TOPICAL REVIEW

Trans-Planckian issues for inflationary cosmology

Robert H Brandenberger\(^1\) and Jérôme Martin\(^2\)

\(^1\) Physics Department, McGill University, 3600 University St, Montreal, QC, H3A 2T8, Canada
\(^2\) Institut Astrophysique de Paris, 98bis Boulevard Arago, F-75014 Paris, France

E-mail: rhb@physics.mcgill.ca and jmartin@iap.fr

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Abstract

The accelerated expansion of space during the cosmological inflation period leads to trans-Planckian issues which need to be addressed. Most importantly, the physical wavelength of fluctuations which are studied at the present time by means of cosmological observations may well originate with a wavelength smaller than the Planck length at the beginning of the inflationary phase. Thus, questions arise as to whether the usual predictions of inflationary cosmology are robust considering our ignorance of physics on trans-Planckian scales, and whether the imprints of Planck-scale physics are at the present time observable. These and other related questions are reviewed in this paper.

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(Some figures may appear in colour only in the online journal)

1. Introduction

The theory of cosmological inflation \[1\] (see also \[2–4\] for related original work) has become the current paradigm of early universe cosmology. Not only did the scenario solve some of the conceptual problems of Standard Big Bang cosmology, the previous paradigm of cosmology, it also provided \[5\] the first explanation for the origin of the large-scale structure of the universe based on causal physics (see also \[2, 6, 7\] for related original work). Better still, inflationary cosmology was predictive. The scenario predicted an almost scale-invariant spectrum of curvature fluctuations which are coherent and passive on scales which in the early universe are larger than the Hubble radius. As has been known since around 1970 \[8, 9\], such a spectrum predicts characteristic oscillations in the angular power spectrum of cosmic microwave (CMB) anisotropy maps, oscillations which were first discovered by the Boomerang experiment \[10\] and later confirmed with high accuracy by the Wilkinson Microwave Anisotropy Probe (WMAP) satellite experiment \[11\].

Inflationary cosmology is based on the assumption that there is a period in the very early universe during which space is expanding at an accelerated rate (typically almost...
Figure 1. Space-time sketch of inflationary cosmology. The vertical axis is time, the horizontal axis corresponds to physical distance. The solid line labelled $k$ is the physical length of a fixed comoving fluctuation scale. The role of the Hubble radius and the horizon are discussed in the text.

exponentially). Figure 1 depicts a space-time sketch of inflationary cosmology. The vertical axis is time, the horizontal axis represents physical distance. The period of inflation begins at a time $t_i$ and ends at time $t_R$. The accelerated expansion of space during this time interval can create a large and almost spatially flat universe from an initial state in which the size of space is microscopic. In this way, inflation solves the ‘size problem’ of Standard Cosmology. In an accelerating universe the contribution of spatial curvature to the total density decreases. Hence, a spatially flat universe is a local attractor in initial condition space [12]³, thus providing a solution to the ‘flatness problem’.

In figure 1 three distance scales are shown. The solid curve which corresponds to an almost constant value during the period of inflation is the Hubble radius $\ell_H(t)$ which is given by the inverse expansion rate of space. The Hubble radius plays an important role in the evolution of cosmological perturbations, the inhomogeneities which give rise to the large-scale structure of the universe and which lead to CMB anisotropies. On length scales smaller than the Hubble radius, matter fluctuations oscillate almost like in flat space-time. These fluctuations freeze out when the wavelength crosses the Hubble radius. On larger scales, the evolution of the perturbations is governed by gravity. The dashed curve which tracks the Hubble radius before the onset of inflation but whose length grows exponentially during the period of inflation is the horizon, the forward light cone from a point at the time of the Big Bang. The horizon is the largest distance that causal information can travel. The fact that the horizon becomes exponentially larger than the Hubble radius provides a solution to the ‘horizon (or homogeneity) problem’ of Standard Cosmology: in the context of inflation, the

³ Note that it is not a global attractor, and hence a certain degree of spatial flatness is required in order to be able to enter into a period of accelerated expansion.
causal horizon can be much larger than any scale which we observe today. In particular this can explain the near isotropy of the CMB.

The third length scale shown in figure 1, the curve labelled by $k$, represents the physical length of a fluctuation mode. The fact that modes well inside the Hubble radius at early times become exponentially larger than the Hubble radius by the end of the period of inflation provides a causal mechanism by which fluctuations are generated in inflationary cosmology. Since classical matter redshifts exponentially during the period of inflation, it is natural to assume that matter approaches the vacuum state. The vacuum, however, is permeated by quantum vacuum perturbations, and hence it was conjectured [2, 6, 7] that the primordial fluctuations are quantum mechanical in nature. The theory of structure formation in inflationary cosmology is based on the quantum theory of cosmological perturbations [13, 14] (see e.g. [15, 16] for in-depth reviews, and [17] for a pedagogical overview). Based on the above discussion it is reasonable to assume that fluctuations begin in their quantum vacuum state [5]. While on sub-Hubble scales, the fluctuation modes undergo quantum vacuum oscillations. They freeze out once the wavelength of the mode becomes larger than the Hubble radius. Thereafter the modes are squeezed while on super-Hubble scales. In particular, all modes acquire the same angle in phase space. They hence enter the Hubble radius at late times as standing waves, leading to the characteristic acoustic oscillations in the angular power spectrum of CMB anisotropies which were mentioned at the beginning of this section. Based on the time-translation symmetry of the inflationary phase, it is expected [6] that the spectrum of the density perturbations produced during inflation is approximately scale-invariant, and the mathematical analysis confirms this result.

The causal generation mechanism of fluctuations described above is the most significant success of inflationary cosmology. However, a second look at the space-time sketch of figure 1 reveals a serious conceptual problem [18, 19]. A sufficiently long period of inflation is required in order for modes which are currently probed in cosmological observations to have a wavelength smaller than the Hubble radius at the beginning of the period of inflation—which is a necessary condition for the validity of the inflationary structure formation scenario. However, if the period of inflation is only slightly longer (70 e-foldings in models in which the energy scale at which inflation takes place is close to the scale of Grand Unification), then the wavelengths of all fluctuation modes which are currently inside the Hubble radius were smaller than the Planck length at the beginning of the period of inflation. We do not understand physics on length scales smaller than the Planck length. The problem for inflationary cosmology is that fluctuation modes emerge from this sub-Planck-wavelength zone of ignorance, as is sketched in figure 2. New physics is required to truly understand the origin and early evolution of the fluctuations.

Based on the geometry of figure 2, there are good reasons to expect that Planck-scale physics can lead to a modification of the spectrum of cosmological perturbations: If we imagine starting off all perturbation modes on a fixed space-like Cauchy surface at the beginning of the period of inflation, then short distance modes will feel the effects of the modified physics for a longer time than long wavelength modes. If the evolution of the modes is not adiabatic on short wavelength scales, then one would expect the spectrum for short wavelength modes to be boosted relative to that of long wavelength modes. This would produce a blue tilt in the spectrum of fluctuations.

The above problem is now called the trans-Planckian problem for inflationary cosmology. While it may be a problem from the point of view of inflationary theory, it can instead
be viewed as a trans-Planckian window of opportunity to probe Planck-scale physics with current cosmological observations. The accelerated expansion of space brings scales into the observable range which we have no chance of probing in accelerator physics.

In the following sections, we will review these trans-Planckian issues for inflationary cosmology. In section 2 we discuss some theoretical approaches, beginning with a brief review of some of the original approaches to studying the sensitivity of the predictions of inflationary cosmology to assumptions about trans-Planckian physics. We continue with a discussion of two more recent realizations of inflation in the context of specific models of trans-Planckian physics. In one case we find dramatic differences compared to the ‘standard results’, in the second case we recover the usual spectrum. We end section 2 with some general remarks. In sections 3 and 4 we discuss observational constraints on the magnitude of trans-Planckian effects in inflationary cosmology. Finally, in section 5 we attempt to draw connections with some broader issues.

2. Theoretical approaches

2.1. Original approaches

2.1.1. Modified dispersion relations. The original work on the trans-Planckian problem for inflation was performed in the context of modified dispersion relations, [19, 21] for the linear fluctuation modes, following works [22, 23] which studied the dependence of black hole radiation on Planck scale physics. This model represents a toy model to study the
unknown effects of trans-Planckian physics, a method which has the practical advantage of remaining within the context of applicability of linear cosmological perturbation theory. Before introducing the basics of the method, we must remind the reader of the relevant formalism.

We are interested in following the scalar metric perturbations, the linearized gravitational fluctuation modes induced by matter perturbations. In the context of General Relativity, the relevant canonical variable (the Mukhanov–Sasaki variable [13, 14]) \( v \) satisfies the following Fourier space equation of motion [15, 17]

\[
v''_k + \left( k^2 - \frac{z''}{z} \right) v_k = 0,
\]

where \( k \) denotes the comoving momentum and a prime stands for the derivative with respect to conformal time \( \eta \). The function \( z(\eta) \) depends on the background cosmology. If the equation of state of the background is time-independent, then \( z(\eta) \) is proportional to the scale factor \( a(\eta) \). The variable \( v \) is related to the curvature fluctuation \( \zeta \) in comoving gauge, a coordinate system in which the matter fluctuation vanishes, via \( \zeta = z^{-1} v \). The variable \( \zeta \) carries vanishing mass dimension.

Of particular interest is the dimensionless power spectrum \( \mathcal{P}_\zeta (k) \) defined via (for a more accurate definition, see equation (23))

\[
\mathcal{P}_\zeta (k) \sim k^3 |\zeta_k|^2.
\]

The power spectrum is called ‘scale-invariant’ if \( \mathcal{P}_\zeta (k) \) is independent of \( k \).

The idea of the modified dispersion relation method is to introduce a non-trivial relation between the physical frequency and momentum of the modes

\[
k^2 \rightarrow k^2_{\text{eff}}(k, \eta) \equiv a^2(\eta) \omega^2_{\text{phys}} \left[ \frac{k}{a(\eta)} \right],
\]

where the function \( \omega_{\text{phys}}(u) \) deviates from its usual form \( \omega_{\text{phys}}(u) = u \) only in the ultraviolet (UV). As will be discussed below, such dispersion relations arise from specific theories of Planck-scale physics such as Hořava–Lifshitz (HL) gravity [24].

The modification of the dispersion relation implies a change in the Hamiltonian of the perturbation modes which has important effects at short wavelengths. The analysis of [19] is based on the following assumptions: first, we consider a space-like hypersurface on which we set up initial conditions. Second, the initial conditions are chosen to represent the lowest energy state of the local Hamiltonian at the initial time. Since this Hamiltonian will deviate from the usual one on short wavelength scales, the initial state will deviate from the usual Bunch–Davies vacuum of the un-modified theory, the deviations increasing towards the UV.

For mild distortions of the dispersion relation such as those considered in [22] where \( \omega^2 \) asymptotes to a constant in the UV, the evolution of the mode functions will be adiabatic (they are the WKB solutions) and track the local vacuum state. In this case, the evolution will lead to the usual scale-invariant spectrum at late times.

On the other hand, if the dispersion relation violates the adiabaticity condition

\[
\left| \frac{3(\omega')^2}{4(\omega^2)^2} - \frac{\omega''}{2(\omega^2)} \right| < 1
\]

then an initial vacuum state will evolve to a state which on large scales differs from the Bunch–Davies vacuum of the unmodified theory. The deviations increase as the frequency increases, thus leading to a steep blue spectrum in violation of the observational constraints (see sections 3 and 4).

