Study of a nonhomogeneous aerogel radiator in a proximity focusing RICH detector

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The use of a nonhomogeneous aerogel radiator, i.e. one consisting of layers with different refractive indices, has been shown to improve the resolution of the Cherenkov angle measured with a proximity focusing RICH detector. In order to obtain further information on the performance of such a detector, a simple model has been used to calculate the resolution and search for optimal radiator parameters.

1. Introduction

As part of the upgrade of the Belle detector at KEK, it is planned to install a ring imaging Cherenkov detector in the forward region of the spectrometer to improve separation of pions and kaons in the momentum range up to 4 GeV/c. The limited available space has led to the decision for a proximity focusing detector using aerogel as radiator\textsuperscript{3}. Different aerogel radiators\textsuperscript{2} as well as different position sensitive photon detectors\textsuperscript{6,7} have been investigated to find an optimal solution.

An idea to further improve the resolution, has recently been proposed and experimentally studied\textsuperscript{5,6}, and was later also discussed in\textsuperscript{7}. By using a nonhomogeneous radiator, i.e. multiple aerogel layers of varying refractive index, one may increase the number of detected photons per per charged particle, but avoid the simultaneous increase in emission point uncertainty that would follow from an increase in the thickness of a homogeneous radiator. This is achieved by suitably choosing the refractive indices of the consecutive layers, so that the corresponding Cherenkov rings either overlap on the photon detector (focusing configuration) or they are well separated (defocusing configuration). Various configurations of aerogel radiators have been experimentally investigated and have shown the expected improvement in resolution\textsuperscript{8}.

In order to achieve optimal performance of the detector in the focusing configuration, we have studied the influence of various radiator parameters, such as difference in refractive index between the layers, their thickness and transmission length, on the resolution of the Cherenkov angle measured for a charged particle of given momentum. Using a simple model to calculate this resolution, we have attempted to find a set of radiator parameters that would produce the best results, i.e. the lowest standard deviation of the measured Cherenkov angle due to monoenergetic pions or kaons.

2. The model

The detector (Fig. 1) has a double layer aerogel radiator of total thickness $D_0$, with the thickness of the downstream radiator layer (labeled 2 in Fig. 1) given as $k_2D_0$. Refractive indices $n_1$ and $n_2 = n_1 + \delta n_2$ correspond to Cherenkov angles $\Theta_1$ and $\Theta_2$. The photon detector, at a distance $L$ from the entry surface of the upstream radiator, has square photosensitive pixels with the side equal to $\Delta$. 

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For a charged particle passing through the two-layer radiator, in general two rings are seen at the plane of the photon detector. We assume that the distribution of Cherenkov photons in each ring is uniform in the distance $R$ from the ring center (Fig. 2). This approximation is good for normal incidence and high transmission lengths of both radiator layers. The two uniform distributions contain $N_1$ and $N_2$ detected photons. The numbers of detected photons are assumed to be:

\[ N_1 = N_0 D_1 \sin^2 \Theta_1 \exp\left(-\frac{D_1}{2\Lambda_1}\right) \quad (1) \]
\[ N_2 = N_0 D_2 \sin^2 \Theta_2 \exp\left(-\frac{D_2}{2\Lambda_2}\right), \]

where $D_2 = k_2 D_0$, $D_1 = (1 - k_2) D_0$ and $\Lambda_1, \Lambda_2$ are the transmission lengths of both aerogel layers. For $N_0$, the figure of merit of Cherenkov counters, we assume a value of 50/cm, which was a typical value from experimental tests of such a configuration [3].

It may be quickly seen that the rms of the distribution of photons from both layers (Fig. 2) is given by:

\[ \sigma_R^2 = \langle R^2 \rangle - \langle R \rangle^2 \quad (2) \]
\[ = \frac{1}{12(N_1 + N_2)^2} \left\{-3[a_1 N_1 + (a_2 + 2d) N_2]^2 \right. \]
\[ + \left. 4(N_1 + N_2)[a_1^2 N_1 + (a_2^2 + 3a_2d + 3d^2) N_2]\right\}, \]

where $a_1$ and $a_2$ are the differences of outer and inner radii for the two rings:

\[ a_1 = D_1 \tan \Theta_1, \quad a_2 = D_2 \tan \Theta_2, \quad (3) \]

and $d$ is the difference between inner radii of the two rings:

\[ d = (L - D_0)(\tan \Theta_{10} - \tan \Theta_{20}) + D_2 \tan \Theta_{12}. \quad (4) \]

Here $\Theta_{10}$ and $\Theta_{20}$ are the values of photon angles after refraction into the air, while $\Theta_{12}$ is the angle of photons from radiator 1 in radiator 2.

