Top pair production at ultra-high energies

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Abstract. The top quark, the heaviest quark and, indeed, the heaviest elementary particle known today, constitutes a novel probe of the long-lived medium in quark-gluon phase which, as expected, can be produced even in light nuclei collisions at ultra-high energies. Some distinctive features are considered for particle production in the top sector in ultra-high energy domain. The antitop-top pair production is studied within the quantum chromodynamics and effective field theory approach used for calculations of total partonic cross sections. Predictions for all observables are computed at NNLO in quantum chromodynamics and at LO in effective field theory. These quantitative results can be important for both the future collider experiments at center-of-mass energy frontier and the improvement of the phenomenological models for development of the cosmic ray cascades in ultra-high energy domain. Thus the study allows the better understanding of heavy particle production and emphasizes the exciting interrelation between the high-energy physics on accelerators and ultra-high energy cosmic ray measurements.

1. Introduction

Among the most challenging problems for the modern physics of fundamental interactions is search for the physics beyond the Standard Model (SM) and the study of the deconfined quark–gluon matter under extreme conditions called also quark-gluon plasma (QGP) which can be created in subatomic particle collisions at high enough energies.

A complete description of any physics beyond the SM requires a new fundamental theory. From a quantum field theory (QFT) perspective potential deviations from the SM expectations can be naturally described within the framework of the effective field theory (EFT). This approach represents a generalized parametrization of various new physics effects on the basis of higher-dimensional operators, suppressed by a sufficiently large matching scale \([1, 2]\). The top quark \((t)\) sector plays an important role in searches for new physics due to largest mass of \(t\) among fundamental particles of the SM and consequent its enhanced sensitivity to hypothetical new heavy particles and interactions.

The multi-TeV energies available at the Large Hadron Collider (LHC) have already opened up the possibility to measure various high-mass elementary particles produced in heavy ion collisions. The Higgs boson \((h)\) and \(t\), the heaviest elementary particles known, constitute a novel probe of QGP. It was suggested to study the behavior of Higgs in QGP with multi-TeV collider elsewhere \([3]\). The study of \(t\) behavior in hot environment created at ultra-high energies opens a unique possibility for investigation of a pre-equilibrium stages of space-time evolution of QGP. In this case \(h\) and \(t\) can be considered as a hard probe of the properties of strongly interacting matter under extreme conditions. The first-ever evidence for the production of top quarks in heavy ion interactions is recently observed in Pb + Pb collisions at \(\sqrt{s_{NN}} = 5.02\) TeV \([4]\). The
estimations obtained recently for global and geometrical parameters indicate the creation of deconfined quark-gluon matter with large enough volume and lifetime in various type, including even light, nuclear collisions with ultra-high energy cosmic ray (UHECR) particles [5].

The one of the most remarkable feature of the modern studies of UHECR is the muon deficit at energies of primary particles above on about 10 PeV [6]. In general, the production of $t$ with subsequent decay may additionally contribute in the muon yields. Furthermore the quark-gluon environment may influence on the dynamics of the processes of $t$ production and decay. That is only qualitative hypothesis which should be verified by calculations for processes with $t$ in final state in vacuum and in QGP. Thus the study of top production at ultra-high energies can be important for high-energy physics and physics of UHECR — that is, they are of an interdisciplinary value.

Top pair production through the strong interaction is studied within the present work. The energy range for protons in laboratory reference system considered in the paper is $E_p = 10^{17} - 10^{21}$ eV which corresponds to the c.m. collision energies $\sqrt{s_{pp}} \simeq 13.7 - 1370$ TeV, where a standard Mandelstam variable in the form of the c.m. collision energy squared is $s_{pp} = 2m_p(E_p + m_p)$, $m_p$ is the proton mass [7]. The justification of the choice of the energy range can be found elsewhere [5,8]. There are numerous results [9,10] for $\sqrt{s_{pp}}$ for the right hand side value is the nominal collision energy for the LHC. Thus the present analysis mostly focuses on the ultra-high energies from $\mathcal{O}(100$ TeV) corresponded for the the Future Circular Collider (FCC) project [11] up to the $\mathcal{O}(1$ PeV) which are close to the right boundary of the energy domain for the Greisen-Zatsepin-Kuzmin limit [12,13] in order of magnitude.

