Conductance fluctuations and weak localization in chaotic quantum dots

E. R. P. Alves and C. H. Lewenkopf
Instituto de Física, Universidade do Estado do Rio de Janeiro,
R. São Francisco Xavier, 524, 20559-900 Rio de Janeiro, Brazil
(March 22, 2002)

We study the conductance statistical features of ballistic electrons flowing through a chaotic quantum dot. We show how the temperature affects the universal conductance fluctuations by analyzing the influence of dephasing and thermal smearing. This leads us to two main findings. First, we show that the energy correlations in the transmission, which were overlooked so far, are important for calculating the variance and higher moments of the conductance. Second, we show that there is an ambiguity in the method of determination of the dephasing rate from the size of the of the weak localization. We find that the dephasing times obtained at low temperatures from quantum dots are underestimated.

A very striking experimental evidence of universal statistical behavior due to quantum coherence and complexity in electronic ballistic transport was recently reported by Huibers and collaborators. They measured the conductance $G$ through a chaotic quantum dot at small bias and low temperatures as a function of an applied magnetic field and the quantum dot shape. For such devices, where the quantum coherence length $\ell_\phi$ and the system size $L$ are such that $\ell_\phi \gg L$, the conductance is expected to parametrically display mesoscopic fluctuations.

To characterize the latter, due to the system complexity, a detailed microscopic theory is neither feasible nor practical. Hence, the indicated theoretical approach should be statistical and tailor made to give the experimental accessible statistical measures such as the conductance distribution $P(G)$, conductance autocorrelation functions, etc.. For ballistic chaotic quantum dots such approach is provided by the random matrix theory (RMT). Indeed, the agreement between the conductance distributions $P(G)$ obtained in Ref. and the corresponding stochastic theory, turned this experiment into a paradigm of the statistical approach.

Early experiments revealed an unexpected aspect to that systems, namely that even at low temperatures the conductance fluctuations significantly deviate from the predictions of the simplest random matrix models. More specifically, we are referring to the suppression of the weak localization peak, which represents the first quantum correction to the classical picture, and to the conductance variance $\text{var}(G)$. The early works were improved and converged to the understanding that even a small loss in quantum coherence affects dramatically the statistical observables. At the quantitative level, some features of the experimental data still remain unexplained.

The main findings presented in this letter are two-fold. First, using an alternative statistical approach we explain the discrepancy between theory and experiment in Ref. for $\text{var}(G)$. This result has important consequences for recent predictions of $\text{var}(G)$ in similar systems. Second, we show that there is an ambiguity in the way the dephasing rates are extracted from the weak localization experimental data in open chaotic quantum dots so far. Within our statistical model we propose a different method, which indicates that the dephasing rates quoted in the literature are overestimated.

The conductance $G = \langle e^2/h \rangle g$ through a two-lead quantum dot is related to the transmission, and hence to the $S$ matrix, by the Landauer formula

$$g(E, X) \equiv T_{21}(E, X) = \sum_{b \in \mathbb{B}} |S_{ba}(E, X)|^2.$$  \hspace{1cm} (1)

Here $g$ is the dimensionless conductance, $T_{21}$ is the transmission of an electron scattered from the incoming lead 1 to the outgoing lead 2, and the labels of the corresponding scattering matrix $S$ indicate the open channels located at each lead. $X$ is a generic parameter such as a gate voltage, which shapes the dot, or an external applied magnetic field $B$. The applicability of the Landauer formula assumes full quantum coherent transport.

