QUANTUM GEOMETRODYNAMICS
IN EXTENDED PHASE SPACE – I.

PHYSICAL PROBLEMS OF INTERPRETATION
AND MATHEMATICAL PROBLEMS OF GAUGE INVARIANCE

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Abstract

The paper is the first of two parts of the work devoted to the investigation of constructing quantum theory of a closed universe as a system without asymptotic states. In Part I the role of asymptotic states in quantum theory of gravity is discussed, that enables us to argue that mathematically correct quantum geometrodynamics of a closed universe has to be gauge-noninvariant. It is shown that a gauge-noninvariant quantum geometrodynamics is consistent with the Copenhagen interpretation. The proposed version of the theory is thought of as describing the integrated system “the physical object + observation means”. It is also demonstrated that introducing the observer into the theory causes the appearance of time in it.

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1. Introduction

The purpose of this work is to explore a possibility of constructing physically (operationally) interpreted quantum geometrodynamics (QGD) of a closed universe by a strict mathematical method without using any assumption not permitting detailed mathematical proofs.

The Wheeler – DeWitt QGD is the extrapolation of conceptions and methods of modern quantum field theory. The validity of them on the scale of the Universe as a whole may arouse doubts. In our paper we attract a special attention to the fact that a closed universe is a system without asymptotic states, in contrast to those normally considered by quantum field theory. In our opinion, taking into account this fact may lead to a theory significantly distinguish from the Wheeler – DeWitt QGD. Namely, we have come to the conclusion that mathematically correct and physically well-grounded QGD of a closed universe is a gauge-noninvariant theory.

The grounds for this conclusion are discussed in the presented below first of the two papers. We show that any gauge-invariant quantum field theory is essentially based on the assumption about asymptotic states. Indeed, to construct a gauge-invariant theory one needs 1) to pass to a description of the system under consideration in terms of “true” physical degrees of freedom, or 2) to impose some selection rules singling out physical state vectors. The first way implies that constraints have to be resolved. For gravitational constraints the latter can be done in the limits of perturbation theory only in asymptotically flat spaces, or in some special cases. As for the second way, in the path integral approach which is adopted in the present investigation as more adequate, selection rules for physical states are equivalent to asymptotic boundary conditions in a path integral, so if the path integral is considered without asymptotic boundary conditions (as it should be in a correct quantum theory of a closed universe) the set of all possible transition amplitudes determined through the path integral inevitably involves gauge-noninvariant ones.

On the basis of the Copenhagen operational interpretation of quantum theory (QT), we establish the discrepancy of the mathematical structure of the gauge-invariant theory to the conditions of observations in...

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a closed universe. In contrast, from the viewpoint of the Copenhagen interpretation, it should be expected that a wave function of the Universe must carry information on geometry of the Universe as well as on a reference system (RS) in which this geometry is studied and which represents the observer in the theory of gravity. All this give rise to the idea of a gauge-noninvariant quantum geometrodynamics of a closed universe. The present work should be thought of as an attempt to give a self-consistent description of a physical object (the Universe) and observation means (a reference system).

The wave function that carries information on geometry of the Universe and on the observer is supposed to satisfy a Schrödinger equation. The path integral approach contains the procedure of derivation of the Schrödinger equation from the path integral in a Lagrangian form and does not require to construct a Hamiltonian form of the theory. For gravity appropriate calculations can be made in the class of differential gauge conditions introducing missing velocities into the Lagrangian. All the calculations are demonstrated explicitly in the second part of our paper taking the Bianchi IX cosmological model as an example.

To approximate the path integral when deriving the Schrödinger equation we use the “extended” set of Lagrangian equations including those for ghosts and a gauge condition. The set of equations can be obtained by varying the Batalin – Vilkovisky (BV) effective action (which reduces to the Faddeev – Popov effective action in the present case). Let us note that at this point the Batalin – Vilkovisky (Lagrangian) formalism turns out not to be equivalent to the Batalin – Fradkin – Vilkovisky (Hamiltonian) one in the sense that the set of Hamiltonian equations in extended phase space (EPS) obtained from the Batalin – Fradkin – Vilkovisky (BFV) action is not equivalent to the extended Lagrangian set of equations. The reason is a different structure of ghost sectors, which, in its turn, results from the fact that the gauge group of gravity does not coincide with the group of canonical transformations generated by gravitational constraints. In the case of a system with asymptotic states the ghost sector plays just an auxiliary role and it is possible to prove the equivalence of the Lagrangian and Hamiltonian schemes in a gauge-invariant sector. However, in the case of a closed universe we are not able, in principle, to single out a gauge-invariant sector by means of asymptotic boundary conditions, and it is clear from general consideration that making use of non-equivalent sets of equations leads to different equations for the wave function of the Universe.

In this situation we give preference to the Lagrangian formulation of the theory. At the same time in the class of gauges mentioned above a Hamiltonian formulation equivalent to the Lagrangian one can be constructed. BRST-transformations in EPS then correspond to gauge transformations of the original theory rather than those generated by the constraints.

In our formulation of Hamiltonian dynamics in EPS the passage to a gauge-invariant theory can be made in a usual manner: all gauge-noninvariant terms in the set of equations in EPS can be excluded by means of asymptotic boundary conditions. One then will come to gauge-invariant Hamiltonian equations complemented by Dirac’s primary and secondary constraints. Thus, from the viewpoint which we advocate here, the Dirac formulation of constrained dynamics seems to correspond to a particular situation when a gauge-invariant sector can be singled out, and so does the BFV approach which inherits many features of Dirac’s scheme. The requirement of BRST-invariance of a state vector leads to the Wheeler – DeWitt equation in the BFV approach but not in the approach to constructing Hamiltonian dynamics in EPS presented here.

In our approach the wave function satisfying the Wheeler – DeWitt equation is a particular solution to the Schrödinger equation answering a special choice of a gauge. Furthermore, bearing in mind that the choice of parametrization of gauge variables and the choice of gauge conditions together fix a reference system, we come to the conclusion that the well-known parametrization noninvariance of the Wheeler – DeWitt equation is in essence an ill-hidden gauge noninvariance. So the Wheeler – DeWitt QGD cannot be thought of as a gauge-invariant theory in a strict sense.

