Efficient Real-time Rail Traffic Optimization: Decomposition of Rerouting, Reordering, and Rescheduling Problem

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Abstract—The railway timetables are designed in an optimal manner to maximize the capacity usage of the infrastructure concerning different objectives besides avoiding conflicts. The real-time railway traffic management problem occurs when the pre-planned timetable cannot be fulfilled due to various disturbances; therefore, the trains must be rerouted, reordered, and rescheduled. Optimizing the real-time railway traffic management aims to resolve the conflicts minimizing the delay propagation or even the energy consumption. In this paper, the existing mixed-integer linear programming optimization models are extended considering a safety-relevant issue of railway traffic management, the overlaps. However, solving the resulting model can be time-consuming in complex control areas and traffic situations involving many trains. Therefore, we propose different runtime efficient multi-stage heuristic models by decomposing the original problem. The impact of the model decomposition is investigated mathematically and experimentally in different rail networks and various simulated traffic scenarios concerning the objective value and computational demand of the optimization. Besides providing a more realistic solution for the traffic management problem, the proposed multi-stage models significantly decrease the optimization runtime.

Index Terms—Mixed-integer linear programming, Problem decomposition, Rail transportation, Railway traffic optimization, Real-time traffic management, Rerouting, Rescheduling, Runtime performance

I. INTRODUCTION

The low specific energy consumption of rail transportation compared to the other sectors is great potential to reduce CO₂ emission [1]–[4]. The climate protection initiatives and the increasing energy prices will enable the railway to play a greater role in the transport of the future, increasing the demand for passenger and freight rail services [5]–[7]. Moreover, increasing infrastructure utilization, competitiveness and decreasing energy consumption and carbon emission is one of the most crucial research and development areas in railway traffic [8]–[10]. The timetable optimization is one of the key components of increasing energy efficiency [11], [12]. However, in many cases, the optimally scheduled trains suffer primary delays due to different traffic disturbances, and the pre-planned timetable would result in conflicting trains [13]. Therefore, the trains must be rescheduled or rerouted and reordered, assigning secondary delays to the trains to solve the real-time railway traffic management problem (rtRTMP) [14]. Nowadays, the conflicts are resolved by human dispatchers according to several generic and regional principles, such as the first-in-first-out (FIFO) policy. However, making an optimal decision within the required short response time is very challenging for a human dispatcher. Hence, automatic railway traffic management and control is an emerging research topic aiming to reduce energy consumption and optimize the capacity usage of rail transportation to satisfy the increasing demands [15], [16].

A. Related Work

Many works tackle the real-time railway traffic management problem with different optimization approaches and techniques to minimize delays or even energy consumption [17], [18]. The models can be divided into microscopic and macroscopic groups according to the levels of the infrastructure representation [18], [19]. The microscopic models include detailed information about the rail network in graphs, as in [20] and [21]. The macroscopic models neglect some lower-level resources such as signals, reducing the complexity of the infrastructure model [22], [23].

The optimization algorithms for the rtRTMP range widely. Mixed-integer linear programming (MILP) is one of the commonly used frameworks, as in [24], [25]. Pellegrini et al. proposed a MILP model in [26], [27], considering the infrastructure at a microscopic, track circuit level. A macroscopic-microscopic approach decomposes the problem into line traffic and station control problems in [28]–[30]. However, since the problem is NP-hard in general, some custom heuristics are introduced in [31], [32] to decrease the computational time of the algorithms. Furthermore, Pellegrini et al. proposed an efficient implementation in [33] by extending their previous work presented in [32] with new valid constraints.

Besides MILP formulation, many other optimization algorithms exist to solve the rtRTMP. A distributed model predictive control (DMPC) is proposed in [34] with similar mixed-integer linear constraints, as in [26]. Ant colony optimization (ACO) is applied in [35] to find the best subset of routing alternatives that serve as an input for the original problem formulated as mixed-integer linear programming. While the differences between ACO and MILP regarding the preliminary
route selection are pointed out in [36], [37]. The lower level control of automatic train operation (ATO) is connected with the traffic management optimization in [38], [39]. While the adaptive train control and speed profile optimization is integrated within an MPC framework in [40]. Törnquist proposed an effective greedy approach in [41] to satisfy the requirement of the available decision time. Other soft computing techniques with lower computational requirements, such as Monte Carlo tree search and reinforcement learning (RL), are used as alternatives to conventional approaches [42], [43].

The different models can also be distinguished according to the objective function. In most of the listed researches (e.g., [32], [33], [35]), the optimization goal is to reduce the delay propagation concerning all trains. However, some papers focus on energy consumption, as in [43]–[45], or a trade-off is made between time and energy efficiency [46]–[48]. Some works aim to increase the comfort of the passengers by minimizing the platform changes and passenger train delays, as in [49], [50], compared to the ones that do not distinguish between freight and passenger trains [51], [52]. Another aspect is to make a trade-off between the delay of all trains reduction and passenger satisfaction by cancelling some transfer connections [53], [54].