Deviations from adiabaticity arise for one branch of the modified dispersion relations proposed in [23]. Specifically, one can consider the example [25, 26]

\[
\omega^2_{\text{phys}} = k^2_{\text{phys}} - 2b_1 k^2_{\text{phys}} + b_2 k^6_{\text{phys}},
\]

5
with $b_1$ and $b_2$ being two positive constants. In the case of this dispersion relation (sketched in figure 7) there can be a region of values of $k_{\text{phys}}$ for which the adiabaticity condition is violated. In this example, adiabaticity is maintained both in the far UV and the infrared (IR).

While we do not expect concrete models of modified physics on trans-Planckian scales to yield dispersion relations which are adiabatic in the far UV, assuming that this takes place in our toy model makes it easy to justify our initial conditions: we can start modes in the adiabatic vacuum in the far UV. As the universe expands, the physical wavenumber will redshift. Modes which begin in the far UV adiabatic regime will experience a time interval of finite duration during which the wavenumber is in the non-adiabatic region. Hence, the wavefunction will obtain corrections from its WKB form. In particular, the amplitude of the final fluctuations will change [25].

To a first approximation (in the deviation of the expansion of space from exponential), the shape of the spectrum will not change since all modes spend the same amount of time in the region of non-adiabaticity (as discussed in section 3, there will be superimposed oscillations in the spectrum). However, in models in which non-adiabaticity is violated at all UV scales (like the ones considered in [19]), short wavelength modes spend more time in the region of non-adiabaticity, and hence the spectrum of cosmological perturbations will acquire a blue tilt whose spectral slope can well exceed current limits.

2.1.2. New physics hypersurface. A more conservative approach to the trans-Planckian problem is simply not to evolve the fluctuation modes during the time period in which their wavelength is smaller than the length scale of the new physics. This corresponds to introducing a time-like new physics hypersurface on which initial conditions are imposed. We expect the wavelength at which the new physics hypersurface is reached to be given by the Planck length (or the string length—in the context of string cosmology—if it is larger).

At this point, the trans-Planckian problem has simply been shifted to the problem of choosing initial conditions on the new physics hypersurface. The most conservative approach is to start modes off in their local adiabatic vacuum. Note, however, that precisely the main point of the analysis of the previous subsection is that new physics acting on trans-Planckian scales can well produce states which are very different from the local adiabatic vacuum at the time when the wavelength equals the new physics scale.

Even continuing with the conservative approach mentioned above, a careful tracking of the mode wavefunctions reveals residual oscillations in the power spectrum [27–31]. The amplitude of these oscillations depends on whether we are computing the spectrum of test scalar fields on a fixed background metric, gravitational waves, or scalar metric fluctuations. In the two former cases the amplitude of the oscillations in the power spectrum is

$$\Delta_{\text{GW}} \sim O(1) \left( \frac{H}{M_C} \right)^2,$$

where $M_C$ is the mass scale of the new physics hypersurface, and in the case of scalar cosmological perturbations we have

$$\Delta_S \sim O(1) \frac{H}{M_C}.$$

Note that within the new physics hypersurface approach, the setting of the initial state is not unique. Whereas the prescription adopted by [27, 28, 31] leads to oscillations of the above-mentioned magnitude, the choice made by [29] and studied in more detail in [32] leads to oscillations with amplitude suppressed by three powers of $H/M_C$. The dependence of the amplitude of the trans-Planckian effects on the definition of the ‘local vacuum state’ was explored in detail in [30].
In the context of the choice of the vacuum state for cosmological perturbations there has been some discussion about vacuum states which are alternatives to the usual Bunch–Davies [33] vacuum. These are the $\alpha$ vacua [34] discussed in the context of cosmological perturbations in [28, 35]. They are in different superselection sectors of states than the Bunch–Davies vacuum, and applied to inflationary cosmology they also lead to oscillations in the power spectrum of fluctuations. The use of such vacua has, however, been criticized on the basis of their instabilities [36, 37] (but see [38]). Our view, however, is that in the context of inflationary cosmology which is not past eternal [39], it makes more sense to restrict attention to states which are in the same superselection sector as the usual vacuum, and which can hence be generated from such a vacuum by local trans-Planckian physics.

2.1.3. Other approaches. In the two previous subsections we discussed two general frameworks for studying trans-Planckian effects on the spectrum of cosmological perturbations. Another general framework is the effective field theory approach. The idea of effective field theory is to integrate out the high energy physics and to derive an effective Lagrangian which only involves observable scales. The application of effective field theory techniques to the trans-Planckian problem of inflation was pioneered in [40] and [41]. However, the expansion of space (in particular the fact that Planck-scale modes are redshifted into the visible sector) leads to challenges which are not present in usual applications of effective field theory, challenges which were not taken into account in [40], where it was claimed that trans-Planckian effects on the spectrum of cosmological perturbations cannot be larger than $O(H/M_C)^2$. However, as shown e.g. in the work of [41], even in the context of effective field theory and local Lorentz invariance it is possible for the pre-inflationary dynamics to produce states which deviate from the standard Bunch–Davies vacuum and for which hence the corrections to the power spectrum are larger.

Some authors have considered general modifications of physics on Planck scales such as modified uncertainty principles [43], space-space non-commutativity [44, 27] and space-time non-commutativity [45] and computed the changes in the spectrum of perturbations in an inflationary cosmology. All of these approaches can be viewed as leading to specific prescriptions of setting the initial conditions on a space-like new physics hypersurface.

If we make the strong assumptions firstly that the microscopic structure of space-time on Planck scales is Lorentz invariant and secondly that the state of the system is in its local vacuum, then it indeed follows that the corrections to the spatial correlation function and hence to the power spectrum are bounded by $O(H/M_C)^2$ [46]. Even if one accepts the first assumption, the second one is a very strong one. We know that (in the context of Einstein gravity with matter fields obeying the usual energy conditions) inflation is singular in the past [39], and there is thus pre-inflationary dynamics which must be taken into account when determining what the initial state of the fluctuation modes is. In particular, perfectly Lorentz-invariant theories can lead to a bouncing universe, e.g. through the addition of quintom matter [47] or in the context of specific string theory backgrounds [48].

2.2. Specific models

2.2.1. Bouncing inflation. In theories which admit non-singular bouncing cosmologies (e.g. HL gravity in the presence of non-vanishing spatial curvature [49]), it is possible to have a post-bounce inflationary phase and, by time reflection symmetry, a contracting deflationary phase (see figure 3 for the corresponding space-time diagram). The spectrum of cosmological perturbations was studied in this context in [50].

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5 These challenges also arise in some non-gravitational contexts, e.g. when studying quantum field theory in strong external electromagnetic fields [42].
Figure 3. Space-time sketch of the bouncing inflation model. The vertical axis is time, the horizontal axis denotes the physical space coordinate. The Hubble radius $H^{-1}$ is constant during the deflationary phase of contraction and during the inflationary phase of expansion. It diverges around the bounce point. The solid (blue) curve labelled $k$ indicates the wavelength of a fluctuation mode which exits the Hubble radius in its Bunch–Davies vacuum state during the radiation phase of contraction at time $t_1(k)$, evolves on super-Hubble scales until it enters the Hubble radius at time $t_3(k) = -t_i(k)$ during the deflationary phase. The time $t_2$ is the transition between radiation contraction and deflation.

Since the universe begins large and cold, it is natural to start with perturbations in their Bunch–Davies vacuum state. In a contracting universe the curvature fluctuation variable $\zeta$ grows on super-Hubble scales (see e.g. [51, 52]). In a radiation phase of contraction the growth rate is proportional to $\eta^{-1}$ (the conformal time $\eta$ is increasing towards 0), and in the deflationary phase $\zeta$ increases as $\eta^3$ ($\eta$ is increasing to $\infty$ in this phase). In this process, the initial vacuum spectrum is transformed into a dimensionless spectrum which scales as

$$P[k, -t_i(k)] \sim k^{-4},$$

where $-t_i(k)$ is the time when the scale $k$ enters the Hubble radius during the deflationary phase.

The evolution of the fluctuations between the time $t_i(k)$ of Hubble radius entry during the contracting phase and Hubble radius exit at the time $t_i(k)$ is symmetric. Hence, the spectrum at Hubble radius exit is the same as at Hubble radius entrance. Since $\zeta$ is conserved on super-Hubble scales in the expanding phase, the final spectrum of curvature fluctuations in the expanding phase is given by (8), i.e. it is an $n_s = -3$ spectrum. This is a highly red spectrum compared to a scale-invariant one ($n_S = 1$ which is indicated by experiments). The redness of the spectrum is due to the extra growth which long wavelength modes undergo since they spend a longer time on super-Hubble scales.

As will be discussed in the following section, a red spectrum is not only constrained by observations, but also by back-reaction. Applied to our model, this implies that the bouncing
model cannot be symmetric about the bounce point. The phase of deflation must be short. As a consequence, one obtains a scale-invariant spectrum in the UV and a red spectrum in the IR.

This model is an example of how trans-Planckian effects can lead to very large deviations from scale-invariance. As in most non-singular bouncing cosmologies, it requires specially tuned matter and initial condition modelling to obtain a scenario in which a period of deflation follows after a radiation phase. One way, as explored in the quintom matter bounce scenarios [47] is to have the pre-bounce radiation come from a scalar field $\phi$ with quartic potential. While the scalar field is oscillating about its ground state, the time-averaged equation of state is that of radiation. In the contracting phase the amplitude of oscillations increases. But once the amplitude exceeds the Planck scale, a deflationary slow-climb phase sets in.

2.2.2. Hořava–Lifshitz inflation. HL gravity is a four space-time-dimensional theory which has been proposed [24] as a power-counting renormalizable theory of quantum gravity. In this theory, the microscopic Lagrangian is not locally Lorentz-invariant. There is a distinguished time direction, and the gravitational Lagrangian contains higher space-derivative terms. As a consequence, space-time diffeomorphism invariance is also lost. The theory is only invariant under the subgroup of spatial diffeomorphisms, and under separate space-independent time reparametrizations. The guiding principle of the construction of the Lagrangian is invariance with respect to the residual symmetries and power-counting renormalizability with respect to an anisotropic scaling

$$x \to b x, t \to b^3 t,$$  \hspace{1cm} (9)

where $b$ is some constant. There are two versions of the theory—the *projectable* version in which the lapse function $N^6$ is a function of only time, and the *non-projectable* version in which $N$ can depend on space and time.

Renormalizability allows up to six-space-derivative terms in the equations of motion. Hence, quite naturally a modified dispersion relation for fluctuation modes results. There is a second major difference in the theory of cosmological perturbations in HL gravity compared to Einstein gravity: because of the reduced symmetry, there is an extra scalar gravitational fluctuation mode (see e.g. [53] for a review of HL gravity). It has been shown [54] that in the case of the projectable version of HL gravity the extra scalar mode is sick (either ghost-like or tachyonic), while in the non-projectable version it can be [55] well-behaved (for suitable choices of the free parameters of the Lagrangian). The extra mode in fact becomes massive in the IR and thus only plays an important role in the UV.

Because of the existence of the extra degree of freedom and because of the modified dispersion relation one might expect that the spectrum of cosmological perturbations in inflationary cosmology in the context of HL gravity would lead to a very different spectrum of cosmological perturbations than in Einstein gravity. However, a careful analysis of this issue [56] has shown firstly that the evolution of the regular fluctuation mode is adiabatic and secondly that the fluctuations induced by the extra degree of freedom are sub-dominant. Hence, up to oscillations in the spectrum whose frequency is too high to be observationally detectable, the spectrum of cosmological perturbations which emerges when starting all modes off at a fixed time in their instantaneous minimum energy state is scale invariant.

