Parametric investigation of functionally gradient wave springs
designed for additive manufacturing

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Abstract
Functionality and design of mechanical springs are simple and limited due to manufacturing constraints of conventional fabrication methods being used for making helical and wave springs. In recent era, design for additive manufacturing has proven its great worth to design and manufacture optimal, complex as well as intricate structures with better mechanical and lightweighting properties. This study aims to investigate the mechanical behavior of functionally gradient wave springs as a function of variation in thickness and morphology of each wave. Functionally gradient wave springs incorporated with different morphology and cross-sections including circular, rectangular and combination of both were designed and printed by keeping mass and height constant to investigate their mechanical properties. Loading–unloading experimentation was conducted within the elastic range (90% of compressible distance) in order to study energy absorption/loss, load-bearing capacity and stiffness of all designs. The experimental results were validated by finite element analysis (FEA) by providing the identical boundary conditions of experimental setup. The results revealed that the stiffness of wave spring having rectangular cross-section is increased significantly, while energy absorption is almost 90% increased due to circular cross-section of waves. Overall, the design with combination of round and rectangular cross-sectional waves has better stiffness and energy absorption properties. For further investigation of mechanical properties due to variation in cross-section of waves, more designs including semi-circular and filleted waves were designed, and FEA of those showed that 786 N of load-bearing capacity is achieved in the wave spring having semicircular cross-section of waves which is double than the wave spring having variable circular cross-section of waves.

Keywords Additive manufacturing (AM) · Functionally gradient wave spring (FGWS) · wave spring · Energy absorption · FEA · Stiffness · Design for AM

1 Introduction
Design for additive manufacturing (DfAM) gives the freedom to design and manufacture the structures which are nearly impossible to design by traditional manufacturing (TM) processes, such as additive manufacturing (AM) of acetabular hip prosthesis cup [1], functionally gradient triply periodic minimal surface (TPMS) structures [2, 3]. The mechanical properties such as energy absorption, stiffness and load-bearing capacity are major considerations for designing a structure. Different types of functionally gradient structures have been designed for efficient energy absorption and concluded that the performance of graded structures can be improved by optimized designs [4]. Lightweight

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structures such as bio-inspired structures are investigated a lot by the researchers because of its excellent energy absorption capacity [5]. Various shapes of bio-inspired aluminum-nested structures were evaluated for the crashworthiness applications as the finite element model was developed and validated against experimental testing [6, 7]. Advanced composite sandwich structure with tubular inserts to quasi-static compression was designed for blast protection and having high energy absorption [8]. Lei Zhang et al. studied the characteristics of metallic triply periodic minimal (TPMS) surface sheet structures under compression and found that TPMS sheet structures (primitive, diamond, Gyroid) have superior mechanical properties than body centered cubic (BCC) lattices [9].

Another type of structures, i.e., auxetic structures can also be used for energy absorption even under crushing load [10, 11]. Other than the structures, researchers have studied different materials to be used for energy absorption such as polystyrene foams which are used for protective helmets [12]. Even foam-filled multi-cell square columns were manufactured and found high-energy absorption efficiency [13, 14]. Functionally graded and multi morphology sheet TPMS lattice structures were designed and manufactured by AM and found that the variations in these caused the betterment of mechanical properties [15, 16]. A similar study has been conducted to study the buckling behavior of AM cellular columns and found that the buckling greatly depends on the distribution of mass [17, 18].

Other than the energy absorbing structures, springs are used as efficient energy absorber. There are a variety of springs according to their shape as well as the type of application of force [19]. Among all the traditionally manufactured springs [19, 20], helical springs have been studied the most for its mechanical behavior [21], finite element analysis of closely coiled helical springs [22], the effect of various parameters [23] as well as variable dimensions [24] were also investigated. The same author designed and investigated the performance of midsole by using the variable dimension helical springs [25]. Also, helical springs were investigated for the static as well as dynamic characteristics in the application of suspensions of railway vehicles [26].

Apart from conventional designs, researchers are working on the designing and performance for composite wave springs due to its high strength-to-weight ratio, corrosion-resistant [27]. M. Taktak et al. developed a dynamic optimization method based on four geometrical properties of the spring as design variables, i.e., wire diameter, middle helix diameter, active coil number and helix pitch, by which mass is greatly reduced which improved design quality along with the mechanical properties [28].

