A note on “Goals, constraints, and transparently fair assignments: A field study of randomization design in the UEFA Champions League”

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Abstract

A paper published in Management Science (Boczoń, M. and Wilson, A. J.: Goals, constraints, and transparently fair assignments: A field study of randomization design in the UEFA Champions League) on 2 September 2022 seems to contain substantial omissions and inaccuracies. This note attempts to correct them.

Keywords: constrained assignment; public draw mechanism; UEFA Champions League

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1 Introduction

A recent article published in Management Science (Boczoń and Wilson, 2022) on 2 September 2022 discusses the problem of draw in sports contests. Even though the efforts of the authors are appreciated and the results are interesting, it seems that the paper contains substantial omissions and inaccuracies, which can be summarised as follows:

- The paper presents the UEFA Champions League Round of 16 draw as a unique field solution to a constrained assignment problem in sports. However, the same problem emerges in the FIFA World Cup draw, the draw of the European Qualifiers
for the FIFA World Cup, the UEFA Euro qualifying draw, and the UEFA Nations League draw. Importantly, the Fédération Internationale de Football Association (FIFA) has adopted the credible and transparent solution of the UEFA for the 2018 FIFA World Cup draw (Guyon, 2018) as suggested by Guyon (2014).

- The paper ignores important previous results on the UEFA Champions League Round of 16 draw (Kiesl, 2013; Klößner and Becker, 2013) and the FIFA World Cup draw (Guyon, 2015).

- The paper fails to mention and evaluate the entirely fair (evenly distributed) algorithms proposed in the literature for the constrained assignment problem (Guyon, 2015; Klößner and Becker, 2013).

All these issues, among others, are detailed below.

2 Main comments

2.1 The uniqueness of the field setting

The authors present the mechanism used in the UEFA Champions League Round of 16 (UCL R16) draw as a unique field solution to an assignment problem in sports. Here, the eight group runners-up should be matched to the eight group winners such that both the association and the group constraints are satisfied. This setting is equivalent to a group draw with two pots, the first containing the runners-up and the second containing the group winners, where the allocation is required to meet the association and the group constraints.

Then the randomisation procedure used by the Union of European Football Associations (UEFA) for the UCL R16 draw is equivalent to the following mechanism:

- Placing the runners-up in the eight groups according to the order in which they are drawn from their urn;
- Labelling the groups in alphabetical order;
- Placing the group winners in the first available group in alphabetical order such that any dead end—a situation when the remaining group winners cannot be assigned with satisfying all draw constraints—is avoided.

This interpretation reveals that the bipartite constraint (each pairing must be between a group winner and the runner-up) is essentially not a constraint because the draw mechanism would be entirely fair in the absence of the other restrictions. Furthermore, the same randomisation procedure is used in the FIFA World Cup draw (FIFA, 2017, 2022), the draw of the European Qualifiers for the FIFA World Cup (UEFA, 2020a), the UEFA Euro qualifying draw (UEFA, 2018, 2022b), and the UEFA Nations League draw (UEFA, 2020b, 2021). However, the draw rules of these competitions are more complicated than the UCL R16 draw due to the higher number of groups and teams, and the more complex sets of constraints.

Consequently, the studies investigating the draw mechanism of the above tournaments are highly relevant for Boczoń and Wilson (2022). Guyon (2014) suggests two tractable procedures for the FIFA World Cup draw that produce balanced, geographically diverse
groups, and are evenly distributed, meaning that all valid assignments of the draw are equally likely. The published paper, Guyon (2015) contains only one of them., but the negligence of this mechanism is hard to defend especially because Guyon (2015, p. 176) says that: “To the best of our knowledge, this is the first time that a random procedure is suggested for the final draw of the FIFA World Cup that is tractable, produces balanced groups, and satisfies the geographic constraint.” Note that the geographic constraint in the FIFA World Cup is more complex than the association and group restrictions in the UEFA CL R16 draw because, besides prohibited games between teams from continents other than Europe, at most two European teams can play in any group.

