The electron and positron acceleration in the first cycle of a laser-driven wakefield is investigated. Separatrices between different types of the particle motion (confined, reflected by the wakefield or ponderomotive potential and transient) are demonstrated. The ponderomotive acceleration is negligible for electrons but is substantial for positrons. An electron bunch, injected as quasimonoeenergetic, acquires a localized energy spectrum with a cut-off at the maximum energy.

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Laser-driven charged particle acceleration is an attractive alternative to cyclic accelerators and linacs, promising to provide much greater acceleration rate with a much more compact facility. At the dawn of the laser technology the “optical maser” was suggested in Ref. [1] to accelerate electrons. The long-living strong Langmuir wave (wakefield), left in the wake of a short intense laser pulse in a low-density collisionless plasma, accelerates duly injected electrons in the Laser Wake-Field Accelerator (LWFA) concept introduced in Ref. [2]. For efficient acceleration of charged particles, the laser pulse must be relativistically strong, i.e., its amplitude \( a_0 = eE_0/m_e\omega c > 1 \). To provide electrons one must use an externally preaccelerated electron bunch or exploit the effect of self-injection due to a longitudinal Langmuir wave-break [3] or a transverse wave-break [4], which dominates when the laser pulse waist becomes comparable with or less than the wakefield wavelength, e.g., due to the pulse self-focusing. The injection of the preaccelerated electron bunch co-propagating with the laser pulse was considered in Ref. [5]. The wave-breaking occurs when the displacement of the plasma electron fluid moving in the wave becomes equal to or larger than the wakefield first cycle. The positron acceleration by a long electromagnetic wave in underdense plasmas was considered in Ref. [6]. The localized electron energy spectrum formation was attributed to electrons accelerated in the wakefield first cycle, overtaking the laser pulse. The so-called “ponderomotive electron acceleration”, suggested in Ref. [7] and analyzed in Refs. [9], describes the charged particle motion at the laser pulse front, thus it is in effect also in the wakefield first cycle. The positron acceleration by a long electromagnetic wave in underdense plasmas was considered in Ref. [10].

In this Letter we revisit theory of the Laser Wake-Field Accelerator and examine the acceleration of charged particles in the first cycle of the wakefield. The electron energy spectrum is calculated in a general case of nonoptimal injection. The role of the ponderomotive acceleration is discussed in the case of electrons and positrons.

In the framework of classical electrodynamics the one-dimensional motion of a particle with charge \(-e\) and mass \(m_e\) in the laser pulse and wakefield is described by the Hamiltonian [11]

\[
\mathcal{H} = \sqrt{m_e^2c^4 + e^2P_\parallel^2 + (eP_\perp + eA_\perp(X))^2} - e\varphi(X),
\]

where \(X = x - v_yt\), \(x\) is the particle coordinate, \(v_y\) is the group velocity of the laser pulse (equal to the wakefield phase velocity), \(0 < v_y < c\); \(P_\parallel\) and \(P_\perp\) are the longitudinal and transverse components of the generalized momentum, \(A_\perp\) is the laser pulse vector-potential, \(\varphi\) is the the wakefield potential. In general, \(P_\perp\) and \(A_\perp\) have two components (along \(x\) and \(z\)). Here we neglect the dispersion of the the laser pulse, assuming that the laser pulse field depends on time and coordinate as \(A_\perp(x - v_yt)\). The Hamiltonian [11] admits a Lie group with generators \(v_y\partial_x + \partial_t, \partial_y, \partial_z\). Correspondingly, the Noether theorem implies the motion integrals:

\[
\mathcal{H} - v_yP_\parallel = m_e c^2 h_0, \quad P_\perp = P_{\perp,0},
\]

where \(h_0\) and \(P_{\perp,0}\) are constants of the particle initial momentum. We introduce dimensionless variables

\[
\beta_{ph} = v_y/c, \quad \Phi(X) = e\varphi(X)/m_e c^2, \quad p_x = P_\parallel/m_e c, \quad a(X) = P_{\perp,0}/m_e c + eA_\perp(X)/m_e c^2.
\]

**Theory of the Laser Wake-Field Accelerator Revisited: Wake Overtaking, Localized Spectrum and Ponderomotive Acceleration**

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(Dated: April, 2005)

The electron and positron acceleration in the first cycle of a laser-driven wakefield is investigated. Separatrices between different types of the particle motion (confined, reflected by the wakefield or ponderomotive potential and transient) are demonstrated. The ponderomotive acceleration is negligible for electrons but is substantial for positrons. An electron bunch, injected as quasimonoeenergetic, acquires a localized energy spectrum with a cut-off at the maximum energy.

