Flux–branes and the Dielectric Effect
in String Theory

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Abstract

We consider the generalization to String and M-theory of the Melvin solution. These are flux p–branes which have (p + 1)–dimensional Poincaré invariance and are associated to an electric (p + 1)–form field strength along their worldvolume. When a stack of Dp–branes is placed along the worldvolume of a flux (p + 3)–brane it will expand to a spherical D(p + 2)–brane due to the dielectric effect. This provides a new setup to consider the gauge theory/gravity duality. Compactifying M–theory on a circle we find the exact gravity solution of the type IIA theory describing the dielectric expansion of N D4–branes into a spherical bound state of D4–D6–branes, due to the presence of a flux 7–brane. In the decoupling limit, the deformation of the dual field theory associated with the presence of the flux brane is irrelevant in the UV. We calculate the gravitational radius and energy of the dielectric brane which give, respectively, a prediction for the VEV of scalars and vacuum energy of the dual field theory. Consideration of a spherical D6–brane probe with n units of D4–brane charge in the dielectric brane geometry suggests that the dual theory arises as the Scherk–Schwarz reduction of the M5–branes (2, 0) conformal field theory. The probe potential has one minimum placed at the locus of the bulk dielectric brane and another associated to an inner dielectric brane shell.

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1 Introduction

The realization that $p$–branes play a fundamental role in the understanding of String and M-theory played a central role in the developments of the past years. In particular, these $p$–branes admit a gravitational description in the low energy supergravity limit of String and M-theory \[1\]. They can carry either the magnetic or electric charges associated with the form gauge potentials of these theories. With the discovery of D–branes in perturbative string theory \[2\] it was possible to use string theory to study black holes \[3\], which later led to the famous AdS/CFT duality \[4, 5, 6\].

Another geometry that appears in the Einstein-Maxwell theory is the so called Melvin Universe \[7\] (see also \[8 – 11\]). This geometry represents a magnetic flux tube in four dimensions. In the cases that the gauge field arises from a Kaluza–Klein compactification the higher dimensional space for the magnetic Melvin solution is flat with some non-trivial identifications \[12, 13, 14\]. This fact led to a construction of the type IIA flux–brane from the compactification of M-theory on a circle \[13, 14\] (see also \[17 – 21\] for further work on the Melvin solution in String theory). This geometry has 8-dimensional Poincaré invariance and therefore is a flux 7–brane (the Melvin fluxtube is a flux string in 4D). It has an additional $SO(2)$ invariance associated to the spherical symmetry in the transverse plane to the brane. The natural question that arises is to generalize this solution to the case of forms with different rank in String and M-theory. This is one of the purposes of this paper.

In analogy with the type IIA flux 7–brane, the flux $p$–branes in $D$–dimensional space–time are associated to a flux of a $(D–p–1)$–form field strength along the transverse space to the brane. If one considers electric variables then they are associated to an electric $(p+1)$–form field strength along the brane worldvolume. This raises the issue of stability of the flux $p$–branes. In fact, one expects to have Schwinger production of spherical $(p–1)$–branes as shown in \[14\]. As such, to find a string theory dual of the flux $p$–branes in the same spirit of the AdS/CFT duality becomes problematic. However, we can try to stabilize the flux $p$–branes. Consider a Ramond–Ramond flux $(p+3)$–brane and place on its worldvolume a stack of $Dp$–branes. Due to the coupling of the $Dp$–branes to the electric $(p+4)$–form field strength the brane will expand to a dielectric 2–sphere forming a D$(p+2)$–brane. In other words, the dielectric effect is at work \[22\]. We shall argue that, in the decoupling limit, the presence of the $Dp$–branes stabilizes the flux $(p+3)$–brane.

To be more precise let us briefly describe the work of Myers on dielectric branes \[22\]. Following general principles such as gauge invariance and T–duality invariance, he found new couplings to the bulk fields in the non–abelian $Dp$–brane action both in the Born–
Infeld and in the Chern–Simons pieces of the action. These new couplings would not be taken into account by just replacing abelian by non–abelian gauge fields in a Dp–brane worldvolume action and by taking a gauge trace. In particular, this work led to a new proposal for the Chern–Simons term of the Dp–brane action bosonic sector with the form

\[
S_{CS} = \mu_p \int \text{Tr} \left( P \left[ e^{2\pi \alpha' i \Phi} \left( \sum A_n e^{-B} \right) \right] e^{2\pi \alpha' F} \right).
\]  

(1.1)

\[P[...]\] denotes the pull–back of the background Kalb–Ramond 2–form \( B \), and Ramond–Ramond \( n \)–form potentials \( A_n \). The novelty of this proposal is the exponential containing the operator \( i \Phi \), denoting interior multiplication by the transverse space vector \( \Phi \) associated to displacements of the branes. Since this reduces the degree of a differential form, it allows for Wess–Zumino couplings of a Dp–brane to Ramond–Ramond forms of degree greater than \( p + 1 \).

A physical effect arising from this new coupling can be seen by considering a collection of \( N \) D0–branes in a constant background electric field described by a Ramond–Ramond 4–form field strength. Then the scalar potential for this collection of branes is

\[
V(\Phi) = - (2\pi \alpha')^2 T_0 \left( \frac{1}{4} \text{Tr} \left( [\Phi^i, \Phi^j]^2 \right) + \frac{i}{3} \text{Tr} \left( \Phi^i \Phi^j \Phi^k \right) F_{tijk} \right),
\]

(1.2)

where \( T_0 \) is the D0–brane mass and \( F_{tijk} = \tilde{E} \epsilon_{ijk} \) along three transverse directions. An analysis of the potential extrema shows that the ground state is described by

\[
\Phi^i = \frac{E}{4 \alpha^i},
\]

(1.3)

where the \( \alpha^i \) belong to the \( N \times N \) irreducible representation of the \( SU(2) \) algebra, with the value for the potential at large \( N \) reading

\[
V_N = - \frac{\pi^2 \alpha'^3/2}{96g} E^4 N^3.
\]

(1.4)

This non–commutative configuration represents a single, somewhat granular, spherical D2–brane with \( N \) D0–branes bound to it. For large \( N \) this sphere has a ‘radius’

\[
s = \frac{\pi}{2} \alpha^i |E| N.
\]

(1.5)

Of course, by T–duality a Dp–brane immersed in a \( (p+4) \) Ramond–Ramond electric field will expand to a D\((p+2)\)–brane with worldvolume geometry \( \mathbb{R}^{p+1} \times S^2 \). The action of the background electric field is to create a dipole moment (and higher multipole moments) with respect to D\((p+2)\)–brane charge. In close analogy with classical electrodynamics Myers
dubbed these branes as Dielectric Branes. Now we see that the source for the external electric field can be taken to be a flux brane.

In a beautiful paper [23], Polchinski and Strassler realized that each of the vacua of the $\mathcal{N} = 1^*$ Super Yang–Mills theory, obtained by adding finite mass terms to the $\mathcal{N} = 4$ theory, is dual to the gravity background associated to a dielectric brane source (further work can be found in [24 – 27]). In this general class of theories one can break all the supersymmetry, for example, by adding a mass term to the gluino. However, in constructing the gravitational dual, they considered a spherical distribution of D3–branes where they placed a D5–brane source, and then solved the gravity equations to first order in the mass perturbation. A natural question that arises is to fully describe the gravitational background for the dielectric branes. We shall address this problem in this paper. By placing a D$p$–brane in a Ramond–Ramond flux $(p + 3)$–brane one expects to generate a dielectric brane. The presence of the D$p$–brane charge will stabilize the system. In fact, after taking the decoupling limit this configuration is expected to be stable since the dual field theory is on the general class of the theories analyzed by Polchinski and Strassler. In particular, we expect tachyons to be absent.

To construct the gravitational background for a dielectric brane, the key idea is to consider the Ramond–Ramond magnetic flux 7-brane that arises from the reduction of M-theory flat space–time with some non–trivial identifications [15, 16]. We know that the double dimensional reduction of the M–theory 5–brane gives the type IIA D4–brane [28]. Hence, it is natural to suspect that implementing the Kaluza–Klein Melvin twisted reduction to the M5–brane will give a D4–brane in a magnetic flux 7-brane, or equivalently, a D4–brane placed in an electric field. Then the coupling of the D4–brane to the dual RR 7–form potential will give rise to the Myers effect. Thus, after the reduction we expect to describe within gravity a D4–brane expanded into a 2–sphere, i.e. a D6–brane.

This paper is organized as follows. In section 2 we make the ansatz for the flux $p$–branes. This geometry has the desired $(p + 1)$–dimensional Poincaré invariance together with $SO(D – p – 1)$ spherical symmetry. The associated $(p + 1)$–form field strength has an electric component along the brane worldvolume. The corresponding system of differential equations does not decouple in terms of non-interacting Liouville systems as it is usually the case for black holes. We investigate the asymptotics of the flux branes geometry.

In section 3 we construct the gravity solution for a D4-brane expanded into a 2–sphere due to the dielectric effect. Firstly we consider the twisted dimensional reduction of the non–extremal M5–brane with a double analytic continuation. The ten–dimensional configuration is interpreted as a D4–brane immersed in a flux 7–brane with maximum magnetic field parameter. Although this value of the magnetic field is unphysically large, we chose,
for the sake of clarity, to consider this case first since the corresponding geometry has all the correct features to be interpreted as a dielectric brane. Then we consider the general case of arbitrary magnetic field which amounts to compactify a rotating M5–brane after a similar double analytic continuation.

Section 4 is devoted to the study of the dielectric brane geometry in the decoupling limit. The resulting geometry has the same asymptotics as the D4-brane geometry in the decoupling limit, which shows that the deformation of the dual theory associated to the coupling of the D4–brane worldvolume theory to the flux 7–brane is irrelevant in the UV. Using the gravitational description we calculate the scalars VEV and vacuum energy of the field theory where the deformation becomes relevant.

In section 5 we probe the dielectric brane geometry using a spherical D6–brane probe with \( n \) units of D4–brane charge. First we consider the probe in the flux 7–brane background and obtain similar results to those of Myers [22], but now taking into account the backreaction of the background electric field on the geometry. This simple case is very useful to understand the stability of the dielectric brane geometry before taking the decoupling limit. We proceed with the study of the probe in the dielectric brane geometry in the decoupling limit. It is seen that either far away or inside the dielectric brane the probe potential has the expected form to be associated with the Scherk–Schwarz [29] reduction of the M5–branes (2, 0) low energy conformal field theory. This potential has a minimum at the locus of the bulk dielectric brane and another minimum in its interior that is associated to an inner dielectric brane shell.

We give our conclusions in section 6.

### 2 Flux–branes

We shall consider the following general action in \( D \)-dimensional space–time

\[
S = \frac{1}{2\kappa^2} \int d^Dx \sqrt{-g} \left[ R - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2d!} e^{\alpha\phi} F^2 \right],
\]

where \( \kappa \) is the gravitational coupling and \( F \) is a generic \( d \)-form field strength. The above action can be regarded as a consistent truncation of either String or M-theory low energy actions, where \( F \) represents any of the field strengths or electromagnetic dual in these theories. Then the equations of motion read

\[
\Box \phi = \frac{a}{2d!} e^{\alpha\phi} F^2 , \quad d \left( e^{\alpha\phi} \star F \right) = 0 , \quad R_{ab} = \tau_{ab} ,
\]
where the tensor $\tau = \tau^\phi + \tau^F$ takes the form

$$
\begin{align*}
\tau^\phi_{ab} &= \frac{1}{2} \partial_a \phi \partial_b \phi , \\
\tau^F_{ab} &= \frac{e^{a\phi}}{2} \left[ \frac{1}{(d-1)!} \mathcal{F}_{a_{c_1} \cdots c_{d-1}} \mathcal{F}^{c_1 \cdots c_{d-1}}_b - \frac{d-1}{(D-2)d!} g_{ab} \mathcal{F}^2 \right].
\end{align*}
\tag{2.3}
$$

In order to describe a flux–brane with $ISO(1,d-1) \times SO(D-d)$ invariance, we shall make the following ansatz

$$
\begin{align*}
ds^2 &= e^{2A(r)} ds^2 \left( \mathbb{M}^d \right) + e^{2C(r)} dr^2 + e^{2B(r)} E^{-2} d\Omega_{\tilde{d}}^2 , \\
\mathcal{F} &= E \epsilon \left( \mathbb{M}^d \right) , \quad \phi = \phi(r) ,
\end{align*}
\tag{2.4}
$$

where $\epsilon \left( \mathbb{M}^d \right)$ is the volume form of $d$–dimensional Minkowski space $\mathbb{M}^d$ and $d\Omega_{\tilde{d}}$ is the metric element on the unit $\tilde{d} = (D - d - 1)$–sphere. We have conveniently multiplied the line element of this sphere by $E^{-1}$ for dimensional reasons. Let us note that the form $\mathcal{F}$ is closed since $E$ is constant and also automatically solves the corresponding equation of motion. Notice that we refer to this electric field as constant due to the independence on the radial coordinate (however it is not covariantly constant). Alternatively, one could consider the electromagnetic dual of $\mathcal{F}$ which reads

$$
\tilde{\mathcal{F}} = E^{1-\tilde{d}} e^{-dA+C+\tilde{d}B} dr \wedge \epsilon \left( S^{\tilde{d}} \right) ,
\tag{2.5}
$$

where $\tilde{\mathcal{F}}$ is a $(D - d)$–form and $\epsilon \left( S^{\tilde{d}} \right)$ is the volume form on the unit $\tilde{d}$–sphere.

