Using Hindsight to Anchor Past Knowledge in Continual Learning

Abstract

In continual learning, the learner faces a stream of data whose distribution changes over time. Modern neural networks are known to suffer under this setting, as they quickly forget previously acquired knowledge. To address such catastrophic forgetting, many continual learning methods implement different types of experience replay, re-learning on past data stored in a small buffer known as episodic memory. In this work, we complement experience replay with a new objective that we call “anchoring”, where the learner uses bilevel optimization to update its knowledge on the current task, while keeping intact the predictions on some anchor points of past tasks. These anchor points are learned using gradient-based optimization to maximize forgetting, which is approximated by fine-tuning the currently trained model on the episodic memory of past tasks. Experiments on several supervised learning benchmarks for continual learning demonstrate that our approach improves the standard experience replay in terms of both accuracy and forgetting metrics and for various sizes of episodic memories.

1. Introduction

We study the problem of continual learning, where a machine learning model experiences a sequence of tasks. Each of these tasks is presented as a stream of input-output pairs, where each pair is drawn identically and independently (iid) from the corresponding task probability distribution. Since the length of the learning experience is not specified a-priori, the learner can only assume a single pass over the data and, due to space constraints, store nothing but a few examples in a small episodic memory. At all times during the lifetime of the model, predictions on examples from all tasks may be requested. Addressing continual learning is an important research problem, since it would enable the community to move past the assumption of “identically and independently distributed data”, and allow a better deployment of machine learning in-the-wild. However, continual learning presents one major challenge, catastrophic forgetting (McCloskey & Cohen, 1989). That is, as the learner experiences new tasks, it quickly forgets previously acquired knowledge. This is a hindrance specially for state-of-the-art deep learning models, where all parameters are updated after observing each example.

Continual learning has received increasing attention from the scientific community during the last decade. The state-of-the-art in algorithms for continual learning fall into three categories. First, regularization-based approaches reduce forgetting by restricting the updates in parameters that were important for previous tasks (Kirkpatrick et al., 2016; Rebuffi et al., 2017; Aljundi et al., 2018; Chaudhry et al., 2018; Nguyen et al., 2018). However, when the number of tasks are large, the regularization of past tasks becomes obsolete, leading to representation drift (Titsias et al., 2019). Second, modular approaches (Rusu et al., 2016; Lee et al., 2017) add new modules to the learner as new tasks are learned. While modular architectures overcome forgetting by design, the memory complexity of these approaches scales with the number of tasks. Third, memory-based methods (Lopez-Paz & Ranzato, 2017; Hayes et al., 2018; Isele & Cosgun, 2018; Riemer et al., 2019; Chaudhry et al., 2019a) store a few examples from past tasks in an “episodic memory”, to be revisited when training for a new task. Contrary to modular approaches, memory-based methods add a very small memory overhead for each new task. Memory-based methods are the reigning state-of-the-art, but their performance remains a far-cry from a simple oracle accessing all the data at once, hence turning the continual learning experience back into a normal supervised learning task. Despite intense research efforts, such gap in performance renders the problem of continual learning an open research question.

Contribution We propose Hindsight Anchor Learning (HAL), a continual learning approach to improve the performance of memory-based continual learning algorithms. HAL leverages bilevel optimization to regularize the training objective with one representational point per class per task, called anchors. These anchors are constructed via gradient ascent in the image space, by maximizing one approximation to the forgetting loss for the current task throughout
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Ranzato, 2017; Hayes et al., 2018; Riemer et al., 2019; Chaudhry et al., 2019a), we consider streams of data that are experienced only once. Therefore, the learner cannot revisit any but a small amount of data triplets chosen to be stored in a small episodic memory $\mathcal{M}$. More specifically, we consider tiny “ring” episodic memories, which contain the last $m$ observed examples per class for each of the experienced tasks, where $m \in \{1, 3, 5\}$. That is, considering as variables the number of experienced tasks $t$ and examples $n$, we study continual learning algorithms with a $O(t)$ memory footprint.

Following Lopez-Paz & Ranzato (2017) and Chaudhry et al. (2018), we monitor two statistics to evaluate the quality of continual learning algorithms: final average accuracy, and final maximum forgetting. First, the final average accuracy of a predictor is defined as

$$\text{Accuracy} = \frac{1}{T} \sum_{j=1}^{T} a_{T,j},$$

(2)

where $a_{i,j}$ denotes the test accuracy on task $j$ after the model has finished experiencing task $i$. That is, the final average accuracy measures the test performance of the model at every task after the continual learning experience has finished. Second, the final maximum forgetting is defined as

$$\text{Forgetting} = \frac{1}{T-1} \sum_{j=1}^{T-1} \max_{i \in \{1, \ldots, T-1\}} \left( a_{i,j} - a_{T,j} \right),$$

(3)

that is, the decrease in performance at each of the tasks between their peak accuracy and their accuracy after the continual learning experience has finished.

Finally, following Chaudhry et al. (2019a), we use the first $k < T$ tasks to cross-validate the hyper-parameters of each of the considered continual learning algorithms. These first $k$ tasks are not considered when computing the final average accuracy and maximum forgetting metrics.

