CP VIOLATION

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Lectures given at the 37th Winter School on Particle Physics,
Schladming, Austria, Febr. 28 - March 7, 1998.

Abstract. The salient features of CP-violating interactions in the standard electroweak theory and in a few of its popular extensions are discussed. Moreover a brief overview is given on the status and prospects of searches for CP non-conservation effects in low and high energy experiments.

1 Introduction

More than 30 years after the discovery of CP non-conservation in the neutral K meson system neither the physics that causes this phenomenon has been clarified nor have other CP- or time-reversal-violating effects been found in laboratory experiments. The standard theory of electroweak interactions can explain the experimental findings by a complex phase in the coupling matrix of the charged weak quark currents [1]. Yet in this theory the key to a deeper understanding of this symmetry violation is hidden behind the mystery of the flavour problem. On the other hand, CP-violating interactions are conceivable which have nothing to do with the fact that there are three generations of quarks and leptons with disparate mass spectra. Interactions of this kind naturally appear in popular and, so far, empirically acceptable extensions of the Standard Model. The question whether CP-symmetry breaking is due to a single “source” – which is most likely the Kobayashi-Maskawa phase [1] – or whether there are several CP-nonconserving interactions which will show up in different physical situations must be resolved experimentally. Another enigma of particle physics – often called “problem number one” – is the dynamics of electroweak symmetry breaking. Very probably these two dark corners are related: clarification of weak gauge symmetry breaking – which must also be achieved by experiments – would also shed light on the origin(s) of CP violation.

In these lectures I shall first review the salient features of CP violation in the Standard Model and in some of its extensions, notably multi-Higgs and supersymmetric extensions. The issue of explicit versus spontaneous CP breaking will also be discussed. Then a brief overview is given on the status and prospects of searches for CP non-conservation effects in weak decays of strange, charmed, and beauty hadrons, on the search for permanent electric dipole moments of particles, and on present and future high energy CP tests at colliders.
2 Models

The discussion in this section rests on the assumption that the Higgs mechanism – which requires at least one elementary scalar field multiplet with non-vanishing ground state expectation value – gives the correct description of electroweak gauge symmetry breaking. A priori, breaking this symmetry does not require elementary Higgs fields; it might have occurred “dynamically” by condensation of (new) fermion bilinears. These vacuum expectation values can have complex phases relative to each other, which induce observable CP violation. I shall not discuss this possibility of CP-breaking, which is not without problematic features, any further here (see, e.g., [2]).

Moreover, the following discussion remains within the context of four-dimensional gauge theories, where CP constitutes a discrete symmetry transformation. In higher dimensional theories, including string theories, CP can be a gauge symmetry which gets spontaneously broken [3].

In the framework of four-dimensional gauge theories with elementary Higgs fields one can also distinguish between two situations:

(a) CP invariance is violated explicitly at the Lagrangian level. That is, in the “Hamiltonian of the world”, $H_{\text{inv}} + H'$, there is a term $H'$ (which by a posteriori reasoning can usually be treated in perturbation theory) for which $[H', U_{\text{CP}}] \neq 0$. Here $U_{\text{CP}}$ is the unitary operator which implements the CP transformation in the Hilbert space of the theory given by $H_{\text{inv}}$.

(b) One may have CP invariance of the full Hamiltonian, $[H, U_{\text{CP}}] = 0$, but this symmetry is spontaneously broken by the ground state, $U_{\text{CP}}|0 > \neq e^{i\phi}|0 >$. This scenario requires more than one Higgs multiplet. In the following we shall discuss both options.

2.1 The Kobayashi-Maskawa mechanism

CP violation in the three-generation Standard Model (SM) of electroweak interactions is related to the fact that Nature has chosen the option that, as far as quarks are concerned, the mass eigenstates are different from the weak interaction eigenstates. (This may also be the case in the lepton sector as recent experimental results on atmospheric neutrinos and their interpretation in terms of neutrino oscillations indicate.) In the weak basis the Yukawa interactions of the quarks with the SU(2) doublet Higgs field are described by two $3 \times 3$ coupling matrices. The requirement of hermiticity of the Hamiltonian does not preclude that these matrices are complex. After having transformed the quark fields from the weak basis to the mass basis, the various pieces of the SM Lagrangian $L_{\text{SM}}$ are diagonal in generation space (in unitary gauge), except for the charged current interactions of quarks,

$$L_{cc} = -\frac{g}{\sqrt{2}} \overline{U}_{L} \gamma^{\mu} V_{KM} D_{L} W_{\mu}^{+} + \text{h.c.},$$

which contains the Cabibbo-Kobayashi-Maskawa (KM) matrix [1], a $3 \times 3$ unitary matrix in generation space. Here $\overline{U} = (\overline{u}, \overline{c}, \overline{t})$ and $D = (d, s, b)^{T}$ are the quark
fields in the mass basis. Five parameters of the CKM matrix elements can be eliminated by a change of phase of the quark fields

\[ u_i \rightarrow e^{i\omega_i} u_i, \quad d_j \rightarrow e^{i\tilde{\omega}_j} d_j \quad \Rightarrow \quad V_{ij} \rightarrow e^{i(\omega_i - \tilde{\omega}_j)} V_{ij}, \]  

where \( i,j = 1,2,3 \) are generation indices. Hence the matrix \( V_{KM} \) has four observable parameters, which may be chosen to be three Euler-type angles and a phase angle \( \delta_{KM} \). If \( \delta_{KM} \neq 0, \pm \pi \), the charged current Hamiltonian \( H_{cc} = -\int d^3x \mathcal{L}_{cc}(x) \) is non-invariant under a CP transformation.

In view of eq. (2), only functions of \( V_{ij} \) which are rephasing-invariant have a physical meaning. The simplest invariants are \( |V_{ij}| \) and \( Q_{ijkl} = V_{ij} V_{kl} V_{il}^* V_{kj}^* \).

For three generations the unitarity of \( V_{KM} \) implies that the \( |\text{Im}(Q_{ijkl})| \) are all equal [4], [5]. (In fact the various unitarity triangles, representing the orthogonality relations of the KM matrix elements, all have the same area which is equal to \( |\text{Im}Q|/2 \).) Therefore, for instance

\[ \text{Im}Q = \text{Im}(V_{ud} V_{cb} V_{ub}^* V_{cd}^*) \]  

is an invariant measure of CP violation à la KM. This expression immediately shows that the strength of KM CP violation is small even if the CP-violating phase angle were maximal. Insertion of the measured values of the moduli of the KM matrix elements into eq. (4) yields that \( |\text{Im}Q| \) is smaller than \( 10^{-4} \). (For a discussion of maximal CP violation in the SM, see [6].)

A deeper understanding of CP violation à la KM requires an answer to the “flavour problem”, i.e., an answer to the question why there are three fermion generations and why is there such a hierarchy in the mass spectra of the u- and d-type quarks, respectively. If any two quarks with the same charge were mass-degenerate, the CP-phase in \( V_{KM} \) could be eliminated by a suitable unitary transformation of the quark fields. This feature of KM CP violation is exhibited by the well-known invariant [5], [7]

\[ J_{CP} = \prod_{i,j} (m_i^2 - m_j^2) \prod_{i,j} (m_i^2 - m_j^2) \quad \text{Im}Q. \]

If neutrinos have non-degenerate masses then there can be KM-type CP violation in the lepton sector as well. For three generations, the leptonic analogue of the KM matrix, \( V_{\text{lept}} \), which then parameterizes the relative strength of the leptonic charged current-induced transitions, can have 1 CP phase angle (3 CP phase angles) if the neutrinos are of the Dirac (Majorana) type.

KM CP violation is observable only in flavour-changing charged current reactions. As is obvious from eqs. (4) and (5), effects are in general small because of small mixing angles involved. In charm hadron and in top quark decays, which are Cabibbo-allowed, even a maximal CP phase in the KM matrix thus leads
only to very small effects. $K$ and $B$ mesons, whose weak decays are at least singly Cabibbo-forbidden, are therefore the objects to test the KM mechanism.

A non-zero KM phase leads only to negligibly small effects in flavour-diagonal amplitudes. For instance it induces tiny electric dipole moments (EDM) of quarks [8] and even tinier ones for charged leptons (see section 4).

