Gravity duals of 5D N=2 SYM from F(4) gauged supergravity

Parinya Karndumri

String Theory and Supergravity Group, Department of Physics, Faculty of Science, Chulalongkorn University, 254 Phayathai Road, Pathumwan, Bangkok 10330, Thailand

and

Thailand Center of Excellence in Physics, CHE, Ministry of Education, Bangkok 10400, Thailand

E-mail: parinya.ka@hotmail.com

Abstract

We study gravity duals of the minimal $N = 2$ super Yang-Mills (SYM) gauge theories in five dimensions using the matter coupled $F(4)$ gauged supergravity in six dimensions. The $F(4)$ gauged supergravity coupled to $n$ vector multiplets contains $4n + 1$ scalar fields, parametrized by $\mathbb{R}^+ \times SO(4, n)/SO(4) \times SO(n)$ coset manifold. Maximally supersymmetric vacua of the gauged supergravity with $SU(2) \times E_{N_f+1}$ gauge group, with $E_{N_f+1}$ being an $n$-dimensional subgroup of $SO(n)$, correspond to five dimensional superconformal field theories (SCFTs) with $N_f$ flavors and $E_{N_f+1}$ global symmetry. Deformations of the $E_{N_f+1}$ SCFTs for $N_f = 0, 1, 2$ that lead to non-conformal $N = 2$ SYM are studied holographically. We explicitly give analytic gravity solutions corresponding to deformations of $E_{N_f+1}$ SCFTs to $SO(2N_f) \times U(1)$, $SU(N_f) \times U(1) \times U(1)$ and $SU(N_f) \times U(1)$ SYM as well as to $E_0$ SYM with no global symmetry.
1 Introduction

Much insight to strongly coupled gauge theories can be gained from studying their gravity duals via the AdS/CFT correspondence [1] and its generalization to non-conformal field theories [2, 3, 4]. One consequence of the AdS/CFT correspondence which has been extensively studied is holographic RG flows. These flows describe deformations of a UV conformal field theory (CFT) to another conformal fixed point or to a non-conformal field theory in the IR. On the gravity side, an RG flow in the dual field theory is described by an asymptotically AdS solution which becomes AdS space in certain limit corresponding to the UV CFT. The gravity solutions interpolate between this AdS space and another AdS space in the case of flows to some IR fixed points. For flows to non-conformal field theories, gravity solutions in the IR will take the form of a domain wall [5]. Furthermore, in flows between CFTs, bulk scalar fields take finite constant values at both conformal fixed points while in flows to non-conformal theories, they are logarithmically divergent.

The above argument leads to gravity duals of various supersymmetric gauge theories in four dimensions, and many important characteristics of the gauge theories such as gaugino condensates and confinements can be successfully described by gravity solutions of five dimensional gauged supergravity, see for example [6, 7, 8]. On the other hand, holographic duals of higher dimensional gauge theories have not much been explored in the literatures. In this paper, we will carry out a similar study for $N = 2$ supersymmetric Yang-Mills (SYM) gauge theories in five dimensions using six-dimensional $F(4)$ gauged supergravity. This should provide the 5-dimensional analogue of the 4-dimensional results in [6, 7, 8].

AdS$_6$/CFT$_5$ correspondence has been studied sometime ago in [9] in which the dual field theory of the matter coupled $F(4)$ gauged supergravity in the bulk has been identified as a singleton field theory on the boundary. The five-dimensional fixed points discovered in [10, 11, 12] have been identified with the maximally supersymmetric vacua of the six-dimensional gauged supergravity. The 5D field theory describes the dynamics of the D4/D8 brane system whose near horizon limit gives rise to AdS$_6$ geometry [13]. At the fixed points, the $SO(2N_f) \times U(1)$ global symmetry of the gauge theory with $N_f < 8$ flavors is enhanced to $E_{N_f+1}$. $E_{6,7,8}$ are the usual exceptional groups and other groups are defined by $E_1 = SU(2)$, $E_2 = SU(2) \times U(1)$, $E_3 = SU(3) \times SU(2)$, $E_4 = SU(5)$ and $E_5 = SO(10)$ [10]. This symmetry enhancement in the case of $SU(2)$ gauge theories has also been shown to appear in the superconformal indices [14].

A number of papers on gauge/gravity correspondence involving 5D gauge theories including a generalization to quiver gauge theories from the ten-dimensional point of view have appeared in [15, 16, 17]. RG flows between 5D quiver gauge theories with $N_f = 0$ have been studied recently in [18] in the ten-dimensional context. Holographic RG flows within in the framework of $F(4)$ gauged supergrav-
gravity have also been studied in [18] and [19]. We will give another flow solutions to 5D non-conformal gauge theories with $N_f = 0, 1, 2$ in the framework of six-dimensional gauged supergravity. As in lower dimensions, this should be more convenient to work with than the ten-dimensional computation and provide a useful tool in the holographic study of $N = 2$ 5D SYM.

The study of gravity duals of 5D gauge theories is not only important in AdS$_6$/CFT$_5$ correspondence but is also useful in the context of AdS$_7$/CFT$_6$ correspondence [21, 22]. This originates from the proposal that $(2, 0)$ gauge theory in six dimensions could be defined in term of 5D SYM. Furthermore, it has been shown that 5D SCFT could be an IR fixed point of $N = 2^*$ gauge theory in four dimensions [23]. Therefore, having gravity duals of 5D SYM could be very useful in understanding the dynamics of M5-branes and gauge theories in other dimensions as well.

The paper is organized as follow. In section 2 we review relevant information about matter coupled $F(4)$ gauged supergravity in six dimensions and formulae used throughout the paper. Some information about the dual field theory is also discussed. Holographic RG flows to non-conformal field theories from the $E_{1,2,3}$ fixed points will be given in section 3, 4 and 5 respectively. All of the solutions can be analytically obtained and might be more useful than the numerical solutions given in some other cases. We end the paper by giving some conclusions and comments in section 6.

2 Matter coupled $F(4)$ gauged supergravity and the dual $N = 2$ Super Yang-Mills

We begin with a brief review of the matter coupled $F(4)$ gauged supergravity in six dimensions. The theory is an extension of the pure $F(4)$ gauged supergravity, constructed long ago in [21], by coupling $n$ vector multiplets to the supergravity multiplet. The resulting theory is elegantly constructed by using the superspace approach in [25, 26, 27]. In the present work, we will need only supersymmetry transformations of fermions and the bosonic Lagrangian involving the metric and scalars. Most of the notations and conventions are the same as those given in [25] and [26] but with the metric signature $(-+++++)$.