3. Hindsight Anchor Learning (HAL)

The current state-of-the-art algorithms for continual learning are based on experience replay (Hayes et al., 2018; Riemer et al., 2019; Chaudhry et al., 2019b). These methods update the model $f_0$ while storing a small amount of past observed triplets in an episodic memory $\mathcal{M} = \{(x', y', t')\}$. For a new mini-batch of observations $\mathcal{B} := \{(x, y, t)\}$ from task $t$, the learner samples a mini-batch $\mathcal{B}_M$ from $\mathcal{M}$ at random, and employ the rule $\theta \leftarrow \theta - \alpha \cdot \nabla_\theta \mathcal{L}(\mathcal{B} \cup \mathcal{B}_M)$ to update its parameters, where

$$\mathcal{L}(A) = \frac{1}{|A|} \sum_{(x, y, t) \in A} \mathcal{L}(f_0(x, t), y)$$

(4)

denotes the average loss across a collection of triplets $A = \mathcal{B} \cup \mathcal{B}_M$. In general, $\mathcal{B}_M$ is constructed to have

The rest of the manuscript then presents our new algorithm HAL (Section 3), showcases its empirical performance (Section 4), surveys the related literature (Section 5), and offers some concluding remarks (Section 6).

2. Continual learning setup

In continual learning, we experience a stream of data triplets $(x_i, y_i, t_i)$ containing an input $x_i$, a target $y_i$, and a task identifier $t_i \in T = \{1, \ldots, T\}$. Each input-target pair $(x_i, y_i) \in \mathcal{X} \times \mathcal{Y}_t_i$ is an identically and independently distributed example drawn from some unknown distribution $P_t(X, Y)$, representing the $t_i$-th learning task. We assume that the tasks are experienced in order ($t_i \leq t_j$ for all $i \leq j$), and that the total number of tasks $T$ is not known a priori. Under this setup, our goal is to estimate a predictor that is anchored to past observations, and that the total number of tasks $T$ is not known a priori. Under this setup, our goal is to estimate a predictor that is anchored to past observations, and that the total number of tasks $T$ is not known a priori. Under this setup, our goal is to estimate a predictor that is anchored to past observations, and that the total number of tasks $T$ is not known a priori. Under this setup, our goal is to estimate a predictor that is anchored to past observations, and that the total number of tasks $T$ is not known a priori. Under this setup, our goal is to estimate a predictor that is anchored to past observations, and that the total number of tasks $T$ is not known a priori. Under this setup, our goal is to estimate a predictor that is anchored to past observations, and that the total number of tasks $T$ is not known a priori. Under this setup, our goal is to estimate a predictor that is anchored to past observations, and that the total number of tasks $T$ is not known a priori. Under this setup, our goal is to estimate a predictor that is anchored to past observations, and that the total number of tasks $T$ is not known a priori. Under this setup, our goal is to estimate a predictor that is anchored to past observations, and that the total number of tasks $T$ is not known a priori. Under this setup, our goal is to estimate a predictor that is anchored to past observations, and that the total number of tasks $T$ is not known a priori.

Results

We compare HAL to EWC (Kirkpatrick et al., 2016), ICARL (Rebiffi et al., 2017), VCL (Nguyen et al., 2018), AGEM (Chaudhry et al., 2019a), experience replay (Hayes et al., 2018; Riemer et al., 2019), and MER (Riemer et al., 2019), across four commonly used benchmarks in supervised continual learning (MNIST permutations, MNIST rotations, split CIFAR-100, and split miniImageNet). In these experiments, HAL achieves state-of-the-art performance, improving accuracy by up to 7.5% and reducing forgetting by almost 23% in the best case. We show that these results hold for various sizes of episodic memories (between 1 and 5 examples per class per task).

We now begin our exposition by reviewing the continual learning setup. The rest of the manuscript then presents our new algorithm HAL (Section 3), showcases its empirical performance (Section 4), surveys the related literature (Section 5), and offers some concluding remarks (Section 6).

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the same size as $\mathcal{B}$, but it can be smaller if the episodic memory $\mathcal{M}$ does not yet contain enough samples.

In the previous, the episodic memory $\mathcal{M}$ reminds the predictor about how to perform at past tasks using only a small amount of data. As such, the behaviour of the predictor on past tasks outside the data stored in $\mathcal{M}$ is not guaranteed. Also, since $\mathcal{M}$ is usually very small, the performance of the predictor becomes sensitive to the choice of samples stored in the episodic memory. Because of this reason, we propose to further fix the behaviour of the predictor at a collection of carefully constructed anchor points $e_t$, one per class per past task $t'$, at each parameter update.

Let us assume that the anchor points $e_t$ are given—we will see later how to construct them in practice. To constrain the change of the predictor at these anchor points, we propose a two-step parameter update rule:

$$
\begin{align*}
\hat{\theta} &\leftarrow \theta - \alpha \cdot \nabla_{\theta} \ell(B \cup B_{\mathcal{M}}), \\
\theta &\leftarrow \theta - \alpha \cdot \nabla_{\theta} \ell(B \cup B_{\mathcal{M}}) \\
&+ \lambda \cdot \sum_{t' < t} (f_{\theta}(e_{t'}, t') - f_{\theta}(e_t, t'))^2 .
\end{align*}
$$

The first step computes a temporary parameter vector $\hat{\theta}$ by minimizing the loss at a minibatch from the current task $t$, and the episodic memory of past tasks (this is the usual experience replay parameter update). The second step employs nested optimization to perform the final update of the parameter $\theta$, which trades-off the minimization of (a) the loss value at the current minibatch and the episodic memory, as well as (b) the change in predictions at the anchor points for all past tasks. The proposed rule not only updates the predictor conservatively, thereby reducing forgetting, but also, as shown in Appendix A, improves the transfer by maximizing the inner product between the gradients on $\mathcal{B} \cup B_{\mathcal{M}}$ and anchor points. In this respect, it bears similarity to gradient-based meta-learning approaches (Finn et al., 2017; Nichol & Schulman, 2018; Riemer et al., 2019).

