S2TD: a Separation Logic Verifier that Supports Reasoning of the Absence and Presence of Bugs

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Abstract. Heap-manipulating programs are known to be challenging to reason about. We present a novel verifier for heap-manipulating programs called S2TD, which encodes programs systematically in the form of Constrained Horn Clauses (CHC) using a novel extension of separation logic (SL) with recursive predicates and dangling predicates. S2TD actively explores cyclic proofs to address the path-explosion problem. S2TD differentiates itself from existing CHC-based verifiers by focusing on heap-manipulating programs and by employing cyclic proof to efficiently verify or falsify them with counterexamples. Compared with existing SL-based verifiers, S2TD precisely specifies the heaps over de-allocated pointers to avoid false positives in reasoning about the presence of bugs. S2TD has been evaluated using a comprehensive set of benchmark programs from the SV-COMP repository. The results show that S2TD is more effective than state-of-art program verifiers and is more efficient than most of them.

Keywords: Separation Logic, Constrained Horn Clauses, Program Verification

1 Introduction

Heap-manipulating programs are often building-blocks of real-world applications. Furthermore, heap-related bugs are the sources of many security vulnerabilities. The correctness of heap-manipulating programs is thus of great importance and yet they are notoriously challenging to verify \cite{18}. Existing verification techniques focus only on either numerical programs or safety verification of heap-manipulating programs. Moreover, existing techniques reason about the presence of memory bugs either by supporting only (in)equality constraints for pointers \cite{70} or by modeling pointers through the theory of arrays \cite{17,29}. None of them could model heap-manipulation precisely so as to effectively and efficiently reason about the presence of heap-related bugs. To bridge this gap, we present S2TD, a novel verifier for heap-manipulating programs. S2TD differentiates itself from existing program verifiers not only by introducing a novel memory model for pointers focusing on heap-manipulating programs but also by adopting a novel approach for verification and falsification of heap-manipulating programs.

Firstly, S2TD is based on the promising framework of Constrained Horn Clause (CHC). Over the years, there has been an increasing interest in logic-based systems using CHC \cite{87,98,54,32,144}, that provide a genuine compound of program verification
techniques. Given a program, a logic-based system partitions the verification process into two phases: (1) generating verification conditions in the form of CHC from the program and (2) checking the properties (e.g., reachability) of the CHC using decision procedures. As a result, logic-based systems can take advantage of rapidly developing satisfiability checking engines, especially SMT solvers \cite{2,23} as the decision procedure.

However, existing logic-based systems provide little support for reasoning about heap-manipulating programs. Fundamentally, this is because they rely on SMT solvers which are primarily based on first-order logic and have not catered to the needs of verifying heap-manipulating programs. Another particular challenge to them is that the encoding of the manipulations over the heaps must be flow-sensitive. For example, the CHC encoded for the two-command consequence \( x = \text{malloc}(...) ; i = x -> \text{val} \) must preserve the order of these two memory actions: first allocating a heap and then reading its content. Otherwise, a false alarm might be raised. To tackle these challenges, S2TD equips CHC with a novel extension of separation logic with recursive predicates and dangling predicates to support precise reasoning about heap-manipulation.

Secondly, S2TD novelly employs cyclic proof techniques to discharge CHC. One way to verify a heap-manipulating program is to verify the program paths one-by-one, which is also known as symbolic execution \cite{44}. Such an approach is ineffective due to the so-called path-explosion problem. Several methods have been proposed to attack this problem, e.g., using function summaries \cite{28}, combining static and dynamic analysis \cite{10,30}, and using interpolation \cite{41}. These methods are however not designed with heap-manipulation in mind. S2TD tackles this problem with cyclic proofs. The idea is to actively explore subsumption relationships between path conditions of different program paths in order to prune them. While such circular reasoning has been applied in safety/termination verification \cite{13,72,69}, this work applies cyclic satisfiability proofs into symbolic execution to reason about the presence (and absence) of bugs.

S2TD proceeds in two phases. In the first phase, it precisely encodes a program into an intermediate representation. The intermediate language is an extension of the existing CHC to support separation logic \cite{39,67}. We further extend the standard separation logic with additional assertions (e.g., dangling predicates). Without the dangling predicates, a reasoning system (e.g., \cite{3,19,20,64,56,63,61}) is unable to represent the de-allocated heaps precisely. Such a system typically over-approximates the semantics of the heaps and produces false positives in general. To the best of our knowledge, S2TD is the first reasoning system supporting such extended separation logic.

In the second phase, the CHC generated in the first phase is solved using a decision procedure, called S2e_{SL}. S2e_{SL} is an instantiation of the general cyclic proof framework presented in \cite{50} for the proposed separation logic. S2e_{SL} can be regarded as a symbolic execution engine for our CHC-based system. It executes the encoded program symbolically and constructs an execution tree. It simultaneously maintains both under- and over-approximation of the CHC and actively explores cyclic proof as described above. It is sound for both verification and falsification. A cyclic proof is constructed in the former case and a counterexample is generated in the latter case.

**Contribution** We make the following contributions: (i) We propose a novel verification system to verify or falsify heap-manipulating programs based on CHC. (ii) We present an effective solver, an instantiation of the general framework in \cite{50}, for the CHC based
on cyclic proofs. (iii) We have implemented S2TD and have applied it to a comprehensive set of heap-manipulating programs. The experimental results show that S2TD is effective and efficient compared with state-of-the-art verifiers.

2 Intermediate Language

We first introduce the intermediate language we use to encode programs as CHC. As shown in Fig. 1, it is based on separation logic with inductive predicates and arithmetic.

The intermediate language formulas and definitions of inductive predicates. A formula $\Phi$ is a disjunction of symbolic heap $\Delta$ where each disjunct models one program path. $\Delta$ is an existentially quantified formula consisting of a spatial constraint $\kappa$ and a pure (non-heap) constraint $\pi$. $FV(\Delta)$ denotes all free variables in the formula $\Delta$. We assume an infinite collection of variables $Var$, a finite collection of data structures $Node$, a finite collection of fields $Fields$, a set of heap addresses $Loc$, a set of non-address values $Val$ i.e. $\mathbb{Z} \subset Val$, $null \in Val$ and $Val \cap Loc = \emptyset$, and a finite set of inductive symbols $P$. $\bar{x}$ denotes a sequence of variables and $\bot \in Val$ is a preserved value denoting the content of a heap cell following de-allocation.

A spatial formula $\kappa$ is either the predicate $\text{emp}$ (asserts an empty heap), a points-to predicate $x \rightarrow c(f:v)$ where $c \in Node$ and $f \in Fields$ (asserts that the pointer $x$ points to singly allocated heap typed $c$ with content $f:v$), an inductive predicate instance $P(v)$ (represents an infinite set of allocated objects which are defined by predicate $P$), or their spatial conjunction. When there is no ambiguity, we discard $f$ and simply write the short form $x \rightarrow c(v)$. A formula is said to be a base formula, denoted by $B$, if it does not contain any inductive predicates. A pure formula $\pi$ can be a formula in Presburger arithmetic $\phi$, a pointer constraint $\alpha$, or a boolean combination of them. The pointer constraint may include (in)equalities, dangling predicate $x \not\rightarrow f$ (i.e., $x$ is not yet allocated or is already de-allocated), or the following three new predicates to simulate memory accesses. $LD(v,f,x,k)$ simulates $k^{th}$ memory access for loading the value at the memory of field $f$ pointed to by $v$ into variable $x$. $ST(v,f,x,k)$ simulates $k^{th}$ memory access for writing the value of variable $x$ into the memory of field $f$ pointed to by $v$. And $DEL(v,k)$ simulates $k^{th}$ memory access for de-allocating memory pointed to by variable $v$. We use $a_1 \neq a_2$ as short forms for $-(a_1 = a_2)$. $\text{res}$ is a reserved variable to denote the returned value of each procedure. Note that in our definition, a pure formula can be negated. However, our encoding only generates positive form of the three simulation predicates.

