Abstract

Memory effects play an important role in the theory of open quantum systems. There are two completely independent insights about memory for quantum channels. In quantum information theory, the memory of the quantum channel is depicted by the correlations between consecutive uses of the channel on a set of quantum systems. In the theory of open quantum systems memory effects result from correlations which are created during the quantum evolution. Here, we study the behavior of the actual speed of the quantum evolution under the effects of the correlated channel. The speed of quantum evolution indicates how a quantum system can be rapidly transformed from an initial state to a secondary state in a quantum evolution. In this work, we consider two correlated channels: Correlated phase damping channel and correlated dephasing channel with colored noise. We will show that the speed of quantum evolution for correlated channel is lower than the speed in uncorrelated channel. We will also show that the speed of quantum evolution in non-Markovian processes and in the presence of dynamical memory effects is greater than speed in the Markovian process and in the absence of dynamical memory effects.

Keywords Speed of quantum evolution · Open quantum system · Correlated noise

1 Introduction

Quantum systems are in fact not separated from their surroundings, and it is almost impossible to provide closed quantum systems. The study of open quantum systems is one of the special topics in quantum information theory. The theory of open quantum systems offers the necessary tools for describing and analyzing the interactions of a favorite system with its environment [1]. In the theory of open quantum systems, various methods are presented to illustrate the environment and its effects on the evolution of the desired system [1, 2]. An interesting method for examining open quantum systems is through the coupling

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strength between the system and its surroundings. If the coupling strength between the system and the environment is weak and the system relaxation time is longer than environment correlation time then there exist the one-way flow of information from the system to the environment. Such a quantum evolution is called Markovian [3] and it can be described by the master equation in the Lindblad form [4–6]. In a more realistic situation, the coupling strength between the system and the environment is strong and the system relaxation time is shorter than environment correlation time. In this case there is a back-flow of information from environment to the system. This type of quantum evolution is called non-Markovian [7–13]. Memory effects in the dynamics of open quantum systems play a fundamental role in various physical phenomena such as quantum biology [14–16], quantum cryptography [17], quantum metrology [18] and quantum control [19]. If one look at the subject in terms of the dynamical memory effects one can divide the quantum evolution into two categories: Markovian processes as a memoryless evolution and Non-Markovian process as a quantum process with memory. Therefore, it can be said that the memory effects will be appeared due to the interaction between the system and environment in non-Markovian regime. It seems natural to assume that the back-flow of information from the environment to the system is related to the existence of memory, Because in this situation future states of the system can depend on its past states as a result of reverse flow of information.

This point of view about the memory effects and non-Markovianity as a typical part of the theory of open quantum systems is completely different from the concept of quantum channels with memory. In order to distinguish between these two points of view, the term “correlated quantum channel” is used to describe quantum channels with memory. The memory of the quantum channel is depicted by the correlations between consecutive uses of the channel on a set of quantum systems [20–22]. Specifically, with memory or memoryless channels display a position in which the consecutive uses of the channels are correlated or independent respectively. In this case, memory is not due to the correlations created during the time evolution of a single quantum system. But it is because of the correlated operation of channels on the system consisting of a set of individual quantum systems. In Ref. [23], the connection between these two insight about memory effects has been studied by Addis et al. They showed how the use of correlated quantum channels can change the dynamical memory effects.

In some literature, the effect of the correlated quantum channel on quantum correlations has been studied. In Ref. [24] the effect of correlated quantum channel has been investigated on the entanglement of X-type state of the Dirac fields in the non-inertial frame. In Ref. [25], the authors have shown that how the correlated channel affects the dynamics of quantum correlations. The behavior of memory-assisted entropic uncertainty relation under the effects of the quantum correlated channels has been investigated in Refs. [26, 27].