In half-maximal $N = (1, 1)$ supersymmetry, the field content of the supergravity multiplet is given by

$$(e_{a \mu}, \psi_{\mu}^{A}, A_\mu^a, B_{\mu \nu}, \chi^{A}, \sigma)$$

where $e_{a \mu}$, $\chi^{A}$ and $\psi_{\mu}^{A}$ denote the graviton, the spin-$\frac{1}{2}$ field and the gravitini, respectively. Both $\chi^{A}$ and $\psi_{\mu}^{A}$ are eight-component pseudo-Majorana spinors with indices $A, B = 1, 2$ referring to the fundamental representation of the $SU(2)_R \sim USp(2)$ R-symmetry. The remaining fields are given by the dilaton $\sigma$, four vectors
\( A_\mu^a, \alpha = 0, 1, 2, 3, \) and a two-form field \( B_{\mu\nu}. \)

A vector multiplet has the component fields

\[(A_\mu, \lambda_\alpha, \phi^\alpha).\]

Each multiplet is labeled by an index \( I = 1, \ldots, n. \) The 4n scalars \( \phi^{\alpha I} \) are described by a symmetric quaternionic manifold \( SO(4, n)/SO(4) \times SO(n). \) The dilaton \( \sigma \) can also be regarded as living in the coset space \( \mathbb{R}^+ \sim O(1, 1). \) As in \( \text{25}, \) it is convenient to decompose the \( \alpha \) index into \( \alpha = (0, r) \) in which \( r = 1, 2, 3. \)

The \( SU(2)_R \) R-symmetry is identified with the diagonal subgroup of \( SU(2) \times SU(2) \sim SO(4) \subset SO(4) \times SO(n). \) A general compact gauge group is then given by \( SU(2) \times G \) with \( \text{dim} \ G = n. \) The gauge group of interest here takes the form of \( SU(2) \times E_{N_f+1}. \) At the conformal fixed points corresponding to \( AdS_6 \) vacua, this will be identified with the \( SU(2)_R \) R-symmetry and the global symmetry \( E_{N_f+1} \) of the dual field theory with \( N = 2 \) supersymmetry and \( N_f \) flavors. The dilaton \( \sigma \) is a singlet under both \( SU(2)_R \) and \( E_{N_f+1}. \) The 4n scalars in the vector multiplets transform as \( (1, \text{dim} E_{N_f+1}) + (3, \text{dim} E_{N_f+1}) \) under \( SU(2)_R \times E_{N_f+1}. \) In this work, we are mainly interested in the \( SU(2)_R \) singlet scalars corresponding to the dual operators of dimension four. On the other hand, the dilaton and \( SU(2)_R \) triplet scalars are dual to operators of dimension three.

The 4n scalars living in the \( SO(4, n)/SO(4) \times SO(n) \) coset can be parametrized by the coset representative \( L^\Lambda_{\Sigma}, \Lambda, \Sigma = 0, \ldots, 3 + n. \) Using the index splitting \( \alpha = (0, r), \) we can split \( L^\Lambda_{\Sigma} \) into \( (L^\Lambda_0, L^\Lambda_1) \) and further to \( (L^\Lambda_0, L^\Lambda_1, L^\Lambda_2). \) The vielbein of the \( SO(4, n)/SO(4) \times SO(n) \) coset \( P_\alpha^I \) can be obtained from the left-invariant 1-form of \( SO(4, n) \)

\[ \Omega^\Lambda_{\Sigma} = (L^{-1})^\Lambda_{\Pi} \nabla L^{\Pi}_{\Sigma}, \]
\[ \nabla L^\Lambda_{\Sigma} = dL^\Lambda_{\Sigma} - f_{\Gamma \Sigma}^\Lambda L^\Gamma_{\Pi} L^\Pi_{\Sigma}, \]  \( \text{via} \)

\[ P_\alpha^I = (P^I_0, P^I_r) = (\Omega^I_0, \Omega^I_r). \]  \( \text{2} \)

The structure constants of the full gauge group \( SU(2)_R \times G \) are denoted by \( f^\Lambda_{\Pi \Sigma}, \) which can be split into \( \epsilon_{rst} \) and \( C_{ijk} \) for \( SU(2)_R \) and \( G, \) respectively. The direct product structure of the gauge group \( SU(2)_R \times G \) leads to two coupling constants \( g_1 \) and \( g_2 \) which, in the above equation, are encoded in \( f^\Lambda_{\Pi \Sigma} \).

In this paper, we are interested in \( n = 3, 4, 11 \) cases and gauge groups \( SU(2)_R \times SU(2), SU(2)_R \times SU(2) \times U(1) \) and \( SU(2)_R \times SU(2) \times SU(3). \) To describe \( SO(4, n)/SO(4) \times SO(n), \) we introduce basis elements of \( (4+n) \times (4+n) \) matrices by

\[ (e^{xy})_{zw} = \delta_{xz} \delta_{yw}, \quad w, x, y, z = 1, \ldots, n + 4. \]  \( \text{3} \)

The \( SO(4), SU(2)_R \) and non-compact generators of \( SO(4, n) \) are accordingly given by

\[ SO(4) : \quad J^{\alpha \beta} = e^{\beta + 1, \alpha + 1} - e^{\alpha + 1, \beta + 1}, \quad \alpha, \beta = 0, 1, 2, 3, \]
\[ SU(2)_R : \quad J^r s = e^{s + 1, r + 1} - e^{r + 1, s + 1}, \quad r, s = 1, 2, 3, \]
\[ Y^{al} = e^{a + 1, l + 4} + e^{l + 4, a + 1}, \quad I = 1, \ldots, n. \]  \( \text{4} \)
Gaugings lead to fermionic mass-like terms and the scalar potential in the
Lagrangian as well as some modifications to the supersymmetry transformations.
We will give only information relevant to the study of supersymmetric RG flows
and refer the reader to \[25\] and \[26\] for more details and complete formulae. The
bosonic Lagrangian for the metric and scalar fields is given by \[26\]

\[
\mathcal{L} = \frac{1}{4} e R - e \partial_{\mu} \sigma \partial^{\mu} \sigma - \frac{1}{4} e P_{I \alpha} P^{I \alpha \mu} - e V
\]

(5)

where \(e = \sqrt{-g}\). The scalar kinetic term is written in term of \(P^{I \alpha} = P^{I \alpha \mu} \partial_{\mu} \phi^i\),
\(i = 1, \ldots, 4n\). For completeness, we also give the explicit form of the scalar potential

\[
V = -e^{2\sigma} \left[ \frac{1}{36} A^2 + \frac{1}{4} \tilde{B}^i \tilde{B}_i - \frac{1}{4} \left( C^t_{i} C_{It} + 4 D^t_{i} D_{It} \right) \right] - m^2 e^{-6\sigma} N_{00}
\]

\[+ me^{-2\sigma} \left[ \frac{2}{3} A L_{00} - 2 \tilde{B}^i L_{0i} \right] \]

(6)

where \(N_{00}\) is the 00 component of the scalar matrix defined by

\[N_{\Lambda \Sigma} = L_{\Lambda}^0 (L^{-1})_{0 \Sigma} + L_{\Lambda}^i (L^{-1})_{i \Sigma} - L_{\Lambda}^I (L^{-1})_{I \Sigma}. \]

(7)

Various quantities appearing in the scalar potential and in the supersymmetry
transformations given below are defined as follow

\[A = \epsilon^{rst} K_{rst}, \quad B^i = \epsilon^{ijk} K_{jk0},\]

\[C^t_{I} = \epsilon^{t I a} K_{I a}, \quad D_{It} = K_{0 It} \]

(8)

(9)

where

\[K_{rst} = g_{1} \epsilon_{lmn} L^l_{i} (L^{-1})_{s}^m L^n_{t} + g_{2} C_{IJK} L^l_{i} (L^{-1})_{s}^j L^K_{t},\]

\[K_{r0} = g_{1} \epsilon_{lmn} L^l_{i} (L^{-1})_{s}^m L^0_{t} + g_{2} C_{IJK} L^l_{i} (L^{-1})_{s}^j L^K_{t};\]

\[K_{it} = g_{1} \epsilon_{lmn} L^l_{i} (L^{-1})_{s}^m L^n_{t} + g_{2} C_{IJK} L^l_{i} (L^{-1})_{s}^j L^K_{t};\]

\[K_{0It} = g_{1} \epsilon_{lmn} L^l_{i} (L^{-1})_{s}^m L^n_{t} + g_{2} C_{IJK} L^l_{i} (L^{-1})_{s}^j L^K_{t}.\]

