OCD factorization for the pion diffractive dissociation into two jets

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Abstract
We present main ideas and obtained results of our recent calculations of Coulomb and QCD contribution to the pion diffractive dissociation into two jets.

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1 Introduction
This talk is devoted to our studies of the pion diffractive dissociation into two jets, being the subjects of Refs. [1], [2]. In this process a highly energetic pion interacts with a nucleus A and produces two jets (di-jets) which in the lowest approximation are formed from the quark-antiquark ($q\bar{q}$) pair

$$\pi A \rightarrow q\bar{q} A.$$ (1)

Primary motivation for studies of this process goes back to Ref. [3], in which it has been conjectured that pion diffraction dissociation on a heavy nucleus $\pi A \rightarrow X A$ is sensitive to small transverse size configurations of pion constituents. It was later argued [4], [5], [6] that selecting a specific hadronic final state that consists of a pair of (quark) jets with large transverse momentum one can obtain important insight into the pion structure as it turns out that the longitudinal momentum fraction distribution of the jets follows that of...
the pion valence parton constituents. A measurement of hard dijet coherent production on nuclei presents, therefore, a possibility of a direct measurement of the pion distribution amplitude and provides striking evidence [7] that this distribution is close to its asymptotic form.

From the theoretical point of view, the principal question is whether the relevant transverse size of the pion \( r_\perp \) (alias the scale of the pion distribution amplitude \( \mu = 1/r_\perp \)) is of the order of the transverse momenta of the jets \( \mu \sim q_\perp \). In this case one can write the scattering amplitude as a convolution of the nonperturbative soft pion distribution amplitude and the skewed gluon distribution in the target with the hard scattering amplitude.

The cross section for this process is largest when the momentum transfer to the target is the smallest one, which means that the large transverse momenta of the quark jet \( (q_{1\perp}) \) and of the antiquark one \( (q_{2\perp}) \) have to balance each other \( q_{1\perp} \simeq -q_{2\perp} \).

The main contribution to the diffractive process (1) is due to Pomeron exchange. However, this process can also occur as result of the electromagnetic interactions between pion and target nucleus (Coulomb exchange). The strenght of the electromagnetic coupling is \( \alpha Z \) (\( \alpha \) is the electromagnetic fine coupling constant). For heavy nuclei \( \alpha Z \) is not small and therefore one can ask about the size of the Coulomb contribution to (1). Since derivation of the Coulomb contribution is technically simpler let us begin from this case.

### 2 Coulomb dissociation of a pion into two jets

The gauge invariant set of diagrams describing the Coulomb contribution to the process (1) is shown in Fig. 1. The wavy line denotes photon exchanged in the t-channel. The large transverse momentum of di-jets results from the
hard gluon exchange denoted by the dashed line. It supplies a hard scale to the process so we neglect the pion mass. The incoming pion has momentum \( p_1, \ p_1^2 = 0 \) and its fraction carried by quark (antiquark) is denoted by \( z \) (\( \bar{z} \equiv 1 - z \)). The nucleus mass is \( M \), its momenta in the final and in the initial states are \( p_2 \) and \( p_3 \), respectively. The large cms energy squared of the process is \( s = (p_1 + p_2)^2 = \bar{s} + M^2 \), and the small momentum transfer squared equals to the photon virtuality \( k^2 \), i.e. \( t = k^2 = (p_2 - p_3)^2 \). The Sudakov decomposition of the photon momentum:

\[
k = \alpha p_1 + \beta p_2' + k_\perp, \quad p_2' = p_2 - \frac{M^2}{\bar{s}}, \quad p_2'^2 = 0,
\]

is fully determined by the kinematics of the process and can be written in terms of the jets variables as

\[
\beta = \frac{M^2_{2j} + (q_1 + q_2)^2}{\bar{s}}, \quad \alpha = -\frac{(q_1 + q_2)^2 + M^2 \beta}{\bar{s}}, \quad k_\perp = q_1 + q_2, \quad (3)
\]

where \( M^2_{2j} \) is the invariant mass of the di-jets. The momentum transfer \( t \) equals \( t = k^2 \approx -(k_\perp^2 + k_{min}^2) \), and according to Eq.(3) its minimal value is

\[
k_{min}^2 = \frac{M^2 M^4_{2j}}{s^2}, \quad \text{where} \quad M^2_{2j} = \frac{q_{1\perp}^2}{z \bar{z}}. \quad (4)
\]

