The Bound-State $\beta^–$–Decay of the Neutron Revisited

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(Dated: January 10, 2020)

This paper is addressed to the analysis of the set of observables of the bound–state $\beta^–$–decay, which can be used for the experimental investigation of contributions of i) interactions beyond the Standard Model (SM) and of ii) the left–handed polarisation state of antineutrinos. For this aim we calculate the branching ratio, probabilities and angular distributions of probabilities of hydrogen in the hyperfine states and of the proton–electron pair in different spinorial states, induced by left–handed and right–handed hadronic and leptonic currents. The branching ratio of the bound–state $\beta^–$–decay is calculated by taking into account radiative corrections. We show that the probabilities of the bound–state $\beta^–$–decay can be good observables for experimental investigations of contributions of interactions beyond the SM, whereas the angular distributions of probabilities are good observables for experimental searches of contributions of the left–handed polarisation state of antineutrinos.

PACS numbers: 12.15.-y, 13.15.+g, 23.40.Bw, 26.30.Jk

I. INTRODUCTION

In Ref. [1] the bound-state $\beta^–$–decay of the neutron $n \rightarrow H + \bar{\nu}_e$, where H is hydrogen, has been revised by taking into account the new value of the axial coupling constant $\lambda = -1.2750(9)$ [2, 3] as well as new effective scalar and tensor weak lepton–nucleon interactions. The amplitude of the continuum-state $\beta^–$–decay of the neutron, calculated in the rest frame of the neutron in the non–relativistic approximation for the proton, including radiative corrections by virtual $\gamma$, $W$ and $Z$–boson exchanges as well as QCD corrections [4–12], is given by [3]

$$M(n \rightarrow pe^- \bar{\nu}_e) = -2m_n \frac{G_F}{\sqrt{2}} V_{ud} \left\{ [\phi^\dagger_p \phi_n] [\bar{u}_e \gamma^0 (C_V + C_A \gamma^5) \nu_{\bar{e}}] \left( 1 + \frac{\alpha}{2\pi} f_{\bar{\nu}_e} (E_{\bar{e}}, \mu) \right) \right. $$

$$- [\phi^\dagger_p \bar{\sigma} \phi_n] \cdot [\bar{u}_e \gamma^\gamma (C_A + C_A \gamma^5) \nu_{\bar{e}}] \left( 1 + \frac{\alpha}{2\pi} f_{\bar{\nu}_e} (E_{\bar{e}}, \mu) \right) + [\phi^\dagger_p \phi_n] [\bar{u}_e (C_S + C_S \gamma^5) \nu_{\bar{e}}] $$

$$+ [\phi^\dagger_p \bar{\sigma} \phi_n] \cdot [\bar{u}_e \gamma^0 \gamma^5 (C_T + C_T \gamma^5) \nu_{\bar{e}}] - \frac{\alpha}{2\pi} g_F (E_e) [\phi^\dagger_p \phi_n] [\bar{u}_e (1 - \gamma^5) \nu_{\bar{e}}] $$

$$+ \frac{\alpha}{2\pi} \lambda g_F (E_e) [\phi^\dagger_p \bar{\sigma} \phi_n] \cdot [\bar{u}_e \gamma^0 \gamma^5 (1 - \gamma^5) \nu_{\bar{e}}] \left\} , \quad (1) \right.$$
where the coupling constants $C_j$ and $C_j$ for $j = V, A, S$ and $T$ describe effective weak interactions, which may be induced by left-handed and right-handed hadronic and leptonic currents (see also [3]) caused by interactions beyond the standard model (SM), for example, supersymmetric interactions [16]. They are related to the coupling constants, analogous to those which were introduced by Herczeg [17], as follows (see also Eq. (2) in Appendix G in Ref. [3]):

\[
C_V = 1 + a_{LL}^h + a_{LR}^h + a_{RR}^h + a_{RL}^h, \\
C_A = -1 - a_{LL}^h - a_{LR}^h + a_{RR}^h + a_{RL}^h, \\
C_A = -\lambda + a_{LL}^h - a_{LR}^h + a_{RR}^h - a_{RL}^h, \\
C_S = A_{LL}^h + A_{LR}^h + A_{RR}^h + A_{RL}^h, \\
C_S = -A_{LL}^h - A_{LR}^h + A_{RR}^h + A_{RL}^h, \\
C_T = 2(\alpha_{LL}^h + \alpha_{RR}^h), \\
\tilde{C}_T = 2(-\alpha_{LL}^h + \alpha_{RR}^h),
\]

where the index $h$ means that the coupling constants are introduced at the hadronic level [3] but not at the quark level as done by Herczeg [17]. In addition, in comparison with Herczeg [17] we have taken away the common factor $G_F V_{ud}/\sqrt{2}$ and defined the coupling constants $a_{LL}^h$ and $a_{LR}^h$ as deviations from the coupling constants of the SM. The SM weak interactions are defined by coupling constants $C_V = -\tilde{C}_V = 1$, $C_A = -\tilde{C}_A = -\lambda$ and $C_S = \tilde{C}_S = C_T = \tilde{C}_T = 0$. The functions $f_{\beta^2}(E_e, \mu)$ and $g_F(E_e)$ in Eq. (11) are equal to [3]

\[
f_{\beta^2}(E_e, \mu) = \frac{3}{2} \ln\left(\frac{m_p}{m_e}\right) - \frac{11}{8} + \ln\left(\frac{\mu}{m_e}\right)[\frac{1}{\beta} \ln\left(\frac{1+\beta}{1-\beta}\right) - 2] + \frac{1}{\beta} L\left(\frac{2\beta}{1+\beta}\right)
- \frac{1}{4\beta} \ln^2\left(\frac{1+\beta}{1-\beta}\right) + \frac{1}{2\beta} \ln\left(\frac{1+\beta}{1-\beta}\right) + C_{WZ},
\]

\[
g_F(E_e) = \frac{\sqrt{1-\beta^2}}{2\beta} \ln\left(\frac{1+\beta}{1-\beta}\right),
\]

where $m_p$ and $m_e$ are the proton and electron masses, $\mu$ is a finite-photon mass regularisation parameter for the regularisation of infrared divergences in virtual one–photon exchanges [4–12] (see also [3]), $\alpha = 1/137.036$ is the fine–structure constant [18], $\beta = \sqrt{E^2 - m_e^2}/E_e$ is the electron velocity and $L(2\beta/(1+\beta))$ is the Spence function [19] (see also [3]). The constant $C_{WZ} = 10.249$ is caused by electroweak boson exchanges and QCD corrections [4–12] (see also Appendix D in Ref. [3]).