In this work, we look at correlated channels from different perspective. We study the behavior of the actual speed of the quantum evolution under the effects of the correlated channel. We will show how classical correlations in the application of quantum channels can affect on the speed of the quantum evolution. The speed of quantum evolution can be determined by various measures. In Ref. [28], Taddei et al. represent a measure based on the quantum Fisher information metric, Del Campo et al. provide a measure founded on the relative purity [29] and in Ref. [30], Paiva-Pires et al. use the Wigner-Yanase (WY) information metric to introduce the measure to quantify the speed of quantum evolution. In this work based on the information geometric formalism we will use quantum Fisher information metric to quantify the speed of quantum evolution. In this work we will show that when there are no correlations between the two applications of the quantum channel, the speed
of the quantum evolution is greater than the conditions in which the correlations exist. This work is organized as follows: In Section 2, we review the concept of the correlated channel and consider the correlated phase damping channel and correlated dephasing channel with colored noise as examples of correlated channels. In Section 3 the notion of the speed of quantum evolution is reviewed. The main results is presented in Section 4. Finally Section 5 includes the conclusion of our main results.

2 Correlated Quantum Channel

First, we review in brief the concept of correlated quantum channels [23, 25–27]. A randomized implementation of the Pauli transformations is given by single-qubit Pauli channel $\Phi_1$ as follows

$$\rho \rightarrow \Phi_1(\rho) = \sum_{i=0}^{3} q_i \sigma_i \rho \sigma_i,$$

(1)

where $\sigma_i$’s are the components of the Pauli operators in x, y, z directions, the $\sigma_0$ indicate $2 \times 2$ identity matrix, $q_i$’s demonstrate the time dependent probability distribution in such a way that $\sum_{i=0}^{3} q_i = 1$. In this work and in our discussion, we will only consider two times consecutive uses of the channel for simplicity. If the channel is assumed to be uncorrelated for two times consecutive uses of the channel. Then the effect of such a channel can be described by autonomous using of channel on the two-qubit state as follows

$$\rho_{AB} \rightarrow \Phi_1(\rho_{AB}) = \sum_{i,j=0}^{3} q_i q_j (\sigma_i \otimes \sigma_j) \rho_{AB}(\sigma_i \otimes \sigma_j),$$

(2)

in this case, the noise is not correlated and $q_{i(j)}$’s are independent probability distributions.

But if there is a classic correlation between the dual repeated uses of the channel then the Pauli channel will act on the two-qubit system as follows

$$\rho_{AB} \rightarrow \Phi_1(\rho_{AB}) = \sum_{i,j=0}^{3} p_{ij} (\sigma_i \otimes \sigma_j) \rho_{AB}(\sigma_i \otimes \sigma_j),$$

(3)

here $p_{ij}$’s are a joint probability distribution. In this case there are no restrictions for the coefficients $p_{ij}$ that are factorized as $p_{i,j} = q_i q_j$. The most important model that considers the role of classical correlations in the application of pauli channels has been proposed by Macchiavello and Palma [20]. In the model which they have provided, the coefficient of probability distribution is defined as follows

$$p_{ij} = (1 - \mu) q_i q_j + \mu q_i \delta_{ij},$$

(4)

where $\mu \in [0, 1]$ represents the degree of classical correlation in the performance of the channel. The non-zero value of $\mu$ stimulates the same Pauli operators to be reused in the second use of the channel with some probability. The channel is completely correlated when $\mu = 1$. In this situation, the same Pauli operators act on both qubits and there is only the second term of the probability distribution, i.e. $p_{i,j} = q_i \delta_{ij}$. When $\mu = 0$ the channel is uncorrelated and the channel has the same form as shown in (2).
2.1 Correlated Phase Damping Dynamical Model

Let us consider a two-level quantum system which interacts with a surrounding bosonic environment. Here the dynamics of single-qubit is described by following Hamiltonian

\[ H = \frac{\omega_0}{2} \sigma_3 + \sum_k \omega_k b_k^\dagger b_k + \sigma_3 \sum_k (g_k b_k^\dagger + g_k^* b_k), \]  

(5)

where \( \sigma_3 \) is the Pauli operator in the z-direction, \( \omega_0 \) shows the two-level system frequency, \( b_k(b_k^\dagger) \) are the annihilation (creation) operators, and \( g_k \) is a constant coefficient which represents the coupling strength between the system and the environment.

