The neutral Higgs self–couplings in the (h)MSSM

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Abstract

We consider the Minimal Supersymmetric extension of the Standard Model in the regime where the supersymmetric breaking scale is extremely large. In this MSSM, not only the Higgs masses will be affected by large radiative corrections, the dominant part of which is provided by the third generation quark/squark sector, but also the various self–couplings among the Higgs states. In this note, assuming that squarks are extremely heavy, we evaluate the next-to-leading order radiative corrections to the two neutral CP–even Higgs self–couplings $\lambda_{HHh}$ and $\lambda_{hhh}$ and to the partial decay width $\Gamma(H \to hh)$ that are most relevant at the LHC. The calculation is performed using an effective field theory approach that resums the large logarithmic squark contributions and allows to keep under control the perturbative expansion. Since the direct loop vertex corrections are generally missing in this effective approach, we have properly renormalised the effective theory to take them into account. Finally, we perform a comparison of the results in this effective MSSM with those obtained in a much simpler way in the so–called hMSSM approach in which the mass value for the lightest Higgs boson $M_h = 125$ GeV is used as an input. We show that the hMSSM provides a reasonably good approximation of the corrected self–couplings and $H \to hh$ decay rate and, hence, it can be used also in these cases.

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1. Introduction

Dedicated analyses of the data collected at the LHC have so far shown excellent agreement between the observed 125 GeV Higgs boson \[1\] and the scalar particle that is predicted in the Standard Model (SM) of particle physics \[2\]. However, it is widely believed that this model is simply an effective theory valid at low energies and that new physics beyond it should manifest itself at a scale not too far from the TeV scale. This new physics should be thus probed at the LHC with higher luminosity and/or at future collider experiments where one should either discover direct evidence of new particles or detect some deviations from SM predictions.

In this respect, reconstructing the Higgs potential at the LHC and eventually at future high energy colliders is a major undertaking both on the experimental and theoretical sides \[3,4\]. The parameters involved in the Higgs potential feature relations among the Higgs masses and their self-couplings that are crucial to determine in order to fully understand the nature of the Higgs particle. Higgs boson pair production probes directly the triple Higgs self–coupling which, in the SM, is entirely fixed in terms of the Higgs mass and the vacuum expectation value. In models with extended Higgs sectors, the value of the self–coupling of the observed Higgs state can not only differ from the SM value but, in addition, other Higgs bosons can be exchanged in the processes in which this light state is doubly produced. Therefore, measuring a deviation in the pair production of the SM–like Higgs boson would point to a non-minimal Higgs sector and, hence, to physics beyond the SM.

Supersymmetry (SUSY) is a typical beyond the SM scenario possessing such an extended Higgs sector. As a matter of fact, in low energy SUSY scenarios, at least two Higgs doublet fields \(H_u\) and \(H_d\) are required to break the electroweak symmetry and to generate masses to the known gauge bosons and fermions. In its simplest incarnation, the Minimal Supersymmetric Standard Model (MSSM), the spectrum consists of five states \[5\]: two charged scalars \(H^\pm\), a CP-odd \(A\) and two CP-even \(h,H\) neutral scalars. The phenomenology of the Higgs sector is described entirely by two input parameters, one Higgs mass that is usually taken to be that of the pseudoscalar \(A\) boson \(M_A\) and the ratio \(\tan\beta\) of the vacuum expectation values of the two doublet fields, \(\tan\beta = v_u/v_d\). However, in the \((M_A,\tan\beta)\) parameter space, the prediction for the mass of the lightest (observed) Higgs boson is at odds with the measured value at the LHC, \(M_h \approx 125\) GeV, unless the large radiative corrections from the other SUSY sectors, most notably from the stop/top sector, are included to raise its mass at the desired value \(6,7\). This renders the MSSM parameter space survey a very complicated task and benchmark scenarios, such as those presented in Refs. \[8,9\], were designed to ease interpretation of the data.

