Forward $\pi^0$ trigger of the deep inelastic + jet probe of BFKL dynamics

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Abstract

We predict the rate of deep inelastic scattering (DIS) events containing an identified forward $\pi^0$ that is expected in the experiments at the HERA electron-proton collider, in order to see if this process can be used as an indicator of the underlying small $x$ dynamics. We determine the background due to deep inelastic events containing forward photons which are fragments of the forward jet. We compare the DIS + $\pi^0$ cross section with that of the DIS + parent jet process.
1. Introduction

The behaviour of the proton structure function $F_2(x, Q^2)$ at small $x$ reflects the behaviour of the gluon distribution, since the gluon is by far the dominant parton in this regime. Perturbative QCD does not predict the absolute value of the parton distributions, but rather determines how they vary from a given input. If, for example, we are given initial distributions at some scale $Q_0^2$, then the DGLAP evolution equations enable us to determine the distributions at higher $Q^2$. DGLAP evolution resums the leading $\alpha_s \ln (Q^2/Q_0^2)$ terms. At sufficiently high electron-proton centre-of-mass energy $\sqrt{s}$ we encounter a second large variable $1/x \sim s/Q^2$, and we must resum the leading $\alpha_s \ln 1/x$ contributions. At leading order the resummation is accomplished by the BFKL equation for the (unintegrated) gluon distribution. The solution of the equation leads to a singular $x^{-\lambda}$ small $x$ behaviour of the gluon distribution, where $\lambda = (3 \alpha_s/\pi)^4 \ln 2$ for fixed $\alpha_s$ and $\lambda \simeq 0.5$ if a reasonable prescription for the running of $\alpha_s$ is assumed and for the treatment of the infrared region. The $x^{-\lambda}$ behaviour of the BFKL gluon feeds through, via the $k_T$-factorization theorem, into the small $x$ behaviour of the structure function $F_2$. Of course in practice we should not expect such a dramatic growth with decreasing $x$, since subleading effects are expected to suppress the effective value of $\lambda$ in the HERA domain.

However, it is difficult to identify the presence of the $\alpha_s \ln 1/x$ terms in the measurements of $F_2$ at HERA even though the data do show a steep rise with decreasing $x$. In fact the rise in the latest precise H1 and ZEUS measurements can be well described by next-to-leading order DGLAP evolution down to $Q^2 \sim 2$ GeV$^2$ and $x \sim 10^{-5}$. The problem in identifying the underlying small $x$ dynamics is due to the parametric freedom that we have in specifying the initial parton distributions. For instance for a non-singular gluon input we can increase the steepness of $F_2$ with decreasing $x$ by simply reducing $Q_0^2$ and increasing the DGLAP evolution length, $\ln(Q^2/Q_0^2)$. Alternatively, we could use (as in the global parton analyses) a singular input form $xg(x, Q_0^2) \sim x^{-\lambda}$, with $\lambda$ chosen to fit the data. We conclude that it is difficult to isolate $\ln 1/x$ effects from measurements of $F_2$. The observable $F_2$ is too inclusive. Rather, we should explore properties of the final state in deep inelastic scattering (DIS).

The classic way to probe the small $x$ behaviour of QCD, which avoids the problem of assuming input parton distributions, is to study deep inelastic $(x, Q^2)$ events which contain an identified forward jet $(x_j, k_{jT}^2)$, see Fig. 1(a). According to BFKL dynamics the differential structure function for DIS + jet events has the following small $(x/x_j)$ behaviour

$$\frac{\partial F_2}{\partial (\ln 1/x_j) \partial k_{jT}^2} = C \alpha_s(k_{jT}^2)x_j \left[ g + \frac{4}{9} (q + \bar{q}) \right] \left( \frac{x}{x_j} \right)^{-\lambda},$$

where the normalisation coefficient $C$ is given in refs. The parton distributions $g, q$ and $\bar{q}$ are to be evaluated at $(x_j, k_{jT}^2)$. The relevant kinematic region is where

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1To be precise we define $x$ to be the Bjorken variable $x \equiv Q^2/2p.q$ where $Q^2 = -q^2$, and $p$ and $q$ are the four momenta of the proton and virtual photon deep inelastic probe respectively.
(i) the jet transverse momentum satisfies $k_{jT}^2 \simeq Q^2$ so as to neutralize the DGLAP evolution, and is sufficiently large so as to suppress diffusion into the infrared region when we solve the BFKL equation at decreasing values of $x/x_j$;

(ii) the jet longitudinal momentum $x_j p$ is as large as is experimentally feasible (and $x$ is as small as possible) so as to be able to probe the region of small $x/x_j$.