2.2 The strong CP problem

At this point it is appropriate to recall the strong CP problem, which is actually not a problem of Quantum Chromodynamics (QCD) in isolation, but of the theory of strong and weak interactions. In QCD topologically non-trivial quantum fluctuations (instantons) induce a parity- and time-reversal-violating term in the QCD quantum action of the form $S_\theta = \left( \frac{\theta g^2}{32\pi^2} \right) \int d^4x G_\mu^a \tilde{G}^{a\mu}$, where $\theta$ is the QCD vacuum angle. This term has observable consequences in flavour-diagonal amplitudes. Observables depend, however, on the parameter

$$\bar{\theta} = \theta - \arg(\det M_q)$$

(6)

where $M_q$ is the non-diagonal mass matrix of the u- and d-type quarks in the weak basis. The experimental upper bound on the neutron EDM implies [9] that $|\bar{\theta}| < 3 \times 10^{-10}$. We lack a deeper understanding why this parameter should be so small. Simply setting $\bar{\theta}$ equal to zero is unsatisfactory because, according to 't Hooft’s naturalness condition [10], it does not increase the symmetries of the SM. After all, CP must not be a good symmetry of the SM if this theory is to explain the observed CP effect in $K^0-\bar{K}^0$ mixing. It requires $\delta_{KM}$ to be of order one and hence one would expect $\arg(\det M_q)$ to be of the same order. So there is apparently severe fine tuning of $\theta$ required to bring $\bar{\theta}$ down to the level of $10^{-10}$. For a more detailed discussion of this problem and of the possible ways out that have been proposed, see [11]. One may take a “just so” attitude and consider $\bar{\theta}$ to be just another one of the uncalculable parameters of the SM that happens to be (very close to) zero. However, many theorists believe that one cannot understand CP violation before one hasn’t solved this problem.

2.3 Extensions of the Standard Model

There are a number of well-known arguments which motivate the belief in new physics beyond the Standard Model, to be discovered in particle physics experiments. Extensions of the SM, even if based on the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$, almost invariably entail a larger non-gauge sector, that is to say, scalar self interactions and Yukawa interactions. In this way quite a number of “new” CP-violating (CPV) interactions for quarks and for leptons are conceivable in a natural way. (In the following, new CP-violating interactions refer to interactions that are not due to the KM phase). In particular CPV interactions with the following features may exist:

(a) Interactions that are unrelated to the mixing of quark generations and the
hierarchy of quark masses. Such interactions induce CP effects also in flavour-diagonal amplitudes.

(b) Higgs-type interactions whose strength increases with the mass of the fermion involved, leading to sizeable effects in the heavy flavour sector.

Explicit CP violation in multi-Higgs models The simplest, phenomenologically viable model with extra CPV besides the KM phase is, perhaps, the extension of the SM by an extra $SU(2)$ Higgs doublet. The two Higgs doublets $\Phi_1, \Phi_2$ are assumed to couple to quarks and leptons in such a way that there are no flavour-changing neutral couplings at the tree-level (see, e.g., [12]). This “natural flavour-conservation constraint” can be enforced by imposing a discrete symmetry. The different implementations of this symmetry define different models (see, for instance, [13]). Apart from complex Yukawa coupling matrices, which lead to the KM phase, the requirement of hermiticity, renormalizability, and $SU(2)_L \times U(1)_Y$ invariance of the Lagrangian does not preclude explicit CPV in the Higgs potential $V_\Phi$. Requiring that the potential breaks the above-mentioned discrete symmetry only softly (that is, by terms with operator dimension less than four) one can have

$$V_\Phi = V_0(\Phi_1, \Phi_2) + [\kappa \Phi_1^\dagger \cdot \Phi_2 + h(\Phi_1^\dagger \cdot \Phi_2)^2 + \text{h.c.}].$$

(7)

Here $V_0$ denotes the CP-invariant part of the potential. A CP transformation,

$$\Phi_{1,2}(x, t) \xrightarrow{CP} e^{i\alpha_{1,2}} \Phi_{1,2}^\star(-x, t),$$

(8)

shows that the term in the square brackets of eq. (7) breaks CP if $\xi = \text{Im}(\kappa^2 h^*) \neq 0$. Note that it is unnatural to assume $\xi = 0$. Even if this were so at tree level, the non-trivial KM phase $\delta_{KM}$, which is needed to explain the observed CPV, would induce a non-zero $\xi$ through radiative corrections.

The spectrum of physical Higgs boson states in the two-doublet models consists of a charged Higgs boson and its antiparticle, $H^\pm$, and three neutral states. As far as CPV is concerned, $H^\pm$ carries the KM phase. It affects the CPV phenomenology of flavour-changing $|\Delta F| = 2$ neutral meson mixing and $|\Delta F| = 1$ weak decays of mesons and baryons. From experimental data on $b \to s + \gamma$ the lower bound $m_{H^+} > 210$ GeV on the mass of this particle was derived [14].

If $\xi$ were zero, the set of neutral Higgs boson states would consist of two scalar (CP=1) and one pseudo-scalar (CP=−1) state. Explicit CPV in the Higgs potential has the consequence that these states mix [15] (note that the mixing occurs already at tree level), leading to three mass eigenstates $\varphi_{1,2,3}$ that no longer have a definite CP parity. That is, they couple both to scalar and to pseudo-scalar quark and lepton currents. The Yukawa interactions read (for ease of notation the same symbol is used for a field and its associated particle state)

$$\mathcal{L}_\varphi = -\left(\sqrt{2}G_F\right)^{1/2} \sum_f \left( a_f m_f \bar{f}f + \tilde{a}_f m_f \bar{f}i\gamma_5 f \right) \varphi.$$

(9)
The sum over the Higgs fields $i = 1, 2, 3$ is implicit. Here $G_F$ is Fermi’s constant, $f$ denotes a quark or lepton field, $m_f$ is the mass of the associated particle, and the dimensionless reduced Yukawa couplings $a_f, \tilde{a}_f$ depend on the parameters of the Higgs potential and on the type of model [16]. From LEP data one infers that the lightest of the three states $\varphi_i$ should have a mass larger than about 50 GeV. (The precise lower bound depends on the parameters of the model.)

In terms of the parameters of eq. (9) CP violation in the neutral Higgs sector occurs if $a_f \cdot \tilde{a}_f \neq 0$. The following generic features arise: (a) The Yukawa interaction (9) leads to CPV in \textit{flavour-diagonal} amplitudes for quarks and for leptons. (b) The induced CP effects are proportional to some power $(m_f)^p$, where one finds $p=1,2,3$ for reactions discussed in sections 4,5 below. For example, neutral $\varphi$ exchange at tree level induces an effective CPV interaction of the form $(\bar{f}f)(\bar{f}f\gamma^5)$ with a coupling strength proportional to $m_f^2$. Neutral $\varphi$ exchange at one-loop in the $\gamma ff, Zff, \text{and } Gqq$ amplitudes ($G$ denotes a gluon) leads to $T$- respectively CP-violating electric, weak, and chromo-electric dipole moment form factors of the fermion involved which are proportional to $m_f^3$ [16]. Potentially large effects can be expected for top quarks.

In models with a more complicated scalar sector, for instance, in models with $n \geq 3$ Higgs doublets, there is more than one charged Higgs particle. The scalar potential can be such that these states mix in a CP-violating way which leads to a complex mass matrix for these bosons. Transforming all fields to their respective mass basis, the interaction of the quarks with the charged Higgs bosons then reads

$$L_{ch} = -(2\sqrt{2}G_F)^{1/2} \sum_i (\alpha_i \bar{U}_L V_{KM} M_D D_R + \beta_i \bar{U}_R M_U V_{KM} D_L) H_i^+ + \text{h.c.}, \quad (10)$$

where $M_{U,D}$ denote the real, diagonal $3\times3$ mass matrices of the u- and d-type quarks, and, in general,

$$\text{Im}(\alpha_i \beta_j^*) \neq 0, \quad (11)$$

due to the complex phases in the mass matrix of the charged Higgs bosons. The interactions of $H_i^\pm$ with leptons are of the same structure. In models where the right-handed quarks $q_R$ couple to several Higgs multiplets one can have additional CP phases such that the reduced Yukawa couplings in eq. (10) satisfy

$$\text{Im}(\alpha_i \alpha_j^*) \neq 0, \quad \text{Im}(\beta_i \beta_j^*) \neq 0 \quad (i \neq j). \quad (12)$$

Charged Higgs exchange with couplings as in eq. (10) induces also CP violation in \textit{flavour-diagonal} amplitudes. For instance, if eq. (11) holds, one- and two-loop contributions to electric dipole form factors of quarks and leptons are induced (see section 4). If eq. (12) holds, CPV chiral-invariant form factors in the $b\bar{b}ZG$ amplitude are generated at one-loop order [17].

