Generating curvature perturbations with or without MSSM flat directions

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Abstract. We consider instant preheating as the mechanism for generating the curvature perturbation at the end of chaotic inflation. Then we examine if inflation could be driven by a minimal supersymmetric standard model (MSSM) flat direction or a sneutrino. Our simple mechanism relaxes some serious constraints that appeared in past studies, making inflation driven by an MSSM flat direction or a sneutrino more plausible.

Keywords: cosmological perturbation theory, inflation, cosmology of theories beyond the SM

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## 1. Introduction

In the standard scenario of the inflationary Universe, the observed density perturbations are assumed to be produced by a light inflaton rolling down its potential during inflation. This ‘standard mechanism’ of generating the curvature perturbation has been investigated by many authors [1]. Although it may be possible to construct some inflationary scenarios with this mechanism, it is not always easy to find a situation where an inflaton field appears naturally in the grand unified theory (GUT) of particle physics, and at the same time has all the required conditions for inflation satisfied without fine-tunings or an additional hidden sector for inflation. In this paper we focus our attention on inflationary models in which an MSSM flat direction or a sneutrino plays the role of an inflaton. As will be presented in section 2, past studies on the MSSM (or sneutrino) inflation were studied on the basis of a conventional mechanism of generating the curvature perturbations related to fluctuations of an inflaton. The conditions required for the coupling constants in the theory were very severe, demanding fine-tunings of the MSSM parameters. Hence, our question in this paper is very simple. Is it possible to solve or relax the above-mentioned conditions with a new mechanism for generating curvature perturbations?

Recently, a new inflationary paradigm has been developed in which the conventional slow-roll picture does not play an essential role in generating the curvature perturbation. Along the lines of this new inflationary paradigm, we will consider a scenario in which the perturbation of a light field is converted into the curvature perturbation at the time of instant preheating that occurs at the end of inflation, when the inflaton kinetic energy is significant. The most important point is that in this new inflationary paradigm the light field is not identified with the inflaton. The light field is decoupled from the inflationary dynamics during inflation, but it plays a significant role at reheating. The idea related
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to such a light field has already been investigated by many authors. Although there have been many attempts in this direction, the most famous examples would be the curvaton models [2, 3]. In the curvaton models, the origin of the large-scale curvature perturbation in the Universe is the late-decay of a massive scalar field that is called the ‘curvaton’. The curvaton paradigm has attracted much attention because it was thought to have obvious advantages. For example, since the curvaton is independent of the inflaton field, there was the hope [4] that the curvaton scenario, especially in models with a low inflationary scale, could cure serious fine-tunings associated with the inflation models. Despite the advantages of the curvaton scenario, Lyth suggested [7] that there is a strong bound for the Hubble parameter during inflation, even when the curvatons are introduced to the model. The bound obtained by Lyth [7] was critical, but it was later suggested by Matsuda [8] that the difficulty could be avoided if an additional inflationary expansion or a phase transition were present [9]. At first this solution seemed quite generic since there could be several phase transitions after inflation, but it was later discovered that this scenario requires many additional components. It is possible to solve the problem [7] with the idea suggested in [8], but then one cannot escape from uncertainties coming from the additional components. Besides the curvatons, the problem of uncertainties is common in almost all the inflationary scenarios that require fields that cannot appear in the standard model (SM). This is one of our motivations to study inflation that occurs in an MSSM or sneutrino direction.

Another idea in this inflationary paradigm was investigated most recently by Lyth [10], where an argument was presented that the density perturbations can be generated at the end of inflation by the fluctuations of the number of e-foldings induced by the fluctuations related to a light scalar field other than the inflaton. An important advantage in the new mechanism is that there are no such stringent conditions coming from the requirement of (1) late-time curvaton dominance, and (2) successful reheating. Using this new idea, we studied, in [12, 13], the generation of curvature perturbations in fast-roll inflationary models. Since the hybrid-type potential appears naturally in brane–antibrane inflation and a light field may generically appear in brane inflationary models, the new mechanism is very useful in brane inflationary models. The most obvious example along these lines would be throat inflation, where a light field related to an enhanced isometry appears at a distance from the moving brane as we have proposed in [12] and [13]. On the other hand, if thebrane motion is relativistic, or the kinetic energy of the inflaton is significant, the so-called ‘instant preheating’ should be significant in such analysis.

