Probing Non-holomorphic MSSM via precision constraints, dark matter and LHC data

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Abstract: In this analysis we explore the phenomenological constraints of models with non-holomorphic soft SUSY breaking terms in a beyond the MSSM scenario having identical particle content. The model referred as NHSSM shows various promising features like the possibility of a strong reduction in electroweak fine-tuning even for a scenario of a heavy higgsino type of LSP, a fact that is unavailable in pMSSM models. The other important aspect is satisfying the muon $g-2$ data even for a small $\tan \beta$ via a small value of coupling $A'_\mu$ associated with the tri-linear non-holomorphic soft term. Thus, a large SUSY contribution to muon $g-2$ is possible even for a significantly large smuon mass $m_{\tilde{\mu}_1}$. The Higgs mass radiative corrections are contributed by both the holomorphic and non-holomorphic trilinear soft parameters $A_t$ and $A'_t$, thus diluting the requirement to have a larger $A_t$ to satisfy the Higgs mass data. The model also provides with valid parameter space satisfying the constraint of $\text{Br}(B \to X_s + \gamma)$ for large values of $\tan \beta$, a scenario unfavourable in pMSSM.
1 Introduction

The discovery of the Higgs Boson at the ATLAS[1] and the CMS [2] experiments of the Large Hadron Collider (LHC) marks the completion of particle searches within the realm of the Standard Model (SM)[3]. The SM is quite successful in explaining electroweak and strong interactions and the associated Higgs mechanism is found to be a viable method for generating masses for fermions and electroweak gauge bosons. Despite its success in explaining most of the observed experimental results, there are many theoretical issues and experimental facts that cannot be addressed while staying within the SM. The gauge hierarchy problem, baryogenesis, the fact that neutrinos have masses, the absence of a dark matter candidate, are a few of the important issues that motivate us to explore Beyond the SM (BSM) scenarios. Models involving Supersymmetry (SUSY) such as the Minimal Supersymmetric Standard Model (MSSM)[4–7] are prominent candidates for BSM physics. However, the fact remains that even after the first few years of running of the Large Hadron Collider (LHC), SUSY is yet to be found. This has obviously put serious constraints on various models of low energy SUSY. In the post-Higgs discovery years, the lighter Higgs boson of MSSM to have a mass of
\( m_h \sim 125 \text{ GeV}[8] \) translates into large radiative corrections[9]. This demands a heavier top squark sector. A large fine-tuning is to be accepted. Furthermore, LHC has pushed up the lower limits of masses of the first two generations of squarks as well as gluino beyond a TeV. At the same time, SUSY models are increasingly being constrained via B-physics related measurements at LHCb. On the other hand, regarding dark matter (DM)[10], the measurements from WMAP/PLANCK[11, 12] for the DM relic density or LUX[13] experiment for DM direct detection have put significant limits. We further emphasize that, the data from the Brookhaven experiment for the anomalous magnetic moment of muon or \((g-2)_{\mu}\) points out a significant deviation (3.2\(\sigma\)) from its SM based evaluation while we note that various uncertainties of the SM contributions to \((g-2)_{\mu}\) are being reduced over the last few years. This leads to stringent constraints on the scalar and gaugino sectors of SUSY models. A combined requirement for satisfying the relic density range from WMAP or PLANCK experiments apart from satisfying the LHC derived sparticle mass[14] bounds particularly creates tension so as to have a reasonably large \(a_{\mu}^{\text{SUSY}}\). Here, \(a_{\mu}^{\text{SUSY}}\) refers to the SUSY contribution to the theoretical evaluation of the muon anomaly \(a_{\mu} \equiv \frac{1}{2}(g-2)_{\mu}\) which is supposed to be equal to the difference between the experimental value and the SM evaluation of the observable. The Higgsinos and Wino are typically required to be heavy for becoming candidates of dark matter. On the other hand, satisfying \((g-2)_{\mu}\) with dominant contributions from the chargino-sneutrino loops demands non-decoupling higgsinos or wino along with a light sneutrino \(\tilde{\nu}_\mu\). We emphasize that in models with gaugino mass universality like minimal supergravity (mSUGRA)[15]/constrained MSSM (CMSSM)[5], the chargino-sneutrino loops dominate in \(a_{\mu}^{\text{SUSY}}[16, 17]\). Thus in the present scenario of sparticle mass limits \(a_{\mu}^{\text{SUSY}}\) is not large enough to explain the observed deviation. In the parameter space of MSSM that is consistent with the Higgs mass data, dark matter relic density, collider limits for scalar and gaugino (electroweakino) masses, it turns out that the primary contribution to \(a_{\mu}^{\text{SUSY}}\) comes from the loop diagrams containing neutralinos and smuons, particularly from the L-R mixing terms that scale with \(M_{1}\mu\tan\beta[18]\). In order to have a larger \(a_{\mu}^{\text{SUSY}}\) so as to account for the deviation the above quantity needs to be large or in other words this restricts the smuon mass \(m_{\tilde{\mu}_1}\) to become large. A comprehensive analysis in a model that identifies the valid region of parameter space satisfying limits from flavor physics such as that from \(\text{Br}(B \to X_s + \gamma)\) and \(\text{Br}(B_s \to \mu^+\mu^-)\) (which we would collectively refer as B-physics constraints), dark matter constraints, while also having a moderate degree of fine-tuning and most importantly that would easily accommodate the \((g-2)_{\mu}\) limits even for a small \(\tan\beta\) all at one go, is undoubtedly important. In this analysis, keeping ourselves contained within the MSSM particle setup we would like to explore whether a consideration of non-holomorphic (NH) soft SUSY breaking terms may be able to reduce the stringency arising out of the \((g-2)_{\mu}\) constraint in particular apart from satisfying all the above mentioned phenomenological requirements.

Away from MSSM, particularly in models with singlet scalars NH soft breaking terms potentially fall in the class of terms that may cause hard SUSY breaking[19–24]. Considering, for example, a hidden sector SUSY breaking scenario like supergravity, one may generically consider a
spontaneous SUSY breaking due by the vacuum expectation value of an auxiliary field $F$ belonging to a chiral superfield $X$. This causes appearance of soft terms in the Lagrangian that are associated with the coupling of $X$ with another chiral superfield $\Phi$ or a gauge field strength superfield $W^\alpha_a$. In a supergravity framework where the mass scale $M$ is large (typically the Planck mass) one obtains the following [20].

$$-L = \left( \frac{1}{M}[XW^{\alpha a}W^\alpha_a]_F + \frac{1}{M}[X\Phi^3]_F + \frac{\mu}{M}[X\Phi^2]_F \right) + \text{c.c.} + \frac{1}{M^2}[X^*X\Phi^*\Phi]_D. \tag{1.1}$$

Here, the parameter $\mu$ is introduced for the soft term so as to follow closely with the usual MSSM notation[5]. Considering the vacuum expectation value $\langle X \rangle = \theta \theta < F >$ and denoting $< F >$ simply by $F$ one has the usual soft terms of MSSM namely the gaugino mass term, the cubic and the analytic scalar squared mass terms and a non-analytic scalar mass term coming out of the above D-term contribution as given below.

$$-L = \left( \frac{F}{M} \lambda^a \lambda_a + \frac{F}{M} \phi^3 + \frac{\mu F}{M} \phi^2 \right) + \text{c.c.} + \frac{|F|^2}{M^2} \phi^* \phi. \tag{1.2}$$

We note that $F/M$ should refer to a weak scale mass which we consider here as the W-boson mass $M_W$. Apart from the above contributions, there can be other D-term contributions that are classified as “maybe soft”[20] such as the ones arising out of $\frac{1}{M^3}[XX^*\Phi^2\Phi^*]_D$ and $\frac{1}{M^3}[XX^*D^\alpha D_{\alpha} \Phi]_D$ that give rise to NH terms in the Lagrangian like $\phi^2 \phi^*$ and $\psi \psi$ both with coefficients $\frac{|F|^2}{M^3} \sim \frac{M_W^2}{M^2}$. These terms in a broader sense may cause quadratic divergence, thus become hard SUSY breaking terms, in scenarios where the visible sector contains a singlet superfield. Nevertheless, the terms are generally highly suppressed in a supergravity type of scenario. Indeed this is why such NH contributions to the Lagrangian are traditionally ignored while discussing models with high scale based SUSY breaking. We must, however, note that, as pointed out in [25], in absence of any gauge singlet field in the visible sector such suppression may not be possible if the supersymmetry breaking effect is communicated to the visible sector at a lower energy. For example, this may happen in scenarios with gauge-mediated supersymmetry breaking[25]. Thus having no gauge singlet superfields MSSM may as well include such NH terms that are soft SUSY breaking in nature. The terms can hardly be ignored in the most general sense. This was discussed or at least pointed out in several works[20–22, 24, 27–31].

