Notes on soft breaking of BRST symmetry in the Batalin-Vilkovisky formalism

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Abstract

We have proved the nilpotency of the operators which describe the gauge dependence of the generating functionals of Green’s functions for the gauge theories with the soft breaking of BRST symmetry in the Batalin-Vilkovisky formalism.

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1 Introduction

In our paper we consider the problems related to the dependence of the Green’s functions on the
gauge in the soft breaking of BRST symmetry in the Batalin-Vilkovisky formalism [1] proposed
in our previous papers [2], [3].

This breakdown in Yang-Mills theories is connected with a restriction of the domain of
integration in the functional integral due to the Gribov horizon [4] and introducing of the
Gribov-Zwanziger action [5],[6]. Note, the investigations for the theories above have been
performed, as a rule, in the Landau gauge only (see, e.g., [7] and references therein).

At the same time, it is well known fact that the physical quantities and, in particular,
S-matrix, can be calculated in different gauges in the framework of the Batalin-Vilkovisky of
quantization of gauge theories, but they must not depend on a choice of the gauge condition.
Recently, in Refs. [2], [3] a generalization of the definition of soft breaking of BRST symmetry
valid for general gauge theories in arbitrary gauges within field-antifield formalism has been
proposed.

The aim of the paper is the detailed elaboration of the properties of operators used in Ref.
[3] for expressing the variations of generating functional of Green’s functions under variation
of the gauge condition.

We will use the condensed DeWitt’s notations [8]. Derivatives with respect to sources and
antifields are taken from the left, while those with respect to fields are taken from the right.
The Grassmann parity of any quantity A in case of its homogeneity is denoted as ε(A).

2 Odd Operators in BV quantization scheme with soft
Breaking of BRST symmetry

Consider the configuration space parameterized by the fields Φ ≡ {Φ^A} = {A^i,...} with
ε(Φ^A) = ε_A, where the dots indicate the full set of additional to A^i fields of this theory
in the BV method in dependence on its reducibility stage. Then for each field Φ^A of this
total configuration space, one should introduce the corresponding antifield Φ^* with opposite
Grassmann parities to that of the corresponding field Φ^A Φ^* ≡ {Φ^*_A} = {A^*_i,...} with
ε(Φ^*_A) = ε_A+1.

In Ref. [3] it was shown that for the generating functional of Green’s functions Z(J, Φ^*),

\[ Z(J, \Phi^*) = \int D\Phi \exp \left\{ \frac{i}{\hbar} \left( S(\Phi, \Phi^*) + J_A \Phi^A \right) \right\} \]  

(1)

with action S(Φ, Φ^*), which is additive extension of the non-degenerate gauge-fixing action
S_{ext}(Φ, Φ^*) by the bosonic functional M(Φ, Φ^*), the variation of Z(J, Φ^*) induced by variation
of the gauge is written in the form

$$\delta Z(J, \Phi^*) = \frac{i}{\hbar} \left[ \left( J_A + M_A \left( \frac{\delta}{\delta \Phi^*_A} \right) \right) \left( \frac{\delta}{\delta \Phi} - \frac{i}{\hbar} M^{A*} \left( \frac{\delta}{\delta \Phi}, \Phi^* \right) \right) \delta \Psi \left( \frac{\delta}{\delta \Phi}, \Phi^* \right) + \delta M \left( \frac{\delta}{\delta \Phi}, \Phi^* \right) \right] Z(J, \Phi^*).$$

(2)

At deriving (2), we have taken into account that functionals $S_{ext}(\Phi, \Phi^*)$ and $(-M)$ satisfy the quantum master-equations of the BV method, $J_A$ appears by the sources to the fields $\Phi^A$, $\varepsilon(J_A) = \varepsilon_A$ and the notations

$$M_A \left( \frac{\delta}{\delta \Phi^*}, \Phi^* \right) = \frac{\delta M(\Phi, \Phi^*)}{\delta \Phi^*} \bigg|_{\Phi \rightarrow \frac{\delta}{\delta \Phi^*}}, \quad M^{A*} \left( \frac{\delta}{\delta \Phi}, \Phi^* \right) = \frac{\delta M(\Phi, \Phi^*)}{\delta \Phi} \bigg|_{\Phi \rightarrow \frac{\delta}{\delta \Phi^*}},$$

were introduced. Here $M = M(\Phi, \Phi^*)$ plays the role of the functional, which describes the soft breaking of BRST symmetry in [3].

