Current-driven skyrmionium in a frustrated magnetic system

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Magnetic skyrmionium is a skyrmion-like topological spin texture that is formed by two concentric circular domain walls. It can be used as a nanometer-scale non-volatile information carrier, which shows no skyrmion Hall effect due to its special internal structure. Here, we report the static and dynamic properties of an isolated nanoscale skyrmionium in a frustrated magnetic monolayer, where the skyrmionium is stabilized by competing interactions. The nanoscale skyrmionium has a size of \( \sim 10 \) nm, which can be further reduced by tuning perpendicular magnetic anisotropy or magnetic field. It is also found that the nanoscale skyrmionium driven by the damping-like spin-orbit torque shows directional motion with a favored Bloch-type helicity. A small driving current can lead to the transformation of an unstable Néel-type skyrmionium to a metastable Bloch-type skyrmionium. A large driving current may result in the distortion and collapse of the Bloch-type skyrmionium. Our results are not only useful for the understanding of skyrmion dynamics in frustrated magnets, but also to provide guidelines for the design of future spintronic devices based on topological spin textures.

Topological spin textures have aroused significant interest within the field of magnetism and spintronics due to their highly possible spintronic applications. Exemplary topological spin textures are skyrmions and skyrmioniums, which can be stabilized in magnetic materials with the Dzyaloshinskii-Moriya (DM) interaction. Recently, a number of studies have suggested that skyrmions can also be found in frustrated magnetic systems, where competing exchange interactions lead to the formation of skyrmions at certain conditions. The magnetic skyrmion stabilized by frustrated exchange interactions shows different static and dynamic properties to that stabilized by DM interactions, such as circular motion and helicity rotation. However, the physical properties of skyrmioniums stabilized by frustrated exchange interactions remain elusive.

A magnetic skyrmion is formed by a circular domain wall, while a magnetic skyrmionium is formed by two concentric circular domain walls. A single isolated skyrmion carries a integer topological charge, which is defined as

\[
Q = \int \mathbf{m}(\mathbf{r}) \cdot \left( \partial_\mathbf{r} \mathbf{m}(\mathbf{r}) \times \partial_\mathbf{r} \mathbf{m}(\mathbf{r}) \right) \, \mathrm{d}^2 r / 4\pi. 
\]

However, the skyrmionium carries a topological charge of \( Q = 0 \) and it can be seen as a combination of two skyrmions with opposite topological charges, i.e., \( Q = +1 \) and \( Q = -1 \). The skyrmionium can be created by electric and optical methods. Because of the absence of topological charge, a rigid isolated skyrmionium shows dynamics that is independent of its topological structure. Hence, the most important feature is that a skyrmionium with \( Q = 0 \) shows no skyrmion Hall effect, which has been regarded as a promising feature for reliable in-line motion in narrow nanotrack systems. As a result, the skyrmionium can be used as a non-volatile information carrier in spintronic applications, such as the racetrack memory.

In this work, we numerically explore the skyrmionium stabilized by frustrated exchange interactions in a magnetic monolayer with certain perpendicular magnetic anisotropy (PMA). We report the static properties and current-driven dynamics of frustrated skyrmioniums with different in-plane spin configurations.

Our simulations are based on the \( J_1-J_2-J_3 \) classical Heisenberg model on a simple monolayer square lattice, where three competing ferromagnetic (FM) and antiferromagnetic (AFM) Heisenberg exchange interactions lead to exchange frustration. The Hamiltonian is expressed as:

\[
\mathcal{H} = -J_1 \sum_{\langle i,j \rangle} \mathbf{m}_i \cdot \mathbf{m}_j - J_2 \sum_{\langle\langle i,j \rangle\rangle} \mathbf{m}_i \cdot \mathbf{m}_j \\
- J_3 \sum_{\langle\langle\langle i,j \rangle\rangle\rangle} \mathbf{m}_i \cdot \mathbf{m}_j - K \sum_i (m_i^z)^2 \\
- \sum_i B \cdot \mathbf{m}_i + \mathcal{H}_{\text{DDI}},
\]

where \( \mathbf{m}_i \) represents the normalized spin at the site \( i \), \( |\mathbf{m}_i| = 1 \), \( \langle i, j \rangle \), \( \langle\langle i, j \rangle\rangle \), and \( \langle\langle\langle i, j \rangle\rangle\rangle \) run over all the nearest-neighbor (NN), next-nearest-neighbor (NNN), and next-next-nearest-neighbor (NNNN) sites in the square-lattice monolayer, respectively. \( J_1, J_2, \) and \( J_3 \) are the coefficients for the NN, NNN, and NNNN Heisenberg exchange interactions, respectively. \( K \) is the PMA constant. \( B \) is the applied magnetic field. \( \mathcal{H}_{\text{DDI}} \) represents the dipole-dipole interaction (DDI).

