Lost in Embedding Space: Explaining Cross-Lingual Task Performance with Eigenvalue Divergence

Haim Dubossarsky\textsuperscript{1}, Ivan Vulič\textsuperscript{1}, Roi Reichart\textsuperscript{2}, Anna Korhonen\textsuperscript{1}
\textsuperscript{1} Language Technology Lab, University of Cambridge
\textsuperscript{2} Faculty of Industrial Engineering and Management, Technion, IIT
\{hd423, iv250, alk23\}@cam.ac.uk roiri@ie.technion.ac.il

Abstract

Performance in cross-lingual NLP tasks is impacted by the (dis)similarity of languages at hand: e.g., previous work has suggested there is a connection between the expected success of bilingual lexicon induction (BLI) and the assumption of (approximate) isomorphism between monolingual embedding spaces. In this work, we present a large-scale study focused on the correlations between language similarity and task performance, covering thousands of language pairs and four different tasks: BLI, machine translation, parsing, and POS tagging. We propose a novel language distance measure, Eigenvalue Divergence (EVD), which quantifies the degree of isomorphism between two monolingual spaces. We empirically show that 1) language similarity scores derived from embedding-based EVD distances are strongly associated with performance observed in different cross-lingual tasks, 2) EVD outperforms other standard embedding-based language distance measures across the board, at the same time being computationally more tractable and easier to interpret. Finally, we demonstrate that EVD captures information which is complementary to typologically driven language distance measures. We report that their combination yields even higher correlations with performance levels in all cross-lingual tasks.

1 Introduction

The effectiveness of joint multilingual modeling and cross-lingual transfer in cross-lingual NLP is critically impacted by the actual languages in consideration (Bender, 2011; Ponti et al., 2019). Characterizing and understanding this cross-language variation is often the first step towards the development of more robust multilingually applicable NLP technology (O’Horan et al., 2016; Bjerva et al., 2019; Ponti et al., 2019). For instance, selecting suitable source languages is a prerequisite for successful cross-lingual transfer of dependency parsers or POS tags (Naseem et al., 2012; Ponti et al., 2018; de Lhoneux et al., 2018).\textsuperscript{1}

Bilingual lexicon induction (BLI) is a cross-lingual task that received plenty of attention in particular as a case study to investigate the impact of cross-language variation on task performance (Søgaard et al., 2018; Artetxe et al., 2018; Alvarez-Melis and Jaakkola, 2018). BLI has become increasingly popular lately as: 1) it serves as a valuable intrinsic evaluation task to measure the quality of cross-lingual word embedding models, showing strong performance correlation with downstream tasks such as natural language inference and cross-lingual information retrieval (Glavaš et al., 2019); 2) it boosts transfer learning for many downstream tasks, especially for resource-poor languages (Duong et al., 2016; Adams et al., 2017; Vulič et al., 2017). Another, more pragmatic reason of BLI popularity lies in its simplicity and reduced resource requirements, which makes it widely applicable across a large number of language pairs. Mapping-based BLI methods are the current state-of-the-art paradigm in low-resource setups. They require only independently trained monolingual word embeddings for the source and target languages, and a small seed dictionary. Based on the seed translations, they learn a global mapping function to induce the translations for all words in the shared cross-lingual space obtained by the mapping (Mikolov et al., 2013; Ruder et al., 2019b).

However, there is a huge variance in BLI performance across different language pairs and BLI

\textsuperscript{1}In another example, with all other factors kept similar (e.g., training data size, domain similarity), the quality of machine translation also depends heavily on the linguistic properties and language proximity of the actual language pair (Lin et al., 2019; Kodagunta et al., 2019).
models; it was empirically verified that for some language pairs BLI performs remarkably well, and for others rather poorly (Søgaard et al., 2018; Vulic et al., 2019). Prior research attempted to explain this variance in performance, and grounded it in the differences between the monolingual embedding spaces themselves. These studies introduced the notion of isomorphism, and argued that it is easier to learn a mapping function between language pairs whose embeddings are approximately isomorphic, than between languages that are not (Barone, 2016; Søgaard et al., 2018). Subsequently, novel methods to quantify the degree of isomorphism were proposed, and were shown to significantly correlate with BLI scores (Zhang et al., 2017; Søgaard et al., 2018; Patra et al., 2019).

In this work, we contribute to this research endeavor and propose a new isomorphism measure: Eigenvalue Divergence (EVD). First, we report much higher correlations with BLI scores than currently used isomorphism measures across a variety of mapping-based BLI approaches. While previous work was limited only to coarse-grained analyses with a small number of language pairs (i.e., < 10), our study is the first large-scale analysis focused on the relationship between (quantifying) isomorphism and BLI performance. We run the analysis across hundreds of diverse language pairs, focusing both on typologically distant pairs as well as on subsets of similar languages. All experimental runs indicate higher correlations when using EVD.

