We obtain several fixed point theorems for hybrid pairs of single-valued and multivalued occasionally weakly compatible maps defined on a symmetric space satisfying a contractive condition of integral type. The results of this paper essentially contain every theorem on hybrid and multivalued self-maps of a metric space as a special case.

Copyright © 2007 M. Abbas and B. E. Rhoades. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

1. Introduction and preliminaries

The study of fixed point theorems, involving four single-valued maps, began with the assumption that all of the maps commuted. Sessa [1] weakened the condition of commutativity to that of pairwise weakly commuting. Jungck generalized the notion of weak commutativity to that of pairwise compatible [2] and then pairwise weakly compatible maps [3]. In the recent paper of Jungck and Rhoades [4], the concept of occasionally weakly commuting maps (owc) was introduced. In that paper, it was shown that essentially every theorem involving four maps becomes a special case of one of the results on owc maps. In this paper, we show that the same is true for the theorems involving four maps, in which two of them are multivalued and for which the contractive condition is of integral type. Branciari [5] obtained a fixed point theorem for a single valued mapping satisfying an analogue of Banach’s contraction principle for an integral-type inequality. Rhoades [6] proved two fixed point theorems involving more general contractive conditions (see also [7–9]). The aim of this paper is to extend the concept of occasionally weakly compatible maps to hybrid pairs of single-valued and multivalued maps in the setting of symmetric space satisfying a contractive condition of integral type. Our results complement, extend, and unify comparable results in the literature.
Consistent with [10–12], we will use the following notations, where $(X,d)$ is a metric space, for $x \in X$ and $A \subseteq X$, $d(x,A) = \inf \{d(y,A) : y \in A\}$, and $CB(X)$ is the class of all nonempty bounded and closed subsets of $X$. Let $H$ be a Hausdorff metric induced by the metric $d$ of $X$, given by

$$H(A,B) = \max \left\{ \sup_{x \in A} d(x,B), \sup_{y \in B} d(y,A) \right\}$$  \hspace{1cm} (1.1)$$

for every $A,B \in CB(X)$.

**Definition 1.1.** Let $X$ be a set. A symmetric on $X$ is a mapping $d : X \times X \to [0,\infty)$ such that

$$d(x,y) = 0 \quad \text{iff} \quad x = y,$$

$$d(x,y) = d(y,x).$$  \hspace{1cm} (1.2)$$

A set $X$ together with a symmetric $d$ is called a symmetric space.

**Definition 1.2.** Maps $f : X \to X$ and $T : X \to CB(X)$ are said to be occasionally weakly compatible (owc) if and only if there exists some point $x$ in $X$ such that $fx \in Tx$ and $fTx \subseteq Tfx$.

The following lemma due to Dube [13] will be used.

**Lemma 1.3.** Let $A,B \in CB(X)$, then for any $a \in A$,

$$d(a,B) \leq H(A,B).$$  \hspace{1cm} (1.3)$$

**Example 1.4.** Let $X = [0,\infty)$ with usual metric. Define $f : X \to X$, $T : X \to CB(X)$ by

$$fx = \begin{cases} 0, & 0 \leq x < 1, \\ 2x, & 1 \leq x < \infty, \end{cases}$$

$$Tx = \begin{cases} \{x\}, & 0 \leq x < 1, \\ [1,1+4x], & 1 \leq x < \infty. \end{cases}$$  \hspace{1cm} (1.4)$$

It can be easily verified that $x = 1$ is coincidence point of $f$ and $T$, but $f$ and $T$ are not weakly compatible there. However, the pair $\{f, T\}$ is occasionally weakly compatible.

## 2. Common fixed point theorems

In this section, we establish several common fixed point theorems for hybrid pairs of single-valued and multivalued maps defined on a symmetric space, which is more general than a metric space. Define $F = \{ \phi : \mathbb{R}^+ \to \mathbb{R}^+ : \phi \text{ is a Lebesgue integral mapping which is summable, nonnegative, and satisfies } \int_0^\epsilon \phi(t) dt > 0, \text{ for each } \epsilon > 0 \}$.

