THE PHASE COHERENCE OF INTERSTELLAR DENSITY FLUCTUATIONS

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ABSTRACT

Studies of MHD turbulence often investigate the Fourier power spectrum to provide information on the nature of the turbulence cascade. However, the Fourier power spectrum only contains the Fourier amplitudes and rejects all information regarding the Fourier phases. Here, we investigate the utility of two statistical diagnostics for recovering information on Fourier phases in ISM column density maps: the averaged amplitudes of the bispectrum and the phase coherence index (PCI), a new phase technique for the ISM. We create three-dimensional density and two-dimensional column density maps using a set of simulations of isothermal ideal MHD turbulence with a wide range of sonic and Alfvénic Mach numbers. We find that the bispectrum averaged along different angles with respect to either the \( k_1 \) or \( k_2 \) axis is primarily sensitive to the sonic Mach number while averaging the bispectral amplitudes over different annuli is sensitive to both the sonic and Alfvénic Mach numbers. The PCI of density suggests that the most correlated phases occur in supersonic sub-Alfvénic turbulence and near the shock scale. This suggests that nonlinear interactions with correlated phases are strongest in shock-dominated regions, in agreement with findings from the solar wind. Our results suggest that the phase information contained in the bispectrum and PCI can be used to find the turbulence parameters in column density maps.

Key words: ISM: structure – magnetohydrodynamics (MHD) – turbulence

1. INTRODUCTION

Magnetohydrodynamic (MHD) turbulence is ubiquitous in space plasma across many orders of magnitude in scale. This includes the solar wind at astronomical unit scales, the interstellar medium (ISM) at parsec scales, and the intercluster medium (ICM) at megaparsec scales (for a review, see Elmegreen & Scalo 2004). In the solar wind, velocity and magnetic perturbations can be directly measured via in situ spacecraft. However, in the ISM and ICM, turbulence must be measured indirectly through line-of-sight tracers such as column density and spectral line profiles (see Lazarian 2009).

The traditional measure of turbulence on Earth, as well as in astrophysical environments, is the spatial Fourier power spectrum. This is because the turbulence energy transfer cascade can be studied by examining the Fourier power spectrum, and the sources and sinks of energy, including the injection and dissipation scales, can be identified. For studies of turbulence in the ISM, the power spectrum of the density and velocity (and its variants such as the structure function and delta variance) has been suggested by several authors to provide information concerning the spatial and kinematic scaling of turbulence, the sonic Mach number, and the injection/dissipation scales (Ossenkopf 2002; Kowal et al. 2007; Burkhart et al. 2009; Federrath & Klessen 2013).

The power spectrum is defined as

\[
P(k) = \sum_{|k-k'|} A(k) \cdot A^*(k'),
\]

where \( k \) is the wavenumber and \( A(k) \) is the Fourier transform of the field under study. Studies of power spectra have played a crucial role for our modern understanding of MHD turbulence (see Goldreich & Sridhar 1995; Lazarian & Vishniac 1999; Cho, Lazarian & Vishniac 2002; Lithwick, Goldreich & Sridhar 2007; Cho & Lazarian 2003; Kowal, Lazarian & Beresnyak 2010). However, such studies have their limitations. Equation (1) demonstrates a critical limitation of the Fourier power spectrum, i.e., it contains only the information on amplitudes and disregards all of the phase information. This is problematic for studies of MHD turbulence because interactions among MHD waves can produce finite correlations of wave phases which are completely missed by the power spectrum. The coherence or randomness of phases in the MHD turbulence cascade is of critical importance for particle transport and our understanding of wave interactions. In light of this, several authors have suggested various techniques which extend beyond the power spectrum to include information on phases.

One such technique that preserves phase information and has been applied in the context of the ISM is the bispectrum. The bispectrum is closely related to the power spectrum. The Fourier transform of the second-order cumulant, i.e., the autocorrelation function, is the power spectrum while the Fourier transform of the third-order cumulant is known as the bispectrum. The bispectrum contains information on both amplitude and phase and has been applied to both MHD simulations (Burkhart et al. 2009; Cho & Lazarian 2009) and observations of neutral hydrogen (Burkhart et al. 2010). These studies found that the bispectrum is a sensitive diagnostic for the sonic and Alfvénic Mach numbers and can describe the behavior of nonlinear mode correlation across spatial scales.

