Research Article

Time Domain Equalizer Design Using Bit Error Rate Minimization for UWB Systems

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Ultra-wideband (UWB) communication systems occupy huge bandwidths with very low power spectral densities. This feature makes the UWB channels highly rich in resolvable multipaths. To exploit the temporal diversity, the receiver is commonly implemented through a Rake. The aim to capture enough signal energy to maintain an acceptable output signal-to-noise ratio (SNR) dictates a very complicated Rake structure with a large number of fingers. Channel shortening or time domain equalizer (TEQ) can simplify the Rake receiver design by reducing the number of significant taps in the effective channel. In this paper, we first derive the bit error rate (BER) of a multiuser and multipath UWB system in the presence of a TEQ at the receiver front end. This BER is then written in a form suitable for traditional optimization. We then present a TEQ design which minimizes the BER of the system to perform efficient channel shortening. The performance of the proposed algorithm is compared with some generic TEQ designs and other Rake structures in UWB channels. It is shown that the proposed algorithm maintains a lower BER along with efficiently shortening the channel.

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1. Introduction

Channel shortening is an equalization technique which forces the effective channel impulse response (combined channel and equalizer) to be confined within a desired temporal window. Channel shortening or time domain equalizers (TEQs) have been used in communication systems since the early 1970s [1–4]. The earlier usage of TEQs was to reduce the number of states in sequence estimation and thus simplify the process. TEQ designs were reinvestigated in the 1990s to mitigate the intersymbol interference (ISI) produced due to inadequate cyclic prefix (CP) in multicarrier modulation (MCM) systems [5–10]. Each of these designs uses a particular cost function, which may be general or system specific, to perform efficient channel shortening. TEQ has also been proposed to simplify multiuser detection in a large set of users [11]. The TEQ in this case eliminates some users’ signals to effectively reduce the size of the user set.

A major problem encountered in UWB systems is to capture enough multipaths through a Rake receiver [12] to maintain a sufficient output signal-to-noise ratio (SNR). An All-Rake (A-Rake) or ideal Rake is not a suitable choice in a dense multipath channel. A Partial-Rake (P-Rake) is easy to implement but provides suboptimum performance. On the other hand, a Selective-Rake (S-Rake) captures a certain number of the strongest multipaths which may not necessarily arrive in successive temporal bins. Therefore, the operational window of the S-Rake may be long enough to cause ISI. Channel shortening can help to mitigate this problem [13–16]. The presence of the TEQ insures that the channel energy is concentrated into the desired number of multipaths that are available in consecutive bins. As a result, loosely speaking, the Rake receiver enjoys the benefits of S-Rake performance or better in the structure of a P-Rake. Improved SNR is also critical in extending the area of coverage. With a TEQ before the Rake reception, the
Rake can be implemented with a smaller number of fingers. This not only simplifies the receiver front end but also the rest of the signal processing and the manufacturing cost involved. Hence, channel shortening in UWB receivers can help in designing a simple and cost effective structure.

UWB communications systems are entirely different from the MCM systems for which a TEQ is commonly proposed. First of all, UWB is a wireless scenario with extremely dense multipath channels. Standard UWB channel models, namely CM1 to CM4 [17], are much more complex than those used in wired line MCM systems, for example, carrier serving area (CSA) loops in asymmetric digital subscriber line (ADSL). Furthermore, to make the UWB receiver design practically simple, a large number of channel taps must be eliminated. This makes the shortened channel window very much smaller than the suppressed channel. Hence, the problem of TEQ design appears in its extreme form. In UWB systems, channel energy capture is crucial to maintain a good output SNR, whereas in most channel. Hence, the problem of TEQ design appears in its extreme form. In UWB systems, channel energy capture is crucial to maintain a good output SNR, whereas in most.

In general, these channel models are chosen. They generally take the following mathematical form:

\[
    h(t) = X \sum_{l=0}^{L} \sum_{k=0}^{K} \alpha_{k,l} \delta(t - T_l - \tau_{k,l}) \quad \text{(1)}
\]

\[
    = \sum_{m=0}^{M-1} h_m \delta(t - \tau_m), \quad \text{(2)}
\]

where \(\alpha_{k,l}\) are the multipath gain coefficients, \(T_l\) is the delay of the \(l\)th cluster, \(\tau_{k,l}\) is the delay of \(k\)th multipath component relative to the \(l\)th cluster arrival time \(T_l\), \(L\) is the number of clusters, \(K\) is the number of multipaths within a cluster, and \(X\) represents the log-normal shadowing associated with multipath amplitudes. Equation (2) is the simplified form of (1) where the multipath gain coefficients \(h_m\) and their arrival times \(\tau_m\) are assumed to have absorbed all the statistical properties of \(X\), \(\alpha_{k,l}\), \(T_l\) and \(\tau_{k,l}\), and the channel contains \(M\) number of multipaths.

We consider an impulse radio (IR) UWB system using pulses \(g(t)\) of width \(T_p\) seconds. In a multiuser environment of \(N_u\) simultaneously active users, the unmodulated signalling waveform of the \(j\)th user is given by

\[
    x_j(t) = \sum_{i=0}^{N_u-1} g(t - iT_c - c_{j,i}T_c), \quad \text{(3)}
\]

where \(N_u\) is the number of pulse repetitions, \(T_f\) is the pulse repetition time, \(T_c\) is the chip duration such that there are \(N_h\) chips within \(T_f\), and \(c_{j,i} \in \{0,1,\ldots,N_h - 1\}\) is the time hopping (TH) sequence for the \(j\)th user.

