Brane World Dynamics and Adiabatic Matter creation

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We have treated the adiabatic matter creation process in various three-brane models by applying thermodynamics of open systems. The matter creation rate is found to affect the evolution of scale factor and energy density of the universe. We find modification at early stages of cosmic dynamics. InGB and RS brane worlds, by choosing appropriate parameters we obtain standard scenario, while the warped DGP model has different solutions. During later stages, since the matter creation is negligible the evolution reduces to FRW expansion, in RS and GB models.

Keywords: Matter Creation; Brane.

1. Matter Creation Scenario

Einstein’s field equations are adiabatic and reversible and hence cannot provide an explanation for the origin of cosmological entropy. But matter constituents may be produced quantum mechanically in the framework of Einstein’s equations. The quantum fluctuations of the gravitational field near singularity, as well as the dynamics of any quantum field living on such a non-stationary background, lead to matter-creating phenomena, which can in turn modify the behavior of the early universe. A detailed microscopic description of matter creation phenomena should be provided by a fully self-consistent quantum field theory on a curved space time.

Many attempts have been made to treat the matter creation process at a phenomenological macroscopic level. The basic idea is that matter creation contributes at the level of the Einstein field equations as a negative pressure term. Prigogine et al. incorporated its effect in the classical Einstein field equations. At the expense of the gravitational field, matter creation can occur only as an irreversible process. The additional pressure term due to the creation depends on the creation rate. The total entropy within a comoving volume is not a constant, due
to the variation of number of particles. This approach explained the average effect induced on the dynamics of an isotropic expanding universe. In this work we study the effect of adiabatic matter creation on brane world models. We haven’t considered the details of the source of created matter. Recently Tetradis has shown that absorption of energy flux from the bulk by the three brane leads to acceleration or deceleration of the universe. We have also derived a phantom field equivalent formulation. This paper is organised as follows. In section 2 we give a brief description of RSII brane world model. In the section 3, thermodynamics of matter creation is explained. In section 4 we describe the effect of matter creation on brane worlds in general, and on RSII in section 5. An equivalent phantom field formulation is given in section 6. In sections 7 and 8 we give the adiabatic matter creation effects on DGP and GB brane models.

2. RSII Brane Model

The idea that our 4-dimensional universe might be a 3-brane embedded in a higher dimensional space-time has attracted much attention over the last several years. Randall and Sundrum (RS) models\textsuperscript{15,16} are the most important. According to the brane world scenario, the physical fields in our 4-dimensional space-time are confined to the three brane, while gravity can freely propagate in the bulk space-time. We assume a Friedmann metric for 4-dimensional spacetime embedded in the bulk 5-dimensional space with a cosmological constant $\Lambda_5$.

The Friedmann equation on the RSII brane is,\textsuperscript{17,18}

\[ H^2 = \frac{\kappa_4^2}{3} \rho \left( 1 + \frac{\rho}{2\sigma} \right) + \frac{\Lambda_4}{3} + \frac{C}{R^4} - \frac{k}{R^2} \]  

(1)

with

\[ \kappa_4^2 = \frac{8\pi G}{3}, \quad \text{and} \quad \frac{\Lambda_4}{3} = \frac{\sigma^2}{36} + \frac{\Lambda_5}{6} \]  

(2)

where $\Lambda_4$ is the cosmological constant on the brane, $\sigma$ is the brane tension, $\rho$ is energy density of ordinary cosmological matter on the brane, $k$ is the curvature index, and the term with $C$ (a constant) is the dark radiation term.\textsuperscript{19} Cosmology constrains the amount of dark radiation to be at most 10 percent of the energy density in photons.\textsuperscript{20} In the following we will assume AdS bulk spacetime, and $C = 0$, $k = 0$, and $\Lambda_4 = 0$

In RSII model the important change in Friedmann’s equation compared to the usual four dimensional form is the appearance of a term proportional to $\rho$. It tells us that if the matter energy density is much larger than the brane tension, i.e. $\rho \gg \sigma$, the expansion rate $H$ is proportional $\rho$. Only in the limit where the brane tension is much larger than the matter energy density, the usual behavior $H \propto \sqrt{\rho}$ is recovered.
3. Thermodynamics of Matter Creation in Cosmology

The Einstein’s field equations with Bianchi identities give, for homogeneous and isotropic universes in closed systems,

$$d(\rho V) = -\tilde{p} dV.$$  \hspace{1cm} (3)

where $\tilde{p}$ is the true thermodynamical pressure.

