The black hole merger event GW150914 within a modified theory of General Relativity

Peter O. Hess

Instituto de Ciencias Nucleares, UNAM, Circuito Exterior,
C.U., A.P. 70-543, 04510, Mexico D.F., Mexico

and

Frankfurt Institute for Advanced Studies, Wolfgang Goethe University,
Ruth-Moufang-Strasse 1, 60438 Frankfurt am Main, Germany

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Abstract

In February 2016 the first observation of gravitational waves were reported. The source of this event, denoted as GW150914, was identified as the merger of two black holes with a about 30 solar masses each, at a distance of approximately 400Mpc. These data where deduced using the Theory of General Relativity. Since 2009 a modified theory was proposed which adds near massive objects phenomenologically the contribution of a dark energy, whose origin are vacuum fluctuations. The dark energy accumulates toward smaller distances, reducing effectively the gravitational constant. In this contribution we show that as a consequence the deduces chirping mass and the luminosity distance are larger. This result suggests that the black hole merger corresponds to two massive black holes near the center of primordial galaxies at large luminosity distance, i.e. large redshifts.

1 Introduction

In February 2016 the first observation of gravitational waves was reported by the LIGO collaboration [1] and in [2] more details of the parameter determination are presented. According to these findings, two black holes, nearly of
equal size of approximately 30 solar masses, merged and released an energy equal to three solar masses. The event happened roughly at a distance of 400Mpc, corresponding to about 1.3 Billion light years.

One of the puzzling results is that the two black holes should have been only 350 km apart. If these two objects were formed in a binary system, the small distance is very difficult to explain, though there are proposals how to reach this state assuming a rapidly rotating primordial star [3].

Since 2009 we have proposed a modification to the Theory of General Relativity (GR), called pseudo-complex General Relativity (pc-GR) [4, 5, 6]. Without going into details, the main differences are: i) That a mass not only curves the space but also changes the vacuum properties, generating an accumulation of dark energy, increasing toward smaller distances $R$. ii) Because of the lack of a complete quantized theory of gravitation, a phenomenological model for the dark energy is proposed, assuming that it increases proportional to $B/R^5$, with $R$ being the radial distance. Finally, we require that iii) there is no event horizon, which provides a lower value to the parameter $B$.

Point i) is justified because semi-classical quantum mechanical calculations reveal that vacuum fluctuations have to be increasingly present toward smaller distances (see, for example, [7]). However, these fluctuations explode at the Schwarzschild radius, rendering the semi-classical approximation meaningless. The reason is the fixed back-ground metric and a back-reaction of the dark energy on the metric is not considered. Point ii) is justified by i) and that a phenomenological approach in physics is often the best way to get answers when an approach of first principles is not available or the system is too complex. Proposing a definite dependence of the dark energy as a function of the radial distance, permits to include its back-reaction on the metric. The $R$-dependence proposed is the simplest one, which adds to the metric a term proportional to $1/R^3$, not entering yet in solar system experiments [8]. Point iii) has a philosophical reason which can of course be questioned: The event horizon is the consequence of the standard GR, however, there is no reason to assume that it has to be so, because other properties, not considered in GR, might come in, introducing modifications to GR in very strong gravitational fields. Furthermore, we find the idea not very attractive that, for an observer nearby, a region of space should be excluded from observation, thus we do not believe that the existence of an event horizon is mandatory even if it is predicted by GR. One also has to consider that up to now no conclusive direct observation of the event horizon is available [9].
The parameter $B$ in the phenomenological theory is chosen such that the metric component $g_{00}$ is always larger than zero. Thus, in this model, the large mass concentration is rather a gray star, though in many aspects it looks like a black hole. This point is very delicate and, for the moment, the reader can accept it as a working hypothesis.

In [6] some observational consequences were discussed, as of detected discrepancies to GR in galactic black holes. In [10] consequences to the observation of accretions disks were presented, marking relevant differences to GR, which may be observationally accessible in near future [11]. One result of [6], relevant for this contribution, is that the orbital frequency of a point particle in a circular orbit does not increase as in GR but near the Schwarzschild radius it deviates from the GR result, shows a maximum and finally reaches zero at two-thirds of the Schwarzschild radius, which is according to the model the approximate position of the surface of the star. Also, in [10] it is shown that the Innermost Stable Circular Orbit (ISCO), as a function of the rotational parameter $a$, first follows the result in GR but at smaller radial distances, until approximately for $a = 0.42$ from which on there is no ISCO and stable circular orbits exist until the surface of the star. In case an accretion disk is present, this produces for large $a$ an additional bright ring in the innermost region. Nevertheless, the gravitational redshift increases such that approaching the surface it gets too large to admit a simple observation of the light emitted from the in-plunging matter.

