Determining neutralino parameters in Left-Right Supersymmetric Models

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Abstract

We report exact analytical expressions relating the fundamental parameters describing the neutralino sector in the context of the left-right supersymmetric model. The method used for such effects is the projector formalism deduced without take into account the Jarlskog’s projector formulae. Also, expressions for the neutralino masses and the neutralino mixing matrix are determined. The results are compared with numerical and analytical ones obtained in similar scenarios in the context of the minimal supersymmetric standard model.

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1 INTRODUCTION

In Ref. [1], based on the Jarlskog’s treatment of the Cabibbo-Kabayashi-Maskawa matrix, the neutralino observables, in the context of the minimal supersymmetric standard (MSSM), were described in terms of projectors. There, exact analytic expressions for the neutralino masses were also obtained by diagonalizing the associated real symmetric neutralino mass matrix. Then, the same formalism was applied to treat a more general case where the associated neutralino mass matrix was given by a complex symmetric matrix Ref. [2]. In this last reference, several CP conserving and violating possible scenarios were considered in the study of the determining parameters of the theory.

The purpose of this work is first to apply the projector formalism [1, 2] to study the existing connections among the fundamental parameters describing the neutralino sector in the context of the left-right supersymmetric (L-R SUSY) model [3]-[4]. Next, to compare the results obtained to the ones obtained in the context of the MSSM [2].

In the L-R SUSY model which is based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the masses and mixing matrices of the neutralinos and charginos are determined by $M_L$, $M_R$, the left-right gaugino mass parameters associated with the gauge group $SU(2)_L$ and $SU(2)_R$ respectively, $M_V$, the gaugino mass parameter associated with the gauge group $U(1)_{B-L}$, $\mu$, the Higgsino mass parameter and the ratio $\tan \theta_k \equiv k_u/k_d$, where $k_u$ and $k_d$ are the vacuum expectation values of the Higgs fields which couple to $d$-type and $u$-type quarks respectively [5]-[12].

In Section 2 we give a brief description of the L-R SUSY model and we write the Lagrangian density describing the neutralino sector in terms of the two component fermion fields and the neutralino mass matrix expressed in terms of the fundamental parameters $M_L$, $\mu$, $\tan \theta_k$, $M_R$ and $M_V$, where $M_L$ and $\mu$ are considered, in general, as complex numbers. In section 3 we compute the exact analytical expressions for the neutralino masses and the corresponding diagonalizing unitary matrix. Also, we plot these masses versus the Higgsino parameter, in both the CP-conserving and CP-violating cases, and we compare the corresponding CP-conserving results with the numerical ones obtained in [11]. In Section 4 the projector formalism [13] for this model is revised. Based on the explicit construction of the diagonalizing neutralino mass matrix, new formulas for the so-called reduced projectors are constructed without appealing to the Jarlskog’s projector formulas [13, 14]. The fundamental properties of these reduced projectors as well as the projectors and the so-called pseudoprojectors [2] are proved. Also, the equivalence of this reduced projectors with those obtained using the Jarlskog’s formulas is proved. In Section 5 using the new reduced projector formulas, we express the complex parameter $M_L$, in terms of the so-called eigenphases.
and the rest of the parameters. Moreover, taking advantage of the mentioned equivalence we
get a novel formula expressing the norm of this complex parameter in terms of its phase and of
the remaining fundamental parameters. An alternative method to disentangle these parameters
are presented in Appendix A. In Section 6 we compare the expected values of the fundamental
parameters in similar scenarios predicted by both the L-R SUSY model and the MSSM. Finally, in
Section 7 we give our conclusions and prospects.

2 A brief description of the Left-Right supersymmetric model

In the L-R SUSY model the full lagrangian is given by

$$L = L_{\text{gauge}} + L_{\text{matter}} + L_Y - V + L_{\text{soft}},$$

where $L_{\text{gauge}}$ contains the kinetic and self-interactions terms for the bosons vector fields $(W^\pm, W^0)_{L,R}$ and $V^0$, and the Dirac Lagrangian of their corresponding superpartners, i.e., the gaugino fields $(\lambda^\pm, \lambda^0)_{L,R}$; $L_{\text{matter}}$ contains the kinetic terms for the fermionic and bosonic matter fields, the Higgs fields and interaction of the gauge and matter multiplets; $V$ is a scalar potential, $L_Y$ (Yukawa Lagrangian) contains the self-interaction terms of the matter multiplets as well as of
the Higgs multiplets, e.g., it contains the self-interaction terms involving the fundamental Higgsino mass parameters $\mu_1 \equiv \mu, \mu_2$ and $\mu_3 : \text{Tr}[\mu_1 (\tau_1 \hat{\phi}_u \tau_1)^T \tilde{\phi}_d], \text{Tr}[\mu_2 (\tau \cdot \hat{\Delta}_L) (\tau \cdot \tilde{\delta}_L) \phi_d]$ and
$\text{Tr}[\mu_3 (\tau \cdot \hat{\Delta}_R) (\tau \cdot \tilde{\delta}_R) \phi_d]$, where $\tau_j, j = 1, 2, 3$ are the usual Pauli matrices, $\tilde{\phi}_d, \hat{\Delta}_L, \hat{\Delta}_R$ and $\tilde{\delta}_L, \tilde{\delta}_R$ are the superpartners of the bi-doublet field $\phi_d$ and the four triplet fields $\Delta_{L,R}$ and $\delta_{L,R}$, respectively, which we will define soon afterward (in the following we will consider $\mu_2 = \mu_3 = 0$); and $L_{\text{soft}}$ is the soft-breaking Lagrangian, involving the fundamental gaugino mass parameters $M_L, M_R$ and
$M_V$, which gives Majorana mass to the gauginos:

$$L_{\text{soft}} = \begin{align*}
M_L (\lambda^a_L \lambda^a_L + \bar{\lambda}^a_L \bar{\lambda}^a_L) \\
+ M_R (\lambda^a_R \lambda^a_R + \bar{\lambda}^a_R \bar{\lambda}^a_R) \\
+ M_V (\lambda^0_V \lambda^0_V + \bar{\lambda}^0_V \bar{\lambda}^0_V). 
\end{align*}$$

The Higgs sector contains two bi-doublet fields,

$$\phi_{u,d} = \begin{pmatrix} \phi^0_u \\ \phi^+_u \\ \phi^0_d \\ \phi^+_d \end{pmatrix}_{u,d} \equiv \left( \frac{1}{2}, \frac{1}{2}, 0 \right),$$

and four triplet fields,

$$\Delta_{L,R} = \begin{pmatrix} \Delta^+ \\ \Delta^0 \\ -\frac{1}{\sqrt{2}} \Delta^+ \end{pmatrix}_{L,R}. $$

\[ \delta_{L,R} = \left( \begin{array}{cc} \frac{1}{\sqrt{2}} \delta^+ & \delta^{++} \\ \delta^0 & -\frac{1}{\sqrt{2}} \delta^+ \end{array} \right)_{L,R}. \]

(2.5)

The Higgs \( \Delta_{L,R} \) transform as \((1, 0, 2)\) and \((0, 1, 2)\) respectively. The triplet Higgs \( \delta_{L,R} \) which transform as \((1, 0, -2)\) and \((0, 1, -2)\) respectively, are introduced to cancel anomalies in the fermionic sector that would otherwise occurs.

In order to generate mass for the gauge bosons we can choose the vacuum expectation values of the Higgs fields in the form \[12\]

\[ \langle \Delta_L \rangle = \langle \delta_{L,R} \rangle = 0, \quad \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \]

(2.6)

\[ \langle \phi_u \rangle = \begin{pmatrix} k_u & 0 \\ 0 & 0 \end{pmatrix}, \quad \langle \phi_d \rangle = \begin{pmatrix} 0 & 0 \\ 0 & k_d \end{pmatrix}. \]

(2.7)

Thus, in a first stage, the spontaneous breaking of \( SU(2)_R \times U(1)_{B-L} \) to \( U(1)_Y \), according to the vacuum expectation value \( \langle \Delta_R \rangle \neq 0 \), given in Eq. (2.6), generates masses for \( W^+_R, W^+_R \) and \( V^0 \). The two neutral states \( W^0_R \) and \( V^0 \) mix yielding the physical field \( Z_R \) and the massless field \( B \). The vacuum expectation value \( v_R \) of the triplet Higgs \( \Delta_R \) has been chosen very big to provide large masses to gauge bosons \( W^+_R \) and \( Z_R \). Next, through the spontaneous breaking of \( SU(2)_L \times U(1)_Y \) into \( U(1)_{em} \), according to the chosen vacuum expectation values \( \phi_{u,d} \) given in Eq. (2.6), the left weak bosons \( W^-_L \) and \( W^-_L \) as well as \( B_\mu \) acquire mass. Once again, the neutral fields mix forming the massless photon \( A_\mu \) and the physical gauge field \( Z_L \). The masses of the right-handed gauge bosons are given by

\[ M_{W_R} = \frac{1}{\sqrt{2}} g_R (k^2_u + k^2_d + v^2_R)^{1/2}, \]

(2.8)

\[ M_{Z_R} = \frac{1}{\sqrt{2}} v_R (g^2_R + 4g^2_V)^{1/2}, \]

(2.9)

whereas the mass of the left-handed ones are given by

\[ M_{W_L} = \frac{1}{\sqrt{2}} g_L (k^2_u + k^2_d)^{1/2}, \]

(2.10)

\[ M_{Z_L} = \frac{1}{\sqrt{2}} [(k^2_u + k^2_d)(g^2_L + 4g'^2_V)]^{1/2}, \]