An inductive predicate is defined as $\text{pred } P(\bar{i}) \equiv \bigwedge_{i=1}^{n} (\Delta_i;l_i)$, where $P$ is the predicate name, $\bar{i}$ is a sequence of parameters and $\Delta_i$ ($i \in \{1...n\}$) are definition rules (branches). Each inductive predicate is associated with an invariant $inv$ representing a superset of all possible models of the predicate. This invariant is generated automatically and used to efficiently prune infeasible executions. Each branch corresponds to
a path in the program captured by path label $l_{i}$, a sequence of controls at the branching statements. This label $l_{i}$ is used to generate the witness to program errors. We use 1 to label the branch which satisfies conditions of branching statements (e.g., then branch) and use 2, otherwise (e.g., else branch). In each definition rule, variables not in $\overline{t}$ are always existentially quantified. Lastly, a CHC $\sigma$ is defined as: $\Delta:l \Rightarrow P(\overline{v})$ where $\overline{v} \subseteq FV(\Delta)$, $\Delta$ is its body (with path label $l$) and $P(\overline{v})$ is its head. A clause without a head is called a query. We note that CHCs with the same head e.g., VC generated for different paths of a procedure $P$ (or a loop) $(\Delta_{1}:l_{1} \Rightarrow P(\overline{v})) \lor (\Delta_{2}:l_{2} \Rightarrow P(\overline{v}))$, is written in the equivalent form of predicate definition as: $P(\overline{v}) \equiv \Delta_{1}:l_{1} \lor \Delta_{2}:l_{2}$.

**Semantics** Formulas of our separation logic fragment are interpreted over pairs $(s,h)$ where $s$ models the program stack and $h$ models the program heap. Formally, we define:

$$\text{Heaps} \overset{\text{def}}{=} \text{Loc} \rightarrow \_ \rightarrow (\text{Node} \rightarrow \text{Fields} \rightarrow \text{Val} \cup \text{Loc}) \quad \text{Stacks} \overset{\text{def}}{=} \text{Var} \rightarrow \text{Val} \cup \text{Loc}$$

The semantics of a formula $\Phi$ is defined by a relation: $s, h \models \Phi$ that forces the stack $s \in \text{Stacks}$ and heap $h \in \text{Heaps}$ to satisfy the constraint $\Phi$. The semantics of all predicates except the dangling predicate and new simulation predicates are standard (e.g., [50] for a reference). The semantics of the new predicates is as follows.

$s \models v \mapsto_{\_} \iff \forall h, c, f \ s.t. \text{dom}(h) = \{s(v)\}, h(s(v))(c, f) = \bot$

$s \models \text{LD}(v, f, x, k) \iff k \in \mathbb{Z}, k \geq 0 \quad \exists h, c, s, h = v \mapsto c(\_)$

$s \models \text{ST}(v, f, x, k) \iff k \in \mathbb{Z}, k \geq 0 \quad \exists h, c, s, h = v \mapsto c(\_)$

$s \models \text{DEL}(v, k) \iff k \in \mathbb{Z}, k \geq 0 \quad \exists h, c, s, h = v \mapsto c(\_)$

### 3 Overall Approach

We show how S2TD works using the illustrative C program shown in Fig. 2(a). The program allocates a null-terminated singly-linked list using a recursive procedure `sll` at line 10. The list contains a list of decreasing non-negative integer numbers. The loop from line 11 to 15 traverses through the list to check if any number stored in the list is negative. For each node in the list, if its value is negative, an error occurs (at line 13). For simplicity, we assume that user assertions are specified in the form of “if (c) then ERROR()”. Given the program, S2TD automatically verifies: (i) whether method ERROR() at line 13 is reachable; and (ii) whether this program is memory safe, i.e., no error caused by de-reference pointers (at line 5, 13 and 14) and free pointers (at line 15). While S2TD could return “no” for all these queries, our experimental results show that the state-of-the-art verification systems (e.g., PredatorHP [45], SeaHorn [34], Cascade [74], ESBMC [22], UAutomizer [50], CPAchecker [7], CBMC [21] and Smack-Corral [35]) could not handle this example.

S2TD has three main components: a front-end parser, a middle-end encoder and the back-end decision procedure S2eb. The parser is based on CIL [58]. It takes the C program as input and transforms it into a core program. We assume that the transformation starts at a special entry method named `main`. Specifically, the parser converts the program into a static single assignment (SSA) form, internalizes global variables and transforms loops into *tail recursive* procedures. For example, the *while* loop from line 11 to 15 is transformed into a recursive procedure `loop11` and the loop is replaced by
arising from either

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We show the flow-sensitive encoding at two levels: inter-procedure and intra-procedure. The former is necessary as it helps to avoid generating infeasible executions (e.g., a trace that never calls \text{sl1} but executes \text{loop11}). Particularly, we annotate every occurrence of inductive predicates, e.g., \(\text{main}(\bar{v})\), with two numbers, e.g., \(\text{main}(\bar{v})^o_u\) where \(o\) is the order of a callee in the calling context and \(u\) is the number of unrolling (for each loop or recursion, whose value is increased by one after being called). For example, in the main procedure as \text{sl1} is always executed before the loop, the sequence number of \text{sl1} is 0 and that of \text{loop11} is 1. These numbers ensure that if a pointer is first allocated

```c
1 struct node { int val; struct node *next; }
2 struct node* all(int i){
3    if(i==0) return NULL;
4    else { struct node *n=(struct node*) malloc(sizeof(struct node));
5        n->val=i; n->next=all(i-1);
6    return n; } }
7 int main(int n){
8    struct node *x;
9    if(n<0) return 0;
10    x=all(n);
11    while(x){
12        struct node *tmp=x;
13        if(x->val<0) ERROR();
14        x=x->next;
15        free tmp; }
16    return 1; }
```

a) An example C program

b) Verification Condition

Fig. 2. An illustrative example

a call of procedure \text{loop11}. The arguments of \text{loop11} include a set of input parameters, which are those variables occurring in the \text{while} condition or the body of the loop, and a set of output parameters, which are those variables that are modified within the loop body (instead of using pass-by-reference variables). Each call of \text{ERROR()} is replaced by: \text{assume}(e=1) where \(e\) is a reserved variable which encodes exit conditions. \(e\) has two possible values: \(e=1\) arising from either \text{ERROR()} or a heap-manipulation violation and \(e=0\) for normal termination. Here, we represent the single kind of error. For a better error explanation, we could consider different kinds of errors using a lattice like in [49].

3.1 Flow-Sensitive Encoding

The encoder takes a core program as input and produces a CHC system. Intuitively, it transforms each procedure into an inductive predicate definition. It is a forward symbolic executor of the form \(\text{exec}(\Delta_{pre}, e) \rightarrow \Delta_{post}\) (a.k.a a Hoare triple \(\{\Delta_{pre}\} e\{\Delta_{post}\}\)), where \(\Delta_{pre}\) denotes pre-states, \(e\) is a program command and \(\Delta_{post}\) depicts post-states.

The engine aborts whenever an error is met. Particularly, for each procedure \(mn(\bar{v})\) with a body \(e\) in the program, the encoder generates the following inductive definition: \(\text{pred } mn(t, \text{res}, e) \equiv \Phi\) such that \(\text{exec}(\text{emp } \land \text{true }, e) \rightarrow \Phi\). The arguments of each generated predicate consist of parameters of the procedure and two additional variables: \(\text{res}\) and \(e\) where \(\text{res}\) encodes the return value of a procedure and \(e\) to capture error status. In the following, we show how to obtain a flow-sensitive encoding.
in \(s11\) and then accessed in the loop, there is no memory error. The unfolding numbers of inductive predicates in generated VCs are initially assigned to 0 if it is not recursive and 1 otherwise and updated by the solver during its execution.

For a flow-sensitive intra-procedural memory access encoding, we introduce a local order, say \(k^{th}\), in a sequence of memory accesses in a procedure over the three new predicates: \(LD(v, f_i, x, k)\), \(ST(v, f_i, x, k)\), and \(DEL(v, k)\). For example, given \(\Delta_{pre}\) and let \(\epsilon\) be a memory read \(x := v \rightarrow f_i\), S2TD generates the following post-state \(\Delta_1 \lor \Delta_2 \lor \Delta_3\):

1. \(\Delta_1 \equiv \Delta_{pre} \land v \neq x \land \epsilon = 1\) encodes a memory error when \(v\) is a dangling pointer.
2. \(\Delta_2 \equiv \Delta_{pre} \land v = \text{null} \land \epsilon = 1\) encodes a memory error when \(v\) equals to \text{null}.
3. \(\Delta_3 \equiv \Delta_{pre} \land v \neq \text{null} \land v \neq x \land \Delta(v, f_i, x, k)\) encodes the memory safety condition when \(v\) has been allocated in the pre-state.

