Thermoelastic enhancement of the magnonic spin Seebeck effect in thin films and bulk samples

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A non-uniform temperature profile may generate a pure spin current in magnetic films, as observed for instance in the spin Seebeck effect. In addition, thermally induced elastic deformations may set in that could affect the spin current. A self-consistent theory of the magnonic spin Seebeck effect including thermally activated magneto-elastic effects is presented and analytical expressions for the thermally activated deformation tensor and dispersion relations for coupled magneto-elastic modes are obtained. We derived analytical results for bulk (3D) systems and thin magnetic (2D) films. We observed that the displacement vector and the deformation tensor in bulk systems decay asymptotically as $u \sim 1/R^2$ and $\varepsilon \sim 1/R^3$, respectively, while the decays in thin magnetic films proceed slower following $u \sim 1/R$ and $\varepsilon \sim 1/R^2$. The dispersion relations evidence a strong anisotropy in the magnetic excitations. We observed that a thermoelastic steady state deformation may lead to both an enhancement or a reduction of the gap in the magnonic spectrum. The reduction of the gap increases the number of magnons contributing to the spin Seebeck effect and offers new possibilities for the thermoelastic control of the Spin Seebeck effect.

I. INTRODUCTION

By virtue of magnetoelastic coupling elastic deformations may trigger a magnetization dynamics and (magneto)elastic waves maybe launched due to spin motion. The study of elastically activated magnetic dynamics in ferro- and antiferromagnetic materials dates back to the late 1950s starting with seminal independent works by A. I. Akhlezer, V. G. Berýakhtar, S.V. Peletminsky \cite{1} and C. Kittel \cite{2}. Further imputes came from the discovery of the magnetoelastic-gap \cite{3, 5} that bears some resemblance to spontaneous symmetry breaking \cite{6}. Since the magnetically excited elastic waves affect in turn the magnetization dynamics the established magnetoelastic gap, being a second order effect, is proportional to the square of the magnetoelastic coupling constant. Thus, the magnetoelastic gap is usually quite small compared to the gap in the magnonic spectrum which is induced for instance by a magnetocrystalline anisotropy or by external field terms.

A thermal heating leading to a steady state elastic deformation may serve as an alternative for activating (magneto)elastic waves that occur in ferromagnetic films and heterostructures \cite{7, 17}. Elasticity involving non-isothermal deformations is part of the well-established field of thermoelasticity \cite{18, 19}. An important question in the context of the present paper is to which extent a steady state thermoelastic deformation influences the magnetoacoustic effect. Due to the grossly different time scales of the dynamics, a steady state thermoelastic deformation is swiftly established (meaning equilibrated with the external thermal bath), and is basically unaffected by the much slower magnetization dynamics. The magnetization dynamics may well be sensitive to thermoelastic deformation, however. We will investigate here the theoretical aspects of thermal magnetoacoustics, i.e. thermally activated magnetoelastic effects with a special focus on phenomena of interest to the active field of spin caloritronics \cite{20, 36}. Experimentally, the utilization of elasticity to steer the magnetic dynamics is meanwhile accessible in a variety of setting. For instance, Rayleigh surface acoustic waves that may couple to spin ordering can be generated by irradiation with laser pulses \cite{37}. This process may well be accompanied by local heating spreading away from the laser spot which in turn may launch temporally a spin Seebeck current. Heating by laser pulses was employed for experiments concerning the time resolved spin Seebeck effect \cite{38}. Simulations for time resolved spin Seebeck effect were presented in Ref. \cite{39}. A comprehensible study of the thermal magnetoacoustic effect should encompass both, heating and elasticity aspects. Heating, for instance by laser pulses leads to a buildup of a nonuniform temperature distribution and possibly a temporal magnonic spin Seebeck effect. Non-isothermal deformations may also contribute to magnetoelastic activation of magnonic spin current. For example, considering that non-isothermal deformation of the thin film may reduce gap in the magnonic spectrum, the spin Seebeck effect may well be modified, for a reduction in the magnonic gap increases the number of magnons contributing to the spin Seebeck effect. In what follows we explore the link between the nonisothermal deformation ($\mathbf{R}$) tensor $\varepsilon_{\xi\zeta}(\mathbf{R})$ and the magnonic energy spectrum $\omega^2(q, \mathbf{R}, \varepsilon_{\xi\zeta}(\mathbf{R}))$ at the wave vector $q$. We derive analytical solutions for the deformation tensor and implement it for the thermally activated magnetoelastic dynamics. We analyze in details the 3D case of a Bulk sample and compare with a 2D case of a thin film. Analytical results are complemented by full numerical micromagnetic simulations. The paper is organized as follows: in section II we intro-
duce the model, in the section III we discuss the generalities of the magneto-thermal effect and derive explicit analytical expressions for the displacement vector and for the deformation tensor for local and non-local heat sources, section IV is dedicated to the dispersion relations for thermally excited magneto-elastic magnonic modes. In section V we present analytical results for the spin wave dispersion in thin films, and in section VI we analyze the results of the micromagnetic numerical calculations followed by a summary and conclusions.

