Sensorless Control of Vehicle-mounted PMSM Based on Improved Sliding Mode Observer

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Abstract: In order to solve the chattering and phase delay problems in the traditional sliding mode observer(SMO), this paper proposes an improved sliding mode observer, which effectively weakens the chattering by using a new piecewise function as the switching function. At the same time, angle compensation is provided for the phase delay caused by the introduction of a low-pass filter, and then the arctangent function is used to extract the rotor position and speed information of the PMSM. Compared with the traditional sliding mode observer, from the simulation results, we can see that the improved control system has obvious advantages in reducing chattering, improving convergence speed, and improving speed estimation accuracy.

1. Introduction
Electric vehicles have entered the market and become a part of the car family. The precise control of the traditional on-board PMSM system must rely on photoelectric encoders and resolvers. However, the installation of these sensors will increase the cost and volume of the motor and reduce the reliability. Based on the above reasons, many companies and organizations have put forward a position sensorless control strategy after in-depth research on PMSM. Compared with other control strategies, SMO control has huge advantages[1].

The SMO reconstructs the motor system state equation according to the PMSM mathematical model, and uses a high-gain nonlinear function to force the system state to enter the sliding mode as quickly as possible and stabilize at the equilibrium point, so that the system state equation is completely equal to the motor mathematical model. To complete the observation of the system state, accurate back EMF information can be obtained through observation[2].

In this control system, the selected function is generally the sgn function, and the switching frequency is not infinitely fast, so that the system state can only be switched near the stable point instead of being stable at the equilibrium point, and chattering problems occur. Therefore, the current research direction on sliding mode control is how to reduce chattering. Because the low-pass filter is used to filter high-frequency harmonics, this causes the problem of phase delay. How to solve this problem is also another important direction of SMO control.

Literature [3] designed a sliding mode control strategy with adaptive gain and a filter with variable cutoff frequency to overcome the traditional SMO problem. At the same time, a variable phase delay compensation method is used to correctly compensate the phase delay of the estimated rotor angular position. Literature [4] replaces the sgn function with the sigmoid function, and applies the full-order sliding mode observation technique to the stator resistance estimation to compensate for the adverse effects of the motor stator resistance change on the entire system. Literature [5] proposed an adaptive synchronous filter for rotor position estimation with quadrature phase-locked loop feedback, which was used to extract the fundamental signal existing in the back electromotive force and effectively suppress
the chattering of the system.
This paper proposes an improved sliding mode observer to solve chattering and phase delay problems. First, a new type of piecewise function is used as the switching function. Second, an angle compensation is added to the estimated rotor position to solve the phase delay problem caused by the introduction of a low-pass filter. Finally, the arctangent function is used to extract the rotor position and speed information of the PMSM, and the improved method proposed is compared with the traditional method through MATLAB/Simulink.

2. PMSM mathematical model
For the convenience of calculation and analysis, the following assumptions are made for the PMSM:

(1) Ignore the magnetic saturation problem of permanent magnets;
(2) Ignore the air gap permeance and the internal permeance of permanent magnets;
(3) There is no damping winding on the rotor;
(4) The stator and rotor magnetic field and the induced back EMF are distributed in accordance with the sinusoid;

According to the above assumptions and regulations, the expression of the PMSM stator voltage equation in the α, β coordinate system is:

\[
\begin{align*}
L_d \frac{d}{dt} i_d &= \left[ R + pL_d \omega_e (L_d - L_q) \right] i_d + e_d - R i_d, \\
L_q \frac{d}{dt} i_q &= \left[ R + pL_q \omega_e (L_q - L_d) \right] i_q + e_q.
\end{align*}
\]

Where \( L_d \) and \( L_q \) are Stator inductance; \( \omega_e \) is the electrical angular velocity; \( p \) is a differential operator; \( \left[ i_d \quad i_q \right]^T \) is the stator current; \( \left[ e_d \quad e_q \right]^T \) is the extended back-EMF; \( \theta_e \) is the angle between the d axis and the \( \alpha \) axis; \( \psi_f \) is the permanent magnet flux of the motor; \( i_d \) and \( i_q \) are the stator current in a two-phase rotating coordinate system;

It can be seen from equation (2) that the back-EMF equation includes the rotor position \( \theta_e \).

The PMSM voltage equation (1) is rewritten as the current state equation as follows (3):

\[
\begin{align*}
\frac{d}{dt} \left[ \begin{array}{c} \dot{i}_d \\ \dot{i}_q \end{array} \right] &= \frac{1}{L_d} \left[ \begin{array}{c} -R -\omega_e (L_d - L_q) \\ \omega_e (L_q - L_d) \end{array} \right] \left[ \begin{array}{c} \dot{i}_d \\ \dot{i}_q \end{array} \right] + \frac{1}{L_d} \left[ \begin{array}{c} u_d \\ u_q \end{array} \right] - \frac{1}{L_d} \left[ \begin{array}{c} e_d \\ e_q \end{array} \right].
\end{align*}
\]