Nevertheless, in a general MSSM framework, it is rather difficult to satisfy the constraint \(M_h \approx 125\) GeV in all cases and, to circumvent this shortcoming, a minimal and almost model independent approach, called the hMSSM \[10,11\], has been put forward. In this framework, by taking the measured mass value \(M_h \approx 125\) GeV as an input, one removes the dependence of the Higgs sector on the dominant radiative corrections and, hence, on the additional SUSY parameters. The hMSSM has been shown to provide a very good description of the MSSM Higgs mass spectra and the mixing angle \(\alpha\) in the CP–even Higgs sector \[9,11\]. As a bonus, it allows us to access the entire \((M_A,\tan\beta)\) parameter space without being in conflict with the LHC data. In particular the low \(\tan\beta\) regime can be probed, at the expense of assuming a very high SUSY scale \(M_{\text{SUSY}}\) such that the radiative corrections (that grow logarithmically with \(M_{\text{SUSY}}\)) allow the mass \(M_h\) to attain the value of 125 GeV.

In the MSSM, not only the Higgs masses and the mixing angle \(\alpha\) are affected by large radiative corrections, but also the various self–couplings between the Higgs states. In the case where the SUSY scale is extremely large, evaluating the radiative corrections through fixed-order perturbative calculations is seriously questionable and using Effective Field Theory (EFT)
methods seems a more appropriate approach; for a recent discussion see Ref. [12]. Indeed, in case of a large mass hierarchy between the SUSY and electroweak scales, EFT techniques resum the large logarithmic contributions and enable to keep under control the perturbative expansions. However, when such techniques are used to compute some physical processes such as decays or production rates, they may miss direct vertex “genuine” corrections, e.g. momentum dependent corrections, that are contained in the fixed-order diagrammatic calculation, which can be significant. This is particularly the case of the triple Higgs couplings and, especially, the decay $H \rightarrow hh$ that involve the self–coupling $\lambda_{Hhh}$ and in which the vertex corrections, e.g. involving top/stop loops, are important [9]. In the hMSSM approach, the effects of the large logarithmic corrections are captured in the neutral Higgs masses and mixing angle but, there also, the genuine vertex corrections should be included.

In this brief note, we combine EFT methods and fixed-order calculations to derive the next-to-leading order (NLO) corrections to the two neutral triple CP–even Higgs couplings $\lambda_{hhh}$ and $\lambda_{Hhh}$ and the rate of the decay mode $H \rightarrow hh$ that are most relevant at the LHC. These are first evaluated in an effective MSSM obtained from matching the full MSSM to an effective two–Higgs doublet model (2HDM) below the scale $M_{\text{SUSY}}$ (where the squarks have been integrated out) and that we renormalise to obtain ultraviolet finite results. We then estimate the size of the additional loop (vertex) corrections and compare with the hMSSM predictions to assess to which extent the two approaches differ. We show that the hMSSM approach provides a reasonably good approximation and thus can be used even in the case of the triple Higgs couplings and the rates for the double production of the $h$ state.

2. The Higgs self–couplings and $H \rightarrow hh$ at NLO

We first require the Higgs boson observed at the LHC to be the lightest Higgs scalar of the MSSM $h$ with a mass $M_h = 125$ GeV. At low $\tan \beta$ and moderate $M_A$ values, since the SUSY scale is defined to be the geometric average of the masses of the two stop partners of the heavy top quark, $M_{\text{SUSY}} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, multi-TeV stop squark masses are required to reach the Higgs mass value above as the leading radiative correction is of the form $M_t^4 \log(M_{t}^2/M_{\text{SUSY}}^2)$ with $M_t$ the top quark mass. Fixed-order NLO calculations performed in this regime are thus not reliable as they lead to too large radiative corrections. In the very large $M_{\text{SUSY}}$ regime, EFT methods lead to much more reliable results since the large logarithms induced by the multi-TeV stops masses are resummed and absorbed into effective couplings. The current state-of-the-art calculations regarding the radiative corrections to the MSSM Higgs boson masses and the mixing angle $\alpha$ combine fixed-order calculations and EFT techniques [7].

