Dipole states in stable and unstable nuclei

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Abstract

A nuclear structure model based on linear response theory (i.e., Random Phase Approximation) and which includes pairing correlations and anharmonicities (coupling with collective vibrations), has been implemented in such a way that it can be applied on the same footing to magic as well as open-shell nuclei. As applications, we have chosen to study the dipole excitations both in well-known, stable isotopes like $^{208}$Pb and $^{120}$Sn as well as in the neutron-rich, unstable $^{132}$Sn nucleus, by addressing in the latter case the question about the nature of the low-lying strength. Our results suggest that the model is reliable and predicts in all cases low-lying strength of non collective nature.

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Giant Resonances (GR) in atomic nuclei lie at excitation energies above the nucleon separation threshold ($\sim$8-10 MeV), have different multipolarity and carry different spin-isospin quantum numbers. They are associated with the elastic, short-time ($\hbar/10$ MeV $\approx 10^{-22}$ s) response of the nuclear system. They have been observed throughout the mass table with large cross sections,

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close to the maximum allowed by sum rule arguments, implying that a large number of nucleons participate in a very collective nuclear motion [11 2].

The giant dipole resonance (GDR) has been the first to be discovered, and is probably the most studied among the various nuclear collective vibrations. In well-bound systems it is often depicted as a coherent oscillations of essentially all protons against all neutrons, boosted by an initial displacement of the centers of the two distributions induced by an external electromagnetic field. The energy of this resonance in MeV, is given by the empirical $80 A^{-1/3}$ law.

In the neutron-rich isotopes, it has been suggested that part of the energy provided by the external field can be used to excite a different kind of motion. In fact, if the valence neutrons occupy orbitals with lower energies and larger radii than the inner, or "core", particles, these orbitals must be somehow decoupled from the others. Consequently, the excitations of the valence neutrons should result in vibrations of these particles against the inner ones, namely in modes with different characteristics than the standard GDR.

Recently [3] a systematic study in the neutron-rich isotopes $^{18-22}\text{O}$, via relativistic Coulomb excitation, has identified sizeable strength (about 10% of the Thomas-Reiche-Kuhn (TRK) sum rule) below 15 MeV, that is, well below the GDR region. The question about the collectivity of this strength is left to theorists. So far, every microscopic model (either within the non-relativistic [4] or relativistic [5] framework) seems to find only non-collective states in the low-lying region. A further natural question which arises, is whether this low-lying dipole strength systematically exists in the medium-heavy nuclei and whether a certain degree of collectivity can develop, with increasing mass number. Another issue is whether the properties of the low-lying dipole can shed light on the (possible) modifications of the nuclear effective Hamiltonian(s) when one goes far from the line of stability.

These questions have been attacked within the most widely used mean field models [6], which however do not take into account the spreading effects, as we will do in this work.

In fact, in nuclei the mean field defines a surface which can vibrate leading to a spectrum of collective low-lying excitations (phonons). The coupling of the nucleons to these dynamical vibrations, a process which goes beyond mean-field, can strongly renormalize the single-particle motion by changing the energy and the occupancy of the levels around the Fermi energy and, eventually, by providing a single-particle spreading width $\Gamma_{s.p.}$. In keeping with the fact that the giant resonances can be viewed as correlated particle-
hole (p-h) or two quasi-particle (2qp) states, the very same mechanism will produce their spreading width $\Gamma_{GR}^\downarrow$, although quantitatively this width is reduced compared to the sum of the particle and hole widths since this is the strong effect of the coherence between the particle and hole motions as shown in Ref. [7].

