Hawking radiation in Reissner-Nordström blackhole with a global monopole via Covariant anomalies and Effective action

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Abstract
We adopt the covariant anomaly cancellation method as well as the effective action approach to obtain the Hawking radiation from the Reissner-Nordström blackhole with a global monopole falling in the class of the most general spherically symmetric charged blackhole (\(\sqrt{-g} \neq 1\)), using only covariant boundary condition at the event horizon.

Keywords: Hawking radiation, Covariant anomaly, Effective action, Covariant Boundary condition

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Introduction:
Hawking radiation is an important and prominent quantum effect arising from the quantization of matter fields in a background spacetime with an event horizon. The radiation is found to have a spectrum with Planck distribution giving the blackholes one of its thermodynamic properties. Apart from the original derivation by Hawking [1, 2], there is a tunneling picture [3, 4] based on pair creations of particles and antiparticles near the horizon and calculates WKB amplitudes for classically forbidden paths. A common feature in these derivations is the universality of the radiation: i.e. Hawking radiation is determined universally by the horizon properties (if we neglect the grey body factor induced by the effect of scattering outside the horizon).

Recently, Robinson and Wilczek ([5]) proposed an interesting approach to derive Hawking radiation from a Schwarzschild-type black hole through gravitational anomaly. The method was soon extended to the case of charged blackholes [6]. Further applications of this approach may be found in [7]-[13]. The basic idea in [5, 6] is that the effective theory near the horizon becomes two-dimensional and chiral. This chiral theory is anomalous. Using the form for two-dimensional consistent gauge/gravitational anomaly, Hawking fluxes are obtained. However the boundary condition necessary to fix the parameters are obtained from a vanishing of covariant current and energy-momentum tensor at the horizon. Soon after, the analysis of [5, 6] was reformulated in [14], [15] in terms of covariant expressions only. The generalization of this approach to higher spin field has been done in [16].

An alternative derivation of Hawking flux based on effective action using only covariant anomaly has been discussed in [17], [18], [19]. This approach is particularly useful since only the exploitation of known structure of effective action near the horizon is sufficient to determine the Hawking flux. An important ingredient in this method is once again to realize that the effective theory near the event horizon becomes two-dimensional and chiral. Another important aspect in this approach is the imposition of covariant boundary conditions only at the horizon.

In this paper, we first adopt the covariant anomaly cancellation approach ([14]) to discuss Hawking radiation from Reissner-Nordström blackhole with a global monopole [21] which is an example of the most general spherically symmetric charged blackhole spacetime (\(\sqrt{-g} \neq 1\)). Finally we adopt the effective action approach [17] to reproduce the same result. However, as in [18], we shall once again ignore effects to the Hawking flux due to scatterings by the gravitational potential, for example the greybody factor [20].

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Hawking radiation from Reissner-Nordström blackhole with a global monopole:

The metric of a general non-extremal Reissner-Nordström blackhole with a global monopole $O(3)$ is given by \[21\]

\[
\text{ds}^2_{\text{string}} = p(r)dt^2 - \frac{1}{h(r)}dr^2 - r^2d\Omega^2
\]

where,

\[
A = \frac{q}{r}dt , \quad p(r) = h(r) = 1 - \eta^2 - \frac{2m}{r} + \frac{q^2}{r^2}
\]

with $m$ being the mass parameter of the blackhole and $\eta$ is related to the symmetry breaking scale when the global monopole is formed during the early universe soon after the Big-Bang \[22\]. The event horizon for the above blackhole is situated at

\[
r_H = (1 - \eta^2)^{-1}[m + \sqrt{m^2 - (1 - \eta^2)q^2}].
\]

Now it has been argued in \[11\] that since the metric (1) is no longer asymptotically flat, so the well known formula

\[
\kappa = \frac{1}{2} \sqrt{-g^{rr}} (g_{tt}, r) \big|_{r=r_H}; \quad g_{tt} = p(r), \quad g^{rr} = -h(r)
\]

for computing the surface gravity for a general spherically symmetric asymptotically flat metric becomes problematic to be applied in the case described by the metric (1) as it does not correspond to the normalized time-like Killing vector. The correct surface gravity of the metric (1) is

\[
\kappa = \frac{1}{2\sqrt{1 - \eta^2}} p'(r_H)
\]

since it corresponds to the normalized time-like Killing vector

\[
j_{(t)}^\mu = (1 - \eta^2)^{-1/2}(\partial_t)^\mu.
\]

