Can bimodality exist without phase transition?

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Abstract

Here we present an explicit counterexample to the widely spread beliefs about an exclusive role of bimodality as the first order phase transition signal. On the basis of an exactly solvable statistical model generalizing the statistical multifragmentation model of nuclei we demonstrate that the bimodal nuclear fragment size distributions can naturally appear in infinite system without a phase transition. It appears at the supercritical temperatures due to the negative values of the surface tension coefficient. The developed statistical model corresponds to the compressible nuclear liquid with the tricritical endpoint located at one third of the normal nuclear density.

1 Introduction

Nowadays the bimodality is considered as an unambiguous signal of the first order phase transition (PT) in finite but large systems. The authors of such beliefs [1, 2, 3] identify each local maximum of the bimodal distribution with a pure phase. For instance, T. Hill justified his assumption on bimodality appearance in finite systems by stating that due to the fact that an interface between two pure phases ‘costs’ some additional energy, the probability of their coexisting in a finite system is less than for each of pure phases [1]. At the same time it is believed [1, 2, 3] that in the thermodynamic limit a bimodality corresponds to a mixed phase only.

Here we give an explicit counterexample based on the exact analytical solution of the constrained statistical multifragmentation model (CSMM) of nuclei in the thermodynamic limit which leads to the bimodal fragment distributions inside of the cross-over region without the phase transition existence. In addition, we develop a realistic equation of state for the liquid phase which, in contrast to the original SMM formulation [4], is a compressible one. The suggested approach obeys the L. van Hove axioms of statistical mechanics. The second important element of the present model is a realistic parameterization of the surface tension temperature dependence which is based on the exact analytical solution of the partition function of surface deformations.

2 CSMM with compressible nuclear liquid in thermodynamic limit

The general solution of the CSMM partition function formulated in the grand canonical variables of volume $V$, temperature $T$ and baryonic chemical potential $\mu$ is given by [5]

$$ Z(V, T, \mu) = \sum_{\{\lambda_n\}} e^{\lambda_n V} \left[ 1 - \frac{\partial \mathcal{F}(V, \lambda_n)}{\partial \lambda_n} \right]^{-1}, \tag{1} $$

where the set of $\lambda_n$ ($n = 0, 1, 2, 3, ..$) are all the complex roots of the equation

$$ \lambda_n = \mathcal{F}(V, \lambda_n), \tag{2} $$
ordered as $\text{Re}(\lambda_n) > \text{Re}(\lambda_{n+1})$ and $\text{Im}(\lambda_0) = 0$. The function $\mathcal{F}(V, \lambda)$ is defined as

$$\mathcal{F}(V, \lambda) = \left(\frac{mT}{2\pi}\right)^{\frac{3}{2}} z_1 \exp\left\{\frac{\mu - \lambda T b}{T}\right\} + \sum_{k=2}^{K(V)} \phi_k(T) \exp\left\{\frac{(p_l(T, \mu) - \lambda T)bk}{T}\right\}.$$  (3)

Here $m \simeq 940$ MeV is a nucleon mass, $z_1 = 4$ is an internal partition (the degeneracy factor) of nucleons, $b = 1/\rho_0$ is the eigen volume of one nucleon in a vacuum ($\rho_0 \simeq 0.17$ fm$^3$ is the normal nuclear density at $T = 0$ and zero pressure). The reduced distribution function of the $k$-nucleon fragment in (3) is defined as

$$\phi_{k>1}(T) \equiv \left(\frac{mT}{2\pi}\right)^{\frac{3}{2}} k^{-\tau} \exp\left[-\frac{\sigma(T)k^{\zeta}}{T}\right],$$  (4)

where $\tau \simeq 1.825$ is the Fisher topological exponent and $\sigma(T)$ is the $T$-dependent surface tension coefficient. Usually, the constant, parameterizing the dimension of surface in terms of the volume is $\zeta = \frac{2}{3}$.

In (3) the exponents $\text{exp}(-\lambda bk)$ ($k = 1, 2, 3, \ldots$) appear due to the hard-core repulsion between the nuclear fragments [5], while $p_l(T, \mu)$ is the pressure of the liquid phase.

We consider the thermodynamic limit only, i.e. for $V \to \infty$ it follows $K(V) \to \infty$. Then the treatment of the model is essentially simplified, since Eq. (2) can have only two kinds of solutions [5], either the gaseous pole $p_g(T, \mu) = T\lambda_0(T, \mu)$ for $\mathcal{F}(V, \lambda_0 - 0) < \infty$ or the liquid essential singularity $p_l(T, \mu) = T\lambda_0(T, \mu)$ for $\mathcal{F}(V, \lambda_0 - 0) \to \infty$. The mathematical reason why only the rightmost solution $\lambda_0(T, \mu) = \max\{\text{Re}(\lambda_n)\}$ of Eq. (2) defines the system pressure is evident from Eq. (1): in the limit $V \to \infty$ all the solutions of (2) other than the rightmost one are exponentially suppressed.

