1 INTRODUCTION

The quark–gluon plasma (QGP) usually is defined as the phase of Quantum Chromodynamics (QCD) in which the quarks and gluons degrees of freedom, that is normally confined within the hadrons, are mostly liberated. The possible phases of the QCD and the precise locations of critical boundaries or points are currently being actively studied. In fact, revealing the QCD phase transition structure is one of the central aims of the ongoing and future theoretical and experimental research in the field of the hot and/or dense QCD [1–4]. It is about thirty years since the study of the hot and dense nuclear matter in the form of the QGP has been started. The experiments at the CERN’s Super Proton Synchrotron (SPS) first tried to create the QGP in the 1980s and 1990s: The first hints of the formation of a new state of matter was obtained from the SPS data in terms of the Fermi liquid picture. The bag model is used, with fixed bag pressure (β) for the nonperturbative part, and the quantum chromodynamics (QCD) coupling is assumed to be constant, i.e., with no dependence on the temperature or the baryon density. The effect of weakly interacting quarks on the QGP phase diagram are shown and discussed. It is demonstrated that the one-gluon exchange interaction for the massless quarks has considerable effect on the QGP phase diagram and it causes the system to reach to the confined phase at the smaller baryon densities and temperatures. The pressure of excluded volume hadron gas model is also used to find the transition phase diagram. Our results depend on the values of bag pressure and the QCD coupling constant. The latter does not have a dramatic effect on our calculations. Finally, we compare our results with the thermodynamic properties of strange quark matter and the lattice QCD prediction for the QGP transition critical temperature.

INTRODUCTION

The quark–gluon plasma (QGP) usually is defined as the phase of Quantum Chromodynamics (QCD) in which the quarks and gluons degrees of freedom, that is normally confined within the hadrons, are mostly liberated. The possible phases of the QCD and the precise locations of critical boundaries or points are currently being actively studied. In fact, revealing the QCD phase transition structure is one of the central aims of the ongoing and future theoretical and experimental research in the field of the hot and/or dense QCD [1–4]. It is about thirty years since the study of the hot and dense nuclear matter in the form of the QGP has been started. The experiments at the CERN’s Super Proton Synchrotron (SPS) first tried to create the QGP in the 1980s and 1990s: The first hints of the formation of a new state of matter was obtained from the SPS data in terms of the Fermi liquid picture, the event–by–event fluctuations, the direct photons and the di-leptons. The current experiments at the Brookhaven National Laboratory’s Relativistic Heavy Ion Collider (RHIC) are still continuing these efforts. In April 2005, the formation of the quark matter was tentatively confirmed by the results obtained at the RHIC. The consensus of the four RHIC research groups was in favor of the creation of the quark–gluon liquid at the very low viscosity [5–8].

Since, in this new phase of matter, the quarks and gluons are in the asymptotic freedom region, one expects that they interact weakly. So, the perturbative methods can be used for such a system. The asymptotic freedom suggests two procedures for the creation of the QGP: (i) The recipe for the QGP at high temperature. If one treats the quarks and gluons as the massless noninteracting gas of molecules, such that the baryon density vanishes, the critical temperature above which the hadronic system dissolves into a system of quarks and gluons (QGP) is $T_c = 140 \text{ MeV}$ [9]. However, the modern lattice QCD calculation estimates the critical temperature, $T_c$, to be about 170 MeV [10]. (ii) The recipe for the QGP at high baryon density. At zero temperature, the critical baryon density required the transition to take place in $n_B = 0.7 \text{ fm}^{-3}$ [9], i.e., four times the empirical nuclear matter density. On these grounds, one should expect to find the QGP in two places in the nature: Firstly, in the early universe, about 10 –5 s after the cosmic Big Bang, or secondly, at the core of super-dense stars such as the neutron and quark stars. This new phase of matter can also be created in the initial stage of the little Big Bang by means of the relativistic nucleus-nucleus collisions in the heavy-ion accelerators [5–8].

