Impact of Electro-Magnetohydrodynamic Nature on Dispersion of solute in the Peristaltic Mechanism

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Abstract. The present article deals with the dispersion of a solute in peristaltic system. This mathematical model provides the analytical solution of electro-magnetohydrodynamic flow of biofluids through a channel. Treating blood as a Jeffrey fluid and considering homogeneous chemical reaction, the closed form of solution of effective dispersion coefficient is obtained by using Taylor’s limiting condition and long wave length approximation. The impacts of physiological parameters such as the electro osmotic velocity, electro kinetic, Jeffrey fluid parameter and Hartmann number on the Peristaltic movement have been investigated. It is found that the average equivalent dispersion coefficient is reduced with respect to the enhancement of homogeneous chemical reaction parameter and Hartmann number. It is pertinent to point out here that the electro osmotic velocity and electro kinetic parameter help to enhance average equivalent dispersion coefficient in comparison with the absence of applied electric field. The present analytical study provides useful information to artificial bio-processors.

1. Introduction

Peristaltic mechanism is a form of material transport due to progressive wave of area contraction or expansion along the length on the boundary of a distensible tube containing fluid. Peristalsis is used by a living body to propel or to mix the contents of the tube such as, in transport of urine from the kidney through the ureters to the bladder, food through the digestive tract, movement of ovum in the fallopian tube etc. The peristalsis system is also used in the application of roller pumps or finger pumps holding the fluid in a flexible tube and is compressed by a series of mechanical fingers by rotating rollers. If the transferred fluid is intended to prevent interaction with the mechanical parts of the pump, these devices are used to pump fluids. Peristaltic movement is used in the nuclear industry to carry the noxious gas. The biomedical applications of this mechanism are heart-lung machine and dialysis machine.

The first attempt with an experimental investigation to study the fluid dynamics aspect of peristaltic mechanism is made by Latham [1]. At the same time Burns and Parkes [2] gave a perturbation solution to the mathematical model of peristaltic transport. Several investigators have analyzed the peristaltic flows of Newtonian or non-Newtonian fluids with different geometrical aspects and assumptions. Gupta and Seshadri [3] have studied peristaltic pumping in the non-uniform tubes and channels and found that the pressure rise over the tube length at any specified flow rate and amplitude ratio is very less in the non-uniform geometry. The significance of a peripheral layer of various viscosity on Peristaltic pumping with Newtonian fluids is investigated by James et al. [4]. Mishra and Ramachandra [5] analyzed the peristaltic transport of a Newtonian fluid in an asymmetric channel and found that the
trapping and reflux layer exist only when cross-section of the channel varies. The simultaneous effect of the magnetic and electrical fields on the peristaltic motion is not examined in the aforementioned mathematical models.

The transport of fluid flow behavior changed by the joined effects of external applied electric and magnetic field is called as electro-magneto hydrodynamics. The concepts of magnetohydrodynamics play a vital role in peristalsis, especially in medical science such as magneto recovery, arterial blood flow, optimization of blood pumping devices, treatment of cancer tumors, and reduction of blood bleeding during surgery. Sud et al [6] studied the influence of a moving magnetic field on blood flow and found that the impact of suitable moving magnetic field accelerates blood velocity. Hayat et al. [7] have investigated the homogeneous-heterogeneous reactions and heat source/sink effects in MHD peristaltic flow of micropolar fluid with Newtonian heating in a curved channel. The effect of magnetic field on peristaltic transport of Nano Eyring-Powell fluid in an asymmetric channel is theoretically investigated by Akbar [8]. Rachid and Quazzani [9] investigated the MHD peristaltic mechanism in the deformable coaxial tubes by using Jeffrey fluid. By considering variable viscosity, Abbasi et al. [10] developed a mathematical model for peristaltic transport of MHD fluid. Kothandapani et al. [11] considered the fluid to be fourth-order and conducting electrically through a transverse magnetic field and observed that the channel asymmetry was created by selecting the peristaltic wave train on the non-uniform walls to have different amplitudes and phases.