II. GENERAL FORMULATION

We study the transversal magnetic dynamics of a magnetoelastically coupled system as it described by the deformation-dependent time evolution of the unit vector field \( \mathbf{M} \). We will work along a Landau-Ginzburg approach starting from the energy functional

\[
H = H_m + U_{mel}(\mathbf{R}).
\]

The magnetic part \( H_m \) can be broken down essentially into the exchange, magnetocrystalline anisotropy, and Zeeman terms, respectively (summation over repeated indexes is assumed)

\[
H_m = A_{\xi\zeta} \frac{\partial \mathbf{M}}{\partial x_\xi} \frac{\partial \mathbf{M}}{\partial x_\zeta} + K_{\xi\zeta} M_\xi M_\zeta - \mathbf{B} \cdot \mathbf{M},
\]

where \( A_{\xi\zeta} \) is the exchange stiffness, \( K_{\xi\zeta} \) quantifies the magnetocrystalline anisotropy energy contribution, and \( \mathbf{B} \) is an external magnetic field. The magnetoacoustic energy density \( U_{mel}(\mathbf{R}) \) reads

\[
U_{mel}(\mathbf{R}) = \frac{B_1}{M_s} M_\xi^2 \epsilon_{\xi\zeta} + \frac{B_2}{M_s^2} M_\xi M_\zeta \epsilon_{\xi\zeta}.
\]

Here \( M_s \) is the saturation magnetization, \( M_\xi(\mathbf{R}) \), \( M_\zeta(\mathbf{R}) \) are the magnetization components along the \( \xi, \zeta = x, y, z \) axes, and \( B_1, B_2 \) are the magnetoelastic constants. The deformation tensor has the explicit form

\[
\epsilon_{\xi\zeta}(\mathbf{R}) = \frac{1}{2} \left( \frac{\partial u_\xi(\mathbf{R})}{\partial x_\zeta} + \frac{\partial u_\zeta(\mathbf{R})}{\partial x_\xi} \right),
\]

where \( u_\xi(\mathbf{R}) \) is the component of the displacement vector. The stress tensor of the system \( \sigma_{\xi\zeta} \) satisfies the relation \( F_\xi = \frac{\partial \sigma_{\xi\zeta}}{\partial x_\zeta} \), where \( F_\xi \) is the component of the external force which is applied on the system. In the absence of an external forces, equilibrium requires that \( \frac{\partial \sigma_{\xi\zeta}}{\partial x_\zeta} = 0 \). The stress and the deformation tensors are interrelated via the algebraic relation

\[
\sigma_{\xi\zeta} = \frac{E}{1+\sigma} \left( \epsilon_{\xi\zeta} + \frac{\sigma}{1-2\sigma} \epsilon_{\zeta\zeta} \right).
\]

Here \( E \) is the elasticity modulus and \( \sigma \) is Poisson’s constant.

III. MAGNETO-THERMAL EFFECTS IN THE 3D BULK SYSTEM

We will be dealing with small amplitude displacements in the 3D bulk system. Proceeding in a standard way, the equation of motion for elastic waves without an applied thermal bias follows as \[19\]

\[
\rho \frac{d^2 \mathbf{u}}{dt^2} = \frac{E}{2(1+\sigma)} \Delta \mathbf{u} + \frac{E}{2(1+\sigma)(1-2\sigma)} \nabla(\nabla \cdot \mathbf{u}).
\]

In the presence of an applied thermal bias \( \nabla T \), one derives the equation of motion for the thermo-elastic waves by adding the temperature term,

\[
\rho \frac{d^2 \mathbf{u}}{dt^2} = \frac{E}{2(1+\sigma)} \Delta \mathbf{u} + \frac{E}{2(1+\sigma)(1-2\sigma)} \nabla(\nabla \cdot \mathbf{u}) - \frac{E \kappa \nabla T}{3(1-2\sigma)}.
\]

\( \kappa \) is the thermal expansion coefficient, and \( \nabla T \) is a temperature gradient which is due to a laser heating, for instance. Eq. \(7\) describes the dynamics of the elastic modes coupled to the magnetization dynamics via magnetoelastic coupling \( U_{mel}(\mathbf{R}) \) (see Eq. \(3\)). Classically, the magnetization dynamics follows the stochastic Landau-Lifshitz-Gilbert equation

\[
\frac{\partial \mathbf{M}(\mathbf{R},t)}{\partial t} = -\gamma \mathbf{M}(\mathbf{R},t) \times \left( \mathbf{H}_{eff} + \mathbf{h} \right) - \frac{A}{\hbar M_s} \mathbf{M}(\mathbf{R},t) \times \nabla^2 \mathbf{M}(\mathbf{R},t) + \frac{\alpha}{M_s} \mathbf{M}(\mathbf{R},t) \times \frac{\partial \mathbf{M}(\mathbf{R},t)}{\partial t}
\]

with the deterministic effective field \( \mathbf{H}^{eff} = -\frac{\partial \mathbf{H}}{\partial \mathbf{M}} \) and \( H = H_m + U_{mel}(\mathbf{R}) \) complemented by a random field \( \mathbf{h}(\mathbf{R},t) \) due to a Gaussian white noise with the autocorrelation function

\[
\langle h_i(t,R) h_j(t',R') \rangle = \frac{2k_B T(\mathbf{R})}{\gamma M_s \delta_i \delta_j} \delta(\mathbf{R} - \mathbf{R'}) \delta(t-t').
\]

Here \( \alpha \) is the Gilbert damping, \( \gamma = 1.76 \cdot 10^{11} \; [1/\text{Ts}] \) is the gyromagnetic ratio, \( T(\mathbf{R}) \) is the local temperature formed in the system and \( M_s \) is the saturation magnetization. The magnonic spin current tensor is evaluated as

\[
J_{\delta j} = \frac{A}{\hbar M_s} \epsilon_{\mu\nu} \mathbf{M}_\mu \nabla_j \mathbf{M}_\nu.
\]

Latin indexes refer to the spatial components while Greek indexes to the spin projections. Since the magnetoelastic term is part of the effective field in Eq. \(8\), it is expected to contribute to the spin current Eq. \(10\). The temporal, spatially nonuniform temperature profile \( \nabla T(\mathbf{R},t) \) can be inferred from the solution of the heat equation
with the appropriate source term $I(R, t)$. Explicitly this equation reads [39]:

$$\frac{\partial T(R, t)}{\partial t} = \frac{k_{ph}}{\rho C} \nabla^2 T(R, t) + I(R, t). \quad (11)$$