2. Formalism for top pair production

The top quark is produced in nuclear collisions predominantly in pairs ($t\bar{t}$) through quantum chromodynamics (QCD) processes, mostly gluon-gluon fusion at ultra-high energies $gg \rightarrow t\bar{t}$. Once produced, it decays very rapidly (within an average distance of $\sim 0.15$ fm) with almost 100% probability into a $W$ boson and a bottom ($b$) quark [4]. Top quark pair production is thereby characterized by final states comprising the decay products of the two $W$ bosons, and two $b$ jets, resulting from the hadronization products of $b$ quarks. The dilepton final states, in which both $W$ bosons decay into leptons ($l$) and the corresponding neutrinos ($\nu$), are the cleanest final states for the $t\bar{t}$ signal measurement [4], despite their relatively small branching fraction $B(t\bar{t} \rightarrow l^+l^-\nu_l\bar{\nu}_lbb) = 5.25\%$ [7], with $l^\pm = e^\pm, \mu^\pm$.

Within the present work the total inclusive $t\bar{t}$ production cross section ($\sigma_{t\bar{t}}^{tot}$) in nuclear interaction

$$A_1 + A_2 \rightarrow t + \bar{t} + X$$

for the collision of the lightest nuclei $A_1 = A_2 = p$ involves the following partonic subprocesses

$$q + \bar{q} \rightarrow t + \bar{t} + X,$$  

$$g + g \rightarrow t + \bar{t} + X,$$  

$$q + g \rightarrow t + \bar{t} + X.$$  

Using the established notations, the $\sigma_{t\bar{t}}^{tot}$ can be written, in particular, as follows

$$\sigma_{t\bar{t}}^{tot} = \sum_{ij} \int_0^{\beta_{max}} \frac{\beta}{\mu_F^2} d\beta \Phi_{ij}(\beta, \mu_F^2) \sigma_{ij}(\beta, m_t, \mu_F^2, \mu_R^2),$$

where $i, j$ run over all initial state partons studied in the present work, $\Phi_{ij}$ is the partonic flux which is a convolution of the densities of partons $i, j$ and includes a Jacobian factor [14]. The dimensionless variable $\beta^2 = 1 - \rho$, with $\rho = 4m_t^2/s$, is the relative velocity of the final state
top quarks having pole mass $m_t$ and produced at the square of the partonic center of mass energy $s$. In general $s = x_1 x_2 s_{pp}$ with $s_{pp}$ is the aforementioned square of the c.m. energy of the colliding nuclei, namely $p$ here, $\forall k = 1, 2$ : $x_k$ is the fraction of the hadronic 4-momentum carried out by one of the incoming partons. The fixed value $x_1 x_2 = 1/9$ is chosen in order to get the well-known relation between $e^+e^-$ and partonic process $s_{e^+e^-} = s$. Here $\mu_{F,R}$ are the renormalization and factorization scales. The total partonic (short-distance) cross section $\hat{\sigma}_{ij}$ for the inclusive production of a heavy quark from partons $i,j$ can be written as an expansion in the strong coupling [15]. For the choice of $\mu_F = \mu_R \equiv \mu$ the NNLO partonic cross section is

$$\hat{\sigma}_{ij}(\beta, m_t, \mu) = \frac{\alpha_s^2}{m_t^2} \left\{ \bar{f}_{ij}^{(0)} + \alpha_s \bar{f}_{ij}^{(1)} + \mu f_{ij}^{(1,1)}(1) + \alpha_s^2 \left[ f_{ij}^{(2)} + \mu f_{ij}^{(2,1)} + \mu^2 f_{ij}^{(2,2)} \right] + \mathcal{O}(\alpha_s^3) \right\},$$

