CORRELATIONS FROM GALACTIC FOREGROUNDS IN THE FIRST-YEAR WILKINSON MICROWAVE ANISOTROPY PROBE DATA

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ABSTRACT

We study a specific correlation in the spherical harmonic multipole domain for cosmic microwave background (CMB) analysis. This group of correlations between \( \Delta l = 2n \), where \( n = 1, 2, \ldots \), is caused by a symmetric signal in the Galactic coordinate system. A phase filter targeting such correlation therefore helps remove the localized bright pointlike sources in the Galactic plane and the strong diffuse component down to the CMB level. We illustrate the significance of these correlations and apply this estimator on some derived CMB maps with foreground residuals. In addition, we show that our proposed estimator significantly damps the phase correlations caused by Galactic foregrounds. This investigation provides understanding of mode correlations caused by Galactic foregrounds, which is useful for paving the way for foreground-cleaning methods for the CMB.

Subject headings: cosmic microwave background — cosmology: observations — methods: data analysis

1. INTRODUCTION

Separation of the cosmic microwave background (CMB) signal from extragalactic and Galactic foregrounds (GF) is one of the most challenging problems for all the CMB experiments, including the ongoing NASA WMAP (Wilkinson Microwave Anisotropy Probe) and the upcoming ESA Planck mission. The GF produces the major (in amplitude) signal in the raw maps, which is localized at a rather small latitude band \( b < 30^\circ \). To avoid any contribution of the GF to the derived CMB map, starting from the COBE (Cosmic Background Explorer) to WMAP experiments, a set of masks and disjoint regions of the map are in use for extraction of the CMB anisotropy power spectrum (Bennett et al. 2003a, 2003b, 2003c; Hinshaw et al. 2003; Tegmark et al. 2003; Eriksen et al. 2004). The question is, what kind of assumption about the properties of the foregrounds should we apply for the data processing and what criteria determine the shape and area of the mask and the model of the foregrounds? To answer these questions we need to know the statistical properties of the GF to determine the strategy of the CMB signal extraction from the observational data sets.

These questions are even more pressing for the CMB polarization. Unlike temperature anisotropies, our knowledge about the polarized foregrounds is still considerably poor. In addition, we have yet to obtain a reasonable truly whole-sky CMB anisotropy map for statistical analysis, while obtaining a whole-sky polarization map seems to be a more ambitious task. Modeling the properties of the foregrounds thus needs to be done for achieving the main goals of the Planck mission: to find the CMB anisotropy and polarization signals for the whole sky with unprecedented angular resolution and sensitivity.

Apart from modeling the foregrounds, Tegmark et al. (2003, hereafter TOH03) propose the “blind” method for separation of the CMB anisotropy from the foreground signal. Their method (see also Tegmark & Efstathiou 1996) is based on minimizing the variance of the CMB plus foreground signal with multipole-dependent weighting coefficients \( \nu(l) \) on the WMAP K–W bands, using 12 disjoint regions of the sky. This leads to their foreground-cleaned map (FCM), which seems to be clean from most foreground contamination, and the Wiener-filtered map (WFM), in which the instrumental noise is reduced by Wiener filtration. It also provides an opportunity to derive the maps for combined foregrounds (synchrotron, free-free, dust emissions, etc.). Both FCMs and WFM shows certain levels of non-Gaussianity (Chiang et al. 2003; Schwarz et al. 2004), which can be related to the residuals of the GF (Naselsky et al. 2004a). Therefore, we believe that it is imperative to develop and refine the “blind” methods for the Planck mission, not only for better foreground separation in the anisotropy maps, but also to pave the way for separating CMB polarization from the foregrounds.

The development of “blind” methods for foreground cleaning can be performed in two ways. One is to clarify the multipole and frequency dependency of various foreground components, including possible spinning dust, for a high multipole range and at the Planck High-Frequency Instrument (HFI) frequency range. The other requires additional information about the morphology of the angular distribution of the foregrounds, including knowledge about their statistical properties in order to construct a realistic high-resolution model of the observable Planck foregrounds.

