Influence of phonon confinement on the optically detected magneto-phonon resonance line-width in parabolic quantum wells

Nguyen Dinh Hien\textsuperscript{1}, Le Dinh\textsuperscript{1}, Vo Thanh Lam\textsuperscript{2}, Tran Cong Phong\textsuperscript{3}

\textsuperscript{1} Center for Theoretical & Computational Physics, Hue University’s College of Education, No. 34, Le Loi Str., Hue City, Viet Nam.
\textsuperscript{2} Sai Gon University, 273 An Duong Vuong Str., Ho Chi Minh, Viet Nam.
\textsuperscript{3} Viet Nam Institute of Educational Sciences, 101 Tran Hung Dao Str., Ha Noi, Viet Nam.

E-mail: dinhhienphysics@gmail.com

Abstract. We investigate the influence of phonon confinement described by the slab mode model on the optically detected magneto-phonon resonance (ODMPR) effect and ODMPR line-width in parabolic quantum wells by using the operator projection. The ODMPR conditions as functions of the well’s parameters and the photon energy are also obtained. The shifts of ODMPR peaks caused by the confined phonon are discussed. The numerical result for a specific quantum well shows that in the two cases of confined and bulk phonons, the line-width (LW) is sensitive to the change of the confinement frequency, and temperature. Furthermore, the influence of slab mode phonon plays an important role and cannot be neglected in reaching the ODMPR line-width for all range of the well’s confinement frequency.

1. Introduction
Magneto-phonon resonance (MPR) effect is known to arise from the resonant coupling between electrons and optical phonons when the separation between two Landau levels approaches the phonon energy. The absorption line-width is well-known as a good tool for investigating the scattering mechanisms of carriers. Hence, it can be used to probe electron-phonon scattering processes. The line-width is defined as the profile of curves describing the dependence of the absorption power on the photon energy or frequency [1]. The line-width has been measured in various kinds of semiconductors, such as quantum wells [2], quantum wires [3], and quantum dots [4]. These results show that the absorption line-width has a weak dependence on temperature and has a strong dependence on the sample size. However, in these studies, the absorption line-width was investigated based on the interaction of electrons and bulk phonons.

Phonon confinement is an essential part of the description of electron-phonon interactions [5]. It causes the increase of electron-phonon scattering rates [6, 7] and modifies the phonon density of states [8]. Phonon confinement affects magneto-phonon resonance mainly through changes in the selection rules for transitions involving subbands of electrons, Landau levels, phonon modes and affects the line-width through changes in the probability of the electron-phonon scattering.

Recently, our group has proposed a method, called the profile method. This method can be used to computationally obtain the line-width from graphs of the absorption power [9]. We used this method to determine the cyclotron resonance line-width and the influence of
phonon confinement on the optically detected electro-phonon resonance line-width in quantum wires [10, 11, 12], and in square quantum wells [13, 14]. However, the absorption line-width in parabolic quantum wells (PQWs) due to confined optical phonon-electron interaction has not been considered in previously published reports.

In the present work, we investigate influence of phonon confinement described by slab mode model on the ODMPR line-width (ODMPRLW) in a PQW. The dependence of the ODMPRLW on the well’s confinement frequency is obtained. The results of the present work are fairly different from the previous theoretical results because the slap mode model of phonon confinement are considered in detail. The paper is organized as follows. In Section 2, we introduce slab mode model of phonon in a PQW. Derivations of analytical expression of the ODMPRLW on the well’s confinement frequency is obtained. The results of the present work are compared with the corresponding energy eigenvalues, $E_{N,n}$, is given by [16]

$$E_{N,n} = (N + 1/2)\hbar \omega_c + (n + 1/2)\hbar \omega_z,$$

where $N = 0, 1, 2, \cdots$ is the Landau level index; $n = 0, 1, 2, \cdots$ is the electric subband quantum number; $\psi_N(x - x_0)$ is the harmonic oscillator wave function centered at $x_0 = -a_c^2k_y$ with $a_c = (\hbar/(m^*\omega_c))^{1/2}$ being the cyclotron radius of the orbit in the $(x, y)$ plane; $L_y$ and $k_y$ are specimen dimension and electron wave-vector in $y$-direction; $\varphi_n(z)$ is the electron wave function in $z$-direction as determined by the parabolic potential $V(z)$, $V(z) = m^*\omega_{z}^2z^2/2$. The corresponding energy eigenvalues, $E_{N,n}$, is given by [16]

