Judgment Aggregation in Multi-Agent Argumentation

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Abstract Given a set of conflicting arguments, there can exist multiple plausible opinions about which arguments should be accepted, rejected, or deemed undecided. We study the problem of how multiple such judgments can be aggregated. We define the problem by adapting various classical social-choice-theoretic properties for the argumentation domain. We show that while argument-wise plurality voting satisfies many properties, it fails to guarantee the collective rationality of the outcome, and struggles with ties. We then present more general results, proving multiple impossibility results on the existence of any good aggregation operator. After characterising the sufficient and necessary conditions for satisfying collective rationality, we study whether restricting the domain of argument-wise plurality voting to classical semantics allows us to escape the impossibility result. We close by listing graph-theoretic restrictions under which argument-wise plurality rule does produce collectively rational outcomes. In addition to identifying fundamental barriers to collective argument evaluation, our results open up the door for a new research agenda for the argumentation and computational social choice communities.

Keywords Argumentation · Agents · Preferences · Social Choice

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1 Introduction

Argumentation has recently become one of the key approaches to automated reasoning and rational interaction in Artificial Intelligence [5,27]. A key milestone in the development of argumentation in AI has been Dung’s landmark framework [15]. Arguments are viewed as abstract entities (a set $\mathcal{A}$), with a binary defeat relation (denoted $\triangleright$) over them. The defeat relation captures the fact that one argument somehow attacks or undermines another. This view of argumentation enables high-level analysis while abstracting away from the internal structure of individual arguments. In Dung’s approach, given a set of arguments and a defeat relation, a rule specifies which arguments should be accepted.

Often, there are multiple reasonable ways in which an agent may evaluate a given argument structure (e.g. accepting only conflict-free, self-defending sets of arguments). Each possible evaluation corresponds to a so-called extension [15] or labelling [8,9]. Different argumentation semantics yield different restrictions on the possible extensions. Most previous research has focused on evaluating and comparing different semantics based on the (objective) logical properties of their extensions [3].

At the heart of most established argumentation semantics is the condition of admissibility: that accepted arguments must not attack one another, and must defend themselves against counter-arguments, by attacking them back. Another more restricted notion is called completeness, and is captured, in terms of labeling, in the following two conditions:

1. An argument is labelled accepted (or in) if and only if all its defeaters are rejected (or out).
2. An argument is labelled rejected (or out) if and only if at least one of its defeaters is accepted (or in).

Otherwise, an argument may be labelled undec. Thus, evaluating a set of arguments amounts to labelling each argument using a labelling function $L : \mathcal{A} \rightarrow \{\text{in, out, undec}\}$ to capture these three possible labels. Any labelling that satisfies the above conditions is also called a legal labelling. We will often use legal labelling and complete labelling interchangeably.

The above conditions attempt to evaluate arguments from a single point of view. Indeed, most research on formal models of argumentation discounts the fact that argumentation takes place among self-interested agents, who may have conflicting opinions and preferences over which arguments end up being accepted, rejected, or undecided. Consider the following simple example.

Example 1 (A Murder Case) A murder case is under investigation. To start with, there is an argument that the suspect should be presumed innocent ($a_3$). However, there is evidence that he may have been at the crime scene at the time ($a_2$), which would counter the initial presumption of innocence. There is also, however, evidence that the suspect was attending a party that day.

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1 This paper is a substantially extended and revised version of [28].
Clearly, $a_1$ and $a_2$ are mutually defeating arguments since the suspect can only be in one place at any given time. Hence, we have a set of arguments $\{a_1, a_2, a_3\}$ and a defeat relation $\rightarrow = \{(a_1, a_2), (a_2, a_1), (a_2, a_3)\}$. There are three possible labellings that satisfy the above conditions:

- $L(a_1) = \text{in}$, $L(a_2) = \text{out}$, $L(a_3) = \text{in}$.
- $L'(a_1) = \text{out}$, $L'(a_2) = \text{in}$, $L'(a_3) = \text{out}$.
- $L''(a_1) = \text{undec}$, $L''(a_2) = \text{undec}$, $L''(a_3) = \text{undec}$.

The graph and possible labellings are depicted in Figure 1.

Example 1 highlights a situation in which multiple points of view can be taken, depending on whether one decides to accept the argument that the suspect was at the party or the crime scene. The question we explore in this paper can be highlighted through the following example, extending Example 1.

Example 2 (Three Detectives) A team of three detectives, named 1, 2, and 3, have been assigned to the murder case described in Example 1. Each detective’s judgment can only correspond to a legal labelling (otherwise, her judgment can be discarded). Suppose that each detective’s judgment is such that $L_1 = L$, $L_2 = L'$, and $L_3 = L'$. That is, detectives 2 and 3 agree but differ with detective 1. These labellings are depicted in the labelled graph of Figure 2.

The detectives must decide which (aggregated) argument labelling best reflects their collective judgment.

Example 2 highlights an aggregation problem, similar to the problem of preference aggregation [2,16,32] and the problem of judgment aggregation on propositional formulae [23,20,22,19]. It is perhaps obvious in this particular example that $a_3$ must be rejected (and thus the defendant be considered guilty), since most detectives seem to think so. For the same reason, $a_1$ must be rejected and $a_2$ must be accepted. Thus, labelling $L'$ (see Example 1) wins by majority. As we shall see in our analysis below, things are not that simple, and counter-intuitive situations may arise. We summarise the main question asked in the paper as follows.
Given a set of agents, each with a specific subjective evaluation (i.e., labelling) of a given set of conflicting arguments, how can agents reach a collective decision on how to evaluate these arguments?

While Arrow’s Impossibility Theorem can be expected to ensue for this problem \textsuperscript{2}, there exist many differences between labellings and preference relations (for which Arrow’s result apply), stemming from their corresponding order-theoretic characterisations. In other words, aggregating preferences assumes that agents submit a full order of preferences over candidates, while in labelling aggregation, agents submit their top labeling for a set of logically connected arguments.

The problem of labelling aggregation is more comparable to the judgement aggregation problem \textsuperscript{23,20,22,19}, by considering arguments as propositions which are logically connected by the conditions of legal labelling. However, one important difference is that in JA, each proposition can have two values: True or False. In labelling aggregation, on the other hand, each argument can have three values: \textit{in}, \textit{out}, or \textit{undec}. This makes labelling aggregation more comparable to 3-value JA \textsuperscript{13,14}.

The 3-value JA problem was proposed by Dokow and Holzman in \textsuperscript{13,14}. However, labelling aggregation is still different from 3-value JA. The main difference is the type of logical connections between arguments/propositions. Argumentation graphs are subject to specific constraints that characterise acceptable labels, and many semantics have been proposed to deal with acceptability of arguments, while 3-value JA only deals with one acceptability semantics, namely propositional satisfiability. Additionally, argumentation essentially abstracts contradictory knowledge bases which is not the case with 3-value JA.

In this paper, we conduct an extensive social-choice-theoretic analysis of argument evaluation semantics by means of labellings. We start by studying the problem of aggregating different individual judgments on how a set of arguments should be evaluated. We introduce the argument-wise plurality rule and study its key properties. We also present key impossibility results on the existence of a good aggregation operators. We then fully characterise the sufficient

\textsuperscript{2} Arrow’s Theorem claims that four quite natural constraints, that capture abstractly the properties of a democratic aggregation process, cannot be simultaneously satisfied.
and necessary conditions under which Collective Rationality is guaranteed for any aggregation operator. We analyse argument-wise plurality rule with respect to this characterisation and we study whether the restriction of the domain to classical semantics would ensure the fulfillment of these conditions. Finally, we provide graph theoretical restrictions under which argument-wise plurality rule produces collectively rational outcomes.

The paper makes three distinct contributions to the state-of-the-art in the computational modelling of argumentation. Firstly, the paper introduces the study of aggregating different individual judgments on how a given set of arguments is to be evaluated. This requires adapting classical social-choice properties to the argumentation domain, and sometimes demands special treatment (e.g. different versions of some properties). The paper highlights that aggregating different argument evaluations can lead to similar paradoxes to those studied in other areas of social choice theory (e.g. the well-known Condorcet Paradox in preference aggregation, and the Discursive Dilemma in the judgment aggregation literature).

The second contribution of this paper is proving the impossibility of the existence of any aggregation operator that satisfies some minimal properties. In doing so, we show impossibility results that concern dealing with ties and producing a collectively rational evaluation of arguments. Moreover, we provide a full characterisation of the necessary and sufficient conditions under which an aggregation operator is guaranteed to produce a rational evaluation of arguments. The impossibility results that we present build on the solid foundation of social choice and judgement aggregation for impossibility results such as Arrow’s celebrated impossibility theorem [1], Sen’s impossibility theorem [30], the Muller–Satterthwaite theorem [24], the Gibbard-Satterthwaite theorem [17,29], and List and Pettit’s impossibility result on judgment aggregation in propositional inference [21]. Hence, as is the case with other aggregation domains, the aggregation paradox in argument evaluation is an example of a more fundamental barrier. These results are important because they give conclusive answers and focus research in more constructive directions (e.g. weakening the desired properties in order to avoid the paradox).

The third contribution of this paper is an extensive analysis of an aggregation rule, namely argument-wise plurality rule. We analyse the properties of argument-wise plurality rule in general, and investigate whether the restriction of the domain of votes to a particular classical semantics would ensure the fulfillment of these conditions. This highlights a novel use of classical semantics, which are originally used to resolve issues in single-agent nonmonotonic reasoning. Finally, we provide graph theoretical restrictions on argumentation frameworks under which argument-wise plurality rule would be guaranteed to produce collectively rational outcomes.

The paper is organised as follows. In section 2, we start by giving a brief background on abstract argumentation systems. Sections 3, 4, and 7 focus on the problem of aggregating sets of judgments over argument evaluation. Sections 5, 8, and 9 focus on introducing and analysing argument-wise plurality rule. We conclude the paper and discuss some related work in Section 10.
2 Background

In this section, we briefly outline key elements of abstract argumentation frameworks. We begin with Dung’s abstract characterisation of an argumentation system [15]. We restrict ourselves to finite sets of arguments.

**Definition 1 (Argumentation framework)** An argumentation framework is a pair $AF = \langle A, \rightarrow \rangle$ where $A$ is a finite set of arguments and $\rightarrow \subseteq A \times A$ is a defeat relation. We say that an argument $a$ defeats an argument $b$ if $(a, b) \in \rightarrow$ (sometimes written $a \rightarrow b$).

![Figure 3](image)

**Fig. 3** A simple argument graph

An argumentation framework can be represented as a directed graph in which vertices are arguments and directed arcs characterise defeat among arguments. An example argument graph is shown in Figure 3. Argument $a_1$ has two defeaters (i.e. counter-arguments) $a_2$ and $a_4$, which are themselves defeated by arguments $a_3$ and $a_5$ respectively.

There are two forms of semantics that define the decision to make about the arguments. One of them is extension-based semantics by Dung [15], which produces a set of arguments that are accepted together. Another equivalent labelling-based semantics is proposed by Caminada [8,9], which gives a labelling for each argument. With argument labellings, we can accept arguments (by labelling them as in), reject arguments (by labelling them as out), and abstain from deciding whether to accept or reject (by labelling them as undec). Caminada [8,9] established a correspondence between properties of labellings and the different extensions. In this paper, we employ the labelling approach.

**Definition 2 (Argument Labelling)** Let $AF = \langle A, \rightarrow \rangle$ be an argumentation framework. An argument labelling is a total function $L : A \rightarrow \{\text{in}, \text{out}, \text{undec}\}$.

We will make use of the following notation.

**Definition 3** Let $AF = \langle A, \rightarrow \rangle$ be an argumentation framework, and $L$ a labelling over $AF$. We define:

- $\text{in}(L) = \{a \in A \mid L(a) = \text{in}\}$
- $\text{out}(L) = \{a \in A \mid L(a) = \text{out}\}$
- $\text{undec}(L) = \{a \in A \mid L(a) = \text{undec}\}$

A labelling $L$ can be represented as $L = (\text{in}(L), \text{out}(L), \text{undec}(L))$. 


