Abstract. We show that the luminosity function of the actively star-forming Lyman break galaxies and the B-band quasar luminosity function at $z = 3$ can be fit reasonably well with the mass function of collapsed galaxy scale dark matter haloes predicted by viable variants of hierarchical cold dark matter dominated cosmological models for lifetimes $t_Q$ of the optically bright phase of QSOs in the range $10^6$ to $10^8$ yr. There is a strong correlation between $t_Q$ and the required degree of non-linearity in the relation between black hole and host halo mass. Such a non-linear relation is motivated by suggesting that the mass of supermassive black holes may be limited by the back-reaction of the emitted energy on the accretion flow in a self-gravitating disc. This would imply a relation of black hole to halo mass of the form $M_{bh} \propto v_{halo}^5 \propto M_{halo}^{5/3}$ and a typical duration of the optically bright QSO phase of the order of the Salpeter time, $\sim 10^7$ yr. The high integrated local mass density of black holes inferred from recent kinematic determinations of black hole masses in nearby galaxies seem to indicate that the overall efficiency of supermassive black holes for producing blue light is lower than was previously assumed. We discuss three possible accretion modes with low optical emission efficiency: (i) accretion well above the Eddington rate, (ii) accretion obscured by dust, and (iii) accretion below the critical rate leading to an advection dominated accretion flow lasting for a Hubble time. We further argue that accretion with low optical efficiency might be closely related to the origin of the hard X-ray background.

1. Motivation

Bright QSO activity in the optical peaks at around redshift $z=2.5$ (Schmidt, Schneider & Gunn 1994; Warren, Hewett & Osmer 1994; Shaver et al. 1996; McMahon, Irwin & Hazard 1997). QSOs are believed to be powered by the accretion onto super-massive black holes at the centre of galaxies (e.g. Rees 1984 and references therein) and a number of authors have linked the changes in QSO activity to changes in the fuel supply at the centre of the host galaxies (Cavaliere & Szalay 1986; Wandel 1991; Small & Blandford 1992). Haehnelt & Rees (1993; HR93 henceforth; see also Efstathiou & Rees 1988) recognized that the peak of QSO activity coincides with the time when the first deep potential wells assemble in plausible variants of hierarchical cosmogonies. The past few years have seen dramatic observational improvements in the detection of galaxies...
out to $z > 3$, transforming our knowledge of galaxy and star formation in the high redshift universe. There are also far more extensive data on the demography of supermassive black holes in nearby galaxies, and on low level activity of AGN both in the optical and the X-ray bands. We discuss here some implications for the formation and evolution of active galactic nuclei, attempting to tie these lines of evidence from high and low redshifts together in a consistent model.

2. Relating the luminosity function of QSOs and star-forming galaxies to the mass function of dark matter (DM) haloes

Steidel and collaborators (Steidel et al. 1992; Steidel et al. 1995; Steidel et al. 1996; Giavalisco et al. 1997) have developed a technique for picking out galaxies in the redshift range $2.5 < z < 3.5$. The detected star-forming population at $z = 3$ bears a close resemblance to local star-burst galaxies. The abundance of these objects is found to be roughly half of that of $L > L^*$ present day galaxies. There are however no secure direct dynamical mass estimates for these Lyman-break galaxies. And little is known at present about the the relation of the masses of these objects to the rate of detected star formation. Strong clustering detected in these Lyman-break systems at $z \sim 3$ leads to an interpretation of these objects as the potential progenitors of massive galaxies at the present epoch.

Several groups have been engaged in the quest for high-redshift quasars. Schmidt, Schneider and Gunn (1994), detected 90 quasars by their Lyman-α emission from the Palomar Transit Grism Survey in the redshift range $2.75 - 4.75$. They find that the space density of $M_B < -26$ quasars decreases by a factor of 2.7 per unit redshift beyond $z = 2.7$. Based on their analysis they conclude that the peak of the co-moving space density distribution of quasars with $M_B < -26$ lies between redshifts $1.7 - 2.7$. The observed decline in the QSO number density could be either due to the real paucity of high-z QSOs or due to obscuration by dust introducing a systematic bias at high redshifts. In a recent paper, Shaver et. al (1996) report the results of the Parkes survey of a large sample of flat spectrum sources in the radio-waveband (that is unaffected by dust) covering roughly 40% of the sky. They claim that the space-density of radio-loud quasars does indeed decline with redshift at $z > 3$, and argue that the same conclusion probably applies to all quasars.

