Disorder-Driven Magnetic Field Dependence of the Internal Field Distribution in the Bragg Glass Phase of Type-II Superconductors

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We use the replica variational method to study the effects of weak point disorder on the variance of the internal field distribution measured in NMR and muon-spin rotation experiments in type-II superconductors. We show that for a simple model there is significant magnetic field dependence which is extrinsic and disorder-driven, and does not have a microscopic (non-s-wave pairing) origin. Results are presented where we examine the dependence of the magnetic field variance upon the strength of the applied external field.

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The study of high temperature superconductors (high-\(T_c\)'s) remains a subject of intense investigation, both in the search of an explanation of its microscopic origin and in the application of phenomenological theories to investigate the effects of thermal fluctuations, random disorder, and transport. It is now generally accepted that high-\(T_c\) superconductors are unconventional \(d\)-wave rather than conventional \(s\)-wave with an energy gap that vanishes along certain directions in momentum space, resulting in nodes in the superconducting gap. Measurement of the temperature dependence of the magnetic penetration depth, \(\lambda(T)\), is one way to probe the non \(s\)-wave nature of the low energy nodal excitations of the superconducting state \([1]\). A detailed understanding of the \(d\)-wave nature of the high-\(T_c\)'s must also include a quantitative description of the dependence of the two fundamental length scales characterizing a superconductor, the penetration depth \(\lambda\) and the coherence length \(\xi_0\), on the strength of an applied magnetic field \(B_0\). An early theoretical investigation of the weak-field response of a \(d_x^2-y^2\) superconductor predicted a direction-dependent nonlinear Meissner effect, associated with the quasiclassical shift of the excitation spectrum due to the superflow created by the screening currents \([2]\). More recently, Amin \textit{et al.} studied nonlinear and nonlocal effects due to the field induced excitations at the gap nodes to predict the temperature and field dependence of an effective penetration depth \(\lambda_{\text{eff}}\) \([3]\).

While understanding the microscopic physics at play in the high-\(T_c\)'s is a formidable challenge, both experimental and theoretical investigations have shown that the magnetic field-temperature \((B-T)\) phase diagram of these materials is also quite a bit more complicated than for conventional superconductors \([4]\). It is well known that the flux lines in a clean \(s\)-wave type-II superconductor form an Abrikosov triangular lattice for fields \(B\) larger than the lower critical field, \(B_{c1}\). However, for high-\(T_c\)'s, this mean field phase diagram is considerably affected by the combined effects of thermal fluctuations and various types of disorder, such as oxygen vacancies, columnar defects, or twin boundaries (see Fig. 1a) \([4]\).

Fig. 1: (a) Schematic phase diagram of high temperature superconductors with point disorder. (b) The generic behavior of the relative displacements correlation function.

Thermal fluctuations lead to melting of the vortex lattice and appearance of a vortex liquid (VL). In addition, extensive experimental, theoretical and numerical work strongly suggest that the Abrikosov vortex lattice in type-II superconductors is destroyed by the presence of disorder and gives place to various superconducting glass-like mixed states \([4]\). If randomness is strong, the underlying translational order of the vortex lattice is completely destroyed giving a vortex glass phase (VG). However, for weak enough randomness, ‘some order’ is expected to survive. Indeed, it has been suggested that there may exist a stable, dislocation-free Bragg glass (BG) phase at low magnetic fields and temperatures in the presence of weak point impurities where the structure factor \(S(k)\) has power law singularities at the Bragg reciprocal vectors \(k = G\) \([5,6]\). For an applied field larger than a critical value \(B^*\) (of order 2 to 20 Teslas, depending on the specific sample considered) \([7]\), the system undergoes a transition to a vortex glass which consists of completely randomly frozen vortex lines with finite width peaks in the structure factor at \(k \approx G\) (see Fig. 1a) \([5,6,8]\).