The post-states \(\Delta_1\), \(\Delta_2\) and \(\Delta_3\) are complete and pairwise disjoint. If \(\Delta_{pre}\) implies that \(v\) is dangling, \(\Delta_1\) is satisfied, but neither \(\Delta_2\) nor \(\Delta_3\). If \(\Delta_{pre}\) implies that \(v\) has been assigned to \text{null}, \(\Delta_2\) or \(\Delta_3\) is satisfied, but neither \(\Delta_1\) nor \(\Delta_3\). Both these scenarios above trigger a memory error. Otherwise, \(\Delta_{pre}\) implies that \(v\) is pointing-to a node, \(\Delta_3\) is satisfied, but neither \(\Delta_1\) nor \(\Delta_2\) and there is no memory error.

**Example Revisited** The CHC for the program shown in Fig. 2a) is presented in Fig. 2b). Each procedure (or loop) is encoded as a predicate definition where each disjunct corresponds to a program path in the procedure. To generate a counter example, we keep track of the program paths using labels. Particularly, each disjunct is attached with a path label (after :) 1 for the \(\text{then}\) (or entering loop) branch and 2 for the \(\text{else}\) (or exiting loop) branch. For example, for procedure \(\text{main}\), the first disjunct of its encoding corresponds to the \(\text{then}\)-branch (with the path label [1]) in Figure 2 whereas the second disjunct corresponds to the \(\text{else}\)-branch (with the path label [2]).

Procedure \(\text{main}\) is encoded with the predicate \(\text{main}\) whose first disjunct encodes the \(\text{then}\)-branch at line 9, and the second disjunct encodes the else-branch. The \(\text{while}\) loop at lines 11-15 is encoded as the predicate \(\text{loop}_{11}\) which has four disjuncts. The first disjunct encodes the branch where the loop condition at line 11 does not hold. The second one encodes the branch where the loop condition holds and there is a dangling-dereference error at the access \(x \rightarrow \text{val}\) at line 13. The third disjunct encodes the if-branch at line 13, i.e., the memory access is safe and the error occurs. The last disjunct encodes the else-branch at line 13, i.e., there is no error. We note that the disjunct corresponding to null-dereference (i.e., \(x = \text{null}\)) is infeasible (and has been discarded) as it contradicts with the loop condition (i.e., \(x \neq \text{null}\)). We further note that the numbers (0 and 1) in the last disjunct on memory reads indicate that the memory access on field \(\text{val}\) must happen before the one on field \(\text{next}\).

**Invariant Generation** After CHCs are generated, S2TD automatically infers for each inductive predicate an over-approximate invariant using the abstract interpretation technique shown in [50]. For example, the invariant of \(s11\) is \(i \geq 0 \land \epsilon = 0\), which is generated through three steps. First, S2TD introduces an unknown predicate \(P(i, \text{res}, \epsilon)\) as a place-holder for the invariant \(\forall i, \text{res}, \epsilon \ldots (i, \text{res}, \epsilon) \Rightarrow P(i, \text{res}, \epsilon)\). Secondly, in the induction step, it unfolds predicate \(s11\) in the left-hand side prior to substituting all occurrences of the predicate \(s11\) with the induction hypothesis above to obtain:

\[(\text{emp} \land \text{res} = \text{null} \land i = 0 \land \epsilon = 0 \lor \exists r. \text{res} \rightarrow \text{node}(i, r) \rightarrow P(i - 1, r, \epsilon) \land i \neq 0) \Rightarrow P(i, \text{res}, \epsilon)\]
Lastly, it over-approximates the heap (e.g., transform \( \text{res} \rightarrow \text{node}(i, r) \) to \( \text{res} \neq \text{null} \)):

\[
(res=null \land i=0 \land \epsilon=0 \lor \exists r. \text{res} \neq \text{null} \land P(i-1, r, \epsilon) \land i \neq 0) \implies P(i, \text{res}, \epsilon)
\]

This constraint is then passed to a fixed point calculator (e.g., the one in [73]) to compute the closure form for \( P \). Similarly, it generates invariant \( 0 \leq \text{res} \leq 1 \) for \( \text{main} \) and \( 0 \leq \epsilon \leq 1 \) for \( \text{loop}_{11} \). Our encoder uses these invariants to prune any infeasible CHC whose body is unsatisfiable. For instance, while encoding the procedure \( \text{main} \), the engine prunes the following CHC \( \sigma_1: \exists \epsilon_1. \text{sll}(n, x, \epsilon_1)^0 \land n \geq 0 \land \epsilon = 1 \land \epsilon_1 = \epsilon \implies \text{main}(n, \text{res}, \epsilon) \).

Due to the inconsistency between the sub-formula \( \epsilon = 1 \) and the over-approximating invariant of the predicate \( \text{sll}(n, x, \epsilon) \) with the sub-formula \( \epsilon = 0 \) of the body of \( \sigma_1 \), this CHC is reduced into \( \sigma_1: \text{false} \implies \text{main}(n, \text{res}, \epsilon) \). Thus, it could be discarded.

**Query Generation** After generating the CHC, we reduce the verification problem into a decision problem i.e., whether there exists a feasible error path starting from \( \text{main} \) or not. Indeed, this encodes the “liveness” property, to ask whether “something good eventually happens” where “good” is an error. Particularly to the example, the problem or not. Indeed, this encodes the “liveness” property, to ask whether “something good eventually happens” where “good” is an error. Particularly to the example, the problem

\( \exists n = 0 \land r \neq \text{null} \land P(i-1, r, \epsilon) \land i \neq 0 \implies P(i, \text{res}, \epsilon) \)

Given \( \Delta_0 \), \( \text{S2e}_{\text{se}} \) constructs a cyclic execution tree which either contains a closed leaf representing a satisfiable base formula or contains only closed leaves which are either unsatisfiable or linked back. Let us illustrate this via our earlier example.

**Example Revisited** \( \text{S2e}_{\text{se}} \) starts with the encoding of \( \text{main} \): \( \Delta_0 \equiv \text{main}(n_0, \text{res}_0, \epsilon_1)^0 \land \epsilon = 1 \).

The execution tree of \( \Delta_0 \) is shown in Fig. 3 where the unsatisfiable nodes are underlined. \( \Delta_0 \) has two children, \( \Delta_{11} \) and \( \Delta_{12} \), obtained by unrolling predicate \( \text{main} \).

\[
\Delta_{11} \equiv \text{emp} \land n_0 < 0 \land \text{res}_0 = 0 \land \epsilon = 0 \land \epsilon_1 = 1 \quad \Delta_{12} \equiv \text{sll}(n_0, x_0, \epsilon_1)^0 \land \text{loop}_{11}(x_0, x, \epsilon_2)^0 \land n_0 \geq 0 \land \pi \epsilon
\]
In this section, we present details on how we encode programs into CHC. The syntax of

\[ sll \equiv \text{null}(x) \land x = x_{\text{null}} \land x = x_{\text{null}} \land x_{\text{null}} = 0 \land \varepsilon = 1 \land \pi = 1. \]

For simplicity, existentially quantified variables are skolemized and path traces (i.e., the labels to mark program paths for witness generation) are discarded. For a flow-sensitive analysis, the unfolding number of predicates \( sll \) and \( \text{loop}_{11} \) in \( \Delta_{12} \) is set to be greater than the unfolding number of predicate \( \text{main} \) in \( \Delta_0 \), and its order is set to be the sum of the order of the predicate in the definition of \( \text{main} \) (i.e., 0 for \( sll \) and 1 for \( \text{loop}_{11} \)) and the order of predicate \( \text{main} \) in \( \Delta_0 \) (i.e., 0). \( \Delta_{11} \) is marked closed as it is unsatisifiable. The unsat core of \( \Delta_{11} \) is underlined above. \( \Delta_{12} \) is then expanded to obtain two children, \( \Delta_{21} \) and \( \Delta_{22} \), obtained by unrolling predicate \( sll \).

\[ \Delta_{21} \equiv x_0 = \text{null}(x_0) \land x_0 = 0 \land x_0 = x_{\text{null}} \land x = x_{\text{null}} \land x_{\text{null}} = 0 \land \varepsilon = 1 \land \pi = 1. \]

\[ \Delta_{22} \equiv x_0 = \text{node}(n_1, r_2) \ast sll(n_1, r_2, x_{\text{null}}) \land x_{\text{null}} = 0 \land n_1 = n_0 - 1 \land n_0 \geq 0 \land \pi = 1. \]

\( sll \) is chosen for unfolding rather than \( \text{loop}_{11} \) as they both have the same unfolding number and the sequence number of the former (0) is smaller than that of latter (1). This ensures the flow-sensitiveness of the execution. Similarly, \( \Delta_{21} \) has three children obtained by unrolling predicate \( \text{loop}_{11} \) as follows.