The Ricci tensor for the above geometry is

$$
\begin{align*}
R_{\mu\nu} &= -e^{2A-2C} \left[ A'' + d(A')^2 - A'C' + \tilde{d}A'B' \right] \eta_{\mu\nu} , \\
R_{ij} &= -e^{2B-2C} \left[ B'' + \tilde{d}(B')^2 - B'C' + dA'B' \right] h_{ij} + E^2 \left( \tilde{d} - 1 \right) h_{ij} , \\
R_{rr} &= \tilde{d} \left[ B'C' - B'' - (B')^2 \right] + d \left[ A'C' - A'' - (A')^2 \right] ,
\end{align*}
\tag{2.6}
$$

where $'$ denotes differentiation with respect to the radial coordinate $r$. We are using coordinates $x^\mu$ along the worldvolume directions of the brane $\mathbb{M}^d$ and $\theta^i$ on the $\tilde{d}$–dimensional unit sphere $S^{\tilde{d}}$. The only non–vanishing component of the tensor $\tau^\phi$ is

$$
\tau^\phi_{rr} = \frac{1}{2} (\phi'')^2 ,
\tag{2.7}
$$
and those of $\tau^F$ are

$$
\tau^F_{\mu\nu} = -\frac{E^2}{2} e^{a\phi} \left( \frac{\tilde{d}}{D - 2} \right) e^{-2(d-1)A} \eta_{\mu\nu},
$$

$$
\tau^F_{ij} = \frac{E^2}{2} e^{a\phi} \left( \frac{d - 1}{D - 2} \right) e^{2B-2dA} h_{ij},
$$

$$
\tau^F_{rr} = \frac{E^2}{2} e^{a\phi} \left( \frac{d - 1}{D - 2} \right) e^{2C-2dA}.
$$

(2.8)

In the ansatz (2.4) there is a freedom of reparametrization of the radial coordinate $r$. We conveniently choose the gauge

$$
dA + \tilde{d}B = C,
$$

(2.9)

in which the equations of motion simplify to

$$
\phi'' = -a\frac{E^2}{2} e^{a\phi+2d\tilde{d}},
$$

$$
A'' = \frac{E^2}{2} \left( \frac{\tilde{d}}{D - 2} \right) e^{a\phi+2d\tilde{d}},
$$

$$
B'' = -\frac{E^2}{2} \left( \frac{d - 1}{D - 2} \right) e^{a\phi+2d\tilde{d}} + E^2 \left( \tilde{d} - 1 \right) e^{2dA+2(d-1)B},
$$

(2.10)

together with a zero-energy constraint from the $rr$ component of Einstein equations. This constrained system of differential equations can be derived from the Lagrangian $L = T - V$ where

$$
T = -\frac{1}{2} (\phi')^2 + d(d - 1)(A')^2 + \tilde{d}(\tilde{d} - 1)(B')^2 + 2dA'B',
$$

(2.11)

and

$$
V = -E^2 \tilde{d}(\tilde{d} - 1)e^{2dA+2(d-1)B} - \frac{E^2}{2} e^{2d\tilde{d}+a\phi},
$$

(2.12)

with the zero-energy constraint $T + V = 0$. In the case of black holes one can usually decouple this system in terms of non-interacting Liouville systems related through the zero energy condition (see for example [30]). However, in this case it is not possible to decouple the system, which makes the solution of the problem much harder. For this reason we were not able to find a general analytic solution but will investigate the asymptotics of the flux branes.
To analyze the differential equation (2.10) we define
\[ f = a \phi + 2\tilde{d} B , \]
\[ g = 2d A + 2 (\tilde{d} - 1) B , \]
\[ h = \left( \frac{\tilde{d}}{D - 2} \right) \phi + a A . \]  

(2.13)

These functions satisfy the following system of differential equations
\[ h'' = 0 , \]
\[ f'' = c_1 e^f + c_2 e^g , \]  
\[ g'' = c_3 e^f + c_4 e^g , \]  

(2.14)

where the constant coefficients \( c_i \) have the form
\[ c_1 = -\frac{E^2}{2} \left[ a^2 + 2\tilde{d}(d - 1) \right] \equiv -\lambda^2 , \]
\[ c_2 = 2\tilde{d} (\tilde{d} - 1) E^2 , \]
\[ c_3 = E^2 , \]
\[ c_4 = 2 \left( \tilde{d} - 1 \right)^2 E^2 . \]  

(2.15)

We shall see that, in the cases of interest in String and M–theory, the constant \( c_1 \) simplifies to \( c_1 = -2E^2 \).

2.1 Dilatonic Melvin

For the usual dilatonic Melvin solution \[31\] one has \( \tilde{d} = 1 \) and the transverse sphere is a circle which is flat. As a consequence \( c_2 = c_4 = 0 \) and the equations (2.14) simplify considerably. In this case \( c_3 f'' = c_1 g'' \), and the general solution is given by
\[ e^f = \left( \frac{\alpha_1}{\cosh \left[ \frac{\alpha_1}{\sqrt{2}} (\lambda r + \alpha_2) \right]} \right)^2 , \]
\[ g = \frac{c_3}{c_1} f + \alpha_3 + \alpha_4 Er , \]
\[ h = \alpha_5 + \alpha_6 Er , \]  

(2.16)

where the \( \alpha_i \)'s are dimensionless constants of integration. Notice that not all these constants are independent because of the zero–energy constraint.

The case we consider here is that of the RR flux 7-brane of type IIA strings studied in \[16\]. In this case we have \( a = -3/2 \) and \( d = 8 \). This solution corresponds to choosing the constants of integration in (2.10) such that
\[ e^f = \frac{1}{\cosh^2 (Er)} , \]
\[ e^g = 1 + e^{2Er} , \]
\[ h = 0 . \]  

(2.17)
Then the coordinate transformation $E \rho = \ln (E \rho / 2)$ will bring the metric, field strength and dilaton field to the form

$$ds^2 = \Lambda^{1/8} \left[ ds^2 \left( \mathbb{M}^8 \right) + d\rho^2 \right] + \Lambda^{-7/8} \rho^2 d\varphi^2 ,$$

$$\mathcal{F} = E \epsilon \left( \mathbb{M}^8 \right) , \quad e^{4\phi/3} = \Lambda \equiv 1 + (E \rho)^2 / 4 .$$

(2.18)

The space–time metric is for $\rho E \ll 1$ approximately flat. We can regard this as the boundary conditions for the flux 7-brane which fixes the constants of integration in (2.16). This fact is also true for the other flux branes and was explored in [33].

The general solution (2.16) contains singular geometries for which the Ricci scalar blows up. These are naked singularities. Only for the flux 7–brane solution (2.18) is the geometry non-singular. One should regard the flux–branes as non-supersymmetric vacua where the energy density of the electromagnetic field spreads to infinity. In this sense the flux branes do not represent localized lumps of energy and are not asymptotically Minkowskian as the usual branes. Far away the energy associated with the constant electric field will dominate.

To analyse the geometry (2.18) we should multiply the metric by the conformal factor $e^{\phi/2}$ to change to the string frame. It turns out that this geometry is quite different than the usual 4D Melvin Universe [7]. In the latter the orbits of $\partial / \partial \phi$ have vanishing length at large radial distance. In the former type IIA case the length of the $\partial / \partial \phi$ orbits $l_\phi$ scales in terms of proper radial distance $u$ as $l_\phi \sim u^{1/3}$. This means that as $u \to \infty$ space-time does not close. Also, this means that while in the 4D Melvin we have a quantization condition for the flux of $\ast \mathcal{F}$ through the transverse space [11], this no longer happens for the IIA flux 7–brane (2.18).

2.2 Flux–branes in type II Strings

Now we turn to the flux branes of type II String theory. We shall consider the case of RR flux branes. In this case the coupling $a = (5 - d)/2$, $\hat{d} = 9 - d$ and the coefficient $c_1$ in (2.15) simplifies considerably to $c_1 = -2E^2$. The cases of NSNS flux branes can be obtained by a S-duality transformation on the IIB flux 2– and 6–branes.

For the RR flux branes the metric functions and dilaton are related to the functions $f$, $g$ and $h$ by the expression (2.13) that reads

$$f = \left( \frac{5 - d}{2} \right) \phi + 2(9 - d) B ,$$

$$g = 2d A + 2(8 - d) B ,$$

$$h = \left( \frac{9 - d}{8} \right) \phi + \left( \frac{5 - d}{2} \right) A .$$

(2.19)
The functions $f$ and $g$ satisfy the system of differential equations

\[
\begin{align*}
    f'' &= -2E^2 e^f + 2(9 - d)(8 - d) E^2 e^g , \\
    g'' &= E^2 e^f + 2(8 - d)^2 E^2 e^g .
\end{align*}
\]

A particular solution to this system of equations can be found by setting $e^g = \zeta e^f$. Then from $f'' = g''$ we must have

\[
\zeta = \frac{3}{2(8 - d)} .
\]

Solving for $f$ we find

\[
e^f = \frac{2}{(25 - 3d)(Er)^2} ,
\]

and we also set $h = 0$. A rather tedious calculation gives the functions $A$, $B$, and $C$ that appear in the Einstein metric (2.4)

\[
\begin{align*}
    2A &= \frac{(9 - d)^2}{8(25 - 3d)} \ln \zeta + \frac{9 - d}{8(25 - 3d)} f , \\
    2B &= \frac{(d - 5)^2}{8(25 - 3d)} \ln \zeta + \frac{25 - d}{8(25 - 3d)} f , \\
    2C &= \frac{(9 - d)(25 - d)}{8(25 - 3d)} \ln \zeta + \frac{25(9 - d)}{8(25 - 3d)} f .
\end{align*}
\]

This solution satisfies the zero–energy constraint defined above. We can change to the string frame, and write the metric in coordinates such that the radial coordinate is the proper radial distance. The final result for the metric and dilaton field is

\[
\begin{align*}
    ds^2 &= \beta^{1/5} (Eu)^{2/5} ds^2 (M^d) + du^2 + \frac{\beta}{\zeta} u^2 d\Omega_{9-d}^2 , \\
    e^{\ln \phi} &= \zeta (Eu)^2 .
\end{align*}
\]

where

\[
\beta = \frac{5^2}{2^3(25 - 3d)} .
\]

This metric describes a geometry with a naked singularity at the origin. However, its asymptotics are those of the RR flux $(d - 1)$–brane [32, 33], for which we expect a smooth geometry. Far away the energy density associated with the electric field dominates and determines the asymptotics[. A comparison with flat space–time is helpful; for example,

\[\text{\footnote{Notice that for } d < 8 \text{ the powers of } u \text{ in the metric (2.24) are independent of } d. \text{ On the other hand for } d = 8, \text{ the Melvin case, the powers of } u \text{ are different.}}\]
the Schwarzschild black hole with negative mass has a naked singularity and is unphysical, however, it converges to flat space–time at infinity where the energy density vanishes. Similarly the solution (2.24) has the correct asymptotics for the flux \((d-1)\)–brane. For \(d < 5\), i.e. for the flux \(p\)-branes with \(p < 4\), the dilaton field converges to zero at infinity and we are in the perturbative string theory regime. For \(d > 5\), the string coupling diverges. The case \(d = 5\) gives the non–dilatonic (and self–dual) flux 4–brane of the type IIB theory. This is analogous to what happens with the D3–brane. Finally, notice that while for the flux 7–brane the flux of \(*\mathcal{F}\) along the transverse space is convergent, for the flux branes analyzed here it diverges.