The calculation of the radiative corrections to the production and decay rates of the lightest Higgs particle in the MSSM are, in turn, less precise. This is particularly true for the important decay modes $H \rightarrow hh$ and $(gg \rightarrow h^*)$ $h^* \rightarrow hh$ that appear in double $h$ production and, thus, probe the CP–even neutral Higgs self–couplings. In this note, we will concentrate on the NLO radiative corrections to the on-shell partial decay width $\Gamma(H \rightarrow hh)$ and to the two Higgs self–couplings $\lambda_{Hhh}$ and $\lambda_{hhh}$ that are most relevant for LHC physics. We will work in an effective field theory obtained from matching the full MSSM to an effective two–Higgs doublet model (2HDM) below the scale $M_{\text{SUSY}}$, where the squarks have been integrated out. The matching is performed using the public code mhEFT and details about the procedure and the code in itself can be found in Ref. [12]. Nevertheless, for simplicity and as a first step, we do not fully match
the MSSM to a general 2HDM and we restrict ourselves to the following scalar potential,

\[
\mathcal{V} = m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + m_{12}^2(H_u \cdot H_d + h.c) + \frac{\lambda_1}{2}|H_d|^4 + \frac{\lambda_2}{2}|H_u|^4 + \lambda_3|H_d|^2|H_u|^2 + \lambda_4|H_u| \cdot |H_d|^2
\]  

(1)

where \(v\) is the “true” vacuum expectation value (vev), \(v = \sqrt{v_u^2 + v_d^2} \simeq 174\,\text{GeV}\). The SU(2) doublets \(H_d, H_u\) with hypercharge \(Y = \mp 1\), respectively, are given by

\[
H_d = \begin{pmatrix} v_d + (\phi_d - i\varphi_d)/\sqrt{2} \\ -\phi_d \end{pmatrix}, \quad H_u = \begin{pmatrix} v_u + (\phi_u + i\varphi_u)/\sqrt{2} \\ 0 \end{pmatrix},
\]  

(2)

In terms of the gauge boson masses which we will use as inputs, a tree-level matching with the MSSM would simply lead to the relations

\[
\lambda_1 = \lambda_2 = \frac{M_Z^2}{2v^2}, \quad \lambda_3 = \frac{2M_W^2 - M_Z^2}{2v^2}, \quad \lambda_4 = -\frac{M_W^2}{v^2}
\]  

(3)

Compared to the general 2HDM we have restricted ourselves only to \(\lambda_1, \ldots, \lambda_4\) quartic couplings but the model is still renormalisable. The reason behind this choice is that at tree-level in the MSSM, SUSY imposes that the \(\lambda_5, \lambda_6, \lambda_7\) quartic couplings are absent\(^4\). However it may well be that models of supersymmetry breaking can provide a direct contribution to these parameters but technically these contributions would not correspond to a soft-SUSY breaking mechanism, as is assumed in the MSSM. Of course, even in such a scenario these additional operators are generated through renormalisation group running but, being absent from the tree level potential, they give rise to finite subleading corrections and, in a first approximation, they can be neglected. In addition, we do not take into account possible higher-dimensional operators that could also modify the Higgs properties. For Higgs phenomenology, the multi-TeV stops masses affect mostly the renormalisation of the \(|H_u|^4\) operator and thus can be absorbed in a redefinition of its Wilson coefficient/coupling (the so-called “\(\lambda_2\)” parameter) using EFT techniques. Matching the full MSSM to an effective 2HDM will thus absorb the largest radiative corrections and NLO calculations within this theory will, a priori, lead to much more stable results.