Importantly, we also show that our findings generalize beyond the core BLI task. We again use EVD scores computed on monolingual fastText Wikipedia embeddings as a proxy measure to characterize language similarity. We show that isomorphism quantified through EVD better correlates and explains greater variance than other isomorphism measures and standard approaches based on typological information (Littell et al., 2017) in cross-lingual transfer experiments such as dependency parsing and POS tagging. We also demonstrate that EVD correlates well with machine translation performance. What is more, our results suggest that EVD captures language (dis)similarity information that is complementary to typological information, and further improvements can be achieved by combining the two.

2 EVD: Eigenvalue Divergence

Word embedding models, and distributional semantic models in general, are grounded in the distributional hypothesis (Harris, 1954; Firth, 1957). That is, they learn the meaning of words according to the words’ occurrence in the context of other words. Consequently, we expect that a word which is more distributed in text and appears in diverse contexts would be “loaded” with more meanings than a word whose usage is more restricted (i.e., it appears in fewer contexts). Therefore, a language whose words are used in more diverse contexts is expected to show greater variance (or covariance) in its d-dimensional embedding space. In contrast, in a language whose usage dynamic is more constrained we expect to see the opposite pattern. We hypothesize that a decomposition of the embedding space spanned by the word vectors using Principal Component Analysis (PCA) should distinguish between these two hypothetical language dynamics.

The covariance of an embedding matrix $X$ is given by $C = X^T X$, which after PCA diagonalization can be written as $VLV^T$.\(^2\) This gives us $V$, a matrix of eigenvectors, and $L$, a diagonal matrix with eigenvalues $\lambda_i$ in a decreasing order.

Essentially, these eigenvalues represent the loading for their corresponding eigenvectors. Thus, having larger values in the original covariance matrix leads to larger eigenvalues on the diagonal of the components that reconstruct the original matrix. Due to their descending order, a slight gradual decline in the eigenvalues means that many eigenvectors are important to reliably reconstruct the originally induced embedding space. On the other hand, a steep decline in the eigenvalues means an equally reliable reconstruction can be achieved with fewer eigenvectors.

We then formally define the Eigenvalue Divergence measure (EVD), the distance between two embedding matrices $L_1$ and $L_2$ corresponding to two languages $L_1$ and $L_2$, as the squared Euclidean distance between their corresponding vectors of eigenvalues after log transformation:

$$EVD(L_1, L_2) = \sum_{i=1}^{d} (\log \lambda^{L_1}_i - \log \lambda^{L_2}_i)^2$$

(1)

$\lambda^{L_1}_i$ and $\lambda^{L_2}_i$, $i = 1, \ldots, d$ are the eigenvalues

\(^2\)We assume that $X$ is mean-centered, that is, column means have been subtracted and are equal to zero.
characterizing the two starting embedding matrices $L_1$ and $L_2$, computed as previously described. We can then define a loose measure of language distance simply as $LD(L_1, L_2) = EVD(L_1, L_2)$. We argue that the EVD measure intuitively captures the degree of complexity in the dynamics of language use between words and their contexts. This notion has already found support in other scientific domains, such as computational neuroscience, where this divergence is used as a marker for statistical dependence between stimuli and determines correlations across diverse stimuli (Stringer et al., 2019).

3 Related Work

Quantifying Isomorphism. Several studies have attempted to validate the isomorphism assumption empirically, developing methods to measure the distance between embedding spaces. Søgaard et al. (2018) proposed a graph-based measure that quantifies isospectrality (IS) of the respective nearest neighbor graphs (created from $L_1$ and $L_2$) via their Laplacian matrices. They argue that Laplacian eigenvalues are compact representations of global properties of the graphs that can consequently capture the degree of (approximate) isomorphism. The IS measure has also been used as the main isomorphism measure in the recent work of Ormazabal et al. (2019), which compared mapping-based versus jointly trained cross-lingual word embeddings.

Patra et al. (2019) propose to use the Gromov-Hausdorff distance (GH) to measure how well two embedding spaces can be aligned under an isometric transformation. The two measures, GH and IS, are most similar to our work in spirit, and we use them as our main baselines throughout the work. The technical descriptions of the two methods are provided in the appendix.

Both distances were reported to have strong correlations with BLI performance, and argue that the isomorphism assumption weakens as the languages involved become more distant (phylogenetically). However, they also share several drawbacks. First, the correlations were computed on a very small number of language pairs (IS: 8 pairs, GH: 10 pairs). Second, they do not scale well computationally. Therefore, for computational tractability, the scores are computed only on the sub-matrices spanning the most frequent subsets of the full embedding spaces (IS: 10k words, GH: 5k words). Finally, we believe that using EVD also leads to increased interpretability in comparison to the two other measures.

Quantifying Language Distance/Similarity. At the same time, distances between language pairs can also be captured through (dis)similarities in their linguistic properties, such as overlap in syntactic features, or proximity along the phylogenetic language tree. The properties are typically extracted from available typological databases such as the World Atlas of Languages (WALS) (Dryer and Haspelmath, 2013) or URIEL (Littell et al., 2017). Such distances were found useful in guiding and informing cross-lingual transfer tasks (Cotterell and Heigold, 2017; Agić, 2017; Lin et al., 2019; Ponti et al., 2019). We show later in §5 that the proposed EVD measure correlates well with cross-lingual transfer scores across several tasks, and its usefulness can be further improved by combining it with the language distance measures based on different linguistic properties.

