A Method of Selecting the Number of Coordinates of a Vector Criterion in a Polyoptimal Decision-Making Process

Bogdan Zak

Polish Naval Academy, Smidowicza 69, 81-127 Gdynia, Poland; b.zak@amw.gdynia.pl; Tel.: +48-261-262-635

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Featured Application: The formulated method of selecting the number of scalar criteria in the vector criterion can be used to design optimal control issues, as well as in solving other decision-making tasks formulated as a multicriteria optimization task.

Abstract: An important problem when formulating multicriteria optimization tasks is the selection of such a number of partial criteria that their meaning, number, and order correspond to the modeled decision situation. The paper proposes a method of selecting the number of criteria in polyoptimal decision-making process. The conditions of completeness, coherence, and nonredundancy that must be met by the vector criterion adopted for the assessment of the decision situation were discussed. Using these conditions, the principles of selecting the number of partial criteria in the vector criterion were specified, specifying the theorem on extending or reducing the scalar criterion. Using the formulated method, a vector quality criterion was selected for the task of controlling the movement of a sailing object. The method proposed in the work can be useful for modeling any decision-making situation in such issues as designing complex systems, controlling complex objects in conditions of uncertainty, and making decisions in other systems.

Keywords: decision making; vector criterion; polyoptimization

1. Introduction

The increasing complexity of control systems, economic and social systems, as well as design problems increases the complexity of related decision problems [1]. They concern, among other things, the decision-making of control of complex systems [2,3], design issues [4–6], political systems [7], organization management [8–13], financial management [14,15], and marketing [16]. Many decision-making problems are characterized by high dimensionality [17], sources of uncertainty, and risk factors [18]. It is important to reconcile conflicting goals, make decisions based on many criteria, and seek compromise [19]. Policy makers are faced with the complexity of decision-making situations and require methods and systems to support decision making [20]. In response to these needs, many solutions have been developed dedicated to selected areas, as well as general purpose methods [21]. In this context, multicriteria decision analysis (MCDA) methods are widely used. In addition to the formal foundations, these methods are characterized by the ability to deal with many conflicting goals, as well as various stakeholders as part of the decision-making process [22]. The dynamic development of MCDA methods has been observed in recent years [23–25] as evidenced by scientific publications appearing in magazines on this subject. However, they differ significantly in many dimensions, such as complexity, way of representing preferences and assessment criteria, type of data aggregation, the possibility of taking into account uncertain data, and the availability of implementation in decision support systems or criteria compensation [26–28].
The huge number of possible MCDA methods allow finding a solution to a properly formulated decision situation. Therefore, an important research issue that has not yet been fully resolved is determining the method for selecting the number of criteria to assess situations and selecting the most satisfactory decision. That is why the formulated research problem is so important for the methods of managing industrial, technological, and social systems. This is an especially important issue in the security and control systems of complex objects in dynamic states in conditions of uncertainty and risk. It follows from a very large number of factors influencing the decisions taken and determining the control, nonscalar criterion functions, and difficulties in determining the direct relationship between the components of the criterion function and these factors. In addition, in these types of systems, especially in conflict situations, there are usually strong restrictions on the time allowed to solve the problem. An additional impediment to their implementation for the decision-maker is the stress resulting from high responsibility for the decisions made and usually a strong limitation in terms of the time they require. The above-mentioned conditions imply the legitimacy of making efforts to model the problems in the form of optimization tasks. Hence, it is desirable to provide computer support for activities performed in individual stages of the decision-making process. In the literature on the subject, there are practically no publications on methods of selecting partial criteria, including the selection of numbers and aspects of assessment. Few publications [29–31] relate primarily to the selection of assessment criteria in the selection process of suppliers in the supply chain. Most often, this problem is solved by experts who arbitrarily define individual partial criteria. This approach may lead to an unnecessary increase in the size of the decision assessment vector or failure to consider relevant criteria. A necessary condition for optimizing decisions is to have a measure of quality assessment [32] formulated to cover all aspects of the problem. For this reason, the main aim of the conducted studies is to formulate a method for selecting the number of criteria to assess the quality of decisions taken.

This article contains five sections. Section 1 introduced the problem, describes the state of the art, and gives the main aim of research. Section 2 presents the problem formulation of the multicriteria optimization of decision tasks. In Section 3, we present the proposed method of selecting subcriteria, as well as theorems and proof used. In Section 4, we present a case study applying the proposed method for formulating vector criterion for an anticollision ship motion control system. All experiments and obtained results are discussed and summarized in Section 5.