\[ \Delta_{31} \equiv \text{emp} \land x_0 = \text{null}(x_0) \land x = x_{\text{null}} \land x_{\text{null}} = 0 \land \varepsilon = 1 \land \pi = 1. \]

\[ \Delta_{32} \equiv \text{emp} \land x_0 = \text{null}(x_0) \land x = x_{\text{null}} \land x_{\text{null}} = 0 \land \varepsilon = 1 \land \pi = 1. \]

\[ \Delta_{33} \equiv \text{loop}_{11}(n_1, r_2) \ast sll(n_1, r_2, x_{\text{null}}) \land x_{\text{null}} = 0 \land n_1 = n_0 - 1 \land n_0 \geq 0 \land \pi = 1. \]

As \( \Delta_{31}, \Delta_{32} \) and \( \Delta_{33} \) are unsat, they are marked as closed. \( \Delta_{22} \) has three children obtained by unrolling predicate \( \text{loop}_{11} \). \( \text{loop}_{11}(x_0, r_2, x_{\text{null}}) \) is chosen but not \( sll(n_1, r_2, x_{\text{null}}) \) as the former has a smaller unfolding number (i.e., 1) than the latter (i.e., 2).

\[ \Delta_{41} \equiv x_0 = \text{node}(n_1, r_2) \ast sll(n_1, r_2, x_{\text{null}}) \land x_0 = x_{\text{null}} \land x_{\text{null}} = 0 \land n_1 = n_0 - 1 \land n_0 \geq 0 \land \pi = 1. \]

\[ \Delta_{42} \equiv x_0 = \text{node}(n_1, r_2) \ast sll(n_1, r_2, x_{\text{null}}) \land x_{\text{null}} = 0 \land n_1 = n_0 - 1 \land n_0 \geq 0 \land \pi = 1. \]

\[ \Delta_{43} \equiv sll(n_1, r_2, x_{\text{null}}) \ast \text{loop}_{11}(n_1, r_2, x_{\text{null}}) \land x_{\text{null}} = 0 \land n_1 = n_0 - 1 \land n_0 \geq 0 \land \pi = 1. \]

\( \Delta_{41} \) and \( \Delta_{42} \) are unsat. \( \Sigma_{\varepsilon_{\text{eq}}} \) detects that \( \Delta_{43} \) is subsumed by \( \Delta_{12} \). \( \Sigma_{\varepsilon_{\text{eq}}} \) then adds a back-link from \( \Delta_{43} \) to \( \Delta_{12} \) to form a cyclic proof [50]. Intuitively, this back-link means that although the path from \( \Delta_{12} \) to \( \Delta_{43} \) can be infinitely unrolled, there is no error detected this way and therefore we can stop unfolding \( \Delta_{43} \). As the cyclic unfolding tree in Fig. 3 is closed, our system proves that the program is safe.

4 CHC Encoding

In this section, we present details on how we encode programs into CHC. The syntax of
mand

order of function calls and is essential for flow-sensitive analysis. We note that our core language does not include loops as all procedures by default.

V returns the fresh form of variables in the precondition. Otherwise, is formalized as follows: exec(Δ₁, e) → Δ₂ where Δ₁ and Δ₂ are its precondition and postcondition, respectively. A procedure mn is encoded by the predicate mn as:

V = \{v, res, ε\}  exec(emp/initV(V);[e]) → Δ₁; l₁  W = fresh(V)

t₀ mn(T) {e} → pred mn(v, res, ε) ≡ (∨ W ⋁ Δ₁; l₁)

The list of arguments of the predicate mn includes the set of parameters of mn and two preserved variables: res for output and ε for error status (initially ε=0). Each program path in mn corresponds to a disjunct constituting the predicate mn. Function fresh(V) returns the fresh form of variables in V which capture the symbolic values of the inputs. initV(V) returns equalities that assign variables in V to the symbolic values in fresh(V). Furthermore, for the symbolic execution of each procedure, we assume that the engine maintains a pair of two numbers n₁ and k, both initially assigned to 0, where n₁ is the next procedure call number (used to preserve the order of the function calls), and k for the next memory access sequence number (used to preserve the order of the memory accesses). Every pointer parameter vᵢ is initially set to be dangling, with the constraint vᵢ → new conjoined with the precondition before executing. Part of the rules for forward symbolic execution are presented in Fig. 5, the remaining rules are shown in the App. The engine halts when it detects an error in the precondition (rule [ERR]). Afterwards, it produces a disjunctive formula (∨ Δᵢ) that precisely captures the post-states of the function where each Δᵢ is a program path in the procedure.

A function/procedure call mn is encoded by two disjoint cases (the rules [CALL-OK] for normal returning call and [CALL-ERR] for error returning call) through an occurrence of the inductive predicate with an increasing order number n₁. This predicate occurrence is spatially conjoined (+) into the precondition of the triple. The correctness of this spatial conjunction + is as follows. If mn allocates new heaps, the spatial conjunction states correctly the separation between the new heaps and the existing heap region in the precondition. Otherwise, mn includes only pure constraint (e.g., emp/π), the correctness is ensured by the axiom: (κ₁ ∧ π₁) + (emp/π) ⇔ κ₁ ∧ π₁ ∧ π [67]. nᵢ captures the order of function calls and is essential for flow-sensitive analysis. The flow-sensitivity

\[ \text{prog ::= datat meth}^* \quad \text{datat ::= data e \{ field;\} } \]
\[ \text{proc ::= t mn ((t v)* \{ e \} } \]
\[ \text{field ::= t v \quad t ::= e \mid \tau} \quad \tau ::= \text{int} \mid \text{bool} \]
\[ e ::= \text{NULL} \mid k \mid v \mid v := e \mid t v \mid mn(v₁,..,vₙ) \]
\[ \text{assume}(\pi) \mid v \rightarrow f \mid \text{new}(v₁,..,vₙ) \mid \text{free}(v) \]
\[ v₁ \rightarrow f := v₂ \mid c₁; c₂ \mid \text{if } v \text{ then } c₁ \text{ else } c₂ \]

**Fig. 4.** A core language \((t: \text{a type}; v: \text{a variable})\)
is ensured by the decision procedure $S2e_{IL}$ such that $S2e_{IL}$ uses this number to choose an inductive predicate for unfolding in a breadth-first manner.

The encoding of memory accesses is one of our main contributions. $\texttt{free}$ is encoded in a way such that it aids for both safety and double free error discovery. Essentially, $\texttt{free}$ is erroneous if the precondition $\Delta$ implies either $v$ is a dangling pointer ($v \not\rightarrow x$) or $v$ has been assigned to $\texttt{null}$ ($v = \texttt{null}$). $\texttt{free}$ is safe if $\Delta$ implies ($v \not\rightarrow x$). The key point to detect double free errors is that the post-condition of the command $\texttt{free}$ must ensure that the latest value of $v$ points to a dangling heap, i.e. $v \not\rightarrow x$.

For memory safety of free and memory access commands (load and store) over the heaps, we delegate the binding to the logic layer where symbolic path traces include explicit points-to predicates. In the cases of memory safety, we add predicates with explicit binding notation, $\texttt{LD}(v,f_i,x,k)$ for memory read and $\texttt{ST}(v,f_i,x,k)$ for memory write, into program states and postpone the binding to the normalization of the error/safe decision procedure. (We recall that $k$ is a globally increasing order among load/store/free operations over the heaps.) Constraints used to encode safety and pointer dereference errors are analogous to the encoding of the $\texttt{free}$ command.

### 5 Decision Procedure

$S2e_{IL}$, the decision procedure in $S2TD$, is presented in Algorithm[1]. It takes a formula $\Delta$ and a place holder $[]$ for a counterexample as inputs. It has two possible outcomes: yes with a counterexample $\xi$ (i.e., a sequence of statements starting from the entry point to the error statement) or no with a cyclic proof. If it does not terminate after some threshold time units, we mark the result as unknown.

