On supersymmetric \(Dp-\bar{D}p\) brane solutions

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Abstract

We analyze in the spirit of hep-th/0110039 the possible existence of supersymmetric \(Dp-\bar{D}p\) brane systems in flat ten dimensional Minkowski space. For \(p = 3, 4\) we show that besides the solutions related by T-duality to the \(D2-\bar{D}2\) systems found by Bak and Karch there exist other ansatz whose compatibility is shown from general arguments and that preserve also eight supercharges, in particular a \(D4-\bar{D}4\) system with \(D2\)-branes dissolved on it and Taub-NUT charge. We carry out the explicit construction in Weyl basis of the corresponding Killing spinors and conjecture the existence of new solutions for higher dimensional branes with some compact directions analogous to the supertube recently found.

1 Introduction

The discovery in type II string theories of cylinder-like branes preserving a quarter of the supersymmetries of the flat Minkowski space-time, the so-called “supertubes” \([1], [2], [3]\) has attracted much attention recently. The stabilizing factor at the origin of their BPS character that prevent them from collapse is the angular momentum generated by the non-zero gauge field living on the brane. The solution in \([1]\) presenting circular section was extended to arbitrary section in \([4]\); supertubes in the matrix model context can be found in \([5], [6]\).

A feature of the supertube is that it has \(D0\) and \(F1\) charges, but not \(D2\) charge. An interesting observation related to this fact was made by Bak and Karch (BK); if we take the elliptical supertube with semi-axis \(a\) and \(b\) in the limit when for example \(a\) goes to infinity that is equivalent to see the geometry near the tube where it looks flat, the

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system should become like two flat branes separated by a distance $b$. But because of the absence of $D2$ charge it is natural to suspect that indeed the system could be a $D2$-$\bar{D}2$ one. The existence of this system as well as systems with arbitrary numbers of $D2$ and $D2$ branes was proved in the context of the Born-Infeld action in reference [7] to higher dimensional brane-antibrane systems. In the course of the investigation we will find, other than the T-dual solutions to that of BK, also new solutions for $p = 3, 4$.

We start by remembering some relevant facts. Let $\{X^M(\xi), M = 0, 1, \ldots, 9\}$ the embedding fields in a ten-dimensional space-time of a $Dp$-brane parameterized by coordinates $\{\xi^\mu, \mu = 0, 1, \ldots, p\}$ and $\{A_\mu(\xi), \mu = 0, 1, \ldots, p\}$ the abelian gauge field living on it, $F = dA$ being the field strength. Let $\epsilon$ be a general Killing spinor of some background $(G_{MN}, B_{MN}, \phi, A^{(p+1)}_{M_1 \ldots M_{p+1}})$ of type II string theory. Then the introduction of the brane in such space will preserve the supersymmetries that satisfy [10]

$$\Gamma \epsilon = + \epsilon$$

(1.1)

where the $\Gamma$-matrix is defined by [3]

$$\begin{align*}
\Gamma &\equiv \frac{1}{|d|} \sum_{n=0}^{[\frac{p+1}{2}]} \frac{1}{2^n n!} f_{\mu_1 \nu_1} \cdots f_{\mu_n \nu_n} \gamma^{\mu_1 \nu_1 \ldots \mu_n \nu_n} \\
\Gamma(0) &\equiv \frac{1}{(p+1)!} \epsilon_{\mu_1 \ldots \mu_{p+1}} \gamma^{\mu_1 \ldots \mu_{p+1}}
\end{align*}$$

(1.2)

In all these expressions the pull-back of the background fields to the brane defined by

$$t_{\mu_1 \ldots \mu_n}(\xi) \equiv T_{M_1 \ldots M_n}(X)|_{X(\xi)} \partial_{\mu_1} X^{M_1}(\xi) \ldots \partial_{\mu_n} X^{M_n}(\xi)$$

(1.3)

for any tensor field $T_{M_1 \ldots M_n}$ as well as $\gamma_\mu \equiv \bar{E}_\mu^A \Gamma_A$ are understood, where $\bar{E}_\mu^A \equiv E_M^A(X)|_{X(\xi)} \partial_\mu X^M(\xi)$ is the “pull-backed” vielbein defined by $g_{\mu \nu}(\xi) = \eta_{AB} \bar{E}_\mu^A(\xi) \bar{E}_\nu^B(\xi)$, $\Gamma_A$ are the flat, tangent space $\Gamma$-matrices in ten dimensions, $d \equiv \det(\delta^\mu_\nu + f^\mu_\nu)$ and $f^\mu_\nu \equiv g^{\mu \rho} (F_{\rho \nu} + b_{\rho \nu}) = g^{\mu \rho} f_{\rho \nu}$ (we take $2 \pi \alpha' = 1$). The induced volume form on the brane is $\epsilon_{\mu_1 \ldots \mu_{p+1}} \equiv \sqrt{|g|} \varepsilon_{\mu_1 \ldots \mu_{p+1}}$, where $\varepsilon_{01 \ldots p} = +1$ in some patch defines an orientation; condition (1.1) with a “−” sign on the rhs corresponds to the anti-brane with the same fields of the brane, since by definition they have opposite orientations. Lastly, in the case of type IIA string theory the spinors must be Majorana, while in the type IIB case we take a pair of Majorana-Weyl spinors with the same chirality (and $\gamma^{\mu_1 \nu_1 \ldots \mu_n \nu_n} \rightarrow 1_2 \otimes \gamma^{\mu_1 \nu_1 \ldots \mu_n \nu_n}$ should be understood in (1.2)).

