Model-free control of non-minimum phase systems and switched systems

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Abstract
This brief presents a simple derivation of the standard model-free control for the non-minimum phase systems. The robustness of the proposed method is studied in simulation considering the case of switched systems.

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1 Introduction

The model-free control methodology, originally proposed by [1], has been widely successfully applied to many mechanical and electrical processes. The model-free control provides good performances in disturbances rejection and an efficient robustness to the process internal changes. The control of non-minimum phase systems has been deeply studied and successful methods have been proposed (e.g. [2] [3] [4] [5] [6] [7] [8] [9]). Since the model-free control can not \textit{a priori} stabilize a non-minimum phase system [1], we propose a possible derivation of the original model-free control law, dedicated to the control of non-minimum phase systems. The dynamic performances are especially tested in the case of switched systems.

The paper is structured as follows. Section II presents an overview of the model-free control methodology including its advantages in comparison with classical methodologies. Section III discusses the application of the modified model-free control, called NM-model-free control, for non-minimum phase systems. Some concluding remarks may be found in Section IV.

2 Model-free control: a brief overview

2.1 General principles

2.1.1 The ultra-local model

We only assume that the plant behavior is well approximated in its operational range by a system of ordinary differential equations, which might be highly nonlinear and time-varying. The system, which is SISO, may be therefore described by the input-output equation

\[ E(t, y, \dot{y}, \ldots, y^{(n)}, u, \dot{u}, \ldots, u^{(\kappa)}) = 0 \]  

(1)

where

\begin{itemize}
    \item $u$ and $y$ are the input and output variables,
    \item $E$, which might be unknown, is assumed to be a sufficiently smooth function of its arguments.
\end{itemize}

Assume that for some integer $n$, $0 < n \leq \iota$, $\frac{\partial E}{\partial y^{(n)}} \neq 0$. From the implicit function theorem we may write locally

\[ y^{(n)} = \mathcal{E}(t, y, \dot{y}, \ldots, y^{(n-1)}, y^{(n+1)}, \ldots, y^{(i)}, u, \dot{u}, \ldots, u^{(\kappa)}) \]

By setting $\mathcal{E} = F + \alpha u$ we obtain the \textit{ultra-local} model.

*See [10] for further details.
Definition 2.1 \[1\] If \(u\) and \(y\) are respectively the variables of input and output of a system to be controlled, then this system admits the ultra-local model defined by:

\[
y^{(n)} = F + \alpha u
\]  \hspace{1cm} (2)

where

- \(\alpha \in \mathbb{R}\) is a non-physical constant parameter, such that \(F\) and \(\alpha u\) are of the same magnitude;
- the numerical value of \(F\), which contains the whole “structural information”, is determined thanks to the knowledge of \(u\), \(\alpha\), and of the estimate of the derivative \(y^{(n)}\).

In all the numerous known examples it was possible to set \(n = 1\) or 2.

2.1.2 Numerical value of \(\alpha\)

Let us emphasize that one only needs to give an approximate numerical value to \(\alpha\). It would be meaningless to refer to a precise value of this parameter.

2.2 Intelligent PI controllers

2.2.1 Generalities

Definition 2.2 \[1\] we close the loop via the intelligent PI controller, or i-PI controller,

\[
u = -\frac{F}{\alpha} + \frac{y^{(n)*}}{\alpha} + C(\varepsilon)
\]  \hspace{1cm} (3)

where

- \(y^*\) is the output reference trajectory, which is determined via the rules of flatness-based control \([11, 12]\);
- \(e = y^* - y\) is the tracking error;
- \(C(\varepsilon)\) is of the form \(K_P\varepsilon + K_I \int \varepsilon\). \(K_P, K_I\) are the usual tuning gains.

Equation \[3\] is called model-free control law or model-free law.

The i-PI controller\[3\] is compensating the poorly known term \(F\). Controlling the system therefore boils down to the control of a precise and elementary pure integrator. The tuning of the gains \(K_P\) and \(K_I\) becomes therefore quite straightforward.
2.2.2 Classic controllers
See [13] for a comparison with classic PI controllers.

