Infrasound generation by turbulent convection

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(Dated: June 22, 2017)

Low frequency acoustic wave generation is studied taking into account the effect of stratification, inhomogeneity of background velocity profile and temperature fluctuations. It is shown that for the typical parameters of convective storms the dipole radiation related to temperature inhomogeneities is at least of the same order as radiation of Lighthill’s quadrupole source. It is also shown that the source related to stratification could have valuable contribution whereas some other sources are shown to be inefficient.

PACS numbers:

I. INTRODUCTION

It is long known that strong convective storms, such as supercell thunderstorms are powerful sources of infrasound [1, 2, 3]. Detailed observations of convective storm generated infrasound [4, 5] provided that at least two different group of infrasonic signals could be identified. The first group, with characteristic period about 1 s, has been found to be in strong connection with prototornadic structures, funnel clouds and tornadoes. Based on coincident radar measurements of tornadoes, which show strong relationship between funnel diameter and infrasound frequency, it is usually supposed that these infrasound waves are generated by radial vibrations of the funnel core [5]. The second group of infrasound signals has the periods from 2 to 60 s. Usually the emission appears about 1 hour before observation of tornado. These waves are not related with tornado itself and are caused by convective processes that precedes tornado formation. The acoustic power radiated by convective storm system could be as high as $10^7$ watts [6]. Although several reasonable mechanisms have been suggested to explain this acoustic radiation, the physical mechanism of the process remains unexplained [2, 4, 7].

Broad and smooth spectrum of the observed infrasound radiation indicates that turbulence is one of the promising sources of the radiation. Lighthill’s acoustic analogy [8] represents the basis for understanding of the generation of sound by turbulent flows. In this approach the flow is assumed to be known and the sound field is calculated as a small by-product of the flow. According to this theory in the case of uniform background thermodynamic parameters interaction of turbulent vortices provides quadrupole source of sound. The acoustic power of the source was estimated by Proudman [9]. But usage of this estimation for the infrasound radiation from convective storms usually leads to the underestimation of the acoustic power [3, 6].

It is also well known that any kind of inhomogeneity of the background flow, such as stratification, shear of the background velocity profile and temperature fluctuations leads to appearance of additional (mainly dipole) sources of sound [10, 11]. Recent developments in this direction led to the formulation of the generalized acoustic analogy [12] that implies: (i) dividing the the flow variables into their mean and fluctuating parts; (ii) substracting out the equation for the mean flow; (iii) collecting all the linear terms on one side of equations and the nonlinear terms on the other side; (iv) treating the latter terms as known source of sound.

In the presented paper we consider acoustic radiation from turbulent convection taking into account mentioned above effects of stratification, inhomogeneity of velocity profile and temperature fluctuations. Performed analysis shows that for the typical parameters of supercell storms the dipole radiation related to temperature inhomogeneities is at least of the same order as radiation of Lighthill’s quadrupole source. It is also shown that the source related to stratification could have valuable contribution whereas some other sources are shown to be very inefficient.

The paper is organized as follows: simple background flow model is constructed in Sec. IV. Linear equation governing the propagation of sound in the flow is obtained in Sec. V. Sources of acoustic radiation are obtained and analyzed in Sec. VI. Summary is given in Sec. VII.
II. BACKGROUND FLOW MODEL

In this section we construct simplified model for background updraft flow. Suppose in the stratified dry atmosphere there exists some region (see Fig. 1) with supply of relatively hot air at some background level and a sink at some height $h$. Assume pressure, density and temperature fields outside this region are $P_0(z), \rho_0(z)$ and $T_0(z)$ respectively. In the case of isentropic atmosphere

$$P_0(z) = P_{0b} \left(1 - \frac{\gamma - 1}{\gamma} \frac{z}{H}\right)^{\gamma/(\gamma-1)},$$  

(1)

where $H \equiv RT_{0b}/g$ is stratification length scale; $P_{0b}$ and $T_{0b}$ are pressure and temperature at the background level; $z$ is vertical coordinate; $g$ is gravity acceleration and $\gamma \equiv c_p/c_v$ is adiabatic index.

In the case of isothermal atmosphere instead of Eq. (1) we have

$$P_0(z) = P_{0b} \exp(-z/H),$$  

(2)

Suppose the temperature of the supplied hot air is $T_{1b} > T_{0b}$. The continuity, Euler, heat and state equations governing the stationary adiabatic updraft motion in the region are:

$$\nabla \rho_1 \mathbf{V}_1 = 0,\quad (3)$$

$$\rho_1 (\nabla \mathbf{V}_1) \mathbf{V}_1 + \nabla P_1 - \rho_1 \mathbf{g} = 0,\quad (4)$$

$$P_1 d\rho_1 - \gamma \rho_1 dP_1 = 0,\quad (5)$$

$$P_1 = \rho_1 RT_1,\quad (6)$$

where $\mathbf{V}_1 = \mathbf{V} + \mathbf{V}_D$; $\mathbf{V}(z) \equiv [0,0,V(z)]$ is updraft motion velocity; $z$ is vertical coordinate; $\mathbf{V}_D(r) = \{\alpha(z)x, \alpha(z)y, 0\}$ is horizontal velocity associated with divergence of updraft motion; and $\rho_1(z), P_1(z), T_1(z)$ are density, pressure and temperature respectively.