At line 1, an execution tree with only one root node $\Delta$ is constructed. The loop (lines 2-11) then iteratively grows the tree while attempting to establish a yes proof.
Algorithm 1: Decision Algorithm \( S_{2e_{SL}} \)

input : \((\Delta[i])\)  
output: (yes, \(\xi\)) or no  
1 \( T \leftarrow (\Delta[i]) \); /* initialize root */  
2 while true do  
3 \( (\text{is\_error}, T) \leftarrow U_{a_{SL}}(T); \) /* base case */  
4 if is\_error then return (yes, get\_error(T));  
5 \( T \leftarrow O_{a_{SL}}(T) \); /* prune infeasible */  
6 \( T \leftarrow \text{link\_back}_{a_{SL}}(T); \) /* induction */  
7 if is\_closed(T) then return no;  
8 else  
9 \( T \leftarrow \text{unfold}_{a_{SL}}(T); \) /* expand */  
10 end  
11 end

or a no proof. In particular, at line 3, it checks whether there is an error or not using a function \( U_{a_{SL}}(T) \). Function \( U_{a_{SL}} \) focuses on those leaf nodes which do not contain inductive predicates (referred to as base formulas). If one of the leaf nodes is proved to be satisfiable, the algorithm returns yes (line 4) together with the satisfiable node as a counterexample (obtained by using function get\_error). Otherwise, those leaf nodes are marked as closed. At line 5, function \( O_{a_{SL}} \) prunes infeasible execution traces. Afterwards, every remaining leaf node is checked to see if it can be linked-back through function \( \text{link\_back}_{a_{SL}} \). At line 7, it checks whether the tree is closed (i.e. all leaf nodes are either marked closed or can be linked-back) and returns no if so. Otherwise, at line 9, it chooses a leaf which is neither marked closed nor linked-back and grows the tree by unfolding the inductive predicate in the node using function \( \text{unfold}_{a_{SL}} \). Afterwards, the same process is repeated until either yes or no is returned or a timeout occurs. In the following, we describe \( O_{a_{SL}}, \text{unfold}_{a_{SL}}, U_{a_{SL}} \) and \( \text{link\_back}_{a_{SL}} \) in more detail.

Function \( O_{a_{SL}} \) checks unsatisfiability of every open leaf node using \( S_{2SAT_{SL}} \). First, \( S_{2SAT_{SL}} \) replaces every inductive predicate by its over-approximated invariant to obtain base formulas. If the base formula does not contain an (over-approximated) error (e.g., \( \varepsilon = 1 \)), it checks whether the base is unsatisfiable and marks it closed if so. Function \( \text{unfold}_{a_{SL}} \) first finds an open leaf node \( \Delta \) in the tree in a breadth-first order. If \( \Delta \) contains multiple occurrences of inductive predicates, the occurrence \( P(\bar{v})_u \) with the smallest \( u \) (i.e. the number of unfoldings) is selected. If there are more than one occurrences that have the same smallest number of unfoldings, the one with the smallest \( o \) is selected. After that, it unfolds \( P(\bar{v})_u \) by spatially conjoining branches of the instance \( P(\bar{v}) \) into the formula. This step also combines the path traces of the branches with the present path trace. For example, suppose \( \Delta \) be \( \exists \bar{w}_0. \kappa \wedge P(\bar{v})_u \wedge \pi \) and \( P(\bar{v}) = \bigvee (\Delta_{i \tau_1}), \) then the unfoldings is \( \exists \bar{w}_0. \kappa \wedge \bigvee (\Delta_{i \tau_1}) \wedge \pi \). The set of new leaves are: \{\( \exists \bar{w}_0. \kappa \wedge \Delta_{i \tau_1} \) for every branch \}. The set of new leaves are: \{\( \exists \bar{w}_0. \kappa \wedge \Delta_{i \tau_1} \) for every branch \}. In the illustrative example, the interprocedural path corresponding to the leaf \( \Delta_{33} \) is \{main, [2]; sll, [1]; loop, [1:1]\}. This trace captures the program path from the entry of the procedure main to the else branch, call function sll once (then branch) and to the body of the while loop once (else branch). For a flow-sensitive analysis, the unfolding and order numbers are updated as follows.
Let $Q(\ell)^{\pi}$ be an occurrence of an inductive predicate in a branch of the definition of $P$. Its unfolding number is set to $u+1$ and its order number to $o_1+o$.

**Normalization**  Finally, the unfolded formula is normalized to remove all ST/LD/DL predicates. First, it detects and prunes infeasible cases. Based on the semantics described in Sect. [2] we propose axioms on the memory access predicates as follows.

\begin{align*}
LD(v.f) &\land \neg v \rightarrow \top \iff \neg \text{false} \land \neg v \rightarrow \top \iff \neg \text{false} \land \neg v \rightarrow \top \iff \neg \text{false} \\
ST(v.f) &\land \neg v \rightarrow \top \iff \neg \text{false} \land \neg v \rightarrow \top \iff \neg \text{false} \land \neg v \rightarrow \top \iff \neg \text{false}
\end{align*}

After that, memory accesses are replayed over the remaining satisfiable paths. Given a sequence of ST/LD/DL memory predicates updating on the same field of a pointer, it first sorts them based on the ordering (i.e., $k$) numbers and then performs the following normalization in the increasing order of the $k$ numbers. For a load $LD(x.f,y,k)$, it binds $y$ to the variable corresponding to the field $f_i$ of the points-to predicate $x$:

\[
\exists \bar{w}.\bar{\kappa} \land x \rightarrow c(f_1:v_1, ... , f_n:v_n) \land \neg x \rightarrow \top \land LD(x.f_i,y,k) \land \pi \\
\iff \exists \bar{w}.\bar{\kappa} \land x \rightarrow c(f_1:v_1, ... , f_n:v_n) \land \neg x \rightarrow \top \land ST(x.f_i,y,k) \land \pi \\
\iff \exists \bar{w}.\bar{\kappa} \land x \rightarrow c(f_1:v_1, ... , f_n:v_n) \land \neg x \rightarrow \top \land \pi \iff \exists \bar{w}.\bar{\kappa} \land \pi.
\]

If the points-to predicate $x \rightarrow e(\cdot)$ does not exist, either $x \rightarrow \top \land \pi = 1$ or $x = \text{null} \land \pi = 1$ must exist and hence, $\Delta$ must have been marked as closed already. To eliminate $ST(x.f_i,y,k)$, we generate a fresh variable $v_i'$ corresponding to the field $f_1$ of the points-to predicate $x$:

\[
\exists \bar{w}.\bar{\kappa} \land x \rightarrow c(f_1:v_1, ... , f_n:v_n) \land \neg x \rightarrow \top \land LD(x.f_i,y,k) \land \pi \\
\iff \exists \bar{w}.\bar{\kappa} \land x \rightarrow c(f_1:v_1, ... , f_n:v_n) \land \neg x \rightarrow \top \land ST(x.f_i,y,k) \land \pi \\
\iff \exists \bar{w}.\bar{\kappa} \land x \rightarrow c(f_1:v_1, ... , f_n:v_n) \land \neg x \rightarrow \top \land \pi \iff \exists \bar{w}.\bar{\kappa} \land \pi.
\]

**Base Case**  Given an unfolding tree $T$, $\forall a \rightarrow 1$ discharges every base formula at open leaf nodes. Let $B$ be a base formula. $\forall a \rightarrow 1$ transforms $B$ into a SMT formula through a reduction function, called $\epsilon \text{XPure}$. We emphasize, applying any of the reductions presented in [1-50, 83-75, 52] which do not take dangling pointers into account would obtain an over-approximated abstraction. Let $B$ be $\exists \bar{w}.\bar{\kappa} \land \pi_{\text{ptr}} \land \pi_a$ where $\pi_{\text{ptr}}$ is a conjunction of equalities and dangling predicates over pointer-typed variables and $\pi_a$ is an arithmetical constraint. Let $\bar{v}$ be the set of variables in $B$. We assume that all these variables are sorted. And $M(\bar{v})$ is a unique integer variable corresponding to $\bar{v}$, $\pi \equiv \epsilon \text{XPure}(B)$ is computed as: (1) if $\bar{v} \rightarrow c(\cdot) \in \kappa$ or $\bar{v} \rightarrow \top \in \pi_{\text{ptr}}$, then $M(\bar{v}) = i \in \pi$; (2) if $\bar{v} = \text{null} \in \pi_{\text{ptr}}$, then $M(\bar{v}) = 0 \in \pi$; (3) if $\bar{v} \rightarrow c(\cdot) \ast \bar{v} \rightarrow c(\cdot) \in \kappa$, then $M(\bar{v}) \neq M(\bar{v}) \in \pi$; (4) if $\bar{v} \rightarrow c(\cdot) \in \kappa$ and $\bar{v} \rightarrow \top \in \pi_{\text{ptr}}$, then $M(\bar{v}) \neq M(\bar{v}) \in \pi$; (5) if $\bar{v} = \bar{v} \in \pi_{\text{ptr}}$, then $\bar{v} = \bar{v} \in \pi$; (6) asserts non-null heap addresses; (3) and (4) capture the semantics of the separation of heap locations; (5) and (6) forwards the constraints on the stack holding by $B$. By induction, we show the following lemma.

