Quantum fluxes at the inner horizon of a spherical charged black hole

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In an ongoing effort to explore quantum effects on the interior geometry of black holes, we explicitly compute the semiclassical flux components \( \langle T_{uu} \rangle_{\text{ren}} \) and \( \langle T_{vv} \rangle_{\text{ren}} \) (\( u \) and \( v \) being the standard Eddington coordinates) of the renormalized stress-energy tensor for a minimally-coupled massless quantum scalar field, in the vicinity of the inner horizon (IH) of a Reissner-Nordström black hole. These two flux components turn out to dominate the effect of backreaction in the vicinity of the IH; and furthermore, their regularization procedure reveals remarkable simplicity. We consider the Hartle-Hawking and Unruh quantum states, the latter corresponding to an evaporating black hole. In both quantum states, we compute \( \langle T_{uu} \rangle_{\text{ren}} \) and \( \langle T_{vv} \rangle_{\text{ren}} \) in the vicinity of the IH for a wide range of \( Q/M \) values. We find that both \( \langle T_{uu} \rangle_{\text{ren}} \) and \( \langle T_{vv} \rangle_{\text{ren}} \) attain finite asymptotic values at the IH. These asymptotic values are found to be either positive or negative (or vanishing in-between), depending on the \( Q/M \) parameter. Note that having a nonvanishing \( \langle T_{vv} \rangle_{\text{ren}} \) at the IH implies the formation of a curvature singularity on its ingoing section, the Cauchy horizon. Motivated by these findings, we also take initial steps in the exploration of the backreaction effect of these semiclassical fluxes on the near-IH geometry.

**Introduction.** The analytically extended Kerr and Reissner-Nordström (RN) metrics, describing respectively spinning and spherical charged isolated black holes (BHs), reveal a traversable passage through an inner horizon (IH) to another external universe [1, 2].

Consider a traveler intending to access this other universe. To do so, she must pass through the BH interior, and in particular, through the IH. What will she encounter along her way? Is her mission doomed to fail? Does this other external universe actually exist? Answering these questions requires one to comprehend the manner in which quantum fields change the internal geometry of BHs. The most renowned phenomenon in which quantum effects profoundly transform the classical spacetime picture is the process of BH evaporation due to Hawking radiation [3, 4]. In fact, already at the classical level, it was demonstrated that introducing matter (or perturbation) fields on BH backgrounds may affect the geometry in a non-trivial manner. A notable example is the null weak [5] curvature singularity that forms along the Cauchy horizon (CH, which is the ingoing section of the IH) in both spinning [6–9] and spherical charged [10–15] BHs. The analogous effect of quantum perturbations is often expected to be significantly stronger [16–18]; but this issue still remains inconclusive, making it the main motivation for this work.

A theoretical framework that lends itself to this problem is the *semiclassical* formulation of general relativity, considering matter fields as quantum fields propagating in a classical curved spacetime, obeying the semiclassical Einstein field equation, given (in units \( G = c = 1 \)) by:

\[
G_{\alpha\beta} = 8\pi \langle T_{\alpha\beta} \rangle_{\text{ren}}. \tag{1}
\]

Here \( G_{\alpha\beta} \) is the Einstein tensor, and the source term \( \langle T_{\alpha\beta} \rangle_{\text{ren}} \) is the renormalized expectation value of the stress-energy tensor (RSET) associated with the quantum field. Note the emergent requirement for self-consistency: spacetime curvature induces a non-trivial stress-energy in the quantum fields, which in turn deforms the spacetime metric — an effect known as backreaction. A possible way to handle this complexity is to break the problem into steps of increasing order in the mutual effect, initially computing \( \langle T_{\alpha\beta} \rangle_{\text{ren}} \) for a fixed, classical background metric. But already at this level, one faces a serious challenge: the computation of the RSET on curved backgrounds.