This example indicates a certain degree of robustness of the standard predictions of inflation even if the underlying physics at the Planck scale is substantially modified.

6 In the Hamiltonian approach to gravity, the lapse function is the fluctuation in the time-time component of the metric.
2.3. General comments

It is interesting to ask to which extent the trans-Planckian problem discussed here is specific to inflationary cosmology or what aspects of the problem arise in any cosmological background. To address this question, let us introduce two paradigms alternative to inflation which can explain the observed almost scale-invariant spectrum of cosmological fluctuations. The first is the string gas cosmology model introduced as a background cosmology in [57] (see also [58]) and shown to lead to a mechanism for producing an almost scale-invariant spectrum of cosmological perturbations in [59, 60]. The second is the matter bounce scenario of [51, 52], a non-singular bouncing cosmology with an initial matter-dominated phase of contraction.

String gas cosmology (see e.g. [61] for a recent review) is based on the realization that a gas of closed strings has a maximal temperature, the ‘Hagedorn temperature’ \( T_H \). At temperatures close to \( T_H \), all of the modes of strings including winding modes are excited. This high temperature phase is called the ‘Hagedorn phase’. It is then reasonable to assume that the universe begins in the Hagedorn phase. Since the temperature of a box of strings is independent of the radius \( R \) of the box for a wide range of values of \( R \) (assuming that the entropy of the system is large), it is not unreasonable to assume that the Hagedorn phase is quasi-static (including fixed dilaton). Eventually (through the decay of string winding modes into string loops) the universe transits into the radiation phase of Standard Cosmology. The time when this happens is called \( t_R \) in analogy to the reheating time at the end of inflation. In the context of this string gas cosmology background, it was realized in [59, 60] that thermal string fluctuations in the Hagedorn phase induce a scale-invariant spectrum of curvature fluctuations at late times with a slight red tilt, and a scale-invariant spectrum of gravitational waves with a slight blue tilt [63]—a key prediction of string gas cosmology with which it can be observationally distinguished from inflationary cosmology.

Figure 4 shows a space-time sketch of string gas cosmology. If the Hagedorn temperature is taken to be similar to the post-inflation temperature (which is typically comparable to the scale of Grand Unification), then the post-reheating evolution in inflationary cosmology is identical to the post-Hagedorn dynamics in string gas cosmology. Working backwards from the present time, it is not hard to see that the physical wavelengths of fluctuations which are observed today are many orders of magnitude larger than the Planck scale at the time \( t_R \) (1 mm is a typical value).

Since in string gas cosmology scales which are observed now never had a wavelength close to the Planck scale, the trans-Planckian problem for cosmological perturbations is absent. The basic problems of quantum field theory in an expanding universe (e.g. time-dependence of the Hilbert space of Fourier modes [64]) still are present, but they are not directly coupled to observations, unlike in inflationary cosmology where they are.

Figure 5 represents a space-time sketch of a matter bounce cosmology. The simplest way to visualize the background space-time is to take a mirror inverse of our expanding Standard Cosmology space-time (which is a contracting universe which begins in a phase of matter-domination) and match it to the expanding phases of Standard Cosmology via a short non-singular bounce phase (see [65] for a recent review of the matter bounce scenario). It goes without saying that new physics (either in the matter or in the gravitational sector) is required to obtain such a non-singular bounce. If the energy scale of the non-singular bounce is comparable to the scale of Grand Unification, then—as is the case in string gas cosmology—the physical wavelengths of scales which are currently probed in cosmological observations never enter the sub-Planck length zone of ignorance, and hence there is no trans-Planckian problem for cosmological perturbations.
Figure 4. Space-time diagram (sketch) showing the evolution of fixed co-moving scales in string gas cosmology. The vertical axis is time, the horizontal axis is physical distance. The solid curve represents the Einstein frame Hubble radius $H^{-1}$ which shrinks abruptly to a micro-physical scale at $t_R$ and then increases linearly in time for $t > t_R$. Fixed co-moving scales (the dotted lines labelled by $k_1$ and $k_2$) which are currently probed in cosmological observations have wavelengths which are smaller than the Hubble radius before $t_R$. They exit the Hubble radius at times $t_i(k)$ just prior to $t_R$, and propagate with a wavelength larger than the Hubble until they reenter the Hubble radius at times $t_f(k)$.

As first realized in [51, 52], initial vacuum fluctuations which exit the Hubble radius during the matter-dominated phase of contraction acquire a scale-invariant spectrum of curvature perturbations on super-Hubble scales. The key point is that curvature perturbations in a contracting universe grow on super-Hubble scales, and in a matter-dominated background the extra growth which long wavelength modes experience because they are super-Hubble for a longer time is exactly the right amount to convert a blue vacuum spectrum into a scale-invariant one. One of the key predictions of the matter bounce scenario is that there is a cutoff scale set by the transition time between matter domination and radiation domination during the contracting period below which the spectrum of perturbations turns blue [66]. In addition, the growth of the curvature fluctuation on super-Hubble scales leads to a particular shape and reasonably large amplitude of the bispectrum [67].

3. Theoretical framework

We have seen in the last sections that trans-Planckian physics could modify the inflationary power spectrum. Since we measure the inflationary power spectrum when we measure the CMB anisotropies, this opens the possibility to test these effects with astrophysical data. In order to carry out this program, we must first understand how to calculate the trans-Planckian corrections. At this point, we meet a first difficulty. Since the physics that controls the shape
of these corrections is by definition unknown, it seems impossible to establish a generic result with regards to the shape of the modified power spectrum. Here we will consider a conservative starting point, namely that the perturbations begin in the expanding phase in their instantaneous minimum energy state (as we have seen before, this is not the most general case).

We have seen before that, if trans-Planckian physics is adiabatic, then the inflationary initial conditions are not modified. This means that the sub-Hubble Mukhanov–Sasaki variable is still given by

\[ v_k(\eta) \approx \alpha_k \frac{4\sqrt{\pi}}{\sqrt{2k}} \frac{\sqrt{2}}{m_{\text{Pl}}} e^{ik\eta} + \beta_k \frac{4\sqrt{\pi}}{\sqrt{2k}} \frac{\sqrt{2}}{m_{\text{Pl}}} e^{-ik\eta} \]  

with \( \alpha_k = 1 \) and \( \beta_k = 0 \). If, however, the physical conditions that prevailed in the trans-Planckian regime were non-adiabatic, then there was particle production and, as a result, \( \beta_k \neq 0 \). This immediately implies that the power spectrum (which is proportional to the square modulus of \( v_k \)) contains super-imposed oscillations. Therefore, even without knowing in detail the trans-Planckian physics, it is possible to establish a generic prediction for the shape of the modified power spectrum. This generic prediction was made for the first time in [19].

However, clearly, if we want to make more precise predictions and, for instance, compute the amplitude, the frequency or the phase of those oscillations, we need to make further assumptions. The most general approach consists in parameterizing the deviations from the standard situation in the initial conditions. Let \( M_C \) be the energy scale above which some new physics is operating. This scale could be the Planck scale or the string scale for instance. A
Fourier mode emerges from the regime where the new effects are relevant when its physical wavelength equals the new length scale introduced in the problem, namely

\[ \lambda(\eta) = \frac{2\pi}{k} a(\eta) = \ell_C \equiv \frac{2\pi}{M_C}, \tag{11} \]

where \( k \) is the comoving wavenumber. It is important to notice that one can have \( \lambda \ll \ell_C \) and, at the same time, \( H \ll m_p \) where \( H \) is the Hubble parameter during inflation. In other words, the Fourier mode wavelength can be much smaller than the new fundamental scale in a regime where spacetime can still be described by a classical FLRW background. The initial conformal time satisfying equation (11) depends on the scale \( k \). At this time, if the trans-Planckian regime was non-adiabatic, then we expect the initial conditions to be modified compared to the standard case. Technically, this is expressed by

\[
\begin{align*}
\upsilon_k(\eta_k) &= \frac{\alpha_k + \beta_k}{\sqrt{2\omega(\eta)}} \frac{4\sqrt{\pi}}{m_p}, \\
\upsilon_k'(\eta_k) &= \sqrt{\frac{\omega(\eta)}{2}} \frac{4\sqrt{\pi}(\alpha_k - \beta_k)}{2m_p},
\end{align*}
\tag{12, 13}
\]

where \( \alpha_k \) and \( \beta_k \) are two complex numbers (already introduced before) satisfying \( |\alpha_k|^2 - |\beta_k|^2 = 1 \). These numbers completely characterize the influence of the new physics on the initial conditions. Again, \( \textit{a priori} \), a complete calculation of \( \alpha_k \) and \( \beta_k \) requires the knowledge of the physics beyond the scale \( M_C \). However, if the ratio \( H/M_C \) goes to zero, then we expect to recover the standard Bunch–Davies situation for which \( \alpha_k = 1 \) and \( \beta_k = 0 \). Therefore, if the expressions of \( \alpha_k \) and \( \beta_k \) are perturbative in \( H/M_C \), then, without loss of generality, one can write

\[
\begin{align*}
\alpha_k &= 1 + y \frac{H}{M_C} + O\left( \frac{H^2}{M_C^2} \right), \\
\beta_k &= x \frac{H}{M_C} + O\left( \frac{H^2}{M_C^2} \right),
\end{align*}
\tag{14, 15}
\]

where \( x \) and \( y \) are two numbers characterizing the perturbative expansion. The modified power spectrum can be then determined perturbatively in terms of the free parameters \( x \) and \( y \). Notice that, strictly speaking, \( x \) and \( y \) can be scale-dependent. Here, for simplicity, and in order not to introduce arbitrary functions in the problem, we assume that they are roughly scale-independent over the range of wavenumbers relevant to the CMB. If this is not the case, then one looses the ability to parameterize the modified power spectrum in a simple way and one would really need a complete theory of the trans-Planckian effects to establish the shape of the corrections. It is also important to recall that \( \alpha_k \) and \( \beta_k \) must satisfy the Wronskian condition.

At leading order in \( H/M_C \), this simply amounts to \( y + y^* = 0 \).

We are now in a position where the modified power spectrum can be derived. Following [32], it can be written as (of course, there is a similar formula for tensor perturbations that we do not show here)

\[
k^3P_l = \frac{H^2}{\pi\epsilon_1 m_p^2} \left[ 1 - 2(C + 1)\epsilon_1 - C\epsilon_2 - (2\epsilon_1 + \epsilon_2) \ln \frac{k}{k_s} 
- 2|x| \frac{H}{M_C} \left[ 1 - 2(C + 1)\epsilon_1 - C\epsilon_2 
- (2\epsilon_1 + \epsilon_2) \ln \frac{k}{k_s} \right] \cos \left[ \frac{2M_C}{H} \left( 1 + \epsilon_1 + \epsilon_1 \ln \frac{k}{a_0 M_C} \right) + \varphi \right]
- |x| \frac{H}{M_C} (2\epsilon_1 + \epsilon_2) \times \sin \left[ \frac{2M_C}{H} \left( 1 + \epsilon_1 + \epsilon_1 \ln \frac{k}{a_0 M_C} \right) + \varphi \right] \right]. \tag{16}
\]
Figure 6. Trans-Planckian power spectra given by equation (16). The blue line corresponds to a vanilla model with $|x| = 0$ and $\epsilon_1 = 1/(2\Delta N)$, $\epsilon_2 = 1/\Delta N$ with $\Delta N \simeq 50$ as predicted for the $m^2\phi^2$ inflationary model. The red line corresponds to a model with the same values for the slow-roll parameters and $H/M_C \simeq 0.002, |x| \simeq 50, \psi = 3$. Finally, the green line represents a model with $H/M_C \simeq 0.001, \psi = 2$ and the same values for the other parameters.