This equation is used to describe an adiabatic evolution of a closed system with a comoving volume $V$. But in the presence of matter creation, the analysis is in the context of open systems.$^{1,21}$ In this case the number of particles $N$ in a given volume is not fixed. For adiabatic transformation,

$$d(\rho V) + p dV - \frac{h}{n} d(nV) = 0.$$  \hspace{1cm} (4)

where $n = \frac{N}{V}$ and $h = \rho + p$ is the enthalpy per unit volume.

In such a transformation, the heat received by the system is due to the change of the number of particles. In our cosmological context, this change is due to the transfer of energy from gravitation to matter. Correspondingly the entropy change $dS$, for adiabatic transformation in open systems is,

$$T dS = \frac{h}{n} d(nV) - \mu (\rho V) = T \frac{s}{n} d(nV)$$  \hspace{1cm} (5)

where $\mu = h - T s$ is the chemical potential and $s = \frac{S}{V}$. Therefore the only particle number variations admitted are such that

$$dN = d(nV) \geq 0$$  \hspace{1cm} (6)

The Eq. (5) can be expressed as,

$$\dot{\rho} = \frac{h}{n} \dot{n}$$  \hspace{1cm} (7)

$$p = \frac{n \dot{\rho} - \rho \dot{n}}{\dot{n}}$$  \hspace{1cm} (8)

where the over dot represents time derivative. The creation of matter corresponds to a supplementary pressure $p_c$, which is negative or zero depending on the presence or absence of particle production. Thus Eq. (5) can be written as,

$$d(\rho V) = -(p + p_c) dV = -\tilde{p} dV$$  \hspace{1cm} (9)

where $p$ is the true thermodynamical pressure and

$$p_c = -\frac{h}{n} \frac{d(nV)}{dV} = -\frac{\rho + p}{n} \frac{d(nV)}{dV}$$  \hspace{1cm} (10)

This scenario is applied to cosmology as follows. In the case of an isotropic and homogeneous universe, we choose, for $V$, the value $V = R^3(\tau)$ where $R$ is the scale factor. The above expression becomes,

$$p_c = -\frac{\rho + p}{3nH} (\dot{n} + 3nH)$$  \hspace{1cm} (11)
The conservation equation becomes,

\[
\dot{\rho} = -3H(\rho + p + p_c) = -3H(\rho + p) + \frac{3H(\rho + p)}{3nH} (\dot{n} + 3nH) \quad (12)
\]

\[
\dot{\rho} = \frac{\dot{n}}{n} (\rho + p) \quad (13)
\]

with

\[
\frac{\dot{S}}{S} = \frac{\dot{N}}{N} = \frac{\dot{n} + 3nH}{n} \quad (14)
\]

The balance equation for particle number density is,

\[
N^i = n u^i, \quad N^a = \dot{n} + 3nH = n\Gamma \quad (15)
\]

with

\[
N = n a^3, \quad \text{and} \quad \Gamma = \frac{\dot{N}}{N} = \frac{\psi}{n} \quad (16)
\]

where \( p \) is thermostatic pressure, \( u^i \) is fluid 4-velocity, \( n \) is particle number density, and \( \Gamma \) is matter creation rate. For \( \Gamma > 0 \), there is particle production, and \( \Gamma < 0 \) corresponds to annihilation.

The above-mentioned macroscopic phenomenological approach is not completely self consistent since it cannot determine the expression for the rate of particle creation, which is an open degree of the scheme.