(10)

Finally, the supersymmetry transformations of \(\chi^A\), \(\lambda^I_A\) and \(\psi^A\) involving
only scalars and the metric are given by

\[
\delta \psi_{\mu A} = D_{\mu} \epsilon_A - \frac{1}{24} \left( A e^\sigma + 6 m e^{-3\sigma} (L^{-1})_{00} \right) \epsilon_{AB} \gamma^A \epsilon^B
\]

\[\frac{1}{8} \left( B e^\sigma - 2 m e^{-3\sigma} (L^{-1})_{00} \right) \gamma^I \epsilon^{AB} \gamma^A \epsilon^B,\]

(11)

\[
\delta \chi_A = \frac{1}{2} \gamma^\mu \partial_\mu \sigma \epsilon_{AB} \epsilon^B + \frac{1}{24} \left[ A e^\sigma - 18 m e^{-3\sigma} (L^{-1})_{00} \right] \epsilon_{AB} \epsilon^B
\]

\[\frac{1}{8} \left( B e^\sigma + 6 m e^{-3\sigma} (L^{-1})_{00} \right) \gamma^I \epsilon^{AB} \gamma^A \epsilon^B,\]

(12)

\[
\delta \lambda^I_A = P^l_{ri} \gamma^\mu \partial_\mu \phi^r \epsilon^{AB} + P^l_{0i} \gamma^\gamma \gamma^I \partial_\mu \phi^r \epsilon^{AB} - \left( 2 i \gamma^I \partial_l + C^I_l \gamma^\sigma \right) \epsilon^{\sigma \gamma A} \epsilon^B
\]

\[- 2 m e^{-3\sigma} (L^{-1})_{0} \gamma^I \epsilon_{AB} \epsilon^B\]

(13)
where $\sigma^C_B$ are Pauli matrices, and $\epsilon_{AB} = -\epsilon_{BA}$. The space-time gamma matrices $\gamma^a$, with $a$ being tangent space indices, satisfy

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab}, \quad \eta^{ab} = \text{diag}(-1,1,1,1,1,1),$$

and $\gamma^7 = \gamma^0\gamma^1\gamma^2\gamma^3\gamma^4\gamma^5$.

On the field theory side, the conformal fixed points correspond to sending the bare gauge coupling to infinity. The fixed points only exist for the theories with $N_f < 8$. At these fixed points, the $SO(2N_f) \times U(1)$ symmetry of the Yang-Mills theories gets enhanced to $E_{N_f+1}$. On the other hand, the $E_{N_f}$ symmetry is broken by a finite value of the gauge coupling $t_0 \sim \frac{1}{g^2} \neq 0$ to $SO(2N_f) \times U(1)$. The $U(1)$ is generated by instantons in the gauge theory. The enhanced global symmetry can also be broken by turning on quark masses $m_i, i = 1, \ldots, N_f$. For example, turning on all equal quark masses $m_i = m$ breaks $E_{N_f+1}$ to $SU(N_f + 1) \times U(1)$. If only $N_f + 1 - p$ of the $m_i$ for $i = p, \ldots, N_f$ are turned on, $E_{N_f+1}$ symmetry will be broken to $E_p$ symmetry. The details on these flows and symmetry breaking patterns can be found in [10, 11] and [12]. In the next sections, we will implement some of these flows on the gravity side.

### 3 RG flows from $E_1$ SCFT

We begin with the simplest possibility with $N_f = 0$ and $SU(2)$ global symmetry. The gravity theory consists of 13 scalars parametrized by $O(1,1) \times SO(4,3)/SO(4) \times SO(3)$ coset space. We are interested in $SU(2)_R$ singlet scalars which are given by $\sigma$ and additional 3 scalars from $SO(4,3)/SO(4) \times SO(3)$. The latter correspond to the non-compact generators $Y_{11}, Y_{12}$ and $Y_{13}$. The coset representative is accordingly written as

$$L = e^{a_1Y_{11}}e^{a_2Y_{12}}e^{a_3Y_{13}}.$$  \hspace{1cm} (15)

The space-time metric is the standard domain wall ansatz

$$ds^2 = e^{2A(r)}dx_{1,4}^2 + dr^2$$  \hspace{1cm} (16)

in which five-dimensional Poincare symmetry is manifest. From now on, the six-dimensional space-time indices will be split as $(\mu, r)$ with $\mu = 0, \ldots, 4$. 

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Using (11), (12) and (13), we find the following BPS equations

\[ a_1' = -2e^{-3\sigma} m \frac{\sinh a_1}{\cosh a_2 \cosh a_3}, \]  
\[ a_2' = -2e^{-3\sigma} m \frac{\cosh a_1 \sinh a_2}{\cosh b_3}, \]  
\[ a_3' = -2e^{-3\sigma} m \cosh a_1 \cosh a_2 \sinh a_3, \]  
\[ \sigma' = -\frac{1}{2} \left[ e^\sigma g_1 - 3e^{-3\sigma} m \cosh a_1 \cosh a_2 \cosh a_3 \right], \]  
\[ A' = \frac{1}{2} \left[ e^\sigma g_1 + e^{-3\sigma} m \cosh a_1 \cosh a_2 \cosh a_3 \right]. \]

where \( ' \) denotes \( \frac{d}{dr} \) and we have used the projection \( \gamma^r e^A = e^A \). The presence of \( \gamma^7 \) in \( \delta \lambda^I_A \) does not impose any condition on \( e^A \) since it appears as an overall factor in all of the BPS equations obtained from \( \delta \lambda^I_A = 0 \). That the bulk gravity solution preserves eight supercharges is to be expected because the minimal SYM in five dimensions has 8 supercharges. The equation for the warp factor \( A(r) \) is obtained from \( \delta \psi^A_\mu, \mu = 0, 1, 2, 3, 4 \). The \( \delta \psi^A_r = 0 \) equation would give the dependence of the Killing spinors on the \( r \)-coordinate as in lower dimensions. We now look at solutions of interest.

### 3.1 Flow to \( SU(2)_R \times U(1) \) SYM

We first study the solution that breaks the \( E_1 \) global symmetry to \( U(1) \). This corresponds to turning on only \( a_3 \) and \( \sigma \). The latter is of course a singlet of the full gauge group \( SU(2) \times E_1 \). With \( a_1 = a_2 = 0 \), equations (17) and (18) are trivially satisfied, and equations (19), (20) and (21) become

\[ a_3' = -2e^{-3\sigma} m \sinh a_3, \]  
\[ \sigma' = \frac{1}{2} \left( -g_1 e^\sigma + 3e^{-3\sigma} m \cosh a_3 \right), \]  
\[ A' = \frac{1}{2} \left( g_1 e^\sigma + e^{-3\sigma} m \cosh a_3 \right). \]

We can solve equation (22) by introducing a new radial coordinate \( \tilde{r} \) such that \( \frac{d\tilde{r}}{dr} = e^{-3\sigma} \). We then find the solution for \( a_3 \)

\[ a_3 = \ln \left[ \frac{1 + e^{-2m\tilde{r} + C_1}}{1 - e^{-2m\tilde{r} + C_1}} \right]. \]

This form is very similar to the solution studied in [6] for the 4D SYM. \( C_1 \) is an integration constant. Combining equations (22) and (23) gives an equation for \( \frac{d\sigma}{da_3} \)

\[ \frac{d\sigma}{da_3} = \frac{1}{4m} \left( e^{4\sigma} g_1 \cosh a_3 - 3m \cosh a_3 \right) \]
whose solution is given by

\[ \sigma = -\frac{1}{4} \ln \left[ \frac{g_1 (3 \cosh a_3 - \cosh(3a_3) + 18C_2 \sinh^3 a_3)}{6m} \right] \]  

(27)

with \( C_2 \) being another integration constant.