$C$ is the phonon heat capacity, $k_{ph}$ is the phononic thermal conductivity, and $\rho$ is the mass density. The imparted energy, e.g. by laser pulses is usually not completely absorbed by the system but is partially dissipated. Thermal loses can be incorporated in a realistic modelling of the laser heating process. For more details we refer to Ref. [39]. It is important to consider the relevant time scales. When the characteristic time scale of the heating process is faster than the magnetization dynamics, (i.e. the phonon relaxation time scale and the time interval between laser pulses are shorter than the precession time $\tau < 1/\gamma H_{eff}$) the magnetic system experiences an effective temperature which is deduced from an average over the much faster time scales. In this case, instead of the coupled set of equations (7), (8) and (11) we can explore a steady state problem. After some algebra, we derive the much faster time scales. In this case, instead of the asymptotic limit of the large $t$, Eq.(15) is quite involved, easy to see that in the asymptotic limit $T\to \infty = T_0$. Upon some involved calculations for the displacement vector we infer

$$u(R) = -\frac{\kappa(1+\sigma)Q}{3C(1-\sigma)} \nabla R \left\{ \frac{2}{\beta^3 R} - e^{-\beta R} \left( \frac{2}{\beta^3 R} + \frac{1}{\beta^2} \right) \right\}. \quad (15)$$

With this relation we obtain an explicit form of the deformation tensor as

$$\varepsilon_{\xi\xi}(R) = -\frac{\kappa(1+\sigma)Q}{3C(1-\sigma)} \delta_{\xi\xi} F_1(\beta R) + \frac{x_\xi x_\xi}{R^2} F_2(\beta R). \quad (16)$$

For brevity we introduced the notations

$$F_1(y) = \left[ \exp(-y)(1+2/y) - 2/y^2(1 - \exp(-y)) \right]/y$$

and

$$F_2(y) = \left[ 6 - 6 \exp(-y) - y \exp(-y)(6 + 3y + y^2) \right]/y^3.$$ 

Formally Eqs. (13)-(16) exhibit a nonphysical divergence in the limit $R \to 0$. We note however, in a coarse-grained approach the unit cell is non-deformable. Therefore, the minimal $R \to 0$ for which the study of the deformation makes sense is larger than the size of the coarse-grained cell $| R \to 0 | > a$ and $a \approx 10$ nm.

While in general case expression for displacement vector Eq.(15) is quite involved, easy to see that in the asymptotic limit of the large $R \to \infty$ we have a decay $1/R^2$.

### B. Extended heat sources

Let us consider an exponential temperature profile matching the numerical solutions of the heat equation with a non-local, i.e. extended heating source $I$. $T(R_1) = Q_1 \exp\left( -\beta(R_1) \right) + T_0$ with the characteristic decay length $\beta$ and $Q_1$ is the density of the heat released by the laser in the vicinity of the point $R_1 = 0$. The temperature in the heating point $(t=0) = Q_1/C + T_0$ and in the asymptotic $T(R_1 \to \infty) = T_0$. Upon some involved calculations for the displacement vector we infer

$$u(R) = \frac{\kappa(1+\sigma)(T'-T_0)}{9(1-\sigma)} \frac{R}{4R_{\text{max}}} \left( 1 - \frac{3R}{4R_{\text{max}}} \right). \quad (17)$$

### C. Linear Temperature profile

Linear temperature profile has particular interest for spin Seebeck experiments [10,12]. We consider linear temperature profile of the following form $T(R) = -(T'-T_0)R/R_{\text{max}} + T'$, where $T' > T_0$, and the temperature at the edges is equal to $T(0) = T'$, $T(R_{\text{max}}) = T_0$. After implementing linear temperature profile, for displacement vector we deduce:

$$u(R) = \frac{\kappa(1+\sigma)(T'-T_0)}{9(1-\sigma)} \frac{R}{4R_{\text{max}}}. \quad (17)$$
and for the deformation tensor we have:

\[ \varepsilon_\varepsilon = \frac{\kappa(1 + \sigma)(T - T_0)}{9(1 - \sigma)} \times \left( \delta_\varepsilon(1 - 3R/4R_{\max}) - \frac{3\varepsilon_x^2}{4R_{\max}R} \right). \]  

(18)

As we see in case of linear temperature gradient asymptotic behavior of the displacement vector and deformation tensor is different and non-monotonic in \( R = \sqrt{x^2 + y^2 + z^2} \). Maximum of the absolute value of the displacement vector corresponds to the case \( |u(R)| = 2R_{\max}/3 \).

IV. DISPERSION RELATIONS FOR THERMAL MAGNETO-ELASTIC SPIN WAVES IN BULK SYSTEMS

Taking into account Eq. [8] and Eq. [13] - Eq. [16] and assuming that the ground state magnetization is aligned parallel to the \( z \) axis we derive the following dispersion relation for the coupled magnetoelastic magnonic modes in the 3D bulk system

\[ \omega^2(q, R) = \left( \frac{2\gamma A_{xx}}{M_s^2} q^2 + K + \gamma H_0 - \frac{2\gamma B_{12}}{M_s^2} (\varepsilon_{zz} - \varepsilon_{xx}) \right) \times \left( \frac{2\gamma A_{xx}}{M_s^2} q^2 + K + \gamma H_0 - \frac{2\gamma B_{12}}{M_s^2} (\varepsilon_{zz} - \varepsilon_{yy}) \right) - \left( \frac{2\gamma B_{12}}{M_s^2} \varepsilon_{xy} \right)^2. \]

(19)

Here \( B_1 = B_2 = B_{12} \) are the magnetoelastic coupling constants.