2. The Problem Formulation

An important property of decision-making optimization tasks is the fact that there is a nonempty set of \(X\) solutions. These can be, for example, the states that will be achieved after making a specific decision. When making a decision, the effects it will bring must be evaluated. It is often difficult to use only one criterion when choosing the right decision, especially in complex situations. For this reason, decisions are evaluated using multiple scalar criteria. This task can be formulated as follows:

\[ F : X \rightarrow \mathbb{R}^N \tag{1} \]

where \(\mathbb{R}\) is the real number set.

The function \(F\) assigns the evaluation to the solution \(x \in X\), as follows:

\[ F(x) = (F_1(x), F_2(x), \ldots, F_n(x), \ldots, F_N(x)) \in \mathbb{R}^N \tag{2} \]

where \(N = \{1, 2, \ldots, n, \ldots, N\}\) is the scalar criteria number set.

We are considering a set of possible solutions \(x_1, x_2, \ldots, x_i, \ldots, x_m \in X\). We assume that \(x_i\) solutions are evaluated using the following criteria:

\[ F_1(x_i), F_2(x_i), \ldots, F_n(x_i), \ldots, F_N(x_i) \in \mathbb{R}^N \tag{3} \]
One of the problems occurring at the stage of formulation of vector optimization tasks is the selection of $N$ scalar criteria so that their meaning, number, and order correspond to the modeled decision situation. Therefore, the task of determining the vector criterion (1) can be formulated so that [2,3]:

$$Q(F^*) = \sup\{Q(F) \mid F \in F\}$$

(4)

where $F$ is the set of acceptable vector criteria and $Q(F)$ is the quality indicator.

When designing the system, the user and the designer determine acceptable scalar criteria by assessing the quality of the adopted decision task solutions. Therefore, user preferences regarding the desired properties of solutions should be taken into account when choosing the number of partial criteria and when choosing the appropriate dominance relationship. Thus, the user can specify their preferences at the stage of formulating a multicriteria optimization task. The correct selection of the number of partial criteria is crucial when modeling a specific decision situation. From the user’s point of view, adopting a negligible partial criterion may lead to proposing solutions with undesirable properties and omitting solutions with interesting features.

3. Method Description

3.1. Conditions for Selecting Subcriteria

The set of $N$ scalar partial criteria $F_1, F_2, \ldots, F_n, \ldots, F_N$ should meet three basic conditions: completeness, consistency, and nonredundancy [5]. The condition of completeness excludes the situation when the user chooses one of two different solutions, where these solutions have equal grades. However, the condition of completeness does not apply to the situation when the user chooses one of the two nondominated solutions in the sense of the dominance relationship.

Let $x, z \in X$, where $X$ is a set of allowable states, and the vector criteria functions with $n$ coordinates, which evaluate these states, are taken the form $F(x)$ and $F(z)$, then the condition of completeness can be formulated as follows:

$$[(F(x) = F(z)) \Rightarrow \text{the user do not prefers } x \text{ rather then } z]$$

(5)

The consistency condition excludes the following situation: two different solutions $x, z \in X$ with identical images $F(x) = F(z)$ are modified. The first solution is modified by “improving” at least one of its partial assessments. Thus, $[F(x^+), F(x)] \in R$, where $R$ is the dominance relation of the polyoptimization task. On the other hand, the modification of the second solution consists of determining a solution with at least one “worse”, in the sense of the dominance relationship, partial indicator. Therefore, $[F(z), F(z^-)] \in R$. It is not advisable for the user to choose the $z^-$ solution instead of the $x^+$ solution. This indicates an incorrect formulation of the criterion function. Formally, the criterion of consistency of a criterion function can be written in the form:

$$[(F(x) = F(z)) \land (F(x^+), F(x^+)) \in R \land (F(z), F(z^-)) \in R \Rightarrow z^+ \text{ for } x, x^+, z, z^- \in X]$$

(6)

The nonredundancy condition of partial criteria in the vector criterion is met when reducing one of the scalar criteria results in the remaining partial criteria not fulfilling the condition of completeness and consistency. Therefore, partial criteria cannot be reduced if they express user preferences. On the other hand, too many partial criteria cannot be taken into account because the nonredundancy condition will not be met.

