Symmetries of 2HDM, different vacua, CP violation and possible relation to a history of time

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Abstract

The same physical reality in Two Higgs doublet model (2HDM) can be described by different Lagrangians. We call this property the reparametrization invariance (in space of Lagrangians) and study corresponding symmetry group and its subgroup describing rephasing invariance. Next we consider the $Z_2$-symmetry of the Lagrangian, which prevents a $\phi_1 \leftrightarrow \phi_2$ transitions, and the different levels of its violation, soft and hard. We argue that the 2HDM with a soft breaking of $Z_2$-symmetry is a natural model in the description of EWSB. We also consider vacuum structure of the 2HDM. We find very simple condition for a CP violation in the Higgs sector. In the Model II for Yukawa interactions we obtain the set of relations among the couplings to gauge bosons and to fermions which allows one to analyze different physical situations (including CP violation) in terms of these very couplings, instead of the parameters of Lagrangian. We discuss possible interaction of Higgs fields of SM or 2HDM with inflationary Higgs field describing exponential expansion of Universe after Big Bang and possible variation in the scenario of beginning of Time.

1 Lagrangian

A spontaneous electroweak symmetry breaking of (EWSB) via the Higgs mechanism is described by the Lagrangian

$$\mathcal{L} = \mathcal{L}_{SM}^{g} + \mathcal{L}_{H} + \mathcal{L}_{Y} \quad \text{with} \quad \mathcal{L}_{H} = T - V. \quad (1a)$$

Here $\mathcal{L}_{SM}^{g}$ describes the $SU(2) \times U(1)$ Standard Model interaction of gauge bosons and fermions, $\mathcal{L}_{Y}$ describes the Yukawa interactions of fermions with Higgs scalars and $\mathcal{L}_{H}$ is the Higgs scalar Lagrangian; $T$ is the Higgs kinetic term and $V$ is the Higgs potential.

In the minimal Standard Model (SM) one scalar isodoublet with hypercharge $Y = 1$ is implemented. Here $\mathcal{L}_{H} = (D_{\mu}\phi)^{\dagger}D_{\mu}\phi - V$, with the Higgs potential $V = \lambda\phi^{4}/2 - m^{2}\phi^{2}/2$ etc. In ref. [1] we study in detail the simplest extension...
of the SM (see \cite{2} for earlier references), with two scalar fields $\phi_i$ being weak isodoublets ($T = 1/2$) with hypercharges $Y = 1$ called the Two-Higgs-Doublet Model (2HDM). The kinetic term of the most general renormalizable Higgs Lagrangian is

$$T = (D_\mu \phi_1)^\dagger(D_\mu \phi_1) + (D_\mu \phi_2)^\dagger(D_\mu \phi_2) + \left[\text{sr}(D_\mu \phi_1)^\dagger(D_\mu \phi_2) + \text{h.c.}\right]$$  \hspace{1cm} (1b)

and the Higgs potential, containing operators of dimension 2 (in mass term) and in terms of fields $T$ isodoublets ($F$ can be described in similar way both in terms of fields $T$ and in terms of fields $\phi$ and in terms of fields $\phi_k$ obtained from $\phi_k$ by a global unitary transformation $\hat{F}$ of $SU(2) \times U(1)$ general reparametrization (RPa) group:

$$V = -\frac{1}{2}\left\{m_{11}^2(\phi_1^1 \phi_1) + m_{22}^2(\phi_2^1 \phi_2) + \left[m_{12}^2(\phi_1^1 \phi_2) + \text{h.c.}\right]\right\}$$  \hspace{1cm} (1c)

$$+\frac{\lambda_1}{2}(\phi_1^1 \phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^1 \phi_2)^2 + \lambda_3(\phi_1^1 \phi_1)(\phi_2^1 \phi_2) + \lambda_4(\phi_1^1 \phi_2)(\phi_2^1 \phi_1)$$  \hspace{1cm} (1d)

$$+\frac{1}{2}\left[\lambda_5(\phi_1^1 \phi_1)^2 + \text{h.c.}\right] + \left\{\lambda_6(\phi_1^1 \phi_1) + \lambda_7(\phi_2^1 \phi_2)\right\}(\phi_1^1 \phi_2) + \text{h.c.}\right\}$$

### 2 Reparametrization and rephasing invariance.

#### 2.1 Reparametrization (RPa) invariance.

Our model contains two fields with identical quantum numbers. Therefore, it can be described in similar way both in terms of fields $\phi_k$ ($k = 1, 2$), used in \[1\], and in terms of fields $\phi'_k$ obtained from $\phi_k$ by a global unitary transformation $\hat{F}$ of $SU(2) \times U(1)$ general reparametrization (RPa) group:

$$\left(\begin{array}{c} \phi'_1 \\ \phi'_2 \end{array}\right) = \hat{F} \left(\begin{array}{c} \phi_1 \\ \phi_2 \end{array}\right), \quad \hat{F} = e^{-i\rho \theta} \left(\begin{array}{cc} \cos \theta e^{i\rho/2} & \sin \theta e^{-i(\tau-\rho/2)} \\ -\sin \theta e^{-i(\tau-\rho/2)} & \cos \theta e^{-i\rho/2} \end{array}\right).$$  \hspace{1cm} (2)

This group splits into proper $SU(2)$ RPa group with parameters $\theta$, $\rho$, $\tau$ (similar to the gauge parameter of gauge theories) and $U(1)$ group describing overall phase freedom, with parameter $\rho_0$.

- This transformation induces the changes of coefficients of Lagrangian (independent on parameter $\rho_0$), $\lambda_i \rightarrow \lambda'_i$ and $m_{ij}^2 \rightarrow (m')_{ij}^2$, $\kappa \rightarrow \kappa'$ with renormalization of fields $\phi'_k$ (RPa transformation of parameters). They are presented at $\kappa = 0$ in Ref. \[1\]. By construction, the Lagrangian of the form \[1\] with coefficients $\lambda_i$, $m_{ij}^2$ and that with new coefficients $\lambda'_i$, $(m')_{ij}^2$ describe the same physical reality. We call this property a RPa invariance in a space of Lagrangians (with coordinates given by its parameters).