4. Effect of Matter Creation on Brane Worlds

The existence of some kind of negative-pressure dark energy is usually assumed to explain the cosmic acceleration. Vacuum energy or a cosmological constant is the simplest dark energy candidate. It has been known that matter creation may lead to negative pressures.\(^{22}\) The modified Friedmann equation on RSII brane world indicates that the universe evolves slowly in the very early times. The process of matter creation may add to the accelerated expansion and thus can compensate to the effect of brane world at early times. A scenario with particle production driving cosmic acceleration was proposed by Zimdahl et al.\(^{23}\) Using a phenomenological macroscopic description of matter creation,\(^1\) we investigate this scenario in the brane world context. However, we do not consider that particle creation as the only source for cosmic acceleration. It has been shown based on the supernova data that matter creation alone does not drive cosmic acceleration.\(^{24}\) Our motivation is the following: the modified Friedmann equation on RSII brane world shows that there is transition for the universe evolution between an early high energy regime characterized by the behavior \( R \propto t^{1/3(1+\omega)} \) and a late low energy regime characterized by the standard evolution \( R \propto t^{2/3(1+\omega)} \).\(^{25,18}\) The matter creation process due to its negative pressure can be the cause of this transition. Our attempt is to consider the possible effects on dynamics of the early universe due the matter creation.
5. Effect of Adiabatic Matter Creation on the RSII Brane Model

The field equations on the RS II brane model are,\(^{26}\)

\[
H^2 = \frac{R^2}{R_0^2} = \frac{\kappa^2}{3} \rho \left(1 + \frac{\rho}{2\sigma}\right) + \frac{\Lambda}{3} + \frac{C}{R^3} - \frac{k}{R^2},
\]

\[
\dot{H} = \frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} = -\frac{\kappa^2}{2} \rho \left(1 + \omega\right) \left(1 + \frac{\rho}{\sigma}\right) - \frac{2C}{R^2} + \frac{\Lambda}{3}.
\]

(17)

(18)

where \(\kappa^2 = 8\pi G\), and \(\omega = \gamma - 1 = \frac{p}{\rho}\) for perfect fluid. With particle creation, creation pressure, Eq. (11), is added to thermostatic pressure \(p\).

\[
p_c = -\left(\rho + p\right) \frac{3nH}{\psi}
\]

(19)

For simplicity we take \(\text{Lambda} = 0\). Also, since the dark radiation term decays rapidly we can put \(C = 0\), Eq. (18) becomes,

\[
\dot{H} = \frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} = -\frac{\kappa^2}{2} \rho \left(1 + \frac{p + p_c}{\rho}\right) \left(1 + \frac{p_c}{\rho}\right)
\]

(20)

The energy conservation equation is unchanged in 5-d setup, which including particle creation, becomes,

\[
\dot{\rho} + 3H (\rho + p + p_c) = 0
\]

(21)

For perfect fluid, using Eq. (19) we get,

\[
\dot{\rho} + 3\frac{\dot{R}}{R} \gamma \rho \left(1 - \frac{\psi}{3nH}\right) = 0
\]

(22)

We take the linear dependence of matter creation rate on Hubble parameter \(H\) as,\(^{27}\)

\[
\psi = \beta 3nH
\]

(23)

where the parameter \(\beta\) is defined in the interval [0, 1] and assumed to be constant. The observational value of \(H_0\) (0.86 \(\leq H_0 \leq 1.91\)) can be translated to the value of \(\beta\). The joint result of SNe Ia and GRB indicate that \(\beta\) is (0.502 \(\pm 0.038\)). The GRB data shows \(\beta\) to be greater than 1/3 (ie.,0.349 \(\pm 0.185\)), and the SNIa data alone shows (0.537 \(\pm 0.040\)).\(^{28}\)