Since the morphology of the CMB and foregrounds is closely related to the phases (Chiang 2001) of \( a_{l,m} \) coefficients from the spherical harmonic expansion \( \Delta T(\theta, \phi) \), this problem can be reformulated in terms of analysis of phases of the CMB and foregrounds, including their statistical properties (Chiang & Coles 2000; Chiang et al. 2002a, 2004; Coles et al. 2004; Naselsky et al. 2003a, 2003b, 2004b).

In Naselsky & Novikov (2005), it is reported that a major part of the GF produces a specific correlation in the spherical harmonic multipole domain at \( \Delta l = 4 \), i.e., between modes \( a_{l,m} \) and \( a_{l+4,m} \). The series of \( \Delta l = 4 \) correlations from the GF requires more investigation. This paper is thus devoted to further analysis of the statistical properties of the phases of the WMAP foregrounds for such correlation. We concentrate on the question as to what the reason is for the \( 4m \) correlation in the WMAP data, and whether such correlation can help us to determine the properties of the foregrounds, in order to separate them from the CMB anisotropies.

In this paper we develop the idea proposed by Naselsky & Novikov (2005) and demonstrate that the pronounced symmetry

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of the GF (in the Galactic system of coordinates) is the main cause not only for the $\Delta l = 4n$ correlation, but also for all even multipoles with some alternation in the sign of the estimator. The estimator designed in Naselsky & Novikov (2005) and the generalized form illustrated in this paper can help us understand GF manifestation in the harmonic domain, leading to the development of a “blind” method for foreground cleaning. In combination with the multifrequency technique proposed in Tegmark & Efstathiou (1996; TOH03), the removal of the $2n$ correlation of phases can be easily used as an effective method of determination of the CMB power spectrum without a Galactic mask and disjoint regions for the WMAP data and especially for the future Planck data. It can serve as a complementary method to the internal linear combination method (Bennett et al. 2003c; Eriksen et al. 2004) and to the TOH03 method as well, in order to decrease the contamination of the GF in the derived maps. Such a kind of correlation should be observed by the Planck mission and will help us to understand the properties of the GF in detail, as it can play a role as an additional test for the foreground models for the Planck mission.

This paper is organized as follows. In § 2 we describe the $d^\Delta_{l,m}$ estimator, which we call the phase filter, for $4n$ correlation in the coefficients $a_{l,m}$ and generalize the filter to correlation between $\Delta l = 2n$. In § 3 we give a detailed account on why such correlations appear. In § 4 we show the maps reconstructed from the filter and discuss the connection between the $2n$ correlation and the WMAP foreground symmetry. We examine the power spectra of the maps from the filter and their correlations in § 5. The conclusion is in § 6.

2. THE $\Delta l = 4n$ CORRELATION AND ITS GENERALIZATION

2.1. The $4n$ Correlation and the Phase Filter

As is shown in Naselsky & Novikov (2005), the Galactic signal reveals the $4n$ correlation. We recapitulate the definition of the estimator, which we call hereafter the “phase filter” (PF), a specific combination of the spherical harmonic coefficients $a_{l,m}$,

$$d^\Delta_{l,m} = a_{l,m} - \frac{|a_{l,m}|}{a_{l+\Delta,m}} a_{l+\Delta,m}, \quad d^\Delta_{l,m=0} = 0,$$  

(1)

where $|m| \leq l$ and the coefficients $a_{l,m} = |a_{l,m}| \exp(i\Phi_{l,m})$ are defined in the standard way,

$$\Delta T(\theta, \phi) = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} |a_{l,m}| \exp(i\Phi_{l,m}) Y_{l,m}(\theta, \phi).$$  

(2)

