Local Polyakov loop domains and their fractality

Hans-Peter Schadler†
In cooperation with
Gergely Endrodi* and Christof Gattringer†

†University of Graz
*University of Regensburg

Mainz, Germany
31.07.2013
Center clusters

- In pure gluodynamics: The **Polyakov loop**

\[ L(x) = \text{Tr} \prod_{t=1}^{N_t} U_4(x, t) \]

is an order parameter for the deconfinement transition.

- \( L(x) \) transforms non-trivially under center transformations.

- **Center domains**: Clusters of spatial points \( x \) where the phase of \( L(x) \) is near the same center element.

- Center domains are reminiscent of Weiss domains.

- The center domains may play a role in QCD phenomenology.
Technical details

- Pure gluodynamics with Wilson gauge action.

- Lattices: $30^3 \times N_t$, $40^3 \times N_t$, $48^3 \times N_t$.

- Fixed scale approach, i.e., we use $N_t$ to drive the temperature.

- Temperature: $0.3 \leq T/T_c \leq 7$.

- Inverse coupling: $\beta = 5.90, 6.20, 6.45$.

- Lattice spacing: $a = 0.112 \text{ fm, } 0.068 \text{ fm, } 0.048 \text{ fm.}$
Histogram of phase and modulus of $L(x)$

$L(x) = \rho(x) \exp(i\theta(x))$
We assign to a spatial point \( x \) the **center sector number** \( n(x) \):

\[
  n(x) = \begin{cases} 
    -1 & \text{for } \theta(x) \in \left[ -\pi + \delta, -\pi/3 - \delta \right] \\
    0 & \text{for } \theta(x) \in \left[ -\pi/3 + \delta, \pi/3 - \delta \right] \\
    +1 & \text{for } \theta(x) \in \left[ \pi/3 + \delta, \pi - \delta \right] 
  \end{cases}
\]

with the real and non-negative parameter

\[
  \delta = f \frac{\pi}{3}, \quad f \in [0, 1).
\]

**Cut parameter** \( f \):

- \( f > 0 \): Sites far from the center elements are removed
- \( f = 0 \): No sites removed
- \( f \to 1 \): All sites removed
Definition of center domains - 2

Two spatial points

\[ x, y = x \pm \hat{\mu} \]

belong to the same cluster if

\[ n(x) = n(y). \]

- Do we observe cluster formation?
- How do these clusters change with temperature?
- What is their fractal dimension?
Visualization of center domains

$T = 0.62T_c$

$T = 0.82T_c$

$T = 0.90T_c$

$T = 0.98T_c$

$T = 1.10T_c$

$T = 1.64T_c$
Percolation probability \((40^3 \times N_t, \beta = 6.2)\)

Number of percolating clusters \(N_{perc}\).
Number of sites in a cluster / weight ($\beta = 6.2, f = 0.30$)

$\langle W \rangle / V$ ... Weight of the largest cluster normalized by the volume.

$\langle W_{np} \rangle$ ... Mean cluster size of non-percolating clusters.
Fractality \((40^3 \times N_t, \beta = 6.2)\)

Fractal dimension \(D\) of largest cluster via box counting method:

\[ N(s) \propto s^{-D}, \]

with \(N(s)\) number of boxes of size \(s\) needed to cover the whole cluster.
Linear cluster extents ($\beta = 6.2$)

(Lattice extent $N_s a : N_s = 30 : 2 \times 1.02, N_s = 40 : 2 \times 1.35, N_s = 48 : 2 \times 1.63 \text{ fm}$)

Radius of a cluster of size $s$:

$$R_s^2 = \sum_{i=1}^{s} \frac{|r_i - r_0|^2}{s},$$

with "center of mass"

$$r_0 = \sum_{i=1}^{s} \frac{r_i}{s}.$$

Avg. distance traveled $d$:

$$d^2 = \sum_s n_s s \bar{R}_s^2.$$

$n_s s$: Probability for a site to belong to a cluster of size $s$.

$\bar{R}_s$: avg. over all $R_s$ for a given $s$. 
Summary:

- We study the formation and properties of center domains.
- Drastic change of properties at the phase transition.
- Many small clusters below the phase transition.
- At $T \geq T_c$ one cluster wins $\Rightarrow$ percolation over the whole lattice.
- Fractality: Below $T_c$ the clusters are highly complex objects; above $T_c$ the fractal dimension becomes $D \rightarrow 3$.
- Results on properties of center clusters may be relevant for phenomenological description of heavy ion collisions.
Linear cluster extents \((\beta = 6.2)\)

\((\text{Lattice extent } N_s a: N_s = 30 : 2 \times 1.02, N_s = 40 : 2 \times 1.35, N_s = 48 : 2 \times 1.63 \text{ fm})\)

Radius of a cluster of size \(s\):

\[
R_s^2 = \sum_{i=1}^{s} \frac{|r_i - r_0|^2}{s}, \quad \text{with "center of mass"} \quad r_0 = \sum_{i=1}^{s} \frac{r_i}{s}.
\]