Feature cognitive model combined by an improved variational mode and singular value decomposition for fault signals

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Abstract: A feature cognitive model combined with an improved variational mode and singular value decomposition is presented to recognise the characteristics of the fault signals from vibration signals of mechanical equipment in this study. Specifically, the variational mode model is constructed firstly to decompose the known fault signals for mechanical equipment with the same load. Singular value decomposition approach is applied to recognise further the inherent modal features of the fault signals and construct the feature set. The supervised learning-support vector machine and the unsupervised learning-fuzzy c-means clustering are used to verify the effectiveness of the presented method. Finally, the provided feature cognitive model is used to recognise the bearing faults to verify its effectiveness. From simulation results, it can be seen that compared to the complete integration empirical mode decomposition (EMD) method, the feature cognitive model combined by an improved variational mode and singular value decomposition can obtain more higher accuracy and larger evaluation coefficients. It is worth mentioning that the presented method can also be applied to recognise the key characteristics of the other signals.

1 Introduction

The vibration acceleration signals collected from the surface of the mechanical equipment are non-stationary, non-linear, complex and variable. They are mixed with other noises. The fault information can be obtained by using signal decomposition or intelligent diagnosis approaches to analyse and decompose the above vibration acceleration signals. In 1998, Huang et al. [1] and Braun and Feldman [2] proposed empirical mode decomposition (EMD). The adaptive mode decomposition method [3–5] based on this method was used widely to analyse the complex non-stationary signals. It decomposed the changing trend of signals into data sequences of different characteristic scales, namely intrinsic mode function (IMF). In 2014, Dragomiretskiy and Zosso [6] proposed a variational mode decomposition (VMD) approach, which can decompose signals adaptively. This method is very suitable for processing non-stationary signals, because it has high decomposition precision, can avoid the problem of mode confusion [7, 8] effectively, and extract the signal accurately with similar central frequency difference method, which can avoid obtaining inaccurate features caused by mode aliasing and insufficient or excessive eigenmode decomposition. In order to improve the accuracy of feature recognition, the SVD model is used to recognise further the features of eigenmode function of VMD and to construct the feature set. The support vector machine (SVM) with the supervised learning function and the fuzzy c-means clustering with unsupervised learning function are used to verify the effectiveness of the presented method. The presented approach is verified by using it to recognise the normal bearing signal, the inner ring fault, the outer ring fault, the rolling element fault. The simulation results are given.

2 Preliminary

VMD [13] defined the IMF as an analytical AM–FM model:

\[ c_k(t) = A_k(t)\cos(\omega_k(t)) \] (1)

where \( c_k(t) \) is the IMF, \( A_k(t) \) is the instantaneous amplitude, and \( \omega_k = \omega_k(t) \) is the instantaneous frequency. Signal \( x(t) \) is decomposed into a series of IMFs \( c_k(t) \). The VMD algorithm iteratively updates the respective IMFs in the frequency domain, and takes the spectrum centre of gravity of the IMF as the centre frequency \( \omega_k \).

It can be assumed that IMF \( c_k(t) \) is sparse if it is compactly clustered around the central frequency \( \omega_k \) in the frequency domain. The \( K \)th decomposition can be considered as the following constrained variational problems:

\[ \min_{\{\phi_k\}} \left\{ \sum_k \left[ \left\| \partial \left( c_k(t) + \frac{1}{\pi t} \right) \right\|_1^2 \right] \right\} \]

s. t. \[ \sum_k c_k(t) = x(t) \] (2)

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where \( \{c_1\} = \{c_2, \ldots, c_l\} \) and \( \{\omega_k\} = \{\omega_1, \ldots, \omega_k\} \) represent the \( k \)th modal component and its corresponding central frequency, respectively. \( K \) is the number of decomposed components. In order to evaluate the bandwidth of each IMF, the analytical signal is constructed by the Hilbert transform to obtain the unilateral spectrum of non-negative frequencies. Then the central frequency band is adjusted to the fundamental frequency band, which is multiplied by an exponential function \( e^{\text{im}\omega t} \). In order to obtain the optimal solution, the above-mentioned constrained variational problem is transformed into a non-constrained variational problem [14].

