Super-Čerenkov Radiation: A new phenomenon useful for RICH Detectors

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(Dated:)

In this contribution the Super-Čerenkov radiation (SCR) as a new phenomenon which includes in a more general and exact form the usual Čerenkov effect is presented. The Super-Čerenkov effect at Čerenkov threshold in the radiators of RICH detectors is investigated. The results on the experimental test of the super-Čerenkov coherence conditions are presented. The SCR-predictions are verified experimentally with high accuracy $\chi^2/n_{dof} = 1.47$ by the data on the Čerenkov ring radii of electron, muon, pion and kaon, all measured with RICH detector. Moreover, it is shown that the Super-Čerenkov phenomenon can explain not only subthreshold ČR but also the observed secondary rings (or anomalous Čerenkov radiation) observed at CERN SPS accelerator. The influence of medium on the particle propagation properties is also estimated and the refractive properties of electrons, muons, pions, in the radiator $\text{C}_2\text{F}_{10}\text{Ar}$ are obtained. So, we proved that the refractive indices of the charged elementary particles in medium are also very important for the RICH detectors, especially at low and intermediate energies.

Introduction. The classical theory of the radiation emitted by charged particles moving with superluminal velocities were traced back to Thomson, Heaviside and Sommerfeld. In fact, Heaviside [1] considered the Čerenkov radiation in a nondispersive medium. He considered this topic many times over the next 20 years, deriving most of the formalism [2] of what is now called Čerenkov radiation (CR). In 1904 Sommerfeld [3] considered radiation from a charge moving in a vacuum at a velocity faster than light velocity ($v > c$) and close approached the formulation of the theory of CR-effect. But Sommerfeld did not apply his results for the particles motion in a refractive (transparent) medium. Moreover, we must add that the ČR have been observed before the Čerenkov and Vavilov by Mallet and other authors but cannot be regarded as the discovery of ČR since the essential characteristic features of this radiation were not revealed and it was not understood that the observed effect is clearly different from luminescence. Consequently, the realizable case of radiation from a charge moving with a constant velocity greater than the phase velocity of light in a dielectric medium was discovered independently in an experiment in 1934 by Čerenkov [4]. So, doing justice [2] to Heaviside and Sommerfeld, we must recall that the classical theory of the ČR phenomenon in a dispersive medium was first formulated by Frank and Tamm [4] in 1937. This theory explained all the main features of the radiation observed by Čerenkov [4]. The quantum theoretical approach to the ČR-problem was developed by many authors (see refs. in [5]). The remarkable properties of the Čerenkov radiation find wide applications in practice especially in high energy physics where it is extensively used in experiments for counting and identifying relativistic particles in the fields of elementary particles, nuclear physics and astrophysics. A short review of Čerenkov radiation and its use for particle identification with threshold and differential counters is presented in Refs.[6]. In the last decades the Čerenkov radiation (CR) is the subject of many studies related to extension to the nuclear media [7]-[9] as well as to other coherent particle emission via Čerenkov-like mechanisms [10]. The generalized Čerenkov-like effects based on four fundamental interactions has been investigated and classified recently in ref. [11]. In particular, this classification includes the nuclear (mesonic, $\gamma$, weak boson)-Čerenkov-like radiations as well as the high energy component of the coherent particle emission via (baryonic, leptonic, fermionic)-Čerenkov-like effects. In 1999, G.L.Gogiberidze, L.K. Gelovani and E.K. Sarkisyan performed the first experimental test [12] of the pionic Čerenkov-like effect in Mg-Mg collisions at 4.3 GeV/c/nucleon and obtained a good agreement with the position and width of the first pionic Čerenkov-like band predicted in ref. [11].

