NONLINEAR EFFECTS IN NUCLEAR CLUSTER PROBLEM

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Some nonlinear aspects of a cluster phenomenon in nuclei are considered using of cubic Nonlinear Schrödinger Equation and Korteveg de Vries Equation. We discuss the following possible nonlinear effects: i) the describing clusters as solitons; ii) an anomalous large angle scattering of $\alpha$-particles by light and intermediate nuclei; iii) stable vortical objects; iv) and dynamical clusterization in the presence of instability.

1. Motivation

Clusters is a very general phenomenon. They exist not only in nuclei and atoms, but also in subnuclear and macro physics (from small drops to atmosphere, stars and galaxes) (see materials and the summary talk of Prof. W. Greiner at CLUS-TERS’93 Conference (Santorini, Greece). There exist very different theoretical methods, developed in these fields. However there are only few basic physical ideas, and most of methods makes deal with nonlinear partial differential equations.

The aim of the present paper is to show that very different effects of nuclear cluster physics could be explained by using of cubic Nonlinear Schrödinger Equation and Korteveg de Vries Equation.

The papers is organized as follows.

In Sec.2 we consider general aspects of nonlinear approach for nuclei. An anomalous large angle scattering of $\alpha$-particles by light and intermediate nuclei are discussed.

A way to descrive clusters as stable nonlinear localized waves (solitons) is discussed in Sec.3. Section 4 is devoted to stable vortical excitations. Dynamical clusterisation and break-up of nuclear system are considered from the point of view of nonlinear dynamics in Sec.5 Last section is a short summary.

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2. Nonlinear Approach for Nuclei

The general philosophy giving birth to the NoSE is a rather simple one. Previous studies into the anomalous large angle scattering (ALAS) of $\alpha$-particles by light and intermediate ($\alpha$-cluster) nuclei that the rise in $d\sigma/d\Omega$ (elastic) for backward angles – together with the measured resonances, $E_x$, and decay widths, $\Gamma_x$, of the dinuclear ($\alpha$-cluster) system – may be described by (the same parameter of) a crude effective surface potential (ESP) arrived at by supplementing the optical model interaction, $V_{OM}$, by a repulsive (hard or soft) core, $V_r$:

$$V_{ESP} = V_{OM} + V_r$$

$V_{OM}$ facilitates the correct evaluation of $d\sigma/d\Omega$ for forward angles while $V_r$ gives the additional potential contribution allowing for an appropriate description of the measured $E_x$, $\Gamma_x$ and of the backward rise in $d\sigma/d\Omega$ as observed for $\theta_{cm} > 90^\circ$.

Similar quasi–molecular features as in these $\alpha$-scattering experiments have also been measured in elastic collisions between heavy ions. Knowing that semi–classical and in particular hydrodynamical approaches work nicely when applied to heavy ion physics, it appears very suggestive to try to model the elastic scattering of two heavy ions in terms of colliding liquid drops with diffuse surfaces.

Assuming that the “membranes”/surfaces of the two drops remain impenetrable, one thus obtain automatically a compression of the densities in the surface regions of the two touching drops. The physically motivated boundary condition is then that this compression — giving rise to a repulsive spring–force proportional to $V_r$ — vanishes for separations

$$r > r_0 = R_i + R_j + (a_i + a_j)/2$$

between the two ions/drops, i.e. $V_r \rightarrow 0$. $R_i$ and $a_i$ denote radius and diffuseness of the i-th ion; $i = 1, 2$. Thus the dynamical interaction between the two nuclei is for $r > r_0$ characterized by their unperturbed/uncompressed asymptotic densities $\rho_{0i}(R_i, a_i)$ — corresponding to the measured (charge) densities — and for $r < r_0$ by the perturbed/compressed densities $\rho_i(r; R_i, a_i)$.

