The Modulation Scheme with Unknown CSI for Analog Network Coding in Multi-Antenna Relay Channel

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Abstract

Analog network coding (ANC) is a hot topic in recent pioneer researches. Two end-sources can exchange information with the help of a relay by making use of the additive nature of electro-magnetic waves in the wireless two-way relay channel (TWRC). The relay amplify and forward the XORed signal to both of the two sources. In this paper, we assume that each source is equipped with a single antenna and the relay is equipped with multiple antennas. Neither the sources nor the relay can acquire the channel state information (CSI). We propose a novel modulation scheme based on differential modulation. The receivers in TWRC can eliminate the self-interference when their transmitting signals are independent. Then we derive the maximum likelihood (ML) detector for the ANC system with differential modulation signal. We compare the system bit error rate (BER) performance with coherent detection and perfect CSI. Theoretical and simulated results show that even without the aid of CSI, the performance of the proposed system is approximately 1-2dB away from the optimal coherent detection scheme.

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1. Introduction

In a typical two-way relay channel (TWRC) \cite{1}, two end-sources exchange their messages with the help of one or multiple relays. For the information exchange between the two end-sources in TWRC, the traditional routing transmission scheme takes four stages. Unlike the traditional routing strategies, in digital network coding (DNC) strategies \cite{2}\cite{3}\cite{4}, three stages are required: two source nodes send their messages separately to the relay node in the first two stages; the relay node decodes and combines the

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messages by some coding algorithms; then it broadcasts the combined message to both of the two source nodes. As a comparison, physical-layer network coding (PNC) scheme [5] which utilizes the additive nature of electro-magnetic (EM) waves requires two stages. In the first stage, source node $A$ and $B$ simultaneously send their messages to relay node $R$. $R$ receives the combined signal, then decodes and re-encodes (using XOR or other linear operations) the signal. In the second stage, $R$ broadcasts the XORed signal of the decoded signals as it does in the DNC scheme. Similar with the PNC scheme, analog network coding (ANC) which is a typical amplify-and-forward (AF) protocol in bidirectional network is widely studied in recent researches [6]-[11]. The ANC protocol requires the same two stages to exchange the information in TWRC. The difference between ANC and PNC schemes is that in the ANC scheme, the relay amplifies and forwards the signal to the two sources without decoding the XORed signal.

In most of the pioneering studies for ANC [6]-[15], the sources and relays are considered to acquire perfect CSI. Thus coherent detection can be performed perfectly at sources and relays. In practice, however, channel state acquisition is a critical problem in the ANC scheme [5]. The CSI acquisition is difficult especially in fast fading environment. In some recent studies [16][17], the channel estimation with pilot is studied. However, it increases the complexity of the communication protocol and the demodulation design for the receivers.

Differential modulation [18][19] is a practical solution for the case in which the CSI acquisition is difficult. The relay and the sources do not need any CSI in the signal transmission and reception with differential modulation. For the ANC scheme, in the first stage, the sources encode the signal differentially. Then, the relay node computes the XORed signal power amplify ratio with its received signal, before it amplifies and forwards the signal. In the second stage, the sources can eliminate the self-interference perfectly with its own differential code. This approach may result in about 3dB performance loss compared to the coherent schemes due to instantaneous detection errors. However, the limitation of the approach in [18][19] is that the relay is equipped with single antenna and the CSI is restricted in the real domain. If the CSI is in the complex domain, the detection scheme proposed in [18][19] cannot demodulate and decode the differential signal. In addition, the approach in [18][19] only considered the single-antenna relay case.

In this paper, we propose a novel ANC modulation and demodulation scheme based on differential modulation where the relay is equipped with multi-antennas. Neither the sources nor the relay are assumed to acquire the CSI. The sources encode their signal differentially. Each receiver in TWRC can eliminate the self-interference by utilizing that the transmitting signals of the two sources are independent. Then we propose the maximum likelihood (ML) detector, by which the sources can demodulate the signal when the channel responses are in the complex domain.

The performance analysis shows that the proposed scheme in this paper is 4-5 dB away from the coherent detection scheme for multi-antenna relay ANC system.