Our papers are organized as following.

In Sec. 2 we briefly remember the paradigm on which the Wheeler – DeWitt QGD is based, and in Sec. 3 we formulate the algorithm of our research. In Sec. 4 the role of asymptotic states in quantum theory of gravity is discussed that leads us to the conclusion that quantum theory of a closed universe, in general, may be not gauge-invariant. In Sec. 5 we show that a gauge-noninvariant QGD is compatible with the

1Here we ignore the related problem of Gribov’s copies, which, in our opinion, needs a special consideration in this context.
Copenhagen interpretation of QT. The interpretation of a reference system fixing the whole information on the evolution of the Universe is given in Sec. 6. The choice of an appropriate formalism for constructing mathematically correct QGD of a closed universe is motivated in Sec. 7. Concluding the first part of the work the gauge-noninvariant extended set of Lagrangian equations is considered, and it is shown that time-ordering in quantum dynamics is a consequence of introducing the observer into QGD equations.

The detailed mathematical proofs of the possibility to realize the proposed approach are given in the second part of the work. All the mathematical operations concerned with deducing a gauge-noninvariant Schrödinger equation and constructing its general solution are carried out in a manifest form for the Bianchi-IX model.

The Bianchi IX cosmological model has been chosen for its mathematical simplicity and physical meaningfulness. It is traditionally used as a test polygon for various theoretical methods in cosmology (see, for example, [7]). The finite number of degrees of freedom enables us to control correctness of used methods under the “pure” conditions without mathematical problems connected to divergences typical for a general quantum field theory with infinite number of degrees of freedom.

In our opinion the approach to QGD of a closed universe proposed by us phenomenologically demonstrates the existence of the problem of searching new fundamental physical principles describing the process of forming the integrated system “a physical object + observation means” and controlling measuring processes in this system and mechanisms of registration of measurement results on physical carriers of measuring devices. Some aspects of this problem are discussed in Conclusions.

2. The many-worlds interpretation of quantum geometrodynamics

The standard QGD is based on the Wheeler – DeWitt equations

\[ \mathcal{T}^\mu |\Psi \rangle = 0, \]

where the operators

\[ \mathcal{T}^0 = \left( -\gamma(3) \right)^{-\frac{1}{2}} p^{ik} \left( \gamma_{il} \gamma_{km} - \frac{1}{2} \gamma_{ik} \gamma_{lm} \right) p^{lm} + \left( -\gamma(3) \right)^{\frac{1}{2}} R(3) + T_{00}^{(\text{mat})}, \]

\[ \mathcal{T}^i = -2 \left( \partial_k p^{ik} + \gamma^i_{lm} p^{lm} \right) + T_{0i}^{(\text{mat})}, \]

\[ p^{ik} \] are the momenta conjugate to the 3-metric \( \gamma_{ik} \), \( \gamma^i_{kl} \) are the three-connections, \( \gamma(3) \equiv \det \| \gamma_{ik} \| \), \( R(3) \) is the 3-curvature, \( T_{\mu\nu}^{(\text{mat})} \) is the energy-momentum tensor of the material fields. Derivation of these equations by quantum-theoretical methods has been discussed by many authors. Because of a number of reasons considered below and analyzed in details for the Bianchi-IX model in Part II of our work, the Wheeler – DeWitt equations are not deducible by correct mathematical methods in the framework of the ordinary quantum theory. In principle, this fact itself is not sufficient to discard the Wheeler – DeWitt theory. The ordinary quantum theory is a phenomenological theory for describing quasilocal (in a macroscopic sense) phenomena. Therefore its extrapolation to the scales of the Universe as a whole is a radical physical hypothesis that may be incompatible correctly with the existing formalism. In this situation it makes sense to analyze the Wheeler – DeWitt theory as it is, without fixing attention on whether a correct way of its construction exists or not.

The most distinctive feature of the Wheeler – DeWitt theory is that there is no quantum evolution of state vector in time. Once adopting the Wheeler – DeWitt theory, one should admit that a wave function satisfying Eqs. (1) describes the past of the Universe as well as its future with all observers being inside the Universe in different stages of its evolution, and all observations to be made by these observers. We should emphasize that the question about the status of an observer in the Wheeler – DeWitt theory is rather specific since there is no vestige of an observer in Eqs. (1). The introduction of the observer into the theory is performed by fixing boundary conditions for a wave function of the Universe, we shall return to them below.
First of all, one should pay attention to another one feature of the Wheeler – DeWitt theory: because of
the status of the wave function of the Universe mentioned above this theory does not use the postulate about
the reduction of a wave packet. The logical coordination of concepts in the Wheeler – DeWitt theory is carried
out within the framework of the many-worlds interpretation of the wave function proposed by Everett \cite{8}
and applied to QGD by Wheeler \cite{9}. The wave function satisfying Eqs. (1) and certain boundary conditions
is thought to be a branch of a many-worlds wave function that corresponds to a certain universe; other
branches being selected by other boundary conditions. Thus, the boundary conditions for the wave function
of the Universe acquire a fundamental meaning for the theory: they fix all actions of an observer through the
whole history of the Universe, i.e. they contain the concentrated information about the continuous reduction
of the wave function in the process of evolution of the Universe including certain observers inside \cite{8, 9}.

Let us discuss the peculiarities of statement of the problems in the Wheeler – DeWitt QGD using the
Bianchi-IX model as an example, space homogeneity of the latter reducing the set of the Wheeler – DeWitt
equations to the only equation

\[ H_{ph} |\Psi\rangle = 0. \] (3)

Gauge invariance is expressed by that the choice of a time coordinate is not made when deriving (more
precisely, when writing down) the Wheeler – DeWitt equation. The equation should be solved under some
boundary conditions. However, carrying out this program one should bear in mind that solutions to this equa-
tion are unnormalizable. The latter is obvious from the following mathematical observations: the Wheeler
– DeWitt equation coincides formally with an equation for the eigenfunction of the physical Hamiltonian
\( H_{ph} \), appropriate to the zero eigenvalue. Meanwhile, nothing prevents us from studying
the whole spectrum of eigenvalues of the operator \( H_{ph} \); then the wave function satisfying Eq. (3) turns out to be normaliz-
able only if the value \( E = 0 \) belongs to a discrete spectrum of the operator \( H_{ph} \).