Let \(\{d_{j,i}\}\) be the data sequence available at the \(j\)th user. We assume that \(\{d_{j,i}\}\) is a wide sense stationary random process with equiprobable symbols. Binary pulse position modulation (BPPM) and binary phase shift keying (BPSK) schemes are considered. Hence, the signal transmitted by the \(j\)th user can be given as

\[
    x_j(t) = \begin{cases} 
    \sum_{i=0}^{N_u-1} g(t - iT_f - c_{j,i}T_c - \Delta d_{j,i}), & \text{BPPM,} \\
    \sum_{i=0}^{N_u-1} d_{j,i} g(t - iT_f - c_{j,i}T_c), & \text{BPSK,}
    \end{cases} \quad \text{(4)}
\]

where \(d_{j,i} \in \{0,1\}\) for BPPM, \(d_{j,i} \in \{-1,1\}\) for BPSK, \(E_j\) is the available power for the \(j\)th user, and \(\Delta\) is the modulation index for BPPM and can be chosen to optimize the performance.

It is reasonable to assume that \(T_p\) is less than the multipath arrival delay bin and no overlapping between the multipath occurs, that is, only resolvable multipaths are considered. A TEQ \(w(t)\) is present at the receiver front end before the Rake reception:

\[
    w(t) = \sum_{n=0}^{N-1} w_n \delta(t - n\tau_n), \quad N \ll M, \quad \text{(5)}
\]

where \(w_n\) is the \(n\)th filter coefficient and \(\tau_n\) is the temporal spacing between any two consecutive filter taps.

### 2. System Architecture

In this paper we use the standard channel models [17], namely CM1 to CM4, to develop the system architecture and evaluate its performance. These channel models are modified versions of the Saleh-Valenzuela (S-V) model [19] and generated to fit different high data rate propagation scenarios. Since we consider a high data rate system in
The received signal from the $j$th user will experience an effective channel of length $M + N - 1$ such that

$$h_j(t) = h_j(t) \ast w(t) = \sum_{m' = 0}^{M-1} h_{j,m'} \delta(t - \tau_{j,m'})$$  \hspace{1cm} (6)

where $h_j(t)$ is the effective channel, $h_j(t)$ is the channel from the $j$th user, "\ast" represents convolution operation, $M' = M + N - 1$ and $\tau_{j,m'}$ is the associated delay.

Therefore, the $j$th user signal at the TEQ output is

$$\bar{x}_j(t) = \begin{cases} \sqrt{E_j} \sum_{i=0}^{N_s - 1} \sum_{m' = 0}^{M-1} h_{j,m'} g \left( t - iT_j - c_{j,i} T_c - \Delta d_{j,i} - \tau_{j,m'} \right) & \text{BPPM,} \\ \sqrt{E_j} \sum_{i=0}^{N_s - 1} \sum_{m' = 0}^{M-1} h_{j,m'} d_{j,i} g \left( t - iT_f - c_{j,i} T_c - \tau_{j,m'} \right) & \text{BPSK.} \end{cases}$$  \hspace{1cm} (7)

The additive white Gaussian noise (AWGN) $n_{TEQ}(t)$ with zero mean and variance $\sigma_{TEQ}^2$ will also be processed through the TEQ and can be considered as filtered noise. Hence the signal available for Rake reception is

$$r(t) = \sum_{j=1}^{N_u} \bar{x}_j(t) + n(t),$$  \hspace{1cm} (8)

where $n(t)$ is the total noise available at the TEQ output.

### 3. Probability of Error Model

We assume that the receiver knows a typical transmitted waveform and uses it as the correlation template. The template waveform $v(t)$ is assumed to be real and synchronized with the TH code of the user of interest and its $m'$th multipath arrival time. This means that the TH code $c_{j,i}$ for the user of interest is known at the receiver. Each finger of the Rake receiver correlates $M'N_u$ multipaths along with the noise. The user $p$ is the user of interest whose TH code is known at the receiver and the $q$th finger of the Rake is under consideration. In this situation, only the $q$th multipath from the $p$th user contributes to the desired signal energy. All other multipaths from the $p$th user can be accounted for self-interference. Whereas, $M' (N_u - 1)$ multipaths from all other users can be regarded as multiple access interference (MAI). The noise, which has now been filtered through the TEQ, is also correlated and contributes through each Rake finger.

