Quantum correlations of observables for two particle states have demonstrated the nonlocal character of the quantum mechanics. However nonlocality can be exhibited even for noncommuting observables of a single particle system. In this paper we show nonlocality of position-momentum correlations of a single particle in the double-slit experiment modeled by an initially correlated Gaussian wavepacket. The positivity or negativity of the Wigner function for the state of the particle at the detection screen is related with the $\sigma_{xp}$ covariances. A Bell’s inequality is constructed from the Wigner function and it is violated for both the positive and negative cases. The case of positive Wigner function is the analogous of the original EPR state for a single particle.

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INTRODUCTION

Entanglement and nonlocality has been extensively studied since Einstein, Podolsky and Rosen (EPR) put forward a gedanken experiment in 1935 questioning the completeness of quantum mechanics in which two particles were entangled simultaneously over a continuum of position and momentum states. The crucial point in EPR reasoning was that the position and momentum of the unmeasured particle were simultaneous realities and thus violated Heisenberg’s uncertainty relation. Nonlocal correlations involving discrete variables were cast later on by Bohm in 1951. The interest in studying nonlocal quantum correlations has grown because they are fundamental resources in field of quantum information science. The Bell’s inequality, derived in 1964, served to prove right EPR’s disturbing action at a distance. It has become an important tool to investigate nonlocality effects in discrete variable systems such as two entangled photons in a cascade experiments and in parametric down-conversion, as well as entangled states of trapped ions. Bell’s inequality violation was also studied for a single particle by considering single photons entangled in momentum and polarization. Recently a loophole free experiment showed the Bell’s inequality violation using electron spins separated by 1.3 kilometres, after the seminal experiment of Aspect and collaborators.

Of particular interest in quantum information tasks is a demonstration of nonlocality in systems described by continuous variables such as the original EPR state or the two-mode squeezed state with the aid of dichotomic observables e.g. pseudo-spins, and parity observables. In it was developed an EPR criterion which could be implemented with momentum and position-like quadrature observables of squeezed states of light. Moreover, in a demonstration of the EPR paradox using position- and momentum-entangled photon pairs produced by spontaneous parametric downconversion. They found that the position and momentum correlations allow the position or momentum of a photon to be inferred from that of its partner with a product of variances $\leq 0.01\hbar^2$, violating the separability bound by two orders of magnitude. The EPR paradox does not represent a true inconsistency because as the measurement involves only one quantity or the other, the position and momentum of the unmeasured particle need not be simultaneous realities.

Although Bell-type experiments involve multiple particles, the non-commuting nature should be independent of whether the system consists of multiple particles or a single particle. Entanglement between two degrees of freedom has been demonstrated in single neutron interferometry experiments. This is interesting because a violation of a Bell inequality would serve to verify the uncertainty principle since it would indicate a definite correlation between, say, position and momentum.

On the other hand quantum correlations of continuous variables can be analyzed in the phase-space using the Wigner function. The latter is an important tool to study nonlocality in continuous-variable systems. Moreover, the Wigner function can be measured in different system configurations and calculated for arbitrary quantum systems. Previously, there have been efforts in understanding the relation between Bell nonlocality and the Wigner function. Such studies show that the positive definite Wigner function of the EPR state can be used with a natural phase-space framework in which the nonlocal character of this state can be studied.

In the simple Gaussian minimum-uncertainty
wavepacket solution for the Schrödinger equation for a free particle, \( \sigma_{xp} \) (hereafter also called \( \sigma_{xp} \) correlations) at \( t = 0 \) are zero but develop at later times as a result of the quantum dynamics \([2, 37]\). However, more complex states such as squeezed states or linear combination of Gaussian states can exhibit initially such correlations \([38–41]\). Such \( \sigma_{xp} \) correlations can be related with phases of the wave function \([2]\). They were also shown to play a role in matter wave slit diffraction: for example, qualitative changes in the interference pattern appear as \( \sigma_{xp} \) correlations develop \([42]\). More specifically, the Gouy phase of matter waves is directly related with these correlations, as studied by the first time in Refs. \([43]\). More recently, it was shown that the maximum of these correlations is related with the minimum number of interference fringes in the double-slit experiment \([44]\). Double-slit experiments have been used to elucidate fundamental aspects of the quantum theory \([45]\). The wave-particle duality has been observed in the double-slit experiment with electrons \([46]\), neutrons \([47]\) and atoms \([48]\). Recently, a controlled electron double-slit diffraction was experimentally realized in which the probability distributions for single- and double-slit arrangements were observed \([49]\).

