 Thermally generated vortices, gauge invariance and electron spectral function in the pseudo-gap regime

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Starting from classical vortex fluctuation picture, we study the single electron properties in the pseudogap regime. We show that it is the gauge invariant Green function of spinon which is directly related to ARPES data in the pseudogap regime instead of the non-gauge invariant one. We find that the random gauge field from the thermally generated vortices completely destroys the coherent spinon motion and leads to collective excitations pertinent to non-Fermi liquid behaviors. The Energy Distribution Curves (EDC) show broad peaks, while the Momentum Distribution Curve (MDC) show rather sharp peaks with Lorenz form. The local density of state at zero energy scales as the inverse of Kosterlize-Thouless length. These results are qualitatively consistent with the ARPES data in the pseudo-gap regime.

Although the superconducting state of high $T_c$ cuprates can be described approximately as pair condensates with a d-wave symmetry and well-defined quasi-particle excitations above them, the anomalous properties of the normal state are still poorly understood. Many experiments such as Angle-Resolved Photoemission (ARPES) and transport characteristics provided evidence for the opening of a gap in the electronic excitation spectrum above the critical temperature $T_c$ up to some characteristic temperature $T^*$ in the underdoped cuprates, this energy gap is dubbed "pseudo-gap." Both ARPES and STM have indicated that in the pseudo-gap regime, the pseudogap possesses the same $d_{x^2−y^2}$ symmetry as the superconducting gap below $T_c$. ARPES detected sharp quasi-particle peak below $T_c$, but only broad spectral weight above $T_c$.

The quasi-particle scattering due to the Volovik effect in the pseudogap regime was discussed in Refs. [14]. As first stressed in Ref. [12], there is also Aharonov and Bohm (AB) phase effect from $hc/2e$ vortices. Although the AB phase effect only provides a periodic potential in the vortex lattice states discussed in [13], it starts to scatter quasi-particles when the vortices are randomly distributed. As pointed out in [13], the elastic scattering due to the AB phase from the disordered vortex array dominate over that from the Volovik effect. In [16], Millis and the author studied the zero temperature quantum phase transition from d-wave superconductor to the underdoped regime driven by the condensation of quantum generated $hc/2e$ vortices. In this paper, we apply the method developed in Refs. [13,14] to study the electron spectral weight in the classical vortex fluctuation regime (the vortex plasma regime in Fig.1) . We derive the important Eq.3 by two approaches. First, replacing vortex density $n_v$ generated by external magnetic field in [13] by the free vortex density $n_f(T)$ above $T > T_{KT}$, we lead to Eq.4 directly from the results in Ref. [14]. Second, starting from the zero temperature effective action developed in Ref. [16], we show that the action reduces to Eq.4 after

In this paper, we are trying to explain the ARPES data near the four nodes $(±0.39\pi, ±0.39\pi)$ from the physical picture proposed by Emery and Kivelson and others: the finite temperature superconducting transition is a Kosterlize-Thouless (KT) transition driven by vortex pair un-binding, namely $T_c = T_{KT}$. Below $T_{KT}$, the phase fluctuation is small, the positive and negative vortex pairs are tightly bound together, above $T_{KT}$, the long-range superconducting coherence is destroyed by strong phase fluctuation which leads to the generation of free vortices with both parity, but the amplitude of the local superconducting order parameter remains non-vanishing up to $T^*$. The quasi-particles in the pseudo-gap regime are the remnants of those in the superconducting phase and are strongly scattered by the free vortices above $T_{KT}$. We are trying to calculate the real electron spectral weight due to these scatterings and compare with the experiments.

![Fig 1: There is a quantum phase transition from d-wave superconductor to the underdoped regime at $x = x_c$. The Quantum Critical (QC), Quantum Disordered (QD) and Vortex plasma regimes are defined in [4]. There is no spin-charge separation in QD and QC regimes. But there is approximate spin-charge separation in the vortex plasma regime (see the text).](image-url)
where the singular gauge transformation and expanding discussed elsewhere \[24\].

This leads to finite local DOS scaled as the inverse of the KT correlation length. These results are qualitatively consistent with the ARPES data in the pseudo-gap regime. The ARPES spectrum in QC and QD regimes will be discussed elsewhere \[24\].

