Enhancing Safe Exploration Using Safety State Augmentation

Aivar Sootla  
Huawei R&D UK  
aivar.sootla@huawei.com

Alexander I. Cowen-Rivers  
Huawei R&D UK,  
Technische Universität Darmstadt  
alexander.cowen.rivers@huawei.com

Jun Wang  
University College London  
jun.wang@cs.ucl.ac.uk

Haitham Bou Ammar  
Huawei R&D UK  
University College London  
haitham.ammar@huawei.com

Abstract

Safe exploration is a challenging and important problem in model-free reinforcement learning (RL). Often the safety cost is sparse and unknown, which unavoidably leads to constraint violations — a phenomenon ideally to be avoided in safety-critical applications. We tackle this problem by augmenting the state-space with a safety state, which is nonnegative if and only if the constraint is satisfied. The value of this state also serves as a distance toward constraint violation, while its initial value indicates the available safety budget. This idea allows us to derive policies for scheduling the safety budget during training. We call our approach Simmer (Safe policy IMproveMEnt for RL) to reflect the careful nature of these schedules. We apply this idea to two safe RL problems: RL with constraints imposed on an average cost, and RL with constraints imposed on a cost with probability one. Our experiments suggest that simmering a safe algorithm can improve safety during training for both settings. We further show that Simmer can stabilize training and improve the performance of safe RL with average constraints.

1 Introduction

Reinforcement learning (RL) is a framework for sequential decision-making that makes minimal prior assumptions about the environment where the agent has to act or make the decisions [41]. The policy for taking the actions is learned through interactions with the environment over time. RL has seen recent successes in playing video games with a computer [28], board games with a human [37] and is on a path toward real-life applications such as video compression [42], and plasma control [19]. There are still, however, some unsolved challenges for a successful deployment of RL such as efficient learning of constrained or safe Markov Decision Processes (MDPs) [4]. The constraints are typically modeled by a discounted sum of nonnegative costs that have to be smaller than some pre-defined value called the safety budget.

Exploration is a crucial component of RL and is still an active area of research [23, 36, 30]. In the context of safe RL, while exploring we do not want to incur the safety cost and constraint violations making exploration a harder task. We can distinguish two main research directions for minimizing constraint violation in safe RL: a model-based approach (which includes partially and fully known environments) and a model-free approach. In the model-free case, which we focus on, the agent would almost certainly visit unsafe regions to learn the safe policy, and therefore it is next to impossible to avoid constraint violations completely. The general goal, in this case, is to minimize the number of violations
during training. For example, use a conservative safety critic and rejection sampling to choose a “safer” action, design a curriculum learning approach, where the teacher resets the student violating safety constraints, and learns to reset the policy if the safety constraint is violated.

In this work, we aim to enhance model-free safe reinforcement learning by augmenting the state-space with one state encapsulating the safety information. The safety state is initialized with the safety budget, the incurred costs are then subtracted over time thus tracking the remaining safety budget. The safety state is nonnegative if and only if the constraint is satisfied and thus serves as a measure of distance to the unsafe region. The initial value of the safety state is the total available safety budget. A similar approach was considered in, which showed many benefits of state augmentation for reinforcement learning with probability one constraints. In this paper, we take a closer look at the purposes of the safety state. First, we claim that safety state augmentation is often crucial for the averaged constrained problem as well and provide examples of such occurrences. In some particular problems, the use of safety state augmentation may potentially be avoided, however, this can be said about any state in the environment. Second, our main suggestion is that scheduling the safety budget at different training epochs can improve the algorithm’s performance. Our claim extends equally over averaged and almost surely constrained reinforcement learning problems. In particular, we claim that scheduling the initial safety budget can lead to reducing safety constraint violations during training for safe RL with probability one constraints. We design two algorithms automatically tuning the safety budget, one is based on a classical control engineering PI controller, while the other uses Q learning to decide on the safety budget.

Related work. Many of the current safe RL methods are extensions of the most successful RL algorithms: Trust region policy optimization (TRPO), proximal policy optimization (PPO), soft actor-critic (SAC), etc. Safe versions of TRPO, PPO, and SAC with a Lagrangian approach were first presented in, which is still considered to be one of the major baselines. A direct extension of TRPO by adding constraints to the trust region update was proposed in. Model-based approaches were also considered cf. and most of them took a Bayesian approach to model one-step transitions. To our best knowledge, the most successful approaches for safe reinforcement learning to date are PID-Lagrangian and LAMBDA. The former views the Lagrangian multiplier update as another control problem and employs a PID controller to it (cf. ). Specifically, the authors link the multiplier update to integral control and add proportional and derivative controllers to achieve a superior behavior. On the other hand, is a model-based approach that uses Bayesian world models to enhance safety. A recent work formulated safe RL as inference resulting in a sample efficient off-policy approach. Some approaches took a slightly different approach to formulate safety, e.g., proposed a two-player framework with the cooperating task agent and safety agents, proposed a safety layer that would be applied after the action is computed using a classical policy. Other formulations of safe reinforcement learning were considered in the literature. For example, proposed to use conditional-value-at risk (CVaR) constraints, while proposed to enforce constraints with probability one. Further, as we discussed above eliminating the number of constraint violations typically requires strong assumptions, e.g., finite state space, knowledge of a partial model or initial safe policy. These results are in the spirit of safe RL with control-theoretic notions, which make significant prior assumptions to guarantee safety. Finally, we note that the closest algorithm in the literature to our method is the curriculum learning approach to Safe RL from. The major difference with our work is its online nature, while pre-train teacher’s policies deciding the curriculum, we do not pre-train our safety budget schedules. Our approach is preferable when a new task needs to be learned with minimal prior information about it, while the method from is preferable when the policy of choosing the constraints can be transferred from another task. We further discuss in Appendix.

2 Simmer: Safe policy improvement for reinforcement learning

2.1 MDP with safety state augmentation

We consider a constrained reinforcement learning setting defined for a Markov Decision Process (MDP) \( M = (\mathcal{S}, \mathcal{A}, \mathcal{P}, r, \gamma) \) with transition probability \( \mathcal{P} : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to [0, 1] \) acting on state \( \mathcal{S} \) and \( \mathcal{A} \) spaces, with the reward \( r : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to \mathbb{R}_+ \) (\( \mathbb{R}_+ \) stands for the nonnegative orthant \([0, +\infty)\)).
and the discount factor $\gamma_c \in (0, 1]$. The MDP is endowed with the following optimization problem

$$
\max_{\pi_t(s, a)} \mathbb{E} \sum_{t=0}^{T-1} \gamma_t r(s_t, a_t, s_{t+1}), \text{ subject to: } g \left( \sum_{t=0}^{T-1} \gamma_t l(s_t, a_t, s_{t+1}) \right) \leq d,
$$

(1)

with the time horizon $T > 0$, the safety discount factor $\gamma_l \in (0, 1]$, the safety cost $l : S \times A \times S \rightarrow \mathbb{R}^n$, and the inequality should be understood as the element-wise comparison. The statistic $g(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is a design choice and there exits many different possibilities (e.g., CVaR, chance constraints etc [14]). In this paper, we consider two most relevant options, from our point of view: a) a constraint with probability one, i.e., $g_{po}(z) = \mathbb{P}(z \geq 0) - 1$, b) a constraint on average $g_{av}(z) = \mathbb{E}z$.

