Peltier effect induced longitudinal resistivity of ideal 2D electron(hole) gas in strong magnetic field

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We demonstrate that in strong quantum limit the thermoelectric Peltier effect could define the longitudinal resistivity of dissipationless two-dimensional electron(hole) gas. The current results in heating(cooling) at first(second) Hall bar sample contact due to Peltier effect. At small current the contacts temperatures are different, the temperature gradient is linear on current. The voltage swing downstream the current is proportional to Peltier effect induced thermopower. As a result, nonzero longitudinal resistivity is measured in experiment. The above effect could exist in 3D case.

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The understanding of electronic transport in solid states subjected into high magnetic fields has undergone a profound revision after the discovery of integral [2] and fractional quantum Hall effect [5] for two-dimensional electron gas( 2DEG ). Both phenomena manifest extraordinary transport behavior as the temperature approaches to zero. The Hall resistivity, \( \rho_{xy} \), is quantized to \( \hbar/e^2 \) with \( i \) being either an integer or rational fraction while longitudinal magnetoresistivity, \( \rho_{xx} \), vanishes.

The main goal of this paper is to present a thermodynamic equilibrium the chemical potential, \( \mu \), for the system conductors+2DEG is constant. Moreover, at small perturbation of thermodynamic equilibrium due to current the chemical potential remains constant. Thus, we conclude that 2D electron gas is, in fact, non-isolated. We argue that an external reservoir of electrons, if exists, could provide the pining of 2DEG chemical potential. Once supposed that \( \mu \) is fixed, it could be shown that 2DEG concentration, \( N \), changes with magnetic field. Indeed, at \( T \to 0 \) one needs \( N = i \Gamma \) electrons in order to occupy \( i \) levels, therefore, the transverse conductivity is given by \( \sigma_{yx} = N e c / H = i e^2 / h \). Accordingly, the external reservoir must offer the possibility of changing \( N \) over sufficiently wide limits in order to achieve observable widths of \( \sigma_{yx} \) plateau. The above idea was put forward first in Ref. [9] where ionized donors at a right distance to heterojunction assumed to serve as such reservoir. Then, the surface states shown in Ref. [7] could play the same role in MOS structures. At a moment, above scenario is abandoned because of Ref. [8] in which was underlined insufficient strength of reservoir in both models. Nevertheless, the experimental results found in Ref. [9] are consistent with reservoir conception. We argue that thermodynamics based argumentation done above is general, thus, we use reservoir idea assuming that 2DEG chemical potential is fixed. The detailed analysis of actual reservoir will be done elsewhere.

Let us consider Hall bar geometry sample(Fig.1) under dc current carrying conditions. The sample is connected by means of two identical extra leads to the current reservoir will be done elsewhere.

Let us consider 2D electron gas in x-y plane(Fig.2) subjected into perpendicular magnetic field \( B \). The 2DEG structure ( MOS, quantum well etc.) is arbitrary, the electrons assumed to occupy the first size quantization subband. Neglecting spin splitting, the energy spectrum is \( \varepsilon_n = \hbar \omega_c (n + 1/2) \),where \( n = 0, 1,... \) is Landau level (LL) number, \( \omega_c = e B / m c \) is the cyclotron frequency, \( m \) is the electron effective mass. For simplicity, we omit LL broadening, thus, the LLs density of states is \( \Gamma = e B / \hbar c \). We will further restrict ourselves within strong quantum limit \( \hbar \omega_c >> kT \). If the case, one could neglect any random short(long) range impurity scattering [4], hence, consider dissipationless( ideal ) 2DEG. Then, for actual \( T \to 0 \) limit we disregard any phonon related effects(\( \sim T^3 \)) in both the bulk and 2DEG.

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\( e \varphi \) is the electrochemical potential, \( \alpha \) is the 2DEG thermopower. For dissipationless 2DEG in strong quantum limit \( \hbar \omega_c \gg kT \) the conductivity tensor, \( \sigma \), is purely off-diagonal. Hence, \( \sigma_{yx} = 1/\rho_{yx} = e^2/h \sum_n f(\varepsilon_n) \) is the transverse conductance at finite temperature, \( f(\varepsilon) \) is the Fermi function. Then, in Eq. (1) \( \kappa \) is the electron related thermal conductivity, \( L = \pi e^2/2k \) is the Lorentz number. Note, in Eq. (1) the diamagnetic contributions satisfy Einstein and Onsager relationships. We emphasize that Eq. (1) could be derived (see Appendix A) by means of drift approach developed in Ref. \([13]\), \([14]\) for 3D case.

