Design and Simulation of Double-loop Sliding Mode Controller for Missile Attitude

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ABSTRACT

Sliding mode control has the advantages of simple controller design, short control time, strong anti-interference ability and easy physical implementation, so it is widely used in missile control. Combining the attitude dynamics and attitude kinematics of the missile, this paper designs two missile attitude control methods by using sliding mode control. The two controllers using the two-loop sliding mode control scheme can well track the expected inputs of the angular velocity and angle. And the controllers can work normally under small disturbance torque. So they have good traceability performance and robustness. In this paper, the missile attitude simulation is used to verify the accuracy, track performance and robustness of the two controllers, and compares the control effects of the two control methods.

KEYWORDS
Missile, Attitude control, Double-loop sliding mode control, Track performance, Robustness.

INTRODUCTION

In the study of linear system theory and linear control system, all the designed controllers work according to the linear control law. Linear control laws may not give full play to the potential of controllers and actuators. In order to achieve the optimal system performance under certain criteria, nonlinear system control is often adopted. For example, in the optimal control, the shortest time control adopts Bang-Bang control. According to certain logic switching rules, this control mode switches the control quantity between the positive value and the negative maximum, which is conducive to maximizing the ability of controller and eliminating errors of system.

The controller of the variable-structure Control system is composed of several different continuous subsystems, and each subsystem has different parameters or different structures. The system switches among these subsystems according to some function rules in the working process to improve the dynamic performance of the system. The structure of sliding mode control (SMC) system is not fixed, and it can changes purposefully to force the system to follow the predetermined state track of sliding mode in the dynamic process, according to the current state of the system (each deviation and its derivatives, etc.).

Sliding mode control system has strong robustness. And it is a parameter-switch-
ing feedback control system. The system can slide on certain sliding curves and surfaces according to the switching conditions, so it is not sensitive to the changes of internal parameters and external disturbance. The system has good robustness and do not require online identification. Meantime, its physical implementation is simple[1].

In this paper, two kinds double-loop sliding mode controllers of missile attitude are designed by sliding mode control theory. Both sliding mode controllers can be represented by the following control block diagram.

DYNAMICS AND KINEMATICS OF MISSILE ATTITUDE

ATTITUDE DYNAMICS

In the missile body coordinate system, the attitude dynamics equation of the missile is as follows[2]:

\[
\begin{align*}
I_x \frac{d\omega_x}{dt} + (I_y - I_z)\omega_y \omega_z &= M_x \\
I_y \frac{d\omega_y}{dt} + (I_z - I_x)\omega_z \omega_x &= M_y \\
I_z \frac{d\omega_z}{dt} + (I_x - I_y)\omega_x \omega_y &= M_z
\end{align*}
\] (1)

Here, \(\omega_x, \omega_y, \omega_z\) are the angular velocity in the missile body coordinate system, \(M_x, M_y, M_z\) are the components of the external force acting on the centroid of missile in the missile body coordinate system, \(I_x, I_y, I_z\) are the three axial components of rotate inertia in the missile body coordinate system.

The simplified aerodynamic torque equation is as follows:

\[
\begin{align*}
M_x &= qS\ell(m_u \omega_x + m_z^0 \delta_z) \\
M_y &= qS\ell(m_u^0 \omega_y + m_z^0 \delta_z) \\
M_z &= qS\ell(m_u \omega_z + m_z^0 \delta_z)
\end{align*}
\] (2)
\( \delta, \phi, \psi, \gamma \) are the angle of rolling rudder angle, yaw rudder angle, pitch rudder angle of missile; \( m^q, m^\phi, m^\psi \) are the handling efficiency of rolling rudder, yaw rudder, pitch rudder; \( m^\phi, m^\psi, m^\gamma \) are the rolling damping torque coefficient, the yaw damping torque coefficient and rolling damping torque coefficient.

