Constraining Red-shift Parametrization Parameters in Brans-Dicke Theory: Evolution of Open Confidence Contours

Ritabrata Biswas
Ujjal Debnath

Department of Mathematics, Bengal Engineering and Science University, Shibpur, Howrah-711 103, India.

Abstract

In Brans Dicke theory of gravity, from the nature of the scalar field-potential considered, the dark energy, dark matter, radiation densities predicted by different observations and the closedness of the universe considered, we can fix our $\omega_{BD}$, the Brans Dicke parameter, keeping only the thing in mind that from different solar system constrains it must be greater than $5 \times 10^5$. Once we have a value, satisfying the required lower boundary, in our hand we proceed for setting unknown parameters of the different dark energy models’ EoS parameter. In this paper we work with three well known red shift parametrizations of dark energy EoS. To constrain their free parameters for Brans Dicke theory of gravity we take twelve point red shift vs Hubble’s parameter data and perform $\chi^2$ test. We present the observational data analysis mechanism for Stern, Stern+BAO and Stern+BAO+CMB observations. Minimising $\chi^2$, we obtain the best fit values and draw different confidence contours. We analyze the contours physically. Also we examine the best fit of distance modulus for our theoretical models and the Supernovae Type Ia Union2 sample. For Brans Dicke theory of gravity the difference from the mainstream confidence contouring method of data analysis is that the confidence contours evolved are not at all closed contours like a circle or a ellipse. Rather they are found to be open contours allowing the free parameters to float inside a infinite region of parameter space. However, negative EoSs are likely to evolve from the best fit values.

Pacs no: 04.60.Pp, 98.80.Qc

1 Introduction

The simplest and best known one among scalar-tensor theories is the Brans-Dicke (BD hereafter) theory of gravity [1]. Scalar Tensor theories of gravity include an extra scalar filed and hereby a potential dependent upon that besides the tensor part considered by the Einstein gravity. Amongst them Brans-Dicke theory of gravity comprises with a constant parameter, named as BD parameter, which regulates the impact of the scalar fields inside the action as well as the field equations etc. The BD theory, being a generalization of general relativity gives the latter back with a high value of $\omega_{BD}$, the BD parameter [2]. Viking space probe says $\omega_{BD}$ should exceed 500 from timing experiments [3]. The best studied binaries with compact objects are the double neutron stars, with the Hulse-Taylor pulsar ($PSR1913 + 16$) as the prototypical case. Unfortunately, in all double neutron-star systems, the masses of the two members of the binary are surprisingly similar and this severely limits the prospects of placing strong constraints on the dipole radiation from them. Indeed, the magnitude of dipole radiation depends on the difference of the sensitivities between the two members of the binaries, and for neutron stars the sensitivities depend primarily on their masses. The resulting constraint imposed on the BD parameter $\omega_{BD}$ by the Hulse-Taylor pulsar is significantly smaller than the limit $\omega_{BD} > 40,000$ set by the Cassini mission [4]. VLBI light deflection theory predicts $\omega_{BD}$ to be $> 3500$.

A massive scalar field has very negligible effect on the motion of celestial bodies provided the mass is large enough with respect to the inverse of the inter body distances. But if the mass is sufficiently small, the

1biswas.ritabrata@gmail.com
2ujjaldebnath@yahoo.com
corresponding potential \( V(\phi) \) can be locally neglected though the coupling function to matter will strongly be constrained by experiment as we will see later on. In case of solar system, the phenomena of precision is a good tool to test from Newton’s law to relativistic correction of it, namely the post-Newtonian relativistic correction of it which is proportional to \( \frac{1}{c^2} \). Parametrized post-Newtonian correction formalism is used to work with parameters of such orders. Two famous parameters among them \( \beta_{PPN} \) and \( \gamma_{PPN} \) introduced by Eddington \([5, 6]\) in the Schwarzschild metric \([-g_{00} = 1 - \frac{2GM}{c^2r} + 2\beta_{PPN} (\frac{2M}{c^2r})^2 + O (\frac{4M}{c^2r})] \) with all the other eights are constrained to be very close to the values of general relativity (in GR \( \beta_{PPN} = \gamma_{PPN} = 1 \)). However, for scalar-tensor theories, the values are not unity any more. The observed value of perihelion shift of mercury implies the bound \([7]\)

\[
|2\gamma_{PPN} - \beta_{PPN} - 1| < 3 \times 10^{-3}.
\]

Lunar Laser Ranging \([8]\) gives the bound

\[
\gamma_{PPN} - 1 = (2.1 \pm 2.3) \times 10^{-5}.
\]

L. Perivolaropoulos in \([9]\) has shown for negligible mass of the field \( \phi \), \( \omega_{BD} \) having the relation \( \gamma = \frac{1 + \omega}{2 + \omega} \) turns to be \( \omega > 4 \times 10^4 \) at the 2\( \sigma \) confidence level. \( \omega > 5 \times 10^4 \) is supported value in some literature \([10]\). The light deflection as measured by Very Long Baseline Interferometry \([11]\) gives the information

\[
|\gamma_{PPN} - 1| < 4 \times 10^{-4}.
\]

The label “Casini” to the impressive recent constraint obtained by measuring the time delay variation to the Cassini spacecraft near solar conjunction \([3]\):

\[
\gamma_{PPN} - 1 = (2.1 \pm 2.3) \times 10^{-5}.
\]

So overall for solar system \( \omega_{BD} > 5 \times 10^4 \) will be supported by all the different observation tools.