Similarly to [39], we augment the safety state information into the state-space by introducing the state $z_t = \gamma^{-1} l(s_t - \sum_{k=0}^{t-1} \gamma_k l(s_k, a_k, s_{k+1}))$, which has the following update $z_{t+1} = (z_t - l(s_t, a_t, s_{t+1}))/\gamma_l$, with $z_0 = d$. Noting that the update is Markovian this state can be easily augmented into the MDP. The variable $z_t$ has the interpretation of the remaining safety budget, and by definition enforcing the constraint on $z_T$ is equivalent to enforcing the constraint on the accumulated cost. Now we can rewrite the safe RL problem as follows:

$$
\max_{\pi_t(s, a, z)} \mathbb{E} \sum_{t=0}^{T-1} \gamma_t r(s_t, a_t, s_{t+1}), \text{ subject to: } g(z_T) \geq 0.
$$

(2)

While [39] considered only the case $g_{po}(z)$, we argue that the case of $g_{av}(z)$ deserves additional attention in the context of safety state augmentation. For completeness, we review the main points of the approach [39] in Appendix. Note that the policy now also depends on the safety state $z$ and this feature deserves a more thorough discussion.

2.2 Do we need the safety state?

It was demonstrated in [39] that having the policy depending on the remaining safety budget is crucial for safe reinforcement learning with probability one constraints. We argue that in many instances it is (at least) beneficial to have this information in the average constrained case as well. As a simple demonstration consider the cartoon in Figure [1]. A robot needs to reach the goal while crossing the hazards region, which is marked by the red circle, and the safety cost is acquired for every time unit spent in the region. Both green and blue paths are safe, i.e., satisfy the constraint. However, at the crossing of these paths robot needs to know which path it took to this state. Switching from the green path to the blue one will lead to a constraint violation. Standard safe RL algorithms will have trouble with such scenarios while adding the safety state solves the issue. We confirm this observation in our experiments.

Figure 1: A robot needs to reach the goal while crossing the hazards region (marked by the red circle) and the safety cost is acquired for every time unit spent in the region. Both green and blue paths are safe, i.e., satisfy the constraint. However, at the crossing of these paths robot needs to know which path it took to this state. Switching from the green path to the blue one will lead to constraint violations.

Let us now discuss how this logic can be mathematically formalized in the case with $g(z) = \mathbb{E}z$. This problem is well studied in the optimal control literature in the context of problems with known dynamics and with terminal or end-point constraints. In the deterministic case, this problem reduces to the standard maximum principle with end-point constraints [45]. The problem was also studied in [46].

1 Parts of the illustration are designed by gstudioimagen, macrovector official, Flaticon and downloaded from http://www.freepik.com
The intuition behind our formulation is quite straightforward. Assume we start with a very strict value function. In both cases, it was shown that the optimal policy depends on the whole state, which in our case includes the safety state $z$. While in some cases one may not need to augment the safety state, in many situations it is critical.

### 2.3 Simmering Safe Reinforcement Learning

In this paper, we propose to use the initial safety budget $d$ as another tuning dial for the algorithms. Specifically, we will exploit the link between the safety budget $d$ with the initial state of the safety state. We argue that adjustment of $d$ during training from some initial value $d^\text{start}$ to the target value $d^\text{target}$ can lead to improved exploration in terms of safety and performance.

Let us now present the mathematical formulation. At every epoch $k$ we pick a test safety budget $d_k$, collect the data set $D_k$, compute the returns $E_{D_k} \sum_{t=0}^{T-1} \gamma_t r(s_t, a_t)$ and the costs $\hat{g}_{D_k}(z_T)$, where the function $\hat{g}_{D_k}(\cdot)$ is the empirical mean or the maximum for the averaged and the probability one constrained problems, respectively. Using this information we aim at solving the following problem at every epoch (but we only apply a pre-determined number of gradient steps):

$$\max_{\pi(s,a,z)} E_{D_k} \sum_{t=0}^{T-1} \gamma_t r(s_t, a_t), \quad \text{subject to:} \quad \hat{g}_{D_k}(z_T) \geq 0, \quad z_0 = d_k$$

(3)

where $s_0 \in S_0$ and $z_0 = d_k$ are the initial states of the Saute MDP. Note that for off-policy algorithms the data set $D_k$ can potentially grow with epochs, while for on-policy algorithms the data set $D_k$ will be emptied on every epoch.

The key steps of our approach are as follows:

- We assume that there is a finite number of test safety budgets, i.e., we will assume that $d_k$ can take the following values $\{d^0, \ldots, d^K\}$ with $d^0 = d^\text{start}$ and $d^K = d^\text{target}$.
- We assume that the value $d^\text{start}$ is such that the task can be solved;
- At for every $k$ we perform only one epoch of optimization solving Problem 3.

Now we can find $d_k$ to achieve our goals and as we will show in the next section the problem of safety during training can be formalized as a two-level decision-making problem. However, we will show that even a naive approach of scheduling $d_k$ can lead to improvement in performance and safety.

### 2.4 Application: Simmering for safety during training

Exploration can often lead to constraint violation during training due to the inherent stochasticity of exploration. While there is a significant effort in research for safe exploration it typically requires significant prior assumptions on MDPs. We propose to embrace the philosophy of classical reinforcement learning and proceed with minimal assumptions. We propose to choose the test safety budget $d_k$ using another decision-making problem:

$$\max_{u_k} \sum_k \text{ReLU} \left(-\hat{g}_{D_k}(z_k^d(d^\text{target}))\right),$$

$$d_{k+1} = \text{clip} \left(d_k + u_k, d^\text{start}, d^\text{target}\right), \quad d_0 = d^\text{start},$$

(4)

where $z_k^d(d^\text{target})$ is the violation of the target constraint obtained as a result of solving one epoch of Problem 3, the action set is $[-\delta d, \delta d]$ and $\delta d$ is pre-determined, $\text{clip}(x, y, z)$ is clipping function defined as follows:

$$\text{clip}(x, y, z) = \begin{cases} x & \text{if } y \leq x \leq z, \\ z & \text{if } z < x, \\ y & \text{if } x < y. \end{cases}$$

(5)

The intuition behind our formulation is quite straightforward. Assume we start with a very strict safety budget $d^\text{start}$ training with which does not lead to significant violations with respect to the
target safety budget $d^{\text{target}}$. Then by gradually increasing the safety budget $d$ from $d^{\text{start}}$ to $d^{\text{target}}$, we can reduce the number of constraint violations during training. We see this formulation as the first step toward eliminating safety violations during training.

We formalized our problem as reinforcement learning over a partially observable process with the non-stationary observations of the learning process. Formulating our problem as a two-level RL problem allows us to use a broad spectrum of tools available in sequential decision-making differing our approach from the existing in the literature. However, solving Problem 4 appears to be a daunting task, especially in the online setting as we intend. Hence, we will employ heuristic solutions, which may prove more effective than the quest to find an optimal solution.

Intuitively, a gradual increase in $d_k$ by assigning scheduling increasing reference values (denoted by $d^{\text{ref}}_k$) would less likely lead to constraint violation due to stricter exploration constraints. However, fixing the schedule a priori limits the ability of the algorithm to react to constraint violations. To alleviate this issue we propose two approaches: PI Simmer (PI-controlled safety budget) and Q Simmer (Online Q-learning with non-stationary rewards).