We calculate the leading asymptotics of the scattering amplitude \( M \) in powers of \( 1/q_{1\perp}^2 \) (at the leading twist level). It is given as the convolution of the hard scattering amplitude \( T_H(u, \mu_F^2) \) with the pion light-cone distribution amplitude \( \phi_\pi(u, \mu_F^2) \)

\[
M = \int_0^1 du \phi_\pi(u, \mu_F^2) T_H(u, \mu_F^2), \quad (5)
\]

where the hard scattering amplitude \( T_H(u, \mu_F^2) \) describes the production of a free \( q\bar{q} \) pair (the di-jets) in collision of the \( t \)-channel photon with \( q \) and \( \bar{q} \) (the later having momenta \( u p_1 \) and \( \bar{u} p_1 \), respectively), collinear to the pion momentum \( p_1 \). The factorization scale \( \mu_F \) is of the order of the di-jets transverse momentum \( q_{1\perp} \).

Eq.(5) describes the factorization procedure in QCD which disentangles the contributions to \( M \) coming from large and small distances. The soft part is described by the pion distribution amplitude [8], [9]

\[
\langle 0|\bar{d}(x)\gamma_\mu\gamma_5 u(-x)|\pi^+(p)\rangle_{x^2 \to 0} = i p_\mu f_\pi \int_0^1 du e^{i(2u-1)(xp)} \phi_\pi(u). \quad (6)
\]
The constant $f_\pi = 131\text{MeV}$ is known experimentally from $\pi \to \mu\nu$ decay.

The hard part, i.e. the amplitude $T_H(u)$ in Eq.(5), is calculable in the perturbative QCD, it is given by four tree diagrams shown in Fig. 1.

The factorization procedure described by Eq. (5) is similar to that used for the hard exclusive processes [8], [9]. Its validity for the quasi-exclusive process (1) can be justified as follows. Let us consider, for example, the diagram Fig.1(b). The large transverse momentum of quark jets flows along the lines $A - B - C$. Therefore their virtualities are much larger than those of the other quark lines $D - A$ and $D - B$. Thus, at leading twist, the quark lines $D - A$ and $D - B$ have to be considered as being on mass shell and this part of the diagram can be factorized out of the hard part given by the highly virtual quark and gluon propagators.

The result for the scattering amplitude corresponding to the diagrams in Fig. 1 is given by the formula (for details of derivation see Ref. [1])

$$M^{\pi^+ + A \to 2j + A} = -i\delta_{ij} f_\pi \frac{2\alpha Z F_{QED}(k^2)}{r_{\pi^2}} \alpha_s(q_{1\perp}) \frac{s}{(k_{\perp}^2 + k_{\perp,min}^2)(q_{1\perp}^2)^2} \frac{\bar{s}}{s} v(q_2) \int_0^1 du \frac{\phi_\pi(u)}{u}.$$  \hspace{1cm} (7)

where $\alpha_s = \frac{g^2}{4\pi}$ and we used the symmetry property $\phi_\pi(u) = \phi_\pi(\bar{u})$. The nucleus was here treated as a scalar particle with the electromagnetic form-factor $F_{QED}(k^2)$.