Considering the following augmented Lagrange function:

\[
L(c_k(t), \{\omega_k\}, \lambda(t)) = \alpha \sum_k \left[ \frac{1}{2} \left( \delta_k \left( \sigma(t) + \frac{1}{N} + c_k(t) e^{\text{im}\omega_k t} \right) \right)^2 \right]_2 + \| x(t) - \sum_k c_k(t) \|_2^2 + \langle \lambda(t), x(t) - \sum_k c_k(t) \rangle
\]

where \( \alpha \) is a penalty parameter, \( \lambda \) denotes the Lagrangian multiplier, and \( \langle \cdot \rangle \) denotes the inner product. In the case of the optimal solution to formula (2), the saddle point formula (3) Lagrange function can be found by means of the sequence iterative suboptimal method, and the original signal is decomposed into \( K \) narrow-band IMF components.

3 Main results

3.1 VMD model

It can be seen from the principle of the VMD algorithm that the number of modal states \( k \) to be decomposed needs to be given firstly. However, different devices have different vibration signals, and the value of \( k \) is also uncertain. In this paper, the centre frequency difference method is used to make sure the number of modal components of the VMD algorithm. The centre frequency [15] of each IMF is an intuitive index, which reflects the nature of IMF. Every IMF has a central frequency value. What is the value of \( K \), you can get how many centre frequencies. The centre frequency can be re-evaluated by changing the value of \( K \), so that the model can be updated.

The specific implementation steps are as follows:

(i) initialise \( k = 1 \);
(ii) when \( k = k + 1 \), an outer loop is executed;
(iii) initialise \( \{c_1\}, \{\omega_1\}, \lambda \) with \( n \);
(iv) when \( n = n + 1 \), the inner loop is executed;
(v) for \( k = 1 : K \), update \( c_k^{n+1}(t), \omega_k^{n+1} \):

\[
c_k^{n+1}(t) = \tilde{\lambda}(\omega) - \sum_{k=1}^{n+1} \langle \tilde{\lambda}(\omega), \omega_k^{n+1} \rangle + (1/2 \sigma_k^{n+1}(t) + 1 + 2\alpha(\omega - \omega_k^n) \sum_{k=1}^{n+1} \omega_k^{n+1} \rangle
\]

\[
\omega_k^{n+1} = \left( \int_0^{\omega} \rho_k^{n+1}(\omega) d\omega \right)^{-1} \left( \int_0^{\omega} \omega_k^{n+1}(\omega) d\omega \right)
\]

(vi) if \( \omega \geq 0 \), update \( \lambda^{n+1} \):

\[
\lambda^{n+1} = \lambda^{n} + \sigma \left( \tilde{\lambda}(\omega) - \sum_{k=1}^{n+1} \omega_k^{n+1}(\omega) \right)
\]

(vii) repeat steps (iv)–(vi) until satisfied

\[
\sum_k \| c_k^{n+1}(t) - c_k^n(t) \|^2_2 < \epsilon
\]

Stop iteration and continue to the next step, otherwise return to step (iv);

3.2 Constructing feature set by using SVD

SVD [16] is an important matrix decomposition method in linear algebra, which can effectively extract the characteristics of matrices. It is constantly applied in various fields such as image compression, article classification, signal noise reduction and fault diagnosis. In this paper, SVD application in signal processing is used to extract the feature vectors of each eigenmode function obtained by VMD algorithm.

Assuming that in the \( A \in \mathbb{R}^{m \times n} \) matrix [17], the rank is represented by \( r [r = \min(m, n)] \), this implies that the orthogonal matrices:

\[
U = [u_1, u_2, \ldots, u_m], \quad U \in \mathbb{R}^{m \times r};
\]

\[
V = [v_1, v_2, \ldots, v_n], \quad V \in \mathbb{R}^{n \times r};
\]

and diagonal matrix:

\[
D = \begin{pmatrix} S & 0 \\ 0 & O \end{pmatrix}
\]

meet the following relationship:

\[
A = UDV^T.
\]

where \( A_{m \times r} \) is the SVD of the matrix. Among them \( O \) is a zero matrix, \( S = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_r) \).