Researchers have investigated for the material selection, design and performance of composite leaf spring which has great importance in the suspension system of automobiles for the damping effect by changing the thickness and width of each leaf to optimize the design for better mechanical properties [29]. Composite as Belleville springs are also studied for automobile application and found a smooth, variable spring rate curve by reducing frictional forces with the use of spacers within the spring stacks; it is also concluded that stacking arrangement of spring, i.e., in series or in parallel, can decrease the spring rate as well as increase the stiffness, respectively, which resulted significantly improved vehicle dynamic performance through reduction of the tire contact patch, force variation and vehicle body acceleration [30].

Another type of spring, i.e., wave spring, is also getting the attention of researchers due to its unique design as well as better performance than helical springs which are commercially available in different designs [31], having multiple applications including midsole for running shoes [32] and spinal implants [33]. Naziftoosi et al. studied the analytical model for presenting composite wave springs and optimized the design by keeping the outer diameter constant while changing the inside diameter and thickness of the strip of the spring. It was concluded that diameter ratio has nonlinear relation with the thickness of the strip as well as with the stiffness of the spring; also the deformation mode in such structure is primarily bending in nature [34]. Ultra-light multifunctional using thermos-elastic processing, metallic foils were shaped into linear wave springs by which the mass of the spring is greatly reduced as compared to conventional spring. He found that under compression, the number of arcs in the spring increases which enhances the load-bearing capacity and spring constant [35]. Extensive literature review revealed that there is hardly any research has been published for manufacturing of wave springs as well as the composite springs including helical and leaf, by AM. Composite springs, as well as wave springs with variable dimensions, are almost impossible to manufacture by traditional machining processes as machines are customized [36]. Haq et al. designed, successfully manufactured variable dimension wave springs along with the variation in geometric shape and investigated the mechanical properties which have been improved [37]. This study only covered the design effect on the mechanical properties but did not investigate further detailed effect of parametric variations on the mechanical performance of such designs.

As mentioned above that the researchers used variation in morphology and parameters or combination of different structures to optimize and improve the mechanical properties, so keeping in view the same approach, the objective of this study is to design and manufacture functionally gradient wave springs in which the circular and flat waves are integrated together having variations in dimensions and in cross-sectional area. These changes were considered to investigate their effect on mechanical properties, i.e., energy absorption, stiffness, load-bearing capacity, stress–strain, and the compression trend. In addition to that, further changes were made to parametrize the wave springs to understand deeply.
the effect on aforementioned properties. Finite element analysis was carried out for all designs to verify the experimental results as well as to support more about our refined design considerations for the improvement of mechanical properties and compression trend.

2 Designing of functionally gradient wave spring

In this study, two main categories of wave springs are included for analysis, one is variable thickness in which the thickness of each wave is different, while the other is composite or function gradient wave springs (FGWS), in which the waves have variation in cross-sectional areas. Both categories are contact wave springs, i.e., the waves are permanently merged with each other, designed by using Solidworks software (Dassault Systems Solidworks Corporation, France) [38] by defining a circle having the diameter equal to the outer diameter of the wave spring, dividing the circle in eight equally distant points having an angle of $45^\circ$ between them and then using derived sketch command. The points were joined by 3D sketch curve to define the path for sweep command. The same as above, a circular cross-sectional area was defined to sweep and got the final shape of the wave. The steps are shown in Fig. 1a.

The difference of design for variable dimension wave springs was the swept profile as the springs were manufactured by either defining the rectangular cross-section of each wave (flat) of the wave spring or by defining the circular cross-section of wave (circular). Finally, assemble the waves by using assembly command and define the surface-to-surface contact between the waves. The nomenclature of wave spring is illustrated in Fig. 1b.

The outer diameter, mass and height of each design were kept constant for the uniform comparison of results for all designs. All the parameters are summarized in Table 1.

The complete assembly of each designed spring is shown in Fig. 2.

