Measurement of the Spin Tune Using the Coherent Spin Motion of Polarized Proton in a Storage Ring

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This paper reports the first spin tune measurement at high energies (24 GeV and 255 GeV) with a driven coherent spin motion. To maintain polarization in a polarized proton collider, it is important to know the spin tune of the polarized proton beam, which is defined as the number of full spin precessions per revolution. A nine-magnet spin flipper has demonstrated high spin-flip efficiency in the presence of two Siberian snakes [1]. The spin flipper drives a spin resonance with a given frequency (or tune) and strength. When the drive tune is close to the spin tune, the proton spin direction is not vertical anymore, but precesses around the vertical direction. By measuring the precession frequency of the horizontal component the spin tune can be precisely measured. A driven coherent spin motion and fast turn-by-turn polarization measurement are keys to the measurement. The vertical spin direction is restored after turning the spin flipper off and the polarization value is not affected by the measurement. The fact that this manipulation preserves the polarization makes it possible to measure the spin tune during operation of a high energy accelerator.

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Introduction--There have been multiple efforts to understand the origin of the nucleon spin structure since the discovery of the spin anomaly by the European Muon Collaboration [2]. Colliders with polarized beams play an important role in the precision experiments to explain the proton spin structure [3,4] and also are important topics in accelerator physics. In this paper, we report the first measurement of driven coherent spin motion in a high energy collider. These techniques of manipulating polarization properties of beams complement the well established techniques of controlling of orbit properties.

To avoid polarization loss from depolarizing resonances during acceleration and at store, high energy polarized proton colliders require full Siberian snakes, which are specially arranged magnets to rotate the spin by 180° around an axis in the horizontal plane [5]. For the Relativistic Heavy Ion Collider (RHIC), a pair of Siberian snakes are installed in each ring. This configuration yields a spin tune \( \nu_s = \frac{1}{2} \) [6], defined as the number of spin precessions per turn. A spin tune of \( \frac{1}{2} \) avoids all depolarizing resonances as long as the vertical betatron tune \( \nu_y \) is not also \( \frac{1}{2} \). However, the higher-order resonances (snake resonances) [7] still can lead to polarization loss. The resonance condition is

\[
\nu_s = k + m \nu_y,
\]

where \( k \) and \( m \) are integers. Snake imperfections and closed orbit errors can also shift the spin tune away from \( \frac{1}{2} \). This shift leads to a shift of snake resonance locations and limits the possible operating parameters of the accelerator [8].

To avoid these higher order resonance conditions, knowing both the vertical betatron tune and the spin tune accurately is important. Betatron tune measurements have been done with various methods in synchrotrons [9]. It is typically measured with coherent turn-by-turn beam oscillations in response to a beam excitation. The spin tune measurement is much harder. A nine-magnet spin flipper has been used in RHIC to flip spin [1]. This is accomplished by sweeping the drive tune of the spin flipper ac dipole across the spin tune with the proper crossing speed. The spin flipper can also be used to drive an artificial spin resonance at a fixed tune. When the drive tune is near the spin tune, the polarization direction is moved away from vertical and is precessing around the vertical direction. The vertical polarization measurement as a function of drive tune in the vicinity of the expected spin tune can give a direct measurement of the spin tune. Examples are shown in Fig. 2 of Ref. [1]. However, such a measurement often results in decoherence of the spin motion and loss of polarization, so it requires several fills of freshly polarized proton beams. It becomes very time consuming if acceleration to higher energy is needed.

In principle, the spin tune can be measured with a similar idea as the betatron tune measurement: measuring the spin response to a driven spin coherence. Such a method can also be non-destructive. A coherent spin precession around the vertical direction can be adiabatically induced by driving the ac spin rotator at a drive tune near the spin tune.

If the undisturbed stable spin direction is vertical, the vertical component of polarization \( P_y \) in the neighborhood of an isolated spin resonance is given by [11]:

\[
P_y = \frac{\nu_s - \nu_{osc}}{\sqrt{\left|\nu_s - \nu_{osc}\right|^2 + |\epsilon|^2}},
\]

where \( \epsilon \) is the strength of the driven spin resonance and \( \nu_{osc} \) is the drive tune. The horizontal component oscillates with \( \nu_{osc} \):

\[
P_x = \frac{\epsilon}{\sqrt{\left|\nu_s - \nu_{osc}\right|^2 + |\epsilon|^2}} \cos(2\pi\nu_{osc}t - \Psi),
\]

