Reduction of the Bethe-Salpeter equation for the scattering amplitude of the particles with spin 1 to system of the integral equations for invariant functions

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Abstract

Bethe-Salpeter equation for the massive particles with spin 1 is considered. The scattering amplitude decomposition of the particles with spin 1 by relativistic tensors is derived. The transformation coefficients from helicity amplitudes to invariant functions is found. The integral equations system for invariant functions is obtained and partial decomposition of this system is performed. Equivalent system of the integral equation for the partial helicity amplitudes is presented.

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The Bethe-Salpeter equation was initially formulated in quantum electrodynamics \[1, 2\] to describe the bound two-body states in the case where neither particle could
be treated as an external-field source. The applicability of this equation was not, however, limited to the quantum electrodynamics framework. The Bethe-Salpeter equation can be formulated in other renormalized models of quantum field theory, such as \( \phi^3, \phi^4 \) and scalar electrodynamics \cite{ref3}. In addition, this equation is applied to describe strong interactions such as the \( \pi N^- \) and \( NN^- \) scattering \cite{ref4, ref5, ref6}, the \( NN - N\Delta \) reactions \cite{ref7, ref8}, and other processes that do not fall within the framework of renormalized theories. This equation is also used in describing the electromagnetic interaction of hadrons, in particular, electron scattering on a deuteron \cite{ref9, ref10}. The properties of scalar and vector mesons within the constituent quark model can also be interpreted in terms of the Bethe-Salpeter equation \cite{ref11, ref12, ref13}. A most frequent application of the Bethe-Salpeter equation is for the spin-0 and spin- \( \frac{1}{2} \) particle reaction amplitudes \cite{ref14, ref15, ref16}. This work considers the Bethe-Salpeter equation for the scattering-reaction amplitude of the vector particles \( 1^- + 1^- \rightarrow 1^- + 1^- \). Mainly, it aims at transforming the Bethe-Salpeter equation for the spin-1 particle scattering amplitude to a system of integral equations for invariant functions. Spin-1 particle scattering corresponds to e.g. vector-meson interactions, vector-meson scattering on a deuteron, or elastic deuteron scattering. The paper is arranged as follows. Section 2 gives a general expression for the \( P^- \) and \( T^- \) invariant amplitude of the reaction \( 1^- + 1^- \rightarrow 1^- + 1^- \). In Section 3, the Bethe-Salpeter equation is reduced to a system of integral equations for invariant functions using the coefficients of the helicity amplitude transformation to invariant functions. Section 4 presents the partial extension of a system of 4D integral equations to obtain a system of 2D integral equations. In Section 5, an alternative system of 2D integral equations is given for partial helicity amplitudes. Appendix provides the coefficients of helicity amplitude transformation to invariant functions and the matrix of the system of 2D integral equations for partial helicity amplitudes.
Let us now consider the general structure of the scattering-reaction amplitude for two spin-1 particles and with a negative intrinsic parity $1^- + 1^- \rightarrow 1^- + 1^-$. Let the initial particle momenta be $k_1$ and $q_1$, the final momenta be $k_2$ and $q_2$, the 4-vector amplitudes of the initial particle be $u$ and $v$, and those for the final ones $u'$ and $v'$. To reveal the amplitude symmetry with respect to the spatial reflection and time inversion, it is convenient to make use of a symmetrical and antisymmetrical combination of momenta:

$$P = k_1 + q_1 = k_2 + q_2, \quad p_1 = \frac{1}{2}(k_1 - q_1), \quad p_2 = \frac{1}{2}(k_2 - q_2).$$

(1)

The invariant variables $s$, $t$ and $u$ are expressed via $P$, $p_1$, and $p_2$ in the following way:

$$s = P^2, \quad t = (k_1 - k_2)^2 = (p_1 - p_2)^2, \quad u = (k_1 - q_2)^2 = (p_1 + p_2)^2,$$

$$s + t + u = 2m_1^2 + 2m_2^2.$$  

(2)

Let us assume the interaction between the particles to be $P$- and $T$- invariant. A general expression for the reaction amplitude with arbitrary spins will then be:

$$T(p_2, p_1; P) = \sum_i f_i(s, t)R^i(p_2, p_1; P),$$

(3)

where $f_i(s, t)$ are the invariant functions depending on the initial- and final-particle 4-momenta via the invariant variables $s$ and $t$ only, and $R^i(p_2, p_1; P)$ are the invariant combinations made of 4-momenta and wave functions of all particles participating in the reaction. To construct the reaction amplitude, we need to determine the number of independent invariant functions entering Eq. (3). It is equal to the number of independent helicity reaction amplitudes taking into account the $P$- and $T$- invariance. The total number of the helicity amplitudes of the reaction $1^- + 1^- \rightarrow 1^- + 1^-$ is equal to
\((2s_1 + 1)(2s_2 + 1)(2s_3 + 1)(2s_4 + 1) = 81\). These amplitudes are related in a manner determined by the \(P-\) invariant interactions

\[
T(\lambda_3, \lambda_4; \lambda_1, \lambda_2) = \eta(-1)^{(\lambda_1-\lambda_2)-(\lambda_3-\lambda_4)}T(-\lambda_3, -\lambda_4; -\lambda_1, -\lambda_2),
\]

(4)

where \(\eta = \eta_1\eta_2\eta_3\eta_4(-1)^{s_3+s_4-s_1-s_2}\), \(\eta_i\), and \(s_i\) are the intrinsic parity and spin of the particles. It is clear that the number of independent helicity amplitudes decreases down to 41. Further restrictions on the number of independent spiral amplitudes follow from the \(T-\) invariance of the interaction. For elastic processes, the \(T-\) invariance gives rise to the following relations between the helicity amplitudes:

\[
T(\lambda_3, \lambda_4; \lambda_1, \lambda_2) = (-1)^{(\lambda_1-\lambda_2)-(\lambda_3-\lambda_4)}T(\lambda_1, \lambda_2; \lambda_3, \lambda_4).
\]

(5)

In this case, the number of helicity amplitudes is decreased to 25. The following helicity amplitudes can be selected as independent ones:

\[
T(1, 1; 1, 1), T(1, 1; 1, 0), T(1, 1; 1, -1), T(1, 1; 0, 1), T(1, 1; 0, 0), T(1, 1; 0, -1),
\]

\[
T(1, 1; -1, 1), T(1, 1; -1, 0), T(1, 1; -1, -1), T(1, 0; 1, 0), T(1, 0; 1, -1), T(1, 0; 0, 1),
\]

\[
T(1, 0; 0, 0), T(1, 0; 0, -1), T(1, 0; -1, 1), T(1, 0; -1, 0), T(1, -1; 1, -1), T(1, -1; 0, 1),
\]

\[
T(1, -1; 0, 0), T(1, -1; 0, -1), T(1, -1; -1, 1), T(0, 1; 0, 1), T(0, 1; 0, 0), T(0, 1; 0, -1),
\]

\[
T(0, 0; 0, 0).
\]

(6)

Now, we have to identify 25 independent invariant spin combinations \(R^i(p_2, p_1; P)\) entering into \(\text{Eq.}(3)\). Since \(s, t\) and \(u\) are \(P-\) and \(T-\) invariant, then \(f_i(s, t)\) is also \(P-\) and \(T-\) invariant. The values \(R^i(p_2, p_1; P) = u'^\mu v'^\nu {\hat R}^i_{\mu\nu\alpha\beta} u^\alpha v^\beta\), where \(u', v', u, v\) are the 4-vector polarizations of final and initial particles, should, therefore, be also \(P-\) and \(T-\) invariant. The following set of the 4th rank 4-tensors satisfying the \(P-\) and \(T-\) invariance could be selected as:

\[
{\hat R}^1_{\mu\nu\alpha\beta} = p_1\mu p_1\nu p_2\alpha p_2\beta, \quad {\hat R}^2_{\mu\nu\alpha\beta} = p_1\mu p_1\nu p_2\alpha p_2\beta + p_1\mu p_1\nu p_2\alpha p_2\beta, \quad {\hat R}^3_{\mu\nu\alpha\beta} = P_\mu p_1\nu p_2\alpha p_2\beta + p_1\mu P_1\nu P_2\alpha p_2\beta.
\]
From Eqs. (4) and (5), it is evident that the relations of the

