Magnetic ratchet effect in black phosphorous

Narjes Kheirabadi
Physics Department, Alzahra University, Vanak, Tehran 1993893973, Iran

Abstract: The magnetic ratchet effect has been studied in black phosphorous (BP) by the use of the Boltzmann kinetic equation that is a semi-classical approach. The Hamiltonian of BP in a parallel magnetic field is derived using the tight-binding model. We consider the effect of the magnetic field on non-linear dynamics in the presence of an ac laser field and spatial inversion asymmetry. We also have shown that for anisotropic 2D materials and BP, the ratchet current response to three different light polarizations: linearly polarized light, circularly polarized light and unpolarized light.

I. INTRODUCTION

As a ratchet machine rotates in one direction, the magnetic ratchet effect is an effect accordingly a dc current will be produced while a semiconductor is under an oscillating electric field of laser radiation under a steady magnetic field. This non-linear effect has been observed or predicted in semiconductors like monolayer [1] and bilayer graphene [2,3], quantum well [4] and Si-MOSFET [5]. In this work, we study the magnetic quantum ratchet effect in black phosphorous (BP) whereby a tunable gate voltage and a substrate, we have broken the symmetry of an anisotropic material; black phosphorous.

The phosphor element has 3p uncoupled electrons in its outer shell. In the phosphorene structure, each phosphor atom has two nearest neighbors; so, in phosphorene structure, each phosphor atom has two strong covalence bands and one free electron. This aspect is similar to the graphene structure despite this fact that in graphene each carbon atom has three covalence bands and one free electron.

In this article, we study the magnetic ratchet effect in the black phosphorous shape of phosphorene. However, for isotropic semiconductors, it has been shown that the response to an ac electric field is a dc produced current. This non-linear effect deduced from linearly polarized light [2]. In this article, we will show that BP produces a dc current that includes responses to linearly polarized light, circularly polarized light and unpolarized light.

II. HAMILTONIAN

The unit cell of BP is depicted in Fig. 1. According to this figure, there are four atoms in the unit cell, two atoms on the bottom layer ($A_1$ and $B_1$), and two atoms on the top layer ($A_2$ and $B_2$).

Intralayer coupling $t_1$, vertical interlayer coupling $t_2$ and skew interlayer couplings, $t_4$ and $t_5$, and $U_1$, $U_2$ and $\delta$ parameters that indicate different on-site energies are depicted in Fig. 1. In addition, the interlayer hopping parameter $t_3$ is the transfer energy of $B_1$ atom of one unit cell with $A_1$ atom of the beside unit cell. Furthermore, the intralayer distance between atoms in one unit cell is $a$, and for $d$ as interlayer distance, $d'$ is the distance between $B_1$ and $A_2$ atoms. We also assume that the interlayer angles are $\alpha = A_1B_1A_1 = B_2A_2B_2 = 98.15^\circ$ and $\beta = A_2B_1A_1 - 90^\circ = 103.69^\circ - 90^\circ = 13.96^\circ$. While, the upper layer is located at $d/2$ and the lower layer is located at $-d/2$ where $d = 6.55 \times 10^{-11}m$. Finally, we assume that BP is under the effect of an in-plane field $\mathbf{B} = (B_x, B_y, 0)$, where its vector potential is $\mathbf{A} = z(B_y, -B_x, 0)$ chosen to preserve translation symmetry in the BP plane.

Because there are four electrons in the unit cell of phosphorene, BP tight-binding Hamiltonian is a $4 \times 4$ matrix, and phosphorene has two conduction bands and two valence bands. To write the tight-binding Hamiltonian of BP in a parallel magnetic field, we use the Peierls substitution. For instance, to determine the Hamiltonian element for a process of hopping between the in-plane A and B sublattices, $H_{AB}$, we have determined the following summation over B sites at the position $\mathbf{R}_{ Bj}$

$$H_{AB} = t_1 \sum_{j=1}^{3} \exp \left( i \mathbf{K} \cdot (\mathbf{R}_{ Bj} - \mathbf{R}_A) - \frac{ie}{\hbar} \int_{\mathbf{R}_{ Bj}}^{\mathbf{R}_A} \mathbf{A} \cdot d\mathbf{l} \right).$$

(1)

