Minimizing cost and backorder risk through customer priority on spares inventory

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Abstract. Spare parts demands usually occur due to the need to replace spare parts on damaged equipment. Uncertain demands and many types of spare parts require complex management. Therefore, the management of spare parts is very challenging. In this study, we developed a spare part inventory method by considering priorities based on customer class. Customers will be grouped according to the number of demands. The number of requests becomes a fill rate priority. Any request that cannot be fulfilled will have backorder risk. This customer prioritization aims to minimize costs and backorder risks. The difference in customer class aims to minimize total inventory costs. Spare parts will be issued based on the queue discipline which is first in first out. The inventory model is solved by the Lagrange multiplier method. Model verification is carried out in real practice in illustrative examples. The results of the research show that customer priorities that are carried out in the provision of the fulfillment of spare parts can reduce costs and backorder risks.

1. Introduction
Along with technological developments and tight competition, companies are required to be able to compete. Competition is not only limited to product prices, but also good quality and service for customers. In this research service for customers in the form of timely fulfillment of demand or product availability when customers need. The purpose of spare parts inventory management is to meet all customer requests based on the target service level and minimize the number of requests that cannot be met. The size of the number of requests that can be fulfilled without the backorder is called fill rate. Fill rate is measured as a whole or for each item of spare parts. Fill rate is used to differentiate each customer class. Demand in each customer class has a different cost if the request is not met by the company.

Several previous studies in spare parts management are investigated from the perspective of information systems, maintenance management and new applications or methods in software \cite{1, 2, 3, 4, 5}. They conducted a review of the spare parts classification literature, inventory forecasting, and investigated the gap between research and practice in spare parts management \cite{6, 7, 8, 9, 12}. Spare parts supply is different from the general inventory. Some special characteristics of spare parts so that it is very difficult to predict: (i) Demand for spare parts due to equipment failure or obsolescence parts; (ii) The number and variety of parts is usually very large requiring accuracy in identifying items in inventory; (iii) There is a risk of obsolete parts so that the amount of Stock Keeping Unit (SKU) storage...
must be correct; (iv) Spare parts for critical equipment, such as health and safety equipment, banking servers and nuclear equipment, etc.

Mathematical models have been developed on the level of repair analysis method in multi-echelon models to control spare parts inventory [1]. Lateral transshipment and emergency delivery are used on their models if needed. The study resulted in a reduction in the average cost of 3% of the previous inventory costs. Furthermore, other authors show that the batch ordering method can solve the problem of spare parts inventory optimally [13]. They discussed a multi-item spare parts system that aims to get a minimum inventory with response time settings.

The emphasis on the service level becomes the priorities for spare parts management to complete the fill rate to get the optimal solution for minimizing inventory investment [6]. The concept of the service level is developed in spare parts inventory in a multi-echelon, multi-item supply chain with time-based customer service level agreements [7].

Researches regarding limited repair capacity in the METRIC model (Multi-Echelon Technique for Recoverable Item Control) have also been developed [11]. The damage that occurs is assumed to follow the Poisson process with a constant average. Simplifying inventory stock analysis was developed through the Palms theorem. However, limited repair capacity can be modified using laterally transshipment between local warehouses, with the Markov process [14]. If other local warehouses do not have stock on hand, emergency replenishment is carried out from the central warehouse. The goal to be achieved is to minimize holding and transportation costs. Completion of optimal costs at lower bound is used by Lagrange relaxation. Subsequent research, they proposed an algorithm to obtain optimal base stock levels with a combination of decomposition and column generation [15].

This research is different from previous research, customers are formed in class to fulfill the demand for spare parts. In this study, the optimization model is an objective function and constraints. Objective function aims to minimize the total cost of spare parts inventory based on METRIC theory. The constraint function is the size of the service level used, the service level size is the fill rate for each item in each customer class. The model formed is solved by Lagrange multiplier.

