Hadronic currents for $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and other decays of interest in TAUOLA

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Abstract

A new set of hadronic form factors, which has been implemented in TAUOLA, is described.

Keywords: Hadronic decays of the Tau lepton, Monte Carlo methods, Resonances, Chiral symmetry.

1. Semileptonic tau decays in TAUOLA

Tau decays including hadrons are a privileged scenario \[1\] to study non-perturbative QCD in a rather clean environment provided by the electroweak half of the process. In Ref. \[2\] it was concluded that the most essential missing step to perform, concerning the study of hadronic tau decays, was the appropriate choice of the hadronic currents. The original version of the Monte Carlo (MC) generator TAUOLA \[3\] used the so-called Kühn-Santamaria (KS) model \[4\], and its extensions, to construct them. Within this model, they are built to fulfll the leading order (LO) result in the low-energy effective field theory of QCD, Chiral Perturbation Theory ($\chi$PT) \[5\], but they violate the next-to-leading order (NLO) one \[6\]. This approach was sufficient and successful twenty years ago, but already the CLEO and Aleph Collaborations realized, later on, departures of the predictions from data, a feature which could be expected taking into account several inconsistencies in later parametrizations of the three meson modes including Kaons \[7\]. This resulted in private versions of the code, with fine-tuned initializations - which sometimes violated basic principles of QCD \[8\] - that were documented in Ref. \[9\]. Nowadays, with the massively increased data samples from the B-factories BaBar and Belle - the most of which have not been analyzed yet, - it is pressing to upgrade the hadronic currents in TAUOLA in order to obtain as much QCD information as possible from experiment; moreover with the perspective of the super-flavour factories producing huge amounts of high-quality data in the near future.

In Ref. \[10\] a new set of form factors for hadronic tau decays based in analytical results obtained from Resonance Chiral Theory ($R_T\chi$PT) \[11\] is documented. In these Proceedings, a description of the implementation of the new modules of the MC program for its user is given in Ref. \[12\]. Here we focus on the hadronic currents themselves.

Lorentz invariance determines the most general decomposition of the hadronic current. This is

$$J^\mu = N[(p_1 - p_2)^\mu F^V(s) + (p_1 + p_2)^\mu F^S(s)],$$

in the two-meson channels, where $F^V(s)$ and $F^S(s)$ are the vector and scalar form factors, given in terms of $s = (p_1 + p_2)^2$, and

$$J^\mu = N[T^\mu_1((p_2 - p_3)^\nu F_1 + (p_1 - p_3)^\nu F_2 + (p_1 - p_2)^\nu F_3) + q^\mu F_4 - \frac{i}{4\pi^2F_2}q^\rho p_1^\nu p_2^\sigma F_3^\rho F_3^\sigma],$$

in the three-meson decays\[1\], where $T^\mu_\nu = g^\mu_\nu - q_\mu q_\nu/q^2$ stands for the transverse projector, and $q^\mu = (p_1 + p_2 + p_3)^\mu$ is the momentum of the hadronic system. Only two among the $F_1$, $F_2$ and $F_3$ (axial-vector) form factors are independent. $F_4$ is the generally suppressed pseudoscalar form factor and $F_4$ is the vector form factor. The kinematical invariants are $q^2$, and two of the

\[1\]The (by-far) most important four-meson tau decay, into four pions, is currently parametrized following Ref. \[13\].
3. $\pi^- \to \pi^- \pi^0 \nu_\tau$

There are different approaches to deal with the diverse energy regimes which are probed through this decay: $\chi PT$ should be a valid description of the data for $s \ll M_\rho$. Computations at NNLO are available both in the $SU(2)$\cite{20} and in the $SU(3)$\cite{21} symmetry cases. In the region $M_\rho \lesssim s \lesssim 1$ GeV, the chiral expansion breaks down and the dominant $\rho(770)$ exchange has to be accounted for. Several approaches have been developed. Among them, matching $\chi PT$ results to vector meson dominance using an Omn`es solution\cite{22} for the dispersion relation\cite{23}, employing an Omn`es solution for the dispersion relation\cite{24} or utilizing the unitarization approach\cite{25}. For larger energies, in the 1-2 GeV region, the excited resonances play an important role and shall be incorporated to the description. Ref.\cite{26} includes the $\rho(1450)$ through a Schwinger-Dyson-like resummation and Refs.\cite{27,28} include a tower of resonances inspired from dual QCD. Since Ref.\cite{23} will be our starting point, let us recall their main features in the following.