\[ R^4_{\mu \nu \alpha \beta} = P^\mu P^\nu P^p P^\gamma + p^\mu p^\nu P^\alpha P^\beta, \]
\[ R^5_{\mu \nu \alpha \beta} = p_{1 \mu} p_{1 \nu} P^\alpha P^\beta, \]
\[ R^6_{\mu \nu \alpha \beta} = p_{1 \mu} P^\nu P^\alpha P^\gamma + p_{1 \mu} p_{1 \nu} P^\alpha P^\beta, \]
\[ R^7_{\mu \nu \alpha \beta} = p_{1 \mu} P^\nu P^\alpha P^\beta + p_{1 \mu} P^\nu P^\gamma P^\alpha, \]
\[ R^8_{\mu \nu \alpha \beta} = p_{1 \nu} P^\alpha P^\beta + p_{1 \nu} P^\gamma P^\alpha, \]
\[ R^9_{\mu \nu \alpha \beta} = p_{1 \mu} P^\nu P^\alpha P^\beta + p_{1 \mu} P^\nu P^\gamma P^\alpha, \]
\[ R^{10}_{\mu \nu \alpha \beta} = P^\mu P^\nu P^\alpha P^\beta, \]
\[ R^{11}_{\mu \nu \alpha \beta} = g_{\mu \nu} p_{2 \alpha} P^\beta + p_{1 \mu} p_{1 \nu} g_{\alpha \beta}, \]
\[ R^{12}_{\mu \nu \alpha \beta} = g_{\mu \nu} p_{2 \alpha} P^\beta + p_{1 \mu} P^\nu g_{\alpha \beta}, \]
\[ R^{13}_{\mu \nu \alpha \beta} = g_{\mu \nu} P^\alpha P^\beta + P^\mu P^\nu g_{\alpha \beta}, \]
\[ R^{14}_{\mu \nu \alpha \beta} = g_{\mu \nu} P^\alpha P^\beta + P^\mu P^\nu g_{\alpha \beta}, \]
\[ R^{15}_{\mu \nu \alpha \beta} = p_{1 \mu} g_{\nu \beta} p_{2 \alpha}, \]
\[ R^{16}_{\mu \nu \alpha \beta} = P^\mu g_{\nu \beta} p_{2 \alpha} + p_{1 \mu} g_{\nu \beta} P^\alpha, \]
\[ R^{17}_{\mu \nu \alpha \beta} = P^\mu g_{\nu \beta} p_{2 \alpha} + P^\mu g_{\nu \beta} P^\alpha, \]
\[ R^{18}_{\mu \nu \alpha \beta} = g_{\mu \nu} P^\alpha P^\beta + p_{1 \mu} g_{\nu \alpha} P^\beta, \]
\[ R^{19}_{\mu \nu \alpha \beta} = g_{\mu \nu} P^\alpha P^\beta + P^\mu g_{\nu \alpha} P^\beta, \]
\[ R^{20}_{\mu \nu \alpha \beta} = g_{\mu \nu} P^\alpha P^\beta + P^\mu g_{\nu \alpha} P^\beta, \]
\[ R^{21}_{\mu \nu \alpha \beta} = g_{\mu \nu} P^\alpha P^\beta + P^\mu g_{\nu \alpha} P^\beta, \]
\[ R^{22}_{\mu \nu \alpha \beta} = g_{\mu \nu} P^\alpha P^\beta + g_{\mu \nu} P^\alpha P^\beta, \]
\[ R^{23}_{\mu \nu \alpha \beta} = g_{\mu \nu} P^\alpha P^\beta + g_{\mu \nu} P^\alpha P^\beta, \]
\[ R^{24}_{\mu \nu \alpha \beta} = g_{\mu \nu} P^\alpha P^\beta, \]
\[ R^{25}_{\mu \nu \alpha \beta} = g_{\mu \nu} g_{\alpha \beta}. \]  

(7)

From Eqs. (4) and (5), it is evident that the relations of the \( P - \) and \( T - \) invariance between the helicity amplitudes for the elastic scattering reaction between the spin 1 - particles coincide, provided the product of their internal parities \( \eta_1 \eta_2 \eta_3 \eta_4 \) equals unity. Note that the number of independent helicity amplitudes also coincides. Hence, the 4-tensor set (Eq. 7) can be used to construct amplitudes for the reactions \( 1^+ + 1^+ \rightarrow 1^+ + 1^+ \), \( 1^+ + 1^+ \rightarrow 1^- + 1^- \), \( 1^+ + 1^- \rightarrow 1^+ + 1^- \).

The 4-tensor set (Eq. 7) is not the only possibility. Consider now an invariant spin combination \( R^{25}(p_2, p_1; P) = u^* \cdot v \cdot v^* \cdot u \). A question arises as to why there are no \( u^* \cdot v^* \cdot u \cdot v \) or \( u^* \cdot v \cdot v^* \cdot u \) spin combinations among the invariant spin combinations (Eq. 8) similar to \( R^{25}(p_2, p_1; P) \). The answer to this question lies in the fact that these spin combinations are not independent, which can be proved using Gram’s determinant. It is well known that in a 4D space any five 4-vectors are linearly related. In our case this implies the Gram’s determinant is zero:

\[ G \left( \begin{array}{c} u, \; u^*, \; P, \; p_1, \; p_2 \\ v, \; v^*, \; P, \; p_1, \; p_2 \end{array} \right) = 0, \]

which contains the polarization vectors of all the particles participating in the reaction,
and all the 4-momenta entering the expression for the 4-tensors $R^i_{\mu\nu\alpha\beta}$. Now, one has to determine the number of independent relations conditioned by Gram’s determinants. Due to the symmetry properties of Gram’s determinants \[17\], any vector permutations within the upper or lower rows are possible - this may result in a change of their common sign only. It is the vector permutation from the upper or lower rows in Gram’s determinant that could give rise to essentially new relations. It should be noted that the permutation between the $P, p_1, p_2$ vectors from the upper and lower rows cannot result in new couplings either, since for any such permutation, there will be two similar 4-vectors in the upper and lower rows, which would result in a identically zero Gram’s determinant. Thus, the remaining permutations are those in the group $u, u^*, v, v^*$. With regard to the symmetry properties of Gram’s determinant, the following permutations occur:

\[
\begin{align*}
(v, u^*) & , (u, v) \\
(u, v^*) & , (u^*, v^*)
\end{align*}
\]

Thus, three Gram’s determinants are equal to zero:

\[
\begin{align*}
G \left( \begin{array}{c} u, u^*, P, p_1, p_2 \\ v, v^*, P, p_1, p_2 \end{array} \right) & = 0, \\
G \left( \begin{array}{c} v, u^*, P, p_1, p_2 \\ u^*, v^*, P, p_1, p_2 \end{array} \right) & = 0,
\end{align*}
\]

Removing brackets, we obtain a homogenous system of three linear equations. This system is, however, of the second rank, since the third linear equation appears to be equal to the difference between the first and second equations. There are two linearly independent relations between the invariant spin combinations, which allows expressing $u^* \cdot v^* u \cdot v$ and $u^* \cdot v v^* \cdot u$ in terms of $R^1 - R^{25}$:

\[
\begin{align*}
u^* \cdot v^* u \cdot v = \frac{4R1}{t (-4m^2 + s + t)} + \frac{R4}{s t (-4m^2 + s + t)} \frac{4}{s t (-4m^2 + s + t)} + \frac{R5}{s t (-4m^2 + s + t)}
\end{align*}
\]
When constructing the amplitude Eq.(3), use can be made of the spin combinations $u^* \cdot v v^* \cdot u$ instead of any other two spin combinations $R^i$ from Eqs.(10) and (11).

Let us write a general expression for the $P-$ and $T-$ invariant helicity amplitude of the reaction $1^- + 1^- \rightarrow 1^- + 1^-$:

$$T_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(p_2, p_1; P) = u^*_3 \cdot p_1 v^*_4 \cdot p_1 u_{\lambda_1} \cdot p_2 v_{\lambda_2} \cdot p_2 f_1 +$$
$$+ (u^*_{\lambda_3} \cdot p_1 v^*_{\lambda_4} \cdot p_1 u_{\lambda_1} \cdot p_2 v_{\lambda_2} \cdot P + u^*_{\lambda_3} \cdot p_1 v^*_{\lambda_4} \cdot P u_{\lambda_1} \cdot p_2 v_{\lambda_2} \cdot p_2) f_2 +$$
$$+ (u^*_{\lambda_3} \cdot p_1 v^*_{\lambda_4} \cdot p_1 u_{\lambda_1} \cdot P v_{\lambda_2} \cdot p_2 + u^*_{\lambda_3} \cdot P v^*_{\lambda_4} \cdot p_1 u_{\lambda_1} \cdot p_2 v_{\lambda_2} \cdot p_2) f_3 +$$
$$+ (u^*_{\lambda_3} \cdot p_1 v^*_{\lambda_4} \cdot p_1 u_{\lambda_1} \cdot P v_{\lambda_2} \cdot P + u^*_{\lambda_3} \cdot P v^*_{\lambda_4} \cdot P u_{\lambda_1} \cdot p_2 v_{\lambda_2} \cdot p_2) f_4 +$$
$$+ u^*_{\lambda_3} \cdot p_1 v^*_{\lambda_4} \cdot P u_{\lambda_1} \cdot p_2 v_{\lambda_2} \cdot P f_5 + (u^*_{\lambda_3} \cdot P v^*_{\lambda_4} \cdot p_1 u_{\lambda_1} \cdot p_2 v_{\lambda_2} \cdot P +}$$
where $\lambda_3, \lambda_4$ and $\lambda_1, \lambda_2$ are the helicities of the vector particles in their initial and final states.

