Surface layer motion in planetary atmosphere containing fog of condensed gases

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Abstract. The article contains a simplified model of a wave motion of the atmospheric surface of planets containing finely dispersed particles of condensed gases, it is assumed that the surface of planets is heated above the saturation temperature of gas condensate, and the surface layers of the foggy atmosphere are strongly cooled. The mechanism of formation and growth of such waves is proposed and justified. It was found that the existence of growing waves on the surface of such an atmosphere is possible, as well as, in the course of time, the formation of a vortex in the atmosphere around the planet. Perturbations of the atmosphere thickness lead to the formation of gravitational waves propagating along its surface. The thickness of the atmosphere at the crest of the wave is greater than that in the trough. While the temperature of the atmosphere under the ridge increases, it decreases under the trough due to shielding of the thermal radiation of the planet. When the crest of a gravitational wave moves, the atmosphere under the trailing edge of the crest has a temperature higher than that under the front edge, since the trailing edge of the crest is heated more intensively by radiation from the surface of the planet. The partial pressure of the vapor of the condensed gases at the rear edge of the ridge is higher than that at the front edge; the work of the pressure difference during the motion of the ridge increases its energy and height. The authors demonstrate the analogy between the mechanisms of wave growth in a foggy atmosphere of planets and the mechanism of wave growth in a thin vapor layer between a strongly heated solid surface or a metal melt and a volatile liquid.

It has been experimentally and theoretically established that when a strongly heated solid body or a metal melt contacts with a volatile liquid on the surface of a liquid, surface waves arise, grow and propagate in a vapor layer [1].

Conditions for the amplitude growth also exist for a wave moving along the surface of the foggy layer of the planetary atmospheres. Figure 1 shows a simplified diagram of the temperature distribution (T) in the foggy part of the atmosphere, depending on the height (Z) above the solid surface.

The average temperature of a foggy atmosphere can be determined by the formula:

$$T_{cp} = 0.5(T_0 + T_1) + \Delta T_{cp},$$  

(1)

Where $T_0$ is the temperature of the planet's surface; $T_1$ is a surface temperature of the foggy atmosphere layer; $\Delta T_{cp}$ is the change in $T_{cp}$ as h changes.

The effect of a change in the thickness of the atmosphere $\Delta h$ on the change in the average temperature $\Delta T_{cp}$ is determined from Figure 1 by the formula:

$$\Delta T_{cp} = 0.5(T_0 - T_1) \frac{\Delta h}{h}$$  

(2)
Such an influence of magnitude on the value is determined by the fact that the periodic motion of the upper layers of the foggy atmosphere rapidly damps with a decrease in $Z$ and at depth $Z \approx \frac{1}{2}h_0$ the wave motion of the surface of the foggy atmosphere has practically no effect on the motion of the atmosphere at this depth; and the temperature change $\Delta T_{cp}$ therefore relates to the same volume of gas from the atmosphere while changing $\Delta h$.

The pressure is represented in the form:

$$P = P_a + P_n,$$

where $P_a$ is the pressure in the absence of fog;

$P_n$ is the partial pressure of the liquid vapor from the fog.

The effect of temperature on the partial pressure of a liquid vapor is determined by the Clapeyron-Clausius formula:

$$\frac{\partial P_n}{\partial x} = \frac{r}{\Delta \nu \cdot T_s} \frac{\partial T_{cp}}{\partial x},$$

Where $r$ is the evaporation heat of the liquid from the droplets of fog;

$\Delta \nu$ is the difference between the specific volumes of liquid from the fog and that of its vapor;

$T_s$ is the saturation temperature of the liquid from the drops;

$x$ is a coordinate, which is parallel to the surface of the planet.

From (2) and (4) for $T_s = T_{cp}$ and $h \approx h_0$:

$$\frac{\partial P_n}{\partial x} = \frac{r(T_0 - T_1)}{\Delta \nu(T_1 + T_0)h_0} \frac{\partial h}{\partial x} \equiv A \frac{\partial h}{\partial x},$$

where

$$A = \frac{r(T_0 - T_1)}{\Delta \nu(T_1 + T_0)h_0}$$

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure1.png}
\caption{Diagram of temperature distribution in the atmosphere: $h_0$ is the thickness of the undisturbed atmosphere; $\Delta h$ is the disturbance value of the atmospheric layer; $Z$ is a coordinate, which is normal to the surface of the planet}
\end{figure}
With increasing thickness of the atmospheric layer $\Delta h$, the temperature and the partial pressure of the liquid vapor from the drops of atmospheric fog increase at the same point in the atmosphere.