For these values of $x_j$ the parton distributions entering [1] are well known from the global parton analyses and so the observation of DIS + jet events offers the opportunity to expose BFKL-type small $x$ dynamics free from the ambiguities associated with the choice of the non-perturbative parton input. In other words we are studying small $x$ dynamics by deep inelastic scattering off a known parton, rather than off the proton. Experimentally, however, the clean identification and kinematic measurement of a forward jet proves to be difficult since we require it to be as close to the proton remnants as possible, that is $x_j$ as large as possible. Nevertheless experimental studies have been attempted and lead to encouraging results [14].

Here we use the improved knowledge of the fragmentation functions to propose that the forward jet is identified through the measurement of a single energetic decay product. As it turns out the $\pi^0$ is the hadron which can be identified in the most forward direction in the detectors at HERA. We use the BFKL formalism to predict the DIS + forward $\pi^0$ rate. The rate will, of course, be suppressed in comparison with the DIS + forward jet rate and it is an experimental issue to see if the advantages of single particle detection as compared to identification of the (parent) jet can compensate for the loss of signal.

The outline of the paper is as follows. In section 2 we present the QCD formalism required to calculate the cross section for the deep inelastic + forward $\pi^0$ process. Then in section 3 we discuss the experimental cuts which we impose and give our numerical predictions for the DIS + $\pi^0$ cross section. Section 4 contains a discussion.

2. The DIS + forward $\pi^0$ cross section

First we recall the derivation of the cross section for the deep inelastic + jet process depicted in Fig. 1(a), which also shows the variables used. The differential cross section is given by [13]

$$\frac{\partial \sigma}{\partial x \partial Q^2} = \int dx_j \int dk_{jT}^2 \frac{4\pi\alpha^2}{xQ^4} \left[ (1 - y) \frac{\partial F_2}{\partial x_j \partial k_{jT}^2} + \frac{1}{2} y^2 \frac{\partial F_T}{\partial x_j \partial k_{jT}^2} \right]$$

(2)

where the differential structure functions have the following leading small $x/x_j$ form

$$\frac{\partial^2 F_i}{\partial x_j \partial k_{jT}^2} = \frac{3\alpha_S(\beta_{jT})}{\pi k_{jT}^4} \sum_a f_a \left( x_j, k_{jT}^2 \right) \Phi_i \left( \frac{x}{x_j}, k_{jT}^2, Q^2 \right)$$

(3)
for $i = T, L$. Assuming $t$-channel pole dominance the sum over the parton distributions is given by
\[
\sum_a f_a = g + \frac{4}{9} (q + \bar{q}).
\] (4)

Recall that these parton distributions are to be evaluated at $(x_j, k_{jT}^2)$ where they are well-known from the global analyses, so there are no ambiguities arising from a non-perturbative input.

The functions $\Phi_i(x/j, k_{jT}^2, Q^2)$ in (3) describe the virtual $\gamma +$ virtual gluon fusion process including the ladder formed from the gluon chain of Fig. 1(a). They can be obtained by solving the BFKL equation
\[
\Phi_i(z, k_{T}^2, Q^2) = \Phi_i(0)(z, k_{T}^2, Q^2) + \frac{3\alpha_s}{\pi} \int_0^1 \frac{dz'}{z'} \int_0^\infty \frac{dk_{T}^2}{k_{T}^2} \left[ \frac{\Phi_i(z', k_{T}^2, Q^2) - \Phi_i(z', k_{T}^2, Q^2)}{|k_{T}^2 - k_{T}^2|} + \frac{\Phi_i(z', k_{T}^2, Q^2)}{\sqrt{4k_{T}^4 + k_{T}^4}} \right].
\] (5)