The contribution of the emission point uncertainty to the error in determination of the transmission length at 400 nm is used for $\Lambda$.\[2\text{In this model we neglect the wavelength dependence of the aerogel transmission length. This approximation turns out to be sufficiently good, provided the transmission length at 400 nm is used for $\Lambda$.}\]
Cherenkov angle due to a single photon is:
\[ \sigma_{\text{emp}} = \frac{\sigma_R}{(L - D_0/2)} \cos^2 \bar{\Theta}, \]  

(5)

where we have denoted by \( \bar{\Theta} \) the average Cherenkov angle. The contribution due to position resolution of the detector, i.e. the pixel size \( \Delta \), is:
\[ \sigma_{\text{det}} = \frac{\Delta}{(L - D_0/2) \sqrt{12}} \cos^2 \bar{\Theta}. \]  

(6)

Other contributions, such as uncertainty of track direction or nonuniformity of density and thickness of the aerogel, we collectively label simply as \( \sigma_{\text{rest}} \). Assuming that the contributions are not correlated, we add them in quadrature and divide by the square root of the number of detected photons to obtain the r.m.s. of the Cherenkov angle for a track, i.e. the parameter that needs to be minimized, as
\[ \sigma_{\text{track}} = \frac{1}{\sqrt{N_1 + N_2}} \sqrt{\sigma_{\text{emp}}^2 + \sigma_{\text{det}}^2 + \sigma_{\text{rest}}^2}. \]  

(7)

The optimization procedure described below refers mainly to the parameters of the aerogel radiator layers such as refractive index, thickness and transmission length. Other parameters, which are determined by our given experimental arrangement, are mainly fixed at values corresponding to the particular detector under study. For example, the radiator-to-photon-detector distance is 20 cm, the photon detector pixel size is set to 6 mm and the contribution from other sources is found to be \( \sigma_{\text{rest}} = 6 \) mrad.

3. Results

3.1. Double layer radiator

First we have checked the equations and the calculation by taking both radiator layers of equal refractive index (\( \delta n_2 = n_2 - n_1 = 0 \)). The refractive index was set to \( n = 1.04 \), which corresponds to a Cherenkov angle of \( \Theta = \Theta_1 = \Theta_2 = 278 \) mrad for 4 GeV/c pions. The transmission length was assumed to be \( \Lambda = \Lambda_1 = \Lambda_2 = 4 \) cm. The photon detector, at a distance of \( L = 20 \) cm from the entry surface of the radiator, has a fixed pad size \( \Delta = 6 \) mm. As suggested by our measurements \( \{3, 5\} \), the contribution from other, not yet completely understood sources, is \( \sigma_{\text{rest}} = 6 \) mrad. The track resolution, given by \( \sigma_{\text{track}} \), was then calculated as a function of the total radiator thickness \( D_0 \). The result, shown in Fig. 3 gives an optimal total thickness of about 2 cm, at which the resolution amounts to \( \sigma_{\text{track}} = 5.4 \) mrad. This is also consistent with the experimental value obtained from our measurements \( \{3\} \).

![Figure 3. Resolution of Cherenkov angle (\( \sigma_{\text{track}} \)) versus the thickness \( D_0 \) of a homogeneous radiator with \( n = 1.04 \) (\( \Theta = 278 \) mrad), \( \Lambda = 4 \) cm, \( L = 20 \) cm, \( \Delta = 6 \) mm and \( \sigma_{\text{rest}} = 6 \) mrad.](image)

In the next step, the difference \( \delta n_2 \) in refractive indices of the two radiators has been varied for the case of \( n_1 = 1.04 \) (\( \Theta_1 = 278 \) mrad) and two equally thick radiators (\( D_0 = 4 \) cm, \( k_2 = 0.5 \)), with attenuation lengths \( \Lambda_1 = \Lambda_2 = 4 \) cm, fixed distance \( L = 20 \) cm, standard pad size of 6 mm and \( \sigma_{\text{rest}} = 6 \) mrad. The minimum of \( \sigma_{\text{track}} \) was found to be about 4.3 mrad at a difference in refractive indices of 0.009, which corresponds to a difference in Cherenkov angle of \( \Theta_2 - \Theta_1 = 29 \) mrad (Fig. 4). We note that the minimum in \( \sigma_{\text{track}} \) is quite broad, a departure of \( \delta n_2 \) by \( \pm 0.002 \) from the optimal value only increases \( \sigma_{\text{track}} \) by about 0.1 mrad.