Dark Energy (DE), assumed to be distributed homogeneously all over in the universe, is a component of the critical density of our current universe as shown by the Cosmic Microwave Background (CMB) and type Ia supernovae (SNIa) observations \([12, 13, 14, 15]\). The cosmic standard candles, type Ia SNe supernovae influence us to think about cosmic acceleration. The Friedmann equation \( \frac{\dot{a}}{a} = -4\pi G (\rho + 3p)/3 \) requires the condition \( (\rho + 3p) < 0 \) for accelerated expansion \( (\dot{a} > 0) \). As density is an ever positive physical quantity, we see the EoS parameter must be negative and also less than \(-1/3\). We give term to such a negative pressure creating substance as DE. DE occupies 73\% of the whole matter-energy of our universe. Theoretically, we can find many proposed DE candidates. In astrophysical sense it is popular to have a redshift parametrization (i.e., taking the redshift \( z \) as the variable parameter of the EoS only) of the EoS as \( p(z) = w(z)\rho(z) \). The EoS parameter \( w \) and its time derivative with respect to Hubble time are currently constrained by the distance measurements of the type Ia supernova and the current observational data constrain the range of EoS as \(-1.38 < w < -0.82 \) \([16]\). Recently, the combination of WMAP3 and Supernova Legacy Survey data shows a significant constraint on the EoS \( w = -0.97^{+0.07}_{-0.09} \) for the DE, in a flat universe \([17]\).

Two mainstream families of redshift parametrizations are there, viz.,

(i) Family I : \( w(z) = w_0 + w_1 \left( \frac{z}{1+z} \right)^n \) and

(ii) Family II : \( w(z) = w_0 + w_1 \left( \frac{z}{1+z} \right)^\gamma \),

where, \( w_0 \) and \( w_1 \) are two undecided parameters, \( n \) is a natural number. We will pick up three particular well known parametrizations:

1. **Linear parametrization:** For \( n = 0 \), family II is known as “Linear parametrization” \( w(z) = w_0 + w_1 z \) \([18]\). Here \( w_0 = -1/3 \) and \( w_1 = -0.9 \) with \( z < 1 \) when Einstein gravity has been considered. This grows increasingly unsuitable for \( z \gg 1 \). Upadhye-Ishak-Steinhardt parametrization \([19]\) can avoid this problem.

2. **CPL parametrization:** For \( n = 1 \), both the families I and II lead to the same parametrization \( w(z) = w_0 + w_1 z \). This ansatz was first discussed by Chevallier and Polarski \([20]\) and later studied more
elaborately by Linder [21]. In Einstein gravity the best fit values for this model while fitting with the SN1a gold data set are \( w_0 = -1.58 \) and \( w_1 = 3.29 \). This parametrization will be shortly named as “CPL Parametrization” after the proposer Chevallier-Polarski-Lindler. There are literature which supports that CPL parametrization has the quantity to catch the dynamics of many DE models and in particular the dynamics of the step like ones [22].

3. JBP parametrization: For family II, \( n = 2 \) gives the parametrization \( w(z) = w_0 + w_1 \left( \frac{z}{1+z} \right)^2 \). A fairly rapid evolution of this EoS allowed so that \( w(z) \geq -1/2 \) at \( z > 0.5 \) is consistent with the supernovae observation in Einstein gravity. We will call this parametrization as “JBP” [22] parametrization.

Study of DE with different EoS value was studied in BD cosmology by several authors. In 5D BD cosmology [24], the authors have shown that the DE component of the universe agrees with the observational data. In [25] considering the holographic energy density as a dynamical cosmological constant in BD theory different future horizon cut-offs are been studied. The work [26] tells the dependence of cosmological parameters considering the holographic energy density as a dynamical cosmological constant in BD theory different future horizon cut-offs are been studied. The work [27] tells the dependence of cosmological parameters considering the holographic energy density as a dynamical cosmological constant in BD theory different future horizon cut-offs are been studied. The work [28] tells the dependence of cosmological parameters considering the holographic energy density as a dynamical cosmological constant in BD theory different future horizon cut-offs are been studied. The work [29] tells the dependence of cosmological parameters considering the holographic energy density as a dynamical cosmological constant in BD theory different future horizon cut-offs are been studied. The work [30] tells the dependence of cosmological parameters considering the holographic energy density as a dynamical cosmological constant in BD theory different future horizon cut-offs are been studied.

Wu and Chen [28] derived observational constraint on the BD model in a flat FLRW universe with cosmological constant and cold dark matter. For cosmic microwave background back ground they had used , they did include the WMAP five year data etc. They found degeneracy for \( \omega_{BD} < 0 \) for few data sets. In [29] the authors used newly published Planck CMB temperature data. The cosmological parameters \( H_0, \omega_{BD}, h^2, \sigma_8 \) etc have been constrained. Fabris et al [30] have studied cosmological solutions for a pressureless fluid in the Brans-Dicke theory exhibit asymptotical accelerated phase for some range of values of the parameter \( \omega_{BD} \), interpolating a matter dominated phase and an inflationary phase. The effective gravitational coupling is negative. The author did test this model against the supernovae type Ia data. The fitting of the observational data is fairly rapid evolution of this EoS allowed so that \( w(z) \geq -1/2 \) at \( z > 0.5 \) is consistent with the supernovae observation in Einstein gravity. We will call this parametrization as “JBP” [22] parametrization.