Obviously in the absence of the magnetoelastic effect \( \varepsilon_\varepsilon(R) = 0 \), the obtained result falls back to the well-known magnonic dispersion relation. Depending on the values of the components of deformation tensor \( \varepsilon_\varepsilon(R) \) the magnetoelastic contribution in the magnonic dispersion relations can be positive or negative. A negative contribution \( -\frac{2\gamma B_{12}}{\mu_0 M_s^2} (\varepsilon_{zz} - \varepsilon_{xx}) < 0 < -\frac{2\gamma B_{12}}{\mu_0 M_s^2} (\varepsilon_{zz} - \varepsilon_{yy}) < 0 \) decreases the magnonic gap, while a positive contribution leads to a narrowing. Thus, the thermal magnetoelastic effect can be used as a tool for reducing the magnonic gap imposed by the magnetoelastic anisotropy or by an external magnetic field. A reduction of the gap naturally increases the spin Seebeck effect since it enhances the number of magnons contributing to the
spin current. We note that $\varepsilon_{\xi\xi}(\mathbf{R})$ is a local quantity and can be different for different $\mathbf{R}$. In some particular cases the magnonic gap can be enhanced and this naturally decreases the spin Seebeck current. After inserting the explicit expression for the deformation tensor $\varepsilon_{\xi\xi}(\mathbf{R})$ in Eq. (19) we deduce

$$
\omega(q, \mathbf{R}) = \frac{1}{\hbar} \sqrt{(Aq^2 + f_R^{(1)}) (Aq^2 + f_R^{(2)}) - f_R^{(3)}}.
$$

(20)

The particular values of the introduced functions $f_R^{(1,2,3)}$ are different for the exponential, the point-like heat source, or for the linear temperature profile.

a) In case of a point-like heat source we find:

$$
f_R^{(1)} = \hbar \left\{ \frac{\kappa(1 + \sigma)}{6\pi M_s^2(1 - \sigma)} \frac{Q}{R^2} \frac{1}{C} B_1(z^2 - y^2) + H_0 \right\},
$$

(21)

$$
f_R^{(2)} = \hbar \left\{ \frac{\kappa(1 + \sigma)}{6\pi M_s^2(1 - \sigma)} \frac{Q}{R^2} \frac{1}{C} B_1(z^2 - y^2) + H_0 \right\},
$$

(22)

$$
f_R^{(3)} = \left\{ \frac{\hbar}{6\pi M_s^2(1 - \sigma)} \frac{Q}{R^2} \frac{1}{C} B_2 xy \right\}^2.
$$

(23)

b) In the case of an exponential temperature profile one finds:

$$
f_R^{(1)} = \hbar \left\{ \frac{2\kappa(1 + \sigma)Q_1(z^2 - y^2)}{3M_s^2R^2(1 - \sigma)} B_1 F_2(\beta R) + H_0 \right\},
$$

(24)

$$
f_R^{(2)} = \gamma \hbar \left\{ \frac{2\kappa(1 + \sigma)Q_1(z^2 - y^2)}{3M_s^2R^2(1 - \sigma)} B_1 F_2(\beta R) + H_0 \right\},
$$

(25)

$$
f_R^{(3)} = \left\{ \gamma \hbar \frac{2\kappa(1 + \sigma)Q_1}{3M_s^2R^2(1 - \sigma)} B_2 F_2(\beta R) \right\}^2.
$$

(26)

c) In the case linear temperature profile we deduce:

$$
f_R^{(1)} = \hbar \gamma \left\{ \frac{B_1}{6R_{\text{max}}M_s^2} \frac{\kappa(1 + \sigma)(T_0 - T^\prime)}{(1 - \sigma)} (x^2 - z^2) + H_0 \right\},
$$

(27)

$$
f_R^{(2)} = \hbar \gamma \left\{ \frac{B_1}{6R_{\text{max}}M_s^2} \frac{\kappa(1 + \sigma)(T_0 - T^\prime)}{(1 - \sigma)} (y^2 - z^2) + H_0 \right\},
$$

(28)

$$
f_R^{(3)} = \left\{ \hbar \gamma \frac{B_2}{6R_{\text{max}}M_s^2} \frac{\kappa(1 + \sigma)(T_0 - T^\prime)}{(1 - \sigma)} xy \right\}^2.
$$

(29)

As we see from Eq. (20) - Eq. (29) the dispersion relation for mixed magnon-phonon modes is rather complex. The dependence on the spatial variable $\mathbf{R}$ is non-uniform with an anisotropic character of the magnonic modes. Note that obtained analytical results correspond to the 3D model, while for the sake of simplicity in numerical calculations we consider 2D model.

In prior to the numerical calculations, we present illustrations to support involved analytical findings. We adopted the material parameters of Nickel: the saturation magnetization is $M_s = 4.8 \times 10^5$ A/m, the exchange
constant is $A_{ex} = 4.6 \times 10^{-12}$ A/m, the damping constant is $\alpha = 0.01$, the mass density is $\rho = 8908$ kg/m$^3$, the heat capacity is $C = 502$ J/(kg K), the thermal conductivity is $k_{ph} = 91$ W/(m K), Young's modulus is $E = 200$ GPa, the Poisson's ratio is $\sigma = 0.31$, and the linear thermal expansion coefficient is $\kappa = 1.3 \times 10^{-5}$ K$^{-1}$. For an exponential temperature profile we set the decay length as $\beta = 5 \times 10^6$ m$^{-1}$, $T_0 = 0$ and $Q_1/C = 50$ K. The result for the exponential temperature profile $T(R_1) = \frac{Q}{\beta} \exp (-\beta | R_1 |) + T_0$ is shown in Fig. 6. As we see the temperature profile is isotropic and symmetric in the $xy$ plane. The temperature is maximal in the area heated by laser and decays exponentially with increasing distance from the laser spot. The symmetry properties of the displacement tensors $u_x$ and $u_y$ for an exponential temperature profile are quite intriguing, see Fig. 6. We clearly observe that the $u_x$ component possesses a mirror symmetry with respect to reflection $y \rightarrow -y$, and $u_y$ is antisymmetric with respect to the reflection $x \rightarrow -x$. Concerning the component $u_y$, the situation is opposite: it is symmetric with respect to the reflection $x \rightarrow -x$ and antisymmetric with respect to $y \rightarrow -y$. In case of the linear temperature gradient see Fig. 6b symmetry properties of the displacement vector are preserved, but maximum is slightly shifted to the edges of the sample $|u(R)| = 2R_{max}/3$. The components of the deformation tensor $\epsilon_{xx}$, $\epsilon_{yy}$ and $\epsilon_{xy}$ are shown in Fig. 7a, Fig. 7b, Fig. 7c and for the linear temperature profile in Fig. 7d. The diagonal components $\epsilon_{xx}$ and $\epsilon_{yy}$ are larger but localized, while the non-diagonal component of the deformation tensor $\epsilon_{xy}$ decays slower with distance and is finite in the whole sample.