In fact, even in two-Higgs doublet models charged Higgs boson exchange can provide CPV independent of the KM phase if one allows for general Yukawa interactions [18] (which imply flavour-changing neutral Higgs boson exchanges at tree-level).
Explicit CP violation in supersymmetric models

In the minimal supersymmetric extension (MSSM) of the Standard Model [19], [20] CP violating phases can arise, apart from the complex Yukawa interactions of the quarks yielding a non-trivial $\delta_{KM}$, from the soft supersymmetry breaking terms. The requirement of gauge invariance and hermiticity of the Lagrangian allows for

i) complex Majorana masses $M_i$ in the gaugino mass terms,

$$-\sum_i (M_i \lambda_i \lambda_i)/2 + \text{h.c.},$$

ii) complex trilinear scalar couplings, that is, complex $3 \times 3$ matrices $A_{q, \ell}$ in generation space which contain the couplings of the scalar quarks and leptons to the Higgs doublets $\Phi_1, \Phi_2$. For instance

$$\bar{\tilde{D}}^L_R A_{d} \Phi_1^\dagger \cdot \tilde{\tilde{Q}}^L + \text{h.c.},$$

and analogous interactions with coupling matrices $A_u$ and $A_{\ell}$. The tilde and the prime denote scalar quark fields in the weak basis, capital letters denote as before vectors in generation space. The label L refers to $SU(2)_L$ doublets, $\tilde{\tilde{Q}}_L^L = (\tilde{\tilde{U}}^L_L, \tilde{\tilde{D}}^L_L)^T$, and the label $R$ in eq. (14) refers to $SU(2)_L$ singlets.

iii) a complex soft term in the Higgs potential

$$\kappa \Phi_1^\dagger \cdot \Phi_2 + \text{h.c.}$$

Motivated by supergravity models it is often assumed that the $A_f$’s are proportional to the Yukawa coupling matrices

$$A_f = A Y_f, \quad f = u, d, \ell,$$

where $A$ is a complex parameter. The observable CP phases of the MSSM are readily counted [21]. Apart from the KM phase and the QCD $\bar{\theta}$ parameter, these are $\text{arg}(AM_i^*)$ and $\text{arg}(\kappa M_i^*)$, where $M_i$ are the gaugino mass terms in eq. (13).

If eq. (16) is not imposed then there are quite a number of independent phases.

After spontaneous symmetry breaking and after having transformed the various fields such that all mass matrices have become real and diagonal, the CP phases have been shifted into the fermion-sfermion-neutralino and -chargino interaction terms. Let us write down here only the gluino interaction, which involves the QCD coupling $g_{QCD}$. One arrives at the CP-violating quark-squark-gluino interaction Lagrangian in the mass basis, which reads for $u$-type quarks

$$\mathcal{L}_{Gu\tilde{u}} = i\sqrt{2} g_{QCD} \sum_{j,l} (e^{-i\varphi_j} \bar{u}_j L \Gamma_j^j \tilde{G}^a \tilde{u}_l + e^{+i\varphi_j} \bar{u}_j R \Gamma_j^j \tilde{G}^a \tilde{u}_l) + \text{h.c.},$$

where $j=1,2,3$, and $l=1,...,6$, $\tilde{G}^a$ denote the gluino fields, $T^a$ are the generators of $SU(3)_c$ in the fundamental representation, and $\Gamma, \Gamma'$ are complex $3 \times 6$ matrices.

\footnote{In order to facilitate the comparison with the above models, the non-SUSY convention, i.e., the same hypercharge assignment for both $SU(2)$ Higgs doublets, $\Phi_i = (\phi_i^+, \phi_i^0)^T$, $(i=1,2)$ is employed here.}
(Recall that for each flavour there are two squark respectively slepton mass eigenstates, which are in general not mass-degenerate.) The $d\bar{d}G$ interaction is of the same form. As already mentioned there are further CP-violating fermion-sfermion-neutralino and -chargino interactions (of similar structure as eq. (17)) with interesting phenomenological consequences.

If eq. (16) holds then the phase $\varphi_q = \varphi_{\tilde{G}} - \varphi_A$ is universal and $\Gamma, \Gamma'$ depend, as far as CP phases are concerned, only on the KM phase. However, in the general case things are really more complex.

As the gluino interactions involve both flavour-diagonal and flavour-changing $\Delta Q = 0$ vertices, $\mathcal{L}_{\tilde{G}q}$ induces CPV effects in neutral meson mixing, in flavour-changing decays of hadrons, and it leads to electric dipole moments (EDM), e.g. of the neutron, of considerable size. The latter constitutes a well-known conflict for the MSSM. The predictions for the neutron EDM come out too large as compared with the experimental upper bound if the CP phases of the soft terms i) ii) and iii) above were of order one and if the squark and gluino masses were about, say, 200 GeV. (See section 4). Therefore it is often assumed in the literature that the CP phases of the soft terms i) ii) and iii) are zero at a very high energy scale, which is usually taken to be a supposed grand unification scale or the Planck scale. Then CP violation at this scale is assumed to come only from the Yukawa couplings, i.e., the KM phase. When evolving the parameters of this constrained MSSM model down to lower energies, the KM phase induces, through renormalization, also small phases in the soft SUSY breaking terms [22], which are phenomenologically acceptable as far as EDMs are concerned. (For a discussion of the CP-violating phases and their phenomenological consequences in the supersymmetric grand unified $SO(10)$ model, see [23].)

What about Higgs sector CPV in supersymmetric extensions of the SM? In the MSSM the tree-level Higgs potential is, schematically, of the form

$$V_{\text{tree}} = V_0(\Phi_1, \Phi_2) + (\kappa \Phi_1^1 \cdot \Phi_2 + \text{h.c.}). \quad (18)$$

As is well-known (see, e.g. [13]) SUSY does not allow for independent quartic couplings in $V_0$. They are proportional to linear combinations of the $SU(2)$ and $U(1)$ gauge couplings squared. Moreover, a term of the form $(\Phi_1^1 \cdot \Phi_2)^2$ and two other quartic terms which are non-invariant under $\Phi_1 \to -\Phi_1$ are absent at tree-level. Hence by suitable adjustment of the phases $\alpha_i$ in eq. (8) a CP transformation on the scalar fields can be implemented such that $\int d^4x V_{\text{tree}}$ is CP-invariant. Thus there is no CPV mixing of the three physical neutral Higgs states at tree level. The CPV interactions in the MSSM discussed above generate a (small) complex coupling $h$ (cf. eq. 7) in the effective potential at one-loop order, and the parameter $\xi$ defined above can now become non-zero. The other quartic terms absent at tree-level will also be induced. Hence there can be CPV mixing at one-loop order of the two CP=1 and the CP= -1 neutral Higgs states in the MSSM. It is, however, expected to be small.

In next-to-minimal SUSY models with two $SU(2)$ Higgs doublet fields, $\Phi_1, \Phi_2$, and a gauge singlet field $N$ the Higgs potential can explicitly violate CP at the
tree level. For instance, this is the case for the potential

$$V_{\text{tree}} = V_{\text{inv}}(\phi_1, \phi_2, N) + (h_1 N^3 + h_2 \phi_1 \cdot \phi_2 N + h_3 \phi_1^* \cdot \phi_2 N^2 + \text{h.c.}), \quad (19)$$

where $h_{1,2,3}$ are arbitrary complex couplings and $V_{\text{inv}}$ is the CP-invariant part of the potential. Appropriate field redefinitions show that there is one observable CPV phase in eq. (19). In this case the three CP=1 and the two CP= – 1 physical neutral Higgs states mix at tree-level [24]. (There is, however, no mixing of the two scalar with the pseudo-scalar component of the two Higgs doublets.)

**Spontaneous CP violation** There is a potential cosmological problem when discrete symmetries are spontaneously broken [25]. When spontaneous CPV occurs in the early universe at some temperature, one expects that domains with different CP signatures (i.e., different signs of the CP-violating phase(s)) are formed. The energy density of the walls which separate these domains dissipate not rapidly enough when the universe expands further. Estimates yield that the energy density associated with these walls today exceeds the closure density of the universe by many orders of magnitude [25]. The problem is avoided if CP got broken before inflation took place. However, the connection to low energy phenomena then becomes very loose.

