Signal Self-nulling Based DOA Estimation Method Using Acoustic Vector Sensor Array

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Abstract. The acoustic vector sensor (AVS) array is a powerful tool for direction of arrival (DOA) estimation of underwater targets. However, traditional DOA estimation algorithms generally suffer from low signal-to-noise ratio (SNR) as well as snapshot deficiency. A novel signal self-nulling based DOA estimation method is proposed. Firstly, achieve the crude estimation of the desired DOA using the traditional algorithms. Secondly, set the observation angular interval around the crudely estimated DOA. Thirdly, make the observation direction vary in the observation angular interval, and for each imaginary DOA, calculate the amplitude response of the minimum variance distortionless response (MVDR) beamformer. Finally, the pseudo spatial spectrum is achieved. Computer simulations verify that the proposed method is efficient in DOA estimation, especially in low SNR and insufficient snapshot data scenarios.

1. Introduction

The acoustic vector sensor (AVS) consists of an omnidirectional acoustic pressure receiver and a dipole-like directional particle velocity receiver [1]. AVS measures the three Cartesian components of the particle velocity as well as the scalar acoustic pressure in sound field synchronously and independently [2]. The main advantage of AVS is that it collects more acoustic information, hence it should outperform the traditional scalar sensor in detection and estimation. Since Nehorai and Paldi first introduce the AVS array measurement model to the signal processing research community [3], diverse types of DOA estimation algorithms have been proposed. Hawkes and Nehorai adapt the Capon approach to AVS array [4]. Wong and Zoltowski link the subspace based methods, which include the estimation of signal parameters via rotational invariance technique (ESPRIT) [5], root multiple signal classification (MUSIC) [6] and self-initiating MUSIC [7] to the AVS array. A 2-D DOA estimation algorithm using the propagator method (PM) is proposed in [8].

In the real ocean environment, the signal-to-noise ratio (SNR) is usually quite low and the snapshot data is usually insufficient. These disadvantages may lead serious performance degradation for DOA estimation using the traditional techniques. To overcome these problems, a number of new algorithms have appeared in the literature [9-14]. Ichige et al. put forward a modified MUSIC algorithm by using both the amplitude and phase information of noise subspace [9]. [10] improved the DOA estimation performance by precisely estimating the noise subspace through signal covariance matrix reconstruction. With the help of optimization method, [11] presented a noise subspace-based iterative algorithm for direction finding. Recently, a few new techniques were combined with DOA estimation, such as the sparse recovery algorithm [12], the sparse decomposition technique [13] and the compressive sensing theory [14].
In [15], Chen et al. took advantage of the phenomenon of signal self-nulling, which exists in the minimum variance distortionless response (MVDR) beamforming, and proposed a novel DOA estimation algorithm. However, all of the methods mentioned in [9-15] are based on the acoustic pressure sensor array, that is to say, only the acoustic pressure is utilized in signal processing. This would cause the port and starboard ambiguity. In this paper, the signal self-nulling based DOA estimation is extended to the AVS array, which can eliminate the ambiguity. What’s more, compared with the conventional DOA estimation methods, the proposed algorithm has a better performance in low SNR or insufficient snapshot data scenarios.

2. Measurement model
We consider a horizontal linear array which consists of $M$ acoustic vector sensors, with a uniform element spacing $d$. Let $K$ mutually uncorrelated narrowband point sources with common centre frequency $\omega$ be located at azimuths $\phi_k$ and elevations $\theta_k$ ($k = 1, 2, \ldots, K$) with respect to the first sensor of the array. In addition, $\phi_k \in [-\pi, \pi)$, $\theta_k \in [0, \pi]$. The AVS array is assumed to be in the far field with respect to all sources, ensuring that the wave fronts at the array are planar.

The source signal at the first sensor is defined as

$$s_k(t) = \eta_k(t) \exp(i\omega t)$$

(1)

where $\eta_k(t)$ is a zero mean random process, which denotes the slowly varying random pressure envelope of the $k$th source signal.

Let $a(\phi_k)$ represent the $M \times 1$ sized steering vector of an equivalent pressure sensor array, i.e., an array with all of the vector sensors being replaced by pressure sensors hypothetically. Thus we have

$$a(\phi_k) = [1, e^{-i2\pi \cos \phi_k/\lambda}, \ldots, e^{-i(2\pi/\lambda)(M-1)\cos \phi_k/\lambda}]^T$$

(2)

where $\lambda$ stands for the wavelength. Besides, let $u_k$ represent the $4 \times 1$ sized response vector of a single AVS to the $k$th source, which is defined as [16]

$$u_k = [1, \cos \phi_k, \cos \theta_k, \sin \phi_k, \sin \theta_k]^T$$

(3)

The output of the $m$th sensor at the moment of $t$ is a $4 \times 1$ sized vector, which is expressed by [4]

$$x_m(t) = \sum_{k=1}^{K} a_m(\phi_k) u_k s_k(t) + n_m(t)$$

(4)

where $a_m(\phi_k)$ is the $m$th element of $a(\phi_k)$, and $n_m(t)$ represents the noise vector of the $m$th sensor. The source signal and noise are statistically independent.