### B. Contributions of the Paper

Mixed-integer linear programming (MILP) allows considering railway traffic regulations by constraining the optimization space with straightforward expressions. Moreover, it provides a global optimum for the problem if the response time is not crucial. Therefore, MILP is a commonly used algorithm to solve the rtRTMP, as shown in the previous section. However, since the problem is NP-hard in general, in the case of large networks and complex scenarios, the runtime of the optimization can exceed the available time. Moreover, we extend the existing models in [26], [32] to consider the safety concept of overlaps. Since the model extension results in high computational cost, a runtime-efficient implementation is essential for optimizing the rtRTMP in a real traffic environment. Some works aim to decrease the computational complexity of MILP by introducing custom heuristics and additional constraints, as in [32], [33]. Preliminary route selection is an alternative approach to accelerating optimization by reducing the number of possible solutions, as in [35]. This paper proposes a multi-stage framework consisting of different MILP models, minimizing the delay propagation by decomposing the rtRTMP into rerouting, reordering, and rescheduling sub-problems. A low-complexity model is implemented to formulate the objective function that focuses on the relation between possible train pairs. This simplified approach provides a fast, sub-optimal solution for the rtRTMP by utilizing the results as constraints.

The result of the simplified model can also serve as an initial condition for the second stage of optimization, reducing the response time significantly when finding the first feasible solution would last longer. Furthermore, we analyzed the optimality condition to obtain the circumstance in which this solution results in a sub-optimal solution. As a result of optimality condition analysis, we developed a new model that, besides decreasing the computational cost, it guarantees the global optimum for the original problem. Similar to the simplified approach, the runtime-efficient global optimum model supports the second stage of the optimization solving the extended problem. The optimization model is introduced in Section II with a brief overview of the existing solutions. The proposed sub-optimal solution is detailed in Section III-A. The optimality condition of the first approach is analyzed in Section III-B and the resulting global optimum model is explained in Section III-C. Finally, the results evaluated in a simulation environment are shown in Section IV.

### II. Model Formulation

The proposed optimization algorithm is formulated as a mixed-integer linear programming problem in line with previous works in [26], [32] that minimizes the delay propagation within a control area. We extended the existing solutions to consider a safety-relevant traffic management regulation, the overlaps. Although the model extension contributes to more feasible solutions in practice, it increases the computational cost handled by the model decomposition. The rtRTMP to be solved is defined by the trains assigned in space and time to the control area. The multi-layer infrastructure model is constructed from a detailed microscopic graph representation of the rail network, as in [55]. The train schedule is given concerning the fundamental elements of the railway infrastructure, the track circuits represented by the corresponding edges of the graph model, on which the presence of a train is automatically detected. The group of track circuits delimited by signals form block sections whose consecutive order compose different routes between the boundaries of the control area. The following nomenclature in accordance with [26], [32] is used for the inputs of the optimization:

- \( T \) set of train circuits and routes of control area
- \( R \) set of track circuits and routes available for train \( r \)
- \( tc_{r,tc} \) dummy track circuits denoting the entry and exit locations of the control area
- \( P_{r,tc} \) preceding and subsequent track circuits of \( tc \) along route \( r \)
- \( b_{r,tc} \) set of track circuits preceding \( tc \) along route \( r \)
- \( re_{r,tc} \) binary indicators with a value of 1 if track circuit \( tc \) involves an opening or closing switch along \( r \), respectively
- \( T \) block section of track circuit \( tc \) on route \( r \)
- \( TC \) reference track circuit to reserve \( tc \) on route \( r \)
- \( R \) two-state signaling system
- \( cl_{r,tc} \) set of trains assigned to the control area
- \( tc_{i,r,tc} \) set of track circuits and routes available for train \( t \)
- \( w_a \) priority weight of train \( t \in T \)
- \( init_t, exit_t \) the time when train \( t \) is scheduled to enter and leave the control area
- \( tc_{in}, tc_{ex} \) the track circuits at where train \( t \) enters and leaves the control area
- \( ds_{i,r,tc}, cl_{r,tc} \) total duration of stay and clearing time of train \( t \) on track circuit \( tc \) along route \( r \)
- \( M \) formation, release, and overlap time large constant
The optimization state space $X$ is formed by continuous variables and binary indicators with the following notations:

- $e_{r,tc}^t$ time when train $t$ enters track circuit $tc$ along $r$
- $d_{r,tc}^t$ delay assigned to train $t$ at track circuit $tc$ on route $r$
- $sR_{tc}^t, eR_{tc}^t$ time when train $t$ starts releasing and reserves track circuit $tc$
- $sO_{tc}^t, eO_{tc}^t$ time when train $t$ starts and finishes releasing track circuit $tc$ due to overlap
- $x_r^t$ route indicator with a value of 1 if train $t$ travels along route $r$, 0 otherwise
- $z_{tc}^t$ stop indicator with a value of 1 if overlap is allocated for the safe stopping expires before train $t$ can utilize track circuit $tc$, 0 otherwise
- $c_{t,tc}^r$ conflict indicator with a value of 1 if train $t$ and $t'$ would collide at track circuit $tc$ without rescheduling
- $y_{r,tc}^t$ precedence indicator with a value of 1 if train $t$ utilizes track circuit $tc$ before $t'$
- $gO_{tc}^t$ overlap precedence indicator with a value of 1 if train $t$ reserves track circuit $tc$ due to overlap before $t'$
- $D^t$ total secondary delay of train $t$ within the control area