We are now ready to start with the analysis of various cases. We will restrict in this paper to work on the flat ten-dimensional Minkowski vacuum of type II string theories, the spinors being constants (in cartesian coordinates) of the type mentioned above.

2 The $D2-\bar{D}2$ system
2.1 The Bak-Karch ansatz

Let us consider a flat D2-brane extended in directions $(X^0, X^1, X^2)$, with field strength $F_{20} = E, F_{12} = B$. The general case with $F_{10} \neq 0$ can be reached from this using the Lorentz invariance $SO(1, 2)$ of the setting by means of a rotation. We can also take $E < 0$ or $E > 0$, it will not be relevant to fix a particular sign. The $\Gamma$-matrix is

$$\Gamma = |1 - E^2 + B^2|^{-\frac{1}{2}} \left( \Gamma_{012} + E \Gamma_1 \Gamma_{11} + B \Gamma_0 \Gamma_{11} \right) \quad (2.1)$$

It is well-known that equation (1.1) has solutions preserving $\frac{1}{2}$ of SUSY, i.e. 16 supercharges, for any constant $F_{\mu\nu}$ [10]. The ansatz introduced in [7] consists in subdividing condition (1.1) in two (or maybe more, if possible) parts in such a way they result compatible; in other words they show we can get novel non-trivial solutions at expenses of SUSY. In particular their solutions, as well as all the solutions presented in this paper, preserve $\frac{1}{4}$ SUSY. We will refer to the two conditions to solve (1.1) with the labels $@$ and $\otimes$; thus we write

$$\Gamma_{@} \epsilon = \epsilon, \quad \Gamma_{@} = E \Gamma_{02} \Gamma_{11} \quad (2.2)$$

$$\Gamma_{@} \epsilon = \epsilon, \quad \Gamma_{@} = s g(B) \Gamma_0 \Gamma_{11} \quad (2.3)$$

where "sg" stands for the sign-function. Equation (2.2) is equivalent to ask for the annihilation of the first two terms in (2.1) when acting on the spinor while (2.3) enforces (1.1) (with a minus sign on the rhs when the $\bar{D}$-brane case is considered). They sign out the presence of dissolved $F1$ in $\bar{e}_2$-direction and $D0$ branes respectively [7]. Consistency conditions for $@$ and $\otimes$ say that

$$\Gamma_{@}^2 \epsilon = \Gamma_{@}^2 \epsilon = \epsilon \quad (2.4)$$

But $\Gamma_{@}^2 = E^2 1$, so we must take $E^2 = 1$ for the electric field, constraint used in writing (2.3). Assumed it, we get $\Gamma_{@}^2 = 1$ and so nothing new is added. Finally the compatibility between the two conditions is assured from the fact that $[\Gamma_{@}; \Gamma_{@}] = 0$. The analysis of these consistency conditions will determine the compatibility conditions and will be the route to be followed to assure the existence of such solutions. From the further properties $tr \Gamma_{@} = tr \Gamma_{@} = tr \Gamma_{@} = 0$ is straightforward to conclude that each condition preserves $\frac{1}{2}$ SUSY and both together $\frac{1}{4}$ SUSY.

2.2 Explicit solution

We can obtain the Killing spinors explicitly working in the Weyl basis, we refer the reader to the appendix for a brief review.

The relevant operators are

$$\Gamma_{@} = E \sigma_2 1 1 1 \sigma_1 \quad , \quad \Gamma_{@} = s g(B) 1 1 1 1 \sigma_1 \quad (2.5)$$

The spinorial space is divided in two 16-dimensional (complex) subspaces with $\Gamma_{@} = \pm 1$; each one is expanded by the vectors

$$\epsilon_{(s_1, \ldots, s_4)}^{(\pm)} = (0 s_1 \ldots s_4) \mp i E (1 s_1 \bar{s}_2 s_3 \bar{s}_4)$$

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The last property is a result of the strict commutation of both operators that allows for the common diagonalisation in each subspace separately. We are of course interested in the subspace with \( \Gamma \varepsilon = +1 \); on it we can introduce the following set of basis vectors

\[
\eta^{(\pm)}(s_{1}s_{2}s_{3}) = \varepsilon(\pm)(s_{1}s_{2}s_{3}) \pm sg(B) \varepsilon^{(\pm)}(s_{1}s_{2}s_{3}),
\]

From here we conclude that the spinors \( \{\eta^{(+)}(s_{1}s_{2}s_{3})\}, \{\eta^{(-)}(s_{1}s_{2}s_{3})\} \) expand the eight-dimensional complex space of Killing vectors for a \( D2 \) (\( \bar{D}2 \)) -brane extended in (012) directions with field strength defined by \( (E_{2} = E, B) \), \( |E| = 1 \), and satisfying conditions (2.2),(2.3). Then a general Killing spinor of the \( D2 \)-brane admits the expansion

\[
\epsilon = \sum_{s_{1},s_{2},s_{3}} \varepsilon^{(s_{1}s_{2}s_{3})} \eta^{(+)}(s_{1}s_{2}s_{3}), \quad \epsilon^{(s_{1}s_{2}s_{3})} \in \mathcal{C}
\]