**Lemma 1.**  Given $s$, we have $s \models \epsilon \text{XPure}(B)$ if and only if there exists $h$, such that $s, h \models B$.

As $\epsilon \text{XPure}(B)$ is in the Presburger arithmetic, satisfiability of $\epsilon \text{XPure}(B)$ is decidable. Furthermore, $\epsilon \text{XPure}(B)$ can be discharged efficiently by an SMT solver.
Table 1. Experimental Results

| Tool       | RQ1: Verification | RQ2: Falsification | RQ3: Overall |
|------------|-------------------|--------------------|--------------|
|            | s | X | t | p | t | s | X | t | p | t | s | X | t | p | t | s | X | t | p | t | s | X |
| SeaHorn [34] | 59 | 1 | 0 | 4 | -58 | 14m37s | 57 | 0 | 1 | 57 | 4m8s | -1 | 18m45s |
| Cascade [47]  | 0 | 0 | 58 | 16 | 0 | 48m45s | 8 | 0 | 50 | 0 | 8 | 13m5s | 8 | 61m50s |
| PredatorHP [45] | 39 | 1 | 16 | 18 | 62 | 64m36s | 38 | 0 | 5 | 15 | 38 | 28m39s | 100 | 95m15s |
| ESBMC [22]   | 39 | 1 | 0 | 34 | 62 | 112m40s | 55 | 0 | 0 | 3 | 55 | 11m33s | 117 | 124m14s |
| CBMC [41]    | 32 | 0 | 1 | 41 | 64 | 130m20s | 54 | 0 | 4 | 54 | 17m56s | 118 | 148m16s |
| Smack-Corral [35] | 44 | 1 | 0 | 29 | 72 | 93m48s | 50 | 0 | 0 | 6 | 50 | 31m48s | 122 | 125m36s |
| CPA-Seq1.9 [11] | 38 | 0 | 13 | 23 | 76 | 434m47s | 42 | 0 | 3 | 13 | 42 | 55m6s | 128 | 489m53s |
| UAutomizer [36] | 49 | 0 | 4 | 21 | 98 | 83m20s | 47 | 0 | 0 | 11 | 47 | 44m20s | 145 | 127m40s |
| CPA-Seq1.4 [7]  | 54 | 0 | 4 | 10 | 108 | 51m41s | 47 | 0 | 1 | 10 | 47 | 25m36s | 155 | 77m17s |
| S2TD        | 59 | 0 | 4 | 11 | 118 | 37m31s | 55 | 0 | 1 | 2 | 55 | 7m31s | 173 | 45m25s |

### Cyclic Proofs

Function \( \text{link}_{\text{back}} \) constructs back-links as follows. For every open leaf node \( \Delta_{\text{bud}} \), \( \text{link}_{\text{back}} \) checks whether there exists an interior node \( \Delta_{\text{comp}} \) such that after some weakening, \( \Delta_{\text{bud}} \) is subsumed by \( \Delta_{\text{comp}} \) and all base formulas in the subtree rooted by \( \Delta_{\text{comp}} \) are unsatisfiable. Particularly, for global infinitary soundness [16,50], \( \text{link}_{\text{back}} \) only considers those \( \Delta_{\text{bud}} \) and \( \Delta_{\text{comp}} \) of the following form.

\[
\Delta_{\text{comp}} \equiv \exists \bar{v}_1, \kappa_{b_1} \cdot P_1(\bar{t}_1)^{n_1}_{b_1} \land \pi_1 \\
\Delta_{\text{bud}} \equiv \exists \bar{v}_2, \kappa_{b_2} \cdot P_1(\bar{t}_1)^{n_2}_{b_2} \land \pi_2
\]

where \( k \geq 1 \), \( n > m \), \( \kappa_{b_1} \) and \( \kappa_{b_2} \) are base formulas. The constraint \( n > m \) implies at least one inductive predicate is unfolded. In particular, function \( \text{link}_{\text{back}}(\Delta_{\text{bud}}, \Delta_{\text{comp}}) \) is implemented as follows. It finds a substitution \( \theta \) such that: (a) for every \( x_j \mapsto c_j(\bar{v}_j) \in \kappa_{b_1} \), there exists \( x'_j \mapsto c_j(\bar{v}_j) \in \kappa_{b_2} \) and \( x_j \mapsto c_j(\bar{v}_j) \equiv x'_j \mapsto c_j(\bar{v}_j) \theta \); (b) similarly, every occurrence of inductive predicates in the companion is linked; (c) \( \pi_2 \theta \Rightarrow \pi_1 \) holds.

### 6 Implementation and Evaluation

S2TD is programmed in OCaml. It uses Z3 [23] for checking satisfiability of first-order logic formulae produced by procedure expure for the base cases. Moreover, the analyzer LFP uses FixCalc [65] to compute an over-approximating invariant of an inductive predicate to prune unsatisfiable nodes in advance via function aS.

In the presence of an erroneous trace, S2TD produces a counterexample annotated with a stack record of a program path \( \xi \) leading to the error. From the counterexample, we extract the error location and generate error witnesses [5] in GraphXML [12]. In the rest of this section, we conduct experiments to answer three research questions (RQ):

- RQ1: Does S2TD verify programs correctly?
- RQ2: Does S2TD falsify program properties correctly?
- RQ3: Is S2TD more effective and efficient than existing tools?

### Experimental setup

We conduct the experiments on a set of challenging programs. Our test subjects include 132 C programs taken from the SV-COMP benchmark repository [4], i.e., all 98 programs in the Recursive category, 24 programs manipulating nested lists and binary trees in the Heap Data Structures category, and 10 generated
by us. Among the 132 programs, 74 of them are safe whereas 58 are erroneous. The programs in the first category implement recursive algorithms like the towers of Hanoi, Fibonacci’s numbers and McCarthy91. The programs in the last category manipulate complex data structures (e.g., parallel lists, reversely doubly-linked list or tll trees), which are yet to be included in [4] due to their complexity. Each program contains at least one loop and/or one recursive method call, and has at least one pure assertion. We set the upper bound of the unfolding number of all predicates in S2TD to 28, i.e., after 28 unfoldings, if the error/safe decision has not been made, $S_{2eul}$ returns $\text{UNKNOWN}$.

We compare our verification system S2TD with the state-of-the-art verification tools participated in the SV-COMP competition including PredatorHP [45], SeaHorn [34], Cascade [74], ESBMC [22], UAutomizer [36], CPA-Seq [7], CBMC [21] and Smack-Corral [35]. Among the tools, SeaHorn is also a CHC-based verification system. PredatorHP runs several instances of Predator [25] in parallel and composes the results into a final verification verdict. Cascade and ESBMC are based on Bounded Model Checking (BMC) and were designed for falsification [74,68,55]. Ultimate Automizer [36] is a model checker based on automata. CPA-Seq [7] builds and refines reachability trees combining multiple techniques like abstract interpretation, predicate abstraction, and shape analysis. Smack-Corral [35] translates LLVM compiler’s intermediate representation into SMT constraints. For CBMC, CPA-Seq1.9, ESBMC, PredatorHP and Ultimate Automizer, we apply the binaries and settings used in Sv-comp 2020 [24]. For the remaining tools, we apply the binaries and settings used in Sv-comp 2016 [4]. All experiments were performed on a machine with an Intel i7-6500U (2.5GHz) 2-processor and 8 GB RAM. The timeout is set to 180 seconds.

**Experimental Results** The results are reported in Table 1. The 1st column shows the verification systems. The rest shows results in relation to the three research questions.

**RQ1** To answer this question, we experiment on 74 safe programs. The first two columns in this part show the number of correct safe ($s\sqrt{\text{\checkmark}}$) and incorrect safe (a.k.a. false positives) ($s\check{\times}$). The next two columns show the number of unknown (unk column) and either timeout or crashes (to column). We rank these tools based on their points (pts column). Following the same rules applied in the SV-COMP competition, we gave $+2$ for one $s\sqrt{\text{\checkmark}}$, $-16$ for one $s\check{\times}$ and 0 for either unknown or timeout or crash. The last column shows the total wall time. The results show that SeaHorn and S2TD produced the highest number of correct safe. However, SeaHorn returned 11 incorrect answers, 9 of which are programs in the second category. This incompleteness is due to the lack of a precise encoding model for complex data structures. PredatorHP, ESBMC and Smack-Corral reported one wrong outcome each. Cascade did not return any incorrect results but it also did not produce any correct ones. S2TD could not decide 15 programs, it returned either $\text{UNKNOWN}$ or timeout, 12 of which are in the first category and 3 are in the second one. These programs contain mutual recursions, constraints outside the linear arithmetic, or non-periodic recurrent relations.