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The simulations are carried out by the object oriented micromagnetic framework (OOMMF) package upgraded with homemade $J_1$-$J_2$-$J_3$ exchange interactions module. The spin dynamics is governed by the Landau-Lifshitz-Gilbert (LLG) equation augmented with the damping-like spin-orbit torque, given as

$$\frac{dm}{dt} = -\gamma_0 m \times h_{\text{eff}} + \alpha \left( m \times \frac{dm}{dt} \right) - u m \times (m \times p), \tag{2}$$

where $h_{\text{eff}} = -\delta H / \delta m$ is the effective field, $t$ is the time, $\alpha$ is the Gilbert damping parameter, and $\gamma_0$ is the absolute gyromagnetic ratio. $u = |(\gamma_0 / \mu_0 \epsilon)| \cdot (j \theta_{\text{SH}}/2 \alpha M_S)$ is the spin torque coefficient, where $h$ is the reduced Planck constant, $\epsilon$ is the electron charge, $\mu_0$ is the vacuum permeability constant, $a$ is the lattice constant, $j$ is the applied current density, $\theta_{\text{SH}}$ is the spin Hall angle, and $M_S$ is the saturation magnetization. $p = \pm \hat{y}$ denotes the spin current polarization direction.

The default geometry of the monolayer consists of $51 \times 51$ spins, and the lattice constant is $a = 0.4$ nm (i.e., the mesh size is $0.4 \times 0.4 \times 0.4$ nm$^3$). The default simulation parameters are given as $J_1 = 30$ meV, $J_2 = -0.8$ (in units of $J_1 = 1$), $J_3 = -0.9$ (in units of $J_1 = 1$), $K_a = 0.01$ (in units of $J_1 / a^2$), $B = 0$ (in units of $J_1 / a^2 M_S = 1$) $\theta_{\text{SH}} = 0.2$, $\alpha = 0.3$, $\gamma_0 = 2.211 \times 10^5$ m A$^{-1}$ s$^{-1}$, and $M_S = 580$ kA m$^{-1}$. We have simulated the metastability diagram, which shows the frustrated skyrmionium can be a metastable state for a wide range of $J_2$, $J_3$, and $K$ parameters (see supplemental material). Note that the minimum required value of $J_3$ for stabilizing skyrmioniums decreases with increasing magnitude of $J_2$ since both $J_2$ and $J_3$ are AFM exchange interactions that compete with FM $J_1$.

As shown in Fig. 1, we first study the static properties of a relaxed isolated skyrmionium in the frustrated magnetic monolayer with PMA of $K = 0.01$, where we set $J_2 = -0.8$, $J_3 = -0.9$, and $B = 0$. The spin texture is parametrized by $m(r) = m(\theta, \phi) = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ with $\phi = Q \psi + \eta$, where $\psi$ is the azimuthal angle in the $x$-$y$ plane ($0 \leq \psi < 2\pi$). For the skyrmionium texture, we assume that $\theta$ rotates $2\pi$ for spins from the skyrmionium center to skyrmionium edge. It is worth mentioning that $\theta$ only rotates $\pi$ for the case of skyrmion. Hence, $Q_\psi = \frac{1}{2\pi} \int_C \psi d\phi$ is the vorticity and $\eta$ is the helicity defined mod $2\pi$, which describe the out-of-plane and in-plane structures of a skyrmionium, respectively. Note that the state with $\eta = 0$ is identical to that with $\eta = 2\pi$.

Fig. 1(a) shows spin configurations of relaxed skyrmioniums with $Q = 0$ but different helicity $\eta = 0, \pi/2, \pi, 3\pi/2$. It should be noted that the skyrmionium has two concentric circular domain walls. The in-plane spin configuration of the inner circular domain wall is directly described by the helicity $\eta$, while the helicity of the outer circular domain wall is always equal to $\eta + \pi$. Namely, once the helicity $\eta$ is given, the in-plane spin configurations of both inner and outer circular domain walls are determined.

