1. INTRODUCTION

As more extrasolar planets are discovered, we are increasingly pressed to describe how planets can form in such a variety of environments. Until just recently, observational selection biases have resulted in the fact that all observed extrasolar planets have been found to orbit within a few AU of their star (Butler et al. 2006). Since it seems unlikely that these planets could have formed in situ (Mayor & Queloz 1995), planet migration is usually invoked (Alibert et al. 2005). Unfortunately, this means that little is known about where—and hence how—the planets originally formed.

In contrast, the technique of direct imaging has presented us with a new set of extrasolar planets that lie far from their star (Kalas et al. 2008; Marois et al. 2008), along with a potential protoplanet (Greaves et al. 2008). Like previous techniques, direct imaging preferentially detects giant planets of several Jupiter masses. Furthermore, planet migration need not be invoked to explain how these planets could form at their observed locations.

One possible mechanism for giant planet formation is core accretion followed by rapid gas accretion (Pollack et al. 1996; Inaba et al. 2003). However, this mechanism has difficulty forming giant planets at large radii. The primary reason for this is that the initial core accretion time scales as \( r^3 \), where \( r \) is the orbital radius of the planet (Ikoma et al. 2000; Kenyon & Bromley 2008). Thus, while it may take \( \sim 1 \) Myr to form a gas giant at 5 AU via core accretion, it would take \( \sim 1 \) Gyr for the same process at 50 AU—far longer than the observed lifetimes of protoplanetary disks (Haisch et al. 2001).

Another mechanism for giant planet formation is disk fragmentation as a consequence of the gravitational instability (Kuiper 1951; Cameron 1978; Boss 1997, see also the recent review by Durisen et al. 2007 and Stamatellos & Whitworth 2009 for recent developments). Provided that the disk surface density is sufficiently large, this mechanism can form giant planetary embryos on timescales of a few orbital periods. However, if the surface density is too large, the disk is unable to cool sufficiently fast for fragmentation to take place at all (Rafikov 2005). The combination of these requirements implies gravitational instability can only form massive planets at large radii.

In this Letter, we consider the planet Fomalhaut b (Kalas et al. 2008), the triple-planet system HR 8799 (Marois et al. 2008), and the potential protoplanet orbiting HL Tau \(^1\) (Greaves et al. 2008). Each of these systems possesses at least one planet with orbital characteristics favored by the disk fragmentation mechanism. By determining the range of surface densities required to form a giant planet with the same semimajor axis as these observed planets, we can infer the range of disk masses needed for the fragmentation mechanism to have operated in these systems.

2. DISK FRAGMENTATION

The stability of a thin, massive disk is controlled by the Toomre (1964) \( Q \) parameter

\[
Q \equiv \frac{c_s \Omega}{\pi G \Sigma},
\]

where \( c_s \) is the isothermal sound speed, \( \Omega \) is the orbital angular frequency (assuming a Keplerian disk), and \( \Sigma \) is the surface density. The disk becomes gravitationally unstable for \( Q \lesssim 1 \). However, even if a disk is gravitationally unstable, it can only fragment if it possesses a sufficiently short cooling time (Gammie 2001; Rice et al. 2003). Specifically, fragmentation will only occur if

\[
t_{\text{cool}} < \frac{\xi}{\Omega},
\]

where \( t_{\text{cool}} \) is the local cooling time for a small, point-source perturbation, and \( \xi \) is a factor of order unity that can depend on

\(^1\) For our purposes, the distinction between planet and protoplanet is irrelevant, and we will use “planet” in both contexts from here on.
the history of the disk (Clarke et al. 2007). We adopt $Q < 1$ and $\xi = 1$ for our fragmentation criteria.

### 2.1. Local Cooling Time

Typically, the effects of cooling have been studied using time-dependent hydrodynamic simulations. Inevitably, these numerical approaches have to employ significant simplification of the radiation field for the sake of computation time (e.g., optically thin cooling or flux limited diffusion). Many of the simulations show that fragmentation does occur given sufficiently high surface densities (Durisen et al. 2007).

In contrast, Rafikov (2005) used an analytic, order-of-magnitude calculation to show that cooling times derived from the equations of radiative transfer were much longer, and that fragmentation thus did not work, except at radii $\gtrsim 100$ AU. Here, we adopt an approach inspired by Rafikov, but with a more complete calculation of the radiative transfer. In brief, we find cooling times that are, in most cases, over an order of magnitude shorter than those given by Rafikov (see D. Nero & J. E. Bjorkman 2010, in preparation for a more complete discussion). As a consequence, we find that fragmentation over a larger range of the outer disk is possible, depending on the details of the system.