The output of the AVS array can be written as

$$X(t) = [x_1(t), x_2(t), \ldots, x_M(t)]^T = [a(\phi_1) \otimes u_1, a(\phi_2) \otimes u_1, \ldots, a(\phi_K) \otimes u_1] S(t) + N(t)$$

(5)

where $S(t) = [s_1(t), s_2(t), \ldots, s_K(t)]^T$ and $N(t) = [n_1^T(t), n_2^T(t), \ldots, n_K^T(t)]^T$. We define the steering vector of the AVS array, which is represented by $\psi(\phi_k)$ as the Kronecker product of $a(\phi_k)$ and $u_k$. That is to say,

$$\psi(\phi_k) = a(\phi_k) \otimes u_k$$

(6)

Thus (5) can be rewritten as

$$X(t) = [\psi(\phi_1), \psi(\phi_2), \ldots, \psi(\phi_K)] S(t) + N(t)$$

$$= \Psi S(t) + N(t)$$

(7)
3. Algorithm

3.1 Signal self-nulling

With regard to MVDR beamforming method, the problem of solving the optimal weight vector \( \mathbf{w} \) can be expressed as

\[
\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_n \mathbf{w}, \quad \text{s.t.} \quad \mathbf{w}^H \psi(\bar{\phi}) = 1
\]  

(8)

where \( \bar{\phi} \) denotes the observation direction, and \( \psi(\bar{\phi}) \) denotes the corresponding observation steering vector. With the help of the Lagrange multiplier approach, \( \mathbf{w} \) can be solved out that

\[
\mathbf{w} = \frac{\mathbf{R}^{-1}_n \psi(\bar{\phi})}{\psi^H(\bar{\phi}) \mathbf{R}_n^{-1} \psi(\bar{\phi})}
\]  

(9)

In (8) and (9), \( \mathbf{R}_n \) stands for the covariance matrix of the noise, i.e. \( E\{\mathbf{N}(t)\mathbf{N}^H(t)\} \), which is usually inestimable. Therefore we replace \( \mathbf{R}_n \) by the estimation of data covariance matrix, which is

\[
\hat{\mathbf{R}} = \frac{1}{N} \sum_{n=1}^{N} \mathbf{X}(n)\mathbf{X}^H(n)
\]  

(10)

where \( N \) denotes the number of snapshots.

The beam amplitude response of MVDR beamformer is defined as

\[
|H(\phi)| = \mathbf{w}^H(\bar{\phi}) \psi(\phi) = \left| \frac{\psi^H(\bar{\phi}) \hat{\mathbf{R}}^{-1} \psi(\phi)}{\psi^H(\bar{\phi}) \mathbf{R}_n^{-1} \psi(\bar{\phi})} \right|
\]  

(11)

Let \( \Phi \) represent the desired direction, i.e., the true DOA. Thus \( \psi(\bar{\phi}) \) denotes the corresponding desired steering vector. Consider a \( \bar{\phi} \)-centred angular interval \( \Phi = [\bar{\phi} - \Delta \phi, \bar{\phi} + \Delta \phi] \), where \( \Delta \phi \) is a small degree. If \( \bar{\phi} \neq \phi \), the MVDR beamformer would treat the desired signal as an interference signal and suppress it, thus in the beam pattern of \( |H(\phi)| \), there will exist a steep null in \( \Phi \). This phenomenon is the so-called signal self-nulling.

On the contrary that if \( \bar{\phi} = \phi \), according to the constraint in (8), \( |H(\phi)| \) will approximately equal to 1 in \( \Phi \). This characteristic of \( |H(\phi)| \) can be exploited in finding the true DOA.

3.2 Proposed Algorithm

When \( \bar{\phi} = \phi \), define the minimum of \( |H(\phi)| \) within the interval \( \Phi \) as \( \hat{H}_{\text{min}} \), which is expressed by

\[
\hat{H}_{\text{min}} = \min_{\phi \in \Phi} \left| \frac{\psi^H(\bar{\phi}) \hat{\mathbf{R}}^{-1} \psi(\phi)}{\psi^H(\bar{\phi}) \mathbf{R}_n^{-1} \psi(\phi)} \right|
\]  

(12)

Meanwhile, when \( \bar{\phi} \neq \phi \), define the minimum of \( |H(\phi)| \) within the interval \( \Phi \) as \( \overline{H}_{\text{min}} \), which is expressed by

\[
\overline{H}_{\text{min}} = \min_{\phi \in \Phi} \left| \frac{\psi^H(\bar{\phi}) \hat{\mathbf{R}}^{-1} \psi(\phi)}{\psi^H(\bar{\phi}) \mathbf{R}_n^{-1} \psi(\bar{\phi})} \right|, \quad \bar{\phi} \neq \phi
\]  