**RQ2** To answer this question, we have experimented on 58 erroneous programs. Columns in this part are similar to that of the safe programs, except that the first two columns show the number of correct error ($e\sqrt{\text{\checkmark}}$) and false negatives ($e\check{\times}$). Following the same
rules applied in the SV-COMP competition, we gave +1 for one e√ and -32 for one e✗. The results show that CHC-based systems performed the best while SeaHorn produced the highest number of correct safe with the shortest running time. Nevertheless, none returned a false negative. S2TD returned 3 timeouts for programs requiring a very large number of unfoldings. For instance, one of them is to compute the Fibonacci number where the input is 25 and requires to invoke 242785 recursive calls. This number of calls is beyond both our algorithm bound and running bound.

RQ3 To answer this question, we sum the points and running time of tools in the previous two parts and report in the respective two columns in the third part of Table 1. The results show that S2TD is the most effective and ranks second in terms of efficiency. Cascade, UAutomizer, CPA-Seq and S2TD did not report any incorrect outcomes. SeaHorn is the fastest in all two categories; however, it was the least in effectiveness. PredatorHP is the third fastest but reported a high number of false positives. These experimental results imply that our proposal, a combination of abstract interpretation, model checking and cyclic proof for CHC, is promising as S2TD is effective and efficient.

7 Related Work and Conclusion

Closest to our work are CHC-based verification systems, e.g. Duality [54], HSF [32,31], SeaHorn [34] and μZ [37]. Details on these systems based on CHC were summarized in [9,8]. As mentioned above, these systems focus on numerical and array programs whereas we target heap-manipulating programs. Our work also relates to cyclic proof verification systems [13,72,69]. While they support only safety verification (i.e. the absence of bugs), ours supports reasoning about both safety and the presence of bugs.

Verification systems support for both verifying and falsifying safety properties include property-directed reachability (PDR) [40,43], interpolation [1], and separation logic [319,64,60,42]. PDR is a sound and relatively complete framework for shape analysis using abstraction predicates. Smallfoot [319] presents foundations in separation logic. GRASShopper [64] and Asterix [60] encode the verification conditions of heap-manipulating programs into an SMT supported form. Our work shows, at the first time, how to employ cyclic proofs for verifying and falsifying the safety properties.

Safety verification systems based on abstract interpretation have been developed for verification (not falsification), e.g. [20,76,25,38,47,51]. These systems over-approximate input programs to verify the absence of errors and may produce false positives. There are related techniques supporting loop-based programs, such as PDR and interpolation (mentioned earlier), loop invariant generation [59,27,53], loop precondition inference using cyclic proofs [15], k-induction based techniques [116]. In contrast to these, we transform the source-code-based loops/recursions into logic-based inductive predicates and rely on a decision procedure to search for cyclic proofs to prune the subsumed execution paths for both safety verification and falsification.

Compare to the symbolic execution for separation logic [3,20,49,56,62,63,61], our system distinguishes itself at the free command. Without the dangling predicate, the operation semantics of this command in these systems is: \{ x \rightarrow c(\_1) \} \text{free } x \{ \text{emp} \}. 
That means, these systems over-approximate the post-state and thus they may produce false positives. We notice the incorrectness separation logic presented in [66] recently which also makes use of negative heaps to reasoning about the presence of bugs. Similar to ours, the operation semantics of this command in this work is: \( \{ x \rightarrow c(v) \} \). The main difference is that as the proposal in [66] is to fix the overapproximation of the frame rule, their negative heap predicate is spatial based. As we use the dangling predicate to assert the guard condition of the heaps, we treat them as pure constraints. Moreover, while the engines in [3,20,56] reduces the compositional verification problem into the validity of entailments, the engine in [62,63,61] generates test inputs from pre-conditions to witness the reachability in bounded programs, our engine is to transform the verification problem into the reachability problem using CHC.

Our solver (S2eSL) relates to decision procedures for satisfiability problems in separation logic [3,5,7,14,64,60,33,75,50,71,52,48,46,40]. The procedure in [48] derived cyclic proofs for word equation problem and [46] proposed to infer summary of inductive predicates for compositional satisfiability solving. Compared with S2SATSL [50], S2eSL additionally supports dangling pointers and memory load/store/del predicates. The decision procedure in [52] reduces the satisfiability problem of an inductive predicate into the satisfiability problem of its numeric part, which requires a separation between the spatial and numeric parts. In contrast to [52] and similarly to [50], our procedure creates back-links based on the weakening and subsumption. Despite of their power, the decision procedures in [52] can not be applied into this work as the flow-sensitiveness forbids the separation of the two domains.

Conclusion We have presented S2TD, a CHC-based verification system using a novel extension of separation logic. Given a C program, our system transforms it into CHC and invokes decision procedure S2eSL to symbolically execute and to discharge these CHC. S2eSL detects bugs with feasible paths, or constructs a cyclic proof to prune subsumed safe paths (so as to prove bug-free programs). We have implemented a prototype and evaluated it on a set of heap-manipulating programs. The experimental results shows that CHC-based approach is a promising approach to software verification for both the absence and the presence of bugs.

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Appendix A  Encoding CHC (Cont)

\[ \textsc{assume} \]
\[ \rho \circ \{ v'/v \mid v \in FV(\pi) \} \]
\[ \text{exec}(\Delta;l, \text{assume } \pi) \sim (\Delta \rho \land \pi);l \]
\[ \text{exec}(\Delta;l, e_1) \sim \Delta_1;l_1 \]
\[ \text{exec}(\Delta_1;l_1, e_2) \sim \Delta_2;l_2 \]
\[ \text{exec}(\Delta;l, \text{if } v \text{ then } e_1 \text{ else } e_2) \sim \Delta_1;l_1 \]
\[ \text{exec}(\Delta;l, \text{if } v \text{ then } e_1 \text{ else } e_2) \sim \Delta_2;l_2 \]

The encoding steps for \textsc{assume}, \text{sequencing}, and \text{if} commands are shown above.

Appendix B  Soundness and Termination

Given a program defined in our core language shown in Figure 4, the program has an error (either \text{assume}(\varepsilon=1) \text{ is reachable or a heap-manipulation is violated}) if and only if there is a satisfiable error formula; and the program has no error if and only if there is a cyclic proof for the generated CHC.

\textbf{Theorem 1.} Given a core program \( P \), (a) if S2TD returns a bug, then there is some input to \( P \) that leads to an error; (b) if S2TD terminates without a bug then there is no input that leads to a null pointer error or double free error; (c) otherwise, S2TD will run forever.

The correctness of the above Theorem relies on \( S2e_{\text{SL}} \). In the following, we analyze the soundness of the solver. Inspired by [50], \( S2e_{\text{SL}} \) is designed for deciding a sound and complete base theory \( L \) augmented with inductive predicates. Furthermore, the \textit{base theory} \( L \) must satisfy the following properties: (i) \( L \) is closed under propositional combination and supports boolean variables; (ii) there exists a complete decision procedure for \( L \). In this work, the theory \( L \) is the fragment of base formulas. The soundness and completeness of \( L \) is shown by Lemma [1].

\textbf{Lemma 2.} \( S2e_{\text{SL}} \) is sound when it returns either yes or no.

This lemma follows Lemma [1] and the soundness of cyclic proofs.
Termination As the number of procedures/loops in a program is finite, our encoding is always terminating. Therefore, S2TD terminates iff \(S2_{\text{at}}\) terminates. In the following, we show that \(S2_{\text{at}}\) terminates in the universal fragment \(\text{SLPA}_{\text{ind+}}\) that is an extension of the decidable fragment presented in [50] with dangling predicates and the heap-manipulation simulation predicates (LD, ST, and DEL).

\(\text{SLPA}_{\text{ind+}}\) is a fragment of separation logic which supports all the syntax shown in Figure 11 except that those definitions of inductive predicates are restricted as follows. An inductive predicate \(P(i)\) is in \(\text{SLPA}_{\text{ind+}}\) if it is in the following form (\(\bar{v}_i \subseteq \bar{w}\) for all \(i \in \{1 \ldots \}\)):

\[
\begin{align*}
\text{pred } P(i) & \equiv B_0 \lor \exists \bar{w}_i. \bigwedge_{i=1}^n \exists x_i \in c_i(d_i) \ast \ast \, \bar{h}_i \ast \bigwedge_{\bar{v}_i \in \bar{w}_i} \bar{v}_i \cdot P(\bar{v}_i) \land \pi_r
\end{align*}
\]

where \(B_0\) is a base formula, and \(\pi_r\) is a conjunction of ordering arithmetical constraints each of which is of the form: either an arithmetical constraint over data fields \(d_i\) or \(v_1 \leq v_2 + k\) or \(v_1 \geq v_2 + k\), \(k\) is an integer number, \(v_1 \in \bar{u} \cup \bigcup_{i=1}^n \{d_i\} \) and \(v_1 \in \bigcup_{i=1}^n \{\bar{v}_i\}\).