In this expression, $\epsilon_1$ and $\epsilon_2$ are the two first slow-roll, or horizon-flow, parameters. The scale $k_*$ is the pivot scale and $a_0$ is the scale factor evaluated at a time $\eta_0$ during inflation (the Hubble parameter which appears in the overall normalization is also evaluated at the same time). This time is arbitrary and does not depend on the scale $k$. For convenience, we choose $k_*/a_0 = M_C$. The quantity $C$ is given by $C \equiv \gamma_E + \ln 2 - 2$ where $\gamma_E$ is the Euler constant. Finally, $\psi$ is the phase of the complex number $x$, that is to say $x \equiv |x| e^{i\psi}$. It is interesting to notice that, at this order, the number $y$ does not appear in the expression for $k^3P_\xi$.

Let us now comment on the power spectrum itself. We see it has the expected shape (see figure 6). There is the usual slow-roll (almost scale-invariant) part, with the overall normalization given by the square of the Hubble parameter (during inflation), measured in Planck units, and divided by the first slow-roll parameter. This slow-roll part is corrected by two oscillatory terms which represent the super-imposed oscillations. It is interesting to notice that these oscillations are logarithmic in $k$. This is due to the fact that the initial conditions have been chosen on the surface $M_C = \text{constant}$ [68, 69]. If, on the contrary, the initial conditions were chosen on a surface of constant time, then the oscillations would be linear. This is for instance what happens in bouncing models. The amplitude of the oscillations is, roughly speaking (taking $|x| \sim 1$), given by $|x|H/M_C$ while the frequency is proportional to $(H/M_C)^{-1}$. Therefore, if the initial state is the instantaneous Minkowski vacuum, the amplitude is just inversely proportional to the frequency. This means that high frequency modes necessarily have a small amplitude. But, of course, there is a priori no reason to assume that the frequency and the amplitude are fully correlated. In addition, as we are going to see, postulating $|x| = 1$ would prevent us finding the best fit.

On general grounds, we expect the ratio $H/M_C$ to be a small number. Indeed, we know from the COBE normalization that $H \lesssim 10^{-5} m_P$. We do not know the scale $M_C$ but reasonable candidates are the Planck scale itself or the string scale $M_S \simeq 10^{-3} - 10^{-1} m_P$. This means that $H/M_C$ is at most of order $\sim 0.01$. Therefore, if $|x| = 1$, the amplitude of the trans-Planckian
corrections is very small and it will be difficult to detect an effect in the CMB, even with an experiment like the Planck satellite.

On the other hand, if $|x| \neq 1$, one can easily reach a level which would be detectable. In this case, we nevertheless meet another issue, namely the backreaction problem \[70, 71\].

If $|x| \neq 1$, we start from a non-vacuum state, a natural assumption if physics beyond the scale $M_C$ is non-adiabatic. But a non-vacuum state means that particles are initially present. Since these particles carry energy density, there is now the danger that this energy density be comparable to or larger than that of the background, (which is $\sim H^2 m^2_{Pl}$), in which case the energy in the particles would prevent the onset of inflation. Clearly, the larger $|x|$ is, the larger the trans-Planckian corrections become, and the more severe is the danger to have a backreaction problem. In fact, it is easy to estimate the regime where there is no backreaction problem. The energy density carried by the ‘trans-Planckian particles’ is

$$\rho_{UV} \simeq M_C^4 |\beta_k|^2 \simeq M_C^2 H^2 |x|^2.$$  \hspace{1cm} (17)

Therefore, the condition $\rho_{UV} \lesssim H^2 m^2_{Pl}$ reduces to $|x| \lesssim m_{Pl}/M_C$ or, using the COBE normalization $H \simeq m_{Pl} 10^{-4} \sqrt{\epsilon_1}$,

$$|x| H_{MC} \lesssim 10^4 \sqrt{\epsilon_1} \left( \frac{H}{M_C} \right)^2.$$  \hspace{1cm} (18)

As a consequence, one can have $|x| H/M_C$ of order 1 and easily satisfy the above inequality (in particular, in inflationary models where the gravity waves contribution is extremely small). Of course, this does not mean that one should see trans-Planckian effects in the CMB. It could very well turn out that $|x|$ is not large enough to compensate the smallness of the ratio $H/M_C$.

Another remark is in order here. In the above calculation, we have used the value of the Hubble parameter during inflation. As noticed in \[72\], if instead one had used its present day value, the no-backreaction condition would have become almost impossible to satisfy. Using the Hubble parameter during inflation assumes that the trans-Planckian production of particles stops after the end of inflation. It is easy to find a situation where this happens. Let us for instance consider the case where the trans-Planckian corrections arise from a modified dispersion relation (see the previous sections). A typical situation is represented in figure 7 (the model and the argument presented here were also studied in \[73\]). The dispersion relation is almost linear for $k_{phys}/m_C \ll 1$ as it should be for obvious phenomenological reasons, and deviates from linearity for $k_{phys}/m_C \gg 1$. In the regimes where $\omega_{phys} \lesssim H$, the adiabatic approximation is violated and particle production takes place. This is of course the case in the regime $k_{phys} \lesssim k_0$ during inflation, this situation corresponding to the case where the wavelength of a Fourier mode is larger than the Hubble radius. But, as can be noticed in figure 7, it is also the case when $k_{phys} \in [k_-, k_+]$ during inflation. In this regime, adiabaticity is violated and the resulting inflationary power spectrum is modified (and, typically, as argued before, acquires super-imposed oscillations). But the Hubble parameter evolves (decreases) during inflation and after. As a consequence, the horizontal red dashed line in figure 7 representing $H/M_C$ goes down in this diagram. At some point, it passes below the minimum of the dispersion relation and particle production stops. In particular, this is the case for the line representing the current Hubble parameter. In such a situation, it is clear that $H_{inf}$ (and not $H_0$) should be used in order to evaluate $\rho_{UV}$.

Moreover, we want $\rho_{UV} \ll H^2 m^2_{Pl}$ because the energy density of the ‘trans-Planckian particles’ could spoil inflation. But, of course, this rests on the prejudice that the corresponding equation of state is different from $p/\rho = -1$, typically that of radiation $p/\rho = 1/3$. When the
dispersion relation is modified, as shown in [72], the equation of state also acquires corrections. And, as was demonstrated in [72], in the case of the dispersion relation in figure 7, the modified equation of state is precisely very close to that of the vacuum. Therefore, even if one has a backreaction problem, it is not obvious that this will prevent inflation from proceeding.

Before turning to the observational constraints, it is also interesting to study how the oscillations in the power spectrum are transferred to the multipole moments. In principle, this calculation can only been done numerically. There is, however, a regime where one can obtain an approximate analytical result. For small $\ell$ (that is to say large angular scales), in the limit $\epsilon_1 M_C / H \gg 1$ and for $\epsilon_2 = -2 \epsilon_1$ (or $n_S = 1$), the multipole moments can be expressed as [74]

$$\ell (\ell + 1) C_\ell \simeq \frac{2 H^2}{25 \epsilon_1 m_{\text{Pl}}^2} (1 - 2 \epsilon_1) \left\{ 1 + \sqrt{\pi |x|} \frac{H}{M_C} \ell (\ell + 1) \left( \frac{\epsilon_1 M_C}{H} \right)^{-5/2} \times \cos \left[ \pi \ell + 2 \frac{M_C}{H} \left( 1 + \epsilon_1 \ln \frac{\epsilon_1}{a H r_{\text{ls}}^3} + \phi - \frac{\pi}{4} \right) \right] \right\},$$

(19)

where $r_{\text{ls}}$ is the distance to the last scattering surface. The first conclusion that can be drawn from the above equation is that super-imposed oscillations in $k$-space are indeed transferred to the $\ell$-space, as the presence of the trigonometric function shows. This type of feature is not wiped out by the transfer function because the oscillations are present everywhere in $k$-space. Moreover, we see that logarithmic oscillations in $k$-space give rise to linear oscillations in real space since the argument of the cosine function scales as $\sim \pi \ell$. In fact, this last property turns out to be an artefact of the approximations used here. Since we are in a situation where

Figure 7. Example of a nonlinear dispersion relation leading to a modified power spectrum. In the regime $k_{\text{phys}} \ll M_C$, the dispersion relation is approximatively linear, as appropriate for standard massless excitations, while in the regime $k_{\text{phys}} \gg M_C$, it strongly deviates from linearity due to trans-Planckian effects. In the regions where $w_{\text{phys}} \ll H$, the adiabatic approximation is violated and trans-Planckian particles production occurs. The horizontal dashed red lines represent the value of the Hubble parameter at different times during the cosmic evolution. In this model, particles production happens during inflation for modes such that $k_{\text{phys}} < k_0$ and $k_{\text{phys}} \in [k_-, k_+]$ and, after inflation, only for modes $k_{\text{phys}} < k_0$, this latter regime taking place in the linear part of the dispersion relation and corresponding to the standard amplification of cosmological perturbations on super-Hubble scales.
the frequency of the oscillations in momentum space goes to infinity, one finds oscillations in $\ell$-space with frequency $\pi$ which is just the maximum frequency possible given that the values of $\ell$ are discrete.

Finally, the amplitude of these oscillations is damped by a factor $(\epsilon_1 M_C/H)^{3/2}$ for very small $\ell$, this effect being compensated by $\ell(\ell + 1)$ for larger $\ell$. However, since the above formula breaks down in the latter regime, it is difficult to reach definite conclusions. One can nevertheless say that we expect the super-imposed oscillations to be absent at very large scales and to appear only at relatively small scales. These considerations are confirmed in figure 8 where multipole moments obtained from a trans-Planckian power spectrum have been displayed. In particular, we see that the super-imposed oscillations only appear at the rise of the first peak but are damped at very large scales.

Having studied how the trans-Planckian effects affect the CMB anisotropy, let us now turn to the question of their possible detection in astrophysical data.

4. Observational constraints

4.1. The power spectrum

We are now ready to discuss the observational constraints on trans-Planckian physics. It is conventional to assert that quantum gravity can be probed by studying the propagation of high energy cosmic rays. However, this is not the only astrophysical method which allows us to constrain trans-Planckian effects. This can also been done with the help of the CMB anisotropies that have been precisely measured by the WMAP satellite [75, 76] (see figure 9). Here, we briefly review the main techniques that have been used to address this question [74, 78, 77] (for related works in the context of inflation without trans-Planckian effects, see [79–83]). The exploration of the parameter space corresponding to equation (16) is usually done using Monte Carlo techniques. Each model is computed using a modified version of the CAMB code [84] and the likelihood is estimated using the COSMOMC code [85].

A first problem met in this kind of analysis is that, in presence of super-imposed oscillations, the calculation of one set of multipole moments $C_\ell$ becomes very time consuming (a few minutes instead of a few seconds). As a consequence, it is not possible to explore
the full parameter space and the analysis must be restricted to the ‘fast parameter space’. This means that the bare cosmological parameters are fixed to their best fit values and that the Monte Carlo exploration is only performed on the ‘primordial parameters’, namely the overall normalization of the spectra, the slow-roll parameters and the parameters describing the oscillations.

The parameters describing the super-imposed oscillations are the amplitude, the frequency and the phase. The parameter describing the amplitude is, as discussed above, $|x|H/M_C$ and it is convenient to take a uniform prior on this parameter in the range $[0, 0.45]$. In order to sample the frequency, one can choose a uniform prior in $[1, 2.6]$ for the parameter $\log(\epsilon_1 M_C/H)$. There is no need to emphasize again the important role played by the fact that $|x| \neq 1$. Taking $|x| = 1$ is not only poorly theoretically justified but, from the data analysis point of view, would render the amplitude and the frequency completely degenerate, a property that can affect a lot the result of the Monte Carlo exploration. Finally, the phase is described by the parameter $\psi = 2M_C(1 + \epsilon_1)/H + \phi$ and one assumes a uniform prior in $[0, 2\pi]$.