Since we will be only interested in computing the by far dominant radiative corrections induced by third generation quarks/squarks to the Higgs decays modes that we are considering, the Yukawa sector of our EFT consists of a type II 2HDM; see Ref. \[13\] for example. In fact, the Yukawa Lagrangian below \(M_{\text{SUSY}}\) should be written as the most general Higgs-fermion Yukawa couplings, and not a type II-like Lagrangian. In particular, at large \(\tan \beta\), it is well-known that wrong Higgs coupling, namely couplings of \(H_u\) to down type fermions, are generated and can be significant. However, for the regime of \(\tan \beta\) that we are interested in (\(1 \leq \tan \beta \leq 10\) as will be seen later), these should not be numerically large and we can neglect them. The trilinear Higgs couplings that will be relevant for our analysis are the following,

\[
\lambda_{hhh} = -3\sqrt{2} \left(v_d s_3^3 \lambda_1 - c_\alpha \left(v_u c_\alpha^2 \lambda_2 + s_\alpha (-v_d c_\alpha + v_u s_\alpha (\lambda_3 + \lambda_4))\right)\right)
\]

(4)

\[
\lambda_{Hhh} = \frac{3v_d s_\alpha s_2 \lambda_1 + 3\sqrt{2}v_u c_\alpha^2 s_\alpha \lambda_2 + \frac{v_d c_\alpha (1 + 3(c_\alpha^2 - 3s_\alpha^2)) + v_u s_\alpha (1 - 3(2c_\alpha^2 + 2s_\alpha^2))}{2\sqrt{2}}}{\lambda_3 + \lambda_4}
\]

(5)

\[
\lambda_{Hhh} = -3\sqrt{2} v_d c_\alpha^2 s_\alpha \lambda_1 + \frac{3v_d s_\alpha s_2 \lambda_2}{\sqrt{2}} + \frac{v_d c_\alpha (1 + 3(c_\alpha^2 - 3s_\alpha^2)) - v_u s_\alpha (1 - 3(2c_\alpha^2 + 2s_\alpha^2))}{2\sqrt{2}}}{\lambda_3 + \lambda_4}
\]

(6)

\(^4\)As well as the bilinear terms \(m_{H_d/c}^2\) and \(m_{t2}^2\) which are only generated once SUSY is (softly) broken and essential for electroweak symmetry breaking.
where we use the abbreviations \( s_\alpha = \sin \alpha \) etc., with \( \alpha \) the mixing angle in the CP–even Higgs sector given by

\[
\tan 2\alpha = \frac{s_{2\beta}(2v^2(\lambda_3 + \lambda_4) - M_A^2)}{2v^2(\lambda_1 c_{2\beta}^2 - s_{2\beta}^2 \lambda_2) - M_A^2 c_{2\beta}} \quad \text{and} \quad \tan \beta = v_u/v_d
\]  

(7)

The only inputs that we need are the four parameters \( \lambda_1, \cdots, \lambda_4, \tan \beta \) and the pseudo–scalar mass \( M_A \). We obtain these parameters from the code \texttt{MhEFT} after running them down to the scale of the pseudoscalar mass \( M_A \). More precisely we set as input values \( \lambda_1, \cdots, \lambda_4 \equiv \lambda_1(M_A), \cdots, \lambda_4(M_A) \) and \( \tan \beta \equiv \tan \beta(M_A) \). As regards the renormalisation of the model, all fields and parameters introduced so far are considered as bare parameters. Shifts are then introduced for the Lagrangian parameters and the fields with the notation that a bare quantity is labeled as \( X_0 \). All bare quantities (\( X_0 \)) are then decomposed into renormalised (\( X \)) and counterterms (\( \delta X \)) quantities as \( X_0 \to X + \delta X \). The counterterms to \( \lambda_1, \cdots, \lambda_4 \) and \( \tan \beta \) will be defined in the \( \overline{\text{MS}} \) scheme. The divergent parts \( \delta \lambda_1^{\overline{\text{MS}}}, \cdots, \delta \lambda_4^{\overline{\text{MS}}} \) can be obtained from the beta functions given, for example, in the Appendix of Ref. [14], retaining only the top and bottom Yukawa contributions. The \( \tan \beta \) counterterm is defined by

\[
\frac{\delta t_{\beta}^{\overline{\text{MS}}}}{t_{\beta}} = \frac{3}{32\pi^2} (Y_b^2 - Y_t^2) C_{UV}
\]  

