Grand Pleromal Transmutation : UV Condensates via Konsishi Anomaly, Dimensional Transmutation and Ultraminimal GUTs.

Charanjit S. Aulakh

Indian Institute of Science Education and Research Mohali,
Sector 81, S. A. S. Nagar, Manauli PO 140306, India
International Centre for Theoretical Physics,
Strada Costiera 11, 34100,Trieste, Italy

Abstract: We argue that theories with gauge Landau poles in the ultraviolet reorganise themselves into a symmetry restored confining high energy phase. This phase is characterised by the formation of physical gauge-singlet condensates that form an ineluctable background present in all the symmetry broken phases allowed in the weak coupling region below the gauge Landau pole. Using consistency requirements relating chiral condensates imposed by the so called Generalized Konishi Anomaly, we show that dimensional transmutation via gaugino condensation in the ultraviolet drives gauge symmetry breaking in a large class of asymptotically strong Supersymmetric gauge theories. For Adjoint multiplet type chiral superfields $\Phi$ (transforming as $r \times \bar{r}$ representations of a non Abelian gauge group $G$), solution of the Generalized Konishi Anomaly (GKA) equations allows calculation of quantum corrected vevs in terms of the dimensional transmutation scale $\Lambda \simeq M_X \frac{8\pi^2}{e^{\pi^2(M_X)^b_0}}$ which determines the G-singlet physical gaugino condensate. Thus the gauge coupling at the perturbative unification scale $M_X$ generates GUT symmetry breaking vevs by non-perturbative dimensional transmutation. This obviates the need for large(or any) input mass scales in the superpotential. Rank reduction can be achieved by including pairs of chiral superfields transforming as either $(Q(r), \bar{Q}(\bar{r}))$ or $(\Sigma((r \otimes r)_{symm}), \bar{\Sigma}((\bar{r} \otimes \bar{r})_{symm}),$ that form trilinear matrix gauge invariants $Q \cdot \Phi \cdot \bar{Q}, \bar{\Sigma} \cdot \Phi \cdot \Sigma$ with $\Phi$. Novel, robust and ultraminimal Grand unification algorithms emerge from the analysis. We sketch the structure of a realistic Spin(10) model, with the 16-plet of Spin(10) as the base representation $r$, which mimics the realistic Minimal Supersymmetric GUT but contains even fewer free parameters. We argue that our results point to a large extension of the dominant and normative paradigms of Asymptotic Freedom/IR colour confinement and potential driven spontaneous symmetry breaking that have long ruled gauge theories.

1aulakh@iisermohali.ac.in
2Senior Associate ICTP, 2013-2019
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1 Introduction

It has been a longstanding dream\cite{1} to provide a mechanism for dynamical generation of the Grand Unification scale from the low energy (i.e. Electro-weak) data. Asymptotic freedom(AF) of the Grand Unified gauge coupling(s) has been a generally unquestioned requirement for acceptable unification models. Some years ago, motivated by the glaring Asymptotic strength(AS) of the couplings of the phenomenologically satisfactory Minimal Supersymmetric Spin(10) GUT model (MSGUT)\cite{2,3}, we proposed\cite{4,5} that this ‘defect’ is actually a signal from the model that it generates its own UV cutoff in the form of a Landau polar scale associated with dynamical symmetry breaking of the Spin(10) gauge symmetry in the degenerate spontaneously broken gauge symmetry phases in the energy range below the scale $\Lambda_{UV}$ where gauge singlet physical(Renormalization Group(RG) invariant) gaugino condensates form due to gauge confinement in the Ultraviolet. Inspired by their defining role in our proposal for robust parameter counting ultra-minimal AS Grand Unification we called such condensates pleromal. We were particularly enthused by the observation that -in contrast to, say, Susy QCD-the nearly exact Supersymmetry at the GUT scale implies that the UV dynamics of the MSGUT, and other AS Susy GUTs, are physically the best justified and most realistic context for the use of the powerful methods \cite{6} for analysing strongly coupled Susy theories.

Due to the novelty of this scenario that seeks to resolve the paradoxes of AS theories that are IR-free but run to strong gauge coupling in the UV, we first sketch our Renormalization Group(RG) based picture of the phase structure of the G-invariant AS gauge theory. We assume-in common with standard Susy GUT scenarios like the Georgi-Dimopoulos model\cite{7}- that Supersymmetric Gaugemodels with Chiral Higgs multiplets can exist in a number of degenerate spontaneously broken(or not) phases at energies around the scale ($M_X$) of convergence of the distinct gauge couplings in their low energy effective theories (which retain light modes only). However, in strong contrast to AF theories, the phase structure of AS theories which possess gauge Landau poles in the UV, often within an order of magnitude above $M_X$, must obviously be completely different. We do not agree with the common attitude that assumes such theories must be inconsistent and buries its head in the sand of a blind taboo against UV strong gauge coupling. It is a broadly applicable scientific truism that in Nature there are no true infinities, but only naive idealisations. The UV gauge Landau poles should thus signal that a phase transition takes place and a new phase described by new field variables and gauge couplings comes into play. The canonical example of QCD and its Susy variants is naturally first to hand and serves admirably as a template. On that basis we expect that at and beyond the gauge Landau pole, the G-coloured particles (i.e. described by gauge variant fields) leave the physical spectrum because their masses run to infinity and a new set of degrees of freedom analogous to the colour singlet Mesons, Baryons, glueballs and various physical condensates of QCD will be required to describe the behaviour of the condensed gauge singlet phase which will form in the strong coupling region. In fact such candidate effective fields have long been identified in Susy YM theories \cite{6,12} as being holomorphic gauge
invariants formed from the Chiral fields present in theory: which are known to parametrize the D-flat moduli space [8] and are thus appropriate to describe the supersymmetric vacua. One expects that a supersymmetric Sigma model involving these gauge singlet but ‘t hooft anomaly matched ‘chiral moduli’ fields describes the UV phase[5]. Be that as it may, the gauge singlet, and therefore automatically symmetry restored, UV phase is not the subject of the investigations in this paper beyond the fact that, again in direct analogy with Susy QCD, we expect gauge invariant, physical, chiral condensates to form in this phase. Once formed, being physical, we expect them to be a physical background that all other phases of the theory must be consistent with. In particular models based on full renormalizable Susy YM Higgs theories(i.e. with all massive modes retained), which are described by gauge variant degrees of freedom but G-invariant Lagrangians, are associated with the various degenerate symmetry broken phases existent at scales below the UV condensation scale. These models must all obey the infinite network of constraints between Susy vacuum condensates first identified in the work of Konishi and Shizuya [9] and then greatly expanded in the work of Cachazo, Douglas, Seiberg and Witten[10] and thereafter. In the symmetry broken phase the G-singlet gaugino condensates can naturally fractionate into different components following the breakup of the gauge generators into the little group H and the coset \( G/H \) such that the total gaugino condensate is still the physical G-singlet condensate formed by the strong UV dynamics. This is in obvious contrast with the symmetry restored phase where any such break up would be meaningless and self contradictory. Thus it should be clear that we do not and cannot advocate that the UV phase itself discriminates among the various types of gauginos as far as their condensates go.

In [5] a toy AS GUT model with gauge group \( SU(2) \) and a single symmetric chiral 5-plet was used to explore the derivation of symmetry breaking vevs from the gauge singlet physical gaugino condensate. The use of the Konishi Anomaly (KA) [9] allowed us to argue that a vev driven by the overall gauge singlet gaugino condensate might well develop. Shortly thereafter, a sophisticated and powerful method based on the Generalized Konishi anomaly (GKA) was invented[10] by Cachazo, Douglas, Seiberg and Witten(CDSW) which allows the fully quantum and non-perturbative calculation of the condensates of the “Chiral Ring” generators \( tr((W_\alpha W^\alpha)^n)\Phi^m|n=0,1,m \in Z_{\geq 0} \) (where \( W_\alpha \) is the gaugino-field strength multiplet, \( \Phi \) the \( N \times \bar{N} \) adjoint multiplet of the Unitary gauge group and the trace is in the N-dimensional fundamental since the adjoint has been written as an \( N \times N \) matrix). Thereafter this method enjoyed a great vogue and was also extended [13, 14] by the addition of pairs of fundamental (“quark”) chiral multiplets leading to either Higgs or “pseudo-confining” vacua, or [15] by other sets of chiral supermultiplets, such as (anti)symmetric representations: which provide more general rank breaking scenarios with several novel and non-trivial features in their strong coupling dynamics.

In this letter, working within the framework of the RG consistent phase structure outlined above we argue that GKA techniques allow fully quantum and non-perturbative calculation of chiral vevs in our [4] asymptotically strong(AS) dynamical symmetry breaking scenario for a vast class of Susy gauge theories with chiral multiplet \( \Phi \) transforming as \( r \otimes \bar{r} \): for any representation \( r \) of any gauge group \( G \). \( \Phi \) may be supplemented by representation pairs \( \{Q, \bar{Q}\}; \{\Sigma, \bar{\Sigma}\} \) that can form singlets \( \bar{Q} \cdot \Phi^a \cdot Q, \bar{\Sigma} \cdot \Phi^a \cdot \Sigma \). Crucially,
Φ, Σ, Ξ can be written as matrices (with rows and columns labelled by indices running over the dimension \(d(r)\) of a general representation \(r\) larger than the fundamental: which we call the base representation of the model). This allows the use[10] of resolvent methods to treat whole collections of condensates in terms of a single resolvent. The basic idea of using a tensor product to define a matrix type representation can be extended to gauge groups other than Unitary groups. In particular it applies to the product of spinorial representations of \(\text{Spin}(N)\). By imposing trace constraints on the matrices Φ, Σ, Ξ that set some, or all but one, irreps contained in the direct products \(r \times \bar{r}, r \times r\) etc to zero one can project out smaller reducible or irreducible contents. However due to the increase in computational load, we defer such refinements to sequels.

The focus in the literature has been on the restricted class of Asymptotically Free(AF) i.e. IR strong models and the derivation of effective Wilsonian superpotentials to describe the (strongly coupled) low energy theory. In the GUT application case one rather wishes to know the Higgs vacuum expectation values(vevs) that must be substituted in the Lagrangian to derive the perturbative spontaneously broken GUT and its effective supersymmetric light mode theory(a.k.a “MSSM”). Thus rather than an effective superpotential for the gaugino condensate fields describing IR condensation in the unbroken subgroup \(H\) that was sought in [10] one rather needs a definition for the quantum vevs and associated “equivalent quantum superpotential” \(W^{(q)}(\Phi, \lambda, v_i^{(q)})\) \((W^{(q)})\) is a (novel) supplementary superpotential -which we introduce for a specific purpose- and is quite distinct from the initial tree level superpotential \(W(\Phi, \lambda...\)) used to define the G-invariant Lagrangian. The extrema of \(W^{(q)}\) are the quantum corrected vevs \(v_i^{(q)}\) computed by the non-perturbative GKA formalism involving solutions for, and contour integrals of, quantum resolvents i.e. by non potential extremisation methods.

On the other hand, since the gaugino condensation occurs above the UV limits of the symmetry broken phases, the issue of calculating supersymmetric condensates for the low energy gauge group is vacuous in scenarios where supersymmetry is broken at TeV scales : as is the case with most realistic Susy GUT models. Thus our focus here is to explore the generation of the GUT scale chiral vevs, in each of the possible symmetry broken phases, by non-perturbative and fully quantum dimensional transmutation, especially when the mass parameters in the superpotential are absent or negligible. We emphasise that the parameters describing (a few) irreducible gauge singlet physical condensates are to be deduced in principle from the gauge singlet dynamics of the UV symmetry restored phase. In practice, however, they are simply unknown dimensionful input parameters playing a vev determining role similar to the mass parameters of standard Susy GUT superpotentials but now within a novel interpretative and calculational framework that seeks to overcome the taboo against AS models by showing that ASGUT symmetry breaking is driven by the (gauge singlet) chiral condensates of the symmetry restored phase that is the only plausible conjecture for their UV phase.

The GKA framework initially introduces a semi-classical vev of the Chiral supermultiplet \(\Phi\) which is diagonal (to zero the D-terms) and carries the critical points \(a_i\) of the superpotential \(W(z)\) placed on the diagonal of \(\Phi\) with multiplicities \((N_i) (\sum_i N_i = N = d(r))\). Then the resolvents \(R(z), T(z)\) that code the infinite set of Chiral Supermultiplet condens-
states have poles at the critical points in the complex $z$-plane. However fully quantum GKA solution for the resolvents reveals that the true vacuum of the theory is instead determined by a branch cut structure (into which semi-classical poles split due to quantum effects) on the complex $z$-plane which needs no reference to semi-classical solutions, but only a set of $\{N_i\}$ which define the unbroken sub-group and which are dynamically invariant.

We propose that the quantum vev is given by a contour integral of $z \mathcal{T}(z)$ around the branch cuts of the “quantum superpotential derivative” $y(z)$ which is a branched function which reduces to $W'(z)$ when quantum effects are set to zero. In the semi-classical case, where the branch cuts collapse to the semiclassical critical points, this obviously yields the semi-classical vevs. In the full quantum case the analysis of [10] shows that the poles are resolved into branch cuts that form the A-cycles of a Riemann surface associated with the factorisation problem of a certain quantum corrected polynomial $(y^2(z))$ specified by the solution of the GKA equations for the resolvents. Correspondingly we introduce the above referred “equivalent quantum superpotential” $W^{(q)}(\Phi, \lambda, \psi^{(q)}_i)$, to make contact with the standard Susy GUT Lagrangian by calculation of the spectrum of the perturbative effective theory based on the quantum vevs $\psi^{(q)}_i$. $W^{(q)}$ is obtained by modifying the coefficients of the original superpotential so that the new vevs are solutions of the F-term conditions of the effective superpotential.