**PI Simmer.** The idea for the PI controller is quite intuitive it takes the error term $e = d^{\text{ref}} - c$ and uses it for action computation. P stands for the proportional control and links the error terms with the action by multiplying the error term by the gain $K$. The proportional part delivers a brute force control by having a large control magnitude for large errors, but it is not effective if the instantaneous error values become small. Proportional control cannot achieve zero error $e$ tracking. To deliver zero error, integral control is typically used, which sums up previous error terms and uses this sum to determine actions instead of the error. PI controller can solve many hard control problems, but there are some implementation and engineering tricks and improvements. For example, if the action values are clipped, integral control can lead to the unwanted phenomenon called integral windup [6]. This is specifically the case when the errors become large leading to large values of the integral, which in turn leads to saturated actions for a long time limiting the ability to react to the environment. We will take the feedback control approach to anti-windup, where the previous saturation errors are fed back and used to determine the current action [6] [1]. Finally, we use a low pass filter for the error terms to avoid reacting to high-frequency fluctuations and acting only on the trends. A reader more familiar with optimization literature will recognize this as a Polyak update. Finally, we propose the following update for the safety budget:

$$\begin{align*}
\text{Error term} & : e_k = d^{\text{ref}}_k - \hat{g}(z_T(d_k)) , \\
\text{Low-pass filter} & : w_k = (1 - \tau)w_{k-1} + \tau e_k, \\
\text{P-part} & : P = K_pe_k, \\
\text{I-part} & : I = K_i \sum_{i=k-T_i}^{k} e_i, \\
\text{Anti-windup} & : AW = K_{aw}(u_{k-1} - u^{\text{raw}}_{k-1}), \\
\text{Raw signal} & : u^{\text{raw}}_k = P + I + AW, \\
\text{Clipping} & : u_k = \text{clip}(u^{\text{raw}}_k, -\delta d, \delta d), \\
\text{New safety budget} & : d_{k+1} = d_k + u_k,
\end{align*}$$

(6)

where $d^{\text{ref}}_k$ is an a priori chosen schedule of the safety budget, $e_k$ is the current constraint violation, $w_k$ is the filtered error term $e_k$, $P$, $I$, $AW$ are the proportional, integral and anti-windup parts of the controller. The gains $K_p$, $K_i$, $K_{aw}$, as well as $T_i$, $\delta d$ and $\tau$ are hyper-parameters. We provide rules-of-thumb for the choice of hyper-parameters in our ablation studies in Appendix.

**Q Simmer.** Consider an MDP with the states $\{d^{\text{ref}}_0, \ldots, d^{\text{ref}}_{K-1}\}$, which for simplicity of notation we denote $\{0, \ldots, K - 1\}$, with the actions $\{a_{-1}, a_0, a_{+1}\}$, where the action $a_{+1}$ moves the state $s = i$ to the state $s = i + 1$, $a_{-1}$ moves the state $s = i$ to the state $s = i - 1$ and the action $a_0$ does not transfer the state. Note that the action $a_{+1}$ is defined for all $i < K - 1$ and the action $a_{-1}$ is defined for all $i > 0$. Our design for the rewards of this MDP is guided by the following intuition. If the current accumulated costs are well below the safety budget $d^{\text{ref}}$, then we are very safe and the safety budget can be further increased. If the accumulated costs are around the safety budget, then we could stay at the same level or increase the safety budget. If the current accumulated costs are well above the safety budget $d^{\text{ref}}$, then the safety budget should be decreased to ensure that the policy is incentivized to be safer.
We are not safe
We are borderline safe
We are very safe

if $s - o \leq -\delta$ : 
if $|s - o| \leq \delta$ : 
if $s - o \geq \delta$ : 

$r = \begin{cases} 
2 & a = -1, \\
-1 & a = 0, \\
-1 & a = 1, 
\end{cases}$ 
$r = \begin{cases} 
-1 & a = -1, \\
1 & a = 0, \\
1 & a = 1, 
\end{cases}$ 
$r = \begin{cases} 
-1 & a = -1, \\
1 & a = 0, \\
2 & a = 1. 
\end{cases}$

where $o$ is the maximum accumulated cost (over an episode) of the safe RL algorithm. We use Q-learning update to learn the Q function:

$$Q(s_t, a_t) = (1 - l_r)Q(s_t, a_t) + l_r(r_t + \max_b Q(s_{t+1}, b))$$

and get the action with $\varepsilon$-greedy exploration strategy:

$$a_t = \begin{cases} 
\arg\max_b Q(s_t, b) & \text{with probability } \varepsilon, \\
\text{random} & \text{with probability } 1 - \varepsilon. 
\end{cases}$$

3 Experiments
3.1 Baselines, environments and code base

Environments: We use the safe pendulum environment defined in [16], we also use the custom-made safety gym environment with deterministic constraints, which we call static point goal [47]. In this environment with 46 states and two actions, a large hazard circle is placed before the goal and forces the agent to go around it to reach the goal similarly to our cartoon in Figure 2. We provide additional details in Appendix. The rest of our tests are performed on the safety gym benchmarks [33].

Baselines: Since Lagrangian PPO (L-PPO) is the base agent for [40] we take it as the base agent for all our experiments. To avoid confusion with terminology from [39], we will refer to an algorithm solving Problem 2 with almost surely constraints as PO PPO (stands for “probability one PPO”) and the Lagrangian PPO solving Problem 2 with average constraints as L-PPO w SA. We also compare to LAMBDA [5] and include their numerical values for CPO, L-PPO, and L-TRPO for the safety gym environments.

Code base: Our code is based on two repositories: safety starter agents [33], and PID Lagrangian [40]. We have implemented the safety state augmentation following the description in [39]. We use default parameters for both code bases unless stated otherwise.

Computational resources: We performed all computations on a PC equipped with 512GB of RAM, two Intel Xeon E5 CPUs, and four 16GB NVIDIA Tesla V100 GPUs.

3.2 Improving Safety During Training for Pendulum Swing-Up

Improving safety during training is more suited for almost surely safe RL and we will take PO PPO as our baseline. In this setting, we can aim to reduce the number of individual trajectories that violate the constraints, and thus we can avoid estimating the statistic $g$. For PI Simmer we chose the following hyper-parameters $K = 0.01$, $K_i = 0.005$, $K_{aw} = 0.01$ and $\tau = 0.995$. We also note that except for an initial burst of violations both our approaches manage to keep the number of violations quite low. Overall we found that low values for $K$ and $K_i$ are beneficial to avoid overreaction to constraint violations. In this case, keeping the value of $K_{aw}$ is advisable as the action saturation does not occur too often. Finally, keeping $\tau$ close to one will force the controller to react to most of the constraint violations.

For Q Simmer we chose $\delta = 1$, $\tau = 0.995$, $l_r = 0.05$, and $\varepsilon = 0.95$. It appears that a fast learning rate here can allow for learning, sufficiently fast forgetting of past rewards, but also to avoid catastrophic forgetting. As we consider a finite state MDP we can avoid using sophisticated techniques for online learning and use the simplest one — tuning learning rate. We chose $\delta$ and $\tau$ to avoid frequent state transitions.

