On the Afferrante-Carbone theory of ultratough peeling

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Abstract

In an interesting theory of ultratough peeling of an elastic tape from a viscoelastic substrate, Afferrante and Carbone (2016) find that, in contrast to the classic elastic Kendall’s theory, there are conditions for which the load for steady state peeling could be arbitrarily large in steady state peeling, at low angles of peeling - what they call ”ultratough” peeling. It is here shown in fact that this occurs near critical speeds where the elastic energy term of Kendall’s equation is balanced by the viscoelastic dissipation. Surprisingly, this seems to lead to toughness enhancement higher than the limit value observed in a very large crack in an infinite viscoelastic body, possibly even considering a limit on the stress transmitted. Kendall’s experiments in turn had considered viscoelastic tapes (rather than substrates), and his viscoelastic findings seem to lead to a much simpler picture. The Afferrante-Carbone theory suggests the viscoelastic effect to be an on-off mechanism, since for large angles of peeling it is almost insignificant, while only below a certain threshold, this ”ultratough” peeling seems to appear. Experimental verification would be most useful.

1. Introduction

Afferrante and Carbone ([1], AC in the following) have given an interesting and very simple extension of the celebrated Kendall solution for the peeling of a tape from an elastic (or rigid) substrate (Kendall, [2]). The importance of the application is evident from pressure-sensitive adhesives to biological attachments in many insects, spiders and lizards or geckos, and we can refer to the reference list in AC or in the review of [3-4]. There is relatively extensive literature on crack propagation in viscoelastic materials
theories and extensive measurements ([5-9]) suggesting steady state subcritical crack propagation occurs with an enhanced work of adhesion $G$ obtained as the product of adiabatic value $G_0$ and a function of velocity of peeling of the crack line and temperature, namely the Gent-Schultz law

$$ \frac{G}{G_0} = 1 + \left( \frac{v}{v_0} \right)^n $$

(1)

where $v_0 = (k a_T^n)^{-1}$ and $k, n$ are (supposed to be) constants of the material, with $0 < n < 1$ and $a_T$ is the WLF factor to translate results at various temperatures $T$ (Williams, Landel & Ferry, [11]). There was originally a difficulty in dealing with viscoelastic cracks ([12-13]) which was solved by [9,10,14], with Barenblatt or Maugis-Dugdale cohesive models zones, which predict the speed dependence of the Gent-Schultz law, but with a limit being given by at very large speeds (elastic fracture)

$$ \frac{G_\infty}{G_0} = \frac{E_\infty}{E_0} $$

(2)

for a material having a relaxed modulus $E_0$ and an instantaneous modulus $E_\infty$. de Gennes [15] in particular suggested a ”viscoelastic trumpet” crack model which, when writing the LEFM (Linear Elastic Fracture Mechanics) condition on both the inner region for the stress intensity factor $K$ as $K^2 = G_0 E_{\infty}$ and in the outside region $K^2 = G_\infty E_0$, leads to a maximum apparent adhesion/toughness enhancement of (2). Notice that this can be possibly of 3 or 4 orders of magnitude ([6]).

The AC theory deals with peeling a tape as in Fig.1, where an angle $\theta$ is referred with respect to the substrate line, and leads to a quite simple exact solution. The perhaps strong assumption made to obtain such a simple solution was that the stress (both pressure and shear) is constant on a strip of dimension $L$ equal to the thickness of the tape, where they also assume the stress is constant (and simply given by equilibrium with the applied load), so they can compute the dissipation in the steady state propagation at velocity $v$. We shall discuss some interesting aspects in the ”ultratough peeling” AC theory.
2. Afferrante-Carbone theory

Considering the general case of an elastic tape of modulus $E$, and a viscoelastic substrate with a single relaxation time $\tau_0$, their eqt.14 gives

$$
\frac{G_0}{EL} = \left[ \frac{1}{2} - \frac{E}{E_0} f_v \left( \frac{v\tau_0}{L} \right) \right] \left( \frac{P}{EL} \right)^2 + \left( \frac{P}{EL} \right) (1 - \cos \theta)
$$

where $P$ is load per unit out-of-plane thickness, and $L$ is the thickness of the tape, and finally $f_v \left( \frac{v\tau_0}{L} \right)$ is a function (see eqt.15 in AC) of exponential integrals which is zero at both zero speed and infinite speed, and otherwise is positive and has a maximum which depends weakly on $E_\infty/E_0$ if $E_\infty/E_0 > 15$.

Eqt.(3) is a quadratic equation for the load which has obviously real solutions only in certain ranges for

$$
\Delta = (1 - \cos \theta)^2 + 4 \frac{G_0}{EL} \left[ \frac{1}{2} - \frac{E}{E_0} f_v \left( \frac{v\tau_0}{L} \right) \right] > 0
$$

and naturally we have to take only positive real roots

$$
\left( \frac{P}{EL} \right)_{1,2} = \frac{- (1 - \cos \theta) \pm \sqrt{\Delta}}{2 \left[ \frac{1}{2} - \frac{E}{E_0} f_v \left( \frac{v\tau_0}{L} \right) \right]} > 0 \text{ when } \Delta > 0
$$
For $v = 0$ or $v = \infty$, the solution returns to well known solution of peeling of Rivlin [16] for rigid tape or Kendall [2] for elastic tape, depending on the tape elastic modulus, but this is obvious since there is no viscoelastic dissipation in these limit cases.