The aim of this paper is the following. If on one hand mean field studies are nowadays performed using sophisticated forces well linked to nuclear matter properties, they do not include the phonon coupling mentioned above. We would like to provide a unified description of the properties of the giant resonances within a self-consistent model which includes such a coupling in both magic and open-shell nuclei. In this sense, the work is a continuation of the old pioneering calculations of Ref. [8], with the very clear improvement coming from the self-consistent use of an effective nucleon-nucleon interaction. Among other limitations, in Ref. [8] pairing was considered in a simplistic way and it was not possibile to calculate the giant resonances along an isotopic chain. As far as the particle-hole channel is concerned, a consistent calculation of the dipole states was performed in Ref. [4]. In this work, a Skyrme-type interaction was employed and the residual interaction between quasiparticles was self-consistently derived. However, the pairing contribution was dropped from the residual interaction. Nowadays, since extrapolations to neutron-rich nuclei are needed, it is desirable to be able to reproduce the lineshape of the giant resonances in different nuclei without \textit{ad hoc} adjustments, neither in the particle-hole nor in the pairing channel. Thus, we have accomplished the implementation a more general model compared to that of [4], in the sense that we now treat consistently also the pairing interaction. We test this new model, that we call QRPA-PC (Quasi-particle Random Phase Approximation plus Phonon Coupling), in the case of dipole. We analyze the well-known $^{208}\text{Pb}$ nucleus already, and also very recently $^{120}\text{Sn}$, investigated in many aspects, and also another stable, yet open shell nucleus, namely $^{120}\text{Sn}$. In both cases we can compare with measured photoabsorption cross sections. Then, we make predictions for the neutron-rich isotope $^{132}\text{Sn}$, for which experimental results should be soon available.

Our starting point is a mean field calculation performed in the HF-BCS (Hartree-Fock-Baarden-Cooper-Schrieffer) plus QRPA approach. As already mentioned, the model is self-consistent, in the sense that a Skyrme force is used in the particle-hole channel, while in the particle-particle one a zero-
range pairing interaction

\[ V_{\text{pair}}(\vec{x}_1, \vec{x}_2) = -V_0 \left[ 1 - \left( \frac{\rho(\vec{x}_1 + \vec{x}_2)}{\rho_c} \right)^\gamma \right] \delta(\vec{x}_1 - \vec{x}_2) \] (1)

is adopted. The parameters of this pairing force are adjusted in such a way that the average pairing gap resulting from the HF-BCS calculation (where the average is made by considering levels within ± 5 MeV from the Fermi energy) agree with the result of the three-point formula (the so-called \( \Delta_{\text{odd}}(N + 1) \) formula [10]). The properties of the excited states are not considered as input to fit the force parameters. For the tin isotopes, using in the p-h channel the SIII force [11], we obtain that the agreement of the average paring gap with the outcome of the three-point formula is within ∼100 keV in \(^{120-126}\text{Sn}\) (if \(\gamma = 1\)) with \(\rho_c = 0.16 \text{ fm}^{-3}\) and \(V_0 = 640 \text{ MeV fm}^3\). In fact, pairing is only included in the case of \(^{120}\text{Sn}\). For \(^{132}\text{Sn}\) and \(^{208}\text{Pb}\), HF-BCS plus QRPA is replaced by HF plus RPA.

The HF-BCS equations are solved in coordinate space. The step and upper limit for the radial coordinate are equal respectively to 0.1 fm and 25 fm. The continuum part of the single-particle spectrum has been discretized by using box boundary conditions. Only the neutron single-particle states \(1h_{11/2}, 2f_{7/2}, 1h_{9/2}, 1i_{13/2}, 3p_{3/2}\) are considered for the BCS equations. The RPA or QRPA equations are solved in the configuration space using the same technique as explained in [4]. We have found that the results are stable and the energy-weighted sum rule (EWSR) is satisfied if an upper cutoff in the two quasi-particle energies is set at 100 MeV.

In this self-consistent mean field model, the spurious states \(0^+\) and \(1^-\) associated respectively with the breaking in the superfluid ground state of the number of particles, and with the translational symmetry, appear at zero energy when the p-h matrix elements are renormalized by a few percent (3%), because of the finite basis size. This little renormalization does not affect significantly the dipole response at any energy. No renormalization of the p-p interaction is needed for the \(0^+\) state.

