Study of the antivibration suspended seat oscillations with quasi-zero stiffness effect under sinusoidal excitation

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Abstract. The relevant task of reducing the vibrations transmitted to a human operator of a construction or road vehicle during operating process is accomplished, among other things, by conducting the research on mathematical models. Oscillations simulation of the human operator's seat antivibration suspension by means of the numerical solution of the ordinary differential equations system remains one of the main methods of the study, used in particular for the discrete mathematical models verification. Therefore, the problem of determining the rational value of the maximum integration step by using the numerical method in solving the systems of the ordinary differential equations describing the operator's anti-vibration suspended seat oscillations is relevant. A discrete mathematical model of a human operator's seat performing the forced vertical oscillations during kinematic excitation of base movements was developed through the use of the differential equation of the translational oscillations of mass on a movable base. The prescribed displacements of the seat base are described by the harmonic oscillation equation. The numerical solution of the ordinary differential equations system is carried out via the built-in ode45 function of the MATLAB mathematical modeling system. Moreover, the parameters of the developed mathematical model are described, the calculation scheme and an example of a static force characteristic including the quasi-zero stiffness region in the middle section of the characteristic are given. The determination accuracy of the maximum acceleration of the seat in the steady-state oscillation mode is shown to decrease when the value of the maximum allowable integration step increases. It is recommended to limit the value of the maximum allowable integration step to one hundredth of a second. Besides, the effect of the values duality of the maximum acceleration and maximum internal movement of the seat relative to its own base with small changes in the base displacement amplitude, which must be taken into account in modeling, is also revealed.

Key-words: vibration protection, oscillations, acceleration, quasi-zero stiffness, integration step

1. Introduction

Construction, road and handling machines are often used in quite severe conditions of high loads, vibrations and shocks [1]. Such operating conditions are typical for such machines as motor graders [2], excavators [3], front-end loaders [4], bulldozers [5], scrapers [6], utility vehicles [7], as well as forklift trucks [8], various hand tools such as jackhammers [9] and brush cutters [10].
Vibrations can cause occupational diseases among equipment operators [11] and reduce the operational performance [12], which is proved by numerous studies [13]. It should be noted that not only construction, road and other processing equipment [14], but also the chassis of tractors and other self-propelled vehicles are subjected to the harmful impact of vibrations [15].

Vibration protection systems and suspensions of construction, road and other self-propelled vehicles cabs are largely capable to neutralize the harmful effects of vibrations on the human operator [16]. However, applying the antivibration suspensions of equipment operators' seats is a more simple, reliable and cost-efficient method of vibration protection [17]. Both passive and active (semi-active) vibration protection systems are applied in seat suspensions [18]. Besides, the pneumatic shells are widely used [19]. Nowadays, when developing the vibration protection systems, one of the promising directions is the use of static force characteristics (the machine force dependence on the displacement) with a section of quasi-zero stiffness [20].

Vibration protection systems with a single translational degree of freedom are developed and utilized mainly for the operator's seats of construction, road and other self-propelled vehicles [21]. The active suspensions with an adjustable static force value are characterized by very good results in reducing vibrations, but high requirements for the energy consumption and high cost of such suspensions, as well as the problem of their potential instability and unreliability prevent the implementation of the active vibration protection systems [22]. In terms of reliability, simplicity and cost-effectiveness, the passive vibration protection systems based only on the machine elements, such as levers, springs, sliders and shock absorbers retain their advantages. In this regard, the most simple systems, for instance structures based on a double Mises truss (figure 1, a) [23] and similar to them are preferred. To date, quite a large number of the mentioned structures have been developed.

Under the kinematic excitation of a vibration-isolated object of a human operator's seat, vibrations of the seat base are examined. In this case, the purpose of vibration protection is to reduce the amplitude of oscillations transferred on the mass of the human operator's seat. Another purpose of vibration protection could be to reduce the vibration acceleration of the mentioned mass. In any case, one of the most significant and currently even necessary stages of the development and improvement of the vibration protection systems is to conduct the studies on mathematical models [10].

The research objective is to develop and study the practical application aspects of dynamic simulation models of passive vibration protection systems with one degree of freedom and with kinematic excitation of the base movement of such a system. It is such systems that can be used for vibration protection of the human operator's seat of construction and road vehicles. The sinusoidal excitation is most widely used in evaluating the models and real-life objects of antivibration suspensions. Consequently, the model should provide the possibility of studying the object displacements under the given type of the kinematic excitation. It is necessary to find out the problems faced by the researchers applying the mathematical models of the vibration protection systems at the stage of the proposed

![Figure 1](image_url)
solutions verification. Various designs of antivibration suspensions can be described using a static force characteristic.