We assume that asymptotic strength results in physical G-singlet gaugino condensation i.e. for the gauge group $G$ as a whole, though, as we will see, in the symmetry broken phases the condensates of gaugino submultiplets lying in the decomposition of the adjoint of $G$ w.r.t. the little group $H$ are distinguishable from each other due precisely to the gauge symmetry breaking. In this work we shall outline the generic features of our proposal applicable to a large class of models with arbitrary gauge group, noting in conclusion only the principal features of its application to a realistic SO(10) model. In short, we suggest a quite novel approach to the hoary problem of GUT symmetry breaking which realises the old dream of symmetry breaking scale determination by dimensional transmutation, not in a engineered perturbative model[1, 11], but generically and robustly in an infinite class of models of a type hitherto largely neglected. Inasmuch as our scenario succeeds in presenting a plausible picture and providing useful and novel calculation techniques for analysing hybrid theories where the deep UV phase G-singlet gaugino condensates drive dynamical symmetry breaking in the lower energy phases, it should help to free AS theories from the limbo to which they have hitherto been banished by a taboo that assumes they are automatically logically inconsistent simply because no method had been found to calculate anything interesting about them. If so it will have much wider implications and ramifications for gauge theories as a whole.

In Section 2. we first review the GKA analysis of CDSW applied to a very restricted type of AF Susy YMH models with $U(N)$ gauge group and adjoint Higgs irrep. Then we discuss the generic features of the phase structure of AF and AS theories with particular attention to the residual effects of physical G-singlet strong coupling condensates in the spontaneous broken supersymmetric phases described by full (i.e. un-truncated) perturbative Susy YMH theories. Having set our assumptions we discuss how the techniques of [10] for Adjoint Multiplets extend to an infinite class of “Generalized Adjoint multiplet
type” (AM) models based on a generalized notion of base-\( r \) tensors transforming as direct products \( r \times \bar{r} \) of the gauge group \( G \). In Section 3, we give a simple example of this extension with gauge group \( SU(3) \) and base representation \( r = 6 \) and illustrate calculations with some numerical results. In Section 4, we discuss how rank breaking may be implemented by introducing additional \( (r, \bar{r}) \) or \( (r \times r, \bar{r} \times \bar{r}) \) pairs of Chiral supermultiplets whose vevs reduce the rank of the little group \( H \). In Section 5, we outline a realistic \( Spin(10) \) GUT model with base representation is the 16 dimensional chiral spinor of \( Spin(10) \). In Section 6, we discuss our results and the outlook for further work from a general viewpoint.

2 Generalized Adjoint models

2.1 Generalized Konishi Anomaly and CDSW formalism for matrix form representations

We begin with a short summary of the basic work [10] on which we rely for our calculations. This work confines itself to consideration of the standard Asymptotically free Super Yang Mills theories which condense in the Infrared. In a supersymmetry preserving vacuum of a super-Yang Mills theory with gauge group \( G \) coupled to a Chiral multiplet \( \Phi \) in an arbitrary representation \( R \) of the gauge group, and with superpotential \( W(\Phi) \), the GKA implies[10, 15, 17] the following relation for condensates of chiral gauge invariants formed from the Gaugino-Field strength Weyl spinor chiral multiplet \( W^A_\alpha, A = 1...dim(G), \alpha = 1, 2 \) and the chiral fields \( \Phi_I, I = 1...dim(R) \):

\[
\langle f_I \partial W \partial \Phi_I \rangle = -\frac{1}{32\pi^2}(W^A_\alpha W^{\alpha B} M^{A J}_I M^{B K}_J \partial f(W_\alpha, \Phi)_K) ; \quad W^{\alpha A} M^{A J}_I \Phi_J = 0 \quad (2.1)
\]

Here \( f(W_\alpha, \Phi)_I \) is an arbitrary chiral variation of the field \( \Phi \) in the representation \( R \) with generators \( M^A \). Repeated indices \( I, J, K \) are summed over \( dim(R) \) values. The important constraint equation whereby the matrix \( W_\alpha \) acting on the “vector form” of the general representation \( \Phi \) is equivalent to zero in the “Chiral Ring” [10] is frequently used in simplifying expressions for chiral expectation values. Henceforth, we drop angular brackets to indicate expectation values of operators since that is all we ever consider.

For the case where \( \Phi \) transforms as the traceful adjoint of \( U(N) \) the method of [10] allowed a complete solution for the generators of the Chiral ring of gauge invariants \( t_{n,m} \sim tr((W_\alpha W^\alpha)^n\Phi^m)(n = 0, 1; m \in Z_{\geq 0} \) ). The solution proceeds by solving for the generating functions of the Chiral Ring generators defined as the resolvents \( tr(W_\alpha W^\alpha)^n(z - \Phi)^{-1} \). These have obvious expansions as power series, valid for large \( z \), whose coefficients are the Chiral Ring Generators \( t_{n,m} \). In [13, 14] the extension to the case with additional (“quark”) superfield pairs \( Q, \bar{Q} \) which can form a singlet with \( \Phi \) as \( Q \cdot \Phi^m \cdot \bar{Q} \) and in [15] the similar case of the Adjoint with conjugate pairs of (anti)symmetric representations \( (\Sigma, \Sigma) \) are resolved. These additional models allow consideration of rank breaking supersymmetric Higgs vacua...
not available with just an adjoint. Thus

\[ R(z) = \kappa \text{tr}(W_\alpha W^\alpha (z - \Phi))^{-1} = \sum_{n=0}^{\infty} \frac{R_n}{z^{n+1}} = \kappa \sum_{n=0}^{\infty} \frac{\text{tr}(\Phi^n W_\alpha W^\alpha)}{z^{n+1}} \equiv \sum_{n=0}^{\infty} R_n z^{n+1} = \kappa \sum_{n=0}^{\infty} \text{tr}(\Phi^n W_\alpha W^\alpha) z^{n+1} \]

\[ T(z) = \text{tr}(z - \Phi)^{-1} = \sum_{n=0}^{\infty} \frac{T_n}{z^{n+1}} \quad ; \quad \kappa = -\frac{1}{32\pi^2} \quad (2.2) \]

\( \text{tr} \) is taken so that the matrix indices run over 1..\( d(r) = N \). The semi classical vacuum of the model is defined by distributing the critical values \( a_i(i = 1..n) W'(a_i) = 0 \) of the superpotential function \( W(z) \) (degree \( n + 1 \)) over the diagonal slots of \( \Phi \) and setting the off-diagonal elements to zero (this ensures minimisation of the D-term contributions to the potential via \( D^A(\Phi, \Phi^\dagger) \sim [\Phi, \Phi^\dagger] = 0 \)). One can extract various interesting quantities, such as the number \( N_i \) of times a critical point \( a_i \) is repeated or the value of gauge invariant chiral ring generators, via integrals of \( z^m \{ R(z), T(z) \} \) etc. around suitable contours \( C_i \).

### 2.2 Renormalization Group and vacuum condensates

Let us begin with the standard RG picture of IR-strong Susy theories used in the GKA formalism for adjoint representation based models described above. Firstly the unbroken symmetry is set already at high energies where the gauge couplings of the little group \( H = \Pi_i U(N_i) \) left unbroken after \( U(N) \to H \) breaking are still perturbative. \( H \) is defined by choosing a set of integers \( \{ N_i, i = 1..n; \sum_{i=1}^{n} N_i = d(r) = N \} \). In the semiclassical limit the \( N_i \) refer to the repetition numbers of the semi-classical critical points \( a_i \) on the \( \Phi \) vev diagonal. However the deep insight provided by the GKA analysis is that the vacuum structure of the fully quantum theory is controlled by branch cuts which may be viewed as the result of bifurcations of the semi-classical critical points \( a_i \) due to quantum corrections. However, more generally, the branch-cut network has a physical meaning in its own right since the determination of the ‘quantum superpotential derivative’ which defines the branch cuts need make no reference to the semi-classical critical points, although, of course, the branch points do coalesce into the semi-classical critical points when the quantum corrections coded in the driving condensates are set to zero by hand. In the IR the non-Abelian gauge couplings become strong and eventually gaugino condensates \( < S_i > \sim W_\alpha A_i W^{\alpha A_i}, A_i = 1..N_i^2 - 1 \) form in each of the non-Abelian sub-sectors.

The matching of the full \( G = U(N) \) theory to the effective theory of the light modes corresponding to the little gauge group \( H \) proceeds along the same lines as the standard AF GUT scenarios. However to handle the difficulties of interpretation of AS models - which have led to this far larger class of models to be obscured by a taboo that considers them intrinsically inconsistent - we need a distinct scenario with a consistent viewpoint regarding the extreme UV phase of the full G-invariant gauge theory. Our position on this issue - follows the maxim that “Nature abhors infinities”. Inconsistencies that arise from extreme idealisations that are too naive or insufficiently imaginative from the point of view of the actual dynamical possibilities of strongly coupled and complex systems are tamed by some novel \( \text{modus naturalis} \). Thus we argue that the growth of the gauge coupling (and thus also of gauge boson masses in the full spontaneously broken but G-invariant theory)
ultimately results in a symmetry restored phase with confined G-colour exactly analogous to QCD in its confining phase.

There are two types of RG behaviour in the standard paradigm, corresponding to the QCD and Electroweak sectors of the Standard Model. The asymptotically free (for not too many quarks) QCD coupling has a Landau Pole in the Infrared at a physical Renormalization Group invariant scale \( \Lambda_{QCD} \approx M_Z \exp \left[ \frac{2 \pi}{b_0 \alpha_3(M_Z)} \right] \). The presence of the Landau pole at \( \Lambda_{QCD} \sim 200 \text{ MeV} \) is interpreted as due to a phase transition whereby the coloured degrees of freedom are no longer appropriate variables to describe physics at energy scales below \( \Lambda_{QCD} \). Rather one must use a description in terms of the colour singlet degrees of freedom corresponding to the observed colourless hadrons (Mesons, Baryons etc).

The quantum vacuum (from study of non-local correlation functions using Operator Product expansion techniques) is known to be filled with various physical symmetry preserving condensates such as quark-antiquark \( < \bar{q}q > \sim \Lambda_{QCD}^3 \), and field strength squared \( < \alpha_3 G_{\mu\nu}^A G^{A\mu\nu} > \sim \Lambda_{QCD}^4 \). Since the right hand side is an RG invariant quantity these condensates must be viewed as real i.e. as present at all scales. Above scales of a few times \( \Lambda_{QCD} \), perturbative QCD (pQCD) with its coloured quark and gluon fields is an increasingly appropriate description of the physics. This is not to say that the (RG invariant) condensates vanish but only that their effects at high energies \( E \) are suppressed by the small value of \( \Lambda_{QCD}/E \). The theory can thus prima facie be defined in terms of the coloured degrees of freedom at any scale from (a few times) \( \Lambda_{QCD} \) up to \( M_{Planck} \) (in which range the QCD coupling is perturbative) and by colour singlet degrees of freedom below \( \Lambda_{QCD} \).

The spontaneously broken Electroweak sector Lagrangian has the full \( SU(2)_L \times U(1)_Y \) gauge symmetry even if it appears in a hidden form due to the choice of a vacuum expectation value to specify the perturbation procedure. The masses of the \( W^\pm \), \( Z_0 \) gauge bosons are parameters of the full broken symmetry theory which is renormalizable and fully consistent and exhibits calculable RG flows for all couplings and masses even in the presence of spontaneous symmetry breaking. The number of gauge couplings is unchanged in the spontaneously broken phase\[18, 19\]. In the electroweak sector, just as in QED, the full spontaneously broken gauge theory is renormalizable and can be used to compute processes at all energies up to possible Landau poles in the far UV. If one chooses, at energies well below the masses of the gauge bosons one may, for simplicity and convenience, use an effective theory that retains a truncated set of light degrees of freedom obtained by integrating out the heavy fields. This yields also non-renormalizable effective (e.g. dimension 6 current-current) operators suppressed by powers of \( E/V_{EW} \). It is crucial that the gauge couplings and non renormalizable operator coefficients in the effective Lagrangian are obtained by matching the effective couplings to the couplings of the full theory which thus determines the low energy effective theory in every crucial detail.

