Couplings of the Rho Meson in a Holographic dual of QCD with Regge Trajectories

Tao Huang\textsuperscript{1,*} and Fen Zuo\textsuperscript{2,1,†}

\textsuperscript{1}Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
\textsuperscript{2}Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, China

Abstract

The couplings $g_{\rho HH}$ of the $\rho$ meson with any hadron $H$ are calculated in a holographic dual of QCD where the Regge trajectories for mesons are manifest. The resulting couplings grow linearly with the exciting number of $H$, thus are far from universal. A simple argument has been given for this behavior based on quasi-classical picture of excited hadrons. It seems that in holographic duals with exact Regge trajectories the $g_{\rho HH}$ universality should be violated. The $\rho$-dominance for the electromagnetic form factors of $H$ are also strongly violated, except for the lowest state, the pion. Quite unexpected, the form factor of the pion is completely saturated by the contribution of the $\rho$. The asymptotic behavior of the form factors are also calculated, and are found to be perfectly accordant with the prediction of conformal symmetry and pertubative QCD.

\textsuperscript{*}Email: huangtao@mail.ihep.ac.cn.
\textsuperscript{†}Email: zuof@mail.ihep.ac.cn.
I. INTRODUCTION

In 1998, Maldacena advocated the famous duality between string theory on Anti-de Sitter (AdS) space and certain conformal field theory (CFT) on the boundary, now known as AdS/CFT correspondence [1]. Based on this duality, it was shown by Polchinski that the power law behavior of the high-energy fixed angle scattering of glueballs in confining gauge theories could be derived nonperturbatively in the dual theory [2], and was found to be precisely as the simple dimensional prediction in QCD [3]. Since then, various perturbations have been introduced to the original AdS background to produce supergravity duals with a mass gap, confinement, and chiral breaking, with the purpose of establishing an exact dual of the true QCD. One can introduce a simple cutoff in the radical coordinate to simulate color confining, and get the so called the hard-wall model [2]. With only one parameter for the cutoff, the lower excited hadron spectrum in this model are found to be perfectly agree with the physical states [4]. Moreover, chiral breaking can be modeled rather well in this model too, with two more parameters: the current quark mass and the quark condensate [5]. One can also introduce fundamental quarks by adding $D7$ branes to the original background, resulting the $D3/D7$ model, in which the chiral breaking can be incorporated rather naturally [6]. However, in both models, the masses of highly excited mesons grow linearly with the exciting number, which conflict with the familiar Regge trajectory [7], where the mass square grows linearly with the exciting number.

The couplings $g_{\rho HH}$ of the $\rho$ meson to any hadron $H$ are shown to be quasi-universal in these AdS/QCD models, even when the $\rho$-dominance is violated [8]. Actually, as argued in Ref. [7], this is an ”accidental” universality, since the contributions of excited vector mesons are of the same order as the rho meson do, but they are sign-alternating and compensate each other, resulting an apparent $\rho$-dominance. However, since the meson spectrum obtained in these models has a wrong dependance on the exciting number, it was argued in Ref. [7] that the $g_{\rho HH}$ universality might be implement in a wrong way.

The asymptotically linear Regge trajectories for the mesons can be obtained in the dual theory by adding a non-constant dilaton field $\Phi$ to the original AdS$_5$ background [9]. The Though ad hoc, the special profile for the dilaton seems to be necessary to guarantee the linear Regge behavior [10]. One can also introduce a gaussian warp factor to the AdS$_5$ metric, which is shown to be equivalent to the previous background when studying the
meson spectrum \[11\]. We would like to discuss if the \(g_{\rho HH}\) universality holds in this modified model, which is often referred as soft-wall model or harmonic oscillator model, or more generally in AdS/QCD models with linear Regge trajectories. We calculate the couplings of \(g_{\rho HH}\) in this modified model and the results show that the couplings actually grow linearly with the exciting number of \(H\), thus far from been universal. We can give a simple explanation for this dependance based on the quasi-classical picture of excited hadrons. Thus it seems that the \(g_{\rho HH}\) universality in Ref.\[8\] is somehow connected to the wrong dependance of the meson mass on the exciting number, and it does not hold any more when exact Regge behavior is obtained. The \(\rho\)-dominance for the electromagnetic form factors of \(H\) are also strongly violated, except for the lowest state, the pion. Quite unexpected, the form factor of the pion is completely saturated by the contribution of the \(\rho\) meson. The asymptotic behavior of the form factors are also calculated, and are found to be perfectly accordant with the prediction of conformal symmetry and pertubative QCD.