2. Problem description and formulation

The demand for each customer class is assumed to be Poisson distribution. Spare parts are called SKU (stock keeping units), used indexes $i = 1, 2, \ldots, I$ and customer classes are given indexes $j = 1, 2, \ldots, J$. Customer class consists of $J \geq 2$. With the difference in customer class, there is a difference in the level of service for each customer class in terms of fulfilling demand. The fill rate for each customer class is denoted by $\beta_{j,\text{obj}}$. Fill rate of customer class 1 is greater than customer class fill rate 2 and so on, written as follows:

$$\beta_{1,\text{obj}} \geq \beta_{2,\text{obj}} \geq \cdots \geq \beta_{J,\text{obj}}.$$  \hspace{1cm} (1)

Replenishment of each spare part is assumed to be independent and identical. Customers from class $j > 1$ have a fill rate that is more than or equal to the fill rate of the customer class $k < j$. Therefore, a minimum inventory of class $j$ customers is more than or at least equal to the minimum inventory that must exist in the customer class, namely:

$$c_{i,1} \leq c_{i,2} \leq \cdots \leq c_{i,j}, \quad c_{i,j} \in \mathbb{N}. \hspace{1cm} (2)$$

In accordance with the METRIC theory, the objective function that will be formed aims to minimize total inventory investment with customer class differences. Based on the METRIC theory, the objective function is formed by the sum of the base stock for each spare part multiplied by the price of the spare parts. If the price of each part $i$ is $p_i$ then the total investment in inventory is:

$$(TC) \text{ Minimize } \sum_{i=1}^{I} p_i c_i,$$

subject to

$$\sum_{i=1}^{I} \frac{m_{i,j}}{M_j} \beta_{i,j}(c_{i,j}) \geq \beta_{j,\text{obj}}, \quad j = 1, 2, \ldots, J$$

$$c_{i,1} \leq c_{i,2} \leq \cdots \leq c_{i,j}, \quad i = 1, 2, \ldots, I$$

$$c_{i,1}, c_{i,2}, \ldots, c_{i,j} \in \mathbb{N}, \quad i = 1, 2, \ldots, I$$

where

$$c_i = c_{i,1} + c_{i,2} + \cdots + c_{i,j}.$$
2.1. Fill rate formulation
In principle, inventory management must be oriented to the efficiency of operations on one side and service level to customers on the other. Fill rate is the percentage of the number of items available when requested by the customer.

Each spare part \(i\) in the customer class \(j\), the amount of inventory for replenishment in the inventory that is still in delivery (pipeline stock) plus the amount of inventory in the physical stock equal to base stock level. After the pipeline arrives in the warehouse, the stock is added to the physical stock. Lead time replenishment for pipeline stock is \(T_i\) and the lead time for each spare part is independent. Inventories in physical stock will only decrease after customer requests are met, assuming a longer stock will be taken first. In this case, the stock in and out of spare part follows the rules of the FIFO (First In First Out) queue. The request is assumed to follow a Poisson distribution with an average arrival time with an exponential distribution. Based on the Palm's theorem and based on M / M / 1, steady state is obtained to calculate the minimum stock in the queue by substituting \(\rho = m_{t,j}T_i\) as follows:

\[
q_{l,c_i,j} = (m_{i,j}T_i)^{c_i} q_{i,0}
\]

c_{i,j} is the number of critical level of part \(i\) items for customer class to \(j\), \(q_{l,c_i,j}\) is the probability \(c_{i,j}\) is in the queue system. \(q_{i,0}\) is the probability of no stock in the queue system, formulated as follows:

\[
q_{i,0} = 1 - m_{i,j}T_i
\]

where

\[
|m_{i,j}T_i| < 1
\]

and

\[
q_{l,c_i,j} = (m_{i,j}T_i)^{c_i}/(1 - m_{i,j}T_i)
\]

We have the fill rate for each spare part:

\[
\beta_{i,j}(c_{i,j}) = q_{l,c_i,j} - m_i
\]

\[
\beta_{i,j}(c_{i,j}) = (m_{i,j}T_i)^{c_i - m_i}(1 - m_{i,j}T_i)
\]

where \(m_i\) is the average demand for part items \(i\).

