Tachyon warm inflationary universe model in the weak dissipative regime

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Abstract

Warm inflationary universe model in a tachyon field theory is studied in the weak dissipative regime. We develop our model for an exponential potential and the dissipation parameter $\Gamma = \Gamma_0 = \text{constant}$. We describe scalar and tensor perturbations for this scenario.

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I. INTRODUCTION

It is well known that several long-standing problems of the Big Bang model (horizon, flatness, monopoles, etc.) may find a natural solution in the framework of the inflationary universe model [1, 2]. One of the successes of the inflationary universe model is that it provides a causal interpretation of the origin of the observed anisotropy of the cosmic microwave background (CMB) radiation and also the distribution of large scale structures [3].

Warm inflation is an alternative mechanism for produce successful inflation and avoiding the reheating period [4]. In warm inflation, dissipative effects are important during inflation, so that radiation production occurs concurrently with the inflationary expansion. The dissipating effects arises from a friction term that describes dissipating the processes of the scalar field into a thermal bath via its interaction with other fields. Also, warm inflation shows how thermal fluctuations during inflation may play a dominant role in the production of initial perturbations. In such models, the density fluctuations arise from thermal rather than quantum fluctuations [5]. Among the most attractive features of these models, warm inflation end at the epoch when the universe stops inflating and ”smoothly” enters in a radiation dominated Big-Bang phase [4]. The matter components of the universe are created by the decay of either the remaining inflationary field or the dominant radiation field [6].

On the other hand, implications of string/M-theory to Friedmann-Robertson-Walker (FRW) cosmological models have been attracted great attention in the late time, in particular, those related to brane-antibrane configurations such as space-like branes [7]. The tachyon field associated with unstable D-branes might be responsible for cosmological inflation in the early evolution of the universe, due to tachyon condensation near the top of the effective scalar potential which could also add some new form of cosmological dark matter at late times [8].

The outline of the paper is a follows. In section II, the dynamics of tachyon warm inflationary model is obtained. In section III, cosmological perturbations are investigated. Finally, in section IV we give some conclusions.
II. TACHYON WARM INFLATIONARY MODEL

As was pointed by Gibbons [9], the energy density, $\rho_{\phi}$, and pressure, $p_{\phi}$, associated with the tachyon field are defined by $\rho_{\phi} = V(\phi) / \sqrt{1 - \dot{\phi}^2}$ and $p_{\phi} = -V(\phi) / \sqrt{1 - \dot{\phi}^2}$, respectively. Here, $\phi$ denotes the tachyon field (with unit $1/m_p$, where $m_p$ represents the Planck mass) and $V(\phi) = V$ is the effective potential associated with this tachyon field. The potential is one that satisfies $V(\phi) \rightarrow 0$ as $\phi \rightarrow \infty$. It has been argued that qualitative tachyonic potential of string theory can be described via an exponential potential of the form [7]

$$V(\phi) = V_0 e^{-\alpha \phi},$$

where $\alpha$ and $V_0$ are free parameters. In the following we will take $\alpha > 0$ (with unit $m_p$). Note that $\alpha$ represents the tachyon mass [10, 11]. In Ref. [8] is given an estimation of these parameters in the limit $A \rightarrow 0$. Here, it was found $V_0 \sim 10^{-10} m_p^4$ and $\alpha \sim 10^{-6} m_p$. We should mention here that the caustic problem with multi-valued regions for scalar Born-Infeld theories with an exponential potential results in high order spatial derivatives of the tachyon field, $\phi$, become divergent [12].

The dynamics of the FRW cosmological model in the warm inflationary scenario, is described by the equations

$$H^2 = \kappa \left[ \rho_{\phi} + \rho_{\gamma} \right] = \kappa \left[ \frac{V}{\sqrt{1 - \dot{\phi}^2}} + \rho_{\gamma} \right],$$

$$\dot{\rho}_{\phi} + 3H (\rho_{\phi} + p_{\phi}) = -\Gamma \dot{\phi}^2 \Rightarrow \frac{\ddot{\phi}}{1 - \dot{\phi}^2} + 3H \dot{\phi} + \frac{V_{,\phi} V}{V} = -\frac{\Gamma}{V} \sqrt{1 - \dot{\phi}^2} \dot{\phi},$$

and

$$\dot{\rho}_{\gamma} + 4H \rho_{\gamma} = \Gamma \dot{\phi}^2,$$

where $H = \dot{a}/a$ is the Hubble factor, $a$ is a scale factor, $\rho_{\gamma}$ is the energy density of the radiation field and $\Gamma$ is a dissipation coefficient, with unit $m_p^5$. Dissipative coefficient is responsible for the decay of the tachyon scalar field into radiation during the inflationary regime [13, 14]. Dissipation coefficient, $\Gamma$ can be assumed as a function of $\phi$ [15], and thus $\Gamma = f(\phi) > 0$ by the second law of thermodynamics. Dots mean derivatives with respect to cosmological time, $V_{,\phi} = \partial V(\phi)/\partial \phi$ and $\kappa = 8\pi/(3m_p^2)$. 
During the inflationary era the energy density associated with the tachyonic field is the order of the potential, i.e. \( \rho_\phi \sim V \), and dominates over the energy density associated with the radiation field, i.e. \( \rho_\phi > \rho_\gamma \).

