PERFORMANCE ANALYSIS AND OPTIMIZATION RESEARCH OF MULTI-CHANNEL COGNITIVE RADIO NETWORKS WITH A DYNAMIC CHANNEL VACATION SCHEME

ZHANYOU MA$^1$, WENBO WANG$^{1,2}$, WUYI YUE$^3$ AND YUTAKA TAKAHASHI$^4$

$^1$School of Science, Yanshan University
Qinhuangdao 066004, China
$^2$First Experimental Primary School of Tongzhou District
Beijing Academy of Educational Sciences, Beijing 101100, China
$^3$Department of Intelligence and Informatics, Konan University
Kobe 658-8501, Japan
$^4$The Kyoto College of Graduate Studies for Informatics
Kyoto 600-8216, Japan

(Communicated by Kok Lay Teo)

Abstract. In order to resolve the issues of channel scarcity and low channel utilization rates in cognitive radio networks (CRNs), some researchers have proposed the idea of “secondary utilization” for licensed channels. In “secondary utilization”, secondary users (SUs) opportunistically take advantage of unused licensed channels, thus guaranteeing the transmission performance and quality of service (QoS) of the system. Based on the channel vacation scheme, we analyze a preemptive priority queueing system with multiple synchronization working vacations. Under this discipline, we build a three-dimensional Markov process for this queueing model. Through the analysis of performance measures, we obtain the average queueing length for the two types of users, the mean busy period and the channel utility. By analyzing several numerical experiments, we demonstrate the effect of the parameters on the performance measures. Finally, in order to optimize the system individually and socially, we establish utility functions and provide some optimization results for PUs and SUs.

1. Introduction. In the traditional static channel allocation method, the system allocates a fixed wireless channel for each server. While the use of this method has enabled the elimination of interference between different servers, it has also led to a high imbalance in channel utilization. Research reports show that licensed channel utilization rates can range from 15% to 85%, but that the utilization of most licensed channels is very low [7]. However, remaining unlicensed channels are used frequently, which leads to an enormous waste of channel resources, and an increased shortage in wireless channel availability.

How to take advantage of channel resources more efficiently has thus become a research focus. Cognitive radio networks (CRNs) have altered the thinking around traditional static channel allocation, and have given rise to the concept of dynamic...
channel distribution. Through the perception of the wireless communication environment, the channels dynamically change the transmission power, carrier frequency, modulation system and some other transmission parameters to adapt to the changes in the environment and improve the channel utilization [1], [2], [3].

In general, there are two types of users and multiple licensed channels in CRNs. Primary Users (PUs) own the exclusive rights to licensed channels, and Secondary Users (SUs) own the ability to sense channels and opportunistically utilize unused licensed channels, which leads to the idea of “secondary utilization” for the licensed channels. Many researchers have investigated the multiple channels model using different strategies. For instance, Yu et al. considered a multi-channel CRN where each SU could only choose to sense a subset of channels, and solved the optimization problem by jointly designing channel-sensing selection [13]. Tumuluru et al. considered an opportunistic spectrum-scheduling strategy for a multi-channel CRN, in which the PU activity and channel quality vary on a slot-by-slot basis, and calculated the expected number of packets by building a Markov chain [10].

Other researchers have concentrated on saving network resources that are wastefully depleted by idle channels. For instance, Zhao et al. investigated a kind of spectrum allocation strategy with a dynamic channel closing scheme, in which parts of the channels could be closed periodically to realize the data transmission control once there are any transmission requests in the system [14]. Wu et al. proposed an optimal resource allocation scheme in a multi-channel CRN, and considered the dynamic channel model and the sensing errors based on a vacation queueing model [12].

Additionally, different strategies using queueing models have been applied in CRNs. For example, Kim considered preemptive priority in CRNs by building a new T-preemptive priority M/G/1 queueing model, and calculated the average waiting time for PUs and SUs [6]. Kaur investigated a centralized CRN, and assumed that the central controller could allocate channels based on the system environment. In addition, he also established an M/M/1 queueing model and an M/G/S/N queueing model respectively, and focused on the response performances for SUs [5]. Lee et al. studied an underlay type cognitive radio network with multiple secondary users, and developed a new analytical model to analyze a random channel access protocol [8]. They also obtained the optimal access probabilities for maximizing the throughput.

Zhao et al. studied a cognitive radio network with multiple SUs [15], [16]. By constructing a three-dimensional Markov chain, they gave the transition probability matrix of the Markov chain, and obtained the steady-state distribution of the system model. Then, they derived some performance measures, compared the Nash equilibrium strategy and the socially optimal strategy for the packets. Katayama et al. considered the effect of overhead sensing on the system performance for cognitive radio networks with channel bonding [4]. They modeled the system on a multidimensional continuous-time Markov chain, and obtained some important performance measures. Finally, they derived the optimal number of bonded sub-channels for the throughput performance.

