The calculation of the river water environmental carrying capacity based on the blind number

Sudi GAO1, Yuan MENG2, Yueying LUO1
1School of Civil and Environmental Engineering, Anhui xinhua university, Hefei, China
2Architecture Engineering Institute, Jinhua polytechnoc, Jinhua, China
Email: gaosudi@163.com, mengyuan@163.com

Abstract: The water environmental carrying capacity is a basic task for water environmental management. Based on the characteristics of uncertainty of water environmental system and the lack of water environmental information, the parameters of river water environmental system are defined as blind numbers. A fuzzy model is established through fuzzifying the parameters to calculate the river water environmental carrying capacity. The blind values of river water environmental carrying capacity are derived by using the model proposed. Study results show that the new model based on the blind number is more scientific and reasonable.

1. Introduction
At present, there are many studies on water environmental capacity. The deterministic model is simple and easy to operate, so it can be widely used. But it does not pay enough attention to the dynamic and uncertainty of the water environment system, so it is difficult to scientifically and accurately reflect the level of the real capacity of the river water. The calculation of river water environmental capacity from the perspective of uncertainty has become the trend of water pollution capacity research[1-3]. The river water environment system is a large system with fuzzy and inaccurate characteristics, so the fuzzy set theory can be used to study the river water environment problem[4,5]. In recent years, some people have tried to use fuzzy technology to simulate and characterize the randomness, fuzziness and inaccuracy of the system or things[5-7]. The blind number theory is used in the study of river water environmental capacity. On the basis of defining the number of blind parameters of water environment system, a blind number model of river water environmental capacity calculation is set up to study the water capacity of water[8-10].

2. Blind number operation[8]

2.1 Blind number
Set up $a_i \in g(I), \alpha_i \in [0,1], i=1,2,...,n$, $f(x)$ is a grey function defined on $g(I)$, and

$$f(x) = \begin{cases} \alpha_i, & x=a_i (i=1,2,...,n) \\ 0, & others \end{cases}$$

(1)

If $i\neq j$, $a_i \neq a_j$ and $\sum_{i=1}^{n} \alpha_i = \alpha \leq 1$, then the function $f(x)$ is a blind number. $\alpha_i$ is the credibility of $f(x)$ and $\alpha$ is the total credibility of $f(x)$.
2.2 The arithmetic of blind numbers

We can suppose that the number of blind numbers \( A \) and \( B \) is the following formula:

\[
A = f(x) = \begin{cases} 
  \alpha_i, & x = x_i (i = 1, 2, \cdots, m) \\
  0, & \text{others} 
\end{cases} 
\]

(2)

\[
B = f(y) = \begin{cases} 
  \beta_i, & y = y_i (i = 1, 2, \cdots, n) \\
  0, & \text{others} 
\end{cases} 
\]

(3)

The operation of blind numbers consists of two parts, one is the operation of probable values, the other is the operation of credibility.

1. Table 1 is called the potential band edge matrix of \( A \) and \( B \), \( x_1, x_2, \ldots, x_n \) and \( y_1, y_2, \ldots, y_n \) are the sequence of possible values of \( A \) and \( B \) respectively. The two lines perpendicular to each other are called the longitudinal axis and the transverse axis. The \( * \) in the table can represent a \( +, -, \times \), or an operator.

2. Table 2 is called the confidence band edge product matrix of \( A \) about \( B \), \( \alpha_1, \alpha_2, \ldots, \alpha_n \) and \( \beta_1, \beta_2, \ldots, \beta_n \) are the reliability sequences of \( A \) and \( B \). Two lines perpendicular to each other are called the longitudinal axis and the transverse axis. The first quadrant elements constitute the m * n matrix, which is called the credibility product matrix of \( A \) and \( B \).

| x_1 | x_1 \cdot y_1 | \ldots | x_1 \cdot y_i | \ldots | x_1 \cdot y_n |
|-----|----------------|------|----------------|------|----------------|
| \cdots | \cdots | \cdots | \cdots | \cdots | \cdots |
| x_i | x_i \cdot y_1 | \ldots | x_i \cdot y_i | \ldots | x_i \cdot y_n |
| \cdots | \cdots | \cdots | \cdots | \cdots | \cdots |
| x_m | x_m \cdot y_1 | \ldots | x_m \cdot y_i | \ldots | x_m \cdot y_n |
| *   |     y_1      | \ldots |     y_i      | \ldots |     y_n      |

| \alpha_1 | \alpha_1 \cdot \beta_1 | \ldots | \alpha_1 \cdot \beta_j | \ldots | \alpha_1 \cdot \beta_n |
| \cdots | \cdots | \cdots | \cdots | \cdots | \cdots |
| \alpha_i | \alpha_i \cdot \beta_1 | \ldots | \alpha_i \cdot \beta_j | \ldots | \alpha_i \cdot \beta_n |
| \cdots | \cdots | \cdots | \cdots | \cdots | \cdots |
| \alpha_m | \alpha_m \cdot \beta_1 | \ldots | \alpha_m \cdot \beta_j | \ldots | \alpha_m \cdot \beta_n |
| \cdots | \beta_1      | \ldots | \beta_j      | \ldots | \beta_n      |

3. Blind number model of river water environmental capacity calculation

The formula for calculating the water capacity of fully mixed water under steady state conditions is expressed as follows:

\[
W = Q(c_s - c) + kc, \quad V = Q(c_s - c) + kc, \quad Q = u / u
\]

(4)

In the formula, \( Q \) — river flow; \( u \) — velocity; \( x \) — river length; \( cs \) — water quality target; \( c \) — pollutant concentration; \( k \) — pollutant degradation coefficient.

