Optimizing structure of complex technical system by heterogeneous vector criterion in interval form

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Abstract. The article examines the methods of development and multi-criteria choice of the preferred structural variant of the complex technical system at the early stages of its life cycle in the absence of sufficient knowledge of parameters and variables for optimizing this structure. The suggested methods takes into consideration the various fuzzy input data connected with the heterogeneous quality criteria of the designed system and the parameters set by their variation range. The suggested approach is based on the complex use of methods of interval analysis, fuzzy sets theory, and the decision-making theory. As a result, the method for normalizing heterogeneous quality criteria has been developed on the basis of establishing preference relations in the interval form. The method of building preferential relations in the interval form on the basis of the vector of heterogeneous quality criteria suggest the use of membership functions instead of the coefficients considering the criteria value. The former show the degree of proximity of the realization of the designed system to the efficient or Pareto optimal variants. The study analyzes the example of choosing the optimal variant for the complex system using heterogeneous quality criteria.

1. Introduction
Complex technical systems, as a rule, are of modular architecture, they are built on the basis of developed miscellaneous hardware and software. There is a large database of the equipment produced by various manufactures, performing the functions necessary for building a certain complex technical system. The choice of the best variant of the system for the definite operation conditions is hindered by the considerable volume of the market of domestic and imported equipment, a large number of structural types, variety of operation conditions, and specific requirements concerning their reliability and efficiency. Moreover, the choice becomes more complicated in practice, as it is sometimes impossible to obtain reliable information on certain components characteristics. This lack of reliable information may occur when the manufacturer is either unwilling to evaluate its produce according to some quality parameters or conceals or provides inaccurate information on equipment characteristics. The designers, while working on the new complex technical systems, have to consider the parameters and characteristics in the form of their variations range.

In the modules and components of a designed complex technical system, the large number of parameters and quality characteristics have fuzzy values and are determined more precisely at the next stages of design.

In this situation, the development of multi-criteria optimization of the structure of the complex technical systems has to be done in the conditions of fuzzy and incomplete input data. The choice of the optimal variant of the structure is performed on the basis of multiple heterogeneous quality criteria that can be given not only in a formalized, quantitative form, but in an indeterminate, linguistic, and partially formalized forms as well.
While optimizing the structure of a radio-electronic system, the obstacles can arise in connection with the problem of comparison of the improvement of one criterion with the decrease in value of the other if they are of heterogeneous character.

The uncertainty of input data will lead to significant errors in calculating the target function, and that, in its turn, influences the choice of optimal modes of the system operation.

2. Problem statement

In multi-criteria optimization it is important to perform simultaneous and uniform consideration of particular criteria described both quantitatively and qualitatively. The most suitable for this task is the representation of parameters and variables of the optimization model in the form of connected areas [1-3] on the scale of possible values for each parameter in particular cases. The input data are given in the form of intervals [4-5], the modelling results will also have interval values, i.e. contain an interval uncertainty.

At the early stages of design, the designers usually lack the precise data on preferential relations between alternative variants, and they can evaluate the degree of preference between pairs of variants by means of some value of the interval [0;1]. This subjective measure of using this or other system variant will be considered as fuzzy preference relation [5-7].

To compare the heterogeneous criteria values, it is necessary to establish fuzzy preference relations in the interval form. To do so, let us introduce the following expressions on the basis of [2,4,5]:

\[ S = \{S_\alpha, \alpha = 1, n\} \] is the set of possible structural variants of a radio-electronic system;

\[ K_i(S_\alpha) = [K_i(S_\alpha); K_i(S_\alpha)] \] is the set of partial optimality criteria given in the interval form, each of the criteria defines corresponding system variant \( S_\alpha \), where \( K_i(S_\alpha) \) is the lower interval limit of criterion assessment, and \( K_i(S_\alpha) \) - the upper interval limit, \( i = 1, \alpha = 1, n \);

\[ K(S_\alpha) = \{K_1(S_\alpha), K_2(S_\alpha), \ldots, K_n(S_\alpha)\} = \left[[K_1(S_\alpha); K_1(S_\alpha)], [K_2(S_\alpha); K_2(S_\alpha)], \ldots, [K_n(S_\alpha); K_n(S_\alpha)]\right] \] is the value of the vector criterion which defines each separate variant of the system.