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The Bethe - Salpeter equation is a relativistic relation for the two-body Green function $G(x'_1, x'_2; x_1, x_2)$:

$$G(x'_1, x'_2; x_1, x_2) = I(x'_1, x'_2; x_1, x_2) + \int K(x'_1, x'_2; x_3, x_4)G(x_3, x_4; x_1, x_2)d^4x_3d^4x_4,$$  

where $x_1, x_2, x'_1, x'_2$ are the initial and final 4D-coordinates of a particle. Thus, the Bethe-
Salpeter equation relates the total Green function for two particles \( G(x_1', x_2'; x_1, x_2) \), representing a sum of all the Feinman diagrams (Fig. 1, left) to a certain topologically derived part of this function \( I(x_1', x_2'; x_1, x_2) \) (Fig. 1 the first term in the right side), which is a sum of all two-body irreducible diagrams in the \( s- \) channel, i.e., the diagrams that could not be split into two linked parts containing points \( x_1, x_2 \) and \( x_1', x_2' \) by breaking two lines in the direction of the \( s- \) channel. The kernel \( K \) of the Bethe-Salpeter equation is explicitly expressed through the sum of two-body irreducible diagrams \( I \) and single-body Green functions for the scattering particles. In Fig. 1, the second term in the right side corresponds to the integral term in Eq.(13). The kernel of the Bethe-Salpeter equation is based on Lagrangian particle interaction. Thus, Eq.(13) provides an expression for the total Feynman diagrams via their two-body irreducible part. Since the kernel and the inhomogeneous term of the equation are derived using the approaches of the perturbation theory, they may only be approximated. That is why it is an approximated equation resulting, when one reduces the manipulations in the sum of irreducible diagrams to the lowest perturbation theory orders, where between the two interacting particles there is a single quantum exchange, which is often taken for the Bethe-Salpeter equation. This
approximation is called a ladder approach. In a momentum space, the Bethe - Salpeter equation (13) has the form of an integral equation for the two-body scattering amplitude $T(p_2, p_1; P)$:

$$T(p_2, p_1; P) = I(p_2, p_1; P) + \int K(p_2, p'; P)T(p', p_1; P) \frac{d^4 p'}{(2\pi)^4},$$  \hspace{1cm} (14)

where $I$ and $K$ are the inhomogeneous term and the kernel of the equation. Equation (14) for the vector-particle scattering amplitude can be written as follows:

$$T_{\mu\nu\alpha\beta}(p_2, p_1; P) = I_{\mu\nu\alpha\beta}(p_2, p_1; P) + \int I_{\mu\nu\gamma}(p_2, p'; P)G^\gamma\gamma\left(\frac{P}{2} + p'\right)G^\eta\eta\left(\frac{P}{2} - p'\right) \times$$

$$\times T_{\gamma\delta\alpha\beta}(p', p_1; P) \frac{d^4 p'}{(2\pi)^4},$$  \hspace{1cm} (15)

where $I$ is the sum of two-body irreducible diagrams and $G$ are the vector particle propagators. In Eq. (15), the 4-vector particle polarizations are omitted, and the sequence of the 4-indices corresponds to the motion opposite the lines shown in Fig. 1. Let us express the amplitude $T_{\mu\nu\alpha\beta}$ and the sum of two-body irreducible diagrams $I_{\mu\nu\alpha\beta}$ as an expansion in tensors $R^i_{\mu\nu\alpha\beta}$:

$$T_{\mu\nu\alpha\beta}(p_2, p_1; P) = \sum_{i=1}^{25} f_i(p_2, p_1; P) R^i_{\mu\nu\alpha\beta}(p_2, p_1; P),$$  \hspace{1cm} (16)

$$I_{\mu\nu\alpha\beta}(p_2, p_1; P) = \sum_{i=1}^{25} g_i(p_2, p_1; P) R^i_{\mu\nu\alpha\beta}(p_2, p_1; P).$$  \hspace{1cm} (17)

Now, proceed from the 4-tensors $T_{\mu\nu\alpha\beta}$ to the helicity amplitudes:

$$T_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(p_2, p_1; P) = \gamma^\mu\nu_{\lambda_3\lambda_4}(p_2; P) T_{\mu\nu\alpha\beta}(p_2, p_1; P) \gamma^\alpha\beta_{\lambda_1\lambda_2}(p_1; P),$$  \hspace{1cm} (18)

where the $\gamma^\mu\nu_{\lambda_3\lambda_4}(p_2; P)$ and $\gamma^\alpha\beta_{\lambda_1\lambda_2}(p_1; P)$ are the products of the helicity 4-vector polarizations of the final and initial particles

$$v^*_{\lambda_3\lambda_4}(p_2; P) = u^*\mu\left(\frac{P}{2} + p_2, \lambda_3\right) v^*\nu\left(\frac{P}{2} - p_2, \lambda_4\right),$$

$$v^\alpha\beta_{\lambda_1\lambda_2}(p_1; P) = u^\alpha\left(\frac{P}{2} + p_1, \lambda_1\right) v^\beta\left(\frac{P}{2} - p_1, \lambda_2\right).$$  \hspace{1cm} (19)
As the independent helicity amplitudes let us choose amplitudes given in Eq.(6) and proceed to the center mass system. Opening brackets for the scalar products in Eq.(18), we obtain a system of 25 linear equations with respect to 25 invariant functions $f_i(p_2, p_1; P)$. This system of linear equations was solved by the Gauss approach using a Mathematica software package. As a result of these manipulations, the invariant functions can be expressed as linear combinations of helicity amplitudes:

$$ f_i(p_2, p_1; P) = \sum_{\{\lambda_3, \lambda_4, \lambda_1, \lambda_2\}} u_{i, \{\lambda_3, \lambda_4, \lambda_1, \lambda_2\}}(p_2, p_1, P) T_{\{\lambda_3, \lambda_4, \lambda_1, \lambda_2\}}(p_2, p_1; P), $$

where transformation coefficients $u_{i, \{\lambda_3, \lambda_4, \lambda_1, \lambda_2\}}(p_2, p_1, P)$ are given in the Appendix. In Eq. (20), the symbol $\{\lambda_3, \lambda_4, \lambda_1, \lambda_2\}$ denotes that summation is not done over all helicity amplitudes but with respect to 25 independent helicity amplitudes only that are given in Eq.(6). The invariant functions $g_i(p_2, p_1; P)$ entering into Eq.(17) can be presented in a similar manner:

$$ g_i(p_2, p_1; P) = \sum_{\{\lambda_3, \lambda_4, \lambda_1, \lambda_2\}} u_{i, \{\lambda_3, \lambda_4, \lambda_1, \lambda_2\}}(p_2, p_1, P) I_{\{\lambda_3, \lambda_4, \lambda_1, \lambda_2\}}(p_2, p_1; P). $$

Having derived the matrix for the helicity amplitude transformation to invariant functions, we may write the Bethe - Salpeter equation (15) as a system of integral equations for the invariant functions $f_i$:

$$ f_i(p_2, p_1; P) = g_i(p_2, p_1; P) + \sum_{j=1}^{25} \sum_{\{\lambda_3, \lambda_4, \lambda_1, \lambda_2\}} u_{i, \{\lambda_3, \lambda_4, \lambda_1, \lambda_2\}}(p_2, p_1, P) v^{\mu \nu \lambda_3 \lambda_4}(p_2, P) \int g_k(p_2, p'; P) \times $$

$$ \times R_{\mu\nu\alpha\beta}(p_2, p'; P) G^{\gamma \delta}(P/2 + p') G^{\gamma \delta}(P/2 - p') R_{\gamma\delta\alpha\beta}(p', p_1; P) f_j(p', p_1; P) \frac{d^4 p'}{(2\pi)^4} v^{\alpha \beta}_{\lambda_1 \lambda_2}(p_1; P). $$

In this equation, the invariant functions $g_i$ and the vector particle propagators are calculated using the perturbation theory based on a Lagrangian interaction, i.e., they may be obtained to a certain approximation. The system of equations (22) can be presented in a more concise form:

$$ f_i(p_2, p_1; P) = g_i(p_2, p_1; P) + \sum_{j=1}^{25} \int K_{ij}(p_2, p', p_1; P) f_j(p', p_1; P) \frac{d^4 p'}{(2\pi)^4}, $$

(23)
where the kernel $K_{ij}(p_2, p', p_1; P)$ is a 4-scalar quantity:

$$K_{ij}(p_2, p', p_1; P) = \sum_{\{\lambda_3, \lambda_4, \lambda_1, \lambda_2\}}\sum_{k=1}^{25} u_i, \{\lambda_3, \lambda_4, \lambda_1, \lambda_2\} (p_2, p_1, P) v^{*\mu*\nu}_{\lambda_3 \lambda_4} (p_2; P) g_k(p_2, p'; P) \times$$

$$\times R^k_{\mu\nu\eta}(p_2, p'; P) G^{\epsilon \gamma}(\frac{P}{2} + p') G^{\eta \delta}(\frac{P}{2} - p') R^j_{\gamma\delta\alpha\beta}(p', p_1; P) v_{\lambda_1 \lambda_2}(p_1; P).$$

Thus, the Bethe-Salpeter equation for the vector-particle scattering amplitude has been reduced to a system of integral equations for the invariant functions $f_i(p_2, p_1; P)$.