In the thermodynamic limit the model has a PT, when there occurs a change of the rightmost solution type, i.e. when the gaseous pole is changed by the liquid essential singularity or vice versa. The PT line $\mu = \mu_c(T)$ is a solution of the equation of ‘colliding singularities’ $p_g(T, \mu) = p_l(T, \mu)$, which is just the Gibbs criterion of phase equilibrium. The properties of a PT are defined only by the liquid phase pressure $p_l(T, \mu)$ and by the temperature dependence of surface tension $\sigma(T)$.

In order to avoid the incompressibility of the nuclear liquid we suggest to employ the following parameterization of its pressure

$$p_l = \frac{W(T) + \mu + a_2(\mu - \mu_0)^2 + a_4(\mu - \mu_0)^4}{b}.  \quad (5)$$

Where $W(T) = W_0 + T^2 W_2$ denotes the usual temperature dependent binding energy per nucleon with $W_0 = 16$ MeV, while the constants $\mu_0 = -W_0$, $a_2 \simeq 1.233 \cdot 10^{-2}$ MeV$^{-1}$ and $a_4 \simeq 4.099 \cdot 10^{-7}$ MeV$^{-3}$ are fixed by the requirement to reproduce the normal nuclear matter properties, i.e. at vanishing temperature $T = 0$ and normal nuclear density $\rho = \rho_0$ the liquid pressure must be zero.

In addition to the new parameterization of the free energy of the $k$-nucleon fragment (3) we propose to use the following parameterization of the surface tension coefficient

$$\sigma(T) = \sigma_0 \left|\frac{T_{cep} - T}{T_{cep}}\right|^{\zeta} \text{sign}(T_{cep} - T),$$  (6)

with $\zeta = \text{const} \geq 1$, $T_{cep} = 18$ MeV and $\sigma_0 = 18$ MeV the SMM. In contrast to the Fisher droplet model [6] and the usual SMM, the CSMM surface tension (6) is negative above the critical temperature $T_{cep}$.
Figure 1: Model phase diagrams are shown in $T - \mu$ plane (a)) and $\rho - p$ plane (b)). In the panel a) a first order PT is shown by the solid curves. The vertical dashed lines display the second order PTs and the black circles correspond to the tricritical endpoints marked as 1 (nuclear matter) and 2 (antinuclear matter). A cross-over occurs along the dotted vertical line of the vanishing surface tension coefficient. The grey areas in the panel b) show the mixed phases of the first order PTs. The isotherms are shown for $T = 11, 16, 17, 18$ MeV from bottom to top. Negative baryonic charge densities correspond to an antimatter.

Now we would like to study the fragment size distribution in two regions of the phase diagram in order to elucidate the role of the negative surface tension coefficient. In order to demonstrate the pitfalls of the bimodal concept of Refs. [2, 3, 1] we study the gaseous phase and the supercritical temperature region, where there is no PT by construction. As one can see from Fig. 2 in the gaseous phase, even at the boundary with the mixed phase, the size distribution is a monotonically decreasing function of the number of nucleons $k$ in a fragment. However, for the supercritical temperatures one finds the typical bimodal fragment size distribution for a variety of temperatures and chemical potentials as one can see from Fig. 2.

A sharp peak at low $k$ values reflects a fast increase of the probability density of dimers compared to the monomers (nucleons), since the intermediate fragment sizes do not have the binding free energy and the surface free energy and, hence, the monomers are significantly suppressed in this region of thermodynamic parameters. On the other hand, it is clear that the tail of fragment distributions in Fig. 2 decreases due to the dominance of the bulk free energy and, hence, the whole structure at intermediate fragment sizes is due to a competition between the surface free energy and two other contributions into the fragment free energy, i.e. the bulk one and the Fisher one.

3 Conclusions

In the present work we showed that the bimodal distributions can naturally appear in infinite system without a PT. At the supercritical temperatures a bimodal distribution is generated by the negative values of the surface tension coefficient. This result is in line with the previously
Figure 2: Fragment size distribution in the gaseous phase is shown for a fixed baryonic chemical potential $\mu = -27.5$ MeV and three values of the temperature $T$. The dotted curve is found exactly at the boundary of gaseous and mixed phases.

discussed role of the competition between the volume and the surface parts of the system free energy.

Also we suggested the new parameterization of the CSMM liquid phase pressure which repairs the two main pitfalls of the original SMM and allows one to consider the compressible nuclear liquid which has the tricritical endpoint at the one third of the normal nuclear density. Surprisingly, the suggested approach to account for the nuclear liquid compressibility automatically leads to an appearance of an additional state that in many respects resembles the physical antinuclear matter.

References

[1] T. L. Hill, *Thermodynamics of small systems*, Dover, New York, 1994.
[2] Ph. Chomaz, F. Gulminelli and V. Duflot, Phys. Rev. E 64, 046114 (2001).
[3] F. Gulminelli, Nucl. Phys. A 791, 165, 2007.
[4] K. A. Bugaev, M. I. Gorenstein, I. N. Mishustin and W. Greiner, Phys. Rev. C 62, 044320, 2000; Phys. Lett. B 498, 144, 2001.
[5] K. A. Bugaev, Acta. Phys. Polon. B 36, 3083, 2005 and reference therein.
[6] M. E. Fisher, Physics 3, 255, 1967.