**ATTITUDE KINEMATICS**

In the ground coordinate system, the attitude kinematics equation of the missile is as follow:

\[
\begin{align*}
\frac{d\vartheta}{dt} &= \omega_\gamma \sin \gamma + \omega_\varphi \cos \gamma \\
\frac{d\psi}{dt} &= \omega_\gamma \cos \gamma - \omega_\varphi \sin \gamma \\
\frac{d\gamma}{dt} &= \omega_z - \tan \vartheta (\omega_\gamma \cos \gamma - \omega_\varphi \sin \gamma)
\end{align*}
\]

Here, \( \vartheta, \psi, \gamma \) are the pitch angle, the yaw angle and the rolling angle.

**DESIGN SLIDING MODE CONTROLLER**

To design a sliding mode variable structure controller, we first choose a suitable switching functions \( x \), so that the determined sliding mode is progressively stable and has good dynamic quality. Secondly, the sliding mode control law \( u(x) \) is designed to satisfy the arrival condition, thereby forming a sliding mode region on the switching surface [3-6]. The sliding mode controllers in this paper are as follows:

**ORDINARY SLIDING MODE CONTROLLER**

**OUTER-LOOP SLIDING MODE CONTROLLER**

To order to design the outer-loop sliding mode controller, the angular velocity vector \( \omega \) in the equation (3) is regarded as the virtual control input \( \omega_c \) to complete the progressive tracking of the desired angular velocity \( \varphi_c \), and then angular velocity command \( \omega_c \) is determined by the inner-loop controller.

Select the sliding mode switching surface of outer-loop controller:

\[
\begin{align*}
s_1 &= \vartheta - \vartheta_e = \vartheta_e \\
s_2 &= \psi - \psi_e = \psi_e \\
s_3 &= \gamma - \gamma_e = \gamma_e
\end{align*}
\]

Choose the sliding mode approach law:

\[
\begin{align*}
\dot{s}_1 &= -\rho_1 \text{sign}(s_1) \\
\dot{s}_2 &= -\rho_2 \text{sign}(s_2) \\
\dot{s}_3 &= -\rho_3 \text{sign}(s_3)
\end{align*}
\]


\( \rho_1, \rho_2, \rho_3 \) are proportional gain factors.

From equation (3):

\[
\begin{align*}
\omega_x &= \dot{y} + \tan \theta \cos \vartheta \dot{\psi} \\
\omega_y &= \dot{\vartheta} \sin \gamma + \dot{\psi} \cos \vartheta \cos \gamma \\
\omega_z &= \dot{\vartheta} \cos \gamma - \dot{\psi} \cos \vartheta \sin \gamma
\end{align*}
\]

Combine equations (4) and (5):

\[
\begin{align*}
\dot{\vartheta} &= \dot{\vartheta}_c - \rho_1 \text{sign}(s_1) \\
\dot{\psi} &= \dot{\psi}_c - \rho_2 \text{sign}(s_2) \\
\dot{\gamma} &= \dot{\gamma}_c - \rho_3 \text{sign}(s_3)
\end{align*}
\]

Combine equations (6) and (7) to obtain \( \omega_x, \omega_y, \omega_z \).

Take \( \omega_{uc} = \omega_x, \omega_{yc} = \omega_y, \omega_{zc} = \omega_z \) as the angular velocity command of the outer-loop to the inner-loop.

In order to avoid the chattering phenomenon of angular velocity command, an outer-loop controller with a boundary layer is applied to approximate the continuous sliding mode. At the same time, in order to guarantee the convergence of the sliding mode surface in finite time, the discontinuous term \( \text{sign}(s) \) is replaced by the continuous saturation function term \( \text{sat}(s) \).

**INNER-LOOP SLIDING MODE CONTROLLER**

After the outer-loop determines \( \omega_c \), the inner-loop controller is used to achieve tracking of \( \omega_c \).

Select the sliding mode switching surface of inner-loop controller:

\[
\begin{align*}
s_4 &= \omega_x - \omega_{uc} \\
s_5 &= \omega_y - \omega_{yc} \\
s_6 &= \omega_z - \omega_{zc}
\end{align*}
\]

Choose the sliding mode approach law:

\[
\begin{align*}
\dot{s}_4 &= -\rho_4 \text{sign}(s_4) \\
\dot{s}_5 &= -\rho_5 \text{sign}(s_5) \\
\dot{s}_6 &= -\rho_6 \text{sign}(s_6)
\end{align*}
\]

Here, \( \rho_4, \rho_5, \rho_6 \) are proportional gain factors.