The converged posteriors, marginalized posteriors (solid black line) and mean likelihood (purple shaded bars), for the amplitude, frequency and the phase, have been plotted in figure 10 given the seven years WMAP data [86]. The marginalized posterior is a quantity which is sensitive not only to the absolute value of the likelihood function but also to the volume occupied in parameter space. Therefore, if a set of parameters leads to a very good fit but is such that it requires a precise fine-tuning of those parameters (that is say, if one slightly detunes the parameters, then the likelihood dramatically drops), then its marginalized probability can be small. On the other hand, the mean likelihood, as its name indicates, is only sensitive to the absolute value of the likelihood function and is independent of volume effects in the parameter space which is being considered.

Let us now describe the results displayed in figure 10. The amplitude of the marginalized probability is peaked over a vanishing value which indicates that the vanilla slow-roll power spectrum (with no trans-Planckian oscillations) is still the favoured model from a Bayesian point of view. One finds that $|x|H/M_C < 0.26$ at $2\sigma$ confidence level. It is, however, also interesting to remark that, as opposed to the marginalized probability, the mean likelihood is peaked over a non-vanishing value. This means that the best fit is actually a model with trans-Planckian oscillations. One finds that $\Delta \chi^2 \approx -11$ compared to the vanilla slow-roll model.
The discrepancy between the marginalized probability and the mean likelihood is interpreted, in agreement with the above discussion, as an indication that volume effects play an important role. In other words, if one goes away from the best fit model, the likelihood function quickly decreases which means that the best fit only occupies a small volume in parameter space. This analysis is confirmed by the frequency posteriors which exhibit a series of peaks at particular frequencies. The fact that those peaks are very narrow indicates that it is necessary to fine tune the frequency parameter in order to find the best fit, which is totally consistent with the fact that the best fit model occupies a small volume and, hence, that the marginalized and mean likelihood posteriors differ. Let us add that, in [87], it was shown that, if the data actually contain super-imposed oscillations, then the likelihood function should precisely exhibit an oscillatory structure similar to the one which seems to emerge from the present analysis (this structure is even more clearly visible on the 3D representation of the posteriors plotted in figure 11). This is an intriguing fact. Finally, the frequency remains basically unconstrained although \( \psi \approx 6 \) seems slightly favoured but, in any case, not in a statically significant way.

We have emphasized previously that the trans-Planckian effects lead to super-imposed oscillations that are logarithmic in \( k \). It is therefore interesting to test whether this particular scale dependence is favoured by the data. This was done in [79] for the WMAP3 data. The idea is to postulate the following power spectrum

\[
 k^3 P_k = P_* \left( \frac{k}{k_*} \right)^{n_s - 1} \left[ 1 - A \cos \left( \frac{\omega}{p} \left( \frac{k}{k_*} \right)^p - 1 + \psi \right) \right],
\]

such that, in the limit \( p \to 0 \), one recovers logarithmic oscillations. Assuming a uniform prior on \( \log p \) in the range \([-5, 0.48]\), one obtains at 2\( \sigma \) confidence level that \( p < 0.68 \). In this sense, one can indeed argue that the logarithmic structure is special.

The above discussion should however be toned down given the fact that the best fit suffers from a severe backreaction problem. Indeed, the best fit is obtained for the following values: \( |x| H/M_c \approx 0.13 \), \( \log (\epsilon/M_c/H) \approx 2 \) and \( \log (\epsilon_1) \approx -3.1 \) (of course, this last result does not imply a detection of primordial gravitational waves since we only explore the fast parameter

Figure 10. Posterior distributions for the three trans-Planckian parameters describing the frequency, amplitude and phase of the super-imposed oscillations, courtesy of [86]. The parameter \( \sigma_0 \) stands for \( H/M_c \). The solid black lines represent the marginalized distributions while the purple shaded bars represent the mean likelihood.
space, see the discussion before). These numbers imply that $H/M_C \simeq 10^{-5}$ and $|x| \sim 10^4$.

It is interesting to notice that the ratio $H/M_C$ has a very natural value and that it was indeed very useful to perform the analysis without the assumption $|x| = 1$. However, equation (18) now reads $|x|H/M_C \lesssim 10^{-4.5}$ and the best fit clearly violates this inequality by three or four orders of magnitude. Of course, this does not invalidate the statistical analysis presented here, it just questions the interpretation of the best fit in terms of super-imposed trans-Planckian oscillations.

So, in conclusion, what is the observational status of the trans-Planckian oscillations? Clearly, the best model remains the slow-roll vanilla model and there is no detection, at a statistically significant level, of super-imposed oscillations. The Bayesian evidence for this model has never been computed (it was computed for the first time for different slow-roll models only recently in [83]) but it seems pretty clear that its estimation would only reinforce this conclusion. Moreover, we do not see any clear trend towards a detection as more and more accurate data set are released. Indeed, for the first year WMAP data, we had $|x|H/M_C < 0.11$, for the three year WMAP data we obtained $|x|H/M_C < 0.38$, which is a ‘better’ result, but the result was $|x|H/M_C < 0.26$ for the seven year data. There are, however, a list of intriguing hints (the best fit is given by a model with super-imposed oscillations, presence of narrow peaks in the likelihood function) that seems to indicate that, maybe, non trivial features are actually present in the data. Of course, if this is the case, it remains to be seen whether there are of primordial origin and whether they can be explained by trans-Planckian physics. But we are of the opinion that these hints are a sufficient motivation to carry out the analysis again when the Planck data become available.

4.2. Non Gaussianities

Let us now study how trans-Planckian effects can modify the conventional predictions for non-Gaussianities. As is now well-established, the level of non-Gaussianity is small in single field inflationary models (with a canonical kinetic term) if the initial state is the Bunch–Davies state. Roughly speaking, as shown for the first time in [88–92] and then further elaborated in
The most general form of the contribution of \[93\] has been to calculate the momentum dependence of the bi-spectrum exactly and to introduce equations (14) and (15) which generically describe the trans-Planckian modifications. The first calculation in this direction was performed in \[94–96\]. It was done for a non-vanishing signal would therefore rule out Gaussian perturbations. For this reason, we are, however, more interested in computing the non-Gaussianity for a state characterized by equations (14) and (15) which generically describe the trans-Planckian modifications.}

In this case, the free theory is still Gaussian and, therefore, the corresponding three-point correlation vanishes. It is only when interactions are taken into account (that is to say beyond the Gaussian approximation) that non-Gaussianity can be produced. This problem has recently been considered by various authors, see \[97–101\], and, in the following, we turn to this question.

A Gaussian distribution has a vanishing three-point correlation function and any detection of a non-vanishing signal would therefore rule out Gaussian perturbations. For this reason, it is interesting to calculate the three point correlation function of the curvature perturbation defined by

$$
\langle \zeta(\eta, x) \zeta(\eta, x) \zeta(\eta, x) \rangle = \int \frac{d^3k_1}{(2\pi)^{3/2}} \int \frac{d^3k_2}{(2\pi)^{3/2}} \int \frac{d^3k_3}{(2\pi)^{3/2}} \times \langle \zeta_k(\eta) \zeta_k(\eta) \zeta_k(\eta) \rangle e^{i(k_1 + k_2 + k_3) \cdot x}.
$$

In the above expression $\zeta_k$ represents the Fourier transform of $\zeta$ (and $k_1 = k_1$ etc). Let us recall that the curvature perturbation $\zeta$ is related to the Mukhanov–Sasaki variable $v$ introduced before by $\zeta = v/(\sqrt{2M_\text{Pl} a \epsilon_1})$. In the framework of the theory of cosmological perturbations of quantum-mechanical origin, it is an operator and it can be written as

$$
\zeta(\eta, x) = \int \frac{d^3k}{(2\pi)^{3/2}} [a_k f_k(\eta) e^{ik \cdot x} + a_k^\dagger f^*_k(\eta) e^{-ik \cdot x}].
$$

Since one has $\zeta_k = (a_k f_k + a_k^\dagger f^*_k)$, the power spectrum can be expressed as (see also equation (2))

$$
k^3 P_v = P_\zeta = \frac{k^3}{2\pi^2} |f_k|^2 = \frac{k^3}{4\pi^2 M_\text{Pl}^2 v_0^2 \epsilon_1^2} |v_k|^2,
$$

where the quantity $k^3 P_v$ was also already introduced before, see for instance equation (16). The most general form of $v_k$ is given by

$$
v_k = \frac{\alpha_k}{\sqrt{2k}} \left( 1 + \frac{1}{ik \eta} \right) e^{-ik \eta} + \frac{\beta_k}{\sqrt{2k}} \left( 1 - \frac{1}{ik \eta} \right) e^{ik \eta}.
$$

Several comments are in order here. Firstly, the coefficients $\alpha_k$ and $\beta_k$ are of course the same coefficients used to evaluate the trans-Planckian power spectrum. The Bunch–Davies vacuum corresponds to the choice $\alpha_k = 1$ and $\beta_k = 0$. Here, we are interested in a situation where the coefficient $f_k$ is of the order of the slow-roll parameters $\eta$.

Although this is very often not properly acknowledged in the literature, the fact that the non-Gaussianity is small for the class of models mentioned above was established in \[88–91\] much before the publication of \[93\]. The main contribution of \[93\] has been to calculate the momentum dependence of the bi-spectrum exactly and to introduce efficient techniques which have then permitted the calculation of non-Gaussianities for other, more complicated, classes of inflationary scenarios.
\( \beta_k \neq 0 \) and a generic parameterization of the deviations from the Bunch–Davies state due to trans-Planckian effects has been presented in equations (14) and (15). Secondly, one can check that the above expression behaves according to equation (10) in the sub-Hubble regime (in fact, we have slightly changed the conventions—no factor \( 4 \sqrt{\pi / m_{Pl}} \) appears anymore—in order to work with notations that are standard in the calculations of non-Gaussianities). Thirdly, equation (24) corresponds to a de Sitter background. This means that we neglect the corrections due to the slow-roll parameters in the mode function (but we took them into account when we computed the power spectrum). These corrections would lead to sub-leading effects in the calculation of the three point correlation function (for a calculation, see for instance [102, 103]). Finally, from equation (24), it is easy to establish the expression of \( f_k \) that will be used later on. One finds

\[
f_k = \frac{i \alpha_k H}{2 M_{Pl} \sqrt{k^3 \epsilon_1}} (1 + i k \eta) e^{-i k \eta} - \frac{i \beta_k H}{2 M_{Pl} \sqrt{k^3 \epsilon_1}} (1 - i k \eta) e^{i k \eta}.
\]

We will also need the derivative of the mode function. One obtains

\[
f'_k = \frac{\alpha_k H}{2 M_{Pl} \sqrt{k^3 \epsilon_1}} k^2 \eta e^{-i k \eta} - \frac{\beta_k H}{2 M_{Pl} \sqrt{k^3 \epsilon_1}} k^2 \eta e^{i k \eta},
\]

where we have neglected terms suppressed by the slow-roll parameters.