Here $\Delta T(\theta, \phi)$ are the whole-sky anisotropies at each frequency band, $\theta$ and $\phi$ are the polar and azimuthal angles of the polar coordinate system, $Y_{l,m}(\theta, \phi)$ are the spherical harmonics, and $|a_{l,m}|$ and $\Phi_{l,m}$ are the moduli and phases of the $l, m$ harmonics. The superscript $\Delta$ in $d^\Delta_{l,m}$ characterizes the shift of the $l$-mode in $d^\Delta_{l,m}$. In Naselsky & Novikov (2005) the authors concentrate on the series of correlation for $\Delta = 4n$, where $n = 1, 2, 3, \ldots$, based on the fact that the signal of the Galaxy lies mostly close to the $\theta = \pi/2$ plane. The phase filter in equation (1) is related to the phases of the multipoles of the $\Delta T(\theta, \phi)$ signal:

$$d^\Delta_{l,m} = |a_{l,m}| \left( e^{i\Phi_{l,m}} - e^{i\Phi_{l+\Delta,m}} \right) = a_{l,m} \left( 1 - e^{i\delta \Phi} \right) = 2a_{l,m} \sin \left( \frac{\delta \Phi}{2} \right) \exp \left[ i \left( \frac{\delta \Phi - \pi}{2} \right) \right].$$  

(3)

where $\delta \Phi \equiv \Phi_{l+\Delta,m} - \Phi_{l,m}$. If $a_{l,m}$ is Gaussian with random phases, then the phase difference $\delta \Phi$ is random as well. The $d_{l,m}$ are then composed of $a_{l,m}$, which are random Gaussian themselves, and a subtraction of two random unit vectors; therefore, $d_{l,m}$ are uncorrelated with each other. The amplitude $\langle |d_{l,m}| \rangle = \langle |a_{l,m}| (1 - \exp(i\delta \Phi)) \rangle = \sqrt{2}/4 |a_{l,m}|$. Hence, the power for $d_{l,m}$, $\langle |d_{l,m}|^2 \rangle$, is twice that for $a_{l,m}$. On the other hand, if the phases are not random, the map synthesized from the phase filter is simply a map of the original $a_{l,m}$ with phases rotated by an angle $(\delta \Phi - \pi)/2$ and the amplitudes decreased by a factor $2|\sin(\delta \Phi/2)|$. For $\delta \Phi \to 0$,

$$d^\Delta_{l,m} \approx 2a_{l,m} e^{-i\pi/2} \sin \left( \frac{\delta \Phi}{2} \right).$$  

(4)

Therefore, for some $a_{l,m}$ modes whose amplitudes are to be decreased significantly by the phase filter, we concentrate on the condition $\delta \Phi \to 0$.

The PF is a nonlinear filter defined in the space of phases, for which an analogy can be drawn with linear filtering for noise or point-source subtraction, such as the Wiener filter, the matched filter (Tegmark & de Oliveira-Costa 1998), and the adaptive top-hat filter (Chiang et al. 2002b). If for some range of multipoles the power of the instrumental noise is negligible in comparison with that of CMB, the PF will reveal the power of the CMB. In general, the signal reconstructed by the PF is a combination of the CMB and instrumental noise.

2.2. Generalization of the PF

We would like to point out that the particular choice of the $\Delta = 4n$ parameter of the filter in Naselsky & Novikov (2005) reflects only partially the properties of Galactic foregrounds and the CMB plus noise signal in the Galactic system of coordinates. The properties of this filter are based on the symmetry of the associated Legendre polynomials and correlation properties of the foregrounds. First, the corresponding angular correlation length for the foregrounds $\Theta_f$ is significantly larger than that for the CMB plus instrumental noise $\Theta_{\gamma}$, i.e., $\Theta_f \gg \Theta_{\gamma}$. Second, we exploit the properties of the Legendre polynomials for generalization of the $\Delta = 4$ filter, taking into account that in the Galactic plane area ($\theta \approx \pi/2$, where $\theta$ is the polar angle), all even Legendre polynomials have maximal values, and their corresponding phases are close to each other or different by $\pi$.

We generalize the PF as

$$d^\Delta_{l,m} = a_{l,m} - (-1)^k \frac{|a_{l,m}|}{|a_{l+2k,m}|} a_{l+2k,m}, \quad d^\Delta_{l,m=0} = 0,$$  

(5)

where $k$ is the order of correlation and the optimal values are 1 and 2. One can see that for $k = 2n$ this filter reproduces the form in Naselsky & Novikov (2005), but for $k = 2n + 1$ the second term of the filter changes signs. Now all the even $\Delta l$ correlations in phases are included in equation (5). We illustrate in § 3 the reason for such generalization.

