Optical properties of MgH₂ measured in situ in a novel gas cell for ellipsometry/spectrophotometry

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The dielectric properties of α-MgH₂ are investigated in the photon energy range between 1 and 6.5 eV. For this purpose, a novel sample configuration and experimental setup are developed that allow both optical transmission and ellipsometric measurements of a transparent thin film in equilibrium with hydrogen. We show that α-MgH₂ is a transparent, colour neutral insulator with a band gap of 5.6 ± 0.1 eV. It has an intrinsic transparency of about 80% over the whole visible spectrum. The dielectric function found in this work confirms very recent band structure calculations using the GW approximation by Alford and Chou.1

I. INTRODUCTION

Among metal-hydrides,1 the magnesium hydrogen system has always occupied a special place. Magnesium reacts reversibly with hydrogen to form MgH₂. It is thus considered to be one of the most important candidates for the reversible storage of hydrogen due to its lightweight, low cost and high hydrogen storage capacity (7.6 wt.% of hydrogen). In spite of the large number of publications on Mg-MgH₂ only little is known about the intrinsic physical properties of this system. The scarcity of data for MgH₂ is mainly due to the experimental difficulties encountered when trying to hydride Mg. Nowadays, a great effort is made to improve the hydrogen ab/desorption kinetics by making nanocrystalline Mg and/or adding e.g. transition metals6,7,8 by ball milling.

Recent theoretical calculations9 reproduce that MgH₂ undergoes various phase transitions10,11 as a function of pressure. All theoretical calculations published so far12,13,14 (using either the local density approximation (LDA) or the generalized gradient approximation (GGA)) predict band gaps between 3.1 and 4.2 eV for α-MgH₂. This is smaller than the few sporadic experimental values reported until now. Krasko mentions a value of 5.16 eV for the band gap from unpublished work by Genossar. He and Pong determined in an indirect way using Penn’s formula15 an average band gap of 5.8 eV. Yamamoto et al. report an optical transmission spectrum for a thin film in which the transmission vanishes at 6.05 eV (205 nm). Apart from that Ellinger et al. found an index of refraction of 1.95 and 1.96 for the ordinary and extraordinary rays at 589.3 nm. The dielectric properties have not been studied at all. This triggered our interest to study the optical properties of MgH₂ in detail.

Another strong reason for our interest in MgH₂ stems from metal-hydride switchable mirrors. In 1996 Hui-berts et al.20 discovered that Y and La thin films change reversibly from shiny, metallic films to transparent, insulating films upon hydrogenation either by changing the surrounding hydrogen gas pressure or the electrolytic cell potential21,22. In 1997 Van der Sluis et al.23 discovered that all rare-earth (RE) metals exhibit such a switchable behaviour. However, all these materials have a characteristic colour in the fully hydrogenated state because their band gap is in the visible (E_g < 3 eV). They showed that alloying with Mg results in colour neutral switchable mirrors. This is very important for applications in e.g. ‘smart’ windows. In 2001 Richardson et al.24 reported that Mg₃Ni (z > 2) also features reversible switching behaviour upon hydrogenation. In all these cases (RE-Mg and MgₓNi), the band gap is shifted to higher energies with increasing magnesium concentration25,26,27. All these alloys disproportionate upon hydrogenation28,29,30. This disproportionation is also known for bulk RE-Mg31,32. The available data suggest that the shift of the band gap to higher energies is due to the formation of MgH₂ which is expected to have a large band gap. At the same time the reflectance in the low hydrogen phase (when the sample is unloaded) increases due to Mg which has a high reflection33,34. At intermediate concentrations the coexistence of Mg and MgH₂ seems to play an important role in the realization of a highly absorbing, black state that is observed during loading and unloading of RE-Mg alloys35,36. It may also play a role in the black state observed in MgₓNi₃7. Thus, to understand the role of MgH₂ in Mg-containing switchable mirrors it is essential to determine the optical properties of MgH₂ thin films.

In this paper we study thin films of magnesium hydride with spectrophotometry and ellipsometry and determine the dielectric function and the optical band gap. For this purpose we use a special substrate geometry and a novel type of optical gas loading cell.
II. EXPERIMENTAL

MgH$_2$ thin films are made in two steps. First Pd capped metallic Mg films are deposited under UHV conditions on an appropriate substrate. The Pd cap layer is necessary to protect Mg against oxidation and to promote hydrogen dissociation and absorption. The films are subsequently loaded with hydrogen under high pressure up to the composition MgH$_2$.

The hydrogenation of Mg to MgH$_2$ is, however, not straightforward as was shown by Krozer, Kasemo and others.$^{27,38,39,40,41}$ Palladium capped Mg films exhibit unusual kinetics due to the formation of a blocking MgH$_2$ layer at the interface between Pd and Mg. The MgH$_2$ layer prevents H from diffusing to the metallic Mg that is still present underneath. The formation of this blocking layer can be circumvented by starting hydrogenation at relatively low (1 mbar) H$_2$ pressure at a temperature of 100°C. Magnesium films with thicknesses up to 150 nm can be fully transformed to MgH$_2$ in this way.$^{39,41}$

A. Film deposition

Thin, polycrystalline films of Mg and Pd are deposited at room temperature in an UHV MBE system with a background pressure of 10$^{-9}$ mbar, using material of typically 99.9 % purity. The magnesium films are evaporated from a Knudsen cell and covered with a 10 nm thick Pd cap layer. These palladium films are deposited from an e-beam evaporation unit. Typically, we deposit films simultaneously on a 10x10 mm$^2$ glassy carbon substrate for Rutherford backscattering spectrometry (RBS), 10x10 mm$^2$ quartz substrates for X-ray diffraction (XRD), resistivity and/or AFM measurements, and on a quartz substrate (Ø 42 mm, Heraeus Suprasil 1) for optical measurements.

B. Film characterization

RBS is used to determine eventual contamination of the films. For this glassy carbon substrates are used in order to separate the Mg signal from the background signal of the substrate. An oxygen contamination between 0.03 ≤ [O]/[Mg] ≤ 0.085 has been found.