To include the coupling with the vibrations, we use the same Hamiltonian of Ref. [12] in the full neutron-proton scheme. The formalism has been described in detail in Ref. [4], to which we confer the reader (cf. in particular Sec. 2 and Appendix A). In the present case the phonons of our model are the states resulting from RPA or QRPA calculations of multipolarity \(1^-\), \(2^+, 3^-\) and \(4^+\) having energy below 30 MeV and exhausting at least 5% of
isoscalar or isovector strength. The properties of the low-lying $2^+$ and $3^-$ states in the Sn isotopes are discussed in ref. [13].

We have checked by means of an explicit calculation for the $^{208}$Pb case, that continuum-RPA gives peak positions quite similar to our discrete RPA, with escape widths smaller than 0.5 MeV, in agreement with the experimental data [2].

We briefly discuss the sensitivity of the dipole results to the particular Skyrme force employed, before analyzing in detail the results obtained using the parametrization SIII. The issue has been studied in Ref. [14]. In that work, it was found by exploiting the $^{208}$Pb case that the RPA centroid energy $E_{-1}$ (defined as $(m_1/m_{-1})^{1/2}$, where $m_k$ is the $k$-th moment of the strength function) of the dipole, has a linear dependence on the quantity $F_D$ associated with the parameters of the Skyrme force in question,

$$E_{-1} = \lambda + \mu F_D. \quad (2)$$

The explicit expression of $F_D$ can be found in Ref. [14], but we remind here that it contains essentially the square root of the product of the bulk symmetry energy $a_\tau$ (sometimes indicated by $J$ or $a_4$), of the ratio between the surface symmetry energy coefficient $a_{ss}$ and $a_\tau$, and of the effective mass $m^*$. The dimensions of $F_D$ are MeV$^{1/2}$.

It is confortable to have found in the present context that the linear relation (2) is valid also for $^{120}$Sn. While in the $^{208}$Pb case, the values $\lambda=11.35$ MeV and $\mu=0.77$ MeV$^{1/2}$ were obtained in [14], in the case of $^{120}$Sn ($^{132}$Sn) we find in the present analysis $\lambda=9.28$ MeV (11.92 MeV) and $\mu=2.34$ MeV$^{1/2}$ (1.46 MeV$^{1/2}$). For $^{120}$Sn, forces like SkP [15], SLy4 [16] and SGII [17] give results for the peak energy which are lower than the experimental ones. This is not the case for the force SIII, and we will use it, since the phonon coupling induces a downward shift of the RPA peak energy.

To be more quantitative, in Table 1 we compare the mean field results obtained with the SIII and SLy4 forces. We report the centroid energies $E_0 = m_1/m_0$ obtained within RPA or QRPA, together with the unperturbed HF or HF-BCS results, and with the phenomenological predictions. We can observe that, for all nuclei we consider, the unperturbed value of the centroid obtained using SIII is similar (yet slightly lower) than that obtained using SLy4, whereas the RPA result is sistematically higher. In fact, while for the two forces the effective mass is about the same ($m^*/m = 0.76$ for SIII and $m^*/m = 0.7$ for SLy4), the repulsive matrix elements obtained with SIII are
nucleus & SIH & SLy4 & $80 A^{-1/3}$ (41 $A^{-1/3}$) \\
$^{208}$Pb & 14.9 (9.4) & 13.4 (9.8) & 13.5 (6.9) \\
$^{120}$Sn (RPA) & 17.1 (10.9) & 15.0 (11.0) & 16.2 (8.3) \\
$^{120}$Sn (QRPA) & 17.3 (11.3) & 15.1 (11.6) & 16.2 (8.3) \\
$^{132}$Sn & 17.0 (10.7) & 15.2 (11.0) & 15.7 (8.1) \\

Table 1: Centroid energies $E_0 = \frac{m_1}{m_0}$ obtained within RPA or QRPA (together with the unperturbed HF or HF-BCS result in parenthesis), compared with the empirical prediction $80 A^{-1/3}$ (41 $A^{-1/3}$).

sistematically larger that those of the SLy4. From the comparison between RPA and QRPA results for the nucleus $^{120}$Sn, we can deduce that the pairing interaction does not play a crucial role, as expected.