The central equation emerging from such considerations together with the use of the Euler equation and some algebra in liaison with the neglect of higher order derivatives is the nonlinear Schroedinger equation (NoSE):

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi + \nabla \Psi - C(1 - \frac{\rho}{\rho_0})\Psi = E\Psi, \quad \rho = |\Psi|^2, \quad \rho_0 = \rho_{01} + \rho_{02}$$

where we use the conventional notation with $C = K/9$ for the nuclear compressibility, $\rho$ and $\rho_0$ denote the pertubated and unpertubated densities respectively. (Usually $K$ is taken to apply to the bulk of nuclear matter while we consider only compressions in the surface regions.)
It turned out that Eq. (1) is a particular solution of a general equation suggested by Bialynsky-Birula and Mysielsky. Using instead of cubic term $\epsilon \log(a | \psi |^2) \psi$ they formulated the nonlinear quantum mechanics without any contradiction in classical quantum mechanics. By the way, Gaehler, Klein and Zeilinger attempted to find the constant $\epsilon$ according to the predictions made in the work. But it was made without any success. Our approach supposes the constant $C = K/9$ or $\epsilon$ (here both constants are the same) to be the compressibility constant. It was confirmed from the values of the constant $K$, obtained from different experiments with the help of this approach ($K \approx 270 MeV$) and also by the recent work of J. Braecher. J. Braecher considered the nonlinear term in the NoSE from the point of view of the information theory. According to the work of Shannon and Weaver, the information $I$ acquired upon measurements of the state $| \psi >$ is proportional to the logarithm of the probability $I = -\log_2(a | \kappa |^2)$ bits, where $a = | \psi_0 >^2$.

The certain amount $\epsilon$ of energy per bit is expended, transferred, stored, associated with the information encoded. That’s why it is necessary to add the term $\epsilon I$ to the standard Hamiltonian:

$$H = H_0 + \epsilon I = \sum_{j=1}^{N} \frac{1}{2m} \nabla_j^2 + U - \epsilon \log_2 (a | \psi |^2)$$  \hspace{1cm} (2)

Now it is possible to show that this constant $\epsilon$ is very close to our compressibility constant. The total energy as the expectation value of the Hamiltonian (2) is:

$$E = < \psi | H end{equation} > = < \psi | H_0 | \psi > + \epsilon < \psi | H_0 | \psi > = E_0 \epsilon < \psi | \log_2 (a | \psi |^2) | \psi >$$  \hspace{1cm} (3)

Defining the entropy for the subsystem:

$$S = K < \psi | \ln (a | \psi |^2) | \psi >$$  \hspace{1cm} (4)

Where $K$ - the Boltzmann constant. Now we obtain the free energy $E_0$, the temperature $T$ and constant $\epsilon$:

$$E_0 = E - TS, \hspace{0.5cm} T = \left( \frac{\partial E}{\partial S} \right)_V, \hspace{0.5cm} \epsilon = KT \ln(2)$$  \hspace{1cm} (5)

The value $\epsilon$ is very close to the constant $C$. If we suppose the value $KT$ for the average kinetic energy of nucleons $\approx 30 MeV$ we obtain the reasonable value of compressibility constant.

Beside NoSE there is another interesting equation, which was applied many times for the prediction of new forms of the motion in nuclei. This is well-known Korteveg de Vries Equation (KdV) equation. KdV equation was applied many times in nuclear physics.

In the work the connection was established between the solution of the KdV equation and the nuclear potential in Schroedinger equation. At the certain circumstances we can see the evolution of the nuclear potential with time.
3. Nonlinear nuclear vortex

In heavy ion collisions a localized stable or long lived excitation might develop. The existence of a fairly well-defined nuclear surface makes it possible to discuss localized excitations in the nuclear surface region.

G.N. Fowler, S. Raha and R.M. Weiner attempted to predict the localized stable surface excitations in peripheral heavy ion collisions like in Rossby waves.

In the work A. Sandulesky and W. Greiner obtained a new coexistence model consisting of the usual shell model and a cluster model, describing a soliton moving on the nuclear surface. They got the excellent description of experimental spectroscopic factors for cluster decays.