where \( \Psi \) is the phase of the drive.
where \( i \) is the \( i \)th orbital revolution and \( \Psi \) is the initial phase offset. Eqs. (8,9) describe the vertical and horizontal components in a perfect accelerator in the presence of a single isolated spin resonance. The ratio of the amplitude part of Eq. (8) \( \hat{P}_x \) and \( \hat{P}_y \) gives the difference between \( \nu_s \) and \( \nu_{osc} \):

\[
\tan \theta_0 = \frac{\hat{P}_x}{\hat{P}_y} = \frac{|\epsilon|}{\nu_s - \nu_{osc}}, \tag{4}
\]

where \( \theta_0 \) is the opening angle of the polarization vector. With known resonance strength \( \epsilon \) from the spin flipper and the drive tune \( \nu_{osc} \), the spin tune \( \nu_s \) can be derived from the measured quantity \( \tan \theta_0 \).

There are two advantages to this technique. First, it is an adiabatic spin manipulation and can preserve the beam polarization. Second, this is a relatively fast measurement. Hence, this technique is ideal for measuring the spin tune at the store energy of a high-energy polarized synchrotron, such as RHIC or a future polarized electron ion collider [11]. The spin tune measurement with coherent spin motion has been used for deuteron beams [12] at low energy (\( \sim 1 \) GeV) in COSY [13], although the coherent spin motion was not driven. Such a measurement is important for the spin manipulation [14] needed for the storage-ring-based electric dipole moment measurement [15]. In this paper we report the first spin tune measurement at high energies (24 GeV and 255 GeV) for protons using driven coherent spin motion.

Requirements for the experiment—For the success of a spin tune measurement using driven coherent spin motion, several conditions have to be met. First, a large enough oscillation amplitude needs to be generated. This requires a strong enough driven spin resonance and a small enough separation between the drive tune and the spin tune. Second, the spin tune spread needs to be small. This allows a drive tune close to the resonance and significantly enhances the signals. Third, the measurement requires a large data sample from the polarimeter and the polarization must be measured as a function of the oscillation phase. Although the vertical component is constant, the horizontal component is changing from turn to turn. To measure such an oscillation accurately, the polarimeter needs to measure the polarization over many turns as a function of the oscillation phase.

The RHIC spin flipper can induce a spin resonance with a strength of \( \epsilon = 0.00057 \). At injection energy, the strength is limited to 0.00024 due to the larger beam size. For this method to work, the separation between \( \nu_s \) and \( \nu_{osc} \) should be similar to the strength to generate a sizable horizontal polarization component.

**Polarimeter and spin vector measurements**—The RHIC p-carbon (pC) polarimeters measure recoil carbon asymmetries in the Coulomb-Nuclear Interference (CNI) region [17-19]. Carbon nuclei are scattered in the plane transverse to the polarized proton direction with an azimuthal angle \( \theta \) distribution

\[
\frac{dN}{d\theta} \propto 1 + a \sin(\theta - \theta_s). \tag{5}
\]

Here \( a = A_N P \), where \( A_N \) is the CNI asymmetry for 100% polarization, which has values \( \sim 10^{-2} \), and \( P \) at RHIC has values \( \sim 0.5-0.8 \). \( \theta_s \) is the azimuthal angle of the spin direction in the plane transverse to the beam direction. The small value of the oscillatory term, \( a \sim \text{few} \times 10^{-3} \), dictates that a large statistical sample is necessary to accurately measure it on top of the flat underlying component.

Figure 1 shows a pC polarimeter layout in the plane transverse to the beam direction. Six silicon strip detectors are arranged as shown approximately 18 cm from the target; they measure energy (\( E_c \)) and time of flight (TOF), allowing selection of the scattered carbon nuclei. An ultra thin carbon target is moved into the beam during polarimeter measurements [20]. The detectors below and above the horizontal plane (Nos. 1, 3, 4 and 6 in Fig. 1) make it possible to measure both vertical and horizontal components of beam polarization.

In operation RHIC is filled with approximately equal numbers of spin up and spin down proton bunches. The carbon hits are recorded with a unique bunch crossing identifier, and hits for up and down bunches are counted separately. A change from spin up to spin down amounts to a shift \( \theta_s \rightarrow \theta_s + \pi \) in Eq. (5). The asymmetry of carbon hits, i.e. the relative difference of up and down bunch hits, normalized by the number of protons on target for each case, as a function of azimuthal angle is then

\[
\text{Asymmetry}|_\theta = a \sin(\theta - \theta_s). \tag{6}
\]