This linear relation implies \(\psi \propto \rho\), at high energy densities. (Recently Freaza et. al.\(^{24}\) have considered a general form for the matter creation in non-brane scenario as, \(\Psi = 3 \beta n H_0 \left(H/H_0\right)^\alpha\), where \(\alpha\) and \(\beta\) are dimensionless constants). Now the solution of Eq. (22) is,

\[
\rho = \rho_0 R^{-q} \text{ where } q = 3\gamma \left(1 - \beta\right)
\]

(24)

Substituting for \(\rho\), and for \(C = k = \Lambda = 0\), Eq. (17) becomes,

\[
\frac{\dot{R}^2}{R^2} - \frac{\kappa^2}{3} \rho_0 R^{-q} - \frac{\kappa^2}{6\sigma} \rho_0^2 R^{-2q} = 0
\]

(25)
Defining \( R^q = X \), this becomes,
\[
\dot{X}^2 = \frac{\kappa^2 \rho_0 \dot{q}^2}{3} X + \frac{\kappa^2 \rho_0^2 \dot{q}^2}{6 \sigma} = A \ X + B
\]  
(26)

Solving we get
\[
t - t_0 = \frac{2}{\sqrt{A}} \sqrt{X + \frac{B}{A}}
\]  
(27)

Using the boundary condition, \( X = R^q = 0 \), when \( t = 0 \), we find the solution for the scale factor as follows,
\[
R = \left( \frac{\kappa^2 \rho_0 \dot{q}^2}{12} t^2 + \frac{\kappa \rho_0 \dot{q}}{\sqrt{6} \sigma} \right)^{\frac{1}{2}}
\]  
(28)

The FRW evolution laws are regained for \( \beta = \frac{1}{2} \), and \( t << \frac{12}{\kappa q \sqrt{6} \sigma} \), which corresponds to high energy densities. Thus we get, for radiation \( R \propto t^{1/2} \), and for dust \( R \propto t^{2/3} \).

Using the phenomenological limit, \( \sigma \geq 1 \ (MeV)^4 \), \( t << \frac{12}{\kappa q \sqrt{6} \sigma} \) gives \( t << 10^{30} \)s. This means that our model is valid for any time in the evolution of the universe. The value of \( q \) depends on \( \beta \). Assuming that the rate of matter creation is insignificant during later stages of the evolution of the universe, the evolution of scale factor reduces to the FRW relation.

### 6. Scalar Field Dynamics

The effect of matter creation may be translated into the dynamics of a minimally coupled scalar field. The effect of negative pressure due to the matter creation is equivalent to a suitable scalar field potential. Simple models of a super accelerated universe include a phantom field, i.e., a minimally coupled scalar field with negative kinetic energy.\(^{30-32}\) It has also been proposed that early inflation and late time acceleration of the universe can be unified in a single theory based on a phantom field.\(^{33}\) Phantom type of matter may also arise from a bulk viscous stress due to particle production.\(^{34}\)

We consider particle production from a phantom field, and derive the corresponding potential. The field equation including the phantom field, \( C \) is,
\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} = 8\pi G \left( \rho_m + P_m \right) U_\mu U_\nu + P g_{\mu\nu} - \partial_\mu C \partial_\nu C + \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} \partial_\alpha C \partial_\beta C \right) \]  
(29)

The RHS can be written as, \( 8\pi G \left( T_{\mu\nu}^{\text{fluid}} + T_{\mu\nu}^{\text{phantom}} \right) \), then the sum \( T_{\mu\nu} = T_{\mu\nu}^{\text{fluid}} + T_{\mu\nu}^{\text{phantom}} \) is conserved by the Bianchi identity but the individual terms are not. This was interpreted in the Steady State theory\(^{35}\) as creation of matter by the creation field in order to maintain a constant matter density.