Next, let us discuss how to choose the anchor points, $e_t$ (one per class per task) as to preserve the performance of current task throughout the entire learning experience. Ideally, the anchor points should attempt to minimize the forgetting on the current task as the learner is updated on future tasks. One could achieve this by letting $e_t$ be an example from the task $t$ that would undergo maximum forgetting during the entire continual learning experience, including past and future tasks. Then, requiring the predictions to remain invariant at such $e_t$, by using Eq. 5, could effectively reduce forgetting on the current task. Mathematically, the desirable $e_t$ for the label $y_t$ is obtained by maximizing the following Forgetting loss:

$$
(e_t, y_t) \leftarrow \arg \max_{(x, y) \sim P_t} \ell(f_{\theta_T}(x, t), y_t) - \ell(f_{\theta_t}(x, t), y_t),
$$

where $\theta_t$ is the parameter vector after training on task $t$ and $\theta_T$ is the final parameter vector after the entire learning experience. Thus, keeping predictions intact on the pair $(e_t, y_t)$ above can effectively preserve the performance of task $t$. However, the idealistic Eq. 6 requires access to (a) the entire distribution $P_t$ to compute the maximization, and (b) access to all future distributions $t' > t$ to compute the final parameter vector $\theta_T$. Both are unrealistic assumptions under the continual learning setup described in Section 2, as the former requires storing the entire dataset of task $t$, and the latter needs access to future tasks.

To circumvent (a), we can recast Eq. 6 as an optimization problem and learn the desired $e_t$ by initializing it at random and using $k$ gradient ascent updates for a given label $y_t$ in the image space ($\mathcal{X} \in \mathbb{R}^D$). The proposed optimization objective is given by:

$$
\begin{align*}
\max_{e_t \in \mathcal{B}} &\left(\ell(f_{\theta_T}(e_t, t), y_t) - \ell(f_{\theta_t}(e_t, t), y_t) - \gamma \left(\phi(e_t) - \phi_t\right)^2\right). \\
\text{Forgetting loss} &- \text{Mean embedding loss}
\end{align*}
$$

where the regularizer, given by mean embedding loss, constrains the search space by trying to push the anchor point embedding towards the mean data embedding. We recall that $\phi$ denotes the feature extractor of the predictor, and $\phi_t$ is the mean embedding (Smola et al., 2007) of all observed examples from task $t$. Since the feature extractor is updated after experiencing each data point, the mean embedding $\phi_t$ are computed as running averages. That is, after observing a minibatch $\mathcal{B} = \{(x, y, t)\}$ of task $t$, we update:

$$
\phi_t \leftarrow \beta \cdot \phi_t + (1 - \beta) \frac{1}{|\mathcal{B}|} \sum_{x \in \mathcal{B}} \phi(x),
$$

where $\phi_t$ is initialized to zero at the beginning of the learning experience. In our experiments, we learn one $e_t$ per class for each task. We fix the $y_t$ to the corresponding class label, and discard $\phi_t$ after training on task $t$. Learning $e_t$ in this manner circumvents the requirement of storing the entire distribution $P_t$ for the current task $t$.

Still, Eq. 7 requires the parameter vector $\theta_T$, to be obtained in the distant future after all learning tasks have been experienced. To waive this impossible requirement, we propose to approximate the future by simulating the past. That is, instead of measuring the forgetting that would happen after the model is trained at future tasks, we measure the forgetting that happens when the model is fine-tuned at past tasks. In this way, we say that forgetting is estimated in hindsight,
using past experiences. More concretely, after training on task $t$ and obtaining the parameter vector $\theta_t$, we minimize the loss during one epoch on the episodic memory $M_t$ to obtain a temporary parameter vector $\theta_{M_t}$ that approximates $\theta_T$, and update $e_t$ as:

$$ e_t \leftarrow e_t + \alpha \cdot \nabla_{e_t} \left( \ell(f_{\theta_{M_t}}(e_t, t), y_t) - \ell(f_{\theta_t}(e_t, t), y_t) - \gamma (\phi(e_t) - \phi_t)^2 \right). $$

This completes the description of our proposed algorithm for continual learning, which combines experience replay with anchors learned in hindsight. We call our approach Hindsight Anchor Learning (HAL) and summarize the entire learning process as follows:

**Hindsight Anchor Learning (HAL)**
- Initialize $\theta \sim P(\theta)$ and $\{e_t \sim P(e)\}_{t=1}^T$ from normal distributions $P(\theta)$ and $P(e)$.
- Initialize $M = \{\}$
- For each task $t = 1, \ldots, T$:
  - Initialize $\phi_t = 0$
  - For each minibatch $B$ from task $t$:
    * Sample $B_{M_t}$ from $M$
    * Update $\theta$ using Eq. 5
    * Update $\phi_t$ using Eq. 8
    * Update $M$ by adding $B$ in a first-in-first-out (FIFO) ring buffer
  - Fine-tune on $M$ to obtain $\theta_{M_t}$
  - Build $e_t$ using Eq. 9 $k$ times
  - Discard $\phi_t$
- Return $\theta$.

### 4. Experiments

We now evaluate the performance of HAL against a variety of baselines on commonly used supervised continual learning benchmarks.

#### 4.1. Datasets and tasks

We perform experiments on four supervised classification benchmarks for continual learning.

- **Permuted MNIST** is a variant of the MNIST dataset of handwritten digits (LeCun, 1998) where each task applies a fixed random pixel permutation to the original dataset. This benchmark contains 23 tasks, each with 1000 samples from 10 different classes.
- **Rotated MNIST** is another variant of MNIST, where each task applies a fixed random image rotation (between 0 and 180 degrees) to the original dataset. This benchmark contains 23 tasks, each with 1000 samples from 10 different classes.
- **Split CIFAR** is a variant of the CIFAR-100 dataset (Krizhevsky & Hinton, 2009; Zenke et al., 2017), where each task contains the data pertaining 5 random classes (without replacement) out of the total 100 classes. This benchmark contains 20 tasks, each with 250 samples per each of the 5 classes.
- **Split miniImageNet** is a variant of the ImageNet dataset (Russakovsky et al., 2015; Vinyals et al., 2016), containing a subset of images and classes from the original dataset. This benchmark contains 20 tasks, each with 250 samples per each of the 5 classes.