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In recent experiments localized energy spectra of electrons accelerated up to 170 MeV were demonstrated [6, 7]. The indications were given that the laser pulse undergo a self-focusing, and the wave-breaking (both the longitudinal and transverse) occurs in the first cycle of the wakefield with the electron self-injection into the acceleration phase. In Ref. [6] the localized electron energy spectrum formation was attributed to electrons accelerated in the wakefield first cycle, overtaking the laser pulse. The so-called “ponderomotive electron acceleration”, suggested in Ref. [7] and analyzed in Refs. [9], describes the charged particle motion at the laser pulse front, thus it is in effect also in the wakefield first cycle. The positron acceleration by a long electromagnetic wave in underdense plasmas was considered in Ref. [10].

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where \(h_0\) and \(P_{\perp,0}\) are constants of the particle initial momentum. We introduce dimensionless variables

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\beta_{ph} = v_y/c, \quad \Phi(X) = e\varphi(X)/m_e c^2, \quad p_x = P_\parallel/m_e c, \quad a(X) = P_{\perp,0}/m_e c + eA_\perp(X)/m_e c^2.
\]
We note that the variable $a(X)$ represents both the particle initial transverse generalized momentum and the laser pulse vector-potential. In the case of $P_{\perp 0} = 0$ it is just the laser dimensionless amplitude. The first integral in (2) in terms of new variables (3) gives the equation

$$h(X, p_x) = \sqrt{1 + p_x^2 + a^2(X)} - \Phi(X) - \beta_{ph} p_x = h_0.$$  (4)

Its solution for $\beta_{ph} < 1$ is written as

$$p_x = \gamma_{ph}^2 \left\{ \beta_{ph}(\Phi(X) + h_0) \pm [\Phi(X) + h_0]^2 - \gamma_{ph}^2 (1 + a^2(X))^{1/2} \right\},$$

(5)

where $\gamma_{ph} = (1 - \beta_{ph}^2)^{-1/2}$; the sign ‘+’ is for $X$ increasing with time and ‘−’ is for $X$ decreasing with time. The particle moving from $X_0$ to $X$ with monotonically increasing $X(t)$ acquires the net kinetic energy

$$E = E_0 + \gamma_{ph}^2 \left\{ \Delta \Phi + \chi_0 + \beta_{ph} [\Delta \Phi + \chi_0]^2 - \gamma_{ph}^2 (1 + a^2(X))^{1/2} \right\} - \chi_0 - \beta_{ph} p_{x0},$$

(6)

where $\Delta \Phi = \Phi(X) - \Phi(X_0)$, $E_0 = \sqrt{1 + p_{x0}^2 + a^2(X_0)} - 1$, $\chi_0 = E_0 + 1 - \beta_{ph} p_{x0}$ and $p_{x0} = p_x(X_0)$.

To exemplify the general property of the system with Hamiltonian $h(X, p_x)$, we show its phase portrait in Fig. 1(c), (e) for the electron with $P_{\perp 0} = 0$ in the case when the circularly polarized quasi-Gaussian laser pulse with amplitude $a(X) = \{ a_0 (\exp(-4 \ln(2)X^2/l_p^2) - 1/16) \}$ at $|X| \leq l_p$, $0$ at $|X| > l_p$, $a_0 = 2$, FWHM size $l_p = 10$ wavelengths, propagates in an ideal Hydrogen plasma with density $n_e = 0.01n_{cr}$, and excites a wakefield, whose potential is described by the Poisson equation, (12),

$$\Phi'' = k_p^2 \gamma_{ph}^3 \beta_{ph} \left\{ (1 + \Phi) \left[ \gamma_{ph}^2 (1 + \Phi)^2 - 1 - a^2(X) \right]^{1/2} - (\mu - \Phi) \left[ \gamma_{ph}^2 (\mu - \Phi)^2 - \mu^2 - a^2(X) \right]^{1/2} \right\},$$

