We establish a nonminimal Einstein-Yang-Mills-Higgs model, which contains six coupling parameters. First three parameters relate to the nonminimal coupling of non-Abelian gauge field and gravity field, two parameters describe the so-called derivative nonminimal coupling of scalar multiplet with gravity field, and the sixth parameter introduces the standard coupling of scalar field with Ricci scalar. The formulated six-parameter nonminimal Einstein-Yang-Mills-Higgs model is applied to cosmology. We show that there exists a unique exact cosmological solution of the de Sitter type for a special choice of the coupling parameters. The nonminimally extended Yang-Mills and Higgs equations are satisfied for arbitrary gauge and scalar fields, when the coupling parameters are specifically related to the curvature constant of the isotropic spacetime. Basing on this special exact solution we discuss the problem of a hidden anisotropy of the Yang-Mills field, and give an explicit example, when the nonminimal coupling effectively screens the anisotropy induced by the Yang-Mills field and thus restores the isotropy of the model.

Keywords: Nonminimal interaction; Yang-Mills-Higgs theory; isotropic cosmological model.

1. Introduction

The discussion of a nonminimal coupling (NMC) of gravity with fields and media has a long history. The most intensely this topic has been studied in connection with the problem of nonminimal coupling of gravity and scalar field, which has numerous cosmological applications. The details of the investigations of this problem.
are discussed, e.g., in the review of Faraoni et al. The development of the theory of NMC of gravity and scalar field $\phi$ has started by the introduction of the term $\xi \phi^2 R$ to the Lagrangian ($R$ is the Ricci scalar). In Ref. 2 the special choice $\xi = 1/6$ has been motivated by the conformal invariance; in Ref. 3 this quantity was considered as an arbitrary parameter of the model. Such a model has been widely used for the cosmological applications, in which $\xi$ played a role of extra parameter of inflation (see, e.g., Refs. 4–9). In Refs. 10–13 the gauge-invariant term $\alpha \Phi^+ \Phi R$ has been introduced instead of $\xi \phi^2 R$ in the context of non-Abelian gauge theory ($\Phi$ is a multiplet of scalar complex Higgs fields interacting with gravity and spinor matter.) Subsequent generalizations have been related to the replacement of $\xi \phi^2$ by the function $f(\Phi^2)$ (see, e.g., Refs. 14–17), as well as, to the inserting of the terms of the type $F(\Phi^2, R)$ both linear and nonlinear in the Ricci scalar, Ricci and Riemann tensors (see, e.g., Refs. 18–20). The idea of nonminimal derivative coupling introduced in Ref. 21 and developed further in Refs. 22–23 has enriched the NMC modeling by the terms $\phi_{ij}$. Nonminimal cosmological models based on the formalism of derivative coupling are the multi-parameter ones and have supplementary abilities for a fitting of observational data. Let us note that the NMC of gravity and scalar field leads to the modifications of both the Klein-Gordon and the Einstein equations, and such modifications are of interest for various inflation scenarios. Thus, the modeling of nonminimal interactions of scalar and gravitational fields is one of the well established and physically motivated branch of modern cosmology. Natural extension of the nonminimal theory from the models with scalar fields coupled to curvature to the models describing scalar fields interacting with gauge fields has the same sound motivation and can disclose new aspects of cosmological dynamics.

The study of the nonminimal coupling of gravity with electromagnetic field has another motivation and another history. In 1971 Prasanna introduced the invariant $R^{ikmn} F_{ik} F_{mn}$ ($R^{ikmn}$ is the Riemann tensor, $F_{ik}$ is the Maxwell tensor) as a possible element of a Lagrangian, and then in Ref. 25 obtained the corresponding nonminimal one-parameter modification of the Einstein-Maxwell equations. In 1979 Novello and Salim proposed to insert the gauge non-invariant terms $R A^k A_k$ and $R^{ik} A_i A_k$ in the Lagrangian ($A_k$ is an electromagnetic potential four-vector). A qualitatively new step has been made by Drummond and Hathrell in Ref. 27, where the one-loop corrections to the quantum electrodynamics (QED) are obtained, which take into account the nonminimal coupling of gravity and electromagnetism. The Lagrangian of such a theory happens to contain three fundamental $U(1)$-gauge-invariant scalars $R^{ikmn} F_{ik} F_{mn}$, $R^{ik} g^{mn} F_{im} F_{kn}$ and $R F_{mn} F^{mn}$ with coefficients reciprocal to the square of the electron mass. This Lagrangian had no arbitrary parameters, but curvature induced modifications of the electrodynamic equations gave the impetus to wide discussions about the formal structure of the nonminimal Lagrangian, basic evolutionary equations, breaking the conformal invariance and the properties of the photons, coupled to curvature in different gravitational backgrounds (see, e.g., Refs. 28–38). The last paper revived, as well, the interest to the
Nonminimal isotropic cosmological model with Yang-Mills and Higgs fields