In order to do a complete calculation of a merger, one has to recur to numerical simulations, which we are for the moment unable to do. However, we follow the procedure as in [1, 2], where the stars are approximated by point masses in a circular orbit relative to each other. More details are explained in the book by M. Maggiore [12].

The paper is organized as follows: In section 2 we resume the relevant equations as used in GR and the differences to pc-GR. Then, the chirping mass is determined and it is shown that within pc-GR it has to be larger than the one deduced in [1, 2]. After this, the luminosity distance is obtained, which also has to be larger. The final conclusion will be that within pc-GR the event GW150914 corresponds to the merger of two massive black holes from the center of two primordial galaxies which also merged before. If this is true, it would not require a binary stellar system with two black holes at a very short distance. The model will be kept simple and the main objective of this contribution is to show that there may exist other interpretations to the source of the gravitational waves observed.
2 The model and its consequences

We will use the approximation of two point masses, as done in [12], valid only as far as the two masses are not near the merging point. In pc-GR the coupling constant depends on their separation $R$, treated as an adiabatic parameter. Because the dependence on $R$ of the coupling constant gets pronounced at small distances and each star has an appreciable extension, this dependence has to be interpreted in average and an exact value for $R$ cannot be given. This will be the reason that near the merging point the model should work less well and that we will present results in dependence of the parameter $R$. Nevertheless, the approximations will be sufficient to describe the general behavior, tendencies and to illustrate the point we want to make. The observer is set at a distance $r >> R$. The total mass is $\tilde{M} = \tilde{m}_1 + \tilde{m}_2$, with $\tilde{m}_k$ ($k = 1, 2$) the masses of the individual participants. We use a tilde to denote the masses involved in pc-GR in order to distinguish to the deduced masses within GR. The reduced mass $\tilde{\mu}$ is given by $\tilde{m}_1\tilde{m}_2/\tilde{M}$. The problem is then equivalent to the motion of a point like mass $\tilde{\mu}$ around a center with mass $\tilde{M}$. Because the sum of the two masses $\tilde{m}_1$ and $\tilde{m}_2$ provoke the accumulation of the dark energy near by, the mass $\tilde{M}$ is relevant, or equivalently the Schwarzschild radius $R_S = \frac{2GM}{c^2}$. This approximation does not include multipole contributions.

In what follows we will resume the main steps which lead to the relation of the chirping mass to the measured frequency and its change per unit time. During the course of the presentation, the differences between GR and pc-GR will be emphasized.

In pc-GR a mass not only curves space but also changes the properties of the vacuum, accumulating dark energy with decreasing radial distance to the mass. This part is treated in a phenomenological manner due to the lack of a full quantized GR. For example, the $g_{00}$ component changes to [4]

\[
g_{00} = 1 - \frac{2G\tilde{M}}{c^2 R} + \frac{B}{2R^3} = 1 - \frac{2G(R)\tilde{M}}{c^2 R},
\]

\[
G(R) = G \left[1 - \frac{b}{4R^2} \left(\frac{G\tilde{M}}{c^2}\right)^2\right] = G \left[1 - \frac{b}{4R} \left(\frac{R_S}{4R}\right)^2\right],
\] (1)

which is equivalent, as shown in the last equation, to an effective dependence of the gravitational coupling constant on the radial distance. When the
limiting value of \( B = b \left( \frac{GM}{c^2} \right) \), with \( b = \frac{64}{27} \) is used, the \( g_{00} \) is zero at two thirds of the Schwarzschild radius (in order to comply with our assumption that there is no event horizon, \( b \) has to be larger but for simplicity we use this value).

Thus, we can introduce an effective gravitational constant \( G(R) = Gg(R) \), where \( g(R) \) acquires the form

\[
g(R) = \left[ 1 - b \left( \frac{R_S}{4R} \right)^2 \right].
\]  

(2)

This has an effect on the dependence of the orbital frequency as a function on the radial distance of a particle in a circular orbit \([10]\):

\[
\omega^2_s = \frac{G]\M}{R^3} F_\omega(R)
\]

\[
F_\omega = 1 - \frac{3b}{4} \left( \frac{R_S}{2R} \right)^2.
\]  

(3)

In standard GR the factor \( F_\omega \) is equal to one. This equation is used in turn to depict the dependence of \( R \) on the orbital frequency \( \omega_s \). Here, we will do it similar and use \( F_\omega \) as a scaling factor.