The reduction of the magnonic gap can be illustrated as follows: The magnonic frequency $\omega(q, R, \epsilon_{\xi \zeta}(R))$ increases with $q$. Suppose the following equation holds: $\omega(q_1, R_1, \epsilon_{\xi \zeta}(R_1)) = \omega(q_2, R_2, \epsilon_{\xi \zeta}(R_2))$ for $q_1 > q_2$. This means, in the vicinity of $R_1$ the magnetoelastic coupling degrades the magnonic frequency, or around $R_2$ increases it. Thus, by the constraint $\omega(q, R, \epsilon_{\xi \zeta}(R)) = const$ we can explore the function $q(R)$ or its inverse function. Using the exponential temperature profile and the analytically derived deformation tensor $\epsilon_{\xi \zeta}(R)$, the dispersion relation is calculated based on Eq. (20) with $H_0 = 3 \times 10^5$ A/m. For a fixed frequency $f = \omega/(2 \pi) = 15$ GHz, the profile of $q(x, y)$ is shown in Fig. 8a. Similar to the temperature profile $T(x, y)$ the symmetry features of the magnon profile manifests an isotropy in $xy$ plane. In the center $(x = 0, y = 0)$, $q$ reaches a minimum. The value of $q$ increases gradually with distance from the center reaching a maximum to decrease near to
the boundary. Since $\omega(q, R, \varepsilon_{zz}(R))$ is fixed, an increase of the wave vector is compensated by a negative contribution of the deformation tensor $\varepsilon_{zz}(R)$ in the magnon dispersion relation. Therefore, the maximum of $q$ for a given fixed frequency $\omega(q, R, \varepsilon_{zz}(R))$ corresponds to a minimum in the magnonic gap. We further calculate the elastic shift of the dispersion relations for different values of the coordinate $x$ and a fixed value of the $y = 0$ coordinate, as shown in Fig. 8(b). As we see, the magneto-elastic effect can either increase the magnonic gap (Fig. 8(b)) or may decrease depending on the geometry of the sample and on the parameters. We note that the value of the gap is a local quantity that depends on $R$.

FIG. 9. 2D thin magnetic film. For $x = 5, 50$ nm and $y = 0$, the dispersion relation calculated using Eq. (35) and the following values of parameters $A_e = 4.6 \times 10^{-12}$ J/m, $\gamma = 1.76 \times 10^{11} /$(T s), $M_s = 4.8 \times 10^5$ A/m, $H_0 = 3 \times 105$ A/m, $B_1 = B_2 = 7.85 \times 10^8$ J/m$^3$, $-J_0/\alpha = 0.01$ K, $\alpha = 5 \times 10^6$ m$^{-1}$, $\kappa = 1.3 \times 10^{-5}$ K$^{-1}$ and $\sigma = 0.31$.

FIG. 10. 2D thin magnetic film. The steady state spatial temperature profile formed in the system due to a laser heating. The result is obtained via a numerical solution of the heat equation. The maximum temperature in the area heated by laser pulses is $T_0 = 40$K in the vicinity of the point $x = 0$ and $y = 0$ and decays gradually to zero with the distance.

V. THERMOELASTIC DISPERSION RELATIONS IN THIN MAGNETIC FILMS

Having explored the 3D case of a bulk system we derive the thermoelastic dispersion relations for a thin 2D magnetic film. The solution of the elasticity equation for the displacement vector reads
FIG. 11. 2D thin magnetic film. Projective plot of the steady state spatial temperature profile formed in the system due to laser heating as follows from the numerical solution of the heat equation.

FIG. 12. 2D thin magnetic film. Steady state spatial profile of the displacement vector (a) $u_x$ and (b) $u_y$.

FIG. 13. 2D thin magnetic film. Steady state spatial profile of the displacement vector (a) $u_x$ at $y = 0$, and (b) $u_y$ at $x = 0$.

$$u(R) = \chi \int (T(R_1) - T_0) \frac{R - R_1}{(R - R_1)^2} d^2R_1,$$ (30)

$$\chi = \frac{\kappa (1 + \sigma)}{6\pi (1 - \sigma)}.$$  

Similar to the 3D case, for 2D thin film we consider a point-like and an extended heat source.

A. Point-like heat source

In particular for the point-like heat source $T(R_1) - T_0 = \frac{Q}{2} \delta(R_1)$ we infer

$$u(R) = \frac{\chi Q R}{C R^2},$$ (31)

while for the deformation tensor we obtain

$$\varepsilon_{\xi\zeta} = \frac{\chi Q}{C} \left( \frac{\delta_{\xi\zeta}}{R^2} - \frac{2\eta_x x_{\xi}}{R^4} \right).$$ (32)
We note that the plane deformation tensor $\varepsilon_{\xi\xi}$ has three independent components: $\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{xy} = \varepsilon_{yx}$.