For example, in Figure 4, we have $L_G = (\{a_3\}, \emptyset, \{a_1, a_2\})$, $L_1 = (\{a_1, a_3\}, \{a_2\}, \emptyset)$, and $L_2 = (\{a_2, a_3\}, \{a_1\}, \emptyset)$.

In the rest of the paper, by slight abuse of notation, when we refer to a labelling $L$ as an extension, we will be referring to the set of accepted arguments $\text{in}(L)$.

Labellings must satisfy the condition that an argument is $\text{in}$ if and only if all of its defeaters are $\text{out}$. An argument is $\text{out}$ if and only if at least one of its defeaters is $\text{in}$. This requirement is captured in the definition of complete labelling.

**Definition 4 (Complete Labelling)** Let $AF = \langle A, \rightarrow \rangle$ be an argumentation framework. A complete labelling is a total function $L : A \rightarrow \{\text{in, out, undec}\}$ such that:

- $\forall a \in A : (L(a) = \text{out} \equiv \exists b \in A$ such that $(b \rightarrow a$ and $L(b) = \text{in}))$; and
- $\forall a \in A : (L(a) = \text{in} \equiv \forall b \in A : (\text{if } b \rightarrow a \text{ then } L(b) = \text{out}))$

Otherwise, $L(a) = \text{undec}$ (since $L$ is a total function). We will use $\text{Comp}(AF)$ to denote the set of all complete labellings for $AF$.

As an example, consider the following.

**Example 3** Consider the graph in Figure 4. Here, we have three complete labellings: $L_G = (\{a_3\}, \emptyset, \{a_1, a_2\})$, $L_1 = (\{a_1, a_3\}, \{a_2\}, \emptyset)$, and $L_2 = (\{a_2, a_3\}, \{a_1\}, \emptyset)$.

![Graph with three complete labellings.](image)

In addition to the complete labelling, there are other semantics which assume further conditions.

**Definition 5 (Other Labellings)** Let $AF = \langle A, \rightarrow \rangle$ be an argumentation framework. Let $L : A \rightarrow \{\text{in, out, undec}\}$ be a complete labelling.

- $L$ is a grounded labelling if and only if $\text{in}(L)$ is minimal, or equivalently $\text{out}(L)$ is minimal, or equivalently $\text{undec}(L)$ is maximal (w.r.t set inclusion).
- $L$ is a preferred labelling if and only if $\text{in}(L)$ is maximal, or equivalently $\text{out}(L)$ is maximal (w.r.t set inclusion).
- $L$ is a semi-stable labelling if and only if $\text{undec}(L)$ is minimal (w.r.t set inclusion).
– \( L \) is a stable labelling if and only if \( \text{undec}(L) = \phi \).

Consider the following example.

**Example 4** Consider the graph in Figure 4. Here, we have the grounded labelling is \( L_G = (\{ a_3 \}, \{ \}, \{ a_1, a_2 \}) \). We have only two preferred labellings: \( L_1 = (\{ a_1, a_3 \}, \{ a_2 \}, \{ \}) \) and \( L_2 = (\{ a_2, a_3 \}, \{ a_1 \}, \{ \}) \). These are also the only stable and semi-stable labellings for this framework.

We refer to the previous semantics as classical semantics. There exist other semantics which we do not consider in this work.

### 3 Aggregation of Argument Labellings

To date, most analyses inspired by Dung’s framework have focused on analysing and comparing the properties of various types of extensions/labellings (i.e. semantics) \([3]\). The question is, therefore, whether a particular type of labelling is appropriate for a particular type of reasoning task in the presence of conflicting arguments.

In contrast with most existing work on Dung frameworks, our concern here is with multi-agent systems. Since each labelling captures a particular rational point of view, we ask the following question: *Given an argumentation framework and a set of agents, each with a legitimate subjective evaluation of the given arguments, how can the agents reach a collective compromise on how to evaluate those arguments?*

Thus, the problem we face is that of judgment aggregation \([21]\) in the context of argumentation frameworks. In particular, taking as an input a set of individual judgments as to how each argument in some argumentation framework \( AF = (A, \rightarrow) \) must be labelled, we need to come up with a collective judgment. Given a set of agents \( Ag = \{1, \ldots, n\} \), if each agent \( i \in Ag \) has a labelling \( L_i \), we need to find an aggregation operator for \( AF \), which we define as a partial function \( F : L(AF)^n \rightarrow L(AF) \), where \( L(AF) \) is the class of labellings of \( AF \). This means that for each \( a \in A \), \( [F(L)](a) \) is the collective label assigned to \( a \), if \( F \) is defined for \( L = (L_1, \ldots, L_n) \).

### 4 Desirable Properties of Aggregation Operators

Aggregation involves comparing and assessing different points of view. There are, of course, many ways of doing this, as extensively discussed in the literature of Social Choice Theory \([10]\). In this literature, a consensus on some normative ideals has been reached, identifying what a ‘fair’ way of adding up

\( ^3 \) We state that the function is partial to allow for cases in which collective judgment may be undefined (e.g. when there is a tie in voting).

\( ^4 \) Throughout the document, if there is no confusion, we will use \( L \) to denote the profile \( (L_1, \ldots, L_n) \). We also use \( L(a) \) to refer to \( (L_1(a), \ldots, L_n(a)) \), where \( a \in A \).
votes should be. So for instance, if everybody agrees, the outcome must reflect that agreement; no single agent can impose her view on the aggregate; the aggregation should be performed in the same way in each possible case, etc. These informal requirements can be formally stated as properties that \( F \) should satisfy \([21][12]\). In all of the following postulates, it is assumed that a fixed argumentation framework \( AF = (A, \rightarrow) \) and a set of agents \( Ag = \{1, \ldots, n\} \) are given. The postulates can be grouped as follows:

\[ \text{Group 1: Domain and co-domain postulates} \]

In judgment aggregation, two postulates that are commonly assumed are those of Universal Domain and Collective Rationality. The former requires that any profile of labellings chosen from a pre-specified set of feasible labellings can be used as input to \( F \) and \( F \) will return an answer. The question is: what do we take to be the set of feasible labellings in our setting? This depends on which semantics we assume is being used. Theoretically we can have a different version of Universal Domain for each semantics. However since complete semantics represent consistent and self-defending points of views, it represents the best counterpart for the logical consistency in Judgment Aggregation:

**Universal Domain** \( F \) can take as input all profiles \( \mathcal{L} = (L_1, \ldots, L_n) \) such that \( \mathcal{L} \in \text{Comp}(AF)^n \)

Similarly we could have a different version of Collective Rationality - one for each semantics - stating that the output of the aggregation should also be feasible. Again, since we focus on complete semantics, we focus on the following version:

**Collective Rationality** For all profiles \( \mathcal{L}, F(\mathcal{L}) \in \text{Comp}(AF) \).

Later, in Section 7, we will break this postulate down into further constituents. Note that although the domain for \( \mathcal{L} \) is not restricted here, we will mostly focus on the case where profiles \( \mathcal{L} \in \text{Comp}(AF)^n \). However, in Subsection 8.2 we will use other semantics as a domain for \( \mathcal{L} \).

**Group 2: Fundamental postulates**

Next we come to the standard property that forms the cornerstone of the usual impossibility results in judgment aggregation. It says the collective label of an argument depends only on the votes on that argument, independent of the other arguments.

**Independence** For any two profiles \( \mathcal{L} = (L_1, \ldots, L_n), \mathcal{L}' = (L_1', \ldots, L_n') \) and argument \( a \in A \), if \( L_i(a) = L_i'(a) \) for all \( i \in Ag \) then \( [F(\mathcal{L})](a) = [F(\mathcal{L}')])(a) \).

\[ ^5 \text{ This style of presentation of postulates was inspired by } [18] \text{ which is on binary aggregation.} \]
The effect of Independence is that aggregation is done “argument-by-argument”. To be slightly more precise, each argument \( a \in A \) essentially has its own aggregation operator \( I_a \) associated to it, that takes an \( n \)-tuple of labels \( x = (l_1, \ldots, l_n) \) as input (representing the “vote” of each agent on the label of \( a \)) and returns another label \( I_a(x) \) as output (the “collective label”) of \( a \). Then \( [F(L)](a) = I_a((L_1(a), \ldots, L_n(a))) \).

Next, we have Anonymity, which says the identity of which agent submits which labelling is irrelevant.

**Anonymity** For any profile \( L = (L_1, \ldots, L_n) \), if \( L' = (L_{\sigma(1)}, \ldots, L_{\sigma(n)}) \) for some permutation \( \sigma \) on \( Ag \), then \( F(L) = F(L') \).

Proposition 1 Let \( F \) be an aggregation operator. Then \( F \) satisfies both Independence and Anonymity iff for each \( a \in A \) there exists a function \( I_a : \mathbb{N}^3 \to \{\text{in}, \text{out}, \text{undec}\} \) such that, for all \( L \) we have \( [F(L)](a) = I_a(#\text{in}, #\text{out}, #\text{undec}) \).

Proof (Outline). The “if” case is straightforward, since permuting the rows does not change the vote distribution and so Anonymity will hold. Independence is also clear.

For the “only if” case, Independence gives us the existence of the function \( I_a \) such that \( [F(L)](a) = I_a(L_1(a), \ldots, L_n(a)) \) and then Anonymity implies that two vectors that have the same vote distribution will give the same results, so we can set \( I_a(#\text{in}, #\text{out}, #\text{undec}) = I_a(L_1(a), \ldots, L_n(a)) \) where \((L_1(a), \ldots, L_n(a))\) is any vote which has \((#\text{in}, #\text{out}, #\text{undec})\) as its distribution.

A weakening of Anonymity is Non-Dictatorship:

**Non-Dictatorship** There is no \( i \in Ag \) such that, for every profile \( L = (L_1, \ldots, L_n) \) we have \( F(L) = L_i \).

Group 3: Unanimity postulates

Next we move to Unanimity, and some other postulates related to it.

**Unanimity** If \( L \) is such that there exist some \( L \) s.t. \( L_i = L \) for all \( i \in Ag \) then \( F(L) = L \).

This postulate is also familiar from judgment aggregation, but the move to 3-valued labellings rather than the 2 usually seen in judgment aggregation opens up the possibility to define other variants of Unanimity, one of which is used by Dokow and Holzman [14], called Supportiveness:
**Supportiveness** For any profile $L$ and all $a \in A$, there exists $i \in Ag$ such that $[F(L)](a) = L_i(a)$.

Supportiveness says that, for each argument $a$ and label $l$, the collective judgment cannot be set to $l$ without at least one agent voting for that $l$. Clearly Supportiveness implies Unanimity.

**Group 4: Systematicity postulates**

Now we come to the Systematicity postulates which deal with neutrality issues across arguments and labels. We can list two variants, both of which imply Independence. We start with the stronger version:

**Strong Systematicity** For any two profiles $L = (L_1, \ldots, L_n)$ and $L' = (L'_1, \ldots, L'_n)$ and arguments $a, b \in A$, and for every permutation $\rho$ on the set of labels $\{\text{in}, \text{out}, \text{undec}\}$, if $\forall i \in Ag : L_i(a) = \rho(L'_i(b))$ then $[F(L)](a) = \rho([F(L')](b))$.

To illustrate Strong Systematicity, consider the example in Figure 5. We have the following three labellings: $L_1 = (\{a\}, \{b\}, \{\}\}$, $L_2 = (\{b\}, \{a\}, \{\}\}$, $L_3 = (\{\}, \{\}, \{a,b\})$.

![Fig. 5 An example illustrating Strong Systematicity.](image)

Consider the profiles $L = (L_1, L_1, L_2, L_3)$ and $L' = (L_3, L_3, L_1, L_2)$. Then, $L(a) = (\text{in}, \text{in}, \text{out}, \text{undec})$ and $L'(a) = (\text{undec}, \text{undec}, \text{in}, \text{out})$. Let $\rho$ be the permutations on labels such that $\rho(\text{in}) = \text{undec}$, $\rho(\text{out}) = \text{in}$, and $\rho(\text{undec}) = \text{out}$. Then, we can see that in this example $\forall i \in Ag : L_i(a) = \rho(L_i(a))$. Strong Systematicity requires that $[F(L')](a) = \rho([F(L)](a))$.

