The lever rounding mechanisms

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Abstract. Mechanisms with complex movement of a curved executive organ rounding around a fixed set line, in the simplest case – direct line or circular arc – can be combined with guiding mechanisms in the analysis and synthesis tasks, if one assumes the center of curvature of the generator of the executive organ to be a guiding point. Moreover, the length of the rounding segment and the parameters of deviation can be also defined through the parameters of the guiding mechanism, whereby sliding in a reciprocal rounding depends on the generator’s curve radius. Unlike the less applied guiding mechanisms lever rounding mechanisms can find a broad application as executive organs in pressure-shaping machines (e.g. forging, molding, cutting, chipping, clamping and compacting), stepping and finned-type propelling devices and other unique mechanisms with a common feature of indifference of the structure and integration of qualities that are non-compatible at a simple movement of an executive organ.

The objective of replication and rounding along the curves is one of the main tasks in the theory of mechanisms and its solution can be realized by means of precision guide and linear rounding [1] and approximate [2] mechanisms. Approximate rounding mechanisms as well as approximate guiding mechanisms have a priority significance for a practical application because they constitute less kinematic couples and, correspondingly, have a lesser rate of altering errors [3].

Let us see as an example a linear rounding mechanism executed on the basis of an elliptic approximate straight-line generating mechanism being a slide-crank mechanism with a guiding point $M$ with an approximate-straight-line segment of trajectory on the continuation of the crank axis. The coupler curve of the mechanism is symmetric, therefore we align its straight-line segment with vertical axis (see Figure 1), whereby its symmetry axis will be aligned with horizontal axis.

![Figure 1. Slide-crank straight-line-generating rounding mechanism.](image-url)
We draw radius arcs \( R_1, R_2, \) и \( R_3 \) out of the guiding point \( M \). Along their movement they curve around the vertical straight lines which horizontal lines \( x_1, x_2 \) and \( x_3 \) are correspondingly equal to \( R_1, R_2 \) and \( R_3 \). The length of the curve section for each straight line equals the length of of the straight-line section of the trajectory of \( M \) point:

\[
L = 2(r_1 \sin \varphi_1 + r_2 \sin \varphi_2)
\]

We assume \( \varphi_1 \) to represent the value of a generalized coordinate which corresponds to the end of approximation half-interval.

The length of the curve section of the arc:

\[
l = R\psi, \text{ where } \psi = 2(\pi - \varphi_2).
\]

Sliding between the reciprocal rounding sections of arc and straight line equals the difference between their values, or algebraic sum:

\[
\varepsilon = L - l = r_1 \sin \varphi_1 + r_2 \sin \varphi_2 + R(\pi - \varphi_2)
\]

(1)

For an elliptic straight-line generating mechanism, or slide-crank mechanism, the function of position is \( \varphi_2 = \pi - \arcsin(r_1 \sin \varphi_1 / b) \). Correspondent functions of position are inserted for other symmetric straight-line generating mechanisms. The radius arc \( R_2 \) on Figure 1 corresponds to the minimum slide, whereby radius arcs \( R_1 \) and \( R_3 \) have a sliding in the process of rounding around the fixed line, at that radius arc \( R_1 \) glides forward and radius arc \( R_3 \) glides backwards. Each of the three cases of sliding can be practically important depending on designation of a particular mechanism.

Thus, for instance, design with a zero summary sliding \( R_2 \) is used in the construction of roll cutting scissors and various squeezing machines and mills, \( R_1 \) design with forward sliding is the ultimate solution for cutting of thick leathers for consumer industry and design \( R_3 \) provides for backward sliding that is useful for moving of a work-piece at its squeezing in an oscillative forging machine and also in design of leg arrangements. The contact point \( K \) between the reciprocal curve of arc and fixed line defines the application point and direction of interaction force, therefore a force analysis is close to the type analysis, except for the calculation of the dependence of interaction force on the deviation in rounding and sliding.

Thus any straight-line guiding design can be used as a straight-line rounding design, be it a linkage or a geared linkage mechanism. Variety of schemes provides for choosing of the best design for every defined use case.

The task of synthesis of a straight-line rounding mechanism can be also divided into two steps: approximate synthesis of a straight-line guiding mechanism, or, as it is illustrated by the previous example, an elliptic straight-line generating mechanism, and determination of a radius of the rounding arc \( R \) depending on the parameters of the necessary sliding at rounding.

The coupler curve of the slide-crank mechanism is known to be a quartic algebraic curve and alignment of symmetric coupler curve with straight line in three points – in the middle of the approximation interval and on its ends - is sufficient to achieve the best approximation. As we examine the coupler curve of a symmetric mechanism, we may regard semi-interval of approximation with a sufficient alignment of two end points. The correspondence connecting the coordinate of the guiding point to the parameters of the mechanism is the design equation [2]. This term is not yet widely used; particular values of correspondence between the sizes of guiding mechanisms are usually stipulated in handbooks without reference to the best approximation [4,5].