The remainder of this paper is organized as follows. The system model and differential modulation scheme are presented in Section 2. Our main contributions on maximum likelihood (ML) detector design without CSI and the BER performance analysis are proposed in Section 3. Simulation results are presented in Section 4. We conclude this paper in Section 5.

The following notations are used in this paper. The boldface letters are used to denote matrices and vectors. Let $\text{tr}(\mathbf{A})$, $\mathbf{A}^T$, $\mathbf{A}^*$, $\mathbf{A}^H$ and $\mathbf{A}^\dagger$ denote the matrix trace, transpose, conjugate, conjugate transpose, and pseudo inverse operations, respectively, for a matrix $\mathbf{A}$. $\| \cdot \|$ denotes the Euclidean norm, and $| \cdot |$ denotes the scalar norm. $\mathbb{C}^{x \times y}$ denotes the space of $x \times y$ matrices with complex-valued elements. The distribution of a circular symmetric complex Gaussian (CSCG) random vector in which each element is distributed with mean $x$ and covariance $\sigma^2$ is denoted by $\mathcal{CN}(x, \sigma^2 \mathbf{I})$, and $\sim$ stands for "distributed as".
2. System Model and Preliminaries

We consider the network with TWRC as shown in Fig. 1. Two end-sources, i.e., \( A \) and \( B \), each with a single antenna exchange their messages in this system with the help of the relay \( R \) with \( M \) antennas. All the nodes are assumed to operate in half-duplex way. The channels between the sources and the relay are assumed as block fading channels. \( A \) and \( B \) transmit signals simultaneously in the first stage. Then \( R \) processes the signals linearly (beamforming) and broadcast it in the second stage. The two stages are assumed as two time slots with equal duration. The synchronization of transmit symbols is assumed as established perfectly. With the help of differential modulation, we assume that neither the sources nor the relay can acquire the CSI. The following analyses in this paper are based on the assumption of perfect timing and frequency synchronization.

Fig. 1. Multi-Antenna Relay ANC System

We use \( c_a(n) \in \mathcal{C} \) and \( c_b(n) \in \mathcal{C}' \) to donate the modulated symbol in the source \( A \) and \( B \), respectively, where \( n = 1, 2, 3, \cdots, N \) is the symbol index. \( \mathcal{C} \) and \( \mathcal{C}' \) are the constellation sets for \( A \) and \( B \), respectively. The differential modulated symbol in the sources are given by

\[
s_a(n) = s_a(n-1)c_a(n), \quad s_b(n) = s_b(n-1)c_b(n),
\]

where \( s_a(n) \sim \mathcal{CN}(0,1) \) and \( s_b(n) \sim \mathcal{CN}(0,1) \) are the transmit symbols from \( A \) and \( B \), respectively.

In the first time slot, \( R \) receives the sum of the signals. The received signal at \( R \) is obtained as

\[
y_R(n) = h_a\sqrt{p_a}s_a(n) + h_b\sqrt{p_b}s_b(n) + u_R(n),
\]

where \( y_R(n) \in \mathbb{C}^{M \times 1} \) is the received signal vector with symbol index \( n \), \( n = 1, 2, 3, \cdots, N \). \( h_a \in \mathbb{C}^{M \times 1} \) and \( h_b \in \mathbb{C}^{M \times 1} \) represent the channel state vectors from \( A \) to \( R \) and from \( B \) to \( R \), respectively, which are constant vectors during the transmission of \( N \) symbols. The elements of \( h_a \) and \( h_b \) follow the same distribution of CSCG and distribute as \( \mathcal{CN}(0,1) \). \( p_a \) and \( p_b \) present the transmit powers of \( A \) and \( B \), respectively. \( u_R(n) \in \mathbb{C}^{M \times 1} \) is the received noise in \( R \) and satisfies \( u_R(n) \sim \mathcal{CN}(0, \sigma^2 I) \). In the second time slot, \( R \) processes the received signal in linear beamforming operation, and broadcasts the processed signal to \( A \) and \( B \). The beamforming process is given by

\[
x_R(n) = \mathbf{A}^H(n)x_R(n), \quad n = 1, 2, \ldots, N,
\]

where \( x_R(n) \in \mathbb{C}^{M \times 1} \) is the transmit signal from \( R \), and \( \mathbf{A} \) is the relay beamforming matrix. We assume that the constraint of the total power of \( M \) antennas on \( R \) is \( P_R \). Thus the transmit power of the relay satisfies

\[
\|\mathbf{A}h_a\|^2 p_a + \|\mathbf{A}h_b\|^2 p_b + \text{tr}(\mathbf{A}^H \mathbf{A}) \sigma^2 \leq P_R.
\]

