New physics effects on temporal and spatial correlations in neutrino oscillations

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Effects of physics beyond the standard model in the neutrino sector are conveniently incorporated through non-standard interaction parameters. Assuming new physics in the form of dimension-6 vector operators, a recent global analysis of neutrino oscillation data including results from COHERENT experiment suggests two favorable new physics scenarios. These are LMA-Light (with normal mass ordering) & LMA-Dark (with inverted mass ordering) sectors of parameters. In this work, we study the effects of these new physics solutions on Leggett-Garg-type (LGtI) inequality which quantifies temporal correlations in the system along with flavor entropy and genuine tripartite entanglement which can be considered as measures of spatial correlations. We show that the violation of LGtI for $E_{\nu}$ energy range between 3-4 GeV in the DUNE experimental set-up can not only be an indication of presence of new physics but such a new physics is expected to be in the form of LMA-Dark sector with inverted ordering. Further, we show that the LMA-Light solution, in general, decreases the values of all measures of quantum correlations in comparison to their SM predictions. On the other hand, the Dark solution can significantly enhance the values of these measures.

I. Introduction

The currently running experiments at the LHC along with the experiments such as BaBar and Belle have provided several engaging evidences of physics beyond the Standard Model (SM) of electroweak interactions. These include hints of lepton flavour universality (LFU) violation in the decays induced by the charged current quark level transition $b \rightarrow c l \nu$ ($l = e, \mu, \tau$) [1] as well as in the neutral current $b \rightarrow s l^+l^-$ ($l = e, \mu$) [2, 3] decays. The preferred Lorentz structure(s) of the possible new physics [4, 12] can be realized through several extensions of the SM.

The effects of new physics can also manifest in the neutrino sector. The experimental facilities in neutrino physics are now tending towards higher precision and have potential to probe such sub-leading effects. This has triggered a considerable interest in the neutrino physics community. The new physics effects in neutrino interactions are conveniently incorporated through effective non-standard interaction (NSI) parameters [13–25].

Considering SM to be the low energy renormalizable approximation, containing only dimension $D \leq 4$ operators, of a higher dimensional theory existing at much higher mass scale $\Lambda$, the new physics effects are assumed to have higher dimensional Lorentz structures ($D > 4$) constructed out of SM fermion fields. In this work, we restrict ourselves to dimension-6 vector operators which may show sub-leading effects in long baseline (LBL) neutrino experiments such as Deep Underground Neutrino Experiment (DUNE).

In a recent analysis, bounds on NSI parameters were obtained by performing a global fit at all relevant data in the neutrino sector. This includes coherent neutrino-nucleus scattering data from COHERENT experiment [23]. In this analysis, two new physics scenarios have been identified as the most favourable solutions to the global data:

1. LMA-Light sector ($0 < \theta_{12} < \pi/4$) with normal ordering (NO),
2. LMA-Dark sector ($\pi/4 < \theta_{12} < \pi/2$) with inverted ordering (IO).

These new physics effects can also affect the temporal and spatial correlations present in the system. The most popular criteria to test spatial quantum correlations is Bell’s inequality. However, till date, it is not clear how such measurements can be performed in the neutrino sector using the current experimental set-ups. On the other hand, the determination of temporal correlations based on the assumptions of macrorealism (MR) and noninvasive measurement (NIM) and usually quantified in terms of Leggett-Garg inequalities (LGI) is experimentally feasible in the context of neutrino oscillations. In fact, violations of a class of such inequalities using data from MINOS and Daya Bay experiments have been demonstrated in refs. [26] and [27], respectively.

In this work we study new physics effects, in particular the impact of two new physics solutions obtained in [25]. In this work, we study the effects of these new physics solutions on temporal correlations in neutrino oscillations quantified in terms of Leggett-Garg-type inequality (LGtI). These inequalities are constructed by replacing the NIM condition by a weaker condition called stationarity [49]. Such inequalities are more suited for the study of temporal correlations in the neutrino sector in comparison to the LGIs as measurement of neutrinos destroys the NIM assumption. Further, LGtIs can be expressed in terms of neutrino survival and transition probabilities [26, 28]. We intended to identify parameter space where violation of LGtI can provide unambiguous signatures of new physics.
Further, we also study NSI effects on spatial correlations quantified in terms of flavor entropy [29, 30] and genuine tripartite entanglement [30]. These are basically measures of entanglement embedded in the system. Moreover, we also analyze correlations of these observables with the neutrino transition probability. We present our results in the context of upcoming LBL DUNE experimental set-up. We show that the violation of LGtI for $\nu_\alpha$ energy range between 3–4 GeV in the DUNE experimental set-up can not only be an indication of presence of new physics but such a new physics is expected to be in the form of LMA Dark sector of $\theta_{13}$ with IO.