(2.11)

where \( g_L, g_R, g_V \) and \( g' = g_R g_V / (g^2_R + 4g^2_V)^{1/2} \) are the coupling constants of the gauge groups \( SU(2)_L, SU(2)_R, U(1)_{B-L} \) and \( U(1)_Y \), respectively.
To find the neutralino masses we must consider the interaction terms between the gauge bosons, the Higgs, and their superpartners. The neutralino particles are produced in two stages of symmetry breaking. The first stage involving the vacuum expectation value $v_R$ of $\Delta_R$ generates masses for three heavy neutralinos $\tilde{\chi}_k^0$, $k = 5, 6, 7$. The second stage involving the vacuum expectation values $k_u$ and $k_d$ of the Higgs $\phi_u$ and $\phi_d$ generate mass for the light neutralinos $\tilde{\chi}_k^0$, $k = 1, \ldots, 4$. The Lagrangian for light neutralinos is given by [12]

$$L_{LN} = -\frac{i}{\sqrt{2}} g_L k_u \tilde{\phi}_{1u}^0 \lambda_L^0 + i \frac{\sqrt{2} g_R g_V}{g_1} k_u \tilde{\phi}_{1u}^0 \lambda_B^0$$

$$- i \sqrt{2} \frac{g_R g_V}{g_1} k_d \tilde{\phi}_{2d}^0 \lambda_B^0 + \frac{i}{\sqrt{2}} g_L k_d \tilde{\phi}_{2d}^0$$

$$+ M_L \lambda_L^0 + \left( \frac{4 M_R g_V^2 + M_V g_R^2}{g_1} \right) \lambda_B^0 \lambda_B^0$$

$$+ 2 \mu \tilde{\phi}_{1u}^0 \tilde{\phi}_{2d}^0 + h.c.,$$

(2.12)

where $\lambda_B^0 = \left( g_R \lambda_R^0 + 2 g_V \lambda_R^0 \right) / g_1$ with $g_1 = \left( g_R^2 + 4 g_V^2 \right)^{1/2}$; $\lambda_{L,R}^0$ and $\lambda_V^0$ are the neutral gaugino fields; and $\tilde{\phi}_{1u}^0$ and $\tilde{\phi}_{2d}^0$ are the neutral Higgsino fields, i.e., the superpartner of the neutral Higgs fields $\phi_{1u}^0$ and $\phi_{2d}^0$, respectively, defined in Eq. (2.3).

The above Lagrangian in matrix form can be written as follows

$$L_{LN} = -\frac{1}{2} (\xi^0)^T N \xi^0 + h.c.,$$

(2.13)

where $N$ is in general a complex symmetric matrix given by

$$N = \left( \begin{array}{cccc}
M_L & 0 & -\frac{1}{\sqrt{2}} g_L k_u & \frac{1}{\sqrt{2}} g_L k_d \\
0 & \frac{4 M_R g_V^2 + M_V g_R^2}{g_1} & \frac{\sqrt{2} g_R g_V}{g_1} k_u & -\frac{\sqrt{2} g_R g_V}{g_1} k_d \\
-\frac{1}{\sqrt{2}} g_L k_u & \frac{\sqrt{2} g_R g_V}{g_1} k_u & 0 & -2 \mu \\
\frac{1}{\sqrt{2}} g_L k_d & -\frac{\sqrt{2} g_R g_V}{g_1} k_d & -2 \mu & 0
\end{array} \right),$$

(2.14)

and the two component fermion field is

$$(\xi^0)^T = (-i \lambda_L^0 - i \lambda_B^0 \phi_{1u}^0 \phi_{2d}^0).$$

(2.15)

### 3 The neutralino masses and the diagonalizing matrix in the left-right supersymmetric model

The two-component light neutralino mass eigenstates $\chi_j^0$ are related to the two component fermion fields given in Eq. (2.15) as

$$\xi_k^0 = \sum_{i=1}^{4} V_{ki} \chi_i^0, \quad k = 1, \ldots, 4,$$

(3.16)
where $V$ is a unitary matrix satisfying

$$
N_D = V^T N V,
$$

and

$$
N_D^2 = V^{-1} N^\dagger N V,
$$

where $(E_j)_{4 \times 4}$ are the basic matrices defined by $(E_j)_{ik} = \delta_{ji} \delta_{jk}$ and $\tilde{\chi}^0_j$ stand for the four component Majorana neutralinos:

$$
\tilde{\chi}^0_j = \begin{pmatrix} \chi_0^j \\ \bar{\chi}_0^j \end{pmatrix}, \quad j = 1, \ldots, 4.
$$

Here, we suppose that the real eigenvalues of $N_D$ are ordered in the following way

$$
m_{\tilde{\chi}_1^0} \leq m_{\tilde{\chi}_2^0} \leq m_{\tilde{\chi}_3^0} \leq m_{\tilde{\chi}_4^0}.
$$

### 3.1 Exact analytical expressions for the neutralino masses

As we have seen in the above section, in the left-right supersymmetric model, the masses, the mixing parameters and the CP-violating properties of the neutralino are determined by the fundamental complex $M_L = |M_L| e^{i\Phi_L}$ and $\mu = |\mu| e^{i\Phi_\mu}$ and real $\tan \theta_k = k_u / k_d$, $M_R$ and $M_V$ parameters. To know the neutralino masses predicted by the present model, we can solve the characteristic equation associated to the Hermitian matrix $H \equiv N^\dagger N$. More precisely, the square root of the positive roots of this characteristic equation corresponds to the physical neutralino masses. The neutralino masses predicted by the present model are known only for the CP-conserving case under the limit of large $M_{L,R}$ or large $|\mu|$ [12], more precisely on the assumptions that $|M_{L,R} \pm \mu| \gg M_{Z_L}$, $M_R > M_V$, and $4g_V^2 M_R + g_R^2 M_V / g_1^2 \simeq 4g_V^2 M_R / g_1^2$. Indeed, a numerical analysis has been implemented to solve the mentioned characteristic equation [12], assuming determined values for the gauge boson masses, couplings constants and taking $\mu$, the higgsino mass parameter, as a free quantity. Here, we put into practice a method [11] [2] giving exact analytic expressions for the neutralino masses.

Starting from Eq. (3.18), we get

$$
(N^\dagger N) V - V N_D^2 = 0.
$$
A more explicit form of this matrix equation is

\[
\begin{align*}
(H_{11} - m^2_{\chi^0_j})V_{1j} &+ H_{12}V_{2j} + H_{13}V_{3j} + H_{14}V_{4j} = 0, \\
H_{21}V_{1j} &+ (H_{22} - m^2_{\chi^0_j})V_{2j} + H_{23}V_{3j} + H_{24}V_{4j} = 0, \\
H_{31}V_{1j} &+ H_{32}V_{2j} + (H_{33} - m^2_{\chi^0_j})V_{3j} + H_{34}V_{4j} = 0, \\
H_{41}V_{1j} &+ H_{42}V_{2j} + H_{43}V_{3j} + (H_{44} - m^2_{\chi^0_j})V_{4j} = 0,
\end{align*}
\]

(3.23)\)

\[ j = 1, \ldots, 4, \text{ where } H_{ij} = \sum_{k=1}^{4} N_{ki}^* N_{kj} : \]

\[
\begin{align*}
H_{11} &= M^2 + |M_L|^2, \\
H_{22} &= 4\kappa^2 M^2 + M^2_{RV}, \\
H_{33} &= 4|\mu|^2 + (1 + 4\kappa^2)M^2 \sin^2 \theta_k, \\
H_{44} &= 4|\mu|^2 + (1 + 4\kappa^2)M^2 \cos^2 \theta_k, \\
H_{12} &= H_{21}^* = -2\kappa M^2, \\
H_{13} &= H_{31}^* = -M \left(2|\mu|e^{i\Phi}\cos \theta_k + |M_L|e^{-i\Phi_L} \sin \theta_k\right), \\
H_{14} &= H_{41}^* = M \left(2|\mu|e^{i\Phi}\sin \theta_k + |M_L|e^{-i\Phi_L} \cos \theta_k\right), \\
H_{23} &= H_{32}^* = 2\kappa M \left(2|\mu|e^{i\Phi}\cos \theta_k + M_{RV} \sin \theta_k\right), \\
H_{24} &= H_{42}^* = -2\kappa M \left(2|\mu|e^{i\Phi}\sin \theta_k + M_{RV} \cos \theta_k\right), \\
H_{34} &= H_{43}^* = -\frac{1}{2}(1 + 4\kappa^2)M^2 \sin(2\theta_k),
\end{align*}
\]

where \( M = g_L M_{ZL} / \sqrt{g_L^2 + 4g^2} \), \( \kappa = g_R g_V / g_1 g_L \), and \( M_{RV} = (4g_V^2 M_R + g_R^2 M_V) / g_1^2 \).