Ignoring this problem, it is nevertheless interesting to investigate multi-Higgs or supersymmetric extensions of the SM with spontaneous CPV at the weak scale. The simplest model in this respect is the original two-Higgs doublet model of T.D. Lee [26], respectively its more recent variants [27], [28], [18]. By construction the Lagrangians of these models, which have the same gauge group and the same particle content – apart from the Higgs sector – as the SM, are CP-invariant. Hence the Yukawa couplings can be taken to be real, without loss of generality. Gauge invariance, hermiticity, and renormalizability imply that the tree level Higgs potential has the general form [26]

$$V = V_0(\phi_1, \phi_2) + (\kappa \phi_1 \cdot \phi_2 + \lambda_1 (\phi_1 \cdot \phi_2)^2 + \lambda_2 (\phi_1^* \cdot \phi_2) (\phi_1^* \cdot \phi_1)$$

$$+ \lambda_3 (\phi_1^* \cdot \phi_2) (\phi_2^* \cdot \phi_2) + \text{h.c.}), \quad (20)$$

where, contrary to eq. (7), the parameters in eq. (20) are real because $V$ is required to be CP-invariant. Minimisation of the potential yields the vacuum expectation values (VEV) of the two Higgs fields (the phase of $\phi_1$ has been adjusted such that the VEV of this field is real)

$$<0|\phi_1^0|0> = v_1/\sqrt{2}, \quad <0|\phi_2^0|0> = v_2 e^{i\theta}/\sqrt{2}, \quad (21)$$

where $v_1, v_2$ are real and positive parameters which have to satisfy the experimental constraint $\sqrt{v_1^2 + v_2^2} = 246$ GeV. There exists a range of parameters of $V$ such that the absolute minimum is characterized by a non-trivial phase [26]

$$\theta = \arccos\left(\frac{2\kappa - \lambda_2 v_1^2 - \lambda_3 v_2^2}{4\lambda_1 v_1 v_2}\right), \quad (22)$$
The necessary condition for this to happen is
\[ \lambda_1 > 0, \quad \left| \frac{2\kappa - \lambda_2 v_1^2 - \lambda_3 v_2^2}{4\lambda_1 v_1 v_2} \right| < 1. \] (23)

If the phase angle \( \vartheta \neq \pm n\pi/2, n = 0,1,2, \ldots \), then the ground state breaks CP spontaneously. It can be shown that there is then no unitary implementation of CP such that the Lagrangian and the vacuum remain invariant [29].

One consequence of spontaneous CPV is, again, CPV mixing of neutral Higgs states. Yet one must account for the observed CPV in \( K^0 - \bar{K}^0 \) mixing, but the Yukawa couplings are real by construction. Therefore the construction principle of “natural flavour conservation” must be given up. The right-handed quark fields \( u_{iR}, d_{iR} \) are coupled both to \( \Phi_1 \) and \( \Phi_2 \), yielding the general Yukawa interactions
\[
\mathcal{L}_Y = -\sum_{k=1}^{2} [Y_d^k \bar{Q}'_L \cdot \Phi_k D'_R + Y_u^k \bar{Q}'_L \cdot \tilde{\Phi}_k U'_R] + h.c.,
\] (24)
where primes denote the weak basis, \( \tilde{\Phi} = i\sigma_2 \Phi \), and \( Y_k^q \) denote four \( 3 \times 3 \) real Yukawa matrices. Expanding around the ground state (21), one obtains the non-diagonal complex mass matrices
\[
M_u = \frac{v_1}{\sqrt{2}} Y_1^u + \frac{v_2 e^{i\vartheta}}{\sqrt{2}} Y_2^u, \quad M_d = \frac{v_1}{\sqrt{2}} Y_1^d + \frac{v_2 e^{i\vartheta}}{\sqrt{2}} Y_2^d.
\] (25)

It follows that \( M_u M_u^\dagger \) and \( M_d M_d^\dagger \) are arbitrary hermitian matrices. Diagonalization of these matrices leads to charged weak quark interactions of the usual form (1) with a complex mixing matrix \( V_{KM} \) whose CP phase depends on \( \vartheta \).

In short, these models have only one CP parameter, namely the “vacuum phase angle” \( \vartheta \), but a rich CP phenomenology:
i) CPV in charged weak current reactions (\( W^\pm \) and \( H^\pm \) exchange) and
ii) CPV mediated by flavour-conserving and by flavour-changing neutral Higgs boson \( \varphi \) exchange.

Note that the observed CPV in \( |\Delta S| = 2 \) \( K^0 - \bar{K}^0 \) mixing is dominantly generated by tree-level \( \varphi \) exchange. \( sd \leftrightarrow \bar{s}d \). In this respect, these models may be regarded as a realization of Wolfenstein’s “superweak hypothesis”. But in general there will be also other CPV \( |\Delta F| = 2 \) tree-level transitions. The problematic feature of these models is that fine tuning of the flavour-changing neutral current couplings (or the appeal to some approximate symmetry in flavour space) and/or rather large \( \varphi \) masses are required in order to avoid conflict with experiment (leaving aside the strong CP problem).

The simplest SM extension with spontaneous CPV and flavour conservation in neutral Higgs particle interactions at tree-level seems to be the three Higgs-doublet model of ref. [30]. CPV originates from two CPV vacuum phase angles \( \vartheta_1, \vartheta_2 \) which lead to CPV neutral Higgs mixing and to a complex mass matrix for the charged Higgs bosons. However, in this model \( V_{KM} \) remains real [31] and the observed CP violation in neutral kaon mixing must be accounted for by
charged Higgs boson exchange (one-loop box diagrams). The model appears to be incompatible with experimental data: One has a hard time explaining why CPV charged Higgs boson exchange generates $\epsilon \approx 10^{-3}$ in the $K$ meson system, but suppresses $\epsilon'/\epsilon$ to the level of $10^{-3}$ or even below that number and the neutron electric dipole moment below $10^{-25}$e·cm.

Is spontaneous CPV a viable concept for supersymmetric extensions of the SM? Let us first consider the MSSM and assume CP invariance of the Lagrangian $\mathcal{L}_{MSSM}$. It is clear from the discussion below eq. (20) that there is no spontaneous CPV at tree level, because the couplings $\lambda_1 = \lambda_2 = \lambda_3 = 0$ in the tree level potential (18). Chargino, neutralino, $t$, and $\tilde{t}$ contributions to the effective potential at one-loop [32] generate small, real couplings

$$\lambda_{1,2,3} \sim g^4/32\pi^2 \sim 5 \times 10^{-4}. \quad (26)$$

If the parameters are such that condition (23) is fulfilled then the model can have a stable ground state [32] which spontaneously breaks CP. It follows from eqs. (23) and (26) that this requires the parameter $\kappa$ to be small, $\kappa = \mathcal{O}(\lambda_1 v^2)$. This implies, however, that the lightest of the three neutral Higgs bosons has a mass of about $m \approx \sqrt{4\lambda_1 v^2 \sin^2 \theta} \approx 5$ GeV, which is incompatible with experimental constraints. Hence this scenario is of no use for the MSSM. (The appearance of a light boson can be understood from the Georgi-Pais theorem [33].)

Spontaneous CPV in the next-to-minimal supersymmetric extension of the SM (see above) was investigated in [34], [35]. Radiative corrections to the tree-level scalar potential (19), with all couplings now being real because of CP invariance, are also essential in this model for the mechanism of spontaneous CPV to work [35]. (The parameters of the potential are constrained by the requirement that the masses of the scalar states must be positive.) Refs. [35] find that in this case the mass of the lightest neutral Higgs boson has an upper bound of about 36 GeV and the sum of the masses of the two lightest neutral Higgs bosons is not much above the mass of the $Z$ boson. It is questionable whether this scenario is still compatible with data from LEP.

| model            | mechanism of CPV      | required non-degeneracy in mass               |
|------------------|-----------------------|-----------------------------------------------|
| SM               | KM                    | u-type quarks, d-type quarks                  |
| massive neutrinos| KM-like               | charged leptons, neutrinos                    |
| multi-Higgs models| neutral Higgs mixing | neutral $\varphi_j$ bosons                   |
|                   | charged Higgs mixing  | charged Higgs bosons $H_j^\pm$               |
| MSSM             | phases in SUSY breaking terms | scalar fermions of a given flavour $\tilde{f}_1, \tilde{f}_2, (\tilde{f} = \tilde{q}, \tilde{t})$ |
Miscellaneous issues As has been discussed in section 2.1, CP violation à la KM requires non-degeneracy of the masses of both u- and d-type quarks. In fact this is a generic feature of CP violation from the non-gauge sector (that is to say, ignoring the $\theta$ term of QCD). For the models discussed above this is exhibited in the table above. Invariants similar to $J_{CP}$ of eq. (5) can be constructed also for the non-KM sources of CPV (cf., for instance, [36]).

So far the only hint for CPV beyond the KM phase comes from attempts to develop scenarios for explaining the baryon asymmetry of the universe on the basis of particle physics models and the big-bang model. It is well-known by now that within this framework baryogenesis requires [37] baryon number violation, C and CP violation, and thermal non-equilibrium, that is to say, an “arrow of time”. Two types of scenarios have been intensely investigated in recent years:
a) Baryogenesis at the electroweak phase transition. Investigations of the nature of the phase transition lead to the conclusion that this scenario does not work within the SM. (For a recent review, see [38].) Even if non-SM interactions exist such that the electroweak phase transition was of first order, it is questionable whether KM CP violation did the job. (For reviews, see [39].) According to present knowledge it seems that, for instance, two-Higgs doublet extensions [39], [40] and the MSSM [41], with CPV as discussed in section 2, still provide viable alternatives.
b) Out-of-equilibrium decays of ultra-heavy Majorana neutrinos [42], with $(B-L)$ violation, at temperatures far above the electroweak phase transition.

