Sending or not sending: Twin-field quantum key distribution with large misalignment error

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Based on the novel idea of twin-field quantum key distribution (TF-QKD, by M. Lucamarini, Z.L. Yuan, J.F. Dynes, & A.J. Shields, Nature 557, pages 400-403 (2018)), we present a protocol named as “sending or not sending TF-QKD” protocol which can tolerate large misalignment error. The revolutionary theoretical breakthrough in quantum communication, TF-QKD, changes the channel-loss dependence of the key rate from linear to square root. However, it demands the challenging technology of long distance single-photon interference, and also, as stated in the original paper, the security proof was not finalized there due to the possible effects of the afterwards announced phase information. Here we show by a concrete Eavesdropping scheme that the afterwards phase announcement do have important effects and the traditional formulas of decoy-state method does not apply to the original protocol. We then present our “sending or not sending” protocol. Our protocol does not take post selection for the bits in Z basis (signal pulses) and hence the traditional decoy-state method directly apply therefore automatically resolves the issue of security proof. Most importantly, our protocol presents a negligibly small error rate in Z-basis because it does not request any single-photon interference in this basis. This makes our protocol greatly improve the tolerable threshold of misalignment error in single-photon interference from the original a few percent to more than 45%. As shown numerically, our protocol exceeds a secure distance of 700 km, 600 km, 500 km, or 300 km even though the single-photon interference misalignment error rate is as large as 15% or 25%, 35%, or 45%.

I. Introduction
Quantum key distribution (QKD)1,2 can in principle present secure private communications with its security guaranteed by principles of quantum physics. With the development of both theory and experiment, it is more and more hoped to be extensively applied in practice, though there are barriers for so. Among all barriers, channel loss of long distance QKD is the major one10,12.

Very recently, a revolutionary theoretical progress was made by Lucamarini et al. They proposed the novel idea of twin-field quantum key distribution (TF-QKD)14 which has historically changed the relationship between key rate and the channel loss from linearly dependent to square root dependent. Consequently, the TF-QKD makes a great breakthrough of a secure distance longer than 500 km. However, as was stated in the original article14, the security is not finally completed because the possible effects of afterwards announcement of the phase information are not taken into consideration. As shown by a concrete Eavesdropping scheme in the appendix, we find that the phase information announced afterwards makes the traditional formulas of the decoy state method5,7 do not apply to the original protocol14. In fact given the scheme in the appendix, Eve can have full information to the key bits while traditional decoy-state method can give a key rate of 50%. Our Eavesdropping scheme shows that the fraction of single-photon bits among all raw bits must be not less than 50%, otherwise Eve may have full information to all bits without causing any disturbance. Although one may naturally turn to the key rate formulas for non-random-phase coherent states to resolve the issue, however, TF-QKD relied on the challenging technology of long distance single-photon interference, which may produce large misalignment error. Here we construct a “sending or not sending” TF-QKD protocol where there is no phase-slice dependent post selection for signal bits. Not only this itself increases the amount of key bits, but also, this makes the traditional calculation formulas for the decoy state method directly apply, the security proof is automatically completed and the less efficient key rate formula for non-phase-random coherent states is not necessary. Most importantly, our protocol can tolerate large misalignment error rate due to the long distance single-photon interference.

II. Sending or not sending protocol

Step 0. At any time window $i$, as requested by the TF-QKD, they (Alice and Bob) take random phase shifts $\delta_A, \delta_B$ to their coherent states accompanied by the reference light which is sent to Charlie. Charlie is also supposed to do appropriate phase compensation, but he is possibly dishonest.

Step 1. At any time window $i$, Alice (Bob) indepen-
dently determines whether it is a decoy window or a signal window. If it is a decoy window, she (he) sends out a decoy pulse (coherent state) with random phase shift $\delta_A$ ($\delta_B$) to Charlie; if it is a signal window, she (he) sends out a signal pulse by probability $\epsilon$ and she does not send anything by probability $1 - \epsilon$ to Charlie.

Note: This sending by small probability $\epsilon$ or not sending by probability $1 - \epsilon$ is the heart of our protocol.