For fixed \( j \), Eq. (3.23) represents a system of homogeneous linear equations depending on only one of the neutralino masses. Thus, the neutralino masses can be determined by solving the characteristic equation associated to this system, that is

\[
X^4 - a X^3 + b X^2 - c X + d = 0,
\]

(3.24)

where

\[
a = |M_L|^2 + 8|\mu|^2 + M^2_{RV} + 2(1 + 4\kappa^2)M^2,
\]

\[
b = \ldots
\]

\[
c = \ldots
\]

\[
d = \ldots
\]
\[ b = (4|\mu|^2 + (1 + 4\kappa^2)M^2)^2 \\
+ M_{RV}^2 (|M_L|^2 + 8|\mu|^2 + 2M^2) \\
+ 8|M_L|^2 (|\mu|^2 + \kappa^2M^2) \\
- 16\kappa^2M^2|\mu|M_{RV}\sin(2\theta_k)\cos \Phi_\mu \\
- 4M^2|\mu||M_L|\sin(2\theta_k)\cos(\Phi_\mu + \Phi_L), \]

\[ c = 16|\mu|^4|M_L|^2 + 4(1 + 4\kappa^2)^2M^4|\mu|^2\sin^2(2\theta_k) \\
+ 16\kappa^2M^2|M_L|^2 (2|\mu|^2 + \kappa^2M^2) \\
+ M_{RV}^2(M^4 + 8|\mu|^2(M^2 + |M_L|^2) + 16|\mu|^4) \\
- 4M^2|\mu||M_L|(4|\mu|^2 + M_{RV}^2)\cos(\Phi_L + \Phi_\mu)\sin(2\theta_k) \\
+ 8\kappa^2M^2M_{RV}[M^2|M_L|\cos \Phi_L \\
- 2|\mu|(4|\mu|^2 + |M_L|^2)\cos \Phi_\mu \sin(2\theta_k)], \]

and

\[ d = 64\kappa^4M^4|\mu|^2|M_L|^2\sin^2(2\theta_k) \\
+ 32\kappa^2M^2|\mu|^2|M_L|M_{RV}\sin(2\theta_k) \\
\times (M^2\cos \Phi_L\sin(2\theta_k) - 2|\mu||M_L|\cos \Phi_\mu) \\
+ 4M^2|\mu|^2M_{RV}^2\sin(2\theta_k) \\
\times (M^2\sin(2\theta_k) - 4|\mu||M_L|\cos(\Phi_L + \Phi_\mu)) \\
+ 16|\mu|^4|M_L|^2M_{RV}^2. \]

Solving Eq. (3.24), we get the exact analytic formulas for the neutralino masses

\[ m_{\tilde{\chi}^0_1}^2, m_{\tilde{\chi}^0_2}^2 = \frac{a}{4} - \frac{\alpha}{2} + \frac{1}{2} \sqrt{\beta - \omega - \frac{\lambda}{4\alpha}}, \]  
(3.25)

\[ m_{\tilde{\chi}^0_3}^2, m_{\tilde{\chi}^0_4}^2 = \frac{a}{4} + \frac{\alpha}{2} + \frac{1}{2} \sqrt{\beta - \omega + \frac{\lambda}{4\alpha}}, \]  
(3.26)
Table 1: Input parameters for scenarios $S_{c_1}$ and $S_{c_2}$. All mass quantities are given in GeV.

| Scenario | $M_R$ | $M_L$ | $k_u$ | $\tan \theta_k$ |
|----------|-------|-------|-------|-----------------|
| $S_{c_1}$ | 300   | 50    | 92.75 | 1.6             |
|          |       |       |       | 4.0             |
| $S_{c_2}$ | 1000  | 250   | 92.75 | 1.6             |
|          |       |       |       | 4.0             |

where

\[
\alpha = \sqrt{\frac{\beta}{2} + \varpi}, \\
\varpi = \frac{\epsilon}{3} + \frac{(2\dot{\gamma})}{3 \epsilon}, \\
\epsilon = (\delta + \sqrt{\delta^2 - 4\gamma^2})^{\frac{1}{2}}, \\
\beta = \frac{a^2}{2} - \frac{4b}{3}, \\
\lambda = a^3 - 4ab + 8c, \\
\gamma = b^2 - 3ac + 12d, \\
\delta = 2b^3 - 9abc + 27c^2 + 27a^2d - 72bd.
\]

3.2 Neutralino masses, numerical results

Let us consider the CP-conserving scenarios $S_{c_1}$ and $S_{c_2}$ described in Tab. 1. This scenarios are similar to the ones studied in Ref. [12] where they have been used to compare the predicted results for the neutralino masses in the left-right SUSY model and the MSSM. Thus, for both scenarios, we consider the coupling constants values $g_R \approx g_L \approx g_V = 0.65$, the gaugino parameters $M_L >> M_V \approx 0.0\text{GeV}$ and the mixing phases $\Phi_L = \Phi_\mu = 0$. Figures 1 and 2 show the behavior of the physical neutralino masses $m_{\tilde{\chi}_i^0}$, $i = 1, \ldots, 3$, versus $\mu$, computed from Eq. (3.26), for the inputs of scenarios $S_{c_1}$ and $S_{c_2}$ with $\tan \theta_k = 1.6$, respectively. Notice that the values of the neutralino mass $m_{\tilde{\chi}_1^0}$ are so big that they cannot be seen. Both Figures reproduce accurately the results of Ref. [12]. We observe the correct size ordering of the neutralino masses, such as required by Eq. (3.21). Also, in both scenarios, we find that for values of $|\mu| \sim 200\text{GeV}$, the neutralino masses $m_{\tilde{\chi}_1^0}$
| Scenario | $|\mu|$ | $M_R$ | $M_L$ | $k_u$ | $\tan \theta_k$ |
|----------|--------|-------|-------|-------|----------------|
| $Snc_1$  | 20     | 248   | 50    | 92.75 | 4.0            |

Table 2: Input parameters for scenario $Snc_1$. All mass quantities are given in GeV.

Figure 1: Neutralino masses $m_{\tilde{\chi}_i^0}$, $i = 1, \ldots, 3$, as functions of $\mu$ for scenario $Sc_1$, assuming $\tan \theta_k = 1.6$.

are approximately $M_L$ and for large values of $|\mu|$, the masses of the neutralinos $m_{\tilde{\chi}_i^0}$, $i = 3, 4$, are heavier than $M_R$. The same analysis is true in the case of scenarios $Sc_1$ and $Sc_2$, where $\tan \theta_k = 4.0$, as we can observe in Figs. 3 and 4. However, comparing Figs. 1 and 3 corresponding to scenarios $Sc_1$ with different values of $\tan \theta_k$, i.e., $\tan \theta_k = 1.6$ and $\tan \theta_k = 4.0$, respectively, we find that for small values of $|\mu|$, the variation of the neutralino masses with respect to $\mu$ in Fig. 3 are smoother than in Fig. 1. This is an important fact to consider when we will study the inverse problem, that is, the determination of the fundamental parameters based on the knowledge of the physical neutralino masses.

Let us now to study the behavior of the neutralino masses $m_{\tilde{\chi}_i^0}$, $i = 1, 2$, respect to the variation of $\Phi_\mu$ and $\Phi_L$. Let us consider two possible CP-violating scenarios $Snc_1$ described in Tab. 2 characterized by two different values of the Higgsino mass parameter, $|\mu| = 20\text{GeV}$ and $|\mu| = 248\text{GeV}$. Figures 5 and 6 show the behavior of the neutralino masses $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_2^0}$, respectively, as a function of $\Phi_\mu$ and $\Phi_L$ for input parameters of scenario $Snc_1$ with $|\mu| = 20\text{GeV}$. Comparing these figures we observe that the variation of the values of $m_{\tilde{\chi}_1^0}$ is bigger than the one of $m_{\tilde{\chi}_2^0}$. Superposing these figures, the corresponding surfaces do not overlap, that is, the size ordering
Figure 2: Neutralino masses $m_{\tilde{\chi}_i^0}$, $i = 1, \ldots, 3$, as functions of $\mu$ for scenario $Sc_2$, assuming $\tan \theta_k = 1.6$.

Figure 3: Neutralino masses $m_{\tilde{\chi}_i^0}$, $i = 1, \ldots, 4$, as functions of $\mu$ for scenario $Sc_1$, assuming $\tan \theta_k = 4.0$.

(see Eq. (3.21)) of the masses $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_2^0}$, is conserved even if in the CP-violating case. Figures 7 and 8 show the behavior of the neutralino masses $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_2^0}$, respectively, as a function of $\Phi_\mu$ and $\Phi_L$ for input parameters of scenario $Snc_1$ with $|\mu| = 248 GeV$. The same considerations as in the previous analysis done for Figs. 5 and 6 are valid in this case. However, we observe that the energy gap between the surfaces in Figs. 7 and 8 is greater than in the case of surfaces of Figs. 5 and 6.

3.3 The eigenvectors forming the matrix $V$

We have found useful to finish this section with the computation of the matrix $V$. A more explicit form of this matrix will allow us to prove some important relations in the next section. The diagonalizing matrix $V$ can be obtained by computing the eigenvectors corresponding to the eigenvalues given in Eq. (3.26). Indeed, by inserting a generic eigenvalue $m_{\tilde{\chi}_j^0}$, into Eq. (3.23)
Figure 4: Neutralino masses $m_{\tilde{\chi}_i^0}$, $i = 1, \ldots, 4$, as functions of $\mu$ for scenario $Sc_2$, assuming $\tan \theta_k = 4.0$.

Figure 5: Neutralino mass $m_{\tilde{\chi}_1^0}$, as a function of $\Phi_\mu$ and $\Phi_L$ for input parameters according to scenario $Snc_1$ with $|\mu| = 20\text{GeV}$. 
Figure 6: Neutralino mass $m_{\tilde{\chi}_1^0}$, as a function of $\Phi_\mu$ and $\Phi_L$ for input parameters according to scenario $S_{nc1}$ with $|\mu| = 20$ GeV.