Recall that already in flat spacetime the stress-energy tensor of a quantum field formally diverges, but this is usually handled through the normal-ordering scheme, which is ill-defined in curved spacetime. The intricate regularization procedure required in curved spacetime, along with its inevitable numerical implementation, has made this computation a decades-lasting hurdle in the study of semiclassical problems. However, the recently developed *pragmatic mode-sum regularization* (PMR) method [19–22], rooted in covariant point-splitting [23, 24], has made this task more accessible. (See, however, earlier works employing other methods, e.g. [26–30]).

The PMR method overcomes the main difficulty in the numerical implementation of point splitting by treating the coincidence limit analytically, through the construction of “modewise” counter-terms. It has been successfully used in recent years to compute both the vacuum expectation value \( \langle \Phi^2 \rangle_{\text{ren}} \) and the RSET for a quantum scalar field \( \Phi \) on various BH exteriors [19–22, 25]. On BH interiors, however, only \( \langle \Phi^2 \rangle_{\text{ren}} \) has been computed in that method so far (initially for Schwarzschild [40], reproducing previous results [22]). Although \( \langle \Phi^2 \rangle_{\text{ren}} \) is not the quantity most relevant for backreaction, it nevertheless provides valuable insights for the computation of the more divergent RSET. In particular, in a recent paper [41], \( \langle \Phi^2 \rangle_{\text{ren}} \) was investigated both numerically and analytically inside RN, with an extensive study of
the vicinity of the IH. The RSET trace (for a minimally-coupled scalar field) was consequently found to diverge at the IH, providing the first clear-cut evidence for the RSET divergence there. The following work is a natural continuation of previous ones, providing novel results for certain key components of the RSET inside a BH — which directly demonstrate the divergence of semiclassical energy-momentum fluxes at the CH. [19]

We hereby consider a spherically-symmetric charged BH, whose geometry is described by the RN metric:

\[ ds^2 = -f(r)dt^2 + \frac{1}{f(r)} dr^2 + r^2 d\Omega^2, \]

where \( d\Omega^2 = d\theta^2 + \sin^2 \theta d\varphi^2 \), and \( f(r) \equiv 1 - 2M/r + Q^2/r^2 \) with mass \( M \) and charge \( Q \). We consider a non-extremal BH, with \( 0 < Q/M < 1 \). The event horizon (EH) and the IH are located at \( r = r_+ \) and \( r = r_- \) respectively, with \( r_{\pm} = M \pm \sqrt{M^2 - Q^2}. \) For later use, we define the two surface gravity parameters, \( \kappa_{\pm} = (r_+ - r_-)/2r_{\pm}^2 \).

Upon this background we introduce an (uncharged) minimally-coupled massless scalar field \( \Phi(x) \), obeying the (covariant) d’Alembertian equation, \( \Box \Phi = 0 \). We decompose the field into modes, which, owing to the symmetries of the metric, may be separated into \( e^{-i\omega t} \), spherical harmonics \( Y_{lm}(\theta, \varphi) \), and a function of \( r \) \([42]\). The latter is encoded in the radial function \( \psi_{\omega l}(r) \), satisfying:

\[ \frac{d^2 \psi_{\omega l}}{dr^2} + \left[ \omega^2 - V_l(r) \right] \psi_{\omega l} = 0, \]

with the effective potential

\[ V_l(r) = f(r) \left[ \frac{l(l+1)}{r^2} + \frac{df/dr}{r} \right]. \]

\( r_* \) is the standard tortoise coordinate defined through \( dr/dr_* = f(r) \), varying from \( r_* \rightarrow -\infty \) at the EH to \( r_* \rightarrow \infty \) at the IH.

In the interior region of the BH, assuming a free incoming wave at the EH, Eq. (2) is endowed with the initial condition

\[ \psi_{\omega l} \approx e^{-i\omega r_*}, \quad r_* \rightarrow -\infty. \]

We consider our field in two quantum states: the Hartle-Hawking (HH) state \([43][44]\), corresponding to a BH in equilibrium with a thermal bath of radiation, and the more physically realistic Unruh state \([45]\), describing an evaporating BH.