In Ref. \[15\], starting from BCS Hamiltonian, performing the singular gauge transformation and expanding \(H_s\) around the node 1 where \(\vec{p} = (p_F, 0)\), Ye obtained \(H_s = H_I + H_c\), where the linearized Hamiltonian \(H_I\) is given by \[17\]:

\[
H_I = v_F(p_x + a_{\nu}^s)^2 + v_D(p_y + a_{\nu}^s)^2 + v_f v_x(\vec{r}) + (1 \rightarrow 2, x \rightarrow y)
\]

where \(v_x = \frac{1}{2} (\vec{v}_A^2 + \vec{v}_B^2) = \frac{\hbar}{2} \nabla \phi - \vec{A}\) is the total superfluid momentum and \(a_{\nu}^s = \frac{1}{2} (v_A^2 - v_B^2) = \frac{1}{2} (\nabla \phi_A - \nabla \phi_B)\) is the internal gauge field. We get the corresponding expression at node 1 and 2 by changing \(v_y \rightarrow -v_y, v_D \rightarrow -v_D\) in the above Eq. For explicit spin SU(2) invariant formulation, see Ref. \[16\].

The curvature term \(H_c\) can be written as:

\[
H_c = \frac{1}{m} \left[ \{\Pi_\alpha, v_\alpha\} + \frac{\vec{\Pi}^2 + \vec{\nu}^2}{2} + \frac{\Delta_0}{2 \epsilon_F} \{\Pi_x, \Pi_y\} \right]
\] (2)

Where \(\vec{\Pi} = \vec{\nu} + \vec{a}_\nu\) is the covariant derivative. \(H_c\) takes the same form for all the four nodes \[13\].

Just like at filling factors at \(\nu = 1/3\) or \(\nu = 1/2\) QH system \[23\], the transport properties can be studied directly in the transformed Hamiltonian. But the single particle electron Green function is much more difficult to calculate \[23\]. In the present problem, the general form of electron annihilation operator was given in Ref. \[16\]:

\[
C_\alpha (\vec{x}) = \sum_{i=1,2} [e^{i\phi/2} e^{i\vec{K}_e} \vec{x} e^{i}\int a^\dagger_{\nu} dx \psi_{i1\alpha} + \epsilon_{\alpha\beta} e^{i\phi/2} e^{i\vec{K}_e} \vec{x} e^{-i}\int a^\dagger_{\nu} dx \psi_{i2\beta}]
\] (3)

where \(i\) is the node index, 1, 2 are \(p-h\) indices and \(\alpha, \beta\) are spin indices.

All the above equations can be applied straightforwardly to the pseudogap regime above \(T > T_{KT}\) where vortices are unbound. There are equal number of free positive and negative vortices generated by thermal fluctuation. On the average, the vanishing of \(v_\alpha\) and \(a_{\nu}^s\) are automatically ensured. Furthermore, the thermal generated vortices are moving around and their smearing effect makes the \(U(1)\) character stronger than their \(Z_2\) character in randomly-pinned static vortex array case. Replacing vortex density \(n_v\) generated by external magnetic field in \[13\] by the free vortex density \(n_f (T)\) above \(T > T_{KT}\), we can describe the effective action by spinons moving in a static magnetic field (there is no feedback effects on the gauge field propagator from the fermions) \[14\]:

\[
\mathcal{L} = \psi^\dagger_{\alpha} \gamma_\mu (\partial_\mu - i a_{\nu}^s) \psi_\alpha + \frac{1}{4n_f} (f^\psi_{\alpha\beta})^2
\] (4)

Where \(\alpha, \beta = 1, 2\) are the space components only; \(a = 1, \ldots, N = 4\) are 4 species of Dirac fermion and \(n_f (T) \sim \xi^{-2}(T)\) where \(\xi(T)\) is the KT correlation length at \(T > T_c\).

In Ref. \[11\], the authors derived an effective action describing the zero temperature quantum phase transition from d-wave superconductor to underdoped regime around the critical doping \(x_c\). At zero temperature, the vortices have to be treated quantum mechanically, one has to perform two singular transformations which are dual to each other to quasi-particles and moving vortices respectively to keep all the possible commutation relations intact. Just like conventional singular gauge transformation \[24\] leads to conventional Chern-Simon (CS) term, the two mutual singular gauge transformations lead to a mutual CS term. In the effective action, quantum generated vortices couple to quasi-particles by the mutual CS term, the vortices are also interacting with a charge \(U(1)\) gauge field \(a_{\mu}\) mediating the long-range current-current logarithmic interactions between the vortices. The \(U(1)\) charge fluctuation leads to mass terms not only for \(v\), but also for the spatial component \(\vec{a}_\nu\). There is no gapless dynamic gauge fluctuations in the Cooper-pair picture. The precise nature of the zero temperature quantum critical point (QCP) between the d-wave superconductor and some unknown charged ordered state is still under investigation. The electron spectral functions in the QC and QD regimes are controlled by this QCP and will be presented elsewhere \[24\].