As a result of preliminary modeling, it is usually possible to determine some basic vector criteria (1). Multicriteria optimization problems often take into account restrictions that “cut off” the value of a criterion function:

$$F_n(x) \leq r_n \text{ for } n = 1, N$$

(7)

where $r_n$ is the value of limitation of the partial criterion $F_n(\cdot)$. 
Each vector criterion $F$ can be modified by removing or adding a partial criterion. Let the relationship below indicate the confidence level for the $F_n$ criterion:

$$\alpha_n = \frac{1}{3}(\alpha_{n,1} + \alpha_{n,2} + \alpha_{n,3})$$

(8)

where $\alpha_{n,1}$, $\alpha_{n,2}$, $\alpha_{n,3} \in [0, 1]$, $\alpha_{n,1}$ is the subjective level of user confidence to the adequacy of the selection of the scalar partial $F_n$ criterion, $\alpha_{n,2}$ is relative accuracy of determining the $F_n$ partial criterion, and $\alpha_{n,3}$ is confidence level of exceeding the limit value $r_n$.

The total confidence level can be determined using the trust function $Q : F \rightarrow R^N$:

$$Q(F) = \frac{1}{N} \sum_{n=1}^{N} \alpha_n = \frac{1}{3N} \sum_{n=1}^{N} (\alpha_{n,1} + \alpha_{n,2} + \alpha_{n,3})$$

(9)

where $Q(F) \in [0, 1]$.

The extended modification of the $F^+$ vector criterion in relation to the base $F$ criterion is understood as a vector criterion created by adding the $F_{N+1}$ partial criterion. Therefore,

$$F^+ : X \rightarrow R^{N+1}$$

(10)

where $F^+(x) = [F_1(x), \ldots, F_n(x), F_{n+1}(x), \ldots, F_N(x), F_{N+1}(x)]$ for $x \in X$.

The concept of reduced modification of the vector criterion $F^-$ in relation to the basic criterion $F$ is understood as the vector criterion created by removing the partial criterion $F_n$. Therefore,

$$F^- : X \rightarrow R^{N-1}$$

(11)

where $F^-(x) = [F_1(x), \ldots, F_{n-1}(x), F_{n+1}(x), \ldots, F_N(x)]$ for $x \in X$.

If the reduction or addition of a partial criterion does not change the subjective confidence levels $\alpha_{n,1}$ for $n = 1, N$, then modification of the vector criterion is effective if the condition:

$$Q(F^{-(+)} > Q(F)$$

(12)

is satisfied.

3.2. Theorems on the Election of a Vector Criterion

Let $F^+(x) = [F_1(x), F_2(x), \ldots, F_n(x), \ldots, F_{N+1}(x)]$ and $\beta_n$ be the confidence level corresponding to the $F_n$ criterion for $n = 1, N + 1$, where the confidence levels $\beta_{n,1}$, $\beta_{n,2}$, $\beta_{n,3}$ are defined for the $F^+$ criterion in the same way as $\alpha_{n,1}$, $\alpha_{n,2}$, $\alpha_{n,3}$ for criterion $F$. Because, from the point of view of the modeled situation, it is important to know the results of introducing or removing a partial criterion, the following statements can be formulated and proved [2,4,5]:

**Theorem 1. (On the introduction of a partial criterion) [2,4,5]**

If $\beta_{i,k} = \alpha_{i,k}$ for $i = 1, n; k = 1, 3$,

If $\beta_{n+1,k} = \alpha_{N+1,k}$ for $k = 1, 3$,

If $\beta_{j+1,k} = \alpha_{j,k}$ for $j = n + 1, N; k = 1, 3$

this extended modification of the vector criterion is effective then and only then if:

$$\alpha_{N+1} > Q(F)$$

(13)
Proof of Theorem 1.

$$Q(F^+) - Q(F) = \frac{1}{3N + 3} \sum_{i=1}^{N+1} (\beta_{i,1} + \beta_{i,2} + \beta_{i,3}) - \frac{1}{3N} \sum_{i=1}^{N} (\alpha_{i,1} + \alpha_{i,2} + \alpha_{i,3})$$

Based on the assumptions of the theorem

$$Q(F^+) - Q(F) = \frac{1}{N+1} (\alpha_1 + \alpha_2 + \ldots + \alpha_n + \alpha_{n+1} + \alpha_{n+2} + \ldots + \alpha_N) - \frac{1}{N} \sum_{i=1}^{N} \alpha_i$$

Let $C = \sum_{i=1}^{N} \alpha_i,$

Then $Q(F^+) - Q(F) = \frac{1}{N(N+1)} (NC + N\alpha_{N+1} - NC - C) = \frac{\alpha_{N+1} - Q(F)}{N+1}.$

Because $N + 1 > 0$ then $Q(F^+) - Q(F) > 0$ when $\alpha_{N+1} > Q(F),$ which ends the proof. $\square$

Conclusion: If $\alpha_{N+1} > Q(F),$ then the partial criterion $F_{N+1}$ can be included in the process of formulating the polyoptimization task, whereas in extreme cases, when $\alpha_{N+1} = 0$ or $Q(F) = 1,$ it makes no sense to consider an additional partial criterion.

