Deflections of continuous reinforced concrete elements

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Abstract. The results of experimental investigations of deflections of continuous reinforced concrete elements are obtained, empirical dependences are deduced. Comparison of the deflection values with the results of calculations by the author's method of NIISK (Bambura A.M.), adapted deformation model of OSACEA, what was realized with the help of the mathematical program MATLAB and also the algorithm of theoretical calculations of the finite element method was developed using the mathematical-graphical environment of the LIRA-SOFT software complex. The results of experimental investigations of deflections of continuous beams were compared with theoretical data (graphical representations of displacements in the form of isofields were obtained). The finite element calculation allowed to monitor the stress-strain state of the test beams at all stages of work, which made it possible to compare the obtained experimental values of the deflections with the designed ones. The performed experiments confirmed the feasibility of taking into account the shear deformations on the supporting sections in determining the deflections of reinforced concrete beams.

1. Introduction
Deformation calculation is of great importance when designing reinforced concrete elements and structures. It is known that the occurrence of one of the boundary states of structures, is characterized precisely by the development of excessive deformations (displacements) from static and dynamic loads.

The estimation of the curvature and displacement of the cross-section is also necessary in determining internal forces in statically indeterminate systems, both in the exploitation stage and before failure, and not only from force effects, but also from temperature fluctuations, shrinkage of concrete and displacement of supports.

2. Analysis of publications
The strength characteristics of reinforced concrete elements depend significantly on the method of their reinforcement and the composition of the concrete, which is confirmed by many studies of various building structures [1-11].
Investigation of continuous reinforced concrete beams was done by V.Ye. Babich [1-5], A.M. Bambura [6] and many others scientists. It is confirmed that the redistribution of forces in the first stages of loading is affected by cracks, and even before the destruction is significantly affected by plastic deformation in concrete and reinforcement.

3. The purpose of research
The purpose of the study is to improve the method of calculation of deflections of continuous reinforced concrete beams, to compare the experimental values of deflections of two-span reinforced concrete beams with its calculated values, calculated by the most common methods and the proposed advanced method of calculation.

4. Research results
Below are the results of experimental and theoretical studies.

4.1. Analysis of the results of the experiment
The deflections of the prototypes [7] were measured at six points: in the middle of spans, under concentrated forces and at the free ends of the beams (Figure 1). The emergence of the first normal cracks and subsequent emergence inclined ones in the shear-span was accompanied by a sharp increase in deflections.

![Figure 1. Photo recording of preparing and conducting the experiment: (a) - installation of dial gauge, affixing strain gauges to determine deformations (linear, angular displacements); (b) - photo recording of the place of installation of the dynamometer for control measurement of the deflection of the continuous beam at the location of the middle support; (c) – conducting the experiment.](image1)

The increase in the magnitude of the deflection during the occurrence of these cracks was not proportional to the increase in the external load. As the bearing capacity was exhausted, the deflection beams increased significantly even with a slight increase in the external load. This is due, on the one hand, to non-linear deformations of compressed concrete with a large percentage of longitudinal reinforcement or with a small amount of stretched reinforcement, and, on the other hand, to shear deformations in the shear span caused by the combined action of the bending moment $M$ and $V$.

The research methodology, the main characteristics of the test specimens-beams and the design schemes of power units are given in [8, 9].

According to the results of the experiment, new data were obtained, namely adequate mathematical models characterizing deflections of continuous beams in the middle of spans and under concentrated forces in the exploitation stage (1), (2) and before destruction (3), (4):

$$
\hat{f}_{p6F} = 1.53 + 0.12X_1 + 0.26X_4 + 0.18X_5 + 0.14X_6^2 - 0.07X_1X_2 + 0.05X_1X_4 + 0.07X_4X_5, \Sigma = 0.22, \sigma = 0.09, \nu = 6.0% 
$$

(1)
The magnitude of the deflections under concentrated forces (2), (4) is most affected by the relative shear-span \( X \). The deflections in the span and under the concentrated forces in the "exploitation stage" depend on the same factors (1), (2). Thus, they increase relative to their average of 1.53 and 0.274 with an increase in the relative shear-span from 1 to 3, respectively, by 15.7 and 314 \%, with an increase in the amount of lower longitudinal reinforcement from 0.0101 to 0.0199 by 34 and 51 \%, with an increase in the amount of upper longitudinal reinforcement from 0.0101 to 0.0199 by 24 and 29 \%.