2. Problem statement
A dynamic system with a concentrated mass \( m \) (the total mass of the seat and human operator i.e. a vibration-protected object) and one translational degree of freedom \( Z \) has been studied. The vertical coordinate of the vibration-protected object mass center in the fixed coordinate system (CS) is denoted by \( z \). Time vertical coordinate derivatives are denoted by the following: \( v = \frac{dz}{dt} \); \( a = \frac{dv}{dt} \). The variable coordinate of the seat base (the base chassis of the vehicle) performing the specified movements is denoted by \( z_{op} \). The seat movements relative to its base are denoted by \( z_1 \) (figure 2).

Thus, although different suspension designs of the passive antivibration systems of seats with one degree of freedom are known, their static force characteristic can be expressed as a three-segment spline, which middle segment is a horizontal straight line (a section of quasi-zero stiffness), while two outermost segments are curved splines of arbitrary order (figure 1, b). In a particular case, the number of segments can be reduced to two or even one describing the entire characteristic at once.

The following symbols were also accepted: \( F \) is the static force generated by the suspension; \( b \) is the suspension damping coefficient; \( A_{mp} \) is the vertical displacements amplitude of the seat base; \( T_0 \) is the period of the seat base oscillations; \( w \) is the circular frequency of the seat base oscillations; \( T_{kon} \) is the final simulation time; \( T_{изм} \) is the measurement time of the vibration characteristics at the final stage of the simulation time; \( dt_{max} \) is the maximum time step for the numerical solution of a system of the differential equations describing the system; \( z_{q1}, z_2 \) are the segments dimensions of the suspension static force characteristic (see figure 1, b).

The base performs the specified sinusoidal movements expressed as a time dependence \( z_{op} = A_{mp} \cdot \sin (w \cdot t) \). There is an obvious dimensional relationship \( z_1 = z - z_{op} \) between the three linear quantities \( z, z_1, \) and \( z_{op} \) shown in figure 2. The equilibrium value of \( z_1 \) is zero.

It is necessary to calculate the maximum acceleration of the vibration-protected mass \( m \) in a fixed coordinate system in a steady-state motion mode which is denoted by \( a_{max} \). Another indicator characterizing the process can be used in the appropriate cases, for example, the average square acceleration of the vibration-protected mass, etc.

The task of a higher hierarchical level is to obtain the dependences of the vibration-protected mass maximum acceleration on the base forced vibrations amplitude \( A_{mp} \) at the fixed values of the seat base oscillations frequency \( w \).
Moreover, another important objective of the research is to study the maximum integration step impact for the numerical solution of the differential equations system $dt_{\text{max}}$ on the results values presented in the form of the maximum accelerations of the vibration-protected mass $a_{\text{max}}$.

3. Theory

Let us introduce the additional notation: $v_i = \frac{dz_i}{dt}$.

The differential equation describing the movements of the vibration-protected mass without taking into consideration the gravity (which is equivalent to the movements of the same mass taking into account the gravity, when the static force characteristic is shifted up along the force axis by the gravity value), can be presented in the form of Cauchy:

$$m \cdot a + b \cdot v_i + F = 0.$$  (1)

If the base coordinate variable $z_{\text{op}}$ is used in the differential equation (1), under the kinematic excitation, this equation takes the following form:

$$a = \frac{-b \cdot d(z - z_{\text{op}})}{dt} - \frac{F}{m} = \frac{-b \cdot (v - A \cdot \omega \cdot \cos(\omega \cdot t))}{m},$$  (2)

where the static force $F$ generated by the vibration protection mechanism, if the three-segment spline shown in figure 1,b is used, is calculated by the dependence:

$$F = \begin{cases} \text{sgn}(z_{11}) \cdot \left( s_1 \cdot |z_{11}|^6 + s_2 \cdot |z_{11}|^4 + s_3 \cdot |z_{11}|^2 + s_4 \cdot |z_{11}| + s_5 \cdot |z_{11}| + s_6 \right) & \text{at } |z_{11}| > \frac{z_{g\text{e}}}{2}; \\ 0 & \text{at } |z_{11}| \leq \frac{z_{g\text{e}}}{2}, \end{cases}$$  (3)

where the parameter $z_{11}$ is calculated by the following formula:

$$z_{11} = \left| z_i \right| - \frac{z_{g\text{e}}}{2}.$$

The constant coefficients $s_1, s_2, s_3, s_4, s_5, s_6$ were defined as the corresponding coefficients of the Hermite spline characterized by the maximum second order derivative [24]:

$$s_1 = \frac{a_{FT} + 6F_0}{2} \frac{z_{z_2}^2}{z_3^3} - \frac{3v_{FT}}{z_2^2} \frac{a_{F0} + 6F_0}{2} \frac{z_{z_2}^2}{z_3^3} + \frac{3v_{F0}}{z_2^2} + \frac{3F_0}{z_2^2};$$

$$s_2 = \frac{3a_{F0} + 18F_0}{2} \frac{z_{z_2}^2}{z_3^3} + \frac{9v_{F0}}{z_2^2} - \frac{a_{FT} z_2 - 7v_{FT} + 15F_0 z_2}{z_3^3} + \frac{v_{F0} z_2}{z_2^2} + \frac{3F_0}{z_2^2};$$

$$s_3 = \frac{a_{FT} z_2^2}{2} - 4v_{FT} z_2 + 10F_0 z_2^2 \frac{3a_{F0} + 18F_0}{2} \frac{z_{z_2}^2}{z_3^3} + \frac{9v_{F0}}{z_2^2} + \frac{3v_{F0} + 9F_0}{z_2^2};$$

$$s_4 = \frac{a_{F0} + 9F_0}{2} \frac{z_{z_2}^2}{z_2^2} + \frac{3v_{F0}}{z_2^2} - \frac{3v_{F0} + 9F_0}{z_2^2}; \quad s_5 = v_{F0}; \quad s_6 = F_0,$$

where $F_0, v_{F0}, a_{F0}$ are the values of zero, first and second derivatives of the vibration protection mechanism static force at the boundary of the quasi-zero stiffness section; $F_T, v_{FT}, a_{FT}$ are the values of zero, first and second derivatives of the vibration protection mechanism static force at the external
boundary of the external segment described by the Hermite spline. This point in the real physical mechanism corresponds to the achievement of the top or bottom bump stops performing the uncushioned stop of the mechanism.

In the Cauchy form, a system of two first-order ordinary differential equations (ODE), to which one second-order ODE (2) is reduced, has the following form:

$$\begin{align*}
    \dot{v} &= -\frac{b}{m} \cdot \left( v - A \cdot \omega \cdot \cos(\omega \cdot t) \right) - F; \\
    \ddot{z} &= v.
\end{align*}$$

(5)

The numerical integration of the ODE system (5) was carried out using the built-in function of ode45 differential equations systems solution of the MATLAB system [25]. In this case, the maximum integration step for using in the ode45 function was set by the MaxStep and accepted the specified value $dt_{\text{max}}$.

ODE systems solution functions in the MATLAB package including ode45 can only output the values of the integrable variables, but not their derivatives, i.e. in this case, the coordinate and velocity values of the vibration-protected mass $z$ and $v$ of the system (5) in the given time interval $[0; T_{\text{kon}}]$. Therefore, after solving the ODE system (5) for calculating the acceleration time values $a$ of the vibration-protected mass, the velocity values $v$ with a uniform step $dt_{\text{max}}$ were first interpolated and then numerical differentiation was carried out according to the following formula [26]:

$$a_i = \frac{(v_i - v_{i-1})}{dt_{\text{max}}}$$

(6)

The maximum acceleration value of the vibration-protected mass $a_{\text{max}}$ in a particular transition process was defined as the maximum absolute value of the time sequence of acceleration $a_i$ obtained by equation (6) in the time interval $[(T_{\text{kon}} - T_{\text{izm}}); T_{\text{kon}}]$ when forced oscillations assumed a steady character.

4. Experimental results

The example of the static force characteristic of the suspension shown in figure 1,b is equivalent to the static characteristic with the same curve shape, which force value in the quasi-zero stiffness segment is different from zero (real suspensions), provided that gravity is not taken into account in the mathematical model. This made it possible to simplify the mathematical formulation of the object and reduce the computational costs of modeling.