paradigm: curvature coupling and equivalence principle, various aspects of which are now discussed (see, e.g., Refs. [39][40]). The QED-motivation of the use of the generalized Maxwell equations can also be found in the papers of Kostelecký and colleagues. [41][42] The effect of birefringence induced by curvature, first discussed in Ref. [27], and some of its consequences for the electrodynamic systems have been investigated in Refs. [43][46] for the case of pp-wave background. The generalization of the idea of nonminimal interactions to the case of torsion coupled to the electromagnetic field has been made in Refs. [47][48] (see, also, Ref. [49] for a review on the problem). To summarize we stress that the study of electrodynamic systems nonminimally coupled to the gravity field poses a natural question about curvature induced variations of photon velocity in the cosmological background. Since the interpretation of observational data in cosmology depends essentially on the velocity of photon propagation during different cosmological epochs, the modeling of nonminimal electrodynamic phenomena seems to be well motivated and interesting from physical point of view.

Concerning the nonminimal Einstein-Yang-Mills (EYM) theory, we can distinguish between two different ways to establish it. The first way is the direct nonminimal generalization of the Einstein-Yang-Mills (EYM) theory. In the framework of this approach Horndeski[50] and Müller-Hoissen[51] obtained the nonminimal one-parameter EYM model from a dimensional reduction of the Gauss-Bonnet action. Now the Gauss-Bonnet models are of great interest in connection with the problem of dark energy (see, e.g., the Gauss-Bonnet model with nonminimal scalar field[52]). Thus, the non-Abelian multi-parameter extensions of nonminimal models are also well motivated, since they give a chance to explain the accelerated expansion of the Universe without addressing to exotic substance. We follow the alternative way, which is connected with a non-Abelian generalization of the nonminimal Einstein-Maxwell theory along the lines proposed by Drummond and Hathrell[27] for the linear electrodynamics. Based on the results of Ref. [53] a three-parameter gauge-invariant nonminimal EYM model linear in curvature is considered. [55][58] Our goal is to formulate a nonminimal Einstein-Yang-Mills-Higgs (EYMH) theory, and this process, of course, also admits different approaches. In fact, the nonminimal EYMH theory should accumulate the ideas and methods both from the nonminimally extended EYM theory and from the nonminimally extended scalar field theory. Initial attempt to develop nonminimal EYMH theory can be found, for instance, in Ref. [59] where the scalar Higgs field is nonminimally coupled with gravity via $\xi \Phi^2 R$ term, and the Higgs field $\Phi$ is included into the Lagrangian of the Yang-Mills field in a composition with a square of the Yang-Mills potential: $\Phi^2 A_{k(a)} A^k_{(a)}$. Such a theory is not gauge-invariant.

In this paper we establish a new six-parameter nonminimal Einstein-Yang-Mills-Higgs model. First three coupling parameters, $q_1$, $q_2$ and $q_3$, describe a nonminimal interaction of Yang-Mills field and gravitational field. The fourth and fifth parameters, $q_4$ and $q_5$, describe the so-called gauge-invariant nonminimal derivative coupling of the Higgs field with gravity. Since the gauge-invariant derivative, $\hat{D}_\mu \Phi^{(a)}$,
contains the potential of the Yang-Mills field, the corresponding nonminimal term is associated with “triple” interaction, namely, gravitational and scalar fields, gauge and scalar fields, and gauge and gravitational fields. The sixth parameter, $\xi$, is the well-known coupling parameter nonminimally connecting gravitational and scalar fields via the term $\xi R \Phi^2$. Of course, this model is only one of a wide class of the nonminimal EYMH models. As for its motivation and possible physical applications, one can see that on the one hand, the interest to a six-parameter nonminimal EYMH model is based on the sound results obtained earlier in the framework of partial nonminimal models (Einstein-Maxwell, Einstein-Yang-Mills and scalar field theories), on the other hand, the six-parameter model under discussion shows new specific solutions of cosmological type, which can not appear in more simple models.

