Algorithm for 2D Mesh Decomposition in Distributed Design Optimization

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ABSTRACT

Optimization of thin-walled structures like an aircraft wing often involves dividing it into numerous localized panels, each characterized by its own set of design variables. In this work, a generalized algorithm to extract localized panels from the two-dimensional (2D) mesh is discussed. The process employs set operations on elemental connectivity information and is independent of nodal coordinates. Thus, it is capable of extracting a panel of any shape given the boundary and can be used during the optimization of a wide range of structures. A method to create stiffeners on the resulting local panels is also developed. The panel extraction process is demonstrated by integrating it into a distributed MDO framework for optimization of an aircraft wing having curvilinear spars and ribs (SpaRibs). A range of examples is included wherein the process is used to create panels on the wing-skin, bounded by adjacent SpaRibs.

KEYWORDS:

Design optimization; Finite element analysis; Meshes and discretization; Modeling and simulation; Meshprocessing; Structural optimization

INTRODUCTION

One of the most important concerns while designing vehicles is to reduce structural weight, which can directly lead to a reduction in fuel consumption. During the era when computers were expensive and not so powerful, structural design was mostly done by hand calculation on simplified mathematical models. However, since the seventies, due to the rapid increase in computational power numerical solution techniques like Finite Element Analysis (FEA) have gained immense popularity. Not only the details and the complexity of the system can be included in the analysis, but multiple disciplines can now be considered while setting up a structural optimization problem. This avenue of research where several disciplines are incorporated into the optimization problem is known as Multidisciplinary Design Optimization (MDO). The major advantage of solving such a problem arises when relevant disciplines are not independent of each other; in other words, the disciplines interact with each other.

Application of MDO for structural design can be traced back to the work of Schmit [1,2,3]. In these work, finite element methods and algorithms for numerical optimization were used. In subsequent years, Starnes, J. and Haftka [4], Haftka et al. [5] and Fulton et al. [6] designed aircraft wings considering constraints on strength, stability
and flutter velocity. With the help of commercial softwares like ANSYS, MDO can be applied to complex 3D spacial structures as well (For example as done by Aru et al.[7]). With the advance in computational power, MDO rapidly gained popularity power in aerospace (Li et al. [8]) and automobile engineering and soon, problems were solved involving a complete three dimensional structures (Devarajan et al. [9]; Singh et al. [10]; Fulton et al. [11]; Kroo et al. [12], Manning [13]; Shihan [14]; Antoine and Kroo[15]; Henderson et al.[16]; Alonso and Colonnio[17]). The processes followed in MDO have either monolithic or distributed architectures (Martins and Lambe[18]). In any MDO, the first step is almost always to describe the system using a set of design variables which are often selected by Design of Experiment (DOE) techniques (An example of DOE can be found in the article by Kumar et al. [19]). The goal is to find the best values for the design variables that minimizes (or maximizes) the objective function while satisfying constraints in several disciplines. For example, in problems involving structural design, the size, shape or topology of the structure are described by a set of design variables. By applying appropriate numerical optimization algorithms, the problem is solved for a set of design variables that gives minimum weight or maximum compliance while satisfying constraint like maximum von Mises stress, minimum buckling factor, maximum displacement etc. In a monolithic architecture, all the design variables and constraints are considered in a single optimization process. This process, commonly known as All-at-Once optimization (Sobieszczanski-Sobieski and Haftka[20]), although simple to implement, is computationally expensive when the number of design variables is large. Such problems involving a large number of design variables are often solved using the other process of MDO i.e. distributed architecture. The distributed MDO architectures involves the decomposition of complex systems into multiple smaller components which are then described by a lower number of design variables and optimized independently. The process is usually implemented using parallel computation which can reduce wall clock time by several times. This process of decomposition of a system into a simple sub-system is often known as global/local design optimization.

