Hazard Analysis of Bidder Collusion in Reverse Auctions Based on Petri Nets

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ABSTRACT Bidder collusion in reverse auctions is a pervasive phenomenon that leads to a series of problems for buyers and their supply chains. This work aims to conduct a hazard analysis on a first-price sealed-bid reverse auction with bidder collusion by applying colored timed Petri nets, in which places and transition nodes represent bidding resources and activities. This work successfully dissects the collusion process, quantifies the harm of collusion, and compares the harm levels. The results of the simulation disclose two main causal factors for bidder collusion information asymmetry and strong relationship between strong bidders. This study also identifies the situations where there is a strong relationship between strong bidders. Based on these findings, several recommendations are proposed to reduce the harm of the ring game in a real reverse auction process. Thus, this study helps the buyers to take control of reverse auctions.

INDEX TERMS Collusion, Petri net, reverse auction, supply chain management.

I. INTRODUCTION

Reverse auctions have become an effective and efficient way for buyers to reduce the purchase price, increase market efficiency, access a large-base of suppliers, and increase the efficiency of the procurement process [1]. In a reverse auction, the desired item is offered by a number of sellers, consequently the buyer controls the market [2]. In an ideal reverse auction, because the lowest bid wins the contract, the sellers as bidders would keep their bid price as a secret and would not disclose any information to other bidders. However, in actual situations, bidders can conspire to rig bids and attempt to control the results of the reverse auction. This bid rigging tactic is referred to as bidder collusion and may cause a deluge of loss to the buyers. Bidder collusion is a pervasive problem [3]. In France there were more than 220 decisions for bidder collusion cases in public procurement between 1991 and 2010 [4]. Collusion can make a profit by destroying the key performance of an auction. Bidder collusion not only reduces the buyers/auctioneers’ profitability due to costs increase [5] but also leads to other serious consequences, including stopping buyers/auctioneers’ production due to shortage of inventory [4]. Even worse, it could destroy a business because it damages the buyers’/auctioneers’ reputation due to the terrible quality of the products. If bidder collusion is common in economic activities, then it creates an unfair phenomenon in society. This impedes socio-economic and all aspects of the development of a society. Hence, it is very important to study how to prevent bidder collusion in a reverse auction.

Although bidder collusion is an important research topic, most of the previous studies in reverse auctions have focused on the bidder’s decision-making models. Researchers have applied many different theories (e.g., game theory and operational research) to test the bidder’s strategies, mechanisms, and behaviors [6]–[10]. However, in the literature, there are few studies related to the auction mechanism on the outcome of bidding performance under collusion by modeling the whole bidding process. To create an environment that discourages collusion in the first stage [11], we need to conduct research to understand the causal factors to prevent bidder collusion in reverse auctions. Therefore, this study asks: what are the causal factors for bidder collusion?

This work aims to conduct a hazard analysis on a first price sealed-bid reverse auction with bidder collusion by modelling the integrated bidding process. In this study, the design of the research model consists of several sub-processes, including
bidder collusion invitation sub-process, ring-membership construction sub-process, bidding sub-process, bidding evaluation sub-process, and bidding performance comparison sub-process. We have employed a Petri net in this study because it is an event-based modeling approach that can formally and statistically describe the possibilities and impossibilities of a system at runtime. Also, timestamps can be added into the colored tokens to reflect the timing event of the sub-processes. In this work, a Petri net model including ‘color’, ‘time’, and ‘hierarchy’ attributes as a hierarchical colored timed Petri net model is constructed to simulate above subprocesses to understand the behavior of strong bidders and weak bidders in different situations. This work compares the cost-down range performance of six reverse auction cases. These six reverse auction cases consist of different numbers of bidders, bidders’ last winning price, and bidders’ last winning share. Consequently, this work can not only find the causal factors of bidder collusion but it can also identify the situations for causing successful bidder collusion. Based on these findings, several recommendations will be proposed to stop the ring game in a real reverse auction process. Thus, this study helps buyers to take control of reverse auctions. It also motivates researchers in bidder collusion research by demonstrating the use of a hierarchical Petri net model.

II. LITERATURE REVIEW
Auctions/reverse auctions are the environment in which collusion is performed. The auction mechanism, organization mode of auctions, number of bidders, bidder strategy, competition situation and bidder’s cost will lead to different bidding behaviors and results. Therefore, at the beginning of model research, it is necessary to review the overall research of auctions/reverse auctions. At the same time, this work embeds the ring game protocol and execution into the whole auction process as a sub-model, making the whole model closer to reality and more accurately explaining operational problems. A colored and timed Petri net is selected as the modeling tool in this work, which will be described in detail in the second part of the review.

A. AUCTIONS IN SUPPLY CHAIN NETWORKS
An auction is defined as a market resource reorganization mechanism that allocates the market resource at the price of a bid with a series of supply chain collaboration items through a set of special auction rules [12]. It is for this important reason that some opportunistic bidders have carried out collusion in the auctions/reverse auction and this has brought great losses to the auctioneers. Many previous auction studies were conducted under a research setting of one seller with multiple buyers as bidders. But in some cases, the setting can be one buyer with multiple sellers as bidders. Under this situation, a buyer selects one or more sellers from a pool of auction participants. The buyer not only considers the price but also 6W + 2H questions of supply chain management (SCM). Therefore, it is important to study auctions in SCM.

In the marketplace, a deluge of economic transactions are executed under four mechanisms of auctions [13], [14], as follows: (1) ascending-bid, (2) descending-bid, (3) first-price sealed-bid, and (4) second-price sealed-bid. For the first two types of auctions, the price gradually increases or decreases until only one bidder wins the auction. The 3rd and 4th types of auctions are closed bids in which each bidder does not know the other’s information. Finally, the object is sold to the bidder who makes the highest bid (the first-price) and the second-highest bid (the second-price), respectively. The 3rd and 4th types of auctions can be applied in a reverse auction where the lowest bid (the first-price) and the second-lowest bid (the second-price) win the purchase contract. Because bidder collusion likely happens in a first-price sealed-bid reverse auction, this study only addresses the issues in this mechanism. Since the first-price sealed-bid auctions have a wider range of applications, this work selects the first-price sealed-bid auctions as the ring game environment. In this bidding mechanism, a comparative study with collusion or not is modeled in order to compare the auction’s performance.

Generally, a reverse auction consists of five stages. Stage one is bidding invitation (existing deadline). A buyer contacts all the potential suppliers to discuss the terms of purchase contract, including product specification, threshold price, credit period, capacity, and lead time. The 2nd stage is the bidding process (existing deadline). Suppliers prepare their tenders with related information. The next stage is bidding close process (existing deadline). Suppliers submit their tenders on or before the tender deadline. Stage four is bidding open process (existing deadline). The buyer opens and fairly uncovers every bidder’s bid. The last stage is a bidding evaluation. The buyer decides the winner set based on the price and trading conditions. From this perspective, each stage of the whole reverse auction process is time-limited, which also provides a foundation for the selection of Petri nets with time constraints in this work.

In 1956, Friedman [7] developed the earliest auction model. Among four types of auction, the first-price sealed-bid auction is the one which has strong restrictions [9]. Therefore, researchers suggested different mechanisms to relax those constraints [15]–[19]. Recently, Skitmore [20] has compared the outcomes of four leading models and found that the predictions of all models are poor. Consequently, there is a need to develop a better mechanism to prevent bidder collusion.

From the auction point of view, mechanism design, bidding language, and the winner determination are the three most popular issues. Accounting for these three issues in a first-price sealed-bid reverse auction process, this study aims to take a hazard analysis by comparing the outcomes of six cases of the integrated bidding process.