Bends in the middle of spans (3) before destruction increase relative to the average value of 3.02: with the increase of the shear-span from 1 to 3 - by 27 \%, with the increase in the amount of lower longitudinal reinforcement from 0.0101 to 0.0199 - by 5.6 \%.

4.2. An advanced method of calculation
The methods for determining the deflections of the investigated elements are based on the incomplete Mohr integral. It is recommended to define deflections by the formula:

\[
\tilde{Y}_{af} = \frac{0.274 + 0.43X_1 + 0.07X_2 + 0.04X_3 + 0.184X_4^2 + 0.014X_5^2 - 0.034X_6X_7 + 0.07X_8X_9}{0.02X_1X_2 + \Sigma}, \quad \Sigma = 0.008, \sigma = 0.018, \nu = 6.4\%,
\]

\[
\tilde{Y}_{sp} = \frac{3.02 + 0.41X_1 + 0.084X_4 + 1.01X_2^2 - 0.19X_3^2 + 0.08X_2X_4 + 0.15X_4X_5}{0.770, \sigma = 0.172, \nu = 5.7\%},
\]

\[
\tilde{Y}_{af} = \frac{1.35 + 1.71X_1 + 0.99X_2 - 0.07X_3^2 - 0.04X_4^2 + 0.13X_2X_4 + 0.126X_4X_5}{0.139, \sigma = 0.073, \nu = 5.4\%}.
\]

\[
(2)
\]

\[
(3)
\]

\[
(4)
\]

\[
(5)
\]

The methods for determining the deflections of the investigated elements are based on the incomplete Mohr integral. It is recommended to define deflections by the formula:

\[
f = f_m + f_q = \int_0^L \frac{1}{r} M_x \left( \frac{1}{r} \right) dx + \int_0^L V_x \times \gamma_x \times dx
\]

where, \( \overline{M} \) and \( \overline{V}_x \) - is the bending moment and the transverse force in the \( i \)-th section what occurred from the action of a single force applied in the direction of the desired displacement in the cross-section "\( i \)" for which the deflection is determined;

\[
\left( \frac{1}{r} \right) - the full value of the curvature of the element in section X from the load from which the deflection is determined;
\]

\[
\gamma_x \times \gamma_x \times dx
\]

\[
\gamma_x \times \frac{1.5V_x \times \varphi_{h2} \times \varphi_{crc}}{G_h \times b \times h_0},
\]

where, \( V_x \) - transverse force in section X from the action of external force; \( \varphi_{h2} \) - correction factor that takes into account the impact of long-term creep; \( G_h \) - concrete shear module; \( \varphi_{crc} \) - coefficient that takes into account the effect of cracks on shear deformation. In the absence of cracks equal to 1, and in the presence of inclined 4.8.

In the presence of normal and inclined cracks this factor is determined by the formulas:

\[
\varphi_{crc}^{sup} = \frac{3(E_h \times I_{rel})_{sup}}{M_{sup} \left( \frac{1}{r} \right)_{sup}},
\]

\[
\varphi_{crc}^{af} = \frac{3(E_h \times I_{rel})_{af}}{M_{af} \left( \frac{1}{r} \right)_{af}},
\]

\[
(7)
\]

\[
(8)
\]
where, \( M_{sp} \), \((1/r)_{sp}\), \( M_{uf} \), \((1/r)_{uf}\) - in accordance, moments and curvatures of the cross-section above the support and under concentrated forces.