The set of RPa transformations for parameters of Lagrangian form representation of RPa group in the 16-dimensional space of Lagrangians with coordinates given by $\lambda_{i-4}$, $Re\lambda_{5-7}$, $Im\lambda_{5-7}$, $m_{11,22}^2$, $Re(m_{12}^2)$, $Im(m_{12}^2)$, $Re\kappa$, $Im\kappa$. A set of physically equivalent Higgs Lagrangians, obtained from each other by mentioned transformations, forms the reparametrization equivalent space (RPaES), being a 3-dimensional subspace of the entire space of Lagrangians. The parameters of Lagrangian can be determined from measurements in principle only with accuracy up to the RPa freedom.

All observable quantities are invariants of RPa transformations (IRpaT). These are, for example, masses of observable Higgs bosons – eigenvalues of
The mass matrix (21) and (19) and eigenvalues of Higgs-Higgs scattering matrices (38). The coefficients of secular equation for diagonalization of these matrices (among them – trace of this matrix and its determinant) can be constructed from these eigenvalues. Therefore, they are also IRPaT’s.

By writing Higgs potential as a sum
\[ Y_{ab}(\phi^*_a \phi_b) + Z_{abcd}(\phi^*_a \phi_b)(\phi^*_c \phi_d) \]
with \( a, b = 1, 2 \), one can construct IRPaT’s at \( \kappa = 0 \) as combinations of products of \( Y \) and \( Z \) summarized on \( a, b \). In this way large series of (generally not independent) IRPaT’s was obtained [3]. The group-theoretical analysis of RPa group allows one to find the complete set of independent IRPaT’s constructed from \( \lambda \)’s and a few more with \( m_{ij}, \kappa \) [4].

2.2 Rephasing (RPh) invariance

It is useful to consider a particular case of the transformations (2) with \( \theta = 0 \) – global rephasing (RPh) transformation of the fields:
\[ \phi \rightarrow e^{-i\rho} \phi, \quad \rho_1 = \rho_0 - \rho/2, \quad \rho_2 = \rho_0 + \rho/2, \quad \rho = \rho_2 - \rho_1. \] (3a)

This transformation leads to a RPh transformation of the Lagrangian:
\[ \lambda_{1,4} \rightarrow \lambda_{1,4}, \quad m^2_{11} \rightarrow m^2_{11}, \quad m^2_{22} \rightarrow m^2_{22}, \]
\[ \lambda_5 \rightarrow \lambda_5 e^{-2i\rho}, \quad \lambda_{6,7} \rightarrow \lambda_{6,7} e^{-i\rho}, \quad m^2_{12} \rightarrow m^2_{12} e^{-i\rho}, \quad \kappa \rightarrow \kappa e^{-i\rho}. \] (3b)

By construction, the Lagrangian of the form (1) with coefficients \( \lambda_i, m^2_{ij} \) and that with coefficients given by eq. (3b) describe the same physical reality. We call this property a RPh invariance. The transformations (3) represent the \( U(1) \) RPh transformation group with parameter \( \rho \). This RPh group is a subgroup of the \( SU(2) \) RPa group.

3 Lagrangian and \( Z_2 \) symmetry

One of the earliest reasons for introducing the 2HDM was to describe the phenomenon of CP violation [5]. The CP violation and the flavour changing neutral currents (FCNC) can be naturally suppressed by imposing on the Lagrangian a \( Z_2 \) symmetry [6], inhibit the \( \phi_1 \leftrightarrow \phi_2 \) transitions, that is the invariance on the Lagrangian under the interchange
\[ \phi_1 \leftrightarrow \phi_1, \phi_2 \leftrightarrow -\phi_2 \text{ or } \phi_1 \leftrightarrow -\phi_1, \phi_2 \leftrightarrow \phi_2. \] (4)

- **The case of exact \( Z_2 \) symmetry** is described by the Lagrangian \( \mathcal{L}_{er} \) with \( \lambda_6 = \lambda_7 = \kappa = m^2_{12} = 0 \) and only one parameter \( \lambda_5 \) can be complex. The RPh transformation (3) with a suitable phase \( \rho \) allows one to get Lagrangian with a real \( \lambda_5 \), within the RPh invariant space.

- **In case of soft violation of \( Z_2 \) symmetry** one adds to the \( Z_2 \) symmetric Lagrangian the term of operator dimension 2, \( m^2_{12}(\phi_1 \phi_2) + \text{h.c.} \), with a generally complex \( m^2_{12} \) (and \( \lambda_5 \) parameter. This type of violation respects the \( Z_2 \) symmetry at small distances (much smaller than \( 1/M \)) in all orders of perturbative series, i.e. the amplitudes for \( \phi_1 \leftrightarrow \phi_2 \) transitions disappear at virtuality \( k^2 \sim M^2 \rightarrow \infty \). That’s why we call it a “soft” violation. The
RPh transformations applied to the Lagrangian with a soft violation of $Z_2$ generate a whole soft $Z_2$ violating Lagrangian family.

The general RPa transformation converts the Lagrangian with exact or softly violated $Z_2$ symmetry $L_s$ to a hidden soft $Z_2$ violation form $L_{hs}$ with $\lambda_6, \lambda_7 \neq 0, \varkappa = 0$. 14 parameters of $L_{hs}$ are constrained since they can be obtained from 9 independent parameters of an initial Lagrangian $L_s$ (+ 3 RPa group parameters), nondiagonal $\varkappa$ kinetic term don’t appear from loop corrections. For such physical system $L_s$ is preferable RPa representation.

