Quantum optical analysis of squeezed state of light through dispersive non-Hermitian optical bilayers

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Abstract
We investigate the propagation of a normally incident squeezed coherent state of light through dispersive non-Hermitian optical bilayers, particularly at a frequency that the bilayers hold parity-time (PT) symmetry. To check the realization of PT-symmetry in quantum optics, we reveal how dispersion and loss/gain-induced noises and thermal effects in such bilayers can affect quantum features of the incident light, such as squeezing and sub-Poissonian statistics. The numerical results show thermally induced noise at room temperature has an insignificant effect on the propagation properties in these non-Hermitian bilayers. Moreover, tuning the bilayers’ loss/gain strength, we show that the transmitted squeezed coherent states through the structure can retain to some extent their nonclassical characteristics, specifically for the frequencies far from the emission frequency of the gain layer. Furthermore, we demonstrate, only below a critical value of gain, quantum optical effective medium theory can correctly predict the propagation of quantized waves in non-Hermitian and PT-symmetric bilayers.

Keywords: dispersive non-Hermitian bilayers, parity-time (PT) symmetry, quadrature squeezing, Mandel parameter

(Some figures may appear in color only in the online journal)

1. Introduction

Bender et al [1–3] introduced a distinct class of non-Hermitian Hamiltonians that commute with the so-called parity-time (PT) symmetric operators. In essence, a PT-symmetric Hamiltonian commutes with the combination of the linear parity operator $\hat{P}$ (i.e. $p \rightarrow -p$ and $r \rightarrow -r$) and the anti-linear time-reversal operator $\hat{T}$ (i.e. $p \rightarrow -p, r \rightarrow r,$ and $i \rightarrow -i$), implying that the quantum potential should satisfy the condition $V(r) = V^*(\mathbf{-r})$ wherein $\mathbf{r}$ and asterisk denote the position vector and complex operators [3]. For optical PT-symmetric systems, the refractive index $n(r)$ obeys the symmetry relation $n(r) = n^*(\mathbf{-r})$, rendering non-Hermitian systems with real eigenvalues [4–7]. The latter relation represents the necessary condition for the so-called exact phase regime. Nonetheless, beyond a critical value of gain/loss strength (i.e. the so-called exceptional point), the system eigenvalues become complex, in which case the system is in a broken PT-symmetric phase [3].

The scattering of purely classical light from PT-symmetric structures has led to several engaging optical effects such as optical switching [8], nonreciprocal propagation [9, 10], coherent perfect absorbers [11], optical isolation...
[12, 13], extraordinary transmission, reflection, and absorption [14–17], topological phase transitions [18], optoelectronic oscillators [19], flat bands generated by symmetries [20], and specifically unidirectional invisibility [21–23]. These unusual functionalities, which are a direct consequence of the $\mathcal{PT}$-symmetry, are essentially classical. Since quantum light as compared with classical light, provides distinct properties, such as reduced noise and strong correlations [24], there is an intriguing possibility that light is treated as a stream of photons rather than classical electromagnetic waves when interacting with these structures. The modifications of the light beam propagating through media give rise to identical effects, such as shift peak and shape distortions of the transmitted pulse [25–27], in both the classical and quantum domains. However, incident light with a nonclassical nature may exhibit some alterations in the squeezing and quantum coherence that can only be described in the framework of full quantum theory [25–35]. Despite the successful implementation of $\mathcal{PT}$-symmetry in the classical systems, some controversial results are proposed in the full quantum regime, such as ultrafast states transformation and the violation of the no-signaling principle [36, 37]. Given these and the apparent compensation of the losses in the $\mathcal{PT}$-symmetric systems, it would be of great interest to analyze the scattering behavior of $\mathcal{PT}$-symmetric structures in detail in a few-photons regime. This investigation may lead to fundamental and finally technological advances.

There are only a few reports devoted to exploring the effects of quantum optics on $\mathcal{PT}$-symmetric systems. Schomerus [38] has shown that a $\mathcal{PT}$-symmetric structure can become a self-sustained radiation source by a quantum noise named micro reversibility-breaking. Then, Agarwal et al [39] have revealed that spontaneous generation can determine the quantum nature of the optical fields in a $\mathcal{PT}$-symmetric structure. Later on, Scheel et al [40] investigated the effect of gain and loss on the photonic quantum states evolving in a $\mathcal{PT}$-symmetric system using the Wigner function. They found that $\mathcal{PT}$-symmetric quantum optics in a loss/gain system is unlikely. Later, Klauck et al [41] reported the observation of two-photon interference in an integrated $\mathcal{PT}$-symmetric optical structure. In that work, they concluded that non-local $\mathcal{PT}$-symmetric quantum mechanics act as a building block for future quantum devices. Meanwhile, Peña et al [42] have addressed the analysis of nonclassical-light generation in a two-mode optical $\mathcal{PT}$-symmetric system. With the above background, we turn to study the propagation of squeezed coherent states through a pair of dispersive dielectric slabs forming a non-Hermitian structure, specifically when this structure is $\mathcal{PT}$-symmetric in a specific frequency.

In this contribution, we use the canonical quantization of the electromagnetic field [26, 28–30, 33–35] in multilayer media to second quantize our system. It prepares the ground for examining the detrimental/beneficial effects of non-Hermitian structures on incident quantum lights. The following questions naturally arise in this context have not been addressed before: (a) How and how much dispersion and loss/gain-induced noises in non-Hermitian structures affect nonclassical properties of the incident light, such as squeezing and sub-Poissonian statistics? Unidirectional invisibility is a captive phenomenon observed in a classical $\mathcal{PT}$-symmetric system. (b) Could it be seen in a few-photons regime too? (c) Can one correctly predict the effective behavior of such structures in quantum optics? To answer these questions, we evaluate the squeezing and the Mandel parameters of the outgoing states from the non-Hermitian bilayer structure and examine the competition between the quantum noises and the loss/gain coefficient in both exact and broken $\mathcal{PT}$-symmetric phases. These parameters can serve as appropriate measures for checking the possibility of implementation of $\mathcal{PT}$-symmetry in quantum optics.

The organization of the manuscript is as follows. In section 2, after introducing the suggested geometry and presenting a summary of the quantum input–output relations, the exact multilayer theory and the quantum optical effective medium theory (QOEMT) [28–30] are used to formulate the transmission and reflection amplitudes and the quantum noise fluxes emitted by the structure. Section 3 is devoted to simulation results analysis, demonstrating the results obtained for the eigenvalues of the scattering matrix and the corresponding transmission and reflection intensities for two types of one-dimensional non-Hermitian bilayers to find the operation regime and anisotropic transmission resonance (ATR) of $\mathcal{PT}$-symmetric bilayer. Also, we find the linear regime that the QOEMT as a ‘linear formalism’ can accurately predict the results of the exact multilayer theory. We assume that squeezed coherent states normally incident upon the bilayer structure. Then, we investigate the quadrature squeezing and photon-counting statistics of the output states of the bilayer structure. Finally, the paper is closed with conclusions in section 4. Appendices A, B, and C provide the details of the elements of the $\mathcal{PT}$-symmetry condition, the scattering matrix, and the effective noise parameters.

