MASS SPECTRUM AND VELOCITY DISPERSIONS
DURING PLANETESIMAL ACCUMULATION

(II - FRAGMENTATION)

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ABSTRACT

The increase of the velocity dispersions which occurs during the growth of the planetesimals (Barge and Pellat, 1991) strongly suggested that fragmentation could come at work in the final stages of the accumulation scenario. Fragmentation has been modeled assuming that mass and energy are redistributed in a simple way between the various fragments; it is found to play a role whose importance is greater than previously believed and depends on the average characteristic size of the primordial planetesimals but not on their mass distribution. In the small size range the mass spectrum is strongly modified with the formation of a small bodies tail but, on the other hand, the growth of the most massive objects is significantly slowed down. With the recent fragmentation model of Housen et al (1991) it is found that 10 kilometer-sized objects can grow into planetary embryos (size of the order of 1000 km) in a time-scale of the order of $10^5$ or $10^7$ years (depending on the fragmentation properties of the planetesimals) whereas 1 kilometer-sized objects cannot on a reasonable time-scale. The formation of planetary embryos much more massive than the rest of the swarm is delayed; nevertheless the time-scale problem for the growth of the outer planets is not revived. The elasticity of the collisions is found to play a negligible role in the investigated size range.
I) Introduction

Catastrophic collisions of the planetesimals are often considered in the literature as plausible, if not unavoidable, events which could have played a role during the accumulation of the planets (Safronov, 1969; Greenberg et al, 1978). Indeed it is likely that the destruction of bodies with size larger than a kilometer could affect the mass spectrum and the velocity dispersion in the whole swarm of planetesimals through mass and energy transfers from the large scales to the small scales. The occurrence of catastrophic collisions has also been suggested to account for a number of observations in the present solar system (Fujiwara, 1982; Davis et al, 1985); they could explain, for example, the existence of some dynamical families in the asteroid belt (Davis et al, 1979) and the irregular shape of Hyperion around Saturn (Farinella et al, 1983).

The collisional destruction of the planetesimals is poorly understood due to the complexity of the physical mechanisms coming into play and also to the lack of knowledge about the material and the internal structure of these bodies. In spite of this, as the resulting mass and energy redistribution could modify the accumulation scenario, fragmentation has to be modeled, even if in a crude way.

The evolution of asteroidal bodies under collisional fragmentation only was the first problem to be investigated (Piotrowski, 1953; Dohnanyi, 1969; Hellyer, 1970). Using a geometrical cross-section, a constant impact velocity and assuming further the fragment mass distribution follows an inverse power law these authors found a steady state solution of the fragmentation equation: except for the largest bodies, the mass distribution is a decreasing power law with exponent 0.8. This result seems in agreement with the size distribution of the asteroids and meteorites and the statistics on the lunar craters.

The accumulation of the planetesimals proceeds from a long term evolution and the assumption of a constant impact velocity is clearly irrelevant; in fact, the occurrence of fragmentation depends on the velocity dispersions of the colliding bodies and fragmentation itself modifies the velocity dispersions through mass and energy redistribution. This self-consistent aspect of the problem (the same as that encountered in the growth of indestructible bodies) must be taken into account in the evolution of the planetesimals.

A model of the planetesimal accumulation which accounts both for accretion and fragmentation has been performed first by the "Moscow School" (Zvyagina et al, 1974; Pechernikova, 1975; Pechernikova et al, 1976). Starting from a power law fragment mass distribution (with exponent 0.8) and assuming the velocity dispersions increase approximately as the escape velocity of the largest body, these authors solve numerically a generalized coagulation equation and found that the mass distribution evolves differently following the size range: at the small scales the mass spectrum can be fitted by a decreasing power law which tends slowly toward an asymptotic solution, whereas at the large scales the time dependence is stronger but no asymptotic behaviour can be established. Unfortunately, estimating the velocity dispersions, these authors omitted to account for the effect of the dynamical friction.

Later Greenberg et al (1978, hereafter G78) and then Wetherill and Stewart (1989, hereafter WS89) considered also the occurrence of fragmentation during planetesimal growth. Both take account of
the dynamical friction: Greenberg et al. in a crude way with their pioneer numerical simulation, Wetherill and Stewart through a formalism developed earlier (Hornung et al., 1985) but used outside its range of validity. Both alike found a velocity dispersion of the largest bodies much smaller than derived with a more elaborate three-body model (Barge and Pellat, 1991) and, by the way, underestimated the actual importance of the fragmentation.

Beaugé and Aarseth (1990) have also accounted for the occurrence of fragmentation in their N-body simulations of the last stage of the planetary formation. Starting from 200 identical lunar-sized bodies they found that large embryos are formed very early in the evolution and that the gravitational perturbations tend to increase the mean eccentricity till fragmentation comes into play; then creation of new embryos stops, those already formed continue to grow while fragments decrease in size. However it is to be noted that their results are not directly comparable with the present ones, on the one hand because their computations are only two dimensional and on the other because they start from lunar-sized bodies which are much stronger against fragmentation than the kilometer-sized first-born planetesimals we started from.

In paper I we studied the simultaneous evolution of mass and velocity dispersions of accreting indestructible planetesimals: the largest bodies were found to grow very fast with an increasing velocity dispersion. The evolution of the largest bodies, in spite of appearances, contrasted with that of Safronov (1969) who neglected completely the dynamical friction and found much longer time scale but it contrasted also with that of Wetherill and Stewart WS89 who claimed that the velocity dispersions of the largest bodies remain always very weak (weaker than our values by more than an order of magnitude). This increase of the velocity dispersions suggested that fragmentation might play a role in the final stages of the planetesimal accumulation.

In this paper we will devise a simple model of the planetesimals fragmentation, based on laboratory results and on a recent model for the collisional disruption of the asteroids, in order to draw nearer to the final stages of the accumulation scenario. Obviously many features of the destruction mechanisms will be neglected as being too detailed for our degree of extrapolation but on the other hand the assumption of purely inelastic collisions will be avoided. This work attempts a self consistent approach of the planetesimal accumulation in which fragmentation is included in the kinetic description of the planetesimal evolution. The accumulation proceeds by a competition between accretion and fragmentation; the evolution of the velocity dispersions account for collisions and encounters as in our previous papers, but also for fragmentation through a simple energy variation rate.

First we reported in section II the results obtained by improving the accretion-code developed in paper I. Section III is devoted to an overview of the fragmentation process and to the determination of the energy thresholds for simple disruption or complete fragmentation of the planetesimals. Section IV illustrates the occurrence of the various possible outcomes of the planetesimal collisions. The mass and energy budget necessary to model fragmentation is performed in section V. The complete set of the generalized evolution equations is given in section VI. The results of the integrations of the evolution equations are reported in section VII for various initial conditions. Finally section VIII is devoted to the conclusions and to the discussion of the results.
II) The growth of indestructible planetesimals

Let us first take a second look at the accretion calculations performed in paper I. The growth of the planetesimals is described in a Lagrangian approach of the coagulation equation, modified in the course of the integration as to account for mass dispersion. From the Lagrangian point of view the evolution of the mass distribution is described with moving batches and the collisional growth process offers two different aspects:

- large bodies sweep up smaller ones in the same way as vapor condensates onto liquid droplets; this mechanism is responsible for batch motion in concerted growth.
- nearly identical bodies coalesce and transfer the newly created entities to neighbouring batches in the same way as liquid droplets coalesce into larger ones; this mechanism is responsible for mass dispersion among batches.

Both aspects were taken into account in our accretion code except at the largest size end, since, if total number of batches is kept constant, it is impossible to transfer the largest bodies to larger batches.