Let us introduce the odd operator $\hat{q}$:

$$\hat{q} = \left( J_A + M_A \right) \left( \frac{\delta}{\delta \Phi^*_A} - \frac{i}{\hbar} M^{A*} \right), \quad (3)$$

which contains non-vanishing terms $M_A \left( \frac{\delta}{\delta \Phi^*}, \Phi^* \right)$, $M^{A*} \left( \frac{\delta}{\delta \Phi}, \Phi^* \right)$ that differs it from the analogous operator considered in [2]. Then $\delta Z(J, \Phi^*)$ in (2) can be written in the form

$$\delta Z(J, \Phi^*) = \frac{i}{\hbar} \left[ \hat{q} \delta \Psi \left( \frac{\delta}{\delta \Phi}, \Phi^* \right) + \delta M \left( \frac{\delta}{\delta \Phi}, \Phi^* \right) \right] Z(J, \Phi^*).$$

Let us prove the nilpotency of the operator $\hat{q}$, i.e. that, $\hat{q}^2 = 0$.

To do this, we will use for the shortness the following notations:

$$M_A = M_A \left( \frac{\delta}{\delta \Phi^*}, \Phi^* \right), \quad M^{A*} = M^{A*} \left( \frac{\delta}{\delta \Phi}, \Phi^* \right).$$

The square of $\hat{q}$ may be directly presented as a sum of four operators

$$\hat{q}^2 = \left[ \left( J_A + M_A \right) \left( \frac{\delta}{\delta \Phi^*_A} - \frac{i}{\hbar} M^{A*} \right) \right]^2 \equiv \sum_{i=1}^{4} D_i =$$

$$= \left( J_A + M_A \right) \frac{\delta}{\delta \Phi^*_A} \left( J_B + M_B \right) \frac{\delta}{\delta \Phi^*_B} - \frac{i}{\hbar} \left( J_A + M_A \right) \frac{\delta}{\delta \Phi^*_B} \left( J_B + M_B \right) M^{B*} -$$

$$- \frac{i}{\hbar} \left( J_A + M_A \right) M^{A*} \left( J_B + M_B \right) \frac{\delta}{\delta \Phi^*_B} + \left( \frac{i}{\hbar} \right)^2 \left( J_A + M_A \right) M^{A*} \left( J_B + M_B \right) M^{B*}.$$ \hspace{1cm} (4)

Consider the first term in the decomposition (4),

$$D_1 = \left( J_A + M_A \right) \frac{\delta M_B}{\delta \Phi^*_A} \frac{\delta}{\delta \Phi^*_B} + \left( J_A + M_A \right) \left( J_B + M_B \right) \frac{\delta}{\delta \Phi^*_B} \frac{\delta}{\delta \Phi^*_A} (-1)^{\varepsilon_A + 1}. \quad (5)$$

In turn, the second summand in $D_1$ (5) has the form

$$(-1)^{\varepsilon_A + 1} \left( J_A J_B + M_A M_B + J_A M_B + (-1)^{\varepsilon_A \varepsilon_B} J_B M_A + \frac{\hbar}{\lambda} M_{AB} \right) \frac{\delta}{\delta \Phi^*_B} \frac{\delta}{\delta \Phi^*_A}, \quad (6)$$
where we have taken into account that \([M_A, J_B] = \frac{i}{\hbar} M_{AB}\), determined as,

\[
M_{AB} = \frac{\delta^2 M(\Phi, \Phi^*)}{\delta \Phi_A \delta \Phi_B} \bigg|_{\Phi \to \frac{\hbar}{i} \frac{\delta}{\delta \Phi}} \quad \text{and} \quad M_{AB} = (-1)^{\epsilon_A \epsilon_B} M_{BA} .
\] (7)

Note, the generalized symmetry properties of the expression in the brackets in (6) and the second derivative with respect to antifields are opposite under replacement of indices \(A \leftrightarrow B\). From this fact it follows that (6) is equal to zero and \(D_1\) term is determined only by the first term. Turning to the summand \(D_2\) we see that after rearranging of the derivatives with respect to antifields \(D_2\) takes the form

\[
D_2 = -\frac{i}{\hbar} \left( J_A + M_A \right) \frac{\delta M_B}{\delta \Phi_A} M_B - \frac{i}{\hbar} \left( J_A + M_A \right) \left( J_B + M_B \right) \frac{\delta M_B}{\delta \Phi_A} (-1)^{(\epsilon_A + 1)\epsilon_B} - \\
- \frac{i}{\hbar} \left( J_A + M_A \right) \left( J_B + M_B \right) M_B \frac{\delta}{\delta \Phi_A} (-1)^{\epsilon_A + 1} .
\] (8)

Then, we can see taking into account of the generalized symmetry property

\[
\frac{\delta M_B}{\delta \Phi_A} = \frac{\delta M_A}{\delta \Phi_B} (-1)^{(\epsilon_A + 1)\epsilon_B} ,
\] (9)

that the second term in \(D_2\) vanishes.