It is necessary to reduce the criteria to a common type convenient for comparison and to find possible variants of building the efficient system structure, where for elements \( S^p_{\alpha_i} \) the conditions should be met that depend on the set task:

\[ \mu_i = \min_{i=1, \alpha_i, p} \left[ K_i(S_\alpha) \right], S^p_{\alpha_i} \in S^p, \quad (1) \]

or

\[ \mu_i = \max_{i=1, \alpha_i, p} \left[ K_i(S_\alpha) \right], S^p_{\alpha_i} \in S^p, \quad (2) \]

for the case when scalar criteria of optimality \( K_i(S_\alpha) = [K_i(S_\alpha); K_i(S_\alpha)] \) have the interval form.

3. Establishing preference patterns in the interval form on the set of structural system variants characterized by heterogeneous scalar criteria of optimality.

When analyzing the sets of pairs \( S_k \) and \( S_j \), which characterize structural variants of system \( S = \{S_\alpha, \alpha = 1, n\} \), similarly to fuzzy preference relations [1,2], it is suggested to introduce fuzzy preference relations in interval form \( R^\alpha K_i(S_k, S_j) \) for \( i^{th} \) partial optimality criterion \( K_i(S_\alpha) = [K_i(S_\alpha); K_i(S_\alpha)] \), \( i = 1, \alpha, \alpha = 1, n \), and for the pair of systems \( (S_k, S_j) \) the interval membership function will be applied: \( \mu^\alpha K_i(S_k, S_j) \). These functions \( \mu^\alpha K_i(S_k, S_j) \) according to studies [1,2,4-6], have the form:
The interval value of the criterion will show the permissible deviation of the quality of the system variant in a certain range.

The distinctive feature of the analyzed approach from the methods of fuzzy sets theory [1,2] is that the interval membership function is determined in the interval of [-1;1].

Thus, a fuzzy preference relation in interval form $R^\alpha$ on set $S_\alpha$ is the subset of the Cartesian product $(S_k \times S_l, \text{where } k = 1, n; l = 1, n; k \neq l)$ characterized by the interval membership function

$$\mu^\alpha K_i(S_k, S_l) : S_k \times S_l \rightarrow [-1;1].$$

The value of function $\mu^\alpha K_i(S_k, S_l)$ is understood as an objective measurement of the degree of performing relation $S_k R^\alpha S_l$ on scalar optimality criterion $K_i(S_\alpha) = \left[ K_i(S_k), K_i(S_l) \right], (i = 1, r, \alpha = 1, n)$ given in the interval form that characterizes each separate system variant $S_\alpha$, where:

$$\mu^\alpha K_i(S_k, S_l) \in [-1;0]$$

is the value that defines maximum degree of losses when accepting system $S_k$, dominating system $S_l$ in scalar interval optimality criterion $K_i$;

$$\mu^\alpha K_i(S_k, S_l) \in [0;1]$$

is the value defining the maximum degree of gain when accepting system $S_k$, dominating system $S_l$ in scalar interval optimality criterion $K_i$;

$$\mu^\alpha K_i(S_k, S_l) \in [-1;0]$$

is the absolute lack of dominance of system $S_k$ over system $S_l$ in scalar interval optimality criterion $K_i$;

$$\mu^\alpha K_i(S_k, S_l) \in [0;1]$$

is the absolute dominance of system $S_k$ over system $S_l$ in scalar interval optimality criterion $K_i$;

$$\left[ \mu^\alpha K_i(S_k, S_l); \mu^\alpha K_i(S_k, S_l) \right] \in [-1;1]$$

is the interval value that defines the degree of gain and the degree of losses when accepting the system $S_k$, dominating system $S_l$ in scalar interval optimality criterion $K_i$.