Our motive for this paper is to study the DE characterized by the redshift parametrization of EoS in BD theory. We will try to constrain the EoS parameters for different data sets. In the next section we will construct the equations for BD theory and the concerned parameters’ expressions. to achieve the value of \( \omega_{BD} \) which will be consistent with the solar system constrains we have calculated different \( \omega_{BD} \) with respect to different \( \alpha \) and chosen an appropriate one. In section 3 we will examine the best fitting values and different sigma contours of \( w_1 \) and \( w_2 \) for Stern Data, Stern+BAO and Stern+BAO+CMB respectively. Our main motive is to find the best fit values for \( w_0 \) and \( w_1 \). We will tally the theoretical bound with the supernova data in the section 4. Finally, we will go for a brief summary in section 5.

2 Basic Equations for Brans-Dicke Theory

The action of the self interacting Brans-Dicke theory reads as [1] (choosing \( c = 1 \))

\[
S = \int d^4x \sqrt{\gamma} \left[ \frac{8\pi}{\phi} \left( \frac{\omega_{BD}}{\phi} \phi^\alpha \phi_{,\alpha} - V(\phi) + 16\pi \mathcal{L}_m \right) \right]
\]

(1)

here \( \phi \) is the BD scalar field, \( \omega_{BD} \) is the BD parameter, \( V(\phi) \) is the self interacting potential. In BD theory \( \frac{1}{\phi} \) exactly resembles with the factor \( G \), the gravitational constant. The action [1] also does match with the low energy string theory action [31] for \( \omega_{BD} = -1 \). The matter content of the universe is composed of DM, DE and the radiation contribution.

From the Lagrangian density [1] we obtain the field equation [31]

\[
G_{\mu \nu} = \frac{8\pi}{\phi} T^m_{\mu \nu} + \frac{\omega_{BD}}{\phi^2} \left[ \phi_{,\mu} \phi_{,\nu} - \frac{1}{2} g_{\mu \nu} \phi_{,\alpha} \phi^{,\alpha} \right] + \frac{1}{\phi} \left[ \phi_{,\mu} \phi_{,\nu} - g_{\mu \nu} \Box \phi \right] - \frac{V(\phi)}{2\phi} g_{\mu \nu}
\]

(2)
and

\[ \Phi = \frac{8\pi T}{3 + 2\omega_{BD}} - \frac{1}{3 + 2\omega_{BD}} \left[ 2V(\phi) - \phi \frac{dV(\phi)}{d\phi} \right] \]  

(3)

where \( T = T_{\mu\nu}g^{\mu\nu} \). Here, \( T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu} \) with 4-velocities \( u^\mu \) obeying \( u_\mu u^\mu = -1 \).

Now, our equation is stated to be following (31)

\[ p = \frac{8\pi \rho_{tot}}{6} + \frac{\omega_{BD}}{6} \left( \frac{\dot{\phi}}{\phi} \right)^2 + \frac{V(\phi)}{6} \]  

and

\[ 2\dot{H} + 3H^2 = -\frac{8\pi \rho_{tot}}{\phi} - \frac{\omega_{BD}}{2} \left( \frac{\dot{\phi}}{\phi} \right)^2 - 2H \frac{\dot{\phi}}{\phi} - \frac{\ddot{\phi}}{\phi} + \frac{V(\phi)}{2\phi} \]  

(6)

where \( \dot{H} = \frac{\dot{a}}{a} \) is the Hubble parameter. If it is assumed that matter is concerned in BD theory the conservation equation is stated to be

\[ \dot{\rho}_{tot} + 3H(\rho_{tot} + p_{tot}) = 0 \]  

(7)

Now, our \( \rho_{tot} \) comprises of densities of dark matter (DM), dark energy (DE) and the density related to the radiation (i.e., \( \rho_{tot} = \rho_{DM} + \rho_{DE} + \rho_{rad} \)). Now, the pressure corresponding these three components are respectively zero, \( p_{DE} \) governed by the EoS of concerned DE considered and one third of the radiation density respectively. These immediately gives us the total densities of the concerned fluids as

\[ \rho_{tot}^{Linear} = \rho_{rad}(1 + z)^4 + \rho_{DM0}(1 + z)^3 + \rho_{DE0}^{Linear} (1 + z)^{3(1 + w_0 - w_1)} \exp \left\{ 3w_1 z \right\} . \]  

(8)

and the same for CPL and JBP parametrization will be

\[ \rho_{tot}^{CPL} = \rho_{rad}(1 + z)^4 + \rho_{DM0}(1 + z)^3 + \rho_{DE0}^{CPL} (1 + z)^3(1 + w_0 + w_1) \exp \left\{ \frac{-3w_1 z}{1 + z} \right\} \]  

(9)

\[ \rho_{tot}^{JBP} = \rho_{rad}(1 + z)^4 + \rho_{DM0}(1 + z)^3 + \rho_{DE0}^{JBP} (1 + z)^3(1 + w_0) \exp \left\{ \frac{3w_1 z^2}{2(1 + z)^2} \right\} \]  

(10)

respectively.

