Purity Calculation Method for Events Samples with Two Identical Particles

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Abstract

This paper studies a method of a two dimensional background calculation for an analysis of events with two particles of the same type registered in experiments in high-energy physics. The standard two-dimensional integration is replaced by an approximation of a specially constructed one-dimensional function. The number of the signal events is found by the subtraction of the background events. It allows calculating the purity of the selection. The procedure does not require a hypothesis about background and signal shapes. Monte Carlo examples of double $J/\psi$ samples are used to demonstrate high performance of the purity calculation method. A comparison of the method with standard two-dimensional fit of the signal revealed a systematic shift of the fit results to lower values.

Keywords: data analysis methods; double $J/\psi$; mass spectrum.
1. Introduction

If we fit by the Gaussian distribution a one-dimensional (1D) signal and the need for the reliable result of the n events statistics, then in the two-dimensional (2D) case one needs for the same fit quality of the signal to have bigger statistics. The task is complicated by the fact that the signal shape is not the Gaussian because the accuracy of the detector reaction depends on the particle kinematics. The additional uncertainty is the background hypothesis which is much harder to build for the two-dimensional case.

The extraction of the cross-section of two similar particles simultaneous production is a common problem in accelerator experiments. The study of such processes is complicated by the smallness of the inclusive cross section of the double production relative to the cross section of single production of the same particle. For example, the cross section of double J/$\psi$ production is more than 3 orders less than that of the single J/$\psi$ production. The opportunity to observe pairs with high invariant masses decreases exponentially with the pair mass. Usually, the main background at selection has a combinatorial origin. We differ 2 types of event selection: 1D and 2D cases when we require one or two similar particles in an event. If we use the Gaussian fit, 2D case requires much higher statistics than 1D case for the same fit quality. An additional complication is the fact that the signal shape could not be the Gaussian because of the dependence of the uncertainty of the detector response on the particle kinematics. Below, we use the example of the selection of events with paired J/$\psi$, to describe the method of the purity calculation in the events with the pair of particles production.

2. Pair selection

In order not to use an abstract description of the method, we use an example of the J/$\psi$ pair selection by high-energy physics detectors to demonstrate the method. Usually, they register decays of J/$\psi$ in two muons of opposite charges(J/$\psi \rightarrow \mu^+ + \mu^-$) [3] - [6]. Both muons come from the same vertex. The invariant mass of the muon pair is $M_{J/\psi}$. We describe the distribution of the $M_{J/\psi}$ value by the probability distribution function (PDF)

$$f_{1D}(M_{J/\psi}).$$

Fig.1 illustrates the distribution which looks like this function in the $2 - 5 GeV/c^2$ mass interval. We constructed the function in Fig.1 to be similar to that in experiments [3] - [6]. We see J/$\psi$ and $\psi(2S)$ peaks over the combinatorial background created by random combinations of muons. The shapes of signals are close to the Gaussian ones which could be deformed by the detector properties. Fig.1 illustrates the signals are constructed from two Gaussian functions: the base one with the mean at the PDG value of the particle mass [1] and an additional one with the mean value below the particle mass. Under the additional Gaussian distribution resides a few percent of all signal events. We use a polynomial of third order to describe the background. In our study, we use the distribution in Fig.1 varying the proportion of signal/background to simulate artificial single J/$\psi$ events in our virtual detector. We select J/$\psi$ candidates as events with the invariant mass in the mass window:

$$M_1 < M_{J/\psi} < M_2.$$
If in some events the detector selects simultaneously two $J/\psi$ candidates, their invariant mass distributions are independent because we do not differ which particle is the first and which is the second. The PDF for the events with double $J/\psi$ candidates is

$$F_{2D}(M_{J/\psi_1}, M_{J/\psi_2}) = f_{1D}(M_{J/\psi_1})f_{1D}(M_{J/\psi_2}).$$

(3)

We use the purity definition of the sample with $J/\psi$ candidates as:

$$P = \frac{\text{signal}}{\text{signal} + \text{background}}.$$  

(4)

Fig. 2 illustrates two examples of the double $J/\psi$ events registration. Events have been simulated using (3) and the shape of the distribution in Fig. 1. The cases differ in terms of the signal amplitude around the $J/\psi$ position and by the background level. For a real detector, these two examples correspond to two different event selections: loose (we do not suppress strongly the combinatorial background) and tight (we suppress the background and simultaneously decrease the signal).