A study of electro-kinetics properties is being performed in the recent scenario and leads to a major branch of modern fluid dynamics [12]. It has a broad range of medical uses including the manufacture and treatment of microfluidics, body fluid flows, chromatography, plasma separation, and colloidal suspension. Using applied electrical fields, Bandopadhyay et al. [13] studied the peristaltic motion of aqueous electrolytes. Tripathi et al. [14] obtained mathematical solutions to the problem of peristaltic flow of Newtonian fluid in a channel under the operation of both magnetic and electrical fields based on the assumptions of low Reynolds number and long wavelength. The effect of electrode on peristaltic electro-kinetic pumping was investigated by McKnight et al. [15]. Goswami et al. [16] examined, through a narrow containment in the form of a deformable tube, the electrokinetically modulated peristaltic transport of power-law fluids. Taking into account the influence of fluid viscosities, power-law index and electroosmosis, the pumping characteristics and the trapping of fluid bolus are studied. In these models, the combined influences of magnetic and electric fields on dispersion of a solute in peristaltic flow of non-Newtonian fluids in a channel by taking into account homogeneous chemical reactions are not investigated.

The dispersion of a soluble matter in fluids has wide application in chemical and biomedical fields especially in the study of blood circulatory system. The study of axial dispersion was initiated by Taylor [17]. He has observed that the solute diffused with an effective dispersion coefficient, relative to a plane moving with the average flow speed, which depends upon the radius of the tube, mean velocity and molecular diffusion coefficient. Following this, Taylor’s results extended by Aris [18] using method of moments with including the impact of axial molecular diffusion.

Several researchers investigated the dispersion of a solute in a viscous fluid under different flow conditions (Fan and Hwang [19], Ghoshal [20], and Shah and Cox [21]). All the investigations mentioned above deal with flows where the solute does not chemically react with the liquid through which it is dispersed. Katz [22], Soloman et al. [23], Gill et al. [24], Gupta et al. [25] and Scherer et al. [26] are some of the workers who have studied Newtonian fluid dispersion by taking into account various aspects of homogeneous and heterogeneous chemical reactions such as steady and unsteady conditions. The impact of homogeneous chemical reaction on non-Newtonian fluids such as Power-law, Bingham and Casson models have been analyzed by Shukla et al. [27]. Philip and Chandra [28] addressed the influence of homogeneous and heterogeneous reactions on a microfluid dispersion of a solvent. Ravi Kiran and Radhakrishnamacharya [29] have investigated the dispersion of a solute in the peristaltic motion of a Jeffrey fluid through a porous medium with slip condition in the presence of both homogeneous and heterogeneous chemical reactions. It is seen from the mathematical models stated above that the simultaneous consequences of magnetic field, electric field, non-Newtonian rheology,
homogeneous chemical reaction on dispersion of a solute in a peristaltic motion through a channel have no longer been investigated.

Among several non-Newtonian models proposed for physiological fluids, Jeffrey model is important because it is possible to deduce Newtonian fluid model as a special case by taking the retardation time ratio is zero. Further, it is speculated that the physiological fluids such as blood exhibit non-Newtonian behaviors during circulation in a living body. This fluid model includes elastic and memory effects exhibited by dilute polymer solutions and biological liquids. The present study analyzes the peristaltic mechanism of Jeffrey fluid in a uniform channel and discussed analytical solutions for the effects of homogeneous chemical reactions and Electro-Magneto-Hydrodynamic on the dispersion of a solute in a Jeffrey fluid.

2. Mathematical formulation of the problem

We consider the peristaltic flow of an incompressible Jeffrey fluid under the effects of electro-magneto hydrodynamics in an infinite uniform channel of width \(2d^*\). The walls of the channel are not rigid, on which are imposed by traveling sinusoidal waves of small amplitude. The Cartesian coordinate system \((x^*, y^*)\) is chosen with the \(x^*\)-axis in the direction of wave propagation and \(y^*\) axis in the direction normal to the mean position of the walls as shown Figure 1.

![Figure 1. Geometry of the problem](image)

The flow is generated within the channel by sinusoidal waves as

\[
y^* = \pm h^* = \pm \left( d_1^* + a_1^* \sin \frac{2\pi}{\lambda^*} (x^* - c^* t^*) \right)
\]

where \(d_1^*\) is the mean width of the channel, \(a_1^*\) is the amplitude of the waves, \(\lambda^*\) is the wavelength, \(c^*\) is the velocity of the wave propagation and \(t^*\) is the time.