2.2. Decomposition of the optimization model
Completion of the problem (TC) is done by changing the constraint function for each customer class \(j = 1, 2, ..., \) be:

\[
\sum_{j=1}^{J} \left( \sum_{i=1}^{I} \frac{m_{i,j}}{M_j} \beta_{i,j}(c_{i,j}) \geq \beta_{j,\text{obj}} \right)
\]

\[
\approx \sum_{j=1}^{J} \left( \sum_{i=1}^{I} \frac{m_{i,j}}{M_j} \beta_{i,j}(c_{i,j}) - \beta_{j,\text{obj}} \geq 0 \right)
\]

\[
\approx \sum_{j=1}^{J} \sum_{i=1}^{I} \left( \frac{m_{i,j}}{M_j} (\beta_{j,\text{obj}} - \beta_{i,j}(c_{i,j})) \right) \leq 0
\]

The constraint function applies to each class, so there is \(j\) Lagrange multiplier. For this reason, the Lagrange multipliers \(\lambda_j M_j\), \(j = 1, 2, ..., \) are used where \(\lambda = (\lambda_1, ..., \lambda_J)\). The objective function (TC) and the constraint function in equation (4) become the following Lagrange function equation:

Minimize \((L(\lambda))\)

\[
\sum_{i=1}^{I} p_i c_i + \left( \sum_{j=1}^{J} \lambda_j M_j \left( \sum_{i=1}^{I} \frac{m_{i,j}}{M_j} (\beta_{j,\text{obj}} - \beta_{i,j}(c_{i,j})) \right) \right)
\]
\[ = \sum_{i=1}^{l} p_i c_i + \left( \sum_{j=1}^{f} \sum_{i=1}^{l} \lambda_i m_{i,j} \left( \beta_{j,\text{obj}} - \beta_{i,j}(c_{i,j}) \right) \right) \]

where

\[
c_{i,1} \leq c_{i,2} \leq \cdots \leq c_{i,j}, \quad i = 1, 2, \ldots, l
\]
\[
c_{i,1}, c_{i,2}, \ldots, c_{i,j}, c_i \in \mathbb{N}, \quad i = 1, 2, \ldots, l
\]

In the equation \( m_{i,j} \left( \beta_{j,\text{obj}} - \beta_{i,j}(c_{i,j}) \right) = m_{i,j} \left( 1 - \beta_{i,j}(c_{i,j}) \right) + m_{i,j} \left( \beta_{j,\text{obj}} - 1 \right); m_{i,j} \left( 1 - \beta_{i,j}(c_{i,j}) \right) \) is the backorder expectation per unit time for each spare \( i \) in the customer class \( j \) and \( m_{i,j}(\beta_{j,\text{obj}} - 1) \) is a constant. Therefore \( \lambda_j \) interprets the backorder cost for customer class \( j \). The Lagrange function \( (L(\lambda)) \) consists of inventory investment, penalty cost for all requests that cannot be immediately fulfilled and constants.

3. Solution procedure

The objective of the problem is to minimize the inventory cost \((TC)\) of each spare part \(i\). We set the completion as follows:

1. The first step to solve the problem \((TC)\) is to get the formula to determine the fill rate value for each part \(i\) in the customer class \(j\) using queuing theory. Furthermore, the Lagrange relaxation method is applied to the objective function and constraint functions.

