A Distributionally Robust Optimization Approach for Unit Commitment in Microgrids

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Abstract—This paper proposes a distributionally robust unit commitment approach for microgrids under net load and electricity market price uncertainty. The key thrust of the proposed approach is to leverage the Kullback-Leibler divergence to construct an ambiguity set of probability distributions and formulate an optimization problem that minimizes the expected costs brought about by the worst-case distribution in the ambiguity set. The proposed approach effectively exploits historical data and capitalizes on the k-means clustering algorithm—in conjunction with the soft dynamic time warping score—to form the nominal probability distribution and its associated support. A two-level decomposition method is developed to enable the efficient solution of the devised problem. We carry out representative studies and quantify the relative merits of the proposed approach vis-à-vis a stochastic optimization-based model under different divergence tolerance values.

Index Terms—distributionally robust optimization, microgrids, unit commitment

I. INTRODUCTION

A microgrid is a cluster of loads, thermal generation resources (TGRs), variable energy resources (VERs), and electric storage resources that operate in coordination to supply electricity in a reliable manner. Typically integrated to its host power system at the distribution level, a microgrid is—for all intents and purposes—a microcosm of a bulk power system that retains most of its innate operational characteristics.

Similar to bulk power systems, the short-term planning of microgrids can be determined via unit commitment (UC) and economic dispatch (ED) decisions [1]. The UC problem seeks minimum cost strategies to determine the commitment statuses of TGRs based on expected load, equipment limitations, and operational policies. The equipment limitations of TGRs and the inter-temporal constraints of microgrid physical asset operations render UC a time-coupled problem, and necessitate that the UC decisions be taken typically one-hour to one-week ahead of operations based on the uncertain data/information available at the time of decision.

The short-term operation of microgrids is fraught with a wide range of sources of uncertainty, including microgrid net load, i.e., microgrid load less VER generation. In the event that a microgrid transacts energy on wholesale markets, the short-term planning is exacerbated by the uncertainty associated with electricity market prices. As such, the judicious short-term planning of microgrids with integrated VERs and potential exposure to volatility in electricity market prices hinges on UC approaches that undertakes an explicit assessment of the uncertainty in net load and market prices.

To engage with uncertainty, most studies in the literature rely on stochastic optimization (SO) or robust optimization (RO) techniques. A major shortcoming of SO is the assumption that the underlying probability distribution of uncertain parameters is known a priori. The veracity of this assumption, however, is highly questionable, as system operators have access to collected data—not to their underlying probability distribution. Indeed, if the assumed probability distribution is incorrect, SO may give rise to a markedly poor out-of-sample performance, which warrants and calls for optimization approaches that is not confined to a pre-specified probability distribution. In contrast to SO, the premise of RO is to disregard completely the probabilistic nature of uncertain parameters and take decisions based on the worst-case scenario, which may result in overly conservative decisions.

Distributionally robust optimization (DRO)—albeit being initially proposed long ago—has recently gained traction as a paradigm that addresses the drawbacks of both SO and RO. Under the DRO paradigm, the probability distribution of uncertain parameters itself is considered to be uncertain and belong to an ambiguity set of probability distributions that may be constructed based on various methods, including using moment information [2], [3], Kullback-Leibler (KL) divergence [4], and Wasserstein distance [5]. Central to DRO is the formulation of an optimization problem that minimizes the expected costs brought about by the worst-case distribution in the ambiguity set. As such, DRO obviates the need to commit to one pre-specified probability distribution and hedges the optimal decisions against adopting a misrepresenting probability distribution. The focus of this paper is the development of a DRO approach for microgrid unit commitment.

A. Related Work

There is a growing body of literature on the application of DRO approaches in the UC problem. In [2], the authors assess the uncertainty associated with wind generation and leverage the KL divergence to propose a DRO model for UC; nevertheless, they do not evaluate the out-of-sample performance of their approaches. The work conducted in [2] utilizes first

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and second moment information to construct the ambiguity set and takes into account the uncertainty in VER generation in the proposed UC model. However, [2] does not consider the uncertainty associated with electricity prices. While [3] harnesses the Wasserstein distance metric, [4] capitalizes on moment information so as to construct ambiguity sets and harnesses the Wasserstein distance metric, [5] capitalizes on the uncertainty associated with electricity prices. While [6] does not consider electricity market prices under a DRO approach. We conduct representative studies and discuss the results. We present our concluding remarks in Section IV with representative studies and discuss the results. We present our concluding remarks in Section V.