With the help of Eq. (1), the current components are:

\[
\begin{align*}
  j_x &= \sigma_{xy} E_y, \\
  j_y &= \sigma_{yx} (E_x - \alpha \nabla_x T) = 0.
\end{align*}
\]  

The longitudinal current, \( j = j_x \), represents the flux of electrons in crossed fields with the drift velocity \( v_d = eE_y/B \), where \( E_y \) is the Hall electric field. The transverse electron flow caused by longitudinal electric field, \( E_x \), is mutually compensated by the one because of temperature gradient, hence \( j_y = 0 \).

We now find out the longitudinal temperature gradient \( \nabla_x T \) caused by Peltier effect. We recall that Peltier effect is generated by the current crossing the contact of two different conductors. At the contact (let say “a” in Fig. (1)) the temperature at the ends of the contacts is heated while the contact “b” is cooled. The contact “a” gains the energy \( \Delta \alpha T \) the leads keeping in mind 2DEG adiabatic cooling conditions. Using Eq. (1), the temperature gradient is given

\[
\frac{dT}{dx} = -\frac{j \Delta \alpha T}{\kappa_{yx}},
\]

thus, linear on current. The voltage swing, \( U \), measured between the ends ”c” and ”d” yields

\[
U = \int_c^d E_x dx = \int_c^d \alpha dT = \Delta \alpha (T_a - T_b).
\]

Here, we disregard the conductors resistances. Finally, the Peltier effect related resistivity, \( \rho = U/jl \), could be written as follows

\[
\rho = \frac{\alpha^2}{\sigma_{yx} (L - \alpha^2)},
\]

where we take into account that for actual case of metallic leads \( \Delta \alpha \approx -\alpha_2 = -\alpha \), then \( l \) is the sample length. Note, Eq. (3) is valid for standard 4-probe Hall bar measurements, 2DHG (Fig. 1, insert b) and in 3D case.

We recall, in strong quantum limit 2DEG thermopower is the universal thermodynamic quantity which is proportional to entropy per one electron: \([15]\), \([16]\).

\[
\alpha = -\frac{8}{e} \int \frac{d^2 \mathcal{N}(\mathcal{E}) f'(\varepsilon)}{\ln 2},
\]

Fig. 1, insert a). Consequently, \( j || q_b = -\kappa_{yx} \nabla_x T \) at the contacts. In Eq. (3) we neglect thermal transfer within the leads keeping in mind 2DEG adiabatic cooling conditions. Using Eq. (1), the temperature gradient is given

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We remind that the total energy flux is continuous at both ”a” and ”b” contacts. Using Eq. (1) one obtains

\[
\begin{align*}
  -\kappa_{yx} \frac{dT}{dx} &= -j T_a (\alpha_2 - \alpha_1), \\
  \kappa_{yx} \frac{dT}{dx} &= -j T_b (\alpha_1 - \alpha_2).
\end{align*}
\]

Here, we take into account that current is known to enter and leave the sample at two diagonally opposite corners (Fig. 1, insert a). Consequently, \( j || q_b = -\kappa_{yx} \nabla_x T \) at the contacts. In Eq. (3) we neglect thermal transfer within the leads keeping in mind 2DEG adiabatic cooling conditions. Using Eq. (1), the temperature gradient is given

\[
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\]

Here, \( \Omega(\mu, T, B) = -kT \sum \ln (1 + \exp((\mu - \varepsilon_n)/kT)) \) is the thermodynamic potential density. For given \( \mu \) thermopower is the universal function of reduced temperature, \( \xi = kT/\mu \), and dimensionless magnetic field \( \hbar c/\mu = \nu^{-1} \), where \( \nu \) is so-called filling factor. Using Eq. (3) we derived (see Appendix B) the asymptotic formulae for 2DEG thermopower which is valid within low temperature and low magnetic field limit \( \nu^{-1}, \xi < 1 \).