Compared to earlier publications we have changed to an explicit notation on masses in kg, the light velocity and the coupling constant. Also the radial distance is now denoted as \( R \), because we distinguish between the relative distance \( R \) of two masses and the position of an observer of a gravitational wave at \( r \). Note that in the orbital frequency an additional factor \( F_\omega(R) \) appears, which produces a maximum at the distance \( R = \sqrt{\frac{5b}{2R_S}} \). Toward smaller distances \( R \), the orbital frequency diminishes until, for \( b = \frac{64}{27} \), it is zero at \( R = \frac{2}{3} R_S \), which is within the model the approximate position when the two stars touch each other. (The sum of the radii of two equal mass gray stars, with mass \( \tilde{m}_1 = \tilde{m}_2 = \frac{M}{2} \), is at approximately twice the value of \( \frac{2}{3} \left( \frac{R_S}{2} \right) \).)

When we consider two point-like masses \( \tilde{m}_1 \) and \( \tilde{m}_2 \), orbiting around each other at a relative distance \( R \), the mass moments [12] are modified within pc-GR to
\[ \tilde{M}_{ij} = \tilde{\mu} x_i x_j = \tilde{\mu} x_i x_j , \quad (4) \]

where the center of mass motion has been set to the origin.

The two additional factors \( g(R) \) (which enters via the factor \( G(R) \) in the amplitude) and \( F_\omega(R) \) will be responsible for modifying the key equations in obtaining the chirping mass and the luminosity distance.

From (3) we obtain

\[ R^2 = \left( \frac{G\tilde{M}}{\omega s^2} \right)^\frac{2}{3} [F_\omega(R)]^\frac{4}{3} \quad (5) \]

and the amplitudes of the gravitational waves change to

\[ h_+(t, \theta, \phi) = \frac{4G\omega^2 R^2}{rc^4} g(R) \frac{1 + \cos^2(\theta)}{2} \cos(2\omega_s t_{\text{ret}} + 2\phi) \]
\[ h_\times(t, \theta, \phi) = \frac{4G\omega^2 R^2}{rc^4} g(R)\cos(\theta)\sin(2\omega_2 t_{\text{ret}} + 2\phi) , \quad (6) \]

with \( t_{\text{ret}} \) is the retarded time and \( g(R) \) enters via \( G(R) \). This expression differs from the GR result [12] by the factor \( g(R) \). When the redshift is taken into account and the expansion of the universe, the \( r \) is changed to the luminosity distance \( \tilde{d}_L \) and to redshifted masses. [12].

To complete the first part, we resume the result for the orbital frequency: Using (5), we get

\[ E_{\text{orbit}} = -\frac{G(R)\tilde{m}_1 \tilde{m}_2}{2R} = -\frac{G\tilde{m}_1 \tilde{m}_2}{2} \left( \frac{\omega_s^2}{GM} \right)^\frac{1}{3} F_\omega^{-\frac{1}{3}}(R) g(R) \]
\[ E_{\text{orbit}} = \left( \frac{G^2 \omega_s^5 \tilde{m}_1^3 \tilde{m}_2^2}{8M} \right)^\frac{1}{3} F_\omega^{-\frac{1}{3}} g(R) , \quad (7) \]

or

\[ E_{\text{orbit}} = -\left( \frac{G^2 \omega_{gw}^2 \tilde{M}_5^5}{32} \right)^\frac{1}{3} F_\omega^{-\frac{1}{3}}(R) g(R) . \quad (8) \]
\[ \tilde{M}_c = \tilde{\mu}^2 \tilde{M}^2 \] is the chirping mass \cite{12} and the frequency of the gravitational wave \( \omega_{gw} = 2\omega_s \).