**Theorem 2.1.** Let $f$, $g$ be self-maps of a metric space $(X,d)$ and let $T$, $S$ be maps from $X$ into $CB(X)$ such that the pairs of $\{f,T\}$ and $\{g,S\}$ are owc. If

$$\int_0^{H(Tx,Sy)} \varphi(t) dt < \int_0^{M(x,y)} \varphi(t) dt,$$  \hspace{1cm} (2.1)$$
where \( \varphi \in F \) and
\[
M(x, y) = \max \{ d(fx, gy), d(fx, Tx), d(gy, Sy), d(fx, Sy), d(gy, Tx) \} \tag{2.2}
\]
for all \( x, y \in X \) for which (2.2) is positive. Then \( f, g, T \) and \( S \) have a common fixed point.

**Proof.** By hypothesis, there exist points \( x, y \in X \) such that \( fx \in Tx, gy \in Sy, fTx \in Tf x, \) and \( gSy \subseteq Sy \). Using the triangle inequality and Lemma 1.3, we obtain
\[
d(f^2x, g^2y) \leq H(Tfx, Sgy).
\]
We first show that \( gy = fx \). Suppose not. Then consider
\[
M(fx, gy) = \max \{ d(f^2x, g^2y), d(f^2x, Tfx), d(g^2y, Sgy), d(f^2x, Sgy), d(g^2y, Tfx) \}
\leq H(Tfx, Sgy).
\tag{2.3}
\]
Condition (2.1) then implies that
\[
\int_0^{H(Tfx, Sgy)} \varphi(t)dt < \int_0^{M(fx, gy)} \varphi(t)dt \leq \int_0^{H(Tfx, Sgy)} \varphi(t)dt,
\tag{2.4}
\]
which is a contradiction and hence \( gy = fx \). Using the triangle inequality, we obtain
\[
d(fx, g^2y) \leq H(Tx, Sfx).
\]
Next, we claim that \( x = fx \). If not, then consider
\[
M(x, fx) = \max \{ d(fx, g^2y), d(fx, Tx), d(g^2y, Sgy), d(gy, Sgy), d(g^2y, Tfx) \}
\leq H(Tx, Sfx).
\tag{2.5}
\]
Condition (2.1) implies
\[
\int_0^{H(Tx, Sgy)} \varphi(t)dt < \int_0^{M(x, fx)} \varphi(t)dt = \int_0^{H(Tx, Sgy)} \varphi(t)dt,
\tag{2.6}
\]
which is again a contradiction and the claim follows. Similarly, we obtain \( y = gy \). Thus \( f, g, T, \) and \( S \) have a common fixed point. \( \square \)

**Theorem 2.2.** Let \( f, g \) be self-maps of the symmetric space \((X, d)\) and let \( T, S \) be maps from \( X \) into \( CB(X) \) such that the pairs of \( \{ f, T \} \) and \( \{ g, S \} \) are owc. If
\[
\int_0^{(H(Tx, Sy))} \varphi(t)dt < \int_0^{M_p(x, y)} \varphi(t)dt,
\tag{2.7}
\]
where \( \varphi \in F \) and
\[
M_p(x, y) = \alpha(d(gy, Tx))^p + (1 - \alpha) \max \{ (d(fx, Tx))^p, (d(gy, Sy))^p, (d(fx, Txy))^{p/2}, (d(gy, Txy))^{p/2}, (d(gy, Tx))^{p/2}, (d(fx, Sy))^{p/2} \},
\tag{2.8}
\]
for all \( x, y \in X \) for which (2.8) is not zero, \( \alpha, \beta \in (0, 1], \) and \( p \geq 1. \) Then \( f, g, T \) and \( S \) have a common fixed point.
Proof. By hypothesis, there exist points $x, y$ in $X$ such that $fx \in Tx, gy \in Sy, fTx \subseteq Tfx,$ and $gSy \subseteq Sgy$. We first show that $gy = fx$. Suppose not. Then consider

$$M_p(fx, gy) = \alpha(d(g^2y, Tfx))^p + (1 - \alpha) \max \{(d(f^2x, Tfx))^p, (d(g^2y, Sgx))^p, (d(f^2x, Tfx))^p/2 (d(g^2y, Tfx))^p/2, (d(g^2y, Tfx))^p/2 (d(f^2x, Sgx))^p/2\}$$