Nevertheless this incomplete description of the evolution is of no consequence if mass dispersion is negligible (as occurs for example when the initial mass distribution is the Safronov exponential we started from in paper I); on the other hand it is a serious limitation when starting from less steeper mass distribution as simple power laws.

The code has been improved by opening new batches at masses larger than the largest bodies; initially empty they fill up gradually as a function of time in the same procedure as in paper I. This improvement was made at the beginning of the evolution when mass dispersion have important consequences (as is the case when the population of the largest bodies is very large).

The other problem encountered in our re-examination of paper I is related to the fact that the thickness of the swarm of planetesimals is inversely proportional to the velocity dispersion. In paper I the variation of the volume of the cloud was taken into account in an incorrect way. In fact, the coagulation coefficient is proportional to the gravitational focusing factor; this has been put right in all subsequent calculations.

Then, with these two improvements of the code, new computations were performed with 12 batches and starting from the same mass spectrum as in paper I; that is:

(i) a Safronov exponential (the same as used by WS89) in which the bodies have a size range between 4.89 km and 14.8 km;

(ii) decreasing power laws in which the size of the bodies ranges from 0.84 km to 1.26 km.

In both cases the calculations were carried out until a single body remains in the largest batch; the results we found have been plotted on figures 1, 2 and 3 and show that this massive body detaches clearly from the rest of the mass spectrum.

Starting from an exponential, the evolution of the planetesimals is quite similar to that obtained in paper I; it is found that the largest bodies behave differently from the rest of the mass spectrum and have two main characteristics:
(i) their velocity dispersion first decreases below the Hill velocity (due to the dynamical friction) and then increases above this limit (due to the viscous stirring),

(ii) their growth, very rapid at the beginning (when the velocity dispersion is very low), tends to slow down (in accordance with the thickening of the swarm).

At the end of the evolution, after $3.10^5$ yrs, a single massive embryo succeeded in capturing approximately 17% of the total mass and its orbit have an eccentricity of the order of $4.10^{-3}$. It must be emphasized that this value of the eccentricity is much larger than found by WS89 or G78.

Starting from power laws, the evolution of the mass distribution is different from that obtained in paper I. Two different cases have been distinguished following the mass of the swarm is initially dominated by the large bodies (exponent equal to 0) or by the small ones (exponent equal to -2) but, in both cases, the evolution is approximately the same:

- the mass distribution relaxes rapidly into a very steep function (even steeper than the Safronov exponential) due to the kinetics of the coagulation process itself. After this transient stage the evolution is similar to that obtained when starting from the Safronov exponential.

- the velocity dispersions, after a quick relaxation toward a near equipartition of the energy, increase as a function of time; as above the largest bodies can reach values greater than the Hill velocity at the end of the evolution.

After $2.10^5$ yrs a single embryo contains some 16% of the total mass and its orbit have an eccentricity of the order of 0.012. The difference we find between these results and those of paper I results mainly from the account of the mass dispersion among the largest bodies in the first part of the integration (a possibility which was excluded in our previous computations).

To summarize, a transient behaviour appears in the beginning of the evolution of the planetesimals: the mass distribution relaxes into a very steep function which gathers most of the total mass into the smallest bodies and, consequently, the distribution of the velocity dispersions relaxes according to a near equipartition of the energy; this behaviour indicates what the initial state of the swarm, compatible with the kinetics under encounters and inelastic collisions, is. Then the evolution is the following:

(i) first the weak velocity dispersions of the largest bodies produces strong gravitational focusing and, so, favours start of "runaway"; (ii) as time elapses, most of the total mass tends to shift from the small scales to the large ones so that gravitational stirring gradually tends to overcome dynamical friction (see paper I). As a result the tendency to "runaway" weakens and the mass tends to stream in a more progressive way to the larger scales.

III) The destruction of colliding planetesimals

Now we will set out some general considerations about the collisional disruption mechanism. To begin with, it is obvious that colliding bodies shatter if the energy released at the impact is strong enough to overcome material’s solid-state cohesion and that the number and the size of the resulting
pieces will depend on the initial kinetic energy. Further informations about the disruption mechanisms come from laboratory experiments and statistics of the asteroid observations.

- The laboratory experiments, performed with small projectiles fired against plane surfaces (Gault et al, 1963; 1969) or against decimetric basalt or mortar targets (Fujiwara et al, 1977; Bianchi et al, 1984; Davis and Ryan, 1990; Ryan et al, 1991; Housen and Schmidt, 1991), bring some general results frequently extrapolated to the much larger scales. In the case of the planetesimals the most useful ones to modeling are:

  (i) the mass distribution of the fragments obeys approximately a decreasing power law with an exponent ranging from 2/3 for barely catastrophic collisions to 1 for supercatastrophic destructions,
  (ii) the velocity distribution of the fragments is roughly fitted by a decreasing power-law with an index equal to -1/6 (Nakamura and Fujiwara, 1991),
  (iii) in the case of core type destructions the relation between impact energy and size of the largest fragments can be fitted by a power law with an exponent equal to 1.24,
  (iv) the actual shape of the target seems unimportant as long as the aspect ratio is of the order unity.

A qualitative physical interpretation of disruptions by high velocity impacts was given by Gault and Wedekind (1969), by Fujiwara (1980) and by Housen and Holsapple (1990). After the impact a compressive wave expands radially into the target and decays very quickly; then, if the body is smaller than a certain size, this wave reflects off the free surface of the target as a complex system of shear and tension waves which, due to the relatively weak tensile strength of rocks, results in multiple spallation and in a destruction of the solid body (see also Fujiwara, 1982 and Fujiwara and Tsukamoto, 1980). This fracture process occurs as a result of the growth and coalescence of pre-existing cracks whose activation depends on the rate at which the material is loaded (Housen and Holsapple, 1990).

However in the laboratory experiments the impact energy never exceeded some $10^3 J$ which is much smaller than the energy necessary for the break up of asteroid like bodies which ranges rather from $10^{15} J$ to $10^{20} J$.

- The observation of asteroid families in the present solar system brings also some useful informations about the very large scale collisions. Indeed the mass distribution of asteroids with strongly correlated orbits has been interpreted by most authors (Anders, 1965; Dohnanyi, 1969; Chapman and Davis, 1975; Ip, 1979) as being produced by a catastrophic collisional evolution. In some cases it has been possible to reconstruct the parent bodies, to give a rough estimate of the impact energy (Fujiwara, 1982; Davis et al, 1985) and even to speculate about the internal structure of the parents.

The mass distribution in the asteroid families have a power law exponent in agreement with the high velocity experiments (Fujiwara, 1980) and moreover the mean relative velocities between the family members are not incompatible with the ejection velocities of the laboratory fragments (Zappala et al; 1984). So extrapolation of experimental datas to the much larger scales, although rather tricky
seems reasonable.

Fujiwara (1980) extrapolated over many order of magnitudes the relation he found between critical size for disruption and impact energy (a power law with an index $\zeta$ between 0.36 and 0.44); then he suggested that the experimental results could be applicable to gravity free bodies even with size as large as 100 km. It must be noticed that, if $\zeta$ is larger than 1/3, the impact strength decreases with the body size; this idea has been used by Farinella et al (1982) to conclude that asteroids could be intrinsically weak as a simple consequence of their large size and corresponds to the assumption that the fragmentation energy is proportional rather to the total surface area of the produced fragments than to their volume. At the time this idea contrasted with the classical assumption of an impact strength independent of the body size (Greenberg et al, 1978; Davis et al, 1979; Fujiwara, 1982).

However very recently Housen and Holsapple (1990) constructed a model, based on a variety of experimental and theoretical evidences, in which the disruption strength depends not only on target size and impact energy but also on strain-rate and impact velocity; this model leads again to the conclusion that large bodies are weaker than small ones, suggesting that asteroids should fracture at much lower specific energy than small-scale experiments would suggest.