It is for this reason that the anomaly cancellation method as well as the effective action approach cannot be immediately used to obtain the consistent formula of the Hawking temperature for the metric (1). Nevertheless, we can do the same analysis in another different way. By rescaling $t \rightarrow \sqrt{1 - \eta^2} \, t$, we can rewrite the metric (1) as

\[
\text{ds}^2 = f(r) dt^2 - h(r)^{-1}dr^2 + r^2d\Omega^2
\]

\[
f(r) = (1 - \eta^2)h(r), \quad h(r) = 1 - \eta^2 - \frac{2m}{r} - \frac{q^2}{r^2}
\]

and immediately derive the expected result for the Hawking temperature $T = f'(r_H)/(4\pi \sqrt{1 - \eta^2})$. Hence, we shall apply the anomaly cancellation method and the effective action approach to the above form of the metric (7). The important point to note is that the determinant of the above metric $\sqrt{-g} \neq 1$.

Anomaly cancellation approach:

With the aid of dimensional reduction procedure one can effectively describe a theory with a metric given by the "$r-t'$" sector of the full spacetime metric (7) near the horizon.

Now we divide the spacetime into two regions and discuss the gauge/gravitational anomalies separately.

Gauge anomaly

Since the spacetime has been divided into two regions, we divide the current $J^\mu$ into two parts. The current outside the horizon denoted by $J_{(o)}^\mu$ is anomaly free and hence satisfies the conservation law

\[
\nabla_\mu J_{(o)}^\mu = 0.
\]

Note that a spherically symmetric asymptotically bounded space-time metric without any loss of generality, can be cast in the form $\text{ds}^2 = g_{tt} dt^2 + g_{rr} dr^2 + r^2 d\Omega^2$. 

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Near the horizon there are only outgoing (right-handed) fields and the current becomes covariantly anomalous and satisfies

\[ \nabla_\mu J^\mu_{(H)} = -\frac{e^2}{4\pi} \tilde{\epsilon}^\rho\sigma F_{\rho\sigma} = \frac{e^2}{2\pi \sqrt{-g}} \partial_r A_t \]  
\hspace{1cm} (9)

where, \( \tilde{\epsilon}^{\mu\nu} = \epsilon^{\mu\nu}/\sqrt{-g} \) and \( \tilde{\epsilon}_{\mu\nu} = \sqrt{-g} \epsilon_{\mu\nu} \) are two dimensional antisymmetric tensors for the upper and lower cases with \( \epsilon^{tr} = \epsilon_{rt} = 1 \).

Now outside the horizon, the conservation equation (8) yields the differential equation

\[ \partial_r (\sqrt{-g} J^r_{(o)}) = 0 \]  
\hspace{1cm} (10)

whereas in the region near the horizon, the anomaly equation (9) leads to the following differential equation

\[ \partial_r \left( \sqrt{-g} J^r_{(H)} \right) = \frac{e^2}{2\pi} \partial_r A_t . \]  
\hspace{1cm} (11)

Solving (10) and (11) in the region outside and near the horizon, we get

\[ J^r_{(o)}(r) = \frac{c_o}{\sqrt{-g}} \]  
\hspace{1cm} (12)

\[ J^r_{(H)}(r) = \frac{1}{\sqrt{-g}} \left( c_H + \frac{e^2}{2\pi} \int_{r_H}^r \partial_r A_t(r) \right) \]  
\hspace{1cm} (13)

where, \( c_o \) and \( c_H \) are integration constants. Now as in [6], writing \( J^r(r) \) as

\[ J^r(r) = J^r_{(o)}(r) \Theta(r - r_H - \epsilon) + J^r_{(H)}(r) H(r) \]  
\hspace{1cm} (14)

where, \( H(r) = 1 - \theta(r - r_H - \epsilon) \), we find

\[ \nabla_\mu J^\mu = \partial_r J^r(r) + \partial_r (\ln \sqrt{-g}) J^r(r) \]  
\[ = \frac{1}{\sqrt{-g}} \partial_r (\sqrt{-g} J^r(r)) \]  
\[ = \frac{1}{\sqrt{-g}} \left[ \left( \sqrt{-g} J^r_{(o)}(r) - J^r_{(H)}(r) \right) + \frac{e^2}{2\pi} A_t(r) \right] \delta(r - r_H - \epsilon) = \frac{e^2}{2\pi} A_t(r) H(r) . \]  
\hspace{1cm} (15)