For simplicity of the calculation, we assume that \( V = \phi_0 \dot{\phi}^n \) and \( \phi = \phi_0 a^\alpha \). To determine \( H \), using equation (5) with equations (8) - (10), we have the following expressions:

**For Linear Parametrization:**

\[ H^2 + \frac{\alpha}{H_0}(1 + z)H_0 + \left[ \frac{\dot{\phi}}{\phi} - \frac{\omega_{BD}}{6} \left( \frac{\dot{\phi}}{\phi} \right)^2 \right] (1 + z)^2 - \frac{V_0}{6H_0^2 \phi_0} \phi_0^{n-1} \frac{1}{(1 + z)^{\alpha(n-1)}} \]

\[ - \frac{8\pi}{3H_0^2 \phi_0} \left( 1 + z \right)^n \left\{ \rho_{rad0}(1 + z)^4 + \rho_{DM0}(1 + z)^3 + \rho_{DE0}^{Linear} (1 + z)^{3(1 + w_0 - w_1)} \exp \left\{ 3w_1 z \right\} \right\} \]

\[ H_0^2 = 0 \]  

(11)

**For CPL Parametrization:**

\[ H^2 + \frac{\alpha}{H_0}(1 + z)H_0 + \left[ \frac{\dot{\phi}}{\phi} - \frac{\omega_{BD}}{6} \left( \frac{\dot{\phi}}{\phi} \right)^2 + \frac{V_0}{6H_0^2 \phi_0} \phi_0^{n-1} \frac{1}{(1 + z)^{\alpha(n-1)}} \right] (1 + z)^2 - \frac{V_0}{6H_0^2 \phi_0} \phi_0^{n-1} \frac{1}{(1 + z)^{\alpha(n-1)}} \]

\[ - \frac{8\pi}{3H_0^2 \phi_0} \left( 1 + z \right)^n \left\{ \rho_{rad0}(1 + z)^4 + \rho_{DM0}(1 + z)^3 + \rho_{DE0}^{CPL} (1 + z)^3(1 + w_0 + w_1) \exp \left\{ \frac{-3w_1 z}{1 + z} \right\} \right\} \]

\[ H_0^2 = 0 \]  

(12)

**For JBP Parametrization:**

\[ H^2 + \frac{\alpha}{H_0}(1 + z)H_0 + \left[ \frac{\dot{\phi}}{\phi} - \frac{\omega_{BD}}{6} \left( \frac{\dot{\phi}}{\phi} \right)^2 \right] (1 + z)^2 - \frac{V_0}{6H_0^2 \phi_0} \phi_0^{n-1} \frac{1}{(1 + z)^{\alpha(n-1)}} \]

\[ - \frac{8\pi}{3H_0^2 \phi_0} \left( 1 + z \right)^n \left\{ \rho_{rad0}(1 + z)^4 + \rho_{DM0}(1 + z)^3 + \rho_{DE0}^{JBP} (1 + z)^3(1 + w_0) \exp \left\{ \frac{3w_1 z^2}{2(1 + z)^2} \right\} \right\} \]

\[ H_0^2 = 0 \]  

(13)
We use the present observational data analysis mechanism for Stern, Stern+BAO and Stern+BAO+CMB observations. The Hubble parameter \( H_0 \), the standard error \( \sigma(\phi(\cdot)) \) shifts (twelve data points) given in observed Hubble data \cite{32} we will proceed. The Hubble parameter at 7 year WMAP data \cite{43}. Now, \( \Omega_{\text{rad}} \) is completely determined by the value of \( V_0 \) which will be taken as a trivial value (= 1). So for our model we have to determine \( \alpha \) (i.e., \( \Omega_{\text{rad}} \)) in such a way that satisfy the observational constraint for \( \omega_{BD} \). It is obvious that \( \alpha \) should be less than 1, else the scalar field will increase abruptly in late universe. For \( \phi_0 = V_0 = 1, n = 2 \) (i.e., \( V(\phi) = \phi^2 \)) we have prepared the chart of \( \omega_{BD} \) vs \( \alpha \) (given in Table 1). We follow that as the value of \( \alpha \) changes from 0.75 to 0.7, the value of \( \omega_{BD} \) exceeds 40,000. From 7.0 to 6.5 it crosses 50,000. Now, beyond that as we decrease \( \alpha, \omega_{BD} \) gets a high value. However we restrict ourself for \( \alpha = 0.5 \) and proceed for the data analysis.

### Table 1: Different values of \( \omega_{BD} \) for different \( \alpha \) chosen.

| \( \alpha \) | \( \omega_{BD} \) |
|-------------|-----------------|
| 0.75        | 38734.6         |
| 0.7         | 44421.7         |
| 0.68        | 47054.5         |
| 0.66        | 49929.6         |
| 0.65        | 51467.5         |
| 0.64        | 53077.9         |
| 0.62        | 56535.0         |
| 0.6         | 60342.9         |
| 0.5         | 86720.9         |

3 Fitting with observational data

Here, we are at the point to fit the observational data with our model. Observed Hubble data at different redshifts (twelve data points) given in observed Hubble data \cite{32} we will proceed. The Hubble parameter \( H(z) \) and the standard error \( \sigma(z) \) for different values of redshift \( z \) are given in Table 2. In the following subsections, we present the observational data analysis mechanism for Stern, Stern+BAO and Stern+BAO+CMB observations. We use the \( \chi^2 \) minimum test from theoretical Hubble parameter with the observed data set and find the best
fit values of unknown parameters for different confidence levels.

| $z$  | $H(z)$ | $\sigma(z)$ |
|------|--------|-------------|
| 0    | 73     | ±8          |
| 0.1  | 69     | ±12         |
| 0.17 | 83     | ±8          |
| 0.27 | 77     | ±14         |
| 0.4  | 95     | ±17.4       |
| 0.48 | 90     | ±60         |
| 0.88 | 97     | ±40.4       |
| 0.9  | 117    | ±23         |
| 1.3  | 168    | ±17.4       |
| 1.43 | 177    | ±18.2       |
| 1.53 | 140    | ±14         |
| 1.75 | 202    | ±40.4       |

Table 2: The Hubble parameter $H(z)$ and the standard error $\sigma(z)$ for different values of redshift $z$.