The critical temperature (critical baryon density) at the zero baryon density (zero temperature) has been obtained simply for the noninteracting, massless up and down quarks and gluons in [9]. Some primary
works in the zero baryon density and in the framework of the bag model have also been presented in [11] (and the references therein). For the more complicated field theoretical approaches, see [12, 13] and the references therein. So, it would be interesting to perform a similar calculation to [9], but with the nonzero interaction, for the region in which both $T$ ≠ 0 and $n_B$ ≠ 0. So, in this work we generalize the above calculations for the region with the finite temperature and baryon density by considering the weakly interacting quarks in the framework of the one-gluon exchange scheme. Since we intend to use the perturbative method in our calculation, we assume that $\alpha_s < 1$. The dependence of $\alpha_s$ on the temperature and baryon density is ignored [14, 15]. We perform calculations beyond the zero hadronic pressure approximation of [9], and the pressure of the excluded volume hadron [16–18] model is also taken into account to find the corresponding QGP transition phase diagram.

So, the paper is organized as follows. In Section 1 we review the derivation of the one-gluon exchange interaction formulas [20] based on the Landau Fermi-liquid picture [21, 22]. Section 2 is devoted to the calculation of the thermodynamic properties of the QGP. The result and its comparison with the strange quark matter [14] are given in Section 3. Finally, a conclusion and summary are presented in Section 4.

1. THE LANDAU FERMI-LIQUID MODEL AND THE ONE-GLUON EXCHANGE INTERACTION

The Landau Fermi-liquid theory describes the relativistic systems such as the nuclear matter under the extreme conditions, the quark matter, the quark–gluon plasma, and other relativistic plasmas. The basic framework of the Landau theory of relativistic Fermi liquids is given by Baym and Chin [21]. In this framework, one can evaluate the energy density of a weakly interacting quarks in QGP by the following formulas:

$$\varepsilon = \varepsilon_{\text{kin}} + \varepsilon_{\text{pot}},$$

$$\varepsilon_{\text{kin}} = \int \rho_p \frac{g_q d^3 p}{(2\pi)^3},$$

$$\varepsilon_{\text{pot}} = \frac{1}{2} \int f_{p,k}^{\text{unpol}} n_p n_k \frac{g_u d^3 p g_d d^3 k}{(2\pi)^3 (2\pi)^3},$$

where $g_q = g_{\text{spin}} \times g_{\text{color}} = 2 \times 3 \times 6 = 36$ is the degeneracy of quarks; $\varepsilon_{\text{kin}}, \varepsilon_{\text{pot}}$ and $n_p$ are the kinetic energy and the potential energies and the familiar Fermi–Dirac liquid distributions of our quasiparticles, respectively. In Eq. (3), $f_{p,k}^{\text{unpol}}$ is the Landau–Fermi interaction function, which is a criterion for the interaction between the two quarks (the “unpol” superscript refers to the unpolarized quark matter [14, 15]). It is related to the two-particle forward scattering amplitude, i.e.,

$$f_{p,k}^{\text{unpol}} = \frac{m_p m_k}{\mu_p \mu_k} M_{p,k}.$$

$M_{p,k}$ is the commonly defined Lorentz invariant matrix element. Now, using the Feynman rules for the QCD, $M_{p,k}$ can be calculated. Since the direct term is proportional to the trace of the Gellman matrices, the color symmetric matrix element is only given by exchange contribution:

$$M_{p,k} = -4 \varepsilon^2 \frac{1}{9} tr \left( \frac{\kappa \kappa}{2} \right)$$

$$\times \bar{u}(k) \gamma_\mu u(p) \bar{u}(p) \gamma^\mu u(k) \frac{-1}{(k - p)^2},$$

where $\varepsilon^2 = 4 \pi \alpha_s$ (our choice is $\alpha_s = 0.2$). Since our system is unpolarized, it is possible to sum over all the spin states and get the average of this quantity as

$$\overline{M}_{p,k} = \frac{2 \varepsilon^2}{9} \frac{m^2_k - k \cdot p}{m^2_q} (k - p)^2.$$

Then for massless up and down quarks,

$$f_{p,k}^{\text{unpol}} = \frac{\varepsilon^2}{9} \frac{1}{|\mu| |\kappa|}.$$

So, the total energy of interacting quarks can be evaluated by using Eqs. (1), (2), (3) and (7). In the next section, the various thermodynamic quantities of the interacting QGP like the internal energy, the pressure, the entropy, etc. are calculated.