As we discussed before, the three point correlation function can only be non-vanishing if interactions are taken into account. In this case, the three point correlation function in momentum space (or bi-spectrum) can be expressed as [93, 104]

\[
\langle \zeta_k(\eta) \zeta_k(\eta) \zeta_k(\eta) \rangle = -i \int_{\eta_{ini}}^{\eta_e} d\tau a(\tau) \langle [\zeta_k(\eta) \zeta_k(\eta) \zeta_k(\eta), H_{int}(\tau)] \rangle,
\]

with \( H_{int} \) being the interaction Hamiltonian while \( \eta_{ini} \) is the time at which the initial conditions are imposed on the modes when they are well inside the Hubble radius. On the other hand, \( \eta_e \) denotes the time when inflation ends. In practice, we always take \( \eta_e \to 0 \). The choice of \( \eta_{ini} \) is more tricky in the trans-Planckian context and will be discussed in more detail in the following. In order to calculate the interaction Hamiltonian, one must compute the action for cosmological perturbations at cubic order (the action of the free Gaussian theory is quadratic). A now standard calculation gives [93, 102–104] (a dot means derivative with respect to cosmic time)

\[
S_3[\zeta] = M_{Pl}^2 \int d\tau d^3x \left[ a^3 \dot{\epsilon}_1 \dot{\zeta}^2 + a \epsilon_1 \dot{\zeta} \right. \\
+ \left. \frac{a^3}{2} \epsilon_1 \dot{\zeta}^2 + \frac{\epsilon_1}{2a} (\dot{\zeta}^2) (\dot{\zeta}^2) + \frac{\epsilon_1}{4a} (\dot{\zeta}^2) (\dot{\zeta}^2)^2 + \mathcal{F} \left( \frac{\delta L_2}{\delta \zeta} \right) \right],
\]

where \( \delta L_2/\delta \zeta \) denotes the variation of the second order action with respect to \( \zeta \), and is given by

\[
\frac{\delta L_2}{\delta \zeta} = \Lambda + H \Lambda - \epsilon_1 \ddot{\zeta},
\]

and the quantities \( \Lambda \) and \( \chi \) are defined by

\[
\Lambda \equiv \frac{a^2 \phi^2}{2 M_{Pl}^2 \Lambda^2} \dot{\zeta} = a^2 \epsilon_1 \dot{\zeta}, \quad \chi \equiv \dot{\Lambda}^2.
\]
The term $\mathcal{F}(\delta L_2/\delta \zeta)$ introduced before refers to the following complicated expression

$$
\mathcal{F}(\frac{\delta L_2}{\delta \zeta}) = \frac{a}{2} \epsilon_1^2 \left( \frac{\delta L_2}{\delta \zeta} \right) \zeta^2 + \frac{2a}{H} \left( \frac{\delta L_2}{\delta \zeta} \right) \zeta
$$

$$
+ \frac{1}{2aH} \left\{ \delta \zeta \left( \frac{\delta L_2}{\delta \zeta} \right) \right\} + \delta^{ij} \left[ \Lambda (\partial_i \zeta) + (\partial^2 \zeta)(\partial_i \chi) \right]
$$

$$
\times \partial_j \left[ \partial^{-2} \left( \frac{\delta L_2}{\delta \zeta} \right) \right] + \delta^{jm} \delta^{jn} \partial_m \partial_n \left[ \partial^{-2} \left( \frac{\delta L_2}{\delta \zeta} \right) \right].
$$

(31)

Moreover, the terms which involve $\delta L_2/\delta \zeta$ can be removed by a suitable field redefinition of $\zeta$ of the following form [93, 102–104]:

$$
\zeta \rightarrow \zeta_n + F(\zeta_n).
$$

(32)

With such a redefinition, the interaction Hamiltonian corresponding to the action $\mathcal{S}_I(\zeta)$, and to be used in equation (27), can be written in terms of the conformal time coordinate $\eta$ (a prime means a derivative with respect to conformal time) as

$$
H_{\text{int}}(\eta) = -M_{\text{pl}}^2 \int d^3x \left[ a \epsilon_1^2 \zeta \zeta^2 + a \epsilon_1^2 \zeta (\partial \zeta)^2 - 2 \epsilon_1 \zeta' (\partial \zeta)' (\partial \chi)ight]
$$

$$
+ \frac{a}{2} \epsilon_1 \epsilon_2 \zeta^2 \zeta' + \frac{\epsilon_1}{2a} \left( \partial \zeta \right)' (\partial \chi) (\partial \chi') + \frac{\epsilon_1}{4a} \left( \partial^2 \zeta \right)' (\partial \chi)^2 \right].
$$

(33)

The three first terms are second order in the slow-roll parameters while the three last ones are third order. Despite the appearance of the slow-roll parameters, this expression is exact at third order in the perturbations.

Having determined the interaction Hamiltonian, one is now in a position where non-Gaussianities can be calculated. For convenience, we redefine the bi-spectrum to

$$
\langle \zeta_{k_1}(\eta) \zeta_{k_2}(\eta) \zeta_{k_3}(\eta) \rangle = \frac{(2\pi)^3}{(2\pi)^2} \mathcal{G}(k_1, k_2, k_3) \delta^{(3)}(k_1 + k_2 + k_3).
$$

(34)

where the delta function ensures momentum conservation. Then, the quantity $\mathcal{G}(k_1, k_2, k_3)$ can be written as

$$
\mathcal{G}(k_1, k_2, k_3) = M_{\text{pl}}^2 \sum_{C=1}^{6} \left[ f_{c_1}(\eta) f_{c_2}(\eta) f_{c_3}(\eta) \mathcal{G}_C(k_1, k_2, k_3) \right.
$$

$$
\left. + f_{c_1}(\eta) f_{c_2}(\eta) f_{c_3}(\eta) \mathcal{G}_C(k_1, k_2, k_3) \right].
$$

(35)

The quantities $\mathcal{G}_C(k_1, k_2, k_3)$ with $C = (1, 6)$ correspond to the six terms in the interaction Hamiltonian (33), and are given by [93]

$$
\mathcal{G}_1(k_1, k_2, k_3) = 2i \int_{\eta_{\text{ini}}}^{\eta_{\text{end}}} d\tau \sqrt{a^2 \epsilon_1} \left( f_{k_1}^* f_{k_2} f_{k_3}^* \right) + \text{two permutations},
$$

$$
\mathcal{G}_2(k_1, k_2, k_3) = -2i \int_{\eta_{\text{ini}}}^{\eta_{\text{end}}} d\tau \sqrt{a^2 \epsilon_1} f_{k_1} f_{k_2}^* f_{k_3}^* (k_1 \cdot k_2 + \text{two permutations}),
$$

$$
\mathcal{G}_3(k_1, k_2, k_3) = -2i \int_{\eta_{\text{ini}}}^{\eta_{\text{end}}} d\tau \sqrt{a^2 \epsilon_1} \left[ f_{k_1} f_{k_2}^* f_{k_3}^* \left( \frac{k_1 \cdot k_2}{k_2^2} \right) \right] + \text{five permutations},
$$

$$
\mathcal{G}_4(k_1, k_2, k_3) = i \int_{\eta_{\text{ini}}}^{\eta_{\text{end}}} d\tau \sqrt{a^2 \epsilon_1} f_{k_1}^* f_{k_2} f_{k_3}^* + \text{two permutations},
$$

$$
\mathcal{G}_5(k_1, k_2, k_3) = \frac{i}{2} \int_{\eta_{\text{ini}}}^{\eta_{\text{end}}} d\tau \sqrt{a^2 \epsilon_1} \left[ f_{k_1}^* f_{k_2}^* f_{k_3}^* \left( \frac{k_1 \cdot k_2}{k_2^2} \right) \right] + \text{five permutations}.
$$

23
\[ \mathcal{G}_6(k_1, k_2, k_3) = \frac{i}{2} \int d^3 \kappa \; a^2 \epsilon_1^3 \left[ f_{k_1}^* f_{k_2}^* f_{k_3} \left( \frac{k_2^2}{k_1^2 k_3^2} \right) (k_2 \cdot k_3) + \text{two permutations} \right]. \]

(36)

Actually, an additional, seventh term arises due to the field redefinition (32), and its contribution to \( \mathcal{G}(k_1, k_2, k_3) \) is found to be

\[ \mathcal{G}_7(k_1, k_2, k_3) = \frac{\epsilon_2}{2} \left( |f_{k_2}|^2 |f_{k_3}|^2 + \text{two permutations} \right). \]

(37)

The other terms in equation (31) do not contribute because they all contain a derivative (time derivative and/or space derivative) and, at the end of inflation, on large super Hubble scales, \( \zeta \) is constant.

The last thing which remains to be done is to define the \( f_{\text{NL}} \) parameter which is conventionally used in the literature to quantify the level of non-Gaussianity. It reads

\[ \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = (2\pi)^{4+3/2} \left( -\frac{3}{10} f_{\text{NL}} \mathcal{P}_S^2 \right) \sum_{i} \frac{k_i^3}{\mathcal{P}_i^2} \delta(k_1 + k_2 + k_3), \]

(38)

where we have neglected the slight scale dependence of \( \mathcal{P}_S \). This definition is introduced to match the formula \( \zeta(\eta, x) = \zeta^G - 3 f_{\text{NL}} (\xi^G)^2 / 5 \) which is used to extract observational bounds on the parameter \( f_{\text{NL}} \) (the coefficient \( 3/5 \) comes from the fact that, during a matter dominated era, the curvature perturbation and the Bardeen potential are related by \( \xi = 5\Phi / 3 \)).

From the above expression, the full bi-spectrum can be calculated. For a slow-roll model with canonical kinetic term, the dominant terms are \( \mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3 \) and \( \mathcal{G}_7 \). This may differ for other type of models. For instance, if slow-roll is violated during a few e-folds, then the main contribution comes from the term \( \mathcal{G}_4 \), see [105–108]. If the kinetic term is not standard, then a new vertex \( \propto \xi^3 \) appears, see [102–104]. Here, we do not give the full bi-spectrum for a slow-roll model since this is not the main goal of this review paper. It can be found in [93, 102–104]. (Notice that it was recently shown that this bi-spectrum is not modified by re/preheting effects in single field inflation, see [109]. Therefore, it actually corresponds to what we should see in the CMB.) We just quote the so-called consistency relation for the squeezed configuration \( k_1 \sim k_2 \gg k_3 \) which reads

\[ f_{\text{NL}}^0 = \frac{5}{12} (n_S - 1), \]

(39)

and expresses nicely the fact that the non-Gaussianity is indeed of the order of the slow-roll parameter (it is also important to recall that the validity of this formula is in fact broader, see for instance [110]).