$$E_{N,n} = (N + 1/2)\hbar \omega_c + (n + 1/2)\hbar \omega_z,$$

with $\omega_c = eB/m^*$, $\omega_z$, and $m^*$ are the cyclotron frequency, well’s confinement frequency, and effective mass of an electron, respectively. In this case, $\varphi_n(z)$ is given by [16]

$$\varphi_n(z) = \frac{1}{\sqrt{2^n n!\sqrt{\pi}a_z}} \exp\left(-\frac{z^2}{2a_z^2}\right)H_n\left(\frac{z}{a_z}\right),$$

with $a_z = (\hbar/(m^*\omega_z))^{1/2}$, and $H_n$ are Hermite polynomials of order $n$.

The matrix element for electron-slab mode phonon interaction in parabolic quantum wells in the extreme quantum limits can be written as [16]

$$|\langle i | H_{e-ph} | f \rangle|^2 = \frac{e^2\hbar \omega_{LO,m,q_\perp} \chi^*}{2\epsilon_0V_0} |V_{m,0,q_\perp}|^2 |J_{N,N'}(u)|^2 |G_{nn'}^{\sigma}\delta_{k_\perp,k_{\perp}+q_\perp}|^2,$$

where

$$|J_{N,N'}(u)|^2 = \frac{n_1^{1/4}n_2^{1/4}}{|n_1^{1/4}e^{-u}u^{n_1-1/4}L_{n_2}^{n_1}(u)|^2}.$$
with \( u = a_q^2 q_\perp^2 / 2 \), \( n_1 = \max\{N, N'\} \), \( n_2 = \min\{N, N'\} \), and \( L_{n_1-n_2}^{n_1-n_2} \) is the Laguerre polynomial. The overlap integral, \( G_{m' n'}^{m \phi} \), is given by

\[
G_{m' n'}^{m \phi} = \int_{-\infty}^{\infty} \varphi^*_n(z) u_{m \phi}(z) \varphi_n(z) dz,
\]

where \( \chi^* = (\chi_{\infty}^{-1} - \chi_0^{-1}) \) with \( \chi_{\infty} \) and \( \chi_0 \) are the high and static-frequency dielectric constants, respectively; \( \epsilon_0 \) is the vacuum dielectric constants, and \( V_0 = SL_z \) is the volume of the system; \( \hbar \omega_{LO_{m \phi}} = \hbar \sqrt{\omega_{LO}^2 - \beta^2 (q_\perp^2 + q_{\parallel}^2)} \) is the energy of confined optical phonon; \( \hbar \omega_{LO} \) is the energy of bulk optical phonon; \( \beta \) is the velocity parameter; \( q_\perp \) is a two dimensional vector in the \((x, y)\) plane of phonon; \( q_m = m\pi/L_z \); \( u_{m \phi}(z) \) is the parallel component of the displacement vector of \( m \)-th phonon mode in the direction of the spatial confinement for the slab mode model \( S \) \[17\]; \( \phi \) are the even (–) and odd (+) slab mode phonons. In the next section, we will use the slab mode model to calculate the optical absorption power in quantum wells. For the slab mode model, we obtain

\[
u_{m+}(z) = \cos\left(\frac{m\pi z}{L_z}\right), \quad m = 1, 3, 5, \ldots,
\]

\[
u_{m-}(z) = \sin\left(\frac{m\pi z}{L_z}\right), \quad m = 2, 4, 6, \ldots.
\]

### 3. Analytical results

We utilize slab mode model for confined phonon to calculate the absorption power in above mentioned PQWs, subjected to an ac electromagnetic field with amplitude \( E_0 \) and frequency \( \omega \). The absorption power is obtained by relating it to the transitions probability of the photon absorption to move to the higher energy levels along with phonon absorption or emission processes as follows \[18\]

\[
P(\omega) = \frac{E_0^2}{2\hbar \omega} \sum |j^\alpha_\omega|^2 \frac{(f_\alpha - f_{\alpha+1})B(\omega)}{(\omega - \omega_0)^2 + [B(\omega)]^2},
\]

where \( |j^\alpha_\omega|^2 = |\langle \alpha | j^+ \alpha \rangle|^2 = (N + 1)(2e^2 \hbar \omega_0) / m^* \), \( f_\alpha \) and \( f_{\alpha+1} \) are the Fermi-Dirac distribution functions of electron at state \( |\alpha \rangle = |N, n, k_\parallel, k_\perp \rangle \) and \( |\alpha + 1 \rangle = |N + 1, n, k_\parallel, k_\perp \rangle \). The term \( B(\omega) \) is given by

