Non-Threshold Quantum Secret Sharing Schemes in the Graph State Formalism

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In a recent work, Markham and Sanders have proposed a framework to study quantum secret sharing (QSS) schemes using graph states. This framework unified three classes of QSS protocols, namely, sharing classical secrets over private and public channels, and sharing quantum secrets. However, most work on secret sharing based on graph states focused on threshold schemes. In this paper, we focus on general access structures. We show how to realize a large class of arbitrary access structures using the graph state formalism. We show an equivalence between \([n, 1]\) binary quantum codes and graph state secret sharing schemes sharing one bit. We also establish a similar (but restricted) equivalence between a class of \([n, 1]\) Calderbank-Shor-Steane (CSS) codes and graph state QSS schemes sharing one qubit. With these results we are able to construct a large class of quantum secret sharing schemes with arbitrary access structures.

Keywords: quantum secret sharing, graph state formalism, quantum codes, CSS codes, quantum cryptography

I. INTRODUCTION

Quantum secret sharing (QSS) \([2,3]\) deals with the problem of sharing classical or quantum secrets using quantum information. Further, the secret sharing protocol could be operated in the presence or absence of eavesdroppers. In \([12]\), a graph state formalism was proposed with a view to unifying all these variants under the same umbrella. This framework was useful in ways other than unifying the various quantum secret sharing protocols. For instance, building upon this framework, researchers have been able to propose new secret sharing protocols \([9]\), and make a connection with the measurement-based quantum computation model \([10]\). More recently, it has motivated research in graph theoretic concepts such as weak odd domination \([6]\).

The graph state framework does in principle include non-threshold access structures for quantum secrets. However, neither \([12]\) nor subsequent works \([6,8,11]\) provide any procedure to explicitly construct schemes with arbitrary access structures in the graph state formalism. The graph state framework in for quantum secret sharing approaches it from a perspective other than quantum error-correction, in contrast to the theory as developed in \([2,4,5]\). But since any secret sharing protocol is ultimately an error-correcting code, the graph state schemes must be equivalent to those based on stabilizer codes and the protocols in \([12]\) must arise from quantum codes. But no results are known in this direction.

The main contribution of this paper is to fill these gaps. We make transparent the connection between the graph state framework and the protocols presented using quantum codes. We show an equivalence between \([n, 1]\) binary quantum codes and graph state protocols sharing one bit. We show a restricted equivalence between a class of \([n, 1]\) CSS codes \([1]\) and protocols sharing one qubit. We also translate many of the schemes developed using quantum codes into those based on the graph state formalism.

We emphasize that our results are constructive and provide concrete details for the construction of the secret sharing schemes as well as the associated details of recovery.

We restrict ourselves to the qubit case in this paper, although as shown in \([11]\) graph state secret sharing schemes can be extended to other alphabet. The general case involving qudits will be explored elsewhere.

II. BACKGROUND

A. Quantum secret sharing

We briefly review the pertinent ideas of quantum secret sharing. We assume that the reader is familiar with quantum codes and the stabilizer formalism \([1,3]\). In a secret sharing scheme, a dealer distributes an encrypted secret to a collection of players. Then certain subsets of players can collaboratively reconstruct the secret. Those subsets which can recover the secret are called authorized sets and those that cannot are said to be unauthorized sets. The collection of authorized sets is called the access structure of the scheme, which we denote as \(\Gamma\). For an access structure to be valid, it must be monotonic, i.e., any set that contains an authorized set must also be an authorized set. An authorized set is said to be minimal if any proper subset of it is unauthorized. The collection of minimal authorized sets is called the minimal access structure. In a threshold scheme with threshold \(k\), any subset consisting of \(k\) or more players can access the secret while those with fewer players cannot. In a general access structure, the authorized sets can be of different sizes and all subsets of that size need not be authorized. A collection of sets \(\Gamma_{\text{gen}}\) is said to generate the access structure \(\Gamma\), if every authorized set contains some element of \(\Gamma_{\text{gen}}\).