In this work, we use the double-slit setup to study nonlocality associated with noncommuting observables for a single particle. This setup enable us to connect the \( \sigma_{xp} \) correlations with the positivity or negativity of the Wigner function. The nonlocal character of correlations for the position and momentum of a single particle is obtained by constructing a Bell’s inequality using the Wigner function. For the sake of calculability we consider an initial Gaussian wavepacket and Gaussian shaped slit apertures. The single particle state at the detection screen will be a superposition of two Gaussian. Hence, after the slits, the single particle state is represented by two parts, which is essential to observe single-particle nonlocal correlations analogous to a two-particle EPR system.

The behavior of \( \sigma_{xp} \) correlations in an initially \(xp\)-correlated Gaussian wavepacket measured by a real parameter \( \rho \) yield crucial information about the Wigner function. Whilst for \( \rho \geq 0 \) the correlation is maximum for a given propagation time (and maximum correlation in turn leads to maximum negativity of the Wigner function) for \( \rho < 0 \) these correlations have a maximum and a minimum for specific propagation times. As we shall see the minima of \( \sigma_{xp} \) correlations will be related with a positive Wigner function. The state at the detection screen with maximum (minimum) \( \sigma_{xp} \) correlations gives the maximum (minimum) value for the Bell inequality violation. Moreover the state at the detection screen (superposition of two Gaussian waves) with minimum \( \sigma_{xp} \) correlations has a positive definite Gaussian shaped Wigner function. A positive Gaussian shaped Wigner function has been used to study the original EPR state \([36]\). As we shall see, the state at the detection screen with minimum \( \sigma_{xp} \) correlations can be considered as analogous of the original EPR state for a single particle.

In section II we model the double-slit experiment with matter waves considering an initially correlated Gaussian wavepacket. The initial wavepacket propagates during the time \( t \) from the source to the double-slit and during the time \( \tau \) from the double-slit to the screen. We calculate the wave functions after the passage through each slit using the Green’s function for the free particle. In section III, we calculate the Wigner function and the \( \sigma_{xp} \) correlations for the state that is a linear combination of the states which passed through each slit. We show that the minimum \( \sigma_{xp} \) correlations are related with a positive definite Wigner function and the maximum \( \sigma_{xp} \) correlations with a Wigner function with negative part. In section IV, we construct a Bell-type inequality for the position and momentum observables using the Wigner function and show the nonlocal character for a single particle. The condition for the production of an analogous EPR state for a single particle in the double-slit setup is also discussed. In section V we draw our concluding remarks.

DOUBLE-SLIT EXPERIMENT MODELED BY INITIALLY CORRELATED GAUSSIAN WAVEPACKET

Consider a classical double-slit experiment with initially correlated Gaussian wavepacket. The initial \( \sigma_{xp} \) correlation will be represented by the real parameter \( \rho \) which can take values in the interval \(-\infty < \rho < \infty\). Assume that such coherent correlated Gaussian wavepacket of initial transverse width \( \sigma_0 \) is produced in the source \( S \) and propagates during a time \( t \) before arriving at a double-slit which splits it into two Gaussian wavepackets. After crossing the grid the wavepackets propagate during a time \( \tau \) before arriving at detector \( D \) in detection screen, where they are recombined. In this model we realize quantum effects only in \( x \)-direction as we can assume that the energy associated with the momentum of the particles in the \( z \)-direction is very high such that the momentum component \( p_z \) is sharply defined, i.e., \( \Delta p_z \ll p_z \). Then we can consider a classical movement in this direction at velocity \( v_z \) and we may write \( z = v_z t \). The sketch of this model is presented in Fig. 1.