3.1 Constraining Tool: $H(z)$-z (Stern) data

We first form the $\chi^2$ statistics as a sum of standard normal distribution as follows: For any data set we will calculate the minimum $\chi^2$, with the formula

$$
\chi^2_{\text{Stern}} = \sum \frac{(H(z) - H_{\text{obs}}(z))^2}{\sigma^2(z)}
$$

(18)

Here, for different redshifts the theoretical and observational values of Hubble parameter is given as $H(z)$ and $H_{\text{obs}}(z)$. The corresponding error term is given as $\sigma(z)$. This is however given in Table 2. In this statistics, the nuisance parameter is given by $H_{\text{obs}}$ which can be safely marginalized. Considering $H_0$ to have a fixed prior distribution we will proceed.

This mechanism has recently been also discussed by several authors in very simple way. Here we shall determine the parameters $w_0$ and $w_1$ from minimizing the above distribution $\chi^2$. The probability distribution function in terms of the parameters $w_0$ and $w_1$ can be written as

$$
L = \int e^{-\frac{\chi^2_{\text{Stern}}}{2}} P(H_0) dH_0
$$

(19)

where $P(H_0)$ is the prior distribution function for $H_0$. We now plot the graph for different confidence levels (like 66%, 90% and 99%).

The values of $w_0$ and $w_1$ (for which we can obtain the least $\chi^2$) are given in the first row of the Table 3. We have plotted the $1\sigma$, $2\sigma$ and $3\sigma$ confidence contours in 1a-c curves. For BD cosmology one new aspect can be followed. Here the contours are not closed. They are open curves. As for example for linear parametrization the less $w_1$, less $w_0$ zones than the best fit $w_0$ and $w_1$ are included in the $1\sigma$ curves. For CPL parametrization the less $w_0$, higher $w_1$ area is included in the $1\sigma$ region. Lastly, for JBP we can see if we choose a particular $w_0$ we need to get high $w_1$ to be in the $1\sigma$ region. Similarly, for a fixed high $w_1$ it is required to take low $w_0$ to be inside the $1\sigma$ contour. The ultimate over all trend tells that we need sufficient negative $w_0$ and $w_1$, which may evolve a negative $w(z)$ ultimately. Linear and CPL parametrizations are quite strict for this negative $w(z)$ fact. But CPL however, allows positive $w(z)$ into $1\sigma$ contour. Nevertheless it has its best fit in negative $w(z)$ area.

3.2 Joint Analysis with Stern+BAO Data Sets

We will follow the pathway shown by Eisenstein et al. for joint analysis, the Baryon Acoustic Oscillation (BAO) peak parameter value. Here we will follow their approach. Sloan Digital Sky Survey (SDSS) survey is one of the first redshift survey by which the BAO signal has been directly detected at a scale $\sim 100 \text{ Mpc}$. For
Figs. 1(a), (b), (c) show that the variation of $w_0$ with $w_1$ for different confidence levels. The 66% (solid, blue, the innermost contour), 90% (dashed, red, next to the inner most contour) and 99% (dashed, black, the outermost contour) contours are plotted in these figures for the $H(z)$-z (Stern) analysis (For Linear, CPL and JBP parameterizations respectively).

low redshift ($0 < z < 0.35$) we will check for the BAO peak to determine the DE parameters. The BAO peak parameters might be defined as

$$A = \sqrt{\Omega_m} \left( \frac{1}{z_1} \int_0^{z_1} \frac{dz}{E(z)} \right)^\frac{1}{2}$$

(20)

Here $E(z) = H(z)/H_0$ is the normalized Hubble parameter, the redshift $z_1 = 0.35$ is the typical redshift of the SDSS sample and the integration term is the dimensionless comoving distance to the redshift $z_1$. The value of the parameter $A$ for the flat model of the universe is given by $A = 0.469 \pm 0.017$ using SDSS data [39] from luminous red galaxies survey. Now the $\chi^2$ function for the BAO measurement can be written as

$$\chi^2_{BAO} = \frac{(A - 0.469)^2}{(0.017)^2}$$

(21)

Now the total joint data analysis (Stern+BAO) for the $\chi^2$ function may be defined by

$$\chi^2_{total} = \chi^2_{Stern} + \chi^2_{BAO}$$

(22)

According to our analysis the joint scheme gives the best fit values of $w_0$ and $w_1$ in the second row of Table 3. Finally we draw the contours $w_0$ vs $w_1$ for the 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) confidence limits depicted in figures 2a to 2c.
Figs. 2(a), (b), (c) show that the variation of $w_0$ with $w_1$ for different confidence levels. The 66% (solid, blue, the innermost contour), 90% (dashed, red, next to the inner most contour) and 99% (dashed, black, the outermost contour) contours are plotted in these figures for the $H(z)$-z (Stern+BAO) analysis (For Linear, CPL and JBP parameterizations respectively).