Theorem 2. (On the removal of the partial criterion) [2,4,5]

If $\beta_{i,k} = \alpha_{i,k}$ for $i = 1, 2, \ldots, n - 1, n + 1, \ldots, N; k = 1, 2,$

then reduced modification of the vector criterion is effective then and only then if:

$$a_N < Q(F)$$

(14)

Proof of Theorem 2.

$$Q(F^-) - Q(F) = \frac{1}{3N - 3} \sum_{i=1}^{N-1} (\beta_{i,1} + \beta_{i,2} + \beta_{i,3}) - \frac{1}{3N} \sum_{i=1}^{N} (\alpha_{i,1} + \alpha_{i,2} + \alpha_{i,3})$$

Based on the assumptions of the theorem

$$Q(F^-) - Q(F) = \frac{1}{N-1} (\alpha_1 + \alpha_2 + \ldots + \alpha_{n-1} + \alpha_{n+1} + \ldots + \alpha_{N-1}) - \frac{1}{N} \sum_{i=1}^{N} \alpha_i$$

Let $C = \sum_{i=1}^{N} \alpha_i,$

Then $Q(F^-) - Q(F) = \frac{1}{N(N-1)} (NC - N\alpha_{N-1} - NC - C) = \frac{\alpha_{N-1} - Q(F)}{N-1}.$

Because $N - 1 > 0$ then $Q(F^-) - Q(F) > 0$ when $\alpha_{N-1} < Q(F),$ which ends the proof. $\square$

Conclusion: If $\alpha_{N-1} < Q(F),$ the partial criterion $F_{N-1}$ can be removed in the process of formulating the polyoptimization task. Therefore, it is advisable to withdraw partial criteria with the lowest confidence levels, and introduce criteria with the highest confidence level after verifying the modified $F^+$ criteria using conditions of completeness, consistency, and nonredundancy.

In practice, the addition or subtraction of a partial criterion causes the decision-maker to verify the value of some subjective confidence levels, while adding it lowers it, while reducing it increases it.

Let $\delta = (\delta_1, \delta_2, \ldots, \delta_n, \ldots, \delta_N)$ denote the vector of subjective confidence levels as a result of adding the $F_{N+1}$ criterion. Therefore, the new, subjective confidence level is $\alpha_{n,1} - \delta_n.$

Theorem 3. (On the introduction of a partial criterion) [2,4,5]
If \( \delta_n \neq 0 \) exists, then the partial criterion \( F_{N+1} \) can be entered into the base criterion \( F \) then and only then, if occurs:

\[
\alpha_{N+1} > Q(F) + \frac{1}{3} \sum_{n=1}^{N} \delta_n
\]  

(15)

**Proof of Theorem 3.** The extended baseline criterion \( F^+ \) should increase the value of the functional \( Q \). Therefore,

\[
Q(F^+) - Q(F) = \frac{1}{N+1} \sum_{n=1}^{N+1} \beta_n - \frac{1}{N} \sum_{n=1}^{N} \alpha_n
\]

where \( \beta_n = \frac{1}{3}[(\alpha_{n,1} - \delta_n) + \alpha_{n,2} + \alpha_{n,3}] \) for \( n = \frac{1}{N} \beta_{N+1} = \frac{1}{3}(\alpha_{N+1,1} + \alpha_{N+1,2} + \alpha_{N+1,3}) = \alpha_{N+1} \).

After substituting the above dependencies and necessary simplifications, we will receive:

\[
Q(F^+) - Q(F) = \alpha_{N+1} - Q(F) - \frac{1}{3} \sum_{n=1}^{N} \delta_n
\]

Hence, \( \alpha_{N+1} > Q(F) + \frac{1}{3} \sum_{n=1}^{N} \delta_n \), which ends the proof. \( \square \)

The criterion \( F^+ \) should also fulfill the condition of completeness. This condition is fulfilled if, for each pair of \( x, z \in X \) such that \( F(x) = F(z) \) and the fulfilled condition \( F_{N+1}(x) = F_{N+1}(z) \), the user does not choose \( x \) or \( z \). If \( F_{N+1}(x) > F_{N+1}(z) \) and the user selects \( x \) before \( z \) while maximizing \( F(\cdot) \), then \( F_{N+1}(\cdot) \) shows the user's preferences.