We introduce the null Eddington coordinates inside the BH, \( u = r_* - t \) and \( v = r_* + t \). The flux components of the RSET, \( \langle T_{uu} \rangle_{\text{ren}} \) and \( \langle T_{uv} \rangle_{\text{ren}} \) are of particular interest \([50]\). The reason is threefold. First and most importantly, as we shall see below, it is these components that prove to be the most significant for backreaction near the CH, with a dramatic accumulating effect on the form of the metric (as opposed to minor local distortions associated with other RSET components). In addition, note that although the classical RN background contains a non-zero stress-energy tensor (of the sourceless electromagnetic field), its \( T_{uu} \) and \( T_{uv} \) components vanish identically, leaving quantum contributions to prevail. Finally, their regularization procedure turns out to be especially manageable. Accordingly, aiming for the “heart” of the RSET in the context of backreaction, this work focuses on the flux components \( \langle T_{uu} \rangle_{\text{ren}} \) and \( \langle T_{uv} \rangle_{\text{ren}} \) in the vicinity of the IH.

In the next section we implement the \( \theta \)-splitting variant of the PMR method \([20][46]\) to obtain expressions for the renormalized semiclassical flux components in both quantum states, revealing notable simplicity when taking the IH limit. We then provide numerical results for a variety of \( Q/M \) values, noting various issues that arise. In the last section, we present a very preliminary analysis of backreaction and the fate of our traveler.

Developing the near-IH flux expressions. In what follows, indices \( U \) and \( H \) correspond to the Unruh and HH states, respectively. As mentioned, we shall only consider the two flux components \( \langle T_{uu} \rangle_{\text{ren}} \) and \( \langle T_{uv} \rangle_{\text{ren}} \), and in order to treat them uniformly we introduce the symbol \( y \), which will stand for either \( u \) or \( v \).

The basic point-splitting expression for the trace-reversed RSET is given in Eq. (2.6) of Ref. \([22]\). In the specific case of interest (i.e. the flux components \( \langle T_{yy} \rangle_{\text{ren}} \) evaluated at \( r \rightarrow r_- \) using \( \theta \)-splitting), two remarkable simplifications occur: (i) the counter-term \( I_{yy}(x, x') \) actually vanishes \([46]\); and (ii) since \( g_{yy} = 0 \), \( T_{yy} \) coincides with its trace-reversed counterpart. The expression then simplifies to

\[ \langle T_{yy} \rangle_{\text{ren}}(x) = \frac{\hbar}{2} \lim_{r \rightarrow r_-} G^{(1)}(x, x'), \]

where \( G^{(1)}(x, x') = \langle \{ \Phi(x), \Phi(x') \} \rangle \), and hereafter \( \{ p(x), q(x') \} \) denotes \( p(x)q(x') + p(x')q(x) \). We can also express \( G^{(1)} \) as

\[ G^{(1)}(x, x') = \sum_{l,m} \int_0^\infty d\omega E_{\omega lm}(x, x'), \]

where the mode contributions \( E_{\omega lm}(x, x') \) inside a RN BH, in the HH state, are given by

\[ E^{H}_{\omega lm}(x, x') = \text{coth} \tilde{\omega} \left[ J_R + J_L + \text{cosh}^{-1} \tilde{\omega} J^{RL} \right] \]

(cf. Eq. (4.3) in \([22]\)) where

\[ J_R = \{ f^{R}_{\omega lm}(x), f^{R*}_{\omega lm}(x') \}, \quad J_L = \{ f^{L}_{\omega lm}(x), f^{L*}_{\omega lm}(x') \} \]

and

\[ J^{RL} = 2\Re \left[ \rho_{\omega l} \left( f^{R}_{\omega lm}(x), f^{L*}_{(\omega)lm}(x') \right) \right]. \]
Here \( \hat{\omega} \equiv \pi \omega / k_+ \), the star denotes complex conjugation, and \( \Re \) stands for the real part. Hereafter, \( \rho_{\omega l}^{up}(r_{\omega l}^{up}) \) represents the reflection (transmission) coefficient for the “up” modes outside the BH \([12]\). The mode functions \( f_{\omega l}^{R,L}(x) \) are given by

