Discussion on the Cosmological Vacuum Energy

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Abstract

The present discussion contribution is some remarks concerning and review of the proposal by one of us to explain the cosmological constant by a/the principle of entropy. Used without further comment this principle of entropy could easily lead to untrustable nonlocalities, but taking into account that the long range correlations are rather to be understood as due to initial condition set up the model for the cosmological constant being small by one of us becomes quite viable.
1 Introduction

In a recent paper one of us [1] proposed to explain the smallness of the cosmological vacuum energy based on the energy limit that general relativity imposes on any given volume. According to Einstein’s theory the maximal amount of energy in a 3-dimensional volume scales with the linear size rather than the volume itself. This energy limit can be equivalently formulated as an upper limit on the entropy contained in the volume, which latter is known as the holographic entropy bound.[2] The entropy bound is well known, for example, for Schwarzschild black holes: the entropy contained within the horizon cannot exceed the quarter of the surface area (in Planck units).

>From this entropy, or equivalently energy, bound it follows that there must exist an ultraviolet cut-off for fields in the region inside any volume [1]. Simply put: the entropy (energy) in the given volume cannot exceed the maximal entropy (energy) of a black hole that fills the given volume. With such a cut-off the zero point energy of any fields, that is the vacuum energy, inside any volume becomes restricted. This in turn limits the cosmological vacuum energy which otherwise would contribute to dark energy (or the cosmological constant) an enormous amount. Such a restriction is not only suitable to reduce dark energy (cosmological constant), but as explained in Ref. [1] it will ensure that the theoretical prediction of the cosmological vacuum energy will exactly match that of the experimentally measured value.

However, it is non-trivial to understand how such an energy cut-off is implemented in quantum field theory. The most naive implementation of such a cut-off would be to restrict the individual degrees of freedom within a given volume independently not to exceed their average energy. Unfortunately, this cut-off would lead to an energy limit that scales with the volume rather than the linear size. Moreover, in case of the cosmological vacuum energy this is experimentally excluded since locally we observe higher energy density systems in our Universe than its average dark energy density. Thus the cut-off has to vary and has to be non-trivially correlated between degrees of freedom of the Universe. This would allow for the existence of small regions with high energy densities while the rest could compensate such that the average never exceeds the ratio of the maximal allowed energy and the volume.

But this sort of cut-off raises another question: How can such a correlated cut-off be consistent with the locality of quantum field theory? If no signal propagates faster than the speed of light, can potentially distant parts of the volume compensate for each other? Naturally, whether such a cut-off is consistent with locality depends on the details of the implementation of the cut-off itself. Most importantly, how the long range correlation between the allowed energy in one region depends on what goes on or what is allowed in an other region. But if the cut-off at one region depends on what goes on at an other, potentially distant, region at the same time, then locality can definitely get into trouble! So it can at best be a tolerable correlation of the cut-off at one locality with what went on somewhat earlier around the Universe in remote regions, otherwise causality is threatened.

To implement such a cut-off while saving locality, one may think in two different ways:

a) One could accept that locality is not necessarily a good principle and the solution necessitates “new physics”. But then one is up to theories like the “complex action theory”

1See Ref. [1] for detailed literature on the subject.
proposed by Ninomiya and the other one of the present authors (HBN).

b) An alternative solution is the use of some cosmic censorship assumption such as the non-existence of "white holes", that is time-reversed black holes. Such an assumption is needed anyway, to maintain the entropy bound [?].

It appears that in quantum field theory the entropy bound holds only if either

- the cut-off is strangely correlated between the degrees of freedom, as suggested by HBN, or

- the limitation of the number of states is not just a limitation due to the cut-off of the theory but due e.g. to some special initial condition. And as an example of the latter - one of us would say more reasonable type of state limitation for the application in question - the cosmic censorship comes in.

In Section 2 we discuss the problems related to the argument that the cosmological vacuum energy is limited by the entropy bound. In Section 3 we put forward an idea of how a cut-off based on the entropy bound could be interpreted or replaced by a cosmic censorship based philosophy. This latter could, at least in a certain sense, be free of the problems with locality or causality. Finally we conclude and look out in the conclusion section [4].