Let us now discuss the features of resistivity given by Eq. (1). We underline that \( \rho \) could be expressed in fundamental units \( h/e^2 \) because \( \Delta \alpha \approx -\alpha \sim k/T \), hence, \( \rho \sim 1/\sigma_{yx} \). Then, according to Eq. (1), \( \rho \) is the universal function of \( \nu, \xi \). In Fig. 2 the resistivity \( \rho \) and off-diagonal component \( \rho_{yx} \) vs \( \nu^{-1} \sim B \) are displayed. When the chemical potential lies between the LLs, \( \Delta \alpha \sim \nu^{-1} \), resistivity \( \rho \) is thermally activated with an activation energy being of the order of magnetic energy. Therefore, near LLs \( \nu^{-1} \) dependence has a series of large peaks. At \( T \rightarrow 0 \) the magnitudes of thermopower and \( \rho \) at half-filled LL \( \nu = 1/2, 3/2, \ldots \) approach the universal values \( \alpha_c \) and \( \rho^c \) independent of the temperature, electron effective mass etc. Note, \( \gamma < 1 \) for fillings \( \nu_c > \sqrt{3}(\ln 2/\pi) = 0.38 \)
i.e. up to half-filled first LL. Then, for higher magnetic fields \( \gamma \approx 1 \) and our basic approach \( T_a, T_b \approx T_0 \) becomes invalid. We underline that at critical fillings \( \nu_c \) the transverse resistivity approaches the universal value, 
\[
\rho_{yx}^c = \frac{\hbar}{4c} \frac{\nu_c}{\nu} \rho_V^c,
\]
as well. Using Eq.(9) one could demonstrate that for \( \nu < \nu_c \) the thermal correction could be presented in the form \( \rho = \rho^c \exp(-\Delta \nu / \nu_0) \), where \( \Delta \nu / \nu_0 \ll 1 \). Here, \( \nu_0(\xi) = 4\nu_0^c / 3 \) is the sample and temperature dependent logarithmic slope. For typical GaAs based 2D electron gas \( n = 10^{13} \text{cm}^{-2} \) at \( B = 0 \) at \( \nu_c = 3/2 \) one obtains \( \nu_0(10) = 0.11 \cdot K \). We stress that above universal \( \rho^c \), \( \rho_{yx}^c \) values could be attributed to so-called "QH transition points" discussed in press \[9,10\]. To test this, in Fig.2 the detailed dependences \( \rho(\nu), \rho_{yx}(\nu) \) in the vicinity of \( \nu_c = 3/2 \) for different temperatures \( \xi \) are presented. Evidently, this set of curves could be collapsed into a single curve since \( \rho, \rho_{yx} \) are universal functions of \( \xi \). We argue, the collapse is governed by a single parameter which could be but linear on temperature. Experimentally, in Ref.\[11,12\] was found that both \( \rho(\nu), \rho_{yx}(\nu) \) dependences are well collapsed. Then, the logarithmic slope \( \nu_0(T) \) found experimentally is linear on \( T \) well down to the lowest temperatures accessed. It should be noticed, however, that the heights of \( \rho \) peaks are lower than ones found in experiment. The above discrepancy could be attributed, for example, to phonon drag related enhancement of thermopower which is omitted in our simple approach.

Let us find out in standard fashion \( \sigma^{-} = (\rho)^{-1} \) the quantities, \( \sigma^x_{xx}, \sigma^y_{yx} \) called the transverse and the longitudinal conductivities respectively:

\[
\sigma^x_{xx} = \sigma^y_{yx} \left( \frac{1}{1 + s^2} \right), \sigma^x_{yx} = \sigma^y_{yx} \left( \frac{1}{1 + s^2} \right),
\]
where \( s = \gamma / (1 - \gamma) \) is the dimensionless parameter. Using Eq.8 one could derive the relationship \( \sigma^2_{xx} + (\sigma^y_{yx})^2 = \sigma^x_{yx} \sigma^y_{yx} \) instead well known semicircle relation. In Fig.3 the \( \sigma_{xx}, \sigma_{yx} \) dependences are plotted in the vicinity \( \nu_c = 3/2 \). Evidently, these curves could be collapsed into a single plot well as \( \rho, \rho_{yx} \) (see above discussion). This result is consistent with experiments.\[9,10\]
We emphasize, in practice \( \sigma^x_{xx}, \sigma^y_{yx} \) are always derived from Hall bar sample resistivity data. In fact, experimentally the interconnection between the components of both resistivity and conductivity tensors is not yet established.\[9,10\] The only within QH plateau the direct Cobino topology measurements\[9,10\] of transverse conductivity demonstrated that \( \sigma^y_{yx} = \sigma_{yx} = 1 / \rho_{yx} \).