The term in the total derivative is cancelled by quantum effects of classically irrelevant ingoing modes. Hence the vanishing of the Ward identity under gauge transformation implies that the coefficient of the delta function is zero, leading to the condition

\[ J^r_{(o)}(r) - J^r_{(H)}(r) + \frac{e^2}{2\pi \sqrt{-g}} A_t(r) = 0 . \]  
\hspace{1cm} (16)

Substituting (12) and (13) in the above equation, we get

\[ c_o = c_H - \frac{e^2}{2\pi} A_t(r_H) . \]  
\hspace{1cm} (17)

The coefficient \( c_H \) vanishes by requiring that the covariant current \( J^r_{(H)}(r) \) vanishes at the horizon. Hence the charge flux corresponding to \( J^r(r) \) is given by

\[ c_o = \sqrt{-g} J^r_{(o)}(r) = -\frac{e^2}{2\pi} A_t(r_H) = -\frac{e^2 q}{2\pi r_H} . \]  
\hspace{1cm} (18)

This is precisely the charge flux obtained in ([11]) using Robinson-Wilczek method of cancellation of consistent gauge anomaly.
Gravitational anomaly

In this case also, since the theory is free from anomaly in the region outside the horizon, hence we have the energy-momentum tensor satisfying the conservation law

\[ \nabla_\mu T^\mu_{(o)\nu} = F_{\mu\nu}J^\mu_{(o)} . \]  \hspace{1cm} (19)

However, the omission of the ingoing modes in the region \( r \in [r_+, \infty] \) near the horizon, leads to an anomaly in the energy-momentum tensor there. As we have mentioned earlier, in this paper we shall focus only on the covariant form of \( d = 2 \) gravitational anomaly given by (5, 6):

\[ \nabla_\mu T^\mu_{(H)\nu} = F_{\mu\nu}J^\mu_{(H)} + \frac{1}{96\pi} \varepsilon_{\nu\mu\rho}\partial^\rho R = F_{\mu\nu}J^\mu_{(H)} + A_\nu . \]  \hspace{1cm} (20)

It is easy to check that for the metric (1), the two dimensional Ricci scalar \( R \) is given by

\[ R = \frac{h f''}{f} + \frac{f h'}{2f} - \frac{f^2 h}{2f^2} \]  \hspace{1cm} (21)

and the anomaly is purely timelike with

\[ A_r = 0 \]
\[ A_t = \frac{1}{\sqrt{-g}} \partial_r N_t^r \]  \hspace{1cm} (22)

where,

\[ N_t^r = \frac{1}{96\pi} \left( h f'' + \frac{f h'}{2} - \frac{f^2 h}{f} \right) . \]  \hspace{1cm} (23)

We now solve the above equations (12, 20) for the \( \nu = t \) component. In the region outside the horizon, the conservation equation (19) yields the differential equation

\[ \partial_r (\sqrt{-g} T^r_{(o)t}) = \sqrt{-g} F_{rt} J^r_{(o)}(r) = c_o \partial_r A_t \]  \hspace{1cm} (24)

where we have used \( F_{rt} = \partial_r A_t \) and (12). Integrating the above equation leads to

\[ T^r_{(o)t}(r) = \frac{1}{\sqrt{-g}} (a_o + c_o A_t(r)) \]  \hspace{1cm} (25)

where, \( a_o \) is an integration constant. In the region near the horizon, the anomaly equation (20) leads to the following differential equation

\[ \partial_r \left( \sqrt{-g} T^r_{(H)t} \right) = \sqrt{-g} F_{rt} J^r_{(H)}(r) + \partial_r (N_t^r) \]
\[ = (c_H + \frac{e^2}{2\pi} \left[ A_t(r) - A_t(r_H) \right]) \partial_r A_t(r) + \partial_r N_t^r(r) \]
\[ = \partial_r \left( \frac{e^2}{2\pi} \left[ \frac{1}{2} A_t^2(r) - A_t(r_H)A_t(r) \right] + N_t^r(r) \right) \]  \hspace{1cm} (26)

where we have used (12) in the second line and set \( c_H = 0 \) in the last line of the above equation. Integration of the above equation leads to

\[ T^r_{(H)t}(r) = \frac{1}{\sqrt{-g}} \left( b_H + \int_{r_H}^r \partial_r \left( \frac{e^2}{2\pi} \left[ \frac{1}{2} A_t^2(r) - A_t(r_H)A_t(r) \right] + N_t^r(r) \right) \right) \]
\[ = \frac{1}{\sqrt{-g}} \left( b_H + \frac{e^2}{4\pi} \left[ A_t^2(r) + A_t^2(r_H) \right] - \frac{e^2}{2\pi} A_t(r_H)A_t(r) + N_t^r(r) - N_t^r(r_H) \right) \]  \hspace{1cm} (27)

where, \( b_H \) is an integration constant.

