An improved algorithm for timing error detection based on all-digital receiver

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Abstract. In view of the wide application of the Gardner algorithm in timing synchronization loop and its limitation of the band-limited signals, an enhanced Gardner timing error detection algorithm, which has a simple structure and can effectively reduce the deviation of the timing error and system self-noise, is proposed in this paper by comparison of the Gardner algorithm and its correction algorithm in both the pattern jitter and timing errors on the basis of the asynchronous sampling. The simulation results show that E-Gardner algorithm has a significant improvement in the performance of QPSK modulation signal’s clock capture and error detection in the case of a small roll-off factor under the Gaussian with Rayleigh fading channel.

1. Introduction

The phase shift keying (PSK) modulation method has been widely applied in a variety of wireless communication systems. In order to improve the performance of the wireless modem, the synchronization technology is introduced. With the development of computers and the digital signal processing technology, the all-digital receiver [1] in the wireless communication has been used extensively. The Doppler frequency shift [2] and the long propagation delay between the mobile terminal and the other terminal inevitably cause sampling deviation and lead to that the sampling point is not necessarily the optimal sampling point. Therefore, many algorithms [3-4] on the timing synchronization of all-digital receiver have been proposed.

This paper presents a timing synchronization scheme based on the QPSK all-digital receiver in wireless communication. Through the application an enhanced Gardner timing error detection algorithm (Evolved of Gardner, referred to as E-Gardner) built on the Gardner correction algorithm, the accurately and efficiently clock error signal detected is back to the front control module for correcting the deviation given rise to by the independent sampling clock sources. As a result, the clocks of the transmitter and receiver maintain a good coherence by adjusting the clock error and finally achieve the goal of the transmitted signal and received signal being synchronized.

2. Timing synchronization loop model

The timing synchronization loop [5] suitable for wireless communication, which mainly consists of cubic interpolation filter, timing error detector, second-order loop filter and timing controller, is researched in this paper on the basis of the Gardner interpolation algorithm using a typical digital PLL structure. According to figure 1, the front end of the timing synchronization loop is a matched filter and usually for a square root raised cosine filter. When the timing error information by the loop
recovering the signal of integer multiples of symbol rate, the signal will be sent to the extraction module to recover the symbol rate.

Figure 1. Timing synchronization loop model

The timing error detection module closely related to the Gardner algorithm is chiefly used to compute timing error deviation making the entire feedback loop a drive signal. When the input signal is $y(n)$, the timing error value of the clock error detector in the classical Gardner algorithm is

$$e(n) = \text{Re} \{ y(n-1/2) [y(n) - y(n-1)] \}$$  \hspace{1cm} (1)$$

where $\text{Re}()$ and $y^*(\cdot)$ respectively indicate the real part and the conjugate of the signal, $e_n$ is a vector, $y(n-1/2)$ indicates the size of timing error, $y(n) - y(n-1)$ represents the direction of the timing error.

The Gardner algorithm has so greater limitation for band-limited signal that ZHANG L etc. have proposed a Gardner correction algorithm [6], and the expression is as follows:

$$e(n) = \text{Re} \{ [y(n-1/2) - \frac{1}{2} y(n) - y(n-1)] [y(n) - y(n-1)] \}$$ \hspace{1cm} (2)$$

This correction algorithm reduces the timing error to a certain extent. Yet the timing error deviation output through the clock error detector is still large for QPSK wireless signal due to the fading characteristics of the Rayleigh channel, which wouldn’t contribute to the locking of the synchronization loop. When the loop can not obtain the timing information, the Gardner correction algorithm will neither decrease the system self-noise nor easily accomplish in the project implementation as a result of the high complexity.

3. E-Gardner algorithm for clock error detection

In order to further improve the system performance and reduce the algorithm complexity, the Gardner correction algorithm has to be improved in the QPSK modulation mode. The basic idea of the E-Gardner algorithm is that we must consider the influence of different adjacent symbols on the intermediate values firstly because of the polarity shift of two adjacent symbols occurring and then how to eliminate this effect to obtain accurate and effective timing error information. When the two adjacent symbols have no polarity transition, the timing synchronization loop would generate the self-noise because of not being able to get the error information. For this reason, we must think of a way about how to reduce the system self-noise. Therefore, it is necessary that we modify the E-Gardner algorithm from two aspects: the timing error deviation and system self-noise.

3.1 The method of reducing deviation for timing error

Although two adjacent symbols have change of polarity and are accurately sampled, timing error output is not zero. What more, the acquisition and tracking performance of the synchronization loop is poor. So we ought to further reduce the timing error on the basis of Gardner correction algorithm.