(2.9)

$$= \alpha(d(g^2y, Tfx))^p + (1 - \alpha)(d(g^2y, Tfx))^p/2 (d(f^2x, Sgx))^p/2 \leq \alpha(H(Tfx, Sgy))^p + (1 - \alpha)(H(Tfx, Sgy))^p = (H(Tfx, Sgy))^p.$$

Condition (2.7) then implies that

$$\int_0^{(H(Tfx, Sgy))^p} \varphi(t) dt < \int_0^{M_p(fx, gy)} \varphi(t) dt \leq \int_0^{(H(Tfx, Sgy))^p} \varphi(t) dt,$$

(2.10)

which is a contradiction, and hence $gy = fx$. Now, we claim that $x = fx$. If not, then since $fx = gy$,

$$M_p(x, fx) = \alpha(d(gfx, Tx))^p + (1 - \alpha) \max \{(d(fx, Tx))^p, (d(gfx, Sfx))^p, (d(fx, Tx))^p/2 (d(gfx, Tx))^p/2, (d(gfx, Tx))^p/2 (d(fx, Sfx))^p/2\}$$

(2.11)

$$= \alpha(d(gfx, Tx))^p + (1 - \alpha)(d(g^2y, Tx))^p/2 (d(fx, Sgy))^p/2 \leq \alpha(H(Tx, Sgy))^p + (1 - \alpha)(H(Tx, Sgy))^p = (H(Tx, Sgy))^p.$$

Condition (2.7) then implies that

$$\int_0^{(H(Tx, Sgy))^p} \varphi(t) dt < \int_0^{M_p(x, gy)} \varphi(t) dt \leq \int_0^{(H(Tx, Sgy))^p} \varphi(t) dt,$$

(2.12)

which is again a contradiction, and the claim follows. Similarly, we obtain $y = gy$. Thus, $f, g, T,$ and $S$ have a common fixed point.

Corollary 2.3. Let $f, g$ be self-maps of a metric space $(X, d)$ and let $T$, $S$ be maps from $X$ into $CB(X)$ such that the pairs of $\{f, T\}$ and $\{g, S\}$ are owc. If

$$\int_0^{H(Tx, Sy)} \varphi(t) dt < \int_0^{M(x, y)} \varphi(t) dt,$$

(2.13)

where $\varphi \in \Gamma$ and

$$M(x, y) = h \max \left\{d(fx, gy), d(fx, Tx), d(gy, Sy), \frac{1}{2} \left[d(fx, Sy) + d(gy, Tx)\right]\right\}$$

(2.14)
for all \(x, y \in X\) for which (2.14) is not zero and \(h \in [0, 1)\). Then \(f, g, T,\) and \(S\) have a common fixed point.

**Proof.** Since (2.14) is a special case of (2.2), the result follows immediately from Theorem 2.1. \(\square\)

Every contractive condition of integral type automatically includes a corresponding contractive condition, not involving integrals, by setting \(\varphi(t) = 1\) over \(\mathbb{R}^+\). Theorem 1 of [14], [15, Theorem 2.3], and [16, Theorem 2] are special cases of Corollary 2.3. Also [17, Theorem 2] and [18, Theorem 1] become special cases of the corollary if we take \(S = T\) and \(f = g\).

**Corollary 2.4.** Let \(f\) be a self-map of the symmetric space \((X, d)\) and let \(T\) be a map from \(X\) into \(CB(X)\) such that \(f\) and \(T\) are owc and for all \(x, y \in X\) for which (2.16) is not zero,

\[
\int_0^{H(Tx, Ty)} \varphi(t) dt < \int_0^{M(x, y)} \varphi(t) dt,
\]

where \(\varphi \in \mathcal{F}\) and

\[
M(x, y) = \max \left\{ d(fx, Ty), \frac{1}{2} [d(fx, Tx) + d(fy, Ty)], \frac{1}{2} [d(fy, Tx) + d(fx, Ty)] \right\}.
\]

(2.15)

Then \(f\) and \(T\) have a common fixed point.

