Glueballs and the Pomeron*

H.B. Meyer\textsuperscript{a}, M.J. Teper\textsuperscript{a}

\textsuperscript{a}Theoretical Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, United Kingdom

We present our latest results on the glueball spectrum of $SU(N)$ gauge theories in 2+1 dimensions for spins ranging from 0 to 6 inclusive, as well as preliminary results for $SU(3)$ in 3+1 dimensions. Simple glueball models and the relation of the even-spin spectrum to the 'Pomeron' are discussed.

1. Introduction

On a Chew-Frautschi plot ($J$ versus $m^2$) the experimentally observed mesons and baryons appear to lie on (nearly) linear and parallel Regge trajectories, $J = \alpha_0 + \alpha' m^2$, with the exchange of the corresponding Regge poles dominating any high energy scattering that involves the exchange of non-trivial quantum numbers. The total cross-section, on the other hand, is related by unitarity to forward elastic scattering and this is dominated by the ‘Pomeron’ which carries vacuum quantum numbers.

The Pomeron trajectory is qualitatively different from other Regge trajectories in that it appears to be much flatter ($\alpha'$ much smaller) and it is not clear what physical particles correspond to integer values of $J$. There are long-standing speculations that these might be glueballs. Simple glueball models supporting this idea are sketched in section 2. Of course it is only in the limit of $SU(N \to \infty)$ that one can expect exactly linear trajectories (no decays) and the leading glueball Regge trajectory to be the Pomeron (no mixing).

The breaking of Lorentz symmetry by the lattice formulation of gauge theories complicates the calculation of the higher $J$ glueball spectrum required to address the question. A cubic lattice respects only a small subgroup of the full rotation group and each irreducible representation (IR) of this subgroup contains states that correspond to different $J$ in the continuum limit. We discuss ways to overcome this difficulty in section 3.1 and present our lattice data in sections 4-6.

2. Glueball models (5 and ref. therein)

In the standard valence quark picture, a high $J$ meson will consist of a $q$ and $\bar{q}$ rotating rapidly around their common centre of mass. For large $J$ they will be far apart and the chromoelectric flux between them will be localised in a flux tube which also rotates rapidly. This picture can be generalised directly to glueballs. We have two rotating gluons joined by a rotating flux tube that contains flux in the adjoint rather than fundamental representation. In this 'adjoint-string' model, one obtains at large $J$ the relation $J = \frac{M^2}{2\pi\sigma_a}$, where $\sigma_a$ is the adjoint string tension.

However for glueballs there is another possibility that is equally natural: a glueball may be composed of a closed loop of fundamental flux. Phonon-like excitations of this closed string can move around it clockwise or anticlockwise and contribute to both its energy and its angular momentum. In (2+1)D for instance, this 'flux-tube' model leads to a Regge trajectory $J = \frac{M^2}{8\pi\sigma_f}$, where $\sigma_f$ is the fundamental string tension.

3. Method

We obtain the spectrum of pure Yang-Mills theories by numerically calculating Euclidean correlation functions of gauge-invariant operators. As usual one needs to ensure that the operators are smooth and extended, so that they have a good projection onto the lighter physical states, and we use an iterative smearing technique for that purpose 6. We use the Wilson action on isotropic lattices, at values of $\beta$ lying in the
scaling region. A 2-level algorithm [4] is implemented that helps reducing the variance on rapidly decaying correlation functions.

### 3.1. Spin identification on the lattice

Consider the eigenstates of the transfer matrix of the lattice field theory. These will belong to the IRs of the cubic rotation group and will not, in general, possess the rotational properties that characterise a continuum state of definite spin. However, due to the absence of relevant gauge-invariant, Lorentz-symmetry breaking operators, space-time symmetry is restored dynamically as $a \to 0$, and each of these states will tend to an energy eigenstate of the continuum theory. As a consequence, in 3+1D one expects to find degeneracy between $2J + 1$ states across the lattice IRs in the continuum limit. In that same limit, operators with continuum-like rotation properties become available on physical length scales.