The purity of events in Fig. 2a is 25% whereas the purity for the sample in Fig. 2b is 35%. By eye, we can assume an opposite situation. We need to know the excess of the double particle events inside the mass window without use of any hypotheses about signal and background shapes.

### 3. Purity of double $J/\psi$ sample

We select events when both muon pairs have the invariant mass inside a mass window around the $J/\psi$ peak. Such events are of 3 types: $RR$ (real $J/\psi$ + real $J/\psi$), $RF$ (real $J/\psi$ + fake $J/\psi$), and $FF$ (fake $J/\psi$ + fake $J/\psi$). Fake $J/\psi$ events come from the combinatorial background. Fig. 2a illustrates two perpendicular “mountain ranges” along the $J/\psi$ mass. They
Figure 2: The scatter plot of $J/\psi$ pairs reconstructed masses. Presented for the mass window $2.85 < M_{J/\psi} < 3.3\text{GeV}$ are a) 200 signal events of total 800 events, b) 70 signal events of total 200 events.

correspond to the background events with one real $J/\psi$ particle and a random muon track combination. In Fig.2, we do not see such phenomenon so evidently. The peak in Fig.2 can be due to the superposition of “mountain ranges.” In fact, concerning the form of a two-dimensional background in Fig.2 we only know that it is a symmetric function around the axis $M_{J/\psi_1} = M_{J/\psi_2}$. Anything else can only be our assumptions with unknown accuracy.

Let us build the distribution of the invariant masses in double particle events when the invariant mass of one candidate is inside the mass window $MW_1$, which we use for double $J/\psi$ selection and the invariant mass of the second candidate is outside the mass window. Fig.3 illustrates an example of the distribution. The distribution defines the function $f_{jff}(M)$ for double particles events with one candidate selected in the mass window $MW_1$. The function $f_{jff}(M)$ outside the mass window is proportional to the 1D PDF because it is a sum of independent subsequences of the random $M_{J/\psi}$ values

$$f_{jff}(M) = 2 \int_{WM_1} f_{1D}(m) dm f_{1D}(M).$$

Using (5), we build the function

$$N_{jff}(M) = \int_{WM_2} f_{jff}(m) dm,$$

where $WM_2$ is an arbitrary mass interval (see Fig.3) and $M$ is its mean value. In practice, it is the sum of events in the interval $WM_2$. If we take $WM_2$ far enough from the $J/\psi$ peak, the function $N_{jff}(M)$ provides double number of types $RF$ and $FF$ events inside the square $[WM_1, WM_2]$. Fig.4 illustrates two functions $N_{jff}(M)$ by points. Excluding the peak region and replacing the function $N_{jff}(M)$ by the fit of its tails, we get the function $n_{jff}(x)$ illustrated in Fig.4 by the dashed line. Taking into account that the number of random muon pairs depends on the mass continuously, we obtain that the value of the function $n_{jff}(M)$ corresponding to the mean value of $WM_1$ is the double number of fake $J/\psi$ pairs of the 1D
projection of all double $J/\psi$ candidates inside the 2D mass window $WM_1$. We use a polynomial to fit tails of $N_{j_{f-f}}(M)$. The polynomial is of the degree producing the best $\chi^2/ndf$ value. If $N_{all}^{2D}$ is the number of candidates inside the 2D mass window $WM_1$, the purity of double $J/\psi$ sample in the 2D window can be calculated by the formula:

$$P_{2D} = 1 - \left( \frac{n_{j_{f-f}}(M_{J/\psi})}{2N_{all}^{2D}} \right)^2.$$  \hspace{1cm} (7)

Figure 3: $J/\psi$ candidate masses in 2D sample. One candidate has the mass inside the mass window $MW_1$, the other has the mass outside of it. $MW_2$ is an arbitrary interval of the length equal to the length of $WM_1$. $X_1$ and $X_2$ are center masses of $MW_1$ and $MW_2$.