Each design has its own variations as in design A1 the maximum thickness of the strip is 3 mm which is the bottom wave while minimum thickness is 1 mm of top wave. Design A2 has uniform thickness variation of 0.5 mm in each wave as having maximum thickness is in the bottom of 2.60 mm

Table 1 Summary of design parameters of each design of wave spring with variations

| n  | Name                              | Diameter (mm) | Width of strip (mm) | Thickness of strip (mm) | Diameter of strip (mm) | Height (mm) | Mass (g) |
|----|-----------------------------------|---------------|---------------------|-------------------------|------------------------|-------------|----------|
| A1 | Variable thickness (variation 1 mm) | 35            | 4                   | 3–1                     | -                      | 17.9        | 14.21    |
| A2 | Variable thickness (variation 0.5 mm) | 35            | 4.5                 | 2.60–1.60               | -                      | 17.2        | 14.62    |
| A3 | Round between flat                | 35            | 5                   | 2.50,1.50               | 2.5                    | 17.8        | 14.20    |
| A4 | Flat between round                | 35            | 6                   | 2                       | 3.2                    | 18.5        | 13.94    |
| A5 | Rectangular with fillet           | 35            | 5                   | 3–2                     | 1.50–1                 | 18.4        | 14.73    |
while the minimum is 1.60 mm of top wave. Design A3 and design A4 are the combinations of flat and circular waves as in design A3 middle wave is round having diameter of 2.5 mm, while top and bottom waves have 1.50 mm and 2.5 mm thickness, respectively; design A4 has flat middle wave with thickness of 2 mm, and bottom and top waves are flat of thickness 3 mm and 2 mm, respectively. By using the design freedom of AM, A5 wave spring is designed which has the variation between the outside thickness and inside thickness of the strip for each wave, provided the fillet at the meeting point of these variable edges. The cross-section area of one of the waves for design A5 is shown in Fig. 3.

3 Materials and methods

3.1 Materials

PA12 (Nylon 12 polymer) material was used to print the parts and properties of the material that were attained by tensile testing. Five tensile standard test specimens of type IV were printed by the same material and by MultiJet fusion, MJF 580 [39] machine. The variation of the thickness and width among the specimens is negligibly small. Tensile testing was performed by MTS Insight universal testing machine (MTS System Corporation, USA). The results of tensile tests are shown in Table 2.

The tensile specimens were tested according to ASTM D638. The results are shown in Fig. 4.

The material properties while taking the average of all specimens are shown in Table 3.

3.2 Methods

AM is a unique method of manufacturing in which complex geometries can be manufactured with better repeatability of samples of each design but with relatively low surface finish. FGWS for this study were manufactured by MultiJet fusion technology (MJF) which is a hybrid AM process that can build parts with much faster printing speed than other AM techniques [40]. MJF, developed in 2014, is a proprietary technology of Hewlett-Packard (HP) Inc. It utilizes an array
of infrared lamps as the energy source to fuse the layer of powder jetted with a fusing agent by inkjet nozzles that can absorb infrared radiation energy. A water-based detailing agent is jetted around contours of the printed parts to inhibit the fusion of powder near the part edges and to improve part resolution, as illustrated in Fig. 5. When printing large parts, the detailing agent is also jetted into specific areas within the large parts to prevent partially excessive thermal accumulation [41].

MultiJet Fusion 580 Color 3D Printer (Hewlett-Packard, USA) [39] has used to manufacture these springs without any support structures. Full color parts are applicable only with this printer, but this aspect of color printing is not considered in this research. All manufactured wave springs are shown in Fig. 6.

The designs for this study are difficult to manufacture by the traditional manufacturing (TM) processes as the thickness is variable for each wave, while in second category flat and round waves are combined together which changes the overall morphology of the spring, but the overall geometric shape of the spring is constant for all the designs. The wave springs fabricated by traditional manufacturing has constant shape and dimensions of raw material as most of the machines are customized on a single setting to manufacture these springs. Hence, the variations in cross-section, shape, width and thickness are impossible while using the same facility. Therefore, to make any changes in the parameters of raw material, customization of the machine needs to be altered which is costlier and sometimes impossible. In comparison with AM wave springs, DfAM enables variations in thickness, width and diameter, even in each wave which can lead the manufacturing with specific strength and overall different morphology wave springs.