3.3 Constructing feature cognitive model combined by an improved variational mode and singular value decomposition

Fault diagnosis based on feature extraction methods of the variational mode and SVDs can be divided into five steps: signal partition, determination of VMD number, sample signal VMD, SVD to construct feature set, SVM training and diagnosis. The exact method is display in Fig. 1:

(i) the vibration signal obtained by the acceleration is divided based on the waveform period in order to obtain the sample signal \( x_i \);
(ii) the parameters of the VMD method are optimised by using the central frequency difference, and the optimal mode decomposition factor \( K \) is calculated and set as the VMD's parameter \( K \);
(iii) VMD of the sample signal \( x_i \) to derive \( k \)th IMF components;
(iv) a SVM is performed on each IMF component to quantify their best features and form the signal's eigenvector;
(v) a SVM is used to train and test feature vectors, and the recorded results are analysed.

4 Simulation results

This paper makes use of the Case Western Reserve University Laboratory's bearing fault data to conduct a test simulation [19]. The drive end bearing and fan end bearing types are SKF6203 and SKF6205, respectively. The bearing to be tested supports the motor's rotating shaft and is single-point damaged by the electric discharge machining. The damage diameters are 0.007, 0.014, and 0.021 inches. The damage points of bearing's the outer ring are located at 3, 6 and 12 o'clock. The experimental data of 1730 r/s, damage diameter of 0.007 inches and sampling frequency of 12 kHz were selected. Forty groups of normal vibration signals, inner ring fault signals, outer ring fault signals (comprising of three kinds of position faults at moments 3, 6 and 12 in time) and rolling element fault signals were extracted from the experimental samples. Six kinds of state signals forming a total of 240 signal groups. Among them, 24 samples were used for training whilst 16 samples were used for testing. The four typical vibration signal's time-domain waveforms are shown in Fig. 2 (red line is normal
4.1 Constructing effective feature set

According to the above data, the original signal is classified into several samples, and the outer ring 6 o'clock fault is taken as an example for analysis and testing. Table 1 shows the centre frequency value under each $K$ value. From Table 1, we get $\omega_k - \omega_{k+1} \leq 2$, $\omega_{k+1} - \omega_k \leq 9$. In order to avoid the over decomposition, the intrinsic mode number $K$ is selected as 6.

The number of modal states in the normal state is 6, the number of modes in the inner ring fault is 4, the number of modes in the rolling element is 4, and the number of modes in the outer ring fault is 6 (6 o'clock position), 6 (3 o'clock position), 4 (12 o'clock position). After determining the number of modal decompositions, $\alpha$ and $\varepsilon$ take default values and 0.3, respectively, for the VMD to reach the decomposition mentioned in [20]. The decomposition results are shown in Fig. 3. Using the above $k$-values, the sample signals are separately subjected to VMDs, each IMF's singular decomposition values are calculated, and the optimal construction feature set is extracted.

In [21], the diagnosis of rotor fault is attained, and the feature extraction method combining complete ensemble empirical mode decomposition with adaptive noise (CEEMDAN) and singular value is used for research. This method is used for comparative experiments in order to further explore the advantages and disadvantages of adaptive decomposition signal feature extraction methods. Of all these, CEEMDAN [22] decomposes the signal and obtains a series of IMF components from high to low frequency. These IMF component parts contain the characteristic information useful for the signal. Taking the aforementioned outer ring 6 o'clock position fault signal as an example, the CEEMDAN algorithm is performed to obtain 12 IMFs. The false IMF discriminant method based on the correlation analysis is selected to identify CEEMDAN's effective mode.