We compare our algorithm to PO PPO in terms of the number of trajectories with constraint violations, and returns, and compare the progression of the schedule $d_{k}^{\text{test}}$. Naturally, some individual trajectories
3.3 Guiding Exploration by Scheduling Safety Constraints

We now turn our attention to safe RL with constraints imposed on average costs. We test the performance of Simmer L-PPO and L-PPO w SA on the swing-up pendulum environment and present training results in Figure 3. Here we use safety starter agents as a base learner for all algorithms.

Figure 3: Comparison of Simmer L-PPO, L-PPO with and without safety state augmentation. Mean returns and cost are computed over a hundred different trajectories obtained for three different seeds. Shaded areas represent standard deviations.

The major observations are that L-PPO w SA delivers almost no constraint violations with respect to the mean cost estimate, and therefore using Q Simmer and PI Simmer becomes redundant. In the meantime, L-PPO has even trouble converging. Note that we have used the same hyper-parameters for all algorithms, which are default parameters in safety starter agents and the learning rate 0.03. While the behavior of the L-PPO algorithm can certainly be improved with tuning, we note that simply augmenting the safety state leads to improved performance as well as stability of the algorithm. Further, we observe that Simmer L-PPO leads to a fewer number of violations, however, the rate of violations for the safety budget 35 is fairly similar to L-PPO w SA. Interestingly, a similar picture occurs with more advanced baselines such as PID Lagrangian and more complicated environments. Here, we used the static point goal environment designed in [47]. The results for PID-L and PID-L w state augmentation are depicted in Figures 4a and 4b, respectively, suggest that the presence of the safety state stabilizes training and leads to a more consistent constraint satisfaction for different safety budgets. Note that hyper-parameters for all the runs are the same for both algorithms. We further apply the naïve simmer approach to both baselines with results depicted in Figure 4c. In both cases, the safety budget takes values 1, 5, 10, 15, and 25, and increased after equal time intervals. Note that in both cases now training curves are quite...
stable, although state augmentation delivers an extra boost. In all our experiments we used the same hyper-parameters for all versions of PID-L, i.e., $K = 0.1$, $K_i = 0.01$, $\gamma_l = 0.99$.

Overall, our experiments suggest that safety during training with constraints imposed on average costs becomes a much easier problem with safety state augmentation. Indeed, even intuitively every outlier trajectory with a large constraint violation should bias the average cost and should instruct the algorithm not to follow this path. We note that in both cases above we have sparse costs, i.e., the agent encounters unsafe regions and incurs costs while navigating toward the goal. While an algorithm without the safety state will receive information on constraint violation after an episode, the safety state would constantly inform the algorithm of the distance toward a violation. This is one of the reasons why safety state augmentation can learn the task better. Note that simmering additionally offers fewer constraint violations while training. These results suggest that simmering safe RL together with state augmentation delivers overall safer solutions with stable training curves.

### 3.4 Tests on safety gym benchmarks

We finally test our approach on more challenging environments from the safety gym benchmark: PointPush1 in Figure 5a, PointButton1 in Figure 5b, PointGoal1 in Figure 5c. Again we use a naïve schedule increasing the safety budget from 10 to 25 in 5 unit increments. We observe that Simmer PID-L and PID-L outperform other baselines significantly in terms of the return while delivering safe policies. Simmer PID-L does perform very similarly to PID-L in terms of returns but generally outperforms PID-L in terms of safety costs and cost rates. We note that our results for PointGoal1 are consistent with the results from [40]. Note that simmering and state augmentation significantly reduce the variance of PID-L. To summarize, Figures 5c, 5b, and 5a demonstrate that after the initial burst in constraint violations, Simmer PID-L keeps the safety violations during training at a low level.

### 4 Conclusion

We augment the safety information into the state space and show how we can effectively use it to improve safe exploration. The safety state is nonnegative if and only if the constraint is satisfied and therefore it can serve as a distance toward constraint violations. We argue that the optimal policy must depend on this information to achieve safe performance. We validate this argument using intuitive examples, references to theoretical results, and experiments. We do not doubt that safety state augmentation is needed for effective safe RL.
Safety state augmentation and simmering show superior performance on pendulum swing-up and static point goal tasks for average constrained problems. We further tuned PID Lagrangian to show very strong performance on safety gym benchmarks, which has not been previously reported. Simmer PID Lagrangian shows competitive performance in terms of returns to PID Lagrangian and outperforms all baselines (including PID Lagrangian) in terms of the cost rate and the costs. Further, using state augmentation for the average constrained problems appears to be quite beneficial as well. Scheduling the safety budget can stabilize safe algorithms without state augmentation, but using both state augmentation and simmering improves performance and safety. We achieve this performance and safety boost at the expense of sample efficiency since we effectively learn a series of safe policies.

Simmering RL algorithms with probability one constraints can significantly reduce safety violations during training. We illustrate this feature by developing two algorithms for safe learning. The first one is based on a PI controller and allows for adjustment of a pre-defined learning schedule depending on the estimated training cost. The second one is based on online Q learning and learns to adjust the safety budget automatically. Both approaches have some advantages and limitations. PI simmering is more effective if there is a reasonable safety budget schedule, while Q simmering requires less prior knowledge to learn.

Our algorithmic development focused on probability one constraints for two reasons. First, since probability one constraints have to be imposed on all trajectories we can simply count the number of violations and do not need to resort to empirical estimates of the mean cost (as we need in the average constrained problem). Second, it appears that in the average constrained case with “simple” environments (such as pendulum swing-up and static point goal), we can track the constraint quite efficiently after the initial (and unavoidable) burst in constraint violations. Thus our experiments suggest that PI Simmer and Q Simmer are simply redundant for the average constrained RL.

Figure 5: Simmer PID-L outperforms all baselines in terms of the cost rate, which is a measure of safety during training. At the same time, Simmer PID-L never performs worse in terms of the returns or safety costs than PID-L, significantly outperforming other baselines. The curves are means and shaded areas are standard deviations computed over five runs with random seeds.
References

[1] Anti-windup control using a PID controller, 2022. https://www.mathworks.com/help/simulink/slref/anti-windup-control-using-a-pid-controller.html (p. 5).

[2] Joshua Achiam, David Held, Aviv Tamar, and Pieter Abbeel. Constrained policy optimization. In International Conference on Machine Learning, pages 22–31, 2017. (p. 2)

[3] Anayo K Akametalu, Jaime F Fisac, Jeremy H Gillula, Shahab Kaynama, Melanie N Zeilinger, and Claire J Tomlin. Reachability-based safe learning with Gaussian processes. In IEEE Conference on Decision and Control, pages 1424–1431, 2014. (p. 2)

[4] Eitan Altman. Constrained Markov decision processes, volume 7. CRC Press, 1999. (p. 1)

[5] Yarden As, Ilnura Usmanova, Sebastian Curi, and Andreas Krause. Constrained policy optimization via Bayesian world models. arXiv preprint arXiv:2201.09802, 2022. (p. 2)

[6] Karl Johan Åström. Advanced PID control, volume 461. The Instrumentation, Systems, and Automation Society, 2006. (p. 2), (p. 5), (p. 3), (p. 4)

[7] Karl Johan Åström and Richard M Murray. Feedback systems. In Feedback Systems. Princeton university press, 2010. (p. 3)