**Proof.** Since (2.16) is the special case of (2.2) with \(S = T\) and \(f = g\), the result follows immediately from Theorem 2.1. \(\square\)

**Corollary 2.5.** Let \(f, g\) be self-maps of a metric space \((X, d)\) and \(T, S\) be maps from \(X\) into \(CB(X)\) such that the pairs of \(\{f, T\}\) and \(\{g, S\}\) are owc and for all \(x \neq y \in X\),

\[
\int_0^{H(Tx, Sy)} \varphi(t) dt < \int_0^{M(x, y)} \varphi(t) dt,
\]

where \(\varphi \in \mathcal{F}\) and

\[
M(x, y) = \alpha d(fx, gy) + \beta \max \{d(fx, Tx), d(gy, Sy)\} + \gamma \max \{d(fx, gy), d(fx, Sy), d(gy, Tx)\},
\]

(2.17)

with \(\alpha, \beta, \gamma > 0\) and \(\alpha + \beta + \gamma = 1\). Then \(f, g, T,\) and \(S\) have a common fixed point.

**Proof.** Since (2.18) is a special case of (2.2), the result follows immediately from Theorem 2.1. \(\square\)

Define \(G = \{g : \mathbb{R}^5 \to \mathbb{R}^+\}\) such that

\(g_1\): \(g\) is nondecreasing in the 4th and 5th variables,

\(g_2\): if \(u, v \in \mathbb{R}^+\) are such that \(u \leq g(v, v, u + v, 0), u \leq g(v, u, v, u + v, 0), v \leq g(u, u, v, u + v, 0),\) or \(u \leq g(v, u, v, u + v, 0),\) then \(u \leq hv\), where \(0 < h < 1\) is constant,
(g₃) if ₚ ∈ ℝ₊ is such that ₚ ≤ ₚ (₀, ₀, ₀, ₀), ₚ ≤ ₚ (₀, ₁, ₀, ₀) or ₚ ≤ ₚ (₀, ₀, ₀, ₀),
then ₚ = ₀.

**Theorem 2.6.** Let ₚ, ₚ be self-maps of the metric space (X, d) and let ₚ, ₚ be maps from X
into CB(X) such that the pairs of {ₚ, ₚ} and {ₚ, ₚ} are owc. If

\[
\int_0^{H(Tgx)} \varphi(t) dt
\]

\[
< g\left( \int_0^{d(tx, gy)} \varphi(t) dt, \int_0^{d(tx, ty)} \varphi(t) dt, \int_0^{d(gy, gy)} \varphi(t) dt, \int_0^{d(gy, ty)} \varphi(t) dt, \int_0^{d(gy, ty)} \varphi(t) dt \right),
\]

where ₚ ∈ F and for all ₓ, ₖ ∈ X for which the right-hand side of (2.19) is not zero, where
ₚ ∈ G, then ₚ, ₚ, ₚ, and ₚ have a common fixed point.

**Proof.** By hypothesis, there exist points ₓ, ₙ ∈ X such that ₚ ∈ Tₓ, ₚ ∈ Sₓ, ₚ Tₓ ⊆ Tₙₓ, and ₚ Sₓ ⊆ Sₙₓ. Also, using the triangle inequality and Lemma 1.3, we obtain d(ₚ, ₚ) ≤
H(Tₓ, Sₓ). First, we show that ₚ = ₚ. Suppose not. Then condition (2.19) implies that

\[
\int_0^{H(Tgx)} \varphi(t) dt < g\left( \int_0^{d(tx, gy)} \varphi(t) dt, 0, 0, \int_0^{d(tx, ty)} \varphi(t) dt, 0, \int_0^{d(gy, ty)} \varphi(t) dt, \int_0^{d(gy, ty)} \varphi(t) dt \right),
\]

which, from (g₃), gives \( \int_0^{H(Tgx)} \varphi(t) dt = 0 \), and hence \( H(Tgx, Sgx) = 0 \), which implies that
d(ₚ, ₚ) = 0. Hence the claim follows. Using the triangle inequality, we obtain d(ₚ, ₚ) ≤
H(Tₙₓ, Sₙₓ). Next, we claim that ₚ = ₚ. If not, then condition (2.19) implies that