That means the magnitude of the correlation coefficient between each IMF and the original signal after the decomposition is calculated as the reliability index of the discrimination. Table 2 depicts the correlation coefficients between the first six IMFs and the original signals. It can be observed that the correlation between IMF1 and IMF2 components is the most significant. With the increase of decomposition components, the correlation coefficient

Fig. 1 Diagnostic flow based on improved VMD

![Diagnostic flow based on improved VMD](image)

Fig. 2 Time-domain waveform of typical signals

![Time-domain waveform of typical signals](image)
Table 1  Centre frequency value of fault signal at 6 o'clock position of outer ring by VMD

| K | $\omega^1$ | $\omega^2$ | $\omega^3$ | $\omega^4$ | $\omega^5$ | $\omega^6$ |
|---|---|---|---|---|---|---|
| 2 | 2290 | 2868 | — | — | — | — |
| 3 | 1881 | 2359 | 2876 | — | — | — |
| 4 | 832 | 2339 | 2805 | 3068 | — | — |
| 5 | 736 | 1931 | 2428 | 2868 | 4272 | — |
| 6 | 546 | 1132 | 2344 | 2795 | 3026 | 4293 |
| 7 | 529 | 1082 | 2406 | 2243 | 2803 | 3034 |
| 8 | 527 | 1073 | 2211 | 2394 | 2756 | 2870 |

| K | $\omega^7$ | $\omega^8$ | $\omega^9$ | $\omega^{10}$ | $\omega^{11}$ | $\omega^{12}$ |
|---|---|---|---|---|---|---|
| 2 | — | — | — | — | — | — |
| 3 | — | — | — | — | — | — |
| 4 | — | — | — | — | — | — |
| 5 | — | — | — | — | — | — |
| 6 | — | — | — | — | — | — |
| 7 | — | — | — | — | — | — |
| 8 | — | — | — | — | — | — |

Table 2  Correlation coefficients between the first six modes

| Number | IMF1 | IMF2 | IMF3 | IMF4 | IMF5 | IMF6 |
|---|---|---|---|---|---|---|
| coefficient | 0.9791 | 0.2302 | 0.0923 | 0.0755 | 0.0498 | 0.0342 |

Table 3  Correlation coefficients between the last six modes

| Number | IMF7 | IMF8 | IMF9 | IMF10 | IMF11 | IMF12 |
|---|---|---|---|---|---|---|
| coefficient | 0.0138 | 0.0012 | 0.0002 | 0.0000 | 0.0002 | 0.0000 |

gradually declines. Table 3 lays out the correlation coefficients between the last six IMFs and the original signals, but the correlation is minute and the fault characteristics are not obvious. The IMF7-IMF12 component is omitted. This implies that IMF1-IMF6 is chosen as the effective component of extraction. The SVD is applied to the first six IMFs, and a feature set based on CEEMDAN and SVD is constructed.

4.2 Fault analysis under the same load components

Six signal states under the same load are selected as the experiment data in this section. There are data of 40 groups, in which the sampling point of each group data is 2048. The following are considered. Label 1 is a normal sample; label 2 is an inner ring fault sample; label 3 is a rolling element fault sample; label 4 is the outer ring fault sample at 6 o’clock position; label 5 is the outer ring fault sample at 3 o’clock position; label 6 is the outer ring fault sample at 12 o’clock position. In this paper, the six states of rolling bearing are extracted by using the improved VMD algorithm and SVD. At the same time, the six states of rolling bearing are identified by using the ‘one to many’ multi-classification SVM in the SVM pattern recognition software package LibSVM developed by Taiwan University’s Professor Lin Zhiren. There are many kernel functions commonly used in SVMs. In this particular context, the radial basis kernel function with few calculation parameters compared with other kernel functions is applied.

The feature set is sent to the SVM for fault identification, and the recognition accuracy is shown in Fig. 4. Meanwhile, the CEEMDAN-based method is used for fault identification, and the recognition accuracy is revealed in Fig. 5. From Fig. 5, it is clearly seen that one sample point of the normal signal is misclassified into a 12 o’clock position fault of label 6’s outer ring, resulting in a slightly lower accuracy. In Fig. 4, each sample can be accurately assigned to the corresponding fault label. The experimental results prove that the feature set's fault recognition rate extracted based on the improved VMD algorithm is higher than that of the complete EMD based on adaptive noise.