In essence, it was revealed (see ref. [4]) by Čerenkov, Tamm and Frank that a charged particle moving in a transparent medium with an refractive index $n_\gamma$, and having a speed $v_x$ greater than phase velocity of light ($v_{\gamma\text{ph}} = 1/n_\gamma$) will emit Čerenkov radiation (CR) at an polar emission angle $\theta_C$ relative to the direction of motion given by the relation (we adopted the system of units $\hbar = c = 1$)

$$\cos \theta_C = \frac{v_{\gamma\text{ph}}}{v_x} = \frac{1}{n_\gamma v_x} \leq 1$$

However, by recent experimental observations of the subthreshold [13] and anomalous Čerenkov radiations [14] it was clarified that some fundamental aspects of the CR can be considered as being still open and that more theoretical and experimental investigations on the CR are needed. So, these new results stimulated new theoretical investigations [15-16] (using the CR correct kinematics) leading to the discovery that CR is in fact only a component (the low energy
component) of a more general phenomenon called by us the Super-$\tilde{\text{\'C}}$erenkov radiation (S$\tilde{\text{\'C}}$R) characterized by the Super-$\tilde{\text{\'C}}$erenkov coherence condition [16]

$$\cos \theta_{\text{SC}} = v_{xph} \cdot v_{\gamma ph} \leq 1$$

(2)

where $v_{xph}$ and $v_{\gamma ph}$ are phase velocities of the charged particle and photon in the medium, respectively. We must underline that the discovery of the subthreshold $\text{\'C}$erenkov radiation was of decisive importance for us to formulate the S$\tilde{\text{\'C}}$R-theory [15-16] which includes the above mentioned new phenomena. Moreover, we can see that the S$\tilde{\text{\'C}}$R coherence condition (2) is obtained in a natural way from the energy-momentum conservation law when the influence of medium on the propagation properties of the charged particle is taken into account. Therefore, the problem of the experimental test of Super-$\tilde{\text{\'C}}$erenkov coherence condition (2) is of great interest not only for the fundamental physics but also for practical applications to the particle detection. Recently such a test was performed [16] by using the Ring Imaging Cherenkov (RICH) detectors.

Now, let us apply the theory of Super-$\tilde{\text{\'C}}$erenkov radiation (S$\tilde{\text{\'C}}$R) to predict some characteristic feature of the subthreshold $\text{\'C}$erenkov-like effects in the RICH detectors.

**The Super-$\tilde{\text{\'C}}$erenkov Radiation (S$\tilde{\text{\'C}}$R) Theory.** Let start with an electromagnetic decay

$$1 \rightarrow \gamma + 2$$

(3)

in a (dielectric, nuclear or hadronic)-medium (we will work in the system of units $\hbar = c = 1$), described in

![Figure 1: A two-body decay in medium: Super-Cerenkov coherence condition.](image)

Fig. 1, where a photon $\gamma$ [with energy $\omega$, momentum $k = \omega \text{Re} n_{\gamma}(\omega)$ and refractive index $n_{\gamma}(\omega)$] is emitted in a (dielectric, nuclear or hadronic) medium from an incident charged particle [with charge $Ze$, energy $E_1$, momentum $p_1 = \text{Re} n_1(E_1)(E_1^2 - M^2)^{1/2}$, rest mass $M$, the refractive index $n_1(E_1)$] that itself goes over into a final particle 2 [with charge $Ze$, energy $E_2$, momentum $p_2 = \text{Re} n_2(E_2)(E_2^2 - M^2)^{1/2}$, rest mass $M$, the refractive index $n_2(E_2)$]. The refractive index $n_x(E_x)$ of any particle $x$ (with the energy $E_x$, momentum $p_x$, rest mass $M_x$) in a medium composed from the constituents $c$ will be described in standard way by the Foldy-Lax formula [17]

$$n^2_x(E_x) = 1 + \frac{4\pi \rho}{E_x^2 - M_x^2} \cdot C(E_x) \mathcal{T}_{xc \rightarrow xc}(E_x)$$

