Improving noise threshold for optical quantum computing with the EPR photon source

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We show that the noise threshold for optical quantum computing can be significantly improved by using the EPR-type of photon source. In this implementation, the detector efficiency $\eta_d$ is required to be larger than 50\%, and the source efficiency $\eta_s$ can be an arbitrarily small positive number. This threshold compares favorably with the implementation using the single-photon source, where one requires the combined efficiency $\eta_d\eta_s > 2/3$. We discuss several physical setups for realization of the required EPR photon source, including the photon emitter from a single-atom cavity.

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Optical quantum computing has raised significant interest in recent years \cite{11, 12, 13, 14, 15, 16, 17, 18, 19, 20}, in particular after the innovative proposal by Knill, Laflamme, and Milburn (KLM), who show that the feed-forward from high-efficiency photon detectors provides the required nonlinearity for the optical gate operations \cite{21}. The architecture of the gate in the original KLM proposal is somewhat complicated and the required detection efficiency is very high for scalable computation \cite{22, 23}. This requirement gets significantly relaxed with an improved approach to optical quantum computing \cite{24, 25}, based on the cluster-state model for quantum computation \cite{26}. The threshold inefficiency for the photon detection (or in general for the photon loss errors) is improved to the 1\% level with this cluster state approach, as estimated in Ref. \cite{27}. The next step of significant improvement has been made recently with the proposal of a clever architecture of tree graphs for efficient correction of the dominant photon loss errors \cite{28}. In this approach, it is shown that the photon loss, as measured by the source efficiency $\eta_s$ and the detection efficiency $\eta_d$, only need to fulfil the threshold requirement $\eta_s\eta_d > 2/3$. The photon loss in the memory or during the optical manipulation can be taken into account by combining their effects with the detection efficiency, which reduces the value of the effective efficiency $\eta_d$.

In this paper, we improve the noise threshold for photon loss in optical quantum computation with a less stringent requirement of $\eta_s > 0$ and $\eta_d > 1/2$. Furthermore, we eliminate the challenging requirement of the number-resolving photon detectors assumed in the previous work \cite{29}. The significant improvement in this paper is achieved with a simple change in the implementation: we use the EPR photon source instead of the single-photon source as usually assumed in optical quantum computing. We then discuss several physical setups for generating the required EPR photon source.

In the cluster-state approach to optical quantum computation, the central task is to create a large-scale graph state that is universal for quantum computation (the single-bit gates are considered to be easy and can be implemented with simple linear optical elements with a high accuracy) \cite{30}. If we require the computation to be inherently robust to the photon loss errors, the underlying graphs for the graph states need to have special architecture as shown in Ref. \cite{31}. These graph states can be generated efficiently through some simple linear-optics quantum gates \cite{32, 33}. In particular, the type-II fusion gates are robust to the photon loss errors \cite{34}. Although these gates are probabilistic in nature, they can lead to efficient buildup of arbitrary graph states \cite{35, 36}.

The type-II fusion gate eats two photons for each application of the gate (the photons are absorbed by the two detectors). To connect two graph states, each of $n$ qubits, with the type-II fusion gate, the output graph state has the qubit number $2n - 2$. To have the qubit number increasing with application of the gates, one needs to have $n \geq 3$. Therefore, one needs to start with graph states initially having three photons, which are just the three-photon GHZ states. Although the three-photon GHZ states have known and well-demonstrated advantage for the coincidence basis \cite{37}, the states there can not be used for optical quantum computation as they only survive in the post-selected Hilbert space which lead to problem in the scaling. For optical quantum computation, a critical requirement is to realize free three-photon GHZ states with the vacuum component as small as possible.

Ref. \cite{38} has shown how to generate the independently degraded (ID) GHZ state from the single-photon source described by the density operator $\rho_s = (1 - \eta_s)|\text{vac}\rangle\langle\text{vac}| + \eta_s|1\rangle\langle1|$, where $|\text{vac}\rangle$ denotes the vacuum component and $\eta_s$ is the source efficiency. The ID state is degraded from the perfect GHZ state with each photon in the state subject to independent loss with the same loss rate $f$. The generated ID GHZ state has an effective loss rate $f = 1 - \eta_s/(2 - \eta_s\eta_d)$ \cite{39}. The ID states can be connected with the type-II fusion, yielding larger graph state with the same effective loss rate $f$. This loss rate $f$, combined with the detection efficiency $\eta_d$ for the final single-bit measurements, need to fulfil the threshold
requirement $(1 - f) \eta_d > 1/2$, which leads to \( \eta_s \eta_d > 2/3 \).