We emphasize that the cooling time we calculate here is for a perturbation, and is not the same as the total disk cooling time employed by Gammie (2001). While the later may be more convenient for numerical hydrodynamic simulations, the former is necessary to properly account for background heating by external illumination (i.e., the host star). The perturbation cooling time determines the onset and initial growth of the instability (in the linear regime), while the total cooling time controls the ultimate (typically nonlinear) completion of the instability. Note, however, that when self-heating is small, the perturbation and total cooling times will be the same within a factor of order unity.

The perturbation cooling time $t_{\text{cool}} = \Delta t/(8\pi \Delta H_0)$, where $\Delta \xi$ is energy per unit area added by the perturbation and $\Delta H_0$ is the frequency-integrated Eddington flux at the disk surface. We consider an anulus within the disk, which we approximate as a plane-parallel atmosphere with finite thickness. For simplicity, we assume that the perturbation is located at the disk mid-plane and that the disk cools equally from its top and bottom surfaces. Under these assumptions, the perturbation cooling time is

$$t_{\text{cool}} = \frac{1}{16 \nu A} - 1 \sigma T^4_{\text{eff}} \frac{1}{\chi_{\text{diff}}} \frac{1}{\Delta B} \frac{\Delta B}{\Delta H_0} d \tau,$$

where $\gamma_A$ is the adiabatic constant for the gas, $\chi_{\text{diff}}$ is the mean opacity (absorption plus scattering), $\tau$ is the optical depth coordinate, $B$ and $\Delta B$ are the depth-dependent Planck function and its perturbation, and $B_m$, $T_m$, and $c_m$ are the Planck function, the temperature, and the isothermal sound speed at the disk mid-plane, respectively. The limits of integration, $\tau_0$ and $-\tau_0$, are the optical depth coordinates at the “top” and “bottom” surface, respectively. Note that we break convention here by placing locations in the disk below the disk mid-plane have negative optical depth coordinates, while those above have positive. Also, note that we have assumed that $\chi_{\text{diff}}$ is approximately constant over the vertical extent of the disk, since the disk is nearly isothermal in the vertical direction. While this is not true for the surface layers, the error is minimal since most of the disk mass—and thus internal energy—is located in the disk interior. Similarly, the vertically isothermal assumption does not apply when accretion is the dominant source of heating (since then there would be a significant vertical temperature gradient); however, at large radii accretion luminosity is usually not the dominant heating mechanism for the disk.

We assume that the relevant physics (i.e., reprocessing/absorption of external radiation along with viscous energy generation) can be preserved by splitting the intensity into two frequency components: diffuse, which corresponds to photons that have been reprocessed and emitted by the disk and external, which corresponds to unabsorbed photons emitted from the central star and potentially scattered in the disk. Assuming gray opacity (i.e., the appropriate mean for each spectrum), the frequency-integrated moments of the transfer equations are

$$\frac{dH^\text{ext}}{dt} = -\frac{\kappa^\text{ext}}{\chi^\text{diff}} J^\text{ext},$$

$$\frac{dJ^\text{ext}}{dt} = -\frac{\chi^\text{ext}}{\kappa^\text{diff}} H^\text{ext},$$

$$\frac{dH^\text{diff}}{dt} = \frac{\kappa^\text{diff}}{\chi^\text{diff}} (B - J^\text{diff}),$$

$$\frac{dJ^\text{diff}}{dt} = -\frac{H^\text{diff}}{\kappa^\text{diff}},$$

where $J$ is the mean intensity, $H$ is the Eddington flux, and $\chi$ and $\kappa$ are the total and absorptive opacity, respectively. We have written all optical depth coordinates in terms of $\tau \equiv \tau^\text{diff} = \tau_{\text{diff}} B^\text{diff} / J^\text{diff}$. The Eddington factors, $J^\text{ext} = K^\text{ext} / J^\text{ext}$ and $J^\text{diff} = K^\text{diff} / J^\text{diff}$, are assumed to be constant with depth. Note that we have included no thermal emission for the external frequency because the disk is typically much cooler than the star.

To find the external radiation, we combine Equations (4) and (5), which have the solution

$$H^\text{ext} = H^\text{ext}_0 \sinh \beta \tau / \sinh \beta \tau_0,$$

where $\beta \equiv \kappa^\text{ext} \chi^\text{diff} / (\chi^\text{diff} J^\text{ext})^2$ and $H^\text{ext}_0$ is the net external surface flux.