2. THE THERMODYNAMIC PROPERTIES OF THE QGP

We begin with calculation of the free ultrarelativistic massless quarks partition function density, which follows its noninteracting gluon contributions. Since we are interested in the heavy-ion processes, it is assumed that the initial state should be the nuclear matter with the equal neutron and proton densities, i.e., $n_p = n_n$. So,

$$n_u = n_d,$$

and the contributions of $u$ and $d$ quarks are equal. Then the fermionic partition function for each quark is written as ($\bar{\psi}$ is the volume)

$$\ln \frac{\Omega_q}{\bar{\Omega}} = \int \ln \left[ (1 + e^{-\beta(\bar{k} - \mu)}) + (1 + e^{\beta(\bar{k} + \mu)}) \right]$$

$$\times \frac{4 \pi g}{(2\pi)^3} \kappa^2 dk.$$

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This integral can be evaluated analytically, i.e.,
\[
\ln \frac{T g}{\gamma} = \frac{4\pi g_s}{3(2\pi)^3} \beta^\gamma \left( \frac{7}{60} \pi^4 + \frac{1}{2} \pi^2 \beta^2 + \frac{1}{4} \beta^4 \right),
\]
where \( \gamma = \mu \beta \) and \( \beta = 1/T \) (in the Boltzmann factor units). Now we are in the position to evaluate the various thermodynamic quantities, such as the energy density, the entropy and the free energy from the above partition function, especially we have
\[
n_q(n, T) = T \left[ \frac{\partial}{\partial \mu} \left( \ln \frac{T g}{\gamma} \right) \right]_T
= \frac{4\pi G_n}{3(2\pi)^3} \left( \pi^2 T^2 \mu + \mu^3 \right),
\]
where \( G_n = 2g_s \) is the quark degeneracy for two, up and down quarks, flavors. All of the thermodynamic quantities are obtained as a function of chemical potential \( \mu \) and temperature \( T \). The temperature is a suitable experimental quantity, but the chemical potential is not; so, it is better to rewrite the thermodynamic quantities as a function of the temperature and the baryon density, instead of the chemical potential. Since the baryon number of a quark is \( l/3 \), we have \( n = 1/3n_q \) and
\[
n = \frac{2}{3\pi^2} \left( \pi^2 T^2 \mu + \mu^3 \right).
\]
Then, the chemical potential can be found as a function of \( n \) and \( T \), from the above equation, i.e.,
\[
\mu(n, T) = \left( \frac{3}{4} \pi^2 n + \frac{9}{16} \pi^4 \right)^{1/3}
+ \left( \frac{3}{4} \pi^2 n - \frac{9}{16} \pi^4 \right)^{1/3}.
\]
So, our thermodynamic quantities become a function of baryon density and temperature.

For the energy density of gluons we have
\[
\varepsilon_g = \int \frac{k}{k^3 - 1} \frac{4\pi g_s k^2 dk}{(2\pi)^3} = g_s \frac{\pi^3}{30} T^4,
\]
where again \( g_s = 16 \) is the degeneracy of gluons.

Our system is ultrarelativistic, so there is a simple relation between the pressure and the energy density:
\[
\mathcal{P} = \frac{1}{3} \varepsilon.
\]

The bag pressure and the vacuum energy contributions should be included, in addition to the quark and gluon contribution energies,
\[
\varepsilon_{QGP}(n, T) = \varepsilon_q(n, T) + \varepsilon_g(T) + \mathcal{B},
\]
\[
\mathcal{P}_{QGP}(n, T) = \mathcal{P}_q(n, T) + \mathcal{P}_g(T) - \mathcal{B},
\]
Having the pressure, the entropy density of system is evaluated,
\[
\mathcal{S}_{QGP}(n, T) = \left( \frac{\partial}{\partial \mathcal{P}} \mathcal{P}_{QGP}(\mu, T) \right)_\mu.
\]