(6)

where $\mu = 2 \ln(\mu/m_t)$ and $\alpha_s(\mu)$ is the MS coupling renormalized with $N_f$ active flavors at scale $\mu^2$ [14] and $N_f = 5$ in this paper. The functions $f_{ij}^{(n,m)}$ depend only on dimensionless parameters $\beta, \rho$. The relation $\mu = m_t$ [16] is used for the preliminary estimations at ultra-high energies below because the NNLO partonic cross section is available namely for this scale for gluon fusion reaction [3] at least [16]. Thus the equation (6) can be rewritten as follows

$$\hat{\sigma}_{ij}(\beta, m_t) = \frac{\alpha_s^2}{m_t^2} \left\{ \bar{f}_{ij}^{(0)} + \alpha_s f_{ij}^{(1)} + \alpha_s^2 f_{ij}^{(2)} + \mathcal{O}(\alpha_s^3) \right\}.$$

(7)

The functions $f_{ij}^{(0)}$ are shown elsewhere [15] and the $f_{ij}^{(1)}$ are known exactly through NLO [17] for the partonic subprocesses [2][3]. One can note the physically motivated fit to the numerically integrated result from [15] are used for the functions $f_{ij}^{(1)}$ within the present work instead of exact complex relations [17]. As shown in [15] the fit agrees with the numerically integrated result to better than 1% and this precision seems quite reasonable and suitable for the aim of the paper, namely for the qualitative study of $t\bar{t}$ production in ultra-high energy domain. The NNLO functions $f_{ij}^{(2)}$ can be found in [14] for the partonic collisions [3], in [16] for [3] and in [18] for $t\bar{t}$ production via quark-gluon interaction [4].

The information about the $\Phi_{ij}$ is limited and model-dependent especially for energy range $\sqrt{s_{pp}} > 0.1$ PeV due to partonic distribution functions at $\mu_F = m_t$ [19]. That amplifies the uncertainties for hadronic cross section [5] significantly. Thus the NNLO partonic cross sections [7] are in the focus of the present work.

In the absence of new resonances, effects of new physics can be described as effective interactions of SM particles at energies below a new physics matching scale $\Lambda$, i.e. effects beyond the SM can be described within EFT approach. Thus the general Lagrangian of EFT is $\mathcal{L}_{\text{EFT}} = \sum_{j=0} \mathcal{L}_j \Lambda^{-j}$, where $\mathcal{L}_0$ is the SM Lagrangian and $\mathcal{L}_{\text{eff}} = \sum_{j=1} \mathcal{L}_j \Lambda^{-j} \equiv$ effective part containing the effects of new physics. There is only operator at dimension five allowed by gauge invariance. Due to the tiny neutrino masses, the scale $\Lambda$ is probably around $10^{15}$ GeV [20]. On the other hand there are many independent dimension-six operators [1][2][21] which provide the leading modification to SM processes at order $\Lambda^{-2}$. Contributions of higher orders in the EFT expansion can be neglected if $\Lambda$ to be larger than the scale can be probed directly. Then the dominant effects are parameterized in terms of Wilson coefficients $C_k$ of dimension-six operators $O_k$ in the effective part $\mathcal{L}_{\text{eff}} = \sum_k (C_k \Lambda^{-2}O_k + \text{h.c.}) + \sum_i C_l \Lambda^{-2}O_l$, where the sum runs over all operators that involve $t$-quarks and non-hermitian operators are denoted as $+\mathcal{O}$ [22]. The lists of the dimension-six operators for top quark production can be found elsewhere [20][22].