The inhomogeneous or driving terms $\Phi_i^{(0)}$ correspond to the sum of the quark box and crossed-box contributions. For small $z$ we have
\[
\Phi_i^{(0)}(z, k_{T}^2, Q^2) \approx \Phi_i^{(0)}(z = 0, k_{T}^2, Q^2) \equiv \Phi_i^{(0)}(k_{T}^2, Q^2).
\] (6)

We evaluate the $\Phi_i^{(0)}$ by expanding the four momentum in terms of the basic light-like four momenta $p$ and $q' \equiv q + xp$. For example, the quark momentum $\kappa$ in the box (see Fig. 1(a)) has the Sudakov decomposition
\[
\kappa = \alpha p - \beta q' + \kappa_T.
\]

We carry out the integration over the box diagrams, subject to the quark mass-shell constraints, and find
\[
\Phi_T^{(0)}(k_{T}^2, Q^2) = 2 \sum_q e_q^2 \frac{Q^2}{4\pi^2} \alpha_s \int_0^1 d\beta \int d^2\kappa_T \left( \beta^2 + (1 - \beta)^2 \right) \left( \frac{\kappa_T^2}{D_1^2} - \frac{\kappa_T(\kappa_T - k_T)}{D_1D_2} \right),
\]
\[
\Phi_L^{(0)}(k_{T}^2, Q^2) = 2 \sum_q e_q^2 \frac{Q^4}{\pi^2} \alpha_s \int_0^1 d\beta \int d^2\kappa_T \beta^2(1 - \beta)^2 \left( \frac{1}{D_1^2} - \frac{1}{D_1D_2} \right),
\] (7)

where the denominators $D_i$ are of the form
\[
D_1 = \kappa_T^2 + \beta(1 - \beta) Q^2
\]
\[
D_2 = (\kappa_T - k_T)^2 + \beta(1 - \beta) Q^2,
\] (8)

assuming massless quarks.
If the QCD coupling $\alpha_S$ is fixed we can solve the BFKL equation (5) and obtain an analytic expression for the leading small $z$ behaviour of the solution. Omitting the Gaussian diffusion factor in $\ln \left( \frac{k^2_T}{Q^2} \right)$ we find

$$\Phi_T(z, k^2_T, Q^2) = \frac{9\pi^2}{512} \frac{2 \sum e_q^2 \alpha_S}{\sqrt{21 \zeta(3) / 2}} \left( k^2_T Q^2 \right)^{\frac{3}{2}} \frac{z^{-\alpha_P + 1}}{\sqrt{\ln(1/z)}} \left[ 1 + O \left( \frac{1}{\ln(1/z)} \right) \right]$$

(9)

$$\Phi_L(z, k^2_T, Q^2) = \frac{2}{9} \Phi_T(z, k^2_T, Q^2)$$

where the Riemann zeta function $\zeta(3) = 1.202$ and the BFKL intercept

$$\alpha_P - 1 = \frac{12 \alpha_S}{\pi} \ln 2.$$  

(10)

Here, however, we follow the approach of [12] and allow the coupling $\alpha_S$ to run. This means that we must numerically solve the BFKL equations for

$$H_i(z, k^2_T, Q^2) \equiv \frac{3\alpha_S(k^2_T)}{\pi} \Phi_i(z, k^2_T, Q^2).$$

(11)

We use the differential form of the equations,

$$\frac{\partial H_i(z, k^2_T, Q^2)}{\partial \ln(1/z)} = \frac{3\alpha_S(k^2_T)}{\pi} k^2_T \int_{k^2_0}^\infty \frac{dk^2}{k^2_T} \left[ H_i(z, k^2_T, Q^2) - H_i(z, k^2_T, Q^2) \right] + \frac{H_i(z, k^2_T, Q^2)}{\sqrt{4k^4_T + k^4_T}}$$

subject to the boundary conditions

$$H_i(z = z_0, k^2_T, Q^2) = H_i^{(0)}(k^2_T, Q^2).$$

(12)

(13)

For the lower limit on the transverse momentum integration we choose $k^2_0 = 1 \text{ GeV}^2$. We start from the “box” expressions, (7), for $H_i^{(0)}$ at $z_0 = 0.1$ and solve (12) to obtain $H_i$ (and $\Phi_i$) for $z < z_0$. In this way we predict the cross section for DIS + jet production from equations (2) and (3).