Based on the channel vacation scheme, we analyzed a preemptive priority queueing system with multiple synchronized working vacations for cognitive radio networks (CRNs) [11]. We then built a three-dimensional Markov process for this queueing model. Through the analysis of performance measures, we obtained the average queue length of the two types of users, the mean busy period and the channel utility.
In this paper, we mainly present channel utility and resource conservation in cognitive radio networks with a dynamic multi-channel vacation scheme controlled by a central controller, and establish a Markov process to evaluate the system performance measures. We would like to consider the performance analyses of the wireless communication networks with interference or a sensing protocol. This paper is an extended version of our conference paper presented in [11] and give some new expressions for additional performance measures, such as the probability that the system is in the busy period and in the working vacation period, the individual benefit function, and the social benefit function. Specifically, in this paper, we add numerical results for some performance measures and optimization analysis to show how to set system parameters to reach the maximum social benefit.

The remainder of this paper is organized as follows. In Section 2, the novel strategy and the queueing model we studied is described. In Section 3, the steady-state distribution of the queueing length is analyzed, and some performance measures are considered. In Section 4, the results of some numerical experiments are given. In Section 5 some individual and social optimization results are shown. In Section 6, conclusions are stated.

2. Channel allocation strategy and system model hypothesis.

2.1. The dynamic channel vacation scheme. In this paper, we consider a CRN consisting of two types of users and multiple licensed channels. The central controller and two types of users interact with information about the licensed channels. In order to utilize the network resource properly, a kind of channel allocation strategy with a dynamic channel vacation scheme is proposed. To ensure the unity of channel state and simplify the system model, we assume that all $c$ channels to be able to enter into a working vacation period to realize data transmission control once there are no remaining transmission requests in the system. In order to understand the channel vacation scheme and the preemptive for the readers easily, $c = 2$ is supposed, the specific description of the CRN with a dynamic channel vacation scheme is shown as Fig. 1.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{The dynamic channel vacation scheme proposed in this paper.}
\end{figure}

According to the dynamic channel vacation scheme, we can divide the system state into three states as follows:

1. Working vacation state. If all channels are empty, they will enter into a working vacation period. In this state, if no user arrives by the time the working vacation period is completed, the channels will begin another working vacation period. However, if some newly arriving users enter the channels
when the system is in a working vacation period, those users will be served at a lower service rate. If there are still some users in the channels at the end of the vacation, the vacation period will stop and a new busy period will begin. At the same time, those users will be served at higher service rates once the busy period begins.

(2) Preemptive priority state. In this state, if there are no channels available but not all channels are used by PUs, the service for SUs will be preempted by a newly arriving PU, and the SU being served will return to the head of the waiting queue.

(3) Blocking state. In this state, all channels are used by PUs. The system will lose any newly arriving PUs because they will be blocked from entering the system.

![Diagram of the running mode of the system.](image)

**Figure 2.** The running mode of the system.

Because of the infinite buffer for the SUs, the SUs are able to access the buffer space directly when waiting for the service if the system is in the second or third state. The specific representation for the CRN with channel vacation scheme is shown as Fig. 2.

The specific actions of the PUs and SUs in this paper are as follows. For the PUs, if there are some empty channels, a newly arriving PU will be served directly. If there are no channels available but not all channels are being used by PUs, the service of SUs is preempted by any newly arriving PUs. If all channels are used by PUs, then any newly arriving PUs will be lost due to the being blocked. For SUs, if there are some empty channels, any newly arriving SU can be served directly. If there are no channels available, any arriving SUs will enter the buffer and wait for the central controller to allot these channels. And if any PUs arrive at the system while SUs are receiving service, the SUs will immediately give up their channels to the PUs and enter the buffer at the head of the queue. We assume that the service order is a FIFO discipline.

2.2. **System model hypothesis.** In this model, we assume that SUs are regarded as class I users, PUs are regarded as class II users, and $c$ licensed channels are
regarded as $c$ servers. At the same time, considering the preemptive priority of PUs to licensed channels, a continuous time three-dimensional Markov process is built, where the class II user has preemptive priority. In order to ultimately enhance the response performance of PUs, an infinite buffer is set for the SUs, and no buffer is set for the PUs. The inter-arrival time, the service time and the vacation time are assumed to be independently sequenced. The specific description for this model is as follows:

(1) The two classes of users arriving at the system follow exponential distributions with parameters $\lambda_1, \lambda_2$ ($\lambda_1, \lambda_2 > 0$) respectively as follows:
\[
\begin{align*}
P\{T_1 \leq x\} &= 1 - e^{-\lambda_1 x}, \ x > 0, \\
P\{T_2 \leq x\} &= 1 - e^{-\lambda_2 x}, \ x > 0
\end{align*}
\]
where $T_1$ and $T_2$ represent the inter-arrival times of SUs and PUs respectively.