For all datasets, the first 3 tasks are used for hyper-parameter optimization (grids available in Appendix C). The learners can perform multiple epochs on these three initial tasks, that are later discarded for evaluation.

#### 4.2. Baselines

We compare our proposed model HAL to the following baselines.

- **Finetune** is a single model trained on the stream of data, without any regularization or episodic memory.
- **ICARL** (Rebuffi et al., 2017) uses nearest-mean-of-exemplars rule for classification and avoids catastrophic forgetting by regularizing over the feature representations of previous tasks using knowledge distillation loss (Hinton et al., 2014).
- **EWC** (Kirkpatrick et al., 2016) is a continual learning method that limits changes to parameters critical to past tasks, as measured by the Fisher information matrix.
- **VCL** (Nguyen et al., 2018) is a continual learning method that uses online variational inference for approximating the posterior distribution which is then used to regularize the model.
- **AGEM** (Chaudhry et al., 2019a) is a continual learning method improving on (Lopez-Paz & Ranzato, 2017), which uses an episodic memory of parameter gradients to limit forgetting.
- **MER** (Riemer et al., 2019) is a continual learning method that combines episodic memories with meta-learning to limit forgetting.
Table 1. Accuracy (Eq. 2) and Forgetting (Eq. 3) results of continual learning experiments. Averages and standard deviations are computed over five runs using different random seeds. When used, episodic memories contain up to one example per class per task. Last two rows are oracle baselines.

| METHOD                  | PERMUTED MNIST | ROTATED MNIST |
|-------------------------|---------------|---------------|
|                          | ACCURACY     | FORGETTING   | ACCURACY     | FORGETTING   |
| FINETUNE                | 53.5 (±1.46) | 0.29 (±0.01) | 41.9 (±1.37) | 0.50 (±0.01) |
| EWC (KIRKPATRICK ET AL., 2016) | 63.1 (±1.40) | 0.18 (±0.01) | 44.1 (±0.99) | 0.47 (±0.01) |
| VCL (NGUYEN ET AL., 2018)  | 51.8 (±1.54) | 0.44 (±0.01) | 48.2 (±0.99) | 0.50 (±0.01) |
| VCL-RANDOM (NGUYEN ET AL., 2018) | 52.3 (±0.66) | 0.43 (±0.01) | 54.4 (±1.44) | 0.44 (±0.01) |
| AGEM (CHAUDHRY ET AL., 2019A) | 62.1 (±1.39) | 0.21 (±0.01) | 50.9 (±0.92) | 0.40 (±0.01) |
| MER (RIEMER ET AL., 2019)   | 69.9 (±0.40) | 0.14 (±0.01) | 66.0 (±2.04) | 0.23 (±0.01) |
| ER-RING (CHAUDHRY ET AL., 2019B) | 70.2 (±0.56) | 0.12 (±0.01) | 65.9 (±0.41) | 0.24 (±0.01) |
| HAL (OURS)               | 73.6 (±0.31) | 0.09 (±0.01) | 68.4 (±0.72) | 0.21 (±0.01) |
| CLONE-AND-FINETUNE       | 81.4 (±0.35) | 0.0           | 87.5 (±0.11) | 0.0          |
| MULTITASK               | 83.0          | 0.0           | 83.3          | 0.0          |

| METHOD                  | SPLIT CIFAR | SPLIT MINIIMAGE NET |
|-------------------------|-------------|---------------------|
|                          | ACCURACY     | FORGETTING   | ACCURACY     | FORGETTING   |
| FINETUNE                | 42.9 (±2.07) | 0.25 (±0.03) | 34.7 (±2.69) | 0.26 (±0.03) |
| EWC (KIRKPATRICK ET AL., 2016) | 42.4 (±3.02) | 0.26 (±0.02) | 37.7 (±3.29) | 0.21 (±0.03) |
| ICARL (REBUFFI ET AL., 2017) | 46.4 (±1.21) | 0.16 (±0.01) | -            | -            |
| AGEM (CHAUDHRY ET AL., 2019A) | 54.9 (±2.92) | 0.14 (±0.03) | 48.2 (±2.49) | 0.13 (±0.02) |
| MER (RIEMER ET AL., 2019)   | 49.7 (±2.97) | 0.19 (±0.03) | 45.5 (±1.49) | 0.15 (±0.01) |
| ER-RING (CHAUDHRY ET AL., 2019B) | 56.2 (±1.93) | 0.13 (±0.01) | 49.0 (±2.61) | 0.12 (±0.02) |
| HAL (OURS)               | 60.4 (±0.54) | 0.10 (±0.01) | 51.6 (±2.02) | 0.10 (±0.01) |
| CLONE-AND-FINETUNE       | 60.3 (±0.55) | 0.0           | 50.3 (±1.00) | 0.0          |
| MULTITASK               | 68.3          | 0.0           | 63.5          | 0.0          |

- **ER-Ring** (Chaudhry et al., 2019b) is a continual learning method that uses a ring buffer as episodic memory.
- **Multitask** is an oracle baseline that has access to all data to optimize Eq. 1, useful to estimate an upper bound on the obtainable Accuracy (Eq. 2).
- **Clone-and-finetune** is an oracle baseline training one independent model per task, where the model for task $t'$ is initialized by cloning the parameters of the model for task $t' - 1$.