A secret sharing scheme is said to be perfect if the unauthorized sets cannot extract any information about the secret. In this paper we only consider perfect secret sharing schemes.

When the secret to be shared is classical, the dealer distributes a set of orthogonal quantum states that encode the secret. The following result, due to Gottesman, states the conditions that must be satisfied by authorized and unauthorized sets for sharing classical secrets through a QSS scheme.
Proposition 1 (Access conditions for classical secrets, [4]). Suppose we have a set of orthonormal states $|\psi_i\rangle$ encoding a classical secret. Then a set $T$ is an unauthorized set iff
\begin{equation}
\langle \psi_i | F | \psi_i \rangle = \epsilon (F)
\end{equation}

independent of $i$ for all operators $F$ on $T$. The set $T$ is authorized iff
\begin{equation}
\langle \psi_i | E | \psi_j \rangle = 0 \quad (i \neq j)
\end{equation}

for all operators $E$ on the complement of $T$.

If we were to share a quantum secret, then the access structure, in addition to being monotonic, must also satisfy the no-cloning theorem [2]. This implies that no two authorized sets are disjoint. In this case the access structure must satisfy the conditions of Proposition 1 for any state in the space spanned by the encoded states, see [4, Theorem 1].

B. Review of graph state formalism for quantum secret sharing

In [12], the quantum secret sharing protocols were classified as follows: i) CC–This protocol deals with the sharing of classical secrets, where we assume that the players have secure channels. ii) CQ–In this protocol we share classical secrets where we assume that the channels between the players are susceptible to eavesdropping. iii) QQ–This protocol shares quantum secrets using quantum channels. In this paper we restrict our attention to CC and QQ protocols.

Let $G$ be a graph with vertex set $V(G)$. We denote the neighbours of a vertex $v \in V(G)$ as $N_v$. We denote the graph obtained by deleting the vertex $v$ from $G$, by $G \setminus v$. The graph state defined on $G$ is denoted $|G\rangle$. Recall that the graph state is a stabilizer state and satisfies $K_v |G\rangle = |G\rangle$, where
\begin{equation}
K_v = X_v \prod_{u \in N_v} Z_u, \text{ for all } v \in V(G).
\end{equation}

We use the notation $K_A = \prod_{i \in A} K_i$. The stabilizer of $|G\rangle$ is denoted as $S(|G\rangle)$.

In the CC quantum secret sharing protocol, the secret bit $s$ is encoded as
\begin{equation}
E : s \mapsto Z_s^A |G\rangle,
\end{equation}

where $Z_s^A = \prod_{i \in A} Z_i^s$. We denote a CC protocol using the graph $G$ and encoding using the set $A$ by $(G, A)$. An authorized set $T$ can recover the secret by either performing a joint measurement of an appropriate operator $M \in S(|G\rangle)$ or by local measurements and combine these results classically (after classical communication), in other words through LOCC.

In the QQ protocol, the dealer needs to add an additional ancilla qubit whose state is the secret to be shared. The dealer then encodes this state by a procedure similar to teleportation. Following this the dealer might have to perform some correction operations on the encoded state to ensure that the secret has been properly teleported. The dealer then distributes the qubits to the players. In this setting, authorized subsets of players can reconstruct the secret by means of suitable nonlocal operations.

In [10], the graph state secret sharing schemes were characterized in terms of graphical conditions. Define the odd neighbourhood of a set $S \subseteq V(G)$ as
\begin{equation}
Odd(S) = \{ v \in V(G) \mid |N_v \cap D| = 1 \mod 2 \}
\end{equation}

Proposition 2 (Authorized sets for CC protocol, [10]). For the CC classical secret sharing protocols $(G, A)$ of [12], the secret can be accessed by a set $S$ if there exists $D \subseteq S$ such that
\begin{equation}
D \cup Odd(D) \subseteq S
\end{equation}
\begin{equation}
|D \cap A| = 1 \mod 2
\end{equation}

Proposition 3 (Unauthorized sets for CC protocol, [10]). For the CC classical secret sharing protocols of [12] on $G$, the secret cannot be accessed by a set $S$ if there exists a $K \in V(G) \setminus S$ such that
\begin{equation}
Odd(K) \cap S = A \cap S
\end{equation}

The authors of [10] proved that these two conditions were sufficient and made the observation that it was open which graphs satisfy them. That these conditions are necessary as well was shown in [3, Lemma 2].

