Conceptual understanding and productive disposition in trigonometry through generative learning

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Abstract. This article aims to analyse how conceptual understanding and productive dispositions in trigonometry through generative learning, using a qualitative research design. Participants of this study were 20 students of the level 1 Mathematics Education Study Program of the University of Swadaya Gunung Jati, who took the Intermediate Selecta Capita course and received trigonometry material. The data analysed through 3 stages, namely data reduction, data display, and conclusion drawing. This study indicates that the student's conceptual understanding can represent the information obtained from the problems given; it is just that when performing algebraic operations still encounter obstacles. Students' constraints in performing algebraic operations on trigonometry can be due to a lack of understanding of the concepts in algebraic operation material. Confidence in studying and providing answers to student problems shows that students' productive disposition is still not optimal.

1. Introduction

In solving a problem in life, there is no application of mathematics in various fields. Because it requires someone's mathematical ability to organize mathematics so that problems can be solved. To learn mathematics successfully, it is necessary to have a term that covers all aspects of mathematics expertise, competence, knowledge, and facilities. Mathematics has an essential role in developing proficiency in mastering mathematical problems and understanding a concept. Conceptual understanding can build a student's cognitive framework so that students' understanding can be optimal [1]. The limited definition of conceptual understanding that is minimal will reflect a broader understanding of conceptual understanding [2].

Trigonometry material is a material that is generally associated with everyday life. However, there are still many concepts from trigonometry that are not understood by students, and there are still students who confused concepts in the use of trigonometry comparison formulas. The trigonometry concept is widely used as prerequisite material such as integral limits, three dimensions, and so on. If the understanding of this concept is not maximal, it will not be easy to learn material related to trigonometry concepts. It is necessary to apply a constructive learning model to support conceptual understanding and productive dispositions in students studying trigonometry. Hakim [3] states that the generative learning model is a learning model that refers to a constructivist philosophy that seeks an active role for students and focuses on motivation, initial conceptions, and learning experiences. In solved trigonometric problems related to triangular angles and trigonometric functions, it is necessary to understand the concept of productive dispositions. Based on the explanation above, it is necessary to have an in-depth
analysis of conceptual understanding and productive dispositions in trigonometry through generative learning.

Conceptual understanding is very important for applying and adapting mathematical ideas into new situations. Applying and adapting mathematical ideas is built by new knowledge and then linked to previous knowledge through conceptual understanding that students have. Students' knowledge is integrating from mathematical ideas that are more than just facts, methods, and procedures [4]. Conceptual understanding according to Laswadi [5] is 1) being able to connect mathematical concepts, 2) able to represent mathematical situations in various ways, 3) knowing about the representation that is suitable for a particular situation. Meaningful learning strategies are an important part of fostering conceptual understanding and student achievement [4]. Students with a good understanding of concepts will get used to and understand the meaning of mathematical language to solve the given mathematical problems. Thus, a conceptual understanding student can organize his knowledge and explain it as a coherent system.

The National Research Council includes productive dispositions as a key component of math proficiency [6]. Productive disposition is the belief and attitude to see mathematics as something that makes sense, is useful, and valuable. It forms the concept of belief in students' self-abilities and believes that persistence will lead to successful and active mathematics learning [6]. Meanwhile, students with productive dispositions view mathematics as a connected conceptual system that can be understood with persistence and effort [7]. This is where mathematics teachers have an important role in encouraging students to maintain a positive attitude towards mathematics, through the way teachers view mathematics and practice the learning process [8]. So that teachers must have a positive disposition to be able to have a positive influence on students [9]. Students with good productive dispositions will play an active role in learning [10], because they have confidence in solving math problems. A person with a productive mathematical disposition will not avoid mathematics because he has natural talent and can deal with difficult math problems [11].

The facts that occur show that few students still have low abilities in solving math problems and are less active in learning; learning strategies are expected to affect the learning process. The learning strategy is a conceptual framework that describes a systematic procedure for achieving learning goals. Learning is a productive activity, where learning actively builds meaning from information that must be learned and mentally rearranges and integrates with existing knowledge [12]. Understanding the material applied in new situations is a form of cognitive processing. One approach that promises to improve metacognitive learning and learning outcomes is encouraging students to engage in generative learning strategies [13]. It is in line with the statement [14] that generative learning strategies such as summarizing, self-explaining, and self-testing can be used to improve learning outcomes.