1 Many attempts have been made to construct realistic models with low inflationary scale [5], where cosmological defects might play an essential role [6]. In spite of the merits of these low-scale inflationary models, an ultimate solution has not yet been found.

2 Our chaotic inflation models can work only if the value of the inflaton is superplanckian. Hence in our scenario we expect that the influence of a multitude of non-renormalizable terms that might appear in the scalar potential is substantial. In our scenario we will consider a tiny Yukawa coupling $y \ll 1$ that can be related to the very weak violation of an approximate symmetry. Hence the above expectation would be conceivable if such higher terms appear with the combination of $\sim |W|^n$ which contains $y^n$, or more generically each combination of the corresponding operator is accompanied by a suppression factor. However, this clearly pushes the inflationary physics beyond the justification of the MSSM.

3 See also [11], where different models for generating the curvature perturbation were proposed along the lines of this new inflationary paradigm. Although we do not mention these alternative ideas, they are equally important.

4 Riotto and Lyth have given another useful discussion on this point [14].
order to solve this problem, we proposed in [15] an alternative mechanism along the lines of this new inflationary paradigm. This is a generic mechanism for generating the curvature perturbation that works with an instant preheating at the end of inflaton. The instant preheating should be equipped with a light field, whose expectation value plays the role of an impact parameter. This alternative mechanism of generating curvature perturbation at the end of inflaton will be quite useful in the MSSM inflationary models, since in such models instant preheating is a natural mechanism of reheating and at the same time there are many flat directions that can play the role of the light field.

In this paper we will examine whether the serious conditions that have been required in past studies can be relaxed by introducing the new mechanism we presented in [15]. It will be helpful to conduct a short review of previous attempts in this direction showing why it was difficult to make an inflationary model on an MSSM (or sneutrino) direction. Conclusions are given in section 3. We will show that one can solve or relax the serious conditions that were found in past studies, by introducing this new mechanism to the theory.

2. New mechanism and fine-tunings

2.1. New mechanism

Generically a multi-field inflationary model is described by the background inflaton fields \( \phi_i(t) \) which evolve according to the system of coupled differential equations

\[
\ddot{\phi}_i + 3H\dot{\phi}_i + \frac{\partial V}{\partial \phi_i} = 0, \quad i = 1, \ldots, n
\]  

(2.1)

and

\[
H^2 = \frac{8\pi}{3M_p^2} \left[ \sum_i \frac{\dot{\phi}_i^2}{2} + V \right].
\]  

(2.2)

Without losing general applicability, we can discuss our mechanism with two orthogonal fields, \( \phi_1 \) and \( \phi_2 \), where \( \phi_1 \) is a conventional inflaton field and \( \phi_2 \) is the light field. The potential \( V(\phi_1, \phi_2) \) is characterized by a hierarchy between the masses of the fields, and can be modelled by

\[
V(\phi_1, \phi_2) = \frac{m_1^2}{2} \phi_1^2 + \frac{m_2^2}{2} \phi_2^2,
\]  

(2.3)

where \( m_1 \approx O(H) \) and \( m_2 \ll m_1. \) We consider the instant preheating model [17] as the process through which the inflaton decays into lighter particles. The typical coupling to the preheat field \( \chi \) is written as

\[
\mathcal{L} = \frac{g}{2}(\phi_1^2 + \phi_2^2)\chi^2,
\]  

(2.4)

which gives a mass to the preheat field. Applying the result obtained in [17], the comoving number density \( n_\chi \) of the preheat field \( \chi \) produced during the first half-oscillation of \( \phi_1 \) becomes

\[
n_\chi \approx \frac{(g|\dot{\phi}_1(t_*)|)^{3/2}}{8\pi^3} \exp \left[ -\frac{\pi g|\phi_2(t_*)|^2}{|\dot{\phi}_1(t_*)|} \right].
\]  

(2.5)