During normal RHIC operation, the main result from the polarimeters is \( a \), which determines the magnitude of the beam polarization. In the present study we focus on \( \theta_s \), the azimuthal angle of the spin vector in the plane transverse to the beam direction, and how it is influenced by the coherent spin motion. To measure the driven coherent spin motion, recoil carbon events need to
be recorded on a turn-by-turn base. Figure 2 shows the spin precession projected onto the $x - y$ plane transverse to the beam direction. The pC polarimeter measures the spin vector projection in this plane. With driven coherent spin motion the spin vector in this plane oscillates over the range shown by the two dashed arrows, with a period equal to that of the driven resonance. The amplitude of the precession $\theta_0$ from Eq. (4); $\theta_{\text{tilt}}$ is an arbitrary offset between vertical and the stable spin direction. From $P_x/P_y$ the spin azimuthal angle $\theta_s$ measured by the pC polarimeter with a possible tilt angle $\theta_{\text{tilt}}$ will follow the precession

$$
\frac{P_x}{P_y} = \tan(\theta_s - \theta_{\text{tilt}}) = \tan \theta_0 \cdot \cos(2\pi \nu_{\text{osc}} i - \Psi).
$$

Note that only the two transverse components of the polarization can be measured. If the spin direction has a significant longitudinal component in addition to the angle $\theta_{\text{tilt}}$, the simple form of Eq. (7) should be modified.

Spin tune measurement results– The experiment was carried out at two different energies, injection at 24 GeV and store at 255 GeV. The revolution frequency in RHIC is about 78.20 kHz. The bunch pattern was 120 bunches in the ring and RHIC bunch crossings were used as a clock signal for the analysis. For these measurements, a signal from the resonance drive was provided to the polarimeter readout, which allowed alignment of the phase of carbon hits within one period of the resonance drive. The drive signal was read with an accuracy of two bunch crossings, whereas the typical period of the drive was $\sim 240$ bunch crossings (for a drive tune near 0.5), so the phase of carbon hits was known to within 1% of a period.

In the experiment, the spin tune was first roughly located by sweeping the drive tune. If the polarization sign flips, the spin tune is covered by the sweep range. This method successively divides the drive tune sweep range in half over the intervals that exhibit a spin-flip. The progressively narrowing tune sweep ranges caused increased polarization loss and eventually converges on a range. This typically gives a narrow range for the spin tune location. Then, with fresh beam, the drive was turned on adiabatically at fixed tune and the driven coherent spin motion was measured with the polarimeter. After the measurement, the drive was turned off adiabatically and the original spin direction was restored.

Figure 3 shows $\theta_s$ versus one cycle of drive phase for one drive setting. For statistical accuracy, the carbon hits were grouped in 6 bins of 40 bunch crossings, spanned nearly one entire drive cycle; the mean spin azimuthal angle $\theta_s$ was measured in each bin. The curve is fit to the function, from rearrangement of Eq. (7):

$$
\theta_s(i) = \theta_{\text{tilt}} + \tan^{-1} (\tan \theta_0 \cdot \cos(2\pi \nu_{\text{osc}} i - \Psi)).
$$

The arbitrary phase offset $\Psi$ depends on the propagation time of proton bunches from the drive to the polarimeter, and the cable delay of the signal from the drive to the polarimeter readout.

The measured $\theta_0$ and derived spin tune for 7 sets of measurements are shown in Table 1. The separation of the drive tune and the spin tune varies from 0.001 to 0.004. For each of the three pairs of measurements done under the same conditions, the results are consistent with each other within the statistical errors. Within statistical errors, the derived spin tune from the driven spin coherence (column 4 of Table 1) agrees with the range obtained from the spin-flip method (column 5 of Table 1).

Among the seven spin tune measurements, five are at injection. The first pair of spin tune measurements were done with two different drive tunes. The experimental errors, the derived spin tune from the driven spin coherence (column 4 of Table 1) agrees with the range obtained from the spin-flip method (column 5 of Table 1).
Although the measurement was 0.499. The experimental data and fit are shown in Fig. 4. The spin tune was also determined as between 0.496 and 0.4965 by a fixed drive tune scan (see Fig. 2 of Ref. [1]). Note that there is a quite large tilt angle of the spin of 0.25 rad at the store energy [21]. The tilt angle is probably due to a nearby orbit bump separating the two RHIC beams [22]. More study is needed to quantify this effect. Even with such a tilt angle, the spin tune still can be measured with a driven spin coherence. The spin tune measured from coherent spin motion is in agreement with the results from the spin-flip and fixed drive tune methods.