Apart from the references shown above related to the issue of absence of quadratic divergence in MSSM in presence of NH soft terms and the possible origin of the terms in relation to a hidden sector SUSY breaking model, we will now briefly refer to a few specific works related to phenomenology. A general analysis with NH soft terms that incorporated renormalization group evolutions with or without using R-parity violation was presented in Ref.[29]. This was followed by a study[30] with NH SUSY breaking terms in an essentially Constrained Minimal Supersymmetric Model (CMSSM)

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1For scenarios with $F$-term SUSY breaking leading to $\frac{1}{\pi^2}$ type of suppression in nonstandard supersymmetry breaking terms including Dirac gaugino mass terms see ref.[26].
setup with all input parameters including the NH ones being given at the gauge coupling unification scale while exploring relevant phenomenological constraints like $\text{Br}(B \to X_s + \gamma)$. Analyses that particularly focussed on the Higgs sector and constraints like $\text{Br}(B \to X_s + \gamma)$ are Refs.[31, 32]. However, in contrast to Ref.[31] that used input parameters given entirely at the unification scale, Ref.[32] analyzed in a mixed set-up where the NH parameters were given at the electroweak scale in an otherwise CMSSM type of setup. Similar mixed input parameters were used in Ref.[28] that discussed reparametrization invariance in special circumstances of the choice of parameters\textsuperscript{2}, spectra as well as the effect on fine-tuning. Ref.[33] may be seen for its emphasis on the Higgs sector while considering CP-violating phases. Nonstandard SUSY breaking was also used in phenomenological studies with R-parity violating NH soft SUSY breaking terms in MSSM framework in Refs.[29, 34, 35].

Unlike all the previous analyses where universal models of CMSSM type were considered with NH parameters being given either at the unification scale or at the electroweak scale, our work on the Non-Holomorphic Supersymmetric Standard Model (NHSSM) will entirely use electroweak scale input parameters similar to what is considered in phenomenological MSSM (pMSSM) model\textsuperscript{3} in relation to MSSM. In this phenomenological NHSSM (pNHSSM) framework, we will explore the extent NH parameters influence on satisfying the $(g - 2)_\mu$ constraint apart from the effect on electroweak fine-tuning and $m_h$ via the associated radiative corrections. The Higgs mass limit is achieved for smaller values of $|A_t|$, the trilinear coupling parameter corresponding to the top-quark in comparison with what is required for MSSM. Furthermore, as we will see such NH parameters do not affect the fine-tuning measure since the Higgs scalar potential would not have any dependence on such parameters\textsuperscript{3}.

We will additionally focus on the low energy processes like $\text{Br}(B \to X_s + \gamma)$ and $\text{Br}(B_s \to \mu^+ \mu^-)$ that also receive a significant amount of contributions from the NH terms. Particularly these constraints may indeed be quite severe for large values of $\tan \beta$ in MSSM. However, the NH terms are able to alter the above branching ratios and a large region of parameter space that would be excluded in MSSM is restored. We will further see that there are valid regions of pNHSSM parameter space that is consistent with correct relic abundance for dark matter (DM) and these may be probed for the Direct and Indirect Detection limits of DM.

Our paper is organized as follows. In Sec.2 we discuss NHSSM and particularly explain the impact of NH terms on the i) Higgs and other scalar sectors, ii) charginos and neutralinos, electroweak fine-tuning and iii) phenomenological aspects related to the constraints coming from dark matter, muon $g - 2$ as well as $\text{Br}(B \to X_s + \gamma)$ and $\text{Br}(B_s \to \mu^+ \mu^-)$. We present the results of our analysis in Sec.3. Finally, we conclude in Sec.4.

\textsuperscript{2}This, however, does not apply to our analysis involving unconstrained NH SUSY breaking parameters.

\textsuperscript{3}See Ref.[37] that appears while this work was being done in which the authors included the NH soft terms in their discussion on Higgsino dark matter while extending MSSM. Our work in NHSSM involves electroweak scale input parameters for all the soft terms similar to pMSSM unlike their analysis with RG evolutions and unification scale input parameters.
2 Non-Holomorphic Supersymmetric Standard Model

We remind that MSSM is considered to have only holomorphic soft SUSY breaking terms. The trilinear soft terms, in particular, are given by as follows [5]

\[- \mathcal{L}_{\text{soft}} \supset \tilde{Q} \cdot H_u A_t \tilde{U} + \tilde{U} \cdot H_d A_b \tilde{D} + \tilde{L} \cdot H_d A_\tau \tilde{E} + h.c. \]  

(2.1)

We have only shown here the dominant terms involving the third generations of fermions. It was shown that in the absence of any Standard Model gauge singlet it is possible to extend the SUSY breaking soft sector by including NH soft SUSY breaking terms, without aggravating any quadratic divergence [19, 22]. Thus the NH soft terms of the NHSSM in general that include trilinear coupling terms as well as a coupling term involving Higgsinos are given by [31, 32],

\[- \mathcal{L}'_{\text{soft}} \supset \tilde{Q} \cdot H_c u A'_t \tilde{U} + \tilde{U} \cdot H_c d A'_b \tilde{D} + \tilde{L} \cdot H_c u A'_\tau \tilde{E} + \mu' \tilde{H}_u \cdot \tilde{H}_d + h.c. \]  

(2.2)

We will now discuss the effect of involving NH terms on several sectors of MSSM sparticle spectra, particularly in the Higgs sector, the squark and slepton sectors, the electroweakinos (charginos and neutralinos) apart from its effect on a fine-tuning measure. We will also include low energy data from precision experiments like \((g-2)_\mu[38-41]\), \(\text{Br}(B \to X_s + \gamma)[42]\), \(\text{Br}(B_s \to \mu^+ \mu^-)[43-45]\) and cosmological observables like dark matter relic density while also taking into account the LHC bounds for sparticles and the Higgs mass data [8].