The conservation equation, assuming Friedmann metric, is,
\[
\dot{\rho} + 3H (\rho + p) = 0
\]  
(30)
We assume the total energy and pressure consist of matter and phantom field components as,

\[ \rho = \rho_m + \rho_\phi \] (31)

\[ \dot{\rho}_m + \dot{\rho}_\phi + 3H(\rho_m + \rho_\phi + p_m + p_\phi) = 0 \] (32)

\[ \dot{\rho}_m + 3H \left( \rho_m + p_m + \frac{1}{3H} \dot{\rho}_\phi + \rho_\phi + p_\phi \right) = 0 \] (33)

To obtain a phenomenological classical description of the essentially quantum creation process, we assume that the creation pressure is proportional to the energy density and pressure of the scalar field as,

\[ p_c = \frac{1}{3H} \dot{\rho}_\phi + \rho_\phi + p_\phi \] (34)

Now the Eq. (33) becomes,

\[ \dot{\rho}_m + 3H (\rho_m + p_m + p_c) = 0 \] (35)

The expression for creation pressure, from Eq. (11) is,

\[ p_c = -(\rho_m + p_m) \frac{\dot{\phi}}{3mH} = -(\rho_m + p_m) \beta \] (36)

For perfect fluid matter, \( p_m = (\gamma - 1) \rho_m \)

\[ \dot{\rho}_m + 3 \frac{\dot{R}}{R} \gamma \rho_m (1 - \beta) = 0 \] (37)

The solution is,

\[ \rho_m = \frac{\rho_m(1)}{R^{q}} \] (38)

where \( q = 3\gamma (1 - \beta) \) For phantom fields,\(^{36,30}\) the energy density,

\[ \rho_\phi = -\frac{1}{2} \dot{\phi}^2 + V(\phi) \] (39)

and pressure,

\[ p_\phi = -\frac{1}{2} \dot{\phi}^2 - V(\phi), \] (40)

where \( V \) is the potential of phantom field. The scalar field equations are,

\[ \frac{\partial}{\partial x^i} (\sqrt{-g} g^{ij} \frac{\partial \phi}{\partial x^j}) = 0 \] (41)

\[ \frac{\partial}{\partial t} (R^3 \dot{\phi}) = 0 \] (42)

The solution is,

\[ \dot{\phi} = \frac{\phi_0}{R^3} \] (43)
Using this solution and Eqs. (24), (25) and (26) in Eq. (23) we get the potential as,

\[ V = \frac{3\gamma \rho_m \beta}{q} \frac{1}{R^q} = \frac{3\gamma \rho_m \beta}{q} \left( \frac{\phi}{\phi_0} \right)^{q/3} \]  

(44)

The modified Friedmann equation, Eq. (1), for very high energy densities becomes,

\[ H^2 = \frac{\dot{R}^2}{R^2} = \frac{\kappa^2}{6\sigma} \rho^2 \]  

(45)

Using Eq. (31) for \( \rho \),

\[ \frac{\dot{R}}{R} = \pm \frac{\kappa}{\sqrt{6\sigma}} (\rho_m + \rho_\phi) \]  

(46)

We can write this equation as,

\[ \frac{\dot{R}}{R} - A\frac{R^q}{R^q} + B = 0 \]  

(47)

where

\[ A = \frac{\kappa \rho_m}{\sqrt{6\sigma}} \left( 1 + 3\gamma \beta \frac{1}{q} \right) = \frac{3\kappa \rho_m \gamma}{q\sqrt{6\sigma}}, \quad B = \frac{\kappa \phi_0^2}{2\sqrt{6\sigma}} \]  

(48)

Defining, \( R^q = X \) where \( q = 3\gamma(1 - \beta) \), the above equation can be integrated as,

\[ \int \frac{dX}{Aq - BqX^{1-6/q}} = \int dt \]  

(49)

This integral can be solved for a given value of \( q \). With \( \beta = 1/2 \), (the value which gives the FRW law for the RSII case), and \( \gamma = 4/3 \) (radiation), \( q \) becomes 2, thus the solution of Eq. (49) is,

\[ t - t_0 = \frac{X}{Aq} - \frac{Bq \arctan \left( \frac{\sqrt{Aq} X}{B} \right)}{(Aq)^{3/2} \sqrt{B}} \]  

(50)

