Program Design for the 3K Planetary Gear Mechanism with more than Three Planet Gears

Qi An¹*, Shuangfu Suo², Jie Yang¹, Changgui Wu³ and Chuanxiang Yan³

¹School of Mechanical Electronic & Information Engineering, China University of Mining & Technology, Beijing, China.
²Machine Design Institute, Department of Mechanical Engineering, Tsinghua University, Beijing, China.
³*Corresponding author e-mail: kuangdaanqi@163.com

Abstract. This paper deals with the gear matching of 3K planetary gears, especially when the number of planetary wheels is more than three. The formulas of gearing ratio, concentricity condition, installation condition and adjacent condition are worked out. The concentric condition error and the adjacent condition error are set respectively in consideration of adjusting the concentricity under the condition of changing the modulus. When the error of the concentric condition is zero, the invariant bit or the height shift is indicated. When the concentric condition error is not zero, the negative shift or the positive displacement is adopted. Adjacency conditions can be given according to the designer's experience. In addition, the maximum size limit is taken into account. By giving these constraints, a computer program can be used to rapidly solve the desired number of combinations of teeth. This paper presents a typical case of seven planets.

1. Introduction

3K planetary gear transmission has the advantages of compact structure, small size, light weight, high bearing capacity, large transmission power and transmission ratio. It is most suitable for short-term discontinuous work mode, where the transmission ratio is relatively large, and where the volume and quality are limited [1]. When designing the 3K planetary gear drive, tooth matching is the most critical step. The traditional 3K planetary gears mainly rely on the query manual, and the manual gives the combination of the number of teeth based on the number of planetary gears is only three or less than three, the increase in the number of planet gears will lead to an increase in the difficulty of teething, which is almost impossible to find in the relevant manuals [2, 3]. At present, computer-aided design has become a tendency. The designer only needs to specify a certain range of gear ratios, the modulus of the gears, the number of planetary gears, and the maximum size of the space to obtain a series of teeth that meet the requirements. Combination and corresponding gear ratio. Greatly reduces the time spent in the process of tooth matching for 3K planetary gears and improves the work efficiency of designers.
2. 3K planetary gear drive Teeth requirements

Figure 1 shows a simplified diagram of a 3K planetary gear drive. In the process of tooth number matching of 3K planetary gears, four basic conditions need to be met: gear ratio conditions, concentric conditions, adjacency conditions, and assembly conditions [4]. The combination of the number of teeth that must satisfy the four conditions at the same time. In addition, there are maximum size conditions depending on the application.

![Figure 1. Transmission diagram of 3K planetary gear](image)

2.1. Transmission ratio conditions

From the diagram of the 3K planetary gear transmission, we can see that the 3K planetary gear train can be seen as a series connection of two 2K-H planetary gears: the composition of planetary gears a, b, c and the composition of planetary gears b, c, d, e [5, 6]. The calculation formula for the gear ratio of the train can be calculated as follows:

\[
i = \frac{1 + \frac{z_b}{z_a}}{1 - \frac{z_b z_c}{z_c z_c}}
\]

(1)

In the formula: i is gear ratio for the planetary gear, \( z_a, z_b, z_c, z_d, z_e \) is the Gear teeth of a, b, c, d, e.

2.2. Concentric conditions

From the above analysis, we can know that the 3K planetary gear train can be regarded as two 2K-H planetary gears in series. The concentricity conditions for the two-series gear mechanism to ensure concentricity are as follows:

\[
\begin{cases}
  z_a + 2z_c = z_b \\
  (z_b - z_c)m_1 = (z_c - z_d)m_2
\end{cases}
\]

(2)

In the formula: \( m_1 \) is the modulus of the gear a, b, c, \( m_2 \) is the modulus of the gear d, e.