As has been pointed out in [20, 22] (see also [3]), the contributions of interactions beyond the SM from the coupling constants $a_{LL}^h$ and $a_{LR}^h$ are absorbed by a redefined axial coupling constant $\lambda_{\text{eff}}$ and CKM matrix element $(V_{ud})_{\text{eff}}$ [3], i.e.

\[
\lambda \rightarrow \lambda_{\text{eff}} = \frac{\lambda - a_{LL}^h + a_{LR}^h}{1 + a_{LL}^h + a_{LR}^h},
\]

\[1\text{ With respect to the definition of the coupling constants in the original paper [17] we have extracted the common factor } G_F V_{ud}/\sqrt{2} \text{ and replaced } a_{LL}^h \text{ and } a_{LR}^h \text{ by } a_{LL}^h \rightarrow a_{LL}^h + (1-\lambda)/2 \text{ and } a_{LR}^h \rightarrow a_{LR}^h + (1+\lambda)/2, \text{ respectively, where the coupling constants } a_{LL}^h \text{ and } a_{LR}^h \text{ describe deviations from the coupling constants of the SM.}
\[ V_{ud} \rightarrow (V_{ud})_{\text{eff}} = V_{ud} (1 + a_{LL}^h + a_{LR}^h). \]  

As has been shown in [3], the axial coupling constant \( \lambda_{\text{eff}} \) is real at the level of order 10\(^{-4} \). After such a redefinition the phenomenological coupling constants \( C_j \) and \( \bar{C}_j \) for \( j = V, A, S \) and \( T \) become

\[
\begin{align*}
C_{V,\text{eff}} &= 1 + \frac{a_{RR}^b + a_{RL}^b}{1 + a_{LL}^b + a_{LR}^b} = 1 + \bar{a}_{RR}^b + \bar{a}_{RL}^b, \\
\bar{C}_{V,\text{eff}} &= -1 + \frac{a_{RR}^b + a_{RL}^b}{1 + a_{LL}^b + a_{LR}^b} = -1 + \bar{a}_{RR}^b + \bar{a}_{RL}^b, \\
C_{A,\text{eff}} &= -\lambda_{\text{eff}} + \frac{a_{RR}^b - a_{RL}^b}{1 + a_{LL}^b + a_{LR}^b} = -\lambda_{\text{eff}} + \bar{a}_{RR}^b - \bar{a}_{RL}^b, \\
\bar{C}_{A,\text{eff}} &= \lambda_{\text{eff}} + \frac{a_{RR}^b - a_{RL}^b}{1 + a_{LL}^b + a_{LR}^b} = \lambda_{\text{eff}} + \bar{a}_{RR}^b - \bar{a}_{RL}^b, \\
C_{S,\text{eff}} &= \frac{A_{LL}^h + A_{LR}^h + A_{RR}^h + A_{RL}^h}{1 + a_{LL}^b + a_{LR}^b} = \bar{A}_{LL}^h + \bar{A}_{LR}^h + \bar{A}_{RR}^h + \bar{A}_{RL}^h, \\
\bar{C}_{S,\text{eff}} &= -\frac{A_{LL}^h - A_{LR}^h + A_{RR}^h + A_{RL}^h}{1 + a_{LL}^b + a_{LR}^b} = -\bar{A}_{LL}^h - \bar{A}_{LR}^h + \bar{A}_{RR}^h + \bar{A}_{RL}^h, \\
C_{T,\text{eff}} &= 2 \frac{a_{LL}^h + a_{RR}^h}{1 + a_{LL}^b + a_{LR}^b} = 2 (\bar{a}_{LL}^h + \bar{a}_{RR}^h), \\
\bar{C}_{T,\text{eff}} &= 2 \frac{-a_{LL}^h + a_{RR}^h}{1 + a_{LL}^b + a_{LR}^b} = 2 (-\bar{a}_{LL}^h + \bar{a}_{RR}^h).  
\end{align*}
\]

Since the velocity of the electron in the hydrogen bound state with principal number \( n \) is equal to \( \beta = \alpha/n \), we take the non-relativistic limit for the electron Dirac spinor in the calculation of the bound-state \( \beta^- \)-decay of the neutron.

## II. BOUND-STATE \( \beta^- \)-DECAY OF NEUTRON AND LEFT-HANDED NEUTRINOS

For the calculation of the amplitude of the bound-state \( \beta^- \)-decay we use the following Dirac wave function for the antineutrino

\[ \psi_{\bar{\nu}_e} = \sqrt{E} \left( \begin{array}{c} \bar{T} \cdot \vec{n} \chi_{\bar{\nu}_e} \\ \chi_{\bar{\nu}_e} \end{array} \right), \]

where \( \vec{n} = \vec{k}/E \) and normalisation equals \( \psi_{\bar{\nu}_e}^\dagger \psi_{\bar{\nu}_e} = 2E \). For the right-handed polarisation states of antineutrinos, corresponding to the left-handed polarisation states of neutrinos, the Pauli wave function \( \chi_{\bar{\nu}_e} \) obeys the equation \( \bar{T} \cdot \vec{n} \chi_{\bar{\nu}_e} = -\chi_{\bar{\nu}_e} \). If the axis of the antineutrino–spin quantisation is inclined relative to the axis of the neutron–spin quantisation with a polar

\[ \text{The Dirac wave function of antineutrinos in the right-handed polarisation state is equal to } u_{\nu_e} = C\bar{\nu}_{\bar{\nu}_e} \text{ or } \psi_{\bar{\nu}_e} = C\bar{\nu}_{\bar{\nu}_e}^T, \text{ where } C = i\gamma^0\gamma^2 \text{ and } T \text{ denotes transposition } [23]. \text{ The wave function } u_{\nu_e} = C\bar{\nu}_{\bar{\nu}_e}^T \text{ is a column matrix function with elements } \sqrt{E} (\varphi_{\nu_e}, \bar{T} \cdot \vec{n} \varphi_{\nu_e}), \text{ where the Pauli spinor wave function } \varphi_{\nu_e} \text{ is equal to } \varphi_{\nu_e} = -i\sigma^2 \chi_{\nu_e}^\dagger \text{ and obeys the equation } \bar{T} \cdot \vec{n} \varphi_{\nu_e} = +\varphi_{\nu_e} \text{ [23].} \]
angle \( \vartheta \), the Pauli wave function \( \chi_{\nu_e} \) is given by

\[
\chi_{\nu_e} = \begin{pmatrix} -e^{-i \varphi} \sin \frac{\vartheta}{2} \\ \cos \frac{\vartheta}{2} \end{pmatrix},
\]  

(7)

where \( \varphi \) is the azimuthal angle. Keeping the leading order contributions in the \( \alpha \)–\( g \)–\( H \) expansion of the amplitude Eq. (11) and following [1], we obtain the amplitude of the transition \( n \rightarrow H + \bar{\nu}_e \), where hydrogen is in the hyperfine \( (ns)_F \) state with hyperfine spin \( F \), in the form

\[
M(n \rightarrow H + \bar{\nu}_e)_{\text{rh}} = G_F (V_{ud})_{\text{eff}} \sqrt{2m_n} \frac{2E_H}{2E} \left( 1 + \frac{\alpha}{2\pi} (f_{\beta^-} - 1) \right) \psi_{(ns)_F}^*(0)
\times \left\{ (1 + g_S) [\varphi_p^\dagger \varphi_n] [\varphi_e^\dagger \chi_{\nu_e}] + (\lambda_{\text{eff}} + g_T) [\varphi_p^\dagger \vec{\sigma} \cdot \vec{\varphi}_n] \right\},
\]

(8)

where the abbreviation “\( \text{rh} \)” means the right–handed polarisation state. Then, \( \varphi_j \) for \( j = p, n, e \) and \( \chi_{\nu_e} \) are Pauli spinorial functions of the proton, neutron, electron and antineutrino, respectively. The term \( (-\alpha/2\pi) \) is the contribution of those terms in Eq. (11), which are proportional to \( (\alpha/2\pi) g_F (E_e) \). Furthermore, \( (f_{\beta^-} - 1) \) and the effective coupling constants \( g_S \) and \( g_T \) are given by

\[
f_{\beta^-} - 1 = \frac{3}{2} \frac{\ell n (m_p/m_e)}{m_e} - \frac{27}{8} + C_{WZ},
\]

(9)

and

\[
g_S = \frac{1}{2} \left( (C_{S,\text{eff}} - \bar{C}_{S,\text{eff}}) + (C_{V,\text{eff}} - 1) - (C_{V,\text{eff}} + 1) \right) = \bar{A}_{LL}^h + \bar{A}_{LR}^h,
\]

\[
g_T = \frac{1}{2} \left( (C_{T,\text{eff}} - \bar{C}_{T,\text{eff}}) - (C_{A,\text{eff}} + \lambda_{\text{eff}}) + (\bar{C}_{A,\text{eff}} - \lambda_{\text{eff}}) \right) = 2\bar{a}_{LL}^h.
\]

(10)

Note that the right–handed leptonic currents with the left– and right–handed hadronic currents do not contribute to the effective coupling constants \( g_S \) and \( g_T \).