[8] Felix Berkenkamp, Riccardo Moriconi, Angela P Schoellig, and Andreas Krause. Safe learning of regions of attraction for uncertain, nonlinear systems with Gaussian processes. In 2016 IEEE 55th Conference on Decision and Control (CDC), pages 4661–4666. IEEE, 2016. (p. 2)

[9] Felix Berkenkamp, Matteo Turchetta, Angela Schoellig, and Andreas Krause. Safe model-based reinforcement learning with stability guarantees. In Advances in Neural Information Processing Systems, pages 908–918, 2017. (p. 2)

[10] Homanga Bharadhwaj, Aviral Kumar, Nicholas Rhinehart, Sergey Levine, Florian Shkurti, and Animesh Garg. Conservative safety critics for exploration. arXiv preprint arXiv:2010.14497, 2020. (p. 3)

[11] Greg Brockman, Vicki Cheung, Ludwig Pettersson, Jonas Schneider, John Schulman, Jie Tang, and Wojciech Zaremba. OpenAI gym. arXiv preprint arXiv:1606.01540, 2016. (p. 3)

[12] Agustin Castellano, Hancheng Min, Juan Bazerque, and Enrique Mallada. Reinforcement learning with almost sure constraints. arXiv preprint arXiv:2112.05198, 2021. (p. 2)

[13] Richard Cheng, Gábor Orosz, Richard M Murray, and Joel W Burdick. End-to-end safe reinforcement learning through barrier functions for safety-critical continuous control tasks. arXiv preprint arXiv:1903.08792, 2019. (p. 2)

[14] Yinlam Chow, Mohammad Ghavamzadeh, Lucas Janson, and Marco Pavone. Risk-constrained reinforcement learning with percentile risk criteria. The Journal of Machine Learning Research, 18(1):6070–6120, 2017. (p. 3)

[15] Yinlam Chow, Ofir Nachum, Edgar Duenez-Guzman, and Mohammad Ghavamzadeh. A Lyapunov-based approach to safe reinforcement learning. In Advances in Neural Information Processing Systems, pages 8092–8101, 2018. (p. 2)

[16] Alexander I Cowen-Rivers, Daniel Palenicek, Vincent Moens, Mohammed Amin Abdullah, Aivar Sootla, Jun Wang, and Haitham Bou-Ammar. SAMBA: Safe model-based & active reinforcement learning. Machine Learning, pages 1–31, 2022. (p. 2), (p. 6), (p. 5)

[17] Gal Dalal, Krishnamurthy Dvijotham, Matej Vecerik, Todd Hester, Cosmin Paduraru, and Yuval Tassa. Safe exploration in continuous action spaces. arXiv preprint arXiv:1801.08757, 2018. (p. 2)

[18] S. Dean, S. Tu, N. Matni, and B. Recht. Safely learning to control the constrained linear quadratic regulator. In American Control Conference, pages 5582–5588, 2019. (p. 2)

[19] Jonas Degrange, Federico Felici, Jonas Buchli, Michael Neuner, Brendan Tracey, Francesco Carpanese, Timo Ewalds, Roland Hafner, Abbas Abdolmaleki, Diego de Las Casas, et al. Magnetic control of tokamak plasmas through deep reinforcement learning. Nature, 602(7897):414–419, 2022. (p. 4)
[20] Benjamin Eysenbach, Shixiang Gu, Julian Ibarz, and Sergey Levine. Leave no trace: Learning to reset for safe and autonomous reinforcement learning. In International Conference on Learning Representations, 2018. (p. 2)

[21] J. F. Fisac, A. K. Akametalu, M. N. Zeilinger, S. Kaynama, J. Gillula, and C. J. Tomlin. A general safety framework for learning-based control in uncertain robotic systems. IEEE Transactions on Automatic Control, 64(7):2737–2752, 2019. (p. 2)

[22] Tuomas Haarnoja, Aurick Zhou, Pieter Abbeel, and Sergey Levine. Soft actor-critic: Off-policy maximum entropy deep reinforcement learning with a stochastic actor. In International conference on machine learning, pages 1861–1870, 2018. (p. 2)

[23] Chi Jin, Akshay Krishnamurthy, Max Simchowitz, and Tiancheng Yu. Reward-free exploration for reinforcement learning. In International Conference on Machine Learning, pages 4870–4879. PMLR, 2020. (p. 1)

[24] Sanket Kamthe and Marc Deisenroth. Data-efficient reinforcement learning with probabilistic model predictive control. In International Conference on Artificial Intelligence and Statistics, pages 1701–1710. PMLR, 2018. (p. 2)

[25] Torsten Koller, Felix Berkenkamp, Matteo Turchetta, and Andreas Krause. Learning-based model predictive control for safe exploration. In IEEE Conference on Decision and Control, pages 6059–6066, 2018. (p. 2)

[26] Zuxin Liu, Zhepeng Cen, Vladislav Izenbaev, Wei Liu, Zhiwei Steven Wu, Bo Li, and Ding Zhao. Constrained variational policy optimization for safe reinforcement learning. arXiv preprint arXiv:2201.11927, 2022. (p. 2)

[27] David Mguni, Joel Jennings, Taher Jafferjee, Aivar Sootla, Yaodong Yang, Changmin Yu, Usman Islam, Ziyan Wang, and Jun Wang. DESTA: A framework for safe reinforcement learning with markov games of intervention. arXiv preprint arXiv:2110.14468, 2021. (p. 2)

[28] Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Alex Graves, Ioannis Antonoglou, Daan Wierstra, and Martin Riedmiller. Playing atari with deep reinforcement learning. arXiv preprint arXiv:1312.5602, 2013. (p. 1)

[29] Motoya Ohnishi, Li Wang, Gennaro Notomista, and Magnus Egerstedt. Barrier-certified adaptive reinforcement learning with applications to brushbot navigation. IEEE Transactions on Robotics, 35(5):1186–1205, 2019. (p. 2)

[30] Deepak Pathak, Pulkit Agrawal, Alexei A Efros, and Trevor Darrell. Curiosity-driven exploration by self-supervised prediction. In International Conference on Machine Learning (ICML), pages 2778–2787, 2017. (p. 1)

[31] Laurent Pfeiffer. Two approaches to stochastic optimal control problems with a final-time expectation constraint. Applied Mathematics & Optimization, 77(2):377–404, 2018. (p. 4)

[32] Kyriakos Polymenakos, Nikitas Rontsis, Alessandro Abate, and Stephen Roberts. SafePILCO: A software tool for safe and data-efficient policy synthesis. In Marco Gribaudo, David N. Jansen, and Anne Remke, editors, Quantitative Evaluation of Systems, pages 18–26. Springer International Publishing, 2020. (p. 2)

[33] Alex Ray, Joshua Achiam, and Dario Amodei. Benchmarking safe exploration in deep reinforcement learning, 2019. (p. 2), (p. 6), (p. a5), (p. a6)

[34] John Schulman, Sergey Levine, Pieter Abbeel, Michael Jordan, and Philipp Moritz. Trust region policy optimization. In International Conference on Machine Learning, pages 1889–1897, 2015. (p. 2)

[35] John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. Proximal policy optimization algorithms. arXiv preprint arXiv:1707.06347, 2017. (p. 2)