### Algorithm 1 Scheduling constraints

1. for $\forall t \in T$ do
2. $e_{tc,\infty}^t = 0$
3. for $\forall r \in R^t$ do
4. for $\forall tc \in TC_r$ do
5. Time constraints:
6. $e_{r,tc}^t \leq M x_r^t$
7. $e_{r,tc}^t \geq e_{r,ref,tc}^t + d_{r,tc}^t x_r^t$
8. if $p_r,tc = t_c, \infty$ then
9. $e_{r,tc}^t = init^t x_r^t$
10. end if
11. if $s_r,tc = t_c, \infty$ then
12. $e_{tc,\infty}^t = e_{tc,\infty}^t + e_{r,tc}^t + d_{r,tc}^t x_r^t$
13. end if
14. Delay constraints:
15. if $bs_r,tc \neq bs_r,s_r,tc$ then
16. $d_{r,tc}^t = e_{r,tc}^t - e_{r,tc}^t + d_{r,tc}^t$
17. else
18. $d_{r,tc}^t = 0$
19. end if
20. $D^t \geq e_{tc,\infty}^t - exit^t$
21. end for
22. end for
23. end for
24. end for

According to [29], the model constraints can be distinguished into the time concerning, delay managing, rolling stock configuration related, and capacity constraints considering the railway traffic regulations. In our model, the scheduling constraints involving the time and delay constraints are detailed in Alg. 1. The time constraints ensure the temporal coherence of train schedules and allow distinguishing the same track circuits belonging to different routes. The delay constraints manage the local delays assigned to the trains at track circuits considering the block sections determined by the signalling system. The $D^t$ total delay as the basis of the objective function is bounded by the difference between the actual and scheduled exit time of the train.

### Algorithm 2 Capacity constraints

1. for $\forall t \in T$ do
2. $\sum_{r \in R} x_r^t = 1$
3. for $\forall tc \in TC^t$ do
4. $sR_{tc}^t = \sum_{r \in R^t \cap tc} e_{r,ref,tc}^t$ for $x_r^t$
5. $eR_{tc}^t = \sum_{r \in R^t \cap tc \in TC_r} e_{r,ref,tc}^t + (eL_{tc,rel}^t f_{tc}^t + rel) x_r^t$
6. for $\forall t' \in T \setminus t$ do
7. $y_{r,tc}^t + y_{r,tc}^t = 1$
8. $sR_{tc}^t \geq eR_{tc}^t - M y_{r,tc}^t$
9. $sR_{tc}^t \geq eR_{tc}^t - M (1 - y_{r,tc}^t)$
10. end for
11. end for
12. end for

The capacity constraints in line 2 of Alg. 2 impose that every train utilizes exactly one route. According to safety regulations, a train $t$ reserves all $tc$ track circuit of $bs_r$ block section with $for$ time in advance before it enters the first track circuit $ref,tc$ of $bs_r$ along route $r$. Moreover, the reservation of a track circuit $tc$ by train $t$ is released only if $tc$ is left by the complete vehicle assembly according to $eL_{tc,rel}$ and the $rel$ release time expires. Finally, the constraints from line 7 to 9 prevent track circuits from being reserved by multiple trains at a given time. Since the focus of this paper is the runtime efficiency, the trains in connection and the constraints due to the change of rolling stock configuration with less impact on the complexity are not considered.

We extend the existing model in Alg. 3 considering that the trains may not be able to stop before signals due to the driver’s reaction time or increased braking distance. In railway traffic control, a safety distance, referred to as overlap, is provided beyond stop signals for the trains forced to stop. According to the constraints in line 6 besides the used block section, the first track circuit following the signal that terminates the section is also reserved for the train. The track circuit is reserved due to overlap until the overlap time $over$ within what the train is assumed to be able to stop safely expires. However, if the train does not stop according to $z_{tc}^t$, the reservation due to overlap finished at time $eO_{tc}^t$ is bounded by $sR_{tc}^t$ in line 8. The conflicting reservation of a track circuit is prevented by the constraints in lines 12–19.

### III. Model Decomposition

The complexity of the extended model consisting of many new integer variables and corresponding constraints causes a significant increase in runtime. Different heuristics and efficient implementations are accelerating the convergence of optimization as in [31]–[33]. The model decomposition allows dividing the original model into sub-problems with lower complexity as in [35]. The proposed decomposition is formulated based on the MILP framework described in Section II provides easy interoperability between the sub-models and the original problem. Therefore, the solutions
of the sub-problems can support the complete MILP model to decrease the computational cost, forming a multi-stage optimization model. The results of the sub-models can form hard constraints in the original problem or serve as an initial condition for the second stage of optimization. Since the models in the first stage of the optimization neglect some constraints of the rRTMP, only the \( x_t^x \) route selection and the \( y_{t',tc}^y \) precedence indicators are used as the solutions to the rerouting and reordering sub-problems. The flowchart of the proposed optimization model is illustrated in Fig. 1.

A. Sub-Optimal Model

The basic idea behind the sub-optimal model is to obtain a feasible solution for the rRTMP close to the global minimum of train delays with a fast response time. Therefore, the original model is simplified by omitting the overlap constraints and reformulating the scheduling and capacity constraints. Delaying a train may affect all the other trains, thereby upsetting the entire schedule. Hence, the rRTMP cannot be managed locally, unlocking the conflicts between the train pairs. However, the total secondary delay is related to the \( d_{t',tc}^y \) delays interpreted between \( \forall \{ t, t' \} \in T \) pairs at track circuit \( tc \) defined as:

\[
d_{t',tc}^{y} = \begin{cases} \hat{e}R_{tc}^y - \hat{s}R_{tc}^{y'}, \text{ if } y_{t',tc}^y > 0 \\ \hat{e}R_{tc}^{y'} - \hat{s}R_{tc}^{y}, \text{ otherwise} \end{cases}
\]

where \( \hat{s}R_{tc}^y \) and \( \hat{e}R_{tc}^{y'} \) denote the times when train \( t \) would start to reserve and release track circuit \( tc \) without delays.