All baselines use the same neural network architectures: a perceptron with two hidden layers of 256 ReLU neurons in the MNIST experiments, and a ResNet18, with three times less feature maps across all layers, similar to Lopez-Paz & Ranzato (2017), in CIFAR and ImageNet experiments. The task identifiers are used to select the output head in the CIFAR and ImageNet experiments, while ignored in the MNIST experiments. Batch size is set to 10 for both the stream of data and episodic memories, across experiments and models. The size of episodic memories is set between 1 and 5 examples per class per task. The results of VCL are compiled by running the official implementation\(^1\), that only works for fully-connected networks, in our continual-learning setup. All the other baselines use our unified code base which is available at [https://bit.ly/2mw8bsE](https://bit.ly/2mw8bsE).

### 4.3. Results

Table 1 summarizes the main results of our experiments when episodic memory of only one example per class per task is used. First, our proposed HAL is the method achieving maximum Accuracy (Eq. 2) and minimal Forgetting (Eq. 3) at all benchmarks. This does not include oracle baselines **Multitask** (which has access to all data simultaneously) and **Clone-and-finetune** (which trains a separate model per task). Second, the relative gains from the second-best method **ER-Ring** to HAL are significant, confirming that the anchoring objective (Eq. 5) allows experience-replay methods to generalize better from the same amount of episodic memory.

Third, regularization based-approaches, such as EWC (Kirkpatrick et al., 2016) and VCL (Nguyen et al., 2018), suffer

\(^1\)https://github.com/nvcuong/variational-continual-learning
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Figure 1. Evolution of Accuracy (Eq. 2) as new tasks are learned. When used, episodic memories contain up to one example per class per task.

Table 2. Accuracy (Eq. 2) results for large (3 to 5 examples per class per task) episodic memory sizes. Here we only compare methods that use an episodic memory. Averages and standard deviations are computed over five runs using different random seeds.

| METHOD          | PERMUTED MNIST | ROTATED MNIST |
|-----------------|---------------|---------------|
|                 | $|M| = 600$    | $|M| = 1000$   |
| VCL-RANDOM      | 55.8 (±1.29)  | 58.5 (±1.21)  |
| AGEM            | 63.2 (±1.47)  | 64.1 (±0.74)  |
| MER             | 74.9 (±0.049) | 78.3 (±0.19)  |
| ER-RING         | 73.5 (±0.43)  | 75.8 (±0.24)  |
| **HAL (OURS)**  | **76.2 (±0.52)** | **78.4 (±0.27)** |

| METHOD          | SPLIT CIFAR | SPLIT MINI IMAGE NET |
|-----------------|-------------|----------------------|
|                 | $|M| = 255$  | $|M| = 425$           |
| ICARL           | 51.7 (±1.41) | -                    |
| AGEM            | 56.9 (±3.45) | 59.9 (±2.64)         |
| MER             | 57.7 (±2.59) | 60.6 (±2.09)         |
| ER-RING         | 60.9 (±1.44) | 62.6 (±1.77)         |
| **HAL (OURS)**  | **62.9 (±1.49)** | **64.4 (±2.15)** |

under the single epoch setup. As noted by Chaudhry et al. (2019a), EWC requires multiple passes over the samples of each task to perform well. The poor performance of VCL is attributed to noisy posterior estimation in the single pass setup. Note that approaches making use of memory (MER, ER and HAL) work significantly better in this setup.

Fourth, ICARL (Rebuffi et al., 2017), another method making use of episodic memory, performs poorly in our setup. From the Table 1, it can be argued that direct training on a very small episodic memory, as done in experience replay, allows the methods to generalize better compared to when the same memory is used indirectly in the knowledge distillation loss (Hinton et al., 2014) as done in ICARL.

Figure 1 shows a more fine grained analysis of average accuracy as new tasks are learned on Permuted MNIST and Split CIFAR. HAL preserves the performance of a predictor more effectively than other baselines.

Table 2 shows the Accuracy of methods employing episodic memory when the size of memory is increased. We use 3 to 5 examples per class per task, resulting in a total memory size from 600 to 1000 for MNIST experiments, and from 255 to 425 for CIFAR and ImageNet experiments. The corresponding numbers for Forgetting are given in Appendix B. HAL consistently improves on ER-Ring and other baselines.

Figure 2 provides the training time of the continual learning baselines on MNIST benchmarks. Although HAL adds an overhead on top of experience replay baseline, it is significantly faster than MER —another approach that makes use of nested optimization to reduce forgetting. However, HAL requires extra memory to store task anchors that, as we will show next, are more effective than additional data samples one can store for experience replay. Overall, we conclude that HAL provides the best trade-off in terms of efficiency and performance.

Ablation Study

We now turn our attention towards two questions; (1) whether for the same episodic memory size in bytes HAL improves over the experience replay baseline, (2) whether fine-tuning on replay buffer is a good approximation of forgetting when the learner is updated on future tasks.
To answer the first question, let $|\mathcal{M}|$ be the total size of episodic memory for all tasks when one example per class per task is stored in the replay buffer. We then run experience replay with double the size of episodic memory (i.e.) storing two examples per class per task instead of one. On the other hand, the episodic memory size in HAL is kept at $|\mathcal{M}|$. This effectively makes the size of memory in bytes taken by experience replay and that of HAL equal as latter requires extra memory to store anchors. Table 3 summarizes the results of this study. For the same memory size in bytes, HAL performs better than experience replay when additional real data samples are stored in the episodic memory. It is surprising that the anchors learned by HAL, initialized from random noise and learned using gradient-based optimization, perform better compared to randomly sampled real data. To understand this, in Figure 3 we visualize HAL’s anchors along with the task data in image and feature space on Permutated MNIST benchmark. From the left of the figure, it can be seen that HAL anchors lie with in the data cluster of a class in the image space suggesting that mean embedding loss in Eq. 9 effectively regularizes against outliers. More interestingly, the figure on the right shows that these anchors lie at or close to the cluster edges in the feature space. In other words, anchor points learned by HAL lie close to the classifier decision boundary. This can explain their effectiveness compared to the real data samples that can lie anywhere in the data cluster in feature space.