In what follows, we explore the link between star-forming galaxies and QSOs at high redshift, assuming that both populations trace the mass function of DM haloes. Using the Press-Schechter formalism we obtain estimates of the space density of DM haloes. The space density of these star-forming galaxies corresponds to those of haloes with masses of $10^{12.5} M_\odot$ and virial velocities of $300 \text{ km s}^{-1}$ (see also Baugh et al. 1997). Further evidence for masses of this order comes from the strong clustering of these galaxies (Steidel et al. 1997, Bagla 1997, Jing & Suto 1997, Frenk et al. 1997, Peacock 1997). A reasonable fit to the luminosity function of these objects can be obtained assuming a linear relation between star formation rate and halo mass, i.e. a constant mass-to-light ratio. A weakly non-linear relation is also consistent with the data and would indeed be required to match the shallow slope of the luminosity function at the high luminosity end reported recently by Bershady et al. (1997). Comparable
fits are obtained for the other CDM variants. Note, however, that the observed $H_\beta$ widths of these galaxies, $\sigma \sim 80$ km s$^{-1}$ (Pettini et al. 1997), might be in conflict with the virial velocities of 300 km s$^{-1}$ quoted above.

The space density of optically selected QSOs at $z=3$ with $M_B < -23$ is smaller than that of the detected star-forming galaxies by a factor of a few hundred. As demonstrated by HR93 the well-synchronized evolution of optically selected QSOs can be linked to the hierarchical growth of DM haloes on a similar timescale if the duration $t_Q$ of the optically bright phase is considerably shorter than the Hubble time. For small $t_Q$ this comes more and more in line with the predicted space density of DM haloes and that of star-forming galaxies at high redshift. However, hardly anything is known about the masses of the host objects of optically selected QSOs and this still leaves considerable freedom in the exact choice of $t_Q$. Following the approach of HR93 we estimate the formation rate of active black holes by taking the positive term of the time derivative of the halo mass function and a simple parameterization for the black hole formation efficiency. It is further assumed that active black holes radiate with a light curve of the form, $L_B(t) = f_B f_{\text{Edd}} L_{\text{Edd}} \exp \left(-t/t_Q\right)$, where $f_{\text{Edd}}$ is the ratio of bolometric to Eddington luminosity and $f_B$ is the fraction of the bolometric luminosity radiated in the B-band.

We obtain reasonable fits for a wide range of lifetimes and for all the CDM variants if we allow ourselves some freedom in the relation between halo mass and black hole mass. There are, however, systematic trends: with increasing lifetime the black hole mass has to become a progressively more nonlinear function of the halo mass and the black hole formation efficiency has to decrease in order to match the luminosity function of QSOs. This is due to the fact that QSOs are identified with rarer and more massive haloes with increasing lifetime, and these fall on successively steeper portions of the halo mass function. In Fig. 1 we plot two specific choices of parameters which we denote as model A and B hereafter (see Haehnelt, Natarajan & Rees (1998) for more details). In the lower panels a QSO lifetime close to the Salpeter timescale $t_{\text{Salp}} = \sigma c T_c / 4 \pi G m_p = 4.5 \times 10^6$ yr and a scaling of black hole mass with halo virial velocity as $M_{\text{bh}} \propto v_{\text{halo}}^5 \propto M_{\text{halo}}^{5/3} (1 + z)^{5/2}$ was assumed ($\epsilon$ is the total efficiency for transforming accreted rest mass energy into radiation). A physical motivation for this particular dependence is discussed later. The upper panel shows the case of a linear relation between the halo and black hole mass advocated by Haiman and Loeb (1997, HL97) which requires a QSO lifetime of less than $10^6$ yr – much shorter than the Salpeter time for usually assumed values of $\epsilon$. In principle $t_Q$ could also depend on mass or other parameters.