Next consider the question of supersymmetric Yang Mills theory and, in particular, Super-QCD without quarks. In this case the gauge bosons and their superpartners (gluinos) are described by a Chiral spinor superfield \( (W^A_\alpha) \). The model is asymptotically free and thus still has an IR Landau pole and associated RG-invariant scale \( \Lambda_{SQCD} \). In the seminal work of Veneziano and Yankielowicz (VY) \[12\] the issue of possible vacuum condensates consistent with the expected unbroken supersymmetry was examined. They argued that,
in contrast to QCD, no condensates (like $\alpha_3 < G^A_{\mu\nu}G^{A\mu\nu} \sim \Lambda_{QCD}^4$) leading to a contribution to the vacuum energy are permitted by unbroken supersymmetry. On the other hand a condensate for the gaugino bilinear $< \lambda^A_\alpha \lambda^{A\alpha} > \sim \Lambda^3$ would not give such Susy breaking contributions and would have all required properties. They proposed an effective Lagrangian implementing the supermultiplet of anomalies known to be operative in the theory in terms of a single gauge singlet Chiral superfield $\mathcal{S} \sim tr(W_\alpha W^\alpha)$ containing the gaugino condensate as the lowest (scalar) component of the supermultiplet. The auxiliary field $M$ of the Chiral supermultiplet $\mathcal{S} = (\phi, \chi, M)$ contains $G^2, G\bar{G}$ and the absence of kinetic terms for the auxiliary field $M$ in the VY Lagrange density $\sim [(\mathcal{S}\mathcal{S}^*)^{(1/3)}]_D + ..$ implies the absence of condensates for $G^2, G\bar{G}$. On the other hand the F-terms in the effective Lagrangian which correctly code all the anomalies (Chiral, scale etc) of the theory are of form $\sim [S(1 - ln(S/\Lambda^3))]_F + h.c.$ and thus predict $<\mathcal{S}> \sim \Lambda^3$. Just as in the QCD case this gaugino condensate is an RG invariant quantity which will be measurable in the low energy correlation functions. Moreover being physical, and in analogy with the QCD case, it would also be present in the weakly coupled, perturbative, high energy regime at energies $E >> \Lambda$ where, however, its effects would scale as $\Lambda/E$ thus be very small.

Now the Asymptotically Strong (or “un-free”) scenario we envisage is an exact complement of this standard type of reasoning applied to the relevant case of an exact or near exact spontaneously broken supersymmetric gauge theory (SBSGT) with a large gauge group (SU(5),SO(10) etc) or/and large Higgs multiplets which make the couplings of the vector bosons approach Landau poles in the ultraviolet. It is important to note that, in the full SBSGT, a renormalization and RG evolution procedure exists to allow definition of the theory at any scale up to the UV Landau poles. Of course for such a theory one may define an effective renormalizable low energy theory by dropping heavy particles and construct ‘exotic’ non-renormalizable operators whose contributions are suppressed by the heavy gauge boson masses. The crucial question for us is now: what effect does the eventual strong coupling have on the SBSGT and in particular on perturbative physics below the Landau polar scale? Notice that as long as we do not insist on asking this question but stick with the full theory at scales below the Landau poles there is no controversy about the definition of the theory. In direct analogy with the QCD case, one expects that above the Landau pole the perturbative degrees of freedom are no longer appropriate. However this trans condensation scale region, with its GUT gauge singlet degrees of freedom, is of no immediate interest to us apart from its implications for the background condensates present in the spontaneously broken phases at energies below the gauge UV Landau pole. The issue is what effect do the physical condensates we expect due to strong coupling have on the physics of the full perturbative spontaneously broken GUT gauge theory whose UV running predicted the Landau pole in first place.

The overall RG scenario we proposed already in [5] consists of a scenario for the phases in three energy regimes that need to be characterised and analysed separately and matched at their common boundaries

1. The UV strong coupled Regime I ($E >> \Lambda_U$) has full “restored but confined” G-symmetry. Wave function renormalizations and spontaneously broken gauge boson
masses become divergent as the Landau pole is approached from below. In analogy with QCD and sQCD, we expect that the gauge symmetry is confined and coloured degrees of freedom exit the physical spectrum. In analogy with AF Susy theories in their strongly coupled phases [6], the dynamics of AS theories in their strong coupled phase would naturally be formulated in terms of the ’t hooft anomaly matched gauge singlet Chiral moduli \( (X_{(n)}) \) of the fundamental \( N = 1 \) Susy Gauge model interacting according to a dynamical superpotential \( W_d(X_{(n)}, \Lambda, m, \lambda) \) (in addition to the tree superpotential of the fundamental theory which depends on the masses and couplings \( m_i, \lambda^{(n)}_i \) of the Chiral \( \Phi_i \) multiplets) which represents the strong coupling dynamics. Moreover the vacuum of this condensed phase would be characterised by the presence of a number of physical condensates (like the gaugino condensate) characterised by the scale \( \Lambda_U \). Inasmuch as these condensates are physical we expect them to remain present at all energy scales so that the other phases of the G-invariant full theory must accommodate them in their own characteristic ways.

2. In the perturbatively coupled supersymmetric intermediate Regime II \( (\Lambda_U > E > M_S) \) the system with G-invariant full Lagrangian may exist in a number of different degenerate phases in each of which the vacuum breaks the gauge G-symmetry of the Lagrangian to a different little group \( H \). According to basic renormalization theory [18, 19] each of these phases is characterised by a single gauge coupling \( g(\mu) \) that runs with the renormalization scale \( \mu \) (using a mass independent renormalization scheme such as MS,MSbar,DRED etc) with the same dimensionless gauge beta function characteristic of the gauge group \( G \) (at higher orders the dimensionless Yukawa couplings also contribute). The dimensionful couplings in each phase are distinct and run with their own beta functions. The overall Lagrangian always has the full gauge G symmetry albeit in the ‘secret symmetry’ sense where the G-variation of the vevs must also be taken into account. The chiral condensates, vevs and masses in these phases must be consistent with the physical condensates engendered by the UV symmetry restored confining phase. The Konsishi anomaly and its generalisations[9, 10] offer us a powerful technology to analyse this consistency and even -see below- to calculate fully quantum symmetry breaking chiral vevs transcending semi-classical potential based estimates. The chiral multiplet vevs calculated in this way may be coded into an “equivalent quantum superpotential” \( W^{(q)} \) whose critical points are the aforementioned fully quantum vevs. This allows us to incorporate the non-perturbative information into the standard Susy SBGT formalism used in GUTs. Note, however, that, even if we assume for simplicity that the effects of the high energy phase on the lower energy phases are confined to what we are capable of analysing using holomorphicity, there remain novel GKA constraints relating multi-point correlators of chiral fields in the GUT theory which have no precedent in standard perturbative GUTs. It remains to be seen if any experimentally testable implications of such correlators can be extracted.

3. Finally in the low energy Regime III much below the perturbative unification scale \( (E << M_X < \Lambda_{UV}) \) the gauge theories corresponding to the spontaneously broken
phases of the Susy GUT (in principle usable at all lower scales if the full spectrum of fields is retained!) each define low energy effective Susy gauge theories of light modes only. These are obtained by integrating out the heavy modes to obtain renormalizable gauge theories of the light modes corrected by exotic non-renormalizable operators in the standard way. The couplings of these effective theories are, of course, to be obtained by matching them to their respective parent spontaneously broken phases defined by fully G-symmetric Lagrangians but with specific $W^{(q)}$ coding the appropriate quantum vevs to be used for spectrum calculation.

Let us emphasise that we nowhere claim to solve the dynamics of the theory in its strongly coupled phase. We only attempt to provide a framework to resolve and even exploit the tension between the strong growth of the gauge coupling with energy above the perturbative unification scale and the apparent success of some AS models\cite{3} in fitting the observed parameters of the low energy theory with a minimal set of parameters. We do this by providing a method to calculate the large vacuum expectation values that define the full spontaneously broken GUT model not in terms of the semi-classical minimization of a fundamental scalar potential with various mass parameters but rather as a consequence of the gauge-singlet gaugino condensates expected to be present at all energy scales due to the strong coupling in the ultraviolet. These condensates are given by RG invariant estimates and are thus physical. The special techniques available in supersymmetric gauge theories allow the extraction of significant information regarding the whole system of condensates. Our aim is firstly to provide a robust underpinning for the very necessity of GUT SSB as a consequence of UV condensation. Secondly we aim to reduce the overall number of free parameters of the massive perturbative GUT model which gives rise to the effective low energy theory. On the one hand we dispense with some or even all of the GUT scale mass parameters of the Lagrangian and on the other we introduce a few additional parameters corresponding to the non-perturbative condensates present at all scales, \textit{which are not calculated by us but taken as input parameters to be scanned over}. The contour integrals around branch cuts in the complex $z$-plane (on which the resolvents $R(z), T(z)$ are defined) which are used to extract the fully quantum GUT symmetry breaking vevs in terms of the input condensate parameters (see below for details) are a far cry from the habitual semi-classical minimization of a scalar field potential. However once we have them in hand we encode them in the form of the parameters of a Spontaneously broken Supersymmetric gauge theory of the standard type specified by a “equivalent superpotential” with the dimensionless Yukawa and gauge couplings that we began with plus a new set of massive parameters containing information about the effect of the non-perturbative condensates for the perturbative GUT theory.

Due to supersymmetry and the attendant holomorphy we have a special situation that \textit{Chiral} condensates (which include expectation values of chiral scalar fields and their products) are related by the powerful infinite sets of constraints of the so called Generalized Konishi Anomaly identities involving the gaugino condensate and its generalizations. Of course we cannot calculate the strong coupling condensates whether Chiral or otherwise directly. However assuming the well motivated \textit{physical, RG invariant} gaugino condensation
at strong coupling we can relate vacuum expectation values of Chiral fields to the gauge singlet UV gaugino condensate. These vevs develop and are determined by the UV Landau polar scale even without any input mass parameters in the superpotential. This scenario self consistently assumes perturbative gauge and Yukawa couplings of the full SBSGT in Regime II. Given that our arguments determine the vevs leading to the gauge boson masses even given just these dimensionless couplings and the gaugino condensate, it is justified to claim that Dimensional transmutation has emerged robustly from the UV strong coupling dynamics. Of course, just as in any Susy GUT, a number of different breaking patterns, or even perhaps even symmetry preservation, is possible at the same vanishing supersymmetric vacuum energy. We can only use the GKA constraints to deepen our understanding of at least this limited (but crucial) set of chiral operators. Moreover even this limited set includes very non-trivial and novel expectations involving multipoint correlators of chiral fields whose implications are so far quite unclear. It is conceivable that there may actually be yet other non-holomorphic distortions of the full perturbative theory due to the strong coupling physics in the UV but, for now, we have nothing to say about such effects. In our view the physics of this quite novel scenario appears to contain no contradiction with the standard picture of Susy GUTs and yields GUT symmetry breaking with very few input parameters. It is thus well worth exploring and reporting even in spite of the many questions, in particular about non-chiral i.e. non-holomorphic operators, that undoubtedly still beg for answers.

2.3 GKA techniques for Generalized Adjoint Multiplets (GAMs)

Our basic observation is that the equations for resolvents derived via the GKA in [10] also hold for the Adjoint Multiplet type (AM) field transforming as $r \times \bar{r}$ for any representation $r$ of any simple/semisimple gauge group $G$ with the sole replacement of the trace ($\text{tr}$) in the fundamental of $SU(N)$ by the trace ($\text{Tr}$) in the representation $r$ of $G$. Note that $d(r) = N$ for the model of [10] but their results generalise easily using the expanded notion of "base-$r$ Adjoint", written as a $d(r) \times d(r)$ matrix, that transforms as $r \times \bar{r}$ rather than as $N \times \bar{N}$. In other words, under a gauge transformation

$$
\Phi' = U_{d(r)} \cdot \Phi \cdot U_{d(r)}^\dagger
$$

where $U_{d(r)}$ are $d(r) \times d(r)$ dimensional Unitary matrices in the representation $r$ of $G$. In general, $r \times \bar{r}$ contains one or more irreps of $G$, besides the singlet and the adjoint always present, and such representations generally carry large $S_2(R) \gg 3C_2(G)$. By constraining the matrix $\Phi$ so as to single out one or more irreps (which are still AS) one can work in terms of irreps of $G$. This complicates the calculations and hence we shall defer such procedures to sequels.

If we define $I[C, F(z)] \equiv \frac{1}{2\pi i} \oint dz F(z)$, then e.g.

$$
N_i = I[C_i, T(z)]
$$

$$
R_0 = I[C_\infty, R(z)] = \kappa S_2(r) W^A W^{\alpha A} = 2 S_2(r) S
$$

$$
R_n = I[C_\infty, z^n R(z)] \quad ; \quad T_k = I[C_\infty, z^k T(z)]
$$

(2.4)
Where $S$ is the gaugino condensate and $S_2(r)$ the index of the representation $r$: 

$$Tr(T^A_r T^B_r) = \delta^{AB} S_2(r).$$

It is obvious that $v_i = a_i = |C_i, z T(z)|/N_i$ extracts the vev $v_i = \Phi_{ii}$ of an $N_i$-fold replicated diagonal $\Phi$ component from the semiclassical $T(z)$ when $C_i$ encloses the critical point $a_i$. This motivates the vev definition in the quantum corrected case when $C_i$ encircles the branch cut connecting the pairs of branch points which are bifurcates of the semi-classical critical points split under the influence of quantum corrections[10].

In the presence of quantum corrections the GKA method of [10] solves for the generating functions $R(z)$ using the position independence and complete factorizability[10] of Chiral ring operator correlators to reduce the GKA (for the case where $\delta \Phi \sim (W_\alpha W^\alpha (z-\Phi)^{-1})$) to a quadratic equation for $R(z):$

$$R(z)^2 - W'(z) R(z) - \frac{1}{4} f(z) = 0 \quad (2.5)$$

where $f(z)$ is a degree $n-1$ polynomial which can be determined and $W(z)$ is just the superpotential (of degree $n+1$) as a function of $z$. Since our entire focus is on the relevance to renormalizable GUTs, we shall only consider cubic superpotentials ($W(z) = \lambda z^3/3 + m z^2/2 + \mu^2 z$). Then $f(z)$ is a linear function $f(z) = f_0 + f_1 z = -4\lambda(R_1 + z R_0) - 4m R_0$. The coefficients $R_0, R_1$ are not determined by the GKA and should be regarded as dynamical moduli of the vacuum manifold of the theory which are to be determined by an appropriate numerical investigation of gaugino condensation at strong coupling. Some information about the main contribution to $R_1/R_0^{4/3}$ can however be gleaned by surveying the GKA constraints numerically.

