Landslide mudflow behaviour: physical and numerical modelling

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Abstract. Landslides constitute one of the natural phenomena that cause the most economic losses and deaths worldwide. After failure occurs, landslides can trigger mudflows. Understanding how mud is transported is very important in infrastructure projects that coincide with hillside areas due to the high risk of this phenomena occurring due to the high slopes, which can imply great risks and produce disasters, generating considerable costs. In this work, the evaluation of a mudflow is presented, from the execution of a scale experiment in the laboratory and its validation from numerical models, considering the material behaviour as a Newtonian fluid and as a non-Newtonian fluid. The physical model was developed using a 3m x 0.5m x 0.7m rectangular channel with dimensions, with slope control. A mud mixture composed of a silty material with 60% of moisture was tested producing a mudflow. Experimental tests were carried out with slopes of 5% and 10%. The numerical models were implemented in ANSYS FLUENT software. At first stage, the numerical model was calibrated with the results of the physical model with a slope of 5% and it was validated with the results of the model for the slope of 10%. Results of the numerical models were compared with the experimental results, and they have shown that these have a great capacity to reproduce what is observed experimentally. In addition, when the material was considerate as a Newtonian fluid, a similar behaviour was found respect to a mudflow as a non-Newtonian fluid, not finding considerable differences in the final deposition length of the flow. The simulation of mudflow, especially multiphase, using CFD is usually a complex process since the boundary conditions and physical and rheological properties of the soil must be correctly defined, considering the contribution of the solid fraction in the behaviour of the numerical model. Nevertheless, despite all the simplifications that a modelling of this type entails, the results are promising to improve the understanding of the phenomenon studied, and its application in risk assessment methodologies for mass movements and their derived effects.

1. Introduction

Mudflows are natural phenomena, which are usually derived from a landslide and present a high impact when they occur [1]. Generally, mudflows are characterized by rapid movement, sudden occurrence, fluid-like movements, and long travel distances from its outlet. Some studies indicate that the occurrence of mud flows is triggered mainly by excess pore pressure or liquefaction processes in the material, generating sudden and high losses in shear resistance, generating a flow of mud[2][3].

Numerical models have been used to predict flow dynamics, providing an alternative way to reproduce the behaviour of a mud flow under their possible physical conditions and quantify the travel
distance and flux speed[4]. Some investigations have used mathematical models of average depth to simulate all stages of the mudflow from its beginning to its disposal, using finite volume method to solve the equations that govern the flow behaviour[5].

Computational fluid dynamics applications (CFD) are often used for different flow problems and phenomena that sometimes cannot be easily described from simple conceptual models[6]. This alternative tool can contribute to the simulation of a mudflow when it is considered as a continuous medium. Investigations from [7][5] have simulated mudflow and debris flow behaviour in pipes and open channels respectively, analysing variables such as the speed and flow volumetric fraction, which would allow predicting the advance length and possible effects in final deposition.

Despite attempts to simulate the behaviour of mud flows, there is still scarce models with solution with CFD tools with properties such as viscosity and soil moisture controlling how the flux moves. [8][9]. Rheological studies have shown the mud as a highly viscous non-Newtonian fluid[10][11]. There are rheological models that simulate the material behaviour as a viscous flow which can be coupled to simulate the advance[12].

This paper presents results of a work in which the behaviour of a mud was studied. The study included the realization of laboratory scale models and numerical models. The scale models consist of a mud flowing in an open channel with an adjustable slope. The numerical models were made using a CFD tool. Comparison of the advance of the flow under different water content conditions and considering within the model a material behaviour as a non-Newtonian fluid or as a Newtonian fluid. The results are finally compared with the experimental observations obtained at the laboratory level.

2. Mathematical Formulation

As part of the modelling process, the differential form of the law of mass conservation and momentum equations or linear momentum that describe a body movement should be used[6]. The turbulent transport process is complex and cannot be calculated with an exact method. This process must be approximated by a turbulence model that allows to relate the amounts of turbulent transport with the mean flow field. In the averaged Reynolds procedure, the Navier-Stokes equations are averaged over a period of time greater than the largest characteristic period of the flow motion obtaining the averaged Reynolds Navier-Stokes equations (RANS) [6][13]:

\[
\frac{1}{t} \frac{\partial U_i}{\partial x_j} = 0 \tag{1}
\]

\[
\frac{\partial U_i}{\partial t} + \frac{\partial (U_i U_j)}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} (\tau_{ij} - \rho u_i u_j) \tag{2}
\]

Where \(U_i\) is the mean flow velocity vector, \(x_i\) the direction in space, \(t\) the time, \(P\) the mean flow pressure, \(\tau_{ij}\) to the mean flow deviating stress tensor, \(u_i\) to the velocity vector the flux due to fluctuations and \(-\rho u_i u_j\) to the Reynolds stress tensor. The latter, cannot be expressed as a function of the mean flow and must be related to known quantities using a turbulence model[13][14].

