Top-quark pair production via polarized and unpolarized protons in the supersymmetric QCD

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ABSTRACT

The QCD corrections to the top-quark pair production via both polarized and unpolarized gluon fusion in $pp$ collisions are calculated in the Minimal Supersymmetric Model (MSSM). We find the MSSM QCD corrections can reach 4% and may be observable in future precise experiments. Furthermore, we studied the CP violation in the MSSM, our results show that the CP violating parameter is sensitive to the masses of SUSY particles (It becomes zero, when the c.m. energy is less than twice the masses of both gluino and stop quarks.) and may reach $10^{-3}$.

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I. Introduction

The minimal supersymmetric model (MSSM) is one of the most interesting extensions of the Standard Model (SM). Therefore testing the MSSM has attracted much interest. As is well known, the MSSM predicts supersymmetric (SUSY) partners to all particles expected by the SM, and searching for their existence is very important.

Since the top-quark was already found experimentally by the CDF and D0 Collaborations at Fermilab, we believe that more and more experimental events including top-quark will be collected in future experiments. That gives us a good chance to study the physics in top-quark pair production from $pp$ or $p\bar{p}$ collisions with more precise experimental results. Because of the heavy mass of the top quark this process provides a test of the SM and possible signals of new physics at high energy.

The dominant subprocesses of top-quark pair production in $pp$ or $p\bar{p}$ colliders are quark-antiquark annihilation and gluon-gluon fusion. The lowest order of those two subprocesses has been studied in Ref. [3]. There it was found that the former subprocess ($q\bar{q}$ annihilation) is more dominant in $p\bar{p}$ collisions when the c.m. energy ($\sqrt{s}$) is near the threshold value $2m_t$, whereas subprocess via $gg$ fusion will be more and more important with increasing c.m. energy, and can become the most dominant one when the c.m. energy is much larger than $2m_t$.

In Ref. [4], the QCD corrections to top-quark pair production in $p\bar{p}$ collisions have been studied in the frame of the SM. It may seem natural that the QCD corrections of those processes in the frame of the MSSM are important for distinguishing those two
models. Recently, the SUSY QCD corrections to top pair production via $q\bar{q}$ annihilation were given in Ref. [5]. The SUSY QCD corrections via unpolarized gluon-gluon fusion were presented by C.S.Li. et. al [6].

It is obvious that the correction from the SUSY QCD is related to the masses of top-quark and SUSY particles. Assuming the SUSY breaking scale at about 1 TeV, the masses of SUSY particles would be smaller than 1 TeV. So we can hope that corrections from SUSY particles are significant, since the heavy mass of the top quark ($m_t = 175.6 \pm 5.5$ GeV (world average)) may be comparable to some of the light SUSY particle masses. Therefore the SUSY QCD correction would give us some significant information about the existence of SUSY particles indirectly.

Recently, the spin structure of the nucleon has been intensely studied by polarized deep inelastic scattering experiments at CERN and SLAC. This knowledge allows us to find a clear signal beyond the SM, if we collect enough events in the process of top-quark pair production from polarized $pp$ or $p\bar{p}$ collisions. In the SM QCD, there is no CP violation mechanism, whereas in the SUSY QCD, the situation may be different. If we introduce phase angle of quark SUSY partners, we can get CP violation in the MSSM QCD [7]. Once we get enough statistics of top-quark pairs from $pp$ or $p\bar{p}$ colliders at higher energy, it will be possible to test CP violation. On the other hand, the spin-dependent parton distributions can be obtained from their polarized structure function data in Ref.[8][10][11]. There one found that the shape of polarized gluon and quark distributions in the nucleon depends on its polarization. Therefore the CP violation effects through the process of
top-quark pair production via $gg$ fusion may be observed in polarized $pp$ or $p\bar{p}$ collisions.

In this work we concentrate on the SUSY QCD corrections to the process $pp \to gg \to t\bar{t}X$ both in polarized and unpolarized colliding beams. In section 2, we give the tree level contribution to subprocess $gg \to t\bar{t}$. In section 3 we give the analytical expressions of the SUSY QCD corrections to $gg \to t\bar{t}$. In section 4 the numerical results of the subprocess $gg \to t\bar{t}$ and the process $pp \to gg \to t\bar{t}X$ are presented. The conclusion is given in section 5 and some details of the expressions are listed in the appendix.

II. The Tree-Level Subprocess

The graphical representation of the process $g(\lambda_1, k_1)g(\lambda_2, k_2) \to t(p_1)\bar{t}(p_2)$ is shown in Fig.1 (a). The Mandelstam variables are defined as usual

\[
\hat{s} = (p_1 + p_2)^2 = (k_1 + k_2)^2 
\]

\[
\hat{t} = (p_1 - k_1)^2 = (k_2 - p_2)^2 
\]

\[
\hat{u} = (p_1 - k_2)^2 = (k_1 - p_2)^2 
\]

so $\hat{s} + \hat{t} + \hat{u} = 2m_t^2$. The amplitude of tree-level diagrams with polarized gluons can be written as:

\[
M_{0}^{(l)} = g_s^2 \epsilon^{\mu, a}(\lambda_1, k_1)\epsilon^{\nu, b}(\lambda_2, k_2)\bar{u}_i(p_1)\Gamma_l\nu_j(p_2), \quad (l = s, t, u) 
\]

with

\[
\Gamma^{(s)} = \frac{T^{abc}_i f_{abc}}{s}[(\not{k}_1 - \not{k}_2)g_{\mu\nu} + (2k_2 + k_1)_\mu \gamma_\nu - (2k_1 + k_2)_\nu \gamma_\mu] 
\]
\[ \Gamma^{(t)} = -iT_{\mu}^{a}T_{\nu}^{b} \gamma_\mu (\slashed{k}_2 - \slashed{p}_2 + m_t) \gamma_\nu \]  
\[ \Gamma^{(u)} = -iT_{\mu}^{a}T_{\nu}^{b} \gamma_\nu (\slashed{k}_1 - \slashed{p}_2 + m_t) \gamma_\mu \]  
(2.6)  
(2.7)

We chose a form in which only physical polarizations of gluon s remained:

\[ \epsilon^{\mu \nu}(\lambda_1, k_i) \epsilon^{\nu}(\lambda_2, k_i) = \frac{\delta_{\lambda_1, \lambda_2}}{2} (-g^{\mu \nu} + \frac{n^\mu k^\nu + n^\nu k^\mu}{(n \cdot k_i)} + i \lambda_1 \epsilon^{\sigma \mu \nu} \frac{k_{ia} n_i}{n \cdot k_i} \)  
(2.8)

where \( n = k_1 + k_2 \), \( \lambda_{1,2} = \pm 1 \). From that, we can get the cross section at the tree-level with both polarized and unpolarized gluons.

III. SUSY QCD corrections (non-SM) to the subprocess \( gg \rightarrow t \bar{t} \)

1. Relevant Lagrangian in the MSSM.

The difference between the MSSM QCD and the SM QCD corrections stems from the interactions of SUSY particles. Thus we can divide SUSY QCD corrections into a standard and a non-standard part. The Lagrangian density of the non-SM part of the SUSY QCD interaction is written as:

\[ L = L_1 + L_2 + L_3 + L_4 \]  
(3.1.a)

Where

\[ L_1 = -i g_s A_\mu^a T^a_{jk} (\bar{q}_L^i \gamma_\mu q^j_L - \bar{q}_R^i \gamma_\mu q^j_R) + (L \rightarrow R) \]  
(3.1.a.2)  
\[ L_2 = -\sqrt{2} g_s T^a_{jk} (\bar{q}_a P_L q^k_L \bar{q}_L^i + \bar{q}^j R P_R \bar{q}_R^k - \bar{q}_a P_R q^k_R \bar{q}_R^i - \bar{q}_R^i P_L \bar{q}_L^k) \]  
(3.1.a.3)  
\[ L_3 = \frac{1}{2} g_s f_{abc} \bar{g}^a \gamma_\mu g^b A_\mu^c \]  
(3.1.a.4)  
\[ L_4 = \frac{1}{6} g_s^2 A_\mu^a A_\nu^b (\bar{q}_L^i q^j_L + \bar{q}_R^i q^j_R) + \frac{1}{2} g_s^2 d_{abc} A_\mu^a A_\nu^{ab} (\bar{q}_L^i T^a_{ij} q^j_L + \bar{q}_R^i T^a_{ij} q^j_R) \]  
(3.1.a.5)
q stands for quark, $\tilde{q}$ for corresponding squark, $\tilde{g}$ for gluino, $P_L$ and $P_R$ for left, right helicity projections, respectively. The mixing between the left- and right-handed stop quarks $\tilde{t}_L$ and $\tilde{t}_R$ can be very large due to the large mass of the top quark, and the lightest scalar top-quark mass eigenstate $\tilde{t}_1$ can be much lighter than the top-quark and all the scalar partners of the light quarks. Therefore the left-right mixing for the SUSY partners of the top quark plays an important role. Here we only considered the SUSY QCD effect from stop-quark, because we assume that other scalar SUSY quarks are much heavier than the stop-quark and hence decoupled. Furthermore we introduce the phase angle $\phi_A$ in the stop mixing matrix. Defining $\theta$ as mixing angle of stop-quark, we have

$$
\tilde{t}_L = e^{-i\phi_A/2} (\tilde{t}_1 \cos \theta + \tilde{t}_2 \sin \theta) \quad (3.a.6)
$$

$$
\tilde{t}_R = e^{i\phi_A/2} (-\tilde{t}_1 \sin \theta + \tilde{t}_2 \cos \theta) \quad (3.a.7)
$$

where we suppose $m_{\tilde{t}_1} \leq m_{\tilde{t}_2}$.

2. Analytical results of the MSSM QCD corrections.

The one-loop SUSY QCD correction diagrams are shown in Fig.1(b). In the following we present only the amplitude expressions of s-channel and t-channel. The amplitude of u-channel can be obtained from the t-channel expression by the following variable exchanges: $t \leftrightarrow u$, $k_1 \leftrightarrow k_2$, $\epsilon^a_\mu(k_1) \leftrightarrow \epsilon^b_\nu(k_2)$ and $T^a \leftrightarrow T^b$. The one-loop diagrams can be divided into three groups: the self-energy diagrams of gluon and top-quark shown in Fig.1(b.1); $gtt$ and $ggg$ vertex correction diagrams shown in Fig.1(b.2); box diagrams shown in Fig.1 (b.3). The ultraviolet divergence is controlled by dimensional regularization ($n = 4 - \epsilon$). The
strong coupling-constants are renormalized by using the modified Minimal Subtraction ($\tilde{M}$S) scheme at charge-renormalization scale $\mu_R$. This scheme violates SUSY explicitly and the $q\bar{q}g$ Yukawa coupling $\hat{g}_s$, which should be the same with the $qqg$ gauge coupling $g_s$ in supersymmetry, takes a finite shift at one-loop order. Therefore we take this shift between $\hat{g}_s$ and $g_s$ as shown in Eq.(3.b.1) into account in our calculation, in order to have the physical amplitudes independent of the renormalization scheme and we subtract the contribution of the false, non-supersymmetric degrees of freedom (also called $\epsilon$ scalars) \cite{12}.

\[
\hat{g}_s = g_s \left[ 1 + \frac{\alpha_s}{4\pi} \left( \frac{2}{3} C_A - \frac{1}{2} C_F \right) \right],
\]

(3.b.1)

where $C_A = 3$ and $C_F = 4/3$ are the Casimir invariants of SU(3) gauge group. The heavy particles (top quarks, gluino, stop-quarks, etc.) are removed from the $\mu_R$ evolution of $\alpha_s(\mu_R^2)$, then they are decoupled smoothly when momenta are smaller than their masses \cite{13}. We define masses of heavy particles as pole masses.