Thermal effects modify Eq. (1) in different ways. First, and most interesting, by increasing the temperature the dynamics in the dot changes, making the coherent single-particle description of the process less realistic. The rich physics involved attracted a lot of attention and a lively debate lately. One way to include such dephasing processes in the theory is provided by the Büttiker phenomenological model. This approach is remarkably successful and its use became customary in the treatment of conductance fluctuation in chaotic dots. It introduces a fictitious voltage probe lead $\phi$, through which there is no net current flow, but allows for electrons to randomize their phases at the reservoir $\phi$. As a result the dimensionless conductance reads

$$g_\phi(E, X) = T_{21} + \frac{T_{2\phi} T_{\phi 1}}{T_{1\phi} + T_{2\phi}},$$  \hspace{1cm} (2)
where the arguments \( E \) and \( X \) are implicit to \( T \).

Temperature also affects the conductance in another (rather trivial) manner: the electrons flowing through a quantum dot are thermally distributed, yielding

\[
G(\mu, X) = \frac{e^2}{\hbar} \int dE \, g_\phi(E, X) \left( -\frac{\partial f_\mu}{\partial E} \right),
\]

where \( f_\mu \equiv \{1 + \exp[(E - \mu)/T]\}^{-1} \) is the Fermi distribution function and \( \mu \) is the chemical potential of the dot. It should be emphasized that in our notation \( g_\phi \) accounts solely for dephasing, while \( G \) is affected both by dephasing and the smearing of the Fermi surface.

Let us consider the simplest statistical measures of \( P(G) \), namely the mean conductance \( \langle G \rangle \) and its variance \( \text{var}(G) \). In experiments \( \langle G \rangle \) is obtained by varying \( \mu \) and/or \( X \), whereas in theory one takes a suitable ensemble averaging over \( g_\phi \). Actually, from Eq. \( \text{(3)} \) it is evident that \( \langle G \rangle = \langle e^2/h \rangle \langle g_\phi \rangle \). The inspection of the conductance autocorrelation function

\[
C(x) = \langle G(\mu, X^+)G(\mu, X^-) \rangle - \langle G(\mu, X) \rangle^2
\]

where \( X^\pm = X \pm x/2 \), reveals that the relation between \( \text{var}(G) = C(0) \) and \( \text{var}(g_\phi) \) is obtained from [19]

\[
C(x) = \left( \frac{e^2}{\hbar} \right)^2 \int_{-\infty}^{\infty} d\omega \, c_\phi(\omega, x) \left[ \frac{d}{dT} \sinh \left( \frac{\omega}{2T} \right) \right]^2
\]

(5)

where \( c_\phi(\omega, x) \) is the dimensionless conductance autocorrelation function defined by

\[
c_\phi(\omega, x) = \langle g_\phi(E^+, X^+)g_\phi(E^-, X^-) \rangle - \langle g_\phi \rangle^2
\]

with \( E^\pm = E \pm \omega/2 \).

So far all theoretical studies [3,10,18] aiming to describe \( P(G) \) used the information theoretical approach. More specifically, one finds the ensemble of \( S \) matrices that maximizes the information entropy and fulfills the symmetries and other constraints of the physical system under consideration. This procedure is very amenable for the analytical calculation of \( P(g_\phi) \) (at fixed \( E \) and \( X \)) but has the limitation of lacking parametric correlations (neither for \( E \) nor for \( X \)) between members of the ensemble. This ingredient is of central importance in obtaining the variance and higher moments of \( G \) as indicated by Eq. \( \text{(3)} \). To circumvent this problem Ref. [4] introduced an heuristic procedure of smearing \( P(G) \), which underestimates \( \text{var}(G)_{\beta=2} / \text{var}(G)_{\beta=1} \).