(13)

According to the previous analysis, it can be drawn that

\[
\hat{H}_{\text{min}} \gg \overline{H}_{\text{min}}
\]  

(14)
(14) indicates that within $\Phi$, if and only if $\vec{\varphi} = \hat{\varphi}$, or the observation steering vector matches the desired steering vector, the minimum of the beam amplitude response reaches a maximum. Therefore we can construct such a worst-case performance optimization problem as

$$\max_{\vec{\varphi}} \min_{\varphi \in \Phi} \frac{\psi^H(\vec{\varphi}) \hat{R}^{-1} \psi(\varphi)}{\psi^H(\vec{\varphi}) \hat{R}^{-1} \psi(\hat{\varphi})} \quad \text{s.t.} \vec{\varphi} \in \Phi \tag{15}$$

We can regard the maximum solving problem in (15) as a spectral peak searching problem, and define the pseudo spatial power spectrum as

$$P(\vec{\varphi}) = \min_{\varphi \in \Phi} \frac{\psi^H(\vec{\varphi}) \hat{R}^{-1} \psi(\varphi)}{\psi^H(\vec{\varphi}) \hat{R}^{-1} \psi(\hat{\varphi})}, \quad \vec{\varphi} \in \Phi \tag{16}$$

The steps of the proposed algorithm are as follows:

1) Achieve the crude estimation of the desired DOA using the traditional algorithms such as MVDR or MUSIC;

2) Set the observation angular interval $\Phi = [\hat{\varphi} - \Delta \varphi, \hat{\varphi} + \Delta \varphi]$;

3) Make the observation direction $\varphi$ vary in $\Phi$, and for each $\varphi$, calculate the amplitude response of the MVDR beamformer, $|H(\varphi)|$, and find the minimum of it;

4) For each $\varphi$, searching the peak of the pseudo spatial power spectrum defined in (15). The corresponding direction is the estimation of true DOA.

### 4. Simulations

Consider an 8-element horizontal linear AVS array, and $d = \lambda / 2$. Assume that two mutually uncorrelated source signals with equal power impinging on the array, and the azimuths of the signals are $30^\circ$ and $60^\circ$ respectively. As we mainly concern on the horizontal DOA, to simply the problem, assume that both elevations are $90^\circ$ and are pre-known so that the array and the sources are in the same horizontal plane. We treat $30^\circ$ as the desired DOA. The observation angular interval is set to be $[25^\circ, 35^\circ]$. The searching step size is $0.01^\circ$.

![Figure 1. SNR=15dB, N=200.](image1.png)

![Figure 2. SNR=-5dB, N=100.](image2.png)

Firstly, we compare the spatial spectra of the proposed method as well as several conventional DOA estimation algorithms, including MVDR, MUSIC, the propagation method based on array data (PM-data), and the propagation method based on covariance matrix (PM-cm). Figure 1 displays the spatial spectra with SNR = 15dB and $N = 200$. It is indicated that for every technique, there exists a peak around $30^\circ$. However, the spectrum peak of the proposed method is much sharper than others. Figure 2 shows the spatial spectra under deteriorated conditions, i.e., SNR = -5dB, $N = 100$. From figure 2, we can find that the spectrum peaks of MUSIC and PM-cm deviate the true DOA and that the
spatial spectra of MVDR and PM-data are nearly flat. Unlike these methods, the spatial spectrum of the proposed method still displays a clear peak around 30°.

Next we adopt 100 times of Monte Carlo trials to assess the DOA estimation performances of the above mentioned algorithms. The root mean square error (RMSE) is defined as

$$\text{RMSE} = \frac{1}{\sqrt{100}} \sum_{m=1}^{100} (\hat{\phi}_m - \phi)^2$$  \hspace{1cm} (17)

where $\hat{\phi}_m$ stands for the estimator of the true DOA in the $m$th Monte Carlo trial. In figure 3, we compare the performances of the five algorithms when SNR ranges from -25 dB to 15 dB, and we set $N$ to be 50. Figure 3 illustrates that all of the performances degrade with SNR decreasing, but the proposed method performs the best under low SNR. In figure 4, we compare the performances of the five algorithms when the number of snapshots ranges from 50 to 500, and we set SNR to be -10dB. It is clearly indicated that the proposed method outperforms the traditional ones.

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{fig3.png}
\caption{RMSE versus SNR, $N=50$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{fig4.png}
\caption{RMSE versus number of snapshots, SNR=-10dB.}
\end{figure}

5. Conclusions

A novel signal sell-nulling based DOA estimation method is proposed in this paper. By calculating the minimum of the MVDR amplitude response for every discrete observation direction in a given angular interval, we can obtain a pseudo spatial power spectrum. The pseudo power spectrum shows a clear peak even when suffers from low SNR or insufficient snapshots. Computer simulation results also verify that the proposed method performs the best when compared with the traditional DOA estimation algorithms.

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