Thus we apply the following prescription, described in detail in [1]. Our operators lie in definite lattice IRs, and we apply the variational method [2] to extract estimates for the eigenstates (in our operator basis) and their masses. In this way we calculate the mass of the lightest state and of several excited states in the given lattice IR. To identify which $J$ each of these states tends to, we Fourier-analyse the angular wave function of the corresponding diagonalised operator. The Fourier coefficients are then extrapolated to the continuum, providing a non-ambiguous spin assignment.

A measurement of this wave function is provided by the correlations between it and a ‘probe’ operator that we are able to rotate to a good approximation by angles smaller than $\pi/2$. We check the rotational properties of the probe by measuring the wave function of the vacuum.

### 4. $SU(2)$ in (2+1)D

We performed simulations for $\beta$ ranging from 6 to 18; detailed results are presented in [5]. In Fig. 1 we plot our continuum $SU(2)$ glueball spectrum in a Chew-Frautschi plot of $m^2/\sigma$ against the spin $J$. We see that the lightest $J = 0, 2, 4, 6$ masses appear to lie on a straight line. If we fit them with a linear function $J = \alpha(t)$, where $\alpha(t) = \alpha_0 + \alpha' t$ and $t = m^2$, then we obtain

$$2\pi\sigma\alpha' = 0.322(16) \quad \alpha_0 = -1.18(11), \quad C.L. = 65\%.$$

### 5. $SU(N)$ in (2+1)D

We use the masses calculated in [6] and, having studied the wave functions of the states with probe operators, relabel as $4^+ -$ the state that is labelled there as the lightest $0^+ -$. Since the $4^- +$ and $4^+ +$ are degenerate in the (infinite volume) continuum limit, this gives us the $0^+ +, 2^+ +$ and $4^+ +$ continuum masses for $N = 2, 3, 4, 5$. A linear fit, $J = \alpha(m^2) = \alpha_0 + \alpha' m^2$, works for all $N$. Interestingly, the lattice result for $\alpha'$ is almost exactly at the midpoint between the two model predictions. We illustrate this fact in Fig. 2.

### 6. $SU(3)$ in (3+1)D: preliminary

We performed simulations at $\beta = 5.9, 6.0, 6.2$ and 6.338. For instance, the spin 4 representation breaks up into lattice IRs as $D_4 = A_1 \oplus E \oplus T_1 \oplus T_2$. We identify the 3rd excited state in the trivial lattice IR as a spin-4-like state; consistently we find a near-degenerate state in the $E$ representation at the smaller lattice spacings. It is our aim to identify the degenerate states in the other IRs in the near future.
Figure 2. The slope $\alpha'$ and intercept ($-\Delta \equiv \alpha_0 + 1$) of the leading Regge trajectory in $2+1$ $SU(N)$ gauge theory.

The continuum extrapolation is taken and the lightest states carrying quantum numbers $J = (2n)^{++}$ are plotted as a Chew-Frautschi plot (fig. 3). The lightest $0^{++}$ glueball does not belong to the leading trajectory: assuming a linear trajectory joining the spin 2 and spin 4, we get

$$\alpha_0 = 1.04(20),$$
$$\alpha' = 0.27(4)/(2\pi\sigma) \simeq 0.22(4) \text{GeV}^{-2}.$$  

The data is thus consistent with the phenomenological parameters of the soft Pomeron ($\alpha_0 = 1.08$, $\alpha' = 0.25 \text{GeV}^{-2}$).

7. Conclusion

In $2+1$ $SU(N = 2)$ gluodynamics, we find a straight leading Regge trajectory, containing the lightest $0^+$ glueball. The slope lies between the predictions of the flux-tube and adjoint-string models. This conclusion seems to remain true in the large $N$ limit.

In the $3+1$ $SU(3)$ case, where our data is preliminary, the lightest $0^{++}$ does not belong to the leading trajectory. If we assume a linear trajectory joining the spin 2 and spin 4, we get parameters that are consistent with the phenomenological parameters of the soft Pomeron. Computing the spin-6 state could provide decisive evidence supporting this assumption.

Thus glueballs do seem to provide a natural interpretation of the Pomeron, although the effect of mixing with mesonic trajectories still needs to be evaluated accurately.

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