We now find out the temperature gradient established in Hall bar sample at, for example, half-filled LL \( \nu_c = 5/2 \). According to Eq.(10) one obtains \( \alpha = k \ln 2 / (e \nu_c) \approx 0.28k / e \), then \( \rho_{yx} = 2 / 5h / e^2 \), thus \( \gamma = 0.02 \). Assuming sample width \( d = 1 \text{mm}, \) length \( l = 3 \text{mm} \) and typical dc current \( I = 10 \text{A} \) one could find the temperature gradient given by Eq.(10) as \( \nabla T \approx c I \rho_{yx} / (Ld) \approx 22 \text{ mK/mm}. \) At helium temperatures \( (T_a - T_b) / T_0 \approx 0.02 << 1 \), hence, our approach is valid. We underline that the temperature gradient is proportional to the ratio \( I / d \), thus, could be the reason of current and sample width dependent scaling observed in\[13,20\] respectively. For example, let the Hall probes(Fig.1, insert a) are shifted by length, \( x_0 \), with respect to middle of the sample. The local temperature established between the probes is \( T_0 - \nabla_x T \cdot x_0 \). At elevated currents, the Hall resistance \( \rho_{xy} \) becomes dependent on local temperature, hence, on sample width \( d \) and current \( I \). The thinner is the sample and (or) higher the current, the higher is the local temperature, thus, broader the interplateau transition. This mechanism appeared to play some role in current-polarity dependent QHE breakdown.\[21\]

Let us estimate the external cooling of the contacts caused by 3D heat leakage. For simplicity, we assume the high field corners(see Fig.1, insert a) as hot(cold) points of radii \( a \). These points are placed into infinite 3D medium. For temperature difference \( \Delta T \) the total heat flux is of the order \( \kappa_{3D} a \Delta T \), where \( \kappa_{3D} \) is the 3D thermal conductivity. This flux is less than the downstream 2DEG heat flux \( \alpha T I \) when \( \kappa_{3D} a / (\sigma_{yx} L d T') = p < 1 \). At \( T = 0.1 \text{K} \) the phonon related thermal conductivity for GaAs based structure is \( 5 \cdot 10^{-4} \text{W/(Km)} \). Assuming the point size being of the order of 2DEG thickness \( a \approx 30 \text{A} \), then \( l / d = 3 \), \( \rho_{yx} \approx 2 / 5h / e^2 \) one obtains \( p < 2 \). We stress, the above estimation gives the upper limit for 3D phonon cooling. Actually, the cooling is much lower because of finite size of sample.

We remind, up to now dc measurements case was discussed. However, our approach could be applied as well for ac case . As was shown in Refs.\[13,20\] at \( B = 0 \) the Peltier effect related resistivity vanishes above certain frequency dependent on inertial processes of thermal diffusion. We recall, in quantizing magnetic field the microscopic current and heat fluxes known to depend on the magnetism of the conducting electrons. Actually, Eqs.(13) describe just average current and heat densities of a bounded topology sample. We argue that diamagnetic surface currents could define the dynamics of thermal processes in 2DEG. Assuming \( m = 0.069 \text{m}_c \) one could estimate the cyclotron frequency \( \omega_c = 2.8 \cdot 10^{10} \text{Hz} \) and magnetic length \( l_H = (\hbar / m_c \omega_c)^{1/2} = 70 \text{A} \) at \( B = 10 \text{T} \). For typical sample size \( \sim 1 \text{mm} \) the transit time of an electron scattered at the sample boundaries is given \( t \sim 1 / (\omega_c l_H) = 5 \cdot 10^{-9} \text{c} \). Accordingly, one could estimate the critical frequency \( f_c \sim 1 / t = 0.2 \text{GHz} \) for thermal dynamics limit.

In conclusion, in strong quantum limit for dissipationless 2DEG(2DHG) under current carrying conditions there exists effectively the longitudinal resistivity caused by Peltier effect. The current results in heating(cooling) at first(second) sample contact due to Peltier effect. At small current the contacts temperatures are different, the temperature gradient is linear on current. The voltage swing downstream the current is proportional to Peltier effect induced thermopower. As a result, nonzero longi-
tudinal resistivity is measured in experiment. This value could be erroneously identified as bulk resistivity of 2D electron gas. The above effect could exists as well in 3D case.

