The impact of DG’s location on the small signal stability of the distribution network

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Abstract. The distribution network has been integrated with more and more distributed generations. Most of these generations are inverter based photo-voltaic plant or DFIG based wind farm. As the proportion of these renewable energies goes high, the stability problems, especially the small signal stability problems gain attention from researchers. Literatures have conducted on the single machine infinite bus system of them, focusing mostly on their dynamic characteristics. This paper analyses the impact of DG’s location on the small signal stability of the distribution network, which includes DGs, network and dynamic load. The conclusion is drawn from the modal analysis, and is verified by time domain simulation.

1. Introduction
There were various kinds of distributed generations (DGs) integrated in the distribution network, for example, the double fed induction generation (DFIG), the cage induction generation, and inverter based photo-voltaic plant, which can help eliminate the emission of the greenhouse gases. Because of the integration of these DGs, the distribution network not only has the load, but also has the power resources, which is completely different from the tradition distribution network in the literature [1]. Also, the power resources introduce the stability problems into the distribution system. Every single DG has its own particular characteristic and control method. Therefore, the impact of various DGs to the distribution system’s small signal stability is different, which is gradually getting the attention of operators and researchers.

Precious small signal stability analysis of various DGs’ dynamic characteristics in the literatures [2-5] and their impacts on the transmission system in the literatures [6-8] are well documented. Some researchers focus on the modal analysis of SMIB, researching the oscillating mode of the DFIG in the literatures [2,3] and the inverter based photo voltaic plant in the literatures [4,5]. The small signal stability of transmission system with these DGs is also researched, researchers in the literature [6] compare the influence of different locations of DFIG to the small signal angular stability; researchers in the literature [7] conduct research on the sub-synchronous oscillations; researchers in the literature [8] investigates the impact of different penetration of PV on the power system small signal stability.

However, the small signal stability of the distribution network considering these DGs is not well researched. The state-space model of an inverter-based micro-grid during autonomous operation is constructed in the literature [9] and its small signal stability is also analyzed. Whereas, the DGs are all assumed to be inverter based power resources, so the dynamic model of DFIG based wind farm is not...
took into account. In the literature [10], the small signal stability of DGs is considered in the reconfiguration of distribution system, while these DGs are all synchronous machine, ignoring the DFIG based wind farm and inverter based photo-voltaic plant.

In this paper, the representative DGs were all took into account, such as the DFIG based small wind farms and the inverter based photo-voltaic plants. All the DGs’ and loads’ dynamic characteristics are both considered in the model. The model of a DFIG includes the drive train, the induction generation and the converters, while ignores the controllers to simplify the overall model. The model of an inverter based photo-voltaic plant includes the controllers and output filter. The model of loads is constructed by assuming all the loads are RL loads. After the model is formed, modal analysis can be carried out. The eigenvalues can indicate the response of the distribution system after a small disturb. Only if all the eigenvalues are located on the left plane of the axis can the system be judged stable. Therefore, the closest eigenvalue to the imaginary axis shows the stability margin of the distribution system.

Considering various control and operate modes of these DGs, their impacts on the stability can be quite different. For example, the inverter based photo-voltaic plant has no rotation part and can response in a short time; while the DFIG based wind farm is based on the induction generation, with the rotor rotating according to the wind speed. Therefore, the dynamic characteristics of the distribution system can be quite different with various DGs. Apart from that, the different locations of them can also make a difference. This paper focuses on the impact of different locations of DGs on the distribution network, using modal analysis to indicate this trend.

This paper is organized as follows. Section 2 constructs the state-space model of the distribution network with DGs. The modal analysis and the results are presented in the section 3. Section 4 gives the conclusions.

2. The dynamic model
The state-space model includes the DFIG model, the inverter based pv plant model and the load model. Then, they are all connected by the network model.

![Figure 1. Structure of DFIG.](image)

**DFIG model.** DFIG serves as the most popular type of wind driven generation because of its flexible operation mode and relative cheap cost. In the super-synchronous state, power is fed from the rotor through the converter to the grid; in the sub-synchronized state, power is transmitted in the opposite direction. In both cases (super-synchronous and sub-synchronous), the stator sends power to the grid, as shown in Figure 1. Its model is complicate when considers the dynamic of the controllers. As the DFIG is a type of induction generation, the time constants of the controllers are much smaller than the drive train. Therefore, the dynamic of the controllers are ignored in this paper. The model built in the literature [11] is adopted:
\[
[\Delta \dot{x}_{\text{DFIG}}] = [A_{\text{DFIG}}][\Delta x_{\text{DFIG}}] + [B_{\text{DFIG}}][\Delta u_{\text{DFIG}}]
\]