Figure 7: Neutralino mass $m_{\tilde{\chi}_1^0}$, as a function of $\Phi_\mu$ and $\Phi_L$ for input parameters according to scenario $S_{nc1}$ with $|\mu| = 248$ GeV.
and dividing each one of these equations by $V_{1j}$, where it is assumed that $V_{1j} \neq 0$, we get

$$
H_{12} \frac{V_{2j}}{V_{1j}} + H_{13} \frac{V_{3j}}{V_{1j}} + H_{14} \frac{V_{4j}}{V_{1j}} - m_{\tilde{\chi}_j^0}^2 = -H_{11},
$$

$$(H_{22} - m_{\tilde{\chi}_j^0}^2) \frac{V_{2j}}{V_{1j}} + H_{23} \frac{V_{3j}}{V_{1j}} + H_{24} \frac{V_{4j}}{V_{1j}} = -H_{21},
$$

$$
H_{32} \frac{V_{2j}}{V_{1j}} + (H_{33} - m_{\tilde{\chi}_j^0}^2) \frac{V_{3j}}{V_{1j}} + H_{34} \frac{V_{4j}}{V_{1j}} = -H_{31},
$$

$$
H_{42} \frac{V_{2j}}{V_{1j}} + H_{43} \frac{V_{3j}}{V_{1j}} + (H_{44} - m_{\tilde{\chi}_j^0}^2) \frac{V_{4j}}{V_{1j}} = -H_{41}.
$$

(3.27)

Solving this system of equations, and taking into account the relation

$$
|V_{1j}|^2 + |V_{2j}|^2 + |V_{3j}|^2 + |V_{4j}|^2 = 1,
$$

(3.28)

it yields the $V_{ij}$ matrix’s component

$$
V_{ij} = \frac{\Delta_{ij}}{\Delta_{1j}} \frac{|\Delta_{1j}| e^{i\theta_j}}{\sqrt{|\Delta_{1j}|^2 + |\Delta_{2j}|^2 + |\Delta_{3j}|^2 + |\Delta_{4j}|^2}}.
$$

(3.29)

when i=1,...,4. Here, the $\theta_j$’s are arbitrary phases, related to the CP eigenphases, which will be fixed by the requirement that $V$ satisfies Eq. (3.17), as we will see in the next section,

$$
\Delta_{1j} = 
\begin{bmatrix}
H_{22} - m_{\tilde{\chi}_j^0}^2 & H_{23} & H_{24} \\
H_{32} & H_{33} - m_{\tilde{\chi}_j^0}^2 & H_{34} \\
H_{42} & H_{43} & H_{44} - m_{\tilde{\chi}_j^0}^2
\end{bmatrix}
$$

(3.30)

and $\Delta_{ij}$, $i = 2, 3, 4$, is formed from $\Delta_{1j}$ by substituting the $(i-1)$th column by $\begin{pmatrix} -H_{21} \\ -H_{31} \\ -H_{41} \end{pmatrix}$. 

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4 The neutralino projectors, pseudoprojectors and CP eigenphases

To describe the neutralino observables we can use the projector formalism [1, 2]. The neutralino projector matrices can be defined as [13]

\[ P_j = P_j^\dagger = V E_j V^{-1}, \]  

so that

\[ P_{j\alpha\beta} = V_{\alpha j} V^*_{\beta j}. \]  

These projectors satisfy the relations

\[ P_i P_j = P_j \delta_{ij}, \quad Tr P_j = 1, \quad \sum_{j=1}^4 P_j = 1, \]  

where \((i, j) = (1 - 4)\) describe the neutralino mass-eigenstate indices. Notice that from Eqs. (3.18) and (4.31) it is possible to write

\[ N^\dagger N = \sum_{j=1}^4 m_{\tilde{\chi}_j}^2 P_j. \]

As in the case of the study of the neutralino projector formalism for complex supersymmetry parameters based on the MSSM [2], here only the projectors are not sufficient to describe the physical observables. For a complete description of physical observables it is also necessary to know the so-called pseudoprojector matrices and CP eigenphases. In the following we implement a method, based on the explicit knowledge of the diagonalizing matrix \(V\) to obtain these quantities and demonstrate some of their properties.

4.1 Reduced projectors

By inserting (3.29) into (4.32), we get

\[ P_{j\alpha\beta} = \frac{p_{j\alpha} P_{j\beta}^*}{1 + |p_{j2}|^2 + |p_{j3}|^2 + |p_{j4}|^2}, \]  

where we define the reduced projectors

\[ p_{j\alpha} \equiv \frac{\Delta_{\tilde{\chi}_j}^{\alpha}}{\Delta^{1j}_{\tilde{\chi}_j}}. \]  

Notice that the expression given in (4.36) is a new version of the reduced projector formula [2]. Indeed, from this last equation, it is clear that \(p_{j1} = 1\). Moreover, from Eq. (4.35) we deduce

\[ P_{j11} = \frac{1}{1 + |p_{j2}|^2 + |p_{j3}|^2 + |p_{j4}|^2}. \]
Thus, inserting this last result into (4.35), we prove the ansatz used in [2]

\[ P_{j\alpha\beta} = P_{j11} p^*_{j\alpha} P_{j\beta}. \] (4.38)

On the other hand, using Eqs. (4.36) and (4.37), we can write the matrix elements of the diagonalizing matrix \( V \) given in Eq. (3.29) in terms of the reduced projectors, that is

\[ V_{\alpha j} = \sqrt{\frac{P_{j11}}{\eta_j}} p^*_{j\alpha}, \] (4.39)

where \( \eta_j \equiv e^{-2i\theta_j} \) stands for the CP eigenphases. As we will see below, this last equation allow us to express the L-R SUSY parameters in terms of the reduced projectors and the eigenphases.

An useful property verified by the reduced projectors \( p_{j\alpha} \) is

\[ P_{j11} \sum_{\beta=1}^{4} p_{j\beta} p^*_{j\beta} = \delta_{ij}, \] (4.40)

which can be directly deduced from Eq. (4.39), taking in account the unitarity of \( V \).

Let us now to define other important matrices. From Eq. (3.17), we can write

\[ N = \sum_{j=1}^{4} m_{\tilde{\chi}_0} V^* E_j V^\dagger = \sum_{j=1}^{4} m_{\tilde{\chi}_0} \bar{P}_j, \] (4.41)

where

\[ \bar{P}_j \equiv V^* E_j V^\dagger \] (4.42)

are the so-called pseudoprojectors [2]. Using Eq. (4.39) and the definition of \( E_j \), the matrix elements of these pseudoprojectors can easily be written in the form

\[ \bar{P}_{j\alpha\beta} = V^*_{\alpha j} V^*_{\beta j} = P_{j11} p_{j\alpha} p^*_{j\beta} \eta_j. \] (4.43)

From this last equation it is clear that \( \bar{P}_j \) is a symmetric matrix, that is \( \bar{P}_j^T = \bar{P}_j \).

Using Eq. (4.43), taking again into account the unitarity of \( V \), and the definition (4.32), we have

\[ (\bar{P}_j^* \bar{P}_k)_{\alpha\beta} = \sum_{\rho=1}^{4} \bar{P}_{j\alpha\rho}^* \bar{P}_{k\rho\beta} \]
\[ = \sum_{\rho=1}^{4} V_{\alpha j} V^*_{\rho j} V^*_{\rho k} V_{\beta k} = \delta_{jk} V_{\alpha j} V^*_{\beta j} = \delta_{jk} P_{j\alpha\beta}, \]

that is, the pseudoprojectors satisfy

\[ \bar{P}_j^* \bar{P}_k = \delta_{jk} P_j. \] (4.44)
In the same way, we can show that

$$\bar{P}_j^* P_k = P_k^t \bar{P}_j = \delta_{jk} \bar{P}_j. \quad (4.45)$$

As we have mentioned in the previous section, the eigenphases $\eta_j$ must be chosen in such a way that the diagonalizing matrix $V$ satisfies Eq. (3.17) or equivalently Eq. (4.41). Inserting Eq. (4.43) into Eq. (4.41) and using the property Eq. (4.40) we get

$$\eta_j m_{\tilde{\chi}^0_j} = \sum_{\alpha=1}^4 N_{\alpha\beta} \frac{p^*_{j\alpha}}{p_{j\beta}} \sum_{\alpha=1}^4 N_{\alpha\beta} \frac{\Delta_{\alpha j}}{\Delta_{\beta j}}. \quad (4.46)$$

This last Equation represents, for fixed $j$, four equivalent relations serving to determine the fundamental parameters of the model, namely $M_L, \mu, M_R, M_V$ and $\tan \theta_k$, in terms of the reduced projectors, the physical neutralino masses, the eigenphases and the L-R SUSY coupling constants. We notice that, starting from Eq. (3.17) and using Eq. (4.39), a more symmetric structure for the eigenphases $\eta_j$ can be reached, that is

$$\eta_j m_{\tilde{\chi}^0_j} = P_{j11} \sum_{\alpha,\beta=1}^4 p^*_{j\alpha} N_{\alpha\beta} p^*_{j\beta}. \quad (4.47)$$

This relation can also be constructed directly from the more fundamental Eq. (4.46), by means of property Eq. (4.40).

