Controlling transport dynamics of confined asymmetric fibers

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Abstract – Transport properties of particles in confining geometries show very specific characteristics as lateral drift, oscillatory movement between lateral walls or the deformation of flexible fibers. These dynamics result from viscous friction with transversal and lateral channel walls inducing strong flow perturbations around the particles that act like moving obstacles. In this paper, we modify the fiber shape by adding an additional, small fiber arm, which leads to T- and L-shaped fibers with only one or, respectively, zero symmetry axes and investigate the transport properties. For this purpose, we combine precise microfluidic experiments and numerical simulations based on modified Brinkman equations. Even for small shape perturbations, the transport dynamics change fundamentally and formerly stable configurations become unstable, leading to non-monotonous fiber rotation and lateral drift. Our results show that the fundamental transport dynamics change with respect to the level of fiber symmetry, which thus enables a precise control of particle trajectories and which can further be used for targeted delivery, particle sorting or capture inside microchannels.

Introduction. – Separating and filtering of particles or micro-organisms as a function of their properties such as flexibility, shape or activity is an important requirement for biomedical or food science applications. Micronscale filters, including micro-pillar arrays [1,2] are commonly used to sort particles with respect to their size or deformability, but face problems of filter clogging or damaging of deformable particles. Recently microfluidic devices, relying on specific transport properties, have been developed to overcome these difficulties [1]. Inertial [2,3] or viscoelastic effects [4] can be used to focus particles at specific positions inside channels and flexible particles are known to migrate away from bounding walls, leading, for example, to a cell-free layer as observed in blood flow [5] or migration of flexible fibers in shear gradients [6]. Some microorganisms use passive reorientation mechanisms to move in gradients, either in the gravitational field (gravitaxis) [7] or in a gradient of viscosity (viscotaxis) [8]. In simple unbounded flows however, when inertia can be neglected, rigid particles only migrate across streamlines when symmetry is broken for example by particle chirality [9].

This situation changes when particles are confined by bounding walls and act as moving obstacles to the flow. The induced strong flow perturbation leads to particle migration, as, for example, lateral drift observed for rigid fibers even at small Reynolds numbers [10]. Recently, theoretical models and experiments have been developed to describe such flows [10–14]. In particular, it has been shown that axisymmetric objects such as fibers or symmetric dumbbells (made of rigid spheres or drops) translate without rotation but drift at a constant angle, and may oscillate between the lateral walls of the channel [11] (fig. 1(a), (b)). Axisymmetric particles with fore-aft asymmetry, such as asymmetric dumbbells, will migrate toward the center of the channel, where they align with the flow [12,14], while laterally unconfined, asymmetric trimers rotate to reach an equilibrium angle [15]. Flexible fibers may be deformed by the viscous forces and will then rotate and align with the flow [6,16]. In all these cases, the particle trajectory is strongly affected by the transversal

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confined, which tunes the magnitude and distribution of the viscous force.

This situation bears some similarities with sedimenting particles where, in particular, there exists a coupling between translation and rotation that depends on the particle geometry, and particles that possess certain symmetries will exhibit specific motions. For example, axially symmetric objects, such as uniform rods, keep their orientation and merely translate without rotating [17]; an axisymmetric object that presents a fore-aft asymmetry, such as a dumbbell composed of spheres of different sizes, will rotate and align with the flow [18]. An asymmetric particle, as for example an L-shaped particle, will rotate until it reaches a stable orientation, at which it will translate without rotating [7]. If in addition the chiral symmetry is broken, the coupling of translational and rotational movement leads to helical trajectories [19,20].

We investigate these effects in detail by studying a model system consisting of a microfiber with increasing degrees of asymmetry transported in a microchannel. By adding a second arm to an initially straight fiber, we create and analyze, both theoretically and experimentally, T-shaped fibers with fore-aft asymmetry and fully asymmetric L-shaped fibers. Examples of these fibers and corresponding chronophotographies of their transport dynamics are given by fig. 1(c)–(g).

Fig. 1: Experimental chronophotographies of transported fibers of different geometries. The fluid flows from left to right. The lateral walls are highlight in red. As reported previously, straight fibers oscillate in the channel either through glancing (a) or reversing (b) [11]. T fibers away from the lateral walls reorient toward a stable orientation parallel to the flow direction (c) or are captured by the wall (d). L fibers first rotate toward a stable orientation and then drift (e)–(g) until they are captured by the lateral wall (g). From top to bottom confinements are ((a), (b)) 0.78, ((c), (d)) 0.82, ((e), (f)) 0.76, and (g) 0.72. Scale bars are 500 μm. The total length of the channels is typically 4 cm. The time interval between successive snapshots is constant for each panel and varies between 1 s and 7 s (such that the fiber is translated about its length between successive images).