Following [1] and keeping only the linear terms in the expansion in powers of \( g_S \) and \( g_T \) we obtain the branching ratio of the bound-state \( \beta^- \)–decay of the neutron

\[
R_{\beta^-} = \left( 1 + \frac{2}{1 + 3\lambda_{\text{eff}}^2} \Re(g_S + 3\lambda_{\text{eff}}g_T) - \left\langle \frac{m_e}{E_e} \right\rangle_{\text{SM}} b_F \right) R_{\text{SM}},
\]

(11)

where \( \left\langle \frac{m_e}{E_e} \right\rangle_{\text{SM}} = 0.6556 \), averaged over the electron–energy density spectrum of the neutron \( \beta^- \)–decay [3]. The Fierz term \( b_F \), defined to linear approximation with respect to the Herczeg coupling constants [3], is given by

\[
b_F = \frac{1}{1 + 3\lambda_{\text{eff}}^2} \Re \left( (C_{S,\text{eff}} - \bar{C}_{S,\text{eff}}) + 3\lambda_{\text{eff}} (C_{T,\text{eff}} - \bar{C}_{T,\text{eff}}) \right) = \frac{2}{1 + 3\lambda_{\text{eff}}^2} \Re(g_S + 3\lambda_{\text{eff}}g_T)(12)
\]

Hence, we may write the branching ratio \( R \) in the form

\[
R_{\beta^-} = \left( 1 + \left( 1 - \left\langle \frac{m_e}{E_e} \right\rangle_{\text{SM}} \right) b_F \right) R_{\text{SM}}.
\]

(13)

As has been pointed out in [3], the Fierz term can be measured from the experimental data on the electron asymmetry \( A_{\text{exp}}(E_e) \) of correlations between the neutron spin and the electron.
3–momentum and the proton–energy spectrum $a(T_p)$ (see also [24]), related to correlations between the 3–momenta of the proton and electron. The branching ratio $R_{SM}$, calculated in the SM, is equal to

$$R_{SM} = 2\pi \alpha^3 \zeta(3) \left(1 + \frac{\alpha}{\pi} (f_{\beta^-} - 1)\right) \frac{m_p + m_e}{m_n} \frac{E^2}{m_e^2 f_n} \sqrt{1 + \frac{E^2}{(m_p + m_e)^2}} = 3.905 \times 10^{-6}. \quad (14)$$

where $\zeta(3) = 1.202$, $E = (m_n^2 - (m_p + m_e)^2)/2m_n = 0.782$ MeV and $f_n = 1.755$ are the Riemann zeta function [19], the antineutrino energy and the phase–space factor of the neutron $\beta^–$–decay rate, including the contributions of the corrections, caused by the “weak magnetism” and the proton recoil as well as radiative corrections [3], respectively.

The contributions of different spinorial states to the helicity amplitudes of the bound-state $\beta^–$–decay as functions of the angles $\vartheta$ and $\varphi$ are given in Table I. 

| $\sigma_n$ | $\sigma_p$ | $\sigma_e$ | $\sigma_{\nu_e}$ | $f$ |
|-----------|-----------|-----------|-----------------|-----|
| $+\frac{1}{2}$ | $-\frac{1}{2}$ | $+\frac{1}{2}$ | $+\frac{1}{2}$ | $(1 + g_S - \lambda_{eff} - g_T) \cos \frac{\vartheta}{2}$ |
| $+\frac{1}{2}$ | $\frac{1}{2}$ | $+\frac{1}{2}$ | $+\frac{1}{2}$ | $-(1 + g_S + \lambda_{eff} + g_T) e^{-i\varphi} \sin \frac{\vartheta}{2}$ |
| $+\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $+\frac{1}{2}$ | $0$ |
| $+\frac{1}{2}$ | $+\frac{1}{2}$ | $+\frac{1}{2}$ | $+\frac{1}{2}$ | $2(\lambda_{eff} + g_T) \cos \frac{\vartheta}{2}$ |
| $-\frac{1}{2}$ | $+\frac{1}{2}$ | $-\frac{1}{2}$ | $+\frac{1}{2}$ | $-2(\lambda_{eff} + g_T) e^{-i\varphi} \sin \frac{\vartheta}{2}$ |
| $-\frac{1}{2}$ | $\frac{1}{2}$ | $+\frac{1}{2}$ | $+\frac{1}{2}$ | $0$ |
| $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $+\frac{1}{2}$ | $(1 + g_S + \lambda_{eff} + g_T) \cos \frac{\vartheta}{2}$ |
| $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $+\frac{1}{2}$ | $-(1 + g_S - \lambda_{eff} - g_T) e^{-i\varphi} \sin \frac{\vartheta}{2}$ |

TABLE I: The contributions of different spinorial states of the interacting particles to the amplitudes of the bound-state $\beta^–$–decay of the neutron and the antineutrino in the state with the wave function Eq. (7); $f$ is defined by $f = (1 + g_S)[\chi_e^\dagger \chi_{\nu_e}^\dagger] | \chi_p^\dagger \varphi_n \rangle + (\lambda_{eff} + g_T)[\chi_e^\dagger \hat{\sigma} \chi_{\nu_e}^\dagger] \cdot [\chi_p^\dagger \hat{\sigma} \varphi_n \rangle$.

Using the results in Table I we get the helicity amplitudes $M(n \rightarrow H_{FMp} + \bar{\nu}_e)_{\sigma_u + \frac{1}{2}}$

$$M(n \rightarrow H_0 + \bar{\nu}_e)_{+\frac{1}{2} + \frac{1}{2}} = +M_0 \frac{1 - 3\lambda_{eff} + g_S - 3g_T}{\sqrt{2}} \cos \frac{\varphi}{2},$$

$$M(n \rightarrow H_{1,+} + \bar{\nu}_e)_{+\frac{1}{2} + \frac{1}{2}} = -M_0 (1 + \lambda_{eff} + g_S + g_T) e^{-i\varphi} \sin \frac{\varphi}{2},$$

$$M(n \rightarrow H_{10} + \bar{\nu}_e)_{+\frac{1}{2} + \frac{1}{2}} = +M_0 \frac{1 + \lambda_{eff} + g_S + g_T}{\sqrt{2}} \cos \frac{\varphi}{2},$$

