MEASURING TIME DEPENDENCE OF DARK ENERGY DENSITY FROM TYPE Ia SUPERNOVA DATA

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ABSTRACT

Observations of high-redshift supernovae imply an accelerating universe that can only be explained by an unusual energy component such as vacuum energy or quintessence. To assess the ability of current and future supernova data to constrain the properties of the dark energy, we allow its density to have arbitrary time dependence, $\rho_X(z)$. This leads to an equation of state for the dark energy, $w_X(z) = p_X(z)/\rho_X(z)$, which is a free function of redshift $z$. We find that current data of type Ia supernovae (SNe Ia) are consistent with a cosmological constant, with large uncertainties at $z \geq 0.5$. We show that $\rho_X(z)/\rho_X(z = 0)$ can be measured reasonably well to about $z = 1.5$ using SN Ia data from realistic future SN Ia pencil beam surveys, provided that the weak energy condition (energy density of matter is non-negative for any observer) is imposed. While it is only possible to differentiate between different models (say, quintessence and $k$-essence) at $z \leq 1.5$ using realistic data, the correct trend in the time dependence of the dark energy density can be clearly detected out to $z = 2$, even in the presence of plausible systematic effects. This would allow us to determine whether the dark energy is a cosmological constant or some exotic form of energy with a time-dependent density.

Subject headings: cosmology: theory — dark matter — supernovae: general

1. INTRODUCTION

Most of the present energy content of our universe is unknown (Bahcall et al. 1999). Distance-redshift relations derived from cosmological standard candles at redshifts between zero and a few are the most sensitive probes of the equation of state of the universe, which allows us to constrain the energy content of the universe.

Type Ia supernovae (SNe Ia) are our best candidates for cosmological standard candles. They can be calibrated to have small dispersion in their intrinsic luminosities (Phillips 1993; Riess, Press, & Kirshner 1995). The data from two independent observational teams, the High-z SN Search (Schmidt et al. 1998) and the Supernova Cosmology Project (Perlmutter et al. 1999), seem to suggest that our universe has a significant vacuum energy content (Garnavich et al. 1998a; Riess et al. 1998; Perlmutter et al. 1999).

While a cosmological constant term (vacuum energy) in the Einstein equations provides the simplest explanation of the current SN Ia data, other forms of energy (quintessence, dark energy, etc.) have also been studied (e.g., White 1998; Garnavich et al. 1998b; Steinhardt, Wang, & Zlatev 1999; Efstathiou 1999; Podariu & Ratra 2000; Waga & Frieman 2000) and are consistent with current data as well. Since cosmology has matured into a phenomenological science at the turn of the new millennium, observational data will dominate aesthetics in the selection of cosmological models.

So far, most cosmologists have assumed time-independent equations of state, i.e., power-law dark energy density (see § 2), in the context of constraining the energy content of the universe. In this paper we allow the dark energy density of the universe to be a free function of redshift, i.e., arbitrary equation of state. We derive constraints on the dark energy density from current SN data and assess future prospects of measuring the dark energy density using simulated data from realistic future SN Ia surveys. We expect a model-independent measurement of the dark energy density as a function of redshift to be very useful in our quest of the unknown energy contents of the universe.

2. PARAMETERIZATION OF THE DARK ENERGY DENSITY

In a smooth Friedmann-Robertson-Walker (FRW) universe, the metric is given by

$$ds^2 = dt^2 - a^2(t)[dr^2 / (1 - kr^2) + r^2(d\theta^2 + \sin^2 \theta d\phi^2)],$$

where $a(t)$ is the cosmic scale factor and $k$ is the global curvature parameter. The cosmological redshift $z$ is given by $1 + z = 1/a$.

To make model-independent measurements of the equation of state, we replace the vacuum energy density $\rho$ with $\rho_X = \rho_X f(z)$ in the total matter density of the universe:

$$\rho(z) = \rho_m(1+z)^3 + \rho_b(1+z)^2 + \rho_X f(z),$$

where the superscript “0” indicates present values, $f(z) = 1$, and $\rho_m - \rho_b - \rho_X = -k/H_0^2$. If the unknown energy is due to a cosmological constant $\Lambda$, $f(z) = 1$. Clearly, the function $f(z)$ is a very good probe of the nature of the unknown energy.