The introduction of the \( F_{N+1} \) criterion may result in failure to comply with the consistency condition. If

\[
F_{N+1}(x^+) \geq F_{N+1}(z^-)
\]

(16)

then the condition of consistency will be maintained.

Taking into account an additional partial criterion will introduce redundancy into the criterion \( F^+ \), provided that it was not the \( F \) criterion. This redundancy can be avoided only after eliminating one of the criteria \( F_n \) for \( n = \frac{1}{N} \). If no such criterion exists, then no additional criterion can be entered. If \( F \) does not meet the nonredundancy condition, the introduction of redundancy does not matter.

If \( \alpha_{N+1} = 0 \) or \( Q(F) = 1 \) under the conditions of completeness, consistency, and nonredundancy, then the criterion \( F_{N+1} \) cannot be considered.

Let \( \delta = (\delta_1, \delta_2, \ldots, \delta_n, \ldots, \delta_N) \) denote the vector of reduction of subjective confidence levels as a result of removing the criterion \( F_k \). Therefore, the new, subjective confidence level is \( \alpha_{n,1} + \delta_n \).

**Theorem 4.** (On the removal of the partial criterion) [2,4,5]

If exists \( \delta_n \neq 0 \), then the partial criterion \( F_k \) can be removed from criterion \( F \) if and only if

\[
\alpha_k < Q(F) - \frac{1}{3} \sum_{n=1}^{N} \delta_n
\]

(17)

**Proof of Theorem 4.** The reduced base criterion \( F^- \) should increase the value of the functional \( Q \). Therefore,

\[
Q(F^-) - Q(F) = \frac{1}{N-1} \sum_{n=1}^{N-1} \beta_n - \frac{1}{N} \sum_{n=1}^{N} \alpha_n
\]

\[
\sum_{n \neq k}^{N}
\]
where \( \beta_n = \frac{1}{3}[(\alpha_{n,1} - \delta_n) + \alpha_{n,2} + \alpha_{n,3}] \) for \( n = 1, N \), \( \beta_k = \frac{1}{3}(\alpha_{k,1} + \alpha_{k,2} + \alpha_{k,3}) = \alpha_k \).

After substituting the above dependencies and necessary simplifications, we will receive:

\[
Q(F^+) - Q(F) = -\alpha_k + Q(F) - \frac{1}{3} \sum_{n=1}^{N} \delta_n > 0
\]

Hence, \( \alpha_k < Q(F) - \frac{1}{3} \sum_{n=1}^{N} \delta_n \), which ends the proof. \( \Box \)

Based on the presented methodology, it is possible to formulate a scheme for formulating a vector criterion when we have a set of scalar criteria that affect the quality of solutions obtained, and thus the effects of decisions.

4. Example of Formulating a Vector Criterion

The issue of choosing the number of criteria used to evaluate decisions is a significant problem in many decision-making tasks. In the paper, as an example of using the presented method of selecting the number of criteria, we will consider the task of choosing a vector criterion in a complex object control system in a conflict situation. The presented principles of selection of the vector quality criterion will be used to formulate a decision task for the needs of designing the ship’s anticollision system. This task can be formulated as follows:

The number of criteria in the vector quality of control indicator used to select the optimal safe trajectory in a collision situation by the ship’s anticollision system should be chosen, assuming:

- the criterion should take into account the aspect of traffic safety and economics,
- gyro compass accuracy ± 3%,
- accuracy of log indications ± 4%,
- radar accuracy when measuring distances ± 4%,
- radar accuracy at speed measurement ± 5%,
- radar accuracy when measuring bearing ± 3%.

Analyzing the process of controlling a sailing object in a collision situation and taking into account the requirements for anticollision systems, we come to the conclusion that the assessment of control should be made in terms of traffic safety, and this aspect of control is the most important. In the set of safe controls, we can look for solutions that provide optimal control regarding the economic aspect.

For the above reasons, the selection of optimal control will be made on the basis of a vector quality indicator which will include both safety-related criteria and criteria related to traffic economics [2–5].