According to (1), the delay resolving the conflict of train \( t \) and \( t' \) at \( tc \) can be computed as the difference of the time when the reservation of the track circuit is finished by the train using it first, and when the other one would start the reservation according to the original timetable. Hence, the schedule variables are not updated in the sub-optimal model, neglecting the capacity constraints in lines [8] - [9] of Alg. 2. Moreover, the delay constraints are reformulated defining the \( d_{t',tc}^y \) delay assigned to train \( t \) at track circuit \( tc \) due to \( t' \) as:

\[
d_{t',tc}^y \geq eR_{tc}^y - sR_{tc}^{y'} - \sum_{\forall r \in R^y; y_{r,tc}^y \in TC^y_r} \sum_{p_r,tc \in P} d_{r,tc}^y \cdot M \cdot y_{r,tc}^y,
\]

\[
d_{t',tc}^y \geq eR_{tc}^y - sR_{tc}^{y'} - \sum_{\forall r \in R^y; y_{r,tc}^y \in TC^y_r} \sum_{p_r,tc \in P} d_{r,tc}^y \cdot M \cdot y_{r,tc}^y,
\]
where the nested sums consider the already served waiting times, preventing multiple delays. For sake of brevity, the non-negativity boundaries of the delays are not given explicitly. Then, the objective function of the sub-optimal model considering the possible train pairs is formed by the reshaped $d^{t',tc}_t$ delay triplets as:

$$f_s \left( d^{t',tc}_t, \delta^t | X \right) = \sum_{t \in T} w^t \left( \sum_{t' \in T: \ \forall t' \neq t} \sum_{t \in TC^t} d^{t',tc}_t + \delta^t \right),$$

(4)

where the $\delta^t$ additional delay resulting from alternative route selection is computed as:

$$\delta^t = \sum_{r \in R^t} \left( e^t_{r,tc} + d^{t',tc}_t x^t_r \right) - e^{t'}_r; \ \ \ s_{r,tc} = tc_{\infty}$$

(5)

The simplified model must provide a feasible solution so that the result can be used in the second stage of the optimization. Therefore, additional constraints related to the precedence indicators are imposed in Alg. 4. The process of changing the precedence indicators between two trains is illustrated in Fig. 2, showing that the order of the trains can change at track circuits $tc$ that include a switch. However, the $y^t_{t',tc}$ precedence indicator may also change when the route of the trains splits at the preceding track circuit $p_{r,tc}$. The constraint in line 6 of Alg. 4 prevents changing $y^t_{t',tc}$ when $tc$ or $p_{r,tc}$ does not contain a switch. Moreover, the precedence indicator $y^t_{t',tc}$ cannot be incremented to 1 only if $tc$ is not used by the $t$ or the overtaken train $t'$ does not use the track circuit preceding $tc$ along the route travelled by $t$ as in Fig. 2. Reversing this constraint, $y^t_{t',tc}$ may change to 0 from 1 when $tc$ is not used by the other train $t'$ or the preceding track circuit of $t$ is not used by $t'$. The expressions from lines 8 to 12 impose these constraints, where $x_{t',tc}$ means that the switch branches the route, while $tc$ with $x_{t',tc} = 1$ closes it, having less subsequent track circuits than preceding ones. According to the constraints in lines 16 - 17, train $t''$ cannot use track circuit $tc$ before $t$ if it enters $tc$ before $t'$ and before $t''$. The precedence $y^t_{t',tc}$ between train $t$ and $t'$ at track circuit $tc$ is set to 0 by inequalities in lines 19 - 20 if $t$ travels along $tc$ while $t'$ does not.

However, besides the constraints in Alg. 4, additional inequalities imposed $\forall \{t, t'\} \in T$ train pairs at their common $TC^{t'} \cap TC^t$ track circuits are needed to ensure that the trains enter the control area when they are scheduled. Since the sub-optimal model does not consider the signalling system, the delay $d^{t',tc}_t$ resolving the conflict between trains $t$ and $t'$ at track circuit $tc$ would be assigned to $t$ at the block section preceding $bs_{r,tc}$. Therefore, delaying train $t$ outside the control area is prevented by setting $d^{t',tc}_t$ to zero if $tc$ belongs to the first block section of $t$’s route as:

$$d^{t',tc}_t \leq M \sum_{r \in R^t: \ \forall t \in TC^t_r} x^t_{r,tc},$$

(6)

where $ebs^{t,tc}_r \in \{0, 1\}$ indicates if the block section $bs_{r,tc}$ of track circuit $tc$ is the first one along route $r$ of train $t$ as:

$$ebs^{t,tc}_r = \begin{cases} 1 & \text{if } bs_{r,tc} = bs_{r,tc_{in}} \\ 0 & \text{otherwise} \end{cases}$$

(7)

