Surface detonation in type Ia supernova explosions?

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Abstract. We explore the evolution of thermonuclear supernova explosions when the progenitor white dwarf star ignites asymmetrically off-center. Several numerical simulations are carried out in two and three dimensions to test the consequences of different initial flame configurations such as spherical bubbles displaced from the center, more complex deformed configurations, and teardrop-shaped ignitions. The burning bubbles float towards the surface while releasing energy due to the nuclear reactions. If the energy release is too small to gravitationally unbind the star, the ash sweeps around it, once the burning bubble approaches the surface. Collisions in the fuel on the opposite side increase its temperature and density and may – in some cases – initiate a detonation wave which will then propagate inward burning the core of the star and leading to a strong explosion. However, for initial setups in two dimensions that seem realistic from pre-ignition evolution, as well as for all three-dimensional simulations the collimation of the surface material is found to be too weak to trigger a detonation.

1. Introduction

Type Ia supernovae are generally associated with the thermonuclear explosion of earth-sized white dwarf (WD) stars – the final stages of the evolution of small and intermediate-mass stars. These dense objects consist of carbon and oxygen and their pressure is dominated by degenerate electrons. In a binary system, the WD can gain mass from its companion by accretion. However, there exists a fundamental limit, the Chandrasekhar mass, beyond which the star is unstable against gravitational collapse. Before reaching this stage, the density at the center of the WD reaches values at which nuclear reactions from carbon to heavier elements ignite. This establishes a stage of convective carbon burning which is finally terminated when a thermonuclear runaway forms a flame. The thermonuclear flame propagates outward and may give rise to an explosion of the star [1, 2]. The process resembles premixed chemical combustion where two solutions – supersonic detonation and subsonic deflagration – are admissible. From observational constraints it is clear that the flame starts out as a subsonic deflagration. The ashes are buoyant and shear instabilities generate strong turbulence. The interaction of the flame with turbulence is a key feature in the model and accelerates the flame. Consequently, one of the outstanding problems is to determine, whether this turbulent flame leads to explosions energetic enough to be consistent with observations, or whether a later transition to a supersonic detonation is both possible and necessary. In the freely expanding medium of SN Ia explosions, such transitions are difficult to achieve.

The exact way of flame ignition is not well determined yet, but previous numerical studies [3, 4] suggest that the ignition process may proceed on only one side of the star’s center in a few patches. While it has been shown that ignitions in multiple spots distributed around the center
of the WD lead to strong explosions [5], one-sided ignitions are expected to release less energy. The effects of the latter scenario on the explosion process are investigated in the present study. It has been speculated that collisions between gravitationally bound laterally expanding regions of fuel and ash could, in principle, give rise to a “gravitationally confined detonation” [6]. This possibility is studied in detail.

2. Numerical model and computation

The code used for simulating the thermonuclear supernova explosions on scales of the WD star has been described in detail previously [7, 8, 9]. Since it is not possible to resolve all relevant spatial scales of the problem, an approach similar to Large Eddy Simulations was chosen.

The modeling of the hydrodynamics is based on the PROMETHEUS [10] implementation of the piecewise parabolic method [11]. Our flame model consists of three components. Flame propagation is followed via a level-set approach. It associates the flame with the zero level set of a signed distance function \( G \) which is evolved in a suitable way [7] to account for the propagation. This flame prescription, however, does not resolve the scales that determine the propagation velocity. In the flamelet regime of turbulent combustion, which applies to the situation in thermonuclear supernovae (possibly with the exception of the very late stages, e.g. [12]) the flame velocity is proportional to the turbulent velocity fluctuations. These are determined from a turbulent subgrid-scale model [13] featuring localized closures to take into account the non-isotropic nature of the problem. The energy release due to nuclear burning is handled in a simplified way. Only five species are considered: carbon and oxygen of the initial WD material, \(^{24}\text{Mg}\) representing intermediate mass elements, alpha particles, and \(^{56}\text{Ni}\) as a representative of iron group nuclei. At fuel densities higher than \(5.25 \times 10^7 \text{g cm}^{-3}\), the material crossed by the flame is burned to nuclear statistical equilibrium, modeled as a mixture of alpha particles and \(^{56}\text{Ni}\). Below, this density burning is incomplete and produces intermediate mass elements. The appropriate amounts of energy are released in the ashes. At densities below \(10^7 \text{g cm}^{-3}\) the reactions cease.