(8)

with \( C_{UV} = 1/\epsilon - \gamma_E + \ln(4\pi) \) and \( Y_f \) the Yukawa coupling of the corresponding fermion \( f \). The shifts on the doublet vevs \( v_u \) and \( v_d \) are related to the counterterm of \( \tan \beta \) through,

\[
(v_u)_0 \to v_u \left(1 - s_{2\beta}^2 \frac{\delta t_{\beta}^{\overline{\text{MS}}}}{t_{\beta}}\right), \quad (v_d)_0 \to v_d \left(1 + c_{2\beta}^2 \frac{\delta t_{\beta}^{\overline{\text{MS}}}}{t_{\beta}}\right)
\]  

(9)

The pseudoscalar Higgs mass is renormalised on-shell and we do not introduce a counterterm for the angle \( \alpha \) since we already consider it as a renormalised quantity, see Ref. [15] for more details. As explained in the latter reference, the renormalisation of the mixing between \( H \) and \( h \) still has to be performed and is transferred into a counterterm to the off-diagonal entry of the CP–even Higgs mass matrix, that we denote as \( \delta M_{hH}^2 \). This counterterm can be obtained from the following equation,

\[
\delta M_{hH}^2 = v^2 s_{2\alpha} \left(s_{2\beta}^2 \delta \lambda_2^{\overline{\text{MS}}} - c_{2\beta}^2 \delta \lambda_1^{\overline{\text{MS}}} \right) + v^2 c_{2\alpha} s_{2\beta} \left(\delta \lambda_3^{\overline{\text{MS}}} + \delta \lambda_4^{\overline{\text{MS}}} \right)
+ 2v^2 \left(s_{2\alpha} \left(c_{2\beta}^2 \lambda_2 - s_{2\beta}^2 \lambda_1 \right) + c_{2\alpha} s_{2\beta} \left(\lambda_3 + \lambda_4 \right)\right) \frac{\delta v}{v} \\
+ \left(4v^2 \left(2s_{2\alpha} c_{2\beta} \left(\lambda_1 + \lambda_2 \right) + c_{2\alpha} \left(\lambda_3 + \lambda_4 \right)\right) - \frac{M_A^2}{2} c_{2\alpha} \right) s_{2\beta} \frac{\delta t_{\beta}^{\overline{\text{MS}}}}{t_{\beta}} \\
+ c_{\beta - \alpha} \left(c_{\beta - \alpha}^2 - 3s_{\beta - \alpha}^2 + 3\right) \frac{\delta T_h}{4\sqrt{2} v} + s_{\beta - \alpha} \left(s_{\beta - \alpha}^2 - 3c_{\beta - \alpha}^2 + 3\right) \frac{\delta T_H}{4\sqrt{2} v}
\]  

(10)

To fully define this counterterm we need to determine the additional counterterms \( \delta v, \delta T_h, \delta T_H \) and we refer to Ref. [15] for their explicit definition. To completely remove the divergencies arising in the one-loop computation of the Higgs-to-Higgs decays, the three field renormalisation constants \( \delta Z_h, \delta Z_H \) and \( \delta Z_{hH/Hh} \) are needed and we again follow the on-shell prescription used in Ref. [15] to define them. We have now all the necessary ingredients to compute the one-loop finite corrections to the trilinear couplings we are interested in.
The $H \rightarrow hh$ partial decay rate is then given by

$$\Gamma(H \rightarrow hh) = \frac{|A(H \rightarrow hh)|^2}{32\pi M_H} \sqrt{1 - \frac{4M_h^2}{M_H^2}} \quad (12)$$