The postulate forces us to give an even-handed treatment to the labels in, out and undec. This makes sense if we consider in, out and undec as three independent labels. However, one might be tempted to consider undec as a middle label between in and out. Hence, the equal treatment might not be desirable in this case. One might suggest a version of Systematicity that treats in and out equally. We define this version (which we call in/out-Systematicity) in a later section. In/out-Systematicity lies in the middle between Strong Systematicity and the following version of Systematicity which can be obtained by restricting the class of permutations, until we only consider the identity.
**Weak Systematicity** For any two profiles \( L = (L_1, \ldots, L_n) \) and \( L' = (L'_1, \ldots, L'_n) \) and arguments \( a, b \in A \), if \( \forall i \in Ag : L_i(a) = L'_i(b) \) then \([F(L)](a) = [F(L')](b)\).

Clearly Independence follows from Weak Systematicity by just setting \( a = b \). If we strengthen Independence to Weak Systematicity then the functions \( I_a \), mentioned earlier, are identical for all arguments.

**Group 5: Monotonicity** postulates

Our final group relates to Monotonicity.

**Monotonicity** Let \( l_a \in \{\text{in}, \text{out}, \text{undec}\} \) be such that given two profiles \( L = (L_1, \ldots, L_i, \ldots, L_{i+k}, \ldots, L_n) \) and \( L' = (L_1, \ldots, L'_i, \ldots, L'_{i+k}, \ldots, L_n) \) (differing only in the labellings of agents \( i, i+1, \ldots, i+k \)) where \( i \in \{1, \ldots, n\} \) and \( k \in \{0, \ldots, n-i\} \), if \( L_i(a) \neq l_a \) while \( L'_i(a) = l_a \) for all \( j \in \{i, \ldots, i+k\} \), then \([F(L)](a) = l_a \) implies that \([F(L')](a) = l_a \).

Monotonicity states that if a set of agents switch their label of argument \( a \) to the collective label of \( a \) then the collective label of \( a \) remains the same.

5 The Argument-Wise Plurality Rule

An obvious candidate aggregation operator to check out is the plurality voting operator \( M \). In this section, we analyse a number of key properties of this operator. Intuitively, for each argument, it selects the label that appears most frequently in the individual labellings.

**Definition 6 (Argument-Wise Plurality Rule (AWPR))** Let \( AF = (A, \rightarrow) \) be an argumentation framework. Given any argument \( a \in A \) and any profile \( L = (L_1, \ldots, L_n) \), then \([M(L)](a) = l_a \in \{\text{in}, \text{out}, \text{undec}\} \) iff

\[
\{|i : L_i(a) = l_a| > \max_{l'_a \neq l_a} \{|i : L_i(a) = l'_a|\}\}
\]

**Example 5 (Three Detectives (cont.))** Continuing on Example 2 applying the argument-wise plurality rule, we have:

\[- \quad [M((L_1, L_2, L_3))](a_1) = \text{out} \]
\[- \quad [M((L_1, L_2, L_3))](a_2) = \text{in} \]
\[- \quad [M((L_1, L_2, L_3))](a_3) = \text{out} \]

Note that \( M \) is defined for all profiles that cause no ties.

5.1 Properties of Argument-Wise Plurality Rule

We now analyse whether AWPR satisfies the properties listed above.
The argument-wise plurality rule operator $M$ satisfies Supportiveness, Anonymity, Strong Systematicity, and Monotonicity.

Proof – Supportiveness: consider any profile $L = (L_1, \ldots, L_n)$. Suppose, towards a contradiction, that for some argument $a$, there exists no agent $i$ such that $L_i(a) = l_a$ where $l_a = [M(L)](a)$. Then $|\{i : L_i(a) = l_a\}| = 0$. But, $|\{i : L_i(a) = l_a\}| > \max_{l_i \neq l_a} |\{i : L_i(a) = l_i\}| > 0$. Contradiction.

Anonymity: consider any profile $L = (L_1, \ldots, L_n)$. $[M(L)](a) = l_a$ if and only if $|\{i : L_i(a) = l_a\}| > \max_{l_i \neq l_a} |\{i : L_i(a) = l_i\}|$ if and only if $|\{i : L_{p(i)}(a) = l_a\}| > \max_{l_i \neq l_a} |\{i : L_{p(i)}(a) = l_i\}|$, which is equivalent to $[M((L_{p(1)}, \ldots, L_{p(i)}, \ldots, L_{p(n)}))](a) = l_a$.

Strong Systematicity: consider, for any two profiles $L = (L_1, \ldots, L_n)$ and $L' = (L'_1, \ldots, L'_n)$, and for any $a, b \in A$, the permutation $p : \{\text{in, out, undec}\} \rightarrow \{\text{in, out, undec}\}$. Suppose, towards a contradiction, that for any $i, L_i(a) = \rho(L_i(b))$, and $[M(L)](a) = l_a$ but $\rho(M(L')|b) \neq \rho(l_a)$. But then, $|\{i : L_i(a) = l_a\}| = |\{i : L_i(b) = \rho(l_a)\}|$ while for any $l'_a \neq l_a$, $|\{i : L_i(a) = l'_a\}| = |\{i : L_i(b) = \rho(l'_a)\}|$. So, if $|\{i : L_i(a) = l_a\}| > \max_{l_i \neq l_a} |\{i : L_i(a) = l_i\}|$, then we have $|\{i : L_i'(b) = \rho(l_a)\}| > \max_{l_i \neq l_a} |\{i : L_i'(b) = \rho(l'_a)\}|$ as well. Contradiction.

Monotonicity: Consider the following two profiles $L = (L_1, \ldots, L_i, \ldots, L_{i+k}, \ldots, L_n)$ and $L' = (L_1, \ldots, L'_i, \ldots, L'_{i+k}, \ldots, L_n)$ (differing only in the labellings of agents $i, i+1, \ldots, i+k$) where $i \in \{1, \ldots, n\}$ and $k \in \{0, \ldots, n-i\}$. Suppose, towards a contradiction, that for a $\in A$ and a label $l_a$, we have that $L_h(a) \neq l_a$ while $L_{h'}(a) = l_a$ for all $h \in \{i, i+1, \ldots, i+k\}$, and we have that $[M(L)](a) = l_a$ while $[M(L')](a) \neq l_a$. But then, $|\{j : L_j(a) = l_a\}| > \max_{l_i \neq l_a} |\{j : L_j(a) = l_i\}|$ in the profile $L$ while in the profile $(L_1, \ldots, L_i, \ldots, L_{i+k}) = L'$, we have $|\{j : L_j(a) = l_a\}| = |\{j : L_j(a) = l_a\}| \cup \{i, i+1, \ldots, i+k\}$ and $|\{j : L_j(a) = l_a\}| \leq |\{j : L_j(a) = l_a\}|$ for every other labelling $l'_a$. Then $|\{j : L_j(a) = l_a\}| > \max_{l_i \neq l_a} |\{j : L_j(a) = l_i\}|$. Contradiction.

Corollary 1 The argument-wise plurality rule operator $M$ satisfies Unanimity, Weak Systematicity, Independence, and Non-Dictatorship.

Proof – Unanimity. Follows from Supportiveness.

Weak Systematicity. Follows from Strong Systematicity.

Independence. Follows from Strong Systematicity.

Non-Dictatorship. Follows from Anonymity.

Note that our definition for Supportiveness is taken over arguments (similar to its counterpart in Judgment Aggregation literature). One should not confuse this definition with Supportiveness taken over labellings i.e. the collective labelling for an AF is supported by at least one agent’s labelling. While AWPR satisfies Supportiveness over arguments, it violates Supportiveness over labellings. It is possible to construct cases in which the aggregated labelling does not coincide with any one of the individual labellings.

To see how, suppose that there are seven arguments $a, b_1, b_2, b_3, c_1, c_2, c_3$, with the attack relation depicted in Figure 6. Suppose we have three agents whose labellings are $L_1, L_2,$ and $L_3$ (as shown in the figure). Then:
\[ M((L_1, L_2, L_3))(b_1) = \text{in}, \quad M((L_1, L_2, L_3))(c_1) = \text{out}, \]
\[ M((L_1, L_2, L_3))(b_2) = \text{in}, \quad M((L_1, L_2, L_3))(c_2) = \text{out}, \]
\[ M((L_1, L_2, L_3))(b_3) = \text{in}, \quad M((L_1, L_2, L_3))(c_3) = \text{out}, \]
and
\[ M((L_1, L_2, L_3))(a) = \text{out}. \]

That is, \( M((L_1, L_2, L_3)) \) is different from \( L_1, L_2 \) and \( L_3 \).

Fig. 6 An example which shows that an aggregated labelling may not coincide with any of the individual labellings.

Despite all these promising results, it turns out that plurality operator violates Universal Domain and Collective Rationality postulates. Since AWPR is not defined for profiles that cause ties, then \( M \) cannot take as input any profile \( \mathcal{L} \in \text{Comp}(AF)^n \). Then, it violates Universal Domain. A weaker version of Universal Domain can be defined.

**No-Tie Universal Domain** An aggregation operator \( F \) can take as input all profiles \( \mathcal{L} = (L_1, \ldots, L_n) \) such that \( \mathcal{L} \) does not cause a tie and \( \mathcal{L} \in \text{Comp}(AF)^n \).

Since there are no restrictions (other than having no ties) on how labellings are defined, AWPR satisfies No-Tie Universal Domain.

As for Collective Rationality, consider the following example.

**Example 6** Suppose argument \( c \) has two defeaters, \( a \) and \( b \), and argument \( a \) (resp. \( b \)) defeats and is defeated by argument \( a' \) (resp. \( b' \)). Suppose we have 3 agents, with votes as shown in Figure 7. We have \( M(\mathcal{L}))(c) = \text{out} \), but it is not the case that \( M(\mathcal{L}))(a) = \text{in} \) or \( M(\mathcal{L}))(b) = \text{in} \).
Interestingly, the above counterexample demonstrates a variant of the discursive dilemma \cite{21} in the context of argument evaluation, which itself is a variant of the well-known Condorcet paradox.

5.2 Introducing Tie-Breaking Rules to Satisfy Universal Domain

As shown earlier, among the ten defined postulates, the argument-wise plurality rule only violates Universal Domain and Collective Rationality which are very fundamental postulates. One might be tempted to try to make AWPR satisfies Universal Domain by adding a deterministic tie-breaking rule to deal with ties. We define a new version of AWPR (a version that deals with ties):

**Definition 7 (AWPR with a Tie-Breaking Rule (AWPR w/TBR ))**

Let $AF = (A, \rightarrow)$ be an argumentation framework. Let $T : \{\text{in}, \text{out}, \text{undec}\}^n \rightarrow \{\text{in}, \text{out}, \text{undec}\}$ be a deterministic tie-breaking rule. Then aggregation operator $M_T$ is defined by setting, for each profile $L$ and argument $a \in A$,

$$[M_T(L)](a) = \begin{cases} [M(L)](a) & \text{if this exists} \\ T((L_1(a), \ldots, L_n(a))) & \text{otherwise} \end{cases}$$

One can show that AWPR w/TBR satisfies Universal Domain since there are no restrictions on how labellings are defined.

Which tie-breaking rule could we use? There are several possibilities. The first one would be to “default” to some fixed label, with an obvious choice being undec. Intuitively, when agents’ votes have a tie, everyone would be happy with undec. However, the use of this rule would result in violating Supportiveness, Strong Systematicity, and Monotonicity as we confirm in the following example. Let $M_u$ be the AWPR (i.e. $M$) but with the use of “choose undec with ties” rule. Consider the example in Figure\ref{counterexample}. We have the following three labellings: $L_1 = (\{a\}, \{b\}, \{\}\}$, $L_2 = (\{b\}, \{a\}, \{\}\}$, $L_3 = (\{\}, \{\}, \{a, b\})$.

- **Supportiveness.** $[M_u((L_1, L_2))]((a) = \text{undec}$ while $L_1(a) \neq \text{undec}$ and $L_2(a) \neq \text{undec}$. 

![Counterexample to Collective Rationality](image)
Fig. 8 An example shows how using AWPR with “choose undec with ties” rule violates Supportiveness, Strong Systematicity, and Monotonicity.