Here, $\mathbf{K} = \mathbf{p}/\hbar$ is the electron wave vector and $d\mathbf{l}$ is the length differential. Consequently, we can show that the
Hamiltonian of BP in the steady magnetic field and in the basis of \((A_1, B_1, A_2, B_2)^T\) is
\[
H = \begin{pmatrix}
U_1 & f_1 + f_3 & f_4 & f_2 + f_5 \\
\delta f_4^* & U_3 + \delta & f_2^* + f_5^* & f_4^* \\
U_2 & f_2 & U_2 + \delta & f_1^* + f_4^* \\
f_2^* + f_5^* & f_4^* & f_1^* + f_3^* & U_2
\end{pmatrix}.
\] (2)

Assuming \(b = ed\mathbf{B}/2\), where \(a_x\) and \(a_y\) are the length of the unit cell into the \(x\) and \(y\) directions, for the bottom layer, we have
\[
f_1 = 2t_1 \cos \frac{a_y (p_y + b_y)}{2\hbar} \exp \left[ -i \left( \frac{p_x - b_y}{\hbar} \right) a \cos \frac{\alpha}{2} \right],
\] (3)
\[
f_3 = 2t_3 \cos \frac{a_y (p_y + b_y)}{2\hbar} \times \exp \left[ -i \left( \frac{p_x - b_y}{\hbar} \right) (2d' \sin \beta + a \cos \frac{\alpha}{2}) \right],
\] (4)
and for the top layer, we have
\[
f_1' = 2t_1 \cos \frac{a_y (p_y - b_y)}{2\hbar} \exp \left[ i \left( \frac{p_x + b_y}{\hbar} \right) a \cos \frac{\alpha}{2} \right],
\] (5)
\[
f_3' = 2t_3 \cos \frac{a_y (p_y - b_y)}{2\hbar} \times \exp \left[ -i \left( \frac{p_x + b_y}{\hbar} \right) (2d' \sin \beta + a \cos \frac{\alpha}{2}) \right].
\] (6)

Furthermore, we have
\[
f_2 = t_2 \exp \left[ -i \left( \frac{p_x}{\hbar} d' \sin \beta \right) \right],
\] (7)
\[
f_4 = 4t_4 \cos \left( \frac{p_x}{\hbar} (d' \sin \beta + a \cos \frac{\alpha}{2}) \right) \cos \left( \frac{p_y}{\hbar} a \sin \frac{\alpha}{2} \right),
\] (8)
\[
f_5 = t_5 \exp \left[ i \left( \frac{p_x}{\hbar} (a_x - d' \sin \beta) \right) \right].
\] (9)

As we mentioned before, because of four free electrons in the unit cell of phosphorene, there are four bands in the band structure of BP. In addition, it is important to work in low-energy regime. To do so, we make a Taylor expansion of \(f_i\) functions in the vicinity of the \(\Gamma\) point. Consequently, we can assume that \(\cos x = 1 - x^2/2\) and \(\exp x = 1 + x + x^2/2\). In addition, we neglect those terms that are quadratic or higher in magnetic field.

III. RATCHET CURRENT IN A TWO DIMENSIONAL MATERIAL

According to perturbation theory, magnetic dependent valence band is
\[
|0\rangle^p = |0\rangle + \frac{1}{E_1 - E_0} |1\rangle
\] (10)
where \(|0\rangle\) and \(|0\rangle^p\) are valence band and perturbed valence band eigenstates, respectively (we can derive the perturbed conduction band, as well). In this equation, \(|1\rangle\) is the conduction band eigenstate, and \(V\) is that part of Hamiltonian which includes the magnetic field. Additionally, \(E_1\) and \(E_0\) are conduction band and valance band energies, respectively.

We assume that the two dimensional material, BP, is under a radiation that is in–plane means that \(E_{\parallel}(t) = E_{\parallel} e^{-i\omega t} + E_{\parallel} e^{i\omega t}\). This in–plane radiation changes the electron distribution function. We use Boltzmann kinetic equation assuming \(V \cdot \partial f/\partial r = 0\) for homogeneous materials, so we have
\[
-e E_{\parallel} \cdot \nabla_p f(p, t) + \frac{\partial f(p, t)}{\partial t} = S\{f\}
\] (11)
where \(-e\) is the charge of electron. Collision integral \(S\{f\}\) is
\[
S\{f\} = \sum_{p'} \{W_{pp'}f(p', t) - W_{p'p}f(p, t)\}.
\] (12)
For a perturbed electron gas, the scattering rate is
\[
W_{p'p} = W_{p'p}^{(0)} + \delta W_{p'p},
\] (13)
where \(W_{p'p}^{(0)}\) is the rate of the electron scattering between unperturbed states, and \(\delta W_{p'p}\) is the change of the scattering rate because of the perturbation.

