One more fitting ($D=5$) of Supernovae red shifts

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In memory of Dr. N. G. Gavrilov

Abstract

Supernovae red shifts are fitted in a simple 5D model: the galaxies are assumed to be enclosed in a giant $S^3$-spherical shell which expands (ultra)relativistically in a (1+4)$D$ Minkowski space. This model, as compared with the kinematical (1+3)$D$ model of Prof Farley, goes in line with the Copernican principle: any galaxy observes the same isotropic distribution of distant supernovae, as well as the same Hubble plot of distance modulus $\mu$ vs redshift $z$. A good fit is obtained (no free parameters); it coincides with Farley’s fit at low $z$, while shows some more luminosity at high $z$, leading to 1% decrease in the true distance modulus (and 50% increase in luminosity) at $z \sim 2$.

The model proposed can be also interpreted as a FLRW-like model with the scale factor $a(t) = t/t_0$; this could not be a solution of general relativity (without inventing some super-dark energy with $w = -1/3$; 5D GR is also unsuitable—it has no longitudinal polarization). However, there still exists the other theory (with $D = 5$ and no singularities in solutions), the other game in the town, which seems to be able to do the job.

1. It is known that supernovae of type 1a (SNe 1a) as a rule can serve as standard candles. To fit the data of SN1a red shifts, the general relativity theory (GR) requires mystical dark energy and a few adjustable parameters. Recently the Hubble diagram was excellently fitted in framework of a very simple model based on the special relativity \[1\] (Farley, 2009). The galaxies are assumed to recede with constant velocities in a usual Minkowski space, $D = 1 + 3$. Taking into account all relativistic effects (time dilation and so on), one can find the luminosity of a SN (at the distance $r$, when the light was emitted) as a function of red shift $z$:

$$L = L_0 \frac{d_*^2}{c^2 t_0^2} \frac{(1 + \beta)^2}{\beta^2(1 + z)^4},$$

where $(1 + z)^2 = \frac{1 + \beta}{1 - \beta}$, $\beta = \frac{z(2 + z)}{2 + 2z + z^2} = \frac{r}{ct_0 - r}$;

$d_*$ = 10 pc, the velocity is $\beta c$, $t_0$ is the time from the big bang; the Hubble constant (as accepted in \[5\]) is

$$H_0 = t_0^{-1} = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}.$$

In magnitudes, the true distance modulus of a SN 1a should be (everywhere log means $\log_{10}$) \[1\]

$$\mu(z) = -2.5 \log(L/L_*) = \mu_0 + 5 \log[\beta/(1 - \beta)] = \mu_0 + 5 \log(z + z^2/2).$$

(1)

Below I consider another model which gives the next solution:

$$\mu(z) = \mu_0 + 5 \log[(1 + z) \ln(1 + z)] = \mu_0 + 5 \log[z + z^2/2 - z^3/6 + \cdots + (-)^n z^n/(n^2 - n) + \cdots];$$

(2)

in both cases, by definition, $\mu(z_*) = 0$, $\mu_0 = -5 \log(z_*) = -5 \log(H_0 d_* / c) \approx 43.3$ ($z_* = \beta_* \ll 1$).

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As Wikipedia says, the (true) distance modulus $\mu = m - M$ is the difference between the apparent magnitude $m$ (corrected for interstellar absorption) and the absolute magnitude $M$ of an astronomical object; the last is defined as the apparent magnitude of an object when seen at a distance of 10 parsecs ($d_*$). To find out more about the issue, it is worth reading the Dark Energy Primer in [2], as well as very interesting remarks on the status of $\Lambda$CDM-model [3] (Lieu, 2007).

The “Union” compilation of SNe Ia data, which includes more than 300 SNe after selection cuts, has been given by Kowalski et al [4]. Even more vast compilation enriched with low-$z$ SNe has been recently given by Hicken et al [5], and I will use the last data (presented in the Table 1 of [5]; the last footnote to this table specifies $H_0$), with the following selection cut imposed on the uncertainties of $\mu$, $\delta \mu$:

$$100 \frac{\delta \mu}{\mu} < 0.64 + z.$$  \hspace{1cm} (3)

Figure 1. Relativistically expanding S$^3$-spherical shell; photons move along the spirals.