A different approach has been given by Kolb et al [16] who assumed \( m_2 \approx m_1. \)
where \( t_* \) is the time when the inflaton \( \phi_1 \) reaches its minimum potential at \( \dot{\phi}_1 = 0 \) and where the light field \( \phi_2 \) may still have an expectation value \( \phi_2(t_*) \neq 0 \). We used \( \dot{\phi}_2 = 0 \) and \( \delta \phi_2 = 0 \) to derive equation (2.5). To obtain an estimate of the curvature perturbation through equation (2.5), we need to write down an expression for \( \frac{\delta n_\chi}{n_\chi} \):

\[
\frac{\delta n_\chi}{n_\chi} = \frac{2\pi g |\phi_2(t_*)|^2 |\delta \phi_2(t_*)|}{|\phi_1(t_*)||\phi_2(t_*)|},
\]

where it is assumed that \( |\delta \phi_2(t_*)| \ll |\phi_2(t_*)| \) so that we can neglect higher terms. To determine the curvature perturbation produced during the decay process of the preheat field \( \chi \), it is sufficient to note that the generated energy density is proportional to the comoving number density \( n_\chi \). Assuming a smooth decay process of the preheat field, the curvature perturbation \( \zeta \) generated during the instant preheating is

\[
\zeta \simeq \frac{\alpha}{n_\chi} \frac{\delta n_\chi}{n_\chi},
\]

where \( \alpha \) is a constant whose numerical value depends on how the density of the produced particle redshifts with the Universe expansion. Since the field \( \phi_2 \) is approximately massless during inflation, the value of the fluctuation is given by \( \delta \phi_2 \simeq H_1/2\pi \simeq V_1^{1/2}/(2\sqrt{3\pi} M_p) \).

In the simplest case of a single-stage inflationary model, the curvature perturbation \( \zeta \) is approximately

\[
\zeta \simeq \frac{\alpha 2\pi g |\phi_2(t_*)|^2 |\delta \phi_2(t_*)|}{|\phi_1(t_*)||\phi_2(t_*)|}.
\]

Since the field \( \phi_2 \) is very light, it is possible to have \( \phi_2(t_i) \simeq \phi_2(t_*) \), where \( t_i \) is the time when the inflaton \( \phi_1 \) starts fast-rolling. As we are considering a case where the kinetic energy of the inflaton field \( \phi_1 \) is significant at the time of preheating,

\[
\dot{\phi}_1(t_*)^2 \simeq m^2 |\phi_1(t_i)|^2 \simeq H_1^2 M_p^2
\]

is a natural consequence. We must also consider the condition for the efficient production of the preheat field \( \chi \), which is written as

\[
m_\chi^2 \simeq g |\phi_2(t_*)|^2 < \dot{\phi}_1(t_*).
\]

Considering the above conditions, we found a relation

\[
\zeta \simeq \frac{\alpha 2\pi g |\phi_2(t_*)|^2 |\delta \phi_2|}{|\phi_1(t_*)||\phi_2(t_*)|} \simeq \frac{\alpha g |\phi_2(t_*)|}{M_p} \simeq \frac{\alpha g^{1/2} \sqrt{m \phi_1(t_i)}}{M_p} \simeq \frac{\alpha g^{1/2} \sqrt{10 m}}{M_p}.
\]

Here \( \alpha \) depends on the redshift of the final product. As we are considering instant preheating in an MSSM direction, the final product is assumed to be massless (\( \alpha = 1/4 \)). The value of \( \phi_2 \) is not a parameter of the underlying theory, but an initial condition at the beginning of inflation. The value of \( \phi_2 \) is important since it determines the value of \( \zeta \). The fate of \( \phi_2 \) is discussed in the appendix.

This is the new mechanism that we will consider in this paper. With this new mechanism for generating curvature perturbations, is it possible to construct a chaotic inflation model with or without MSSM directions and to solve or relax the conditions obtained in past studies on the MSSM and sneutrino inflation?
2.2. Quartic potential

There may be a rather peculiar possibility for chaotic inflation with the MSSM direction, which is to use a $D$-flat but not an $F$-flat direction [19]. In this scenario we consider the quartic potential

$$V(\phi) = \frac{\lambda}{4} \phi^4,$$

(2.12)

where $\lambda$ is a dimensionless coupling constant. The value of the inflaton field when a fluctuation denoted by $k$ exits the horizon is related to the number of e-foldings $N_k$ that have elapsed after the horizon exit as

$$\phi_k \simeq \sqrt{8N_k M_p}.$$

(2.13)