We already see the difference to the bulk system. Instead of $1/|\mathbf{R}^2|$ for the bulk system (Eq.13), for the 2D thin magnetic film the displacement vector decays slower $1/|\mathbf{R}|$. The same applies to the deformation tensor Eq.32.

The magnetoacoustic energy density of the thin film has the form

$$U_{mel}(R) = \frac{B_1}{M_s^2}(M_x^2 \varepsilon_{xx} + M_y^2 \varepsilon_{yy}) + \frac{2B_2}{M_s^2} M_x M_y \varepsilon_{xy},$$

and the effective magnetoacoustic field is

$$H_{x \text{eff}} = -\frac{2B_1}{M_s^2} \varepsilon_{xx} M_x - \frac{2B_2}{M_s^2} \varepsilon_{xy} M_y,$$

$$H_{y \text{eff}} = -\frac{2B_2}{M_s^2} \varepsilon_{yx} M_x - \frac{2B_1}{M_s^2} \varepsilon_{xx} M_y.$$

Utilizing Eq.8, Eq.33, and Eq.34 and assuming that the ground state magnetization is aligned parallel to the $z$ axis we derive the following dispersion relation of the coupled magnetoelastic magnonic modes in the thin films as

$$\omega(q, R) = \frac{1}{\hbar} \sqrt{\left(A q^2 + g_D^1\right) \left(A q^2 + g_D^2\right) - g_D^3},$$

$$g_D^1 = \hbar \gamma \left(H_0 + \frac{2B_1}{M_s^2} \varepsilon_{xx}\right),$$

$$g_D^2 = \hbar \gamma \left(H_0 + \frac{2B_1}{M_s^2} \varepsilon_{xy}\right),$$

$$g_D^3 = \left(\hbar \gamma \frac{2B_2}{M_s^2} \varepsilon_{xy}\right)^2.$$

As we see from Eq.35 the dispersion relation is defined by the external field $H_0$ and the deformation tensor $\varepsilon_{\xi\xi}$.

### B. Extended heat source

In order to explore the effect of the extended heat source we solve the heat equation (11):

$$\frac{\partial^2 T(x, y)}{\partial x^2} + \frac{\partial^2 T(x, y)}{\partial y^2} = -aI(x, y), a = \frac{\rho C}{k_{ph}}$$

We adopt the source term $I = I_0 e^{-\alpha (x+y)}$, with a positive characteristic decay constant $\alpha > 0$ and the following boundary conditions: $x > 0, y > 0$. Then, the stationary solution of Eq.36 reads

$$T(x, y) = -\frac{aI_0}{2\alpha^2} e^{-\alpha(x+y)} + T_0,$$

$$\alpha > 0, x > 0, y > 0.$$

Taking into account Eq.37 for the displacement vector and the deformation tensor we deduce following solutions

$$u_x(x, y) = -\frac{\alpha I_0}{2\alpha^2} \int_0^\infty \int_0^2 \frac{e^{-\alpha(x+y)} (x - x_1) dx_1 dy_1}{(R - R_1)^2},$$

$$u_y(x, y) = -\frac{\alpha I_0}{2\alpha^2} \int_0^\infty \int_0^2 \frac{e^{-\alpha(x+y)} (y - y_1) dx_1 dy_1}{(R - R_1)^2},$$

and

$$\varepsilon_{xx} = -\varepsilon_{yy} =$$

$$= \frac{\alpha I_0}{2\alpha^2} \int_0^\infty \int_0^2 \frac{e^{-\alpha(x+y)} (x - x_1)^2 - (y - y_1)^2 dx_1 dy_1}{(R - R_1)^4},$$

$$\varepsilon_{xy} = \varepsilon_{yx} =$$

$$= \frac{\alpha I_0}{2\alpha^2} \int_0^\infty \int_0^2 \frac{e^{-\alpha(x+y)} (x - x_1)(y - y_1) dx_1 dy_1}{(R - R_1)^4},$$

$$(R - R_1)^2 = (x - x_1)^2 + (y - y_1)^2.$$

The obtained result is quite involved in general and can be made transparent by numerical simulations. In the isotropic case, the problem simplifies and we obtain an analytical solution in a closed form.

### C. Extended isotropic heat source

We assume that source term is isotropic $I = I_0 e^{-\alpha R}$ and the temperature is a function of $R$ only. Utilizing polar coordinates and performing the integration over angle we obtain from Eq.36

$$\frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial T(R)}{\partial R} \right) = -aI(R).$$

We adopt the boundary condition $T(R \rightarrow \infty) = T_0$ and find the stationary solution of the heat equation in the following form:

$$T(R) - T_0 = -\frac{I_0}{\alpha^2} \left( \Gamma(0, \alpha R) + e^{-\alpha R} \right).$$

Here $\Gamma(\alpha, z) = \int_z^\infty t^{\alpha-1} e^{-t} dt$ is the incomplete Gamma function.

Taking into account the temperature profile Eq.41, for the displacement vector and the deformation tensor we deduce the following solutions

$$u_x = -\chi \frac{I_0}{\alpha^2} \int_0^\infty \int_0^{2\pi} \frac{\Gamma(0, \alpha R') + e^{-\alpha R'}}{R^2 + R'^2 - 2RR' \cos(\theta - \theta')} R' dR' d\theta',$$

$$u_y = -\chi \frac{I_0}{\alpha^2} \int_0^\infty \int_0^{2\pi} \frac{\Gamma(0, \alpha R') + e^{-\alpha R'}}{R^2 + R'^2 - 2RR' \cos(\theta - \theta')} R' dR' d\theta'.$$
After performing the integration for the displacement vector and the deformation tensor we arrive at the expression

\[
\begin{align*}
  u_x &= -\chi \frac{3\pi I_0 x}{2\alpha^4 R^2}, \\
  u_y &= -\chi \frac{3\pi I_0 y}{2\alpha^4 R^2},
\end{align*}
\]

and

\[
\begin{align*}
  \varepsilon_{xx} &= -\varepsilon_{yy} = \chi \frac{3\pi I_0 x^2 - y^2}{\alpha^4 R^4}, \\
  \varepsilon_{xy} &= \varepsilon_{yx} = \chi \frac{3\pi I_0 xy}{\alpha^4 R^4}.
\end{align*}
\]