\[
(L(\lambda)) = \sum_{i=1}^{l} p_i c_i + \sum_{j=1}^{f} \sum_{i=1}^{l} \lambda_i m_{i,j} \left( \beta_{j,\text{obj}} - \beta_{i,j}(c_{i,j}) \right) 
\]

\[
= \sum_{i=1}^{l} \left( p_i c_i + \sum_{j=1}^{f} \lambda_i m_{i,j} \left( \beta_{j,\text{obj}} - \beta_{i,j}(c_{i,j}) \right) \right) 
\]

\[
= \sum_{i=1}^{l} \left( p_i c_i + \sum_{j=1}^{f} \lambda_j \left( m_{i,j} \left( \beta_{j,\text{obj}} - 1 + 1 - \beta_{i,j}(c_{i,j}) \right) \right) \right) 
\]

\[
= \sum_{i=1}^{l} \left( p_i c_i + \sum_{j=1}^{f} \lambda_j \left( m_{i,j} \left( 1 - \beta_{i,j}(c_{i,j}) \right) + m_{i,j}(\beta_{j,\text{obj}} - 1) \right) \right) 
\]

\[
= \sum_{i=1}^{l} \left( p_i c_i + \sum_{j=1}^{f} \lambda_j m_{i,j} \left( 1 - \beta_{i,j}(c_{i,j}) \right) + \sum_{j=1}^{f} \lambda_j m_{i,j}(\beta_{j,\text{obj}} - 1) \right) 
\]

\[
= \sum_{i=1}^{l} \left( p_i c_i + \sum_{j=1}^{f} \lambda_j m_{i,j} \left( 1 - \beta_{i,j}(c_{i,j}) \right) \right) + \sum_{j=1}^{f} \left( \sum_{i=1}^{l} m_{i,j} \right)(-1)\lambda_j(1 - \beta_{j,\text{obj}}) 
\]

\[
= \sum_{i=1}^{l} \left( p_i c_i + \sum_{j=1}^{f} \lambda_j m_{i,j} \left( 1 - \beta_{i,j}(c_{i,j}) \right) \right) + \sum_{j=1}^{f} M_j(-1)\lambda_j(1 - \beta_{j,\text{obj}}) 
\]

\[
= \sum_{i=1}^{l} \left( p_i c_i + \sum_{j=1}^{f} \lambda_j m_{i,j} \left( 1 - \beta_{i,j}(c_{i,j}) \right) \right) - \sum_{j=1}^{f} M_j \lambda_j(1 - \beta_{j,\text{obj}}) 
\]

\[\text{(5)}\]
2. The second step, the optimization is done with Lagrange multiplier. The process of optimizing with the Lagrange multiplier can be decomposed into an optimization problem for one type of spare parts.

3. The third step; analyze the convexity of the function, whether it has a minimum value at a certain point. Furthermore, to obtain a solution of the objective function, a partial derivative is performed.

4. We substitute the fill rate item for each spare part in each customer class.

5. The fifth step is to look for partial derivative of decomposition functions to get critical levels and backorder costs for each spare part in each customer class.

\[
\frac{\partial l_i}{\partial c_{i,1}} = p_i + \frac{\partial}{\partial c_{i,1}} \left[-\lambda_1 m_{i,1} (m_{i,1} T_i)^{c_{i,1}-m_i} (1 - m_{i,1} T_i)\right]
\]

\[
0 = p_i + \left(1 - m_{i,1} T_i\right) \left(-\lambda_1 m_{i,1}\right) \frac{\partial}{\partial c_{i,1}} \left[(m_{i,1} T_i)^{c_{i,1}-m_i}\right]
\]

\[
0 = p_i - (1 - m_{i,1} T_i) (\lambda_1 m_{i,1}) (m_{i,1} T_i)^{c_{i,1}-m_i} \log(m_{i,1} T_i)
\]

\[
\frac{\partial l_i}{\partial c_{i,2}} = p_i + \frac{\partial}{\partial c_{i,2}} \left[-\lambda_2 m_{i,2} (m_{i,2} T_i)^{c_{i,2}-m_i} (1 - m_{i,2} T_i)\right]
\]

\[
0 = p_i + (1 - m_{i,2} T_i) (-\lambda_2 m_{i,2}) \frac{\partial}{\partial c_{i,2}} \left[(m_{i,2} T_i)^{c_{i,2}-m_i}\right]
\]