The vortex state of high-\(T_c\)'s leads to a spatially varying magnetic field \(B(r)\) inside the superconductor which may be investigating using various techniques. The muon spin rotation (\(\mu\)SR) technique is an almost ideal method for studying the magnetic field distribution in the mixed
state, and magnetic field and temperature dependence of $\lambda(B_0)$ and $\xi_0(B_0)$ [9]. In a $\mu$SR experiment the inhomogeneous magnetic field is sensitively monitored by implanting muons into the sample, one at a time, where the muon’s spin precess with a Larmor frequency which is directly proportional to the local magnetic field $B(r)$. The muon-spin precession signal, which is obtained by detecting the muons positron-decay pattern, reflects the distribution of the Larmor precession frequencies from all the muon stopping sites, hence forming an asymmetric $\mu$SR lineshape [9]. The width of this lineshape and, correspondingly, the width of the internal field distribution, is roughly proportional to $1/\lambda^2$. A parametrization of the experimental lineshape based on a theoretical lineshape produced by an effective London model that includes both the cut-off effects arising from the finite-size of the vortex core and a phenomenological description of the effects of broadening caused by static and random distortion of the vortex lattice and/or nuclear dipolar magnetic fields allows for a determination of effective $\chi^d(B_0)$ and $\chi^s_0(B_0)$ [9–11].

$\mu$SR experiments on YBa$_2$Cu$_3$O$_{6+\delta}$ (YBCO) [9,11] have reported a magnetic field $B_0$ dependence of the $\mu$SR lineshape. This has been so far attributed to a microscopic and intrinsic magnetic field dependence of the London penetration length $\lambda$ and coherence length $\xi_0$ that could possibly be due to the underlying $d$–wave symmetry of the superconducting pairing state of YBCO [3]. This brings up the following question: Could the magnetic field dependence of the $\mu$SR lineshape and the field dependence ascribed to the extracted effective $\chi^d(B_0)$ and $\chi^s_0(B_0)$ be partially extrinsic and unrelated to the underlying microscopic $d$–wave physics, but rather due to the fact that the lineshape ought to change from (somewhat) asymmetric in the Bragg glass state to more or less a Gaussian shape in the vortex glass phase? Indeed, such field-driven evolution of the asymmetry (referred to as skewness) of the $\mu$SR lineshape has been reported for BSCCO [12,13] and YBCO$_{6.60}$ [14] high-$T_c$ materials. In this paper, we study the applied magnetic field $B_0$ dependence of the variance of the field distribution, and consequently $\chi^d(B_0)$, caused by weak point disorder. To this end, we use replica variational solutions of the displacement correlations of the flux lines in the presence of weak point disorder, and calculate the magnetic field variance, which is related to the structure factor $S(k)$ of the vortex system. We note, however, that the vortex-vortex displacement correlation functions of the weakly disordered vortex lattice is anisotropic and may have nonuniversal exponents [15]. Consequently, we use an isotropic model to simplify calculations and expose the basic underlying physics at stake. Here, we only study the effect of disorder; the role of thermal fluctuation effects will be discussed elsewhere.

The influence of the randomness on the translational correlation function has been discussed by many authors [4]. It was originally discussed by Larkin [16] using a model in which weak random forces act independently on each vortex. This model predicts an exponential decay of the translation correlations on length scales larger than a disorder dependent Larkin length, $R_L$. However, Bouchaud et al. pointed out that at large scales the lattice starts behaving collectively as an elastic manifold in a random potential with many metastable states and that the exponential decay does not hold beyond the Larkin length $R_L$ [17]. They used a variational replica field theory to study the pinning of vortex lattice by impurities they found a power law roughening of the lattice with stretched exponential decay of the translational correlation function for length scale beyond $R_L$ [17]. Later, Giamarchi and Le Doussal [5] showed that while disorder produce algebraic growth of displacement correlations at short scales, periodicity takes over at large scales and results in a growth of displacements that is at most logarithmic in $x$. The results for the displacement correlation function are summarized in Fig. 1 (b). For length scales smaller than $R_L$ the model is equivalent to Larkin’s model with correlations $\propto x$ (in three dimension) and which corresponds to the replica symmetric part of the variational solution. For $R_L < x < R_c$, the system is in the random manifold (RM) regime [17]. In this regime, replica symmetry is broken due the various metastable vortex configuration states and relative displacements correlate as $x^{2\nu}$, where $\nu = 1/6$ in variational approximation. For $x > R_c$, the periodicity of the lattice becomes important and one enters the asymptotic Bragg glass logarithmic regime [5,6].