The energies of relaxed skyrmioniums with $\eta = 0, \pi/2, \pi, 3\pi/2$ are given in Fig. 1(b). The total energy and all constituting energy terms depend on the helicity. The Bloch-type skyrmioniums with $\eta = \pi/2, 3\pi/2$ are more energetically stable than the Néel-type skyrmioniums with $\eta = 0, \pi$. By analyzing the energy difference between the Bloch-type and Néel-type skyrmioniums, it is found that the total energy difference is mainly induced by the DDI, which naturally favors Bloch-type spin configurations (see supplemental material). If we do not consider the effect of DDI, the energy of skyrmionium will be independent of $\eta$ (see supplemental material).

We also study the effect of PMA on the static profile of a relaxed skyrmionium. Figure 2 shows the spin configuration of a Bloch-type skyrmionium with $\eta = \pi/2$, which is relaxed in the frustrated magnetic monolayer for different $K$. It can be seen that the size of skyrmionium decreases with increasing $K$. The diameter of the outer circular domain wall of the skyrmionium is $\sim 9.2$ nm at $K = 0.01$, while it is only $\sim 4.4$ nm at $K = 0.10$. In particular, when $K \geq 0.11$, the skyrmionium with $Q = 0$ is transformed to a skyrmion with $Q = -1$, which is a result of the annihilation of the inner circular domain wall. A further increase in $K$ will lead to the annihilation of the skyrmion. The total energy of the skyrmionium also increases with $K$, however, when the skyrmionium is transformed to a skyrmion, the total energy suddenly drops, indicating the skyrmion is more energetically stable than the skyrmionium. On the other hand, we also study the effect of external perpendicular magnetic field $B$ on the static profile of a relaxed skyrmionium, where we found that the skyrmionium...
FIG. 2. Total energy $E_{\text{Total}}$ and out-of-plane spin component $m_z$ as functions of PMA $K$. The initial state is a relaxed Bloch-type skyrmionium with $\eta = \pi/2$ at the monolayer center. Here, $J_2 = -0.8$, $J_3 = -0.9$, and $B = 0$. Insets show zoomed top views of relaxed states at selected values of $K$.

FIG. 3. (a) Velocities $v$ for isolated Bloch-type skyrmioniums with $\eta = \pi/2, 3\pi/2$ driven by the damping-like spin-orbit torque as functions of the driving current density $j$. The velocity at different $j$ is measured when steady motion is attained. Here, $J_2 = -0.8$, $J_3 = -0.9$, $K = 0.01$, and $B = 0$. (b) Trajectories for isolated Bloch-type skyrmioniums with $\eta = \pi/2, 3\pi/2$ driven by the damping-like spin-orbit torque. Here, $j = 5$ MA cm$^{-2}$. The red and blue arrows indicate the direction of motion. Inset shows top views of current-driven skyrmioniums with $\eta = \pi/2, 3\pi/2$.

nium size can be adjusted by the perpendicular magnetic field within certain range (see supplementary material).

As the Bloch-type skyrmioniums in the frustrated magnetic monolayer are more stable than Néel-type ones, we continue to investigate their dynamics driven by the damping-like spin-orbit torque (see Eq. 2). Here, we assume that the damping-like spin-orbit torque is generated by utilizing the spin Hall effect of a heavy-metal substrate.$^{6,8,10,12,64}$ For the sake of simplicity, we ignore the effect of the field-like torque, as it cannot drive skyrmions and skyrmioniums into directional motion.$^{19}$

Figure 3(a) shows the velocities of Bloch-type skyrmioniums with $\eta = \pi/2, 3\pi/2$ driven by the damping-like spin-orbit torque. The Bloch-type skyrmionium with $\eta = \pi/2$ moves toward the $-y$ direction, while the one with $\eta = 3\pi/2$ moves toward the $+y$ direction [see Fig. 3(b)]. Namely, the direction of motion is parallel to the spin polarization direction $\mathbf{p} = +\hat{y}$, and also depends on the helicity of skyrmionium. However, the velocity is independent of helicity. The Bloch-type skyrmioniums with $\eta = \pi/2$ and $\eta = 3\pi/2$ show identical current-velocity relation, where the velocity is proportional to the driving current density $j$. It is worth mentioning that the Bloch-type skyrmioniums move at speed of $\sim 14.3$ m s$^{-1}$ driven by the current density $j = 10$ MA cm$^{-2}$. However, as reported in Ref. 51, the skyrmioniums stabilized by DM interactions in conventional FM systems driven by $j = 10$ MA cm$^{-2}$ could reach a speed of $\sim 92.2$ m s$^{-1}$, which is faster than the frustrated skyrmioniums. The reason is that the speed induced by the same current density increases with the size of skyrmion.$^{51}$ In this work, the diameter of the outer circular domain wall of the frustrated skyrmionium at $j = 10$ MA cm$^{-2}$ is $\sim 9$ nm, while it is $\sim 90$ nm in Ref. 51. The larger size of skyrmionium leads to higher speed, however, it may result in lower storage density. On the other hand, it should be noted that the skyrmionium in the frustrated magnetic monolayer driven by a moderate damping-like spin-orbit torque (i.e., $j = 1 \sim 10$ MA cm$^{-2}$) shows directional motion instead of circular motion, which is different from frustrated skyrmions driven by the damping-like spin-orbit torque.$^{27,56}$