\[
B(\omega) = \frac{e^2 \hbar \omega_{LO_{m \phi}} \chi^*}{8\pi \hbar \epsilon_0 L_z} \sum_{N', n', m, \phi=\pm} \frac{|G_{m' n'}^{m \phi}|^2}{(f_{N+1, n} - f_{N, n})} \int_0^\infty q_\perp dq_\parallel \frac{|J_{N, n'}(u)|^2}{a_{m \phi} q_\perp^2 + b_{m \phi}^2 E_z^2} \times \frac{1}{E_z^2} \left\{ [(1 + N_q) f_{N+1, n} - N_q f_{N, n'}(1 - f_{N+1, n})] \delta(E_1^-) + [(1 + N_q) f_{N, n'}(1 - f_{N+1, n}) - N_q f_{N, n'}(1 - f_{N, n'})] \delta(E_1^+) \right\} + \frac{e^2 \hbar \omega_{LO_{m \phi}} \chi^*}{8\pi \hbar \epsilon_0 L_z} \sum_{N', n', m, \phi=\pm} \frac{|G_{m' n'}^{m \phi}|^2}{(f_{N+1, n} - f_{N, n})} \int_0^\infty q_\perp dq_\parallel \frac{|J_{N+1, n'}(u)|^2}{a_{m \phi} q_\perp^2 + b_{m \phi}^2 E_z^2} \times \frac{1}{E_z^2} \left\{ [(1 + N_q) f_{N, n'}(1 - f_{N, n}) - N_q f_{N, n'}(1 - f_{N', n'})] \delta(E_2^-) + [(1 + N_q) f_{N, n}(1 - f_{N', n'}) - N_q f_{N', n'}(1 - f_{N, n})] \delta(E_2^+) \right\},
\]

with \( N_q \) is the Planck distribution function for slab mode phonon at the state \( |q \rangle = |m, q_\parallel, q_\perp \rangle \),

\[
E_1^+ = \hbar \omega + (N' - N - 1) \hbar \omega_c + (n' - n) \hbar \omega_z \pm \hbar \omega_{LO_{m \phi}},
\]

\[
E_2^+ = \hbar \omega + (N - N') \hbar \omega_c + (n - n') \hbar \omega_z \pm \hbar \omega_{LO_{m \phi}},
\]
the Dirac delta functions \( \delta(E^\pm_1), \ell = 1, 2 \) in Eq. (10) are replaced by Lorentzians of width \( \gamma^S_{N,N'}, \) and \( \gamma^S_{N+1,N'} \), namely [19]

\[
\delta(E^\pm_1) = \frac{1}{\pi} \frac{\gamma^S_{N,N'}}{(E^\pm_1)^2 + (\gamma^S_{N,N'})^2}, \quad \delta(E^\pm_2) = \frac{1}{\pi} \frac{\gamma^S_{N+1,N'}}{(E^\pm_2)^2 + (\gamma^S_{N+1,N'})^2},
\]

where

\[
(\gamma^S_{N,N'})^2 = \frac{\hbar^2 \omega_{\text{LO}, m,q} \chi^*}{8\pi^2 \hbar^2 \epsilon_0 V_0} (N_0 + \frac{1}{2} \pm \frac{1}{2}) \sum_{m',\phi=\pm} |G^{m\phi}_{n'\alpha}|^2 \int_{q_L}^{\infty} dq_\perp dq_\parallel J_{N,N'} a^{S}_{m\phi} g^{S}_{\parallel} d^{2} + \frac{\beta^{5}_{m\phi}}{L^2}
\]

\[
(\gamma^S_{N+1,N'})^2 = \frac{\hbar^2 \omega_{\text{LO}, m,q} \chi^*}{8\pi^2 \hbar^2 \epsilon_0 V_0} (N_q + \frac{1}{2} \pm \frac{1}{2}) \sum_{m',\phi=\pm} |G^{m\phi'}_{n\alpha}|^2 \int_{q_L}^{\infty} dq_\perp dq_\parallel J_{N+1,N'} a^{S}_{m\phi'} g^{S}_{\parallel} d^{2} + \frac{\beta^{5}_{m\phi}}{L^2}
\]

We can see that these analytical results appear very involved. However, physical conclusion can be drawn from graphical representations and numerical results, obtained from adequate computational methods.