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In the system of equations (23), integration was carried out over four independent variables $d^4 p' = dp'dp'_1 dp'_2 dp'_3 = dp'_0 p^2 dp' d\Omega'$. Using a partial expansion of the invariant function $f_i$, we can integrate with respect to the solid angle $\Omega'$ and reduce the system of 4D integral equations, Eq.(23), to a system of 2D integral equations for the coefficients of invariant function expansion with respect to the spherical harmonics. Let us assume that the z-axis of the center mass system is directed along the initial vector $p_1 = \frac{1}{2}(k_1 - q_1)$ and the final vector $p_2 = \frac{1}{2}(k_2 - q_2)$ corresponds to scattering at an angle $\theta$ in a plane with a zero azimuthal angle $\phi$. The invariant functions $f_i(p_2, p_1; P)$ depend on the 4-vectors $p_1, p_2, P$ via the invariants $s$ and $t$ only that are independent of the azimuthal scattering angle $\phi$. The expansion of the invariant functions $f_i(p_2, p_1; P)$ and $g_i(p_2, p_1; P)$ with respect to spherical harmonics will have the form:

$$f_i(p_2, p_1; P) = \sum_l \frac{1}{|p_2|} f^l_i(20, |p_2|, p_{10}, |p_1|; P) Y^{\theta} (\theta, 0),$$

$$g_i(p_2, p_1; P) = \sum_l \frac{1}{|p_2|} g^l_i(20, |p_2|, p_{10}, |p_1|; P) Y^{\theta} (\theta, 0).$$

Although the values $p_{10}, |p_1|$ and $p_{20}, |p_2|$ are not independent for free particles, they are written in an explicit form in the arguments of $f^l_i$ and $g^l_i$ for the case where 4-momenta of
the particles are located off the mass surface. The expansion of the kernel $K_{ij}(p_2, p', p_1; P)$ with respect to spherical harmonics has the form:

$$K_{ij}(p_2, p', p_1; P) = \sum_{l, l', m} \frac{1}{|P||p|} K_{ij}^{l0, l'm}(p_2, |p_2|, |p'|, |p_1|; P) Y_l^* (\theta, 0) Y_{l'}^m (\theta', \phi'). \quad (26)$$

In this case, as above, we have written the temporal components of the 4-momenta $p_1$ and $p_2$ as the arguments of $K_{ij}^{l0, l'm}$, which are independent variables when the 4-momenta are not located on the mass surface. Having substituted the expansions over the spherical harmonics for $f_i$, $g_i$ and $K_{ij}$ into the system of equations Eq.(23), taking into account that the invariant functions $f_j(p', p_1; P)$ do not depend on the azimuthal angle $\phi'$ of the intermediate momentum $p'$, we will arrive at a system of 2D integral equations for $f^l_i$:

$$f^l_i(p_{20}, |p_2|, p_{10}, |p_1|; P) = g^l_i(p_{20}, |p_2|, p_{10}, |p_1|; P) + \sum_j \sum_{l'} \int K_{ij}^{l0, l'm}(p_2, |p_2|, |p'|, |p_1|; P) f^l_j(p_0, |p'|, p_{10}, |p_1|; P) \frac{dp_0 dp_1 |p'|}{(2\pi)^4}. \quad (27)$$

The functions $f^l_i$ entering into Eq.(27) are related to the partial reaction amplitudes $T_{J, L, S, L', S'}$, where $J$ is the total momentum, and $L$, $S$ and $L'$, $S'$ are the orbital momentum and the total spin of the initial and final states, respectively. The formalism for the helicity [18] is, therefore, more preferable since it makes it possible to obtain a system of equations for the partial helicity amplitudes of the reaction $1^- + 1^- \rightarrow 1^- + 1^-$. The partial helicity amplitudes of the reaction $T_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}$, unlike the function $f^l_i$, possess a direct physical meaning.

Let us express the spin-1 particle scattering tensor as an expansion over the helicity
amplitudes:

\[ T_{\mu\nu\alpha\beta}(p_2, p_1; P) = \sum_{\lambda_1, \lambda_2, \lambda_3, \lambda_4} T_{\lambda_3, \lambda_4, \lambda_1, \lambda_2}(p_2, p_1; P) u_\mu^*(\frac{P}{2} + p_2, \lambda_3) v_\nu^*(\frac{P}{2} - p_2, \lambda_4) \times \]

\[ \times u_\alpha^*\left(\frac{P}{2} + p_1, \lambda_1\right) v_\beta^*\left(\frac{P}{2} - p_1, \lambda_2\right) , \]  

(28)

where summation is carried out over 81 helicity amplitudes. The same representation will be valid for the tensor \( I_{\mu\nu\alpha\beta} \), which is the sum of two-body irreducible diagrams:

\[ I_{\mu\nu\alpha\beta}(p_2, p_1; P) = \sum_{\lambda_1, \lambda_2, \lambda_3, \lambda_4} I_{\lambda_3, \lambda_4, \lambda_1, \lambda_2}(p_2, p_1; P) u_\mu^*(\frac{P}{2} + p_2, \lambda_3) v_\nu^*(\frac{P}{2} - p_2, \lambda_4) \times \]

\[ \times u_\alpha^*\left(\frac{P}{2} + p_1, \lambda_1\right) v_\beta^*\left(\frac{P}{2} - p_1, \lambda_2\right) . \]  

(29)

Upon contraction of the scattering tensor, Eq.(28), with the helicity 4-vectors of the initial and final particles, due to orthogonality of the latter, we will readily obtain the desired helicity amplitudes. Contrary to the invariant functions \( f_i(p_2, p_1; P) \), the helicity amplitudes \( T_{\lambda_3, \lambda_4, \lambda_1, \lambda_2}(p_2, p_1; P) \) depend not only on the invariants \( s \) and \( t \) but also on the azimuthal scattering angle \( \phi \). As shown before, only 25 (Eq.(6)) out of 81 helicity amplitudes are independent, taking into account the \( P- \) and \( T- \) invariance restrictions. Equations (28) and (29) can, therefore, be written in the form:

\[ T_{\mu\nu\alpha\beta}(p_2, p_1; P) = \sum_{\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}} T_{\{\lambda_3, \lambda_4, \lambda_1, \lambda_2\}}(p_2, p_1; P) U_{\{\lambda_3, \lambda_4, \lambda_1, \lambda_2\} \mu\nu\alpha\beta}(p_2, p_1; P) , \]

\[ I_{\mu\nu\alpha\beta}(p_2, p_1; P) = \sum_{\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}} I_{\{\lambda_3, \lambda_4, \lambda_1, \lambda_2\}}(p_2, p_1; P) U_{\{\lambda_3, \lambda_4, \lambda_1, \lambda_2\} \mu\nu\alpha\beta}(p_2, p_1; P) , \]  

(30)

where summation is performed over 25 helicity amplitudes given in Eq.(6), while the tensors \( U_{\{\lambda_3, \lambda_4, \lambda_1, \lambda_2\} \mu\nu\alpha\beta} \) are the sum products of the helicity vector polarizations of the initial and final particles including the \( P- \) and \( T- \) invariance relations, Eqs.(4) and (5).

For instance, the tensor \( U_{\{11, 11\} \mu\nu\alpha\beta} \) is equal to

\[ U_{\{11, 11\} \mu\nu\alpha\beta} = u_\mu^*(1)v_\nu^*(1)u_\alpha^*(1)v_\beta^*(1) + u_\mu^*(-1)v_\nu^*(-1)u_\alpha^*(-1)v_\beta^*(-1) . \]  

(31)
The other tensors $U_{\{\lambda_3 \lambda_4, \lambda_1 \lambda_2\}} \mu \nu \alpha \beta$ in Eq. (30) can be presented in the same way. In this formulation, the tensors $T_{\mu \nu \alpha \beta}$ and $I_{\mu \nu \alpha \beta}$ will satisfy the requirements on the $P-$ and $T-$ invariance.

Let us take the following vectors as the helicity 4-vectors of polarization:

$$u^\mu(k, 1) = \sqrt{\frac{1}{2}} \exp(i \phi) \left(0, -\cos(\theta) \cos(\phi) + i \sin(\phi), -i \cos(\phi) - \cos(\theta) \sin(\phi), \sin(\theta)\right),$$

$$u^\mu(k, 0) = \left(\frac{|k|}{m}, \frac{k_0 \cos(\phi) \sin(\theta)}{m}, \frac{k_0 \sin(\theta) \sin(\phi)}{m}, \frac{k_0 \cos(\theta)}{m}\right),$$

$$u^\mu(k, -1) = \sqrt{\frac{1}{2}} \exp(-i \phi) \left(0, \cos(\theta) \cos(\phi) + i \sin(\phi), -i \cos(\phi) + \cos(\theta) \sin(\phi), -\sin(\theta)\right),$$

where $\theta$ and $\phi$ are the polar and azimuthal angles of the vector particle 4-momentum.

For arbitrary 4-momenta, not necessarily lying on the mass surface, these 4-vectors of polarization satisfy the relation

$$k \cdot u(k, \lambda) = 0, \quad u(k, \lambda) \cdot u^*(k, \lambda') = ((1 + N(k))\delta_{\lambda 0} - 1)\delta_{\lambda \lambda'},$$

where $N(k) = \frac{|k|^2 - k_0^2}{m^2}$. From these formula it is clear that 4-vectors of polarization, Eq.(32), are orthogonal, and for $\lambda = \pm 1$ they are normalized with respect to -1. It is only in the case of longitudinal polarization $\lambda = 0$ that 4-vectors of polarization are normalized to $N(k)$ that is equal to -1, if the 4-momentum of the particle is located on the mass surface.