The paper is organized as follows. In Sec. 2 we formulate the nonminimal EYMH model, which contains six phenomenological coupling parameters, and establish the nonminimally extended Yang-Mills, Higgs and Einstein equations. In Sec. 3 we apply the introduced master equations to the spacetime with constant curvature and obtain the specific relationships between coupling constants, which turn the extended equations for the gauge field and scalar field into identities. In Subsec. 3.3 we discuss the exact solutions to the nonminimal EYMH equations attributed to the isotropic cosmological model with Yang-Mills field, characterized by hidden anisotropy.

2. The formalism of the nonminimal EYMH theory

2.1. Minimal EYMH theory and basic definitions

The minimal Einstein-Yang-Mills-Higgs theory can be formulated in terms of the action functional

$$S_{(EYMH)} = \int d^4x \sqrt{-g} \left\{ \frac{R}{\kappa} + \frac{1}{2} F_{mn}^{(a)} F^{mn}_{(a)} - D_m \Phi^{(a)} D^m \Phi^{(a)} + V(\Phi^2) \right\},$$  

(1)

where $g = \det(g_{ik})$ is the determinant of a metric tensor $g_{ik}$, $R$ is the Ricci scalar, Latin indices run from 0 to 3. The symbol $\Phi^{(a)}$ denotes the multiplet of the Higgs scalar fields, $V(\Phi^2)$ is a potential of the Higgs field and $\Phi^2 \equiv \Phi^{(a)}\Phi^{(a)}$. Let us mention that there are two formal variants to introduce the cosmological constant into the action (1): first, explicitly as an additional term $\frac{2\Lambda}{\kappa}$, second, as a term $V(0)$ in the decomposition

$$V(\Phi^2) = \frac{2\Lambda}{\kappa} + \mu \Phi^2 + \omega \Phi^4 + \ldots$$  

(2)

Below we consider the second variant. Following Ref. [60], Section 4.3, we consider the Yang-Mills field $F_{mn}$ and the Higgs field $\Phi$ taking values in the Lie algebra of the gauge group $SU(n)$:

$$F_{mn} = -i \mathcal{G} t_{(a)} F^{(a)}_{mn}, \quad A_m = -i \mathcal{G} t_{(a)} A^{(a)}_m, \quad \Phi = t_{(a)} \Phi^{(a)}.$$  

(3)

Here $t_{(a)}$ are the Hermitian traceless generators of $SU(n)$ group, the constant $\mathcal{G}$ is the strength of the gauge coupling, $F^{(a)}_{mn}$, $A^{(a)}_m$ and $\Phi^{(a)}$ are real fields ($A^{(a)}_m$ represents
the Yang-Mills field potential) and the group index \((a)\) runs from 1 to \(n^2 - 1\). The symmetric tensor \(G_{(a)(b)} \equiv 2 \text{Tr} \ t_{(a)} t_{(b)}\) plays a role of a metric in the group space so that, e.g., \(\Phi_{(a)} \equiv G_{(a)(b)} \Phi^{(b)}\). The Yang-Mills fields \(F^{(a)}_{mn}\) are connected with the potentials of the gauge field \(A^{(a)}_{i}\) by the well-known formula (see, e.g., Refs. [60]-[62])

\[
F^{(a)}_{mn} = \nabla_{m} A^{(a)}_{n} - \nabla_{n} A^{(a)}_{m} + \mathcal{G} f^{(a)}_{(b)(c)} A^{(b)}_{m} A^{(c)}_{n}.
\]