Theorem 2. \(S2_{\text{at}}\) terminates for \(\text{SLPA}_{\text{ind+}}\).

To show that \(S2_{\text{at}}\) always terminates for \(\text{SLPA}_{\text{ind+}}\), we essentially prove that for each leaf (respectively a bud) of the unfolding tree derived for an unsatisfiable formula if it has not been classified as no yet, it must be linked back to an interior node (respectively a corresponding companion) to form a cyclic proof in finite steps. We remark that in the following, we only consider companions which are descendant of a bud. We recap that a bud is linked to a companion if free pointer variables of points-to predicates and predicate instances are exhaustively matched and fixed points of all numeric projections of buds are computable. In addition, the function \(\text{init}_{\text{back}_{\text{at}}}\) does not rely on constraints over data fields. We use \(\pi_{df}\) to denote the constraint on variables of data fields of points-to predicates. We show \(S2_{\text{at}}\) always terminates for \(\text{SLPA}_{\text{ind+}}\) through three steps. First, we show that the termination of \(S2_{\text{at}}\) does not rely on the pure constraints over data fields of points-to predicates.

Lemma 3. Given a bud \(\Delta_{\text{bud}} \equiv \exists \bar{w}_2 \cdot \Delta_2 \land \pi_{df}^{\bar{w}_2}\) if there exists a companion \(\Delta_{\text{comp}} \equiv \exists \bar{w}_1 \cdot \Delta_1 \land \pi_{df}^{\bar{w}_1}\) such that the subformula \(\exists \bar{w}_2 \cdot \Delta_2\) can be linked back to the subformula \(\exists \bar{w}_1 \cdot \Delta_1\), then \(\Delta_{\text{bud}}\) can be linked back to \(\Delta_{\text{comp}}\).

Proof. We assume \(\exists \bar{w}_2 \cdot \Delta_2\) can be linked back to \(\exists \bar{w}_1 \cdot \Delta_1\). As so, the heap in bud can be weakened and matched with the heap in the companion with some substitutions \(\rho\). And \(\pi_{df}^{\bar{w}_2}\) after weakened and substituted to become \(\pi_{df}^{\bar{w}_1}\), at the last step we need to prove \(\pi_{df}^{\bar{w}_2} \implies \pi_{df}^{\bar{w}_1}\). Suppose that \(\pi_{df}^{\bar{w}_2} \equiv \pi_{df}^{\bar{w}_1} \land \pi_{df}^{\bar{w}_1}\) where \(\text{FV}(\pi_{df}^{\bar{w}_1}) \subseteq w_2\) and \(\text{FV}(\pi_{df}^{\bar{w}_1})\) are observable variables. Similarly, suppose that \(\pi_{df}^{\bar{w}_1} \equiv \pi_{df}^{\bar{w}_1} \land \pi_{df}^{\bar{w}_1}\) where \(\text{FV}(\pi_{df}^{\bar{w}_1}) \subseteq w_1\) and \(\text{FV}(\pi_{df}^{\bar{w}_1})\) are observable variables.

As \(\Delta_{\text{comp}}\) is still open (it is not unsatisfiable) and \(\pi_{df}^{\bar{w}_1}\) is in a decidable fragment, there exists \(s\) such that \(s \models \pi_{df}^{\bar{w}_1}\). This implies that there exists \(s_1\) such that \(s_1 \subseteq F_1\), \(s_1 \models \exists \bar{w}_1 \cdot \pi_{df}^{\bar{w}_1}\). This means true \(\implies \exists \bar{w}_1 \cdot \pi_{df}^{\bar{w}_1}\) (1).

Note if \(\Delta_{\text{comp}}\) is a descendant of \(\Delta_{\text{bud}}, \pi_{df}^{\bar{w}_2}\) must be of the form \(\pi_{df}^{\bar{w}_2} \equiv \pi_{df}^{\bar{w}_1} \land \pi_{df}^{\bar{w}_1} \land \ldots \land \pi_{df}^{\bar{w}_2}\).

Thus, \(\pi_{df}^{\bar{w}_2} \implies \pi_{df}^{\bar{w}_1}\).

From (1) and (2), we have \(\pi_{df}^{\bar{w}_2} \implies \pi_{df}^{\bar{w}_1}\). \qed
Secondly, we prove that $S2e_{SL}$ terminates for formulae including spatial-based inductive predicates, those predicates whose parameters are pointer-typed. The quantifier elimination of function $\text{sat}(B)$ implies that the satisfiability problem in this fragment does not rely on existentially quantified variables. In other words, this problem is based on free variables. As the number of these free variables in a formula is finite, checking satisfiability in this fragment is decidable.

**Lemma 4.** $S2e_{SL}$ terminates for a system of spatial-based inductive predicates.

**Proof.** In the following, we use $\bar{v}$ to denote a sequence of variables. Furthermore, we use $v_i$ to denote the $i^{th}$ variable in the sequence $\bar{v}$. Two sequence $\bar{v}$ and $\bar{t}$ have the same sequence of observable variables if for all valid position $k$ then $v_k$ is an observable variable or null and $t_k=v_k$.

Suppose we are constructing a proof for the input $\Delta_0$ where $\Delta_0=\exists \bar{w} \cdot B \cdot P_1(v_1) \ldots * P_N(v_N)$ and $\Delta_0$ has $m$ observable variables. We are computing the longest path from $\Delta_0$ to a leaf $\Delta_{bud}$ such that $\Delta_{bud}$ can be linked back to $\Delta_0$. In the worst case, this back-link is established if (i) all points-to predicates are linked, (ii) inductive predicates are linked and (iii) (dis)equalities constraints between them are identical. The complexity result is computed based on the following three facts.

1. There are $O(2^m)$ possibilities of the spatial conjunction of points-to predicates.
2. There are $O(2^m)$ possibilities of sequence of parameters of each inductive. And we have $O(N)$ inductive predicates.
3. The number of equality conjunctions over $m$ observable variables (including null) is $2m^2$. Thus, there are $O(2^{2m^2})$ possibilities of equalities.

Thus, the longest path of an unfolding tree derived for $\Delta_0$ is: $O(2^m \times 2^{2m^2} \times (2^m)^N)$.

Lastly, we show that $S2e_{SL}$ always terminates for formulas which contain inductive predicates with shape, pure constraints over data fields and periodic relations over arithmetical parameters. We show that given a formula in $SLPA_{ind}^+$, $S2e_{SL}$ can always construct a cyclic proof in finite time.

The main theorem is shown based on the following.

1. The termination does not rely on those constraints on data fields of points-to predicates (lemma 3).
2. $S2e_{SL}$ runs in $O(2^m \times 2^{2m^2} \times (2^m)^N)$ to link shape part of a leaf to one of its descendant (lemma 4).
3. We now show the implication checking on pure formulas always hold after shape parts have been linked. For simplicity, we consider one arithmetical parameter $n$ of one inductive definition with periodic relation $R$. Let $\pi_n$ be the pure formula of the input relevant to $n$, $S$ be the periodic set of $R$ and $inv_n$ be its corresponding Presburger formula. There are two following subcases:
   - $\pi_n \land \nabla_n \implies \text{false}$. This case is detected by over-approximation step.
   - Otherwise, $\pi_n \land \nabla_n$ is satisfied. We notice that $\pi_n \land \nabla_n$ must imply $\nabla_n$. In a path, the arithmetical part of a $SLPA_{ind}^+$ bud is a disjunct of the unfolding from its descendant. The relation $R$ is well-founded relation. If $R$ is an increasing
relation, applying $R$ over $\pi_n \land \nabla_n$ will be bounded by $\nabla_n$. Thus, $S_2e_{SL}$ can link the pure parts in finite time. If $R$ is a decreasing relation, the periodic set $S'$ derived for this disjunct is a subset of the set derived for its descendant. Thus, the implication at the last step of function $\text{link\_back}_{e_{SL}}$ always holds. That is, $\text{link\_back}_{e_{SL}}$ can always link the arithmetical part of such above leaf to its descendant nodes. Hence, $S_2e_{SL}$ always terminates.