[36] Philipp Schwartenbeck, Thomas FitzGerald, Ray Dolan, and Karl Friston. Exploration, novelty, surprise, and free energy minimization. Frontiers in psychology, 4:710, 2013. (p. 1)

[37] David Silver, Aja Huang, Chris J Maddison, Arthur Guez, Laurent Sifre, George Van Den Driessche, Julian Schrittwieser, Ioannis Antonoglou, Veda Panneershelvam, Marc Lanctot, et al. Mastering the game of go with deep neural networks and tree search. Nature, 529(7587):484–489, 2016. (p. 1)
[38] Thiago D Simão, Nils Jansen, and Matthijs TJ Spaan. Alwayssafe: Reinforcement learning without safety constraint violations during training. In Proceedings of the 20th International Conference on Autonomous Agents and MultiAgent Systems. International Foundation for Autonomous Agents and Multiagent Systems, 2021. (p. 2)

[39] Aivar Sootla, Alexander I. Cowen-Rivers, Taher Jafferjee, Ziyan Wang, David Mguni, Jun Wang, and Haitham Bou-Ammar. SAUTE RL: Almost surely safe reinforcement learning using state augmentation, 2022. (p. 2), (p. 3), (p. a3), (p. a6)

[40] Adam Stooke, Joshua Achiam, and Pieter Abbeel. Responsive safety in reinforcement learning by pid lagrangian methods. In International Conference on Machine Learning, pages 9133–9143. PMLR, 2020. (p. 2), (p. 5), (p. 7), (p. 8), (p. a6)

[41] Richard S Sutton and Andrew G Barto. Reinforcement learning: An introduction. MIT press, 2018. (p. 1)

[42] MuZero Applied Team. Muzero’s first step from research into the real world. In DeepMind blog. Deepmind, Feb 2022. https://www.youtube.com/watch?v=ZTnySFuJGiM. (p. 1)

[43] Matteo Turchetta, Felix Berkenkamp, and Andreas Krause. Safe exploration in finite Markov decision processes with Gaussian processes. In Advances in Neural Information Processing Systems, pages 4312–4320, 2016. (p. 2)

[44] Matteo Turchetta, Andrey Kolobov, Shital Shah, Andreas Krause, and Alekh Agarwal. Safe reinforcement learning via curriculum induction. arXiv preprint arXiv:2006.12136, 2020. (p. 2), (p. a2)

[45] Richard B Vinter. Optimal control. Springer, 2010. (p. 3)

[46] Akifumi Wachi, Yanan Sui, Yisong Yue, and Masahiro Ono. Safe exploration and optimization of constrained MDPs using Gaussian processes. In AAAI Conference on Artificial Intelligence, 2018. (p. 2)

[47] Qisong Yang, Thiago D Simão, Simon H Tindemans, and Matthijs TJ Spaan. WCSAC: Worst-case soft actor critic for safety-constrained reinforcement learning. In AAAI Conference on Artificial Intelligence., 2021. (p. 2), (p. 6), (p. 7), (p. a5)

Appendices

A1 Related work

A1.1 Safe Reinforcement Learning Using Curriculum Induction

A1.2 RL with probability one constraints

A2 Additional algorithm details

A2.1 PI-Simmer

A2.2 Q Simmer

A3 Implementation details

A4 Further Experiments

A4.1 Safety gym benchmarks

A4.2 Ablation for PI Simmer

A4.3 Ablation for Q Simmer

a1
A1 Related work

A1.1 Safe Reinforcement Learning Using Curriculum Induction

Here we review the work from [44]. Consider the Safe RL problem $\mathcal{M} = (\mathcal{S}, \mathcal{A}, \mathcal{P}, r, D)$ with the following objective:

$$\max_{\pi} \mathbb{E}_{\rho^\pi} \sum_{t=0}^{T} r(s_t, a_t, s_{t+1})$$

s.t. $\mathbb{E}_{\rho^\pi} \sum_{t=0}^{T} \mathbb{I}(s_t \in D) \leq \kappa$

where $\rho^\pi$ is the distribution of trajectories induced by $\pi$, $\mathbb{I}$ is the indicator function and $D$ is the unsafe set. Note that this safe RL problem is less general than the standard formulation of safe RL.

The authors introduce a teacher-student hierarchy. The student tries to learn a safe policy, while the teacher is guiding the student through interventions $\mathcal{I}$. The interventions $\mathcal{I}$ are represented by pairs $(\mathcal{D}_i, T_i)$ that modify the safe RL problem into a student’s problem $\mathcal{M}_i = (\mathcal{S}, \mathcal{A}, \mathcal{P}_i, r_i, \mathcal{D}_i)$, where we make the following modifications to the safe RL problem. The state transitions are modified as $\mathcal{P}_i(s'|s, a) = \mathcal{P}(s'|s, a)$ for all $s \not\in \mathcal{D}_i$ and $\mathcal{P}_i(s'|s, a) = T_i(s'|s)$ for all $s \in \mathcal{D}_i$. This means that the teacher changes the probability transition for the student if they enter the set $\mathcal{D}_i$. The reward is also modified as well: $r_i(s, a, s') = r(s, a, s')$ if $s \not\in \mathcal{D}_i$ and $r_i(s, a, s') = 0$ if $s \in \mathcal{D}_i$. Therefore, the student does not get any reward in the unsafe set. The student incorporates the interventions into their objective as follows:

$$\max_{\pi} \mathbb{E}_{\rho^\pi} \sum_{t=0}^{T} r_i(s_t, a_t, s_{t+1})$$

s.t. $\mathbb{E}_{\rho^\pi} \sum_{t=0}^{T} \mathbb{I}(s_t \in \mathcal{D}_i) \leq \kappa_i$

$$\mathbb{E}_{\rho^\pi} \sum_{t=0}^{T} \mathbb{I}(s_t \in \mathcal{D}_i) \leq \tau_i,$$

where $\kappa_i, \tau_i$ are intervention-specific tolerances set by the teacher.

In order to learn the teacher’s policy the following constraints are followed:

- The unsafe set is contained in the intervention set $\mathcal{D} \subseteq \mathcal{D}_i$
- If $\kappa_i + \tau_i \leq \kappa$, then the set of feasible policies of the student is a subset of the set of feasible policies of the safe RL problem $\Pi_{\mathcal{M}_i} \subset \Pi_{\mathcal{M}}$.

The teacher learns when to intervene and switch between different interventions. The teacher is modeled by a POMDP $(\mathcal{S}^T, \mathcal{A}^T, \mathcal{P}^T, \mathcal{R}^T, \mathcal{O}^T)$, where the state-space is the set of all student policies $\mathcal{S}^T = \bigwedge_{\mathcal{M}_i}$ (not only feasible ones), the action space is the space of all interventions $\mathcal{A}^T = \mathcal{I}$, i.e., the teacher chooses the index $i$ for the student problem, the state transitions $\mathcal{P}^T : \bigwedge_{\mathcal{I}} \times \mathcal{I} \times \bigwedge_{\mathcal{M}_i} \rightarrow [0, 1]$ is governed by the student’s algorithms, the observation space $\mathcal{O}^T = \Phi$ is the space of evaluation statistics of student policies $\pi$. Finally the reward function is defined through the policy improvement $\mathcal{R}^T(n) = \hat{V}(\pi_{n,i}) - \hat{V}(\pi_{n-1,i})$, where the index $i$ denotes the student. The total reward for an episode is $\hat{V}(\pi_{N,i}) = \sum_{n=1}^{N} \mathcal{R}^T(n)$. Now the policy is transferred from the trial $n - 1$ to next $n$. Note also the student index $i$ can be understood as an episode of learning teacher’s policy in this case.