We assume that the motion of the foggy layer of the atmosphere can be approximately described by the equations of Euler’s motion:

$$\begin{align*}
\frac{\partial V_x}{\partial t} &= -\frac{1}{\rho_x} \cdot \frac{\partial P_x}{\partial x} - A \cdot \frac{\partial h}{\partial x} = -\frac{1}{\rho_x} \cdot \frac{\partial}{\partial x} (P_u + A \cdot h) \quad (6) \\
\frac{\partial V_z}{\partial t} &= -\frac{1}{\rho} \cdot \frac{\partial P_u}{\partial z} + g, \quad \frac{\partial P_u}{\partial z} \equiv 0 \quad (7) \\
\frac{\partial V_x}{\partial x} + \frac{\partial V_z}{\partial z} &= 0, \quad (8)
\end{align*}$$

where $\rho$ is the density of atmospheric gas; $g$ is the gravity acceleration.

The velocities $V_x$ and $V_z$ are represented in the form:

$$
V_x = \frac{\partial \varphi}{\partial x} \quad (9) \\
V_z = \frac{\partial \varphi}{\partial z} \quad (10)
$$

Where $\varphi$ is the potential, which is sought in the form:

$$
\varphi = (B \cdot e^{\omega z} + C \cdot e^{\omega z}) \cos(\kappa \cdot x - \omega \cdot t) \quad (11)
$$

Where $\kappa$ is the wave number; $\omega$ is the angular frequency; $B, C$ are constants determined from the boundary conditions; $\omega = 2\pi \cdot f$; $\kappa = \frac{2\pi}{\lambda}$; $\lambda$ is the wavelength; $f$ is the frequency.

The dispersion relationships, i.e. the relation between $\omega$ and $\kappa$ is found from the boundary conditions on the free surface of the foggy atmosphere. The boundary condition:

$$P_a + P_u = P_0 \text{ where } z = h, \quad (12)$$

Where $P_0$ is the gas pressure above the fog.

From (6), (11):

$$-\rho \frac{\partial \varphi}{\partial t} - \rho \cdot g \cdot h - A \cdot h = P_0 \quad (13)$$

We differentiate with respect to $t$, using (11) and (13):

$$\rho \cdot \omega^2 - (\rho \cdot g + A) \cdot \kappa = 0. \quad (14)$$

Phase velocity of wave propagation over the surface of a foggy atmosphere:

$$c = \frac{\omega}{\kappa} = (\frac{\rho \cdot g + A \cdot \kappa}{\rho})^{\frac{1}{2}} \quad (15)$$
For example, in the conditions of the Earth, for water fog: \( r = 2.2 \times 10^6 \frac{J}{kg} \); \( T_0 \approx 300K \); \( T_1 \approx 200K \); \( \Delta v \approx 0.5 \text{ m}^3 / \text{kg} \); \( h_0 \approx 10^4 \text{ m} \); \( \rho \approx 0.5 \text{ kg} / \text{m}^3 \); \( g \approx 10 \text{ m/s}^2 \), the values of \( A \) and \( \rho g \) are: \( A \approx 88 \); \( \rho g \approx 5 \). It can be seen that the quantity \( A >> \rho g \) and therefore its influence on the phase velocity of wave propagation over the surface of a foggy atmosphere under the Earth conditions is much greater than the influence of gravity (as well as for frequency and wave number).

Thus, it is shown that on the surface of a foggy part of the atmosphere containing particles of condensed gases, waves can propagate. These waves modulate the average temperature of a foggy atmosphere in such a way that at that place of the surface of the atmosphere where the crest of the wave passes and its therefore thicker than in other places, the average temperature of the atmosphere, as well as the mist drops, is at the largest level, and therefore the partial pressure of the vapor of the liquid from the fog is also at the highest level. The front and rear edges of the crest of such a wave have different temperatures: the temperature of the leading edge is less than the temperature of the rear one; since it takes some time to warm up the leading edge to the temperature of the ridge. It also takes time for the trailing edge to cool and reduce its temperature to the temperature of the trough. Therefore, the partial pressure of the vapor of the liquid from the fog at the trailing edge of the wave crest is greater than that at the leading edge. The emerging work of the pressure difference on both sides of the wave crest, as it moves, increases the energy of the wave and the magnitude of its crest. The growth of the wave amplitude is limited by the dissipation of the wave energy due to internal friction in the atmosphere. When nonlinear components in the equations of motion are taken into account, as well as the motion of fog-free atmospheric layers in numerical calculations, a constant component of the mass transfer of the atmosphere parallel to the surface of the planet appears. This can lead to the appearance of an atmospheric vortex around the planet, which has such a foggy atmosphere.

The scale of the atmosphere of the planets where such waves can develop is \( \approx 10^7 \text{ m} \), the scale of the vapor film upon contact of the metal melt with water is \( \approx 10^6 \text{ m} \), the difference in scales is \( 10^{13} \), but the mechanism for the appearance and propagation of surface waves with such a large difference in scales is the same.

References
[1] Avakimyan N N 2004 *Investigation of the instability processes at the liquid-vapor boundary near a highly superheated surface* (Krasnodar: PhD thesis) p 147.
[2] Matveyev L T 2000 Fundamentals of general meteorology *Phys. of the atmosphere* (SPb: Gidrometeoizdat) p 751.