- In general case the terms of the operator dimension 4, with generally complex parameters $\lambda_6, \lambda_7$ and $\varkappa$, are added to the Lagrangian with a softly violated $Z_2$ symmetry. In the case of the true hard violation of $Z_2$ symmetry this Lagrangian cannot be transformed to the exact or softly violated $Z_2$ symmetry form by any RPa transformation, the $Z_2$ symmetry is broken at both large and small distances in any scalar basis.

The mixed kinetic terms can be eliminated by the nonunitary transformation (rotation + renormalization), e.g.

$$(\phi_1', \phi_2') \rightarrow \left( \frac{\sqrt{\varkappa} \phi_1 + \sqrt{\varkappa} \phi_2}{2\sqrt{|\varkappa|(1 + |\varkappa|)}}, \frac{\sqrt{\varkappa} \phi_1 - \sqrt{\varkappa} \phi_2}{2\sqrt{|\varkappa|(1 - |\varkappa|)}} \right).$$  \tag{5}

However, in presence of the $\lambda_6$ and $\lambda_7$ terms, the renormalization of quadratically divergent, non-diagonal two-point functions leads anyway to the mixed kinetic terms (e.g. from loops with $\lambda_6^* \lambda_1, 3, 5$ and $\lambda_7^* \lambda_2, 5$). It means that $\varkappa$ becomes nonzero at the higher orders of perturbative theory (and vice versa a mixed kinetic term generates counter-terms with $\lambda_6, 7$). Therefore all of these terms should be included in Lagrangian on the same footing, i.e. the treatment of the true hard violation of $Z_2$ symmetry without $\varkappa$ terms is inconsistent. The parameter $\varkappa$ is running like $\lambda$’s. (This term does not appear if parameters $\lambda_i$ are constrained by relations of hidden soft violation of $Z_2$ symmetry.) Therefore, the diagonalization is scale dependent, and the Lagrangian remains off-diagonal in fields $\phi_{1,2}$ even at very small distances in any RPa representation. Such theory seems to be unnatural.

Although in [1] and in this paper we present relations for the case of hard violation of $Z_2$ symmetry at $\varkappa = 0$, the loop corrections can change results significantly. Such treatment of the case with true hard violation of $Z_2$ symmetry is as incomplete as in most of the papers considering this "most general 2HDM potential".

4 Vacua

The extremes of the potential define the vacuum expectation values (v.e.v.’s) $\langle \phi_{1,2} \rangle$ of the fields $\phi_{1,2}$ via equations:

$$\frac{\partial V}{\partial \phi_i}|_{\phi_i = \langle \phi_i \rangle} = 0.$$  \tag{6}

These equations have trivial electroweak symmetry conserving solution $\langle \phi_i \rangle = 0$ and electroweak symmetry violating solutions, discussed below. With accuracy to the choice of $z$ axis in the weak isospin space, and using the overall phase freedom of the Lagrangian to choose one v.e.v. real, in the minimal SM
such equation has single EWSB solution
\[
\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = m/\sqrt{2\lambda}.
\] (7)

With the same accuracy the most general electroweak symmetry violating solution of (6) can be written in a form
\[
\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ v_2 e^{i\xi} \end{pmatrix}.
\] (8)

To describe these extremes it is useful to denote
\[
x_1 = (\phi_1^\dagger \phi_1), \quad x_2 = (\phi_2^\dagger \phi_2), \quad x_3 = (\phi_3^\dagger \phi_3),
\]
y_1 = \langle x_1 \rangle, \quad y_2 = \langle x_2 \rangle, \quad y_3 = \langle x_3 \rangle, \quad Z = y_3^* y_3 - y_1 y_2.

It is easy to check that \( \partial x_1 / \partial \phi_2 = \partial x_2 / \partial \phi_1 = 0 \) and
\[
x_3 \left( \frac{\partial x_1}{\partial \phi_1} \right) - x_1 \left( \frac{\partial x_3}{\partial \phi_1} \right) = x_3 \left( \frac{\partial x_3}{\partial \phi_1} \right) - x_2 \left( \frac{\partial x_1}{\partial \phi_1} \right) = 0,
\]
x_3 \left( \frac{\partial x_3}{\partial \phi_1} \right) - x_1 \left( \frac{\partial x_3}{\partial \phi_1} \right) = x_3 \left( \frac{\partial x_3}{\partial \phi_1} \right) - x_2 \left( \frac{\partial x_1}{\partial \phi_1} \right) = x_3 x_3 - x_1 x_2.

Now the extremum condition (6) can be rewritten as
\[
\langle x_3 \left( \frac{\partial V}{\partial \phi_1} \right) - x_1 \left( \frac{\partial V}{\partial \phi_1} \right) \rangle = Z \left( \lambda_1 y_1 + \lambda_3 y_2 + \lambda_5 y_3 + \lambda_7 y_2 y_3 - \frac{m_1^2}{2} \right) = 0,
\]
\[
\langle x_3 \left( \frac{\partial V}{\partial \phi_1} \right) - x_2 \left( \frac{\partial V}{\partial \phi_1} \right) \rangle = Z \left( \lambda_1 y_1 + \lambda_3 y_2 + \lambda_5 y_3 + \lambda_7 y_2 y_3 - \frac{m_2^2}{2} \right) = 0,
\]
\[
\langle x_3 \left( \frac{\partial V}{\partial \phi_1} \right) - x_1 \left( \frac{\partial V}{\partial \phi_2} \right) \rangle = Z \left( \lambda_2 y_1 + \lambda_3 y_2 + \lambda_5 y_3 + \lambda_7 y_3 - \frac{m_2^2}{2} \right) = 0.
\] (10)

Therefore, two opportunities can be realized, in dependence of zero or nonzero value of \( Z = y_3^* y_3 - y_1 y_2 \). Depending on the parameters of potential, these solutions describe either saddle point or a minimum of the potential. The condition for minimum is that all eigenvalues of Higgs mass matrix are positive, and vacuum energy of one of these states is smaller than of second.