In the London limit, the field of point vortices sitting at arbitrary positions $r_m = (x_m, y_m, z_m)$ for $\lambda >> d$, where $d$ is the distance between layers, varies slowly between layers, and is given by [18–20]:

$$B(r) = \int \frac{d^3k}{(2\pi)^3} b(k) \sum_m \exp[i k \cdot (r - r_m)]$$

$$b(k) = \frac{d\phi_0(\lambda^2 k^2 \pm k_z k_z)}{k^2 (1 + \lambda^2 k^2)} ,$$

where $k = (k_x, k_y, k_z)$.

Brandt [20] had used Eq. (1) to study the magnetic field variance in layered superconductors. Brandt showed that perturbation of a lattice of rather stiff flux lines increases the field variance [21]. However, the fluctuations of vortex line segments or vortex pancakes of highly flexible flux lines may decrease the $\mu$SR linewidth [20,22].

The general expression for the magnetic field variance, $\delta^2 = \left \langle \left \lbrace B(r) - \left \langle B(r) \right \rangle \right \rbrace^2 \right \rangle$, where $[\ldots], < \ldots >$, and $\rightarrow$ denote space, thermal, and disorder average, respectively, can be written as

$$\delta^2 = \left \langle \int \frac{d^3k}{(2\pi)^3} |b(k)|^2 \frac{1}{V} \sum_m \exp(-i k \cdot r_m) |^2 \right \rangle ,$$

where $V$ is the volume of the sample. The lattice sum is the structure factor of the vortex-point arrangement. One can rewrite Eq. (2) as
\[ \delta^2 = \frac{\rho_0}{d} \int \frac{d^3k}{(2\pi)^3} |b(k)|^2 S(k), \]  

(3)

where \( \rho_0 = B_0/\phi_0 \) is the density of vortices and the structure factor \( S(k) \) is given by

\[ S(k) = \frac{(B_0/\phi_0)}{d} \sum_G \int d^3x \frac{e^{i(k-\mathbf{G}) \cdot x}}{1 + \lambda^2 x^2} C_G(x). \]  

(4)

Here \( G \) is reciprocal lattice vector and \( C_G(x) \) is the transversal correlation function, which for simple isotropic case has the following form [5,6]

\[ C_G(x) = \exp \left\{-\frac{G^2}{2} \left( |u(x)|^2 \right) \right\}, \]  

(5)

where \( u(x) \) is the displacement of the flux line from its equilibrium position.

The above equation for \( \delta^2 \) has in general both perpendicular, \( \delta_\perp \), and \( \delta_\parallel \), components. Before analyzing the effect of impurities on the magnetic field variance, we first discuss a simple limiting case to estimate the order of different components. If the point vortices in each layer are assumed to be randomly positioned, then one simply has

\[ \delta^2 = \int \frac{d^3k}{(2\pi)^3} \frac{(B_0/\phi_0 d)}{(1 + \lambda^2 k^2)^2} \left[ 1 + \frac{1}{k_\perp^2 + \lambda^2} \right]. \]  

(6)

The first term gives the fluctuations of \( B_\perp \), and the second term the fluctuations of \( B_\parallel \). A cutoff has been used due to the factor \( 1/k_\perp^2 \), for which the Josephson length \( \lambda_J = d\lambda_c/\lambda \) was inserted by replacing \( 1/k_\perp^2 \) by \( 1/(k_\perp^2 + \lambda^2) \) [20]. Integrating over \( \mathbf{k} \) leads to

\[ \delta^2 = \frac{B_0 d \phi_0}{8\pi \lambda^2} \left[ 1 + \frac{\cos^{-1}(\lambda J/\lambda)}{\sqrt{\lambda J^2 - 1}} \right]. \]  

(7)

For \( \lambda J << \lambda \) the second term which comes from the random perpendicular fluctuations of \( \mathbf{B}(\mathbf{r}) \) is small. Consequently, and also for sake of simplicity, the perpendicular contribution shall be disregarded in the rest of the paper.