After changing to \( \tilde{r} \) coordinate and using \( a_3 \) solution, we find that the combination \((24)+(23)\) becomes

\[ A' + \sigma' = \frac{2m (e^{4m\tilde{r}} + e^{2C_1})}{e^{2C_1} - e^{4m\tilde{r}}} . \]  

(28)

The solution to this equation can be readily found to be

\[ A = 2mr + \ln \left( 1 - e^{C_1 - 2mr} \right) + \ln \left( 1 + e^{C_1 - 2mr} \right) - \sigma \]  

(29)

where we have neglected the additive integration constant to \( A \) by absorbing it into the rescaling of the \( x^\mu \) coordinates. To identify the maximally supersymmetric vacuum at \( \sigma = a_3 = 0 \) with the \( N = 2 \) SCFT, we have to set \( g_1 = 3m \). In the above solutions, we have not done this in order to keep the solutions in a generic form. Note also that if we try to truncate \( \sigma \) out by setting \( \sigma = 0 \), equation \((23)\) will imply \( a_3 = 0 \). Therefore, to obtain a non-trivial solution, we must keep \( \sigma \) non-vanishing.

An RG flow to a non-conformal field theory with only the dilaton \( \sigma \) in pure \( F(4) \) gauged supergravity has been studied in [18]. The resulting solution is interpreted as the analogue of the Coulomb branch flow. In this work, we have generalized this solution to the case of matter coupled gauged supergravity. As \( r \rightarrow \infty, \sigma, a_3 \rightarrow 0 \), we see that \( \tilde{r} \sim r \rightarrow \infty \). In this limit, we obtain the maximally supersymmetric \( AdS_6 \) background with \( A \sim 2mr = \frac{\tilde{r}}{L} \) where the \( AdS_6 \) radius in the UV is given by \( L = \frac{2m}{g_1} \). From the above solutions, the behavior of \( \sigma \) and \( a_3 \) near the UV point with \( g_1 = 3m \) is readily seen to be

\[ a_3 \sim e^{-2mr} = e^{-\frac{\tilde{r}}{2}}, \quad \sigma \sim a_3^3 \sim e^{-6mr} = e^{-\frac{3\tilde{r}}{2}} . \]  

(30)

We see that \( a_3 \) corresponds to a deformation by a relevant operator of dimension \( \Delta = 4 \) while \( \sigma \) describes a deformation by a vacuum expectation value of operator of dimension \( \Delta = 3 \).

From the solution, we see that \( a_3 \) is singular when \( \tilde{r} \rightarrow \frac{C_1}{2m} \). In this limit, we find \( a_3 = -\ln \left( 2m\tilde{r} - C_1 \right) + \ln 2 \) and

\[ \sigma = \frac{3}{4} a_3 - \frac{1}{4} \ln \left[ \frac{9C_2 - 2}{8} \right] \]

\[ = \frac{3}{4} \ln \left( 2m\tilde{r} - C_1 \right) - \frac{1}{4} \ln(9C_2 - 2) \]  

(31)

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The warp factor $A$ near $\tilde{r} \to \frac{C_1}{2m}$ is given by

$$A = \frac{1}{4} \ln (2m\tilde{r} - C_1) + \text{constant} = \frac{1}{4} \ln (2mr - C) + \text{constant} \quad (32)$$

where we have used the relation $(2m\tilde{r} - C_1)^\frac{1}{4} = \frac{1}{4} (2mr - C)$, near $\tilde{r} \sim \frac{C_1}{2m}$, with a new integration constant $C$. With all irrelevant constants neglected, the full metric becomes a domain wall

$$ds^2 = (2mr - C)^\frac{2}{13} dx_{1,4}^2 + dr^2. \quad (33)$$

According to the Domain-Wall/Quantum-Field-Theory (DW/QFT) correspondence, this background is dual to a non-conformal SYM theory in five dimensions.

### 3.2 Flow to $SU(2)_R$ SYM

If the other scalars $a_1$ and $a_2$ are non-vanishing, the solution will break the $E_1$ global symmetry, completely. It is now more difficult to solve all five BPS equations, but it turns out that these equations admit analytic solutions.

To obtain the solution, we consider $A$, $\sigma$, $a_1$ and $a_2$ as functions of $a_3$. Combining equations (18) and (19), we find

$$\frac{da_2}{da_3} = \frac{\tanh a_2}{\sinh a_3 \cosh a_3}. \quad (34)$$

This is easily solved by

$$a_2 = \ln \left[ \frac{e^{2a_3+C_1} - e^{C_1} + \sqrt{(1 + e^{2a_3})^2 + e^{2c_1} (e^{2a_3} - 1)}}{1 + e^{2a_3}} \right] = \sinh^{-1} \left( e^{C_1} \tanh a_3 \right). \quad (35)$$

Similarly, by solving equations (17) and (19), we obtain

$$a_1 = \sinh^{-1} \left( \frac{e^{C_2} \sinh a_3}{\sqrt{1 - e^{2C_1} + (1 + e^{2C_1})(2eC_2 - a_3)}} \right). \quad (36)$$

Using the $a_1$ and $a_2$ solutions and the new radial coordinate $\tilde{r}$, we find the solution for $a_3$

$$a_3 = -\frac{1}{2} \cosh^{-1} \left[ \frac{e^{2C_2} + 2e^{2C_1} - 2 + 4 \tanh^2 (2m\tilde{r} - C_3)}{2 + 2e^{2C_1} + e^{2C_2}} \right]. \quad (37)$$
We can similarly solve for $\sigma$ as a function of $a_3$. The solution is given by

$$\sigma = \frac{1}{4} \ln \left[ 3m \left( \tilde{A}^2 + \tilde{B}^2 \right)^2 \csch^6 a_3 \left( 36 \tilde{A}^2 C_4 \left( \tilde{A}^2 + \tilde{B}^2 \right)^2 \sinh^3 a_3 \left( \tilde{A}^2 \cosh(2a_3) + \tilde{B}^2 \right) - 2 \left( 3\tilde{A}^2 + \tilde{B}^2 - 2\tilde{A}^2 \cosh(2a_3) \right) \left( \tilde{A}^2 \cosh(2a_3) + \tilde{B}^2 \right)^{3/2} \right) \right]$$

$$- \frac{1}{4} \ln \left[ 1296 \tilde{A}^4 C_4^2 g_1 \left( \tilde{A}^2 + \tilde{B}^2 \right)^4 \left( \tilde{A}^2 \cosh(2a_3) + \tilde{B}^2 \right) - 4g_1 \csch^6 a_3 \left( \tilde{A}^4 \cosh(4a_3) + \tilde{A}^4 + \tilde{A}^2 \left( \tilde{B}^2 - 3\tilde{A}^2 \right) \cosh(2a_3) - 3\tilde{A}^2 \tilde{B}^2 - \tilde{B}^4 \right)^2 \right].$$

(38)

We have defined two new constants $\tilde{A} = \sqrt{2 + 2e^{2C_1} + e^{2C_2}}$ and $\tilde{B} = \sqrt{2 - 2e^{2C_1} - e^{2C_2}}$ for convenience.