Note: A coherent state of intensity $x$ and global phase $\rho$ is a linear superposition of photon number states $\{|k\rangle\}$ of $\sqrt{\exp(i\rho)} = \sum_{k=0}^{\infty} e^{-(\pi/2)/\sqrt{k!}}|k\rangle$. In a signal window, if Alice or Bob decides to send, she (he) shall always send a coherent state of intensity $\mu'$. For example, at a certain time when they both determined signal windows, if Alice decides to send while Bob decides not to send, the two-mode state from this time window is $\sqrt{\exp(i\rho_A)} \otimes |\sqrt{\exp(i\rho_B)}\rangle$; if both of them decide to send, the two-mode state is $\sqrt{\exp(i\rho_A)} \otimes |\sqrt{\exp(i\rho_B)}\rangle$; if both of them decide not to send, the state at that time window is $|0\rangle$. Here $\rho_A, \rho_B$ are global phases of the coherent states. They are known to Eve because Alice and Bob also send strong reference pulses accompany the weak coherent light. States from a decoy window can have different intensities. If at a certain time both of them have chosen decoy window and both of them have happened to choose the same intensity $\mu$, the two-mode coherent state from this time window is $\sqrt{\exp(i\rho_A)} \otimes \sqrt{\exp(i\rho_B)}$. In the protocol, Charlie is supposed to do phase compensation, trying to change the global phases so as $\rho_A, \rho_B$ into the same value $\rho$. If Charlie does this perfectly, the states from each side after the compensation have the same global phases. For example, state $\sqrt{\exp(i\rho_A)} \otimes \sqrt{\exp(i\rho_B)}$ will be changed into $\sqrt{\exp(i\rho_A)} \otimes \sqrt{\exp(i\rho_B)}$ after a perfect phase compensation by Charlie.

Step 2. Charlie is supposed to measure all twin-fields with a beam-splitter after taking phase compensation and announce the measurement outcome.

Note: We define an effective event by the following criterion: (i) If Charlie announces only one detector counting corresponding to a time window $i$ when both of them have determined a signal window, it is an effective event; (ii) If Charlie announces only one detector counting corresponding to a time window $i$ when both of them have determined a decoy window, used the same intensity of coherent states, and in that time window, the pre-chosen values $\delta_A, \delta_B$ satisfy

$$1 - \cos(\delta_A - \delta_B) \leq |\lambda|. \quad (1)$$

Here the value $\lambda$ is determined by the size of phase slice [14] chosen by Alice and Bob. Whenever an effective event happens, a bit in the corresponding basis is recorded.

Step 3. They announce each one’s decoy windows and signal windows. They also announce details for intensities of pulses sent from decoy windows and values $\delta_A, \delta_B$ they each have used.

Note: We define a Z-window as a time window when both Alice and Bob have determined a signal window. We name states from such Z-windows as states in Z-basis, or simply Z-pairs, Z-states. Effective events happen in Z-basis as named as Z-bits. Given that $\delta_A$ value ($\delta_B$ value) is randomized, whenever Alice or Bob sends a coherent state of intensity $\mu'$, it can be equivalently regarded as a density matrix of $\sum_{k=0}^{\infty} e^{-\mu' k!} |k\rangle\langle k|$, which is a classical mixture of different photon number states only. Hence we can define Z-windows as a subset of Z-windows when only one party of Alice and Bob decides to send and she (he) actually sends a single-photon state. In a Z1-window, the two-mode single photon state sent out is either $|z_0\rangle = |01\rangle$ or $|z_1\rangle = |10\rangle$. We shall call them as Z1 states or Z1 pairs. Also, effective events caused in Z1-windows are named as Z1-bits. Furthermore, we define an X-window as a time window when 1) both of them have chosen the decoy window, 2) both of them have chosen the same intensity for the coherent state to send, and 3) the random phase $\delta_A, \delta_B$ chosen for the window satisfy Eq. (1). We name the two-mode states from X-windows as states in X-basis, or simply X-pairs or X-states, and an X-bit is a bit caused by X pair. Also, as shown later, states of X pairs can be regarded as a probabilistic mixture of different photon-number states, with the two-mode single-photon ingredient $|\Psi\rangle|\Psi\rangle$, and $|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i(\delta_A - \delta_B) + i(\delta_B - \delta_A)}|1\rangle)$. Therefore we can define an X1-window as an X-window when they send a (two-mode) single-photon state. We also name those states from X1-windows as X1-pairs or X1-states, and the bits caused X1-pairs or X1-bits, though they can know the number of these windows and bits by calculation. If we only consider Z1-windows and X1-windows, the states set here is similar to that in a BB84 protocol [11].

Step 4. They randomly choose some Z-bits to do error test. By this they can know the bit-error rate in Z-basis, $E^Z$. They discard the test bits and the remaining Z-bits will be distilled for the final key.

Note: For any effective event happens in Z-basis, Alice (Bob) judges the bit value in this way: if she (he) has decided to send out a signal pulse, she (he) denotes a bit value 1 (0); if she (he) has decided not to send, she denotes a bit value 0 (1). One can see strictly, if an effective event happens while both Alice and Bob have decided not to send, or both of them have decided to send, a wrong bit in Z-basis is created. Because in such a case, the bit value denoted by Alice is different from the bit value denoted by Bob.