The observation made by BK is that a \( \bar{D}2 \) with different fields \( (E_{2} = E, -B) \), \( |E| = 1 \), must have the same Killing spinors. This fact follows immediately from the equality \( \eta^{(-)}(s_{1}s_{2}s_{3})|_{-B} = \eta^{(+)}(s_{1}s_{2}s_{3})|_{B} \). Therefore we should be able to put arbitrary (parallel) number of \( D2 \)-branes and \( \bar{D}2 \)-branes of the characteristics defined above and such configurations must preserve \( \frac{1}{4} \) SUSY with the corresponding Killing spinors given by (2.8).

Finally it is worth to spend some words about the Majorana condition to be imposed. In a Majorana basis where all the \( \Gamma \)-matrices are real (or purely imaginary) the constraint is straightforward because we can take \( D = 1 \) in such a basis. But it is not so in a Weyl basis; this is a price to be paid for working in a setting where computations are relatively easy to handle in any dimension. We get from imposing (A.5) on the spinor (2.8)

\[
\varepsilon^{(s_{1}s_{2}s_{3})} = i sg(BE)(-)^{1+s_{1}+s_{3}} \varepsilon(\bar{s}_{1}\bar{s}_{2}\bar{s}_{3})
\]

This completes the characterization of the Killing spinors of the BK solution.

After sketched with this known example the route to follow we move to higher dimensional cases.

### 3 The \( D3-\bar{D}3 \) system

Let us consider a flat D3-brane extended in directions \( (X^{0}, X^{1}, X^{2}, X^{3}) \) with field strength

\[
(f_{\mu}^{\nu}) = \begin{pmatrix}
0 & E_{1} & E_{2} & 0 \\
E_{1} & 0 & 0 & 0 \\
E_{2} & 0 & 0 & B \\
0 & 0 & -B & 0
\end{pmatrix}
\]

(3.1)
where we have taken with no loss of generality the electromagnetic fields in the plane (12). The $\Gamma$-matrix is then given by

$$
\Gamma = \begin{pmatrix}
    d & -1/2 \\
    -1/2 & (i \sigma_2 \otimes \Gamma)_{0123} - \sigma_1 \otimes (E_1 \Gamma_{23} + E_2 \Gamma_{31} - B \Gamma_{01}) - E_1 B i \sigma_2 \otimes 1_{32}
\end{pmatrix}
$$

(3.2)

The equation to solve (1.1) is written as

$$
\Gamma \left( \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} \right) = \left( \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} \right)
$$

(3.3)

where $(\epsilon_1, \epsilon_2)$ are Majorana-Weyl spinors of the same chirality. A minus sign on the rhs applies for the antibrane. We have found two possible solutions to (3.3).

### 3.1 Solution I

Consistency of $\otimes$ will require the constraint $E_1^2 + E_2^2 = 1$ which implies $d = B^2 E_2^2$ and then $B$ and $E_2$ non zero, fact that we will assume; the ansatz is

$$
\Gamma_{\otimes} \epsilon_1 = \epsilon_1, \quad \Gamma_{\otimes} \epsilon_2 = -\epsilon_2 \quad ; \quad \Gamma_{\otimes} = -E_1 \Gamma_{01} - E_2 \Gamma_{02}
$$

(3.4)

$$
\Gamma_{\overline{\otimes}} \epsilon_1 = \epsilon_2, \quad \Gamma_{\overline{\otimes}} = \frac{sgn(B)}{|E_2|} (E_1 1 + \Gamma_{01})
$$

(3.5)

with a minus sign in (3.3) for the $\overline{D}3$-brane. The compatibility for both constraints requires that

$$
\Gamma_{\otimes} \Gamma_{\overline{\otimes}} \Gamma_{\otimes}^{-1} \epsilon_1 = -\Gamma_{\overline{\otimes}} \epsilon_1
$$

(3.6)

A short computation shows

$$
\{ \Gamma_{\otimes}, \Gamma_{\overline{\otimes}} \} = \frac{2 E_1 \text{sgn}(B)}{|E_2|} (\Gamma_{\otimes} - 1)
$$

(3.7)

that says that in the subspace with $\Gamma_{\otimes} = 1$ to which $\epsilon_1$ belongs (3.6) is obeyed.