(2) The service times $S_1, S_2$ follow exponential distributions with parameters $\mu_1, \mu_2$ ($\mu_1, \mu_2 > 0$) as follows:
\[
\begin{align*}
P\{S_1 \leq x\} &= 1 - e^{-\mu_1 x}, \ x > 0, \\
P\{S_2 \leq x\} &= 1 - e^{-\mu_2 x}, \ x > 0
\end{align*}
\]
where $S_1, S_2$ represent the service times of SUs and PUs when the channels are in busy periods respectively.

(3) The vacation time $V$ follows an exponential distribution with parameter $\theta$, and the service times $S_1^{(v)}, S_2^{(v)}$ follow exponential distributions with parameters $\mu_1^{(v)}, \mu_2^{(v)}$ ($\mu_1^{(v)}, \mu_2^{(v)} > 0$) in a working vacation period as follows:
\[
\begin{align*}
P\{V \leq x\} &= 1 - e^{-\theta x}, \ x > 0, \\
P\{S_1^{(v)} \leq x\} &= 1 - e^{-\mu_1^{(v)} x}, \ x > 0, \\
P\{S_2^{(v)} \leq x\} &= 1 - e^{-\mu_2^{(v)} x}, \ x > 0
\end{align*}
\]
where $S_1^{(v)}, S_2^{(v)}$ represent the service times of SUs and PUs when the channels are in working vacation periods respectively.

3. Performance analysis of system.

3.1. Model analysis. According to the above description, let $N_1(t), N_2(t)$ be the number of SUs and PUs in the system at instant $t$, and $J(t)$ be the server state at instant $t$. Define
\[
J(t) = \begin{cases} 
0, & \text{the instant } t \text{ is in the working vacation period} \\
1, & \text{the instant } t \text{ is in the busy period.}
\end{cases}
\]

Then, the three-dimensional stochastic process \{(N_1(t), N_2(t), J(t)), t \geq 0\} is a Markov process with the state space:
\[
\Omega = \{(0, 0, 0)\} \cup \{(0, l, j), 1 \leq l \leq c, j = 0, 1\} \\
\cup \{(i, l, j), i \geq 1, 0 \leq l \leq c, j = 0, 1\}.
\]

All possible states: $(i, 0, 0), (i, 1, 0), (i, 1, 1), ..., (i, c, 0), (i, c, 1)$ are called level $i$, where $i \geq 1$. Specifically, level 0 has states: $(0, 0, 0), (0, 1, 0), (0, 1, 1), ..., (0, c, 0), (0, c, 1)$. 
Using the lexicographical ordering for the states, the state transition rate matrix of the process can be written as:

\[
Q = \begin{bmatrix}
A_{0,0} & A_{0,1} \\
A_{1,0} & A_{1,1} & A_2 \\
A_{2,1} & A_{2,2} & A_2 \\
& & & \ddots & \ddots & \ddots \\
& & & & A_{c-1,c-1} & A_{c-1,c} & A_2 \\
& & & & A_{c,c-1} & A_{c,c} & A_2 \\
& & & & & & \ddots & \ddots & \ddots \\
\end{bmatrix}
\]

(1)

where \(A_{0,0}, A_{0,1}, A_{1,0}, \ldots, A_{c-1,c-1}, A_{1,1}, \ldots, A_{c,c}, A_2\) represent the state transition rate between levels. Specifically, \(A_{0,1}\) represents the state transition rates from level 0 to level 1. \(A_{i,i}\) represents the state transition rates from level \(i\) to level \(i\), where \(0 \leq i \leq c-1\), and \(A_{c,c}\) represents the state transition rates from level \(i\) to level \(i\), where \(i \geq c\). \(A_{j,j-1}\) represents the state transition rates from level \(j\) to level \(j-1\), where \(1 \leq j \leq c-1\), and \(A_{c-1,c-1}\) represents the state transition rates from level \(j\) to level \(j-1\), where \(j \geq c\). \(A_2\) represents the state transition rates from level \(k\) to level \(k+1\), where \(k \geq 1\). We define symbols \(\delta^{(v)}_{ij}, \delta^{(v)}_{ij}, \delta^{(v)}_{ic}, \delta^{(v)}_{ic}, \delta^{(v)}_{ic}, \delta^{(v)}_{ic}, \delta^{(v)}_{ic}, \delta^{(v)}_{ic}\) as follows:

\[
\begin{align*}
\delta^{(v)}_{ij} &= -(\lambda_1 + \lambda_2 + \theta + i(m^{(v)}_1 + j(m^{(v)}_2)), 1 \leq i \leq c-1, 1 \leq j \leq c-1, \\
\delta^{(v)}_{ij} &= -(\lambda_1 + \lambda_2 + i(m_1 + j(m_2)), 1 \leq i \leq c-1, 1 \leq j \leq c-1, \\
\delta^{(v)}_{ic} &= -(\lambda_1 + \theta + i(m^{(v)}_1 + c(m^{(v)}_2)), 1 \leq i \leq c-1, \\
\delta^{(v)}_{ic} &= -(\lambda_1 + \theta + i(m_1 + c(m_2)), 1 \leq i \leq c-1, \\
\delta^{(v)}_{ic} &= -(\lambda_1 + \lambda_2 + \theta + i(m^{(v)}_2)), 1 \leq i \leq c-1, \\
\delta^{(v)}_{ic} &= -(\lambda_1 + \lambda_2 + \theta + i(m_2)), 1 \leq i \leq c-1, \\
\delta^{(v)}_{ic} &= -(\lambda_1 + \theta + c(m^{(v)}_2)), \\
\delta^{(v)}_{ic} &= -(\lambda_1 + \theta + c(m_2)).
\end{align*}
\]

The detailed matrices are as follows:

\[
A_2 = \begin{bmatrix}
\lambda_1 \\
\lambda_1 \\
\ddots \\
\lambda_1
\end{bmatrix}, \quad A_{0,1} = \begin{bmatrix}
\lambda_1 & 0 & 0 \\
0 & \lambda_1 & 0 \\
0 & \ddots & \ddots \\
0 & 0 & \lambda_1
\end{bmatrix},
\]

\[
A_{1,0} = \begin{bmatrix}
\mu_1^v & 0 & 0 \\
\mu_1 & 0 & 0 \\
\mu_1 & 0 & \ddots \\
\mu_1 & 0 & \ddots & 0 \\
\mu_1 & 0 & \ddots & \ddots & 0 \\
\mu_1 & 0 & \ddots & \ddots & \ddots & 0 \\
\mu_1 & 0 & \ddots & \ddots & \ddots & \ddots & 0 \\
\mu_1 & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\
\mu_1 & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0
\end{bmatrix},
\]
ANALYSIS AND OPTIMIZATION OF MULTI-CHANNEL COGNITIVE RADIO NETWORKS

\[ A_{i,i-1} = \begin{bmatrix}
  i\mu_1v & i\mu_1 & i\mu_1 & \cdots & i\mu_1v \\
  i\mu_1 & i\mu_1v & i\mu_1 & \cdots & i\mu_1v \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  i\mu_1v & i\mu_1 & i\mu_1 & \cdots & i\mu_1v \\
  i\mu_1 & i\mu_1v & \cdots & \cdots & i\mu_1 \\
\end{bmatrix} \]

where \( 2 \leq i \leq c \).

\[ A_{0,0} = \begin{bmatrix}
  -(\lambda_1 + \lambda_2) & \lambda_2 & 0 & \cdots & 0 \\
  \mu_2^{(v)} & \delta_{11}^{(v)} & \theta & \lambda_2 & \cdots & 0 \\
  \mu_2 & 0 & \delta_1 & \lambda_2 & \cdots & 0 \\
  2\mu_2^{(v)} & 0 & \delta_2^{(v)} & \theta & \lambda_2 & \cdots & 0 \\
  2\mu_2 & 0 & \delta_2 & 0 & \lambda_2 & \cdots & 0 \\
  \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
  (c-1)\mu_2^{(v)} & 0 & \delta_{c-1}^{(v)} & \theta & \lambda_2 & \cdots & 0 \\
  (c-1)\mu_2 & 0 & \delta_{c-1} & 0 & \lambda_2 & \cdots & 0 \\
  c\mu_2^{(v)} & 0 & \delta_{c}^{(v)} & \theta & \cdots & \cdots & \cdots & \cdots \\
  c\mu_2 & 0 & \delta_{c} & \cdots & \cdots & \cdots & \cdots & \cdots \\
\end{bmatrix}, \]

where \( 1 \leq i \leq c \).

According to the block tri-diagonal structure of the transition rate matrix, we can show that \( \{(N_1(t), N_2(t), J(t)), t \geq 0\} \) is a quasi birth and death process. When the Markov process is positive recurrent, the steady-state distribution is indicated as follows:

\[ \pi_{i,l,j} = \lim_{t \to \infty} P\{N_1(t) = i, N_2(t) = l, J(t) = j\}, \quad (i, l, j) \in \Omega, \]

\[ \Pi = (\pi_0, \pi_1, \pi_2, \ldots) \]

where

\[ \pi_0 = (\pi_0,0,0, \pi_0,0,1,0, \pi_0,0,1,1, \ldots, \pi_0,c,0, \pi_0,c,1), \]

\[ \pi_i = (\pi_i,0,0, \pi_i,0,1, \pi_i,1,0, \pi_i,1,1, \ldots, \pi_i,c,0, \pi_i,c,1), \quad i \geq 1. \]