The paper is organized as follows. In the next section we give a brief review of the soft-wall model and derive the string modes for the scalar hadron states. In sec.III we give explicit results of \(g_{\rho HH}\) and show that the \(g_{\rho HH}\) universality doesn’t hold any more in this model. Then we go on to discuss \(\rho\)-dominance and asymptotic behavior of the form factors in Sec.IV and V, and give a simple summary in the last section.

II. A BRIEF REVIEW OF SOFT-WALL MODEL AND STRING MODES FOR SCALARS

We are in position to work in the background proposed in \[9\]. That is, the AdS metric is given by

\[
ds^2 = e^{2A(z)}(dz^2 + \eta_{\mu\nu}dx^\mu \, dx^\nu)
\]

with \(A(z) = -\log z\) and turn on a non-constant dilaton field given by \(\Phi(z) = z^2\). We are going to calculate the \(\rho\)'s couplings \(g_{\rho HH}\) to the scalar states \(H\) as a function of the excitation level \(a\) and the twist \(\tau\) of the corresponding operator, as in Ref.\[8\]. We have chosen to consider the twist but not the naive dimension due to the arguments in Ref.\[12\]. First, we have to solve the equation of motion for the scalar field \(s_H(z)\) dual to \(H\). Actually this has been done in discussing the spectrum of the scalar glueball \[13\]. It is found that after
applying a Bogoliubov transformation

\[ \psi(z) = e^{-\frac{B(z)}{2}}s(z) \]

with \( B(z) = \Phi(z) - 3A(z) \), the function \( \psi(z) \) satisfies one dimensional Schrödinger equation:

\[ -\psi''(z) + V_H(z)\psi(z) = -q^2\psi(z). \]  

(2)

For the normalizable mode \( q^2 = -m^2 \) gives the mass of the corresponding hadron state. The potential \( V_H(z) \) is given by

\[ V_H(z) = z^2 + 15/4z^2 + 2 + m_5^2/z^2, \]  

(3)

with \( m_5^2 = \tau(\tau - 4) \) the AdS mass of the dual field. Then one can easily get the eigenfunctions

\[ \psi_a(z) = e^{-z^2/2}z^{\tau-3/2} \sqrt{\frac{2a!}{(\tau + a - 2)!}} L_a^{\tau-2}(z^2), \]  

(4)

where \( L_a^m \) are associated Laguerre polynomials, and the corresponding eigenvalues are \( m^2 = 4a + 2\tau \). After making the Bogoliubov transformation we get the original mode function:

\[ s_a(z) = z^{\tau} \sqrt{\frac{2a!}{(\tau + a - 2)!}} L_a^{\tau-2}(z^2). \]  

(5)

Certainly \( s_a(z) \rightarrow z^\tau \) as \( z \rightarrow 0 \) as it should be. Notice that if supposing that the variation of \( \tau \) is due to the excitation of the orbital momentum as in Ref.\[4\], \( \tau = 2 + L \), one can get the linear dependence of the meson mass square on the orbital momentum

\[ m^2 = 4a + 2L + 4 \]  

(6)

though the slopes for the radical number and the orbital momentum are different. As in Ref.\[4\], we can identify the \( L = 0 \) states with pion and its radical exciting states. Since the chiral breaking has not been included, their masses are degenerate with the corresponding vector mesons \[9\].

On the other hand, the solutions for the \( \rho \) mesons have been derived in Ref.\[9\], and are given by

\[ v_n(z) = z^2 \sqrt{\frac{2n!}{(1 + n)!}} L_n^1(z^2) \]  

(7)

with squared masses of the \( \rho s \) \( m_n^2 = 4(n + 1) \), and the decay constants:

\[ F_{\rho_n}^2 = \frac{1}{g_5^2} [v_n''(0)]^2 = \frac{8(n + 1) g_5^2}{g_5^2} \]  

(8)

where \( g_5 \) is the 5D coupling of the dual vector field for rho. Here \( n \) is supposed to be the radical number and \( n = 0 \) gives the \( \rho \) meson.
III. THE $\rho$ COUPLINGS $g_{\rho HH}$

Now we can calculate the couplings $g_{\rho HH}$ using the formula [14]:

$$g_{\rho HH} = g_5 \int \mu_H(z) v_0(z) s_H(z)^2$$

(9)

with $\mu_H(z)$ in the above background given by

$$\mu_H(z) = e^{-B(z)} = e^{-\Phi(z)+3A(z)}.$$  

(10)