Neglecting the second term in comparison with the first, and \( t_0 = 0 \) we get,

\[ R = 2A \frac{1}{2} \frac{3\kappa \rho_m \gamma}{\sqrt{6\sigma}} t^{1/2} \]  

(51)

Substituting in Eq. (43) and solving for \( \phi \) we get,

\[ t = \frac{4\phi_0^2 (6\sigma)^3}{(3\kappa \rho_m \gamma)^3} \frac{1}{\phi^2} \]  

(52)

Using Eqs. (44), (51) and (52), we get the expression for the potential in terms of phantom field as,

\[ V = \frac{9\beta \kappa^4 (\rho_m \gamma)^5}{32\sigma^2 \phi_0^5} \phi^2 \]  

(53)

This potential of the phantom field gives the same evolution for the scale factor as produced by the adiabatic matter creation, in the high density regime. In this sense the phantom field model is equivalent to the adiabatic matter creation model discussed previously in section 4.
7. Gauss-Bonnet brane model

Gauss-Bonnet (GB) combination arises as the leading order for quantum corrections in the heterotic string effective action. In five dimensions it represents the unique combination of curvature invariants that leads to second-order field equations, linear in the second derivatives in the metric tensor, and it is ghost-free.\textsuperscript{37–44} The GB correction to the Einstein-Hilbert action will disrupt mildly the degeneracy between tensor and scalar perturbations.\textsuperscript{45} Recently there is a renewed interest upon this scenario. Here we consider the effect of adiabatic matter creation in this scenario.

In the RSII scenario with a GB term, the effective Friedmann equation describing the motion of brane containing general perfect fluid may be derived from a generalization of the Birkhoff’s theorem.\textsuperscript{46} The 5-dimensional bulk action for the GB braneworld scenario is given by,

\[ S = \frac{1}{2\kappa_5^2} \int_M d^5x \sqrt{-g} \left( R - 2\Lambda^{(5)} + \alpha [GB] \right) + S_{\partial M} + S_m \]

where $[GB] = (R^2 - 4 R_{AB} R^{AB} + R_{ABCD} R^{ABCD})$ is the Gauss-Bonnet term, $\alpha \geq 0$ with dimension (mass)$^2$ is the Gauss-Bonnet coupling, $\Lambda^{(5)} < 0$ is the bulk cosmological constant and $\kappa_5^2 = 8\pi M_5^{-3}$ determines the 5d Planck scale. The $S_{\partial M}$ is the boundary term that is required to cancel normal derivatives of the metric tensor which arises when varying the action with respect to the metric, and $S_m$ is the action for matter confined on the brane. We assume a Z2 symmetry across the brane. We will consider the case that a perfect fluid matter source with density $\rho$ is confined to a brane with tension $\sigma$.

A new constant is defined, $b = (1 + \frac{4}{3} \alpha \Lambda^{(5)})^{3/2}$. Since $\Lambda^{(5)} < 0$, there is an upper bound on $\alpha$, $\alpha \leq \frac{3}{4|\Lambda^{(5)}|} = \alpha_u$. In order that the standard Friedmann equation be recovered at sufficiently low energy scales ($\rho << \sigma$), $\kappa_4^2 = \frac{8\pi}{M_4^2} = \frac{\kappa_5^2}{6\beta^2}$, where $M_4$ is the 4d Planck scale. Introducing a dimensionless variable $\chi$, the modified Friedmann equations are

\[ \rho + \lambda = \left( \frac{\sigma b^{1/3}}{3 \alpha \kappa_4^2} \right)^{1/2} \sinh \chi \]

and,

\[ H^2 = \frac{1}{4\alpha} \left[ b^{1/3} \cosh \left( \frac{2\chi}{3} \right) - 1 \right] \]

where $\sigma$ is the brane tension. In the high energy approximation, $\rho >> \sigma$, we can write it as,

\[ H^2 = A \rho^{2/3} \]

In order for that the 4d effective cosmological constant vanishes, the brane tension should satisfy $\sigma = \frac{1}{4} \frac{b^{1/3}}{\alpha \kappa_4^2}$. We can see that $\sigma \approx |\Lambda^{(5)}|/\kappa_4^2$ for any $0 \leq \alpha \leq \alpha_u$. 