The explicit solution can be worked out; first we introduce $(E_\pm = E_1 \pm i E_2)$

$$
\epsilon^{(\pm)}_{(s_1 \ldots s_4)} = (0 s_1 \ldots s_4) \mp (-)^{\sum_{k=1}^4 s_k} (1 s_1 s_2 s_3 s_4)
$$

$$
\Gamma_{\otimes} \epsilon^{(\pm)}_{(s_1 \ldots s_4)} = \pm \epsilon^{(\pm)}_{(s_1 \ldots s_4)}
$$

$$
\Gamma_{\overline{\otimes}} \epsilon^{(\pm)}_{(s_1 \ldots s_4)} = -i \text{sgn}(B E_2) \epsilon^{(-)}_{(s_1 \ldots s_4)}
$$

(3.8)

The last line shows the compatibility of both constraints; the imposition of (3.7) gives for both spinors the following general form

$$
\epsilon_1 = \sum_{s_1, \ldots, s_4} \epsilon^{(s_1 \ldots s_4)}_{(s_1 \ldots s_4)} \epsilon^{(\pm)}_{(s_1 \ldots s_4)}
$$

$$
\epsilon_2 = -i \text{sgn}(B E_2) \sum_{s_1, \ldots, s_4} \epsilon^{(s_1 \ldots s_4)}_{(s_1 \ldots s_4)} \epsilon^{(-)}_{(s_1 \ldots s_4)}
$$

(3.9)
The complex parameters $\epsilon^{(s_1...s_4)}$ expand a 32-dimensional space; however the spinors must be Weyl and Majorana. Because $\Gamma_{11}(\epsilon^{(+)}_{(s_1...s_4)}) = (-)^{1+\sum_{k=1}^{4}s_k}$ we must constraint this value to $+1 (-1)$ if we decide to take both of them left- (right) handed, so one index, e.g. $s_4$, will be fixed by this condition. Finally the Majorana condition

$$e^{(s_1...s_4)^*} = E_+ (-)^{s_2+s_4} e^{(s_1...s_4)}$$

(3.10)

shows that the solution has indeed 8 supercharges. Conditions (3.4), (3.5) can be interpreted as dissolved fundamental strings in the plane (12) at angle $\arctan \frac{E_2}{E_1}$ and $D1$-branes in the $\tilde{e}_1$-direction respectively. This solution for a $D3$-brane can be extended to a $\bar{D}3$-brane with the same Killing spinors by reverting the direction of the magnetic field, $B \rightarrow -B$, as in the BK solution. In fact it is easy to see that we can reach it by T-dualizing in $\tilde{e}_1$ and boosting with $\beta = -E_1$ in that direction, remaining with the dual $D2$-brane in the $(023)$-hyperplane, electric field $E = sg(E_2)$ in $\tilde{e}_2$ and magnetic field $B$.

### 3.2 Solution II

Here we will work out a new ansatz. The idea is to ask for the cancellation of the field-dependent part in (3.2). Compatibility of this ansatz imposes the constraints $E_1 = 0$ and $B^2 = E_2^2 \neq 0$, i.e. it is just a solution for orthogonal electric and magnetic fields with the same module. The corresponding conditions are

$$\Gamma_{@} \epsilon_i = \epsilon_i, \ i = 1,2 \ , \ \Gamma_{@} = -sg(B E_2) \Gamma_{03}$$

(3.11)

$$\Gamma_{@} \epsilon_1 = \epsilon_2, \ \Gamma_{@} = -\Gamma_{0123}$$

(3.12)

The further properties $\Gamma_{@}^2 = -\Gamma_{@}^2 = 1$, $[\Gamma_{@}; \Gamma_{@}] = 0$ assure the existence of solutions to these equations.

To get the Killing spinors associated to these systems we first solve $@$ through

$$\epsilon^{(s_1...s_4)}_{(s_1...s_4)} = (s_1 \ldots s_4 0) + sg(B E_2) (-)^{1+\sum_{k=2}^{4}s_k} (s_1 \bar{s}_2 s_3 s_4 1)$$

$$\Gamma_{@} \epsilon^{(+)}_{(s_1...s_4)} = \epsilon^{(+)}_{(s_1...s_4)}$$

$$\Gamma_{@} \epsilon^{(+)}_{(s_1...s_4)} = i sg(B E_2) (-)^{s_1} \epsilon^{(+)}_{(s_1...s_4)}$$

(3.13)

Then (3.12) yields the general solution

$$\epsilon_1 = \sum_{s_1,...,s_4} \epsilon^{(s_1...s_4)} \epsilon^{(+)}_{(s_1...s_4)}$$

$$\epsilon_2 = i sg(B E_2) \sum_{s_1,...,s_4} (-)^{s_1} \epsilon^{(s_1...s_4)} \epsilon^{(+)}_{(s_1...s_4)}$$

(3.14)

As with the Solution I a Weyl left (or right) condition $(-)^{1+\sum_{k=1}^{4}s_k} = +1 (-1)$ must be imposed; together with the Majorana constraint

$$\epsilon^{(s_1...s_4)^*} = (-)^{1+s_2+s_4} \epsilon^{(s_1...s_4)}$$

(3.15)
it is shown that the solution preserves $\frac{1}{2}$ SUSY. Furthermore conditions (3.11) and (3.12) correspond to the SUSY's preserved by a p-p wave moving on $\tilde{e}_3$ direction and a D3-brane in (0123), the existence of such configuration of intersecting branes being known since time ago (see for example reference [11]); the SUSY of the D3-brane configuration results then only sensitive to that of the constituents induced by the background fields.