The governing equations for the flow of an incompressible Jeffrey fluid are stated as

\[
\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0
\]
\[
\rho^* u_1^* \frac{\partial u_1^*}{\partial x^*} + v_1^* \frac{\partial v_1^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{\partial \tau_{xx}^*}{\partial x^*} + \frac{\partial \tau_{xy}^*}{\partial y^*} - \sigma^* B_0^* u_1^* + \rho e^* E_x^*
\]  
(3)

\[
\rho^* v_1^* \left( u_1^* \frac{\partial v_1^*}{\partial y^*} + v_1^* \frac{\partial u_1^*}{\partial x^*} \right) = -\frac{\partial p^*}{\partial y^*} + \frac{\partial \tau_{yy}^*}{\partial y^*} + \frac{\partial \tau_{xy}^*}{\partial x^*}
\]  
(4)

where

\[
\tau_{xx}^* = \left( \frac{2\mu^*}{1 + \lambda_1^*} \right) \left[ 1 + \lambda_2^* \left( v_1^* \frac{\partial}{\partial y^*} + u_1^* \frac{\partial}{\partial x^*} \right) \right] \frac{\partial u_1^*}{\partial x^*}
\]  
(5)

\[
\tau_{yy}^* = \left( \frac{2\mu^*}{1 + \lambda_1^*} \right) \left[ 1 + \lambda_2^* \left( v_1^* \frac{\partial}{\partial y^*} + u_1^* \frac{\partial}{\partial x^*} \right) \right] \frac{\partial v_1^*}{\partial y^*}
\]  
(6)

\[
\tau_{xy}^* = \left( \frac{\mu^*}{1 + \lambda_1^*} \right) \left[ 1 + \lambda_2^* \left( v_1^* \frac{\partial}{\partial y^*} + u_1^* \frac{\partial}{\partial x^*} \right) \right] \left( \frac{\partial u_1^*}{\partial y^*} + \frac{\partial v_1^*}{\partial x^*} \right)
\]  
(7)

where \( u_1^* (x^*, y^*, t^*) \) and \( v_1^* (x^*, y^*, t^*) \) are the velocity components in the \( x^* \) and \( y^* \) directions respectively, \( p^* \) is the pressure, \( \rho^* \) is the density, \( \sigma^* \) is the electrical conductivity, \( \lambda_1^* \) is the ratio of the relaxation to retardation times, \( \lambda_2^* \) is the delay time, \( \mu^* \) is the viscosity of the fluid, \( B_0^* \) is the applied uniform magnetic field, \( \rho e^* \) is the net charged density, \( E_x^* \) is the component of external electric field in the axial direction, \( \tau_{xx}^*, \tau_{yy}^* \) and \( \tau_{xy}^* \) are the components of the stress tensor.

The boundary conditions are

\[ u_1^* = 0 \quad \text{at} \quad y^* = \pm h^* \]  
(8)

Based on the theory of electrostatic, the potential function \( \phi_1^* (x^*, y^*) \) and the free electric charge density \( \rho e^* \) are related by the following expression as

\[
\nabla^2 \phi_1^* (x^*, y^*) = \frac{\partial^2 \phi_1^*}{\partial x^*^2} + \frac{\partial^2 \phi_1^*}{\partial y^*^2} = \frac{\rho e^*}{\alpha_2}
\]  
(9)

Eq. (9) is termed as Boltzmann equation together with the boundary condition and the net charged density

\[ \phi_1^* = \phi_0^* \quad \text{at} \quad y^* = \pm h^* \]  
(10)

\[ \rho e^* = e_0 (n^+ - n^-) = -2z_0^2 e_0^2 n_0^2 \phi_1^* (y^*) \]  
(11)

where \( \alpha_2 \) is the dielectric constant, \( \phi_0^* \) is the potential on the wall, \( z_0, n_0, e_0, k_B, T_e, n^+, n^- \) are the ions valence, concentration of ions, the electronic charge, the Boltzmann constant, the local absolute temperature of the fluid, the density number of cation and anions respectively. By adopting the Debye-Huckel approximation, the linear form of the above Eq. (9) becomes
\( \alpha_1^2 \phi_1^* (y^*) = \frac{\partial^2 \phi_1^*}{\partial y^2} \)  \hfill (12)

where \( \alpha_1^2 \) represents the Debye-Huckel parameter.