Next let us consider the process where the forward jet fragments into $\pi^0$’s as shown schematically in Fig. 1(b). We are looking at the case where the $\pi^0$ is collinear with the parent quark jet. This means that if the $\pi^0$ carries a fraction $x_\pi$ of the proton’s longitudinal momentum, then it carries a fraction $z = x_\pi / x_j \ (0 \leq z \leq 1)$ of the parent jet’s longitudinal momentum and its transverse momentum $k_{\pi T} = z k_{j T}$. In order to calculate the cross section for DIS + $\pi^0$ production we have to convolute the DIS + jet cross section with the $\pi^0$ fragmentation functions. We obtain

$$\frac{\partial \sigma_\pi}{\partial x_\pi \partial k_{\pi T}} = \int_{x_\pi}^1 dz \int dx_j \int dk_{\pi T} \left[ \frac{\partial \sigma_q}{\partial x_j \partial k_{\pi T}^2} D_q^\pi(z, k_{\pi T}^2) + \sum_q \left( \frac{\partial \sigma_q}{\partial x_j \partial k_{\pi T}^2} D_q^\pi(z, k_{\pi T}^2) \right) \right] \times \delta(x_\pi - z x_j) \delta(k_{\pi T} - z k_{j T})$$

(14)
where the sum over \( q \) runs over all quark and antiquark flavours. The partonic differential cross sections can be obtained from (2) and (3) by substituting for the sum over the parton distributions \( \sum f_a \) either the gluon distribution \( g \) or the quark or antiquark distribution \( \frac{4}{9}q \) or \( \frac{4}{9}\bar{q} \) respectively. In analogy to choosing \( z_0 = 0.1 \) in (13) we impose the constraint \( x/x_\pi < 0.1 \), i.e. \( x/x_j < 0.1 \) since \( x_\pi < x_j \), on the \( x_j \) integration here. The functions \( D_\pi^0(z,k_{\pi T}^2) \) and \( D_q^0(z,k_{\pi T}^2) \) in (14) give the probability that a gluon or quark jet fragments into a \( \pi^0 \) carrying a fraction \( z \) of the parent jet’s momentum. We assume that these fragmentation functions satisfy leading order DGLAP evolution equations. Note that SU(2) isospin symmetry implies that

\[
D_i^\pi^0(z,k_{\pi T}^2) = \frac{1}{2} \left( D_i^{\pi^+}(z,k_{\pi T}^2) + D_i^{\pi^-}(z,k_{\pi T}^2) \right)
\]

for all partons \( i = q, g \). Therefore (14) describes the average of the cross sections for \( \pi^+ \) and \( \pi^- \) production. We use the parametrizations of the leading order charged pion fragmentation functions obtained by Binnewies et al. [15]; their analysis treated light, s, c, and b quarks independently for the first time.

### 3. Predictions for the DIS + \( \pi^0 \) cross section

We use (14) to calculate the event rate for deep inelastic scattering in which, in addition to the scattered electron, the \( \pi^0 \) is measured in the final state. To ensure that the \( \pi^0 \) is really a fragment of the forward jet (and does not come from the quark-antiquark pair which form the quark box) we require the \( \pi^0 \) to be emitted in the forward hemisphere in the virtual photon-proton centre-of-mass frame. If we express the pion four momentum in terms of Sudakov variables

\[
k_\pi = x_\pi p + \beta_\pi q' + k_{\pi T}
\]

then the forward hemisphere requirement is

\[
x_\pi > \beta_\pi.
\]

Since the outgoing pion satisfies the on-mass-shell condition \( k_\pi^2 = m_\pi^2 \approx 0 \) we have

\[
\beta_\pi = \frac{x}{x_\pi} \frac{k_{\pi T}^2}{Q^2}.
\]

Then (17) gives

\[
x_j > x_\pi > \sqrt{x k_{\pi T}^2 / Q^2}.
\]

We thus have an implicit lower limit on the \( x_j \) integration in (14), which is generally stronger than the condition \( x_j > 10x \) imposed on the solution of the BFKL equation.