2. Let us consider a 5D cosmological model in which our Universe is a sphere $S^3$, or more exactly a spherical shell of some thickness, which expands in a 5D Minkowski space. It is supposed that both matter and light are kept or confined inside this shell, as in some kind of a wave guide, and that the speed of expansion is close to the unit (put $c = 1$):

$$R = BT \approx T, \quad \Gamma = (1 - B^2)^{-1} \gg 1.$$  

Caps letters relate to the privileged reference frame, to the main observer resting in the center of this sphere. In this frame, the radial and angle components of speed of a photon, $V_R$ and $V_\varphi$, read (after Lorentz’s transformation from a comoving reference system; for comoving observer, the shell thickness is large enough to neglect the light dispersion):

$$\begin{pmatrix} 1 \\ V_R \\ V_\varphi \end{pmatrix} = \begin{pmatrix} \Gamma & \Gamma B & 0 \\ \Gamma B & \Gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \omega \\ 0 \\ \omega \end{pmatrix}; \quad \Omega = \Gamma \omega, \quad V_R = B, \quad V_\varphi = \Gamma^{-1}. \hspace{1cm} (4)$$

Hence the light move along the spirals in accordance with the next equations:

$$\frac{dR}{dT} = B, \quad R \frac{d\varphi}{dT} = \frac{1}{\Gamma}; \quad R = BT, \quad \varphi = \frac{1}{\Gamma B} \ln \frac{T}{T_i}, \quad \frac{R}{R_i} = e^{\Gamma B \varphi}. \hspace{1cm} (5)$$

Figure 1 illustrates this and shows two photon paths for a semi-relativistic case, with $\Gamma B = 1$. It is clear that time dilation, with respect to the main observer, is the same for all comoving observers (for all galaxies), $\Delta t_i = \Delta T / \Gamma$; so the mutual time dilation concerned with light, or red shift $z$, $1 + z = \Delta t_0 / \Delta t_i$, can be easily derived:

$$1 + z = \frac{\Delta T_0}{\Delta T_i} = \frac{R_0}{R_i} = e^{\Gamma B \varphi}. \hspace{1cm} (6)$$

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Figure 2. Observed SN magnitudes and equations (1) and (2) vs redshift; \( \mu_0 = 43.3 \).

It means that the angle \( \varphi \) is very small, \( \varphi \sim \Gamma^{-1} \ll 1 \) (at least for \( z < 10 \)).

A host of photons radiated by a SN at \( R_i \) forms at \( R_0 \) a sphere of radius [see Eq. \( \text{[9]} \) and Fig. 1]

\[
r = R_0 \sin \varphi \approx \Gamma t_0 \varphi = t_0 \ln(1 + z);
\]

hence the luminosity of SN (i.e. the power flow to a detector of unit surface) should look as follows:

\[
L \propto \frac{1}{(1 + z)^2 \ln^2(1 + z)};
\] (7)

here the factor \((1 + z)^{-2}\) accounts for reduction of both the energy of photons and the frequency of their occurrence in a detector, see also [1]. This equation leads easily to (2); Figures 2 and 3 show the observed data [5] and the model curves (1) and (2).

Figure 3. The low-z region; curves (1) and (2) look the same.
It is also interesting to estimate the observed total $z$-density, $f_{\text{obs}}(z) = dM_{\text{obs}}/dz$ (that is, baryon density). Let $\rho(T)$ be the (3d volume) density of matter; then one can find

$$\Delta M_{\text{obs}} \sim \rho R^3 \varphi^2 \Delta \varphi, \quad f_{\text{obs}} \propto \rho(z) \ln^2(1+z)/(1+z)^4. \quad (8)$$

Different assumptions about $\rho(z)$ can be made and tested (the very possibility that our Universe is a non-isolated system is very amazing):

(a) the total number of baryons is a constant; then

$$\rho T^3 = \text{const}, \quad \rho \propto (1+z)^{-3};$$

(b) the number of baryons depends on the 4d volume swept by the spherical shell (if it is single); some perturbations hit the shell and produce new matter (and light as well); in this case (if the rate of production does not change)

$$\rho T^3 \propto T^4, \quad \rho \propto (1+z)^{-1};$$

(c) perhaps another simple regime, $\rho \sim \text{const}$ or maybe $\rho T^3 \sim T$, is more rational (say, if the shell follows another one).

3. It is worth adding a few words about the theory which in my opinion can support the proposed 5D model.

Although some variants of Riemann-squared modified gravities (e.g. $R_{\mu
u}G^{\mu
u}$-gravity, with equations of forth order) have a longitudinal polarization related to the Ricci scalar, these theories are not appropriate [6]: the polarizations related to the Weyl tensor (and responsible for gravity, tidal forces) are linearly unstable.

Therefore, my main candidate is the theory of frame field, $h^a_{\mu}$, also known as Absolute Parallelism (AP). AP benefits from its very high symmetry group which includes both global symmetries of Special Relativity (this global subgroup defines the space signature) and local symmetries, the pseudogroup $\text{Diff}(D)$, of General Relativity theory. AP has the unique variant (no free parameters; with the unique $D$, $D = 5$) where solutions of general position are eternal and free of arising singularities [7]:

$$E_{ab} = L_{ab\nu;\nu} - \frac{1}{3}(f_{ab} + L_{ab\mu\nu} \Phi_{\nu}) = 0, \quad (9)$$

where

$$L_{ab\nu} = L_{a[b\nu]} = \Lambda_{ab\nu} - S_{ab\nu} - \frac{2}{3} h_{ab[\mu} \Phi_{\nu]};$$

$$\Lambda_{ab\nu} = 2h_{a[b\nu]}, \quad S_{ab\nu} = 3\Lambda_{[ab\nu]}, \quad \Phi_{\mu} = \Lambda_{ab\mu}, \quad f_{ab\nu} = 2\Phi_{[b\nu;\mu]} = 2\Phi_{[b\nu;\mu]} \quad (10)$$