The new physics effects in the context of quantum correlations were first studied in [31] where the NSI effect on a measure of quantum coherence was studied. While this work was in preparation, the article [32] appeared on the arXiv where NSI effects on LGI was studied. They showed that LGI violation can be enhanced as compared to the standard scenario for specific choices of NSI parameters. In this work we study LGtI under the effects of NSI, however, apart from the study of suppression and enhancement in the value of LGtI parameter over the SM value, we focus on identifying the parameter space where one can get unequivocal imprints of new physics. Further, we study NSI effects on spatial correlations as well.

The Plan of this work is as follows. In Sec. II, we illustrate the dynamics of neutrino oscillations within SM interaction as well in the presence of NSI. We also define the measures of temporal and spatial quantum correlation used in this work. Then in Sec. III we present and explain our results. Finally, we conclude in Sec. IV.

II. Formalism

In this section, we present the theoretical framework of our analysis. We start with the dynamics of neutrino oscillations under the effect of both SM interaction and NSI in subsection II A and II B respectively. Then in subsection II C we define the correlation measures used in this work.

A. Neutrino oscillation in matter

Let us consider that the neutrino is produced initially in the $\nu_\alpha$ flavour at time $t = 0$ ($\alpha = e, \mu, \tau$). The flavour state is related to the mass eigenstate $\nu_i(t = 1, 2, 3)$ by the so called $3 \times 3$ unitary matrix $U$ (PMNS matrix) as

$$| \nu_\alpha \rangle = \sum_{i=1}^{3} U_{\alpha i}^{*} | \nu_i \rangle,$$  \hspace{1cm} (1)

Time evolved mass eigenstates at time $t$ can be represented by $| \nu_i(t) \rangle = e^{-i H_m t} | \nu_i \rangle = e^{-i E_i t} | \nu_i \rangle$, where $H_m$ is the Hamiltonian of neutrino propagation in mass basis and $E_i$ are the eigenvalues corresponding to $\nu_i$. Then the time evolution of the flavor state is given as,  \hspace{1cm} 

$$| \nu_\alpha(t) \rangle = e^{-i H_f t} | \nu_\alpha \rangle = U_f(t) | \nu_\alpha \rangle$$ \hspace{1cm} (2)

where, $H_f = U H_m U^\dagger$, is the Hamiltonian of neutrino oscillation in flavour basis.

The Hamiltonian $H_f$ in the flavour basis, when neutrino propagates in matter, is given as

$$H_f = H_{vac} + H_{mat} = U \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} U^\dagger + A \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$  \hspace{1cm} (3)

where $A = \pm \sqrt{2} G_F N_e$ is standard matter potential, $G_F$ is the Fermi constant and $N_e$ is the electron number density. The sign of $A$ is positive for neutrinos and negative for anti-neutrinos. The ultra-relativistic limit , $t = L$ is applied for neutrino experiments. Following the framework of [33], the flavour evolution operator can be obtained as

$$U_f(L) = e^{-i H_f L} = \phi \prod_{a=1}^{3} e^{-i L \lambda_a} \frac{1}{3 \lambda_a^2 + C_1} \times \left[ (\lambda_a^2 + C_1) L + \lambda_a \tilde{T} + \tilde{T}^2 \right],$$  \hspace{1cm} (4)

where $T = H_m - tr(H_m)/3$ is the traceless matrix, $\phi = \exp(-iL \text{tr}(H_m)/3)$ and $\tilde{T} = U T U^\dagger$. Further, $\lambda_a$ ($a = 1, 2, 3$) are the eigenvalues of $T$-matrix and $C_1 = \text{Det}(T) \text{tr}(T^{-1})$.