3. WHY DO THE CORRELATIONS OF THE EVEN $\Delta l$ APPEAR?

In this section we describe the general properties of the $\Delta = 2n$ periodicity of the Galactic signal, taking into account the symmetry of the Legendre polynomials $P_{l,m}$ and $P_{l+\Delta,m}$ and the correlation of the Galactic foregrounds and the CMB plus noise signals for even $\Delta$. To illustrate the properties of the $d^\Delta_{l,m}$, we adopt the following model of the signal without losing generality. We assume for simplicity that all the pixels in the map are of equal area.
We use a polar system of coordinates, and the \( \theta \) and \( \phi \) mark the position of the \( j \)th pixel in the map. Let us assume that the Galactic signal is localized in the \( \theta \)-direction and is confined in the belt region with half-width \( \omega \). We single out the set of pixels \( p \) of this region and denote it as \( S = \{ p(\theta_j, \phi_j); \pi/2 - \omega \leq \theta_j \leq \pi/2 + \omega \} \); thus, the map of the Galactic signal in \( S \) is

\[
\Delta T(\theta, \phi) = T_\theta \delta^D(\cos \theta - \cos \theta_j) \delta^D(\phi - \phi_j),
\]

where \( \delta^D \) is the Dirac \( \delta \)-function and the amplitude of the signal at pixel \( (\theta_j, \phi_j) \) is \( T_\theta \). Note that we do not specify the localization in the \( \phi \)-direction of the signal in \( S \). In addition, we assume that the \( T_\theta \) is the sum of Galactic foreground signal \( T_\theta^f \), CMB \( T_{\text{CMB}} \), and instrumental noise \( T_\theta^n \), where we denote \( T_\theta = T_{\text{CMB}} + T_\theta^n \).

It is important to note that the statistical properties of the foreground \( T_\theta^f \) are different from \( T_\theta \), in terms of both amplitudes and pixel-pixel correlations \( \langle T_\theta^f T_\theta \rangle \). In particular, we assume \( T_\theta^f \gg T_\theta \) within \( S \), while \( T_\theta^f \ll T_\theta \) outside \( S \). Using such a proposed model of the signal in the map, we can obtain the corresponding \( a_{l,m} \) coefficients of the spherical harmonic expansion,

\[
a_{l,m} = \sqrt{\frac{2l + 1}{4\pi} \frac{(l - m)!}{(l + m)!}} \sum_j T_j \bar{P}_l^m(\cos \theta_j) e^{-im\phi_j},
\]

which can be represented as a sum of foreground \( F_{l,m} \) coefficients and the CMB plus noise coefficients \( c_{l,m} \).

In order to understand the nature of the \( 2n \) periodicity of the Galactic foreground, let us rewrite equation (1) in terms of coefficients \( a_{l,m} \) from equation (7),

\[
d_{l,m} = \sqrt{\frac{2l + 1}{4\pi} \frac{(l - m)!}{(l + m)!}} \sum_j e^{-im\phi_j} G_{l,\Delta=2n}^m(\cos \theta_j),
\]

where

\[
G_{l,\Delta=2n}^m(\cos \theta_j) = P_l^m(\cos \theta_j) - (-1)^n P_{l+2n}^m(\cos \theta_j) M_{\Delta=2n}^m,
\]

\[
M_{\Delta}^m = \left[ \frac{\sum_{\mu,\nu} T_\mu^\mu T_{\nu}^\nu P_l^m(\cos \theta_\mu) P_l^m(\cos \theta_\nu) e^{im(\phi_\mu - \phi_\nu)}}{\sum_{\mu,\nu} T_\mu^\mu T_{\nu}^\nu P_{l+2n}^m(\cos \theta_\mu) P_{l+2n}^m(\cos \theta_\nu) e^{im(\phi_\mu - \phi_\nu)}} \right]^{1/2}.
\]