III. GRAPH STATE SCHEME FOR GENERAL ACCESS STRUCTURES

A. Classical secrets

In this section we make a connection between the CC protocol in the graph state formalism and the standard error correction model. We establish a correspondence between all graph state schemes sharing one bit and $[[n, 1]]$ binary quantum codes. This provides an alternative characterization of the access structure of the CC secret sharing protocols. Further, Theorem 1 also generalizes the results of [12], which only uses CSS codes derived from self-dual codes.

Theorem 1. Let $Q$ be an $[[n, 1, d]]$ quantum code with stabilizer matrix
\begin{equation}
S = \begin{bmatrix}
I_r & A_1 & A_2 & B & 0 & C \\
0 & 0 & 0 & D & I_{n-r-1} & E
\end{bmatrix},
\end{equation}

where $\text{diag}(B + CA_2^t) = 0$. Then the graph $G$ with the adjacency matrix $A_G$
\begin{equation}
A_G = \begin{bmatrix}
B + CA_2^t & A_1 & A_2 \\
A_1^t & 0 & 0 \\
A_2^t & 0 & 0
\end{bmatrix},
\end{equation}

gives rise to a CC quantum secret sharing protocol with $(G, A)$, where $A = \text{supp}([C^t \ E^t \ 1])$. A generating set for the access structure is given by
\begin{equation}
\Gamma_{gen} = \{ \text{supp}(g) | g \text{ is an encoded } Z \text{ operator} \}.
\end{equation}
Proof. One choice of logical $X$ and $Z$ operators for $Q$ is given by
\[
\begin{bmatrix}
X \\
Z
\end{bmatrix} = \begin{bmatrix}
0 & E^t & 1 & C^t & 0 & 0 \\
0 & 0 & 0 & A_2 & 0 & 1
\end{bmatrix}.
\]

Let $|\overline{0}\rangle$ be the state stabilized by $S$ and $Z$ and $|\overline{T}\rangle = |X\rangle|0\rangle$. Then $I^{\otimes n} - |\overline{0}\rangle\langle X\rangle|\overline{0}\rangle = Z^\perp_A|G\rangle$. Therefore, up to local Clifford unitaries, the basis states of the CC secret sharing scheme induced by $(G, A)$ and the basis states of $Q$ are equivalent. Therefore the secret can be recovered if we can distinguish between the states $|\overline{0}\rangle$ and $|\overline{T}\rangle$. Consider We can rewrite the encoding for the CC protocol in terms of the basis states of $Q$ as follows:
\[E : s \mapsto \begin{bmatrix} X \\ Z \end{bmatrix} |\overline{0}\rangle = |\psi_s\rangle.\]

If $\omega \subseteq \{1, \ldots, n\}$ contains the support of an encoded $Z$ operator, then we can recover the secret because $\begin{bmatrix} X \\ Z \end{bmatrix} |\overline{0}\rangle = (-1)^{\omega} |\overline{0}\rangle$. Thus the support of any encoded $Z$ operator gives an authorized set.