In this model the dynamics of single-qubit system is characterized by the following time-local master equation [31]

\[ \mathcal{L}(\rho(t)) = \frac{\gamma(t)}{2} (\sigma_3 \rho(t) \sigma_3 - \rho(t)), \]

(6)

where \( \gamma(t) \) is the time-dependent dephasing rate. In this model, the off-diagonal elements of the density matrix of single-qubit system decay with the decoherence factor \( e^{-\Gamma(t)} \) during the quantum process, while the diagonal elements remain unchange because due to \([H, \sigma_z] = 0\), there is no transition between energy levels. when the temperature of the environment is zero, \( \Gamma(t) \) is given by

\[ \Gamma(t) = 4 \int d\omega J(\omega) \frac{1 - \cos \omega t}{\omega^2}, \]

(7)

here, \( J(\omega) \) represents the spectral density of the environment [31]. Here we consider the Ohmic-like spectral density for the environment

\[ J(\omega) = \omega_c^{1-s} \omega^s e^{-\frac{\omega}{\omega_c}}, \]

(8)

where \( \omega_c \) is the cutoff frequency and \( S \) is Ohmicity parameter. Given the value of Ohmlicity parameter, the environment is sub-Ohmic \((s < 1)\), Ohmic \((s = 1)\), and super-Ohmic \((s > 1)\). It is worth noting that in this model, the non-Markovian range is determined by Ohmicity parameter. Based on the results obtained in reference [11], when \( s \in [2.5, 5.5] \) the dynamics are non-Markovian. This model can be described by the following Kraus operators

\[ D_1(t) = \sqrt{\frac{1 + e^{-\Gamma(t)}}{2}} \sigma_0, \quad D_2(t) = \sqrt{\frac{1 - e^{-\Gamma(t)}}{2}} \sigma_3. \]

(9)

For the above considered model, time-dependent coefficients \( q_i \)'s can be obtained as follows

\[ q_0 = \frac{1 + e^{-\Gamma(t)}}{2}, \quad q_1 = q_2 = 0, \quad q_3 = \frac{1 - e^{-\Gamma(t)}}{2}. \]

(10)

Using (4), the correlated channel in (3) can be written in Kraus form as

\[ \Phi(\rho) = p_{03} (\sigma_0 \otimes \sigma_3) \rho (\sigma_0 \otimes \sigma_3) + \\
+ p_{30} (\sigma_3 \otimes \sigma_0) \rho (\sigma_3 \otimes \sigma_0) + \\
+ p_{00} (\sigma_0 \otimes \sigma_0) \rho (\sigma_0 \otimes \sigma_0) + \\
+ p_{33} (\sigma_3 \otimes \sigma_3) \rho (\sigma_3 \otimes \sigma_3). \]

(11)
2.2 Correlated Dephasing Model with Colored Noise

Here, we consider the interaction between a single-qubit system and environment which has the property of a random telegraph signal noise. The dynamics of single-qubit is described by time dependent Hamiltonian

\[ H(t) = \sum_{k=1}^{3} \Gamma_k(t) \sigma_k, \]

(12)

where \( \sigma_k \)'s are the Pauli operators in \( (x, y, z) \) directions respectively, \( \Gamma_k(t) \)'s are random variable which follow the statistics of a random telegraph signal. \( \Gamma_k(t) \) depends on the random variable \( n_k(t) \) as \( \Gamma_k(t) = \alpha_k n_k(t) \). Where \( n_k(t) \) has a Poisson distribution with an average value equal to \( t/2\tau_k \) and \( \alpha_k \)'s are coin-flip random variables that can have values \( \pm \alpha_k \) randomly. Here, we have dephasing model with colored noise when \( \alpha_1 = \alpha_2 = 0 \) and \( \alpha_3 = \alpha \). In this case, the dynamics can be described using the following Kraus operators

\[ D_1(t) = \sqrt{1 + \Lambda(v)/2} \sigma_0, \quad D_2(t) = \sqrt{1 - \Lambda(v)/2} \sigma_3, \]

(13)

where \( \Lambda(v) = e^{-v}[\cos(\mu v) + \sin(\mu v)/\mu], \mu = \sqrt{(4\alpha \tau)^2 - 1} \) and \( v = t/2\tau \). Here, the range of the \( \tau \) quantifies an interval in which the channel is non-Markovian. Based on the results presented in Ref. [11], if \( \tau \geq 1/2 \) the quantum evolution is non-Markovian. In the same way as mentioned before, one can obtain time-dependent coefficients \( q_i \) as

\[ q_0 = \frac{1 + \Lambda(v)}{2}, \quad q_1 = q_2 = 0, \quad q_3 = \frac{1 - \Lambda(v)}{2}. \]

(14)

Similar to what was done in Section 2.1, one can capture the Kraus form of this correlated channel, as is in (11).