\[
\int_0^{H(Tgx, Sy)} \varphi(t) dt < g\left( \int_0^{d(Tgx, Sy)} \varphi(t) dt, 0, 0, \int_0^{d(Tgx, Sy)} \varphi(t) dt, 0, \int_0^{d(Syx, Sy)} \varphi(t) dt, \int_0^{d(Syx, Sy)} \varphi(t) dt \right),
\]

which, from (g₃), gives \( H(Tgx, Sy) = 0 \), which implies that d(ₚ, ₚ) = 0. Hence the
claim follows. Similarly, it can be shown that ₚ = ₚ which proves the result. □

A control function ₚ is defined by ₚ : ℝ₊ → ℝ₊ which is continuous monotonically
increasing, ₚ(₂) ≤ ₂ ₚ(₁) and ₚ(₀) = ₀ if and only if ₚ = ₀. Let ₚ : ℝ₊ → ℝ₊ be such that
ₚ(t) < ₚ for each ₚ > ₀.

**Theorem 2.7.** Let ₚ, ₚ be self-maps of the metric space (X, d) and let ₚ, ₚ be maps from X
into CB(X) such that the pairs of {ₚ, ₚ} and {ₚ, ₚ} are owc. If

\[
\int_0^{\Phi(H(Tgx, Sgx))} \varphi(t) dt < \Phi\left( \int_0^{M(x, y)} \varphi(t) dt \right),
\]

(2.22)
where $\varphi \in F$ and

$$M(x, y) = \max \left\{ \Phi(d(fx, gy)), \Phi(d(fx, Tx)), \Phi(d(gy, Sy)), \frac{1}{2} \left[ \Phi(d(fx, Sy)) + \Phi(d(gy, Tx)) \right] \right\}$$

(2.23)

for all $x, y \in X$ for which (2.23) is not zero. Then $f, g, T$ and let $S$ have a common fixed point.

Proof. By hypothesis, there exist points $x, y$ in $X$ such that $fx \in Tx, gy \in Sy, fTx \subseteq Tfx,$ and $gSy \subseteq Sgy.$ Also, using the triangle inequality, we obtain $d(fx, gy) \leq H(Tx, Sy).$

First, we show that $H(Tx, Sy) = 0.$ Suppose not. Then consider

$$M(x, y) = \max \left\{ \Phi(d(fx, gy)), 0, 0, \frac{1}{2} \Phi(2H(Tx, Sy)) \right\} = \Phi(H(Tx, Sy)).$$

(2.24)

Condition (2.22) implies that

$$0 < \int_{0}^{\Phi(H(Tx, Sy))} \varphi(t) dt < \Psi \left( \int_{0}^{M(x, y)} \varphi(t) dt \right) < \int_{0}^{\Phi(H(Tx, Sy))} \varphi(t) dt,$$

(2.25)

which is a contradiction. Therefore $H(Tx, Sy) = 0,$ which implies that $d(fx, gy) = 0.$ Hence the claim follows. Using the triangle inequality, we obtain $d(fx, f^2x) \leq H(Tfx, Sy).$ Next, we claim that $H(Tfx, Sy) = 0.$ If not, then consider

$$M(fx, y) = \max \left\{ \Phi(d(f^2x, gy)), 0, 0, \frac{1}{2} \Phi(2H(Tfx, Sy)) \right\} = \Phi(H(Tfx, Sy)).$$

(2.26)

Then condition (2.22) implies that

$$0 < \int_{0}^{\Phi(H(Tfx, Sy))} \varphi(t) dt < \Psi \left( \int_{0}^{M(fx, y)} \varphi(t) dt \right) < \int_{0}^{\Phi(H(Tfx, Sy))} \varphi(t) dt,$$

(2.27)

which is a contradiction. Therefore $H(Tfx, Sy) = 0,$ which implies that $d(fx, f^2x) = 0.$ Hence the claim follows. Similarly, it can be shown that $gy = g^2y,$ which proves the result.