On the other hand for the $\bar{D}_3$-branes (3.12) has a minus sign on the rhs; what is the same it is replaced by $\Gamma^{\bar{a}} e_2 = \epsilon_1$, i.e. both spinors are interchanged, operation that obviously leaves invariant (3.11). In view of the existence of the $O(2)$ automorphism group in type IIB string theory which rotates the supersymmetry generators we are tempted to conjecture the existence of systems of branes and antibranes in arbitrary number provided that on each one live fields $(E_2, B)$ with $|E_2| = |B|$ but otherwise arbitrary, the only condition common to all of them being to have the same $sg(B E_2)$.

4 The $D4$-$\bar{D}4$ system

Now we take a flat D4-brane extended in directions $(X^0, X^1, X^2, X^3, X^4)$

$$ (f^\mu_{\nu}) = \begin{pmatrix} 0 & E_1 & 0 & E_2 & 0 \\ E_1 & 0 & B_1 & 0 & 0 \\ 0 & -B_1 & 0 & 0 & 0 \\ E_2 & 0 & 0 & 0 & B_2 \\ 0 & 0 & 0 & -B_2 & 0 \end{pmatrix} \tag{4.1} $$

We again do not lose generality doing so because the anti-symmetric matrix $(f^i_{\ j}) = (F_{ij}), i, j = 1, \ldots, 4,$ can be put in the standard form $i \sigma_2 \otimes \begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix}$ by means of a $SO(4)$-rotation; a $SO(2) \times SO(2)$ is then left over that we fix by putting the electric field $E_i \equiv F_{i0}$ to the form in (4.1). The $\Gamma$-matrix is

$$ \Gamma = |d|^{-\frac{1}{2}} (\Gamma_{11} \Gamma_{01234} + E_1 \Gamma_{234} + E_2 \Gamma_{124} - B_1 \Gamma_{034} - B_2 \Gamma_{012} + E_1 B_2 \Gamma_{11} + E_2 B_1 \Gamma_{14} \Gamma_{11} - B_1 B_2 \Gamma_0 \Gamma_{11}) \\ d = 1 - E_1^2 - E_2^2 + B_1^2 + B_2^2 - E_1^2 B_2^2 - E_2^2 B_1^2 \tag{4.2} $$

We have found two solutions to equation (1.1).

4.1 Solution I

Consistency of $\otimes$ will require the constraint $E_1^2 + E_2^2 = 1$ which we will assume; the ansatz is as in (2.2), (2.3) where the relevant operators are

$$ \Gamma_{\otimes} = (E_1 \Gamma_{01} + E_2 \Gamma_{03}) \Gamma_{11} \tag{4.3} $$

\[\text{More generically, any anti-symmetric matrix in } d\text{-dimensions can be written as } B = k^d \, d \, k \text{ where } k \in SO(d)/SO(2)^d \text{ and } d = i \sigma_2 \otimes \text{diag}(B_1, \ldots, B_{d/2}) \text{ (a 0 is added if } d \text{ is odd). It can be shown that in the general case when the magnetic field matrix } (B_{ij}) = (F_{ij}) \text{ has inverse there is no boost compatible with } (4.1). \text{ However it is possible in this case to go to a frame where there is no electric field, being the boost velocity } \beta = -B^{-1} \tilde{E}, \text{ if the condition } \beta^2 = \tilde{E}^t (-B)^{-1} \tilde{E} < 1 \text{ is fulfilled.}\]
\[ \Gamma_{\otimes} = d^{1/2} \left( -B_1 \Gamma_{034} - B_2 \Gamma_{012} + (E_1 B_2 \Gamma_2 + E_2 B_1 \Gamma_4 - B_1 B_2 \Gamma_0) \Gamma_{11} \right) \quad (4.4) \]

where \( d = B_1^2 B_2^2 + B_1^2 E_1^2 + B_2^2 E_2^2 > 0 \). Assuming \( \otimes \) is obeyed, the consistency conditions \((\Gamma_{\otimes} \otimes - 1) \epsilon = [\Gamma_{\otimes} ; \Gamma_{\otimes}] \epsilon = 0\) follow from

\[
\begin{align*}
\Gamma_{\otimes}^2 &= 1 + \frac{2B_1 B_2}{d} \Gamma_{1234} (\Gamma_{\otimes} - 1) \\
[\Gamma_{\otimes} ; \Gamma_{\otimes}] &= 2 (E_1 B_2 \Gamma_2 + E_2 B_1 \Gamma_4) (\Gamma_{11} (1 - \Gamma_{\otimes}) \quad (4.5)
\end{align*}
\]

From standard arguments we should get again a solution preserving \( 1/4 \) SUSY.