As for the operator \( H_{ph} \), explicit form of which will be presented in Part II of the paper, it has a
continuous spectrum. In this situation one faces the alternative: 1) to declare the Bianchi-IX model to be
meaningless and to put the question about searching for such models, whose operator \( H_{ph} \) has a discrete
spectrum line at \( E = 0 \), or 2) to refuse to normalize the wave function of the Universe enlarging more,
by that, the discrepancy between QGD and ordinary quantum theory. In the Wheeler – DeWitt QGD the
second way is chosen that does not contradict in principle to the statement about the status of the wave
function of the Universe mentioned above \cite{8, 9}.

What should the Wheeler – DeWitt theory be taken for by an individual local observer? Obviously – for
a paradigm fixing a certain way of thinking that in principle cannot be verified or overthrown experimentally.
The reference to the fact that in the classical limit of the Wheeler – DeWitt theory one can obtain the Einstein
equations, conclusions from which can be compared with cosmological observations, is not an argument, since
it is obvious in advance that there exist an infinite number of ways to make a quantum generalization of the
classical theory of gravity based on mathematically correct procedures but on adopting another paradigm.

The Wheeler – DeWitt paradigm may contain a deep sense that is inaccessible for understanding yet.
However, it is clear that its existence does not deprive another approach to QGD problems of a sense; for
instance, an approach based on adopting another interpretative paradigm or an approach based on procedures
claiming in a greater measure for mathematical strictness then those used in the Wheeler – DeWitt theory.

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2 Strictly speaking, unnormalizability of the wave function of the Universe means that there is no probabilistic interpretation
of it, which is quite natural, because the results of the continuous reduction of a wave packet are involved in the boundary
conditions but not in the structure of a superposition. Nevertheless, attempts were made by many authors to retrieve
a probabilistic interpretation of QGD (see, for example, \cite{10}) analyzing its physical and mathematical contents by methods of
the modern gauge field theory. In such an approach mathematical correctness of the analysis, thorough study of the question,
whether the used mathematical procedures do exist, take on fundamental significance. Exactly that very problem will be
discussed in details in our papers. The conclusions of our investigation are: mathematically correct procedures exist at least
for systems with finite number of degrees of freedom, however, such procedures lead to a theory radically distinguished from
the Wheeler – DeWitt QGD by its mathematical form as well as by its physical content.
3. Our research algorithm

In our work the following research algorithm is realized.

1. A transition amplitude between any two states of the Universe expressed through a path integral is adopted as a basic object of QGD (that predetermines the probabilistic interpretation of the theory).

2. We take notice of the circumstance that a closed universe has no asymptotic states. Imposing asymptotic boundary conditions in the path integral is not correct in this case. In this situation we do not see any foundations to for a statement of gauge invariance of the theory.

3. Instead we state the problem of constructing a wave function of the Universe containing information about a physical object as well as about a RS (fixed by a gauge) in which the object is studied. The explicit solution of the problem has been obtained for the Bianchi-IX cosmological model: this model enables one to investigate the structure of the general solution to the gauge-noninvariant Schrödinger equation.

4. We pay attention to the fact that such a wave function of the Universe corresponding to the observation conditions in a closed universe answers the Copenhagen interpretation of QT.

5. We take into account the notion about a RS, formulated by Landau and Lifshitz [11].

6. A parameter used for fixing information about observation means (a reference system) in the wave function of the Universe is proved to have a mathematical status of an eigenvalue of the gravitational Hamiltonian.

7. The question of existence of the Wheeler – DeWitt equation is analyzed from the two positions: a) we find a particular solution to the dynamical Schrödinger equation that corresponds to the zero eigenvalue of the gravitational Hamiltonian and to a factored EPS; the way of deriving the Wheeler – DeWitt equation in this case shows, however, that the parametrization noninvariance of the Wheeler – DeWitt equation [12] is an ill-hidden gauge noninvariance since any change of parametrization is equivalent to a new gauge; b) we consider the derivation of the Wheeler – DeWitt equation as a consequence of the requirement for a state vector to be BRST-invariant. In the BFV approach the requirement of BRST-invariance leads to the Wheeler – DeWitt equation, but we argue that the assumption about asymptotic states is implied in the BFV scheme.

8. For the simplified Bianchi-IX model with one of the gravitation degrees of freedom being frozen out the exact solutions of gauge-noninvariant conditionally-classical equations as well as the Schrödinger equation are found. These solutions demonstrate explicitly the status of an eigenvalue of the gravitational Hamiltonian as a governing parameter that regulates properties of a cosmological solution.

9. Within the framework of the worked out QGD version we propose the hypothesis about the Universe creation from “Nothing” as a reduction of a singular state wave function.

4. The assumption about asymptotic states in quantum theory of gravity

The problem of constructing quantum theory of gravity can be formulated after adopting some postulates fixing its physical contents. As a rule such postulates are 1) the identification of gravitational field with a system possessing two field degrees of freedom; 2) gauge invariance of observable quantities. In this Section we shall argue that to construct a theory based on these two postulates one should appeal to an additional assumption, namely, the assumption about asymptotic states. So, there is no ground to make doubt about
the legitimacy of the adopted physical postulates in the graviton S-matrix theory. However, in a general case without asymptotic states, which includes quantum geometrodynamics of a closed universe, a more general approach has to be considered that may lead to gauge-noninvariant quantum theory of gravity.