The wavefunctions at the left 1(+) and right slit 2(−) are given by \([12]\)

\[
\psi(x, t, \tau) = \int_{-\infty}^{\infty} dx_j \int_{-\infty}^{\infty} dx_t G_2(x, t + \tau; x_j, t) \\
\times F(x_j \pm d/2)G_1(x_j, t; x_i, 0)\psi_0(x_i), \quad (1)
\]

where
free nonrelativistic propagators for a particle of mass $m$. The wavepacket propagates during a time $t$ before attaining the double-slit and during a time $\tau$ from the double-slit to the detector $D$ in the screen of detection. The slit transmission functions are taken to be Gaussian of width $\beta$ and separated by a distance $d$.

\[ G_1(x_j, t; x_i, 0) = \sqrt{\frac{m}{2\pi i\hbar t}} \exp\left[\frac{-im(x_j - x_i)^2}{2\hbar t}\right], \quad (2) \]

\[ G_2(x, t + \tau; x_j, t) = \sqrt{\frac{m}{2\pi i\hbar \tau}} \exp\left[\frac{-im(x - x_j)^2}{2\hbar \tau}\right], \quad (3) \]

\[ F(x_j \pm d/2) = \frac{1}{\sqrt{\beta \sqrt{\pi}}} \exp\left[\frac{-im(x_j \pm d/2)^2}{2\beta^2}\right], \quad (4) \]

\[ \psi_0(x_i) = \frac{1}{\sqrt{\sigma \sqrt{\pi}}} \exp\left[-\frac{x_i^2}{2\sigma^2} + \frac{i\rho x_i^2}{2\sigma^2}\right]. \quad (5) \]

The kernels $G_1(x_j, t; x_i, 0)$ and $G_2(x, t + \tau; x_j, t)$ are the free nonrelativistic propagators for a particle of mass $m$, $F(x_j \pm d/2)$ describes the double-slit transmission functions which are taken to be Gaussians of width $\beta$ separated by a distance $d$. The parameter $\rho$ ensures that the initial state is correlated. In fact, we obtain for the initial state $\psi_0(x_i)$ that the uncertainty in position is $\sigma_x = \sigma_0 \sqrt{2}$, whereas the uncertainty in momentum is $\sigma_p = (\sqrt{1 + \rho^2})\hbar/\sqrt{2\sigma_0}$ and the $\sigma_{xp}$ correlations are $\sigma_{xp} = \hbar \rho/2$. For $\rho = 0$ we have a simple uncorrelated Gaussian wavepacket with $\sigma_{xp} = 0$. In order to obtain analytic expressions for the wavefunction, Wigner function and $\sigma_{xp}$ correlations in the screen of detection we use a Gaussian transmission function instead of a top-hat transmission function, because both a Gaussian transmission function represents a good approximation to the experimental reality and it is mathematically simpler to treat than a top-hat transmission function.

After some algebraic manipulations, we obtain the following result for the wavefunction that passed through slit $1(+)$$\psi_1(x, t, \tau) = \frac{1}{\sqrt{B(\sqrt{\pi})}} \exp\left[-\frac{(x + D/2)^2}{2B^2}\right] \times \exp\left(\frac{-imx^2}{2\hbar B} + i\Delta x + i\theta + i\mu\right), \quad (6)\]

where

\[ R(t, \tau) = \frac{\left(\frac{1}{\beta} + \frac{1}{\beta} + \frac{\hbar}{\pi}\right)^2}{\frac{1}{\beta} + \frac{\hbar}{\pi} + \frac{\hbar}{\pi}} + \frac{\hbar}{\pi} + \frac{\hbar}{\pi}, \quad (7) \]

\[ C = \left[\frac{\tau_0^2}{\tau} + \frac{\tau_0^2 \rho^2}{\tau} + \frac{\tau_0^2 \rho^2}{\tau} + \frac{\tau_0^2 \rho^2}{\tau} + \frac{\tau_0^2 \sigma_0^2}{\beta}\right], \quad (8) \]

\[ B^2(t, \tau) = \frac{\left(\frac{\beta}{\pi} + \frac{\rho}{\pi}\right)^2}{\left(\frac{\beta}{\pi} + \frac{\rho}{\pi}\right)^2 \left(\frac{\beta}{\pi} + \frac{\rho}{\pi}\right)^2}, \quad (9) \]