We believe that the attempts to improve the original Goldreich & Sridhar (1995) theory by adding effects such as dynamical alignment (Boldyrev 2005, 2006), polarization (Beresnyak & Lazarian 2006), and non-locality (Gogoberidze 2007) have not resulted in improving the model, as the new simulations show that the original theory holds. We believe that the problems of the initial testing of the theory that initiated these attempts were related to the fact that the MHD turbulence is less local compared to its hydrodynamic counterpart (Beresnyak & Lazarian 2010). As a result, higher-resolution simulations are necessary to avoid the influence of the bottleneck effect on the spectrum.
Another technique to investigate phase information, used thus far in the context of the solar wind community, is the so-called phase coherence index (PCI, see Hada et al. 2003; Koga & Hada 2003; Koga et al. 2007; Chian et al. 2008, 2010). The PCI employs two surrogate data sets: one in which the phase information in an image or signal is randomized and another in which the phase information is perfectly correlated. These surrogate data share the same power spectrum with each other and the original data, however, they have different phases. A comparison of the original data and the surrogates provides us with insight into the level of phase coherence or randomness. PCI has been applied to solar wind observations and simulations, but until now there has not been an application to MHD turbulence in the context of the ISM or ICM.

In this paper, we investigate the application of the bispectrum and PCI to MHD simulations geared toward ISM observations. In principle, the starting point for the definition of both the bispectrum and the phase coherence technique is the Fourier transform, \( A(k) \). We can describe the power by computing \( P(k) \) and the phase distribution \( \phi(k) = \tan^{-1}(\text{Im}(A(k))/\text{Re}(A(k))) \).

In particular, we are interested in the physical processes that cause the nonlinear phases to be correlated or uncorrelated. That is, are phase techniques sensitive to the amplitude of turbulence fluctuations, the magnetic field strength, or the sonic Mach number? Most importantly, we seek to understand whether or not phase techniques could be useful for observations of ISM turbulence and focus our analysis on fluctuations in two-dimensional (2D) column density maps. This paper is organized as follows. In Section 2, we discuss the MHD simulations used for the study of the bispectrum and PCI. In Section 3, we discuss our results for a new averaging procedure to the 2D isocontours of the bispectrum. In Section 4, we present the first application of the PCI to simulations of ISM MHD turbulence. Finally, in Sections 5 and 6, we discuss our results and then our conclusions.

2. SIMULATIONS

We use a database of three-dimensional (3D) numerical simulations of isothermal compressible (MHD) turbulence with a resolution of 512\(^3\) presented in a number of past works (e.g., Kowal et al. 2007; Burkhart et al. 2009, 2013). We refer to these works for the details of the numerical set-up and here provide a short overview.

We use the isothermal MHD code detailed in Cho & Lazarian (2003) and vary the input values for the sonic \((M_s = v/c_s)\), where \(v\) is the flow velocity and \(c_s\) is the sound speed) and Alfvénic Mach number \((M_A = v/\lambda,\) where \(\lambda\) is the Alfvén speed). The code is a third-order-accurate ENO scheme which solves the ideal MHD equations in a periodic box with purely solenoidal driving. The magnetic field consists of the uniform background field and a turbulent field, i.e., \(B = B_{\text{sim}} + b\). Initially \(b = 0\).

In total, we have 14 simulations with a resolution of 512\(^3\). The simulations have sonic Mach numbers ranging from the range \(M_s \approx 0.5\)–10. There are two different magnetic field values used in this investigation: \(M_A \approx 0.7\) (sub-Alfvénic) and \(M_A \approx 2.0\) (super-Alfvénic). The simulations are all normalized to have mean values of unity in both density and column density.

3. BISPECTRUM

The bispectrum technique characterizes and searches for nonlinear interactions and departures from Gaussianity, which makes it a useful technique for studies of MHD turbulence in the context of the ISM and solar wind. This is because, as turbulence eddies evolve, they transfer energy from large scales to small scales generating a hierarchical turbulence cascade as \(k_1 + k_2 \) interact to form \(k_3\). For incompressible flows, under Kolmogorov’s assumptions, this can be expressed as \(k_1 \approx k_2 = k\) and \(k_3 \approx 2k\). Nonlinear wave–wave interactions take place more strongly in compressible and magnetized flows, and in that case we can have \(k_1 \neq k_2\). The utility of the bispectrum or other three-point statistics is that they can characterize nonlinear interactions in both Fourier amplitude and phase (see Barnett 2002; Masahiro & Bhuvnesh 2004).