In the familiar case of asymptotically free Susy YM with Adjoint Chiral Higgs the gauge coupling runs to a Landau pole in the infrared at a (RG invariant) scale $\Lambda$ approximated by the one loop value (exact for the Wilsonian gauge coupling)

$$\Lambda = \mu e^{b_0/3} \quad (2.6)$$

where $b_0 = -2N$ for the adjoint-SYM. There are good arguments[12] to support the conjecture that strong coupling causes the development of a gauge singlet, RG invariant and physical gaugino condensate at this scale

$$S \equiv \frac{K}{2} W_\alpha A \nabla^A = b \Lambda^3 \quad (2.7)$$

Where the constant $b$ is scheme dependent and may be chosen to be 1[10].

Our core assumption is that physical gaugino condensation also occurs when the gauge coupling runs to a Landau pole in the ultraviolet i.e. for the case $b_0(R) = S_2(R) - 3C_2(G) > 0$. Note that $S_2(r \times \bar{r}) = 2d(r) S_2(r)$, which grows fast with $d(r)$, so that $b_0(r \times \bar{r}) > 0$ for most base representations $r$. We employ it to deduce, via the GKA relations and consistency conditions, the symmetry breaking quantum vevs of $\Phi$ that define the effective low energy theory as functions of the basic gaugino condensates and the superpotential parameters. The analysis is performed at a scale where all the degrees of freedom of the Super YMH theory are retained with the RG invariant gaugino condensates as a given
background. Scale dependent quantities such as superpotential parameters and vevs should be regarded as defined at such an intermediate scale where the gauge and superpotential couplings are still perturbative. Note that the beta function for the cubic superpotential coupling is \( \beta_\lambda = \lambda (b_0^2 - |\lambda|^2 - g^2 d(G) S_2(R) / d(R)) \) where \( b_0 \) is a positive constant. Thus the strong divergence of \( g^2 \) in the ultraviolet can drive \( d\lambda / dt \) negative. We will not here try to explore the \( \lambda \)-dependence near the Landau pole but confine ourselves to the influence of the gaugino condensate in the perturbative region.

By assumption, a G-siblinglet physical gaugino condensate \( S \) for the group as a whole develops in the ultraviolet. This gaugino condensate then requires (via the GKA and its solution) that the fields develop vevs (calculated below) which break the gauge symmetry in a manner dictated by the placement of these quantum corrected vevs \( v_i^{(q)} \) on the diagonal of the \( \Phi \) vev in any way we choose: thus defining a variety of degenerate spontaneously broken supersymmetric phases. This placement of vevs determines the little group \( H \) in practice. The different gaugino bilinears condense in a pattern determined by the vev placement. The assumption that one may choose the \( N_i \) (subject to \( d(r) = \sum_i N_i \)) to be fixed integers is an important constraint on dynamical symmetry breaking.

The solution of the GKA equation for \( R(z) \) is

\[
R(z) = \frac{1}{2} (W'(z) - \sqrt{W'(z)^2 + f(z)}) \equiv \frac{1}{2} (W'(z) - y(z))
\]

\[
y^2(z) = W'(z)^2 + f(z) ; \quad f(z) \equiv 4 \kappa \text{Tr}(W^\alpha (W'(\Phi) - W'(z))(z - \Phi)^{-1}) \quad (2.8)
\]

The GKA equation obtained for \( \delta \Phi = (z - \Phi)^{-1} \) yields

\[
(2R(z) - W'(z))T(z) - \frac{c(z)}{4} = 0
\]

\[
4 \text{Tr}(W'(\Phi) - W'(z))(z - \Phi)^{-1} \equiv c(z) \quad (2.9)
\]

Like \( f(z) \), \( c(z) \) is a polynomial of degree \( n - 1 \). Thus

\[
T(z) = -\frac{1}{4} \frac{c(z)}{\sqrt{W'(z)^2 + f(z)}} \equiv -\frac{c(z)}{4 y(z)} \quad (2.10)
\]

For a cubic superpotential \( (n = 2) \) the expansion of \( T(z) \) for large \( z \) and the above solution for \( T(z) \), yields \( c_0 = -4\lambda T_1 - 4 m d(r), c_1 = -4 \lambda d(r), T_0 \equiv d(r) \). The square root of the polynomial \( y^2(z) = W'(z)^2 + f(z) \) encodes the ‘quantum’ superpotential derivative, which is distinct from the classical one if \( R_{0,1} \neq 0 \). The higher coefficients \( R_n, T_n \) which are not present in the solutions \( y(z), T(z) \) are determined in terms of \( R_{0,1,...,n-1}, T_{0,1,...,n-1} \) by the GKA relations. For the cubic case \( (n = 2) \) the unknowns are thus \( R_{0,1}, T_1 \). The gaugino condensate \( R_0 = 2S_2(R)S = 2S_2(R) \Lambda^3 \) sets the scale of all other condensates and is estimated directly from the running of the perturbative gauge coupling in the full theory. Thus, for numerical work, we can conveniently rescale all our expectation values and integration variables to dimensionless forms using units of \( R_0^{1/3} \).
For a cubic superpotential $R(z), T(z)$ the first few dependent $R_n, T_n$ are:

\[
R_2 = -\frac{\mu^2 R_0 R_1 \lambda}{\lambda} ; \quad R_3 = \frac{1}{\lambda^2} (m \mu^2 R_0 + m^2 R_1 + R_0^2 \lambda - \mu^2 R_1 \lambda) \\
T_2 = -\frac{1}{\lambda^2} (d(r) \mu^2 m T_1) ; \quad T_3 = \frac{1}{\lambda^2} (d(r) \mu^2 + m^2 T_1 + 2 \lambda d(r) R_0 - \lambda \mu^2 T_1) \\
R_4 = \frac{1}{\lambda^3} (-m^2 \mu^2 R_0 - m^3 R_1 + \lambda \mu^4 R_0 - \lambda m R_0^2 + 2 \lambda m \mu^2 R_1 + \lambda^2 2 R_0 R_1)  \\
T_4 = \frac{1}{\lambda^3} (-d(r) m^2 \mu^2 - m^3 T_1 + \lambda d(r) \mu^4 - 2 \lambda d(r) m R_0 + 2 \lambda m \mu^2 T_1 + 2 \lambda^2 d(r) R_1 + 2 \lambda^2 R_0 T_1)  \\
(2.11)
\]

The superconformal case where $m = \mu = 0$ is particularly simple and interesting:

\[
R_2 = 0 ; \quad R_3 = \frac{R_0^2}{\lambda} ; \quad T_2 = 0 ; \quad T_3 = \frac{2 d(r) R_0}{\lambda} ; \quad R_4 = \frac{2 R_0 R_1}{\lambda} \\
R_5 = \frac{R_1^2}{\lambda} ; \quad T_4 = \frac{2 (d(r) R_1 + R_0 T_1)}{\lambda} ; \quad T_5 = \frac{2 R_1 T_1}{\lambda}  \\
(2.12)
\]

If, restoring angular brackets for a moment, we separate $\Phi(x) = \langle \Phi \rangle + \tilde{\Phi}(x)$ where $\tilde{\Phi}$ has zero vev, we see that $T_1 = T_r \langle \Phi \rangle = \sum N_i v_i^{(q)}$ but

\[
T_2 = \langle T_r \Phi(x)^2 \rangle = \langle T_r \Phi(0)^2 \rangle = T_r \langle \Phi \rangle^2 + \langle T_r (\Phi(0)^2) \rangle \geq 0 \\
\Rightarrow \sum N_i v_i^{(q)} \geq - \langle T_r (\Phi(0)^2) \rangle \\
(2.13)
\]

This sort of interplay between non-gauge invariant quantum correlators and non-gauge invariant vevs can yield gauge invariant resolvent coefficients $R_n, T_n$. Thus although we shall use vacuum expectation values deducible from $T(z)$ to calculate masses and define an effective theory with spontaneously broken gauge group at low energies, we must keep in mind that due to the strongly non-perturbative physics at high energies the GKA relations imply a host of strong constraints on higher order chiral correlators whose implications for the effective theory remain to be explained. The phenomenological implications of these constraints for correlators involving quantum fields which are known to describe light particles, like SM fields, are not clear to us. We regard this hybrid of a perturbative effective theory with strong correlations due to underlying microscopic strong coupling as one of the most puzzling and intriguing implications of our results: which may provide fresh insight on how to think about vacua arising non-perturbatively in strongly correlated systems.

We argue below that consistency of the GKA relations with the choice of $N_i$ also determines $T_1$ in terms of contour integrals involving $y(z)$ and the integer repetition numbers $N_i$ of the vevs $v_i^{(q)}$, $i = 1..n$ on the diagonal of $\Phi$ which, following [10], we assume to be free inputs stable against quantum deformation. This only leaves $R_0$ and $R_1 = \kappa T_r \Phi W_\alpha W^\alpha$ undetermined. We shall see below that the GKA equations offer insight and an estimate even for this dynamical modulus: at least in cases where the effects of the condensation are primarily encoded in the quantum vevs and gaugino condensates.

2.4 Determination of Quantum vevs $v_i^{(q)}$

The essence of the analysis of [10] in the AF case is that the influence of gaugino condensation modifies the polynomial $y^2(z)$ such that its zeros are bifurcates $a_i^{(q)}, a_i^{(q)'},$ of the
critical points \( a_i \) \((i = 1..n)\) \(|W'(a_i) = 0\). These pairs are branch points of \( y(z) \) and are connected by branch cuts. They merge when the quantum condensates \( R_0, R_1 \) are sent to 0 and \( y(z) \) reverts to its classical value \( W'(z) \). The polynomial \( y^2(x) \) defines a two-sheeted Riemann surface (of genus 1 when \( n = 2 \) since \( y^2 \) is then quartic). The contours \( C_i \) enclosing the semi-classical critical points \( a_i \) now become contours \( A_i \) enclosing branch cuts running between the corresponding pairs of branch points \( a_i^{(q)}, a_i^{(nq)} \). Thus \( \{A_i, i = 1..n\} \) become\[^{10}\] a (once redundant) basis for the A-cycles of the Riemann surface defined by \( y^2(z) \).

The consistency of these contour integrals in the quantum case involve integrals around the A-cycles:

\[
R_{0i} = -\frac{1}{4\pi i} \oint_{A_i} dz \ y(z) = -\frac{1}{2} I[A_i, y(z)]
\]

\[
R_{1i} = -\frac{1}{4\pi i} \oint_{A_i} dz \ z \ y(z) = -\frac{1}{2} I[A_i, z \ y(z)]
\]

\[
v_i^{(q)} = -\frac{1}{8\pi i N_i} \oint_{A_i} dz \ \frac{z \ c(z)}{y(z)} = -\frac{1}{4N_i} I[A_i, z \ \frac{c(z)}{y(z)}]
\]

(2.14)

It is clear that as \( f(z) \to 0 \) the vevs approach their classical values \( v_i^{(q)} \to a_i \). As emphasised in \[^{10}\], the sets of integers \( N_i \) which invariantly specify the little group \( H \) should not change even when evaluated (using the solution for the resolvents) as

\[
N_i = -\frac{1}{8\pi i} \oint_{A_i} dz \ \frac{c(z)}{y(z)}
\]

(2.15)

In contrast to the \( r = N, \Phi \sim N \times \overline{N} \) case studied in the AF case in \[^{10}\], the sub-condensates \( R_{0i} \) correspond not to unbroken subgroup factors’ gaugino condensates but instead to certain combinations of the gauginos of the unbroken gauge sub-group \( H \) and the \( G/H \) coset gaugino condensates. The combination relevant for \( R_{0i} \) can be identified by evaluating \( TrT_i^{(r)} T_r^{(r)} \) where in \( T_d^{(r)} \) only the unit diagonal elements in the sector corresponding to the cycle \( A_i \) are retained and the others are set to zero. This is illustrated explicitly with an example in the next section. The consistency of these contour integrals with the Laurent expansion even in the quantum corrected case is ensured by the fact that

\[
\sum_{i} \oint_{A_i} z^n(y(z))^{\pm m} dz = \oint_{C_{\infty}} z^n(y(z))^{\pm m} dz = \oint_{C_{\infty}} dz' z^{-(n+2)}(y(1/z'))^{\pm m} \quad (2.16)
\]

since the region enclosed by the curves \( \cup_j A_j \cup (-C_{\infty}) \) is free of singularities or branch cuts.