B. MODELING WITH PETRI NETS
Petri nets [21] have been applied in many areas, such as workflow [22], evaluation and event management [23], communications [24], electronics [24], chemistry logistics [25],
single-arm cluster tool with wafer revisiting [26], manufacturing systems [27]–[48], and supervisory control of discrete event systems [49]–[52]. Based on the original paradigm, researchers have extended the Petri net to address its limitations. Therefore, there are now many different versions of Petri nets, including generalized stochastic Petri nets (GSPNs) [53], timed Petri nets (TPNs) [54], colored Petri nets (CPNs) [55], colored timed Petri nets (CTPNs) [56], batch deterministic stochastic Petri nets (BDSPNs) [57], and deterministic and stochastic Petri nets (DSPNs) [58]. Accordingly, there are different simulation tools such as INA [59], TINA [60], CPN [55], and ExSpect [61].

Among the versions of Petri nets, CPNs are widely employed. In a CPN, the colors of tokens specify the notion of token types [62]. Based on the firing rules, a token can be fired. By a transition firing, the color of a token can be changed with a complex data value [63]. Timing information can be added to CPNs as a timestamp for reflecting a real timing event. Thus, a CTPN model is a timed CPN model. By adding time constraints, CTPNs can cope with multiple processes including concurrency, distributed nature, and synchronization [64].

Specifically, CPNs are conducted with high-level programming languages based on the ability of basic Petri nets. They are widely used in modeling systems in which communication, synchronization, and resource sharing play a critical role. Given a place node, all tokens must have token colors that belong to a specified type, which is called a color [62].

In a CPN, each token is bundled with a color, presenting the common identity of tokens. A transition executing some behaviors can fire the token/tokens to the output place based on its related firing rules. The color affixed to a token may be changed by a transition firing, which often represents a complex data value [63].

According to [63], a CPN is defined as a 5-tuple \( CPN = (P, T, C, W, M_0) \), where

1) \( P \) is a set of places;
2) \( T \) is a set of transitions;
3) \( P \cap T = \emptyset \), \( P \cup T \neq \emptyset \);
4) \( C \) is the colored-function defined from \( P \cup T \) into nonempty sets;
5) \( W \) is the incidence-function defined on \( P \times T \) such that \( W(p, t) \in [C(p) \rightarrow \mathbb{Z}]_1 \) for all \( (p, t) \in P \times T \), where \( \mathbb{Z} \) denotes integers; and
6) \( M_0 \), the initial marking, is a function defined on \( P \), such that \( M_0(p) \in [C(p) \rightarrow \mathbb{N}]_1 \) for all \( p \in P \), where \( \mathbb{N} \) denotes the set of nonnegative integers.

The elements of \( C(p) \) and \( C(t) \) are called colors. A place \( p \) is an input place (output place) for a transition \( t \). If \( W(p, t)(c')c'' < 0(W(p, t)(c')c'' > 0) \) for at least one pair of colors \( c' \in C(t) \) and \( c'' \in C(p) \). To formalize the firing rule, a weighted set of transitions is a function defined on \( T \) such that \( X(t) \in [C(t) \rightarrow \mathbb{Z}]_1 \) for all \( t \in T \).

Fig. 1 illustrates a CPN with a net structure, colored tokens, and transition rules.

In the net structure, the initial marking is \( (2, 1, 0) \), where two colored tokens including 32 and 35 are in place \( p_1 \), and one token is in place \( p_2 \). Firing transition \( r \) will remove tokens from places \( p_1 \) and \( p_2 \), and then deposit the token valued as \((x - y)\) into the output place \( p_3 \) conditional on the guard of \( [x > y] \).

The following literature review will focus on the application of Petri nets in supply chain discrete events. Due to the large number of references, Table 1 lists the key studies to show their relevance to this work.

In 2011, Zhang et al. [62] conducted a literature review on Petri nets’ applications in SCM and found only one study that used an auction Petri net model in manufacturing (Nandula and Murali [65]). Most recently, Zhang et al. compared the performance of first-price and sealed-bid auctions with online dynamic auctions [66] and combinatorial auctions [67]. Therefore, using Petri nets to study auctions is rare.

Viswanadham and Raghavan [68] solved the decoupling point location problem in supply chains considering both the procurement process and delivery logistics using GSPNs. Their research also compared the performance of two production planning and control mechanisms, including make-to-stock (MTO) and the assemble-to-order (ATO) based on the framework of integrated GSPN. Therefore, the Petri net method is suitable for comparing different situations. Given that this study attempts to analyze the hazard levels of six cases, so this study considers the Petri net method.

Zhang et al. embed programming solver into transition nodes of a CTPN [67] and they compared the supply chain coordination performance of a first-price sealed-bid multi-object sequential auction with a first-price sealed-bid combinatorial auction. Their study showed how the CTPN tool can be used to graphically describe every stage of a reverse auction, as well as the bidding data and decision algorithms in each one. Therefore, this study follows Zhang et al. to apply the CTPN tool for studying first-price sealed-bid reverse auctions.

To adapt to the development of global supply chain and change the current agricultural product supply mode, Liu and Wu [69] conducted a series of research on relevant countermeasures based on the modeling method of Petri net and Markov chain. They showed that Petri net method can be used to divide the agricultural supply chain transition and analyze the operational efficiency of each link, so as to put forward the integrated distribution model which changes
TABLE 1. List of key literature.

| Ref. NO. | Author          | Year | Title                                                                 | Contribution                                                                 |
|---------|-----------------|------|----------------------------------------------------------------------|------------------------------------------------------------------------------|
| [7]     | Friedman        | 1956 | A competitive bidding strategy                                        | a classic bidding strategy model is constructed                              |
| [62]    | X. Zhang et al. | 2011 | Petri net based application for supply chain management: An overview  | an overview of Petri net application in supply chain is given                 |
| [66]    | Zhang et al.    | 2017 | Performance analysis of reverse auction mechanisms based on Petri nets | comparisons of the performance of first-price and sealed-bid auctions with online dynamic auctions based on CTPN is presented |
| [67]    | Z. Li, et al.   | 2018 | Comparative analysis of sequential and combinatorial auctions based on Petri nets | comparisons of the performance of first-price and sealed-bid auctions with combinatorial auctions based on CTPN is presented |
| [68]    | Raghavan, et al.| 2000 | Performance analysis and design of supply chains: A Petri net approach | the decoupling point location problem in supply chains is considered both the procurement process and delivery logistics using GSPNs is modeled |
| [69]    | Liu, et al.     | 2018 | A Petri net based supply chain system                                | a series of researches on relevant countermeasures based on Petri net and Markov chain are conducted |
| [71]    | MF. Blos, et al.| 2018 | A framework for designing supply chain disruptions management considering productive systems and carrier viewpoints | a framework to model, analyse, control and monitor the robust and global SC based on Petri net is presented considering both PSs and carrier viewpoints |
| [73]    | I. Outmal, et al.| 2016 | Modeling and performance analysis of a closed loop supply chain using first order hybrid Petri nets | a green supply chain based on first-order hybrid Petri nets to map and analyze the complexity and dynamics is modeled |
| [76]    | S. Du, et al.   | 2018 | The collaborative system workflow management of industrial design based on hierarchical colored Petri-net (HCPN) | a management process of the industrial design collaborative system based on hierarchical colored Petri net (HCPN) is modeled |
| [77]    | L. Lu, et al.   | 2017 | The risk management of perishable supply chain based on coloured Petri net modeling | CPNs can accurately describe information classification and information transmission on the condition of integrated information management in the discrete event in a supply chain |

the previous decentralized management model. Furthermore, Petri net methods have recently been applied in supply chain finance [70], supply chain disruptions management [71], and RFID-enabled supply chains [72]. Because the Petri net has commonly been applied to analyze different supply chain coordination problems, this study also attempts to employ the Petri net method.

Researchers have used various varieties of Petri nets to model the supply chains. For example, Outmal et al. [73] modeled a green supply chain based on first-order hybrid Petri nets to map and analyze its complexity and dynamics. Liu et al. designed a new Petri net (PNCA) for conflict analysis in supply chain which assists decision makers in finding suitable solutions to supply chain conflicts [74]. Jennifer et al. used hierarchical Petri nets to discover the conflict in a supply chain before they occur and introduce an unforeseen load on existing systems [75]. Du et al. [76] developed a hierarchical colored Petri net (HCPN) to model the management process of the industrial design collaborative system. Lu et al. regarded Petri nets as information description tool with absolute advantage and conclude that CPN can accurately describe information classification and information transmission on the condition of integrated information management in the discrete event in a supply chain [77].