The thickness of the film is measured with a DekTak$^3$ or a Tencor Alpha step 200 mechanical stylus profilometer. The surface structure, both before and after hydrogen loading, is investigated with a NanoScope III atomic force microscope (AFM), operating in tapping mode using silicon cantilevers. The scanned areas are typically 1x1 and 5x5 µm$^2$ from which the root-mean-square (RMS) roughness is determined. The thickness and roughness values from these techniques are used as input parameters in the modeling of the transmission, reflection and ellipsometric data (see Sec. III).

Some samples are contacted ultrasonically with four 30 µm aluminium wires to monitor the resistivity with the Van der Pauw method$^{44}$ during loading with hydrogen.

X-ray experiments are carried out with Cu Ko radiation in a Rigaku ‘Rotaflex’ RU 200 or Bruker D8 Discover X-ray diffractometer to monitor the transformation of hcp Mg to rutile MgH$_2$ in a θ-2θ mode.

C. Optical techniques

Optical reflection and transmission measurements at room temperature (RT) are carried out in a Perkin Elmer Lambda 900 spectrophotometer in the range 0.5 < hω < 6.7 eV (2500 > λ > 185 nm). The specular and total transmission is recorded while the spectrophotometer is purged with argon or nitrogen in order to reduce absorption by O$_2$ in the ultraviolet (UV), and H$_2$O in the infrared (IR). The quartz substrates (without film) and Pd samples are measured in reflection geometry from the top side (i.e. the metallic side) at near normal incidence (8°) in an absolute reflection unit (using a so-called VN geometry).

Ellipsometry measurements (at RT) in the energy range 1.0 to 6.5 eV (1240 > λ > 190 nm) are carried out in a rotating analyzer variable-angle spectroscopic ellipsometer (VASE, J.A. Woollam Co. Inc.), using the WVASE32 software program for data acquisition and analysis. This instrument measures the ratio of the complex Fresnel reflection coefficients $R$ of parallel (p) and perpendicular (s) polarized light. This ratio defines the ellipsometric angles $\Psi(\omega)$ and $\Delta(\omega)$ according to

$$\frac{R_p}{R_s} = \tan(\Psi(\omega)) \exp(i\Delta(\omega)) \quad (1)$$

Three angles of incidence (60, 65 and 70°) are used to obtain adequate sensitivity over the whole spectral range. Standard deviation and ellipsometric data ($\Psi, \Delta$) are recorded at each data point as an average of at least 100 revolutions, and up to 4000 revolutions of the analyzer for the most critical data.

D. Semi-cylindrical substrate

As the Pd cap layer on top of the very transparent MgH$_2$ layer is strongly absorbing, ellipsometry cannot be carried out from the Pd-side. Thus, ellipsometry measurements need to be performed from the ‘backside’, through the substrate. Flat substrates would need to be so thick that reflections from the front and backside of the substrate can be well separated. With a 2 mm diameter of the light beam the substrate must be thicker than 3 mm. However, at large angles of incidence the intensity loss in the light beam is substantial due to reflections at the air/substrate interface. Furthermore, at energies close to the limit of the ellipsometer (6-6.5 eV),
FIG. 1: Semi-cylindrical quartz glass substrate for ellipsometry and transmission measurements. The upper figure is a side view of the substrate, the one on the bottom a top view. A perspective view is given in Fig. 2. The angle of incidence of light can be varied between 55 and 75°. On the flat part of the substrate three samples with different Mg thicknesses are deposited. They are all covered with a 12 nm thick Pd cap layer.

the intensity of the light beam is diminishing quickly. These limitations can be avoided with a semi-cylindrical substrate.

This substrate is designed for normal incidence of light at the ambient/substrate interface and oblique incidence at the internal substrate/MgH$_2$ interface (see Fig. 1). For the normal incident approximation to be valid, the diameter of the semi-cylindrical substrate must be large compared to the size of the light beam (2 mm). For practical reasons we choose a semi-cylindrical substrate with a diameter of 42 mm. The top part of the semi-cylindrical substrate is cut away parallel to the base surface to enable transmission measurements. This design allows the angle of the incident light onto the sample to vary between 55° and 75° from the normal. Both the flat and the semi-cylindrical substrates are made of quartz glass (Heraeus Suprasil 1). This material is transparent deep into the UV beyond the limit of our ellipsometer and spectrophotometer.

On the top of the large flat face we deposit, under exactly the same condition, three films with differing Mg thicknesses (see Fig. 1). This allows us to analyze compositionally identical films. This method makes the determination of the dielectric function from ellipsometric data more reliable.

FIG. 2: Exploded view of the optical gas loading cell for in situ ellipsometry and transmission measurements in a hydrogen environment. In this sketch the cell is shown with the semi-cylindrical substrate (see Fig. 1) as sample window at the front side. Alternatively, a flat window such as the one at the backside of the cell, can be used as sample substrate.

E. Optical gas loading cell and high pressure loading chamber

In order to measure the optical properties of MgH$_2$ and PdH$_x$ in situ in equilibrium with hydrogen at various pressures, we designed an experimental setup consisting of three parts: i) a substrate/window (see Fig. 1), ii) an optical gas loading cell, and iii) a high pressure loading chamber. Components ii) and iii) are described below in more detail. The setup also includes a special substrate holder with a sliding mask for the deposition of the films, and a holder to attach the optical gas loading cell to the ellipsometer.

A semi-cylindrical or a flat substrate with the Pd-capped MgH$_2$ films deposited on top functions as window in the optical gas loading cell (see Fig. 2). For ellipsometry the sample/window is illuminated through the substrate to measure the ‘backside’ of the film. For in situ transmission measurements a window is mounted on the opposite side of the cell as well. The sample can be exposed to a controlled hydrogen gas atmosphere during the measurements via two tubes connected to a vacuum pump and hydrogen gas cylinder. The cell is designed for vacuum, but works also reliably up to a few bar hydrogen pressure.