Concentric conditions can be guaranteed by gear shifting, so an error limit can be set here:

\[
\begin{cases}
  |z_a + 2z_c - z_b| < err1 \\
  |(z_b - z_c)m_1 - (z_c - z_d)m_2| < err1
\end{cases}
\]

(3)

In the formula: \( err1 \) Concentric condition error is given by the user. When zero is taken by \( err1 \), it means that the concentric condition is not satisfied by the dislocation.
2.3. Assembly conditions
Regarding the assembly problem, we discuss here in two situations.

(1) The number of gear teeth, a, b, e can be divided by the number of planets, as follows:

\[
\begin{align*}
\frac{z_a}{n_w} &= K_1 \\
\frac{z_b}{n_w} &= K_2 \\
\frac{z_e}{n_w} &= K_3
\end{align*}
\]  
(4)

Or use the following formula:

\[
\begin{align*}
\frac{z_a + z_e}{n_w} &= K_4 \\
\frac{z_e}{n_w} &= K_5
\end{align*}
\]  
(5)

In the formula: \(n_w\) is the number of planets. \(K_1, K_2, K_3, K_4, K_5\) are all positive integers.

The number of gear teeth a, b, e cannot be divisible by the number of planets. That is, the case of the above formula, \(K_1, K_2, K_3\) are not a positive integer, namely the following formula:

\[
\frac{z_a + z_e}{n_w} + \left(1 - \frac{z_d}{z_e}\right) \left(E_A + n - \frac{z_a}{n_w}\right) = K_3
\]  
(6)

Make sure that \(E_A, n, K_3\) is an integer; when \(\frac{z_a}{n_w}\) is an integer, the value of \(E_A = \frac{z_a}{n_w}\), \(n\) will take from a non-zero positive integer. When \(\frac{z_a}{n_w}\) is not an integer, \(E_A\) is a positive integer slightly larger than \(\frac{z_a}{n_w}\), \(n\) is also takes a value from a non-zero positive integer.

2.4. Adjacent conditions
The adjacency condition is calculated based on the larger planets, as shown in the following formula:

\[
\max \left\{(z_c + 2)m_1, (z_d + 2)m_2\right\} < (z_a + z_e)m_1 \sin \frac{\pi}{n_w}
\]  
(7)

That is, the following formula is satisfied

\[
(z_a + z_e)m_1 \sin \frac{\pi}{n_w} - \max \left\{(z_c + 2)m_1, (z_d + 2)m_2\right\} > err2
\]  
(8)
err2 is an adjacency condition error, generally greater than or equal to 0.5, The specific value is given by the user.

2.5. **maximum Dimensions condition**
In some applications, for the 3K planetary gear assembly with space position limitations, the planetary gear can be estimated based on the tooth root circle diameter of the largest tooth outer ring. The formula can be expressed as:

$$\max(m_1(z_b + 2), m_2(z_c + 2)) < D$$

Through the above five constraints using computer programming can be quickly found the combination of the number of teeth and the corresponding gear ratio of $z_a$, $z_b$, $z_c$, $z_d$, $z_e$. Through the efficiency of the transmission system, further optimization can be made to finally obtain the required teething results.

3. **3K planetary gear tooth program flow**

3.1. **program solving problem description**
As shown in Figure 2, known modulus $m_1$ of a, b, c and also known modulus $m_2$ of d, e, Number of planets $n_w$. The range of transmission ratios, maximum dimensions D of the 3K planetary gears, Concentric condition error $err_1$ and adjacency condition error $err_2$ are also known. Find all tooth combinations that satisfy the condition $z_a$, $z_b$, $z_c$, $z_d$, $z_e$, And find the corresponding gear ratio $i$. Finally, the corresponding given parameters are output, as well as the tooth number combination and transmission ratio determined.