(4)

where $\rho$ is the density of constituents $c$, $C(E_x)$ is a coherence factor, $\mathcal{T}_{xc \rightarrow xc}(E_x)$ is the averaged forward xc-scattering amplitude. $C(E_x) = 1$ when the medium constituents are randomly distributed. The phase velocity $v_{phx}(E_x)$ of any particle $x$ in medium is modified as follows:

$$v_{xph}(E_x) = \frac{E_x}{p_x} = \frac{v_{xph}^{vac}(E_x)}{\text{Re} n_x(E_x)}$$

(5)

Now, using the energy-momentum conservation
\[ E_1 = \omega + E_2, \quad \vec{p}_1 = \vec{k} + \vec{p}_2 \]

we obtain

\[ \cos \theta_{1\gamma} = v_{1ph}(E_1)v_{\gamma ph}(\omega) + \frac{1}{2p_1k} [-D_1 + D_2 - D_\gamma] \]

\[ \cos \theta_{12} = v_{1ph}(E_1)v_{2ph}(E_2) + \frac{1}{2p_1p_2} [-D_1 - D_2 + D_\gamma] \]

where \( D_x, x = B_1, B_2, \gamma, \) are the mass shell relations in medium for x-particle and are given by

\[ D_x = E_x^2 - p_x^2 \]

We note that the second terms from the right side of Eqs (7)-(8) can be considered as quantum correction to the first one. So, the *semiclassical angles* are given by: \( \cos \theta_{1\gamma} = v_{1ph}(E_1)v_{\gamma ph}(\omega), \cos \theta_{12} = v_{1ph}(E_1)v_{2ph}(E_2), \) respectively.

(a) *Semiclassical Theory of SCR.* A semiclassical theory of SCR can be developed step by step in similar way with the classical theory of CR [4] and here we present only some final results for the case of a transparent nondispersive medium. So, if in the Maxwell equations we introduce the duality relation \( v_{ph} = v_{ph}^{-1}(E_x) \) in the charge and current distribution, then it is easy to obtain that the classical intensity of the Čerenkov radiation can now be written in a more exact form

\[
\frac{dN}{d\omega}(SCR) = Z^2 \alpha L \sin^2 \theta_{1\gamma} \\
= Z^2 \alpha L \left[ 1 - v_{1ph}^2(E_1)v_{\gamma ph}^2(\omega) \right],
\]

for \( p_1 \approx p_2, \) where \( \frac{dN}{d\omega}(SCR) \) is the number of photons emitted in the energy interval: \( (\omega, \omega + d\omega), \) and \( L \) is the length of path. Hence, at Čerenkov threshold we have

\[
\frac{dN}{d\omega}(SCR)|_{\text{C.R.th.}} \approx Z^2 \alpha L \left[ 1 - \left( \frac{1}{\text{Re} n_1(E_1)} \right)^2 \right]
\]

Therefore, the existence of the subthreshold Čerenkov radiation can be obtained for \( \text{Re} n_1(E_1) > 1. \)

(b) *Quantum Theory of SCR* [16]. Now, we start with a two-body spin \( (1/2^+ \rightarrow \gamma + 1/2^+) \) decay in a (dielectric, nuclear, or hadronic) medium where the propagation properties of all three particles (see Fig. 1) are changed according to the eqs. (4)-(5). Next, for simplicity we consider that the same interaction Hamiltonian as in the ČR-theory with some modifications of the source fields in medium can describe the coherent \( \gamma \)-emission in all SCR-sectors. Then, we obtain that the intensity of the *Super-Čerenkov radiation* for transparent (nonabsorbtent) media can be written in the following general form

\[
\frac{d^2 N_{SCR}}{dt d\omega} = \frac{\alpha Z^2}{v_1} \frac{1}{|n_{B_1}|^2|n_{B_2}|^2|n_{\gamma}|^2} \frac{k}{\omega} \frac{dk}{d\omega} S \cdot \Theta(1 - \cos \theta_{SC})
\]

where \( N_{SCR} \) is the total number of the SCR-photons, \( \Theta(1 - \cos \theta_{SC}) \) is Heaviside step function, while the spin factor \( S \) is given by