| Constraining Tool | Name of the Dark energy model | Best fit values of $w_0$, $w_1$ and $\chi^2$ |
|-------------------|--------------------------------|-----------------------------------------------|
|                   |                                | $w_0$  | $w_1$  | $\chi^2$  |
| Stern             | Linear                         | -1.68326 | -2.94136 | 7.30381 |
|                   | CPL                            | -2.23223 | -0.0250015 | 7.32474 |
|                   | JBP                            | -1.58429 | -5.6347 | 7.31521 |
| Stern+BAO         | Linear                         | -1.68687 | -3.27207 | 768.128 |
|                   | CPL                            | -2.26669 | -0.02538 | 768.14821 |
|                   | JBP                            | -1.70844 | -2.37717 | 768.144 |
| Stern+BAO+CMB     | Linear                         | -1.64718 | -4.35378 | 9962.81 |
|                   | CPL                            | -2.52016 | -0.0292668 | 9963.47 |
|                   | JBP                            | -1.59007 | -6.00811 | 9962.82 |

Table 3: Best fit values of $w_0$, $w_1$ and $\chi^2$ for Linear, CPL and JBP parametrizations models of dark energy in Stern, Stern+BAO and Stern+BAO+CMB observations.

Like Stern data analysis, for Stern+ BAO also we get the open contours while drawing the different $\sigma$ curves. For Liner parametization the contours are open downwards and a bit more oblique towards the negative $w_0$ axis(if we compare with the Stern case). Which immediately tells us for this case if $w_1$ is comparatively low (though positive!) we can vary $w_0$ as we wish. The best fit value lie in the third quadrant. For CPL parametization the scenario remains exactly the same. The best fit $w_0$ and $w_1$ are in the second quadrant of the ($w_0$, $w_1$) space. JBP parametization also requires a third-quadrant situated best fit for the minimum $\chi^2$. Here $w_0$ is bounded at right like linear parametization case. The sigma contours have a negative slope, i.e., as we decrease our $w_0$ we can increase $w_1$ as well to stay inside the 1$\sigma$ contour.
Figs. 3(a), (b), (c) show that the variation of $w_0$ with $w_1$ for different confidence levels. The 66% (solid, blue, the innermost contour), 90% (dashed, red, next to the inner most contour) and 99% (dashed, black, the outermost contour) contours are plotted in these figures for the $H(z)$-$z$ (Stern+BAO+CMB) analysis (For Linear, CPL and JBP parameterizations respectively).

### 3.3 Joint Analysis with Stern + BAO + CMB Data Sets

In this subsection, we shall follow the pathway, proposed by some author [40, 41, 42], using Cosmic Microwave Background (CMB) shift parameter. One interesting geometrical probe of DE can be determined by the angular scale of the first acoustic peak through angular scale of the sound horizon at the surface of last scattering which is encoded in the CMB power spectrum. It is not sensitive with respect to perturbations but are suitable to constrain model parameter. The CMB power spectrum first peak is the shift parameter which is given by

$$ R = \sqrt{\Omega_m} \int_{0}^{z_2} \frac{dz}{E(z)} $$

where $z_2$ is the value of redshift at the last scattering surface. From WMAP7 data of the work of Komatsu et al. [43] the value of the parameter has obtained as $R = 1.726 \pm 0.018$ at the redshift $z = 1091.3$. Now the $\chi^2$ function for the CMB measurement can be written as

$$ \chi^2_{CMB} = \frac{(R - 1.726)^2}{(0.018)^2} $$

Now when we consider three cosmological tests together, the total joint data analysis (Stern+BAO+CMB) for the $\chi^2$ function may be defined by

$$ \chi^2_{TOTAL} = \chi^2_{Stern} + \chi^2_{BAO} + \chi^2_{CMB} $$

Now the best fit values of $w_0$ and $w_1$ for joint analysis of BAO and CMB with Stern observational data support the theoretical range of the parameters given in the third row of Table 3. The 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) contours are plotted in figures 3a - 3c.

At a glance, the sigma contours for Stern+BAO+CMB analysis resemble a bit with the Stern+BAO case. Suppose for Linear parametrization, here also the curves are open downwards in ($w_0, w_1$) space. The slope is sufficiently negative. Best fit is situated at the third quadrant. All over a negative EoS is indicated. Though for CPL and JBP it may be concluded that the models in BD cosmology will not mind if we make our $w_1$ positive and with a high magnitude. But of course these will be very particular cases or exceptions which may not be physical.
Figs. 4(a), (b), (c) show the variation of $\mu(z)$ with $z$ for Linear, CPL and JBP parameterizations respectively (Solid lines). The dots denote the Union Sample.

4 Redshift-Magnitude Observations from Supernovae Type Ia

The Supernova Type Ia experiments provided the main evidence for the existence of DE. Since 1995, two teams of High-$z$ Supernova Search and the Supernova Cosmology Project have discovered several type Ia supernovas at the high redshifts [13, 12]. The observations directly measure the distance modulus of a Supernovae and its redshift $z$ [14]. Now, take recent observational data, including SNe Ia which consists of 557 data points and belongs to the Union2 sample [46].