Because of the large number of enterprises gathered in the supply chain procurement bidding process, the information flow presents the characteristics of interactivity, complexity and centrality. Compared to other analytical methods, CTPNs can classify information flow well. They can reduce the size of Petri nets to an acceptable range with good expandability. At the same time, because each process node of the supply chain has time limit, it is necessary to add time element. Therefore, CTPNs become an idea tool for modeling auction processes. During the collusion invitation process, a stipulated time is set to invite vendors to submit their tenders. Thus, the characteristics of CTPNs are appropriate for simulating bidder collusion invitation. By using CTPNs to study the first-price sealed-bid reverse auctions, CTPNs can accurately present the five steps of a reverse auction with a time limit for each step. During the simulation process, CTPNs can visually show the processes in every time unit as a vidicon and the colors of tokens show the changes of bidding states of every bidder, as happened in a real scenario. Therefore, this work attempts to apply CTPNs to model, analyze, and evaluate the performance of two heterogeneous multi-object reverse auction mechanisms.

III. RESEARCH DESIGN

Although many collusion studies have focused on the issues of collusion detection, collusion equilibrium strategy, auction design, and effect factors, it is important to cope with collusion behavior. However, there have only been a few studies on modeling a procurement bidding auction under bidder collusion with related interactive behavior in a cartel, determining the sponsor of the ring game and the cartel membership. This study attempts to apply Petri nets to model and investigate...
TABLE 2. Notation of model in Section IV.

| notation | set/variable | meaning |
|----------|--------------|---------|
| $n$      | $n = 1, 2, \ldots, n$ | the quantity of bidders |
| $\delta$ | -            | annual loan interest rate |
| $p_s$    | -            | average bidder’s bid |
| $L$      | $L = \{l_i | i = 1, 2, \ldots, n\}$ | bidders’ last winning price |
| $S$      | $S = \{s_i | i = 1, 2, \ldots, n\}$ | bidders’ last winning share |
| $D$      | $D = \{d_i | i = 1, 2, \ldots, n\}$ | bidders’ credit period |
| $C$      | $C = \{c_i | i = 1, 2, \ldots, n\}$ | industry average costs assessed by bidders |
| $c_0$    | $c_0 = \{c_{0i} | i = 1, 2, \ldots, n\}$ | the actual costs of bidders considering holding costs, shortage |
| $Q$      | $Q = \{q_i | i = 1, 2, \ldots, n\}$ | the negotiation price before bid |
| $Y$      | $Y = \{y_i | i = 1, 2, \ldots, n\}$ | the capacity of bidders |
| $\lambda$ | $\lambda = \{\lambda_i | i = 1, 2, \ldots, n\}$ | the every bidder’s estimated number of rivals |
| $X$      | $X = \{x_i | i = 1, 2, \ldots, n\}$ | the current bid (decision variable of bid decision-making method) |
| $H$      | $H = \{h_i | i = 1, 2, \ldots, n\}$ | the current winning share (decision variable of winner decision-making method) |

the bidding performances of first-price sealed-bid reverse auctions with and without collusion. First, a Petri net model of a first-price sealed-bid reverse auction without the probability of cartel excluding the constraints of production capacity is constructed. Then, a Petri net model of a first-price sealed-bid reverse auction under collusion is constructed. In addition to the basic processes of first-price and sealed-bid reverse auction, collusion invitation sub-process, ring-membership construction sub-process, and collusion bidding sub-process are embedded into the time interval of the bidding process. Next, several programming methods are implanted in the rules, including the bidder’s bid solving with or without collusion and the collusion vote decision-making method. Finally, a hierarchical Petri net model is employed to compare cost-down range performances of two auctions with and without collusion.

To reveal the harm of collusion in auctions and some collusion activities according to the bidding data, this study compares two reverse auctions and simulates 1000 groups of results by modeling a comparison rule through a transition node. These 1000 experiments mainly focus on the IT industry in the headset, battery, built-in antenna, external antenna, charger, power button and other relatively competitive market. The supply market of these industries does not belong to a monopoly or an oligopoly, and it has the prerequisite of bidding and the high-quality scene foundation of model simulation experiment. Some supplementary analyses according to the results of simulation are performed. The rest of this work is organized as follows. Sections 4 and 5 propose Petri net models for a first-price sealed-bid reverse auction with collusion or not, respectively. The contrastive simulation is conducted in Section 6 and Section 7 concludes this research.

IV. A PETRI NET MODEL OF STATIC FIRST-PRICE SEALED-BID REVERSE AUCTION WITHOUT COLLUSION
This section briefly describes the previously published Petri net model (see reference [66] for details). On the basis of understanding the model, ring game behavior is added in the next Petri net model, which makes the latter model quite different from the former model, and thus has a significant impact on the bidding performance.

A. ASSUMPTIONS
In this model, a total of 11 assumptions are considered based on many practical factors. According to the assumptions in [66] (see Appendix), Assumptions 1), 2), 6), and 9) ensure an independent information circumstance with the bid, so every bidder makes its decision in a private and rational manner to maximize its profit without any influence from the other bidders. Assumptions 3), 4), 5), 7), 10), and 11) are the constraints of a static first-price sealed-bid reverse auction. Assumption 8) guarantees the comparability between the current and last cost-down performances.

B. NOTATIONS
A total of 13 variables are considered in this model. These variables describe the number of bidders, the estimated average bid price, the actual cost of each bidder, and the historical and current conditions of the successful bid (see Table 2).

C. DESCRIPTION OF A STATIC FIRST-PRICE SEALED-BID REVERSE AUCTION PROCESS
Step 1: A buyer invites vendors who are interested in a purchase contract to participate in the reverse auction process, to negotiate with price, capacity, credit period, and so on. These vendors may have different true costs, estimated industry average costs, capacity, last time winning price, and share.

Step 2: Then, comparing the negotiation prices with threshold price, the vendors with higher negotiation prices belong to Type-1 vendors and the vendors with lower negotiation prices are type-2 vendors.

Step 3: The buyer sets the stipulated time and invites only type-2 vendors to submit their tenders with a sealed form before the stipulated time. All the bids are stored in a dark box (or database) to ensure that nobody knows the bids.

Step 4: After the stipulated time, the buyer opens the dark box and makes a winner-decision based on total costs minimization. At the same time, the buyer needs to check all the winners’ capacity with its demand.
Step 5: The buyer calculates the cost down range and compares the current price with the last price.

Steps 1 and 2 are the bid invitation sub-process. Step 3 is the bidding submission sub-process. Step 4 is the bidding close sub-process and Step 5 is the bidding evaluation sub-process.

D. PETRI NET MODEL OF A STATIC SEALED-BID REVERSE AUCTION

In this study, a CTPN model is constructed that involves a set of concurrent processes formed by a number of temporally related tasks, which are executable by bidders and bundled bidding data. The event rules are formulated according to the assumptions. The behavior process of reverse auctions is set by taking transition nodes to run the auction process rules and by regarding the colored timed tokens as process conditions/results. The time constraints are formulated by referring to the availability of each resource in resource sets and the tasks in the processes.

The interpretations of the Petri net are presented as Tables 3, 4, 5, and 6.

