Intracavity frequency-doubled solid-state lasers as light sources in the visible spectral range have a high technical potential because of their efficiency as well as their favorable ratio of compactness and output power [1]. Indeed, the intracavity frequency conversion design makes the nonlinear processes highly efficient, however, under multimode operation disadvantageous dynamical instabilities of the system, commonly known as the green problem, are often observed in such kind of lasers. The instabilities originate from a coupling of the individual resonator modes due to the processes of sum frequency generation inside the nonlinear crystal and spatial hole burning within the laser-active media. Any kind of fluctuation of the output intensity can be a serious obstacle to many types of technical applications. Therefore, many people in industry and science are working on the stabilization of such kind of lasers.

Since the first scientific discussion of the green problem by Baer [2] a number of proposals were made concerning the suppression of the fluctuations by means of different optical modifications of the resonator (e.g. ring resonator, mode selecting etalon) [3,4,5]. An alternative approach was proposed by Roy et al. [6]. Leaving the optical setup of the system unchanged they exploited the existence of unstable steady states to perform a stabilization of the laser output intensity on a periodic orbit utilizing the so called occasional proportional feedback, known as OPF. As far as the application is concerned, it is more desirable to achieve a constant output intensity instead of a stable oscillating one. Nevertheless, it was demonstrated that it is possible to control the output intensity of the laser by a pure electronic feedback in principle.

In this letter theoretical aspects and experimental results of an alternative control method are presented. From a rate equation model [4] we derived a control scheme [7] that is able to stabilize the laser output by feeding back a control signal \( u(t) \). The signal \( u(t) \) can be derived from the sum intensities of the two orthogonal directions of the linear-polarized infrared light intensities which are emitted by the laser. More precisely, \( u(t) \) has the form

\[
    u(t) = k_x(s_x^0 - s_x(t)) + k_y(s_y^0 - s_y(t)),
\]

where the light intensities \( s_x \) and \( s_y \) correspond to the sum of the individual modes polarized in the orthogonal directions \( x \) and \( y \), respectively. The directions \( x \) and \( y \) correspond to the polarization eigenvectors of the laser cavity and depend on the birefringence and the length of the crystals inside the laser cavity. \( s_x^0 \) and \( s_y^0 \) are the time-independent sum intensities at the steady state and \( k_x \) and \( k_y \) are two control parameters. First experimental results on such kind of stabilization have been reported in [8].

The approach to form the feedback signal like Eq. (1) results from an analysis of rate equations, which consists of two ODEs for each individual lasing mode. Each set of ODEs for the individual laser modes incorporates a set parameter which are in general different from each other. The parameters are experimentally not accessible, therefore, for simplicity (w.l.o.g.) and to treat the system analytically we have chosen for all the laser modes the same set parameters. It can be shown, that the equations modelling the dynamics of the sum intensities \( s_x \) and \( s_y \) break off from the total system of equations describing the dynamics of each individual mode intensities in vicinity of the fixed point [9]. The stability of each individual laser mode intensity is reduced to the problem of the stability of the sum intensities \( s_x \) and \( s_y \). By using a control signal \( u(t) \) in accordance with Eq. (1) and choosing proper values for the control parameters \( k_x \) and \( k_y \) one can theoretically achieve and maintain the stability of the fixed point intensities as shown by Pyragas et al. [7]. In the following we will give evidence that this control strategy also works experimentally in agreement with the theoretical predictions.

The experimental setup of our intracavity frequency-doubled Nd:YAG laser is shown schematically in Fig. 1. A collimated beam of an infrared pump laser diode is focused on one
end of a cylindrical Nd:YAG crystal. The optical resonator is formed by the planar HR coated surface of the Nd:YAG crystal and a concave output mirror. A type II phase-matched KTP crystal is placed inside the cavity and serves as the optical nonlinear element for generation of radiation at the second harmonics at $\lambda = 532$nm. Due to the optical density of the Nd:YAG and the KTP crystal of about 1.82 and 1.8, respectively, the optical length of the resonator is about 43mm.