The conclusions which one can draw from Eq.(7) are the following. Since the behaviour of the pion distribution amplitude at the end-points is known [8], [9]: $\phi_\pi(u) \sim u$ for $u \to 0$, and $\phi_\pi(u) \sim \bar{u}$ for $\bar{u} \to 0$, therefore the integral over the momentum fraction fraction $u$ is well defined what confirms that factorization holds for the process we discuss. Let us emphasize that the integral of $\phi_\pi(u)$ over $u$ in Eq. (7) generates only an overall factor. Therefore the dependence of the amplitude $M$ on $\bar{z}$ is universal, i.e. it doesn’t depend on the shape of the pion distribution amplitude. Moreover, let us note that the amplitude (7) vanishes for $z = e_u/(e_u - e_d) = 2/3$. Due to the opposite signs of the electric charges of the pion constituents, $e_u = 2e/3, e_d = -e/3$, the contribution of the diagrams (a) and (b) in Fig. 1 cancels the one of diagrams (c) and (d).

The result (7) differs essentially from the corresponding formula (78) of Ref. [6]. We predict the universal $\bar{z}$ dependence of the scattering amplitude $M$, independent of the shape of $\phi_\pi(u)$. Contrary to that, in Ref. [6] the amplitude is proportional to the lowest pion Fock state wave function, $M \sim \Psi_\pi(z, q_{1\perp})$ (in our notation). This disagreement between two results is in our opinion re-
lated to unproper treatment of the hard gluon exchange in Ref. [6]. Within the QCD factorization approach, which we followed in this study, the hard gluon exchange has to be treated as a part of the hard scattering block $T_H$. In Ref. [6] the hard gluon exchange is considered as the high transverse momentum tail of the wave function $\Psi(\pi, q_{1\perp})$.

Our estimates of the magnitude of the Coulomb effect show that it is very small in comparison to the QCD contribution (for details see Ref. [1]). But we cannot directly compare our predictions with E791 data since their absolute normalization unfortunately is not reported.

3 QCD contribution to the pion dissociation into two jets

The kinematics of the process is shown in Fig. 2. We restrict ourselves to scattering from a single nucleon having initial momentum $p_2$ and the final one $p'_2$. Fig. 2 expresses also graphically the factorization formula for the amplitude of hard dijet production in QCD

$$M_{\pi\rightarrow 2\text{jets}} = \int_0^1 dz' \int_0^1 dx_1 \phi_\pi(z', \mu_F^2) T_H(z', x_1, \mu_F^2) F^{g}_\zeta(x_1, \mu_F^2).$$

in which $\phi_\pi(z', \mu_F^2)$ is again as in the Coulomb case (5) the pion distribution amplitude, and $F^{g}_\zeta(x_1, \mu_F^2)$ is the non-forward (skewed) gluon distribution [10–12] in the target nucleon or nucleus. (The asymmetry parameter $\zeta$ is fixed by the process kinematics.) $T_H(z', x_1, \mu_F^2)$ is the hard scattering amplitude and $\mu_F$ is the (collinear) factorization scale.
Since at high energies the scattering amplitudes corresponding to Pomeron exchange are dominated by their imaginary parts we calculated only cut diagrams. They are built of tree-level on-shell scattering amplitudes and their form is strongly constrained by gauge invariance. The existing cut diagrams can be grouped into the four gauge-invariant contributions shown in Fig. 4a–d, which differ by the position of the hard gluon that provides the large momentum transfer to the jets. For example, in Figs. 4a and 4b it is assumed that the hard gluon exchange appears to the left of the cut and to the right, respectively; typical diagrams are shown in Figs. 3a and 3c. The two remaining contributions in Fig. 4c and Fig. 4d take into account the possibility of real gluon emission in the intermediate state. The filled circles stand for the effective vertices describing the gluon radiation. The calculations are performed in Feynman gauge and for details of them I refer to Ref. [2].