If $\omega$ does not contain the support of a logical $Z$ operator, then it is an unauthorized set. Let $\Gamma$ be the secret stabilization of $S$ and $Z$, then $I^{\otimes n} - |\overline{0}\rangle\langle X\rangle|\overline{0}\rangle$ is detectable, therefore $\langle \psi_{\alpha} | T | \psi_{\alpha} \rangle = 0$. If $T \in S$, then $\langle \psi_{\alpha} | T | \psi_{\alpha} \rangle = 1$. If $T \in C(S) \setminus S$ and does not contain the support of an encoded $Z$ operator, then it must be an encoded $X$ or $Y$ operator. Since $|\psi_{\alpha}\rangle = \begin{bmatrix} X \\ Z \end{bmatrix} |\overline{0}\rangle$, we have $\langle \psi_{\alpha} | T | \psi_{\alpha} \rangle = \langle \overline{0} | \begin{bmatrix} X \\ Z \end{bmatrix} ^\dagger T \begin{bmatrix} X \\ Z \end{bmatrix} | \overline{0} \rangle = 0$. We used the fact that $T |\overline{0}\rangle = \alpha |\overline{T}\rangle$ for some $\alpha \in \mathbb{C}$. Therefore, by Lemma $\ref{lem:unauthorized}$ $\omega$ is unauthorized. This shows that the access structure generated by $\Gamma_{\text{gen}}$ is complete and must coincide with the access structure as defined by Propositions $\ref{prop:encoding}$ and $\ref{prop:decoding}$. \hfill $\square$

Remark 1. The requirement that $B + CA_2 = 0$ is not a restriction because any such code can be transformed by local Clifford unitaries to a code which satisfies this condition. These two codes will lead to the same access structure.

Our theorem gives a succinct characterization of the access structure, we just need to specify the stabilizer generators and the encoded $Z$ operator. All the authorized sets can then be enumerated easily. There is no need to further check any other conditions on the sets. But note that our characterization does not give the minimal access structure but rather a generating set for the access structure. If we want to obtain the minimal access structure then we only need to look at those encoded $Z$ operators which are also minimal in the sense they do not properly contain any other encoded $Z$ operator within their support.

Corollary 2. Let $G$ be a connected graph with adjacency matrix $A_G$ as in equation (10), where $A_1$, $A_2$ are chosen arbitrarily and $B + CA_2$ is symmetric. Let $A$ be an arbitrary subset of one of the bipartitions of the graph. Then $(G, A)$ is a CC quantum secret sharing scheme.

Two special cases of are worth highlighting because of their importance.

\begin{itemize}
  \item (i) $A_1 = 0$, then it can be seen that we are covering in effect all possible graphs. Thus every graph leads to a CC quantum secret sharing scheme.
  \item (ii) $B + CA_2 = 0$. This corresponds to the situation where $G$ is bipartite. These secret sharing schemes are precisely those arising from an $[[n, 1]]$ CSS code.
\end{itemize}

If $A_2 = 0$, then the access structure is trivial: the minimal access structure contains a singleton set.

The framework as developed in $\cite{4}$ makes it possible to use mixed states for sharing classical secrets. At the present it is not clear how to include those schemes in the graph state formalism, since graph states are by definition pure and they lead to pure state quantum secret sharing schemes. Recall that a pure state scheme is one in which a pure state is encoded into a pure state. In a mixed state scheme a pure state could be encoded into a mixed state. Such schemes could be more efficient than the pure state schemes.

\section{B. Quantum secrets}

In this section, we use the graph state formalism to construct QQ quantum secret sharing schemes for general access structures. Every secret sharing scheme includes a step where the dealer encrypts the secret before distributing the shares. In $\cite{11, 12}$, this was broken down into the following steps. In the first step, the dealer prepares a graph state over the dealer’s qubit and the players qubits. Then an ancilla qubit prepared in the secret state is entangled with the dealer’s qubit. Then the ancilla and dealer’s qubits are measured in the Bell basis leading to an encoded teleportation onto the players’ qubits. In this paper we simplify these steps by involving only one additional qubit. We make use of the teleportation scheme to encode into a quantum code using graph states, see $\cite{3, 7}$.