Among the criteria for evaluating control from the standpoint of security, one can distinguish the following criteria:

- The shortest approach distance \( D_{min}^j \) determined by the equation:

\[
F_1(x) = D_{min}^j = \frac{y_j V_{wy}^j - x_j V_{wx}^j}{V_w^j}
\]

where \( V_w^j \) is the relative speed of the \( j \)-th object relative to its own ship, \( V_{wx}^j, V_{wy}^j \) are the relative coordinates of relative speed, \( x_j, y_j \) are the appropriate coordinates of the location of the \( j \)-th object in the rectangular coordinate system associated with its own ship;

- Time remaining until reaching the closest distance is determined by the equation:

\[
F_2(x) = T_{D_{min}}^j = \frac{y_j V_{wy}^j + x_j V_{wx}^j}{(V_w^j)^2}
\]
Collision risk indicator is determined by the equation:

\[ F_3(x) = r_j = \frac{D_b T_b}{\sqrt{\epsilon_1 (D_{min}^j T_b)^2 + \epsilon_2 (D_b T_j D_{min}^j)^2}} \]  \hspace{1cm} (20)

where \( D_b \) is the distance from the ship to the \( j \)-th object, \( T_b \) is the safe time to complete the maneuver, \( \epsilon_1, \epsilon_2 \) are weight factors depending on the type of body of water, degree of visibility at sea, and dynamic length and width of the ship and draft;

Collision risk angle is defined by the equation:

\[ F_4(x) = \delta_j^0 = \arcsin \frac{D_j}{D_j} \]  \hspace{1cm} (21)

where \( D_j \) is the distance from the ship to the \( j \)-th object,

Aspect of the \( j \)-th object is determined by the equation:

\[ F_5(x) = A_j = q_j^0 = \pi + N_j - \psi_j \]  \hspace{1cm} (22)

where \( N_j \) is the bearing on the \( j \)-th object, \( \psi_j \) is the course of the \( j \)-th object.

The criteria for assessing control from an economic point of view include the following criteria:

Time lost for anticollision maneuver is determined by the equation:

\[ F_6(x) = T_t = \frac{VS - V_z \cos(\psi_z - \psi)}{V_z V} \]  \hspace{1cm} (23)

where \( V_z, \psi_z \) are the set speed and course of the ship before a collision situation, \( V, \psi \) are the current speed and course of the ship, and \( S \) is the distance traveled;

Fuel consumption per anticollision maneuver is determined by the equation:

\[ F_7(x) = \Delta Z_p = a(V_z^3 - V^3) - b(V_z^2 - V^2) + c(V_z - V) \]  \hspace{1cm} (24)

where \( a, b, c \) are coefficients approximating the fuel consumption curve, depending on the type of propulsion system, ship’s hull, and hydromechanical conditions of the basin;

Loss of road for anticollision maneuver

\[ F_8(x) = \Delta S = \int_{t_0}^{t_k} [V_z - V \cos(\psi_z - \psi)] dt \]  \hspace{1cm} (25)

where \( t_0, t_k \) are the start and end time of the anticollision maneuver, respectively;

Deviation from the set trajectory is determined by the equation:

\[ F_9(x) = \Delta y = \int_{t_0}^{t_k} V \sin(\psi_z - \psi) dt \]  \hspace{1cm} (26)

Deviation from the set course:

\[ F_{10}(x) = \Delta \psi = \psi_z - \psi \]  \hspace{1cm} (27)

For specified scalar quality criteria (18)–(27), subjective confidence levels \( \alpha_{n,1} \) were adopted, which are the arithmetic mean of confidence levels estimated by experts. However, the values of
confidence levels \( a_{n,2} \) resulting from the accuracy of determining partial criteria are calculated from the relationship:

\[
    a_{n,2} = 1 - \delta F_n(x)
\]

(28)

where \( \delta F_n(x) \) is the relative error value of the \( n \)-th partial criterion.

Confidence levels \( a_{n,3} \) resulting from restrictions imposed on partial criteria in the form of:

\[
    F_n(x) \leq r_n \text{ or } F_n(x) \geq r_n
\]

are determined from the equation:

\[
    a_{n,3} = \frac{\max F_n(x)}{r_n} \text{ or } a_{n,3} = \frac{r_n}{\min F_n(x)}
\]

(30)

Confidence levels \( a_{n,3} \) for the adopted partial criteria are determined with the following limitations:

\[
    \min F_1(x) \geq D_b, \min F_2(x) \geq T_b, \min F_3(x) \geq F_3, \min F_4(x) \geq F_4,
\]

\[
    \min F_5(x) \geq F_5, \max F_6(x) \leq 1.05T_t, \max F_7(x) \leq 1.1Z_{pl},
\]

\[
    \min F_8(x) \geq F_8, \min F_9(x) \geq F_9, \min F_{10}(x) \geq F_{10}
\]