It is obvious that the formula reflects the amount of pollutants that the river can bear in a given case, but because of the instability of the river, the changes in the flow and flow rate of the river cannot represent the water environmental capacity of the river, so it cannot reflect the actual water level of the water.

The river flow \( Q \), pollutant concentration \( C \) and flow velocity \( u \) are respectively constructed as blind numbers. It can characterize the randomness and fuzziness of the system or things. They are
A blind number model for calculating water environmental capacity can be obtained by substituting the above blind number into formula (4).

\[
W = \frac{1}{4} \left[ f(\{Q_1, Q_p\}, f_i(Q)) \left\{ c_r, \{c_i, c_p\}, f_i(c) \right\} \right] + \frac{k}{c_s} \left( \frac{\{Q_1, Q_p\}, f_i(Q)}{\{u_1, u_p\}, f_i(u)} \right)
\]  

\[W = \frac{1}{4} \left( f(Q) (15 - f(c)) + \frac{64000 \times 10^{-5} \times 1.16 \times 15 \times 12000 \times f(Q)}{f(u)} \right)\]  

\[= 86.4 \times \left( f(Q) (15 - f(c)) + 0.21 \times f(Q) / f(u) \right)
\]  

4. Calculation example

The Huaihe River Basin has a long tributary of about 12km. According to the measured data of the annual flow, flow rate and COD\(_{\text{Mn}}\) concentration in 2007, the pollutant degradation coefficient of the river section is \(k=1.16 \times 10^{-6} \text{ /s}\), and the water quality control target of the river section is \(c_s=15\text{mg/L}\). The following formula can be obtained by constructing blind parameters.

The possible value of water environmental capacity and its corresponding credibility can be obtained by using the algorithm of blind number. The interval of possible values can be converted to intersected interval values, and a 9 order blind number of the difference between the upper and lower limits of each interval is finally obtained. We can see the Table 3 below.

| possible value | reliability | possible value | reliability |
|----------------|-------------|----------------|-------------|
| [600,1600]     | 0.15        | [5600,6600]    | 0.06        |
| [1600,2600]    | 0.26        | [6600,7600]    | 0.01        |
| [2600,3600]    | 0.24        | [7600,8600]    | 0.01        |
| [3600,4600]    | 0.19        | Others         | 0           |
| [4600,5600]    | 0.08        |                |             |

After transformation, the possible range of water environmental capacity and its corresponding reliability can be obtained, as shown in Table 4 below.

| possible value (kg/d) | reliability | possible value (kg/d) | reliability |
|-----------------------|-------------|-----------------------|-------------|
| [43300,129700]        | 0.15        | [475300,561700]       | 0.06        |
| [129700,216100]       | 0.26        | [561700,648100]       | 0.01        |
| [216100,302500]       | 0.24        | [648100,734500]       | 0.01        |
| [302500,388900]       | 0.19        | Others                | 0           |
| [388900,475300]       | 0.08        |                       |             |

5. Conclusion

In the calculation of water environmental capacity, the deterministic steady-state model is usually used to calculate the water environmental capacity. Although the calculation process of the deterministic model is relatively simple, it ignores the fuzziness, randomness, fuzziness and unascertained of the river water environment system.

The calculation model of river water environment capacity under blind information is established by blind number theory from the angle of various uncertainties of water environment system. According to the model, it can not only obtain the possible range of possible value of the river water environment capacity, but also obtain the corresponding confidence values in each interval. Therefore, it is more scientific and reasonable than the deterministic method.

It is of practical significance to study river water environmental capacity by using blind number theory. This method is applicable not only to the study of river environmental capacity, but also to the
calculation of water environmental capacity of other water bodies, such as lakes and reservoirs.

Acknowledgements:
This work was financially supported by the quality engineering project of Anhui Xinhua University (NO.2015xqjdx01) and the Natural Science Research Projects of Anhui Educational Commission (NO.KJ2016A302).

References
[1] CARDWELL H, ELLS H. Stochastic dynamic programming models for water quality management[J]. Water Resources Research, 1993,29(4):800-820.
[2] Li Ruzhong. Progress and trend analysis of theoretical models of water quality assessment. Journal of Hefei University of Technology, 2005,28(4):369–373(in Chinese)
[3] Chen Shouyi, Xiong Deqi. Fuzzy set theory and model of lake eutrophication. Journal of Lake Sciences, 1993,5(2):140–155(in Chinese)
[4] LEE Chih-sheng, CHANG Shui-Ping. Interactive fuzzy optimization for an economic and environmental balance in a river system[J]. Water research, 2005, 39:200-240.
[5] Ronald EG, Robert EY. Analysis of the error in the standard approximation used for multiplication of triangular and trapezoidal fuzzy numbers and the development of a new approximation [J]. Fuzzy Sets and Systems, 1997, 91(1):1-13.
[6] Wu Kaiya, Jin Juliang, Pan Zhengwei. Urban flood vulnerability contact number assessment model based on triangular fuzzy number cut set[J]. Journal of Hydraulic Engineering, 2010, 41(6) : 710-728.
[7] Ronald EG, Robert EY. A parametric representation of fuzzy numbers and their arithmetic operators[J]. Fuzzy Sets and Systems, 1997,91(2):185-202.
[8] MICHAEL H. On the implementation of fuzzy arithmetical operations for engineering problems[A]. Proceedings of 18th International Conference of the North American Fuzzy Information Processing Society—NAFIPS'99, New York, USA, 1999:460-468.
[9] Silvert W. Ecological impact classification with fuzzy sets[J]. Ecological Modelling, 1997,96(2):1-15.
[10] KENTEL E, AREL M M. 2D Monte Carlo versus 2D Fuzzy Monte Carlo health risk assessment[J]. Stochastic Environmental Research and Risk Assessment, 2005,19(1):80-100