In the case of the planetesimals it must be emphasized that no reliable data exist about the fragmentation or even the composition of these bodies. They could be constituted from some primitive carbonaceous chondritic material resulting from the aggregation of solar system rocky condensates whose mechanical properties would be comparable to the terrestrial basalt. Moreover the planetesimals, as a number of asteroids in the present solar system, could be coated with a thick regolith layer or again strongly fractured by the repeated collisions; so the propagation of the compressive waves into their interiors would be very different from that occurring in the experimental cases. Nevertheless some idea becomes to emerge about the large scale collisional fragmentation. In recent year Housen et al (1991) conducted new fragmentation experiments simulating these large scale catastrophic events (targets with size larger than some 100 km) by applying an appropriate overpressure onto small decimetric targets; they found that the specific energies for fragmentation are of the same order of magnitude as those estimated from the observations of the Themis, Eos and Koronis asteroids families.

It is also to be noted that during the destruction of bodies with size larger than 100 km, the gravitational attraction between the resulting pieces must be also taken into account as being able to hinder seriously the dispersal of the fragments. Thus, after a break-up event, it is necessary to distinguish between two cases: the simple disruption of the body cohesion and the complete "fragmentation", that is a shattering of the body followed by a dispersion of the resulting pieces as independent fragments.

1) Disruption threshold

Following Housen et al (1991) the energy threshold for the disruption of a planetesimal $m_i$ can be written:
\[ Q_D(i) = \left[ Q_o \left( \frac{m_i}{m_o} \right)^a + G_o \left( \frac{m_i}{m_o} \right)^b \right] \left( \frac{\Delta V}{V_A} \right)^{0.35} \]  \tag{3.1}

where \( a = 0.92, \ b = 1.55, \ \Delta V \) is the impact velocity and \( V_A \) is the average relative velocity in the asteroid belt (\( \simeq 5 \text{ km} \text{s}^{-1} \)); \( Q_o = 1.5 \times 10^3 \ m_o^{0.92} \) is scaled from experimental values only and \( G_o = 7.7 \times 10^{-7} \ m_1^{1.55} \) is scaled both from the new experimental values of Housen et al. (1991) and from the disruption specific energies estimated by Fujiwara (1982) from a reconstruction of parents bodies for the Themis, Eos and Koronis Hirayama families (for a kilometer-sized bodies \( Q_o \simeq 2.1 \times 10^{15} J \) and \( G_o \simeq 2.3 \times 10^{14} J \)). The first term in (3.1) corresponds to the energy necessary for the break up when the body is considered as a gravity free solid whereas the second one, introduced first by Davis et al. (1985), is the energy necessary to overcome material strengthening due to the gravitational compressive load. Indeed, due to the variation of the pressure with depth, the interiors are more difficult to disrupt than the near surface and the intensity of the tension waves must exceed the material strength plus the compressive load of the upper layers.

Obviously in the case of the planetesimals the effective strength is highly uncertain due to the lack of knowledge on their actual composition and internal structure. They could be brittle bodies made either of weakly bounded regolith (G78) or of deeply cracked material but, on the contrary, they could be much more stronger if constituted by a thick regolith layer upon a solid core (Hartmann, 1978). At any rate the energy threshold (equation (3.1)) deduced from the above mentioned considerations is, at present, the most detailed expression available to describe planetesimal evolution; it will be used through the rest of the paper.

Now, let us check the two simplifying assumptions we have made as to model a catastrophic event (that is when the impact energy is large enough to shatter the target) in a tractable way:

- both impacting bodies shatter in various fragments;
  this assumption is appropriate in the case of identical bodies, but must be discussed in the opposite case. If the projectile is shattered and the target is not, two cases have to be distinguished: (i) if the target mass is large enough to capture the various pieces, the collision results in an accretion of the projectile onto the target (this happened, for example, in the meteor crater event); (ii) if the target cannot reaccumulate a large fraction of the projectile mass the assumption is weak and tends to underestimate the importance of the fragmentation.

- the disruption occurs with the same crushing law for the two impacting bodies;
  the destruction of the two bodies is assumed to produce two different cores and two species of fragments regardless of a "mixing" of the ejectas; this is a convenient but a rough assumption.

2) Fragmentation threshold

If the two impacting planetesimals are massive enough the self gravity of their various pieces can be sufficient for fragments not to escape; then collisions result in accretion with the formation of a single body rather than in fragmentation. Bodies issuing from the reaccumulation of a large number of various pieces could exist among the present asteroids and have been evoked many
times in the literature (Davis and Chapman, 1977; Davis et al, 1979; Farinella et al, 1982) but whether reaccumulation of fragments can be efficient enough to produce ”piles of rubble” is still an open question. On the other hand the continuous bombardment by the smaller bodies (and the partial reaccumulation due to the repeated collisions) leads certainly to a regolith layer which, if thick enough, could modify seriously the efficiency of the sticking and disruption mechanisms (Hartmann, 1978).

The total amount of energy necessary for a disruption followed by an ejection of independent fragments will be estimated, in the same way as in the work of Zvyagina et al (1974), with the help of total energy conservation before and after the destruction. The energy budget for the fragmentation of two bodies $m_i$ and $m_j$ will be settled in the following way:

- before the impact: $K_o + U_i + U_j$ ,

where $K_o$ is the initial kinetic energy and $U_i = 3Gm_i^2/5R_i$ is the gravitational potential energy of the body $m_i$;

- after the impact: $K_1 + (1 - \chi)K_o + U_F(i, j)$ ,

where $U_F(i, j)$ is the potential energy of the dispersed fragments and $K_1$ their final kinetic energy; $\chi$ is the fraction of the initial kinetic energy transferred to the various pieces. After Fujiwara the most probable value of the $\chi$ factor in the case of asteroidal bodies is only some ten percent; in all subsequent calculations we will take $\chi = 0.1$.

Finally, energy conservation leads to: $K_1 = \chi K_o + U_i + U_j - U_F(i, j)$.

The dispersal of the fragments will be assumed to occur if their final kinetic energy is greater than zero. This is obviously an approximate condition which neglects any mass dependence in the fragment velocity (in fact, small pieces are easier to disperse than large ones); it is satisfied if the impact energy is strong enough to overcome an amount of energy equal to:

$$\frac{1}{\chi}(U_F(i, j) - U_i - U_j).$$

(3.2)

On the other hand the potential energy of the dispersed fragments will be estimated from the continuous mass spectrum $n(m)$ through the relation:

$$U_F(i, j) \simeq -\int dm\, n(m)U(m).$$

(3.3)

As discussed above the differential mass distribution of the fragments is approximately given by the power law:

$$n(m) = n_o \left(\frac{m}{m_o}\right)^{-q},$$

(3.4)

with $q = 1 + \alpha$ and $\alpha \simeq 2/3$; then we find approximately:

$$U_F(i, j) \simeq \frac{U_o}{3} \left(\frac{m_i + m_j}{m_o}\right)^{5/3}$$

(3.5)
where $U_o = -3Gm_o^2/5R_o$ is the gravitational potential of a typical body (of the order of $10^{13} J$ for a 1 km body). As a result the energy threshold for the fragmentation of the two bodies $m_i$ and $m_j$ will be defined as:

$$Q_F(i, j) = Q_D(i) + Q_D(j) - \frac{U_o}{\chi} \left[ \left( \frac{m_i}{m_o} \right)^{5/3} + \left( \frac{m_j}{m_o} \right)^{5/3} - \frac{1}{3} \left( \frac{m_i + m_j}{m_o} \right)^{5/3} \right].$$  \quad (3.6)

The second part of the r.h.s. of this equation represents the additional gravitational binding energy to be exceeded for a dispersal of the shattered bodies into independent fragments.