To study the phases of the QGP, one should concentrate on the QGP and the hadronic pressures, \( \mathcal{P}_{QGP} \) and \( \mathcal{P}_{hadrons} \), respectively, i.e., for \( \mathcal{P}_{QGP} < \mathcal{P}_{hadrons} \), the system is in the confinement phase and the quarks and gluons are inside the bag, but for \( \mathcal{P}_{QGP} > \mathcal{P}_{hadrons} \), the quarks and gluons pressures can overcome to the bag and the hadronic pressures and the system is in the de-confinement phase, i.e., the QGP phase is created. So, the phase diagram can be extracted by solving the following equation:
\[
\mathcal{P}_{QGP}(n, T)
= \mathcal{P}_q(n, T) + \mathcal{P}_g(T) - \mathcal{B} = \mathcal{P}_{hadrons}(n(\mu), T),
\]
where in this work, both \( \mathcal{P}_{hadrons} = 0 \) and \( \mathcal{P}_{hadrons} \neq 0 \) cases are considered. For \( \mathcal{P}_{hadrons} \neq 0 \), the excluded volume effect for the nuclear matter equation of state is used [16–18] to calculate the hadronic pressure (Eq. (29) of [17]), i.e.,
\[
\mathcal{P}_{hadrons}(n(\mu), T)
\]
\[
= \mathcal{P}_{hadrons}(\mu, T) \approx \frac{\mathcal{P}_{ideal}}{1 + \nu n(\mu)}.
\]
where \( \mathcal{P}_{ideal} \) is the pressure of free ideal nucleonic (Fermion) matter [18, 19] (here, degeneracy is 4, \( m = 939 \) MeV, \( \varepsilon = 4(4/3)\pi^3 \) and \( r = 1.2 \) fm, e.g., see Eq. (9) of [18]).

Now, the interaction energy between the quarks can be calculated by using the Landau Fermi-liquid model and the results derived by us in the previous section. The potential energy density of the interacting massless up or down quarks is found by using Eqs. (3) and (7),
\[
\varepsilon_{pot}(\mu, T)
\]
\[
= 1 - \int \frac{1}{2} \mathcal{E}_g \frac{1}{p} \mathcal{E}_g \frac{d^4 p d^4 q}{(2\pi)^5},
\]
where again \( g_s = 6 \) is the quark degeneracy of one flavor. By using the Fermi–Dirac distribution and the value of each quark flavor potential energy, i.e., Eq. (21), an analytical formula for the total potential energy density of the two quark flavors is written as follows:
\[
\varepsilon_{pot}(\mu, T)
\]
\[
= \frac{\alpha T^4}{\pi^2} \left( \frac{\pi^4}{9} + 2 \frac{\pi^2 (\mu \beta)^2}{3} + \frac{\mu \beta}{\pi} \right).
\]
Note that the potential energy for massless quarks is always positive. So, the interaction between quarks
inside the bag is repulsive and it helps the interacting quarks and gluons to penetrate from the bag more easily, rather than the noninteracting case, and furthermore, the one-gluon exchange interaction, because of its repulsive properties, makes the conditions easier for the system to make the transition to the QGP phase.

The internal energy density of the QGP is evaluated by performing the summation over the interacting and noninteracting parts of the energy density of quarks, the vacuum energy and the gluon energy density which were calculated before, and having that, the other thermodynamic quantities of the QGP are found.

As was pointed out before, it is assumed that the bag (hadronic) pressure is the one used in [9], i.e., $\bar{p} = 208$ MeV fm$^{-3}$ (zero), in order to compare our phase diagram with this reference, and since it is intended to compare our results with [14], the QCD coupling constant was chosen to be $\alpha_s = 0.2$. On the other hand, for the nonzero hadronic pressure, the baryonic chemical potential and the bag pressure are varied to reach the critical temperature of the lattice QCD predictions [10], i.e., $T_c = 170$ MeV.