Here \(\nabla_{m}\) is a covariant spacetime derivative, the symbols \(f^{(a)}_{(b)(c)}\) denote the real structure constants of the gauge group \(SU(n)\). The gauge-invariant derivative is defined according to the formula (Ref. [60], Eqs. (4.46, 4.47))

\[
\tilde{D}_{m} \Phi^{(a)} \equiv \nabla_{m} \Phi^{(a)} + \mathcal{G} f^{(a)}_{(b)(c)} A^{(b)}_{m} \Phi^{(c)}.
\]

For the derivative of arbitrary tensor defined in the group space we use the following rule [62]:

\[
\tilde{D}_{m} Q^{(a)}(\cdots)_{(d)} \equiv \nabla_{m} Q^{(a)}(\cdots)_{(d)} + \mathcal{G} f^{(a)}_{(b)(c)} A^{(b)}_{m} Q^{(c)}(\cdots)_{(d)} - \mathcal{G} f^{(c)}_{(b)(d)} A^{(b)}_{m} Q^{(a)}(\cdots)_{(c)} + \cdots
\]

The commutator and anticommutator of the generators \(t_{(a)}\) take the form

\[
[t_{(a)}, t_{(b)}] = i f^{(c)}_{(a)(b)} t_{(c)},
\]

\[
\{t_{(a)}, t_{(b)}\} \equiv t_{(a)} t_{(b)} + t_{(b)} t_{(a)} = \frac{1}{n} G_{(a)(b)} I + d^{(c)}_{(a)(b)} t_{(c)},
\]

where \(d^{(c)}_{(a)(b)}\) are the completely symmetric coefficients and \(I\) is the unitary matrix. The metric \(G_{(a)(b)}\), the structure constants \(f^{(c)}_{(a)(b)}\) and the coefficients \(d^{(c)}_{(a)(b)}\) are supposed to be constant tensors in standard and covariant manner [62]. This means that

\[
\partial_{m} G_{(a)(b)} = 0, \quad \partial_{m} f^{(a)}_{(b)(c)} = 0, \quad \partial_{m} d^{(a)}_{(b)(c)} = 0,
\]

\[
\tilde{D}_{m} G_{(a)(b)} = 0, \quad \tilde{D}_{m} f^{(a)}_{(b)(c)} = 0, \quad \tilde{D}_{m} d^{(a)}_{(b)(c)} = 0.
\]

### 2.2. Nonminimal extension of the Lagrangian

Any version of nonminimal generalization of the Lagrangian of the EYMH theory is based on the choice of the set of admissible invariants. The classification of the Yang-Mills fields based on the invariant polynomials in the \(F^{(a)}_{ik}\) tensor has been made in Ref. [63]. Nonlinear constitutive equations for Yang-Mills field along a line of the Born-Infeld theory has been first discussed in Ref. [65] (see, also, e.g., Ref. [66]). Possessing the tensorial quantities \(F^{(a)}_{ik}, \Phi^{(a)}_{i}, \tilde{D}_{i} \Phi^{(a)}, R^{ikmn}, R^{ik}, R\) one can construct a variety of gauge invariant scalars both minimal and nonminimal. This procedure has been discussed in the framework of scalar field theory and electrodynamics (see Introduction and references therein). Einstein-Yang-Mills-Higgs theory possesses an extended set of basic elements for such a representation, thus, a number of candidates to be included into a Lagrangian is much bigger. For instance, in order to couple the group indices \((a)\) and \((b)\) in the product \(F^{(a)}_{ik} F^{(b)}_{mn}\), we can use, first, the
standard convolution procedure, based on metric \( G_{(a)(b)} \), second, the projections onto \( \Phi_{(a)}\Phi_{(b)}, \Phi_{(a)}\hat{D}_j\Phi_{(b)} \) or \( \hat{D}_j\Phi_{(a)}\hat{D}_k\Phi_{(b)} \), third, the convolution with symmetric tensors \( d_{(a)(b)(c)}\Phi^{(c)}, d_{(a)(b)(c)}\hat{D}_j\Phi^{(c)} \), or with antisymmetric tensors \( f_{(a)(b)}^{(c)}\Phi^{(c)}, f_{(a)(b)}^{(c)}\hat{D}_j\Phi^{(c)} \). The corresponding examples of the scalar invariants, admissible for including into the nonminimal Lagrangian are

