Does Modified Gravity Predict Fast Stellar Bars in Spiral Galaxies?

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Abstract

The evolution of disk galaxies in modified gravity is studied by using high-resolution N-body simulations. More specifically, we use the weak field limit of two modified gravity theories, that is, nonlocal gravity and scalar–tensor–vector gravity, known as MOG, and ignore the existence of a dark matter (DM) halo. We construct the same models as in the standard DM model and compare their dynamics with the galactic models in modified gravity. It turns out that there are serious differences between galactic models in these different viewpoints. For example, we explicitly show that the galactic models in modified gravity host faster bars compared to the DM case, but the final stellar bars are weaker in modified gravity. These facts are not new and have already been reported in our previous simulations for exponential galactic models. Therefore, our main purpose is to show that the above-mentioned differences, with an emphasis on the speed of the bars, are independent of the initial density profile of the adopted disk and halo. To do so, we employ different profiles for the disk and halo and show that the results remain qualitatively independent of the initial galactic models. Moreover, a more accurate method has been used to quantify the kinematic properties of the stellar bars. Our results imply that, contrary to the DM models, bars in modified gravity are fast rotators that never leave the fast-bar region until the end of the simulation.

Unified Astronomy Thesaurus concepts: Spiral galaxies (1560); Disk galaxies (391); Galaxy evolution (594); Scalar-tensor-vector gravity (1428); Non-standard theories of gravity (1118)

1. Introduction

It is well understood that the dynamics of spiral galaxies are substantially influenced by the existence of dark matter (DM) particles. More specifically, the DM halo is the main mass component of each galaxy and plays a central role in many secular phenomena in the disk. For example, there is a direct correlation between the stellar bar evolution and the DM halo properties. It is well established in the literature that a rigid halo can suppress the bar instability and prevent the formation of strong bars (Ostriker & Peebles 1973). In contrast, a responsive halo does not necessarily suppress the instability. The transfer of angular momentum between the halo and the disk can expedite bar formation (Athanassoula 2002). The pattern speed of the bar is another important observable that is directly related to the properties of the surrounding halo. Because of dynamical friction between the halo particles and the bar, the pattern speed decreases with time. This behavior is seen in many galactic numerical simulations; for example, see Debattista & Sellwood (2000). As expected, cosmological simulations also confirm the pattern speed slowdown (Algorry et al. 2017). Therefore we may conclude that simulations in the particle DM paradigm predict slow bars.

On the other hand, it is well known in the literature that almost all of the observed pattern speeds are fast (Corsini 2011; Garma-Oehmichen et al. 2020). Furthermore, it seems that bars are formed fast and remain as fast rotators during the cosmological timescale (Pérez et al. 2012). This fact is considered to be a challenge to the standard cosmological model. Of course, some proposals do address this problem. For example, it has been shown in Hui et al. (2017) that ultralight, fuzzy DM particles lead to substantially less dynamical friction. Consequently, this kind of DM particle does not decrease the bar pattern speed via dynamical friction.

Based on the fact that DM particles have not yet been detected, there is another approach, modified gravity, to the above-mentioned challenge. Modified gravity theories aimed at replacing DM particles with viable and self-consistent modifications to general relativity (GR) have a long history (Clifton et al. 2012). In this paper, we will discuss two theories: nonlocal gravity (NLG), proposed by Hehl & Mashhoon (2009) to include nonlocal features of gravitation, and the scalar–tensor–vector theory of gravity, known as MOG, proposed to address the DM problem (Moffat 2006). Besides modified Newtonian dynamics (MOND; Milgrom 1983), these theories are the main alternative models to DM in the literature. Note that these theories are different in nature and phenomenology from the other branch of modified theories of gravity that try to address the dark energy enigma in cosmology.

Although NLG and MOG are based on completely different postulates and motivations, their weak field limit is similar. Both theories lead to Yukawa-like corrections to the gravitational force between point masses. These corrections help to explain the flat rotation curves of spiral galaxies, as well as the mass discrepancy in galaxy clusters. For more details, see Rahvar & Mashhoon (2014) and Moffat & Rahvar (2013, 2014). The formation and evolution of stellar bars in these theories have been investigated in Roshan & Rahvar (2019) and Roshan (2018) using high-resolution simulations. For low-resolution but comprehensive sets of simulations in MOG, we refer the reader to Ghafourian & Roshan (2017). Since there is no DM halo in galactic models in modified gravity, there is no significant dynamical friction in the system. Consequently, Roshan & Rahvar (2019) and Roshan (2018) show that the bars are faster compared to the standard case and the pattern speed is almost constant with respect to time. This fact appears as a serious difference between modified gravity and particle DM. Combined with the relevant observations, this deviation between these different viewpoints may help to
distinguish between particle DM and modified gravity in galactic scales. However, simulations presented in Roshan & Rahvar (2019) and Roshan (2018) have three main restrictions, and it is necessary to resolve them to ensure that modified gravity, unlike the standard picture, predicts the existence of fast bars in spiral galaxies.

Let us briefly mention the three restrictions. (1) Fast bars appear in simulations of modified gravity with exponential disks. This does not necessarily mean that bars are fast in other density profiles. In other words, it is necessary to show that the main results of our previous simulations are model-independent. (2) There is no bulge component in the simulations. For more realistic galactic models, it is necessary to include the bulge component. (3) We know that gas plays a crucial role in the secular evolution of galactic disks. Therefore, hydrodynamic simulations are required in order to strengthen and confirm the results obtained from N-body simulations.

Our main motivation in this paper is to investigate the first restriction by constructing different galactic models. In other words, we show that our previous results are model-independent. As mentioned, this is a crucial test in order to ensure the significant deviations between modified gravity and particle DM. To do so, we first briefly review the main features of NLG and MOG in Section 2. Then we discuss the numeric methods, the initial conditions, and, in particular, the GALAXY code in Section 3. Furthermore, we present the results of a series of N-body simulations in Section 4. Finally, we end with the discussion and conclusion in Section 5.

2. Yukawa Corrections: Manifestation of Nonlocality or Coupled Proca Vector Field?

As already mentioned, NLG and MOG induce Yukawa corrections to the gravitational force between point masses. Note that to investigate the dynamics of a spiral galaxy that lies in the Newtonian regime, we need the weak field limit of the gravitational theories. This point combined with the fact that there is no screening behavior in NLG and MOG causes a substantial simplicity in our analysis. In other words, one may conveniently linearize the field equations of the theories and find the modified version of the Poisson equation. Then it is straightforward to find the gravitational force between point particles. The absence of screening behavior allows us to use the superposition principle to find the gravitational field of an extended mass distribution. This kind of linearization is not allowed in $f(R)$ gravity theory, where the inherent scalar degree of freedom of the theory possesses a screening mechanism (Hu & Sawicki 2007).