### 4.1 \( u \neq 0 \) Solution, charged vacuum

We denote by charged vacuum solution appeared at
\[
Z = y_2^* y_3 - y_1 y_2 \neq 0 \quad \Rightarrow \quad u \neq 0.
\] (11)

In this case the v.e.v.’s are given by equations followed directly from (10)
\[
\lambda_1 y_1 + \lambda_3 y_2 + \lambda_5 y_3 + \lambda_7 y_3 = m_1^2/2,
\]
\[
\lambda_2 y_1 + \lambda_3 y_2 + \lambda_5 y_3 + \lambda_7 y_3 = m_2^2/2,
\] (12)
\[
\lambda_4 y_3 + \lambda_5 y_3 + \lambda_3 y_1 + \lambda_7 y_2 = m_2^2/2.
\]

With these \( y_i \) the Higgs potential (11) can be written via \( \bar{x}_i = x_i - y_i \) as \( E_{\text{vac}}^c \) is a vacuum energy
\[
V = \lambda_1 \bar{x}_1^2/2 + \lambda_2 \bar{x}_2^2/2 + \lambda_3 \bar{x}_1 \bar{x}_2 + \lambda_4 \bar{x}_3^2 + \lambda_5 \bar{x}_3 \bar{x}_1 + \lambda_7 \bar{x}_3 \bar{x}_2 + \text{h.c.} + E_{\text{vac}}^c.
\] (13)

In this case it is not possible to split the gauge boson mass matrix into a neutral and charged sector, the interaction of gauge bosons with fermions will not conserve electric charge, photon become massive, etc. Certainly, this case does not realized in our World.
4.2 $u = 0$ solution, physical (neutral) vacuum
We consider below (except final section) only solution of extremum condition (1), obeying a condition for $U(1)$ symmetry of electromagnetism,
\[
Z = y_3^2 y_1 - y_1 y_2 = 0 \Rightarrow \langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}.
\]
(14)
The other standard notations are $v_1 = v \cos \beta$, $v_2 = v \sin \beta$, with SM constraint $v = (\sqrt{2} G_F)^{-1/2} = 246 \text{GeV}$.

The rephasing of fields shifts the phase difference $\xi$ as $\xi \rightarrow \xi - \rho$.

Let us take some Lagrangian describing our model and calculate v.e.v.'s.

Than, by making the RPh transformation with $\rho = \xi$, we get the real vacuum Lagrangian with real $v_2$ and with parameters, supplied for a moment by subscript $rv$:
\[
\lambda_{1-4, rv} = \lambda_{1-4}, \quad \lambda_{5, rv} = \lambda_5 e^{-2i\xi}, \quad \lambda_{6, rv} = \lambda_6 e^{-i\xi}, \quad \lambda_{7, rv} = \lambda_7 e^{-i\xi},
\]
(15)
\[
\varpi_{rv} = \varpi e^{-i\xi}, \quad m_{12, rv}^2 = m_{12}^2 e^{-i\xi}.
\]
The set of real vacuum Lagrangians forms a subspace in the entire RPaES. In different points of this subspace the tan $\beta$ values are different.

The following combinations of parameters and new quantities are useful:
\[
\lambda_{3, rv} + \lambda_{4, rv} + Re \lambda_{5, rv} = \lambda_{345, rv}, \quad \frac{v_1}{v_2} \lambda_{6, rv} \pm \frac{v_2}{v_1} \lambda_{7, rv} = \lambda_{67, rv},
\]
(16)
\[
m_{12, rv}^2 = 2 v_1 v_2 (v + i \delta).
\]
The minimum conditions (9) for this form of Lagrangian are written as
\[
(m_{11}^2 - Re m_{12, rv}^2) v_1/2 - \lambda_1 v_1^2 + \lambda_{345} v_2^2 + Re (3 \lambda_6 v_1 v_2 + \lambda_7 v_2^2/v_1) = 0,
\]
\[
(m_{12}^2 - Re m_{12, rv}^2 v_1/2 - \lambda_2 v_2^2 + \lambda_{345} v_1^2 + Re (3 \lambda_7 v_1 v_2 + \lambda_6 v_1^2/v_2) = 0,
\]
\[
Im m_{12, rv}^2 = 2 v_1 v_2 \delta = v_1 v_2 2 m_{12, rv}^2 (\lambda_5 + \lambda_7^2).
\]
(17)

We prepare calculations below for real vacuum potential, describing it in terms of $v_1, v_2$ instead of three quadratic parameters $m_{11, 22, 12}^2$ (17). In this way $\nu \propto Re m_{12, rv}^2$ is single free parameter in addition to $v_{1, 2}$ while $\delta \propto Im m_{12, rv}^2$ is expressed via $Im(\lambda_{5-7, rv})$ (17).

5 Physical Higgs representation
A standard decomposition of the fields $\phi_{1, 2}$ in the component fields is
\[
\phi_1 = \left( v_1 + \eta_1 + i \chi_1 \right) / \sqrt{7}, \quad \phi_2 = \left( v_2 + \eta_2 + i \chi_2 \right) / \sqrt{7}.
\]
(18)
At $\varpi = 0$ such decomposition conserve a diagonal form of kinetic terms for fields $\phi^+_1$, $\chi_1$, $\eta_1$. The mass-squared matrix is transformed to the block diagonal form by a separation of the massless Goldstone boson fields, $G^0 = \cos \beta \chi_1 + \sin \beta \chi_2$ and $G^\pm = \cos \beta \varphi^\pm_1 + \sin \beta \varphi^\pm_2$, and the charged Higgs boson fields $H^\pm$ with mass $M_{H^\pm}$,
\[
H^\pm = -\sin \beta \varphi^+_1 + \cos \beta \varphi^+_2, \quad M_{H^\pm}^2 = \left[ \nu - \frac{\lambda_4 + Re \lambda_5 + Re \lambda_7^2}{2} \right] v^2.
\]
(19)
5.1 Neutral Higgs sector

By definition $\eta_{1,2}$ are the standard $C-$ and $P-$ even (scalar) fields. The field

$$A = -\sin\beta \chi_1 + \cos\beta \chi_2,$$

(20)
is $C-$odd (which in the interactions with fermions behaves as a $P-$ odd particle, i.e. a pseudoscalar). In other words, the $\eta_{1,2}$ and $A$ are fields with opposite CP parities (see e.g. [2] for details).