The renormalized amplitude corresponding to all SUSY QCD one-loop corrections (as shown in Fig.1) can be split into the following components:

\[
\delta M = \delta M_s + \delta M_v + \delta M_{\text{box}} + \delta M_d.
\]

(3.b.2)

where $\delta M_s$, $\delta M_v$, $\delta M_{\text{box}}$ and $\delta M_d$ are the one-loop amplitudes corresponding to the self-energy, vertex, box correction diagrams and the decoupling part, respectively. The $\delta M_d$ stems from the decoupling of the heavy flavors from the running strong coupling, and is
given explicately by (see also [12] [13]):

\[
\delta M_d = M_0 \left( \alpha_s \left( \mu \right) \pi \right) \left[ 1 \frac{1}{24} \log \left( \frac{\mu^2}{m_t^2} \right) + \frac{1}{12} \log \left( \frac{\mu^2}{m_t^2} \right) + \frac{1}{2} \log \left( \frac{m_t^2}{m_b^2} \right) \right]
\]

(3.b.3)

3. Self-energy corrections to the amplitude.

The amplitude of self-energy diagrams \( \delta M_s \) (Fig. 1. (b.1)) can be decomposed into \( \delta M_g \) (gluon self-energy) and \( \delta M_q \) (top-quark self-energy), i.e.

\[
\delta M_s = \delta M_g + \delta M_q = \delta M_g + \delta M_g^{(t)} + \delta M_g^{(u)} + \delta M_q^{(u)}.
\]

(3.c.1)

The amplitudes \( \delta M_g^{(s)}, \delta M_g^{(t)} \) and \( \delta M_g^{(u)} \) are for s-, t- and u-channel, respectively. They can be expressed as:

\[
\delta M_g^{(s)} = \frac{1}{2} M_0^{(s)} \left[ \Pi(k_1^2) + \Pi(k_2^2) + 2\Pi(s) \right],
\]

(3.c.2)

\[
\delta M_g^{(t)} = \frac{1}{2} M_0^{(t)} \left[ \Pi(k_1^2) + \Pi(k_2^2) \right],
\]

(3.c.3)

\[
\delta M_g^{(u)} = \frac{1}{2} M_0^{(u)} \left[ \Pi(k_1^2) + \Pi(k_2^2) \right],
\]

(3.c.4)

where \( M_0 \) is the tree-level amplitude defined in Eq (2.4).

\[
\Pi(k^2) = -\frac{2}{g^2} \left( T_F \left( \bar{B}_0 + 4 \bar{B}_1 + 4 \bar{B}_{21} \right) \left\{ k, m_{t_1}, m_{t_2} \right\} + T_F \left( \bar{B}_0 + 4 \bar{B}_1 + 4 \bar{B}_{21} \right) \left\{ k, m_{t_2}, m_{t_1} \right\} - 4 C_A (\bar{B}_1 + \bar{B}_{21}) \left\{ k, m_{\bar{g}}, m_{\bar{g}} \right\} - \frac{4}{3} C_A \right).
\]

(3.c.5)

where \( C_F = \frac{4}{3}, T_F = \frac{1}{2}, C_A = 3 \) are invariants in the SU(3) color group, \( B_i \) and \( B_{ij} \) are Passarino-Veltman two-point functions [14] [15]. The definitions of \( \bar{B}_0, \bar{B}_1 \) and \( \bar{B}_{21} \) are listed in Appendix A. The amplitude \( \delta M_g^{(t)} \) is written as:

\[
\delta M_g^{(t)} = \frac{-ig_2^2 T_{a \bar{u}} T_{b \bar{v}}}{(s-m^2)} \epsilon^{\mu, a}(k_1) \epsilon_{\nu, b}(k_2) \bar{u}_i(p_1) \gamma_\mu \left( \hat{k}_2 - \hat{p}_2 + m_t \right) \left[ \hat{S}_{kl}(k_2 - p_2) \right] \left( \hat{k}_2 - \hat{p}_2 + m_t \right) \gamma_{\nu} v_j(p_2).
\]

(3.c.6)
Here we define

\[
\hat{\Sigma}_{kl}(p) = C_F(H_L p P_L + H_R p P_R - H^S_L P_L - H^R_S P_R)\delta_{kl}
\]  

(3.c.7)

with

\[
H_L = \frac{g^2}{8\pi^2}x_1 x_3 B_1[p, m\tilde{g}, m_{\tilde{t}_1}] + (m_{\tilde{t}_1} \to m_{\tilde{t}_2}, x_i \to y_i) + \frac{1}{2}(\delta Z_L + \delta Z^*_L),
\]

(3.c.8)

\[
H_R = \frac{g^2}{8\pi^2}x_2 x_4 B_1[p, m\tilde{g}, m_{\tilde{t}_1}] + (m_{\tilde{t}_1} \to m_{\tilde{t}_2}, x_i \to y_i) + \frac{1}{2}(\delta Z_R + \delta Z^*_R),
\]

(3.c.9)

\[
H^S_L = \frac{g^2}{8\pi^2}x_2 x_3 m\tilde{g} B_0[p, m\tilde{g}, m_{\tilde{t}_1}] + (m_{\tilde{t}_1} \to m_{\tilde{t}_2}, x_i \to y_i) + \frac{1}{2}m_t(\delta Z_L + \delta Z^*_L) + \delta m_t,
\]

(3.c.10)

\[
H^R_S = \frac{g^2}{8\pi^2}x_1 x_4 m\tilde{g} B_0[p, m\tilde{g}, m_{\tilde{t}_1}] + (m_{\tilde{t}_1} \to m_{\tilde{t}_2}, x_i \to y_i) + \frac{1}{2}m_t(\delta Z_R + \delta Z^*_R) + \delta m_t.
\]

(3.c.11)

Where we abbreviate \( \phi = \phi_A, x_1 = \cos \theta e^{-i\phi}, x_2 = \sin \theta e^{i\phi}, x_3 = \cos \theta e^{i\phi}, x_4 = \sin \theta e^{-i\phi}, y_1 = \sin \theta e^{-i\phi}, y_2 = -\cos \theta e^{i\phi}, y_3 = \sin \theta e^{i\phi}, y_4 = -\cos \theta e^{-i\phi}, \) and \( \theta \) is mixing angle of stop-quarks, see Eq (3.a.6 \sim 7).

The explicit expressions of the top-quark wave-function renormalization constants have the following forms:

\[
\delta Z_L = -\frac{g^2}{8\pi^2}(x_1 x_3 Re[B_1] - \frac{m_t^2}{m_t^2}(x_1 x_4 - x_2 x_3)Re[B_0])
+ m_t^2(x_1 x_3 + x_2 x_4)Re[B_1^*]
- m_t m\tilde{g}(x_2 x_3 + x_1 x_4)Re[B_0^*]|_{p'=m_t^2},
\]

(3.c.12)

\[
\delta Z_R = -\frac{g^2}{8\pi^2}(x_2 x_4 Re[B_1] + m_t^2(x_1 x_3 + x_2 x_4)Re[B_1^*])
- m_t m\tilde{g}(x_2 x_3 + x_1 x_4)Re[B_0^*]|_{p'=m_t^2},
\]

(3.c.13)

\[
\delta m_t = \frac{g^2}{16\pi^2}((x_1 x_3 + x_2 x_4)m_t Re[B_1]
- (x_2 x_3 + x_1 x_4)m\tilde{g} Re[B_0])|_{p'=m_t^2},
\]

(3.c.14)

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We use the following abbreviations: $B'_{i,ij}[p,m_1,m_2] = \frac{\partial B_{i,ij}[p,m_1,m_2]}{\partial p^i}$.

4. Vertex-corrections to the amplitude.

The amplitudes for vertex diagrams can be expressed as:

$$\delta M_v^{(l)} = g_s\epsilon^\mu, (k_1)\epsilon^\nu, (k_2)\bar{u}_i(p_1)\Lambda^{(l)}_{\nu, (ij)}(p_2), \quad (l = s, t, u), \quad (3.d.1)$$

where

$$\Lambda^{(s)} = -\frac{T^c_{ij}}{s} \left[\Lambda^{(3g)}_{\mu, \rho}(k_1, k_2)\right] \gamma^\rho$$

$$-\frac{i}{s} f_{abc}[k_1 - k_2] \rho g_{\mu \nu} + (2k_2 + k_1)_{\mu} g_{\nu \rho}$$

and

$$\Lambda^{(t)} = \frac{i}{t - m_t^2} \left\{ T^b_{mj} \left[ \Lambda^{a}_{\mu, (im)}(p_1, k_1 - p_1) \right] (k_2 - k_2 + m_t) \gamma^\muight.$$  

$$+ T^a_{im} \gamma^\mu(k_2 - k_2 + m_t) \left[ \Lambda^{a}_{\nu, (mj)}(k_2 - p_2, p_2) \right] \right\}. \quad (3.d.2)$$

The functions $\Lambda^{(3g)}_{\mu, \rho}$ and $\Lambda^{a}_{\mu, (ij)}$ are listed in Appendix B.

5. Box-corrections to the amplitude.

The box diagram corrections in the t-channel (Fig.1(b.3)) are given as follows:

$$\delta M_{\text{box}}^{(l)} = 2g_s^2\epsilon^\mu, (k_1)\epsilon^\nu, (k_2)\bar{u}_i(p_1)((T^c T^a T^b T^c)_{ij} F_{\mu \nu}^{(1)})$$

$$- i f_{bcd}(T^c T^a T^d)_{ij} F_{\mu \nu}^{(2)} - f_{a cm} f_{b md} (T^c T^d)_{ij} F_{\mu \nu}^{(3)}$$

$$- (T^c (T^a T^b + T^b T^a) T^c)_{ij} F_{\mu \nu}^{(4)} v_j(p_2), \quad (3.e.1)$$

Where $f_{abc}$ is defined as $[T^a, T^b] = if_{abc} T^c$. The form factors $F_{\mu \nu}^{(i)} (i = 1 - 4)$ correspond to the kernel of the four Feynman diagrams in Fig.1(b.3) respectively and are given explicitly in Appendix C.

6. Total cross section.
Collecting all terms in Eq (3.b.2), we can get the total cross section:

\[
\sigma(\lambda_1, \lambda_2) = \sigma_0(\lambda_1, \lambda_2)(1 + \delta \sigma(\lambda_1, \lambda_2)) = \frac{1}{16\pi s} \int_{t^-}^{t^+} dt \sum_{\text{spins}} |M_0|^2 + 2Re(M_0^\dagger \delta M) \]

where \( t^\pm = (m_t^2 - \frac{1}{2} s) \pm \frac{1}{2} s \beta_t \), \( \beta_t = \sqrt{1 - 4m_t^2/s} \), and the spin sum is performed only over the final top-quark pair when we considered polarized gluons.