The weak field limit of NLG and MOG has been reviewed in Roshan & Rahvar (2019) and Roshan (2018), respectively. Also, for more details one may see Rahvar & Mashhoon (2014) and Roshan & Abbassi (2014). Therefore we do not repeat the details here. However, it is important to mention that both theories introduce the following corrections to the Newtonian gravitational force between point particles:

$$f = - \frac{m_1 m_2 G}{r^2} (1 + \alpha - \alpha(1 + \beta r) e^{-\mu r}) \mathbf{r}$$

(1)

where $\alpha$ and $\mu$ are constant parameters, and $\beta = \mu/2$ in NLG and $\beta = \mu$ in MOG. These parameters have been fixed by fitting rotation curve data of spiral galaxies. It is interesting that although both theories lead to the same functionality for the gravitational force, the origin of the Yukawa corrections is completely different. In MOG, besides the metric tensor, there are two extra scalar fields and an extra Proca vector field which is coupled to matter. In this case, $\alpha$ and $\mu$ are related to the current background values of the scalar fields. Furthermore, $\mu$ plays the role of the mass of the vector field. These parameters, in principle, are functions of time, although Jamali et al. (2018), using the dynamical system approach for the cosmology of MOG, have shown that $\mu$ does not vary significantly during the thermal history of MOG. Thus, for simplicity, we assume that they are constant. It is also necessary to mention that recently Green & Moffat (2019) have postulated that $\alpha$ and $\mu$ should be decreasing functions of the mass of the galaxy to explain the radial acceleration relation discovered by McGaugh et al. (2016).

On the other hand, in NLG, $\alpha$ and $\mu$ appear in the weak field limit as a manifestation of the nonlocality in gravitational interaction. Interestingly, nonlocal features of gravity can effectively appear as a source for strengthening the gravitational force (Hehl & Mashhoon 2009). Note that modified gravity models that try to address the DM problem should increase the gravitational force without postulating the existence of DM particles.

We emphasize that by NLG we mean the NLG theory introduced by Hehl & Mashhoon (2009). This theory is directly based on the interesting similarity between Maxwell’s equations for electrodynamics and those of GR when written in the teleparallel formulation. This similarity is extremely helpful for adding nonlocal features to gravity in a way similar to that already developed in electrodynamics; for a comprehensive discussion, see Mashhoon (2017). There are different types of NLG theories in the literature based on different viewpoints and motivations. For a pioneering work in this direction, see Deffayet & Woodard (2009).

In NLG, the free parameters $\alpha$ and $\mu$ also vary with time. However, for the sake of simplicity, we assume that these parameters are constant. Of course, for a more careful analysis, it is necessary to take into account the time dependence of these parameters. However, implementation of this requirement in the N-body simulations is not a simple task from a technical point of view. On the other hand, NLG is not an action-based theory, and in practice, when dealing with real physical systems, some substantial mathematical and conceptual complexities appear in the analysis. For a comprehensive review of NLG, we refer the reader to the book by Mashhoon (2017). For recent developments in the theoretical aspects of the theory, see Puetzfeld et al. (2019) and Puetzfeld & Obukhov (2020).

3. Numerical Method

To study the evolution of disks under the effect of a DM halo or modified gravity, we use GALAXY, a high-resolution N-body code (Sellwood 2014) that is ideal for investigating collisionless stellar systems (Sellwood 2019). The code constructs initial conditions for the systems in equilibrium and then evolves them according to time by using a leapfrog method. The models under study are flat disks without any central bulge. As we mentioned earlier, simulations in modified gravity do not employ a DM halo. So in the absence of the bulge, the models in modified gravity are considered to be one-component systems. However, for the Newtonian model, we use a spherical live DM halo, whose properties will be described in the next section. In the DM model, we use a
hybrid mesh consisting of a spherical three-dimensional (S3D) mesh plus a cylindrical polar three-dimensional (P3D) mesh, for the DM halo and the baryonic disk, respectively, while in the single-component models of modified gravity, MOG and NLG, only a P3D mesh is used.

3.1. Initial Conditions

As mentioned earlier, the main purpose of this work is to investigate the model-independent of previously reported results on the difference in disk evolution in modified gravity and DM. Because previous studies focused on exponential (EXP) disks and Plummer halos (Roshan 2018; Roshan & Rahvar 2019), to ensure the independence of the results for the adopted disk/halo models, we first changed the disk’s density distribution from EXP to Kuzmin–Tommer (KT) while we keep the live Plummer halo model unchanged (LPH model). In the next step, we change both the halo and disk profiles. In this case, we employ a KT disk and a live isothermal halo (LIH model). To investigate further, we also consider another halo model with the Hernquist profile (LHH model). This halo model has a cusp in the inner radii, which increases the initial rotational velocity in the inner radii and influences the evolution of the disk. As we will discuss comprehensively, the dynamical friction plays a central role in discriminating between DM and modified gravity. On the other hand, it is natural to expect that this mechanism works differently in the cored and cuspy halo models. Therefore, to draw a complete and consistent picture, it seems necessary to compare both the cored and cuspy types with modified gravity models. For this purpose, we include the LHH model in the simulations.

In this study, the KT disk, model 1 of Toomre (1963), has a surface density of the form

$$\Sigma(R) = \frac{M_d}{2\pi R_d^2} \left[ \frac{R}{R_d} \right]^2 + 1 \right]^{-3/2},$$

(2)

where $R$ is the radial coordinate in the cylindrical coordinate system $(R, \phi, z)$. The Plummer halo, a polytrope model of index 5, has the density

$$\rho(r) = \frac{3M_h}{4\pi b^3} \left[ 1 + \left( \frac{r}{b} \right)^2 \right]^{-5/2},$$

(3)

while for the simple cored Isothermal halo model (Evans 1993), the density is given by

$$\rho(r) = \frac{V_h^2}{4\pi G \alpha b^3} \left[ 3 + x^2 \right].$$

(4)

The Hernquist halo model (Hernquist 1990) also has density of the form

$$\rho(r) = \frac{M_h c}{2\pi r(r + c)^3}.$$

(5)

In the above equations, $M_d$ and $M_h$ are the disk and halo mass, respectively; $R_d$ is the radial disk scale length; $a$, $b$, and $c$ are characteristic radii of the halos; and $x = r/a$. Furthermore, $V_h$ in the isothermal sphere is $V_h = \sqrt{GM_h/a}$. Note also that the system of units is considered so that $M_d = R_d = G = 1$, where $G$ is the gravitational constant. Therefore, the units of velocity and time can be written as $V_0 = (GM_d/R_d)^{1/2}$ and $\tau_0 = R_d/V_0 = (R_d^3/GM)^{1/2}$. As a suitable example, selecting $R_d = 2.6$ kpc and $\tau_0 = 10$ Myr yields $M_d \approx 4 \times 10^{10} M_\odot$ and $V_0 \approx 254$ km s$^{-1}$.