The best approximation corresponds to the maximum possible quantity of interpolation points on the interval of approximation, i.e. it can’t be more than three points, but it can be less, still in this case we arrive at increase of the deviation or growth of the dimensions of the mechanism. For example, coupler curves of slot and crank or oscillating crank mechanism are sextic algebraic curves, that is why one should have at least three interpolation points on every semi-interval of approximation for the best approximation. Apart from the best there is a possibility of an intermediate approximation with two interpolation points on semi-interval of approximation, and even with one interpolation point, in
which case the approximation is the worst (in terms of V.S. Karelin [2]). At that the ratio of interpolation points is, correspondingly, growing.

Design equation for the guiding mechanism can be found with help of symmetric equations for curves to which the arc of the crank circle is approaching on the section of change of generalized coordinate corresponding to the semi-interval of approximation [3].

For example, crank-sliding straight-line guiding mechanism is defined as approximated elliptic straight-line generating mechanism because the circular arc of crank in this mechanism approaches elliptic arc within a certain section [2]. It is rooted in the simplest mechanism of ellipsograph consisting of crank and two sliders with particular guiding lines that are mutually transverse in particular case (see Figure 2).

![Figure 2. On the approximate synthesis of an elliptic straight-line generating mechanism.](image)

Any point on a crank describes ellipse with semi-axes being equal to the sections dividing the crank. One of the sliders, for example, in point $M$, is dropped out, whereby point $B$ of the crank, that describes ellipse with semi-axes $a$ and $b$, is flexibly connected to crank arm (this method is called conversion of a mechanism by introduction of a passive connection). Deviation of elliptic arc from the trajectory of point $B$ – circular arc $r$ – results in deviation $\Delta$ of the guiding point $M$ from the straight line that is aligned with vertical axis.

Therefore the approximate synthesis of elliptic straight-line generating mechanism can be performed by means of interpolation of elliptic arc towards the circular arc. Ellipse and circle are quadric algebraic curves, therefore there have to be four interpolation points on the approximation interval, as well as in the case of approximation of coupler curve of slide-crank mechanism (quartic) to the straight line.

Given this, the equation of scheme of an elliptic straight-line generating mechanism is constituted with use of a symmetric equation of an ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$  \hspace{1cm} (2)

We adopt the convention that $AB=r$, $BC=b$, $BM=a$ and define coordinates of point $B$ through the parameters of slide-crank mechanism:

$$X_B = a - r + r\cos\varphi$$

$$Y_B = r\sin\varphi.$$  \hspace{1cm} (3)
Coordinates are defined out of the condition that the elliptic arc on the section of approximation has three common points with the circular arc – in the middle of the section of approximation and on its ends. Symmetric property of the coupler curve provides for definition of one common point at the end of semi-interval at the set value of crank arm rotation \( \varphi \) by alignment of coordinates of the circle and ellipse at \( \varphi = 0 \).

After coordinates of point \( B \) are imported into the elliptic equation (2) we receive the dependence between parameters of linear guided mechanism at the best approximation:

\[
(a - r + r \cos \varphi)^2/a^2 = 1 - (r \sin \varphi)^2/b^2,
\]

from which the distance to the guiding point \( M \) at the set \( r \) and \( b \) is:

\[
a = \frac{r \cdot b \cdot (1 - \cos \varphi)}{b - \sqrt{b^2 - r^2 \sin^2 \varphi}}.
\]

The design equation of an elliptic approximate straight-line generating mechanism is derived and can be used for its designing. At any set parameters of slide-crank mechanism in continuation of the crank according to the derived formula we can find a point with approximate straight line section of coupler curve. Moreover, there is another same point in continuation of the crank from the side of the slider; the distance to the point is defined according to the same formula, but the length of straight line section on its trajectory is less.

The quality of approximation being subject to the main condition of synthesis is represented as deviation function:

\[
\Delta_{\text{max}} = X_M(\varphi_{\text{max}})
\]

The function can be expressed by the parameters of the mechanism by means of an enclosed vector polygon \( ABMO \) (see Figure 2), where the horizontal coordinate of point \( M \) corresponding to the maximum deviation \( \Delta_{\text{max}} \) is:

\[
X_M = a - r + a \sqrt{1 - \frac{r^2 \sin^2 \varphi}{b^2}}
\]

(4)

Derived function:

\[
\dot{X}_M = \frac{a \cdot r^2 \cos \varphi}{b^2 \sqrt{1 - \frac{r^2 \sin^2 \varphi}{b^2}}}
\]

We set this part to zero thus receiving:

\[
\varphi_{\Delta_{\text{max}}} = \arcsin \frac{b^4 - a^4 r^2}{(b^2 - r^2 a^2) r^2}
\]

(5)

Function \( X_M \) reaches its maximum at this value of argument \( \varphi \). Substituting the value \( \varphi_{\Delta_{\text{max}}} \) to (4) we define the maximum deviation in trajectory of point \( M \) from the straight line on the set section of approximation.