We introduce the following dimensionless variables:

\[
x = \frac{x^*}{\lambda}, y = \frac{y^*}{d_1}, u_1 = \frac{u_1^*}{c^*}, h = \frac{h^*}{d_1}, \phi_1 = \frac{\phi_1^*}{\phi_0^*}, \alpha_1 = \frac{\alpha_1^*}{d_1^*}, \delta = \frac{\delta^*}{\lambda^*}
\]

\[
t = \frac{c^* t^*}{\lambda}, v_1 = \frac{v_1^*}{c^* \delta^*}, L = \frac{d_1^* L^*}{c^* \lambda^* \mu^*}, P = \frac{d_1^* p^*}{\delta^* \mu^*}, \text{Re} = \frac{\rho c^* d_1^*}{\mu^*}, \beta_1^2 = \frac{d_1^*}{\alpha_1^*} \hfill (13)
\]

Making use of these variable in Eqs. (2) to (7) and using the assumption of long wave length and low Reynolds number, the dimensionless form of governing equations is stated as

\[
\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \hfill (14)
\]

\[
\left( 1 + \frac{\lambda_1}{\lambda} \right) \frac{\partial^2 u_1}{\partial y^2} - M_1^2 u_1^2 - \frac{\partial p}{\partial x} - u_e \beta_1^2 \phi_1 (y) \hfill (15)
\]

where \( \lambda_1 \) is the Jeffrey fluid parameter, \( M_1^2 \) is the Hartmann number.

\[
u_e = \frac{E_x^* \alpha_2^*}{c^* \mu^*} \hfill (16)
\]

is the ratio of the Hedmhohz-smoluchowski velocity to the wave velocity and \( \beta_1^2 \)

represent non-dimensional form of width of electro kinetic.

The non-dimensional boundary conditions are

\[ u_1 = 0 \quad \text{at} \quad y = \pm h \hfill (16) \]

The dimensionless form of Eq. (12) and its boundary conditions are

\[
\frac{\partial^2 \phi_1}{\partial y^2} - \beta_1^2 \phi_1 = 0 \hfill (17)
\]

\[ \phi_1 = 1 \quad \text{at} \quad y = \pm h \hfill (18) \]

Solving Eq. (17) along with boundary conditions, the dimensionless potential distribution can be reduced to

\[
\phi_1 (y) = \frac{\cosh (\beta_1 y)}{\cosh (\beta_1 h)} \hfill (19)
\]

Substituting Eq. (19) into Eq. (15), we get
\[
\left[ \frac{\partial^2}{\partial y^2} - (1 + \lambda_1)(M_1^2) \right] u_1 = \left(1 + \lambda_1 \right) \left[ p' - u_e \beta_1^2 \left( \frac{\cosh(\beta_1 y)}{\cosh(\beta_1 h)} \right) \right]
\]  
(20)

where \( p' = \frac{\partial p}{\partial x} \).

Solving Eq. (20) under the boundary conditions (16), we get

\[
u_1(y) = A \cosh(\alpha y) - \frac{p'}{(M_1^2)} \left( \frac{1 + \lambda_1}{\beta_1^2 - \alpha^2} \right) \left( \frac{\cosh(\beta_1 y)}{\cosh(\beta_1 h)} \right)
\]

(21)

where

\[
A = \frac{\frac{p'}{(M_1^2)} + \left( \frac{1 + \lambda_1}{\beta_1^2 - \alpha^2} \right)}{\cosh(\alpha h)} \quad \text{and} \quad \alpha^2 = (1 + \lambda_1)(M_1^2)
\]

(22)

The mean velocity is defined as

\[
\bar{u}_1 = \frac{1}{2h} \int_{-h}^{h} u_1(y) dy
\]

(23)