The comoving distance $r$ is given by (Weinberg 1972)

$$r(z) = cH_0^{-1} \frac{S(\kappa T)}{\kappa}, \quad \kappa \equiv |\Omega_k|^{1/2},$$

$$\Gamma(z; \Omega_m, \Omega_X, F) = \int_0^z dz' \frac{1}{E(z')},$$

$$E(z) \equiv [\Omega_m(1+z')^3 + \Omega_X f(z') + \Omega_b(1+z')^2]^{1/2},$$

where

$$S(x) = \begin{cases}
\sinh (x), & \Omega_k > 0, \\
 x, & \Omega_k = 0, \\
\sin (x), & \Omega_k < 0.
\end{cases}$$

The angular diameter distance is given by $d_A(z) = r(z)/(1+z)$, and the luminosity distance is given by $d_L(z) = (1+z)^2 d_A(z)$.
Einstein’s equations in an FRW metric, together with the first law of thermodynamics, give us
\[
(1 + z) \frac{d\rho}{dz} = 3(\rho + p) .
\]  
(6)

For unknown energy \( \rho_X(z) \), we find
\[
\rho_X(z) = \rho_X^0 f(z) , \quad p_X(z) = \rho_X^0 \left[ \frac{1}{3}(1 + z) f'(z) - f(z) \right] .
\]  
(7)

Now we can write the equation of state of the unknown energy as
\[
w(z) \equiv \frac{p_X(z)}{\rho_X(z)} = \frac{1}{3} (1 + z) \frac{f'(z)}{f(z)} - 1 .
\]  
(8)

A constant equation of state corresponds to \( f(z) \propto (1 + z)^\alpha \), where \( \alpha \) is a constant. The values \( \alpha = 0, \alpha = 3, \) and \( \alpha = 4 \) correspond to a cosmological constant, matter, and radiation, respectively.

To obtain accelerated expansion, we need \( \rho + 3p < 0 \). Since \( \rho_X + 3p_X = \rho_X^0 \left[ (1 + z) f' - 2f \right] \), this implies \( \alpha < 2 \) for \( f(z) \propto (1 + z)^\alpha \).

The weak energy condition states that for all physically reasonable classical matter, the energy density of matter as measured by any observer is nonnegative (Wald 1984). For a perfect fluid, the weak energy condition will be satisfied if and only if
\[
\rho + p \geq 0 .
\]  
(9)

This leads to (see eq. [6])
\[
f'(z) \geq 0 .
\]  
(10)

The weak energy condition imposes strong constraints on the jointly estimated cosmological parameters \( \Omega_m, \Omega_X, \) and the dark energy density \( f(z) \).

Note that the comoving distance \( r(z) \) depends on the equation of state \( X \) through
\[
\Gamma(z) \equiv \int_0^z dz \left[ \Omega_m(1 + z)^3 + \Omega_X f(z) + \Omega_k(1 + z)^2 \right]^{-1/2}
\]
\[
= H_0 \int_0^r \frac{dr}{\sqrt{1 - kr^2}} .
\]  
(11)

Hence,
\[
\Gamma'(z) = \left[ \Omega_m(1 + z)^3 + \Omega_X f(z) + \Omega_k(1 + z)^2 \right]^{-1/2}
\]
\[
= \frac{H_0}{(1 + \Omega_k r^2)^{1/2}} \frac{dr}{dz} .
\]  
(12)

To measure \( f(z) \) directly from the data, we need to evaluate the derivative of the distance \( r(z) \) with respect to redshift \( z \).

To avoid taking derivatives of noisy data, we can parameterize \( f(z) \) with its values at \( n \) equally spaced redshifts \( z_i \) and assume that \( f(z) \) is given by linear interpolations at other values of \( z \). We write
\[
f(z) = \left( \frac{z - z_i}{z_i - z_{i-1}} \right) f_{i-1} + \left( \frac{z - z_{i-1}}{z_i - z_{i-1}} \right) f_i , \quad z_{i-1} < z \leq z_i ,
\]
\[
z_0 = 0 , \quad z_n = z_{\text{max}} ; \quad f_0 = 1 , \quad f_n = f_{n-1} ,
\]  
(13)

where \( f_i (i = 1, 2, \ldots, n - 1) \) are independent variables to be estimated from the data.