The bispectrum can be defined as

\[
B(k_1, k_2) = \sum_{k_1-k_2} A(k_1) \cdot A(k_2) \cdot A^*(k_1 + k_2),
\]

where \(k_1\) and \(k_2\) are the wave numbers of two interacting waves, and \(A(k)\) is the original data with a finite number of elements with \(A^*(k)\) representing the complex conjugate of \(A(k)\). We refer to Burkhart et al. (2009) for more details about the numerical calculation of the bispectrum. The final result of our calculation is a 2D isocontour image of bispectral amplitudes as a function of the wave vectors \(k_1\) and \(k_2\) (all angular information in the bispectral triangles is averaged out).

The bispectrum of density and column density was studied in Burkhart et al. (2009) using 2D isocontour plots. Burkhart et al. (2009) found that simulations with higher sonic Mach number and an increased magnetic field produced a higher mode correlation across a larger range of scales. In particular, Burkhart et al. (2009) found that this result applied to both 3D density and observable 2D column density. Due to the bispectrum’s sensitivity to the sonic Mach number and magnetic field when applied to column density, Burkhart et al. (2010) performed a follow-up study on the HI column density map of the SMC in order to constrain the nonlinear interaction of turbulence in atomic hydrogen gas. Burkhart et al. (2010) also compared the bispectrum to other statistical methods for obtaining turbulence parameters.

The issues faced in the above-mentioned studies concerning the use of the bispectrum of column density maps for finding \(M_s\) and \(M_A\) is that the 2D isocontour maps of observations are difficult to compare to simulations. In this paper, one of our aims is to condense the information provided by the bispectral amplitudes into a more readily understandable one-dimensional (1D) form. In order to achieve this, we focus on two different averaging procedures in order to distill the information in the 2D isocontour images of the bispectral amplitudes into a 1D plot. We use both averaging along different annuli for a given \(R^2 = k_1^2 + k_2^2\) and angular averaging of all values along a line with a given angle \(\alpha\), as measured from the \(k_1\) axis.\(^4\) Figure 1 shows an example of both averaging procedures on a 2D isocontour map of the bispectrum amplitudes.

Figure 2 shows the averaged bispectral amplitudes of column density with LOS in the X-direction with an angular averaging of all values along a line with a given angle \(\alpha\). We do not find the averaging procedure to be sensitive to the LOS direction.

\(^4\) Note that it does not matter if we average with respect to the \(k_1\) or \(k_2\) axis since the bispectral amplitudes are symmetric about \(k_1 = k_2\).
Radial averaging averages all of the bispectral amplitude values for a given $R^2 = k_1^2 + k_2^2$ (example of red circular curve that intersects $k_1 = k_2 = 33$). Angular averaging of the bispectral amplitudes averages all of the bispectral amplitudes for a given angle ($\alpha$) as measured from zero (e.g., the three red radial lines shown as an example).

![Figure 1](image1.png)

**Figure 1.** Example of the two bispectral averaging procedures used in this paper. Radial averaging averages all of the bispectral amplitude values for a given $R^2 = k_1^2 + k_2^2$ (example of red circular curve that intersects $k_1 = k_2 = 33$). Angular averaging of the bispectral amplitudes averages all of the bispectral amplitudes for a given angle ($\alpha$) as measured from zero (e.g., the three red radial lines shown as an example).

Angular averaging of the bispectral amplitudes averages all of the bispectral amplitudes for a given angle ($\alpha$) as measured from zero (e.g., the three red radial lines shown as an example). As expected, past $\alpha = 45^\circ$ the averaged values of the isocountours are the same because the bispectrum is symmetric about the $k_1 = k_2$ axis. Furthermore, also as expected, the highest amplitude occurs at the averaging along $k_1 = k_2$ line, which is equivalent to setting $\alpha = 45^\circ$.