4. Numerical results and Discussion

To clarify the obtained analytical results we numerically evaluate the absorption power, \( P(\omega) \), for a specific GaAs/AlAs PQW. The absorption power is considered to be a function of the photon energy. The parameters used in our computational evaluation are as follows [20]: \( \chi_\infty = 10.9, \chi_0 = 12.9, m^* = 0.067 \times m_0 \) (\( m_0 \) being the mass of free electron), \( E_0 = 5.0 \times 10^9 \) Vm\(^{-1} \), \( \hbar \omega_{\text{LO}} = 36.250 \) meV, and \( \beta = 4.73 \times 10^3 \) ms\(^{-1} \). In this case, for example, we have \( \hbar \omega_{\text{LO}, m,q} = \hbar \sqrt{\omega_{\text{LO}}^2 - \beta^2 (q_\perp^2 + q_\parallel^2)} = 36.2467 \) meV \( \approx \hbar \omega_{\text{LO}} \). It means that optical phonons are actually weakly dispersive, confined phonon energies in the different levels (\( \hbar \omega_{\text{LO}, m,q} \)) differ from bulk phonon ones (\( \hbar \omega_{\text{LO}} \)) very little [14, 21]. The following results are obtained in the extreme quantum limit, and assuming that only the lowest electric subbands are occupied by the electrons: \( n = 0, n' = 0, 1 \) for confined electron and Landau levels \( N = 0, N' = 1 \).

Figure 1. a) Dependence of the absorption power in PQW on the photon energy for two different models of phonon: bulk phonons (the solid curve) and confined phonons (the dashed curve). Here, \( T = 300 \) K, \( B = 20.97 \) T, and \( \omega_z = 0.5 \omega_{\text{LO}} \). Figure b) is at the fourth peak in Fig. a).

Figure 1a) shows the dependence of the absorption power on the photon energy for bulk phonons (solid curve), confined phonons described by the slap model (dashed curve). The
result is calculated for a parabolic quantum well, at $T = 300$ K, $B = 20.97$ T, and $\omega_z = 0.5\omega_{LO}$. There are four peaks in each curve, which correspond to the ODMPR for intra-subband transitions (0-0) and inter-subband transitions (0-1). For intra-subband transitions, the ODMPR peaks satisfy the condition $h\omega = (N' - N)h\omega_c + (n - n')h\omega_z \pm h\omega_{LO}$ for inter-subband transitions.

- The first peak corresponds to the photon energy $h\omega = 18.125$ meV, which satisfies the condition $h\omega = (N' - N)h\omega_c - h\omega_z \pm h\omega_{LO}$, i.e., $18.125$ meV $= (1 - 0) \times 36.250 - (0 - 1) \times 0.5 \times 36.250$ meV $- 36.250$ meV $= 36.250$ meV. This is the condition for ODMPR with $P = N' - N = 1$. It describes the transitions of an electron between both the Landau levels $N = 0, N' = 1$ and the inter-subband $n = 0, n' = 1$ by absorbing a photon with energy $h\omega$ along with the absorption of a phonon with energy $h\omega_{LO}$.

- The second peak corresponds to the photon energy $h\omega = 36.250$ meV, which satisfies the condition $h\omega = (N' - N)h\omega_c$. Therefore, this peak is called cyclotron resonance one with $P = 1$. It describes the transition of an electron from the initial Landau level $N = 0$ to the another Landau level $N' = 1$ by absorbing a photon with energy $h\omega$.

- The third peak corresponds to the photon energy $h\omega = 72.500$ meV, which satisfies the condition $h\omega = (N' - N)h\omega_c + h\omega_{LO}$, i.e., $72.500$ meV $= (1 - 0) \times 36.250$ meV $+ 36.250$ meV. This is the condition for ODMPR with $P = 1$. This condition implies that an electron in the Landau level $N = 0$ can move to an another Landau level $N' = 1$ by absorbing a photon with energy $h\omega$ along with the emission of a phonon with energy $h\omega$.

