Cosmology of SUSY Q-balls

Alexander Kusenko

Department of Physics and Astronomy, University of California, Los Angeles, CA 90095-1547
and
RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA

Abstract

Supersymmetric extensions of the Standard Model predict the existence of Q-balls, some of which can be entirely stable. Both stable and unstable Q-balls can play an important role in cosmology. In particular, Affleck–Dine baryogenesis can result in a copious production of stable baryonic Q-balls, which can presently exist as a form of dark matter. Formation and decay of unstable Q-balls can also have some important effects on baryogenesis and phase transitions.
Cosmology of SUSY Q-balls

Alexander Kusenko
Department of Physics and Astronomy, University of California, Los Angeles, CA 90095-1547
E-mail: kusenko@ucla.edu

Supersymmetric extensions of the Standard Model predict the existence of Q-balls, some of which can be entirely stable. Both stable and unstable Q-balls can play an important role in cosmology. In particular, Affleck–Dine baryogenesis can result in a copious production of stable baryonic Q-balls, which can presently exist as a form of dark matter. Formation and decay of unstable Q-balls can also have some important effects on baryogenesis and phase transitions.

1 Non-topological solitons in MSSM

In a class of theories with interacting scalar fields \( \phi \) that carry some conserved global charge, the ground state is a Q-ball, a lump of coherent scalar condensate that can be described semiclassically as a non-topological soliton of the form

\[
\phi(x, t) = e^{i\omega t} \tilde{\phi}(x).
\]

Q-balls exist whenever the scalar potential satisfies certain conditions that were first derived for a single scalar degree of freedom with some abelian global charge and were later generalized to a theory of many scalar fields with different charges. Non-abelian global symmetries and abelian local symmetries can also yield Q-balls.

It turns out that all phenomenologically viable supersymmetric extensions of the Standard Model predict the existence of non-topological solitons associated with the conservation of baryon and lepton number. If the physics beyond the standard model reveals some additional global symmetries, this will further enrich the spectrum of Q-balls. The MSSM admits a large number of different Q-balls, characterized by (i) the quantum numbers of the fields that form a spatially-inhomogeneous ground state and (ii) the net global charge of this state.

First, there is a class of Q-balls associated with the tri-linear interactions that are inevitably present in the MSSM. The masses of such Q-balls grow linearly with their global charge, which can be an arbitrary integer number. Baryonic and leptonic Q-balls of this variety are, in general, unstable with respect to their decay into fermions. However, they could form in the early universe through the accretion of global charge or, possibly, in a first-order phase transition.
The second class of solitons comprises the Q-balls whose VEVs are aligned with some flat directions of the MSSM. The scalar field inside such a Q-ball is a gauge-singlet combination of squarks and sleptons with a non-zero baryon or lepton number. The potential along a flat direction is lifted by some soft supersymmetry-breaking terms that originate in a “hidden sector” of the theory at some scale \( \Lambda_s \) and are communicated to the observable sector by some interaction with a coupling \( g \), so that \( g \Lambda \sim 100 \text{ GeV} \). Depending on the strength of the mediating interaction, the scale \( \Lambda_s \) can be as low as a few TeV (as in the case of gauge-mediate SUSY breaking), or it can be some intermediate scale if the mediating interaction is weaker (for instance, \( g \sim \Lambda_s/m_{\text{Planck}} \) and \( \Lambda_s \sim 10^{10} \text{ GeV} \) in the case of gravity-mediated SUSY breaking). For the lack of a definitive scenario, one can regard \( \Lambda_s \) as a free parameter. Below \( \Lambda_s \) the mass terms are generated for all the scalar degrees of freedom, including those that parameterize the flat direction. At the energy scales larger than \( \Lambda_s \), the mass terms turn off and the potential is “flat” up to some logarithmic corrections. If the Q-ball VEV extends beyond \( \Lambda_s \), the mass of a soliton is no longer proportional to its global charge \( Q \), but rather to \( Q^{3/4} \). This allows for the existence of some entirely stable Q-balls with a large baryon number \( B \) (B-balls). Indeed, if the mass of a B-ball is \( M_B \sim (1 \text{ TeV}) \times B^{3/4} \), then the energy per baryon number \( (M_B/B) \sim (1 \text{ TeV}) \times B^{-1/4} \) is less than 1 GeV for \( B > 10^{12} \). Such large B-balls cannot dissociate into protons and neutrons and are entirely stable thanks to the conservation of energy and the baryon number. If they were produced in the early universe, they would exist at present as a form of dark matter.