However, the delay $d^{t',tc}_t$ formulated by $t$ and $t'$ neglects the effect of other trains. Supposing train $t'$ leaves the control area before $t$ enters it at the same track circuit $tc_{in} = tc_{ex}$ according to the original timetable, the delays assigned to $t'$ due to another $t'' \in T$ may change the precedence of $t$ and $t'$. Therefore, the delay accumulated by $t'$ up to the block section where $t$ enter the control area is tackled by the constraint imposed in (9). Moreover, delaying a train $t'$ traveling in the same direction as $t$ at the track circuit $tc_{in}$ forces $t$ to enter control area later then it is scheduled according to the timetable. Therefore, the $d^{t',tc}_{t''}$ delays assigned to $t'$ at track circuit $tc$ due to the conflict with with another $t''$ at track circuits $s_{t,tc}$ is prevented by the expression in (9) if $tc$ belongs to the first block section of $t$ and $t'$ uses $tc$ before $t$. For sake of brevity, the counter-pair of the constraints in (6), (8), (9) related to the $d^{t',tc}_{t''}$ delay resolving the conflict of $t$ and $t'$, and the $d^{t',tc}_{t''}$, $d^{t',tc}_{t''}$ propagated delays are neglected.

$$s_{Rtc} \geq e_{Rtc}^{t'}/ tf_{Rtc} \sum_{r \in R^t: \ \forall t \in TC^t_r} d^{t',tc}_t - M y^t_{t',tc} -$$

$$M \left( 1 - \sum_{r \in R^t: \ \forall t \in TC^t_r} x^t_{r,tc} ebs^t_{r,tc} \right) -$$

$$M \left( 1 - \sum_{r \in R^t: \ \forall t \in TC^t_r} x^t_{r,tc} ebs^t_{r,tc} \right)$$

(8)

$$s_{Rtc} \geq e_{Rtc}^{t'/t} \sum_{r \in R^t: \ \forall t \in TC^t_r} d^{t',tc}_t - M y^t_{t',tc} -$$

$$M \left( 1 - \sum_{r \in R^t: \ \forall t \in TC^t_r} x^t_{r,tc} ebs^t_{r,tc} \right) -$$

$$M \left( 1 - \sum_{r \in R^t: \ \forall t \in TC^t_r} x^t_{r,tc} ebs^t_{r,tc} \right)$$

(9)

### B. Optimality Condition Analysis

The conditions in which the model detailed in the previous section provides a sub-optimal solution is investigated and given explicitly in this section. Since the sub-optimal model tackles the rerouting and reordering problem as well, the optimality condition of the reordering is discussed first with fixed route selection. An example scenario involving three trains on a network modelling two connected stations is illustrated in Fig. 3 when the solution of the model is sub-optimal in terms of train precedence. The platforms represented by dashed lines are 400 m in length with 100 and 40 km/h maximum velocity on the lower and upper branches, respectively. The track circuits, including the switches, have 200 m length along each route, while $tcS$ connecting the stations is 1200 m long, and the maximum speed allowed on them is uniformly set to 100 km/h. The trains with $w^t = 1$ identical priorities would travel with constant speed in accordance with Fig. 3 but $t_1$...
that is scheduled to dwell two additional minutes on platform tc7. According to the sub-optimal model, train t3 highlighted with red should wait for both t1 and t2 at track circuit tc3, and t2 is delayed at tc3 due to the two-minute dwell of t1. Therefore, the objective value \( D_X(1) \) of the solution X1 is:

\[
D_X(1) = d_{t1,tc8}^{t3} + d_{t2,tc8}^{t3} + d_{t1,tc3}^{t2} .
\]  

However, since the \( d_{t1,tc3}^{t2} \) delay assigned to t2 is propagated to t3 as well, the \( D_X(1) \) total delay of the trains would be:

\[
D_X(1) = 2 d_{t1,tc3}^{t2} + d_{t2,tc3}^{t2} .
\]  

In the optimal solution X2, the green train t1 is overtaken by t2 at track circuit tc4, also giving the precedence to t3 with \( D_X(2) < D_X(1) \) global objective value:

\[
D_X(2) = 2 d_{t3,tc8}^{t1} + d_{t2,tc8}^{t1} .
\]  

Since however, the objective value \( D_X(2) \) of the global optimum X2 according to the sub-optimal model as:

\[
D_X(2) = d_{t3,tc2}^{t1} + d_{t2,tc2}^{t1} + d_{t3,tc2}^{t1} > D_X(1) ,
\]  

the solution X1 is sub-optimal in terms of total delay.

In general, the solution X of the reordering or the rerouting problems neglecting the delay propagation is sub-optimal compared to the original model without overlaps if:

\[
\begin{align*}
\exists y_{t1,tc} \in X' \forall \{t, t'\} \in T, \forall t, t' \in TC' & \land TC' \ni \exists x_{t,tc} \in X' \forall t \in T, \forall r \in R .
\end{align*}
\]

satisfying the constraints in Alg. 2 and the resulting \( d_{t,tc}^{t'} \) delays according to (2) and (3) meet the following criteria:

\[
\begin{align*}
 & f_s \left( d_{t,tc}^{t'} , \delta | X' \right) > f_s \left( d_{t,tc}^{t} , \delta | X \right) , 
 & f_s \left( d_{t,tc}^{t}, \delta' | X' \right) \leq f_s \left( d_{t,tc}^{t'}, \delta' | X \right) .
\end{align*}
\]

where \( d_{t,tc}^{t'} \) denotes the delay assigned to train t at track circuit tc due to the conflict with \( t' \) at tc, including the delay propagation as:

\[
\sum_{r \in R, t \in TC} d_{t',tc}^{t} = eR_{tc}^{t} - sR_{tc}^{t} + \Delta_{t',tc} - \Delta_{t,tc}^{t}. 
\]  