Hence, using a standard calculation [1], the Cosmic Background Explorer (COBE) normalization is given by

$$\frac{\sqrt{\lambda}}{8} \left(\frac{\phi_k}{M_p}\right)^3 \simeq 5.2 \times 10^{-4},$$

(2.14)

which means that normalization of the primordial density fluctuation requires a tiny value for the coupling constant $\lambda \sim 10^{-13}$ [1]. One way to solve this problem is to expect an approximate symmetry, such as the flavour symmetry or $R$-parity in the MSSM. If the inflaton field $\phi$ is related to a $D$-flat direction in the MSSM, one can obtain the quartic inflaton potential considering the Yukawa interaction that lifts the $D$-flat direction. Then the coupling constant $\lambda$ is given by $\lambda \simeq y^2$, where the Yukawa coupling constant is denoted by $y$. In the present case we have to consider an approximate symmetry that protects the tiny Yukawa coupling. Actually, smaller $\lambda$ is more favourable if it is related to the $R$-parity violating Yukawa coupling. We will revisit this issue and give more details in the appendix.

We think it is important to notice here that instant preheating would be a dominant mechanism for reheating in the MSSM inflationary model. It was discussed by Kasuya et al [19] that instant preheating related to Yukawa couplings is efficient in this model. In this case it was suggested that the effect of a $A$-term is so small that the inflaton exhibits almost straight-line motion on the complex plane. Hence the disturbance of the homogeneous motion caused by the $A$-term was neglected. In a brane inflationary scenario of ‘trapped inflation’ [20], it has been discussed that open strings stretched between branes play the role of a preheat field. In terms of the brane worldvolume fields the preheat field was identified with massive gauge boson that becomes massless at enhanced symmetry point (ESP). In the scenario of trapped inflation the W boson was assumed to be a stable excitation; however, in an MSSM inflationary model gauge bosons can decay into lighter particles. If there is something like a Heisenberg symmetry that protects a flat direction from obtaining $O(H)$ mass during inflation, and also if this direction gives mass to the corresponding preheat field, the impact parameter for the instant preheating $|\phi_2|$ is non-zero and also it has a Gaussian fluctuation that exits the horizon during inflation. This is the required situation of our inflationary model. The curvature perturbation generated with the instant preheating becomes important in our model.

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6 See the appendix for more details.

7 $R$-parity is exact in the MSSM. In this case we are expecting something beyond the MSSM.

8 In some other cases the amplitude of a oscillation might be much smaller than that in an MSSM chaotic inflation, and the oscillation might be free from efficient reheating induced by instant preheating. A typical example would be found in the analysis of Affleck–Dine baryogenesis [18].
Despite the novelty of the idea, the actual calculation of the new mechanism is straightforward. The fluctuation of the number density of the preheat field takes precisely the same form as equation (2.6), but the kinetic energy $\dot{\phi}_1$ is different from equation (2.9). Due to the quartic term in the inflaton potential, $\dot{\phi}_1$ is given by
$$\dot{\phi}_1 \simeq \lambda |\phi_1(t_i)|^4 \simeq 10^4 \lambda M_p^4.$$  
(2.15)
This relation changes the curvature perturbation obtained in equation (2.11) as
$$\zeta \simeq \frac{\alpha^2 \pi g |\delta \phi_2|}{|\phi_1(t_*)|} \frac{|\phi_2(t_*)|}{|\phi_2(t_*)|} \simeq \frac{\alpha g |\phi_2(t_*)|}{M_p} < \alpha g^{1/2} 10^{1/4} \lambda^{1/4}.$$  
(2.16)
Hence, the curvature perturbation generated at the end of inflation can be fitted to WMAP data if $\lambda > 10^{-24} g^{-2}$, which gives the lower bound for $\lambda$. On the other hand, the `standard' curvature perturbation (2.14) generated by an inflaton fluctuation becomes large if $\lambda > 10^{-13}$. Hence we still need to consider a fine-tuning $10^{-24} < \lambda < 10^{-13}$ for $g \sim 1$ to suppress the unwanted contribution from the inflaton. As we have presented above, it is possible to use the new mechanism to generate the curvature perturbation in MSSM inflation. The obstacle in the above attempt was the large curvature perturbation generated by an inflaton when $\lambda > 10^{-13}$. In the $D$-flat inflationary model, we found that this problem is solved if a fine-tuning is introduced to the coupling constant $\lambda$. Although a kind of fine-tuning is required for the coupling constant $\lambda$, the experimental upper bounds obtained from phenomenological considerations are consistent with $10^{-24} < \lambda < 10^{-13}$ [19] if $\lambda$ is related to an $R$-parity violating Yukawa coupling. We are expecting that $\lambda$ is protected by an approximate symmetry, which may (or may not) explain the smallness of $\lambda$. As a result, the new mechanism `relaxes' the severe condition for $\lambda$ if it is related to the weak violation of the approximate $R$-parity.