Csató (2022d) presents how the randomisation procedure is connected to a well-known problem in computer science (generating all permutations of a given sequence), quantifies the (un)fairness of the 2018 FIFA World Cup, and evaluates the distortions caused by the draw procedure in the probability of qualification for the knockout stage for each nation. Csató (2022b) shows how a careful choice of pot labels can reduce the extent of uneven distribution (see also Section 2.4). Roberts and Rosenthal (2022) aim to find mechanisms for the general constrained assignment problem that follow the uniform distribution over all feasible assignments. The authors suggest two procedures by using balls and bowls—but they require computer draws at one or several stages, which raises the suspicion of rigging. Both algorithms can be tried interactively at http://probability.ca/fdraw/. To be fair to Boczoń and Wilson (2022), Roberts and Rosenthal (2022) have been published only after Boczoń and Wilson (2022) have been accepted. On the other hand, it is worth noting that Csató (2022b), Csató (2022d), and Roberts and Rosenthal (2022) cite a former version (Boczoń and Wilson, 2018) of Boczoń and Wilson (2022).

The FIFA World Cup draw offers an especially instructive case study. The draw of the 1990 (Jones, 1990), 2006 (Rathgeber and Rathgeber, 2007), and 2014 (Guyon, 2015) World Cups were seriously unfair due to a strange policy of ensuring that the unseeded South-American teams are not drawn against seeded South-American teams. Therefore, the French mathematician Julien Guyon has suggested using the procedure of the UCL R16 draw in the FIFA World Cup draw (Guyon, 2014) (this recommendation is missing from the published version of Guyon (2015)). Fortunately, FIFA has heard the message and adopted the credible and transparent solution of the UEFA for the 2018 FIFA World Cup draw (Guyon, 2018). Furthermore, as has turned out in the spring of 2022 (after Boczoń and Wilson (2022) have been accepted), the same procedure has been used for the 2022 FIFA World Cup draw—albeit, FIFA has made an annoying mistake in the allocation of the teams into pots (Csató, 2022c).

2.2 The near-optimality of the UEFA procedure

The authors do not cite two papers that examined the UCL R16 draw already in 2013 (Kiesl, 2013; Klößner and Becker, 2013). Kiesl (2013) shows the uneven distribution of the draw procedure for the 2012/13 season and provides some uniformly distributed—but uninteresting to watch—mechanisms. Nonetheless, the author argues that there is no need to improve the randomisation procedure used by the UEFA. Kiesl (2013) also proves by Hall’s marriage theorem why the existence of a feasible assignment is guaranteed. The relationship between the UCL R16 draw and Hall’s marriage theorem is discussed in Haigh (2019, Section 3.6), too.

The main findings of Klößner and Becker (2013), based on the Champions League seasons played between 2008/09 and 2012/13, can be summarised as follows:
• Under the currently used draw procedure, it is impossible that every feasible assignment has the same probability. Even though almost all deviations between UEFA and uniform probabilities are quite small in both absolute and relative magnitudes, even these small differences in the pairwise probabilities can change the order of the most likely opponents for certain teams (Section 3).

• The huge amount of money at stake translates the small probability differences into quite powerful monetary effects: in almost every season, there are teams whose expected revenues decline by more than 10 thousand euros because of the imperfect randomisation procedure used by the UEFA, while other teams unduly profit by similar amounts (Section 4).

• Dropping the association constraint would significantly improve the distortions of the draw (Section 5.1).

• An alternative mechanism, which is both fascinating for fans and able to produce the right probabilities, is provided: a random matching is generated and a fixed number of swap moves carried out in an appropriate way (Section 5.3).

However, the initial assignment of this algorithm can only be obtained by a computer draw, which is not transparent.

Even though Guyon (2014) and Guyon (2015) discuss the FIFA World Cup draw, the proposed evenly distributed procedures based on drawing the continents first and the teams second can be easily adapted for the UCL R16 draw by drawing the countries first, followed by the teams. These mechanisms use only balls and bowls, hence, they are entirely transparent.