2.1. Turbulence models

There are several models of turbulence that CFD tools use for their solution. The best known is the standard \(k-\varepsilon\) model [13]. Given the complexity of turbulence in most engineering flow problems, including the use of simple formulas, it is possible to develop similar transport equations to accommodate the turbulent quantity \(k\) and other turbulent quantities as the rate turbulent energy dissipation \(\varepsilon\)[13]. Based on the standard model, the RNG and RAE (Realizable) models were developed for \(k\) and \(\varepsilon\) [13][14].
Realizable model, which corresponds to the model to be used in this research, was developed by [15]. This model positively satisfies the constraints on the related quantities in the calculation of Reynolds stresses, according to the physics of turbulent flows. The model transport equations for $k$ and $\varepsilon$ in the workable model are:

\[
\rho \frac{\partial k}{\partial t} + \rho U_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_j} \left( \mu + \frac{\mu_t}{\sigma_k} \frac{\partial k}{\partial x_j} \right) + \tau_{ij} \frac{\partial U_i}{\partial x_j} - \rho \varepsilon \tag{3}
\]

\[
\rho \frac{\partial \varepsilon}{\partial t} + \rho U_i \frac{\partial \varepsilon}{\partial x_i} = \frac{\partial}{\partial x_j} \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x_j} \right) + \rho C_1 S \varepsilon - \rho C_2 \frac{\varepsilon^2}{k} + \sqrt{\nu \varepsilon} \tag{4}
\]

The model constants correspond to $C_2 = 1.9, \sigma_\varepsilon = 1.2, \sigma_k = 1.0, C_1 = \text{max} [0.43; (\eta/\eta+5)]$ [15].

\[
\eta = \frac{S k}{\varepsilon} \tag{5}
\]

\[
S = \sqrt{2S_{ij}S_{ij}} \tag{6}
\]

2.2. Multiphase flow

A multiphase system is defined as the combination of the solid liquid and gas phases [16]. To correctly model the sludge flow, a multiphase air-mud model must be implemented that considers two phenomena: the free-flowing surface or the air-mud interface. The VOF (Volume of fluid) model and the mixture model are shown below.

2.2.1. VOF Model. The VOF (Volume of fluid) model can model two or more immiscible fluids by solving a single set of moment equations and tracking the volume fraction of each of the fluids over the entire domain. Typical applications include prediction of stream rupture, movement of large bubbles in a liquid, movement of liquid after a dam failure, and constant or transient monitoring of any liquid-gas interface [17].

To simulate multiphase flow, this model uses a unique set of conservation equations for the full flow domain. This set must consider the differences in the fluids involved properties, as well as the surface tension at the interfaces. Eq.(1) does not change, and the flow motion is given by the Navier-Stokes equations for a compressible fluid presented in Eq.(2)[18]. An additional term is added to explain the surface tension present at the air-sludge interface [16]:

\[
T = \int \sigma k' n'^{\beta} (x - x') ds'
\]

Where $\sigma$ is the surface tension coefficient, $k'$ is free surface curvature in the 2D domain, $n'$ the unit vector on the surface, $\delta \beta$ is the Dirac delta function, where $\beta = 2$ for 2D domains and $\beta = 3$ for 3D domains, $x$ is the point on the coordinate axis where the equation is evaluated and $x'$ is the point of the free surface, such that if $x = x'$ then the function $\delta$ is different from zero, guaranteeing that the term $T$ is evaluated only at the fluid interface, $ds'$ is free surface differential. In Eq. (1) and (2) $\rho$ is replaced by [16]:

\[
\rho_m = \alpha \rho_w + (1 - \alpha) \rho_a \tag{8}
\]

Where $\alpha$ is the water volumetric fraction, defined as the ratio between the mud volume and the total mixture volume, $\rho_w$ is water density, and $\rho_a$ air density. Finally, it is necessary to have $t$ an additional transport equation for volumetric fraction of the fluid:

\[
\frac{\partial \alpha}{\partial t} + \frac{\partial}{\partial x_i} (\alpha U_i) = 0 \tag{9}
\]
2.2.2. **Mixture Model.** A mixture model can be used for this application, when there is a suspension of the primary phase corresponding to a continuous flow, in this case, the mudflow that flows over a surface, and a secondary phase or dispersed phase (air presence). The mass and momentum conservation equations for the air-mud mixture, and the continuity equation for the dispersed phase are used [16] [18].