We illustrate this procedure through an example. Consider the graph shown in Fig. $\ref{fig:graph}$. Pick any vertex of the graph, say we pick 0. The dealer prepares this qubit in the secret state to be shared. Then this qubit is entangled with the qubits in $N_0$ using controlled-Z gates. Then we measure the dealer’s qubit in the $\sigma_x$ basis. If we measure 0, then the secret has been encoded as desired, otherwise, we need to apply a correction of the encoded Z on the state. The qubits are then distributed to the players.

Consider the secret being encoded into $Z_3^\perp|G \setminus 0\rangle$, where $A = \{4, 5, 7\}$. Then it can be verified that all the minimal authorized sets given in $\Gamma_{0, \text{min}}$ satisfy both equations (6) and (7).

\[
\Gamma_{0, \text{min}} = \left\{ \{1, 2, 7\}; \{1, 3, 5\}; \{1, 4, 6\}; \{2, 3, 4\}; \{2, 5, 6\}; \{3, 6, 7\}; \{4, 5, 7\} \right\}.
\]

Lemma 3. Let $G$ be a bipartite graph with adjacency matrix $A_G$ given by
\[
A_G = \begin{bmatrix} 0 & P \\ P^t & 0 \end{bmatrix}, \quad \text{where } PP^t = I.
\]

Then for any set $D \subseteq B$, where $B$ is in one of the bipartitions, (i) $\text{Odd}(\text{Odd}(D)) = D$. (ii) $|\text{Odd}(D) \cap N_i| = 0 \mod 2$ for any $i \in B \setminus D$. 

Proof. Let \( g \in F_2^3 \) be such that \( \text{supp}(g) = D \). Then \( \text{Odd}(D) = \text{supp}(gA_G) \) and \( \text{Odd}(\text{Odd}(D)) = \text{supp}(gA_GA_G) = \text{supp}(g) \). Let \( h \in F_2^3 \) be such that \( \text{supp}(h) = N_i \). Let \( h \) correspond to the \( i \)th row in \( A_G \). Note that \( \text{Odd}(D) = \text{supp}(A_Gg_i) \). To show that \( (\text{Odd}(D)) \cap N_i = 0 \pmod{2} \), it is enough to prove that \( bA_Gg_i = 0 \). This is zero because \( h \) is orthogonal to all but the \( i \)th column of \( A_G \). \( \square \)

Theorem 4. Let \( G \) be a bipartite graph whose adjacency matrix \( A_G \) is given by

\[
A_G = \begin{bmatrix}
0 & P \\
P^t & 0
\end{bmatrix}, \text{ where } P P^t = I.
\]

(15)

Then for every vertex \( i \) we can define a perfect QQ quantum secret sharing scheme from \( G \). The encoding for the quantum secret sharing scheme is given by

\[
E : a[0] + b[1] \mapsto a[G \setminus i] + bZ_{N_i}[G \setminus i].
\]

(16)

A generating set for the access structure \( \Gamma_i \) is given by the following

\[
\Gamma_{i,\text{gen}} = \left\{ D \cup \text{Odd}(D) \setminus i \ \bigg| \ D \subseteq V_r \right\}
\]

\[
D \cap N_i = 1 \pmod{2}
\]

(17)

where \( V_r \) is the bipartition of vertices of \( G \) that does not contain \( i \). The encryption and recovery of the secret are as shown in Fig. 2 and 3 respectively.

![Fig. 1.](image1.png)

**FIG. 1.** (Color online) A general QQ secret sharing scheme from a bipartite graph. All qubits except the dealer’s qubit are prepared in the \(|+\rangle \) state, while the dealer’s qubit \((0)\) is prepared in the secret state. Then we apply CZ gates along the edges of \( G \). The dealer’s qubit is then measured in the \( \sigma_x \) basis. A correction operator is applied if we measure one.

![Fig. 2.](image2.png)

**FIG. 2.** Encrypting the secret state \(|\psi\rangle\) for QQ secret sharing using teleportation. The operator \( K'_j = X_j \prod_{k \in N_j \setminus i} Z_k \) is such that \( j \in N_i \). It is applied only if the measurement outcome is 1.