- **Strong Systematicity.** Let ρ be the permutations on labels such that ρ(in) = in, ρ(out) = undec, and ρ(undec) = out (i.e. just swap out and undec), and consider $[M_u((L_1, L_3))](a)$. Note that we have $L_1(a) = \rho(L_1(a))$ and $L_3(a) = \rho(L_3(a))$. Strong Systematicity requires $[M_u((L_1, L_3))](a) = \rho([M_u((L_1, L_2))](a)) = \rho(undec) = out$. However, $[M_u((L_1, L_3))](a) = undec$.

- **Monotonicity.** We have $[M_u((L_1, L_1, L_2, L_2))](a) = undec$. If first agent switches from $L_1$ to $L_3$ (where $L_3(a) = undec$), then Monotonicity requires that $[M_u((L_3, L_1, L_2, L_2))](a) = undec$. However, $[M_u((L_3, L_1, L_2, L_2))](a) = out$.

The violation of Supportiveness, Strong Systematicity, and Monotonicity is mainly caused by the special treatment of undec. However, treating undec in a special way might not be a problem in some cases. In these cases, one should at least maintain the neutrality between in and out. Hence, one can redefine the previous three properties for the convenience of $M_u$. Following, we define in/out-Supportiveness, in/out-Systematicity, and in/out-Monotonicity.

**in/out-Supportiveness** For any profile $L = (L_1, \ldots, L_n)$ and $a \in A$, if $[F(L)](a) \neq undec$ then there exists some agent $i$ such that $[F(L)](a) = L_i(a)$.

**in/out-Systematicity** For any 2 profiles $L = (L_1, \ldots, L_n)$ and $L' = (L'_1, \ldots, L'_n)$ and arguments $a, b \in A$, and for every undec-preserving permutation ρ on the set of labels {in, out, undec} (i.e. ρ(undec) = undec), if $\forall i \in Ag : L_i(a) = \rho(L'_i(b))$ then $[F(L)](a) = \rho([F(L')](b))$.

**in/out-Monotonicity** Let $l_a \in \{in, out\}$ be such that given two profiles $L = (L_1, \ldots, L_i, \ldots, L_{i+k}, \ldots, L_n)$ and $L' = (L_1, \ldots, L'_i, \ldots, L'_{i+k}, \ldots, L_n)$ (differing only in the labellings of agents $i, i+1, \ldots, i+k$) where $i \in \{1, \ldots, n\}$ and $k \in \{0, \ldots, n-i\}$, if $L_j(a) \neq l_a$ while $L'_j(a) = l_a$ for all $j \in \{i, \ldots, i+k\}$, then $[F(L)](a) = l_a$ implies that $[F(L')](a) = l_a$. 
The following result confirms that \( M_u \) does indeed satisfy these modified postulates.

**Proposition 2** \( M_u \) satisfies \textit{in/out-Supportiveness}, \textit{in/out-Systematicity} and \textit{in/out-Monotonicity}.

**Proof (Outline)**

- **\textit{in/out-Supportiveness}:** If \( [M_u(L)](α) ≠ \text{undec} \) then it must be the case that \( [M_u(L)](α) = [M(L)](α) \) and so there must be at least one \( i \) such that \( [M_u(L)](α) = L_i(α) \).

- **\textit{in/out-Systematicity}:** Let \( ρ \) be an \textit{undec}-preserving permutation on \{\textit{in}, \textit{out}, \textit{undec}\} such that \( L_i(α) = ρ(L_i(b)) \) for all \( i \in Ag \). There are two possibilities for \( ρ \). Either (i) it equals the identity, or (ii) it just swaps \textit{in} and \textit{out}.

  - In case (i) we know the set - and hence the number - of agents who vote \textit{in} (respectively \textit{out}, \textit{undec}) for \( α \) in \( L \) is the same as the set - and hence the number - of agents who vote \textit{in} (respectively \textit{out}, \textit{undec}) for \( b \) in \( L' \). Given this it is clear that \( [M_u(L)](α) = [M_u(L')](b) \), i.e., (since \( ρ \) is the identity) \( [M_u(L)](α) = ρ([M_u(L')](b)) \).

  - In case (ii) we know the set - and hence the number - of agents who vote \textit{in} (respectively \textit{out}, \textit{undec}) for \( α \) in \( L \) is the same as the set - and hence the number - of agents who vote \textit{out} (respectively \textit{in}, \textit{undec}) for \( b \) in \( L' \). This means that \textit{in}, respectively \textit{out}, \textit{undec}, is a plurality winner for \( α \) in \( L \) iff \textit{out}, respectively \textit{in}, \textit{undec}, is a plurality winner for \( b \) in \( L' \). So either there is a plurality winner for both \( α \) in \( L \) and for \( b \) in \( L' \), in which case we obtain \( [M_u(L)](α) = ρ([M_u(L')](b)) \), or there is neither a plurality winner for \( α \) in \( L \), nor for \( b \) in \( L' \), in which case \( [M_u(L)](α) = \text{undec} = [M_u(L')](b) = ρ([M_u(L')](b)) \) (this last step using the fact that \( ρ \) is \textit{undec}-preserving).

- **\textit{in/out-Monotonicity}:** Let \( L, L' \) be as in the statement of the postulate and suppose \( [M_u(L)](α) ∈ \{\textit{in}, \textit{out}\} \). Then \( [M_u(L)](α) = [M(L)](α) \) by definition of \( M_u \). Using the fact that \( M \) satisfies Monotonicity we know \( [M(L')](α) = [M(L)](α) \). Since a plurality winner for \( α \) exists in \( L' \), we must have \( [M_u(L')](α) = [M(L')](α) \) and so \( [M_u(L)](α) = [M_u(L)](α) \) as required.

Two more possible tie-breaking rules would be to let one agent decide in case of a tie, or to introduce a special symbol to represent a tie label. An example of the first case might exist in some committees where the chairperson resolves ties. Unfortunately, the use of this rule obviously violates \textit{Anonymity}.

Let \( M_d \) be the AWPR (i.e. \( M \)) but with the use of “let the same agent decides” rule.

- **Anonymity.** Consider the example in Figure 8. Let the first agent always decides in case of ties. Then, we have \( [M_d(L_1, L_2)](α) = \text{in} \). Now \textit{Anonymity} requires that \( [M_d(L_2, L_1)](α) = \text{in} \). However, \( [M_d(L_2, L_1)](α) = \text{out} \).

The second case is to use a special tie-symbol (such as \( * \)) to represent a tie label. This was suggested by Dokow and Holzman [13], but for the case...
of binary labelling, where * resembles abstention. However, one can extend it to the ternary case: Use * as a labelling for an argument whenever a tie happens. Then, check for each argument with *, whether it has only one valid labelling (given its defeaters labellings). Keep repeating the process until every argument with * can have more than one valid labelling. However, this process is not guaranteed to terminate and it obviously violates Independence. Another procedure can be to choose one fixed label to fill every *, but this implies a special treatment for one of the labels in, out, or undec, and hence a violation of Strong Systematicity.

As we see, all the three rules violate either Strong Systematicity or Anonymity. However, the three previous rules are not exclusive, there can be different others. Then, an interesting question is “Does there exist an operator that satisfies Universal Domain, Strong Systematicity, and Anonymity?” In the next section, we show that the answer for this question is negative.

6 The Impossibility of Good Aggregation Operators

In the previous section, we analysed a particular judgment aggregation operator (namely, argument-wise plurality rule). We showed that while it satisfies most key properties, it fails to satisfy Universal Domain and Collective Rationality. Then, we tried to make the AWPR satisfy Universal Domain by adding a deterministic tie-breaking rule. This produced a new operator, which we named AWPR w/TBR. However, we failed to suggest a good tie-breaking rule that satisfies both Strong Systematicity and Anonymity. In this section we show that, depending on the argumentation framework and the number of agents, it is impossible to have such a rule.

**Theorem 2** There exists an argumentation framework AF such that, for any set of agents that is divisible by three, there exists no labelling aggregation operator satisfying Universal Domain, Anonymity and Strong Systematicity.

*Proof* It is enough to assume an AF that contains at least one argument a which can feasibly take on any label, i.e. there exist complete labellings \( L_{\text{in}} \), \( L_{\text{out}} \) and \( L_{\text{undec}} \) over AF such that \( L_{\text{in}}(a) = \text{in} \), \( L_{\text{out}}(a) = \text{out} \) and \( L_{\text{undec}}(a) = \text{undec} \). Divide \( n \) agents into 3 groups \( G_1, G_2, G_3 \) of equal size. By Universal Domain, all profiles consisting of legal labellings are valid input. Assume a profile in which everyone in \( G_1 \) provides labelling \( L_{\text{in}} \), everyone in \( G_2 \) provides \( L_{\text{out}} \) and everyone in \( G_3 \) provides \( L_{\text{undec}} \). For now let’s denote this profile by \( L = (G_1 : L_{\text{in}}), (G_2 : L_{\text{out}}), (G_3 : L_{\text{undec}}) \). Now, assume for contradiction that \( F \) is an aggregation operator for AF satisfying Universal Domain, Anonymity and Strong Systematicity. Let \( \rho : \{\text{in, out, undec}\} \to \{\text{in, out, undec}\} \) be any permutation on the set of labels such that \( \rho(l) \neq l \) for all labels \( l \) (for instance, \( \rho(\text{in}) = \text{out}, \rho(\text{out}) = \text{undec}, \rho(\text{undec}) = \text{in} \)), and let \( L' \) denote the profile

\(^6\) Note that we do not consider the use of a non-deterministic tie-breaking rule since it has its own issues too, such as producing different outcomes given the same profile.
\([\{G_1 : L_{\rho(\text{in})}, G_2 : L_{\rho(\text{out})}, G_3 : L_{\rho(\text{undec})}\}]\). Since \(L_i'(a) = \rho(L_i(a))\) for all \(i \in A\), Strong Systematicity implies \([F(L')](a) = \rho([F(L)])(a)\). However, we chose \(\rho\) s.t. \(\rho(l) \neq l\). Hence, \([F(L')](a) \neq [F(L)](a)\). But Anonymity implies \([F(L)](a) = [F(L')](a)\). Contradiction. Hence no such \(F\) can exist.

The previous result can be read in two ways: First, the AWPR cannot be made to satisfy Universal Domain without violating Strong Systematicity or Anonymity. Second, there exists no aggregation operator that satisfies Universal Domain, Strong Systematicity and Anonymity.

Note that the previous theorem was stated for a set of agents divisible by three. Hence, one might wonder whether we could rule out the possibility of 3-way ties, by assuming \(n\) cannot be a multiple of three. This is possible, but then with even number of agents, we can show that there is still a large class of AFs which do not have an operator satisfying those three postulates without violating Collective Rationality.

**Theorem 3** There exists an argumentation framework \(AF\) such that, for any set of agents of even cardinality, there exists no labelling aggregation operator satisfying Universal Domain, Anonymity, Strong Systematicity and Collective Rationality.

Proof It is enough to assume an AF that contains at least one argument \(a\) that can feasibly take on just two out of the three possible labels. For concreteness suppose \(a\) can only take on labels out and undec. (An example of such a framework and an argument can be seen in the proof of Theorem 3 below, in which \(c\) can only be either out or undec). Let \(L_{\text{undec}}\) and \(L_{\text{out}}\) be two complete labellings such that \(L_{\text{undec}}(a) = \text{undec}\) and \(L_{\text{out}}(a) = \text{out}\). Divide the agents into two groups \(G_1, G_2\) of equal size. By Universal Domain, all profiles consisting of legal labellings are valid input, so assume a profile in which everyone in \(G_1\) provides labelling \(L_{\text{undec}}\) and everyone in \(G_2\) provides \(L_{\text{out}}\).