Step 5. They use the announced data from X pairs
to calculate the counting rate (yield) $s_1$ for $X_1$-windows (which is also the value for $Z_1$-windows). The number of bits created in $Z_1$-windows can be directly calculated from this value. Also, by observing error rates of $X$ pairs of intensity $\mu$, $E_\mu^X$, they can calculate the upper bound value of flipping rate of $X_1$-bits by $e_{1X} = \frac{s_1 E_\mu^X}{\mu}$. Asymptotically, the phase-flip rate $e_{1ph}$ for $Z_1$ bits is $e_{1ph} = e_{1X}$. Note: In the protocol, Charlie does the beam-splitter measurement\textsuperscript{14} after he takes the phase compensation. There are two output ports of the beam-splitter: right detector and left detector. They use the following criterion to judge a right bit or a wrong bit in $X$-basis: A right $X$-bit is the left (right) detector clicking caused by an $X$-pair with positive (negative) value of $\cos(\delta_A - \delta_B)$. A wrong $X$-bit is the right (left) detector clicking caused by a $X$-pair with positive (negative) value of $\cos(\delta_A - \delta_B)$. Given the observed error rate in $X$-basis and $s_1$, the phase-flip error rate $e_{1ph}$ for $Z_1$-bits can be obtained because asymptotically it is just the error rate of those single-photon-caused $X$-bits, as shown in the appendix. Note that, although they know the number of $X_1$-bits, they don’t know which ones are $X_1$-bits and hence quantity $e_{1X}$ cannot be directly observed, it can be only calculated by the formula above.

Note: Also, as one can easily see, if Charlie does the phase compensation perfectly, the out of beam-splitter measurement\textsuperscript{14} will produce a small observed error rate in $X$-basis, if $|\lambda|$ is small in the post-selection criterion Eq.\textsuperscript{(1)}. Charlie does not have to be honest or do the compensation perfectly. But this will only change the observed error rate in $X$-basis rather than the security of the protocol.

**Step 6.** They distill the final key with an asymptotic key rate formula

$$N_f = n_1 - n_1 H(e_{1ph}) - n_1 f H(E^Z)$$  \hspace{1cm} (2)

$N_f$: number of final bits, $n_1$: number of remaining $Z_1$-bits after error test in Step 4; $n_1$: number of remaining $Z$-bits after error test in step 4, $H(x) = -x \log x - (1-x) \log(1-x)$: binary entropy function, and $f$: error correction efficiency factor. The formula can be equivalently written in the following form of key rate per time window:

$$R = p_s^2 \cdot \left[2e(1-e)\mu' e^{-\mu' s_1} \left(1 - H(e_{1ph}^Z)\right) - S_Z f H(E^Z)\right]$$  \hspace{1cm} (3)

where $S_Z$ is the observed counting rate of $Z$-windows, $p_s$ is the probability that Alice (Bob) determines a time window to be a signal window in the protocol.

**III. numerical simulation**

In our protocol, we use the traditional formulas for the decoy-state method. Since we don’t need any post selection in $Z$ basis and we only need sending or not sending, there is no misalignment error in this basis. This makes the protocol be able to work with large misalignment from the single-photon interference in X basis. The results of numerical simulation are summarized in Fig.1 and Fig.2. In the calculation, we have assumed a detector with dark count rate of $10^{-11}$, and the detection efficiency of 80%. An error correction coefficient of 1.1 is set in our calculation. Here, we have only considered the asymptotic result and we have set the phase slice infinitely small. We can do so because in our case we take no post selection in $Z$ basis. And, at each data, point, we have optimized $\epsilon$ and the signal pulse intensity so as to obtain the best key rate. We can see that our protocol is so robust to misalignment errors that it can exceed a secure distance of nearly 300 km even with the misalignment error rate of 45%. It exceeds a secure distance of 700 km or 600 km even though the single-photon misalignment error rate is as large as 15% or 25%. Also, fixed at the distance to be 500 km, the key rates are shown
with different misalignment errors. The largest tolerable error rate can be 35%. These results show that our protocol by far breaks the existing a-few-percent threshold of single-photon misalignment error rate of for a larger-than-0 secure distance. When there is no misalignment error, our protocol exceeds a secure distance of more than 800 km.

IV. Security analysis

One may argue that there are afterwards announcement of phase information for decoy pulses, how to guarantee the validity of traditional decoy-state method here, e.g., Eq. (2). Since the phase shift information of signal pulses are never announced, we can regard signal pulses as classical mixture of different photon number states. What we want to know is the number of single-photon-caused bits and their phase-flip rate from signal bits. Once we know the facts, they do not change by any action outside the lab. Consider a virtual protocol where Alice and Bob secretly decided the random phase shift values prior to the protocol. In such a case, our calculations at Step 6 above is obviously solid. Note that the values of single-photon counts and phase-flip error rate are objective facts which do not change by any outside actions. After Alice and Bob know the fact, they can announce the phase information of all decoy pulses. But they can also choose to first announce the phase information and then calculate the crucial values for the signal bits, because no one knows at which time they have done the calculation. In such a case, they do not need to predetermine the random phase values, they just use the protocol we proposed above. Also, there is a similar story in the MDI-QKD: the bases information can not be announced before the states are measured. But it can be announced afterwards, for, the X-basis states are only used to know the phase-flip value of those qubits in Z basis.