In the gauge-invariant quantum theory of gravitational field with two degrees of freedom the correspondence principle in the most rigorous form is adopted: the classical pre-modes of quantum equations of motion and selection rules of physical state vectors are equations of motion and constraints in the canonical dynamics of systems with constraints. For a classical gravitational field the various representations of this dynamics, which are equivalent up to canonical transformations, are offered by Dirac [13], Arnowitt, Deser, Misner [14], Faddeev [15]. However, it is impossible to construct an appropriate quantum theory using canonical quantization formalism because of the degeneracy of operator equations, since in the theory of gravity there are no canonical or any other local gauges completely removing the degeneracy of the Einstein equations under the diffeomorphism group transformations. The Feynman formalism of path integration is more adequate: it allows one to control the procedure of selection of gauge orbit representatives and contains a method of residual degeneracy compensation.

The full set of the constraint equations $T_\mu = 0$ and local gauge conditions $f^\mu = 0$ are explicitly solvable only within the limits of perturbation theory in asymptotically flat spaces, where the effect of asymptotic dynamical splitting off the three-dimensionally transversal gravitational waves from the so-called “nonphysical” degrees of freedom takes place. In this only case the constraint equations enable one to express explicitly gravitational variables in terms of the “true” gravitational degrees of freedom. The procedure of solving the constraints can be reproduced exactly in the framework of the path integral approach (Faddeev and Popov, [15, 16]). It results in a path integral that can be skeletonized on the extremals of the action of a system with two field degrees of freedom. In a general case (without dynamic separation of three-dimensionally transversal modes), which QGD of a closed universe belongs to, the similar operations are mathematically impracticable.

In a general case one should use the ghost field technique to compensate residual degrees of freedom not fixed by a local gauge condition. The more powerful approach was suggested by Batalin, Fradkin and Vilkovisky (BFV) [3, 4, 5, 6], see also [18]. The structure of an effective action is uniquely determined by algebra of gauge transformation, the latter, in its turn, depending on a chosen parametrization.

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In the canonical formalism the algebra of the transformations generated by the constraints (4) is open, since one of the structure functions in the relations
is known to be closed, so that the effective action constructed in accordance with the Batalin – Vilkovisky method \[2\] will be reduced to the Faddeev – Popov form,

\[
S_{FP} = \int \left( -\frac{1}{2\kappa} \sqrt{-g} g^{\mu\nu} R_{\mu\nu} + \lambda_{\mu} f^\mu + \overline{\theta}_\nu \hat{M}_\nu^{\mu} \theta^\mu \right) \, d^4 x, \tag{7}
\]

where \( \hat{M}_\nu^{\mu} \) is the Faddeev – Popov operator of the equation for residual transformation parameters

\[
\hat{M}_\nu^{\mu} \xi^\mu = 0; \tag{8}
\]

\[
\hat{M}_\nu^{\mu} = \frac{\delta f^\nu}{\delta h_{\rho\sigma}} (-\partial_{\rho} h_{\sigma\lambda} + \delta_{\rho}^\sigma h_{\lambda\lambda} + \delta_{\rho}^\sigma h_{\lambda\lambda} \partial_{\lambda}). \tag{9}
\]

We assume that a path integral is determined as a mathematical object if the way of its evaluation, i.e. its skeletonization, is given. To skeletonize the path integral one should use a full set of extremal equations obtained by varying an appropriate effective action including equations for ghosts and gauge conditions. Two sets of equations obtained by varying the actions \[4\] and \[6\] are not equivalent, and, in general, extremal equations corresponding to various parametrizations will not be equivalent since ghost sectors will be different. One can talk only about the equivalence of physical (gauge-invariant) sectors. To separate a physical sector one should consider particular solutions to extremal equations (trivial solutions for ghosts and Lagrange multipliers of a gauge-fixing term). These solutions, as a rule, are singled out by asymptotic boundary conditions. The same boundary conditions ensure BRST-invariance of the path integral and thus play the role of selection rules for physical states \[18\].

The full system of extremal equations is gauge-noninvariant; therefore, the set of all transition amplitudes determined through the path integral with the appropriate effective action will necessarily involve gauge-noninvariant amplitudes. To select gauge-invariant amplitudes one needs

1. to use the mentioned above particular solutions to the extremal equations corresponding to trivial solutions for ghosts and Lagrange multipliers;

2. to eliminate from the solutions of extremal equations gauge-dependent coordinate effects existing in case of any local gauge condition and described by the functions \[5\].

The problem is that the coordinate effects can be eliminated just locally (in the vicinity of space-time point), therefore the second operation mentioned above can be really performed only within the limits of perturbation theory in asymptotically flat spaces. In the other version of perturbation theory gauge-noninvariant extremal equations are used with the particular choice of asymptotic states (the vacuum of ghosts, 3-scalar and 3-vector gravitons). In this case a gauge-invariant physical vector is factored out from the state vector of the system, thus ensuring gauge invariance of S-matrix. Hence, in asymptotically flat spaces two gauge-invariant versions of the theory based on the path integral with BRST-invariant effective action and the path integral evaluated on the extremals of the action of a system with two field degrees of freedom are equivalent by computational capabilities as well as by physical results obtained within the limits of perturbation theory. These two versions are therefore completely consistent.

In a general case (when transitions between nonasymptotic states are under consideration) the procedures mentioned above cannot be carried out. Imposing the asymptotic boundary conditions is not correct in this case. When there are no asymptotic states we have no foundation for a statement about gauge invariance of the theory. Strictly speaking, we even cannot declare the path integral to be BRST-invariant since BRST-invariance of boundary conditions is also required \[18\] so that the boundary conditions cannot be chosen arbitrarily. The properties of the theory should be analyzed within this theory itself. Using gauge-noninvariant extremals makes it clear that the answer will be gauge-noninvariant and the problem is, whether it is possible in general to extract a gauge-invariant information from it. Our investigation (see Part II) shows that one cannot do it – the fact, which we shall try to understand from the point of view of the Copenhagen interpretation of quantum theory and, in particular, of the Copenhagen interpretation of QGD.
5. Gauge noninvariance and the integrity principle

The Copenhagen integrity principle in its most states that a quantum object has no properties itself; it gets and exhibits some properties compatible with the complementarity principle only in a certain semiclassical macroscopic situation \[19, 20\]. The main representative of the latter is a measuring device; so, in equivalent terms, we mean the integrity of the physical object and observation means (OM).