**TABLE 3. Interpretation of the places in the Petri net in Fig. 2.**

| place | description                      |
|-------|----------------------------------|
| e₀    | Vendor resource                  |
| e₀₀   | Vendor resource                  |
| e₁    | Vendor unable to reach the threshold price |
| e₂    | Vendor able to reach the threshold price |
| e₃    | Bid                              |
| e₄    | Winner set                       |
| e₄₀   | Loser set                        |
| e₅    | The result of cost-down range    |

**TABLE 4. Interpretation of the transitions in the Petri net in Fig. 2.**

| transition | description                          |
|------------|--------------------------------------|
| t₁         | Negotiation and filtration           |
| t₂         | Section “Bid-decision making method” |
| t₃         | Section “Winner-decision making method” |
| t₄         | Calculate the cost-down range        |

**TABLE 5. Colored value interpretation of the places in the Petri net in Fig. 2.**

| place | colored token value                     |
|-------|----------------------------------------|
| e₀    | 1*(ID, L, S, C, C₀, λ, Y)++            |
| e₀₀   | 1*(ID, L, S, C, C₀, λ, Y)++            |
| e₁    | 1*(ID, L, S, C, C₀, λ, Q, D, Y)++      |
| e₂    | 1*(ID, L, S, C, C₀, λ, Q, D, Y)++      |
| e₃    | 1*(ID, C₀, Q, D, Y, X)@++              |
| e₄    | 1*(ID, X, H)                           |
| e₄₀   | 1*(ID, X, H)                           |
| e₅    | Cost-down range                        |

**TABLE 6. Colored timed interpretation of transitions in Petri net in Fig. 2.**

| transition | transition rule description               |
|------------|------------------------------------------|
| t₁         | Guard: [Q < threshold price] [11,12]     |
| t₂         | “Bid-decision making method” [21,22]    |
| t₃         | “Winner-decision making method” [31,32] |
| t₄         | Calculate the cost-down range            |

1) BID-DECISION MAKING METHOD

Based on Assumption 2, every bidder aims to maximize the corresponding profit. Hence, a profit function (objective function) is the output. From the bidder’s perspective, their own actual costs are known. The unit profit is the difference between the selling price/bid (X) and actual costs (C₀).

\[
\text{unit profit} = (X - C₀) \quad (1)
\]

According to Friedman’s probability of winning [7], the function \( P(X) \) is represented as:

\[
P(X) = \exp[-\lambda(1 - \sum_{v=0}^{b} \frac{1}{v!} \left( \frac{aX}{C} \right)^v e^{-aX/c})] \quad (2)
\]

The winning of probability \( P(X) \) is the probability of being the lowest bidder with a bid of X. When the rivals are known, \( P(X) \) is simply the product of the probabilities of defeating each of the known rivals [7]. Actually, every bidder cannot get the number of rivals. Friedman [7] introduced a concept of an “average” bidder by combining all previous ratios of an opposition bid to the decision maker’s cost estimate and obtaining one distribution function. In Eq. (2), \( a \) and \( b \) are constants by the probability density function of the ratio of the average bidder’s bid to one bidder’s cost estimate, which has a \( \gamma \) distribution, \( c \) represents the industry average costs assessed by bidders, and \( \lambda \) is the number of rivals estimated by each bidder.

According to Dye and Hsieh [78], the seller’s quantity of winning represented as a function of the price-dependent demand as indicated in Assumption 11) can be formulated as follows.

\[
\text{quantity of winning} = \alpha - \beta X \quad (3)
\]
where $\alpha$ and $\beta$ are two nonnegative variables, representing the total demand scale and coefficient of price-dependent, respectively, in this echelon.

Profit = (unit profit) * (quantity of winning) * (probability of winning)

By Eqs. (1), (2), and (3), profit can be represented by:

$$\Pi = \left\{ \exp\left[-\lambda(1 - \sum_{v=0}^{b} \frac{1}{v!} \left( \frac{aX}{C} \right)^{v} e^{-aX/c}}) \right] \right\} \times (X - C_0) \times (\alpha - \beta X)$$

Note that the capacities should be enough to support corresponding winner share quantity, denoted by $t_i \leq \alpha - \beta * x_i$. As a rational bidder indicated in Assumption 9), every bidder's bid is larger than its actual costs, as denoted by $x_i > c_0i$. A bid-decision model follows.

Maximize $\Pi = \left\{ \exp\left[-\lambda(1 - \sum_{v=0}^{b} \frac{1}{v!} \left( \frac{aX}{C} \right)^{v} e^{-aX/c}}) \right] \right\} \times (X - C_0) \times (\alpha - \beta X)$

Subject to:

$t_i \leq \alpha - \beta * x_i$

$x_i > c_0i$

2) WINNER-DECISION MAKING METHOD

According to Step 5, the buyer knows every bidder’s bid. Resulting from the bid decision-making model, the value of $X$ is obtained and the current winning share, denoted by $H$, is a decision variable.

First, the unit purchasing price is calculated. For instance, two bidders bid with same value as $4.5 and different credit periods. Which one is better? The auctioneer converts the same two bids into “present net value”, denoted by $x_i(1 + \delta)^{-d}$ in finance, based on the annual interest rate, denoted by $\delta$, and every bidder’s credit period, denoted by $d$. Consequently, the weighted average of unit purchasing price can be found as follows.

$$\sum_{i=1}^{n} x_i(1 + \delta)^{-d} h_i$$

(5)

In a supply chain, every partner not acting as an end customer is both a buyer and a seller. The buyer’s downstream demand is also price-dependent according to Assumption 11), and the auctioneer’s purchasing price is decided by the weighted average of the winners’ bids, as denoted by $\sum x_i h_i$, which leads to the buyer’s downstream demand, as follows.

$$\alpha' - \beta' \sum_{i=1}^{n} x_i h_i$$

(6)

where, $\alpha'$ and $\beta'$ are two nonnegative variables, representing the total downstream demand scale and the coefficient of price-dependent, respectively, in this echelon.

Then, the total costs ($TC$) are the product of demand and unit purchasing price formulated as follows.

$$TC = (\alpha' - \beta' \sum_{i=1}^{n} x_i h_i) \sum_{i=1}^{n} x_i(1 + \delta)^{-d} h_i$$

(7)

As the constraints, the sum of every bidder’s winning share is equal to 1, denoted by $\sum_{i=1}^{n} h_i = 1$, and the winner’s capacity should support its winning quantities, denoted by $t_i \leq (\alpha' - \beta' \sum_{i=1}^{n} x_i h_i) h_i$. Consequently, a winner-decision making method is formulated as follows.

Minimize $TC = (\alpha' - \beta' \sum_{i=1}^{n} x_i h_i) \sum_{i=1}^{n} x_i(1 + \delta)^{-d} h_i$

subject to: $t_i \leq (\alpha' - \beta' \sum_{i=1}^{n} x_i h_i) h_i$

$$n \times h_i = 1$$

Finally, the cost-down range (CDR) issue can be solved. According to the definition of cost-down, the aim of the issue is to find the difference between the current and last prices. As a ratio, the difference should be divided by last price, as follows.

$$CDR = \frac{[(\text{Last Price}) - (\text{Current Price})]}{(\text{Last Price})} \times 100\%$$

where the weighted average of last unit price is denoted by $\sum_{i=1}^{n} l_i s_i$ based on the last bid (l) and the last winning share (s), while the weighted average of current unit price can be denoted by $\sum_{i=1}^{n} x_i h_i$ conditional on current bid (x) and current winning share (h). The cost-down range (CDR) is formulated as follows.

$$CDR = \frac{(\sum_{i=1}^{n} l_i s_i - \sum_{i=1}^{n} x_i h_i)}{\sum_{i=1}^{n} l_i s_i} \times 100\%$$

(8)

V. A PETRI NET MODEL OF A STATIC FIRST-PRICE SEALED-BID REVERSE AUCTION WITH COLLUSION

A. ASSUMPTIONS

Collusion activities are the key difference between the first-price sealed-bid reverse auction with collusion or not, which results in series of additional assumptions, as follows.

1) The agreement of participant to a ring membership is dependent on the profit that they could receive compared with the profit that they could get without collusion. If the profit is larger than the profit in an auction without collusion, then they will agree, and vice versa.

2) There are at least two bidders. This assumption ensures the operation probability of an auction and collusion.

3) When an auction is closed, the result of the winner will be declared by the auctioneer. All the bidders will know the
information of the last winner with its bid and the bidding ranking of every bidder. This is very important to a cartel sponsor.