3. Local demography of black holes

The last few years have seen tremendous progress in establishing the existence of supermassive black holes, there are now a number of excellent cases (including that of our own Galaxy) where observations strongly imply the presence of a relativistic potential well (Watson & Wallin 1994; Miyoshi et al. 1995; Genzel et al. 1997). Magorrian et al. (1997; Mag97 henceforth) published a sample of about thirty estimates for the masses of the putative black holes in the bulges of nearby galaxies. Mag97 confirm previous claims of a strong correlation between
Figure 1. Model A: The B-band QSO luminosity function at $z = 3$ for 2 cosmological models computed using the time derivative of the space density of DM halos. A bolometric correction factor of 6.0. In the lower panel: a non-linear relation with $\alpha = 5/3$ between the BH mass and the halo mass with a QSO lifetime of $1 \times 10^7$ yr was assumed and in the upper panel a linear relation between the accreting BH mass and the DM halo mass with a shorter lifetime for the QSO was assumed $1 \times 10^6$ yr [the same parameters as the best-fit model explored in Haiman & Loeb 1997]. The over-plotted data points are from Boyle et al. (1988).
bulge and black hole mass (Kormendy & Richstone 1995). A linear relation of the form, $M_{bh} = 0.006 M_{bulge}$, was obtained by Mag97 as a best fit. However, considering the large scatter a mildly non-linear relation would probably also be consistent with the data. We would further like to note here that a linear relation between black hole to bulge mass does not necessarily imply a linear relation between black hole and halo mass and as we will argue later a non-linear relation might be more plausible. Fugukita, Hogan & Peebles (1997) estimate the total mass density in stellar bulges as $0.001 \leq \Omega_{bulges} \leq 0.003 h^{-1}$ and together with the above ratio of black hole to bulge mass we get,

$$\rho_{bh} = 3.3 h \times 10^6 \frac{M_{bh}}{M_{bulge}/0.006} \frac{\Omega_{bulge}/0.002 h^{-1}}{M_{\odot} Mpc^{-3}}.$$

Considering the complicated selection biases of the Mag97 sample, the small sample size and possible systematic errors in the black hole mass estimates this number is still rather uncertain. Van der Marel (1997) e.g. emphasizes the sensitivity of black hole mass estimates to the possible anisotropy of the stellar velocity distribution and argues that the Mag97 mass estimates might be systematically too high. Nevertheless, as pointed out by Phinney (1997; see also Faber et al. 1996) $\rho_{bh}(\text{nearby galaxies})$ exceeds the mass density in black holes needed to explain the blue light of QSOs purely by accretion onto super-massive black holes,

$$\rho_{acc}(\text{QSO}) = 1.4 \times 10^5 \left( f_B \epsilon / 0.01 \right)^{-1} M_{\odot} Mpc^{-3},$$

by a factor of about ten unless the value of $f_B \epsilon$ is smaller than usually assumed (Soltan 1982, Chokshi & Turner 1992). While a few years ago it seemed difficult to discover the total mass in black holes necessary to explain the blue light emitted by QSOs at high redshift, black hole detections in nearby galaxies now suggest that accretion onto supermassive black holes may actually be rather inefficient in producing blue light.

4. Constraints on the accretion history of supermassive black holes

There are three options to explain the apparently large value of

$$\rho_{bh}(\text{nearby galaxies})/\rho_{acc}(\text{QSO})$$

(i) $\rho_{bh}(\text{nearby galaxies})$ is strongly overestimated, or (ii) $f_B \epsilon$ during the optically bright phase is smaller than previously assumed, or (iii) supermassive black holes do not gain most of their mass during the optically bright phase. A plausible solution with $f_B \epsilon$ significantly smaller than 0.01 is discussed later in this section. We first explore the third possibility somewhat further. The typical mass of a black hole at the end of the optically bright phase of duration $t_Q$ exceeds that accreted during this phase by a factor $M_{bh}/M_{acc} = f_{Edd}^{-1} t_{Salp}/t_Q$. This factor should be larger than 1 and smaller than $\rho_{bh}(\text{nearby galaxies})/\rho_{acc}(\text{QSO})$ and therefore,

$$1 \leq f_{Edd}^{-1} \epsilon^{-1} (t_Q/4.5 \times 10^8 \text{ yr})^{-1} \leq 25 h (f_B \epsilon / 0.01) (\rho_{bh}/3.3 h \times 10^6 Mpc^{-3}).$$

The question when supermassive black holes gained most of their mass is therefore closely related to $t_Q$ and $f_{Edd}$. For bright quasars, $f_{Edd}$ must be $> 0.1$;
otherwise excessively massive individual black holes would be required. Furthermore, \( f_{\text{Edd}} \) will always be smaller than unity even if the ratio of the accretion rate to that necessary to sustain the Eddington luminosity, \( \dot{m} \), greatly exceeds unity. This is because a “trapping surface” develops at a radius proportional to \( \dot{m} \), within which the radiation advects inwards rather than escapes. In consequence, the emission efficiency declines inversely with \( \dot{m} \) for \( \dot{m} > 1 \) (Begelman 1978).