The explicit solution can be obtained as it was made in the precedent cases; first we introduce a basis for the subspace of spinors obeying \( \otimes \),

\[
\begin{align*}
\epsilon^{(+)}_{(s_1 \ldots s_4)} &= (s_1 \ldots s_4 0) - E_1 (\bar{s}_1 s_2 s_3 s_4 1) - E_2 (-)^{s_1} (s_1 \bar{s}_2 s_3 s_4 1) \\
\Gamma_{\otimes} \epsilon^{(+)}_{(s_1 \ldots s_4)} &= + \epsilon^{(+)}_{(s_1 \ldots s_4)}
\end{align*}
\]

In terms of them we can express a basis which also diagonalize \( \Gamma_{\otimes} \),

\[
\begin{align*}
\eta^{(\pm)}_{(s_1 s_2 s_3)} &= \epsilon^{(+)}_{(s_1 s_2 s_3)} \pm a(s) \epsilon^{(+)}_{(s_1 s_2 s_3)} \pm b(s) \epsilon^{(\pm)}_{(s_1 s_2 s_3)} \\
\Gamma_{\otimes} \eta^{(\pm)}_{(s_1 s_2 s_3)} &= \pm \eta^{(\pm)}_{(s_1 s_2 s_3)} \quad (4.6)
\end{align*}
\]

where

\[
\begin{align*}
a(s) &= \frac{E_1 B_1}{\sqrt{d}} (B_2 + i (-)^{s_1 + s_2 + s_3}) \\
b(s) &= \frac{E_2 B_2}{\sqrt{d}} ((-1)^{s_1} B_1 + i (-)^{s_2 + s_3}) \quad (4.8)
\end{align*}
\]

So we conclude that \( \{\eta^{(+)}_{(s_1 s_2 s_3)}\} \) is a basis of Killing spinors for (parallel superposition of) \( D4 \)-branes, while \( \{\eta^{(-)}_{(s_1 s_2 s_3)}\} \) it is for a \( \bar{D}4 \)-brane (with the same fields as the \( D4 \)-brane). However to get brane-antibrane BPS systems the preserved supersymmetries must coincide. Condition \( \otimes \) implies that both \( D4 \) and \( \bar{D}4 \) branes must have the same electric field, but \( \otimes \) implies that the magnetic fields must have opposite signs and not only this, direct inspection of (4.4) (or (4.7)) shows that \( B_1 B_2 = 0 \) necessary must hold. This gives two possible solutions, related by permutations of the planes (12) and (34).

**Case** \( B_1 = 0; \ d = B_2^2 E_2^2 \)

Then \( E_2 \) and \( B_2 \) are non zero; the \( \bar{D}4 \)-brane will have \( E_2 \) and \(- B_2 \) fields as said. From (4.7), (4.8) the Killing spinors reduce to

\[
\eta^{(+)}_{(s_1 s_2 s_3)} = \epsilon^{(+)}_{(s_1 s_1 s_2 s_3)} + i s g(E_2 B_2) (-)^{s_2 + s_3} \epsilon^{(+)}_{(s_1 s_1 s_2 s_3)} \quad (4.9)
\]

**Case** \( B_2 = 0; \ d = B_1^2 E_1^2 \)

Now \( E_1 \) and \( B_1 \) must be non zero; the \( \bar{D}4 \)-brane will have \( E_1 \) and \(- B_1 \) fields. The Killing spinors reduce to

\[
\eta^{(+)}_{(s_1 s_2 s_3)} = \epsilon^{(+)}_{(s_1 s_1 s_2 s_3)} + i s g(E_1 B_1) (-)^{s_1 + s_2 + s_3} \epsilon^{(+)}_{(s_1 s_1 s_2 s_3)} \quad (4.10)
\]
Again we can identify (1.3) and (1.4) with $F1$ charge in (01) and $D2$-brane charge in (034) respectively; these solutions are T-duals to the Solutions I just considered (for example in the last case, by means of a T-duality in $\tilde{c}_3$, a boost in that direction with $\beta = -E_2$ and further T-duality in $\tilde{c}_3$ we go back to BK solution). The Majorana condition for a generic Killing spinor (2.8) reads
\[ \epsilon^{(s_1 s_2 s_3)} = (-)^{1+s_1+s_3} \epsilon^{(s_1 s_2 s_3)} \] (4.11)

### 4.2 Solution II

Here we present a new solution giving a $D4$-$\bar{D}4$ system. The ansatz (this time consists in imposing the cancellation of the part of (1.2) even in the fields by itself. Consistency imposes the constraint
\[ B_1^2 B_2^2 - B_1^2 B_2^2 E_2^2 - B_2^2 E_1^2 = 1 \] (4.12)
which replaces the $\bar{E}^2 = 1$ constraint of the BK-type solutions and that we will assume henceforth. Let us note that $|B_1 B_2| \geq 1$, in particular $B_{ij}$ must be non singular; also from (1.12) $d = 2 - E_1^2 - E_2^2 + B_1^2 + B_2^2 \geq 2$.