For \( n = 2 \), \( T(z) = (d(r)(z\lambda + m) + \lambda T_1)y^{-1} \), and we can solve the definition of \( N_1 \) to obtain a consistency condition of the assumption\[^{10}\] that the value of the integrals giving \( N_1 \) around the critical points \( a_i \) do not change under quantum corrections. From eqn(2.15) we get

\[
T_1 = \frac{2\pi i N_1 - \oint dz (\lambda d(r)z + m d(r)) y^{-1}}{\oint dz \lambda y^{-1}}
\]

(2.17)
The equation for $N_2$ gives nothing fresh because of the complementarity of the integrals around $C_\infty$ and the union of the A cycles. When $n > 2$ we can similarly obtain equations for $T_1, ..., T_{n-1}$ by using the definitions of $N_1, ..., N_{n-1}$ and solving the $n - 1$ linear equations for the $T_i$.

The dynamically determined condensate $R_1$ (at or near the perturbative unification scale where the whole symmetry breaking evaluation of the low energy effective Lagrangian is performed) is required to proceed further. $R_1$ can be rigorously determined in terms of $\Lambda$ and the superpotential parameters only by a numerical calculation of condensates in the full Super YMH theory. However in favourable cases (see the section on Rank Breaking models below) even $R_1$ can emerge determined in terms of $R_0$.

In the pure $r \times \bar{r}$ case, analysis of the GKA equations also yields insight and constraints upon the form of $R_1$. Closed form evaluation of the elliptic or hyper-elliptic type integrals involved is difficult since $\lambda, R_1$ are, in general, complex. Numerical evaluation of expressions for $T_1, v_i^{(q)}, R_0, R_1$ (in units of $R_0^{1/3} \sim \Lambda$) for a range of values of $\lambda, R_1$ and a choice of $N_i$ is straightforward. An important class of possible (even necessary, if the dynamics is to support a picture where the main effect of the condensation for defining the effective perturbative theory is coded in quantum vacuum expectation values $v_i^{(q)}$ of the field $\Phi$ ) solutions are those where $\sum_j R_{0j} v_j^{(q)}$ closely approximates $R_1 = \sum_j R_{1j}$ i.e. for which the dimensionless semi-classicality parameter is small:

$$\delta_{SC}(R_1) \equiv \frac{|R_1 - \sum_j R_{0j} v_j^{(q)}|}{R_1} << 1$$

While we cannot calculate the dimensionless condensate $R_1/R_0^{4/3}$ we do find that there are regions of the dimensionless parameter space $(\lambda, R_1/R_0^{4/3}) \in \mathbb{C}^2$ where $\delta_{SC}$ is very small. Such regions in the parameter space are thus candidate vacua where the gaugino condensate and the quantum corrected vevs encode most of the dynamical information relevant for defining the effective perturbative theory below the scale of dynamical gauge symmetry breaking. We emphasise that $\delta_{SC}$ is merely a numerical measure useful in understanding the types of solutions obtained while numerically scanning the parameter space of the theory (including the values of the non-perturbative physical condensates such as $R_{0,1}$) while evaluating the quantum vevs. It is not any sort of constraint imposed on the supersymmetric dynamics to ensure its consistency, but merely a parameter that indicates that the quantum dynamics is in some sense close to semiclassical when it is small.

### 2.4.1 The Equivalent Quantum Superpotential $W^{(q)}(z)$

We propose that the vevs $v_i^{(q)}$ be used to define a consistent effective Lagrangian for calculating mass spectra in the spontaneously broken theory by modifying the semiclassical superpotential $W(z)$ to a quantum modified or effective superpotential $W^{(q)}(z)$ by changing the coefficients in $W(z)$ so that the vevs obtained are zeros of $W^{(q)}(z)$ i.e. $W^{(q)}(v_i^{(q)}) = 0$. Thus, for example for a cubic superpotential, we take:

$$W^{(q)}(z) \equiv \mu_i^{(q)} z + \frac{m_i^{(q)}}{2} z^2 + \frac{\lambda}{3} z^3$$

$$\mu_i^{(q)} \equiv \lambda v_i^{(q)} v_i^{(q)} ; \quad m_i^{(q)} \equiv -\lambda (v_i^{(q)} + v_i^{(q)})$$

(2.19)
Classically \( v_\pm = \frac{-m_{\pm} \sqrt{m_{\pm}^2 - 4a^2}}{2a} \) and one recovers the original superpotential. This proposal has the virtue that the cubic coupling describing the interactions has not been modified and the changes made are only in the soft super-renormalizable couplings. Use of such a superpotential will ensure that the super-Higgs effect and spectrum calculations using \( v_i^{(q)} \) are consistent.

For evaluating the contour integrals around the \( A_4 \) cycles we should first define the square root branched function \( g(z) \) to lie unambiguously on the first sheet:

\[
y(z) \equiv \lambda \prod_{i=1}^{2n} |(z - z_i)|^{\frac{1}{2}} \prod_{i=1}^{n} e^{\frac{1}{2} i \theta(\frac{z_{2i-2} - z_{2i-1}}{z_{2i-1} - z_{2i-2}})} \tag{2.20}
\]

where \( \theta(z) \in (-\pi, \pi] \) is the quadrant wise correct polar angle of the complex number \( z \). Then the integral over the \( A_4 \)-cycle which encloses the branch cut running from \( z_{2i} \) to \( z_{2i-1} \) is achieved by (P.V. denotes principal value and the function \( g(z) \) should be such that the contribution from the end circles around the branch points is zero)

\[
\oint_{A_4} dz \, g(z) = 2 \, \text{P.V.} \int_0^1 dx \, (z_{2i-1} - z_{2i})g(x(z_{2i-1} - z_{2i}) + z_{2i}) \tag{2.21}
\]

If the dynamical behaviour supports the emergence of an effective spontaneously broken perturbative theory then we expect that \( \sum_j R_{0j} v_j \simeq R_1 \) upto small quantum corrections due to irreducible three point correlation functions of two gaugino superfields and \( \Phi \). We can scan the parameter space of superpotential couplings together with \( R_1 \) (in units of \( R_0^{1/3} \)) to see if there are parameter regions where \( \delta_{SC}(R_1) << 1 \). Using parameters from such regions of the parameter space presumably illustrates the behaviour of the effective theory we may expect: even without doing the difficult dynamical calculation of \( R_1/R_0^4 \). We show instances of such parameter regions in an explicit example below (see Table 1.).

### 3 A Simple Example

As a simple example of condensation in AM-AS systems, consider a (traceful) AM \( \Phi \) transforming as \( R \sim 6 \times 6 \) of \( SU(3) \), i.e. the base representation is \( r = 6_{SU(3)} \). In this case \( d(r) = 6, S_2(6) = 5/2, d(R) = 36, S_2(R) = 30, b_0 = +21 \). Note also that \( 6 \times 6 = 1 + 8 + 27, S_2(27) = 27 \), so that the 27--plet irrep alone could be used as the AM-AS system with \( b_0 = 18 \). We here examine symmetry breaking \( G = SU(3) \rightarrow H = SU(2) \times U(1)_Y, Y = \text{Diag}(1,1,-2) \) driven by gluino condensation in the UV. The 6-plet of \( SU(3) \) which is the symmetric two (fundamental) index tensor of \( SU(3) \), decomposes under \( H \) as

\[
6_{\alpha \beta} \equiv 3_{(\alpha \beta)}^{[2]} + 2_{3\alpha}^{[-1]} + 1_{3\alpha}^{[-4]} \quad \alpha, \beta = 1,2 (T_3 = \pm 1/2, Y = +1) \tag{3.1}
\]

while the \( SU(3) \) triplet index 3 has \( T_3 = 0, Y = -2 \). \( SU(2) \) irreps are identified by dimension and \( Y \) quantum numbers are given in square brackets. Then

\[
6 \times 6 = (1 + 3 + 5)[0] \oplus (1 + 3)[0] \oplus 1[0] \oplus \cdots \tag{3.2}
\]
The diagonal $3 \times 3, 2 \times 2, 1 \times 1$ blocks of $\Phi$ are occupied by representations that are $Y$ singlets and contain $SU(2)$ singlets so that

$$\langle \Phi \rangle = \text{Diag}(V_1 \mathcal{I}_3, V_2 \mathcal{I}_2, V_3)$$

(3.3)

where $V_1, I = 1..3$ (not all equal) are chosen from $\{v_i^{(q)}; i = 1..n = 2\}$ and $\mathcal{I}_n$ is the $n$-dimensional identity matrix. $\langle \Phi \rangle$ breaks the gauge group $SU(3)$ to $SU(2) \times U(1)_Y$. Three possibilities are $A : V_1 = V_2 \neq V_3; B : V_1 = V_3 \neq V_2; C : V_2 = V_3 \neq V_1$ and in each case either the equal pair or the single vev can take the value $V_1^{(q)}$.

Fermion mass terms from the superpotential arise in the pattern $\lambda \psi^I_I \psi^I_J (-(v_1^{(q)} + v_2^{(q)}) + (V_I + V_J)); I, J = 1..6$. Clearly if $V_I, V_J$ are distinct i.e. $V_I + V_J = v_1^{(q)} + v_2^{(q)}$ the mass term will vanish. In case $A$ this can happen for 5 index pairs (1/2, 3/4/5 paired with 6), Case $B$ for 8 base rep index pairs (4/5 paired with 1/2/3/6) and in Case $C$ for 9 (1/2/3 with 4/5/6). Thus these are the numbers of off-diagonal index pairs (out of the total of 15) which remain massless. Since we expect Dirac partners only for the 4 gauginos of the coset $SU(3)/(SU(2) \times U(1)_Y)$, pseudo Goldstone(PG) multiplets arise. The 4 coset gauginos transform as two doublets of $SU(2)$ with $Y = \pm 3$ and pair up with 2 doublet partners from the above enumerated massless pairs of conjugate fields. The 6 diagonal pairs are of course always massive. For case $A \psi^{33}_{\bar{3}3}(\psi^{\bar{3}3}_{33})$ teams up with $\lambda^2_3(\lambda^3_{\bar{3}3})$ respectively while $\psi^{33}_{\bar{3}3}(\psi^{\bar{3}3}_{33})$ remain massless. However we shall see that introduction of rank breaking fields gives these putative PG (PPG) multiplets a mass.

3.1 Spontaneous Fractionation of the Gaugino condensate

The physical gaugino condensate defined by the UV condensed phase is $G$-invariant and thus cannot discriminate between different gauge equivalent gauginos. However in any given spontaneously broken phase a distinction between gauginos associated with the little group ($H$) and coset ($G/H$) generators is meaningful. The ur-gaugino condensates ($R_{0.1}$ for GAM model) are physical $G$-singlet quantities that are backgrounds relevant for the computation of quantum vevs in the spontaneously broken phases of the system in Regime II. We cannot compute them and thus they are input dimensionful parameters for the phases decided by the vevs of the chiral supermultiplets. On the other hand the CDSW formalism computes contour integrals of the resolvents $R(z), T(z)$ (that can be solved for in terms of the input parameters $R_{0.1}, \lambda$ etc) around the A-cycles of the Riemann surface defined by the quantum dynamics or, in more prosaic terms, around the branch cuts defined by the “quantum superpotential derivative” ($y(z) = \sqrt{W'(z)^2 + f(z)}$ in the GAM case). In the IR strong models the coset ($G/H$) gauginos decouple from the low energy physics and these integrals give the gaugino condensates $S_i$ corresponding to the different $U(N_i) \in H$ factors.

**Now in the AS case the situation is radically different.** All the gauginos are still in play in the full spontaneously broken gauge theory of Regime II i.e. at and below Grand Unified scales $\sim M_X$. However the SSB pattern coded in the $\{N_i\}$ partition of $d(r)$ assumed as dynamically invariant input implies that the integrals of the resolvent $R(z)$ will now yield the condensates of certain characteristic combinations of gaugino bilinears since
the contour integral $\oint_{A_i} dz \text{Tr}(T_A^r T_B^r (z - \Phi)^{-1})$ will give a non-zero result only for the $i$-th vev/branch cut. This implies that only certain combinations of the gauginos will contribute to the integral computed using the solution for $R(z)$. The peculiar indirect manner in which the little group is specified by the placement of $v_i^{(q)}$ on the diagonal of $< \Phi >$ determines the combinations. Thus for example in Case A where $\Phi = \text{Diag}(V_1 I_3, V_3)$ the relevant combinations are determined by the partial traces defined by including a projector on to the subspace corresponding to the cycle $A_i$:

$$W_\alpha^A W^{\alpha B} \text{Tr} T^A_A (r) T^B_B (r) \text{Diag}(I_3, 0) = \frac{5}{2} \bar{W}^2 + 2 \sum_{A=4}^7 W_A^2 + \frac{7}{6} W_8^2$$

$$W_\alpha^A W^{\alpha B} \text{Tr} T^A_A (r) T^B_B (r) \text{Diag}(\phi_5, 1) = 0 \bar{W}^2 + \frac{1}{2} \sum_{A=4}^7 W_A^2 + \frac{4}{3} W_8^2$$  \hspace{0.5cm} (3.4)

The gaugino sub-condensate patterns are described by the values of the SU(2) triplet condensate $\kappa \bar{W}^2$, SU(3)/(SU(2) × U(1)$_Y$) coset condensate $\kappa \sum_{A=4}^7 W_A^2$ and the hypercharge gaugino condensate $\kappa W_8^2$. The total of the coefficients of each gaugino squared is equal to $S_2(6_{SU(3)}) = 5/2$ as expected, confirming that the division corresponds to a fractionalization of the gauge singlet condensate following the SSB pattern of the phase in question. The computation of the trace can be easily carried out by setting up the 6 × 6 generators of SU(3) in the 6-plet representation using the symmetric 6-plet generators obtained from the symmetrized tensor product: $T^{(6)}_{\alpha (ij) (kl)} = (1/2)(\delta_{(j} T^{(3)}_{\alpha (i) k}) + \delta_{(k} T^{(3)}_{\alpha (j) i})$. These combinations are denoted compactly by giving the coefficients for each of these vevs: In Case A (5/2,2,7/6) for $V_1 = V_3$ and (0,1/2,4/3) for $V_2$. Case B: coefficients (2,5/4,7/3) for $V_1 = V_3$ and (1/2,5/4,1/6) for $V_2$ and in Case C (1/2,7/4,3/2) for $V_2 = V_3$ and (2,3/4,1) for $V_1$.