### 4.2 Explicit form of the reduced projectors

According to Eq. (4.36), to obtain the explicit form of the reduced projectors in terms of the fundamental parameters of the theory only we need to know the explicit form of quantities $\Delta_{\alpha j}^*$. For fixed $j$, they are given by

$$\Delta_{1j}^* = -4\kappa^2 (1 + 4\kappa^2) M^4 (m_{\tilde{\chi}^0_j}^2 - 4|\mu|^2 \sin^2(2\theta_k))$$

$$+ M^2 (m_{\tilde{\chi}^0_j}^2 - 4|\mu|^2) [m_{\tilde{\chi}^0_j}^2 - M_{RV}^2]$$

$$+ 8\kappa^2 m_{\tilde{\chi}^0_j}^2 + 16\kappa^2 M_{RV} |\mu| \cos \Phi_\mu \sin(2\theta_k)]$$

$$- (m_{\tilde{\chi}^0_j}^2 - M_{RV}^2) (m_{\tilde{\chi}^0_j}^2 - 4|\mu|^2)^2, \quad (4.48)$$

$$\Delta_{2j}^* = -2\kappa (1 + 4\kappa^2) M^4 [m_{\tilde{\chi}^0_j}^2 - 4|\mu|^2 \sin^2(2\theta_k)]$$

$$+ 2\kappa M^2 [m_{\tilde{\chi}^0_j}^2 - 4|\mu|^2]$$

$$\times \left\{ m_{\tilde{\chi}^0_j}^2 + M_{RV} |M_L| e^{-i\Phi_L} + 2|\mu| \sin(2\theta_k) \right\} \times [M_L e^{-i(\Phi_L + \Phi_\mu)} + M_{RV} e^{i\Phi_\mu} \right\}, \quad (4.49)$$
\[ \Delta_{3j}^* = 2M^3 \left\{ |\mu| \cos \theta_k \cos(2\theta_k) \left[ 4\kappa^2 M_RV |M_L| e^{i(\Phi_{\mu} - \Phi_L)} \right] \\
- (m_{\tilde{\chi}_j}^2 + 1 + 4\kappa^2 - M_{\tilde{\chi}_j}^2) e^{i\Phi_{\mu}} \right\}^2 - 2m_{\tilde{\chi}_j}^2 \sin \theta_k \\
\times (m_{\tilde{\chi}_j}^2 - 8|\mu|^2 \cos^2 \theta_k) \left( M_{\tilde{\chi}_j}^2 - |M_L| e^{-i\Phi_L} \right) \\
+ M(m_{\tilde{\chi}_j}^2 - 4|\mu|^2) (m_{\tilde{\chi}_j}^2 - M_{\tilde{\chi}_j}^2) \\
\times (|M_L| e^{-i\Phi_L} \sin \theta_k + 2|\mu| e^{i\Phi_{\mu}} \cos \theta_k) \right\} \] (4.50)

\[ \Delta_{4j}^* = 2M^3 \left\{ |\mu| \sin \theta_k \cos(2\theta_k) \left[ 4\kappa^2 M_RV |M_L| e^{i(\Phi_{\mu} - \Phi_L)} \right] \\
- (m_{\tilde{\chi}_j}^2 + 1 + 4\kappa^2 - M_{\tilde{\chi}_j}^2) e^{i\Phi_{\mu}} \right\}^2 - 2m_{\tilde{\chi}_j}^2 \cos \theta_k \\
\times (m_{\tilde{\chi}_j}^2 - 8|\mu|^2 \sin^2 \theta_k) \left( M_{\tilde{\chi}_j}^2 - |M_L| e^{-i\Phi_L} \right) \\
- M(m_{\tilde{\chi}_j}^2 - 4|\mu|^2) (m_{\tilde{\chi}_j}^2 - M_{\tilde{\chi}_j}^2) \\
\times (|M_L| e^{-i\Phi_L} \cos \theta_k + 2|\mu| e^{i\Phi_{\mu}} \sin \theta_k). \right\} \] (4.51)

The formulas (4.48, 4.51) allow us to express, through the reduced projectors, all the essential quantities of the model in terms of the original parameters.

### 4.3 Consistence with the Jarlskog’s formula

Using the projector properties (4.33) and some ones associated to the coefficients of the characteristic polynomial (4.24), we can write the projectors \( P_j \) in terms of the neutralino masses and the \( H \) matrix, in the Jarlskog’s forme [7]:

\[
\begin{align*}
P_1 &= \frac{(m_{\tilde{\chi}_4}^2 - H)(m_{\tilde{\chi}_3}^2 - H)(m_{\tilde{\chi}_0}^2 - H)}{(m_{\tilde{\chi}_4}^2 - m_{\tilde{\chi}_3}^2)(m_{\tilde{\chi}_3}^2 - m_{\tilde{\chi}_0}^2)(m_{\tilde{\chi}_0}^2 - m_{\tilde{\chi}_4}^2)}, \\
P_2 &= \frac{(m_{\tilde{\chi}_4}^2 - m_{\tilde{\chi}_3}^2)(m_{\tilde{\chi}_3}^2 - m_{\tilde{\chi}_0}^2)(m_{\tilde{\chi}_0}^2 - m_{\tilde{\chi}_4}^2)}{(m_{\tilde{\chi}_4}^2 - H)(m_{\tilde{\chi}_3}^2 - H)(m_{\tilde{\chi}_0}^2 - H)}, \\
P_3 &= \frac{(m_{\tilde{\chi}_4}^2 - m_{\tilde{\chi}_3}^2)(m_{\tilde{\chi}_3}^2 - m_{\tilde{\chi}_0}^2)(m_{\tilde{\chi}_0}^2 - m_{\tilde{\chi}_4}^2)}{(m_{\tilde{\chi}_4}^2 - H)(m_{\tilde{\chi}_3}^2 - H)(m_{\tilde{\chi}_0}^2 - H)}, \\
P_4 &= \frac{(m_{\tilde{\chi}_4}^2 - m_{\tilde{\chi}_3}^2)(m_{\tilde{\chi}_3}^2 - m_{\tilde{\chi}_0}^2)(m_{\tilde{\chi}_0}^2 - m_{\tilde{\chi}_4}^2)}{(m_{\tilde{\chi}_4}^2 - m_{\tilde{\chi}_3}^2)(m_{\tilde{\chi}_3}^2 - m_{\tilde{\chi}_0}^2)(m_{\tilde{\chi}_0}^2 - m_{\tilde{\chi}_4}^2)}. \\
\end{align*}
\] (4.52)

A more useful expression for these projectors is obtained if we define

\[
P_j = \frac{\tilde{P}_j}{\Delta_j}, \quad (4.53)
\]
where
\[ \tilde{\Delta}_j = -3m_{\tilde{\chi}_j}^8 + 2a m_{\tilde{\chi}_j}^6 - b m_{\tilde{\chi}_j}^4 + d. \] (4.54)

Indeed, by performing some algebraic manipulations we get
\[
\tilde{P}_{j\alpha\beta} = -m_{\tilde{\chi}_j}^6 H_{\alpha\beta} + m_{\tilde{\chi}_j}^4 (a H_{\alpha\beta} - H_{\alpha\beta}^2)
+ m_{\tilde{\chi}_j}^2 (a H_{\alpha\beta}^2 - b H_{\alpha\beta} - H_{\alpha\beta}^3) + d \delta_{\alpha\beta}.
\] (4.55)

Now, combining Eqs. (4.38) and (4.53), we deduce the expression
\[
p_{j\alpha} = P_{j1\alpha} = \tilde{P}_{j1\alpha} = \tilde{P}_{j11},
\] (4.56)
which can also be considered as a definition for the reduced projectors.

Equations (4.36) and (4.56) are equivalent expressions for the reduced projectors when we substitute into them the exact analytical masses \(m_{\tilde{\chi}_j}\) given in (3.26). Thus, combining these equations and comparing the expressions (4.49-4.51) with the corresponding \(\tilde{P}_{j1\beta}, \beta = 2, 3, 4\), computed from Eq. (4.55), we can show that
\[
\tilde{P}_{j1\alpha} = m_{\tilde{\chi}_j}^2 \Delta^*_{\alpha j}, \quad \forall \alpha = 1, \ldots, 4,
\] (4.57)
with
\[
\tilde{P}_{j11} = M^4[m_{\tilde{\chi}_j}^2 - 4|\mu|^2 \sin^2(2\theta_k)]m_{\tilde{\chi}_j}^2(1 + 4\kappa^2)
- 16\kappa^4|M_L|^2 - M_{RV}^2 - 8\kappa^2|M_L|M_{RV}\cos\Phi_L
- M^2(m_{\tilde{\chi}_j}^2 - 4|\mu|^2)\{2|\mu||M_L|\sin(2\theta_k)
\times [2(m_{\tilde{\chi}_j}^2 - M_{RV}^2)\cos(\Phi_L + \Phi_\mu)
- 8|M_L|M_{RV}\kappa^2 \cos\Phi_\mu]
+ m_{\tilde{\chi}_j}^2[(m_{\tilde{\chi}_j}^2 - M_{RV}^2) - 8\kappa^2|M_L|^2]\}
- |M_L|^2(m_{\tilde{\chi}_j}^2 - M_{RV}^2)(m_{\tilde{\chi}_j}^2 - 4|\mu|^2)^2.
\] (4.58)

Indeed, Eqs. (4.57), for \(\alpha = 1, 2, 3\), constitute an identity whereas for \(\alpha = 1\) it constitutes an useful equivalence, as we will show in the next section.