2. Physical structure and background theory

2.1. Geometry

Consider a dispersive non-Hermitian bilayer structure that is composed of a pair of gain ($g$) and loss ($l$) slabs of identical thicknesses $l_g = l_l = l$ and surrounded by vacuum (figure 1). For simplicity, we assume that quantum states of light, impinging at normal incidence from either side of the bilayer structure. The arrows normal to the $x$–$y$ plane and the bosonic operators show the input and output modes. A simple schematic of the balanced homodyne detection system including, a beam splitter, two detectors, and a local oscillator (LO) field, is depicted on the right-hand side of the bilayer. Let us assume that the permittivity of the gain (loss) layer, $\varepsilon_{g(l)}$, can be described by the one-resonance Lorentz model as

$$\varepsilon_{g(l)}(\omega) = \varepsilon_{bg(l)} - \frac{\alpha_{g(l)} \omega \gamma_{g(l)} \gamma_{bg(l)}}{\omega^2 - \omega_0^{g(l)} + \omega \gamma_{g(l)}}, \tag{1}$$

where $\varepsilon_{bg(l)}$ is a real number representing the background permittivity of the gain (loss) medium, $\omega$ is the input light
where $A$ is the area of quantization in the $x$–$y$ plane and $\hat{a}_{R(L)}^{(j)}(z,\omega)$ represents the corresponding annihilation operator associated with the right (R) and left (L) propagating modes for the $j$th medium ($j = 1, 2, 3,$ and $4$). Taking the Hermitian adjoint of equation (3)——i.e. $\hat{E}_{++}^{(j)}(z,t) = \hat{E}_{++}^{(j)}(z,t)$——results in the negative frequency component of the electric field operator. Using the quantum input–output relations, the bosonic annihilation operators of the output modes on the left ($z = -l$) and right ($z = l$) boundaries (figure 1), $\hat{a}_{L}^{(1)}(-l,\omega)$ and $\hat{a}_{R}^{(4)}(+l,\omega)$, can be expressed in terms of their input counterparts on the opposite boundaries——i.e. $\hat{a}_{L}^{(4)}(+l,\omega)$ and $\hat{a}_{R}^{(1)}(-l,\omega)$——and also the noise operators, $\hat{F}_{R(L)}(\omega)$ [29],

$$
\begin{pmatrix}
\hat{a}_{L}^{(1)}(-l,\omega) \\
\hat{a}_{R}^{(4)}(+l,\omega)
\end{pmatrix} = S \begin{pmatrix}
\hat{a}_{L}^{(1)}(-l,\omega) \\
\hat{a}_{R}^{(4)}(+l,\omega)
\end{pmatrix} + \begin{pmatrix}
\hat{F}_{L}(\omega) \\
\hat{F}_{R}(\omega)
\end{pmatrix},
$$

where

$$S \equiv \begin{pmatrix}
\upsilon_{L} & t \\
t & \upsilon_{R}
\end{pmatrix} = A_{22}^{-1} \begin{pmatrix}
-A_{21} & 1 \\
A_{11} A_{22} - A_{12} A_{21} & A_{12}
\end{pmatrix},$$

is a $2 \times 2$ scattering matrix describing the multiple transmissions and reflections within the bilayer structure. Analogous to the classical optics, $t$ represents the transmission amplitude and, $\upsilon_{L}$ and $\upsilon_{R}$ denote the reflection amplitudes through and from the bilayer binaries at $z = -l$ and $l$. The explicit forms of $\hat{F}_{R(L)}(\omega)$ and the matrix elements $A_{mn}$ ($m, n = 1, 2$) are given in appendix B. Note that the optical input and output annihilation operators in equation (4) satisfy bosonic commutation relations of the form,

$$[\hat{a}_{R}^{(1)}(\omega), \hat{a}_{L}^{(1)}(\omega')] = [\hat{a}_{L}^{(4)}(\omega), \hat{a}_{R}^{(4)}(\omega')] = 0,$$

$$[\hat{a}_{L}^{(1)}(\omega), \hat{a}_{R}^{(1)}(\omega')] = \delta(\omega - \omega'),$$

where $\delta(\omega - \omega')$ is the Dirac delta function.

### 2.2. The exact multilayer theory

Here, we use the second quantization formalism of electromagnetic field in the presence of dissipative and amplification media. Accordingly, the positive frequency component of the electric field $\vec{E}(z,t)$ is given by [26, 28–30, 33–35],

$$
\begin{align*}
\hat{E}_{++}^{(j)}(z,t) = & \int_{0}^{\infty} d\omega \sqrt{\hbar \omega/4\pi \varepsilon_0 c A} \\
& \times \left[ \hat{a}_{R}^{(j)}(z,\omega) e^{i\omega z/c} + \hat{a}_{L}^{(j)}(z,\omega) e^{-i\omega z/c} \right] e^{-i\omega t},
\end{align*}
$$

where $A$ is the area of quantization in the $x$–$y$ plane and $\hat{a}_{R(L)}^{(j)}(z,\omega)$ represents the corresponding annihilation operator associated with the right (R) and left (L) propagating modes for the $j$th medium ($j = 1, 2, 3,$ and $4$). Taking the Hermitian adjoint of equation (3)——i.e. $\hat{E}_{++}^{(j)}(z,t) = \hat{E}_{++}^{(j)}(z,t)$——results in the negative frequency component of the electric field operator. Using the quantum input–output relations, the bosonic annihilation operators of the output modes on the left ($z = -l$) and right ($z = l$) boundaries (figure 1), $\hat{a}_{L}^{(1)}(-l,\omega)$ and $\hat{a}_{R}^{(4)}(+l,\omega)$, can be expressed in terms of their input counterparts on the opposite boundaries——i.e. $\hat{a}_{L}^{(4)}(+l,\omega)$ and $\hat{a}_{R}^{(1)}(-l,\omega)$——and also the noise operators, $\hat{F}_{R(L)}(\omega)$ [29],