4 Experimental setup

Monolingual Word Embeddings. For EVD, GH and IS computations, we rely on publicly available 300-dimensional monolingual word embeddings pretrained on Wikipedia using the fastText method (Bojanowski et al., 2017) for all languages in our analyses. The embeddings are length-normalized and trimmed to the 200k most frequent words.

4.1 Isomorphism and Language Similarity

We probe the following isomorphism measures, discussed in §2 (EVD) and §3 (IS and GH). Remember that they can be also used as proxy measures of language similarity. We compute a full PCA transformation (i.e., no dimensionality reduction) for EVD, and calculate the EVD score following Eq. (1). For IS and GH, we replicate the exact experimental setup as prior work (see the appendix for further details and short descriptions of the two baseline methods): we compute the IS score over the top 10k most frequent words in Wikipedia is much cleaner or even hand-curated, and adheres to the rules of standard language (Grave et al., 2018). Note that we use Wikipedia embeddings to compute the three distance scores, while some of the BLI results reported below were obtained using different word embeddings which were trained on other corpora, including the Common Crawl data.
each respective monolingual space (Søgaard et al., 2018; Ormazabal et al., 2019), while the GH score is computed over the top 5k words from each monolingual space (Patra et al., 2019).⁴

Linguistic Properties. We also rely on three precomputed measures of language distance based on the URIEL typological database (Littell et al., 2017)⁵. Genetic distance (GEN) is derived from the hypothesized phylogenetic tree of language descent. Syntactic distance (SYN) is computed based on the syntactic structures of the languages from the WALS database (Dryer and Haspelmath, 2013). Geographic distance (GEO) is obtained from the locations where languages’ are spoken; see the work of Littell et al. (2017) for more details.

4.2 Bilingual Lexicon Induction

Mapping-based Methods. We compute correlations with BLI scores already available in related literature, e.g., from Glavaš et al. (2019) and Vulić et al. (2019). However, in order to extend the breadth of the analysis to more languages, we also run more BLI experiments using several well-established mapping-based approaches. 1) PROC is the standard supervised method from prior work (Artetxe et al., 2016; Smith et al., 2017) that learns a mapping by solving the orthogonal Procrustes problem (Schönemann, 1966). 2) VECMAP-SUPER is another standard supervised method that additionally applies a variety of preprocessing and post-processing steps (e.g., whitening, dewhitenning, symmetric re-weighting) before and after learning the mapping matrix (Artetxe et al., 2018). 3) VECMAP-UNSUPER is a fully unsupervised method based on the “similarity of monolingual similarities” heuristic to extract the seed dictionary from monolingual data. It then uses an iterative self-learning procedure to improve on the initial noisy dictionary (Artetxe et al., 2018). For more technical details on the fully unsupervised model, we refer the reader to prior work and recent studies (Ruder et al., 2019a; Vulić et al., 2019). The two methods based on the VECMAP framework⁶ have shown very competitive and robust BLI performance across a wide range of language pairs in recent comparative analyses (Glavaš et al., 2019; Vulić et al., 2019; Doval et al., 2019).

Language Pairs and Other BLI Studies. We analyze the results from two previous studies that report BLI scores for a large number of language pairs, but which do not delve deeper into the connection between isomorphism and BLI performance. First, Glavaš et al. (2019) run BLI experiments on 28 language pairs spanning 8 different languages. They derive their training and test dictionaries (5k and 2k translation pairs, respectively) automatically from Google Translate.

Vulić et al. (2019) run BLI experiments on 210 language pairs spanning 15 typologically diverse languages. Their training and test dictionaries (5k and 2k translation pairs) are derived from PanLex (Baldwin et al., 2010; Kamholz et al., 2014). We complement the original 210 pairs from Vulić et al. (2019) with additional 210 language pairs of 15 closely related (European) languages using dictionaries extracted from PanLex following the procedure of Vulić et al. (2019). Here, the aim is to probe if we can also capture more subtle/smaller language difference with the isomorphism measures.⁷

Finally, we also analyze the BLI results on the 108 language pairs available in the MUSE benchmark (Conneau et al., 2018). For the BLI experiments on MUSE and similar language pairs (PanLex), we produce scores with the three previously described mapping-based methods, relying on pretrained fastText Wikipedia embeddings. In sum, our analyses are conducted on three different BLI benchmarks that span 556 language pairs and cover both related and distant languages, and we analyse both supervised and unsupervised state-of-the-art mapping-based methods.⁸

BLI Evaluation Measure. Following prior work (Glavaš et al., 2019), our primary BLI evaluation

⁴https://github.com/joelmoniz/BLISS ⁵https://pypli.org/project/lang2vec ⁶https://github.com/artetxem/vecmap ⁷We run the additional BLI experiments on 210 language pairs composed of the following European languages: English, German, Dutch, Swedish, Danish (Germanic languages); Italian, Portuguese, Spanish, French, Romanian (Romance); Croatian, Polish, Russian, Czech, Bulgarian (Slavic). For the lists of language pairs involved in previous BLI studies, we refer the reader to prior work (Conneau et al., 2018; Glavaš et al., 2019; Vulić et al., 2019).

