CMB photons travel from the last scattering surface, when the primary CMB has been generated, along the surface of the light cone to us. During their travel, they are affected by many secondary effects such as the integrated Sachs-Wolfe effect and CMB lensing. These CMB secondary effects modify the CMB primary power spectrum adding degeneracies and decreasing the sensibility to primordial parameters. The possibility to reconstruct the primary CMB anisotropies will allow us to have a more direct observable to test the physics of the early universe.

We propose to study the imprint of features in the primordial power spectrum with the primary CMB after the subtraction of the reconstructed ISW signal from the observed CMB temperature angular power spectrum. We consider the application to features models able to fit two of the large scales anomalies observed in the CMB temperature angular power spectrum: the deficit of power at \( \ell \sim 2 \) and at \( \ell \sim 22 \).

This method allows to improve significantly the constraints on the features parameters up to 16\% for models predicting a suppression of power of the quadrupole and up to 27\% for models with features at \( \ell \sim 22 \), assuming instrumental sensitivity similar to the Planck satellite (depending on the goodness of the ISW reconstruction). Furthermore, it gives the opportunity to understand if these anomalies are attributed to early- or late-time physics.

I. INTRODUCTION

Although observations show how a spatially flat \( \Lambda \)CDM model with a tilted power-law spectrum of primordial density fluctuations provides a good fit to CMB temperature and polarization anisotropies [1], there are interesting hints for new physics beyond the \( \Lambda \)CDM model based on slow-roll inflation in the WMAP [2] and Planck data [3, 4], such as anomalies in the large angular scale pattern of CMB temperature anisotropies [2–18].

Anomalies in the CMB angular power spectra, as well as in the dark matter power spectrum, are predicted by several theoretically well motivated mechanisms that occur during inflation; these mechanisms support deviations from a simple power-law for the primordial power spectrum, connected with the violation of the slow-roll phase, and provide a better fit to the CMB data at \( \sim 2\sigma \).

In Fig. 1 it is plotted the comparison between the best-fit CMB temperature power spectrum for the standard \( \Lambda \)CDM model and the best-fits for some features models [4] which improve the fit of CMB data. Although the difference between these models, the cosmic-variance restricts our ability to discriminate between them even with a perfect measure of the CMB anisotropies.

The situation improves if well-suited data in addition to the CMB temperature anisotropies are available:

- CMB \( E \)-mode polarization have been highlighted as a possible way to constrain primordial features with high confidence thanks to the narrower transfer functions compared to the ones of the CMB temperature [19–22].

- The opportunity to look elsewhere for the imprint of primordial features, as in the matter power spectrum, is a unique chance to improve our current understanding of these possible anomalies; see for instance [29, 32].

- Combined search for primordial features in the power spectrum and bispectrum is another promising way to test such models thanks to the imprints on higher-order correlators [28, 33, 38].

In this paper, we propose a new further method to im-
prove the current understanding of the large scales CMB anomalies based on the possibility to subtract the reconstructed integrated Sachs-Wolfe (ISW) signal from the observed CMB temperature angular power spectrum in order to constrain models with features in the primordial power spectrum with the primary CMB. After subtracting the ISW signal, possible by cross-correlating CMB maps with tracer maps of the matter density fluctuation, we have the opportunity to test the CMB angular power spectrum dominated by the SW contribution at the largest scales like the primary CMB signal generated at the last scattering surface.

ISW, such as CMB lensing deflection, can be considered as foreground contribution to the primary CMB signal. They can be used to further study the information content from some late-time physics (dark energy and small scales matter perturbation for instance), but they also can play an important role in evolving gravitational potential wells associated with overdensities and lose energy on passing through underdensities [39]. This effect mainly contributes to large angular scales and therefore can be used to study the Hubble radius during radiation domination.