B. Non Standard Interaction in neutrino oscillation

In addition to the standard interactions, the neutrino dynamics can also be affected by NSI. Effects of NSI can be visible for long baseline experiments, such as DUNE, where neutrinos pass through the Earth’s material medium over a long distance ($\sim 1300$ km). NSI can be classified in two types: charged current (CC)-NSI and neutral current (NC)-NSI. CC-NSI mainly affects neutrino production and detection processes [34, 35], while NC-NSI affects the neutrino propagation in matter via coherent forward elastic scattering [25]. The effect of incoherent scattering is neglected in case of Earth matter density $\rho \sim 2.8$ gm/cc, as the mean free path for the process is much larger than Earth’s diameter when the neutrino energy is lower than $\sim 10^5$ GeV [26]. The CC-NSI is strictly constrained, at least by an order of magnitude in comparison to the NC-NSI [35], due to bounds coming mainly from the Fermi constant, CKM unitarity, pion decay and the kinematic measurements of the masses of the gauge bosons $M_Z$ and $M_H$. Hence in many analyses, the NSI parameters at the source and detectors are set to zero.

SM can be considered the lower energy effective theory of some higher dimensional theory. Therefore, the effective Lagrangian can be expressed in terms of higher
dimensional (\(d\)) non-renormalizable operators (\(O_{i,d}\)),
\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda} \sum_i C_{i,5} \mathcal{O}_{i,5} + \frac{1}{\Lambda^2} \sum_i C_{i,6} \mathcal{O}_{i,6} + \ldots \quad (5)
\]
Here \(\Lambda\) is the scale of new physics and \(C_i\)'s are the Wilson coefficients encapsulating the short-distance physics. Beyond SM, dimension-5 Weinberg operator is the first higher dimensional operator which can generate small neutrino mass after electroweak symmetry breaking.

\[
\mathcal{L}_{\text{CC--NSI}} = 2\sqrt{2} G_F \epsilon_{\alpha \beta}^{f \mathcal{L}} (\nu_{\alpha} \gamma^\mu P_L \beta)(\bar{f} \gamma_\mu P_L f) + 2\sqrt{2} G_F \epsilon_{\alpha \beta}^{f \mathcal{R}} (\nu_{\alpha} \gamma^\mu P_L \beta)(\bar{f} \gamma_\mu P_R f),
\]
\[
\mathcal{L}_{\text{NC--NSI}} = 2\sqrt{2} G_F \epsilon_{\alpha \beta}^{f \mathcal{L}} (\nu_{\alpha} \gamma^\mu P_L \nu_\beta)(\bar{f} \gamma_\mu P_L f) + 2\sqrt{2} G_F \epsilon_{\alpha \beta}^{f \mathcal{R}} (\nu_{\alpha} \gamma^\mu P_L \nu_\beta)(\bar{f} \gamma_\mu P_R f).
\]

\(P_{L,R} = (1 \mp \gamma^5)/2\) are left and right handed chirality operators. \(\epsilon_{\alpha \beta}^{f \mathcal{L}}\) and \(\epsilon_{\alpha \beta}^{f \mathcal{R}}\) are the dimensionless Wilson coefficients which give relative strength of NSI for CC and NC, respectively. For CC-NSI, \(f \neq f'\) and \(f, f' = u, d\) while for NC-NSI, \(f = e, u, d\).

The concept of NSI was first introduced in \[23\] in terms of flavour changing neutral current (FCNC) as shown in Eq. (4). In the limit \(\epsilon_{\alpha \beta} \to 0\), SM result is restored. When \(\epsilon_{\alpha \beta} \sim 1\), the new physics effects have the same strength as SM weak interaction. \(\epsilon_{\alpha \beta} \neq 0\) for \(\alpha \neq \beta\) implies lepton flavour violation (LFV) and \(\epsilon_{\alpha \beta} \neq \epsilon_{\beta \alpha}\) shows lepton flavour universality violation (LFUV). For neutrino oscillation in matter, vector part of NSI, \(\epsilon_{\alpha \beta}^M = \epsilon_{\alpha \beta}^{f \mathcal{L}} + \epsilon_{\alpha \beta}^{f \mathcal{R}}\), is relevant.