Theorem 1 of [19] and [20, Theorem 1] become special cases of Theorem 2.7 with $\Phi(x) = 1.$

Remark 2.8. It is natural to ask if integral contractive conditions are indeed generalizations of corresponding contractive conditions not involving integrals. We illustrate this fact with an example. In [6, Theorem 4], a unique fixed point was established for a self-map of complete metric space $X$ satisfying the integral condition

$$\int_{0}^{d(Tx, Ty)} \varphi(t) dt \leq h \int_{0}^{M(x, y)} \varphi(t) dt,$$

(2.28)
for all \( x, y \in X \), where \( 0 \leq h < 1 \) and
\[
M(x, y) = \max \{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}.
\] (2.29)

It was also assumed that there was a point in \( X \) with bounded orbit.

If there exists points \( x, y \) in \( X \) for which \( d(Tx, Ty) \geq M(x, y) \), then one obtains a contradiction to (2.28). Therefore for all \( x, y \) in \( X \),
\[
d(Tx, Ty) < M(x, y).
\] (2.30)

Even if one assumes the continuity of \( T \), Taylor [21] has shown that there exists a map as \( T \) satisfying (2.30), with bounded orbit, but which does not possess a fixed point.

Acknowledgment

The first author gratefully acknowledges support provided by Lahore University of Management Sciences (LUMS) during his stay at Indiana University Bloomington as a Post doctoral Fellow.

References

[1] S. Sessa, “On a weak commutativity condition of mappings in fixed point considerations,” *Publications de l’Institut Mathématique*, vol. 32(46), pp. 149–153, 1982.
[2] G. Jungck, “Compatible mappings and common fixed points,” *International Journal of Mathematics and Mathematical Sciences*, vol. 9, no. 4, pp. 771–779, 1986.
[3] G. Jungck, “Common fixed points for noncontinuous nonself maps on nonmetric spaces,” *Far East Journal of Mathematical Sciences*, vol. 4, no. 2, pp. 199–215, 1996.
[4] G. Jungck and B. E. Rhoades, “Fixed point theorems for occasionally weakly compatible mappings,” *Fixed Point Theory*, vol. 7, no. 2, pp. 287–296, 2006.
[5] A. Branciari, “A fixed point theorem for mappings satisfying a general contractive condition of integral type,” *International Journal of Mathematics and Mathematical Sciences*, vol. 29, no. 9, pp. 531–536, 2002.
[6] B. E. Rhoades, “Two fixed-point theorems for mappings satisfying a general contractive condition of integral type,” *International Journal of Mathematics and Mathematical Sciences*, vol. 2003, no. 63, pp. 4007–4013, 2003.
[7] A. Aliouche, “A common fixed point theorem for weakly compatible mappings in symmetric spaces satisfying a contractive condition of integral type,” *Journal of Mathematical Analysis and Applications*, vol. 322, no. 2, pp. 796–802, 2006.
[8] P. Vijayaraju, B. E. Rhoades, and R. Mohanraj, “A fixed point theorem for a pair of maps satisfying a general contractive condition of integral type,” *International Journal of Mathematics and Mathematical Sciences*, vol. 2005, no. 15, pp. 2359–2364, 2005.
[9] X. Zhang, “Common fixed point theorems for some new generalized contractive type mappings,” *Journal of Mathematical Analysis and Applications*, vol. 333, no. 2, pp. 780–786, 2007.
[10] S. B. Nadler Jr., “Multi-valued contraction mappings,” *Pacific Journal of Mathematics*, vol. 30, pp. 475–488, 1969.
[11] S. V. R. Naidu, “Fixed points and coincidence points for multimaps with not necessarily bounded images,” *Fixed Point Theory and Applications*, vol. 2004, no. 3, pp. 221–242, 2004.
[12] S. L. Singh and S. N. Mishra, “Coincidences and fixed points of nonself hybrid contractions,” *Journal of Mathematical Analysis and Applications*, vol. 256, no. 2, pp. 486–497, 2001.
[13] L. S. Dube, “A theorem on common fixed points of multi-valued mappings,” Annales de la Société Scientifique de Bruxelles, vol. 89, no. 4, pp. 463–468, 1975.

[14] A. Azam and I. Beg, “Coincidence points of compatible multivalued mappings,” Demonstratio Mathematica, vol. 29, no. 1, pp. 17–22, 1996.