\[
\tilde{f}_{\omega l}^R = e^{-i\omega t}\psi_{\omega l}(r) \quad \text{and} \quad \tilde{f}_{\omega l}^L = e^{i\omega t}\psi_{\omega l}(r),
\]

where \( \tilde{f}_{\omega l}^R \) and \( \tilde{f}_{\omega l}^L \) are the aforementioned radial function solving Eq. \([2]\) with the boundary condition \([1]\). (For more details see \([12]\).)

A similar expression exists for the Unruh-state counterpart, \( E_{\omega l}^U \). In what follows, we shall describe the analysis for the HH state only. For the Unruh state the analysis is similar and we shall merely quote finite results below (with the more detailed derivation deferred to \([49]\)).

We are interested in the asymptotic behavior at the CH, where the effective potential \( V_{IH} \) vanishes like \( f \propto r - r_- \). Hence the radial equation \([2]\) for \( \psi_{\omega l} \) admits the general asymptotic solution \( \tilde{A}_{\omega l} e^{-i\omega r} + \tilde{B}_{\omega l} e^{-i\omega r} \), which in turn implies

\[
\tilde{f}_{\omega l}^R \cong \tilde{A}_{\omega l} e^{i\omega u} + \tilde{B}_{\omega l} e^{-i\omega v}, \quad \tilde{f}_{\omega l}^L \cong \tilde{A}_{\omega l} e^{i\omega v} + \tilde{B}_{\omega l} e^{-i\omega u}.
\]

Equations \([5,6]\) yield

\[
\langle T_{uu} \rangle_{ren}^H (x) = \frac{\hbar}{2} \lim_{r_+ \to r_-} \lim_{r_- \to 0} \sum_{l,m} \int_0^\infty \! d\omega \frac{E_{\omega l}^{H}}{E_{\omega l}^{H}} \langle x, x', y y' \rangle \, \left| A_{\omega l} \right|^2.
\]

It is interesting to inspect the form of \( E_{\omega l}^{H} \) within the near-IH approximation \([7]\). Consider, for example, the contribution coming from the \( J^R \) term. Focusing for concreteness on the \( y = u \) case, we readily see that the \( \partial_{uu} \) operator annihilates the terms depending on \( v \) in Eq. \([7]\). Also, \( r_+ = f/2 \propto r - r_- \) vanishes at \( r \to r_- \), altogether yielding

\[
J_{uu}^R = \frac{[\omega]}{4\pi r_-} \left\{ Y_{l m}(\theta, \varphi), Y_{l m}^\star(\theta', \varphi) \right\} \left| A_{\omega l} \right|^2.
\]

In the right hand side (RHS) we have substituted \((u', v', \varphi') = (u, v, \varphi)\), as we use \( \theta \)-splitting. Remarkably, although \( J_{uu}^R \) does contain terms like \( e^{i\omega(v+u)} = e^{2i\omega r} \), at the IH limit, \( J_{uu}^R \) is free of such oscillatory terms — and is in fact entirely independent of \( r_+ \) and \( t \). This simplification occurs for all three “\( J \)” terms in the expression for \( E_{\omega l}^{H} \). Combining their contributions and summing over \( m \), one readily obtains

\[
\langle T_{uu} \rangle_{ren}^H = \hbar \lim_{\delta \theta \to 0} \sum_{l=0}^\infty \frac{2l+1}{8\pi} P_l(\cos \delta \theta) F_{l}^H,
\]

where \( \delta \theta \equiv \theta' - \theta \), and \( F_{l}^H = \int_0^\infty \! d\omega \hat{E}_{\omega l} \).

(see fuller derivation in \([48]\)).