where the amplitude $A$ is the sum of the tree level plus one loop amplitudes $A = A_0 + A_1$. The amplitude $A_0$ is simply given by the coupling $\lambda_{Hhh}$ in eq. (5). The one-loop amplitude $A_1$ corresponds to,

$$A(H \rightarrow hh) = \left(1 + \delta Z_h + \frac{1}{2} \delta Z_H + \frac{\delta \lambda_{Hhh}}{\lambda_{Hhh}}\right) \lambda_{Hhh} + \frac{1}{2} \delta Z_{hh} \lambda_{hh} + \delta Z_{hh} \lambda_{HHh} + \Lambda_{hhhh}(M_h^2, M_h^2, M_h^2)$$

(13)

where $\Lambda_{hhhh}$ is the unrenormalised one-loop proper vertex. $\delta \lambda_{Hhh}$ is the counterterm to the coupling given by eq. (5), obtained after performing the shifts on the input parameters defining it. The two other trilinear Higgs couplings were given in eqs. (4),(6).

We define the finite one-loop correction to the triple $h$ self–coupling by,

$$\lambda_{hhh}^{(1)}(M_h^2, M_h^2, 4M_h^2) = \left(1 + \frac{3}{2} \delta Z_h + \frac{\delta \lambda_{hhh}}{\lambda_{hhh}}\right) \lambda_{hhh} + \frac{3}{2} \delta Z_{hh} \lambda_{HHh} + \Lambda_{hhhh}(M_h^2, M_h^2, 4M_h^2)$$

(14)

where the first line is for the counterterm contribution and the second line for the unrenormalised proper vertex correction. Again, the counterterm $\delta \lambda_{hhh}$ is obtained after performing the appropriate shifts on eq. (4).

To compute these one-loop observables our renormalisation program has been implemented in the SloopS code \[15,16\], to perform our numerical investigation, to which we now turn.

3. Numerical analysis

To perform our numerical analysis, we have implemented the Lagrangian defined by eq. (1) within SloopS, a code for the automated generation and evaluation of any cross section for any model. SloopS is an interface between two packages, the LanHEP program \[17\], where the Lagrangian of the model is defined as well as the one-loop shifts on the parameters and the bundle FormCalc/FeynArts/LoopTools \[18\] that we will call FFL for short. LanHEP automatically derives the Feynman rules of the model, including the counterterm contribution. The generated model files are then passed to the FFL bundle which takes care of computing at the one-loop level the observables. Some studies have already been performed with SloopS in Higgs phenomenology in the MSSM \[15\] and recently in the next-to-MSSM (NMSSM) \[19\].

Our procedure to compute the partial decay width and the one-loop correction to the triple $h$ couplings at NLO within the effective field theory that we defined in eq. (1) is the following. First, we perform a scan over the parameters $\tan \beta$ and $M_A$ with the code $MhEFT$ in order to obtain a mass for the lightest MSSM Higgs boson $M_H = 125 \pm 5 \text{GeV}$, where 5 GeV is assumed to be a very gross estimate of the theoretical uncertainty in the determination of this mass in the MSSM. These two parameters are scanned in the following ranges, $\tan \beta \in [1; 10]$ and $M_A \in [240; 600] \text{GeV}$. These intervals represent a regime where the decay $H \rightarrow hh$ is phenomenologically relevant. Indeed, far from the $t\bar{t}$ threshold, $M_H \gg 2M_t$, the decay mode $H \rightarrow t\bar{t}$ becomes largely dominant and all the other modes are then irrelevant.
With the help of the \texttt{MhEFT} code, we have performed two scans with different SUSY common mass scales, $M_{\text{SUSY}} = 10$ and 50 TeV and with all sfermion soft masses assumed to be equal to this value. The trilinear soft SUSY-breaking terms are set to $A_{\text{h/\tau}} = 5$ TeV while the trilinear soft SUSY-breaking term $A_t$ is defined through the stop mixing parameter $X_t = A_t - \mu \cot \beta$ and is set to $X_t/M_{\text{SUSY}} = \sqrt{6}$, corresponding to the so-called maximal mixing scenario. The higgsino mass parameter $\mu$ and the gaugino soft mass terms $M_1$ and $M_2$ which play a minor role have been set to $\mu = M_1 = M_2 = 2$ TeV. The parameters $\lambda_1, \cdots, \lambda_4$ are then extracted at the scale $M_A$ and fed into \texttt{SloopS} for the computation of the NLO correction to the trilinear Higgs couplings. The top Yukawa coupling is defined from the top pole mass which is set to $M_t = 172.5$ GeV and similarly for the bottom Yukawa coupling with $M_b = 4.62$ GeV. Since we work in an effective theory where all heavy sfermions have been integrated out, their loop contributions (in particular the $\hat{t}$ and $\hat{b}$ ones) are already encoded into the parameters $\lambda_1, \cdots, \lambda_4$. Thus, we only included the top and bottom contribution into the relevant loops for computing the NLO corrections as all other contributions are sub-dominant.