Writing the energy-momentum tensor as a sum of two contributions (6)

\[ T^\nu_t(r) = T^\nu_{(o)t}(r) \theta(r - r_H - \epsilon) + T^\nu_{(H)t}(r) H(r) \]  \hspace{1cm} (28)
we find
\[\nabla_{\mu}T^{\mu}_{\nu} = \partial_{\nu}(\ln\sqrt{-g})T^{\nu}_{\nu}(r)\]
\[= \frac{1}{\sqrt{-g}}[\partial_{\nu}(\sqrt{-g})T^{\nu}_{\nu}(r)]\]
\[= \frac{1}{\sqrt{-g}} \left[ -\frac{e^{2}}{2\pi}A_{t}(r_{H})\partial_{t}A_{t}(r) + \left( \sqrt{-g}(T_{(o)\nu}(r) - T_{(H)\nu}(r)) + \frac{e^{2}}{4\pi}A_{t}^{2}(r) + N_{t}^{r}(r) \right) \delta(r - r_{+} - \epsilon) \right.\]
\[\left. + \partial_{\nu} \left( \frac{e^{2}}{4\pi}A_{t}^{2}(r) + N_{t}^{r}(r) \right) \right] \]
\[\] (29)

where we have substituted the value of \(c_{0}\) from (18) in the last line.

Now the first term in the above equation is a classical effect coming from the Lorentz force. The term in the total derivative is once again cancelled by quantum effects of classically irrelevant ingoing modes.

The quantum effect to cancel this term is the Wess-Zumino term induced by the ingoing modes near the horizon. Hence the vanishing of the Ward identity under diffeomorphism transformation implies that the coefficient of the delta function in the above equation vanishes

\[T_{(o)\nu}^{r} - T_{(H)\nu}^{r} + \frac{1}{\sqrt{-g}} \frac{e^{2}}{4\pi}A_{t}^{2}(r) + N_{t}^{r}(r) = 0.\] (30)

Substituting (25) and (27) in the above equation, we get

\[a_{0} = b_{H} + \frac{e^{2}}{4\pi}A_{t}^{2}(r_{H}) - N_{t}^{r}(r_{H}).\] (31)

The integration constant \(b_{H}\) can be fixed by imposing that the covariant energy-momentum tensor vanishes at the horizon. From (27), this gives \(b_{H} = 0\). Hence the total flux of the energy-momentum tensor is given by

\[a_{0} = \frac{e^{2}}{4\pi}A_{t}^{2}(r_{H}) - N_{t}^{r}(r_{H}) = \frac{e^{2}q^{2}}{4\pi r_{H}^{2}} + \frac{1}{192\pi}f'(r_{H})h'(r_{H}),\]
\[= \frac{e^{2}q^{2}}{4\pi r_{H}^{2}} + \frac{1}{192\pi}f^{2}(r_{H}).\] (32)

This is precisely the Hawking flux obtained in ([11]) using Robinson-Wilczek method of cancellation of consistent anomaly.

**Effective action approach:**

As we have already mentioned earlier, with the aid of dimensional reduction technique, the effective field theory near the horizon becomes a two dimensional chiral theory with a metric given by the \(r-t\) sector of the full spacetime metric (7) near the horizon.

We now adopt the methodology in [17]. For a two dimensional theory the expressions for the anomalous (chiral) and normal effective actions are known [23]. We shall use only the anomalous form of the effective action for deriving the charge and the energy flux. The current and the energy-momentum tensor in the region near the horizon is computed by taking appropriate functional derivative of the chiral effective action. Next, the parameters appearing in the solution is fixed by imposing the vanishing of covariant current and energy-momentum tensor at the horizon. Once these are fixed, the charge and the energy flux are obtained by taking the asymptotic \((r \to \infty)\) limit of the chiral current and energy-momentum tensors. We also use the expression for the normal effective action to establish a connection between the chiral and the normal current and energy-momentum tensors.