The aircraft wing is a complex structure consisting of the outer aerofoil shell known as the wing-skin, and internal stiffening elements: the ribs and spars. The global/local optimization of wing design has been used by several research groups including Cimpa et al. [21], Yang et al. [22] and Barkanov et al. [23]. Even though the availability of computational power makes the exploration of large design space feasible, there has always been a concern in the industry about manufacturing limitations. It is often very expensive to produce unconventional designs using conventional manufacturing processed. However, with the invention of 3D printing techniques, the manufacturing industry is likely to be revolutionized over the next few decades. A new additive manufacturing technique known as Electron Beam Free Form Fabrication or EBF3, in short, has recently been developed by Taminger and Hafley[24] at NASA Langley Research Center to fabricate metallic structures of complex shapes, which now can be printed with significant precision. This technology inspired Kapania et al. at Virginia Tech (Locatelli et al. [25]; Locatelli et al. [26]; Jrad et al.[27]; Devarajan et al.[28]; Miglani et al.[29]) to propose the use of curvilinear stiffening elements to reduce structural weight and achieve desirable aeroelastic properties for aircraft. The EBF3GLWingOpt is one of the several optimization frameworks that is being developed at Virginia Tech to optimize aircraft structures using curvilinear spars and ribs (SpaRibs). It performs global/local optimization of high aspect ratio cantilever transport aircraft wing for multiple constraints including stress, buckling and crippling. The wing geometry and mesh are
generated using commercial software, MSC.PATRAN and MSC.NASTRAN is used for static and buckling analysis. The framework is written in python environment and it enables the use of parallel processing.

In the original version of EBF3GLWingOpt, written by Liu et al., the Linked-shape method proposed by Locatelli et al. [27] is used to create SpaRibs in each of the wing-boxes using a limited number of design variables (which specified the shape of the SpaRibs). The upper and the lower skin of the wing are divided into local panels using the intersection of the SpaRibs and the stiffeners are attached to each of the panels. The thickness of each of the local panels, the stiffener height and thickness are considered as design variables. The optimization framework could only explore limited design space where SpaRibs could start at the leading-edge spar and end at the trailing-edge spar. In order to overcome this limitation, De et al. [30] proposed the Extended-space method to create SpaRibs. By this method not only the constraint on the starting and ending point of the SpaRibs was removed but also SpaRibs crossing the junction of the inner and outer wing-box can be created. In addition, the algorithm to extract local panel from the finite element model as used the original version of the EBF3GLWingOpt framework was dependent on nodal coordinate coordinates of the nodes and could extract only panels with four edges from the finite element model. Moreover, the idea of dividing a surface into local panels and assigning a thickness design variable to each of the panels is very generalized and can be applied not only to aircraft wings but different other structures including automobile, ships, buildings etc. Thus, the need was felt to develop a generalized algorithm to break a finite element model into local panels. In the following work, such an algorithm to divide a surface mesh (consisting of triangular elements) has been discussed. The process can be implemented on the CRM wing for any SpaRibs configuration as well as other shell-structures like the fuselage or automobile frame with minor modifications. This algorithm is not dependent on nodal coordinates and is purely based on set operations performed on the element connectivity matrix.

The article is organized into three parts. First, a detail description of the developed algorithm to extract the local panel is given. Second, the capability of the method is described using different examples. In the first example, the method is implemented on a simple rectangular wing-box to create local panels from the top and bottom skin. In the second example, the method is integrated with the EBF3GLWingOpt framework and demonstrates its effectiveness for a range of SpaRibs profiles with examples. Finally, a method of laying down stiffeners on the panels after they are generated using the algorithm and performing optimization is demonstrated before the concluding article.

METHODOLOGY

The goal is to develop an algorithm to split a 2D finite element surface mesh into local panels given the boundary nodes of the panel. To do so, the elements interior to the local panel along its outer edge, i.e., the element which shares the boundary nodes, need to be determined. Once these boundary nodes are identified, all nodes and elements interior to the local panel can be found using the connectivity matrix information, following the process mentioned in the flowchart given in Figure 1. Information about the nodal coordinates is not required for this process. Figure 2 shows a general 2D surface mesh from which local panels are needed to be generated. Figure 2a shows the nodes (marked with black dots) are the boundary of the panel. Figure 2b shows, in orange, the boundary elements and interior to the panel. Using the process which is described in Figure 1, all elements of the panel (shown in orange in
Figure 2c) are determined. The list of elements and list of nodes of the local panel are stored in the lists: ELEMENT_LIST and NODE_LIST, respectively.

**Figure 1. Algorithm to find Local Panel (De et al. 2018)**
Figure 2. Determination of elements constituting local panel: (a) Boundary nodes of panel marked with black dots; (b) Boundary elements and interior starting node (marked with a red dot); (c) Elements comprising the local panel.