Problem formulation and methods. – Figures 2(a), (b) depict the configuration of an L fiber in the channel. The fiber is of square cross-section with width and height h, and ℓa and ℓb denote the lengths of the long and short arms, respectively. We will refer to ℓa/h and ℓb/h as the non-dimensional lengths or aspect ratios of the long and short fiber arms, respectively, and quantify the asymmetry by ℓa/ℓb, which is bounded by unity (ℓb = ℓa, symmetric fiber with two arms of equal length) and ℓa/h (ℓb = h, I fiber). Starting from ℓa = ℓb, the asymmetry is first enhanced when decreasing the length of the short arm with respect to the long arm before the symmetry is restored when the short arm vanishes and a straight (I) fiber is attained.

The fiber is placed in a rectangular channel of height H and width W, and subjected to a pressure-driven flow fully characterized by its mean velocity U0. Due to the Hele-Shaw–like geometry of the channel (the aspect ratio W/H of the channel varies from 15 to 70), the flow in the xy plane is a plug flow, except close to the side walls and in the vicinity of the fiber, where the transverse vorticity is confined within boundary layers of characteristic thickness H. In the z-direction the flow is Poiseuille-like. The transversal and lateral confinements of the fiber are quantified by β = h/H and ξ = ℓa/W, respectively. As the channel is assumed to be of infinite length, the state of the fiber is completely described by the lateral position of its center of mass y and the orientation angle θ, together with the corresponding velocities ˙y and ˙θ. Moreover, we
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![Diagram](image)

Fig. 2: (a) Geometry of an L-shaped fiber in the channel. The pressure-driven flow is sketched in blue. (b) Image treatment and definition of angles. Using MatLab procedures we derive from the picture of the particle its skeletonized shape (blue) and its center of mass (yellow circle). The scale bar is 200 μm. (c) Evolution of the dimensionless rotation velocity $\dot{\theta}$ and its center of mass (yellow circle). The scale bar is 200 μm from the picture of the particle its skeletonized shape (blue). (d) Experimentally observed evolution of the orientation angle $\theta$ as a function of the dimensionless time (dark blue). A fiber initially in its equilibrium orientation hits an obstacle (yellow stripe) and deviates from the equilibrium orientation. It then rotates back to the latter. The reorientation dynamics close to the equilibrium orientation (gray area) are well adjusted by an exponential fit (red curve in the zoom). The fitting method is detailed in the SM.

define the drift angle $\alpha$ by $\tan \alpha = \dot{y}/\dot{x}$. Note that there exists a mirror symmetry and that the mirror image of the fiber shown in figs. 2(b) corresponds to opposite fiber chirality.

The experimental methods, theoretical model, and numerical techniques are extensively described and verified in [11] and [16] and are outlined only briefly here. Fibers of controlled shape, position and confinement are fabricated using the stop-flow microscope-based projection photo-lithography method developed by Dendukuri et al. [21]: a pulse of UV-light passing through a mask illuminates the microchannel filled with a photosensitive solution (PEG-DA $M_\alpha = 575$ g/mol and Darocur 1173, viscosity $\mu = 68$ mPa s) to create a rigid particle which is then transported in the non-crosslinked solution, with a pressure-driven flow imposed by a syringe pump (Nemesys, Cetoni). The presence of inhibition, i.e., uncured, layers of constant thickness $b = 6 \pm 1.6\mu$m in the vicinity of the channel walls [22] determines the fiber height $h = H - 2b$, and thus the transversal confinement $\beta$. For each channel height, we adjust the mask geometry to ensure a square cross-section of side $h$ and a constant aspect ratio $\ell_\beta/h = 5$. With this method, the asymmetry $\ell_\alpha/\ell_\beta$ is tuned by changing the length of the long arm $\ell_\alpha/h$. Due to the low value of the Reynolds number $Re \sim 10^{-3}$, inertia will be neglected in this work. This also implies that the dynamics does not explicitly depend on the flow velocity.