In quantum theory of gravity the integrity principle turns our attention to the fact, that a gauge condition fixes a RS, i.e. it is directly related to OM and can be operationally interpreted. Another indisputable but not less important fact is that there exist no inertial RS in the theory of gravity. (The special case of an island system, that can be investigated in an asymptotically inertial RS, should be considered separately, see below). In a general case one deals with noninertial reference systems, their properties being represented by inertial fields. From the point of view of an observer inside a gravitating system the existence of the inertial fields affect the results of measurements. It is clear, that physical processes in measuring instruments creating macroscopic background for a quantum object depend on the inertial fields. For this reason the integrity principle does not contradict ideologically to the assumption about specific quantum correlations between the properties of inertial and true gravitational fields. The information about these very correlations turns out to be contained in the gauge-noninvariant amplitudes.

Of course, appealing to the integrity principle is a way to make a phenomenological prediction and the question remains what is the nature of the discussed correlations. It is obvious in advance that nonlinearity of the gravity theory itself does not ensure the existence of unremovable quantum gauge-noninvariant effects. An absolutely new element of physical reality that has no analog in the classical theory must take part in their formation. One can guess that only a nonperturbative gravitational vacuum with broken symmetry under transformations of diffeomorphism group could be such an element. The vacuum condensate fixing symmetry breaking must arise as a consequence of continuous distribution of OM inside the gravitating system. It will be shown below that this notion of gravitational vacuum can be expressed in a mathematical language.

The operational interpretation of quantum gauge-noninvariant effects allows to single out a particular physical situation, in which these effects should not exist. Let us consider an island universe – a quasilocalized coagulate of gravitational fields – and an observer who is about at spatial infinity from it. The properties of the observer’s RS asymptotically approaches to the properties of an inertial system. The observer’s subject of investigation is a graviton scattering. The properties of the system “gravitons + detectors” ensure that the measuring devices do not affect dynamical quantum phenomena. The role of the devices is restricted to a wave packet reduction taking place in a space-time region asymptotically far from the interaction region of quantum subsystems. The detectors located on bodies forming an asymptotically inertial RS cannot fix anything but three-dimensionally transversal gravitational waves. It is obvious that in this situation experimental data should be described by a gauge-invariant S-matrix. Mathematically this circumstance is taken into account by appealing to selection rules singling out from the full set of the amplitudes only those corresponding to observable gauge-invariant phenomena characterized by the vacuum of ghosts, 3-scalar and 3-vector gravitons.

The case of QGD of a closed universe is absolutely different. Firstly, in a closed universe there is no asymptotic state in which three-dimensional transversal gravitational waves would be dynamically split off a longitudinal gravitational field describing the expansion of the universe as a whole. Secondly, inertial fields are an unremovable reality for an observer inside a closed universe. Having no possibility to eliminate the inertial fields by a global transformation of coordinates and time, the observer is not able in principle to make measurements in such a way that the chosen RS properties would not affect measurement results. The absolute ban on such measurements is imposed by the equivalence principle. On the other hand, in accordance with the operational conception of quantum theory expressed in its Copenhagen interpretation, a state vector describes results of measurements carried out on a quantum object under real conditions created by a certain OM. Therefore, the wave function of the Universe must carry information on its geometry as well as on a noninertial RS in which this geometry is studied. It should be expected that QGD of a closed universe constructed in a mathematically correct manner will contain the wave functions of this type only. It will be shown in Part II that QGD of the model Bianchi-IX allowing one to make explicitly all the calculations.
6. The operational interpretation of a reference system in a closed universe

According to the above the task of QGD is, firstly, to find an operationally interpreted gauge-noninvariant wave function of the Universe and, secondly, to extract information from the wave function about properly geometry of the Universe as well as in what degree its properties depend on those of OM (a RS). This approach makes us put a question about a certain physical notion of an object which one could consider as an OM carrier in a closed universe.

As it was shown by Landau and Lifshitz [11], full information about dynamical geometry can be obtained directly in experimental way (without theoretical reconstruction) only in a RS disposed on infinite number of bodies filling the whole space. Each of the bodies should be equipped by arbitrarily going clock. The choice of a certain RS within this class is realized by choosing defined operations to co-ordinate clocks disposed on various bodies. Of course, the equations fixing the choice (gauge conditions) are noninvariant under the transformations of group of space-time symmetry (diffeomorphism group) covering all possible reference systems. It is worth paying attention that a special place of time in the theory becomes obvious when operationally interpreting the Landau-Lifshitz RS.

The co-ordination of clocks is performed by setting the metric components

$$g_{0\mu}(x) = \chi_\mu \left[ \gamma^{ik}(x) \right],$$

where $$\chi_\mu$$ is some functional of the 3-metric; $$\gamma^{ik} = -g^{ik}$$. A special role of $$g_{0\mu}$$ in the operational procedure is bound up with the lack of generalized velocities $$\dot{g}_{0\mu}$$ in the gravitational Lagrangian. The latter circumstance indicates that dynamics of $$g_{0\mu}$$ is a joint prerogative of a physical object and OM.

It is worth emphasizing that any gauge condition aims at fixing Eq. (10). In the classical theory (10) can either be given directly before integrating the Einstein equations, or (when using gauges unsolvable explicitly for $$g_{0\mu}$$) be found as a result of integration of the Einstein equations. The choice of a way to specify a RS does not make real significance since the classical theory is gauge-invariant by its mathematical structure. The same concerns the quantum theory of a gauge-invariant S-matrix.