3. THE RESULTS OF THE QGP CALCULATION

We begin by presenting the calculated free energy per baryon for both the QGP and the strange quark matter [14] in Figs. 1 and 2, as a function of baryon density (temperature) at the two different temperatures (baryon densities). The free energy for the QGP is larger than those of strange quark matter, since we know that the strange quark matter should be more stable than the QGP and therefore the strange quark matter free energy should be smaller than that of the QGP. As one should expect, the free energy increases (decreases) by increasing the baryon density (temperature). While the QGP has less temperature dependence with respect to the strange quark matter, they have similar density dependence at fixed temperature.

Similar comparisons are made for the entropies in Figs. 3 and 4. The entropy per baryon for the QGP is an increasing (decreasing) function of temperature (baryon density), and it is smaller than that of the strange quark matter [14]. Again, their dependence on the density is the same, but they behave especially differently at larger temperatures.

The plots of the equation of states of both the QGP (without the effect of constant bag pressure) and the strange quark matter as a function of baryon density at two different temperatures are given in Fig. 5. The QGP equation of state is much harder than that of
strange quark matter at the same baryon density and temperature.

The pressure of weakly interacting QGP for two different QCD coupling constants, and the noninteracting QGP (without the effects of constant bag pressure) at zero temperature as a function of baryon density is shown in Fig. 6. The increase in the interaction strength makes the pressure to rise, and therefore at the smaller baryon densities, the pressure of quarks becomes equal to the bag pressure. So, the interaction facilitates the quarks transition to the deconfined phase at lower density. The QCD coupling constant also plays the same role; i.e., it will reduce the transition density.

Finally, the phase diagrams for both the interacting and the noninteracting QGP are shown in Fig. 7 for $P_{\text{hadron}}(n(\mu), T) = 0$. The one-gluon exchange interaction, which is repulsive, causes to get the QGP at the smaller baryon densities and temperatures. As was pointed out before, the reason is very simple, the repulsive interaction between quarks helps them to escape from the bags. So, the formation of the QGP happens much easier for the interacting quarks than for the noninteracting one. But, the critical temperature is about 140 MeV, which is much less than the lattice QCD suggestion of 170 MeV. In Fig. 8, the hadronic pressure has also been taken into account (see Eq. (19) and [20]). The slashed area is the forbidden region; i.e., for the bag pressure approximately larger than 200 MeV, there is no critical temperature with the zero baryonic chemical potential (density). With the bag pressure about 441 MeV (note that the bag pressure estimated to be as large as 500 MeV [4]) and the low baryonic chemical potential (density), it is possible...
able to get results close to the lattice QCD prediction, i.e., $T_c = 170$ MeV.

4. CONCLUSION AND SUMMARY

In conclusion, the one-gluon exchange interaction was used to evaluate the strength of potential energy of the QGP in the Fermi-liquid model. By calculating the QGP partition function, the different thermodynamic properties of the QGP as a function of baryon density and temperature for the both interacting and noninteracting cases were discussed. It was found that the QGP internal and free energies are much larger than those of strange quark matter. On the other hand, if we consider the massive quarks like the strange quarks in the QGP, the potential energy becomes a negative quantity, but for the massless quarks it is always a positive quantity; therefore, the internal and free energy densities of strange quark matter become smaller than the QGP ones. We have seen how the one-gluon exchange interaction for the massless quarks affects the phase diagram of the QGP and causes the system to reach the deconfined phase at the smaller baryon densities and temperatures. Our results depend on the values of bag pressure and the QCD coupling constant. The latter does not have a dramatic effect on our results. The increase of the hadronic and bag pressure can improve our results toward the lattice QCD calculations. In the future works, we could adjust our phase diagram to get the relation between the bag constant and the QCD coupling constant. On the other hand, it is possible to generalize our method for the nonconstant QCD coupling and the bag pressure. Finally, we can also add the interaction between the gluons to our present calculations.

ACKNOWLEDGMENTS

We would like to acknowledge the Research Council of University of Tehran and Institute for Research and Planning in Higher Education for the grants provided for us.

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