For this choice of the helicity 4-vectors of polarization, the scattering tensor $T_{\mu \nu \alpha \beta}$ satisfies the relations:

$$T_{\mu \nu \alpha \beta} k_1^\alpha = 0, \quad T_{\mu \nu \alpha \beta} q_1^\beta = 0,$$

$$T_{\mu \nu \alpha \beta} k_2^\mu = 0, \quad T_{\mu \nu \alpha \beta} q_2^\nu = 0,$$

where $k_1$, $q_1$, $k_2$, $q_2$ are the 4-momenta of the initial and final particles. From Eq. (34) it follows that the fraction of the vector particle propagator proportional to the product of
its 4-momenta $k^\mu k^\nu$ will become zero when being contracted with the tensors $T_{\mu\nu\alpha\beta}$ and $I_{\mu\nu\alpha\beta}$. Hence, upon convolution with $T_{\mu\nu\alpha\beta}$ and $I_{\mu\nu\alpha\beta}$, the contribution will come only from a fraction of the propagator that is proportional to $g^{\mu\nu}$. Note that the Bethe-Salpeter equation now acquires the following form:

$$T_{\mu\nu\alpha\beta}(p_2, p_1; P) = I_{\mu\nu\alpha\beta}(p_2, p_1; P) + \int I_{\mu\nu\eta}(p_2, p'; P) T^{\eta \alpha\beta}(p', p_1; P) \times$$

$$\times \frac{1}{D(\frac{P}{2} + p') D(\frac{P}{2} - p')} \left(\frac{d^4 p'}{(2\pi)^4}\right).$$

(35)

where $D(\frac{P}{2} + p')$ and $D(\frac{P}{2} - p')$ are the renormired denominators of the vector particle propagators. Using the representations of Eq.(28) and (29) for the tensors $T_{\mu\nu\alpha\beta}$ and $I_{\mu\nu\alpha\beta}$ and the relations of orthonormalization of Eq. (33) for the 4-vectors of polarization, we may write Eq.(35) as

$$T_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(p_2, p_1; P) = I_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(p_2, p_1; P) + \sum_{\lambda', \lambda''} \int I_{\lambda_3 \lambda_4, \lambda' \lambda''}(p_2, p'; P) T_{\lambda' \lambda'', \lambda_1 \lambda_2}(p', p_1; P) \times$$

$$\times \left(\frac{(1 + N(\frac{P}{2} + p')) \delta_{\lambda'0} - 1)}{D(\frac{P}{2} + p') D(\frac{P}{2} - p')} \left(\frac{d^4 p'}{(2\pi)^4}\right)\right).$$

(36)

In Eq. (36), there is no summation with respect to the 4-indices, instead, the summation is carried out over the intermediate-particle helicities. This allows us to move from the 4D system of integral equations (Eq.(36)) to a 2D system of integral equations via the expansion of the helicity amplitudes with respect to the partial spiral amplitudes and integration over the angles. The above-mentioned expansion has the following form: [19]:

$$T_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(p_2, p_1; P) = \sum_{J,M} \frac{2J + 1}{4\pi} D_{\lambda_3 - \lambda_4, M}(n_2) D_{\lambda_1 - \lambda_2, M}(n_1) \times$$

$$\times T_{J^*}(p_{20}, \|p_2\|, p_{10}, \|p_1\|; P).$$

(37)

$$I_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(p_2, p_1; P) = \sum_{J,M} \frac{2J + 1}{4\pi} D_{\lambda_3 - \lambda_4, M}(n_2) D_{\lambda_1 - \lambda_2, M}(n_1) \times$$

$$\times I_{J^*}(p_{20}, \|p_2\|, p_{10}, \|p_1\|; P).$$

(38)
In these expressions, the $D-$ function arguments are taken as

$$D^J_{\Lambda M}(n) = D^J_{\Lambda M}(\phi, \theta, 0) = e^{iM\phi}d^J_{\Lambda M}(\theta),$$

(39)

where $\theta$ and $\phi$ are the polar and azimuthal angles of the unit vector $n$. Using the orthogonality properties of the $D-$ functions, we may integrate with respect to the angular variables in Eq.(36). As a result, the system of 4D integral equations for the helicity amplitudes, Eq.(36), will be reduced to a system of 2D integral equations for the partial helicity amplitudes:

$$T^J_{\lambda_3, \lambda_4, \lambda_1, \lambda_2}(p_{20}, |p_2|, p_{10}, |p_1|; P) = I^J_{\lambda_3, \lambda_4, \lambda_1, \lambda_2}(p_{20}, |p_2|, p_{10}, |p_1|; P) +$$

$$+ \sum_{\lambda', \lambda''} \int I^J_{\lambda_3, \lambda_4, \lambda', \lambda''}(p_{20}, |p_2|, p_0', |p'|; P)T^J_{\lambda', \lambda'', \lambda_1, \lambda_2}(p_0', |p'|, p_{10}, |p_1|; P) \times$$

$$\times \frac{(N(p_0' + p'))\delta_{\lambda_0'} - 1)((N(p_0' - p'))\delta_{\lambda_0'} - 1)|p'|^2 dp_0' dp'|(2\pi)^4. \quad (40)$$

The partial spiral amplitudes satisfy the relations for the $T-$ and $P-$ invariance:

$$T^J_{\lambda_3, \lambda_4, \lambda_1, \lambda_2} = \eta_1 \eta_2 \eta_3 \eta_4 (-1)^{s_3 + s_4 - s_1 - s_2} T^J_{-\lambda_3, -\lambda_4, -\lambda_1 - \lambda_2},$$

$$T^J_{\lambda_3, \lambda_4, \lambda_1, \lambda_2} = T^J_{\lambda_1, \lambda_2, \lambda_3, \lambda_4}. \quad (41)$$

This is the reason why only 25 out of 81 partial helicity amplitudes with this $J$ will be independent. Let us select the partial helicity amplitudes with the same helicity as in Eq.(6) as 25 independent amplitudes and number them from 1 to 25. Now, the system of 2D integral equations can be written in terms of 25 independent partial amplitudes as follows:

$$T^J_i(p_{20}, |p_2|, p_{10}, |p_1|; P) = I^J_i(p_{20}, |p_2|, p_{10}, |p_1|; P) +$$

$$+ \sum_{j=1}^{25} \int K^J_{i, j}(p_{20}, |p_2|, p_0', |p'|; P)T^J_j(p_0', |p'|, p_{10}, |p_1|; P) \frac{|p'|^2 dp_0' dp'|}{D(p_0' + p')D(p_0' - p') (2\pi)^4}. \quad (42)$$

The values $K^J_{i, j}$ are expressed via $I^J_i$ and the normalization multipliers $N$. Their explicit representation is given in Appendix.
This work was supported by the Russian Basic Research Foundation (grants N N 01-02-17276 and 02-02-06866) and the RF Ministry of Education (grant N E00-3, 3-110).

Appendix

In the general case, the matrix for the transformation of 25 helicity amplitudes to 25 invariant functions has 625 elements. Some of these elements are considered to be cumbersome mathematical expressions. In this appendix we present the transformation matrix elements for scattering of the vector particles having equal masses $m_1 = m_2 = m$. In this case, out of 625 elements, 361 are equal to zero, and the remaining 264 non-zero elements are related by numerous symmetry relations. The non-zero elements may be classified into 63 groups with their elements differing in the constant factors only. For the sake of space, the spiral amplitude indices corresponding to Eq.(6) were numbered from 1 to 25.

Given below are the explicit expressions for the non-zero transformation matrix elements $u_{i,j}$, where $i$— is the index of the invariant function $f_i$, and $j$— is the helicity amplitude index.

\[
\begin{align*}
    u_{11} &= \frac{1}{2}u_{13} = \frac{1}{2}u_{17} = u_{19} = -u_{23} = u_{27} = -u_{33} = u_{37} = 2u_{43} = 2u_{47} = 2u_{63} = \\
    &= 2u_{67} = \frac{-8 \csc(\theta)^2}{(-4 \, m^2 + s)^2}, \\
    u_{41} &= u_{49} = \frac{\csc(\frac{\theta}{2})^2 \sec(\frac{\theta}{2})^2}{-8 \, m^2 \, s + 2 \, s^2}, \\
    u_{51} &= u_{81} = \frac{-\left((-4 \, m^2 + s + (4 \, m^2 - 2 \, s) \, \cos(\theta)) \, \csc(\frac{\theta}{2})^2 \sec(\frac{\theta}{2})^2\right)}{2 \, s \, (-4 \, m^2 + s)^2},
\end{align*}
\]
\[ u_{61} = \frac{(-4 m^2 + s \cos(\theta)) \csc\left(\frac{\theta}{2}\right)^2 \sec\left(\frac{\theta}{2}\right)^2}{2 s (2 m^2 + s)^2}, \]
\[ u_{71} = -u_{91} = \frac{(-1 + 3 \cos(\theta)) \csc\left(\frac{\theta}{2}\right)^2 \sec\left(\frac{\theta}{2}\right)^2}{4 (2 m^2 s - s^2)}, \]
\[ u_{101} = \frac{-(\cos(\theta) + 4 (2 m^2 - 4 s^2) \cos(\theta)) \csc\left(\frac{\theta}{2}\right)^2 \sec\left(\frac{\theta}{2}\right)^2}{8 s^2 (2 m^2 + s)^2} \times \csc\left(\frac{\theta}{2}\right)^2 \sec\left(\frac{\theta}{2}\right)^2, \]
\[ u_{111} = u_{119} = -u_{117} = -u_{121} = u_{181} = -u_{189} = -u_{1817} = u_{1821} = \frac{\csc\left(\frac{\theta}{2}\right)^2 \sec\left(\frac{\theta}{2}\right)^2}{8 m^2 - 2 s}, \]
\[ u_{121} = u_{129} = -u_{1217} = -u_{1221} = -u_{131} = -u_{139} = u_{1317} = u_{1321} = u_{191} = u_{199} = -u_{1917} = \]
\[ = u_{1921} = -u_{201} = u_{209} = u_{2017} = -u_{2021} = \frac{\cos(\theta) \csc\left(\frac{\theta}{2}\right)^2 \sec\left(\frac{\theta}{2}\right)^2}{-16 m^2 + 4 s}, \]
\[ u_{141} = u_{149} = -u_{1417} = -u_{1421} = u_{211} = -u_{219} = -u_{2117} = u_{2121} = \frac{\cos(\theta) \csc\left(\frac{\theta}{2}\right)^2 \sec\left(\frac{\theta}{2}\right)^2}{-32 m^2 + 8 s}, \]
\[ u_{151} = \frac{1}{2} u_{153} = u_{1517} = -2 u_{161} = -u_{163} = -2 u_{1617} = u_{221} = \frac{1}{2} u_{227} = u_{2217} = 2 u_{231} = u_{237} = \]
\[ = 2 u_{2317} = \frac{\sec\left(\frac{\theta}{2}\right)^2}{-4 m^2 + s}, \]
\[ u_{171} = \frac{1}{2} u_{173} = u_{1717} = u_{241} = \frac{1}{2} u_{247} = u_{2417} = \frac{-\left((-4 m^2 + s + 3 s^2) \cos(\theta)\right)}{4 s (4 m^2 + s)} \sec\left(\frac{\theta}{2}\right)^2, \]
\[ u_{251} = \frac{1}{2} u_{253} = \frac{1}{2} u_{257} = u_{259} = u_{2517} = u_{2521} = \frac{1}{2}, \]
\[ u_{52} = u_{58} = 2 u_{62} = 2 u_{64} = 2 u_{66} = 2 u_{68} = 4 u_{78} = u_{84} = u_{86} = -4 u_{96} = \frac{8 \sqrt{2} m \csc(\theta)}{\sqrt{s} (4 m^2 + s)^2}, \]
\[ u_{72} = -u_{94} = \frac{2 \sqrt{2} m \left(8 m^2 - 3 s\right) \csc(\theta)}{s^2 (4 m^2 + s)^2}, \]
\[ u_{74} = u_{76} = -u_{92} = -u_{98} = \frac{-2 \sqrt{2} m \csc(\theta)}{s^2 (4 m^2 + s)}, \]
\[ u_{102} = u_{104} = \frac{8 \sqrt{2} m \left(3 m^2 - s\right) \cot(\theta)}{s^2 (4 m^2 + s)^2}, \]
\[ u_{122} = -u_{128} = -u_{1211} = -u_{1215} = -u_{134} = u_{136} = -u_{1318} = u_{1320} = u_{164} = u_{166} = \]
\[ = u_{1618} = u_{1620} = u_{192} = u_{198} = -u_{1911} = u_{1915} = -u_{204} = -u_{206} = u_{2018} = u_{2020} = -u_{232} = \]
\[-u_{238} = -u_{2311} = u_{2315} = \frac{2 \sqrt{2} m \csc(\theta)}{\sqrt{s} (-4 m^2 + s)},\]