$$
\begin{align*}
\begin{pmatrix}
\hat{a}_{L}^{(1)}(-l,\omega) \\
\hat{a}_{R}^{(4)}(+l,\omega)
\end{pmatrix} = S \begin{pmatrix}
\hat{a}_{L}^{(1)}(-l,\omega) \\
\hat{a}_{R}^{(4)}(+l,\omega)
\end{pmatrix} + \begin{pmatrix}
\hat{F}_{L}(\omega) \\
\hat{F}_{R}(\omega)
\end{pmatrix},
\end{align*}
$$

where

$$S \equiv \begin{pmatrix}
\upsilon_{L} & t \\
t & \upsilon_{R}
\end{pmatrix} = A_{22}^{-1} \begin{pmatrix}
-A_{21} & 1 \\
A_{11} A_{22} - A_{12} A_{21} & A_{12}
\end{pmatrix},$$

is a $2 \times 2$ scattering matrix describing the multiple transmissions and reflections within the bilayer structure. Analogous to the classical optics, $t$ represents the transmission amplitude and, $\upsilon_{L}$ and $\upsilon_{R}$ denote the reflection amplitudes through and from the bilayer binaries at $z = -l$ and $l$. The explicit forms of $\hat{F}_{R(L)}(\omega)$ and the matrix elements $A_{mn}$ ($m, n = 1, 2$) are given in appendix B. Note that the optical input and output annihilation operators in equation (4) satisfy bosonic commutation relations of the form,

$$[\hat{a}_{R}^{(1)}(\omega), \hat{a}_{L}^{(1)}(\omega')] = [\hat{a}_{L}^{(4)}(\omega), \hat{a}_{R}^{(4)}(\omega')] = 0,$$

$$[\hat{a}_{L}^{(1)}(\omega), \hat{a}_{R}^{(1)}(\omega')] = \delta(\omega - \omega'),$$

where $\delta(\omega - \omega')$ is the Dirac delta function.

### 2.3. The QOEMT

In classical optics, we can describe a sub-wavelength bilayer structure of figure 1 by a single effective medium, in which the refractive index is spatially homogeneous ($n_{\text{eff}}$). Nonetheless, one should modify the simplified model to include quantum noises via the effective noise photon distribution $N_{\text{eff}}$ in quantum optics, accompanying the classical effective refractive index $n_{\text{eff}}$ (appendix C). The modified model is the so-called QOEMT [28–30]. We here briefly present the essentials needed for understanding this paper. Replacing the bilayer of figure 1 with a single effective slab of thickness $2l$, equation (4) reduces to

Figure 1. 3D representation of a non-Hermitian bilayer structure consists of a gain and a loss slab with an identical thickness of $l$ along the $z$-direction. The arrows normal to the $x$–$y$ plane and the bosonic operators show the input and output modes. A balanced homodyne detection is shown on the right-hand side of the structure. LO refers to the local oscillator field which is assumed to be coherent and much stronger than the input signal.

beam frequency, $\omega_{0g}(l)$, $\gamma_{g}(l)$, and $\alpha_{g}(l)$ denote the emission (absorption) center frequency, the corresponding linewidth, and gain (loss) coefficient. Due to the causality principle, we consider $\alpha_{1} > 0$ and $\gamma_{1} > 0$ for the loss slab. For the gain slab, we take $\alpha_{g} < 0$ and $\gamma_{g} > 0$. To guarantee the structure to be $\mathcal{PT}$-symmetric, we require an exact balance between the gain and loss of two slabs as following:

$$\text{Re}[\varepsilon_{g}(\omega)] = \text{Re}[\varepsilon_{l}(\omega)] \quad \text{and} \quad \text{Im}[\varepsilon_{g}(\omega)] = -\text{Im}[\varepsilon_{l}(\omega)].$$

(2)

We refer the readers interested in detailed descriptions of the $\mathcal{PT}$-symmetry condition to appendix A. An example of a practical experimental proposal for this structure could be the plasmonic metamaterial, as suggested by [45–47], grown on a lossless glass substrate wherein the quantum noise flux vanishes.
Table 1. Physical parameters for the gain and loss layers constituting the bilayer of figure 1, used in the simulations [43, 44].

| Symbol | Definition | Size   | Unit   |
|--------|------------|--------|--------|
| εbl   | Loss layer background dielectric constant | 2, 3.22 | —      |
| εbg   | Gain layer background dielectric constant | 2, 2 | —      |
| γl    | Loss layer absorption linewidth | 67 | 140 Trad s⁻¹ |
| γg    | Gain layer emission linewidth | 67, 67 | Trad s⁻¹ |
| ωl0   | Loss layer absorption radian frequency | 1000 | 1200 Trad s⁻¹ |
| ωg0   | Gain layer emission radian frequency | 1000 | 1000 Trad s⁻¹ |

3.3. Linear regime

To describe the propagation of a nonclassical wave through the given bilayer, we must take care of the gain coefficient to obtain physically sound results. If the gain strength is large enough, the wave amplitude increases rapidly after a completed one round-trip and tends to saturate due to the reduction of the population inversion in the gain medium. The so-called gain-saturation phenomenon is a nonlinear effect associated with the round-trip parameter, characterizing a traveling wave that returns to its original position after going through two reflections at the right and left surfaces of the bilayer [27, 48],

\[ \eta(\omega) = \frac{(n_{\text{eff}}(\omega) - 1)^2}{(n_{\text{eff}}(\omega) + 1)^2} \exp(4i\omega n_{\text{eff}}(\omega)/c). \]  

(10)

Considering equations (9) and (10), one can easily show that the amplitudes \( r_{\text{eff}}(\omega) \) and \( t_{\text{eff}}(\omega) \) have poles at frequencies for which the round-trip parameter satisfies the following conditions, simultaneously:

\[ |\eta(\omega)| = 1, \]  

(11a)

and

\[ \arg[\eta(\omega)] = 0. \]  

(11b)

Here, we focus on the linear regime of the interaction of the optical field of the incident wave with the gain slab. In particular, a sufficient condition for obtaining analytic relations for the reflection and transmission amplitudes, \( r_{\text{eff}} \) and \( t_{\text{eff}} \), is \( |\eta(\omega)| < 1 \).

Using equation (10), we have calculated the magnitude of the round-trip parameter, \( |\eta(\omega)| \), as a function of the loss coefficient, \( \alpha_1 \), for the two types of bilayers. The solid (dashed) curve in figure 2 represents the results for set 1 (2) material. A careful inspection of the data depicted by the solid curve reveals that the condition \( |\eta(\omega)| < 1 \) fails for \( \alpha_1 = \alpha_2 \geq 147 \), making analytic relations for amplitudes \( r_{\text{eff}} \) and \( t_{\text{eff}} \) unachievable and prediction of the exact multilayer theory by the QOEMT as a ‘linear formalism’ inaccurate for the bilayer composed of set 1 material. The dashed curve in figure 2 shows the condition \( |\eta(\omega)| < 1 \) holds, for the entire range of given \( \alpha_1 \), and hence, enabling the QOEMT to predict the results of the exact multilayer theory accurately for the bilayer composed of set 2 material.