2.1 Influence of non-holomorphic terms on the scalar sector

The NH trilinear coupling parameters may cause a significant amount of change in the masses of squarks and sleptons. For example, the mass matrix for the up type of scalar quark, in general, is given by [31, 32].

\[ M^2_{\tilde{u}} = \begin{pmatrix} m^2_{\tilde{Q}} + \left( \frac{1}{2} - \frac{3}{2} \sin^2 \theta_W \right) M^2_Z \cos 2\beta + m^2_u - m_u (A_u - (\mu + A'_u) \cot \beta) & -m_u (A_u - (\mu + A'_u) \cot \beta) & m^2_u + \frac{3}{2} \sin^2 \theta_W M^2_Z \cos 2\beta + m^2_u \\ m^2_u - m_u (\mu + A'_u) \cot \beta & m^2_u + \frac{3}{2} \sin^2 \theta_W M^2_Z \cos 2\beta + m^2_u & m_u (A_u - (\mu + A'_u) \cot \beta) \end{pmatrix}. \]  

(2.3)

Similar matrices for a slepton or a down-type of squark may be written as given below.

\[ M^2_{\tilde{e}} = \begin{pmatrix} M^2_{l_L} + (T^2_{3L} - Q_e \sin^2 \theta_W) \cos 2\beta + m^2_e - m_e (A_e - (\mu + A'_e) \tan \beta) & -m_e (A_e - (\mu + A'_e) \tan \beta) & M^2_{l_R} + M^2_Z Q_e \sin^2 \theta_W \cos 2\beta + m^2_e \\ -m_e (A_e - (\mu + A'_e) \tan \beta) m_e & m^2_e + \frac{3}{2} \sin^2 \theta_W M^2_Z \cos 2\beta + m^2_e & M^2_{l_L} + (T^2_{3L} - Q_e \sin^2 \theta_W) \cos 2\beta + m^2_e \end{pmatrix}. \]  

(2.4)

Clearly, \( \mu \) of MSSM that contributes to L-R mixing of squark is replaced by \( \mu + A'_u \) in NHSSM. The contributions of the NH terms will thus be more effective for i) low tan \( \beta \) in the case of up type of squarks and ii) large tan \( \beta \) in case of down type of squarks or sleptons.

An effect on top-squark sector is transmitted to the Higgs mass radiative corrections. We remind that in the MSSM framework the discovery of the Higgs boson with a mass of 125.09 \( \pm \) 0.24GeV [8] is translated into a large radiative corrections to the mass of the lighter neutral CP-even Higgs...
boson $h$. The above requirement pushes up the masses of the top-squarks in MSSM which in turn indicates the need for a large value of $|A_t|$ so as to have a larger Left-Right mixing. Considering the top-stop loops which constitute the most contributing terms, the above radiative corrections in NHSSM read as follows.

$$\Delta m_{h,\text{top}}^2 = \frac{3g_2^2 \tilde{m}_t^4}{8\pi^2 M_W^2} \ln \left( \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{\tilde{m}_t^2} \right) + \frac{X_t^2}{m_{\tilde{t}_1} m_{\tilde{t}_2}} \left( 1 - \frac{X_t^2}{12 m_{\tilde{t}_1} m_{\tilde{t}_2}} \right).$$  \hspace{1cm} (2.5)

Here, $X_t = A_t - (\mu + A'_t) \cot \beta$. Clearly $A'_t = 0$ corresponds to the MSSM result. Here $\tilde{m}_t$ refers to the running top-quark mass that includes corrections from the electroweak, QCD and SUSY QCD effects. The maximal mixing scenario refers to $X_t = \sqrt{6} M_S[5, 6]$ where $M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$. We note that unlike MSSM, with a suitable combination of the signs of $A_t$ and $A'_t$, it is possible to limit $|A_t|$ so as to satisfy the Higgs mass constraint.

We shall now discuss the effect of considering NHSSM on the Electroweakino (Chargino-Neutralino) sector and fine tuning.

### 2.2 Electroweakinos in NHSSM

The NH parameter $\mu'$ modifies both the chargino and neutralino mass matrices. Essentially the charginos and neutralinos now have higgsino components corresponding to a higgsino mass of $|\mu - \mu'|$. Thus in NHSSM the neutralino mass matrix reads[31, 32]:

$$M_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -M_Z \cos \beta \sin \theta_W & M_Z \sin \beta \sin \theta_W \\ 0 & M_2 & M_Z \cos \beta \cos \theta_W & -M_Z \sin \beta \cos \theta_W \\ -M_Z \cos \beta \sin \theta_W & M_Z \cos \beta \cos \theta_W & 0 & -(\mu - \mu') \\ M_Z \sin \beta \sin \theta_W & -M_Z \sin \beta \cos \theta_W & -(\mu - \mu') & 0 \end{pmatrix}. \hspace{1cm} (2.6)$$

Similarly for the chargino matrix we have[31, 32]:

$$M_{\tilde{\chi}^\pm} = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin \beta \\ \sqrt{2} M_W \cos \beta & -(\mu - \mu') \end{pmatrix}. \hspace{1cm} (2.7)$$

We note that the LEP bound on the lighter chargino mass will essentially apply to $|\mu - \mu'|$ instead of $\mu$. However, as we will see below the fine-tuning measure is still dependent on $\mu$ rather than $\mu'$ and this provides with a quite unique signature of a possibility of having low fine-tuning irrespective of the nature of dark matter considered in NHSSM. For example, one can have a bino-like LSP even for very low $\mu$.

### 2.3 Electroweak fine tuning in pNHSSM

The nonholomorphic trilinear parameters are associated with charged or colored scalars whereas the parameter $\mu'$ is associated with fermions (higgsinos). Thus with no influence of the nonholomorphic

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\(^4\)Precision measurements indicate that the discovered Higgs Boson is consistent with being SM-like[8]. With the above in perspective we consider the decoupling regime of MSSM Higgs characterized by $M_Z^2 \ll M_A^2[9]$. 

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soft breaking terms on the neutral scalar potential, the latter is same as that of MSSM as given below.

\[ V = (m_{H_u}^2 + \mu^2)|H_u^0|^2 + (m_{H_d}^2 + \mu^2)|H_d^0|^2 - b(H_u^0 H_d^0 + h.c.) + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2, \]  

(2.8)

where \( \mu \) is the bilinear Higgs mixing parameter of the superpotential, \( m_{H_u}, m_{H_d} \) are scalar mass parameters and \( b \) is the Higgs mixing parameter within the SUSY breaking soft sector. After minimization with respect to the vacuum expectation values (vevs) of the neutral Higgs scalars, one finds, the well-known relations for electroweak symmetry breaking \([5, 6]\)

\[ \frac{m_Z^2}{2} = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - |\mu|^2, \]  

(2.9)

and,

\[ \sin 2\beta = \frac{2b}{m_{H_d}^2 + m_{H_u}^2 + 2|\mu|^2}. \]  

(2.10)

Clearly, at the tree-level one requires a fine cancellation between the two terms in the right-hand side of Eq.2.9 coming from SUSY breaking parameters \( m_{H_d}, m_{H_u} \) and supersymmetry preserving parameter \( \mu \) for obtaining the left-hand side namely \( m_Z^2/2 \) where \( m_Z \) refers to the measured value of 91.2 GeV. The degree of cancellation broadly indicates a measure of the fine-tuning at the electroweak scale. We consider here a general definition of electroweak fine tuning (EWFT) that uses log derivatives\([46–48]\), namely

\[ \Delta p_i = \left| \frac{\partial \ln m_Z^2(p_i)}{\partial \ln p_i} \right|, \]  

(2.11)

where \( p_i \equiv \{\mu^2, b, m_{H_u}, m_{H_d}\} \) are the parameters that determine the tree-level value of Z boson mass. The total EWFT at low scale is given by,

\[ \Delta_{Total} = \sqrt{\sum_i \Delta_{p_i}^2}. \]  

(2.12)

It turns out that with large \( \tan \beta \) the most important terms are \( \Delta(\mu) \sim \frac{4\mu^2}{m_Z^2} \) and \( \Delta(b) \sim \frac{4M^2_B}{m_Z^2 \tan^2 \beta} \). The above expressions show that even for a moderately large \( \tan \beta \), a small value of EWFT demands a lower value of \( \mu \). However, very small values of \( \mu \) are excluded in MSSM due to LEP bound of lighter chargino mass and we will see that this has an important significance in relation to the fine tuning in NHSSM in Sec.3. We however point out that for small \( \tan \beta \) and very small \( \mu \) (typically much smaller than the above chargino mass limit) situation may arise where \( \Delta(m_{H_u}) \) and \( \Delta(m_{H_d}) \) become larger than \( \Delta(\mu) \)[49]. As a result even for negligible values of \( \mu \) one may obtain finite EWFT as we will see in Sec.3.3.