2 Trouble for the Entropy Principle

In a physical system obeying the laws of thermodynamics the extensive thermodynamical variables, such as energy and entropy, typically scale with the volume containing the system. Since in quantum systems the energy, in turn, typically scales with the number of degrees of freedom, the latter is usually thought to grow with the volume. Considering a system of fields defined within a volume, without any special restrictions on the degrees of freedom, it is clear that the number of states can grow as an exponential of the volume. In field theory in any local region of space the energy density can reach that of the highest energy accelerators and beyond, and the number of degrees of freedom in a given volume is unlimited. Because of this field theory does not respect the Bekenstein-Hawking entropy bound or the Schwarzschild energy limit. As we saw before, the entropy bound as a naive, uncorrelated cut-off on any given volume is out of question in field theory. The entropy and energy bounds can only be consistent with field theory, they can only allow reaching energy densities well tested in science and in daily life, if the corresponding cut-off is highly correlated between the degrees of freedom. Thus the nature of the cut-off is such that it imposes a strong restriction on the allowed states.

About a decade ago Bousso extended the Bekenstein-Hawking entropy bound into a covariant entropy conjecture [2]. While Bousso sharpened the definition of the interior of the surface in which the entropy is limited by the enclosing area, in his derivation he also made the crucial assumption that there should be no singularities in the interior in question. (Cf. page 9 of [2].) Bousso also pointed out that "Because the conjecture is manifestly time reversal invariant, its origin cannot be thermodynamic, but must be statistical. It thus places a fundamental limit on the number of degrees of freedom in nature."
In case of black holes, for example, the non-singularity assumption appears to be an obvious necessity. Otherwise one can imagine white holes channeling entropy into the volume that is independent of the surface area of the black hole. This could easily violate the entropy bound.

2.1 Time reversal thinking on the number of states problem

It is natural to get an idea of our problem for the “entropy principle” by thinking for a moment in the time reversal way:

If we think about how one of the to a volume-behaving entropy enourmously many states could have come about, we may produce the answer by thinking time-reversed: What would happen if we started with a typical state taken out of the situation with a volume proportional entropy, and reversed the Hubble expansion to be a Hubble contraction. That would mean a situation with a very high energy density over a very large extension and would of course correspond to a world that were already to be considered inside a black hole. Also it would have already so much entropy that it would be too much for a/one black hole. Rather what such a system would develop into would be many many black holes. As such a collapsing universe with a lot of energy density develops the energy density gets even bigger and after some time there will be many relatively small subregions which have both too much energy and too much entropy to avoid being black holes. So at some stage it would develop into an approximately smooth distribution of “small” black holes. This would mean a kind of piecewise collaps - even before the naively calculated total collaps, when the general size of this universe would go to zero. One could say that this in naive sense calculated collaps due to the radius going to zero never gets realized, because the piecewise collaps into seperate black holes takes over effectively and forms a collaps at an earlier stage.

Now time reversing this scenario back to the real world, its means that the majority of the to the volume behaving entropy corresponding states are of such a nature, that they could only be formed from an earlier stage of the Universe containing enourmously many “small” “white holes” rather than comming from a genuine Big Bang or other single or few singularity picture as usual cosmology tells.

Since the “white holes” - meaning as we just used here the time reversed black holes - are precisely the most important example of what a cosmic censorship principle should forbid, it is clear that the majority of states in the volume-based entropy scenario are cosmic censorship forbidden states. In this way there is at least the hope that it is the cosmic censorship that can bring the number of states down to match the Bekenstein-Hawking-area law.

