Time evolution of spin accumulation and spin current in a magnetic domain wall

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Abstract. Time evolution of the spin accumulation and spin current in a magnetic domain wall was studied by solving the Boltzmann equation of the spin accumulation. Assuming the spatially homogeneous electric current density, we find that the time evolution of the longitudinal spin current is characterized by the momentum relaxation time of the conducting electrons. On the other hand, the time evolution of the transverse spin accumulation and spin current are characterized by the spin-flip relaxation time.

Spin-transfer torque induced in the magnetic multilayer and magnetic domain wall has been enormously studied because of its potential applications to spin-electronics devices such as non-volatile magnetic random access memory \cite{1} and racetrack memory \cite{2}. The concept of spin-transfer torque was first proposed by Slonczewski \cite{3} and independently by Berger \cite{4} in 1996 for magnetic multilayer systems. Recently, it has been revealed that Slonczewski torque (the adiabatic torque) in a domain wall is not sufficient to interpret the experimental data \cite{5,6}. In 2004, Zhang and Li estimated the spin transfer torque in the domain wall by solving a phenomenological diffusion equation of the spin accumulation, and showed the existence of the non-adiabatic torque \cite{7}. In Zhang and Li theory, the ratio of the magnitude of the adiabatic and non-adiabatic torque is characterized by the precession frequency of the spin accumulation around the localized magnetization and the spin-flip scattering time, respectively, and the non-adiabatic torque is two orders of magnitude smaller than the adiabatic torque. However, in Zhang and Li theory, the spin accumulation is assumed to be static and spatially homogeneous. Recently, we studied the spin accumulation by solving the Boltzmann equation in a steady state, and showed that the magnitude of the non-adiabatic torque is increased with decreasing the thickness of the domain wall \cite{8}. For a domain wall whose thickness is much smaller than the spin-flip length, the magnitude of the non-adiabatic torque is only one order of magnitude smaller than the adiabatic torque.

In this paper, we study the time evolution of the spin accumulation and spin current by solving the Boltzmann equation of the spin accumulation. Assuming the spatially homogeneous electric current density, we find that the time evolution of the longitudinal spin current is characterized by the momentum relaxation time, which is on the order of $10^{-6}$ ns. On the other hand, the time evolution of the transverse spin accumulation and spin current are characterized by the spin-flip scattering time, which is on the order of $10^{-4}$ ns.
We consider an electron transport in one dimensional magnetic nanowires with a 180° linear magnetic domain wall lying over \(-d/2 \leq x \leq d/2\). We assume that the spin transport is described by the sd model in which the conducting electron interacts with the localized spin angular momentum via the Hamiltonian \(\hat{H}_{sd} = -(J/2) \hat{\sigma} \cdot \hat{S}\), where \(J\) is the sd coupling constant, \(\hat{\sigma}\) is the vector of the Pauli matrices and \(\hat{S} = (0, -\sin \theta, \cos \theta)\) is the unit vector along the direction of the localized spin angular momentum. The angle \(\theta\) is given by 0 for \(x < -d/2\), \((\pi/2)/(x + d/2)\) for \(-d/2 \leq x \leq d/2\) and \(\pi\) for \(x > d/2\). We employ the rotating frame in which the basic unit vectors are defined as \(\hat{e}_x = \alpha^{-1}\hat{S} \times (\partial \hat{S}/\partial x)\), \(\hat{e}_y = -\alpha^{-1}\partial \hat{S}/\partial x\) and \(\hat{e}_z = \hat{S}\), where \(\alpha = d\theta/dx\).

The spin accumulation is obtained by solving the Boltzmann equation for the Wigner function \(\hat{F}(x, p_x, t) = \{f(x, p_x, t) + g(x, p_x, t)\} \cdot \hat{\sigma}\), where \(f(x, p_x, t)\) and \(g(x, p_x, t)\) are the charge and spin distribution functions, respectively. The accumulation \(\mathbf{s}\) and spin current \(\mathbf{j}\) are defined as

\[
\mathbf{s} = \int \frac{d^3 p}{(2\pi\hbar)^3} \mathbf{g}, \tag{1}
\]

\[
\mathbf{j} = \int \frac{d^3 p}{(2\pi\hbar)^3} v_x \mathbf{g}, \tag{2}
\]