3.2. Eigenvalues and ATR

In this subsection, we turn to the eigenvalues of the scattering matrix (5) to explore the \( P^T \)-symmetry exceptional (phase breaking) points. At these points, the \( P^T \)-symmetry system transit from the exact symmetry phase with real eigenvalues to the broken symmetry phase with complex eigenvalues. Eigenvalues of the scattering matrix (5) for the bilayer of figure 1 are:

\[ \lambda_{1(2)} = \frac{(\lambda_{12} - \lambda_{21}) \pm \sqrt{(\lambda_{12} - \lambda_{21})^2 + 4\lambda_{11}\lambda_{22}}}{2\lambda_{22}}. \]  

(12)
For the $\mathcal{PT}$-symmetric bilayer both eigenvalues satisfy the relation $|\lambda_1| = |\lambda_2| = 1$ with the unimodular magnitude—i.e. $|\lambda_1| = \lambda_2 = 1$, and the inverse moduli—i.e. $|\lambda_1| = |\lambda_2|^{-1} > 1$, characterizing the exact and the broken symmetry phases, respectively [49, 50]. Using equation (12), we first calculate the scattering matrix eigenvalues as a function of the loss coefficient $\alpha_l$ at the frequency $\omega_{\mathcal{PT}/\omega_{\mathcal{PT}}}$ = 1 for set 1 (figure 3(a)). As can be observed from this figure, the exceptional point occurs at $\alpha_l = 890$, below (beyond) which this bilayer (set 1) is in the exact (broken) symmetry phase regime.

Then, we obtain a similar plot for the second bilayer (set 2) at $\omega_{\mathcal{PT}/\omega_{\mathcal{PT}}} = 1.58$ (figure 3(b)). As can be seen from the inset shown in this figure, only for $\alpha_l = 2$ ($|\alpha_l| = 20.86$), at which $|\lambda_1| = |\lambda_2| = 1$ the non-Hermitian bilayer operates in the exact phase regime, recalling that for $\alpha_l \neq 2$, this bilayer (set 2) is not a $\mathcal{PT}$-symmetric medium. Using the definitions for the reflection intensity (reflectance) from the right (left) side, $R_{\mathcal{PT}} = |R_{\mathcal{PT}}|^2$—i.e. at $z = +(-)l$,—and the transmission intensity (transmittance) through the non-Hermitian bilayer, $T = |T|^2$, and the symmetric property of the scattering matrix in equation (5), there is a generalized conservation law, $|T| - 1 = (R_L R_R)^{1/2}$ when the structure is $\mathcal{PT}$-symmetric with

\[ \phi_l = \begin{cases} \phi_R & \text{for } T < 1 \\ \phi_R - \pi = \phi_l - \pi / 2 & \text{for } T > 1 \end{cases} \]  

When $T = 1$, the generalized conservation relation results in ATR—i.e. the zero reflection occurs only for the incidence from one side and not the other, while the scatterings from both sides conserve the flux [43, 49]. For an accidental degeneracy (i.e. $R_R = R_L = R$), the flux of an incident wave conserves from both sides (i.e. $|T| - 1 = R$). Using equations (4) and (5), we have calculated the behaviors of $T$, $R_R$, $R_L$, $\phi_l$, and $\phi_l$ versus the loss coefficient $\alpha_l$, for the bilayer structure made of materials represented by set 1 (figure 4(a)) and set 2 (figure 4(b)).

The inset in figure 4(a) shows the loss coefficient values ($\alpha_l \approx 24$ and 114) at which ATRs occur, and both $\phi_R$ and $\phi_l$ undergo a 180° phase change. In other words, in the range of $24 < \alpha_l < 114$ the scattering from both sides of the structure are super-unitary (i.e. $T > 1$) while in other ranges of $\alpha_l$ ($\alpha_l < 24$ and $\alpha_l > 114$) are sub-unitary (i.e. $T < 1$). Besides, there is an accidental degeneracy at $\alpha_l \approx 52$. A careful inspection of the data shown in figure 4(b) reveals that the transmittance is always smaller than unity, demonstrating a lack of ATR at $\alpha_l = 2$ where the conservation law satisfies.

3.3. The quadrature squeezing

In this and the following subsection, we assume that a continuous-mode squeezed coherent state of light, $|\text{L}_R\rangle$ and quantum vacuum state, $|\text{L}_L\rangle$ incident from the left and right sides of the bilayer structure. The squeezed coherent state is the state with minimum uncertainty, in which the quantum fluctuations in one of the quadrature components may fall below the vacuum level at the expense of the other. We can
define the squeezed coherent state of light, produced by a
degenerate parametric amplifier pumped at the frequency of
2 Ω, by the action of the squeezed operator [26, 28–30],
\[
\hat{S}\{\xi(\omega), \phi_\xi(\omega)\} = \exp \left[ \int_0^{\Delta \omega} d\omega \xi(\omega) \hat{a}_R^{(1)}(\omega) \hat{a}_R^{(1)}(2\Omega - \omega) - h.c. \right], \tag{14}
\]
and then the displacement operator,
\[
\hat{D}\{\rho(\omega), \phi_\rho(\omega)\} = \exp \left[ \int_0^{\Delta \omega} d\omega \rho(\omega) e^{-i\phi_\rho(\omega)} \hat{a}_R^{(1)}(\omega) - h.c. \right], \tag{15}
\]
on the vacuum state \(|0\rangle\), as follows:
\[
|R\rangle = \hat{D}\{\rho(\omega), \phi_\rho(\omega)\} \hat{S}\{\xi(\omega), \phi_\xi(\omega)\} |0\rangle, \tag{16}
\]
where \(\xi(\xi')\) and \(\phi_\xi(\phi_\xi')\) represent the strength and phase of the squeezed coherent state, and \(\rho(\omega)\) and \(\phi_\rho(\omega)\) are the amplitude and phase of the coherent component of the state \(|R\rangle\). The quadrature squeezing of the scattered light on the right-hand side of the structure can be measured by a balanced homodyne detector, as depicted in figure 1. In this way, we mix the outgoing light from the bilayer and a strong coherent LO by a 50:50 beam splitter. Considering the detector to be ON during a sufficiently long-time-interval (i.e. a narrow-bandwidth homodyne detector), the variance of the difference between the photocurrents in the two arms of this detector at a finite temperature \(T\) is given by:
\[
\left\langle \left(\Delta \hat{E}_L\right)^2\right\rangle_{\text{out}} = 1 + 2\left(\hat{F}_R^0(\omega)\right) + T\left\{2\sinh^2(\phi) - \Re(e^{2i(\phi_0 - \phi_\ell/2)} \sinh 2\phi)\right\} + R_R\left\{2\sinh^2(\phi') - \Re(e^{2i(\phi_0 - \phi_\ell/2)} \sinh 2\phi')\right\}, \tag{17}
\]
where \(\phi_0\) and \(\phi_\ell\) represent the phase and frequency of the LO field and \(\hat{F}_R^0(\omega)\) is the average flux of the noise photons, which is given by (equation B7) for the exact multilayer theory and equation (C5) for the QOEMT. The quadrature component of the scattered light is squeezed if the variance (equation (17)) is less than unity. In our calculations, we consider \(\xi = 0.2\) and \(\phi = 2\phi_0 - 5\), with the squeezing of \(\left\langle (\Delta \hat{E}_L)^2\right\rangle_{\text{in}} = 0.926\) for the squeezed coherent state \(|R\rangle\) and \(\xi' = 0\) for vacuum state \(|L\rangle\) [26].