In Table 2, the results for the centroid energy without and with the phonon coupling are shown. In the case of $^{208}$Pb, the centroid energy after the coupling is 14.4 MeV, to be compared with the value 14.9 MeV at the level of RPA. In the case of $^{120}$Sn, the centroid energy, at the level of the complete calculation is the same value of the QRPA calculation. The same is true for $^{132}$Sn.

The integral of the strength as a function of the upper integration limit, that is, the cumulated value of the EWSR, is shown in Fig. 1. This cumulated value is shown as a fraction of the total expected value, which is the TRK sum rule multiplied by the enhancement factor $(1 + \kappa)$. The values of the total EWSR in MeV·fm$^2$ (and of $\kappa$ in parenthesis) are, for $^{120}$Sn, $^{132}$Sn and $^{208}$Pb respectively: 2448 (0.399), 2566 (0.397) and 4202 (0.408). One can see that, in order to exhaust more than 90-95% of the EWSR in the (Q)RPA-PC calculation, one has to reach 30 MeV in the case of $^{208}$Pb and 40 MeV in the case of the Sn isotopes. We stress that, while the shape of the strength distribution and the GDR peak position change when the particle-vibration coupling is taken in account, the centroid and the EWSR values are much less affected.

In the Figures 2-4, we show the calculated photoabsorption cross sections, together with the experimental results for the two stable nuclei. The values of energies and the widths obtained from a lorentzian fit of our cross sections are summarized in Table 3.

We can see that the agreement with the experimental results is good.
for both $^{208}\text{Pb}$ and $^{120}\text{Sn}$. This gives us confidence to use the model for a prediction of the photoabsorption in the unstable nucleus $^{132}\text{Sn}$. In this case the width is about 6 MeV, like in $^{120}\text{Sn}$, but the cross section is very fragmented.

In the low-lying energy region, we obtain strength in the three nuclei, as we discuss below. The cumulated value of the EWSR below 12 MeV is shown in Fig. For $^{208}\text{Pb}$, a comparison is possible with the data of Ref. Our calculation gives a total integrated strength $B(\text{E1})=0.53 \text{e}^2\text{fm}^2$ (below 6 MeV) and $B(\text{E1})=1.27 \text{e}^2\text{fm}^2$ (up to 8 MeV), whereas the experimental data are $B(\text{E1})=0.52 \text{e}^2\text{fm}^2$ (below 6 MeV) and $B(\text{E1})=0.80 \text{e}^2\text{fm}^2$ (up to 8 MeV). The states carrying this strength are mainly single-particle excitations, so the value of their energy is affected by the single-particle energy spectrum. On the other hand, the global $B(\text{E1})$ distribution is well reproduced qualitatively and quantitatively.

Coming to the Sn isotopes, the low-lying dipole strength grows faster, at the level of (Q)RPA, in $^{120}\text{Sn}$. The phonon coupling affects more $^{132}\text{Sn}$, reversing somewhat the trend. Also from Figs. and one can see that in $^{132}\text{Sn}$ four definite peaks exhaust the strength below 10 MeV, whereas in $^{120}\text{Sn}$ only a kind of smooth background is visible.

We have analyzed the structure of the four lying below 10 MeV in $^{132}\text{Sn}$. In RPA, the lowest peak at 8.44 MeV is mainly (78%) associated with the $3s_{1/2} \rightarrow 3p_{3/2}$ single-particle transition, and absorbs only 0.2% of the EWSR. There are two higher peaks which lie at 8.61 MeV and 9.53 MeV and absorb respectively 0.5% and 0.3% of the EWSR: they involve an admixture of the $2d_{3/2} \rightarrow 3p_{1/2}$ and $3s_{1/2} \rightarrow 3p_{1/2}$ transitions. Other small states involve the $d \rightarrow f$ transitions but there is no peak to which more than two or three components contribute. The pattern is very similar if we move to the interaction SLy4. This situation is somewhat at variance with the findings of Ref., in which the lowest state emerging from the relativistic RPA has four or five configuration well mixed and it exhausts alone about 1.4% of the EWSR. We conclude that an experiment aimed to explore the existence of either a well defined, relatively collective peak or a fragmented pattern in the low-lying energy region can really discriminate among microscopic self-consistent models.