The necessary and sufficient condition that the Markov process \( \{(N_1(t), N_2(t), J(t)), t \geq 0\} \) is positive recurrent is the matrix quadratic equation

\[ R^2 A_{c,c-1} + RA_{c,c} + A_2 = 0, \]
which has a minimal non-negative solution \( \mathbf{R} \), a spectral radius \( \text{sp}(\mathbf{R}) < 1 \), and a \((2c^2 + 4c + 1)\)-dimensional stochastic matrix:

\[
B[\mathbf{R}] = \begin{bmatrix}
A_{0,0} & A_{0,1} & & \\
A_{1,0} & A_{1,1} & A_2 & \\
A_{2,1} & A_{2,2} & & \\
& & & \\
& & & \\
& & & \\
& & & \\
& & & \\
& & & \\
A_{c-1,c-2} & A_{c-1,c-1} & A_2 & \\
& A_{c,c-1} & RA_{c,c-1} + A_{c,c}
\end{bmatrix}
\] 

(2)

has a left-zero vector. When the Markov process is positive recurrent, its steady-state distribution satisfies:

\[
\begin{align*}
\pi_0, \pi_1, \ldots, \pi_c B[\mathbf{R}] &= 0 \\
\sum_{i=0}^{c-1} \pi_i e + \pi_c (I - \mathbf{R})^{-1} e &= 1 \\
\pi_i &= \pi_c R^{i-c}, \quad i \geq c
\end{align*}
\] 

(3)

where the left-zero vector means that when a vector multiplies the left side of a matrix the product is equal to the zero vector, this vector is called a left-zero vector of the matrix, i.e., \( x \mathbf{A} = 0 \), then \( x \) is a left-zero vector of \( \mathbf{A} \). And \( e \) is an appropriate dimensional column vector with all elements being equal to one.

The proof of Eq. (3) can be obtained by using the equilibrium equation \( \Pi \mathbf{Q} = \mathbf{0} \) and the matrix-geometric solution method presented in [9]. The main step is given as follows. Firstly, according to \( \mathbf{Q} \) and \( B[\mathbf{R}] \), we can prove that \( B[\mathbf{R}] \) is a \((2c^2 + 4c + 1)\)-dimensional square matrix and \( B[\mathbf{R}] e = 0 \). Secondly, because the Markov process satisfies the steady state equation \( \Pi \mathbf{Q} = \mathbf{0} \), we can obtain the matrix geometric distribution \( \pi_i = \pi_c R^{i-c}, \quad i \geq c \). Thirdly, according to the normalization condition, we can get \( \sum_{i=0}^{c-1} \pi_i e + \pi_c (I - \mathbf{R})^{-1} e = 1 \).

In fact, only when matrices \( A_{c,c-1}, A_{c,c}, A_2 \) have the special structural forms of an upper triangular matrix and a lower triangular matrix or their structures to be relatively simple, we can obtain the analytical expression of \( \mathbf{R} \). However, in this paper, matrices \( A_{c,c-1}, A_{c,c}, A_2 \) have general matrix forms and their expressions are relatively complex. Therefore, getting an analytic expression of the rate matrix \( \mathbf{R} \) directly will become more difficult. It is difficult to obtain more intuitive and explicit condition of the stability using parameters of the system. Therefore, we usually need to derive the recursion expression of the rate matrix \( \mathbf{R} \) and calculate the numerical solution by using an algorithm with Matlab program.

3.2. Performance measures. According to the results from Sect. 3.1, we can obtain the corresponding formulas for the performance measures in terms of the data loss rate, the average queueing length of SUs and PUs, the throughput of PUs, and the channel utility.

(1) The average queueing length \( E(L_1) \) is defined as the number of SUs in the system. The average queueing length \( E(L_2) \) is defined as the number of PUs.

The average queueing length \( E(L_1) \) and \( E(L_2) \) are given by:

\[
E(L_1) = \sum_{k=0}^{\infty} k P(L_1 = k) = \sum_{k=1}^{\infty} \sum_{j=0}^{c} k \pi_{k,i,j} \]

\[
E(L_2) = \sum_{i=0}^{c} i P(L_2 = i) = \sum_{i=1}^{c} i \left( \sum_{j=0}^{1} \sum_{k=0}^{\infty} \pi_{k,i,j} \right)
\]
(2) The mean busy period $E(B)$ is defined as the mean time that the system state takes to go from busy to empty. According to Little’s Law, the mean busy period $E(B)$ is given by:

$$E(B) = E(L_1) \frac{1}{\mu_1} + E(L_2) \frac{1}{\mu_2}.$$  

(3) The data loss rate $P_d$ is defined as the probability that PUs are lost due to the blocked state. The data loss rate $P_d$ is given by:

$$P_d = \sum_{k=0}^{\infty} \sum_{j=0}^{1} \pi_{k,c,j}.$$  

(4) The probability $P_b$ and $P_{wv}$ are defined as the probability that the system is in the busy period and in the working vacation period. The probability $P_b$ and $P_{wv}$ are given by:

$$P_b = \sum_{i=1}^{c} \pi_{0,i,1} + \sum_{k=1}^{\infty} \sum_{i=0}^{c} \pi_{k,i,1},$$