Coma ”,” and semicolon ”;” denote partial derivative and usual covariant differentiation with symmetric Levi-Civita connection, respectively; our choice is $\eta_{ab} = \text{diag}(-1,1,1,1,1)$, then $g_{ab} = \eta_{ab} h^a_{\mu} h^b_{\nu}$, and for any $D$, $h = \text{det} h^a_{\mu} = \sqrt{-g}$.

One should retain the identities: $\Lambda_{a[b\nu;\lambda]} \equiv 0$, $h_{a\lambda} \Lambda_{abc;\lambda} \equiv f_{cb} (= f_{cb} h^b_{\mu} h^b_{\nu})$, $f_{[b\nu;\mu;\lambda]} \equiv 0$.

The equation $E_{ab\mu} = 0$ gives ‘Maxwell-like equation’ (for brevity $\eta_{ab}$ and $g^{\mu\nu} = h^a_{\mu} h^b_{\nu}$ are omitted in contractions):

$$(f_{ab} + L_{ab\nu} \Phi_{\nu})_{,\mu} = 0, \quad f_{ab\nu} = (S_{ab\lambda} \Phi_{\lambda})_{,\nu} \ (= -\frac{1}{2} S_{ab\lambda} f_{\nu;\lambda}, \text{see below}). \quad (11)$$

Really (11) follows from the symmetric part, because skewsymmetric one gives just the identity; note also that the trace part becomes irregular (the principal derivatives vanish) if $D = 4$ (the forbidden number):

$$2E_{[b\nu]} = S_{ab\lambda;\lambda} = 0, \quad E_{[b\nu;\mu;\nu]} = 0; \quad E_{ab\mu} = E_{ab\mu} h^a_{\mu} h^b_{\nu} = \frac{4-D}{3} \Phi_{\mu;\nu} + (\Lambda^2) = 0.$$

The system (9) remains compatible under adding $f_{ab\nu} = 0$, see (11); this is not the case for another covariant, $S, \Phi$, or Riemannian curvature, which relates to $\Lambda$ as usually:

$$R_{ab\mu\nu} = 2h_{ab;\nu;\nu} h^a_{\mu} h^b_{\lambda} = \frac{1}{2} S_{ab\lambda} - \Lambda_{ab\mu\nu}.$$
At first sight this theory does not have a longitudinal polarization because in linear approximation
\[ R \approx -2\Phi_{a,a} \approx 0; \]
that is, Ricci scalar does not provide a polarization.

However in the case of high symmetry (like spherical symmetry or plane longitudinal wave, i.e. isotropic it the tangential dimensions), skew-symmetric tensors tend to become zero: \( S_{\mu\nu\lambda} \equiv 0 \), and integration of Eqs. (11) gives also \( f_{\mu\nu} = 0 \). So, there is a ‘scalar field’ \( \psi \),
\[ \psi_{,\mu} = \Phi_{,\mu}, \]
which is responsible for the longitudinal polarization.

4. What is the truth and can we find it in physics? Okay, take a bit more concrete question: should the true fundamental theory (its existence can explain the usefulness of phenomenological theories and models) have some clear and sensible distinction from enumerable false theories? If so, the true theory, the true description of Nature, has to have no free parameters; all *fundamental constants* should be long-lasting parameters of a solution, not a theory.2 [And I am with those who think that quantum mechanics (which still needs ‘classical crutches’ to make predictions; psi-function, amplitude has no intrinsic, all-sufficient, independent meaning or ontology) is not the only possible (nor the best) starting point for the way to a deep insight into the origins of Nature.]

The unique variant of AP is a very promising 5D theory, with topological charges and quasi-charges; their phenomenology can look, at some conditions and to a certain extent, like a 4D quantum field theory.

Despite clear lack of development, the theory can give qualitative but important explanations (lepton flavors et cet.) as well as predictions (which can be falsified in future LHC experiments):

1. the absence of spin zero elementary particles (there are no \( SO_3 \)-symmetrical quasi-charges);
2. there is no room for supersymmetry and dark matter [the gravitation theory should be instead replaced with \( (R^{\mu\nu}G_{\mu\nu} - 0.6\langle R_{,\mu}\rangle R) \)]—like modified gravity.

References

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\[2\] The string theory (it is not yet a ready theory; it’s not a clear road, just a direction) still has one free parameter—the string tension. (Nature should keep the value in every point? In which storage?)