The decomposition (13) results in the symmetric mass–squared matrix $\mathcal{M}$ in the $\eta_1$, $\eta_2$, $A$ basis

$$\mathcal{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & M_{23} \\ M_{13} & M_{23} & M_{33} \end{pmatrix},$$

(21)

$$M_{11} = \left[ c_\beta^2 \lambda_1 + s_\beta^2 \nu + s_\beta^2 Re(\lambda_{67}^+/2 + \lambda_{67}^-) \right] v^2,$$

$$M_{22} = \left[ s_\beta^2 \lambda_2 + c_\beta^2 \nu + c_\beta^2 Re(\lambda_{67}^+/2 - \lambda_{67}^-) \right] v^2,$$

$$M_{33} = \left[ \nu - Re(\lambda_5 - \lambda_{67}^+/2) \right] v^2 = M_A^2,$$

$$M_{12} = - \left[ \nu - \lambda_{345} - Re(3\lambda_{67}^+/2) \right] c_\beta s_\beta v^2,$$

$$M_{13} = - (\delta + Im\lambda_{67}^+/2) s_\beta v^2; \quad M_{23} = - (\delta - Im\lambda_{67}^-/2) c_\beta v^2,$$

where $c_\beta = \cos\beta$, $s_\beta = \sin\beta$. Note that $M_{33}$ is equal to the mass squared of the CP–odd Higgs boson in the CP conserving case $M_A^2$.

The masses squared $M_i^2$ of the physical neutral states $h_{1-3}$ are eigenvalues of the matrix $\mathcal{M}$. These states are obtained from fields $\eta_1$, $\eta_2$, $A$ by a unitary transformation $R$ which diagonalizes the matrix $\mathcal{M}$:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ A \end{pmatrix}, \quad R = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix},$$

(22)

with $R M R^T = \text{diag}(M_1^2, M_2^2, M_3^2)$. All observable Higgs fields $h_i$, $H^\pm$, their masses and couplings are RPA independent, in contrast with original fields $\phi_{1,2}$. The useful 2-step diagonalization procedure is described in [1].

With radiative corrections the physical states $h_i$ become unstable, they have no asymptotic states. Mass matrix for these Higgs bosons become non-hermitian. This effect can be neglected when the widths of Higgs bosons are much smaller than the mass splitting. If one of masses $M_i$ is close to another mass, a reasonable description of the masses and couplings is given by an approximation in which a (complex) matrix of polarization operators is added to the mass matrix [21]. Full treatment of this problem demands a subtle theoretical analysis.

5.2 Criterium for CP violation

In general, the Higgs eigenstates $h_i$ [22] have no definite CP parity since they are mixtures of fields $\eta_{1,2}$ and $A$ having opposite CP parities. Just this mixing provides a CP nonconservation within the Higgs sector since the interaction of these Higgs bosons with matter explicitly violates the CP–symmetry.

The eq. (21) shows that such mixing is absent and the CP does not violated if and only if $M_{13} = M_{23} = 0$. The explicit form for these terms [24] shows
that these two conditions can be valid (at $\sin 2\beta \neq 0$) if and only if $\lambda_5 \neq 0$ and $m^2_{12}$ are real. In accordance with \[10\] it means that the CP violation is absent if all coefficients in potential of a real vacuum form are real. Vice versa, the complexity of some parameters of the potential in a real vacuum form is a sufficient condition for CP violation in the Higgs sector. Simple but cumbersome calculation shows that similar conclusion is valid also for for $\sin 2\beta = 0$. For an arbitrary form of Lagrangian the necessary and sufficient condition for CP violation in the Higgs sector can be written as complexity at least one of combinations
\[
\lambda_5^* (m^2_{12})^2, \quad (\lambda_6^* + \lambda_7^*) m^2_{12}, \quad \lambda_6^* \lambda_7.
\] (23)

Each this quantity is not RPa invariant one but these forms are very simple. (For the soft $Z_2$ violated potential one should be $\text{Im} \lambda_5^* (m^2_{12})^2 \neq 0$ – cf. \[8\]).

The RPa invariant conditions for CP violation \[9\], \[3\] are more complex.

\[\nabla\] In MSSM, etc. CP symmetry can be violated by interaction of Higgs fields with different scalar squarks, etc. In this case the CP violated terms (like $\text{Im} \lambda_{5-7}$, etc.) must be added in Lagrangian for renormalizability.

### 6 Couplings to gauge bosons and fermions

Below we use in principle measurable relative couplings – ratios of the couplings of each neutral Higgs boson $h_i$ to the corresponding SM couplings
\[
\chi^{(i)}_V = g^{(i)}_V / g^{SM}_V .
\] (24)

for the gauge bosons $W$ or $Z$ and the quarks or leptons ($j = W, Z, u, d, \ell...$).

- The gauge bosons $V$ ($W$ and $Z$) couple only to the CP–even fields $\eta_1, \eta_2$.

For the physical Higgs bosons $h_i$, \[22\] one obtains
\[
\chi^{(i)}_V = \cos \beta R_{i1} + \sin \beta R_{i2}, \quad V = W \text{ or } Z.
\] (25)

#### 6.1 Yukawa interaction

The general form of Yukawa interaction couples 3-family vector of the left-handed quark isodoublets $Q_L$ with 3-family vectors of the the right-handed field singlets $d_R$ and $u_R$ and Higgs fields $\phi_i$. It allows large FCNC effects and lead to true hard violation of $Z_2$ symmetry via loop effect (see \[1\] for details).