(4) There are Strong bidders and Weak bidders. There are strong and weak bidders according to their dominant positions of prices. The last winner is definitely the strong bidder, and the other bidders with similar prices varied from 0%-1% are also strong bidders, who could be potential competitors to the last winner. The weak bidders are the other bidders besides the strong bidders.

(5) The second-lowest bid holder with a bid closed to the lowest holder, the last winner, is the cartel sponsor. A bidder is called a weak bidder if its price or truthful cost is more than 1% more than the last winner. Consequently, the weak bidder is still in a weak position in cutting prices. If a weak bidder wishes to construct a ring membership, then it could not invite weak bidders, leading to a failure of collusion without winning the auction based on high cost and high bid. If it invites strong bidders to the cartel, then strong bidders would not agree according to the Assumption (1), which leads to a situation adverse to its profit also based on the high cost of weak bidders. As the strongest bidder besides the last winner, it attempts to conduct a strong-strong union situation instead of oppressively competition. In other words, the ring membership is organized by the competitor’s strong bidders.

(6) Besides the cartel sponsor and the last winner, the other strong bidders will be invited by the cartel sponsor to avoid the competition and reach a higher profit.

(7) As the strong bidders in the industry, the industrial average cost and the approximate cost of weak bidders are well known by these strong bidders. Consequently, the submitted bid of the cartel sponsor should be lower than the lowest bid of weak bidders to ensure the winner’s right of an auction under collusion. In this case, the cartel sponsor submits 1.5% lower than the lowest bid of weak bidders, the other ring member should then submit a bid higher than that of the cartel sponsors and lower than its own negotiation price.

(8) Ring membership cannot be constructed if any ring member does not agree.

Making a profit is the simple and general aim for doing business, so the first assumption is that profit pursuit is the driving force of the collusion. The significance of the auction lies in the competition under the environment of sufficient asymmetric information. Therefore, it is in line with the bidding logic to take two or more bidders as described by the second assumption. The third assumption illustrates one of the information basis and important sources of collusion, as well as the “learning” mechanism. The fourth assumption provides the subject of collusion, which is a profitable group and the subject in competition. This assumption is in line with the collusion logic. It also indicates that those suppliers who can reduce the price belong to the property of companion, which is very similar to reality. The fifth and sixth assumptions provide an idea of the serial bid initiator; that is, the strongest bidder hopes to obtain the order through cooperation rather than competition. However, this cooperation cannot be achieved if a ring member does not agree, so the eighth assumption is used to reflect this possible situation. As a peer, each bidder has a relatively clear understanding of the average cost, competitors and industry profits, so the ring team needs to offer lower than the lowest price among the remaining bidders, which is the seventh assumption.

B. NOTATIONS
There are some notations in the model, which are given in Table 7.

C. DESCRIPTION OF A STATIC FIRST-PRICE SEALED-BID REVERSE AUCTION WITH COLLUSION PROCESS
In the following, a series of questions are illustrated. (1) Why does a bidder wish to organize a cartel or ring membership as a cartel sponsor?
(2) Who will be invited by the cartel sponsor? And, what is the reason behind this invitation?
(3) Why does the invited bidder agree with the ring membership?
(4) What about the behavior of every cartel member? In other words, what is the bidding decision-making method of ring membership?
(5) What about the behavior of other bidders not involved the ring game?

Fig. 3 shows the process of static first-price sealed-bid reverse auction with collusion. A description of the five steps of the process follows.

Step 1: A collusive idea arises from a strong bidder who becomes the cartel proponent. The proponent knows the other strong bidders. During the period from tender publication to bid submission deadline, the proponent invites the other strong bidders to perform a cartel in the aim to get more profit. The proponent introduces its collusion plan, including the cartel designated bid, the profit allocation mechanism, temporary bond costs paid to the third party, and every ring member’s supporting action.

Step 2: When other strong bidders have received the cartel invitation from the proponent, they compare the profit with and without collusion. If the profit with collusion is higher than the profit without collusion, then the strong bidders agree.
TABLE 7. Notation of model in Section V.

| notation | set/variable | meaning                        |
|----------|--------------|--------------------------------|
| $n$      | $n = 1, 2, n$| number of strong bidders       |
| $m$      | $m = 1, 2, m$| number of weak bidders         |
| $k$      | $k = 1, 2, k$, where $k = n + m$| number of all the bidders |
| $\lambda_1$ | $\lambda_1 = \{\lambda_{i1} | i = 1, 2, \ldots, n\}$ | the strong bidder’s estimated number of rivals |
| $\lambda_2$ | $\lambda_2 = \{\lambda_{i2} | i = 1, 2, \ldots, m\}$ | the weak bidder’s estimated number of rivals |
| $\lambda$ | $\lambda = \{\lambda_{i} | i = 1, 2, \ldots, n\}$ | the every bidder’s estimated number of rivals |
| $P_a$    |              | average bidder’s bid           |
| $H$      | $H = \{h_i | i = 1, 2, \ldots, k\}$ | bidders’ last winning price    |
| $R$      | $R = \{r_i | i = 1, 2, \ldots, k\}$ | bidders’ last winning share   |
| $C$      | $C = \{c_i | i = 1, 2, \ldots, k\}$ | the industry average costs assessed by bidders |
| $C_0$    | $C_0 = \{c_0 | i = 1, 2, \ldots, k\}$ | the actual costs of bidders considering holding costs, shortage costs, and opportunity costs |
| $Q$      | $Q = \{q_i | i = 1, 2, \ldots, k\}$ | the negotiation price in pre-auction |
| $X_h$    |              | the cartel designated bid      |
| $X_e$    |              | the ring member’s collusive bid|
| $C_e$    |              | the temporary bond paid by ring member to neutral third party |
| $C_a$    |              | the cartel member’s average costs involving the distribution of interests among the members of the group |
| $X$      | $X = \{x_i | i = 1, 2, \ldots, k\}$ | the current bid in a first-price sealed-bid auction without collusion (decision variable of bid decision-making method) |
| $B$      | $B = \{b_i | i = 1, 2, \ldots, k\}$ | the current winning share      |
| $V$      |              | the strong bidder’s vote results, if a strong bidder agrees to join the cartel, its value of $v$ is equal to 1 and vice versa |
| $ID$     |              | the bidder’s number            |
| $CDR$    |              | the cost-down range of bidding performance |

to join the cartel, and vice versa. Note that the ring game will only begin if every strong bidder votes ‘yes.’

**Step 3:** If a cartel is constructed, then a designated bidder represents the cartelar’s bid. The other strong bidders submit relatively higher prices. The weak bidders also submit their bids. On the condition of non-collusion, every strong bidder is an individual entity with other rivals.

**Step 4:** When the auction finishes, the auctioneer opens the bid set, checks all the bids, and announces the winner of the auction.

**Step 5:** The buyer calculates the cost down range and compares the current price with the last price.

Step 1 is the collusion invitation sub-process. Step 2 is the collusion decision-making sub-process. Step 3 is the bidding submission sub-process. Step 4 is the bidding close sub-process and Step 5 is the bidding evaluation sub-process.

This action must be completed within the time specified in each stage of the reverse auction, otherwise all the work will be meaningless.

**D. PETRI NET MODEL OF A STATIC SEALED-BID REVERSE AUCTION WITH COLLUSION PROCESS**

The Petri net model of a first-price sealed-bid reverse auction with collusion is relatively complex. There are seven transition nodes from $t_{c1}, t_{c2}, t_2$ to $t_6$ conducting tasks of collusion policy conveying, collusion participating decision-making, bidding process with collusion, bidding process without collusion, bidding evaluation process, and cost-down range calculation. The critical and varied transition nodes execute tasks considering a ring game different from the auction without collusion. $t_{c1}$ is the beginning of the ring game with token valued of collusion sponsor and related bidding data, while $t_{c2}$ is the collusion participating decision-making process with the output of the vote of agreement of collusion participation. If the ring membership is established with $V = 1$, then the bidding task will execute the transition $t_3$. However, if the construction of ring membership fails, then the bidding task will execute transition $t_2$ where all the bidders submit their bid set within the deadline of bidding close process. The interpretations of the Petri net are presented as Tables 8, 9, 10, and 11. In particular, Table 8 and Table 10 show the collusion initiation, collusive agreement decision-making, bidding and bid evaluation in a first-price sealed-bid reverse auction with collusion. In addition, Table 8 and Table 10 are input and output of these transition nodes.