However, the spectral function in the vortex plasma regime around \(T_{KT}\) in Fig.1 are determined by the classical hydro-dynamics of vortices which is the focus of the present paper. The vortices being treated classically \[19\], their commutation relations can be neglected, the dual singular gauge transformation in the vortices is not necessary, the Berry phase term for the boson (the linear derivative term) can also be neglected \[19\]. More specifically, only the time component of the charge gauge field \(a_0\) is kept to mediate the long-range density-density logarithmic interaction between the vortices. Obviously, \(a_0\) couples to the vortices the same way as the superfluid velocity \(v_{a}\) couples to the spinon, its fluctuation leads to a mass term for the \(v - v\) correlation. However, being orthogonal to the spatial component \(\vec{a}_\nu\), \(a^\nu - a^\nu\) correlator remains gapless and is given by the Maxwell term
in Eq.[4]. This is exactly consistent with the results first derived in [1].

It is important to point out that what measured by ARPES is the spectral function of the real electron in Eq.3 which carry both spin and charge, instead of that of the spinon ψ which carry spin only. It is vital to emphasize that the electron Green function \( G(\vec{x},t) = \langle C_\alpha(\vec{x},0)|C_\alpha^\dagger(\vec{x},t) \rangle \) is gauge invariant under the internal gauge transformation [14], but that of the spinon ψ is not! We make the following gauge invariant mean field approximation for the electron Green function:

\[
G(\vec{x},t) = \langle e^{i\phi(\vec{x})/2} e^{-i\phi(0)/2} > \\
[ e^{i\vec{K} \cdot \vec{x}} \psi_{1\alpha}(\vec{x},t) e^{i \int_0^\infty a^\dagger(x) dx \psi_{1\alpha}^\dagger(0,0) >} + e^{-i\vec{K} \cdot \vec{x}} \psi_{1\alpha}(\vec{x},t) e^{-i \int_0^\infty a^\dagger(x) dx \psi_{1\alpha}(0,0) >}]
\]

As shown in Ref. [15], it is vital to make a mean field approximation to respect the exact T symmetry. Very similarly, in the pseudo-gap regime, it is equally important to make a mean field approximation to respect the exact gauge invariance. Any approximation violating the gauge invariance will lead to completely wrong conclusions! The vortices being treated classically, only the \( \omega = 0 \) component is kept. So the vortices fluctuate much slowly than the fermions which are treated completely quantum mechanically. We expect the spin-charge separation indicated in Eq.5 is a good approximation in the vortex hydro-dynamic regime. However, as shown in [16], there is no spin-charge separation at \( T = 0 \) and QD regimes (Fig.1). This is in contrast to \( Z_2 \) gauge theory [23] where there is true spin-charge separation at \( T = 0 \).

The first factor in Eq.(5) is the classical correlation function of the vortices, at \( T > T_{KT} \), it decays exponentially:

\[
G_v(\vec{x}) = \langle e^{i\phi(\vec{x})/2} e^{-i\phi(0)/2} > \sim e^{-x/\xi}
\]

The second factor in Eq.(5) is the gauge invariant Schwinger Green function of the spinon in a given configuration of a static gauge field:

\[
[G_s^{inv}(\vec{x},t)]_{11} = \langle \psi_{1\alpha}(\vec{x},t) e^{i \int_0^\infty a^\dagger(x) dx \psi_{1\alpha}^\dagger(0,0) >}
\]

The physical meaning of Schwinger gauge invariant Green function has been painfully sought. Remarkably, it is this function directly involved in the ARPES in the pseudogap regime in high temperature superconductor [22]! It is well known that the infrared divergence in the gauge fluctuation in Eq.3 plagues the non-gauge invariant spinon Green function \( G_s(\vec{x},t) = \langle \psi_{1\alpha}(\vec{x},t) \psi_{1\alpha}^\dagger(0,0) > \). As shown in [14], it leads to divergent quasi-particle scattering rate \( 1/\tau_1 \). However, the infra-red divergence has no physical meaning and was shown to be cancelled by the vertex correction in the gauge invariant transport time \( \tau_{tr} [13] \). Very similarly, this divergence will disappear in the gauge invariant \( G_s^{inv}(\vec{x},t) \) which directly determine the ARPES data. The motion of non-relativistic particle in random magnetic field has been studied in Ref. [24]. It was found that the particle’s propagation in random magnetic field ceases to be coherent and \( G_s^{inv}(\vec{x},t) \) develops branch cut singularities. Therefore quasi-particle picture completely breaks down which is a hallmark of non-Fermi liquid picture. Studying massless Dirac fermion in random magnetic field is a much more complicated problem, we leave it to future publication [24]. Instead, we can look at a much simpler problem of Dirac fermion interacting with dynamic fluctuating gauge field. This simpler problem has Lorenz invariance at \( T = 0 \) which dictates \( G_s^{inv}(k) \sim i k \gamma_{\mu}/k^2 - 2\eta \) with \( k = (\vec{p}, i\omega_n) \). It can be explicitly written as