The final result for the imaginary part of the amplitude for dijet production from a nucleon reads

\[ \text{Im} \, M = -i \, s \, f_{\pi} \, \alpha_s^2 \, \frac{4 \, \pi^3}{N_c^2 \, q_1^4} \, \bar{u}(q_1) \gamma_5 \gamma_5 \, \frac{p_2}{s} \, v(q_2) \, I \, \delta_{i,j} \]  

\[ (9) \]

\[ I = \int_0^1 dz' \, \phi_{\pi}(z', \mu^2) \left\{ \left( \frac{z \bar{z}}{z' \bar{z}'} + 1 \right) \left[ C_F \left( \frac{z \bar{z}}{z' \bar{z}'} + 1 \right) + \frac{1}{2N_c} \left( z' + \bar{z}' \right) \right] \right. 
\times \left[ \frac{\Theta(z' - z)}{(z' - z)} \, F_{\zeta} \left( \zeta \frac{z' \bar{z}}{z' - z}, \mu^2 \right) + \frac{\Theta(z - z')}{(z - z')} \, F_{\zeta} \left( \zeta \frac{z' \bar{z}}{z - z'}, \mu^2 \right) \right] \]
Let us discuss conclusions which follow from this result. We begin by noting that similarly as in the Coulomb contribution, the scattering amplitude $M$ is not directly proportional to the pion distribution amplitude $\phi_\pi(z, \mu^2)$ but it is a rather complicated convolution of this quantity and the corresponding coefficient function. The singularity at $z' = z$ of the integrand in (10) is present in the contributions in Fig. 4c,d which include real gluon emission in the intermediate state. The logarithmic integral $\int dz'/|z-z'| \sim \ln s$ is nothing but the usual energy logarithm that accompanies each extra gluon in the gluon ladder. We can simplify the integrand in (10) for $z' = z$, to get

$$I\bigg|_{z'=z} = 4N_c \phi_\pi(z) \int \frac{dz'}{z' - z} F_\zeta^\pi(z', z) \simeq 4N_c \phi_\pi(z) \int \frac{dy}{y} F_\zeta^\pi(y, q^2_\perp).$$  (11)

For a flat gluon distribution $F_\zeta^\pi(y) \sim \text{const}$ at $y \to 0$, and the integration gives const $\cdot \ln 1/\zeta$ which is the above mentioned logarithm. The r.h.s. of (11) with the factor $2N_c/y$ appearing in (11) can be interpreted as the relevant limit of the DGLAP-type evolution equation [11]

$$q^2_\perp \frac{\partial}{\partial q^2_\perp} F_\zeta(x = \zeta, q^2_\perp) =$$
$$= \frac{\alpha_s}{2\pi} \int \frac{dy}{\zeta} P_{\zeta g}^g(\zeta, y) F_\zeta(y, q^2_\perp) \simeq \frac{\alpha_s}{2\pi} \int \frac{dy}{\zeta} \frac{2N_c}{y} F_\zeta(y, q^2_\perp).$$  (12)

The quantity on the l.h.s. of (12) defines what can be called the unintegrated non-forward gluon distribution and the physical meaning of Eqs. (11) and 12 is that in the region $z' \sim z$ hard gluon exchange can be viewed as a large transverse momentum part of the gluon distribution in the proton, cf. [5].

Next, consider the contribution to the imaginary part of the amplitude of the dijet production coming from the end-points $z' \to 0$ and $z' \to 1$:

$$I\bigg|_{\text{end-points}} = \left( N_c + \frac{1}{N_c} \right) z\bar{z} \int_0^1 dz' \frac{\phi_\pi(z', \mu^2)}{z'^2} F_\zeta(\zeta, \mu^2).$$  (13)

Since $\phi_\pi(z') \sim z'$ at $z' \to 0$, the integral over $z'$ diverges logarithmically. This divergence indicates that the collinear factorization conjectured in (8) is generally not valid. Remarkably, the divergent integral containing the pion distribution amplitude is just a constant and does not involve any $z$-dependence.
Therefore, the longitudinal momentum distribution of the jets in the nonfactorizable contribution is calculable and, as it turns out, has the shape of the asymptotic pion distribution amplitude $\phi_{as}^\pi(z) = 6z\bar{z}$.

Our result (10) containing the end-point singularities does not agree with the result of independent calculations done in [13] by means of the method which the light-cone dominance assumed from the very begin. We intend to clarify the origin of this disagreement in a forthcoming publication.

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