(7)

where the prime denotes differentiation with respect to the $X$ coordinate, $k_p = \omega_{pe}/c$ and $\mu = m_i/m_e = 1836$ is the ion-to-electron mass ratio. The potential $\Phi$, corresponding to the longitudinal electric field, and the electron and ion densities as well as the laser pulse envelope are shown in Fig. 1(a), (b). We consider a circularly polarized pulse to ensure the dependence of $a^2$ on $X = x - v_p t$ and existence of motion integrals (2) thus avoiding complication that the laser pulse electromagnetic wave phase velocity is equal to $v_{ph, lms} = 1/v_{ph} > 1$. We choose the finite quasi-Gaussian pulse shape to emphasize the existence of the “ponderomotive” separatrix (see below). The pulse length is taken to be less than, but not too much, the (relativistic) wakefield wavelength so as to make the wakefield excitation efficient enough to preserve the effect of the laser field on the particle motion inside the first period of the wakefield and to simplify formulae below.

Each orbit $\{ X(t), p_x(t) \}$ of the electron in the $(X, p_x)$-plane is a segment of a level curve of the function $h(X, p_x)$. The $(X, p_x)$-plane is divided into basins of a finite motion, where the particle is trapped by the wakefield potential, and two basins of infinite motion. The basins are separated from each other by special orbits, separatrices, which join at singular points situated on the curve $p_{x0}(X) = \beta_{ph} \gamma_{ph} \sqrt{1 + a^2(X)}$. On this line the square root in the right hand side of Eq. (4) vanishes: $(\Delta \Phi + \chi_0)^2 - \gamma_{ph}^{-2} (1 + a^2(X)) = 0$.

An electron started from the singular point $X_s$ acquires the maximum kinetic energy at the top $X_t$ of the separatrix, as one can easily calculate from Eq. (4) substituting $X_0 = X_s$, $X = X_t$, $p_{x0} = p_{x0}(X_s)$. If the laser pulse length is shorter than a half of wakefield wavelength, then in the first period of the wakefield the points $X_s$ and $X_t$ correspond, respectively, to the local minimum and maximum of the wakefield potential, $\Phi(X_s) = \Phi_{min}$, $\Phi(X_t) = \Phi_{max}$. So the maximum kinetic energy on the separatrix is

$$E_{max} = \gamma_{ph}^2 \left( \Delta \Phi_{max} + \beta_{ph} \left[ \Delta \Phi_{max}^2 + 2 \gamma_{ph}^{-1} \Delta \Phi_{max} \right]^{1/2} \right) + E_{min},$$

(8)

where $\Delta \Phi_{max} = \Phi_{max} - \Phi_{min}$, $E_{min} = \gamma_{ph} - 1$. If the laser pulse length is much shorter than the wakefield wavelength and $\gamma_{ph} \gg 1$, we have $E_{max} \approx 2 \gamma_{ph}^2 \Delta \Phi_{max} + \gamma_{ph} - 1$. The lowest value of the potential $\Phi$ is reached when the laser pulse sweeps the greatest possible amount of electrons (in 1D – a half of all the electrons per wakefield period), $\Phi_{min} \geq -1 + 1/\gamma_{ph}$, and the highest value is limited by the ion responce, $\Phi_{max} \leq \mu (1 - 1/\gamma_{ph})$. Knowing the minimum of the solution to
Eq. (4), one can find its maximum; in the case of a sufficiently short and intense laser pulse \((l_p \ll \lambda_{ef}, a \gg 1)\), Eq. (4) gives \(\Phi_{\text{max}} = (1 - \gamma_{ph}^{-1})(2\gamma_{ph}\mu + \mu - 1)/(2\gamma_{ph} + \mu - 1)\), which in the limit \(m_i \gg m_e\) tends to \(2\gamma_{ph} - 1 - \gamma_{ph}^{-1}\). If the laser pulse has the optimal length, then \(\Phi_{\text{max}} \approx a^2/2\) for \(a \lesssim \sqrt{\Gamma}\), \((13)\).