Using an average \( \langle \ldots \rangle \) over the time, assuming that the rate of change in time of the background metric is small compared to the frequency of the gravitational wave, the energy loss per unit time and solid angle is given by

\[
\frac{dE}{dt d\Omega} = \frac{c^3 r^2}{16\pi G g(R)} (h_+^2 + h_\times^2) \\
= \frac{2c^5}{\pi G} \left( \frac{G\tilde{M}_c \omega_{gw}}{2c^3} \right)^{\frac{10}{3}} g(R) F^\frac{4}{3}(R) \tag{9}
\]

and using \( \frac{dE}{dt} = -\frac{dE_{orb}}{dt} \), we obtain the time derivative of the circular frequency of the gravitational wave \cite{12}

\[
\frac{2}{3} \omega_{gw}^3 \frac{d\omega_{gw}}{dt} = \frac{32c^5}{5} \left( \frac{G\tilde{M}_c \omega_{gw}}{2c^3} \right)^{\frac{10}{3}} \left( \frac{32}{G^2 \tilde{M}_c^2} \right)^{\frac{1}{3}} F^\frac{5}{3}(R) \\
= \frac{12}{5} 2^{\frac{1}{3}} \left( \frac{G\tilde{M}_c}{c^3} \right)^{\frac{5}{3}} \omega_{gw}^{\frac{11}{3}} F^\frac{5}{3}(R). \tag{10}
\]

Using for the frequency \( f_{gw} = \frac{\omega_{gw}}{2\pi} \), we obtain

\[
\frac{df_{gw}}{dt} = \frac{96}{5} \pi^\frac{2}{3} \left( \frac{G\tilde{M}_c}{c^3} \right)^{\frac{5}{3}} f_{gw}^{\frac{11}{3}} F^\frac{5}{3}(R) \tag{11}
\]

and solving for the chirping mass, as it appears in \cite{1, 2}, shifting the additional factors to the side of the chirping mass, we obtain

\[
\mathcal{M}_c = \frac{\tilde{M}_c F_{\omega}(R)}{\frac{c^2}{G} \left[ 5 \cdot 8 \frac{df_{gw}}{dt} \left( \frac{c^3}{3} \frac{df_{gw}}{dt} \right) \right]^\frac{2}{3}}. \tag{12}
\]

Note, that the factor \( F_{\omega} \) on the left decreases with lower distances \( R \) and that the value on the right is obtained from the observed frequency and its change in time of the gravitational wave. If (12) is of the order of \( \mathcal{M}_c = 30 \) \cite{1} (we
are not taking the exact deduced values because we are only interested in the
general behavior), the chirping mass \( \tilde{M}_c \) on the left has to be larger. When
the factor approaches zero, the chirping mass tends to infinity. Of course, our
model of two point masses, orbiting each other, and the approximation of an
effective gravitational constant, has to be taken with care and very probably
is inaccurate near the merging point of the two masses. Nevertheless, the
result shows that within pc-GR the chirping mass is probably larger and does
not correspond to two stars with 30 solar masses each.

Using for \( b \) the limiting values, the factors \( g(R) \) and \( F_\omega(R) \) have the form

\[
\begin{align*}
g(R) &= 1 - \frac{1}{3} \left( \frac{2R_S}{3R} \right)^2 \\
F_\omega(R) &= 1 - \left( \frac{2R_S}{3R} \right)^2.
\end{align*}
\] (13)

In a second step, we determine which value the luminosity distance has
to acquire, without contradicting the observation, i.e., we will not derive its
value but rather present an estimation:

Assuming a flat universe and the evolution of the dark energy as a function
in time is equal to the present best known models (in [13] other scenarios are
discussed), the luminosity distance can be determined as a function of the
redshift in a standard manner (using \( \Omega_0 = 0.3036 \) and \( H_0 = 68.14 \) [14]). Here,
we are interested to estimate the luminosity distance required in order to get
the same amplitude for the gravitational wave. Note, that the amplitude of
the gravitational wave is given by (see Eq. (6))

\[
A = \frac{4G\tilde{\mu}\omega_s^2 R^2}{r c^4} g(R).
\] (14)