where \([\Delta x_{\text{DFIG}}] = [\Delta i_{ds}, \Delta i_{qs}, \Delta v_{d}^T, \Delta v_{q}^T, \Delta \omega_d, \Delta \omega_q, \Delta \theta_m, \Delta \theta_i, \Delta i_{dg}^T, \Delta i_{qq}^T]^T\), \(i_d, i_q\) are the stator d-q currents, respectively; \(v_d, v_q\) are the d-q voltages between the transient reactance, respectively; \(\omega_d, \theta_m, \omega_q\) are the turbine, shaft twist angle and generator speed, respectively; \(i_{ds}, i_{qs}\) are the GSC d-q currents, respectively. \([\Delta u_{\text{DFIG}}] = [\Delta v_{d}, \Delta v_{q}, \Delta T_m, \Delta v_{dg}, \Delta v_{qq}, \Delta i_{dg}, \Delta i_{qq}, \Delta i_{dq}, \Delta i_{qo}, \Delta i_{d}, \Delta i_{q}]^T\), \(v_{dg}, v_{qu}\) are the d-q rotor voltages, respectively; \(T_m\) is the input torque; \(v_{dg}, v_{qu}\) the d-q GSC output voltages, respectively;

**Inverter based pv plant model.** Comparing to the DFIG model, the time constant of inverter controller is relatively low, so the model of the controller should be taken into account. In this paper, the voltage of the capacitor behind the inverter is considered to be stable. Therefore, the inverter based pv plant can be deemed to a voltage-source inverter. Figure 2 shows the structure of it, and the state space model in the literature [9] is obtained:

\[
[\Delta \dot{x}_{\text{inv}}] = [A_{\text{inv}}][\Delta x_{\text{inv}}] + [B_{\text{inv}}][\Delta u_{\text{inv}}]
\]

where \([\Delta x_{\text{inv}}] = [\Delta x_{id}, \Delta x_{iq}, \Delta x_{id}, \Delta x_{iq}, \Delta i_{id}, \Delta i_{iq}, \Delta v_{id}, \Delta v_{iq}, \Delta i_{od}, \Delta i_{qq}, \Delta i_{dq}, \Delta i_{qo}]^T\), \(x_{id}, x_{iq}\) are the state variables in the voltage control process; \(x_{od}, x_{qq}\) are the state variables in the voltage control process; \(i_{id}, i_{iq}\) are the output variables of the voltage controller; \(v_{id}, v_{iq}, i_{od}, i_{qq}\) are the output voltage and current variables; \([\Delta u_{\text{inv}}] = [v_{od}^*, v_{iq}^*, v_{od}^*, v_{iq}^*, v_{od}^*, v_{iq}^*]^T\), \(v_{od}^*, v_{iq}^*, v_{od}^*, v_{iq}^*\) are the target voltages.

**Figure 2.** The structure of inverter.

The distribution network model. In a distribution system, there are various DGs and loads. The DFIG and inverter model are built on their own reference frame, which is different from each other. To build a distribution network model, the transformation technique in the literature [12] is needed.

**Figure 3.** Reference frame transformation.
\[
[f_{DQ}] = \begin{bmatrix}
\cos \delta_i & -\sin \delta_i \\
\sin \delta_i & \cos \delta_i
\end{bmatrix} \begin{bmatrix} f_{d,i} \
 f_{q,i}
\end{bmatrix}
\]

where the DQ coordinate system denotes the common coordinate system, the \(d, q\) coordinate system denotes the system of each distributed generation. Assuming all the loads are RL loads, and taking the current of the distribution lines and loads as state variables, the model of the distribution network can be obtained as follows:

\[
\begin{bmatrix}
\Delta i_{DFIG} \\
\Delta i_{inv} \\
\Delta i_{line} \\
\Delta i_{load}
\end{bmatrix} = A \begin{bmatrix}
\Delta x_{DFIG} \\
\Delta x_{inv} \\
\Delta i_{line} \\
\Delta i_{load}
\end{bmatrix} + B \begin{bmatrix}
\Delta u_{DFIG} \\
\Delta u_{inv}
\end{bmatrix}
\]

The state-space model of distribution network is given in (3), and the eigenvalues of matrix \(A\) denote oscillation modes of the distribution network, the smallest of those eigenvalues denotes the stability margin of the system.

### 3. Eigenvalue and modal analysis

The small signal stability analysis is conducted on the IEEE 33-bus system in Figure 4, where a DFIG is connected to the node 2, a pv based inverter is connected to the node 22, and another pv based inverter is connected to the node 18, all nodes are connected with RL loads, and such a system is named basic situation in this paper. The line label is the same as the line’s end node label.