Finally, to answer the second part we assume a non-continual setup where at each step the learner has an oracle access to all future tasks. After training on task $t$, the learner is fine-tuned on all future tasks and anchor points are subsequently learned by optimizing idealistic Eq. 7. The results are reported in Table 4. It can be seen from the table that the proposed HAL performs very close to the non-continual oracle baseline. This suggests that HAL’s approximation of forgetting when the learner is updated on future tasks by replaying past data is effective in many existing continual learning benchmarks.

5. Related work

In continual learning (Ring, 1997), also called lifelong learning (Thrun, 1998), a learner addresses a sequence of changing tasks without storing the complete datasets of these tasks. This is in contrast to multitask learning (Caruana, 1997), where the learner assumes simultaneous access to data from all tasks. The main challenge in continual learning is to avoid catastrophic interference (McCloskey & Cohen, 1989; McClelland et al., 1995; Goodfellow et al., 2013), that is, the learner forgetting previously acquired knowledge when learning new tasks. The state-of-the-art methods in continual learning can be categorized into three classes.

First, regularization approaches discourage updating parameters important for past tasks (Kirkpatrick et al., 2016; Aljundi et al., 2018; Nguyen et al., 2018; Zenke et al., 2017). While efficient in terms of memory and computation, these approaches suffer from brittleness due to feature drift as the number of tasks increases (Titsias et al., 2019). Additionally, these approaches are only effective when we can perform multiple passes over each dataset (Chaudhry et al., 2019a), a case deemed unrealistic in this work.

Second, modular approaches use different parts of the prediction function for each new task (Fernando et al., 2017; Aljundi et al., 2017; Rosenbaum et al., 2018; Chang et al., 2018; Xu & Zhu, 2018; Ferran Alet, 2018). Modular approaches do not scale to a large number of tasks, as they require searching over combinatorial space of module ar-

![Figure 2](image-url)

**Figure 2.** Training time (s) of MNIST experiments for the entire continual learning experience. MER and HAL both use meta-learning objectives to reduce forgetting.

**Table 3.** Comparison of HAL with experience replay. ER-Ring and HAL use one example per class per task in the episodic memory, whereas ER-Ring-2|$|\mathcal{M}|$ uses two examples per class per task in the memory. Averages and standard deviations are computed over five runs using different random seeds.

| METHOD       | PERMUTED MNIST | SPLIT CIFAR |
|--------------|----------------|-------------|
|              | ACCURACY       | FORGETTING  | ACCURACY       | FORGETTING  |
| ER-RING      | 70.2 ±(0.56)   | 0.12 ±(0.01)| 56.2 ±(1.93)  | 0.13 ±(0.01) |
| ER-RING-2    | 71.9 ±(0.31)   | 0.11 ±(0.01)| 58.6 ±(2.68)  | 0.12 ±(0.01) |
| HAL          | 73.6 ±(0.31)   | 0.09 ±(0.01)| 60.4 ±(0.54)  | 0.10 ±(0.01) |

**Table 4.** Performance comparison of HAL with Oracle where the learner has access to all the future tasks to exactly quantify forgetting of an anchor. Averages and standard deviations are computed over five runs using different random seeds.

| ANCHOR TYPE | PERMUTED MNIST | SPLIT CIFAR |
|-------------|----------------|-------------|
| HAL         | 73.6 ±(0.31)   | 0.09 ±(0.01)| 60.4 ±(0.54)  | 0.10 ±(0.01) |
| ORACLE      | 73.9 ±(0.41)   | 0.09 ±(0.01)| 61.1 ±(0.94)  | 0.09 ±(0.01) |

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Figure 3. t-SNE visualization of images and anchors (HAL) in the image space (left) and the feature space (right) on Permuted MNIST benchmark for a single task. Anchor points are exaggerated in size for the purpose of better visualization. The left plot shows that anchor points lie within the data cluster of a class, whereas the right plot shows that, in the feature space, anchor points lie close to the edge of the cluster of a class or near decision boundaries.

There are other definitions of continual learning, such as the one of task-free continual learning. The task-free formulation does not consider the notion of tasks, and instead works on undivided data streams (Aljundi et al., 2019a;b). We have focused on the task-based definition of continual learning and, similar to many recent works (Lopez-Paz & Ranzato, 2017; Hayes et al., 2018; Riemer et al., 2019; Chaudhry et al., 2019a), assumed that only a single pass through the data was possible.

Finally, our gradient-based learning of anchors bears a similarity to (Simonyan et al., 2014) and (Wang et al., 2018). In Simonyan et al. (2014), the authors use gradient ascent on class scores to find saliency maps of a classification model. Contrary to them, our proposed hindsight learning objective optimizes for the forgetting metric, as reducing it is necessary for continual learning. Dataset distillation (Wang et al., 2018) proposes to encode the entire dataset in a few synthetic points at a given parameter vector by a gradient-based optimization process. Their method requires access to the entire dataset of a task for optimization purposes. We, instead, learn anchors in hindsight from the replay buffer of past tasks after training is finished for current task. While Wang et al. (2018) aim to replicate the performance of the entire dataset from the synthetic points, we focus on reducing forgetting of an already learned task.