$$P_{wv} = \sum_{k=0}^{\infty} \sum_{i=0}^{c} \pi_{k,i,0}.$$  

(5) The channel utility $P_u$ is defined as the probability that the channels are occupied in the process of data communication. The channel utility $P_u$ is given by:

$$P_u = \min_{c} \left\{ E(L_1) + E(L_2) \right\}$$

where, when $E(L_1) + E(L_2) < c$, some channels are idle, then $\min_{c} \left\{ E(L_1) + E(L_2) \right\} = E(L_1) + E(L_2)$. And when $E(L_1) + E(L_2) \geq c$, all channels are busy, therefore $\min_{c} \left\{ E(L_1) + E(L_2) \right\} = c$.

4. **Numerical results.** In this section, we provide some numerical results to describe the effect of parameters on performance measures. Taking $\lambda_1 = 6$, $\mu_1 = 3$, $\mu_1^{(v)} = 0.5$, $\mu_2^{(v)} = 1$, $\theta = 3$.

![Figure 3. The relation of $E(L_1)$ to $\mu_2$ and $c$.](image)
The vacation rate $\theta$
The average queue length $E(L_2)$

$\mu_2 = 6$
$\mu_2 = 8$
$\mu_2 = 10$

Figure 4. The relation of $E(L_2)$ to $\mu_2$ and $\theta$.

In Fig. 3, taking $\lambda_2 = 10$, we find that the average queueing length $E(L_1)$ of the SUs increases with an increase of $\mu_2$. That is mainly because as $\mu_2$ increases, the PUs can be served more quickly, which leads to less opportunity for the SUs to occupy the channels due to the preemptive priority for the PUs, therefore the average queueing length of the SUs also increases. When $\mu_2$ is unchanged, $E(L_1)$ decreases with an increase in the value $c$. This is mainly because as $c$ increases, so the SUs have more chances to depart. Hence, the average queueing length of the SUs decreases.

In Fig. 4, taking $c = 4$, we can find that the average queueing length $E(L_2)$ of the PUs decreases as their vacation rate $\theta$ increases. That is mainly because as $\theta$ increases, the PUs can be served more chances due to the preemptive priority for the PUs, therefore the average queueing length of the PUs also decreases. And when $\theta$ is unchanged, the average queueing length of the PUs decreases as the serving rate $\lambda_2$ increases. This is mainly because as $\mu_2$ increases, the PUs have more chances to depart, then the average queueing length of the PUs decreases.

In Table 1, taking $\mu_2 = 6$, the average busy period $E(B)$ increases with an increase in $\lambda_2$. When $\lambda_2$ is unchanged, the average busy period decreases as the value $c$ increases. This is mainly because an increase of the value $c$ allows more PUs to access service and reduces the opportunity for servers to be in an idle state; therefore, the average busy period decreases.

| $c$ | $\lambda_2 = 6$ | $\lambda_2 = 7$ | $\lambda_2 = 8$ | $\lambda_2 = 9$ | $\lambda_2 = 10$ |
|-----|----------------|----------------|----------------|----------------|----------------|
| 3   | 0.4900         | 0.4983         | 0.5052         | 0.5109         | 0.5156         |
| 4   | 0.4678         | 0.4793         | 0.4891         | 0.4975         | 0.5046         |
| 5   | 0.4594         | 0.4731         | 0.4852         | 0.4957         | 0.5049         |

In Fig. 5, taking $\mu_2 = 8$, the data loss rate $P_d$ increases with an increase in $\lambda_2$. This is mainly because an increase in the arrival rate of the PUs leads to an increased likelihood of more channels being in the blocking state. Hence, the data
loss rate $P_d$ increases. On the other hand, when $\lambda_2$ is fixed, the data loss rate $P_d$ decreases with an increase in the channel number $c$. This is because the more channels there are, the more opportunity there is for the PUs to be offered service.

In Fig. 6, taking $\mu_2 = 8$, as the arrival rate $\lambda_2$ increases, the change trend for the probability $P_b$ exhibits two stages. During the first stage, $P_b$ decreases with an increase in $\lambda_2$. During the second stage, $P_b$ increases with an increase in $\lambda_2$. On the other hand, when $\lambda_2$ is fixed, the probability $P_b$ decreases with an increase in the channel number $c$. This is because the more channels there are, the more opportunity there is for the service to rest.

In Fig. 7, as the arrival rate $\lambda_2$ increases, the change trend for the probability $P_{wv}$ exhibits two stages. During the first stage, $P_{wv}$ increases with an increase in $\lambda_2$. During the second stage, $P_{wv}$ decreases with an increase in $\lambda_2$. On the other hand, when $\lambda_2$ is fixed, the probability $P_{wv}$ increases with an increase in the channel number $c$.

