condition for which temperature \(\theta_i\) unknown and heat flux \(\varphi_i\) is imposed, second kind condition for which heat flux \(\varphi_i\) is unknown and temperature \(\theta_i\) is imposed, third kind condition \(\varphi_i=f(\theta_i)\).

Here, we consider the 3D domains \(\Omega_1\), \(\Omega_2\) and \(\Omega_3\). For \(\Omega_1\), \(\Omega_2\) and \(\Omega_3\) with boundary \(\partial \Omega_1=\Gamma_{10}\cup\Gamma_{12}\) and \(\partial \Omega_2=\Gamma_{20}\cup\Gamma_{c}\cup\Gamma_{23}\cup\Gamma_{21}\) and \(\partial \Omega_3=\Gamma_{30}\cup\Gamma_{32}\). In the minchannel (Fig. 5, where both \(\theta_i\) and \(\varphi_i\) are unknown (\(T_{\text{surface}}, \phi_{\text{surface}}\) here), we have an equation per element \(\Gamma_i\) with two unknowns:

| Flux | Condition | Boundary |
|------|-----------|----------|
| \(-\lambda \frac{\partial \theta(x)}{\partial n}\) | \(f(T_{\text{ext}})\) | \(x \in \Gamma_{10}\) |
| \(-\lambda \frac{\partial \theta(x)}{\partial n}\) | \(\varphi_{12}\) | \(x \in \Gamma_{12}\) |
| \(-\lambda \frac{\partial \theta(x)}{\partial n}\) | \(f(T_{\text{ext}})\) | \(x \in \Gamma_{20}\) |
| \(-\lambda \frac{\partial \theta(x)}{\partial n}\) | \(\phi_{\text{surface}}\) | \(x \in \Gamma_{c}\) |
| \(-\lambda \frac{\partial \theta(x)}{\partial n}\) | \(\varphi_{23}\) | \(x \in \Gamma_{23}\) |
| \(-\lambda \frac{\partial \theta(x)}{\partial n}\) | \(\phi_{21} = -\varphi_{12}\) | \(x \in \Gamma_{21}\) |
| \(-\lambda \frac{\partial \theta(x)}{\partial n}\) | 0 | \(x \in \Gamma_{30}\) |
| \(-\lambda \frac{\partial \theta(x)}{\partial n}\) | \(\phi_{32} = -\varphi_{23}\) | \(x \in \Gamma_{32}\) |

4. The Solution Method

Taking into account the BEM formulation, the discrete form of the \((N+N')\) BIE (Equ. 3) leads to a system of simultaneous equation:

\[
AX = B
\]

\(A\) is a matrix of dimension \((N+N';M)\), \(X\) the vector of the \(M\) unknowns with \((\phi_{\text{surface}}, T_{\text{surface}})\) and \(B\) a vector of dimension \((N+N')\) is containing a linear combination of the measurements. In this vector, we find the contribution of the heat sources. Our system presents 255 equations for 270 unknowns. Assuming that the difference between \(AX\) and \(B\) can be considered as Gaussian distributed, we can find \(\hat{X}\) as the solution of the Ordinary Least Squared problem:

\[
\hat{X} = \arg\left\{\min\left(||AX - B||^2\right)\right\}
\]

Thus, we observe for the system numerical resolution an instability of solution \(\hat{X}\) with regards to the measurements the errors introduced into the vector \(B\). As a result, we need to obtain a stable solution of this system by using regularization methods such as Tikhonov regularization method [13] or the truncated SVD solution- Hansen [14]. In order to smooth the solution, we use in our study the truncated SVD solution method and used by some authors - Martin and Dulikravitch [9]. Matrix \(A(N+N';M)\) can be approximated by \(\hat{A}\), a product of squares orthogonal matrices \((U\ and \ V)\) and \(W\), the diagonal matrix of the singular values \(\omega_j\):

\[
\hat{A} = UWV^T
\]
where estimated solution can be formulate:

$$
\hat{X} = \left( V \text{Diag} \left( \frac{1}{\omega_j} \right) U^T \right) B
$$

(10)

$A$ is ill-conditioned when some singular values $\omega_j \to 0 \ (1/\omega_j \to \infty)$. As a result, the measurements errors contained in vector $B$ are increased. In our case, we have fewer linear equations $M$ than unknowns $N$, so that we are not expecting a unique solution. There is an $N-M$ dimensional family of solutions. Thus, the SVD decomposition will yield $N-M$ zero or negligible $\omega_j$’s. Indeed, there are additional zero from any degeneracy in our $M$ equations. We must be sure that you find this many small $\omega_j$, and zero, which will give the particular solution vector.