Uniaxial compression testing (loading–unloading) was performed for the three specimens of each design of wave springs. The printed samples were tested by MTS Insight universal testing machine (MTS System Corporation, USA) at room temperature. The crosshead speed was 300 mm/min which was high for compression testing in order to check the energy absorption during loading and energy released during unloading by the springs. To test the specimens, compressible distance for each design had been calculated by measuring the distance between each wave of each design as it is the maximum distance up to which a spring can be compressed and all the waves are fully in-contact with each other. Each design has different compressible distances as recorded in Table 4. For safety and uniformity of testing, each specimen has been compressed up to 90% of its compressible distance. The testing setup is shown in Fig. 7.

The weight of each specimen had been recorded, and average weight was considered for the calculations and analysis. By comparing the dimensions, it was found that there is negligible variation in the overall dimensions for designed and manufactured designs due to porosity, post-processing and sticking of residual powder. The summary of all parameters of each designed wave spring is presented in Table 4. Each spring was tested up to 10 cycles

| Sample # | Width (mm) | Thickness (mm) | Yield offset (%) | Peak Load (N) | Peak Stress (MPa) | Strain at break (mm/mm) | Modulus (MPa) | Stress at offset yield (MPa) |
|----------|------------|----------------|------------------|---------------|-------------------|-------------------------|---------------|----------------------------|
| 1        | 3.23       | 6.13           | 0.2              | 844           | 42.6              | 0.08                   | 1535.6        | 28.1                       |
| 2        | 3.33       | 6.23           | 0.2              | 857           | 41.3              | 0.083                  | 1457.8        | 26.0                       |
| 3        | 3.18       | 6.12           | 0.2              | 805           | 41.4              | 0.082                  | 1479.5        | 26.4                       |
| 4        | 3.2        | 6.17           | 0.2              | 836           | 42.4              | 0.083                  | 1466.1        | 26.0                       |
| 5        | 3.25       | 6.14           | 0.2              | 806           | 40.4              | 0.082                  | 1456.3        | 24.6                       |

Table 2 Results of all tensile specimen show uniformity

| Sample # | Width (mm) | Thickness (mm) | Yield offset (%) | Peak Load (N) | Peak Stress (MPa) | Strain at break (mm/mm) | Modulus (MPa) | Stress at offset yield (MPa) |
|----------|------------|----------------|------------------|---------------|-------------------|-------------------------|---------------|----------------------------|
| 1        | 3.23       | 6.13           | 0.2              | 844           | 42.6              | 0.08                   | 1535.6        | 28.1                       |
| 2        | 3.33       | 6.23           | 0.2              | 857           | 41.3              | 0.083                  | 1457.8        | 26.0                       |
| 3        | 3.18       | 6.12           | 0.2              | 805           | 41.4              | 0.082                  | 1479.5        | 26.4                       |
| 4        | 3.2        | 6.17           | 0.2              | 836           | 42.4              | 0.083                  | 1466.1        | 26.0                       |
| 5        | 3.25       | 6.14           | 0.2              | 806           | 40.4              | 0.082                  | 1456.3        | 24.6                       |

Table 3 Material properties calculated after tensile testing

| Density (g/cm³) | Young’s modulus (MPa) | Poisson’s ratio | Yield strength (MPa) | Ultimate strength (MPa) |
|----------------|-----------------------|----------------|----------------------|-------------------------|
| 1.01           | 1479                  | 0.33           | 26                   | 41                      |

Fig. 4 Engineering tensile stress–strain curve for PA 12 material
of loading/unloading because literature review suggested that loading–unloading are required to ensure a steady-state hysteresis which means a less than 3% change in hysteresis loop (area of hysteresis loop) between the two consecutive cycles of loading–unloading [43] which was attained in 8th cycle in our research. The graphs which are presented in this research were based on average value of load and compression while stiffness for each designed spring is calculated by using Eq. (1)

\[
\text{Stiffness of spring}(k) = \frac{F}{x}
\]  

(1)

The calculated stress is based on the cross-sectional area, while in variable dimension wave spring, the least area was considered for the stress calculations as it will be higher on the smaller areas.