In the presence of NSI, the Hamiltonian in flavour basis given in Eq. (5) is modified as,
\[
\mathcal{H}_f = U \begin{pmatrix}
0 & 0 & 0 \\
\frac{\Delta_{23}}{2} & 0 & \frac{\Delta_{12}}{2} \\
0 & 0 & \frac{\Delta_{23}}{2}
\end{pmatrix} U^+ + \mathcal{A} \begin{pmatrix}
1 + \epsilon_{ee}(x) & \epsilon_{eu}(x) & \epsilon_{et}(x) \\
\epsilon_{ee}(x) & \epsilon_{mu}(x) & \epsilon_{mt}(x) \\
\epsilon_{ee}(x) & \epsilon_{mu}(x) & \epsilon_{mt}(x)
\end{pmatrix},
\]
where \(\epsilon_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij}\) and \(\delta\) is the CP violating phase. Due to the consideration of complex NSI parameters, there appears extra phase factor \(\phi_{\alpha \beta}\) which can affect the correct estimation of \(\delta_{CP}\). To get rid of this difficulty, PMNS matrix is specifically chosen as given in Eq. (11). This has been discussed extensively in \[23\]. Another difficulty arises due to CPT symmetry under which the vacuum Hamiltonian has to transform

However, the required new physics scale is \(\sim 10^{13}\) GeV for the generation of neutrino mass of the order of 1 eV, which is beyond the energy range of LHC \[37\, 39\]. Operators of dimension-6 and 8 are studied extensively in \[20\, 40\]. In our work, we are focusing on lepton number conserving dimension-4 four-fermion operators which can significantly affect neutrino oscillations through NSI \[41\]. Lagrangian for CC and NC-NSI are represented using the dimension-6 operators as following \[42\].
tant solution ($\pi/4 < \theta_{12} < \pi/2$) with large values of NSI. When matter effect is included along with NSI, to restore the CPT invariance the following transformation is required

$$\sin \theta_{12} \leftrightarrow \cos \theta_{12},$$

$$\Delta m^2_{31} \rightarrow -\Delta m^2_{31} + \Delta m^2_{21},$$

$$\delta \rightarrow \pi - \delta,$$

$$\epsilon_{ee} - \epsilon_{\mu\mu} \rightarrow - (\epsilon_{ee} - \epsilon_{\mu\mu}) - 2,$$

$$\epsilon_{\tau\tau} - \epsilon_{\mu\mu} \rightarrow - (\epsilon_{\tau\tau} - \epsilon_{\mu\mu}),$$

$$\epsilon_{\alpha\beta} \rightarrow - \epsilon_{\alpha\beta}.$$  \hfill (12)

It has been estimated in the global data analysis that LMA sector ($\theta_{12} \approx 34^\circ$) favours $\epsilon_{\alpha\beta} \sim 0$ while LMA-Dark solution is favourable for large value of NSI (flavour diagonal) $O(\epsilon) \sim 1$. Standard matter potential lifts the degeneracy. Hence the solar neutrino and KAMLAND data could determine the octant of $\theta_{12}$ which showed the best fit for LMA solution [44].

In a recent analysis of COHERENT experiment [45], it has been shown that LMA-Dark solution is discarded for a broad range of NSI parameters where the mediator mass is above $\sim 10$ MeV. However, a mediator of mass $\sim 10$ MeV is able to produce sufficiently large value of NSI [46–47].

C. Quantum correlation quantities

Here we will briefly discuss some of the spatial as well as temporal quantum correlation measures used in this work.

1. Flavour Entropy: It is a standard measure of entanglement for a tripartite system which is defined as the sum of the von Neumann entropy of the reduced density matrix obtained by taking the trace over each one of the subsystems involved. Therefore, this measure can be considered as an absolute entanglement measure for a tripartite system since its nonzero value ensures the existence of the nonzero entanglement at least in one bipartition. Flavour entanglement entropy for the three flavor neutrino oscillation system can be defined as [29, 30],

$$S \left( |U_{fij}|^2 \right) = -3 \sum_{j=1}^{3} |U_{fij}|^2 \log_2 \left( |U_{fij}|^2 \right) -3 \left( 1 - |U_{fij}|^2 \right) \log_2 \left( 1 - |U_{fij}|^2 \right),$$  \hfill (13)