The truncated matrix can be built up like $W_t$ and the estimate solution vector $\hat{X}_t$ is function of the new truncated matrix:

$$
\hat{X}_t = \left( V W_t^{-1} U^T \right) B
$$

(11)

$W_t^{-1}$ is a diagonal matrix of $N$ dimension with the $p < N$ non zero terms. We observe like in the regularization method by modifications of the functions to be minimised (for example Tikhonov) a smoothing of the solution and an increase of the residuals. The presence of these low singular values is a result of the ill-conditioned character of matrix $A$ results. They are a consequence of linear dependent equations: indication of a strong correlation between the unknown factors. The truncation level is determined by the technique known as the “L”-curve [14]. The optimal value is in the bend of the L where the best compromise between stable results and low residuals (on the distinct corner separating the vertical and the horizontal part of the curve). The numerical procedure can be found in the LAPACK or in Numerical recipes [15]. In our problem, the truncation number is around $10^7$.

As a result, the estimated vector $\hat{X}$ contains all the estimated surface temperatures ($T_{\text{surface}}$) and the surface heat flux ($\phi_{\text{surface}}$) as a function of the location $x$. Then, we calculate the local heat transfer coefficient in the minichannel knowing the liquid saturation temperature ($T_{\text{sat}}$):

$$
\hat{h} = \frac{\hat{\phi}_{\text{surface}}(x)}{T_{\text{surface}}(x) - T_{\text{sat}}}
$$

(12)

Indeed, the inverse heat condition problem is ill-posed and very sensitive to the measurements errors.

5. Sensitivity study

We test the influence of an error on the location of the thermocouples. Actually, the locations of the sensors are given using digital pictures which were taken before the cast the cement rod. During this process, the coordinates of the thermocouples may have changed so that it can influence the solution. From our study, we assume that there is no influence of the location with the $(x,y)$ location since the heat transfers are located on the $z$-axis. Then, we use the modified temperature distribution using the wrong locations and compare the result to the original one (in Fig. 6) using correct location. We can see that the profiles don’t fit so that our inverse problem is very sensible to the thermocouple locations. Indeed, the $z$-coordinate is a critical parameter since the different profiles present important variations from one to another. To solve this problem, we used a synchrotron at ESRF located in Grenoble (France) to perform a tomography at $20 \mu m$ of resolution of our cement rods. When we compare the real locations of the thermocouples and the heating wires with the one we have before we cast the cement rod (Fig. 7), we have $6\%$ error on the $z$-axis, $10\%$ on the $y$-axis and $4\%$ on the $x$-axis. Within these new locations, we are decreasing the error due to the locations of the sensors and this is very important particularly during the inversion. The minimization of the residuals is better.
Compared to the original profile, we obtain some temperature errors of 6 degrees which leads to an error of 13% on the estimated heat transfer coefficient.

The final system modeled is made up of two domains, the cement and the inconel. We made this choice because the polycarbonate had no influence on the studied zone (adiabatic condition). A variation in the different coefficients on the edge of the device does not change the output temperature profiles. Besides, concerning the heat conduction, we work with a cement rod to have a significant temperature gradient, necessary for the inversion and as we work in a steady state we didn’t take into account the accumulation term due to the heat capacity.

6. Inverse results

Fig. 8 shows the evolution of the wall temperature along the minichannel. First, the profile increases along the main axis for the three gravity levels. Secondly, under microgravity conditions, the temperature level is a little bit lower than in normal and hyper gravity. Thirdly, the wall temperature differences between the channel inlet and the channel outlet is 36 °C. In general observations, it seems that the curve presents 3 behaviors: we observe that the temperature rises first at the minichannel inlet, then there is a flat profile in the middle zone, which corresponds to the phase change process and the temperature increases in the outlet channel. On Fig. 9, the local heat transfer coefficient along the main axis of the channel is plotted for several microgravity conditions. Whatever the gravity level is, the heat transfer coefficient decreases with the $x$ location and remains constant along the channel. Besides, the values obtained in microgravity are higher whatever the channel abscissa. At the inlet, it reaches 16000 W.m$^{-2}$.K$^{-1}$ while in hypergravity and normal gravity it is around 8000 W.m$^{-2}$.K$^{-1}$. Indeed, the heat transfer coefficient are more important and generally, microgravity conditions lead to a larger bubble size which is accompanied by a deterioration in the heat transfer rate. For low vapor quality, gravity influence is not negligible and for larger than 30%, the observed influence of the gravity level is independent of the fluid velocity. During the microgravity period, in inlet of the minichannel ($x=20$ mm), the heat transfer coefficient is very high with a value around 16000 W.m$^{-2}$.K$^{-1}$ thought in hyper and normal gravity, it is equal to 8000 W.m$^{-2}$.K$^{-1}$. Another result shows the decrease along the $x$ length in the flow direction.
7. Conclusion
As pointed in the introduction, the studied phenomena are transient most of the time. Thus to deal with more experimental data from the PF63 campaign during microgravity, we need to use a transient formulation. The BEM formulation is very similar to the steady problem. In transient case, A is a matrix of dimension $((N+N)', M)$ multiplied by the number of the time step, which makes the regularization using SVD more complicated. Nevertheless some routines provides by SCALAPACK will be able to solve this problem using parallel computers.

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