The ISW contribution to the CMB temperature fluctuations in direction \( \hat{n} \) is a secondary anisotropy in the CMB caused by the passage of CMB photons through evolving gravitational potential wells

\[
\frac{\delta T_{\text{ISW}}}{T} (\hat{n}) = - \int dz \ e^{-\tau(z)} \left[ \frac{d\Phi}{dz}(\hat{n}, z) + \frac{d\Psi}{dz}(\hat{n}, z) \right],
\]

where \( \Phi \) and \( \Psi \) are the gravitational potentials in the longitudinal gauge and \( e^{-\tau(z)} \) is the visibility function. On large scales, in a dark-energy-dominated universe, CMB photons gain energy when they pass through the decaying potential wells associated with overdensities and lose energy on passing through underdensities [39]. This effect mainly contributes to large angular scales and therefore at low multipoles, i.e. \( \ell \lesssim 100 \), since there is a little power in the potentials at late times on scales that entered the Hubble radius during radiation domination.

\[ C_{\ell}^{\text{primary}} = C_{\ell}^{TT} - C_{\ell}^{\text{ISW}}, \]

\[ \mathcal{N}_{\ell}^{\text{primary}} = \mathcal{N}_{\ell}^{TT} + \mathcal{N}_{\ell}^{\text{ISW}}, \]

where \( \mathcal{N}_{\ell}^{\text{ISW}} \) is the noise of the ISW angular power spectrum after the reconstruction.

ISW is generally reconstructed by cross-correlating CMB temperature angular power spectrum with LSS galaxy surveys [40] or other LSS tracers such as CMB lensing [41], termal Sunyaev-Zeldovich [42], intensity mapping emission lines [43] and clusters of galaxies [44].

In order to quantify the errors from the reconstruction of the ISW signal by cross-correlating the CMB with one or more LSS tracers, we consider the standard theoretical signal-to-noise ratio (defined according to [45, 46]) to build the noise angular power spectra for three different cases: a 3σ level reconstruction of the ISW signal, compatible with the significance obtained in [47] by cross-correlating the Planck temperature map with a compilation of publicly available galaxy surveys [47]; a 6σ significance expected for next-generation of LSS galaxy surveys [43, 45]; an ideal case with a perfect reconstruction (∼ 10σ) of the late-time ISW signal with \( \mathcal{N}_{\ell}^{\text{ISW}} \approx 0 \) in Eq. [3].

### III. FISHER FORECAST FORMALISM

With these definitions in hand, we can proceed to perform a Fisher matrix analysis for CMB angular power spectra (temperature and E-mode polarization) [49–53]

\[ \mathcal{F}_{\alpha\beta}^{\text{CMB}} = \frac{1}{2} \text{tr} \left[ C_{\alpha\beta} C^{-1} C_{\beta\alpha} C^{-1} \right], \]

where

\[ C = \begin{bmatrix} C_{\ell}^{TT} & C_{\ell}^{TE} \\ C_{\ell}^{TE} & C_{\ell}^{EE} \end{bmatrix}, \]

Here \( C_{\ell}^{X} \) is the sum of the theoretical spectrum \( C_{\ell}^{X} \) and the effective noise \( N_{\ell}^{X} \), which is given by the inverse noise weighted combination of the instrumental noise deconvolved with the beams of different frequency channels. For the temperature and polarization angular power spectra, a noise power spectrum with Gaussian beam profile [49] has been used

\[ N_{\ell}^{X} = \sigma_{X} b_{\ell}^{-2}. \]

Here \( b_{\ell}^{2} \) is the beam window function, assumed Gaussian, with \( b_{\ell} = e^{-\ell(\ell+1)/2\theta_{\text{FWHM}}^{2}/16\ln 2} \); \( \theta_{\text{FWHM}} \) is the full width half maximum (FWHM) of the beam in radians; \( \sigma_{T} \) and \( \sigma_{P} \) are the square of the detector noise level on a steradian patch for temperature and polarization, respectively.
IV. MODELS OF FEATURES IN THE PRIMORDIAL POWER SPECTRUM

We consider three inflation models that generate features in the primordial power spectrum (see Fig. 1): the cutoff model \([54]\), which reproduces a suppression of power at large scales, and two models which lead to localized features in the primordial power spectrum, i.e. the kink model \([55]\) and the step model \([56]\). Following \([27]\), the fiducial spectra are centred at their best-fit parameters from Planck 2015 TT+lowP data \([4]\) for each parameterization. The primordial power spectrum can be written as the standard power-law \(P_{R,0}\), modulated by the contribution dues to the violation of slow-roll

\[
P_R(k) = P_{R,0}(k) \cdot P_{R,\chi}(k),
\]

\[
P_{R,0}(k) = A_k \left( \frac{k}{k_*} \right)^{n_s-1},
\]

where \(A_k\) is the amplitude of the curvature power spectrum, \(n_s\) is the scalar spectral index and the pivot scale is fixed at \(k_* = 0.05\) Mpc\(^{-1}\).