As already mentioned in the introduction of Sec. 11 hydrogenation of Mg to MgH$_2$ can be successfully achieved at moderate temperatures (100°C) by starting at low hydrogen pressure. Therefore, to be sure that our thin films of Mg are completely transformed to MgH$_2$ we start loading with a H$_2$ pressure of 1 mbar and increase it in steps (within a few hours) up to 100 bar H$_2$. To do this the optical gas loading cell is mounted inside a high pressure loading chamber (see Fig. 3). This chamber is
made of stainless steel and proof-pressurized to 200 bar. The design is such that the pressure in the high pressure chamber is everywhere the same, and it is not necessary to expose the sample to the ambient after loading at 100 bar H$_2$. It is possible to release hydrogen and close the optical gas loading cell with valve (3) when a pressure of 1 bar hydrogen or less is reached. With valve (2) closed as well, the system is disconnected from the gas tubes (1) (while still filled with H$_2$). Afterwards, valve (2) is opened to equalize the pressure outside and inside the high pressure chamber. Then, the chamber is opened, and the valve on top of the optical gas cell (number (4)) is closed. The cell can then be disconnected from the lid of the high pressure loading chamber while still filled with hydrogen. The high pressure loading chamber is also equipped with an electrical feed-through (5) with several wires to permit measurements of the resistivity of a sample during hydrogenation and of the temperature inside the chamber with a RhFe100 sensor. During hydrogen loading of a Mg film the total chamber can be resistively heated up to 100°C.

### III. RESULTS

#### A. Sample characterization

Since Mg is a metal and MgH$_2$ an insulator, the time evolution of hydrogenation can be followed in situ in real time by monitoring the change of the resistivity\cite{20}. This allows us to optimize the hydrogen pressure (starting at low pressures and increasing it stepwise to 100 bar H$_2$) in such a way that no impenetrable MgH$_2$ layer is formed at the interface between Pd and Mg. For practical reasons we have mounted an extra sample in the high pressure loading chamber for resistivity measurements.

The resistivity of this as-deposited 150 nm Mg film covered with 15 nm of Pd at RT is 6.5 $\mu\Omega$cm (the literature value for bulk Mg at 20°C is 4.4 $\mu\Omega$cm\cite{21}). The reflection of this as-deposited film is high in the visible and near-infrared regions (~80%). Both the low resistivity and the high reflection indicate the good quality of the film. After loading the resistivity reaches 680 $\mu\Omega$cm under 100 bar H$_2$ at 100°C. Since MgH$_2$ is an insulator one would at first sight expect a much higher value. The moderate resistivity found experimentally is, however, due to the metallic Pd cap layer that short-cuts the MgH$_2$ layer. Moreover, at a temperature of 100°C Mg and Pd may interdiffuse to form a Mg-Pd alloy\cite{22,23}. This intermixing has been suggested for Pd capped Y as well\cite{24} and was conclusively shown with photoelectron spectroscopy recently\cite{25}. RBS showed an intermixing of Mg and Pd in our films as well. This can be due to either alloying, interface roughening or both. The net result is that a relatively Pd-rich Pd-Mg alloy is formed on top of MgH$_2$ that absorbs some hydrogen but does not become insulating and this causes the shortcut.

In the as-deposited metallic state, hcp Mg has a preferential growth direction, and only the (002) reflection is present in the X-ray diffractogram (see Fig. 4(a)). Loading a thin Mg film in 1 bar H$_2$ at 100°C does not transform Mg completely to MgH$_2$ (see Fig. 4(b)). Loading at 100 bar and 100°C, on the other hand, left no traces of metallic Mg. Only the peaks corresponding to the tetragonal structure of the rutile type\cite{26} of α-MgH$_2$ are observed (see Fig. 4(c)). Such a preferred growth direction is not observed for MgH$_2$ where weak signals from the (110), (101) and (200) peaks can be seen. Rocking curves around the (002) Mg peak and the (110) MgH$_2$ peaks show that our samples are polycrystalline.

AFM measurements revealed a significant difference between the as-deposited Pd covered Mg film and the fully hydrogenated films (see Fig. 5). Mg expands by 32% in volume when transforming from hcp Mg to rutile MgH$_2$\cite{27}. Since the film is clamped by the substrate it cannot expand laterally and all the expansion must take place out-of-plane. With AFM we indeed noted an increase in the RMS roughness from 5 nm to 14 nm. It can be seen as well that our top layer of Pd is cracked. With a mechanical stylus profilometer we found a corresponding increase of the thickness of the film from 113 to
FIG. 4: X-ray diffraction spectra of a 188 nm Mg/10 nm Pd film (a) as-deposited, (b) loaded up to 1 bar H\textsubscript{2} and (c) of a 150 nm Mg/15 nm Pd film loaded up to 100 bar H\textsubscript{2}. The large background is due to the quartz substrate.  

162 nm. This 43\% increase is larger than the expected 32\% volume expansion because the mechanical stylus has a tip radius of 12.5 \( \mu \)m, and hence cannot probe the deep valleys seen on the AFM image. During ellipsometry we look through the substrate at the backside of our films and not from the top side as with AFM. Nevertheless, we found that surface roughening of our Pd top layer needed to be taken into account when modeling the ellipsometric data.

B. Transmission and band gap of MgH\textsubscript{2}

The optical transmission of Pd capped MgH\textsubscript{2} films is measured \textit{in situ} in 1 bar H\textsubscript{2} using the gas loading cell (see Fig. 2). Figure 6 shows the total, specular and diffuse transmission of a 150 nm thick MgH\textsubscript{2} film capped with 12 nm Pd, loaded at 100 bar H\textsubscript{2} and 100\(^\circ\)C. The total transmission is measured with the optical gas loading cell placed at the entrance port of the integrating sphere in the spectrophotometer. In this experiment a flat 3 mm thick quartz glass substrate is used. Since we look at our film from the substrate side, the MgH\textsubscript{2} layer is situated 3 mm away from the port of the integrating sphere. The specular transmission is measured with the sample in the sample compartment, using the direct detector to monitor the signal. The difference between these two signals is the diffuse (scattered) transmission. It is probably due to the rough surface (see Sec. III A and Fig. 5(b)) of our loaded samples. This diffuse transmission, \( T_d \), has a strong wavelength dependence and is proportional to

\[
T_d \propto \frac{1}{\lambda^4} \propto (\hbar \omega)^4,
\]

where \( \lambda \) is the wavelength of light. \( \hbar \) It is clear from Fig. 6(b) that the diffuse transmission decreases strongly above the band gap as the film starts to absorb light. The optical band gap \( E_g \), can be estimated from the intersection of a fitted \( (\hbar \omega)^4 \) curve to the data and an extrapolation of the flank of the absorption edge. Using this so called ‘Rayleigh method’ we find \( E_g = 5.61 \) eV for this sample and \( E_g = 5.67 \) eV for a second sample.