Specifically, in the protocol Alice takes a random phase shift $\delta_A$ to her coherent state and Bob takes a random phase shift $\delta_B$ to his coherent state. Consider a two-mode weak coherent state prepared in an X-window $|\sqrt{\rho e^{i\delta_A+i\delta_B}}| \otimes |\sqrt{\rho e^{i\delta_B+i\delta_B}}|$. Here the global phases $\rho_A$ and $\rho_B$ cannot be regarded as random phases because they also send the strong reference pulses. First, we introduce the new independent variables $\delta_{\pm} = (\delta_B \pm \delta_A)/2$. Integrating the two-mode state of X-pairs on variable $\delta_+$ over the range of $[0, 2\pi)$, we obtain a classical mixture in the convex form $\sum_k p_k |\Psi_k\rangle \langle \Psi_k|$ with $|\Psi_k\rangle$ being the state of total photon number $k$ for the two-mode state $|\Psi_k\rangle$. For example,

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|01\rangle + e^{i(\delta_B-\delta_A)+i(\rho_B-\rho_A)}|10\rangle), \quad (4)$$

$$|\Psi_2\rangle = \frac{1}{2} |11\rangle + \frac{1}{2\sqrt{2}} e^{i(\delta_B-\delta_A+\rho_B-\rho_A)} |02\rangle + \frac{1}{2\sqrt{2}} e^{i(\delta_A-\delta_B)+i(\rho_A-\rho_B)} |20\rangle,$$

and so on. Second, consider the condition of Eq. (1) for state $|\Psi_1\rangle |\Psi_1\rangle$. Suppose $\Delta \in [0, \pi/2]$ and $\cos \Delta = 1 - |\lambda|$. Since values of $\delta_A$, $\delta_B$ are pre-chosen randomly, the values of $\delta_B - \delta_A$ is actually uniformly distributed in the ranges of $[-\Delta, \Delta]$ and $[\pi - \Delta, \pi + \Delta]$ if we request the condition of Eq. (1). Therefore, in Eve’s eyes, the (un-normalized) density operator for an $X_1$-pair is

$$\int_{-\Delta}^{\Delta} |\Psi_1\rangle \langle \Psi_1| dy + \int_{\pi-\Delta}^{\pi+\Delta} |\Psi_1\rangle \langle \Psi_1| dy \quad (5)$$

and $y = \delta_B - \delta_A = 2\delta_-$. This gives rise to

$$\frac{1}{2} (|01\rangle \langle 01| + |10\rangle \langle 10|) \quad (6)$$

Obviously, this is also the density matrix of $Z_1$-pairs and hence $X_1$-windows and $Z_1$-windows must have the same value of $s_1$, counting rate (yield) of the single-photon states. By decoy-state method, we can calculate the yield of $X_1$-windows and also estimate its error rate. This gives the yield of $Z_1$-windows, and also, as shown in the appendix, gives the phase-flip error rate of $Z_1$-bits, $e_1^{ph}$. Having shown the validity of the decoy-state method, we only need to show the security proof for perfect single-photon states here. We shall use the reduction, starting from virtual protocol with quantum entanglement distillation.

VI. Concluding remark

In conclusion, following the novel idea of TF-QKD [14], we proposed the sending or not sending TF-QKD protocol. Our protocol does not need to announce the phase information of signal pulses and hence the traditional decoy-state formulas can be directly applied. The single-photon interference is not needed in $Z$ basis thus the error rate in $Z$ basis can be negligibly small. This makes the protocol be tolerable to a fairly large error rate in $X$ basis where single photon interference must be done. Numerical simulation shows that the protocol can exceed a secure distance of 800 km without misalignment error, and more than 700 km with a misalignment error of 15%. Even though the misalignment error for the single-photon interference is as large as 25%, the protocol can still reach a secure distance of more than 600 km. Thanks to the revolutionary progress made by TF-QKD proposed in [14].

Note added: After we announced our Eavesdropping scheme on the [arXiv:1805.02272], it was then suggested using different key rate formulas directly pointing to non-random-phase coherent states [16, 17].