Making use of the exact multilayer theory and the QOEMT, we have plotted the numerical values of the variance \(\left\langle (\Delta \hat{E}_L)^2\right\rangle_{\text{out}}\) as a function of the loss coefficient \(\alpha\) for the non-Hermitian bilayers composed of set 1 and set 2 materials. Figures 5(a) and (b) show the corresponding results at temperatures \(T = 0\) (magenta) and 300 K (blue) when the LO frequency equals the bilayers’ \(\mathcal{PT}\)-symmetric resonance frequencies. The solid and dashed curves show the results obtained from the exact multilayer theory, and the dotted and dash-dotted curves represent those of the QOEMT.

A careful inspection of the data shown in figure 5(a) indicates that the QOEMT can accurately predict the behavior of the \(\mathcal{PT}\)-symmetric bilayer (set 1) for \(\alpha \leq 147\). Moreover, the variance \(\left\langle (\Delta \hat{E}_L)^2\right\rangle_{\text{out}}\) is always greater than unity. In other words, the incident squeezed coherent state of frequency \(\omega_{LO} = \omega_0\), after transmitting through the \(\mathcal{PT}\)-symmetric bilayer (set 1) in the given range of \(\alpha\) does not remain squeezed. Physically, unlike the loss slab that does not contribute to the noise flux at zero temperature, the contribution from the gain slab due to the perfect population inversion at \(\omega_0\) results in a maximized noise flux via the spontaneous emission of the atoms. This quantum noise contribution is significant enough to dominate the negative values of the transmission-related term (i.e. the third term on the right-hand side in equation (17)), resulting in \(\left\langle (\Delta \hat{E}_L)^2\right\rangle_{\text{out}} > 1\). Furthermore, this inspection reveals that the data obtained for \(\theta = 0\) and 300 K coincide, indicating the insignificance of the thermal noise at room temperature for the given incident frequencies.

Moreover, we observe that the quadrature squeezing of the output state transmitted through this particular bilayer structure differs from the input state at ATR points (see figure 4). This phenomenon is an apparent manifestation of broken \(\mathcal{PT}\)-symmetry in quantum optics of this bilayer structure, which agrees with the results of [28, 29]. Moreover, it shows in contrast to the classical results of [21], the given bilayer in the presence of normally incident coherent squeezed light is no longer unidirectionally invisible.

Examining the data shown in figure 5(b), we observe excellent agreements between the data for QOEMT and the exact multilayer theory for the non-Hermitian bilayer composed of
set 2 material for $\omega_{LO}/\omega_{bg} = 1.58$ over the entire range of given $\alpha_1$ and at both temperatures. Moreover, the inset in this figure reveals that for $\alpha_1 \leq 18$ the transmitted light is not squeezed because in the given range $\langle (\Delta \hat{E})^2 \rangle_{\text{out}} \geq 1$. In these ranges of $\alpha_1$, where $R_R \approx R_L \approx 0$ and $T \approx 1$ (figure 4(b)), the non-Hermitian bilayer behaves like a lossless/gainless slab, and hence the noise flux becomes vanishingly small. Nonetheless, unlike the case for set 1, the transmission-related term with a small positive value contributes significantly to the squeezing, leading to $\langle (\Delta \hat{E})^2 \rangle_{\text{out}} > 1$ for set 2. For $\alpha_1 > 18$, this dominant term becomes negative due to the presence of the $\alpha_1$-dependent phase, $\phi$, resulting in $\langle (\Delta \hat{E})^2 \rangle_{\text{out}} < 1$. In other words, the incident squeezed coherent state of frequency $\omega_{LO} = 1.58\omega_{bg}$ retains its squeezed feature after transmitting through the non-Hermitian bilayer composed of set 2 material with $\alpha_1 > 18$.

For $\alpha_1 > 800$ and beyond, for either type of non-Hermitian bilayers (set 1 and 2), as can be seen in figure 4, the corresponding transmittance becomes insignificant ($T \to 0$), and both reflectances ($R_R$ and $R_L$) become significant. Hence, both bilayers behave like mirrors, and the corresponding scattering states measured with the balanced homodyne detector approach that of the vacuum state—i.e. $\langle (\Delta \hat{E})^2 \rangle_{\text{out}} \to 1$, showing contributions to the output quadrature variance from the noise- and transmission-related terms in equation (17) are insignificant.