Finally, adding (20) to (21) and changing the variable from $r$ to $a_3$, we find a simple equation for $A$

$$\frac{dA}{da_3} + \frac{d\sigma}{da_3} = - \coth a_3$$

(39)

whose solution is

$$A = -\sigma - \ln(\sinh a_3).$$

(40)

Near the UV point, we find $r \sim \tilde{r} \to \infty$, $a_1 \sim a_2 \sim a_3 \sim e^{-\frac{r}{4}}$ and $\sigma \sim e^{-\frac{a_3}{4}}$. The solution for $A$ then gives $A \sim 2mr = \frac{\tilde{r}}{4}$. The flow is again driven by turning on operators of dimension four corresponding to $a_{1,2,3}$ and a vev of a dimension three operator dual to $\sigma$.

It can be checked by expanding (37) that $a_3 \to \infty$ for $2m\tilde{r} \to \tilde{C}$ where we have collectively denoted all constant terms from the expansion by $\tilde{C}$. The behavior of $a_3$ near this point is $a_3 \sim -\ln(2m\tilde{r} - \tilde{C})$. Although $a_3$ blows up when $2m\tilde{r} \sim \tilde{C}$, $a_1$ and $a_2$ remain finite. The behavior of $\sigma$ is $\sigma \sim -\frac{3}{4}a_3 = \frac{3}{4}\ln(2m\tilde{r} - \tilde{C})$. The warp factor is given by $A \sim -\frac{1}{4}a_3 \sim \frac{1}{4}\ln(2m\tilde{r} - \tilde{C})$. In term of the old radial coordinate $r$, we find the IR metric

$$ds^2 = (2mr - C)^{\frac{4}{3}} dx_{1,4}^2 + dr^2$$

(41)

where we have used the relation $2mr - C = \frac{1}{13}(2m\tilde{r} - \tilde{C})^{\frac{4}{3}}$ and absorbed the multiplicative constant to the scaling of $x^\mu$ coordinates.

### 3.3 Flow to $SU(2)_{\text{diag}}$ SYM

In this subsection, we will look at an RG flow with $SU(2)_{\text{diag}} \sim (SU(2)_R \times E_1)_{\text{diag}}$ singlet scalars. Some non-supersymmetric $AdS_6$ vacua and holographic RG flows interpolating between these critical points and the maximally supersymmetric
AdS$_6$ have been studied in [19]. In this work, we will give a supersymmetric flow to a non-conformal field theory.

There is only one singlet scalar under $SU(2)_{\text{diag}}$ from $\frac{SO(4,3)}{SO(4) \times SO(3)}$, see the detail in [19]. The coset representative can be written as

\[ L = e^{a(Y_{21} + Y_{32} + Y_{41})}. \] (42)

The supersymmetry transformations of $\psi_{\mu}^A$, $\chi^A$ and $\lambda^I_A$ give the following BPS equations

\[ a' = -e^\sigma \sinh(2a) \left( g_1 \cosh a - g_2 \sinh a \right), \] (43)

\[ \sigma' = \frac{1}{2} e^{-3\sigma} \left[ 3m + e^{4\sigma} \left( g_2 \sinh^3 a - g_4 \cosh^3 a \right) \right], \] (44)

\[ A' = \frac{1}{2} e^{-3\sigma} \left[ m + e^{4\sigma} \left( g_1 \cosh^3 a - g_2 \sinh^3 a \right) \right]. \] (45)

Note that for non-singlet scalars of $SU(2)_R$, the $E_1$ coupling $g_2$ appears.

In order to solve the above equations, we will treat $\sigma$ and $A$ as functions of $a$.

\[ \frac{d\sigma}{da} = \frac{3me^{-4\sigma} - g_1 \cosh^3 a + g_2 \sinh^3 a}{2 \sinh(2a) \left( g_1 \cosh a - g_2 \sinh a \right)} \] (46)

which can be solved by

\[ \sigma = \frac{1}{4} \ln \left[ \frac{6m \cosh(2a) + C_1 \sinh(2a)}{2g_1 \cosh a - 2g_2 \sinh a} \right]. \] (47)

We can check that as $a \to 0$ and $g_1 = 3m$, $\sigma \to 0$ as expected for the UV point. This is the case for any value of $C_1$. To solve for $a$ from equation (43), it is convenient to define a new coordinate $\tilde{r}$ via $e^\sigma = \frac{d\tilde{r}}{dr}$. Only in this case, $\tilde{r}$ is defined by $e^\sigma = \frac{d\tilde{r}}{dr}$. In all other cases, we always have $e^{-3\sigma} = \frac{d\tilde{r}}{dr}$.

With this new variable, we can solve for $\tilde{r}$ as a function of $a$. The resulting solution is given by

\[ 2g_1 g_2 \tilde{r} = g_2 \ln \coth \frac{a}{2} - 2g_1 \tan^{-1} \left[ \tanh \frac{a}{2} \right] + 2\sqrt{g_1^2 - g_2^2} \tan^{-1} \left[ \frac{g_1 \tanh \frac{a}{2} - g_2}{\sqrt{g_1^2 - g_2^2}} \right] \] (48)

where we have neglected the additive integration constant.

Taking the combination (44)-3× (45) with (43), we can rewrite equation for $A$ as

\[ \frac{d\sigma}{da} - \frac{3}{2} \frac{dA}{da} = \frac{g_1 \sinh a + g_2 (1 - \cosh a)}{g_1 \cosh a - g_2 \sinh a}. \] (49)

The solution is readily obtained to be

\[ A = \frac{1}{3} \left[ \sigma + \ln \sinh(2a) + \ln(g_1 \cosh a - g_2 \sinh a) \right]. \] (50)
From the above solutions, we can find the behavior of $a$, $\sigma$ and $A$ near the UV point, $a = \sigma = 0$. In this limit, $\tilde{r} \sim r \to \infty$, we find $a \sim \sigma \sim e^{-6mr} = e^{-\tilde{r}}$ and $A \sim 2mr = \frac{\tilde{r}}{2}$. This indicates that the flow is driven by vacuum expectation values of operators of dimension three. This is to be expected since it has been pointed out in [19] that the flow driven by turning on the operators dual to $\sigma$ and $a$ corresponds to a non-supersymmetric flow to a non-supersymmetric IR fixed point.