The non-canonical contribution to \(P_R\) for the cutoff model is given by

\[
P_{R,\text{cutoff}}(y) = 1 - e^{-y^{c}},
\]

\[
y \equiv \frac{k}{k_c},
\]

for the kink model by

\[
P_{R,\text{kink}}(y) = 1 + \frac{9}{2} A_{\text{kink}}^2 \left( \frac{1}{y} + \frac{1}{y^3} \right)^2
\]

\[
+ \frac{3}{2} A_{\text{kink}} \left( 4 + 3 A_{\text{kink}} - 3 \frac{A_{\text{kink}}}{y^4} \right)^2 \frac{1}{y^2} \cos(2y)
\]

\[
+ 3 A_{\text{kink}} \left( 1 - \frac{1 + 3 A_{\text{kink}}}{y^2} - \frac{3 A_{\text{kink}}}{y^4} \right)^2 \frac{1}{y} \sin(2y),
\]

\[
y \equiv \frac{k}{k_{\text{kink}}},
\]

and for the step model by

\[
P_{R,\text{step}}(y) = \exp \left\{ I_0(y) + \ln \left[ 1 + I_1^2(y) \right] \right\},
\]

\[
y \equiv \frac{k}{k_{\text{step}}},
\]

where the first- and second-order parts are

\[
I_0(y) = A_{\text{step}} W'(y) D \left( \frac{y}{x_{\text{step}}} \right),
\]

\[
\sqrt{2} I_1(y) = \frac{\pi}{2} \left( 1 - n_s \right) + A_{\text{step}} X'(y) D \left( \frac{y}{x_{\text{step}}} \right),
\]

where a prime denotes d/d\(\ln y\), and the damping envelope is

\[
D(y) = \frac{y}{\sinh y}.
\]

FIG. 2. Derivatives of the CMB temperature angular power spectrum with respect to the features parameters for the cutoff (top panels), kink (central panels), step (bottom panels) models. The red lines refer to the derivative of the full observed CMB temperature angular power spectrum and the green ones refer to the derivative of the primary spectra.

The window functions are

\[
W(y) = \frac{3 \sin(2y)}{2y^3} - \frac{3 \cos(2y)}{y^2} - \frac{3 \sin(2y)}{2y},
\]

\[
X(y) = \frac{3}{y^3} \left( \sin y - y \cos y \right)^2.
\]

See Refs. \([27] [54] [56]\) for a clear description of the models.
After the ISW removal, the signal of the CMB temperature anisotropies decreases at the large angular scales. On these scales where the instrumental noise is negligible, this effect is compensated by an effective lower cosmic-variance since \( \sqrt{2/(2\ell + 1)} C_\ell \), i.e.

\[
\frac{C_\ell^{TT}}{C_\ell^{TT} + N_\ell^{TT}} \approx \frac{C_\ell^{primary}}{C_\ell^{primary} + N_\ell^{primary}},
\]

assuming a negligible \( N_\ell^{ISW} \). On the other hand, the primary CMB is more sensitive to the variation of the cosmological parameters connected with the primordial power spectrum. It is possible to see this effect by looking at the derivatives of the CMB temperature anisotropies. In Fig. 2, the derivatives of the CMB primary anisotropies respect to the features parameters are always bigger in amplitude compared to the derivatives of the observed CMB temperature anisotropies.

We consider two different configurations of current CMB experiment: a representative of current CMB measurements by considering the Planck 143 GHz channel full mission sensitivity and angular resolution as given in \( f_{sky} = 0.7 \) and a CMB cosmic-variance limited experiment, both with \( f_{sky} = 0.7 \). Results are collected in Tab. II.