[15] T. Kamran, “Common coincidence points of R-weakly commuting maps,” International Journal of Mathematics and Mathematical Sciences, vol. 26, no. 3, pp. 179–182, 2001.

[16] O. Hadzic, “Common fixed point theorems for single-valued and multivalued mappings,” Review of Research. Faculty of Science. Mathematics Series, vol. 18, no. 2, pp. 145–151, 1988.

[17] H. Kaneko and S. Sessa, “Fixed point theorems for compatible multi-valued and single-valued mappings,” International Journal of Mathematics and Mathematical Sciences, vol. 12, no. 2, pp. 257–262, 1989.

[18] H. Kaneko, “A common fixed point of weakly commuting multi-valued mappings,” Mathematica Japonica, vol. 33, no. 5, pp. 741–744, 1988.

[19] T. H. Chang, “Common fixed point theorems for multivalued mappings,” Mathematica Japonica, vol. 41, no. 2, pp. 311–320, 1995.

[20] P. K. Shrivastava, N. P. S. Bawa, and S. K. Nigam, “Fixed point theorems for hybrid contractions,” Varahmihir Journal of Mathematical Sciences, vol. 2, no. 2, pp. 275–281, 2002.

[21] L. E. Taylor, “A contractive mapping without fixed points,” Notices of the American Mathematical Society, vol. 24, p. A-649, 1977.

Mujahid Abbas: Department of Mathematics, Indiana University, Bloomington, IN 47405-7106, USA
Current address: Department of Mathematics, Lahore University of Management Sciences, Lahore 54792, Pakistan
Email address: mujahid@lums.edu.pk

B. E. Rhoades: Department of Mathematics, Indiana University, Bloomington, IN 47405-7106, USA
Email address: rhoades@indiana.edu
Special Issue on
Space Dynamics

Call for Papers

Space dynamics is a very general title that can accommodate a long list of activities. This kind of research started with the study of the motion of the stars and the planets back to the origin of astronomy, and nowadays it has a large list of topics. It is possible to make a division in two main categories: astronomy and astrodynamics. By astronomy, we can relate topics that deal with the motion of the planets, natural satellites, comets, and so forth. Many important topics of research nowadays are related to those subjects. By astrodynamics, we mean topics related to spacecraft dynamics.

It means topics where a satellite, a rocket, or any kind of man-made object is travelling in space governed by the gravitational forces of celestial bodies and/or forces generated by propulsion systems that are available in those objects. Many topics are related to orbit determination, propagation, and orbital maneuvers related to those spacecrafts. Several other topics that are related to this subject are numerical methods, nonlinear dynamics, chaos, and control.

The main objective of this Special Issue is to publish topics that are under study in one of those lines. The idea is to get the most recent researches and published them in a very short time, so we can give a step in order to help scientists and engineers that work in this field to be aware of actual research. All the published papers have to be peer reviewed, but in a fast and accurate way so that the topics are not outdated by the large speed that the information flows nowadays.

Before submission authors should carefully read over the journal's Author Guidelines, which are located at http://www.hindawi.com/journals/mpe/guidelines.html. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/ according to the following timetable:

|                          |             |
|--------------------------|-------------|
| Manuscript Due           | July 1, 2009|
| First Round of Reviews   | October 1, 2009|
| Publication Date         | January 1, 2010|

Lead Guest Editor

Antonio F. Bertachini A. Prado, Instituto Nacional de Pesquisas Espaciais (INPE), São José dos Campos, 12227-010 São Paulo, Brazil; prado@dem.inpe.br

Guest Editors

Maria Cecilia Zanardi, São Paulo State University (UNESP), Guaratinguetá, 12516-410 São Paulo, Brazil; cecilia@feg.unesp.br

Tadashi Yokoyama, Universidade Estadual Paulista (UNESP), Rio Claro, 13506-900 São Paulo, Brazil; tadashi@rc.unesp.br

Silvia Maria Giuliani Winter, São Paulo State University (UNESP), Guaratinguetá, 12516-410 São Paulo, Brazil; silvia@feg.unesp.br