Modeling Free Surface Elevation around Tandem Piers of the Longitudinal Bridge by Computational Fluid Dynamics

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Abstract. This study explores the impacts of tandem piers of longitudinal bridges on the free surface elevation of river flow by computational fluid dynamics (CFD). Water flow around a single pier is simulated to develop the Kármán vortex street model, which also verifies the effectiveness of CFD simulations in this study. Then, the influence mechanism of span on the free surface elevation was analyzed in the light of the streamline of flow at $y=0.4m$ in different times. If the span is shorter than 20D, including 20D, the superposition of Kármán Vortex Streets obviously affect the fluctuation of streamline. The leap height of the first pier of different models is very close. If the span is shorter than 40D, the leap height decreases by degrees and finally reaches stability. When the span is greater than 40D, the leap height from the 2\textsuperscript{nd} pier to the last one is close.

Keywords: Longitudinal bridge; Tandem piers; Mountainous regions; Free surface elevation; CFD.

1. Introduction
Over the last twenty years, the Chinese government has constructed a great quantity of highways. In order to reduce the impact of freeway construction on the natural environment, more and more longitudinal bridges are adopted. Compared to the traditional bridges that cross the rivers, the piers of longitudinal bridges are built on the river bed. This will help to reduce the impact on the environment caused by the construction of embankment and tunnel. Therefore, nearly 10 kilometers of longitudinal bridges were contained in the Xi-Han expressway of China as shown in Figure. 1.

![Longitudinal bridge piers in valleys.](image)

\textbf{Figure 1.} Longitudinal bridge piers in valleys.

The backwater or leap height is one of the main factors determining the elevation of a bridge. Compared with conventional bridges, the longitudinal bridges have such a large number of piers, which may
2. Model Setup

2.1. Governing Equations

Continuity

\[ V_F \frac{\partial p}{\partial t} + \frac{\partial}{\partial x}(\rho u A_x) + R \frac{\partial}{\partial y}(\rho v A_y) + \frac{\partial}{\partial z}(\rho w A_z) = R_{SOR} \]  

(1)

X-momentum

\[ \frac{\partial u}{\partial t} + \frac{1}{V_F}\left( u A_x \frac{\partial u}{\partial x} + v A_y R \frac{\partial u}{\partial y} + w A_z \frac{\partial u}{\partial z} \right) - \xi A_x v^2 \frac{1}{x V_F} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + G_x + f_x \]  

(2)

Y-momentum

\[ \frac{\partial v}{\partial t} + \frac{1}{V_F}\left( u A_x \frac{\partial v}{\partial x} + v A_y R \frac{\partial v}{\partial y} + w A_z \frac{\partial v}{\partial z} \right) - \xi A_y w^2 \frac{1}{y V_F} = \frac{r}{p} \frac{\partial p}{\partial y} + G_y + f_y \]  

(3)

Z-momentum

\[ \frac{\partial w}{\partial t} + \frac{1}{V_F}\left( u A_x \frac{\partial w}{\partial x} + v A_y R \frac{\partial w}{\partial y} + w A_z \frac{\partial w}{\partial z} \right) = - \frac{1}{\rho} \frac{\partial p}{\partial z} + G_z + f_z - b_z \]  

(4)

\[ V_F \frac{\partial F}{\partial t} + \nabla(AUF) = 0 \]  

(5)

where \( V_F \) = open volume ratio to flow, \( (u, v, w) \) = velocity components in x, y, and z direction, respectively, \( R_{SOR} \) = source function, \( (A_x, A_y, A_z) \) = fractional areas, \( (G_x, G_y, G_z) \) = gravitational force, \( (f_x, f_y, f_z) \) = viscosity acceleration, \( (b_x, b_y, b_z) \) = flow losses in x, y, z directions, respectively. When the velocity is 0, mass injection is shown on the right side of equations 2 to 4. A is average area of flow, U is average velocity, F is volume fluid function. If the fluid fills of the entire cell, the F value is 1, and it’s 0 when empty. The FLOW-3D software uses two methods for simulation. Firstly, the VOF can show the behavior of water. Secondly, the FAVOR can be used for rigid volumes and free surface modeling.