Here \( R^2 = x^2 + y^2 \) is the in-plane radius vector. In order to obtain the dispersion relations in a closed form, we use Eq. (44) - Eq. (46) in Eq. (35). Plots of the displacement vector, the deformation tensor and the dispersion relation in the case of a 2D extended heat source are presented in Fig. 9. As we see the magnon dispersion relation is local and is different in the different areas of the film.

D. Linear temperature profile in the thin magnetic films

We again consider the linear temperature profile of the following form \( T(R) = -(T' - T_0) R/R_{\text{max}} + T' \), where \( T' > T_0 \), and the temperature at the edges is equal to \( T(0) = T'' \), \( T(R_{\text{max}}) = T_0 \). After implementing linear temperature profile, for displacement vector we deduce:

\[
\begin{align*}
  u_x &= -\chi(T_0 - T') \frac{\pi R_{\text{max}}^2 x}{6 R^2}, \\
  u_y &= -\chi(T_0 - T') \frac{\pi R_{\text{max}}^2 y}{6 R^2},
\end{align*}
\]

and for the deformation tensor:

\[
\begin{align*}
  \varepsilon_{xx} &= -\varepsilon_{yy} = \chi(T_0 - T') \frac{\pi R_{\text{max}}^2 x^2 - y^2}{6 R^4}, \\
  \varepsilon_{xy} &= \varepsilon_{yx} = \chi(T_0 - T') \frac{\pi R_{\text{max}}^2 xy}{6 R^4}.
\end{align*}
\]

As we see linear temperature profile in the thin magnetic films lead to the same type of asymptotic decay, \( 1/R \) for dismament vector and \( 1/R^2 \) for the deformation tensor.

VI. NUMERICAL RESULTS

The numerical simulations are performed for a two dimensional Nickel film. The external field with \( H_z = 4.5 \times 10^5 \) A/m is applied along the \( x \) axis leading to a uniform ground state. A two-dimensional ferromagnetic thin film with a length of 1000 nm is aligned along \( x \) axis and has a width of 1000 nm in \( y \) direction. We also assume that the source term that enters in the heat equation and describes the effect of the laser heating is a local function \( I(R,t) = \delta(R - R_0) \) corresponding to the case when the intensity of the laser field is spatially localized. This approximation is similar to the local heat source model discussed analytically in the previous sections. We assume the heating laser spot is around \((x = 0 \text{ nm}, y = 0 \text{ nm})\). The thermal effect of the laser heating is described by the heat equation Eq. (11) using the fixed boundary conditions

\[
\begin{align*}
  T(x = 500\text{nm}) &= 0, \\
  T(x = -500\text{nm}) &= 0, \\
  T(y = 500\text{nm}) &= 0, \\
  T(y = -500\text{nm}) &= 0.
\end{align*}
\]

The value of the temperature \( T_0 \) formed in the area of the heating laser depends on the laser intensity \( I \). For the boundary conditions Eq. (52) implemented numerically in the heat equation Eq. (11) we obtain the stable spatial temperature profile. As inferred from Fig. 10...
the temperature profile $T(x, y)$ shows an exponential decay with the distance from $T_0(x = 0, y = 0)$. This numerical result for the temperature profile is in a good agreement with the assumptions used for the analytical solution. The exponential character of the decay of the temperature profile is more evident from Fig. 11.

The heat diffusion induces a thermal gradient $\nabla T$ and an elastic deformation in the system. Using the equation of elasticity Eq. (7) we find numerically the temperature profile $T(x, y)$ and calculate the components of the displacement vector for the following fixed boundary conditions

$$
\begin{align*}
\mathbf{u}(x = 500\text{nm}) &= (0, 0, 0), \\
\mathbf{u}(x = -500\text{nm}) &= (0, 0, 0), \\
\mathbf{u}(y = -500\text{nm}) &= (0, 0, 0), \\
\mathbf{u}(y = 500\text{nm}) &= (0, 0, 0).
\end{align*}
$$

Fig. 15. 2D thin magnetic film. The steady state spatial profile of the components of the magnonic spin current tensor in the absence of a magneto-elastic effect (a) $J_{x}^{ME}$ and (b) $J_{y}^{ME}$ when $T_0 = 80K$.

Fig. 16. 2D thin magnetic film. The steady state spatial profile of the components of magnonic spin current tensor (a) $J_{x}^{ME}$ at $y = 0$, and (b) $J_{y}^{ME}$ at $x = 0$ with and without the magneto-elastic term. Obtained analytical results. Namely, we clearly see that the $u_x$ component exhibits the mirror symmetry with respect to the reflection $y \rightarrow -y$, and is antisymmetric with respect to the reflection $x \rightarrow -x$. Concerning the component $u_y$, the situation is reversed: symmetry is given for $x \rightarrow -x$ and antisymmetry for $y \rightarrow -y$. The components of the deformation tensor $\epsilon_{xx}$, $\epsilon_{yy}$ and $\epsilon_{xy}$ are shown in Fig. 7, Fig. 8, Fig. 9. The diagonal components $\epsilon_{xx}$ and $\epsilon_{yy}$ are larger but localized, while the non-diagonal component of the deformation tensor $\epsilon_{xy}$ decays slower and remains finite in the whole sample.