Let us now consider the slot and crank mechanism. The equation of scheme of a conchoidal straight-line generating mechanism is derived by the similar way while we place its scheme in rectangular (Cartesian) axes axis \( XOY \) in such a manner that the rack is aligned with horizontal axis ans the set straight line – with vertical axis (see Figure 3).
Figure 3. On the synthesis of conchoidal straight-line generating mechanism.

Crank circle of this mechanism on the set section of approximation is aligned with the loop that is created by the internal branch of conchoid, at that its baseline is aligned with the $Y$ axis and the conchoidal equation for this case of positioning is as follows:

$$y^2 - (x - a)^2 x^2 = l^2 (x - a)^2.$$

We import the coordinates of point $B$ into this equation and introduce the parameter $b = AC$:

$$x = l - r + r \cos \phi,$$
$$y = r \sin \phi,$$
$$a = l - r - b.$$

The following design equation for scheme of conchoidal straight-line generating mechanism is acquired on rearrangement:

$$l = \frac{r(1 - \cos \phi)}{1 - \frac{b + r \cos \phi}{\sqrt{b^2 + 2rbcos \phi + r^2}}}.$$

Because the coupler curve of slot and crank mechanism is a sextic curve the given design equation corresponds to *intermediate* approximation, which means that the acquired coordinate $l$ of the mechanism will be a bit bigger than at the best approximation. If this is not critical, the equation can be successfully used at synthesis of a conchoidal straight-line generating mechanism with regard for the set permitted deviation from rectilinearity.

Deviation in trajectory of point $M$ from the straight line is defined as in the first case:

$$X_M = l - r - r \cos \phi - l \cos \arcsin \frac{r \sin \phi}{\sqrt{b^2 + 2rbcos \phi + r^2}}.$$

It is impossible to explicitly express crank angle corresponding to the maximum deviation:

$$(b^2 + 2rbcos \phi + r^2)^2 = lr(r + b \cos \phi).$$

To search for the maximum of a function in this case one has to use numerical techniques.

The above mentioned examples are the most elementary cases of application of symmetric equations, when the relation between the parameters of mechanism and parameters of equation of a curve is obvious or can be easily established after an insignificant reexpression of equation form.

Unlike the elliptic straight-line generating mechanism conchoidal straight-line generating mechanism has a bigger forward stroke angle, or crank angle $2 \phi$, corresponding to the approximation
interval, at a very insignificant deviation, because the conchoid loop is close to the circular arc $r$ on a significant section. This provided for creation of a leg mechanism on the base of two conchoidal straight-line generating mechanisms (see Figure 4) with their crank arms being installed in a reverse phase (i.e. $2\varphi = \pi$) and bearing surface areas of buckled arch bearings are shaped by the circular arcs being drawn out of the guiding point $M$.

This off-road machine is specially designated to stamp trenches and channels, including irrigation channels, in soil, because the tracks of buckled arch bearings overlay and stepping is made in a natural “hill to toe” way.

It is more convenient to apply the following generalized method for synthesis of multi-lever and triple point geared linkage linear guiding mechanisms (when the degree of curve is unknown or assumed as a transcendent equation) with account for the symmetry of coupler curve.

Position of guiding point $M$ at any moment of time defines the guiding vector $\rho$ (see Figure 5) drawn out of the crank pivot point $O$ positioned in $XOY$ grid:

$$\rho = r_1 + r_2 + \cdots + r_i.$$  

Coordinates of the guiding point $M$:

$$X_M = X_0 + r_1 \cos \varphi_1 + r_2 \cos \varphi_2 + \cdots + r_i \cos \varphi_i$$

$$Y_M = Y_0 + r_1 \sin \varphi_1 + r_2 \sin \varphi_2 + \cdots + \varphi_i \sin \varphi_i$$

where $r_1, r_2, \ldots, r_i$ are the vectors of links of the guiding chain; $\varphi_2 = f(\varphi_1), \ldots, \varphi_i = f(\varphi_1)$ are the swing functions of the correspondent links.

Guiding mechanism is positioned in the $XOY$ grid in such a way that the symmetry axis of its coupler curve is aligned with the horizontal axis and approximate-straight-line section of trajectory is aligned with the vertical axis. The condition of triple point approximation will be observed for a
symmetric mechanism if the horizontal coordinate $X_0$ has equal value in two positions - at $\phi_1 = \phi_1^0$ and $\phi_1 = \phi_1^{\max}$, that we shall further designate as eigen $\phi_1$.