In our discussion on EWFT we must, however, remember that, we should include the principal corrections due to one-loop radiative effects due to top-stop loops and this leads to\([49]\),

\[ \delta m_{H_u}^2 = -\frac{3y_t^2}{8\pi^2}(m_{t_L}^2 + m_{t_R}^2 + |A_t^2|) \log \left( \frac{\Lambda}{m_{t_L}} \right). \]  

(2.13)
Albeit this depends on the choice of the cut-off scale $\Lambda$. The requirement of a large $A_t$ in the post-Higgs@125 GeV scenario increases $\delta m^2_{H_u}$, which specially shows the need to include the one-loop corrections shown above. This obviously enhances the EWFT, although we will not include this effect in our EWFT measure. We must, however, point out that minimizing the Higgs potential near a scale where the logarithmic term vanish in Eq.2.13 may reduce the requirement of inclusion of the one-loop radiative corrections significantly as it was discussed in the context of hyperbolic branch and electroweak fine-tuning Ref.[50].

As noted before, a small value of $\mu$ is consistent with smaller EWFT. However, the lighter chargino mass bound from LEP limits $\mu$ hence to the EWFT measure not to become too small. In NHSSM the higgsino content of electroweakinos of Eqs.2.6 and 2.7 depends on $|\mu - \mu'|$, thus EWFT may become small irrespective of the mass of electroweakinos.

2.4 Low Energy Constraints viz. $(B \rightarrow X_s + \gamma)$ and $(B_s \rightarrow \mu^+ \mu^-)$ in NHSSM

It is known that rare B-decays within the SM like $(B \rightarrow X_s + \gamma)$, $(B_s \rightarrow \mu^+ \mu^-)$ that are helicity suppressed may have large contributions from the radiative corrections due to superpartners in the loops. The SM contributions to $(B \rightarrow X_s + \gamma)$ almost saturate the experimental data. Thus any BSM correction should be low enough to accommodate the difference of the SM and the experimental results, which is albeit small. SUSY parameter space is thus strongly constrained via cancellation of dominantly contributing diagrams, while the contributions individually may be large. One must also remember that there are next to leading order (NLO) contributions that can also be quite significant, specially for large values of $\tan \beta$. In the SM the dominant radiative corrections come from $t-W$ loops. In MSSM the significantly contributing diagrams involve $t-H^\pm$ and $\tilde{t}-\tilde{\chi}^\pm$ loops. The contributions from the former loops share the same sign with the $t-W$ loop contributions of the SM. In NHSSM, the soft terms from the NH trilinear coupling $A'_t$ and the bilinear higgsino coupling $\mu'$ have significant influences on the SUSY diagrams associated with the above flavor related processes. In contrast to MSSM, loops involving left-right mixing of top-squarks in NHSSM are associated with the factor $A_t - (\mu + A'_t) \cot \beta$ [31, 32], whereas for the contributions involving the higgsino loops, $\mu$ is replaced by the difference $(\mu - \mu')$[31, 32].

Regarding the constraint from $\text{Br}(B_s \rightarrow \mu^+ \mu^-)$ that is typically stringent for large $\tan \beta$ and small pseudoscalar mass in MSSM models, it turns out that the available parameter space that survives after imposing the $\text{Br}(B \rightarrow X_s + \gamma)$ constraint, is not affected much when one imposes the constraint from $\text{Br}(B_s \rightarrow \mu^+ \mu^-)$.

2.5 Muon $g-2$

The anomalous magnetic moment of muon $(a_\mu = \frac{1}{2}(g-2)_\mu)$ is an extremely important constraint for new physics[16]. The experimental data $(\equiv a_\mu^{\text{exp}})$[38, 39] shows more than $3\sigma$ level of deviation from the SM prediction $(\equiv a_\mu^{\text{SM}})$[40, 41]. The difference of the two values for a BSM contribution
amounts to:

$$\Delta a_\mu = a^{\text{exp}}_\mu - a^{\text{SM}}_\mu = (29.3 \pm 9.0) \times 10^{-10}. \quad (2.14)$$

The above result leads to the following $2\sigma$ and $1\sigma$ limits for $a^{\text{SUSY}}_\mu \equiv \Delta a_\mu$, where $a^{\text{SUSY}}_\mu$ refers to the contributions to the muon magnetic moment coming from the loop level diagrams involving SUSY particles. The limits of $a^{\text{SUSY}}_\mu$ becomes:

$$11.3 \times 10^{-10} < a^{\text{SUSY}}_\mu < 47.3 \times 10^{-10} \quad (2\sigma) \quad (2.15)$$

and,

$$20.3 \times 10^{-10} < a^{\text{SUSY}}_\mu < 38.3 \times 10^{-10} \quad (1\sigma). \quad (2.16)$$

The Feynman diagrams containing chargino-sneutrino and neutralino-smuon loops produce the most dominant SUSY contribution to $a^{\text{SUSY}}_\mu$ [16, 17]. In a bino like LSP scenario with $M_1 < \mu \ll M_2$, significantly large $a^{\text{SUSY}}_\mu$ is achievable in MSSM via the presence of very light smuon [18]. This is particularly true when lighter chargino is not so light or in other words $\mu$ is not very small. However, LHC is increasingly pushing up the masses of sleptons and this would require a large value of $\mu$ in order to accommodate the muon $g - 2$ data[18] in MSSM. A large $\mu$ is obviously not desirable in the context of EWFT. Moreover, such lower values of $m_{\tilde{\mu}_1}$ may be disfavored by the LHC data. Unlike MSSM, $a^{\text{SUSY}}_\mu$ can be enhanced significantly in NHSSM because of additional terms proportional to $A'_\mu \tan \beta$. This is true even in a very natural scenario (i.e. with small EWFT) characterized by low $\mu$ along with relatively heavier smuons.

3 Results

The focus of our analysis in this section would be the important features of NHSSM in relation with MSSM, particularly the effects of considering NHSSM on i) Higgs boson mass, ii) flavor violating processes like $\text{Br}(B \to X_s + \gamma)$ and $\text{Br}(B_s \to \mu^+\mu^-)$ iii) SUSY contributions to muon $g - 2$, iv) dark matter relic density and v) electroweak fine-tuning. Regarding the last two points we will particularly demonstrate the fact that NHSSM allows to have a higgsino-like DM with a sufficiently low electroweak fine-tuning, a feature almost impossible to get in MSSM. We will additionally show a few benchmark points consistent with the various above constraints.