\[u_{142} = u_{144} = -u_{146} = -u_{148} = -u_{1411} = -u_{1415} = u_{1418} = -u_{1420} = \frac{1}{2} u_{174} = \frac{1}{2} u_{176} = \frac{1}{2} u_{1718} = \]

\[= \frac{1}{2} u_{212} = u_{214} = u_{216} = u_{218} = -u_{2111} = u_{2115} = -u_{2118} = -u_{2120} = \frac{1}{2} u_{242} = \frac{1}{2} u_{248} = \]

\[= \frac{1}{2} u_{2411} = -\frac{1}{2} u_{2415} = \frac{\sqrt{2} m \cot(\theta)}{\sqrt{s} (-4 m^2 + s)},\]

\[u_{53} = -2 u_{73} = u_{87} = 2 u_{97} = \frac{\sqrt{2} m \cot(\theta)}{\sqrt{s} (-4 m^2 + s)},\]

\[u_{57} = 2 u_{77} = u_{83} = -2 u_{93} = \frac{(-4 m^2 + s + (-4 m^2 + 2 s) \cos(\theta)) \csc(\frac{\theta}{2})^2 \sec(\frac{\theta}{2})^2}{s (-4 m^2 + s)^2},\]

\[u_{103} = u_{107} = \frac{(-4 m^2 + s + 4 m^2 \cos(\theta)) (-4 m^2 + s + (-4 m^2 + 2 s) \cos(\theta)) \csc(\frac{\theta}{2})^2 \sec(\frac{\theta}{2})^2}{4 s^2 (-4 m^2 + s)^2},\]

\[u_{157} = 2 u_{157} = 2 u_{1521} = 2 u_{167} = 4 u_{169} = 4 u_{1621} = u_{223} = 2 u_{229} = 2 u_{2221} = -2 u_{233} = -4 u_{239} =\]

\[= -4 u_{2321} = -\frac{2 \csc(\frac{\theta}{2})^2}{-4 m^2 + s},\]

\[u_{177} = 2 u_{179} = 2 u_{1721} = u_{243} = 2 u_{249} = 2 u_{2421} = \frac{(-4 m^2 + s + 4 m^2 \cos(\theta)) \csc(\frac{\theta}{2})^2}{2 (4 m^2 s - s^2)},\]

\[u_{105} = \frac{32 m^4 - 16 m^2 s}{(-4 m^2 s + s^2)^2},\]

\[u_{145} = u_{1419} = -u_{1722} = -u_{1724} = -u_{2112} = -u_{2114} = -u_{2410} = -u_{2416} = \frac{4 m^2}{-4 m^2 s + s^2},\]

\[u_{106} = u_{108} = -\frac{1}{2} u_{1013} = -\frac{1}{2} u_{1023} = \frac{8 \sqrt{2} m^3 \cot(\theta)}{s^2 (-4 m^2 + s)^2},\]

\[u_{510} = u_{614} = -2 u_{710} = 2 u_{714} = u_{822} = -2 u_{914} = 2 u_{922} = -8 m^2 \sec(\frac{\theta}{2})^2}{s (-4 m^2 + s)},\]

\[u_{109} = \frac{(8 m^4 - 8 m^2 s + s^2 - 2 (4 m^2 - s) s \cos(\theta) - 8 m^4 \cos(2 \theta)) \csc(\frac{\theta}{2})^2 \sec(\frac{\theta}{2})^2}{8 s^2 (-4 m^2 + s)^2},\]

\[u_{1010} = u_{1022} = -\frac{2 m^2 (4 m^2 - s + (4 m^2 - 2 s) \cos(\theta)) \sec(\frac{\theta}{2})^2}{s^2 (-4 m^2 + s)^2},\]