The shape of the 2D mass window in used examples is a square. The described above method allows calculating the purity of 2D samples for the case of the selection with the 2D mass window being a circle. It is only needed to use the weight $w$ and the $WM_1$ mass window of the $2R_{MW}$ size around the $J/\psi$ mass or the calculation of the function $f_{j_{f-f}}(M)$. $R_{MW}$ is the radius of the circle mass window used to select $J/\psi$. The weight $w$ is

$$w = \frac{\sqrt{R_{MW}^2 - (m_{J/\psi} - M_{J/\psi})^2}}{R_{MW}},$$  \hspace{1cm} (8)

where $m_{J/\psi}$ is the $J/\psi$ mass [1] and $M_{J/\psi}$ is the mass of the $J/\psi$ candidate which is inside the mass window $WM_1$.

4. Checks using Monte Carlo methods

The contamination of the background in a $J/\psi$ pair sample increases as the square of the mass window size because the background is approximately uniform. The signal grows much slower because of its peaked shape. This means that the signal/background ratio depends stronger on the size of the mass window as compared to the single particle selection. The knowledge of the sample purity can help find the golden mean between the minimal background
Figure 4: $N_{J\psi ff}$ and $n_{J\psi ff}$ functions. The function $N_{J\psi ff}(M)$ is shown by points. The function $n_{J\psi ff}(M)$ is the dashed line. The horizontal line height is the number of selected $J/\psi$ candidates in the 2D window. The vertical line shows the mean value of the invariant mass in $MW_1$. a) and b) correspond to selection cases illustrated in Fig.2 and mass window $2.85 < M_{J/\psi} < 3.3 GeV/c^2$.

and good signal. We performed MC studies using a generic simulation of the detector response to check results of the previous section. Using (1), we generated the statistics ($\sim 1000$) of 2D MC samples defined by the mean purity and the event number inside the base mass window $2.85 < M_{J/\psi} < 3.3 GeV/c^2$. Simulations are in the range of $1.5 < M_{J/\psi} < 5 GeV/c^2$. To each sample, the described above method has been applied. Table 1 illustrates purities and signal values found for the two selections a) and b) in Fig.2 and different mass windows. We give the average and root mean square (RMS) values. The data confirm the sensitivity of the suggested method to the purity change and show high agreement between MC and reconstructed values.

### Table 1: Purity of double $J/\psi$ different selection samples.

| version | window, GeV/c² | purity, % | signal, events |
|---------|----------------|-----------|----------------|
|         |                 | MC        | our method     | MC              | our method     |
| a       | 2.85 – 3.3      | 24.9 ± 1.6| 24.0 ± 3.1     | 199 ± 16        | 193 ± 29       |
|         | 2.95 – 3.2      | 39.9 ± 2.5| 39.7 ± 3.0     | 159 ± 13        | 153 ± 26       |
| b       | 2.85 – 3.3      | 36.6 ± 3.4| 36.6 ± 4.3     | 70 ± 8          | 70 ± 13        |
|         | 2.95 – 3.2      | 50.8 ± 4.8| 51.8 ± 6.2     | 56 ± 8          | 57 ± 11        |

A comparison of our results with the standard fitting of the signal + background sum [6] is an important check of our method. In the simulation of the detector reaction, we use the signal specified by 6 parameters (parameters of 2 Gaussians) and the background given by 4 parameters of the polynomial. In the inverse task, we leave only 2 free parameters: signal and background values. We take the parameters defining their shapes from MC and fix them in
the fit. This corresponds to exact knowledge about the detector response. The 2D fit is in the square of $2.3 < M_{J/\psi} < 3.9 \text{GeV}/c^2$. We use average values of MC sample as initial approach for the fit.