With the above methodology in mind, we write down the anomalous (chiral) effective action (describing the theory near the horizon) [23]

\[\Gamma_{(H)} = -\frac{1}{3}z(\omega) + z(A)\] (33)
where \( A_\mu \) and \( \omega_\mu \) are the gauge field and the spin connection and

\[
z(v) = \frac{1}{4\pi} \int d^2 x \, d^2 y \, \epsilon^{\mu\nu} \partial_\mu v_\nu(x) \Delta^{-1}_g(x, y) \partial_\nu \left( \epsilon^{\rho\sigma} + \sqrt{-g} g^{\rho\sigma} \right) v_\sigma(y)
\]  

(34)

where \( \Delta_g = \nabla^\mu \nabla_\mu \) is the laplacian in this background.

The energy-momentum tensor is computed from a variation of this effective action. To get their covariant forms in which we are interested, one needs to add appropriate local polynomials [23]. Here we quote the final result for the chiral covariant energy-momentum tensor and the chiral covariant current [23]:

\[
T^\mu_\nu = \frac{e^2}{4\pi} D^\mu BD_\nu B + \frac{1}{4\pi} \left( \frac{1}{48} D^\mu GD_\nu G - \frac{1}{24} D^\mu D_\nu G + \frac{1}{24} \delta^\mu_\nu R \right)
\]

(35)

\[
J_\mu = -\frac{e^2}{2\pi} D^\mu B
\]

(36)

where \( D_\mu \) is the chiral covariant derivative

\[
D_\mu = \nabla_\mu - \tilde{\epsilon}_\mu^\nu \nabla_\nu = -\tilde{\epsilon}_\mu^\nu D_\nu.
\]

(37)

Also \( B(x) \) and \( G(x) \) are given by

\[
B(x) = \int d^2 y \, \sqrt{-g} \Delta^{-1}_g(x, y) \epsilon^{\mu\nu} \partial_\mu A_\nu(y)
\]

(38)

\[
G(x) = \int d^2 y \, \Delta^{-1}_g(x, y) \sqrt{-g} R(y)
\]

(39)

and satisfy

\[
\nabla^\mu \nabla_\mu B = -\partial_\nu A_\nu(r) \quad , \quad \nabla^\mu \nabla_\mu G = R.
\]

(40)

The solutions for \( B \) and \( G \) read

\[
B = B_o(r) - at + b; \quad \partial_\nu B_\nu = \frac{1}{\sqrt{f} h} (A_t(r) + c)
\]

(41)

\[
G = G_o(r) - 4pt + q; \quad \partial_\nu G_\nu = -\frac{1}{\sqrt{f} h} \left( \sqrt{\frac{h}{f}} f' + z \right)
\]

(42)

where \( a, b, c, p, q, z \) are constants of integration.

By taking the covariant divergence of (35) and (36), we get the anomalous Ward identities [20] and [43]. In the region away from the horizon, the effective theory is given by the standard effective action \( \Gamma \) of a conformal field with a central charge \( c = 1 \) in this blackhole background [23] and reads:

\[
\Gamma = \frac{1}{96\pi} \int d^2 x d^2 y \, \sqrt{-g} R(x) \frac{1}{\Delta_g} (x, y) \sqrt{-g} R(y) + \frac{e^2}{2\pi} \int d^2 x d^2 y \epsilon^{\mu\nu} \partial_\mu A_\nu(x) \frac{1}{\Delta_g} (x, y) \epsilon^{\rho\sigma} \partial_\rho A_\sigma(y).
\]

(43)

The covariant energy-momentum tensor \( T^{\mu\nu}_{(o)} \) and the covariant gauge current \( J^\mu_\nu \) in the region outside the horizon are given by

\[
T^{\mu\nu}_{(o)} = \frac{2}{\sqrt{-g} \delta g^{\mu\nu} \delta e^{\mu\nu}} \frac{\delta \Gamma}{\delta g^{\mu\nu}}
\]

\[
= \frac{1}{48\pi} \left( 2 g_{\mu\nu} R - 2 \nabla_\mu \nabla_\nu G + \nabla_\mu G \nabla_\nu G - \frac{1}{2} g_{\mu\nu} \nabla^\rho G \nabla_\rho G \right)
\]

\[+ \frac{e^2}{\pi} \left( \nabla_\mu B \nabla_\nu B - \frac{1}{2} g_{\mu\nu} \nabla^\rho B \nabla_\rho B \right)
\]

(44)

\[
J^\mu_\nu = \frac{\delta \Gamma}{\delta A_\mu} = \frac{e^2}{\pi} \epsilon^{\mu\nu} \partial_\nu B
\]

(45)
and satisfy the normal Ward identities (19) and (8).