Another problem to be taken into account is that at HERA pions can only be detected if they are emitted at a large enough angle to the proton beam. This also ensures that there is no contamination from pions produced in the proton remnant. We require

\[
\theta_{\pi p} > \theta_0.
\]
In Fig. 2 we show the relation between the pion kinematic variables for different choices of the minimum angle $\theta_0$ defined in the HERA frame. We find that pions with large longitudinal momentum fraction $x_\pi$ are only emitted at small angles $\theta_{\pi p}$. To reach larger $x_\pi$ for a given $\theta_{\pi p}$ we can measure pions with larger $k_{\pi T}^2$ but at a depleted event rate. In the same figure we also plot the boundary given by the hemisphere cut, (17), for $x = 6 \times 10^{-4}$ and $Q^2 = 20$ GeV$^2$, which acts as a lower limit on the allowed kinematic region. We will use $\theta_0 = 5^\circ$ for the main presentation of our results (although later we compare the predictions with those obtained with $\theta_0 = 7^\circ$).

Now we are in the position to give numerical predictions for the cross section for the DIS $+ \pi^0$ production using (14) and implementing the cuts that we just discussed. Recall that it follows from (17) that the cross section for $\pi^0$ production equals the average of the cross sections for $\pi^+$ and $\pi^-$ production. Therefore the results we will show in the following multiplied by a factor of 2 will be valid for charged pion production. Throughout the analysis we assumed three flavours of massless quarks. In Fig. 3 we plot the $x$ dependence of this cross section integrated over bins of size $\Delta x = 2.10^{-4}$ and $\Delta Q^2 = 10$ GeV$^2$ for three different $Q^2$ bins, namely 20-30, 30-40, 40-50 GeV$^2$. Here we required that $x_\pi > 0.05$ and $3 < k_{\pi T} < 10$ GeV and used the fragmentation functions [15] at scale $k_{\pi T}^2$. We compare the results obtained when BFKL small $x$ resummation is included with the case when gluon radiation is neglected. In the first case the strong $x$ dependence of the cross section is driven by the small $z$ behaviour of the $\Phi_i$ and therefore there is a strong enhanced increase with decreasing $x$. For example, if we compare the cross section for $x \approx 5 \times 10^{-4}$ in the two cases, we find that the results are about a factor 7 larger when the BKFL resummation is included than when it is neglected. This enhancement is the signature of BFKL soft gluon resummation. In fact the BFKL behaviour should be identified via the shape in $x$ rather than the value of the cross section, since the latter is subject to normalisation uncertainties [12]. In Fig. 4 we show the cross section (in fb), for the same cuts as in Fig. 3, in various bins in $x$ and $Q^2$ which are accessible at HERA. We find that the cross section drops off very rapidly with $Q^2$ which means that we can reach the highest values for the bins with $10 < Q^2 < 20$ GeV$^2$ and $x$ very small.

Of course the DIS $+ \pi^0$ cross section depends on the values chosen for the cuts. In table 1 we show the effect of changing the limits on the $k_{\pi T}$ integration. Since the cross section decreases with increasing $k_{\pi T}$ it is more sensitive to the lower limit on the $k_{\pi T}$ integration than to the upper limit.
Table 1: The DIS + $\pi^0$ cross section in the bin $10^{-3} < x < 1.2 \times 10^{-3}$, $20 < Q^2 < 30$ GeV$^2$ as calculated in Fig. 4, but for different choices of the limits of the integration over the transverse momentum of the $\pi^0$.

| $k_{\pi T,\text{min}}$ [GeV] | $k_{\pi T,\text{max}}$ [GeV] | $\sigma$ [pb] |
|-----------------------------|-------------------------------|-------------|
| 3                           | 8                            | 0.23        |
| 3                           | 10                           | 0.26        |
| 5                           | 10                           | 0.18        |

To obtain the results shown in Figs. 3 and 4 we used the pion fragmentation functions at scale $k_{\pi T}^2$. In Table 2 we show the cross section for the deep inelastic + $\pi^0$ process in the bin $10^{-3} < x < 1.2 \times 10^{-3}$, $20 < Q^2 < 30$ GeV$^2$ calculated imposing the same constraints and including BFKL soft gluon resummation but evaluating the pion fragmentation functions at the scales $\frac{1}{2} k_{\pi T}^2$, $k_{\pi T}^2$ and $2 k_{\pi T}^2$. The values demonstrate the scale ambiguity in the prediction of the cross section.