Since the laser pulse has a finite duration, the “runaway” separatrix exists, a segment of the level curve \(h(X, p_x) = 1/\gamma_{ph} - \Phi_{\text{min}}, \) Fig. (1c). If an electron beam is injected exactly onto this separatrix, it asymptotically overtakes the laser pulse and becomes monoenergetic with the final energy, as it follows from Eq. (13).

\[ E_I = \gamma_{ph}^2(\Phi_{\text{min}} + \beta_{ph}[\Phi_{\text{min}}^2 + 2\gamma_{ph}^{-1}|\Phi_{\text{min}}|^2]^{1/2}) + E_{\text{inj}}, \]  

where \(|\Phi_{\text{min}}| = -\Phi_{\text{min}} > 0\). In the limit \(\gamma_{ph} \gg 1\), this energy can be much higher than the required minimum injection energy. If, additionally, the wakefield is strongly nonlinear \((a \gg 1)\), \(\Phi_{\text{min}}\) tends to its lowest value \(-1 + 1/\gamma_{ph}\) and we have \(E_{I,\text{max}} \approx 2\gamma_{ph}^2 + \gamma_{ph} - 1\).

In the first period of the wakefield behind the laser pulse there is also the “confined” separatrix, a segment of the level curve \(h(X, p_x) \approx 1/\gamma_{ph}\) (the exact value is discussed below). It encloses a basin of orbits of electrons which are trapped inside the potential well moving along with the laser pulse. Between the “confined” and “runaway” separatrices lies a bunch of reflecting orbits. On such an orbit an electron starts with the longitudinal momentum \(p_x^0\) in the range \(\beta_{ph}\gamma_{ph} > p_x^0 > \beta_{ph}\gamma_{ph} + \gamma_{ph}^2(\beta_{ph}|\Phi_{\text{min}}| - [\Phi_{\text{min}}^2 + 2\gamma_{ph}^{-1}|\Phi_{\text{min}}|^2]^{1/2}) \geq 0\) at \(t \to -\infty\). Then it is accelerated by the first cycle of the wakefield, reaching the maximum energy defined by Eq. (13), where the momentum \(p_x^0 \to 0\), \(p_x^0 = p_x^+\), \(a(X_0) = \Phi(X_0) = 0\). Finally, the electron overtakes the laser pulse. Its longitudinal momentum \(p_x^+\) and kinetic energy \(E^+\) increase as

\[ p_x^+ = p_x^- + 2\gamma_{ph}^2(\beta_{ph} - v_x^-), \]

\[ E^+ = E^- + 2\beta_{ph}\gamma_{ph}^2(\beta_{ph} - v_x^-) < E_I, \]

where \(\Gamma^- = [1 + (p_x^-)^2]^{1/2}, \ E^- = \Gamma^- - 1, \ v_x^- = p_x^-/\Gamma^- < \beta_{ph}\). The same equations describe an elastic rebound of a relativistic particle from the wall moving at a speed \(\beta_{ph}\).

Yet another, the third, “ponderomotive” separatrix exists in the vicinity of the laser pulse front \(X_f = l_p\), Fig. (1c), (e). It joins the second, “confined”, separatrix at the point \((X_p, p_x(X_p))\) defined by equation \(a(X_p)a'(X_p) = \gamma_{ph}\Phi(X_p)/\sqrt{1 + a^2(X_p)}\) and so the exact value of the Hamiltonian for both separatrices is \(h_p = h(X_p, p_x(X_p))\). The third separatrix encloses a thin basin of orbits with \(1/\gamma_{ph} < h(X, p_x) < h_p = h(X_p, p_x(X_p))\), going from \(X = +\infty\) at
FIG. 2: (color). The energy spectrum of the electron bunch scattered about the top $X_t$ of the separatrix for the bunch initial energy spread $\Delta E = 0$ (a) and for $\Delta E = 0.02E_m$ (b). The distribution $dN/dE$ unit is $\frac{3N_b}{2E_m} \frac{e^{(\Delta E/E_m)^{1/2}}}{1 - (1 - \Delta E/E_m)^{3/2}}$.