4 Results and discussion

Five designs were printed having three samples of each, and the experimental results are recorded for load-bearing capacity, energy absorption, and energy loss in each designed wave spring. FEA has been performed for all designs to compare the experimental results with the simulation results in broader spectrum.

4.1 Compression testing (loading–unloading)

Design A1 is a variable thickness wave spring, which shows linear behavior as the load transformation is smooth with respect to compression. A2 design is also variable thickness of 0.5 mm as compared to A1 design having variation of
1 mm. A2 design has more stiffness and show higher load-bearing capacity. Also, the results show a sudden increase in load capacity once all waves are compressed fully because of material setting property, i.e., maximum densification within the elastic region of the material. Design A3 has lower load-bearing capacity as compared to other designs and once all waves are compressed, material tends to move outside from the middle wave as this wave is round, so this variable morphology round between flat waves enables the sudden increase in load-bearing capacity. Design A4 has also smooth and nearly linear behavior while having moderate load-bearing capacity as well as showing good stiffness.

### Table 4 Dimensions of printed designs along with the testing criteria for compression testing

| Design name | Avg. mass (g) | Diameter (mm) | Height (mm) | Compressible distance (mm) | 90% of compressible distance (mm) | Strain end point |
|-------------|---------------|---------------|-------------|----------------------------|----------------------------------|-----------------|
| A1          | 12.9          | 34.4          | 17.6        | 5.5                        | 5.0                              | 0.28            |
| A2          | 13.1          | 34.5          | 17.1        | 5.1                        | 4.6                              | 0.27            |
| A3          | 13.0          | 35.1          | 17.8        | 5.6                        | 5.0                              | 0.28            |
| A4          | 13.5          | 34.8          | 18.4        | 5.5                        | 5.0                              | 0.27            |
| A5          | 13.6          | 34.3          | 18.5        | 5.3                        | 4.8                              | 0.26            |

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**Fig. 7** (a) Experimental setup showing direction of application of compressive force. (b) As per testing criteria, 90% compressed designs
Design A5 can bear the highest amount of load and exhibits high stiffness properties. In all designs, under the loading, the first wave tends to compress fully, and then it transfers the load to the lower subsequent waves, so for each wave, the upper wave behaves like a base or rigid body. In this way, load is transferred to all waves and vice versa during unloading. The compression trend for all the designed springs is shown in Fig. 7b.

The experimental results for all the designs for cycle 1 and cycle 10 are shown in Fig. 8.

By comparing the load-bearing capacity of all designs, it is found that design A5 and design A2 bears maximum load because the thickness variation of wave is less than all other designs, while A3 design has the lowest load-bearing capacity because of buckling phenomena in the middle round wave. Also, it is noted that each structure shows the linear...
behavior until all the waves are completely in contact; after that point, there is sudden increase which is shown as a spike which is more evident in 10th cycle of loading–unloading as shown in Fig. 9b which show the material setting. The comparison of each design for 1st and 10th cycle is shown in Fig. 9a, b respectively.

The design A3 and A4 with round waves have shown slight change in compression as A3 design has one round wave which results more energy loss while A4 design has two round waves which results in more load-bearing capacity than others. The comparison of design A1 and design A3 after 90% compression is shown in Fig. 10.

The above comparison shows that the middle wave, i.e., round wave, behaves as a solid body to absorb more energy. A clear difference of compression has been highlighted which explains that round wave is responsible for absorbing more energy.

Stiffness of each design is calculated at 3 mm of compression to compare the stiffness of all designs and analyze the effect of variable morphology of spring. The comparison of stiffness of each design is shown in Fig. 11 which shows that A2 and A5 have almost the same stiffness which is maximum as compared to other designs because of the minimum variation in thickness of waves.

### 4.2 Energy absorption

Energy loss in each spring has been calculated for compression test of 1st and 10th cycle. The comparison of energy loss for each design is presented in Fig. 12 which shows more energy loss during 1st cycle of loading–unloading because structure will always absorb more energy at the start to develop micro-cracks and cavities which later leads to permanent deformation.