Another estimate for \( E_g \) can be obtained from the absorption edge of the transmission spectra using the Lambert-Beer law, \( T(\omega) = T_0 \exp[-\alpha(\omega)d] \), where \( \alpha \) is the absorption coefficient, \( d \) the film thickness and \( T_0 \) contains the transmission of the Pd cap layer and the quartz substrate. In the region of the absorption edge \( T_0 \) can be considered as constant in our films (see Figs. 7).
FIG. 6: (a) Total (solid line), specular (dashed line) and diffuse (dashed dotted line) transmission as a function of photon energy for a 150 nm thick MgH$_2$ film capped with 12 nm Pd and loaded at 100$^\circ$C in 100 bar of hydrogen. (b) Detail of the diffuse transmission. A fit using Eq. 2 and an extrapolation of the transmission edge are shown. The intersection of these two curves gives an estimate of 5.61 eV for the optical band gap.

The frequency dependence of $\alpha$ near the band edge is related to the optical gap through,

$$\alpha(\omega) \propto \frac{(\hbar \omega - E_g)^\nu}{\hbar \omega}. \quad (3)$$

For direct, allowed (forbidden) transitions $\nu = \frac{1}{2}$ ($\nu = \frac{3}{2}$) and for indirect, allowed (forbidden) transition $\nu = 2$ ($\nu = 3$). In amorphous material it has been found that $\nu = 2$ gives the best results. Combining these equations gives

$$\ln T(\omega) = \ln T_0 - C \frac{(\hbar \omega - E_g)^\nu}{\hbar \omega}. \quad (4)$$

and a fit to the spectra near the transmission edge in the interference-free region.

Applied to the total transmission this 'Tauc procedure' gives a gap of 5.48 ± 0.05 eV using $\nu = 2$. It was also possible to get a Tauc fit with $\nu = 3$ and $\nu = \frac{3}{2}$. However, using $\nu = 3$ gives values that are too low compared to the 'Rayleigh procedure' and we might be fitting an interference fringe instead of the absorption edge. For $\nu = \frac{3}{2}$ the quality of the fit is not as good and we obtain a gap of 5.75 ± 0.05 eV from the total transmission.

Since the diffuse transmission increases rapidly at small wavelengths (i.e. with increasing energy), it is clearer where the absorption starts in this spectrum than in the total transmission. Therefore, we conclude that the band gap of MgH$_2$ is 5.6 ± 0.1 eV.

C. Ellipsometry

1. Modeling strategy

Extracting the dielectric function of a layer from ellipsometric data on samples like ours, which consists of several thin layers on a substrate, is a complex task. The complicated inversion of the ellipsometric data to the dielectric function is, however, greatly simplified if the optical properties of each individual layer is measured in separate experiments. For this reason we adopted the following strategy to determine the dielectric function of MgH$_2$.

We start by investigating the dielectric properties of the quartz substrate. The second step is to evaluate the optical properties of the hydrogenated Pd cap layer. This is done by investigating a 12 nm thick film of Pd on quartz, hydrogenate it in 1 bar H$_2$ using the optical gas loading cell and measure it in the ellipsometer. In a third step we study the properties of the hydrogenated interface region between Pd and MgH$_2$ carefully since earlier experiments with Mg-Pd thin films showed that interdiffusion starts already at 100$^\circ$C. This is done by investigating a 'Pd-Mg' alloy layer consisting of 10 nm Mg on a quartz substrate covered with 10 nm Pd. This sample is loaded with hydrogen at 100$^\circ$C and 100 bar and measured in the ellipsometer. The final step is to measure the total stack (quartz, MgH$_2$, Pd-Mg) and to extract the optical properties of MgH$_2$ using the optical properties of all the other layers.

An optical model is defined for each sample and in the fitting procedure the difference between the calculated (cal) and the measured (exp) ($\Psi$, $\Delta$)-values (see Sec. II C) are weighted with the experimental standard deviation $\sigma$ and fitted with a Levenberg-Marquardt algorithm to minimize the mean squared error (MSE) according to...
where \( N \) is the number of \((\Psi, \Delta)\)-pairs, \( M \) is the number of fitting parameters, and the indices \( \lambda \) and \( \Theta \) denote data points at different wavelengths and angles. In most cases also normal incidence transmission data, \( T \), are used to improve the accuracy of the determination of the dielectric function. Then, a third term is included in the summation in Eq. (5).

In the modeling we take into account experimental errors in incident angle and angular spread due to the substrate design, and film thickness non-uniformity. It is difficult to model both the thickness and the dielectric properties simultaneously in ellipsometry. Thus, we allow the layer thicknesses to vary only slightly around our measured thickness values during fitting. The output from the modeling consists of the best-fit value of the Lorentz-Drude parameters (see Eq. (6)) and their \( 90\% \) confidence intervals.

### 2. Optical constants of the glass substrate

The optical constants of the quartz substrates (both flat and semi-cylindrical) and the quartz window used in the optical gas loading cell (see Fig. 2) are determined using optical reflection and transmission measurements and ellipsometry. This is straightforward, and our results match the tabulated values of the refractive index from the manufacturer as well as those of quartz glass cited in Ref. 60 (see Fig. 7a). The extinction coefficients being not tabulated in Ref. 60 are assumed to be zero. Our results on the extinction coefficient show a slight absorption near and above 6.5 eV, but still below \( 10^{-7} \) (see Fig. 7b). The corresponding dielectric function is used in the consecutive modeling of the metal hydride layers.