$$+ \left( 1 - |U_{fij}|^2 \right) \log_2 \left( 1 - |U_{fij}|^2 \right),$$  \hfill (14)

where $i = 1, 2, 3$ corresponding to initial neutrino flavour $\alpha = e, \mu, \tau$ and $U_f$ is the neutrino evolution operator. Minimum value of $S = 0$ i.e., no entanglement condition is obtained if any one of $P_{\alpha\beta} = 1$ and the maximum value or upper bound, $S = 2.75$, of this parameter can be approached when $P_{\mu\mu} = P_{\mu\tau} = P_{\tau\mu} = \frac{1}{3}$, i.e., all the three flavors are equally probable.

2. Genuine tripartite Entanglement: Another measure of tripartite entanglement can be defined as cube of the geometric mean of von Neumann entropies of each bipartite section and can be expressed as following [30]

$$G \left( |U_{fij}|^2 \right) = \Pi_{j=1,2,3} H \left( |U_{fij}|^2 \right),$$  \hfill (15)

where $i = 1, 2, 3$ corresponding to $\alpha = e, \mu, \tau$ and $H(x) = -x \log_2(x) - (1 - x) \log_2(1 - x)$. The nonzero value of this measure can be obtained only when all the subsystems are entangled with each other. $G$ will be zero if any of the subsystems is not entangled with the rest of the system.

3. Leggett-Garg type Inequality (LGI) ($K_3$): The above two measures of entanglement can be considered as measures of spatial correlation in a given system. Leggett-Garg inequalities (LGI), based on the assumptions of macro-realism (MR) and noninvasive measurement (NIM) is considered to capture temporal correlations in a system. The LGI parameter is basically a linear combination of autocorrelation functions $C(t_i, t_j) = \frac{1}{4} \text{Tr} \{ [\hat{Q}(t_i), \hat{Q}(t_j)] \rho(t_0) \}$ with $\rho(t_0)$ being the initial state of a given system at time $t = 0$ and can be written as [43]

$$K_3 = C(t_1, t_2) + C(t_2, t_3) - C(t_1, t_3) \leq 1.$$  \hfill (16)

Here, $\hat{Q}$ is a dichotomic observable, i.e., $\hat{Q} = \pm 1$ with $\hat{Q} = +1$ if the system is found in the target state and $\hat{Q} = -1$ otherwise. Measurement of neutrinos destroys the NIM assumption. Hence the weaker condition of stationarity is applied to relax this assumption [49]. Due to the stationarity condition, functions $C(t_i, t_j)$ now depend only on the time difference $t_j - t_i$. The $K_3$ quantity can be written as [26, 28, 30]

$$K_3 = 2C(0, t) - C(0, 2t) \leq 1,$$  \hfill (17)

for $t_1 = 0$ and $t_2 - t_1 = t_3 - t_2 \equiv t$. The inequality $K_3 \leq 1$ in Eq. (17) is known as Leggett-Garg-type inequality which is more suited, in comparison to the LGI inequalities, from the point of view of experiential measurements in the context of neutrino oscillations. For oscillating neutrino-system, being produced in $|\nu_\alpha\rangle$ state, the dichotomic observable can be defined as $\hat{Q}(t) = 2 |\nu_\beta\rangle \langle \nu_\beta| - 1$ with completeness condition $\sum_{\beta} |\nu_\beta\rangle \langle \nu_\beta| = 1$, $\beta = e, \mu, \tau$ with $\hat{Q} = +1$, if neutrino is detected in $|\nu_\beta\rangle$ state and $-1$, if it has other flavour. This leaves us with the expression of $K_3$ in terms of probability $P_{\alpha\beta}$ as [28]

$$K_3 = 1 + 2P_{\alpha\beta}(2L, E) - 4P_{\alpha\beta}(L, E),$$  \hfill (18)
Here we have applied the condition $t = L$ for ultrarelativistic neutrinos. It was shown in $^{26}$ that the parameter $K_3$ can be determined experimentally by making use of the condition $P_{\alpha\beta}(2L,E) = P_{\alpha\beta}(L,E)$ by suitable choice of $E$ and $E$.