To have only the soft violation of $Z_2$ symmetry, each right-handed fermion should couple to only one scalar field, either $\phi_1$ or $\phi_2$ \[6\], \[10\].

#### 6.2 Model II

We consider first most popular opportunity (realized also in MSSM) named as Model II (cf. \[2\] for classification). In this Model the physical reality allows the description, where the fundamental scalar field $\phi_1$ couples to $d$-type quarks and charged leptons $\ell$, while $\phi_2$ couples to $u$-type quarks and this interaction is diagonal (or almost diagonal) in family index $k$

\[
-\mathcal{L}_Y^{II} = \sum g_{uk} \bar{Q}_{Lk}\phi_1 d_{Rk} + \sum g_{uk} \bar{Q}_{Lk}\phi_2 u_{Rk} + \sum g_{uk} \bar{\ell}_{Lk}\phi_1 \ell_{Rk} + \text{h.c.}
\] (26)
The suitable choice of phases in RPh transformations makes all Yukawa parameters real. As it was written above, different forms of Lagrangian can have different values of \( \tan \beta \). To underline that we use the mentioned Lagrangian, we will supply (only in this section) quantity \( \beta \) by a subscript II, \( \beta \to \beta_{II} \). (The RPa transformation makes Model II property of Lagrangian hidden and changes \( \tan \beta \).)

The relative Yukawa couplings of the physical neutral Higgs bosons \( h_i \), \( i = u, d – \text{type} \) and for all \( d – \text{type} \) quarks (and charged leptons). They are expressed via elements of the rotation matrix \( R \) as

\[
(R_{II}) : \quad \chi_u^{(i)} = \frac{R_{i2} - i \cos \beta_{II} R_{i3}}{\sin \beta_{II}}, \quad \chi_d^{(i)} = \frac{R_{i1} - i \sin \beta_{II} R_{i3}}{\cos \beta_{II}}. \quad (27)
\]

In the cases of weak CP violating and soft \( Z_2 \)-violation the relative coupling of the neutral scalar \( h_i \) to the charged Higgs boson is expressed via the couplings of this neutral Higgs boson to the gauge bosons and fermions:

\[
\chi_{H^+}^{(i)} = \left( 1 - \frac{M^2}{2M_H^2} \right) \chi_V^{(i)} + \frac{M^2 - \nu^2}{2M_H^2} \Re \left( \chi_u^{(i)} + \chi_d^{(i)} \right). \quad (28)
\]

\( \nabla \) The unitarity of the mixing matrix \( R \) allows one to obtain a number of relations between the relative couplings of neutral Higgs particles. These relations are very useful in phenomenological analyzes.

First, the quantity \( \tan \beta_{II} \) (coincidency with the ratio \( v_2/v_1 \) only in a Model II form of Lagrangian) is described via the basic couplings to \( h_i \) as

\[
\cot^2 \beta_{II} = \left( \frac{\chi_V^{(i)}}{\chi_u^{(i)}} - 1 \right)^2 = -\frac{|\chi_u^{(i)}|^2 - 1}{\chi_d^{(i)} - \chi_V^{(i)}} \sum_i \Im \chi_u^{(i)} = 2. \quad (29)
\]

1. The pattern relation for each neutral Higgs particle \( h_i \) (for CP conserving case see [11, 12]):

\[
(\chi_u^{(i)} + \chi_d^{(i)}) \chi_V^{(i)} = 1 + \chi_u^{(i)} \chi_d^{(i)}. \quad (30)
\]

2. A vertical sum rule for all three neutral Higgs bosons \( h_i \) [15]:

\[
\sum_{i=1}^3 (\chi_j^{(i)})^2 = 1 \quad (j = V, d, u). \quad (31)
\]

For couplings to the gauge bosons this sum rule takes place independently on a particular form of the Yukawa interaction.

3. A horizontal sum rule [13] for each neutral Higgs boson \( h_i \):

\[
|\chi_u^{(i)}|^2 \sin^2 \beta_{II} + |\chi_d^{(i)}|^2 \cos^2 \beta_{II} = 1. \quad (32)
\]

These sum rules guarantee that the cross section of production of each neutral Higgs boson \( h_i \) of the 2HDM by one of types of quarks cannot be lower than that for the SM Higgs boson with the same mass [13].

4. Besides, the linear relation follows directly from Eqs. (22), (27):

\[
Re \left( \cos^2 \beta_{II} \chi_u^{(i)} + \sin^2 \beta_{II} \chi_u^{(i)} \right) = \chi_u^{(i)}, \quad \text{Re} \left( \cos^2 \beta_{II} \chi_d^{(i)} - \sin^2 \beta_{II} \chi_d^{(i)} \right) = 0. \quad (33)
\]

5. The relation between CP violated parts of Yukawa couplings is obtained by exclusion of \( \beta_{II} \) from the equations (22), (27).
\[(1 - |\chi_d^{(i)}|^2) \text{Im} \chi_u^{(i)} + (1 - |\chi_u^{(i)}|^2) \text{Im} \chi_d^{(i)} = 0. \quad (34)\]

A number of applications of this set of relations is discussed in [1].

∇ The observable quantities correspond the Lagrangian with radiative corrections (RC). Then one can treat the relations (30)–(34) as obtained from the renormalized parameters. For each relative coupling (24) the RC are included in both: the couplings of the 2HDM (in the numerator) and those of SM (in the denominator). The largest RC to the Yukawa \(\phi \bar{q}q\) couplings are the one–loop QCD corrections due to the gluon exchange. They are identical in the SM and in the 2HDM and cancel in ratios \(\chi_u\) and \(\chi_d\). The same is valid for purely QED RC to all basic couplings as well as for electroweak corrections including virtual \(Z\) or \(W\) contributions.

The electroweak RC containing Higgs bosons in the loops are different in the SM and 2HDM, their values depend on the parameters of 2HDM. These type of RC may modify slightly some relations presented above. However, it is naturally to expect that these RC are small (< 1 %) except for some small corners of parameter space.