The simultaneous equation (8) and (9) has:
\[
\begin{align*}
\dot{\omega}_x &= \dot{\omega}_x - \rho_x \text{sign}(s_x) \\
\dot{\omega}_y &= \dot{\omega}_y - \rho_y \text{sign}(s_y) \\
\dot{\omega}_z &= \dot{\omega}_z - \rho_z \text{sign}(s_z)
\end{align*}
\]  
(10)

From equation (1) and (2):

\[
\begin{align*}
\delta_x &= \frac{1}{q_{\text{Slm}}} [ -q_{\text{Slm}} \cdot \omega_x + (I_x - I_y) \omega_y + I_Y \omega_z ] \\
\delta_y &= \frac{1}{q_{\text{Slm}}} [ -q_{\text{Slm}} \cdot \omega_y + (I_y - I_z) \omega_z + I_Z \omega_x ] \\
\delta_z &= \frac{1}{q_{\text{Slm}}} [ -q_{\text{Slm}} \cdot \omega_z + (I_z - I_x) \omega_x + I_X \omega_y ]
\end{align*}
\]  
(11)

The simultaneous equation (10) and (11) has: \( \delta_x, \delta_y, \delta_z \).

Take \( \delta_z = \delta_3 \) as attitude rudder command.

In order to avoid the chattering phenomenon of control torque command, the inner-loop controller with a boundary layer is also applied to approximate the continuous sliding mode. At the same time, the discontinuous term \( \text{sign}(s) \) is also replaced by the continuous saturation function term \( \text{sat}(s) \).

**SLIDING MODE CONTROLLER WITH INTEGRAL TERM**

The second sliding mode controller is similar to the first controller. They are different at sliding mode switching surface. So only the sliding mode switching surface and approach law of the inner and outer loop controllers are given below.

**OUTER-LOOP SLIDING MODE CONTROLLER**

Assume \( \Phi = \begin{bmatrix} \vartheta \\ \gamma' \\ \gamma \end{bmatrix} \), \( \Phi_c = \Phi_c - \Phi \).

Select the sliding mode switching surface of outer-loop controller:

\[
S_i = \Phi_c + C_i \int_0^t \Phi_c(\tau)
\]  
(12)

Choose the sliding mode approach law:

\[
\dot{S}_i = -K_i S_i - Q_i \text{sat}(S_i)
\]  
(13)

Here, \( C_i, K_i, Q_i \) are three-dimensional orthogonal and diagonal matrix.

The simultaneous equation (12) and (13) has:

\[
\dot{\Phi} = \Phi_c + C_i \Phi_c + K_i S_i + Q_i \text{sat}(S_i)
\]  
(14)
The rest of the formula derivations are the same as the ordinary outer-loop sliding mode controller. 

**INNER-LOOP SLIDING MODE CONTROLLER**

\[
\Omega = \begin{bmatrix}
\omega_x \\
\omega_y \\
\omega_z 
\end{bmatrix}, \quad \Omega_r = \Omega - \Omega_c
\]

Assume \( \Omega_r = \Omega - \Omega_c \).

Select the sliding mode switching surface of inner-loop controller:

\[
S_2 = \Omega_r + C_z^T \Omega_r(t)
\]  
(15)

Choose the sliding mode approach law:

\[
\dot{S}_2 = -K_2 S_2 - Q_z \text{sat}(S_2)
\]  
(16)

Here, \( C_z, K_2, Q_z \) are three-dimensional orthogonal and diagonal matrix.

The simultaneous equation (15) and (16) has:

\[
\Omega = \Omega_r + C_z \Omega_r + K_2 S_2 + Q_z \text{sat}(S_2)
\]  
(17)

The rest of the formula derivations are the same as the ordinary inner-loop sliding mode controller.

**ESTABLISHING MISSILE ATTITUDE CONTROL SIMULATION MODELS**

Taking a certain type of missile as the research object, the simulation model is built by Simulink. The model data of missile can be found in the simulation model.