2.3. Quadratic potential

Perhaps the straightforward extension of the previous example would be to use a quadratic potential instead of the quartic potential. The known example in this direction is sneutrino inflation. Let us first show the known problems and the required fine-tunings in sneutrino inflation, and then show how one can escape from these problems using the new mechanism. The sneutrino is the scalar supersymmetric partner of a heavy singlet neutrino in the minimal seesaw model of neutrino masses. The possibility that inflation was driven by a sneutrino has been discussed by many authors [21]. Following the past studies, we consider chaotic inflation with a $V = \frac{1}{2} m^2 \phi^2$ potential. The number of e-foldings $N_e$ is given by
$$N_e \simeq \frac{1}{4} \frac{\phi^2}{M_p^2} \simeq 50,$$  
(2.17)
which gives the value of $\phi$ at the beginning of inflation, $\phi(t_i) \simeq \mathcal{O}(10) M_p$. The scale of the inflaton potential is normalized by the WMAP data on density fluctuations,
$$\zeta \simeq \frac{V^{1/2}}{2 \sqrt{6 \pi M_p^2} \epsilon^{1/2}} \simeq 10^{-5},$$  
(2.18)

Notice that there are only upper bounds for the couplings related to the $R$-parity violating terms, while there are strict lower bounds for the $R$-parity conserving Yukawa terms. See the appendix for more details.
where $\epsilon \equiv (\frac{1}{2})M_p^2\langle (dV/d\phi)/V \rangle^2$ is a slow-roll parameter. From the above equations we can find the required mass as $m \simeq 10^{13}$ GeV, which is within the range of heavy singlet sneutrino masses. The problem in this scenario is that the reheating temperature $T_R \simeq 10^{13}$ GeV is much larger than the bound obtained from the thermal production of gravitinos [22]. It may be possible to make $T_R$ much smaller than the bound, if the neutrino Yukawa coupling $Y_\nu$ is much smaller than unity. For example, the reheating temperature becomes as low as $T_R \simeq 10^8$ GeV if the neutrino Yukawa coupling is $|Y_\nu Y_\nu^T| \simeq 10^{-12}$.

Otherwise, one should expect an additional late-time entropy production [1, 23, 24], which would be the secondary inflationary expansion that reduces unwanted particles. However, if there is a late-time entropy production, the baryon number asymmetry of the Universe must be produced after the entropy production, which induces another problem in cosmology that may or may not be solved by introducing additional ingredients to the theory [25].

Let us show how this serious condition can be relaxed when the new mechanism is taken into account. Our idea is very simple. Instead of introducing a tiny Yukawa coupling constant, we consider a smaller mass for the sneutrino inflaton. Then, as we have mentioned above for the $D$-flat inflationary model, we can assume that reheating is induced by instant preheating. Any flat direction that gives a mass for the preheat field can play the role of the impact parameter $\phi_2$, provided that the direction is flat. Then the dominant contribution comes from equation (2.11) if the inflaton mass is smaller than $10^{13}$ GeV. On the other hand, from equation (2.11) the ratio between $m$ and $M_p$ is bounded from below, and is given by

$$\frac{m}{M_p} > 10^{-11} \text{ g}^{-1}. \quad (2.19)$$

This suggests that the inflaton mass $m$ can be as light as $O(10^7 \text{ GeV})$ for $g \simeq 1$. In the present case the bound for the sneutrino mass is $10^7 \text{ GeV} < m \leq 10^{13}$, which is much looser than the condition found in the previous study and is suitable for solving the gravitino problem in sneutrino inflation.