\[u_{59} = u_{69} = u_{79} = u_{89} = u_{99} = \frac{(-4 m^2 + s + 4 m^2 \cos(\theta)) \csc(\frac{\theta}{2})^2 \sec(\frac{\theta}{2})^2}{2 s (-4 m^2 + s)},\]
\[ u_{211} = -u_{320} = -2u_{411} = -2u_{420} = \frac{16 \sqrt{2} m \left( \cot(\theta) - \csc(\theta) \right) \csc(\theta)^2}{\sqrt{s} \left( -4 m^2 + s \right)^2}, \]
\[ u_{511} = -2u_{611} = -2u_{620} = u_{820} = \frac{-4 \sqrt{2} m \sec(\frac{\theta}{2}) \tan(\frac{\theta}{2})}{\sqrt{s} \left( -4 m^2 + s \right)^2}, \]
\[ u_{711} = -u_{920} = -\frac{m \left( 8 m^2 - 3 s + (8 m^2 + s) \cos(\theta) \right) \csc(\frac{\theta}{2}) \sec(\frac{\theta}{2})^3}{\sqrt{2} s^\frac{3}{2} \left( -4 m^2 + s \right)^2}, \]
\[ u_{720} = -u_{911} = \frac{m \left( 4 m^2 - s + (4 m^2 - 3 s) \cos(\theta) \right) \csc(\frac{\theta}{2}) \sec(\frac{\theta}{2})^3}{\sqrt{2} s^\frac{3}{2} \left( -4 m^2 + s \right)^2}, \]
\[ u_{1011} = u_{1020} = -\frac{\sqrt{2} m \cos(\theta) \left( 2 m^2 - s + 2 m^2 \cos(\theta) \right) \csc(\frac{\theta}{2}) \sec(\frac{\theta}{2})^3}{s^\frac{3}{2} \left( -4 m^2 + s \right)^2}, \]
\[ u_{1012} = \frac{4 m^2 \left( -4 m^2 + s + (4 m^2 - 2 s) \cos(\theta) \right) \csc(\frac{\theta}{2})^2}{\left( -4 m^2 + s \right)^2}, \]
\[ u_{215} = u_{318} = 2u_{415} = -2u_{418} = \frac{-16 \sqrt{2} m \csc(\theta)^2 \left( \cot(\theta) + \csc(\theta) \right)}{\sqrt{s} \left( -4 m^2 + s \right)^2}, \]
\[ u_{515} = -2u_{615} = 2u_{618} = -u_{818} = \frac{4 \sqrt{2} m \cot(\frac{\theta}{2}) \csc(\frac{\theta}{2})^2}{\sqrt{s} \left( -4 m^2 + s \right)^2}, \]
\[ u_{715} = u_{918} = \frac{m \left( 1 + 3 \cos(\theta) \right) \csc(\frac{\theta}{2})^3 \sec(\frac{\theta}{2})}{\sqrt{2} \sqrt{s} \left( -4 m^2 + s \right)^2}, \]
\[ u_{718} = u_{915} = -\frac{m \left( -4 m^2 + s + (4 m^2 + s) \cos(\theta) \right) \csc(\frac{\theta}{2})^3 \sec(\frac{\theta}{2})}{\sqrt{2} s^\frac{3}{2} \left( -4 m^2 + s \right)^2}, \]
\[ u_{713} = -u_{913} = \frac{-16 \sqrt{2} m^3 \csc(\theta)}{s^\frac{3}{2} \left( -4 m^2 + s \right)^2}, \]
\[ u_{1014} = \frac{-4 m^2 \left( 4 m^2 - s + 4 m^2 \cos(\theta) \right) \sec(\frac{\theta}{2})^2}{s^2 \left( -4 m^2 + s \right)^2}, \]
\[ u_{1015} = -u_{1018} = \frac{\sqrt{2} m \cos(\theta) \left( -2 m^2 + s + 2 m^2 \cos(\theta) \right) \csc(\frac{\theta}{2})^3 \sec(\frac{\theta}{2})}{s^\frac{3}{2} \left( -4 m^2 + s \right)^2}, \]
\[ u_{1016} = u_{1024} = \frac{2 m^2 \left( -4 m^2 + s + 4 m^2 \cos(\theta) \right) \csc(\frac{\theta}{2})^2}{\left( -4 m^2 + s + s^2 \right)^2}, \]
\[ u_{117} = -\frac{(-3 + \cos(\theta)) \csc(\frac{\theta}{2})^2 \sec(\frac{\theta}{2})^4}{\left( -4 m^2 + s \right)^2}. \]
\[ u_{21} = -u_{317} = \frac{2 \sec \left( \frac{\theta}{2} \right)^4}{\left( -4 m^2 + s \right)^2}, \]
\[ u_{417} = \frac{(4 m^2 - 3 s + (4 m^2 + s) \cos(\theta)) \csc \left( \frac{\theta}{2} \right)^2 \sec \left( \frac{\theta}{2} \right)^4}{4 s \left( -4 m^2 + s \right)^2}, \]
\[ u_{517} = u_{817} = \frac{(2 m^2 - 3 \cos(\theta) + (-2 m^2 + s) \cos(2 \theta)) \csc \left( \frac{\theta}{2} \right)^2 \sec \left( \frac{\theta}{2} \right)^4}{4 s \left( -4 m^2 + s \right)^2}, \]
\[ u_{617} = \frac{(8 m^2 - 5 s + 2 (4 m^2 + s) \cos(\theta) - s \cos(2 \theta)) \csc \left( \frac{\theta}{2} \right)^2 \sec \left( \frac{\theta}{2} \right)^4}{8 s \left( -4 m^2 + s \right)^2}, \]
\[ u_{717} = -u_{917} = \frac{(-12 m^2 + s + 8 (-2 m^2 + s) \cos(\theta) - (4 m^2 + s) \cos(2 \theta)) \csc \left( \frac{\theta}{2} \right)^2 \sec \left( \frac{\theta}{2} \right)^4}{16 s \left( -4 m^2 + s \right)^2}, \]
\[ u_{1017} = \frac{(-4 m^4 - 16 m^2 s + 3 s^2) \cos(\theta) + (8 m^4 - 4 m^2 s + 3 s^2) \cos(2 \theta)) \csc \left( \frac{\theta}{2} \right)^2 \sec \left( \frac{\theta}{2} \right)^4 +}{16 s^2 \left( -4 m^2 + s \right)^2} \]
\[ + \frac{(4 (2 m^4 - 3 m^2 s + s^2 - m^4 \cos(3 \theta))) \csc \left( \frac{\theta}{2} \right)^2 \sec \left( \frac{\theta}{2} \right)^4}{16 s^2 \left( -4 m^2 + s \right)^2}, \]
\[ u_{419} = \frac{32 m^2 \csc(\theta)^2}{s \left( -4 m^2 + s \right)^2}, \]
\[ u_{719} = -u_{919} = \frac{-16 m^2 \cot(\theta) \csc(\theta)}{s \left( -4 m^2 + s \right)^2}, \]
\[ u_{1019} = \frac{-16 m^2 \left( -m^2 + s + m^2 \cos(2 \theta) \right) \csc(\theta)^2}{s^2 \left( -4 m^2 + s \right)^2}, \]
\[ u_{121} = \frac{(3 + \cos(\theta)) \csc \left( \frac{\theta}{2} \right)^4 \sec \left( \frac{\theta}{2} \right)^2}{\left( -4 m^2 + s \right)^2}, \]
\[ u_{221} = -u_{321} = \frac{-2 \csc \left( \frac{\theta}{2} \right)^4}{\left( -4 m^2 + s \right)^2}, \]
\[ u_{421} = \frac{-((-4 m^2 + 3 s + (4 m^2 + s) \cos(\theta)) \csc \left( \frac{\theta}{2} \right)^4 \sec \left( \frac{\theta}{2} \right)^2)}{4 s \left( -4 m^2 + s \right)^2}, \]
\[ u_{521} = u_{821} = \frac{((-2 m^2 + s + 3 s \cos(\theta) + 2 m^2 \cos(2 \theta)) \csc \left( \frac{\theta}{2} \right)^4 \sec \left( \frac{\theta}{2} \right)^2}{4 s \left( -4 m^2 + s \right)^2}, \]
\[ u_{621} = \frac{-((8 m^2 + s + (-8 m^2 + 6 s) \cos(\theta) + s \cos(2 \theta)) \csc \left( \frac{\theta}{2} \right)^4 \sec \left( \frac{\theta}{2} \right)^2)}{8 s \left( -4 m^2 + s \right)^2}, \]
\[ u_{721} = -u_{921} = \frac{(-4 m^2 + s + (4 m^2 + s) \cos(\theta)) \csc \left( \frac{\theta}{2} \right)^4}{4 s \left( -4 m^2 + s \right)^2}, \]
\[
\begin{align*}
    u_{1021} &= \left( (-4m^4 + s^2) \cos(\theta) + (-8m^4 + 4m^2 s + s^2) \cos(2\theta) + 2 \left( 4m^4 - 2m^2 s + s^2 + 2m^4 \cos(3\theta) \right) \right) \frac{\csc(\frac{\theta}{2})^4 \sec(\frac{\theta}{2})^2}{16s^2 (-4m^2 + s)^2}, \\
    u_{1025} &= \left( 16m^4 \right) \left( -4m^2 s + s^2 \right)^2
\end{align*}
\]

In these formulas \( \theta \) is the scattering angle and \( s \) is the squared total energy in the center of mass system.

Out of 625 elements of the matrix \( K_{i,j}^J \), 420 appear to be equal to zero. Presented below are 205 non-zero elements of the \( K_{i,j}^J \). The order of numbering corresponds to the order in which the helicity amplitudes are given in Eq. (6).