⁸We report all results for each BLI method, dictionary and language pair: https://tinyurl.com/skn5cfl. We also analyse BLI scores for 39 language pairs reported by (Zhang et al., 2019), which include another BLI method: RCSLS (Joulin et al., 2018). We do not include those results for brevity, as we observe exactly the same correlation patterns. RCSLS has also been benchmarked in the BLI task by Glavaš et al. (2019), and we analyze these scores later in Table 1.
measure is Mean Reciprocal Rank (MRR); we note that identical findings emerge from running the correlation analyses based on Precision@1 scores in lieu of MRR.

4.3 Downstream Tasks
Following the large-scale nature of our BLI analyses, we also run similar analyses on several downstream tasks that comprise a large number of (both similar and distant) language pairs. We rely on the results from the recent study of Lin et al. (2019) which focused on cross-lingual transfer performance for tasks such as dependency parsing, POS tagging, and machine translation. We use the six distance measures from §4.1 as measures of language distance $LD(L_1, L_2) = X(L_1, L_2)$, where $X = \{EVD, IS, GH, GEN, SYN, GEO\}$, and investigate to what extent performance in these tasks correlates with different distance measures.

Machine Translation. Lin et al. (2019) report BLEU scores when translating 54 source $L_1$ languages into English as the target language in two settings: 1) directly translating from $L_1$ to English (54 BLEU scores in total); 2) in a transfer learning setting, one of the 54 languages is the source language and another one is the transfer/pivot language, which resulted in a total of $54 \times 53 = 2,862$ language pairs. Running a correlation analysis with the six language distance metrics for the first setting is straightforward, whereas each MT run in the transfer setting involves three languages: source, transfer and target (English). We therefore define the decisive distance between source and target languages $L_1$ and $L_2$ as the shortest path either directly or through the pivot $L_p$: $LD(L_1, L_2) = \min(LD(L_1, L_2), LD(L_1, L_p) + LD(L_p, L_2))$, and use it in the analyses.

Dependency Parsing. Again, we base our analysis on the cross-lingual zero-shot parser transfer results of Lin et al. (2019): the standard biaffine dependency parser (Dozat and Manning, 2017; Dozat et al., 2017) is trained on the training portions of Universal Dependencies (UD) treebanks from 31 languages (Nivre et al., 2018), and is then used to parse the test treebank of each language, now used as the target language. We use the reported Labeled Attachment Scores (LAS) for all combinations of 31 languages, resulting in 930 source-target pairs.

POS Tagging. We also use POS tagging accuracy scores reported by Lin et al. (2019). These scores span 26 low-resource target languages and 60 transfer languages which measure the utility of each transfer language to each of the 26 target languages in POS tagging. We use a sample of 935 language pairs for the correlation analysis, as only 17 low-resource target languages and 55 transfer languages have their pretrained monolingual embeddings.

For further details regarding the models used to compute the scores for the downstream task, we refer the interested reader to the work of Lin et al. (2019) and the accompanying repository: https://github.com/neulab/langrank.

4.4 Correlation Analyses and Statistical Tests
We report Pearson’s correlation coefficients for each of the six distance measures for all cross-lingual tasks: BLI as well as the three downstream tasks. This allows us to investigate their individual degree of association in a particular task, and test which of the different distances is most important to predict performance in each task.

However, this simple analysis is not sufficient to account for the the multitudinous and complex interdependencies between the six distance measure themselves, and more importantly how these interact with the downstream tasks’ performances. For that purpose, we analyse the results in a standard linear stepwise regression model:

$$ Y = \beta_0 + \beta_1 x_1 + \ldots + \beta_n x_n + \epsilon \quad (2) $$

in which task performance, $Y$, is predicted using a set of regressors, $\beta_1, \ldots, \beta_n$, that are added to the model incrementally only if their marginal addition to predicting $Y$ is statistically significant. This method allows us to investigate which of the six distance measures overlap, and which complement each other so they could be used in tandem to better account for cross-lingual task performance. In order to allow meaningful comparison to the simple (single variable) correlations, we describe the results of this regression analysis as a “dummy” variable, $\hat{r}$, which represents the combined contribution of all statistically significant regressors as a unified correlation coefficient.

5 Analyses and Results
The results are summarized in Table 1 and Table 2. The first main finding is that the proposed
Table 1: Correlation scores with BLI performance using the six distance measures on three standard BLI datasets \(a, b, c\), each with three mapping-based BLI methods. Results for the best distance measure in each column are in \textbf{bold}. \(a\) Results for 210 language pairs taken directly from Vulić et al. (2019) + results for the additional 210 language pairs we trained for this study; \(b\) 108 language pairs (trained); \(c\) Results for 28 language pairs taken from Glavaš et al. (2019). \(d\) Stepwise regression unified correlation coefficient (superscripts indicate which of the six distances were found statistically significant and are included in the regression model). *EVD scores are computed on the first 40 eigenvalues which we empirically find to give slightly better results on BLI.