In order to avoid the influence of label on fault identification, another unsupervised identification algorithm fuzzy c-means that is the clustering algorithm is adopted. This algorithm is a perfect
combination of fuzzy theory and c-mean algorithm by mathematicians. The authors in [15] describe its principle in detail, and applies the algorithm combined with the improved ISOMAP algorithm to the fault identification of roll bearing, which solves the problem of partial overlap in fault identification.

In this paper, three evaluation indexes of the algorithm are used to analyse and verify the advantages and disadvantages of the two feature extraction methods. The three evaluation indexes are Hamming closeness, classification coefficient $F$ and average fuzzy entropy $H$. Closeness is a concept in fuzzy mathematics, in which the Hamming closeness is used to evaluate the effect of classification intuitively. Then calculate the Hamming closeness formula of test samples $A_i$ and $B_i$ in each state as follows:

$$N(B_i, A) = 1 - \frac{1}{n} \sum_{k=1}^{n} |B_i(x_k) - A_i(x_k)|$$  \hspace{1cm} (8)

classification coefficient $F$:

$$F = \frac{1}{n} \sum_{j}^{b} \sum_{i}^{a} u_{ij}$$ \hspace{1cm} (9)

average fuzzy entropy $H$:

$$H = -\frac{1}{n} \sum_{j}^{b} \sum_{i}^{a} u_{ij} \ln u_{ij}$$ \hspace{1cm} (10)

The Hamming closeness histograms in Figs. 6 and 7 clearly demonstrate that the test samples obtained by the feature extraction method based on VMD–SVD and the clustering centre are closer to 1. However, the closeness of the test samples obtained by the feature extraction method based on CEEMDAN–SVD and the cluster centre is significantly smaller.

Furthermore, from the classification coefficient in Table 4, the former has a high classification coefficient and the average fuzzy entropy is closer to 0; whereas the latter has a low classification coefficient and a large average fuzzy entropy. Consequently, the feature extraction method based on variational mode and SVDs can extract fault feature information more accurately, making the fault identification easier.

5 Conclusion

A feature cognitive model combined by an improved variational mode and SVD is presented to recognise the characteristics of the fault signals from vibration signals [10–12] of mechanical equipment in this paper:

(i) The constructed variational mode model can decompose the known fault signals for mechanical equipment with the same load. The SVD approach can effectively recognise the inherent modal features of the fault signals.

(ii) Concerning the extraction of roll bearing vibration signals, the adaptive processing method as reported by the VMD is verified from different angles through supervised and unsupervised learning. The algorithm advantages are more discernible for signal separation and extraction of effective features.

(iii) Although the complete EMD with adaptive noise is continuously improved on the basis of EMD, it is still inferior to the VMD in the feature extraction effect, and always relies on the EMD algorithm.

(iv) The VMD algorithm has a rigorous mathematical foundation. Its feature extraction is highly adaptive, but there are still some inadequacies, for instance the fact that the parameters need to be set artificially. There is no theoretical basis for the parameter setting requirements, and further exploration is necessary.

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References

[1] Huang, N.E., Shen, Z., Long, S.R., et al.: ‘The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis’, Proc. R. Soc. A, Math. Phys. Eng. Sci., 1998, 454, pp. 903–995

[2] Braun, S., Feldman, M.: ‘Decomposition of non-stationary signals into varying time scales: some aspects of the EMD and HVD methods’, Mech. Syst. Signal Process., 2011, 25, (7), pp. 2608–2630

[3] Park, M., Kim, D., Oh, H.-S.: ‘Quantile-based empirical mode decomposition: an efficient way to decompose noisy signals’, IEEE Trans. Instrum. Meas., 2015, 64, (7), pp. 1802–1813

[4] Xu, Y., Wang, D., Zhang, W., et al.: ‘Detection of ventricular tachycardia and fibrillation using adaptive variational mode decomposition and boosted-CART classifier’, Biomed. Signal Process. Control, 2018, 39, pp. 219–229