### 3. Optical properties of PdH\(_x\)

A 12 nm thick Pd film, deposited on quartz is exposed to 1 bar hydrogen at RT in the optical gas loading cell and investigated in the ellipsometer (see Fig. 8 for the experimental and fitted data). The Pd hydride, PdH\(_x\), that is formed is a strongly absorbing metal. Its dielectric function \( \epsilon(\omega) = \epsilon_1 + i\epsilon_2 \) can be adequately parameterized with a Lorentz-Drude (LD) model:

\[
\epsilon(\omega) = \epsilon_\infty - \sum_{i=1}^{N} \frac{\omega_{\text{p},i}^2}{\omega^2 + i\omega/\tau_i} + \sum_{j=1}^{M} \frac{f_j}{\omega_j^2 - \omega^2 - i\Gamma_j \omega} \quad (6)
\]

where the constant \( \epsilon_\infty \) accounts for excitations far above 6.5 eV; the \( N \) Drude terms describe the free-carrier response with \( \omega_{\text{p},i} \) the plasma frequency of the \( i \)-th Drude term and \( \tau_i \) the relaxation time; the \( M \) Lorentz terms represent the effect of interband transitions with \( f_j \) the intensity of the \( j \)-th oscillator, \( \omega_j \) its energy, and \( \Gamma_j \) its broadening. The relation between the dielectric function and the refractive index, \( n \) and extinction coefficient, \( k \), is: \( \epsilon_1 = n^2 - k^2 \) and \( \epsilon_2 = 2nk \). The LD parameters obtained for PdH\(_x\) in 1 bar H\(_2\) are given in Table I.

In addition to the sample for ellipsometry, an identical sample on a flat substrate is prepared for measurements in the spectrophotometer. The transmission and absolute reflection are determined for this sample in 40 mbar hydrogen (4\% H\(_2\) in Ar). At higher pressures it is not
TABLE I: Lorentz-Drude parameters and their 90% confidence intervals of a 12 nm thick PdHₓ layer in 1 bar H₂ obtained from ellipsometric data (see Fig. 8). MSE = 7.4, εᵢ∞ = 1.266 ± 0.371. All parameters are in eV.

|   | ωₚ,i | 1/τᵢ | (j) | ωⱼ | √fⱼ | Γⱼ  |
|---|------|------|-----|-----|-----|-----|
| (1) | 3.389 ± 0.901 | 0.1892 ± 0.019 | (1) | 3.418 ± 0.124 | 8.588 ± 4.6 | 5.195 ± 0.691 |
| (2) | 8.656 ± 2.02 | 1.775 ± 0.147 | (2) | 6.878 ± 0.119 | 11.32 ± 4.63 | 7.713 ± 0.519 |
| (3) | 3.418 ± 0.124 | 8.588 ± 4.6 | (3) | 9.669 | 11.96 | 0.5601 |

FIG. 8: Experimental and fitted ellipsometric data, Ψ, (a) and Δ (b) for a 12 nm thick Pd film in 1 bar H₂ used to determine the dielectric function of PdHₓ (see Fig. 10). The mean square error corresponding to the fit is 7.4.

possible to use the optical gas loading cell when we determine the absolute reflection since we must look directly at the Pd sample and not via the substrate. To obtain a hydrogenated Pd sample, the whole spectrophotometer is purged in Ar containing 4% H₂, corresponding to a partial pressure of 40 mbar H₂. This is the highest H₂ concentration we can use in the (open) spectrophotometer. In Fig. 9 the experimental and fitted data are displayed, the LD parameters obtained for PdHₓ in 40 mbar H₂ are given in Table II.

The difference in the MSE (see Table II) between the 1 bar and 40 mbar measurements is mainly due to the different substrate geometry, semi-cylindrical vs. flat substrate. We assign the major part of the difference in the MSE to the cylindrical incident and exit surface of the semi-cylindrical substrate and to the fact that the ellipsometry measurements have been performed from the ‘backside’ of the sample through the substrate. Note that this does not change the dielectric function, it merely gives a larger spread in the input data for the analysis.

In Fig. 10 we show the resulting dielectric function at...
TABLE II: Lorentz-Drude parameters and their 90% confidence intervals of a 12 nm thick PdH\textsubscript{x} layer in 40 mbar H\textsubscript{2} (4\% H\textsubscript{2} in Ar) obtained from reflection and transmission data (see Fig. 9). MSE = 1.04, \(\epsilon_{\infty} = 1.280 \pm 0.241\). All parameters are in eV.

| (i) | \(\omega_{p,i}\) | \(1/\tau_i\) | (j) | \(\omega_j\) | \(\sqrt{\tau_j}\) | \(\Gamma_j\) |
|-----|----------------|-------------|-----|-------------|--------------|-----------|
| (1) | 5.156 \pm 0.955 | 0.001 \pm 0.0241 | (1) | 4.131 \pm 0.111 | 9.815 \pm 4.17 | 7.104 \pm 0.581 |
| (2) | 8.382 \pm 2.65 | 2.238 \pm 0.323 | (2) | 7.623 \pm 0.249 | 7.793 \pm 4.14 | 0.927 \pm 0.235 |

FIG. 10: Real (a) and imaginary (b) parts of \(\epsilon(\omega)\) for the PdH\textsubscript{x} films as determined from ellipsometry in 1 bar H\textsubscript{2} (see Fig. 8) and reflection and transmission measurements in 40 mbar H\textsubscript{2} (4\% H\textsubscript{2} in Ar) (see Fig. 9). For comparison, the dielectric function found by Rottkay et al.\textsuperscript{61} for PdH\textsubscript{x} in 4\% H\textsubscript{2} and the dielectric function of pure Pd according to Ref. \textsuperscript{60} are included.

The plasma frequency, \(\omega_p\), of PdH\textsubscript{x} in 1 bar H\textsubscript{2} is slightly larger than the one in 40 mbar H\textsubscript{2}. Because \(\omega_p \propto \sqrt{n_c}\) with \(n_c\) the charge carrier density, there are more free charge carriers in PdH\textsubscript{x} in 1 bar than in 40 mbar H\textsubscript{2}. Values for the (optical) resistivity \(\rho\) can be derived from the Drude parameters using

\[
\rho_{\text{opt}} = \frac{1}{\epsilon_0 \omega_p^2 \tau},
\]

with \(\epsilon_0\) the vacuum permittivity, \(\omega_p\) the plasma frequency and \(\tau\) the electron relaxation time. This optical resistivity calculated for the dominant Drude term no. 2 shows the same trend as the plasma frequency itself: In 40 mbar H\textsubscript{2} PdH\textsubscript{x} has a resistivity of 235 \(\mu\Omega\text{cm}\), in 1 bar 178 \(\mu\Omega\text{cm}\). However, bulk Pd has a resistivity of 10.53 \(\mu\Omega\text{cm}\) and PdH\textsubscript{x} has a maximum resistivity of about 20 \(\mu\Omega\text{cm}\) when \(x = 0.7\) at RT.\textsuperscript{62} Our much larger resistivities are probably due to the fact that the 12 nm thick Pd film consists of somewhat disconnected islands. Hydrogen absorption causes the Pd islands to expand which decreases the resistivity between them.\textsuperscript{63,64} Thus, we have a sort of percolation effect and the resistivity is lower in 1 bar H\textsubscript{2} than in 40 mbar H\textsubscript{2} contrary to bulk PdH\textsubscript{x}.

