Fuzzy environment replacement model

M Balaganesan¹ and K Ganesan¹*
¹ Department of Mathematics, SRM Institute of Science and Technology, Kattankulathur, Chennai, Tamil Nadu, India
E-Mail: balaganesanm11@gmail.com, ganesank@srmist.edu.in

Abstract. This paper focuses on replacement of equipment in fuzzy environment. Fuzzy set theory is the main tool to emphasize the vagueness parameters which involving in many real life applications. All imprecise costs involved in this fuzzy replacement model are taken as triangular fuzzy numbers. The proposed method gives the optimal replacement time of the replacement problem in fuzzy nature and it is validated using a real life numerical example.

1. Introduction
In our real world scenario Replacement of equipment is the most important in economic decision analysis. The theory of replacement analysis is used in engineering economics to determine an optimal decision for maintenance and replacement purposes.

If the production facility is new, then machine will work with full operating efficiency. With passage of time and due to usage, if it is become old then the operating efficiency of the facility may gradually decreases. To regain it, maintenance should be needed. When the first maintenance is attended, its performance will reduce slightly and in the second maintenance, it will reduce more than the previous one. Like this the facility deteriorate, and at the end the efficiency of the machine reduces to some desired level. Therefore, it won’t be economical because the maintenance cost will be so high at the same time a production cost per unit also increases. Therefore, the replacement of the facility is due at this stage.

In our real life situation due to uncertainty the decision makers deal with Zadeh’s [1] fuzzy set theory, it will be more suitable one to determine the optimal replacement time of given equipment in a tolerable range instead of a crisp number. The authors such as Bellman [2], Dreyfus [3], Mahdavi and Mahdavi [4], and Zhao et.al [5] were discussed different kinds of replacement models in classical nature.

Afterwards Chang [6] classified fuzzy strategic replacement. Balaganesan and Ganesan [7, 8] analyzed fuzzy and intuitionistic fuzzy economic life of equipment with change in money value. Al-Najjar and Alsyouf [9] discussed fuzzy approach in multiple criteria decision making maintenance model.

Biswas and Pramanik [10, 11], El-Kholy and Abdelalim [12] used fuzzy ranking methods to determine economic life of equipment in crisp nature with does not change in money value and change in money value. Esogbue and Hearnes [13] discussed replacement models via fuzzy set theoretic framework. In general, the authors have discussed the fuzzy replacement models by converting them into equivalent classical models. But this paper gives the replacement time of equipment under fuzzy nature. Here we did not convert it in to its classical version.
The rest of our work is organized as follows. Fuzzy set and its Basics, arithmetic operations of triangular fuzzy numbers are discussed in section 2. Algorithm for the proposed model is discussed in section 3. A real life example is dedicated to understand the replacement model in section 4. Conclusion part in Section 5.

2. Preliminaries

Definition 2.1.
A fuzzy set $\tilde{A}$ defined on the set of real numbers $\mathbb{R}$ is said to be a fuzzy number, if its membership function $\tilde{A}: \mathbb{R} \rightarrow [0,1]$ has the following characteristics:

(i) $\tilde{A}$ is convex, (i.e.) $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, \lambda \in [0,1]$, for all $x_1, x_2 \in \mathbb{R}$

(ii) $\tilde{A}$ is normal, (i.e.) there exists an $x \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x) = 1$

(iii) $\tilde{A}$ is piecewise continuous.

Definition 2.2.
A fuzzy number $\tilde{A}$ on $\mathbb{R}$ is a triangular fuzzy number if its membership function $\tilde{A}: \mathbb{R} \rightarrow [0,1]$ has the following characteristics:

\[ \mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \]

We denote this triangular fuzzy number as $\tilde{A} = (a_1, a_2, a_3)$. We use $F(\mathbb{R})$ to denote the set of all triangular fuzzy numbers defined on $\mathbb{R}$.

Definition 2.3.
A triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3) \in F(\mathbb{R})$ can also be represented as a pair $\tilde{A} = (\underline{a}, \bar{a})$ of functions $\underline{a}(\gamma), \bar{a}(\gamma)$, for $0 \leq \gamma \leq 1$ which satisfies the following requirements:

(i) $\underline{a}(\gamma)$ is a bounded monotonic increasing left continuous function.