\[
\frac{1}{2} \mathcal{R}^{ikmn}_{(I)} F_{ik}^{(a)} F_{mn}^{(b)} \left[ G_{(a)(b)} + d_{(a)(b)(c)}\Phi^{(c)}\Phi_1^2 + \Phi_{(a)}\Phi_{(b)}\Phi_2^2 \right] + (\hat{D}_i\Phi_{(a)})(\hat{D}^j\Phi_{(b)})\Psi_3^2 + d_{(a)(b)(c)}\Phi^{(c)}(\hat{D}^i\Phi_{(b)})(\hat{D}_l\Phi_{(h)})\Psi_4^2 + \ldots \right] , \quad (10)
\]

\[
\mathcal{R}^{ikmn}_{(II)} F_{ik}^{(a)} \left[ f_{(a)(b)(c)}(\hat{D}_m\Phi^{(b)})(\hat{D}_n\Phi^{(c)})\Psi_5^2 + \ldots \right] , \quad (11)
\]

\[
\mathcal{R}^{ikmn}_{(III)} \left\{ g_{im}(\hat{D}_k\Phi^{(a)})(\hat{D}_n\Phi^{(b)}) \left[ G_{(a)(b)} + d_{(a)(b)(c)}\Phi^{(c)}\Phi_6^2 + \Phi_{(a)}\Phi_{(b)}\Phi_7^2 + \ldots \right] + (\hat{D}_i\Phi^{(a)})(\hat{D}_k\Phi^{(b)})(\hat{D}_m\Phi^{(c)})(\hat{D}_n\Phi^{(d)}) f_{(a)(b)(c)(d)}^{(h)} \Psi_8^2 + \ldots \right\} . \quad (12)
\]

Here \( \Psi_1, \Psi_2, \ldots, \Psi_8 \) are arbitrary functions of their argument, and the tensors \( \mathcal{R}^{ikmn}_{(I)}, \mathcal{R}^{ikmn}_{(II)} \) and \( \mathcal{R}^{ikmn}_{(III)} \) are considered to be appropriate linear combinations of the Riemann tensor and its convolutions with phenomenological coupling constants \( q_1, q_2, \ldots, q_j \). These constants are treated to be independent and have a dimensionality of area. Below the examples of such tensors are presented explicitly.

In this paper we restrict ourselves to the consideration of a Lagrangian, which satisfy the following requirements: the EYMH Lagrangian is a gauge invariant curvature scalar in a spacetime linear in a spacetime curvature, quadratic in the Yang-Mills field strength tensor \( F^{(a)}_{ik} \) and depending on the first derivative of the Higgs field only. In addition, in this paper we consider the convolutions of the standard type only, i.e., the terms including \( G_{(a)(b)}F^{(a)}_{ik}F^{(b)}_{mn}, G_{(a)(b)}\Phi^{(a)}\Phi^{(b)} \), etc. We intend to consider more sophisticated models in future papers.

### 2.3. Explicit example of nonminimal gauge-invariant Lagrangian

Consider now an action functional

\[
S_{(NMEYMH)} = \int d^4x\sqrt{-g} \left\{ \frac{R}{\kappa} + \frac{1}{2} F^{(a)}_{ik} F^{(a)}_{ik} - \hat{D}_m\Phi^{(a)}\hat{D}^m\Phi_{(a)} + V(\Phi^2) \right\} + \frac{1}{2} \mathcal{R}^{ikmn}_{(I)} F^{(a)}_{ik} F^{(b)}_{mn(a)} - \mathcal{R}^{mn}_{(II)} \hat{D}_m\Phi^{(a)}\hat{D}_n\Phi_{(a)} + \xi R\Phi^2 \right\} , \quad (13)
\]

where the tensors \( \mathcal{R}^{ikmn} \) and \( \mathcal{R}^{mn} \) are defined as follows:

\[
\mathcal{R}^{ikmn}_{(I)} = \frac{q_1}{2} R (g^{im}g^{kn} - g^{in}g^{km}) + \frac{q_2}{2} (R^{im}g^{kn} - R^{in}g^{km} + R^{kn}g^{im} - R^{km}g^{in}) + q_3 \mathcal{R}^{ikmn}_{(II)} , \quad (14)
\]
\[ \mathbb{R}^{mn} = q_4 R g^{mn} + q_5 R^{\prime mn} . \] (15)

This action describes a six-parameter nonminimal Einstein-Yang-Mills-Higgs model, and \( q_1, q_2, \ldots, q_5, \xi \) are the constants of nonminimal coupling.