In this case $N = \sqrt{7}$ in Eq.(1), and the scalar form factor is zero in the $SU(2)$ symmetry limit.\cite{29} The vector form factor, at NLO in $\chi PT$, is

$$ F^V(s) = 1 + \frac{2L_2(\mu)}{F_2^\pi} - \frac{s}{96\pi F_2^\pi} \left[ A_\rho(s) + \frac{1}{2} A_K(s) \right], \quad (3) $$

with

$$ A_\rho(s) = \text{Log} \left( \frac{m_\rho^2}{\mu^2} \right) + \frac{8m_\rho^2}{s} - \frac{5}{3} + \sigma_p^2 \text{Log} \left( \frac{\sigma_p + 1}{\sigma_p - 1} \right), $$

$$ \sigma_p = \sqrt{1 - \frac{4m_\rho^2}{s}}. \quad (4) $$

The computation in $RVT$, within the antisymmetric tensor formalism, reads

$$ F^V(s) = 1 + \frac{F_V G_V}{F_2^\pi} - \frac{s}{M_\rho^2 - s}. \quad (5) $$

When $F^V(s) \underset{s \to 0}{\to} 0$ is required, the condition $F_V G_V = F_2^\pi$ is found, which yields the vector meson dominance prediction ($\mu \sim M_\rho$)

$$ F^V(s) = \frac{M_\rho^2}{M_\rho^2 - s} \Rightarrow L'_\rho = \frac{F_2^\pi}{2M_\rho^2}. \quad (6) $$

The matching of Eqs.(5) and (6) is straightforward

$$ F^V(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi F_2^\pi} \left[ A_\rho(s) + \frac{1}{2} A_K(s) \right]. \quad (7) $$

\footnote{\cite{30} Even when first order isospin violating corrections are included it does not contribute.}
When unitarity and analyticity properties are required, the Omnès solution emerges

$$F^V(s) = \frac{M_0^2}{M_\rho^2 - s} \exp\left\{ -\frac{s}{96\pi F_\rho^2} \left[ A_\rho(s) + \frac{1}{2} A_K(s) \right] \right\} .$$

(8)

Not surprisingly, the $\rho(770)$ off-shell width is related to the imaginary part of the same loop function

$$\Gamma_{\rho}(s) = -\frac{M_0^2}{96\pi F_\rho^2} \text{Im}\left\{ A_\rho(s) + \frac{1}{2} A_K(s) \right\} .$$

(9)

A possible solution to avoid double counting of the imaginary parts, which was adopted in Ref. [23], is

$$F^V(s) = \frac{M_0^2}{M_\rho^2 - s - iM_\rho \Gamma_{\rho}(s)} e^{\text{Re}\left\{ A_\rho(s) + \frac{1}{2} A_K(s) \right\}} .$$

(10)

Eq. (10) reproduces $\chi PT$ at NLO, vanishes at $s \to \infty$, has $SU(2)$ symmetry built-in and complies with analiticity and unitarity constraints up to first order in the expansion of the exponential. This description was successfully confronted to data using only one parameter, $M_0$. Present data have become much more precise and the Belle results [29] point to an interference pattern between excited resonances in this decay. All this motivates us to include [30], analogously, the contribution of the excited resonances [$\rho' = \rho(1450)$ and $\rho'' = \rho(1700)$ in this case] while keeping these nice properties [31]

$$F^V(s) = \frac{M_0^2}{M_\rho^2 - s - iM_\rho \Gamma_{\rho}(s)} \left\{ e^{\text{Re}\left\{ A_\rho(s) + \frac{1}{2} A_K(s) \right\}} + \frac{s\gamma e^{\phi_1} + \delta e^{\phi_2}}{M_\rho^2 - s - iM_\rho \Gamma_{\rho}(s)} \right\} .$$

(11)

The parameters $\gamma$ and $\delta$ are related to $R_T$ couplings for the excited resonances [as $\gamma_K$ in Eq. (13)], the $\Gamma_{\rho}(s)$ and $\Gamma_{\rho'}(s)$ widths are modeled as decays to two pions, and the phases $\phi_1$ and $\phi_2$ should vanish, at least, as $1/N_C$. Eq. (11) corresponds to what is included in TAUOLA right now [4]. SU(2) breaking has only been coded partially, through the kinematical and loop functions. Electromagnetic corrections [28, 32] have been considered [30] but not incorporated to the MC yet.