According to the relation of two best sampling points of two adjacent symbols and their mid-points, the impact of adjacent symbols on the intermediate values should be taken into account, which can be deduced as follows:
\[ y(n - 1/2) \approx y(nT - T/2 - \tau) = A \sum_{i} c_i g(nT - T/2 - iT - \tau) = A c_n g(-T/2 - \tau) + A c_{n-1} g(T/2 - \tau) + \]
\[
\sum_{j=-n+1}^{n} A c_j g(T - (nT - T/2 - \tau)) = A c(n) g(-T/2 - \tau) + A (n-1) g(T/2 - \tau)
\]
\[ (3) \]

In the case of the raised cosine filter impulse response \( g(t) \) being symmetrical and normalization time offset \( \tau \) being statistically independent, according to the minimum mean square error (MMSE) criterion, so we can get:
\[
A c(n) g(-T/2 - \tau) + A c(n-1) g(T/2 - \tau) \approx g(-T/2) [c(n) + c(n-1)]
\]
\[ e(n) = \frac{g(-T/2)}{g(0)} [y(n) + y(n-1)] = \frac{2\cos(\alpha\pi/2)}{\pi(1-\alpha^2)} [y(n) + y(n-1)]
\]
\[ (4) \]

When using the intermediate value to calculate the timing error, the non-zero value \( y(n-1/2) \) in equation (3) should be subtracted. By using the equation \( y(1) \approx y(T/2 + \tau) \approx c(1) \cdot g(t-T) = c(1) \cdot g(0) \), \( c(n) = y(n)/g(0) \), \( y(n) \) and \( y(n-1) \) substitute for \( c(n) \) and \( c(n-1) \). So timing error without parameter estimation can be expressed as:
\[
e(n) = \text{Re} [\{y(n-1/2) - \frac{2\cos(\alpha\pi/2)}{\pi(1-\alpha^2)} [y(n) + y(n-1)]\} [y(n) - y(n-1)]]
\]
\[ (5) \]

If the shaping filter impulse response \( g(t) \) is unknown, a simple feedback can be used to calculate the multiplication factor such as \( 2\cos(\alpha\pi/2)/\pi(1-\alpha^2) \) in formula (4), and timing error’s size \( y(n-1/2) - 2\cos(\alpha\pi/2)/\pi(1-\alpha^2) [y(n) + y(n-1)] \) should be near zero. So timing error with parameter estimation can be expressed as:
\[
e(n) = \text{Re} [\{y(n-1/2) - \tilde{\xi}(n)[y(n) + y(n-1)]\} [y(n) - y(n-1)]]
\]
\[ (6) \]

Where \( \tilde{\xi}(n) = \tilde{\xi}(n-1) + \omega (y(n-1/2) - \tilde{\xi}_{n-1} [y(n) + y(n-1)]) \), \( \omega \) is the parameter of step length.

### 3.2 The method of reducing self-noise for synchronization loop

In the case of ideal sampling, mid-point sample values between the optimal sampling points of two adjacent symbols should be zero. While the channel is the Gaussian with Rayleigh, the intermediate sample values should be dealt with as 3.1 to receive more accurate timing error. However, when the polarity of two adjacent symbols varies, the synchronization loop will produce self-noise.

For the sake of reducing the system self-noise, the error detector output should be zero when adjacent symbols have no polarity jump. Otherwise the timing error value is not zero. So the sign function \( \text{sign}(\cdot) \) must be used in the E-Gardner algorithm. As for the sign function \( \text{sign}(\cdot) \), when \( x \geq 0 \) \( \text{sign}(x) = 1 \), or else \( \text{sign}(x) = -1 \). Thus \( [\text{sign}(y(n-1)) - \text{sign}(y(n))] \) is zero when there is the same polarity, on the contrary, \( [\text{sign}(y(n-1)) - \text{sign}(y(n))] \) is not zero.

### 3.3 E-Gardner algorithm’s expression

When the QPSK signals are transmitted in the form of consecutive frames, the polarity of all adjacent symbols is not necessarily the same. For this situation, in order to improve the performances of the timing error detector, the timing jitter, capture time etc., we shall be obliged to eliminate the impact value brought about by adjacent symbols as shown in (5) and (6). At the same time, the sign function \( \text{sign}(\cdot) \) is utilized as show in 3.2 to diminish the system self-noise. E-Gardner algorithm is put forward based on comprehensive considering the two suggestions mentioned above. So we get the complete E-Gardner algorithm’s expression:
\[
e(n) = [y_i(n - 1/2) - \frac{2\cos(\alpha\pi/2)}{\pi(1-\alpha^2)} [y_i(n) + y_i(n-1)] [\text{sign}(y_i(n)) - \text{sign}(y_i(n-1))]
\]
\[ + [y_o(n - 1/2) - \frac{2\cos(\alpha\pi/2)}{\pi(1-\alpha^2)} [y_o(n) + y_o(n-1)] [\text{sign}(y_o(n)) - \text{sign}(y_o(n-1))]
\]
\[ (7) \]
4. Simulation results and performance analysis

The various particles in the atmosphere will scatter or reflect the large number of radio signals when the wireless signals spread, which belongs to the situation of non-direct signal from transmitter to the receiver. So we simulate the performances of the E-Gardner algorithm using Matlab under the Gaussian with Rayleigh channel. The parameters of the system are set: the modulation mode is QPSK; SNR is 20dB; the symbol rate is 7.5MHz; the loop operating frequency is 30MHz; the carrier frequency is 1.66GHz; the normalized time offset is 0.2; the equivalent loop noise bandwidth $B_n = 7.5 \times 10^4$ Hz; the loop filter coefficient $c_1 = 7.07 \times 10^3$, $c_2 = 1.22 \times 10^7$; the roll-off factor of the raised cosine is 0.2; the Doppler shift [7] (considering only the uplink) and the rate of change are not greater than 528Hz and 24.6Hz/s separately.