The values of these angles correspond to the mid-point and end of the approximation interval; the horizontal coordinate of point $M$ equals zero:

$$X_M = X_0 + r_1\cos\phi_1 + r_2\cos\phi_2 + \cdots + r_i\cos\phi_i = 0.$$  

Using the expression we deduce the horizontal coordinates of point $O$ in two positions:

$$X_0 = -r_1\cos\phi_1^0 - r_2\cos\phi_2^0 - \cdots - r_i\cos\phi_i^0;$$  

$$X_0 = -r_1\cos\phi_1 - r_2\cos\phi_2 - \cdots - r_i\cos\phi_i.$$  

The difference of the right parts equals to zero:

$$r_1\cos\phi_1^0 + r_2\cos\phi_2^0 + \cdots + r_i\cos\phi_i^0 - r_1\cos\phi_1 - r_2\cos\phi_2 - \cdots - r_i\cos\phi_i = 0; \quad (6)$$

We arrive at the generalized design equation for a symmetric straight-line guiding mechanism providing for a definition of an unknown parameter of its design out of the condition of triple spot interpolarity.

Approximation section length equals to two ordinates of point $M$:

$$L = 2|Y_M| = 2|Y_0 + r_1\sin\phi_1 + r_2\sin\phi_2 + \cdots + r_i\sin\phi_i|. \quad (7)$$

The current deviation of guiding point $M$ from the straight line between interpolation nodes is defined by horizontal coordinate of point $M$:

$$\Delta = r_1\cos\phi_1^0 + r_2\cos\phi_2^0 + \cdots + r_i\cos\phi_i^0 - r_1\cos\phi_1 - r_2\cos\phi_2 - \cdots - r_i\cos\phi_i.$$  \hspace{1cm} (8)

The equations (6), (7) and (8) represent a generalized system for synthesis of straight-line-generating mechanisms with symmetric coupler curve. For mechanisms having various designs the equations are distinguished only by the type of swing function $\phi_2 = f(\phi_1), . . . , \phi_i = f(\phi_1)$.

For linear rounding mechanism one also needs to define sliding between the circular arc of radius $R$, drawn on the crank plane out of the guiding point $M$, and straight line that is parallel to the straight-line section of coupler curve of the point $M$.

In the general case when the input parameters for synthesis are crank angle $\phi_1$, maximum deviation from the set trajectory $\Delta_{\max}$, length of approximation section $L$ and sliding $\varepsilon$ between the reciprocally rounding straight line and radial arc, the synthesis of a symmetric linear rounding mechanism is reduced to solution of a system of non-linear equations (1), (6), (7) and (8) in unknown $r_1$, $r_2$, $b$ and $R$, whereby one of the known methods in equation (8) defines the maximum value of deviation $\Delta_{\max}$.

After the discussion of linear rounding mechanisms, for the purposes of getting the whole picture, we shall add circular rounding mechanisms, thus, specifically, geared linkage automotive brake mechanism in which the brake cheek, that is connected to the satellite, curves over the position of operating surface of the brake drum during the operating movement. Curvature of operating area equals to the initial radius of carrying wheel, while curvature of brake cheek equals to the initial radius of satellite, therefore such brake is called centroidal. Testing of an experimental sample showed that the efficiency coefficient of the centroid brake is 20 times higher than the corresponding indicator for disk brake at comparable stability, i.e. the characteristics of brake torque remains unchanged at reversal of drum rotation. Moreover, the design allows for self-amplification and self-weakening of brake torque thus affecting its stability, if necessary.

As can be seen from above, the advantage of centroidal brake with a complex rounding movement of brake cheek consists of a combination of high efficiency, that is impossible for disk brake, with high stability [6].

Integration of useful qualities, that are inaccessible at a simple (stepping or rotating) movement of executive organ, is also inherent in other mechanisms with complex rounding movement of executive organ. Thus, for instance, roll cutting scissors combine constancy of cutting angle with a constant
overlapping, mechanism of the backup of an oscillative forging machine is simultaneously shifting the work piece along the axis and drafts it. Jaw crusher with a complex rounding jaw movement combines the constancy of output slit size and a free output of crushing products at minimal abrasive wear of operating surface of jaws. In case of propelling leg machines integration is realized in combination of high cross-country ability with a relative tightness of the propelling machine; finned-type propelling device combines efficiency of a volumetric propelling piston-type machine with continuity of screw-type mechanism, approaching in its input-output ratio and type of movement to the most advanced aquatic propelling device – whale tail.

We obtained about thirty invention certificates and patents for new technological solutions in the class of lever rounding mechanisms since 1985. Every solution has proved its functional capability and a significant superiority over the executive mechanisms with simple movement of operating organ. The suggested means of their synthesis make new prospect mechanisms available for further design works in design practice and in educational processes.

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