(31)

Subjective confidence levels determined by experts and other confidence levels determined based on the above relationships are presented in Table 1. Table 1 also presents the values of the total confidence levels of individual scalar criteria, which were determined from relationship (8).

| \( F_1(x) \) | \( F_2(x) \) | \( F_3(x) \) | \( F_4(x) \) | \( F_5(x) \) | \( F_6(x) \) | \( F_7(x) \) | \( F_8(x) \) | \( F_9(x) \) | \( F_{10}(x) \) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( a_{n,1} \)  | 0.98            | 0.98            | 0.85            | 0.78            | 0.76            | 0.93            | 0.92            | 0.95            | 0.95            | 0.93            |
| \( a_{n,2} \)  | 0.92            | 0.96            | 0.884           | 0.87            | 0.85            | 0.95            | 0.892           | 0.97            | 0.955           | 0.94            |
| \( a_{n,3} \)  | 1.0             | 1.0             | 1.0             | 0.97            | 0.97            | 0.95            | 0.9             | 0.95            | 0.95            | 0.85            |
| \( a_n \)      | 0.9667          | 0.98            | 0.9113          | 0.8733          | 0.86            | 0.9433          | 0.904           | 0.9567          | 0.9517          | 0.9067          |

As a result of the analysis of the proposed partial criteria, a vector base criterion was formulated that includes two scalar criteria related to traffic safety and two scalar criteria assessing the economic aspect of control. The criterion is presented in the form of:

\[
    F(x) = [F_1(x), F_2(x), F_6(x), F_7(x)]
\]

(32)

The total confidence level determined from (9) for the baseline criterion is \( Q(F) = 0.9485 \).

Using Theorem 1, we check the possibility of extending the base vector criterion to the other defined scalar criteria. This condition is not met for criteria \( F_3(x), F_4(x), F_5(x), \) and \( F_{10}(x) \) because there are conditions such that \( Q(F) > a_3 = 0.9113, \ Q(F) > a_4 = 0.8733, \ Q(F) > a_5 = 0.86, \) and \( Q(F) > a_{10} = 0.9067; \) therefore, these scalar criteria according to Theorem 1 cannot be entered into the vector criterion.

For scalar criteria \( F_8(x) \) and \( F_9(x) \), there are conditions such that \( a_8 = 0.9567 > Q(F) \) and \( a_9 = 0.9517 > Q(F) \). In this case, we check by entering these scalar criteria into the vector criterion whether the conditions resulting from Theorem 3 are met.

Both criteria are criteria from the economics assessment group control; therefore, the first two coordinates of the reduction vector assume a value of zero.
The criterion assessing road loss and the criterion assessing the deviation from a given trajectory affect both time lost and fuel consumption; therefore, the reduction vector will take the form:

$$\delta = [0, 0, 0.012, 0.01]$$

For such a reduction vector, we check the fulfillment of the condition resulting from Theorem 3 for criteria $F_8(x)$ and $F_9(x)$. Theorem 3 shows that there is the relation:

$$\alpha_8 = 0.9567 > Q(F) + \frac{1}{3} \sum_{j=1}^{4} \delta_j = 0.9485 + 0.0073 = 0.9558$$

$$\alpha_9 = 0.9517 < Q(F) + \frac{1}{3} \sum_{j=1}^{4} \delta_j = 0.9485 + 0.0073 = 0.9558$$

Therefore, only criterion $F_8(x)$ can be entered into the vector criterion, because only it meets the condition of Theorem 3. Therefore, the vector criterion function will take the form:

$$F(x) = [F_1(x), F_2(x), F_6(x), F_7(x), F_8(x)]$$

For a modified vector criterion, using (9) we determine the total confidence level, which is $Q(F) = 0.95014$.

For the new vector criterion, we check the possibility of removing the scalar criterion that meets the condition resulting from Theorem 2. It follows that $\alpha_1 > Q(F), \alpha_2 > Q(F), \alpha_8 > Q(F)$; however, $\alpha_6 < Q(F), \alpha_7 < Q(F)$ occur for the criteria $F_6(x)$ and $F_7(x)$. Therefore, according to Theorem 2, they can be removed from the vector criterion.