2.3. **Viscosity Models**

Because the material behaves like a non-Newtonian fluid, it is important to consider a rheological behaviour of the material as a complement to model development. Since the mudflow, when characterized as a highly viscous material, can present variations in its physical properties that influence its progress across any surface[11] and when material behaviour is not linear but obeys a mathematical model which describes the relationship between the shear stress and the rate of deformation of the material, this model is explained by the equation:

\[ \tau = \tau_y + K \gamma^n \]  \hspace{1cm} (10)

Where \( \tau \) is the shear stress, \( \tau_y \) is the elastic limit, \( K \) is the apparent viscosity, \( \gamma \) is the rate of shear deformation, and \( n \) describes the rate of change in viscosity. Meanwhile, Newtonian flows show a linear behaviour, and depending on properties such as their density and consistency, there may be changes in their slope.

3. **Materials and Methods**

For this research, scale laboratory tests and numerical models were made. using the following methodology.

3.1. **Soil classification**

For the laboratory models, a slurry prepared with silt and water in different proportions was used. The soil is from residual origin and it was characterized by the following tests: particle size distribution (ASTM D422), consistency limits (ASTM D423 and ASTM D424) and specific gravity (ASTM D854). The results are described in Table 1.

| Soil                  |     |
|-----------------------|--|
| Liquid Limit (%)      | 43 |
| Plastic Index (%)     | 13 |
| Specific gravity      | 2.71 |
| US sieve N°10-opening: 2 mm (% finer) | - |
| US sieve N°40-opening: 425 µm (% finer) | - |
| US sieve N°200-opening: 75 µm (% finer) | 84.25 |
| USCS classification (ASTM D2487) | ML |

3.2. **Viscosity determination**

The dynamic viscosity value was obtained for each slurry prepared. For this, soil mud with different water content was prepared, to obtain a viscosity curve as a function of moisture, considering that Stokes' law is satisfied. Mudflows were prepared at 60%, 65%, 70% and 80% moisture. Test procedure is shown in Figure 1 and is calculated with the following expression:

\[ \eta = \frac{2 \ast g \ast (\rho_p - \rho_m) \ast r^2}{9 \ast \left( \frac{L}{r} \right)} \]  \hspace{1cm} (11)
Where $\eta$ is the material viscosity in [Pa * s], $g$ is the gravity acceleration [m / s$^2$], $\rho_p$ the sphere density [kg / m$^3$], $\rho_m$ mud density [kg / m$^3$], $r$ is the sphere radius [m], $L$ the length that the sphere travels vertically along the material, and $t$ is the time [s] elapsed in which the sphere crosses the length $L$ [m].

![Figure 1. Viscosity test with Stokes law](image)

### 3.3. Laboratory Model

To carry out the laboratory tests, an open channel 3.0 m long, 0.5 m high and 0.7 m width was made, with slope control at the bottom and a gate 0.25 m from the beginning (Figure 2). Tests were carried out with slopes of 5% and 10% for sludge at 60% and 80% moisture. The material was placed with the gate closed and then opened to flow as shown in the Figure 3. After the test, the deposition heights were measured with a laser distance meter along an advance length $L$, which was also measured, based on the work carried out by [19]. To measurement with laser distance meters, a grid system was established in the upper part of the channel, using 10 cm x 10 cm cells (Figure 4). Through this system, a distance meter was in the centre of the cell to verify the deposition heights. Deposition height was calculated as $HL = H - n$.

![Figure 2. Channel geometry](image)
3.4. Numerical simulation
Mud flow from the channel was calculated using a student version CFD package, in this case FLUENT 2020 R2. A geometric model was developed for the flow profile through section D shown in Figure 4, and a numerical model was assigned to simulate the flow behaviour. The model was calibrated for a 5% slope and validated for a 10% slope. Mass, momentum, energy, and turbulence equations were solved sequentially, through a segregated formulation, using techniques based on finite volume method. The calculation must be carried out iteratively until reaching a convergent solution [6][13][17].

