The Effect of the Nonlinearities on Gaussian Pulses Propagation in Photonic Crystal Fiber

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Abstract: Nonlinear effects are attributed to the dependence of the susceptibility on the electric field, which becomes important at high field strengths, in optical fibers impose different limitations on the communications link, and an understanding of such effects is almost a prerequisite for actual light wave system designers. On the other hand, they offer a variety of possibilities for all-optical signal processing, amplification and regeneration, in the nonlinear regime, is introduced and shows the influence and consequences of the nonlinear effects of the propagation Gaussian pulse in photonic crystal fibers. In this paper, one reviews the effects - both detrimental and potentially beneficial - of optical nonlinearities in photonic crystal fibers.

Keywords: Kerr effect, Raman scattering, Brillouin scattering, Nonlinear Schrödinger equation.

1. Introduction:

Photonic crystal fibers (PCFs) are fibers with an internal periodic structure made of capillaries, filled with air, laid to form a hexagonal lattice. Light can propagate along the fiber in defects of its crystal structure. A defect is realized by removing one or more central capillaries. PCFs are a new class of optical fibers. Combining properties of optical fibers and photonic crystals they possess a series of unique properties impossible to achieve in classical fibers [1]. The interaction of optical signal with different medium leads to nonlinear optical effects [2]. Nonlinearities of fibers mode of photonic crystals are divided into two classes first class due to refractive index modulation of silica by changes of intensity in signal or Kerr effect. Such effects lead to self-phase modulation (SPM), cross-phase modulation (XPM), and four wave modulation (FWM). The second class is nonlinearities because of stimulated scattering processes, Stimulated Raman Scattering (SRS), Stimulated Brillouin Scattering (SBS) [3]. Linearity’s because both classes responded to the materials from dynamic or
static, are modified by the large optical fields; such response is usually written as follows [4]:

\[ P = X^{(1)}e + X^{2}ee + X^{3}eee + \ldots \]  

\( P \) is the medium polarization, \( X^{(n)} \) is the nth order susceptibility represented and \( e \) is the electric field.

Where \( X^{(n)}( \ n = 1,2,3,4,\ldots ) \), are higher order susceptibilities at optical frequencies. \( X^{(2)} \) equals to zero in medium with optical isotropy. nonlinearities Various types can be expressed in terms of imaginary and real parts of \( X^{(n)} \). Real part related to refraction index while imaginary part associated with gain [5-7].

2. Effects of Nonlinear in Photonic Crystal Fibers:

2.1. The Kerr effect:

It is usually to attribute nonlinear effects due to the susceptibility dependence on electric field when the later has high strength. As a result, vector of the total polarization \( (P) \) can be written in the frequency domain as expansion of a power series in vector of the electric field [8]:

\[ P(R, \omega) = \varepsilon_{0} \left[ X^{(1)}e + X^{(2)}ee + X^{(3)}eee + \ldots \right] = P_{L}(R, \omega) + P_{NL}(R, \omega) \]  

\( X^{(j)} \), \( j = 1,2,3,\ldots \), are tensors of ranks \((j+1)\). the polarization linear part \((P_{L})\) are determined by this linear susceptibility while second and higher orders determined the nonlinear polarization, \( P_{NL} \). \( X^{(3)} \) implies that refractive index relies on the high intensity\((I)\) where [9]:

\[ n = n_{0} + n_{2}I \]  

\( n_{0} \) is index the linear refractive \((n_{0}=(1 + X^{(1)})^{1/2})\) and \( n_{2} \) is index of the nonlinear refractive \((n_{2} = \frac{3X^{(3)}}{4\varepsilon_{0}c}n_{0}^{2})\) [9]. \( n_{2} \) is known as the Kerr coefficient.

There are main effects due to Kerr nonlinearity are:

(i) Self-phase modulation (SPM):

This signal phase traversing down the fiber which varies with distance as given in this following expression [10]:

\[ \varphi = n_{0}k_{0}z + \gamma P(t)z \]
Where $\varphi$ is the phase, $k$, $z$ are represents the vacuum wave number and the fiber length respectively, $\gamma = n_2 2\pi/\lambda A_{eff}$, $A_{eff}$ is the effective area of guided mode, $\lambda$ is the wavelength for user light, $P(t)$ is the pulse input power [11].

(ii) Cross-phase modulation:

Evolution of the nonlinear phase of a signal at a frequency $\omega_i (i = 1, 2, 3)$, as a result of two or more signals with frequencies of different carrier transmitted inside a photonic crystal, depends on the power of the other signals [10]. The signal’s nonlinear phase shifts at $\omega_i$ becomes [12]:

$$\phi_{i}^{NL} = \gamma z [P_i + 2 \sum_{i \neq j} P_j] \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 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acoustic phonons participate [25]. Fig (1) is Schematic representations of both scattering phenomena.

![Schematic representations of both scattering phenomena.](image)

Figure (1): Stimulated Raman and Brillouin scattering [26].