We use the Hamiltonian approach to the statistical \( S \)-matrix instead [24]. Both frameworks are equivalent for the calculation of \( \text{var}(g_\phi) \) [25], but not for \( \text{var}(G) \). The resonant \( S \)-matrix is given by

\[
S(E, X) = \mathbb{1} - 2\pi iW^\dagger \frac{1}{E - H(X) + i\pi W W^\dagger} W
\]

(7)

where \( H \) is taken as a member of the Gaussian orthogonal (unitary) ensemble for systems where time-reversal symmetry is manifest (absent). For simplicity we take the case of \( N \) open channels at each lead. Since the \( H \) matrix, of dimension \( M \), is statistically invariant under orthogonal (\( \beta = 1 \)) or unitary (\( \beta = 2 \)) transformations, the statistical properties of \( S \) depend only on the mean resonance spacing \( \Delta \) determined by \( H \) and on \( W \), the coupling matrix elements between resonances and channels. Those enter the theory through \( y_c = \pi^2(W^\dagger W)_c/\langle M \Delta \rangle \) contained in the so called sticking coefficients \( P_c = 4y_c/(1+y_c)^2 \). The later quantify the transmission through a given channels \( c \), being maximal for \( P_c = 1 \). By assuming the channels to be equivalent we can drop the index \( c \). For open quantum dots the transmission is large and consequently it is assumed that \( P \approx 1 \). In addition, we consider \( N_c \) open channels at the voltage probe lead, each of them with a sticking coefficient \( p \). The loss of phase coherence is modeled by the single parameter \( P_\phi = pN_c \), with \( N_c \gg 1 \) [19]. The later can be converted in a dephasing width \( \Gamma_\phi = \Delta P_\phi/2\pi \), from which the dephasing time \( \tau_\phi = \hbar/\Gamma_\phi \) is extracted.

In this approach the parametric correlations are automatically taken into account, but due to technical reasons it is very difficult to proceed analytically, unless \( N \gg 1 \) and \( \beta = 2 \) [9]. On the other hand, numerical simulations can be implemented straightforwardly. For each realization of \( H \) we invert the propagator and compute \( S \) for values close to the center of the band, \( E = 0 \), where the \( \Delta \) is approximately constant. The dimension of \( H \) was fixed to be \( M = 200 \), taken as a compromise between having a reasonable wide energy window to work with and not slowing too much the computation. For each case under consideration we obtained good statistics for \( P(g_\phi) \) and \( c_\phi(\omega, x) \) with \( 10^4 \) realizations.

We find that for the case of experimental interest, \( N = 1 \) and \( P_\phi \neq 0 \), the numerically computed dimensionless autocorrelation function \( c_\phi(\omega, x) \) is quite similar to the one obtained in the semiclassical regime (\( N \gg 1 \)) and \( P_\phi = 0 \) [19,22,23], namely

\[
c_\phi(\omega, x) \approx \frac{\text{var}(g_\phi)}{[1 + (x/X_c)^2] + (\omega/\Gamma)^2}.
\]

(8)

Our results, shown for the \( \beta = 1 \) case in Fig. 1 (\( \beta = 2 \) gives essentially the same agreement), scale according to

\[
\Gamma = \Gamma_0 + \frac{\Gamma_\phi}{\beta} = \Delta \left( \frac{2NP + P_\phi}{\beta} \right)
\]

(9)

and \( X_c = \kappa \sqrt{2NP + P_\phi} \), where \( \kappa \) is system specific and depends on the kind of parametric variation. It is worth mentioning that for \( P_\phi = 0 \) there is additional work [24] showing that Eq. \( \text{(8)} \) is a good approximation for any \( N \). For simplicity we take \( P = 1 \) for the moment.
where \( \bar{x} = x/X_c \). For \( C(0) = \text{var}(G) \), the above expression is nicely approximated (within 15%) by

\[
\text{var}(G) = \left( \frac{e^2}{h} \right)^2 \frac{\text{var}(g_\phi)}{1 + 2T/\Gamma} \quad \text{(11)}
\]

as shown in Fig. 3. In Eq. (11) the dependence of \( \text{var}(G) \) on \( \beta \) is implicit in both \( \text{var}(g_\phi) \) and \( \Gamma_\phi \). Figure 3 shows that these considerations reconcile the theory with the experimental data. The information theoretical approach underestimates the ratio \( \text{var}(G)_{\beta=2}/\text{var}(G)_{\beta=1} \), because it lacks the temperature correction given by \( (1 + 2T/\Gamma_{\beta=2})/(1 + 2T/\Gamma_{\beta=1}) \). Notice that we do not introduce any additional fitting parameter in our theory.