$$M(n \rightarrow H_{1,-1} + \bar{\nu}_e)_{+\frac{1}{2} + \frac{1}{2}} = 0,$$

$$M(n \rightarrow H_{00} + \bar{\nu}_e)_{-\frac{1}{2} + \frac{1}{2}} = +M_0 \frac{1 - 3\lambda_{eff} + g_S - 3g_T}{\sqrt{2}} e^{-i\varphi} \sin \frac{\varphi}{2},$$

$$M(n \rightarrow H_{1,1} + \bar{\nu}_e)_{-\frac{1}{2} + \frac{1}{2}} = 0,$$

$$M(n \rightarrow H_{10} + \bar{\nu}_e)_{-\frac{1}{2} + \frac{1}{2}} = -M_0 \frac{1 + \lambda_{eff} + g_S + g_T}{\sqrt{2}} e^{-i\varphi} \sin \frac{\varphi}{2},$$

$$M(n \rightarrow H_{1,-1} + \bar{\nu}_e)_{-\frac{1}{2} + \frac{1}{2}} = +M_0 (1 + \lambda_{eff} + g_S + g_T) \cos \frac{\varphi}{2}, \quad (15)$$
where \( M_0 \) is given by

\[
M_0 = G_F (V_{ud})_{\text{eff}} \sqrt{2m_e E_H} \left( 1 + \frac{\alpha}{2\pi} (f_{\beta^-} \bar{t} - 1) \right) \psi_{\text{ns}}^* (0). 
\]  

(16)

Following we define the angular distributions of the production of hydrogen in the hyperfine states with \( F = 0 \) and \( F = 1 \) in the bound-state \( \beta^- \)-decay of the polarised neutron

\[
4\pi \frac{dW^{(\pm)}_{F=0}(\theta)}{d\Omega} = \left\{ \frac{1}{8} \frac{(1 - 3\lambda_{\text{eff}})^2}{1 + 3\lambda_{\text{eff}}^2} + \frac{3}{4} \frac{(1 + \lambda_{\text{eff}})(1 - 3\lambda_{\text{eff}})}{(1 + 3\lambda_{\text{eff}}^2)^2} \Re(\lambda_{\text{eff}} g_S - g_T) + \frac{1}{8} \frac{1}{1 + 3\lambda_{\text{eff}}^2} |g_S - 3g_T|^2 \\
- \frac{1}{8} \frac{(1 - 3\lambda_{\text{eff}})^2}{1 + 3\lambda_{\text{eff}}^2} \left[ (1 - 3\lambda_{\text{eff}})(|g_S|^2 + 3|g_T|^2) + 4\Re(g_S + 3\lambda_{\text{eff}} g_T) \Re(g_S - 3g_T) \right] \\
+ \frac{1}{2} \frac{(1 - 3\lambda_{\text{eff}})^2}{(1 + 3\lambda_{\text{eff}}^2)^3} \left[ \Re(g_S + 3\lambda_{\text{eff}} g_T) \right]^2 \right\} (1 \pm \cos \theta),
\]

\[
4\pi \frac{dW^{(\pm)}_{F=1}(\theta)}{d\Omega} = \left\{ \frac{3}{8} \frac{(1 + \lambda_{\text{eff}})^2}{1 + 3\lambda_{\text{eff}}^2} - \frac{3}{4} \frac{(1 + \lambda_{\text{eff}})(1 - 3\lambda_{\text{eff}})}{(1 + 3\lambda_{\text{eff}}^2)^2} \Re(\lambda_{\text{eff}} g_S - g_T) + \frac{1}{8} \frac{1}{1 + 3\lambda_{\text{eff}}^2} |g_S + g_T|^2 \\
- \frac{3}{8} \frac{(1 + \lambda_{\text{eff}})^2}{1 + 3\lambda_{\text{eff}}^2} \left[ (1 + \lambda_{\text{eff}})(|g_S|^2 + 3|g_T|^2) + 4\Re(g_S + 3\lambda_{\text{eff}} g_T) \Re(g_S + g_T) \right] \\
+ \frac{3}{2} \frac{(1 + \lambda_{\text{eff}})^2}{(1 + 3\lambda_{\text{eff}}^2)^3} \left[ \Re(g_S + 3\lambda_{\text{eff}} g_T) \right]^2 \right\} (1 \pm \frac{1}{3} \cos \theta).
\]

(17)

The upper indices \((\pm)\) in \( W^{(\pm)}_{F}(\theta) \) correspond to the neutron polarisation, \( \theta = \pi - \vartheta \) is the angle between the neutron polarisation and the 3–momentum of hydrogen.

From Table I we define the angular distributions of the probabilities \( W_1^{(\pm)}(\theta), W_2^{(\pm)}(\theta), W_3^{(\pm)}(\theta) \) and \( W_4^{(\pm)}(\theta) \) of the neutron decay into the bound \((pe^-)\) spinorial states \( |+1/2\rangle_p + 1/2\rangle_e, |+ 1/2\rangle_p - 1/2\rangle_e, |-1/2\rangle_p + 1/2\rangle_e \) and \( |-1/2\rangle_p - 1/2\rangle_e \) (see [25]), respectively. For the neutron spin polarisation \( \sigma_n = +1/2 \) the angular distributions of the probabilities \( W_1^{(\pm)}(\theta), W_2^{(\pm)}(\theta), W_3^{(\pm)}(\theta) \) and \( W_4^{(\pm)}(\theta) \) are equal to

\[
4\pi \frac{dW_1^{(\pm)}(\theta)}{d\Omega} = \left\{ \frac{1}{4} \frac{(1 + \lambda_{\text{eff}})^2}{1 + 3\lambda_{\text{eff}}^2} - \frac{1}{4} \frac{(1 + \lambda_{\text{eff}})(1 - 3\lambda_{\text{eff}})}{(1 + 3\lambda_{\text{eff}}^2)^2} \Re(\lambda_{\text{eff}} g_S - g_T) + \frac{1}{4} \frac{1}{1 + 3\lambda_{\text{eff}}^2} |g_S + g_T|^2 \\
- \frac{1}{4} \frac{(1 + \lambda_{\text{eff}})^2}{1 + 3\lambda_{\text{eff}}^2} \left[ (1 + \lambda_{\text{eff}})(|g_S|^2 + 3|g_T|^2) + 4\Re(g_S + 3\lambda_{\text{eff}} g_T) \Re(g_S + g_T) \right] \\
+ \frac{1}{4} \frac{(1 + \lambda_{\text{eff}})^2}{(1 + 3\lambda_{\text{eff}}^2)^3} \left[ \Re(g_S + 3\lambda_{\text{eff}} g_T) \right]^2 \right\} (1 + \cos \theta),
\]

\[
4\pi \frac{dW_2^{(\pm)}(\theta)}{d\Omega} = \left\{ \frac{1}{4} \frac{(1 - \lambda_{\text{eff}})^2}{1 + 3\lambda_{\text{eff}}^2} + \frac{1}{4} \frac{(1 - \lambda_{\text{eff}})(1 + 3\lambda_{\text{eff}})}{(1 + 3\lambda_{\text{eff}}^2)^2} \Re(\lambda_{\text{eff}} g_S - g_T) + \frac{1}{4} \frac{1}{1 + 3\lambda_{\text{eff}}^2} |g_S - g_T|^2 \\
- \frac{1}{4} \frac{(1 - \lambda_{\text{eff}})^2}{1 + 3\lambda_{\text{eff}}^2} \left[ (1 - \lambda_{\text{eff}})(|g_S|^2 + 3|g_T|^2) + 4\Re(g_S + 3\lambda_{\text{eff}} g_T) \Re(g_S - g_T) \right] \\
+ \frac{1}{4} \frac{(1 - \lambda_{\text{eff}})^2}{(1 + 3\lambda_{\text{eff}}^2)^3} \left[ \Re(g_S + 3\lambda_{\text{eff}} g_T) \right]^2 \right\} (1 - \cos \theta),
\]