Another important point that we can see from the above analysis is that the inflaton field $\phi_1$ can now be identified with an MSSM flat direction itself. In this case the inflaton field $\phi$ would be the heaviest direction which is rather heavier than the lightest direction $\phi_2$ so that inflation ends before the light field $\phi_2$ starts to roll down the potential. If the dominant part of the curvature perturbation is generated at the end of inflation, which occurs if $m < 10^{13}$ GeV, the scale of the potential is not normalized by equation (2.18).

10 Weak inflation [23] is a mechanism for such late-time entropy production and is supposed to have the Hubble parameter not much larger than $O(\text{TeV})$. There are many ideas for weak inflation and other kinds of late-time entropy production; however, the most important realization would be the thermal inflation model. The idea of thermal inflation has been given by Lyth and Stewart in [24].

11 When one makes predictions in the framework of the MSSM, one encounters parameter freedom which is mainly due to soft SUSY-breaking terms. The predictive power of the model may be increased if one restricts this freedom, which is the hypothesis called ‘universality of the soft terms’. Under this assumption one is left with five free parameters. However, the MSSM with the universal soft masses might not work in practice. Our inflationary scenario supports non-universal soft masses with at least one parameter appearing at $O(10^7) \text{ GeV}$. This is an interesting possibility that requires further study. See also [26] in which models with split supersymmetry have been proposed. Alternatively, one may consider some extension of the MSSM to include a heavy scalar field.
2.4. \textit{A}-term

We would like to make some comments on other inflationary models of the MSSM flat direction. Recently it has been pointed out that an MSSM flat direction might support slow-roll inflation with an initial field value much less than the Planck scale $M_p$ and the tree-level potential

$$V(\phi) = m_\phi^2 \phi^2 + A \cos(n\theta + \theta_0) \frac{\phi^{n+3}}{M^n}. \quad (2.20)$$

This potential may have a secondary minimum at $\phi_{2\text{nd}} = \phi_0 \simeq (m_\phi M_p^{-3})^{1/(n-2)} \ll M_p$, provided that the coefficient of the $A$-term satisfies the condition

$$A \geq A_c \equiv 2\sqrt{2(n-1)m_\phi}. \quad (2.21)$$

Moreover, if $A$ takes the critical value $A_c$ the first and the second derivatives of $V$ vanish at $\phi_0$. If $A = A_c$, the potential near the saddle point is now very flat along the real direction, and it becomes a successful inflaton candidate in the MSSM potential [27]. On the other hand, one might think the condition $A = A_c$ is a kind of fine-tuning that must be explained by the underlying theory, making the above discussion not within the MSSM. We may agree with these critical comments; however, the motivation to make inflationary scenarios on the MSSM direction is still very strong in this scenario. If the inflaton sector belongs to an unknown (hidden) sector, then there would be too many uncertainties which really hamper the progress not only in particle cosmology but also in GUT phenomenology. On the other hand, if the MSSM flat direction could play the role of an inflaton, there will be a chance to prove by some observations the existence of such fine-tunings. If the existence of such fine-tunings may be proved by observations, one will be forced to consider a theory behind the MSSM that can explain such fine-tunings.