The results, some of which are presented in figure 3, have been obtained from the computational experiments using the developed mathematical model. The constant parameters of the vibration protection system and transition process accepted the following values: $b=20$ N/(m/s); $A_{mp}=0.1$ m; $T_{a}=5$ s; $T_{kon}=300$ s; $T_{izm}=50$ s; $z_{0}=0.1$ m; $z_{2}=0.1$ m; $F_{0}=0$ N, $v_{F0}=0$ N/m, $a_{F0}=0$ N/m$^2$, $F_{T}=1000$ N, $v_{FT}=20000$ N/m, $a_{FT}=0$ N/m$^2$. 
Figure 3. The examples of the acceleration dependencies: (a) is the total time dependence of the acceleration at \( dt_{\text{max}} = 0.1 \) s; (b) is the functional dependence of the maximum acceleration on the time step \( dt_{\text{max}} \); (c) is the peaks fragment of the acceleration time dependence at \( dt_{\text{max}} = 0.1 \) s; (d) is the local maximum acceleration at \( dt_{\text{max}} = 0.001 \) s; (e) is the peaks fragment of the acceleration time dependence at \( dt_{\text{max}} = 0.001 \) s; (f) is the local maximum acceleration at \( dt_{\text{max}} = 0.001 \) s.

Figure 4. The dependences examples of the maximum acceleration (a) and maximum amplitude of the seat displacement relative to its own base (b) on the base vibrations amplitude.
The maximum integration step in this series of experiments took the variable values in the following range of \( dt_{\text{max}} = [0.001; 0.1] \). In the second series of experiments, the maximum integration step was assumed to be constant and equals to \( dt_{\text{max}} = 0.001 \) s. The variable parameter was the seat base forced vibrations amplitude varied in the range of \( A_{\text{mp}} = [0.0005; 0.3] \) m with a step of 0.0005 m. The other parameters took the values given above.

The results of the second series of computational experiments are shown in figure 4.

5. Results discussion

The dependences shown in figures 3 and 4 are part of a larger amount of results obtained at other parameters values, particularly at the lower values of the base forced oscillations period \( T_0 \). Increasing the maximum integration step \( dt_{\text{max}} \) from 0.001 to 0.1 s causes an additional error in defining the maximum acceleration in the range from 3 to 4%.

The change in the forced vibrations amplitude of the seat base \( A_{\text{mp}} \) in the considered limits exceeding the value of the quasi-zero stiffness section \( z_{\text{qz}} \) results in some uncertainty in the values of both the maximum acceleration and displacement \( z_{\text{1}} \) when changing the forced oscillations amplitude. In the region of the vibration protection mechanism \( z_{\text{1}} \) internal displacement expanding beyond the limits of the static power characteristics quasi-zero section (i.e. in the region of the high amplitudes), multiple abrupt changes of the seat maximum acceleration values \( a_{\text{max}} \) at a sufficiently small amplitude increment are observed. The same applies to the maximum value of the internal displacements of the vibration protection mechanism \( z_{\text{1}} \). Moreover, both maximum and minimum values of the mentioned parameters change according to the certain curvilinear dependencies (see figure 4). Therefore, abrupt changes in these parameters occur within the certain limits reaching 50% at the relatively small amplitudes.

6. Conclusions

The forced oscillations of a single-mass mechanical system with one translational degree of freedom were studied on the developed mathematical model. This model describes the vertical vibrations of a human operator's seat of a construction vehicle. The list of the system considered parameters includes the human operator and seat mass, the viscous friction coefficient of the vibration isolation mechanism, the static force characteristics of the vibration isolation mechanism, the kinematic excitation parameters of the seat base sinusoidal oscillations, the maximum allowable integration step. The built-in ode45 function of the MATLAB system was used to solve the differential equations system.

The maximum allowable step of the numerical integration of the differential equations system was found out to have a significant impact on the accuracy of the obtained seat acceleration values. When the maximum allowable step of the numerical integration is increased to 0.1 s, the error of the seat maximum acceleration in the steady-state mode reaches from 3 to 4%.

It is appropriate to limit the maximum allowable step of the numerical integration of the differential equations system to 0.01 s.

The maximum acceleration and maximum amplitude of the seat displacements relative to its own base is characterized by the uncertainty or more precisely the duality of the obtained values depending on the base oscillations amplitude. It is shown in the abrupt significant changes in the values of the maximum acceleration and maximum amplitude of the seat displacements with small variations in the base oscillations amplitude. Moreover, the same uncertainty is observed not only for the maximum acceleration, but also for the root-mean-square acceleration of the seat. Increasing the total modeling time (at the constant and sufficient measurement time of the vibration characteristics at the final stage of 50 s) does not eliminate this uncertainty and hardly reduces the parameter values jumps. At the same time, both values of the seat acceleration (maximum and minimum) are expected to increase as the base oscillation amplitude increases. This dependence in a wide range of the argument values changing is close to the linear one. The resulting uncertainty must be taken into consideration when the computational and experimental testing of the theoretical conclusions obtained in the form of the
analytical expressions for the vibration protection systems with one degree of freedom is performed and if the test is performed on a simulation mathematical model in the form of solving ODE systems.

7. References
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