As indicated above the dominant channel is [3] for top pair production through the strong interaction at ultra-high energies. Within the approach of $\mathcal{L}_{\text{eff}}$ at order $\Lambda^{-2}$ the Feynman diagrams can be found in [20] for the gluon channel [3]. The operator $O_{tG}$ changes the SM $gtt$ coupling, and also generates a new $gGtt$ interaction, $O_G$ affects the three-point gluon vertex
in QCD and $O_{G}$ generates a new diagram with an s-channel Higgs boson \cite{20}. The notation $\hat{\sigma}_{gg}^{(0),\text{QCD}}$ is used for the LO cross section of the subprocess $[3]$ within pure QCD and $\hat{\sigma}_{gg}^{(0),\text{eff}}$ — for the LO cross section of the partonic interaction $g + g \to t + \bar{t}$ provided by the contribution of effective part of the Lagrangian $L_{\text{EFT}}$, i.e. by the higher dimension operators at order $\Lambda^{-2}$. Then the LO partonic $tt$ production cross section due to gluon fusion $[3]$ within EFT with leading modification to SM process ($\hat{\sigma}_{gg}^{(0),\text{EFT}}$) is \cite{20}

$$
\hat{\sigma}_{gg}^{(0),\text{EFT}} = \hat{\sigma}_{gg}^{(0),\text{QCD}} + \hat{\sigma}_{gg}^{(0),\text{eff}} \\
= \frac{\alpha_s^3 v}{m_t^2} f(0) + \Re C_{tG} \frac{\alpha_s^{3/2} v}{6 \Lambda^2} \sqrt{\frac{\pi \rho}{2s}} \left(8 \ln \frac{1 + \beta}{1 - \beta} - 9 \beta\right),
$$

where $v = 246$ GeV is the vacuum expectation value (VEV) of scalar Higgs field $\phi$, the set $C_G = C_{\phi G} = 0$ is used here because the Higgs-gluon interaction, i.e. top pair production through $g + g \to h \to t + \bar{t}$ is not considered in the present work and $O_{G}$ is strongly constrained by multi-jet production and its contribution can be neglected for $\hat{\sigma}_{gg}^{(0),\text{eff}}$ at the sensitivity reached for $[3]$ channel \cite{22}. Thus the top-gluon dipole operator $O_{tG}$ is the only contribution from the effective part of the Lagrangian $L_{\text{EFT}}$ to the leading partonic subprocess $[3]$ and one can expect a high sensitivity to $O_{tG}$ in inclusive top pair production \cite{22}. The Wilson coefficient $C_{tG}$ is suggested pure real, $C_{tG} \in [0.24; 0.57]$ in units of $(\text{TeV}/\Lambda)^2$ is used below and the range for $C_{tG}$ corresponds to the 95% confidence level from the full global top fit with halved theoretical uncertainties \cite{22}.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{energy_dependence.png}
\caption{Energy dependence of NNLO partonic cross sections $\hat{\sigma}_{ij}$ within QCD. The solid line shows $\hat{\sigma}_{q\bar{q}} \times 10$, the dashed line corresponds to the channel $[3]$, the dotted line — to the $q + g \to t + \bar{t}$ and thick solid line is the sum of the channels $[2,3,4]$.}
\end{figure}