In the framework of semiclassical nonlinear nuclear hydrodynamics, we have presented a type of vortical excitations new for nuclear physics. In the paper we began to consider a pure vortical motion in nuclear systems excluding the usual approximation of smallness of the excitation amplitude and the additional assumptions about the shape of the nuclear system. We considered the two-dimensional analog of a nuclear disk, a plane nuclear vortex, a new type of a pure vortical state of incompressible inviscid nuclear matter. It is a finite area of constant vorticity in a plane within a uniformly rotating contour. These states can be considered as the generalization of the elliptic Kirchhoff vortex. Recently we generalized this type of solution to the similar three-dimensional picture and consider a new possible class of nonlinear Euler type equations on a nuclear surface. We have found a class of uniformly rotating solutions to nonlinear Euler type equations on a sphere. The equations of motion describing a localized vortex on a spherical nuclear surface - bounded region of constant vorticity, surrounded by irrotational flux - are derived. These excitations may be linked with hot spots created in peripheral heavy ion collisions.

Let us consider the next type of nonlinear excitations in the region of nuclear surface, as a quadrupole deformation of the nuclear potential:

\[ U = U_0 \text{sech}^2 \left( \frac{r - R}{a} \right) = 2aU_0 \frac{df\left(2(r - R)/a\right)}{dr} \]  \hspace{1cm} (6)

The evolution of this perturbation having the soliton form with time is given by the KdV equation:

\[ U_t - 6UU_x + \beta U_{xxx} = 0, \]  \hspace{1cm} (7)

where the function \( U \) is the potential of Schroedinger equation

\[ \beta \psi_{xx} - U \psi = 0, \hspace{1cm} \beta = \hbar^2/2m \]  \hspace{1cm} (8)

On the other hand, this perturbation in the form of soliton can decay on the certain quantity of solitons with the small amplitude. If we fix the KdVE in the form

\[ U_t + UU_x + \beta U_{xxx} = 0, \]  \hspace{1cm} (9)

its solution will be in the form:

\[ U(x) = U_0 \text{sech} \left( \frac{r_0}{2\beta} \right) \]  \hspace{1cm} (10)
Now we apply the similarity principle. The initial condition we take in the form:

\[ U(x,0) = U_0 \phi(x,t). \]  

(11)

With new variables

\[ \eta = U/U_0, \quad x = X/l, \quad \tau = U_0 t/l, \]  

(12)

we obtain the new KdVE:

\[ \eta_t + \eta_{xx} + \sigma^{-2} \eta_{xxx} = 0, \]  

(13)

where \( \sigma = l(U_0 \beta)^{1/2} \).

In the work \[21\] the critical value \( \sigma = \sqrt{12} \) was obtained. With increasing number \( \sigma \) the corresponding perturbation breaks up into a larger number of solitons. The number of this solitons was determined by Karpman \[22\]

\[ N = 2a \sqrt{\frac{U_0}{6\beta}}. \]  

(14)

For the scattering of \( \alpha \)-particles with kinetic energies in the range 4 and 12 MeV we can get for the nucleus \( {}^{12}\text{C} \) the value \( N \sim 5 \). It corresponds to the number of existing states generated by the surface potential well: \( K^+ = 0, 2, 4, 6, 8 \). Gardner et al. \[23\] showed, that the amplitude of solitons determined by eigenvalues \( A = 2E_n \).

The potential having the soliton shape gives the following spectrum \[24\]:

\[ E_n = -\frac{\hbar^2}{8m\Delta} \left[ -(1 - 2n) + (1 + 8m \Delta U_0 \hbar^{-2})^{1/2} \right]. \]  

(15)

For our example \( (n = 0, 2, 4, 6, 8) \) we can get the following values (MeV): \(-15.57, -8.65, -1.75, 5.19, 12.11 \) respectively. The smaller eigen value the larger soliton amplitude. Knowing the eigen values one can construct the total potential \[25\].

This expression (6) provides the key to the understanding of the behaviour of the deformation length for different targets and for different projectiles. In a recent paper by Koster et al. \[26\] the empirical formula

\[ (\beta_2 r_0)_{\text{nucleons}} = \frac{Z}{A_T} (\beta_2 r_0)_{\text{protons}} + \frac{N}{A_T} (\beta_2 r_0)_{\text{neutrons}} \]  

(16)

is given to explain the different values of the deformation length for even isotopes of \( \text{Pd} \).