In the second time slot, the received signals in \( A \) and \( B \) can be expressed as

\[
y_a(n) = h_a^T x_R(n) + u_a(n) = h_a^T \mathbf{A}h_a^* s_a^*(n) + h_a^T \mathbf{A}h_b^* s_b^*(n) + h_a^T \mathbf{A}u_R^*(n) + u_a(n),
\]

\[
y_b(n) = h_b^T x_R(n) + u_b(n) = h_b^T \mathbf{A}h_b^* s_b^*(n) + h_b^T \mathbf{A}h_a^* s_a^*(n) + h_b^T \mathbf{A}u_R^*(n) + u_b(n),
\]

where \( u_a(n) \) and \( u_b(n) \) represent the transmit powers of \( A \) and \( B \), respectively.
where $y_a(n) \in \mathbb{C}^{1 \times 1}$ and $y_b(n) \in \mathbb{C}^{1 \times 1}$ are the received signal in $A$ and $B$, respectively. $u_a(n) \in \mathbb{C}^{1 \times 1}$ and $u_b(n) \in \mathbb{C}^{1 \times 1}$ are noise in $A$ and $B$, respectively, and satisfy $u_a(n), u_b(n) \sim \mathcal{CN}(0, \sigma^2)$. $h^T_a \mathbf{A} u^*_R(n)$ is the noise forwarded by the relay, and satisfies $h^T_a \mathbf{A} u^*_R(n) \sim \mathcal{CN}(0, \|h^T_a \mathbf{A}\|^2 \sigma^2 \mathbf{I})$. As $A$ and $B$ are symmetrical nodes in the mathematical expressions, we analyze the decoding scheme and the performance only for the source $A$ in the following sections.

3. Maximum-Likelihood Detector

3.1. Self-Interference Elimination

In the second time slot of the ANC relay, source $A$ cancels the self-interference and decodes the signal from source $B$. For simplicity, we write the received signal in source $A$ as

$$y_a(n) = v s_a^*(n) + u_a(n),$$  

(7)

where $v = h^T_a \mathbf{A} s_b \sqrt{\rho_b}$ and $u = h^T_a \mathbf{A} s_b \sqrt{\rho_b}$. $w_a(n)$ is the noise accompanied with the signal and is given as

$$w_a(n) = h^T_a \mathbf{A} u^*_R(n) + u_a(n).$$  

(8)

To cancel the self-interference, we estimate $v$ with the following signal as

$$y'_a(n) = y_a(n) s_a(n) = v |s_a(n)|^2 + u s_a^*(n) s_a(n) + w_a(n) s_a(n).$$  

(9)

As $s_a(n)$ and $s_b(n)$ are two independent signals, without loss of generality, we have $\mathbb{E}[s_a^*(n) s_a(n)] \approx 0$. Thus the parameter $v$ is estimated by

$$v = \frac{\mathbb{E}[y'_a(n)]}{\mathbb{E}[|s_a(n)|^2]}.$$  

(10)

Then we derive the signal without self-interference as

$$\tilde{y}_a(n) = y_a(n) - v s_a^*(n)$$

$$\approx u s_a^*(n) + w_a(n).$$  

(11)

3.2. Differential Demodulation

As we have derived the signal without self-interference, we can demodulate the target signal from source $B$ with the using of differential signal phase. Thus we have that the signal in (11) can be derived as

$$\tilde{y}_a(n) = \tilde{y}_a(n - 1) c_b^*(n) + w_a(n) - w_a(n - 1) c_b^*(n).$$  

(12)