Denote the resulting profile by \(\mathcal{L} = ([G_1 : L_{\text{undec}}], [G_2 : L_{\text{out}}])\) and assume for contradiction that \(F\) is an aggregation operator for this AF that satisfies Universal Domain, Anonymity, Strong Systematicity and Collective Rationality. Let \(\rho\) be the permutation that swaps undec and out, i.e., \(\rho(\text{undec}) = \text{out}\) and \(\rho(\text{out}) = \text{undec}\), and let \(\mathcal{L}' = ([G_1 : L_{\text{out}}], [G_2 : L_{\text{undec}}])\). By Anonymity we know \([F(\mathcal{L})](a) = [F(\mathcal{L}')](a)\). Then it cannot be that \([F(L)](a) = \text{undec}\), for if so then Strong Systematicity would imply \([F(L')](a) = \rho(\text{undec}) = \text{out} \neq [F(L)](a)\), and similarly it cannot be that \([F(L)](a) = \text{out}\). Thus we must have \([F(L')](a) = \text{in}\). But by Collective Rationality \([F(L')](a) \in \{\text{undec, out}\}\). Contradiction.

The careful reader can realize that Collective Rationality can be substituted with Supportiveness in the previous theorem. As for the proof, the last sentence becomes: “Thus we must have \([F(\mathcal{L})](a) = \text{in}\). But by Supportiveness \([F(\mathcal{L})](a) \in \{\text{undec, out}\}\). Contradiction”.

However, one might argue that Strong Systematicity is quite a strong condition. Treating in, out, and undec differently can be tolerated. Then, it is
interesting to ask: “Does there exist an operator that satisfies Universal Do-
main, Weak Systematicity, and Anonymity?” We certainly have no answer for
this question in the meantime. However, we can show that such an operator,
if it exists, is not a perfect one. Following, we show that an operator that
satisfies Universal Domain, Weak Systematicity, and Anonymity, if it exists,
would violate Collective Rationality and/or Unanimity.

**Theorem 4** There exists an argumentation framework $AF$ such that, for any
set of agents of even cardinality, there exists no labelling aggregation opera-
tor satisfying Universal Domain, Weak Systematicity, Anonymity, Collective
Rationality, and Unanimity.

**Proof** Consider the following argumentation framework. An argument $c$ is de-
feated by two arguments $a$ and $b$ which defeat each others.

Consider the two labellings $L = (\{a\}, \{b, c\}, \{\})$ and $L' = (\{b\}, \{a, c\}, \{\})$.
Assume, towards a contradiction, there exists an aggregation operator $F$ that
satisfies Universal Domain, Collective Rationality, Weak Systematicity, Anonymity
and Unanimity.

By Universal Domain, we may consider any profile consisting of legal la-
bellings. Consider the two profiles $L = (L, \ldots, L, L', \ldots, L')$ and $L' = (L', \ldots, L', L, \ldots, L)$.
That is, in $L$ half the agents give $L$ and the other half give $L'$, and then in $L'$
the agents switch from $L$ to $L'$ and vice versa.

By Unanimity we know

$$[F(L)](c) = \text{out}. \quad (2a)$$

By Weak Systematicity we also know $[F(L)](a) = [F(L')] (b)$. But since $L$ and
$L'$ are permutations of each other we know $F(L) = F(L')$ by Anonymity and
so we obtain

$$[F(L)](a) = [F(L')](b). \quad (2b)$$

But there is no complete labelling simultaneously satisfying (2a) and (2b).
Contradiction. Hence no $F$ can exist.

One might note that all of the above theorems exploit the use of profiles
that include ties. What if we relax Universal Domain to No-Tie Universal Do-
main. Following, we show that an aggregation operator which satisfies No-Tie
Universal Domain (but not necessarily Universal Domain) cannot also satisfy Weak Systematicity, Anonymity, Collective Rationality, and Supportiveness.

**Theorem 5** There exists an argumentation framework AF such that, for any set of agents that is divisible by three, there exists no labelling aggregation operator satisfying No-Tie Universal Domain, Weak Systematicity, Anonymity, Collectiv Rationality, and Supportiveness.

**Proof** Consider the following argumentation framework. An argument \(a\) is defeated by two arguments \(b\) and \(c\). Argument \(b\) (resp. \(c\)) defeats and is defeated by argument \(b'\) (resp. \(c'\)).

Consider the three labellings \(L_1 = (\{b, c', \}, \{a, b', c\}, \})\), \(L_2 = (\{b', c\}, \{a, b, c'\}, \})\) and \(L_3 = (\{a, b', c'\}, \{b, c\}, \})\).

Assume, towards a contradiction, there exists an aggregation operator \(F\) that satisfies No-Tie Universal Domain, Collective Rationality, Weak Systematicity, Anonymity and Supportiveness.

By No-Tie Universal Domain, we may consider any profile consisting of legal labellings as long as it does not cause a tie. We consider here three agents, but the same proof can be shown for any set of agents that is divisible by three. Consider the three profiles \(L = (L_1, L_2, L_3)\), \(L' = (L_1', L_2', L_3') = (L_3, L_1, L_2)\) and \(L'' = (L_1'', L_2'', L_3'') = (L_2, L_3, L_1)\).

Since \(\forall i, L_i(a) = L_i'(c)\), then by Weak Systematicity we know:

\[ F(L)(a) = F(L')(c) \]  (3a)

But since \(L\) and \(L'\) are permutations of each other we know \(F(L) = F(L')\) by Anonymity and so we obtain

\[ F(L)(c) = F(L')(c) \]  (3b)

From Eq.3a and Eq.3b

\[ F(L)(a) = F(L)(c) \]  (3c)

Similarly, since \(\forall i, L_i(b) = L_i''(c)\), then by Weak Systematicity we know:
\[ F(L)(b) = F(L')(c) \quad (3d) \]

But since \( L \) and \( L'' \) are permutations of each other we know \( F(L) = F(L'') \) by Anonymity and so we obtain
\[ F(L)(c) = F(L'')(c). \quad (3e) \]

From Eq. 3d and Eq. 3e
\[ F(L)(b) = F(L)(c). \quad (3f) \]

From Eq. 3e and Eq. 3f
\[ F(L)(a) = F(L)(b) = F(L)(c). \quad (3g) \]

The last equation suggests that a, b, and c have the same collective labelling. However, by Collective Rationality, the only legal labelling that satisfy Eq. 3g is \text{undec}: \[ F(L)(a) = F(L)(b) = F(L)(c) = \text{undec}. \quad (3h) \]

However, \( F \) satisfies Supportiveness by assumption. Contradiction.

One can draw a connection between this result and the previous one. Relaxing Universal Domain to No-Tie Universal Domain, introduces another impossibility result, in which Unanimity is replaced with the stronger postulate Supportiveness.

The above impossibility results highlight a major barrier to reaching good collective judgment about argument evaluation in general. These build on the solid foundation of social choice and judgement aggregation for impossibility results such as Arrow’s celebrated impossibility theorem on preference aggregation [1] and List and Pettit’s impossibility theorem on judgment aggregation in propositional logic [21]. In our context, the result means that an aggregation on argument evaluation can only be achieved at a cost to some few basic desirable properties. Unfortunately, there is no escape from violating these conditions or accepting irrational aggregate argument labellings without somewhat lowering our standards in terms of desirable criteria.

7 Collective Rationality Postulates

In this section, we characterise Collective Rationality in terms of conditions that need to be satisfied by profiles. To do this, we need to go back to the conditions of legal (i.e. complete) labelling, since we have to ensure that these conditions are satisfied in the outcome of an aggregation operator. For ease of reference, we reformulate the postulates for legal labelling here (from Definition [4] following the formulation in [9] Proposition 1):

- \( \forall a \in A : \text{if } L(a) = \text{in} \text{ then } \forall b \in A : ( \text{ if } b \rightarrow a \text{ then } L(b) = \text{out} ); \) and
- \( \forall a \in A : \text{if } L(a) = \text{out} \text{ then } \exists b \in A \text{ such that } ( b \rightarrow a \text{ and } L(b) = \text{in} ); \)
What we need to do now is identify conditions on the domain of the aggregation operator that guarantee these postulates. So, to guarantee that the postulate holds, we need some restrictions.

We define the following condition, which we call \textit{IN-Collective Rationality (IN-CR)}. Intuitively, it requires that if an argument \(a\) is collectively accepted by the agents, then the agents must collectively reject all counter-arguments against \(a\).

\textbf{IN-Collective Rationality (IN-CR)} For any profile \(L\) and \(a \in A\), if \([F(L)](a) = \text{in}\) then \(\forall b \in A, (b \rightarrow a \text{ and } L(b) = \text{undec}); \text{ and } \exists b \in A : (b \rightarrow a \text{ and } L(b) = \text{in})\).

IN-CR, however, can be further characterised by two parts: IN-CR1 and IN-CR2.

\textbf{IN-CR1} For any profile \(L\) and \(a \in A\), if \([F(L)](a) = \text{in}\) then \(\nexists b \in A, \text{ such that } b \rightarrow a \text{ and } [F(L)](b) = \text{in}\).

\textbf{IN-CR2} For any profile \(L\) and \(a \in A\), if \([F(L)](a) = \text{in}\) then \(\nexists b \in A, \text{ such that } b \rightarrow a \text{ and } [F(L)](b) = \text{undec}\).

An aggregation operator satisfies IN-CR if and only if it satisfies both IN-CR1 and IN-CR2. Note that IN-CR1 represents the “collective” counterpart of the condition of conflict-freeness which is usually agreed on as a minimal reasonable condition in argument evaluation.

We present now the \textit{OUT-Collective Rationality (OUT-CR)} condition. Intuitively, this condition means that if an argument \(a\) is collectively rejected by the agents, then the agents must also collectively agree on accepting at least one of the counter-arguments against \(a\). In other words, the agents’ individual attacks on \(a\) are not arbitrary, but must exhibit some minimal form of coordination.

\textbf{OUT-Collective Rationality (OUT-CR)} For any profile \(L\) and \(a \in A\), if \([F(L)](a) = \text{out}\) then \(\exists b \in A, \text{ such that } b \rightarrow a \text{ and } [F(L)](b) = \text{in}\).

The final condition we require has to do with the label \text{undec}. We present now the \textit{UNDEC-Collective Rationality (UNDEC-CR)} condition. An argument must be labelled \text{undec} if and only if: (i) it is not the case that all of its defeaters are \text{out}, that is, at least one of its defeaters is \text{undec}; and (ii) none of its defeaters is \text{in}.

\textbf{UNDEC-Collective Rationality (UNDEC-CR)} For any profile \(L\) and \(a \in A\), if \([F(L)](a) = \text{undec}\) then:

1. \(\nexists b \in A, \text{ such that } b \rightarrow a \text{ and } [F(L)](b) = \text{in}\).
2. \(\nexists b \in A, \text{ such that } b \rightarrow a \text{ and } [F(L)](b) = \text{undec}\).

Similarly, UNDEC-CR can be further characterised by two parts: UNDEC-CR1 and UNDEC-CR2.
For any profile \( \mathcal{L} \) and \( a \in \mathcal{A} \), if \( [F(\mathcal{L})](a) = \text{undec} \) then \( \nexists b \in \mathcal{A} \), such that \( b \rightarrow a \) and \( [F(\mathcal{L})](b) = \text{in} \).

UNDEC-CR2 For any profile \( \mathcal{L} \) and \( a \in \mathcal{A} \), if \( [F(\mathcal{L})](a) = \text{undec} \) then \( \exists b \in \mathcal{A} \), such that \( b \rightarrow a \) and \( [F(\mathcal{L})](b) = \text{undec} \).

An aggregation operator satisfies UNDEC-CR if and only if it satisfies both UNDEC-CR1 and UNDEC-CR2.

**Proposition 3** An argument aggregation operator \( F \) satisfies Collective Rationality if and only if for each profile \( \mathcal{L} = (L_1, \ldots, L_n) \) in its domain, it satisfies the IN-CR, OUT-CR, and UNDEC-CR conditions.

### 8 Plurality Rule with Classical Semantics

In this section, we analyse the performance of AWPR with respect to Collective Rationality when agents’ labellings are restricted to some classical semantics (i.e. complete, grounded, stable, semi-stable, and preferred). This investigation gives an entirely novel meaning to classical semantics in social choice settings. Rather than simply being compared by their logical rigour from the perspective of a single agent, semantics are compared based on the extent to which they facilitate collectively rational agreement among agents.

