Research on the Diesel Engine with Sliding Mode Variable Structure Theory

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Abstract. This study constructed the nonlinear mathematical model of the diesel engine high-pressure common rail (HPCR) system through two polynomial fitting which was treated as a kind of affine nonlinear system. Based on sliding-mode variable structure control (SMVSC) theory, a sliding-mode controller for affine nonlinear systems was designed for achieving the control of common rail pressure and the diesel engine’s rotational speed. Finally, on the simulation platform of MATLAB, the designed nonlinear HPCR system was simulated. The simulation results demonstrated that sliding-mode variable structure control algorithm shows favourable control performances which are overcoming the shortcomings of traditional PID control in overshoot, parameter adjustment, system precision, adjustment time and ascending time.

1. Introduction
Electronically controlled diesel engine has undergone three major development stages [1]. The first-generation position-control diesel engine used electronic governor to replace the mechanical governor, but cannot change the injection characteristics of the cam injection system. The second-generation product is time-control diesel engine, which employed magnetic value for addressing the problems in fuel injection timing and duration. However, pilot injection and multi-injection can be difficult to realize because of injection pressure. The third-generation product, which is widely applied at present, is the common-rail diesel engine maintaining a high fuel injection pressure. This is also referred to as time/pressure-control diesel engine, which can overcome the limitations of cam outlines. The fuel injection pressure is irrelevant to rotational speed and load [2].

Common-rail pressure is generally controlled by closed-loop proportion-integration-differentiation (PID) controller [3]. Classical PID controllers show simple structures and need not construct the system’s accurate mathematical model. However, it is fairly difficult to seek out the appropriate proportional band, integration time and differentiation time simultaneously so as to achieve the optimal control effects. Moreover, in some engineering applications such as simple temperature or pressure systems, once the parameters of PID is changed, the system’s control effect can hardly be restored to factory settings.

Variable-structure control is an advance control algorithm based on accurate mathematical model. As a comprehensive method for nonlinear control systems, variable-structure control is practical and can acquire a series of favourable properties and qualities [4]. Sliding mode variable structure control (SMVSC) is a control strategy for variable-structure control systems, which is also a comprehensive method in modern control theory based on phase plane method. The basic idea is that the system’s
motion first reaches a certain switching surface, then, the motion follows a sliding mode and gradually approaches the origin, and finally, the control goal can be realized when the sliding motion is endowed with good character.

At present, the main control method of diesel engine running parameters is PID. This paper used Sliding mode variable structure control algorithm to study the nonlinear diesel engine.

2. Control of diesel engine

In a high-pressure common rail (HPCR) system without accumulator-type injector and boosting piston, the fuel oil pressure in public oil passage can be directly controlled to maintain a high level (over 100 MPa). Fuel injection quantity and injection timing are regulated via the electromagnetically-controlled three-way valve or two-way valve. Specifically, fuel injection quantity and injection timing change with the variation of backpressure, which is controlled by the three-way valve or two-way valve. The pressure of high-pressure oil rail is related to the circulating fuel supply quantity [5].

2.1. Establishment of the nonlinear diesel engine system model

The common-rail pressure is subjected to the effects of fuel supply quantity of the high-pressure fuel pump, fuel return flow and fuel injection quantity of the injector. Accordingly, for the convenience in the design of control law and the related stability analysis, the common-rail pressure model for a diesel engine can be simplified. The relationship among common-rail pressure, fuel injection flow, fuel supply flow and fuel return flow can be written as:

\[
\frac{dP_{cr}}{dt} = \frac{K}{V} (Q_1 - Q_2 - Q_3)
\]  

(1)

where \(Q_1\) denotes the fuel supply flow of a HVCR pump, \(Q_2\) denotes the injector’s fuel injection flow, \(Q_3\) denotes fuel return flow of the pressure control valve (PCV), \(P_{cr}\) denotes the fuel pressure in the HVCR pipe, \(K\) denotes the diesel’s elastic modulus and \(V\) denotes the high-pressure common rail volume.

For accurately acquiring the quantitative relations among common rail pressure, the engine’s rotational speed and fuel injection quantity, a 6L16/24 diesel was selected for the experiments on a HPCR testing platform. The fuel supply and return data at different common-rail pressures of 80MPa, 100MPa, 120MPa and 140MPa were recorded for further analysis.