B. Contributions of the paper

The general contributions and novel aspects of this paper are as follows:

1) We develop a new DRO approach for microgrid UC using KL divergence. To the best of our knowledge, this is the first study that jointly evaluates the uncertainty associated with microgrid net load and electricity market prices under a DRO approach. We conduct representative studies and demonstrate the effectiveness of the proposed approach on real-world data.

2) Our studies provide valuable insights into the influence of divergence tolerance, and hence the degree of conservatism, on the out-of-sample performance.

3) We present a methodology that leverages k-means clustering and soft dynamic time warping (SDTW) score so as to construct the nominal probability distribution and its support. The presented methodology lends itself to the joint study of the uncertainty in net load and electricity market prices through multidimensional clusters without unduly exacerbating the computational burden.

4) We provide a tractable reformulation of the developed DRO problem and present a two-level decomposition method in conjunction with an iterative algorithm that enables its solution by off-the-shelf solvers. The presented algorithm is amenable to parallelization on the basis of scenarios.

This paper contains four additional sections. In Section II, we develop the mathematical formulation of the proposed UC approach and present the construction of the nominal probability distribution and its support. We present an iterative decomposition method in Section III for the solution of the proposed optimization problem. We illustrate the capabilities and effectiveness of the proposed DRO framework in Section IV with representative studies and discuss the results. We present our concluding remarks in Section V.

II. MATHEMATICAL FORMULATION

We devote this section to the delineation of the mathematical models utilized in the proposed DRO approach. We discretize the time-axis and adopt 1 hour as the smallest indecomposable unit of time and 24 hours as the scheduling horizon. We define the study period by the set $\mathcal{H} := \{h: h = 1, \ldots, 24\}$.

A. Problem Formulation

The proposed DRO approach explicitly represents the uncertainty associated with net load and wholesale electricity market price over the study period. We define by $\bar{\xi}$ the random matrix on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $\Omega$ is a sample space, $\mathcal{F}$ is a set of subsets of $\Omega$ that is a $\sigma$–algebra, and $\mathbb{P}$ is a probability distribution on $\mathcal{F}$. The random matrix $\bar{\xi} \in \Xi \subset \mathbb{R}^{24 \times 2}$ denotes the uncertain net load and electricity price over the study period, where $\Xi$ denotes the support of the probability distribution $\mathbb{P}$. We assume that $\mathbb{P}$ has a finite support taking $S$ realizations that we equivalently refer to as scenarios, i.e., $|\Xi| = S < \infty$. The construction of $\Xi$ is detailed in Section II.B. For each realization $\omega \in \Omega$, $\bar{\xi}$, we write the relation $\bar{\xi}^{\omega} = [\eta^{\omega}, \lambda^{\omega}]$, where $\eta^{\omega} \in \mathbb{R}^{24}$ and $\lambda^{\omega} \in \mathbb{R}^{24}$ represent the net load and electricity price values over the 24 hours of the study period $\mathcal{H}$, respectively. We denote by $\xi^{\omega}[h] = [\eta^{\omega}[h], \lambda^{\omega}[h]]$ the row $h$ of $\xi^{\omega}$, which represents the net load ($\eta^{\omega}[h]$) and electricity price ($\lambda^{\omega}[h]$) in hour $h \in \mathcal{H}$ for scenario $\omega$. We denote by $p^{\omega}$ the probability assigned to the scenario $\omega$ by the probability distribution $\mathbb{P}$.

The proposed formulation is based on a two-stage decision mechanism that mimics the order in which $UC$ and $ED$ decisions are taken.

\[
\begin{aligned}
&\text{minimize} & & \sum_{h \in \mathcal{H}} \sum_{g \in \mathcal{G}} c_g^v v_g[h] + c_g^u u_g[h] \\
&\text{subject to} & & v_g[h] \geq u_g[h] - u_g[h-1], \quad \forall g \in \mathcal{G}, \forall h \in \mathcal{H}, & (2) \\
& & & u_g[h] - v_g[h-1] \leq u_g[v], \quad \forall v \in \mathbb{N} \text{ such that} \\
& & & h \leq v \leq \min\{h + 1 - T_g, 24\}, \forall g \in \mathcal{G}, & (3)
\end{aligned}
\]
\[ u_g[h-1] - u_g[h] \leq 1 - u_g[\nu], \forall \nu \in \mathbb{N} \text{ such that} \]
\[ h \leq \nu \leq \min\{h+1, T_g^\delta\}, \forall g \in \mathcal{G}, \quad (4) \]
\[ u_g[h], v_g[h] \in \{0, 1\}, \forall g \in \mathcal{G}, \forall h \in \mathcal{H} \quad (5) \]