\[
0 = p_i - (1 - m_{i,2} T_i) (\lambda_2 m_{i,2}) (m_{i,2} T_i)^{c_{i,2}-m_i} \log(m_{i,2} T_i)
\]

\[
\frac{\partial l_i}{\partial c_{i,3}} = p_i + \frac{\partial}{\partial c_{i,3}} \left[-\lambda_3 m_{i,3} (m_{i,3} T_i)^{c_{i,3}-m_i} (1 - m_{i,3} T_i)\right]
\]

\[
0 = p_i + (1 - m_{i,3} T_i) (-\lambda_3 m_{i,3}) \frac{\partial}{\partial c_{i,3}} \left[(m_{i,3} T_i)^{c_{i,3}-m_i}\right]
\]

\[
0 = p_i - (1 - m_{i,3} T_i) (\lambda_3 m_{i,3}) (m_{i,3} T_i)^{c_{i,3}-m_i} \log(m_{i,3} T_i)
\]

\[
\frac{\partial l_i}{\partial c_{i,j}} = p_i + \frac{\partial}{\partial c_{i,j}} \left[-\lambda_j m_{i,j} (m_{i,j} T_i)^{c_{i,j}-m_i} (1 - m_{i,j} T_i)\right]
\]

\[
0 = p_i + (1 - m_{i,j} T_i) (-\lambda_j m_{i,j}) \frac{\partial}{\partial c_{i,j}} \left[(m_{i,j} T_i)^{c_{i,j}-m_i}\right]
\]

\[
0 = p_i - (1 - m_{i,j} T_i) (\lambda_j m_{i,j}) (m_{i,j} T_i)^{c_{i,j}-m_i} \log(m_{i,j} T_i)
\]

\[
\frac{\partial l_i}{\partial \lambda_1} = m_{i,1} (1 - (m_{i,1} T_i)^{c_{i,1}-m_i} (1 - m_{i,1} T_i)) = 0
\]

\[
\frac{\partial l_i}{\partial \lambda_2} = m_{i,2} (1 - (m_{i,2} T_i)^{c_{i,2}-m_i} (1 - m_{i,2} T_i)) = 0
\]

\[
\frac{\partial l_i}{\partial \lambda_3} = m_{i,3} (1 - (m_{i,3} T_i)^{c_{i,3}-m_i} (1 - m_{i,3} T_i)) = 0
\]

\[
\frac{\partial l_i}{\partial \lambda_j} = m_{i,j} (1 - (m_{i,j} T_i)^{c_{i,j}-m_i} (1 - m_{i,j} T_i)) = 0
\]

Equation (10) - (13) is solved to obtain the optimal values of \(c_{i,1}, c_{i,2}, c_{i,3}, \ldots, c_{i,j}\) as follows:

\[
m_{i,1} (1 - (m_{i,1} T_i)^{c_{i,1}-m_i} (1 - m_{i,1} T_i)) = 0
\]

\[
1 - (m_{i,1} T_i)^{c_{i,1}-m_i} (1 - m_{i,1} T_i) = 0
\]

\[
(m_{i,1} T_i)^{c_{i,1}-m_i} (1 - m_{i,1} T_i) = 1
\]
\[(m_{i,1}T_i)^{c_{i,1} - m_i} = \frac{1}{1 - m_{i,1}T_i}\]
\[c_{i,1} - m_i = \log m_{i,1}T_i \frac{1}{1 - m_{i,1}T_i}\]
\[c_{i,1} = m_i - \log m_{i,1}T_i (1 - m_{i,1}T_i)\]
\[c_{i,1} = m_i - \log (1 - m_{i,1}T_i)\]
\[c_{i,2} = m_i - \log (1 - m_{i,2}T_i)\]
\[c_{i,3} = m_i - \log (1 - m_{i,3}T_i)\]
\[c_{i,j} = m_i - \log (1 - m_{i,j}T_i)\]