\[
\begin{align*}
    K_{1,1}^J &= I_1^J, \\
    K_{1,2}^J &= N_2I_2^J, \\
    K_{1,3}^J &= I_3^J, \\
    K_{1,4}^J &= N_1I_4^J, \\
    K_{1,5}^J &= N_1N_2I_5^J, \\
    K_{1,6}^J &= N_1I_6^J, \\
    K_{1,7}^J &= I_7^J; \\
    K_{1,8}^J &= N_2I_8^J, \\
    K_{1,9}^J &= I_9^J; \\
    K_{2,2}^J &= I_1^J, \\
    K_{2,8}^J &= -I_9^J, \\
    K_{2,10}^J &= -N_2I_2^J, \\
    K_{2,11}^J &= -I_3^J, \\
    K_{2,12}^J &= -N_1I_4^J, \\
    K_{2,13}^J &= -N_1N_2I_5^J; \\
    K_{2,14}^J &= -N_1I_6^J, \\
    K_{2,15}^J &= -I_7^J, \\
    K_{2,16}^J &= -N_2I_8^J; \\
    K_{3,3}^J &= I_1^J, \\
    K_{3,7}^J &= I_9^J, \\
    K_{3,11}^J &= -N_2I_2^J, \\
    K_{3,15}^J &= N_2I_8^J, \\
    K_{3,17}^J &= I_3^J, \\
    K_{3,18}^J &= N_1I_4^J, \\
    K_{3,19}^J &= N_1N_2I_5^J, \\
    K_{3,20}^J &= N_1I_6^J, \\
    K_{3,21}^J &= I_7^J; \\
    K_{4,4}^J &= I_1^J, \\
    K_{4,6}^J &= -I_9^J, \\
    K_{4,12}^J &= -N_2I_2^J, \\
    K_{4,14}^J &= -N_2I_8^J, \\
    K_{4,18}^J &= I_3^J, \\
    K_{4,20}^J &= -I_7^J, \\
    K_{4,22}^J &= -N_1I_6^J, \\
    K_{4,23}^J &= -N_1N_2I_5^J, \\
    K_{4,24}^J &= -N_1I_6^J; \\
    K_{5,5}^J &= I_1^J + I_9^J, \\
    K_{5,13}^J &= N_2(I_8^J - I_2^J), \\
    K_{5,19}^J &= I_3^J + I_7^J, \\
    K_{5,23}^J &= N_1(I_6^J - I_4^J), \\
    K_{5,25}^J &= N_1N_2I_5^J; \\
    K_{6,4}^J &= -I_9^J, \\
    K_{6,6}^J &= I_1^J, \\
    K_{6,12}^J &= -N_2I_2^J, \\
    K_{6,14}^J &= -N_2I_8^J, \\
    K_{6,18}^J &= -I_7^J, \\
    K_{6,20}^J &= I_3^J, \\
    K_{6,22}^J &= -N_1I_6^J, \\
    K_{6,23}^J &= N_1N_2I_5^J, \\
    K_{6,24}^J &= -N_1I_4^J; \\
    K_{7,3}^J &= I_9^J, \\
    K_{7,7}^J &= I_1^J, \\
    K_{7,11}^J &= N_2I_8^J, \\
    K_{7,15}^J &= -N_2I_2^J, \\
    K_{7,17}^J &= I_7^J, \\
    K_{7,18}^J &= -N_1I_6^J, \\
    K_{7,19}^J &= N_1N_2I_5^J, \\
    K_{7,20}^J &= -N_1I_4^J; \\
    K_{8,2}^J &= -I_9^J, \\
    K_{8,8}^J &= I_1^J, \\
    K_{8,10}^J &= -N_2I_8^J, \\
    K_{8,11}^J &= I_7^J, \\
    K_{8,12}^J &= -N_1I_6^J, \\
    K_{8,13}^J &= N_1N_2I_5^J, \\
    K_{8,14}^J &= -N_1I_4^J, \\
    K_{8,15}^J &= I_3^J, \\
    K_{8,16}^J &= -N_2I_2^J;
\end{align*}
\]
\[
\begin{align*}
K_{9,1}^J &= I_9^J, \quad K_{9,2}^J = -N_2 I_8^J, \quad K_{9,3}^J = I_7^J, \quad K_{9,4}^J = -N_1 I_6^J, \quad K_{9,5}^J = N_1 N_2 I_5^J, \quad K_{9,6}^J = -N_1 I_4^J, \\
K_{9,7}^J &= I_3^J, \quad K_{9,8}^J = -N_2 I_2^J, \quad K_{9,9}^J = I_1^J; \\
K_{10,2}^J &= -I_2^J, \quad K_{10,8}^J = -I_8^J, \quad K_{10,10}^J = -N_2 I_{10}^J, \quad K_{10,11}^J = -I_{11}^J, \quad K_{10,12}^J = -N_1 I_{12}^J, \\
K_{10,13}^J &= -N_1 N_2 I_{13}^J, \quad K_{10,14}^J = -N_1 I_{14}^J, \quad K_{10,15}^J = -I_{15}^J, \quad K_{10,16}^J = -N_2 I_{16}^J; \\
K_{11,3}^J &= -I_2^J, \quad K_{11,7}^J = I_8^J, \quad K_{11,11}^J = -N_2 I_{10}^J, \quad K_{11,15}^J = N_2 I_{16}^J, \quad K_{11,17}^J = I_{11}^J, \quad K_{11,18}^J = N_1 I_{12}^J, \\
K_{11,19}^J &= N_1 N_2 I_{13}^J, \quad K_{11,20}^J = N_1 I_{14}^J, \quad K_{11,21}^J = I_{15}^J; \\
K_{12,4}^J &= -I_2^J, \quad K_{12,6}^J = -I_8^J, \quad K_{12,12}^J = -N_2 I_{10}^J, \quad K_{12,14}^J = -N_2 I_{16}^J, \quad K_{12,18}^J = I_{11}^J, \quad K_{12,20}^J = -I_{15}^J, \\
K_{12,22}^J &= -N_1 I_{12}^J, \quad K_{12,23}^J = -N_1 N_2 I_{13}^J, \quad K_{12,24}^J = -N_1 I_{14}^J; \\
K_{13,5}^J &= I_8^J - I_2^J, \quad K_{13,13}^J = N_2 (I_{16}^J - I_{10}^J), \quad K_{13,19}^J = I_{11}^J + I_{15}^J, \quad K_{13,23}^J = N_1 (I_{14}^J - I_{12}^J), \\
K_{13,25}^J &= N_1 N_2 I_{13}^J; \\
K_{14,4}^J &= -I_8^J, \quad K_{14,6}^J = -I_2^J, \quad K_{14,12}^J = -N_2 I_{16}^J, \quad K_{14,14}^J = -N_2 I_{10}^J, \quad K_{14,18}^J = -I_{15}^J, \quad K_{14,20}^J = I_{11}^J, \\
K_{14,22}^J &= -N_1 I_{14}^J, \quad K_{14,23}^J = N_1 N_2 I_{13}^J, \quad K_{14,24}^J = -N_1 I_{12}^J; \\
K_{15,3}^J &= I_8^J, \quad K_{15,7}^J = -I_2^J, \quad K_{15,11}^J = N_2 I_{16}^J, \quad K_{15,15}^J = -N_2 I_{10}^J, \quad K_{15,17}^J = I_{15}^J, \quad K_{15,18}^J = -N_1 I_{14}^J, \\
K_{15,19}^J &= N_1 N_2 I_{13}^J, \quad K_{15,20}^J = -N_1 I_{12}^J, \quad K_{15,21}^J = I_{11}^J; \\
K_{16,2}^J &= -I_8^J, \quad K_{16,8}^J = -I_2^J, \quad K_{16,10}^J = -N_2 I_{16}^J, \quad K_{16,11}^J = I_{15}^J, \quad K_{16,12}^J = -N_1 I_{14}^J, \quad K_{16,13}^J = N_1 N_2 I_{13}^J, \\
K_{16,14}^J &= -N_1 I_{12}^J, \quad K_{16,15}^J = I_{11}^J, \quad K_{16,16}^J = -N_2 I_{10}^J; \\
K_{17,3}^J &= I_3^J, \quad K_{17,7}^J = I_7^J, \quad K_{17,11}^J = N_2 I_{11}^J, \quad K_{17,15}^J = N_2 I_{15}^J, \quad K_{17,17}^J = I_{17}^J, \quad K_{17,18}^J = N_1 I_{18}^J, \\
K_{17,19}^J &= N_1 N_2 I_{19}^J, \quad K_{17,20}^J = N_1 I_{20}^J, \quad K_{17,21}^J = I_{21}^J; \\
K_{18,4}^J &= I_3^J, \quad K_{18,6}^J = -I_7^J, \quad K_{18,12}^J = N_2 I_{11}^J, \quad K_{18,14}^J = -N_2 I_{15}^J, \quad K_{18,18}^J = I_{17}^J, \quad K_{18,20}^J = -I_{21}^J, \\
K_{18,22}^J &= -N_1 I_{18}^J, \quad K_{18,23}^J = -N_1 N_2 I_{19}^J, \quad K_{18,24}^J = -N_1 I_{20}^J; \\
K_{19,5}^J &= I_3^J + I_7^J, \quad K_{19,13}^J = N_2 (I_{11}^J + I_{15}^J), \quad K_{19,19}^J = I_{17}^J + I_{21}^J, \quad K_{19,23}^J = N_1 (I_{20}^J - I_{18}^J), \\
K_{19,25}^J &= N_1 N_2 I_{19}^J; \\
K_{20,4}^J &= -I_7^J, \quad K_{20,6}^J = I_3^J, \quad K_{20,12}^J = -N_2 I_{15}^J, \quad K_{20,14}^J = N_2 I_{11}^J, \quad K_{20,18}^J = -I_{21}^J, \quad K_{20,20}^J = I_{17}^J.
\end{align*}
\]
In these formulas

\[ K_{20,22}^J = -N_1 I_{20}^J, \quad K_{20,23}^J = N_1 N_2 I_{19}^J, \quad K_{20,24}^J = -N_1 I_{18}^J; \]
\[ K_{21,3}^J = I_7^J, \quad K_{21,7}^J = I_3^J, \quad K_{21,11}^J = N_2 I_{15}^J, \quad K_{21,15}^J = N_2 I_{11}^J, \quad K_{21,17}^J = I_{21}^J, \quad K_{21,18}^J = -N_1 I_{10}^J; \]
\[ K_{21,19}^J = N_1 N_2 I_{19}^J, \quad K_{21,20}^J = -N_1 I_{18}^J, \quad K_{21,21}^J = I_{17}^J; \]
\[ K_{22,4}^J = -I_4^J, \quad K_{22,6}^J = -I_6^J, \quad K_{22,12}^J = -N_2 I_{12}^J, \quad K_{22,14}^J = -N_2 I_{14}^J, \quad K_{22,18}^J = -I_{18}^J, \quad K_{22,20}^J = -I_{20}^J; \]
\[ K_{22,22}^J = -N_1 I_{22}^J, \quad K_{22,23}^J = -N_1 N_2 I_{23}^J, \quad K_{22,24}^J = -N_1 I_{24}^J; \]
\[ K_{23,5}^J = I_6^J - I_4^J, \quad K_{23,13}^J = N_2 (I_{14}^J - I_{12}^J), \quad K_{23,19}^J = I_{20}^J - I_{18}^J, \quad K_{23,23}^J = N_1 (I_{24}^J - I_{22}^J), \]
\[ K_{23,25}^J = N_1 N_2 I_{23}^J; \]
\[ K_{24,4}^J = -I_6^J, \quad K_{24,6}^J = -I_4^J, \quad K_{24,12}^J = -N_2 I_{14}^J, \quad K_{24,14}^J = -N_2 I_{12}^J, \quad K_{24,18}^J = -I_{20}^J, \quad K_{24,20}^J = -I_{18}^J; \]
\[ K_{24,22}^J = -N_1 I_{24}^J, \quad K_{24,23}^J = N_1 N_2 I_{23}^J, \quad K_{24,24}^J = -N_1 I_{22}^J; \]
\[ K_{25,5}^J = 2 I_5^J, \quad K_{25,13}^J = 2 N_2 I_{13}^J, \quad K_{25,19}^J = 2 I_{19}^J, \quad K_{25,23}^J = 2 N_1 I_{23}^J, \quad K_{25,25}^J = N_1 N_2 I_{25}^J. \]

In these formulas \( N_1 = \frac{|k'|^2 - k_0'^2}{m^2} \) and \( N_2 = \frac{|q'|^2 - q_0'^2}{m^2} \) are the normalization coefficients for the vector particles whose 4-momenta are \( k' = (k'_0, k') \) and \( q' = (q'_0, q') \).

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