As an illustration, consider the draw in the quarterfinals of the 2021/22 UEFA Champions League with three English (E), three Spanish (S), one French (F), and one German (G) club, as well as the association constraint. The number of valid assignments is 42, however, drawing one of them randomly is perhaps not transparent and certainly uninteresting. On the other hand, there are only three valid allocations of countries: the F-G and three E-S pairs (6 cases), or the F/G-S, E-F/G, and two E-S pairs (18 cases each). Since the probability of the last two allocations with two E-S pairs is three times the probability of the first with three E-S pairs (in the former, the English/Spanish teams are not identical), the draw should be started with an urn of seven balls containing the possible allocations of countries.\footnote{If putting more balls for a given allocation raises the suspicion of rigging, it can be avoided with fixing the English or Spanish teams at this stage such that the three balls are labelled by the names of the clubs.} Depending on the outcome of this draw, either an unconstrained 2 × 3 assignment problem should be solved for the English and Spanish teams, or an unconstrained 2 × 2 assignment problem follows the choice of one English club and one Spanish club randomly. The procedure is fair (evenly distributed), uses a small number of bowls and balls, and allows for a nice television show of limited length.

In view of these findings, it is a bit misleading to state that “although marginally better randomizations are possible, the tournament’s transparency first procedure under our objective resembles the fairest possible lottery over the constrained assignments” (Boczoń and Wilson, 2022, p. 2), or “the chosen procedure comes very close to achieving the fairest possible outcome” (Boczoń and Wilson, 2022, p. 15).
2.3 The significance of the problem

According to Csató (2022e), draw restrictions can be effective in reducing the probability of an incentive incompatible situation. Thus, a remarkable trade-off exists between the number of constraints and their monetary and distortive effects.

2.4 Analysing the effect of a reversed draw order

The third part of Boczoń and Wilson (2022, Proposition 2) uncovers that the UCL R16 draw is asymmetric as the runners-up are drawn first. The role of the draw order in the UCL R16 draw has already been recognised by Klöffner and Becker (2013, Footnote 19). Although starting the draw with the group winners instead of the runners-up is verified to have only marginal effects in the 2017/18 (Guyon, 2017) and 2019/20 (Guyon, 2019) seasons, it would be interesting to see the fairness distortions (Boczoń and Wilson, 2022, Figure 4) when the group winners are drawn first because (1) this reform of the UCL R16 draw has no price; and (2) the publication of such an analysis is almost impossible after Boczoń and Wilson (2022) appeared.

2.5 The implications of the association constraint

Boczoń and Wilson (2022, Section 4.1) thoroughly analyse the monetary and the distortive effects of the association constraint in the UCL R16 draw. Nonetheless, this draw restriction also influences the outcome of the tournament: since the best teams are usually concentrated in some associations, they benefit from avoiding each other in the Round of 16. The implications are studied via both theoretical and simulation models in Csató (2022a).

3 Refinements

- Page 3: Anbarcı et al. (2015) have been published in 2021 (Anbarcı et al., 2021).
- Footnote 11: In the new format of the UEFA Champions League from the 2024/25 season, the top eight teams in the league of 36 teams qualify for the Round of 16 in the knockout stage, and the teams finishing in the 9th to 24th positions compete in a two-legged play-off to enter the Round of 16 (UEFA, 2022a). Therefore, the bipartite constraint is meaningless since groups do not exist. The association constraint will probably continue to hold as it is currently required in the UEFA Europa League and the UEFA Europa Conference League, where the Round of 16 in the knockout stage is contested by the eight automatically qualified group winners and the eight winners of the knockout round play-offs.
- Footnote 12: Seeding in the group stage is not only determined by the teams’ league ranking and the value of their UEFA club coefficients as the Champions League titleholder and, from the 2018/19 season, the Europa League titleholder are automatically placed in the first (strongest) pot (Csató, 2021, Chapter 2.3). However, Engist et al. (2021) find no evidence that seeding itself contributes positively to the team’s success in the tournament.
4 Conclusion

Economists seem to be increasingly interested in analysing sports rules. If they want to publish these studies in top journals, it is worth exploring what has already been done in the tournament design literature. Some survey articles (Kendall and Lenten, 2017; Lenten and Kendall, 2021; Wright, 2009, 2014) and a recent book (Csató, 2021) can be recommended as a starting point. It also seems that journals focusing on the quantitative aspects of sports such as the Journal of Quantitative Analysis in Sports or the Journal of Sports Analytics ought to be checked to find all relevant previous results.

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