t \to -\infty$ with $p_x < \beta \gamma$ and reflecting back with increased $p_x > \beta \gamma$ at $t \to +\infty$. In contrast to orbits between the “confined” and “runaway” separatrices, where particles are reflected by the wakefield potential, the orbits enclosed by the third separatrix belong to electrons which are reflected (accelerated) by the ponderomotive force of the laser pulse. Such reflection is possible because the laser pulse has the speed $v_p < 1$ and the wakefield potential $\Phi(X)$ always grows slower than the $a(X)$ on the laser pulse front. Using series expansions of functions $a(X)$ and $\Phi(X)$ about the point $X = X_f$, $a(X_f + \xi) = a_1 \xi + a_2 \xi^2/2 + o(\xi^3)$, $\Phi(X_f + \xi) = a_1^2 \beta^2 \mu^2 (1 - \mu^2) (a_1^3 \xi^4/24 + a_1 a_2 \xi^5/40) + o(\xi^6)$, where $a_1 = a'(X_f)$, $a_2 = a''(X_f)$, we can estimate the “ponderomotive” basin thickness, which is the energy difference between the upper and lower branches of the “ponderomotive” separatrix

$$\Delta E_p = 2 \beta \gamma \sqrt{\frac{\hbar^2}{\gamma^2} - \frac{\gamma_{ph}^2}{\gamma^2}} \approx -2\sqrt{3} \frac{\beta \gamma}{k_p} a'(X_f) \frac{1}{2} + 2\sqrt{2} \frac{\beta \gamma}{5k_p} a''(X_f)$$

(12)

at $|a'(X_f)| \ll 1$, $|a''(X_f)| \ll 1$, and $\mu > 1$, $\gamma_{ph} > 1$. The ponderomotive acceleration affects only those electrons that move in the same direction as the laser pulse and whose velocity is slightly less than $\beta \gamma$. The acceleration gain turns out to be rather small, because the ponderomotive and electrostatic potentials almost completely compensate each other. However, it is still not zero even with the ideal Gaussian pulse; the maximum effect is reached when the laser pulse has a sharp front.

We examine the energy spectrum change of an electron bunch injected into the first period of the wakefield wave onto the “runaway” separatrix, Fig. 1(c). When a relatively long, initially quasi-monoenergetic, bunch is injected from the singular point $X_s$ and accelerated in the first period of the wakefield wave, its particles are distributed along the “runaway” separatrix with some density $N(X)$. As a result, the particle energy spectrum broadens from the initial energy $E_{inj} = \gamma_{ph} - 1$ to the cut-off (maximum) energy $E_m$. Besides these two limits, the spectrum can have a peculiarity at $E_t$, Eq. (b). Near the top of the separatrix the particle energy has a parabolic dependence on $X$, $E(X) \simeq E_m (1 - (X - X_t)/\Lambda^2)$, where $\Lambda^2 = -2E_m/\gamma''(X_t)$, as it follows from Eqs. (a), (b). Hence the energy spectrum near the cut-off energy is

$$\frac{dN}{dE} = \frac{N(X)}{dE/dX} \simeq \frac{\Delta N(X_t)}{2E_m (E_m - E)}$$

(13)

where $E < E_m$. Assuming that the density $N(X)$ of the particle distribution along the separatrix is smooth enough at $X = X_t$, we see that the spectrum has an integrable singularity. If the particles are arranged uniformly on the separatrix, the spectrum, despite its singularity, has rather large spread, e. g. a half of the particles occupies the energy interval $3E_m/4 \leq E \leq E_m$.

In general, the bunch has some initial energy spread $\Delta E$. During acceleration it occupies some region around the separatrix in the plane $(X, E)$. To estimate the bunch spectrum, we assume that the particles are distributed uniformly between two orbits which we approximate by parabolas $E_1(X) = E_m - \Delta E - E_m X^2/\Lambda^2$ and $E_2(X) = E_m - E_m X^2/\Lambda^2$. Correspondingly, the particle distribution function is

$$f(X, E) = \frac{3N_b \theta[X + \Lambda] \theta[\Lambda - X] \theta[E - E_1] \theta[E_2 - E]}{4\Lambda E_m (1 - (1 - \Delta E/E_m)^{3/2})}$$

(14)

where $\theta$ is the Heaviside step function ($\theta[\xi] = 1$ for $\xi \geq 0$ and $= 0$ for $\xi < 0$), $N_b$ is the number of particles in the
bunch. Then the energy spectrum is

$$\frac{dN}{dE} = \Lambda f(X, E) dX = \frac{3N_b \sqrt{E_m - E} - \sqrt{E_m - \Delta E - E}}{2 \sqrt{E_m^{3/2} - (E_m - \Delta E)^{3/2}}}.$$

(15)

where the square root must be set to zero if its argument is negative. In the limit $\Delta E \to 0$ this spectrum tends to $dN/dE = N_b \theta(E_m - E)/2\sqrt{E_m (E_m - E)}$, which is equivalent to Eq. (13) at $N_b = \Lambda N(X)$. The sum of the spectrum over the range of $E_m$ varying from $E_m - \Delta E$ to $E_m$, weighted with $\sqrt{E_m}$ yields Eq. (15). Both the spectra (14) and (15) are shown in Fig. 2.