### 2.3 Flux–branes in M-theory

In M-theory there are flux 3– and 6–branes, both non–dilatonic. We have \(D = 11\) and \(a = 0\). The functions \(f\) and \(g\) are related to those appearing in the metric by
\[
\begin{align*}
    f &= 2(10 - d)B , \\
    g &= 2dA + 2(9 - d)B .
\end{align*}
\]

These functions satisfy the system of differential equations
\[
\begin{align*}
    f'' &= -2E^2 e^f + 2(10 - d)(9 - d)E^2 e^g , \\
    g'' &= E^2 e^f + 2(9 - d)^2E^2 e^g .
\end{align*}
\]

To find the asymptotics of the M flux–branes we set \(e^g = \eta e^f\), which gives
\[
\eta = \frac{3}{2(9 - d)} .
\]

In the case of the flux 3–brane \((d = 4)\) the function \(f\) reads
\[
e^f = \frac{1}{8(Er)^2} ,
\]

and the metric functions \(A\), \(B\), and \(C\) become
\[
\begin{align*}
    2A &= \frac{1}{4} \ln \eta + \frac{1}{24}f , \\
    2B &= \frac{1}{6}f , \\
    2C &= \ln \eta + \frac{7}{6}f .
\end{align*}
\]

This gives the following metric written in terms of the proper radial distance coordinate \(u\):
\[
ds^2 = \left(\frac{2}{9}\right)^{1/4} (Eu)^{1/2} ds^2 (\mathbb{M}^4) + du^2 + \frac{20}{27} u^2 d\Omega_6^2 .
\]

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For the flux 6–brane \((d = 7)\) we have

\[
e^{f} = \frac{2}{7(Er)^2},
\]

and

\[
2A = \frac{1}{7} \ln \eta + \frac{1}{21} f, \quad 2B = \frac{1}{3} f, \quad 2C = \ln \eta + \frac{4}{3} f.
\]

The corresponding metric element can be written in the form

\[
ds^2 = \left( \frac{7}{18} \right)^{1/7} (Eu)^{2/7} ds^2(M^7) + du^2 + \frac{14}{27} u^2 d\Omega_3^2.
\]

As for the flux \(p\)–branes of String theory with \(p < 7\), the flux along the transverse space diverges for the M flux–branes.

### 2.4 Stability of the flux–branes

The flux \(p\)–branes described above are not stable. They will decay through the nucleation of spherical \((p-1)\)–branes as described generally in [14]. If we compactify \((p-1)\) directions of the flux \(p\)–branes then they will decay through the usual Schwinger production of \((p-1)\) brane/anti–brane pairs. Similarly to the RR flux 7–brane case [16], one can consider a \((p-1)\)–brane probe at the core of the flux \(p\)–brane and calculate the action for the instanton associated with this decay process. This gives the well known result for the nucleation rate \(\Gamma\)

\[
\Gamma \sim e^{-I}, \quad I = \frac{M_{p-1}}{|E|},
\]

where \(M_{p-1}\) is the mass of a \((p-1)\)–brane. It is expected that this calculation can be reproduced using the Euclidean Quantum gravity approximation for the nucleation of brane pairs. One would need to find the instanton for the nucleation process with the same asymptotics of the flux–branes described above. This calculation could confirm the expected periodicity of the electric (or magnetic) field parameter. As explained in [16], the existence of a maximum electric field for a generic \(p\)–form is expected on the basis of String duality and of the analysis of the RR flux 7–brane case from a M-theory perspective. An interesting physical interpretation for this maximum electric field was given in [33]: since the typical distance for nucleation is of order \(1/E\), for larger values of \(E\) the black hole horizons will touch and the pair production will cease to exist.

A very interesting question is to consider the string theory duals of the flux brane geometries in some decoupling limit. This can be problematic because, as explained above, these geometries are not stable, which makes the duality difficult to establish (see [33] for
a discussion of this point). However, one can try to stabilize the flux branes. As explained in the Introduction this can be done by considering the dielectric effect in String theory [22].

Consider the case of a RR flux \((p+3)\)–brane and place a stack of \(N\) \(Dp\)-branes along its worldvolume. Then the \(Dp\)-branes will couple to the electric RR \((p+4)\)-form field strength expanding into a \(D(p+2)\)-brane with geometry \(\mathbb{M}^{p+1} \times S^2\). Now, the presence of the \(N\) \(Dp\)–branes changes the asymptotics of the geometry. Far away, we have the geometry for the flux–brane together with a charge due to the \(N\) \(Dp\)-branes. We would then need to find an instanton with these asymptotics, representing the instability of space–time. We know from the perturbative String theory description of the dielectric effect that this system is locally stable, and therefore such an instanton represents a quantum tunneling effect. Moreover, we shall argue that, the geometry in the decoupling limit has the same asymptotics as the usual D–branes without external electric field. Therefore, in this limit the instanton instability no longer exists and the configuration is stable. Note that the case of \(p = 3\) is nothing but a D3–brane expanding to a spherical D5–brane due to the dielectric effect. These type of configurations have already made their appearance in the gauge theory/gravity duality of the theories of the type analyzed by Polchinski and Strassler [23], which are stable.

In the following sections we shall treat the case of \(p = 4\), where one can find the exact gravitational description by using the M-theory reduction of the M5–brane to the type IIA theory and hence confirming the aforementioned expectations.

### 3 Dielectric branes

Because of the complexity of the gravitational background presented below it is important to set our conventions for the bosonic sector of the eleven–dimensional supergravity action:

\[
S = \frac{1}{2\kappa_{11}^2} \left\{ \int d^{11}x \sqrt{-g} \left[ R - \frac{1}{2 \cdot 4!} \mathcal{F}^2 \right] + \frac{1}{6} \int \mathcal{F} \wedge \mathcal{F} \wedge \mathcal{A} \right\},
\]

where \(\kappa_{11}\) is the eleven–dimensional gravitational coupling and \(\mathcal{F} = d\mathcal{A}\) with \(\mathcal{A}\) a 3–form field potential. Reduction to the type IIA theory is achieved through the ansatz

\[
ds_{11}^2 = e^{-2\phi/3} ds_{10}^2 + e^{4\phi/3} \left( dx^{11} + \mathcal{A}_a dx^a \right)^2, \quad \mathcal{A} = \mathcal{A}_3 + \mathcal{B} \wedge dx^{11}.
\]

We shall present the construction of the dielectric branes in two steps. First we consider a D4–brane placed in a flux 7–brane with maximum magnetic (or electric) field. This value of the magnetic field is unphysically large but the solution is simpler and retains all the correct features. Then we consider the case of arbitrary magnetic field.
3.1 Maximal magnetic field

Consider the non–extremal M5–brane solution obtained by a double analytic continuation from the usual solution. The metric and 3–form potential read

\[
\begin{align*}
\mathcal{A} &= -r_H^3 \sinh \alpha \cosh \alpha \cos^3 \theta \, d\tilde{\varphi} \wedge \epsilon (S^2) ,
\end{align*}
\]

where \(0 \leq \tilde{\varphi} < 2\pi, 0 \leq \theta \leq \pi/2\). The unusual parameterization of the transverse 4–sphere is standard for rotating black holes [34]. In this coordinate system the transverse space naturally splits into the \(\theta = 0\) three–plane orthogonal to the \(\theta = \pi/2\) two–plane. The functions \(f\) and \(H\) have the form

\[
H = 1 + \left( \frac{R}{r} \right)^3 \equiv 1 + \left( \frac{r_H}{r} \right)^3 \sinh^2 \alpha , \quad f = 1 - \left( \frac{r_H}{r} \right)^3 .
\]

The M5–brane charge quantization gives the condition

\[
r_H^3 \sinh \alpha \cosh \alpha = \pi N l_P^3 ,
\]

where \(l_P\) is the eleven–dimensional Planck length and \(N\) the number of M5–branes. In order to avoid a conical singularity, the Euclidean time direction \(\tau\) has periodicity given by

\[
2\pi R_{11} = \frac{4\pi}{3} r_H \cosh \alpha ,
\]

and it is related to the ten–dimensional type IIA string coupling and tension by \(R_{11} = g\sqrt{\alpha'}\).

If we compactify along the killing vector \(\partial/\partial \tau\) there will be a set of fixed points at \(r = r_H\), spanning a 4–sphere in the transverse space. The reduced space will be singular on such a 4–sphere. One can instead compactify along the killing vector field

\[
q = \frac{\partial}{\partial \tau} + B \frac{\partial}{\partial \tilde{\varphi}} , \quad B = \frac{1}{R_{11}} ,
\]

which corresponds to the maximum value for the magnetic field. Notice that the parameter \(B\) is related to the electric field \(E\) in the two previous sections by \(E = 2B\). The fixed points of this isometry are at \(r = r_H, \theta = 0\) corresponding to a 2–sphere on the transverse space (see figure 1). At each point of this 2–sphere the action of the isometry is the same as for a Kaluža–Klein monopole. Hence, the reduced space will be singular on a 2–sphere that we identify with the D4–branes expanded into a D6–brane. Asymptotically space–time will
look like the flux 7–brane with maximal magnetic field parameter $B = 1/R_{11}$, together with the D4–brane charge. We have chosen this particular value for $B$ because, as will be seen below, any other choice would lead to a conical singularity for the ten–dimensional geometry. For this value of the magnetic field, perturbative string theory will hold only for $r \ll R_{11}$, while the eleventh direction remains unobservable for $r \gg R_{11}$. We shall consider this unphysical case first because it is much simpler and retains all the features of the general case. In the next subsection we shall allow for general and physical values of the magnetic field by considering the rotating M5–brane geometry.

To perform the reduction we change to the azimuthal angle $\varphi = \tilde{\varphi} - B\tau$, then $\tau$ has period $2\pi R_{11}$ and $\varphi$ has the standard period of $2\pi$. A straightforward calculation gives the following type IIA background fields:

$$
\begin{align*}
\quad & ds_{10}^2 = \left( \frac{\Sigma}{H} \right)^{1/2} ds^2 (\mathbb{M}_5^5) + \left( \frac{\Sigma H^2}{f} \right)^{1/2} \left[ \frac{dr^2}{f} + r^2 \left( d\theta^2 + \cos^2 \theta \, d\Omega_2^2 \right) \right] + \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad + \left( \frac{H}{\Sigma} \right)^{1/2} f \, r^2 \sin^2 \theta \, d\varphi^2 , \\
& e^{2\phi} = \Sigma^{3/2} H^{-1/2} , \quad B = -r_H^3 B \sinh \alpha \cosh \alpha \cos^3 \theta \epsilon (S^2) , \\
& A_1 = B \Sigma^{-1} H r^2 \sin^2 \theta \, d\varphi , \quad A_3 = -r_H^3 \sinh \alpha \cosh \alpha \cos^3 \theta \, d\varphi \wedge \epsilon (S^2) .
\end{align*}
$$

(3.8)

The function $\Sigma$ has the form

$$
\Sigma \equiv f + H( Br \sin \theta )^2 ,
$$

(3.9)

and $B = 1/R_{11} = 1/(g\sqrt{\alpha'})$. Notice that $g$ is the asymptotic value of the string coupling along the $\theta = 0$ three–plane.
3.1.1 The ten–dimensional geometry

The above solution describes a geometry with 5–dimensional Poincaré invariance as it is appropriate to describe a D4–brane. In fact, by integrating $F_4$ over the four–sphere we obtain the total D4–brane quanta of charge

$$N = \frac{1}{(2\pi)^3 g_\alpha^{3/2}} \int F_4 = \frac{r_H^3 \sinh \alpha \cosh \alpha}{\pi g_\alpha^{3/2}}.$$  (3.10)

To interpret this geometry consider first its asymptotics. For $r \gg R, r_H$ we obtain the metric, dilaton field and 1–form potential for the flux 7–brane solution

$$ds_{10}^2 = \Lambda^{1/2} \left\{ ds^2 \left( M^5 \right) + dr^2 + r^2 \left( d\theta^2 + \cos^2 \theta d\Omega_2^2 \right) + \frac{r^2 \sin^2 \theta}{\Lambda} d\varphi^2 \right\},$$

$$e^{2\phi} = \Lambda^{3/2}, \quad A_1 = r^2 \sin^2 \theta B \Lambda^{-1} d\varphi,$$  (3.11)

where $\Lambda = 1 + (Br \sin \theta)^2$. A simple calculation shows that the dual of the 2–form RR field strength is

$$F_8 = 2B \epsilon \left( M^5 \right) \wedge \epsilon \left( E^3 \right),$$  (3.12)

where $\epsilon (E^3)$ is the volume form on the $\theta = 0$ three–plane. This supports the interpretation of the solution as a D4–brane immersed in a constant RR 8–form electric field, i.e. on a flux 7–brane.