Returning to the general expression Eq. (2), using Eqs. (3), (4) and (5), and considering only the \( z \) component, and integrating first over all (six) angles and \( k \) we find

\[ \delta^2 \approx \delta_z^2 = \frac{B_0^2}{2\lambda^2} \sum_G \int_0^\infty xe^{-x/\lambda} C_G(x) \sin(Gz) dx. \]  

(8)

The above equation can then be used to study the variance of the magnetic field. The simple case of \( C_G(x) = 1 \) corresponds to the zero disorder case, in which integrating over \( x \) leads to the London model with \( \delta^2 = \frac{B_0^2}{\lambda^2} \sum_G \phi_0^2 1/(1 + \lambda^2 G^2)^2 \). For finite disorder, recall that the relative displacement correlation function has different behavior in the Larkin, RM, and logarithmic regimes (Fig. 1b). Therefore, we have to carry the integration of Eq. (8) for these three \( x \)-dependent regime of \( C_G(x) \).

Using expression (9) of Ref. [6] for \( R_c \),

\[ R_c = \frac{2a_0/\phi_0 c_{44}}{c_{44}^4 c_{66}}. \]  

(9)

where \( a_0 \) is the lattice spacing, \( \xi_0 \) is superconducting coherence length, \( U_p \) is a typical pinning energy per unit length along \( z \), and \( c_{44} \) and \( c_{66} \) are elastic constants. Considering \( c_{44} = c_{66} = e_0/(4a_0^2) \) with the vortex line tension \( e_0 = (\phi_0/4\pi\lambda^2) \), and \( c = \ln(\gamma d/\xi_0) \), \( \gamma \) is the anisotropy ratio, one gets

\[ R_c = \frac{2c_{44}/\gamma d\xi^0}{\pi^2} \left[ \frac{a_0}{\phi_0} \right]^4 \left( \frac{\xi_0}{U_p} \right)^2. \]  

(10)

The Larkin length is also given by \( R_L \approx (\xi_0/a_0)R_c \). Taking parameters for YBCO \( \gamma = 5 \), \( \lambda = 1200A, d = 10A, \xi_0 = 10A, \) and \( U_p/\xi_0 = 1/40 \), we see that \( R_L \approx 5A \) and \( R_c \approx 10^4A \) (for \( B_0 \approx 1T \)). Since \( R_L << a_0 \), \( R_c >> a_0 \), and \( R_c >> \lambda \), the effects of Larkin and logarithmic regions are very small, the main contribution to the integral Eq. (8) comes from the RM regime as we have confirmed by explicit calculation. Since the flux line displacement correlations \( \langle u(x) - u(0) \rangle^2 \sim \xi_0^2 (x/R_L) \) in the Larkin regime and \( \approx \langle \log(Ax/R_c) \rangle \) in the logarithmic regime \( (A \) and \( B \) are constants [5]), \( \delta \) can be calculated exactly in those two regimes and the contributions of these two regimes are indeed negligible.

We note that using these numerical values, the crossover field \( H_{cross} = \pi c_4/(\gamma d^2) \) is of order of \( 10^2T \) and the Bragg glass to vortex glass transition magnetic field \( H_M(T = 0) = ((\pi c_L)^4/(16\pi)^{1/3} \pi^2)) (e_0/U_p)^2 H_{c2}^{1/3} H_{cross}^{1/3} \), where \( c_L \) is the Lindemann criterion, is \( \sim 12T \) (for \( H_{c2} = 100T \) and \( c_L \approx 0.12 \) [5,6]. Similar values but \( U_p/\xi_0 = 1/60 \) leads to \( H_M(T = 0) \approx 20T \), and if \( \gamma \approx 35 \) and \( U_p/\xi_0 = 1/40 \) one obtains \( H_M(T = 0) \approx 4T \) and \( H_{cross} \approx 18T \) (we have used these three different cases in Fig. 2).