There are three sources of energy for the diffuse radiation: (1) absorption of external radiation $-dH^\text{diff} / d\tau$, (2) accretion luminosity $L_{\text{acc}}$ with surface flux $H_0 = (dL_{\text{acc}} / dA) / 8\pi$, and (3) the point-source perturbation at the mid-plane $\Delta H_0 \delta(\tau)$. Thus, the flux transported by the disk is $H^\text{diff} = H_0(\tau / \tau_0) + \Delta H_0 \delta(\tau)$.

From Equation (7), we can now obtain the diffuse mean intensity

$$J^\text{diff} = H_0 \left( \frac{\tau_0^2 - \tau^2}{2 f^\text{diff} \tau_0^2 + \frac{1}{g^\text{diff}}} \right) - H_0^\text{ext} \left( \frac{\cosh \beta \tau_0 - \cosh \beta \tau}{\beta f^\text{diff} \sinh \beta \tau_0 + \frac{1}{g^\text{diff}}} \right) + \Delta H_0 \left( \frac{\tau_0 - \tau}{\frac{1}{f^\text{diff}} + \frac{1}{g^\text{diff}}} \right),$$

where $g^\text{diff} = H^\text{diff}(\tau_0) / f^\text{diff}(\tau_0)$ is the second Eddington factor.

Finally, the temperature may now be found from radiative equilibrium, Equation (6), which gives

$$B = J^\text{diff} + \frac{\chi^\text{diff}}{\kappa^\text{diff}} \frac{dH^\text{diff}}{d\tau},$$

(10)
as well as
\[ \Delta B = \Delta H_0 \left( \frac{v_0 - |v|}{f \text{diff}} + \frac{1}{g \text{diff}} + 2 \lambda \text{diff} \right). \]

Given the temperature and its perturbation, we calculate the cooling time from Equation (3). In terms of the surface density, \( \Sigma = 2 \tau_0 / \chi \text{diff} \), the cooling time
\[ t_{\text{cool}} \propto \Sigma^a, \]
where the exponent \( a \) depends on both the disk optical depth and the ratio of heating from the star to self-heating from accretion (see D. Nero & J. E. Bjorkman 2010, in preparation), but is generally in the range 0–2. In the negligible self-heating regime \( (H_0 \to 0, B/B_m \to 1) \), Equation (3) reduces to
\[ t_{\text{cool}} = \frac{1}{16} \frac{c_m^2}{\gamma_A - 1} \left( \frac{\chi \text{diff}^2}{f \text{diff}} + \frac{\Sigma}{g \text{diff}} + \frac{1}{k \text{diff}} \right) \cdot \frac{\Sigma}{\chi \text{diff}^2}. \]  

In the optically thick limit \( (\tau_0 > 1) \), \( f \text{diff} = 1/3 \), Equation (13) further simplifies to
\[ t_{\text{cool}} \approx \frac{3}{128} \frac{c_m^2}{\gamma_A - 1} \frac{\Sigma}{\chi \text{diff}}. \]

Note that this expression is directly comparable to the optically thick limit of Equation (4) in Rafikov (2005), with the exception that our numerical factor \( 3/128 \) is over an order of magnitude smaller than his \( 1/4 \), thus leading to significantly reduced cooling times. This difference from Rafikov’s result is much larger than the difference between total and perturbation cooling times. It instead arises from his assumption that several factors were of order unity, which is true individually; however, the cumulative effect of his assumptions results in his making a large overestimate.

Our analytic expression for the cooling time, Equation (14), assumes no accretion luminosity and large optical depth. While it is probably safe to assume that self-heating is small at large disk radii, it is not as certain that the optically thick limit will hold. For this reason, we use the full solution of Equation (3) when calculating \( t_{\text{cool}} \), rather than assuming that this limit holds. Nonetheless, Equation (14) makes for a good estimate in most cases where \( \Sigma \) is large enough to satisfy \( Q < 1 \) in the outer disk.

2.2. Fragment Masses

When determining if disk fragmentation is a viable mechanism for forming giant planets, another point to consider is the issue of producing the proper planetary mass of a few \( M_{\text{Jupiter}} \). While a full treatment of this problem is beyond the scope of this Letter, we provide a toy model to argue that this is likely to be the case.

If \( Q \lesssim 1 \) at some radius \( r \), the disk becomes gravitationally unstable. Supposing \( m \) spiral arms form, each arm has local surface density
\[ \Sigma_{\text{arm}} = \frac{w \, r}{m \, R}, \]
where \( \Sigma \) is the original surface density of the previously axisymmetric disk, \( R \) is the current width of the spiral arms, and \( w \) is a constant that depends on the winding angle. For simplicity, we assume that most of the disk mass is confined to the spiral arms, while the space between the arms is effectively empty.