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
 & \multicolumn{3}{|c|}{PanLex\textsuperscript{a}} & & \multicolumn{3}{|c|}{MUSE\textsuperscript{b}} & & \multicolumn{3}{|c|}{GoogleTrans\textsuperscript{c}} \\
 & PROC & SUPER & UNSUPER & PROC & UNSUPER & PROC & UNSUPER & PROC & RCSLS & UNSUPER \\
\hline
1. EVD & \textbf{-744} & \textbf{-762} & \textbf{-737} & \textbf{-600} & \textbf{-601} & \textbf{-604} & \textbf{-631} & \textbf{-625} & \textbf{-586} & \\
2. GH & -412 & -408 & -461 & -432 & -325 & -278 & -503 & -494 & -527 & \\
3. IS & -508 & -528 & -490 & -422 & -424 & -344 & -330 & -333 & -343 & \\
4. GEN & -553 & -551 & -410 & -532 & -624 & -340 & -498 & -495 & -520 & \\
5. SYN & -565 & -522 & -379 & -466 & -640 & -436 & -568 & -575 & -434 & \\
6. GEO & -619 & -664 & -617 & -570 & -537 & -350 & -263 & -251 & -375 & \\
\hline
\end{tabular}

Table 2: Correlations of language distances and performance in downstream tasks \(a, b, c, d\). Results for best distance measure are in \textbf{bold}. \(a\) Results on 54 language pairs in the standard direct \(L_1 \rightarrow \) English translation task; \(b\) Results on 2,862 pairs for the MT task with pivot/transfer languages (see §4.3); \(c\) Results in the zero-shot dependency parsing task for 930 language pairs. \(d\) POS tagging results on 935 language pairs. \(e\) Stepwise regression unified correlation. \(p < .01\) due to the small number of language pairs.

\begin{tabular}{|c|c|c|c|c|}
\hline
 & MT\textsuperscript{a} & MT\textsuperscript{b} & DEP\textsuperscript{c} & POS\textsuperscript{d} \\
\hline
1. EVD & -558 & -550 & -791 & -561 & \\
2. GH & -146 & -194 & -697 & -443 & \\
3. IS & -129 & -163 & -666 & -395 & \\
4. GEN & -454 & -474 & -560 & -124 & \\
5. SYN & -657 & -642 & -397 & -053 & \\
6. GEO & -262 & -298 & -582 & -186 & \\
\hline
\end{tabular}

Bilingual Lexicon Induction. All distance measures show moderate to high correlations with BLI performance, with EVD showcasing the strongest correlation scores. This effect is attested with all training and test BLI dictionaries (PanLex, MUSE and Google Translate) (see §4.2 before). Owing to the large and different number of language pairs tested, this further demonstrates EVD’s robustness.

This pattern clearly indicates that all distances capture some important properties of embedding space similarity that are useful for BLI. However, the different distances might capture similar properties of the embedding space, or different ones, which could not be teased apart using the simple correlation analysis. The stepwise regression analysis addresses this point exactly, and reveals that
while the three embedding distances (EVD, GH, IS) capture overlapping properties of embedding space similarity, the linguistic distances (GEN, SYN, GEO) seem to capture complementary aspects of language similarity that are useful for this task. This conclusion is supported by the unified correlation coefficient (bottom row in Table 1) which indicates that for all different datasets and settings, EVD is only complemented with typological distance measures and not with the other two isomorphism-based measures. When combining EVD with the three typological distances, the regression model reaches impressive correlation scores, with >.85 for the PanLex dataset; with 420 language pairs, PanLex is the most comprehensive dataset in our study.

**Machine Translation.** Only EVD, GEN, and SYN show moderate to high correlation scores with MT performance (Table 2). Furthermore, although EVD does not have the highest correlation with MT performance, it still holds the strong advantage over GH and IS. In fact, it seems that EVD is the only isomorphism measure that captures some useful language similarity properties needed for this task. Moreover, we again observe the synergistic effect when combining EVD with typologically informed language distances, as shown by the unified correlation coefficient of the stepwise regression (bottom line in Table 2). This pattern of results was found for both the direct setting and the transfer setting (columns MT\textsuperscript{a} and MT\textsuperscript{b} in Table 2, respectively).

**Dependency Parsing.** All six distance measures display strong correlations with performance in the cross-lingual dependency parsing transfer. For the isomorphism measures, this association is not surprising given the fact that the dependency parser was trained with the same embeddings on which the different distances were computed. EVD is the most informative language distance measure by a large margin. Typologically driven distances fall behind the three isomorphism measures: however, combining GEN, SYN, and GEO with EVD leads to substantially improved correlation scores, while the same effect is not achieved when combining them with IS and GH through the stepwise regression analysis.

**POS Tagging.** Only the three isomorphism measures show moderate to high correlations with performance in the POS tagging task, with EVD again topping the other two. Although statistically significant, the unified correlation of EVD and IS adds only very little to EVD, pointing to the partially overlapping nature of the two distances.