Additionally, according to the golden rule rate, the transition rate between \(p\) and \(p'\) states under a scattering potential, \(V_{p'p}\), is
\[
W_{p'p} = \frac{2\pi}{\hbar} |\langle p' | V_{p'p} | p \rangle|^2 \delta(\epsilon_p - \epsilon_{p'}).
\] (14)
where angular brackets indicate an average over impurity positions. Considering static impurities, we can write the following equation for \(V_{p'p}\)
\[
V_{p'p} = \sum_{j=1}^{N_{imp}} \tilde{Y} u(r - R_j)
\] (15)
where \(N_{imp}\) is the number of impurities, \(u(r - R_j)\) describes the spatial dependence of the impurity potential, and \(\tilde{Y}\) is a dimensionless matrix describing structure. We neglect interference between different impurities, and we use the Fourier transform of the impurity potential
\[
\tilde{u}(q) = \int d^2r u(r)e^{-iq \cdot r}/\hbar.
\] (16)
In the scattering rate, we perform a harmonic expansion of the impurity potential as described in the following equation
\[
| \tilde{u}(p' - p) |^2 = \sum_{m'\nu} \nu_m e^{im'(\phi' - \phi)}.
\] (17)
where \(\phi\) is the momentum direction and \(\nu_{-m} = \nu_m\) because it is an even function of \((\phi' - \phi)\). To determine the current by the Boltzmann kinetic equation, (Eq. [14]) and
the harmonic expansion method, we consider that $f(p, t)$ is a series with two indices $(n, m)$

$$f(p, t) = \sum_{n,m} f_n^m \exp(im\phi - im\omega t), \quad (18)$$

where $m$ and $n$ are integers. Multiplying the Boltzmann equation by a factor $\exp(-i\phi + il\omega t)$, where $j$ and $l$ are integers. Integrating over a period $2\pi$ of angle $\phi$ and a period of time, $t$, lead to coupled equations between different harmonic coefficients

$$(\tau_{ij}^{-1} - il\omega)f_j^l = \alpha_{ij}^l - f_{ij}^{l+1} + \bar{\alpha}_{ij}^{l-1} + \eta_{ij}^l + f_{ij}^{l+1} + \delta S_j^i.$$ \quad (19)

For isotropic materials in the absence of magnetic field

$$\tau_{ij}^{-1} = \sum_{p'} W_{p'p}[1 - \cos(j[\phi' - \phi])], \quad (20)$$

is the relaxation time of the $j$th angular harmonic of the electron distribution function. However, for an anisotropic 2DEG like BP, it is \[6, 7\]

$$\tau_{ij}^{-1}(\xi, p) = \frac{2\pi}{h} \sum_{p'} [p' \mid H \mid p]^2 \delta(\epsilon_p - \epsilon_{p'}) \times \left\{ 1 - \frac{\langle \xi, \nu(p') \rangle \tau_{ij,p'}{\nu(p)p}}{\langle \xi, \nu(p)p \rangle \tau_{ij,p}} \right\}, \quad (21)$$

where $\xi$ is the unit matrix of the electric field and $\nu$ is the group velocity. In addition, operators in Eq. [19] are

$$\alpha_j = \frac{e(E_x - iE_y)}{2} \left( -\frac{j}{\rho} + \frac{\partial}{\partial \rho} \right), \quad (22)$$

$$\bar{\alpha}_j = \frac{e(E_x^* - iE_y^*)}{2} \left( -\frac{j}{\rho} + \frac{\partial}{\partial \rho} \right), \quad (23)$$

$$\eta_j = \frac{e(E_x + iE_y)}{2} \left( \frac{j}{\rho} + \frac{\partial}{\partial \rho} \right), \quad (24)$$

$$\bar{\eta}_j = \frac{e(E_x^* + iE_y^*)}{2} \left( \frac{j}{\rho} + \frac{\partial}{\partial \rho} \right). \quad (25)$$

The factors $\delta S_j^i$ in Eq. [19] describe the correction to scattering caused by the magnetic field.