Then, in order to illustrate our model, the specific energies $Q_D^*$ and $Q_F^*$ have been plotted on figure 4 as a function of the target size (assuming an average impact velocity $\Delta V = V_A$); the results of the experimental tests of Housen et al (1991) and the specific disruption energies estimated by Fujiwara (1982) from the observations of the Themis, Eos and Koronis Hirayama families have been reported on the same figure. As noted Housen et al the reason that the family datas lay above the $Q_D^*$ threshold is that reaccumulation of collisional debris increases the size of the largest remnant. This is likely why the $Q_F^*$ fragmentation threshold, which accounts roughly for reaccumulation effects, seems to give better agreement between the model and the family datas.

Obviously the gravitational attraction between projectile and target is also of importance before impact since it enhances both collisional cross section and impact velocity; in our model the gravitational focusing is described only in a two-body formalism (as in our previous papers) but, on the other hand, the increase of the impact energy is neglected.

Now, before to model the way in which mass and energy are redistributed among fragments, let us first discuss the occurrence of the various collision outcomes as a function of the impact energy.

**IV) The outcomes of the planetesimal collisions**

An alternative to the catastrophic outcomes is obviously the rebound of the two bodies with a change of momentum and a loss of energy. This case was studied in our previous paper with the set up of a critical velocity $u_R$ which determines whether the two bodies stick one another or rebound. The energy threshold above which rebounds are expected is derived in a straightforward way from the expression of $u_R$ and writes:

$$K_R = \frac{(1 - \gamma_o^2)}{\gamma_o^2} \frac{1}{2} m_{ij}^* T_{esc}$$  \quad (4.1)

where $m_{ij}^*$ is the reduced mass of the colliding bodies; $\gamma_o$ is the restitution coefficient and $T_{esc}$ is the square escape velocity.

Now in order to discuss more easily the occurrence of the various outcomes of a planetesimal collision, the disruption threshold $Q_D$ will be assumed independent of the impact velocity (that is we will take $\Delta V = V_A$ in equation (3.1)).

1) The various outcomes
Clearly the different outcomes of a planetesimal collision can be distinguished when the impact energy is compared with the above defined energy thresholds; nevertheless the discussion will be easier in term of the velocities we will define the square velocity thresholds:

\[ T_\nu = \frac{2E_\nu}{m_{ij}^2} \]  

where \( E_\nu \) stands for one of the energies \( K_R, Q_D \) or \( Q_F \).

If one of the two bodies is much smaller than the others \( (m_j << m_i) \) these expressions reduce to:

\[ T_R \simeq \frac{(1 - \gamma_o^2)}{\gamma_o^2} T_{esc} \]  

\[ T_D \simeq \left[ \frac{Q_o}{m_o} \left( \frac{m_i}{m_o} \right)^{a-1} + \frac{G_o}{m_o} \left( \frac{m_i}{m_o} \right)^{b-1} \right] \frac{m_i}{m_j} \]  

\[ T_F \simeq T_D + \frac{1}{5} \frac{T_{esc} m_i}{\chi m_j} \]

and remain available in order of magnitude even for identical bodies. In the following it will be convenient to name target the large body and projectile the small one.

The three velocity thresholds have been plotted on figures 5 as a function of the projectile to target radius ratio; for a given size of the target, they delimit distinct regions which correspond to the various outcomes of a collision.

- In the case of the large targets (sizes of the order of 100 km; cf figure 5c) four distinct regions appear. In the upper part, the region of the strong impact velocities, collisions result in a fragmentation whereas in the lower part they result in a sticking. What happens in the intermediate zone is two fold:

  (i) in the region between the disruption and the fragmentation limits the two bodies are disrupted but reaccumulate in a single body (a "rubble pile"),

  (ii) in the region between the rebound and the disruption limits the two bodies can rebound after the impact; however the existence of this region depends clearly on the restitution coefficient and on the target mass.

- In the case of the small targets (figures 5a and 5b) the velocity thresholds are lower and the conclusions are similar; the rebound region is wider but on the other hand the reaccumulation region is narrower or inexistent since gravitational effects are less important.

Obviously, in this simple sketch, erosion and partial reaccumulation are omitted; moreover complete reaccumulation is taken into account only in a rough way through the definition of the composite energy threshold \( Q_F \). In fact, it is clear that the crushing law itself should account for gravity induced mass segregation.

Now we will mention some peculiarities of our model:
(i) at the frontier between reaccumulation and rebound, self gravity plays a crucial role and the outcome of a two body collision is uncertain since an impact could result either in two "cracked bodies" or in a single weakly agglomerated structure (this unstable situation disappears if the bodies have sizes smaller than some 10 km);

(ii) at the frontier between reaccumulation and sticking, collisions result either in a single body or in a rubble pile;

(iii) at the intersection between the disruption and the rebound thresholds, collisions result either in rebound or in sticking or again in reaccumulation; this situation (which occurs if the projectile mass reach a critical value $m_c$) have some remote similarity with the coexistence in thermodynamics of the three different phases of a pure element.

However we must keep in mind that such unstable situations emerge from a naive discussion of a simple model and, so, could remain only speculations.

Anyway this simple sketch illustrates easily the occurrence of the various outcomes and the fact that the accretion of two planetesimals results either from simple sticking or from fragment reaccumulation. On the other hand the extent of the rebound region depends on a critical value of the mass which, normalized to the target mass, writes:

$$\frac{m_{c}(i,j)}{m_i} = \frac{\gamma_0^2}{1 - \gamma_0^2} 3 G_o \left( \frac{m_i}{m_o} \right)^{\frac{2}{3} - b} \left[ 1 + \frac{Q_o}{G_o} \left( \frac{m_o}{m_i} \right)^a \left( \frac{1}{T_i} \right) \right].$$  \hspace{1cm} (4.6)

It is wider for stronger elasticities or smaller targets. Figures 5, with a restitution coefficient equal to 0.2 (a probable value in the case of the planetesimals, see for example G78) shows that the rebound region can be very wide for the smallest bodies.

2) The characterization of the various outcomes

Now, in order to proceed further, we will define a characteristic function for the occurrence of each collision outcome (accretion, rebound or fragmentation respectively) as a function of the impact velocity. The simplest way to achieve this is to use step functions of the square velocity dispersions $T_i$.

The accretion will be characterized by the function $\Phi_A$ defined with the help of the exhaustive rule as:

$$\Phi_A = 1 - \Phi_F - \Phi_R,$$  \hspace{1cm} (4.7)

where $\Phi_F$ and $\Phi_R$ are the characteristic functions for the occurrence of fragmentation or rebound (respectively) and are given by:

$$\Phi_F(i,j) = H \left( \frac{T_i + T_j}{T_F} \right).$$  \hspace{1cm} (4.8)

and
\[ \Phi_R(i, j) = H \left( \frac{T_i + T_j}{T_D} \right) H \left( \frac{T_i + T_j}{T_R} \right), \]  

(4.9)

where \( H \) denotes the Heaviside function which vanishes for arguments smaller than 1.

V) Mass and energy budget of the fragmentation

In paper I planetesimals were assumed indestructible spheres which can only stick one another; their mass distribution was described with a finite number of populations defined by a mass \( m_i \) \((i = 1, \ldots, N)\) and a velocity dispersion \( \sqrt{T_i} \). Now planetesimals are allowed to fragment but the same discrete description of the mass spectrum and of the velocity dispersions will be kept up. The mass evolution due to the fragmentation will be modeled under the assumption that the mass lost by a planetesimal is redistributed into fragments of a planetesimal size; that is, each fragment is assumed to belong to one of the \( N \) populations defined in the previous step.