Both scalar criteria that we can remove are criteria from the group assessing the economics of control; therefore, the first two coordinates of the reduction vector take a value equal to zero, and the other three values are different from zero. The reduction vector takes the form $\delta = [0, 0, 0.012, 0.017, 0.015]$. For such a reduction vector, we check the fulfillment of the condition resulting from Theorem 4 for criteria $F_6(x)$ and $F_7(x)$. It follows that:

$$\alpha_6 = 0.9433 > Q(F) - \frac{1}{3} \sum_{j=1}^{4} \delta_j = 0.95014 - 0.01467 = 0.93547$$

$$\alpha_7 = 0.904 < Q(F) - \frac{1}{3} \sum_{j=1}^{4} \delta_j = 0.95014 - 0.01467 = 0.93547$$

According to Theorem 4, the criterion $F_7(x)$ is removed from the vector criterion. Then, the vector criterion will take the form

$$F(x) = [F_1(x), F_2(x), F_6(x), F_8(x)]$$

For this criterion, the value of the total confidence level is $Q(F) = 0.961675$. After checking the condition resulting from Theorem 1, we cannot introduce additional scalar criterion into this vector criterion. Thus, the process of forming the vector criterion for the ship’s anticollision motion control system has been completed.

5. Conclusions

Decision making is an indispensable element of our private and professional lives. Such decisions may concern, for example, acceptance of a technical solution, choosing the best solution among the available options, or choosing the most appropriate control systems in conditions of uncertainty and risk. The above issues are related to making engineering decisions involving the search for optimal
solutions; therefore, the task must be a so-called well-defined problem. Thus, the designer is faced with the task of formulating an optimization problem that includes formulating criteria for the assessment of decisions, actions, or effects of controlling any system. For this reason, the paper proposes a method for selecting the number of criteria to assess the quality of decisions taken, which can be used in modeling decision-making situations, especially in engineering issues related to controlling complex systems. In such cases, depending on the time horizon and external conditions, making the optimal decision may involve difficulties, and is often associated with the risk of making a mistake. Therefore, when designing the control device, the designer must formulate a vector criterion function that allows the analysis of many variants in a limited time.

Formulated in this work:

- The conditions of completeness, coherence, and nonredundancy that must be met by the vector criterion adopted for the assessment of the decision situation, ensuring the selection of the number of scalar criteria in the vector criterion relevant to the modeled decision situation;
- A theorem of introducing the scalar criterion into the vector criterion, and a theorem of reducing the number of criteria in the vector criterion for variables and constant user preferences, ensuring the selection of such a number of criteria that user preferences are taken into account in the modeled decision situation;
- Claims about increasing or reducing the dimension of the vector criterion allowed in order to create a method of forming the vector criterion for any decision task in which the scalar criteria with their confidence levels are specified.

The presented method was used to formulate a decision task for the needs of a marine anticollision system. A specific vector criterion based on the formulated method ensures the selection of safe control in a collision situation of the ship’s movement and is optimal in the economic aspect.

The application of the presented method to model the decision situation requires defining a set of scalar criteria assessing the quality of the decision made. If these are measurable values, then the relative accuracy $a_{n,2}$ can be determined, e.g., resulting from the measurement accuracy as well as the confidence level of exceeding the limit value $a_{n,3}$. The confidence level $a_{n,1}$ is a subjective quantity determined by experts. It should be emphasized that this parameter is one of the three confidence levels used to assess the confidence of a given scalar criterion. In addition, the subjectivity of the adopted values can be minimized by increasing the number of experts participating in the assessment. The adopted method ensures finding a vector criterion function, the form of which excludes its further modification in accordance to the presented theorems. The solution obtained in the presented example is intuitively correct because in the final form, the vector criterion takes into account the most important indicators assessing traffic safety, i.e., the smallest approach distance and the time remaining to reach the smallest approach distance, as well as the two most important indicators related to traffic economics, i.e., lost time and loss roads for anticollision maneuver. The remaining indicators taken into account during the selection procedure of the optimal vector indicator are based on indicators that were adopted in its final form, and thus it seems intuitive to not take them into account.

It should be stated that the formulated method of selecting the number of scalar criteria in the vector criterion can be used to formulate optimal control issues, as well as in solving other decision-making tasks formulated as multicriteria optimization tasks. However, the applicability of this approach is limited to decision-making issues for which we can define scalar criteria and specify their confidence levels. Despite this, the presented approach does not limit the number of scalar criteria that can be used to assess the analyzed decision situation, nor their complexity.

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