7.1. Mater Creation on GB Model

Following a similar calculation as in RS II model in section 5, with a substitution \( X = R^q \) in Eq. (57), we get

\[
\dot{X}^2 = \frac{4q^2 \rho_0^{2/3} A}{9} X
\]  

The solution with the boundary condition, \( X = R^q = 0 \) when \( t = 0 \), gives

\[
t = 2 \sqrt{\frac{X}{A}}
\]

\[
R = \left( \frac{q^2 \rho_0^{2/3} A}{9} \right)^{\frac{2}{q}} t^\beta
\]

where \( q = 3\gamma \left( 1 - \frac{\psi}{3nH} \right) \) and \( \beta = \frac{\psi}{3nH} \).

This evolution is for high energy densities \( t << 1 \), since we considered the modified Friedmann equation for that regime. The case of \( \beta = 1/2 \), which was considered in RSII model, gives for radiation and dust as \( R \propto t^{3/2} \) and \( R \propto t^2 \) respectively. These are modified evolution equations compared to standard cases without matter creation.

8. DGP Brane Model

An alternative scenario to the RS II brane model was proposed by Dvali, Gabadadze and Porrati (DGP). In this model the extra dimension is infinitely large. The key feature is the presence of a four-dimensional curvature scalar in the action. The DGP model predicts that 4D Newtonian gravity on a brane world is regained at distances shorter than a given crossover scale \( r_c \) (high energy limit), whereas 5D effects become manifest above that scale (low energy limit). The interesting feature is that the effective cosmological constant on the brane can be extremely reduced in contrast to the case of the Randall-Sundrum model even if a bulk cosmological constant and a brane tension are not fine-tuned. Also, the model can explain late-time acceleration without having to invoke a cosmological constant or quintessential matter. Here we consider a generalized DGP brane model of Sahni and Shtanov.

The action of the generalized DGP model is,

\[
S = S_{\text{bulk}} + S_{\text{brane}}
\]  

where,

\[
S_{\text{bulk}} = \int d^5 X \sqrt{-g^{(5)}} \left[ \frac{1}{2\kappa_5^2} R^{(5)} + L_m^{(5)} \right], \quad \text{and,}
\]

\[
S_{\text{brane}} = \int d^4 x \sqrt{-g} \left[ \frac{1}{\kappa_5^2} K^{\perp} + L_{\text{brane}} (g_{\alpha\beta}, \chi) \right]
\]
Here $\kappa_5^2$ is the 5-dimensional gravitational constant, $R^{(5)}$ and $L_m^{(5)}$ are the 5-dimensional curvature scalar and the matter Lagrangian in the bulk, respectively. $x^\mu (\mu = 0, 1, 2, 3)$ are the induced 4-dimensional coordinates on the brane, $K^\pm$ is the trace of extrinsic curvature on either side of the brane and $L_{brane}(g_{\alpha\beta}, \chi)$ is the effective 4-dimensional Lagrangian, which is given by a generic functional of the brane metric $g_{\alpha\beta}$ and matter fields $\chi$ on the brane.

The brane Lagrangian consists of the following terms,

$$L_{brane} = \frac{1}{2\kappa_4^2} R - \sigma + L_m$$

(64)

where $\sigma$ is the brane tension.