\[
4\pi \frac{dW_3^{(\pm)}(\theta)}{d\Omega} = \left\{ \frac{\lambda_{\text{eff}}^2}{1 + 3\lambda_{\text{eff}}^2} - \frac{2\lambda_{\text{eff}}}{(1 + 3\lambda_{\text{eff}}^2)^2} \Re(\lambda_{\text{eff}} g_S - g_T) + \frac{1}{1 + 3\lambda_{\text{eff}}^2} |g_T|^2 \\
- \frac{\lambda_{\text{eff}}^2}{(1 + 3\lambda_{\text{eff}}^2)^3} \left[ \Re(g_S + 3\lambda_{\text{eff}} g_T) \right]^2 \right\} (1 + \cos \theta),
\]

\[
4\pi \frac{dW_4^{(\pm)}(\theta)}{d\Omega} = \left\{ \frac{\lambda_{\text{eff}}^2}{1 + 3\lambda_{\text{eff}}^2} - \frac{2\lambda_{\text{eff}}}{(1 + 3\lambda_{\text{eff}}^2)^2} \Re(\lambda_{\text{eff}} g_S - g_T) + \frac{1}{1 + 3\lambda_{\text{eff}}^2} |g_T|^2 \\
- \frac{\lambda_{\text{eff}}^2}{(1 + 3\lambda_{\text{eff}}^2)^3} \left[ \Re(g_S + 3\lambda_{\text{eff}} g_T) \right]^2 \right\} (1 - \cos \theta).
\]

(17)
\[ -\frac{\lambda_{\text{eff}}}{(1 + 3\lambda_{\text{eff}}^2)^2} \left[ \lambda_{\text{eff}} (|g_S|^2 + 3|g_T|^2) + 4\text{Re}(g_S + 3\lambda_{\text{eff}} g_T) \text{Re}(g_T) \right] \\
+ \frac{4\lambda_{\text{eff}}^2}{(1 + 3\lambda_{\text{eff}}^2)^3} \left[ \text{Re}(g_S + 3\lambda_{\text{eff}} g_T) \right]^2 \left(1 - \cos \theta\right), \]

\[ 4\pi \frac{dW_4^{(+)} (\theta)}{d\Omega} = 0, \]  

(18)

For the neutron spin polarisation \( \sigma_n = -1/2 \) the angular distributions of the probabilities \( W_1^{(-)} (\theta), W_2^{(-)} (\theta), W_3^{(-)} (\theta) \) and \( W_4^{(-)} (\theta) \) are

\[ \frac{dW_1^{(-)} (\theta)}{d\Omega} = \frac{dW_4^{(+)} (\pi - \theta)}{d\Omega}, \]
\[ \frac{dW_2^{(-)} (\theta)}{d\Omega} = \frac{dW_3^{(+)} (\pi - \theta)}{d\Omega}, \]
\[ \frac{dW_3^{(-)} (\theta)}{d\Omega} = \frac{dW_2^{(+)} (\pi - \theta)}{d\Omega}, \]
\[ \frac{dW_4^{(-)} (\theta)}{d\Omega} = \frac{dW_1^{(+)} (\pi - \theta)}{d\Omega}. \]  

(19)

In comparison with [1] we have expanded the angular distributions up to second order in scalar and tensor couplings. We have found that to linear approximation with respect to the coupling constants \( g_S \) and \( g_T \) the bound–state \( \beta^- \)–decay of the free neutron is sensitive to the effective coupling constant \( \text{Re}(\lambda_{\text{eff}} g_S - g_T) \) only. The terms of order \( |g_S|^2, |g_T|^2 \) and \( \text{Re}(g^*_S g_T) \) contain more complicated combinations of the coupling constants \( g_S \) and \( g_T \). However, both of these contributions contain no information on right–handed leptonic currents.

III. BOUND–STATE \( \beta^- \)–DECAY OF NEUTRON AND RIGHT–HANDED NEUTRINOS

The absence of contributions of right–handed leptonic currents in the probabilities of the bound–state \( \beta^- \)–decay, calculated in section [1], is not a surprise, because we have used there the wave functions of antineutrinos in the right–handed polarisation state, corresponding to the left–handed polarisation state of neutrinos. Being multiplied by the projection operator \( P_R = (1 + \gamma^5)/2 \), appearing in the right–handed leptonic currents, the wave function of antineutrinos \( \bar{v}_{\nu_e} \) in the right–handed polarisation state with \( \bar{\sigma} \cdot \vec{n} \chi_{\bar{v}_{\nu_e}} = -\chi_{\bar{v}_{\nu_e}} \) gives a vanishing contribution, i.e. \( P_R \bar{v}_{\nu_e} = 0 \).

In this section, we assume that antineutrinos can have also left–handed polarisation state that is possible if antineutrinos (neutrinos) are massive. According to [18], a mass of the electron neutrino (antineutrino) should not exceed a few eV. Since in the bound–state \( \beta^- \)–decay \( E = Q_{\beta^-} = 0.782 \text{ MeV} \) [1], one can neglect the antineutrino mass with respect to the antineutrino energy \( E \) and use the Dirac wave function Eq. [16]. However, the Pauli wave function \( \chi_{\bar{v}_{\nu_e}} \) of antineutrinos in the left–handed polarisation state should obey the equation \( \bar{\sigma} \cdot \vec{n} \chi_{\bar{v}_{\nu_e}} = +\chi_{\bar{v}_{\nu_e}} \). If the axis of the antineutrino–spin quantisation is inclined relative to the

\[ \chi_{\bar{v}_{\nu_e}} = C\bar{v}_{\nu_e}, \]

where the Pauli spinor wave function \( \varphi_{\bar{v}_{\nu_e}} = -i\sigma^2 \chi_{\bar{v}_{\nu_e}} \) obeys the equation \( \bar{\sigma} \cdot \vec{n} \varphi_{\bar{v}_{\nu_e}} = -\varphi_{\bar{v}_{\nu_e}} \).
axis of the neutron–spin quantisation with a polar angle $\vartheta$, the Pauli wave function $\chi_{\bar{\nu}_e}$ is given by

$$\chi_{\bar{\nu}_e} = \begin{pmatrix} \cos \frac{\vartheta}{2} \\ e^{+i\varphi} \sin \frac{\vartheta}{2} \end{pmatrix}. \quad (20)$$