The effect of viscoelasticity at intermediate speeds is contained in the effective toughness written from (3) as

\[
\frac{G}{G_0} = 1 + \frac{1}{G_0/(EL)} \frac{E}{E_0} f_v \left( \frac{v \tau_0}{L} \right) \left( \frac{P}{EL} \right)^2
\]  

(6)

where $P$ is the solution of (3) at the equilibrium condition.

AC find that some ranges of the peeling angle and material combinations exist for which "steady state peeling may occur for arbitrarily large values of the applied force", which is what they call "ultratough peeling". In fact, we find that this is not entirely true, as there is a limit load and this is not infinite, although it is much larger than for other angles of peeling. Above this limit load, we expect the system to turn unstable, and the delamination grow at much higher speeds.

We consider the case $\frac{E_\infty}{E_0} = 10$, $\frac{E}{E_0} = 1.5$ and $\frac{G_0}{EL} = 10^{-4}$ which is also a case considered in the AC paper for their illustrative purposes. We report in Fig.2a the lower of the two solutions of the load. We find that this corresponds to a toughness enhancement larger than the infinite system one (in the figure is appears as $G_{\text{max}}/G_0 \simeq 40$ but a closer look revails that in fact $G/G_0 \simeq 140$). This is certainly bigger than $\frac{E_\infty}{E_0} = 10$. Although there is no rigorous proof that the infinite crack in the infinite system has the largest possible enhancement, this sounds physically quite intuitive since a finite system has simply a smaller region which can dissipate, and is also suggested by theories about crack in finite size systems like de Gennes [15]. Naturally, there is no attempt to model a true crack singularity in the AC theory, and this should be further investigated.

There seems to be a dramatic effect of the peeling angle in Fig.2, since for $\theta > \theta_{th}$ the toughness enhancement is in fact almost insignificant at all speeds, and nothing remotely close to the order we expect for a large crack in infinite size specimen, that is $E_\infty/E_0$, and see the orders of magnitude difference in Fig.2a, so one could use the Kendall solution. The viscoelastic effect really seems to be an on-off effect depending on the angle.

Also, as AC say "Notice equilibrium at higher loads could be, in principle, established provided the system is forced to move on the upper branch of the
And these high low solutions are reported in Fig.2b where it is found that they correspond to a toughness enhancement of orders of magnitude larger than the infinite system one. These solutions look really unrealistic.

Fig.2 - Enhancement \( \frac{G}{G_0} - 1 \) for the AC peeling theory taking as function of the peeling velocity \( v \) (a) low load solution (b) large load solution. Case \( \frac{E}{E_0} = 1.5 \). Peeling angles \( \theta = \pi/64, \pi/8, \pi/4, \pi/2 \) (black, blue, red, green colors). \( \frac{G_0}{EL} = 10^{-4}, \frac{E_i}{E_0} = 20 \).

AC find the "ultratough" enhancement when \( E/E_0 \) is not too large. However, the exact threshold depends on the peeling angle. For \( E/E_0 = 1.5 \), ul-
tratough peeling will occur for $\theta < \theta_{th} = 0.14\text{rad}$, but for higher $E/E_0 > 1.5$, $\theta_{th}$ will increase and the range will be larger.

3. Limits of the solutions on physical grounds

We could disregard the very large load solutions by adding a cutoff on the force on physical grounds, as a limit to the stress which can be carried by the tape (or the substrate): as discussed by Kendall (1975) for elastomers or polymer materials can be of the order of the elastic modulus, $\sigma_c = E$, where in turn $E$ is of the order of $E_0$ and $E_\infty >> E_0$ means this condition is also an even stronger limit for the substrate: this results in

$$\frac{P}{EL} < 1 \quad (7)$$

This however will not limit much the maximum amplification. Indeed, taking $\frac{E}{E_0} f_v \left(\frac{\nu \tau_0}{L}\right) \simeq 1$, $\left(\frac{P}{EL}\right)^2 \simeq 1$, i.e. considering the limit on the cohesive stress (for the higher of the two solution in the quadratic equation)

$$\left(\frac{G}{G_0}\right)_{\text{max,1}} \simeq \frac{1}{G_0/(EL)} \quad (8)$$

(where the subscript ”1” stands for the higher of the two solutions of the quadratic equation), which can of course go to infinity even after the regularization.