The algorithm to determine the boundary elements

The Mesh-continuity algorithm is easy to apply once the elements along the outer boundary of the panel are determined. The general process to find these boundary elements is complicated and will be discussed in detail in this section with a simple example where the objective is to split a rectangular plate (with holes) that has been meshed with triangular elements, into two panels. *Since the process does not depend on the nodal coordinates, it would work on any mesh created by a linear transformation of this flat-plate.* The process consists of two steps. The first step is to determine all elements along the outer periphery of the panels. The second step is to determine the elements with edges along the ‘dividing curve,’ which lies inside the finite element mesh. In Figure 3, the outer periphery of the panel and dividing curve are shown.
Figure 3. The periphery of the panel and the internal dividing curve.

_The algorithm to determine elements outer periphery_

The plate shown in Figure 4 is needed to be split along Curve 2 (Curve 3 is the next curve in the family; Curve 1 is the previous one). The first elements along the periphery of the panels, will be found. In the algorithms described in this section, CHECKPOINT_NODE and CHECKPOINT_ELEMENT refer to the node and element respectively with respect to which the positions of the next node determined. ELEMENT_LIST is a list containing lists of the form: [Element reference number, Connectivity Nodes] while NODE_LIST is a list of the reference number of the nodes. All of them are initialized as null lists. The algorithm is described in Figure 5 using a flowchart.

Figure 4. Application to splitting a plate.
Figure 5. Algorithm to find elements along the outer boundary.
The nodes along Curve1 are already known. Thus, finding the elements along this curve is straightforward. All it needs is to find the elements that have at least two nodes along Curve1. The elements in ELEMENT_LIST and those along Curve1 are marked in red in Figure4. If the CHECKPOINT_NODE is assigned N5, it means we are moving along the elements marked in green, which is not the correct path. It also means that we have started with the element among E1 and E2 which lies outside the panel. If we move in the wrong path, it requires the re-initialization of ELEMENT_LIST and NODE_LIST as a null list and starts the process again starting from the correct element among E1 and E2. The process described above cannot be used to find elements interior to the local panel with edges along Curve2 as it will include elements outside the local panel as well (elements shown in yellow in Figure4), because all these elements have edges along Curve2. To find the elements located inside the panels, another algorithm known as Mid-element Algorithm (MEA) has been developed, which is described in the flowchart in Figure6.

![Flowchart Diagram](image)

**Figure 6. Flowsheet diagram showing the middle element algorithm (MEA).**

While implementing the Mid-element algorithm (MEA), we collect all the elements and nodes in the same lists, i.e., ELEMENT_LIST and NODE_LIST, respectively. Similar to the process used to determine elements along the periphery, we use two variables CHECKPOINT_NODE and CHECKPOINT_ELEMENT for node and element with respect to which the next node and element are found. The algorithm is applied along Curve 2 considering the nodes...
in order. The reference number of the nodes and the element interior to the panel along the curve, Curve 2, in order from right to left, are $Ni$ and $Ei$, respectively. The stopping criteria are when $\text{CHECKPOINT\_NODE} = N4$.

However, before implementing the MEA, it is important to correctly choose the first element i.e. $E1$. It must be an element interior to the panel. This can be determined with the help of the current $\text{CHECKPOINT\_ELEMENT}$, which contains only the elements marked with red in Figure 7. We initiate another vector, $\text{CHECKPOINT\_ELEMENT\_1}$ and a null list, $\text{ELEMENT\_LIST\_1}$. The process is given in Figure 8. Here the ‘Mid-element algorithm’ (MEA) is applied to the flat plate example:

![Figure 7. Using the Middle-element algorithm to find elements colored yellow and blue in Local panel.](image)

![Figure 8. Algorithm to find the correct first element for MEA.](image)
The elements included in the list \texttt{ELEMENT\_LIST} after implementation of MEA include elements above Curve 2 (marked by yellow and blue in Fig.8 and elements form the periphery of the panel above Curve 2 (marked in red in Figure8). Once these elements along the outer periphery are determined, the Mesh-continuity algorithm described at the start of this section can be used to find all the elements belonging to the panel.