The corresponding conditions are as in (2.2), (2.3) where the relevant operators are
\[ \Gamma_{@} = -B_1 E_2 \Gamma_{0123} - B_2 E_1 \Gamma_{0134} - B_1 B_2 \Gamma_{1234} \] (4.13)
\[ \Gamma_{@} = d^{-\frac{1}{2}} (E_1 \Gamma_{234} + E_2 \Gamma_{124} - B_1 \Gamma_{034} - B_2 \Gamma_{012}) \] (4.14)

Consistency of the ansatz follows from $\Gamma_{@} = 1 - \frac{1}{d} (\Gamma_{@} - 1)^2$, $[\Gamma_{@};\Gamma_{@}] = 0$, which shows that there should exists solution preserving eight supersymmetries. In order to write it we introduce a basis in which $\Gamma_{@}$ is diagonal
\[ \tilde{\epsilon}_{(s_1 \ldots s_5)} = (s_1 \ldots s_5) - i \alpha_1 (-)^{\sum_{k=2}} (s_1 s_2 s_3 s_4 s_5) \]
\[ \Gamma_{@} \tilde{\epsilon}_{(s_1 \ldots s_5)} = s g(B_1 B_2) (-)^{s_1 + s_2} \tilde{\epsilon}_{(s_1 \ldots s_5)} \] (4.15)
where $\alpha_1 = \frac{sg(B_1 B_2) B_2 E_1}{1 + |B_1 B_2|}$, $\alpha_2 = \frac{sg(B_1 B_2) B_1 E_2}{1 + |B_1 B_2|}$. From here it is clear that a basis for the space obeying $\Gamma_{@}\epsilon = \epsilon$ consists of the spinors
\[ \tilde{\epsilon}_{(s_1 s_2 s_3 s_4 s_5)} \equiv \epsilon_{(s_1 \cdots s_5)} (-)^{s_1} = sg(B_1 B_2)(-)^{s_2} \] (4.16)

With the definitions
\[ A_1 = \frac{B_1 (1 + B_2^2 - E_2^2) + sg(B_1 B_2) B_2 (1 + B_1^2 - E_1^2)}{\sqrt{d} (1 + |B_1 B_2|)} \]
\[ A_2 = \frac{E_1 E_2 (B_2 - sg(B_1 B_2) B_1)}{\sqrt{d} (1 + |B_1 B_2|)} \] (4.17)

$((A_1^2 + A_2^2 = 1))$ the Killing spinors result
\[ \eta_{(s_1 s_2 s_3)} = \epsilon_{(s_1 s_2 s_3)}^{(+)} + i sg(B_1 B_2) (A_1 (-)^{\sum_{k=1}^3} \epsilon_{(s_1 s_2 s_3,0)}^{(s_k)} + A_2 (-)^{\sum_{k=2}^3} \epsilon_{(s_1 s_2 s_3,0)}^{(s_k)}) \]
\[ \Gamma_{\partial} \eta_{(s_1 s_2 s_3)}^{(\pm)} = \pm \eta_{(s_1 s_2 s_3)}^{(\pm)} \] (4.18)

It is easy to see that the $\bar{D}4$-brane solution (corresponding to have a minus sign in the (\(\partial\))-condition) will have the same Killing spinors provided that it has both electric and magnetic fields with opposite signs wrt that of the $D4$-brane. Therefore (2.8) with $\eta_{(s_1 s_2 s_3)}^{(+)}$ given in (4.18) is the general Killing spinor of such brane-antibrane systems.

It is worth to note however that (4.12) implies \(0 < ||B^{-1}E||^2 = 1 + \left(\frac{E_1}{B_1}\right)^2 + \left(\frac{E_2}{B_2}\right)^2 = 1 - \frac{1}{B_1 B_2} < 1\), and therefore from the footnote at the beginning of this Section we know that there exists a boost with \(\vec{\beta} = -B^{-1}E\) that eliminates the electric field; a further rotation will lead us to the case \(\vec{E} = \vec{0}\) (with different $B_1$, $B_2$). The Majorana condition in this case looks like in (4.11).

5 Conclusions

We have studied in the context of the Born-Infeld effective action the existence of supersymmetric, presumably stable solutions of $D2$, $D3$ and $D4$-branes preserving a quarter of the supersymmetries of the flat background in which they are embedded. These results allow to conjecture at the light of the compatible ansatz in solutions II, the existence of configurations other than the T-dual to those presented here, for $D5$ and $D6$ branes, and from here for $D7 - D\bar{D}7$ and $D8 - D\bar{D}8$ and so on. An explicit prove of these facts along the lines followed here should be straightforward.

In the case of the $D4$ brane we note that equations (4.13), (4.14) imply the existence of Taub-NUT charge and a sort of $D2$-brane charge for this solution, no $D4$-brane charge is present; the $D4-D\bar{D}4$ systems should represent genuine bound states of these components. What is more, it is plausible that a five dimensional supertube-like solution exists, leading in a certain limit to the brane-antibrane system much as it happens with the supertube.

Another interesting open problem is certainly to find the explicit form of the corresponding supergravity solutions since as it is stressed in reference [2] the low energy analysis presented here does not assure by itself the complete absence of instabilities. To this goal the knowledge of the world-volume fields as well as the explicit form of the Killing spinors (although they do not take into account the back-reaction) could be of great help. All these items are under current investigation [12].