Fuel supply flow mainly depends on the rotational speed of the high-pressure fuel pump which equals to half of the diesel’s rotational speed \(n\) and the common rail pressure \(P_{cr}\). Therefore, the following expression can be derived:

\[
Q_1 = k_1 n + c_1
\]  

(2)

Then, a quadratic polynomial of \(P_{cr}\) relative to \(k_1\) and \(c_1\) can be acquired through fitting, which can be written as:

\[
k_1 = -0.0000000392P_{cr}^2 + 0.00000273P_{cr} + 0.001065
\]  

(3)

\[
c_1 = 0.0000581P_{cr}^2 - 0.00356P_{cr} - 0.072
\]  

(4)

If the injector’s fuel injection flow \(q\) is set as the circulating fuel injection flow, the following expression can be acquired:

\[
Q_2 = q
\]  

(5)

The fuel return quantity increases with the rising of HPCR pressure. In this study, by neglecting the other factors of the equipment, fuel return quantity is directly determined by HPCR pressure:

\[
Q_3 = 0.00416P_{cr}
\]  

(6)

According to equations (1)-(6), the mathematical model of a HPCR nonlinear system can be described as:
\[ \frac{dP_{cr}}{dt} = (a_1 P_{cr}^2 + a_2 P_{cr} + a_3)n + a_4 P_{cr}^2 + a_5 P_{cr} + a_6 + a_7q \]  
where \( a_1 = -0.00392, \ a_2 = 0.273, \ a_3 = 106.5, \ a_4 = 5.81, \ a_5 = -35.6, \ a_6 = -720, \ a_7 = 929.6 \)

Obviously, the differential equation not only includes the nonlinear quadratic component of HPCR pressure but also connects with the diesel’s rotational speed and fuel injection quantity.

The differential equation of the rotational speed control system is also required. Similarly, using curve fitting method, the circulating fuel injection quantity data and average indicating pressure \( P_i \) at different rotational speeds (900 r/min, 1000 r/min, 1100 r/min and 1200 r/min, respectively) were analyzed. The average indicating pressure can be expressed as:

\[ P_i = k_2q + c_2 \]  
Through quadratic polynomial fitting, the following expression can be derived:

\[ k_2 = -0.0024025 n^2 + 6.9n + 4873.3 \]  
\[ c_2 = 0.00003346 n^2 - 0.107n - 71.5294 \]

The variation of diesel rotating speed directly depends on the difference between output torque \( M \) and load torque \( M_l \). According to the D’Alembert fundamental equation of motion [6], the following expression can be acquired:

\[ J \frac{dw}{dt} = M - M_l \]  
Assuming that the characteristic of load torque \( M_l \) is the vehicle’s stable road characteristic, the following expression can be derived:

\[ M_l = l \cdot n^2 \]  
Let \( l = 0.00007 \) and \( J = 5 \text{ kg} \cdot \text{m}^2 \), the nonlinear mathematic model for the diesel’s rotational speed controlling can be written as:

\[ \frac{dn}{dt} = b_1 n^2 + b_2n + b_3 + (b_4 n^2 + b_5n + b_6)q \]  
where \( b_1 = -0.0001, \ b_2 = -0.144, \ b_3 = -114.7, \ b_4 = -0.003, \ b_5 = 8.4, \ b_6 = 5296 \)

According to equations (7) and (13), the mathematic model of a nonlinear HPCR and speed control system can be written as:

\[
\begin{align*}
\frac{dP_{cr}}{dt} &= (a_1 P_{cr}^2 + a_2 P_{cr} + a_3)n + a_4 P_{cr}^2 + a_5 P_{cr} + a_6 + a_7q \\
\frac{dn}{dt} &= b_1 n^2 + b_2n + b_3 + (b_4 n^2 + b_5n + b_6)q \\
\end{align*}
\]

This model is characterized by the quadratic component with nonlinearity and includes two state variables to be controlled, namely, common rail pressure and the diesel’s rotational speed.

2.2. Design of a sliding mode controller (SMC)

For the system as described in equation (14), the state variable can be set as:

\[ x_1 = \int_0^t e_1 \, dt, \quad x_2 = e_1 = P_{cr0} - P_{cr}, \]

where \( P_{cr0} \) denotes the preset common rail pressure and \( e_1 \) denotes the pressure difference.

\[ x_3 = \int_0^t e_2 \, dt, \quad x_4 = e_2 = n_0 - n, \]

where \( n_0 \) denotes the preset value of the diesel’s rotational speed and \( e_2 \) denotes the difference of rotational speed.
Therefore, the state equation of the system can be rewritten as:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -(a_1 P_{cr}^2 + a_2 P_{cr} + a_3) n - (a_4 P_{cr}^2 + a_5 P_{cr} + a_6) - a_7 q \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -(b_1 n^2 + b_2 n + b_3) - (b_4 n^2 + b_5 n + b_6) q
\end{align*}
\]  
(15)

