Majid, Shahn; Tomašić, Ivan

On braided zeta functions. (English) Zbl 1258.14030

Bull. Math. Sci. 1, No. 2, 379-396 (2011).

The general setting of the paper under review is the analysis of braided zeta functions in $q$-deformed geometry. In this framework, the authors define a zeta function for any rigid object in a ribbon braided category. In the ribbon case, they define braided Hilbert series for objects in an abelian braided category.

The organization of the interesting paper is as follows: 1. Introduction. 2. Classical $\zeta$-functions for finite sets. 3. Braided dimension. 4. Braided zeta function of $\mathbb{C}^n$. 5. Braided Hilbert series.

We recall that Dedekind has defined a zeta function for polynomials over prime finite fields. This zeta function is trivial and equal to $1 - \frac{1}{1 - p^z}$. However, combining the zeta function with the Chebyshev-Möbius inversion formula we obtain the number of monic irreducible polynomials over $\mathbb{F}_p$ of natural degree $m$.

Riemann and Dedekind zeta functions are first examples of motivic zeta functions.

Following the motivic approach by J. Krajiček and T. Scanlon [Bull. Symb. Log. 6, No. 3, 311–330 (2000; Zbl 0968.03036)], one can attach to schemes over a field $k$ motivic zeta functions. In the introduction, the author of the paper under review consider how the motivic version might be generalized for noncommutative geometry. Section 3 is devoted to the study of the categorical rank or ‘braided dimension’ of an object in a braided category and the authors’ variant of the multiplicative categorical rank in the case of a ribbon braided category.

In Section 4, the authors establish the main result of the paper, formulas for zeta in the ribbon braided category of finite-dimensional $U_q(sl_n)$-modules ($q$ is generic) and compute $\zeta_t(\mathbb{C}^n)$ in this case. They show that this coincides with $\zeta_t(\mathbb{C}^n)$ where $\mathbb{C}^n$ is the $n$-dimensional representation in the category of $U_q(sl_2)$-modules and that this equality of two braided zeta functions is equivalent to the classical Cayley-Sylvester formula for the decomposition into irreducible of the symmetric tensor products $S^j(V)$ for $V$ an irreducible representation of $sl_2$. The authors obtain functional equations for the associated generating function. The case of $\zeta_t(C_q[S^2])$ is also discussed.

In Section 5, the interpretation of the braided $\zeta$-function in the $q$-deformed case for generic $q$ as a braided Hilbert series is considered.

The technique of the paper under review is category-theoretic, combinatorial and representation-theoretic, involving (among others) results of V. Drinfeld [Quantum Groups, Proc. ICM-1986, 798–820 (1987)], S. Majid and Y. Soibelman [Commun. Math. Phys. 137, No. 2, 249–262 (1991; Zbl 0726.17011)], and from the book [S. Majid, Foundations of quantum group theory. Cambridge: Cambridge Univ. Press. (2000; Zbl 0857.17009)].

Reviewer: Nikolaj M. Glazunov (Kyiv)

MSC:

14G10 Zeta functions and related questions in algebraic geometry (e.g., Birch-Swinnerton-Dyer conjecture)
58B32 Geometry of quantum groups
81R50 Quantum groups and related algebraic methods applied to problems in quantum theory
18D10 Monoidal, symmetric monoidal and braided categories (MSC2010)

Keywords:

$q$-deformed geometry; quantum group; motivic zeta function; finite field; braided category; ribbon category

Full Text: DOI arXiv

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