### 6 Further Discussion and Conclusion

In this study we have revisited the (approximate) isomorphism hypothesis: it has originally been postulated to explain the performance variance observed in the BLI task (Zhang et al., 2017; Søgaard et al., 2018; Patra et al., 2019). The hypothesis posits that it is easier to learn a mapping between two languages if they “are used to convey thematically similar information in similar contexts” (Barone, 2016). We have used different methods to evaluate the similarities between languages captured either through quantitative isomorphism measures or typologically driven linguistic properties, and have tested the isomorphism hypothesis on a much larger scale than ever before. In fact, our analyses span thousands of language pairs, and we extend the analyses beyond the core BLI task to several cross-lingual downstream tasks. The results demonstrate the importance of similarity between languages to the performance levels on cross-lingual tasks, and affirm the validity of the isomorphism hypothesis.

This study has introduced a novel method, dubbed Eigenvalue Divergence (EVD), to quantify isomorphism between two monolingual embedding spaces. The EVD measure can by proxy also be used as a general method to capture language (dis)similarity. The method, which is based on a standard PCA decomposition of the languages’ embedding spaces, provides a language distance which is easier to interpret and compute in comparison to other standard isomorphism measures such as GH and IS. We also empirically demonstrate that EVD outperforms GH and IS across all tasks and settings it was tested on.\footnote{Less formally, one should prefer EVD over the other isomorphism measures not only due to its empirically validated superior performance, but also on the basis of parsimony (i.e., Occam’s razor).} Moreover, the usefulness of EVD has also been proven in an ensemble which combines it with typological distance measures: we show that combining EVD and the three linguistic distances accounts for a substantial portion of task performance in almost all tasks in our analysis (i.e., BLI, MT, and dependency parsing).
Importantly, while some of the tasks in our study (e.g., BLI and dependency parsing) do rely on the actual embeddings to train and test their models, other tasks (e.g., MT and POS tagging) do not, and are instead trained directly on the corpora. At the same time the superiority of EVD that we report, either alone or in combination with other distance measures, is maintained throughout the four different tasks, regardless of the embedding usage (or the lack of it). The results lend support to the idea that EVD may capture aspects of language similarities that are grounded in fundamental differences between the languages, and are not limited only to superficial differences between embedding spaces.

Finally, this study addresses the question of why certain language pairs are able to transfer well in cross-lingual tasks and others do not. Overall, our results strongly support the idea that similarity between languages, especially as evaluated using EVD, plays a key role in explaining cross-lingual task performance. Although we present only correlation analyses to back up our main claims, the fact that all analyses converge to the same result helps us to corroborate this conclusion. At the same time, this study does not focus on how this insight could be used to facilitate better transfer learning, which we leave for future research.

Acknowledgments

The work of IV and AK is supported by the ERC Consolidator Grant LEXICAL: Lexical Acquisition Across Languages (no 648909) awarded to AK. HD is supported by the Blavatnik Postdoctoral Fellowship Programme.

References

Oliver Adams, Adam Makarucha, Graham Neubig, Steven Bird, and Trevor Cohn. 2017. Cross-lingual word embeddings for low-resource language modeling. In Proceedings of EACL, pages 937–947.

Željko Agić. 2017. Cross-lingual parser selection for low-resource languages. In Proceedings of the NoDaLiDa 2017 Workshop on Universal Dependencies (UDW 2017), pages 1–10.

David Alvarez-Melis and Tommi Jaakkola. 2018. Gromov-Wasserstein alignment of word embedding spaces. In Proceedings of EMNLP, pages 1881–1890.

Mikel Arteste, Gorka Labaka, and Eneko Agirre. 2016. Learning principled bilingual mappings of word embeddings while preserving monolingual invariance. In Proceedings of EMNLP, pages 2289–2294.

Mikel Arteste, Gorka Labaka, and Eneko Agirre. 2018. A robust self-learning method for fully unsupervised cross-lingual mappings of word embeddings. In Proceedings of ACL, pages 789–798.

Timothy Baldwin, Jonathan Pool, and Susan Colowick. 2010. PanLex and LEXTRACT: Translating all words of all languages of the world. In Proceedings of COLING (Demo Papers), pages 37–40.

Antonio Valerio Miceli Barone. 2016. Towards cross-lingual distributed representations without parallel text trained with adversarial autoencoders. In Proceedings of the 1st Workshop on Representation Learning for NLP, pages 121–126.

Emily M. Bender. 2011. On achieving and evaluating language-independence in NLP. Linguistic Issues in Language Technology, 6(3):1–26.

Johannes Bjerva, Robert Östling, Maria Han Veiga, Jörg Tiedemann, and Isabelle Augenstein. 2019. What do language representations really represent? Computational Linguistics, 45(2):381–389.

Piotr Bojanowski, Edouard Grave, Armand Joulin, and Tomas Mikolov. 2017. Enriching word vectors with subword information. Transactions of the Association for Computational Linguistics, 5:135–146.