The first-stage problem (1)-(5) seeks to determine the binary commitment \((u_g[h])\) and start-up \((v_g[h])\) variables of the TGRs over the study period, while taking into account the minimum uptime \((3)\) and downtime \((4)\) constraints of the TGRs. We represent all first-stage decision variables by the vector \(x\) comprising \(u_g[h]\) and \(v_g[h]\). The first-stage decisions are taken before the realization of the uncertain net load values and electricity prices with the objective (1) to minimize the startup costs plus the worst-case expected power generation and purchase costs.

A salient feature of the proposed DRO approach is to capitalize on an ambiguity set of probability distributions denoted by \(\mathcal{P}\) to study the uncertainty associated with net load and electricity prices. Such an approach ensures that all probability distributions that belong to the set \(\mathcal{P}\) be assessed and the optimal first-stage decisions be taken based on the expected costs brought about by the worst-case distribution. We elaborate on the construction of the set \(\mathcal{P}\) in Section II-B.

The function \(Q(x^1, \xi^\omega)\) in (11) denotes the uncertain power generation and purchase costs. For a specific vector of first-stage decision variables \(x^1\) and a realization \(\xi^\omega\), \(Q(x^1, \xi^\omega)\) is evaluated by solving the following second-stage problem:

\[
Q(x^1, \xi^\omega) := \sum_{h \in \mathcal{H}} \sum_{g \in \mathcal{G}} \rho_p^g p_g[h] + \lambda^\omega[h] p_b[h] \quad (6)
\]

subject to

\[
u_g[h] p_g^m \leq p_g[h] \leq u_g[h] p_g^M, \quad \forall g \in \mathcal{G}, \forall h \in \mathcal{H} \quad (7)
\]

\[
\sum_{g \in \mathcal{G}} p_g[h] + p_b[h] - p_s[h] = \eta^\omega[h], \quad \forall h \in \mathcal{H}, \quad (8)
\]

\[
p_b[h], p_s[h] \geq 0, \forall h \in \mathcal{H}. \quad (9)
\]

The second-stage problem (6)-(9) seeks to determine the power dispatch of TGRs \((p_g[h])\), purchased power from the wholesale electricity market \((p_b[h])\) and the spilled power \((p_s[h])\) over the study period with the objective (6) to minimize the power generation and purchase costs. The second-stage problem takes into account the TGR output limits (7) and power balance constraint (8). We define by \(y\) the vector of all second-stage variables comprising \(p_g[h]\), \(p_b[h]\), and \(p_s[h]\).

**B. Ambiguity Set Construction Methodology**

We devote this subsection to the methodology undertaken to construct \(\mathcal{P}\). We denote by \(N_D\) the number of days for which historical net load and electricity market price data are initially considered. We utilize the k-means algorithm to partition the \(N_D\) number of multidimensional time series to \(S\) multidimensional clusters so as to assign each time series to the cluster with the nearest cluster centroid. To this end, we utilize the SDTW score to measure the similarity between time series data, which—when applied jointly with k-means clustering—was reported to deliver better results for time series clustering tasks vis-à-vis the Euclidean distance [6].

We note that the computational complexity of (1)-(5) gets aggravated with increasing number of uncertain parameters and scenarios. As such, we specifically aim at the joint representation of uncertain net load and market prices by multidimensional clusters, which affords the capability to simultaneously assess the uncertainty associated with net load and electricity prices without undue computational burden.

We use each of the constructed \(S\) clusters to form each of the \(S\) scenarios of the nominal probability distribution. For each cluster \(\omega\), we utilize the cluster centroid to represent the realization \(\xi^\omega\) and construct the support \(\Xi := \{\xi^\omega: \omega = 1, \ldots, S\}\). We denote by \(N_{\omega}\) the number of time series assigned to cluster \(\omega\) and—for the nominal probability distribution \(P_o\)—assign the probability for the scenario \(\omega\) as \(\pi^\omega_{P_o} = \frac{N_{\omega}}{N_{D}}, \omega = 1, \ldots, S\).