The indicated curvatures are determined by the average values of the relative deformations of reinforcement and concrete in the areas between the cracks. Before the appearance of the plastic hinge over the middle support (Figure 2a), the deflection caused by the moment can be determined by the simplified formula:

\[
f_m = \frac{M}{3} \left[ \frac{1}{r_{uf}} (d + \lambda) + \frac{1}{r_{sup}} (a - \lambda) \right],
\]

where, \( \lambda = \frac{M_{sp} \times a}{M_{sp} + M_{uf}} \), which takes into account the proportionality of the change of moments and the curvature of the beam invariably along the length of the section without pre-stressing the reinforcement.

(a) Figure 2. The design scheme of the investigated element (two-span beams): (a) - in the elastic stage of the work of materials in determining the curvature; (b) - the left part of the beam after the occurrence of the plastic hinge above the middle support.

The deflections caused by displacement deformations are proposed to be determined by the formula:

\[
f_q = \int_0^l \frac{\rho_{y,q}}{G \times b \times h} \left( V_B' \times a \times \phi_{qc}^{w^{a}} + V_d' \times d \times \phi_{qc}^{w^{d}} \right),
\]

after the appearance of the plastic hinge (Figure 2b):
\[ \lambda = \frac{M_{ef} \times c}{\sqrt{M_{ef}^2 + M_{af}^2}} \]  

(11)

Deflections are then determined by formulas (9) and (10).

To determine the maximum deflection, a single force is applied in the middle of the spans. To determine deflections in the short-term action of loading in relatively short beams (\( L < 10h \)) use formula (5), at \( L \geq 10h \) exposure to the second addition of formula (5), can be neglected.