The energy absorbed by each spring is calculated by calculating areas under the loading (energy applied) and energy returned (unloading) curves and substituting the values Eq. (2):

\[
\text{Energy loss} = \frac{\text{Energy Applied} - \text{Energy Returned}}{\text{Energy Applied}} \times 100
\]  

A3 and A4 showed higher energy loss during the 1st cycle, while A1 had the lowest energy loss during 10th cycle, but overall, there is not much difference in terms of energy loss. The energy loss and energy return are highest in design A3, while the minimum energy return is in design A1. While comparing the energy loss in design A1 and design A3 as shown in Fig. 13, it is found that once the morphology of waves is changed, i.e., combination of rectangular and round cross-sectional waves, energy loss as well as energy return is more, that is why A1 having the rectangular cross-sectional waves (variable thickness) has less energy loss than A3 having variable morphology waves. The energy loss for cycles 7, 8, 9, and 10 is almost same in all the designs which shows that these designs become stable in terms of energy loss after 5th cycle of loading–unloading.

### 4.3 Stress–strain analysis

Stress is calculated for each spring for cycle 1 and cycle 10 which shows that design A4 has the highest stress while design A1 has linear stress–strain behavior. By comparing the stresses in each design, it is found that A4 has maximum stress as the top and bottom waves have round cross-sectional area, while middle wave has rectangular cross-sectional area, and also top and bottom waves have more material, i.e., this spring can bear more stress as the top wave will fully compressed then it will transfer stress to the middle and bottom wave. A3 has minimum stress as the thickness variation is more; subsequently contact area is more which results in lower stress. By comparing the 10th and 1st cycle of loading–unloading, lower in stress values is noted because the micro-cavities on the surface of each wave have been deformed gradually till the 10th cycle which causes...
the less stress to withstand in each design. Comparison of
stress strain curve for 1st and 10th cycle for each design is
shown in Fig. 14.

5 Validation of experimental results

FEA has been performed on ANSYS 19.2 for all printed
designs, and the nonlinear properties of PA 12 material were
used for the simulation. It is assumed that the parts manufac-
tured by MJF are nearly isotropic [40, 42]. Frictional con-
tacts have been applied between waves [44] as well as plate
to wave with friction coefficient of 0.20 [45]. The simulation
setup was identical with experimental setup as boundary
condition was the same by fixing the bottom plate and apply-
ing displacement from the top plate.

Figure 15 illustrates the comparison of experimental
and simulation results for the loading curve as well as
unloading curve for each printed sample. Overall, the
comparison showed close agreement between experimen-
tal and simulation results. The experimental and simu-
lation results of linear region of each spring are same,
but there was small deviation in nonlinear region which
can be due to buckling, the contact area for the graded
dimension or the residual powder remain inside the waves
during compression testing. Design A3 has the highest
deviation in nonlinear region because of the difference
of cross-sectional areas of the waves as middle wave hav-
ing circular while top and bottom waves have rectangular
cross-sectional areas, due to which the middle wave tends
to slide or move out which resulted the deviation in experi-
mental and FEA results.

Stress distribution of each design was shown in Fig. 16
which highlights that stress is evenly distributed in each
designed wave spring.

The compression pattern of simulation as well as for
experiment was almost identical as Fig. 17 shows compari-
on of simulation and experimental pattern for A4 design.

5.1 Further parametrisation

The experimental and simulation results have close agree-
ment, also noting that these morphological changes contrib-
ute significantly in the variation of mechanical properties so
need to be investigated more; hence the following supple-
mentary designs for FGWS were designed by making some
more parametric variations. Design A6 is almost same as
design A2 as the only difference is variation in thickness of

Fig. 11 Comparison of stiffness of all designs showing variation
between 1st and 10th cycle of loading–unloading of each designed
spring

Fig. 12 Energy loss for each design in 1st and 10th cycle of loading–
unloading

Fig. 13 Comparison of energy loss in design A1 and design A3 in
each cycle
each wave which is three times more in design A6. Design A7 is related to design A1, having round waves instead of flat as it was in design A1. Design A8 and design A9 are the modified shapes of design A4 as they have the combination of flat and round waves, i.e., in design A8 has top and bottom round waves of same diameter but thicker than the middle flat wave while A9 having top and bottom wave of same diameter but thinner than the middle flat wave. Design A10 and design A11 are variant forms of design A3 as in all the variation of thickness of waves is from the middle wave, i.e., the middle wave is thicker than the top and the bottom wave. A10 has all flat waves, while A11 has all round waves.