A. Appendix

For 2DEG within zero approximation with respect to scattering the force, \( \mathbf{F} \perp \mathbf{B} \) results in macroscopic drift of electrons with the drift velocity \( \mathbf{v}_d = -\frac{e}{m} [\mathbf{F} \times \mathbf{B}] \). The dissipationless current and entropy densities are:

\[
\mathbf{j} = -eN \mathbf{v}_d = \frac{e}{B^2} N [\mathbf{N} \mathbf{F} \times \mathbf{B}],
\]

\[
\mathbf{j}_s = sN \mathbf{v}_d = -\frac{e}{eB^2} [\mathbf{S} \mathbf{F} \times \mathbf{B}],
\] for nonuniform electrochemical potential in x-y plane. The macroscopic current and heat flux density, \( \mathbf{q}_h = T\mathbf{j}_s = q + \mathbf{j}/c, \) are given

\[
\mathbf{j} = -\frac{eN}{B^2} [\mathbf{E} \times \mathbf{B}] - \frac{cS}{B^2} \nabla \mathbf{T} \times \mathbf{B},
\]

\[
\mathbf{q}_h = \frac{cTS}{B^2} [\mathbf{E} \times \mathbf{B}] + \frac{c}{eB^2} T^2 \partial \nabla T \partial T \int_{-\infty}^{\mu} \frac{\Omega d\mu'}{[\mathbf{V} \mathbf{T} \times \mathbf{B}]}.
\]

Eq.(A-2) coincide with Eq.(B). Note, Eq.(A-2) could be generalized as \[2\]

\[
\mathbf{j} = \frac{e}{B^2} \text{rot}((Ne\mathbf{\varphi} + \Omega)\mathbf{B}),
\]

\[
\mathbf{q}_h = -\frac{e}{eB^2} T \left( \frac{\partial}{\partial T} \right)^{\mu-\infty} \text{rot}((Ne\mathbf{\varphi} + \Omega)\mathbf{B}) d\mu.
\]

B. Appendix

We now find out asymptotic equations for 2D thermopower and conductivity \( \sigma_{yx} \) within low temperature and magnetic field limit \( \nu^{-1}, \xi < 1 \). Using well known Poisson formulæ

\[
\sum_{n_0}^{\infty} \varphi(n) = \int_{a}^{\infty} \varphi(n) dn + 2 \text{Re} \sum_{k=1}^{\infty} \int_{a}^{\infty} \varphi(n) e^{2\pi i km} dm,
\]

\[(B-1)\]

where \( m_0 - 1 < a < m_0 \), the 2DEG thermodynamic potential \( \Omega(\mu, \xi, \nu) \) is given

\[
\Omega = n_0 \mu \xi^2 \left(-F_1(1/\xi) + 2\pi^2 \sum_{k=1}^{\infty} (-1)^k \cos(2\pi k \nu) \frac{r_k}{r_k \sinh(r_k)} \right).
\]

(B-2)

Here, \( n = \frac{m}{2\pi^2 \xi^2 \mu} \) is the 2DEG concentration at \( \mathbf{B} = 0 \) (spin is neglected), \( r_k = 2\pi^2 \xi^2 \nu k \sim kT/\hbar \omega \) is the dimensionless parameter. Therefore, \( F_\nu(y) = \int_{0}^{y} (1 + \exp(x - y))^{-1} dx \) is the Fermi integral. Finally, 2DEG entropy \( S \), concentration \( N \) and are

\[
N = n\xi \left(F_\nu(1/\xi) + 2\pi \sum_{k=1}^{\infty} (-1)^k \sin(2\pi k \nu) \right) \sinh(r_k),
\]

\[
S = k\nu \left(\frac{d}{d\xi}(F_\nu(1/\xi)) - 2\pi^2 \xi \sum_{k=1}^{\infty} (-1)^k \Phi(r_k) \cos(2\pi k \nu) \right),
\]

where \( \Phi(z) = \frac{1 - z \coth(z)}{z \sinh(z)} \). From Eq.(B-3), one could find out \( \alpha = -S/eN \) and \( \sigma_{yx} = Ne\nu/eB \).

FIG. 1. Quantum Hall experimental setup. Inserts: current flow in a) 2DEG b) 2DHG Hall bar samples.

FIG. 2. Transverse magnetoresistance \( \rho_{yx} \) and thermal correction \( \rho \) (scaled by factor 10) given by Eq.(B) vs \( \nu^{-1} \) for \( \xi = 0.01, 0.02, 0.04, 0.08 \). Insert: Low field dependence \( \rho(\nu^{-1}) \) for \( \nu = 0.01 \) found out by means of Eqs.(B-3).

FIG. 3. Transverse magnetoresistance \( \rho_{yx} \), thermal correction \( \rho \) and conductivities \( \sigma_{yx} \), \( \sigma_{yx}^* \) nearby half-filled LL \( \nu_c = 3/2 \) (enlarged section (a) in Fig.(2)) for \( \xi = 0.015, 0.02, 0.025, 0.04 \)

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\[ \text{grad}(T) > 0 \]

\[ \text{grad}(T) < 0 \]
Fig. 3