Then, using the exact multilayer theory, we numerically calculated $\langle (\Delta \hat{E})^2 \rangle_{\text{out}}$ versus $\omega_{LO}/\omega_{bg}$, for both non-Hermitian bilayers at $\theta = 0$ K, for a set of given $\alpha_1$ in the $PT$-symmetric range (figure 6). Figure 6(a) shows the plots for the bilayer of set 1, obtained for the specific values of loss coefficients whose significance was discussed in sections 3.1 and 3.2—i.e. $\alpha_1 = 24$ (dotted), 52 (dashed), 114 (dash-dotted), 147 (solid line), and 890 (asterisks). As can be observed from figure 3(a), each of the four $\alpha_1$ for the structures under study is in the exact symmetry phase (i.e. $\alpha_1 < 890$) while for the case $\alpha_1 = 890$, the system is in a broken $PT$-symmetric phase. The inset shows for each given $\alpha_1$ there is an upper limit to $\omega_{LO}/\omega_{bg}$ below which the quadrature squeezing becomes $\langle (\Delta \hat{E})^2 \rangle_{\text{out}} < 1$. Those upper limits are $\omega_{LO}/\omega_{bg} = 0.73$ (dotted), 0.64 (dashed), 0.55 (dash-dotted), 0.52 (solid line), and 0.29 (asterisks). Below these upper limits, the contribution of the transmission-related term is dominant and negative enough to make $\langle (\Delta \hat{E})^2 \rangle_{\text{out}} < 1$. As the incident frequency approaches to $\omega_{bg}$, the contribution from this term gradually becomes positive, and increases until $\omega_{LO} = \omega_{bg}$ along with the dominant noise flux contribution maximizes the variance $\langle (\Delta \hat{E})^2 \rangle_{\text{out}}$, for all the given loss values except for $\alpha_1 = 890$. Near $\omega_{LO} = \omega_{bg}$, the behavior of all curves is consistent with the transmission intensities. Except for $\alpha_1 = 890$, each plot exhibits a peak around frequency $\omega_{LO} = \omega_{bg}$. One can attribute the dip about $\omega_{LO} = \omega_{bg}$ for $\alpha_1 = 890$ (asterisk) to the insignificant transmission in a wide range of frequencies. Any further increase in frequency (i.e. $\omega_{LO}/\omega_{bg} > 1$) causes the contributions from the noise flux and the transmission-related term to drop to zero and a small positive value, respectively, eventually resulting in a variance which is slightly greater than unity.

Figure 6. The variance $\langle (\Delta \hat{E})^2 \rangle_{\text{out}}$ as a function of $\omega_{LO}/\omega_{bg}$ at $\theta = 0$ K for the non-Hermitian bilayer composed of (a) set 1 material for $\alpha_1 = 24$ (dotted), 52 (dashed), 114, (dash-dotted), 147 (solid line), and 890 (asterisks); (b) set 2 material for $\alpha_1 = 2$. The inset in each part shows a zoomed-in portion in the frequency range where $\langle (\Delta \hat{E})^2 \rangle_{\text{out}} \leq 1$, enhancing the visibility of the squeezed regime.

3.4. The Mandel parameter

To study the photon-counting statistics of the transmitted squeezed coherent state of the light through the bilayer structure, then we analyze the Mandel parameter [26]

$$Q \equiv \left\langle \left( \Delta \hat{N} \right)^2 - \left\langle \hat{N} \right\rangle \right\rangle / \left\langle \hat{N} \right\rangle, \quad (18a)$$

where

$$\langle \left( \Delta \hat{N} \right)^2 \rangle = \left\langle \hat{N}^2 \right\rangle - \left\langle \hat{N} \right\rangle^2,$$

and

$$\hat{N} = \int_0^{T_0} dt \hat{a}_R^{(4)}(t) \hat{a}_R^{(4)}(t) \quad (18b)$$

is the number of photons that reach the detector over the time interval $T_0$ in region 4. The positive, zero, and negative values of the parameter $(Q)$ refer to the super-Poissonian, Poissonian, and sub-Poissonian distributions.
Instead of the balanced homodyne detection in the prior subsection, here we consider a photo-count detector with a Gaussian filter function of bandwidth $\sigma_\text{H}$ and center frequency $\omega_\text{0g}$, in which $\sigma_\text{H} \ll \omega_\text{0g}$, to measure the Mandel parameter. Using the input–output relation (4) together with (18), after some exhausting manipulations, the Mandel parameter at finite temperature becomes

$$Q = Q_0 \left\{ \left( 1 - R_\text{K} \right)^2 + T^2 \left[ 1 + \sinh^2 \xi \left( \cosh 2\xi - 2 \right) + 2\sigma_\text{H} \sqrt{\pi} |\rho|^2 \left( 2\sinh^2 \xi + \sinh 2\xi \cos (2\phi_\xi - \phi_\xi) - 2 \right) + 2T \left( 1 - R_\text{K} \right) \left( \sinh^2 \xi + 2\sigma_\text{H} \sqrt{\pi} |\rho|^2 - 1 \right) \right] \right\}$$

$$\times \left\{ T \left( \sinh^2 \xi + 2\sigma_\text{H} \sqrt{\pi} |\rho|^2 \right) + \left\langle \hat{F}_R (\omega) \hat{F}_R (\omega') \right\rangle \right\}^{-1}$$

(19)

where $Q_0 = \sigma_\text{H} \omega_\text{0g}/4 \pi^{3/2}$. Following [26] we consider the Mandel parameter of $Q_0/Q_0 = -0.33$ for an incident squeezed coherent state $|R_i\rangle$, and presume $2\sigma_\text{H} \pi^{1/2} |\rho|^2 = 25$ and $\phi_\xi = (2\phi_\xi - \pi)$. In this manner, we prepare an incident squeezed coherent state with a sub-Poissonian statistic, demonstrating a nonclassical state. In figure 7, we depict the normalized Mandel parameter ($Q/Q_0$) versus $\alpha_l$, obtained for the non-Hermitian bilayer composed of (a) set 1 at $\omega_\text{PT}/\omega_\text{0g} = 1$ and (b) set 2 at $\omega_\text{PT}/\omega_\text{0g} = 1.58$ at temperatures $\theta = 0$ (magenta) and 300 K (blue). The spectra depicted by solid lines and dashes show the results obtained from the exact multilayer theory, and those displayed by dots and dash-dotted represent those of the QOEMT.

The inset in figure 7(a) shows that for $\alpha_l < 4.9$ the transmitted light is sub-Poissonian (i.e. $Q/Q_0 < 0$), although is not squeezed (see figure 5(a)). One may attribute this behavior to the insignificant reflectances ($R_\text{L} = R_\text{L} = 0$) and the transmittance of $T = 1$ (see figure 4(a)), in addition to the vanishing noise flux for the given range of $\alpha_l$. Knowing these facts, one can easily simplify equation (19), yielding $Q/Q_0 < 0$. Thus, the optical $\mathcal{PT}$-symmetric bilayer with $\alpha_l < 4.9$ can preserve sub-Poissonian statistics of incident quantum states at the output. For $\alpha_l > 4.9$, $Q/Q_0 > 0$, resulting in super-Poissonian output states. Similar to the results of figure 5(a), for data for temperatures ($\theta = 0$ and 300 K), here too coincide, implying an insignificant thermal noise effect at room temperature at incident frequency. Moreover, to our expectation, the QOEMT cannot accurately predict the behavior of the $\mathcal{PT}$-symmetric bilayer (set 1) for $\alpha_l > 147$ (figure 5(a)).