Our strategy will be based on the following approach. Since, by Proposition 3, Collective Rationality arises iff IN-CR, OUT-CR, and UNDEC-CR are satisfied, it is enough to check whether AWPR satisfies those properties.

#### 8.1 Complete Semantics

Since the complete semantics generalizes other classical semantics, we provide analysis for it first. Every property that is satisfied by AWPR with the complete semantics would be also satisfied by AWPR for the other other classical semantics.

Note that we have already shown earlier that AWPR violates Collective Rationality (for some \( \mathcal{AFs} \)) when agents’ labellings are restricted to complete semantics. In this section, we dig further to show which of the Collective Rationality parts are violated.

It is very interesting to see that, as the lemma below shows, when agents collectively accept an argument, the structure of the AWPR will ensure that they will not collectively accept any of its defeaters:

**Lemma 1** AWPR satisfies IN-CR1. Using the argument-wise plurality rule, given any profile \( \mathcal{L} = (L_1, \ldots, L_n) \), if an argument \( a \) is collectively accepted, none of its defeaters will be collectively accepted. Formally, if \( [M(\mathcal{L})](a) = \text{in} \) for some arbitrary \( a \in \mathcal{A} \), then \( \nexists b \in \mathcal{A} \), such that \( b \rightarrow a \) and \( [M(\mathcal{L})](b) = \text{in} \).
Proof Suppose that \( |M(L)(a) = \text{in}| \) holds. By definition:

\[
|\{i : L_i(a) = \text{in}\}| > |\{i : L_i(a) = \text{out}\}| \quad (4a)
\]

Since each \( L_i \) is a legal labelling, an agent who votes \( \text{in} \) for \( a \) must also vote \( \text{out} \) for each defeater of \( a \). Therefore:

\[
\forall b \rightarrow a \quad |\{i : L_i(b) = \text{out}\}| \geq |\{i : L_i(a) = \text{in}\}| \quad (4b)
\]

We want to show that: \( \exists b \in A \) such that \( b \rightarrow a \) and \( |M(L)(b) = \text{in}| \). Assume (towards contradiction) that the contrary holds. That is, \( \exists b' \in A \) such that \( b' \rightarrow a \) and \( |M(L)(b') = \text{in}| \). Then:

\[
|\{i : L_i(b') = \text{in}\}| > |\{i : L_i(b') = \text{out}\}| \quad (4c)
\]

Since every agent who voted \( \text{in} \) for \( b' \) would have voted \( \text{out} \) for \( a \), we have:

\[
|\{i : L_i(a) = \text{out}\}| \geq |\{i : L_i(b') = \text{in}\}| \quad (4d)
\]

By Eq.4c and Eq.4d:

\[
|\{i : L_i(a) = \text{out}\}| > |\{i : L_i(b') = \text{out}\}| \quad (4e)
\]

while from Eq.4b and Eq.4e we have that:

\[
|\{i : L_i(a) = \text{out}\}| > |\{i : L_i(a) = \text{in}\}| \quad (4f)
\]

But this contradicts Eq.4a and the assumption that \( |M(L)(a) = \text{in}| \).

It is important to recognise that Lemma 1 is a non-trivial result. It shows that, with AWPR, the postulate IN-CR1 is satisfied. This means, as we mentioned earlier, that AWPR satisfies the “collective” version of conflict-freeness, a condition that is usually agreed on as a minimal reasonable condition in argument evaluation. This comes “for free” as a result of the intrinsic structure of the individual labellings, leading to coordinated votes. Note, however, that the postulate is not fully satisfied. Although Lemma 1 guarantees that a collectively accepted argument will never have a collectively accepted defeater, it does not guarantee IN-CR2 that none of its defeaters will be collectively undecided. This is demonstrated in the following remark.

Remark 1 AWPR violates IN-CR2. If an argument is collectively accepted, some of its defeaters might be collectively undecided.

Proof Suppose argument \( c \) has two defeaters, \( a \) and \( b \). Suppose we have 7 agents, with votes as shown in Figure 9. Clearly, while \( c \) is collectively accepted because \( |M(L)(c) = \text{in}| \), one of its defeaters is not collectively rejected because \( |M(L)(b) = \text{undec}| \).

As we saw earlier in Example 6, OUT-CR is violated by AWPR.

Remark 2 AWPR violates OUT-CR. If an argument is collectively rejected, it is not guaranteed that one of its defeaters will be collectively accepted.
The following remark shows that there are no intrinsic guarantees for satisfying $\text{UNDEC-CR1}$.

**Remark 3** AWPR violates $\text{UNDEC-CR1}$. If an argument is collectively undecided, it is possible that one of its defeaters will be collectively accepted.

**Proof** Suppose argument $c$ has two defeaters, $a$ and $b$. Suppose we have 7 agents. Suppose the votes are as shown in Figure 10. We have $[M(\mathcal{L})](c) = \text{undec}$ with 4 votes, but we have $[M(\mathcal{L})](a) = \text{in}$ with 3 votes, thus violating the postulate.
Similarly, the remark below shows that UNDEC-CR2 is not intrinsically guaranteed.

**Remark 4** AWPR violates UNDEC-CR2. If an argument is collectively undecided, it is possible that none of its defeaters will be collectively undecided.

**Proof** Suppose argument $c$ has two defeaters, $a$ and $b$. Suppose we have 3 agents, with votes as shown in Figure 11. Clearly, we have $[M(L)](c) = \text{undec}$, but we have $[M(L)](a) = \text{out}$ and $[M(L)](b) = \text{out}$, which would have required $c$ to be in.

![Figure 11](image)

**Fig. 11** Three votes collectively undecided about $c$, but not collectively undecided about any of its defeaters $a$ or $b$.

So far, we undertook a detailed investigation of which of the properties (defined in the previous section) that characterise Collective Rationality are intrinsically satisfied due to structural properties of the argument complete labelling domain under plurality rule. Our finding is quite disappointing. With the exception of the IN-CR1 shown in Lemma 1, the remarks above show a bleak picture.

**8.2 Other Classical Semantics**

Throughout this work, and especially in the previous part, we have assumed (sometimes implicitly) that each agent is required to choose one possible complete labelling. This is a valid requirement since each possible complete labelling represents a valid self-defending viewpoint, that satisfies the widely accepted admissibility criterion. However, some may argue that other classical semantics (considered as a domain to choose from) might be more desirable in some cases. For example, sometimes, it might be more desirable for individuals to have more committed viewpoints (i.e. to maximize in-labelled or
out-labelled arguments), or to be restricted to labellings that minimize uncertainty (i.e., to minimize undec-labelled arguments).

Since the classical semantics, we consider here, add more restrictions to the complete labelling, using any of these semantics can be considered as a domain restriction. In other words, choosing other classical semantics implies a relaxation for No-Tie Universal Domain. Then, one may ask, “if agents were restricted to other classical semantics, would the argument-wise plurality rule find an escape route from the impossibility results?”. If that were the case, different semantics would find a novel field of application in the aggregation of labellings.

In this subsection, we provide an analysis for the grounded, stable, semi-stable, and preferred semantics as more restricted forms of labellings to choose from. We mainly show whether the Collective Rationality would be satisfied under any of these semantics. Note that the stable semantics would restrict the topology of the graph, since stable semantics do not always exist.

The following proposition looks trivial but, as we will see, it is the most positive result in this subsection.

**Proposition 4** If for every argument, agents can only vote for the grounded labelling, then $M$ satisfies IN-CR1, IN-CR2, OUT-CR, UNDEC-CR1 and UNDEC-CR2. Equivalently, $M$ satisfies Collective Rationality.

**Proof** There always exists exactly one grounded extension [15]. Since complete extensions and complete labellings have a one-to-one relationship [8, Theorem 1], then there always exists one grounded labelling (the grounded labelling is also a complete labelling). Hence, if agents are restricted to grounded semantics, all agents will vote for the (only) grounded labelling for each argument. By Unanimity, the collective labelling of every argument is the grounded labelling of this argument, which is a valid (complete) labelling. The Collective Rationality is trivially satisfied. By Proposition 3, IN-CR, OUT-CR, and UNDEC-CR are all satisfied. Equivalently, IN-CR1, IN-CR2, OUT-CR, UNDEC-CR1 and UNDEC-CR2 are all satisfied.

As a corollary for Lemma 1, when agents votes are restricted to stable (respectively semi-stable or preferred) labellings, AWPR satisfies IN-CR1.

**Corollary 2** When agents can only vote for stable (respectively semi-stable or preferred) labellings, AWPR satisfies IN-CR1

**Proof** From Lemma 1, if agents can only vote for complete labellings, then AWPR satisfies IN-CR1. Since every stable (respectively semi-stable or preferred) labelling is a complete labelling, then when agents votes are restricted to these semantics, AWPR satisfies IN-CR1.

**Lemma 2** When agents can only vote for a stable labelling, AWPR satisfies IN-CR2. If an argument is collectively accepted, none of its defeaters is collectively undecided.
Proof Suppose, towards a contradiction, that there exists an argument that is collectively accepted and one of its defeaters is collectively undecided. Then, by Supportiveness, there exists one submitted labelling (by some agent) in which this argument is undecided. However, agents are only allowed to submit a stable labelling, and stable labellings have no argument labelled undecided. Contradiction.

Remark 5 When agents can only vote for stable (respectively semi-stable or preferred) labellings, AWPR violates OUT-CR. If an argument is collectively rejected, it is possible that none of its defeaters is collectively accepted.

Proof See Example 6 for a counterexample.

Lemma 3 When agents can only vote for a stable labelling, AWPR satisfies UNDEC-CR (i.e. it satisfies both UNDEC-CR1 and UNDEC-CR2). If an argument is collectively undecided, none of its defeaters is collectively accepted, and at least one of its defeaters is collectively undecided.

Proof Since in stable labelling no argument is labelled undecided, by Supportiveness, there is no argument that is collectively undecided. Then, this lemma holds.

We continue with the semi-stable and preferred semantics.

Remark 6 When agents can only vote for a semi-stable (respectively preferred) labelling, AWPR violates IN-CR2. If an argument is collectively accepted, it is possible that one of its defeaters is collectively undecided.

Proof Suppose argument $c_4$ has two defeaters, $a_4$ and $c_6$. Suppose we have 7 agents, with votes as shown in Figure 12. Clearly, while $c_4$ is collectively accepted because $[M(L)](c_4) = \text{in}$, one of its defeaters, namely $a_4$, is collectively undecided because $[M(L)](a_4) = \text{undec}$.

Fig. 12 A counterexample shows how, given semi-stable (respectively preferred) semantics, AWPR violates IN-CR2 and UNDEC-CR1.
Remark 7 When agents can only vote for a semi-stable (respectively preferred) labelling, AWPR violates UNDEC-CR1. If an argument is collectively undecided, it is possible that one of its defeaters is collectively accepted.

**Proof** Suppose argument \( d \) has two defeaters, \( c_4 \) and \( c_5 \). Suppose we have 7 agents, with votes as shown in Figure 12. Clearly, while \( d \) is collectively undecided because \( [M(L)](d) = \text{undec} \), one of its defeaters, namely \( c_4 \), is collectively accepted because \( [M(L)](c_4) = \text{in} \).

Remark 8 When agents can only vote for a semi-stable (respectively preferred) labelling, AWPR violates UNDEC-CR2. If an argument is collectively undecided, it is possible that none of its defeaters is collectively undecided.

**Proof** Suppose argument \( c \) has two defeaters, \( a_4 \) and \( b_3 \). Suppose we have 3 agents, with votes as shown in Figure 13. Clearly, while \( c \) is collectively undecided because \( [M(L)](c) = \text{undec} \), none of its defeaters is collectively undecided.

Fig. 13 A counterexample shows how, given semi-stable (respectively preferred) semantics, AWPR violates UNDEC-CR2.

To sum up, the only restriction that would satisfy the Collective Rationality is the grounded semantics (Proposition 4). This is trivially true because only one grounded labelling exists. However, stable semantics violates Collective Rationality only because it violates OUT-CR. As for the semi-stable and preferred semantics, they only satisfy IN-CR1, a property they inherit from the complete semantics. Refer to Table 1 for a summary of the results we have found.