Burkhart et al. (2009) found increased bispectral amplitudes for simulations with higher sonic Mach number since these simulations depart strongly from Gaussian distributions. Figure 2 reflects this finding in a more compact way. The higher the sonic Mach number, the larger the bispectral amplitudes are, and therefore the higher the average is. This is true regardless of whether the simulations are super-Alfvénic or sub-Alfvénic, and we do not see a strong difference between the left and right panels of Figure 2 which show different magnetic field strengths. Thus, we can conclude that angular averaging of bispectral isocountour values along a given $\alpha$ is sensitive to the sonic Mach number regardless of the Magnetic field strength.

![Figure 2](image2.png)

**Figure 2.** Bispectral amplitudes of column density along the magnetic field (X-direction) which are radially averaged along different angles ($\alpha$). We use angular bin sizes of one degree and plot angles between $0^\circ$ and $90^\circ$.

and show only the direction parallel to the mean magnetic field. As expected, past $\alpha = 45^\circ$ the averaged values of the isocountours are the same because the bispectrum is symmetric about the $k_1 = k_2$ axis. Furthermore, also as expected, the highest amplitude occurs at the averaging along $k_1 = k_2$ line, which is equivalent to setting $\alpha = 45^\circ$.

Similar to averaging over different $\alpha$, averaging the bispectral amplitudes over different annuli shows a strong sensitivity to the sonic Mach number. Column density maps produced from simulations with larger sonic Mach number show higher values of the averaged bispectral amplitudes. However, comparing the left and right panels in Figure 3, it is clear that averaging along different annuli also shows a sensitivity to the magnetic field strength. Simulations with $M_A = 2.0$ show slightly larger averaged amplitudes as compared with simulations with $M_A = 0.7$ for a large range of different sonic Mach numbers. These findings persist regardless of the LOS direction and we do not find this diagnostic to be very sensitive to the LOS with respect to the mean magnetic field orientation, and hence we show only the LOS parallel to the mean field direction. Thus, we can conclude that averaging the bispectral amplitudes over different annuli is sensitive to both the sonic and Alfvénic Mach numbers.

It is particularly encouraging that both averaging procedures have slightly different sensitivities to the parameters of the turbulence. The annuli averaging is sensitive to both sonic and Alfvénic Mach numbers, while the angular averaging is sensitive only to the sonic Mach number. In both cases, the LOS does not seem to play a significant role in the overall amplitudes. These differences will allow researchers to break the degeneracy in the bispectrum’s sensitivity to multiple turbulence parameters.

4. THE PHASE COHERENCE

4.1. Phase Coherence Technique

The PCI was first introduced in Hada et al. (2003) and Koga & Hada (2003) in order to evaluate the degree of phase coherence among Fourier modes. In essence, this technique involves the construction of two surrogate data sets from an original N-dimensional data set. In particular, given an original sequence of data, henceforth denoted as ORG (e.g., time series data or fluctuations of density along a position axis), we can...
where $\langle \ldots \rangle$ denote the ensemble average, $X$ corresponds to the 2D coordinate in the intensity plane, and $R$ corresponds to the 2D lag between the correlating points. In other words, $L$ is the absolute value of the first-order structure function of the intensity field. For practical purposes, instead of taking the ensemble averaging, the spatial averaging should be applicable in the assumption of the homogeneity of the statistics. Then, averaging can be performed over the intensity distribution over the plane, i.e., $\langle |I(\mathbf{X} + \mathbf{R}) - I(\mathbf{X})| \rangle_X$, where the symbol $\langle \ldots \rangle_X$ denotes the spatial averaging. Equation (3) generalizes the path length, which was defined in Koga et al. (2008) for time series to two-dimensional data sets available from astrophysical observations.