[5] Kou, K.I., Li, H.: ‘Greedy adaptive decomposition of signals based on nonlinear Fourier atoms’, J. Wavelets Multiresolution Inf. Process., 2016, 14, (3), p. 1650014

[6] Dragomiretskiy, K., Zosso, D.: ‘Variational mode decomposition’, IEEE Trans. Signal Process., 2014, 62, pp. 531–544

[7] Anesh, C., Kumar, S., Hisham, P.M., et al.: ‘Performance comparison of variational mode decomposition over empirical wavelet transform for the classification of power quality disturbances using support vector machine’, Procedia Comput. Sci., 2015, 46, pp. 372–380

[8] Yao, J., Xiang, Y., Qian, S., et al.: ‘Noise source identification of diesel engine based on variational mode decomposition and robust independent component analysis’, Appl. Acoust., 2017, 116, pp. 184–194

[9] Li, Z., Jiang, Y., Guo, Q., et al.: ‘Multi-dimensional variational mode decomposition for bearing-crack detection in wind turbines with large driving-speed variations’, Renew. Energy, 2018, 116, (Part B), pp. 55–73

[10] Wu, Q., Lin, H.: ‘Daily urban air quality index forecasting based on variational mode decomposition, sample entropy and LSTM neural network’, Sustain. Cities Soc., 2019, 50, p. 101657

[11] Fang, J., Wen, Z., Gu, H., et al.: ‘Seismic random noise suppression method based on variational mode decomposition’, Pet. Geophys. Explor., 2019, 54, (4), pp. 757–767

[12] Li, J., Zhu, S., Wu, Q.: ‘Monthly crude oil spot price forecasting using variational mode decomposition’, Energy Econ., 2019, 83, pp. 240–253

[13] Lv, Z.: ‘Research on early fault diagnosis method of rotating machinery based on variational mode decomposition and optimization of multi-core support vector machine’, PhD thesis, Chongqing University, 2016, pp. 51–53

[14] Xin, L., Wang, J., Su, X.: ‘Early fault diagnosis of gear based on VMD and minimum entropy deconvolution’, Machinery & Manufacture, 2019, 6, pp. 50–54

[15] Wang, Y.-p., Li, S.-s., Ge, J.-H., et al.: ‘Fault identification of rolling bearing by equidistance mapping and fuzzy c-means’, J. Harbin Univ. Sci. Technol., 2019, 24, pp. 41–47

[16] Zhang, A., Huang, J., Wei, J., et al.: ‘Study on fault diagnosis of planetary gear box based on EMD-SVD and PNN’, Mech. Transm., 2018, 42, pp. 160–165

[17] Shao, X., Xin, Y.: ‘A scheduling algorithm based on the singular value decomposition heuristic method in a distributed manufacturing system’, Expert Syst., 2019, 36, p. e12433

[18] Xu, Y., Cao, S., Pan, X., et al.: ‘Random noise attenuation using a structure-oriented adaptive singular value decomposition’, Acta Geophys., 2019, 67, pp. 1091–1106

[19] Liu, C., Wu, Y., Zhen, C.: ‘Fault diagnosis of rolling bearing based on variational mode decomposition and fuzzy c-means clustering’, Chin. J. Electr. Eng., 2015, 35, pp. 3358–3365

[20] Chen, S., Yang, Y., Dong, X., et al.: ‘Warped variational mode decomposition with application to vibration signals of varying-speed rotating machineries’, IEEE Trans. Instrum. Meas., 2019, 68, (6), pp. 2755–2767

[21] Yue, X., Liu, F., Zhou, X., et al.: ‘Rotor fault diagnosis based on CEEMDAN singular value entropy and SVM’, Manuf. Technol. Mach. Tools, 2018, 674, (8), pp. 87–92

[22] Shi, Z., Gu, C., Cao, L., et al.: ‘Study on rotor fault diagnosis of steam turbine based on CEEMDAN and CBBO-SVM’, Thermal Power Eng., 2018, 33, (1), pp. 69–74