2 Fragmentation of Affleck–Dine condensate into Q-balls

Several mechanisms could lead to formation of B-balls and L-balls in the early universe. First, they can be produced in the course of a phase transition. Second, thermal fluctuations of a baryonic and leptonic charge can, under some conditions, form a Q-ball. Finally, a process of a gradual charge accretion, similar to nucleosynthesis, can take place. However, it seems that the only process that can lead to a copious production of very large, and, hence, stable, B-balls, is fragmentation of the Affleck-Dine condensate.

At the end of inflation, the scalar fields of the MSSM develop some large expectation values along the flat directions, some of which have a non-zero baryon number. Initially, the scalar condensate has the form given in eq. (1) with \( \phi(x) = \text{const} \) over the length scales greater than a horizon size. One can think of it as a universe filled with Q-matter. The relaxation of this condensate
to the potential minimum is the basis of the Affleck–Dine (AD) scenario for baryogenesis.

It was often assumed that the condensate remains spatially homogeneous from the time of formation until its decay into the matter baryons. This assumption is, in general, incorrect. In fact, the initially homogeneous condensate can become unstable and break up into Q-balls whose size is determined by the potential and the rate of expansion of the Universe. B-balls with $12 < \log_{10} B < 30$ can form naturally from the breakdown of the AD condensate. These are entirely stable if the flat direction is “sufficiently flat”, that is if the potential grows slower than $\phi^2$ on the scales or the order of $\bar{\phi}(0)$. The evolution of the primordial condensate can be summarized as follows:

- Affleck-Dine condensate
  - baryons
  - unstable (decay)
  - related
- Dark Matter
  - stable

This process has been analyzed analytically in the linear approximation. Recently, some impressive numerical simulations of Q-ball formation have been performed; they confirm that the fragmentation of the condensate into Q-balls occurs in some Affleck-Dine models. The global charges of Q-balls that form this way are model dependent. The subsequent collisions can further modify the distribution of soliton sizes.

3 SUSY Q-balls as dark matter

Conceivably, the cold dark matter in the Universe can be made up entirely of SUSY Q-balls. Since the baryonic matter and the dark matter share the same origin in this scenario, their contributions to the mass density of the Universe are related. Therefore, it is easy to understand why the observations find $\Omega_{DARK} \sim \Omega_B$ within an order of magnitude. This fact is extremely difficult to explain in models that invoke a dark-matter candidate whose present-day abundance is determined by the process of freeze-out, independent of baryogenesis. If this is the case, one could expect $\Omega_{DARK}$ and $\Omega_B$ to be different by many orders of magnitude. If one doesn’t want to accept this equality as fortuitous, one is forced to hypothesize some *ad hoc* symmetries that could
relate the two quantities. In the MSSM with AD baryogenesis, the amounts of dark-matter Q-balls and the ordinary matter baryons are related. One predicts $\Omega_{DARK} = \Omega_B$ for B-balls with $B \sim 10^{26}$. This size is in the middle of the range of Q-ball sizes that can form in the Affleck–Dine scenario.

The value $B \sim 10^{26}$ is well above the present experimental lower limit on the baryon number of an average relic B-ball, under the assumption that all or most of cold dark matter is made up of Q-balls. On their passage through matter, the electrically neutral baryonic SUSY Q-balls can cause a proton decay, while the electrically charged B-balls produce massive ionization. Although the condensate inside a Q-ball is electrically neutral, it may pick up some electric charge through its interaction with matter. Regardless of its ability to retain electric charge, the Q-ball would produce a straight track in a detector and would release the energy of, roughly, 10 GeV/mm. The present limits constrain the baryon number of a relic dark-matter B-ball to be greater than $10^{22}$. Future experiments are expected to improve this limit. It would take a detector with the area of several square kilometers to cover the entire interesting range $B \sim 10^{22}...10^{30}$.