In other words, fragmentation removes mass from a given population and adds bodies to the populations of smaller masses (in accordance with the fragment distribution); so, in our model, planetesimal accumulation will proceed under the competition between capture and ejection of smaller bodies.

Now we will examine in more details the mass and the energy budget.

1) The mass budget

In order to make the model still more tractable we will further assume that catastrophic destructions are only core type, that is fragmentation leads always to a single residual body (the "core") and to smaller fragments; in the case of supercatastrophic collisions this is a rough assumption and the "core" will be defined, with an abuse of the standard language, as the largest fragment.

a) The core

The mass of the residual body which remains after the ejection of the fragments is an important parameter which determines the amount of the mass loss. The laboratory experiments have shown that, for decimetric targets, the mass of the core depends on the relative impact energy per unit mass through a power law relation which writes approximately:

\[ m_{\text{core}} \propto m_t \left( \frac{K}{m_t + m_p} \right)^\lambda \]  

(5.1)

where \( \lambda \simeq 1.24 \), \( m_t \) is the mass of the target and \( m_p \) the mass of the projectile. For these centimeter-sized gravity-free bodies the mass of the core is approximately proportional to \([K/Q_D(i, j)]^\lambda\).

In the case of the planetesimals gravitational effects can become important through the lithostatic stresses and the self-gravity of the fragments; then catastrophic collisions are less destructive than for gravity-free bodies with the formation of larger cores and the production of smaller amounts of fragment mass. In order to account roughly for the reaccumulation effects and, in the absence
of any better assumption, we will extrapolate the above relation to the case of the planetesimals replacing directly the disruption threshold $Q_D$ by the fragmentation threshold $Q_F$.

Then, if the impact energy $K_{ij}$ is greater than or equal to the fragmentation threshold $Q_F(i, j)$, the ratio of the mass of the core to the mass of the parent planetesimal will be defined by the relation:

$$\frac{m_{ij}^c}{m_j} = \frac{1}{2} \left( \frac{K_{ij}}{Q_F(i, j)} \right)^{-\lambda}. \quad (5.2)$$

The greatest value of this ratio, when $K_{ij}$ is just equal to the fragmentation threshold, is 1/2 as assumed in G78. Obviously for the largest planetesimals this relation is quite speculative since, in fact, the mass of the core depends in a complex way on ejection dynamics and reaccumulation processes; however this weakness of the model is not a crucial one.

On the other hand, if $K_{ij}$ is smaller than $Q_F$, no fragmentation can occur and the erosion mechanisms (as for example ejection from impact craters) will be neglected as unimportant as regard to the evolution as a whole.

Then, from the mass of the core it is straightforward to obtain the mass loss which writes:

$$m_j - m_{ij}^c = m_j R_{ij} \quad (5.3)$$

where $R_{ij}$ define a "loss matrix".

The above expressions have been derived only for head-on collisions which are less probable events than oblique ones. The impact experiments performed by Fujiwara and Tsukamoto (1980) indicate that oblique impacts are less efficient than head-on impacts for the disruption of the target. In the case of the asteroids, Davis et al (1985) accounted for this effect and derived a mean expression for the modified core mass; the overestimate due to the head-on approximation is important for the very catastrophic events. In the planetesimal case the obliquity of the impacts will be taken into account approximately replacing, after Davis et al (1985), the mass of the core $m_{ij}^c$ by the simple expression:

$$3 \left( \frac{m_{ij}^c}{m_j} \right)^{2/3} - 2 \left( \frac{m_{ij}^c}{m_j} \right). \quad (5.4)$$

b) The fragments

The fragment mass distribution is commonly fitted by a power law whose exponent ranges approximately from $2/3$ for barely catastrophic collisions (with $m_c/m_j \simeq 1/2$) to 1 for completely catastrophic collisions (with $m_c/m_j << 1$). G78 found that the expression:

$$b_{jk} = \left( 1 + \frac{m_{ik}^c}{m_j} \right)^{-1} \quad (5.5)$$

where $m_j$ is the target mass and $m_k$ the projectile mass, can reproduce satisfactorily the exponent of this power law.
The dependence of this exponent upon \( Q_F \) (through equation (5.2)) indicates that the way in which we have accounted for self gravity modifies indirectly the mass distribution of the fragments. It is obvious, as noticed above, that the self gravity can distort the mass distribution of the ejecta through selective capture but, on the other hand, it is uncertain that our rough model can describe appropriately such effect; anyway a more detailed description of this problem is out of the scope of the present paper.

With this power law relation the number of the fragments \( m_i \) which comes from the destruction of a planetesimal \( m_j \) after an impact with another one \( m_k \) will be written:

\[
G_i^j(k) = g_{jk} \left( \frac{m_i}{m_o} \right)^{-b_{jk}}
\]  

(5.6)

where the normalization parameter \( g_{jk} \), derived from the conservation of the mass before and after the impact, writes:

\[
g_{jk} = \frac{m_j}{m_{ej}} R_{jk}
\]  

(5.7)

in which:

\[
m_{ej} = \sum_{l=1}^{l_{max}} m_l \left( \frac{m_l}{m_o} \right)^{-b_{jk}}
\]  

(5.8)

is proportional to the total mass of the ejected fragments; \( l_{max} \) (the index of the largest fragments) is obtained as the maximum value of the index for which \( m_{c}/m_l < 1 \). The above expressions are defined only if \( i < j \) (fragments are smaller than the target); on the other hand as the fragments of the smallest planetesimals are outside our discrete mass spectrum we will set up the relation \( G_i^1(k) = 0 \) for any value of \( i \) and \( k \).

It will be also convenient to write the total number of the fragments \( m_i \) produced in a single collision as:

\[
G_i^{jk} = G_i^j(k) + G_k^i(j).
\]  

(5.9)

2) The energy budget

The laboratory experiments (Gault and Heitowit, 1963; Fujiwara, 1980; 1982) provide some scarce informations on the ejecta velocity distribution; they indicate that the core have a very low velocity whereas the smallest fragments, which originate mainly from the surface of the target, have the largest ones; in a more general way it seems that the larger the fragment the smaller the ejection velocity. Such conclusions could support roughly the idea of an equipartition of the energy (still available after the disruption) between the various fragments, as was suggested by Wiesel (1978) and Fujiwara (1980), in fact the experimental results of Nakamura and Fujiwara (1991) seem to show that the velocity distribution of the fragments could be fitted by a power-law with an index equal to -1/6. So, concerning the partition of the kinetic energy among the ejectas, we will make the two following assumptions:
(i) the core have a zero relative energy (as assumed by Fujiwara (1982)),
(ii) the fragments share between them all the energy still available after the disruption of the parent body proportionally to the 2/3 power of their mass.

Obviously the velocity distribution of the ejectas issued from a large body depends strongly on self gravity effects; however, as mentioned above, the account of the ejecta motion will be omitted as being an additional complication unjustified as regard to the roughness of all the other assumptions.

3) The evolution of the smallest bodies

In the above model it is clear that a problem arises with the smallest bodies since, strictly speaking, their fragments cannot be included into any discrete description of the mass spectrum. In order to get round this problem we modified somewhat our integration procedure in the following way. For a given time step the smallest planetesimals are assumed to fragment only into identical bodies (with a core size); then, at the beginning of the next step, the mass and the energy of this "transient population" are reprocessed into the new population of the smallest bodies.

It is clear that this rough description of the evolution of the smallest bodies is one of the most severe limitation of our model, especially in the case of the supercatastrophic collisions.

VI) The generalized evolution equations

The fragmentation of the planetesimals will be included in the accumulation scenario by addition of new collision terms to the evolution equations derived in paper I (equations (4.7), (4.8) and (4.14) of that paper).