The amplitude of the bound–state $\beta^-$–decay of the neutron with antineutrinos in the left–handed polarisation state is given by

$$M(n \to H + \bar{\nu}_e)_{lh} = G_F (V_{ud})_{\text{eff}} \sqrt{2 m_n} 2E_H 2E \left( 1 + \frac{\alpha}{2\pi} f_{\beta_e} \right) \times \left\{ \bar{g}_S [\varphi_p^\dagger \varphi_n] [\varphi_p^\dagger \chi_{\bar{\nu}_e}] + \bar{g}_T [\varphi_p^\dagger \bar{\sigma} \varphi_n] \cdot [\varphi_p^\dagger \bar{\sigma} \chi_{\bar{\nu}_e}] \right\} \psi^*_{(ns)}(0), \quad (21)$$

where the abbreviation “lh” means the left–handed polarisation state. The coupling constants $\bar{g}_S$ and $\bar{g}_T$ are equal to

$$\bar{g}_S = \frac{1}{2} \left( (C_{S,\text{eff}} + \bar{C}_{S,\text{eff}}) + (C_{V,\text{eff}} + \bar{C}_{V,\text{eff}}) \right) = \bar{A}_{RR}^h + \bar{A}_{RL}^h + \bar{a}_{RR}^h + \bar{a}_{RL}^h,$$

$$\bar{g}_T = \frac{1}{2} \left( (C_{T,\text{eff}} + \bar{C}_{T,\text{eff}}) - (C_{A,\text{eff}} + \bar{C}_{A,\text{eff}}) \right) = 2\bar{a}_{RR}^h - \bar{a}_{RR}^h + \bar{a}_{RL}^h. \quad (22)$$

One may see that the coupling constants $\bar{g}_S$ and $\bar{g}_T$ are defined in terms of the contributions of the right–handed leptonic currents and left(right)–handed hadronic currents only. The coupling constants $\bar{a}_{RR}^h$ and $\bar{a}_{RL}^h$ can in principle be induced by exchanges of electroweak $W_R^\pm$ bosons, causing effective low–energy current–current interactions $(V+A)_{\text{leptonic}}(V+A)_{\text{hadronic}} \quad [26, 28]$. Of course, the contributions of these interactions can be screened by scalar and tensor interactions with coupling constants $\bar{A}_{RR}^h, \bar{A}_{RL}^h$ and $2\bar{a}_{RR}^h$.

The contributions of different spinorial states to the helicity amplitudes of the bound–state $\beta^-$–decay of the neutron with the antineutrino in the left–handed polarisation state as functions of the angles $\vartheta$ and $\varphi$ are given in Table II.

| $\sigma_n$ | $\sigma_p$ | $\sigma_e$ | $\sigma_{\bar{\nu}_e}$ | $f$ |
|------------|------------|------------|------------------------|-----|
| $+\frac{1}{2}$ | $+\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $(\bar{g}_S - \bar{g}_T) e^{+i\varphi} \sin \frac{\vartheta}{2}$ |
| $+\frac{1}{2}$ | $+\frac{1}{2}$ | $+\frac{1}{2}$ | $+\frac{1}{2}$ | $(\bar{g}_S + \bar{g}_T) \cos \frac{\vartheta}{2}$ |
| $+\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $0$ |
| $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $2\bar{g}_T e^{+i\varphi} \sin \frac{\vartheta}{2}$ |
| $-\frac{1}{2}$ | $+\frac{1}{2}$ | $+\frac{1}{2}$ | $+\frac{1}{2}$ | $2\bar{g}_T \cos \frac{\vartheta}{2}$ |
| $-\frac{1}{2}$ | $+\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $0$ |
| $+\frac{1}{2}$ | $+\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $(\bar{g}_S + \bar{g}_T) e^{+i\varphi} \sin \frac{\vartheta}{2}$ |
| $+\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $(\bar{g}_S - \bar{g}_T) \cos \frac{\vartheta}{2}$ |

TABLE II: The contributions of different spinorial states of the interacting particles to the amplitudes of the bound-state $\beta^-$–decay of the neutron and the antineutrino in the state with the wave function Eq. (20); $f$ is defined by $f = \bar{g}_S [\varphi_p^\dagger \chi_{\bar{\nu}_e}] [\varphi_p^\dagger \varphi_n] + \bar{g}_T [\varphi_p^\dagger \bar{\sigma} \chi_{\bar{\nu}_e}] \cdot [\varphi_p^\dagger \bar{\sigma} \varphi_n]$. 
Using the results in Table II we get the helicity amplitudes $M(n \rightarrow H_{F_{MP}} + \bar{\nu}_e)_{\sigma_{u,-\frac{1}{2}}}$

\[
M(n \rightarrow H_{00} + \bar{\nu}_e)_{\frac{1}{2},-\frac{1}{2}} = +M_0 \frac{\bar{g}_s - 3\bar{g}_T}{\sqrt{2}} e^{+i\varphi} \sin \frac{\vartheta}{2},
\]

\[
M(n \rightarrow H_{1,+1} + \bar{\nu}_e)_{\frac{1}{2},-\frac{1}{2}} = +M_0 (\bar{g}_S + \bar{g}_T) \cos \frac{\vartheta}{2},
\]

\[
M(n \rightarrow H_{10} + \bar{\nu}_e)_{\frac{1}{2},-\frac{1}{2}} = +M_0 \frac{\bar{g}_S + \bar{g}_T}{\sqrt{2}} e^{+i\varphi} \sin \frac{\vartheta}{2},
\]

\[
M(n \rightarrow H_{1,-1} + \bar{\nu}_e)_{\frac{1}{2},-\frac{1}{2}} = 0,
\]

\[
M(n \rightarrow H_{00} + \bar{\nu}_e)_{-\frac{1}{2},-\frac{1}{2}} = -M_0 \frac{\bar{g}_s - 3\bar{g}_T}{\sqrt{2}} \cos \frac{\vartheta}{2},
\]

\[
M(n \rightarrow H_{1,+1} + \bar{\nu}_e)_{-\frac{1}{2},-\frac{1}{2}} = 0,
\]

\[
M(n \rightarrow H_{10} + \bar{\nu}_e)_{-\frac{1}{2},-\frac{1}{2}} = +M_0 (\bar{g}_S + \bar{g}_T) \cos \frac{\vartheta}{2},
\]

\[
M(n \rightarrow H_{1,-1} + \bar{\nu}_e)_{-\frac{1}{2},-\frac{1}{2}} = +M_0 (\bar{g}_S + \bar{g}_T) e^{+i\varphi} \sin \frac{\vartheta}{2}.
\] (23)

Taking into account the contributions of the left-handed polarisation states of antineutrinos for the angular distributions we obtain the following expressions

\[
4\pi dW^{(\pm)}_{F=0}(\theta) = \left\{ \frac{1}{8} \frac{(1 - 3\lambda_{\text{eff}})^2}{1 + 3\lambda_{\text{eff}}^2} + \frac{3}{4} \frac{(1 + \lambda_{\text{eff}})(1 - 3\lambda_{\text{eff}})}{(1 + 3\lambda_{\text{eff}}^2)^2} \frac{\text{Re}(\lambda_{\text{eff}} g_S - g_T) + \frac{1}{8} \frac{1}{1 + 3\lambda_{\text{eff}}^2} |g_S - 3g_T|^2}{|\bar{g}_S - 3\bar{g}_T|^2 (1 + \cos \theta)} \right\} (1 + \cos \theta) + \frac{1}{8} \frac{1}{1 + 3\lambda_{\text{eff}}^2} \frac{\text{Re}(\lambda_{\text{eff}} g_S - g_T) + \frac{1}{8} \frac{1}{1 + 3\lambda_{\text{eff}}^2} |g_S + g_T|^2}{|g_S + g_T|^2 (1 + \cos \theta)} \right\} (1 + \frac{1}{3} \cos \theta)
\] (24)