Assume that $p_{j,m'}^{(q)}$ represents the cross-correlation between the template and the received waveform associated with the $m'$th multipath from the $j$th user at the $q$th Rake finger for any of the modulation schemes:

$$p_{j,m'}^{(q)} = \int_{T_j} v(t) g(t) dt,$$  \hspace{1cm} (9)

where the integral is evaluated over one pulse repetition period, therefore, the index $i$ has been dropped. Similarly, $\sigma_j^2$ is the power of the filtered noise $n(t)$ available at the $q$th Rake finger output, such that

$$\sigma_j^2 = \mathbb{E} \left[ \left( \int_{T_j} v(t)n(t)dt \right)^2 \right].$$  \hspace{1cm} (10)

Since the actual separation between the Rake fingers is negligible, the channel coefficients from a particular user to any Rake finger can be assumed to be the same. Thus, the contribution of the $j$th user signal power at the $q$th Rake finger output due to multipath channel can be given as

$$r_j^{(q)} = E_j N_s^2 \sum_{m'' = 0}^{M-1} \left( h_{j,m'} p_{j,m'}^{(q)} \right)^2.$$  \hspace{1cm} (11)

The power available at the $q$th finger output due to the received signal from all users and correlated noise is

$$r_j^{(q)} = N_s^2 \sum_{j=1}^{N_u} E_j \sum_{m'' = 0}^{M-1} \left( h_{j,m'} p_{j,m'}^{(q)} \right)^2 + \sigma_j^2.$$  \hspace{1cm} (12)

Hence, the total received power is the summation of all Rake fingers’ output as given below:

$$r = N_s^2 \sum_{q=0}^{M-1} \left\{ \sum_{j=1}^{N_u} E_j \sum_{m'' = 0}^{M-1} \left( h_{j,m'} p_{j,m'}^{(q)} \right)^2 + \sigma_j^2 \right\}.$$  \hspace{1cm} (13)

As the $p$th user is the user of interest, the TEQ shortens the channel for this user only. In this case, the total received power in (13) can be rewritten in terms of the desired signal $\sigma_j^2$, self-interference $\sigma_{ij}^2$, multiple access interference (MAI) $\sigma_j^2$, and the total noise $\sigma_i^2$ as follows:

$$r = E_p N_s^2 \sum_{q=0}^{M-1} \left( h_{p,q} p_{p,q} \right)^2 + \sigma_j^2,$$

$$+ E_p N_s^2 \sum_{q=0}^{M-1} \sum_{m'' = 0}^{M-1} \left( h_{p,m'} p_{p,m'} \right)^2 + \sigma_{ij}^2,$$

$$+ N_s^2 \sum_{q=0}^{M-1} \sum_{j=1}^{N_u} \sum_{m'' = 0}^{M-1} \left( h_{j,m'} p_{j,m'} \right)^2 + \sigma_j^2,$$

where $\sigma_j^2 = \sum_{q=0}^{M-1} \sigma_q^2$.

Let the TEQ shorten the channel to a window of $\ell$ consecutive multipaths. The TEQ length $N$ and $\ell$ are inversely related. For a fixed length TEQ, reducing the value
of $\ell$ may deteriorate its performance. Also, the location of the shortened window within the effective channel should be chosen to optimize the performance. The shortened channel window may theoretically be anywhere in the effective channel. This window basically provides the strongest multipaths of the effective channel in consecutive temporal bins. This also avoids any possibility of ISI which remains in the actual S-Rake design. Suppose, the shortened channel window appears from $h_{p, \ell}$ to $h_{p, \ell + \ell - 1}$. With this assumption, the desired signal energy will also be available over the same $\ell$ taps in the effective channel. Now, the contribution of the multipaths from the $p$th user beyond the shortened window can be regarded as the residual interference. Hence,

$$\sigma_p^2 = \sigma_{\text{win}}^2 + \sigma_{\text{wall}}^2,$$  \hspace{1cm} (15)

where $\sigma_{\text{win}}^2 = E_p N_0^2 \sum_{q=0}^{z-1} \left(h_{p,q} \rho_p(q)\right)^2$ and $\sigma_{\text{wall}}^2 = E_p N_0^2 \sum_{q=0}^{z-1} \sum_{m=0}^{M-z-1} \left(h_{p,q} \rho_p(q)\right)^2$. The instantaneous probability of error for the $p$th user can now be given as

$$P_e = Q\left(\frac{\sigma_{\text{win}}^2}{\sigma_{\text{wall}}^2 + \sigma_p^2 + \sigma_j^2 + \sigma_t^2}\right),$$  \hspace{1cm} (16)

where $Q(\cdot)$ represents the complementary Gaussian distribution function.

We refer to the term $\sigma_{\text{win}}^2/\sigma_{\text{wall}}^2 + \sigma_p^2 + \sigma_j^2 + \sigma_t^2$ in (16) as shortening signal-to-interference and noise ratio (SSINR) represented by $\gamma_p$. Optimization of this term will not only shorten the channel but also optimize the BER of the system. It is important to note that a common standard Gaussian approximation (SGA) approach is used when MAI is considered. The proposed method turns out to minimize the instantaneous BER in the low to moderate SNR region where the SGA is accurate. Let each user have unit power transmit available, that is, $E_1 = E_2 = \cdots = E_j = \cdots = E_{N_u} = 1$, then

$$\gamma_p = \frac{N_0^2 \sum_{q=0}^{z-1} \left(h_{p,q} \rho_p(q)\right)^2}{N_t^2 \left[A + B + C + D\right] + \sigma_t^2},$$  \hspace{1cm} (17)

where

$$A = \sum_{q=0}^{z-1} \left(h_{p,q} \rho_p(q)\right)^2,$$

$$B = \sum_{q=z+\ell}^{M-1} \left(h_{p,q} \rho_p(q)\right)^2,$$

$$C = \sum_{q=0}^{M-1} \sum_{m=0}^{M-1} \sum_{m'=0}^{M-1} \left(h_{p,m} \rho_{p,m'}(q)\right)^2,$$

$$D = \sum_{q=0}^{M-1} \sum_{j=1}^{N_u} \sum_{j'=1}^{N_u} \sum_{m=0}^{M-1} \sum_{m'=0}^{M-1} \left(h_{j,m} \rho_{j,m'}(q)\right)^2.$$