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A Appendix

We briefly summarize conventions and formalism used in the text (see for example [13]).
We start with the representation of the anti-commutation relation \( \{b; b^\dagger\} = 1 \) in a two-dimensional space expanded by the vectors \( |s> \) where \( s = 1 \) (spin up) or \( s = 0 \) (spin down). In this basis we take \( |1> \rightarrow (1) \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |0> \rightarrow (0) \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \). Then the representation of the fermion algebra is given by the matrices

\[
b = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad b^\dagger = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}
\]  

(A.1)

Then \( \Gamma_1 \equiv b + b^\dagger = \sigma_1 \) and \( \Gamma_2 \equiv -i (b - b^\dagger) = -\sigma_2 \) satisfy \( \{\Gamma_M; \Gamma_N\} = 2 \delta_{MN}, M, N = 1, 2 \), where \( \{\sigma_i, i = 1, 2, 3\} \) are the Pauli matrices. Thus \( \sigma_1(s) = \bar{s}, \sigma_2(s) = i(-)^s(s), \sigma_3(s) = (-)^s(s) \), where we define \( \bar{s} \equiv 1 - s \). The number operator \( N \equiv b^\dagger b \) obeys \( N|s> = s|s> \).

The Weyl basis for the vector space of spinors in \( d = 10 \) can be constructed as the tensor product of five copies of the space defined above, and consists therefore of 32 vectors denoted by \((s_1) \otimes \ldots \otimes (s_5) \equiv (s_1 \ldots s_5)\). The euclidean Clifford algebra \( \{\Gamma^M; \Gamma^N\} = 2 \delta_{MN} \) is realized in this space by the matrices

\[
\Gamma_{2k-1} = (-)^{k-1} \sigma_3 \otimes \ldots \otimes \sigma_3 \otimes \sigma_1 \otimes 1 \otimes \ldots \otimes 1 \\
\Gamma_{2k} = (-)^k \sigma_3 \otimes \ldots \otimes \sigma_3 \otimes \sigma_2 \otimes 1 \otimes \ldots \otimes 1
\]  

(A.2)

where \( k = 1, \ldots, 5 \), and \( \sigma_1 \), respectively \( \sigma_2 \), is placed in the \( k \)-position. We will omit in general the tensor product symbol \( \otimes \). By definition, \( \Gamma_0 \equiv i \Gamma_{10} = -i \sigma_3 \sigma_3 \sigma_3 \sigma_3 \sigma_2 \), and then \( \{\Gamma_M; \Gamma_N\} = 2 \eta_{MN}, M, N = 0, 1, \ldots 9 \), where \( (\eta_{MN}) = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \), \( 1_n \) standing for the \( n \times n \) identity matrix.

The chirality matrix is defined by

\[
\Gamma_{11} \equiv i \Gamma_1 \ldots \Gamma_{10} = \Gamma_1 \ldots \Gamma_9 \Gamma_0 = \sigma_3 \sigma_3 \sigma_3 \sigma_3 \sigma_3 \\
\Gamma_{11} (s_1 \ldots s_5) = (-)^{\sum_{k=1}^{5} s_k} (s_1 \ldots s_5)
\]  

(A.3)

from where it follows that states with number of spins down even (odd) have positive (negative) chirality.

As usual we denote \( \Gamma_{M_1 \ldots M_n}, n \leq 10 \), to the completely antisymmetric product of the \( n \Gamma \)-matrices, i.e. \( \Gamma_{1 \ldots n} = \Gamma_1 \ldots \Gamma_n \). In particular \( S_{MN} \equiv S(X_{MN}) = \frac{1}{2} \Gamma_{MN} \) gives the spinorial representation of the Lorentz generators satisfying the standard algebra

\[
[X_{MN}, X_{PQ}] = \eta_{MQ} X_{NP} + \eta_{NP} X_{MQ} - (M \leftrightarrow N)
\]  

(A.4)

However this representation is reducible due to the fact that \( [\Gamma_{11}; S_{MN}] = 0 \). This leads to define \( S^{(\pm)} \equiv \frac{1}{2} (1 \pm \Gamma_{11}) S_{MN} \); thus \( S_{MN} = S_{MN}^{(+)} + S_{MN}^{(-)} \) decomposes in two irreducible representations with \( \Gamma_{11} = +1 \) (positive chirality) and \( \Gamma_{11} = -1 \) (negative chirality) of the Lorentz algebra, or Weyl-left and Weyl-right spinors space respectively.

A Majorana condition is a reality condition; to be able to define it in some fixed basis we need a matrix \( D \) satisfying \( D^{-1} S_{MN} D = S^*_{MN} \) in such a way that if \( \Psi \) is a spinor, e.g.
a 32-dimensional vector (in $d = 10$) transforming linearly in the spinorial representation of the Lorentz group, then $\Psi^c \equiv D \Psi^*$ also it is; $\Psi^c$ is called the conjugate spinor \[14\]. Then it has sense to impose the Majorana condition

$$\Psi^c \equiv D \Psi^* = \Psi$$  \hspace{1cm} (A.5)

In the Weyl basis a matrix $D$ verifying $D^{-1} \Gamma_M D = \Gamma_M^*$, $M, N, = 0, 1, \ldots , 9$, can be taken as

$$D \equiv -\Gamma_2 \Gamma_4 \Gamma_6 \Gamma_8 = \sigma_1 \sigma_2 \sigma_1 \sigma_2 1$$  \hspace{1cm} (A.6)

It is worth to note that if $D$ is such a matrix in a basis $\{|\alpha >, \alpha = 1, \ldots , 32\}$, under a change of basis $|\alpha > = P_{\alpha} \beta |\beta >'$ it transforms as $D' = P^{-1} D P^*$.

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