2.2. Model Setup without Bridge Piers

As shown in Figure 1(a), the models of river and tandem piers can be established. In this field, the diameter of the pier is 1.0m and the span is 25.0m. A common fixed-bed model is established to simulate significantly influence the backwater or leap height in the front of the bridge pier and the safety of the bridge, especially during the flood season. Many experimental and numerical studies have investigated free surface elevation in open channels, including hydrodynamics, the characteristics of free surface turbulence[1-10]. For bridge engineering, researchers mainly study the characteristics of the water flow around the cylindrical pier and the backwater caused by it, which is very important to the determination of the bridge elevation. Tian et al. [11] explored the characteristics of the water flow around the cylindrical pier. He et al. [12] explored the impacts of bridge piers on the water flow. Li et al. [13] explored the relationship between the field around the bridge and the pier configuration. Gu [14] and Meneghini et al. [16] explored the interference between two circular cylinders. Sumner et al. [15] explored the flow pattern identification for two staggered piers. Wisner et al. [17], Charbeneau et al. [18], Chen et al. [19], Li et al. [20], Wang et al. [21], Duan et al. [22], Xia et al. [23], Li et al. [24], Zhang et al. [25] studied the model and the method for the computation of backwater caused by bridge piers. For piers arranged in tandem, some research has been conducted to explore the relationship between pile spacing and local scour, like Igarashi [26], Wang et al. [27], Rutuja et al. [28], Mubeen et al. [29], Palau et al. [30], Hassan et al. [31] and Milad et al. [32]. Existing studies mainly focused on the backwater characteristics around a single pier or pier group. Research on the free surface elevation around tandem piers of longitudinal bridge were rarely reported. This study systematically explored the flow of free surface elevation around tandem piers of longitudinal bridge using the CFD code Flow-3D. Based on the Fractional Area/Volume Obstacle Representation (FAVOR), it has a particularly powerful meshing capability, which can explain the complex boundaries. It is dedicated to the research of free surface and multi-phase applications, serving many industries, such as open channel hydraulics, biomedical equipments and other fields [33-40]. As a typical model for tracking water surface, fluid volume (VOF) can be used to solve the nonlinear Navier-Stokes equation. And FAVOR can figure out the area of flow obstacles. This will provide insights into the impacts of bridge piers on the water flow. Therefore, it can provide an important reference for our scientific research work. However, numerical simulation can not fully reflect all the problems that may occur in actual working conditions, and it needs to be solved by combining model tests or other means.
the river and the scale is set at 1:25. Under this condition, in the model, the diameter is 4cm. Based on the research results of Sarker[41] and Breuer[42], some parameters will affect the results, in this study, the width of the calculation domain is set to 20D and the initial water depth is set to 4D(D is the pier diameter) to avoid that. The parameters are shown in Table 1.

| Item                  | Width of channel(cm) | Height of channel(cm) | Average velocity of cross-section(cm/s) | Initial flow depth(cm) | Pier diameter(cm) | Shape of cross-section |
|-----------------------|----------------------|-----------------------|----------------------------------------|------------------------|-------------------|-----------------------|
| Parameters            | 20D (80)             | 6D (24)               | 50                                     | 4D (16)                | 4                 | Rectangle             |

Based on these parameters, the numerical model is established. The water is regarded as incompressible, and the Renormalized group model is adopted. According to the research of Sarker [41], the grid size is set to D/20 (0.2cm). To simulate the actual situation, the upstream boundary conditions (the velocity, or the pressure) for calculating the domain should be sufficiently developed. According to the model parameters in Table 1, a 50-meter long channel without pier is calculated. The boundary conditions of Xmin(Upstream) and Xmax (Downstream) are respectively velocity (50m/s) and outflow; the boundary conditions of Ymin and Ymax are both wall; and the boundary conditions of Zmin and Zmax are respectively wall and pressure. The water depth is set to 16cm and the average velocity is set to 50cm/s. Simulations show that the flow reach stability within 400 seconds, and the residual meet the requirement (residual is less than 1×10^{-2}%). The calculation time should be set to 400 seconds. In the channel without pier, the flow velocity in different cross sections is shown in Figure. 2. Which remains unchanged from x= 49m to the end of the channel, which means that the flow has been fully developed. To sum up, the velocity distribution at x= 50m should be set as the boundary conditions of the entrance with pier.