\hspace{1cm} (18)

\section{BER Optimization Algorithm}

The maximization of (17) can be classified into the category of single Rayleigh quotient optimization \cite{20, 21}. Any existing approach can be used to find the optimum solution if the BER is defined in a proper matrix form. Therefore, we first derive the BER in a form which is suitable for optimization. To the knowledge of the authors no such expression is available in the literature for UWB systems. To represent $\gamma_p$ in the matrix form we define the following terms.

Let $w = [w_0, w_1, \ldots, w_{N-1}]^T$ be the TEQ vector. $H_{\text{TEQ}} = [h_{j,0}, h_{j,1}, \ldots, h_{j,M-1}]^T$ is the effective channel vector for the $j$th user such that $H_{\text{TEQ}} = H_j w$, where $H_j$ is the convolution matrix of the $j$th user channel $h_j = [h_{j,0}, h_{j,1}, \ldots, h_{j,M-1}]^T$. Similarly, $H_{\text{wall}}$ is a submatrix of $H_j$ containing $\ell$ consecutive rows from the $j$th to $(\ell + 1)$th row and $H_{\text{wall}}$ contains the rest of the rows. The correlation vector for all multipaths from the $j$th user at the $q$th finger is $\rho_j^{(q)} = [\rho_j^{(q)}, \rho_j^{(q)}, \ldots, \rho_{j,M-1}^{(q)}]$. The vector for the noise entering the TEQ is $N_{\text{TEQ}}$ and $N_{\text{TEQ}}$ is the corresponding convolution matrix. Therefore, $N_{\text{TEQ}} w$ is the filtered noise processed through the TEQ. The correlation amplitude of the filtered noise at each finger is $\sigma_q = [\sigma_0, \sigma_1, \ldots, \sigma_{M-1}]$. Hence, each term in (16) can be written in the matrix form as follows:

$$\sigma_{\text{win}}^2 = w^T \left(H_{\text{TEQ}}^H \Lambda_{\text{win}} \Lambda_{\text{win}}^H H_{\text{TEQ}}\right) w = w^T D_{\text{win}} w,$$

$$\sigma_{\text{wall}}^2 = w^T \left(H_{\text{wall}}^H \Lambda_{\text{wall}} \Lambda_{\text{wall}}^H H_{\text{wall}}\right) w = w^T D_{\text{wall}} w,$$

$$\sigma_{p,q}^2 = w^T \left(\sum_{q=0}^{M-1} H_{p,q}^H \Lambda_{p,q} \Lambda_{p,q}^H H_{p,q}\right) w = w^T \left(\sum_{q=0}^{M-1} S_q \right) w,$$

$$\sigma_t^2 = w^T \left(\sum_{q=0}^{M-1} \sum_{j=1}^{N_u} H_j^H \Lambda_{j,q} \Lambda_{j,q}^H H_j\right) w = w^T \left(\sum_{q=0}^{M-1} \sum_{j=1}^{N_u} M_{j,q}\right) w,$$

$$\sigma_q^2 = w^T N_{\text{TEQ}} \Lambda_{q} \Lambda_{q}^T N_{\text{TEQ}} w = w^T N w,$$

\hspace{1cm} (19)

where $H_{p,q}$ is the $q$th row removed version of $H_p$, $\Lambda_{p,q} = \text{diag}[\rho_p^{(q)}, \rho_p^{(q)}, \ldots, \rho_p^{(q)}]$, $\Lambda_{p,q} = \text{diag}[\rho_p^{(0)}, \rho_p^{(0)}, \ldots, \rho_p^{(M-1)}]$, $\Lambda_{j,q} = \text{diag}[\rho_j^{(q)}, \ldots, \rho_j^{(q)}, \rho_j^{(q)}]$, and $\Lambda_{j,q} = \text{diag}[\rho_j^{(q)}]$ and $\Lambda_{q}$ is a matrix such that $(N_{\text{TEQ}} w)^T \Lambda_{q} = \sigma_q$.

Substituting (19) in (17) we get

$$\gamma_p = \frac{w^T D_{\text{win}} w}{w^T D_{\text{wall}}^\top \sum_{q=0}^{M-1} S_q^\top + \sum_{q=0}^{M-1} \sum_{j=1}^{N_u} M_{j,q} + (1/N^2) N} w.$$

\hspace{1cm} (20)

It is to be noted that the assumptions in the previous discussion are very general. Specially, the correlation term for
each multipath from any user at any Rake finger is different. Practically, this situation is expected and it is basically a result of nonorthogonal TH codes and imperfect time synchronization between the users and the Rake. Nonorthog- 416 onality of TH codes allows the system to accommodate more users with near optimum performance. If a perfect time 417 synchronicity exists between the $p$th user and the receiver, (17) through (20) can be simplified. In this case, any of the qth Rake fingers will produce the same correlation term with the corresponding multipath from the $p$th user:

$$
\rho_p^{(1)} = \rho_p^{(2)} = \cdots = \rho_p^{(M-1)} \triangleq \rho_p.
$$

(21)