The major difference of our work is its online nature, while [44] pre-train teacher’s policies deciding the curriculum, we do not pre-train our safety budget schedules $\mathcal{d}_k$ explicitly, but use rules-of-thumb to determine the parameters of our online learning procedures. Our approach is preferable when a new task needs to be learned with minimal prior information about it, while the method from [44] is preferable when the policy of choosing the constraints can be transferred from another task. We also note that we do not reset our environment, but let it train further if the constraints are violated. Hence our approach can further be improved by adding reset policies similarly to [44] and [20].
A1.2 RL with probability one constraints

We have introduced the safety state to the environment as follows:
\[ s_{t+1} \sim P(\cdot \mid s_t, a_t) \]
\[ z_{t+1} = (z_t - l(s_t, a_t))/\gamma_l \]
This means we have
\[ \gamma_l z_{t+1} - z_t = -l(s_t, a_t) \]
and
\[ \gamma^{t+1}_l z_{t+1} - \gamma^t_l z_t = -\gamma^{t+1}_l l(s_t, a_t), \]
\[ \gamma^t_l z_t - \gamma^{t-1}_l z_{t-1} = -\gamma^{t-1}_l l(s_{t-1}, a_{t-1}), \]
\[ \vdots \]
\[ \gamma^1_l z_1 - z_0 = -l(s_0, a_0), \]
\[ z_0 = d. \]

Now we can sum up the left and the right hand side and obtain:
\[ \gamma^{t+1}_l z_{t+1} = d - \sum_{k=0}^{t} \gamma^k_l l(s_k, a_k). \]

This means that the constraint can now be rewritten as \( g(z_T) \geq 0 \) resulting in the following optimization problem.
\[
\max_{\pi(\cdot)} E \sum_{t=0}^{T-1} \gamma^t r(s_t, a_t, s_{t+1}), \quad \text{subject to } g(z_T) \geq 0.
\] (A1)

If we consider the problem with probability one constraints then as a constraint we get \( z_T \geq 0 \) almost surely (or equivalently with probability one). It is noted that \( z_T \geq 0 \) almost surely is equivalent to \( z_t \geq 0 \) for all \( t \geq 0 \). Now the reward is reshaped to get:
\[ \hat{r}(s_t, s_{z_t}, a_t, s_{t+1}, z_{t+1}) = \begin{cases} r(s_t, a_t, s_{t+1}) & \text{if } z_t \geq 0, \\ -\Delta & \text{if } z_t < 0. \end{cases} \] (A2)

This problem can be solved using off-the-shelf RL algorithms for any finite \( \Delta \). Finally, it was noted in [39] that with \( \Delta \to +\infty \), the problem converges to safe RL with probability one constraints.

The main difference to [39] is that we investigate the effect of the safety state on safety during training for both probability one constrained RL and average constrained RL.

A2 Additional algorithm details

A2.1 PI-Simmer

First, we discuss our design for the PI controller and discuss the necessary parts for it. Our main source for this discussion are the control engineering textbooks [7, 6]. The idea for the PI controller is quite intuitive it takes the error term \( d_{\text{test}} - c \) and uses it for action computation. P stands for the proportional control and multiplies the error term by the gain \( K \) proportionally linking the error terms with actions. The proportional part delivers a brute force control by having a large control magnitude for large errors, but it is not effective if the instantaneous error values become small. In fact, a proportional control cannot achieve zero error tracking, which is achieved by integral control and summing previous error terms. If the action values, however, are saturated integral control can lead to the unwanted phenomenon called wind-up and catastrophic effects [5]. This is specifically
Figure A1: Block-diagram of our PI controller. The arrow signifies the direction of the signal, the triangles mean multiplication with a constant, the rest of the block mean the following operations: “LPF” stands for low-pass filter (or the Polyak’s update), “Clip” stands for clipping the values to pre-defined minimum and maximum values, \( \Sigma \) stands for a sum of signals and \( \int \) stands for the integral of the signal over time. Main components of our controller: proportional gain \( K \), integral gain \( K_i \) for the integrator (marked as \( \int \) ) anti-windup gain \( K_{aw} \).

the case when the errors become large leading to large values of the integral, which in turn leads to saturated actions for a long time. There are several ways of dealing with wind-up, e.g., resetting the integral, and limiting the integration time, but we will take the feedback control approach where the previous saturation errors are fed back and used to determine the current action \([6, 1]\). Finally, we use a low pass filter for the error in order to avoid reacting to high-frequency fluctuations and acting only on the trends.

We propose the following update for the safety budget:

\[
\begin{align*}
\text{Error term: } & e_k = d^\text{est}_k - \hat{g}(z_T(d^\text{est}_k)) , \\
\text{I-part: } & w_k = (1 - \tau)w_{k-1} + \tau e_k , \\
\text{P-part: } & P = K_p e_k , \\
\text{Anti-windup: } & I = K_i \sum_{i=k-T_i}^k e_i , \\
\text{Raw signal: } & AW = K_{aw}(u_{k-1} - u^\text{raw}_{k-1}) , \\
\text{New safety budget: } & u_k = \text{clip}(u^\text{raw}_k, -\delta d, \delta d) , \\
\text{where } & \delta d \text{ is the current constraint violation, } u_k \text{ is the filtered error term } e_k , \\
& P, I, AW \text{ are the proportional, integral and anti-windup parts of the controller. The gains } K_p, K_i, K_{aw} \text{, as well as } T_i, \\
& \delta d \text{ and } \tau \text{ are hyper-parameters.}
\end{align*}
\]

A2.2 Q Simmer

Our design for the Q learning approach is guided by the intuition depicted in Figure A2 and presented in what follows. If the current accumulated costs are well below the safety budget \( d^\text{ref} \), then we are very safe and the safety budget can be further increased. If the accumulated costs are around the safety budget, then we could stay at the same level or increase the safety budget. If the current accumulated costs are well above the safety budget \( d^\text{ref} \), then the safety budget should be decreased to ensure that the policy is incentivized to be safer.

Consider an MDP with the states \( \{d_0, \ldots, d_{K-1}\} \), for simplicity of notation we consider the state space \( \{0, \ldots, K-1\} \) with the actions \( \{a_{-1}, a_0, a_{+1}\} \), where the action \( a_{+1} \) moves the state \( s = i \)
We're very safe!
We can risk it!
We're not safe!
No risk!
We're borderline safe...
Can we risk?
We're very safe!
We can risk it!