A two-dimensional Brinkman model supplemented by a model flow profile in the gap between the fiber and the transversal channel walls is utilized to determine the flow around the fiber and the resulting force and torque distribution on the fiber. Note that the velocity vector is uniquely determined by the particular state $(y, \theta)$, as inertial effects are negligible and the problem is thus reversible in time. The flow equations are solved with COMSOL MULTIPHYSICS and verified by separate calculations with the ULAMBATOR code [23]. Detailed information on the model can be found in previous works [11,16] and in the supplementary material Supplementarymaterial.pdf (SM).

Results. – We first focus on the equilibrium position reached by the fiber far from the lateral walls, i.e., $\xi \to 0$. While for an I fiber all orientations are stable, i.e., the fiber keeps a constant orientation and translates without rotating, the L fiber reorients toward a stable orientation, and then drifts at a constant angle (figs. 1(e), (f)). We obtain numerically the evolution of the rotation speed $\dot{\theta}$ as a function of $\theta$ for an asymmetric L fiber with $\ell_\alpha/h = 10$, $\ell_\beta/h = 5$, and $\beta = 0.76$ (fig. 2(c)). There are two fixed points separated by $180^\circ$: a stable one ($\theta = \theta_s = -47^\circ$) and an unstable one ($\theta = \theta_u = 133^\circ$). For all initial orientations, the fiber rotates monotonously until the stable orientation $\theta_s$ is reached. Note that close to the unstable orientation $\theta_u$, a small variation can change the direction of rotation as depicted in figs. 1 (e), (f); although both fibers start at very similar configurations ($\theta \simeq 130^\circ$), the first fiber is rotating counterclockwise while the second is rotating in the opposite direction until they both reach the stable orientation. A fiber pushed out of the equilibrium orientation by an obstacle in the channel, fig. 2(d), rotates immediately back to the equilibrium position, showing the attractive strength of the stable fixed point. Close to the stable orientation, the rotation velocity $\dot{\theta}$ depends approximately linearly on $\theta$ (dashed line in fig. 2(e)), i.e., the angle increases/decreases exponentially in time to reach its stable value $\theta_s$. We use this result to determine $\theta_s$ experimentally with an exponential fit (fig. 2(d)), as detailed in the SM.

The stable orientation angle $\theta_s$ and the corresponding drift angle $\alpha_s$ are uniquely defined by fiber geometry and transversal confinement. Figure 3 shows both quantities as determined in the experiment for different lengths of the long arm $\ell_\alpha/h$, i.e., for varying asymmetry, and compared to the numerical results. While the drift angle varies only slightly in the examined regime, the stable orientation angle $\theta_s$ evolves non-monotonously with increasing length of the long arm $\ell_\alpha/h$: it first increases,
reaches a maximum at $\ell_a/h \sim 7.5$, and then decreases again. A variation of the transversal confinement impacts both $\theta_s$ and $\alpha_s$, as visualized by the gray shaded regions in fig. 3. While the effect of $\beta$ is almost negligible for $\beta < 0.6$, the orientation and drift angles increase rapidly with increasing confinement. In particular, we found that increasing the transversal confinement increases the rotation speed $\dot{\theta}$, i.e., the higher (lower) the transversal confinement is, the faster (slower) the fiber approaches the stable orientation (see SM).

In order to obtain a general overview of the dependence of $\theta_s$ and $\alpha_s$ on the fiber geometry, we systematically compute these quantities in the parameter space $\{\ell_a/h, \ell_b/h\}$ and visualize the result in a map as shown in fig. 4. Due to symmetry and for the sake of compactness, we restrict the space to $\ell_a \leq \ell_b$ and combine the two maps for $\alpha_s$ (top) and $\theta_s$ (bottom) in one. The black line corresponds to $\ell_a = \ell_b$, where the fiber is symmetric and is transported with $\theta_s = -45^\circ$ along the flow direction, i.e., $\alpha_s = 0^\circ$. For $\ell_a > \ell_b$ the L fiber is fully asymmetric as long as $\ell_b > h$, and the map of stable orientations $\theta_s$ can be divided into two distinct regions. For large values of the aspect ratios $\ell_a/h$ and $\ell_b/h$, the isolines of constant $\theta_s$ approach straight lines, i.e., $\theta_s$ depends on the asymmetry ratio $\ell_a/\ell_b$ only and is independent of the fiber height $h$ and thus the fiber aspect ratios. In this region, increasing asymmetry (decreasing $\ell_a/\ell_b$) leads to a decrease of $\theta_s$, as evidenced by the white dots and the corresponding fiber shapes.