Also, with perfectly orthogonal TH codes, all other multipaths from the $p$th and the other users will have the same correlation with the template at any Rake finger:

$$
\rho_{p,m',q}^{(q)} \mid m' \neq q = \rho_{j,m',q}^{(q)} \mid j \neq p \triangleq \rho_x.
$$

(22)

But, this phenomenon will reduce the number of users that can be accommodated, unless $N_p$, $T_c$, or $N$, are varied. Using (21) and (22), we can rewrite (17) as

$$
\tilde{y}_p = N_p^2 \rho_p^2 \sum_{q=0}^{\ell-1} \rho_{q,p}^2 \left( \sum_{q=0}^{\ell-1} \rho_{q,p}^2 \right) + \sigma_r^2,
$$

(23)

where

$$
E = \rho_p^2 \sum_{q=0}^{\ell-1} \rho_{q,p}^2 + \sum_{q=\ell}^{\ell-1} \rho_{q,p}^2,
$$

$$
F = \rho_x^2 \left( \sum_{q=0}^{\ell-1} \sum_{m'=0}^{M-1} \rho_{q,m',q}^2 + \sum_{j=1}^{N} \sum_{m'=0}^{M-1} \rho_{j,m',q}^2 \right).
$$

(24)

Alternatively, in matrix form we have

$$
\tilde{y}_p = \rho_p^2 \cdot w^T \hat{D}\text{win} w,
$$

$$
\text{w}^T \left( \rho_p^2 \cdot \hat{D}\text{wall} + \rho_x^2 \sum_{q=0}^{\ell-1} \tilde{S}_q + \rho_x^2 \sum_{j=1}^{N} \sum_{m'=0}^{M-1} \tilde{M}_{j,m'} + (1/N^2) N \right) w
$$

(25)

where

$$
\hat{D}\text{win} = H_{p,\text{win}}^T H_{p,\text{win}}, \quad \hat{D}\text{wall} = H_{p,\text{wall}}^T H_{p,\text{wall}}, \quad \tilde{S}_q = H_{q,p}^T H_{p,q} \rho_{q,p}^2 \text{ and } \tilde{M}_{j,m'} = H_{j,m'}^T H_j.
$$

From (20) and (25), it is important to note that the contribution of the noise to the SSINR can be reduced by choosing a large value of $N$. Also, if the TH codes of the users are sufficiently orthogonal, we have $\rho_p \gg \rho_x$, which makes MAI significantly small.

Designing a TEQ which minimizes the BER of the system as shown in (16) is equivalent to maximizing the SSINR $y_p$ or $\tilde{y}_p$ as in (20) or (25). The optimization of (20) or (25) is a traditional constrained optimization problem. It poses an optimization [8] to maximize $w^T B w$ with $w^T A w = 1$, where $B = D_{\text{win}}$ or $\rho_p^2 \tilde{D}\text{win}$ and $A = (D_{\text{wall}} + \sum_{q=0}^{\ell-1} \tilde{S}_q + \sum_{j=1}^{N} \sum_{m'=0}^{M-1} \tilde{M}_{j,m'} + (1/N^2) N)$ or $(\rho_x^2 \cdot \hat{D}\text{wall} + \rho_x^2 \sum_{q=0}^{\ell-1} \tilde{S}_q + \rho_x^2 \sum_{j=1}^{N} \sum_{m'=0}^{M-1} \tilde{M}_{j,m'} + (1/N^2) N)$. Hence,

$$
w_{opt} = \left( \sqrt{A} \right)^{-1} \hat{a}_{\text{max}},
$$

(26)

where $\hat{a}_{\text{max}}$ is the eigenvector corresponding to maximum eigenvalue of $(\sqrt{A})^{-1} B (\sqrt{A})^{-1}$ and $\sqrt{A}$ is the Cholesky factor of $A$.

The above optimization, as used in many other TEQ designs [7–9], is performed iteratively to choose the best location of the shortened channel window in the effective channel. The iterative process slides the shortened window from the beginning till the end of the effective channel and chooses the location where the cost function is maximum. It is also possible to define a particular location of the shortened window, but it may not necessarily be an optimum solution.

5. Performance Analysis

In contrast to the proposed TEQ design, the MSSNR design [7, 8] was basically developed for a single user and noiseless system. When this TEQ is used in a multiuser and AWGN environment, its performance is severely deteriorated. It is important to note that in the case of a noiseless single user system, if the BER is estimated before the Rake reception, $y_p$ reduces to the shortening signal-to-noise ratio (SSNR) as defined in [7]. In other words, maximum SSNR (MSSNR) designs [7, 8] optimize the BER before the actual signal detection in any system. This is the reason, though they shorten the channel effectively, but perform poorly in terms of BER as shown in [18]. In the considered system with orthogonal TH codes, the amplitude of the qth Rake finger output due to the qth multipath from the pth user is $\rho_p H_{p,q} \cdot w$ or collectively for all multipaths at their corresponding fingers is $\rho_p H_{p} \cdot w$. At the same time, each multipath causes the self-interference on the remaining $M-1$ Rake fingers. The amplitude of the self-interference at the fingers other than the qth finger due to the qth multipath is $\sum_{m'=0}^{M-1} \rho_p H_{p,m'} \cdot w$. Collectively, we can stack the self-interference vectors due to each multipath as follows:

$$
H_{\text{self}} = \rho_x \left[ \begin{array}{c} H_{p}[0, :] \\ \vdots \\ H_{p}[M-1, :] \end{array} \right] \sum_{m'=0}^{M-2} H_{p}[M'-1, :]
$$