**Table 1.** Comparison of calculated and experimental values of deflections of the test beams.

| № experiment | Experimental values | Calculated values |
|--------------|---------------------|------------------|
|              | Deflections in the middle of the span \( f_{ef} \) | \( f_{sp} \) | \( f_{uf} \) | \( f_{sp} \) | \( f_{uf} \) | \( f_{sp} \) | \( f_{uf} \) | \( f_{sp} \) | \( f_{uf} \) | \( f_{sp} \) | \( f_{uf} \) |
|              | under concentrated force \( f_{cp} \) | \( f_{uf} \) | \( f_{uf} \) | \( f_{uf} \) | \( f_{uf} \) | \( f_{uf} \) | \( f_{uf} \) | \( f_{uf} \) | \( f_{uf} \) | \( f_{uf} \) | \( f_{uf} \) |
| 1            | 4.85                | 4.32             | 5.82             | 5.03             | 4.01             | 3.37             | 4.81             | 4.04             | 4.98             | 4.54             | 1.57             | 1.57             |
| 2            | 3.47                | 0.44             | 1.22             | 0.14             | 4.28             | 0.57             | 5.14             | 0.68             | 4.01             | 0.45             | 3.31             | 1.40             |
| 3            | 3.53                | 0.51             | 1.04             | 0.13             | 2.17             | 0.28             | 2.60             | 0.34             | 3.64             | 0.39             | 0.56             | 0.47             |
| 4            | 4.31                | 4.09             | 4.84             | 4.19             | 3.16             | 2.65             | 3.79             | 3.18             | 4.88             | 4.43             | 1.80             | 1.80             |
| 5            | 3.52                | 0.55             | 1.87             | 0.22             | 3.69             | 0.48             | 4.43             | 0.58             | 3.98             | 0.43             | 0.26             | 1.39             |
| 6            | 4.46                | 4.22             | 5.90             | 5.07             | 4.70             | 3.97             | 5.64             | 4.76             | 4.79             | 4.34             | 2.81             | 3.75             |
| 7            | 4.30                | 4.10             | 4.68             | 4.07             | 2.46             | 2.04             | 2.95             | 2.45             | 4.78             | 4.38             | 1.62             | 1.62             |
| 8            | 3.48                | 0.50             | 0.80             | 0.09             | 2.81             | 0.37             | 3.37             | 0.44             | 3.67             | 0.42             | 0.63             | 0.84             |
| 9            | 3.53                | 0.54             | 1.57             | 0.20             | 2.91             | 0.38             | 3.49             | 0.46             | 4.12             | 0.39             | 0.91             | 1.09             |
| 10           | 3.99                | 3.47             | 4.77             | 4.14             | 3.64             | 3.11             | 4.37             | 3.73             | 4.10             | 3.44             | 2.79             | 2.09             |
| 11           | 4.05                | 3.91             | 5.50             | 4.71             | 3.46             | 2.84             | 4.15             | 3.41             | 4.62             | 4.22             | 1.67             | 1.67             |
| 12           | 2.99                | 0.43             | 1.04             | 0.11             | 3.91             | 0.51             | 4.69             | 0.61             | 2.95             | 0.35             | 2.38             | 4.08             |
| 13           | 3.38                | 0.54             | 1.28             | 0.15             | 3.04             | 0.39             | 3.65             | 0.47             | 3.69             | 0.42             | 0.90             | 1.20             |
| 14           | 4.00                | 3.80             | 6.03             | 5.15             | 4.42             | 3.67             | 5.30             | 4.40             | 4.43             | 4.15             | 0.99             | 0.99             |
| 15           | 4.04                | 3.48             | 5.01             | 4.37             | 3.16             | 2.68             | 3.79             | 3.22             | 4.29             | 3.72             | 1.52             | 1.52             |
| 16           | 3.54                | 0.53             | 0.95             | 0.11             | 3.34             | 0.45             | 4.01             | 0.54             | 3.93             | 0.41             | 1.30             | 1.95             |
| 17           | 4.44                | 4.03             | 5.39             | 4.66             | 3.50             | 2.93             | 4.20             | 3.52             | 4.96             | 4.50             | 2.22             | 2.22             |
| 18           | 3.62                | 0.61             | 1.29             | 0.15             | 3.16             | 0.41             | 3.79             | 0.49             | 3.83             | 0.42             | 0.29             | 1.55             |
| 19           | 3.08                | 1.29             | 3.51             | 1.63             | 3.10             | 1.42             | 3.72             | 1.70             | 3.19             | 1.38             | 1.58             | 1.58             |
| 20           | 2.96                | 1.23             | 3.30             | 1.52             | 3.82             | 1.78             | 4.58             | 2.14             | 3.30             | 1.43             | 1.83             | 2.44             |
| 21           | 3.06                | 1.33             | 3.38             | 1.56             | 3.34             | 1.54             | 4.01             | 1.85             | 3.33             | 1.46             | 1.71             | 1.71             |
| 22           | 2.98                | 1.33             | 3.38             | 1.56             | 3.34             | 1.54             | 4.01             | 1.85             | 3.32             | 1.46             | 1.37             | 1.37             |
| 23           | 3.10                | 1.27             | 3.39             | 1.57             | 3.63             | 1.70             | 4.36             | 2.04             | 3.34             | 1.40             | 1.66             | 2.22             |
| 24           | 2.94                | 1.31             | 3.14             | 1.44             | 3.03             | 1.38             | 3.64             | 1.66             | 3.23             | 1.47             | 1.20             | 1.44             |
| 25           | 2.83                | 1.39             | 3.74             | 1.71             | 3.85             | 1.77             | 4.62             | 2.12             | 3.07             | 1.38             | 1.45             | 1.93             |
| 26           | 2.83                | 1.27             | 3.09             | 1.44             | 2.90             | 1.35             | 3.48             | 1.62             | 3.12             | 1.28             | 1.28             | 1.28             |
| 27           | 3.02                | 1.33             | 3.38             | 1.56             | 3.34             | 1.54             | 4.01             | 1.85             | 3.27             | 1.42             | 1.11             | 1.33             |

The coefficient of variation \( v = \frac{\sigma}{h_b} \times 100 \) %

- 50%  39%  25%  49%  22%  17%  12%  10%  61%  81%
A comparison of the calculated deflection values before failure with the experimental data is shown in Table 1.

The results of the calculations of displacements in the LIRA-SOFT are presented in the form of isofield in Figures 3-5.