Having briefly reviewed the standard situation, let us now turn to the evaluation of the bispectrum in a situation where \( \beta_3 \neq 0 \). In principle, one could just restart from equation (28). However, it was noticed in [93] that the first three terms in this equation (that is the say the three dominant terms, second order in the slow-roll parameters) can be reduced to

\[ \mathcal{S}_{\text{l}}[\xi] = M_3^3 \int d^3 x \; 4 a^2 H \epsilon_1^2 \xi^2 \partial^2 \xi', \]

(40)

up to higher order terms in the slow-roll parameters and terms involving \( \delta L_2 / \delta \xi \). This can be achieved by performing various integrations by part. The calculation is then reduced to the calculation of a single vertex. This means that the expression (35) is still valid but that the three vertices \( \mathcal{G}_1, \mathcal{G}_2 \) and \( \mathcal{G}_3 \) are now replaced by a single vertex \( \mathcal{G}_{123} \) given by

\[ \mathcal{G}_{123} = 8 i \left( \sum_{i=1}^{3} \frac{1}{k_i^2} \right) \int d^3 \kappa \; a^2 H \epsilon_1^2 f_{k_1}^* f_{k_2}^* f_{k_3}^*. \]

(41)

In addition, since we have seen that \( \mathcal{G}_4, \mathcal{G}_5 \) and \( \mathcal{G}_6 \) are in fact sub-dominant in the slow-roll approximation, we only have to calculate the above vertex. Using the expressions (25) and (26) for the mode function and its derivative, one arrives at
\[ \left\langle \zeta_{k_1}(\eta_1) \zeta_{k_2}(\eta_2) \zeta_{k_3}(\eta_3) \right\rangle = (2\pi)^{-3/2} \delta(k_1 + k_2 + k_3) \frac{(-1)^H}{8M_p^4} \frac{k_1^2 k_2^2 k_3^2}{\prod_{i=1}^{3} k_i^3} \left( \sum_{i=1}^{3} \frac{1}{k_i^2} \right) \times \left[ (\alpha_{k_1} - \beta_{k_1})(\alpha_{k_2} - \beta_{k_2})(\alpha_{k_3} - \beta_{k_3}) \int_{\eta_{in}}^{\eta_{out}} \frac{d\tau}{\eta_{in}} \left( \frac{\alpha_{k_1} e^{i\eta_{in} \tau}}{\alpha_{k_1}} - \frac{\beta_{k_1} e^{-i\eta_{in} \tau}}{\beta_{k_1}} \right) \right] \]

We can check that, if \( \beta_k = 0 \) in the above equation, together with the vertex \( \mathcal{G}_7 \), one recovers the consistency relation (39). If \( \beta_k \neq 0 \) however, interesting new effects appears. For instance, the bi-spectrum contains the following term

\[ \left\langle \zeta_{k_1}(\eta_1) \zeta_{k_2}(\eta_2) \zeta_{k_3}(\eta_3) \right\rangle \supset \alpha_{k_1}^* \alpha_{k_2}^* \alpha_{k_3}^* \int_{\eta_{in}}^{\eta_{out}} \frac{d\tau}{\eta_{in}} e^{i(k_1 - k_2 + k_3)} \]

We see that, in the squeezed limit \( k_1 = k_2 \gg k_3 \), there is an enhancement of the signal by a factor \( k_1/k_3 \gg 1 \) [100]. One can therefore hope to constrain excited states (and hence trans-Planckian physics) with the help of the future non-Gaussianity measurements.

In fact, it is straightforward to calculate the full bi-spectrum for the generic parameterization given by equations (14) and (15). One obtains

\[ \left\langle \zeta_{k_1}(\eta_1) \zeta_{k_2}(\eta_2) \zeta_{k_3}(\eta_3) \right\rangle = (2\pi)^{-3/2} \delta(k_1 + k_2 + k_3) \frac{H^4}{8M_p^4} \frac{k_1^2 k_2^2 k_3^2}{\prod_{i=1}^{3} k_i^3} \left( \sum_{i=1}^{3} \frac{1}{k_i^2} \right) \times \frac{2}{k_T} \left\{ 1 - \cos(k_T \eta_{in}) - 3|x| \frac{H}{M_C} \cos \phi + 3|x| \frac{H}{M_C} \cos(k_T \eta_{in} + \phi) \right. \]

\[ \left. -3|x| \frac{H}{M_C} \frac{k_T}{k_1 + k_2 - k_3} \cos \phi + 3|x| \frac{H}{M_C} \frac{k_T}{k_1 + k_2 - k_3} \cos \phi \right. \]

\[ \left. \times \cos([k_1 + k_2 - k_3] \eta_{in} - \phi) - 3|x| \frac{H}{M_C} \frac{k_T}{k_1 + k_2 - k_3} \cos \phi \right. \]

\[ \left. +3|x| \frac{H}{M_C} \frac{k_T}{k_1 + k_2 + k_3} \cos([k_1 - k_2 + k_3] \eta_{in} - \phi) \right. \]

\[ \left. -3|x| \frac{H}{M_C} \frac{k_T}{k_1 + k_2 + k_3} \cos \phi + 3|x| \frac{H}{M_C} \frac{k_T}{k_1 + k_2 + k_3} \cos \phi \right. \]

\[ \left. \times \cos((-k_1 + k_2 + k_3) \eta_{in} - \phi) \right\}. \]

where we have defined \( k_T \equiv k_1 + k_2 + k_3 \). This expression is valid at leading order in \( \beta_k \). When \( |x| \to 0 \), one recovers the usual result. Let us notice that, in this case \( \eta_{in} \) should be rotated to the complex plane and sent to infinity. With this usual procedure which singles out the vacuum state of the theory, the term \( \cos(k_T \eta_{in}) \) also vanishes. The above expression is quite complicated but we see that the bi-spectrum is enhanced when \( k_i \equiv \sum_{j \neq i}^3 k_j - 2\delta \), vanishes [98]. In order to have a clearer view of the result, it is interesting to consider the squeezed limit again. In the limit \( k_1 = k_2 \gg k_3 \), the above equation takes the form

\[ \left\langle \zeta_{k_1}(\eta_1) \zeta_{k_2}(\eta_2) \zeta_{k_3}(\eta_3) \right\rangle^{\text{sl}} = (2\pi)^{-3/2} \delta(k_1 + k_2 + k_3) \frac{H^4}{8M_p^4} \frac{k^3}{\prod_{i=1}^{3} k_i^3} \times \left[ \epsilon_1 + 6\epsilon_1 |x| \frac{H}{M_C} \frac{k}{k_3} \cos(k_3 \eta_{in} - \phi) \right] \]
This simple expression captures the main features of the result. We see that the corrections are proportional to $|x|H/M_C$. We also see the enhancement in the squeezed limit, proportional to $k/k_3 \gg 1$. This brings us to the question of the initial time $\eta_{ini}$. We have seen that, in the minimal approach, the modes are created at different times, when their wavelength equals the new fundamental scale $M_C^{-1}$, see equation (11). In an inflationary background, this implies that $\eta_{ini} = \eta_k \approx M_C/(kH)$, that is to say the initial time becomes scale dependent. However, when one computes the various vertices that contribute to the bi-spectrum, one has to integrate the product of three mode functions with different wavenumbers $k_1, k_2$ and $k_3$ from $\eta_{ini}$ to zero, see the previous expression for $G_{123}$ for instance. Then comes the question of which wavenumber should be considered in the expression of $\eta_{ini} = \eta_k \approx M_C/(kH)$? Let us notice, however, that if the scaling $\eta_{ini} \propto M_C/(kH)$ is correct (whatever the precise definition of $k$ is), then the frequency of the oscillatory term in the bi-spectrum becomes $\propto M_C/H$ in a way which is very similar to the super-imposed oscillations found in the power spectrum, see equation (16).

Finally, let us remark that once the bi-spectrum has been determined in momentum space, one still needs to perform the 2D projection in order to obtain the $f_{NL}$ seen in the sky. This is a highly non trivial procedure [98, 100]. According to these references this could lead to an observable $f_{NL}$ which, therefore, could constrain trans-Planckian physics.

5. Broader challenges

5.1. UV and IR cutoff issues

Let us now return to some more general issues related to trans-Planckian physics in cosmology. The trans-Planckian problem for cosmological perturbations is related to basic challenges to the applicability of effective field theory in an expanding background.

In free quantum field theory, the starting point is canonical quantization of the fields. In an expanding background (see [33] for an overview of quantum field theory in curved space-times), the basic modes which are quantized are the plane wave modes which have constant wavelength in comoving coordinates and whose physical wavelength is hence increasing. Even in Minkowski space-time, quantum field theory requires regularization and renormalization in order to obtain finite answers for observables. In the context of gravity, pure quantum field theory must break down on wavelengths smaller than the Planck scale since waves with such large wavelengths would trigger the collapse of the background space-time into a gas of black holes. This problem is intimately related to the cosmological constant problem which arises when considering quantum field theory in gravity. Hence, in the presence of gravity, the UV cutoff is not just a computational tool, but it as an issue with definite physical significance.

The problem in an expanding background is that the UV cutoff is expected to correspond to a fixed scale in physical coordinates, whereas the basic modes of a quantum field have constant comoving wavelength. Thus, as already discussed in [64], a fixed physical UV cutoff implies that the Hilbert space of the quantum field must be time-dependent. As the universe expands, more and more new modes are required to describe the physics.

The above UV problem arises in any cosmological space-time. What is special to inflationary cosmology (as opposed to the alternatives which we have mentioned in section 2) is that these new modes are inflated to a wavelength which at the current time is observable.

For massless fields, there are also IR divergences. Since gravitational waves are massless, potential IR divergences must be addressed in cosmology.

---

8 Similar issues arise in applications to black hole backgrounds and to non-gravitational external field background.
In cosmology, there is a natural scale which separates UV and IR modes—the Hubble scale $H^{-1}(t)$, where $H(t)$ is the expansion rate of space at time $t$. On sub-Hubble scales matter fluctuations oscillate as they do in flat space-time. However, on super-Hubble scales the oscillations are frozen out and plane wave fluctuation modes become standing waves whose amplitude evolves in time as determined by the gravitational background. Furthermore, a local observer (local in space and time) has a range of view limited by the Hubble radius. Hence, the Hubble radius determines the separation between UV and IR modes.

The increasing physical wavelength of the basic modes of a quantum field also leads to an IR problem: in the case of an accelerating universe such as in inflationary cosmology, the phase space of IR modes increases. Even in the alternatives to inflation which were mentioned in section 2 this problem arises. In fact, the increase of the phase space of IR modes is a necessary condition to have a causal theory of structure formation since we need to make sure that scales which are being observed in cosmology today (and which were in the IR sea, i.e. had a wavelength greater than the Hubble radius for most of the late time universe) start out with a wavelength which is sub-Hubble. In inflationary cosmology, the phase space of IR modes increases during the inflationary phase, in string gas cosmology it grows at the end of the Hagedorn phase, and in a bouncing cosmology it grows during the contracting phase.

Infrared divergences can be ‘cured’ by imposing an IR cutoff. Such an IR cutoff can be well justified on physical grounds: it corresponds to setting to zero the contribution of modes which are super-Hubble at the initial time, and whose values would depend on dynamics before that time. In an expanding universe, this cutoff must be fixed in comoving coordinates, in contrast to the UV cutoff which is fixed in physical coordinates (for a discussion of this point see e.g. [111]).

Since the phase space of IR modes is increasing relative to the phase space of UV modes, the magnitude of the IR effects will be time-dependent. A possible consequence of this will be discussed in the following subsection.

As mentioned above, the growth of the sea of IR modes occurs in any causal theory of structure formation. What is special in inflationary cosmology is that the sea becomes populated by modes which initially were on trans-Planckian scales. Thus, in inflationary cosmology the trans-Planckian problem for cosmological fluctuations and the IR issues become coupled, which they are not in string gas cosmology or in a matter bounce scenario.

The effects of IR modes on the spectrum of cosmological perturbations in inflationary cosmology has been studied recently in a large number of works [112]. A first point to consider is that the effects of IR modes is different in an inflationary universe (a cosmology with a finite duration de Sitter phase) compared to what happens in exact de Sitter space. As recently studied in [113], the functional form of the IR terms is different in the two cases, the extra terms arising in an inflationary universe being multiplied by ‘slow-roll parameters’ of inflationary dynamics.

As first pointed out in [114] and later discussed in depth in [115, 116], it is crucial to measure the effects in terms of a physical clock as opposed to in terms of a background quantity which itself obtains corrections from the fluctuations. In the case of purely adiabatic fluctuations, it turns out that the effects of IR modes are not physically measurable [117, 118]. On the other hand, there are effects of IR modes of entropy fluctuations which are physically measurable [113].