To quantify the dc current caused by an ac electric field, it is necessary to determine time–independent asymmetric parts of the distribution function; $f_{j+1}^{-1}$ terms. We assume that electrons are trapped in a huge box with length $L$ and under a periodic potential. For $\delta f = f_1^0 \exp(i\phi) + f_2^1 \exp(-i\phi)$, the current density is

$$J = -\frac{g}{L} \sum_{p} eV_g \delta f,$$ \quad (26)

where $g$ is the spin degeneracy ($g = 2$).

IV. BP

We assume that the band dispersion is equal to $\epsilon$, density of states per spin per unit area is $\Gamma$, and the group velocity of trapped electrons is $V_g$ where $V_g = \nabla_\rho \epsilon$. We break the symmetry of BP by considering different amounts for $U_1, U_2$ and $\delta$ factors (Fig. [1]). Depending on these three factors, we have a general form for $\delta W$.

This general form that is linear in magnetic field and momentum is

$$\delta W_{pp} = \frac{2\pi n_{imp}}{L^2} |\tilde{u}(p' - p)|^2 \delta(\epsilon_{p'} - \epsilon_p) \times \left\{ \text{Constant} + N_1 bb_p k(\cos \phi + \cos \phi') \right\} + N_2 b\phi x k(\sin \phi + \sin \phi'), \quad (27)$$

where $n_{imp} = N_{imp}/L^2$ is the density of impurities, $bb_i = b_i/h$, $k = p/h$. Constant, $N_1$ and $N_2$ are three prefactors that change by the change of on–site energies and disorder types.

To estimate Constant, $N_1$ and $N_2$ prefactors, we discuss about the problem numerically. The BP constants have been selected based on Ref. [8], we select $U_1 = 0$, and we consider different values for $U_2$ and $\delta$ [9]. $U_2$ is a tunable factor that shows the difference between the potential of the top and bottom layers [2, 9]. We consider $\delta$ in the range of 0 to $20\text{meV}$, $U_2$ in the range of 0 to $40\text{meV}$, then we calculate $\text{Constant}$, $N_1$ and $N_2$ based on such a selection for three different disorder types. In the symmetric case, where the top and bottom layers are under the effect of disorder, the disorder matrix is a unit matrix. In such a disorder type, we can show that the ratchet current is equal to zero. Consequently, the symmetry of the top and bottom layer should be broken by disorder or substrate to have a nonzero ratchet current. For a BP in which the symmetry is broken by $U_1, U_2, \delta$ and disorder type, if $I$ consider that the bottom layer is disordered, the disorder matrix is

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$ \quad (28)

Additionally, considering disorder on the top layer means that the disorder matrix is equal to

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$ \quad (29)

Accordingly, we can consider that the $\text{Constant}$ is equal to 0.25 in order of $10^{-2}$. For these two disorder types, for conduction and valance bands, we can show that considering $\delta = U_2 = 0$ will deduce to zero ratchet current. For any amount of $\delta$ factor, for nonzero $U_2$ amount, $N_1$
is nonzero. However for $\delta = 0$ or $U_2 = 0$, $N_2$ is equal to zero.

According to the Eq. \[27\] $\delta W$ is proportional to Constant $+ N_1 b_\gamma k (\cos \phi + \cos \phi') + N_2 b \phi k (\sin \phi + \sin \phi')$. Consequently, a current will be produced as the result of such a scattering rate by the in-plane magnetic field.

The relevant $\delta S^I_j$ factors, that are the change of the collision integral because of in–plane magnetic field are

$$
\begin{align*}
\delta S^I_0 &= 0, \\
\delta S^I_1 &= \Delta (N_1 B_y + i N_2 B_z) f^I_1, \\
\delta S^I_{-1} &= \Delta (N_1 B_y - i N_2 B_z) f^I_{-1}, \\
\delta S^I_{2} &= \Delta (N_1 B_y + i N_2 B_z) f^I_{2}, \\
\delta S^I_{-2} &= \Delta (N_1 B_y - i N_2 B_z) f^I_{-2},
\end{align*}
$$

(30)

where

$$
\begin{align*}
\Delta &= \frac{e d \pi N_{imp}}{2h^2} \Omega \gamma (e)p, \\
\Omega &= - (\nu_0 - \nu_2).
\end{align*}
$$

(31)

(32)