In the IR, from the $a$ solution, we have $a \sim -\ln(g_1 \tilde{r} - \tilde{C})$ for some constant $\tilde{C}$. $\sigma$ also blows up since $\sigma \sim \frac{1}{4}a \sim -\frac{1}{3}\ln(g_1 \tilde{r} - \tilde{C})$ as $\tilde{r} \sim \frac{\tilde{C}}{3m}$. where we have used the relation $g_1 = 3m$. In this limit, the warp factor becomes $A \sim -\frac{1}{4}a + \text{constant} \sim \frac{1}{4}\ln(g_1 \tilde{r} - \tilde{C})$ which gives a domain wall metric

$$ds^2 = (3mr - C)\frac{2}{3}dx^2_{1,4} + dr^2$$

where we have dropped the multiplicative constant to $A$ and expressed the $\tilde{r}$ coordinate in term of the original $r$ via the relation $3mr - C = \frac{2}{3}(3m\tilde{r} - \tilde{C})^{\frac{2}{3}}$ with another integration constant $C$. This should describe the IR $\tilde{N} = 2$ SYM in five dimensions.

4 RG flows from $E_2$ SCFT

We then move to $F(4)$ gauged supergravity coupled to four vector multiplets and $E_2 \sim SU(2) \times U(1)$ gauge group. There are 16 scalars parametrized by $SO(4,4)/SO(4) \times SO(4)$ coset. Together with the dilaton $\sigma$, there are five $SU(2)_R$ singlet scalars. The coset representative can be written as

$$L = e^{a_1 Y_{11}} e^{a_2 Y_{12}} e^{a_3 Y_{13}} e^{a_4 Y_{14}}.$$  

Using the projector $e^{\sigma} e^A = e^A$, we can derive the following BPS equations

$$a_1' = -\frac{2me^{-3\sigma} \sinh a_1}{\cosh a_2 \cosh a_3 \cosh a_4},$$

$$a_2' = -\frac{2me^{-3\sigma} \sinh a_2 \cosh a_1}{\cosh a_3 \cosh a_4},$$

$$a_3' = -\frac{2me^{-3\sigma} \cosh a_3 \cosh a_2 \sinh a_3}{\cosh a_4},$$

$$a_4' = -\frac{2me^{-3\sigma} \cosh a_1 \cosh a_2 \cosh a_3 \sinh a_4}{\cosh a_4},$$

$$\sigma' = \frac{1}{2} \left[ 3me^{-3\sigma} \cosh a_1 \cosh a_2 \cosh a_3 \cosh a_4 - g_1 e^\sigma \right],$$

$$A' = \frac{1}{2} \left[ me^{-3\sigma} \cosh a_1 \cosh a_2 \cosh a_3 \cosh a_4 + g_1 e^\sigma \right].$$

We are interested in the RG flows with the symmetry breaking $E_2 \to E_1$, $E_2 \to U(1) \times U(1)$, $E_2 \to U(1)$ and the completely broken $E_2$. The procedure is essentially the same as in the previous section, so we will neglect some details and simply give the solutions.
4.1 Flow to $SU(2)_R \times SU(2)$ SYM

In order to preserve $SU(2) \subset SU(2) \times U(1)$, only $a_4$ is allowed to be non-vanishing. The above equations reduce to three simple equations

$$a'_4 = -2m e^{-3\sigma} \sinh a_4,$$

$$\sigma' = \frac{1}{2} \left(3m e^{-3\sigma} \cosh a_4 - g_1 e^\sigma\right),$$

$$A' = \frac{1}{2} \left(me^{-3\sigma} \cosh a_4 + g_1 e^\sigma\right).$$

By introducing a new radial coordinate $\tilde{r}$ via $\frac{d\tilde{r}}{dr} = e^{-3\sigma}$ as in the previous section, we find the solutions

$$a_4 = \ln \left[1 + \frac{e^{C_1 - 2m\tilde{r}}}{1 - e^{C_1 - 2m\tilde{r}}}\right],$$

$$\sigma = \frac{1}{4} \ln \left[\frac{12m \sinh^3(C_1 - 2m\tilde{r})}{g_1 \left(9 \cosh(C_1 - 2m\tilde{r}) - 3 \sinh[3(C_1 - 2m\tilde{r})] + 72m^3 C_2 e^{-2C_1}\right)}\right],$$

$$A = 2m\tilde{r} - \sigma + \ln \left(e^{3(C_1 - 2m\tilde{r})} - 1\right).$$

Near the UV point, $a_4$, $\sigma$ and $A$ behave as

$$a_4 \sim e^{-2m\tilde{r}}, \quad \sigma \sim e^{-6m\tilde{r}}, \quad A \sim 2m\tilde{r}.$$

At the IR, we find

$$a_4 \sim -\ln(2m\tilde{r} - C_1), \quad \sigma \sim \frac{3}{4} \ln(2m\tilde{r} - C_1),$$

$$A \sim \frac{1}{4} \ln(2m\tilde{r} - C_1),$$

and the IR metric in the $r$ coordinate becomes

$$ds^2 = (2mr - C)\tilde{r}^2 dx_{1,4}^2 + dr^2$$

with a new integration constant $C$.

4.2 Flow to $SU(2)_R \times SO(2) \times U(1)$ SYM

In this subsection, we will give the solution for the flow to SYM with $SU(2)_R \times SO(2) \times U(1) \sim SU(2)_R \times U(1)^2$ symmetry. To find this solution, we set $a_1 = a_2 = a_4 = 0$. The BPS equations give the following solutions, in term of $\tilde{r}$ coordinate,

$$a_3 = \ln \left[\frac{e^{2m\tilde{r}} + e^{2C_1}}{e^{2m\tilde{r}} - e^{2C_1}}\right],$$

$$\sigma = \frac{1}{4} \ln \left[\frac{12m \sinh^3[2(C_1 - m\tilde{r})]}{g_1 \left(9 \cosh[2(C_1 - m\tilde{r})] - 6 \sinh[6(C_1 - m\tilde{r})] + 18m^3 C_2 e^{-2C_1}\right)}\right],$$

$$A = -2m\tilde{r} + \ln \left(e^{4m\tilde{r}} - e^{4C_1}\right) + \ln \left(e^{12C_1} + e^{12m\tilde{r}} - 9e^{4(C_1 + 2m\tilde{r})} - 9e^{4(C_1 + 2m\tilde{r})} - 36C_2 m^3 e^{4C_1 + 6m\tilde{r}}\right).$$

13
Near the UV point, we find $a_3 \sim e^{-2mr}$, $\sigma \sim e^{-6mr}$ and $A \sim 2mr$. In the IR, $\tilde{r} \to \frac{C_1}{m}$, the solutions become

$$a_3 \sim -\ln(m\tilde{r} - C_1), \quad \sigma \sim \frac{3}{4} \ln(m\tilde{r} - C_1),$$

$$ds^2 = (mr - C)^{\frac{4}{13}} dx_{1,4}^2 + dr^2, \quad (m\tilde{r} - C)^{\frac{13}{4}} = \frac{13}{4} (mr - C). \quad (67)$$