MHD turbulence is anisotropic in terms of velocity and magnetic field (see Goldreich & Sridhar 1995; Cho & Lazarian 2003). The anisotropies are also present for the density field, but their value may be reduced for supersonic turbulence (see Cho & Lazarian 2003; Beresnyak et al. 2005; Kowal, Lazarian & Beresnyak 2007). In this paper, where we provide the first exploration of this technique, we will not consider the turbulence anisotropy. Therefore, the lag in our case is calculated as

$$L(R) = \langle |I(\sqrt{x^2 + R^2}) - I(X)| \rangle_X. \quad (4)$$

When the phases of a given data set are correlated, the path length will be smaller than in situations where the phases are randomized. Because of this, the PCI can be defined as

$$C(\phi)(R) = \frac{L_{\text{PRS}}(R) - L_{\text{ORG}}(R)}{L_{\text{PRS}}(R) - L_{\text{PCS}}(R)} \quad (5)$$

and provides an evaluation of the degree of coherence in the phases. If the original data have randomized phases, then $C(\phi)$ should be roughly zero. If $C(\phi)$ is close to unity, then this indicates that the phases are nearly completely correlated.

In practice, we averaged together five random Gaussian realizations to construct the phases of the PRS data set. Along with the original data (ORG), we use one realization of the PCS (all phases are set to unity). We calculate the first-order structure function of the ORG, PRS, and PCS data sets (i.e., we apply Equation (3)) and then compute the phase coherence given in Equation (5). A schematic of the ORG, PCS, and PRS data sets is shown in Figure 4.

4.2. The Phase Coherence of Density

For the 3D density, the definition of lag can be easily generalized:

$$L(r) = \langle |n(\sqrt{x^2 + r^2}) - n(x)| \rangle_x, \quad (6)$$

where the density $n$ is averaged over the volume with coordinates $x$ for the given 3D distance between the points.

We plot the phase coherence, $C(\phi)(r)$, versus $r$, of the density in Figure 5. The left panel shows simulations with sub-Alfvénic turbulence, while the right panel shows simulations with super-Alfvénic turbulence. Different sonic Mach number runs are denoted with different colors and line styles.

The phase coherence is peaked at unity for simulations with highly correlated phases and approaches zero when the phases are random. The largest values of the phase coherence occur at the smallest lag values, and decrease and eventually saturate at higher lag values. The saturation occurs roughly at lag $= 40$ for

![Figure 4.](https://example.com/fig4.png)
all of the simulated boxes with smaller values than this being well within the dissipation range of our simulations. The peak of the PCI occurs roughly on the shock scale of a few pixels in length. In the inertial range of the simulations, the phase coherence is roughly constant across different lag values. This is in contrast to the power spectrum (or the analogous structure function), which decrease as a power law in the inertial range but lacks any phase information.

As in other phase analysis techniques, such as the bispectrum, $C_{\phi}(r)$ has a strong dependence on the sonic Mach number of the simulation. Subsonic simulations (shown with black dotted lines) have phase distributions that are closer to random ($C_{\phi}(r) \approx 0.35$). This is not surprising since subsonic turbulence has many statistical features in common with a Gaussian distribution, i.e., low bispectral amplitudes (see Burkhart et al. 2009), similar topological features (see Chepurnov et al. 2008), and a lognormal PDF (see Kowal et al. 2007). As the sonic Mach number increases, the phase coherence also increases. In the inertial range, there is nearly no overlap between different MHD runs for different sonic Mach numbers (for the same $M_0$), making the phase coherence a sensitive diagnostic of the sonic Mach number.

A comparison between the left and right panels of Figure 5 also shows that, like the bispectrum, the phase coherence is also sensitive to the magnetic field information present in the data. Simulations with a higher mean magnetic field (i.e., sub-Alfvénic simulations) show enhanced values of $C_{\phi}$. This suggests that the phase coherence might also be used to assess the Alfvénic nature of the gas when the sonic Mach number is known.

To demonstrate the above point, we average the values of $C_{\phi}$ over lag values from 40 to 100 and plot the average $C_{\phi}$ versus sonic Mach number of the simulation in Figure 6. The average phase coherence increases with increasing sonic Mach number and levels off near $C_{\phi}(\tau) = 1$ for Mach numbers greater than 8. This suggests that the phase coherence will be a sensitive diagnostic of sonic Mach number out to $M_s \approx 8$, however, for larger sonic Mach numbers, the phase coherence approaches unity. This is similar to the findings of Koga et al. (2008), who found higher phase coherence values at Earth’s bow shock. Additionally, there is a slight degeneracy with the strength of the magnetic field, which becomes even more apparent when averaging the phase coherence. A stronger magnetic field produces density fluctuations with more correlated phases.