To make a reasonable comparison, we constrain our models to start their evolution from a similar state. Therefore, the models are constructed such that the initial rotational velocities of the four models are as close as possible. Furthermore, the initial velocity dispersions of the models have a very similar trend. To do this, the employed free parameters of the DM halo density and modified gravity should be varied until the same initial state for the baryonic matter is achieved in all models. Notice that we use the same parameters for the disk density in all models.

The truncation radius of the KT disk is chosen to be $r = 6R_d$, where the density starts to taper smoothly from $r = 5R_d$ by using a cubic function. For our Plummer model, the halo mass is chosen as $M_h = 3.6M_d$ and the halo scale length as $b = 8.5R_d$, where the truncation radius is at $r = 3.4b$. In the Isothermal model, the halo mass is $M_h = 1.7M_d$ and its scale length is $a = 6R_d$ with truncation radius $r = 10a$. In the Hernquist model, the adopted halo mass is $M_h = 14M_d$, the scale length is $c = 19R_d$ and the truncation radius is $r = 8c$.

As mentioned earlier, the cusp in the Hernquist halo results in a relatively higher rotational velocity in the inner radii, but, with this choice of parameters, the rotation curve in the outer radii matches the other models very well. The LHH model seems too massive compared to the other models. However, regarding Gauss’s theorem, at least at the beginning of the simulation, only the DM mass inside a sphere with radius $6R_d$, that is, $M(6R_d)$, contributes to the gravitational force. It is easy to show that $M(6R_d) \approx 0.8$, 0.7, and 0.85 for the LIH, LPH, and LHH models, respectively. In other words, in all of the models, the DM mass fraction inside $r = 6R_d$ is around 0.4. Therefore, one may ensure that all of the models start with suitable initial conditions.

For the NLG model, the values selected for the free parameters are $\alpha = 10.9$ and $\mu = 0.0045R_d^{-1}$. Also, for the MOG model we employ $\alpha = 2.8$ and $\mu = 0.031R_d^{-1}$. The resulting rotational velocities of our models are illustrated in Figure 1.

For all of the models, the Toomre parameter $Q$ (Toomre 1964) is considered to be 1.5. This value is high...
enough for our models to protect the disk from local instability and fragmentation.

It should also be mentioned that for every active component (DM halo/disk) in our main simulations, we have adopted \( N = 2 \times 10^6 \) particles. However, to make sure that the results are independent of the particle number, we have performed the simulations for a higher number of particles (Section 4.6).

Recall that the adopted mesh for the disk and DM halo is cylindrical polar (P3D) and spherical (S3D), respectively. The grid size selected for our S3D mesh is 1001 \( \times \) 2501, where the first number is the number of radial grid shells and the second is the radius of the grid outer boundary. For the P3D mesh, we have 193 \( \times \) 224 \( \times \) 125, where the numbers demonstrate the number of mesh points in the radial, azimuthal, and vertical directions, respectively. Also, “lscale,” which is the number of mesh points on the length unit, is chosen to be 12.5. Furthermore, the softening length in the P3D mesh is \( \epsilon = 0.16R_d \). This choice of mesh points is enough to certify that the majority of particles do not leave the mesh during the simulations. This property is checked for all of the models, and less than 2% of the particles escape the mesh before the end of the simulation.

It is necessary to mention that the computation of the potential is achieved by using sectoral harmonics \( 0 \leq m \leq 8 \) in the P3D mesh and surface harmonics \( 0 \leq l \leq 8 \) in the S3D. More details can be found in the online manual.3

The duration of our simulations is selected to be \( \tau = 800\tau_0 \) and the time step is \( \Delta\tau = 0.01\tau_0 \). It should also be mentioned that three different time zones with factors of 1, 2, and 4 \( \Delta\tau \) are selected to account for the evolution of particles in regions of different density; shorter time steps are required for particles in denser regions. However, to ensure that the results are not affected by the choice of time step, we also checked the results for lower values of \( \Delta\tau \) (Section 4.6). Moreover, the grid is recentered every 16 steps to avoid numerical artifacts.

According to the selected options, the code computes the initial positions and velocities of the particles to form the initial equilibrium state of the system. Note that if the model consists of more than one component, it is necessary to find the distribution function (DF) in the composite system and then choose the particles from the resulting DF. Accounting for this matter in our DM model, we used the Compress algorithm (Young 1980; Sellwood & McGaugh 2005) implemented in the code, which finds the potential and changes the halo density so that the equilibrium condition in the presence of the disk is met. The initial positions and velocities of the system are then obtained.

4. Results

To study the systems under consideration, we start from the initial conditions described in the previous section, and then the evolution of the system is monitored with time. To better understand the effect of our modified gravity theories in contrast to the DM halo, we study the following quantities: the growth of the bar, its pattern speed and power spectra, the evolution of the Toomre parameter, which is a measure of disk’s heating, the vertical thickness of the disk, and finally the \( R \) parameter. This parameter quantifies the pattern speed of the bars. There are explicit observations for \( R \) that could be useful in contrasting different gravitational models. A comparison

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3 http://www.physics.rutgers.edu/~sellwood/manual.pdf
\( \tau \approx 400 \), and then it continues to grow until the end of the simulation. However, note that although the bar strengths in these two DM models have an overall similar trend and reach almost the same value at final states, the initial behavior in the formation and growth of the bar is not independent of the adopted halo profile. One can infer that, in comparison to the Plummer model, the bar starts to form in the Isothermal model faster and at an earlier time, which matches MOG in early stages. However, a clear discrepancy between the evolution of the disks in the modified gravity and cored DM models is visible after about \( \tau \approx 200 \), regardless of the selected mass model.

The behavior of the LHH model, which includes a cuspy DM halo at the beginning of the simulation, shows some differences from the other DM models. From Figure 2, it is clear that the growth rate of bar formation in the LHH model is higher. Moreover, the value of bar strength remains around its initial peak for a longer duration in comparison to the other halo models. Then, at about \( \tau \approx 250 \), another phase in the evolution of the bar amplitude in the LHH model starts, in which the bar weakens. This period is also long-lasting in comparison to the other DM models. At about \( \tau \approx 500 \), the bar amplitude of the LHH model starts to grow again. However, at \( \tau \approx 800 \), the bar in LHH is clearly weaker than in the LPH and LIH models. Since the evolution of the LHH model does not seem to be completed at this stage, we continued its evolution up to \( \tau = 1600 \) in the subplot of Figure 2. As one can see, the growing phase of the bar amplitude in this model continues to the end of the simulation. According to this figure, it could be concluded that although the details in the evolution of a cuspy DM model are different from a cored DM model and it shows some delays, its overall behavior does not change and is still very different from the models of modified gravity.