**Charge and Energy Flux:**

In this section we calculate the charge and the energy flux by using the expressions for the anomalous covariant gauge current (36) and anomalous covariant energy-momentum tensor (35). We will show that the results are the same as that obtained by the covariant anomaly cancellation method.

Using the solution for \( B(x) \) (38) and (37), the \( \mu = r \) component of the anomalous (chiral) covariant gauge current (36) becomes

\[
J_r(r) = e^2 2 \pi \sqrt{\frac{h}{f}} [A_t(r) + a + c].
\]

(46)

Now implementation of the boundary condition namely the vanishing of the anomalous (chiral) covariant gauge current at the horizon, \( J^r(r_H) = 0 \), leads to

\[
a + c = -A_t(r_H).
\]

(47)

Hence the expression \( J^r(r) \) reads

\[
J^r(r) = e^2 2 \pi \sqrt{\frac{h}{f}} [A_t(r) - A_t(r_H)].
\]

(48)

Now the charge flux is given by the asymptotic \( (r \to \infty) \) limit of the anomaly free covariant gauge current (45). Now from (9), we observe that the anomaly vanishes in this limit. Hence the charge flux is abstracted by taking the asymptotic limit of the above equation multiplied by an overall factor of \( \sqrt{-g} \). This yields

\[
c_0 = (\sqrt{-g}J^r)(r \to \infty) = (\sqrt{\frac{f}{h}}J_r^r)(r \to \infty) = -e^2 2 \pi A_t(r_H) = -e^2 q 2 \pi r_H.
\]

(49)

which agrees with (18).

We now consider the normal (anomaly free) covariant gauge current (45) to establish its relation with the chiral (anomalous) covariant gauge current (48). The \( \mu = r \) component of \( J^r(\infty) \) is given by

\[
J^r(\infty) = e^2 2 \pi \sqrt{\frac{h}{f}} a.
\]

(50)

The asymptotic form of the above equation (50) must agree with the asymptotic form of (48). This yields (using (47)):

\[
a = c = -\frac{1}{2} A_t(r_H).
\]

(51)

Using the above solutions in (48) and (50) yields (49) and

\[
\sqrt{-g}J^r(\infty) = \sqrt{\frac{f}{h}} J^r(\infty) = -e^2 2 \pi A_t(r_H).
\]

(52)

The above expressions (49) and (51) yields the equation between the chiral (anomalous) and the normal energy-momentum tensors (16).

Now we focus our attention on the gravity sector. Using the solutions for \( B(x) \) (38) and \( G(x) \) (39), the \( r - t \) component of the anomalous (chiral) covariant energy-momentum tensor (35) becomes

\[
T_{rt}(r) = \frac{e^2}{4 \pi} \sqrt{\frac{h}{f}} [A_t(r) - A_t(r_H)]^2 + \frac{1}{12 \pi} \sqrt{\frac{h}{f}} \left[ p - \frac{1}{4} \left( \sqrt{\frac{h}{f}} f' + z \right) \right]^2
\]

\[
+ \frac{1}{24 \pi} \sqrt{\frac{h}{f}} \left[ \sqrt{\frac{h}{f}} f' \left( p - \frac{1}{4} \left( \sqrt{\frac{h}{f}} f' + z \right) \right) + \frac{1}{4} h f'' - \frac{f'}{8} \left( \frac{f}{f'} \right) f' - h' \right].
\]

(53)

\(^2\)This is true since the anomaly in the asymptotic limit \( (r \to \infty) \) vanishes as can be readily seen from (9).
Now implementing the boundary condition namely the vanishing of the covariant energy-momentum tensor at the horizon, \( T^t_t(r_H) = 0 \), leads to

\[
p = \frac{1}{4} \left[ z \pm \sqrt{f'(r_H)h'(r_H)} \right] ; \quad f'(r_H) \equiv f'(r = r_H) . \tag{54}
\]

Using either of the above solutions in \((53)\) yields

\[
T^t_t(r) = \frac{e^2}{4\pi} \sqrt{\frac{h}{f}} [A_t(r) - A_t(r_H)]^2 + \frac{1}{192\pi} \sqrt{h} \left[ f'(r_H)h'(r_H) - \frac{2h}{f} f'' + 2hf'' + f'f' \right] . \tag{55}
\]

The energy flux is now given by the asymptotic \((r \to \infty)\) limit of the anomaly free energy-momentum tensor \((44)\). Now from \((20)\), we observe that the anomaly vanishes in this limit. Hence the energy flux is abstracted by taking the asymptotic limit of the above equation multiplied by an overall factor of \(\sqrt{-g}\).