The sequence \( F_{l}^H \) appearing in Eq. \([9]\) approaches a non-vanishing constant \( \beta \equiv F_{\infty}^H \). One can show \([48]\), analytically, that \( \beta = (\kappa_-^2 - \kappa_+^2) / 24\pi^2 r_-^2 \). After the limit \( \delta \theta \to 0 \) is taken (using the methods of Ref. \([20]\); see also \([48]\)) we obtain the final result

\[
\langle T_{uu} \rangle_{ren}^H = \langle T_{uu} \rangle_{ren}^H = \sum_{l=0}^\infty \frac{2l+1}{8\pi} \Delta F_{l}^U.
\]

The upper “-” index in this final result indicates the IH limit.

The analogous expression for the Unruh state is \([48]\):

\[
\langle T_{yy} \rangle_{ren}^U = \langle T_{yy} \rangle_{ren}^U + \sum_{l=0}^\infty \frac{2l+1}{8\pi} \Delta F_{l}^U(\rho_{\omega l}^{up} A_{\omega l} B_{\omega l})
\]

(10)

Note that the two Unruh-state flux components are not independent: From energy-momentum conservation, \( 4\pi r_-^2 \left( \langle T_{uu}(x) \rangle_{ren}^U - \langle T_{vv}(x) \rangle_{ren}^U \right) \) is constant (it is actually the Hawking outflux); see Supplemental Material \([48]\).

Numerical results. Recalling the Wronskian relation \( |\rho_{\omega l}^{up}|^2 = 1 - |\rho_{\omega l}^{up}|^2 \), the final expressions \([11,12]\) for the near-IH fluxes in both quantum states reveal simple dependence on \( A_{\omega l}, B_{\omega l} \) and \( \rho_{\omega l}^{up} \). We numerically compute \( A_{\omega l} \) and \( B_{\omega l} \) by integrating the radial equation \([2]\) from \( r_+ \) to \( r_- \) (and \( \rho_{\omega l}^{up} \) likewise, by solving the radial equation outside the BH). We then compute the three flux quantities \( \langle T_{yy} \rangle_{ren}^H \) (that is \( \langle T_{yy} \rangle_{ren}^H, \langle T_{uu} \rangle_{ren}^H \) and \( \langle T_{vv} \rangle_{ren}^H \)) at the IH, as prescribed in Eqs. \([11,12]\). Further numerical details may be found in \([48]\). We find exponential convergence of both the integral over \( \omega \) (entailed in \( F_{l}^H, \Delta F_{l}^U \)) and the sum over \( l \), for all three quantities \( \langle T_{yy} \rangle_{ren}^U \), as they attain well-defined finite values. Note that a finite non-vanishing \( \langle T_{vv} \rangle_{ren}^U \) implies a curvature singularity at the CH, since transforming to a regular Kruskal-like coordinate \( V = -e^{-r_-^2} \) yields \( \langle T_{vv} \rangle_{ren}^H \propto e^{2u_-^2} \to \infty \).

Remarkably, the three fluxes \( \langle T_{uu} \rangle_{ren}^U \) may be either positive or negative, depending on \( Q/M \). We find that sufficiently close to extremality all three flux components become negative, whereas further away from extremality they are all positive. Whether the diverging \( \langle T_{vv} \rangle \) is positive or negative would have a crucial effect on the nature of the tidal deformation (contraction vs. expansion), a point to be expanded hereafter.
Figure 1 displays the three flux quantities $\langle T^{--}_{uu}\rangle_{\text{ren}}$ in the range $0.96 < Q/M < 1$, exhibiting the transition from positive to negative values at around $Q/M \sim 0.97$. More precisely, the change of sign occurs at $Q/M$ values of $q^U_v \approx 0.9650$, $q^U_u \approx 0.9671$ and $q^H_y \approx 0.9675$ for $\langle T^{--}_{uu}\rangle_{\text{ren}}$, $\langle T^{--}_{uu}\rangle_{\text{ren}}$, and $\langle T^{--}_{yy}\rangle_{\text{ren}}$, respectively.

Figure 2 displays the three flux quantities in a wider range $0.1 \leq Q/M < 1$. Note the very rapid increase in the fluxes as $Q/M$ decreases. This is perhaps not too surprising, since a decrease in $Q/M$ implies an (even faster) decrease in $r_{-}/M$, and correspondingly an increasing curvature at the IH.