The resulting couplings are:

$$F_\rho g^{(2)}_{\rho a a}/m_\rho^2 = 1, 3, 5, 7, 9, ...$$

(11)

for $a = 0, 1, 2, 3, 4$; and

$$F_\rho g^{(\tau)}_{\rho 00}/m_\rho^2 = 1, 2, 3, 4, 5, ...$$

(12)

for $\tau = 2, 3, 4, 5, 6$. It seems that $g_{\rho HH}$ actually grows linearly with the exciting number $a$ and the twist dimension $\tau$ of $H$. This is quite different from the behavior of $g_{\rho HH}$ in the hard-wall and the D3/D7 model considered in Ref. [8], where $g_{\rho HH}$ are shown to be quasi-universal and lie within a narrow band near $m_\rho^2/F_\rho$. Notice that in both the models there the meson masses grow linearly with the radical number, so the $g_{\rho HH}$ universality in these models seems to be connected to this wrong dependance. Our results show that when the dependance is corrected, the $g_{\rho HH}$ universality doesn’t hold any more.

To make this more clear, let’s analyze the integral in Eq.(9) carefully. As shown in Ref.[8], the $g_{\rho HH}$ universality in generic AdS/QCD models is based on two facts. First, there is a maximal value $z = z_{\text{max}}$ beyond which scale-invariance is badly broken; second, being the lowest mode of a conserved current, the $\rho$ meson should be structureless and have no nodes, and must satisfy Neumann boundary conditions at $z = z_{\text{max}}$. In other words, $v_0(z)$ will always increase monotonously with $z$ and reach it’s maximal value at $z = z_{\text{max}}$. Thus the integral in Eq.(9) in always dominated by the contributions in the region $z \sim z_{\text{max}}$ where $v_0(z)$ varies slowly and has a typical value $\hat{v}_0(z)$. Replacing $v_0(z)$ by $\hat{v}_0(z)$ in Eq.(9) and using the normalization condition for $s_H(z)$, we finally get the universal coupling $g_{\rho HH}$ and can actually prove that it equals to the expected value $m_\rho^2/F_\rho$ from $\rho$-dominance [8]. However, in the soft-wall model, there isn’t an absolute cutoff $z_{\text{max}}$. Based on the quasi-classical arguments in Ref.[7], one can expect that this cutoff should be proportional to the length
$L_n = M_n/\sigma$ of the flux tube of the excited meson, where $\sigma$ is the tension of the flux tube. Note $v_0(z) \sim z^2$ and the linear relation of mass square $m_n^2 \sim n$, one should then replace $v_0(z)$ by $z^2 \sim L_n^2 \sim n$. Thus one gets a linearly increasing coupling $g_{\rho HH}$.

IV. VIOLATION OF THE VECTOR MESON DOMINANCE

It is well known that, the $\rho$ meson coupling universality can be induced from the VMD hypothesis, that is the $\rho$ meson gives the dominant contribution to the electromagnetic form factor of the various hadrons. Now since the $g_{\rho HH}$ universality doesn’t hold any more, we can conclude that the $\rho$ meson dominance must be violated at the same time. Then we go ahead to calculate the couplings of the excited $\rho$ mesons with $H$. The calculation is straightforward, we just need to replace the $\rho$ mode in Eq.(9) by the corresponding exciting $\rho$ states. For the radical exciting states, the results are as follows: for $n = 0, 1, 2, 3, 4, ...$

$$F_{\rho_n}g_{\rho,00}^{(2)}/m_{\rho_n}^2 = 1, 0, 0, ...$$
$$F_{\rho_n}g_{\rho,11}^{(2)}/m_{\rho_n}^2 = 3, -4, 2, 0, 0...$$
$$F_{\rho_n}g_{\rho,22}^{(2)}/m_{\rho_n}^2 = 5, -14, 22, -18, 6, 0, 0...$$

For the orbital exciting states, we have:

$$F_{\rho_n}g_{\rho,00}^{(2)}/m_{\rho_n}^2 = 1, 0, 0, ...$$
$$F_{\rho_n}g_{\rho,00}^{(3)}/m_{\rho_n}^2 = 2, -1, 0, 0, ...$$
$$F_{\rho_n}g_{\rho,00}^{(4)}/m_{\rho_n}^2 = 3, -3, 1, 0, 0, ...$$

with all the neglected terms vanishing. From the result follows that, the electromagnetic form factor of the pion is completely saturated by the contribution of the $\rho$ meson, given exactly the phenomenologically successful VMD fit

$$F_{\pi}^{VMD}(Q^2) = 1/(1 + Q^2/m_{\rho}^2)$$

The VMD in the electromagnetic form factor of the pion was also found in the hard-wall model in Ref.[13] in considering chiral symmetry breaking. Except for the pion, there is no evidence of VMD in the form factors of the other hadrons. Actually, the contribution of some exciting state may exceed that of $\rho$ by several times. The vector meson dominance is certainly violated very badly, just as what we have predicted from the previous calculation.
However, the contributions of various $\rho$ mesons indeed show a sign-alternating feature, which is supported by QCD analysis \[7\].