Similar to the results of figure 5(b), data in figure 7(b) shows the Mandel parameter for the non-Hermitian bilayer composed of set 2 material, obtained from both models (the exact and QOEMT) at for both temperatures ($\theta = 0$ and 300 K), coincide. Moreover, the same data reveal non-Hermitian bilayer of set 2 can maintain the sub-Poissonian characteristic of the incident state at the output, over the entire range of given $\alpha_l$ ($Q/Q_0 < 0$). Contrary to set 1, here we see at the $\omega_\text{PT}/\omega_\text{0g} = 1.58$ that is far from the emission frequency of the gain layer in the given set, the quantum noise is infinitesimal. In other words, some optical non-Hermitian bilayers can preserve particular nonclassical features of incident quantum states, such as sub-Poissonian statistics at their output, when the incident frequency is far from the emission frequency of the gain layer.

Finally, varying the input signal frequency, we calculated the Mandel parameter for a squeezed coherent state transmitted through the non-Hermitian bilayer composed of (a) set 1 and (b) set 2 with the particular loss coefficients, the same as those used in figure 6, at $\theta = 0$ and 300 K (figure 8).
Table 2. The normalized frequencies (ω′/ω₀g and ω″/ω₀g) for which the output from the bilayer of set 1 transits between sub- and super-Poissonian statistics, for the α₁ range given in figure 8(a).

| α₁ | 24  | 52  | 114 | 147 | 890 |
|----|-----|-----|-----|-----|-----|
| ω′/ω₀g | 0.936 | 0.902 | 0.854 | 0.836 | 0.645 |
| ω″/ω₀g | 1.068 | 1.1 | 1.168 | 1.196 | 1.534 |

The dotted, dashed, dash-dotted and solid curves depict data related to α₁ = 24, 52, 114, and 147, respectively, and asterisks correspond to α₁ = 890. As can be observed from this figure, there is a frequency range over which the output light is super-Poissonian (i.e. ω < ω′ and ω″), and two other ranges over which the bilayer (set 1) retains the sub-Poissonian feature of the incident squeezed coherent state at its output (i.e. ω < ω′ and ω > ω″). The frequencies ω′ and ω″ corresponding to different α₁ values are tabulated in table 2.

From figure 8(b), we can observe that in the frequency range of 0.943 < ω/ω₀g < 1.061, the incident squeezed coherent state becomes super-Poissonian at the output of the non-Hermitian bilayer of set 2 and remains to be sub-Poissonian out of this frequency range.

4. Conclusion

We have investigated the normal propagation of the squeezed coherent state of light through dispersive non-Hermitian optical bilayers, in the framework of the second quantization formalism, particularly at a specific frequency that the bilayers hold PT-symmetric. In this investigation, we have calculated the output quadrature variance and the Mandel parameter at the output of each bilayer versus its loss coefficient, keeping the incident signal frequency fixed at the corresponding frequency where PT-symmetry holds. Our results show that one cannot implement PT-symmetry in the quantum optics domain as far as the squeezing feature of outgoing light is concerned, which agrees well with the results of [33]. In this sense, the PT-symmetric bilayer is not unidirectionally invisible that conflicts with the classical results of [21].

Moreover, we have repeated the same calculations and varied the input frequency for particular sets of loss coefficients related to the PT-symmetric regimes of the bilayers. Numerical results show, for frequencies far from the emission frequency of the gain layer, the non-Hermitian bilayers can maintain the squeezing feature and sub-Poissonian statistics of incident squeezed coherent states to some extent at the bilayers’ outputs. The set 2 bilayer retains its nonclassical characteristics of the incident states better than set 1. Because, unlike the latter, the former bilayer, behaves like a loss-dominated system—i.e. Im r₂α > 0—for a set of given α₁, consistent with the results in [40, 41].

The numerical results also show thermally induced noise at room temperature has an insignificant effect on the propagation properties in these non-Hermitian bilayers. Furthermore, we have demonstrated only below a critical value of gain coefficient (linear regime), the QOEMT can correctly predict the propagation of quantized waves in non-Hermitian and PT-symmetric bilayers.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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Appendix A. PT-symmetry condition

Substituting PT-symmetry conditions (2) in (1), we can relate the gain and loss parameters:

\[ |\alpha_ε| = |\alpha_1\omega_0\gamma_1| \frac{\left(\omega^2 - \omega_0^2 + \frac{2\pi}{\eta_1} \left(\omega^2 - \omega_0^2\right)^2 + \gamma_1^2\omega^2\right)}{\left(\omega^2 - \omega_0^2\right)^2 + \gamma_1^2\omega^2}. \]  

(A1)

Combining equation (A1) with the imposed condition on the real part of permittivity in equation (2), after some manipulation, we get:

\[ \Delta \varepsilon = \varepsilon_{bg} - \varepsilon_{bg} = \frac{\alpha_1\omega_0\gamma_1 \left(\omega^2 - \omega_0^2 + \frac{2\pi}{\eta_1} \left(\omega^2 - \omega_0^2\right)^2 + \gamma_1^2\omega^2\right)}{\left(\omega^2 - \omega_0^2\right)^2 + \gamma_1^2\omega^2}. \]  

(A2)

For set 1 material (i.e. \(\Delta \varepsilon = 0\)), from (equation A2) the frequency at which PT-symmetry condition is fulfilled becomes [43]:

\[ \omega = \sqrt{\frac{\omega_0^2\gamma_0^2 + \omega_0^2\gamma_1^2}{\gamma_0^2 + \gamma_1^2}} \equiv \omega_{PT}. \]  

(A3)

For a case, in which \(\omega_{bg} = \omega_{bg}\) and \(\gamma_{bg} = \gamma_{bg}\), equation A1 gives \(\varepsilon_{bg} = \varepsilon_{bg}\) for all frequencies and (equation A2) reduces to \(\omega_{PT} = \omega_{bg}\). Nonetheless, for \(\Delta \varepsilon \neq 0\), in general, there are at most two roots for the frequency at which the PT-symmetry condition is satisfied. In particular for set 2 material (i.e. \(\Delta \varepsilon = 1.22\)) \(\omega = \omega_{PT} = 1.58\omega_{bg}\) solely for \(\alpha_1 = 2\) and \(\varepsilon_{bg} = 20.86\).