If there is a longitudinal component of the stable spin direction, the stable spin direction has an angle of \( \theta_0 \) with respect to the \((x,y)\) plane. For a rotation around the \( x \)-axis, the transverse polarization ratio becomes

\[
\tan(\theta_s - \theta_{\text{tilt}}) = \frac{\tan \theta_0 \cos \phi}{\cos \theta_t + \sin \theta_t \tan \theta_0 \sin \phi}
\]

(9)

where \( \phi = 2\pi \nu_{\text{osc}} \ell - \Psi \). Expanding the function to the first order of \( \theta_t \),

\[
\tan(\theta_s - \theta_{\text{tilt}}) \approx \tan \theta_0 \cos \phi (1 - \theta_t \tan \theta_0 \sin \phi) = \tan \theta_0 \cos \phi - \frac{\theta_t}{2} \tan^2 \theta_0 \sin 2\phi
\]

(10)

The second term is in the 2nd harmonic of the phase angle. From the shapes of the data points in Figs. 3 and 6, this component is small. In addition, for the second term to be small, \( \theta_t \) must satisfy \( \theta_t \tan \theta_0 \ll 1 \). The \( \theta_t \) from the data fits are consistent with zero within one sigma of the statistical error. The largest case is \( \theta_t = 0.66 \pm 0.06 \) rad while \( \theta_0 \) is in the range of 0.06 to 0.27, which satisfies the constraint. The resulting difference in \( \theta_0 \) with such a small \( \theta_t \) is in the order of \( 10^{-3} \) and the effect on spin tune is negligible.

**Conclusion**—Driven coherent spin motion has been used to measure the spin tune in RHIC at 24 GeV and 255 GeV. The results show that the spin tune can be measured by driven spin coherence when the tune separation is quite large.

Data and fit from the second set are shown in Fig. 5. The spin tune was also determined as between 0.496 and 0.4965 by a fixed drive tune scan (see Fig. 2 of Ref. [1]). Note that there is a quite large tilt angle of the spin of 0.25 rad at the store energy [21]. The tilt angle is probably due to a nearby orbit bump separating the two RHIC beams [22]. More study is needed to quantify this effect. Even with such a tilt angle, the spin tune still can be measured with a driven spin coherence. The spin tune measured from coherent spin motion is in agreement with the results from the spin-flip and fixed drive tune methods.

If there is a longitudinal component of the stable spin direction, the stable spin direction has an angle of \( \theta_0 \) with respect to the \((x,y)\) plane. For a rotation around the \( x \)-axis, the transverse polarization ratio becomes

| set | \( \theta_0 \) (rad) | \( \nu_{\text{osc}} \) | \( \nu_\ell \) from coherence | \( \nu_\ell \) from flip |
|-----|------------------|----------------|---------------------------|---------------------------|
| 1   | 0.273±0.039     | 0.499         | 0.4999±0.0002             | 0.4975±0.5              |
| 2   | 0.134±0.015     | 0.498         | 0.4998±0.0002             | 0.4975±0.5              |
| 3   | 0.109±0.015     | 0.5004        | 0.5026±0.0003             | 0.5022±0.5025           |
| 4   | 0.132±0.021     | 0.5009        | 0.5027±0.0003             | 0.5022±0.5025           |
| 5   | 0.062±0.015     | 0.499         | 0.4951±0.0010             | 0.491-0.495             |
| 6   | 0.263±0.033     | 0.494         | 0.4961±0.0003             | 0.495-0.4965            |
| 7   | 0.174±0.024     | 0.493         | 0.4962±0.0005             | 0.495-0.4965            |

**TABLE I**: Spin tune measurement results. The first five cases are at 24 GeV and the last two cases are at 255 GeV. The precession amplitude angle \( \theta_0 \) is in the second column. The third column is the drive tune of the ac dipoles. The derived spin tune from driven coherence is in the forth column. Last column is the spin tune range from spin flipper operation.

![Fig. 4: Measured spin azimuthal angle as a function of driven oscillation phase at 24 GeV with drive tune as 0.499.](image1)

![Fig. 5: Measured spin azimuthal angle as a function of driven oscillation phase at 255 GeV with drive tune as 0.494.](image2)
is small enough. For it to work, the drive tune needs to be close to the spin tune, which requires a small spin tune spread. In RHIC, where a pair of Siberian snakes are used, the small spin tune spread was achieved by the reduction of dispersion slope difference at the two Siberian snakes [23, 24]. These experimental results prove that it is possible to routinely measure the spin tune of polarized proton beams—the most important polarized beam parameter—which will lead to more stable and optimized operation of a high-energy polarized collider, such as RHIC or a future polarized electron ion collider.

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