Let us examine if our mechanism can help $A$-term inflation. Since $V'$ appears in the calculation of the number of $e$-foldings, the fine-tuning related to $V'$ does always remain. On the other hand, another condition for the $\epsilon$-parameter given by the COBE normalization $V^{1/4}/\epsilon^{1/4} = 0.027 M_p$ does not appear if the curvature perturbation is generated by an alternative. The most significant condition for the $A$-term inflationary scenario comes from the spectral index $n \simeq 1 + 2\eta$ [28] which is related to the $\eta$-parameter, but it does not matter if the curvature perturbation is generated by an alternative. As we have stated above, our mechanism requires an additional field $\phi_2$ that is light ($m_2 \ll H_I$) during inflation. Since $A$-term inflation starts with $m_0 \ll M_p$, the Hubble parameter during inflation is $H_I \ll m_\phi$. For example, for $\phi_0 \simeq 10^{-4} M_p$ the potential is $V(\phi_0) \sim m_\phi^2 \phi_0^2$, and the Hubble constant is $H_I \sim 10^{-4} m_\phi$. Hence our mechanism requires a light field whose mass satisfies the condition $m_2 \ll 10^{-4} m_\phi$, which introduces another kind of problem to the model. Therefore, for successful generation of the spectrum in $A$-term inflation one can choose either to consider fine-tuning for the shift in the mass\footnote{See figures in [28]. $m_\phi$ is the universal soft mass for the components of the inflaton direction. Fine-tunings for the radiative corrections are included.} $\delta^2/m_\phi^2 \sim 10^{-18}$, or to introduce a kind of hierarchy $m_2/m_\phi \ll 10^{-4}$. In the latter case, considering equation (2.11) we found that the inflaton mass must be as large as $m_\phi > 10^{10}$ GeV, and a weak fine-tuning is still required for $\delta^2/m_\phi^2$ to have $N_e \sim 50$. 

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3. Conclusions

A new mechanism for generating the curvature perturbation at the end of inflation is discussed in this paper. The dominant contribution to the primordial curvature perturbation may be generated by this new mechanism, which converts the perturbation related to a light field into curvature perturbation during the period of instant preheating. The light field is ‘not’ the inflaton field. Based on this new inflationary paradigm, we considered the possibility that chaotic inflation is driven by an MSSM flat direction. We also considered sneutrino inflation as an example for a quadratic inflationary model. Our simple mechanism relaxes some serious constraints that appeared in past studies, making chaotic inflation on an MSSM and a sneutrino direction more plausible than ever before.

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Appendix. Cosmological parameters and MSSM couplings

A.1. Basic cosmological parameters

We consider an arbitrary monomial potential which consists of a single power of $\phi$:

$$V = \lambda_\alpha \frac{\phi^\alpha}{M_p^\alpha - 4}. \quad (A.1)$$

Here $\alpha$ is a positive integer. The slow-roll parameters are

$$\epsilon = \frac{\alpha^2 M_p^2}{2 \phi^2},$$
$$\eta = \alpha (\alpha - 1) \frac{M_p^2}{\phi^2}. \quad (A.2)$$

Assuming that inflation ends when $\eta = 1$, we obtain $\phi_e^2 = \alpha (\alpha - 1) M_p^2$. Then the value of $\phi$ is given by

$$\phi_k = \sqrt{\phi_e^2 + 2 \alpha N_k M_p} = \sqrt{\alpha (\alpha - 1 + 2 N_k) M_p} \simeq \sqrt{2 N_k \alpha M_p}, \quad (A.3)$$

when the cosmological scale related to the number of e-foldings $N_k$ exits the horizon$^{13}$.

It would be important to note here that the chaotic inflation models can work only if the value of the inflaton is superplanckian. This is evident in our present discussions. At such displacements one expects a multitude of non-renormalizable terms to appear and become important in the scalar potential, whose influence on inflation is substantial. This clearly pushes inflationary physics beyond the justification of the MSSM. Therefore, our present model can be applied to the MSSM field contents while it requires physics beyond the justification of the MSSM.

$^{13}$ Using the exact expression we find $\epsilon = \alpha / (2 (\alpha - 1 + N_k))$, which suggests that the approximate expression $\epsilon \simeq \alpha / 2 N_k$ deviates by a few per cent from the exact value. The exact value should be used for the spectral index.
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The COBE normalization is given by
\[ \sqrt{\frac{\lambda}{\alpha}} \left( \frac{\phi}{M_p} \right)^{\alpha/2+1} \simeq 5.2 \times 10^{-4}. \] (A.4)

The COBE normalization corresponds to \( m \simeq 10^{13} \) GeV for the quadratic potential \( V = m^2 \phi^2/2 \), and to \( \lambda \simeq 10^{-13} \) for the quartic potential \( V = \lambda \phi^4/4 \) [1].