\[
G_s^{inv}(\vec{p},i\omega_n) = \frac{1}{E^2_p - (i\omega_n)^2} \left( \frac{i\omega_n + v_f p_x}{v_f p_y} \frac{v_f p_y}{i\omega_n - v_f p_x} \right)
\]

where \( E^2_p = (v_f p_x)^2 + (v_f p_y)^2 \) and \( \eta \) is the anomalous dimension which was calculated to one loop in \( 1/N \) expansion to be \( \eta \sim 0.27 \) in Ref. [23].

Taking analytic continuation to the real frequency and then taking imaginary part, we get the spectral function \( A(\vec{k},\omega) = -\text{Im} G(\vec{k},\omega + i\epsilon) \) of the electron near \( \vec{K} \) at temperature \( T \):

\[
A(\vec{k},\omega) = \int \frac{d^2\vec{p}}{(2\pi)^2} \frac{\sin(\pi\eta)}{\sqrt{(\vec{k} - \vec{p})^2 + \xi^{-2}(T)}} \frac{\omega + v_f p_x}{\omega^2 - E^2_p} \frac{1}{1 - \eta} \times \theta(\omega - E_p) - \theta(-\omega - E_p)
\]

with the similar expression at \( -\vec{K} \).

It can be shown that

\[
A(\vec{k},\omega \rightarrow 0) = \frac{\sin(\pi\eta)|\omega|^{2+2\eta}}{4\pi(k^2 + \xi^{-2}(T))} \\
A(\vec{k},\omega \rightarrow \infty) = \frac{\sin(\pi\eta)|\omega|^{-1+2\eta}}{4\pi\eta}
\]

Precise energy distribution curve (EDC) ( at fixed \( \vec{k} \) ) and momentum distribution curve (MDC) ( at fixed \( \omega \) ) for \( \xi(T) \sim 10 \) are obtained numerically and drawn in Fig.2.
Fig 2: Energy distribution curve (EDC) (at fixed $k$) and Momentum distribution curve (MDC) (at fixed $\omega$).

It is easy to see that just as random static gauge field, the dynamically fluctuating gauge potential also destroys the coherent quasi-particle picture. The spinon in the pseudo-gap regime loses its identity and replaced by collective excitations, in contrast to its counter-part in the superconducting state. The Volovik effect is short-ranged as shown in [2], its effect is far less dramatic than that of the random gauge field which completely destroys the quasi-particle picture. Our interpretation of the ARPES data is in sharp contrast with that of $Z_2$ gauge theory [24]. In this theory, the gauge field is absent due to the generation of double strength $hc/e$ vortices which leads to spin charge separation in the long length scale at $T = 0$.

From the long time behavior of $G(\tau) \sim G^{inv}_2(\tau) \sim 1/\tau^{2+2\eta}$, we find the local density of state (DOS) $N(\omega) \sim \omega^{3+2n}$. However, as shown in [13] the Volovik effect will generate finite DOS at zero energy $N(\omega = 0, T) \sim \sqrt{\omega T} \sim \xi^{-1}(T)$. So the EDC curves in Fig. 2 should approach a small finite value at very small frequency due to the Volovik effect.

A few caveats should be pointed out before comparing Fig.1 with the available ARPES data in the node directions: (1) The mean field approximate spin-charge separation Eq.3 is only valid at vortex hydro-dynamic regime (Fig.1). It is only $30 - 40K$ above $T_{KT}$ consistent with the recent experiments by Corson et al. Raising temperature higher will move into QC regime. There is no spin-charge separation at $T = 0$. A different calculation is needed to calculate ARPES spectrum in QC and QD regimes [24] (2) Conceptually, the gauge invariant Green function should be calculated in a static gauge field instead of in a dynamic gauge field. However, we do not expect the technical calculation in static gauge field will lead to dramatically different EDC and MDC than those in Fig.2, because unlike the gauge non-invariant Green function, the gauge invariant one is free of infra-red divergences. Therefore, we expect Fig.2 qualitatively describe the electron spectral weight in the vortex hydro-dynamic regime if the phase fluctuation scenario is the correct one.

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