At small values of the aspect ratios ($\leq 10$), the isolines exhibit a different tendency: changing $\ell_a$ or $\ell_b$ while keeping $\ell_a/\ell_b$ constant leads to different values of $\theta_s$, i.e., $\theta_s$ depends not only on the fiber asymmetry, but also on the fiber aspect ratios. Keeping for example $\ell_b/h$ constant and changing $\ell_a/h$ (red dots in fig. 4), as we did in the experiments shown in fig. 3, leads to a non-monotonous evolution of the orientation angle. The crossover between these two regimes can be evidenced by following the isoline for $\theta_s = -46^\circ$ (black dots and corresponding shapes). Along this line, the fiber shape first changes significantly at small fiber arm aspect ratios, but finally reaches an homothetically similar shape at large values of the aspect ratio.

For asymmetric fibers, which form an angle $\theta_s$ with the flow direction, there is a lateral drift whose magnitude depends on a combination of both equilibrium angle and asymmetry. For a fixed $\ell_b/h$, changing $\ell_a/h$ (red dots) leads to a small change in drift angle although the
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orientation angle varies significantly. On the contrary, for a given orientation (black dots), changing the shape leads to a significant change in drift angle. In the limit of large fiber arm aspect ratios, changing the asymmetry $a/h$ (white dots) leads to significant variations in orientation and drift angle.

These findings demonstrate that it is not possible in general to rationalize the fiber transport using only the fiber asymmetry $a/h$, but that the two-dimensional geometry of the fiber has to be taken into account instead of a one-dimensional skeleton.

The overall fiber dynamics is given by a combination of rotation toward $\theta_s$, drift, and interaction with the lateral walls. For a more global view, we plot the trajectories in the configuration space spanned by $\theta$ and $y$ for I, T and L fibers in fig. 5. The experiments (right panel) are obtained for a range of parameters (transversal confinement $0.7 \leq \beta \leq 0.8$, lateral confinement $0.12 \leq \xi \leq 0.8$, and long arm aspect ratio $10 \leq a/h \leq 20$). In the left panel, we present the fiber trajectories obtained numerically for a single set of parameters ($\xi = 0.5$, $\beta = 0.76$ and $a/h = 10$) for qualitative comparison. The qualitative trends are well reproduced; differences may arise from the different values of the lateral confinement $\xi$ as will be discussed later.

As already presented previously [11], I fibers show mostly permanent lateral oscillations, clockwise around the fixed point at $\theta = 0^\circ$ (red trajectory, fig. 1(a)) or anti-clockwise around the fixed point at $\theta = \pm 90^\circ$ (green trajectory, fig. 1(b)) except if they are located very close to the wall (blue trajectory). If we reduce the level of symmetry by analyzing a T-shaped fiber, the picture changes significantly. The stable fixed points at the center of the channel for $\theta = 0^\circ, \pm 90^\circ, \pm 180^\circ$ are reduced to one at $\theta = 0^\circ$. Apart from trajectories ending at this configuration (red trajectory, fig. 1(c)), there are also attracting points at the lateral channel walls (blue and green trajectories, the latter corresponds to fig. 1(d)). While approaching these final positions, the T fiber may exhibit non-monotonous lateral movement as it is rotating. Moreover, the interaction with the lateral walls can lead to a change of rotation direction, as two of the four unstable fixed points observed for I fibers are still present (highlighted by red dots in fig. 5). The green trajectory (fig. 1(d)) corresponds to an example of such a non-monotonous evolution of the orientation angle.

For the fully asymmetric L fibers, the centrally located fixed points finally disappear completely and the only permanent configurations correspond to the fiber being captured by one of the walls, with the sharp edge pointing toward the channel center. Even though there exist trajectories pointing away from this configuration, they finally lead back to it and the fiber cannot escape. The stable and unstable orientations observed without lateral confinement do not appear as fixed points, as they lead to lateral drift, but as ridges with the trajectories either leading away from or attracting to. In other words, after rotating toward its stable orientation, the L fiber drifts toward the wall where it will remain stuck. In some cases (light-red area), the fiber is captured by the opposite wall during the reorientation. We highlight the trajectories of the three cases presented in fig. 1(e)–(g). The fiber, initially close to the unstable orientation, leaves the orientation by rotating either clockwise (green curve, fig. 1(e)) or anti-clockwise (red curve, fig. 1(f)) to reach equilibrium. When approaching the wall with the sharp edge toward it (blue curve, fig. 1(g), (f)), the fiber is first repelled, rotates until reaching its equilibrium angle, and finally drifts across the channel width to reach the other wall where it will remain. Note that for a mirrored fiber of opposite chirality, there exists a mirror-symmetric diagram where most fibers collect at the opposite wall.