### 4.3 Flow to SU(2)$_R \times$ U(1) SYM

We then consider the flow that breaks SU(2) $\times$ U(1) global symmetry to U(1). In this case, we turn on both $a_3$ and $a_4$. This leads to more complicated equations due to the coupling between $a_4$ and $a_3$. We will regard $a_4$ as a new variable and find the following solutions for $a_3$, $\sigma$ and $A$

$$a_3 = \sinh^{-1} \left[ e^{C_1} \tanh a_4 \right],$$

$$\sigma = -\frac{1}{4} \ln \left[ \frac{g_1}{6\sqrt{2m}} \left[ 72C_2 \sinh^3 a_4 (1 + e^{2C_1}) 
          - 2 \cosh a_4 \left[ (1 + e^{2C_1}) \cosh(2a_4) - e^{2C_1} - 2 \right] \sqrt{2 + 2e^{2C_1} \tanh^2 a_4} \right] \right],$$

$$A = -\sigma - \ln \sinh a_4. \quad (68)$$

The solution of $a_4$ in term of $\tilde{r}$ is given by

$$\tilde{r} = \frac{1}{2m} \tanh^{-1} \sqrt{\frac{1 + \cosh(2a_4) + 2e^{2C_1} \sinh^2 a_4}{2}}. \quad (69)$$

At the UV point, we find the expected behavior $a_3, a_4 \sim e^{-2mr}$, $\sigma \sim e^{-6mr}$ and $A \sim 2mr$. For the IR, we consider the behavior of the solutions as $a_4 \to \infty$. In this limit, the $a_4$ solution becomes $a_4 \sim -\ln(2m\tilde{r} - \tilde{C})$ for some constant $\tilde{C}$. The behavior of $a_3$, $\sigma$ and $A$ is given by

$$a_3 \sim \text{constant}, \quad \sigma \sim \frac{3}{4} \ln(2m\tilde{r} - \tilde{C}), \quad A \sim \frac{1}{4} \ln(2m\tilde{r} - \tilde{C}). \quad (70)$$

With the relation $2mr - C = \frac{13}{4} (2m\tilde{r} - \tilde{C})^{\frac{13}{4}}$, the metric in the IR then takes the form of a domain wall

$$ds^2 = (2mr - C)^{\frac{13}{4}} dx_{1,4}^2 + dr^2. \quad (71)$$

### 4.4 Flow to SU(2)$_R$ SYM

We now quickly look at the flow breaking the $E_2$ symmetry, completely. Finding the solution in this case amounts to solving all of the six BPS equations. This
however turns out to be not difficult. The resulting solutions for $a_i$, $\sigma$ and $A$ are given by

\[
\begin{align*}
a_3 &= \sinh^{-1}(e^{C_1} \tanh a_4), \\
a_2 &= \sinh^{-1} \frac{e^{C_2} \sinh a_4}{\sqrt{1 - e^{2C_1} + (1 + e^{2C_1}) \cosh(2a_4)}}, \\
a_1 &= \sinh^{-1} \frac{e^{C_3} \sinh a_4}{\sqrt{2 - 2e^{2C_1} - e^{2C_2} + (2 + 2e^{2C_1} + e^{2C_2}) \cosh(2a_4)}}, \\
\sigma &= \frac{1}{4} \ln \left[96\sqrt{2}m\sqrt{4 + \alpha^2 - \alpha^2 \text{sech}^2 a_4}\right] - \frac{1}{4} \ln \left[96\sqrt{2}m\sqrt{4 + \alpha^2 - \alpha^2 \text{sech}^2 a_4}\right] - \sqrt{2} \text{sech} a_4 \left(3\alpha^4 + (\alpha^2 + 4)^2 \cosh(4a_4) + 16\alpha^2 - 4 (\alpha^4 + 6\alpha^2 + 8) \cosh(2a_4) - 48\right) + \left[13 \right], \\
A &= -\sigma - \ln \sinh a_4, \\
a_4 &= \frac{1}{2} \cosh^{-1} \left[\frac{8 \tanh^2(2m\tilde{r} - C_5) + \alpha^2 - 4}{\alpha^2 + 4}\right] \quad (72)
\end{align*}
\]

where $\alpha = \sqrt{4e^{2C_1} + 2e^{2C_2} + e^{2C_3}}$. At the UV fixed point and in the IR, the solutions become, respectively,

\[
\begin{align*}
a_{1,2,3,4} &\sim e^{-2mr}, \quad \sigma \sim e^{-6mr}, \quad A \sim 2mr, \quad (73)
\end{align*}
\]

and

\[
\begin{align*}
a_4 &\sim -\ln(2m\tilde{r} - \tilde{C}), \quad a_{1,2,3} \sim \text{constant}, \\
\sigma &\sim \frac{3}{4} \ln(2m\tilde{r} - \tilde{C}), \quad A \sim \frac{1}{4} \ln(2m\tilde{r} - \tilde{C}), \\
ds^2 &= (2mr - C)^2 dx_{1,4}^2 + dr^2. \quad (74)
\end{align*}
\]

All of the flows given above are driven by turning on operators of dimension 4 and a vev of a dimension 3 operator.

## 5 RG flows from $E_3$ SCFT

Finally, we consider $F(4)$ gauged supergravity coupled to 11 vector multiplets with gauge group $SU(2)_R \times E_3 \sim SU(2)_R \times SU(2) \times SU(3)$. The $SU(2)$ structure constants are still given by $\epsilon_{IJK}$. We will also need the $SU(3)$ structure constants whose explicit form is given by

\[
\begin{align*}
f_{123} &= 1, \quad f_{458} = f_{678} = \frac{\sqrt{3}}{2}, \\
f_{147} &= -f_{156} = f_{246} = f_{257} = f_{345} = -f_{367} = \frac{1}{2}. \quad (75)
\end{align*}
\]
All of them are totally anti-symmetric in the three indices.

Apart from $\sigma$, there are 11 scalars which are singlet under $SU(2)_R$ and parametrized by the coset representative of $SO(4, 11)/SO(4) \times SO(11)$. They correspond to the non-compact generators $Y_{1i}$, $i = 1, \ldots, 11$. We will denote the first three scalars by $b_i$, $i = 1, 2, 3$ and the remaining eight ones by $a_i$, $i = 1, \ldots, 8$. The former are singlets under $SU(3)$ and the latter are singlets under $SU(2)$.

The BPS equation for any $a_i$ and $b_i$ is satisfied by setting the corresponding scalar to zero. We will not find the solution with all eleven scalars non-vanishing, so the BPS equations for eleven scalars $b_i$ and $a_i$ will not be shown. In the following subsections, only the relevant equations for scalar of interest in each case will be given. This case has a much richer structure, and we can have symmetry breaking patterns $E_{N_f+1} \rightarrow SO(2N_f) \times U(1), SU(N_f) \times U(1) \times U(1)$ and $E_3 \rightarrow E_2, E_1, SU(3)$.