It would be fascinating to relate the observed CP violation in the laboratory, which so far amounts to the $\epsilon$ parameter of neutral $K$ meson mixing, to the fact that the visible universe contains matter, rather than anti-matter, with a baryon-to-photon ratio of $n_B/n_\gamma \sim 10^{-10}$. However, as suggested by the investigations of scenario b), it may be that the CP-violating interactions which were at work in the early universe are irrelevant for reactions that can be explored in, say, an atomic physics experiment, at a $B$ meson factory, or even at the LHC. In any case, it is challenging enough to search for CPV phenomena in laboratories on the earth. In the following a number of those phenomena which are predicted by the KM mechanism and/or by some non-KM sources of CP violation are discussed.

3 Weak Decays

Observable CP violation à la KM requires quarks whose weak decays are Cabibbo suppressed. That is not the case for $c$ and $t$ quarks. Therefore CP searches involving these quarks will predominantly test for new interactions.

3.1 Kaons and Hyperons

CP violation in the kaon system manifests itself in the very existence of the decays $K_L \to 2\pi$ and in a non-zero CP asymmetry, $\delta$, between the rates of
the the semi-leptonic decays $K_L \to \pi^\mp e^\pm \nu$. From these observations it can be inferred that CPV takes place in the $|\Delta S| = 2$ $K^0-\bar{K}^0$ mixing amplitude. The strength of $|\Delta S| = 2$ CPV is characterized by the $\epsilon$ parameter. One has $\delta \approx 2\text{Re}(\epsilon) = 3.27(12) \times 10^{-3}$. The KM mechanism can naturally explain the order of magnitude of this number – given the moduli of the CKM matrix elements measured in other decays. Recall that CPV in the SM is small because of small inter-generation mixing angles involved (cf. eq. (4)). The parameter $\epsilon$ is determined in the SM by the ratio of the imaginary part to the real part of the well-known box diagram mixing amplitude. (To be precise, of its dispersive part). A simple counting of the CKM matrix elements involved shows that $\epsilon_{SM} \sim 10^{-3} \sin \delta_{KM}$.

The present experimental status of whether there is also “direct” $|\Delta S| = 1$ CP violation in the $K^0 \to 2\pi$ amplitudes is inconclusive [43], [44]. New experiments [45] aim at measuring the corresponding observable, namely $\text{Re}(\epsilon'/\epsilon)$, at the level of $10^{-4}$. On the theoretical side considerable effort has been spent over the last years to calculate the next-to-leading order QCD corrections to the effective weak Hamiltonian within the SM, to pursue various approaches in determining weak matrix elements, and to get a handle on the various uncertainties involved in the prediction of $\epsilon'/\epsilon$. Recent detailed reviews [46], [47] of the current status estimate this quantity within the SM $\sim a$ few $\times 10^{-4}$. There are considerable uncertainties of this estimate due to (a) cancellations of the QCD penguin and electroweak penguin diagram contributions to $\epsilon'$, (b) uncertainties in the knowledge of some SM parameters, notably the mass of the strange quark, and (c) methodic uncertainties in calculating the non-leptonic weak decay matrix elements.

The present high statistics kaon decay experiments [45] can also search for and investigate several rare $K$ decays. For instance, in the case of the decay $K_L \to \pi\pi e^+ e^-$, there is a CP asymmetry in the distribution $d\Gamma/d\phi$ ($\phi$ is the angle between the normal vectors of the $e^+ e^-$ and $\pi\pi$ planes) generated by the observed CP violation in $K^0-\bar{K}^0$ mixing. The asymmetry is predicted to be rather large, about 14 percent [48].

Hyperon decays also offer a possibility to search for CP violation in $\Delta S = 1$ decays – although detectable effects require, very probably, non SM CP-violating interactions. Consider for instance the decay of polarized $\Lambda \to p\pi^-$ and $\bar{\Lambda} \to \bar{p}\pi^+$. The differential decay distributions are proportional to $(1 + \alpha_{\Lambda} \omega_{\Lambda} \cdot \hat{p}_p)$ and $(1 + \alpha_{\bar{\Lambda}} \omega_{\bar{\Lambda}} \cdot \hat{p}_{\bar{p}})$, respectively, where $\omega$ is the hyperon polarization vector and $\hat{p}$ is the (anti) proton direction of flight in the hyperon rest frame. The spin analyser quality factor $\alpha$, which is parity-violating, is generated by the interference of S and P wave amplitudes. CP invariance requires that $\alpha_{\Lambda} = -\alpha_{\bar{\Lambda}}$. Hence a CP observable is

$$A_A = \frac{\alpha_{\Lambda} + \alpha_{\bar{\Lambda}}}{\alpha_{\Lambda} - \alpha_{\bar{\Lambda}}}. \quad (27)$$

Note that $A_A$ is CP-odd but T-even, i.e., even under the reversal of momenta and spins. Hence a non-zero asymmetry (27) requires, apart from CP phases, also absorptive parts in the amplitudes. Neglecting isospin $I = 3/2$ contributions, an
approximate expression for $A_A$ is given by (see, for instance ref. [49])

$$A_A \simeq -\tan(\delta_{1/2}^P - \delta_{1/2}^S) \sin(\varphi_{1/2}^P - \varphi_{1/2}^S),$$

(28)

where $\delta_{1/2}^{S,P}$ and $\varphi_{1/2}^{S,P}$ are the S,P wave final state phase shifts and weak CP phases for the isospin $I = 1/2$ amplitudes, respectively.

In the Standard Model CP violation in $\Delta S = 1$ hyperon decays is induced by penguin amplitudes. Extensions of the SM may add charged Higgs penguin, gluino penguin contributions, etc. Predictions for hyperon CP observables like $A_A$ are usually obtained [50], [51], [52] as follows: within a given model of CP violation one computes first the effective weak $\Delta S = 1$ Hamiltonian at the quark level. (In the SM its next-to-leading order QCD corrections are known [46].) The strong phase shifts $\delta_{1/2}^{S,P}$ are extracted from experimental data. The usual strategy in determining the weak phases $\varphi_{1/2}^{S,P}$ is to take the real parts of the matrix elements $<\pi p|H_{\text{eff}}|\Lambda>_{I=1/2}^{S,P}$ from experiment, whereas the CPV part is computed using various models for hadronic matrix elements. Although the theoretical uncertainties are quite large one may conclude [51], [52] from these calculations that within the SM the asymmetry $A_A$ is about $4 \times 10^{-5}$. Contributions from non SM sources of CP violation can yield larger effects, but are constrained by the $\epsilon'$ and $\epsilon$ parameters from $K$ decays. He and Valencia conclude that $|A_A^{\text{non-SM}}|$ cannot exceed a few $\times 10^{-4}$.

Data from a high statistics hyperon experiment [53] (E871) at Fermilab are presently being analysed. The decay chain $\Xi^- \to \Lambda\pi^- \to p\pi^+\pi^-$ and the corresponding decay chain for $\bar{\Xi}^+$ will be used. They $\Xi$'s will be produced unpolarized. Then the $\Lambda$ polarization is given by $\omega_\Lambda = \alpha_\Xi \hat{p}_\Lambda$, where $\hat{p}_\Lambda$ is the $\Lambda$ direction of flight in the $\Xi$ rest frame. E871 measures the asymmetry

$$A = \frac{\alpha_A \sigma_{\Xi} - \alpha_A \sigma_{\Xi}}{\sigma_A \sigma_{\Xi}} \simeq A_A + A_{\Xi}. $$

(29)

$A_{\Xi}$ is estimated to be smaller than $A_A$ because of smaller phase shifts. E871 expect to produce about $10^9$ events. They aim at a sensitivity $\delta A \simeq 10^{-4}$. If an effect will be observed at this level it will be, in view of the above, most probably of non SM origin.

3.2 Charm

$D^0 - \bar{D}^0$ mixing and associated CP violation in the $\Delta C = 2$ mixing amplitude, and direct CP violation in the $\Delta C = 1$ charm decay amplitudes are predicted to be very small in the SM.