For \( 0.1 \leq f_{\text{Edd}} \leq 1 \) the possible range for \( t_Q \) is,

\[
2h^{-1} \times 10^6 (f_B/0.1)^{-1} (\rho_{\text{bh}}/3.3h \times 10^6 \, Mpc^{-3})^{-1} \text{yr} \leq t_Q \leq 4.5 \times 10^8 (\epsilon/0.1) \text{yr}.
\]

If \( t_Q \) is very short (as in model B) and \( f_B \epsilon \) is not significantly smaller than 0.01 it seems inevitable that supermassive black holes have acquired most of their mass before the optically bright phase. We would like to point out here that a value of \( \rho_{\text{bh}} \) as large or larger than we infer from Mag97 is actually needed for short \( t_Q \).

For the remainder of this section we assume that \( t_Q \) is of order the Salpeter time. The ratio of accreted mass to total mass at the end of the optically bright phase is then equal to \( f_{\text{Edd}}^{-1} \). If \( f_{\text{Edd}} \sim 1 \) during the optically bright phase (and if \( f_B \epsilon \) is not significantly smaller than 0.01) then the corresponding gain in mass by a factor \( \rho_{\text{bh}} \) (nearby galaxies)/\( \rho_{\text{acc}} \) (QSO) indicated by Mag97 has to occur after the optically bright phase. As the accretion should not be optically bright the most plausible options are advection dominated accretion flows (ADAFs) and dust-obscured accretion (Narayan & Yi 1995, Fabian et al. 1997 and earlier references cited therein). ADAFs require low accretion rate with \( \dot{m} < m_{\text{crit}} \) where \( m_{\text{crit}} = 0.3\alpha_{\text{ADAF}}^2 t_{\text{ADAF}}/t_{\text{Salp}} \) and \( \alpha \) is the Shakura-Sunyaev viscosity parameter. There is therefore a maximum growth factor for the black hole mass density due to ADAFs \( \sim 3.0\alpha_{\text{ADAF}}^2 t_{\text{ADAF}}/t_{\text{Salp}}(\epsilon = 1) \) and

\[
\alpha_{\text{ADAF}} > 0.3 \left[ \epsilon \rho_{\text{bh}} \right] (\rho_{\text{bh}})_{\text{QSO}}^{-0.5} \text{would be required even if the accretion lasts all the way from } z=3 \text{ to } z=0.
\]

If, however, \( f_{\text{Edd}} \sim 0.1 \) (and \( t_Q \sim t_{\text{Salp}} \)) then the gain in mass by a factor \( \rho_{\text{bh}} \) (nearby galaxies)/\( \rho_{\text{acc}} \) (QSO) has to occur before the optically bright phase as in the case of small \( t_Q \). For ADAFs this would require \( \alpha_{\text{ADAF}} \sim 1 \) and is therefore hardly plausible. In this case dust-obscured accretion would be the only viable option.

5. Possible accretion histories with low optical efficiency

Two possible accretion histories with a low overall efficiency for producing blue light are sketched in Fig. 3 - the solid curves describe an accretion history where most of the mass is accreted during the ADAF phase while the dashed curves are for an accretion history where the black hole gains most of its mass during a short-lived early phase with with \( \dot{m} > 1 \). The figure shows mass accretion rate in units of the Eddington accretion rate, the mass relative to the final mass and the optical and hard X-ray luminosity. The accretion rate is constant at the beginning with \( \dot{m} > 1 \). The mass is therefore linearly rising and \( \dot{m} \) decreases. The spectral energy distribution for accretion with \( \dot{m} > 1 \) is rather uncertain (as indicated by the three parallel lines for \( \dot{m} > 1 \) in the two bottom panels) and
Figure 2. Two accretion histories with low overall optical emission efficiencies are illustrated here.
should depend on the absorbing column and the dust content of the outer parts of the self-gravitating disc and/or the host galaxy. The sharp drop of $\dot{m}$ marks the onset of the back-reaction on the accretion flow and either the start or the peak of the optical bright phase (with a rather inefficient production of hard X-rays). Once the accretion rate has fallen below the critical rate for an ADAF (indicated by the dashed lines in the top panel) the spectral energy distribution will change to one peaked in the hard X-ray waveband.