2.3 A first academic example: a stable monovariable linear system

Introduce as in [1, 10] the stable transfer function

\[
\frac{(s + 2)^2}{(s + 1)^3}
\]  

(4)

2.3.1 A classic PID controller

We apply the well known method due to Broïda (see, e.g., [14]) by approximating System 4 via the following delay system

\[
K e^{-\tau s} \\
(T s + 1)
\]

\(K = 4, T = 2.018, \tau = 0.2424\) are obtained thanks to graphical techniques. The gain of the PID controller are then deduced [14]:

\[K_P = \frac{100(0.4 + T)}{120K\tau} = 1.8181, K_I = \frac{1}{1.33K\tau} = 0.7754, K_D = \frac{0.35T}{K} = 0.1766.\]

2.3.2 i-PI.

We are employing \(\dot{y} = F + u\) and the i-PI controller

\[u = -[F]_e + \dot{y}^* + C(\varepsilon)\]

where

- \([F]_e = [\dot{y}]_e - u,\)
- \(\dot{y}^*\) is a reference trajectory,
- \(\varepsilon = y^* - y,\)
- \(C(\varepsilon)\) is an usual PI controller.

2.3.3 Numerical simulations

Figure 1(a) shows that the i-PI controller behaves only slightly better than the classic PID controller (Fig. 1(b)). When taking into account the other hand the ageing process and some fault accommodation there is a dramatic change of situation: Figure 1(c) indicates a clear cut superiority of our i-PI controller if the ageing process corresponds to a shift of the pole from 1 to 1.5, and if the previous graphical identification is not repeated (Fig. 1(d)).
2.3.4 Some consequences

- It might be useless to introduce delay systems of the type

\[ T(s)e^{-Ls}, \quad T \in \mathbb{R}(s), \quad L \geq 0 \]  

for tuning classic PID controllers, as often done today in spite of the quite involved identification procedure.

- This example demonstrates also that the usual mathematical criteria for robust control become to a large irrelevant.

- As also shown by this example some fault accommodation may also be achieved without having recourse to a general theory of diagnosis.

Figure 1: Stable linear monovariable system (Output (–); reference (- -); denoised output (. .)).
3 Control of non-minimum phase systems

We explain in this section, how to derive the model-free control law (3) in order to stabilize and guarantee certain performances for non-minimum phase systems. We will show that the proposed control law is also robust to disturbances and switched models.

3.1 Discrete model-free control law for non-minimum phase systems

Firstly, consider the discretized model-free control law, which is typically used for a digital implementation.

**Definition 3.1** \([13]\) For any discrete moment \(t_k, k \in \mathbb{N}\), one defines the discrete controller \(i\)-PI.

\[
u_k = u_{k-1} - \frac{1}{\alpha} \left( y^{(n)}|_{k-1} - y^{*(n)}|_{k} \right) + C(\varepsilon)|_k
\]

where

- \(y^*\) is the output reference trajectory;
- \(\varepsilon = y^* - y\) is the tracking error;
- \(C\) is a usual corrector PI where \(K_P, K_I\) are the usual tuning gains.

The discrete intelligent controller is also called discrete model-free control law or discrete model-free law.

Non-minimum phase systems are characterized by negative zero(s). Such zero can be approximated by a delay since \(e^{-Ts} \approx 1 - Ts\) using a Taylor expansion. To compensate the effect of the delay, that may destabilize the control, we take the derivative of the output \(y\) instead of the output \(y\) to create the measurement feedback. This way allows to anticipate the variations of \(y\) and finally cancel the disturbances associated to the presence of the delay.

We define consequently the \(i^*\)-PI controller for non-minimum phase systems.

**Definition 3.2** For any discrete moment \(t_k, k \in \mathbb{N}\), one defines the discrete controller \(i^*\)-PI for non-minimum phase systems. \(\lambda\) and \(\delta_j\) are real coefficients.

\[
u_k = \mathcal{G}(\varepsilon) \left\{ u_{k-1} - \sum_{j=1}^{n} \delta_j \left( \lambda y^{(j)}|_{k-1} - y^{*(j)}|_{k} \right) \right\}
\]

where
• \( y^* \) is the output reference trajectory;

• \( \varepsilon = y^* - y \) is the tracking error;

• \( G(\varepsilon) \) is called a gain function and is either a pure gain or an integrator.

The discrete intelligent controller is also called discrete NM-model-free control law or discrete NM-model-free law.