Constraint (7) considers the signaling system by delaying train t at track circuit tc due to the blocks on section tc along route r. The propagation variables \( \Delta_{t',tc}^{t} \) and \( \Delta_{t,tc}^{t} \) in (7) increasing and decreasing the \( d_{t,tc}^{t} \) delay of train t at track circuit tc with respect to \( t' \) is quantified by the following equations:

\[
\begin{align*}
\Delta_{t',tc}^{t} &= \sum_{t' \in T : t' \neq t, t' \neq t' \land t' \in TC'} \sum_{t' \in R, t \in TC} d_{t',tc}^{t} , 
\Delta_{t,tc}^{t} &= \sum_{t' \in T : t' \neq t, t' \neq t' \land t' \in TC'} \sum_{t' \in R, t \in TC} d_{t',tc}^{t} .
\end{align*}
\]

so that substituting \( d_{t,tc}^{t} = \sum_{t' \in T : t' \neq t, t' \neq t' \land t' \in TC'} x_{t',tc}^{t} \) in Alg. 1 and Alg. 2 results in a feasible solution. Since the subtrahend \( \Delta_{t',tc}^{t} \) in (7) decreases the objective value, false delays may occur, leading to an infeasible solution according to the original model. This problem is tackled by the global optimum model detailed in the following section.

\section*{C. Global Optimum Model}

The proposed global optimum model extends the formulation of \( d_{t,tc}^{t} \) in the sub-optimal model involving the delay propagation according to the optimality condition analysis as:

\[
\begin{align*}
\sum_{r \in R, t \in TC} d_{t',pr,ref,tc}^{t} & \geq eR_{tc}^{t} - sR_{tc}^{t} + \Delta_{t',tc}^{t} - \Delta_{t,tc}^{t}, \\
\sum_{r \in R, t \in TC} d_{t,pr,ref,tc}^{t} & \leq eR_{tc}^{t} - sR_{tc}^{t} + \Delta_{t,tc}^{t} - \Delta_{t',tc}^{t} .
\end{align*}
\]

where the last two terms neglect to delay train t at track circuit tc where t has priority over \( t' \) or it does not use tc. However, false values decreasing the objective value may occur due to the \( \Delta_{t',tc}^{t} \) total delay accumulated by train t to track circuit tc.

The false delays result in an infeasible solution is prevented.
by the constraints in Alg. 5. Besides the delay propagation, the global optimum model considers the signaling system of the control area concerning the block sections. Therefore, the potential conflict of trains \( t \) and \( t' \) at track circuit \( tc \) is resolved at the track circuit \( P_{r,ref,tc} \), preceding the corresponding block section \( bs_{r,tc} \), and the rescheduling at track circuits lacking a signal is prevented by the constraints in lines 4 - 5 of Alg. 5. The inequalities in lines 6 - 7 impose that the delay assigned to train \( t \) due to the conflict with \( t' \) at track circuit \( tc \) can only be nonzero if \( t' \) is reported to use \( tc \) before \( t \). According to lines 8 - 9, the trains cannot be delayed on track circuits they do not use. A conflict between two trains may arise if a block section is reserved by both of them. Hence, the false delay assigned to train \( t \) at track circuit \( tc \) due to \( t' \) is avoided by the constraints in lines 11 and 14 when the block section \( bs_{r,nc,tc} \) of the subsequent track circuit \( s_{r,tc} \) does not intersect the \( r' \) route of \( t' \). Finally, according to expression in lines 17 - 18 if train \( t \) has to wait for multiple trains at a given track circuit, the delay assigned to it is computed from the schedule of the last train to pass. Although this model provides a global optimum for the original problem in [26], [32], it is not guaranteed for the proposed model of the rRTMP extended with the overlap constraints. The global optimum model can result in a sub-optimal solution \( X' \) when a feasible alternative solution \( X'' \) in accordance with (14) so that:

\[
\begin{align*}
    f_s \left( \tilde{d}_{t',tc}^t \cdot \delta^t \shuffle X' \right) & \geq f_s \left( \tilde{d}_{t',tc}^t \cdot \delta^t \shuffle X'' \right), \\
    f_s \left( \tilde{d}_o^t \cdot \delta^t \shuffle X' \right) & \leq f_s \left( \tilde{d}_o^t \cdot \delta^t \shuffle X'' \right),
\end{align*}
\]

(21)

where \( \tilde{d}_o^t \cdot \delta^t \shuffle X' \) denotes the delay that has to be assigned to train \( t \) to resolve the conflict with \( t' \) at track circuit \( tc \) considering the overlaps according to Alg. 3 as:

\[
\tilde{d}_o^t \cdot \delta^t = \tilde{d}_{t',tc}^t + y_{t',tc}^t \left( 1 - c_{t',tc}^t \right) \left( eR_{tc}^t - sO_{tc}^t \right) + c_{t',tc}^t \left( eO_{tc}^t - sR_{tc}^t \right) + \left( sO_{tc}^t - eO_{tc}^t \right) y_{t',tc}^t.
\]

(23)

The optimality of the proposed global optimum model concerning the extended rRTMP is investigated experimentally in the following section.