3. Traditional SMO model
The SMO reconstructs the motor system state equation according to the PMSM mathematical model, and uses a high-gain nonlinear function to force the system state to enter the sliding mode as quickly as possible and stabilize at the equilibrium point, so that the system state equation is completely equal to the motor mathematical model. Complete the observation of the system status. The construction of a traditional SMO is shown in equation (4):

\[
\frac{d}{dt} \left[ \begin{array}{c} \dot{i}_d \\ \dot{i}_q \end{array} \right] = \frac{1}{L_d} \left[ \begin{array}{c} -R -\omega_e (L_d - L_q) \\ \omega_e (L_q - L_d) \end{array} \right] \left[ \begin{array}{c} \dot{i}_d \\ \dot{i}_q \end{array} \right] + \frac{1}{L_d} \left[ \begin{array}{c} u_d \\ u_q \end{array} \right] - \frac{k}{L_d} \left[ \begin{array}{c} \text{sgn}(\dot{i}_d - i_d) \\ \text{sgn}(\dot{i}_q - i_q) \end{array} \right]
\]

Where \( k \) is the sliding mode gain factor; \( \dot{i}_d \) and \( \dot{i}_q \) are Observed value of stator current;

By making the difference between equation (4) and equation (3), the error equation of the observer can be obtained as shown in equation (5):

\[
\frac{d}{dt} \left[ \begin{array}{c} \dot{e}_d \\ \dot{e}_q \end{array} \right] = \frac{1}{L_d} \left[ \begin{array}{c} -R -\omega_e (L_d - L_q) \\ \omega_e (L_q - L_d) \end{array} \right] \left[ \begin{array}{c} \dot{e}_d \\ \dot{e}_q \end{array} \right] + \frac{1}{L_d} \left[ \begin{array}{c} u_d \\ u_q \end{array} \right] - \frac{k}{L_d} \left[ \begin{array}{c} \text{sgn}(\dot{i}_d - i_d) \\ \text{sgn}(\dot{i}_q - i_q) \end{array} \right]
\]
\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} \tilde{i}_a \\ \tilde{i}_\beta \end{bmatrix} &= \frac{1}{L_d} \begin{bmatrix} -R & -\omega_d (L_d - L_q) \\ \omega_d (L_d - L_q) & -R \end{bmatrix} \begin{bmatrix} \tilde{i}_a \\ \tilde{i}_\beta \end{bmatrix} + \frac{1}{L_d} \begin{bmatrix} e_a - k \cdot \text{sgn} \tilde{i}_a \\ e_\beta - k \cdot \text{sgn} \tilde{i}_\beta \end{bmatrix} \\
\end{align*}
\]

(5)

Where \( \tilde{i}_a = (\hat{i}_a - i_a) \) and \( \tilde{i}_\beta = (\hat{i}_\beta - i_\beta) \) are current observation error;

In IPMSM \( L_d = L_q = L_s \), Therefore, equation (5) can be simplified as:

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} \tilde{i}_a \\ \tilde{i}_\beta \end{bmatrix} &= \begin{bmatrix} -R & 0 \\ 0 & -R \end{bmatrix} \begin{bmatrix} \tilde{i}_a \\ \tilde{i}_\beta \end{bmatrix} + \frac{1}{L_s} \begin{bmatrix} e_a - k \cdot \text{sgn} \tilde{i}_a \\ e_\beta - k \cdot \text{sgn} \tilde{i}_\beta \end{bmatrix} \\
\end{align*}
\]

(6)

When the current error approaches zero, you can get:

\[
\begin{bmatrix} e_a \\ e_\beta \end{bmatrix} = \begin{bmatrix} k \cdot \text{sgn} \tilde{i}_a \\ k \cdot \text{sgn} \tilde{i}_\beta \end{bmatrix}
\]

(7)

After obtaining the observed back-EMF information, it can be known from the equation (2) that the back-EMF equation can obtain the required information by obtaining the arctangent and differentiation of the \( \alpha \) axis and the \( \beta \) axis, as shown in equation (8):

\[
\begin{align*}
\dot{\hat{\theta}} &= -\tan^{-1} (\hat{\omega}_a / \hat{\omega}_b) \\
\hat{\omega}_e &= \frac{d\hat{\theta}}{dt}
\end{align*}
\]

(8)

Where \( \dot{\hat{\theta}} \) is the estimated value of the motor rotor position; \( \hat{\omega}_e \) is the estimated value of the motor rotor speed;

Its structural block diagram is shown in Figure 1 below:

**Fig. 1** Block diagram of traditional SMO

4. **Improved SMO model**

4.1 **Improved piecewise function**

In order to obtain a smooth back EMF and suppress the chattering generated by the sign function, a new type of piecewise function is designed in this paper to replace the sign function. The new piecewise function is as follows:
\[ h(x) = \begin{cases} 
1 & x \geq \Delta \\
x^2 + z\Delta & 0 \leq x \leq \Delta \\
-x^2 - z\Delta & -\Delta < x < 0 \\
-1 & x \leq -\Delta 
\end{cases} \] (9)

Where \( x \) is the current observation error; \( \Delta \) is the boundary layer thickness; \( z > 0 \). By selecting a reasonable value of \( z \), chattering can be further reduced.