\section{3. Parameter Estimation}

The measured distance modulus for an SN Ia is
\[
\mu^{\text{obs}}_p = \mu^{\text{th}}_p + \epsilon^{\text{obs}} ,
\]  
(14)

where \( \mu^{\text{th}}_p \) is the theoretical prediction
\[
\mu^{\text{th}}_p = 5 \log \left[ \frac{d_L(z_i)}{\text{Mpc}} \right] + 25
\]  
(15)

and \( \epsilon^{\text{obs}} \) is the uncertainty in the measurement, including observational errors and intrinsic scatters in the SN Ia absolute magnitudes.

Denoting all the parameters to be fitted as \( s \), we can estimate \( s \) using a \( \chi^2 \) statistic, with (Riess et al. 1998)
\[
\chi^2(s) = \sum_i \frac{\left[ \mu^{\text{obs}}_p(z_i | s) - \mu^{\text{th}}_p(z_i | s) \right]^2}{\sigma^{2}_{\text{obs}}_i + \sigma^{2}_{\text{meas},i}} \equiv \sum_i \frac{\left[ \mu^{\text{obs}}_p(z_i | s) - \mu^{\text{th}}_p(z_i | s) \right]^2}{\sigma^2} ,
\]  
(16)

where \( \sigma^{\text{meas}}_i \) is the estimated measurement error of the distance modulus and \( \sigma^{\text{meas}}_i \) is the dispersion in the distance modulus due to the dispersion in galaxy redshift, \( z_g \), due to peculiar velocities and uncertainty in the galaxy redshift (for the Perlmutter et al. 1999 data, the dispersion due to peculiar velocities is included in \( \sigma^{\text{meas}}_i \)). Since
\[
\sigma^{\text{meas}}_i = \frac{5}{\ln 10} \frac{1}{d_L} \frac{d^2}{d_L^2} \frac{d^2}{dz^2} \sigma^2 ,
\]  
(17)

\( \sigma^{\text{meas}}_i \) depends on the parameters \( s \). The probability density function (PDF) for the parameters \( s \) is
\[
p(s) \propto \exp \left( - \frac{\chi^2}{2} \right) .
\]  
(18)

The normalized PDF is obtained by dividing the above expression by its sum over all possible values of the parameters \( s \).

In order to impose the weak energy condition \( f'(z) \geq 0 \), we compute the PDFs on an \( N \)-dimensional grid for \( N \) parameters. The PDF of a given parameter \( s_i \) is obtained by integrating over all possible values of the other \( N - 1 \) parameters. To reduce the computation time, we can integrate over the Hubble constant \( H_0 \) analytically and define a modified \( \chi^2 \) statistic, with
\[
\tilde{\chi}^2 \equiv \chi^2 - C_1 \frac{C_2}{C_1 + \frac{2}{5} \ln 10} ,
\]  
(19)

where
\[
\chi^2 \equiv \sum_i \frac{1}{\sigma^2} \left[ \mu^{\text{th}}_p - \mu^{\text{obs}}_p \right]^2 ,
\]
\[
C_1 \equiv \sum_i \frac{1}{\sigma^2} \left[ \mu^{\text{th}}_p - \mu^{\text{obs}}_p \right] ,
\]
\[
C_2 \equiv \sum_i \frac{1}{\sigma^2} ,
\]  
(20)

where
\[
\mu^{\text{th}}_p \equiv \mu_p (h = h^*) = 42.384 - 5 \log h^* + 5 \log [H_0 r(1 + z)] .
\]  
(21)

We take \( h^* = 0.65 \). Our results are independent of the choice of \( h^* \).
After experimenting with a number of different techniques, we developed an adaptive iteration method of estimating $f(z)$ based on the requirement that $f'(z) \geq 0$ (i.e., the weak energy condition is satisfied). Starting with the initial guess of $f(z) = f(z = 0) = 1$ (a cosmological constant), we iteratively build up $f(z)$ as parameterized by equation (13) while minimizing $\chi^2$.

4. CONSTRAINTS OF $\rho_X(z)$ FROM CURRENT SNe Ia DATA

Wang (2000b) has combined the data of the High-z SN Search team (Schmidt et al. 1998; Garnavich et al. 1998a; Riess et al. 1998) and the Supernova Cosmology Project (Perlmutter et al. 1999), yielding a total of 92 SNe Ia. Using the method described in the previous section, we estimate $\Omega_m$, $\Omega_X$, and $f(z)$ simultaneously by minimizing the modified $\chi^2$ (see eq. [19]).