The relic Q-balls can accumulate in neutron stars and can lead to their ultimate destruction over a time period from one billion years to longer than the age of the Universe. If the lifetime of a neutron star is in a few Gyr range, the predicted mini-supernova explosions may be observable.

4 B-ball baryogenesis

An interesting scenario that relates the amounts of baryonic and dark matter in the Universe, and in which the dark-matter particles are produced from the decay of unstable B-balls was proposed by Enqvist and McDonald.

5 Phase transitions precipitated by solitosynthesis

In the false vacuum, a rapid growth of non-topological solitons can precipitate an otherwise impossible or slow phase transition. Let us suppose the system is in a metastable false vacuum that preserves some U(1) symmetry. The potential energy in the Q-ball interior is positive in the case of a true vacuum, but negative if the system is in the metastable false vacuum. In either case, it grows as the third power of the Q-ball radius $R$. The positive contribution of the time derivative to the soliton mass can be written as $Q^2/\int \bar{\phi}^2(x)d^3x \propto R^{-3}$, and the gradient surface energy scales as $R^2$. In the true vacuum, all three contributions are positive and the Q-ball is the absolute minimum of energy (Fig.). However, in the false vacuum, the potential energy inside the Q-ball is negative and goes as $\propto -R^3$. As shown...
Figure 1: Energy (mass) of a soliton as a function of its size. In the true vacuum, Q-ball is the global minimum of energy (solid curve). In the false vacuum, if the charge is less than some critical value, there are two solutions: a “stable” Q-ball, and an unstable “Q-bounce” (dashed curve 1). In the case of a critical charge (curve 2), there is only one solution, which is unstable.

in Fig. 1, for small charge $Q$, there are two stationary points, the minimum and the maximum. The former corresponds to a Q-ball (which is, roughly, as stable as the false vacuum is), while the latter is a critical bubble of the true vacuum with a non-zero charge.

There is a critical value of charge $Q = Q_c$, for which the only stationary point is unstable. If formed, such an unstable bubble will expand.

If the Q-ball charge increases gradually, it eventually reaches the critical value. At that point Q-ball expands and converts space into a true-vacuum phase. In the case of tunneling, the critical bubble is formed through coincidental coalescence of random quanta into an extended coherent object. This is a small-probability event. If, however, a Q-ball grows through charge accretion, it reaches the critical size with probability one, as long as the conditions for growth are satisfied. The phase transition can proceed at a much faster rate than it would by tunneling.

6 Conclusion

Supersymmetric models of physics beyond the weak scale offer two plausible candidates for cold dark matter. One is the lightest supersymmetric particle, which is stable because of R-parity. Another one is a stable non-topological soliton, or Q-ball, carrying some baryonic charge.

SUSY Q-balls make an appealing dark-matter candidate because their formation is a natural outcome of Affleck–Dine baryogenesis and requires no
unusual assumptions.

In addition, formation and decay of unstable Q-balls can have a dramatic effect on baryogenesis, dark matter, and the cosmic microwave background. Production of unstable Q-balls in the false vacuum can cause an unusually fast first-order phase transition.

References

1. G. Rosen, J. Math. Phys. 9 (1968) 996; *ibid.* 9 (1968) 999; R. Friedberg, T. D. Lee, and A. Sirlin, Phys. Rev. D13 (1976) 2739; S. Coleman, Nucl. Phys. B262 (1985) 263.

2. A. Kusenko, Phys. Lett. B405 (1997) 108 [hep-ph/9704273].

3. A. M. Safian, S. Coleman and M. Axenides, Nucl. Phys. B297 (1988) 498. A. M. Safian, Nucl. Phys. B304 (1988) 392. M. Axenides, Int. J. Mod. Phys. A7 (1992) 7169. M. Axenides, E. Floratos and A. Kehagias, Phys. Lett. B444 (1998) 190 [hep-ph/9810230].