### 4.3. The Phase Coherence of Column Density

We apply the phase coherence to 2D images of column density maps created from the 3D density distributions in order to test the applicability of the PCI for observations of the ISM. We average the values of $C_{\phi}$ over lag values from 40 to 100. Super-Alfvénic simulations are denoted with red stars while sub-Alfvénic simulations are denoted with black plus symbols.

Figure 5. Phase coherence $C_{\phi}(r)$ of density vs. $r$. The left panel shows simulations with sub-Alfvénic turbulence while the right panel shows simulations with super-Alfvénic turbulence. Different sonic Mach number runs are denoted with different colors and line styles. The color scheme is the same as in Figure 2, where the yellow, magenta, orange, and green lines show supersonic simulations, red and blue lines show transsonic simulations, and the black line shows our subsonic run.

Figure 6. Averaged values of $C_{\phi}$ vs. the sonic Mach number for 3D density fields. We average values of $C_{\phi}$ over lag values from 40 to 100. Super-Alfvénic simulations are denoted with red stars while sub-Alfvénic simulations are denoted with black plus symbols.
all three LOS orientations, the lack of sensitivity in the column density PCI to magnetic field and LOS become even more apparent. Nevertheless, the PCI of the column density is still sensitive to the overall sonic Mach number of the gas and the shock scale, making this technique useful for determining these ISM turbulence parameters from column density maps.

The procedure that we applied for the column density is directly applicable to observed intensities. Similar to the procedures above, one can use the intensity distribution, take its Fourier spectrum, and randomize/correlate the phases for the Fourier components. The output of such studies would be the dependencies of $C_{\phi}$, which reveal the properties of the underlying MHD turbulence. As this technique uses phase information, its contribution is complementary to the studies of MHD turbulence using the power spectra of PDF information (see Burkhart et al. 2009).

5. DISCUSSION

Phase information is usually not used in studies of turbulence, which generally focus on the Fourier power spectrum, i.e., the Fourier amplitudes. Studies of power spectra resulted in substantial progress of our understanding of interstellar turbulence. For instance, the Big Power Law in the Sky (Armstrong et al. 1995; Chepurnov & Lazarian 2010), as well as studies of random densities (see Elmegreen & Scalo 2004, for a review) and velocities (see Lazarian 2009 for a review; Chepurnov et al. 2015), provided convincing evidence for the existence and importance of turbulence in the interstellar media.

Nevertheless, power spectra as well as their real-space counterparts, namely, structure functions and correlation functions (see Monin & Yaglom 1972), cannot provide a full description of the turbulent field. This led to extensive studies of alternative measures of turbulence. Probability distribution functions of column densities, including different measures obtained with them, e.g., skewness and kurtosis (Burkhart et al. 2009, 2010), Tsallis statistics (Esquivel & Lazarian 2010; Toffelmire, Burkhart & Lazarian 2011), and dispersion (Burkhart & Lazarian 2012), were considered as tools to obtain the properties of turbulence from observations. Together with the techniques for the anisotropy of turbulence studies (Lazarian et al. 2002; Esquivel & Lazarian 2011; Heyer et al. 2008; Burkhart et al. 2014) and intermittency studies (Padoan et al. 2004; Kowal et al. 2007), they present an impressive toolbox for quantitative studies of interstellar turbulence.

At the same time, it is known that nonlinear interactions among MHD waves are likely to produce a finite correlation of the wave phases, and therefore the phases should be studied. Recently, several studies have promoted the study of phase information for ISM MHD turbulence. These include three-point statistics, such as the bispectrum studied here and in Burkhart et al. (2009, 2010). Furthermore, it was recently shown in Correia et al. (2016) that the principle component analysis (PCA; see Heyer & Schloerb 1997; Heyer et al. 2008) is sensitive to the phase information. Intermittency studies also utilize phase information (see Cho & Lazarian 2003; Kowal et al. 2007).