According to this behavior, one may conclude that the comparison of bar evolution in our modified gravity/DM models shows two different phases. In the first phase, which is common in all five models, an initial strong nonaxisymmetric \( m = 2 \) mode forms in the disk that is short-lived and dissipates after a short period of time. As we mentioned, at least for the MOG case, it is a bit difficult to discriminate between modified gravity and particle DM in this phase.

On the other hand, the second phase, which starts at intermediate times \( \tau \gtrsim 200 \), reveals some significant differences between the modified gravity and DM models. The bar continues with an almost constant value in modified gravity, while in the cored DM models it almost constantly grows until the end of the simulation. On the other hand, in the cuspy DM model, it shows a period of constancy with a low value of bar amplitude and then, similar to the cored models, starts to grow until the end of the simulation. The other distinctive feature of bar amplitude in our models is the oscillations, which are only present in modified gravity models. This behavior might be due to the presence of different beating modes, which will be discussed in Section 4.5. However, one should be careful about the artifacts that are due to the choice of the center at which the quantities are being calculated. To be specific, let us assume that the center of the grid does not coincide with the centroid of the stellar bar. Then it is straightforward to verify that the bar amplitude given by \( A_2/A_0 \) would oscillate with roughly twice the frequency of the bar rotation \( (\Omega_\star) \). However, as we will show in the next subsection, this is not the case, and the oscillation period is not short enough to be one-half the rotational period of the bar. On the other hand, the GALAXY code uses McGlynn’s method to find the position of the particle centroid (McGlynn 1984). With a suitable choice of the method’s single parameter, this method is more sensitive to the internal condensation of the particles, where the bar is formed, and puts less weight on distant particles. Therefore, the code appropriately finds the centroid of the bar. This means that the oscillations are not artifacts and reveal a real feature. Furthermore, as we already mentioned, the grid is recentered every 16 time steps in order to minimize the numeric artifacts.

The same general behavior of the disks under modified gravity/DM halo was also presented in Roshan & Rahvar (2019), where the same gravitational models are studied for different mass profiles. Therefore, it could be concluded that, ignoring the first stage of evolution, the significant differences between bar evolution in modified gravity and DM remain unchanged when we vary the initial mass densities. Of course, the time evolution of \( A_2(\tau) \) is not enough, and we still need to explore the other important properties of the stellar bar.

For a better comparison between our models, 2D projections of the particles in each model are presented in Figures 3–7. According to the face-on projection of these figures, the bar in the DM models seems to be more elongated than in modified gravity models. For the LHH model, the evolution is completed in a longer period of time. Other features apparent in these figures will be discussed in the next sections.

### 4.2. Bar Pattern Speed

Another quantity that behaves differently in modified gravity and DM models is the bar pattern speed. Unlike in observations, where the required information should be derived from only one snapshot, in numerical simulations, the evolution of the disk and the rotation of its bar are measured more easily. Therefore, away from the complications of the observational methods, one can simply observe the bar rotation in the simulations to calculate the bar pattern speed \( \Omega_p \). The evolution of this quantity with time is demonstrated in Figure 8.

From this figure, it is apparent that the pattern speed in the models of modified gravity is higher than in the DM models. This quantity decreases at a very slow rate in modified gravity, while it has a significant drop in the DM models. The decline in the bar’s pattern speed in the cored DM models begins at \( \tau \approx 200 \), while the bar strength starts to increase and the second phase in the evolution of the bar amplitude has begun. This is because the halo is responsive and the bar slows down because of the dynamical friction caused by DM particles. On the other hand, for the LHH case, similar to the bar amplitude evolution, a constancy phase is visible in the pattern speed, which lasts until about \( \tau \approx 500 \), and then the second phase of the drop in pattern speed begins. In this period, the dynamical friction starts to act more effectively. However, such behavior is not observed in any mass models of the current and previous simulations of modified gravity (Tiret & Combes 2007; Roshan & Rahvar 2019). In other words, as expected, there is no dynamical friction in modified gravity models. Of course, the dynamical friction experienced by the bar via disk particles is negligible. Notice that the previously mentioned oscillations also appear in the pattern speed of the modified gravity models. Before moving on to discuss the \( R \) parameter, which is of observational importance, let us discuss the LHH model with more emphasis on the amount of dynamical friction in this model.
4.2.1. Cusp/Core Effect in the LHH Model

To better understand the behavior of the LHH model, we studied the Hernquist DM halo density during the evolution of the system. As one can see in Figure 9, at the beginning of the simulation, a cusp exists in the inner regions of the LHH model (black curves). As the simulation continues, the secular evolution in the system transforms this cusp to a core that flattens further during the simulation. It is clear from the gray curves in Figure 9 that such a transformation is absent in the initially cored Plummer halo model, which is almost unchanged during the simulation time.

As discussed in Section 4.1, the initial rise in the bar amplitude of the LHH model is followed by a period in which the bar is weakened. According to the projected face-on snapshots of Figure 7, it is visible that during this period, there is an almost spherical configuration at the center, that is, a bulge, which might have formed due to the gravitational effect of the cuspy halo. On the other hand, it is well established that a massive bulge has stabilizing effects against global instabilities; for example, see Sellwood & Evans (2001). This symmetric configuration results in the suppression of the bar amplitude and also reduces the dynamical friction between the halo and disk particles because there is no substantial bulk motion to
cause significant dynamical friction. We expect that a moving object inside a medium feels more friction than an object that rotates without any linear motion. Therefore, the pattern speed remains almost constant. On the other hand, the weak bar activity eventually destroys the cusp; see Figure 9. This mechanism has been reported in the literature to address the so-called core-cusp problem (El-Zant et al. 2004). Consequently, the bar starts to grow, as one can see in Figure 2. This directly means that the dynamical friction would rise and the pattern speed will decrease accordingly; see Figure 8.

It is worth mentioning that we also checked another cuspy model, Navarro–Frenk–White (NFW), alongside its MOG and NLG counterpart models, but with a different initial rotation curve. From these simulations, we also found similar results for the evolution of the disk under the effect of the cuspy DM halo of NFW. Because of this similarity, we omit these simulations for the sake of brevity.

4.3. \( R \) Parameter

Another useful quantity that is related to the bar pattern speed and has observational importance is the \( R \) parameter, defined as

\[
R = \frac{D_L}{a_B},
\]

(7)

where \( D_L \) is the corotation radius and \( a_B \) is the bar semimajor axis. Observations indicate that regardless of the Hubble type,
bars in disk galaxies appear to be fast rotators with $1 < R < 1.4$ (Aguerri et al. 2003, 2015; Cuomo et al. 2019; Garma-Oehmichen et al. 2020). However, hydrodynamical simulations in the $\Lambda$CDM paradigm report slow bars in disk galaxies with $R > 1.4$ (Algorry et al. 2017). This is the case also in isolated disk simulations; for example, see Debattista & Sellwood (1998). There are some proposals to address this discrepancy in the context of the DM hypothesis. For example, it is claimed that ultralight axionic DM particles suppress the dynamical friction at galactic scales and consequently leads to fast bars (Hui et al. 2017). However, the current debate on this problem in DM models has not yielded a final conclusion. On the other hand, this issue is simply resolved in modified gravity because of the absence of dynamical friction.