Let us introduce the relations of rigid interval preference of system $S_k$ over system $S_l$ and define it by membership function $\mu^\alpha_o K_i(S_k, S_l)$ that characterize the intensity of dominance of the system $S_k$ over the system $S_l$ in $i$-th particular interval optimality criterion as follows:

$$\mu^\alpha_o K_i(S_k, S_l) = \mu^\alpha K_i(S_k, S_l) - \mu^\alpha K_i(S_l, S_k) - \mu^\alpha K_i(S_k, S_l) - \mu^\alpha K_i(S_k, S_l) - \mu^\alpha K_i(S_k, S_l) =$$

$$= \left( \min \left[ \mu^\alpha K_i(S_k, S_l) - \mu^\alpha K_i(S_l, S_k) - \mu^\alpha K_i(S_k, S_l) - \mu^\alpha K_i(S_k, S_l) - \mu^\alpha K_i(S_k, S_l) \right] : \max \left[ \mu^\alpha K_i(S_k, S_l) - \mu^\alpha K_i(S_l, S_k) - \mu^\alpha K_i(S_k, S_l) - \mu^\alpha K_i(S_k, S_l) - \mu^\alpha K_i(S_k, S_l) \right] \right)$$

Let us introduce the relation of interval non-dominance of system $S_k$ over system $S_l$ and determine its by membership function $\mu^\alpha_{nd} K_i(S_k, S_l)$, as an addition to $\mu^\alpha_o K_i(S_k, S_l)$, in the form:

$$\mu^\alpha_{nd} K_i(S_k, S_l) = \begin{cases} 1, & \text{if } \mu^\alpha_o K_i(S_k, S_l) < 0 \\ 1 - \mu^\alpha_o K_i(S_k, S_l), & \text{if } \mu^\alpha_o K_i(S_k, S_l) \geq 0 \end{cases}$$

The membership function of the set of non-dominated systems $\mu^\alpha_o K_i(S_k)$ will determine the degree of non-dominance of system $S_k$ by any other system for the $i$-th scalar partial optimality criterion [1-3] and will have the form:


\[ \mu_{D}^{*}K_{i}(S_k) = \min \mu_{ND}K_{i}(S_k, S_j). \]  

(6)

Membership function \( \mu_{D}^{*}K_{i}(S_k) \) will show the degree of proximity of the system variant for \( i^{th} \) partial optimality criterion.

Value \( \mu_{D}^{*}K_{i}(S_k) \) is regarded as a preference measure that ensures objective and adequate comparison of complex systems which is characterized by heterogeneous criteria in the interval form and determines the priority value in the process of choosing the optimal system variant [1,3].

4. Sample multi-criteria selection of the optimal variant of a radio-electronic system.
Radio-electronic systems belong to the class of complex technical systems and their design is not an easy task.

As it has already been mentioned above, at the early stages of design, it is often the case that a big number of the criteria, for example, of the examined complex radio-electronic systems are given as the variations ranges of different values. These may be the working frequencies range, frequency passband, range of action in different modes, operation limits of radio-electronic system parameters, weight, costs, etc. The parameters that determine reliability, interference immunity, possibility for improvement, application efficiency can also be represented by means of some subjective characteristics in the interval form.

Let us analyze the example of three system variants, which are characterized by four quality subjective criteria given in Table 1.

| Criteria | System variants |
|----------|-----------------|
| \( K_1(S_o) \) - cost (roubles, thousands) | \( S_1 \) | \( S_2 \) | \( S_3 \) | \( m_i \) |
| \( K_2(S_o) \) - reliability, mean time between failures (hours, thousands) | [500;700] | [400;900] | [550;650] | 1000 |
| \( K_3(S_o) \) - interference immunity (degree) | [40;70] | [60;90] | [50;80] | 100 |
| \( K_4(S_o) \) - mass (kg) | [5;7] | [4;6] | [6;8] | 10 |

The criteria are presented in the interval form. It is necessary to select the optimal system variant, satisfying the following conditions:

\[ K_1(S_o) = \min_{a \in I} \left[ K_1(S_a) \right], \quad K_2(S_o) = \min_{a \in I} \left[ K_2(S_a) \right] \]  

(7)

\[ K_3(S_o) = \max_{a \in I} \left[ K_3(S_a) \right], \quad K_4(S_o) = \max_{a \in I} \left[ K_4(S_a) \right] \]  

(8)

The criteria have heterogeneous character, different measurement scales \( m_i \) and ranges of quality deviation. Apart from that, conditions (7) are the polar opposites of conditions (8). The analysis results are transferred into a special evaluative matrix. When comparing systems \( S_k \) and \( S_l \), \( k \)-systems should go in rows and \( l \)-systems in columns.