With \( \Gamma = \Gamma_0 = \text{const.} \) and using the exponential potential given by Eq. (1), we find that the slow roll parameter become

\[
\varepsilon = -\frac{\dot{H}}{H^2} = \frac{1}{6\kappa} \left[ \frac{V_\phi}{V} \right]^2 \frac{1}{V} = \frac{1}{6\kappa V_0 e^{-\alpha \phi}}. \tag{5}
\]

Assuming the set of slow-roll conditions, \( \dot{\phi}^2 \ll 1 \), and \( \ddot{\phi} \ll 3H(1 + r)\dot{\phi} \sim 3H\dot{\phi} \), the Hubble parameter is given by \( H(\phi) = \sqrt{\kappa V_0} e^{-\alpha \phi/2} \), where the rate \( r \) becomes

\[
r = \frac{\Gamma}{3HV} = \frac{m_p \Gamma_0}{\sqrt{24\pi}} \frac{1}{V_0^{3/2}} e^{3\alpha \phi/2} < 1, \tag{6}
\]

and parameterizes the dissipation of our model. For the weak (or high) dissipation regime, \( r < 1 \) (or \( r \gg 1 \)).

The evolution of \( \dot{\phi} \) during this scenario is governed by the expression \( \dot{\phi} = -V, \phi/3HV \). In the following, the subscripts \( i \) and \( f \) are used to denote the beginning and the end of inflation.

Using Eq. (2), the total number of e-folds at the end of warm inflation results as

\[
N_{\text{total}} = -3\kappa \int_{\phi_i}^{\phi_f} \frac{V_i^2}{V_i} d\phi = 3\kappa \frac{\alpha^2}{a^2} [V_i - V_f], \tag{7}
\]

where the initial tachyonic field satisfies \( \phi_i < \phi_f \), since \( V_i > V_f \). Rewriting the total number of e-folds in terms of \( V_f \) and \( V_i \), and using that \( \varepsilon_f \simeq 1 \), we find \( V_i = (2N_{\text{total}} + 1)V_f \). Since, the \( N_{\text{total}} \) parameter could assume appropriate values (at least 60) in order to solved standards cosmological puzzles. To do this, we need the following inequality must be satisfied: \( V_i > 10^2 V_f \).

### III. THE PERTURBATIONS

In this section we will describe scalar perturbations in the longitudinal gauge, and then we will continue describing tensor perturbations.

By using the longitudinal gauge in the perturbed FRW metric, we write

\[
ds^2 = (1 + 2\Phi)dt^2 - a(t)^2(1 - 2\Psi)\delta_{ij}dx^i dx^j, \tag{8}
\]
where $\Phi = \Phi(t, x)$ and $\Psi = \Psi(t, x)$ are gauge-invariant variables introduced by Bardeen [17]. Since that we need the non-decreasing adiabatic and isocurvature modes on large scale $k \ll aH$, (which turn out to be weak time dependent quantities), when $k$ is expressed in the momentum space, and combining with the slow roll conditions we may define $\Phi$, $\delta \phi$, $\delta \rho$, and $v$ (we omit the subscript $k$ here) [18] by following equations

$$\Phi \simeq \frac{4\pi}{M_p^2} \left( \frac{V \dot{\phi}}{H} \right) \left[ 1 + \frac{\Gamma}{4HV} + \frac{\Gamma \dot{\phi}}{48H^2V} \right] \delta \phi,$$

$$\left[ 3H + \frac{\Gamma}{V} \right] (\delta \dot{\phi}) + \left[ (\ln(V))_{,\phi} + \dot{\phi} \left( \frac{\Gamma}{V} \right)_{,\phi} \right] \delta \phi \simeq \left[ \phi \frac{\Gamma}{V} - 2(\ln(V))_{,\phi} \right] \Phi,$$

$$\delta \rho \simeq \frac{\dot{\phi}^2}{4H} [\Gamma_{,\phi} \delta \phi - 3\Gamma \Phi] \implies \frac{\delta \rho}{\rho} \simeq \frac{\Gamma_{,\phi}}{\Gamma} \delta \phi - 3\Phi,$$

and

$$v \simeq -\frac{k}{4aH} \left[ \Phi + \frac{\delta \rho}{4\rho} + \frac{3\Gamma \dot{\phi}}{4\rho} \delta \phi \right].$$

Here $v$ appears from the decomposition of the velocity field $\delta u_j = -\frac{\dot{\phi}}{k} e^{ikx} (j = 1, 2, 3)$ [17].