FIG. 1: The construction of a free three-photon GHZ state based on the EPR photon source. The input modes 1 and 2, 3 and 4, 5 and 6 are in an imperfect EPR state with vacuum components. The photons in the modes 2, 4, and 6, first go through polarization beam splitters (PBS), 45°-degree polarization rotators, and horizontal (H) polarizers, and then are detected by single photon detectors. If each detector registers a photon, the modes 1, 3, and 5 are projected onto the GHZ state.

Here, instead of the single-photon source, we start with an imperfect EPR state with the source efficiency \( \eta_s \), described by the density operator,

\[
\rho_{EPR} = (1 - \eta_s)|\text{vac}\rangle\langle \text{vac}| + \eta_s|EPR\rangle\langle EPR|
\]

where \( |EPR\rangle = (|H_1 H_2\rangle + |V_1 V_2\rangle)/\sqrt{2} \) denotes the standard EPR state. We can generate a three-photon GHZ state

\[
|GHZ\rangle_{135} = 1/\sqrt{2}(|H_1 H_3 H_5\rangle + |V_1 V_3 V_5\rangle)
\]

with the setup shown in Fig. 1, using three pairs of the imperfect EPR state \( \rho_{EPR} \). The process is probabilistic and it succeeds if the three detectors each register a horizontally polarized photon. In this case, we need to have at least one photon coming from each input state \( \rho_{EPR} \), so the vacuum component in \( \rho_{EPR} \) only influences the success probability, and has no contribution to the final state when the process succeeds. The generated GHZ state has no vacuum component (and thus no photon loss with the above loss rate \( f = 0 \)), and these GHZ states can be used to build up large-scale graph states with the type-II fusion gates. So the threshold requirement now is given by \( \eta_d > 1/2 \), which is independent of the source efficiency \( \eta_s \) in the initial state \( \rho_{EPR} \). We only require \( \eta_s > 0 \), so that the preparation of the GHZ states succeeds with a finite probability given by \( \eta_s^3 \eta_d^3/32 \) (note that the success probability is \( \eta_s^3 \eta_d^3/256 \) for the case of single-photon source). The finite success probability for preparation of the GHZ state does not affect the scaling and only leads to a constant overhead for overall quantum computation. Notice also that in the setup shown in Fig. 1, the photon detectors do not need to resolve the photon numbers, as no more than one photon can hit each detector in the event of "success". This is different from the case of single-photon source, where more challenging number-resolving photon detectors need to be assumed.

Now we discuss several physical setups for possible implementation of the EPR photon source described in Eq. (1). The photon pairs generated from the spontaneous parametric down conversion (SPDC) are usually written in the form of Eq. (1) \[13,14\]. However, there is an important point that needs to be clarified. For the photon pairs generated from the SPDC, there is a small probability to get two (or more) EPR pairs. Although this double EPR probability is small, it leads to a serious problem. The density operator for the photon pairs from the SPDC can be written in the form

\[
\rho_s = (1 - \eta_s)|\text{vac}\rangle\langle \text{vac}| + \eta_s|EPR\rangle\langle EPR| + (x\eta_s^2/2)(|EPR\rangle\langle EPR|)^{\otimes 2} + ..., \tag{3}
\]

where for a Possionian distribution \( x = 1 \) (which is typically the case for the SPDC). If we input three of this type of states to the setup shown in Fig. 1, after detection on the modes 2, 4, 6, we can analyze the output state from the modes 1, 3, 5. We assume the source efficiency \( \eta_s \ll 1 \). In this case, up to the order of \( \eta_s^4 \) (any orders lower than this can not give the three counts on the detectors 2, 4, 6), the following terms can make a contribution to the registered photon counts: (i) \( |EPR\rangle, |EPR\rangle, |EPR\rangle \) (one EPR pair from each of three inputs); (ii) \( |\text{vac}\rangle, |EPR\rangle, |EPR\rangle \otimes 2 \) and its permutations (one input is in the vacuum whereas another input has double EPR pairs). So, conditional on a photon count registered on each of the three detectors, the output state for the three modes 1,3,5 is given by (unnormalized)

\[
\rho_{out} = |GHZ\rangle_{135}\langle GHZ| + (x/2)(1 - \eta_s)|H_1 V_1 H_5\rangle\langle H_1 V_1 H_5| + |H_1 V_1 V_5\rangle\langle H_1 V_1 V_5| + |H_3 V_3 H_1\rangle\langle H_3 V_3 H_1| + |H_3 V_3 V_5\rangle\langle H_3 V_3 V_5| + |H_5 V_5 V_5\rangle\langle H_5 V_5 V_5| + |H_5 V_5 H_3\rangle\langle H_5 V_5 H_3| + O(\eta_s). \tag{4}
\]