Coming in from the large $r$ region one first finds a metric singularity at $r = r_H$. But unlike the usual D4–brane solution, the immersion in the magnetic flux brane makes the geometry smoother on this hypersurface. Evaluating the Ricci scalar of (3.8) on the horizon $r = r_H$ yields

$$R = \frac{9 B^2 r_H^2 (5 \sin^2 \theta \sinh^2 \alpha - 2 \cos^2 \theta \cosh^2 \alpha) - 6}{r_H^5 B^3 \sin^3 \theta \cosh^4 \alpha}.$$  (3.13)

Thus, one realizes that the flux brane smooths out the irregular horizon, except in the $\theta = 0$ three–plane. One might therefore think that we can travel through the horizon along the two–plane $\theta = \pi/2$. However, the metric component $g_{\varphi \varphi}$ for the azimuthal angle in this two–plane becomes zero at $r = r_H$. This should therefore be faced as the locus of the origin on this two–plane. Thus, the spacetime is geodesically complete and there is a singular horizon at $\theta = 0, r = r_H$. This horizon spans a $E^4 \times S^2$ hypersurface representing the expansion of the D4–brane into a D6–brane. Furthermore, the locus $r = r_H, 0 < \theta \leq \pi/2$, is a regular three–dimensional surface in the transverse space with the dielectric two–sphere as a boundary. This is represented in figure 2.

\[\text{This solution can be brought to the form of that in section two by the coordinate transformation } v = r \cos \theta \text{ and } \rho = r \sin \theta.\]
Figure 2: Geometry of the hypersurface $M$, defined as the restriction to the transverse space of the intersection of $r = r_H$ with a partial Cauchy surface. For $B = 0$, $M$ is a point in the transverse space and can be thought of as the physical singularity associated to the locus of the D4–branes. As $B$ is turned on, $M$ expands into a regular three-space bounded by a singular $S^2$, which can be thought of as the locus of the D4–branes which have expanded into a D4–D6–bound state. In the next section we will be able to smoothly interpolate between these two configurations.

Before we have a closer look at the geometry near the dielectric brane, let us discuss the issue of a possible conical singularity in the $\theta = \pi/2$ two–plane due to the shift in the symmetry axis introduced in the compactification from M–theory. A simple analysis on this two–plane shows that for non–vanishing $B$ and close to $r = r_H$, $\varphi$ would need to be identified with period

$$\frac{4\pi}{3} B \cosh \alpha r_H = 2\pi B R_{11} .$$

(3.14)

Since $\varphi$ has periodicity $2\pi$ the conical singularity is avoided by setting $B = 1/R_{11}$. This shows why we have chosen such value for the magnetic field parameter earlier.

Now we want to see in more detail that the D4–branes have expanded to a D6–brane with $N$ D4–branes bound to it and with worldvolume geometry $M^5 \times S^2$. First consider the usual solution describing a ‘flat’ bound state of D4–branes within D6–branes. The bulk fields are

$$ds_{10}^2 = F^{-1/2}ds^2(M^5) + F^{1/2} \tilde{F}^{-1}ds^2(E^2) + F^{1/2}ds^2(E^3) ,$$

$$e^{2\phi} = F^{-1/2} \tilde{F}^{-1} , \quad A_1 = -\mu \cos \delta \cos \eta d\varphi ,$$

$$B = \tan \delta \tilde{F}^{-1} \epsilon (E^2) , \quad A_3 = -\mu \sin \delta \tilde{F}^{-1} \cos \eta d\varphi \wedge \epsilon (E^2) .$$

(3.15)
The metric functions read
\[ F = 1 + \frac{\mu}{\rho} , \quad \tilde{F} = \sin^2 \delta + F \cos^2 \delta , \]
(3.16)
and \((\eta, \varphi)\) are the usual angles on a 2–sphere. These fields are obtained by T–duality at an angle starting from the D5–brane solution \([35, 36]\). Near the ‘core’, i.e. for \(\rho \ll \mu\), this solution has the form
\[ ds_{10}^2 = \sqrt{\frac{\rho}{\mu}} ds^2 \left( M^5 \right) + \frac{1}{\cos^2 \delta} \sqrt{\frac{\rho}{\mu}} ds^2 \left( E^2 \right) + \sqrt{\frac{\mu}{\rho}} ds^2 \left( E^3 \right) , \]
\[ e^{2\phi} = \frac{1}{\cos^2 \delta} \left( \frac{\rho}{\mu} \right)^{3/2} , \quad A_1 = -\mu \cos \delta \cos \eta d\varphi , \]
(3.17)
\[ B = \tan \delta \cos^2 \delta \rho \epsilon \left( E^2 \right) , \quad A_3 = -\frac{\sin \delta}{\cos^2 \delta} \rho \cos \eta d\varphi \wedge \epsilon \left( E^2 \right) . \]

We wish to compare these fields with the ones of solution \((3.8)\) near the ‘core’ \(r = r_H, \theta = 0\). It is convenient to introduce the dimensionless radial coordinate
\[ \lambda^2 = \frac{2}{3} \left( \frac{\tilde{r}}{r_H} \right)^2 , \]
(3.18)
where \(\tilde{r}^2 = r^2 - r_H^2\). Then the metric in \((3.8)\) becomes
\[ ds_{10}^2 = \frac{3 \sqrt{\lambda^2 + \theta^2}}{2} \left\{ \frac{ds^2 \left( M^5 \right)}{\cosh \alpha} + r_H^2 \cosh \alpha \left( d\lambda^2 + d\theta^2 + d\Omega_2^2 + \frac{\theta^2 \lambda^2}{\theta^2 + \lambda^2} d\varphi^2 \right) \right\} . \]
(3.19)
The metric in the \(\theta, \lambda, \varphi\) three–space is conformally flat. This is made explicit by the coordinate transformation
\[ \frac{4\rho}{3r_H} = \lambda^2 + \theta^2 , \quad \frac{2\rho}{3r_H} \sin \eta = \lambda \theta . \]
(3.20)
Moreover, with the identifications
\[ \frac{3}{r_H \cosh^2 \alpha} \leftrightarrow \frac{1}{\mu} , \quad \cosh \alpha \leftrightarrow \frac{1}{\cos \delta} , \]
(3.21)
we find the ‘near–core’ metric, together with the dilaton and gauge fields to have the form
\[ ds_{10}^2 = \sqrt{\frac{2}{\mu}} ds^2 \left( M^5 \right) + \frac{1}{\cos^2 \delta} \sqrt{\frac{\rho}{\mu}} r_H^2 d\Omega_2^2 + \sqrt{\frac{\mu}{\rho}} ds^2 \left( E^3 \right) , \]
\[ e^{2\phi} = \frac{1}{\cos^2 \delta} \left( \frac{\rho}{\mu} \right)^{3/2} , \quad B = \tan \delta \frac{\rho}{2 \cos^2 \delta} \epsilon \left( S^2 \right) , \]
\[ A_1 = \mu \cos \delta (1 - \cos \eta) d\varphi , \quad A_3 = \frac{\sin \delta}{\cos^2 \delta} \rho (1 - \cos \eta) r_H^2 \epsilon \left( S^2 \right) \wedge d\varphi . \]
(3.22)
Comparison with (3.17) reveals that near the ‘core’, the metric, dilaton field and RR 1–form potential are the same, except for the crucial replacement of $ds^2(E^2) \rightarrow r_H^2 d\Omega_2^2$, which is exactly what one would expect from the spherical D4–D6–branes bound state in the dielectric effect. For this reason it is natural to regard $r_H$ as the radius of the dielectric brane. The 2–form and 3–form potentials $B$ and $A_3$ are not in the same gauge orbit as those in the solution (3.17). This fact is expected because these fields have components along the 2–sphere. We have checked that both $B$ and $A_3$ in (3.22) solve their equations of motion:

$$d\left(\star [e^{-2\phi} H - A_1 \wedge \tilde{F}_4]\right) = -\frac{1}{2} F_4 \wedge F_4 ,$$

where $\tilde{F}_4 = F_4 - H \wedge A_1$.

Since anti–podal points on the 2–sphere have opposite D6–brane charge, the total charge vanishes. From the source term in the RR 1–form potential equation of motion one finds the usual quantization of the D6–brane tension:

$$\mu \cos \delta = \frac{r_H \cosh \alpha}{3} = \frac{g \sqrt{\alpha'}}{2} N_6 .$$

Using the relation (3.6) one sees that $N_6 = 1$. Hence, the solution (3.8) is interpreted as the ground state of dielectric branes described in terms of $SU(2)$ irreducible representations, as explained in the Introduction.

Another consistency check is to view the system as a bound state of a D6–brane with $N$ D4–branes due to a flux of the $U(1)$ worldvolume gauge field (or gauge equivalently of the Kalb–Ramond 2–form field $B$). A simple calculation shows that

$$\Phi = -\frac{1}{2\pi \alpha'} \int_{\partial M} B = 2\pi N .$$

Finally, the radius of the dielectric sphere $r_H$ is

$$r_H \sim \frac{\sqrt{\alpha'}}{g} N = \alpha' B N ,$$

in agreement with the scaling behavior in the brane picture (1.3). We shall return to this point below.

### 3.2 Arbitrary magnetic field

The above construction is not general because the magnetic field has an unphysically large value $B = 1/R_{11}$. We chose to present it as a warm up to the general case because it is
simpler and encodes the correct physics. To obtain an arbitrary value for the magnetic field parameter one needs to consider a version of the rotating M5–brane solution \[37, 38\] obtained after a double analytic continuation. The corresponding eleven–dimensional background fields are

\[
ds^2 = H^{-1/3} \left[ ds^2(M^5) + f \left( d\tau + \frac{2ml \cosh \alpha}{r^3 \Delta \tilde{f}} \sin^2 \theta \, d\tilde{\varphi} \right)^2 \right] + \\
+ H^{2/3} \left[ \frac{d\tau^2}{f} + r^2 \left( \Delta \, d\theta^2 + \sin^2 \theta \frac{\Delta \tilde{f}}{f} \, d\tilde{\varphi}^2 + \cos^2 \theta \, d\Omega_2^2 \right) \right],
\]

\[(3.27)\]

The functions \(H, \Delta, f\) and \(\tilde{f}\) have the form

\[
H = 1 + 2m \sinh^2 \alpha, \quad \Delta = 1 - \frac{l^2 \cos^2 \theta}{r^2}, \\
f = 1 - \frac{2m}{\Delta r^3}, \quad \tilde{f} = \frac{r^3 - l^2 r - 2m}{r^3 \Delta}.
\]

\[(3.28)\]

This solution has three independent parameters, namely \((m, \alpha, l)\), which after reduction will be related to the number of D4–branes \(N\), the string coupling \(g\) and the magnetic field \(B\). Notice that this double analytically continued solution is static. The initial rotation is mapped into a ‘twist’ along what will become the internal direction.