According to the Wittrock model (generative learning), meaningful learning consists of four main components: generator, motivation, attention, and memory. Generating refers to the relationship between the knowledge learned and the knowledge that students will learn. Motivation refers to a student's desire to understand a lesson. Attention refers to the process of relevant student knowledge, and memory refers to students' knowledge, experience, and beliefs about the material. Emphasizing the role of students as active learners, making the Wittrock model a form of learning with meaningful learning theory that is influential, relevant, and provides an introduction to learning methods and learning strategies that aim to improve student understanding. Generative learning strategies are considered a process where teachers actively involve students in thinking, reasoning, guessing, understanding, and applying mathematics through reading, copying, creating, constructing, and solving problems [15]. This study aimed to analyze in depth how the conceptual understanding and productive dispositions of students on trigonometric material through generative learning.

2. Methods
This study used a qualitative research method; 20 participants were Level 1 students in the Mathematics Education Study Program at the Gunung Jati Swadaya University. Participants are students who take the Kapita Selekte Intermediate course and get trigonometry material. During the Covid 19 virus
pandemic, this research took place by learning an online system using a Learning Management System (LMS) and still applying a generative learning model without reducing the research’s purpose. To find a picture of how students’ conceptual understanding and productive disposition is done through tests with trigonometry material, including 1) trigonometric comparisons on right triangles, 2) trigonometric comparisons for related angles, 3) simplifying trigonometric shapes and proving trigonometric identities. Conceptual understanding of data collection techniques in this study through the study of documentation in documents trigonometric test results. Data on the productive disposition is obtained through questionnaires and interviews through discussion forums during online learning.

Adopting a research framework, according to Abdalla, Oliveira, Azevedo & Gonzalez [16], the data were analyzed through 3 stages, namely data reduction, data display, and conclusion drawing. The data reduction stage was carried out by examining the conceptual understanding indicator on the test document while checking the productive disposition through the questionnaire results and interview results. The data display stage is carried out to represent each indicator of conceptual understanding and productive disposition. The conclusion stage is the giving of meaning descriptively to the analyzed test and interview documents.

3. Results and Discussion

Capita Selekta lectures are given through generative learning with stages 1) exploration, 2) focusing, 3) introduction to concepts, and 4) application to be actively involved in lecture-discussion forums. In the exploration, stage students are directed to explore knowledge by being given a stimulus in the form of activities or assignments to problems related to the concept of trigonometry. In the focusing stage, students are formed in groups to practice cooperating, help friends, respect friends’ opinions, exchange ideas, and dare to ask questions. The concept introduction stage by giving students practice exercises so that students better understand steadily the concept of trigonometry in the last stage, which is the application stage, students are asked to solve problems with the trigonometric concepts they have. The three trigonometric problems were analyzed in-depth about how conceptual understanding and productive disposition in trigonometry. The problem is given in Figure 1.

In the first problem given, there is some understanding of students’ concepts seen from the way they represent; in Figure 2 (a), students understand the concept of trigonometric comparisons on right triangles. They are demonstrated by students’ ability to connect and represent the information provided in the picture and be able to solve problems correctly. In Figure 2 (b), it can be seen that students are good enough to connect the problem with the idea of first determining the value of $\tan 37^\circ = \frac{3}{4}$ used to determine the length of $BC$ side by using $\tan \angle BDC$.

Figure 2 shows the lack of students’ conceptual understanding of the concept of trigonometric comparisons on right triangles. Students are wrong in connecting and representing the information given in the picture; the value of $\tan 37^\circ = \frac{3}{4}$ means that 3 and 4 are the length of the $\triangle BCD$ side, so that new trigonometric comparisons can be made without $\tan 53^\circ = \frac{x}{4}$ where $x$ is the length of the side in question.
Conceptual understanding refers to the understanding that enables students to learn new ideas by connecting them with their knowledge. Following the National Research Council [15], the knowledge learned with understanding will provide the basis for generating new knowledge and solving new problems. From the results of tests conducted on 20 students, researchers conducted data reduction to obtain test results that could be analyzed following conceptual understanding indicators. Errors in connecting and representing the information provided, resulting in students unable to solve the problems provided.