\[
S = \left[ \frac{p_1^2}{E_1 + M} + \frac{p_2^2}{E_2 + M} + 2 \frac{(\vec{e}_k \cdot \vec{p}_1)(\vec{e}_k \cdot \vec{p}_2) - (\vec{e}_k \times \vec{p}_1)(\vec{e}_k \times \vec{p}_2)}{(E_1 + M)(E_2 + M)} \right]
\]

where the vector \( \vec{e}_k \) is the photon polarization vector for a given photon momentum \( \vec{k} \). As in the usual case of ČR-theory, for a given vector \( \vec{k} \) we choose two orthogonal photon spin polarization directions, corresponding to a polarization vector \( \vec{e}_k \) perpendicular and parallel to the decay plane \( Q \) given by the vectors \( \vec{p}_1 \) and \( \vec{k} \), respectively. Then, from eq. (13) we get the following expressions of the spin factors \( S^\perp \) and \( S^{\parallel} \):

\[
S^\perp = \frac{(E_1 + M)(E_2 + M)}{4E_1E_2} \left[ \frac{\vec{p}_1}{E_1 + M} - \frac{\vec{p}_2}{E_2 + M} \right]^2
\]

\[
S^{\parallel} = v_1 \text{Re} n_1 \ v_2 \text{Re} n_2 \ \sin \theta_{1\gamma} \ \sin \theta_{2\gamma}
\]

where \( v_i, i=1,2 \) are the corresponding particle velocities in vacuum.
Now, one can see that Heaviside function $\Theta(1 - \cos \theta_{SC})$ is 1 at least in two physical $\gamma$-energy regions defined by the inequality (see Fig. 2): $\cos \theta_{2\gamma} \equiv \theta_{2\gamma} \approx \theta_{1\gamma}$, while the high $\gamma$–energy sector is that where $\theta_{SC} \equiv \theta_{2\gamma} \approx \theta_{1\gamma}$. Therefore, the limiting low $\gamma$–energy SCR-sector can be identified as extended $\gamma$–Čerenkov domain where the condition: $v_{\gamma ph}(\omega)v_{1ph}(E_2) \leq 1$ is fulfilled, while limiting high $\gamma$–energy SCR-sector can be identified as extended source-Čerenkov-like domain, in the sense that the charged particle spontaneously decays into a high $\gamma$–energy photon fulfilling a generalized Čerenkov-like relation: $v_{1ph}(\omega)v_{2ph}(E_2) \leq 1$.  

Figure 2: Schematic description of the Super-Čerenkov radiation.

Figure 3: Unified description of two Čerenkov-like limits of the Super-Čerenkov radiation when $n_1(E_1) = 1$. 
Of course, in the limit \( n_1(E_1) \to 1 \) these physical regions are going (see Fig. 3) in the corresponding ČR-like domains where the respective conditions: \( v_{\gamma ph}(\omega) \leq v_1 \) and \( v_{2ph}(E_2) \leq v_1 \), are satisfied. In conclusion the Super-Čerenkov effect cannot be confused with the usual Čerenkov radiation since the SCR can be considered as a continuous two-body decay in medium which includes not only two generalized Čerenkov-like phenomena but also their interference effects. The relations (12)-(15) includes in a general and unified way all the main predictions of the Super-Čerenkov radiation from which the results from Table 1 are obtained as two particular limiting cases.