To calculate the $R$ parameter, at the first step one should find the corotation radius, the radius at which the rotational velocity of the pattern is equal to that of the particles (stars) in the disk. Using the pattern speed $\Omega_p$ introduced in the previous section, it is easy to find the corotation radius.

However, calculating bar length $a_B$ is not a straightforward task, neither in real galaxies nor in simulations. Various methods have been proposed to measure this quantity. For example, visual estimation (Kormendy 1979; Martin 1995), ellipse fitting to galaxy isophotes (Wozniak et al. 1995), and Fourier decomposition on the surface brightness of the galaxy.

**Figure 5.** Projected positions of the particles in an MOG disk.
Elmegreen & Elmegreen (1985; Ohta et al. 1990; Aguerri et al. 2000) are among the methods that have been widely used in the literature.

In order to distinguish between the behavior of our models regarding $R$, we used the Fourier decomposition method in finding $a_B$. In this scheme, Fourier components of the intensity ($I_m$) are calculated, and the bar ($I_b$) and interbar ($I_{ib}$) regions are introduced as

$$ I_b = I_0 + I_2 + I_4 + I_6 $$

and

$$ I_{ib} = I_0 - I_2 + I_4 - I_6, $$

respectively, where $I_m$ is the $m$th component of the Fourier decomposition. Finding the ratio of $I_b/I_{ib}$ throughout the disk, Aguerri et al. (2000) show that the outer radius of $a_B$ using this method in numerical simulations is less than 4%, but it reaches $\approx 8\%$ for very thin bars.

By applying the above technique, the $R$ parameter could be determined for our models. However, from the projected

Figure 6. Projected positions of the particles in an LIH disk.
face-on view of the models, it is clear that there are some stages, especially at the beginning of our simulations, where strong spiral waves exist in the system. Similar to the bar amplitude, these spiral arms affect the calculations of the bar length and yield a higher value for this parameter. This could result in artificially smaller values for the $\nu$ parameter. To avoid this effect, we consider time intervals of $\Delta\tau = 40$, choose the minimum value of the bar length in each interval, and then report the related $\nu$ parameter (Hilmi et al. 2020). The results, namely the bar length and the $\nu$ parameter, are presented in Figure 10. It is clear from the left panels that in the second half of the simulation time in all of the DM models, the bar length increases with time. On the other hand, from the right panels, we see that the $\nu$ parameter is also an increasing quantity. This directly shows that the rate of change in the corotation radius of these models is higher than in the bar length.

As you may see in Figure 10, the bar is in the slow regime ($R > 1$) for all of the DM models, and also the $R$ parameter shows a growing trend. According to the previous studies, this behavior is expected because of the dynamical friction between the bar and the DM particles. However, in the modified gravity models, namely NLG and MOG, it is obvious that $R$ remains almost constant in the fast-bar regime ($1 < R < 1.4$), and the bar retains its rotation speed. It is apparent that the NLG model results in slightly higher values of the $R$ parameter than in MOG. It should also be mentioned that this property seems to be independent of the adopted disk/halo mass models. As seen in Figure 10, a similar trend in the LPH, LIH, and LHH models.
is apparent, although note that the delay in the evolution of the LHH model is also seen in the plots of bar length and $R$ parameter.

Therefore, regarding the results of pattern speed and $R$, we conclude that models under the effect of modified gravity show better compatibility with the observational results in comparison to DM halo models.

4.4. Vertical Structure of the Disk

To have a more precise view of the evolution of our models, we studied the disk’s behavior in the vertical direction. First, we focus on the inner radii, at which the bar dominates, and check the vertical behavior of the models with time. Then we will study the radial behavior of each model’s thickness at a final time of $\tau \approx 800$. The two lower panels of each part in Figure 11 illustrate the time evolution of the mean $\langle z \rangle$ and rms $\sqrt{\langle z^2 \rangle}$ thickness of the disk at two inner radii $r = 1.1$ and $r = 2.1$, respectively. In this figure, the top left, top right, middle left, middle right, and bottom parts present the results for NLG, MOG, LIH, LPH, and LHH, respectively. In all of the models studied in this work, except the LHH model, growth in rms thickness is visible at $\tau \approx 100$. Similar to the results of the previous sections, the rms thickness in the LHH model shows a delay in its evolution and starts its initial increase at about $\tau \approx 200$. However, in comparison to the MOG and DM models, NLG shows a sharper and more distinct increase, while for the other models, the growth happens at a slower rate. This is compatible with the higher initial bar growth rate of NLG, which was discussed in Section 4.1. Note that the evolution of the vertical behavior of the disk is linked to the bar strength evolution. Comparing the results of the rms thickness to bar amplitude in our modified gravity models, we see that immediately after the buckling instability, that is, $\tau \approx 150$ in NLG and $\tau \approx 200$ in MOG, the bars enter the oscillatory regime around a constant value. On the other hand, a second step-like behavior is visible in both the LIH and LPH models at $\tau \approx 400$. Such a second step-like behavior is not present in the rms curve of $r = 1.1$ for the LHH model, but a smooth growth in $r = 2.1$ is visible at around $\tau \approx 800$, which seems to be compatible with the slight change in the slope of the bar amplitude curve after this time.

It should be stressed that each step-like behavior in rms thickness can be interpreted as the manifestation of the buckling instability. During this instability, the bar is weakened as the result of vertical resonances (Raha et al. 1991). It is interesting that in our DM models LPH and LIH this instability happens twice. The first buckling happens at the same time when there is a tangible reduction in the bar amplitude. However, note that for all of the DM models, the first buckling is stronger, and its effect on the bar amplitude is more explicit. For example, the rms thickness of the LIH model at $r = 1.1$ varies from 0.5 to about 2 during the first buckling, an enhancement by a factor of 4, but grows only from about 2.5 to 4, that is, by a factor of 1.6, in the second buckling. Consequently, the second buckling is not accompanied by a rapid reduction in the bar magnitude. Considering the fact that $A_2$ is measured using the projected position of the particles in the $x - y$ plane, in the second buckling the bar thickens in the vertical direction, and there is no substantial change in its projected width and length measured in the $x - y$ plane. From Figure 11, it is also apparent that in the DM models the trend of rms thickness at larger radii (dashed curves) in the second half of the simulation differs from the smaller radii (solid curves) and grows at an almost constant rate, while it stays almost constant at smaller radii. This could be because a peanut shape is formed in our DM models. Although the disk thickens during the first buckling, the peanut shape appears more explicitly after the second buckling in the LPH and LIH models. However, in the LHH model, although there is a smooth second buckling, there is a vivid peanut showing up after the first buckling. To see this fact more clearly, refer to the plots of the particles’ projected positions in different planes of each model in Figures 3 and 6. In modified gravity models, there is only one buckling, and although we see a sudden increase in the rms thickness, there is no explicit twofold symmetric peanut.
The mean thickness, which is presented in the middle panel of each part in Figure 11, is a measure for the asymmetric distribution of particles around the \( z = 0 \) plane. A sudden change around the \( z = 0 \) plane in the \( \langle z \rangle \) diagram happens at the same time as the growth in rms thickness.