Appendix A: Security Analysis: virtual entanglement distillation and reduction

Consider a virtual protocol where there are two parties: Alice and Bob are one party and Charlie (Eve) is the other party. We shall use they to represent Alice and Bob if there isn’t any confusion. Initially, they (Alice and Bob) prepare two-photon pair states. In any pair, there is a $\chi$-photon, the first photon...
in the pair which will be sent to Eve, and a $\Phi$-photon, the second photon which will be stored locally by Alice and Bob in virtual protocols. A pair state is:

$$|T_2\rangle = \frac{1}{\sqrt{2}}(|\chi^0\rangle|\Phi^0\rangle + |\chi^1\rangle|\Phi^1\rangle)$$

(7)

where

$$|\chi^0\rangle = \frac{1}{\sqrt{2}}(|z_0\rangle + e^{i\delta_0+i\phi}|z_1\rangle), \quad |\chi^1\rangle = \frac{1}{\sqrt{2}}(|z_0\rangle - e^{i\delta_0+i\phi}|z_1\rangle);$$

and $\delta_0$ is randomly chosen by Alice and Bob with the condition

$$\cos \delta_0 \geq 0,$$

(8)

$\phi$ is a random phase unknown to Alice and Bob but known to Charlie (Eve). States $|\Phi^0\rangle$ and $|\Phi^1\rangle$ are perfect entangled states of

$$|\Phi^0\rangle = \frac{1}{\sqrt{2}}(|z_0\rangle + |z_1\rangle), \quad |\Phi^1\rangle = \frac{1}{\sqrt{2}}(|z_0\rangle - |z_1\rangle).$$

Note: Values of $\phi$ are known to Eve but not necessarily known to Alice and Bob.

**Virtual protocol 1.** They divide all pairs into 3 random sets, $I, V, C$. Set $C$ contains pairs randomly chosen from all pairs. To each photon pairs in $C$, $\delta_0$-values are randomly chosen from the ones satisfying

$$1 - \cos \delta_0 \leq |\lambda|.$$

(9)

The $\delta_0$-values for photon pairs not in $C$ are also chosen randomly, and those pairs happen to satisfy $1 - |\cos \delta_0| \leq |\lambda|$ are labeled by set $\mathcal{V}$ and those pairs happen to satisfy Eq. (9) are labeled by set $I$. Here $|\lambda|$ value actually determines the size of phase slice [14]. They send all $\chi$-photons to Eve. It is easy to show that, those $\chi$-photons from whatever sets $I, V$, or $C$ are all identical to Eve, because they all have the same density matrix

$$\frac{1}{2}(|z_0\rangle\langle z_0| + |z_1\rangle\langle z_1|).$$

Therefore Eve cannot treat them differently according to which set they are from. Charlie then announces his measurement outcome, note that he can be dishonest.

**Definition:** If Charlie announces only one detector clicking and the clicking is due to a $\chi$-photon from set $\mathcal{V}$ or set $C$, we call it an effective event. The corresponding pair, $\chi$-photon, and $\Phi$-photon in the same pair are called an effective pair, effective $\chi$-photon and effective $\Phi$-photon, respectively.

**Definition:** They measure all effective $\Phi$-photons labeled by set $\mathcal{V}$ in $X$-basis (i.e., $|\Phi^0\rangle, |\Phi^1\rangle$). They compare Charlie’s announced measurement outcome of effective photons from set $\mathcal{V}$ with their own measurement outcome of effective $\Phi$-photons in set $\mathcal{V}$. The mismatching rate is the error rate in $X$ basis, for effective $\Phi$-photons in set $\mathcal{V}$. We denote this by $e^X_1$. Specifically we can use a binary string $\kappa$ to represent Charlie’s announced outcome of effective $\chi$-photons from set $\mathcal{V}$ and another binary string $\kappa'$ to represent the measurement outcome of those effective $\Phi$-photons in set $\mathcal{V}$. The mismatching rate between these two strings is the error rate $e^X_1$ for effective $\Phi$-photons in set $\mathcal{V}$ in $X$-basis. Asymptotically, the error rate in $X$-basis of effective $\Phi$-photons in set $\mathcal{V}$ must be equal to that in set $C$. Say, $e^X_1$ for effective $\Phi$-photons in set $\mathcal{V}$ must be equal to the error rate in $X$-basis for effective $\Phi$-photons in set $C$. Therefore, to know the error rate for set $C$, they do not need measure $\Phi$-photons in set $\mathcal{V}$, they just use $e^X_1$ for set $\mathcal{V}$. In another terminology, $e^X_1$ is also the **phase-flip** error rate $e^{ph}_1$ in $Z$-basis for effective $\Phi$-photons in set $C$. If there were no channel noise and Charlie is honest, they could judge each $\Phi$-photon’s state ($|\Phi^0\rangle$ or $|\Phi^1\rangle$) exactly just by hearing Charlie’s announcement. However, in actual case, there could be channel noise and Charlie could be dishonest. They can preliminarily judge states of each effective $\Phi$-photons according to Charlie’s announced outcome for effective $\chi$-photons from $C$, and then distill entanglement from effective $\Phi$-photons in set $C$ with parameter $e^{ph}_1$. After distillation, they obtain high quality entangled states with a rate $1 - H(e^{ph}_1)$. They measure the distilled $\Phi$-photons in $Z$ basis (i.e., $|z_0\rangle, |z_1\rangle$), and obtain a secret string with rate $1 - H(e^{ph}_1)$. Note that, if they only want to obtain the final secure string rather than the entangled states from effective $\Phi$-states of $C$, they only need those operations in $Z$ basis in the “entanglement distillation”, operations in other basis and computation to find out the error positions are not needed. Therefore, for the purpose of making a secure final string only, they can actually measure all their $\Phi$-photons of set $\mathcal{C}$ in $Z$-basis in the very beginning, and they only need the value $e^{ph}_1$ which is known from effective events due to set $\mathcal{V}$. They do privacy amplification to the classical bits as the outcome from effective $\Phi$-states in set $\mathcal{C}$ measured in $Z$-basis in the beginning. Also, it makes no difference to Eve if they measure each $\Phi$-photons in set $\mathcal{V}$ in the very beginning in basis $X$-basis, (i.e., $|\Phi^0\rangle, |\Phi^1\rangle$). In the initial pairs, $\chi$-photons and $\Phi$-photons are perfectly entangled, after this measurement, they know the states of each $\chi$-photons in set $\mathcal{V}$ exactly, ($|\chi^0\rangle$ or $|\chi^1\rangle$). To know the error rate value $e^X_1$, they only need to compare Charlie’s announced measurement outcome and their own measurement outcome of effective $\chi$-photons from set $\mathcal{V}$.