3 Can we Rescue the Cosmological Constant Derivation?

At first there might seem to be a chance to rescue the work of one of us on deriviving the cosmological constant being small derivation by just saying:
Taken as simply an ultraviolet cut off straight away it looks dangerous for locality with the correlations in the cut off needed to make the cut off match the entropy principle. However, if we now interprete the cut off as mainly due to the initial conditions occurring due to say cosmic censorship requirements it would sound much more acceptable, since it would no longer even threaden the locality in a genuine sense. You cannot really in a world, which has come from a development, in which a cosmic censorship principle were valid, send messages faster than light or the like. At least we have no experimental evidence against that we should live in a world with no white holes (unless though perhaps Big Bang itself should be considered a white hole or a cosmic censorship violating event). So a cut off considered due to a cosmic censorship would seemingly not be against what we could believe. However, then the problem would be: The main job of the cut off which we should obtain due to the entropy principle were to limit the zero-point energy of the quantized field theory of the world, say of the Standard Model. At first one would think that the cosmic censorship and other agents that could influence the initial state would not really influence the zero-point energy! One would say this because when we think of initial conditions caused by such influences as cosmic censorship or from inflations and development whatever, then one has in mind that all those high frequency modes which are at a certain time not excited by onshell particles will fluctuate nevertheless “peacefully” in their zero-point fluctuation way. We so to speak normally imagine that the zero point fluctuations for the high frequency modes just are there as in vacuum for all the frequencies higher than the ones relevant for the state being realized. In this philosophy the initial state and thus the cosmic censorship would not get true access to influence the cutting off of the high frequency modes. If we cannot get the cosmic censorship influence the higher frequencies zero mode fluctuation of course the above discussion and proposal to use cosmic censorship would not help.

3.1 But could initial state effects possibly influence zero-point fluctuations?

But now really the question is: Shall we take it for a good argument that zero-point oscillations of very high frequencies are organized to be present as soon as we reach temperatures where they are no longer excited? At first one would again say: yes, it is reasonable that the high frequency modes would fall to their zero point fluctuation level but no longer as the Universe expands with a very strong Hubble expansion and effectively the excitation of a mode is moved from one mode to a lower one due to this expansion. The zero point oscillation cannot be reduced by the Hubble expansion and the higher frequency modes would seemingly have to stay in their zero point fluctuation.

But are we not more and more dreaming about a phantacy world of high frequencies which never according to the entropy principle should even have a chance to be realized? If one turned the philosophy a bit around one would say: We have this phantastic dream of there existing a number of possible states of the Universe system which is the number of states corresponding to an entropy going with the volume of the Universe, but on the other we know from entropy principle or essentially equivalently from the cosmic censorship that it is only a very tiny minority of these states that have a true chance to be realized. In a way it would be most sensible if in an ontological way only the states that have at least
the chance corresponding to the entropy principle would exist in a sense of being present in the most fundamental theory. But if that were so then all these phantasy states making up the to volume proportional entropy ought not to be there. That would of course make it more strange to worry about the zero point energy involved in the many high energy modes which can essentially ever be excited, or at least almost never. It should namely be had in mind that at colliders like LHC we do excite very high modes which are normally - i.e. in the almost empty universe - never excited.

Shall we really imagine that in the fundamental theory at the ontological level which we shall may be once find the degrees of freedom relevant for the LHC are in some strange way being built up from some degrees of freedom that at first looked like being made for a bit microwaves in a low frequency passing far behind the Moon? Shall we really imagine that ontologically at the end the degrees of freedom are being shuffled around so that when the LHC needs some more degrees of freedom it collects them up from perhaps big distances away? Although it sounds a great challenge to construct just a model showing that such an idea is possible in a local way, it may not be totally excluded since either a clever way may be found or nature might at the root of it not respect our usually expected principles of locality.

4 Conclusion

We have discussed some problems with the model of one of us solving the cosmological constant problem - of the surprisingly small size of the cosmological constant found experimentally - by using the entropy principle (of the entropy only going as the surrounding area). The major problem is really a problem with the entropy principle rather than only with the proposed solution to the cosmological constant. You namely cannot interpret the entropy principle at all as a restriction given on the number of states as due to some conventional cut off. So either you must say that the entropy principle has nothing to do with the number of states allowed by an ultraviolet cut off - but is say a question of the initial state only (perhaps via cosmic censorship)- or we must be satisfied by an ultraviolet cut off that at least at first looks rather complicated with one would say mysterious correlations. It may be that these “mysterious correlations” could sound sensible from a speculative fundamental physics point of view.