In correspondence with the integrity principle it is necessary to realize the conception of joint and self-consistent evolution of a physical object and OM in QGD of a closed universe. The simplest phenomenological hypothesis is that this evolution is going according to the laws of quantum Hamilton dynamics. When constructing a gauge-noninvariant QGD this hypothesis becomes a postulate limiting the class of admissible gauges. A gauge is thought to be admissible if it extends the phase space of gravitational variables, maintains the theory being local and does not introduce derivatives of higher orders to the theory. A gauge condition that satisfies these requirements looks like

$$f^{\mu\nu} \dot{g}_{0\nu} = \chi^\mu,$$

where $$f^{\mu\nu} (g^{\lambda\sigma})$$, $$\chi^\mu (g_{0\nu}, g_{0\nu, i}, \gamma^{ik}, \dot{\gamma}^{ik}, \gamma^{ik}, \gamma^{ik, l})$$ are algebraic forms of the variables indicated. We shall confine attention to gauges of the class (11) because we do not see any technical possibility to go over to Eq. (10) within the framework of the path integral approach to QGD of a closed universe when gauges unsolvable explicitly for $$g_{0\mu}$$, in particular, canonical gauge, are under consideration.

The gauges (11) introducing missing generalized velocities to a Lagrangian, enable us to go over to a Hamiltonian theory in EPS. In such a theory the indeterminacy principle is valid for all metric components, that, in turn, allows to deduce a Schrödinger equation for the wave function of the Universe directly from the Hamilton operator equations and commutation relations. Exactly the same equation can be obtained for the wave function of the Universe defined through the path integral with the effective action in EPS will be demonstrated in Part II by direct calculations.

Let us turn to discussing a physical nature of an object that is supposed to be an OM carrier in a closed universe. The conception formulated by Landau and Lifshitz show that a medium with the following properties must be considered as a RS in the theory of gravity:
1. The medium fills the whole space, i.e. it is continual.

2. A periodic process occurs inside the medium, its characteristic can be used for choosing metric measurement standards.

3. The symmetry of the medium under diffeomorphism group transformations is broken.

There is no medium with the enumerated properties in classical physics. However, one can notice that quantum field theory and particle physics give examples of objects with similar properties. We mean non-perturbative vacuum condensate like Higgs or quark-gluon condensates. The vacuum condensate is a semi-classical medium (though it can have an internal quantum structure); in the spectrum of its excitations, as a rule, there are (pseudo) Goldstone modes which, in principle, can be considered as metric measurement standards; at last, the condensate formation is accompanied by symmetry breaking.

Adoption of the QGD operational interpretation conception allows to predict that its formalism contains the effect of the origin of a special gauge-noninvariant gravitational vacuum condensate breaking space-time symmetry under diffeomorphism group transformations. Various states of the condensate distinguished by residual symmetry groups correspond to various Landau – Lifshitz RS (means of observation upon the Universe as a whole). The information about physical processes going inside the Universe affects the quantum-wave Goldstone structures of the excited condensate, i.e. these structures can be used to record results of measurements.

Appealing to the Landau-Lifshitz RS makes the statement about the gauge noninvariance of quantum theory of gravity be almost obvious. Indeed, a formal transformation of coordinates meaning a transition to another gauge, physically corresponds to removing OM from the whole space of the Universe and replacing them by other OM. From the point of view of the integrity principle that declares existence of unremovable connections between the properties of an object and OM, it seems to be incredible that such an operation performed on the whole Universe scale, would not result in changing its quantum properties.

7. The wave function of a closed universe and quantization scheme

As was argued in Sec. 4, we have no ground to declare that a wave function of a closed universe must be gauge-invariant. It was demonstrated in Sec. 5 that introducing a wave function containing information about geometry of the Universe as well as about a RS does not contradict to the Copenhagen interpretation of QT. Moreover, in Sec. 2 it was shown that the theory based on the Wheeler – DeWitt equation is just a paradigm which cannot be founded on general QT principles.

With all the above in mind, we cannot require for a wave function of a closed universe to satisfy the Wheeler – DeWitt equation. At the same time, independently on our notion about gauge invariance or non-invariance of the theory, the wave function has to obey some Schrödinger equation. Only after constructing the wave function of a closed universe satisfying the Schrödinger equation, we shall be able to investigate the question, under what conditions this wave function could obey the Wheeler – DeWitt equation as well.

The Feynman approach contains the procedure of derivation of a Schrödinger equation from a path integral in the Lagrangian form. According to the physical situation, we consider the path integral without asymptotic boundary conditions.

In this situation we face the alternative: to work with the path integral with the Faddeev – Popov effective action (7), or consider the BFV form of a path integral with following integrating out all momenta and passing on to a path integral over extended configurational space. These path integrals should be skeletonized on different gauge-noninvariant sets of equations that will lead to nonequivalent results. In the present paper we choose the Lagrangian formulation of the theory based on the action (7) as a starting point for our investigation. Indeed, it has been already mentioned in Sec. 5 that for a system without asymptotic states one cannot declare the BFV effective action (7) to be BRST-invariant. But the BFV scheme is broken.

\[^3\] In the theory of quantum transitions between asymptotic states it is obvious that the replacement of bodies on which an asymptotically inertial RS is realized and replacement of detectors disposed on these bodies cannot change a physical situation. Therefore such a theory has to be gauge-invariant.
if we cannot ensure the BRST-invariance of the action: the Fradkin – Vilkovisky theorem is not valid then. So there are no grounds to think that the Hamiltonian formalism for the system without asymptotic states ought to be constructed along the BFV line. The BFV approach was developed originally for constructing a relativistic S-matrix of a constrained system. The purpose of its authors was to build the formalism equivalent to the Dirac quantization scheme [3]. One should bear in mind that even at the classical level Dirac’s formulation for gravity is a theory the group of transformations of which does not coincide with gauge group in the Lagrangian formalism.

Though the path integral approach does not require to construct a Hamiltonian formulation before deriving the Schrödinger equation, it implies that the Hamiltonian formulation can be constructed. As was pointed out in Sec. 6, it is possible to do it in the class of gauges (11). The Hamiltonian can be obtained in a usual way (according to the rule $H = p\dot{q} - L$, where $(p, q)$ are the canonical pairs of EPS) by introducing momenta conjugate to all degrees of freedom including gauge ones. Thus we get the Hamiltonian formulation in EPS equivalent to the Lagrangian formulation. We exploit the idea of extended phase space in the sense that gauge and ghost degrees of freedom are treated on the equal base with other variables and constrained equations and gauge conditions will be included to the set of Hamiltonian equations in EPS. In the second part of our paper we shall consider a model form of the gauge condition (11) for the Bianchi IX cosmology. Before quantizing the model we shall discuss the Lagrangian and Hamiltonian formulations and compare the latter with the BFV one (see Sec. 10, 11). It will be shown that Dirac’s constraints correspond to a particular form of gauge-noninvariant Hamiltonian equations in EPS.