From the observations, the luminosity distance $d_L(z)$ determines the dark energy density and is defined by

$$d_L(z) = (1 + z)H_0 \int_0^z \frac{dz'}{H(z')}$$

(26)

the apparent magnitude $m$ of a supernova and its redshift $z$ are directly measured from the observations. The apparent magnitude $\mu$ (the distance modulus - distance between absolute and apparent luminosity of a distance object- for Supernovae) is related to the luminosity distance $d_L$ of the supernova by the relation:

$$\mu(z) = 5 \log_{10} \left[ \frac{d_L(z)/H_0}{1\, Mpc} \right] + 25$$

(27)

The best fit of distance modulus as a function $\mu(z)$ of redshift $z$ for our theoretical model and the Supernova Type Ia Union2 sample are drawn in figure 4a, 4b and 4c. It is very clear that for low redshifts $z < 0.4$ CPL and JBP are efficient enough to explain the Observational data (in the background of the BD cosmology). The linear parametrization case is efficient enough upto $z = 0.2$. Then it is over determined and after $z = 0.6$ it is under determined.

5 Brief Summary

Though, this is the time to describe the outcomes of this work in brief, we must say some important results of existing literature. Fabris et al [30] have predicted that the consideration of $\Lambda$CDM model might give the lowest $\chi^2$. It is true that their main motive was to determine $H_0$, and ultimately, the value of which was determined around 0.6 which quite resembles with the pre-predicted values of $H_0 (= 0.72 \pm 0.05)$ [14]. For Brans-Dicke (BD) cosmology, they have speculated the best value of $\omega_{BD}$ to be $-1.5$ (remarkably, this is conformally equivalent to
In this work, we determine the value of this parameter from the density factors of different components of the universe and a few distinct parameters of Brans Dicke cosmology itself. Using this value of $\omega_{BD}(=86720.9)$ our main concern is to set the unknown parameters of the different DE models' EoS parameter. So far, we have found the best fit values of two unknown parameters $w_0$ and $w_1$ (in redshift parametrization of DE) in the background of BD cosmology. One important point to be signed is that previous study indicate that structures can form in the Brans-Dicke model considered here during all the evolution of the universe, after the radiative phase, even the gravitational coupling is, at large scale, repulsive. We have however fixed the $H_0$ at the beginning and wanted to find out the confidence intervals of those parameters which explicitly determine the nature of DE/ the DE EoS. Most strange thing is always we got a open confidence contour (66%, 90%, 99%). To do this we take twelve point red-shift vs Hubble’s parameter data and perform $\chi^2$ test. We present the observational data analysis mechanism for Stern, Stern+BAO and Stern+BAO+CMB observations. Minimizing $\chi^2$, we obtain the best fit values of DE redshift parametrization parameters and draw different confidence contours. Though the best fit values, found out from different analysis pointed out towards a negative EoS at $z = 0$. Mathematically, it was showing a large range (actually unbounded) ordered pair of $(w_0 , w_1)$ is allowed to stay in the 1$\sigma$ confidence contour. Hypothetically, it will not mind if we keep our parameter at any place in that range. We will be still in the confidence level. But, the values taken by the parameters on their own, i.e., the physical values of them are not forced to stay anywhere at that range. Rather, they will chose their own positions! Preferably that will be inside the specified zone. While giving the confidence range, BD is giving enough liberty. As a modification of Einstein gravity it is very interesting nature to follow. Finally, we examine the best fit of distance modulus for our theoretical models and the Supernova Type Ia Union2 sample and we found that for low redshifts $z < 0.4$, all the parameterizations are efficient enough to explain the Observational data in the BD cosmology.

The concluding lines in a nutshell should be: In BD theory, while all the observational data supported values of Hubble’s parameter, dark energy, dark matter, radiation densities etc have been considered in an closed universe with a potential proportional to the square of the scalar field present inside it, if the scalar field is proportional to the square root of the scale factor of the universal expansion, we get almost negative EoS-s for the fluid present inside the universe (for three particular fluids with redshift parametrizations of their EoS). Unlike the other gravity theory results we do not get a closed 1$\sigma$ confidence contour for the parametric values (considering the redshift parametrization). We get a open curve, tendency of which says it is preferable to get negative parametric values which ultimately would evolve negative EoS strictly indicating the negative pressure inside the BD universe. Inclusion of a scalar field has such an impression upon the inside-fluid’s EoS that in spite of being confined inside a short region, infinite values of EoS are likely to have.

Acknowledgement :

UD thanks to CSIR, Govt. of India for providing research project grant (No. 03(1206)/12/EMR-II) and RB also thanks to above CSIR project for awarding Research Associate fellowship.