We examine the properties of equation (9) at the coordinates \( \theta_\mu, \theta_\nu \approx \pi/2 \), for which \( P_l^m(\cos \theta_j) \approx \frac{2^n \sqrt{\Gamma(n+1+(l-m)/2)\Gamma(-n-(l-m+1)/2)}}{\Gamma(n+1+(l-m)/2)\Gamma(-n-(l-m+1)/2)} \).

\[
P_{l+2n}^m(0) = \frac{2^n \sqrt{\Gamma(n+1+(l-m)/2)\Gamma(-n-(l-m+1)/2)}}{\Gamma(n+1+(l-m)/2)\Gamma(-n-(l-m+1)/2)}.
\]

The properties of the Legendre polynomials \( P_{l+2n}^m(0) \) depend on the \( n, l, \) and \( m \). First, for \( n = 0 \) only even \( l + m \) contribute to the \( P_l^m(0) \), while all the odd \( l + m \) formally correspond to

\[
P_{l+\Delta}^m(0) = (-1)^n P_l^m(0) \prod_{j=1}^{\Delta} \frac{j+(l-m+1)/2}{\prod_{j=1}^{\Delta} [j+(l-m)/2]},
\]

Substituting equation (11) into equation (9), we obtain \( M_{\Delta}^m \) depends on the Galactic plane at \( \theta = \pi/2 \) vanishes because of the design of the PF.
However, in reality the Galactic foregrounds stretch well beyond the Galactic plane, and having equation (9) we can investigate how they can be removed by the PF.

For that we discuss the properties of the Legendre polynomials for \( \theta_\nu, \theta_\nu \neq \pi/2 \) in equation (9), but \( \theta_\nu \gg 1 \). In this case we use the asymptotic form of the Legendre polynomials,

\[
P^m_{l+\Delta}(\cos \theta) \simeq \frac{2}{\sqrt{\sin \theta}} \frac{\Gamma(l + \Delta + m + 1)}{\Gamma(l + \Delta + 3/2)} \times \cos \left( \left( l + \frac{1}{2} \right) \theta + \frac{1}{2} m\pi - \frac{\pi}{4} \right) \times \left\{ 1 - \tan (2n\theta) \tan \left( \left( l + \frac{1}{2} \right) \theta + \frac{1}{2} m\pi - \frac{\pi}{4} \right) \right\}.
\]

once again, if \( \theta = \pi/2 - \delta \), where \( \delta \ll \pi/2 \), then from equation (12) we get

\[
\cos (2n\theta) \simeq (-1)^n \cos (2n\delta) \sim (-1)^n, \tan (2n\theta) \sim -\tan (2n\delta) \ll 1, \text{ and } G^l_{\theta,\phi=2\pi} \rightarrow 0 \text{ in equation (9)}.
\]

4. THE \( D(\theta, \phi) \) MAP FROM \textit{WMAP} DATA

As the PF breeds a new set of \( a_{l,m} \), we can also synthesize such processed \( a_{l,m} \) into a whole-sky map. A nontrivial aspect of the PF is that it significantly decreases the brightest part of the Galaxy image in the \textit{WMAP} K–W maps. In the following analysis we use a particular case \( n = 2 \) so that \( \Delta = 4 \), although it can be demonstrated that for other values the results of the analysis do not change significantly as long as \( \Delta \leq l_{\text{noise}} \), where \( l_{\text{noise}} \) is the multipole number in the spectrum for which the instrumental noise starts dominating over the GF signal. The optimal values, however, to best filter the GF are \( n = 1, 2, 3, \) and \( 4 \).

We show in Figure 1 how the PF transforms the GF image in the \textit{WMAP} K–W maps.\(^3\) These maps are synthesized from the \( d^A_{l,m} \) from the \textit{WMAP} K–W bands for \( \Delta = 4 \) and \( l_{\text{max}} = 512: \)

\[
D(\theta, \phi) = \sum_{l=2}^{l_{\text{max}}} \sum_{m=1}^{l} d^A_{l,m} Y_{l,m}(\theta, \phi).
\]

Note that the temperature amplitudes are significantly reduced in each map. It should be emphasized that the \( D(\theta, \phi) \) map is not a temperature anisotropy map, as the phases are altered.