Table 2 illustrates values for the mass window $2.85 < M_{J/\psi} < 3.3 \text{GeV}/c^2$ reconstructed by both methods. Just like in Table 1, average and RMS values are given. Different signal and background conditions have been used in the used MC examples. It allows us to check the reliability of the purity calculation method in a wide region. Table 2 illustrates high performance of the method provided in all regions of purities and signals. The 2D fit method, which uses full information about signal and background shapes, produces better dispersion at low purities. At high purities, we do not observe an advantage of the 2D fit method. Moreover, it produces strong systematic shift which reduces real values of the signal. The investigation of the reason of this phenomenon is beyond the scope of the present study. We can only say that at high purities, the mean value of $\chi^2$ divided by the number of degrees of freedom is significantly less than one. In contrast to the 2D fit method, the purity calculation method suggested in this study does not require a hypothesis about the background and signal shapes to calculate the purity of the 2D sample.

| version | purity, % | signal, events |
|---------|-----------|----------------|
|         | MC  | 2D fit | our method | MC  | 2D fit | our method |
| a       | 5.1 ± 0.8 | 4.8 ± 0.6 | 5.1 ± 1.8 | 41 ± 7 | 39 ± 5 | 42 ± 15 |
|         | 15.0 ± 1.3 | 14.2 ± 1.0 | 14.9 ± 2.7 | 122 ± 12 | 119 ± 10 | 120 ± 24 |
|         | 30.0 ± 1.7 | 30.6 ± 1.6 | 29.3 ± 3.3 | 242 ± 16 | 245 ± 18 | 236 ± 32 |
|         | 50.1 ± 1.8 | 51.9 ± 1.7 | 48.3 ± 2.0 | 403 ± 20 | 417 ± 25 | 390 ± 28 |
|         | 75.1 ± 1.5 | 68.4 ± 6.2 | 74.1 ± 1.4 | 600 ± 23 | 547 ± 54 | 592 ± 27 |
|         | 95.0 ± 0.1 | 90.8 ± 4.0 | 95.2 ± 0.1 | 759 ± 18 | 723 ± 51 | 759 ± 18 |
| b       | 5.0 ± 0.2 | 5.5 ± 1.5 | 4.9 ± 2.5 | 11 ± 3 | 11 ± 3 | 10 ± 6 |
|         | 15.0 ± 2.5 | 16.1 ± 2.5 | 14.4 ± 3.9 | 31 ± 6 | 32 ± 6 | 30 ± 10 |
|         | 30.0 ± 3.3 | 26.6 ± 3.0 | 29.3 ± 4.5 | 61 ± 8 | 53 ± 8 | 59 ± 13 |
|         | 50.1 ± 3.5 | 33.4 ± 3.4 | 50.1 ± 3.8 | 101 ± 10 | 67 ± 10 | 101 ± 14 |
|         | 74.9 ± 3.0 | 63.9 ± 5.2 | 75.7 ± 2.5 | 150 ± 11 | 128 ± 15 | 151 ± 14 |
|         | 95.0 ± 1.6 | 81.7 ± 4.0 | 95.9 ± 1.0 | 188 ± 7 | 163 ± 12 | 189 ± 8 |

5. Conclusion

The purpose of this paper was to present the method of the two dimensional background calculations for an analysis of events with two particles of the same type observed by detectors of high-energy physics. The standard two-dimensional integration is replaced by an approximation of a specially constructed one-dimensional function. The number of the signal events is found by the subtraction of the background from the number of the total selected events. It allows calculating the purity value of the selected events sample. The procedure does not require a hypothesis about shapes of the background and signal events. High performance of the purity calculation method is demonstrated using generic Monte Carlo examples of double J/ψ samples. It has been demonstrated that the 2D fit of the signal like two-dimensional Gaussian by known shape functions of the signal and background could produce a significant systematic shift. The shift is bigger at low statistics. For this reason, it is important to check the fit result by the suggested method.
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