### 6.3 Model I

We consider also, for completeness, this model, in which all right handed fermions are coupled to one Higgs field \(\phi_1\). The general RPa transformation makes this property hidden, changing simultaneously \(\tan \beta\). We supply the parameter \(\beta\) for the Model I form of Lagrangian by subscript \(I\).

The corresponding Model I Yukawa Lagrangian is similar to that (26) with only change \(\phi_2 \rightarrow \phi_1\). For this form of Lagrangian we have

\[(M I) : \quad \chi_u^{(i)} = \chi_d^{(i)} \equiv \chi_f^{(i)} = \frac{[R_{i2} - i \cos \beta_I R_{i3}]}{\sin \beta_I}. \quad (35)\]

In this case only one of methods for finding of \(\beta_I\) via observable quantities among series presented in (29) works, \(\cot^2 \beta_I = \sum_i \left(\text{Im} \chi_u^{(i)} \right)^2\), just as vertical sum rules (31). Other relations written for Model II ((30), (32)–(34) don’t work for this Model.

### 7 Constraints for Higgs Lagrangian

#### 7.1 Positivity (vacuum stability) constraints.

To have a stable vacuum, the potential must be positive at large quasi–classical values of fields \(|\phi_k|\) (positivity constraints) for an arbitrary direction in the \((\phi_1, \phi_2)\) plane. These constraints were obtained for the case of soft \(Z_2\) violation (see e.g. [14], [16]), they are

\[\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0, \quad \lambda_3 + \lambda_4 - |\lambda_5| + \sqrt{\lambda_1 \lambda_2} > 0. \quad (36)\]
7.2 Minimum constraints

The condition for vacuum describes the extremum of potential but not obligatory the minimum. The minimum constraints are the conditions ensuring that above extremum is a minimum for all directions in \((\phi_1, \phi_2)\) space, except of the Goldstone modes (the physical fields provide the basis in the coset). This condition is realized if the mass-matrix squared for the physical fields is positively defined: \(M^2_{h_{1-3}}\), \(M^2_{H^{\pm}} > 0\). Note that the change of sign of \(M^2_{H^{\pm}}\) with all positive neutral mass squares correspond to the transition from physical vacuum to the charged vacuum.

7.3 Tree level unitarity constraints

The quartic terms of Higgs potential \((\lambda_i)\) lead, in the tree approximation, to the s–wave Higgs-Higgs and \(W_L W_L\) and \(W_L H\), etc. scattering amplitudes for different elastic channels. These amplitudes should not overcome unitary limit for this partial wave – that is the tree-level unitarity constraint.

The unitarity constraint was obtained first for the minimal SM, with Higgs potential \(V = (\lambda/2)(\phi^\dagger \phi - v^2/2)^2\). Such constraints for the 2HDM with a soft \(Z_2\) violation and CP conservation were derived in \(17\). In the general CP nonconserving case unitarity constraints are written in ref. \(18\) as the bounds for the eigenvalues of the high energy Higgs–Higgs-scattering matrix \(S_{Y \sigma} = 16\pi \Lambda_{Y \sigma}\) for the different quantum numbers of an initial state: total hypercharge \(Y\) and weak isospin \(\sigma\) \((38)\). (In each case left upper \(2 \times 2\) corner presents scattering matrix for \(Z_2\)–even states and right–down corner — for \(Z_2\)–odd states, while coefficients \(\lambda_6, \lambda_7\) describe mixing among these states.)

The eigenvalues of these matrices can be found as roots of equations of the 3-rd or 4-th degree. It is useful to start diagonalization from corners of

\[
\begin{align*}
\Lambda_{Y=2, \sigma=1} & = \begin{pmatrix}
\lambda_1 & \lambda_5 & \sqrt{2}\lambda_6 \\
\lambda_5 & \lambda_2 & \sqrt{2}\lambda_7 \\
\sqrt{2}\lambda_6 & \sqrt{2}\lambda_7 & \lambda_3 + \lambda_4
\end{pmatrix}, \\
\Lambda_{Y=2, \sigma=0} & = \lambda_3 - \lambda_4, \\
\Lambda_{Y=0, \sigma=1} & = \begin{pmatrix}
\lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\
\lambda_2 & \lambda_7 & \lambda_6 & \lambda_5 \\
\lambda_3 & \lambda_6 & \lambda_7 & \lambda_4 \\
\lambda_4 & \lambda_5 & \lambda_6 & \lambda_7
\end{pmatrix}, \\
\Lambda_{Y=0, \sigma=0} & = \begin{pmatrix}
3\lambda_1 & 2\lambda_3 + \lambda_4 & 3\lambda_6 & 3\lambda_5^* \\
2\lambda_3 + \lambda_4 & 3\lambda_2 & 3\lambda_7 & 3\lambda_6^* \\
3\lambda_6 & 3\lambda_7 & \lambda_3 + 2\lambda_4 & 3\lambda_5^* \\
3\lambda_5 & 3\lambda_6 & 3\lambda_7 & \lambda_3 + 2\lambda_4
\end{pmatrix}.
\end{align*}
\]
these matrices, corresponding to fixed values of the $Z_2$ parity. This particular diagonalization transform $\Lambda Y \sigma$ to the form with diagonal elements coincident with eigenvalues found in [17] (for soft $Z_2$ violation without CP violation) with sole change $\lambda_5 \to |\lambda_5|$. Let us remind that all diagonal matrix elements $M_{ii}$ of a Hermitian matrix $||M_{ij}||$ with maximal and minimal eigenvalues $\Lambda_{\pm}$ lie between them, $\Lambda_{+} \geq M_{ii} \geq \Lambda_{-}$. It means that the mentioned corrected constraints from [17] form necessary conditions for unitarity. These constraints are enhanced due to terms describing hard $Z_2$ violation.