Table 2: The DIS + $\pi^0$ cross section in the bin $10^{-3} < x < 1.2 \times 10^{-3}$, $20 < Q^2 < 30$ GeV$^2$ calculated imposing the same cuts as for Fig. 4, but evaluating the fragmentation functions at the three different scales $\frac{1}{2} k_{\pi T}^2$, $k_{\pi T}^2$ and $2 k_{\pi T}^2$.

| fragmentation scale | $\sigma$ [pb] |
|---------------------|----------------|
| $\frac{1}{2} k_{\pi T}^2$ | 0.31 |
| $k_{\pi T}^2$        | 0.26 |
| $2 k_{\pi T}^2$      | 0.23 |

Since $\pi^0$'s are measured through their decay into two photons there is a background from events in which the parent jet fragments into a photon which is emitted collinearly, see Fig. 5. In analogy to (14) the corresponding cross section is given by

$$
\frac{\partial \sigma}{\partial x_\gamma \partial k_{\gamma T}^2} = \int_{x_\gamma}^1 d' z \int dx_j \int dk_{j T}^2 [\frac{\partial \sigma}{\partial x_j \partial k_{j T}^2} D^\gamma (z, k_{\gamma T}^2) + \sum_q \left( \frac{\partial \sigma}{\partial x_j \partial k_{j T}^2} - D^\gamma (z, k_{\gamma T}^2) \right) \times \delta (x_\gamma - z x_j) \delta (k_{\gamma T} - z k_{j T})]. \quad (21)
$$

We estimated this background using the fragmentation functions of Owens [16] and found that it is 1-2 % of the cross section for $\pi^0$ production.

Considering the smallness of the background from photons which are fragments of the forward jet, a comment on the errors on the calculation of the cross section for pion production is due here. From the numerical point of view there is an error from the Monte-Carlo integration used to evaluate (14) which is of the order of 5 %. To our knowledge the errors on the pion
fragmentation functions are of the order of a few percent for quarks and 30 - 40 \% for gluons \[^{17}\] Since the dominant contribution to the cross section for pion production comes from the fragmentation of gluons we expect these errors on the fragmentation functions to result in an error of at most 25 \% on the cross section. The parametrizations of the fragmentation functions describe the DGLAP evolution correctly up to 10 \% \[^{15}\]. We found that our results are more sensitive to the normalisation of the fragmentation functions than to their shape.

Finally let us compare our predictions for DIS + forward $\pi^0$ production as shown in Fig. 4, with the corresponding cross sections for the DIS + forward jet events, the process originally proposed by Mueller as the probe of small x dynamics. In order to quantify the suppression due to the fragmentation of the jet into the $\pi^0$ we integrate the DIS + jet differential structure functions given in (3) over the same domains of $x_j$ and $k^2_{jT}$ that we used for $x_\pi$ and $k^2_{\pi T}$ for the DIS + $\pi^0$ predictions. To be precise we integrate over the region $3 < k_{jT} < 10$ GeV and $\theta_{jp} > 5^\circ$ with a hemisphere cut for the jet in analogy to (17), that is $x_j > \beta_j$. The upper and lower numbers in Fig. 6 compare the DIS + $\pi^0$ with the DIS + jet cross section in the various bins of $x$ and $Q^2$. We see that the fragmentation of the forward jet into a $\pi^0$ meson costs a factor of about 40 in the suppression in event rate. Whether this loss of event rate is compensated by the advantage of identifying a forward $\pi^0$ as compared to a jet (adjacent to the proton remnants) is an experimental question. Table 3 offers a guide to the possible gain using the $\pi^0$ signal. For instance if we were able to identify $\pi^0$ mesons down to $5^\circ$ in angle and 5 GeV in $k_T$ with the same accuracy as jets down to $7^\circ$ in angle and 7 GeV in $k_T$ then we would gain back a factor of $4^2$. Moreover if we were to add in the DIS + forward $\pi^\pm$ signal then we gain an extra factor of 3. Table 3 also shows that in the HERA regime, where we need to take $x_j$ sufficiently large (say $x_j > 0.05$) to make $x/x_j$ small, the low $k_T$ events are kinematically forbidden by the cuts. For example for $\theta_0 = 7^\circ$ we find that $k_T > 5.0$ GeV, while for $\theta_0 = 5^\circ$ we have $k_T > 3.6$ GeV.