Our results are displayed in Figures 1 and 2 for the $H \to hh$ partial width and Figures 3 and 4 for the NLO correction to the triple $h$ coupling.

![Figure 1](image1.png)

*Figure 1*: Left: Decay width (in GeV) $\Gamma(H \to hh)$ computed at NLO in the $(M_A, \tan \beta)$ plane. Center: Relative size of the one-loop corrections to $\Gamma(H \to hh)$ in percent. Right: Relative difference in $\Gamma(H \to hh)$ between the predictions from the hMSSM approach in eq. (15) and our computational procedure. In all three cases, we set $M_{\text{SUSY}} = 50$ TeV.

The left panels in Figs. 1 and 2 represent the total NLO decay width $\Gamma(H \to hh)$ obtained with our computational framework. The central panels are for the relative one-loop corrections to the tree-level decay widths and in the right panels, we perform a comparison with the prediction for $\Gamma(H \to hh)$ derived from the hMSSM approach \cite{10,11}. In this approach the trilinear coupling of the heavy scalar to two light ones reads, in units of $M_Z^2/2v^2$ (in passing we also give the expression for the triple light Higgs coupling to be discussed below),

$$\lambda_{Hhh} = 2s_{2\alpha}s_{\alpha+\beta} - c_{2\alpha}c_{\alpha+\beta} + 3\frac{\Delta M_{22}^2}{M_Z^2}c_{2\alpha}c_{\alpha}$$

$$\lambda_{hhh} = 3c_{2\alpha}s_{\alpha+\beta} + 3\frac{\Delta M_{22}^2}{M_Z^2}s_{\alpha+\beta}$$

where $\Delta M_{22}^2$ is obtained from the known value of $M_h$:

$$\Delta M_{22}^2 = \frac{M_h^2(M_A^2 + M_Z^2 - M_h^2) - M_A^2M_Z^2c_{\beta}}{M_Z^2c_{\beta}^2 + M_A^2s_{\beta}^2 - M_h^2}$$

(17)
Both Figures 1 and 2 are limited from below and above by the constraint on the mass $M_h$ and exhibit qualitatively the same features. In Fig. 2, we see that lowering $M_{\text{SUSY}}$ down to 10 TeV allows for a larger $\tan\beta$ range than in Fig. 1 in which $M_{\text{SUSY}} = 50$ TeV. Nevertheless this enables us to probe smaller values of $\tan\beta$.