8 2HDM and observations

Some possible observation will be clear signal in favor of difference of our world from that described by minimal SM and in the attempt to check whether EWSB is given by 2HDM. (i) If more than one Higgs boson will be observed. (ii) If – in the case of observation of single Higgs boson – the strong difference in the couplings of Higgs boson with matter from SM predictions will be observed.

The most difficult for analysis is the case of realization of SM-like physical picture: the lightest Higgs boson $h_1$ is similar to the Higgs boson of the SM while other Higgs bosons escape observation being too heavy (or weakly coupled with matter).

Heavy Higgs bosons in 2HDM. Besides, many authors assume in addition that masses of other Higgs bosons $M$ are close to the scale of new physics, $M \sim \Lambda$, and that the theory should possess an explicit decoupling property, i.e. the correct description of the observable phenomena must be valid for the (unphysical) limit $M \to \infty$ [19, 16]. (This property – independence from phenomena at $p > \Lambda$ – is necessary feature of any consistent theory describing phenomena at $p \ll \Lambda$ but only if limit $\Lambda \to \infty$ has physical sense [20].) The 2HDM allows also for another realization of the SM-like physical picture.

Looking for mass matrix one can see that the large masses of Higgs particles may arise from large parameters $\nu$ or $\lambda$'s, or both. Obviously, large values of $\lambda$'s may be in conflict with unitarity constraints, which is not the case for large $\nu$. The case $\nu \gg |\lambda_i|$ correspond to a decoupling regime, while the case of small $\nu$ and not very high $|\lambda_i|$ allows quite another realization of SM-like scenario. Both these opportunities were analyzed in detail in [1].

This analysis allows one to show that the natural set of parameters of 2HDM correspond to the case of soft violation of $Z_2$ symmetry and $|\nu|, |\lambda_5| \ll |\lambda_{1-4}|$. From this point of view the decoupling case of 2HDM with $\nu \gg |\lambda_i|$ is unnatural.

The 2HDM with natural set of parameters (not in the decoupling case) and SM can be distinguished via observation of Higgs boson production at Photon Collider [21].
9 Possible relation to a history of time

The modern description of the beginning of time contains assumption about SM Higgs mechanism of EWSB. In the hot primitive medium after Big Bang the effective Higgs potential of SM is added by term $cT^2 \phi^2/2$. It changes standard mass term $-m^2 \phi^2/2$ so that the v.e.v. of Higgs field $\langle \phi \rangle$ with growth of temperature decreases as $\sqrt{m^2 - cT^2/\sqrt{\lambda}}$. At the temperature $T_c \approx m/\sqrt{\lambda}$ (determined with accuracy to quantum corrections) we have phase transition.

After Big Bang, when $T > T_c$, we had $\langle \phi \rangle = 0$, EWSB was not broken, particles were massless, providing exponential inflatory expansion of Universe. After cooling to $T < T_c$ we come to our world with massive particles, etc. and nonzero vacuum energy $E_{\text{vac}}$ – see for details references in \[22\].

One can imagine two opportunities for this picture.

• First, the inflation mechanism is given by Higgs field, responsible for EWSB. In this case, possible existence of two vacua in 2HDM opens new opportunity in the history of time. Here in the hot medium the effective potential (1a) is added by terms

\[
[ c_{11}\langle \phi_1^* \phi_1 \rangle + c_{12}\langle \phi_1^* \phi_2 \rangle + c_{12}^*\langle \phi_2^* \phi_1 \rangle + c_{22}\langle \phi_2^* \phi_2 \rangle ] \frac{T^2}{2}. \tag{39}
\]

They change mass terms of our Lagrangian so that immediately after Big Bang the Universe expands inflatory in the same manner as in minimal SM. The subsequent fate of Universe depends on values of parameters.

In one case at the growth of time the EWSB vacuum $\langle \phi_1 \rangle = \langle \phi_2 \rangle = 0$ at some critical temperature is transformed directly to the neutral vacuum (14). In this case transformation of Universe are completely the same as those discussed in respect of minimal SM.

In the other case at the growth of time the EWSB vacuum $\langle \phi_1 \rangle = \langle \phi_2 \rangle = 0$ at some critical temperature $T_{c1}$ is transformed first in the charged vacuum and only with subsequent growth of time at some temperature $T_{c2} < T_{c1}$ the charged vacuum is transformed into well known neutral vacuum (14). The life of Universe in the period when $T_{c2} < T < T_{c1}$ can be quite unusual. In this stage the medium is absolutely non-transparent for light (photon is massive), the transformations of particles are quite different from modern, the $C$ violation for particles (vacuum is charged) can leave after second phase transition an Universe with residual CP violation and influence for baryon asymmetry, etc. Besides, some small domains of charged phase appeared from fluctuations in one of phase transitions can leave up to our time, influencing for modern observations. Some of these opportunities can be excluded quickly by first analysis, but the other must be studied in future in detail.

• Second, inflation can be related to a specific inflaton Higgs field $\phi_0$ with varying in time v.e.v. $\langle \phi_0 \rangle = U_0(t)$. This field should interact with Higgs field responsible for EWSB like (39), the effective Higgs potential is added by term

\[
[ a_{11}\langle \phi_1^* \phi_1 \rangle + a_{12}\langle \phi_1^* \phi_2 \rangle + a_{12}^*\langle \phi_2^* \phi_1 \rangle + a_{22}\langle \phi_2^* \phi_2 \rangle ] \frac{U_0^2}{2}, \tag{40}
\]

where coefficients $a_{ij}$ can be both positive and negative. Therefore, during inflation effective mass term of the EWSB Higgs field varies with time as $m_{\phi_0}^2 \rightarrow m_{\phi_0}^2 - c_{ij}T^2 - a_{ij}U_0^2(t)$. It can results in even more complex sequence of phase
transitions than that discussed above (e.g., with restoration of $SU(2) \times U(1)$ symmetry in some intermediate period).

Both these opportunities should be analyzed in future.

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