### Table 1

| Object | \( M_* \) (M⊙) | \( R_* \) (R⊙) | \( T_* \) (K) | \( r_{\text{planet(s)}} \) (AU) |
|--------|----------------|----------------|-------------|------------------|
| Fomalhaut b | 2.0 | 1.8 | 8500 | > 101.5* |
| HR 8799 b, c, d | 1.5 | 1.8 | 8200 | 68, 38, 24b |
| HL Tau b | 0.3 | 0.6 | 3700 | 65c |

Notes.

* Chiang et al. (2009).
* Marois et al. (2008).
* Greaves et al. (2008).

In addition, we assume \( w \sim 1 \), corresponding to moderate winding.

If \( t_{\text{cool}} \gtrsim 1/\Omega \), then the spiral arms are pressure supported and are stable against fragmentation. However, if \( t_{\text{cool}} \lesssim 1/\Omega \), then the arms are instead supported by centrifugal forces. As \( R \) continues to decrease, they will fragment radially once the centrifugal support is lost. Balancing self-gravity against the centrifugal support, \( \Omega^2 R = \pi G \Sigma_{\text{arm}} \), fragmentation occurs when

\[ R < R_f \equiv r_f^2 \sqrt{\frac{\pi \Sigma}{m \, M_*}}. \]

where \( M_* \) is the stellar mass. The fragment mass is \( \pi R_f^2 \Sigma_{\text{arm}} \), so assuming \( m \sim \pi \), we find a characteristic fragment mass

\[ M_f \sim 1 \, M_{\text{Jupiter}} \left( \frac{\Sigma}{10 \text{ g cm}^{-2}} \right)^{3/2} \left( \frac{M_*}{M_{\odot}} \right)^{-1/2} \left( \frac{r}{100 \text{ AU}} \right)^3 \]

which is consistent with our requirement to produce Jupiter-mass planets.

3. DISK MASS LIMITS

The condition that both the Toomre \( Q \) and the cooling time be sufficiently small (Equations (1) and (2)) can be used to place limits on what disk surface densities lead to fragmentation. To be gravitationally unstable, the Toomre \( Q \) condition requires that the surface density be larger than a maximum, \( \Sigma_{\text{min}} \), while the cooling condition imposes a maximum surface density, \( \Sigma_{\text{max}} \). Therefore, local disk fragmentation is only possible if \( \Sigma_{\text{min}} < \Sigma < \Sigma_{\text{max}} \). It of course follows that for fragmentation to be possible at all, \( \Sigma_{\text{min}} \) must be less than \( \Sigma_{\text{max}} \). This limits the range of radii where fragmentation is even a possibility.

In Figure 1, we plot \( \Sigma_{\text{min}} \) and \( \Sigma_{\text{max}} \) for the systems Fomalhaut, HR 8799, and HL Tau, using the parameters listed in Table 1. In all cases we used an accretion rate of \( 10^{-6} \, M_{\odot} \, \text{yr}^{-1} \), a mean molecular weight for the disk of \( \mu_g = 2.33 \), and an adiabatic gas constant of \( \gamma_A = 1.43 \). The temperature is determined from Equation (10), using a flared disk model with a power-law scale height \( h \propto r^{5/4} \), which determines the angle of incidence of the external radiation. We use the dust opacity of Cotera et al. (2001) and the Rosseland mean gas opacity of Helling et al. (2000). We note that the stellar parameters in Table 1 are not necessarily appropriate if planet formation occurs during the Class 0/1 phase when the star is significantly more luminous. However, our results are relatively insensitive to this effect, since \( T_m \) is only weakly dependent on the stellar luminosity.

Each system presented here has a planet that might have formed via disk fragmentation, assuming that the local surface
found the hydrodynamical simulations of V orobyov & Basu (2009) plotted as vertical dotted lines.

For conservative, we are using the smallest radius found by Chiang disk fragmentation mechanism. Also note that, in order to be \( \Sigma \), density had a value between \( \Sigma_{\text{min}} \) and \( \Sigma_{\text{max}} \) for at least a few orbital periods. By assuming a power law for the surface density, \( \Sigma \propto r^{-p} \), we can calculate a range of disk masses (interior to \( r_{\text{planet}} \)) that satisfies this condition.