The terms proportional to \( x \) in this equation comes from the contribution of the case (ii), and the last term
$O(\eta_e)$, which is negligible when $\eta_e$ is small, comes from the higher order contributions (remember the photon detectors are not number resolving). The state is not a 1D GHZ state, and the terms proportional to $x$ have no photon in some mode while two photons in the other mode. These terms, after a series of type-II diffusion gates, lead to complicated error patterns for the final graph state, which is difficult to correct with the photon detectors. So the state cannot be used for optical quantum computation unless $x$ is small, which requires sub-Possionian distribution in the input state $\rho_\sigma$ in Eq. (3). For the conventional SPDC, unfortunately it has Poissonian distribution with $\rho_\sigma$ unless it has a small excitation probability and decay of the atom to other atomic levels, the photon source is described by $\rho_{EPR}$ in Eq. (1) with a finite source efficiency, and we can tolerate a large amount of error due to this finite efficiency as we explained before. For this setup, we have the double excitation probability $x = 0$, as with a short pulse, a single atom can emit only a single photon.

$|\rho_\sigma| = \frac{1}{\pi} \int d^2 \sigma \rho_{\sigma}(\sigma) |\sigma|^2$, where $\rho_{\sigma}$ is the distribution with $\sigma \in \mathbb{C}^2$ and $|\sigma|^2$ is the probability that the state is a state with $\sigma$. So the state can not be used for optical quantum computation unless $x$ is small, which requires sub-Possionian distribution in the input state $\rho_\sigma$ in Eq. (3). For the conventional SPDC, unfortunately it has Poissonian distribution with $\rho_\sigma$ unless it has a small excitation probability and decay of the atom to other atomic levels, the photon source is described by $\rho_{EPR}$ in Eq. (1) with a finite source efficiency, and we can tolerate a large amount of error due to this finite efficiency as we explained before. For this setup, we have the double excitation probability $x = 0$, as with a short pulse, a single atom can emit only a single photon.

$\rho_{EPR} = \rho_{\sigma} \otimes |\sigma\rangle \langle \sigma|$, where $\rho_{\sigma}$ is the distribution with $\sigma \in \mathbb{C}^2$ and $|\sigma|^2$ is the probability that the state is a state with $\sigma$. So the state cannot be used for optical quantum computation unless $x$ is small, which requires sub-Possionian distribution in the input state $\rho_\sigma$ in Eq. (3). For the conventional SPDC, unfortunately it has Poissonian distribution with $\rho_\sigma$ unless it has a small excitation probability and decay of the atom to other atomic levels, the photon source is described by $\rho_{EPR}$ in Eq. (1) with a finite source efficiency, and we can tolerate a large amount of error due to this finite efficiency as we explained before. For this setup, we have the double excitation probability $x = 0$, as with a short pulse, a single atom can emit only a single photon.
preparation scheme shown in Fig. 1, we detect the mode 2 (which has a larger photon loss) and output the mode 1. When the input EPR pairs are represented by the general state \( \rho_c \) in Eq. (6), we can derive the output state for the modes 1, 3, and 5 after detection on the modes 2, 4, and 6 in Fig. 1. After some tedious but straightforward calculations, we find that the final output state is given by

\[
\rho_{\text{out}} = (1 - f)^3 |GHZ\rangle\langle GHZ| + \frac{f(1-f)^2}{2}(|H_3H_5\rangle\langle H_3H_5| + |V_1V_3\rangle\langle V_1V_3| + |H_1H_5\rangle\langle H_1H_5| + |V_1V_5\rangle\langle V_1V_5| + |H_3V_5\rangle\langle H_3V_5| + \frac{f^2(1-f)}{2}|H_1\rangle\langle H_1| + |V_1\rangle\langle V_1| + |H_3\rangle\langle H_3| + |V_3\rangle\langle V_3| + |H_5\rangle\langle H_5| + |V_5\rangle\langle V_5| + f^3|\text{vac}\rangle\langle \text{vac}|.
\]  

(7)

This is exactly an independently degraded (ID) GHZ state with the loss probability \( f = \frac{p_2}{p_2 + p_3} \). This ID-GHZ states can be used to construct large scale graph states with the same loss probability by applying the type-II fusion gate. So the threshold requirement becomes \((1-f)\eta_d > 1/2\). If we take the detector efficiency \( \eta_d = 75\%\), the ratio \( p_2/p_3 \) is required to be \( p_2/p_3 < 1/2 \) (note that the vacuum component \( p_0 \) can be arbitrarily large). It is pretty routine to achieve such a requirement with the state of the art cavity technology [19, 20].

In summary, we have shown that with use of an imperfect EPR photon source, the threshold on photon loss for optical quantum computation can be significantly improved and we can eliminate the requirement of using the number-resolving photon detectors. We discuss physical setups where the required EPR photon source can be implemented. In particular, the single-dipole cavity provides a clean EPR source with no double excitations, and the requirements are pretty realistic with the state-of-the-art cavity technology.

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