In Fig. 8, taking $\lambda_2 = 10$, we find that the channel utility $P_u$ decreases with an increase in $\mu_2$. This is mainly because an increase in the service rate of the PUs allows users to be serviced quicker, and channels have more chances to be in an idle state.
The arrival rate $O_2$

The probability $P_{wv}$

$c=4$
$c=5$
$c=6$

The serving rate $P_2$

The channel utility $P_u$

$c=5$
$c=6$
$c=7$

Figure 7. The relation of $P_{wv}$ to $\lambda_2$ and $c$.

Figure 8. The relation of $P_u$ to $\mu_2$ and $c$.

state; therefore the channel utility decreases. When $\mu_2$ is unchanged, the channel utility decreases with an increase in the number of servers. The more channels there are, the more idle channels there will be.

5. Individual and social optimization. Firstly, we discuss the individual optimization for each user. We assume that $R_1$ and $R_2$ represent the rewards which the users could obtain when per SU or PU served; $C_1$ and $C_2$ represent the cost to users per waiting time in the system; $w_1$ and $w_2$ represent the unit waiting time of SUs and PUs; $U_{I1}$ and $U_{I2}$ represent the individual benefit of SUs and PUs respectively. The individual benefit $U_{Ii}$ is given as

$$U_{Ii} = \frac{\mu_i}{\lambda_i} R_i - w_i C_i, \quad i = 1, 2$$

where $w_i = E(L_i)/\lambda_i$.

In Fig. 9, taking $c = 4, \lambda_2 = 8, R_1 = 60, C_1 = 150$, when $\mu_2$ is unchanged, the individual benefit $U_{I1}$ decreases with an increase in $\theta$. But when $\theta$ is unchanged, the
The vacation rate $\theta$

The individual benefit $U_{1I}$

$\mu_2=4$
$\mu_2=6$
$\mu_2=8$

Figure 9. The relation of $U_{1I}$ to $\mu_2$ and $\theta$.

The vacation rate $\theta$

The individual benefit $U_{2I}$

$\mu_2=4$
$\mu_2=6$
$\mu_2=8$

Figure 10. The relation of $U_{2I}$ to $\mu_2$ and $\theta$.

individual benefit $U_{1I}$ increases with an increase in $\mu_2$. That is mainly because as $\mu_2$ increases, the average queueing length of the SUs decreases, then the individual benefit $U_{1I}$ increases.

In Fig. 10, taking $c = 4, \lambda_2 = 8, R_2 = 130, C_2 = 400$, when $\mu_2$ is unchanged, the individual benefit $U_{1I}$ decreases with an increase in $\theta$. However, when $\theta$ is unchanged, the individual benefit $U_{2I}$ increases with an increase in $\mu_2$. That is mainly because as $\mu_2$ increases, the average queueing length of the PUs decreases, then the individual benefit $U_{2I}$ increases.

In order to analyze the social optimization of the system, we give some assumptions as follows:

1. $R_0$ represents the average reward when the users have been served.
2. $C$ represents the average cost to users per waiting time.
3. $C_p$ represents the average cost when a channel is in a busy period.
(4) $C_d$ represents the potential loss when PUs are lost due to the blocking state. 
(5) $U_s$ represents the social benefit.

Secondly, the social benefit $U_s$ of the system is given as:

$$U_s = \lambda_s \left( \frac{\mu_s}{\lambda_s} R_0 - w_s C \right) - cC_p - C_d P_d$$

where $w_s = (w_1 + w_2) / 2, \lambda_s = (\lambda_1 + \lambda_2) / 2, \mu_s = (\mu_1 + \mu_2) / 2$.

This social optimization arrival rate $\lambda_s^*$ with the maximum social welfare $U_s^*$ can be denoted as follows:

$$\lambda_s^* = \arg \max_{0 < \lambda_s < 1} \left\{ \lambda_s \left( \frac{\mu_s}{\lambda_s} R_0 - w_s C \right) - cC_p - C_d P_d \right\}.$$  

Taking $R_0 = 23, C = 1.7, C_p = 15, C_d = 48$. In Fig. 11, taking $\mu_2 = 6$, we find that when $c$ is unchanged, the social benefit $U_s$ increases with an increase in $\lambda_2$ firstly, then the social benefit decreases with the continuous increase in $\lambda_2$. When $\lambda_2$ is unchanged, the social benefit decreases with the increase of $c$. This is mainly because with an increase in $c$, the channel utility decreases and there are more resources that will be wasted; hence, the cost will increase and the benefit will decrease. We also can find that when $c = 4$, and $\lambda_2 = 8.2$ the social benefit can reach its maximum. When $c = 6$, and $\lambda_2 = 7.7$ the social benefit can reach its maximum. Finally, when $c = 8$, and $\lambda_2 = 7.5$ the social benefit can also reach its maximum.

In Fig. 12, taking $c = 4$, we can find that the social benefit $U_s$ decreases with an increase in $\mu_2$, and the social benefit increases with an increase in $\theta$. That is mainly because as $\theta$ increases, the average queueing length of the PUs decreases, therefore the social benefit $U_S$ increases.