An alternative approach to using the Omnès solution consists in employing an $n$-subtracted dispersion relation where the relevant phaseshift, $\delta_1(s)$, is obtained as $\text{Im} F^V(s)/\text{Re} F^V(s)$. In this procedure [13], unitarity and analyticity are satisfied to all orders with [30]

$$F^V(s) = \frac{M_0^2 + s(\gamma e^{\phi_1} + \delta e^{\phi_2})}{M_\rho^2 \left[ 1 + \xi_\rho \text{Re} \left\{ A_\rho(s) + \frac{1}{2} A_K(s) \right\} \right] - s - iM_\rho \Gamma_{\rho}(s)}$$

$$- \frac{M_\rho^2 \left[ 1 + \xi_{\rho'} \text{Re} A_{\rho'}(s) \right] - s - iM_{\rho'} \Gamma_{\rho'}(s)}{\text{Re} \left\{ A_{\rho'}(s) + A_K(s) \right\}} .$$

(12)

in which $\xi_\rho = \frac{e^{\phi_1}}{96\pi F_\rho^2}$ and $\xi_{\rho'} = \frac{e^{\phi_2}}{96\pi F_{\rho'}^2}$ (analogously for $\xi_{\rho''}$). The result for the resummation in Ref. [13] has been employed in the denominator of the $\rho$ contribution.

4. Other two meson $\tau$ decay channels

The $\tau^+ \to K^- K^0 \nu_\tau$ decays are again described only in terms of the vector form factor to an excellent degree of approximation. The current parametrization in TAUOLA follows the Guerrero-Pich formula, see Eq. (12) [23, 34]. There is also an option to use Eq. (11). Further developments in $F^V_{K^0}(s)$ will be immediately translated to $F^V_{K^+}(s)$. The vector form factor in the $\tau^- \to (K\pi)^-\nu_\tau$ decays is currently coded following Ref. [35]

$$F^V_{K\pi}(s) = \left( \frac{M_{K^-}^2 + s\gamma_{K\pi}}{M_{K^-}^2 - s - iM_{K^-} \Gamma_{K^-}(s)} - \frac{s\gamma_{K\pi}}{M_{K^0}^2 - s - iM_{K^0} \Gamma_{K^0}(s)} \right)$$

$$\times \exp\left\{ -\frac{s}{128\pi^2 F_{\rho}^2} \text{Re} \left\{ A_{K\pi}(s) + A_{K\rho}(s) \right\} \right\} .$$

(13)

The function $A_{PQ}(s)$ is [5]

$$A_{PQ}(s) = -\frac{192\pi^2}{s} \left( s M_{PQ}(s) - L_{PQ}(s) \right) ,$$

(14)

in the notation of Gasser and Leutwyler. An option will be given to switch between this form factor and the one in Ref. [13]. The scalar form factor is fundamental in this decay channel to achieve a precise description of the decay data [17] at low values of $s$. Moreover, it is essential to understand CP violation in this channel, which has been reported recently [38]. TAUOLA is ready [3] to handle such $\tau^+$ and $\tau^-$ distinguishing terms. The implementation of $F^V_{K\pi}(s)$ [39] is documented in Ref. [40].
5. Three meson $\tau$ decay channels

The $\tau^- \to (\pi\pi)^-\nu_\tau$ and $\tau^- \to (K\pi)^-\nu_\tau$ decays have been coded following Refs. [41]. One- and two-resonance exchange diagrams were considered within $R_f T$ and the appropriate short-distance behaviour was required, yielding sets of compatible relations among the Lagrangian couplings in both decays (including $F_f G_V = F^2_V$, as in the two meson tau decays). The $\rho'$ resonance was introduced phenomenologically to improve the description of the data in the $\tau^- \to (\pi\pi)^-\nu_\tau$ decays. The progress with respect to the earlier description given by the KS model can be appreciated in Fig. 1 of Ref. [42]. The inclusion of final state interactions (FSI) in these decays is under study [40]. It should improve the agreement with data in the $d\Gamma/ds_{ij}$ distributions, specially at low values of $s_{ij}$.

6. Conclusions

A set of form factors based on $R_f T$ calculations, corresponding to 88% of the hadronic width of the $\tau$ lepton, has been implemented in TAUOLA. They are ready for precise confrontation with data gathered at Belle and BaBar (and future Belle II & Frascati superB facilities). In order to obtain the maximum possible information from experiments, the theory input to the MC has to be as accurate as possible with known properties respected ($\chi^2 PT$ results at low energies, smooth behaviour of the form factors at short distances, unitarity, analyticity, ...). Still, there are improvements to be done in all modes: appropriate inclusion of SU(2) breaking in the $\pi^-\pi^0$ channel, stabilization of $F_2^{\rho'}(s)$, inclusion of excited resonances in the $K\pi\tau$ modes, and addition of FSI (mainly the $\sigma$ effect) in the $3\pi$ mode.

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