Figure A2: Intuition for Q simmer reward shaping: first the costs c are passed through a low-pass filter getting \( \hat{c} \).

to the state \( s = i + 1 \), \( a_{-1} \) moves the state \( s = i \) to the state \( s = i - 1 \) and the action \( a_0 \) does not transfer the state. Note that the action \( a_{i+1} \) is defined for all \( i < K - 1 \) and the action \( a_{-1} \) is defined for all \( i > 0 \). The reward for this MDP is non-stationary and defined as follows:

We are not safe if \( s - o \leq -\delta : \)

\[
\begin{cases} 
2 & a = -1, \\
1 & a = 0, \\
-1 & a = 1, 
\end{cases}
\]

We are borderline safe if \( |s - o| \leq \delta : \)

\[
\begin{cases} 
-1 & a = -1, \\
1 & a = 0, \\
1 & a = 1, 
\end{cases}
\]

We are very safe if \( s - o \geq \delta : \)

\[
\begin{cases} 
-1 & a = -1, \\
1 & a = 0, \\
2 & a = 1. 
\end{cases}
\]

(A4)

(A5)

(A6)

where \( o \) is the maximum accumulated cost (over an episode) of the constrained algorithm. We use Q-learning update to learn the Q function:

\[
Q(s_t, a_t) = (1 - l_r)Q(s_t, a_t) + l_r(r_t + \max_b Q(s_{t+1}, b))
\]

(A7)

and get the action with \( \varepsilon \)-greedy exploration strategy:

\[
a_t = \begin{cases} 
\arg\max_b Q(s_t, b) & \text{with probability } \varepsilon, \\
\text{random} & \text{with probability } 1 - \varepsilon.
\end{cases}
\]

(A8)

### A3 Implementation details

**Pendulum Swing-up.** Our first environment is a safe pendulum swing up defined in [16] and built upon the Open AI Gym environment [11]. The reward is defined as

\[
r(s, a) = 1 - \frac{\theta^2 + 0.11\dot{\theta}^2 + 0.001a^2}{\pi^2 + 6.404},
\]

while the safety cost

\[
l = \begin{cases} 
1 - \frac{|\theta - \delta|}{50} & \text{if } -25 \leq \theta \leq 75, \\
0 & \text{otherwise},
\end{cases}
\]

where \( \theta \) is deviation of the pole angle from the upright position. Effectively, we want to force the pendulum to swing-up from one side.

**Safety Gym.** We use several safety gym environments [33]. We use the benchmark environments PointGoal1, PointButton1, PointPush1, CarGoal1, CarPush1, but we also use a custom-made one from [47]. In the custom-made environment called static point goal, a large static hazard region is
placed in front of the goal. This is similar to our motivational example in Figure 1. This environment confirms our intuition. We use two robots, the point robot has 46 states and 2 actions, while the car robot has 56 states and 2 actions.

**Code base and hyper-parameters.** Our code is based on two repositories: safety starter agents [33], and PID Lagrangian [40]. We have implemented the safety state augmentation following the description in [39]. We use default parameters for both code bases unless stated otherwise. The changed hyper-parameters are presented in Table A1. The simmering schedule changes the safety budget every 200 epochs, where the length of the epoch depends on the batch size and is presented in Table A1.

| Parameter | Static PointGoal | PointGoal1 | PointButton1 | PointPush1 | CarGoal1 | CarPush1 |
|-----------|------------------|------------|--------------|------------|----------|----------|
| Batch Size | 128              | 128        | 320          | 512        | 256      | 512      |
| $K_p$     | 0.1              | 0.01       | 0.01         | 0.03       | 5        | 0.05     |
| $K_i$     | 0.01             | 0.05       | 0.03         | 0.01       | 0.01     | 0.001    |
| $\gamma_c$ | 0.99             | 0.995      | 0.995        | 0.995      | 0.995    | 0.995    |
| Steps per epoch | 13312          | 13312      | 33280        | 53248      | 26624    | 53248    |

### A4 Further Experiments

#### A4.1 Safety gym benchmarks

We conduct further experiments on safety gym benchmarks with results depicted in Figure A3. For the CarPush1 environment, we observe that PID Lagrangian [40] has worse performance in terms of costs and cost rate, but manages to recover the returns by the end of training. For CarGoal1 the results are similar but PID Lagrangian performs better in terms of return, but again the cost rate and the cost performance are worse.

Figure A3: Simmer PID-L outperforms all baselines in terms of the cost rate, which is a measure of safety during training. At the same time, Simmer PID-L never performs worse in terms of the returns or safety costs than PID-L, significantly outperforming other baselines. The curves are means and shaded areas are standard deviations computed over five runs with random seeds. Note that we do not have baseline results for CarPush1.

#### A4.2 Ablation for PI Simmer

First, we conduct the ablation study over the parameter of the low pass filter $\tau$ in Figure A4a, while we set $K = 0.1$, $K_i = 0.005$, $K_{aw} = 0.01$. We notice that smaller values of $\tau$ deliver small
deviations from the schedule, while larger values have more freedom to decide on an appropriate schedule. However, it is noticeable that the runs with small \( \tau \) (0.001 and 0.005) are constantly acquiring constraint violations, which happens less often for large values of \( \tau \) (1 and 0.995). We believe this is because the controller with smaller values of \( \tau \) ignores (filters out) spurious constraint violations, but with larger values of \( \tau \) this is not happening. At the same time, PI Simmer with large \( \tau \) changes the safety budget quite aggressively in the early stages due to constant constraint violations, which can be avoided if a small \( \tau \) is chosen. Ultimately the choice of \( \tau \) is up to the user, but we would recommend starting with large values.

We observed that it is probably most prudent to choose a small value for \( K \) as our deviations from schedule \( d^*_k \) are bounded and there is no need for drastic changes in the safety budget that proportional control can achieve. Indeed, we set \( K_{aw} = 0.1, K_i = 0.005, \tau = 0.995 \) and change the parameter \( K \) and the results in Figure A4c supports our claims.

We now turn our attention to the gain \( K_i \) of the integral control, while we set \( K = 0.1, K_{aw} = 0.01, \tau = 1 \). We stress that the choice of the gains \( K_i, K_{aw} \) is somewhat interconnected, i.e., with increasing \( K_i \) the gain \( K_{aw} \) should increase as well to counteract saturation. However, it appears the choice of \( K_i \) can have a more significant effect on performance. Figure A4b suggests that large values of \( K_i \) lead to aggressive changes in the safety budget and, more importantly, later lead to saturation in control limiting the ability of PI Simmer to react to violations. Smaller values of \( K_i \) allow for a balanced approach.

Finally, ablation over \( K_{aw} \) in Figure A4d with \( K = 0.1, K_i = 0.04, \tau = 1.0 \) shows that while large values of \( K_{aw} \) decrease the risk of saturation they also diminish the efficacy of integral control. On the other hand, with zero anti-windup gain, the actions are always saturated leading to more constraint violations without any reaction from the controller. Therefore, again a balanced approach is preferable.

### A4.3 Ablation for Q Simmer

We perform ablation on the following parameters the learning rate \( l_r \), the Polayk’s update \( \tau \), and the reward threshold \( r_{thr} \). Figure A5a suggest that large learning rates preferable. This is perhaps due to the ability to fast learning, but also fast forgetting may be important since the process is non-stationary. Ablation with respect to \( \tau \) is performed in Figure A5b suggesting that higher values of \( \tau \) are preferable, but \( \tau = 1 \) forces the algorithm to oscillate between two adjacent values too often. Similarly the value for \( r_{thr} \) needs to be chosen so that oscillations between adjacent safety budget does not occur as Figure A5c suggests.
Figure A4: Ablation for PI Simmer
Figure A5: Ablation for Q Simmer