6. Conclusions. In this paper, a dynamic channel vacation scheme in cognitive radio networks is considered in order to conserve network resources and to guarantee the QoS for the users. Based on the working principle of a dynamic channel vacation scheme and a preemptive priority discipline for PUs in cognitive radio networks, we presented a preemptive priority M/M/$c$ queueing model with multiple synchronized
working vacations, and established a three-dimension Markov process for the system. By using a matrix-geometric solution method, we derived the steady-state distribution of the queue length. Then, some numerical experiments are discussed to illustrate the effect of the parameters on the system measures. Finally, some optimization results were analyzed to find the optimal parameters for producing the greatest social benefit.

Acknowledgments. This work was supported in part by the National Natural Science Foundation (Nos. 61973261 and 61872311), Natural Science Foundation of Hebei Province (Nos. A2020203010 and A2018203088), Key Foundation of Higher Education Science and Technology Research of Hebei Province (No. ZD2017079), China, and was supported in part by MEXT and JSPS KAKENHI Grant (No. JP17H01825), Japan.

REFERENCES

[1] Y. Chen, P. Liao and Y. Wang, A channel-hopping scheme for continuous rendezvous and data delivery in cognitive radio network, Peer-to-Peer Networking and Applications, 9 (2016), 16–27.

[2] L. Chouhan and A. Trivedi, Performance study of a CSMA based multi-user MAC protocol for cognitive radio networks: Analysis of channel utilization and opportunity perspective, Wireless Networks, 22 (2016), 33–47.

[3] S. Jin, X. Yao and Z. Ma, A novel spectrum allocation strategy with channel bonding and channel reservation, KSII Transactions on Internet and Information Systems, 9 (2015), 4034–4053.

[4] H. Katayama, H. Masuyama, S. Kasahara and Y. Takahashi, Effect of spectrum sensing overhead on performance for cognitive radio networks with channel bonding, Journal of Industrial and Management Optimization, 10 (2014), 21–40.

[5] P. Kaur, A. Khosla and M. Uddin, Markovian queuing model for dynamic spectrum allocation in centralized architecture for cognitive radios, IACSIT International Journal of Engineering and Technology, 3 (2011), 96–101.

[6] K. Kim, T-preemptive priority queue and its application to the analysis of an opportunistic spectrum access in cognitive radio networks, Computers and Operations Research, 39 (2012), 1394–1401.
[7] P. Kolodzy, Spectrum policy task force: Finding and recommendations, *International Symposium on Advanced Radio Technologies*, 96 (2003), 392–393.

[8] S. Lee and G. Hwang, A new analytical model for optimized cognitive radio networks based on stochastic geometry, *Journal of Industrial and Management Optimization*, 13 (2017), 1883–1899.

[9] M. Neuts, *Matrix-Geometric Solutions in Stochastic Models: An Algorithmic Approach*, The Johns Hopkins Universit Press, Baltimore, MD, 1981.

[10] V. Tumuluru, P. Wang and D. Niyato, A novel spectrum-scheduling scheme for multi-channel cognitive radio network and performance analysis, *IEEE Transactions on Vehicular Technology*, 60 (2011), 1849–1858.

[11] W. Wang, Z. Ma, W. Yue and Y. Takahashi, Performance analysis of a dynamic channel vacation scheme in cognitive radio networks, In *Proceedings of the 13th International Conference on Queueing Theory and Network Applications*, (2018), 183–190.

[12] K. Wu, W. Wang, H. Luo, G. Yu and Z. Zhang, Optimal resource allocation for cognitive radio networks with imperfect spectrum sensing, *2010 IEEE 71st Vehicular Technology Conference*, 9 (2010), 1–4.

[13] H. Yu, W. Tang and S. Li, Joint optimal sensing time and power allocation for multi-channel cognitive radio networks considering sensing-channel selection, *Science China Information Sciences*, 57 (2014), 1–8.

[14] Y. Zhao, S. Jin and W. Yue, Performance optimization of a dynamic channel bonding strategy in cognitive radio networks, *Pacific Journal of Optimization*, 9 (2013), 679–696.

[15] Y. Zhao and W. Yue, Performance evaluation and optimization of cognitive radio networks with adjustable access control for multiple secondary users, *Journal of Industrial and Management Optimization*, 15 (2019), 1–14.

[16] Y. Zhao and W. Yue, Cognitive radio networks with multiple secondary users under two kinds of priority schemes: Performance comparison and optimization, *Journal of Industrial and Management Optimization*, 13 (2017), 1475–1492.

Received October 2018; revised July 2020.

E-mail address: mzhy55@ysu.edu.cn
E-mail address: wangwenbo931118@163.com
E-mail address: yue@konan-u.ac.jp
E-mail address: takahashi@i.kyoto-u.ac.jp