The state equation of a kind of affine nonlinear system can be written as:

\[
\frac{d}{dt} x = f(x) + g(x) u \quad x \in \mathbb{R}^n \quad u \in \mathbb{R}
\]  
(16)

where \( f(x) \) and \( g(x) \) are two function vectors

\[
\begin{bmatrix}
  f_1(x) \\
  f_2(x) \\
  f_3(x) \\
  f_4(x)
\end{bmatrix} =
\begin{bmatrix}
  x_2 \\
  -(a_1 P_{cr}^2 + a_2 P_{cr} + a_3) n \\
  -(a_4 P_{cr}^2 + a_5 P_{cr} + a_6) \\
  x_4 \\
  -(b_1 n^2 + b_2 n + b_3)
\end{bmatrix}
\]

\[
\begin{bmatrix}
  g_1(x) \\
  g_2(x) \\
  g_3(x) \\
  g_4(x)
\end{bmatrix} =
\begin{bmatrix}
  0 \\
  -a_7 \\
  0 \\
  -(b_4 n^2 + b_5 n + b_6)
\end{bmatrix}
\]

This equation is linear for the control variable \( u \). In a similar form, the system can be rewritten as:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -(a_1 P_{cr}^2 + a_2 P_{cr} + a_3) n - (a_4 P_{cr}^2 + a_5 P_{cr} + a_6) - a_7 u \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -(b_1 n^2 + b_2 n + b_3) - (b_4 n^2 + b_5 n + b_6) u
\end{align*}
\]  
(17)

Therefore, we can derive that:

\[
\begin{bmatrix}
  f_1(x) \\
  f_2(x) \\
  f_3(x) \\
  f_4(x)
\end{bmatrix} =
\begin{bmatrix}
  x_2 \\
  -(a_1 P_{cr}^2 + a_2 P_{cr} + a_3) n \\
  -(a_4 P_{cr}^2 + a_5 P_{cr} + a_6) \\
  x_4 \\
  -(b_1 n^2 + b_2 n + b_3)
\end{bmatrix}
\]

\[
\begin{bmatrix}
  g_1(x) \\
  g_2(x) \\
  g_3(x) \\
  g_4(x)
\end{bmatrix} =
\begin{bmatrix}
  0 \\
  -a_7 \\
  0 \\
  -(b_4 n^2 + b_5 n + b_6)
\end{bmatrix}
\]

where \( u = q \), \( P_{cr} = P_{cr0} - x_2 \), \( n = n_0 - x_4 \)

Assuming \( s = s(x) = s(x_1, x_2, x_3, x_4) \) denotes the switching function in sliding-mode variable structure controlling, the following expression can be written as:

\[
s(x) = c_1 x_1 + c_2 x_2 + c_3 x_3 + x_4
\]  
(18)

\[
u_{eq}(x) = -\left[ \frac{\partial}{\partial x} s(x) g(x) \right]^{-1} \frac{\partial}{\partial x} s(x) f(x) = \frac{c_1 x_2 - c_2 A_1 n - c_2 A_2 + c_3 x_4 - c_4 B_1}{-c_2 A_1 - B_2}
\]  
(19)

where \( A_1 = a_1 P_{cr}^2 + a_2 P_{cr} + a_3 \), \( A_2 = a_4 P_{cr}^2 + a_5 P_{cr} + a_6 \), \( B_1 = b_1 n^2 + b_2 n + b_3 \), \( B_2 = b_4 n^2 + b_5 n + b_6 \)

By substituting equation (19) into equation (16), the sliding-mode motion equation can be described as:

\[
\begin{bmatrix}
  \frac{d}{dt} x = f(x) - g(x) \left[ \frac{\partial}{\partial x} s(x) g(x) \right]^{-1} \frac{\partial}{\partial x} s(x) f(x) \\
  s(x) = 0
\end{bmatrix}
\]  
(20)
SMVSC can be solved by reaching law control. In this study, the simplest constant reaching law was used in the solution which is $ds/dt = -\varepsilon \text{sgn}(s), \quad \varepsilon > 0$. Therefore, the reaching law can be rewritten as:

$$\varepsilon \text{sgn}(s) = \sum_{i=1}^{n} c_i f_i(x) + \left[\sum_{i=1}^{n} c_i g_i(x)\right]u$$