The infrared and green spectral components of the total output intensity $I$ was separated by a spectral selective beam splitter and the sum intensities $s_x$ and $s_y$ at 1064nm were separated by polarizing beamsplitter. Special care was taken to operate the laser in the TEM$_{00}$ mode of the laser resonator. In general, with increasing pump power the number of oscillating longitudinal resonator modes increases. The dependence of the intensities $I_k$ of the individual modes on the pump power $P_{LD}$ is complicated and is mainly governed by the process of spatial hole burning [4]. Spatial hole burning originates from competition of modes for the same locally exited states inside the active media and results for instance in the fading of a laser mode with increasing pump power in favor of an other one. In order to decide which kind of nonlinear mode coupling takes place inside the KTP crystal, i.e. sum frequency generation or frequency doubling, the mode spectrum at $\lambda = 532$nm can serve as a good indicator. The ratio between sum frequency generation and frequency doubling can be controlled by rotating the main axes of the birefringent KTP crystal [4]. For the following experiments a constellation with only sum frequency generation and frequency doubling was chosen.

Two photodetectors were used in order to detect the temporal changes of the sum intensities of the modes polarized in the two directions. The voltage signals of the detectors saved in a computer by means of an analog-to-digital converter (100MSa/s, 12bit). Two typical time traces are shown in the diagrams (a) and (b) of Fig. 2. The fluctuations displayed in Fig. 2(a) are regular and were observed at a relative pump power of $w = 1.89$. The mode configuration was $[1, 4]$ (one mode polarized in x direction and four modes polarized in y direction). For the measurement in Fig. 2(b) the relative pump power $w$ was increased to a value of 2.07. All other system parameters remained unchanged. The corresponding mode configuration has changed to $[1, 5]$. Irregular fluctuations can be seen. Note that the time traces of the sum intensity $s_x$ correspond to the dynamical behavior of a single mode in both diagrams and that the oscillations of $s_x$ and $s_y$ are out of phase indicating a mode competition process, known as the antiphase dynamics. For increasing pump power, the characteristic frequencies of the dynamics increase, following a square root behavior. The characteristic frequency is below 200kHz for all the measurements presented here.

The electric circuit used to implement the proportional feedback control is displayed schematically in Fig. 3 Instead of calculating the actual difference $s_x - s_y$ (where * has to be $x$ or $y$) as written in Eq. 1 which is complicated, because in the experiment the values of the fixed point intensities $s_{x0}$ are not known, we use a trick to estimate the difference by using high pass filtered signal. This approximation is valid because the fixed point intensity is equal to the time average of the intensity fluctuations. This procedure is discussed in more detail in [7]. For this purpose we have used two high pass filters with a cutoff frequency of about 1kHz. Subsequently, the two signals are weighted with a variable gain amplifier ($-1 \leq K_{x,y} \leq 1$) and afterwards added up. The gain factors $g_x$ and $g_y$ of the amplifiers were adjusted by means

![FIG. 1: Schematic drawing of the intracavity frequency doubled Nd:YAG laser pumped by a laser diode.](image1)

![FIG. 2: Measured time series of the infrared sum intensities $s_x$, bold line, and $s_y$, light line, polarized in $x$- and $y$-direction, respectively, at two different relative pump powers $w$ ((a) $w = 1.89$ and (b) $w = 2.07$).](image2)

![FIG. 3: Schematic diagram of the control feedback loop, to perform the stabilization by proportional feedback](image3)
of digital potentiometers and controlled remotely via a personal computer. A third amplifier with variable gain $K$ was used. Overall we end up with a voltage signal of the laser.

Lower than 30 degrees at the highest characteristic frequency the control circuit was about 300kHz and the phase shift was connected in parallel to the laser diode. The -3dB bandwidth of the current of the laser diode was modulated by a bypass point. In order to feed the control voltage back into the laser, only important during the time, where the control is switched.