Using the expressions from studies [1,2,4-8], let us determine \( \mu_{D}^{*}K_{i}(S_k, S_l) \) and \( \mu_{D}^{*}K_{i}(S_k, S_l) \), for each criterion that are then entered into Table 2.

| System variants Sk | System variants Sl |
|--------------------|--------------------|
| \( \mu_{D}^{*}K_{i}(S_k, S_l) \) | \( \mu_{D}^{*}K_{i}(S_k, S_l) \) |
| S1 | S2 | S3 | S1 | S2 | S3 |
| S1 | - | [-0.2; 0.1] | [-0.05; 0.05] | - | -0.1 | 0 |
| S2 | [-0.1; 0.2] | - | [-0.15; 0.25] | 0.1 | - | 0.1 |
| S3 | [-0.05; 0.05] | [-0.25; 0.15] | - | 0 | -0.1 | - |

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Using expressions from [4, 5], let us find values \( \mu_{\mu_0} K_1(S_1, S_1) \) and \( \mu_{\mu_0} K_1(S_1, S_1) \) for each criterion that are then entered into Table 3.

| System variants \( S_k \) | \( \mu_{\mu_0} K_1(S_k, S_k) \) | \( \mu_{\mu_0} K_1(S_k, S_1) \) | \( \mu_{\mu_0} K_1(S_1, S_k) \) | \( \mu_{\mu_0} K_1(S_1, S_1) \) |
|--------------------------|----------------------|----------------------|----------------------|----------------------|
| S1                       | -                    | -0.4                 | -0.2                 | 1                    |
| S2                       | [0.2; 0.2]           | [0.1; 0.1]           | 0.4                  | -0.2                 |
| S3                       | [0.1; 0.1]           | [0.1; -0.1]          | 0.2                  | -0.2                 |

\( \mu_{\mu_0} K_3(S_k) \) for each criterion, the data are entered into Table 4.

| System variants \( S_k \) | \( \mu_{\mu_0} K_3(S_k) \) | \( \mu_{\mu_0} K_3(S_1) \) | \( \mu_{\mu_0} K_3(S_2) \) | \( \mu_{\mu_0} K_3(S_3) \) |
|--------------------------|----------------------|----------------------|----------------------|----------------------|
| S1                       | 0.6                  | 0.8                  | 0.8                  | 0.6                  |
| S2                       | 0.8                  | 1                    | 0.7                  | 0.3                  |
| S3                       | 0.8                  | 0.6                  | 1                    | 0.6                  |

Thus, all interval optimal criteria are presented in a uniform pattern which is suitable for comparison. When analyzing \( \mu_{\mu_0} K_3(S_k) \) and \( \mu_{\mu_0} K_3(S_1) \), let us compar pairs of systems \( S_k \) and \( S_1 \), rejecting the inefficient variants, it is possible to come to the target variant in the end.

5. Discussion and results
The range of possible solutions of output variables of the optimization model, calculated with the interval methods, can often be quite wide. However, this problem can partially be solved in case there is an apriori objective, or heuristic information on the distribution of possible values within the

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interval. This also allows one to further enlarge the opportunities of quantitative analysis of the optimization, which allows, by means of comparing the couples of alternative variants, to choose the optimal structure of a complex technical system. One of the forms of representation of the possible distribution of values within the interval is the form of fuzzy sets. The use of membership function makes this approach subjective, but it does not require a huge set of data sampling which significantly simplifies the process of decision-making at the early stages of designing a complex technical system.

The numerical expression of the analyzed criteria in the form of interval values allows to show the degree of deviation of the system variant, ranging from the minimum to the maximum. In this situation it is important to set value $m_i$. If necessary, the following can be used as $m_i$: limit values of optimality criteria and the reference system; limit values of optimality criteria that are to be achieved in optimization problems; in control problems - limit values of controlled parameters, and others.

6. Conclusion
The conducted study allowed one to determine preferential relations among various alternative choices of a system variant on the basis of heterogeneous criteria of optimality, presented in partially formalized and interval forms. The process of choosing the optimal variant can be conducted with the use of a heterogeneous vector criterion of optimality. The suggested approach can be employed in solving various applied decision-making problems. Higher efficiency of the suggested method can be achieved by applying it at the early stages of designing complex technical systems, as well as in problems of comparative analysis of existing products of high-technology engineering.

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