In spite of the aforementioned studies, this is the first paper, as far as we are aware of, which is entirely focused on using information about phases to provide simple measures that can be used to study interstellar turbulence. For instance, the bispectrum is a rich measure which presents a two-dimensional distribution. In this paper, we condensed the information provided by the bispectrum by presenting two simple functions, which describe the radial and azimuthally averaged measures of bispectrum. In addition, we presented the measures of the PCI, which can provide another way to characterize the phase information related to turbulence. We find that in shock-dominated turbulence, the PCI approaches unity, which is in agreement with solar wind studies such as that of Koga et al. (2008). This is equally true for the bispectrum, where the correlation of wave modes increases for supersonic, highly magnetized turbulence, as first discussed in Burkhart et al. (2009) for the ISM.

Our present paper continues the trend of bringing statistical techniques that were developed in other areas into studies of
interstellar turbulence. For instance, PCI, similar to Tsallis statistics, was first used for solar wind studies (Burlaga et al. 2006, 2007, 2009). This is the case for the PCI. At the same time, the analysis of the bispectrum is our own attempt to reduce the dimensionality of the bispectrum output. We strongly advocate the viewpoint that these diagnostics are not to be used in isolation, but rather multiple diagnostic tools should be used together (e.g., the power spectrum, bispectrum, Tsallis, PCI, PCA, etc.) in order to discern the parameters of magnetized turbulence in a reliable way.

In the exploratory study above, we have limited our studies to obtaining the sonic and Alfvén Mach numbers of turbulent flows. We expect that the phase information can be even more important as a means of statistical diagnostics of self-gravitating turbulence in molecular clouds (see Burkhart et al. 2015 as an example of such studies). Additionally, there is the question of obtaining the magnetic field orientation via statistical diagnostics. Our studies suggest that the bispectrum, as defined in the present work as well as in Burkhart et al. (2009), which averages over all $k_1$ and $k_2$, is insensitive to the orientation of the magnetic field. This is because we are averaging over all of the bispectrum triangle configurations. In a future work, we plan to explore definitions of the bispectrum (e.g., Slepian & Eisenstein 2016) which preserve angle information in the triangles, which may better illuminate the preferred directions. Additionally, an angular dependency on PCI would be an interesting extension of the technique, especially in the context of a calculation of PCI parallel and perpendicular to the plane of sky magnetic field (from polarization observations), similar to the 3D structure function analysis of Cho & Lazarian (2003).

With the advent of new telescopes and precision measurements, we expect many of the interstellar turbulence techniques to be used for studying turbulence in galaxy clusters. Additionally, our suggestion of bispectrum averaging may be useful for studying turbulence beyond the interstellar turbulence domain and could also be used for cosmological studies which already employ the bispectrum.

6. CONCLUSIONS

We investigated the utility of two statistical diagnostics for recovering information on the Fourier phases in the ISM using a set of simulations of MHD turbulence with a larger range of sonic and Alfvénic Mach numbers. In particular, we focused our study on a new averaging procedure for the bispectrum isocontour amplitudes in order to distill the information in the isocontours into a 1D form. We also introduce the PCI, which is a new technique for studies of the density fluctuations in the ISM.

We find the following.

1. The bispectrum averaged along different angles with respect to either the $k_1$ or $k_2$ axis is primarily sensitive to the sonic Mach number.
2. Averaging the bispectral amplitudes over different annuli is sensitive to both the sonic and Alfvénic Mach numbers.
3. Higher sonic Mach number and larger magnetic field produce density structures which have more correlated phase behaviors compared to a random Gaussian distribution of phases.
4. We find that in shock-dominated turbulence, the PCI approaches unity, in agreement with solar wind studies.
5. The PCI of density is sensitive to both the sonic and Alfvénic Mach numbers. However, when applied to column density maps, the PCI is sensitive only to the sonic Mach number. The peak amplitude of the PCI indicates the shock scale in supersonic turbulence.