The quantization of the M5–brane charge gives the relation

\[
2m \cosh \alpha \sinh \alpha = \pi N l p^3.
\]

\[(3.29)\]

The ‘Euclidean angular velocity’ and the radius of the compact direction \(\tau\) are

\[
\Omega = \frac{l}{\cosh \alpha (r_H^2 - l^2)},
\]

\[(3.30)\]

and

\[
R_{11} = g \sqrt{\alpha'} = \frac{4m \cosh \alpha}{3r_H^2 - l^2},
\]

\[(3.31)\]

respectively. The constant \(r_H\) is the maximum zero of the cubic equation

\[
H^3 - l^2 r_H = 2m,
\]

\[(3.32)\]

and corresponds to the locus of the horizon in the usual Lorentzian version of the solution. Explicitly the location of \(r_H\) can be expressed as

\[
r_H = m^{1/3} \left[ \left( 1 + \sqrt{1 - \frac{l^6}{27m^2}} \right)^{1/3} + \left( 1 - \sqrt{1 - \frac{l^6}{27m^2}} \right)^{1/3} \right],
\]

\[(3.33)\]

20
which for $l^3/m << 1$ is
\[ r_H \simeq (2m)^{1/3} \left( 1 + \frac{l^2}{(2m)^{2/3}} \right). \tag{3.34} \]

In particular, notice that in the double analytically continued solution, $r_H$ grows from the static value ($r_H^3 = 2m$), whereas it would decrease in the usual Lorentzian solution. Since at $\theta = \pi/2$ the ergosurface is at $r^3 = 2m$, in the analytically continued solution there is no ergoregion outside the horizon. The Euclidean continuation $l \to il$, maps the ergoregion inside the $r = r_H$ surface. Therefore, coming from infinity, the first metric singularity will be at the zero of $\tilde{f}$ in the analytically continued solution rather than at the ergosurface $f = 0$ of the usual Lorentzian geometry.

The 4–sphere at $r = r_H$ is the set of fixed points of the Killing vector field
\[ k = \frac{\partial}{\partial \tau} - \Omega \frac{\partial}{\partial \tilde{\varphi}}. \tag{3.35} \]

If we reduce along this direction the compactified space–time will have a 4–sphere singularity at $r = r_H$. If instead we reduce along
\[ q = \frac{\partial}{\partial \tau} + \left( \frac{1}{R_{11}} - \Omega \right) \frac{\partial}{\partial \tilde{\varphi}}, \tag{3.36} \]
the set of fixed points will be a 2–sphere at $r = r_H$, $\theta = 0$. The resulting solution will describe a D4–brane immersed in a flux 7–brane with magnetic field parameter given by
\[ B = \frac{1}{R_{11}} - \Omega = \frac{3r_H + l}{2r_H(r_H + l) \cosh \alpha}, \tag{3.37} \]
where we have used the previous relations for $\Omega$ and $R_{11}$ together with the cubic equation for $r_H$. Defining the azimuthal coordinate $\varphi = \tilde{\varphi} - B\tau$, the eleventh direction $\tau$ will have periodicity $R_{11} = g\sqrt{\alpha'}$ at fixed $\varphi$. Then, the reduction to the type IIA theory yields the following fields:
\[ ds_{10}^2 = \left( \frac{\Sigma}{H} \right)^{1/2} ds^2 \left( M^5 \right) + (\Sigma H)^{1/2} \left[ \frac{dr^2}{\tilde{f}} + r^2 \left( \Delta d\theta^2 + \cos^2 \theta d\Omega_2^2 \right) \right] + \]
\[ + \left( \frac{H}{\Sigma} \right)^{1/2} \tilde{f} r^2 \sin^2 \theta \Delta d\varphi^2, \]
\[ e^{2\phi} = \frac{\Sigma^{3/2}}{H^{1/2}}, \quad B = -\frac{2m \sinh \alpha \cos^3 \theta}{\Delta} \left[ \frac{l}{r^2} + \left( 1 - \frac{l^2}{r^2} \right) B \cosh \alpha \right] \epsilon \left( S^2 \right), \tag{3.38} \]
\[ A_1 = \Psi \Sigma^{-1} d\varphi, \quad A_3 = -\frac{2m \sinh \alpha \cosh \alpha \cos^3 \theta}{\Delta} \left( 1 - \frac{l^2}{r^2} \right) d\varphi \wedge \epsilon \left( S^2 \right), \]
where we have introduced
\[ \Sigma \equiv f \left( 1 + \frac{2mlB \cosh \alpha \sin^2 \theta}{r^3 \Delta f} \right)^2 + H(Br \sin \theta)^2 \Delta \frac{\tilde{f}}{\tilde{f}}, \]  
and
\[ \Psi \equiv \frac{\Sigma - f}{B} - \frac{2ml \cosh \alpha \sin^2 \theta}{r^3 \Delta}. \]  
Again, this solution describes a geometry with 5–dimensional Poincaré invariance. The number of D4–branes is
\[ N = \frac{1}{(2\pi)^3 g \alpha'^3/2} \int F_4 = \frac{\sinh \alpha}{2\pi \alpha'} \left( 3r_H^2 - l^2 \right). \]  
Also, asymptotically one recovers the flux 7–brane solution (3.11).

3.2.1 Analysis of Parameters

As pointed out in the discussion above, the supergravity solution depends on three parameters. It is convenient to choose as independent parameters \( r_H, l \) and \( \alpha \), with \( m \) implicitly defined by equation (3.32). Positivity of the parameter \( m \) requires \( r_H > |l| \). In terms of these parameters the physical quantities \( g, B \) and \( N \) are defined by the equations (3.31), (3.37) and (3.41), respectively. We have already seen that the \( l = 0 \) case corresponds to an unphysically large magnetic field \( B = 1/R_{11} \). The physical regime where \( B \) can be tuned to an arbitrary small value, \( B \ll 1/R_{11} \), corresponds to the case \( r_H - l \ll r_H \). In this limit, the physical parameters become
\[ N \simeq \frac{r_H^2 \sinh \alpha}{\pi \alpha'}, \]  
\[ B \simeq \frac{1}{r_H \cosh \alpha}, \]  
\[ g \sqrt{\alpha'} \simeq 2 (r_H - l) \cosh \alpha. \]  

A careful analysis of the parameters shows that, for fixed \( g \) and \( B \), the number of \( D4 \)–branes \( N \) is bounded above by a critical number \( N_{\text{crit}} (g, B) \). In the limit just described, this can be shown simply by noting that
\[ \pi \alpha' N B^2 \simeq \frac{\sinh \alpha}{1 + \sinh^2 \alpha} \leq \frac{1}{2}, \]  
so that
\[ N_{\text{crit}} \simeq \frac{1}{2\pi \alpha' B^2}. \]
Similarly, for fixed \( g \) and \( N \) there is a critical magnetic field \( B_{\text{crit}} \simeq 1/\sqrt{2\pi \alpha'}N \). Notice that the existence of an upper bound on the D4–brane charge is another manifestation of the stringy exclusion principle \([39]\).

Finally, let us note that the map \((r_H, l, \alpha) \to (g, N, B)\) is generically two–to–one, except at the special points where the parameters \( N, B \) acquire their critical value. In particular for a given set \((g, N, B)\) there will be generically two values of the dielectric sphere radius \(r_H\). The physical interpretation of this fact will follow.

### 3.2.2 Pure Melvin and Pure D4–brane Limits

Before we consider the general case, let us analyse the geometry for \( l = r_H \). Then the parameter \( m \) vanishes and we recover the flux 7–brane solution written in spheroidal oblate coordinates, which can be related to the usual cartesian coordinates by the transformation

\[
\begin{pmatrix}
y_1 \\
y_2 + iy_3 \\
y_4 + iy_5
\end{pmatrix} = \begin{pmatrix}
r \cos\theta \cos\psi \\
r \cos\theta \sin\psi e^{ix} \\
\sqrt{r^2 - r_H^2} \sin\theta e^{i\phi}
\end{pmatrix}.
\tag{3.45}
\]

To recover flat space one should send \( \alpha \to \infty \), since from (3.37) the magnetic field \( B \) vanishes in such limit.

Next, we consider the limit of vanishing magnetic field keeping the total number of D4–branes \( N \) and the string coupling \( g \) fixed. For \( B \) small enough we will always be in the region of parameter space \( r_H - l \ll r_H \) defined in the previous subsection. More precisely inverting the relations (3.42) we have

\[
\begin{align*}
r_H &\simeq \frac{\pi \alpha' N B}{l} \to 0 , \\
\frac{r_H - l}{r_H} &\simeq \frac{1}{2} g \sqrt{\alpha' B} \to 0 , \\
\cosh \alpha &\simeq \frac{1}{\pi \alpha' NB^2} \to \infty .
\end{align*}
\tag{3.46}
\]

In this limit \( f \to 1 \) and the forms \( A_1 \) and \( B \) in (3.38) vanish. The resulting solution is that of the single–center D4–brane geometry, as expected since for vanishing magnetic field the D4–branes will seat on top of each other. Conversely, starting with the single–center D4–brane geometry, which is a special case of (3.38), we can turn on a magnetic field and observe the expansion of the system into a spherical dielectric brane.

\[\text{[Before the revised version of this paper came out, a pre–print [40] appeared that independently realized, from the analysis of the parameters of our solution, the existence of two radii and of a critical number of branes.]\]
3.2.3 General Case

Similarly to the maximal magnetic field case, the geometry described by (3.38) has a metric singularity at \( r = r_H \). The geometry has exactly the same features as described by Figure 2. The locus \( r = r_H \) is the origin on the \( \theta = \pi/2 \) two-plane, and the D6–brane worldvolume with geometry \( M^5 \times S^2 \) is represented by \( r = r_H \) and \( \theta = 0 \). The three-dimensional space \( r = r_H, \, 0 < \theta \leq \pi/2 \) in the transverse space represents the interior of the dielectric two–sphere.

The analysis of the geometry near \( r = r_H, \, \theta = \pi/2 \) shows that to avoid a conical singularity the following condition must hold

\[
3m + l^2 r_H = lr_H^2 + 2mr_H B \cosh \alpha .
\]  

(3.47)

Using the cubic equation for \( r_H \), this yields relation (3.37) and explains why we chose this particular value for the magnetic field parameter.

Now we show that the geometry near \( r = r_H, \, \theta = 0 \) describes a spherical D6–D4–brane bound state as appropriate for the dielectric effect. In this limit the functions appearing in the metric become

\[
H \simeq \cosh^2 \alpha , \quad \Delta \simeq \frac{2m}{r_H^3} + \frac{l^2}{r_H^2} \left( \frac{\theta^2 + \tilde{r}^2}{r_H^2} \right) ,
\]

\[
f \simeq \frac{3r_H^2 - l^2}{4mr_H} \tilde{r}^2 + \frac{l^2 r_H}{2m} \theta^2 , \quad \tilde{f} \simeq \frac{3r_H^2 - l^2}{4mr_H} \tilde{r}^2 ,
\]

(3.48)

where as before we defined the radial coordinate \( \tilde{r}^2 = r^2 - r_H^2 \). It is convenient to change to the new coordinates

\[
\rho = \frac{\tilde{r}^2}{2r_H} + \frac{3r_H^2 - l^2}{4r_H} \theta^2 , \quad \rho \sin \eta = \sqrt{\frac{3r_H^2 - l^2}{2r_H^2}} \tilde{r} \theta .
\]

(3.49)

With the identification

\[
\mu \leftrightarrow \frac{2m \cosh^2 \alpha}{3r_H^2 - l^2} , \quad \cosh \alpha \leftrightarrow \frac{1}{\cos \delta} ,
\]

(3.50)

we obtained exactly the same metric, dilaton field and RR 1–form and 3–form potentials as in the ‘near–core’ geometry for the maximal magnetic field case (3.22). The 2–form Kalb-Ramond field is now

\[
\mathcal{B} = \frac{\tan \delta}{\cos^2 \delta} \left( \frac{1}{\mu} \rho \sin^2 (\eta/2) + \frac{lr_H \cos^2 \delta}{m} \rho \cos^2 (\eta/2) \right) r_H^2 \epsilon(S^2) .
\]

(3.51)
As before we verified that the equations of motion (3.23) for the 2 and 3–form are obeyed for this ‘near–core’ solution. Again, therefore, the interpretation of a spherical bound state holds, as appropriate for the dielectric effect.

The quantization condition on the D6-brane tension gives

\[ \mu \cos \delta = \frac{2m \cosh \alpha}{3r_H^2 - l^2} = \frac{g\sqrt{\alpha'}}{2}, \]

which together with the equation for \( R_{11} = g\sqrt{\alpha'} \) in (3.31) gives \( N_6 = 1 \). Hence, as for the maximal magnetic field we consider the case with a single D6–brane. Viewing the system as a bound state of a D6–brane with \( N \) D4–branes requires a flux quantization of the Kalb–Ramond 2–form field \( B \) along the two-sphere. Indeed a straightforward calculation yields the same result as in (3.25).