This yields

\[
a_0 = (\sqrt{-g} T^t_t)(r \to \infty) = \left(\sqrt{\frac{f}{h}} T^t_t\right)(r \to \infty) = \frac{e^2}{4\pi} A_t(r_H)^2 + \frac{1}{192\pi} f'(r_H)h'(r_H)
\]

\[
= \frac{e^2 q^2}{4\pi r_H^2} + \frac{1}{192\pi} f'(r_H)h'(r_H) \tag{56}
\]

which correctly reproduces the energy flux \((52)\).

We now consider the normal (anomaly free) energy-momentum tensor \((44)\) to establish its relation with the chiral (anomalous) energy-momentum tensor \((55)\). The \(r-t\) component of \(T^\mu_{\nu(o)}\) is given by

\[
T^t_t(o)(r) = \frac{e^2}{\pi} \sqrt{\frac{h}{f}} a[A_t(r) + c] - \frac{1}{12\pi} \sqrt{\frac{h}{f}} z p
\]

\[
= -\frac{e^2}{2\pi} \sqrt{\frac{h}{f}} A_t(r_H)[A_t(r) - \frac{1}{2} A_t(r_H)] - \frac{1}{12\pi} \sqrt{\frac{h}{f}} z p \tag{57}
\]

where we have used \((51)\). Once again since the anomaly in the asymptotic limit \((r \to \infty)\) vanishes as can be readily seen from \((20)\), the asymptotic form of the above equation \((57)\) must agree with the asymptotic form of \((53)\). This yields:

\[
p = -\frac{z}{4} . \tag{58}
\]

Solving \((54)\) and \((58)\) gives two solutions for \(p\) and \(z\):

\[
p = \frac{1}{8} \sqrt{f'(r_H)h'(r_H)} ; \quad z = -\frac{1}{2} \sqrt{f'(r_H)h'(r_H)}
\]

\[
p = -\frac{1}{8} \sqrt{f'(r_H)h'(r_H)} ; \quad z = \frac{1}{2} \sqrt{f'(r_H)h'(r_H)} . \tag{59}
\]

Using either of the above solutions in \((53)\) and \((57)\) yields \((55)\) and

\[
\sqrt{-g} T^t_t(o)(r) = \sqrt{\frac{f}{h}} T^t_t(o)(r) = -\frac{e^2}{2\pi} A_t(r_H)[A_t(r) - \frac{1}{2} A_t(r_H)] + \frac{1}{192\pi} f'(r_H)h'(r_H) . \tag{60}
\]

The above expressions \((55)\) and \((60)\) yields the equation between the chiral (anomalous) and the normal energy-momentum tensors \((30)\).

**Discussions:**

In this paper, we studied the problem of Hawking radiation from Reissner-Nordström blackhole with a global monopole using covariant anomaly cancellation technique and effective action approach. The point to note in the anomaly cancellation method is that Hawking radiation plays the role of cancelling
gauge and gravitational anomalies at the horizon to restore the gauge/diffeomorphism symmetry at the horizon. An advantage of this method is that neither the consistent anomaly nor the counterterm relating the different (covariant and consistent) currents, which were essential ingredients in [6], were required. On the other hand, in the effective action technique, we only need covariant boundary conditions, the importance of which was first stressed in [17]. Another important input in the entire procedure is the expression for the anomalous (chiral) effective action (which yields anomalous Ward identity having covariant gauge/gravitational anomaly). The unknown parameters in the covariant current and energy-momentum tensor derived from this anomalous effective action were fixed by a boundary condition—namely the vanishing of the covariant current and energy-momentum tensor at the event horizon of the blackhole. Finally, the charge and the energy flux were extracted by taking the \( r \rightarrow \infty \) limit of the chiral covariant current and energy-momentum tensor. The relation between the chiral and the normal current and energy-momentum tensors is also established by requiring that both of them match in the asymptotic limit which is possible since the gauge/gravitational anomaly vanishes in this limit.

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