The above-stated analysis concerns the phase portrait of a negatively charged particle (electron). In this paragraph we make digression considering the case of a positively charged particle, positron. In this case formulae (4)-(6) remain valid with the substitution $\Phi \to -\Phi$. The phase portrait of the positron in the same wakefield and laser field as above is shown in Fig. 1(d). In the wakefield, the electron’s points of equilibrium correspond to the positron’s singular points (for sufficiently short laser pulse). The positron injected from the singular point into the second cycle of the wakefield returns back to the same singular point. In the first half-cycle of the wakefield, in contrast to the case of the electron, both forces acting on the positron – the wakefield electrostatic force and the laser pulse ponderomotive force – pull the positron in the same direction (“forward”). Therefore we see a wide “ponderomotive” basin in which the orbits with initial momentum $\beta_p\gamma_{ph} > p_{x,\text{pos}} > \gamma_{ph}^2 (\beta_{ph}(\Phi_{p,\text{pos}} + \gamma_{ph}^{-1}\sqrt{1 + a_{p,\text{pos}}^2}) - [\Phi_{p,\text{pos}} + \gamma_{ph}^{-1}\sqrt{1 + a_{p,\text{pos}}^2}]^2 - \gamma_{ph}^{-2})^{1/2}$ are accelerated up to the energy

$$E_{p,\text{pos}}^{+} = \gamma_{ph}^2 \left( \Phi_{p,\text{pos}} + \gamma_{ph}^{-1}\sqrt{1 + a_{p,\text{pos}}^2} \right)$$

$$+ \beta_{ph} \left[ (\Phi_{p,\text{pos}} + \gamma_{ph}^{-1}\sqrt{1 + a_{p,\text{pos}}^2})^2 - \gamma_{ph}^{-2} \right]^{1/2}$$

(16)

in accordance with Eqs. (10), (11). Here the limiting values of the positron momentum and energy in the “ponderomotive” basin are defined via the wakefield potential $\Phi$ and the laser amplitude $a$ taken at the singular point $X_{p,\text{pos}}$, a non-trivial solution to the equation

$$a(X)a'(X) + \gamma_{ph}\Phi'(X)\sqrt{1 + a^2(X)} = 0.$$

(17)

The momentum of the lower branch of the “ponderomotive” separatrix is negative at $\Phi_{p,\text{pos}} + \gamma_{ph}^{-1}\sqrt{1 + a_{p,\text{pos}}^2} > 1$, then the positron initially at rest is accelerated up to momentum $2\beta_{ph}\gamma_{ph}^2$ and energy $2\beta_{ph}^2\gamma_{ph}^2$, in accordance with Eqs. (10), (11). Thus even the “background” positrons, introduced externally or created in the laser-plasma interaction, are substantially accelerated. In the limit of a long laser pulse ($t_p \gg \lambda_{w,\text{t}}$), the maximum energy (16) becomes $\approx (1 + \beta_{ph})\gamma_{ph}^2 a_0$, since $\Phi_{p,\text{pos}} \approx a_0$ in this limit.

In conclusion, in the first cycle of the Langmuir wave in the wake of the short relativistically strong laser pulse electrons have at least three separatrices: on the “runaway” separatrix the electron overtakes the wakefield and the laser pulse, on the “confined” separatrix it moves together with the laser pulse and the “ponderomotive” separatrix places quite tight limit for the ponderomotive acceleration. In contrast to electrons, positrons see the wakefield and the laser pulse ponderomotive force acting in the same direction. It is shown that the energy spectrum of the initially mono-energetic particle bunch spread about the top of the separatrix has a typical localized shape $\propto (E_m - E)^{-1/2}$ with a sharp cut-off.

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