**SIMULATION MODEL OF ORDINARY SLIDING MODE CONTROLLER**

The simulation model of ordinary sliding mode controller has been shown below. Meantime, the structure diagram of the inner-loop and outer-loop controller is also given below.

![Simulation diagram of the sliding mode controller.](image-url)
The simulation model of sliding mode controller with integral term is similar to ordinary sliding mode controller. So we only give the structure diagram of the inner-loop and outer-loop controller.
SIMULATION RESULTS OF MISSILE ATTITUDE CONTROL

According to the simulation model built in Simulink, the control simulation results of two kinds of sliding mode controllers are obtained and compared. In the simulation, the step signal is used to investigate the control quality of the controller, the sinusoidal signal is used to investigate the track performance of the model output, and the external disturbance torque is added to investigate the robustness of the controller. The disturbance torque:

\[ M_d = [10\sin\pi, 10\sin\pi, 10\sin\pi] N\cdot m \]

SIMULATION RESULTS OF ORDINARY SLIDING MODE CONTROLLER

In the ordinary sliding mode controller, the desired attitude angle command is a step signal. At the same time, the equivalent attitude angle of the rudder is limited to the range of [-20, 20], and the external disturbance torque is added to the controller. The control parameters are adjusted multiple times to obtain the following simulation results.

Figure 7. Attitude angle of the ordinary sliding mode controller in step input.

Figure 8. Attitude angular velocity of the ordinary sliding mode controller in step input.

Figure 9. Angle of the rudder of the ordinary sliding mode controller in step input.
Take the sinusoidal signal as the desired attitude angle command. The control parameters are adjusted some times to obtain the following simulation results.

![Figure 10. Attitude angle of the ordinary sliding mode controller in sinusoidal input.](image)

**SIMULATION RESULTS OF SLIDING MODE CONTROLLER WITH INTEGRAL TERM**

Under the same conditions as ordinary sliding mode controller, the parameters of the sliding mode controller with integral term are adjusted multiple times to obtain the following simulation results.

![Figure 11. Attitude angle of the sliding mode controller with integral term in step input.](image)

![Figure 12. Attitude angular velocity of the sliding mode controller with integral term in step input.](image)
Take the sinusoidal signal as the desired attitude angle command. The control parameters are adjusted some times to obtain the following simulation results.

CONCLUSION

From the simulation results of the sinusoidal input in the previous section, it can be seen that both control systems have good output tracking performance and strong robustness. However, it can be seen from the simulation results of the step signal input that the ordinary sliding mode controller has a shorter adjustment time, but it has an overshoot, and the sliding mode controller with integral term has a longer adjustment time, no overshoot, and a smooth transition. Considering that the missile attitude will not be abrupt, and the angle of the rudder is limited, the above two control methods can be used as the control law of the missile attitude.

REFERENCES

1. X. X. Hu, L. G. Wu, and G. H. Hu, “Adaptive sliding mode tracking control for a flexible air-breathing hypersonic vehicle,” Journal of the Franklin Institute, 349(2), pp. 559-577, 2012.
2. X. G. Li and Q. Fang, “Flight dynamics of winged missiles,” Northwestern Polytechnical University Press, 2005.(in Chinese)
3. F. K. Yeh, “Sliding-mode-based contour-following controller for guidante and autopilot systems of launch vehicles,” Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering, 227(G2), pp.285-302, 2013.
4. H. B. Sun, S. H. Li and C. Y. Sun, “Robust adaptive integral-sliding-mode fault-tolerant control for airbreathing hypersonic vehicles,” Proceedings of the Institution of
Mechanical Engineers, Part I: Journal of Systems and Control Engineering, 226(10), pp. 1344-1355, 2012.
5. Y. Deng, X. D. Liu and Y. J. Wu, “Analysis of the maximum available rudder deflection of ballistic missile in reentry,” Control Engineering, 21(5), pp. 648-652, 2014. (in Chinese)
6. J. Jia and Q. Jing, “Reentry attitude double-loop SMC of the spacecraft and its logic selection,” Space Control, 24(3), pp. 25-28, 2006. (in Chinese)