Then, \( \delta n_2 \) was set to 0.009 and, with all the
other parameters left unchanged, the relative thickness of aerogel 2 was varied. The minimum is at \( k_2 = 0.44 \) as may be seen in Fig. 5. We observe that the variation of \( \sigma_{\text{track}} \) with \( k_2 \) is weak: it stays within 3% of the minimum value over a broad interval \( 0.35 < k_2 < 0.55 \).

A plot of \( \sigma_{\text{track}} \) depending on both relative thickness \( k_2 \) and difference of refractive indices \( \delta n_2 \), gives a minimum of 4.2 mrad at \( \delta n_2 = 0.009 \) and \( k_2 = 0.44 \) (Fig. 5). If, in addition to \( \delta n_2 \) and \( k_2 \), also the total radiator thickness \( D_0 \) is varied, \( \sigma_{\text{track}} \) has the same minimal value (4.2 mrad) at the thickness of \( D_0 = 3.2 \) cm.

Table 1 shows the optimized parameters for different values of the transmission length. It is seen that aerogels with good transmission may considerably improve the resolution. Table 2 gives the optimized parameters for the case that 5 cm more space is available between radiator and photon detector. The beneficial effect of these 5 cm of additional space is comparable to the effect of perfect transmission.

We have therefore seen, that for the limited
Table 1
Optimized parameters for different Λ and for \( n_1 = 1.04 \), \( L = 20 \) cm, \( Δ = 6 \) mm and \( σ_{\text{rest}} = 6 \) mrad. The parameters are mainly explained in the text except for \( N \) and \( σ_0 \), which are the number of photons per track and the single photon resolution, respectively.

| \( \Lambda \) | \( \delta n_2 \) | \( D_0 \) (cm) | \( k_2 \) | \( σ_0 \) (mrad) | \( N \) | \( σ_{\text{min}}^{\text{track}} \) (mrad) |
|---|---|---|---|---|---|---|
| \( \Lambda = 4 \) cm | 0.007 | 3.2 | 0.45 | 12.5 | 9.0 | 4.2 |
| \( \Lambda = 3 \) cm | 0.006 | 3.0 | 0.45 | 12.2 | 7.7 | 4.4 |

Table 2
Optimized parameters for \( L = 25 \) cm, \( n_1 = 1.04 \), \( Δ = 6 \) mm and \( σ_{\text{rest}} = 6 \) mrad.

| \( \Lambda \) | \( \delta n_2 \) | \( D_0 \) (cm) | \( k_2 \) | \( σ_0 \) (mrad) | \( N \) | \( σ_{\text{min}}^{\text{track}} \) (mrad) |
|---|---|---|---|---|---|---|
| \( \Lambda = 4 \) cm | 0.006 | 3.5 | 0.45 | 10.8 | 9.4 | 3.5 |

space between radiator and photon detector (\( L = 20 \) cm), for given transmission length (\( Λ = 4 \) cm) and pixel size (\( Δ = 6 \) mm) and for the given contribution of other sources to \( σ_{\text{track}} \) (\( σ_{\text{rest}} = 6 \) mrad), one may achieve an improvement in resolution of about \( 1.2 \) mrad (i.e. from \( 5.4 \) mrad to \( 4.2 \) mrad) by optimizing the thicknesses and refractive indices of two consecutive aerogel layers. We note that a similar improvement was also observed in experimental tests of such a counter. Further improvements may be achieved with aerogels of better transmission or with additional space available for the detector.

3.2. Multiple layer radiator

The calculation has been extended to the case when the radiator consists of more than two layers. Table 3 shows the optimized parameters for 3 and 4 layers compared to the dual radiator. It is evident that the improvement of resolution is primarily due to an increased radiator thickness, and consequently to the measured number of photons per track, while the single photon resolution \( σ_0 \) remains approximately constant.

A simple estimate of \( σ_{\text{track}} \) may be obtained just by dividing the contribution due to emission point uncertainty by the number of layers and adding the other (fixed) contributions in quadrature. The curves in Fig. 7 are obtained by such a simplified procedure. It is seen that the results of optimization using the model described in section 2. of this paper and represented by points in Fig. 6 agree quite nicely with such a simple estimate. From Fig. 8 we also see that an increased number of layers leads to an increase in optimal overall radiator thickness. For this simplified model we plot in Fig. 8 the track resolution \( σ_{\text{track}} \) at the optimized radiator thickness as a function of the number of layers.