A survey of 24 circumstellar disks by Andrews & Wilkins (2007) found \( p \approx 0.0-1.0 \) with an average of \( p \approx 0.5 \), while the hydrodynamical simulations of Vorobyov & Basu (2009) found \( p \approx 1.0-2.0 \) with an average around \( p \approx 1.5 \). Disk mass limits \( M_d \) for \( p = 0.0, 0.5, 1.0, \) and 1.5, along with the more fundamental surface density limits are given in Table 2. We also provide the characteristic fragment mass (approximate planet mass) \( M_f \) from Equation (17) that we would expect from the disk fragmentation mechanism. Also note that, in order to be conservative, we are using the smallest radius found by Chiang et al. (2009) for Fomalhaut b. Using one of their better fits (e.g., 115 AU) will decrease our lower disk mass limit by a few percent, increase our upper disk mass limit by \( \approx 30\% \) (which would make fragmentation slightly easier), and increase the characteristic fragment mass by \( \approx 50\% \).

4. Discussion

We have refined the calculations of Rafikov (2005) and found cooling times over an order of magnitude shorter. We have used these cooling times, along with the observed stellar parameters of Fomalhaut, HR 8799, and HL Tau, to test the viability of the disk fragmentation mechanism. We found that in each of these systems, at least one planet could have formed in situ as the result of fragmentation, assuming the disk mass interior to those planets fell within a particular range as indicated in Table 2.

While the ranges in Table 2 only span a factor of a few, this is not by itself a significant limitation. Even if the local surface density is above the upper instability limit, fragmentation may still occur since the surface density must eventually drop through the unstable regime as the disk evolves and dissipates. The caveat is that the surface density needs to evolve on a timescale longer than an orbital period so that there can be sufficient time to fragment.

Our minimum disk masses for Fomalhaut b, HR 8799 b, and HL Tau b are about an order of magnitude larger than those inferred from observations (Andrews & Wilkins 2007). Note, however, that this is a problem for all planet formation models in general. Even core accretion models require an enhanced surface density (although to a somewhat lesser extent; Pollack et al. 1996; Inaba et al. 2003). One possible mechanism for increasing the surface density is mass loading from an infalling envelope (Vorobyov & Basu 2006). Conversely, current estimates of disk masses may be too low because they depend on: (1) the extrapolation of surface densities in the outermost regions of the disk to the inner disk and (2) the rather uncertain dust opacity. For example, larger dust grains would require larger disk masses to fit the observed spectral energy distributions (Andrews & Wilkins 2007).

As further evidence for underestimated disk masses, numerical hydrodynamical simulations by Vorobyov (2009) found disk masses much higher than those of Andrews & Wilkins (2007). In particular, stars like Fomalhaut and HR 8799 can support disks as large as \( 0.5 M_\odot \), while HL Tau could have a disk as massive as \( 0.1 M_\odot \), all of which are within our limits for disk fragmentation. We caution, however, that our choice of opacity model can have a major effect on our results. For example, decreasing the dust opacity raises the temperature and decreases the cooling time in the outer disk, resulting in disk fragmentation at smaller radii. On the other hand, increasing the opacity would have the opposite effect.

Regardless of the above considerations, HR 8799 c and d are too close to their parent star to have formed in situ via fragmentation under the conditions modeled here. Appealing to chronically overestimated dust opacity can only get us so far. Dropping the opacity by an order of magnitude brings HR 8799 c into the
fragmentation zone, but still leaves HR 8799 d out. Likewise, twiddling other parameters can also move the fragmentation radius inward, but reaching the required 24 AU with reasonable parameters does not seem possible. We therefore conclude that HR 8799 c and d likely did not form in situ as the result of disk fragmentation (of course, they could have formed at larger radii where the disk is more likely to fragment and migrated inward).

For those planets that could form by disk fragmentation, we find characteristic fragment masses (approximate planet masses) of a few \( M_{Jupiter} \) for the lower end of unstable disk surface density. Our estimates are mostly consistent with expectations, although Chiang et al. (2009) found Fomalhaut b to have an upper mass limit of 3 \( M_{Jupiter} \), which is 60% lower than our lowest estimate of 5 \( M_{Jupiter} \). Nonetheless, considering the crudeness of Equation (17), we are not convinced that this discrepancy rules out Fomalhaut b from having formed as a result of disk fragmentation.

In conclusion, given our current uncertainty of typical disk masses and dust opacities, we cannot rule out planet formation from disk fragmentation. Of the three currently known systems that might possess planets formed in this manner, all are able to produce at least their outermost giant planet via fragmentation, given a large enough disk mass at some point in their history.

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