We next leverage the KL divergence to construct an ambiguity set of probability distributions \(\mathcal{P}\) around the nominal probability distribution \(P_o\):

\[
\mathcal{P} := \{P: \sum_{\omega=1}^{S} \pi^\omega_P \log \left( \frac{\pi^\omega_P}{\pi^\omega_{P_o}} \right) \leq \rho \} \quad (10)
\]

\[
\sum_{\omega=1}^{S} \pi^\omega_P = 1 \quad (11)
\]

\[
\pi^\omega_P \geq 0, \forall \omega \in \Omega \} \quad (12)
\]

Central to the construction of \(\mathcal{P}\) is the divergence tolerance \(\rho\), which serves to adjust the degree conservatism of the constructed ambiguity set. When \(\rho = 0\), \(\mathcal{P}\) shrinks to a singleton that contains only the nominal distribution \(P_o\). On the flip side, as \(\rho \rightarrow \infty\), \(\mathcal{P}\) admits all probability distributions, which may result in overly conservative decisions.

For notational brevity, we present the KL divergence-based microgrid unit commitment (KL–MUC) formulation as:

\[
\text{KL–MUC : minimize } c \cdot x + \max_{P \in \mathcal{P}} \sum_{\omega \in \Omega} \pi^\omega_P \mathcal{Q}(x, \xi^\omega) \quad (13)
\]

subject to \(x \in \mathcal{X}, \quad (10) - (12)\).

where \(\mathcal{X}\) represents the feasibility region of \(x\) defined by the constraints (2)-(5).

**III. SOLUTION METHOD**

In this section, we present a method based on Benders’ decomposition to ensure the efficient solution of KL–MUC. We take the dual of the inner maximization problem in KL–MUC and assign the dual variables \(\zeta\) and \(\mu\) to the constraints (10) and (11), respectively, which, as per [7], yields
the following convex mixed-integer nonlinear reformulated KL–MUC (KRKL–MUC) problem:

\[
\text{RKL–MUC :} \begin{align*}
\text{minimize} & \quad c \cdot x + \mu + \rho \zeta + \zeta \sum_{\omega=1}^{S} \pi_{\omega}^{x} e^{R(\omega,m,\zeta)-1} \\
\text{subject to} & \quad x \in \mathcal{F}, \\
& \quad \zeta \geq 0,
\end{align*}
\]

where \(e^{R(\omega,m,\zeta)-1}\) is a nonlinear convex function. For notational brevity, we define the following functions:

\[
R(\omega,m,\zeta) := \sum_{\omega=1}^{S} \pi_{\omega}^{x} e^{R(\omega,m,\zeta)-1},
\]

\[
R(x,\zeta,\mu) := \sum_{\omega=1}^{S} \pi_{\omega}^{x} e^{R(\omega,x,\zeta,\mu)-1}.
\]

We decompose RKL–MUC to a lower-bounding master problem (MP) and an upper-bounding subproblem (SP).

\[
\text{MP :} \begin{align*}
\text{minimize} & \quad c \cdot x + \mu + \rho \zeta(\nu) + \theta(\nu) \\
\text{subject to} & \quad x(\nu) \in \mathcal{F}, \\
& \quad \zeta(\nu) \geq 0, \\
& \quad \theta(\nu) \geq \alpha(\nu) \cdot (x(\nu) - x(j)) + \beta(\nu) \cdot (\mu(\nu) - \mu(j)) + \gamma(\nu) \cdot (\zeta(\nu) - \zeta(j)) + R(x(j),\zeta(j),\mu(\nu)), \\
& \quad j = 1, \ldots, \nu - 1,
\end{align*}
\]

where (23) represents the Benders’ optimality cuts that serve to approximate from below the function \(R(x,\zeta,\mu)\). At each iteration \(\nu\), the candidate optimal variables \((x(\nu),\mu(\nu),\zeta(\nu))\) evaluated by the MP are fixed as \(x(\nu) \leftarrow x(\nu), \mu(\nu) \leftarrow \mu(\nu),\) and \(\zeta(\nu) \leftarrow \zeta(\nu)\). Note that \(R(x,\zeta,\mu)\) is a nonlinear convex function. To ease the computational burden, instead of minimizing \(R(\cdot)\) in the SP, we adopt the linear program \(Q(\cdot,\mu,\zeta)\) presented in (6)-(9) as the SP [7] and leverage the chain rule along with the optimal solutions of SP to evaluate the optimality cuts. At each iteration \(\nu\), the SP is defined for each scenario \(\omega\) as:

\[
\text{SP :} \begin{align*}
\text{minimize} & \quad \sum_{h \in \mathcal{H}} \sum_{g \in \mathcal{G}} c_{g} p_{g}[h] + \lambda^{\omega}[h] p_{g}[h] \\
\text{subject to} & \quad x_{f}^{\omega} = x_{f} \leftarrow \overline{\mathcal{C}}(\nu,\mu,\zeta(\nu)).
\end{align*}
\]