According to the five time-limited stages of reverse auction mentioned in the literature review, the relevant stages are...
TABLE 8. Interpretation of the places in the Petri net in Fig. 4.

| place | description |
|-------|-------------|
| s1    | 1st strong bidder |
| s2    | 2nd strong bidder |
| s3    | 3rd strong bidder |
| w1    | 1st weak bidder |
| w2    | 2nd weak bidder |
| e1    | Collision policy |
| e2    | collusive bidder subset |
| e3    | Individual strong bidder |
| e4    | Bid set |
| e5    | Loser set |
| e6    | Winner set |
| e6'   | History weighted average price |
| e7    | Cost-down range (CDR) |

TABLE 9. Interpretation of the transitions in the Petri net in Fig. 4.

| transition | description |
|------------|-------------|
| tc1        | Collusion policy conveying |
| tc2        | Collusion participating decision-making |
| t2         | Bidding process without collusion |
| t3         | Bidding process with collusion |
| t4         | History weighted average price calculation |
| t5         | Cost-down range calculation |
| t6         | Bidding evaluation process |

TABLE 10. Colored value interpretation of the places in the Petri net in Fig. 4.

| place | token value |
|-------|-------------|
| s1    | ID, Xh, C, e, n, Cc, C0, 1, 2, Q, H, R |
| s2    | ID, C0, C, Q, H, R |
| s3    | ID, C0, C, Q, H, R |
| w1    | ID, C0, C, Q, H, R |
| w2    | ID, C0, C, Q, H, R |
| e1    | ID, Xh, c, 1, 20@Timeinterval |
| e2    | ID, Y = 1, Xh, Xe@Delay |
| e3    | ID, C0, C, Q, @Delay |
| e4    | ID, X, B@DeadlineTime |
| e5    | ID, X, B |
| e6    | ID, X, B |
| e6'   | ID, H, R |
| e7    | Real |

attached to time labels in both Table 6 and Table 11. These periods follow the logic of the reverse auction stage with values in order of logic flow.

1) THE COLLUSIVE AGREEMENT OF FIRST-PRICE SEALED-BID REVERSE AUCTION

Generally, the first-price sealed-bid reverse auction collusive agreement includes the cartel designated bid, the profit allocation mechanism, temporary bond costs paid to the third party, and every ring member’s supporting action.

a: COLLUSIVE BID SOLVING

Based on Assumption 2), the objective of every bidder is profit maximization. From the view of the proponent (one of the strong bidders), the unit profit is the difference between selling price/bid ($X_h$) and truthful costs ($C_0$), that can be represented by:

$$\text{Unitprofit} = (X_h - C_0)$$  \hspace{1cm} (9)

According to Friedman’s probability of winning [7], the function $P(x)$ is represented as:

$$P(x_h) = \exp[-\lambda_2(1 - \sum_{v=0}^{b} \frac{1}{v!} \left\{ \frac{ax_h}{C} \right\}^v e^{-ax_h/c})]$$  \hspace{1cm} (10)

The winning of probability ($P(x_h)$) is the probability of being the lowest bidder with a bid of $X_h$. When the rivals are known, $P(x_h)$ is simply the product of the probabilities of defeating each of the known rivals [7]. Actually, every bidder cannot obtain the information about the numbers of rivals. Friedman [7] introduced the concept of an “average” bidder by combining all the previous ratios of an opposition bid to the decision maker’s cost estimate and obtaining one distribution function. In Eq. (11), $a$ and $b$ are constants in the probability density function of the ratio of the average bidder’s bid to one bidder’s cost estimate which has a $\gamma$ distribution, $c$ denotes the industry average cost estimated by bidders, and $\lambda_2$ is the proponent’s estimated number of rivals excluding the strong bidders. Under the condition of collusion, collusive bidder subset consisting of all the strong bidders can be regarded as one bidder who interacts with weak bidders on behalf of cartel organization. Based on the concept of strong and weak bidders, the probability of winning of cartel increases significantly.

Profit = (unit profit) * (probability of winning)

By Eqs. (10) and (11), the total profit of proponent with collusion can be represented by:

$$\Pi(X_h) = (X_h - C_0) \times \exp[-\lambda_2(1 - \sum_{v=0}^{b} \frac{1}{v!} \left\{ \frac{ax_h}{C} \right\}^v e^{-ax_h/c})]$$  \hspace{1cm} (11)

Note that as a rational bidder, its bid is larger than its actual costs, and the bid should not be higher than the negotiation price in pre-auction.
Thus, the collusive solving bid of the proponent is formulated as follows:

$$\text{Maximize } \Pi(X_h) = (X_h - C_0) \times \exp[-\lambda (1 - \sum_{v=0}^{b} \frac{1}{v!} \left( \frac{ax_h}{C} \right)^v e^{-ax_h/c} ]$$

Subject to $X_h > C_0$

$$X_h \leq Q.$$

\[ (12) \]

### b: SIDE-PAYMENT (S-P)

If a designated strong bidder wins the contract, then the total profit will be equally allocated to each ring member. The total profit of cartel is the difference between the bid of winning and cartel members’ average costs

$$\Pi'(X_h) = (X_h - C_a)$$

Based on the notation of $n$, there are $n$ strong bidders/ring members. Consequently, the side-payment of every ring member can be presented as

$$\text{side - payment } = \frac{X_h - C_a}{n}$$

(13)

It is worth noting that ring-members bond themselves with paying the temporary cost ($C_e$) to neutral third-party, and this bond will return when the corresponding ring-member conducts with the cartel rule.

### c: BID SOLVING WITHOUT COLLUSION

From the perspective of profit maximization, every individual bidder bids when its profit reaches the maximum value. As in the analysis process of the collusive bid, the unit profit is the difference between selling price/bid ($X$) and truthful costs ($C_0$), which can be represented by:

$$\text{Unit profit } = (X - C_0)$$

(14)

The formulation of the probability of winning is the same as that used in the collusive solving bid section, excluding the difference of the number of rivals and the bid valuation, which is represented as:

$$P(X) = \exp[-\lambda (1 - \sum_{v=0}^{b} \frac{1}{v!} \left( \frac{ax}{C} \right)^v e^{-ax/c} ]$$

(15)

The variables and parameters involving in the probability of winning function are basically the same as those in the collusive bid solving section. Note that the number of rivals is denoted by $\lambda$, the estimated number of competitive bidders including strong and weak bidders.

According to the profit equation denoted by Profit = (unit profit) *(probability of winning), by substituting Eqs. (16) and (10) into the profit equation, a profit function without collusion can be formulated as follows.

$$\Pi(X) = (X - C_0) \times \exp[-\lambda (1 - \sum_{v=0}^{b} \frac{1}{v!} \left( \frac{ax}{C} \right)^v e^{-ax/c} ]$$

(16)

Consequently, a bid solving without collusion is formulated as follows.

$$\text{Maximize } \Pi(X) = (X - C_0) \times \exp[-\lambda (1 - \sum_{v=0}^{b} \frac{1}{v!} \left( \frac{ax}{C} \right)^v e^{-ax/c} ]$$

Subject to: $X > C_0; \quad X \leq Q.$

(17)

Before the cartel construction and related rule is conducted, every bidder may have an individual bid without collusion. Then, when strong bidders excluding the proponent receive the invitation of a ring game, they will compare the collusion effect on their profit. If they vote ‘yes,’ then they will bid consistently with the cartel rule (e.g. $X_r > X_h$) to ensure that the designated bidder can win the contract. Conversely, even if one of the strong bidders votes ‘no,’ then the ring-membership cannot be constructed and they will submit an individual bid without collusion. Obviously, the collusion decision-making should be formulated to guide the result of votes, which will be described in the next subsection.