Equation (6) - (10) is solved to obtain the backorder costs for each spare part in each customer class as follows:

\[0 = p_i - (1 - m_{i,1}T_i) (\lambda_1 m_{i,1}) (m_{i,1}T_i)^{c_{i,1} - m_i} \log (m_{i,1}T_i)\]
\[p_i = (1 - m_{i,1}T_i) (\lambda_1 m_{i,1}) (m_{i,1}T_i)^{c_{i,1} - m_i} \log (m_{i,1}T_i)\]
\[\lambda_1 = \frac{m_{i,1} (1 - m_{i,1}T_i) (m_{i,1}T_i)^{c_{i,1} - m_i} \log (m_{i,1}T_i)}{p_i}\]
\[\lambda_2 = \frac{p_i}{m_{i,2} (1 - m_{i,2}T_i) (m_{i,2}T_i)^{c_{i,2} - m_i} \log (m_{i,2}T_i)}\]
\[\lambda_3 = \frac{p_i}{m_{i,3} (1 - m_{i,3}T_i) (m_{i,3}T_i)^{c_{i,3} - m_i} \log (m_{i,3}T_i)}\]
\[\lambda_j = \frac{p_i}{m_{i,j} (1 - m_{i,j}T_i) (m_{i,j}T_i)^{c_{i,j} - m_i} \log (m_{i,j}T_i)}\]

4. Illustrative examples
In this section, we consider a planning horizon a company engaged in server machine maintenance has demand data for 10 parts in 1 year. The division of the customer class is used to calculate the total demand per year. Classification criteria for customer classes are determined by the number of requests with the following rules: (1) Class 1: customer with a total demand of more than 1,300 items per year; (2) Class 2: customers with a total demand of more than 1,200 to 1,300 items per year; (3) Class 3: customers with a total demand of more than 1,150 to 1,200 items per year; (4) Class 4: customers with a total demand of fewer than 1,150 items per year; (5) Class 5: customers with a total demand of fewer than 1,100 items per year.

The demand data is tested to determine the distribution for each spare part. Distribution of demand for each SKU of spare parts to guide customer class. Kolmogorov Smirnov's test results revealed that the overall demand per SKU data in each customer class can be said to be Poisson distributed with a Poisson distribution rank of 1, 2 and 3. The data for each SKU can be seen in Table 1.
Table 1. The data of spare parts.

| No | Demand | Unit price (Rp) | Cost per SKU (Rp) | Lead time (day) |
|----|--------|----------------|------------------|-----------------|
| 1  | 125    | 999,900        | 124,987,500      | 7               |
| 2  | 211    | 145,000        | 30,595,000       | 5               |
| 3  | 1,030  | 130,000        | 133,900,000      | 1               |
| 4  | 170    | 599,900        | 101,983,000      | 7               |
| 5  | 785    | 120,000        | 94,200,000       | 2               |
| 6  | 800    | 199,900        | 159,920,000      | 1               |
| 7  | 180    | 700,000        | 126,000,000      | 6               |
| 8  | 960    | 49,000         | 47,040,000       | 1               |
| 9  | 460    | 150,000        | 69,000,000       | 2               |
| 10 1,320 | 120,000 | 150,000,000 | 1 | |

Total Cost (Rp) 1,037,625,500

Table 2. The results of the optimization of spare parts with the Lagrange multiplier method.

| SKU-i  | Critical level | Total | Total Cost (Rp) |
|--------|----------------|-------|-----------------|
| c_{l,1} | c_{l,2} | c_{l,3} | c_{l,4} | c_{l,5} | | | |
| 1  | 20  | 20  | 21  | 21  | 22  | 104 | 115,988,400 |
| 2  | 37  | 39  | 41  | 41  | 197 | 27,260,000 |
| 3  | 204 | 204 | 204 | 204 | 1,020 | 131,300,000 |
| 4  | 3  | 30  | 32  | 32  | 129 | 76,187,300 |
| 5  | 119 | 139 | 149 | 149 | 696 | 93,360,000 |
| 6  | 159 | 159 | 159 | 159 | 759 | 158,320,800 |
| 7  | 33  | 33  | 34  | 34  | 168 | 106,400,000 |
| 8  | 190 | 190 | 190 | 190 | 851 | 46,354,000 |
| 9  | 90  | 91  | 91  | 91  | 453 | 67,050,000 |
| 10 | 246 | 246 | 246 | 247 | 247 | 1,232 | 138,720,000 |