**RESULTS AND DISCUSSION**

*Demonstration on rectangular wing-box*

The method is demonstrated first on a simple example. The rear wing-box of Boeing HSCT N+2 Wing containing 7 \textit{SpaRibs} in the span-wise direction and 8 \textit{SpaRibs} in the chord-wise direction is constructed in MSC.PATRAN, as shown in Figure9a. Figure 9b shows a local panel extracted from the top skin from the wing-box using the method.

![Figure 9](image.png)

\textbf{Figure 9.} (a)Mesh of rear wing-box of Boeing HSCT N+2 wing using triangular elements; (b) Local panel from upper skin of Boeing N+2 HSCT wing-box.
Integration of algorithm with EBF3GLWingOpt

The algorithm can be integrated with any distributed MDO framework if the purpose is to optimize the thickness of 2D surfaces. EBF3GLWingOpt is one such MDO framework being developed at Virginia Tech by Kapania et al. to optimize commercial aircraft wing with curvilinear spars and ribs (SpaRibs). The shape of the SpaRibs, as well as thickness distribution of the wing-skin and the SpaRibs, are considered for optimization.

The algorithm is integrated with the EBF3GLWingOpt framework to create a local panel on the upper and lower skin of the NASA CRM Wing (high-aspect ratio transport aircraft wing) for a range of SpaRibs configurations possible using the Extended-space Method. The wing-skin is first divided by the set of SpaRibs, which replace the ribs. Each of the sections is then divided by the family of SpaRibs that replace the spars. The boundary nodes are the nodes common to the wing-skin and the SpaRibs, thus bounding the local panel. Since the algorithm is completely

Figure 10. (a) Internal layout of a commercial aircraft wing (with one internal Spar and 37 straight Ribs); (b), (c) Top-skin of wing being split into local panels and enlarged view of selected local panels; (d) Internal layout of the wing with curvilinear SpaRibs (concave upward); (e) Internal layout of the wing with curvilinear SpaRibs (convex upward); (f), (g) Top-skin of wing being split into local panels using its intersection with SpaRibs (concave upward) and enlarged view of selected local panels; (h), (i) Top-skin of wing being split into local panels using its intersection with SpaRibs (convex upward) and enlarged view of selected local panels.
based on set operations performed on the connectivity matrix and independent of nodal coordinates, the algorithm can be used in a variety of problems requiring the generation of local panels. The process is also independent of the element reference numbers, and it works no matter in whatever order the elements are distributed. Depending on the shape of the bounding SpaRibs, the panel can have any number of edges greater than three. Another advantage of this method is that the boundary nodes of each of the local panels are already determined, and the information is stored to be used in imposing boundary conditions during the optimization process. The proposed algorithm has been implemented on wings with different SpaRibs profiles and is found to be useful for SpaRibs of different orientations and curvatures. Figure 10a shows the internal spars and ribs layout of a commercial aircraft wings. The wing contains one inner spar and 37 straight ribs. Figures10b and c show the top skin of the wing which is divided into local panels using the algorithm and some of the local panels created (marked in orange in b), respectively. Figure 10d and e show the internal layout of a wing with curvilinear spars and ribs (SpaRibs) of two different profiles, respectively. The local panels created from top skin using SpaRibs shown in Figure 10d and e are represented in Figure 10f, g and Figure 10h, i, respectively.

Creating Stiffened Panels and Optimization

Shell structures like the skin of a wing are usually laid with blade-stiffeners (which are similar to spars but without full depth), which help to reduce overall weight. In an aircraft wing, the stiffeners are usually placed along the span-wise direction from the wing-root to the wing-tip, as shown in Figure 11(a-c).

![Image](image.png)

Figure 11. (a) Upper skin (b) Blade stiffeners (c) Lower skin.

It is challenging to determine the parts of stiffeners that are attached to each of the panels isolated for distributed optimization. An approach is developed where stiffeners on the top and bottom skin of the wing and are meshed with the quad elements. The element size is chosen to be larger than the height of the stiffeners to ensure that the stiffeners are represented by a chain of quad elements with each element sharing two common nodes with the skin. The nodes belonging to each of the local panels are already known by the method of creating local panels using the Mesh-continuity algorithm. MSC.NASTRAN “.bdf” file for the stiffeners is generated and read to find the quad elements with nodes common with the nodes shared with the local panels. These elements form the stiffeners for respective local panels. An example of such a stiffened panel is shown in Figure 12.
If the cross-section of the stiffeners is rectangular, and isotropic material is used for both stiffener and the win-skin, it is reasonable to assign 3 design variables to each of the local panels: one for the thickness of the panel, one for stiffener height and one for stiffener width. In such case the total number of design, \( N_{\text{variable}} \) variables is given by:

\[
N_{\text{variable}} = 3(N_{\text{Spars}} + 1)(N_{\text{Ribs}} + 1)
\]  

(1)

where, \( N_{\text{Spars}} \) and \( N_{\text{Ribs}} \) are the number of spars and the number of ribs, respectively.