IV. Numerical results

We denote \( \hat{\sigma}_0 \) for the Born cross section and \( \hat{\sigma} \) for the cross section including one-loop SUSY QCD corrections of subprocess \( gg \rightarrow t\bar{t} \), and define its relative correction as \( \hat{\delta} = \frac{\hat{\sigma} - \hat{\sigma}_0}{\hat{\sigma}_0} \). For polarized gluon fusions, \( \hat{\sigma}_{++} \), \( \hat{\sigma}_{--} \) and \( \hat{\sigma}_{+-} \) are the cross sections with positive, negative and mixed polarization of the gluons, respectively. In order to inspect the CP violating effects we introduce the CP-violation parameter for the subprocess defined by \( \hat{\xi}_{CP} = \frac{\hat{\sigma}_{++} - \hat{\sigma}_{--}}{\hat{\sigma}_{++} + \hat{\sigma}_{--}} \). The possible SUSY QCD effects in \( gg \rightarrow t\bar{t} \) should be observed in \( pp \) colliders. By analogy we can define also the relative correction and CP violating parameter for the process \( pp \rightarrow gg \rightarrow t\bar{t} \) as \( \delta = \frac{\sigma - \sigma_0}{\sigma_0} \) and \( \xi_{CP} = \frac{\sigma_{++} - \sigma_{--}}{\sigma_{++} + \sigma_{--}} \), respectively. The SUSY QCD contribution to the process \( p(P_1, x)p(P_2, y) \rightarrow gg \rightarrow t\bar{t}X \) (\( x,y \) are polarizations of protons) can be obtained by convoluting the subprocess with gluon distribution functions.

\[
\sigma(s) = \int dx_1 dx_2 G(x_1, Q) G(x_2, Q) \delta(s, \alpha_s(\mu))
\]

with \( k_1 = x_1 P_1 \), \( k_2 = x_2 P_2 \) and \( \tau = x_1 x_2 = \hat{s}/s \). \( G(x_i, Q)(i = 1, 2) \) are gluon distribution functions of protons. We take \( Q = \mu_R = 2m_t \).
In order to get results of top quark pair production from polarized \( pp \) collisions, we need to consider the polarized gluon distributions in protons. The cross sections of polarized \( pp \rightarrow gg \rightarrow t\bar{t}X \) can be written as

\[
\sigma(x, y) = \Sigma_{\lambda_1, \lambda_2=\pm} \int dx_1 dx_2 G^{x\lambda_1}(x_1, Q) G^{y\lambda_2}(x_2, Q) \hat{\sigma}_{\lambda_1, \lambda_2}(\hat{s}, \alpha_s(\mu)) \tag{4.2}
\]

where \( x \) and \( y \) are the polarizations of incoming protons and \( \lambda_1, \lambda_2 \) are the polarizations of gluons inside protons. \( G^{x\lambda_1}(x, Q), G^{y\lambda_2}(x, Q) = G^{\pm}(x, Q) \) for equal (+) and opposite (-) polarization, \( G^{+}(x, Q) \) and \( G^{-}(x, Q) \) are polarized gluon distribution functions in the proton.

We used unpolarized proton structure functions of Glück et al. \[9\] in our numerical calculations. For the polarized proton structure functions, we use the evolution equations of Glück et al. \[10\] with input parameters from the paper of Stratmann et al. \[11\] (Next-To-Leading-Order). Since the structure functions are one of the least certain input in our calculation, we checked the result against other set, i.e. the polarized structure functions \( G^{\pm}(x, Q) \) of Brodsky et al. \[8\] (Using Leading-Order only). This tests the stability of our results against the particular form of the input structure functions. The two different sets of input are compared in Fig.2, which gives the relative SUSY QCD correction \((\delta)\) and \(\xi_{CP}\) versus c.m. energy \(\sqrt{s}\) for the process \(pp \rightarrow gg \rightarrow t\bar{t}X\). Though the SUSY QCD corrections from the two sets of structure functions are not too different, for \(\delta\), there is some noticeable change for \(\xi_{CP}\). Because \(\xi_{CP}\) depends strongly on the c.m. energy of the subprocess \(gg \rightarrow t\bar{t}\) (shown in Fig.3(b)), a small modification of structure functions may lead to a large change of \(\xi_{CP}\). Thus we can infer that the NLO-QCD calculation

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is required and the precise numerical prediction does depend on the reliability of the structure functions.

The SUSY QCD relative corrections are about $2\% \sim 4\%$ and decrease with increasing c.m. energy from Fig.2. These correction effects are within reach of future precision experiments and provide a possible discrimination of the SM and the MSSM effects. From Fig.2(c) we can see that the CP violation parameter $\xi_{CP}$ can be $10^{-3}$. Therefore, CP violation in this process stemming from the SUSY QCD can in principle be tested in future precision experiments. That would help us to learn more about the sources of CP violation.

In order to explore the effects of the SUSY QCD correction for future arrangements of optimal experimental conditions, we also investigate the subprocess $gg \rightarrow t\bar{t}$.

The relative SUSY QCD correction and CP violating parameter versus c.m. energy ($\sqrt{s}$) for different polarization gluons are plotted in Fig.3 (a \sim c) with $m_{\tilde{g}} = 200 \, GeV$, $m_{\tilde{t}_1} = 250 \, GeV$, $m_{\tilde{t}_2} = 450 \, GeV$, and $\theta = \phi = 45^\circ$. In Fig.3(a) $\hat{\delta}_{++}$ and $\hat{\delta}_{--}$ are drawn in solid line and dashed line, respectively. $\xi_{CP}$ as a function of c.m. energy is depicted in Fig.3(b) and $\hat{\delta}_{+-}$ as function of $\sqrt{s}$ is plotted in Fig.3(c). Each curve in Fig.3(a) has an obvious peak near the position of the threshold of top pair production. That large enhancement is the combined effect of the threshold, when $\sqrt{s}$ is just larger than $2m_t = 350 \, GeV$, and the resonance when $\sqrt{s} \sim 2m_{\tilde{g}} = 400 \, GeV$. The small spikes around the position of $\sqrt{s} = 900 \, GeV$, there $\sqrt{s} \sim 2m_{\tilde{t}_2} = 900 \, GeV$, shows also the resonance effect. Although Fig.3(a) shows that $\hat{\delta}_{++}$ and $\hat{\delta}_{--}$ approach equal values when
the c.m. energy is far beyond its threshold value $2m_t$, the quantitative difference between $\hat{\delta}_{+-}$ and $\hat{\delta}_{++}$ still exists in the whole energy range plotted in these figures. Fig.3(b) shows also that $\hat{\xi}_{CP}$ will be zero, if the c.m. energy is below the threshold of SUSY particles in the loop (i.e. $\sqrt{s} \leq 2m_{\tilde{g}} = 400$ GeV in Fig.3(b)). This is reasonable because only beyond this point can we have absorptive terms which give contributions to $\hat{\xi}_{CP}$. $\hat{\xi}_{CP}$ has obvious resonance effect in the regions around $\sqrt{s} \sim 2m_{\tilde{g}} = 400$ GeV and $\sqrt{s} \sim 2m_{\tilde{t}_i}, (i = 1, 2) = 500$ GeV, 900 GeV. We also find that the two stop quarks give opposite contributions to $\hat{\xi}_{CP}$ and when their masses are degenerate $\hat{\xi}_{CP}$ will vanish. When the c.m. energy $\sqrt{s}$ is larger than 1 TeV, $\hat{\xi}_{CP}$ will be near zero, because the contributions from the two stop quarks will cancel each other. Therefore a quantitative strong change of $\hat{\xi}_{CP}$ as function of c.m. energy can be an indication for the signals of stop quarks and gluino.

$\hat{\sigma}(\pm, \pm)$ and $\hat{\xi}_{CP}$ as functions of $m_{\tilde{g}}$ are shown in Fig.4 (a) and Fig.4 (b), respectively. In Fig.4 we take $\sqrt{s} = 500$ GeV, $m_{\tilde{t}_1} = 100$ GeV, $m_{\tilde{t}_2} = 450$ GeV, $\theta = \phi = 45^\circ$. We can see from Fig.4(b) that $\hat{\xi}_{CP}$ changes its sign when $m_{\tilde{g}}$ is near $m_t = 175$ GeV. The curves in Fig.4(a)(b) show again the resonance effect when $\sqrt{s} \sim 2m_{\tilde{g}} = 500$ GeV, note that for each line there is a steep change of the value of $\hat{\sigma}(\pm, \pm)$ or $\hat{\xi}_{CP}$ around the position of $m_{\tilde{g}} = 250$ GeV.

Dependences of relative correction $\hat{\delta}_{\pm \pm}$ and $\hat{\xi}_{CP}$ for the subprocess $gg \rightarrow t\bar{t}$ on $m_{\tilde{t}_1}$ are plotted in Fig.5 (a) and Fig.5 (b). $\hat{\delta}_{\pm \pm}$ and $\hat{\xi}_{CP}$ as functions of $m_{\tilde{t}_2}$ are shown in Fig.6 (a) and Fig.6 (b), respectively. In all figures of Fig.5 and Fig.6, we take the common
parameter set with $\sqrt{s} = 500 \text{ GeV}$, $m_{\tilde{g}} = 200 \text{ GeV}$ and $\theta = \phi = 45^\circ$. In Fig.5, we set $m_{\tilde{t}_2} = 450 \text{ GeV}$, whereas $m_{\tilde{t}_1} = 100 \text{ GeV}$ in Fig. 6. We find that $\hat{\xi}_{CP}$ in fact increases with mass splitting of stop-quarks (i.e. $m_{\tilde{t}_2} - m_{\tilde{t}_1}$) and when $m_{\tilde{t}_1} = m_{\tilde{t}_2}$, $\hat{\xi}_{CP}$ is equal to zero. The resonance effect of stop quarks, when $\sqrt{s} \sim 2 m_{\tilde{t}_i} (i = 1, 2)$, is superimposed on the curves in Fig.5 (a)(b) and Fig. 6(a)(b) around the positions of $m_{\tilde{t}_1} = 250 \text{ GeV}$ in Fig.5(a)(b) and $m_{\tilde{t}_2} = 250 \text{ GeV}$ in Fig.6(a)(b), respectively. Around those points the relatively sharp changes of the values of $\hat{\xi}_{CP}$ and the relative corrections are shown in these figures.

Finally, the dependence of $\hat{\delta}_{\pm \pm}$ and $\hat{\xi}_{CP}$ on the phase $\phi$ is shown in Fig.7 (a) and (b). In Fig.7, we take $\sqrt{s} = 500 \text{ GeV}$, $m_{\tilde{g}} = 200 \text{GeV}$, $\theta = 45^\circ$ and $m_{\tilde{t}_1} = 150 \text{ GeV}$. We find that $\hat{\xi}_{CP}$ is directly proportional to $\sin (2\phi)$ and reaches its maximal value when $\phi = \frac{\pi}{4}$.

IV. Conclusion

In this work we have studied the one-loop supersymmetric QCD corrections to the subprocess $gg \to t\bar{t}$ and process $pp \to gg \to t\bar{t}X$. The calculations show that the SUSY QCD effects are significant. The absolute values of the corrections are about $2\% \sim 4\%$, so they may be observable in future precision experiments. Furthermore, we find $\xi_{CP}$ depends strongly on masses of SUSY particles and can reach $10^{-3}$ when we take plausible SUSY parameters.