respectively. We assume that the localized spin angular momentum varies slowly in space compared to the Fermi wavelength \(\lambda_F\), i.e., \(\alpha \ll 2\pi/\lambda_F\). Then, up to the first order of \(\alpha\), the Boltzmann equation of \(f(x, p_x, t)\) and \(g(x, p_x, t)\) are given by [9, 10]

\[
\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + ev_x E_0 \frac{\partial f}{\partial \varepsilon} = -\frac{f^{(1)}}{2T} + \frac{\beta g^{(1)}}{2T}, \tag{3}
\]

\[
\frac{\partial g_z}{\partial t} + v_x \frac{\partial g_z}{\partial x} = -\frac{g_z^{(1)}}{2T} + \frac{\beta f^{(1)}}{2T} - \frac{2g_y^{(1)}}{\tau_{sf}}, \tag{4}
\]

\[
\frac{\partial g_x}{\partial t} + v_x \frac{\partial g_x}{\partial x} - \omega_J g_y = -\frac{g_x^{(1)}}{2T} - \frac{2g_y^{(1)}}{\tau_{sf}}, \tag{5}
\]

\[
\frac{\partial g_y}{\partial t} + v_x \frac{\partial g_y}{\partial x} - \alpha v_x g_z + \omega_J g_x = -\frac{g_y^{(1)}}{2T} - \frac{2g_y^{(1)}}{\tau_{sf}}, \tag{6}
\]

where \(E_0\) is the applied electric field, \(T = (1/T_1 + 1/T_1)^{-1}\) is the average of the momentum relaxation time of the spin electrons \(T_s\), \(\beta = (T_1 - T_1)/(T_1 + T_1)\) is the spin polarization factor, \(\omega_J = J/\hbar\) is the precession frequency of the spin accumulation around the localized spin angular momentum and \(\tau_{sf}\) is the spin-flip scattering time. \(f^{(0)}\) is the charge distribution function in equilibrium. According to Šimáněk and Rebei [9, 10], we assume that the non-equilibrium components of \(f\) and \(g\) are decomposed as \(f^{(1)}(x, p_x, t) + g^{(1)}(x, t)\) and \(g^{(1)}(x, p_x, t) + g^{(1)}(x, t)\), where \(f^{(1)}\) and \(g^{(1)}\) represent the linear response to the electric field and \(\tilde{f}^{(1)}\) and \(\tilde{g}^{(1)}\) represent the local variation of the chemical potential [11]. It should be noted that the spin accumulation and spin current defined by Eqs. (1) and (2) are determined by \(\tilde{g}^{(1)}\) and \(\tilde{g}^{(1)}\), respectively. In following we apply the diffusion approximation \(\int v_x^2 g^{(1)} d^3 p/(2\pi\hbar)^3 = \int v_x^2 \tilde{g}^{(1)} d^3 p/(2\pi\hbar)^3 \approx (v_F^2/3)\mathbf{s}\) to the Boltzmann equation.

As shown by Šimáněk and Rebei [10], up to the first order of \(\alpha\), the longitudinal (collinear to \(\hat{S}\) spin accumulation in a domain wall can be neglected. On the other hand, the longitudinal spin current acts as the source of the transverse (perpendicular to \(\hat{S}\) spin accumulation and spin current, as shown below. The time evolution of the longitudinal spin current is estimated by solving Eqs. (3) and (4). Assuming the spatially homogeneous electric current density \(j_e\) and using the Ohm’s law, we find that the longitudinal spin current \(j_z\) is given by

\[
j_z = \beta \frac{j_e}{e} \left[ 1 - \frac{(1 + \beta)}{2\beta} \exp \left\{ \frac{-(1 - \beta)}{2T} t \right\} + \frac{(1 - \beta)}{2\beta} \exp \left\{ \frac{-(1 + \beta)}{2T} t \right\} \right], \tag{7}
\]
The equations of motion of the transverse spin accumulations, $s_x$ and $s_y$, are obtained by integrating Eqs. (5) and (6) over the momentum space, and are given by

$$\frac{\partial s_x}{\partial t} + \frac{\partial j_x}{\partial x} - \omega_j s_y + \frac{2}{\tau_{sf}} s_x = 0,$$

$$\frac{\partial s_y}{\partial t} + \frac{\partial j_y}{\partial x} + \omega_j s_x + \frac{2}{\tau_{sf}} s_y = \alpha j_z,$$

$$\frac{\partial j_x}{\partial t} + \frac{D}{T} \frac{\partial s_x}{\partial x} - \omega_j j_y + \frac{1}{2T} j_x = 0,$$

$$\frac{\partial j_y}{\partial t} + \frac{D}{T} \frac{\partial s_y}{\partial x} + \omega_j j_x + \frac{1}{2T} j_y = 0,$$

where $D = v_F^2 T / 3$ is the diffusion coefficient. We solve Eqs. (8)-(11) numerically with Eq. (7). The boundary conditions of the transverse spin accumulation and spin current are that they vanish in the limit of $|x| \to \infty$.