We now switch our attention to the question of how to determine the dephasing rate in open chaotic quantum dots. There are three main proposed strategies [25,11]. Let us start addressing the one based on the weak localization peak. As shown \( \langle G \rangle = (e^2/h) \langle g_\phi \rangle \), allows one to read the average dimensionless conductance directly from the empirical data. In turn, provided that the leads are ideal (\( P = 1 \)), the weak localization peak, defined as

\[
\delta g = \langle g_\phi \rangle_{\beta=2} - \langle g_\phi \rangle_{\beta=1} \quad \text{(12)}
\]

is in direct relation to \( \Gamma_\phi \). The problem is that in actual experiments \( P < 1 \). Thus, \( \delta g \) is a function not only of \( \Gamma_\phi \) but of \( P \) as well. An inspection of Fig. 3 obtained from our simulations for \( N = 1 \), shows that by fixing \( \delta g \) (as obtained from the experiment) and decreasing \( P \) by a small factor always increases \( P_\phi \). The effect becomes small for \( P_\phi \gg 1 \), but is rather large for \( P_\phi \lesssim 1 \). In this situation, reducing \( P \) by 5% decreases \( P_\phi \) by as much as 100%. The dephasing time \( \tau_\phi \) increases accordingly.
Hence, $\delta g$ does not uniquely fixes $P_\phi$. This ambiguity can be eliminated by using the experimental $\langle g_\phi \rangle$ for $\beta = 1$ and 2 to fix both coefficients $P$ and $P_\phi$. The data from Ref. [10] indicate that the correction to $P_\phi$ is significant.

There are two other methods to extract $\tau_\phi$ from experiments dealing with chaotic quantum dots. Both are related and rely on the study of the parametric dependence of the conductance. Based on a semiclassical argument, it was proposed that the study of the conductance autocorrelation as a function of an external magnetic field, $C(\mathbf{x}) = B_\mathbf{e}/B_c$, has a simple dependence on $\Gamma_\phi$, namely $\langle B_\mathbf{e}/\phi_0 \rangle^2 = \kappa(2NP + P_\phi)$. (Our numerical results support this relation, as depicted in Fig. [1].) By measuring $B_\mathbf{e}(T)$ one can thus obtain $\tau_\phi(T)$. The problem here is that $C(\mathbf{x})$ changes its functional dependence when going from the $T \gg \Delta$ to the $T \ll \Delta$ limit [19], which can jeopardize the determination of $B_\mathbf{e}$ for $T \approx \Delta$. Alternatively, the width of the Lorentzial dip of the average conductance around $B = 0$, can also be used as $\langle g_\phi(B) \rangle = \langle g_\phi \rangle_{\beta=2} - \delta g/[1 + (B/B_c)^2]$. Both methods were recently shown to give consistent results with the weak localization one, at least for $T \geq \Delta$ [1]. This is in apparent contradiction with our claim that there is an ambiguity in the weak localization peak method. However, differences are only expected for small values of $P_\phi$, where the parametric methods were not employed. Moreover, since $N = 1$ and $P \approx 1$ give $B_\mathbf{e} \propto \sqrt{2 + P_\phi}$, the latter become evidently inaccurate for $P_\phi \ll 1$.

In conclusion, we presented a detailed statistical study of conductance fluctuations in chaotic quantum dots. We solved the only serious discrepancy between theory and experiment, giving a stronger support to the statistical approach incorporating dephasing. In addition, we pointed out some problems in the quantitative assertion of $\tau_\phi$ from the data, and propose an alternative solution.

E. R. P. Alves acknowledges financial support by CAPES. This work was partially funded by CNPq and PRONEX-Brazil.

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