1) The fragmentation collision terms

In some sense the fragmentation of the planetesimals is similar to a simple creation-disparition process; the associate mass and energy variation rates allow to derive easily the various collision terms which will be used to complete our model.

- On the one hand fragmentation increases the number of the small bodies at the expense of the mass of the large ones. The new collision terms which account for this mass redistribution will be written:

\[
\left( \frac{dn_i}{dt} \right)_{\text{Frag}} = \frac{1}{2} \sum_{j,k} \omega_{jk}n_k f_{jk} G_{jk}^{ij}
\]

\[
\left( \frac{dm_i}{dt} \right)_{\text{Frag}} = - m_i \sum_j \omega_{ij} n_j f_{ij} R_{ij}
\]

where \( \omega_{ij} = (R_i + R_j)^2 T(0)^{1/2} \) is the geometrical collision rate which accounts for the evolution of the cloud thickness and \( f_{ij} = (1 + \Theta_{ij}) \Phi_F(i,j) \) is a fragmentation coefficient in which the \( \Theta_{ij} \) are the well known Safronov parameters.
The first term (6.1) corresponds to a creation rate; that is to a total number of fragments, with mass \( m_i \), issued per second from all possible pairs of colliding planetesimals. The second term (6.2) corresponds to the mass loss of the \( i \)th population which results from the catastrophic collisions between a target \( m_i \) and any smaller projectile.

- On the other hand fragmentation also contributes to the evolution of the planetesimal kinetic energy. The energy variation rate of a given population has been derived from a simple balance between the energy supplied by fragments \( m_i \) issued from larger bodies and the energy loss which results from the disparition of the destructed bodies; we obtained:

\[
\left( \frac{dK_i}{dt} \right)_{\text{Frag}} = \frac{1}{2} \sum_{jk} \omega_{jk} n_j n_k \epsilon_{jk} f_{jk} G_{ij}^{jk} - \frac{1}{2} n_i m_i T_i \sum_j \omega_{ij} n_j f_{ij} \frac{m_{ij}^*}{m_i} \tag{6.3}
\]

where:

\[
\epsilon_{jk} = \frac{\chi}{\sum_{jk} F_{jk}} \frac{1}{2} m_{jk}^*(T_j + T_k) \left( \frac{m_i}{m_o} \right)^{\frac{3}{4}} \tag{6.4}
\]

is the kinetic energy each fragment carries away after the partition (in accordance with the velocity distribution of the fragments) of the energy still available after the disruption and with also:

\[
\Sigma_{jk}^{F} = \sum_i G_{ij}^{jk} \left( \frac{m_i}{m_o} \right)^{\frac{3}{4}} \tag{6.5}
\]

2) The complete set of the equations

When both accretion and fragmentation operate the evolution equations for the mass and the energy write:

\[
\frac{dn_i}{dt} = - \sum_{j} n_i n_j \omega_{ij} a_{ij} C_{ij} + \frac{1}{2} \sum_{j,k} n_k \omega_{jk} f_{jk} G_{ij}^{jk} \tag{6.6}
\]

\[
\frac{dm_i}{dt} = \sum_{j} \omega_{ij} n_j (m_j a_{ij} A_{ij} - m_i f_{ij} R_{ij}) \tag{6.7}
\]

\[
\frac{dK_i}{dt} = \left( \frac{dK_i}{dt} \right)_{\text{Acc}} + \left( \frac{dK_i}{dt} \right)_{\text{Frag}} + \left( \frac{dK_i}{dt} \right)_{\text{Enc}} \tag{6.8}
\]

in which the contributions of the accretion and of the encounters, reported from paper I, are given by:

\[
\left( \frac{dK_i}{dt} \right)_{\text{Acc}} = \frac{m_i}{2\sqrt{2}} \sum_{j=1}^{N} A_{ij} \frac{\omega_{ij} n_j m_j}{(m_i + m_j)^2} [(m_j T_j - m_i T_i) - (m_i + m_j) T_i] I \Phi_A(i,j) \tag{6.9}
\]

and
\[
\left( \frac{dK_i}{dt} \right)_{\text{Enc}} = 2\sqrt{2} m_i \sum_{j=1}^{N} \frac{\omega_{ij} n_j m_j}{(m_i + m_j)^2} \Theta_{ij}^2 L_{ij} \left[ (m_j T_j - m_i T_i) H + \frac{1}{2} m_j (T_i + T_j) J \right]
\]  

(6.10)

in the above equations we used the following matricial notations:

\[ A_{ij} = \begin{cases} 1, & \text{if } j \leq i ; \\ 0, & \text{otherwise}. \end{cases} \]

for the mass accretion,

\[ C_{ij} = \begin{cases} 1, & \text{if } j \geq i ; \\ 0, & \text{otherwise}. \end{cases} \]

for the capture of bodies,

and we defined also the accretion coefficient: \( a_{ij} = (1 + \Theta_{ij}) \Phi_A(i, j) \).

VI) Planetesimal growth with allowance for fragmentation

In paper I colliding planetesimals were only allowed to stick one another and integration was stopped when the increasing velocity dispersions reached 500 \( k \text{ms}^{-1} \), a value assumed large enough for fragmentation to play a role.

Then, in section II, the new accretion calculations were carried out (see figures 1, 2 and 3) until a single body remains in the largest batch; below, the results of these calculations, in which no fragmentation comes into play, will be used to drive meaningful comparisons.

Now each collision outcome have an occurrence (defined in section IV) which changes as a function of time according to the evolution of the velocity dispersions. The integration of the generalized evolution equations has been performed with the same procedure as in paper I (except the above modifications necessary to account for the fragmentation of the smallest bodies) and with a resolution on the mass distribution given by 12 different batches. In the course of the computations, conservation of mass and energy (kinetic energy partitioned between the various ejecta) were controlled at regular time intervals.

In order to proceed with increasing complexity and to make appropriate comparisons with the results obtained for indestructible bodies, we will distinguish between two cases according to the elasticity of the collisions.

1) The case of completely inelastic collisions

The relaxation of the mass spectrum, set out in section II, reduces the range of initial mass distributions to very steep functions. However initial conditions differ also by the choice of the characteristic size of the primordial planetesimals; these bodies which resulted from the agglomeration of the dust particles in the first age of the protoplanetary nebula (a sedimentation stage followed by a gravitational instability after the traditional Safronov-Goldreich and Ward’s scenario) could have an
average size of the order of some kilometers at the earth distance. In order to discuss the importance of this initial size, it will be convenient to distinguish between two different cases, each one corresponding to a well defined characteristic size: 1 km or 10 km.

In the beginning, the velocity dispersions are too small for fragmentation to operate and the evolution of the planetesimals looks like that of the indestructible bodies investigated above (this appears clearly when figures 1, 2 and 3 are compared with the figures 6, 7 and 8). Then, as the velocity dispersions increase as a function of time, fragmentation comes gradually into play and modifies the course of the evolution.

- Primordial planetesimals with a characteristic size of the order of 10 km

In the size range of the Safronov exponential mass spectrum, two cases have been distinguished following the disruption energy $Q_D$ is assumed dependent or independent of the impact velocity. In both cases calculations were carried out until a single body (with a size of the order of 1000 km) remains in the largest batch. Fragmentation is found to play a role only at the end, in relation with the large values of the velocity dispersions. The number of bodies and the velocity dispersions obtained in these two cases are plotted on figures 6 when the disruption energy is assumed independent of the impact velocity and on figures 7 when the opposite assumption holds.

Fragmentation comes into play after a rapid growth stage (the size of the largest bodies increase by a factor 100 in a time-scale of the order of $3 \times 10^5$ yrs) and modifies the evolution of the mass spectrum in the following way:

- a tail comes at the small size end; it contains a fraction of the total mass which ranges from 7% to 17%, following $Q_D$ is independent or dependent of the impact velocity, respectively; at the end, the tail evolves faster and faster due to the increasing importance of the destruction processes.