Fig. 14 Stress–strain comparison for all designs (a) for 1st cycle and (b) for 10th cycle

Fig. 15 The comparison of experimental and FEA results for the printed samples
waves. Design A12 and design A13 are related to design A5, i.e., design A12 has variable thickness from inside to outside and provided the curvature between these within each wave, while design A13 has variable thickness of top face and bottom face provided between the curvature which are opposite in both faces within each wave. All the designs, i.e.,

![Stress distribution of each spring](image)

**Fig. 16** Stress distribution of each spring which shows equally distribution across the surface without any concentration points.

![Simulation and experimental pattern of compression](image)

**Fig. 17** Simulation and experimental pattern of compression for design A4 showed uniformity.
Table 5  Design parameters along with variations for the supplementary designs

| Design # | Name                                                   | Diameter (mm) | Width of strip (mm) | Thickness of strip (mm) | Diameter of strip (mm) | Height (mm) | Mass (g) |
|----------|--------------------------------------------------------|---------------|---------------------|-------------------------|------------------------|-------------|----------|
| A6       | Variable thickness (variation 1.5 mm)                  | 35            | 3.75                | 3.50–0.5                | -                      | 17.49       | 14.04    |
| A7       | Round (variation 1 mm)                                 | 35            | -                   | -                       | 3.2–1.2                | 18.9        | 13.12    |
| A8       | Flat wave between constant (maximum) round waves       | 35            | 4.5                 | 1                       | 3                      | 18.9        | 13.67    |
| A9       | Flat wave between constant (minimum) round waves       | 35            | 5.5                 | 2                       | 1.5                    | 17.34       | 13.20    |
| A10      | Flat waves with thickness variation from the middle     | 35            | 4.0                 | 3–2                     | -                      | 18.7        | 14.5     |
| A11      | Round waves with variation from middle                  | 35            | -                   | -                       | 3.20–2.70              | 18.8        | 13.78    |
| A12      | Semicircular cross-sectional waves (same curve)        | 35            | 5                   | 3–2                     | -                      | 18.6        | 14.8     |
| A13      | Semicircular cross-sectional waves wave spring (opposite curve) | 35            | 5                   | 3–2                     | 1.5–1                  | 18.8        | 14.6     |

Fig. 18  Complete assemble of all supplementary designs along with the cross-sectional view of each wave
design A1 till design A13, have the same designed mass and simulated with the same criteria for the ease in comparison among all designs. The description along with the variations for the supplementary designs are summarized in Table 5.

The complete assemble of all supplementary designs along with the variation are shown in Fig. 18.

The parametric change such as more variations in thickness and replacing flat waves by round waves is to study the compression trend by making the top wave soft and precedent waves much stiffer to get the different distinct stages in the same curve of load and compression. Also, the round waves always improve the energy absorption of structure, so the variation of diameter is considered for the effect on energy absorption as well as buckling phenomena. Researchers have designed the combination of different lattice structures to manufacture hybrid structures which has improved the mechanical properties of the structure [16], the same approach is used to design and manufacture the springs by the combination of round and flat waves to study the effect of stacking behavior of these waves on mechanical properties such as stiffness and energy absorption. Morphology of flat surfaces was changed to semi-flat surfaces as in design A12 and design A13 to enhance the effect of fillet by designing the curved surfaces for providing the smooth transformation of force as well as smooth contact point between the surfaces.

![Fig. 19 FEA results along with the compression of each wave for designs (a) A6, (b) A7, (c) A8, and (d) A9](image_url)
The FEA of the above designed springs have been performed and discussed in two groups on the basis of their behavior depicted in load vs compression curves, i.e., 1st group comprises of design A6 to design A9, while second group is of design A10 to design A13. FEA results for the 1st group is shown in Fig. 19.