(27)

Similarly, the amplitude of MAI is $\frac{1}{N} \sum_{m'=0}^{M-1} \rho_x H_{p} \cdot w$ and that of the noise is $(1/N) A_{\text{TEQ}}^T N_{\text{TEQ}} w$. Hence, the MSSNR TEQ
attempts to optimize a Rayleigh quotient derived from the following matrix:

\[ \mathbf{X} = \rho_p \mathbf{H}_p + \rho_x \left( \mathbf{H}_{\text{self}} + \sum_{q=0}^{M-1} \sum_{j=1}^{N_q} \mathbf{H}_j \right) + \frac{1}{N} \mathbf{A}_n^T \mathbf{N}_{\text{TEQ}}. \]  

(28)

Now, the MSSNR design defines a window of \( \ell \) consecutive rows within the matrix \( \mathbf{X} \). The shortened channel window in this case not only contains the desired signal power but also the self-interference, MAI and the noise. The optimum TEQ is

\[ \mathbf{w}_\text{mssnr} = \arg \max_{\mathbf{w}} \frac{\mathbf{w}^T (\mathbf{X}^T_{\text{win}} \mathbf{X}_{\text{win}}) \mathbf{w} \cdot \mathbf{w}^T (\mathbf{X}^T_{\text{wall}} \mathbf{X}_{\text{wall}}) \mathbf{w}}{\mathbf{w}^T \mathbf{w}} \]  

(29)

where \( \mathbf{X}_{\text{win}} \) is a partition of \( \mathbf{X} \) having any consecutive \( \ell \) rows, \( \mathbf{X}_{\text{wall}} \) is the remaining part and the term optimized can be referred to as \( \gamma_p^{\text{mssnr}} \).

The unwanted power within the window can be given as

\[ \phi = N_s^2 \sum_{q=0}^{M-1} \sum_{j=1}^{N_q} \sum_{m=0}^{z} \sum_{z'=0}^{z-1} \rho_{p,q}^2 \rho_x^2 \sigma_w^2 + z \sigma_y^2. \]  

(30)

It is evident from (29) and (30) that in an attempt to maximize the cost function given in (29), the MSSNR TEQ also enhances \( \phi \), that is, the self-interference, MAI and the noise available within the window. On the other hand, the proposed TEQ keeps the unwanted power terms to their minimum.

Let \( \beta \) be a measure of the extent to which the available power is compressed within the shortened window by the MSSNR TEQ. The value of \( \beta \) will always lie between 0 and 1. A higher value represents a more efficient TEQ which can be achieved using a larger \( N \). Hence, the MSSNR TEQ will compress \( \beta \phi \) portion of the available unwanted power \( \phi \) within the window during the optimization. If we compare \( \gamma_p \) as defined in (23) to \( \gamma_p^{\text{mssnr}} \) as in (29), it is clear that the enhanced unwanted power \( \beta \phi \) will actually contribute to reduce the SSINR. Therefore, the denominator of (23) will always be increased by a term \( \beta \phi \) when the MSSNR TEQ is used. The SSINRs of both the TEQs can now be compared as follows:

\[ \gamma_p = \left( \frac{\beta \phi}{k_d} + 1 \right) \gamma_p^{\text{mssnr}}, \]  

(31)

where \( k_d \) is the denominator of (23).

This shows that the SSINR of the proposed TEQ will always be greater than MSSNR TEQ in a multiuser and/or AWGN environment. In a single user and noise-free system, both of them will have same performance if the self-interference is neglected. It is also interesting to note that making the MSSNR TEQ more efficient in terms of \( \beta \) by increasing the value of \( N \) will further worsen its performance.

The performance of the proposed TEQ in comparison to the S-Rake or P-Rake depends upon the length of the shortened channel window \( \ell \) and the TEQ length \( N \). This comparison is more statistical than analytical. The energy capture performance, that is, \( \beta \) for the proposed algorithm can be improved either by increasing \( N \) or \( \ell \). If we keep on increasing the value of \( \ell \) with a fixed \( N \), first of all, it is contradicting to the aim of the proposed TEQ design. Secondly, the performance of the S-Rake will start approaching the A-Rake upper bound. Whereas, the performance of the P-Rake will exhibit the same tendency but rather slowly. Eventually, at a large value of \( \ell \) we will see a “cross-over” point after which the performance of the proposed TEQ will become inferior. This phenomenon is shown in Figure 1 which is evaluated for the CM1 profile. At the top right corner of the Figure 1, a triangular-shaped region is visible where the S-Rake performance plane emerges above the proposed TEQ plane. At this region \( \ell \approx 50 \) and \( N = 32 \). The performance of the proposed TEQ can still be improved by increasing \( N \) and the proposed TEQ performance plane again comes up. Obviously, the cost is the system complexity. Therefore, \( \ell \) and \( N \) dictate a tradeoff between performance and complexity of the proposed TEQ.