In the second problem given, most students make mistakes in connecting and representing, this is because they do not understand the concept of trigonometric comparisons for related angles. Figure 4 (a) shows students are not able to represent the information provided, so the value of $\cos \alpha = \frac{-3}{\sqrt{13}}$ they are looking big $\angle \alpha$. Once known $\angle \alpha = 48^\circ$, substituted into the problem given. Furthermore, in figures 4 (b) and 4 (c), students are able to use the concept of trigonometric comparisons for related angles, but in completion they lack understanding of the concept of algebraic operations.

The third problem was that most students could connect and represent the trigonometric equations given and prove the trigonometric identity had no difficulty. In Figure 5 (a), the student changes tan, cot, "and" cosec to the inverse form of these angles then performs algebraic operations so that the problems given prove to be the same between the left and right sides. Unlike the case in Figure 5 (b), which shows students only change the trigonometric form of the denominator on the left side, then perform algebraic operations so that the equations for the left and right segments are proven to be the same. The process carried out by students in Figure 5 (a) and Figure 5 (b) clearly illustrates that the understanding of student concepts is perfect, as evidenced by the fact that students can connect and represent the information obtained to solve problems smoothly.

In Figure 5 (c), students can represent the information obtained from the problems given; it is just that when doing algebra operations, they still encounter problems. Students' constraints in performing algebraic operations on trigonometry can be due to a lack of understanding of the concepts in algebraic operation material.
Figure 4. Mistaken conceptual understanding of problem 2.

Figure 5. Mistaken conceptual understanding of problem 3.

Students who show productive disposition will pay attention to mathematics in real life and apply mathematics. When students build strategic competencies in learning, their attitudes and beliefs will be more positive in learning mathematics. From the questionnaire results, 14 out of 20 students had confidence that they had studied trigonometry material very well, even though at first, they experienced problems with the online learning process. Online learning is still carried out using generative learning models. With stages 1) exploration, 2) focusing, 3) introduction of concepts, and 4) application to be actively involved in lecture-discussion forums. Based on the discussions conducted online, the researchers saw that not all students actively provided comments on the focusing stage questions. To find out more about why this could happen, the researchers conducted telephone interviews with active and less active students in the discussion.
The following are the results of conversations with students who were active in the discussion forum:
*Researcher*: Can you take part in trigonometric learning?
*Student 1*: Yes, trigonometry material is easy to understand, although sometimes it stops in some complex questions
*Researcher*: When discussing in lectures, can you express your opinion even though you don't know your opinion is right or wrong?
*Student 1*: Yes I always try to comment on the problem being discussed, as long as I understand the concept.
*Researcher*: Do you feel challenged when you are asked about trigonometry?
*Student 1*: yes, each question has a different way of solving and also a different level of difficulty, so that is a challenge for me.
*Researcher*: Do you feel confident that you can solve the trigonometry questions given during the exam?
*Student 1*: Yes, because I feel I already understand the concepts given in the lecture.

It can be seen that when a student understands the concept of the given trigonometry material, he will be very confident of being able to solve the problems given about trigonometry.

The following are the results of conversations with students who are not active in discussion forums:
*Researcher*: Can you take part in trigonometric learning?
*Student 1*: No, because I am confused to understand trigonometry material online.
*Researcher*: When discussing in lectures, can you express your opinion even though you don't know your opinion is right or wrong?
*Student 1*: No, because I don't know what I want to ask and answer.
*Researcher*: Do you feel challenged when you are asked about trigonometry?
*Student 1*: I'm not very sure.
*Researcher*: Do you feel confident that you can solve the trigonometry questions given during the exam?
*Student 1*: I'm still not so sure because I'm a person who forgets.

The limited understanding of the material makes students reluctant to be active in discussions, not only that, students are not sure that they can solve any trigonometric problems that are given.

4. Conclusion
In solving the problems given, most students give answers following the understanding of the concepts they have. However, because learning is done online in full, there are limitations in transferring material to understand students' concepts is not optimal. Conceptual understanding of conceptual been able to represent information obtained from the problems given, it's just that when performing algebraic operations, still encounter obstacles. The obstacles experienced by students in performing algebraic operations on trigonometry, can be due to the lack of understanding of algebraic operating material. Confidence in learning and giving answers to students' problems shows that students' productive disposition is still not optimal.

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