3.1 Impact of non-holomorphic soft parameters on $m_h$

The effects of nonholomorphic parameters particularly $A'_\mu$ on radiative corrections to the CP-even lighter Higgs boson mass $m_h$ as enumerated in Eq.2.5 is particularly prominent for smaller $\tan \beta$. Keeping this in mind, we choose $\tan \beta = 10$ and show the extent of variation of $m_h$ due to varying $A'_\mu$ and $A'_t$. This is displayed in Fig.1(a). We assume a 3 GeV window in $m_h$ leading to the following range[51].

$$122.1 \leq m_h \leq 128.1 \text{ GeV}. \quad (3.1)$$
We note that the above uncertainty that has been widely used arise from renormalization scheme related dependencies, scale dependence, problems in computing higher order loop corrections up to three loops or the uncertainty in the experimental value of top-quark mass\(^5\). Our choice of electroweak scale parameters\(^6\), and their ranges, that may produce the right amount of radiative corrections to Higgs mass are as given below,

\[
10 \leq \mu \leq 1000 \text{ GeV}, \
-3000 \leq A_t \leq 3000 \text{ GeV}, \
-2000 \leq \mu' \leq 2000 \text{ GeV}, \
-3000 \leq A'_t \leq 3000 \text{ GeV}.
\]

(3.2)

The values of relevant strong sector input like \(M_3\), and the third generation of scalar mass parameters are fixed at 1.5 TeV and 1 TeV respectively that lead to physical states like that of the gluino or the top-squarks to have masses above the LHC limits. All other trilinear couplings are set to zero. Finally, without losing any generality we do the analysis for a fixed choice of gaugino masses namely, \(M_1 = 150 \text{ GeV}, M_2 = 250 \text{ GeV}\). We compute the spectrum using SPheno\(^{[53]}\) [v.3.3.3] while implementing the model from SARAH\(^{[54]}\) [v.4.4.4]. The sparticle mass limits are also taken into account\([14]\). The ranges mentioned in Eq.3.2 correspond to the results of Sec.3.1 and 3.2.

\(\text{Figure 1. Fig.1(a) shows the variation of } m_h \text{ against } A_t \text{ for the scanning ranges of Eq.3.2 for } \tan \beta = 10. \) The magenta and cyan colored regions correspond to NHSSM and MSSM respectively. Fig.1(b) is same as Fig.1(a) except with \(\tan \beta = 40\). The green lines (dashed) represent the lower limit of Eq.3.1 for \(m_h\).

\(^5\)We also remind the reader the additional issue of uncertainty of about 2.8 GeV in \(m_t^{\text{pole}}\) as argued in Ref.\([52]\).

\(^6\)The parameters are given at the scale of the geometric mean of the top-squark parameters before mixing. The relevant SM parameters used are \(m_t^{\text{pole}} = 173.5 \text{ GeV}, m_b^{\text{MS}} = 4.18 \text{ GeV} \) and \(m_\tau = 1.77 \text{ GeV}\).
In order to probe NHSSM signatures on the Higgs boson mass we plot both the NHSSM and MSSM specific parameter points in Fig.1(a). The magenta colored points in Fig.1(a) correspond to the NHSSM scenario where variation due to relevant holomorphic and non-holomorphic parameters are as referred in Eq.3.2. We isolate the MSSM specific parameter points in the cyan colored region by a choice of $A'_t = \mu' = 0$. Clearly, focusing on the non-maximal region of $m_h$ and a given value of $A_t$ we note that the lighter Higgs boson mass may have a $2-3$ GeV amount of enhancement/decrease, a signature of NHSSM. Additionally, compared to MSSM, NHSSM is able to provide with correct ranges of $m_h$ for a significantly lower value of $|A_t|$. We must, however, note that for smaller $|A_t|$ less than a TeV or so, the contribution from $A'_t$ to the radiative corrections to the Higgs mass is hardly large enough so as to satisfy the lower limit of Eq.3.1. Needless to mention a choice of a heavier third generation of squark would easily enhance $m_h$ close to its upper bound of Eq.3.1. Now, we would like to focus on $\tan \beta = 40$. Since the contribution of $A'_t$ is suppressed by $\tan \beta$ (Eq.2.3), we expect only a marginal impact on $m_h$. This is evident from the appearance of only a small spread around the cyan region of Fig.1(b). Although we have considered $A'_b$ to be vanishing, it may be noted that a significantly larger value of $A'_b$ may lead to non-negligible contribution towards $m_h$ via the effect of sbottom loops. This effect is indeed enhanced by $\tan \beta$ and depends on the off-diagonal quantity $X_b$, where $X_b = \{A_b - (\mu + A'_b) \tan \beta\}^7$. Similar contribution arises also from the stau loops in the presence of large $A'_t$ in the large $\tan \beta$ regime. However for our analysis of Fig.1 with $\mu < 1$ TeV, we hardly expect any significant contribution to $m_h$ from the sbottom loops since the prefactor of $\tan \beta$ in the off-diagonal sbottom mass matrices is not too large with the given range of parameter regions considered in this analysis. This is also true for stau loops. Apart from the effect of NH trilinear parameters particularly $A'_t$ on $m_h$ via top-squarks in the loops, we must remember that the other NH parameter $\mu'$ may play an important role via the chargino loop contributions to the Higgs boson mass[57, 58]. The latter contributions that are intrinsically negative are essentially independent of $\tan \beta$[57, 58]. As a result, in the region near $A_t = 0$ in Fig.1(b) where $A'_t$ is not able to influence on $X_t$ because of suppression via $\tan \beta$, we find a spread of the magenta points toward the smaller direction of $m_h$. That the effect does not depend on $\tan \beta$ is manifested in the similar region of Fig.1(a). Thus, larger values of $\mu - \mu'$ may cause a decrease in $m_h$ for both values of $\tan \beta$ as used in the figure. Although enhancement of $m_h$ due to NH parameters is not very significant in the large $\tan \beta$ limit, the impact of the above parameters on particularly the low energy phenomenological constraints like $\text{Br}(B \to X_s + \gamma)$ is extremely important as we will see in Sec.3.2.

3.2 Effects of non-holomorphic parameters on SUSY contributions to $\text{Br}(B \to X_s + \gamma)$ and $\text{Br}(B_s \to \mu^+\mu^-)$

In this subsection, we would like to discuss the results of including the constraints of $\text{Br}(B \to X_s + \gamma)$ and $\text{Br}(B_s \to \mu^+\mu^-)$ on the NHSSM parameter space. The experimental limits on $\text{Br}(B \to X_s + \gamma)$

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7See [56] and references therein along with references for the NHSSM and the MSSM.
at $3\sigma$ level reads\[42\]

$$2.77 \times 10^{-4} \leq \text{Br}(B \to X_s + \gamma) \leq 4.09 \times 10^{-4}.$$  \hspace{1cm} (3.3)

On the other hand, the recent constraints from $\text{Br}(B_s \to \mu^+\mu^-)$ at $3\sigma$ level results into\[43–45\]

$$0.8 \times 10^{-9} \leq \text{Br}(B_s \to \mu^+\mu^-) \leq 5 \times 10^{-9}.$$  \hspace{1cm} (3.4)

Figure 2. The variation of $m_h$ against $A_t$ for the scanning ranges of Eq.3.2, with $\tan \beta = 10$ and $\tan \beta = 40$. The magenta and cyan colored regions correspond to NHSSM and MSSM respectively. The green lines (dashed) represent the lower limit of Eq.3.1 for $m_h$. Furthermore, we impose the constraints from $\text{Br}(B \to X_s + \gamma)$ and $\text{Br}(B_s \to \mu^+\mu^-)$ on the resulting spectrum. Clearly, for $\tan \beta = 40$ a large region of parameter space in MSSM with large $A_t$ is excluded by these constraints. However, NHSSM essentially recovers the large $A_t$ regions consistent with the Higgs mass as well as the $B$-physics constraints.

Fig: 2(a) for $\tan \beta = 10$ shows the effect of imposing the $B$-physics constraints on the parameter space of MSSM and NHSSM that are displayed with different colors. Both the MSSM (cyan) and NHSSM (magenta) parameter regions of Fig: 2(a) hardly show any change when $B$-physics constraints are imposed in comparison to Fig.1(a) where the same were not included. Since $\tan \beta$ is not large one does not expect any significant degree of change in the MSSM parameter space when $\text{Br}(B \to X_s + \gamma)$ constraint is applied because the SUSY contribution of the same approximately scales with $\tan \beta$\[55\]. The scaling behavior also holds good in NHSSM.