| Table 1: The SCR main predictions. |
|-----------------------------------|
| Name | (SCR) Low γ-energy sector (see LE in figs. 2-3) | (SCR) High γ-energy sector (see HE in figs. 2-3) |
| 1. coherence relation | \( v_{\gamma ph} \cdot v_{x ph} \leq 1 \) | \( v_{\gamma ph} \cdot v_{x ph} \leq 1 \) |
| 2. coherence angle | \( \cos \theta_{SC} = v_{\gamma ph} \cdot v_{xph} \) since | \( \cos \theta_{SC} = v_{1ph} \cdot v_{2 ph} \) since |
| Since \( \theta_{SC} = \theta_1 \) | \( \theta_{SC} = \theta_2 \approx \theta_{1/2} \) |
| 3. threshold velocity | \( v_{x thr}(SC) = \frac{1}{n_1^{1/2}} \) | \( v_{x thr}(SC) = \frac{1}{n_1^{1/2}} \) |
| 4. emission angle | \( \theta_{max}^{SC} = \arccos \left( \frac{1}{n_1^{1/2}} \right) \) | \( \theta_{max}^{SC} = \arccos \left( \frac{1}{n_1^{1/2}} \right) \) |
| 5. spectrum | \( \frac{dN}{d\omega} = \alpha L z^2 \sin^2 \theta_{SC} \) | \( \frac{dN}{d\omega} = \frac{L Z^2}{2} \) ** |
| 6. polarization | 100% (|e||Q) | 100% (|e||Q)** |

The results of quantum theory presented here are complete for the Super-Čerenkov radiation (SCR) produced by any spin-1/2 particle [such as \((e^\pm, \mu^\pm, \tau^\pm)\)-leptons, \((p, \Sigma^+, \Xi^-, \Omega^-)\)-baryons etc.] moving in a (dielectric, nuclear or hadronic)-medium, with its phase velocity satisfying the Super-Čerenkov coherence condition (2).

**Experimental tests and predictions for Super-Čerenkov radiation.** The problem of the experimental test of *Super-Čerenkov coherence condition (2)* is of great interest not only for the fundamental physics but also for practical applications to the particle detection. Such a test can be performed by using a Ring Imaging Čerenkov (RICH) detectors (see ref. [18]).

![Figure 4: Schematic description of a RICH detector.](image)

In RICH detectors, particles pass through a radiator, and a spherical mirror focuses all photons emitted at \( \theta_{SC} \) along the particle trajectory at the same radius \( r_{SC} = (R/2) \tan \theta_{SC} \) on the focal plane. Photon sensitive detectors placed at the focal plane detect the resulting ring images in the RICH detector. So, RICH-counters are used for identifying and tracking charged particles. Čerenkov rings formed on a focal surface of the RICH provide information about the velocity and the direction of a charged particle passing the radiator. The particle’s velocities are related to the Čerenkov angle \( \theta_C \) or to the Super-Čerenkov \( \theta_{SC} \) by the relation (1) (or (2), respectively). Hence, these angles are determined by measuring the radii of the rings detected with the RICH. In ref. [18] a \( C_7 F_{10} Ar(75 : 25) \)
filled RICH-counter read out was used for measurement of the Čerenkov ring radii. Fig. 5a shows the experimental values of the ring radii of electrons, muons, pions and kaons measured in the active area of this RICH-detector. The saturated light produced from electrons was a decisive fact to take an index of refraction \( n_\gamma = 1.00113 \) for the radiator material. The absolute values for excitation curves of electron, muon, pion and kaon, shown by dashed curves in Fig. 5a, was obtained by using this value of refractive index in formula: 

\[
r_C(p) = (R/2) \tan \theta_C(p).
\]

The solid curves show the individual best fit of the experimental ring radii with eq.. 

\[
r_{SC}(p) = (R/2) \tan \theta_{SC}(p) \text{(see Table 1)}.
\]

Table 2: The best fit parameters of experimental ring radii with the Super-Čerenkov prediction.

| Particle | Number of exp. data | \(10^3 \cdot a^2 \text{ (GeV/c)}\) | \(\chi^2/\nu\text{dof}\) |
|----------|---------------------|----------------------------------|---------------------|
| e        | 6                   | -0.081±0.101                     | 0.468               |
| \(\mu\)  | 4                   | 1.449±0.098                      | 3.039               |
| \(\pi\)  | 7                   | 2.593±0.167                      | 0.234               |
| \(K\)    | 1                   | 21.140±2.604                     | \(<10^{-14}\)       |
| All data | 18                  | \((a/m)^2 = 0.1211 ± 0.0053\)    | 1.47                |