- The fourth peak corresponds to the photon energy $h\omega = 90.625$ meV, which satisfies the condition $h\omega = (N' - N)h\omega_c - (n - n')h\omega_z + h\omega_{LO}$, i.e., $90.625$ meV $= (1 - 0) \times 36.250 - (0 - 1) \times 0.5 \times 36.250$ meV $+ 36.250$ meV $= 36.250$ meV. This is the condition for ODMPR with $P = 1$. It describes the transitions of an electron between both the Landau levels $N = 0, N' = 1$ and the inter-subband $n = 0, n' = 1$ by absorbing a photon with energy $h\omega$ along with the emission of a phonon with energy $h\omega_{LO}$.

![Figure 2](image_url)

**Figure 2.** Dependence of the LW for two different models of phonon. The dashed curve and the solid curve correspond to the case of confined phonons and bulk phonons, respectively: a) on the temperature. Here, $B = 20.97$ T and $\omega_z = 0.5\omega_{LO}$. b) on the well’s confinement frequency. Here, $T = 300$ K, and $B = 20.97$ T.

Figure 2a) shows that the LW increases with increasing temperature in both models of phonon. This is reasonable because the possibility of the electron-phonon scattering rises as the temperature increases. It is also seen that the LW of the ODMPR peak in the case of confined phonon varies faster and has a larger value than the bulk one. This is because when phonons are confined the probability electron-phonon scattering is increased.

Figure 2b) shows that the LW increases with increasing well’s confinement frequency for
both models of the phonon. This can be explained physically by a increases in the possibility of electron-phonon scattering when the well’s confinement frequency increases. Furthermore, the LW for the confined phonon case varies faster and has a larger value than it does for the bulk phonon case. Thus, in all range of the confinement frequency, the influence of phonon confinement plays an important role and cannot be neglected in reaching the ODMPR line-width.

5. Conclusions
We have calculated analytical expressions for the absorption power in parabolic quantum wells due to confined electrons and LO slab mode-confined phonons interaction. From the graphs of the absorption power, we obtained the LW as a profile of curves. Computational results show that in the cases of both bulk and confined phonons, the LW increases with temperature and well’s confinement frequency. In addition, the LW for the confined phonon case has a larger value and varies faster than it does for the bulk phonon case. Thus, in all range of the well’s confinement frequency, the influence of phonon confinement plays an important role and cannot be neglected in reaching the ODMPR line-width. This result is the same as that obtained in a two-dimensional system of ref. 7 which is verified by theory and experiments.

Acknowledgments
This work was supported by MOET-Vietnam in the scope research project coded of B2014-45-01.

References
[1] Cho Y J, Choi S D 1994 Phys. Rev. B 49 14301
[2] Spector H N, Lee J, Melman P 1986 Phys. Rev. B 34 2554
[3] Weman H, Sirigu L, Karlsson K F, Leifer K, Rudra A, Kapon E 2002 Appl. Phys. Lett. 81 2839
[4] Matthiasen C, Vamivakas A N, Atatre M 2012 Phys. Rev. Lett. 108 090602
[5] Bennett C R, Guven K, Tanatar B 1998 Phys. Rev. B 57 3994
[6] Nishiguchi N 1995 Phys. Rev. B 52 5279
[7] Rudin S, Reinecke T L 1990 Phys. Rev. B 41 7713
[8] Svizhenko A, Balandin A, Bandyopadhyay S, Stroscio M A 1998 Phys. Rev. B 57 4687
[9] Phong T C, Phuc H V 2011 Mod. Phys. Lett. B 25 1003
[10] Phong H V, Suy N V, Suoi N K 2012 Superlattices Microstruct. 52 16
[11] Phong T C, Phuc H V, Phong T C 2014 Physica E 56 102
[12] Phong T C, Phong L T T, Phuc H V, Vinh P T 2013 J. Korean Phys. Soc. 62 305
[13] Phong T C, Phong T C, Phong T C 2014 Physica E 56 102
[14] Phong T C, Phong L T T, Hien N D, Lam V T 2015 Physica E 71 79
[15] Sang Chil Lee et al 2005 Physica E 28 402
[16] Bhat J S, Mulimani B G, Kubakaddi S S 1994 Phys. Rev. B 49 16459
[17] Licari J J, Evrard R 1977 Phys. Rev. B 15 2254
[18] Kang N L, et al. 1995 J. Phys.: Condens. Matter 7 8629
[19] Chaubey M P, Van Vliet C M 1986 Phys. Rev. B 33 5617
[20] Musale M, Constantious N C 1993 Phys. Rev. B 48 11128
[21] Hien N D, Dinh L, Lam V T, Phong T C 2016 Journal of Physics: Conference Series 726 012012