\[ \Delta(t, \tau) = \frac{\tau \sigma_0^2 d}{2\tau_0^2 \beta^2 B^2}, \quad (10) \]

\[ D(t, \tau) = \frac{d(1 + \frac{\beta}{\pi})}{1 + \frac{\beta^2}{\pi}}, \quad (11) \]

\[ \theta(t, \tau) = \frac{md^2}{8\hbar \beta^4} \left[\frac{\beta}{\pi} + \frac{\rho}{\pi}\right]^2 + \frac{m}{\beta^2} \left(\frac{1}{\beta} + \frac{1}{\beta}\right), \quad (12) \]

\[ \mu(t, \tau) = -\frac{1}{2} \arctan \left[\frac{t + \tau(1 + \rho^2)}{\tau_0 \left(1 - \frac{\tau \sigma_{xp}^2}{\rho \rho_{xp}}\right) + \rho (t + \tau)}\right], \quad (13) \]

\[ b(t) = \frac{\sigma_0}{\tau_0} \left[\frac{\tau_0^2 + \tau_0^2 + 2\tau_0 \rho + t^2 \rho^2}{\tau_0} \right], \quad (14) \]

and

\[ r(t) = \frac{\left(\frac{\beta}{\pi} + \frac{\rho}{\pi}\right)^2}{\left(\frac{\beta}{\pi} + \frac{\rho}{\pi}\right)^2 \left(\frac{\beta}{\pi} + \frac{\rho}{\pi}\right)^2}, \quad (15) \]

For the right slit $2(−)$, we have just to substitute the parameter $d$ with $−d$ in the expressions corresponding to the wave passing through the first slit. Here, the parameter $B(t, \tau)$ is the beam width for the propagation through one slit, $R(t, \tau)$ is the radius of curvature of the wavefronts for the propagation through one slit, $b(t)$ is the beam width for the free propagation and $r(t)$ is the radius of curvature of the wavefronts for the free propagation.

$D(t, \tau)$ is the separation between the wavepackets produced in the double-slit. $\Delta(t, \tau)$ is a phase which varies linearly with the transverse coordinate. $\theta(t, \tau)$ and $\mu(t, \tau)$ are the time dependent phases and they are relevant only if the slits have different widths. $\mu(t, \tau)$ is the
Gouy phase for the propagation through one slit. Differently from the results obtained in Ref. [44], all the parameters above are affected by the correlation parameter $\rho$. $\tau_0 = m\sigma_0^2/\hbar$ is one intrinsic time scale which essentially corresponds to the time during which a distance of the order of the wavepacket extension is traversed with a speed corresponding to the dispersion in velocity. It is viewed as a characteristic time for the “aging” of the initial state [12, 44] since it is a time from which the evolved state acquires properties completely different from the initial state.

### WIGNER FUNCTION AND $\sigma_{xp}$ CORRELATIONS

Having obtained the wavefunctions in the previous section, we calculate the Wigner function at the detection screen. Then we use the Wigner function to evaluate the $\sigma_{xp}$ correlations. We observe that the $\sigma_{xp}$ correlations have only a maximum. We also notice that the Wigner function for the state with minimum $\sigma_{xp}$ correlations does not present a negative part and thus it represents a classical state in this sense. On the other hand, the Wigner function for the state with $\rho \geq 0$ presents a negative part for some values of position $x$ and momentum $p$ and thus signalize quantum state [51]. The state with negative correlation parameter $\rho < 0$ is known as contractive state [52] which, for a free quantum particle, was introduced by Yuen [53] in an attempt to evade the standard quantum limit for repeated position measurements.

The Wigner function of the phase-space quasiprobability distribution in one-dimension configuration space is defined as [54]

$$ W(x, k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dy e^{-iky} \psi^* \left( x - \frac{y}{2} \right) \psi \left( x + \frac{y}{2} \right), (16) $$

where

$$ \psi(x, t, \tau) = \frac{\psi_1(x, t, \tau) + \psi_2(x, t, \tau)}{\sqrt{2 + 2 \exp \left[ -\frac{D^2}{4\hbar^2} - \Delta^2 B^2 \right]}. (17) $$

is the normalized wavefunction at the screen of detection of the double-slit experiment.