Figure 1 shows the dimensionless dark energy density $f(z)$ (as parameterized by eq. [13]) measured from this set of 92 SNe Ia. It is consistent with $f(z) = 1$ (a cosmological constant), with large uncertainty beyond $z \gtrsim 0.5$. Note that none of the error bars extend beneath $f(z) = 1$ because we have imposed the weak energy condition, i.e., $f'(z) \geq 0$, which implies that $f(z) \geq f(z = 0) = 1$. The estimated values of $\Omega_m = 0.3$ (0.9) and $\Omega_X = 1.7$ (0.4, 2.2) are consistent with previous results (Garnavich et al. 1998a; Riess et al. 1998; Perlmutter et al. 1999; Wang 2000). The errors are estimated from the ranges of parameters for which $\chi^2 = \chi^2_{\text{min}} + 1$.

Wang (2000b) found that when fitted to a model with a cosmological constant as the dark energy, flux averaging changes the best-fit model to this data set of 92 SNe Ia. Without flux averaging, the best-fit model is a closed universe with $\Omega_m = 0.7 \pm 0.4$ and a vacuum energy density fraction of $\Omega_X = 1.2 \pm 0.5$, consistent with the estimated values in Figure 1. The flux-averaged data yield $\Omega_m = 0.3 \pm 0.6$ and $\Omega_X = 0.7 \pm 0.7$. This difference may have resulted from the data containing large redshift-dependent uncertainties, which would have caused the results from the unbinned data to be biased. The effect of flux averaging on the best-fit model assuming an arbitrary dimensionless dark energy density $f(z)$ will be studied elsewhere.

Future cosmic microwave background (CMB) space missions MAP (Bennett et al. 1997) and Planck (De Zotti et al. 1999), together with the galaxy redshift surveys SDSS (Gunn 1999) and 2df (Dalton et al. 2000), will give us exquisitely accurate measurements of the geometry of the universe and the matter density in the universe (Eisenstein, Hu, & Tegmark 1999; Turner & Tyson 1999; Wang, Spergel, & Strauss 1999). SN data can provide the unique probe on the nature of dark energy by allowing us to measure how the dark energy density varies with time.

Current CMB anisotropy measurements seem to indicate that we live in a flat universe (de Bernardis et al. 2000; Balbi et al. 2000). Cluster abundances strongly suggest a low matter density universe (Bahcall, Lubin, & Dorman 1995; Carlberg et al. 1996; Bahcall & Fan 1998). The values of $\Omega_m = 0.3$ and $\Omega_X = 0.7$ provide the best-fit model to current observational data. We will use $\Omega_m = 0.3$ and $\Omega_X = 0.7$ for our simulated data in the rest of this paper.

5. MEASURING $\rho_X(z)$ FROM FUTURE SNe Ia DATA

A large number of SNe Ia at $z \gtrsim 1$ are critical in resolving the important systematic uncertainties of SNe Ia as cosmological standard candles, such as dust (Aguirre 1999), gravitational lensing (Kantowski, Vaughan, & Branch 1995; Wambsganss et al. 1997; Holz 1998; Metcalf & Silk 1999; Wang 1999; Barber et al. 2000), and luminosity evolution (Drell, Loredo, & Wasserman 1999; Riess et al. 1999; Wang 2000b), and in making SNe Ia useful probes of the dark energy content of the universe. The most efficient method of obtaining a large number of SNe Ia at $z \gtrsim 1$ is conducting a supernova pencil beam survey on a dedicated large-aperture telescope with a square degree field of view (Wang 2000a).

To study how well SN data can probe the dark energy density, let us consider two hypothetical dimensionless dark energy densities $f_d(z)$ and $f_z(z)$, given by

\[
\begin{align*}
 f_d(z) &= \frac{e^{1.5z}}{(1 + z)^{1.5}}, \\
 w_d(z) &= -1 + 0.5z, \\
 f_z(z) &= \exp[0.9(1 - e^{-z})], \\
 w_z(z) &= 0.3(1 + z)e^{-z} - 1.
\end{align*}
\]

We have chosen $f_d(z)$ and $f_z(z)$ to represent quintessence ($dw_d/dz > 0$) and k-essence ($dw_z/dz < 0$) models, respectively (Caldwell, Dave, & Steinhardt 1998; Armendariz-Picon, Mukhanov, & Steinhardt 2000). Note that $f_d(z)$ and $f_z(z)$ satisfy the weak energy condition $f'(z) \geq 0$; they give an accelerating universe for $z \lesssim 1.33$ and $z \lesssim 2$, respectively.