![IEEE 33-bus system](image)

**Figure 4.** IEEE 33-bus system.

Assuming that all the renewable energies are operating in their rated power, we can acquire the eigenvalues that denote the oscillation modes and the corresponding participation factors. Then, dominate state variables of each oscillation modes can also be obtained.

| Eigenvalues | Damping factor | Damped frequency | Dominated state variables |
|-------------|----------------|-----------------|--------------------------|
| \(\lambda_1, \lambda_2\) | -0.32 \(\pm\) 1.00 | 0.30 | 0.16 | \(i_{load,29}\) |
| \(\lambda_3, \lambda_4\) | -11.10 \(\pm\) 40.53 | 0.26 | 6.45 | \(v_{od,1}\) |
| \(\lambda_5, \lambda_6\) | -11.11 \(\pm\) 40.43 | 0.26 | 6.43 | \(v_{od,2}\) |
| \(\lambda_7\) | -5.98 | 1 | 0 | \(\omega_p, v_q\) |

*Table 1. Representative oscillation modes of the basic situation.*
Table 1 displays some representative oscillation modes and corresponding dominated state variables. The dynamic characteristic of $load_{29}$ denotes the stability margin of the system, whose eigenvalue is the biggest among all the eigenvalues. The dominated state variable of $pv_1$ and $pv_2$ is the output voltage, and that of DFIG is the generator speed and the voltage between the transient reactance, which denotes a electromechanical mode.

In order to compare the influence of different locations of pvs and DFIG on the small signal stability, they are changed as follows: a. fixing the locations of the pvs and changing the location of DFIG from node 3 to node 17, and from node 25 to node 32; b. fixing the location of $pv_2$, and exchanging the location of $pv_1$ and DFIG, then changing the location of $pv_1$ from node 3 to node 17, and from node 25 to node 32. The dominate eigenvalues are displayed in the Figure 5 and Figure 6:

**Figure 5.** The dominate eigenvalue trajectories of DFIG (a) and distribution system (b).

**Figure 6.** The dominate eigenvalue trajectories of $pv_1$ (c) and distribution system (d).

The x-coordinate denotes the node label, and the y-coordinate denotes the real part of the dominate eigenvalues of DFIG, pv and the distribution system. The following conclusions can be obtained from Figure 5 and Figure 6:

From Figure 5(a), as the location of DFIG goes towards the terminal of the system, namely from node 3 to node 17, the real part of the dominate state variable increases from about -5.7 to about -5.25, and the changing ratio is about 8.6%. In contrast, as location of the inverter based pv plant goes towards the terminal of the system, the real part of the dominate state variable remains stable, with the
range from -11.123 to -11.106, and the changing ratio is about 0.15\%, which is far less than that of DFIG. The absolute value of the DFIG’s dominate eigenvalue decreases significantly as the location moving towards the terminal of the system, while the value of pv1 doesn’t change much. Therefore, DFIG is more sensitive than the pv to the location. The closer to the terminal of the network, the less stable of the DFIG, while the stability of pv doesn’t change significantly.

Figure 5(b) is very similar to Figure 6(d), the biggest stability margin exists when the DFIG or pv connected to the node 29. It’s worth noting that the heaviest load of the system is connected to the node 29, which can be obtained from the parameters of the IEEE 33 systems. Therefore, no matter what kind of energy connected to the system, the one closer to the heaviest load node can improve the stability margin significantly.

The x-coordinate denotes the simulation time, and the y-coordinate denotes the rotor speed of DFIG. The conclusion can be verified by theoretical analysis and time domain simulation: Firstly, the pv plant is based on the electronic devices with no rotation part. The electronic devices can respond quickly and need no reactive power to establish a magnetic field. Therefore, the pv plant can operate well no matter where it connected. Secondly, the heaviest RL load can be more stable if it connected near to the power source, which can provide enough power and good voltage level. Lastly, the dynamic characteristic of DFIG can be verified by time domain simulation. From Figure 7, assuming that wind speed changes at time zero, the red line denotes the rotor speed of DFIG when it is connected to the node 2, and the blue line denotes the rotor speed of DFIG when it is connected to the node 17. It shows that the DFIG connected to the node 2 has smaller oscillation amplitude and get stable more quickly. Therefore, the closer is DFIG to the terminal of the network, the less stable of it.

4. Conclusions
This paper builds the model of a distribution system with various DGs, such as the DFIG based wind farm and the inverter based photo-voltaic plant. Considering the different operational mechanisms of the DGs, their impact on the small signal stability will also be different. This paper conducts research on the impact of the different locations of these DGs on the small signal stability, getting the result that The closer to the terminal of the network, the less stable of the DFIG, while the stability of pv doesn’t change significantly. Apart from this, the closer of the DGs to the heaviest load, the more stable of the system.

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