State preservation by repetitive error detection in a superconducting quantum circuit

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Quantum computing becomes viable when a quantum state can be protected from environment-induced error. If quantum bits (qubits) are sufficiently reliable, errors are sparse and quantum error correction (QEC) is capable of identifying and correcting them. Adding more qubits improves the preservation of states by guaranteeing that increasingly larger clusters of errors will not cause logical failure—a key requirement for large-scale systems. Using QEC to extend the qubit lifetime remains one of the outstanding experimental challenges in quantum computing. Here we report the protection of classical states from environmental bit-flip errors and demonstrate the suppression of these errors with increasing system size. We use a linear array of nine qubits, which is a natural step towards the two-dimensional surface code QEC scheme7, and track errors as they occur by repeatedly performing projective quantum non-demolition parity measurements. Relative to a single physical qubit, we reduce the failure rate in retrieving an input state by a factor of 2.7 when using five of our nine qubits and by a factor of 8.5 when using all nine qubits after eight cycles. Additionally, we tomographically verify preservation of the non-classical Greenberger–Horne–Zeilinger state. The successful suppression of environment-induced errors will motivate further research into the many challenges associated with building a large-scale superconducting quantum computer.

The ability to withstand multiple errors during computation is a critical aspect of error correction. We define nth-order fault tolerance to mean that any combination of n errors is tolerable. Previous experiments based on nuclear magnetic resonance8,9, ion traps10 and superconducting circuits11–13 have demonstrated multi-qubit states that are first-order tolerant to one type of error. Recently, experiments with ion traps and superconducting circuits have shown the simultaneous detection of multiple types of errors14,15. All of these experiments demonstrate error correction in a single round; however, quantum information must be preserved throughout computation using multiple error-correction cycles. The basics of repeating cycles have been shown in ion traps16 and superconducting circuits17. Until now, it has been an open challenge to combine these elements to make the information stored in a quantum system robust against errors which intrinsically arise from the environment.

The key to detecting errors in quantum information is to perform quantum non-demolition (QND) parity measurements. In the surface code, this is done by arranging qubits in a chequerboard pattern—with data qubits corresponding to the white squares (blue in Fig. 1), and measurement qubits to the black squares (green in Fig. 1)—and using these ancilla measurement qubits to repetitively perform parity measurements to detect bit-flip (X) and phase-flip (Z) errors18,19. A square chequerboard with \((4n + 1)\) qubits is nth-order fault tolerant, meaning that at least \(n + 1\) errors must occur to cause failure in preserving a state if fidelities are above a threshold. With error suppression factor \(A > 1\) and more qubits, failure becomes increasingly unlikely with probability \(e \propto -1/A^{n+1}\) (assuming independent errors). The surface code is highly appealing for superconducting quantum circuits as it requires only nearest-neighbour interactions, single and two-qubit gates, and fast repetitive measurements with fidelities above a lenient threshold of approximately 99%. All of this has recently been demonstrated in separate experiments19,20.

The simplest system demonstrating the basic elements of the surface code is a one-dimensional chain of qubits, as seen in Fig. 1a. It can run the repetition code, a primitive of the surface code, which corrects bit-flip errors on both data and measurement qubits. The device shown in Fig. 1b is a chain of nine qubits, which allows us to experimentally test both first- and second-order fault tolerance. It consists of a superconducting aluminium film on a sapphire substrate, patterned into Xmon transmon qubits21 with individual control and readout. The qubits are the cross-shaped devices; they are capacitively coupled to their nearest neighbours, controlled with microwave drive and frequency detuning pulses, and measured with a dispersive readout scheme. The device consists of five data qubits and four measurement qubits in an alternating pattern; see Supplementary Information for details.

To detect bit-flips, we determine the parity of adjacent data qubits by measuring the operator ZZ. We do this using an ancilla measurement qubit, and performing single- and two-qubit quantum gates (Fig. 1c). The operator measurement can have two values: +1 for states \(|00\rangle\) and \(|11\rangle\), and −1 for \(|01\rangle\) and \(|10\rangle\). Therefore, errors can be detected as they occur by repeating this operator measurement and noting changes in the outcome. Importantly, this measurement does not destroy the quantum nature: given input \(|a(00) + b(11)\rangle\) the result will be +1 and the quantum state remains, with similar behaviour for other Bell-like superposition states. In the repetition code, simultaneous measurements of these operators enable multiple bit-flip errors to be detected.

We now discuss how bit-flip errors, which can occur on any qubit and at any time, are identified. The quantum circuit of the repetition code is shown in Fig. 2a, for three cycles (in time) and nine qubits. This is the natural extension of the schematic in Fig. 1c, optimized for our hardware (Supplementary Information). Figure 2a illustrates four distinct types of bit-flip errors (stars): measurement error (gold), single-cycle data error (purple), two-cycle data error (red), and a data error after the final cycle (blue). Controlled-NOT (CNOT) gates propagate bit-flip errors on the data qubit to the measurement qubit. Each of these errors is typically detected at two locations if in the interior and at one location if at the boundary; we call these ‘detection events’. The error connectivity graph22 is shown in Fig. 2b, where the grey lines indicate every possible pattern of detection events that can arise from a single error. The last column of values for the ZZ operators in Fig. 2b are constructed from the data qubit measurements, so that errors between the last cycle and data qubit measurement can be detected (Supplementary Information).