Note that in the case of the scalar perturbations tachyon and radiation fields are interacting. Therefore, isocurvature (or entropy) perturbations are generated, besides the adiabatic ones. This occurs because warm inflation can be considered as an inflationary model with two basics fields. In this context, dissipative effects themselves can produce a variety of spectral ranging between red and blue [5], thus producing the running blue to red spectral as suggested by WMAP five-year data.

The above equations can be solved taking $\phi$ as an independent variable instead of $t$. With the help of Eq. [2] we find

$$\left( 3H + \frac{\Gamma}{V} \right) \frac{d}{dt} = \left( 3H + \frac{\Gamma}{V} \right) \dot{\phi} \frac{d}{d\phi} = -(\ln(V))_{,\phi} \frac{d}{d\phi},$$

and introducing an auxiliary function $\varphi$ given by

$$\varphi = \frac{\delta \phi}{(\ln(V))_{,\phi}} \exp \left[ \int \frac{1}{(3H + \frac{\Gamma}{V})} \left( \frac{\Gamma}{V} \right)_{,\phi} d\phi \right],$$

we obtain the following equation for $\varphi$

$$\frac{\varphi_{,\phi}}{\varphi} = -\frac{9}{8} \left( \frac{\Gamma}{V} + 2H \right) \frac{\Gamma_{,\phi}(\ln(V))_{,\phi}}{12H(3H + \frac{\Gamma}{V})} \frac{(\ln(V))_{,\phi}}{V}.$$
Solving Eq. (14) for $\Gamma = \Gamma_0 = \text{constant}$ and using Eq. (13) and condition $r < 1$, we find that
\[ \delta \phi = C (\ln(V))_\phi \exp[\Im(\phi)], \]
where $C$ is an integration constant and
\[ \Im(\phi) = -\frac{9}{2} \int H(\ln(V))_\phi \left[ \frac{(\Gamma_0/V + 2H)}{(\Gamma_0/V + 3H)^2} \right] d\phi, \]
(15)
or equivalently
\[ \Im(\phi) = -\frac{9}{2} \int \frac{H}{V} \left[ \frac{(\Gamma_0/V + 2H)}{(\Gamma_0/V + 3H)^2} \right] dV. \]
(16)

In this way, the expression for the density perturbations for $\Gamma = \Gamma_0 = \text{constant}$, becomes
\[ \delta H = \frac{2 m_p^2 \exp[-\Im(\phi)]}{5 (\ln(V))_\phi} \delta \phi. \]
(17)

We noted here that in the case $\Gamma = 0$, Eq. (17) is reduced to $\delta H \sim V \delta \phi / (H \dot{\phi}) \sim H \delta \phi / \dot{\phi}$, which coincides with expression obtained in cool inflation.

The fluctuations of the tachyon field are generated by thermal interaction with the radiation field, instead of quantum fluctuations. Therefore, we may write in the case $r < 1$, that $(\delta \phi)^2 \simeq H T_r/2 m_p^4 \pi^2$, where $T_r$ is the temperature of the thermal bath [19].

On the other hand, from Eqs. (2) and (3), under slow roll approximations and a quasi-stable state, i.e. $\dot{\rho}_\gamma \ll \Gamma \dot{\phi}^2$, together with $\rho_\gamma = \sigma T_r^4$, ($\sigma$ the Stefan-Boltzmann constant) we get
\[ T_r = \left[ \frac{\Gamma_0}{36 \sigma H^3} \frac{V'}{V^2} \right]^{1/4}, \]
and from Eq. (17), we find
\[ \frac{d \ln \delta^2_H}{d \phi} = \left[ -2 \frac{d \Im(\phi)}{d \phi} + \frac{V'}{8V} \right]. \]
(18)

The scalar spectral index $n_s$, is defined by
\[ n_s - 1 = \frac{d \ln \delta^2_H}{d \ln k}, \]
(19)
where the interval in wave number and the number of e-folds are related by $d \ln k(\phi) \simeq d N(\phi)$. By using Eqs. (11), (18) and (19), we get
\[ n_s - 1 \simeq -\frac{17 \alpha^2}{24 \kappa V}. \]
(20)

Note that $n_s - 1$ is $-\frac{\alpha^2}{24 \kappa V}$ bigger than that obtained in the tachyonic cold inflation case, where $n_s - 1 \approx -2 \alpha^2/(3 \kappa V)$ [20]. The warm inflation expression for $n_s$ in the weak dissipative regimen for the standard case was done in Ref. [21].