The results show that there is an obvious difference between the corrections for the protons polarized with parallel spin and that with anti-parallel spin. Hence there is a
possibility to study spin-dependence in the frame of the MSSM QCD.

We also presented and discussed the results of the subprocess $gg \rightarrow t\bar{t}$. We find that when the c.m. energy passes through the value $2m_{\tilde{g}}$ or $2m_{\tilde{t}_i}$ ($i = 1, 2$), the value of the CP violating parameter $\hat{\xi}_{CP}$ changes strongly. If c.m. energy is less than both $2m_{\tilde{g}}$ and $2m_{\tilde{t}_i}$ ($i = 1, 2$), $\hat{\xi}_{CP}$ will be zero. If in future experiments a sharp change in $\hat{\xi}_{CP}$ is found with $\sqrt{s}$ running from low c.m. energy to high c.m. energy, it would be interpreted as a signal of SUSY particles. Furthermore, because the CP violating parameter $\hat{\xi}_{CP}$ is sensitive on the mass of gluino (as shown in Fig.4 (b)) and the mass splitting of stop-quarks $m_{\tilde{t}_2} - m_{\tilde{t}_1}$ (as shown in Fig.5 and Fig.6), we can also get information of SUSY particles from precise measurements of $\hat{\xi}_{CP}$.

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Appendix

A. Loop integrals:

We adopt the definitions of two-, three- and four-point one-loop Passarino-Veltman integral functions of reference[14][15].

1. The two-point integrals are:

\[ \{ B_0; B_\mu; B_{\mu \nu} \}(p, m_1, m_2) = \frac{(2\pi\mu)^4}{i\pi^2} \int d^n q \frac{\{1; q_\mu; q_\mu q_\nu\}}{[q^2 - m_1^2][(q + p)^2 - m_2^2]}, \quad (A.a.1) \]

The function \( B_\mu \) is proportional to \( p_\mu \):

\[ B_\mu(p, m_1, m_2) = p_\mu B_1(p, m_1, m_2) \quad (A.a.2) \]

Similarly we define:

\[ B_{\mu \nu} = p_\mu p_\nu B_{21} + g_{\mu \nu} B_{22} \quad (A.a.3) \]

We denote \( \bar{B}_0 = B_0 - \Delta \), \( \bar{B}_1 = B_1 + \frac{1}{2} \Delta \) and \( \bar{B}_{21} = B_{21} - \frac{1}{3} \Delta \). with \( \Delta = \frac{2}{\epsilon} - \gamma + \log(4\pi) \), \( \epsilon = 4 - n. \) \( \mu \) is the scale parameter.

2. The three-point integrals are:

\[ \{ C_0; C_\mu; C_{\mu \nu}; C_{\mu \nu \rho} \}(p, k, m_1, m_2, m_3) = \]

\[ -\frac{(2\pi\mu)^4}{i\pi^2} \int d^n q \frac{\{1; q_\mu; q_\nu; q_\rho; q_\mu q_\nu q_\rho\}}{[q^2 - m_1^2][(q + p + k)^2 - m_3^2][q^2 - m_1^2][(q + p)^2 - m_2^2]}. \quad (A.a.4) \]

We define form-factors as follows:

\[ C_\mu = p_\mu C_{11} + k_\mu C_{12} \]
\[ C_{\mu\nu} = p_\mu p_\nu C_{21} + k_\mu k_\nu C_{22} + (p_\mu k_\nu + k_\mu p_\nu)C_{23} + g_{\mu\nu}C_{24} \]

\[ C_{\mu\nu\rho} = p_\mu p_\nu p_\rho C_{31} + k_\mu k_\nu k_\rho C_{32} + (k_\mu p_\nu p_\rho + p_\mu k_\nu p_\rho + p_\mu p_\nu k_\rho)C_{33} + \]

\[ (k_\mu k_\nu p_\rho + p_\mu k_\nu k_\rho + k_\mu p_\nu k_\rho)C_{34} + (p_\mu g_{\nu\rho} + p_\nu g_{\mu\rho} + p_\rho g_{\mu\nu})C_{35} + \]

\[ (k_\mu g_{\nu\rho} + k_\nu g_{\mu\rho} + k_\rho g_{\mu\nu})C_{36} \quad (A.a.5) \]

3. The four-point integrals are:

\[
\{D_0; D_{\mu}; D_{\mu\nu}; D_{\mu\nu\rho}; D_{\mu\nu\rho\sigma}\}(p, k, l, m_1, m_2, m_3, m_4) =
\]

\[
\frac{(2\pi\mu)^{4-n}}{i\pi^2} \int d^n q \frac{\{1; q_\mu; q_\mu q_\nu; q_\mu q_\nu q_\rho; q_\mu q_\nu q_\rho q_\sigma\}}{[q^2 - m_1^2][(q + p)^2 - m_2^2][(q + p + k)^2 - m_3^2][(q + p + k + l)^2 - m_4^2]}, \quad (A.a.6)\]

Again we define form-factors of D functions:

\[ D_{\mu} = p_\mu D_{11} + k_\mu D_{12} + l_\mu D_{13} \]

\[ D_{\mu\nu} = p_\mu p_\nu D_{21} + k_\mu k_\nu D_{22} + l_\mu l_\nu D_{23} + \{pk\}_{\mu\nu} D_{24} + \{pl\}_{\mu\nu} D_{25} + \{kl\}_{\mu\nu} D_{26} + g_{\mu\nu} D_{27} \]

\[ D_{\mu\nu\rho} = p_\mu p_\nu p_\rho D_{31} + k_\mu k_\nu k_\rho D_{32} + l_\mu l_\nu l_\rho D_{33} + \{kpp\}_{\mu\nu\rho} D_{34} + \]

\[ \{lpp\}_{\mu\nu\rho} D_{35} + \{pkk\}_{\mu\nu\rho} D_{36} + \{pll\}_{\mu\nu\rho} D_{37} + \{llk\}_{\mu\nu\rho} D_{38} + \]

\[ \{kll\}_{\mu\nu\rho} D_{39} + \{pkl\}_{\mu\nu\rho} D_{310} + \{pg\}_{\mu\nu\rho} D_{311} + \{kg\}_{\mu\nu\rho} D_{312} + \{lg\}_{\mu\nu\rho} D_{313} \quad (A.a.7) \]

where

\[ \{pk\}_{\mu\nu} = p_\mu k_\nu + k_\mu p_\nu \]

\[ \{pkl\}_{\mu\nu\rho} = p_\mu k_\nu l_\rho + l_\mu p_\nu k_\rho + k_\mu l_\nu p_\rho \]

\[ \{pg\}_{\mu\nu\rho} = p_\mu g_{\nu\rho} + p_\nu g_{\mu\rho} + p_\rho g_{\mu\nu} \quad (A.a.8) \]
The numerical calculation of the vector and tensor loop integral functions can be traced back to the four scalar loop integrals $A_0$, $B_0$, $C_0$, and $D_0$ in Ref.[14][15] and the references therein.

B. Vertex corrections:

The 3-gluon-vertex can be written as: (a,b,c are the color indices of the external gluons)

$$\Lambda^{(3g)}_{\mu\nu\rho}(k_1,k_2) = \frac{i g^3}{16\pi^2} \left\{ Tr(T^b T^c T^a) \left[ \Lambda^{(1)}_{\mu\nu\rho}(k_1,k_2) \right] + i f^{cmn} f^{ant} f^{blm} \left[ \Lambda^{(2)}_{\mu\nu\rho}(k_1,k_2) \right] \right\},$$

(A.b.1)

the vertex functions $\Lambda^{(1)}_{\mu\nu\rho}$, $\Lambda^{(2)}_{\mu\nu\rho}$ are expressed as follows:

$$\Lambda^{(a)}_{\mu\nu\rho}(k_1,k_2) = f^{(a)}_1 g_{\mu\nu} k_{1\rho} + f^{(a)}_2 g_{\mu\nu} k_{2\rho} + f^{(a)}_3 g_{\mu\nu} k_2 \rho + f^{(a)}_4 g_{\mu\nu} k_2 \mu + f^{(a)}_5 k_{1\mu} k_{1\rho} k_{2\mu} + f^{(a)}_6 k_{1\mu} k_{2\mu} k_{2\mu}$$

(A.b.2)

where $a = 1, 2$, and the $f^{(1)}_i$, $f^{(2)}_i$ are given in terms of the Passarino-Veltman functions with internal stop lines $C^{(1)}_{ij} (= C_{ij}[-k_1, -k_2, m_{t_i}, m_{t_i}, m_{t_i}])$ and internal gluino lines $C^{(2)}_{ij} (= C_{ij}[-k_1, -k_2, m_{\tilde{g}}, m_{\tilde{g}}, m_{\tilde{g}}])$. For simplicity, we abbreviate the definite part of C integral functions (using the definitions of [14][15]) as follows: $\bar{C}^{(a)}_{24} = C^{(a)}_{24} - \frac{1}{2} \Delta$, $\bar{C}^{(a)}_{35} = C^{(a)}_{35} + \frac{1}{2} \Delta$, $\bar{C}^{(a)}_{36} = C^{(a)}_{36} + \frac{1}{12} \Delta$. ($a = 1, 2$)

$$f^{(1)}_1 = -8 \bar{C}^{(1)}_{24} - 8 \bar{C}^{(1)}_{35},$$

$$f^{(1)}_2 = -4 \bar{C}^{(1)}_{24} - 8 \bar{C}^{(1)}_{35},$$

$$f^{(1)}_3 = -8 \bar{C}^{(1)}_{36},$$

$$f^{(1)}_4 = -4 \bar{C}^{(1)}_{24} - 8 \bar{C}^{(1)}_{36}.$$
\begin{align*}
 f_5^{(1)} &= 4C_{12}^{(1)} + 12C_{23}^{(1)} + 8C_{33}^{(1)}, \\
 f_6^{(1)} &= 4C_{12}^{(1)} + 8C_{22}^{(1)} + 4C_{23}^{(1)} + 8C_{34}^{(1)},
\end{align*}

and

\begin{align*}
 f_1^{(2)} &= -8m_g^2C_{0}^{(2)} - 4m_g^2C_{11}^{(2)} - 16\bar{C}_{24}^{(2)} + 12\bar{e}_{C_{24}^{(2)}} - 8\bar{C}_{35}^{(2)} + 6\bar{e}_{C_{35}^{(2)}} \\
 &\quad + 8k_1 \cdot k_2C_{12}^{(2)} + 16k_1 \cdot k_2C_{23}^{(2)} + 8k_1 \cdot k_2C_{34}^{(2)}, \\
 f_2^{(2)} &= -4m_g^2C_{11}^{(2)} - 8\bar{C}_{35}^{(2)} + 6\bar{e}_{C_{35}^{(2)}} + 8C_{23}^{(2)} k_1 \cdot k_2 + 8C_{33}^{(2)} k_1 \cdot k_2, \\
 f_3^{(2)} &= 4m_g^2C_{0}^{(2)} - 4m_g^2C_{12}^{(2)} + 8\bar{C}_{24}^{(2)} - 6\bar{e}_{C_{24}^{(2)}} - 8\bar{C}_{36}^{(2)} + 6\bar{e}_{C_{36}^{(2)}} \\
 &\quad + 8k_1 \cdot k_2C_{22}^{(2)} + 8k_1 \cdot k_2C_{34}^{(2)}, \\
 f_4^{(2)} &= -4m_g^2C_{0}^{(2)} - 4m_g^2C_{12}^{(2)} - 8\bar{C}_{24}^{(2)} + 6\bar{e}_{C_{24}^{(2)}} - 8\bar{C}_{36}^{(2)} + 6\bar{e}_{C_{36}^{(2)}} \\
 &\quad + 8k_1 \cdot k_2C_{12}^{(2)} + 8k_1 \cdot k_2C_{22}^{(2)} + 8k_1 \cdot k_2C_{23}^{(2)} + 8k_1 \cdot k_2C_{34}^{(2)}, \\
 f_5^{(2)} &= -8C_{12}^{(2)} - 24C_{23}^{(2)} - 16C_{33}^{(2)}, \\
 f_6^{(2)} &= -8C_{12}^{(2)} - 16C_{22}^{(2)} - 8C_{23}^{(2)} - 16C_{34}^{(2)}
\end{align*}