In Fig. 4 the theoretical domains of stability are displayed as shaded areas in the plane of the control parameters $(k_x, k_y)$ according to theory. The data were taken for two different exemplary situations with values of the relative pump power of $w = 1.38$ and $w = 2.07$ and mode configurations $[1, 2]$ and $[1, 4]$ in case of Fig. 4(a) and Fig. 4(b), respectively. The parameters have been chosen to be as close as possible to the performed experiments. Please note, that the mode configuration is the only possibility in the model to have a handle on the mean sum intensity ration of the two polarization directions, because in the model all parameters for the different modes are chosen to be the same for simplicity. In the experiment it is more likely that the asymmetry of the intensities is caused by different sets of parameters for the different modes but they are experimentally not accessible. Most remarkable is the wedge-like shape of the two regions of stability of infinite size for larger $k_x$ and more negative $k_y$ values. Increasing the pump power, the angle between the two border lines of the region decreases.

In Fig. 5 two measured domains of stability observed at different relative pump powers are displayed. We varied the two parameters $(k_x, k_y)$ systematically by taking data at each combination of a hundred values in each direction and, therefore, scanned the parameter plane. As a measure for the stability, the standard deviation of the sum intensity $s_v$ was chosen, which directly relates to the amplitude of the fluctuations.

Darker regions correspond to lower values of the standard deviation. Thus, the black regions mark the domain with nearly no fluctuations, that is, the domain in which the laser has a constant output power. For the second measurement corresponding to Fig. 5(b), the pump power was increased to a value of $w = 2.07$. At this pump power the fluctuations were highly irregular with a dynamic range of 100%. This means, the laser switched on and off. The mode configuration was $[1, 2]$ as well as (b) at a relative pump power $w = 2.07$ and the mode configuration $[1, 4]$. The color codes the standard deviation of the laser intensity output (the darker the color, the lower the standard deviation, i.e., black is closed to zero deviation). The two straight bold lines in both of the diagrams are added to guide the eye.

To understand this behavior we have to consider an important issue, which we have neglected up to now. The region of stability was calculated using linear stability analysis by only looking at the real part of the eigenvalue without taking into consideration the imaginary part of the eigenvalue. Looking at the imaginary part of the eigenvalue we notice that the imaginary part increases if $k_y$ becomes more and more negative. From the experimental point of view it is obvious that the feedback signal is not modelled correctly. In any experimental situation the bandwidth of the feedback loop is restricted which was not considered in the model — in the model the
The cutoff frequency of the low-pass filter can be modelled by adding an additional differential equation to the rate equations introduced in [7]. The additional equation reads as follows:

$$\frac{du_l}{dt} = \omega_c (k_{x}(s_x^0 - s_x(t)) + k_{y}(s_y^0 - s_y(t)) - u_l),$$  

where $\omega_c$ denotes the cutoff frequency of the low pass filter. Using this form of the feedback signal and performing the stability analysis we obtain the results presented in Fig. 6. For the simulation a cutoff frequency of 300kHz was used, which corresponds to the experimental situation. It can be seen that now all essential features of the experimentally observed region of stability can now be reproduced. This finding shows that an increased bandwidth gives the possibility to stabilize the laser even further to higher pump powers up to values important for technical applications.

In conclusion we presented experimental evidence that an intracavity frequency-doubled solid state laser can be stabilized by a proportional feedback of the two sum intensities of the orthogonal polarized modes to the pump power. By refining an earlier model it was possible to understand the shape and dependence on system parameters of the region of stability in the control parameter plane. The presented result aim in the direction what to do for an application of the method in a laser with more realistic values of the pump power and, therefore, of the intensity of the light output. To extend the applicability of the method to a hundred times higher pump powers the bandwidth has to be increased by the factor ten. This can be easily achieved. We are convinced that these findings are of great importance for technical application of such a kind of laser.

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