### d: COLLUSION DECISION-MAKING

Profit is the key for collusion decision-making. In other words, an invited strong bidder makes the collusion decision comparing the profit in both the collusive and the non-collusive cases. If the profit with the cartel is larger than without, then it prefers the ring game, and vice versa. Thus, the collusion-based and non-collusion-based profit functions should be formulated. The strong bidder votes when it has made a decision on whether or not to join the cartel. $V = 1$ (vote as “yes”) or $V = 0$ (vote as “no”). Consequently, the collusion decision-making can be represented as follows.

$$V = \begin{cases} V = 1 & \text{side - payment } > \Pi(X) \\ V = 0 & \text{side - payment } \leq \Pi(X) \end{cases}$$

(17)

### e: SIMULATION RESULTS

To demonstrate the collusion harm to the performance of a first-price sealed-bid reverse auction, six experiments were performed by using simulation data that were based on the bidding for a 3.5mm earphone bundled with a music player. The results of the experiments are reported in Table 12, which lists the different performance of cost-down range with CDRc and CDR representing a first-price sealed-bid reverse auction with or without ring membership, respectively. The harm level is the value of $\text{CDR} - \text{CDR}_c$. Relatively new sets denoted by $X_r, NPV_r, B, profit, agree, and CDR$ represent the set under collusion of the bid price, net present price, current share, profit, vote for agreement of ring membership based on the collusion sub-model, and cost-down range performance under collusion. In contrast to the first-price sealed-bid reverse auction without ring membership, the results of simulation reflect the changes caused by the ring membership based on the collusion sub-model. First, the strong bidders with a competitive bid to submit are the last two bidders, while the weak bidders without competition are not in the
TABLE 12. Simulation results of hazard analysis.

|   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|
|   | N | L | S | Q | C₀ | X₀ | H | profit | N | PV | P₀ | λ | CDR | X₀ | N | PV₀ | B | profit | agree | CDR₀ | harm |
| 1 | 3.5 | 0.3 | 1.1 | 2.8 | 2.94 | 0 | 0 | 2.8 | 2.8 | 0 | 0 | 0 | 26.1% | 2.94 | 2.85 | 0 | 0 | NO | - | - |
| 2 | 3.5 | 1.1 | 3.0 | 2.8 | 2.94 | 0 | 0 | 2.8 | 2.8 | 0 | 0 | 0 | 26.0% | 2.94 | 2.82 | 0 | 0 | NO | - | - |
|   | 3.7 | 1.1 | 3.0 | 2.8 | 2.94 | 0 | 0 | 2.8 | 2.8 | 0 | 0 | 0 | 26.0% | 2.94 | 2.82 | 0 | 0 | NO | - | - |
|   | 3.5 | 1.1 | 3.0 | 2.8 | 2.94 | 0 | 0 | 2.8 | 2.8 | 0 | 0 | 0 | 26.0% | 2.94 | 2.82 | 0 | 0 | NO | - | - |
|   | 3.7 | 1.1 | 3.0 | 2.8 | 2.94 | 0 | 0 | 2.8 | 2.8 | 0 | 0 | 0 | 26.0% | 2.94 | 2.82 | 0 | 0 | NO | - | - |
|   | 3.5 | 1.1 | 3.0 | 2.8 | 2.94 | 0 | 0 | 2.8 | 2.8 | 0 | 0 | 0 | 26.0% | 2.94 | 2.82 | 0 | 0 | NO | - | - |
|   | 3.7 | 1.1 | 3.0 | 2.8 | 2.94 | 0 | 0 | 2.8 | 2.8 | 0 | 0 | 0 | 26.0% | 2.94 | 2.82 | 0 | 0 | NO | - | - |
|   | 4.5 | 1.1 | 3.0 | 2.8 | 2.94 | 0 | 0 | 2.8 | 2.8 | 0 | 0 | 0 | 26.0% | 2.94 | 2.82 | 0 | 0 | NO | - | - |
|   | 3.5 | 1.1 | 3.0 | 2.8 | 2.94 | 0 | 0 | 2.8 | 2.8 | 0 | 0 | 0 | 26.0% | 2.94 | 2.82 | 0 | 0 | NO | - | - |

In this work, a total of 1000 cases have been simulated, which can help us find the rule of collusion. Table 12 lists three groups of reverse auctions (case 1-case 3) involving four bidders and three groups of reverse auctions (case 4-case 6) involving two bidders. We will draw relevant conclusions from the detailed data and explanations of these six cases. Table 12 compares reverse auctions with collusion or not. The set “L” to the set “CDR” correspond to a first-price sealed-bid reverse auction without collusion. The set “X” to the set “CDR” corresponds to that with the ring game. Whether or not to make a together-conspired bidding depends on whether the ring member considers it profitable, and “harm” is the loss to the bidder equaling to the difference between the normal cost-down range and the one involving collusion.

In many simulation results, six cases have been selected to explain in detail. In these six cases, 3.5mm headset is the subject matter. From Case 1 to Case 3 is four bidders involved, while the rest cases are two bidders involved. Since the earphone is a general configuration for a period of time, there is a need for periodic purchasing auction. These six cases all contain historical bidding data, such as the last winner, the winner’s bid, and the winner’s share etc. These historical data forms different information environment for bidding and collusion in this stage. Furthermore, it produces different cartel results and auction performance. In fact, the collusion must be conducted on the basis of familiarity or self-perceived familiarity with the subject matter, target price, average cost, number of bidders, etc., or it will lead to legal disputes.

In case 1, the last winner (as a strong bidder) with the last bid value of “$3.3” is invited by the second-lowest bidder (another strong bidder) to construct a ring membership to make their own profit higher than their profit without collusion. The actual costs for another strong bidder and another two weak bidders are “$2”, “$2.8”, and “$2.8”, respectively, ring membership submits the lowest bid.

VI. A HIERARCHICAL PETRI NET MODEL FOR COMPARISON

After introducing two types of first-price sealed-bid reverse auctions with collusion or not, this section analyzes the harm caused by collusion behavior with a worse cost-down range performance. Two input place nodes in Fig. 5 including the place of $e_{5_0}$ and $e_{7_0}$ are outputs of the Petri net model of a static first-price sealed-bid reverse auction without collusion in Fig. 2, represented by the place of $e_{5_0}$, and output of the Petri net model of a static first-price sealed-bid reverse auction with collusion, represented by $e_{7_0}$ in Fig. 4, respectively. A hierarchical Petri net model employing transition $t$ compares the performance of two different auctions and fires two kinds of tokens to places of “1” and “0”, as follows.

In many simulation results, six cases have been selected to explain in detail. In these six cases, 3.5mm headset is the subject matter. From Case 1 to Case 3 is four bidders involved, while the rest cases are two bidders involved. Since the earphone is a general configuration for a period of time, there is a need for periodic purchasing auction. These six cases all contain historical bidding data, such as the last winner, the winner’s bid, and the winner’s share etc. These historical data forms different information environment for bidding and collusion in this stage. Furthermore, it produces different cartel results and auction performance. In fact, the collusion must be conducted on the basis of familiarity or self-perceived familiarity with the subject matter, target price, average cost, number of bidders, etc., or it will lead to legal disputes.

In case 1, the last winner (as a strong bidder) with the last bid value of “$3.3” is invited by the second-lowest bidder (another strong bidder) to construct a ring membership to make their own profit higher than their profit without collusion. The actual costs for another strong bidder and another two weak bidders are “$2”, “$2.8”, and “$2.8”, respectively, ring membership submits the lowest bid.
value of “$2.91” \((NPV = 2.78)\) to win the auction and has a side-payment as “$392.25”. Consequently, \(V = 0\), meaning that ring membership is not accepted because this profit is less than the profit for the strong bidder without collusion.