The code is efficiently parallelized on the basis of domain decomposition in an MPI implementation. While two-dimensional simulations can be run easily on a multi-processor desktop computer, three-dimensional simulations require a supercomputer. The code has been used on several machines and proved good scaling behavior. The three-dimensional simulations described here were carried out on a \([512]^3\) cells computational grid required about 20,000 CPU hours each on the NCCS Jaguar Cray XT3 supercomputer. Simulations of this size were performed on 256 processors.

3. Simulations

In our model, the flame is artificially ignited. Therefore the initial flame configuration is an undetermined parameter. Since it is not constrained tightly by complementary studies, several numerical experiments with asymmetric ignitions have been performed. We considered initial flame configurations in form of a single spherical bubble placed at different radii off-center of the WD, more complex bubble-substructures and perturbations from perfect sphericity, and (on average) teardrop-shaped initial flames extending down to the center of the star or even overshooting it.

In Fig. 1, the deflagration flame was ignited in a single spherical bubble 200 km off-center. This bubble is filled with hot ashes of lower density than the surrounding fuel and therefore is buoyant. As it burns and floats, the flame shape evolves from a sphere to a torus in the first tenths of a second (upper left panel of Fig. 1). Continuing towards the surface, the torus becomes distorted by growing features that eventually connect.

The energy released during the flame’s transit from the center to the surface is too small to explode the WD, which remains gravitationally bound. Once the bubble of ash breaks out, it
starts to sweep around the core, colliding on the diametrically opposite side of the star. In the collision, the temperature and the density of unburnt material are increased.

The question is whether this increase is sufficient to initiate a detonation. A detonation wave would propagate inwards burning the core of the star and leading to an explosion. Clearly, the strength of the collision depends upon the initial parameters mentioned above, all of which alter the energy release in the nuclear burning and hence the expansion of the WD prior the the surface material collision. These parameters were tested in about 20 two-dimensional simulations. Since such simulations impose an artificial cylindrical symmetry and, moreover, turbulence behaves differently in 2D and 3D simulations, neither the nuclear energy release nor the collision strength are expected to be realistic. Therefore, the 2D simulations merely provide a way of exploring the dependencies. Five three-dimensional simulations were carried out in addition to assess what might realistically happen in a supernova.
4. Results and conclusions

Initiation of a detonation wave in the collision region depends on the temperature reached, the density, and the spatial extent of the collision region. For our 2D simulations ignited in a spherical bubble, an inverse correlation exists between nuclear energy release prior to collision and the maximum temperature reached (cf. Fig. 2). This correlation arises naturally from the expansion of the star due to the energy release. The more material is burnt in the flame on its way to the surface, the more the star expands and the weaker is the collision.

The energy release depends on the displacement of the initial flame from the center. Initiating the flame at larger radii leads to less burning and thus to stronger collisions of the surface material. The collision strength is very sensitive to perturbations of the initial flame from perfect sphericity, either directly by imposing more complex initial configurations (teardrop-like shapes, multiple initial flame kernels) or by different numerical resolution and resulting discretization errors.

To trigger a detonation, a collision temperature over $2 \times 10^9$ K is necessary. However, this threshold temperature needs to be reached in fuel with density higher than $\sim 3 \ldots 10 \times 10^6$ g cm$^{-3}$. Therefore, the maximum temperature in the collision region at a density exceeding these values was measured. The results are plotted in Fig. 3. From this measurement, we find that a detonation initiation is only possible in those 2D models that start with a spherical initial flame placed at unrealistically large radii ($R \geq 200$ km) off-center of the WD star (cf. Fig. 3).

3D simulations are cooler in the collimation region for the same nuclear energy release. However, except for one model (included in Figs. 2 and 3) the 3D simulations release more energy in the nuclear burning than the 2D simulations. In the model shown in Fig. 1, the energy release amounted to $\sim 2.8 \times 10^{50}$ erg and the temperature in the collision region barely exceeded $10^9$ K. For the one-sided teardrop-shaped initial flame the energy release was even greater and one single-bubble ignition and the two-sided teardrop-ignition unbound the star. All 3D simulations clearly failed to trigger a detonation.

We conclude, that although a detonation due to the colliding surface material is in principle possible for specific (and artificial) ignition configurations, it is unlikely to happen in nature.
models that remain gravitationally bound, failures to initiate a detonation will lead to pulsations of the WD star. This may be a second chance for triggering a detonation ("pulsational delayed detonation scenario", [14]) and will be addressed in a follow-up study.

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