Let us first comment on the total decay width $\Gamma(H \rightarrow hh)$ which is displayed in the left panels of Fig. 1 and 2. In both cases we observe that $\Gamma(H \rightarrow hh)$ is favored for low values of $\tan\beta$ and $M_A$ and decreases more steeply with respect to increasing values of $\tan\beta$ than with respect to $M_A$. The panels in the center of Fig. 1 and 2 show that our perturbative calculation is relatively under control, with the NLO corrections reaching their maximum for low values of $\tan\beta$ and higher values for $M_A$. Although the radiative corrections are substantial, they are much more reasonable than if we had computed $\Gamma(H \rightarrow hh)$ in the plain MSSM. Indeed, in this case, they can reach several hundreds of percent, jeopardizing the validity of a pure fixed-order calculation in this regime of low $\tan\beta$. In the right panels where we performed a comparison between our approach and the hMSSM one, we can see that both predictions agree well and the hMSSM indeed captures the bulk of the radiative corrections in this regime of low $\tan\beta$ values and moderate $M_A$ value for the $H \rightarrow hh$ decay.

We next turn to the discussion of the one-loop corrections to the triple $h$ coupling, where our results are displayed in Fig. 3 and Fig. 4. We only present the relative one-loop corrections on the left panel and in the right panel we perform a comparison with the prediction from the hMSSM obtained from eq. (16).

The results for this coupling exhibit the same features as for the $H \rightarrow hh$ decay rate. Here also, the effective approach allows to keep the radiative correction under control in the region where the heavy Higgs decay is phenomenologically interesting, i.e. low $\tan\beta$ and $M_H \lesssim 350$ GeV. For both decays we observe that the size of the loop corrections grow mainly with the pseudoscalar mass $M_A$ and are almost independent of $\tan\beta$. Again, the main visible difference between Fig. 3 and Fig. 4 is that for the former lower values can be probed because of the light Higgs mass constraint. The right panels of Fig. 3 and Fig. 4 display the comparison between the hMSSM prediction and the one obtained from the procedure detailed previously. In this case also, the hMSSM prediction is very close to the more complete calculation performed here.
Figure 3: Left: Relative size of the one-loop corrections to $\lambda_{hhh}$ in percent. Right: Relative difference in $\lambda_{hhh}$ between the predictions from the hMSSM approach in eq. (16) and our computational procedure. In both cases, we set $M_{\text{SUSY}} = 50$ TeV.

Figure 4: Same as in Fig.3 except that $M_{\text{SUSY}} = 10$ TeV.
4. Conclusion

In this note, we have considered the neutral Higgs boson self-couplings $\lambda_{hhh}$ and $\lambda_{Hhh}$ and performed a comparison of their predicted values in both the hMSSM and an effective MSSM approaches, in a regime where the SUSY scale $M_{\text{SUSY}}$ is extremely large. The use of an effective MSSM theory instead of the full MSSM is necessary at very high $M_{\text{SUSY}}$ as large logarithmic corrections involving this scale and corresponding to the squark masses appear and need to be resummed in order to obtain reliable results. To this purpose, we have matched the full MSSM to an effective theory which corresponds to a restricted (but still rather general) renormalisable two-Higgs-doublet model of type II. To include also the genuine direct or vertex corrections due to the third generation quarks, which are absent in a renormalisation group improved calculation, we have renormalised the effective theory in the $\overline{\text{MS}}$ scheme.

We have then shown that the NLO radiative corrections are well under control but they can be substantial for large $M_A$ values. Nevertheless for the partial width of the process $H \rightarrow hh$, the corrections are small in the regions where the decay is phenomenologically relevant. The comparison with the hMSSM predictions for the neutral self-couplings revealed that this simple approach still provides a reasonably good approximation (the deviations are smaller than 10% and are even less in general) and, hence, the hMSSM approach can be used not only to determine the MSSM Higgs masses and the mixing angle $\alpha$, as shown in previous studies, but also to evaluate these Higgs self-couplings.

In a future work, we plan to refine our analysis by including in the effective MSSM theory the subleading contributions of the gauge and scalar bosons and of the gauginos and higgsinos. We will also improve our predictions by matching the full MSSM to a general 2HDM, thereby including also the subleading contributions coming from the $\lambda_{5,6,7}$ Lagrangian parameters and their renormalisation. In addition, an extension of the analysis to the other Higgs self-couplings, in particular those involving the pseudoscalar and charged Higgs states, is foreseen.

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