### III. Results and discussion

Here, we study various correlation measures in the context of DUNE experiment ($L = 1300$ km, $E = 1 - 10$ GeV) $^{51}$. In Fig. 1, we show predictions for the quantity $K_3$ (upper panel), genuine tripartite entanglement ($G$) (middle) and flavor entropy ($S$) (lower panel) in the $E - \delta$ plane for the case of SM interaction + NO (first column), NSI interaction with LMA-Light sector of $\theta_{12}$ and NO (second column), SM interaction + IO (third column) and LMA-Dark sector of $\theta_{12}$ with IO (fourth column). The NSI parameter-values (within 1$\sigma$ interval) have been taken from the global analysis of neutrino oscillation and coherent neutrino scattering (COHERENT experiment) data as given in Tab. II (where $\epsilon'_{3\alpha\beta}$ is neglected) $^{25}$, while we make use of the values of standard neutrino oscillation parameters from a recent analysis $^{52}$ and is given in table I. The constant matter density potential $\rho = 2.8$ gm/cc, appropriate for DUNE experiment, has been used.

The quantity $K_3$ has been plotted for the $\nu_\mu \rightarrow \nu_e$ channel. It can be seen that for almost all range of neutrino energy $E > 4$ GeV, $K_3$ exceeds the value 1 for both SM and NSI scenarios (for NO) for $\delta \in [0, 2\pi]$, i.e., the LGtI is always violated in this range of $E$ and $\delta$. Also, the violation is evident in the range $E < 2$ GeV as well. However, it can be seen that the NSI scenario, LMA-Light + NO, tends to decrease the violation of LGtI as compared to that in case of SM interaction. The maximal value of $K_3$ in SM scenario is $\geq 1.05$ (at around $4 \text{ GeV} \geq E \geq 8.5 \text{ GeV}$ & $0 \leq \delta \leq 3\pi/2$ and $\delta$ near $2\pi$) while due to NSI (LMA-Light + NO) effect, the maximum value is always less than 1.05 in the entire parameter space (except in a small region around 1 GeV).

In case of inverted ordering with SM interaction, LGtI is violated ($i.e., K_3 > 1$) for a small range of energy $E \leq 2$ GeV and for $E > 8$ GeV (except for a narrow region near $\sim 3\pi/2$). On the other hand, for LMA-Dark + IO solution, $K_3$ reaches the value > 1.05 in the range approximately 3 GeV $\leq E \leq 7$ GeV for all values of CP-phase $\delta$. Here, it is noticeable that $K_3$ is violated, (i.e., greater than 1) at $E \approx 3$ GeV only in case of LMA-Dark + IO among all four scenarios. Hence, observation of violation of LGtI at $E \approx 3$ GeV can represent a signature of new physics as well as it will favour the LMA-Dark solution (for all $\delta$).

Genuine tripartite entanglement is described in the middle figures. It is seen that in case of SM interaction + NO, large entanglement ($> 0.231$) is visible for $4 \text{ GeV} \leq E \leq 5.5 \text{ GeV}$ for $0 \leq \delta \leq \pi$ and $7\pi/4 \leq \delta \leq 2\pi$, while the maximum value ($\geq 0.33$) is approached at around 2 GeV. Here also the NSI scenario, LMA-Light + NO, is seen to decrease the genuine entanglement present in the system as compared to the SM interaction. It is clear that at $E \approx 4$ GeV for almost all values of $\delta$ for LMA-Light + NO, $0.132 \leq G \leq 0.165$.

In case of SM interaction with IO, the genuine entanglement decreases drastically at around $E \approx 2$ GeV which is evident from the plots in the second row and third column of Fig. 1, while for LMA-Dark + IO, $G$ is again increased in the range $4 \text{ GeV} \leq E \leq 5$ GeV and is largely increased around 2 GeV roughly for all values of $\delta$. On the other hand, in case of flavor entropy $S$, represented in lower panel of Fig. 1, no significant change has been observed due to NSI effects both in case of NO and IO. These features of $G$ and $S$ quantities show that NSI effect can alter the genuine entanglement present in the system, while the residual entanglement will remain almost unchanged.