In terms of the soliton approach the compatibility condition for solitons in different media can be written \[28\]

\[ A \delta = A' \delta' = \text{const} \]  

(17)

where \( A \) and \( A' \) correspond to the formula (17) and

\[ \delta = \frac{\hbar^2}{2\mu}, \quad \delta' = \frac{\hbar^2}{2\mu'}. \]
and
\[ \mu = \frac{A_{p,n} \cdot A_T}{A_{p,n} + A_T} \quad \mu' = \frac{A_{p,n} \cdot A'_T}{A_{p,n} + A'_T}. \]

The product \( A\delta \) should be the same for different isotopes of the target nucleus. The resulting prediction is that the deformation length in the case of inelastic scattering of \( \alpha \)-particles on different Pd isotopes is should be largest for the heaviest one for the \( 2^+ \) state.

Thus we interpreted two equations NoSE and KdVE respectively to the problem of clustering. This work is only the first approximation to this problem because we used only one dimension. To our mind the next step should be the application of Kadomtsev-Petviashvili equation to this problem.

The most promising thing is the application of the Similarity principle to the linear alpha-chain states in nuclei.

4. Solitons as the clusters

What are clusters? Clusters are spatially correlated nucleons. What kind of forces are between clusters and clusters and nuclei? Japanese theoreticians showed\[30\] that there are four typical "di-molecules" in nature (molecules consisting from atoms, nuclei, nuclei plus hyperon, nucleons) with the same kind of potential. We consider the typical potential interaction between \( ^4\text{He} \) atoms.

In the approximation one can consider \( ^4\text{He} \) atoms as hard spheres with the radius \( 2.7\text{ A} \). Minimum says that atoms have a tendency to develop into crystal. But the warm energy does not permit it.

In the different energy scales the situations is very similar.

The origination of the attractive part of the potential is due to the exchange of nucleons in the case of nuclear potential\[31\].

We shall consider the repulsive core. The best understanding can be obtained in the micro and macro approaches. The existence of a repulsive core in the potential interaction of two complex nuclei follows from Pauli principle which forbids their mutual interpenetration. Let us give one example – the interaction of two alpha-particles. In this interaction the repulsive core is manifested as a result of which quasimolecular state the nucleus forms. The \( \alpha + \alpha \) phase shifts were obtained from the different experiments. The phase shift \( \delta_2 \) begins to increase rapidly from an energy about 1 MeV and \( \delta_4 \) from an energy about 6 MeV. This indicates an extension with a fairly abrupt boundary in the exterior region with \( r \leq 4\text{ fm} \) (the impact parameter \( b = \lambda l \) is approximately 4 fm at \( E_{cm} = 1 \text{ MeV} \) and 3 fm at \( E_{cm} = 6 \text{ MeV} \)). This character of the scattering can be well described by a phenomenological potential with the repulsive core.

In terms of the NoSE we can estimate the value of a hard core. For this purpose it is necessary to rewrite the NoSE.

\[ -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} U + (E - V)U + \beta U^3 = 0. \quad (18) \]
After the factorization at the wave function 

\[ U = \psi_0 f, \]

\[ f = 0, \ \text{at} \ x = 0, \]

\[ f = 1, \ \text{at} \ x \to \infty \]

we obtain the NoSE

\[ \frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi + \alpha \psi + \beta \psi^3 = 0. \] (19)

Then using such approaches \( \alpha \to \infty, U = \psi_0, \psi_0^3 = -\alpha/\beta \) we have

\[ \frac{\hbar^2}{2m \alpha} \frac{d^2}{dx^2} \psi + \frac{\beta}{\alpha} \psi^3 = 0. \] (20)

In the compact form this equation can be transformed into another equation

\[ -\bar{\xi}^2 \frac{d^2}{dx^2} f - f + f^3 = 0. \] (21)

Multiplaying this equation on the derivative \( df/dx \) we can obtain a solution

\[ f = t h[x/\sqrt{2}\bar{\xi}]. \] (22)

The solution is very similar on the correlation function of Bose gas with hard spheres:

\[ f = 0, \ \text{for} \ r < \delta, \]

\[ f = t h[r/\delta - 1]. \]

