ORIGINAL ARTICLE

Geometrical optimization of centerless grinding process by profiled workrest

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Abstract
This work presents a method to optimize centerless grinding geometrical configuration. It is based on the regulation of the horizontal axes of operating and regulating wheels of the machine, coupled with an opportune blade profile, allowing a continuous selection of workrest angle and workpiece height, without requiring blade substitution and/or manual interventions. The regulation of workrest angle and height cannot be done independently: their relationship is defined during blade profile design. In machines with two independent axes for regulating wheel and blade horizontal displacement, this regulation can be performed in process or in the setup phase. This results in optimal processing parameter choice leading to improved quality and shorter processing time as well as reduced setup time.

Keywords Centerless grinding · Profiled workrest · Rounding stability

1 Introduction

The centerless grinding process is characterized by a particular workpiece (WP) holding system. The WP is supported on three points by the grinding wheel, the control wheel and the workrest, also called supporting blade (see Fig.1). This configuration makes the process prone to roundness errors, depending on the initial setup of the machine [17]: WP center can oscillate and create a polyhedral profile instead of perfectly rounded WP (lobing effect).

To a first approximation, with given grinding and rubbing wheels and workpiece diameters, the lobing effect depends only on the geometric setup of the workpiece, i.e., workrest angle ($\gamma$) and WP height ($h_w$) [9]. Any imperfection on WP profile interacts with both the regulating wheel and workrest producing complex orbiting motions of WP center. These oscillations are reflected in an irregular material removal that, on its turn, increases WP imperfections. This phenomenon is known as rounding mechanism instability, as introduced in the pioneeristic work of [6]. After defining the basic geometrical relationships of the process, it is possible to use different stability criteria to analyze the theoretical instability produced by different configurations. These stability criteria allow the computation of maps that can be used to select the $\gamma - h_w$ pair with the lowest lobes growth rate, thus satisfying the roundness requirements for most practical situations as in [18]. A more comprehensive simulation-based analysis, including multiple technological constraints, is provided by Cui et al. in [5]. Recent researches consider, in the frequency-domain stability analysis, also nonlinear effects, such as contact loss between WP and grinding wheel [4, 14].

In general, several machine setup and process optimization solutions have been proposed in literature [10, 19, 20]. Most of them embed rounding mechanism stability analysis. A paradigmatic one is presented in [11]: it employs stability criteria together with a knowledge-based heuristic optimization algorithm which aims at finding the optimal combination of variables that maximizes a Performance Index where greater or lower weight is applied to different variables, based on the needs of process accuracy or productivity. With the objective of finding a global solution for the process setup considering both the interrelation between cutting and
wear mechanism and rounding instabilities, Barrenetxa et al. developed more sophisticated models and optimization algorithms [2], which found industrial applications [3].

However, the above-mentioned approaches, based on stability maps for the process configuration, present various drawbacks:

1. The stable zones are rather rare and narrow.
2. Due to other technological constraints (work piece spinning, cycle time reduction, need to obtain workpiece rotation even during air cutting, etc.), it could be impossible to operate in a stable region.
3. The level of stability (“negative growth rate”) indicates the capability of obtaining a good roundness in short time by canceling the lobes already present on the raw workpiece. In the usual centerless grinding process the best growth rates are only slightly negative; hence the smoothing process is rather slow.
4. The change of γ implies the substitution of the blade with another one with that specific angle, which is a time-consuming task.
5. As the number of blades is obviously limited, γ can be selected only among a discrete set of values, usually 3 (20 deg, 30 deg, and 40 deg). This limitation is reflected in a poor optimization of process configuration.

Seeking for a more flexible solution, Klocke et al. proposed the adoption of a functional workrest blade [13]. The workrest functionalities presented in that article include the possibility to adjust the blade position in vertical and horizontal directions, change its orientation [15], add active damping and monitoring sensors.

In the track of the above-mentioned work, this paper presents a solution to change rapidly workrest angle and WP height. It is based on the adoption of a profiled workrest, while the contact point between the profile and the WP is piloted by moving operating and control wheel via the machine axes. This enhanced configuration strategy allows the possibility to regulate angle γ over a continuous interval without changing the workrest blade or adding supplementary axes, thus overcoming limitations 4 and 5 in a simpler way than [13]. Moreover, the same solution suggests the application of a stability improvement technique, based on the disruption of waviness regeneration mechanism (overcoming limitations 1, 2, and 3).