4. Conclusion

In principle, the DIS + jet measurement should be an excellent way of identifying the BFKL soft gluon resummation effects at HERA. It turns out, however, that it is experimentally quite difficult to measure a forward jet so close to the proton remnants. We therefore suggested studying the fragmentation of this forward jet into a single energetic decay product, the $\pi^0$. This should be easier to measure. (The DIS + $\pi^0$ signal can, of course, be supplemented by also observing jet fragmentation into $\pi^\pm$ mesons). We found that when we include BFKL dynamics in the calculation of the cross section it leads to the characteristic steep rise with decreasing $x$.

\[^2\]Since $x_\pi = zz_j$ with $\langle z \rangle \sim 0.4$, our choice of cut-off on $x_\pi$, that is $x_\pi > 0.05$, is probably too conservative \[^{13}\]. If we were to take $x_\pi > 0.02$ then the $\pi^0$ production rates are all increased. For example if we choose $Q^2 \sim 25$ GeV\(^2\) then for $x = 5.10^{-4}$ and $10^{-3}$ the results are enhanced by additional factors of 6 and 5 respectively. Also the recent DIS + jet data \[^{19}\] lie about a factor of 1.7 above the values that would be obtained from our choice of input, so the DIS + $\pi^0$ rates are expected to be further enhanced by such a factor.
Table 3: The DIS $+ \pi^0$ and DIS $+ \text{jet}$ cross sections in the bin $10^{-3} < x < 1.2 \times 10^{-3},
20 < Q^2 < 30 \text{ GeV}^2$ as calculated for Fig. 6, but integrated over domains with different choices of the minimum angle $\theta_0$ between the proton and the $\pi^0$ or the jet, and of the minimum transverse momentum $k_{T,\text{min}}$ of the $\pi^0$ or the jet.

| $k_{T,\text{min}}$ [GeV] | $\theta_0$ | $\sigma_{\pi^0}$ [pb] | $\sigma_j$ [pb] |
|------------------------|-----------|----------------|-----------|
| 3                      | 5°        | 0.26           | 10.3      |
| 3.5                    | 5°        | 0.26           | 10.3      |
| 5                      | 5°        | 0.18           | 8.0       |
| 7                      | 5°        | 0.07           | 3.7       |
| 3.5                    | 7°        | 0.08           | 3.4       |
| 5                      | 7°        | 0.08           | 3.4       |
| 7                      | 7°        | 0.04           | 2.0       |

$x$. The disadvantage of using the DIS $+ \pi$ process is that the event rate is lower than for DIS $+ \text{forward jet}$. We quantified the suppression which arises from this jet to $\pi$ fragmentation (see also footnote 2). It is an experimental question as to whether the loss of event rate can be compensated by the more forward domain accessible for $\pi$ detection and the more accurate measurement of the kinematic variables possible for pions as opposed to jets. We presented sample results for different acceptance cuts to help provide an answer.

Since $\pi^0$'s are measured via the two photon decay, there is a background to the deep-inelastic $+ \pi^0$ measurement from events in which the parent jet fragments into a photon which is being emitted collinearly to the jet. We found that this background is about 1-2 %. We conclude that deep-inelastic $+ \text{pion}$ events should be a good way of probing small $x$ dynamics at HERA.

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Figure Captions

Fig. 1 Diagrammatic representation of (a) a deep inelastic + forward jet event, and (b) a deep inelastic \((x, Q^2)\) + identified forward \(\pi^0 (x_\pi, k_{\pi T})\) event.

Fig. 2 The relation between the \(\pi^0\) kinematic variables for DIS \(+\pi^0\) events with \(x = 6 \times 10^{-4}\) and \(Q^2 = 20\,\text{GeV}^2\) for various choices of the angle \(\theta_0\) in (20). In the HERA \((27.6 \times 820\,\text{GeV})\) laboratory frame the pion angle \(\theta_{\pi p}\) to the proton direction is not uniquely specified by \((x, Q^2; x_\pi, k_{\pi T}^2)\). Varying the remaining azimuthal angle transforms the lines of constant \(\theta_{\pi p}\) into narrow bands in the \(x_\pi, k_{\pi T}^2\) plane. Here we averaged over the azimuthal degree of freedom. The plot is insensitive to variations of \(x, Q^2\) over their relevant intervals. The continuous lines are the upper boundary on the allowed kinematic region for different choices of \(\theta_0\). The dashed line represents the lower boundary given by the hemisphere cut, (17), for \(x = 6 \times 10^{-4}\) and \(Q^2 = 20\,\text{GeV}^2\).