In the SM direct CPV may be significant only for singly Cabibbo suppressed decays. In this case one has at the quark level two contributions to the decay amplitude, namely the usual “tree” amplitude and the penguin amplitude, that have different weak phases. At the hadron level the decay amplitude is of the form $A e^{i\delta_A} + B e^{i\delta_B}$, where $\delta_{A,B}$ are strong interaction phase shifts. This leads to a CP asymmetry
\[ A_D = \frac{\Gamma(D \to f) - \Gamma(\bar{D} \to \bar{f})}{\Gamma(D \to f) + \Gamma(\bar{D} \to \bar{f})} \propto \text{Im}(AB^*) \sin(\delta_B - \delta_A). \tag{30} \]

Buccella et al. [55] have investigated \( A_D \) within the SM for a number of Cabibbo suppressed channels. They calculated the strong phase shifts for the respective channels by assuming dominance of the nearest resonance. For some modes, for instance \( D^+ \to K^{*0}K^+ \) and \( D^+ \to \rho^+\pi^0 \) they find \( A_D \sim 10^{-3} \). In some extensions of the SM like non-minimal supersymmetry [56] or left-right-symmetric models [57] \( A_D \) can be larger by about one order of magnitude. Moreover, asymmetries of the same order could also be generated in these models for Cabibbo allowed and doubly Cabibbo suppressed channels.

\( D^0 - \bar{D}^0 \) mixing is very small in the SM, \( x = \Delta m_D/\Gamma_D << 10^{-2} \). However, quite a number of extensions of the SM, for instance multi-Higgs or supersymmetric extensions, can lead to \( x \sim 10^{-2} \). In these models it is quite natural that there is (new) CP violation associated with \( \Delta C = 2 \) mixing. It is mostly these expectations [58] from SM extensions that nourish the hope of observable mixing and observable indirect and direct CP violation in proposed high statistics charm experiments [54] with \( 10^8 \) to \( 10^9 \) events.

### 3.3 Beauty

High statistics experiments with the aim of measuring CPV rate asymmetries in \( B \) decays will provide, in the years to come, the decisive tests of the KM mechanism [47], [59]. These asymmetries are characterized by the angles – conventionally called \( \alpha, \beta, \) and \( \gamma \) - of a well-known CKM unitarity triangle, which visualises the following orthogonality relation of the CKM matrix elements:

\[ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \tag{31} \]

Several fits (see, e.g., [60], [61] and for a more recent analysis [47]), using as input the value of \( \epsilon \) parameter of the \( K \) system, \( B_d^0 - \bar{B}_d^0 \) mixing, etc., have been performed to constrain these angles. These fits yield in particular \( 0.3 \leq \sin(2\beta) \leq 0.8 \), supporting the expectation that CP violation outside the \( K \) system will first be observed through an asymmetry between the rates of \( B_d^0 \) and \( \bar{B}_d^0 \to J/\Psi + K_S \). The integrated rate asymmetry, which can be calculated in a clean way (that is to say, almost without uncertainties due to hadronic final state interaction phases), is proportional to \( \sin(2\beta) \).

Similarly the time integrated rate asymmetry of \( B_d^0, \bar{B}_d^0 \to \pi^+\pi^- \) is related to \( \sin(2\alpha) \). However, apart from the fact that these modes have very small branching ratios, there is an uncertainty in the prediction of the CP asymmetry because of penguin diagrams contributing to the decay amplitudes. In principle this uncertainty can be eliminated by an isospin analysis [62]. (Recall that there is no QCD penguin contribution to the \( I = 3/2 \) component of the \( B_d \to \pi\pi \) amplitude.) The method requires measuring \( B_d^0 \to \pi^+\pi^- \), \( \pi^0\pi^0 \) and the conjugated channels, and \( B^{+} \to \pi^+\pi^0 \). It will be difficult to carry out.
The CP parameter $\sin(2\gamma)$ is for instance related to the time integrated asymmetry of the rates $B_0^0, B_0^s \to \rho K_S$. However, that is not a clean and feasible way of extracting $\sin(2\gamma)$: firstly because these modes have very small branching ratios and secondly because of theoretical complications in view of penguin contributions. One proposed alternative is as follows [63]: From the measured decay rates one has to determine the moduli of the decay amplitudes for $B^+ \to D^0 K^+, \bar{D}^0 K^+, D_{1,2} K^+$ and for the charge conjugated channels. ($D_{1,2}$ are the CP-even and odd eigenstates.) From the two triangle relations relating the three complex amplitudes for $B^+$ and for $B^-$, respectively, one can obtain $\sin^2 \gamma$ up to an ambiguity which can in principle also be resolved. (For other proposals to measure the angles $\alpha$ and $\gamma$, see, e.g., the review [47].)

According to the KM mechanism for the three generation SM $\alpha + \beta + \gamma = \pi$. A deviation from this relation would provide evidence for new CP-violating interactions [64]. (If the sum of these angles turns out to be $\pi$, note that this does not necessarily imply absence of new CPV effects in the $B$ system.) Of course, more specific searches for new CPV in the $B$ system can be made, for instance by investigating CP observables that are predicted to be small in the SM, e.g., the asymmetry in the rate for $B^0_\tau \to J/\Psi + J/\Psi$ and its conjugated mode and, likewise, the rate asymmetry for $B^{\pm} \to J/\Psi \pm K^{\pm}$. (For investigations of non-KM CPV in $B$ decays, see, e.g., [65].)

4 Electric Dipole Moments

The searches for permanent electric dipole moments (EDM), for instance of the neutron or of an atom with non-degenerate ground state are known to be a very sensitive means to trace new CPV interactions. Recall that a non-zero EDM of a non-degenerate stationary state would signal P and T violation, that is, CP violation assuming CPT invariance. Consider the expectation value of the EDM operator $D = \int d^3 x \rho(x)$, where $\rho$ is the charge density operator, in a particle state $|j\rangle$ of definite total angular momentum at rest. Rotational invariance tells us that

$$< j | D | j > = d < j | J | j >,$$

where $J$ is the total angular momentum operator. With respect to parity and time-reversal transformations one has

$$D \xrightarrow{P} -D, \quad D \xrightarrow{T} D,$$

$$J \xrightarrow{P} J, \quad J \xrightarrow{T} -J.$$

Hence $d \neq 0$ signals P and T violation. This argument applies not only to elementary particles but to atoms and molecules as well, as long as the the stationary state under consideration has no energy degeneracies besides those due to rotational invariance. (For an elaborate discussion, see [66].) The experimental signature for an EDM is a linear Stark effect in an external electric field.
A non-zero atomic EDM $d_A$ could be due to a non-zero electron EDM $d_e$, non-zero nucleon EDMs, P- and T-violating nucleon-nucleon, and/or electron-nucleon interactions. Schematically,

$$d_A = R_A d_e + C_{AN}^e + C_{AN}^N. \tag{35}$$

It has been shown long ago [67] that paramagnetic atoms can have large enhancement factors $R_A$. (See also [68] for a recent review.) More recent atomic physics calculations [69] obtained for instance for Thallium the factor $R_{Tl} \simeq -585$ with an estimated error of about 10%. For Thallium one has to good approximation $d_{Tl} \simeq d_e R_{Tl} + C_{Tl}^e$. The nuclear contributions can be neglected for the following reasons: The nuclear ground state of $^{205}$Tl has spin 1/2 and therefore cannot have a nuclear quadrupole moment. A potential (small) contribution of a Schiff moment of the Thallium nucleus is irrelevant at the present level of experimental sensitivity. From the experimental upper bound [70] on $d_{Tl}$ and with $R_{Tl}$ the upper bound $|d_e| < 4 \cdot 10^{-27} e$ cm was derived [70].

Very precise experimental upper bounds were obtained on the EDMs of certain diamagnetic atoms, in particular for mercury [71]. The mercury EDM, like that of other diamagnetic atoms, is not sensitive to $d_e$ but to the Schiff moment of the $^{199}$Hg nucleus which at the quark-parton level would be due to non-zero (chromo) EDMs of quarks and/or P- and T-violating quark-quark or gluonic effective interactions. As the transition from the level of partons to the level of a nucleus involves large uncertainties the experimental limits on the EDMs of diamagnetic atoms are difficult to interpret in terms of microscopic models of CP violation [72].

Experimental searches for a non-zero EDM of the neutron at Grenoble [73] and at Gatchina [74] have lead to the upper limit $|d_n| < 9 \cdot 10^{-26} e$ cm.

Theoretical predictions of the EDM of the electron – or of other leptons – usually constitutes a straightforward problem of perturbation theory because models of CPV are weak coupling theories a posteriori. However, a firm numerical prediction within a given extension of the SM would require knowledge of parameters like masses and couplings of new particles, apart from CP phases. The calculation of $d_n$ and of T-violating nucleon-nucleon interactions, etc. involves in addition methodological uncertainties. For a given model of CPV one can usually construct with reasonable precision the relevant effective P- and T-violating low energy Hamiltonian at the quark gluon level which contains (chromo) EDM operators of quarks, the $GG$ and $GGG$ operators, etc. The transition to the nucleon/nuclear level, that is, the computation of T-violating hadronic matrix elements involves large uncertainties. In computing/estimating the neutron EDM naive dimensional estimates, the quark and the MIT bag model [75], sum rule techniques [76], [77], [78], and experimental constraints on the quark contribution to the nucleon spin [79] have been used.