Figures 1 (a) and 1 (b) show the time evolution of the transverse spin accumulations at $x=0$ where the time $t$ is renormalized by the spin-flip scattering time $\tau_{sf}$. The parameters are taken to be $J = 1.0$ eV, $\beta = 0.5$, $l_{mfp} = 3.0$ nm, $\tau_{sf} = 10^{-4}$ ns, $j_e = -5.0 \times 10^7$ A/cm$^2$ and $d = 30$ nm. The Fermi velocity is determined by the Fermi energy $\varepsilon_F$ via $v_F = \sqrt{2\varepsilon_F/m}$, where $\varepsilon_F$ and $m$ are taken to be 5.0 eV and the value of the free electrons, respectively. As shown in Figs. 1 (a) and 1 (b), the time evolution of the transverse spin accumulation is characterized by the spin-flip scattering time $\tau_{sf}$. According to Eqs. (10) and (11), the time evolution of the transverse spin current is determined by the transverse spin accumulation. Thus, the time
evolution of the transverse spin current is also characterized by the spin-flip scattering time. Figs. 1 (c) and 1 (d) show the spatial variation of the spin accumulation $s_x$ and $s_y$, respectively. The solid and dotted lines correspond to the spin accumulation at $t = 0.1\tau_{sf}$ and in a steady state ($t \to \infty$), respectively. As shown in Figs. 1 (c) and 1 (d), the spin accumulation is induced at the boundaries of the domain wall at $t \ll \tau_{sf}$, and spread into and out to the domain wall.

The physics behind the above results are as follows. We neglect the longitudinal spin accumulation because the magnitude of it is higher order of the small parameter $\alpha$ [10]. Then we find that $g_z^{(1)} = 0$ and the spin-flip scattering time $\tau_{sf}$ plays no role on the time evolution of the longitudinal spin current; see Eq. (4). The currents of the spin-up and spin-down electrons flow through the domain wall independently, and the relaxation of the motion of them are characterized by $T_1 = 2T/(1-\beta)$ and $T_1 = 2T/(1+\beta)$, respectively. Thus, the time evolution of the longitudinal spin current is characterized by the momentum relaxation time $T$. On the other hand, the transverse spin accumulations are on the first order of $\alpha$ [8]. The relaxations of the transverse spin accumulations are characterized by the spin-flip scattering time, as shown in Eqs. (8) and (9). The origin of the transverse spin accumulation is the spatial gradient of the localized spin angular momentum, $\partial S/\partial x$, which is proportional to $\alpha$. The electrons injected into the domain wall change the direction of their spins to the direction of $S(x = \pm d/2)$ at the boundaries of the domain wall at first, and then change the direction of spins to the directions of $S(x = -d/2 < x < d/2)$ in the wall. Thus, the spin accumulation is first induced at $x = \pm d/2$.

We add the comments about the models of the angle of the magnetization $\theta$. There’re two models about the angle $\theta$; one is the linear model, i.e., $\theta = (\pi/d)(x+d/2)$ [9, 10], and the other is the tangent model, i.e., $\theta = \pi - 2\arctan[\exp(x/d)]$ [13, 14]. The linear model we used corresponds to a rigid domain wall while the tangent model corresponds to a soft domain wall. The spatial distribution of the transverse spin accumulation in the tangent model is presented in Ref. [14].

In conclusion, we studied the time evolution of the spin accumulation and spin current in a magnetic domain wall by solving the Boltzmann equation of the spin accumulation. We find that the time evolution of the longitudinal spin current is characterized by the momentum scattering time while the time evolution of the transverse spin accumulation and spin current is characterized by the spin-flip scattering time $\tau_{sf}$. We also find that the transverse spin accumulation is induced at the boundaries of the domain wall at $t \ll \tau_{sf}$ and spread into and out to the wall.

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