In the absence of errors, there are two possible patterns of sequential measurement results. If a measurement qubit’s neighbouring data...
qubits are in the $|000\rangle$ or $|111\rangle$ state, the measurement qubit will report a string of identical values. If the data qubits are in the $|01\rangle$ or $|10\rangle$ state, the measurement qubit will report alternating values, as measurement is QND. Single data bit-flip errors make the measurement outcomes switch between these two patterns. For example, if the measurement outcomes for three cycles are 0, 0 and 1, this indicates a retention of genuine quantum entanglement. In the case of two detection events, which indicate a likely data qubit error in the first cycle, we find elements away from the ideal positions. By applying the recovery operation in post-processing (a single bit-flip on the blue data qubit) we can restore the state. Energy relaxation, the most likely cause of the detected bit-flip error, induces both bit-flip and phase-flip errors. The bit-flip error is corrected and the diagonal terms are preserved, but any phase-flip error remains uncorrected, reducing the off-diagonal terms and fidelity to 59%. We note that genuine entanglement is preserved. Conditional tomography for every configuration can be found in Supplementary Information.

The data in Fig. 3 clearly show that the one-dimensional repetition code algorithm does not necessarily destroy the quantum nature of the state. It allows for preserving the quantum state in the case of no errors, and correcting bit-flip errors otherwise. This preservation is achieved purely through error detection and classical post-processing, like for the full surface code, avoiding the need for dynamic feedback with quantum gates. For the remainder, we investigate the logical basis states individually, as tomographic reconstruction cannot be done fault-tolerantly.

We now address the critical question of how well our implementation of the repetition code protects logical states over many cycles. The process flow is illustrated in Fig. 4a. We start by initializing the data qubits in either of the logical basis states: $|0\rangle = |000\rangle$ or $|1\rangle = |111\rangle$. We then run the repetition code algorithm for $k$ cycles, and finish by measuring the state of all data qubits. We repeat this 90,000 times to gather statistics. The classical measurement results are converted into detection events, which are processed using minimum-weight perfect matching to generate corrections (see Supplementary Information). These corrections are then applied to the measured data qubit output events. We find a state fidelity of 78% in the case of no detection events, indicating a retention of genuine quantum entanglement. In the case of two detection events, which indicate a likely data qubit error in the first cycle, we find elements away from the ideal positions. By applying the recovery operation in post-processing (a single bit-flip on the blue data qubit) we can restore the state. Energy relaxation, the most likely cause of the detected bit-flip error, induces both bit-flip and phase-flip errors. The bit-flip error is corrected and the diagonal terms are preserved, but any phase-flip error remains uncorrected, reducing the off-diagonal terms and fidelity to 59%. We note that genuine entanglement is preserved. Conditional tomography for every configuration can be found in Supplementary Information.

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to see if the input is recovered. Owing to the topological nature of errors in space and time, we either recover the logical state, or the bit-wise inverse (see Supplementary Information). The fidelity of the repetition code algorithm is defined by the success rate of this recovery. In our inverse (see Supplementary Information). The fidelity of the repetition code is greater than 0.98, indicating a non-zero off-diagonal elements, indicating some bit-flip errors are attributed to the statistically increasing number of odd parity measurement qubit errors. A successful recovery converts the measured data qubit outcomes into detection events and matched to find likely errors, see Fig. 2. A successful recovery converts the measured data qubit state into the input state. Memory fidelity versus time and cycles for a single physical qubit (black) and the five- (blue) and nine- (red) qubit repetition code. Note that energy relaxation decays from a fidelity of 1 to 0, whereas the repetition code decays from a fidelity of 1 to 0.5. Five qubit data sampled from nine qubit data, see Supplementary Information. The average physical qubit lifetime (‘data qubit avg.’) is $T_1 = 29 \mu s$, and after eight cycles we see an improvement in error rate by a factor of 2.7 (blue arrow at right) for five qubits (5 qubit RC), and 8.5 (red arrow at right) for nine qubits (9 qubit RC) when using the repetition code. This indicates a $A$ parameter of 3.2 (green arrow) for our system after eight cycles. Average number of detection events per measurement qubit (open symbols), versus cycle number, for experiments consisting of eight cycles. We see an increasing average rate of detection events (black line) with increasing cycle number. This can be attributed to the statistically increasing number of odd parity ZZ measurements, see text. Grey regions indicate initial and final data qubit state leakage. Initial logical states of all 0s or 1s have even parity for all ZZ operators, maintaining the initial measurement qubit |0\rangle state. A bit-flip error on a data qubit, statistically more likely with increasing cycle number, will cause the nearby ZZ operators to have odd parity. This will flip measurement qubits between the |0\rangle and |1\rangle state at each cycle, making them susceptible to energy relaxation and hence increasing the rate of detection events (see Supplementary Information). Figure 4 demonstrates state preservation through error correction. We emphasize that we correct errors that intrinsically arise from the measurement qubit. Additionally, we see larger repetition codes leading to greater error suppression. This is evidence for the system operating with fidelities above the repetition code threshold. As the error rates...
grow with cycle number, the many-cycle behaviour of the repetition code must be explored to ensure that the system remains above threshold. The ratio of the errors for the \( n = 1 \) and \( n = 2 \) cases after eight cycles suggests \( A = 3.2 \), but larger system sizes are needed to infer this accurately for large \( n \) and verify that the logical error rate is suppressed exponentially as \( \text{logical error rate} \propto 1/A^{n-1} \), as desired.

Our demonstration that information can be stored with lower error in logical states than in single physical qubits shows that the basic physical processes required to implement surface code error correction are technologically feasible. We hope that our work will help to accelerate research into the many outstanding challenges that remain, such as the development of two-dimensional qubit arrays with scalable wiring and four-qubit QND parity checks, improving gate and measurement fidelities\(^\text{27} \), and investigating the many-cycle behaviour of error correction schemes.

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