Next, for the term \(D_3\) we have after sequence of the transformations

\[
D_3 = -\frac{i}{\hbar} \left( J_A + M_A \right) M_A \frac{\delta}{\delta \Phi_B} J_B - \frac{i}{\hbar} \left( J_A + M_A \right) M_A M_B \frac{\delta}{\delta \Phi_B} = \\
= -\frac{i}{\hbar} \left( J_A + M_A \right) \left( \frac{\hbar}{i} M_A + J_B M_A (-1)^{\epsilon_B(\epsilon_A + 1)} \right) \frac{\delta}{\delta \Phi_B} - \\
- \frac{i}{\hbar} \left( J_A + M_A \right) M_A M_B \frac{\delta}{\delta \Phi_B} = \\
= - \left( J_A + M_A \right) M_A \frac{\delta}{\delta \Phi_B} - \frac{i}{\hbar} \left( J_A + M_A \right) \left( J_B + M_B \right) M_A \frac{\delta}{\delta \Phi_B} (-1)^{\epsilon_B(\epsilon_A + 1)} ,
\] (10)

where we have used the notation

\[
M_A = \frac{\delta^2 M(\Phi, \Phi^*)}{\delta \Phi_A \delta \Phi_B} \bigg|_{\Phi \to \frac{\hbar}{i} \frac{\delta}{\delta \Phi}} .
\] (11)

At last, transforming the fourth summand \(D_4\) in (4) as follows,

\[
D_4 = \left( \frac{i}{\hbar} \right)^2 \left( J_A + M_A \right) \left( \frac{\hbar}{i} M_A + J_B M_A (-1)^{\epsilon_B(\epsilon_A + 1)} \right) M_B + \\
+ \left( \frac{i}{\hbar} \right)^2 \left( J_A + M_A \right) M_A M_B = \\
= \frac{i}{\hbar} \left( J_A + M_A \right) M_B + \left( \frac{i}{\hbar} \right)^2 J_B M_A M_B (-1)^{\epsilon_B} + \\
+ \left( \frac{i}{\hbar} \right)^2 \left( \frac{i}{\hbar} M_A + J_B M_A (-1)^{\epsilon_B} \right) M_A M_B (-1)^{\epsilon_B(\epsilon_A + 1)} + \\
+ \left( \frac{i}{\hbar} \right)^2 \left( J_A + M_A \right) M_A M_B .
\]
we have finally,
\[
D_4 = \frac{1}{\hbar} \left( J_A + M_A \right) M_{AB}^{A*} M^{B*} + \left( \frac{i}{\hbar} \right)^2 J_B J_A M_{A*} M^{B*} (-1)^{\varepsilon_B} + \frac{i}{\hbar} M_{AB} M^{A*} M^{B*} (-1)^{\varepsilon_B} + \left( \frac{i}{\hbar} \right)^2 J_B M_A M_{A*} M^{B*} (-1)^{\varepsilon_B} - \left( \frac{i}{\hbar} \right)^2 J_A M_{AB} M^{B*} M^{A*} (-1)^{\varepsilon_A} + \left( \frac{i}{\hbar} \right)^2 M_{AB} M_{A*} M^{B*} (-1)^{\varepsilon_B} (-1)^{\varepsilon_B}. \tag{12}
\]

It is easy to see that the second, third and sixth terms in the last expression identically vanish due to the generalized symmetry properties under changing of the indices $A \leftrightarrow B$. Indeed, the quantities $(J_B J_A)$, $M_{AB}$ are generalized-symmetric ones, whereas $(M_{A*} M^{B*} (-1)^{\varepsilon_B})$ is generalized-antisymmetric, and $(M_{A*} M^{A*})^2 \equiv 0$. Next, the sum of the fourth and fifth terms is equal to zero. Therefore $D_4$ is reduced to the first term in (12).