For T and L fibers we can quantify the percentage of fibers being trapped at one of the channel walls or localized at the channel center. For T fibers, the corresponding sets of initial configurations which end up in the channel center (light green) or at the walls (light red or blue) are highlighted in fig. 5. Consequently, the areas of these regimes represent the statistical probability of finding a fiber with random initial orientation and lateral position at the channel center or wall. In the presented case, approximately 40% of the fibers end in the channel center and the rest are captured by the lateral walls. Looking at the trajectories ending at the walls, we can see that they correspond to the case where the fiber is captured by the wall during reorientation toward the central position (fig. 1(d)). This means that we can tune this ratio by varying the lateral confinement $\xi$. In fact, for $\xi = 0.3$, we find approximately 73% of the fibers in the channel center. Decreasing the lateral confinement thus increases the probability of finding a T fiber in the middle of the channel. It has to be noted, however, that close to the end points at the walls, there exist trajectories which lead away again from the wall. Assuming some experimental noise, this can easily lead to a higher amount of fibers finally ending in the channel center than obtained from a noise-free theoretical study. Turning to L fibers, most of them end at the same lateral wall (approximately 86% for $\xi = 0.5$, light-blue area), which is determined by the sign of drift angle at stable orientation of a fiber without lateral confinement. Analogue to the T fiber, the partition of fibers being captured by the opposite wall can be reduced (increased) by decreasing (increasing) the lateral confinement (approximately 93% of the fibers at one wall for $\xi = 0.3$). See SM for details.

These findings show that it is possible to control the lateral position of fibers transported in confining microchannels by tuning their geometry, in particular their symmetry properties, and/or the confinement, i.e., the channel size. I fibers (two axes of symmetry) are mostly laterally oscillating, or tend to end in the channel center due to small imperfections (damped oscillation) [11]. T fibers (left-right symmetry, fore-aft asymmetry) are either pushed toward the channel center or toward the lateral walls, depending on the level of lateral confinement.

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Fig. 5: Trajectory map in the configuration space spanned by orientation angle $\theta$ and lateral position $y$ for (a) an I fiber, (b) a T fiber, and (c) an L fiber for numerics (left) and experiments (right). The physical accessible space is shaded in color. For T fibers, the space of trajectories ending at the channel center is shaded in light green. The space of trajectories ending at the wall at $y = 0.5$ (respectively, $y = -0.5$) is shaded in light blue (respectively, light red). Stable and unstable fixed points are highlighted in green and red, respectively. Experiments: transversal confinement $0.7 \leq \beta \leq 0.8$, lateral confinement $0.12 \leq \xi \leq 0.8$, and arm lengths $10 \leq \ell_a/h \leq 20$, $\ell_b/h = 5$. Numerics: long arm aspect ratio $\ell_a/h = 10$, lateral confinement $\xi = 0.5$ and transversal confinement $\beta = 0.76$. The short arms of the T and L fibers have half the length of the long arm, i.e., $\ell_b/h = 5$. Some numerical and experimental trajectories are highlighted for comparison.

L fibers (fully asymmetric) are always captured by the lateral walls. Depending on their chirality being left- or right-handed, most of the fibers end hereby at one or the other wall. The dynamics is discussed in the SM. The reorientation time, i.e., the time needed to reach the equilibrium orientation in the absence of lateral walls, strongly depends on the transversal confinement $\beta$ but is typically $\tau \sim 10 – 100 \ell_a/U$ (see fig. 2 of the SM), indicating that a fiber reaches its equilibrium position after traveling a few tens of its length. For fibers of length $\ell_a \sim 100 \mu m$, ...
equilibrium is thus reached after a few centimeters. In addition, for laterally confined channels, starting from a broad initial distribution, all fibers have reached the end point of their trajectories (i.e., at the wall or at the center of the channel) after 100–1000ℓa (see fig. 4 of the SM). Typically, a channel of length 4 cm is sufficient for fibers to reach their final configuration (fig. 1). This dynamics can be tuned by adjusting both transversal (β) and lateral (ξ) confinements. The presented results thus open new routes toward the design of sorting devices or filters as a function of, for example, micro-organism shape. In addition they can be used to design targeted delivery or particle capture applications in microfluidic devices.

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