Another notable feature is the decay of the fluxes as extremality is approached. We intend to analytically address the near-extremal domain (characterized by $|Q/M - 1| \ll 1$) in a future paper [47]. Remarkably, we find excellent qualitative agreement between our analytical expressions (to be presented in [47]) and the numerical data illustrated on the rightmost part of Fig. [47].

The steep drop at about $Q/M \sim 9671$ and $q^H_y \sim 9675$ for $r_{-}/M$, exhibits the transition that semiclassical backreaction is negligible — and hence the actual backreacted geometry should be well approximated by the RN metric — as long as we are not too close to the CH.

This approximation is indeed valid in the HH state. Note, however, that it fails to apply globally in the Unruh state, because already at the EH the mass parameter drifts significantly as the BH evaporates. For this reason, in the backreaction analysis below we shall focus on the HH state, and the more subtle extension to the Unruh state is left for future research.

To address backreaction, we write the general spherically-symmetric metric in double-null coordinates as $-e^\sigma du dv + r^2 d\Omega^2$. The two unknown metric functions, $r(u,v)$ and $\sigma(u,v)$, are to be determined from the semiclassical Einstein equation [47]. This system contains constraint equations, which are two independent ODEs (one along each null direction) that involve the flux components $\langle T^{--}_{yy}\rangle_{\text{ren}}$ only; and evolution equations, which are two coupled PDEs involving $\langle T^{--}_{uu}\rangle_{\text{ren}}$ and $\langle T^{--}_{yy}\rangle_{\text{ren}}$. Our analysis will mainly rely on the two constraint equations, which we write uniformly as

$$r_{,yy} - r_{,v}\sigma_{,y} = -4\pi r \langle T^{--}_{yy}\rangle_{\text{ren}} .$$

To proceed, we shall now restrict the analysis to the weak-backreaction domain, in which $r, \sigma_y$ and $\langle T^{--}_{yy}\rangle_{\text{ren}}$ (but not $r_y$) are still well approximated by their RN background values. Furthermore, we shall focus on the near-CH portion of this domain [51]. In this region, we may replace the RHS by the constant $-4\pi r \langle T^{--}_{yy}\rangle_{\text{ren}}$, and $\sigma_y$ by $-\kappa_-$ (its near-CH value in RN). We obtain a trivial linear ODE for $r_y$, which is easily solved. After an exponentially decaying term ($\propto e^{\sigma}$) is dropped, we are left with

$$r_{,y} \equiv -4\pi (r_{-}/\kappa_-) \langle T^{--}_{yy}\rangle_{\text{ren}} .$$

This result expresses a small but steady asymptotic drift of $r(u,v)$ in both null directions. In the long run (i.e. at sufficiently large $u$ and/or $v$) this drift results in a major deviation of $r$ from its RN value, which in turn leads us away from the domain of weak backreaction.

To discuss the physical implications of this result, recall our infalling traveler, as she approaches the CH (located at $v \to \infty$). We emphasize that although the near-CH drift in $r$ is very “slow” in terms of $v$ (i.e. $r_v \ll 1$), it actually marks a catastrophic physical effect for our infalling traveler — which arrives the CH at a finite proper time [52]. The nature of this physical effect will crucially depend on the sign of $\langle T^{--}_{yy}\rangle_{\text{ren}}$ and hence on the value of $Q/M$ (associated with the original RN background): For $Q/M < q^H_y$, $\langle T^{--}_{yy}\rangle_{\text{ren}} > 0$ and correspondingly our

Backreaction near the CH. The semiclassical backreaction, being $\propto \hbar/M^2 = (m_p/M)^2$ (where $m_p$ denotes the Planck mass), is basically an extremely weak effect for macroscopic BHs. For instance, for astrophysical BHs it is typically $< 10^{-75}$. However, these effects accumulate (and become singular) near the CH, as we shortly discuss. For this reason, it is natural to take the viewpoint that semiclassical backreaction is negligible — and hence the actual backreacted geometry should be well approximated by the RN metric — as long as we are not too close to the CH.
traveler will undergo sudden contraction. However, for 
\( Q/M > q_0^2 \), \( \langle T_{\nu \nu} \rangle_{\text{ren}} \) is negative — implying an abrupt expansion on approaching the CH.