IV. EXPERIMENTAL RESULTS

The proposed models and the multi-stage optimization workflow is evaluated in two different rail networks and various scenarios. The infrastructure model of the two control areas illustrated in Fig. 3 is constructed manually in a Matlab GUI implemented for this purpose. The dashed lines in Fig. 4 represent the platforms, and the signals are located at the boundaries of the track circuits. The network in Fig. 4 consists of 17 track circuits and eight routes, representing three stations connected by single rail sections. The infrastructure in Fig. 4 also includes 17 track circuits, forming 12 routes between the four entry and exit locations. The details of the track circuits, including their total length and speed limit, are summarized in Table IV. Two hundred scenarios are generated randomly in both networks, one hundred involving three trains and the other one hundred with four trains. The parameters defining the initial state and properties of the trains are uniformly distributed between the limits given in Table III. The trains can enter the control area at either of the entry locations, or may have already been in the network at the beginning of the optimization. The trains that entered the control area before, referred to as initial trains, are drawn with a given \( P_{in} \) discrete probability. The location of the initial trains is selected from the track circuits with even probability. The trains impose to wait with probability \( P_{dw} \) must dwell some uniformly distributed random time at either of the platforms. The randomly generated scenarios must be feasible according to the original model defined in Section II extended with the overlap constraints. The proposed models are evaluated in terms of the objective value and runtime of the optimization. The performance of the proposed models is given relatively to the original models averaging the results of the \( N \) different scenarios with the following metrics:

\[
\begin{align*}
    P_{obj} &= \frac{\sum_{i=1}^{N} \left( 1 + f(X_i) - f(\hat{X}_i) \right)}{\sum_{i=1}^{N} f(X_i)} \cdot 100, \\
    P_{run} &= \frac{1}{N} \sum_{i=1}^{N} r_{ti} \cdot 100,
\end{align*}
\]

(24)

(25)

where \( f(X_i) \), \( f(\hat{X}_i) \), \( r_{ti} \) and \( \hat{r}_{ti} \) denotes the objective value and runtime of the proposed and reference model in the \( i \)-th scenario. The runtime performance is given simply by the average ratio of the proposed and reference model in (25). However, since the objective value can be zero, the metric in (24) is defined based on the difference between the proposed and reference model and the average objective value of the reference model. The objective value is computed according

![Fig. 3. Example scenario resulting in sub-optimal solution](image-url)
According to the first stage is applied, and then the precedence stage is assessed in two ways. First, only the route selection extracted from the solution of the first stage, the second the preliminary solution. Since multiple information can be variables and constraints related to the overlaps without using reference model of the second stage is extended with the first stage provided by the proposed models. Therefore, the with overlap constraints in Alg. 3 using the solution of the solve the rescheduling sub-problem of the rtRTMP extended overlaps. The second stage of the optimization is intended to rescheduling are not bijective sub-problems, considering the runtime of the sub-optimal model. The reordering and by the runtime of the sub-optimal model, increasing the response time (9). In this case, the model is evaluated as if it provides the simplified formulation of the delay propagation in (8) and may provide an infeasible solution in some cases due to the constraining the precedence indicators as:

\[ f(X) = \sum_{t \in T} w_t D_t. \]  

Both proposed models solve either the rerouting and reordering sub-problems of the rtRTMP, determining the local delays of the trains based on the original schedule and resulting precedence indicators as:

\[
\begin{aligned}
    d_{t',tc}^f &= \max_{t' \in T, \forall t' \neq t} \left( e_{R_{t',tc}}^t - s_{R_{t',tc}}^t - \sum_{p_{tc} \in P_{t',tc}} d_{p_{tc}}^t + \sum_{t' \in R^t} \sum_{p_{tc} \in P_{t',tc}} d_{p_{tc}}^{t'} x_{r,t'}^{t'} y_{r,t'}^{t'} sig_{r,tc'} \right). \\
\end{aligned}
\]  

In Eq. (27), \( sig_{r,tc} \in \{0,1\} \) indicates if \( tc \) terminates its \( bs_{r,tc} \) block section with a signal along route \( r \) as:

\[
    sig_{r,tc'} = \begin{cases} 
    1 & \text{if } bs_{r,tc} \neq bs_{r,rs,tc} \\
    0 & \text{otherwise}
    \end{cases}
\]  

Therefore, the first stage of the optimization is compared directly with the original model neglecting the overlaps. Despite the complexity of the optimization is decreased significantly by both of the proposed models. In most circumstances, the runtime of the sub-optimal model is around 50% of the reference model in the first stage of the optimization. Even in the most complex scenarios, the proposed model is faster by 39% on average. Despite the significant runtime improvement, the maximum overhead of the model simplification regarding the objective value is 25%. The 52–43% runtime decrease in the second optimization stage is even more significant using the preliminary solution of the first stage provided by the sub-optimal model. However, since constraining the precedence between the trains besides the route selection leads more frequently to an infeasible solution considering the overlaps, sometimes it even increases the runtime. Although the global optimum model does not decrease the response time of the first stage as much as the sub-optimal one, it always provides the same solution as the reference model 27–46% faster. Even the objective value in the second stage does not deteriorate more than 17%. Despite the complexity of the global optimum model, it is slightly more efficient than the sub-optimal one with respect to the runtime due to the faster convergence and lower probability of an infeasible solution.