It should be noted that if the initial state is a relaxed but unstable Néel-type skyrmionium with $\eta = 0$ or $\eta = \pi$, a small value of damping-like spin-orbit torque acting on the skyrmion will lead to the transition of the unstable Néel-type helicity to metastable Bloch-type helicity. As shown in Fig. 4(a), when a small driving current of $j = 1$ MA cm$^{-2}$ is applied, the skyrmionium with $\eta = 0$ is transformed to a skyrmion with $\eta = 3\pi/2$. Also, the skyrmionium with $\eta = \pi$ is transformed to a skyrmion with $\eta = \pi/2$ [see Fig. 4(b)]. The total energy of the system decreases during the current-induced helicity transition [see Fig. 4(c)], which indicates Bloch-type skyrmioniums are more energetically favorable.

As shown in Fig. 5, we also demonstrate that a large driving current can result in the distortion and annihilation of metastable Bloch-type skyrmioniums during their motion. Figure 5(a) shows that when a large driving current of $j = 20$ MA cm$^{-2}$ is applied, the metastable Bloch-type skyrmionium with $\eta = \pi/2$ moves toward the $-y$ direction. However, the spin configuration of its outer circular domain wall is distorted soon after the injection of the driving current. At $t = 270$ ps, the outer and inner circular domain walls touch each other and the skyrmionium structure is therefore transformed into a topological trivial bubble with $Q = 0$. The topological trivial bubble with $Q = 0$ continues to move and shrink, which is ultimately annihilated at $t = 640$ ps. The final state of the system is the ground-state FM state. During the large-current-induced distortion and annihilation of the Bloch-type skyrmionium, the total energy of the system and the out-of-plane spin component decreases and increases with time, respectively [see Fig. 5(b)]. It is noteworthy that the total energy of the system slightly increases before the destruction of the
skyrmionium structure [see Fig. 5(b) inset], which means the metastable Bloch-type skyrmionium is protected by an energy barrier.

In conclusion, we have studied the static and dynamic properties of an isolated skyrmionium in a frustrated magnetic monolayer. The frustrated skyrmionium is stabilized by competing interactions, including FM and AFM exchange interactions, PMA, and DDI. The DM interaction is not required to stabilize the skyrmionium in the frustrated magnetic monolayer. It is found that the skyrmionium energy depends on its helicity. The Bloch-type skyrmioniums with \( \eta = \pi/2, 3\pi/2 \) are metastable, while the Néel-type skyrmioniums are unstable. We find the size of a metastable skyrmionium can be adjusted by tuning the PMA or applying a perpendicular magnetic field. The advantage of the frustrated skyrmionium is that its size could be smaller than 10 nm, which is good for increasing information storage density. We also investigated the dynamics of skyrmionium driven by the damping-like spin-orbit torque. It is found that the Bloch-type skyrmioniums can be driven into steady linear motion by a moderate current density, where the direction of motion depends on the skyrmionium helicity and the velocity increases with the driving current density. We also demonstrated that a small driving current can lead to the transformation of an unstable Néel-type skyrmionium to a metastable Bloch-type skyrmionium, and a large driving current can result in the distortion and annihilation of the Bloch-type skyrmionium during its motion. We believe our results are useful for understanding the skyrmionium properties in frustrated magnetic spin systems, and could provide guidelines for the design of information storage and processing devices based on skyrmioniums.

See supplemental material for the metastability diagram of an isolated frustrated skyrmionium and more simulation results. Supplementary Video 1 shows the current-driven motion of a frustrated Bloch-type skyrmionium. Supplementary Video 2 shows the current-induced helicity transition of a frustrated skyrmionium. Supplementary Video 3 shows the current-induced distortion and destruction of a frustrated skyrmionium.

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