4. Optical constants of the double layer Pd/Mg

To investigate the optical properties of the partially interdiffused Pd-Mg top layer, we deposit a layer of 10 nm Mg capped with 10 nm Pd on quartz. This Pd-Mg film is then exposed to hydrogen at 100\(^{\circ}\)C and 100 bar (together with the thick Mg film covered by Pd (see Fig. 1)). After hydrogenation the optical properties are determined in the optical gas loading cell at RT. This sample is treated as consisting of two layers: a Mg-rich Pd-Mg alloy on the substrate covered with a Pd-rich Pd-Mg alloy on top. As starting values for the fitting procedure we use the PdH\textsubscript{x} dielectric function determined above for the Pd-rich Pd-Mg top layer, and combine it with voids in a Bruggeman effective medium approximation (EMA) to take surface roughness into account.\textsuperscript{65} A Lorentz-Drude model is used for the second layer, the Mg-rich Pd-Mg alloy. Ellipsometric data for three angles of incidence (55, 60 and 65\(^{\circ}\)) and normal incidence transmission data are then combined in a multiple data type fit. All data are measured on the same sample and during fitting the layer optical functions and thicknesses are coupled. The final iteration results in a fit with a MSE of 8.7. The optical properties of this double layer are then used as starting values for the top layer of the thicker MgH\textsubscript{2} film.
5. Dielectric function of MgH$_2$

In the evaluation of the optical properties of MgH$_2$ we analyze ellipsometric and transmission data of a 124 nm thick MgH$_2$ film capped with 12 nm PdH$_x$ (as measured with a mechanical stylus profilometer in the hydrogenated state). In addition to these data, transmission data of a compositionally identical film, but with a thickness of 162 nm (when hydrogenated) are included in the modeling. These three data sets are evaluated in three parallel, coupled models simultaneously. The main features can be modeled using two Lorentz oscillators at the high energy side of the measured spectra, at 6.4 and 6.9 eV. These oscillators mark the beginning of the conduction band.

The optical parameters of the capping layer, consisting of the Pd-rich Pd-Mg alloy on top of the Mg-rich Pd-Mg alloy, are initially fixed to the parameter values obtained in Section III C. The only parameters of the top layers which are allowed to vary are the thicknesses since the diffusion of Pd into MgH$_2$ may be larger than the 10 nm in the thin Pd/Mg double layer. In the final iteration a global fit is used in which all LD parameters are allowed to change. The final MSE is 17.17. Table III gives the LD parameter values from the final iteration for MgH$_2$, the Pd-rich Pd-Mg cap layer and the Mg-rich Pd-Mg cap layer.

The plasma frequencies obtained for the two top layers give us a clue about their composition. Since the plasma frequency of the top layer ($\omega_p = 14.35$ eV) is much larger than the one of the lower cap layer ($\omega_p = 6.672$ eV), the top layer has a larger charge carrier density and is thus more metallic than the lower one. This indicates that the top layer is formed by a metallic Pd-Mg alloy. The lower layer contains some insulating MgH$_2$ as well. The optical resistivity (see Eq. 7) of the top layer is 38 $\mu\Omega$cm compared to an electrical resistivity of 63 $\mu\Omega$cm (at RT) for the total stack as measured after loading. Thus, the top layer is indeed shunting the resistivity measurements of MgH$_2$. Since PdH$_x$ has an optical resistivity of 178 $\mu\Omega$cm it is clear that the top layer contains some metallic Mg as well which has a much lower resistivity (6.5 $\mu\Omega$cm).

The total thickness of the stack obtained from ellipsometry is 130.6 nm after hydrogenation. With the stylus profilometer we found a thickness of 136 nm. However, as mentioned before the profilometer gives a value that is too large. Before hydrogenation the thickness was 95 nm. This would mean an increase of 37.5% instead of the theoretical 32% volume expansion.

In Fig. 11(a), (b) the experimental and fitted values of $\Psi$ and $\Delta$ are given, in (c), (d) the experimental and fitted transmission curves are shown. Finally, Fig. 12(a), (b) shows the real and imaginary part of $\epsilon(\omega)$ obtained for $\alpha$-MgH$_2$.

![Image of the optical properties of MgH$_2$](image)

**FIG. 11:** Experimental and fitted data for a 124 nm thick MgH$_2$ film covered with 12 nm Pd used to determine the dielectric function of MgH$_2$ (see Fig. 12); (a) ellipsometric data for $\Psi$, (b) ellipsometric data for $\Delta$, (c) optical transmission (d) transmission of a sample consisting of 150 nm MgH$_2$/12 nm Pd. All experimental data are modelled simultaneously in the fitting procedure as described in Sec. III C. The mean square error of the fit is 17.17.

IV. DISCUSSION

In Table IV, both experimental and theoretical values for the band gap of MgH$_2$ are given. Hartree-Fock (HF) calculations are not included since this method overestimates the band gap considerably. Very recent and not yet published theoretical work by Herzig $^1$, Auluck $^2$, and Alford and Chou on MgH$_2$ is included.