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9 Obviously, the expansion of space leads to a damping of these oscillations.
5.2. IR effects

In the previous subsection we have discussed IR effects on cosmological fluctuations. What about IR effects on the background? From the point of view of cosmological perturbations, in the same way that a mode with wavenumber \( k' \) can combine with a mode with wavenumber \( k - k' \) to cause a second order correction to the mode with wavenumber \( k \), modes with wavenumbers \( k \) and \(-k\) lead to a second order correction to the cosmological background. We call this a ‘back-reaction’ effect.

In a long-standing research program, Tsamis and Woodard have been studying the back-reaction of IR gravitational waves on the cosmological background [119]. They find a secular instability of the de Sitter background, on the basis of which they proposed a quantum gravitational model of inflation [120].

In [121], the back-reaction effect of scalar metric fluctuations in the de Sitter phase of an inflationary cosmology was investigated (see [122] for a review of this method). The starting point was the following ansatz for metric and matter:

\[
g_{\mu\nu}(x, t) = g_{\mu\nu}^0(t) + g_{\mu\nu}^{(1)}(x, t),
\]

\[
\varphi(x, t) = \varphi^0(t) + \varphi^{(1)}(x, t),
\]

(46)

where \( g_{\mu\nu}^0(t) \) and \( \varphi^0(t) \) form the cosmological background and \( g_{\mu\nu}^{(1)}(x, t) \) and \( \varphi^{(1)}(x, t) \) are the linear cosmological perturbations (the linearization parameter is the amplitude of the cosmological perturbations).

This ansatz does not satisfy the Einstein equations beyond linear order. There are quadratic corrections to both the background and the fluctuations. Here we are interested in the corrections to the background (see [123] for an analysis of the back-reaction on the fluctuations using the same formalism). We can absorb the quadratic corrections to the background metric into a new metric \( g_{0,br}^{\alpha\beta}(t) \) which equals the original background metric plus quadratic corrections which take into account that (46) does not solve the Einstein equations to quadratic order.

To obtain the equations of motion for \( g_{0,br}^{\alpha\beta}(t) \) we insert (46) into the full Einstein equations and expand to quadratic order. To extract the equation of motion for the modified background metric we take the spatial average of the resulting equations. There are terms quadratic in the linear metric fluctuations which we move to the right-hand side of the equations of motion to define an effective energy–momentum tensor \( \tau_{\mu\nu} \) for cosmological perturbations (see also [124, 125] where this method was first developed, albeit in a different context). The equation of motion for \( g_{0,br}^{\alpha\beta}(t) \) which includes the quadratic corrections of first order fluctuations becomes

\[
G_{\mu\nu}[g_{0,br}^{\alpha\beta}] = 8\pi G[T_{\mu\nu}^{(0)} + \tau_{\mu\nu}],
\]

(47)

where \( G_{\mu\nu} \) denotes the Einstein tensor, \( T_{\mu\nu}^{(0)} \) is the energy–momentum tensor of matter, and \( G \) stands for Newton’s gravitational constant.

In terms of the linear metric and matter fluctuations, the effective energy–momentum tensor takes the form

\[
\tau_{\mu\nu} = \left(T_{\mu\nu}^{(2)} - \frac{1}{8\pi G}G^{(2)}_{\mu\nu}\right),
\]

(48)

where the first term contains the terms quadratic in the linear matter fluctuations, and the second term those quadratic in the linear metric perturbations. There are also products of metric and matter fluctuations which appear in the first term (see [121] for the full expressions).

The effective energy–momentum tensor \( \tau_{\mu\nu} \) obtains contributions from perturbations of all wavelengths. The contributions of all Fourier modes add up linearly. We can thus define an UV and an IR part which consist, respectively, of the contributions of sub-Hubble and super-Hubble modes. In the de Sitter limit of inflation, the UV part is time-independent (this follows
both by symmetry and by explicit computation). On the other hand, due to the increasing phase space of IR modes, the IR part grows in time and hence dominates in the late time limit.

We now sketch the evaluation of the IR part of $\tau_{\mu\nu}$. To be specific, we assume that matter takes the form of a scalar field $\phi$. We work in longitudinal gauge in which the metric and matter including fluctuations take the form:

$$\text{d}s^2 = a^2[(1 + 2\Phi)\text{d}\eta^2 - (1 - 2\Phi)\text{d}x^2], \quad \phi = \phi_0 + \delta\phi.$$  

Note that the metric and matter fluctuations $\Phi$ and $\delta\phi$, respectively, are related by the Einstein constraint equations

$$\delta\phi = -\frac{2V}{V'}\Phi,$$  

where $V(\phi)$ is the potential energy function of $\phi$. To obtain the leading terms contributing to the IR part of $\tau_{\mu\nu}$ we can neglect spatial gradient terms and work to leading order in the slow-roll approximation for $\phi$. With these approximations the IR part of $\tau_{\mu\nu}$ takes the form of a cosmological constant with effective energy density

$$\rho^{(\text{br})} = \rho^{(0)} \simeq \left(\frac{2V^2}{V'^2} - 4V\right)\langle\Phi^2\rangle.$$  

For simple inflationary models [e.g. $V(\phi) = \frac{1}{2}m^2\phi^2$]

$$\rho^{(\text{br})} < 0$$  

and hence the back-reaction of IR modes takes the form of a negative contribution to the cosmological constant whose magnitude grows in time since the phase space of IR modes is increasing.

We may conjecture that the back-reaction of IR modes thus leads to a dynamical relaxation mechanism for a bare positive cosmological constant $\Lambda$ [122] (for a recent review on the cosmological constant problem, see [126]). We begin with a large bare $\Lambda$ (in the presence of matter). The presence of $\Lambda$ will lead to a period of inflation. This, in turn, will cause a sea of IR modes to build up. Then, back-reaction will set in. The effect of IR modes will lead to an effective cosmological constant which determines the full background metric which takes the form

$$\Lambda_{\text{eff}}(t) = \Lambda - |\rho^{(\text{br})}(t)|,$$  

and shows the onset of an instability.

Naive extrapolation of the back-reaction effect to large times would lead to $\Lambda_{\text{eff}}(t) = 0$ at some time $t = t_{BR}$. However, long before this happens the perturbative approach will break down. Assuming that the instability which is seen to leading order survives to a non-perturbative treatment, an interesting scenario emerges [122]: the back-reaction contribution ceases to increase in magnitude once $\Omega_{\Lambda}(t) < \Omega_m(t)$ (where $\Omega_X$ is the contribution of the energy density of the substance $X$ to the critical density), since at that time the acceleration of space ceases and the IR sea ceases to grow. The universe, however, is still expanding, and hence the matter energy density decreases. Thus, the effective cosmological constant once again raises its head. The upshot of this dynamics is that there is a dynamical fixed point with

$$\Omega_{\Lambda}(t) \sim \frac{1}{2}.$$  

(53)

More precisely, oscillations of $\Omega_{\Lambda}(t)$ about that value are expected. Thus, the back-reaction of long wavelength cosmological perturbations has the potential of providing a dynamical relaxation mechanism for a large bare cosmological constant which explains dark energy and has no coincidence problem.
Note that the above mechanism is not in conflict with causality since only modes are considered which are inside the horizon\(^{10}\). The effect can be locally described in terms of a time-dependent change in the spatial curvature constant \(k\) and \(\Lambda\) [127].

The key concern is that the above calculations have been performed as a function of a background time \(t\). As conjectured in [114] and verified in [115, 116], in the case of purely adiabatic fluctuations (the energy densities of all components of matter being proportional) the leading IR back-reaction effect discussed above can be entirely absorbed by a second order time-reparametrization. In other words, if we calculate observables such as the local Hubble expansion rate as a function of a physical clock variable as opposed to the background time \(t\), there is no leading order effect of the IR fluctuations. For adiabatic fluctuations, the only physical clock is the matter field \(\phi\), and we find [115]

\[
H^2(\phi, \Phi) = H^2(\phi, 0). \quad (54)
\]

However, in the presence of multi-field systems with entropy fluctuations, it can be shown that the leading IR back-reaction effects are physically measurable [128] (see also [129]). If \(\chi\) is an entropy field (e.g. a field which represents the temperature of the CMB), then

\[
H^2(\chi, \Phi) \neq H^2(\chi, 0). \quad (55)
\]

Thus, there appears to be evidence for an instability of the de Sitter phase of inflationary cosmology. What about de Sitter space itself? We will end with a very brief literature survey of work on this topic.

### 5.3. Stability of de Sitter

De Sitter space-time is a classical solution of Einstein’s field equations in the presence of a positive cosmological constant. According to the classical ‘no-hair’ theorems [130, 131] the expanding branch is classically stable (the contracting branch is classically unstable because of the growth of fluctuations in such a phase).

As discussed in the previous subsection, there is perturbative evidence that in the presence of matter and entropy fluctuations even the expanding branch of de Sitter space is semi-classically unstable. This question has been studied for a long time. Based on studies of the renormalization of the energy–momentum tensor of a massless scalar field in de Sitter space there have been claims of instability [132, 133]. There are also claims of instability due to particle production [134, 135], due to a thermodynamic instability [136] and due to a conformal anomaly [137]. The work of [119] mentioned in the previous subsection supports the claim of semi-classical instability.

What about non-perturbative statements? In the context of four space-time dimensional gravity there have been long-standing claims that de Sitter space is unstable [138] (see also [139]). However, these claims have been disputed in [140].

There has been some interesting recent work on the stability issues of de Sitter space. In the context of three space-time dimensional pure gravity it has been shown that the method used to construct the partition function of quantum gravity which works in anti-de-Sitter space breaks down in the presence of a positive cosmological constant [141]. In the context of the higher spin theory limit of string theory is has also been shown that de Sitter space does not seem to be a solution at the quantum level [142].

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\(^{10}\) Recall that in inflationary cosmology the horizon is larger than the Hubble radius by a factor which exponentially increases during inflation.
6. Conclusions

We have seen that the exponential expansion of space during the expanding branch of de Sitter space-time, and consequently also during an inflationary phase of an expanding FRWL universe, leads to important trans-Planckian issues. In particular, fluctuation modes which are being probed in current cosmological observations originate with wavelengths smaller than the Planck length at the onset of the phase of exponential expansion. Since the physics on trans-Planckian scales is unknown, this raises conceptual questions regarding to the robustness of the usual calculations in inflationary cosmology.

One aspect of this problem, namely the fact that in an expanding space comoving modes continuously cross the boundary of the trans-Planckian zone of ignorance, is common to all expanding cosmologies. What is special to inflationary cosmology is that scales which are currently explored in cosmological observations emerge from this trans-Planckian sea. This is not the case in the two other cosmological paradigms mentioned earlier, namely the matter bounce scenario and string gas cosmology.

These issues have been discussed in section 2 of this review. Consequences for cosmological observations have been discussed in sections 3 and 4. Possibly the most important point is that due to the accelerated increase in the wavelength of scales, trans-Planckian physics can in fact be tested with current cosmological observations. We have seen that under the assumption that at some initial time all modes are in their local vacuum state, then oscillations in the spectrum of CMB anisotropies are induced. The strongest constraint on the amplitude of these oscillations comes from back-reaction considerations, namely from demanding that the produced particles do not destabilize the inflationary background. Such oscillations can be searched for in current CMB anisotropy data. With current results, a vanilla inflationary model without oscillations is still an excellent fit to the data.

The accelerated expansion of space during an inflationary period leads to more conceptual problems. The increase of the wavelength of fixed comoving modes relative to the Hubble radius leads to a rapid increase of the phase space of such infrared (IR) modes. This may—in the presence of entropy fluctuations—lead to large IR effects, possibly even to a de-stabilization of the de Sitter phase.

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