The dual variable \(\overline{\mathcal{C}}(\nu)\) associated with the constraint represents the negative of the sensitivity of (24) to \(x_{f}\). We remark that the SP for each scenario \(\omega\) is a continuous problem as the elements of \(x_{f}^{\omega}\) are not constrained to be binary. For each scenario \(\omega\), we evaluate the terms:

\[
\begin{align*}
\overline{\mathcal{C}}(\nu) &= \zeta(\nu) e^{R(x_{f},\mu,\zeta(\nu))-1} \overline{\mathcal{C}}(\nu), \\
\overline{\mathcal{C}}(\nu) &= \frac{\partial R(x_{f},\mu,d)}{\partial \zeta(\nu)} = (1 - e^{R(x_{f},\mu,d,\zeta(\nu))}) e^{-R(x_{f},\mu,d,\zeta(\nu))-1},
\end{align*}
\]

and compute the terms \(\alpha(\nu), \beta(\nu), \gamma(\nu)\) in the Benders’ optimality cuts in (23) as follows:

\[
\alpha(\nu) = \sum_{\omega=1}^{S} \pi_{\omega}^{x} \overline{\mathcal{C}}(\nu), \\
\beta(\nu) = \sum_{\omega=1}^{S} \pi_{\omega}^{x} \overline{\mathcal{C}}(\nu), \\
\gamma(\nu) = \sum_{\omega=1}^{S} \pi_{\omega}^{x} \overline{\mathcal{C}}(\nu).
\]

Note that, since KL–MUC has a relatively complete recourse and the KL divergence does not necessitate a feasibility cut, feasibility cuts are not required in the MP. We emphasize that, since the dual variables can be evaluated independently for each scenario, the presented method lends itself to parallelization on the basis of scenarios. We succinctly represent the decomposition algorithm in Algorithm 1.

**Algorithm 1 Decomposition algorithm for RKL–MUC**

1. Initialize \(x \leftarrow 0\).
2. Solve SP. Set \(Q^{M} \leftarrow \max(\{Q(x,\zeta^{\omega}) : \omega \in \Omega\})\)
3. Initialize UB \(\leftarrow -\infty\), LB \(\leftarrow 0\), \(\nu \leftarrow 0\), \(\theta(\nu) \leftarrow 0\).
4. while UB – LB \(\geq\) TOL do
5. Solve MP. Determine \(x(\nu),\zeta(\nu),\mu(\nu),\) and \(\theta(\nu)\) so that \(Q^M - \mu(\nu) \leq R^M\), \(LB \leftarrow \theta(\nu)\).
6. Solve SP. Determine \(\alpha(\nu), \beta(\nu), \gamma(\nu), R(x(\nu),\zeta(\nu),\mu(\nu))\). UB \(\leftarrow R(x(\nu),\zeta(\nu),\mu(\nu)). \nu \leftarrow \nu + 1\).
7. end while

We point out that \(\alpha(\nu), \beta(\nu), \gamma(\nu)\) contain the term \(e^{R(\cdot)}\) in the exponent, which renders the proposed method prone to overflowing errors during its execution. As such, we expressly stipulate a computational upper bound on \(\overline{\mathcal{C}}(\nu)\) denoted by \(K^{M}\). Nevertheless, in lieu of of relying on \(e^{R(\cdot)} \leq K^{M}\) to bound \(\overline{\mathcal{C}}(\nu)\), we impose a more restrictive upper bound, viz. \(\frac{Q^{M} - \mu(\nu)}{\zeta(\nu)} \leq K^{M}\). In contrast to [7] that evaluates \(\overline{\mathcal{C}}(\nu)\) in the SP and necessitates additional iterations to compute a new \(z(\nu)\) in the event that \((x(\nu),\mu(\nu),\zeta(\nu))\) evaluated by the MP prompts \(\overline{\mathcal{C}}(\nu)\) to be greater than \(K^{M}\), our proposed upper bound ensures that \((x(\nu),\mu(\nu),\zeta(\nu))\) determined by the MP satisfy \(\overline{\mathcal{C}}(\nu) \leq K^{M}\) \(\forall \omega \in \Omega\), thereby precluding additional iterations.