As was discussed in section 2.1, the KM phase induces only tiny CP-violating effects in flavour-diagonal amplitudes. Hence the SM predicts tiny particle EDMs (barring the strong CP problem of QCD; i.e., assuming $\theta = 0$). A typical estimate [75] for the neutron is $|(d_n)^{KM}| < 10^{-30} e$ cm. In the SM with massless neutrinos
CPV in the lepton sector occurs only as a spill-over from the quark sector: estimates \[80\], \[81\] yield \(|(d_e)^{KM}| < 10^{-37}\,e\text{ cm} \). Quite a number of other CPV interactions are conceivable that lead to neutron and electron EDMs of the same order of magnitude as the present experimental upper bounds. (For reviews, see \[75\], \[80\], \[82\].) Multi-Higgs extensions of the SM can contain neutral Higgs particles with indefinite CP parity (cf. section 2.3). Exchange of these bosons induces quark and lepton EDMs already at one loop. For light quarks and leptons the dominant effect occurs at two loops \[83\]. In two-Higgs doublet extensions \[84\], \[85\] of the SM with maximal CPV in the neutral Higgs sector and a light neutral Higgs particle with mass of order 100 GeV neutron and electron EDMs as large as \(10^{-25}\,e\text{ cm}\) and \(a\,\text{few}\times 10^{-27}\,e\text{ cm}\), respectively, can be induced. Contributions from charged Higgs boson exchanges can have a similar order of magnitude \[86\].

In the minimal supersymmetric extension of the SM (MSSM) there are in general, apart from the KM phase, extra CP phases due to complex soft SUSY breaking terms (cf. section 2.3). These phases are not bound to be small a priori. They generate quark and lepton EDMs and chromo EDMS of quarks at one-loop order \[87\], \[79\], \[90\] which can be quite large. (Unless the gaugino, squark or slepton masses are close \[88\] to 1 TeV which causes, however, other problems.) In particular, the prediction for the electron, which is not clouded by hadronic uncertainties, is \(d_e \simeq 10^{-25}\sin \phi_e\,e\text{ cm}\) for neutralino and \(\tilde{e}\) masses of the order of 100 GeV. That means the leptonic SUSY phase \(\phi_e\) must be quite small, \(\phi_e \sim 0.01\), which seems unnatural in the generic MSSM case. The constrained versions of the MSSM, mentioned in section 2.3, lead to substantially smaller predictions for the neutron and electron EDMs \[22\], \[89\].

In supersymmetric grand unified theories the small phase problem eases by construction, too. In the SO(10) model considered in refs. \[23\], \[90\] the phases in the soft terms are assumed to be zero at the Planck scale. Unification of the quarks and leptons of a generation into a single multiplet leads, apart from the KM phase, to extra CKM phases entering the fermion-sfermion gaugino (higgsino) interactions at the weak scale. GIM cancellations lead to a smaller \(d_n\) and \(d_e\) than in the generic MSSM – but \(d_e\) can be close to its experimental upper bound.

Clearly, the present experimental EDM bounds have an impact on the parameter spaces of popular extensions of the SM. In particular the bound on \(d_e\) is important in view of the “theoretically clean” predictions. Further improvement of experimental sensitivity is highly desirable. As to future low-energy T violation experiments: A number of proposals \[68\], \[91\], \[92\] have been made to improve the experimental sensitivity to \(d_e\) and to the EDMs of certain atoms by factors of 10 to 100. The Berkeley Tl experiment \[70\] will improve its sensitivity to \(d_e\) significantly. An experiment is underway \[91\] with the paramagnetic molecule YbF, which is very challenging but has the incentive of having a high sensitivity to \(d_e\). There is also a new idea \[93\] to measure the neutron EDM with substantially improved sensitivity.

The present experimental sensitivity to EDMs of quarks and leptons from
the second and third fermion generation is typically of the order $10^{-16}$ to $10^{-18} \, \text{e cm}$ (see below and [68]). Although this is orders of magnitude larger than the present limit on $d_e$ it constitutes nevertheless interesting information. Some CP-violating interactions, for instance CPV Higgs boson or leptoquark exchange, lead to EDMs in the heavy flavour sector that are much larger than $d_e$ or $d_n$.

5 High Energy Searches

Many proposals and studies for CP symmetry tests in high energetic $e^+e^-$, $p\bar{p}$, and $pp$ collisions have been made (see [94], [95], [96], [99], [100] for early studies). In particular the production and decay of $\tau$ leptons, $b$, and $t$ quarks are suitable for this purpose, as it allows for searches of new CPV interactions that become stronger in the heavy flavour sector. Contributions from the KM phase to the phenomena discussed below are negligibly small. Typically one pursues statistical tests with suitable asymmetries or correlations. Consider a reaction where the initial and the final states are eigenstates of CP. This means that the various contributions to the scattering amplitude $T$, and the observables associated with this reaction, can be classified as being even or odd under a CP transformation. CP tests are to be made with CP-odd observables $O_{CP}$ which change sign under a CP transformation. If the scattering amplitude of the reaction is affected by CPV interactions in a significant way, $T = T_{inv} + T_{CPV}$, then the interference of the CP-invariant and the CPV part generates a non-zero expectation value

$$< O_{CP} > = \frac{\int d\sigma O_{CP}}{\int d\sigma} \neq 0.$$  

(36)

Because an unpolarized $f\bar{f}$ state is a CP eigenstate in its c.m. frame it can be shown [97] that unpolarized (and transversely polarized) $e^+e^-$ and $p\bar{p}$ collisions allow for “theoretically clean” CP symmetry tests: in these cases $< O_{CP} >$ cannot be faked by CP-invariant interactions as long as the phase space cuts are CP-blind. The “self conjugate” situation discussed above can be realized in these cases by comparing data from the reaction $i \rightarrow f$ with those of the CP-conjugated one $i \rightarrow \bar{f}$. In the case of $pp$ collisions potential contributions from CP-invariant interactions to an observable being used for a CP symmetry test (for instance, T-odd\(^2\) observables will in general receive contributions from QCD absorptive parts) must be carefully discussed.

In order to maximize the sensitivity to CPV couplings it is often useful to consider so-called optimal observables [98] that maximize the signal-to-noise ratio. For a given reaction and a given model of CPV – or a model independent description of CPV using effective Lagrangians or form factors – with only one or a few small parameters these observables can be constructed in a straightforward fashion.

\(^2\) Recall that “T-odd” refers to being odd under the reversal of momenta and spins. The initial and final state are not interchanged.
5.1 $e^+e^- \to \tau^+\tau^-$

CPV effects in tau lepton production with $e^+e^-$ collisions and in $\tau$ decay were discussed in [99], [100], [101], [102], [103], [104], [105], [106], [107], [108]. CPV in $e^+e^- \to \tau^+\tau^-$ can be traced back to non-zero EDM and weak dipole moment (WDM) form factors [100], [101] $d_\tau^Z(s)$ and $d_\tau^Z(s)$, respectively, where $s = E_{c.m.}^2$. These form factors induce a number of CP-odd tau polarization asymmetries and spin-spin correlations, for instance a non-zero $d_\tau^Z(s)$ (more precisely, the real part of that form factor) leads to a difference in the polarizations of $\tau^+$ and $\tau^-$ orthogonal to the scattering plane. Because the taus auto-analyse their spins through their parity-violating weak decays the tau polarization asymmetries and spin-spin correlations transcribe to a number of CP-odd angular correlations $<\mathcal{O}_{CP}>$ among the final states from $\tau^+\tau^-$ decay.

In their pioneering work the OPAL and ALEPH collaborations [109], [110], [111], [112] at LEP have demonstrated that CP tests in high energy $e^+e^-$ collisions can be performed with an accuracy at the few per mill level. In the meantime the four LEP experiments measured a number of CP-odd correlations in $e^+e^- \to \tau^+\tau^-$. They turned out to be consistent with zero. From these results upper limits on the real and imaginary parts of the WDM form factors were derived. The combined upper limit on the real part is [113] $|Re d_\tau^Z(s = m_Z^2)| < 3.6 \cdot 10^{-18} e\text{ cm (95\% CL)}$.