**Virtual protocol 2.** They first measure those $\Phi$-photons in set $\mathcal{C}$ in $Z$-basis (i.e., $|z_0\rangle, |z_1\rangle$), and also measure $\Phi$-photons in set $I$ and $V$ in $X$-basis. They then send all $\chi$-photons to Eve. They will do virtual entanglement distillation from those effective $\Phi$-photons in set $C$. This protocol and the earlier protocol are identical to Eve because the states she receives in the two protocols are all the same. They can obtain a secret string by the same rate. This protocol has removed quantum memory already. More-
over, $\Phi$-photons in all sets are actually not needed, Alice and Bob can directly prepare $\chi$-photon states as if these states were outcome from the collapse of $\Phi$-photons in the same pairs. So, we can convert the protocol here to the following simpler one:

**Virtual protocol 3.** At any time window $i$, they send a single-photon state randomly chosen from set $C$ containing states $|z_0\rangle$, $|z_1\rangle$ only, and set $V \cup I$ containing states $|\chi^0\rangle$, $|\chi^1\rangle$ only and with value of $\delta_0$ being randomly chosen. For any state $|\chi^0\rangle$, $|\chi^1\rangle$, it belongs to set $V$ if $1 - |\cos \delta_0| \leq |\lambda|$, and set $I$ otherwise. Using the data announced by Charlie for effective photons in set $V$, one can estimate the quantity $e_1^X$ which also represents the phase flip error rate $e_1^{ph}$ for effective states from set $C$. Secret key can thus be distilled from bits caused by effective states from set $C$, with a rate $1 - H(e_1^{ph})$.

**Further reduction.** Actually, states for set $V$ and set $I$ can be post selected. They are spatially separated and they cannot in determine in advance the set $V$. So, they can do this equivalently by post selection: Alice takes a random phase shift $\delta_A$ to her coherent state $|\sqrt{\rho}e^{i\phi}\rangle = e^{-\mu/2} \sum_k e^{i\delta_A} (\sqrt{\mu})^k |k\rangle$ and Bob takes a random phase shift $\delta_B$ to his coherent state $|\sqrt{\rho}e^{i\phi}\rangle = e^{-\mu/2} \sum_k e^{i\delta_B} (\sqrt{\mu})^k |k\rangle$. After taking the random phase shift of $\delta_A$ and $\delta_B$, the two mode weak coherent state prepared by them is $|\sqrt{\rho}e^{i\phi}\rangle = |\sqrt{\rho}e^{i\phi}\rangle \otimes |\sqrt{\rho}e^{i\phi}\rangle$. If we only consider the ingredient of the single-photon state in the two mode state (omitting the overall phase factor of $e^{i\phi}\rangle$), it is the state $\frac{1}{\sqrt{2}} |01\rangle |10\rangle e^{i\delta_0 + i\phi} = \frac{1}{\sqrt{2}} (|z_0\rangle \pm e^{i\phi} |z_1\rangle)$, where $\cos \delta_0 = |\cos (\delta_B - \delta_A)|$ and $\phi = \rho_B - \rho_A$. As was shown in the main text, since here the traditional decoy-state method applies, we can only consider this single-photon ingredient for a secure key rate in the protocol. After Charlie completes announcement of his measurement outcome, they announce the values of $\delta_A$, $\delta_B$, setting $\cos \delta_0 = |\cos (\delta_A - \delta_B)|$, i.e., $\delta_0 = \delta_B - \delta_A$ if $\cos (\delta_B - \delta_A) > 0$ and $\delta_0 = \delta_B - \delta_A + \pi$ if $\cos (\delta_B - \delta_A) \leq 0$ in applying the criterion to label the states for set $V$ or $I$. They can then calculate $e_1^X$ which is also the phase-flip rate for those qubits in $Z$-basis (states in set $C$).