4. Discussion

Even looking at the smaller of the two solutions of the quadratic equation, this large amplifications occur when: (i) the coefficient of the quadratic equation of the AC theory becomes nearly zero i.e.

$$\frac{E}{E_0} f_v \left(\frac{\nu \tau_0}{L}\right) \simeq \frac{1}{2} \quad (9)$$

and (ii) the angle of peeling is low, which makes also the coefficient of the linear term of the quadratic equation close to zero. Under these conditions,
physically (i) suggests we are considering the case where viscoelastic dissipation nearly exactly cancels the elastic energy terms of the Kendall equation, so that one finds the Rivlin equation load

\[ P = \frac{G_0}{1 - \cos \theta} \]  

(10)

and this leads to the toughness enhancement

\[ \frac{G}{G_0} = 1 + \frac{E}{E_0} f_v \left( \frac{v \tau_0}{L} \right) \frac{G_0}{EL} \left( \frac{1}{1 - \cos \theta} \right)^2 \]  

(11)

which at \( \theta = \frac{\pi}{64} \text{rad} \), \( \frac{E}{E_0} f_v \left( \frac{v \tau_0}{L} \right) \simeq \frac{1}{2} \) and \( \frac{G_0}{EL} = 10^{-4} \) is of the order of the \( \frac{G}{G_0} \simeq 140 \) we are observing in Fig.2a. Hence, (ii) suggests that the Rivlin load grows unbounded. Again, assuming the limit \( \frac{P}{EL} = 1 \), the Rivlin equation corresponds to the limit \( \frac{G_0}{EL} = (1 - \cos \theta) \) and hence the maximum toughness enhancement at the critical velocities where \( \frac{E}{E_0} f_v \left( \frac{v \tau_0}{L} \right) \simeq \frac{1}{2} \) is

\[ \left( \frac{G}{G_0} \right)_{\text{max,2}} = 1 + \frac{1}{2} \frac{1}{1 - \cos \theta} \]  

(12)

(where the subscript ”2” stands for the lower of the two solutions of the quadratic equation) and hence even this can grow still unbounded even if we limit the transmitted stress.

4.1. Kendall’s theory and experiments

Let us return to the classical elastic Kendall’s solution and experiments [2]. At large peeling angles, Kendall did not need to modify the Rivlin solution (”no extension” theory), and since the tape in Kendall’s experiment is viscoelastic, Kendall measured from the Rivlin solution the toughness as a function of speed

\[ P_{\text{Rivlin}} (v) = \frac{G (v)}{1 - \cos \theta} \]  

(13)

The case of low angles is exactly where the Kendall solution is needed, as the Rivlin solution predicts ever increasing load and in fact infinite loads at zero peel angles — so Kendall is the regularized version of the Rivlin solution, predicting instead much lower loads. Kendall made exactly experiments at low angles, a force was applied to the film and the crack speed
was determined. The angle in the experiment was therefore adjusted until the crack speed was $80\,\mu m/s$ so to correspond to an adhesive energy of $P_{\text{Rivlin}}(80\,\mu m/s) = G(80\,\mu m/s) = 5\,N/m$ as determined from peeling tests at angle $\pi/2$, i.e. equation (13). Kendall verified this equation for angles as low as $10^\circ$ where the tape was ethylene propylene rubber adhering on glass. This verified very well Kendall’s equation which therefore we can interpret already as a "viscoelastic tape" solution as

$$\frac{G(v)}{EL} = \frac{1}{2} \left( \frac{P(v)}{EL} \right)^2 + \left( \frac{P(v)}{EL} \right) (1 - \cos \theta) \tag{14}$$

where $G(v)$ is what has been measured using (13). This solution shows no critical speeds, and finds a unique (positive) solution

$$\left( \frac{P(v)}{EL} \right)_{1,2} = -(1 - \cos \theta) + \sqrt{(1 - \cos \theta)^2 + 2 \frac{G(v)}{EL}} > 0 \tag{15}$$

so at low angles we simply have

$$\left( \frac{P(v)}{EL} \right)_{\theta=0} = \sqrt{2G(v)} \frac{1}{EL} = \sqrt{2 [P(v)]_{\theta=\pi/2}} \tag{16}$$

where we used that using $G(v)$ from the peeling tests at angle $\pi/2$, i.e. equation (13) $G(v) = [P(v)]_{\theta=\pi/2}$ so nothing extraordinary seems to occur at low angles with respect to toughness enhancement. On the contrary, if toughness increases at large angles by a factor, say, of 1000, then we expect a reduced toughness enhancement at low angles because of the square root in (16). This is not exactly found in Kendall’s figures, since the same range of loads is found at $\theta = \pi/2$ and $\theta = 0$ in the experiments, so this requires some further clarification. But the AC theory predicts a much different result anyway, since the toughness enhancement at low angles is dramatically higher than at large angles, as in results like fig.2.

5. Conclusions

We have discussed that the very simple "ultratough peeling" theory of Afferrante and Carbone [1] leads to two possible equilibrium solutions for the load at a given peeling speed. The low load solution gives in some ranges very high toughness enhancement, possibly already much larger even than
that expected in a crack in an infinite system. More problematic appears the high load solution which, even considering a limit stress carried by tape and substrate, will not avoid some solutions to have unbounded toughness enhancement. The viscoelastic effect seems to induce an on-off mechanism, where for large angles of peeling it predicts solutions extremely close to the Kendall elastic one, while only below a certain angle threshold, this "ultratough" peeling regime appears. A qualitative discussion of the viscoelastic experiments of Kendall has been added, and his treatment of the viscoelastic effects (although these were relative to a viscoelastic tape, and not substrate).

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7. References

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