5 General disentangle formula of \(M_L\) in terms of the eigenphases

From Eq. (4.46), choosing \(\beta = 1\) and using (2.14), we get
\[
\eta_j m_{\tilde{\chi}_j} = M_L - M \frac{\sin \theta_k \Delta_{\beta j} - \cos \theta_k \Delta_{\alpha j}}{\Delta^*_{\alpha j}}.
\] (5.59)
Inserting (4.50) and (4.51) into (5.59) and solving a linear algebraic equation for the parameters of the CP-conserving scenario, we get

\[
M_L = \frac{\Delta^*_i m_{\chi^0_j}}{D_j} \eta_j + \frac{2M^2}{D_j} \left\{ |\mu| e^{-i\phi_\mu} \sin(2\theta_k) \times (m^2_{\chi^0_j} - M^2_{RV})(m^2_{\chi^0_j} - 4|\mu|^2) \right. \\
+ 2\kappa^2 M^2 M_{RV}[m^2_{\chi^0_j} - 4|\mu|^2 \sin^2(2\theta_k)] \bigg\} = A_j \eta_j + B_j,
\]

where

\[
A_j = \frac{\Delta^*_i m_{\chi^0_j}}{D_j} = -\frac{m_{\chi^0_j}}{D_j} \left\{ (m^2_{\chi^0_j} - M^2_{RV})(m^2_{\chi^0_j} - 4|\mu|^2)^2 \right. \\
+ 4\kappa^2 (1 + 4\kappa^2) M^4 |m^2_{\chi^0_j} - 4|\mu|^2 \sin^2(2\theta_k)| \\
- M^2 (m^2_{\chi^0_j} - 4|\mu|^2)[8\kappa^2 m^2_{\chi^0_j} + (m^2_{\chi^0_j} - M^2_{RV})] \\
+ 16\kappa^2 |\mu| M_{RV} \cos \Phi_\mu \sin(2\theta_k)] \bigg\},
\]

\[
B_j = \frac{2M^2}{D_j} \left\{ |\mu| e^{-i\phi_\mu} \sin(2\theta_k) \times (m^2_{\chi^0_j} - M^2_{RV})(m^2_{\chi^0_j} - 4|\mu|^2) \right. \\
+ 2\kappa^2 M^2 M_{RV}[m^2_{\chi^0_j} - 4|\mu|^2 \sin^2(2\theta_k)] \bigg\}
\]

and

\[
D_j = -\left\{ (m^2_{\chi^0_j} - M^2_{RV})(m^2_{\chi^0_j} - 4|\mu|^2)^2 - 8\kappa^2 M^2 \right. \\
\times (m^2_{\chi^0_j} - 4|\mu|^2)[m^2_{\chi^0_j} + 2|\mu| M_{RV} \cos \Phi_\mu \sin(2\theta_k)] \\
+ 16\kappa^4 M^4 [m^2_{\chi^0_j} - 4|\mu|^2 \sin^2(2\theta_k)] \bigg\}.
\]

Equation (5.60) allow us to determine the behavior of \(|M_L|\) and \(\Phi_L\) in terms of the eigenphases \(\eta_j\) and the physical masses \(m_{\chi^0_j}\), when the rest of fundamental parameters are fixed. We notice that this equation has been obtained without use the Jarlskog’s projector formula (4.52) or its equivalent (4.55). The method used to obtain it is direct and it is based essentially on the fact that \(\Delta_{ij}\) is independent of \(|M_L|\) and of \(\Phi_L\).

Figure 9 shows the behavior of \(|M_L|\) as a function of the neutralino masses \(m_{\chi^0_j}\), for input parameters of the CP-conserving scenario \(S\overline{c}_{3u}\), given in Tab. 3 (as before, we assume \(g_L = g_R = g_V = 0.65\), and \(k_u = 92.75\)). We observe that for small values of the neutralino masses, i.e., for masses of order 200 GeV approximately, the size of \(|M_L|\) becomes the same in both cases, the
| Scenario | $|\mu|$ | $M_R$ | $M_V$ | $\tan \theta_k$ | $\eta_j$ |
|-----------|--------|-------|-------|-----------------|---------|
| $Sc_{3a}$ | 248    | 500   | 50    | 30              | 1       |
|           |        |       |       |                 | -1      |
| $Sc_{3b}$ | 248    | 500   | 500   | 30              | 1       |
|           |        |       |       |                 | -1      |
| $Sc_{3c}$ | 500    | 500   | 50    | 30              | 1       |
|           |        |       |       |                 | -1      |
| $Sc_{3d}$ | 248    | 500   | 50    | 10              | 1       |
|           |        |       |       |                 | -1      |

Table 3: Input parameters for scenarios $Sc_{3a}, \ldots, Sc_{3d}$. All mass quantities are in GeV.

Figure 9: Graph of $|M_L|$, using the general formula (5.60), for inputs of scenario $Sc_{3a}$, with $\eta_j = 1$ (solid line) and $\eta_j = -1$ (dashed line), as a function of the physical neutralino masses $m_{\tilde{\chi}_j^0}$.
Figure 10: Graph of $|M_L|$, using the general formula (5.60), for inputs of scenario $Sc_3b$, with $\eta_j = 1$ (solid line) and $\eta_j = -1$ (dashed line), as a function of the physical neutralino masses $m_{\tilde{\chi}_j^0}$.

Figure 11: Graph of $|M_L|$, using the general formula (5.60), for inputs of scenario $Sc_3c$, with $\eta_j = 1$ (solid line) and $\eta_j = -1$ (dashed line), as a function of the physical neutralino masses $m_{\tilde{\chi}_j^0}$.

Figure 12: Graph of $|M_L|$ as a function of the physical neutralino masses $m_{\tilde{\chi}_j^0}$, using the general formula (5.60), for inputs of scenario $Sc_3d$. The curves are: $\tan \theta_k = 10, \eta_j = 1$ (light solid); $\tan \theta_k = 10, \eta_j = -1$ (light dashed); $\tan \theta_k = 50, \eta_j = 1$ (heavy solid) and $\tan \theta_k = 50, \eta_j = -1$ (heavy dashed).
scenario $S_{c_3a}$ with $\eta_j = 1$ and the scenario $S_{c_3a}$ with $\eta_j = -1$. Let us now consider the scenario $S_{c_3b}$, which is the same as the scenario $S_{c_3a}$, except for the value of $M_V$ which have been increased from 50 GeV to 500 GeV. In this case, the common value of $|M_L|$ in both scenarios, i.e., $S_{c_3b}$ with $\eta = \pm 1$, is found to be $|M_L| \approx 300$ GeV, in the region of small physical neutralino masses of the order of 300 GeV, as we can see from Fig 10. Figure 11 shows the behavior of $|M_L|$ as a function of the neutralino masses $m_{\tilde{\chi}_j^0}$, for input parameters of the CP-conserving scenario $S_{c_3c}$, given in Tab. 3. This scenario differs from the scenario $S_{c_3a}$ in the value of $\mu$ which have now been taken $|\mu| = 500$ GeV. We observe that the curves corresponding to the input parameters of scenario $S_{c_3c}$ with different eigenphases values, i.e., $\eta_j = \pm 1$, intersect when $m_{\tilde{\chi}_j^0} \approx 412.17$ GeV, giving the common value of $|M_L| \approx 412.93$ GeV, which is bigger than the corresponding common values of $|M_L|$ computed in the previous scenarios. On the other hand, if we compare the curves representing the behavior of $|M_L|$ as a function of the physical neutralino masses for different values of the parameter $\tan \theta_k$, we don’t observe important differences between them when the values of this last parameter is chosen in the range 30-50, however, if we compare the mentioned curves for values of $\tan \theta_k$ chosen, for instance, in the range 10-30 or 10-50, we observe that for small neutralino masses $m_{\tilde{\chi}_j^0}$, the values of $|M_L|$ approach from the right to the given value of $m_{\tilde{\chi}_j^0}$, and this approach is more significative for big values of $\tan \theta_k$ than for small ones when $\eta_j = 1$ and viceversa, this approach is more significative for small values of $\tan \theta_k$ than for big ones when $\eta_j = -1$. This means that the value of the light neutralino mass which provides the value of $|M_L|$ which is independent of the eigenphases $\eta_j = \pm 1$, increases when $\tan \theta_k$ augments. The above mentioned behavior of the parameters is verified by seen the plots in Fig. 12, where we plot $|M_L|$ versus $m_{\tilde{\chi}_j^0}$, for inputs of scenario $S_{c_3d}$ with $\tan \theta_k = 10$ and $\tan \theta_k = 50$.

5.1 An alternative way to obtain $|M_L|$

When $\alpha = 1$, Eq. (4.57) combined with Eqs. (4.48) and (4.58), allow us to express the norm of $M_L$ in terms of the rest of the fundamental parameters $\Phi_L$, $|\mu|$, $\Phi_\mu$, $\tan \theta_k$ and $M_{RV}$ and the physical masses $m_{\tilde{\chi}_j^0}$. Indeed, inserting (4.48) and (4.58) into (4.57) and solving a quadratic algebraic equation for $|M_L|$, we get

$$|M_L| = \frac{-B_j \pm \sqrt{B_j^2 - 4D_j(C_j - m_{\tilde{\chi}_j^0}^2 \Delta_{1j}^*)}}{2D_j},$$

(5.64)
Table 4: Input parameters for scenario $S_{C4}$. All mass quantities are in GeV.

| Scenario | $|\mu|$ | $M_R$ | $M_V$ | $\tan \theta_k$ | $\Phi_L$ |
|----------|--------|-------|-------|----------------|----------|
| $S_{C4}$ | 248 | 500 | 50 | 30 | 0 | $\pi$ |

Table 4: Input parameters for scenario $S_{C4}$. All mass quantities are in GeV.

where

$$B_j = -4M^2 \{ |\mu| (m_{\tilde{\chi}_j^0}^2 - M_{RV}^2) (m_{\tilde{\chi}_j^0}^2 - 4|\mu|^2) \}
\times \cos(\Phi_L + \Phi_\mu) \sin(2\theta_k) + 2\kappa^2 M^2 M_{RV}$$
$$\times [m_{\tilde{\chi}_j^0}^2 - 4|\mu|^2 \sin^2(2\theta_k)] \cos (\Phi_L), \quad (5.65)$$

$$C_j = -M^2 m_{\tilde{\chi}_j^0}^2 (m_{\tilde{\chi}_j^0}^2 - M_{RV}^2) (m_{\tilde{\chi}_j^0}^2 - 4|\mu|^2) + M^4 [m_{\tilde{\chi}_j^0}^2 (1 + 4\kappa^2) - M_{RV}^2]$$
$$\times [m_{\tilde{\chi}_j^0}^2 - 4|\mu|^2 \sin^2(2\theta_k)] \quad (5.66)$$

and $D_j$ is given in Eq. (5.63).