3. Results

Figure 1 shows the energy dependence of NNLO partonic cross sections $\hat{\sigma}_{ij}$ within QCD. The solid line shows $\hat{\sigma}_{q\bar{q}} \times 10$, the dashed line corresponds to the channel $[3]$, the dotted line — to the $q + g \to t + \bar{t}$ and thick solid line is the sum of the channels $[2,3,4]$. As expected the contribution of top pair production through $q\bar{q}$ annihilation $[2]$ is negligible in comparison with $\hat{\sigma}_{ij}$ for the channels with incoming gluon in all energy domain under study. Futhermore $\hat{\sigma}_{q\bar{q}}$ decreases with collision energy even at NNLO level in contrast with $\hat{\sigma}_{gg}$ and $\hat{\sigma}_{qg}$ which increase smoothly with growth of $\sqrt{s_{pp}}$. The sum of QCD NNLO partonic cross sections for the channels $[2,3,4]$ reaches the value
on order 0.1 nb at $\sqrt{s_{pp}} \gtrsim 0.5$ PeV and this value exceeds on about two times the corresponding quantity at $\sqrt{s_{pp}} \approx 0.1$ PeV. These results together with the recent calculations [19, 23] allow the qualitative expectation the total inclusive $t\bar{t}$ production cross section (5) at NNLO to be at level around or in order of magnitude $0.1$ µb for the highest $E_p \approx 10^{20} - 10^{20.5}$ eV measured in UHECR. As discussed above the improving of the precision for estimations of $\Phi_{ij}$ is important for deducing the quantitative estimation of $\sigma_{t\bar{t}}$ at ultra-high energies $\sqrt{s_{pp}} > 0.1$ PeV with higher accuracy and consequently for more defined physical conclusions.

The energy dependence for channel (4) is presented in figure 2 for terms $\hat{\sigma}_{gg}^{(0), QCD}$ and $\hat{\sigma}_{gg}^{(0), eff}$ in (8) as well as for the sum $\hat{\sigma}_{gg}^{(0), EFT}$. As seen all LO partonic cross sections decrease with the increase of the collision energy as $\propto s^{-1}$ at qualitative level. Such behavior is well-known for the functions $f_{ij}^{(0)}$ which vanish both at threshold ($\beta \to 0$) and in high energy ($\rho \to 0$) domain [24]. Figure 2 shows that the contribution from the new physics effects at LO level decrease rapidly with $s$ as well. The value of $\hat{\sigma}_{gg}^{(0), eff}$ is smaller in around of order of magnitude than $\hat{\sigma}_{gg}^{(0), QCD}$ from the SM amplitudes at corresponding $s$. The LO partonic cross section from the corrections induced by new physics is $\hat{\sigma}_{gg}^{(0), eff} \sim 10$ fb at collision energy for FCC project $\sqrt{s_{pp}} \approx 0.1$ PeV and $\hat{\sigma}_{gg}^{(0), eff} \sim 0.1$ fb at $\sqrt{s_{pp}} \approx 1$ PeV. Despite of the relatively small values of all partonic cross sections under consideration the visible excess is observed for the aggregated cross section $\hat{\sigma}_{gg}^{(0), EFT}$ over SM term $\hat{\sigma}_{gg}^{(0), QCD}$ at confidence level (CL) larger than 1 s.d. (standard deviation) driven by the uncertainty of the Wilson coefficient $C_{tG}$. This excess is for all energy range $\sqrt{s_{pp}} \gtrsim 14$ TeV studied within the present work. For calculation levels higher than LO one can expect at qualitative level the increase of the difference between $\hat{\sigma}_{gg}^{(m>0), EFT}$ and $\hat{\sigma}_{gg}^{(m>0), QCD}$ with the growth of $s$ due to the amplification of the effects of physics beyond SM. Future theoretical developments are important for the verification of this qualitative hypothesis especially the high-order calculations for corrections induced by new physics.

The study of single top production is in the progress for ultra-high energy domain.

4. Summary

Top pair production through the strong interaction processes is studied for ultra-high energy domain. The partonic cross sections are evaluated to NNLO level calculation within SM.
quantities increase smoothly on collision energy within QCD for the channels with incoming gluon. The sum of QCD NNLO partonic cross sections reaches the value on order 0.1 nb at $\sqrt{s_{pp}} \gtrsim 0.5$ PeV. For the channel of the top pair production through gluon fusion the energy dependence is derived for LO cross sections driven by SM processes and within EFT approach with take into account dimension-six operators. LO partonic cross section decreases rapidly down to the values on order 0.1–1.0 fb at highest energies $\mathcal{O}(1$ PeV) under consideration. Despite of the relatively small values of all LO cross sections the corrections due to new physics provide visible difference between EFT and SM values for the quantities for gluon channel.

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