Then the ML detector is obtained as

$$\hat{c}_b(n) = \arg \max_{c_b(n)} \text{Re} \{\tilde{y}_a^*(n - 1) c_b(n) \tilde{y}_a(n)\}.$$  

(13)

3.3. BER Performance Analysis

In our proposed system model, we do not limit the modulation schemes. Thus the modulation schemes such as M-PSK are compatible in our system. Without loss of generality, we analyze the BER performance of BPSK in this section. Then the bit error probability is derived as

$$BER_{dp} = \frac{1}{2} e^{-\gamma n},$$  

(14)
where $\gamma_b = E_b/N_0$. From (12), we note that there are two parts of noise in the received signal which is caused by the differential signal. In our proposed system, the end-to-end link SNR is defined as

$$\gamma^M_{\alpha} = \frac{\|h^T_{\alpha} A h^{*}_{\alpha}\|^2 p_b}{2 (\|h^T_{\alpha} A\|^2 + 1) \sigma^2},$$

(15)

$$\gamma^M_{b} = \frac{\|h^T_{b} A h^{*}_{b}\|^2 p_a}{2 (\|h^T_{b} A\|^2 + 1) \sigma^2}.$$

(16)

For BPSK modulation, we have $\mathbb{E}[|s_{\alpha}(n)|^2] = \mathbb{E}[|s_b(n)|^2] = 1$. The end-to-end link SNRs with BPSK modulation are calculated as

$$\gamma^M_{\alpha,BPSK} = \frac{\|h^T_{\alpha} A h^{*}_{\alpha}\|^2 p_b}{2 (\|h^T_{\alpha} A\|^2 + 1) \sigma^2},$$

$$\gamma^M_{b,BPSK} = \frac{\|h^T_{b} A h^{*}_{b}\|^2 p_a}{2 (\|h^T_{b} A\|^2 + 1) \sigma^2}.$$

(17)

Then we have the BER performance of our proposed system with BPSK modulation as

$$BER_{dp,BPSK}^c = \frac{BER_{dp,BPSK}^c + BER_{dp,BPSK}^b}{2} = e^{-\frac{\gamma^M_{\alpha,BPSK}}{4}} + e^{-\frac{\gamma^M_{b,BPSK}}{4}}.$$

(18)

4. Numerical Results

In this section, we present the numerical results of the differential modulation scheme, and we compare our proposed scheme with existing beamforming schemes [9]. For simplicity, we assume that the sources perform BPSK modulation. The channel response vector between $R$ and $A$ is a randomly generated CSCG vector which satisfies $h_{\alpha} \sim \mathcal{CN}(0, I)$. Similarly, we have $h_b \sim \mathcal{CN}(0, I)$, where $h_{\alpha}$ and $h_b$ are independent. The block length $N$ is set to $N = 100$. The power is assumed to be allocated equally in $A$, $B$ and $R$. Thus we set $p_a = p_b = P_s = 1$ and $P_R = 1$. The system SNR in simulations is defined as $P_S/\sigma^2$.

![Fig. 2. BER Performance of the System](image-url)
The numerical results in Fig.2 show that the BER performance of our proposed system is about 1-2 dB away from the coherent detection with perfect CSI. Fig.3 shows that the analytical results are coherent with the theoretical analyses in Section 3.3. The little performance loss of analytical results, which is about 0.5dB, is caused by the imperfect independent source bits.

5. Conclusion

In this paper, we have proposed a novel modulation scheme for multi-antenna relay ANC system based on differential modulation where neither the relay nor the sources need to acquire the CSI. The receivers can eliminate the self-interference when their transmitting signals are independent. Then we derived the maximum-likelihood detector. We demonstrated the performance of the proposed beamforming scheme and the ML detector with numerical results, which show that even if no CSI is acquired the performance of the proposed scheme is only approximately 1-2 dB away from the optimal coherent detection scheme with perfect CSI.

References

[1] Shannon C E, Two-way Communication Channels, Proc. 4th Berkeley Symp. Mathematical Statistics Probability, Berkeley, CA, 1961: 611-644.