The procedure of derivation of the Schrödinger equation enables one to control the correctness of mathematical operations. It is remarkable that all mathematical expressions appear to be well-definite, in contrast to attempts of deriving a gauge-invariant Schrödinger equation (see Sec. 14). Such attempts inevitably lead to divergent path integrals, the divergence being entirely due to the fact that dynamics of gauge variables is not fixed.

The procedure gives an explicit form of the Hamiltonian operator which corresponds to the Hamiltonian in EPS (up to the ordering problem). Thus the Lagrangian and Hamiltonian formulations and the quantization procedure are consistent in this approach.

As one can see, the “extended” set of Lagrangian equations obtained by varying the effective action (7) is of importance in our consideration. So, before proceeding to our program for the Bianchi IX model, we shall discuss its properties more profoundly. It will be shown that the proposed modification of QGD gives rise to the appearance of time in quantum geometrodynamics.

8. The extended set of the Einstein equations and the appearance of time in quantum geometrodynamics

Let us consider the gauge

$$\partial_0 (\sqrt{-g} g^{0\mu}) = 0,$$

which belongs to the class (11) and expands the phase space of gravitational variables. The transition amplitude depends on the action with gauge-fixing and ghost terms

$$S = \int \left( -\frac{1}{2\kappa} \sqrt{-g} R + \sqrt{-g} L_{\text{mat}} + \lambda_\mu \partial_0 (\sqrt{-g} g^{0\mu}) + \tilde{\theta}_\nu \hat{M}_\mu^\nu \theta^\mu \right) d^4x,$$

where $L_{\text{mat}}$ is the Lagrangian of nongravitational physical fields;

$$\hat{M}_\mu^\nu = -\partial_\mu (\sqrt{-g} g^{0\nu} \partial_0) + \delta^\sigma_\nu \partial_0 (\sqrt{-g} g^{0\sigma} \partial_\sigma) + \delta_\mu^0 \partial_0 (\sqrt{-g} g^{\nu\sigma} \partial_\sigma)$$

is the Faddeev – Popov operator [9], corresponding to the gauge (12); $\lambda_\mu$ are the Lagrange multipliers which will be further referred to as condensate variables. The variations of the action (13) yields the gauge condition (12), the ghost equations

$$\hat{M}_\mu^\nu \theta^\mu = 0, \quad \hat{M}_\mu^+ \theta_\nu = 0,$$
and the gauged Einstein equations

$$\frac{1}{\kappa} \left( R^\nu_\mu - \frac{1}{2} \delta^\nu_\mu R \right) = T^\nu_\mu^{(mat)} + T^\nu_\mu^{(obs)} + T^\nu_\mu^{(ghost)}, \quad (15)$$

where

$$T^\nu_\mu^{(obs)} = - \left( g^{0\nu} \delta^0_\mu + g^{\nu\sigma} \delta^0_\mu - g^{0\sigma} \delta^\nu_\mu \right) \partial_0 \lambda_\sigma; \quad (16)$$

$$T^\nu_\mu^{(ghost)} = \bar{\theta}_\mu,\sigma g^{0\nu} \theta^0_\sigma + \bar{\theta}_\rho,\sigma g^{\nu\sigma} \theta^0_\mu - \bar{\theta}_\sigma,\nu g^{0\rho} \theta^0_\sigma - \bar{\theta}_\sigma,0 g^{0\nu} \theta^0_\mu - \bar{\theta}_0,\nu g^{0\rho} \theta^0_\sigma - \bar{\theta}_0,0 g^{0\nu} \theta^0_\mu \quad (17)$$

are the quasi-energy-momentum tensor (quasi-EMT) of the OM and ghosts, respectively. It is supposed, that equations for nongravitational fields are considered together with (12), (14), (15). For the right hand side of (13) it results from the Bianchi identities that

$$\partial_\nu \left( \sqrt{-g} T^\nu_\mu \right) - \frac{1}{2} \sqrt{-g} g_{\nu\sigma,\mu} g^{\sigma\lambda} T^\nu_\lambda = 0. \quad (18)$$

For $T^\nu_\mu^{(mat)}, T^\nu_\mu^{(ghost)}$ Eq. (18) holds identically on the equations of motion; the substitution of the quasi-EMT of the OM in (18) enables us to obtain the equations which demonstrate explicitly the dynamics of the condensate variables:

$$\partial_\nu \left( \sqrt{-g} T^\nu_\mu^{(obs)} \right) - \frac{g^{0\lambda}}{g^{00}} \partial_0 \lambda_\lambda \partial_\mu \left( \sqrt{-g} g^{00} \right) = - \partial_\nu \left( \sqrt{-g} g^{0\nu} \partial_0 \lambda_\mu \right) -$$

$$- \delta^0_\mu \partial_\nu \left( \sqrt{-g} g^{0\nu} \partial_0 \lambda_\lambda \right) + \partial_\mu \left( \sqrt{-g} g^{0\nu} \partial_0 \lambda_\lambda \right) - \frac{g^{0\lambda}}{g^{00}} \partial_0 \lambda_\lambda \partial_\mu \left( \sqrt{-g} g^{00} \right) = 0. \quad (19)$$

In quantum theory Eqs. (12), (14), (13) are used for approximating a path integral or considered as operator equations in the canonical approach. The set of equations has a particular solution, where

$$\theta^\mu = 0; \quad \bar{\theta}_\nu = 0; \quad \lambda_\mu = 0; \quad (20)$$

$$\frac{1}{\kappa} \left( R^\nu_\mu - \frac{1}{2} \delta^\nu_\mu R \right) = T^\nu_\mu^{(mat)}. \quad (21)$$