This analysis still needs to be extended in two important directions: (i) to the domain of strong backreaction, and (ii), from the HH state to the more realistic Unruh state. Both extensions appear to be feasible. Preliminary results seem to indicate that the steady drift of \( r \) seen in the weak-backreaction domain extends (with certain modifications) to the strong-backreaction domain — as well as to the Unruh state. We hope to address these issues in the future.

Discussion. Motivated by long-standing expectations that semiclassical effects may drastically influence the interior geometry of spinning or charged BHs, this work focused on the RSET flux components (for a minimally-coupled massless scalar field), in the vicinity of the IH, on a fixed RN background. We presented novel results for the flux components in the Unruh and HH states for various \( Q/M \) values. Both flux components \( \langle T_{\nu \nu} \rangle_{\text{ren}} \) and \( \langle T_{V V} \rangle_{\text{ren}} \) — in both quantum states — exhibit finite asymptotic values at the IH limit. Recall, however, that a non-vanishing finite \( \langle T_{\nu \nu} \rangle_{\text{ren}} \), implies unbounded curvature (and unbounded tidal force) at the CH \( (v \to \infty) \), because the corresponding Kruskal-like component \( \langle T_{V V} \rangle_{\text{ren}} \) then diverges as \( e^{2\kappa - v} \).

Our numerical results indicate that all flux components change their signs at around \( Q/M \approx 0.97 \), being negative for larger \( Q/M \) and positive (and typically much larger) for smaller \( Q/M \) values. The sign may have crucial implications to the nature of the tidal effect: catastrophic contraction (for \( \langle T_{\nu \nu} \rangle_{\text{ren}} > 0 \)) vs. expansion (for \( \langle T_{\nu \nu} \rangle_{\text{ren}} < 0 \)).

In the near-extremal domain the problem of near-IH flux computation lends itself to analytical treatment, leading to excellent agreement with our numerical results (see [17]).

We also made initial steps towards analyzing the semiclassical backreaction effects of the fluxes on the near-CH geometry, in the HH state. The result expressed in Eq. [18] hints for drastic deformation of the area coordinate \( r \) on approaching the CH. However, the discussion provided here was rather preliminary. It should be extended, as mentioned, beyond the weak-backreaction domain, as well as to the Unruh state.

Other obvious extensions are in order. First, it would be worthwhile to generalize the analysis to all RSET components, and also to the entire interior domain \( r_- < r < r_+ \). More importantly, this investigation should be extended from scalar to the more realistic electromagnetic quantum field — and in addition, from the spherical RN background to the astrophysically much more relevant background of a spinning BH.

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See Supplemental Material for derivation of \( \hat{E}_{\nu}^{\mu} \) as it appears in Eq. (10) (section 1), derivation of the “plateau value” \( \beta \equiv F_{\nu \rightarrow \infty} \) (section 2), an analogous treatment for the Unruh state flux expressions (section 3) and some basic numerical parameters (section 4).

See also [39], but note that the unusual quantum state constructed there does not allow investigating the anticipated semiclassical CH divergencies.

Note that the fluxes do not contribute to the RSET trace, which was shown to diverge in [41].

This is the region in which \( e^{-\kappa_{\nu} (u+v)} \) is already \( \ll 1 \) (hence in the RN background \( r, \sigma, y \) and \( \langle T_{yy} \rangle_{ren} \) are well approximated by their near-CH values); and correspondingly, the drift effect in \( r \) is already present — but still hasn’t accumulated much.

In particular, recall that \( dr/d\tau \propto dv/d\tau \propto e^{\kappa_{\nu} - v} \).