The formula for $|M_L|$ given in Eq. (5.64), constitutes an alternative to the one given in Eq. (5.60), serving to study the behavior of $|M_L|$ as a function of the phase $\Phi_L$, the physical mass $m_{\tilde{\chi}_j^0}$, and the rest of the fundamental parameters. For instance, let us consider the possible CP-conserving scenario $S_{C4}$ given in Tab. 4. In this case, the behavior of $|M_L|$ in terms of one of the physical mass $m_{\tilde{\chi}_j^0}$, is shown in Figs. 13 and 14. It is clear that superposing Figures 13 and 14 we reconstruct Figure 9. Comparing these figures, we also observe that, in the CP-conserving case, when $\Phi_\mu = 0$, the eigenphase values $\eta_j = \pm 1$ correspond to the $M_L$ phase values $\Phi_L = \pm \pi$, respectively. The same considerations are valid when we take $\Phi_\mu = \pi$. That is, in the CP-conserving case, when all the parameters but $m_{\tilde{\chi}_j^0}$ are fixed, the choice of the two different values $\Phi_L = 0, \pi$ in Eq. (5.64), correspond to the choice of the two possible values of the eigenphases $\eta_j = 1, -1$, in Eq. (5.60).

### 6 Determining L-R SUSY parameters

In this section we investigate the behavior of $|M_L|$ and $\Phi_L$ when the eigenphases $\eta_j, j = 1, 2$, change. We concentrate in two possible scenarios $S_{nc2}$ and $S_{nc3}$, described in Table 5 for fixed
Figure 13: Graph of $|M_L|$ as a function of the physical mass $m_{\tilde{\chi}_0^i}$ for input parameters of scenario $S_{C4}$ with $\Phi_L = 0$. Here, according Eq. (5.64), the graphs with the $+$ and $-$ signs are represented in solid and dashed lines, respectively.

Figure 14: The same as in Fig. 13 but with $\Phi_L = \pi$. 
In the cases where we assume that the physical masses \(m_{\chi_0^0}\) and \(m_{\chi_2^0}\) as well as \(\mu\) and \(M_R\) are known. The \(M_V\) parameter would eventually be allowed to vary but in this case we assume that it has a fixed value in each one of the mentioned scenarios.

Figure 15 shows the behavior of the norm of \(M_L\), calculated from Eq. (5.60), as a function of the eigenphase \(\eta_1\), with input parameters of scenario Snc2 when \(m_{\chi_1^0} = 164.36\) GeV. This is a scenario, similar to the Sp1-type scenario used in [2] in the context of the MSSM, characterized by a big rate between \(k_u\) and \(k_d\), i.e., \(\tan \theta_k = 30\). For small values of \(\Phi_\mu\), we observe small differences among the plots of \(|M_L|\). For all the plots shown in this Figure, the mean value of \(|M_L|\) is 165.75 GeV approximately and the maximum amplitude difference of them is 0.6 GeV approximately.

Figures 16 and 17 show the dependence of the mixing phase \(\Phi_L\) and the relative phase \(\phi_L - \text{Arg}(\eta_1)\), respectively, calculated from Eq. (5.60), with respect to the eigenphase \(\eta_1\), in the Snc2 scenario with \(m_{\chi_1^0} = 164.36\) GeV. We observe, for all the cases \(\Phi_\mu = 0, \frac{\pi}{6}, \frac{\pi}{4}, \pi\), a linear dependence between \(\Phi_L\) and \(\text{Arg}(\eta_1)\). Thus, \(\Phi_L \approx \text{Arg}(\eta_1)\) when \(\text{Arg}(\eta_1) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\), \(\Phi_L \approx \pi + \text{Arg}(\eta_1)\) when \(\text{Arg}(\eta_1) \in (\pi, 2\pi)\) and \(\Phi_L \approx -\pi + \text{Arg}(\eta_1)\) when \(\text{Arg}(\eta_1) \in \left(\frac{\pi}{2}, \pi\right)\).

Figure 18 shows the behavior of the norm of \(M_L\), calculated from Eq. (5.60), as a function of the eigenphase \(\eta_2\), with input parameters of scenario Snc2 when \(m_{\chi_2^0} = 241.94\) GeV. In this case, the mean amplitude difference of \(|M_L|\) for the different plots is greater than before, 120 GeV approximately. However, in the region of small \(\Phi_\mu\) and \(\text{Arg}(\eta_j), j = 1, 2\), the results are closely similar, that is, the values of \(|M_L|\) concentrate in the range 150 GeV – 170 GeV. Figures 19 and 20 show the dependence of the mixing phase \(\Phi_L\) and of the relative phase \(\phi_L - \text{Arg}(\eta_2)\), respectively, calculated from Eq. (5.60), with respect to the eigenphase \(\eta_2\), for input parameters of scenario Snc2. As before, for all the cases \(\Phi_\mu = 0, \frac{\pi}{6}, \frac{\pi}{4}, \pi\), we observe the same linear dependence between \(\Phi_L\) and \(\text{Arg}(\eta_1)\) practically. However, the non exact linearity implies differences in the behavior of \(|\Phi_L|\) when it is measured with respect to \(\text{Arg}(\eta_2)\) and \(\Phi_L\), as we can see by comparing Figures 18 and 21.

Let us now assume other possible scenario, Snc3, described in Table 5, where \(|\mu| = 150\) GeV and \(\tan \theta_k = 4\). In this case, either the physical mass are given by \(m_{\chi_1^0} = 156.238\) GeV or \(m_{\chi_2^0} = 236.39\) GeV, the same as in the case of scenario Snc2, there exist practically a linear dependence between the eigenphase and the mixing phase \(\Phi_L\). Thus, a description of \(|M_L|\) in terms of \(\Phi_L\) is similar to the one based on the eigenphases. Figure 22 shows the behavior of \(M_L\) with respect to the phase \(\Phi_L\), computed from Eq. (5.64), according to Snc3 scenario with \(m_{\chi_1^0} = 156.238\) GeV.
### Table 5: Input parameters for scenarios $Snc_2$ and $Snc_3$

All mass quantities are in GeV and all angles are in radians.

| Scenario | $|\mu|$ | $\Phi_u$ | $m_{\tilde{\chi}_1^0}$ | $m_{\tilde{\chi}_2^0}$ | $M_R$ | $M_V$ | $\tan \theta_k$ |
|----------|--------|----------|----------------|----------------|-------|-------|----------------|
| $Snc_2$  | 248    | $0$      | 164.36         | 241.94         | 300   | 20    | 30            |
|          |        | $\pi/8$  |                |                |       |       |               |
|          |        | $\pi/6$  |                |                |       |       |               |
|          |        | $\pi$    |                |                |       |       |               |
| $Snc_3$  | 150    | $0$      | 156.24         | 236.79         | 300   | 50    | 4.0           |
|          |        | $\pi/8$  |                |                |       |       |               |
|          |        | $\pi/6$  |                |                |       |       |               |
|          |        | $\pi$    |                |                |       |       |               |

Figure 15: Norm of $M_L$ as a function of $\text{Arg}(\eta_1)$, according to scenario $Snc_2$, for $m_{\tilde{\chi}_1^0} = 164.36 \text{ GeV}$, $\Phi_\mu = 0$ (heavy solid), $\pi/8$ (light solid), $\pi/6$ (heavy dashed), $\pi$ (light dashed).
Figure 16: Mixing parameter $\Phi_L$ as a function of $\text{Arg}(\eta_1)$ with the same set of fixed parameters used in Fig. 15.

Figure 17: Relative difference between $\Phi_L$ and $\text{Arg}(\eta_1)$ as a function of $\text{Arg}(\eta_2)$, as observed from Fig. 16.

Figure 18: Norm of $|M_L|$ as a function of $\eta_2$ in the case of scenario $Snc_2$, with $m_{\tilde{\chi}_2^0} = 241.94$ GeV, $\Phi_\mu = 0$ (heavy solid), $\pi/8$ (light solid), $\pi/6$ (heavy dashed), $\pi$ (light dashed).
Figure 19: Mixing parameter $\Phi_L$ as a function of $\text{Arg}(\eta_2)$ with the same set of fixed parameters used in Fig. 18.

Figure 20: Relative difference between $\Phi_L$ and $\text{Arg}(\eta_2)$ as a function of $\text{Arg}(\eta_2)$, as observed from Fig. 19.

Figure 21: Norm of $|M_L|$ as a function of $\Phi_L$, computed from Eq. (5.64), for scenario $Sn\chi^2$, with $m_{\tilde{\chi}_2^0} = 241.94$ GeV, $\Phi_{\mu} = 0$ (heavy solid), $\pi/8$ (light solid), $\pi/6$ (heavy dashed), $\pi$ (light dashed).
Figure 22: Norm of $|M_L|$ as a function of $\Phi_L$, computed from Eq. (5.64), for scenario $Snc_3$, with $m_{\tilde{\chi}_1^0} = 156.24$ GeV, $\Phi_\mu = 0$ (heavy solid), $\pi/8$ (light solid), $\pi/6$ (heavy dashed), $\pi$ (light dashed).