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REFERENCES

Armstrong, J. W., Rickett, B. J., & Spangler, S. R. 1995, ApJ, 443, 209
Barnett, A. G. 2002, PhD thesis, The Univ. of Queensland
Beresnyak, A., & Lazarian, A. 2010, ApJL, 722, L110
Beresnyak, A., & Lazarian, A. 2006, ApJL, 640, L175
Beresnyak, A., Lazarian, A., & Cho, J. 2005, ApJL, 624, L93
Burkhart, B., Collins, D. C., & Lazarian, A. 2015, ApJ, 808, 48
Burkhart, B., Falceta-Gonçalves, D., Kowal, G., & Lazarian, A. 2009, ApJ, 693, 250
Burkhart, B., & Lazarian, A. 2012, ApJL, 755, L19
Burkhart, B., Lazarian, A., Leão, I. C., de Medeiros, J. R., & Esquivel, A. 2014, ApJ, 790, 130
Burkhart, B., Ossenkopf, V., Lazarian, A., & Stutzki. J. 2013, ApJ, 771, 122
Burkhart, B., Stanimirović, S., Lazarian, A., & Kowal, G. 2010, ApJ, 708, 1204
Boldyrev, S. 2005, ApJL, 626, L37
Boldyrev, S. 2006, PhRvL, 96, 115002
Burlaga, L. F., Ness, N. F., & Acuña, M. H. 2006, ApJ, 642, 584
Burlaga, L. F., Ness, N. F., & Acuña, M. H. 2007, APT, 668, 1246
Burlaga, L. F., Ness, N. F., & Acuña, M. H. 2009, ApJL, 691, L82
Chepurnov, A., Burkhart, B., Lazarian, A., & Stanimirović, S. 2015, ApJ, 810, 33
Chepurnov, A., Gordon, J., Lazarian, A., & Stanimirović, S. 2008, ApJ, 688, 1021
Chepurnov, A., & Lazarian, A. 2010, ApJ, 710, 853
Chian, A. C., Miranda, R. A., Koga, D., et al. 2008, NPGeo, 15, 567
Chian, A. C., Miranda, R. A., Rempel, E. L., Saiši, Y., & Yamada, M. 2010, PhRvL, 104, 254102
Cho, J., & Lazarian, A. 2003, MNRAS, 345, 325
Cho, J., & Lazarian, A. 2009, ApJL, 701, 236
Cho, J., Lazarian, A., & Vishniac, E. T. 2002, ApJ, 564, 291
Correia, C., Lazarian, A., Burkhart, B., Pogosyan, D., & De Medeiros, J. R. 2016, ApJ, 818, 118
Elmegreen, B. G., & Scalo, J. 2004, ARA&A, 42, 211
Esquivel, A., & Lazarian, A. 2005, ApJL, 631, 320
Esquivel, A., & Lazarian, A. 2010, ApJL, 710, 125
Esquivel, A., & Lazarian, A. 2011, ApJL, 740, 117
Federrath, C., & Klessen, R. 2013, ApJL, 763, 51
Gogoberidze, G. 2007, PhPl, 14, 022304
Goldreich, P., & Sridhar, S. 1995, ApJL, 438, 763
Hada, T., Koga, D., & Yamamoto, E. 2003, SSRv, 107, 473
Heyer, M., Gong, H., Ostriker, E., & Brunt, C. 2008, ApJ, 680, 420
Heyer, M. H., & Peter Schloerb, F. 1997, ApJ, 475, 173
Karga, D., & Hada, T. 2003, SSRv, 107, 495
Koga, D., Chian, A. C., Miranda, R. A., & Rempel, E. L. 2007, PhRvL, 75, 046401
Koga, D., Chian, A. C., Hada, T., & Rempel, E. L. 2008, RSTA, 366, 1864
Kowal, G., Lazarian, A., & Beresnyak, A. 2007, ApJ, 658, 423
Lazarian, A. 2009, SSRv, 143, 357
Lazarian, A., Pogosyan, D., & Esquivel, A. 2002, in ASP Conf. Proc., Seeing Through the Dust: the Detection of HI and the Exploration of the ISM in Galaxies (San Francisco, CA: ASP), 276
Lazarian, A., & Vishniac, E. T. 1999, ApJL, 517, 700
Lithwick, Y., Goldreich, P., & Sridhar, S. 2007, ApJL, 655, 269
Masahiro, T., & Bhuvnesh, J. 2004, MNRAS, 348, 897
Ossenkopf, V. 2002, A&A, 391, 295
Padoan, P., Jimenez, R., Nordlund, Å, & Boldyrev, S. 2004, PhRvL, 92, 191102
Slepian, Z., & Eisenstein, D. 2016, MNRAS, 455, 31
Tofflemire, B. M., Burkhart, B., & Lazarian, A. 2011, ApJ, 736, 60