The region of validity for the Melvin background requires that \( R_{11} \ll r \ll 1/B \). The former condition arises by considering length scales much larger than the compactification scale and the latter by requiring small string coupling. Let us again take \( g \) and \( N \) fixed, and vary \( B \). As we already noted in section 3.2.1, \( 0 < B < B_{\text{crit}} \). To work within the region of parameter space \( r_H - l \ll r_H \), we will require that \( B_{\text{crit}} \ll 1/R_{11} \). Since in this case \( B_{\text{crit}} \simeq 1/\sqrt{2\pi\alpha'N} \), this requirement is equivalent to the mild condition \( \sqrt{N} \gg g \).

Now we can calculate the radius of the dielectric brane using the gravitational approximation. Using the relations (3.42) for \( B \) and \( N \) in terms of \( r_H \) and \( \alpha \) in the limit \( r_H - l \ll r_H \), we find the quartic equation for \( r_H \)

\[ r_H^4 - \left( \frac{r_H}{B} \right)^2 + (\pi\alpha'N)^2 = 0. \]

(3.53)

As mentioned before one finds two solutions for \( r_H \) in terms of the physical parameters,

\[ r_H^2 = \frac{1}{2B^2} \left( 1 \pm \sqrt{1 - (2\pi\alpha'NB^2)^2} \right). \]

(3.54)

For small magnetic field parameter, \( B << B_{\text{crit}} \), we obtain

\[ r_- = \pi\alpha'NB, \quad r_+ = \frac{1}{B}. \]

(3.55)

We shall come back to these results in section 5. Here we shall mention just that \( r_H = r_- \) is exactly the radius found by Myers in his approach, and that \( r_H = r_+ \) is not a stable solution.

### 3.2.4 M-theory Orbifold and Multiple D6–branes

A small extension of the previous construction gives rise to a dielectric brane which can be regarded as a bound state with \( N_6 \) D6–branes. The only modification is to consider
an orbifold compactification of M-theory. More precisely, we mod out the $\tau - r$ plane by a discrete $\mathbb{Z}_{N_6}$ identification. This is implemented by modifying the eleven–dimensional period via

$$R_{11} = g\sqrt{\alpha'} = \frac{4m \cosh \alpha}{3r_H^2 - l^2} \frac{1}{N_6},$$

which replaces the previous relation (3.31). The magnetic field parameter is still related to $(r_H, l, \alpha)$ by (3.37), while the total number of D4–branes becomes

$$N = \frac{N_6}{2\pi \alpha'} \left(3r_H^2 - l^2\right) \sinh \alpha .$$

In fact, given a ten–dimensional solution with parameters $(r_H, l, \alpha)$ only the ratio $N/N_6$ is determined. Only in eleven dimensions the individual values of $N$ and $N_6$ are singled out.

To see that $N_6$ should be interpreted as the number of D6–branes we look at the geometry close to the dielectric sphere. The associated D6–brane tension is

$$\mu \cos \delta = \frac{2m \cosh \alpha}{3r_H^2 - l^2} = \frac{g\sqrt{\alpha'}}{2} N_6 ,$$

which, as claimed, should be identified with a dielectric configuration with multiple D6–branes.

The analysis of the radius $r_H$ follows the one at the end of the previous section with the replacement $N \to N/N_6$. In particular, the dielectric radius $r_- = \pi \alpha' B N/N_6$ is in agreement with the brane picture in terms of reducible $SU(2)$ representations (formed from $N_6$ irreducible representations each of integer dimension $N/N_6$) [22]. Let us remark that the maximal case $N_6 = N$ is described by the same ten–dimensional geometry as the case $N = 1$, and hence outside the scope of the gravitational description.

For simplicity, in the remainder of this paper we shall restrict ourselves to the single D6–brane case.

### 3.2.5 Gravitational Energy

Having placed the D4–branes in the magnetic field background associated with the flux 7–brane it is natural to ask what is the gravitational energy associated with such configuration. Thus we are led to compute the spacetime energy. In order to get a finite result we will use the ‘reference background subtraction’ method [41], with the flux 7–brane as the reference background. This method yields the right result for the Ernst black hole in the Melvin Universe [9], a situation somewhat analogous to, although much simpler than, the present case. The mass calculated using this method should be interpreted as the dielectric
brane mass placed in a flux 7–brane. This mass is associated with the D4–branes mass together with the interacting energy with the flux 7–brane.

For the dielectric brane geometry described by (3.38) the relevant energy expression is

\[
E = -\frac{1}{8\pi G_{10}} \left[ \int_{S_7} \sqrt{h} N K d^8 x - \int_{S_7} \sqrt{h_0} N_0 K_0 d^8 x \right].
\]  

(3.59)

Here, we have denoted by \( h_{\alpha\beta} \) the induced metric on a co-dimension two surface which can be thought of as a \( r = \text{const.} \) (as \( r \to \infty \)) section of a spacelike hypersurface. \( K \) is the trace of the second fundamental form on \( S_7 \) and \( N = \sqrt{-g_{00}} \) is the lapse function. The quantities with a zero subscript refer to the flux 7–brane background. Explicitly the trace of the second fundamental form reads

\[
K = \frac{1}{2\sqrt{g_{rr}}} \left( h_{\alpha\beta} h_{\alpha\beta,r} \right).
\]  

(3.60)

Then a straightforward computation leads to the result

\[
E = \frac{\pi V_4}{3G_{10}} \left( \frac{3}{2} R^3 + 5m \right),
\]  

(3.61)

where \( V_4 \) stands for the volume of \( E^4 \) and we used the value for the unit 4–sphere volume \( \Omega_4 = 8\pi^2/3 \).

To relate this expression to the D4–branes mass plus interaction with the flux 7–brane, consider the limit of small magnetic field defined in section 3.2.1. Writing \( R^3 \) and \( 2m \) as

\[
R^3 = 2m \sinh^2 \alpha = 2m \left( \cosh^2 \alpha - 1 \right),
\]

\[
2m = r_H \left( r_H^2 - l^2 \right) \approx 2r_H^3 \frac{r_H - l}{r_H},
\]  

(3.62)

and using the relations (3.46), together with the formulae for the gravitational constant and D–brane tension

\[
16\pi G_{10} = (2\pi)^7 g^2 \alpha'^4, \quad T_p = \frac{1}{(2\pi)^p g \alpha'^{(p+1)/2}},
\]  

(3.63)

we arrive at the result

\[
E \simeq N M_4 + \frac{2\pi^2 \alpha'^2}{3} B^4 N^3 M_4.
\]  

(3.64)

Here \( M_4 = V_4 T_4 \) is the mass of one D4-brane in flat space. The second term should be identified with the interaction between the D4-branes and the flux 7–brane that results in the dielectric configuration. One could think that this energy is higher than the mass of \( N \) D4–branes and therefore the solution is unstable. Notice, however, that the vacuum, i.e. the reference background, is not flat space but the flux 7–brane which does not allow for a configuration of \( N \) D4–branes placed on top of each other with the corresponding flat space mass.
4 Decoupling Limit

The gravity/gauge theory duality holds in the limit when closed strings in the bulk and open strings on the D–brane decouple [4]. On the gauge theory side we expect that the coupling of the D4–branes to the flux 7–brane will generate a relevant deformation on the D4–brane worldvolume theory, controlled by the magnetic field $B$. Of course, the most interesting case would be the one involving D3–branes expanding to a spherical D5–brane, which is analogous to the one presented in this paper.

The gravitational dual is obtained by taking the decoupling limit on the geometry (3.38). This limit corresponds to sending $\alpha' \to 0$ keeping fixed the parameters

$$g_{Y_M}^2 = (2\pi)^2 g' \alpha'$$, \quad $U_0 = \frac{rH}{\alpha'}$, \quad $a = \frac{l}{\alpha'}$, \quad (4.1)

and the energy scale

$$U = \frac{r}{\alpha'}$$. \quad (4.2)

In this limit, the extremality parameter $\alpha \to \infty$ so that $\alpha' \cosh \alpha$ remains finite with value

$$\kappa = \frac{g_{Y_M}^2}{8\pi^2} \left( \frac{3U_0^2 - a^2}{U_0^3 - a^2U_0} \right)$. \quad (4.3)

Moreover, the magnetic field parameter $B$ is kept fixed, and can be expressed as

$$B = \frac{(2\pi)^2 (3U_0 + a) (U_0 - a)}{g_{Y_M}^2 3U_0^2 - a^2}$. \quad (4.4)

A straightforward calculation shows that the background fields have the form

$$ds_{10}^2 = \alpha' \left\{ \left( \frac{\Sigma}{H} \right)^{1/2} ds^2 (\mathbb{M}^5) + (\Sigma H)^{1/2} \left[ \frac{dU}{f} + U^2 \left( \Delta d\theta^2 + \cos^2 \theta d\Omega_2^2 \right) \right] + \right.$$

$$+ \left( \frac{H}{\Sigma} \right)^{1/2} \tilde{f} U^2 \sin^2 \theta \Delta d\varphi^2 \right\}$, \quad (4.5)

$$g^2 e^{2\phi} = \frac{g_{Y_M}^4}{(2\pi)^4} \Sigma^{3/2} H^{-1/2}$, \quad $A_1 = \Psi \Sigma^{-1} d\varphi$, \quad (4.5)

$$\mathcal{B} = -\alpha' \frac{g_{Y_M}^2 N \cos^3 \theta}{4\pi} \Delta \left[ B + \frac{a - \kappa Ba^2}{\kappa} \frac{1}{U^2} \right] \epsilon (S^2)$, \quad (4.5)

$$\mathcal{A}_3 = -\alpha' \frac{g_{Y_M}^2 N \cos^3 \theta}{4\pi} \Delta \left( 1 - \frac{a^2}{U^2} \right) d\varphi \wedge \epsilon (S^2)$, \quad (4.5)
where the functions $H, \Sigma$ and $\Psi$ now read

$$H = \frac{g_{YM}^2 N}{4\pi \Delta U^3},$$

$$\Sigma = f \left(1 + \frac{g_{YM}^2 aBN \sin^2 \theta}{4\pi \Delta f U^3}\right)^2 + \frac{g_{YM}^2 NB^2}{4\pi} \frac{\sin^2 \theta \tilde{f}}{U \tilde{f}},$$

$$\Psi = \frac{\Sigma - f}{B} - \frac{g_{YM}^2 aN}{4\pi \Delta U^3} \sin^2 \theta,$$

with

$$\Delta = 1 - \frac{a^2 \cos^2 \theta}{U^2}, \quad f = 1 - \frac{2m_0}{\Delta U^3}, \quad \tilde{f} = \frac{U^3 - a^2 U - 2m_0}{\Delta U^3}.$$ (4.7)

The constant $m_0$ is defined through the cubic equation $2m_0 = U_0^3 - a^2 U_0$.

Consider first the asymptotics of the above solution. For

$$U \gg U_0, \quad g_{YM}^2 NB^2$$ (4.8)

(recall that $U_0 > |a|, m_0^{1/3}$) it is easy to see that one obtains the same asymptotics as for the decoupling limit of the D4–brane geometry defined in [42]. This fact supports the result announced at the end of section two: if one places $N$ D$p$–branes in a flux $(p + 3)$–brane and takes the decoupling limit of [42] keeping the magnetic field fixed, the resulting geometry will have the same asymptotics as for the decoupled D$p$–brane geometry. From the field theory point of view, this means that the couplings of the D–branes to the flux–branes – e.g. the Myers coupling – become irrelevant in the UV. This fact was used in the analysis of an associated class of $\mathcal{N} = 1$ supersymmetric 4–dimensional theories by Polchinski and Strassler in [23].

Next we want to analyze the region of validity of the type IIA supergravity description. We will work in what follows with $g_{YM}^2 B \ll 1$, which, as we shall see, is the relevant regime to have a useful supergravity solution in the large $N$ limit. In this case

$$U_0 \simeq \pi BN, \quad \frac{U_0 - a}{U_0} \simeq \frac{g_{YM}^2 B}{8\pi^2},$$ (4.9)

and condition (4.8) simply becomes

$$U \gg U_0.$$ (4.10)

First we recall some facts from [42] on the gravitational description of the pure D4–brane near horizon geometry. The relevant quantity is the effective dimensionless ’t Hooft coupling $\lambda_{eff} (U) = g_{YM}^2 NU$, in terms of which the region of validity of supergravity is

$$1 \ll \lambda_{eff} \ll N^{4/3}.$$ (4.11)
Figure 3: The region of validity of the gravitational approximation, in terms of the variables $v = U \cos \theta / (2\pi)$ and $\rho = U \sin \theta / (2\pi)$. There are curvature corrections around $U = U_0$ and $\theta = 0$ (point P), where the dielectric brane is placed. Inside the dielectric sphere, i.e., for $U = U_0$ and $0 < \theta \leq \pi/2$, there is a large region surrounding the center of the sphere (point Q at $\theta = \pi/2$) where the curvature is small. Far away the appropriate description is eleven-dimensional.