(ii) $\bar{a}(\gamma)$ is a bounded monotonic decreasing left continuous function.

(iii) $\underline{a}(\gamma) \leq \bar{a}(\gamma)$, $0 \leq \gamma \leq 1$.

It is also represented by $\tilde{A} = (a_0, a_*, a^*)$ where $a_* = (a_0 - \underline{a}), a^* = (\bar{a} - a_0)$ are called the left fuzziness index function and the right fuzziness index function respectively. For an arbitrary triangular fuzzy number $\tilde{A} = (\underline{a}, \bar{a})$ the number $a_0 = \left( \frac{\underline{a}(1) + \bar{a}(1)}{2} \right)$ is said to be a location index number of $\tilde{A}$.
2.1. Arithmetic Operations on Triangular Fuzzy Numbers
For any two triangular fuzzy number $\tilde{A} = (a_0, a_*, a^*)$, $\tilde{B} = (b_0, b_*, b^*)$, the arithmetic operations on the fuzzy numbers are defined by

(i) Addition: $\tilde{A} + \tilde{B} = (a_0 + b_0, a_* + b_*, a^* + b^*)$

(ii) Subtraction: $\tilde{A} - \tilde{B} = (a_0 - b_0, a_* - b_*, a^* - b^*)$

(iii) Multiplication: $\tilde{A} \times \tilde{B} = (a_0 \times b_0, a_* \times b_*, a^* \times b^*)$

(iv) Division: $\tilde{A} \div \tilde{B} = (a_0 \div b_0, a_* \div b_*, a^* \div b^*)$

We used the triangular fuzzy ranking concept discussed by Krishnaveni and Ganesan [9]

3. Triangular fuzzy replacement model
The aim of this paper is to determine the optimum replacement year of an item whose maintenance (running) cost increases with time, the money value remains static during that period and the time is a discrete variable.

Here, $\bar{R}_n$ – Fuzzy Running cost for $n$ hours

$\tilde{C}$ – Fuzzy capital cost of the item

$\tilde{S}_n$ – Fuzzy scrap value of the item in $n$ time duration

Annual cost for $n$ hours = $\bar{R}_n + \tilde{C} - \tilde{S}_n$

The total cost incurred on the item is $\tilde{P}_n = \sum_{i=1}^{n} \bar{R}_i + \tilde{C} - \tilde{S}_n$

The average total cost is $\tilde{W}(n) = \frac{\tilde{P}(n)}{n} = \frac{\sum_{i=1}^{n} \bar{R}_i}{n} + \frac{\tilde{C}}{n} - \frac{\tilde{S}_n}{n}$

$\tilde{W}(n)$ is minimum if $\Delta \tilde{W}(n - 1) < 0 < \Delta \tilde{W}(n)$ is satisfied.

$$\Delta \tilde{W}(n) = \tilde{W}(n + 1) - \tilde{W}(n)$$

$$= \left[ \frac{\sum_{i=1}^{n+1} \bar{R}_i}{n + 1} + \frac{\tilde{C} - \tilde{S}_n}{n + 1} \right] - \left[ \frac{\sum_{i=1}^{n} \bar{R}_i}{n} + \frac{\tilde{C} - \tilde{S}_n}{n} \right]$$

$$= \frac{\bar{R}_{n+1}}{n + 1} \left[ \frac{\sum_{i=1}^{n+1} \bar{R}_i - \sum_{i=1}^{n} \bar{R}_i}{n(n + 1)} + \frac{\tilde{C} - [(n + 1)\tilde{S}_n - n\tilde{S}_{n+1}]}{n(n + 1)} \right]$$

For minimum $\tilde{W}(n)$, $\Delta \tilde{W}(n - 1) < 0 < \Delta \tilde{W}(n)$

$$\Rightarrow \frac{\bar{R}_{n+1}}{n + 1} > \frac{\sum_{i=1}^{n} \bar{R}_i - [(n + 1)\tilde{S}_n - n\tilde{S}_{n+1}]}{n(n + 1)}$$
Thus, \( \tilde{R}_{n+1} > \frac{P}{n} > \tilde{R}_n \) is used to determine the optimum replacement period.