- the growth of the largest bodies slows down but a single embryo succeeds in accreting 17% of the mass of the swarm; the growth time scale is of the order of $3.7 \times 10^5$ yrs or $6.2 \times 10^7$ yrs, following $Q_D$ is independent or dependent of the impact velocity, respectively; the growth is slower and slower as fragmentation becomes more and more efficient.

On the other hand the evolution of the velocity dispersions is only weakly modified as compared to the accretion of indestructible planetesimals: the small bodies of the tail have velocity dispersions joining up smoothly with the rest of the spectrum and the largest ones can reach the Hill velocity. Nevertheless the final values of the velocity dispersions are slightly smaller than in the case of indestructible bodies.

- Primordial planetesimals with a characteristic size of the order of 1 km

In the size range of the power law mass distributions defined in section II, planetesimals are more brittle bodies and only the case of a disruption energy independent of the impact velocity has been investigated.

Fragmentation is found to modify the evolution of the mass distribution much more strongly than in the 10 km size range; this is obviously due to the fact that one-kilometer-sized planetesimals are the most brittle bodies (this appears clearly on Fig. 4). The calculations were performed in the same way as above; the resulting number of bodies and velocity dispersions are plotted on figures
8 as a function of mass and for various time steps. The evolution have approximately the same
trend as in the ten-kilometer size range but fragmentation takes place earlier. The rapid growth
stage, in which the size of the largest bodies increases by a factor 100, lasts only some \(10^4\) yrs but
on the other hand, the stage in which fragmentation comes into play is much more longer. After
\(4.10^7\) yrs, the largest batch contains 12% of the total mass, that is approximately 16 bodies with a
size of the order of 670 km, and the tail only some 7%. A great amount of the total mass is found
in the intermediate scales.

As expected from the previous case, if the disruption energy is assumed dependent on the impact
ergy the planetesimals are still more brittle and the importance of the catastrophic collisions is
still stronger.

2) The case of weakly elastic collisions

The inelasticity of the planetesimal collision is likely very large due to the surface composition
(presence of a regolith layer) but also to the internal structure of these bodies. However, in order
to test the importance of this parameter, we have set up the restitution coefficient \(\gamma_0\) equal to 0.2,
a value in the range of the largest values assumed by G78.

A priori considerations suggest that rebounds could have important consequences on the evolution
of the planetesimals and figures 2 testify also that rebounds can operate in some range of the impact
velocity. For example, rebounds are expected to modify the evolution of the mass spectrum at the
small size end because they could hinder the growth of the small bodies and their own capture by
the larger ones; so, the mass distribution should be expected steeper than for completely inelastic
collisions. It is also to be noted that fragmentation, which results in the formation of small body
populations, could strengthen the importance of the rebounds. In fact computations performed in
the same way as above seem to show that, in the investigated size range, the inelasticity of the
collisions play a negligible role.

VIII) Discussion and conclusions

This work improves and completes the model of the planetesimal accumulation started in paper
I. Mass dispersion among the largest bodies takes an important part in the evolution of initially
gentle mass distributions. For example, simple power laws are found to relax rapidly into a very
steep function. Such a relaxation of the mass distribution comes from the kinetics of the coagu-
lation process and is similar, in some respect, to the relaxation of the distribution of the velocity
dispersions which is driven by the dynamical friction (see paper I).

This transient behaviour of the mass and velocity distributions, whose detailed study is out of the
scope of the present work, shows that the first stage of the evolution of the planetesimals is nearly
independent of the initial conditions; the swarm itself supplies us rapidly with appropriate initial
conditions. Afterwards the evolution of the planetesimals proceeds approximately in the same way
as in paper I when we started from an exponential mass distribution: a rapid growth of the largest
bodies with a simultaneous increase of the velocity dispersions (such behaviour occurs because
gradually gravitational stirring can overcome dynamical friction). This increase of the velocity
dispersions strongly suggests that fragmentation have to play a role in the accumulation scenario
of the planets.

When catastrophic destructions are possible outcomes of the two-body planetesimal collisions, the
evolution of the mass distribution proceeds, in qualitative agreement with the work of Beaugé and
Aarseth (1990), into two different stages. During the first one accretion is the dominant process and
results in the rapid growth of much larger bodies; during the second one fragmentation counteracts
the effect of the accretion and the evolution of the mass distribution have two different aspects:
- at the small scales the mass spectrum stretch itself with the formation of a tail, a behaviour in
agreement with the results of G78 and of WS89 (see Figs. 6);
- at the large scales ”brittle” planetesimals grow on a longer time scale than indestructible ones;
this new aspect of the accumulation is consistent with the conclusion of Beaugé and Aarseth (1990)
according to which the formation of new embryos stalls when fragmentation begin to operate.
Both the extent of the tail and the growth time-scale are found to depend on the fragmentation
properties of the planetesimals.

Obviously during this second stage, the velocity dispersions evolve in relation with the changes of
the mass spectrum into two different ways:

- accretion tends to transfer most of the total mass from the small scales to the larger ones and,
so, keeps going the gravitational stirring;
- fragmentation tends to transfer most of the total cross-section of the inelastic collisions to the
smaller scales and, thus, controls the cooling of the swarm.

The velocity dispersions evolve more slowly as when fragmentation is not allowed but their distri-
bution among the various populations remains approximately the same.

It results from this two fold evolution that the collisions with the small bodies, although the most
probable ones, cannot contribute significantly to the growth of the planetesimals as the small body
populations contain only a small amount of the total mass. On the other hand, due to the increasing
velocity dispersions, fragmentation is at work up to a maximum size which increases as a function
of time; when this size reaches the intermediate scales of the mass spectrum a significant amount
of the total mass is affected and streams much less rapidly to the large scales. These two reasons
explain why planetesimals grow more slowly when fragmentation comes at work.

The second stage of the evolution also depends on the characteristic size of the primordial plan-
etesimals:

- when set up of the order of 10 km, fragmentation cannot prevent the growth of a single planetary
embryo (with some 17% of the total mass and a radius of the order of 1000 km) which remains
embedded in a swarm of much smaller bodies,
- when set up of the order of 1 km, fragmentation is easier and operates earlier than for 10-kilometer
sized bodies but the evolution is much more slowly.

As a result catastrophic collisions tend to slow down significantly the growth of planetary embryos.
The growth time-scales depend both on the initial size of the primordial planetesimals and on their
effective strength all along their growth.

Following the classical formation model (Safronov, 1969; Goldreich and Ward, 1973), the characteristic size of these primordial bodies is of the order of some kilometer at the earth distance but increases with increasing distances from the sun (some hundred kilometers at the Uranus distance); if so, catastrophic collisions could slow down the growth of the embryos much more effectively in the inner parts of the solar system than in its outer parts. It must be pointed out that such conclusion is consistent with the accumulation scenario of the giant planet cores and of the outer planets.

Following the recent fragmentation model of Housen et al (1991), the effective strength of asteroidal bodies is likely velocity dependent. As appears on figures 6 and 7 such a dependence have a significant consequence on the accretion time-scale (it is of the order of $10^5 \text{ yrs}$ or $10^7 \text{ yrs}$ following the strength is velocity dependent or not, respectively). However, as regard to the crudeness of the present model, this slowing down of the growth of the planetary embryos does not seems strong enough for the time-scale problem of the formation of the outer planets to be reopened. Nevertheless this could be the case if the characteristic size of the primordial planetesimals was smaller than one kilometer; then the formation of the planetesimals would play a much more important role than previously believed since the size of the bodies build during this stage could control the growth time-scale of the planetary embryos and, so, the fate of protoplanetary clouds evolution.