In view of the derivations above, we have the following final representations for $D_1$, $D_2$, $D_3$, $D_4$,
\[
D_1 = \left( J_A + M_A \right) \frac{\delta M_B}{\delta \Phi_A^*} \frac{\delta}{\delta \Phi_B^*}, \tag{13}
\]
\[
D_2 = -\frac{i}{\hbar} \left( J_A + M_A \right) \frac{\delta M_B}{\delta \Phi_A^*} M^{B*} - \frac{i}{\hbar} \left( J_A + M_A \right) \left( J_B + M_B \right) M^{B*} \frac{\delta}{\delta \Phi_A^*} (-1)^{\varepsilon_A}, \tag{14}
\]
\[
D_3 = -\left( J_A + M_A \right) M_{B*} \frac{\delta}{\delta \Phi_B} - \frac{i}{\hbar} \left( J_A + M_A \right) \left( J_B + M_B \right) M_{A*} \frac{\delta}{\delta \Phi_B} (-1)^{\varepsilon_B}, \tag{15}
\]
\[
D_4 = \frac{i}{\hbar} \left( J_A + M_A \right) M_{A*} M^{B*}. \tag{16}
\]

From the Eqs. (13)-(16) we immediately obtain, first, the sum of the operator $D_4$ and the first term in $D_2$ is equal to zero, second, the sum of the operator $D_1$ and the first term in $D_3$ vanishes, third, the sum of the second terms in both operators $D_2$ and $D_3$ is equal to zero.

Thus, our statement, that $\hat{q}^2 = 0$ is completely proved.

As the consequence, we have simultaneously proved the nilpotency of the operator $\hat{Q}$, which is unitarily related to $\hat{q}$
\[
\hat{Q} = \exp -\frac{i}{\hbar} W \hat{q} \exp \frac{i}{\hbar} W = \left( J_A + M_A \left( \frac{\delta W}{\delta J} + \frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^* \right) \right) \frac{\delta}{\delta \Phi_A^*}. \tag{17}
\]

Remind [3], the operator $\hat{Q}$ the dependence of the generating functional of connected Green’s functions $W(J, \Phi^*) = \frac{1}{i} \ln Z(J, \Phi^*)$ on the variation of the gauge in the representation
\[
\delta W(J, \Phi^*) = \hat{Q} \delta \Phi \left( \frac{\delta W}{\delta J} + \frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^* \right) + \delta M \left( \frac{\delta W}{\delta J} + \frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^* \right). \tag{18}
\]

At last, for the generating functional of vertex Green’s functions (effective action), which is obtained from $W(J, \Phi^*)$ by means of the Legendre transformation with respect to sources
\(J_A, \ (\Gamma(\Phi, \Phi^*) = W(J, \Phi^*) - J_A\Phi^A)\) with average fields \(\Phi^A = \frac{\delta W}{\delta J_A}\), the local (for \(M = 0\)) representation for the odd and nilpotent operator \(\hat{s}_q\) being equal to \(\hat{Q}\), but acting on the space of average fields and antifields will be valid as well,

\[
\delta \Gamma(\Phi, \Phi^*) = \hat{s}_q < \delta \Psi > + < \delta M > .
\]

(19)

The angle brackets in the expressions (19) denote the averaging of the quantities and operators with respect to the functional \(\Gamma(\Phi, \Phi^*)\), considered in details in [2], [3], whereas the operator \(\hat{s}_q\) itself has the form, presented in fact in [3], with using the left derivative with respect to \(\Phi^A\),

\[
\hat{s}_q = - \left( \frac{\delta \Gamma}{\delta \Phi^A} - \hat{M}_A \right) \frac{\delta}{\delta \Phi_A^*} - (-1)^{\varepsilon_A} \left( \frac{\delta \Gamma}{\delta \Phi_A^*} - \hat{M}_A^* \right) \frac{\delta}{\delta \Phi^A}
\]

\[
- \frac{i}{\hbar} \left[ \hat{M}_A \frac{\delta \Gamma}{\delta \Phi_A^*} + \frac{\delta \Gamma}{\delta \Phi_A} \hat{M}_A^* - \hat{M}_A \hat{M}_A^* , \Phi^B \right] \frac{\delta}{\delta \Phi_B} .
\]

(20)

where the sign \([ , ]\) means for supercommutator.

The nilpotency of the operators \(\hat{q}, \hat{Q}, \hat{s}_q\) proved here repeats the properties of theirs analogs in the Ref. [2], but in case of more general regularization than dimensional one and takes fundamental character reflecting the presence of the BRST symmetry in the theory but with its soft breaking.

**Acknowledgement**

The authors are thankful to P.M. Lavrov for useful discussions. The work of O.V. Radchenko is supported by the RFBR grant 12-02-31820. The work of A.A. Reshetnyak was supported by the RFBR grant, project Nr. 12-02-00121 and by LRSS grant Nr.224.2012.2.

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