For the conventional inflationary scenario the spectral index \( n \) is given by
\[ n \simeq 1 - 6 \epsilon + 2 \eta. \] (A.5)

In our present case the spectral index is given by \[ n_{PR} \simeq 1 - 2 \epsilon_H, \] (A.6)
where the slow-roll parameter \( \epsilon_H \) is
\[ \epsilon_H \equiv -\frac{\dot{H}}{H^2}. \] (A.7)

As we are considering a very light field \( \phi_2 \), the corrections from \( \phi_2 \) are negligible. Notice that \( n_{PR} \) does depend on the \( \epsilon \)-parameter of the inflaton potential, but it does not depend on the \( \eta \)-parameter of the inflaton potential. This is very important for the discussion of the fine-tuning in the A-term inflation model. If there is no slow-roll field other than the inflaton, the spectral index is \( n_{PR} \simeq 1 - \alpha/2N_k \). Of course there is an ambiguity in \( n_{PR} \) since one cannot simply disregard the corrections to \( \epsilon_H \) from other fields which might still be rolling down the potential and contribute to \( \epsilon_H \) at the time not so long after the onset of the inflationary expansion.

A.2. MSSM parameters

We first consider the \( D \)-flat direction lifted by the conventional Yukawa terms that conserve \( R \)-parity. We denote the MSSM superfields as \( Q(3, 2, 1_6), U(3^*, 1, -2_3), D(3^*, 1, 1_3), L(1, 2, -\frac{1}{2}), E(1, 2, \frac{1}{2}) \) and \( H_u(1, 2, \frac{1}{2}), H_d(1, 2, -\frac{1}{2}) \), where the quantum numbers for the \( SU(3)_C \times SU(2)_L \times U(1)_Y \) gauge group are shown in the parentheses. The relevant terms are given by
\[ W = (Y_U)_{ij} Q_i U_j H_u + (Y_D)_{ij} Q_i D_j H_d + (Y_E)_{ij} L_i E_j H_d, \] (A.8)
where the indices \( i \) and \( j \) denotes the generation. If we consider chaotic inflaton with the quartic potential of the \( Q_i U_j H_u \) direction, the coupling constant \( \lambda \) is given by \( \lambda \simeq |(Y_U)_{ij}|^2 \).

Since we are expecting small coupling \( \lambda \leq 10^{-13} \) for the quartic inflaton potential, there is the condition \( |Y_{ij}| \leq 10^{-7} \) that must be satisfied by the corresponding Yukawa coupling. This condition is so severe that naively the above Yukawa couplings cannot satisfy the condition. However, besides the tree-level contributions, there could be radiative corrections coming from off-diagonal (flavour-violating) elements. It has been discussed by Kasuya et al [19] that these corrections may reduce the expected value of the Yukawa couplings to be within the allowed range of the quartic inflaton scenario.

Besides the \( R \)-parity conserving couplings that we have shown above, there could be many kinds of \( R \)-parity violating Yukawa terms\(^{14}\). The experimental bounds suggest that

\(^{14}\) See [30] for experimental bounds.
the magnitude of the $R$-parity violating Yukawa couplings are typically below $\sim O(10^{-7})$. Hence, the $R$-parity violating terms are the possible candidates of the MSSM inflaton potential if the curvature perturbation is generated by instant preheating.

One might think that in the present scenario we expanded the parameter space of chaotic inflation towards the wrong direction, since this work allows $\lambda$ to be smaller than but not larger than $\lambda \sim 10^{-13}$. However, there are strict experimental upper bounds on the Yukawa couplings related to the violation of the approximate $R$-parity. Hence the smaller value is more favourable if the inflaton is identified with such directions.

### A.3. Fate of the light field $\phi_2$

In the above analysis we assumed that the expectation value of the light field $\phi_2$ is approximately a constant during inflation. Due to the exponential factor in $n_{\chi}$, there is an upper bound for the expectation value of $\phi_2$,

$$\phi_2 < \sqrt{\dot{\phi}_1} \sim V_I^{1/4}. \tag{A.9}$$

Since in the present case the reheating temperature becomes as large as $T_R \sim V_I^{1/4}$, the thermal effects from the plasma strongly affect the dynamics of the flat direction $\phi_2$ evolution after instant reheating. Hence in the present case $\phi_2$ evaporates soon after reheating.

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