Therefore the replacement policy is,

(i) Do not replace the equipment if the next period’s cost is less than the weighted average of previous costs.

(ii) Replace the equipment if the next period’s cost is greater than the weighted average of previous costs.

3.1 Algorithm for Fuzzy maintenance model

The algorithm to determine the fuzzy replacement time period (See \( \tilde{R} \)). Here the parameters are triangular fuzzy numbers.

4. Example

Consider the problem discussed by Biswas et.al. [10] with fuzzy cost and the resale value of the trucks in rupees are \( \tilde{C} = \langle 8100000, 8150000, 8200000 \rangle \), \( \tilde{S} = \langle 202000, 204000, 206000 \rangle \) and the running fuzzy costs per hundred hours are given in Table 1 (Take Rs. 10000 = 1 unit and 100 hours = 1 unit). Determine the time period which is profitable to replace the truck with a new one?

| Time(n) | \( \tilde{R}_n \) | Parametric form of \( \tilde{R}_n \) |
|---------|-----------------|-----------------------------------|
| 1       | \( < 13,14,14.5 > \) | \( < 14, 1-\gamma, 0.5-0.5\gamma > \) |
| 2       | \( < 25,25.5,26 > \) | \( < 25.5, 0.5-0.5\gamma, 0.5-0.5\gamma > \) |
| 3       | \( < 30,31.5,32 > \) | \( < 31.5, 1.5-1.5\gamma, 0.5-0.5\gamma > \) |
| 4       | \( < 37.5,38.5,39 > \) | \( < 38.5, 1-\gamma, 0.5-0.5\gamma > \) |
| 5       | \( < 45,46.5,48 > \) | \( < 46.5, 1.5-1.5\gamma, 1.5-1.5\gamma > \) |
| 6       | \( < 60,61.5,63 > \) | \( < 61.5, 1.5-1.5\gamma, 1.5-1.5\gamma > \) |
| 7       | \( < 90,92.5,95 > \) | \( < 92.5, 2.5-2.5\gamma, 2.5-2.5\gamma > \) |
| 8       | \( < 110,125,140 > \) | \( < 125, 15-15\gamma, 15-15\gamma > \) |
| 9       | \( < 170,185,195 > \) | \( < 185, 15-15\gamma, 10-10\gamma > \) |
| 10      | \( < 220,230,237.5 > \) | \( < 230, 10-10\gamma, 7.5-7.5\gamma > \) |
### Table 2. Table determining the fuzzy optimal replacement time.

| Time (n) | $\tilde{\eta}_n$ | $\tilde{\xi}_n$ | $\sum \tilde{\eta}_n$ | $C_i - \tilde{\xi}_n + \sum \tilde{\eta}_n$ | $W(n) = \frac{\tilde{y}(n)}{n}$ | $R[\tilde{W}(n)]$ |
|----------|------------------|-----------------|------------------------|---------------------------------|--------------------------------|-----------------|
| 1        |                  |                 |                        |                                 |                                | 808.6           |
|          | $< 14, 1.0, 0.5, 0.5 >$ |                 |                        |                                 |                                |                 |
|          | $< 4, 0.2, 0.2, 0.2 >$ |                 |                        |                                 |                                |                 |
|          |                  |                 |                        |                                 |                                |                 |
| 2        |                  |                 |                        |                                 |                                | 417.05          |
|          | $< 25.5, 0.5, 0.5, 0.5 >$ |                 |                        |                                 |                                |                 |
|          | $< 20.4, 0.2, 0.2 >$ |                 |                        |                                 |                                |                 |
|          |                  |                 |                        |                                 |                                |                 |
| 3        |                  |                 |                        |                                 |                                | 288.53          |
|          | $< 31.5, 1.5, 1.5, 0.5 >$ |                 |                        |                                 |                                |                 |
|          | $< 20.4, 0.2 >$ |                 |                        |                                 |                                |                 |
|   |   |   |   |   |
|---|---|---|---|---|
| 4 | <38.5, 1.5γ, 0.5-0.5γ> | <20.4, 0.2-0.2γ> | <109.5, 1.5-1.5γ> | 226.03 |
| 5 | <46.5, 1.5-1.5γ, 1.5-1.5γ> | <204.0, 0.2-0.2γ> | <156, 1.5-1.5γ> | 190.12 |
| 6 | <61.5, 1.5-1.5γ, 1.5-1.5γ> | <204.0, 0.2-0.2γ> | <217.5, 1.5-1.5γ> | 168.68 |
| 7 | 157.8 |
| --- | --- |
| 92.5, 2.5-2.5γ | 20.4, 0.2-0.2γ |
| 310.2, 2.5-2.5γ | 1104.6, 0.2-0.2γ |