Focusing now on the RM regime contribution to \( \delta \), the displacement correlation function \( \langle u(x) - u(0) \rangle^2 \) is of order of \( (a_0/\pi)^2 (x/R_c)^{2/\nu} \), where \( \nu = 1/6 \) [5,6,17]. The magnetic field variance can then be calculated from Eq. (8):

\[ \delta_{RM}^2 = \sum_G \frac{B_0^2}{2\lambda G} \int_0^\infty xe^{-x/\lambda} C_G(x) \sin(Gz) dx, \]  

(11)

where \( a = R_L/\lambda, b = R_c/\lambda, \eta_G = 8G^2/(3K_0^2) \). The behavior of the magnetic field variance from numerical integration (shown in Fig. 2).

One notes that the magnetic field variance increases by increasing either magnetic field or the strength of the impurities. Since \( R_c \) is very small and \( R_L \) is very large, we can approximately replace the limit of integral from 0 to \( \infty \), and do the integral analytically. The results of analytical integration show the same behavior for the magnetic field variance, in agreement with numerical integration.
and [3]. As mentioned above, the microscopic effects such as those discussed in Refs. [2] and [3] are not quantitatively described in a rigorous manner by the phenomenological Gaussian broadening parameter [9–11]. Possibly, the phenomenology exposed in our calculation above may already be handled to some extend by the usage of a phenomenological Gaussian broadening parameter. However, more theoretical, numerical and experimental work is needed to assess whether or not this is the case. To conclude, we have shown that there can be significant magnetic field dependence for the magnetic field variance in the Bragg phase which is extrinsic and disorder driven and has no microscopic $d$–wave origin. We hope that our preliminary study will motivate further work.

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Fig. 2: Behavior of the variance $\delta$ of the internal magnetic field as a function of the strength of the applied magnetic field for different strength of the disorder.

We now briefly discuss our results in the context of $\mu$SR measurements [9,11]. In Ref. [11], an increase of $\lambda_{\text{eff}}(B_0)$ at $T = 4$ K between 25 $\text{A}$ to 50 $\text{A}$ per Tesla was reported. This corresponds to an effective narrowing of $\mu$SR linewidth. Above, in Fig. 2, we found a broadening of the internal magnetic field distribution by approximately 5–10 Gauss per Tesla of applied field $B_0$. To compare the magnitude of our field induced linewidth effect to the experiment, we invoke the formula for $\delta$ that applies to that of a perfect Abrikosov triangular vortex lattice in the London limit $\delta_{\text{NL}} \sim 0.061 \Phi_0/\lambda^2$, where $\Phi_0 = 2 \times 10^{-15}$ Tm$^2$ [9]. We find that an increase of $\lambda_{\text{eff}}$ of between 25 $\text{A}$ to 50 $\text{A}$ reported in Fig. 3 of Ref. [11] corresponds to a decrease of the width of the internal magnetic field distribution by about 4–8 Gauss per Tesla of applied field. In other words, the effect we report is of the same magnitude as that reported in $\mu$SR experiments [11] but ascribed therein to the intrinsic nature of the field dependence of the microscopic $\lambda$. The fact that we get a contribution of opposite trend (i.e. broadening) to that observed in experiments is irrelevant. Our results suggests that the extrinsic modification of the internal field distribution as the system evolves from the Bragg glass to the vortex glass is of the same order as a contribution that may be of microscopic origin. Consequently, a quantitative description of the $\mu$SR data that incorporates both intrinsic and extrinsic effects is desirable in order to expose in a quantitative manner any microscopic effects such as those discussed in Refs. [2] and [3]. As mentioned above, the $\mu$SR data are typically parametrized by including a phenomenological Gaussian broadening parameter [9–11]. Possibly, the phenomenology exposed in our calculation above may already be handled to some extend by the usage of a phenomenological Gaussian broadening parameter. However, more theoretical, numerical and experimental work is needed to assess whether or not this is the case. To conclude, we have shown that there can be significant magnetic field dependence for the magnetic field variance in the Bragg phase which is extrinsic and disorder driven and has no microscopic $d$–wave origin. We hope that our preliminary study will motivate further work.

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