Where \( \delta \) – is the radius of a hard sphere. In terms of this approach the radius of the hard core is:

\[ R_0 = \sqrt{2} \xi = \sqrt{2} \frac{\hbar^2}{\sqrt{2m(E - V)}} \frac{\hbar^2}{\sqrt{m(E - V)}}. \] (23)

The microscopic estimates of the value of the hard core were made in the work of V.N.Bragin and R.Donangelo[32] and in the work of A.Tohsaki-Suzuki and K.Naito[33]. The last authors used RGM. In this case it is very evident to expect the appearance of the hard core. In the first work the authors used the Brukner theory according to which the total energy of a system of interacting fermions can be written as a functional of the local single-particle energy density. They have computed the heavy ion potential in the sudden approximation as

\[ V(r) = \int \left[ \xi(\rho_1 + \rho_2) - \xi(\rho_1) - \xi(\rho_2) \right] d\tau. \] (24)

The potential which was obtained in this approach is highly repulsive in the nuclear interior quite an opposite behavior to that of the folding model calculations. Summing up, it is necessary to say, that the momentum and energy dependent hard
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or soft core appears in the microscopic calculations of two interacting nuclei due to Pauli principle. On this base this core can be taken in account also in the macroscopic approaches.

Keeping in mind the structure of the potential for nucleon–nucleon collision (the repulsive and attractive parts) we applied the inverse field method, which will be described in the next section. Within the inverse mean field method solitons are taken to model of elastic $\alpha + \alpha$ collisions in a TDHF–like fashion. Attention is drawn to common points of this approach with TDHF. The analytical formula for the phase-shift within this approach yields a nice correspondence to experiment.

5. Nuclear instability and soliton theory

An existence of solitary waves is determined by two essential factors, namely, nonlinearity and dispersion. Both the factors, which are responsible for the stability of a wave, are connected in their turn with two different types of instability. A localized pulse will tend to spread out due to dispersion terms of the equations of motion. The nonlinearity which is responsible for the formation of solitary waves, on the other hand, automatically leads to their destruction, if it is alone. Both instabilities may compensate each other and lead to stable solutions (solitons).

Let us look from these points of view at multifragmentation phenomena. The formation and breakup of a highly excited and compressed nuclear system is the most striking process observed in intermediate-energy heavy ion reactions. How does such a system expand and finally disassemble when passing through a regime of dynamical instabilities? What is the mechanism of the clustering (stable light and intermediate mass fragment production)? Quite a variety of models have been developed to discuss this question (see, for instance, the recent review and Proceedings). The dynamical clusterization in the presence of instabilities is the focus of attention of the intermediate energy heavy ions physics. Ten years ago multifragmentation has been associated with the onset of the spinodal instability. This instability is associated with the transit of a homogeneous fluid across a domain of negative pressure, which leads to its breaking up into droplets of denser liquid embedded in a lower density vapor. Since the spinodal instability can occur in an infinite system, it can be called the bulk or volume instability. On the other hand, it physically means that pressure depends on density, that is just a nonlinearity in terms of density.

Recently, it has been pointed out that a new kind of instability (sheet instability) may play an important role in multifragmentation. This new instability can be assigned to the class of surface instabilities of the Rayleigh-Taylor kind. System escapes from the high surface energy of the intermediate complex by breaking up into a number of spherical fragments with less overall surface. At the same time, it physically means the existence of the gradient terms of the equations of motion, i.e the dispersion.

The spinodal instability and the Rayleigh-Taylor instability may compensate each other and lead to stable quasi-soliton type objects. In the next Sec. we present this physical picture using a simple analytical model proposed to describe
the time evolution of compressed nuclear systems.

An analysis of stability of nonlinear dynamical systems and an analysis of nonlinear evolution of initial complex states is a traditional goal of Soliton Theory. The inverse methods to integrate nonlinear evolution equations are often more effective than a direct numerical integration. Let us demonstrate this statement for a very simple case.

The type of systems under our consideration are uncharged slabs of nuclear matter. The slabs are finite in the \( z \) coordinate and infinite and homogeneous in two transverse directions.