**Real protocol.** In the real protocol, since Alice and Bob are spatially separated, there is no way for them to prepare state $|10\rangle$ or $|01\rangle$ exactly without information leakage. Note that for security they cannot discuss on who has sent or not sent a photon now. What we can do is to set a small probability $\epsilon$ for each of them to send a signal pulse in signal windows. Besides $Z_1$ window, there are other types of $Z$-windows. We just use the tagged model here: A $Z$-bit caused by a $Z$-window is regarded as a tagged bit if it is not $Z_1$-window. Thus we have the calculation formula in the main text.

**Appendix B: Eavesdropping scheme based on afterwards announced phase information of signal states.** Earlier, we showed that our protocol can apply the traditional decoy-state method directly because the phase information of signal states is never announced. But, if it were announced and it took a role in bit value, then there were Eavesdropping schemes effectively attacking the secret bits. Here we show this by a specific scheme. Consider the original TF-QKD protocol[14] as shown in Fig.3. Suppose coherent state of intensity $\mu$ is used by each sides for signal pulses. The pulse pairs are phase modulated before being sent out for Charlie. The phase modulation includes the coding phase (0 or $\pi$) at each sides and the random phase shift we assume to be $\rho$ at both sides[14]. After modulation, the states of signal pulse pairs are two-mode coherent states $|\psi^+\rangle = |\sqrt{\rho}e^{i\phi}\rangle - |\sqrt{\rho}e^{i\phi}\rangle$ for bit value 0 and $|\psi^-\rangle = |\sqrt{\rho}e^{i\phi}\rangle - |\sqrt{\rho}e^{i\phi}\rangle$ for bit value 1, which will cause clicking of detector $D0$ only; and also $|\phi^+\rangle = |\sqrt{\rho}e^{i\phi}\rangle$ for bit value 0 and $|\phi^-\rangle = |\sqrt{\rho}e^{i\phi}\rangle$ for bit value 1, which will cause the clicking of detector $D1$ only. Note that the strong reference light is controlled by Eve, here we have assumed the reference phase to be 0 for conciseness. Eve applies the following scheme: **Step 0.** Eve can set whatever channel transmittance. For simplicity, we assume Eve sets the channel transmittance to be 1 here. Consider Fig.1. Before the twin pulses enter the beam splitter, Eve (Charlie) just honestly does whatever as requested by the the TF-QKD protocol. **Step 1** Eve, takes non-destructive crude measurement to project the output light from the beam splitter to vacuum or non vacuum subspace. Suppose she obtains non-vacuum, she stores the detected state and continue the attacking scheme. **Step 2** Eve takes a crude measurement to project the stored state either to the subspace $S = \{||1\rangle, ||2\rangle\}$ or to the subspace $S = \{|3\rangle, |4\rangle, |5\rangle, \cdots \}$. Suppose the outcome is $S$, she stores the state and continues. **Step 3** Eve, takes the following unitary transformation to her stored state above: $|1\rangle \rightarrow \sqrt{\mu} |1\rangle + \sqrt{1 - \mu} |m_0\rangle$, $|2\rangle \rightarrow |2\rangle$ where $|m_0\rangle$ is a state orthogonal to both $|1\rangle$ and $|2\rangle$. Eve, takes a crude measurement which collapses the stored state in Step 3 either to state $|m_0\rangle$ or the subspace $S$ spanned by the Fock states $\{|1\rangle, |2\rangle\}$. Suppose she obtains subspace $S$ in step 3, she stores the state and announces which detector ($D0$ or $D1$) has counted. She wait until Alice and Bob’s announcement, then goto Step 5.
sults in favor of her attacking in those non-trace-preserving maps. The point is that, at any step, if Eve doesn’t obtain the measurement outcome in favor of her, she just announces that she has not detected anything.

**Step 5** After Alice and Bob announce the value of $\rho$, bases of each pulse pairs, and which pulses are decoy pulses and which pulses are signal pulses, Eve can takes a phase shift operation to her stored state, changing it into one of the following 2 states corresponding on bit value 0 or 1 of the incident pulse pair: $\frac{1}{\sqrt{2}}(|1\rangle \pm |2\rangle)$. This enables Eve to know the bit value for sure without causing any noise by a projective measurement.