We also apply the PF to the \textit{WMAP} foreground maps at the Q, V, and W bands. These foreground maps are the sum of synchrotron, free-free, and dust emissions. As these foreground maps...
maps do not contain the CMB signal and instrumental noise, they allow us to estimate the properties of the GF in detail.

In Figure 2 we plot the $D(\theta, \phi)$ maps for the Q-, V-, and W-band foregrounds ($\Delta = 4$) for the multipole range $l \leq 46$. This range is determined by the resolution of the WMAP foreground maps ($l \leq 50$). As one can see from these maps, the GF follows $4n$ multipole correlation, which removes the brightest part of the signal from $[-1.31, 8.78]$ mK for the Q band, from $[-0.54, 3.82]$ mK for the V band, from $[-0.33, 2.30]$ mK for the W band, and from $[-0.24, 1.72]$ mK for the $D(\theta, \phi)$ map, the difference between the V and W foregrounds. Note that these limits are related to the brightest positive and negative spots in the maps, while diffuse components have significantly smaller amplitudes. To show the high-resolution $D(\theta, \phi)$ map, which characterizes the properties of the foregrounds in the V and W bands, in Figure 3 we plot the map of the difference V – W bands and the corresponding $D(\theta, \phi)$ map for $l \leq 50$. Note that the V – W map does not contain the CMB signal, but for high $l$ the properties of the signal are determined by the instrumental noise.

Fig. 6.—The $C_s(l)$ and $S_i(l)$ trigonometric moments for the cross-correlation of phases between the TOH03 FCM and WFM (top pair). The solid line represents the limit when the phases are identical. The middle pair is the $C_s(l)$ and $S_i(l)$ trigonometric moments for the FCM with a phase difference $\xi_{s,4,m} - \xi_{s,m}$. The bottom pair is $C_s(l)$ and $S_i(l)$ trigonometric moments for the WFM with a phase difference $\xi_{s,4,m} - \xi_{s,m}$.

5. THE POWER SPECTRUM AND CORRELATIONS OF THE $D$ MAP

5.1. What Is Constructed from the $d_{l,m}^\Delta$?

To characterize the power spectrum of the $D(\theta, \phi)$ maps we introduce the definitions

$$D(l) = \left(\frac{1}{2l + 1}\sum_{m=-l}^{l} |d_{l,m}^\Delta|^2\right),$$

$$G(l) = \frac{l(l+1)}{2\pi}D(l).$$
As we have pointed out in § 2.1, if the derived \( d_{l,m}^{p} \) signal is Gaussian, then \( D(l) \) contains all the statistical properties of the signal and is twice the power spectrum of the underlying field. For non-Gaussian signal, \( D(l) \) contains the diagonal elements of the correlation matrix. From Figure 4 it can be clearly seen that for WMAP foregrounds, especially for the V and W bands, the power spectra of \( D(\ell, \phi) \) are significantly smaller than the power of the CMB, which we estimate simply by using the power of the TOH03 FCM, transformed by the PF as

\[
D_{\text{FCM}}(l) = \frac{1}{2l + 1} \sum_{m=-l}^{l} \left| c_{l,m} \right|^2 \left| c_{l+\Delta,m} \right|^2, \tag{16}
\]

assuming that the FCM is fairly clean from the foreground signal. An important point of analysis of the WMAP foregrounds is that for the V and W bands the PF significantly decreases the amplitude of the GF, practically by 1–2 orders of magnitude below the CMB level.

The most intriguing question related to 4\( n \) correlation of the map derived from the WMAP V and W band signals is, what is reconstructed by the \( d_{l,m}^{p} \)? The next question is why the power spectra of \( d_{l,m}^{p} \) is the same at the range of multipoles \( l \leq 100 \) (where we can neglect the contribution from instrumental noise at both channels and the differences of the antenna beams). The equivalence of the powers for these two signals, shown in Figure 5, suggests that these derived maps are related to pure CMB signal (which we assume to be frequency independent).