The other quantity related to the disk’s stability is the Toomre parameter \( Q \), for which the time evolution has been illustrated in the upper panel of each part in Figure 11. We should recall that, to avoid local collapse and instability of the disk, the condition of \( Q > 1 \) should be fulfilled. From Figure 11, it can be seen that for all of our models, the \( Q \) parameter in small radii has a lower value than in larger radii. Starting from \( Q \approx 1.5 \) at \( \tau = 0 \), this parameter in MOG and NLG increases rapidly to \( Q \approx 2 \) in \( R = 1.1 \) and remains almost constant until the end of the simulation. However, in the same radius in the LIH and LPH models, \( Q \) starts from 1.5 but reaches about 3 at the end of the simulation. On the other hand, the \( Q \) parameter in the LHH model behaves similarly in the other DM models at this radius until \( \tau \approx 800 \) and then continues to grow smoothly to about 4 at the end of its evolution.

The modified gravity and DM models show another difference at larger radii. Although they start from the same initial value for \( Q \), NLG and MOG show a sharper growth at an earlier stage of the evolution. In the NLG model, this growth happens at about \( \tau \approx 50 \) and reaches \( \approx 5 \) and remains constant until the end of the simulation. Also in the MOG model, we see that the growth starts at \( \tau \approx 50 \). However, the growth rate is not as fast as in NLG and lasts until \( \tau \approx 200 \), where the \( Q \) parameter reaches an almost constant value \( \approx 4 \). On the other hand, in the DM models, this growth is smoother and starts from the second half of the simulation (\( \tau \approx 400 \) for the LPH and LIH models and \( \tau \approx 800 \) for the LHH model), where the second buckling instability happens in the disk. The final value of the \( Q \) parameter at the end of simulation reaches \( \approx 7 \) in all of the DM models.

To have a better view of the overall behavior of the disk in the vertical direction, we plotted the mean and rms thickness in terms of radius for all of the models at \( \tau = 800 \) in the left and right panels of Figure 12, respectively. The rms thickness in the LIH and LPH models is higher than in modified gravity models only at radii \( 1 \lesssim R \lesssim 2 \). This peak in rms thickness is a result of the peanut shape that occurs in the DM models. The peak in the LHH model at \( \tau = 800 \) has a lower value and happens at a smaller radius. On the other hand, the rms thickness in the MOG and NLG models rises almost smoothly from the inner to outer radii and in most of the disk has higher values than in DM models. In other words, ignoring the inner parts, the disks are thicker in modified gravity models.

From Figure 12 (left), it can also be seen that the mean thickness in NLG and MOG shows a deviation from zero. This means that, in accordance with the conclusion of Tiret & Combes (2007), our MOG and NLG models are flared and warped. This could also be seen in the edge-on projections of the disks (Figures 4 and 5).

It is necessary to mention that the vertical behavior of the modified gravity models considered in this work shows some similarities to MOND (Tiret & Combes 2007), but there are some differences. For example, the peanut shape explicitly appears in MOND simulations presented in Tiret & Combes (2007), whereas we do not confirm its existence in NLG and MOG. These differences are related to the distinct nature of these models. There is a fundamental acceleration \( a_0 \approx 10^{-10} \text{m s}^{-2} \) in MOND, beyond which these deviations from Newtonian dynamics appear. On the other hand, NLG’s corrections originate in the nonlocal features of gravitation. There is no fundamental acceleration scale in this theory. The field equations are also different in these theories. In other words, although the gravitational potential \( \Phi \) satisfies a linear differential equation in NLG and MOG, MOND’s field equation is nonlinear. Therefore, in principle, we do not expect the completely same behavior for them in galactic dynamics.
Figure 11. Time evolution of Toomre parameter (upper panel), vertical mean thickness (middle panel), and vertical rms thickness (lower panel) of the disk for two different radii. Results from NLG (top left/blue), MOG (top right/green), LIH (middle left/orange), LPH (middle right/black), and LHH (bottom/magenta) simulations are specified in each part. Each quantity is plotted for a smaller radius of $R = 1.1$ (solid line) and a larger radius of $R = 2.1$ (dashed line). Notice that the LHH model is plotted for a longer period.
Further investigations on the comparison between these modified gravity models are required to reveal their viability using N-body galactic simulations.

All of the discussion presented in this subsection can be reduced to the fact that at the inner radii, in which the bar is dominating, modified gravity models lead to disks with smaller thickness; that is, the bars in DM models seem to be thicker and show stronger peanut configurations. However, the disk in modified gravity models at larger radii is more warped and flared and shows an overall higher thickness in comparison to the DM models. This is of great importance in the sense that it implies that the morphology of the disks could be different in modified gravity than in DM models. It is necessary to mention that in a similar phenomenology, it is recently claimed that the vertical structure of the galaxies could help to discriminate between modified gravity and DM (Lisanti et al. 2019a, 2019b).

As the final remark in this section, let us recall that the above-mentioned fact dealing with the thickness of the disks at small radii has also been reported in Roshan (2018) and Roshan & Rahvar (2019) for different mass profiles. Therefore, regarding the main purpose of this paper, it seems that the emergence of disks with smaller thickness at small radii compared to DM models is a model-independent feature in modified gravity models. Let us add our new finding to the previous results: disks at large radii are thicker in modified gravity models. In other words, ignoring the central region, the galactic disks seem thicker in modified gravity models compared to the standard picture.

4.5. Power Spectrum

In Figure 13 we have plotted the contours of the power spectrum with respect to the radius in the second phase of the simulations for $\tau > 200$. This figure helps us to understand the oscillating behavior of the bar magnitude. It should be emphasized that although we use different modified gravity model parameters $\alpha$ and $\mu$ compared to Roshan (2018), we see the same oscillatory behavior reported in that paper. In Figure 13, the top left and top right panels belong to NLG and MOG, respectively. In both models, we see at least two distinct frequencies in the power spectrum. This means that there are two density waves with different frequencies propagating on the surface of the disk. Naturally, this yields an oscillatory behavior in the physical properties of the bar. Interestingly, the frequencies of these waves in NLG and MOG are close to each other. This is expected in that the pattern speeds of the bars in these models are also close to each other.