The average runtime decrease varies between 27% and 78%
Algorithm 5 Global optimum delays

1: for \( \forall t \in T \) do
2:   for \( \forall t' \in T : t' \neq t \) do
3:     for \( \forall t'' \in T : t'' \neq t' \) do
4:       \( \bar{d}_{t',tc} = M \sum_{r \in R^t} x^t_r \)
5:       \( \bar{d}^t_{t,tc} = M \sum_{r \in R^t} x^t_r \)
6:       \( \sum_{r \in R^t: t \in TC} \bar{d}^t_{t,tc} \leq M(1 - y^t_{t,tc}) + M \left( 1 - \sum_{r \in R^t: tc \in TC} x^t_r \right) \)
7:       \( \sum_{r \in R^t: t \in TC} \bar{d}^t_{t,tc} \leq M(1 - y^t_{t,tc}) + M \left( 1 - \sum_{r \in R^t: tc \in TC} x^t_r \right) \)
8:       \( \sum_{r \in R^t: t \in TC} \bar{d}^t_{t,tc} \leq M \sum_{r \in R^t: tc \in TC} x^t_r \)
9:       \( \sum_{r \in R^t: t \in TC} \bar{d}^t_{t,tc} \leq M \sum_{r \in R^t: tc \in TC} x^t_r \)
10: for \( \forall r \in R^t: t \in TC_r \) do
11:     \( d^t_{t,tc} \leq M \sum_{r \in R^t: tc \in TC} x^t_r \)
12:     end for
13: for \( \forall r \in R^t: t \in TC_r \) do
14:     \( d^t_{t,tc} \leq M \sum_{r \in R^t: tc \in TC} x^t_r \)
15:     end for
16: for \( t'' \in T : t'' \neq t, t'' \neq t' \) do
17:     \( \sum_{r \in R^t: t \in TC} \bar{d}^t_{t,tc} \leq M \left( 2 - y^t_{t',tc} - y^t_{t''} \right) + M \left( 2 - \sum_{r \in R^t: tc \in TC} x^t_r \right) \)
18:     \( \sum_{r \in R^t: t \in TC} \bar{d}^t_{t,tc} \leq M \left( 2 - y^t_{t',tc} - y^t_{t''} \right) \)
19:     end for
20: end for
21: end for
22: end for

Fig. 5. Runtime comparison between the proposed and reference models

Table I

|          | Sub-optimal | Global optimum |
|----------|-------------|---------------|
| 1st      | Route       | Precedence    |
| 2nd      | Route       | Precedence    |
| 3 Trains | 125         | 120           |
|          | 109         | 102           |
|          | 48          | 54            |
|          | 31          | 30            |

Table II

|          | Sub-optimal | Global optimum |
|----------|-------------|---------------|
| 1st      | Route       | Precedence    |
| 2nd      | Route       | Precedence    |
| 3 Trains | 118         | 113           |
|          | 115         | 112           |
|          | 50          | 55            |
|          | 48          | 47            |

in the different circumstances. However, the runtime complexity trend of the proposed algorithms illustrated in Fig. 5 is even more favourable. The more complex the scenario is, the more significant the impact of the proposed models solving the problem with the highest 15-second runtime more than 35 times faster than the original model. At the same time, the runtime peak of the sub-optimal and global optimum models are less than 1.5 and 0.75 seconds, respectively, neglecting the overlap constraints. In the second stage of the optimization, the 244-second maximum response time of the reference model, is decreased by 96–99.9% by using the preliminary route
selection and precedence indicators reported by the global optimum model. The objective value of the proposed models compared to the reference depicted in Fig 6 does not show the same tendency as the runtime complexity. In most cases, the solution of the sub-optimal model differs slightly from the reference model. Although the train delays weighted by their priorities are twice as higher as the reference in some scenarios neglecting the overlap constraints, the relative performance of the proposed model does not change with the objective value of the reference model. Besides granting the solution that minimizes the delay propagation without overlaps, the global optimum model also provides good input for the second stage of the optimization, resulting in similar objective values to the reference.

V. CONCLUSION

The reinterpreted delays resolving the local conflict between two trains allow the formulation of the proposed models based on the original timetable without any feedback. The simplified sub-optimal model provides a fast solution for the original problem without the overlap constraints. Despite neglecting the delay propagation, the performance of the optimization does not degrade significantly on average. The global optimum model reduces the average runtime almost as much as the sub-optimal one, ensuring the best solution in terms of the optimization objective. The MILP model of the real-time railway traffic management problem extended with the safety-critic regulation of the overlaps may not be able to resolve the conflicts within the desired response time. While the proposed multi-stage optimization workflow, decomposing the rerouting, reordering, and rescheduling problems, is characterized by a relatively low increase of the objective value, it significantly reduces the complexity of the problem by using the preliminary solution of the proposed models. Furthermore, the higher runtime of the reference model, the more efficient decrease can be achieved by the proposed models, resulting in a much more favourable complexity class. Therefore, the multi-stage optimization workflow with the sub-optimal and global-optimum models can sufficiently solve the extended real-time railway problem.
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