The control rate $u$ can be solved according to Eq. (21):

$$u^+ = \frac{\varepsilon + c_1 x_2 - c_2 A_1 n - c_2 A_2 + c_3 x_4 - c_4 B_1}{-c_2 a_7 - B_2} \quad s > 0$$

$$u^- = \frac{-\varepsilon + c_1 x_2 - c_2 A_1 n - c_2 A_2 + c_3 x_4 - c_4 B_1}{-c_2 a_7 - B_2} \quad s < 0$$

3. Software simulations

The diesel engine’s rotational speed was controlled by a SMVSC. Firstly, based on the identified deviation of rotational speed, the control input $u$ was obtained, i.e., the circulating fuel injection quantity $q\, (g)$. Secondly, using both the circulating fuel injection quantity and the feedback rotational speed as the known quantities, the average indicating pressure can be acquired by looking up the MAP charts [7], and the output torque was calculated by subtracting the average friction pressure from the average indicating pressure. Finally, according to D’Alembert principle, the actual rotational speed was acquired based on the difference between the output torque $M$ and the load torque $M_l$.

As to the controlling of HPCR pressure, the control input $u$ was firstly acquired based on the pressure deviation. Then, both the control input $u$ and the feedback pressure were adopted as the known quantities, and fuel injection pulse width, the opening degree of PCV valve and the actual HPCR pressure were obtained by looking up the MAP charts of the common rail pressure of the diesel engine. The simulation program of the system is shown in Figure 1. Simulation results of the common rail pressure and speed of the diesel engine are shown in Figures 2-7.

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**Figure 1.** Simulation structure figure of the common rail and speed system.
According to the simulation results of the vehicle’s stable road characteristic, For SMVSC, the rotational speed overshoot was much smaller than PID control, especially in the initial stage of diesel engine starting, and speed fluctuated within 10 r/min in the stable stage. Additionally, the diesel engine’s transient speed-regulating rate was smaller than 10%, the stable speed-regulation rate was about 0%, and the fluctuation ratio of rotational speed was smaller than 1%. Under a sudden load-up of 100%, the stabilization time of 5% error was smaller than 4 seconds.
For the common rail pressure control, SMVSC varies within a range of 5%. The simulation results fully proved the feasibility of variable structure control theory in investigating the common rail and rotational speed systems of nonlinear diesel engine. Using variable structure control method, the system’s nonlinear characteristics were well maintained.

4. Conclusions
In previous studies, the diesel engine’s rotational speed, common rail pressure, fuel viscosity temperature and cooling water temperature were generally controlled by traditional PID strategy. Although the control structure was simple, the control precision, stability, robustness and control performances were far from ideal.

The fundamental difference between sliding-mode variable structural control and conventional control lies in the control’s discontinuity, i.e., a kind of switching characteristics that can make the system structure change at any time. The control characteristics can force the system structure to change purposefully in a leaping mode during the transient process according to the system state at that moment (deviation and its all-order derivatives), and make up-and-down motions slightly but frequently along the specified state trajectory under certain conditions. Using sliding model variable structure control method, system shows a series of advantages including fast response, small overshoot and strong robustness.

References
[1] Bahram Bahri and Azhar Abdul Aziz 2013 Misfiring cycle pressure measurement for diesel-converted HCCI engine 2013 3rd International Conference on Instrumentation Control and Automation pp 45-50
[2] An Xiaohui and Liu Bolan 2010 Electronic control unit development for unit pump diesel engine 2010 International Conference on Optoelectronics and Image Processing vol 1 pp 479-482
[3] Farouk N and Leghimi 2012 Speed control system on marine diesel engine based on a self-tuning fuzzy PID controller Research Journal of Applied Sciences, Engineering and Technology vol 4 chapter 6 pp 686-690
[4] Xu F and Chen H 2015 Fast Nonlinear model predictive control on FPGA using particle swarm optimization IEEE Transactions on Industrial Electronics vol 63 chapter 5 pp 310-321
[5] Fakhari V and Ohadi A 2013 A robust adaptive control scheme for an active mount using a dynamic engine model Journal of Vibration & Control vol 21 chapter 11 pp 1-23
[6] Sun Z and Kuo T W 2010 Transient control of electro-hydraulic fully flexible engine valve actuation system IEEE Transactions on Control Systems Technology vol 18 chapter 3 pp 613-621
[7] Wang Y and Chen B 2011 Simulation research on intelligent control for propulsion system of large ship Oceans IEEE vol 5 chapter 7 pp 1-7

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