In the bottom left and bottom right panels, we have shown the power spectra for the LIH and LPH models. The vertically aligned contours directly show that there is no constant frequency in the system. In other words, the frequency of the bar is varying with time. We know that it is a decreasing function with respect to time because of the dynamical friction. This feature is seen in both DM models. Note that the noisy concentration of the contours at large distances does not necessarily mean that there are several density waves because the main concentration of the baryonic matter is limited to smaller radii, as you may see in Figures 3 and 6. However, as also reported in previous studies, disks in modified gravity expand to larger radii. Therefore, the analysis of the power spectrum would be fully relevant at these radii.

As another test of the oscillatory behavior in our modified gravity models, we calculated the bar amplitude in the inner region of the disk, at $r \lesssim 4$, where the bar mode dominates. As mentioned earlier, it is clear from Figure 13 that there are two distinct modes at radii $r < 4$ and $r > 4$ in these two models. The results are compared to the bar amplitude that was previously calculated for all of the disk particles (gray curves).
in Figure 14. The value of the $m = 2$ mode in the outer disk is also presented in this figure. All of the values are scaled to the total number of the particles (mass) in each model. As is clear in Figure 14, the amplitude of the oscillations in the NLG and MOG models considerably changes by considering the particles inside $r \approx 4$. It could be concluded that the oscillations in the bar amplitude and therefore in pattern speed are directly related to the existence of the second mode in the outer disk.

4.6. Tests on Reliability of the Results

To ensure that the results do not suffer from numeric artifacts, we have performed some tests. The conservation of
energy is checked to be less than 2% until the end of the simulation. To make sure the main results are not affected by shot noise, we checked the results by changing the number of particles from 2 to 10 million. For illustration, some of the results are reported in Figure 15. As you may see in Figure 15 (left), similar to the models with 2 million particles, the initial peak in the bar amplitude happens earlier in NLG models. A similar behavior is seen in the LHH model, but the peak in LHH is slightly lower than in the NLG model, a result that is compatible with the result of the low-resolution simulation. This initial increase happens later and with a slower rate in the MOG model and then in the DM models, as was also seen previously. On the other hand, the drop of bar amplitude in the MOG model with \( N = 10^7 \) is smoother, but at later stages, both modified gravity models reach an equal amount of bar amplitude again. Furthermore, the second phase of the increase in the bar amplitude is clearly seen in all of the DM models.

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Figure 14. Reduction of the oscillations in bar amplitude by eliminating the effect of the modes in the outer radii of the NLG and MOG disks in the left and right panels, respectively.

Figure 15. Results of the evolution of bar amplitude (left) and pattern speed (right) for our models with \( N = 10^7 \) particles.

energy is checked to be less than 2% until the end of the simulation. To make sure the main results are not affected by shot noise, we checked the results by changing the number of particles from 2 to 10 million. For illustration, some of the results are reported in Figure 15. As you may see in Figure 15 (left), similar to the models with 2 million particles, the initial peak in the bar amplitude happens earlier in NLG models. A similar behavior is seen in the LHH model, but the peak in LHH is slightly lower than in the NLG model, a result that is compatible with the result of the low-resolution simulation. This initial increase happens later and with a slower rate in the MOG model and then in the DM models, as was also seen previously. On the other hand, the drop of bar amplitude in the MOG model with \( N = 10^7 \) is smoother, but at later stages, both modified gravity models reach an equal amount of bar amplitude again. Furthermore, the second phase of the increase in the bar amplitude is clearly seen in all of the DM models. Note that the constancy phase in the evolution of the LHH model is shorter and the increasing phase starts faster in comparison to the low-resolution simulation. Figure 15 (right) illustrates the evolution of pattern speed in our models with \( N = 10^7 \) particles. As is apparent, the value of pattern speed in our modified gravity models is considerably higher than in the DM models. On the other hand, the fast drop in the LIH model and the convergence of the NLG and MOG models happen later in comparison to the \( N = 2 \times 10^6 \) case. However, it is clear that the main differences in the characteristics of DM and modified gravity models remain unaltered. Such a change in the pace of disk evolution as a result of the change in the number of particles, has also been reported in the previous studies (Ghafourian & Roshan 2017).

Altering the time step from 0.01 to 0.005 leads to the same results as in our main simulations. This means that the time evolution is already converged and we have used a suitable choice for the time step. Furthermore, considering a higher resolution by changing the mesh dimensions to be \( 240 \times 256 \times 135 \), and changing lscale from 12.5 to 14.5, keeps the main results unaltered.

5. Discussions and Conclusions

In this paper, we made a comparison of the evolution of disks under the effect of two modified gravity models, NLG and MOG, and three DM halo models consisting of an LPH and an LIH, which are considered as cored DM halos, and a live Hernquist halo (LHH), which has a cusp at the beginning of the simulation. The same baryonic disk, namely the Kuzmin–Toomre profile, was adopted for all five models, and the initial conditions of the models were constructed in a way
that they start their evolution from a similar state. Therefore, the initial rotational velocity, velocity dispersions, and initial Toomre parameter of the models are selected to be as identical as possible. However, the dynamical evolution of the models does not follow the same track.

The formation of the stellar bar in the NLG model is faster and has a higher amplitude. However, after the buckling instability, it experiences a rapid reduction and remains almost constant until the end of the simulation. The same trend is followed by the MOG model, but the rate of bar formation is slower and it has a lower amplitude. On the other hand, it has been illustrated that for the DM models, the initial rise in the bar amplitude is lower and slower than in the NLG case, but after a drop in its value (buckling instability), it starts to grow again. The evolution in the low-resolution LHH model is slower in comparison to the other halo models. The suppression of the bar amplitude continues for a longer period. However, in about one-half of their simulation time, all of the DM models undergo another buckling. The outcome of this buckling instability in the LPH and LIH models is the appearance of a stellar peanut configuration. Then they start growing again at an almost constant rate. According to this behavior and regardless of the initial mass profiles, we could conclude that the system under DM influence, in comparison to modified gravity models under consideration, would result in stronger bars after a limited evolution time. It should be mentioned that, although the second buckling has not been observed in our previous simulations with the EXP disk profile, the larger thickness of the disks in DM models at the inner regions, in comparison to modified gravity models, is independent of the adopted disk/halo model.