6. Conclusion

We introduced a bilevel optimization objective, dubbed anchoring, for continual learning. In our approach, we learned one “anchor point” per class per task, where predictions are requested to remain invariant by the means of nested optimization. These anchors are learned using gradient-based optimization, and represent points that would maximize the forgetting of the current task throughout the entire learning experience. We simulate the forgetting that would happen during the learning of future tasks in hindsight, that is, by taking temporary gradient steps across a small episodic memory of past tasks. We call our approach Hindsight Anchor Learning (HAL). As shown in our experiments, an-
Choring in hindsight complements and improves the performance of continual learning methods based on experience replay, achieving a new state of the art on four standard continual learning benchmarks.

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Appendix

Section A describes the approximate update performed by anchoring objective (Eq. 5 in the main paper). Section B reports the Forgetting metric (Eq. 3) for bigger episodic memories. Section C provides the grid considered for hyper-parameters. Section D gives pseudo-code for HAL.

A. Approximate Update by Anchoring Objective

Here we will use a Taylor series expansion to approximate the update performed by anchoring objective (Eq. 5 in the main paper). In particular, we are interested in the regularization part of the anchoring objective that involves nested update. We refer to this gradient as $g\mid\ddagger$. We follow similar arguments as (Nichol & Schulman, 2018).

Let $\theta_0$ be the parameter vector before the temporary update in the anchoring objective (Eq. 5). Also, let $\ell_{ce}$ and $\ell_{L2}$ be the cross-entropy and L2 losses, respectively. We use the following definitions:

$g_0 = \ell'_{ce}(\theta_0)$ (gradient of cross-entropy loss at initial point on $B \cup B_M$)

$H_0 = \ell''_{ce}(\theta_0)$ (Hessian of cross-entropy loss at initial point on $B \cup B_M$)

$g_1 = \ell'_{L2}(\theta_0)$ (gradient of L2 loss at initial point on anchors)

$H_1 = \ell''_{L2}(\theta_0)$ (gradient of L2 loss at initial point on anchors)

Let $U_0 = \theta_0 - \alpha g_0$ be the operator giving a temporary update in the two-step process of (Eq. 5), and let $\theta_1$ be the temporary update itself ($i.e.$ $\theta_1 := U_0$ (note that $\tilde{\theta}$ is used in the main paper instead of $\theta_1$)). The $g\mid\ddagger$ is given by:

$$g\mid\ddagger = \frac{\partial}{\partial \theta_0} \ell_{L2}(U_0)$$

$$= U'_0 \cdot \ell'_{L2}(\theta_1)$$

$$= (I - \alpha H_0) \cdot \ell'_{L2}(\theta_1), \quad (10)$$

where the second step is obtained by using chain rule. Now, if we calculate the first order Taylor series approximation of $\ell'_{L2}(\theta_1)$,

$$\ell'_{L2}(\theta_1) = \ell'_{L2}(\theta_0) + \ell''_{L2}(\theta_0) \cdot (\theta_1 - \theta_0) + O(||\theta_1 - \theta_0||^2)$$

$$= \overline{g}_1 + \overline{H}_1 \cdot (\theta_0 - \alpha \overline{g}_0 - \theta_0) + O(\alpha^2)$$

$$= \overline{g}_1 - \alpha \overline{H}_1 \cdot \overline{g}_0 + O(\alpha^2), \quad (11)$$

where in the second step we substituted the value of $\theta_1$. By putting Eq. 11 in Eq. 10 and after some simplification we get:

$$g\mid\ddagger = \overline{g}_1 - \alpha (\overline{H}_1 \cdot \overline{g}_0 + \overline{H}_0 \cdot \overline{g}_1) + O(\alpha^2). \quad (12)$$

This form is very similar to the second-order MAML gradient formulation, Eq. 25 in (Nichol & Schulman, 2018). Further simplification of the inner product terms between Hessian and gradients yields inner product between the gradients $\overline{g}_0$ and $\overline{g}_1$. This shows that similar to MAML (Finn et al., 2017), Reptile (Nichol & Schulman, 2018) and MER (Riemer et al., 2019), anchoring objective, as described in Eq. 5 of the main paper, maximizes the inner product between the gradients. However, unlike the other meta-learning approaches, in anchoring objective, these gradients correspond to different loss functions, cross-entropy and L2 losses on data from current task and episodic memory, and HAL anchors, respectively.
B. More Results

Table 5 shows the Forgetting of methods employing episodic memory when the size of memory is increased. We use 3 to 5 examples per class per task, resulting in a total memory size from 600 to 1000 for MNIST experiments, and from 255 to 425 for CIFAR and ImageNet experiments.

Table 5. Forgetting (Eq. 3) results for large (3 to 5 examples per class per task) episodic memory sizes. Here we only compare methods that use an episodic memory. Averages and standard deviations are computed over five runs using different random seeds.

| METHOD       | PERMUTED MNIST | ROTATED MNIST |
|--------------|----------------|---------------|
|              | $|M| = 600$     | $|M| = 1000$   |
| VCL-RANDOM   | 0.39 (±0.01)   | 0.36 (±0.01)  | 0.37 (±0.01)   |
| AGEM         | 0.20 (±0.01)   | 0.19 (±0.01)  | 0.41 (±0.01)   |
| MER          | 0.14 (±0.01)   | 0.09 (±0.01)  | 0.12 (±0.01)   |
| ER-RING      | 0.09 (±0.01)   | 0.07 (±0.01)  | 0.15 (±0.01)   |
| HAL (ours)   | 0.07 (±0.01)   | 0.05 (±0.01)  | 0.12 (±0.01)   |

| METHOD       | SPLIT CIFAR    | SPLIT MINIIMAGENET |
|--------------|----------------|---------------------|
|              | $|M| = 255$     | $|M| = 425$          |
| ICARL        | 0.13 (±0.02)   | 0.13 (±0.02)        | -                  |
| AGEM         | 0.13 (±0.03)   | 0.10 (±0.02)        | 0.10 (±0.02)       |
| MER          | 0.11 (±0.01)   | 0.09 (±0.02)        | 0.12 (±0.02)       |
| ER-RING      | 0.09 (±0.01)   | 0.06 (±0.01)        | 0.07 (±0.02)       |
| HAL (ours)   | 0.08 (±0.01)   | 0.06 (±0.01)        | 0.06 (±0.01)       |