A feasible SN pencil beam survey (either from ground\(^2\) or from space), with a square degree field of view and an effective observational period of 1 yr, can yield almost 2000 SNe Ia out to $z = 2$ (Wang 2000a). Let us combine the data from the SN pencil beam survey with SN data at smaller redshifts, so that there are a minimum of 50 SNe Ia per 0.1 redshift interval at any redshift. This yields a total of 1966

\(^2\) The SNe Ia at $z \gtrsim 1.5$ will likely require follow-up spectroscopy from space.
Figure 2 shows that the parameterization of \( f(z) \) with \( n = 10 \) yields less biased estimates than with \( n = 6 \). This is as expected, since \( f(z) \) is more accurately parameterized as one increases \( n \). However, the errors in the estimates increase with \( n \) as well. One must choose an optimal \( n \) such that \( f(z) \) is adequately parameterized, while the errors on the estimated amplitudes of \( f(z) \) are not too large to be useful. We have experimented with \( n > 10 \) parameterizations of \( f(z) \) and found that \( n = 10 \) is a good choice.

The biased estimates of the dark energy density \( f(z) \) and \( f_{\text{sys}}(z) \) in Figure 2 are mainly due to the bias in the estimate of \( \Omega_m \). Figure 3 shows \( f(z) \) and \( f_{\text{sys}}(z) \) estimated assuming that we know \( \Omega_m = 0.3 \), with the same line types as in Figure 2. Even with this ideal assumption, it is only possible to differentiate marginally between the two models.

Figure 4 shows the effect of adding a systematic shift of \( dm_{\text{sys}}(z) \) to 100 random data sets with a realistic dispersion of 0.2 mag. The line types are the same as in Figure 2. We have added a systematic shift in \( \mu_0 \) of 0.01 z mag (Fig. 4a) and 0.05 z mag (Fig. 4b). Clearly, systematic shifts can significantly increase the bias in the estimate of \( \Omega_m \) and the estimates of \( f(z) \) for \( z \gtrsim 1.2 \), while having little effect on the estimates of \( f(z) \) at \( z \lesssim 1.2 \). Systematic errors as a function of \( z \) may arise from intrinsic properties of the supernovae varying with \( z \) such as progenitor evolution or dust characteristics changing with metallicity of the universe. Accuracy may also be limited by the observations themselves in the case of \( k \)-corrections or selection biases. The estimated errors from these sources in the current surveys are between 2% and 10% (Schmidt et al. 1998). It is perhaps not realistic to expect to measure both \( \Omega_m \) and \( f(z) \) to high redshift accurately from SN Ia data alone.

It is clear from Figure 4 that even in the presence of plausible systematic effects, we can expect to measure the...
time variation in the dark energy density \( f(z) \) with reasonable accuracy to a redshift of about 1.2. The bias and the errors in the estimated \( f(z) \) increase substantially beyond \( z = 1.2 \).

We only applied our adaptive iteration method to 100 random samples and with a resolution of \( \Delta \Omega_m = 0.02 \) because this method takes several hours per sample on a fast Sun workstation in finding the best-fit \( f(z) \) and \( \Omega_m \). It is work planned for the future to improve this promising method for application to much larger numbers of random samples, as well as adding \( \Omega_k \) as an estimated parameter.

6. IMPLICATION FOR DARK ENERGY MODELS

Recently, there has been a great deal of activity in exploring the possibilities of the existence of exotic dark energy (Peebles & Ratra 1988; Frieman et al. 1995; Caldwell et al. 1998; Sahni & Wang 2000) in the universe. While the present observational data are consistent with the dark
energy being a cosmological constant, they do not rule out alternatives in the form of various scalar fields.

It is important that we measure the time dependence of the dark energy density. If the dark energy density is measured to be constant in time within reasonable uncertainties, a cosmological constant should be favored, and more theoretical efforts should be directed toward the derivation of a cosmological constant from first principles. At the very least, this places a strong constraint on the classes of scalar field models for the dark energy. On the other hand, if the time dependence of the dark energy density is established by the observational data, we would come to the exciting discovery of new physics in the universe.