By solving the integration in equation (16) we obtain the following result

$$ W(x, k) = W_1(x, k) + W_2(x, k) $$

$$ + \frac{2}{\pi \alpha^2} \exp \left[ -\frac{(x^2 + \left( k - \frac{m x}{\hbar R} \right)^2}{B^2} \right] $$

$$ \times \cos \left( k - \frac{m x}{\hbar R} D + 2\Delta x \right), (18) $$

where

$$ W_1(x, k) = \frac{1}{\pi \alpha^2} \exp \left[ -\frac{(x + \frac{D}{2})^2}{B^2} \right] $$

$$ \times \exp \left[ -\left( k - \frac{m x}{\hbar R} - \Delta \right)^2 B^2 \right], (19) $$

$$ W_2(x, k) = \frac{1}{\pi \alpha^2} \exp \left[ -\frac{(x - \frac{D}{2})^2}{B^2} \right] $$

$$ \times \exp \left[ -\left( k - \frac{m x}{\hbar R} + \Delta \right)^2 B^2 \right], (20) $$

and

$$ \alpha = 2 + 2 \exp \left[ -\frac{D^2}{4\hbar^2} - \Delta^2 B^2 \right]. (21) $$

The result displayed by equation (15) is composed by the terms $W_1(x, k)$ and $W_2(x, k)$ which are the Wigner functions for a particle that passed through slit 1 and 2, respectively, as well as an interference term. The Wigner function and the $\sigma_{xp}$ correlations are related by [55]

$$ \sigma_{xp} = \int p W(x, p) dx dp \int W(x, p) dx dp. (22) $$

where $p = h k$. After some algebraic manipulation we obtain

$$ \sigma_{xp}(t, \tau) = \frac{m B^2}{2 R} + \frac{m D^2 / R}{4 + 4 \exp \left[ -\frac{D^2}{4\hbar^2} - \Delta^2 B^2 \right]} - \frac{\hbar \Delta D}{2} $$

$$ - \frac{(m \Delta^2 B^4 / R)}{1 + \exp \left[ \frac{D^2}{4\hbar^2} + \Delta^2 B^2 \right]. (23) $$

In the following, we plot the curve for the $\sigma_{xp}$ correlations as a function of the time $t/\tau_0$ for neutrons. The reason to consider neutrons relies in their experimental reality, which is most close to our model for interference with completely coherent matter waves, although we still have loss of coherence as discussed in Ref. [56]. We adopt the following parameters: mass $m = 1.67 \times 10^{-27}$ kg, initial width of the packet $\sigma_0 = 7.8 \mu$m (which corresponds to the effective width of $2\sqrt{2}\sigma_0 \approx 22 \mu$m), slit width $\beta = 7.8 \mu$m, separation between the slits $d = 125 \mu$m and de Broglie wavelength $\lambda = 2$ nm. These same parameters were used previously in double-slit experiments with neutrons by A. Zeilinger and collaborators. [17]. In Fig. 2, we show the correlations as a function of $t/\tau_0$ for $\tau = 18\tau_0$ and $\rho = -1.0$. We use a negative value of the correlation parameter $\rho$ in order to obtain $\sigma_{xp}$ correlations with a point of minimum and of maximum. For the parameters above we calculate the points of minimum and maximum correlations and obtain, respectively, $t_{\text{min}} = 0.49\tau_0$ and $t_{\text{max}} = 1.36\tau_0$. 
any time different of negative portion occurs for negative portion. We also observe that the maximum negation (23), i.e., D(B) ~ mD^2/2R and D(x) ~ mD^2/4R. Therefore, we have the following conditions

\[ B^2(t_{min}) \gg D^2(t_{min}), \quad D^2(t_{max}) \gg B^2(t_{max}). \]  

Since \( B(t, \tau) \) is the width of the wavepacket and \( D(t, \tau) \) the separation between the wavepackets at the screen, the region of overlap between the two packets is bigger for minimum \( \sigma_{xp} \) correlations as compared to maximum correlations. When we apply the conditions of equation (24) in the terms of equation (13) we observe that the interference term contribute to the Wigner function more than the terms \( W_1(x, k, t) \) and \( W_2(x, k, t) \).