In the following discussion we discuss what kinds of combinations are presented in the \( d_{l,m}^{p} \) between amplitudes and phases of the CMB signal and the foregrounds between the WMAP V and W bands. As was mentioned in § 2, the PF is designed as a linear estimator of the phase difference \( \Phi_{l+\Delta,m} - \Phi_{l,m} \) if the phase difference is small. Let us introduce the model of the signal at each band \( d_{l,m}^{p} = c_{l,m} + x_{l,m} \), where \( c_{l,m} \) is the frequency-independent CMB signal and \( x_{l,m} \) is the sum over all kinds of foregrounds for each band \( j \) (synchrotron, free-free, dust emission, etc.).

According to the investigation above on the foreground models, it is realized that without the ILC (internal linear combination) signal the \( d_{l,m}^{p} \) estimation of the foregrounds, especially for the V and W bands, corresponds to the signal

\[
d_{l,m}^{\Delta(\text{CMB})} = c_{l,m} - \frac{\left| c_{l,m} \right|}{\left| c_{l+\Delta,m} \right|} c_{l+\Delta,m}. \tag{18}
\]

In terms of moduli and phases of the foregrounds at each frequency band,

\[
F_{l,m} = \left| F_{l,m} \right| \exp(i\Phi_{l,m}), \quad c_{l,m} = \left| c_{l,m} \right| \exp(i\xi_{l,m}), \tag{19}
\]

where \( \Phi_{l,m} \) and \( \xi_{l,m} \) are the phases of foreground and the CMB, respectively. From equation (19) we get

\[
d_{l,m}^{\Delta,f} = F_{l,m} \left( e^{i\Phi_{l,m}} - e^{i\Phi_{l+\Delta,m}} \right), \tag{20}
\]

and practically speaking, we get \( \Phi_{l,m} = \Phi_{l+\Delta,m} \). Thus, taking the 2\( n \) correlation into account, we can conclude that it reflects directly strong correlation of the phases of the foregrounds, determined by the GF. Moreover, if any foreground-cleaned CMB maps derived from different methods were to display the 2\( n \) correlation of phases, it would be evident that foreground residuals still determine the statistical properties of the derived signal.

5.2. 2\( n \) Phase Correlation of the \( D \) Map

One of the basic ideas for comparison of phases of two signals is to define the trigonometric moments for the phases \( \xi_{l,m} \) and \( \Psi_{l,m} \) as

\[
\text{Cs}(l, l') = \frac{1}{\sqrt{l}} \sum_{m=1}^{l} \cos(\xi_{l,m} - \Psi_{l,m}), \quad \text{Si}(l, l') = \frac{1}{\sqrt{l}} \sum_{m=1}^{l} \sin(\xi_{l,m} - \Psi_{l,m}) \tag{21}
\]

where \( l \leq l' \). We apply these trigonometric moments to investigate the phase correlations for the TOH03 FCM and WFM. For that, we simply substitute \( l = l' \) in equation (21) and define \( \xi_{l,m} \) as the phase of the FCM and \( \Psi_{l,m} \) as that of the WFM. The result of the calculations is presented in Figure 6.

From Figure 6 it can be clearly seen that the FCM has strong \( \Delta l = 4 \) correlations starting from \( l \approx 40 \), which rapidly increase.
for \( l > 40 \), while for the WFM these correlations are significantly damped. However, the \( d_{l,m}^2 \) estimator allows us to clarify the properties of phase correlations for the low multipole range. The idea is to apply the \( d_{l,m}^2 \) estimator to the FCM and WFM and to compare the power spectra of the signals obtained before and after that.

According to the definition of the \( d_{l,m}^2 \) estimator, the power spectrum of the signal is given by equation (16), which now has the form

\[
D(l) = \frac{2}{l} \sum m |c_{l,m}|^2 \left[ 1 - \cos (\xi_{l+\Delta,m} - \xi_{l,m}) \right].
\]
The last term in equation (22) corresponds to the cross-correlation between the $l, m$ and $l + 4, m$ modes, which should vanish for Gaussian random signals after averaging over the realization. For a single realization of the random Gaussian process, this term is nonzero because of the same reason as the well-known “cosmic variance,” implemented for estimation of the errors of the power spectrum estimation (see Naselsky et al. 2004a). Thus,

$$D(l) \approx \frac{2}{7} \sum_m |c_{l,m}|^2,$$  \hspace{1cm} (23)

and the error of $D(l)$ is of the order of

$$\frac{\Delta D(l)}{D(l)} \approx \frac{2}{\sqrt{l + 1/2}}.$$  \hspace{1cm} (24)