The boundary condition at the boundary of regions 1 and 2 implies

$$P_1(z) = P_0(z).$$  

(7)

Eq. (3) provides for $\alpha(z)$

$$\alpha(z) = \frac{\partial_x (\rho_1 V)}{2\rho_1}, \quad (8)$$

For other variables straightforward calculations yield

$$\rho_1(z) = \rho_{0b} \frac{T_{0b}}{T_{1b}} \left(1 - \frac{\gamma - 1}{\gamma} \frac{z}{H}\right)^{1/(\gamma-1)},$$  

(9)

$$T_1(z) = T_{1b} \left(1 - \frac{\gamma - 1}{\gamma} \frac{z}{H}\right),$$  

(10)

$$V^2(z) = 2g z \left(\frac{T_{1b}}{T_{0b}} - 1\right),$$  

(11)

for isentropic atmosphere

$$\rho_1(z) = \rho_{0b} T_{0b} \exp\left(-\frac{z}{\gamma H}\right),$$  

(12)

$$T_1(z) = T_{1b} \exp\left(-\frac{(\gamma - 1)z}{\gamma H}\right),$$  

(13)

$$V^2(z) = 2g \left[\frac{T_{1b}}{T_{0b}} \frac{\gamma H}{\gamma - 1} \left(1 - \exp\left(-\frac{\gamma - 1}{\gamma} \frac{z}{H}\right)\right) - z\right],$$  

(14)

for isothermal atmosphere.

Eqs. (3) and (11) yields

$$\frac{V}{|V_D|} \sim \frac{H}{L},$$  

(15)

where $L$ is horizontal length scale of region 1. Therefore if $H \gg L$ we conclude that

$$V \gg |V_D|.\quad (16)$$

Considered model of background flow do not include horizontal rotation of the updraft flow as well as horizontal wind with vertical shear. Existence of the latest one is known to be necessary for the formation of the strongest convective storm systems, supercell storms [3, 14]. However, as it will be shown below none of these flows can have valuable direct influence on the infrasound generation.

III. LINEAR OPERATOR FOR ACOUSTIC WAVES

Next step of the study in the framework of mentioned above generalized acoustic analogy is derivation of the linear equation for acoustic wave propagation. Taking into account Eq. (16) linearized continuity, Euler and state equations can be written as

$$D_t \left(\frac{\rho}{\rho_1}\right) + \nabla (\rho_1 \mathbf{v}) = 0,$$  

(17)

$$\bar{D}_t \mathbf{v}_x + \partial_x \frac{p}{\rho_1} = 0,$$  

(18)

$$\bar{D}_t \mathbf{v}_y + \partial_y \frac{p}{\rho_1} = 0,$$  

(19)
perturbations are very inefficient sources of sound. The acoustic power is proportional to $e^{-1/2M^2}$ and $e^{-\pi\delta^2/2M}$ for instability waves and continuous spectrum perturbations respectively. In the last expression $\delta$ is the ratio of length scales of energy containing vortices and background velocity inhomogeneity $(\nabla/\partial_t)$. In the case of supercell thunderstorm $M \sim 0.1 - 0.15$ and $\delta \sim 10^{-2}$, therefore both linear mechanisms have negligible acoustic output and attention should be payed to sources of sound related to nonlinear terms and entropy fluctuations that will be studied in the next section.

Propagation of infrasound in the atmosphere was intensively studied by different authors (see, e.g., [13, 14] and references therein) and will not be considered in this paper.

IV. SOURCES OF INFRASOUND

For obtaining the sources of acoustic waves generated by turbulent convection we follow standard technique of acoustic analogy. Dividing the flow variables into their mean and fluctuating parts keeping the nonlinear terms in Eqs. [17–20], and noting that in general in the flow there can exist entropy fluctuations, so replacing Eq. [24] by

$$\frac{p}{P_1} - \frac{\rho}{\rho_1} = \frac{s}{c_v},$$

where $s$ is entropy fluctuation, after straightforward calculations we obtain following equation

$$LQ = S_l + S_s + S_V + S_T,$$

where

$$S_l = \frac{\partial_t[\partial_x(\partial_z(v_i v_x)) + \partial_y(\partial_z(v_i v_y))]}{\bar{D}_t}\left[\partial_z - \frac{V}{c_s}D_t\right]\partial_i(v_i v_z),$$

$$S_s = \bar{D}_t\partial_i(v_i v_z)\left[\frac{\partial_x}{\rho} - \frac{V^2}{c_s^2}\partial_z\left(\frac{1}{c_s^2}\right)\right],$$

$$S_V = -\bar{D}_t\frac{V\partial_z V}{c_s^2}\partial_i(v_i v_z),$$

$$S_T = \left[\partial_zD_t\left(\frac{s\partial_zP_1}{\rho_1} - \frac{V\partial_z\rho_1}{\rho_1} - \frac{V^2\rho_1}{\rho_1}\right)\right].$$

are source terms. In the derivation of Eq. [20] all the nonlinear terms containing $\rho_i$ has been omitted, due to the fact that these terms are known to be much weaker sources of sound then the terms containing $v_i v_j$ in the low Mach number flows [16].