Similarly, the gtt vertex functions are composed of left-handed and right-handed contributions plus a counterterm: (We define $a$ as color index of external gluon and $i, j$ as colors of external top-quarks)

\begin{align*}
 \Lambda_{\mu,ijkl}^{(a)}(p_1, p_2) &= -\frac{g^2}{16\pi^2} T_{ij}^{\mu a} \{(2C_F - C_A)(\Lambda_{\mu}^{(LL)}(p_1, p_2)P_L + \Lambda_{\mu}^{(LR)}(p_1, p_2)P_R) \\
 &\quad + C_A(\Lambda_{\mu}^{(LL)}(p_1, p_2)P_L + \Lambda_{\mu}^{(LR)}(p_1, p_2)P_R) \} \\
 &\quad + (m_{\tilde{t}_i} \to m_{\tilde{t}_j}, x_i \to y_i) + \Lambda_{\mu}^{(CT)}
\end{align*}

The expressions of $\Lambda_{\mu}^{(n)}$, $n = 1L, 1R, 2L, 2R$ are given as following:

\begin{align*}
 \Lambda_{\mu}^{(n)}(p_1, p_2) &= h_1^{(n)} \gamma_{\mu} + h_2^{(n)} p_{1\mu} + h_3^{(n)} p_{2\mu} + h_4^{(n)} \not{p}_1 p_{1\mu} \\
 &\quad + h_5^{(n)} \not{p}_1 p_{2\mu} + h_6^{(n)} \not{p}_2 p_{1\mu} + h_7^{(n)} \not{p}_2 p_{2\mu} + h_8^{(n)} \gamma_{\mu} \not{p}_1 \\
 &\quad + h_9^{(n)} \not{\gamma}_{\mu} \not{p}_2 + h_{10}^{(n)} \gamma_{\mu} \not{p}_1 \not{p}_2
\end{align*}

We define

\begin{align*}
 C_0^{(3)}, C_{ij}^{(3)} &= C_0, C_{ij}[-p_1, -p_2, m_{\tilde{t}_i}, m_{\tilde{t}_j}, m_{\tilde{t}_i}]
\end{align*}
\[ C_{ij}^{(4)}, C_{ij}^{(4)} = C_0, C_{ij}[-p_1, -p_2, m_\tilde{g}, m_i, m_\tilde{g}] \]

Then we can get \( h_i^{(m)} \) as follows: \( i = 1, 2, \ldots, 10 \)

\[
\begin{align*}
    h_1^{(1L)} &= -2x_2x_4C_{24}^{(3)} \\
    h_2^{(1L)} &= x_2x_3m_\tilde{g}(C_0^{(3)} + 2C_{11}^{(3)}) \\
    h_3^{(1L)} &= x_2x_3m_\tilde{g}(C_0^{(3)} + 2C_{12}^{(3)}) \\
    h_4^{(1L)} &= x_2x_4(C_0^{(3)} + 3C_{11}^{(3)} + 2C_{21}^{(3)}) \\
    h_5^{(1L)} &= x_2x_4(C_0^{(3)} + C_{11}^{(3)} + 2C_{12}^{(3)} + 2C_{23}^{(3)}) \\
    h_6^{(1L)} &= x_2x_4(C_{12}^{(3)} + 2C_{23}^{(3)}) \\
    h_7^{(1L)} &= x_2x_4(C_{11}^{(3)} + 2C_{22}^{(3)}) \\
    h_8^{(1L)} &= h_9^{(1L)} = h_{10}^{(1L)} = 0
\end{align*}
\]

and

\[
\begin{align*}
    h_1^{(2L)} &= x_2x_4(-m_\tilde{g}^2C_0^{(4)} - 2C_{24}^{(4)} + \epsilon C_{24}^{(4)}) \\
        &+ x_2x_4p_1^2(C_1^{(4)} + C_{21}^{(4)}) + 2x_2x_4p_1\dot{p}_2(C_{12}^{(4)} + C_{23}^{(4)}) \\
        &+ x_2x_4p_2^2(C_{12}^{(4)} + C_{22}^{(4)}) \\
    h_2^{(2L)} &= 2x_2x_3m_\tilde{g}^2C_{11}^{(4)} \\
    h_3^{(2L)} &= 2x_2x_3m_\tilde{g}^2C_{12}^{(4)} \\
    h_4^{(2L)} &= -2x_2x_4(C_{11}^{(4)} + C_{21}^{(4)}) \\
    h_5^{(2L)} &= -2x_2x_4(C_{12}^{(4)} + C_{23}^{(4)}) \\
    h_6^{(2L)} &= -2x_2x_4(C_{11}^{(4)} + C_{23}^{(4)})
\end{align*}
\]
\[ h^{(2L)}_7 = -2x_2x_4(C_{12}^{(4)} + C_{22}^{(4)}) \]
\[ h^{(2L)}_8 = h^{(2L)}_9 = x_2x_3m_\gamma^2C_0^{(4)} \]
\[ h^{(2L)}_{10} = x_2x_4(C_{11}^{(4)} - C_{12}^{(4)}) \]

\( h_i^{(1R)} \) and \( h_i^{(2R)} \) can be obtained by exchanging \( x_1 \leftrightarrow x_2 \) and \( x_3 \leftrightarrow x_4 \) in \( h_i^{(1L)} \) and \( h_i^{(2L)} \). (\( i = 1, 2, ..., 10 \))

The counter terms are given by:
\[
\Lambda^{(CT)}_\mu = -C_F \bar{q} \gamma_\mu T^a_3 \gamma_\mu \left[ (\delta Z_L + \delta Z_R^\dagger) P_L + (\delta Z_R + \delta Z_R^\dagger) P_R \right] \quad \text{(A.b.10)}
\]

The wave function renormalization constants can be obtained from Eq.(3.c.12) and Eq.(3.c.13).

C. Box corrections:

Finally, we list the four form factors \( F^{ti\mu\nu}_{kL} \) as given in Eq.(3.e.1) in terms of Passarino-Veltmann functions. First, we define \( F^{tiL}_{kL} \) and \( F^{tiR}_{kR} \) by:
\[
F^{(ti)}_{\mu\nu} = \frac{i\delta^2}{16\pi^2} P_R \left[ \gamma_\mu \gamma_\nu F_{1}^{(tiR)} + \gamma_\nu \gamma_\mu F_{2}^{(tiR)} \right.
+ \gamma_\mu \gamma_\nu F_{3}^{(tiR)} + \gamma_\nu \gamma_\mu F_{4}^{(tiR)}
+ \gamma_\mu \gamma_\nu F_{5}^{(tiR)} + \gamma_\nu \gamma_\mu F_{6}^{(tiR)}
+ \gamma_\mu \gamma_\nu F_{7}^{(tiR)} + \gamma_\nu \gamma_\mu F_{8}^{(tiR)}
+ \gamma_\mu \gamma_\nu F_{9}^{(tiR)} + \gamma_\nu \gamma_\mu F_{10}^{(tiR)}
+ \gamma_\mu \gamma_\nu F_{11}^{(tiR)} + \gamma_\nu \gamma_\mu F_{12}^{(tiR)}
+ \gamma_\mu \gamma_\nu F_{13}^{(tiR)} + \gamma_\nu \gamma_\mu F_{14}^{(tiR)}
+ \gamma_\mu \gamma_\nu F_{15}^{(tiR)} + \gamma_\nu \gamma_\mu F_{16}^{(tiR)}
+ \gamma_\mu \gamma_\nu F_{17}^{(tiR)} + \gamma_\nu \gamma_\mu F_{18}^{(tiR)}
+ \gamma_\mu \gamma_\nu F_{19}^{(tiR)} + \gamma_\nu \gamma_\mu F_{20}^{(tiR)}
\left. + (P_R \rightarrow P_L, F_{k}^{(tiR)} \rightarrow F_{k}^{(tiL)}) \right), (k = 1 \sim 20, \ i = 1 \sim 4). \quad \text{(A.c.1)}
\]

In the following we only give the expressions of \( F^{tiR}_{k}(k = 1, 2, ..., 20 \ and \ i = 1 \sim 4) \). The expressions of \( F^{tiL}_{k} \) can be obtained from \( F^{tiR}_{k} \) by exchanging \( x_1 \leftrightarrow x_2 \) and \( x_3 \leftrightarrow x_4 \).
Furthermore, the form factors in the u-channel are given by

\[ F_{\mu\nu}^{(u)}(k_1, k_2, p_1, p_2) = F_{\nu\mu}^{(u)}(k_2, k_1, p_1, p_2) \]  

(A.c.2)

The expressions of \( F_{k}^{(t1R)} \) (\( k = 1 \sim 20 \)) are given as below:

\[
F_1^{(t1R)} = F_2^{(t1R)} = -2x_1x_4m_5D_{27}^{(1)} + 2x_2x_4m_tD_{31}^{(1)} + 2(x_1x_3 - x_2x_4)m_tD_{313}^{(1)},
\]

(A.c.3)

\[
F_3^{(t1R)} = 4x_1x_3(D_{31}^{(1)} - D_{312}^{(1)}),
\]

\[
F_4^{(t1R)} = -4x_1x_3(D_{27}^{(1)} + D_{312}^{(1)}),
\]

\[
F_5^{(t1R)} = 4x_1x_3(D_{27}^{(1)} + D_{311}^{(1)} - D_{313}^{(1)}),
\]

\[
F_6^{(t1R)} = -4x_1x_3D_{313}^{(1)},
\]

\[
F_7^{(t1R)} = F_8^{(t1R)} = 2x_1x_3(D_{313}^{(1)} - D_{312}^{(1)}),
\]

\[
F_9^{(t1R)} = 4x_1x_3(D_{13}^{(1)} - D_{12}^{(1)} - D_{22}^{(1)} - D_{23}^{(1)} - D_{24}^{(1)})
+ D_{25}^{(1)} + 2D_{26}^{(1)} - D_{36}^{(1)} + D_{38}^{(1)} - D_{39}^{(1)} + D_{310}^{(1)},
\]

\[
F_{10}^{(t1R)} = 4x_1x_3(D_{37}^{(1)} - D_{39}^{(1)} + D_{38}^{(1)} - D_{310}^{(1)}),
\]

\[
F_{11}^{(t1R)} = F_{12}^{(t1R)} = F_{13}^{(t1R)} = F_{14}^{(t1R)} = 0,
\]

\[
F_{15}^{(t1R)} = 4x_1x_3m_5(D_{13}^{(1)} - D_{23}^{(1)} + D_{25}^{(1)} + D_{26}^{(1)} - D_{39}^{(1)} + D_{310}^{(1)})
- 4x_1x_4m_5(D_{0}^{(1)} + D_{11}^{(1)} + D_{12}^{(1)} - D_{13}^{(1)} + D_{24}^{(1)} - D_{26}^{(1)})
+ 4x_2x_4m_5(D_{11}^{(1)} - D_{13}^{(1)} + D_{21}^{(1)} + D_{23}^{(1)} + D_{24}^{(1)} - 2D_{25}^{(1)}
- D_{26}^{(1)} + D_{34}^{(1)} + D_{39}^{(1)} - 2D_{310}^{(1)}),
\]

\[
F_{16}^{(t1R)} = 4x_1x_3m_tD_{25}^{(1)} + D_{26}^{(1)} - D_{35}^{(1)} + D_{37}^{(1)} - D_{39}^{(1)} + D_{310}^{(1)}
+ 4x_1x_4m_5(D_{11}^{(1)} - D_{12}^{(1)} + D_{21}^{(1)} - D_{24}^{(1)} - D_{25}^{(1)} + D_{26}^{(1)})
- 4x_2x_4m_5(D_{21}^{(1)} - D_{24}^{(1)} - D_{25}^{(1)} + D_{26}^{(1)} + D_{31}^{(1)}
- D_{34}^{(1)} - 2D_{35}^{(1)} + D_{37}^{(1)} - D_{39}^{(1)} + 2D_{310}^{(1)}),
\]