Fig:2(b) for $\tan \beta = 40$, shows that the constraints of Eqs.3.3 & 3.4 exclude a large amount of MSSM parameter region (cyan) when $|A_t|$ is large. This is indeed expected with the scaling behavior with respect to $\tan \beta$ as mentioned above in regard to the $\text{Br}(B \to X_s + \gamma)$ constraint.

\footnote{For further discussion see \[56, 59\] and references therein.}
The region with large $A_t$ along with $\mu A_t < 0$ ($\mu$ is scanned over positive value as in Eq.3.2) is discarded via the lower bound of Eq. 3.3 whereas the region with $\mu A_t > 0$ is disallowed via the upper bound of Eq. 3.3. Thus, a large $|A_t|$ regions become unavailable in MSSM which in turn causes $m_h$ to go below the lower limit of Eq.3.1. Certainly, $m_h$ can be increased via increasing the third generation of scalar mass that would enhance the Higgs mass radiative corrections.

In contrast to MSSM, we find that the magenta region corresponding to NHSSM includes parameter points that satisfy the Higgs mass bounds in addition to the B-physics constraints. Thus large values of $A_t$ (with preference to negative region) correspond to valid parameter zones simply because of the fact that the role played by $A_t$ is effectively replaced by $A_t - A'_t \cot \beta$ in NHSSM (see Sec.2.4). Thus a scan over $A'_t$ even for a large value of $\tan \beta$ in NHSSM is able to accommodate appreciably large values of $|A_t|$ consistent with the Higgs mass as well as B-physics constraints.

3.3 Electroweak fine-tuning and higgsino dark matter

Typically a higgsino dominated dark matter with a mass around a few hundred GeV produces extremely large annihilation cross section. Apart from the LSP pair annihilation there is a substantial amount of $\tilde{\chi}_1^0 - \tilde{\chi}_1^\pm$ coannihilation. A larger higgsino content in a primarily bino dominated $\tilde{\chi}_1^0$ such as what one obtains in the focus point[62, 63]/hyperbolic branch scenario[50] may produce the right relic abundance satisfying the experimental constraint of DM relic density. However, this is highly constrained by the direct detection of DM experiments like LUX [13]. In MSSM, a highly higgsino dominated LSP satisfies the DM relic density limits as given by Eq.3.5 from PLANCK [12] data for an LSP mass of $\sim 1$ TeV[64, 65].

$$0.092 \lesssim \Omega_{\tilde{\chi}_1^0} h^2 \lesssim 0.138. \quad (3.5)$$

Certainly, with a 1 TeV higgsino mass the electroweak fine-tuning estimate $\Delta T_{total}$ of Eq.2.12 becomes generally large. One, on the other hand, is able to isolate significantly the EWFT measure from LSP mass in NHSSM simply because of the fact that the Higgs potential (Eq.2.8) does not depend on the nonholomorphic Higgsino parameter $\mu'$, whereas the higgsino content of the LSP is determined via the difference of $\mu$ and $\mu'$.

We now scan the NHSSM parameter space focusing on a higgsino dominated LSP. We select a fixed value of 3 TeV for all the following masses namely $M_1, M_2$ and $m_A$ along with all the squark and slepton mass parameters in a scenario of varying $\mu - \mu'$ as mentioned below.

$$-3 \text{ TeV} \leq \mu \leq 3 \text{ TeV},$$

$$-3 \text{ TeV} \leq \mu' \leq 3 \text{ TeV}. \quad (3.6)$$

---

9Specifically, see Eq.2.28 and 2.30 of Ref.[55] in relation to the $Br(B \to X_s + \gamma)$ constraint. Refs.[60, 61] may be seen for similar other analyses in MSSM.

10See however Ref.[66] where the authors considered specific GUT scenarios with non-universal gaugino masses.
We note that the LEP limit on lighter chargino mass is translated in NHSSM as $|\mu - \mu'| \gtrsim 100$ GeV and we probe an LSP mass zone up to 1.5 TeV so that $\tilde{\chi}^0_1$ remains sufficiently higgsino dominated in its composition whereas we do not include the s-channel $A$-boson annihilation region. Furthermore, in order to have the Higgs mass in the correct range, we vary the trilinear parameters $A_t$ and $A'_t$ as given below.\[ ^{11} \]

\[ -3 \text{ TeV} \leq A_t \leq 3 \text{ TeV}, \]
\[ -3 \text{ TeV} \leq A'_t \leq 3 \text{ TeV}. \]

(3.7)

The parameter ranges of Eq.3.6 and 3.7 apply to the discussion of the present subsection only. In Fig.3 we plot $\Delta_{Total}$ (Eq.2.12) vs $m_{\tilde{\chi}^0_1}$ for MSSM and NHSSM cases drawn in blue and brown colors respectively. The MSSM part of the analysis corresponds to vanishing $\mu'$ and $A'_t$ and appears as a thin blue line in the middle. The relic density limits are satisfied in the vertical strip shown in magenta or green corresponding to NHSSM and MSSM cases. The lighter chargino mass limit disallows the LSP mass to go below 100 GeV (white region in the left). Fig.3(a) shows that $\Delta_{Total}$ for NHSSM can either be larger or smaller than the MSSM specific values. The larger $\Delta_{Total}$ region, of course, occurs when $|\mu|$ is larger while the lower region corresponds to smaller $|\mu|$. The LSP mass is essentially same as $|\mu - \mu'|$. Fig.3(b) shows a similar result for $\tan \beta = 40$. The lowest value of $\Delta_{Total}$ satisfying the DM relic density constraint of Eq.3.5 for MSSM is about 500, whereas the same for NHSSM for $\tan \beta = 10$ is about 50 rather than being vanishingly small.\[ ^{12} \]

On the other hand, $\Delta_{Total}$ for $\tan \beta = 40$ can indeed approach zero for vanishingly small $\mu$. The above difference of small $\mu$ behavior of $\Delta_{Total}$ is indeed consistent with the discussion of Sec. 2.3. We have used micrOMEGAs for relic density computation.

We note that a higgsino type of LSP generally satisfies the LUX data and it may be probed via XENON1T.

3.4 Constraint from muon $g - 2$ in relation to large $a^\text{SUSY}_\mu$ in NHSSM

In this subsection, we would like to demonstrate a novel signature of NHSSM on $a^\text{SUSY}_\mu$ by showing the degree of influence of the NH trilinear parameter $A'_\mu$. We would particularly stress on the fact that even a small value of $A'_\mu$ like 50 GeV can cause a tremendous change in $a^\text{SUSY}_\mu$ when compared to the corresponding MSSM scenario. Clearly, this is possible when the neutralino-smuon loops dominate over the chargino-sneutrino loops in their contribution to $a^\text{SUSY}_\mu$. Keeping this in mind we study the effect of Muon $g - 2$ constraint on the $m_{\tilde{\mu}_1} - m_{\tilde{\chi}^0_1}$ plane while selecting a low range for $M_1$ satisfying $M_1 < \mu < M_2$, corresponding to fixed values of $\mu$ and $M_2$ namely, $\mu = 500$ GeV and $M_2 = 1500$ GeV. The scanning of NHSSM parameter space with $\mu' = 0$ is considered in a background of fixed squark and stau masses set at 1 TeV, while choosing $A_t = -1.5$ TeV with all $\mu'$.