Frédéric Chazal, David Cohen-Steiner, Leonidas J Guibas, Facundo Mémoli, and Steve Y Oudot. 2009. Gromov-Hausdorff stable signatures for shapes using persistence. In Computer Graphics Forum, volume 28, pages 1393–1403.

Alexis Conneau, Guillaume Lample, Marc’Aurelio Ranzato, Ludovic Denoyer, and Hervé Jégou. 2018. Word translation without parallel data. In Proceedings of ICLR.

Ryan Cotterell and Georg Heigold. 2017. Cross-lingual character-level neural morphological tagging. In Proceedings of EMNLP, pages 748–759.

Yerai Doval, Jose Camacho-Collados, Luis Espinosa-Anke, and Steven Schockaert. 2019. On the robustness of unsupervised and semi-supervised cross-lingual word embedding learning. CoRR, abs/1908.07742.

Timothy Dozat and Christopher D. Manning. 2017. Deep biaffine attention for neural dependency parsing. In Proceedings of ICLR.

Timothy Dozat, Peng Qi, and Christopher D. Manning. 2017. Stanford’s graph-based neural dependency parser at the CoNLL 2017 shared task. In Proceedings of the CoNLL 2017 Shared Task: Multilingual Parsing from Raw Text to Universal Dependencies, pages 20–30.
David Kamholz, Jonathan Pool, and Susan M. Colowich. 2013. \textit{WALS Online}. Max Planck Institute for Evolutionary Anthropology, Leipzig.

Long Duong, Hiroshi Kanayama, Tengfei Ma, Steven Bird, and Trevor Cohn. 2016. Learning crosslingual word embeddings without bilingual corpora. In \textit{Proceedings of EMNLP}, pages 1285–1295.

John Rupert Firth. 1957. A synopsis of linguistic theory, 1930-1955. \textit{Studies in Linguistic Analysis}.

Goran Glavaš, Robert Litschko, Sebastian Ruder, and Ivan Vulić. 2019. How to (properly) evaluate cross-lingual word embeddings: On strong baselines, comparative analyses, and some misconceptions. In \textit{Proceedings of ACL}, pages 710–721.

Edouard Grave, Piotr Bojanowski, Prakhar Gupta, Armand Joulin, and Tomas Mikolov. 2018. Learning word vectors for 157 languages. In \textit{Proceedings of LREC}, pages 3483–3487.

Zellig S. Harris. 1954. Distributional structure. \textit{Word}, 10(23):146–162.

Armand Joulin, Piotr Bojanowski, Tomas Mikolov, Hervé Jégou, and Edouard Grave. 2018. Loss in translation: Learning bilingual word mapping with a retrieval criterion. In \textit{Proceedings of EMNLP}, pages 2979–2984.

David Kamholz, Jonathan Pool, and Susan M. Colowick. 2014. PanLex: Building a resource for panlingual lexical translation. In \textit{Proceedings of LREC}, pages 3145–3150.

Sneha Kudugunta, Ankur Bapna, Isaac Caswell, and Orhan Firat. 2019. Investigating multilingual NMT representations at scale. In \textit{Proceedings of EMNLP-IJCNLP}, pages 1565–1575.

Miryam de Lhoneux, Johannes Bjerva, Isabelle Augenstein, and Anders Søgaard. 2018. Parameter sharing between dependency parsers for related languages. In \textit{Proceedings of EMNLP}, pages 4992–4997.

Yu-Hsiang Lin, Chian-Yu Chen, Jean Lee, Zipui Li, Yuyan Zhang, Mengzhou Xia, Shruti Rijhwani, Junxian He, Zhisong Zhang, Xuezhe Ma, Antonios Anastasopoulos, Patrick Littell, and Graham Neubig. 2019. Choosing transfer languages for cross-lingual learning. In \textit{Proceedings of ACL}, pages 3125–3135.

Patrick Littell, David R. Mortensen, Ke Lin, Katherine Kairis, Carlisle Turner, and Lori Levin. 2017. UREIL and lang2vec: Representing languages as typological, geographical, and phylogenetic vectors. In \textit{Proceedings of EACL}, pages 8–14.

Tomas Mikolov, Quoc V Le, and Ilya Sutskever. 2013. Exploiting similarities among languages for machine translation. \textit{arXiv preprint arXiv:1309.4168}.

Tahira Naseem, Regina Barzilay, and Amir Globerson. 2012. Selective sharing for multilingual dependency parsing. In \textit{Proceedings of ACL}, pages 629–637.

Joakim Nivre, Mitchell Abrams, Željko Agić, Lars Ahrenberg, Lene Antonsen, Katya Aplonova, Maria Jesus Aranzabe, et al. 2018. Universal Dependencies 2.3.

Helen O’Horan, Yevgeni Berzak, Ivan Vulić, Roi Reichart, and Anna Korhonen. 2016. Survey on the use of typological information in natural language processing. In \textit{Proceedings of COLING}, pages 1297–1308.

Aitor Ormazabal, Mikel Artetxe, Gorka Labaka, Aitor Soroa, and Eneko Agirre. 2019. Analyzing the limitations of cross-lingual word embedding mappings. In \textit{Proceedings of ACL}, pages 4990–4995.