The idea to improve rounding stability by regeneration mechanism disruption is not new. In [3], for instance, the disruption is attained by the application of continuous spindle speed variation technique (for the control wheel). By disrupting the mechanism of WP profile regeneration, this technique can profitably prevent regenerative chatter occurrence due to wheel-WP relative structural dynamics. On the other side, the inherent geometrical instability due to rounding mechanism remains untackled. Even though it is well known that different lobes number arise or are canceled according to grinding geometrical setup [8, 16], a systematic analysis of the combined effect of different configurations is missing. Whereas spindle speed variation entails a variable time-delay in the process dynamic equations, the stability analysis is carried out by means of time-domain simulation. Hitherto, no other attempts to prevent instability exploiting regeneration disruption have been undertaken. As a matter of fact, while in milling process the disruption of regeneration mechanism can be attained in different ways (e.g., variable pitch angles, serrated cutters, variable helix angle of solid end mill, etc.), in centerless grinding the possibilities to disrupt the geometrical rounding instability is rather limited.

On these premises, it is relevant to show how the proposed reconfiguration technique can be applied in-process for implementing a continuous or step-wise variation of geometrical parameters. Actually, this variation breaks the regeneration phenomenon associated to rounding mechanism, producing a general improvement of the geometrical stability. As the varying parameters entail algebraic equations with a variable delay, which do not usually offer a closed-form solution, a discretization approach is adopted to study workpiece evolution in terms of waviness growth and attenuation, as suggested in [1, 12]. Then, the stability properties of the resulting system can be treated with the usual criteria for discrete domain.

The paper is structured as follows. In Section 2, the proposed configuration method is presented. It will be shown how a profiled workrest can be exploited to vary the support angle γ and WP height h_w (not independently, if an additional degree of freedom for WP height regulation is not available). The basic relationships between wheels axes position and the resulting (γ, h_w) pair are provided. Furthermore, different types of profile are presented (convex, concave and piece-wise) and their properties in terms of allowable γ and h_w spans are analyzed. Section 3
explains the use of the proposed reconfiguration method. It introduces the concept of multistage approach enabled by the possibility of easily varying process geometry, which permits the attainment of very low gain factor for the evolution of waviness components in the workpiece profile. In the case of two \((\gamma, h_w)\) pairs, their selection can be eased by proper maps, whose example is provided for sake of clarity. Section 4 reports discussion and conclusions.

2 Fast configuration method

Let the schematic of Fig. 2 be considered (the rubbing wheel has not been drawn for sake of clarity), where \(R\) and \(r\) are the grinding wheel and workpiece radius, respectively. Provided the blade profile \(f(x)\), the \(\alpha\) angle and workpiece height \(h_w\) depend on the position of the contact point between blade and workpiece \((x_0, y_0)\). In particular,

\[
\gamma = \tan^{-1} f'(x_0),
\]

with \(f'(x_0) = \left. \frac{df(x)}{dx} \right|_{x=x_0} \) (1)

while the height \(h_w\) (coinciding with the \(y_w\) coordinate of the workpiece center) can be derived by geometrical relationships. Once the \((\gamma, h_w)\) pair is chosen, it is possible to determine the operating and rubbing wheels axial positions (respectively \(x_s\) and \(x_c\)) realizing the desired configuration. In general, \(h_w\), \(\gamma\), \(x_s\), and \(x_c\) can be expressed with respect to the contact point position along the curvilinear abscissa \(s\) of the given blade profile:

\[
L(s) \triangleq \begin{cases} 
    h_w = h_w(s) \\
    \gamma = \gamma(s) \\
    x_s = x_s(s) \\
    x_c = x_c(s)
\end{cases}
\]

Hence, \(L(s)\) completely defines the configuration of the centerless grinding process, both in terms of process parameters \(h_w\) and \(\gamma\) and in terms of axes coordinates to be commanded to obtain the desired \(h_w\) and \(\gamma\).