| 8 | 153.7 |
| --- | --- |
| 125, 15-15γ, 15-15γ | 20.4, 0.2-0.2γ |
| 435, 15-15γ, 15-15γ | 1229.6, 0.2-0.2γ |

| 9 | 157.18 |
| --- | --- |
| 185, 15-15γ, 10-10γ | 20.4, 0.2-0.2γ |
| 620, 15-15γ, 15-15γ | 1414.6, 0.2-0.2γ |
From the above Table 2, we conclude that the average cost per hundred hours is minimum at the end of 800 hours and the total average cost is Rs. $< 153.5+0.2\gamma, 153.7, 153.9-0.2\gamma >$ (Rs.10000 = 1 unit). Therefore the owner should expect to replace the truck to a new one at the end of 800 hours. Using our proposed method, the truck owners having the following advantage to choose their $\gamma \in [0,1]$.

Table 3. Decision maker’s choice, $\gamma \in [0,1]$.

| Values of $\gamma$ | Total average cost (Rs.10000 = 1unit) | Values of $\gamma$ | Total average cost (Rs.10000 = 1unit) |
|-------------------|--------------------------------------|-------------------|--------------------------------------|
| $\gamma = 0$      | $< 153.5, 153.7, 153.9 >$           | $\gamma = 0.55$   | $< 153.61, 153.7, 153.79 >$          |
| $\gamma = 0.1$    | $< 153.52, 153.7, 153.88 >$         | $\gamma = 0.6$    | $< 153.62, 153.7, 153.78 >$          |
| $\gamma = 0.2$    | $< 153.53, 153.7, 153.87 >$         | $\gamma = 0.65$   | $< 153.63, 153.7, 153.77 >$          |
| $\gamma = 0.25$   | $< 153.55, 153.7, 153.85 >$         | $\gamma = 0.7$    | $< 153.64, 153.7, 153.76 >$          |
| $\gamma = 0.3$    | $< 153.56, 153.7, 153.84 >$         | $\gamma = 0.75$   | $< 153.65, 153.7, 153.75 >$          |
| $\gamma = 0.35$   | $< 153.57, 153.7, 153.83 >$         | $\gamma = 0.8$    | $< 153.66, 153.7, 153.74 >$          |
| $\gamma = 0.4$    | $< 153.58, 153.7, 153.82 >$         | $\gamma = 0.85$   | $< 153.67, 153.7, 153.73 >$          |
| $\gamma = 0.45$   | $< 153.59, 153.7, 153.81 >$         | $\gamma = 0.9$    | $< 153.68, 153.7, 153.72 >$          |
| $\gamma = 0.5$    | $< 153.6, 153.7, 153.8 >$           | $\gamma = 1$     | $< 153.70, 153.7, 153.70 >$          |

4.
In order to maximize the profit as well as minimize the expenses in Table 3, many industrial firms and military organizations are constantly concerned with the problem of determining the optimal replacement of durable or obsolete equipment. This paper provides a mathematical technique for explicitly incorporate this model under imprecise nature without converting to its crisp version. The proposed method is very easy to understand and very simple to calculate the optimum replacement time of the equipment. Biswas and Pramanik [10, 14] discussed the problem in fuzzy nature but their results in crisp nature. In real life problems Crisp value never be the solution for an imprecise situation, so that a numerical example is solved to show that the vagueness in the solution obtained by our proposed method without disturbing the imprecise nature. This work may be extended to triangular intuitionistic fuzzy replacement model with money value change and does not change with time.

5. References

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