and

\[
4\pi dW^{(\pm)}_{F=1}(\theta) = \left\{ \frac{1}{4} \frac{(1 + \lambda_{\text{eff}})^2}{1 + 3\lambda_{\text{eff}}^2} - \frac{1}{2} \frac{(1 + \lambda_{\text{eff}})(1 - 3\lambda_{\text{eff}})}{(1 + 3\lambda_{\text{eff}}^2)^2} \frac{\text{Re}(\lambda_{\text{eff}} g_S - g_T) + \frac{1}{4} \frac{1}{1 + 3\lambda_{\text{eff}}^2} |g_S + g_T|^2}{|\bar{g}_S + g_T|^2 (1 + \cos \theta)} \right\} (1 + \cos \theta) + \frac{1}{4} \frac{1}{1 + 3\lambda_{\text{eff}}^2} \frac{\text{Re}(\lambda_{\text{eff}} g_S - g_T) + \frac{1}{4} \frac{1}{1 + 3\lambda_{\text{eff}}^2} |g_S - g_T|^2}{|\bar{g}_S + g_T|^2 (1 - \cos \theta)} \right\} (1 + \frac{1}{3} \cos \theta),
\]

\[4\pi dW^{(+)}_{F=1}(\theta) = \left\{ \frac{1}{4} \frac{(1 - \lambda_{\text{eff}})^2}{1 + 3\lambda_{\text{eff}}^2} + \frac{1}{2} \frac{(1 - \lambda_{\text{eff}})(1 + 3\lambda_{\text{eff}})}{(1 + 3\lambda_{\text{eff}}^2)^2} \frac{\text{Re}(\lambda_{\text{eff}} g_S - g_T) + \frac{1}{4} \frac{1}{1 + 3\lambda_{\text{eff}}^2} |g_S - g_T|^2}{|\bar{g}_S + g_T|^2 (1 - \cos \theta)} \right\} (1 + \frac{1}{3} \cos \theta),
\]

\[
4\pi dW^{(+)}_{F=2}(\theta) = \left\{ \frac{1}{4} \frac{(1 - \lambda_{\text{eff}})^2}{1 + 3\lambda_{\text{eff}}^2} + \frac{1}{2} \frac{(1 - \lambda_{\text{eff}})(1 + 3\lambda_{\text{eff}})}{(1 + 3\lambda_{\text{eff}}^2)^2} \frac{\text{Re}(\lambda_{\text{eff}} g_S - g_T) + \frac{1}{4} \frac{1}{1 + 3\lambda_{\text{eff}}^2} |g_S - g_T|^2}{|\bar{g}_S + g_T|^2 (1 - \cos \theta)} \right\} (1 + \frac{1}{3} \cos \theta),\]
given by probabilties depend only on the effective coupling constant \( \Re(\lambda) \). Herczeg’s phenomenological coupling constants, introduced at the hadronic level \([3]\), these handed baryon currents, iii) we have found that to linear approximation with respect to in the right–handed polarisation state interact only with the scalar and tensor left(right)–hand baryon currents.

For the calculation of the angular distributions Eqs.(24), (25) and (26) we have neglected the contributions of order \( \alpha g_s^2/\pi \), \( \alpha |\bar{g}_T|^2/\pi \) and \( \alpha \Re(\bar{g}_S g_T)/\pi \).

The angular distributions of the probabilities \( W_1^{(-)}(\theta) \), \( W_2^{(-)}(\theta) \), \( W_3^{(-)}(\theta) \) and \( W_4^{(-)}(\theta) \) of the neutron bound \( \beta^– \)–decay with the neutron in the spin polarisation state \( \sigma_n = -\frac{1}{2} \) are given by

\[
\frac{dW_1^{(-)}(\theta)}{d\Omega} = \frac{dW_4^{(+)}(\pi - \theta)}{d\Omega}, \\
\frac{dW_2^{(-)}(\theta)}{d\Omega} = \frac{dW_3^{(+)}(\pi - \theta)}{d\Omega}, \\
\frac{dW_3^{(-)}(\theta)}{d\Omega} = \frac{dW_2^{(+)}(\pi - \theta)}{d\Omega}, \\
\frac{dW_4^{(-)}(\theta)}{d\Omega} = \frac{dW_1^{(+)}(\pi - \theta)}{d\Omega}.
\]

For the calculation of the angular distributions Eqs.(24), (25) and (26) we have neglected the contributions of order \( \alpha g_s^2/\pi \), \( \alpha |\bar{g}_T|^2/\pi \) and \( \alpha \Re(\bar{g}_S g_T)/\pi \).

**IV. CONCLUSION**

We have revisited the bound-state \( \beta^- \)–decay of the free neutron and addressed the paper to the analysis of the set of observables, which can be used for experimental searches of contributions of interactions beyond the SM and the left–handed polarisation state of antineutrinos. For this aim we have analysed the dependence of the probabilities of the bound-state \( \beta^- \)–decay of the neutron and their angular distributions on phenomenological coupling constants \([13, 15, 17]\), describing the most general weak effective lepton–nucleon interactions.

In comparison with the results, obtained in \([1]\): i) we have analysed the probabilities of the bound–state \( \beta^- \)–decay of the neutron by using the most general phenomenological lepton–baryon weak interaction for the neutron \( \beta^- \)–decay, ii) we have shown that antineutrinos in the right–handed polarisation state interact only with the scalar and tensor left(right)–handed baryon currents, iii) we have found that to linear approximation with respect to Herczeg’s phenomenological coupling constants, introduced at the hadronic level \([3]\), these probabilities depend only on the effective coupling constant \( \Re(\lambda_{\text{eff}} g_S - g_T) \), which carries
no information about weak interactions, caused by right-handed leptonic and left(right)-handed hadronic currents, iv) we have calculated the contributions of the left-handed polarisation state of antineutrinos and found that the effective scalar $\bar{g}_S$ and tensor $\bar{g}_T$ coupling constants are caused by contributions of the vector and axial-vector baryon currents and left(right)-handed baryon scalar and tensor currents of interactions beyond the SM, v) we have calculated the probabilities of the production of hydrogen in the hyperfine states with $F = 0$ and $F = 1$ and the probabilities of the production of the $(pe^-)$ pairs in the spinorial states $|+1/2\rangle_p + 1/2\rangle_e$, $|+1/2\rangle_p - 1/2\rangle_e$, $|-1/2\rangle_p + 1/2\rangle_e$ and $|-1/2\rangle_p - 1/2\rangle_e$ and their angular distributions to second order of the effective coupling constants of interactions beyond the SM for antineutrinos in the right(left)-handed polarisation state, vi) we have found that in the probabilities under consideration the contributions of the left-handed polarisation state of antineutrinos can be screened by the contributions of their right-handed polarisation state. This implies that the probabilities of the bound-state $\beta^-$-decay $W_F^{(\pm)}$, $W_1^{(\pm)}$, $W_2^{(\pm)}$, $W_3^{(\pm)}$ and $W_4^{(\pm)}$ can be used as observables for experimental investigations of contributions of interactions beyond the SM without specification of the contributions of the polarisation states of antineutrinos and vii) we have shown that the angular distributions of most of these probabilities are good observables for the detection of contributions of the left-handed polarisation state of antineutrinos.