### 3. Mathematical simulations:

Pulse propagation in photonic crystal fiber governed by Nonlinear Schrodinger Equation [27]:

\[
\frac{\partial A}{\partial z} = \frac{\alpha}{2} A + i (\beta(\omega) - \beta_0) A + i \gamma |A|^2 A \quad \ldots \ldots \ldots (7),
\]

where \(A\) represents the optical field envelope, \(\alpha\) is the attenuation constant, \(\beta\) propagation constant at any frequency \(\omega\), and \(\beta_0\) is the constant of propagation about frequency of the center, and \(\gamma\) is the nonlinearity coefficient [28]:

\[
\gamma(\omega) = \frac{\omega n_2}{c} \left( \frac{\iint |E|^2 dx dy}{\iint |E|^4 dx dy} \right)^2 \quad \ldots \ldots \ldots \ldots (8)
\]

Where \(E\) is the electric field, In the derivation of the nonlinear Schrodinger equation (NLSE), it is assumed that the materials nonlinear response of optical field incident occurs instantaneously. In reality ultra-short pulses usually causes delay in the nonlinear responses in the material as a result of Raman effect. NLSE can be written as:

\[
\frac{\partial A}{\partial z} = - \frac{\alpha}{2} A + i (\beta(\omega) - \beta_0) A + i \gamma \mathcal{F} \left( A \int_0^\infty R(t') \left| A(z, t - t') \right|^2 dt' \right) \ldots \ldots (9)
\]

Equation (9) represents the Generalized Nonlinear Schrodinger Equation (GNLSE) in the frequency domain [28]. \(R(t)\) is the nonlinear response function and \(\mathcal{F}\) denotes the Fourier transform. \(R(t)\) can be written as:

\[
R(t) = (1 - f_R)(\delta(t)) + f_R h_R (\tau_1^2 + \tau_2^2) \tau_1 e^{-t/\tau_2} \sin \left( \frac{t}{\tau_1} \right) \quad \ldots \ldots (10)
\]
\( f_R \) represents the fractional contribution of delayed response to the Nonlinear Polarization. The first term on the right hand side of equation (10) represents the instantaneous electronic response while the second term is related to the delayed Raman response. For silica, the following formula agrees with experimental observations:

\[ h_R = (\tau_1^{-2} + \tau_2^{-2})\tau_1 e^{-t/\tau_2} \sin\left(\frac{t}{\tau_1}\right) \] .......................................................... (11)

Where the Raman response function, \( h_R \), contains information on the vibration of molecules of silica as light passes through fiber.

Although analytical value of \( h_R \) is available, the experimentally calculated values are [28]:

\[ f_R = 0.18, \tau_1 = 12.2 fs, \tau_2 = 32 fs. \]

4. Result and Discussion:

The first present numerical result obtained by using the parameter of the PCF is neglect \( \alpha \) and have a balance between the effects from \( \beta_2 \) and the come from \( \gamma \), where \( \alpha \) is the attenuation, \( \beta_2 \) is the dispersion in the group velocity, and \( \gamma \) is the nonlinear effects. Assuming no attenuation, no dispersion, and no nonlinear effects the Gaussian pulse traverse the length of the fiber with no effect, no change of the pulse with time occurs as shown in figure (2) below which explained the pulses intensity with the distance and time, wherein the figure (3) represent three dimension sketch to explain the propagation pulse shape within the PCF, figure (4) represent the pulse intensity with whoever the distance and time, one observed the packet shape continue without change through the propagation inner the fiber with the distance and time. This study is based on solving nonlinear Schrodinger equation using Split-Step Fourier Method (SSFM) [2] and Matlab system.
Figure (2): The propagated Gaussian pulse if we neglect $\alpha$ and have a balance between the effects come from $\beta_2$ and the come from $\gamma$.

When the Gaussian pulse entering to the PCF as if this material is not suffering from attenuation effect or the attenuation effects ($\alpha = 0.0003\,\text{dB/km}$) [29] and the group velocity dispersion (GVD) for this material is ($\beta_2 = 0.0006\,\text{ps}^2/\text{km}$) [30], by taken a different value of nonlinear factor ($\gamma$) which are $\gamma = 0.0002\,\text{W}^{-1}\text{km}^{-1}$, $\gamma = 0.002\,\text{W}^{-1}\text{km}^{-1}$, and $\gamma = 0.02\,\text{W}^{-1}\text{km}^{-1}$ [31]. As shown in figure (3).
Figure (3): show propagation pulse through the PCF, and picture for pulse intensity change with distance and time when:

\[ \alpha = 0.0003 \frac{dB}{km}, \beta_2 = 0.0006 \frac{ps^2}{km}, \text{ where } (a) \gamma = 0.0002 W^{-1} km^{-1}, \]
\[ (b) \gamma = 0.002 W^{-1} km^{-1}, (c) \gamma = 0.02 W^{-1} km^{-1} \]

To explain the behavior the pulse with time inter the PCF orphan draw the behavior time for amplitude pulse in two and three value along the PCF, a while inter the pulse and in the half distance and at finale the PCF (L) as show in figure (4). And at same the value in figure (3) noticed the amplitude pulse decreased with increased the distance inter the PCF over and above expansion of width.

Figure (4): Shows Change the intensity of Gaussian pulse through of PCF with the time, when \( (\beta_2 = 0.0002 \frac{ps^2}{km}, \alpha = 0.00002 \frac{dB}{km}) \) where (a) \( \gamma = 0.00002 \ W^{-1} km^{-1} \),
(b)\( \gamma = 0.0002 W^{-1} km^{-1} \), (c)\( \gamma = 0.002 W^{-1} km^{-1} \).
5. Conclusion:

The SPM, XPM and FWM effects that have been discussed, have an important role in PCF communication systems. The transmitted signal quality is determined by the nonlinear effects of PCF. Various values of nonlinear effects ($\gamma$) were taken in PCF which have been processed, the effects of ($\gamma$) change on pulse was also described with the distance and time. The pulse also disappears by increasing ($\gamma$) values. The first study which explained the change of pulse intensity with the distance and time, where the second explained the behavior of time for the pulse, explicates the pulse capacity decreased with increase the distance through the PCF over and above the width is expands of the pulse.

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