We have studied two dark energy models $f(z)$ and $w(z)$ (see eq. [22]), representing two general classes of dark energy models, quintessence (Caldwell et al. 1998) and $k$-essence (Armendariz-Picon et al. 2000). This allows us to examine how well different models can be differentiated by realistic data, as well as the robust determination of the time dependence of the dark energy density.

Realistic future SN data, as described in the previous section, have the potential of determining the time dependence of the dark energy density. This will clearly have a dramatic impact on models of the dark energy.

7. CONCLUSIONS

To access the prospects of measuring the time variation in the equation of state, we have developed a promising adaptive iteration method that is powerful in extracting the dark energy density $f(z) = \rho_X(z)/\rho_X(z=0)$ from realistic data. This method is based on the requirement that the weak energy condition (energy density of matter is non-negative for any observer) is satisfied.

We have found that current SN Ia data are consistent with a cosmological constant, with large uncertainties at $z \gtrsim 0.5$. We show that $\Omega_m$ (assuming a flat universe) and the dimensionless dark energy density $f(z) = \rho_X(z)/\rho_X(z=0)$ can be measured reasonably well to about $z = 1.5$ using SN Ia data from realistic future SN Ia pencil beam surveys, provided that the weak energy condition (energy density of matter is nonnegative for any observer) is imposed. For $z \gtrsim 1.5$, the errors increase significantly, while the estimates become more biased, making it impossible to differentiate between different models (say, quintessence and $k$-essence). However, the correct trend in the time dependence of the dark energy density can be clearly detected out to $z = 2$, even in the presence of plausible systematic effects. This would allow us to determine whether the dark energy is vacuum energy or some exotic form of energy with a time-dependent density.

The simulated data we used are for an SN pencil beam survey (either from ground or from space) with a square degree field of view and for an effective observational period of 1 yr (Wang 2000a), combined with SN data at smaller redshifts, so that there are a minimum of 50 SNe Ia per 0.1 redshift interval at any redshift. Although the dispersion of 0.20 mag (intrinsic plus observational) assumed in our simulated data is appropriate for ground-based surveys, we expect our results to apply qualitatively to space-based SN pencil beam surveys as well because our assumed dispersion of 0.20 mag is dominated by intrinsic dispersion (about 0.17 mag).

At the completion of this lengthy numerical study of the feasibility of measuring the time dependence of the dark energy density from realistic SN Ia data, we became aware of the recent paper by Maor, Brustein, & Steinhardt (2001). They claimed that distance-redshift relations derived from SNe Ia and similar classical measures are poor methods for resolving the time dependence or measuring the amplitude of the equation of state $w_X(z) = p_X(z)/\rho_X(z)$ and, consequently, no useful information can be obtained about the future fate of the universe. Our work has confirmed the difficulty of measuring the properties of the dark energy from realistic SN Ia data. However, we have found their assessment to be overly pessimistic. Their work indicates that it is impossible to tell whether the equation of state $w_X(z)$ varies in time (see also Barger & Marfatia 2001), but knowing that the dark energy density $\rho_X(z)$ varies in time is sufficient to rule out vacuum energy as dark energy, thus giving support to exotic dark energy models. Our work has shown that one can indeed clearly detect the time dependence of $\rho_X(z)$ using realistic future SN Ia data.

The main problem reported by Maor et al. (2001) was the "smearing" effect of the multi-integral relation between the luminosity distance $d_L(z)$ and the equation of state $w_X(z)$. Instead of $d_L(z)$ and $w_X(z)$, our analysis uses the time derivative of the comoving distance $r'(z)$ and the dimensionless dark energy density $f(z)$. Thus, our results are less affected by the smearing effect. Our method will be refined and made more efficient, and it should become quite useful in analyzing future SN data.

In view of our results, it is important that reasonable yet substantial observational efforts are devoted to future SN Ia surveys, for example, a dedicated 4 m telescope for an SN pencil beam survey (Wang 2000a), combined with surveys of nearby SNe Ia. The total cost of such surveys would be modest compared to the great scientific return, the determination of the systematic uncertainties of SNe Ia as cosmological standard candles, and the measurement of the time dependence in the dark energy density of the universe to constrain fundamental physics.

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