All the sources presented in Eq. [20] are well known in aeroacoustics. Consequently, we do not repeat calculations of acoustic power (i.e., acoustic energy generated in
a unit time) of each source term and adopt well known results [10, 11, 16, 21] for the problem under consideration. The main idea of mentioned methods of acoustic power calculations is the following: in the low Mach number flows convective propagation effects have minor influence on sound generation process and therefore all the terms containing $V$ on the left hand side of Eq. (26) can be neglected. This circumstance allows one to obtain simple estimations of acoustic power of the sources.

In the absence of background flow ($V = 0$) the first term $S_t$ reduces to the Lighthill’s quadrupole source [10]. Its acoustic power

$$N_t \sim \frac{\rho \nu^8}{c_s^5} F, \quad (31)$$

where $\nu$ is rms of turbulent velocity fluctuations and $F$ is total volume of turbulent flow. This expression was used in ref. [10] to estimate acoustic power of supercell storm.

The term $S_s$ is dipole source caused by the stratification. The influence of stratification on acoustic wave generation has been studied intensively in the context of solar physics [11, 20]. Related acoustic power

$$N_s \sim \frac{\rho \nu^6 \bar{v}}{c_s^2 H^2} F = \frac{c_s^2}{\omega^2 H^2} N_t, \quad (32)$$

where $\omega = \bar{v}/l$ is the characteristic frequency of emitted acoustic waves. Substituting $c_s = 340$ m/s, $H = 10^4$ m and $\omega \sim 0.1$ s$^{-1}$, we obtain $N_s \sim 0.1 N_t$, i.e., the power of this source is weaker than acoustic power of Lighthill’s quadrupole source, but in principle considered dipole source could have valuable contribution to the total acoustic power.

The source term $S_v$ is related to inhomogeneity of updraft velocity. It is dipole source similar to $S_s$. Corresponding acoustic power

$$N_v \sim \frac{\rho |V|^4 \nu^6}{H^2 c_s^5} F = M^4 N_s, \quad (33)$$

is negligibly small in comparison with $N_s$ for low Mach number flows (for supercell storms $M \sim 0.1 - 0.15$).

The last term $S_T$ is caused by entropy fluctuations. The most strong acoustic source of this kind is so-called thermo acoustical source [11] that is related to small scale inhomogeneities of background density (and therefore temperature) fluctuations, that produce dipole source. The physics of this kind of acoustic radiation is the following: "hot spots" or "entropy inhomogeneities" behave as scattering centers at which dynamic pressure fluctuations are converted directly into sound. The acoustic power

$$N_T \sim \frac{\rho \Delta T^2 \nu^6}{l T^2 c_s^5} F, \quad (34)$$

where $\Delta T$ denotes the rms of temperature fluctuations.

Taking for supercell storm [14] $\bar{v} \sim (3 - 10)$ m/s; $\Delta T \sim (3 - 10)°K$ and $T = 270°K$ we see that for the typical parameters of supercell storm the dipole radiation related to temperature inhomogeneities is at least of the same order as radiation of Lighthill’s quadrupole source.

In the considered model no attention was payed to the horizontal wind with vertical shear and rotation of the updraft flow. These assumptions allow us to obtain relatively simple equations. These flows could not make valuable changes in the acoustic output estimations. Indeed, consideration of the wind and rotation can lead to appearance in the right hand side of Eq. (26) additional dipole terms similar to $S_v$ that has negligible acoustic radiation, and also additional quadrupole source term known as "shear noise" [16]. Acoustic power of the latter one is also much weaker compared to Lighthill’s quadrupole source in low Mach number flows.

In the presented paper we study dry atmosphere and therefore influence of humidity on the sound generation in the convective storms has not been considered. We intend to study this influence in the future.

V. SUMMARY

The simple model of the background flow has been constructed and the problem of acoustic radiation from turbulent convection taking into account the effects of stratification, inhomogeneity of velocity profile and temperature fluctuations has been considered in the framework of acoustic analogy. Performed analysis shows that for the typical parameters of supercell storms the dipole radiation related to temperature inhomogeneities is at least of the same order as radiation of Lighthill’s quadrupole source. It is also shown that the source related to stratification could have valuable contribution whereas dipole and quadrupole sources related to the inhomogeneity of background velocity profile are shown to be very inefficient.

Acknowledgments

Authors are grateful to Ming Xue for valuable help.

The research described in this publication was made possible in part by Award No. 3315 of the Georgian Research and Development Foundation (GRDF) and the U.S. Civilian Research and Development Foundation for the Independent States of the Former Soviet Union (CRDF). The research was supported in part by ISTC grant No. G 553.
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