**IV. CASE STUDY AND RESULTS**

In this section, we carry out representative studies to illustrate the application and effectiveness of the proposed DRO approach. We consider a microgrid with an integrated TGR and a PV panel. The source code and simulation scripts for the case study are provided in [8]. The load and PV generation dataset [9] contains measurements for an anonymous house in New York collected from June 1, 2019 to August 31, 2019. To ensure consistency, we consider the locational marginal prices at the N.Y.C. bus in the New York Independent System Operator network cleared in the day-ahead market for the said time period and add a surcharge to the prices so as to reflect the rates available to residential customers [10].

We start out by the construction of the scenarios. We utilize the data collected from June 1, 2019 to July 31, 2019 and
deploy the methodology described in Section II-B to assign each data point to \( S \) clusters. To determine \( S \), we examine the percentage of variance captured for different values of \( S \) and observe that the point of diminishing returns (i.e., the so-called elbow) is reached at \( S = 8 \), with which 88.06% of the total variance is captured and capturing an additional 10% of the variance requires 36 more clusters.

We draw on the solution method described in Section III to solve the RKL – MUC problem. We perform our implementations in Pyomo using Gurobi 9.0.2 as the solver with the optimality tolerance gap \( TOL = 10^{-5} \) on a 2.6 GHz Intel Core i7 CPU with 16 GB of RAM. In Section II-B, the divergence tolerance \( \rho \) was brought out as a key determinant in ambiguity set construction. As such, we probe the influence of \( \rho \) by solving the RKL – MUC problem with each of the following values: \( \rho = 0, 0.2, 0.4, 0.6, 0.8, 1.0 \). To carry out comparative assessments, we develop the following equivalent stochastic formulation of the KL – MUC problem, which serves as a benchmark for our experiments:

\[
\begin{align*}
\text{SUC:} \quad & \min_{x} \quad c \cdot x + \sum_{\omega \in \Omega} \pi_p(x, \xi^\omega), \\
\text{subject to} \quad & x \in \mathcal{X},
\end{align*}
\]

and employ the L-shaped algorithm for its solution.

While RKL – MUC and SUC problems are solved using the constructed scenarios, their feasibility must be assessed on real-life data that were not part of the scenarios. To this end, we capitalize on the data collected from August 1, 2019 to August 31, 2019 to form the out-of-sample dataset and empirically investigate the out-of-sample performance of the RKL – MUC formulation for each of the investigated six values of \( \rho \), as well as that of the SUC problem. To do so, for each of the seven setups, we fix the optimal first-stage decision variables obtained using the constructed scenarios and evaluate the total cost by providing each setup with the data points of the out-of-sample dataset.

We present in Fig. 1 the total cost under the RKL – MUC and SUC formulations. At the outset, we note that the SUC solution tallies with that of the RKL – MUC solution for \( \rho = 0 \), which validates our computations, as when \( \rho = 0 \), the ambiguity set contains solely the nominal probability distribution and so the KL – MUC formulation reduces to the SUC formulation. We remark upon the fact that, for all considered \( \rho \) values, the total cost under the RKL – MUC formulation is less than or equal to that under the SUC formulation. We further observe that the total cost decreases as \( \rho \) increases from \( 0 \) to \( 0.6 \). These observations bring out the benefit of taking into account additional probability distributions other than the nominal probability distribution and make clear that the nominal probability distribution need not be taken at face value. This notwithstanding, the total cost slightly picks up as \( \rho \) increases above \( 0.6 \), which may be accounted for by the fact that the assignment of increasingly large values to \( \rho \) permits the incorporation of probability distributions that assign markedly high probabilities to adverse scenarios in the ambiguity set, which are evidently not reflected in the out-of-sample dataset.

![Fig. 1. Out-of-sample performances under RKL – MUC and SUC](image)

**V. CONCLUSION**

In this paper, we propose a DRO approach for microgrid unit commitment under net load and electricity price uncertainty. Our approach takes full advantage of the copious amounts of data imparted by the deployment of information and communication technologies as per the smart grid paradigm. The methodology leveraged to construct scenarios affords the capability to conjointly study the uncertainty associated with net load and electricity price without aggravating the computational burden. The hallmark of our approach is to minimize the worst-case expected costs over an ambiguity set of probability distributions constructed using the KL-divergence, which enables us to hedge the optimal decisions against adopting a misrepresenting probability distribution. The case studies conducted on real-world data demonstrate the effectiveness of the proposed approach.

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