\]

22
\[
F_{17}^{(I R)} = 4x_1 x_3 m_t (D_{27}^{(1)} - D_{39}^{(1)}) - 4x_1 x_4 m_\bar{\delta}(D_{25}^{(1)} - D_{26}^{(1)}) + 4x_2 x_4 m_t (D_{35}^{(1)} - D_{37}^{(1)}) + D_{39}^{(1)} - D_{37}^{(1)} - D_{39}^{(1)}),
\]
\[
F_{18}^{(I R)} = 4x_1 x_3 (-D_{27}^{(1)} + D_{24}^{(1)} - D_{28}^{(1)} + D_{26}^{(1)} + D_{34}^{(1)}) - D_{35}^{(1)} - D_{36}^{(1)} + D_{37}^{(1)} + D_{38}^{(1)} - D_{39}^{(1)}),
\]
\[
F_{19}^{(I R)} = 4x_1 x_3 (-D_{23}^{(1)} + D_{26}^{(1)} - D_{39}^{(1)} + D_{38}^{(1)}),
\]
\[
F_{20}^{(I R)} = 4x_1 x_3 m_t (-D_{23}^{(1)} - D_{39}^{(1)}) + 4x_1 x_4 m_\bar{\delta}(D_{13}^{(1)} + D_{26}^{(1)}) + 4x_2 x_4 m_t (D_{23}^{(1)} - D_{25}^{(1)} + D_{38}^{(1)} - D_{310}^{(1)}),
\]

where we denote \(D_i^{(1)}, D_{ij}^{(1)}, D_{ijk}^{(1)} = D_i, D_{ij}, D_{ijk} \{-p_1, k_1, k_2, m_\bar{\delta}, m_i, m_i, m_i\} \).

The expressions of \(F_k^{(I R)} \) \((k = 1 \sim 20)\) are as follows:

\[
F_1^{(I R)} = 2x_1 x_3 m_t (D_{27}^{(2)} + D_{313}^{(2)}) + 2x_1 x_4 m_\bar{\delta} D_{27}^{(2)} - 2x_2 x_4 m_t (D_{27}^{(2)} + D_{311}^{(2)}), \quad \text{(A.c.4)}
\]
\[
F_2^{(I R)} = 2x_1 x_3 m_t D_{313}^{(2)} + 2x_1 x_4 m_\bar{\delta} D_{27}^{(2)} - 2x_2 x_4 m_t D_{313}^{(2)},
\]
\[
F_3^{(I R)} = 4x_1 x_3 (-D_{27}^{(2)} - D_{311}^{(2)} + D_{312}^{(2)}),
\]
\[
F_4^{(I R)} = 4x_1 x_3 (D_{312}^{(2)} - D_{313}^{(2)}),
\]
\[
F_5^{(I R)} = 8x_1 x_3 (D_{27}^{(2)} + D_{313}^{(2)}) + 2x_1 x_3 m_t^2 (D_{12}^{(2)} + D_{11}^{(2)}) + 2x_1 x_3 m_t^2 (D_{12}^{(2)} - D_{13}^{(2)} - 2D_{21}^{(2)} - D_{23}^{(2)} - D_{25}^{(2)} - D_{31}^{(2)} - D_{37}^{(2)}) + 2(x_2 x_3 + x_1 x_4) m_\bar{\delta} m_t (D_{11}^{(2)} + D_{12}^{(2)}) - 2x_2 x_4 m_t^2 (D_{11}^{(2)} - D_{13}^{(2)} + D_{21}^{(2)} - D_{25}^{(2)}) + 4x_1 x_3 k_1 \cdot p_1 (D_{12}^{(2)} + 2D_{24}^{(2)} + D_{34}^{(2)}) + 4x_1 x_3 k_1 \cdot p_2 (D_{13}^{(2)} + D_{25}^{(2)} + D_{26}^{(2)} + D_{310}^{(2)}) - 4x_1 x_3 p_1 \cdot p_2 (D_{13}^{(2)} + 2D_{25}^{(2)} + D_{35}^{(2)}),
\]
\[
F_6^{(I R)} = 2x_1 x_3 m_\bar{\delta}^2 D_{13}^{(2)} - 2x_1 x_3 m_t^2 (D_{23}^{(2)} + D_{25}^{(2)} + D_{33}^{(2)} + D_{35}^{(2)}) + 2(x_2 x_3 + x_1 x_4) m_\bar{\delta} m_t D_{13}^{(2)} + 2x_2 x_4 m_t^2 (D_{23}^{(2)} - D_{25}^{(2)}) + 4x_1 x_3 k_1 \cdot p_1 (D_{26}^{(2)} + D_{310}^{(2)}) + 4x_1 x_3 k_1 \cdot p_2 (D_{23}^{(2)} + D_{39}^{(2)}) - 4x_1 x_3 p_1 \cdot p_2 (D_{23}^{(2)} + D_{37}^{(2)}) + 8x_1 x_3 D_{313}^{(2)},
\]
\[
F_7^{(I R)} = 2x_1 x_3 (D_{27}^{(2)} + D_{312}^{(2)}),
\]
\[
F_8^{(I R)} = 2x_1 x_3 D_{312}^{(2)}.
\]
\[ F_{9}^{(12R)} = 4x_1x_3(D_{12}^{(2)} - D_{13}^{(2)} + D_{22}^{(2)} + D_{24}^{(2)} - D_{25}^{(2)} - D_{26}^{(2)} + D_{36}^{(2)} - D_{310}^{(2)}), \]

\[ F_{10}^{(12R)} = 4x_1x_3(D_{38}^{(2)} - D_{310}^{(2)}), \]

\[ F_{11}^{(12R)} = F_{12}^{(12R)} = 0, \]

\[ F_{13}^{(12R)} = 2x_1x_3m_t(-D_{12}^{(2)} + D_{13}^{(2)} - D_{24}^{(2)} + D_{25}^{(2)}) + 2x_1x_4m_{\bar{g}}(D_{0}^{(2)} + D_{11}^{(2)}) - 2x_2x_4m_t(D_{11}^{(2)} - D_{12}^{(2)} + D_{21}^{(2)} - D_{22}^{(2)}), \]

\[ F_{14}^{(12R)} = 2x_1x_3m_t(D_{12}^{(2)} - D_{26}^{(2)}) - 2x_1x_4m_{\bar{g}}D_{13}^{(2)} - 2x_2x_4m_t(D_{25}^{(2)} - D_{26}^{(2)}), \]

\[ F_{15}^{(12R)} = 4x_1x_3m_t(D_{12}^{(2)} - D_{13}^{(2)} - D_{23}^{(2)} + D_{24}^{(2)} - D_{25}^{(2)} + D_{26}^{(2)} - D_{37}^{(2)} + D_{310}^{(2)} + 4x_1x_4m_{\bar{g}}(D_{12}^{(2)} - D_{13}^{(2)} + D_{24}^{(2)} - D_{25}^{(2)}) - 4x_2x_4m_t(D_{12}^{(2)} - D_{13}^{(2)} + 2D_{24}^{(2)} - 2D_{25}^{(2)} + D_{34}^{(2)} - D_{35}^{(2)}), \]

\[ F_{16}^{(12R)} = 4x_1x_3m_t(D_{12}^{(2)} - D_{13}^{(2)} + D_{24}^{(2)} - 2D_{25}^{(2)} + D_{26}^{(2)} - D_{35}^{(2)} + D_{310}^{(2)} + 4x_1x_4m_{\bar{g}}(-D_0^{(2)} - 2D_{11}^{(2)} + D_{12}^{(2)} - D_{21}^{(2)} + D_{24}^{(2)}) + 4x_2x_4m_t(D_{11}^{(2)} - D_{12}^{(2)} + 2D_{21}^{(2)} - 2D_{24}^{(2)} + D_{31}^{(2)} - D_{34}^{(2)}), \]

\[ F_{17}^{(12R)} = 4x_1x_3m_t(-D_{23}^{(2)} + D_{26}^{(2)} - D_{37}^{(2)} + D_{39}^{(2)}) + 4x_1x_4m_{\bar{g}}(-D_{13}^{(2)} - D_{25}^{(2)} + D_{26}^{(2)}) + 4x_2x_4m_t(D_{23}^{(2)} - D_{26}^{(2)} + D_{35}^{(2)} - D_{310}^{(2)}), \]

\[ F_{18}^{(12R)} = 4x_1x_3(D_{22}^{(2)} - D_{24}^{(2)} - D_{34}^{(2)} + D_{36}^{(2)}), \]

\[ F_{19}^{(12R)} = 4x_1x_3(-D_{23}^{(2)} + D_{26}^{(2)} - D_{38}^{(2)} - D_{39}^{(2)}), \]

\[ F_{20}^{(12R)} = 4x_1x_3m_t(-D_{23}^{(2)} + D_{26}^{(2)} - D_{33}^{(2)} + D_{39}^{(2)}) + 4x_1x_4m_{\bar{g}}(-D_{23}^{(2)} + D_{26}^{(2)}) + 4x_2x_4m_t(D_{23}^{(2)} - D_{26}^{(2)} + D_{37}^{(2)} - D_{310}^{(2)}), \]

where \(D_i^{(2)}, D_{ij}^{(2)}, D_{ijk}^{(2)} = D_i, D_{ij}, D_{ijk}[-p_1, k_1, -p_2, m_{\bar{g}}, m_{\bar{t}}, m_{\bar{t}}, m_{\bar{g}}].\)
The expressions for $F_k^{(13R)}$ ($k = 1 \sim 20$) are written as:

\[
F_1^{(13R)} = 2x_1x_3m_1(D_{27}^{(3)} + 2D_{313}^{(3)}) + x_1x_3m_1m_2^2(D_0^{(3)} + D_3^{(3)}) \\
- x_1x_3m_1^2(D_0^{(3)} + 2D_{11}^{(3)} - D_{13}^{(3)} + D_{21}^{(3)} + 2D_{33}^{(3)} + 2D_{33}^{(3)} - 2D_{37}^{(3)}) \\
+ 2x_1x_4m_5D_2^{(3)} + x_1x_4m_5^2D_0^{(3)} \\
-x_1x_4m_5^2m_0^2(D_0^{(3)} + 2D_{11}^{(3)} - 2D_{13}^{(3)} + D_{21}^{(3)} + 2D_{23}^{(3)} - 2D_{25}^{(3)}) \\
+ 2x_1x_4m_5(D_2^{(3)} + 2D_{311}^{(3)} - 2D_{313}^{(3)}) + x_2x_4m_5^2(D_{11}^{(3)} - D_{13}^{(3)}) \\
- x_2x_4m_5^2(D_{11}^{(3)} - D_{13}^{(3)} + 2D_{21}^{(3)} + 2D_{23}^{(3)} - 4D_{25}^{(3)} + D_{31}^{(3)} - 2D_{33}^{(3)} - 3D_{35}^{(3)} + 4D_{37}^{(3)}) \\
+ x_1x_3m_1^2 \cdot p_1(D_{12}^{(3)} - D_{13}^{(3)} - D_{23}^{(3)} + D_{24}^{(3)} + D_{33}^{(3)} - D_{37}^{(3)} - D_{39}^{(3)} + D_{310}^{(3)}) \\
+ x_2x_1m_5k_1 \cdot p_1(D_{11}^{(3)} + D_{12}^{(3)} - 2D_{13}^{(3)} + D_{23}^{(3)} + D_{24}^{(3)} - D_{25}^{(3)} - D_{26}^{(3)}) \\
+ x_2x_1m_5k_1 \cdot p_1(D_{11}^{(3)} - D_{13}^{(3)} + D_{21}^{(3)} + 2D_{25}^{(3)} \\
+ D_{24}^{(3)} - 3D_{25}^{(3)} - D_{26}^{(3)} + D_{33}^{(3)} - D_{35}^{(3)} + D_{37}^{(3)} + D_{39}^{(3)} - 2D_{310}^{(3)}) \\
+ x_1x_3m_1k_1 \cdot p_2(-D_{11}^{(3)} - D_{26}^{(3)} + D_{33}^{(3)} - D_{35}^{(3)}) \\
+ x_1x_4m_5k_1 \cdot p_2(-D_{13}^{(3)} + 2D_{23}^{(3)} - D_{26}^{(3)}) \\
+ x_1x_4m_5k_1 \cdot p_2(D_{23}^{(3)} - D_{25}^{(3)} - D_{33}^{(3)} + D_{37}^{(3)}) \\
+ x_1x_3m_1p_1 \cdot p_2(D_{13}^{(3)} + D_{23}^{(3)} - D_{37}^{(3)}) \\
+ x_1x_3m_1p_1 \cdot p_2(-D_{23}^{(3)} + D_{25}^{(3)} + D_{33}^{(3)} + D_{35}^{(3)} - D_{37}^{(3)}),
\]

(A.e.5)

\[
F_2^{(13R)} = -2x_1x_4m_2D_{27}^{(3)} - 2x_1x_3m_1D_{313}^{(3)} - 2x_2x_4m_1(D_{27}^{(3)} + D_{311}^{(3)} - D_{313}^{(3)}),
\]

\[
F_3^{(13R)} = 4x_1x_3(D_{27}^{(3)} + 2D_{312}^{(3)} - 3D_{313}^{(3)}) + 2x_1x_3m_2^2(-D_{0}^{(3)} + D_{11}^{(3)} - D_{13}^{(3)}) \\
+ 2x_1x_3m_1^2(D_{13}^{(3)} - D_{21}^{(3)} + 2D_{25}^{(3)} - D_{31}^{(3)} + 2D_{33}^{(3)} + 3D_{35}^{(3)} - 4D_{37}^{(3)}) \\
- 2x_2x_3m_1m_3D_0^{(3)} - 2x_2x_3m_1m_3D_0^{(3)} - 2x_2x_4m_1^2(D_{0}^{(3)} - D_{11}^{(3)}) \\
+ 4x_1x_3k_1 \cdot p_1(D_{23}^{(3)} + D_{24}^{(3)} - D_{25}^{(3)} - D_{26}^{(3)}) \\
- D_{33}^{(3)} + D_{34}^{(3)} + 2D_{37}^{(3)} + D_{39}^{(3)} - 2D_{310}^{(3)}) \\
+ 4x_1x_3k_1 \cdot p_2(-D_{25}^{(3)} + D_{26}^{(3)} + D_{33}^{(3)} + D_{37}^{(3)} + D_{39}^{(3)} - D_{310}^{(3)}) \\
+ 4x_1x_3p_1 \cdot p_2(D_{33}^{(3)} + D_{35}^{(3)} - 2D_{37}^{(3)}),
\]

\[
F_4^{(13R)} = 4x_1x_3D_{312}^{(3)} - 2x_1x_3m_2^2D_0^{(3)} \\
+ 2x_1x_3m_2^2(D_{11}^{(3)} + D_{21}^{(3)} + 2D_{23}^{(3)} - 2D_{25}^{(3)}) \\
- 2x_2x_3m_1m_3D_0^{(3)} - 2x_1x_4m_1m_3D_0^{(3)} \\
- 2x_2x_4m_1^2(D_{0}^{(3)} + D_{11}^{(3)}) + 4x_1x_3k_1 \cdot p_1(-D_{23}^{(3)} + D_{25}^{(3)}) \\
+ 4x_1x_3k_1 \cdot p_2(-D_{23}^{(3)} + D_{26}^{(3)}) + 4x_1x_3p_1 \cdot p_2(D_{23}^{(3)} - D_{25}^{(3)}),
\]

25
\[ F_6^{(13R)} = 4x_1x_3(-D_{11}^{(3)} + D_{13}^{(3)}) + 2x_1x_3m_2^3D_0^{(3)} - 2x_1x_3m_2^3(D_{13}^{(3)} + D_{25}^{(3)}) + 2x_2x_3m_2m_5^3(D_0^{(3)} + D_{11}^{(3)} - D_{13}^{(3)}) + 2x_1x_4m_4m_5^3(D_0^{(3)} + D_{11}^{(3)} - D_{13}^{(3)}) + 2x_2x_4m_4^2D_0^{(3)} + D_{13}^{(3)} - D_{25}^{(3)}) + 4x_1x_3k_1 \cdot p_2(D_{25}^{(3)} - D_{26}^{(3)}), \]

\[ F_6^{(13R)} = -8x_1x_3D_{13}^{(3)} - 2x_1x_3m_2^3D_0^{(3)} + 2x_1x_3m_2^3(D_{25}^{(3)} + 2D_{33}^{(3)} + D_{35}^{(3)} - 2D_{37}^{(3)}) - 2x_2x_3m_2m_5^3D_{13}^{(3)} - 2x_1x_4m_4m_5^3D_{13}^{(3)} - 2x_2x_4m_4^2D_0^{(3)} + D_{13}^{(3)} + D_{25}^{(3)}) + 4x_1x_3k_1 \cdot p_1(D_{23}^{(3)} - D_{25}^{(3)} - D_{33}^{(3)} + D_{37}^{(3)} + D_{39}^{(3)} - D_{310}^{(3)}) + 4x_1x_3k_1 \cdot p_2(-D_{33}^{(3)} + D_{39}^{(3)}) + 4x_1x_3p_1 \cdot p_2(D_{33}^{(3)} - D_{37}^{(3)}), \]

\[ F_7^{(13R)} = -4x_1x_3(D_{12}^{(3)} - D_{13}^{(3)}) + x_1x_3m_2^3(D_0^{(3)} - D_{12}^{(3)} + D_{13}^{(3)}) + x_1x_3m_2^3(-D_{11}^{(3)} + D_{12}^{(3)} - D_{13}^{(3)} - D_{21}^{(3)}) + 2D_{24}^{(3)} - 2D_{26}^{(3)} - 2D_{34}^{(3)} - D_{35}^{(3)} + 2D_{37}^{(3)} + 2D_{39}^{(3)} - 2D_{310}^{(3)}) + (x_2x_3 + x_1x_4)m_4m_5^3D_0^{(3)} + x_2x_4m_4^2(D_0^{(3)} + D_{11}^{(3)}) + 2x_1x_3k_1 \cdot p_1(-D_{22}^{(3)} - D_{23}^{(3)} + 2D_{26}^{(3)} + D_{33}^{(3)} - D_{35}^{(3)} + D_{37}^{(3)} + D_{39}^{(3)} - D_{310}^{(3)}) + 2x_1x_3k_1 \cdot p_2(D_{33}^{(3)} + D_{38}^{(3)} - 2D_{39}^{(3)}) + 2x_1x_3p_1 \cdot p_2(D_{25}^{(3)} - D_{26}^{(3)} - D_{33}^{(3)} + D_{37}^{(3)} + D_{39}^{(3)} - D_{310}^{(3)}), \]

\[ F_8^{(13R)} = 2x_1x_3(D_{27}^{(3)} + D_{32}^{(3)} - D_{310}^{(3)}), \]

\[ F_9^{(13R)} = 4x_1x_3(D_{22}^{(3)} + D_{23}^{(3)} - D_{25}^{(3)} - D_{26}^{(3)} + D_{36}^{(3)} - D_{38}^{(3)} - D_{39}^{(3)} - D_{310}^{(3)}), \]

\[ F_{10}^{(13R)} = 4x_1x_3(D_{25}^{(3)} - D_{26}^{(3)} - D_{37}^{(3)} - D_{38}^{(3)} + D_{39}^{(3)} + D_{310}^{(3)}), \]

\[ F_{11}^{(13R)} = 2x_1x_3m_2(-D_{25}^{(3)} + D_{26}^{(3)}) + 2x_1x_4m_5^3(-D_{11}^{(3)} + D_{12}^{(3)}) + 2x_2x_4m_4m_5^3D_{12}^{(3)} - D_{13}^{(3)} + 2D_{24}^{(3)} + D_{25}^{(3)} - D_{26}^{(3)}), \]

\[ F_{12}^{(13R)} = 2x_1x_3m_2D_{13}^{(3)} + D_{26}^{(3)} + 2x_1x_4m_5^3D_{12}^{(3)} - D_{13}^{(3)} + 2D_{24}^{(3)} + D_{25}^{(3)} - D_{26}^{(3)}), \]

\[ F_{13}^{(13R)} = 2x_1x_3m_2(D_{12}^{(3)} - D_{13}^{(3)} + D_{24}^{(3)} - D_{25}^{(3)} + 2x_1x_4m_5^3(D_{11}^{(3)} - D_{13}^{(3)}), \]

\[ 2x_2x_4m_4m_5^3D_{12}^{(3)} - D_{13}^{(3)} + 2D_{24}^{(3)} + D_{25}^{(3)} - D_{26}^{(3)}), \]

\[ F_{14}^{(13R)} = -2x_1x_3m_2(D_{13}^{(3)} + D_{26}^{(3)}) - 2x_1x_4m_5^3D_{13}^{(3)} + 2x_2x_4m_4m_5^3D_{12}^{(3)} - D_{24}^{(3)} + D_{25}^{(3)} - D_{26}^{(3)}), \]