In order to avoid mechanical interference between the blade and the wheels, \(h_w\) and \(\gamma\) can vary only in a limited interval. Exploring the \(s\) space, the interference-free interval \([s_{\min}, s_{\max}]\) is found and, then, projected into the \(h_w\) and \(\gamma\) spaces, thus obtaining the limiting intervals \([h_{w_{\min}}, h_{w_{\max}}]\) together with \([\gamma_{\min}, \gamma_{\max}]\), respectively. Basically, depending on wheels and workpiece diameter, profile shape \(f(x)\) and the maximum WP height \(h_{w_{0}}\) (obtained for \(s = 0\), denoted by \(s_{0}\)), only a portion of the blade profile can be used.
In Fig. 3, the kinematic of the regulation is illustrated, assuming a fixed blade. The shaped workrest can have a convex or concave profile (see Fig. 4). Hints on how to select between these two categories are provided in the following. The convex profile has no inferior bounds in terms of curvature radius, as the contact always occurs in one point only. By decreasing the radius of curvature (which means having a thinner blade), it is possible to obtain large variations of $\gamma$ angle. On the other side, by moving the contact point towards high $\gamma$ angles, the workpiece undergoes a large movement away from the operating wheel, forcing it to follow; thus, if the diameter of the piece is too small, the operating wheel interferes with the blade before touching the piece, making machining impossible. Besides, a thin blade is usually more compliant and could generate dynamic issues.

With the concave profile, by moving the contact point towards low $\gamma$ angles, thanks to a counter-rotation, the overall displacement of the workpiece is limited. Therefore, there are less problems of interference between the working wheel and the blade, especially for pieces with small diameters. On the other side, concave profile cannot have a radius of curvature smaller than that one of the workpiece; otherwise there would be 2 points of contact between the two bodies. For pieces with a large diameter, the radius of curvature of the blade must be large (which means having a wider blade), and this limits the excursion of the possible values of $\gamma$ angle. The selection of the proper blade profile, depending on workpiece diameter, aims to increase the extent of $\gamma$ angle and $h_w$ variation ranges and, therefore, the chance of being able to reach the optimal set-up condition.

Besides concave/convex profiles, a piece-wise profile with discrete values of $\gamma$ angles can be also adopted. It corresponds to having multiple workrests in one blade. An example of piece-wise profile with 2 discrete values $\gamma_1$ and $\gamma_2$ is illustrated in Fig. 5.

The proposed approach includes the special case where the blade profile $f(x)$ is a straight line, thus degenerating in the usual blade. In this case, $\gamma$ angle cannot be regulated (i.e., $\gamma(s) = \text{const}$). Anyway, the method is still able to suggest a way to regulate workpiece height (even though in a limited range) using machine controlled axes, without intervening manually on the workrest.

Provided a workrest profile $f(x)$, the relationship $x_s(\gamma)$, $x_c(\gamma)$ and $h_w(\gamma)$ can be computed by solving the contact constraints. Namely, remembering Eq. (1), the following system must be solved:

$$
\begin{align*}
&x_w = x_0 + r \sin \gamma \\
&y_w = h_w = f(x_0) + r \cos \gamma \\
&x_s = x_w - \sqrt{(R + r)^2 - y_w^2} \\
&x_c = x_w + \sqrt{(R_c + r)^2 - y_w^2} \\
&\text{s.t.} x_s(s_{\text{min}}) < x_s < x_s(s_{\text{max}})
\end{align*}
$$

where $R_c$ is the radius of the control wheel, $x_s$ the horizontal position of control wheel, and $x_c$ the workrest profile independent variable that satisfies Eq. (1). The first of Eq. (3) yields directly $h_w(\gamma)$. Then, by substitution, $x_s(\gamma)$ and $x_c(\gamma)$ can be obtained as well.
In order to study the limits in terms of allowable $\gamma$ and $h_w$ induced by geometrical interference, the inverse relationships $h_w(x_s)$, $\gamma(x_s)$ and $\gamma(h_w)$ can be studied. Just pursuing a qualitative intuition, in Fig. 6, these allowable function ranges are plotted for a sample concave profile at different $h_w$, height and compared with the straight line case of the common workrest. Curves length is limited to respect the above-mentioned interference constraints. In Fig. 7, the complementary case of convex blade profile is plotted.

Some observations can be made on the plots, supporting the statements done in this section of the different profile designs:

- The concave and convex shaped blades allow a significant variation of $\gamma$, easily covering the range spanned in the industrial practice (20 deg–40 deg).
- A convex blade allows a larger movement of the grinding wheel, compared with a concave blade, without producing interference.
- The height variation required to obtain the full blade angle range is perceptually limited (especially with the concave profile), when compared with the variation obtained by the usual manual blade regulation.
- Even with a traditional straight blade is possible to change the workpiece height, by moving the control wheel with respect to the supporting blade, while, obviously, the $\gamma$ angle remains unchanged.