Another parameter which may severely affect the system performance is the duration of the Rake’s operational window. The proposed BER minimization TEQ, the MSSNR TEQ, and the P-Rake are identical in this sense as each of them looks for a certain number of multipaths arriving in consecutive temporal bins. In contrast, the S-Rake searches for the equivalent number of the strongest multipaths which may not necessarily arrive consecutively. This search may be long enough to cause ISI. Figure 2 shows the duration of Rake operational window for different values of \( \ell \) in different channel profiles. It is clearly visible that the proposed TEQ reduces the Rake operational window by roughly 12 ns in CM1 to 40 ns in CM4 scenarios, while capturing more signal energy and maintaining a lower BER as shown in Section 6. This phenomenon is also helpful in increasing the data rate of the system without causing ISI.

6. Complexity Analysis

A very important issue is the relative complexity of the proposed solution. One can think that the simplification in the Rake structure is now transformed into the complexity of the proposed TEQ design. In fact, the proposed solution can be considered as a TEQ followed by a P-Rake. Since the P-Rake does not need a search algorithm for the arriving multipaths, its complexity is negligible as compared to the TEQ complexity. Hence, the overall complexity of the proposed solution, that is, TEQ plus P-Rake, actually lies in the TEQ design. In this section, we briefly analyze the complexity of the proposed TEQ with the S-Rake design. The comparison can be made on different sets of criteria. Here we compare both designs for initial evaluation on the basis of the number of multipaths collected, that is, \( \ell \). The complexity of the proposed TEQ lies in calculating the parameters used in (25) and then performing the optimization. The complexity of the S-Rake lies in searching a subset of \( \ell \) strongest multipaths in a channel which is \( M \) multipaths long.
The conventional S-Rake algorithm [22] defines the signal to interference and noise ratio (SINR) for every multipath. In our system model, this SINR for the $q$th multipath from the $p$th user at the $q$th Rake finger can be written as

$$\gamma_p^{(q)} = \frac{h_{p,q}^2}{\rho_p^2 \sum_{m=0}^{M-1} h_{p,m}^2 + \rho_q^2 \sum_{j=1 \atop j \neq p}^M \sum_{m=0}^{M-1} h_{j,m}^2 + (1/N_p^2) \tilde{\sigma}_q^2},$$

for $q = 0, 1, \ldots, M - 1$, (32)

where $\tilde{\sigma}_q^2 = E[\int_{t_0}^{t_f} v(t) n_{TEQ}(t) dt^2]$.

For the S-Rake, (32) must be computed for $M$ multipaths from $N_u$ users at $M$ fingers of the Rake. In total, there are $N_u M$ values of $h_{p,q}$ that must be squared, leading to $N_u M$ multiplies. Also, the term $h_{p,q}^2$ must be multiplied by $\rho_q^2$ for $N_u M$ combinations of $p$ and $q$, and the numerator must be divided by the denominator for $N_u M$ combinations of $p$ and $q$. Everything else in the (32) requires much fewer computations and can be ignored. Thus, (32) requires $O(2N_u M)$ multiplies and $O(N_u M)$ divisions, or $O(3N_u M)$ operations.

For the proposed design, (25) must be computed. Efficient techniques utilizing reuse of computations [23] can reduce the complexity of evaluating $D_{\text{whd}}$ and $D_{\text{wall}}$ to $O(N(N + \ell))$. The other terms are mostly summations, and are generally cheaper than $O(N(N + \ell))$. Thus, maximizing the Rayleigh quotient, which is $O((1/3)N^3)$ [23], is more complex than computing the Rayleigh quotients in (25), and the overall complexity is $O((1/3)N^3)$.

Another issue is memory use. The S-Rake stores all the values of $\gamma_p^{(q)}$ and the related index $q$. In fact, the memory usage is directly proportional to the duration of the operational window of the S-Rake. This is another disadvantage of S-Rake's long operational window as shown in Figure 2. It is evident that the complexity of the S-Rake increases in dense multipath channels (large $M$) and with increasing number of users (large $N_u$). If both values increase simultaneously, the complexity grows in a quadratic fashion. The complexity of the proposed TEQ is independent of the channel length and the number of users but it grows with cubic power of the TEQ length. Therefore, the TEQ length must be chosen very carefully. For a numerical example, the CM3 profile is roughly 250 taps long at a sampling rate of 0.5 ns. In a multiuser system with $N_u = 20$ and the TEQ length of $N = 32$, the complexity of the S-Rake is $O(15000)$ whereas the complexity of the proposed TEQ is roughly $O(11000)$. In fact, in dense multipath channels the complexity of both designs is comparable, but when it comes to the memory usage the proposed TEQ outperforms the S-Rake. As depicted in Figure 2, the operational window of S-Rake is 2 to 6 times larger than the operating window of the proposed TEQ. Hence, the S-Rake needs 2 to 6 times more memory from CM1 to CM4 channels.
7. Simulation Results