For the particle refractive index we used the parametrization

\[
n_\gamma^2(p) = 1 + a^2/p^2,
\]

\[
v_x = p/\sqrt{p^2 + m^2}
\]

where \( p \) is the particle momentum in the vacuum. In the paper [16] we fitted all the 18 experimental data on the ring radii from ref.[18] with our Super-Čerenkov prediction formula (see fig. 4)

\[
r_{SC}(p/m) = \frac{R}{2} \tan \theta_{SC}
\]

and we obtained the following consistent result (see Fig. 5b). The best fit parameters are as follow: \((a/m)^2 = 0.12109 ± 0.00528\) and \(\chi^2/\nu\text{dof} = 1.47\), where \(\nu\text{dof} = 18\) is the number of degree of freedom (dof). The \(r_{SC}(p/m)\) scaling function [16] together with all experimental data on the ring radii of the electron, muon, pion and kaon, are plotted as a function of the scaling variable \((p/m)\) in Figs. 5b. Now combining eqs.(10) and (16) we obtain

\[
\frac{dN}{d\omega}(S\tilde{cran}R) = Z^2\alpha L \left[ 1 - \frac{1}{n_\gamma^2} \left( \frac{p^2 + m^2}{a_x m_x^2} \right) \right]
\]

Therefore, at ČR threshold we get:

\[
\frac{dN}{d\omega}(S\tilde{cran}R)_{|_{CRthr}} \approx Z^2\alpha L \left[ 1 - \frac{1}{n_\gamma^2} \right]
\]

or

\[
\frac{dN}{d\omega}(S\tilde{cran}R)_{|_{CRthr}} \bigg|_{v_1=1} = \frac{a_x}{m_x^2} \frac{n_\gamma^2}{1 + \frac{a_x}{m_x^2}} = 0.1301 ± 0.088
\]

since

\[
p_{thr}(\tilde{cran})/m = \frac{1}{(n_\gamma^2 - 1)^{1/2}} = 21.029
\]

\[
p_{thr}(S\tilde{C})/m = \frac{1}{(n_\gamma^2 - 1)^{1/2}} = 18.477
\]

Moreover, we can estimate the \(\theta_{SC}\) and the SČ-ring radius \(r_{SC}\) at Čerenkov thresholds and we obtain:

\[
\theta_{SC}(v_{Cthr}) = \arctan \left[ \frac{a_x}{m_x} \left( n_\gamma^2 - 1 \right)^{1/2} \right] = 0.948 ± 0.021(\text{deg})
\]
\[ r_{SC}(\bar{CR} - thr) = \frac{R}{2} \left[ \left( \frac{a_x}{m_x} \right)^2 (n_x^2 - 1) \right]^{1/2} = (1.466 \pm 0.055) \, \text{cm} \] (22)

Figure 5: Experimental test of SCR- coherence condition (2) by using (17) and Čerenkov ring radii obtained by Debbe et al [18] with a RICH detector. a) SCR-prediction (solid curve) and CR-prediction (dashed curve). b) The SCR-scaling (solid curve) and CR-scaling (dashed curve) compared with ring radii (see text).

Figure 6: The relative Yields (a) and their scaling property (b) for the SCR-effect predicted by eqs. (10), (16) and the fitted constant \((a/m)^2 = 0.12109\).

In Figs. 6 we present the results for SCR-Yields defined as \(Y_{SCR}(p) = N_{SCR}(p)/N_{SCR}(\infty)\). Also the difference
The yield difference $Y_{SCR}(p) - Y_{CR}(p)$ is calculated and given in Fig. 7. What is interesting is that $D_T(p)$ is maximum at Čerenkov threshold (see eq. (20)).