### 3.2 Numerical investigation of semi-classicality

In Table 1 we give instances of the calculation of the vacuum expectation values for this model taking $m = \mu = 0$, for simplicity, for which $\delta_{\text{SC}}$ defined in eqn(2.18) is indeed small and the semiclassical approximation $R_1 \simeq \sum_j R_{0j} v_j$ is good. Note that the nonzero values of $v_i^{(q)}$ obtained are in units of $R_0^{1/3}$ and there are no nonzero vevs at the classical level. It is convenient to rescale to dimensionless (hatted) variables in units of $R_0^{1/3}$ thus: $z = \hat{z} R_0^{1/3}$, $f_0 = \hat{f}_0 R_0^{1/3}$ and so on. It is easy to find values of the free parameter $(\lambda, \hat{R}_1)$ sets for which accurate semi-classicality could be attained. However a numerical calculation of the condensate values actually obtained in the strong coupling region is beyond our abilities at this stage.

### 4 Rank Reduction

Since the elements of $\Phi \sim r \otimes \bar{r}$ with vevs are neutral w.r.t. all the Cartan subalgebra generators the gauge group rank cannot decrease due to symmetry breaking in any purely AM type model. However GUT models (such as those based on $SO(10)$) with rank $\geq 5$...
require rank reduction to break the gauge symmetry to the SM gauge group which has rank 4. In the Minimal SO(10) GUT [2, 3], just such a rank reduction is achieved by including a pair of conjugate representations \((126, \overline{126})\) whose role is precisely to break \(SO(10) \rightarrow SU(5)\). The authors of [10] have also provided [13, 14] an analysis for Adjoint-SYM with Flavours, i.e. super \(SU(N_c)\) YM with an adjoint as well as \(N_f\) pairs of Quark fundamental -anti-fundamental \(N_c\)-plets \(Q_f, \overline{Q}_f, f = 1...N_f\). Such models possess “Higgs vacua” with \(<Q_f>, <\overline{Q}_f> \neq 0, f = 1...n\), which imply rank reduction \(N_c - 1 \rightarrow N_c - n - 1\) which is unachievable with AM type fields alone.

Consider \(N_f = 1\) i.e. with a pair of complex representations \(Q, \overline{Q}\) transforming as \(r, \bar{r}\) added to AM \(\Phi \sim r \times \bar{r}\). We modify the superpotential while maintaining renormalizability by adding gauge invariant terms \((\Delta W = W_Q = -\eta \bar{Q} \cdot \Phi \cdot Q)\). We have omitted a Quark mass term by shifting \(\Phi\). The model then admits semi-classical “Higgs Vacua” in which parallel components of the complex multiplets \(Q, \overline{Q}\), say \(Q_1, \overline{Q}_1\), obtain vevs (of equal magnitude to cancel the D term contributions) leading to a lowering of the rank of the little group by one:

\[
<\Phi_{11}> = z_1 = 0 \quad ; \quad <Q_1> = \sigma, \quad <\overline{Q}_1> = \bar{\sigma} \quad ; \quad |\sigma| = |\bar{\sigma}| \quad ; \quad \sigma\bar{\sigma} = \frac{W'(0)}{\eta}
\]  

(4.1)

Since \(W_{A}^{\alpha}(T^A)_{ij}Q_j = W_{A}^{\alpha}(T^A)_{ij}\overline{Q}_i \simeq 0\) in the Chiral ring, the loop equation containing \(R^2(z)\) is unchanged from the pure AM case(equations (2.5,2.8)) but the equation for \(T(z)\) is

| \(N_1\) | \(d(r)\) | \(\lambda\) | \(\hat{R}_1\) | \(\hat{t}_1\) | \(\hat{v}_1\) | \(\hat{v}_2\) | \(\bar{R}_0^{(1)}\) | \(\bar{R}_0^{(2)}\) | \(\delta_{SC}\) |
|------|------|------|------|------|------|------|------|------|------|
| 4    | 6    | -1.582 - \(0.295i\) | .247 + \(0.374i\) | -.0328 + \(0.443i\) | -2.20 + \(0.630i\) | .424 - \(1.038i\) | .772 - \(1.192i\) | .228 + \(1.192i\) | .000 |
| 4    | 6    | 1.381 - \(0.0680i\) | -1.001 + \(0.487i\) | -3.122 + \(4.241i\) | -7.82 + \(2.72i\) | .904 + \(0.312i\) | 1.124 - \(0.293i\) | -1.124 + \(0.293i\) | .0009 |
| 2    | 6    | 0.4 | 0.595 + \(0.995i\) | -3.540 + \(4.186i\) | 0.201 + \(2.72i\) | -.986 - \(0.312i\) | 0.566 - \(0.293i\) | .434 + \(0.293i\) | .003 |
| 2    | 6    | 0.4 | 0.595 - \(1.005i\) | -3.489 - \(4.169i\) | 0.180 - \(2.704i\) | -0.962 + \(0.310i\) | 0.566 + \(0.299i\) | .434 - \(0.299i\) | .002 |

**Table 1.** Illustrative values obtained by searching for values of \(\hat{R}_1\) that minimize \(\delta_{SC}\) given \(N_1, \lambda\) for the classically mass scale free model with \(d(r) = 6\) and symmetry breaking \(SU(3) \rightarrow SU(2) \times U(1)_Y\). All dimensionful quantities are in units of \(R_0^{1/3} = 5^{1/3}\Lambda\) and the vacuum expectation values are due purely to quantum effects. The occurrence of such regions of parameter space supports the possibility of solutions where the quantum corrections driven by the gluino condensate are captured by the quantum vevs in the sense that \(R_1 \simeq \sum R_0^{(i)} v_j\).
modified to:

\[(2R(z) - W'(z))T(z) + \eta S(z) - \frac{c(z)}{4} = 0\]

\[S(z) = \bar{Q} \cdot \frac{1}{z - \Phi} \cdot Q \equiv \sum_{n=0}^{\infty} \frac{S_n}{z^{n+1}} \quad (4.2)\]

The GKA for \(\delta Q \equiv \frac{1}{z - \Phi} \cdot Q\) or \(\delta \bar{Q} \equiv \frac{1}{z - \Phi} \cdot Q\) give

\[S(z) = -R(z) + \eta \bar{Q} \cdot Q \quad (4.3)\]

Then if we impose

\[\frac{1}{2\pi i} \oint_{C_{z_1}} T(z) dz = 1 \quad (4.4)\]

we find

\[\bar{Q}_1 Q_1 = \frac{W'(0) + y(0)}{2\eta} \quad (4.5)\]

Which reverts to its classical value \(W'(0)/\eta\) as \(y(z) \rightarrow W'(z)\) i.e. in the absence of quantum effects a.k.a gluing condensate. The vevs of \(Q, \bar{Q}\) lead to useful mass matrix contributions.

For example for a SU(3) model based on the 6-plet (i.e. \(\phi \sim 6 \otimes 6\)) discussed above we take

\[Q_6 = Q_{(33)} = \sigma, \quad \bar{Q}_6 = \bar{Q}_{(33)} = \bar{\sigma}\]

Dirac-pair \(Q_{(33)}, \bar{Q}_{(33)}\) with gauginos \(\lambda_1^\alpha, \lambda_1^\beta\) respectively. Moreover they modify the masses of the putative pseudo-Goldstone (PPG) multiplets. For example in case A where \(V_{(\alpha \beta)} = V_{(33)} \neq V_{(33)}\), the PPGs \(\Phi_{(\alpha \beta)}, \Phi_{(\bar{\alpha} \bar{\beta})}\) Dirac-pair with \(\bar{Q}_{(\alpha \beta)}, Q_{(\bar{\alpha} \bar{\beta})}\) and become massive.

We can define an effective cubic superpotential that works to reproduce the quantum vevs along the same lines as in the pure adjoint case by modifying the parameters of the cubic superpotential so as to support a Higgs vacuum solution with \(\Phi_{11} = 0\) even in the quantum case but with:

\[\lambda^{(q)} = \lambda ; \quad m^{(q)} = -2\lambda (v_+^{(q)} + v_-^{(q)})\]

\[\mu_{(q)}^2 = \lambda v_+^{(q)} v_-^{(q)} ; \quad \eta^{(q)} = \frac{Q_1 Q_1}{\mu_{(q)}^2} \quad (4.6)\]

\[\lambda^{(q)} = \lambda ; \quad m^{(q)} = -2\lambda (v_+^{(q)} + v_-^{(q)})\]

4.1 \(r \times r\) based rank reduction

Besides rank reduction based on complex representations \(\sim r, \bar{r}\), one can also consider more complicated scenarios based upon pairs of representations \(\sim (r \times r), (\bar{r} \times \bar{r})\) which prove useful in realistic scenarios (see the next section). Following and extending [15, 16], we survey the solution of the resolvent system for this case. Introduce a pair of chiral supermultiplets \(\Sigma_{ij} = \Sigma_{ji}, \Sigma_{ij} = \Sigma_{ji}; i, j = 1...d(r)\) transforming as \((r \times r)_{symm}, (\bar{r} \times \bar{r})_{symm}\) and an additional superpotential

\[W_{\Sigma} = -\eta \Sigma_{ij} \Phi_j^{(q)} \Sigma_{ki} \quad (4.7)\]
As in the $Q\bar{Q}\Phi$ case there is a semiclassical Higgs vacuum where one conjugate component pair from $\Sigma, \bar{\Sigma}$ (say $(\Sigma_{MM}, \bar{\Sigma}^{MM})$) gets a vev:

$$\Sigma_{MM} = \sigma ; \quad \bar{\Sigma}^{MM} = \bar{\sigma} ; \quad |\sigma| = |\bar{\sigma}|$$

$$\Phi_M^M \equiv z_\sigma = 0 ; \quad \sigma \bar{\sigma} = \frac{W'(0)}{\eta} \quad (4.8)$$

For example in the case of the SU(3) model based on the 6-plet $S_{ij} = S_{ji}$, $\Sigma, \bar{\Sigma}$ are $6 \times 6$ symmetric matrices and we can break $SU(3) \rightarrow SU(2)$ by

$$\Phi = \text{Diag}(V_1, V_1, V_1, V_2, 0) \quad ; \quad V_i \in \{v^{(q)}_+, v^{(q)}_-\}$$

$$\Sigma = \text{Diag}(0, 0, 0, 0, 0, \sigma) \quad ; \quad \bar{\Sigma} = \text{Diag}(0, 0, 0, 0, 0, \bar{\sigma})$$

$$\mathcal{G} = \{(11), (22), (12), (13), (23), (33)\} \quad (4.9)$$

Since $S_{(33)}$ is an SU(2) singlet but has $Y = -4$ it is clear that the vevs of $\sigma, \bar{\sigma} \neq 0$ reduce the rank by 1. As in the case with $d(r)$-plet rank-breaker pairs $\bar{Q}, Q$ the PPG spectrum becomes massive due to the rank breaking vevs.