Substituting Eq. (21) in Eq. (23), we get

\[
\bar{u}_1 = \frac{A \sinh(\alpha h)}{\alpha h} - \frac{p'}{(M_1^2)} \left( \frac{1 + \lambda_1}{\beta_1^2 - \alpha^2} \right) \tanh(\beta_1 h)
\]

(24)

As pointed out by Gupta and Gupta [25], the relative fluid velocity \( u_x \) can be obtained as

\[
u_x = u_1 - \bar{u}_1
\]

(25)

\[
u_x = A \cosh(\alpha y) - \frac{A \sinh(\alpha h)}{\alpha h} - \left( \frac{1 + \lambda_1}{\beta_1^2 - \alpha^2} \right) \left[ \beta_1 \left( \frac{\cosh(\beta_1 y)}{\cosh(\beta_1 h)} \right) - \tanh(\beta_1 h) \right]
\]

(26)

3. Analytical Solution of the Diffusion with a homogeneous chemical reaction

Following Taylor [17] and Gupta and Gupta [25], the dispersion equation for the substance's concentration under isothermal conditions is as follows:

\[
\frac{\partial C^*}{\partial t} + u_1^* \frac{\partial C^*}{\partial x} = D^* \left( \frac{\partial^2 C^*}{\partial y^2} \right) - k_1^* C^*
\]

(27)

where \( D^* \) denotes the coefficient diffusion, \( C^* \) indicates the fluid concentration and \( k_1^* \) represents the first order chemical reaction rate constant. Introducing \( x_1^* \) and assuming \( \bar{u}_1^* \approx \bar{c}^* [29] \), in Eq. (27) becomes

\[
\frac{\partial C^*}{\partial t} + \left( u_1^* - \bar{u}_1^* \right) \frac{\partial C^*}{\partial x_1} = D^* \left( \frac{\partial^2 C^*}{\partial y^2} \right) - k_1^* C^*
\]

(28)
Suppose that the Taylor’s limiting condition is valid [17] that is, the partial equilibrium state in any cross section of the channel is well recognized, Eq. (28) may be reduced to

\[ (u_1^* - \bar{u}_1^*) \frac{\partial C^*}{\partial x_1} = D^* \left( \frac{\partial^2 C^*}{\partial y^2} \right) - k_1^* C^* \]  

(29)

where \( \frac{\partial C^*}{\partial t^*} = 0 \) ((Gupta and Gupta [25])

Eq. (29) becomes

\[ \frac{\partial^2 C^*}{\partial y^2} - \left( \alpha_c \right)^2 C^* = \left( \alpha_5 \right) u_x \frac{\partial C^*}{\partial x_1} \]  

(30)

where \( \alpha_5 \left( = \frac{d^* u^*}{\alpha^* D^*} \right) \) and \( \alpha_c^2 \left( = \frac{k_1^* d^*}{D^*} \right) \) is the homogeneous chemical reaction parameter.

The boundary conditions for concentration \( C^* \) are

\[ \frac{\partial C^*}{\partial y} = 0 \quad \text{for} \quad y = \pm h = \pm \left( 1 + a_1 \sin \left( 2\pi x_1 \right) \right) \]  

(31)

Assuming that \( \frac{\partial C^*}{\partial y} \) is independent of \( y \) and solving Eq. (30) with the help of the boundary conditions (31), the expression for \( C^* \) may be obtained as

\[ C^*(y) = A_1 \cosh(\alpha_c y) + \frac{\alpha_5 \left( \frac{\partial C^*}{\partial x_1} \right) \left( \frac{A}{\alpha^2 - \alpha_c^2} \right) \cosh(\alpha y) + \left( \frac{A}{a h \alpha_c} \right) \sinh(\alpha h) - \left( \alpha \frac{1}{a_1} \right) u_x \beta_1 \left( \frac{1}{\beta_1^2 - \alpha^2} \right) \left( \frac{\beta_1}{\beta_1^2 - \alpha_c^2} \right) \cosh(\beta_1 y) + \frac{\tan(\beta_1 h)}{h \alpha_c^2} \right)}{\alpha_c \sinh(\alpha_c h)} \]  