Indeed, for example, for the neutron in the spin state $\sigma_n = +\frac{1}{2}$ the angular distributions of the probabilities of the production of hydrogen in the hyperfine state with $F = 0$ and of the $(pe^-)$ pairs in the spinorial states $|+1/2\rangle_p + 1/2\rangle_e$, $|+1/2\rangle_p - 1/2\rangle_e$ and $|-1/2\rangle_p + 1/2\rangle_e$ at a fixed angle $\theta = 0$ and $\theta = \pi$ we obtain

\[
4\pi \frac{dW_F^{(\pm)}(\theta)}{d\Omega} \bigg|_{\theta=0} = \frac{1}{4} \frac{1}{1 + 3\lambda_{\text{eff}}^2} |\bar{g}_S - 3\bar{g}_T|^2 = 4.25 \times 10^{-2} |\bar{g}_S - 3\bar{g}_T|^2, \\
4\pi \frac{dW_1^{(\pm)}(\theta)}{d\Omega} \bigg|_{\theta=\pi} = \frac{1}{2} \frac{1}{1 + 3\lambda_{\text{eff}}^2} |\bar{g}_S + \bar{g}_T|^2 = 8.50 \times 10^{-2} |\bar{g}_S + \bar{g}_T|^2, \\
4\pi \frac{dW_2^{(\pm)}(\theta)}{d\Omega} \bigg|_{\theta=0} = \frac{1}{2} \frac{1}{1 + 3\lambda_{\text{eff}}^2} |\bar{g}_S - \bar{g}_T|^2 = 8.50 \times 10^{-2} |\bar{g}_S - \bar{g}_T|^2, \\
4\pi \frac{dW_4^{(\pm)}(\theta)}{d\Omega} \bigg|_{\theta=\pi} = 2 \frac{1}{1 + 3\lambda_{\text{eff}}^2} |\bar{g}_T|^2 = 34.03 \times 10^{-2} |\bar{g}_T|^2. \tag{27}
\]

The numerical coefficients are calculated for $\lambda_{\text{eff}} = -1.2750 \ [3]$. The probability $W_{F=1}^{(\pm)}$ and its angular distribution $dW_{F=1}^{(\pm)}(\theta)/d\Omega$ are not sensitive to the contributions of the right(left)-handed polarisation states of antineutrinos, caused by interactions beyond the SM. Then, the probability $W_4^{(\pm)}$ of the production of the $(pe^-)$ pair in the spinorial state $|-1/2\rangle_p - 1/2\rangle_e$ and its angular distribution $dW_4^{(\pm)}(\theta)/d\Omega$ vanish for all polarisation states of antineutrinos. For the neutron in the spin state with $\sigma_n = -\frac{1}{2}$ the angular distributions of the corresponding probabilities are given in Eq.(26).

It is important to notice that the effective scalar $\bar{g}_S$ and tensor $\bar{g}_T$ coupling constants, which can be measured from the angular distributions Eq.(27), depend on the coupling constants of lepton interactions with vector and axial-vector baryon currents, caused by right-handed electroweak bosons, and the left(right)-handed scalar and tensor baryon currents (see Eq.(22)). In this case it seems that it should be hard to distinguish the contributions of interactions with the vector and axial-vector baryon currents, caused by the right-handed
electroweak bosons like those in the electroweak models with left–right symmetries [26, 28], from the interactions with the left(right)–handed scalar and tensor baryon currents. The available experimental data on the correlation coefficients and the lifetime of the neutron $\beta^-$–decay [3] cannot be used for an estimate of the contributions of the scalar and tensor interactions beyond the SM. As has been shown in [3] the electron–neutron spin asymmetry $A_{\text{exp}}(E_e)$ and the proton recoil energy spectrum $a(T_p)$, where $T_p$ is a kinetic energy of the decay proton, can be used for measurements of the axial coupling constant $\lambda_{\text{eff}}$ and the Fierz term $b_F$. Then, the antineutrino–neutron spin asymmetry $B_{\text{exp}}(E_e)$ and the proton–recoil–neutron spin asymmetry $C_{\text{exp}}$ can give an information about the real parts of the scalar $g_S$ and tensor $g_T$ coupling constants. In turn, the experimental data on the lifetime of the neutron $(\tau_n)_{\text{exp}}$, taken together with the experimental data on the axial coupling constant $\lambda_{\text{eff}}$ and the Fierz term $b_F$ measured from the electron–neutron spin asymmetry $A^W(E_e)$ and the proton recoil energy spectrum $a(T_p)$, are able to give the information about the CKM matrix element $V_{ud}$. Since available experimental data on the electron–neutron spin asymmetry $A^W(E_e)$, the proton recoil energy spectrum $a(T_p)$, the antineutrino–neutron spin asymmetry $B_{\text{exp}}(E_e)$ and the proton–recoil–neutron spin asymmetry $C_{\text{exp}}$ are not precise enough and were not elaborated together with the aim to extract the experimental value of the axial coupling constant and the experimental constraints on the scalar $g_S$ and tensor coupling $g_T$ constants we propose to make an estimate of the Fierz term $b_F$ by using the theoretical value of the neutron lifetime $(\tau_n)_{\text{th}} = 879.6(1.1)$ s, calculated in [3], and the world average value $(\tau_n)_{\text{exp}} = 880.1(1.1)$ s [13]. Using the results, obtained in [3] (see Chapter X of Ref. [3]) we get

$$b_F = -\frac{1}{\langle m_e/E_e \rangle} \frac{(\tau_n)_{\text{exp}} - (\tau_n)_{\text{th}}}{(\tau_n)_{\text{th}}} \pm \frac{\sqrt{2}}{(\tau_n)_{\text{th}}^2} \Delta \tau_n = (-0.87 \pm 2.70) \times 10^{-3}, \quad (28)$$

where $\langle m_e/E_e \rangle = 0.6556$ [3] and $\Delta \tau_n = 1.1$ s. At $\lambda_{\text{eff}} = -1.2750$ for $\text{Re}(g_S + 3\lambda_{\text{eff}}g_T)$ we get the following estimate

$$\text{Re}(g_S + 3\lambda_{\text{eff}}g_T) = (-2.56 \pm 7.93) \times 10^{-3}. \quad (29)$$

Since to order $10^{-3}$ the right-hand side (r.h.s) of Eq. (29) is commensurable with zero, we may only assume that $\text{Re}(g_S + 3\lambda_{\text{eff}}g_T) \sim 10^{-5}$ or even smaller. Hence, an expected value of the effective coupling constant $\text{Re}(\lambda_{\text{eff}}g_S - g_T)$ is $\text{Re}(\lambda_{\text{eff}}g_S - g_T) \sim 10^{-4}$. This results in that $10^{-5}$ is an expected order of the contribution to the angular distribution $dW^{(\pm)}_{F=0}(\theta)/d\Omega$ of the term, proportional to $\text{Re}(\lambda_{\text{eff}}g_S - g_T) \sim 10^{-4}$, relative to the main one. Thus, if the coupling constants of the right–handed leptonic currents $|\tilde{g}_S|$ and $|\tilde{g}_T|$ are of order $|\tilde{g}_S| \sim |\tilde{g}_T| \sim 10^{-4}$ the contribution of the left–handed polarisation state of antineutrinos is of order $10^{-9}$ or even smaller.

V. ACKNOWLEDGEMENT

We are grateful to Hartmut Abele for fruitful discussions. This work was supported by the Austrian “Fonds zur Förderung der Wissenschaftlichen Forschung” (FWF) under the
contracts I689-N16, I534-N20 PERC and I862-N20.

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