The evolution of the pattern speed in our four models also shows a similar trend in NLG and MOG, which is considerably different from all of the DM models. The pattern speed of the bar in modified gravity models has some initial fluctuations and then shows small oscillations around an almost constant value. However, in DM models, it starts falling with an almost constant rate after $\tau \approx 200$ for the LIH and LPH models and $\tau \approx 500$ for the LHH model, which continues to the end of the simulation. We conclude that the effect of dynamical friction, which regulates the evolution of the bar and is mainly triggered by the live DM halo that surrounds the bar, is not present in modified gravity models. This difference between modified gravity and live DM halo models is vividly visible in the time evolution of the $R$ parameter. As the main result of this paper, we confirm that the modified gravity models predict fast bars in contrast to the DM models. Of course, this means that modified gravity is in better agreement with observation. More importantly, this fact is independent of the initial disk’s mass distribution.

Investigation of the vertical behavior of our models in the inner regions reveals the existence of the buckling instability in all of the models. The buckling instability is more efficient in the DM models in the sense that the final value of rms thickness of the DM models is higher than in NLG and MOG at the end of the simulation. It is necessary to reiterate that this difference is also reported in Roshan (2018) and Roshan & Rahvar (2019) for exponential disks. On the other hand, studying the vertical behavior of the models in the outer regions illustrates that this parameter has higher values in our modified gravity models in comparison to the DM models. As presented in the edge-on views, the modified gravity models are more warped and flared, which is compatible with the previous results presented in Tiret & Combes (2007).

According to the measured parameters, it could be concluded that apart from the early stages of the disk evolution, the general behavior of the bar instability, pattern speed, and also vertical structure does not change significantly by changing the disk surface density in modified gravity models. Furthermore, the substantial differences with disks surrounded by DM remain unchanged. In other words, we emphasize that, in analogy to the results presented in Roshan (2018) and Roshan & Rahvar (2019), the difference in the evolution of the disk galaxies under the effect of modified gravity and a live DM halo is independent of the employed density profile of the baryonic disk and DM halo.

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References

Aguerri, J. A. L., Debattista, V. P., & Corsini, E. M. 2003, MNRAS, 338, 465
Aguerri, J. A. L., Méndez-Abreu, J., Falcón-Barroso, J., et al. 2015, A&A, 576, A102
Aguerri, J. A. L., Muñoz-Tuñón, C., Varela, A. M., & Prieto, M. 2000, A&A, 361, 841
Aldering, D. G., Navarro, J. F., Abadi, M. G., et al. 2017, MNRAS, 469, 1054
Athanassoula, E. 2002, ApJL, 569, L83
Athanassoula, E., & Misiriotis, A. 2002, MNRAS, 330, 35
Clifton, T., Ferreira, P. G., Padilla, A., & Skordis, C. 2012, PhR, 513, 1
Corsini, E. M. 2011, MSAIS, 18, 23
Cuomo, V., Lopez Aguerri, J. A., Corsini, E. M., et al. 2019, A&A, 632, A51
Debattista, V. P., & Sellwood, J. A. 1998, ApJL, 493, L5
Debattista, V. P., & Sellwood, J. A. 2000, ApJ, 543, 704
Deyafyet, C., & Woodard, R. P. 2009, JCAP, 2009, 023
Elmegreen, B. G., & Elmegreen, D. M. 1985, ApJ, 288, 438
El-Zant, A. A., Hoffman, Y., Primack, J., Combes, F., & Shlosman, I. 2004, ApJL, 607, L75
Evans, N. W. 1993, MNRAS, 260, 191
Garma-Oechmichen, L., Cano-Díaz, M., Hernández-Toledo, H., et al. 2020, MNRAS, 491, 3655
Ghaforian, N., & Roshan, M. 2017, MNRAS, 468, 4450
Green, M. A., & Moffat, J. W. 2019, PDU, 25, 100323
Hehl, F. W., & Mashhoon, B. 2009, PhLB, 673, 279
Hernquist, L. 1990, ApJ, 356, 359
Hilmi, T., Minchev, I., Buck, T., et al. 2020, arXiv:2003.05457
Hu, W., & Sawicki, I. 2007, PhRvD, 76, 064004
Hui, L., Ostriker, J. P., Tremaine, S., & Witten, E. 2017, PhRvD, 95, 043541
Jamali, S., Roshan, M., & Amendola, L. 2018, JCAP, 2018, 048
Kormendy, J. 1979, ApJ, 227, 714
Lisanti, M., Moschella, M., Outezguine, N. J., & Slone, O. 2019a, PhRvD, 100, 083009
Lisanti, M., Moschella, M., Outezguine, N. J., & Slone, O. 2019b, arXiv:1911.12365
Martin, P. 1995, AJ, 109, 2428
Mashhoon, B. 2017, Nonlocal Gravity, 167
McGaugh, S. S., Lelli, F., & Schombert, J. M. 2016, PhRvL, 117, 201101
McGlynn, T. A. 1984, ApJ, 281, 13
Milgrom, M. 1983, ApJ, 270, 365
Moffat, J. W., & Rahvar, S. 2013, MNRAS, 426, 1439
Moffat, J. W., & Rahvar, S. 2014, MNRAS, 441, 3724
Ohta, K., Hamabe, M., & Wakamatsu, K.-I. 1990, ApJ, 357, 71
Ostriker, J. P., & Peebles, P. J. E. 1973, ApJ, 186, 467
Pérez, I., Aguerri, J. A. L., & Méndez-Abreu, J. 2012, A&A, 540, A103
Puetzfeld, D., Obukhov, Y. N. 2020, arXiv:2001.11073
Puetzfeld, D., Obukhov, Y. N., & Hehl, F. W. 2019, PhRvD, 99, 104013
Raha, N., Sellwood, J. A., James, R. A., & Kahn, F. D. 1991, Nat, 352, 411
Rahvar, S., & Mashhoon, B. 2014, PhRvD, 89, 104011
Roshan, M. 2018, ApJ, 854, 38
Roshan, M., & Abbassi, S. 2014, PhRvD, 90, 044010
Roshan, M., & Rahvar, S. 2019, ApJ, 872, 6
Sellwood, J. A. 2014, arXiv:1406.6606
Sellwood, J. A. 2019, GALAXY: N-body Simulation Software for Isolated, Collisionless Stellar Systems, Astrophysics Source Code Library, ascl:1904.002
Sellwood, J. A., & Evans, N. W. 2001, ApJ, 546, 176
Sellwood, J. A., & McGaugh, S. S. 2005, ApJ, 634, 70
Tiret, O., & Combes, F. 2007, A&A, 464, 517
Toomre, A. 1963, ApJ, 138, 385
Toomre, A. 1964, ApJ, 139, 1217
Turk, M. J., Smith, B. D., Oishi, J. S., et al. 2011, ApJS, 192, 9
Wozniak, H., Friedli, D., Martinet, L., Martin, P., & Bratschi, P. 1995, A&AS, 111, 115
Young, P. 1980, ApJ, 242, 1232