6. Faint X-ray sources and the hard X-ray background

As pointed out by many authors, the X-ray emission of optically selected QSOs is too soft to explain the hard X-ray background. Di Matteo & Fabian (1996) and Yi & Boughn (1997) argued that the emission from ADAFs has a spectral shape similar to the hard X-ray background. Fabian et al. (1997) suggested that this might also be true for dust-obscured accretion. It is therefore tempting to link the rather large value of $\rho_{bh}(\text{nearby galaxies})/\rho_{acc}(\text{QSO})$ inferred from Mag97 to the origin of the hard X-ray background and the recently detected large space density of faint X-ray sources (Almaini et al. 1996; Hasinger et al. 1997, Schmidt et al. 1997, McHardy et al. 1997, Hasinger 1998). The presence of extremely low-level optical AGN activity in a large fraction of galaxies reported by Ho, Filippenko & Sargent (1997) would also fit in nicely with such a picture. The efficiency of ADAFs is $\epsilon_{\text{ADAF}} = 0.1 (\alpha/0.3)^2 \dot{m}/\dot{m}_{\text{crit}}$ and decreases rapidly for small $\alpha$ and small $\dot{m}/\dot{m}_{\text{crit}}$. If the hard-X-ray background was produced by ADAFs onto supermassive black holes in ordinary galaxies this requires a value of $\rho_{bh}(\text{nearby galaxies})$ as high as we infer from Mag97, a large value of $\alpha$ and a value of $\dot{m}$ below but still close to $\dot{m}_{\text{crit}}$ lasting for a Hubble time for the majority of supermassive black holes. At faint flux levels and high redshifts a possible star-burst contribution to the total spectral energy distribution will become more and more important in the optical and probably also the soft X-ray. It is interesting to note here that the recently detected number counts in the sub-mm wave-band by SCUBA could be evidence for dust-obscured accreting AGN at high redshifts (Blain et al. 1998; Almaini et al. 1998).

7. Conclusions

The optical QSO luminosity function at $z \sim 3$ can be plausibly matched with the luminosity function of star forming galaxies at the same redshift and the mass function of DM haloes predicted by a range of variants of CDM cosmogonies believed to comply with observational constraints in the low-redshift universe. This is possible for lifetimes of optically bright QSOs anywhere in the range $10^6$ to $10^8$ yr. There is a correlation between the lifetime and the required degree of non-linearity in the relation between black hole and halo mass. The non-linearity has to increase for increasing lifetime. Predicted host halo masses, host galaxy luminosities, and the clustering strength all increase with increasing lifetime and further observations of these offer our best hope of constraining the duration of the optically bright phase of QSOs.

The present-day black hole mass density implied by the integrated luminosities of optically bright QSO may be significantly smaller than that inferred from
recent black hole estimates in nearby galaxies for generally assumed efficiencies for producing blue light. We have discussed three possibilities for how and when this mass could be accreted in an optically inconspicuous way: (i) in the early stages of accretion at rates far above the Eddington rate, (ii) by accretion where optical emission is obscured by dust, or (iii) in the late stages of accretion at a rate below the critical rate for an advection dominated accretion flow with an Shakura-Sunyaev parameter of $\alpha_{\text{ADAF}} > 0.3$.

Acknowledgments. The Isaac Newton Institute for Mathematical Sciences and specifically the program on the Dynamics of Astrophysical Discs are gratefully acknowledged for providing a lively scientific environment. Thanks are due to my collaborators Martin Haehnelt and Martin Rees for permission to present results from our joint work.