The performance of the proposed BER minimization TEQ is compared with the MSSNR TEQ [7, 8], A-Rake, P-Rake, and S-Rake [22] in CM1, CM2, CM3, and CM4 environments. All users are provided with random semiorthogonal TH codes and employ TH-BPPM and/or TH-BPSK. Channel coefficients are generated at a sampling rate of 2 GHz with $M = 175$ for CM1, 200 for CM2, 252 for CM3, and 420 for CM4. P- and S-Rake are capturing the first $\ell$ and the strongest $\ell$ multipaths, respectively. A-Rake is providing a lower bound by capturing all the multipaths and gathering the total available signal energy except the self-interference. First-order Gaussian derivative pulses of $T_p < 0.5$ ns with center frequency 3 GHz are used. The transmit antenna effects are modeled via random low pass
filtering which changes the shape of the transmitted pulse to the second order Gaussian pulse. The modulation index $\Delta$ is 2 ns and the chip duration $T_c$ is 5 ns. Other system parameters, for example, $\ell$, $N_u$, $N_h$, and $N_s$ are either kept constant to a certain value or varied in different simulations.

Extensive simulations were performed to test the capabilities of the proposed BER minimization TEQ design. The results are generated by averaging the performance parameter through Monte Carlo simulations. As an SGA approach is used, all the simulations for BER are performed in low to moderate SNR range. Since the performance depends upon many factors, each factor is considered individually.

Figure 3 shows the BER of the system versus SNR. There were 10 users in the system, including the user of interest. The length of the TEQ was $N = 32$ and the shortened channel window was $\ell = 10$ taps long. The performance is evaluated for both the modulation schemes, that is, TH-BPPM and TH-BPSK with $N_h = 25$ and $N_s = 7$. The TH-codes are semiorthogonal for all receivers except the A-Rake. The performance of an ideal A-Rake is used as lower bound with an assumption that the TH-codes of other users are perfectly orthogonal, resulting in zero correlation with the $p$th user template. It is observed that the performance of the proposed TEQ and other receiver structures is almost the same for both modulation schemes. The proposed TEQ clearly maintains a lower BER in all channel models along with efficiently shortening the channel.

In Figure 4, all other parameters are the same as in the previous figure, except the SNR which is now fixed at 15 dB. The TEQ length is varied from 32 to 64 with an increment of 8. As, the performance of all the receiver structures is found to be the same with TH-BPPM and TH-BPSK, in rest of the results we evaluate the performance only for TH-PPM. Figure 4 is actually a comparison between the MSSNR TEQ and the proposed TEQ as the A-Rake, S-Rake, and P-Rake are not affected by the TEQ length. Only A-Rake’s performance is shown for reference. An increasing value of $N$ improves the performance of the proposed TEQ, specially in less dense channels, but, as stated earlier, at the cost of increased receiver complexity. Therefore, the TEQ length $N$ can be considered as a designer parameter. If the system is needed to operate at a certain BER in a particular propagation environment, Figure 4 can help in choosing the suitable value of $N$.

The proposed TEQ performs an optimization in which it tries to keep the MAI at its minimum, while the MSSNR TEQ does not include MAI and hence is incapable of handling a multiuser system. Hence, as shown in Section 4, it enhances the noise and MAI which falls within the shortened channel window. Therefore, as expected, the proposed TEQ is not significantly effected by the increasing number of users as shown in Figure 5. On the other hand, the performance of MSSNR TEQ gradually degrades as the number of users increases. All other system parameters are the same as in the previous case. The performance of A-Rake, S-Rake and P-Rake is found stable because of semiorthogonal TH codes and therefore not shown.

Figure 6 depicts the energy capture performance of the different receiver structures against the length of shortened
channel window $\ell$. The value of $\ell$ varies from 2 to 20. This figure is basically a two-dimensional image of Figure 1 and is drawn for all the four channel models. The range of $\ell$ is selected so that the Rake receiver design is practically simplified and benefits of the TEQ can be seen. As mentioned in Section 6, if the value of $\ell$ is further increased, S-Rake performance will supersede the proposed TEQ. A similar cross-over point can be seen for the MSSNR TEQ. But, larger values of $\ell$ directly contradict the aim of proposing TEQ at the receiver front end and hence are not considered here. Energy capture for A-Rake (not shown) is a straight line parallel to the $x$-axis and close to unity. The small gap to perfection is due to self-interference.

8. Concluding Remarks

In this paper, we consider a realistic UWB scenario with all the main factors which may affect the Rake receiver performance. We derive an expression for the BER of the this system in the presence of a TEQ at the receiver front end. Based on the derived formula, we propose a TEQ design which directly attempts to optimize the BER of the system while pushing the effective channel energy within the desired temporal window. We compared the BER performance of the proposed design with P-Rake, S-Rake, and MSSNR TEQ with A-Rake as realistic lower bound. It is shown that the proposed TEQ performs better than the MSSNR TEQ, S-Rake, and P-Rake and is confirmed through simulations.
All the major factors which may affect the performance of the proposed TEQ are simulated and discussed. It is shown that the proposed TEQ outperforms the considered MSSNR TEQ and the Rake architectures in any performance aspect. Especially, the proposed TEQ maintains a lower BER while shortening the dense multipath channels to a desired small temporal window. Hence, with the proposed TEQ design, an UWB Rake receiver can be designed with significantly less number of fingers/correlators without compromising the receiver performance in terms of the BER. This will also simplify the receiver architecture and analysis that follow the Rake.

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