The Design of the Phase Transformer Based on Transformation Optics

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Abstract. Based on transformation optics we theoretically propose a design of phase transformer. The arbitrary cylindrical wave to plane wave is implemented by the impedance-matched nonmagnetic slab with the kind of phase transformer. Firstly, indirect transformation method (ITM) is applied to design the transformer, second, the transformer is designed that can convert the cylindrical wave to plane wave directly by ITM. The results are further validated by numerical simulations.

1.  Introduction
Within the past few years, the new research area of transformation optics based on the form-invariance of Maxwell’s equations under coordinate transformations has been arousing great interest [1, 2]. It provides a general method to manipulate electromagnetic waves by materials, usually metamaterials [3]. According to the requirement of the devices, we can design the materials that required and it provides the great flexibility to design photonic devices or control the light [4-5]. Using it a host of striking electromagnetic devices has been demonstrated, of which the first is the invisibility cloak [6-8]. Following cloak is a large number of work focused on controlling the path of light tray, such as light concentrator / absorber [9, 10], beam expanders / compressors [11, 12], beam shifter [13, 14], and beam bends [15, 16]. At the same time many researchers applied it to molding the phase, such as phase rotaters [17] and phase transformers [18-20].

Recently, the study on phase transformer is more widely. In this work, we propose the design of phase transformer based on equal optical path length, applying this method one can convert the arbitrary cylindrical wave to plane wave, hence it is more convenient to design the phase transformer. According to the optical path length [21, 22], any transformation can be achieved, such as planar to planar wavefront transformation, planar to curved wavefront transformation and curved to curved wavefront transformation [21]. The phase transformer in this method can convert the cylindrical wave to plane wave only considering the equal optical path length, without regard to the details of transformation.

2.  The basic formulation
Let us begin with the method of Ref. [21-23] and apply it to convert any cylindrical wave front into plane one. We follow the principle of equal optical path length to attain the phase transformer. For simplicity we consider the two-dimensional wave problem.
Figure 1. Schematic diagram of the space transformation to cylindrical wave front into plane phase front by (a) direct transformation method in literature and (b) the indirect transformation method in the present work. AE, a profile of the spatial separation of the two wavefronts B’C’ and A’E’, is taken to be transformed into a plane BC, the coordinate of O is (x, r).

To convert cylindrical wave front into plane phase front the usual approach in literature is to transform the incident wave surface to the desired wave shape directly, i.e. from A’E’ to B’C’, as shown in Fig. 1(a), which is named direct transformation method (DTM) [21-23]. There the virtual space OA’E’D is transformed into the physical one OB’C’D. To obtain a planar device we have proposed an indirect transformation method (ITM) based on the principle of equal optical path length [22-23]: the profile of the spatial separation between the original and desired wavefronts is converted to a plane surface, i.e. from AE to BC, as shown in Fig. 1(b). Here it is the virtual region OAED that is transformed into the flat physical one OBCD. Obviously the effective optical path lengths involved in ITM and DTM are equal, thereby leading to the same function.

Let the incident wave front A’E’ be denoted as \( x = g_i(y) \) and the outgoing one B’C’ as \( x = g_0(y) \). Thus the spatial distortion from OAED to OBCD can be written as a mapping,

\[
x' = \frac{cx}{\Delta}, y' = y, z' = z
\]  

(1)

Where c and d are the width and height of the slab respectively and

\[
\Delta = a + g_i(y) - g_0(y)
\]  

(2)

Corresponding to the profile of spatial separation between the two wavefronts AE. According to transformation optics, the permittivity tensor \( \varepsilon \) and permeability tensor \( \mu \) in the transformed coordinate system are connected with the original \( \varepsilon_0 \) and \( \mu_0 \) by the relationships \( \varepsilon = J \varepsilon_0 J^T / \det(J) \), \( \mu = J \mu_0 J^T / \det(J) \) where \( J^T = \partial x' / \partial x \) is the Jacobian transformation matrix [7]. Applying Eq. (1) gives a general result of the relative material parameters for the conversion between two arbitrarily curved wavefront:
Where $\Delta' = \frac{\partial \Delta}{\partial y}$ and the original space is considered as vacuum. Notice that the superscripts of coordinate variables have been omitted for simplicity.

It is important to point out that in deriving Eq. (2), AE should be shifted so as not to intersect the $y$ axis to avoid the singularity in $\varepsilon$ and $\mu$ in the slab.

3. Applying the above method to design the phase transformer
Applying the above method, a wavefront can be converted into any desired one through a slab with $\varepsilon$ and $\mu$ determined by Eq. (3). In the following we discuss phase transformation to illustrate the method. Incidentally, similar wavefronts could have been generated by other media using different methods somewhere in literature, e.g. conventional lens or transformation media, but our principal emphasis is on the new general method used to design flat media to induce phase shift and bend wavefront.

Suppose a cylindrical wave to be a plane wave by the slab. Without loss of generality, then

$$\Delta = -x_1 + \sqrt{rr^2 - (y - rr)^2}$$

And using Eq. (3) one obtains the relative material parameters for the slab:

$$\varepsilon_{xx} = \frac{x^2(y - rr)^2 + c^2[rr^2 - (y - rr)^2]}{c[rr^2 - (y - rr)^2][-x_1 + \sqrt{rr^2 - (y - rr)^2}]}$$

$$\varepsilon_{yy} = \varepsilon_{xx} = \frac{x(y - rr)}{c\sqrt{rr^2 - (y - rr)^2}}$$

$$\varepsilon_{zz} = \varepsilon_{yy} = \varepsilon_{xx} = \frac{-x_1 + \sqrt{rr^2 - (y - rr)^2}}{c}$$

To test the effectiveness of the ITM and the proposed design, suppose $c = 0.5m$, $rr = 0.5m$, $x_1 = 0.2m$, wavelength of incident $\lambda = 0.06m$, the electric field distribution obtained from a finite-element simulation is shown in Figure 2.

**Figure 2.** Normalized E-field patterns in the slab (a) the transformation without the transformer. (b) the transformation with the transformer.
We see that the propagation of electromagnetic wave in the free space in Fig.2 (a) is not changed, it means that the outgoing wavefront is the same as the incident wavefront both cylindrical wave. Interestingly, by comparing (a) and (b) we find that the incident wavefront is transformed from the cylindrical wave to the plane wavefront by the slab in Fig.2 (b). The incident wavefront is gradually flattened inside the slab. The exit beam is totally plane wavefront with the transformer. Meanwhile, the incident is set as point light source in Fig.2 (a) and (b).

4. Conclusion
Indirect transformation method (ITM) is applied to design the phase transformer, which is independent of device shape. The phase transformer can transform cylindrical wave front into plane phase front, the simulation result is well.

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