Practically, simulations show that \( n = 2 \) is sufficient to ensure at least the stability of the model-free control closed-loop. Therefore, (7) is written:

\[
u_k = G(\varepsilon) \left\{ u_{k-1} - \delta_2 \left( \lambda \left. \frac{d^2y}{dt^2} \right|_{k-1} - \left. \frac{d^2y^*}{dt^2} \right|_k \right) - \delta_1 \left( \lambda \left. \frac{dy}{dt} \right|_{k-1} - \left. \frac{dy^*}{dt} \right|_k \right) \right\}
\]

(8)

For the following applications, we choose the gain function as an integrator, with a \( K_i \) constant, such that:

\[
G(\varepsilon) = K_i \int_0^t \varepsilon \, dt
\]

(9)

### 3.2 Applications

Consider the systems \( \Sigma_1, \Sigma_2, \Sigma_3 \) and \( \Sigma_4 \), which are minimum and non-minimum phase systems, and which are described respectively by the state-space representations:

\[
\Sigma_1 := \begin{cases} 
\dot{x} = \begin{pmatrix} 0 & -1000 \\ 100000 & -5000 \end{pmatrix} x + \begin{pmatrix} 2.10^4 \\ 0 \end{pmatrix} u \\
y = (-10 \ 1) x 
\end{cases}
\]

(10)

\[
\Sigma_2 := \begin{cases} 
\dot{x} = \begin{pmatrix} 0 & -900 \\ 80000 & -3500 \end{pmatrix} x + \begin{pmatrix} 2.10^4 \\ 0 \end{pmatrix} u \\
y = (-13 \ 1) x 
\end{cases}
\]

(11)

\[
\Sigma_3 := \begin{cases} 
\dot{x} = \begin{pmatrix} 0 & -900 \\ 80000 & -3500 \end{pmatrix} x + \begin{pmatrix} 2.10^4 \\ 0 \end{pmatrix} u \\
y = (+13 \ 1) x 
\end{cases}
\]

(12)

\[\text{Depending on the application, a pure gain can be enough to ensure good tracking performances.}\]

\[\text{The possibility of reducing } n \text{ will be studied in a future work.}\]
\[ \Sigma_4 := \begin{cases} \dot{x} = \begin{pmatrix} 0 & -400 \\ 70000 & -1500 \end{pmatrix} x + \begin{pmatrix} 2 \times 10^4 \\ 0 \end{pmatrix} u \\ y = \begin{pmatrix} +5 & 1 \end{pmatrix} x \end{cases} \] (13)

The unitary step response of these systems is presented Fig. 2.

![Step response graph](image)

Figure 2: Step responses of the systems \( \Sigma_1 \), \( \Sigma_2 \) and \( \Sigma_4 \).

The following figures present some preliminary examples of the application of the \( i^* \)-PI control. Figure 3 presents the tracking of an exponential reference for the system \( \Sigma_1 \) (with a focus on the beginning of the transient). Figures 4 and 5 show the response \( y \) of the controlled system when respectively \( \Sigma_1 \) switches to \( \Sigma_2 \) and with the addition of a sinusoidal disturbance on the variable \( u \). Figures 6 and 7 present the control of switched systems; in particular the commutation from a non-minimum phase system to a minimum phase system. Figures 8 and 9 present the tracking of a sinusoidal reference when systems switch. The case where \( \Sigma_1 \) switches to \( \Sigma_3 \) has been already studied in [16]. We investigated the application of the model-free control in a microgrid environment under load / transfer function changes. These changes imply substantial modifications of the controlled models.
(a) Transient with stabilization.

(b) Focus on the beginning of the transient.

Figure 3: Tracking of an exponential reference.
Figure 4: Tracking of an exponential reference; $\Sigma_1$ switches to $\Sigma_2$ at $t = 5$ ms.

Figure 5: Tracking of an exponential reference with a sinusoidal disturbance added on $u$ such that $\tilde{u} = 5 \cos \left( \frac{2\pi}{5 \times 10^{-3}} t \right)$.
Figure 6: Tracking of an exponential reference; $\Sigma_1$ switches to $\Sigma_3$ at $t = 5$ ms.

Figure 7: Tracking of an exponential reference; $\Sigma_1$ switches to $\Sigma_4$ at $t = 1.5$ ms.
4 Concluding remarks

We presented some simulation results that confirm the fact that the NM-model-free control or i*-PI controller, designed for the control of non-minimum phase systems,
ensure good tracking performances. We evaluated the performances in presence of disturbances and in the case of switched systems. In particular, the NM-model-free control is able a priori to control both minimum and non-minimum-phase systems. The proposed control law seems to have the same properties than the original model-free control [1] for which its performances have been successfully proved in simulation when controlling switched systems (e.g. [5]). Further work will concern the study of the stability of the NM-model-free control method and its applications to networked systems.

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