Figure 3. Isofield displacements according to the results of LIRA-SOFT calculations for test specimen 1 with a big shear-span: (a) - displacement $X(x)$; (b) - displacement of $Z(x)$.

Figure 4. Isofield displacements according to the results of LIRA-SOFT calculations for test specimen 25 with an average shear-span: (a) - displacement $X(x)$; (b) - displacement of $Z(x)$. 
Figure 5. Isofield displacements according to the results of LIRA-SOFT calculations for test specimen 2 with a small shear-span: (a) - displacement $X(x)$; (b) - displacement of $Z(x)$.

5. Conclusions
The performed experiments confirmed the feasibility of taking into account the shear deformations on the supporting sections in determining the deflections of reinforced concrete beams, and their contribution is 25-30 % of the total deflection of the beam. Therefore, it is recommended to define the deflections as the sum of the deflections caused by the moment and shear deformations. In this case, before and after the appearance of the plastic hinge, it is necessary to take into account the indicator of the proportionality of the moments change and the curvature of the beam with constant cross-section along the full length without pre-stressing the reinforcement. Adequate mathematical models obtained from experimental researches characterize the deflections of the studied beams in the mid-spans and under concentrated forces in the maintenance stage and before failure.

The simulated deformation technique with the finite element method allows to predict sufficiently accurately the deformability of continuous beams in different span sections ($\nu=10...22\%$), at the same time, the normative results of deflections show their poor convergence ($\nu=25...89\%$).

References
[1] Babich V E 2001 Experimental studies of the work of continuous reinforced concrete beams. *Creation and application of high-efficiency resource-saving technologies, machines and complexes (Materials of the Int. Sc. and Tech. Conf.*)* (Mogilev: MSTU) 268-269
[2] Babich V Ye 2005 The stressed-deformed and durability of not cutting reinforce-concrete beams at disposable and repeated loadings: *Dis. cand. tech. sc.* (Poltava: PoltNTU) 210
[3] Babich V Ye 1999 The stress-strain state of continuous reinforced concrete beams, taking into account the complete diagram of concrete deformation *Scientific Bulletin of Civil Engineering* (Kharkiv: KTUBA) 7 101-107
[4] Babich V Ye 2003 On the boundary conditions of the redistribution of effort in continuous reinforced concrete beams *Bulletin of the Ukrainian State University of Water Management*
and Environmental Management (Rivne) 6(19) 196-201

[5] Babich V Ye 2012 Practical method of calculation of deflections of reinforced concrete beams according to DSTU B V.2.6-156:2010 Municipal economy of cities (Kharkiv: KhNUUE named O M Beketov) 101 532-540

[6] Davydenko A I, Bambara A N, Belyaeva S Yu and Prisyajnyuk N N 2007 On the calculation of the strength of sections inclined to the longitudinal axis of the element using a complete program of deformation of concrete Mechanics and physics of destruction of building materials and structures (Karpenko Physico-Mechanical Institute of the National Academy of Sciences of Ukraine) (Lviv: Kamenyar) 7 209-216

[7] Dorofeev V S, Karpiuk V M and Krantovska O M 2010 Strength, crack resistance and deformability of continuous reinforced concrete beams (monograph) (Odesa: Even) 175

[8] Krantovska O, Petrov M, Ksonshkevych L, Synii S and Sunak P 2018 Improved engineering method for calculating the strength of the supporting areas of reinforced concrete elements MATEC Web of Conf. 230 02014

[9] Krantovska O, Petrov M, Ksonshkevych L, Orešković M, Synii S and Ismailova N 2019 Numerical simulation of the stress-strain state of complex-reinforced elements Technical J. (Varaždin: University North) 13 (2) 110-115

[10] Babych E M and Andriichuk O V 2017 Strength of Elements with Annular Cross Sections Made of Steel-fiber-Reinforced Concrete Under One-Time Materials Science (New York) 52 (4) 509-513

[11] Andriichuk O, Babich V, Yasyuk I and Uzhehov S 2017 The influence of repeated loading on work of the steel fiber concrete drainage trays and pipes on the roads MATEC Web of Conf. 116 02001