3 Speed of Quantum Evolution

The speed of quantum evolution determines how a quantum system can be rapidly transformed from a primary state to a secondary state in a quantum process. The speed of quantum evolution can be defined by various measures. Taddei et al. proposed a measure based on the quantum Fisher information metric [28], Del Campo et al. consider a measure founded on the relative purity [29] and in Ref. [30], Paiva-Pires et al. use the Wigner-Yanase (WY) information metric to introduce the measure for quantifying the speed of quantum evolution.

Based on the unified form of Riemannian metric the squared infinitesimal distance between two neighboring states \( \rho \) and \( \rho + d\rho \) is given by

\[ (dl)^2 = g_{\rho}(d\rho, d\rho). \]

(15)

Given the (15), the speed of quantum evolution \( \rho(t) = \Phi_t[\rho(0)] \) is given as follows

\[ v(t) = \frac{dl}{dt} = \sqrt{g(t)}, \]

(16)

where \( g(t) = g_{\rho(t)}(\dot{\rho}(t), \dot{\rho}(t)) \). In this work based on the information geometric formalism we will use quantum Fisher information metric to quantify the speed of quantum evolution.
For sake of simplicity, we use an alternative fidelity definition, as the distance measure of two quantum states to describe the speed of quantum evolution [32]

$$F(\rho(0), \rho(t)) = \frac{\text{Tr}[\rho(0), \rho(t)]}{\sqrt{\text{Tr}[\rho(0)^2] \text{Tr}[\rho(t)^2]}}. \quad (17)$$

Due to the fact that the second derivative of the fidelity with respect to $t$ is proportional to the quantum Fisher information, following relation is hold for the quantum Fisher information metric [33]

$$g(t) = -2 \frac{d^2}{dt^2} F(\rho(0), \rho(t)). \quad (18)$$

4 Main Results

After a brief discussion we have had so far in the case of classical correlated channels, now we are in a position to study the effect of these correlations on the speed of quantum evolution. In this work, we reviewed two correlated channels: Correlated phase damping channel and correlated dephasing channel with colored noise. Let us consider the set of two-qubit states with the maximally mixed marginal states that is called Bell-diagonal as an initial state

$$\rho_0 = \frac{1}{4} (\sigma_0 \otimes \sigma_0 + \sum_{k=1}^{3} c_k \sigma_k \otimes \sigma_k), \quad (19)$$

where $c_k$’s are real numbers that can take values within the range $-1 \leq c_k \leq 1$.

**Speed of quantum evolution for correlated phase damping noise:** One can easily verify by straightforward calculations that the shape of the Bell-diagonal states does not change under correlated phase damping noise i.e. The evolving state will be in the form below

$$\rho_t = \frac{1}{4} (\sigma_0 \otimes \sigma_0 + \sum_{k=1}^{3} c_k(t) \sigma_k \otimes \sigma_k), \quad (20)$$

where, the real coefficients $c_k(t)$ can be easily obtained as follows

$$c_1(t) = \Theta(t, \mu)c_1, \quad c_2(t) = \Theta(t, \mu)c_2, \quad c_3(t) = c_3, \quad (21)$$

where $\Theta(t, \mu) = \mu + (1 - \mu)e^{-2\Gamma(t)}$. From (17), one can easily obtain the fidelity between initial state and evolved state at time $t$ as

$$F(\rho_t, \rho_0) = \frac{1 + c_2^2 + \Theta(t, \mu) \sum_{k=1}^{2} c_k^2}{\sqrt{(1 + \sum_{k=1}^{3} c_k^2)(1 + c_3^2 + \Theta(t, \mu) \sum_{k=1}^{2} c_k^2)}}. \quad (22)$$

Now one can calculate the speed of quantum evolution by taking the second derivative of the fidelity and following the procedure which presented in Section 3.