The basic equations for the static mean-field description of the slabs are the following

\[
\psi_{k_{\perp}n}(x) = \frac{1}{\sqrt{\Omega}} \psi_n(z) \exp(i k_{\perp} r), \quad \epsilon_{k_{\perp}n} = \frac{\hbar^2 k_{\perp}^2}{2m} + e_n,
\]

\[-\frac{\hbar^2}{2m} \frac{d^2}{dz^2} \psi_n(z) + U(z) \psi_n(z) = e_n \psi_n(z), \]

\[\rho(x) \rightarrow \rho(z) = \sum_{n=1}^{N_0} a_n \psi_n^2(z), \quad (25)\]

\[A \rightarrow A = (6 A \rho_N^2 / \pi)^{1/3} = \sum_{n=1}^{N_0} a_n, \quad a_n = \frac{2m}{\pi \hbar^2} (e_F - e_n),\]

\[E \rightarrow \frac{\hbar^2}{2mA} \left( \sum_{n=1}^{N_0} a_n \int_{-\infty}^{\infty} \left( \frac{d\psi_n}{dz} \right)^2 dz + \frac{\pi}{2} \sum_{n=1}^{N_0} a_n^2 \right) + \frac{1}{A} \int_{-\infty}^{\infty} \mathcal{E}[\rho(z)] dz, \]

where \( r \equiv (x, y), k_{\perp} \equiv (k_x, k_y), \) and \( \Omega \) is the transverse normalization area. \( N_0 \) is the number of occupied bound orbitals.

The dynamical description will be done in the framework of the inverse mean field method. One can found the details of this approach in[49,50]. We concentrate here only on essentials.

The evolution of a system is given by the famous hydrodynamical Korteweg-de Vries equation (KdV) for the mean-field potential \( U(z, t) \)

\[\sum_{n=1}^{N} \frac{\partial U}{\partial (S_n t)} = 6U \frac{\partial U}{\partial z} - \frac{\hbar^2}{2m} \frac{\partial^3 U}{\partial z^3}, \quad (26)\]

where \( S_n \) are constants which are determined by the initial conditions.

General solution of KdV Eq. (26) can be derived in principle via direct methods numerically. This way is to assign a functional of interaction \( \mathcal{E} \) (as usual an effective density dependent Skyrme force), a total number of particles (or a thickness of a slab \( A \)) and to solve Hartree-Fock equations to derive a spectrum of the single particle states \( e_n \) and wave functions \( \psi_n(z, 0) \), the density profile \( \rho(z, 0) \) and the one-body potential \( U(z, 0) \) for the initial compressed nucleus. Then, one calculate an evolution of the one-body potential with the help of Eq. (26).
However, there is an inverse method to solve KdV Eq. (26). The main advantage of this way is to reduce the solution of the nonlinear KdV Eq. (26) to the solution of the linear Schroedinger - type equation

\[ -\frac{\hbar^2}{2m} \frac{d^2}{dz^2} \psi_n(z,t) + U(z,t)\psi_n(z,t) = e_n \psi_n(z,t), \]  

(27)

and linear integral Gelfand - Levitan - Marchenko equation to derive the function $K(x,y)$

\[ K(x,y) + B(x+y) + \int_x^\infty B(y+z)K(x,z)dz = 0. \]  

(28)

The kernel $B$ is determined by the reflection coefficients $R(k)(\epsilon_k = \hbar^2k^2/2m)$, and by the $N$ bound state eigenvalues $e_n = -\hbar^2\kappa_n^2/2m$. $N$ is the total number of the bound orbitals.

\[ B(z) = \sum_{n=1}^{N} C_n^2(\kappa_n) + \frac{1}{\pi} \int_{-\infty}^{\infty} R(k) \exp(ikz)dk. \]

The coefficients $C_n$ are uniquely specified by the boundary conditions

\[ C_n(\kappa_n) = \lim_{z \to \infty} \psi_n(z) \exp(\kappa_n z), \]

and the wanted single particle potential is given by

\[ U(z,t) = -\frac{\hbar^2}{m} \frac{\partial}{\partial z} K(z,z). \]

The time $t$ is included in Eqs. (27,28) only as a parameter, so it has been omitted in the above formulas.