\[ ^{11} \text{However, while doing a generic study on fine-tuning we do not impose any explicit constraints like Higgs mass (Eq.3.1) or B-physics limits (Eqs.3.3,3.4).} \]

\[ ^{12} \text{We essentially agree with the analysis of Ref.[37].} \]
Figure 3. The variation of $\Delta_{\text{Total}}$ against $m_{\tilde{\chi}_1^0}$ for the scanning ranges of Eqs.3.6 and 3.7, with $\tan \beta = 10$ and $\tan \beta = 40$. The NHSSM and MSSM are shown in brown and blue colors respectively. The scan does not include the Higgs mass range of Eq.3.1. The relic density limits of Eq.3.5 are satisfied in the vertical strip shown in magenta or green corresponding to NHSSM and MSSM cases. It is evident from the figures that EWFT can be significantly lower in NHSSM in a region with higgsino-like LSP providing the required relic abundance.

other trilinear parameters being set to zero. The range of variation considered for $M_1$ and the first two generation of slepton masses $M_{\tilde{l}}^i$ are as follows.

$$100 \text{ GeV} < M_1 < 400 \text{ GeV},$$

$$100 \text{ GeV} < M_{\tilde{l}}^i < 1000 \text{ GeV}.$$  \hfill (3.8)

Fig.4 shows the parameter points in the $m_{\tilde{\mu}_1} - m_{\tilde{\chi}_1^0}$ plane corresponding to the MSSM scenario for $\tan \beta = 10$ and 40 where we isolate the degree of satisfying the Muon $g - 2$ constraint at 1$\sigma$, 2$\sigma$ and 3$\sigma$ levels as shown in blue, green and brown colors respectively. The upper limits of $m_{\tilde{\mu}_1}$ at 1$\sigma$ level in Fig.4(a) and Fig.4(b) are about 125 GeV and 260 GeV respectively. Thus one requires very light $\tilde{\mu}_1$ in order to have Muon $g - 2$ within 1$\sigma$ limits. This will drastically change in NHSSM as we will see in the following figures.

Fig.5 for NHSSM shows the parameter points in the same plane corresponding to $\tan \beta = 10$ and 40 for a fixed value of $A'_\mu = 50$ GeV. Even with such a small value of $A'_\mu$ we see that the upper limits of $m_{\tilde{\mu}_1}$ at 1$\sigma$ level jumping to 420 GeV and 500 GeV respectively. Clearly, the contribution to $a_{\mu}^{\text{SUSY}}$ is visibly substantial when we compare the above with the results of the MSSM case of Fig.4. The analysis is further extended for $A'_\mu = 300$ GeV in Fig.6. The same upper limits of $m_{\tilde{\mu}_1}$ are now 750 GeV and 800 GeV, almost impossible to reach within MSSM whatsoever while assuming the dominating loops to involve neutralinos rather than charginos.
Thus NHSSM can easily accommodate the stringent muon $g-2$ constraint even with a small amount of NH trilinear coupling $A'_\mu$ by allowing larger smuon masses. Apart from the above in relation to the effect of the combined constraints from B-physics and Muon $g-2$, one finds that NHSSM can accommodate the large $\tan \beta$ regimes (that naturally increases $a_\mu^{\text{SUSY}}$) easily in comparison with the MSSM scenario. Table 1 compares MSSM and NHSSM spectra for two benchmark points in detail. Both the points for NHSSM satisfy all the relevant constraints, whereas the corresponding MSSM points do not necessarily satisfy the same.

![Figure 4](image-url)  

**Figure 4.** Scattered plot of $m_{\tilde{\chi}^0_1}$ against $m_{\tilde{\mu}_1}$ for $\tan \beta = 10$ and 40 in MSSM for $\mu = 500$ GeV and $M_2 = 1500$ GeV. All the squark and stau mass parameters are set at 1 TeV along with choosing vanishing MSSM trilinear couplings except $A_t$ which is set at $-1.5$ TeV, favorable to have a correct Higgs boson mass. The Higgs mass constraint of Eq.3.1 is however not imposed. The blue, green and brown regions correspond to satisfying the muon $g-2$ constraint within $1\sigma$, $2\sigma$ and $3\sigma$ limits respectively.

Finally, we will comment on the possible effect of considering a negative sign of $\mu$. In regard to Sec.3.1 the part involving the radiative corrections to Higgs boson mass is controlled by $X_t = A_t - (\mu + A'_t) \cot \beta$. Hence for large $\tan \beta$ the contribution coming from the part involving $\mu$ and $A'_t$ is suppressed. On the other hand, for smaller values of $\tan \beta$ ($= 10$) the above contribution is relatively larger. However, since $A_t$ as well as $A'_t$ both are scanned with a larger range ($-3$ TeV to $3$ TeV) than what is used for $\mu$ (up to $1$ TeV), a negative $\mu$ would not lead to a much different result in the given plane of the figures. The conclusion is also similar for Sec.3.2 where both signs of $A_t$ and $A'_t$ are used with a larger range than what is used for $\mu$. For fine-tuning estimate given in Sec.3.3, the result would be essentially unchanged since the measure depends on $\mu$ quadratically unless $\tan \beta$ is small and $\mu$ is vanishingly small. In regard to $a_\mu^{\text{SUSY}}$ where a fixed sign of gaugino masses are used in our analysis of Sec.3.4, use of a negative $\mu$ would require appropriate values of $A'_\mu$ so as to satisfy the experimental data. This is via the contribution from the neutralino-smuon...
Figure 5. Scattered plot of $m_{\tilde{\chi}^0_1}$ against $m_{\tilde{\mu}_1}$ in NHSSM for the scanning ranges of Eq.3.8, with $\tan \beta = 10$ and 40 for $A'_{\mu} = 50$ GeV. The color scheme along with the relevant MSSM parameters are same as in Fig.4.

Figure 6. Scattered plot of $m_{\tilde{\chi}^0_1}$ against $m_{\tilde{\mu}_1}$ in NHSSM for the scanning ranges of Eq.3.8, with $\tan \beta = 10$ and 40 for $A'_{\mu} = 300$ GeV. The color scheme along with the relevant MSSM parameters are same as in Fig.4.

loops to $\alpha_{\mu}^{SUSY}$. 
Table 1. Benchmark points for NHSSM. Masses are shown in GeV. Only the two NHSSM benchmark points shown satisfy the phenomenological constraint of Higgs mass, dark matter relic density along with direct detection cross section, muon anomaly, \( \text{Br}(B \to X_s + \gamma) \) and \( \text{Br}(B_s \to \mu^+ \mu^-) \). The associated MSSM points are only given for comparison and do not necessarily satisfy all the above constraints.