In continue, we assume that the scattering happens because of the short range scattering $u(r - R_j) = u_0 \delta (r - R_j)$ \[22\]. Hence, we can show that the corresponding in–plane current is

$$
\begin{align*}
J_x &= M_{1,x} [B'_y (|E_x|^2 - |E_y|^2) - B'_y (E_x E_y^* + E_y E_x^*)] \\
&\quad + M_{2,x} B'_y |E|^2 + M_{3,x} B'_y (E_x E_y^* - E_y E_x^*), \\
J_y &= M_{1,y} [B'_y (|E_x|^2 - |E_y|^2) + B'_y (E_x E_y^* + E_y E_x^*)] \\
&\quad - M_{2,y} B'_y |E|^2 + M_{3,y} B'_y (E_x E_y^* + E_y E_x^*),
\end{align*}
$$

(33)

(34)

where $B'_y = N_1 B_y$ and $B'_z = N_2 B_z$. Furthermore, $M_{ij}$ coefficients are responses to different light polarizations in the current. $M_1$ is response to linearly polarized light, $M_2$ is response to unpolarized light, and $M_3$ is response to circularly polarized light. We can show that for BP and anisotropic 2D materials

$$
\begin{align*}
M_{1,i} &= - \frac{ge^3}{4L^2} \sum_p V_{g,i} \tau_{1,i} \tau_{2,i} \Delta \left( - \frac{1}{p} + \frac{\partial}{\partial p} \right) \frac{2 \tau_{1,i}}{\omega^2 \tau_{1,i}^2 + 1} \left| \frac{\partial f_0}{\partial \epsilon} \right| p, \\
M_{2,i} &= - \frac{ge^3}{4L^2} \sum_p V_{g,i} \tau_{1,i} \left( 2 \frac{\partial}{\partial p} \right) \frac{(2 \tau_{1,i} \tau_{2,i}) (1 - \omega^2 \tau_{1,i} \tau_{2,i}) \left| \frac{\partial f_0}{\partial \epsilon} \right| p}{(1 + \omega^2 \tau_{1,i}^2) (1 + \omega^2 \tau_{2,i}^2)} p, \\
M_{3,i} &= - \frac{ge^3}{4L^2} \sum_p V_{g,i} \tau_{1,i} \left( 2 \frac{\partial}{\partial p} \right) \frac{(2 \omega \tau_{1,i} \tau_{2,i}) (\tau_{1,i} + \tau_{2,i}) \left| \frac{\partial f_0}{\partial \epsilon} \right| p}{(1 + \omega^2 \tau_{1,i}^2) (1 + \omega^2 \tau_{2,i}^2)} p.
\end{align*}
$$

(35)

(36)

(37)

V. DISCUSSION

To determine the $M$ factors and the current, we assume that the eigenvalues of the system are based on the Ref. \[10\]. Accordingly,

$$
\frac{\partial}{\partial p} = C_{BP} \frac{\partial}{\partial \epsilon}
$$

(38)

and

$$
C_{BP} = - \frac{2}{\hbar^2} \left[ \frac{\epsilon^2}{E_g} + (\eta_0 + \nu_0) \right].
$$

(39)

where $E_g = E_x - E_v$, $\gamma = 0.480eV$, $\eta_0 = 0.038eV$ and $\nu_0 = 0.030eV$ are from Ref. \[10\]. The anisotropic phosphorene has an origin band gap of $2eV$, and it can be potentially tuned \[11\]. Hence, we can show that

$$
\begin{align*}
M_{1,i} &= - \frac{ge^3}{2} C_{BP} \tau_{1,i} \omega^2 + 1 \left[ V_{g,i} \tau_{1,i} \Gamma (\tau_{1,i} \tau_{2,i} A) + C_{BP} \frac{\tau_{1,i} \tau_{2,i}}{\tau_{1,i} \omega^2 + 1} \right], \\
M_{2,i} &= - \frac{ge^3}{2} C_{BP} \tau_{1,i} \tau_{2,i} \left( 1 - \omega^2 \tau_{1,i} \tau_{2,i} \right) \left( 1 + \tau_{1,i}^2 \omega^2 \right) \left( 1 + \tau_{2,i}^2 \omega^2 \right) \left( V_{g,i} \tau_{1,i} \Gamma (\tau_{1,i} \tau_{2,i} A) + C_{BP} \frac{\tau_{1,i} \tau_{2,i}}{\tau_{1,i} \omega^2 + 1} \right), \\
M_{3,i} &= - \frac{ge^3}{2} C_{BP} \left( V_{g,i} \tau_{1,i} \Gamma (\tau_{1,i} \tau_{2,i} A) + C_{BP} \frac{\tau_{1,i} \tau_{2,i}}{\tau_{1,i} \omega^2 + 1} \right),
\end{align*}
$$