(32)

where

\[ A_1 = - \frac{\alpha_5 \left( \frac{\partial C^*}{\partial x_1} \right) \left( \frac{A}{\alpha^2 - \alpha_c^2} \right) \sinh(\alpha h) - \left( \alpha \frac{1}{a_1} \right) u_x \beta_1^3 \left( \frac{1}{\beta_1^2 - \alpha^2} \right) \left( \frac{\beta_1}{\beta_1^2 - \alpha_c^2} \right) \tan(\beta_1 h)}{\alpha_c \sinh(\alpha_c h)} \]  

(33)

\[ L = \alpha_c \sinh(\alpha_c h) \]

The volumetric rate of the solute is

\[ Q = \int_{-h}^{h} C^*(y) u_x dy \]  

(34)

Substituting Eqs. (26) and (32) in Eq. (34), we obtain the volumetric rate \( Q \) as

\[ Q = -2 \left( \alpha_5 \left( \frac{\partial C^*}{\partial x_1} \right) F \right) \]  

(35)
where

\[
F = \frac{1}{L} \left[ \frac{M_0 \beta_1 \tanh(\beta_1 h)}{a^2 - c^2} \right] \left[ \frac{AM_0 M_3}{2} \right] + \frac{M_0 A \sinh(a h) \sinh(a_c h)}{a_c} \left[ \frac{\alpha c h}{\beta_1 h} \right] + \frac{M_0 \alpha h}{\beta_1 h} \left[ \frac{\beta_1}{\beta_1^2 - a^2} \right] + \frac{\alpha}{\beta_1^2 - a^2} \left[ \frac{\beta_1}{\beta_1^2 - a^2} \right]
\]

\[
+ \frac{\alpha c h}{\beta_1 h} \left[ \frac{M_0 A \sinh(a h) \tanh(\beta_1 h)}{\alpha c h} \right] + \frac{M_0 A \sinh(a h) \tanh(\beta_1 h)}{\alpha c h} + \frac{\alpha}{\beta_1^2 - a^2} \left[ \frac{\beta_1}{\beta_1^2 - a^2} \right]
\]

\[
+ \frac{A^2 \sin^2 h(a h)}{2 \beta_1 h a c^2} \left[ \frac{h^2 \sinh(2 \beta_1 h)}{\beta_1^2 - a^2} \right] + \frac{M_0 \alpha h}{\beta_1 h} \left[ \frac{\beta_1}{\beta_1^2 - a^2} \right] + \frac{\alpha}{\beta_1^2 - a^2} \left[ \frac{\beta_1}{\beta_1^2 - a^2} \right]
\]

\[
M_0 = \frac{1 + \lambda_1}{\beta_1^2 - a^2}
\]

\[
M_2 = \alpha_c \cosh(a h) \sinh(a_c h) - \alpha \cosh(a_c h) \sinh(a h)
\]

\[
M_3 = \alpha \cosh(\beta_1 h) \sinh(a h) - \beta_1 \cosh(a h) \sinh(\beta_1 h)
\]

\[
M_4 = \alpha_c \cosh(\beta_1 h) \sinh(a_c h) - \beta_1 \cosh(a_c h) \sinh(\beta_1 h)
\]

\[
h = 1 + a_2 \sin(2 \pi x_1), x_1 = \frac{x^*}{L^*}, a_2 = \frac{a_1^*}{d_1^*} \text{ is the amplitude ratio}
\]

When we compare Eq. (35) with Fick’s law of diffusion, the effective dispersion coefficient \( D_1 \) is given by

\[
D_1 = 2 \left( \alpha_5 \right) F
\]

Let the average of \( F \) be \( F_1 \) and is defined by

\[
F_1 = \int_0^1 F dx_1
\]