Assuming a perfect reconstruction of the ISW, we found that for the cut-off model the errors decrease by 6% on \( \lambda_c \) and by 16% on \( \log_{10} (k_c \text{ Mpc}^{-1}) \) for an experiment with Planck's sensitivity. For the kink model the errors improve by a 17% on \( A_{kink} \) and by 10% on \( \log_{10} (k_{kink} \text{ Mpc}^{-1}) \). The step model is the one that benefits more from this method, the errors improve by a 27% on \( A_{step} \), by 25% on \( \log_{10} (k_{step} \text{ Mpc}^{-1}) \) and by 19% on \( \ln (x_{step}) \).

The case with the subtraction of the 3\( \sigma \) and of the 6\( \sigma \) detected ISW does not lead to any improvements for both the cut-off and the kink models. Instead, for the step model there is still a reduced improvement of 5% on the amplitude and 10% on the scale parameter, even for these cases with injected noise from the ISW reconstruction.

Feature models which predict departures from the standard power-law primordial power spectrum will benefit from having better measurements of large angular scale CMB E-mode polarization at the cosmic-variance level. However, the cut-off model affects the largest angular scales reproducing a suppression of power at \( \ell < 30 \) in temperature and \( \ell < 10 \) in the E-mode polarization. For this reason, the relative improvement does not change when we consider a cosmic-variance limited CMB experiment. The instrumental noise on the E-mode polarization is small even for Planck on such scales. In this case the improvement from the subtraction of the ISW signal is very small, \( \sim 5\% \), even for the case of perfect ISW reconstruction for all the three considered models.

**VI. CONCLUSION**

In conclusion, this method shows an alternative way to improve the constraints for features models even without better measurements of CMB polarization and this approach performs well for the step model which fits the deficit in power at \( \ell \approx 20 \sim 30 \), improving the contraints by 5 – 27% on the amplitude and by 10 – 25% on the scale parameters.

Finally, even if the final improvement for realistic cases of ISW subtraction could lead to small differences in terms of constraining power on the parameters of these features models, the subtraction of the ISW signal could lead to a change in the pattern of the largest scales of the CMB temperature anisotropies changing the shape of the features. For instance, if an anomaly vanishes after the subtraction of the ISW signal to the CMB temperature, then a primordial explanation would be eliminated.

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| Model    | Parameters          | full CMB | 3\( \sigma \) ISW | 6\( \sigma \) ISW | primary CMB |
|----------|---------------------|---------|-----------------|-----------------|-------------|
| cut-off  | \( \lambda_c \)     | 0.218/0.176 | 0.340/0.234 | 0.264/0.196 | 0.204/0.162 |
|          | \( \log_{10} (k_c \text{ Mpc}^{-1}) \) | 0.371/0.325 | 0.564/0.440 | 0.418/0.357 | 0.310/0.280 |
| kink     | \( A_{kink} \)     | 0.0466/0.0334 | 0.0779/0.0430 | 0.0534/0.0360 | 0.0387/0.0296 |
|          | \( \log_{10} (k_{kink} \text{ Mpc}^{-1}) \) | 0.0962/0.0530 | 0.129/0.0564 | 0.105/0.0549 | 0.0866/0.0529 |
| step     | \( A_{step} \)     | 0.257/0.125 | 0.344/0.130 | 0.247/0.126 | 0.187/0.120 |
|          | \( \log_{10} (k_{step} \text{ Mpc}^{-1}) \) | 0.0368/0.0165 | 0.0418/0.0174 | 0.0336/0.0168 | 0.0275/0.0160 |
|          | \( \ln (x_{step}) \) | 0.362/0.189 | 0.471/0.199 | 0.364/0.192 | 0.293/0.183 |

TABLE I. 68% constraints on the features parameters for a Planck-like CMB experiment (left) and a cosmic-variance limited one (right). Constraints are given for the standard case (full CMB) and after ISW subtraction by considering different levels of ISW detection.
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