References

[1] Brans, C., Dicke, R., H.: *Phys. Rev* 124, 925(1961).
[2] Barrow, J. D., Maeda, K.: *Nucl Phys. B.* 341, 294(1990).
[3] Reasenberg, R. D. et al: *ApJ* 234, L219(1979).
[4] Bertotti, B., Iess, L., Tortora, P.: *Nature* 425, 374(2003).
[5] Eddington, A.S.: *The mathematical Theory of Relativity, Cambridge University Press* (1922); Nordtvedt, K.: *Phys. Rev.* 169, 1017(1968); Will, C.M., Nordtvedt, K.: *Astrophys. J.* 177, 757 (1972); Will, C.M.: *Theory and Experiment in Gravitational Physics, Cambridge Univ. Press* (1993).
[6] Will, C.M.: *Living Rev. Rel.* 4, 4 (2001), gr-qc/0103036
[7] Shapiro, I.I. :- in General Relativity and Gravitation 12, edited by N. Ashby, D.F. Bartlett, and W. Wyss, Cambridge University Press (1990), p. 313.
[8] Williams, J.G., Newhall, X.X., Dickey, J.O. :- Phys. Rev. D 53, 6730 (1996).
[9] Perivolaropoulos, L. :- Phys.Rev.D 81, 047501(2010).
[10] Moon, T., Oh, P.:- arXiv 1302.3061v1.
[11] Eubanks,T.M., Martin, J.O., Archinal, B.A.et al.:- Bull. Am. Phys. Soc., Abstract No. K 11.05(1997), unpublished; draft at [ftp://casa.usno.navy.mil/navnet/postscript/prd15.ps] (1999); Shapiro, S.S., Davis, J.L., Lebach, D. E., Gregory, J.S.:- Phys. Rev. Lett. 92, 121101(2004).
[12] Riess, A. G. et al :- [Supernova Search Team Collaboration], Astron. J. 116, 1009(1998)[arXiv:9805201(astro-ph)].
[13] Perlmutter, S. et al :- [Supernova Cosmology Project Collaboration], ApJ 517, 565(1999)[arXiv:9812133(astro-ph)].
[14] Spergel, D. N. et. al.: Astrophys. J. Suppl. 148 175(2003).
[15] Knop, R. A. et. al.: Astrophys.J. 598 102(2003).
[16] Melchiorri, A., Mersini, L., Trodden, M. :- Phys. Rev. D 68 043509(2003).
[17] Seljak, U., Slosar, A., McDonald, P. :- JCAP 0610 014 (2006).
[18] Cooray, A. R., Huterer, D. :- Astrophys. J. 513 L95(1999).
[19] Upadhye, A., Ishak, M., Steinhardt, P. :- Phys Rev D 72 063501(2005).
[20] Chevallier, M., Polarski, D. :- Int. J. Mod. Phys. D 10 213(2001).
[21] Linder, E. V. :- Phys. Rev. Lett. 90 091301(2003).
[22] Linden, S., Virey, J. -M. :- Phys. Rev. D 78 023526(2008).[arXiv:0804.0389].
[23] Jassal, H. K., Bagla, J. S., Padmanabhan, T. :- MNRAS 356 L11(2005).
[24] Errahmani, A ; Ouali, T :- Phys. Lett. B 641 357(2006).
[25] Kim, H., Lee, H. W., Myung, Y. S. :- Phys.Lett. B 628 11(2005).
[26] Arik, M., Calik, M.C.:- Mod.Phys.Lett.A 211241(2006).
[27] Kim, H. :- Mon.Not.Roy.Astron.Soc. 364 813(2005).
[28] Wu, F.-Q., Chen, X. :- PRD 82, 083003 (2010).
[29] Li, Y.-C., Wu, F.-Q., Chen, X.- [arXiv:1305.0055 [astro-ph.CO]].
[30] Fabris, J.C., Goncalves, S.V.B., Ribeiro, R. de Sa :- Grav.Cosmol. 12 49(2006).
[31] Sen, S.,Sen, A. A. :- Phys. Rev. D 63 124006(2001); Sen, S., Seshadri, T. R. :- Int. J. Mod. Phys. D 12 445 (2003); Chakraborty, W., Debnath, U. :- Int. J. Theor. Phys. 48 232 (2009); Faraoni, V. :- Phys. Rev. D 62 023504 (2000); Saini, T. D.,Raychaudhury, S., Sahni, V.,Starobinsky, A. A. :- Phys. Rev. Lett. 85 1162 (2000).
[32] Stern, D. et al, 2010, JCAP 1002, 008.
[33] Wu, P. and Yu, H., 2007, Phys. Lett. B 644, 16.
[34] Thakur, P., Ghose, S. and Paul, B. C., 2009, Mon. Not. R. Astron. Soc. 397, 1935.
[35] Paul, B. C., Ghose, S. and Thakur, P., [arXiv:1101.1360v1 [astro-ph.CO]].
[36] Paul, B. C., Thakur, P. and Ghose, S., [arXiv:1004.4256v1 [astro-ph.CO]].
[37] Ghose, S., Thakur, P. and Paul, B. C., [arXiv:1105.3303v1 [astro-ph.CO]].
[38] S. Chakraborty, U. Debnath and C. Ranjit, Eur. Phys. J. C, 72 2101 (2012).
[39] Eisenstein, D. J. et. al. :- Astrophys. J. **633**, 560(2005).
[40] Bond, J. R. et. al.: 1997, Mon. Not. Roy. Astron. Soc. 291, L33.
[41] Efstathiou, G., Bond, J. R. :-MNRAS 304, 75(1999).
[42] Nessaeris, S., Perivolaropoulos, L.:– JCAP 0701, 018(2007).
[43] Komatsu, E. et al: Astrophys. J. Suppl. **192**, 18(2011).
[44] Kowalaski et. al. :- Astrophys. J. **686**, 749(2008).
[45] Alberto Vazquez, J., Bridges, M., Hobson, M.P., Lasenby, A.N. :- [arXiv:1205.0847[astro-ph.CO]].
[46] Amanullah, R. et al:– Astrophys. J. **716**, 712(2010).