By considering a combination of the loop equations for GKA variations $\delta \Phi = \kappa W_\alpha W^\alpha (z \pm \Phi)^{-1}$ and $\delta \Sigma = 2\kappa W_\alpha (z - \Phi)^{-1} \cdot \Sigma \cdot (W^\alpha (z + \Phi)^{-1})^T$ and using the Chiral ring constraints $W^\alpha_{(i} \Sigma_{kj)} = 0$ (and similarly for $\bar{\Sigma}$) one derives[15] the loop equation (notation $\bar{F}(z) \equiv \bar{F}(-z)$)

$$R(z)^2 + \bar{R}(z)^2 + R(z)\bar{R}(z) - W'(z)R(z) - \bar{W}'(z)\bar{R}(z) = r_1(z) \equiv \frac{f(z) + \bar{f}(z)}{4} \quad (4.10)$$

For a cubic superpotential $r_1 = f_0/2 = -2\kappa Tr(\lambda \Phi + m)W_\alpha W^\alpha$. Due to branch cuts in the $z$ plane (that emerge further on) the resolvent function $R(z)$ is not even in $z$. Introducing a new resolvent (the analogue was automatically zero in the previous case due to the Chiral ring constraint $W_\alpha \cdot Q \simeq 0$):

$$U(z) \equiv \kappa \Sigma \cdot \frac{W_\alpha W^\alpha}{z - \Phi} \cdot \Sigma$$

we obtain a modified equation for $R^2$:

$$R^2 - W'(z)R = \frac{f(z)}{4} - \eta U(z) \quad (4.12)$$

and a similar equation for $\bar{R}$. One can then show [15, 16] that by substituting $R(z) = \omega(z) + \omega(z), R(z) = \omega(-z) + \omega(z); \omega(z) \equiv (2W'(z) - \bar{W}'(z))/3$, eqn(4.10) simplifies to just $\omega^2 + \omega^2 + \omega(\omega) = r(z) = r_0(z) + r_1(z); r_0 \equiv (W^2 + \bar{W}^2 + W \bar{W}')/3$. Then we obtain $\omega = u_1(z), \bar{\omega} = u_3(z), (-\omega + \bar{\omega}) = u_2(z)$ where $u_\alpha, \alpha = 1, 2, 3$ are solutions of a cubic equation of a special form

$$u^3(z) - r(z)u(z) - s(z) = \prod_{a=1}^{3}(u - u_\alpha(z)) = 0 \quad (4.13)$$
Here \( s(z) = s_0(z) + s_1(z) \) and \( s_0, s_1 \) are polynomials of degree 3\( n, 2n - 2 \) which can be explicitly calculated [15] given \( W(z) : \)

\[
    \begin{align*}
    s_0(z) &= \omega_r \bar{\omega}_r (\omega_r + \bar{\omega}_r) = \frac{2}{27} (\lambda z^2 + \mu^2)((\lambda z^2 + \mu^2)^2 - 9 m^2 z^2) \\
    s_1(z) &= 2\eta (m U_0 + \lambda U_1) + \frac{1}{4} (\omega_r f(z) + \bar{\omega}_r f(z)) \\
    &= 2\eta (m U_0 + \lambda U_1) - \frac{2}{3} (m R_0 (\mu^2 - 2\lambda z^2) + R_1 (\mu^2 + \lambda z^2)) \quad (4.14)
    \end{align*}
\]

Thus one finds

\[
    \omega(z) = e^{-\frac{2\pi i}{3}} \omega_+ + e^{\frac{2\pi i}{3}} \frac{r(z)}{3\omega_+} ; \quad \omega_+ = \left( \frac{s(z)}{2} + \sqrt{\frac{s^2}{4} - \frac{r(z)^3}{27}} \right)^{\frac{1}{3}} \quad (4.15)
\]

In spite of the cube root, the Riemann surface branching structure for \( \omega(z) \), \( \bar{\omega}(z) \) is still two sheeted provided \( \omega_- (z) = \omega_+ (-z) = \frac{\omega(z)}{\overline{\omega(z)}} \). The third root \( \omega(z) + \bar{\omega}(z) \) occupies an isolated ‘singleton sheet’. Since \( s(z), r(z) \) are even polynomials the condition on \( \omega_\pm \) can be satisfied provided \( \sqrt{\Delta} \equiv \sqrt{\frac{s^2}{4} - \frac{r(z)^3}{27}} \) is an odd function. This can be ensured by imposing a constraint fixing a higher \( R_n \) coefficient in terms of a lower \( R_n \) coefficient. Writing \( \Delta(z) = z^2 Q(z^2) = z^2 P(z) \), one finds that for non-zero \( m, \mu \) the polynomial \( P(z) \) defining the branch cuts and Riemann surface is quartic in \( z^2 \) i.e. even and of degree 8 in \( z \) and has 4 square-root branch cuts defining a Riemann surface of genus 3. Contour integrals around these branch cuts play the same role as in the pure AM case. Thus we expect 4 possible quantum vevs when the tree level superpotential is cubic, even though at tree level there are just two semi-classical vevs \( v_\pm \neq 0 \) besides the vanishing vev of \( \Phi_{11} \). This indicates that the effective superpotential in the general case with \( m, \mu \neq 0 \) will need to be quintic in \( \Phi \).

Since the analysis becomes quite involved for the general case we here present the explicit solution of the resolvent system only for \( m = \mu^2 = 0 \). In some sense this solution is more interesting since it eliminates all explicit mass scales completely so that all masses arise purely by dimensional transmutation and the classical theory will be superconformal. Moreover, since \( \Delta(z) \) is sextic and \( P(z) \) is quartic, there are only two branch cuts and thus two quantum vevs. A cubic quantum effective super-potential can still be defined as in the earlier cases studied.

We now trace the determination of resolvent coefficients \( \{U, R, T, S\}_n \) using the available GKA equations. Firstly the expansion of eqn(4.11) and then eqn(4.10) for large \( z \) determines \( U_n, R_{2n+3}, n = 0, 1, 2, \ldots \) in terms of \( R_{2n}, R_1 : \)

\[
    \begin{align*}
    U_0 &= \frac{\lambda R_2}{\eta} ; \quad U_1 = -\frac{R_0^2}{2\eta} \\
    U_2 &= \frac{\lambda R_4 - 2R_0 R_1}{\eta} ; \quad U_3 = \frac{R_1^2 - 2R_2 R_0}{2\eta} \\
    R_3 &= \frac{R_2^2}{2\lambda} ; \quad R_5 = \frac{(3R_1^2 + 2R_0 R_2)}{2\lambda} \ldots \quad (4.16)
    \end{align*}
\]
Next imposing $\Delta(0) = 0$ fixes $R_1$ and with $\Delta(z) = z^2 P_4(z)$ we have ($s_{1,2} = \pm 1$)

$$R_1 = \left( -\frac{27}{32 \lambda} R_0^3 \right)^{1/4}$$

$$P_4(z) = -\frac{1}{432} R_0^2 \lambda^4 (108(2R_0^3)^{1/4} + 9(2\lambda R_0)^{1/4} z^2 + 16z^4 \lambda^{1/4})$$

$$= -\frac{\lambda^4 R_0^3}{27} \prod_{s=\pm} (z - z^{(s)})(z - z^{(-s)}) ; \quad z^{(s_1,s_2)} = \frac{s_1}{4} \left( \frac{R_0}{\sqrt{2} \lambda} \right)^{1/4} \sqrt{-9 + s_2 15 i \sqrt{15}}$$

Thus the two branch cuts in this (degenerate) case run between $z^{+},z^{+\pm}$ and $z^{-},z^{-\pm}$. We emphasize that the determination of $R_1$ in terms of $R_0$ is a novel consequence of adding $\Sigma, \bar{\Sigma}$. This simplifies the numerical analysis significantly since-after rescaling to dimensionless form- only the dimensionless coupling $\lambda$ remains free. Contrast this with the pure AM case where $R_1$ was to be dynamically determined.

Now we can also obtain all the even coefficients $R_{2n}, n = 1, 2, ...$ by expanding the cubic equation for $\omega(z) = R(z) - \omega_0(z)$ for large $z$. This gives

$$R_2 = 9 \left( \frac{R_0^5}{2^3 3^2 \lambda^2} \right)^{1/4} ; \quad R_4 = -\frac{1233}{512} \left( \frac{R_0^7}{4 \lambda^4} \right)^{1/4} ; \quad ...$$

Finally we define the pure rank-breaker resolvent $S(z) \equiv \bar{\Sigma} \cdot (z - \Phi)^{-1} \cdot \Sigma$. As in the $\Phi QQ$ case one derives the system of GKA resolvents

$$\eta S(z) = \frac{c(z)}{4} + (W'(z) - 2R(z))T(z)$$

(4.19)

where $f(z), c(z)$ are as before. Thus given $T(z)$ one can derive $S(z)$. To find $T(z)$ we use the equation [15] which is the analogue of eqn(4.10) derived using the same $\Phi$ variations but with $\kappa W_\alpha W^\alpha$ factors omitted:

$$\frac{c(z) + \bar{c}(z)}{4} = (2R(z) - W'(z))T(z) + (2\bar{R}(z) - \bar{W}(z))\bar{T}(z)$$

$$+(R(z)\bar{T}(z) + \bar{R}(z)T(z)) + \frac{2\bar{R}(z) - R(z)}{z}$$

(4.20)

Motivated by the solution of the $\bar{Q}Q\Phi$ case where the corresponding equation differs only by the absence, on the r.h.s., of the mixing (third) term and the factor of 2 in the fourth term and has solution $T(z) = (2R - W'(z))^{-1}(R - \eta Q) + c(z)/4$ we propose

$$T(z) = \frac{1}{(2R - W'(z) + R)} \left( \frac{c(z)}{4} + \frac{2R(z)}{z} + \zeta(z) \right)$$

(4.21)

where $\zeta(z)$ is to begin with an arbitrary odd function of $z$. However the behaviour of $T(z)$ as $z \to \infty$ allows only $\zeta(z) = -\lambda T_2/z$. Here $c(z) = -4\lambda(T_1 + T_0 z)$. By expanding the solution eqn(4.21) for large $z$ we get $T_{n>3}$. If, following the pattern of the Higgs vacuum solution in the $\bar{Q}Q\Phi$ case we demand that the residue of $T(z)$ at $z = 0$ be unity, corresponding to the rank breaking Higgs vacuum, we determine $T_2$:

$$T_2 = \left( -\frac{R_0^2}{2 \lambda^2} \right)^{1/4} ; \quad T_3 = \frac{(T_0 - 2) R_0}{\lambda} \quad ; \quad T_4 = \frac{R_0 T_1}{\lambda} - 3(3T_0 - 2) \left( \frac{R_0^3}{32 \lambda^4} \right)^{1/4}$$

(4.22)
Just as in the $QQ\Phi$ case eqn(4.21) gives the correct $T(z)$ in the semiclassical limit where $R,\bar{R} \to 0$. Of course the semi-classical limit is trivial in this massless case in the sense that all vevs are then zero. The quantum superpotential derivative is now

$$y(z) \equiv W'(z) - 2R(z) - \bar{R}(z) = -(2e^{\frac{2i\pi}{3}} + e^{\frac{2i\pi}{3}})\omega_+(z) - (2e^{\frac{-2i\pi}{3}} + e^{\frac{-2i\pi}{3}})\omega_-(z)$$

(4.23)

Note that the square root branching structure of $y$ is now hidden inside the expressions for $\omega_{\pm}$ which contain $\sqrt{P(z)}$. Where $P(z)$ is the quartic ($m = \mu = 0$)/octic ($m, \mu$ non-zero) polynomial which defines the branch cuts. The resolvent for $S(z)$ is also determined via eqn(4.19) once $R(z), T(z)$ are known so that the coefficients $S_n$ of $z^{-n-1}$ in the large $z$ expansion can be read off. Thus we get

$$\eta S_0 = \lambda T_2 = (-\frac{R_0^2\lambda}{2})^{1/3} \quad ; \quad \eta S_1 = -R_0(T_0 + 2)$$

$$\eta S_2 = R_0T_1 - 3(T_0 - 2)(\frac{R_0^4}{32\lambda})^{1/3}$$

(4.24)

and so on. The residue of $\eta S(z)$ at $z = 0$ is $(-4\lambda R_0^2)^{1/3}$.

Since $T_0 = d(r)$ we have only $R_0, T_1$ left undetermined. The former is set by the gaugino condensation $S = \Lambda^3$. The same argument as for the case with the Adjoint gives

$$T_1 = \frac{2\pi iN_1 - \oint dz (\lambda d(r)z + (\lambda T_2 - 2R)/z) y^{-1}}{\oint dz \lambda y^{-1}}$$

(4.25)

where $y(z)$ was given explicitly above. Thus with input parameters $\lambda, N_1$ ($N_2 \equiv d(r) - N_1 - 1$) and using units of $R_0^{3/2} \sim \Lambda$ for dimensionful quantities we can evaluate the quantum vevs by performing the contour integrals numerically. Details will be given in a sequel.

Although we also defer detailed consideration of the case with $m, \mu \neq 0$ to the sequel it is important to underline that it presents new features not observed in the degenerate case described above. One finds $\sqrt{\Delta} = z\sqrt{Q_4(z^2)} = z\sqrt{P_8(z)}$ so that one has 4 rather than 2 branch cuts. This opens the possibility of cases where the quantum spontaneous symmetry breaking includes vev patterns with no semi-classical antecedent. With cubic $W(z)$ and thus two critical points, the contour integrals around the 4 branch cuts define 4 different vevs : which may be placed at will on the diagonal of the quantum corrected vev of $\Phi$ giving a symmetry breaking pattern without a semi-classical analog. Thus the corresponding “perturbative effective quantum superpotential” will need to be quintic rather than cubic.

5 Realistic MSGUT type model

We next come full circle and consider our motivating problem: the gauge UV Landau pole in the successful MSGUT[3]. To illustrate how the AS dynamics permits novel realistic GUT scenarios with dimensional transmutation and dynamical symmetry breaking, we propose a Spin(10) gauge model with 3 matter 16-plets and a Higgs structure generated by base representation $r = 16$. We take $\Phi \sim 16 \times \overline{16} = 1 + 45 + 210, \Sigma \sim 16 \times 16 = 10 + 120 + 126, \overline{\Sigma} = \overline{16} \times \overline{16} = 10 + 120 + \overline{126}$. As before we note that one may choose to work
with just the irreps of the MSGUT, or some extended set thereof, by applying projectors to select only the irreps one wishes to keep at the cost of an increase in calculational overhead. The required SO(10) decompositions w.r.t. $G_{PS}$ are explicitly available in [20]. But we will not work with projections here. The superpotential for the complete model as

$$ W = \frac{m}{2} Tr \Phi^2 + \frac{\lambda}{3} Tr \Phi^3 + \mu^2 Tr \Phi - \eta \sum \Phi \cdot \Sigma $$

$$ + h_{AB} \Psi_A \cdot \sum \Psi_B + h'_{AB} \Psi_A \cdot \Sigma \cdot \Psi_B $$

(5.1)

where $\Psi_A, A = 1, 2, 3$ are the three matter 16-plets and it is understood that in the last term only the real 10-plet and 120-plet parts of the tensor product will be present since there is no invariant between two matter 16-plets ($\Psi_A$) and a 126-plet. Notice the remarkable economy of AM type couplings. We recall our proposal[21] to further reduce the number of matter Yukawa couplings by making $\Sigma, \Sigma$ carry the generation indices. We do not discuss the matter couplings further except to note that as per our arguments the Konsishi anomaly will also force the development of large trilinear condensates involving $\Psi_A \sum \Psi_B, \Psi_A \Sigma \cdot \Psi_B$, even provided the solution has no 16-plet vevs. The theoretical and phenomenological implications of such condensates are not clear to us.