As already mentioned above the tau EDM and WDM form factors can be much larger than the electron EDM. There are a number of SM extensions where the dominant contributions to these form factors are one-loop effects, being not suppressed by small fermion masses. In these models one has \( d_\tau = e \frac{\delta}{m_Z} \) with \( \delta \) of order $\alpha/\pi$. For multi Higgs models one finds [106] that $d_\tau$ can reach $10^{-20} e\text{ cm}$, whereas CPV scalar leptoquark exchange [106] can lead to $d_\tau$ as large as $3 \cdot 10^{-19} e\text{ cm}$. In [114] the EDM and WDM form factors were computed in the minimal supersymmetric extension of the SM. These authors obtained $d_\tau^Z$ of order $10^{-21} e\text{ cm}$.

5.2 $e^+e^- \to b \bar{b} \text{ gluon(s)}$

CP violation in this neutral current reaction would signal new interactions. At the parton level these interactions would affect correlations among parton momenta/energies and parton spins. While the partonic momentum directions can be reconstructed from the jet directions of flight the spin-polarization of the $b$ quark cannot, in general, be determined with reliable precision due to fragmentation. This implies that useful CP observables are primarily those which originate from partonic momentum correlations [95]. With these correlations only chirality-conserving effective couplings can be probed with reasonable sensitivity. Several correlations were proposed and studied [95], [115], [116], [117]. This situation is in contrast to $\tau^+\tau^-$ and $t\bar{t}$ production (see below) where the fermion polarizations can be traced in the decays. That is why in these cases searches for CPV dipole form factors, which are chirality-flipping, can be made with good precision.
In the framework of SU(2)$_L$-invariant effective Lagrangians it can be shown that chiral invariant CPV effective $Z\bar{b}bG$ interactions of dimension $d = 6$ (after spontaneous symmetry breaking) exist \cite{95}, \cite{97}. In multi-Higgs extensions of the SM these interactions can be induced to one-loop order \cite{17}. They remain non-zero in the limit of vanishing $b$ quark mass. Note that these CPV effective interactions are chiral-invariant and flavour-diagonal which is a remarkable feature. A dimensionless coupling $\hat{h}_b$ associated with these interactions \cite{116} turns out to be of the order of a typical one-loop radiative correction, i.e., a few percent if CP phases are maximal. This coupling could be larger in models with excited quarks.

At the $Z$ resonance the above reaction provides an excellent possibility to probe for this type of interactions. The ALEPH collaboration \cite{118} has made a CP study with their sample of $Z \rightarrow \bar{b}bG$ events. They obtained a limit of $|\hat{h}_b| < 0.59$ at 95% CL.

### 5.3 Top Quarks and Higgs Bosons

Because of their extremely short lifetime top quarks decay on average before they can hadronize. This means that the spin properties of $t$ quarks can be inferred with good accuracy from their weak decays. (The SM predicts that $t \rightarrow W b$ is the main decay mode.) Like in the case of the tau lepton a number of $t$ spin-polarization and spin-spin correlation effects may be used to search for non-SM physics. Because of their heavy mass, top quarks – once they are available in sufficiently large numbers – will be a good probe of the electroweak symmetry breaking sector through their Yukawa couplings. In particular they will be a good probe of Higgs sector CP violation. Many CP tests involving top quarks have been proposed. These proposals include $t\bar{t}$ production in high energy $e^+e^-$ collisions \cite{119}, \cite{16}, \cite{120}, \cite{121}, \cite{122}, \cite{123}, \cite{124}, \cite{125}, \cite{126} and in $pp$ and $pp$ collisions \cite{127}, \cite{128}, \cite{129}, \cite{130}, \cite{131}, \cite{132}, \cite{133}, \cite{134} at Tevatron and LHC energies, respectively. (As already mentioned, in the latter case no genuine CP tests in the way described above can be made. One must carefully discuss and compute potential fake effects.) Useful channels for these tests are the final states from semi-leptonic decay of both $t$ and $\bar{t}$ and those from semi-leptonic (non-leptonic) $t(\bar{t})$ decay plus the charge conjugated channels. (The charged lepton from semi-leptonic $t$ decay is known to be the most efficient $t$ spin analyzer. Non-leptonic $t$ decays, on the other hand, allow for reconstruction of the top momentum.) Observables $O_{CP}$ include triple correlations, energy asymmetries, etc. and their optimized versions. Computations of $<O_{CP}>$ have been made in a model-independent way using effective Lagrangians, form factor parameterizations of the $t$ production and decay vertices, and within several extensions of the SM, notably two-Higgs doublet and supersymmetric extensions. At the upgraded Tevatron one can reach an interesting sensitivity to the chromo EDM form factor of the top of about $\delta q_t^{\text{chromo}} \simeq 10^{-19} e\text{cm}$. Multi-Higgs extensions of the SM can induce top EDM, WDM, and chromo EDM form factors of this order of magnitude \cite{16}, \cite{135}. The minimal SUSY extension of
the SM leads to smaller predictions for these form factors [121], [136]. EDM and WDM form factors could be searched for most efficiently in $e^+e^- \to t\bar{t}$ not far above threshold [119], [121], [124]. It was shown [121] that, within two-Higgs doublet extensions of the SM, neutral Higgs sector CP violation induces effects at the percent level in this reaction.

A possibility to check for CPV Yukawa couplings of the $t$ quark would be associated $t\bar{t}$ Higgs boson production. CP effects can be large [137], but the cross sections are quite small.

If neutral Higgs boson(s) $\phi$ will be discovered and at least one of them can be produced in reasonably large numbers then the CP properties of the scalar sector could be determined directly by checking whether $\phi$ has $J^{PC} = 0^{++}$, $0^{--}$, or whether it has undefined CP parity as predicted by multi-Higgs extensions of the SM with Higgs sector CPV. A number of suggestions and theoretical studies in this respect were made [138], [139], [140], [141], [142], [143], [144], [145], [146], [147], [148]. (Some of them follow the text book descriptions of how to determine the CP parity of $\pi^0$.) In the fermion-antifermion decay of a neutral Higgs particle with undefined CP parity CP violation occurs at tree level and manifests itself in spin-spin correlations [138]. One of them is CP-odd and can be as large as 0.5. These correlations could be traced in $\phi \to \tau^+\tau^-$ and, for heavy $\phi$, in $\phi \to t\bar{t}$; for instance, when $\phi$ is produced in high-energetic electron positron collisions. In the case of LHC production, $pp \to \phi + X \to t\bar{t} + X$, interference with the non-resonant $t\bar{t}$ background diminishes the effect [129], [138].

A “Compton collider” realized by backscattering laser photons off high energy $e^-$ or $e^+$ beams would be an excellent tool to study Higgs bosons [149] by tuning the beams to resonantly produce $\phi$. The CP properties of $\phi$ could be checked by appropriate asymmetries and correlations [142], [146], [147].

6 Summary

The gauge theory paradigm, which describes physics so well up to the highest energy scales presently attainable, suggests that, if there is physics beyond the Standard Theory, there can be a number of different types of CP-violating interactions which manifest themselves in different physical situations. Hence searches for CP violation effects should be made in as many particle reactions as possible. Present experimental investigations of $K$ decays and of hyperon decays search for “direct” CP violation in $|\Delta S| = 1$ weak transitions at the level of $10^{-4}$. While an effect of this order of magnitude can be induced by the KM phase in $K$ decays, it would point towards a new source of CPV in the case of hyperon decays. However, in order to be eventually able to discriminate better between different models of CPV improved calculations of hadronic matrix elements both for $K \to 2\pi$ and for non-leptonic hyperon decays are needed. The decisive tests of the KM mechanism will hopefully be provided by the $B$ meson factories in the years to come. The searches for a neutron EDM, atomic EDMs, or other T-violation effects in atoms or molecules remain a unique low energy
window to physics beyond the SM. Searches of non-SM CP violation can also be made at present and future high energy colliders. Experiments at LEP have already demonstrated that high-energy CP tests can attain sensitivities at the sub-percent level. Specifically, if Higgs sector CPV exists, effects of up to a few percent are possible in the top quark system. Moreover, when Higgs boson(s) will be discovered and eventually produced in large numbers it is also conceivable to study their CP properties directly.

While at present CP non-conservation may still be considered, from an agnostic point of view, as a curious and small effect of mysterious origin in the neutral kaon system, one can be optimistic that, in view of the activities outlined above, we will have a clearer understanding of the cause of this symmetry violation in the not too distant future.

Acknowledgements

I am indebted to E. Commins, E.A. Hinds, S.K. Lamoreaux, and K.B. Luk for information about their present and planned experiments. Moreover, I wish to thank A. Brandenburg and P. Uwer for discussions, and the organizers of the 1998 Schladming Winter School, W. Plessas and his colleagues, for inviting me to this pleasant meeting.

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