V. THE ASYMPTOTIC BEHAVIOR OF THE ELECTROMAGNETIC FORM FACTOR

Another issue we want to emphasize is the asymptotic behavior of the form factor. This can be easily obtained from the previous results and the decomposition formula \[14\]:

$$F_H(q^2) = \sum_n F_{\rho n} g_{\rho n H H} \frac{q^2 + m^2_{\rho n}}{q^2}$$ \(16\)

We find, for the hadrons created by the lowest twist operators, all the radical states exhibit the same asymptotic behavior exactly:

$$F_{a a}^{(\tau=2)}(q^2) \rightarrow (\tau - 1) m^2_\rho / q^2.$$ \(17\)

This is obvious from the VMD for the pion, but not so straight for the other radical states. The asymptotic behavior of the pion has been analyzed utilizing the Light-Cone wave function dual to the string modes in this model, and the same results was obtained in Ref. \[16\] and further in Ref. \[17\]. For the higher twist states, the results are radical number dependant, and we give the results for the first few states only:

$$F_{00}^{(\tau=3)}(q^2) \rightarrow (\tau - 1) \frac{m^2_\rho}{q^2} \left( \frac{m^2_{\rho_1}}{q^2} \right)$$ \(18\)

$$F_{11}^{(\tau=3)}(q^2) \rightarrow (\tau - 1) \frac{m^2_\rho}{q^2} \left( \frac{m^2_{\rho_2}}{q^2} \right)$$

$$F_{22}^{(\tau=3)}(q^2) \rightarrow (\tau - 1) \frac{m^2_\rho}{q^2} \left( \frac{m^2_{\rho_3}}{q^2} \right)$$

$$F_{00}^{(\tau=4)}(q^2) \rightarrow (\tau - 1) \frac{m^2_\rho}{q^2} \left( \frac{m^2_{\rho_1} m^2_{\rho_2}}{q^2} \right)$$

$$F_{11}^{(\tau=4)}(q^2) \rightarrow (\tau - 1) \frac{m^2_\rho}{q^2} \left( \frac{m^2_{\rho_1} m^2_{\rho_2}}{q^2} \right)$$

$$F_{22}^{(\tau=4)}(q^2) \rightarrow (\tau - 1) \frac{m^2_\rho}{q^2} \left( \frac{m^2_{\rho_1} m^2_{\rho_2}}{q^2} \right)$$

All these coincide with the prediction $1/q^{(2\tau-2)}$ from the conformal symmetry \[2\] and the pertubative QCD analysis \[3\]. From the above relations one can guess the asymptotic
behavior for generic states with radical number $n$ and arbitrary twist $\tau$:

$$F_{nn}^{(\tau)}(q^2) \rightarrow (\tau - 1) \left( \frac{m^2_{\rho}}{q^2} \right)^{\tau - 1} \prod_{k=n}^{k=n+\tau-3} \frac{m^2_{p_k}}{q^2}. \quad (19)$$

Or using $m^2_{p_n} = 4n + 4$ this can be simplified as:

$$F_{nn}^{(\tau)}(q^2) \rightarrow (\tau - 1) \left( \frac{n + \tau - 2)!}{(n + 1)!} \right) \left( \frac{m^2_{\rho}}{q^2} \right)^{\tau - 1}. \quad (20)$$

For $n = 0$ this reduces to

$$F_{nn}^{(\tau)}(q^2) \rightarrow (\tau - 1)! \left( \frac{m^2_{\rho}}{q^2} \right)^{\tau - 1}. \quad (21)$$

which has been derived analytically in Ref. [17].

VI. SUMMARY

In this paper we employ the AdS/QCD models with linear Regge trajectories and calculate the couplings of $g_{\rho HH}$ in this typical holographic model, which is often referred as soft-wall model or harmonic oscillator model. The calculated result of the couplings $g_{\rho HH}$ shows that $g_{\rho HH}$ grows linearly with the radical number $n$ and the twist of $H$, and thus are far from being universal. A simple argument has been given for this behavior based on quasi-classical picture of excited hadrons. As an inference, the vector meson dominance is strongly violated, except for the pion, of which the form factor is completely saturated by the $\rho$ meson. The contributions of various $\rho$ mesons indeed show a sign-alternating feature, which is supported by QCD analysis [7]. The asymptotic behavior of the form factor for various states are studied and are in accordance with the expectation from conformal symmetry.

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