Anomalous ČR: The anomalous Čerenkov ring observed recently by Vodopianov et al. [14] at SPS accelerator at CERN Pb-beam can be considered also as experimental signature of the HE-Super-Čerenkov component (see Fig. 4) since both Pb and high-energy $\gamma$ after spontaneous emission of a photon can produce secondary anomalous ČR-rings. One of the characteristic feature of the predicted by SCR-effect (see Fig. 4) is that the secondary anomalous ČR-rings must have a constant inclination angle $\alpha$ relative to beam direction given by the relation

$$\cos \alpha = \cos \theta_{12} = v_{1ph}(E_1) \cdot v_{2ph}(E_2)$$

(23)

This first important condition is verified experimentally with high accuracy by four from seven anomalous ČR-rings observed in ref. [14]. Therefore, these CR-anomalous rings can be interpreted as being produced by the photons emitted by secondary Pb (see again Fig. 4). Then, it is easy to show that the Čerenkov angles $\cos \theta'$ must be given by a relation of form:

$$\cos \theta'_{2\gamma} = v_{\gamma ph}(\omega) \cdot v_{2ph}(E_2) = \frac{1}{n_\gamma} \cdot \frac{1}{n_2 v_2}$$

(24)

Therefore, the velocities higher than unity inferred by Vodopianov et al. [14] from their anomalous ČR-rings, must be divided by the refractive index $n_2 > 1$ of the secondary Pb. Consequently, their anomalous CR-rings [14] cannot be interpreted as being produced by tachyons.

![Figure 7: The relative (a) Yields differences $Y(SCR) - Y(CR)$ and (b) scaling of Yields differences.](image)

Summary and Conclusions. In this contributed paper the Super-Čerenkov effect as a new dual coherent particle production mechanism is presented. The main results and conclusions can be summarized as follows:

(i) The Super-Čerenkov phenomenon can be considered as a continuous two-body decay in medium which is possible only in two distinct limiting physical regions where the Super-Čerenkov coherence condition (2) can be fulfilled. One of them is at very low $\gamma$-energies (LE) where

$$v_{1ph}(E_1) \geq v_{\gamma ph}(\omega), \text{ (extended } \gamma - \hat{C}R \text{ region)}$$

(25)

(see Figs. 2-3 and predictions in Table 1), and, a second region at very high $\gamma$-energies (HE) where

$$v_{2ph}(E_2) \geq v_{1ph}(E_1), \text{ (extended } 2 - \hat{C}R \text{ -Čerenkov-like region)}$$

(26)
(see Figs. 2-3 and predictions in Table 1) where $E_{zi}$ and $E_{zf}$ are the particle energy before and after $\gamma$-emission.

(ii) The experimental test of the SCR-coherence condition: $v_{\phi ph}(\omega) = 1$, is performed by using the data of Debye et al. [18] on Čerenkov ring radii of electrons, muons, pions and kaons in a RICH detector (see Fig. 4). The results on this experimental test of the super-coherence conditions are presented. These SCR-predictions are verified experimentally with high accuracy: $\chi^2/dof = 1.47$ (see Fig. 5). The scaling law of the ring radii and Yields (see Fig. 5b) predicted by the SCR-effect are also experimentally confirmed with high accuracy.

(iii) The inferred SCR-yield at just CR-threshold, is of order of magnitude $Y(SCR) = 1.301$ (see eqn. 19).

(iv) The influence of medium on the particle propagation properties is investigated and the refractive properties of electrons, muons, pions, in the radiator $C_4 F_8 Ar$ are obtained. The refractive indices for this radiator at $p_{lab} = 1 GeV$ are as follows: $n_e = 1.001449 \pm 0.000098$, $n_x = 1.0012593 \pm 0.000167$, $n_K = 1.0214 \pm 0.0026$, $n_p = 1.1066 \pm 0.046$. So, we proved that the refractive indices of the particles in medium are also very important for the RICH detectors, especially at low and intermediate energies.

(v) We proved that the anomalous Čerenkov rings observed recently by Vodopianov et al.,[14] at SPS accelerator at CERN Pb-beam can also be considered as an experimental signature of the HE-Super-Čerenkov component (see Fig.3).

Finally, we remark that new and accurate experimental measurements of Čerenkov ring radii, as well as for the anomalous HE-component of SCR effect are needed.

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