5.1 RG flow to $SU(2)_R \times SO(4) \times U(1)$

In the symmetry breaking $E_3 \rightarrow SO(4) \times U(1) \sim SU(2) \times SU(2) \times U(1)$, $SU(3)$ is broken to $SU(2) \times U(1)$. There is only one singlet under $SU(2) \times U(1) \subset SU(3)$ since the $SU(3)$ adjoint representation decomposes as

$$8 = 3_0 + 1_0 + 2_3 + 2_{-3}$$

under $SU(2) \times U(1)$. This singlet corresponds to the non-compact generator $Y_{111}$. As mentioned above, we will denote this scalar by $a_8$. The BPS equations for $a_8$, $\sigma$ and $A$ are given by

$$a_8' = -2me^{-3\sigma} \sinh a_8, \quad \sigma' = \frac{1}{4}e^{-3\sigma} (6m \cosh a_3 - 2g_1e^{4\sigma}), \quad A' = \frac{1}{4}e^{-3\sigma} (2m \cosh a_8 + 2g_1e^{4\sigma}).$$

(76)

With the usual $\tilde{r}$ coordinate, the corresponding solutions are easily obtained

$$a_8 = \ln \left[ \frac{e^{2m\tilde{r}} - e^{2C_1}}{e^{2m\tilde{r}} + e^{2C_1}} \right],$$

$$\sigma = \frac{1}{4} \ln \left[ \frac{12m \sinh^3 [2(C_1 - m\tilde{r})]}{g_1 (9 \cosh [2(C_1 - m\tilde{r})] \cosh [6(C_1 - m\tilde{r})] - 144C_2 m^3 e^{-2C_1})} \right],$$

$$A = -2m\tilde{r} - \sigma + \ln (e^{4m\tilde{r}} - e^{4C_1}).$$

(77)

The behavior of the solutions near the UV point is given by

$$a_8 \sim e^{-2m\tilde{r}}, \quad \sigma \sim e^{-6m\tilde{r}}, \quad A \sim 2m\tilde{r}.$$
For the IR behavior, we find

\begin{align*}
a_8 &\sim \ln(m\tilde{r} - C_1), \quad \sigma \sim \frac{3}{4} \ln(m\tilde{r} - C_1), \\
A &\sim \frac{1}{4} \ln(m\tilde{r} - C_1).
\end{align*}

(79)

The IR metric in term of the \( r \) coordinate is

\begin{equation}
ds^2 = (mr - C)^2 dx^2_{1,4} + dr^2.
\end{equation}

(80)

5.2 RG flow to \( SU(2)_R \times SU(2) \times U(1) \times U(1) \)

We now further break one factor of \( SU(2) \) to \( U(1) \). This can be achieved by turning on one more scalar which we will call \( b_3 \). The residual symmetry is then \( SU(2)_R \times SU(2) \times U(1) \times U(1) \). The four BPS equations are as follow

\begin{align*}
b_3' &= -2me^{-3\sigma} \frac{\sinh b_3}{\cosh a_8}, \\
a_8' &= -2me^{-3\sigma} \cosh b_3 \sinh a_8, \\
\sigma' &= \frac{1}{2} \left( 3me^{-3\sigma} \cosh b_3 \cosh a_8 - g_1 e^{\sigma} \right), \\
A' &= \frac{1}{2} \left( me^{-3\sigma} \cosh b_3 \cosh a_8 + g_1 e^{\sigma} \right).
\end{align*}

(81)

The solutions to these equations are given by

\begin{align*}
b_3 &= \sinh^{-1} \left( e^{C_1} \tanh a_8 \right), \\
\sigma &= -\frac{1}{4} \ln \left[ \frac{g_1}{6\sqrt{2}m} \left[ 72C_2(1 + e^{2C_1}) \sinh^3 a_8 \\
&\quad -2 \cosh a_8 (\cosh(2a_8) + 2e^{2C_1} \sinh^2 a_8 - 2) \sqrt{2 + 2e^{2C_1} \tanh^2 a_8} \right] \right], \\
A &= -\sigma - \ln \sinh a_8, \\
\ddot{r} &= \frac{1}{2m} \tanh^{-1} \left[ \cosh a_8 \sqrt{1 + e^{2C_1} \tanh^2 a_8} \right].
\end{align*}

(82)

Near the UV point, we find the expected behavior

\begin{equation}
a_8 \sim b_3 \sim e^{-2mr}, \quad \sigma \sim e^{-6mr}, \quad A \sim 2mr.
\end{equation}

(83)

The IR behavior of the solutions is given by

\begin{align*}
a_8 &\sim -\ln(2m\tilde{r} - \tilde{C}), \quad \sigma \sim \frac{3}{4} a_8 \sim \frac{3}{4} \ln(2m\tilde{r} - \tilde{C}), \\
b_3 &\sim \text{constant}, \quad A \sim \frac{1}{4} \ln(2m\tilde{r} - \tilde{C}).
\end{align*}

(84)
for a constant $\tilde{C}$. We then find the IR metric

$$ds^3 = (2mr - C)^{\frac{2}{3}}dx_{1,4}^2 + dr^2$$

(85)

where $C$ is another integration constant from solving $\frac{d\tilde{r}}{dr} = e^{-3\sigma}$.

For the flow to $SU(2)_R \times SU(3)$ SYM, the solutions are the same as those given in subsection 3.2 with $a_{1,2,3}$ replaced by $b_{1,2,3}$. Similarly, solutions breaking the $E_3$ global symmetry to $SU(2) \times U(1)$ are given by the solutions of subsection 4.4 with $a_{1,2,3}$ and $a_4$ replaced by $b_{1,2,3}$ and $a_8$, respectively. The flow that completely breaks the $E_3$ symmetry would involve solving for all $a_i$ and $b_i$ which we will not attempt to do here.

As in the previous sections, the flows are driven by operators of dimension 4 and vev of a dimension 3 operator.

6 Conclusions

We have studied various holographic RG flows from matter coupled $F(4)$ gauged supergravity. These flows describe deformations of the UV $N = 2$ SCFTs in five dimensions to non-conformal $N = 2$ SYM theories in the IR. The UV theories are $E_{N_f+1}$ with $N_f = 0, 1, 2$ fixed points of $N = 2$ five dimensional SYM. We have explored possible symmetry breaking patterns $E_{N_f+1} \rightarrow SO(2N_f) \times U(1), SU(N_f) \times U(1) \times U(1)$ and $SU(N_f) \times U(1)$ and interpreted the solutions as RG flows driven by turning on operators of dimension 4 in a vacuum with non-zero vev of a dimension 3 operator dual to the six-dimensional dilaton. These solutions might be useful to the study of strongly coupled $N = 2$ SYM in five dimensions.

It is interesting to holographically compute various characteristics of the 5D gauge theories such as the Wilson loops as done in [28]. It could be useful to do this computation directly in six-dimensional framework similar to the four dimensional gauge theories studied in [6, 7]. The solutions found in this paper would hopefully be useful in this aspect and other holographic calculations.

It is presently not known how to embed the six-dimensional $F(4)$ gauged supergravity coupled to $n$ vector multiplets although the pure $F(4)$ gauged supergravity and the theory coupled to 20 vector multiplets are known to originate from massive type IIA compactification on warped $S^4$ and $K3$, respectively [29, 30]. The embedding of $F(4)$ gauged supergravity in type IIB theory via the non-abelian T-duality has been proposed recently in [31]. This might also provide another mean to embed the six-dimensional gauged supergravity in higher dimensions. It would be interesting to find such an embedding which in turn can be used to uplift the solutions found here and in [19] to ten dimensions. This could provide some insight to the dynamics of D4/D8-brane system.

There is an issue of singularities which are typical in flows to non-conformal field theories. Physical and unphysical singularities can be classified by using
the criterion given in \[32\]. We have not checked this criterion for the solutions given here. However, using the scalar potential given in \[19\], at least the flow to $SU(2)_{\text{diag}}$ SYM in the case of flows from $E_1$ SCFT is physical provided that we set $g_2 = g_1 = 3m$. This is because the scalar potential $V \to -\infty$ as $\sigma, a \to \infty$. Since the potential is bounded above, the solution is physical by the criterion of \[32\]. It is interesting to also check other solutions and see whether they are physical in some range of the parameters in the solutions. And, if they are not, what pathologies in the gauge theories are. We hope to come back to these issues in future works.

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