We observe that the dependence of \( σ_{\text{track}} \) on radiator thickness at the minimum is relatively weak. It is also not symmetric with respect to the minimum, i.e. an increase in overall thickness produces a smaller change in \( σ_{\text{track}} \) than an equal decrease (Fig. 5 and Fig. 6). It would therefore be advisable to use a thickness, which at higher charged particle momenta is somewhat greater than the value required to minimize \( σ_{\text{track}} \). This
Table 3
Optimized parameters for one, two, three and four radiator layers and with fixed \(n_1 = 1.04\), \(\Lambda = 4\) cm, \(L = 20\) cm, \(\Delta = 6\) mm and \(\sigma_{\text{rest}} = 6\) mrad; \(k_i\) is the relative thickness of the layer \(i\), and \(n_1 + \delta n_i\) is the corresponding refractive index.

|       | single layer | two layers | three layers | four layers |
|-------|--------------|------------|--------------|-------------|
| \(\delta n_2\) | 0.007        | 0.007      | 0.008        |             |
| \(\delta n_3\) |             | 0.014      | 0.015        |             |
| \(\delta n_4\) |             |            | 0.022        |             |
| \(k_2\)      | 0.45         | 0.34       | 0.28         |             |
| \(k_3\)      |              | 0.27       | 0.23         |             |
| \(k_4\)      |              |            | 0.19         |             |
| \(D_0\) (cm) | 1.9          | 3.2        | 4.4          | 5.6         |
| \(\sigma_0\) (mrad) | 12.8        | 12.5      | 12.6         | 12.8        |
| \(N\)        | 5.7          | 9.0        | 11.9         | 14.7        |
| \(\sigma_{\text{track}}^{\text{min}}\) (mrad) | 5.4         | 4.2        | 3.7          | 3.3         |

Figure 7. The resolution in Cherenkov angle (\(\sigma_{\text{track}}\)) as a function of total radiator thickness with the number of layers of differing refractive index as parameter. The curves are obtained by simply dividing \(\sigma_{\text{emp}}\) with the number of layers. The points are calculated as described in the previous section. The fixed parameters are: \(n_1 = 1.04\), \(\Lambda = 4\) cm, \(L = 20\) cm, \(\Delta = 6\) mm and \(\sigma_{\text{rest}} = 6\) mrad.

Figure 8. The track resolution \(\sigma_{\text{track}}\) at optimal \(D_0\) versus the number of layers obtained with the simple estimate described in the text. The fixed parameters are: \(n_1 = 1.04\), \(\Lambda = 4\) cm, \(L = 20\) cm, \(\Delta = 6\) mm and \(\sigma_{\text{rest}} = 6\) mrad.
would produce a negligible loss of resolution at high momenta, but would represent a valuable increase in the number of detected photons at low momenta, where the difference in Cherenkov angle between pions and kaons is large anyway (or where kaons are below the threshold for Cherenkov radiation). This low sensitivity on the thickness of individual radiator layers (Fig. 5) also has a practical advantage; it permits equal thicknesses of the different layers, thus simplifying the production process.

Finally we note that the quantity we are actually interested in is the separation between pions and kaons at a given momentum, $s_{\pi K} = (\bar{\Theta}_\pi - \bar{\Theta}_K)/\sigma_{\text{track}}$. Since in addition to $\sigma_{\text{track}}$ also the average Cherenkov angle $\bar{\Theta}$ varies with $k_i$, $n_i$ and $D_0$, these parameters assume different values if $s_{\pi K}$ is optimized instead of $\sigma_{\text{track}}$. The total radiator thickness is, e.g. somewhat smaller if $s_{\pi K}$ is maximized ($D_0 = 3.0$ cm, 3.7 cm and 4.3 cm for the two, three and four layer configuration, respectively). It turns out, however, that the resulting $\sigma_{\text{track}}$ and $s_{\pi K}$ are almost equal in both cases.

4. Conclusion

We have shown that with a nonhomogeneous aerogel radiator in a proximity focusing Cherenkov ring imaging detector, one may, by adjusting the refractive index of consecutive radiator layers, reduce the contribution of emission point uncertainty to the overall Cherenkov angle resolution of a charged particle track. In the particular case of the RICH detector studied for the Belle upgrade, the limiting factors are then the lack of space and the pixel size. We have also seen that the optimal resolution is not very sensitive to minor variations of refractive index or thickness of individual aerogel layers. This is a very welcome property, since it somewhat reduces the stringent demands on the aerogel production process.

In our previous experimental studies, we have seen that there also exists an important contribution ($\sigma_{\text{rest}}$) from factors not yet understood. These will be the subject of further study, as well as the investigation into possibilities of improving the aerogel production procedure in order to obtain the best possible transmission and homogeneity of individual aerogel layers. However, already at this stage, the resolution in Cherenkov angle of charged particles seems to meet the requirements of a 4σ separation of pions and kaons in the 1-4 GeV/c momentum range.

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