The 5-D bulk space includes only a cosmological constant $\Lambda^{(5)}$. It is a generalized version of Dvali-Gabadadze-Porrati model, which is obtained by setting $\sigma = 0$ as well as $\Lambda^{(5)} = 0$. From the field equation induced on the brane, the effective 4-dimensional cosmological constant on the brane is,

$$\Lambda = \frac{\Lambda^{(5)}}{2} + \frac{1}{2}\kappa_4^2 \left( R - \frac{4}{3} \rho \right)$$

(65)

At high energy limit, $\rho >> \rho_0$, the modified Friedmann equation Eq. (65) reduces to

$$H^2 + \frac{k}{R^2} \approx \frac{\kappa_4^2}{3} \left( \rho + \frac{\delta}{\kappa_4^2 R^4} \sqrt{2\eta \rho_0 \rho} \right)$$

(66)

We will use this evolution law to investigate the effects of adiabatic matter creation.

### 8.1. Matter Creation on DGP Model

We assume the expression, $\psi = \beta 3n H$, for matter creation. Defining, $q = 3\gamma (1 - \beta)$, and inserting the solution, $\rho = \rho_0 R^{-q}$, of the conservation equation (with modified creation pressure term) in Eq. (68), we get

$$\frac{\dot{R}^2}{R^2} - \frac{\rho_0 \kappa_4^2}{3} R^{-q} - \frac{\delta \kappa_4^2 \sqrt{2\eta \rho_0}}{3} R^{-q/2} = 0$$

(69)
Defining $R^{q/2} = X$, we get

$$\frac{\dot{R}}{R} = \frac{\dot{X}}{X^{q/2}}$$

(70)

Substituting this in the above equation,

$$\frac{\dot{X}^2}{X^2 q^2 / 4} - \frac{\kappa_4^2 \rho_0}{3} \frac{1}{X^2} - \delta \frac{\kappa_4 \sqrt{2} \eta \rho_0}{3} \frac{1}{X} = 0$$

(71)

This equation can be written as,

$$\dot{X}^2 = AX + B$$

(72)

Where $A = \frac{\delta \kappa_4^2 \sqrt{2} \eta \rho_0}{3} \frac{q^2}{4}$ and $B = \frac{\kappa_4^2 \rho_0}{3} \frac{q^2}{4}$. The solution of Eq. (72) is

$$t - t_0 = \frac{2}{\sqrt{A}} \sqrt{X + B}$$

(73)

With the boundary condition, $X = R^q = 0$, when $t = 0$, the constant $t_0$ becomes $-\frac{2}{\sqrt{A}}$. Therefore we get,

$$R = \left( \delta \frac{q^2 \sqrt{2} \eta \rho_0}{12 \mu^2} t^2 + \frac{q}{2} \sqrt{\frac{\rho_0}{3 \mu^2}} t \right)^{\frac{1}{q}}$$

(74)

where $q = 3\gamma \left(1 - \frac{\beta}{3n_H}\right)$, $\delta = \pm 1$ and $\beta = \frac{\psi}{n_H}$.

For $\beta = \frac{1}{2}$ (special solution in the case of RSII), and for $t << \frac{\sqrt{\mu}}{q \eta}$, which corresponds to high energy densities, we get,

for radiation, $R \propto t$  \hspace{1cm} (75)

for dust, $R \propto t^{4/3}$  \hspace{1cm} (76)

These are new evolution equations different from FRW laws. For $t >> \frac{\sqrt{\mu}}{q \eta}$ (later stage) the rate of matter creation is assumed to be insignificant, thus the evolution of scale factor is unaffected.

9. Discussion

The adiabatic matter creation rate is found to affect the evolution of scale factor and energy density of the universe, in all the cases we have considered. The modified Friedmann equation on brane world shows that the universe evolves slowly in the very early times. Thus there is a transition of the universe evolution between an early high-energy regime and a late low energy regime. The negative pressure due to the matter creation process can explain this transition. For the RSII model the standard FRW evolution law can be obtained for a matter creation rate $\beta = 1/2$. This value is consistent with the observed data. We have derived a scalar field equivalent formulation using a phantom field. A potential is derived which would give the dynamics same as that produced by adiabatic matter creation. In the case
of Gauss-Bonnet and warped DGP models there are modifications in evolution of the universe with matter creation. For later stages of the evolution of the universe, the rate of matter creation is insignificant, thus the evolution of scale factor is unchanged.

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