Total 5,747 960,940,500

The total initial inventory cost of Rp 1,037,625,500. The demand fulfillment target for each customer class as follows: \( \beta_{1,OBJ} = 0.98, \beta_{2,OBJ} = 0.96, \beta_{3,OBJ} = 0.95, \beta_{4,OBJ} = 0.92, \beta_{5,OBJ} = 0.90. \)

Optimization results using MATLAB software for 10 SKUs can be seen on Table 2 and Table 3.

The solution of \( I_i \) is a solution for \( (TC) \) due to \( I_i \) where requirement meets:

\[
c_{l,1} \leq c_{l,2} \leq \cdots \leq c_{l,J} \leq c_{l,f} \quad i = 1, 2, \ldots, I
\]

\[
c_{l,1}, c_{l,2}, \ldots, c_{l,J} \in \mathbb{N}, \quad i = 1, 2, \ldots, I
\]

The minimum total inventory that must be stored in the warehouse is 5,747. This amount is the accumulation of minimum inventory for each customer class from 10 types of spare parts with a minimum cost of Rp 960,940,500 and backorder risk Rp 764,342. The total cost prior to optimization by the Lagrange method is Rp. 1,037,625,500 to obtain a reduction in costs of 7.4%.
Table 3. Lambda value of 10 SKUs in each customer class.

| SKU-i | \( \lambda_1 \) | \( \lambda_2 \) | \( \lambda_3 \) | \( \lambda_4 \) | \( \lambda_5 \) | Total |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|-------|
| 1     | 81,183          | 48,729          | 43,261          | 24,420          | 13,929          | 211,522 |
| 2     | 14,077          | 8,128           | 4,358           | 3,636           | 1,744           | 31,943 |
| 3     | 2,157           | 1,153           | 719             | 609             | 304             | 4,942  |
| 4     | 145,953         | 49,606          | 32,875          | 21,459          | 19,476          | 269,369 |
| 5     | 817             | 541             | 577             | 343             | 932             | 3,210  |
| 6     | 1,649           | 1,103           | 1,182           | 733             | 462             | 5,129  |
| 7     | 100,669         | 43,909          | 31,796          | 25,470          | 13,419          | 215,263 |
| 8     | 648             | 404             | 231             | 238             | 136             | 1,657  |
| 9     | 2,981           | 1,910           | 1,219           | 1,358           | 797             | 8,265  |
| 10    | 8,122           | 2,302           | 1,199           | 894             | 525             | 13,042 |
| **Total** |                |                 |                 |                 |                 | **764,342** |

5. Conclusion
Spare parts stock needs to be well coordinated because minimal costs are the goal of meeting customer needs. A large number of SKUs spare parts ordered by customers vary greatly. Consequently, the spare parts inventory plan needs to be coordinated by making the customer class based on the number of requests. However, the fill rate and critical level are also of concern in making decisions. Likewise, the inventory arrangement of spare parts issued must pay attention to certain queuing systems that relate to the expiration date and lead time. In this study, we aim to fill in the gaps in the literature in the context of minimizing costs and backorder risk by coordinating the number of orders in the customer class. We develop optimization models by setting class priorities from customer demand. The difference in the fill rate depends on the suitability of the customer class division. Numerical results revealed that the proposed model was able to reduce costs and backorder risks. The scope for future research, multi-objective function with different spare part queue properties may be used to compute customer priority.

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