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The rate $r < 1$ (characteristic of the weak dissipative regime) allows to establish a condition for the ratio $\Gamma_0/\alpha^3$. This condition in terms of the scalar index is given by

$$\frac{\Gamma_0}{\alpha^3} < 3 \left( \frac{17}{24} \right)^{3/2} \frac{1}{\kappa (1 - n_s)^{3/2}}.$$  \hspace{1cm} (21)

Here, we have used Eqs. (6) and (20). In particular, for $n_s \simeq 0.97$ the above inequality is well supported for $\Gamma_0 < 42 \alpha^3 m_p^2$.

The generation of tensor perturbation during inflation would produce stimulated emission in the thermal background of the gravitational wave [18]. From expression (17), we may write the tensor-scalar ratio $R(k) = (A_g^2/P_R)$ as

$$R(k_0) = \frac{48 \pi}{m_p^2} \left[ \frac{\varepsilon H^3}{T_r} e^{2 \Theta(\phi)} \coth \left( \frac{k}{2T} \right) \right] \bigg|_{k=k_0},$$  \hspace{1cm} (22)

where we have used expressions $\delta_H \equiv 2 P_R^{1/2}/5$ and $A_g^2 = 32 \coth[k/2T]/(3m_p^4)$, whith $k_0$ is referred to as the pivot point.

From the combination of WMAP five-year data [3] with the Sloan Digital Sky Survey (SDSS) large scale structure surveys [22], there is found an upper bound $R(k_0=0.002 \text{ Mpc}^{-1}) < 0.28(95\% CL)$, where $k_0=0.002 \text{ Mpc}^{-1}$ corresponds to $l = \tau_0 k_0 \simeq 30$, with the distance to the decoupling surface $\tau_0= 14400$ Mpc. SDSS measures galaxy distributions at red-shifts $a \sim 0.1$ and probes $k$ in the range $0.016 \ h \text{ Mpc}^{-1} < k < 0.011 \ h \text{ Mpc}^{-1}$. Also WMAP five-year results gives the values for the scalar curvature spectrum $P_R(k_0) \equiv 25 \delta_H^2(k_0)/4 \simeq 2.4 \times 10^{-9}$ and the scalar-tensor ration $R(k_0) = 0.095$. Using the WMAP five-year data and choosing the parameters $T \simeq T_r \simeq 0.24 \times 10^{16} \text{ GeV}$ and $k_0 = 0.002 \text{ Mpc}^{-1}$. We obtained from Eqs. (17) and (22), that $V(\phi_0) \sim 10^{-12} m_p^4$ and $\alpha \sim 10^{-5} m_p$. Also, we would like to note that from Eq. (21) when the dissipative effects have a negligible influence ($\Gamma_0 < 10^{-14} m_p^5$) warm inflation occurs in the so-called weak dissipative regime for $n_s \simeq 0.97$.

IV. CONCLUSIONS

In this paper we have investigated the tachyonic warm inflationary scenario in the weak dissipative regime. Our specific model is described by an exponential scalar potential where the dissipation coefficient, $\Gamma = \Gamma_0 = \text{constant}$. In relation to the corresponding perturbations, we found a general relation for the density perturbation expressed by Eq. (17). The
tensor-scalar ratio is modified by a temperature dependent factor via stimulated emission into the existing thermal background (see Eq. (22)).

Using the WMAP five-year data, we have found some constraints for the parameters appearing in our model. For example, the potential becomes of the order of $V(\phi_0) \sim 10^{-12}m_p^4$ when it leaves the horizon, at the scale of $k_0 = 0.002\text{Mpc}^{-1}$, and the parameter $\alpha \sim 10^{-5}m_p$. From Eq. (21) we obtained a constrain from the dissipative parameter $\Gamma = \Gamma_0 = const. < 42\alpha^3m_p^2$ for $n_s = 0.97$. In particular, for $\alpha \sim 10^{-5}m_p$ the dissipation coefficient $\Gamma = \Gamma_0 = const. < 10^{-14}m_p^5$ in order that weak dissipative regime occurs.

Dissipative effects play a crucial role in producing the entropy mode; they can themselves produce a rich variety of spectra ranging between red and blue. The possibility of a spectrum which runs from blue to red is particularly interesting because it is not commonly seen in inflationary models, which typically predict red spectral. Models of inflation with dissipative effects and models with interacting fields have much more freedom than single self-interacting scalar field to fit the observational data. Summarizing, we have been successful in described tachyon warm inflationary model for characterize the early epoch of the universe in the weak dissipation regime.

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