4.1 Outlook and hope

It looks that our discussion is driving us in the direction of asking how much reality there is at the fundamental level in the zero point fluctuations of the various fields. For instance in last years discussions there were a contribution by one of us(H.B.N.) Moulaka and Nagao and Norma Mankoc Borstnic relating to the quantum mechanics philosophy going back to De Broglie. The crux of the matter is that the quantum system has a position even when it is not in a position eigenstate! Translated into field theory we might take this to mean that the fields have values even when they are not in an eigenstate field values. This is of course crazy and in disagreement with Heisenberg uncertainty principle, but for Bohm and De Broglie the philosophy is different. If we bought the theory of Bohm and
De Broglie for fields we would not have to believe that there were truly (ontologically) zero point fluctuations, but could leave it as a more difficult physical question to be answered by a deeper understanding of the more fundamental theory we are looking for. But to by such a means get the contribution to the cosmological constant from the high frequency modes be negligible as is hoped to get rid of the cosmological constant problem it would be needed that this hoped for theory behind (or at the end at the most fundamental level) would put the energy of the high frequency modes to be lower than what is possible in quantum mechanics with its minimum at the zero point energy for a harmonic oscillator. We would have to put the harmonic oscillator corresponding to the high frequency modes of the fields - for which we want to get rid of the contribution to the cosmological constant - to have both zero momentum and zero position rather exactly! For Heisenberg impossible, but for Bohm and De Broglie the question is more to be studied with more details added.

4.2 The more Private hope using the Complex Action Model

Since as the discussion above has shown the proposed model for solving the cosmological constant problem has been threadened - although not definitively killed on that ground - by locality principle. If it should at the end turn out to be indeed needed to give up such principles and for instance go to a model like the “complex action model” by one of us (H.B.N.) and Ninomiya - originally based on ideas developed by H.B.N. and Don Bennett - which were the model used in the above mentioned discussion contribution from last year by Moultake et. al. we might use such a model to suggest what should be the classical values of the high frequency fields. In other words we might now asks in the complex action model for how the in this model essentially classically standing fields behave (it means they do not respect Heisenberg in the way we ask for their fundamental values). We can almost immediately guess the answer: In the complex action model the guiding principle is that the initial conditions get set so as to minimize the imaginary part of the action. Now this must looking at world as it is mean that to have the vacuum we live in is extremely favourable to lower this imaginary part of the action. Then presumably if the “God”(having such a minimization principle arranging the quantity the imaginary part of the action $S_I$ to be minimal is almost like having a “God” in quotation marks governing the world to make “His” deficit $S_I$ as small as possible, preferably negative) behind the governing of the Universe were so keen to make so much vacuum, “He” should be even more keen to push the vacuum the last little bit by putting the fields that in our usual vacuum picture are in their zero point fluctuation states the last bit so as to have both momentum and position go to the bottom. If the imaginary part of the action $S_I$ is just a reasonably smooth function(al) of the field configurations and their conjugate momenta and it seems that the most beloved state ( by “God”, meaning giving the most favoured meaning low $S_I$) is the vacuum then if it were possible almost certainly the classical replacement for the vacuum having the fields exactly zero would have an even lower $S_I$ and thus be even more beloved! So our complex action model would indeed predict that the values of the fields - only being allowed by De Broglie and Bohm - would be so that the zero point enrgy would be killed at the fundamental level! That would as the reader can immediately understand be wonderful for the cosmological constant model we have discussed: we suggested in the last years discussion that the complex action
model could function approximately as a model behind the Bohm-De Broglie picture, and now that the prediction from complex action would then be that the vacuum fields would be - at least when not too much disturbed - be put to zero exactly (contrary to what Heisenberg uncertainty would allow, but that is O.K. in Bohm-De Broglie and in complex action interpreted the right way as being “by hindsight”, i.e. including knowledge collected by a measurement) as well as the conjugate momentum to the field modes in question.

Now using the complex action might however be an almost too high price in the sense that Ninomiya and one of us (H.B.N.) already have an article suggesting that this complex action model is good for helping with the cosmological constant problem [?]. In the kind of thinking in the articles seeking to solve cosmological constant problem in the complex action model or related models previously the philosophy were however quite a bit different in as far as in that sort of works it would rather be assumed that there is presumably very big bare cosmological constant, which simply gets adjusted by essentially the already mentioned “God” in quotation marks so as to minimize the imaginary part of the action. If “He” for some reason should want a Universe avoiding collabs but not expanding faster than necessary “He” could easily arrive to vote for a small cosmological constant. But if “He” has power to adjust the bare cosmological constant, “He” hardly need to for that reason go into adjusting zero-point-fluctuating modes, but “He” according to the above does it anyway.

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References

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