To summarize, in order to form a planetary system, the growth of massive embryos should take place before catastrophic collisions become too effective; that is accretion should bring most of the total mass to the largest scales before fragmentation have time to shift most of the total cross-section to the small scales.

At the end of our integration some large bodies are found to drop out of the mass distribution; this fact seems consistent with the idea (Safronov, 1966; Lissauer and Safronov, 1991) that the largest bodies to be captured by the proto-earth should have masses of the order of $10^{-2}$ or $10^{-3}$ earth masses as to explain planetary spins.

Now, we will also mention the implicit assumptions made in the above calculations:
- a 3-body approximation of the accretion rate was omitted since in paper I such a refinement was found unimportant in the overall evolution of the planetesimals.
- integration was pursued till a single body remains in the population of the largest bodies; in fact it is clear that a problem arises in such a situation since present theory is inappropriate to describe the change in the velocity dispersions of the small bodies. Nevertheless the work of Lecar and Aarseth (1986) supports the idea that, at the end of the evolution, long range perturbations between planetary embryos allow secular increase of the velocity dispersion and, so, can avoid premature isolation.

Finally let us make two comments about the formalism used in this paper:
- The basic statistical assumptions are well suited in the beginning of the evolution (since the initial number of bodies is very large) but become poorer and poorer at the end, when the population of the largest bodies reduces strongly; then (as pointed out by Safronov, 1969), if the mass of the embryos is of the order of the mass of all other bodies inside a given zone, strong perturbations of
the small body orbital motions are expected and the formalism must be changed.

- The accumulation process neglects the spatial fluctuations in the number density of the planetesimals; in this homogeneous mean-field description the accretion calculations are easier but the planetary embryos can form anywhere in the system.

The next step we have begun to investigate is the study of non-homogeneous accumulation processes and angular momentum transfers, two underlying questions in the final stage of the accumulation scenario.

Obviously other constraints for planet formation are also to be found in the present high velocity dispersions of the asteroid belt, likely in connection with the date of birth of a massive Jupiter core.

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. Figs.1 : Cumulative number of bodies and velocity dispersions as a function of mass for different time intervals \((\theta = t/t_0)\) starting from a Safronov exponential mass spectrum. The collisions are purely inelastic, the same gas drag as in WS89 is assumed. The characteristic parameters have the following values: \(m_o = 1.15 \times 10^{16} \, kg\), \(V_o = 15.5 \, ms^{-1}\), \(N_o = 5.51 \times 10^7\) and \(t_o = 1.22 \times 10^5 \, yrs\) (obtained from a surface density \(\sigma = 224 \, kgm^{-2}\)).

- Fig.1a : Evolution of the cumulative number of bodies

- Fig.1b : Evolution of the velocity dispersions

. Figs.2 : Cumulative Number of bodies and velocity dispersions as a function of mass for different time intervals \((\theta = t/t_o)\) if the mass is initially in the large bodies (the initial mass distribution is a power law with an index \(a = 0\).). The collisions are purely inelastic and the same gas drag as in WS89 is assumed. The characteristic parameters have the following values: \(m_o = 1.6 \times 10^{13} \, kg\) (corresponding to a radius of 1 km), \(V_o = T_o^{1/2} = 1.73 \, ms^{-1}\), \(N_o = 4 \times 10^{10}\) and \(t_o = 1.36 \times 10^4 \, yrs\).

- Fig.2a : Evolution of the cumulative number of bodies

- Fig.2b : Evolution of the velocity dispersions

. Figs.3 : Cumulative Number of bodies and velocity dispersions as a function of mass for different time intervals \((\theta = t/t_o)\) if the mass is initially in the small bodies (the initial mass distribution is a power law with an index \(a = -2\).). The collisions are purely inelastic, the same gas drag as in WS89 is assumed. The characteristic parameters have the following values: \(m_o = 1.6 \times 10^{13} \, kg\) (corresponding to a radius of 1 km), \(V_o = T_o^{1/2} = 1.73 \, ms^{-1}\), \(N_o = 4 \times 10^{10}\) and \(t_o = 1.36 \times 10^4 \, yrs\).

- Fig.3a : Evolution of the cumulative number of bodies

- Fig.3b : Evolution of the velocity dispersions

. Fig.4 : Specific energies required to disrupt \(Q_D^*\) or to fragment \(Q_F^*\) bodies. We have assumed an impact velocity of 5\(kms^{-1}\) and an energy transfer rate \(\chi = 0.1\). In the right part of curve \(Q_D^*\) the target cannot be disrupted and in between curve \(Q_D^*\) and curve \(Q_F^*\) the target is disrupted but the ejection of the fragments is impossible.

. Figs.5 : The velocity thresholds \(T_R^{1/2}, T_D^{1/2}, T_F^{1/2}\) for the occurrence of accretion, rebound or fragmentation, respectively, as a function of the projectile to target radius ratio (see text).

- Fig.5a : Case of the small bodies. Rebounds are very effective whereas reaccumulation is inexistent.

- Fig.5b : Case of the intermediate bodies. In the small strip between the two decreasing curves reaccumulation could began to play a role. Accretion corresponds either to a sticking or to a
- Fig.5c: Case of the large bodies. In the region between the two decreasing curves reaccumulation is likely to play a role whereas rebounds are much less effective.

- Figs.6: Cumulative number of bodies and velocity dispersions as a function of mass for different time intervals ($\theta = t/t_o$) starting from a Safronov exponential mass spectrum. Both accretion and fragmentation are allowed; the disruption energy is assumed independent of the impact velocity; the same gas drag as in WS89 is assumed. The characteristic parameters have following values: $m_o = 1.15 \times 10^{16} \text{ kg}$, $V_o = 15.5 \text{ ms}^{-1}$, $N_o = 5.51 \times 10^7$ and $t_o = 1.22 \times 10^5 \text{ yrs}$ (obtained from a surface density $\sigma = 224 \text{ kgm}^{-2}$).

- Fig.6a: Evolution of the cumulative number of bodies
- Fig.6b: Evolution of the velocity dispersions

- Figs.7: Cumulative number of bodies and velocity dispersions as a function of mass for different time intervals ($\theta = t/t_o$) starting from a Safronov exponential mass spectrum. Both accretion and fragmentation are allowed; the disruption energy is assumed to depend on the impact velocity; the same gas drag as in WS89 is assumed. The characteristic parameters have following values: $m_o = 1.15 \times 10^{16} \text{ kg}$, $V_o = 15.5 \text{ ms}^{-1}$, $N_o = 5.51 \times 10^7$ and $t_o = 1.22 \times 10^5 \text{ yrs}$ (obtained from a surface density $\sigma = 224 \text{ kgm}^{-2}$).

- Fig.7a: Evolution of the cumulative number of bodies
- Fig.7b: Evolution of the velocity dispersions

- Figs.8: Cumulative Number of bodies and velocity dispersions as a function of mass for different time intervals ($\theta = t/t_o$) if the mass is initially in the small bodies (the initial mass distribution is a power law with an index $a = -2$). Both accretion and fragmentation are allowed; the disruption energy is assumed independent of the impact velocity; the same gas drag as in WS89 is assumed. The characteristic parameters have the following values: $m_o = 1.6 \times 10^{13} \text{ kg}$ (corresponding to a radius of 1 km), $V_o = T_o^{1/2} = 1.73 \text{ ms}^{-1}$, $N_o = 4 \times 10^{10}$ and $t_o = 1.36 \times 10^4 \text{ yrs}$.

- Fig.8a: Evolution of the cumulative number of bodies
- Fig.8b: Evolution of the velocity dispersions