To execute the numerical model in the CFD tool according to the 2D channel geometry, meshes were made for each case of slope and material properties. The characteristics of the meshes are presented in Table 2. To guaranteed that the properties of the material will not be affected by the resolution of the mesh in the air domain, the zone where the material is found in its initial condition, the cells were refined to 1x10-3 respect to the rest of the domain that has air, with 2 x10-3 cell sizes, as shown in the Figure 5. A refinement was also made along the bottom of the channel in order to avoid that when the air is replaced by the flow the properties of the material are modified.

| Slope | Water content | Elements Number |
|-------|---------------|----------------|
| 5%    | 60%           | 62970          |
| 10%   | 60%           | 68310          |
| 5%    | 80%           | 65830          |
| 10%   | 80%           | 71130          |
Figure 5 shows the boundary conditions for the flow. The perimeter zone over which mud flows is established as a wall type, with a shear condition without slip. A pressure outlet limit condition is used. A manometric atmospheric pressure of 0 Pa is established and a reflux air volume fraction of 1 is defined to ensure that in cases of backflow, only air enters the domain[18]. In addition, the areas where the material and air are in their initial condition are selected, to proceed to configure the numerical model.

![Mesh design. 10% Slope](image)

**Figure 5.** Mesh design. 10% Slope

4. **Results and discussion**

After performing the procedures described in Section 3, the following results were obtained:

4.1. **Viscosity tests**

**Figure 6** presents results obtained from the described laboratory test, to obtain the value of the mud viscosity under different water contents and relationship with density.

![Viscosity and density variation according to water content. Case: Newtonian fluid](image)

**Figure 6.** Viscosity and density variation according to water content. Case: Newtonian fluid

Results show a decrease in the viscosity value when the water content increases, this, because a greater amount of water decreases attraction between particles, which decreases the shear force. This decreases material strength to flow, and therefore implies greater advance lengths. When the material reaches moistures greater than 70%, the viscosity has a rate of decrease that has a lower slope, which implies a lower decrease for muds with high water volumes. Thus, for moistures greater than 70 %, a
mudflow could present a behaviour very similar to a Newtonian fluid, which could produce well results without considering rheological models.

Density of the material is proportional to viscosity, when, to higher densities, greater resistance to flow is observed, which implies higher viscosity values. Material behaviour as a viscous flow is important because the parameters that describe this behaviour are the input elements in the model calibration process in FLUENT software.

4.2. Channel test
Results allow to affirm that the slope had no influence on the flow advance, while the water content influenced the travelled distance. For moisture of 60%, the length was 0.9 and 1.1m. The results obtained in the laboratory are shown in the Figure 8 as discontinuous lines.

4.3. Numerical models
Models, considering the material as a non-Newtonian fluid were carried out using the Herschel-Bulkley rheological model. For this, regression analyses of the viscosity tests were carried out to estimate a rheological model to 60% and 80% moistures. Results are presented in Figure 7 and summarized in Table 3. In both materials the value of the shear stress increases with time, and elastic limit in steady state was plotted for each strain rate, defining Ty at a zero-strain rate. Results were adjusted with that proposed by Herschel-Bulkley.

![Figure 7. Material rheological behaviour with Herschel-Bulkley model](image)

In the material with 80% of moisture, compared to the material with 60% of moisture, a behaviour with a lower growth rate is observed, due to the amount of water it has, which reduces the internal shear forces and allows a greater material fluidity. On the other hand, material with 60% moisture shows a greater increase in shear force as the rate of deformation increases. Although this material already behaves like a fluid because it exceeds its liquid limit by 17%, it still has internal forces that allow slower movement and could be more resistant to the material flow.