The bound $1 \ll \lambda_{\text{eff}}$ comes from the requirement of small curvature, whereas small string coupling gives the bound $\lambda_{\text{eff}} \ll N^{4/3}$. We want the asymptotic region $U \gg U_0$ described above to be still within the SUGRA regime, and for this to hold we simply require that $\lambda_{\text{eff}}(U_0) \ll N^{4/3}$ or

$$g_{YM}^2 B \ll N^{-2/3}.$$  \hspace{1cm} (4.12)

When the above bound holds, as we move away from the dielectric brane, we reach the asymptotic region before we arrive at the region of large string coupling, where Type IIA supergravity ceases to hold and where we enter the eleven-dimensional M-theory region (see Figure 3). As we move in towards the dielectric brane to lower values of $U$, we eventually reach a region of large curvature, where $\alpha'$ corrections to supergravity are important. This region is localized around the spherical D6 brane, which, in the $U$-$\theta$ plane, is located at $U = U_0$ and $\theta = 0$ (point P in Figure 3). To analyze the extent of the region of large curvature, we focus our attention, in particular, on the region inside the dielectric sphere, around $U = U_0$ and $\theta = \pi/2$ (point Q in the Figure). This point is separated from the spherical brane by a distance (in energy units) of order $U_0$, and therefore the curvature
is small if the effective coupling satisfies \( \lambda_{\text{eff}}(U_0) \gg 1 \), or
\[
g^2_{YM} B \gg N^{-2} .
\]
When the above holds, the region of large curvature around point P does not extend all the way to the interior of the dielectric sphere to point Q and we are in the situation described in Figure 3. This last bound was obtained with a heuristic argument, but it can be confirmed by computing the curvature at point Q and requiring it to be small in units of \( \alpha' \).

### 4.1 Scalars VEV and vacuum energy

The reason for the above analysis is to justify the validity of the gravitational calculation of both the radius and potential energy of the dielectric brane in the decoupling limit. These are related to the VEV for the scalars of the field theory and the energy of the vacuum. Notice that, however, as it stands our field theory is not renormalizable and should be regarded as a low energy effective description of a deformation of the compactified (2,0) six–dimensional conformal theory associated to the M5–branes.

First recall that the unperturbed theory is free in the IR, and that the scalars have vanishing expectation value in the vacuum. The deformation, proportional to \( B \), is on the other hand IR relevant, and changes the vacuum structure. More precisely, the VEV for the scalars is given by
\[
U_0 = 2\pi \langle \Phi \cdot \Phi \rangle^{1/2} . \tag{4.14}
\]
This is defined implicitly in terms of \( B \), \( N \) and \( g^2_{YM} \). Provided \( U_0 \) is much larger than the thin layer around the dielectric brane, where curvature corrections are important, the gravitational approximation is accurate. Now we consider the limit of small \( B \) defined above. Then to linear order in \( B \) we have
\[
U_0 \simeq \pi B N \, . \tag{4.15}
\]
which exactly corresponds to the Myers result, including the numerical factor. This value corresponds to the radius \( r_H = r_- \) of section 3 after taking the decoupling limit. The other value \( r_H = r_+ \) in (3.55) scales to infinity in the decoupling limit.

The calculation of the vacuum energy uses the ‘reference background subtraction’ method explained in section 3.2.5. Now, the appropriate reference vacuum is the D4–brane geometry in the decoupling limit. A straightforward calculation gives
\[
\mathcal{E} = \frac{4V_4 m_0}{3\pi g^4_{YM}} . \tag{4.16}
\]
For small magnetic field we have, to leading order in $B$,

$$\mathcal{E} \simeq \frac{V_4}{6g_{YM}^2} B^4 N^3,$$

(4.17)

which is exactly the same energy as the interacting energy term in (3.64) after taking the decoupling limit. This vacuum energy has the same scaling behavior as the vacuum energy $V_N$ calculated by Myers in the flat space case (1.4). Let us note, however, that the above result for the energy includes, from the field theory point of view, two contributions. The first comes from the change in the vacuum energy coming from the new relevant operators proportional to $B$ (the graphs with no external legs). The second contribution comes from the graphs with external legs at zero momentum, since the vacuum value of the scalar fields is not zero in the dielectric brane configuration.

To better interpret these results notice first that we are considering only leading terms in $B$ (even if our expressions for $U_0$ and $\mathcal{E}$ are valid to all orders). The 3–point vertex deformation in the Lagrangian that is linear in $B$ has the form of (1.2) and it is the Myers coupling of the D4–brane to the electric field on the flux brane. The deformation, as argued before, is relevant in the IR, and therefore graphs of all orders will contribute to deep IR problems like vacuum structure (recall that the unperturbed theory becomes free in the IR, since the effective coupling $\lambda_{eff} (U)$ goes to zero as $U \to 0$). We are, on the other hand, in a position in which we can fine–tune the magnetic field to an arbitrarily small value, and therefore we can see results in perturbation theory in $B$ regarding IR physics. We therefore conclude that (1.14) and (4.16) are strong coupling (in $B$) results for the vacuum structure of the perturbed theory, given a small deformation in the UV. We will say a bit more on the form of the high–energy Lagrangian and deformation in the next section, using a probe computation in the decoupled geometry. It would be very interesting to use these gravity predictions to match some field theory results.

5 Probing Dielectric Branes

In this section we will study the flux–brane and dielectric brane geometries using a D–brane probe. Let us start by discussing a $D6$–brane probe in the presence of a flux 7–brane. Following Myers $^{[22]}$, we will take the probe geometry to be $M^5 \times S_2$, with $n$ units of $D4$–brane charge arising from a constant $U (1)$ flux on the two–sphere. The action for a $D6$–brane probe is

$$S = S_{BI} + S_{WZ},$$

(5.18)
where
\[
S_{BI} = -T_6 \int d^7 \sigma \, e^{-\Phi} \sqrt{-\det (\hat{g} - B + 2\pi \alpha' F)} ,
\]
\[
S_{WZ} = T_6 \int \sum A_{p+1} \wedge e^{-B+2\pi \alpha' F} .
\]
(5.19)

The worldvolume magnetic field \( F \) has the form \(-\frac{1}{2} n \epsilon(S_2)\), and the position of the probe is parameterized by the two coordinates \( r \) and \( \theta \), together with the azimuthal coordinate \( \phi \) which, by rotational symmetry, does not enter in the following expressions for the probe potential. Using the flux 7–brane background (3.11), one arrives at the potential
\[
V = 4\pi T_6 V_4 \left[ \Lambda^{1/2} \Lambda (r \cos \theta)^4 + (\pi \alpha' n)^2 - \frac{2}{3} B (r \cos \theta)^3 \right] ,
\]
(5.20)

with \( \Lambda = 1 + (Br \sin \theta)^2 \). At \( \theta = 0 \), the above potential is exactly the one found by Myers neglecting the back–reaction of the external electric field on the geometry. This potential has two critical points which are placed at \( \theta = 0 \) and \( r = r_\pm \) with
\[
r_\pm^2 = \frac{1}{2B^2} \left( 1 \pm \sqrt{1 - (2\pi \alpha' nB^2)^2} \right) .
\]
(5.21)
The point \( r_- \) is a minimum of the potential while the other point at \( r_+ \) is a saddle point. For small \( B \), more precisely for \( B \ll 1/\sqrt{\alpha' n} \), we have
\[
r_- \simeq \pi \alpha' n B , \quad r_+ \simeq \frac{1}{B} .
\]
(5.22)

It is simple to check that both extrema are local minima with respect to variations of \( \theta \), and therefore the full geometry induced by the external electric field confines the probe to the core of the flux–brane. We recall that, if one neglects the backreaction and follows the computation of Myers, one finds a potential with flat directions, corresponding to moving the probe in the directions orthogonal to the electric field. The full flux–brane geometry stabilizes the probe, effectively setting \( \theta = 0 \) (\( \Lambda = 1 \)). Therefore \( r_- \) is a true minimum of the full potential. This analysis exactly reproduces the one from supergravity in section 3.2.3, suggesting that the gravity solution with \( r_H = r_+ \) is not a minimum of the gravitational action. The true minimum is unstable through quantum tunneling, but this process is suppressed in the decoupling limit. In fact we have already seen that, in the decoupling limit, the local maximum is not seen anymore, making the local minimum global. We shall confirm these expectations in the following.

Let us also note that, as one increases the electric field \( B \), the two extrema \( r_- \) and \( r_+ \) approach each other, and above a critical electric field
\[
B_{\text{crit}}^2 = \frac{1}{2\pi \alpha' n} ,
\]
(5.23)

33
the potential does not exhibit any extrema. This analysis precisely matches the one from supergravity, which was discussed in section 3.2.1.

Finally, let us note that the above results, although derived in the flux 7–brane case, are actually valid for a general flux \( p \)-brane, and this can be checked by expanding the flux brane background fields around its core.

### 5.1 The probe potential

In this section we repeat the above probe computation in the general background of the dielectric brane. In particular, we will specialize to the decoupled geometry \((4.5)\), which is most relevant for the field theory/gravity duality. In this case, the RR form fields \( \mathcal{A}_1 \) and \( \mathcal{A}_3 \) are both non–vanishing, and the forms \( \mathcal{A}_5 \) and \( \mathcal{A}_7 \) in the Wess–Zumino part of the probe action are determined by electro–magnetic duality. More precisely, if one defines the field strength \( \mathcal{F}_{p+2} = d\mathcal{A}_{p+1} \), and the gauge invariant field strength \( \tilde{\mathcal{F}}_{p+2} = \mathcal{F}_{p+2} - H \wedge \mathcal{A}_{p-1} \), one has\(^7\)

\[
\tilde{\mathcal{F}}_6 = *\tilde{\mathcal{F}}_4 , \quad \tilde{\mathcal{F}}_8 = - * \tilde{\mathcal{F}}_2 .
\]

For the background \((4.5)\), a rather tedious computation gives the following result for \( \mathcal{A}_5 \) and \( \mathcal{A}_7 \)

\[
\mathcal{A}_5 = \alpha'^2 a_5 \epsilon(\mathbb{M}^5) , \quad \mathcal{A}_7 = \alpha'^3 a_7 \epsilon(\mathbb{M}^5) \wedge \epsilon(S_2) ,
\]

where \( a_5 \) and \( a_7 \) are functions only of \( r, \theta \) and are given by

\[
a_5 = - \frac{1}{H} - \frac{Ba}{\kappa} \sin^2 \theta , \quad a_7 = \frac{2}{3}BU^3 \cos^3 \theta - \frac{2m_0 B \cos \theta}{\Delta} \left[ \frac{\sin^2 \theta (1 - a \kappa B \sin^2 \theta)}{\Delta} + (1 - a \kappa B) \left( 1 - \frac{1}{3} \cos^2 \theta \right) \right] .
\]

We recall that the constant \( \kappa \), the limit of \( \alpha' \cosh \alpha \) in the decoupling limit, is given in \((4.3)\) and that the magnetic field parameter \( B \) is given in \((4.4)\). Finally, it is convenient to express the Kalb–Ramond field as

\[
\mathcal{B} = \alpha' b \epsilon(S_2) ,
\]

\[
b = - \frac{2m_0 \kappa \cos^3 \theta}{\Delta} \left[ B \kappa \frac{a - a^2 B \kappa}{U^2} \right] .
\]