It is generally accepted that the approximation of a path integral using (20), (21) leads to a gauge-invariant version of the quantum theory. Correspondingly, in the canonical approach the equations of motion are the \( (\hat{\epsilon}) \)-Einstein equations (21); all the other equations make sense only in operating on a state vector, i.e. they are selection rules for physical states. Actually, as was already mentioned, one can extract gauge-invariant effects in the theory based on the gauge-noninvariant set of equations (12), (14), (15) as well; to do it when constructing S-matrix it is sufficient to identify asymptotic states with the vacuum states of ghost, 3-scalar and 3-vector graviton fields. (This procedure is realized in the standard operator formulation of perturbation theory). Using the particular solutions (20), (21) inevitably makes one perform mathematically incorrect operations with divergent path integrals. Thus there remains no ground for using (20), (21) except appealing to the strong formulation of the correspondence principle that is typical for the Dirac approach.

If one puts \( \mu = 0 \) in the equation (19) for $T^\nu_\mu^{(obs)}$ it gives the continuity equation

$$\partial_\nu \left( \sqrt{-g} T^\nu_0^{(obs)} \right) = 0,$$

indicating that when extending phase space a new integral of motion appears in the physical system,

$$\int \sqrt{-g} T^0_0^{(obs)} d^3x = - \int \sqrt{-g} g^{00} \partial_0 \lambda_0 d^3x = -E. \quad (22)$$
In the gauge (12), $\sqrt{-g} g^{00}$ is a function of space coordinates only, therefore from (22) one concludes that the quantity $\partial_0 \lambda_0$ contains a constant component

$$(\partial_0 \lambda_0)_{(\text{cond})} = \frac{E}{V}, \quad V = \int \sqrt{-g} g^{00} d^3 x = \text{const},$$

that we propose to refer to as the energy density of the gravitational vacuum condensate (GVC). Thus, the existence of the gauge-noninvariant integral of motion (22) means that in the present theory gravitational vacuum breaking down the symmetry of the system under diffeomorphism group transformations shows itself as a real physical subsystem. Further, the Hamiltonian constraint of the Einstein theory $H_{(ph)} = 0$ is replaced by the constraint $H = E$, $H_{(ph)}$ being a Hamiltonian of gravitational and material fields and $H$ being a Hamiltonian in EPS. The constraint $H = E$ after the quantization procedure becomes the condition on a state vector

$$H |\psi\rangle = - \int d^3 x \sqrt{-g} T_{0(\text{obs})}^0 |\psi\rangle$$

instead of the Wheeler – DeWitt equation (3). The equation (23) states the spectra coincidence of the Hamiltonian operator in EPS and the operator

$$- \int d^3 x \sqrt{-g} T_{0(\text{obs})}^0$$

This equation is compatible with the Schrödinger equation in EPS and does not limit the Hamiltonian spectrum by the unique zero eigenvalue. The comparison with the Schrödinger equation allows us to identify the operator (24) in the Schrödinger representation with the operator $i \partial / \partial t$ (in the Heisenberg representation, as we saw, (24) is the operator corresponding to the integral of motion). We shall refer to the tensor $T_{\mu(\text{obs})}^\nu$ as a quasi-EMT of the condensate.

The problem of time, so typical of the Wheeler – DeWitt QGD, does not arise in the proposed modification of QGD; there is no necessity to introduce time in an unnatural way, by declaring a scale factor or a scalar field to be a parameter replacing a time variable (see, for example, [21, 22]). The proposed formulation of QGD, however, enables not just to avoid the problem of time, but to point out the source of time appearance in QGD. The obtained result shows that the origin of time as a physical concept in a quantum closed universe is bound up with an observer’s presence in it. The time appears owing to our pointing an instrument to measure it, introducing into consideration a subsystem of the Universe – the GVC, that could be regarded as the carrier of OM (a reference system). In principle, we get the possibility to examine the Universe evolution in time considering sequential quantum transitions between states with certain eigenvalues of the operator (24), the quantity $E$, defining a full energy of the GVC in a closed universe. The problem of time in the Wheeler – DeWitt QGD, in our opinion, results from using incorrect mathematical procedures, breaking a quantum integrity of the system “a physical object + OM”.

The equation (19) reveals in the GVC appropriated to the gauge (12) the presence of both mentioned above components which are necessary for performing the function of a RS: a continual component breaking the symmetry under diffeomorphism group and a periodic one that can give metric measurement standards. It results from the fact that the equation (19) for the parameter $\partial_0 \lambda_0$ of the quasi-EMT $T_{\mu(\text{obs})}^\nu$ after substituting $\partial_0 \lambda_0 = \partial_\sigma \chi$ can be reduced to the d’Alembert equation

$$\partial_\mu \left( \sqrt{-g} g^{\mu\nu} \partial_\nu \chi \right) = 0,$$

its general solution containing both components, a constant and wave one interacting with the metric.

Up to some extent of accuracy, the quantum field $\chi$ can be interpreted as a Goldstone mode of the GVC collective excitations. It is worth noting that the association of the field $\chi$ precisely with solutions to the d’Alembert equation (24) describing massless particles propagating with the speed of light is a direct consequence of our choice of the concrete gauge (12). At all the other gauges extending phase space objects like the GVC and its collective excitations would appear as well. However, the latter ones would not propagate with the speed of light; in some cases their velocities are less than the speed of light, and superlight in other cases. We do not know the principles of integrated system formation; probably, the properties of the field
\( \chi \) reflected in Eq. (25) might serve as a phenomenological criterion in this situation. It is easy to see from Eq. (19) that the interactions between the field \( \chi \) quanta and the ordinary matter are as weak as similar interactions of gravitational waves. Exactly for this reason the wave structures of a field \( \chi \), in principle, may be used for the registration of information about the Universe evolution. The phenomenological character of the discussed theory and the lack of physical principles regulating formation of the integrated system “a physical object + OM”, of course, do not allow us to put the question about the concrete mechanism of the information registration. This problem, seemingly, is a prerogative of a future theory.

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