![Fig. 3.](image3.png)

**FIG. 3.** Reconstructing the secret state \(|\psi\rangle\) for QQ secret sharing given an authorized set \( D \) as in equation (17). The operator \( K_D = \prod_{j \in D} K_j = \prod_{j \in D} X_j \prod_{k \in \text{Odd}(D)} Z_k \) and \( U_D = \prod_{j \in D} Z_j \prod_{k \in \text{Odd}(D) \setminus i} X_k \).

**Proof.** We shall prove this theorem in parts. For convenience, we shall ignore the normalization factors for quantum states.

(i) Encryption of the secret: Assume that the secret to be encoded is \(|\psi\rangle = a[0] + b[1]\). Then it can be easily verified that in Fig. 2 the state \(|\psi\rangle(G \setminus i)\) is transformed to the following state prior to measurement (up to normalization):

\[
|0\rangle(a[G \setminus i] + bZ_{N_i}[G \setminus i]) + |1\rangle(a[G \setminus i] - bZ_{N_i}[G \setminus i])
\]

If we measure zero, then we get the desired state but if we measure one, then we have to apply the correction operator \( K'_j = X_j \prod_{k \in N_j \setminus i} Z_k \) for any \( j \in N_i \). This operator anti-commutes with the operator \( Z_{N_i} \), but stabilizes the state \(|G \setminus i\rangle\), therefore it acts as a correction operator to give the state in equation (17).

(ii) Recovery: Before we show that \( D \cup \text{Odd}(D) \setminus i \) is authorized, we need the following result. Note that \( K_D = \prod_{j \in D} K_j \), therefore \( \text{supp}(K_D) = D \cup \text{Odd}(D) \) and \( i \in \text{supp}(K_D) \). Because of the fact \( PP^t = I \), the element \( U_D = X_i U_D' = \prod_{j \in D} Z_j \prod_{k \in \text{Odd}(D)} X_k \in S(|G\rangle) \). Let us write \(|G\rangle\) as

\[
|G\rangle = |0\rangle(G \setminus i) + |1\rangle Z_{N_i}[G \setminus i],
\]

Then it follows that

\[
U_D|G\rangle = |1\rangle U_D'|G\setminus i\rangle + |0\rangle U_D'Z_{N_i}[G \setminus i].
\]

Hence, \( U_D' Z_{N_i} \) stabilizes \(|G \setminus i\rangle\).

With respect to the recovery observe that the set \( D \cup \text{Odd}(D) \) as in Eq. (17) satisfies the requirements of Proposition 2, therefore if we trace through the circuit given in Fig. 3 the state transforms as follows:

\[
|+\rangle|\psi\rangle = (|0\rangle + |1\rangle)(a[G \setminus i] + bZ_{N_i}[G \setminus i])
\]

\[
\xrightarrow{c-K_D} |0\rangle(a[G \setminus i] + bZ_{N_i}[G \setminus i]) + |1\rangle(aK_D[G \setminus i] + bKDZ_{N_i}[G \setminus i])
\]

\[
\xrightarrow{H} a[0]G \setminus i + b[1]Z_{N_i}[G \setminus i]
\]

\[
\xrightarrow{c-U_D'} a[0]G \setminus i + b[1]G \setminus i
\]

\[
= (a[0] + b[1])[G \setminus i]
\]

where we used the fact that \( U_D' Z_{N_i} \) stabilizes \(|G \setminus i\rangle\). Thus \( D \cup \text{Odd}(D) \setminus i \) is able to reconstruct the quantum secret \(|\psi\rangle\). The no-cloning theorem now implies that the complement of this set is unauthorized.

(iii) Completeness of \( \Gamma_{i,\text{gen}} \): Now we show that the access structure as defined in Eq. (17) is complete in the sense that every authorized set contains some element of \( \Gamma_{i,\text{gen}} \).