4.1 Closed-loop Simulation

When the carrier frequency offset is 10KHz and the E-Gardner algorithm is exploited, the constellation diagram of receiving side is shown as figure 2. Figure 3 is the constellation diagram when the carrier phase deviation is 30°. The simulation results demonstrate the QPSK signals received carry out the timing synchronization, from which the E-Gardner algorithm is irrelevant with the carrier frequency offset and phase offset. This is also the major reason that we choice the Gardner algorithm as the fundamental algorithm.

Figure 2. Carrier frequency offset

![Figure 2. Carrier frequency offset](image)

Figure 3. Carrier phase offset

![Figure 3. Carrier phase offset](image)

Figure 4 and figure 5 are respectively convergence curves of the Gardner correction algorithm and the E-Gardner algorithm. When 5000 symbols are continuously sent out in the form of frames, as shown in figure 4, the steady-state error $u_k$ of the numerically controlled oscillator (NCO) is basically stabilized after about 1400 symbols. While entering the tracking state, $u_k$ has the large fluctuation near 0.5. However, the steady-state error $u_k$ reaches a stable convergence after about 1000 symbols in figure 5 and has little jitter when $u_k$ is in the tracking state. By contrast, we have found that the E-Gardner algorithm is capable of entering the tracking stage more rapidly and has the less pattern jitter in the tracking state than the Gardner correction algorithm.

Figure 4. Correction algorithm’s convergence

![Figure 4. Correction algorithm’s convergence](image)

Figure 5. E-Gardner algorithm’s convergence

![Figure 5. E-Gardner algorithm’s convergence](image)
output value of the timing error detection (TED) is in the range of \([-0.005, 0.005]\) according to figure 6. While the TED output value is about from \(-0.0025\) to \(0.0025\) in figure 7. By comparison, the improvement of the E-Gardner algorithm’s timing jitter is not obvious, but the timing error output value is roughly half of the Gardner correction algorithm. So the E-Gardner algorithm has promoted greatly in the stability of the entire synchronization loop.

The performance of the timing synchronization can be analyzed by the bit error rate (BER) [8] in the demodulated QPSK signal. Figure 8 is the statistical curve for the BER. When \(E_b / N_0\) is in \([-2, 10]\) range, the bit error rate’s curves have downward trend with the increase of the signal-to-noise (SNR) ratio. By contrast, it is found that the BER curve for the E-Gardner algorithm is very close to the theoretical curve. When the SNR is 10dB, the BER of the Gardner correction algorithm is about \(2.8 \times 10^{-3}\), while BER for the E-Gardner algorithm is \(1.2 \times 10^{-3}\), the latter performance improvement is approximately 4dB by calculating. Thus the E-Gardner algorithm has better BER performance compared with the Gardner correction algorithm.

4.2 Open-loop simulation

S-curve can directly reflect whether a timing error detection algorithm’s property is good or not. The smoother and steeper S-curve is, the better the performance of the corresponding algorithm will be. So we need an open-loop simulation to further verify the performance of the E-Gardner algorithm. The extracted timing error is not fed the loop filter and the timing controller module in the open-loop simulation. In other words, the loop has no feedback control, and only obtains the S characteristic curve shown in figure 9 that expresses the average of the TED corresponding the normalized time offset \(\tau\). The clock error detector’s gain \(K_d\) called S curve’s slope at the zero can be acquired when the curve can be approximated as a straight line near zero. Then \(K_d\) can be expressed as follows:

\[
K_d = \frac{\{E[e(\tau)]\}_{\tau=0.05} - E[e(\tau)]_{\tau=-0.05}}{0.1}
\]

(8)
By comparing S curves in figure 12, the conclusion is that the S-curve’s slope of the E-Gardner algorithm is larger than the Gardner correction algorithm. Therefore, E-Gardner algorithm has better performance and higher sensitivity.

5. Conclusion
This paper studies the timing synchronization algorithm based on the all-digital receiver. For the envelope fluctuation of the transmitted QPSK wireless signals, which makes the Gardner correction algorithm the larger timing error and the system self-noise, an enhanced E-Gardner algorithm proposed is applied to the timing synchronization loop of the QPSK signals. The simulation results show us that this enhanced algorithm is capable of attaining the rapid timing synchronization under the high dynamic conditions and can provide the performance boost of several decibels. On the other hand, the application of the sign function in the E-Gardner algorithm changes the multi-bit multiplication into the single-bit strobe’s operation. Thereby the E-Gardner algorithm is able to decrease the computational complexity and cut down the consumption of system resources. Lastly, the E-Gardner algorithm is not sensitive to the carrier phase and can overcome the difficulties of the Doppler acceleration etc. existing in the wireless communication, which has a much higher application value especially in the all-digital demodulation of the wireless communication system.

Acknowledgement
This research was financially supported by the science and technology development project of Shizuishan city (Grant No. 2018BDE23023).

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