References

Almaini O., Lawrence, A., Boyle B.J., 1998, preprint, submitted to MNRAS
Almaini O., Fabian A.C., 1997, MNRAS, 288, L19
Almaini O., Boyle B.J., Shanks T., Griffiths R.E., Roche N., Stewart G.C., Geogantopoulos I., MNRAS, 1996, 282, 295
Bagla J. S., 1997, submitted to MNRAS, astro-ph/9711081
Barnes J., Efstaithiou G., 1987, ApJ, 319, 575
Baugh C.M., Frenk C.S., Lacey C., 1997, submitted to ApJ, astro-ph/970311
Begelman M. C., 1978, MNRAS, 184, 53
Bershady M., Majewski S.R., Koo D.C., Kron R.G., Munn A., 1997, ApJ, in press, astro-ph/9709216
Blain, A. W., Smail, I., Ivison, R., & Kneib, J-P., 1998, MNRAS, in press astro-ph/9806062.
Bond J.R, Arnett W.D., Carr B.J., 1984, ApJ, 280, 825
Cavaliere A., Szalay A., 1986, ApJ, 311, 589
Cavaliere A., Vittorini V., 1997, in Müller V. et al., 1997, Proc. 12th Potsdam cosmology workshop., preprint
Chokshi A., Turner E. L., 1992, MNRAS, 259, 421
Di Matteo T., Fabian A. C., 1997, MNRAS, 286, 393
Efstaithiou G. P., Rees M. J., 1988, MNRAS, 230, 5p
Efstathiou G. P., Bond J. R., White S. D. M., 1992, MNRAS, 258, 1p
Faber S.M. et al., 1997, AJ, 114, 1771
Fabbian A.C., Barcons X., Almaini O., Iwasawa K., MNRAS, preprint
Fugukita M., Hogan C.J., Peebles P.J.E. 1997, submitted to ApJ, astro-ph/9712020
Genzel R., Eckart A., Ott T., Eisenhauer F., 1997, MNRAS, 201, 219
Giavalisco M., Steidel C. S. Macchetto F. D., 1996, ApJ, 470, 189
Haehnelt, M., Natarajan, P., & Rees, M. J., 1998, MNRAS in press.
Haehnelt M.G., Rees M.J., 1993, MNRAS, 263, 168 (HR93)
Haehnelt M.G., Steinmetz M., 1997, MNRAS, in press
Haiman Z., Loeb A., 1997, submitted to ApJ, astro-ph/9710208 (HL97)
Hasinger G., 1998, Astron. Nachr, in press, preprint
Hasinger G., Burg R., Giacconi R., Schmidt J., Trümper J., Zamaroni G., 1997, AA in press, astro-ph/9709142
Jenkins A. et al., 1997, submitted to ApJ, astro-ph/9709010
Jing J.P., Suto Y., 1997, submitted to ApJ, astro-ph/9710090
Kaiser N., 1984, ApJ 284, L9
La Franca F., Andreani P., Cristiani S., 1997, ApJ, in press, astro-ph/9711048
Magorrian J., et al., 1997, submitted to AJ, astro-ph/9708072
McHardy, et al., 1997, MNRAS, in press, astro-ph/9703163
Miyoshi M., Moran M., Hernstein J., Greenhill L., Nakai N., Diamond P., Inoue N., 1995, Nature, 373, 127
Narayan R., Yi I., 1995, ApJ, 452, 710
Natarajan P., Sigurdsson S., Silk J., 1997, MNRAS, 298, 577
Peacock J.A., 1997, KNAW Colloquium, The most distant radio galaxies, astro-ph/9712068
Phinney E. S., 1997, talk presented at the IAU Symposium 186 Kyoto 1997
Rees M. J., 1984, ARAA, 22, 471
Schmidt M., Schneider D. P., Gunn J. E., 1994, AJ, 110, 68
Schmidt M. et al., 1997, AA, in press, astro-ph/9709144
Shaver P., Wall J. V., Kellermann, K. I., Jackson C. A., Hawkins M. R. S., 1996, Nature, 384, 439
Silk J., Rees M.J., 1998, A&A, in press
Shlosman I., Begelman M.C., Julian F., 1990, Nat., 345, 679
Small T. A., Blandford R. D., 1992, MNRAS, 259, 725
Soltan A., 1982, MNRAS, 200, 115.
Steidel C. S., Hamilton D., 1992, AJ, 104, 941
Steidel C. S., Pettini M., Hamilton D., 1995, AJ, 110, 2519
Steidel C. S., Giavalisco M., Pettini M., Dickinson M., Adelberger K. L, 1996, ApJ, 462, L17
Steidel C. S., Adelberger K., Dickinson M., Giavalisco M., Pettini M., 1997, ApJ, in press, astro-ph/9708125
van der Marel R. P., 1997, in Sanders D.B., Barnes J., eds, IAU Symposium 186 Kyoto 1997. Kluwer, astro-ph/9712076
Wandel A., 1991, AA, 241, 5
Warren S.J., Hewitt P.C., Osmer P.S., 1994, ApJ, 421, 412
Watson W.D., Wallin B.K., 1994, ApJ, 432, L35
White S.D.M., Efstathiou G., Frenk C., 1993, MNRAS, 262, 1024
Yi I., Boughn S.P., 1997, submitted to ApJ, astro-ph/9710147