\[ F_{14}^{(13R)} = -2x_1x_3m_2(D_{13}^{(3)} + D_{26}^{(3)}) - 2x_1x_4m_5^3D_{13}^{(3)} + 2x_2x_4m_4m_5^3D_{12}^{(3)} - D_{24}^{(3)} + D_{25}^{(3)} - D_{26}^{(3)}), \]

\[ 26 \]
\[ F_{15}^{(13R)} = -4x_1 x_3 m_t (D_{12}^{(3)} - D_{23}^{(3)} + D_{24}^{(3)} + D_{25}^{(3)} - D_{39}^{(3)} + D_{310}^{(3)}) \\
+ 4x_1 x_4 m_{f(B)} (-D_{11}^{(3)} - D_{12}^{(3)} + D_{13}^{(3)} - D_{24}^{(3)} + D_{26}^{(3)}) \\
+ 4x_2 x_4 m_t (-D_{13}^{(3)} + D_{23}^{(3)} - D_{21}^{(3)} - D_{23}^{(3)} \\
- D_{24}^{(3)} + 2D_{25}^{(3)} + D_{26}^{(3)} - D_{34}^{(3)} - D_{39}^{(3)} + 2D_{310}^{(3)}), \\
F_{16}^{(13R)} = -4x_1 x_3 m_t (D_{12}^{(3)} - D_{13}^{(3)} + D_{24}^{(3)} - D_{25}^{(3)} - D_{35}^{(3)} + D_{37}^{(3)} - D_{39}^{(3)} + D_{310}^{(3)}) \\
+ 4x_1 x_4 m_{f(B)} (-D_{11}^{(3)} + D_{13}^{(3)} + D_{21}^{(3)} - D_{24}^{(3)} - D_{25}^{(3)}) \\
+ 4x_2 x_4 m_t (D_{21}^{(3)} - D_{24}^{(3)} - D_{25}^{(3)} + D_{26}^{(3)} \\
+ D_{31}^{(3)} - D_{34}^{(3)} - 2D_{35}^{(3)} + D_{37}^{(3)} - D_{39}^{(3)} + 2D_{310}^{(3)}), \\
F_{17}^{(13R)} = 4x_1 x_3 m_t (D_{13}^{(3)} + D_{26}^{(3)} - D_{37}^{(3)} + D_{39}^{(3)}) \\
+ 4x_1 x_4 m_{f(B)} (D_{13}^{(3)} - D_{25}^{(3)} + D_{26}^{(3)}) + 4x_2 x_4 m_t (-D_{35}^{(3)} + D_{37}^{(3)} - D_{39}^{(3)} + D_{310}^{(3)}), \\
F_{18}^{(13R)} = 4x_1 x_3 (D_{22}^{(3)} - D_{24}^{(3)} + D_{25}^{(3)} - D_{26}^{(3)} - D_{34}^{(3)} + D_{35}^{(3)} + D_{36}^{(3)} \\
- D_{37}^{(3)} - D_{38}^{(3)} + D_{39}^{(3)}), \\
F_{19}^{(13R)} = 4x_1 x_3 (D_{23}^{(3)} - D_{26}^{(3)} - D_{38}^{(3)} + D_{39}^{(3)}), \\
F_{20}^{(13R)} = 4x_1 x_3 m_t (D_{3}^{(3)} + D_{23}^{(3)} + D_{26}^{(3)} + D_{39}^{(3)}) \\
+ 4x_1 x_4 m_{f(B)} (D_{13}^{(3)} + D_{26}^{(3)}) + 4x_2 x_4 m_t (-D_{23}^{(3)} + D_{25}^{(3)} - D_{39}^{(3)} + D_{310}^{(3)}), \\
\text{where } D_{i}^{(3)}, D_{ij}^{(3)}, D_{ijk}^{(3)} = D_{i}, D_{ij}, D_{ijk}[-p_{1}, k_{1}, k_{2}, m_{f}, m_{g}, m_{g}, m_{g}]. \\
The F_{k}^{(14R)} (k = 1 \sim 20) \text{ are written explicitly as:} \\
F_{1}^{(14R)} = F_{2}^{(14R)} \\
= \frac{1}{2} ((x_1 x_3 m_t (C_{11} - C_{12}) \\
+ x_2 x_4 m_t C_{12} - x_2 x_3 m_{f(B) C_{0}} [-p_{1}, p_{1} + p_{2}, m_{g}, m_{g}, m_{g}])) \\
F_{i}^{(14R)} = 0, (i = 3, 4, \ldots 20). \\
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Figure Captions

**Fig. 1** Feynman diagrams at the tree-level and one-loop level in the SUSY QCD for the $gg \to t\bar{t}$ subprocess. Fig.1 (a): Tree level diagrams. Fig.1 (b.1): Self-energy diagrams (for top-quark and gluon). Fig.1 (b.2): Vertex diagrams (including tri-gluon and gluon-top-top interactions). Fig.1 (b.3): Box diagrams (only t-channel). Dashed lines represent $\tilde{t}_1, \tilde{t}_2$ in Fig.1 (b).

**Fig. 2** (a) relative corrections to polarized and unpolarized cross sections of the $t\bar{t}$ production process in pp colliders as a function of $\sqrt{s}$ with input structure functions of Brodsky et al. \cite{8}(LO); 
(b) relative corrections to polarized and unpolarized cross sections of the $t\bar{t}$ production process in pp colliders as a function of $\sqrt{s}$ with input structure functions of Glück et al. \cite{9}, \cite{10}, \cite{11}(NLO), in both above figures, solid line for the MSSM QCD correction with unpolarized protons, dashed line for the MSSM QCD correction with proton(+)proton(+) polarization, dotted line for the MSSM QCD correction with proton(−)proton(−) polarization and dot-dashed line for the MSSM QCD correction with proton(+)proton(−) polarization;
(c) the CP-violating parameter $\xi_{CP}$ as a function of $\sqrt{s}$, solid line for input structure functions of Glück et al(NLO), dashed line for input structure functions of Brodsky et
Fig. 3 (a) relative corrections to the cross section of the $t\bar{t}$ production subprocess, $\hat{\delta}_{\pm\pm}$ as a function of $\sqrt{s}$, solid line for the MSSM QCD correction with $\text{gluon}(+)\text{gluon}(+)$ polarization and dashed line for the MSSM QCD correction with $\text{gluon}(-)\text{gluon}(-)$ polarization. (b) the CP-violating parameter $\hat{\xi}_{CP}$ of the subprocess as a function of $\sqrt{s}$. (c) relative corrections to the cross section of the subprocess $\hat{\delta}_{+-}$ as a function of $\sqrt{s}$.

$m_{\tilde{g}} = 200 \, GeV$, $m_{\tilde{t}_1} = 250 \, GeV$, $m_{\tilde{t}_2} = 450 \, GeV$ and $\theta = \phi = 45^\circ$.

Fig. 4 (a) cross section of the $t\bar{t}$ production subprocess via gg fusion, $\hat{\sigma}_{\pm\pm}$ as a function of $m_{\tilde{g}}$, solid line for the MSSM QCD correction with $\text{gluon}(+)\text{gluon}(+)$ polarization and dashed line for the MSSM QCD correction with $\text{gluon}(-)\text{gluon}(-)$ polarization. (b) the CP-violating parameter $\hat{\xi}_{CP}$ of the subprocess as a function of $m_{\tilde{g}}$.

$m_{\tilde{g}} = 200 \, GeV$, $m_{\tilde{t}_1} = 250 \, GeV$, $m_{\tilde{t}_2} = 450 \, GeV$ and $\theta = \phi = 45^\circ$.

$m_{\tilde{g}} = 200 \, GeV$, $m_{\tilde{t}_1} = 250 \, GeV$, $m_{\tilde{t}_2} = 450 \, GeV$ and $\theta = \phi = 45^\circ$.

Fig. 5 (a) relative corrections to the cross section of the $t\bar{t}$ production subprocess via gg fusion, $\hat{\delta}_{\pm\pm}$ as a function of $m_{\tilde{t}_1}$, solid line for the MSSM QCD correction with $\text{gluon}(+)\text{gluon}(+)$ polarization and dashed line for the MSSM QCD correction with $\text{gluon}(-)\text{gluon}(-)$ polarization. (b) the CP-violating parameter $\hat{\xi}_{CP}$ of the subprocess as a function of $m_{\tilde{t}_1}$.

$m_{\tilde{g}} = 200 \, GeV$, $m_{\tilde{t}_1} = 250 \, GeV$, $m_{\tilde{t}_2} = 450 \, GeV$ and $\theta = \phi = 45^\circ$.

$m_{\tilde{g}} = 200 \, GeV$, $m_{\tilde{t}_2} = 450 \, GeV$, $\sqrt{s} = 500 \, GeV$ and $\theta = \phi = 45^\circ$.

Fig. 6 (a) relative corrections to the cross section of the $t\bar{t}$ production subprocess via gg fusion, $\hat{\delta}_{\pm\pm}$ as a function of $m_{\tilde{t}_2}$, solid line for the MSSM QCD correction with $\text{gluon}(+)\text{gluon}(+)$ polarization and dashed line for the MSSM QCD correction with $\text{gluon}(-)\text{gluon}(-)$
polarization. (b) the CP-violating parameter $\hat{\xi}_{CP}$ of the subprocess as a function of $m_{\tilde{t}_2}$.

$m_{\tilde{g}} = 200 \text{ GeV}, m_{\tilde{t}_1} = 100 \text{ GeV}$ and $\sqrt{s} = 500 \text{ GeV}$ and $\theta = \phi = 45^\circ$.

**Fig. 7** (a) relative corrections to the cross section of the $t\bar{t}$ production subprocess via gg fusion, $\hat{\delta}_{\pm\pm}$ as a function of $\phi$, solid line for the MSSM QCD correction with $gluon(+)gluon(+$) polarization and dashed line for the MSSM QCD correction with $gluon(-)gluon(-)$ polarization. (b) the CP-violating parameter $\hat{\xi}_{CP}$ of the subprocess as a function of $\phi$.

$m_{\tilde{g}} = 200 GeV, m_{\tilde{t}_1} = 150 GeV, m_{\tilde{t}_2} = 450 GeV, \sqrt{s} = 500 GeV$ and $\theta = 45^\circ$. 


\[ \varepsilon^{a}_{\mu}(k_1) \rightarrow t(p1) \text{ Fig.1 (a)} \]

\[ \varepsilon^{b}_{v}(k_2) \rightarrow t(p2) \]

\[ g \text{ Fig.1 (b-1)} \]
Fig. 1 (b-2)
Fig. 2(c)
Fig. 1 (b-3)
Fig. 4(a)

$\sigma(gg \rightarrow t\bar{t})$ (pb)

mass of gluino (GeV)
Fig. 4(b)

\[ \xi_{CP} (gg \rightarrow t \bar{t}) \]

mass of gluino (GeV)

\[ \xi_{CP} \]

\[ 0.02 \]

\[ 0.015 \]

\[ 0.01 \]

\[ 0.005 \]

\[ 0 \]

\[ -0.005 \]

\[ -0.01 \]

\[ -0.015 \]

\[ -0.02 \]

\[ -0.025 \]

100

150

200

250

300

350

400
Fig. 5(b)

$\xi_{CP} (g g \rightarrow t \bar{t})$

mass of stop1 (GeV)
Fig. 6(a)
Fig. 7(b)

\[ \xi_{CP}(gg \rightarrow t \bar{t}) \]