### 3 Process reconfiguration use

In the common practice, after having defined the basic geometrical relationships of the process, it is possible to use different stability criteria to analyze the theoretical instabilities, produced by different configurations. These stability criteria allow the computation of maps that can be used to select the ($\gamma$, $h_w$) pair with the lowest lobes growth rate, thus satisfying the roundness requirements for most practical situations ([7, 13, 21]). The curve $\gamma(h_w)$, outlined in the previous paragraph, corresponds to a trajectory in the stability map, a trajectory mainly parallel to the $\gamma$ axis. While the selection of a new stable zone of the diagram generally implies the substitution of the blade or its manual height adjustment, with the proposed method the selection of a stable region is pursued by a simple and fast wheel axes command, attainable by any machine mounting the profiled workrest, if equipped with autonomous motion axes moving the two wheels (or one wheel and the blade support, in the horizontal direction).

As a matter of fact, the possibility of regulating in continuous the angle $\gamma$ (with the associated $h_w$ variation), according to $\gamma(x_s)$ and $h_w(x_s)$ functions, overcomes the former limitations. More interestingly, it enables the implementation of a multistage approach based on a piecewise or continuous variation of the ($\gamma$, $h_w$) pair during the process. It can be demonstrated that the combination of stages
with different \((\gamma, h_w)\) pairs allows to reduce the growth rate, or gain, thus allowing a fast achievement of a satisfactory WP roundness.

The WP profile is discretized in \(P\) points, via a radial \(z\)-buffer vector \(v\). The final profile \(v_f\) produced by a process composed by \(N\) stages, can be computed applying a linear transformation \(M\) on the initial profile \(v_i:\)

\[
v_f = M(\gamma_1, \ldots, \gamma_N, h_{w1}, \ldots, h_{wN})v_i
\]

The effect of the transformation on waviness components in \(v_i\) can be evaluated by \(M\) eigenvalues: the process attenuates all waviness components if the module of all eigenvalues is lower than 1, i.e., the spectral radius of \(M\) is lower than 1.

If the multistage process is composed by 2 stages only, the gains associated to the waviness critical components (lobes) can be represented by a 2D map. In Fig.8 the gain surface log \((\rho(\gamma_1, \gamma_2))\) is depicted, where the other parameters of the 2 stages, described in the caption, have been kept fixed. It can been observed that a proper selection of \((\gamma_1, \gamma_2)\) can lead to a spectral radius much lower than 1, thus guaranteeing a good WP roundness (for instance, the pair \((30\,\text{deg}, 35\,\text{deg})\)).

4 Conclusions

In this paper, a solution to change rapidly workrest angle and workpiece height is proposed. It is based on the adoption of a profiled workrest, while the contact point between the profile and the workpiece is selected by moving the operating and control wheel axes (or the regulating wheel and the blade support axes, depending on the machine). This enhanced configuration strategy allows to regulate
the work rest angle over a continuous interval without changing the workrest blade (if the workpieces diameter does not change too much), thus increasing process set-up efficiency. Moreover, the same solution suggests the application of a novel stability improvement technique, based on the disruption of waviness regeneration mechanism by dividing the process into different stages, characterized by different work rest angles and workpiece heights.

The proposed approach can be applied to existing centerless grinding machines, if they are equipped with two independent controlled axis, able to change the relative position of the wheels and the blade. The Numerical Control should be upgraded, defining the proper kinematic relationships, in order to easily operate the process in the desired point of the blade profile. Additionally, if a multistage process is desired, the involved machine axis should be able to deliver the required positioning accuracy while moving along the blade profile, in order to obtain the expected material removal in every stage. Hitherto, the present work is purely theoretical. A patent request has been deposited and an extensive experimental campaign will be carried out, in cooperation with a centerless grinding machine builder and a blade provider.

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Authors’ contributions Marco Leonesio developed the configuration methodology and conceived the multistage stability analysis. Jeremi Wojcicki had the fundamental idea of profiled workrest and contributed to the configuration methodology development. Giacomo Bianchi had methodology and conceived the multistage stability analysis. Jeremi Wojcicki had the fundamental idea of profiled workrest and contributed to the configuration methodology development. Giacomo Bianchi had the idea of workpiece height regulation by workrest profile exploitation and contributed to the multistage stability analysis. All authors edited, read, and approved the final manuscript.

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Data availability The work is purely theoretical. The author can share the Matlab code used to compute the gain map of Fig. 8: Matlab code.

Declarations

Ethical approval Not applicable.

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Competing interests The authors declare that they have no competing interests.

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