Even though the total number of design variables can be large, in the EBF3GLWingOpt framework, three design variables associated with each panel is optimized at once. The isolated panel can be assumed to be “simply-supported,” and the panel thickness, stiffener height and profile can be optimized considering constraint on stress and buckling factor. In the EBF3GLWingOpt framework, the maximum von Mises stress \( (\sigma_{vm})_{\text{max}} \) and the first buckling eigenvalue \( (\lambda_p) \) are computed using MSC.NASTRAN Sol 101 (linear static solver) and MSC.NASTRAN Sol 105 (buckling solver) respectively.

The optimization problem for each of the panel can be stated as follows:

**Objective:** \( \text{min}(\text{Weight}) \)

**Constraint:**

\[
\lambda_p > 1.05
\]

\[
(\sigma_{vm})_{\text{max}} < \sigma_y
\]

\[
(t_{\text{stiff}})_{\text{min}} \leq t \leq (t_{\text{stiff}})_{\text{min}}
\]

\[
(t_{\text{stiff}})_{\text{min}} \leq t_{\text{stiff}} \leq (t_{\text{stiff}})_{\text{max}}
\]

\[
(h_{\text{stiff}})_{\text{min}} \leq h_{\text{stiff}} \leq (h_{\text{stiff}})_{\text{max}}
\]

(2)

where, \( t, t_{\text{stiff}}, h_{\text{stiff}} \) are panel thickness, stiffener thickness, and stiffener height respectively and \( \sigma_y \) is material yield stress.
The optimal value of $t, t_{stiff}, h_{stiff}$ can be obtained by gradient-based optimization or by using the approximate method as used by Jrad et al. [27]. In each iteration, the global wing model is constructed by assembling the optimized local panels and the new global deformation field is computed by running MSC.NASTRAN Sol 144 (static aeroelastic solver). From the deformation field, boundary conditions for the panels are computed for the next iteration. After every iteration, the percentage difference between the weight of the new design and the design from the previous iteration is calculated. Convergence is said to have been achieved when the difference falls within a certain user-specified limit. The optimization process is shown in Figure 13.

![Diagram](image)

Figure 13. Distributed design optimization of Aircraft Wing using EBF3PanelOpt (Jrad et al. [27]).

It is worth mentioning that, the analysis of isolated stiffened panel could further be improved by isogeometric analysis [28-33].

Although in this global-local optimization framework, the satisfaction of buckling and stress constraints is ensured for the local panel, it does not automatically lead to the satisfaction of stress and buckling constraints for the entire wing. It was found that even though weight convergence can be achieved, the global buckling constraint is often not satisfied. This drawback was overcome and the algorithm was successfully implemented by De et al. [34] to perform distributed optimization of transport aircraft wing considering 6 degrees and -2 degree angle of attack at Mach 0.85 and 35000 ft cruising altitude.
CONCLUSIONS

This paper gives a detailed description of a new method to create stiffened local panels from two-dimensional finite element models. The capability of the algorithm used in the original EBF3GLWingOpt framework to create local panels was limited to the generation of four-edged panels with two edges along adjacent spars from the wing-skin of the CRM Wing. The new methodology is based on performing set operations on the element connectivity data of the wing’s finite element model and is independent of the nodal coordinate. It can be used to create local panels of any shape and size and can be used for structures other than the NASA CRM wing. Elements of the stiffeners attached to each of the local panels are determined, and stiffened panels are created. Once the local panels are determined, the structure is ready for distributed MDO, as was shown by De et al. [35-37]. Although the algorithm has been demonstrated on the skin of a commercial aircraft wing, it is a much-generalized method and can pretty much be used to decompose a thin structure meshed with triangular shell elements. One potential application is optimization of the side frame of truck chassis with regions of variable thickness [38-41].

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