4. K. Lee, J. A. Stein-Schabes, R. Watkins and L. M. Widrow, Phys. Rev. D39 (1989) 1665; T. Shiromizu, T. Uesugi and M. Aoki, Phys. Rev. D59 (1999) 125010 [hep-ph/9811420].

5. D. A. Demir, Phys. Lett. B450 (1999) 215 [hep-ph/9810453].

6. A. Kusenko, Phys. Lett. B404 (1997) 285 [hep-th/9704073].

7. K. Griest and E. W. Kolb, Phys. Rev. D40 (1989) 3231; J. A. Frieman, A. V. Olinto, M. Gleiser and C. Alcock, Phys. Rev. D40 (1989) 3241.

8. A. Kusenko, Phys. Lett. B406 (1997) 26 [hep-ph/9705361].

9. J. A. Frieman, G. B. Gelmini, M. Gleiser and E. W. Kolb, Phys. Rev. Lett. 60 (1988) 2101; K. Griest, E. W. Kolb and A. Maaslootti, Phys. Rev. D40 (1989) 3529; J. Ellis, J. Hagelin, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B125, 275 (1983).

10. G. Dvali, A. Kusenko, and M. Shaposhnikov, Phys. Lett. B417 (1998) 99 [hep-ph/9707423].

11. A. Kusenko, M. Shaposhnikov, and P. G. Tinyakov, Pisma Zh. Eksp. Teor. Fiz. 67 (1998) 229 [hep-ph/9801041].

12. A. Kusenko and M. Shaposhnikov, Phys. Lett. B418 (1998) 46 [hep-ph/9709492].

13. S. Khlebnikov and I. Tkachev, [hep-ph/9902272].

14. I. Affleck and M. Dine, Nucl. Phys. B 249 (1985) 361; M. Dine, L. Randall and S. Thomas, Phys. Rev. Lett. 75 (1995) 398; Nucl. Phys. B 458 (1996) 291.

15. M. Laine and M. Shaposhnikov, Nucl. Phys. B532 (1998) 376 [hep-ph/9804237].
16. A. Kusenko, V. Kuzmin, M. Shaposhnikov, and P. G. Tinyakov, Phys. Rev. Lett. 80 (1998) 3185 [hep-ph/9712212].
17. A. Kusenko, M. Shaposhnikov, P. G. Tinyakov, and I. I. Tkachev, Phys. Lett. B423 (1998) 104 [hep-ph/9801212].
18. K. Enqvist and J. McDonald, Phys. Lett. B425 (1998) 309 [hep-ph/9711514]; Nucl. Phys. B538 (1999) 321 [hep-ph/9803380]; Phys. Rev. Lett. 81, 3071 (1998) [hep-ph/9806213]; Phys. Lett. B440 (1998) 59 [hep-ph/9807269]; Phys. Rev. Lett. 83 (1999) 2510 [hep-ph/9811412]; hep-ph/9908316; K. Enqvist, these Proceedings.
19. S. Kasuya and M. Kawasaki, Phys. Rev. D61 (2000) 041301 [hep-ph/9909508]; S. Kasuya, these Proceedings.
20. M. Axenides, S. Komineas, L. Perivolaropoulos and M. Floratos, [hep-ph/9910388].
21. D. B. Kaplan, Phys. Rev. Lett. 68 (1992) 741.
22. I. A. Belolaptikov et al., astro-ph/9802223; M. Ambrosio et al. [MACRO Collaboration], hep-ex/9904031; B. C. Choudhary [MACRO Collaboration], hep-ex/9905023; G. Giacomelli and L. Patrizii, DFUB-98-30 Prepared for 5th ICTP School on Nonaccelerator Astroparticle Physics, Trieste, Italy, 29 Jun - 10 Jul 1998. E. Aslanides et al. [ANTARES Collaboration], astro-ph/9907432.
23. M. Axenides, E. G. Floratos, G. K. Leontaris and N. D. Tracas, Phys. Lett. B447 (1999) 67.