Here are details of the state evolution for the non-trace-preserving map above. Suppose at Step 1 detector $D0$ counts only, the incident state can be either $|\psi^+\rangle$ or $|\psi^-\rangle$. If the incident state is $|\psi^+\rangle$, the stored states $\{|\psi_1^+\rangle\}$ at the end of each Steps $\{i\}$ are: $|\psi_1^+\rangle = N_1 \sum_{k=1}^{\infty} \frac{\sqrt{\pi e^{\mu} / k}}{\sqrt{kN}} |k\rangle$; $|\psi_2^+\rangle = N_2(\sqrt{N}|1\rangle + \mu e^{\mu}|2\rangle)$; $|\psi_3^+\rangle = \frac{1}{\sqrt{2}}(|1\rangle + e^{\mu}\langle 2|)$). $|\psi_5^+\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$). All parameters $N_1, N_2, N_4$ are normalization factors.

Similarly, given the incident states $\{|\psi^-\rangle\}$, we can also calculate time evolution of $\{|\psi^-\rangle\}$ at each Steps $\{i\}$, and we obtain: $|\psi_1^-\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$. This means $|\psi_5^-\rangle$ and $|\psi_5^+\rangle$ are orthogonal to each other and Eve can know the corresponding bit value for sure. In the same way, one can easily show that Eve can also obtain full information of bit values without causing disturbance.

In the Eavesdropping above, the fraction of bits caused by single-photon state is 50% among all raw bits. According to the key rate formula (Eq.(2)) of Ref. [14], TF-QKD will present a key rate of 50% from raw key to final key although the actual key rate is obviously 0. This means the key rate formula does not match the protocol itself. The root of the problem is that Eve can make use of post announced phase information of signal states there. Given that protocol, one have to apply a different key rate formula.

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[1] C.H. Bennett and G. Brassard, in Proc. of IEEE Int. Conf. on Computers, Systems, and Signal Processing (IEEE, New York, 1984), pp. 175-179.
[2] N. Gisin, G. Ribordy, W. Tittel, et al., Rev. Mod. Phys. 74, 145 (2002); N. Gisin and R. Thew, Nature Photonics, 1, 165 (2006); M. Dusek, N. Lütkenhaus, M. Hendrych, in Progress in Optics VVVX, edited by E. Wolf (Elsevier, 2006); V. Scarani, H. Bechmann-Pasquinucci, N. J. Cerf, et al., Rev. Mod. Phys. 81, 1301 (2009).
[3] H.-K. Lo, M. Curty, and B. Qi, Phys. Rev. Lett. 108, 130503 (2012).
[4] S.L. Braunstein and S. Pirandola, Phys. Rev. Lett. 108, 130502 (2012).
[5] W.-Y. Hwang, Phys. Rev. Lett. 91, 057901 (2003).
[6] X.-B. Wang, Phys. Rev. Lett. 94, 230503 (2005).
[7] H.-K. Lo, X. Ma, and K. Chen, Phys. Rev. Lett. 94, 230504 (2005).
[8] A. Rubenok, J. A. Slater, P. Chan, I. Lucio-Martinez, and W. Tittel, Phys. Rev. Lett. 111, 130501 (2013).
[9] Y. Liu, T.-Y. Chen, L.-J. Wang et al., Phys. Rev. Lett. 111, 130502 (2013).
[10] Y.-H. Zhou, Z.-W. Yu, X.-B. Wang, Phy. Rev. A 93, 042324 (2016).
[11] L.C. Comandar, M. Lucamarini, B. Fröhlich, et al., Nature Photonics 10, 312 (2016).
[12] H.-L Yin, T.-Y Chen, Z.-W Yu, et al., Phy. Rev. Lett. 117, 190501 (2016).
[13] C. Wang, Z.-Q. Yin, S. wang, W. Chen, G.-C. Guo, Z.-F. Han, Optica, 4, 1016 (2017).
[14] M. Lucamarini, Z.L. Yuan, J.F. Dynes, & A.J. Shields, Nature, 557, pages 400-403 (2018)
[15] H. Inamori, N. Lütkenhaus, and D. Mayers, European Physical Journal D, 41, 599 (2007), which appeared in the arXiv as quant-ph/0107017; D. Gottesman, H.K. Lo, N. Lütkenhaus, et al., Quantum Inf. Comput. 4, 325 (2004).
[16] X.F. Ma, P. Zeng, and H.Y. Zhou, arXiv: 1805.05538
[17] K. Tamaki, H.-K. Lo, W.Y. Wang, and M. Lucamarini, arXiv: 1805.05511