The band gap of 5.6 $\pm$ 0.1 eV determined in this work for $\alpha$-MgH$_2$ is close to values mentioned sporadically in literature. The value 5.16 eV was obtained in an indirect way from X-ray photoelectron spectroscopy (XPS) data using Penn’s formula. $^{17}$ Yamamoto et al. $^{18}$ measured the specular optical transmission of thin layers of MgH$_2$ cov-
TABLE III: Lorentz-Drude parameters and their 90% confidence intervals of a 124 nm MgH$_2$ film covered with 12 nm Pd obtained from ellipsometric and transmission data (see Fig. 11). MSE = 17.17 and $\epsilon_\infty = 1.595 \pm 0.092$. All parameters are in eV.

| Material                  | Method     | $E_g$ (eV) | Ref. |
|---------------------------|------------|------------|------|
| $\alpha$-MgH$_2$          | expt.      | 5.16       | 15   |
|                           | UV-absorption | 5.8       | 16   |
|                           | EELS       | >3.7$^a$   | 68   |
|                           | XPS        | >3.4$^a$   | 69   |
|                           | Transmission | 6.05$^b$ | 18   |
|                           | Ellipsometry/Transmission | 5.6 ± 0.1 | this work |
|                           | calc.      | 3.06       | 12   |
|                           | LDA        | 3.4        | 13   |
|                           | LDA        | 3.45 (4.34) | 66   |
|                           | LDA        | 3.3        | 67   |
|                           | LDA        | 3.10       | 1    |
|                           | GGA        | 3.78       | 14   |
|                           | GGA        | 4.2        | 9    |
|                           | sX-LDA     | 5.71 (6.41) | 66   |
|                           | GWA        | 5.25 (6.11) | 1    |
| $\gamma$-MgH$_2$          | calc.      | 4.3        | 9    |
| $\beta$-MgH$_2$           | calc.      | 0.23       | 70   |
|                           | GGA        | 2.35       | 71   |

$^a$If there are no charging effects, the band gap would be twice the indicated value.

$^b$This is the photon energy where the transmission vanishes.

MgH$_2$ film, $d = 115.9$ nm

| $(i)$ | $\omega_p,i$ | $1/\tau_i$ | $(j)$ | $\omega_j$ | $\sqrt{f_j}$ | $\Gamma_j$ |
|-------|--------------|-----------|-------|-------------|--------------|------------|
| (1)   | 6.4          |           | (1)   | 5.516       | 0.6454       |            |
| (2)   | 6.9          |           | (2)   | 7.689 ± 0.519 | 0.01223 ± 0.109 |            |

Top cap layer: Pd-rich Pd-Mg alloy (with 53% voids), $d = 15.6$ nm

| $(i)$ | $\omega_p,i$ | $1/\tau_i$ | $(j)$ | $\omega_j$ | $\sqrt{f_j}$ | $\Gamma_j$ |
|-------|--------------|-----------|-------|-------------|--------------|------------|
| (1)   | 14.35 ± 0.062 | 1.055 ± 30.6 | (1)   | 2.964 ± 0.122 | 5.208 ± 0.559 | 1.702 ± 9.03 |
| (2)   | 8.5          |           | (2)   | 17.408 ± 12.2 | 0.01366 ± 4.96 |            |

Lower cap layer: Mg-rich Pd-Mg alloy, $d = 4.1$ nm

| $(i)$ | $\omega_p,i$ | $1/\tau_i$ | $(j)$ | $\omega_j$ | $\sqrt{f_j}$ | $\Gamma_j$ |
|-------|--------------|-----------|-------|-------------|--------------|------------|
| (1)   | 6.672 ± 0.865 | 0.003029 ± 8.29 | (1)   | 3.293 ± 0.0824 | 24.61 ± 0.420 | 14.09 ± 7.06 |

FIG. 12: Real (a) and imaginary (b) parts of $\epsilon(\omega)$ for MgH$_2$ determined from ellipsometry and transmission data (see Fig. 11). The real and imaginary parts calculated by Alford and Chou using the GW approximation are shown for comparison. The transmission of a 100 nm MgH$_2$ film in vacuum calculated with the dielectric functions shown in (a) and (b) is displayed in the lower panel (c). The theoretical gap is approximately 0.5 eV too large.
As can be seen in Table IV, LDA calculations give a band gap that is systematically too low. This is a well-known feature of this approximation. Similarly, the GGA used in two other papers\textsuperscript{9,14} to calculate the density of states of MgH\textsubscript{2}, underestimates the band gap. Our band gap is closest to the theoretically calculated gaps of Herziger\textsuperscript{68} using screened-exchange-LDA (sX-LDA) and Alford and Chou\textsuperscript{69} using the GW approximation (GWA). Furthermore, it is interesting to point out that our experimentally found gap $E_g = 5.6 \pm 0.1$ eV is very close to the difference in ionization energy between Mg and H: 5.952 eV.

Back in 1955 Ellinger et al.\textsuperscript{70} determined the refractive index of $\alpha$-MgH\textsubscript{2} at 589.3 nm (2.107 eV) and found $n = 1.95$ and 1.96 for the ordinary and extraordinary rays, respectively. We find $n = 1.94$ and $k = 7.6 \times 10^{-3}$ at the same energy which is very close.

Both Auluck\textsuperscript{67} using LDA and Alford and Chou\textsuperscript{71} using LDA and GWA have calculated the band structure and dielectric function for $\alpha$-MgH\textsubscript{2}. To obtain the dielectric function only direct transitions are taken into account. The only difference between the LDA and GWA curves is the energy position. The dielectric function obtained for $\alpha$-MgH\textsubscript{2} with GWA agrees quite well with our measured values (see Fig. 12(a),(b)), while the LDA curve, as expected, is shifted to too low energies. This indicates that a scissors-operation that shifts the conduction band rigidly with respect to the valence band works well to correct LDA calculations.\textsuperscript{72,73} The energy at which both $\varepsilon_1$ and $\varepsilon_2$ exhibit a marked increase is, however, slightly different for the experiment and the GWA calculation. sX-LDA seems to overestimate the band gap since it gives even larger values for the (in)direct gap than GWA.

Figure 12(c) shows the optical transmission for a 100 nm thick MgH\textsubscript{2} film in vacuum as calculated with our experimental dielectric function and the one calculated by Alford and Chou. As can be seen MgH\textsubscript{2} has an intrinsic transparency of about 80% over the entire visible spectrum. The difference in energy between the absorption edges of the two transmission spectra and the dielectric functions is about 0.5 eV. Thus, GWA overestimates the optical gap by 0.5 eV. The direct gap determined from the band structure using GWA is 6.11 eV. Subtracting 0.5 eV from 6.11 eV gives a value of 5.61 eV which is within the error margin of our experimentally found gap. More information about the dielectric function and band structure of MgH\textsubscript{2} by Alford and Chou will be published elsewhere.