To evaluate qualitatively the range of possible non-Gaussianity of the FCM and WFM, in Figure 7 we plot the function $F(l) = 2D(l) - 2C(l) - D(l) + C(l)$ for the FCM and WFM, in which we mark the limits $\pm 2\sqrt{l}$. As one can see, the ranges of multipoles with potential non-Gaussianity are $l = 3-4$, $l = 21-24$, and $l \approx 100-150$ for the WFM. Nonrandomness on some of the multipole modes is mentioned in Chiang & Naselsky (2006).

At the end of this section we would like to demonstrate that application of the PF to maps with foreground residuals, such as the FCM, provides additional cleaning. In Figure 8 we present the $C(l)$ and Si$(l)$ trigonometric moments for the FCM with the shift of the multipoles $l' = l + 2$. One can see that the $\Delta = 2$ correlation of phases is strong (practically, they are at the same level as the $\Delta = 4$ correlations). However, after $d_{l,m}^2$ filtration these correlations are significantly decreased.

The implementation of the PF to the non-Gaussian signal significantly decreases these correlations. The properties of the PF described can manifest themselves more clearly in terms of images of the CMB signal. In Figure 9 we plot the results of the maps with $d_{l,m}^2$ implemented on the FCM and WFM, in order to demonstrate how the PF works on the non-Gaussian tails of the derived CMB maps. In Figure 9 we can clearly see that the morphology of the $D(l, \phi)$ maps is the same, and the difference between $D_{WFM}(l, \phi)$ and $D_{WFM}(l, \phi)$ is related to point-source residuals localized outside the Galactic plane (see the third panel). A direct subtraction of the WFM from the FCM reveals significant contamination of the GF residuals and non-Galactic point sources (the third from the bottom and bottom maps). The second from the bottom map corresponds to the difference between $D_{FCM}(l, \phi)$ and $D_{WFM}(l, \phi)$, for which the amplitudes of the signal are represented in the color-bar limit $\pm 0.1$ mK. One can see that the GF is removed down to the noise level. In combination with the phase analysis we can conclude that the implementation of the PF looks promising as an additional cleaning of the GF residuals and can help investigate the statistical properties of derived CMB signals in more detail.

6. CONCLUSION

In this paper we examine a specific group of correlations between $l$, which is used as an estimation of the statistical properties of the foregrounds in the WMAP maps. These correlations among phases in particular are closely related to symmetry of the GF (in the Galactic coordinate system). An important point of the analysis is that for the foregrounds the correlations of phases for the total foregrounds at the V and W bands have a specific shape when $\Psi_{l,m} = \Psi_{l+\Delta m}$, where $\Delta = 4n$ and $n = 1, 2, 3, \ldots$. These correlations can be clearly seen in the W band of the WMAP data sets down to $l_{\max} = 512$ and must be taken into account for modeling of the foreground properties for the upcoming Planck mission. We apply the PF to the TOH03 FCM, which contains strong residuals from the GF, and show that these residuals are removed from the $D_{WFM}(l, \phi)$ map. Moreover, in that map the statistics of the phases display statistics closer to Gaussian than the original FCM (no correlation of phases between different $l$, $m$ modes except between $l + \Delta$, $m$ and $l$, $m$, which is chosen as a basic one, defined by the form of the PF).

In this paper we do not describe in detail the properties of the signal derived from the PF by the WMAP V and W bands. Further developments of the method, including multifrequency combination of the maps and CMB extraction by the PF, will be in a separate paper. To avoid misunderstanding and confusion, here we stress again that any $D(l, \phi)$ maps synthesized from the $d_{l,m}^2$ are by no means the CMB signals (since the phases of these signals are not the phases of true CMB), and the true CMB can be obtained after multifrequency analysis, which is the subject of the forthcoming paper.

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