In the following, we will show that how it is possible to reduce the speed of quantum evolution by the classical correlations in the use of phase damping noise. In Fig. 1, we plot the time evolution of the speed of quantum evolution when the environment is sub-Ohmic with $s = 0.1$. It is necessary to note that in this case, the evolution is Markovian and there is no dynamical memory effect. We consider the initial Bell-diagonal state with coefficients $c_1 = c_2 = c_3 = 0.1$. In Fig. 1 the speed of quantum evolution is represented for four distinct values of the correlation parameter $\mu$. Here it can be seen that by increasing the
correlation parameter $\mu$ the speed of quantum evolution is reduced. This means that if there exist correlation in the channel, the value of the speed of quantum evolution is lower than that in which there is no correlation i.e. $\mu = 0$.

In Fig. 2, we display the time evolution of the speed of quantum evolution when the environment is super-Ohmic with $s = 3.5$. Here we should note that by choosing this value for Ohmicity parameter the memory effects will be revealed and the dynamic is non-Markovian. We consider the same initial state as we choose in Fig. 1 i.e. $c_1 = c_2 = c_3 = 0.1$. In Fig. 2 the speed of quantum evolution is plotted for four different values of the correlation parameter $\mu$. As can be seen from Fig. 2, increasing the correlation parameter $\mu$ reduces the speed of quantum evolution.

By comparing Figs. 1 and 2, one can see that the value of the speed of quantum evolution in non-Markovian processes and in the presence of dynamical memory effects is greater than it in the Markovian process and in the absence of dynamical memory effects.
Speed of quantum evolution for correlated dephasing channel with colored noise: In this model, it can also be easily shown that the shape of the Bell-diagonal states does not change under correlated dephasing channel with colored noise i.e. The evolving state will be in the Bell-diagonal form. In this model the real coefficients \( c_k(t) \) can be easily obtained as

\[
c_1(t) = \Gamma(v, \mu)c_1, \quad c_2(t) = \Gamma(v, \mu)c_2, \quad c_3(t) = c_3,
\]

where \( \Gamma(v, \mu) = \mu + (1 - \mu)\Lambda^2(v) \) and \( v = t/2\tau \). By following the same procedure used for the correlated phase damping noise, one can calculate the speed of quantum evolution for correlated dephasing channel with colored noise. Here we will show that the speed of the quantum evolution is reduced by the classical correlations in the use of dephasing channel with colored noise.

In Fig. 3, we plot the time evolution of the speed of quantum evolution when \( \tau = 0.1 \). Here the evolution is Markovian and there is no dynamical memory effect. We consider the same initial state as we choose in correlated phase damping noise model. In Fig. 3 the speed of quantum evolution is plotted for four distinct values of the correlation parameter \( \mu \). As can be seen from Fig. 3 the speed of quantum evolution is reduced by increasing the correlation parameter \( \mu \). Thus, when there exist correlation in the channel, the value of the speed of quantum evolution is lower than that in which there is no correlation i.e. \( \mu = 0 \).

In Fig. 4, we plot the time evolution of the speed of quantum evolution when \( \tau = 4 \). Here we should note that by choosing this value for scaled time \( \tau \) the memory effects will be revealed and the dynamic is non-Markovian. We consider the same initial state as we choose in Fig. 3. In Fig. 4 the speed of quantum evolution is plotted for four different values of the correlation parameter \( \mu \). As can be seen from Fig. 4, increasing the correlation parameter \( \mu \) reduces the speed of quantum evolution.

By comparing Figs. 3 and 4, one can see that the value of the speed of quantum evolution in non-Markovian processes and in the presence of dynamical memory effects is greater than it in the Markovian process and in the absence of dynamical memory effects.

![Fig. 3](Color online)Speed of quantum evolution under correlated dephasing channel with colored noise is plotted as a function of time for different correlation parameter \( \mu \) in Markovian regime
5 Conclusion

In conclusion, we investigated the speed of quantum evolution when the system is affected by a classically correlated channel. In this work, correlated phase damping channel and correlated dephasing channel with colored noise were considered as examples of correlated channels. In this work, the Bell-diagonal state was considered as the initial state of the system that was affected by correlated noise. We observed that as much as the classical correlation of the channel increases, the speed of the quantum evolution decreases. In this work we have investigated the effect of both Markovian processes as a memoryless evolution and Non-Markovian process as a quantum process with memory on the speed of quantum evolution. We showed that the speed of quantum evolution in non-Markovian processes and in the presence of dynamical memory effects is greater than speed in the Markovian process and in the absence of dynamical memory effects.

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