The analysis of the RPA-PC wave functions of the states is slightly more difficult and less transparent, also because of the use of a complex Hamiltonian. However, we can state that the wave functions do not acquire more collectivity. This is confirmed by considering the transition densities, which
Table 2: Centroid energies $E_0 = \frac{m_1}{m_0}$ in MeV, obtained in the (Q)RPA and (Q)RPA-PC calculations. Only in the case of $^{208}$Pb we observe an appreciable shift when the phonon coupling is introduced. In the other two cases the downward shift is smaller than 100 keV.

| nucleus | RPA | QRPA-PC |
|---------|-----|---------|
| $^{208}$Pb | 14.9 | 14.4 |
| $^{120}$Sn | 17.1 | 17.1 |
| $^{132}$Sn | 17.0 | 17.0 |

Table 3: Values of the peak energy (width) in MeV calculated in (Q)RPA-PC (calculated by means of a lorentzian fit to the cross sections) in comparison with the experimental values.

| nucleus | QRPA-PC | exp. |
|---------|---------|------|
| $^{208}$Pb | 13.1 (3.7) | 13.46 (3.9) |
| $^{120}$Sn | 15.7 (5.3) | 15.4 (4.9) |
| $^{132}$Sn | 15.5 (5.8) | - |

are reported in Fig. 6 and are associated indeed to the RPA-PC calculation. The transition density associated with the the GDR peak at 13.7 MeV is compared with that of the low-lying state at 9.7 MeV. This latter has many nodes while that of the GDR displays the expected shape of collective type. The low-lying transition densities is also dominated by the neutron contribution at the surface and this fact is of course relevant for any consideration concerning its excitation by means direct reactions. The neutron character of the low-lying state at the surface is even more pronounced in the complete calculation, than in simple RPA.

To judge the collectivity of the low-lying strength, one may think about using the so-called “cluster sum rule” $S_{clus}$ [13]. However, as stated in many occasions, its use is justified only provided an unambiguous decoupling of excitations of core and valence particles is present in the physical systems. Already from the above discussion, and in particular from the analysis of the wave functions, it is clear that this is not the case for $^{132}$Sn: all the excess neutrons contribute to the low-lying strength and not only the least bound ones.
In conclusion, we have implemented with the present work a complete model which includes states of four quasi-particles nature in the description of the vibrational nuclear states. The model makes self-consistent use of a Skyrme force in the particle-hole channel and of a zero-range, density-dependent pairing force in the particle-particle channel. We have applied this model to the description of the the dipole strength in different systems, reaching satisfactory results. In $^{120}$Sn and $^{208}$Pb we obtained agreement with the experimental data, for the peak energy and width of the GDR (and for the pygmy states recently studied in $^{208}$Pb). In $^{132}$Sn, we have predictions which show a large fragmentation of the main GDR and the absence of collective states in the low-lying region. This latter result, obtained using two different effective forces, is somehow at variance with the outcome of relativistic RPA studies \[. However, it is consistent with the discussion in Ref. \[, where it is argued that the soft dipole strength, observed in halo light nuclei, should decrease in skin nuclei, due to the coupling to the GDR. These remarks point to the importance of clear-cut experimental data which are able to identify or exclude the presence of a low-lying collective dipole in medium-heavy nuclei.
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Figure 1: Exhaustion of the dipole EWSR in the three nuclei considered in the present paper. The full line refers to the complete (Q)RPA calculation, while the dashed line to the (Q)RPA calculation.
Figure 2: Photoabsorption cross section for $^{208}$Pb.

Figure 3: Photoabsorption cross section for $^{120}$Sn.
Figure 4: Photoabsorption cross section for $^{132}$Sn.
Figure 5: Low-lying dipole strength in the three isotopes studied. The figure has the same pattern of Fig. 1, but it refers only to the interval 8–12 MeV.
Figure 6: Transition densities in $^{132}$Sn at the level of RPA-PC. The lower panels correspond to the GDR at 13.5 MeV while the two upper panels to the low-lying state at 9.7 MeV. Proton/neutron (isoscalar/isovector) transition densities are shown in the left (right) side.