According to equation (9), an improved SMO can be constructed as shown in equation (10):

\[
\frac{d}{dt} \begin{bmatrix} \hat{i}_a \\ \hat{i}_c \end{bmatrix} = \begin{bmatrix} -R & 0 \\ L_s & -R \end{bmatrix} \begin{bmatrix} \hat{i}_a \\ \hat{i}_c \end{bmatrix} + \frac{1}{L_s} \begin{bmatrix} \hat{u}_a \\ \hat{u}_c \end{bmatrix} - \frac{k}{L_s} \begin{bmatrix} h(\hat{i}_a - i_a) \\ h(\hat{i}_c - i_c) \end{bmatrix} \] (10)

By making the difference between equation (10) and equation (3), the observer error equation can be obtained as shown in equation (11):

\[
\frac{d}{dt} \begin{bmatrix} \tilde{i}_a \\ \tilde{i}_c \end{bmatrix} = \begin{bmatrix} -R & 0 \\ L_s & -R \end{bmatrix} \begin{bmatrix} \tilde{i}_a \\ \tilde{i}_c \end{bmatrix} + \frac{1}{L_s} \begin{bmatrix} c_a - k h(\hat{i}_a) \\ c_c - k h(\hat{i}_c) \end{bmatrix} \] (11)

4.2. Phase compensation

In order to obtain a smooth observation of the back EMF, a low-pass filter needs to be used to filter out other harmonics. The expression of the back EMF processed by the low-pass filter is shown in equation (12)

\[
\begin{bmatrix} \hat{e}_a \\ \hat{e}_c \end{bmatrix} = \begin{bmatrix} \omega_c & 0 \\ 0 & \omega_c \end{bmatrix} \begin{bmatrix} h(\hat{i}_a) \\ h(\hat{i}_c) \end{bmatrix} \] (12)

Where \( \omega_c \) is the cutoff frequency. An angle compensation can be added after formula (8), and then formula (13) can be obtained

\[
\begin{bmatrix} \hat{\theta} \\ \hat{\omega}_c \end{bmatrix} = -\tan^{-1}(\hat{E}_a / \hat{E}_c) + \Delta \theta \\
\hat{\omega}_c = d\hat{\theta} / dt \] (13)

Where \( \Delta \theta \) is the phase compensation angle \( \Delta \theta = \arctan(\hat{\omega}_c / \hat{\omega}_a) \): Its structural block diagram is shown in Figure 2 below:

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**Fig.2** Block diagram of improved SMO
5. Simulation analysis

Use MATLAB/Simulink to compare and analyze the traditional sliding mode control strategy and the improved sliding mode control strategy. The motor parameters used in the simulation are shown in Table 1 below.

| Parameter          | Numerical value | Parameter          | Numerical value |
|--------------------|-----------------|--------------------|-----------------|
| Stator resistance  | 2.875 Ω         | DC side voltage    | 311 V           |
| $R$                |                 | $U_{dc}$ (V)       |                 |
| Stator inductance  | 8.5 mH          | operating frequency| 10000 Hz        |
| $L$                |                 | $f_{pwm}$ (Hz)     |                 |
| Flux linkage       | 0.175 wb        | Moment of inertia  | 0.001 (kg m²)   |
| $\psi_f$ (wb)      |                 |                   |                 |
| Number of pole pairs | 4               | Damping coefficient $B$ | 0               |

Set the initial speed to 800r/min and the simulation time to 0.1s. When the simulation time is 0.05s, a load torque of 5N*M is applied and the speed changes to 1000r/min. To compare the simulation performance of the two observers, the results are shown in Figure 3 to Figure 7.

Fig.3  Variation curve of rotor position estimated value and actual value of traditional SMO and improved SMO

Fig.4  Variation curve of the estimated value and actual value of traditional SMO and improved SMO
It can be seen from Figure 3 that there is a significant phase delay between the estimated value of the rotor position of the traditional SMO and the actual value, and the improved version perfectly solves this problem. It can be seen from Figure 4 that the improved speed observation using the improved piecewise function is relatively more stable, and the amplitude of the sudden change in the torque is relatively small, and it is more stable after reaching the target speed. When the speed changes, the stable state can be reached more quickly and the fluctuation range is obviously reduced after reaching the stability. It can be seen from Fig. 5 that after reaching the target speed, the traditional SMO's speed estimation error continuously changes between -5 and 10r/min, while the improved speed estimation error is stabilized at 0r/min after a short adjustment. After a sudden change in torque and speed, it quickly returns to a stable state after a short change.

6. Conclusion
In order to solve the chattering and phase delay problems in the traditional sliding mode observer, this paper uses a new type of piecewise function as the switching function, which effectively weakens the chattering. Since a low-pass filter is used in the sliding mode observer to filter out the high-frequency harmonics in the back electromotive force, which will cause a phase delay problem and affect the accuracy of the rotor position, a compensation angle is added to solve this problem. Finally, the arctangent function is used to extract the rotor position and speed information of the PMSM, and it runs under Simulink. Compared with the traditional sliding mode observer, from the simulation results, we can see that the improved control system is reducing chattering and improving convergence speed. It has obvious advantages in improving the accuracy of speed estimation.

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