Substituting \( r \) by the luminosity distance \( \tilde{d}_L \), this translates to

\[
A = \frac{4G^\frac{5}{3}}{c^4} \frac{\tilde{\mu} \tilde{M}_c^\frac{2}{3} \omega_s^\frac{2}{3}}{\tilde{d}_L} F_\omega^\frac{2}{3}(R) g(R)
\]

\[
= \frac{4G^\frac{5}{3} \tilde{M}_c^{\frac{2}{3}} \omega_s^\frac{2}{3}}{c^4} \frac{\tilde{d}_L^\frac{2}{3}}{d_L} F_\omega^\frac{2}{3}(R) g(R) .
\] (15)
Demanding that in pc-GR the same amplitude is reproduced as in GR, leads to the relation

\[
\frac{\tilde{M}_c^3}{d_L} F_\omega^2(R) g(R) = \frac{M_c^3}{d_L},
\]

where \(d_L\) is the luminosity distance as deduced in [1, 2]. Using (12) and resolving for \(\tilde{d}_L\), leads finally to

\[
\tilde{d}_L = d_L \frac{g(R)}{F_\omega(R)}.
\]

Note, that the function \(g/F_\omega\) becomes very large for smaller relative distances \(R\) of the two massive objects.

![Figure 1: The factor to the chirping mass in Eq. (12).](image)

The inverse of \(F_\omega(R)\), plotted in Fig. 2, gives the factor by which the reported value \(M_c\) has to be multiplied in order to obtain \(\tilde{M}_c\). For a given
Table 1: Sample values for three distance values $x = \frac{R}{R_S}$, where $R_S$ is the Schwarzschild radius for the combined mass $\tilde{M}$. The first column lists the values of $x$. The second column tabulates the factor $F(R)$ by which the chirping mass $\tilde{M}_c$ in pc-GR is multiplied in order to obtain the observational deduced value $M_c$. The third column lists the factor by which the experimental deduced luminosity distance $d_L = 400$ Mpc has to be multiplied in order to obtain the luminosity distance $\tilde{d}_L$ within pc-GR. The last column gives the approximate redshift factor $z$.

| $x = \frac{R}{R_S}$ | $F(R)$ | factor for $\tilde{d}_L$ | $z$ (approx.) |
|---------------------|--------|-------------------------|--------------|
| 0.6682              | 0.00458| 145.8                   | $\approx 6$ |
| 0.6690              | 0.00667| 96.1                    | $\approx 4$ |
| 0.7000              | 0.09297| 7.5                     | $\approx 0.5$|

value of $x = \frac{R}{R_S}$ a corresponding new chirping mass and a luminosity distance $\tilde{d}_L$ in Eq. (17) is obtained.

For example, for $x \approx 0.6682$ the factor for the chirping mass is approximately 218, i.e. a new chirping mass of about $\tilde{M}_c = 6540$. The luminosity distance is then $\tilde{d}_L = 400$ Mpc $\times 146 = 58400$, which corresponds in the standard model of a flat universe to a redshift of a little less than $z = 6$, the limit where the first galaxies may have formed (in [15] galaxies with a redshift between 4 to 4.5 have been observed). Remember that the two massive objects touch (merge) at $x = 2/3 = 0.66$. For larger $R$, the final masses decrease and also the value of $z$. For smaller relative distances the masses increase to very large values, however the redshift factor is then too large, such that galaxies did not exist.

One has also to keep in mind that the model presented is very simple and larger masses at moderate $z$-values cannot be excluded.

Some sample values for different distances $x = \frac{R}{R_S}$ are listed in Table 1.

3 Conclusions

We have applied a modified theory of GR, called pc-GR, which adds near great masses a dependence on a dark energy, acting repulsively. We resumed the differences of the equations in pc-GR compared to GR, used to obtain the
amplitude of the gravitational wave and the chirping mass. The main result is that the chirping mass is larger than the reported one in [1, 2] and that the luminosity distance corresponds to the remote past of the universe. The system, producing the gravitational wave, would then consist of two large black holes, which were formerly at the center of two merging primordial galaxies. This interpretation seems to us more plausible than assuming a rapidly rotating super-massive star, which fissions in two very large chunks, undergoing at the same time a collapse as a supernova. Also the formation of two black holes from super-massive stars in a binary system at such short distances is questionable. Maybe, here we have a case where the first observation of gravitational waves hints to a needed modification of GR in very strong fields.

The model applied is very simple and results in a realistic treatment may change such that higher masses with a smaller luminosity distance may be possible. The present contribution has the objective to show that other possibilities exist for the interpretation of the initial object of GW150914.

Just now, another event was reported [16] with a chirping mass of approximately 9 solar masses, at 440 Mpc. All interpretations given here are also applicable to this new event, i.e. that it should be a merger of two massive black holes, following a merger of two primordial galaxies. A further possible event is reported in [17], together with the two other events. Again our conclusions is the same that one observed the merger of large black holes.

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