Numerical models were made in FLUENT, as described in section3.4, and parameters obtained from the material as Newtonian and non-Newtonian fluid was incorporated (Table 3). Figure 8 shows the advance distance and deposition heights obtained with CFD tool as continuous lines. The results obtained in the channel are also presented.
Table 3. Material Rheological characteristics to input in Fluent models.

| Model | Slope (%) | Moisture (%) | Material Type | Fluid Type   | Density (kg/m³) | Viscosity (Pa·s) | Yield Strength (kPa) | Consistency | Herschel Bulkley Exponent |
|-------|-----------|--------------|---------------|--------------|----------------|------------------|----------------------|-------------|--------------------------|
| 1     | 5%        | 60%          | Non-Newtonian | Non-Newtonian| 1487.20        | N/A              | 100                  | 80          | 0.45                     |
| 2     | 5%        | 80%          | Non-Newtonian | Non-Newtonian| 1424.45        | N/A              | 15.5                 | 5           | 0.45                     |
| 3     | 5%        | 60%          | Newtonian     | Newtonian    | 1487.20        | 90.230           | N/A                  | N/A         | N/A                      |
| 4     | 5%        | 80%          | Newtonian     | Newtonian    | 1424.45        | 0.887            | N/A                  | N/A         | N/A                      |
| 5     | 10%       | 60%          | Non-Newtonian | Non-Newtonian| 1487.20        | N/A              | 100                  | 80          | 0.45                     |
| 6     | 10%       | 80%          | Non-Newtonian | Non-Newtonian| 1424.45        | N/A              | 15.5                 | 5           | 0.45                     |
| 7     | 10%       | 60%          | Newtonian     | Newtonian    | 1487.20        | 90.230           | N/A                  | N/A         | N/A                      |
| 8     | 10%       | 80%          | Newtonian     | Newtonian    | 1424.45        | 0.887            | N/A                  | N/A         | N/A                      |

Figure 8. Laboratory test and numerical models

Mudflow advance distance and deposition height are consistent with the material rheological behaviour, where a greater advance of the flow is observed when it has 80% compared to the mudflow with 60% moisture. Because the analysis is based on the advance distance and deposition heights, also, the behaviour of the material with a humidity of 80% on both slopes is very similar.

Numerical models for considering the flux as a Newtonian or non-Newtonian fluid, are similar for the entire deposition length. This could be due to the low variation in the growth rate in the shear force that the material presents under 80% moisture, where the variation of the shear force between particles it is small. In this way, the variations can be observed according to the channel slope value, where approximately 18% more material accumulates in the last 0.2 m of channel when its slope is 10%.

When the material presents 60% of moisture, it is observed that for 5% slope, mudflow reaches a 0.9 m approximately and 1.06 m for 10% slope, due to the influence of gravity. Each modelling process
reproduces what is obtained at the laboratory level, obtaining small differences respect to flow final height of deposition and the advance distance.

Comparing Figure 8a with Figure 8b, for a slope of 5% and a material with 60% humidity, it is observed that when considering a rheological model to simulate the material viscous behaviour, a better reproduction of the real phenomena is obtained, respect to a linear viscous behaviour. In addition, flow advance distance, when the material is considered as a Newtonian fluid is 7 cm greater than laboratory test, while, considering the material as a non-Newtonian fluid, the difference in the advance distance is 2cm. The same occurs when comparing Figure 8c with Figure 8d for slopes of 10% and material with humidity 60%. Then, is more advisable to use a rheological behaviour of the material in numerical models.

5. Conclusions
The “k-ε Realizable” turbulence model simulates the mudflow respect to results obtained in the experimental tests. Considering the material as a non-Newtonian fluid or as a Newtonian fluid within the model, there are some variations in some coordinates of the flow compared to the final heights of deposition and advance distance, however, that difference is not important, considering that mudflow occurs over a controlled surface. In this way, for study cases, where there are important variations in the surface where the flow advances, the difference in the results obtained by both models could increase, the, the turbulence model considering the mudflow as a Newtonian fluid could present problems in materials with moisture below 70%, where there are greater variations in the shear force.

For mudflows with silty soils with a moisture content greater than 70%, behaviours with lower rates of change in linear shear force are expected, because viscosity decreases with water content, which implies a greater drag of solids to length of a surface and longer feed lengths. In this way, the reduction of the material internal shear forces with this moisture, make the flow begin to have a behaviour very similar to that of a non-Newtonian fluid, reducing the flow advance variation within numerical models.

It is recommended to use a model that reproduces the mudflow advance distance detonated from a slide. To validates the model effectiveness, since the numerical results obtained and Compared with the experimental execution, show that there is a flow reproduction in the movement and final deposition, which can be constituted as a powerful tool for these phenomena reproduction to estimate and the risk prevention in urban infrastructure.

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