In Fig. 2, we presented the correlation of all the three nonclassicality measures $K_3$, $G$ and $S$ with probability $P(\nu_\mu \rightarrow \nu_e)$. Here red and green dots illustrate respectively, the cases of SM and NSI interactions. It is

| Parameters | LMA+NO ($\sim 1\sigma$ interval) | LMAD+IO ($\sim 1\sigma$ interval) |
|------------|---------------------------------|-----------------------------------|
| $\sin^2 \theta_{23}$ | [0.54, 0.61] | [0.540.61] |
| $|\Delta m_{31}^2| \times 10^{-3} \text{ eV}^2$ | [2.45, 2.55] | [2.475, 2.575] |
| $\epsilon_{ee} - \epsilon_{\mu\mu}$ | [-0.5, 0.25] | [-2.5, -1.75] |
| $|\epsilon_{ee} - \epsilon_{\mu\mu}|$ | [0, 0.1] | [-0.2, 0] |
| $|\epsilon_{e\tau}$ | [0, 0.1] | [0, 0.1] |
| $|\epsilon_{\mu\tau}$ | [0, 0.2] | [0, 0.025] |
| $\phi_{e\mu}$ | [67.5°, 281.25°], [0°, 90°], [247.5°, 360°] |
| $\phi_{e\tau}$ | [0°, 360°] |
| $\phi_{\mu\tau}$ | [0°, 360°] |
| $\delta$ | [180°, 315°], [213.75°, 360°] |

**TABLE II:** 1$\sigma$ interval of NSI parameters taken from Ref. $^{25}$.
clear that in case of NO, the maximum value of probability $P(\nu_\mu \rightarrow \nu_e)$ can be $\sim 0.085$ in case of SM interaction, while NSI (LMA Light) sector suppresses this probability up to the value $\approx 0.065$. Similarly, LGtI will be more violated in the case of SM interaction (i.e., $K_3$ reaches up to 1.1), whilst, NSI (LMA Light) condition decreases the violation of LGtI ($K_3^{max} \sim 1.02$).

For IO, LMA-Dark sector of parameters enhances the value of $P(\nu_\mu \rightarrow \nu_e)$ up to $\sim 0.115$ as well as it increases the violation of LGtI, $K_3^{max} \sim 1.22$. While, in the SM
scenario, a relatively lower value of both $P(\nu_\mu \to \nu_e) \sim 0.065$ and $K_2 \sim 1.02$ can be obtained.

Similar kind of features are visible in case of tripartite entanglement $G$. In case of NO, maximal value ($\sim 0.35$) of $G$ can be approached corresponding to the maximum value of probability $\sim 0.08$, in case of SM interaction. The LMA Light scenario reduces the value of $G$ up to around 0.3 for $P(\nu_\mu \to \nu_e)_{\text{max}}^{\text{L}} \approx 0.06$. In case of IO, $G_{\text{max}}^{\text{SM}} \approx 0.32$ can be achieved for $P(\nu_\mu \to \nu_e)_{\text{max}}^{\text{IO}} \approx 0.065$ for SM interaction. On the other hand, for LMA-Dark sector $G_{\text{max}}^{\text{L}} \approx 0.5$ for $P(\nu_\mu \to \nu_e)_{\text{max}}^{\text{L}} \approx 0.115$. Also, the maximum value of flavor entropy $S$ remains in the range $2 - 2.5$ (specifically $\sim 2.3$) for all the four possible scenarios.

IV. Conclusions

We study the effects of NSI on temporal correlations quantified in terms of LGtI as well as spatial correlations quantified in terms of flavor entropy and genuine tripartite entanglement in the oscillating neutrino system. We find that, in case of normal mass ordering, LGtI violation of its classical bound in the presence of SM interaction is large in comparison to the NSI interactions. Reverse is the case for inverted mass ordering, i.e., LGtI-violation is enhanced for the LMA-D scenario over the SM interaction. Similar features have been observed in case of genuine entanglement measure, while the flavor entropy, a measure of residual entanglement, remains almost unaltered. An interesting result of this work is that if LGtI is violated at $E \approx 3$ GeV, then this would be possible only for LMA-Dark solution with IO. This indicates that if LGtI is found to violate the classical bound at $E \approx 3$ GeV, it would be an indication of new physics. In the wake of observational implications, we have also presented correlations between oscillation probability and various correlation measures.

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