Appendix B. The elements of the scattering matrix

The quantum noise terms originating from the absorption and amplification within the loss and gain layers can be written as [29],

\[ \hat{F}_{RL}^{(L)}(\omega) = D^{(2)}\hat{c}_{RL}^{(2)}(\omega) + D^{(3)}\hat{c}_{RL}^{(3)}(\omega), \]  

(B1)

in which

\[ D^{(j)} = A_{22}^{-1}\left( B^{(j)}_{11}A_{22} - A_{13}B^{(j)}_{21} B^{(j)}_{12}A_{22} - A_{12}B^{(j)}_{22} \right), \]  

(B2)
represents a $2 \times 2$ matrix that arises from the amplification and absorption inside the gain ($j=2$) or loss ($j=3$) layers, with no classical analogue. Notice that for the bilayer structure under study, $D^{(1)} = D^{(2)} = 0$. Here, the matrix $A$ satisfies the following relation:

$$A = T^{(3)} R^{(3)} T^{(2)} R^{(2)} T^{(1)},$$

wherein $R^{(1)}$ is a $2 \times 2$ diagonal matrix of elements $R^{(1)}_{ij} = 1 / |R^{(1)}_{22}| = \exp (-n_j' \omega l / c)$, and the elements of the matrix $T^{(1)}$ are given by,

$$T^{(1)}_{ij} = \sqrt{R^{(1)}_{ij} \frac{n_{j'+1} + n_j}{2n_j}} \exp [i (n_j' - n_{j'+1}) \omega z_j / c],$$

with the commutation relations:

$$[\hat{c}^{(j)}_{R(L)} (\omega), \hat{c}^{(j')}_{R(L)} (\omega')] = 2i e^{+n_j' \omega' l / c} f_{R(L)}^{(j)} (\omega'),$$

with the commutation relations:

$$[\hat{c}^{(j)}_{R(L)} (\omega), \hat{c}^{(j')}_{L(R)} (\omega')] = -2 \frac{n_j''}{n_j'} e^{+n_j' \omega' l / c} \sinh \left( \frac{n_j'' \omega l}{c} \right) \delta (\omega - \omega'),$$

with the commutation relations:

$$\langle \hat{F}_L (\omega) \hat{F}_R (\omega') \rangle = \delta (\omega - \omega') \times 2 \sum_{j=1}^{3} \{ \sgn (\text{Im} \varepsilon_j (\omega)) \sinh (n_j' \omega l / c) \times \left( \frac{\langle D^{(j)} (|B^{(j)}|) e^{-|B^{(j)}|^2 c^2 \omega l / c} \rangle}{\langle |B^{(j)}|^2 \rangle} - \left| n_j'' \right| \sin (n_j'' \omega l / c) \times \left( \frac{|D^{(j)}|^2 e^{+n_j'' \omega' l / c} |B^{(j)}|^2 e^{-n_j'' \omega' l / c} \rangle}{\langle |B^{(j)}|^2 \rangle} \right) \} \times \{ N_{\text{th}} (\omega, \theta) \text{Im} \varepsilon_j (\omega) + (N_{\text{th}} (\omega, \theta) + 1) \Theta [-\text{Im} \varepsilon_j (\omega)] \},$$

where $\theta$ and $\Theta$ represent the temperature and the step function and $N_{\text{th}} (\omega, \theta) = \exp (\hbar \omega / k_B \theta) - 1^{-1}$ is the mean number of thermal photons emitted by the bilayer structure, in which $\hbar$ and $k_B$ are the reduced Planck’s and Boltzmann’s constants. Notice that equation (B7) is valid for the normal incidence at any given temperature.

### Appendix C. The effective noise parameters

The right (R) and left (L) components of the quantum effective noise $F_{\text{eff}} (\omega)$ in the quantum input–output relation (7) are given by [29]:

$$F_{\text{effL}} (\omega) = \frac{-2i \sqrt{2} n_{\text{eff}} e^{2i \omega' l / c}}{\langle n_{\text{eff}+1} \rangle^2 - \langle n_{\text{eff}-1} \rangle^2 \exp [4i n_{\text{eff}} \omega l / c] \times \left( \langle n_{\text{eff}-1} \rangle e^{+i n_{\text{eff}} \omega l / c} \int_{-l}^{l} dz' e^{-i n_{\text{eff}} \omega l / c} f_{\text{effL}} (z', \omega) \right) + \langle n_{\text{eff}+1} \rangle \int_{-l}^{l} dz' e^{+i n_{\text{eff}} \omega l / c} f_{\text{effL}} (z', \omega) \right),$$

with the commutation relations:

$$\langle \hat{F}_{\text{effL}} (\omega) \hat{F}_{\text{effL}} (\omega') \rangle = \left( 1 - \left| r_{\text{eff}} \right|^2 - \left| t_{\text{eff}} \right|^2 \right) \delta (\omega - \omega'),$$

$$\langle \hat{F}_{\text{effL}} (\omega) \hat{F}_{\text{effR}} (\omega') \rangle = - \left( r_{\text{eff}}^* t_{\text{eff}} + r_{\text{eff}}^* t_{\text{eff}} \right) \delta (\omega - \omega').$$

Furthermore, by using equations (C1)–(C4), the flux of noise photons emitted by the effective slab at finite temperature $\theta$ can be written as,
\[ \langle F_{\text{eff}}^{R}(L) | (\omega') F_{\text{eff}}^{R}(L') | \omega' \rangle \]
\[ = \left( 1 - |r_{\omega}^{g} - |r_{\omega'}^{g} |^{2} \right) \delta (\omega - \omega') \]
\[ \times \left\{ N_{\text{eff}}(\omega, \theta) \Theta \left( \text{Im} n_{\text{eff}}^{2}(\omega) \right) \right. \]
\[ - \left( N_{\text{eff}}(\omega, |\theta|) + 1 \right) \Theta \left( -\text{Im} n_{\text{eff}}^{2}(\omega) \right) \left\} , \right. \]
\[ \text{(C5)} \]

where

\[ N_{\text{eff}}(\omega, \theta) = -\frac{1}{2} + \frac{1}{2} \sum_{j=g,l} p_{j} \left| \frac{\text{Im} n_{\text{eff}}^{2}(\omega)}{n_{\text{eff}}^{2}(\omega)} \right| (2N_{\text{th}}(\omega, |\theta|) + 1) , \]
\[ \text{(C6)} \]

is the effective noise photon distribution. Here, \( p_{j} = J_{j}(I_{g} + I_{l}) \), \( (j = g, l) \) are the volume fractions of the layers and equals 1/2 for the bilayer structure of figure 1. Moreover, the effective refractive index \( n_{\text{eff}} \) can be obtained from the following Bloch dispersion relation in the long-wavelength limit,

\[ \cos (2n_{\text{eff}} \omega / c) = \cos (n_{g} \omega / c) \cos (n_{l} \omega / c) \]
\[ - \frac{1}{2} \left( \frac{n_{g}}{n_{g}} + \frac{n_{l}}{n_{l}} \right) \sin (n_{g} \omega / c) \sin (n_{l} \omega / c) . \]
\[ \text{(C7)} \]

Here, \( n_{g} \) and \( n_{l} \) are the refractive indices of the gain and loss layers that support the Bloch waves.

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