![Figure 2](image)

Figure 2. The profiles of velocity at flow surface in different cross sections without pier(t=400s).

2.3. Model Setup with Bridge Pier

The Xi-Han expressway adopts a 4-lane dual carriageway design. The longitudinal bridge sections adopt separate carriageway design, which have only one bridge pier with the diameter of 1.0m. In the light of JTG D60-2015, the standard span of pier within 50 meters are 0.52m, 0.64m, 0.80m, 1.2m, 1.6m and 2.0m. The position of the model in the river is 0.4m. Based on the research of Sarker [41], the space between the water inlet and the first cylinder should be greater than 5D to reduce the influence of parameters such as the water depth and the size of the calculation domain on the calculation results. As shown in Figure. 3, the length of the domain will vary with the span of the pier. The boundary conditions and the layout are shown in Figure 3.
3. Results and Discussion

3.1. Model Simulations

3.1.1. Free surface elevation profile of a single pier. In this study, firstly, the flow characteristics of a pier were simulated to prove the authenticity of the simulation results. As shown in the Figure 3, the boundary conditions and the layout of the model with a single pier are consistent with the model. The pier is located at x = 50.3m and y = 0.4m.

In the light of the simulation results, the flow can be stabilized within 50 seconds, and the residuals (less than 1×10^{-2}) can meet the requirement. Therefore, 50 seconds is set as the simulation time. The conclusions are shown in Figures 4, 5, and 6.

The free surface elevation at t = 50s is shown in Figure 4. When the water flows past the bridge piers, a swirling flow and Kármán vortex street will be formed. As a result, the free surface and velocity change sharply after the bridge piers. After the fluid flows away from the bridge pier, the effect decreases by degrees, and the water flow finally reaches stability. The free surface profile of flow with a single pier at y=0.4m and t=50s is shown in the Figure 4(b). The length affected by the blocking and circling flow is from x=50.00m to x=50.50m, which is 0.50m (12.50D) (that is 0.30m (7.50D) in front of the pier and 0.20m (5D) behind the pier), the second is the recovery area with a length of 60cm (15D), ranging from x = 50.50m to x = 51.10m, and then followed by the stable area which is longer than 20D.
(b) The profiles of free surface elevation in X-direction with a single pier at y=0.4m and t=50s.

Figure 4. The profiles of free surface elevation with a single pier. The zoomed velocity around the pier is shown in the Figures. 5 (a) and (b). The following phenomena can be drawn: the horseshoe vortex, a wake vortex behind the pier, a flow separation on both sides of the pier. As shown in Figure. 5(b), the water flow is separated at an angle of about 95. The shear stress field on the surface of the river bed is shown in Figure 6. From this we can find that the maximum shear stress occurs around 45°-70° on the side of the pier. Based on the studies of Summer [43], this position is also where erosion begins.

Figure 5. Velocity field around the pier at z=0.002m and t=50s (unit: m/s).

Figure 6. Contours of ESS on the riverbed at z=0.002m and t=50s (unit: Pa).

According to the study of Roulund [44], there are some flow characteristics when the vertical pile is fixed on the bed, such as bottom boundary layer, the new boundary layer in front of the pier, and the horseshoe vortex which can be observed in Figures. 5 and 6. This can prove the accuracy of the simulation in this study.

3.1.2. The flow free surface around tandem piers. Tandem piers with different spans are built in the river model 6, and the same parameter setups are used here. Each model has a free surface elevation with a different span at t=50s as shown in Figure 7.
Figure 7. The free surface elevation at t=50s with different spans.
From the above figures, the following conclusions can be drawn. Firstly, the maximum of free surface elevation in each model are almost the same, and the influence of each bridge pier on the flow depth near each pier becomes weaker and weaker along the flow direction in each model. Secondly, the influence of each bridge pier on the lateral range of depth changing increases gradually along the flow direction in each model, the closer to the downwards of the river is, the larger the influence on the lateral range is. Thirdly, the fluctuation of the flow increases gradually along the flow direction in each model. Fourthly, compared with the free surface elevation of each model to that without piers, the shorter the span is, the larger the differences in the free surface elevation are. If the span is shorter than 20D (0.80m) (including 20D), the free surface elevation between the first two piers are close, and that between the
subsequent piers decreases gradually. With a span longer than 40D (1.60m), the free surface elevation between piers are close in different conditions.