\(^7\)Throughout the paper we are using the following convention for Hodge duality. If \( A, B \) are \( p \)-forms, then \( A \wedge * B = (A \cdot B) \omega \), where \( \omega \) is the volume form and \( A \cdot B \) is the inner product of the forms.
For static configurations the potential energy \( V = V_{BI} + V_{WZ} \) of the brane probe is given by the Born–Infeld and Wess–Zumino pieces

\[
V_{BI} = \frac{V_4}{4\pi^3g_{YM}^2} \frac{\Sigma^{1/2}}{H} \sqrt{\Sigma H U^4 \cos^4 \theta + (b + \pi n)^2},
\]

\[
V_{WZ} = \frac{V_4}{4\pi^3g_{YM}^2} \left[ -a_7 + a_5 (b + \pi n) \right].
\]

The dimensionless quantities which parameterize the decoupled geometry solution are \( N \) and \( g_{YM}^2 B \). As explained in section 4, we shall work in the small \( B \) regime \( g_{YM}^2 B \ll 1 \), i.e. within the scope of the gravitational approximation. In this limit, \( U_0 \) and \( (U_0 - a)/U_0 \) are given by (4.9), and the constant \( \kappa \) simplifies to

\[
\kappa \simeq \frac{1}{\pi B^2 N}.
\]

The functions \( a_5, a_7 \) and \( b \) are then expressed in terms of the physical quantities \( N, B \) and \( g_{YM}^2 \) and simplify to (here we have only kept the first change in the fields due to \( B \))

\[
a_5 \simeq -\frac{4\pi}{g_{YM}^2 N} U^3 + \frac{4\pi^3 N B^2}{g_{YM}^2} U \cos^2 \theta + \cdots,
\]

\[
a_7 \simeq \frac{2}{3} B U^3 \cos^3 \theta + \cdots,
\]

\[
b \simeq -\frac{1}{4\pi} N B g_{YM}^2 \cos^3 \theta + \cdots.
\]

### 5.1.1 Potential for large \( U \)

Now let us analyse the potential \( V \) for large values of the radial coordinate \( U \). An expansion in powers of \( 1/U \) yields (keeping, for each term in the expansion in \( U \), the leading term in \( B \))

\[
V = \frac{2V_4}{ng_{YM}^2} \left[ v^4 - \frac{2}{3} \left( nB \right) v^3 + \frac{1}{4} \left( nB \right)^2 \rho^2 + \cdots \right],
\]

where

\[
v = \frac{U}{2\pi} \cos \theta, \quad \rho = \frac{U}{2\pi} \sin \theta,
\]

are the radial coordinates along the \( \theta = 0 \) three–plane and the \( \theta = \pi/2 \) two–plane, respectively. Note that the above potential has the same form as the improved Myers potential (5.20). In fact, one can obtain the leading terms in the potential \( V \) in (5.31) by starting with the Myers potential (5.20) and taking the decoupling limit \( \alpha' \to 0 \) with \( U = r/\alpha' \) fixed. The \( \rho^2 \) mass term in \( V \) then comes from the expansion of \( \Lambda^{1/2} \).
The potential $V$ can be understood in relation to the dual effective low energy action which governs the physics on the $n$ $D4$–branes in the presence of a flux 7–brane. From the field theory point of view, the gravity construction in section 3 corresponds to a twisted compactification of the $(2,0)$ low energy conformal field theory on a stack of $M5$–branes. More precisely, recall that the usual compactification of the $(2,0)$ theory on a circle flows in the IR to the maximally supersymmetric $U(N)$ Yang–Mills theory on the $D4$–branes worldvolume. The gravity analysis of section 3 corresponds to a Scherk–Schwarz circle compactification [23] which breaks supersymmetry by twisting the fields with an $R$–symmetry rotation (breaking the $R$–symmetry group $SO(5)$ to a $SO(3) \times SO(2)$ subgroup, which is also evident in the supergravity solution).

To analyze the above reduction in more detail, it is convenient to first consider the analogous situation of the Scherk–Schwarz reduction on a circle of $\mathcal{N} = 4$, $D = 4$ SYM. The situation is similar since both theories are superconformal with the same number of supercharges and with similar $R$–symmetry groups, whose action rotates the scalars of the theories. We will use in the sequel this analogy since the $\mathcal{N} = 4$, $D = 4$ SYM Lagrangian is known, whereas the $(2,0)$ theory Lagrangian is only known for the abelian case [13, 14, 15].

Divide the scalars of the theory in $\Phi^m$, $m = 1, 2$ and $\Phi^A$, $A = 3, \cdots , 6$. As we go around the compactification circle, we rotate the scalars $\Phi^m$ in the 1–2 plane by an angle $2\pi BR_c$, while leaving the others fixed (here $R_c$ is the compactification radius). The non trivial part of the Scherk–Schwarz reduction for the bosonic fields comes from the kinetic term for the $\Phi^m$ scalars in the compact direction

$$-\frac{1}{2g_{YM}^2} \Tr (D_c \Phi^m D_c \Phi^m) \to -\frac{R_c}{g_{YM}^2} \Tr \left( \frac{1}{2} B^2 \Phi^m \Phi^m + i B\epsilon_{mn} \Phi^m [\Phi^n, \Phi^c] \right) ,$$

(5.33)

where the scalar $\Phi^c$ comes from the dimensional reduction of the gauge field, and where $g_{YM}^2/R_c$ is the square of the Yang–Mills coupling for the 3–dimensional reduced theory.

Let us move back to the $(2,0)$ compactification (now $A = 3, 4, 5$ since there are only 5 scalars). At low energies, we expect the deformed Lagrangian to be

$$\mathcal{L}_{SYM} - \frac{1}{2g_{YM}^2} \Tr \left( \frac{1}{2} B^2 \Phi^m \Phi^m + \frac{i}{3} B\epsilon_{ABC} \Phi^A [\Phi^B, \Phi^C] \right) .$$

(5.34)

The mass term for the scalars $\Phi^m$ above is exactly the same as in (5.33), if we consider a standard kinetic term for the scalars of the underlying $(2,0)$ theory. The other term in (5.34) is the Myers coupling, and is similar to the second term in (5.33). The difference in the index structure comes from the fact that the gauge field in the $(2,0)$ theory is not a 1–form but rather a matrix–valued 2–form, whose coupling to the matter fields is not
known in the non-abelian case. Therefore we predict that the Scherk–Schwarz reduction of the non-abelian \((2,0)\) theory gives rise to the Myers coupling in \((5.34)\).

To conclude this discussion we match exactly the Lagrangian \((5.34)\) to the probe potential \((5.31)\), following \cite{23}. This is easily done by the ansatz

\[
\Phi^1 + i\Phi^2 = e^{i\phi} \rho \cdot 1_{n\times n}, \quad \Phi^A = \frac{2}{n} v \cdot \alpha^A_{n\times n},
\]

(5.35)

where \(\alpha^A_{n\times n}\) is the \(n\)-dimensional representation of the \(SU(2)\) algebra, normalized to \(4\alpha^A\alpha^A = (n^2 - 1) \cdot 1\) (or \([\alpha^A, \alpha^B] = i\epsilon_{ABC}\alpha^C\)). The coefficients in \((5.35)\) are fixed by the conditions

\[
\Phi^m\Phi^m = \rho^2 \cdot 1_{n\times n}, \quad \Phi^A\Phi^A = v^2 \cdot 1_{n\times n}.
\]

(5.36)

5.1.2 Potential inside the dielectric brane

We proceed to analyse the probe potential around the center of the dielectric brane. More precisely, we will expand the potential around \(U = U_0\) and \(\theta = \pi/2\). Recall that inside the dielectric brane the radial coordinate in the three-plane is \(U_0 \cos \theta\). On the other hand, the radial coordinate in the transverse two-plane needs to be modified. It turns out, as one can guess from the coordinate transformation \((3.45)\), that the appropriate definitions for the radial coordinates \(v\) and \(\rho\) are now

\[
v = \frac{U}{2\pi} \cos \theta, \quad \rho = \frac{\sqrt{U^2 - U_0^2}}{2\pi} \sin \theta.
\]

(5.37)

In the limit of large \(U\) considered before we recover \((5.32)\). Moreover, in terms of the above radial variables, the probe potential inside the dielectric brane has \textit{exactly} the same form, to leading order in the deformation parameter \(B\), as in \((5.31)\).

Now we can easily see, from the form of the potential \(V\) around the center of the dielectric sphere, that the probe has one stable minimum at

\[
U = U_0, \quad \cos \theta = \frac{\pi nB}{U_0} \simeq \frac{n}{N}.
\]

(5.38)

This correspond to a dielectric shell of \(n\) \(D4\)-branes inside the larger shell of \(N\) \(D4\)-branes associated to the background geometry. This configuration corresponds to a vacuum of \((5.34)\) formed by a reducible \(SU(2)\) representation. Moreover, as the probe approaches the outer shell at \(U = U_0, \theta = 0\), one finds a second minimum corresponding to the fusion of the probe to the background dielectric brane, as can be seen by the plot of the full potential \((5.28)\) in Figure 4. Very close to the dielectric brane there will be curvature corrections to
Figure 4: The two minima of the potential for $g_{YM}^2 B = 0.01$ and $n/N = 0.3$. In the top graph the dimensionless radial coordinate $x$ is defined, for $0 \leq x < 1$, by $U = U_0$, $x = \cos \theta$ and it is the radial coordinate inside the dielectric sphere. For $x > 1$, we have $x = U/U_0$ and $\theta = 0$ as appropriate for the radial coordinate outside the dielectric sphere on the $\theta = 0$ three–plane. The other two plots zoom into the minima whose interpretation is given in the main text.

To conclude we summarize our findings of this section. Motivated by the gravity construction, the dual field theory is defined as a Scherk–Schwarz compactification of the $(2,0)$ theory. From the point of view of the effective Lagrangian, the Scherk–Schwarz reduction introduces the IR relevant couplings in (5.34) defined at the compactification energy scale. The probe computation, based on the gravity solution (5.34), matches exactly the Lagrangian (5.34) in the region of small curvature, which corresponds to the strongly coupled regime of the undeformed SYM theory. This region actually extends to the interior of the background dielectric brane, where again the probe potential has exactly the same form to leading order in $B$. As one approaches the dielectric brane, the supergravity approximation breaks down. This corresponds to the deep infrared regime of the theory.
where the deformation terms in (5.34) become relevant. The corrections to the classical VEV of the scalars can be seen from the gravity prediction for the dielectric radius \( U_0 \) which differs from its classical value \( \pi N B \) by \( g_{YM}^2 \) corrections.

\section{Conclusion}

In this paper we have shown the existence of a new type of branes is String and M-theory. These are flux \( p \)-branes with \( (p+1) \)-dimensional Poincaré invariance. They are characterized by a flux of an associated form field strength through the transverse space to the brane. To find the analytic solution for the flux branes would require solving a system of differential equations that can be casted in the form of interacting Liouville systems. The asymptotics of the flux brane geometry are the same as the asymptotics of a particular singular analytic solution.

The flux branes are non-supersymmetric and non-stable vacua of String and M-theory. We argued that for a RR flux \( (p+3) \)-brane, this vacuum can be stabilized by placing \( N \) D\( p \)-branes on the flux brane. This leads to the expansion of the D–brane to a dielectric brane, rendering the system classically stable but unstable under quantum tunneling. After taking the decoupling limit the configuration is stable both classically and quantum mechanically.

With the help of the M-theory Kaluza–Klein reduction to the type IIA theory, we were able to construct the gravity solution for a D4–brane expanded into a dielectric 2–sphere due to the presence of a flux 7–brane. This is the first exact gravity solution for the dielectric effect in the literature. Previous work has considered a spherical configuration of D3–brane charge, with a uniform source of 5-brane charge, and then solved the gravity equations perturbatively far away from the sources \cite{23}. A new venue of research is to consider \( N \) D3–branes placed on RR and/or NSNS flux 6–branes. This configuration should be of the type studied in \cite{23}.

The analysis of the probe potential in the decoupled geometry gave us some hints for what the Scherk–Schwarz reduction of the M5–branes \((2,0)\) low energy conformal field theory should be. We were successful in reproducing the mass term in the dual field theory that arises from the reduction, but the precise origin of the Myers cubic term remains unclear. We think this issue deserves further investigation since the full ‘non–abelian’ action for the M5–branes effective theory remains unknown.
Note Added

While this work was in progress two pre–prints [32, 33] appeared that have some overlap with the material presented in section 2 of this paper. In particular, they investigate the asymptotics [32, 33] and the ‘near–core’ [33] behavior of the solutions to the flux brane equations (2.14) that we had independently derived.

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