[2] Yeung R W, and Zhang Z, Distributed Source Coding for Satellite Communications, IEEE Trans. Inform. Theory,1999, 45(4): 1111-1120.

[3] Katti S, Rahul H, Hu W J, et al. XORs in the Air: Practical Wireless Network Coding, IEEE/ACM Trans. Networking, 2008, 16(3):497-510.
[4] Yuen C, Chin W H, Guan Y L, et al. Bi-Directional Multi-Antenna Relay Communications with Wireless Network Coding, IEEE Vehicular Technology Conference (VTC), Singapore, 2008, 11(14): 1385-1388.

[5] Zhang Shengli, Liew SC, Lam P P, Physical-layer Network Coding, Proc. ACM Annual Int. Conf. on Mobile Computing and Networking (ACM MOBICOM), California, USA, September 2006.

[6] Katti S, Gollakota S, Katabi D, Embracing Wireless Interference: Analog network coding, in Proc. ACM SIGCOMM, Kyoto, Japan, Aug. 2007: 397-408.

[7] Popovski P, Yomo H, Wireless Network Coding by Amplify-and-Forward for Bi-Directional traffic Flows, IEEE Commun. Lett., Jan. 2007, 11(1): 16-18.

[8] Chen Chen, Xiang Haige, The Throughput Order of Ad Hoc Networks with Physical-layer Network Coding and Analog Network Coding'. Proc. IEEE Int. Conf. Commun. (ICC), Beijing, China, May 2008.

[9] Zhang R, Liang YC, Chai C C, et al, Optimal Beamforming for Two-Way Multi-Antenna Relay Channel with Analogue Network Coding', IEEE J. Select. Areas Commun', June 2009, 27(5): 699-712.

[10] Han Yang, Ting S H, Ho C K, et al. Performance Bounds for Two-Way Amplify-and-Forward Relaying, IEEE Trans. Wireless Commun., Jan 2009, 8(1): 432-439.

[11] Yang Jing, Fan Pingzhi, Duong T Q, et al. Exact Performance of Two-Way AF Relaying in Nakagami-m Fading Environment, IEEE Trans. Wireless Commun., March 2011, 10(3): 980-987.

[12] Gacanin H, Adachi F, Channel Capacity of Analog Network Coding in a Wireless Channel, Proc. IEEE 20th International Symposium on Personal, Indoor and Mobile Radio Commun. (PIMRC), Tokyo, Japan, 2009: 1722-1726.

[13] Lu Kejie, Fu Shengli, Qian Yi, Capacity of Random Wireless Networks: Impact of Physical-layer Network Coding, Proc. IEEE Int. Conf. Commun. (ICC), Beijing, China, May 2008.

[14] Lu Kejie, Fu Shengli, Qian Yi, et al. On Capacity of Random Wireless Networks with Physical-layer Network Coding, IEEE J. Select. Areas in Commun., June 2009, 27(5): 763--772.

[15] Gacanin H, Adachi F, Broadband Analog Network Coding, IEEE Trans. Wireless Commun., 9 (5), May 2010: 1577-1583.

[16] Gacanin H, Sjodin T, Adachi F, On Channel Estimation for Analog Network Coding in a Frequency-Selective Fading Channel, EURASIP Journal on Wireless Commun. and Networking, vol. 2011, Article ID 980430, 12 pages, 2011, doi:10.1155/2011/980430.

[17] Roemer F, Haardt M, Tensor-Based Channel Estimation and Iterative Refinements for Two-Way Relaying with Multiple Antennas and Spatial Reuse, IEEE Trans. Signal Processing, 58 (11), Nov 2010: 5720-5735.

[18] Song Lingyang, Li Yonghui, Huang Anpeng, et al. Differential Modulation for Bidirectional Relaying With Analog Network Coding, IEEE Trans. Signal Processing, July 2010, 58(7): 3933-3938.

[19] Song Lingyang, Hong Guo, Jiao Bingli, Joint Relay Selection and Analog Network Coding using Differential Modulation in Two-Way Relay Channels, IEEE Trans. Vehicular Tech., July 2010, 59 (6): 2932-2939.