Assume that there exists some set \( A \) which is authorized but not generated by \( \Gamma_{i,\text{gen}} \). The complement of
In this paper we have only considered secret sharing schemes with arbitrary access structures using the graph state formalism. These results also elucidate the connection between graph state framework and quantum secret sharing schemes based on quantum codes [2, 4].

IV. CONCLUSION

We showed how to construct quantum secret sharing schemes which are not ideal. Such a need arises because there are some schemes that are not ideal.

Remark 2. In this paper we have only considered secret sharing schemes where the share distributed to each party is of the same dimension as the dimension of the secret. Such schemes are said to be ideal. The graph state framework has not been used to study schemes which are not ideal. Such a need arises because there are some schemes that are not ideal.

Our results make it possible to answer some questions related to the graph state formalism very easily as exemplified by the following theorem.

Theorem 6. There do not exist any graph state QQ secret sharing protocols for \((k, 2k − 1)\) if \(k \geq 4\).

Proof. In [13], it was shown that every \((k, 2k − 1)\) quantum threshold secret sharing scheme is an \([2t − 1, 1, t]_q\) quantum MDS code. In [1], it was shown that there do not exist any \([n, 1]\) binary quantum MDS codes of length greater than 5. It follows therefore, there are no (pure state) QQ quantum threshold schemes of length \(2k−1\) greater than 5, equivalently \(k \geq 4\).

The existence of quantum threshold schemes was studied at great length in [9]. Through the connection to quantum codes we are able to shed light on this issue, immediately improving upon the lower bound in [9, Corollary 4].
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[1] A.R. Calderbank, E.M. Rains, P.W. Shor, and N.J.A. Sloane. Quantum error correction via codes over GF(4). IEEE Trans. Inform. Theory, 44:1369–1387, 1998.
[2] R. Cleve, D. Gottesman, and H.-K. Lo. How to share a quantum secret. Phys. Rev. Lett., 83(3):648–651, 1999.
[3] D. Gottesman. Stabilizer codes and quantum error correction. Caltech Ph. D. Thesis, eprint: quant-ph/9705052 1997.
[4] D. Gottesman. Theory of quantum secret sharing. Phys. Rev. A, 61(042311), 2000.
[5] M. Grassl. Variations on encoding circuits for stabilizer quantum codes. In Proceedings Third International Workshop Coding and Cryptology, Lecture Notes in Computer Science, pages 142–158, 2011.
[6] S. Gravier, J. Javelle, M. Mhalla, and S. Perdrix. On weak odd domination and graph-based quantum secret sharing, 2011. eprint:arXiv:1112.2495
[7] M. Hein, J. Eisert, and H. J. Briegel. Multiparty entanglement in graph states. Phys. Rev. A, 69(062311), 2004.
[8] M. Hillery, V. Buzek, and A. Berthame. Quantum secret sharing. Phys. Rev. A, 59(3):1829–1834, 1999.
[9] J. Javelle, M. Mhalla, and S. Perdrix. New protocols and lower bound for quantum secret sharing with graph states.
[10] E. Kashefi, D. Markham, M. Mhalla, and S. Perdrix. Information flow in secret sharing protocols. 2009. eprint:arXiv:0909.4479
[11] A. Keet, B. Fortescue, D. Markham, and B. C. Sanders. Quantum secret sharing with qudit graph states. eprint:arXiv:1004.4619, 2010.
[12] D. Markham and B. Sanders. Graph states for quantum secret sharing. Phys. Rev. A, 78(042309), 2008.
[13] K. Rietjens, B. Schoenmakers, and P. Tuyls. Quantum information theoretical analysis of various constructions for quantum secret sharing. In Proc. 2005 IEEE Int'l Symposium on Information Theory, Adelaide, Australia, pages 1598–1602, 2005.
[14] P. Sarvepalli and A. Klappenecker. Sharing classical secrets with Calderbank-Shor-Steane codes. Phys. Rev. A, 80(022321), 2009.
[15] A. Smith. Quantum secret sharing for general access structures. eprint:arXiv:quant-ph/0001087 2000.