Yet unpublished calculations indicate that the optical properties of both $\alpha$- and $\gamma$-MgH\textsubscript{2} are close to each other.\textsuperscript{67,74} Vajeeston et al.\textsuperscript{75} find a gap of 4.2 and 4.3 eV for $\alpha$- and $\gamma$-MgH\textsubscript{2}, respectively with GGA; Bastide et al.\textsuperscript{76} and Bortz et al.\textsuperscript{77} have found that the structures of the two different phases are closely related and that the density is almost the same as well as the H-H distances. Therefore, our dielectric function for $\alpha$-MgH\textsubscript{2} is probably a very good approximation for $\gamma$-MgH\textsubscript{2} as well. This is important to model the optical properties of switchable mirrors since in fully hydrogenated Y-Mg alloys Nagen- gast et al. found that fcc YH\textsubscript{3} coexists with $\gamma$-MgH\textsubscript{2}.\textsuperscript{82} $\beta$-MgH\textsubscript{2} seems to be very different both structurally and optically. Its density is much larger and the calculated band gap for this material turns out to be considerably smaller than that of both $\alpha$- and $\gamma$-MgH\textsubscript{2} (see Table IV). The augmented plane wave (APW) calculations underestimate the band gap considerably, the GGA value of 2.35 eV (again by Vajeeston\textsuperscript{75}) is much more reliable. The difference between the calculated band gap using GGA and the one measured for $\alpha$-MgH\textsubscript{2} is 1.4 eV. Assuming that the same scissors-operation can be applied we expect an experimental gap of 5.7 eV for $\gamma$-MgH\textsubscript{2} and 3.75 eV for $\beta$-MgH\textsubscript{2}.

We now compare MgH\textsubscript{2} to related materials such as MgF\textsubscript{2}, MgO, MgS, MgSe, and other alkaline-earth and alkali hydrides. In Table V the band gaps of these materials are listed. For all of them it turns out that LDA underestimates the measured gap. Again, as for MgH\textsubscript{2}, the GW approximation seems to give a very good agreement between experiment and calculation for MgO, LiH and LiD. The same is true for the alkali halides.\textsuperscript{78} MgH\textsubscript{2} and CaH\textsubscript{2} seem to be very similar. Both materials are wide band gap insulators and have a valence band that

### Table V: Literature data for the band gap, $E_g$, of materials closely related to MgH\textsubscript{2} both experimental (expt.) and calculated (calc.).

| Material   | Method  | $E_g$ (eV) | Ref. |
|------------|---------|-----------|-----|
| MgF\textsubscript{2} | expt. | Reflectance | 12.4 | 71 |
|           | calc. | tight-binding, pseudopotentials | 12.8 | 75 |
| MgO       | expt. | Reflectance | 7.77 | 76 |
|           | calc. | LDA | $\sim$ 5 | 77 |
|           |       | GWA | 7.8 | 78 |
| MgS       | calc. | LDA | 2.6 | 79 |
|           |       | LDA (corrected) | 4.59 | 80 |
| MgSe      | expt. | 5.6 | 81 |
| MgTe      | expt. | 4.7, 3.6 | 81 |
| CaH\textsubscript{2} | expt. | XPS | $\sim$ 5$^a$ | 82 |
|           | calc. | SRPES | $\sim$ 5$^a$ | 83 |
| SrH\textsubscript{2} | expt. | XPS | $\sim$ 5$^a$ | 82 |
| BaH\textsubscript{2} | expt. | XPS | $\sim$ 5$^a$ | 82 |
| LiH       | expt. | 4.99 | 85 |
|           | calc. | GWA | 5.24 | 85 |
| LiD       | expt. | 5.04 | 78 |
|           | calc. | LDA | 2.84 | 78 |
|           |       | GWA | 5.37 | 78 |

$^a$Charging effects are taking into account in the experiments.

$^b$If there are no charging effects, the band gap would be twice as large.
is predominantly determined by hydrogen orbitals.\textsuperscript{12,13,84} For MgO and MgS both valence and conduction bands are determined by the anions (O or S)\textsuperscript{12,13,84} as well and again the same holds for the alkali halides.\textsuperscript{86}

Since both the valence and conduction band of MgH\textsubscript{2} are formed by H states, the band gap of the alkaline-earth hydrides is expected to be almost independent of the metal as is the case for the alkali halides.\textsuperscript{12,13,84,86} The remaining dependence results from the influence of the metal-ion on the lattice parameter, and the influence of hybridization of the conduction band states with metal-ion s- and d-states. From XPS experiments by Franzen et al.\textsuperscript{86} it seems that the onset of transitions in the valence band regions start for CaH\textsubscript{2}, SrH\textsubscript{2} and BaH\textsubscript{2} all at about 2.5 eV. Since charging effects are taking into account, this gives a gap of about 5 eV quite close to what we have found for α-MgH\textsubscript{2}. It is striking that LiH and LiD also have a gap of 5.0 eV.

V. CONCLUSIONS

In this study we use a novel experimental setup for optical transmission and ellipsometry measurements. This setup facilitates greatly measurements of the optical properties and dielectric function of metal hydrides in a hydrogen environment. It is possible to control the gas pressure from 1 mbar to 100 bar, and thus the composition of metal hydrides over a wide range. The temperature, pressure and resistivity are monitored in situ during hydrogenation of a sample. We determine the dielectric properties of α-MgH\textsubscript{2} and PdH\textsubscript{2} and find that MgH\textsubscript{2} is a transparent, colour neutral insulator with a band gap of 5.6 ± 0.1 eV. The transparency over the whole visible spectrum is ∼80% (for a 100 nm thick film). The experimentally determined dielectric function in the photon energy range between 1 and 6.5 eV of α-MgH\textsubscript{2} is in very good agreement with very recent calculations using the GW approximation. If we assume that calculations of the band gap for γ- and β-MgH\textsubscript{2} underestimate the experimental gap by the same amount as in α-MgH\textsubscript{2} we expect an experimental gap of 5.7 eV for γ-MgH\textsubscript{2} and 3.75 eV for β-MgH\textsubscript{2}.

In a coming publication we shall show that the dielectric function of MgH\textsubscript{2} determined here can be used to explain the large optical absorption (the so-called ‘black state’) of Mg-based alloys where metallic Mg and insulating MgH\textsubscript{2} nanodomains coexist.

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