As for Cases 2 and 3 in the results of simulation, ring membership is accepted because of a higher profit resulting from bidder auction. In Case 2, the actual costs of two weak bidders and two strong bidders are “$3.7”, “$3.7” and “$2”, “$2”, respectively. Thus, the weak bidders could not submit a bid price to compete with strong bidders. Consequently, the cartel sponsor can submit the negotiation price in pre-auction as the bid price of “$3.4” (which is just a little lower than the actual cost of the lowest weak bidder “$3.7”) and gets a higher profit of “$629.69” than the profit of without collusion “$440”. However, the ring game behaviour of the cartel causes damage to the performance of the cost-down range in an auction from 30.3% dropping to 2.9%, which means that in a collusive bidding auction, the auctioneer/buyer pays 27.4% more than the normal cost from a normal auction (without collusion).

In Case 3, the lowest actual cost of the weak bidders is $3.5 (5.41% lower than Case 2); the negotiation price will be $3.2 (5.88% lower than Case 2); the performance of cost-down range will be 8.6%. Therefore, the harm to an auction will be lower when the actual cost of the weak bidders is lower. The harmful effect on performance difference is 21.7%. From there, the relativity between the strong bidder and the weak bidder is significant for the cost-down range.

The other three cases (Cases 4, 5, and 6) are all collusive. There are two strong bidders, which is often the case when only two evenly matched suppliers are left to bid after several rounds of bidding. They only estimate the number of rivals and do not make any judgments of the weak bidders’ actual costs. The results of the simulation show that in this case, the ring membership wins the bid with a higher probability. The harm caused by ring membership can obviously be reached. In Case 1, there is no collusion and no damage stemming, and the other three cases lead to harmful effects on performance difference with 18.9%, 13.1%, and 30.3%. These three cases illustrate the importance of large numbers of homogeneous competition in procurement bidding.

Case 2 has the same bidding initial data as Case 3, but the different collusion strategies result in diverse returns and cost-down range. It is worth noting that only two bidders participated in Case 4, Case 5, and Case 6. This situation is a first-price sealed-bid reverse auction where each bidder is unaware of how many bidders are involved in this case. However, after announcing many rounds of the winner’s data, the ring game partner can guess the number of bidders with high probability, and then adjust their own collusion strategies. In this sense, the harm to the auctioneer mainly depends on the degree of familiarity of the collusion partner with the bidding information. In other words, if the bidding information is not derived from historical data to a large extent, but is generated according to the random changes of the competitive environment, then the environment for the ring game is very unfavorable. The auctioneer can change the cartel planer’s pre-judgment of the bidding by changing the bidding information including the number of bidder, bidding objects, the auction mechanism and so on. Through this approach, the auctioneer can achieve the goal of preventing or minimizing the harm of collusion.

VII. CONCLUSION
A. THE CASUAL FACTORS OF BIDDER COLLUSION
Bidder collusion can appear in an auction and is one of the most important challenges for bidding auctions in a supply chain. This study develops a CTPN model and successfully demonstrates how to use CTPNs to simulate six situations of a first-price sealed-bid reverse auction. The results of six experiments using simulation data conclude that cost-down range performance is always damaged by the behavior of a ring game and the harm caused by collusion is different because of the following causal factors.

First, information asymmetry is a causal factor for bidder collusion. In an ideal case, the information for every bidder should be independent and symmetrical. However, in the real case, strong bidders can easily guess the lowest cost of weak bidders but weak bidders find it difficult to know the cost of strong bidders. Therefore, there is information asymmetry between strong bidders and weak bidders. In a bidder collusion case, strong bidders take this advantage. They not only hold competitive bids but also share the information with ring members to conducting collusion.

Second, the relationship between strong bidders is another causal factor. If two weak bidders collaborate with each other to conduct a ring game, then they usually may not win the bid because their actual costs are both high. Then, if a weak bidder attempts to invite a strong bidder into a cartel, the strong bidder may usually refuse because the collusion lowers its profit from collusion. As is assumed, ring membership is given to strong bidders who hold competitive bids, actual costs that are close to each other, and whose information is well known. A strong-cartel with strong bidders is an assumption corresponding to logic. The relationship between every bidder depends on following situations.

1) If the last bid of every strong bidder is very close to its actual cost, then the ring game is meaningless because of the lower side-pay.

2) If there is a large price space between the last bid and its actual cost for every strong bidder in a price battle, then there is a high probability that bidder collusion may exist.

3) If the number of strong bidders is very small, whatever the numbers of weak bidders, then the cartel will have 100% probability to win the bid if there are only two strong bidders in a reverse auction with a given situation (e.g. one of the strong bidders is forbidden because of its quality).

To study bidding auctions in the supply chain, Zhang et al. [66] used CTPNs to develop models for...
analysing sequential and combinatorial auctions. This work supplements Zhang et al.’s [67] study by comparing the hazard levels of six cases of the integrated bidding process. It contributes to a mechanism that can be further applied to study similar problems in different types of auctions. This study provides valuable insights for supply chain coordination in auction operations.

### B. IMPLICATIONS FOR PRACTICE

Several solutions can be recommended to prevent bidder collusion. According to the causal factors, including a transformation of information circumstance of a reverse auction, an introduction of a new bidder to an auction can cause the catfish effect on the whole bidders set, leading to new information with new independence and symmetry of information. Meanwhile, information asymmetry is one of the causes of bidder collusion. The buyers can set a communication platform for sellers to share market information. This can increase the transparency of the information among weak bidders. In case weak bidders can also estimate the cost of strong bidders, then the strong bidders will have less intention to set a ring game.

The results of cases 2 & 3 indicate that when the actual cost of the weak bidders is lower, the harm to an auction will be lower. The buyers can provide sellers with a list of global raw materials suppliers. This helps weak bidders to seek a lower cost of materials to reduce their costs. Under certain circumstances, for some important components, buyers can bargain with component suppliers to obtain a better price for all of the bidders.

The results of cases 4, 5, & 6 indicate that when the negotiation price is lower, the harm to an auction will be lower. Therefore, setting the negotiation price to a reasonable level can prevent an unreasonably high forbiddance bid. This is a valid way to stop bidder collusion based on the claim of a bid being equal to or lower than the negotiation price, which could become a very efficient measure to decrease the probability of bidder collusion.

If buyers apply these methods (including often changing the information circumstance, recruiting new bidders, and revising price requirements), then this will lead to an unknown bidding environment and will confuse the setting of a ring game.

### C. LIMITATIONS

The limitations of this study are that it only considered two strong bidders because three or more strong bidders would be unprofitable, it assumed that strong bidders can estimate the costs of the weaker bidder because they have more information sources and bargaining power with material suppliers, and finally it did not consider the level of margin associated with the industry.

### APPENDIX

Section IV.A-Assumptions

1) It is of information symmetry between bidders/sellers and of asymmetry between sellers and buyers. That is to say, every bidder only holds its private information (business secret), and it cannot get any information of other competitors. However, a buyer knows some information that is unknown to bidders, such as the number of bidders, capacity of every bidder, and negotiated price.

2) The objective of every bidder is profit maximization.

3) Each rival is likely to bid as it has done in the past and the behavior is imperceptible and never changed by others.

4) Share auctions: it is admissible that a winner set must include the bidder with the lowest price, and the second lowest price may be a member of a winner set, which is dependent on the actual demand of the buyer and the capacity of every bidder.

5) Price sensitive demand for every bidder and a buyer. Scale of demand is the quantity of contract which is the demand forecasting agreement of a long term, and a bidder predicts price sensitive demand by itself for more precise order.

6) Every bid is independent.

7) Single bidding object and multi-units: there is only one bidding object with multiple quantities in the reverse auctions.

8) In order to ensure the fairness of the cost-down range calculation, same capacities of each bidder have been put into two reverse auction mechanisms.

9) Each bidder is rational.

10) Discriminatory auctions: this means pay-your-bid for your share.

11) The seller’s quantity of winning can be represented in function of the price-dependent demand.

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