C. Hyper-parameter Selection

In this section, we report the hyper-parameters grid considered for experiments. The best values for different benchmarks are given in parenthesis.

- **Multitask**
  - learning rate: [0.003, 0.01, 0.03 (CIFAR, miniImageNet), 0.1 (MNIST perm, rot), 0.3, 1.0]

- **Clone-and-finetune**
  - learning rate: [0.003, 0.01, 0.03 (CIFAR, miniImageNet), 0.1 (MNIST perm, rot), 0.3, 1.0]

- **Finetune**
  - learning rate: [0.003, 0.01, 0.03 (CIFAR, miniImageNet), 0.1 (MNIST perm, rot), 0.3, 1.0]

- **EWC**
  - learning rate: [0.003, 0.01, 0.03 (CIFAR, miniImageNet), 0.1 (MNIST perm, rot), 0.3, 1.0]
  - regularization: [0.1, 1, 10 (MNIST perm, rot, CIFAR, miniImageNet), 100, 1000]

- **AGEM**
  - learning rate: [0.003, 0.01, 0.03 (CIFAR, miniImageNet), 0.1 (MNIST perm, rot), 0.3, 1.0]
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- **MER**
  - learning rate: \([0.003, 0.01, 0.03 \text{ (MNIST, CIFAR, miniImageNet)}, 0.1, 0.3, 1.0]\)
  - within batch meta-learning rate: \([0.01, 0.03, 0.1 \text{ (MNIST, CIFAR, miniImageNet)}, 0.3, 1.0]\)
  - current batch learning rate multiplier: \([1, 2, 5 \text{ (CIFAR, miniImageNet)}, 10 \text{ (MNIST)}]\)

- **ER-Ring**
  - learning rate: \([0.003, 0.01, 0.03 \text{ (CIFAR, miniImageNet)}, 0.1 \text{ (MNIST perm, rot)}, 0.3, 1.0]\)

- **HAL**
  - learning rate: \([0.003, 0.01, 0.03 \text{ (CIFAR, miniImageNet)}, 0.1 \text{ (MNIST perm, rot)}, 0.3, 1.0]\)
  - regularization \((\lambda)\): \([0.01, 0.03, 0.1 \text{ (MNIST perm, rot)}, 0.3 \text{ (miniImageNet)}, 1 \text{ (CIFAR)}, 3, 10]\)
  - mean embedding strength \((\gamma)\): \([0.01, 0.03, 0.1 \text{ (MNIST perm, rot, CIFAR, miniImageNet)}, 0.3, 1, 3, 10]\)
  - decay rate \((\beta)\): 0.5
  - gradient steps on anchors \((k)\): 100

**D. HAL Algorithm**

Algorithm 1 provides pseudo-code for HAL.
Algorithm 1 Training of HAL on sequential data $D = \{D_1, \cdots, D_T\}$, with total replay buffer size ‘mem$_{sz}$’, learning rate ‘$\alpha$’, regularization strength ‘$\lambda$’, mean embedding decay ‘$\beta$’, mean embedding strength ‘$\eta$’.

1: procedure HAL($D$, mem$_{sz}$, $\alpha$, $\lambda$, $\beta$)
2: $M \leftarrow \{\}$ * mem$_{sz}$
3: $\{e_t, \cdots, e_T\} \leftarrow \{\}$
4: for $t \in \{1, \cdots, T\}$ do
5:   $\phi_t \leftarrow 0$
6:   for $B \sim D_t$ do
7:     $B_M \sim M$
8:     $\theta \leftarrow \theta - \alpha \cdot \nabla_{\theta} \ell(B \cup B_M)$
9:     $\phi_t \leftarrow \phi_t - \alpha \cdot \nabla_{\phi_t} \left(\ell(B \cup B_M) + \lambda \cdot \sum_{t' < t} (f_{\theta}(e_{t'}, t') - f_{\tilde{\theta}}(e_{t'}, t'))^2\right)$
10:    $M \leftarrow$ UpdateMemory($M, B$)
11: end for
12: $e_t, \theta \leftarrow$ GetAnchors($M, \theta, \phi_t, \eta$)
13: end for
14: return $\theta, M$
15: end procedure

1: procedure GetAnchors($M, \theta_t, \phi_t, \gamma$)
2: $\theta \leftarrow \theta_t$
3: for $B_M \sim M$ do
4:   $\theta \leftarrow \theta - \alpha \cdot \nabla_{\theta} \ell(B_M)$
5: end for
6: $\theta_M \leftarrow \theta$
7: $e_t \leftarrow$ rand()
8: for $1, \cdots, k$ do
9:   $e_t \leftarrow e_t + \alpha \cdot \nabla_{e_t} \left(\ell(f_{\theta_M}(e_t, t), y_t) - \ell(f_{\theta_2}(e_t, t), y_t) - \gamma(\phi(e_t) - \phi_t)^2\right)$
10: end for
11: return $e_t, \theta_t$
12: end procedure
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