We implement the numerically deduced temperature profile $T(x, y)$ and the elastic displacement profile $\mathbf{u}(x, y)$, in the stochastic LLG equation Eq. 8. The existence of the thermal gradient formed in the system due to the laser heating may lead to the emergence of magnonic spin current and a longitudinal spin Seebeck effect. However, on top of this standard effect, the thermal heating leads to a thermal activation of the deformation tensor. Due to the magneto-elastic interaction the thermally activated deformation tensor contributes to the
Therefore, it reduces the gap in the magnon spectrum. magnetization dynamics and modifies the net magnonic current. For, \( T_0 = 80 \text{K} \), the components of the magnonic spin current tensor \( J_x^{\text{ME}} = \frac{2 \gamma A_x}{\mu_0 M_s^2} (M_y \partial_x M_z - M_z \partial_x M_y) \) and \( J_y^{\text{ME}} = \frac{2 \gamma A_y}{\mu_0 M_s^2} (M_y \partial_y M_z - M_z \partial_y M_y) \) are plotted in Fig. [14]

Fig. [15] evidences that the magnon current emerges in the area heated by the laser pulses and propagates according to the formed thermal bias (which has been established swiftly on the magnon time scale). The \( J_x^{\text{ME}} \) and \( J_y^{\text{ME}} \) components of the spin current tensor are negative for \( x > 0 \) and \( y > 0 \), respectively becoming positive in the rest of the sample. A similar abrupt switching of the displacement vector \( u_x \) and \( u_y \) we observe in Fig. [12](b). To highlight the role of the elastic term on the magnonic spin current we plot the magnonic current in the presence/absence of the magneto-elastic contribution for the same thermal gradient. Fig. [15] shows the profile of the magnonic spin current, in particular the tensor components \( J_x^{\text{ME}} \) and \( J_y^{\text{ME}} \). They have qualitatively the same behavior in the presence or in the absence of the magneto-elastic coupling. However, as shown in Fig [16] the value of \( J_y^{\text{ME}} \) is obviously becoming larger with the elastic term, while the change in \( J_x^{\text{ME}} \) due to the elastic term is marginal. We conclude so that elasticity leads to the enhancement of the magnonic spin Seebeck current in certain directions. Also, the bias temperature is important. Raising \( T_0 \) and hence \( u \), the increase in the magnonic spin current \( J_x^{\text{ME}} \) and the elastic enchantment of \( J_y^{\text{ME}} \) are evident, as shown in Fig. [17]

To understand the selective enhancement of the magnonic spin current induced by the magneto-elastic coupling we inspect the \( x \)-component of the magneto-elastic effective field \( H_{\text{ME}-x}^{\text{eff}} = -\frac{1}{\beta} \left( \delta U_{\text{me}}(R)/\delta M(R) \right)_{x} \) in Fig. [18](a) which shows that the effective magneto-elastic field is directed opposite to the external field. Therefore, it reduces the gap in the magnon spectrum.
FIG. 20. 2D thin magnetic film. The $x$ component of the backward spin current $I^x_{\text{fl}}$ in the presence and absence of the thermoelastic effect. The temperature $T_d$ of the detecting bar is uniform, while the temperature of the magnet is inhomogeneous. The equilibrium magnetization is along $+x$.

By changing the equilibrium magnetization along $+y$, the distribution of the effective magneto-elastic filed is changed (Fig. 18(b)), and the corresponding magnonic spin current, i.e. $J^{xy}_{M}$, is selectively enhanced. This feature is further testified by Fig. 19. The maximal temperature in the center of the laser intensity is equal to $T_0 = 80$ K. For the experimental observation for the thermoelastic effect, we suggest exploiting the magnetization sensitive thermoelastic effect, and detecting the spin Seebeck effect for the different direction of the equilibrium magnetization.

At the typical Spin Seebeck experiment, spin current is measured by means of inverse spin Hall effect and Platinum detecting bar. The total net current $I = I_{\text{sp}} + I_{\text{fl}}$ consists of two contributions: spin pumping current $I_{\text{sp}}$ flowing from magnetic insulator towards detecting bar, and a backward spin current $I_{\text{fl}} = -M_s \mathbf{m} \times \mathbf{h}$ injected from the detecting bar into the magnetic insulator. Here $\mathbf{h}$ is the random magnetic field related to the Platinum detecting bar. The interesting question is whether the thermoelasticity plays an important role in the backward spin current. In order to answer this question, we calculate the backward spin current $I_{\text{fl}}$ in the presence and absence of the thermoelastic effect. The result of the numerical calculations is shown in Fig. 20. As we thermoelasticity see has no influence on the backward spin current.

VII. CONCLUSIONS

We studied the spin Seebeck effect in bulk samples and thin ferromagnetic films and explored the influence of the thermoelastic steady state deformation on the magnonic spin current. For a particular temperature profile in the system, we obtained analytical expressions for the thermoelastic deformation tensor. We derived analytical results for the 3D bulk system and 2D thin magnetic film as well. We observed that the displacement vector and the deformation tensor in bulk systems decay asymptotically as $u \sim 1/R^2$, and $\varepsilon \sim 1/R^3$, respectively. The decay in thin magnetic films is slower being $u \sim 1/R$, and $\varepsilon \sim 1/R^2$. We found that due to the magnetoelastic coupling, the thermoelastic deformation tensor has a significant impact on the magnetization dynamics. We derived analytical expressions for the dispersion relations for thermoelastic magnons highlighting a principle difference between the thermoelastic and the magneto-elastic effects. Magnetoelastic effects always enhance the magnetoelastic gap in the magnonic spectrum. Thermoelastic steady state deformation may lead either to an enchantment or to a reduction in the gap of the magnonic spectrum. A reduction of the gap increases the number of magnons contributing to the spin Seebeck effect offering so a thermoelastic control of the spin Seebeck effect.

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