Fig. 3 The cross section, \(\langle \sigma \rangle\) in pb, for deep inelastic \(+\pi^0\) events integrated over bins of size \(\Delta x = 2 \times 10^{-4}, \Delta Q^2 = 10\,\text{GeV}^2\) which are accessible at HERA for \(\pi^0\)'s with transverse momentum \(3 < k_{\pi T} < 10\) GeV where the constraints \(x_\pi > 0.05, \theta_{\pi p} > 5^\circ,\) and the hemisphere cut, (17), were imposed. The fragmentation functions were evaluated at scale \(k_{\pi T}^2\). The \(\langle \sigma \rangle\) values are plotted at the central \(x\) value in each \(\Delta x\) bin and joined by straight lines. The \(x\) dependence is plotted for three different \(\Delta Q^2\) bins, namely \((20,30), (30,40)\) and \((40,50)\,\text{GeV}^2\). The continuous curves show \(\langle \sigma \rangle\) calculated with \(\Phi_i\) obtained from the BFKL equation. The corresponding \(\langle \sigma \rangle\) values calculated neglecting soft gluon resummation and just using the quark box approximation \(\Phi_i = \Phi_i^{(0)}\) are plotted as dashed curves. For clarity a dotted vertical lines joins each pair of curves belonging to the same \(\Delta Q^2\) bin.

Fig. 4 The cross section, \(\langle \sigma \rangle\) in fb, for deep inelastic \(+\pi^0\) events in various \((\Delta x, \Delta Q^2)\) bins which are accessible at HERA, and integrated over the region \(3 < k_{\pi T} < 10\) GeV, \(\theta_{\pi p} > 5^\circ,\) \(x_\pi > 0.05,\) and subject to the hemisphere cut, (17). The fragmentation functions were evaluated at scale \(k_{\pi T}^2\). The values in brackets are the cross sections obtained when using only the quark box approximation \(\Phi_i = \Phi_i^{(0)}\). Therefore the difference between the two numbers shown in one bin is the enhancement due to BFKL soft gluon resummation. Recall that (15) implies that the results shown for the DIS \(+\pi^0\) cross section here equal the average of the cross sections for \(\pi^+\) and \(\pi^-\) production. The curves are the boundaries of the acceptance regions at HERA given by \(8^\circ < \theta_e < 172^\circ, E_e > 5\,\text{GeV}\) and \(0.1 < y < 0.9\).

Fig. 5 Diagrammatic representation of the background to deep-inelastic \(+\pi^0\) events arising from photons which are fragments of the forward jet.

Fig. 6 The upper and lower numbers are respectively the DIS \(+\pi^0\) and DIS \(+\text{jet}\) cross sections (in pb) in various bins of \(x\) and \(Q^2\). For the pion the cuts are those given in Fig. 4, and exactly the same cuts are used for the forward jet.
(a) DIS + jet

(b) DIS + $\pi^0$

Fig. 1
\[ \theta_0 = 1^\circ \]
\[ \theta_0 = 2^\circ \]
\[ \theta_0 = 5^\circ \]
\[ \theta_0 = 10^\circ \]
\[ \theta_0 = 20^\circ \]

Fig. 2
Fig. 3

\[ \langle \sigma \rangle (\text{pb}) \]

\[ 20 < Q^2 < 30 \text{ GeV}^2 \]

\[ (\Phi) \]

\[ (\Phi^{(0)}) \]

\[ (30, 40) \]

\[ (40, 50) \]

\[ 5 \times 10^{-4} \] - \[ 10^{-3} \]
DIS + π^0 cross section (in fb) in various Δx, ΔQ^2 bins

Fig. 4
$Q^2$

$x, k$

$\cdots$

$x_j, k_{jT}$

$\gamma$

$x_{\gamma}, k_{\gamma T}$

Fig. 5