3.2. The Streamline of Flow at y=0.4m
To explore the influence mechanism of the free surface elevation as a function the span of piers, the streamline of flow colored by the velocity at y=0.4m in the x-direction with a span of 0.52m were generated and shown in Figure 8.

At the beginning (t=1s), water flows around each pier in turn, which is the same as the observation around a single pier. Streamlines are basically parallel to each other, except for the streamlines near the bridge pier, which are bent due to the circling flow around the pier. After the circling flow of all the piers completed, the characteristics of the streamlines, such as the shape, the width, and the density of the streamlines, between piers are almost the same. The streamlines downwards the last pier recoveries and returns to parallel gradually as shown in Figure 8(t=4s). Subsequently, as the time increases, the Kármán Vortex Streets is generated after each pier in the flow direction in turn. With the development of the Kármán Vortex Streets generated by each pier, superposition of Kármán Vortex Streets occurs from the upstream to the downstream, morphological characteristics of Kármán vortex streets generated by subsequent piers are affected by Kármán vortex streets from the previous piers, except for that generated by the first pier. Such influences are accumulative, and becomes more and more serious at the downstream of the flow, which will sharply impact the characteristics of the streamline, including the lateral range and the shape of the streamline, especially in the area near the bottom of the channel, as shown in Figure 8(t=8s, t=14s and t=27s). Finally, the streamline reaches stability at t= 50s as shown in Figure 8(t=50s).

From the above figures, the following conclusions can be drawn. Firstly, The range of streamline affected by the superposition of Kármán Vortex streets at the end of the channel will decrease as the span increases. The influence of the span on the elevation of the free surface will become more obvious as the span decreases. Secondly, with a span shorter than 20D(including 20D), the pier is located in the Kármán Vortex Street affected area of the precedent pier, so the streamline of flow is disturbed before recovery, which causes the free surface elevation around the piers downwards the x-direction is affected and the range of circling flow behind the pier becomes larger. Thirdly, with a span longer than 20D, the Kármán vortex streets generated by each pier are relatively independent, the superposition of Kármán Vortex Streets have less effect on the fluctuation of streamline and the elevation, especially on the range of area behind each pier.

3.3. The Influence of Span on the Leap Height
The backwater or leap height is one of the main factors determining the elevation of a bridge. Compared with general bridges, the longitudinal bridges have a large number of piers, which have a more significant effect on the backwater or leap height in the front of the pier. The flow is supercritical flow in this study, so the leap height will be the main factor that affect the bridge elevation. The Ratio of the leap height of each pier with different span to that of a single pier \( R_{LH} \), the leap height of a single pier is 0.1936m. From the Figure 9, the following conclusions can be reached. Firstly, the \( R \) of each pier is gradually reduced along the flowing direction in each model. The \( R \) of the first pier in each model are close to each other and are about 1.042 times of that of a single pier independent of the span.
This is consistent with the finding reported by Wang et al. (2016). Secondly, when the span is smaller than 1.60m, the leap height decreases by degrees and finally reaches stability. When the span is greater than 1.60m, the leap height gets close from the 2nd pier to the last one, especially when the span is 2.00m, the leap height of the last three piers are very close.

Figure 9. $R_{lh}$ of each pier in different models.

4. Conclusions
From the above research, the conclusions can be drawn as follows:
1. The shorter the span is, the greater the differences between the models with piers and without piers are.
2. In this study, if the span is shorter than 20D (including 20D), the superposition of Kármán Vortex Streets has obviously impact on the fluctuation of streamline and the free surface elevation. With a span longer than 20D, the Kármán vortex streets generated by each pier are relatively independent, the superposition of Kármán Vortex Streets have less impact on the fluctuation of streamline and the free surface elevation.
3. Leap height in front of the pier is one of the main factors determining the elevation of a bridge. The leap height of the first pier of different models is very close. If the span is shorter than 1.60m, the leap height decreases by degrees and finally reaches stability. If the span is longer than 1.6m, the leap height gets close from the 2nd pier to the last one, especially when the span is 2.0m, the leap height of the last three piers are very close.

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