We follow the conventions and use the results of [20] to explicitly calculate the decomposition of Spin(10) invariants. Here $\mu = \bar{\mu}, 4; \bar{\mu} = 1, 2, 3$ refer to the SU(4) indices of the Pati-Salam maximal subgroup $G_{422} = SU(4) \times SU(2)_L \times SU(2)_R \subset SO(10)$. Barred mid-greek indices ($\bar{\mu}$ etc) are colour indices. The fundamental doublet indices of $SU(2)_L(SU(2)_R)$ are referred to as $\alpha, \beta (\hat{\alpha}, \hat{\beta}) = 1, 2(1, 2)$. It is convenient to order the elements of the 16-plet according to their SM quantum numbers (denoted compactly by the relevant MSSM left-chiral fermion symbol) as

$$ 16 = \psi_{\bar{\mu}, \alpha}(4, 2, 1) \oplus \psi_{\bar{\mu}}^\mu(\bar{4}, 1, 2) = \{ \nu^c[1, 1, -1/2, 1](4^*, \hat{2}), e^c[1, 1/2, 1](4^*, \hat{1}) \} $$

$$ u^c[3, 1, -1/2, \frac{1}{3}](\bar{\mu}^*, \hat{2}), d^c[3, 1, 1/2, -\frac{1}{3}](\bar{\mu}^*, \hat{1}), \bar{L}[1, 2, 0, -1](4, \alpha), Q[3, 1, 0, \frac{1}{3}](\bar{\mu}, \alpha) \} $$

(5.2)

where we have also given the dimensions/quantum numbers w.r.t $[SU(3), SU(2)_L, T_{3R}, B \sim L]$ and the $G_{422}$ indices of each left-chiral matter field. Apart from matter 16-plet vevs, which must vanish in viable vacua, the vev patterns which can develop will follow the earlier discussion

$$ \Phi = Diag(V_{4^*2} = V_1, V_{4^*1} = V_2, V_{\bar{\mu}^*2} = V_3 \mathcal{I}_3, V_{\bar{\mu}^*1} = V_4 \mathcal{I}_3, V_{4^*1} = V_5 \mathcal{I}_2, V_{\bar{\mu}^*2} = V_6 \mathcal{I}_6) $$

(5.3)

with $\Phi_{11} \equiv \Phi_{\nu^c, \nu^\mu} = V_1 = 0$ as per the 16-plet labels introduced above. Then the rank breaking vevs will be $\Sigma = Diag(\sigma, \check{\theta}_{15})$ i.e. $\Sigma_{4^*2, 4^*1} = \sigma, \Sigma = Diag(\check{\sigma}, \check{\theta}_{15})$, i.e. $\Sigma_{4^*2, 4^*1} = \check{\sigma}$, all other component vevs zero. If we insist on a cubic tree level superpotential for $\Phi$ and set also $m = \mu = 0$ then in addition to the vanishing singleton vev in the rank breaking($\Phi_{\nu^c, \nu^\mu} = \Phi_{\nu^c, \nu^\mu}$) sector we will have only two possible vevs emerging from the pair of branch cuts that develop. This case will the easiest to analyse. Moreover the effective potential can then also be chosen to be just cubic.

The symmetry breaking patterns corresponding to the various vev distribution possibilities can be easily worked out. The easiest way of identifying the unbroken symmetry is
to look at the gaugino masses that arise for a given distribution of $v_{±}^{(q)}$ over the 5 vevs $V_{2−6}$ with $V_1 = 0$, $σ, ṡ ≠ 0$ fixed by the Higgs vacuum structure necessary to reduce the gauge group rank from 5 to 4 without breaking the Standard Model. Firstly it is clear that the nonzero vevs $σ, ṡ$ break $SO(10) \rightarrow SU(5)$ since 16 decomposes as $16 = 10_1(u^c, e^c, Q_L) + \bar{5}_{−3}(d^c, L_L) + 1_{5}(ν^c)$ w.r.t. $SU(5) \times U(1)_X \times X = 3(B − L) − 4T_{3R}$. Thus the vevs of $Σ, \bar{Σ}$ give masses to the 21 gauginos of the coset $SO(10)/SU(5)$ ( $\lambda_κ^1, \lambda_κ^2, \lambda_α^4, \lambda_α^6, \lambda_α^8, \lambda_κ^4, \lambda_κ^6$ are 22 independent fields but $Λ_Y, Y = 2T_{3R} + B − L$ remains massless). The remaining 12 coset gauginos of $SU(5)/G_{321}$ obtain masses unless $V_2 = V_3 = V_6$ and $V_4 = V_5$. The logic of these conditions is transparent once we note that $16 × 16 = 1 + 45 + 210$ and 45 and 210 each contain $SU(5) × U(1)_X$ singlets corresponding to the diagonal terms of the product $5(−3) × 5(3) = 1(0) + ...$ in the 45-plet getting equal vevs ($V_4 = V_5$) and the diagonal terms of product $10(1) × 10(−1) = 1(10) + ...$ in the 210-plet getting equal vevs $V_2 = V_3 = V_6$. Thus any distribution of the quantum vevs $v_{±}^{(q)}$ over the last 5 ($Φ_{11} = Φ_{νc}, νc = 0$ for the Higgs vacuum solution we are focussed on here) diagonal blocks of $Φ$ that violate these equalities will break $SO(10) \rightarrow G_{123}$. For instance $Φ = Diag(0, v_{±}^{(q)}I_4, v_{±}^{(q)}I_{11})$ has $V_2 = V_6, V_3 = V_6$ both non zero while $Φ = Diag(0, v_{±}^{(q)}, v_{±}^{(q)}I_{14})$ has $V_3 = V_6 = 0$ but $V_2 = V_6 ≠ 0$. Thus both break to the same SM little group even though the mass patterns are dissimilar.

The mass spectrum is straightforward to evaluate given the contractions ($A_{\muναβ}$ is the (6,2,2) of $G_{422}$ and $A_{\muναβ}$ is its $SU(4)$ dual) [20]

$$
16 \cdot 16^* = 16_μα(16^*)_μ^* γ_α + 16_μ^*α(16^*)_μ γ_α
$$

$$
16(ψ)^*16^*(φ^*)45(A) = 2A^2_κ(ψ_μβφ^*_κ β, ψ_μ^*βφ^*_κγ) − 2A^2_γψ^*_αψ^*_βγ μ − 2A^2_γψ^*_αψ^*_βγ μ^* γ μ
$$

which can be easily deduced from equations (116)-(117) of [20] provided we consistently identify

$$
\overline{16}_μ = ε_αβ 16^*μ^* γ_β, \overline{16}_μ = −ε_αβ 16^*μ^* γ_β
$$

(5.5)

The analysis, while tedious due to the reducible reps used, is straightforward and similar to the case of the Adjoint of $SU(N)$ with a symmetric representation except for the crucial fact that the 16-plet is not the fundamental of Spin(10). The evaluation of the resolvents and quantum vevs proceeds along the lines discussed above leading again to spontaneous breaking $SO(10) \rightarrow G_{321}$ via dimensional transmutation. Details will be given in the sequel. We note that the general features of the model are similar to the MSGUT except for the extreme economy with regard to superpotential parameters since the cubic potential for $Φ$ has just 3 (if $m ≠ 0$) complex parameters and just one in the massless case. Since the matter Yukawa couplings $h_{AB}, h^*_{AB}$ include couplings of two 16-plets to both 10, $\overline{126}$-plets (symmetric Yukawas) and to 120-plets(antisymmetric Yukawas) they are general $3 × 3$ complex matrices and thus have ample scope for fitting the observed matter fermion mass parameters and mixings. Note again the new feature, not present in the MSGUT, that $Σ ≈ 16 × 16 = 10 + 126 + 120$ and it is possible to couple the 10, 120 pleats generated in this way to the matter 16 bilinear although $16 \cdot 16 \cdot 126 ≡ 0$ as before.
6 Discussion

Using Generalized Konishi Anomaly relations obeyed by gluino and scalar condensates in the supersymmetric vacua we have shown that asymptotically strong Supersymmetric Yang Mills Higgs theories with matrix type Higgs multiplets transforming as general matrix type base $r$ tensors of $G$ provide a calculable implementation of spontaneous symmetry breaking of the fundamental gauge group, including rank reduction, via dimensional transmutation. This breaking is driven by the formation of a G-singlet RG invariant physical condensate in the microscopic/high energy Regime I strong coupled phase : which therefore forms an ineluctible background at all larger length scales. As such they provide a robust and novel method of making sense of AS Susy GUT models and justify the surmise that the AS exhibited by phenomenologically successful and minimal models such as the MSGUT is a signal of nontrivial UV behaviour that makes the theory consistent and yields a sensible low energy limit. This demonstration calls for deep revision of our notions of the relation between strong coupling behaviour in the microscopic theory and a phenomenologically acceptable low energy effective theory.

For any given YM gauge group $G$, the number of asymptotically free models is strictly limited, whereas we have shown that the number of asymptotically strong models with sensible low energy limits is essentially unlimited. Thus our approach points the way to a vast expansion of admissible microscopic theories beyond the narrow set of currently canonical AF type models. The signal successes of QCD and the amiable ease of analysis of AF models have led, over the half century since their discovery and dominance, to the hardening of a Dogma that sees AF as the necessary condition for a field theory to be physically sensible and relevant as a fundamental microscopic theory. On the other hand we continue, especially in Condensed matter Physics, to be challenged by the need to tackle quantum systems that are strongly correlated or massively entangled at the microscopic level. The success of the AdS-CFT conjecture, and the Seiberg-Witten analysis of monopole condensation leading to confinement in $N = 2$ supersymmetric YM theories, in providing fruitful working paradigms in manifold non-supersymmetric strongly coupled field contexts, even though the original contexts which allowed calculation were specifically supersymmetric, suggests that even our analysis which is rooted in supersymmetry may, in the long run, motivate a more broad minded view of the way in which microscopic condensation due to strong coupling can generate sensible low energy behaviour, such as the effective perturbative gauge model we found after GUT dynamical SSB, even when Supersymmetry is absent. After all, the strong coupling dynamics underlying satisfaction of the “kinematic” GKA constraints must enforce the development of vevs driven by the physical G-invariant gaugino condensate present at all scales. This phenomenon may well persist even when one moves off the supersymmetric point in coupling space, and even, perhaps, for “small” structural differences w.r.t the fields present. These matters require the development of Lattice methods applicable to AS theories for their definitive resolution. The recent development of Lattice methods applicable to supersymmetric gauge theories encourages us to hope that such methods will be developed. Workers on the lattice will then have a plethora of AS toy models to choose from. For example even the behaviour of
our original $SU(2)$ model which is a projection of a symmetric $3 \times 3$ matrix $\Phi$ corresponding to a triplet of $SU(2)$ as the base representation $r$ awaits investigation. We also note that discovery of weakly coupled models dual to the AS models of the type we have suggested would open up fruitful avenues towards a deeper understanding of the possibilities we have almost blindly raised on the basis of the Generalized Konishi Anomaly alone.

We have emphasised that in Susy GUTs, in sharp contrast to say SQCD, the smallness of the ratio of the Susy breaking scale $M_S$ to the GUT scale $M_X$ implies that Supersymmetry may be assumed to be essentially exact at the scales where the theory becomes strongly coupled. Nevertheless it is clear that the issue of supersymmetry breaking must be tackled for such AS GUT models to make contact with reality. The soft supersymmetry breaking terms typically invoked in the MSSM and Susy GUTs, can be introduced by spurion Chiral supermultiplets that take fixed values thus breaking supersymmetry ($S_F = \theta^2 A, S_M = \theta^2 M_3, S_D = \theta^2 \bar{\theta}^2 m_2^2$ and so on ). Since the Konishi anomaly relations involve the lowest components of Chiral multiplets the new terms should not affect the GKA relations directly. Thus after carrying out the GUT SSB using the quantum vevs computed by using the CDSW formalism and thus achieving robust dimensional transmutation we define a consistent “equivalent superpotential ”. Using the heavy-light spectrum evaluated therefrom we can define an effective low energy supersymmetric model (with exotic operators), add in the RG run down small susy breaking terms for light fields, and proceed as usual to study electroweak breaking and low energy phenomenology.

One novel and mysterious implication of the GKA relations is that even the superpotential terms containing matter chiral multiplets(along with Higgs multiplets) must participate at least in trilinear condensates with superheavy values, even though vevs for the smatter fields are phenomenologically unacceptable. The phenomenological implications of such three point correlators are not clear to us. Perhaps such novel quantum background contaminations of the perturbative theory will eventually yield novel signals of the dynamical symmetry breaking origin of GUT spontaneous symmetry breaking.

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