4. Results and Discussion

In this section, we have discussed the influence of Electro-Magnetohydrodynamic nature on dispersion of solute in the peristaltic mechanism via graphs. The impact of various physical parameters on the average equivalent dispersion coefficient \( F_1 \) such as the electro kinetic \( \beta \), the Jeffrey fluid parameter \( \lambda_1 \), the homogeneous chemical reaction parameter \( \alpha_c \), the electro osmotic velocity \( u_e \) and Hartmann number \( M^2 \) are studied. Integral involved in the solution of average dispersion coefficient is evaluated numerically by Simpson’s \( \frac{1}{3} \) rule.
Figures 2 and 3 show the variation of the average equivalent dispersion coefficient $F_1$ with respect to Jeffrey fluid parameter $\lambda_1$ for distinct values of the width of electro kinetic $(\beta = 0, 1, 2)$ and electro osmotic velocity ($u_e = -2, 0, 2$). It is noticed from Figure 2 that for a given value of the width of electro kinetic ($\beta$), the average equivalent dispersion coefficient $F_1$ is decreased with the enhance in Jeffrey fluid parameter ($\lambda_1$) and its rate of reduce is found to be higher for the greater value of $\lambda_1$ while it is minor when the smaller value of $\lambda_1$. One of the important outcomes is that the electro kinetic width support to increase the magnitude of the average equivalent dispersion coefficient $F_1$. Figure 3 reveals that the enhance in electro osmotic velocity provide a powerful improvement in the dispersion process. This is due to the fact that the enhancing nature of electro osmotic velocity tends to boost the velocity of the fluid which, in turn, force to increasing the average equivalent dispersion coefficient $F_1$.

Figure 2. The variation of $F_1$ with $\lambda_1$ for different values of $\beta$ with fixed $\alpha_c = 0.5, a_1 = 0.1, u_e = 2, M^2 = 5, p' = 10$.

Figure 3. The variation of $F_1$ with $\lambda_1$ for different values of $u_e$ with fixed $\alpha_c = 0.5, a_1 = 0.1, \beta = 1, p' = 10, M^2 = 5$. 
The influence of the magnetic field on the average equivalent dispersion coefficient $F_1$ is exposed in Figure 4. It exposes that the amount of the strength of the exterior force due to the applied magnetics field, which means that the average equivalent dispersion coefficient $F_1$ reduces with respect to enhance in Hartmann number $M^2 = 0, 5, 10$. It is obvious that the fluid velocity decreases with enhance the values of $M^2$. This is because of the existence of the transverse magnetic field encouraging a Lorentz force like to the drag force that acts in the reverse direction of the fluid movement, thus causing the average equivalent dispersion coefficients $F_1$ to diminish. It is related to point out here that the average equivalent dispersion coefficient $F_1$ declines naturally and more nonlinearly with the enhance in Jeffrey fluid parameter $\lambda_1$ in the case of small energy of applied magnetics force (low value of $M^2$) as related to that of the higher value of $M^2$.

![Figure 4. The variation of $F_1$ with $\lambda_1$ for different values of $M^2$ with fixed $\alpha_c = 0.5, a_1 = 0.1, \beta = 1, \rho' = 10, u_e = 2$.](image)

Figure 5. Details the significance of homogeneous chemical reaction parameter ($\alpha_c$) in the dispersion process. It is of interest to mention that the average equivalent dispersion coefficients $F_1$ is decreased with respect to the improvement of homogeneous chemical reaction parameter. Physically enhance in $\alpha_c$ leads to rising number of moles of solute undergoing chemical reaction and this incident decreases the fluid velocity beginning slowdown in the dispersion process.
Figure 5. The variation of $F_1$ with $\lambda_1$ for different values of $\alpha_c$ with fixed $M^2 = 5, \beta = 1, p' = 10, a_1 = 0.1, u_e = 2$.

5. Conclusion

In the present article, we have investigated the problem of electro-magneto hydrodynamic dispersion of a solute in peristaltic flow of Jeffrey fluid through a uniform channel. The first order homogeneous chemical reaction has been studied under Taylor’s limiting condition and long wavelength approximation. It is pertinent to pin point out that the new information regarding the effects of magnetic and electric fields on the average equivalent dispersion coefficient have been, for the first time, analyzed and added to the literature. It is seen that average equivalent dispersion coefficient decreases as the chemical reaction parameter and Hartmann number increases. Further it is enhanced with increasing electro kinetic and the electro osmotic velocity parameters. The present analytical study provides useful information to the circulatory system handled by artificial bio-processors in the medical field.

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