The results shown in Table 1 regarding the classical semantics are disappointing. However, possible future work may investigate for a set of graphs with some graph-theoretic properties, whether one semantics would be superior to another. In other words, for any graph \( g \in G \) (where \( G \) is a set of graphs with some properties), if semantics \( S_1 \) violates Collective Rationality then semantics \( S_2 \) violates Collective Rationality. In this case, we might say \( S_1 \) is at least as good as \( S_2 \) for the set of graphs \( G \), from the point of view of judgment aggregation.
9 Restricting the Domain of Argumentation Graphs to Satisfy Collective Rationality

In an earlier section, we showed that, out of the ten defined postulates, AWPR only violates Universal Domain and Collective Rationality. Then, we investigated whether AWPR can satisfy Universal Domain by introducing a tie-breaking rule. In Section 7, we characterised Collective Rationality in its constituents and, in Section 8, we checked how close AWPR is to satisfying them. In this section, we investigate whether AWPR can satisfy Collective Rationality by restricting the argumentation framework to graphs with certain graph theoretical properties. We show that graphs consisting of disconnected issues (a notion we define below) and graphs in which arguments have limited defeaters (in some sense) guarantee collectively rational outcomes when the AWPR is used.

9.1 Disconnected Issues

The notion of “issue” was defined in [7] in order to quantify disagreement between graph labellings. In this section, we use this notion to provide a possibility result.

Crucial to the definition of the “issue” is the concept of “in-sync”. Two arguments $a$ and $b$ are said to be in-sync if the (complete) label of one cannot be changed without causing a change of equal magnitude to the label of the other.

**Definition 8 (in-Sync $\equiv [7]$)** Let $\text{Comp}(AF)$ be the set of all complete labellings for argumentation framework $AF = \langle A, \rightarrow \rangle$. We say that two arguments $a, b \in A$ are in-sync ($a \equiv b$):

$$a \equiv b \text{ iff } (a \equiv_1 b \lor a \equiv_2 b)$$ (5)

where:

| Semantics   | IN-CR | IN-CR2 | OUT-CR | UNDEC-CR | UNDEC-CR |
|-------------|-------|--------|--------|----------|----------|
| Grounded    | Yes   | Yes    | Yes    | Yes      | Yes      |
| Stable      | Yes   | Yes    | No     | Yes      | Yes      |
| Semi-stable | Yes   | No     | No     | No       | No       |
| Preferred   | Yes   | No     | No     | No       | No       |
| Complete    | Yes   | No     | No     | No       | No       |

Table 1 The Collective Rationality properties that are satisfied/violated by AWPR given different semantics.
\[ a \equiv_1 b \iff \forall L \in \text{Comp}(AF) : L(a) = L(b). \]
\[ a \equiv_2 b \iff \forall L \in \text{Comp}(AF) : (L(a) = \text{in} \Leftrightarrow L(b) = \text{out}) \land (L(a) = \text{out} \Leftrightarrow L(b) = \text{in}) \]

This relation forms an equivalence relation over the arguments, and the equivalence classes are called "issues". An argumentation framework \( AF \) consists of a set of issues. One extreme case is when \( AF \) has only one issue (that is all the arguments in graph are in-sync), the other extreme case is when \( AF \) contains a set of issues each of which contains one argument (that is, no pair of arguments are in-sync).

**Definition 9 (Issue [7])** Given the argumentation framework \( AF = \langle A, \rightarrow \rangle \), a set of arguments \( B \subseteq A \) is called an issue iff it forms an equivalence class of the relation \( \text{in-Sync} (\equiv) \).

For example, in Figure 14, the graph consists of three issues, namely \( \{a_1\} \), \( \{a_2, a_3\} \), and \( \{a_4, a_5\} \).

\[
\begin{align*}
\text{Fig. 14} \text{ An example about issues.}
\end{align*}
\]

The following lemma is crucial in showing the main result of this subsection. We show that if the defeaters of an argument belong to the same issue as the argument, then the collective labelling of this argument chosen by AWPR is always a legal labeling.

**Lemma 4** Let \( AF = \langle A, \rightarrow \rangle \) be an argumentation framework. Let \( a \in A \) be an argument in this framework. If every defeater of \( a \) (call it \( b \)) belong to the same issue of \( a \) (i.e. \( \forall b \in a^- : b \equiv a \)), then AWPR would always produce a legal collective labelling for \( a \).

**Proof** Let \( b_1, \ldots, b_m \in A \) such that \( b_j \in a^- \) and \( a \equiv b_j \ \forall j = 1, \ldots, m \). Then, for every complete labelling \( L \):

\[
\begin{align*}
L(a) = \text{out} \Leftrightarrow L(b_1) = \text{in} \Leftrightarrow \ldots \Leftrightarrow L(b_m) = \text{in} \quad (6a) \\
L(a) = \text{in} \Leftrightarrow L(b_1) = \text{out} \Leftrightarrow \ldots \Leftrightarrow L(b_m) = \text{out} \quad (6b) \\
L(a) = \text{undec} \Leftrightarrow L(b_1) = \text{undec} \Leftrightarrow \ldots \Leftrightarrow L(b_m) = \text{undec} \quad (6c)
\end{align*}
\]

From Equations 6a, 6b and 6c, for every labelling profile \( L = (L_1, \ldots, L_n) \):

\[
\begin{align*}
|\{i : L_i(a) = \text{out}\}| &= |\{i : L_i(b_1) = \text{in}\}| = \ldots = |\{i : L_i(b_m) = \text{in}\}| \quad (6d) \\
|\{i : L_i(a) = \text{in}\}| &= |\{i : L_i(b_1) = \text{out}\}| = \ldots = |\{i : L_i(b_m) = \text{out}\}| \quad (6e)
\end{align*}
\]
\[ \{i : L_i(a) = \text{undec}\} = \{i : L_i(b_1) = \text{undec}\} = \ldots = \{i : L_i(b_m) = \text{undec}\} \]

From Equations 6d, 6e, and 6f:

\[ [M(L)](a) = \text{out} \iff [M(L)](b_1) = \text{in} \iff \ldots \iff [M(L)](b_m) = \text{in} \]  
(6g)

\[ [M(L)](a) = \text{in} \iff [M(L)](b_1) = \text{out} \iff \ldots \iff [M(L)](b_m) = \text{out} \]  
(6h)

\[ [M(L)](a) = \text{undec} \iff [M(L)](b_1) = \text{undec} \iff \ldots \iff [M(L)](b_m) = \text{undec} \]  
(6i)

From Equations 6g, 6h, and 6i, AWPR satisfies \text{IN-CR}, \text{OUT-CR}, and \text{UNDEC-CR} with respect to \( a \) in this case. Then, \( a \) is always legally collectively labelled by AWPR if every defeater of it is in the same issue as \( a \).

Given the previous lemma, we show that if the argumentation framework consists of a set of disconnected issues, then AWPR satisfies \text{Collective Rationality} for this framework.

\textbf{Theorem 6} For every \( AF = \langle A, \rightarrow \rangle \) that consists of a set of disconnected components, each of which forms an issue, the argument-wise plurality rule would always produce collectively rational outcomes.

\textbf{Proof} Since \( AF \) consists of a set of disconnected issues, then \( \forall a \in A, a \) has the following property: \( \forall b \in A \) such that \( b \in a^- \) then \( b \equiv a \). From Lemma 4, \( a \) is always legally collectively labelled by AWPR. Then AWPR satisfies \text{Collective Rationality} for this \( AF \).

9.2 Limited Defeaters

The notion of “Justification Status” is defined in [33]. Intuitively, the justification status of an argument is the set of possible labellings that this argument can take.

\textbf{Definition 10 (Justification Status [33])} Let \( AF = \langle A, \rightarrow \rangle \) be an argumentation framework, and \( a \in A \) some argument. The justification status of \( a \) is the outcome yielded by the function \( J S : A \rightarrow 2^{\{\text{in, out, undec}\}} \) such that \( J S(a) = \{L(a) | L \in \text{Comp}(AF)\} \).

There are six possible justification statuses. Neither \( \emptyset \) nor \( \{\text{in, out}\} \) is a possible justification statuses. The latter is shown in the following theorem:

\textbf{Theorem 7 ([33, Theorem 2])} Let \( AF = \langle A, \rightarrow \rangle \) be an argumentation framework, and \( a \in A \) some argument. If \( AF \) has two complete labellings \( L_1 \) and \( L_2 \) such that \( L_1(a) = \text{in} \) and \( L_2(a) = \text{out} \), then there exists a labelling \( L_3 \) such that \( L_3(a) = \text{undec} \).

The following lemma shows that an argument with one of its defeaters belong to the same issue as long as all the other defeaters of this argument have the justification status of \( \{\text{out}\} \).
Lemma 5 Let $AF = \langle A, \rightarrow \rangle$ be an argumentation framework, and $a, b \in A$ two arguments such that $b \rightarrow a$. If the following holds:

\[
\forall c \neq b : (c \rightarrow a \Rightarrow JS(c) = \{\text{out}\})
\]  

(7a)

Then $a$ and $b$ belong to the same issue (i.e. $a \equiv b$). Moreover, $a$ is always legally collectively labelled by AWPR.

Proof One can show that:

\[
L(a) = \text{out} \iff L(b) = \text{in}
\]  

(7b)

\[
L(a) = \text{in} \iff L(b) = \text{out}
\]  

(7c)

\[
L(a) = \text{undec} \iff L(b) = \text{undec}
\]  

(7d)

Hence, $a \equiv b$.

Moreover, in a similar way to Lemma 4 one can show that, for every possible profile $L = (L_1, \ldots, L_n)$, the following holds:

- If $[M(L)](a) = \text{out}$ then $[M(L)](b) = \text{in}$ ($b \in a^-$).
- If $[M(L)](a) = \text{in}$ then $[M(L)](b) = \text{out}$, and by Unanimity, $\forall c \neq b : (c \rightarrow a \Rightarrow [M(L)](c) = \text{out})$.
- If $[M(L)](a) = \text{undec}$ then $[M(L)](b) = \text{undec}$ ($b \in a^-$), and by Supportiveness, $\forall c \neq b : (c \rightarrow a \Rightarrow [M(L)](c) \neq \text{in})$.

Hence, $a$ is always legally collectively labelled by AWPR.

Corollary 3 Let $AF = \langle A, \rightarrow \rangle$ be an argumentation framework, and $a \in A$ an argument. If $|a^-| = 1$ then $a$ is always legally collectively labelled.

Proof From Lemma 4 $a$ is always legally collectively labelled by AWPR.

Now we present the main theorem for this subsection. It says if all arguments have limited defeaters then AWPR always produces legally collective labelings. The limitation of the defeaters is characterised in both their number and their justification statuses.

Theorem 8 Let $AF = \langle A, \rightarrow \rangle$ be an argumentation framework. If $AF$ does not contain an argument $a \in A$ such that all the following hold:

1. $a$ has at least two defeaters (i.e. $|a^-| \geq 2$),
2. At least two defeaters’ justification statuses share undec (i.e. $\exists b, c \in a^- \text{ s.t. } b \neq c \text{ and } \text{undec} \in JS(b) \cap JS(c)$).

then, AWPR satisfies Collective Rationality for this $AF$.

Proof Suppose, towards a contradiction, that AWPR violates Collective Rationality for the $AF$ described above. Then, there exists some argument $a$ that is illegally collectively labelled. Note that if $|a^-| = 0$ (i.e. $a$ has no defeaters), then $a$ will be collectively labelled in (by Unanimity), and if $|a^-| = 1$ (i.e. $a$ has only one defeater), then from Corollary 3 $a$ would be legally collectively labelled. Hence, $|a^-| \geq 2$ (i.e. $a$ has at least two defeaters).
Since AWPR satisfies IN-CR1 by Lemma 4. Then, AWPR violates either IN-CR2, OUT-CR, UNDEC-CR1, or UNDEC-CR2. We will show that in each of these cases, there exists two defeaters whose justification statuses share undec:

- **AWPR violates IN-CR2** i.e. \([M(L)](a) = \text{in}\) and \(\exists b \in a^-\) s.t. \([M(L)](b) = \text{undec}\). Then, by Supportiveness, \(\text{undec} \in JS(b)\). Since \(|a^-| \geq 2\), then \(\exists c \neq b\) s.t. \(c \in a^-\). Suppose \(c \neq b\) s.t. \(c \in a^-\) \(\Rightarrow \text{undec} \notin JS(c)\). Then, by Theorem 8, \(\forall c \in a^-\) either \(JS(c) = \{\text{in}\}\) or \(JS(c) = \{\text{out}\}\). However, if \(c \in a^-\) s.t. \(JS(c) = \{\text{in}\}\) then \(JS(a) = \{\text{out}\}\). By Unanimity, a would be legally collectively labelled. The only left option is \(c \neq b\) s.t. \(c \in a^-\) \(\Rightarrow JS(c) = \{\text{out}\}\). From Lemma 5, a would be legally collectively labelled. Contradiction. Then \(\exists c \neq b\) s.t. \(c \in a^-\) and \(\text{undec} \in JS(c)\).