3.2. **Diagram of procedure flow chart**
The computer program flow chart is as follows:

![Figure 2. Flow chart of program design](image)

The gear cycle selection block diagram is as follows:
4. Example for program with teeth
We use the 3K planetary gear with a number of planet gears of 7 to perform program tothing calculations. The input parameters are as follows:

\[ i_{\text{max}} = 500, i_{\text{min}} = 200, n_r = 5, m_1 = 1, m_2 = 1, \text{err1} = 0, \text{err2} = 0.5, D = 200 \]

\[ z_{\text{num}} = 60, z_{\text{num}} = 100, z_{\text{num}} = 60, z_{\text{num}} = 60, z_{\text{num}} = 100 \]

\[ z_{\text{num}} = 12, z_{\text{num}} = 12, z_{\text{num}} = 12, z_{\text{num}} = 12, z_{\text{num}} = 12 \]

We can get the result of running the program as:

| m1, m2: | 1.0, 1.0 |
| nw: | 5 |
| iMin, iMax: | 200, 500 |
| linjie: | 0.5 |
| D: | 200 |
| err2: | 0.0 |

| za | zb | ze | i |
|-----|----|----|---|
| 0: | 25 | 85 | 30 | 92 | 95 | 29 | 84 | 1 |
| 1: | 27 | 93 | 33 | 32 | 92 | 224 | 201 | 599999999999997 |
| 2: | 28 | 94 | 33 | 32 | 93 | 219 | 21428571428635 |
| 3: | 29 | 95 | 33 | 32 | 94 | 213.93103448275824 |
| 4: | 30 | 96 | 33 | 32 | 95 | 209.0000000000003 |
| 5: | 31 | 97 | 33 | 32 | 96 | 204.3870677419334 |
| 6: | 31 | 99 | 34 | 33 | 98 | 214.96774193548444 |
| 7: | 32 | 98 | 33 | 32 | 97 | 200.0625000000003 |
From the results of the operation, it can be seen that under the condition of gear teeth constrained by gear ratio, concentricity, assembly, and adjacency conditions, after the calculation, there are 10 different transmission schemes for selection for the same gear ratio. According to the calculation formula of the 3K planetary gear efficiency [7]. After calculation, we finally determined that the gear transmission efficiency of scheme 1 is the highest, which is our optimal solution. The procedure with teeth has unparalleled advantages compared to manual teeth.

The modeling and analysis and physical processing were carried out based on the calculation results of the matching program. Figure 4 is a digital map of a 3K-type planetary gear reducer, and Figure 5 is a physical map of a 3K-type planetary gear decelerator. After testing, within the allowable range of error, the program obtained by the matching program meets the design requirements.

5. Conclusion

Through computer programming methods, it is well solved that the sum of the number of sun gear teeth is not an integral multiple of the number of planet gears. The set of tooth numbers solved includes the combination of the number of teeth in the literature. As long as the speed of the computer is fast enough, the input conditions can be arbitrarily changed to obtain a combination of the number of teeth required to meet the requirements. In addition, it is also considered that the teeth are satisfied by changing the modulus to satisfy the concentric condition and to consider the condition of the displacement. Through verification, the combination of tooth numbers meets four basic conditions. This program can also provide reference and reference for the teeth of ordinary planetary reducer.

Acknowledgments

This work was financially supported by National key research and development plan fund.

References

[1] Manglik V K, Elements of Mechanical Engineering, Katson Publishing House, 1985.
[2] Cun-Guang L U, Duan Q H, The Unit Analysis Method of 3K Type Planetary Gear Trains, J. Machine Design & Research, 2009, 25(6) 22-24.
[3] Karaszewski W, Fundamentals of machine design, Mir Publishers, 1976.
[4] Bo D U, Fei Y, Liu X Y, The tooth number calculation and program design for 3K(III) planetary gear transmission, J. Machinery, 2014.
[5] Hsieh L C, Tang H C, Chen T H, The Kinematic Design of 2K Type Planetary Gear Reducers with High Reduction Ratio, J. Applied Mechanics & Materials, 2013, 421 40-45.
[6] Nerstad K A, Windish W E, Planetary transmission, J. 1987.
[7] Lee H K, Sang H L, Kim M S, Development of a Design and Analysis Program for Automatic Transmission Applications to Consider the Planetary Gear Noise and Its Adaptation, J. 2015, 25(7) 487-495.