In design A6, the behavior of the springs is consisted of four sub-stages. In the 1st stage, the load is nearly zero till the fully compression of 1st wave, and then load is transferred to second wave which resulted the load-bearing capacity linearly increased till the fully compressed second wave. After that the load is transferred to third wave which results sudden increase in the load-bearing capacity as the stiffness of the waves is increased as shown in Fig. 19a. The same phenomena can be seen in design A7 which consists of round waves instead of rectangular waves. The only difference between the behavior of design A6 and A7 is the distinct stages of compression in load versus compression graph as in design A6, four different stages while in design A7 is three, also design A6 has more load-bearing capacity than A7.

The design of A8 and A9 consists of FGWS, in which central wave is of rectangular cross-sectional while top and bottom are round cross-sectional waves. In these designs, initially, spring shows linear behavior, and then its behavior changes to nonlinear. It is also noted that, in design A8, the top and bottom waves are stiffer than the middle wave, so during the linear stage, the top and bottom waves compressed simultaneously, and then the middle wave compressed, while design A9 is vice versa of design A8 as in design A9 the top and bottom waves are softer than the middle one, so the spring start compressing form the top as 1st wave is fully compressed and then it transfers loads to lower subsequent waves.

The non-behavior has different potential advantages in different applications in which the load needs to be reduced gradually or dynamically at different stages [46]. Previously, the same authors investigated the stiffness and energy absorption of variable dimension helical springs by changing the means diameter of the spring, pitch, and wire diameter and found that the spring performance improved as compared to simple helical springs [24]. The studies have
been made for this nonlinear behavior of springs for different applications such as in robot mechanisms [47] and electrostatic actuators [48] and also nonlinear springs are modelled by using the rubber material [49]. FEA results for the stress distribution in designs for group 1 are shown in Fig. 20.

In continuation to the FEA results of the 2nd group of supplementary designed wave springs, A10 design initially shows linear and then nonlinear behavior once all waves were fully compressed, showing high stiffness, while A11 design does not have high stiffness, lower load-bearing capacity, but energy loss is more in this particular designs as can be seen by the width of hysteresis loop of this design. In both design A11 and A12 the variable dimensions are from middle wave, i.e., the maximum thickness or maximum diameter of the strip is for the middle due to which compression took place from top and bottom simultaneously.

A12 design shows the smooth transfer of force and have load-bearing capacity less than design A10 and greater than design A11. Semi-flat wave cross-sectional area increases the stiffness of the spring, which can be seen in design A13, which results to high load-bearing capacity of all the supplementary designs. The load-bearing curve has smooth bend which shows the stability of the spring as well as smooth transmission of load among the waves. The FEA results are shown in Fig. 21.

FEA results reveals that the stress distribution in the second group of supplementary designs is also uniform along all the surfaces of waves as shown in Fig. 22.
Conclusions

In this study, FGWS with variable dimensions are successfully designed and manufactured by AM which proves the versatility, freedom of design, and flexibility of manufacturability of AM. The composite wave combination, i.e., rectangular cross-sectional waves with round cross-sectional, resulted in improved energy absorption capacity; its nonlinear behavior has multiple applications in the industry. Also, the variation in the thickness of the strip is directly proportional to the load-bearing capacity and stiffness while lesser variation in the diameter of the strip for circular waves, which results to more energy absorption in the spring. The flat waves in the wave spring are responsible for higher stiffness which is increased up to 40% compared to circular waves, while the circular waves give more energy absorption as it is increased up to 10% compared to flat waves. It was also noted that if the spring has variation from the middle, the compression will start from top wave and bottom simultaneously and load is transferred to middle wave once top and bottom wave is fully compressed.

By comparing all the designs, FGWS proved to be best of all due to smooth transmission of load from one wave to another as well as better energy absorption capacity. Also, semi flat waves, i.e., design A13, proved to be better than flat waves for the construction of the wave spring as having higher load-bearing capacity.

By keeping in view, the above research, FGWS needs to be studied for the designing of dampers with variable damping coefficient in the same application and can be designed customized wave springs for high-speed train where load is varied according to the acceleration and also can be utilized as energy absorbent in automobiles.

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Availability of data and material Data is available on request from the authors.

Declarations

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Conflict of interest  The authors declare no competing interests.

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