- **AWPR violates OUT-CR** i.e. \([M(L)](a) = \text{out}\) and \(\exists b \in a^-\) s.t. \([M(L)](b) = \text{in}\). We have three cases:

  1. \(\exists b \in a^-\) s.t. \([M(L)](b) = [M(L)](c) = \text{undec}\). Then, by Supportiveness, \(\text{undec} \in JS(b) \cap JS(c)\).

  2. \(\exists b \in a^-\) s.t. \([M(L)](b) = \text{undec}\) and \(\forall c \in a^-\) \([M(L)](c) = \text{out}\). Then, by Supportiveness, \(\text{undec} \in JS(b)\). Since \(|a^-| \geq 2\), then \(\exists c \neq b\) s.t. \(c \in a^-\). Suppose \(c \neq b\) s.t. \(c \in a^-\) \(\Rightarrow \text{undec} \notin JS(c)\). Then, by Supportiveness and Theorem 5, the only left option is \(c \neq b\) s.t. \(c \in a^-\) \(\Rightarrow JS(c) = \{\text{out}\}\). From Lemma 6, a would be legally collectively labelled. Contradiction. Then \(\exists c \neq b\) s.t. \(c \in a^-\) and \(\text{undec} \in JS(c)\).

  3. \(\forall b : (b \in a^- \Rightarrow [M(L)](b) = \text{out})\). Then, by Supportiveness, \(\forall b \in a^-\) we have \(out \in JS(b)\). Since \([M(L)](a) = \text{out}\), by Supportiveness, \(out \in JS(a)\). Then, \(\exists b \in a^-\) s.t. \(in \in JS(b)\). By Theorem 6, \(JS(b) = \{\text{in, out, undec}\}\). Suppose \(c \neq b\) s.t. \(c \in a^-\) \(\Rightarrow \text{undec} \notin JS(c)\). In a similar way as before we find that \(\exists c \neq b\) s.t. \(c \in a^-\) and \(\text{undec} \in JS(c)\).

- **AWPR violates UNDEC-CR1** i.e. \([M(L)](a) = \text{undec}\) and \(\exists b \in a^-\) s.t. \([M(L)](b) = \text{in}\). By Supportiveness, \(in \in JS(b)\). Note that if \(JS(b) = \{\text{in}\}\) then \(JS(a) = \{\text{out}\}\). Then, by Unanimity, \([M(L)](a) = \text{out}\). Contradiction. Then, either \(JS(b) = \{\text{in, undec}\}\) or \(JS(b) = \{\text{in, out, undec}\}\). Suppose \(\forall c \neq b: (c \in a^- \Rightarrow \text{undec} \notin JS(c)\)\). In a similar way as before we find that \(\exists c \neq b\) s.t. \(c \in a^-\) and \(\text{undec} \in JS(c)\).

- **AWPR violates UNDEC-CR2** i.e. \([M(L)](a) = \text{undec}\) and \(\forall b : (b \in a^- \Rightarrow [M(L)](b) = \text{out})\). By Supportiveness, \(\text{undec} \in JS(a)\). Then, \(\exists b \in a^-\) s.t. \(\text{undec} \in JS(b)\). Suppose \(\forall c \neq b: (c \in a^- \Rightarrow \text{undec} \notin JS(c)\). In a similar way as before we find that \(\exists c \neq b\) s.t. \(c \in a^-\) and \(\text{undec} \in JS(c)\).

Note that the condition given on \(AF\) in Theorem 8 is neither a generalisation nor a special case of the condition on \(AF\) in Theorem 5. Example 7 shows an \(AF\) that satisfies the condition in Theorem 5 (i.e. disconnected issues), but violates the condition in Theorem 8 (i.e. limited defeaters), while Example 8 shows an \(AF\) that satisfies the condition in Theorem 8 (i.e. limited defeaters), but violates the condition in Theorem 6 (i.e. disconnected issues).
Example 7 Note that the argumentation framework in Figure 15 satisfies the condition in Theorem 6. All the arguments \( a, b, c, d, \) and \( e \) are in the same issue, so this AF consists of disconnected issues (only one issue in this case). However, this AF violates the condition in Theorem 8, since argument \( a \) is defeated by two arguments \( b \) and \( c \), each of these defeaters has a justification status of \( \{ \text{in, out, undec} \} \), and so their justification statuses share \( \text{undec} \).

Example 8 Note that the argumentation framework in Figure 16 satisfies the condition in Theorem 8. The only argument that is defeated by more than one argument is argument \( a \) which has two defeaters \( b \) and \( c \). Moreover, \( \text{undec} \notin JS(c) \), so \( \text{undec} \notin JS(b) \cap JS(c) \). However, this AF violates the condition in Theorem 6, since it contains two connected issues. The first issue is \( \{a, b, d\} \) and the second issue is \( \{c, e\} \).

10 Discussion & Conclusion

In this paper, we presented an extensive analysis of social-choice-theoretic aspects of Dung’s highly-influential argumentation semantics. We introduced and explored the following question:

Given a set of agents, each with a specific subjective evaluation (i.e. labelling) of a given set of conflicting arguments, how can agents reach a collective decision on how to evaluate those arguments?

In this context, the specific original contributions of this paper can be summarised as follows:
1. Defining the multi-agent collective argument evaluation problem, including adapting various desirable postulates from the classical judgment and preference aggregation literature.

2. Analysing the “argument-wise plurality rule”, proving many of its key properties, showing its failure to satisfy “Universal Domain” and “Collective Rationality”.

3. Proving general impossibility results on the existence of any argument evaluation aggregation operator that satisfies some minimal properties. Then fully characterizing conditions under which an aggregation operator does satisfy Collective Rationality.

4. Analysing the argument-wise plurality rule with respect to the characterised conditions.

5. Showing that using other classical semantics as a domain restriction for the argument-wise plurality rule is not enough (except for the grounded semantics) for the fulfillment of the characterised conditions—and characterising exactly where each semantics fails.

6. Showing graph theoretic restrictions on the argumentation framework would help argument-wise plurality rule to satisfy “Collective Rationality”.

Argumentation-based semantics have mainly been compared on the basis of how they deal with specific benchmark problems that reflect specific logical structures from the point of view of a single omniscient observer (e.g. argument graph structures with odd-cycles, floating defeaters etc.). Recently, it has been argued that argumentation semantics must be evaluated based on more general intuitive principles. Our work can be seen as a contribution in this direction, focusing on issues relating to multi-agent preferences.

The closest work to the present paper is Caminada and Pigozzi. In their work, they propose three aggregation operators, namely sceptical, credulous and super credulous. Although the operators satisfy Collective Rationality, they violate Independence. These operators are also more applicable to scenarios with small numbers of agents where individuals are expected to publicly defend the group decision (e.g. a jury). Argument-wise plurality rule, on the other hand, can be applied to classical scenarios with a large group of agents where some individuals might naturally disagree with the opinion of the group. Additionally, unlike our work, their work focuses on the proposed operators with only little attention to the general aggregation problem. Only four postulates are proposed, namely Universal Domain, Collective Rationality, Anonymity, and Independence, and there are no general impossibility results that holds for any operator.

The social choice theoretic Arrovian properties have been analyzed in the context of social argument justification in [31]. An extended argumentation framework $AF^n = (A, \rightarrow_1, \ldots, \rightarrow_n)$ is defined, where each $\rightarrow_i$, $1 \leq i \leq n$, is a particular attack relation among the arguments in $A$, representing different attack criteria. Then, the authors define an aggregate argumentation framework $AF^* = (A, F(\rightarrow_1, \ldots, \rightarrow_n))$, where $F(\rightarrow_1, \ldots, \rightarrow_n)$ is an attack relation obtained by the aggregation of the individual attack criteria $\rightarrow_1, \ldots, \rightarrow_n$, via
different kinds of mechanisms (e.g. majority voting, qualified voting and mechanisms that can be described by classes of decisive sets). The aggregation of individual attack criteria cannot be assimilated to the kind of mechanisms proposed here. In [31] an individual may sanction an attack between two given arguments while another individual may not, which in terms of labellings means that for the same pair of arguments there may exist two possible labellings, say (in, out) and (in, in). This is impossible in our setting. Hence, the Arrovian properties (e.g. Collective Rationality) are conceived in an essentially different way.

In [6] the authors analyse the problem of aggregating different individual argumentation frameworks over a common set of arguments in order to obtain a unique socially justified set of arguments. One of the procedures considered there is one in which each individual proposes a set of justified arguments and then the aggregation leads to a unique set of socially justified arguments. The argument-wise plurality rule mechanism proposed here fits this procedure for the special case in which individually justified arguments are simply the sets of arguments labelled in for each individual.

There is much work on using an individual agent’s preferences to help evaluate arguments (e.g. based on given priorities over rules [25]). But this line of work does not address the preferences of multiple agents and how they may be aggregated. In other related work, Bench-Capon [4] associates arguments with values they promote or demote, and considers different audiences with different preferences over those values. Such preferences determine whether particular defeats among arguments succeed. Thus, one gets different argument graphs, one for each audience. Bench-Capon uses this to distinguish between an argument’s subjective acceptance with respect to a particular audience, and its objective acceptance in case it is acceptable with respect to all possible audiences. Our work differs in two important ways. Firstly, in our framework, an agent (or equivalently, an audience) does not have preferences over individual arguments, but rather preferences over how to evaluate all arguments collectively (i.e. over labellings). Secondly, our concern here is not with how individual agents (or audiences) accept an argument, but rather on the possibility of achieving important social-choice properties in the final aggregated argument labelling.

In relation to aggregation, Coste-Marquis et al explored the problem of aggregating multiple argumentation frameworks [11]. Each agent’s judgment consists of a different argument graph altogether. This contrasts significantly with our work, in which agents do not dispute the argument graph, but rather how it must be evaluated/labelled. Our setting is more akin to a jury situation, in which all arguments have been presented by the prosecution and defense team, and are visible to the jury members. The jury members themselves do not introduce new arguments, but are tasked with aggregating their individual judgments about the arguments presented to them.

Finally, we refer to the work of Rahwan and Larson [26] on strategic behaviour when arguments are distributed among agents, and where these agents may choose to show or hide arguments. Thus, their interest is in how agents
contributing to the construction of the argument graph itself, which is then evaluated centrally by the mechanism (e.g. a judge). In contrast, our present paper is concerned with how agents individually cast votes on how to evaluate each argument in a given fixed graph.

Our results on the aggregation of different argument evaluations by multiple agents provide a new approach for conflict-resolution in multi-agent systems. While much research has focused on the aggregation of simpler preferences (e.g. binary preference statements expressing preorders) and its associated paradoxes, our research focuses on the aggregation of more complex preferences, expressed as argumentative points of view.

Our work also opens new research problems for the computational social choice community. As is the case with other aggregation domains, the aggregation paradox in argument evaluation is an example of a fundamental barrier. Thus the impossibility results are important because they give conclusive answers and focus research in more constructive directions (e.g. weakening the desired properties in order to avoid the paradox). Another promising research direction is characterising the computational complexity of checking whether a given argumentation graph would violate desirable properties like collective rationality. An algorithmic agenda would complement this research by providing efficient algorithms for such problems. Strategic manipulation, by misreporting one’s true vote, is also an important area of investigation, especially when such manipulations are exercised by coalitions of agents.

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