(40)

(41)

(42)

Here, derivatives are related to the energy and all of the variables are evaluated on the the Fermi surface: $E_f = h^2 \pi n / m_d$; $n$ is the carrier density in BP \[12\] and $m_d = \sqrt{m_{xx} m_{yy}}$ where $m_{xx} = 0.8m_0$, $m_{yy} = 0.7m_0$ and $m_0$ is electron free mass. In addition, we have $V_{g,x} = -2p_s(\gamma^2 + E_g \eta_0)/E_g h^2$ and $V_{g,y} = -2p_s \nu_0 / h^2$. We also assumed that density of states, $\Gamma$, is constant and it is equal to $m_d / \pi h^2$. We also assume that the scattering time are independent of energy \[12\]. Consequently, we have

$$
\begin{align*}
M_{1,i} &= - \frac{ge^3}{2} C_{BP} \tau_{1,i} \tau_{2,i} \left( 1 - \omega^2 \tau_{1,i} \tau_{2,i} \right) \left( 1 + \tau_{1,i}^2 \omega^2 \right) \left( 1 + \tau_{2,i}^2 \omega^2 \right) \left( V_{g,i} \tau_{1,i} \Gamma (\tau_{1,i} \tau_{2,i} A) + C_{BP} \frac{\tau_{1,i} \tau_{2,i}}{\tau_{1,i} \omega^2 + 1} \right), \\
M_{2,i} &= - \frac{ge^3}{2} C_{BP} \frac{\tau_{1,i} \tau_{2,i}}{\tau_{1,i} \omega^2 + 1} \left( 2 V_{g,i} \gamma \tau_{1,i} \tau_{2,i} A \Gamma (\tau_{1,i} \tau_{2,i} A) + C_{BP} \frac{\tau_{1,i} \tau_{2,i}}{\tau_{1,i} \omega^2 + 1} \right),
\end{align*}
$$

(43)

(44)
\[ M_{3,i} = \frac{g e^3}{2} C_{BP} \Gamma \left( \frac{\omega \tau_{2,i}^2}{1 + \tau_{1,i}^2 \omega^2} \right) \left( 1 + \tau_{1,i}^2 \omega^2 \right) \left( 1 + \tau_{2,i}^2 \omega^2 \right) \]

\[ 2V_{g,i} + C_{BP} p_i \left( V_{g,i} p_i \right) \].  \hspace{1cm} (45)

These prefactors are not equal to zero, consequently, BP and anisotropic 2D materials have responses to three types of radiation: unpolarized light, linearly polarized light and circularly polarized light. However, in bilayer graphene, and other isotropic 2D materials, there is only a response to linearly polarized light \[2\].

Furthermore, based on the direction of momentum relaxation time and group velocity, the current will change. Besides, dependent on the place of disorder that is on top or bottom layers, the effect of an applied magnetic field or on the place of disorder that is on top relaxation time and group velocity, the current will change. Hence, the current density caused by the applied \(B_2\) is in order of \(nAm^{-1}\) and the current density caused by the applied \(B_y\) is in order of \(\mu Am^{-1}\).

For the conduction band the magnitude of \(N_1\) and \(N_2\) prefactors are similar to the valance band. For instance, in the case of conduction band where \(\delta = 0.02eV\), and for \(0 < U_2 < 0.04, N_1\) prefactor decreases linearly between \(0\) and \(-1.6 \times 10^{-4} \text{A}^2\), and \(N_2\) prefactor increase linearly from \(0\) to \(2.5 \times 10^{-8} \text{A}^2\).

VI. CONCLUSION

We consider BP material to study the ratchet current in anisotropic materials. The tight-binding Hamiltonian of BP in a parallel magnetic field has been derived. Moreover, the semi-classical Boltzmann kinetic equation is used to derive the direct current in BP under the in-plane magnetic field. Even though isotropic materials have a nonzero response to linearly polarized light, for anisotropic material under asymmetric disorder or substrate ratchet current includes the response to three types of radiations means linearly polarized ligh, circularly polarized light, and unpolarized light.