The state with minimum \( \sigma_{xp} \) correlations is a superposition of two Gaussian wavefunctions but has a Gaussian shaped Wigner function exhibiting only positive values. This is analogous of the original EPR state for a single particle. Most interesting here is verify whether these states violate a Bell inequality enabling us to demonstrate the nonlocal character for a single particle correlation, just as for the analogous original EPR state, with possibility of measurement in the double-slit setup. In order to show nonlocal character for a single particle we construct in the next section its Bell’s inequality.

**NONLOCAL CORRELATIONS AND BELL INEQUALITY**

Since Bell type inequalities can be found for the correlations of spin and polarization of a single particle the same should be true for the correlations of other observables such as position \( \hat{x} \) and momentum \( \hat{p} \) operators as in the original EPR paper [1]. In order to attest the nonlocal correlations for these observables, in this section we construct the Bell’s inequality using the Wigner function of the wavefunction at the screen of detection in the double-slit experiment. We obtain the Bell’s inequality violation for both positive and negative Wigner function which characterizes nonlocal correlations of position and momentum observables of a single particle. We define the correlation function as in Ref. [5]. We consider the following four combinations of \( x \) and \( k \): \( x_1 = x_3, x_2 = x_4 = x, k_1 = k_2 \) and \( k_3 = k_4 = k \). With these quantities we construct the combination

\[ B(t, x, k) = |W(x_1, k_1) + W(x, k_1)| + |W(x_1, k) - W(x, k)|, \]  

which was pointed out as similar to the CHSH inequality [5].

We observe that the terms \( W_1(x, k) \) and \( W_2(x, k) \) in equation (18) can be neglected in comparison with the interference term. Therefore, the interference causes the Bell’s inequality violation in equation (24) namely a nonlocal correlation. In [5] it was shown that the CHSH inequality violation stems from an interference effect as well.

In Fig. 4 we show the Bell inequality as a function of \( x \) and \( k \) for the parameters of neutrons with \( x_1 = 1 \mu m \) and \( k_1 = 10^3 \text{ m}^{-1} \). In Fig. 4(a) we have the Bell inequality at the time for which the \( \sigma_{xp} \) correlations are
minimal and in Fig. 4(b) we show the Bell inequality for the time for which these correlations are maximal. We show only values exceeding the bound imposed by local theories. We observe that the state for \( t_{\text{max}} \) violates the Bell inequality more than the state for \( t_{\text{min}} \) for specific values of \( x \) and \( k \). We also observe that the maximum value of the Bell inequality for the state when the time is \( t_{\text{min}} \), the original EPR state, is \( B \approx 2.19 \). The same value was obtained in Ref. [36] using another system.

![Bell inequality graph](image)

**CONCLUSIONS**

We showed that the \( \sigma_{xp} \) correlations at the screen of detection in the double-slit experiment can be maxima and minima depending of the time evolution from the source to the double-slit. We obtained that minimal correlations are possible only if the initial state is a contractive state, i.e., the state with negative coefficient of correlation. We observed that there is a connection between the \( \sigma_{xp} \) correlations and the positivity/negativity of the Wigner function. The minimal correlations are associated with a positive definite Wigner function whereas correlations other than those are associated with negative parts in the Wigner function. The maximal correlations are associated with maximal negative parts in the Wigner function. We used the Wigner function to construct a Bell-type inequality for the noncommuting position and momentum observables. The Bell’s inequality is violated for positive and negative Wigner function for some values of the position and momentum variables. Therefore, we have shown the nonlocal character of a single particle in the double-slit experiment. By choosing a certain set of parameters the maximum Bell’s inequality violation is \( B = 2.59 \) and is obtained when the Wigner function has greater negative part. On the other hand, when the Wigner function is positive definite the maximum Bell’s inequality violation is \( B = 2.19 \). The case of positive Wigner function is the analogous of the original EPR state for a single particle. Therefore, our results display nonlocality of the position and momentum observables for a single particle which can be tested in the double-slit experiment.

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