The general solution, $U(z,t)$, should naturally contain both, contributions due to the continuum of the spectrum and to its discrete part. There is no way to obtain the general solution $U(z,t)$ in a closed form. Eqs. (27,28) have to be solved only numerically.

However in the case of reflectless ($R(k) = 0$) symmetrical ($U(−z,0) = U(z,0)$) potentials one can derive the following basic relations

\[ U(z,t) = -\frac{\hbar^2}{m} \frac{\partial^2}{\partial z^2} \ln(\det |M|) = -\frac{2\hbar^2}{m} \sum_{n=1}^{N} \kappa_n \psi_n^2(z,t), \]

\[ \psi_n(z,t) = \sum_{n=1}^{N} (M^{-1})_{nt} \lambda_t(z,t), \]

\[ \lambda_n(z,t) = C_n(\kappa_n) \exp(-\kappa_n z + 2\hbar^2\kappa_n^3 Snt/m)), \]  

(29)
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\[ M_{nl}(z,t) = \delta_{nl} + \frac{\lambda_n(z,t)\lambda_l(z,t)}{\kappa_n + \kappa_l}, \]

\[ C_n(\kappa_n) = \left(2\kappa_n \prod_{l \neq n}^{N} \frac{\kappa_n + \kappa_l}{\kappa_n - \kappa_l} \right)^{1/2}. \]

So, the wave functions, potential and the density profile are completely defined by the bound state eigenvalues.

The first step is to solve the Schroedinger Eq. (27) for the initial potential \( U(z,0) \), which is suitable to simulate compressed nuclear system or to simulate this state with the help of spectrum. Then one calculates the evolution of \( \rho(z,t) \) and \( U(z,t) \) with the help of Eqs. (29).

Although there is definitely some progress in the application of the inverse methods to nuclear physics they are not yet too popular. As illustration of these methods, we considered a one-dimensional three-level system in details.

A three-level system may be useful for modelling the evolution of light nuclei, for instance, of oxygen.

For large \( z \) and \( t \), the time-dependent one-body potential and the corresponding density distributions are represented by a set of stable solitary waves. The energy spectrum of an initially compressed system completely determines widths, velocities and the phase shifts of the solitons.

The number of waves is equal to the number of occupied bound orbitals. Thickness ('number' of particles) of an \( n \)-wave is equal to \( a_n \).

Reflecting terms (\( R(k) \neq 0 \)) of \( U(z,t) \) cause ripples (oscillating waves of a small amplitude) in addition to the solitons in the final state.

The initially compressed system expands so that for large times one can observe separate density solitons and ripples ('emissions'). This picture is in accordance with the TDHF simulation of the time evolution of a compressed \( O^{16} \) nucleus. The disassembly shows collective flow and clusterization. It is important to note that the clusterization was not observed in the absence of the self-consistent mean-field potential, i.e. this confirms our supposition that the nonlinearity is very important for the clustering.

It is necessary to note that the present model is too primitive in order to describe a real breakup process. However this model can be used to illustrate an inverse mean-field method scheme, a nonlinear principle of superposition and the idea that nonlinearity and dispersion terms of the evolution equation can lead to clusterization in the final channel.

Certainly there are a lot of open questions relative to the presented approach. The most crucial concerns the generalization to 3+1 dimensional model with a finite temperature. One possible way to do it keeping simplicity of the approach would be to use an information theory. Such investigations are in progress.
6. Summary

It is shown that very different effects of nuclear cluster physics could be explained in the frame of Korteweg de Vries Equation and cubic Nonlinear Schrödinger Equation.

The NoSE has been shown to be a very useful tool for the analysis of experimental data involving the elastic interaction of light and heavy ions. Its novel future is that it determines fairly accurately the compressibility modulus of nuclear matter. The problem of dynamical instability and clustering are considered from the points of view of the soliton conception and using KdV type equation for a mean-field potential. Nonlinear nuclear vortex is discussed. Due to the soliton conception it is understandable the behavior of the quadrupole deformation parameters for different isotopes of one nucleus.

7. References

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