Figure 23: The same inputs as in Fig. 22 but considering the neutralino mass $m_{\tilde{\chi}_2^0} = 236.79$ GeV.

Comparing with Fig. 15 constructed in similar conditions according to scenario $Snc_2$, in this case we observe a greater dispersion of the values of $|M_L|$ when $\Phi_\mu$ vary. For small phases $-1 \leq \Phi_L \leq 1$ and $0 \leq \Phi_\mu \leq \pi/8$, the values of $M_L$ lies in the range $163$GeV – $166$GeV, approximately. Figure 23 shows the behavior of $M_L$ with respect to the phase $\Phi_L$, computed from Eq. (5.64) for input parameters of scenario $Snc_3$ with $m_{\tilde{\chi}_2^0} = 236.39$GeV. Similarly, in this case, the values of $|M_L|$ in the mentioned lies in the range $155$GeV – $185$GeV, approximately. Thus, the value of $|M_L|$ must be localized in the intersection of these regions, i.e. it is determined more accurately in the case of scenarios where the mass $m_{\tilde{\chi}_1^0}$ is a known quantity.
7 Conclusions

In this paper we have studied the implications of a complex symmetric neutralino mass matrix in the context of left-right SUSY model. This matrix was described by seven real parameters $|M_L|$, $\Phi_L$, $|\mu|$, $\Phi_\mu$, $M_R$, $M_V$ and $\tan \theta_k$. To find analytical expressions for the physical masses $m_{\tilde{\chi}^0_j}$, $j = 1, \ldots, 4$, of the neutralinos and some connecting relations among the parameters, at the tree level, we have diagonalized this matrix by constructing the corresponding diagonalizing unitary matrix. The masses, obtained by solving the associated characteristic polynomial to this problem, have been ordered by sizes and plotted as a function of the Higgsino parameter $|\mu|$, and also as a function of the mixing phases $\Phi_\mu$ and $\Phi_L$. In the CP-conserving case, when all except the $\mu$ parameter were fixed, according to the possible scenarios studied in Ref. [5], we observe that there is not intersection between the different curves representing the behavior of the neutralino masses a function of $\mu$. In the CP-violating case, considering two possible scenarios, similar to the previous ones but where $|\mu|$ was fixed and $\Phi_\mu$ and $\Phi_L$ were allowed to vary, we observe that there is not overlapping between the surfaces representing the behavior of the neutralino masses $m_{\tilde{\chi}^0_1}$ and $m_{\tilde{\chi}^0_2}$.

The inverse problem consisting to determine the mixing parameters $|M_L|$ and $\Phi_L$ in terms of the rest of fundamental parameters have been solved using the projector formalism without appeal to the Jarlskog’ projector formula. In this way, the so-called reduced projectors have been expressed essentially in terms of the minors of the determinant of the matrix formed from the product between the original mass matrix and its adjoint. Thus, the $M_L$ parameter has been disentangled and expressed in terms of the eigenphases by solving a simple linear algebraic equation, in contrast to the standard treatment where you need to solve a system of six linear equations with six unknowns (see Appendix A). Moreover, combining the novel definition of the reduced projectors with the Jarlskog’ formula and then solving a quadratic algebraic equation we have obtained a new formula expressing the norm of $M_L$ in terms of the mixing parameter $\Phi_L$ and of the rest of the fundamental parameters. This last formula provide a description for the behavior of $|M_L|$ in terms of $\Phi_L$ equivalent to the one in terms of the eigenphases.

In the treatment of the inverse problem, in the CP-violating case, we have considered two scenarios, the first one similar to the $Sp1$–type considered in [2] in the context of the MSSM, characterized by a big rate between $k_u$ and $k_d$ and the second one characterized by a relatively small rate between $k_u$ and $k_d$, with similar conditions to those studied in [12] but adapted to the CP-violating case. In both scenarios, we have observed that the value of $|M_L|$ can be determined more accurately if we know the the mass of the lighter neutralino.
A similar analysis can be carried out for the chargino sector. This sector is more difficult to treat using the projector technique because the corresponding chargino mass matrix is not symmetric and requires two unitary matrices to diagonalize it. This analysis is underway and will be reported in a separate communication.

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A The standard method

In this section we demonstrate the equivalence between the method implemented in the above section and the one using the Jarlskog’s formula (4.52), or well Eq. (4.55). The method using the Jarlskog’s formula to express $M_L$ in terms of the eigenphases and of the rest of the fundamental parameters has been used in reference Ref.[2], in the case of the MSSM.

Equation (4.46), for fixed $j$, represent a system of four complex algebraic equations serving to determine the six fundamental L-R SUSY parameters and corresponding eigenphase and physical neutralino mass in terms of the reduced projectors. The explicit form of this system of equations is obtained by inserting Eq. (2.14) into Eq. (4.46), we give

\[
\eta_j m_{\tilde{\chi}_j^0} = M_L - M (\sin \theta_k p^*_{j3} - \cos \theta_k p^*_{j4}) \\
= \frac{M_{RV} p^*_{j2} + 2 \kappa M (\sin \theta_k p^*_{j3} - \cos \theta_k p^*_{j4})}{p_{j2}} \\
= M \sin \theta_k (2 \kappa p^*_{j2} - 1) - 2 \mu p^*_{j4} \\
= M \cos \theta_k (1 - 2 \kappa p^*_{j2}) - 2 \mu p^*_{j3}.
\]

The inverses of these equations determines the fundamental LRSUSY parameters in terms of $p_{j\alpha}$,
that is
\[
M_L = \eta_j m_{\tilde{\chi}_j^0} + M (\sin \theta_k p_{j3}^* - \cos \theta_k p_{j4}^*),
\]
\[
M_{RV} = \frac{p_{j2} \eta_j m_{\tilde{\chi}_j^0} - 2 \kappa M (\sin \theta_k p_{j3}^* - \cos \theta_k p_{j4}^*)}{p_{j2}},
\]
\[
\mu = \frac{M (\sin \theta_k p_{j4} + \cos \theta_k p_{j3})(1 - 2 \kappa p_{j2}^*)}{|p_{j3}|^2 - |p_{j4}|^2},
\]
where the complex neutralino mass of $\tilde{\chi}_j^0$ is given by
\[
\eta_j m_{\tilde{\chi}_j^0} = -M \frac{(\sin \theta_k p_{j3}^* + \cos \theta_k p_{j4}^*)(1 - 2 \kappa p_{j2}^*)}{|p_{j3}|^2 - |p_{j4}|^2}.
\]

Also, as $M_{RV}$ is a real quantity, from (A.72) we obtain
\[
\tan \theta_k = -\frac{\text{Im}[p_{j4}(p_{j2}^*)^2] + 2 \kappa (|p_{j4}|^2 - |p_{j2}|^2 - |p_{j3}|^2) \text{Im}[p_{j3}p_{j2}^*]}{\text{Im}[p_{j3}(p_{j2}^*)^2] + 2 \kappa (|p_{j3}|^2 - |p_{j2}|^2 - |p_{j4}|^2) \text{Im}[p_{j3}p_{j2}^*]}.
\]

The complex reduced projectors $p_{j2}, p_{j3}$ and $p_{j4}$ can be computed from (A.68-A.70) without considering an explicit dependence of $|M_L|$ and $\Phi_L$. Solving this system, equivalent to six linear equations with six real unknowns, we get
\[
p_{j2} = \frac{1}{2 \kappa} + (m_{\tilde{\chi}_j^0}^2 - 4 \mu^2) Z_j^*,
\]
\[
p_{j3} = 2 \kappa M (m_{\tilde{\chi}_j^0}^2 \eta_j^* \sin \theta_k Z_j + 2 \mu |e^{i \Phi_j} \cos \theta_k Z_j^*),
\]
\[
p_{j4} = -2 \kappa M (m_{\tilde{\chi}_j^0} \eta_j^* \cos \theta_k Z_j + 2 \mu |e^{i \Phi_j} \sin \theta_k Z_j^*),
\]
where
\[
Z_j = Z_{j1} - m_{\tilde{\chi}_j^0} \eta_j Z_{j2},
\]
with
\[
Z_{j1} = \frac{1}{2 \kappa D_j} \left\{ (m_{\tilde{\chi}_j^0}^2 - M_{RV}^2)(m_{\tilde{\chi}_j^0}^2 - 4 \mu^2) - 4 \kappa^2 M^2 [m_{\tilde{\chi}_j^0}^2 + 2 M_{RV} |\mu| e^{i \Phi_j} \sin(2\theta_k)] \right\}
\]
and
\[
Z_{j2} = -\frac{2 \kappa M^2}{D_j} [M_{RV} + 2 \mu e^{i \Phi_j} \sin(2\theta_k)].
\]
Thus inserting (A.77) and (A.78) into (A.71), we obtain
\[
M_L = m_{\tilde{\chi}_j^0} \eta_j + 2 \kappa M^2 \times (m_{\tilde{\chi}_j^0} \eta_j Z_j^* + 2 |\mu| e^{-i \Phi_j} \sin(2\theta_k) Z_j)
\]
\[
= A_j \eta_j + B_j,
\]
where $A_j$ and $B_j$ are given in Eqs. (5.61) and (5.62), respectively.
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