| Parameters | MSSM | NHSSM | MSSM | NHSSM |
|------------|------|-------|------|-------|
| \( m_{1,2,3} \) | 472, 1500, 1450 | 472, 1500, 1450 | 243, 250, 1450 | 243, 250, 1450 |
| \( m_{Q_3}/m_{\tilde{Q}_3}/m_{D_3} \) | 1000 | 1000 | 1000 | 1000 |
| \( m_{Q_2}/m_{\tilde{Q}_2}/m_{D_2} \) | 1000 | 1000 | 1000 | 1000 |
| \( m_{Q_1}/m_{\tilde{Q}_1}/m_{D_1} \) | 1000 | 1000 | 1000 | 1000 |
| \( m_{L_3}/m_{\tilde{L}_3} \) | 2236 | 2236 | 1000 | 1000 |
| \( m_{L_2}/m_{\tilde{L}_2} \) | 592 | 592 | 1000 | 1000 |
| \( m_{L_1}/m_{\tilde{L}_1} \) | 592 | 592 | 1000 | 1000 |
| \( A_t, A_b, A_{\tau} \) | -1500, 0, 0 | -1500, 0, 0 | -1368.1, 0, 0 | -1368.1, 0, 0 |
| \( A'_{t}, A'_{\mu}, A'_{\tau} \) | 0, 0, 0 | 2234, 169, 0 | 0, 0, 0 | 3000, 200, 0 |
| \( \tan \beta \) | 10 | 10 | 40 | 40 |
| \( \mu \) | 500 | 500 | 390.8 | 390.8 |
| \( \mu' \) | 0 | -175 | 0 | 1655.5 |
| \( m_A \) | 1000 | 1000 | 1000 | 1000 |
| \( m_{\tilde{g}} \) | 1438.9 | 1439.1 | 1438.9 | 1438.9 |
| \( m_{\tilde{t}_1}, m_{\tilde{t}_2} \) | 894.4, 1151.2 | 865.5, 1154.9 | 907.8, 1154.9 | 907.8, 1154.9 |
| \( m_{\tilde{b}_1}, m_{\tilde{b}_2} \) | 1032.4, 1046.2 | 1026.3, 1045.1 | 1013.2, 1051.2 | 1013.2, 1051.2 |
| \( m_{\tilde{\mu}}, m_{\tilde{\mu}} \) | 596.4, 596.3 | 573.5, 595.9 | 502.0, 497.1 | 502.0, 497.1 |
| \( m_{\tilde{\tau}_1}, m_{\tilde{\tau}_2} \) | 2237.1, 2238.5 | 2237.1, 2238.5 | 985.4, 997.2 | 985.4, 997.2 |
| \( m_{\tilde{\chi}^\pm_1}, m_{\tilde{\chi}^\pm_2} \) | 504.2, 1483.6 | 677.6, 1484.7 | 244.6, 421.0 | 244.6, 421.0 |
| \( m_{\tilde{\chi}^0_1}, m_{\tilde{\chi}^0_2} \) | 488.6, 509.0 | 464.0, 680.6 | 231.3, 249.9 | 231.3, 249.9 |
| \( m_{\tilde{\chi}^0_3}, m_{\tilde{\chi}^0_4} \) | 522.6, 1483.5 | 683.2, 1484.7 | 1253.3, 1253.7 | 1253.3, 1253.7 |
| \( m_{H^\pm} \) | 1011.9 | 1005.8 | 955.7 | 955.7 |
| \( m_{H^0}, m_{h} \) | 1008.1, 121.4 | 984.8, 122.8 | 948.0, 122.4 | 948.0, 122.4 |
| \( \text{Br}(B \to X_s + \gamma) \) | \( 3.00 \times 10^{-4} \) | \( 3.01 \times 10^{-4} \) | \( 2.01 \times 10^{-4} \) | \( 4.05 \times 10^{-4} \) |
| \( \text{Br}(B_s \to \mu^+ \mu^-) \) | \( 3.40 \times 10^{-9} \) | \( 3.45 \times 10^{-9} \) | \( 5.06 \times 10^{-9} \) | \( 1.65 \times 10^{-9} \) |
| \( a_{\mu} \) | \( 1.94 \times 10^{-10} \) | \( 22.3 \times 10^{-10} \) | \( 34.8 \times 10^{-10} \) | \( 35.8 \times 10^{-10} \) |
| \( \Omega_{\chi^0_1 h^2} \) | 0.035 | 0.095 | 0.0114 | 0.122 |
| \( \sigma_{\chi^0_1 \chi^0_1} \) in pb | \( 4.01 \times 10^{-9} \) | \( 3.47 \times 10^{-10} \) | \( 6.79 \times 10^{-9} \) | \( 3.15 \times 10^{-12} \) |
4 Conclusion

In MSSM the superpotential is a holomorphic function of superfields and one considers soft SUSY breaking terms that are also holomorphic function of fields. However a SUSY theory devoid of an SM gauge singlet allows non-holomorphic soft SUSY breaking terms in the Lagrangian, and this has been used in various beyond the MSSM scenarios analyzing neutrino physics, leptogenesis, CP violation etc. In this analysis, we focus on the relevant phenomenological constraints while considering non-holomorphic soft SUSY breaking terms in a beyond the MSSM scenario with identical particle content as that of MSSM. Our work on Non-Holomorphic Supersymmetric Standard Model (NHSSM) uses electroweak scale input parameters similar to what is considered in the pMSSM model. This is unlike the previous analyses where only the non-holomorphic parameters were given at the electroweak scale while other soft parameters belonged to the grand unification scale or all the input parameters were given at the aforesaid scale.

We particularly analyze NHSSM specific effects on the Higgs mass radiative corrections, electroweak fine-tuning, electroweakino spectra, the constraint due to \( \text{Br}(B \to X_s + \gamma) \) and \( \text{Br}(B_s \to \mu^+ \mu^-) \) and the novel signature of NHSSM that enhances \( a_\mu^{\text{SUSY}} \) so that it can easily accommodate the Muon \( g-2 \) limits even for larger smuon masses or small \( \tan \beta \). In the context of radiative corrections to the Higgs boson mass it is seen that for \( \tan \beta = 10 \) and a given value of \( A_t \), the variation of NHSSM parameters, particularly due to \( A'_t \), the trilinear nonholomorphic parameter may easily cause a change in \( m_h \) by 2-3 TeV in either direction positive or negative. In other words, NHSSM does not necessarily require large values of \( |A_t| \) in order to produce the right amount of radiative corrections to Higgs mass, since both \( A_t \) and \( A'_t \) contribute toward the corrections. The radiative contributions to \( m_h \) due to \( A'_t \) is suppressed by \( \tan \beta \) leading to a quite small effect for \( \tan \beta = 40 \). However, for large \( \tan \beta \) NHSSM makes parameter space with large \( |A_t| \) to become valid via its effects on \( \text{Br}(B \to X_s + \gamma) \). We note that with squark and gluino masses assuming just above the LHC limits and \( \tan \beta = 40 \) the constraints from \( \text{Br}(B \to X_s + \gamma) \) in MSSM eliminates the large \( |A_t| \) zones altogether so that \( m_h \) goes below 122.1 GeV, the lower limit of Higgs mass considered in the analysis.

The non-holomorphic terms in NHSSM through their contributions toward \( \text{Br}(B \to X_s + \gamma) \) allows a significant part of the large \( |A_t| \) region to become valid. It is known that a higgsino type of LSP in MSSM that is supposed to satisfy the PLANCK data on DM relic density has a mass of around 1 TeV. In pMSSM this obviously increases the electroweak fine-tuning due to the sufficiently large value of \( \mu \). In contrast, NHSSM is able to produce a drastic reduction of the electroweak fine-tuning measure even for such a large mass of higgsino. The dependence of electroweak fine-tuning on \( \mu \) rather than \( \mu' \), the bilinear Higgs nonholomorphic parameter whereas the fact that electroweakino masses are related to the difference of \( \mu \) and \( \mu' \) indeed isolates the two sectors\(^{13} \). The electroweak fine-tuning can either decrease or increase depending on the relative contributions of \( \mu \) and \( \mu' \) to the difference \( \mu - \mu' \).

\(^{13}\)As mentioned before, we agree with the result of the recent analysis of ref.[37] in this regard.
Regarding the \((g - 2)_\mu\) constraint, NHSSM is able to significantly enhance \(\alpha_\mu^{\text{SUSY}}\) even for a small \(\tan \beta\) via a small value of the associated trilinear coupling parameter \(A'_\mu\). This is true even for a significantly large smuon mass \(m_{\tilde{\mu}}\). This is indeed a very novel feature of NHSSM. Just a small amount \(A'_\mu\) like 50 GeV even for a small \(\tan \beta\) (= 10) may significantly alter the MSSM predictions on lighter smuon mass that would satisfy the \((g - 2)_\mu\) constraint at 1\(\sigma\) level. This has the potential to cause a significant change in predictions involving SUSY models in general.

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