Comment on Stochastic Polyak Step-Size: Performance of ALI-G

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Abstract

This is a short note on the performance of the ALI-G algorithm (Berrada et al., 2020) as reported in (Loizou et al., 2021). ALI-G (Berrada et al., 2020) and SPS (Loizou et al., 2021) are both adaptations of the Polyak step-size to optimize machine learning models that can interpolate the training data. The main algorithmic differences are that (1) SPS employs a multiplicative constant in the denominator of the learning-rate while ALI-G uses an additive constant, and (2) SPS uses an iteration-dependent maximal learning-rate while ALI-G uses a constant one. There are also differences in the analysis provided by the two works, with less restrictive assumptions proposed in (Loizou et al., 2021). In their experiments, (Loizou et al., 2021) did not use momentum for ALI-G (which is a standard part of the algorithm) or standard hyper-parameter tuning (for e.g. learning-rate and regularization). Hence this note as a reference for the improved performance that ALI-G can obtain with well-chosen hyper-parameters. In particular, we show that when training a ResNet-34 on CIFAR-10 and CIFAR-100, the performance of ALI-G can reach respectively 93.5% (+6%) and 76% (+8%) with a very small amount of tuning. Thus ALI-G remains a very competitive method for training interpolating neural networks.

1 Context

For context, we provide a summary on the Adaptive Learning-Rate for Interpolation with Gradients (ALI-G) algorithm. More details can be found in (Berrada et al., 2020).

Loss Function. We consider a supervised learning task where the model is parameterized by $w \in \mathbb{R}^p$. The objective function is expressed as an expectation over $z \in Z$, a random variable indexing the samples of the training set:

$$f(w) \triangleq \mathbb{E}_{z \in Z} [\ell_z(w)], \quad (1)$$

where each $\ell_z$ is the loss function associated with the sample $z$ and is assumed to be non-negative: $\forall w \in \mathbb{R}^p, \ell_z(w) \geq 0$.

Regularization. We incorporate regularization (if any) as a constraint on the feasible domain: $\Omega = \{w \in \mathbb{R}^p : \phi(w) \leq r\}$ for some value of $r$. For example, for $\ell_2$ regularization: $\Omega = \{w \in \mathbb{R}^p : \|w\|^2_2 \leq r\}$, for which the projection is given by a simple rescaling of $w$.

Problem Formulation. The learning task can be expressed as the problem (P) of finding a feasible vector of parameters $w_\star \in \Omega$ that minimizes $f$:

$$w_\star \in \arg\min_{w \in \Omega} f(w). \quad (2)$$

Interpolation Assumption. We assume that the problem at hand satisfies the interpolation assumption, i.e. we assume that there exists a solution $w_\star$ that simultaneously minimizes all individual loss functions:

$$\forall z \in Z, \ell_z(w_\star) = 0. \quad (3)$$

ALI-G Update. The hyper-parameters of ALI-G are a maximal learning-rate $\eta$ (kept constant), and a small constant $\delta$ for numerical stability. Then given a sample $z_t$ drawn at iteration $t$, the ALI-G update can be written as:

$$w_{t+1} = \Pi_{\Omega} \left( w_t - \min \left\{ \frac{\ell_{z_t}(w_t)}{\|\nabla \ell_{z_t}(w_t)\|^2 + \delta}, \eta \right\} \nabla \ell_{z_t}(w_t) \right). \quad (4)$$
Remark on Hyper-Parameters. In [Loizou et al., 2021] the dependence of the hyper-parameters of ALI-G on the smoothness parameter is stated to be “limiting the method’s practical applicability”. This is not correct: in order to establish our theoretical convergence results we do indeed assume some dependence on the problem properties for our choices of $\eta$ (maximal learning-rate) and $\delta$ (denominator constant). However, in practical applications we simply set $\delta$ to a low value (typically $\delta = 10^{-5}$) and treat it much like the $\epsilon$ numerical constant in Adam [Kingma and Ba, 2013]. Furthermore, in ALI-G only the maximal learning-rate $\eta$ is tuned, and it is then kept constant throughout training: this constitutes a major improvement over the most common practice of using a manually designed learning-rate schedule.

Remark on Momentum. No momentum is used for ALI-G in the experiments of [Loizou et al., 2021], with the justification that ALI-G with momentum does not come with proofs of convergence. While ALI-G with momentum is indeed not guaranteed to converge, it can still be used in practice, much like most optimization algorithms (including the Stochastic Polyak Step-Size) are applied to non-convex deep learning problems without formal guarantees of convergence. Finally, we point out that the optimal choice of learning-rate depends on the use of momentum. Hence, to compensate for the deactivation of momentum for a method would call for careful tuning of the learning-rate.

2 Empirical Performance on CIFAR Data Sets

In [Loizou et al., 2021] ALI-G is employed without momentum and with a learning-rate set to 0.1, which was not selected for best performance. This set of experiments shows that the performance of ALI-G can be greatly improved with only a small amount of tuning. We use the official code of [Loizou et al., 2021] at https://github.com/IssamLaradji/sps and run ALI-G on the task of learning a ResNet on CIFAR-10 and CIFAR-100. We find that using momentum (with the common value of 0.9) and a maximal $\ell_2$ norm for regularization (with a value of 100) gives results significantly better than those reported in [Loizou et al., 2021], as shown in Table 1.

| Data Set  | Method (Reference) | Hyper-Parameters Used | Validation Accuracy |
|-----------|--------------------|-----------------------|---------------------|
|           |                    | Learning-Rate | Momentum | Max $\ell_2$ norm |        |
| CIFAR-10  | ALI-G (Loizou et al., 2021) | 0.1 | 0 | $\infty$ | 87.5%  |
|           | ALI-G (this note   )       | 0.1 | 0.9 | 100    | 93.5%  |
| CIFAR-100 | ALI-G (Loizou et al., 2021) | 0.1 | 0 | $\infty$ | 68%    |
|           | ALI-G (this note   )       | 0.1 | 0.9 | 100    | 76%    |

Table 1: Performance of ALI-G with well-chosen hyper-parameters (this note), compared to the performance reported in [Loizou et al., 2021]. As can be observed, a small amount of tuning leads to large improvements (+6% and +8% on CIFAR-10 and CIFAR-100).

Summary. In this note we reaffirm the practical applicability of ALI-G, and we demonstrate that with well-chosen hyper-parameters ALI-G can obtain state-of-the-art results for training interpolating neural networks without manually designed learning-rate schedules.

References

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