Barun Patra, Joel Ruben Antony Moniz, Sarthak Garg, Matthew R. Gormley, and Graham Neubig. 2019. Bilingual lexicon induction with semi-supervision in non-isometric embedding spaces. In \textit{Proceedings of ACL}, pages 184–193.

Edoardo Maria Ponti, Helen OHoran, Yevgeni Berzak, Ivan Vulić, Roi Reichart, Thierry Poibeau, Ekaterina Shutova, and Anna Korhonen. 2019. Modeling language variation and universals: A survey on typological linguistics for natural language processing. \textit{Computational Linguistics}, 45(3):559–601.

Eduardo Maria Ponti, Roi Reichart, Anna Korhonen, and Ivan Vulić. 2018. Isomorphic transfer of syntactic structures in cross-lingual NLP. In \textit{Proceedings of ACL}, pages 1531–1542.

Sebastian Ruder, Anders Søgaard, and Ivan Vulić. 2019a. Unsupervised cross-lingual representation learning. In \textit{Proceedings of ACL: Tutorial Abstracts}, pages 31–38.

Sebastian Ruder, Ivan Vulić, and Anders Søgaard. 2019b. A survey of cross-lingual word embedding models. \textit{Journal of Artificial Intelligence Research}, 65:569–631.

Peter H Schönenmann. 1966. A generalized solution of the orthogonal Procrustes problem. \textit{Psychometrika}, 31(1):1–10.

Samuel L. Smith, David H.P. Turban, Steven Hamblin, and Nils Y. Hammerla. 2017. Offline bilingual word vectors, orthogonal transformations and the inverted softmax. In \textit{Proceedings of ICLR}.

Anders Søgaard, Sebastian Ruder, and Ivan Vulić. 2018. On the limitations of unsupervised bilingual dictionary induction. In \textit{Proceedings of ACL}, pages 778–788.

Carsen Stringer, Marius Pachitariu, Nicholas Steinmetz, Matteo Carandini, and Kenneth D. Harris. 2019. High-dimensional geometry of population responses in visual cortex. \textit{Nature}, 571:361–365.

Ivan Vulić, Goran Glavaš, Roi Reichart, and Anna Korhonen. 2019. Do we really need fully unsupervised cross-lingual embeddings? In \textit{Proceedings of EMNLP}, pages 4398–4409.
Ivan Vulić, Nikola Mrkšić, and Anna Korhonen. 2017. Cross-lingual induction and transfer of verb classes based on word vector space specialisation. In Proceedings of EMNLP, pages 2536–2548.

Meng Zhang, Yang Liu, Huanbo Luan, and Maosong Sun. 2017. Earth mover’s distance minimization for unsupervised bilingual lexicon induction. In Proceedings of EMNLP, pages 1934–1945.

Mozhi Zhang, Keyulu Xu, Ken-ichi Kawarabayashi, Stefanie Jegelka, and Jordan Boyd-Graber. 2019. Are girls neko or шоjo? cross-lingual alignment of non-isomorphic embeddings with iterative normalization. In Proceedings of ACL, pages 3180–3189.

### A Appendix

**Gromov-Hausdorff Distance (GH)** This distance measures the worst case distance between two metric spaces \(X\) and \(Y\) with a distance function \(d\), formulated as follows:

\[
\mathcal{H}(X, Y) = \max \{ \sup_{x \in X} \inf_{y \in Y} d(x, y), \sup_{y \in Y} \inf_{x \in X} d(x, y) \} \tag{3}
\]

It measures the distance between the nearest neighbours that are farthest apart. The Gromov-Hausdorff distance (GH) then minimizes this distance over all isometric transforms \(X\) and \(Y\):

\[
\mathcal{GH}(X, Y) = \inf_{f, g} \mathcal{H}(f(X), g(Y))
\]

Computing \(\mathcal{GH}\) directly is computationally intractable in practice, but it can be tractably approximated by computing the Bottleneck distance between the metric spaces (Chazal et al., 2009). As shown by Patra et al. (2019), GH also seems to correlate well with the degree of isomorphism between two embedding spaces.

**Isospectrality (IS)** After length-normalizing the vectors, we compute the nearest neighbour graphs of a subset of \(N\) words, and then calculate the Laplacian matrices \(L_1\) and \(L_2\) of each graph. For \(L_1\), the smallest \(k_1\) is then sought such that the sum of its \(k_1\) largest eigenvalues \(\sum_{i=1}^{k_1} \lambda_{1i}\) is at least 90% of the sum of all its eigenvalues. The same procedure is used to find \(k_2\). We then define \(k = \min(k_1, k_2)\). The final IS measure \(\Delta\) is then the sum of the squared differences of the \(k\) largest Laplacian eigenvalues: \(\Delta = \sum_{i=1}^{k} (\lambda_{1i} - \lambda_{2i})^2\). The lower \(\Delta\), the more similar are the graphs and, consequently, the more isomorphic are the two embedding spaces. The IS measure was advocated as a suitable measure of isomorphism in the work of Søgaard et al. (2018).