2.3.1. Nonminimal extension of the Yang-Mills field equations

The variation of the action \( S_{(NMEYMH)} \) with respect to the Yang-Mills potential \( A^{(a)}_i \) yields

\[ \hat{D}_k H^{ik}_{(a)} = -\mathcal{G}(\hat{D}_k \Phi^{(b)}) f_{(a)(b)(c)} \left( g^{ik} + \mathbb{R}^{ik} \right) . \] (16)

Here the tensor \( H^{ik}_{(a)} \) is defined as

\[ H^{ik}_{(a)} = \left[ \frac{1}{2} (g^{im} g^{kn} - g^{in} g^{km}) + \mathcal{R}^{ikmn} \right] G_{(a)(b)} F^{(b)}_{mn} . \] (17)

This equation looks like Maxwell equation for the medium with the susceptibility tensor \( \mathcal{R}^{ikmn} \) and the current vector \( \mathcal{G}(\hat{D}_k \Phi^{(b)}) f_{(a)(b)(c)} \left( g^{ik} + \mathbb{R}^{ik} \right) \) induced by the Higgs field.

2.3.2. Nonminimal extension of the Higgs field equations

The variation of the action \( S_{(NMEYMH)} \) with respect to the Higgs scalar field \( \Phi^{(a)} \) yields

\[ \hat{D}_m \left( \hat{D}^m \Phi^{(a)} + \mathbb{R}^{mn} \hat{D}_n \Phi^{(a)} \right) = -\xi R \Phi^{(a)} - V'(\Phi^2) \Phi^{(a)} . \] (18)

This equation can be rewritten in the form

\[ \hat{D}_m \Psi^{m(a)} = -\left[ \xi R + V'(\Phi^2) \right] \Phi^{(a)}, \quad \Psi^{m(a)} = \hat{D}^m \Phi^{(a)} + \mathbb{R}^{mn} \hat{D}_n \Phi^{(a)} , \] (19)

and can be considered as scalar analog of (16) and (17).

2.3.3. Master equations for the gravitational field

In the nonminimal theory linear in curvature the equations for the gravity field related to the action functional \( S_{(NMEYMH)} \) take the form

\[ \left( R_{ik} - \frac{1}{2} R g_{ik} \right) \cdot (1 + \kappa \xi \Phi^2) = \kappa \xi \left( \hat{D}_i \hat{D}_k - g_{ik} \hat{D}_m \hat{D}^m \right) \Phi^2 + k T^{(NMYMH)}_{ik} . \] (20)

The principal novelty of these equations, in comparison with the well-known equations for nonminimal scalar field, is associated with the third, fourth, etc., terms in the decomposition

\[ T^{(NMYMH)}_{ik} = T^{(YM)}_{ik} + T^{(H)}_{ik} + q_1 T^{(I)}_{ik} + q_2 T^{(II)}_{ik} + q_3 T^{(III)}_{ik} + q_4 T^{(IV)}_{ik} + q_5 T^{(V)}_{ik} . \] (21)
The first term $T^{(YM)}_{ik}$:

$$T^{(YM)}_{ik} = \frac{1}{4} g_{ik} F^{(a)}_{mn} F^{(a)}_{mn} - F^{(a)}_{in} F^{(a)}_{kn}, \quad (22)$$

is a stress-energy tensor of pure Yang-Mills field. The second one, $T^{(H)}_{ik}$, is

$$T^{(H)}_{ik} = \hat{D}_i \Phi^{(a)} \hat{D}_k \Phi^{(a)} - \frac{1}{2} g_{ik} \hat{D}_m \Phi^{(a)} \hat{D}^m \Phi^{(a)} + \frac{1}{2} V(\Phi^2) g_{ik}, \quad (23)$$

is a stress-energy tensor of the Higgs field. The definitions of other five tensors are related to the corresponding coupling constants $q_1, q_2, \ldots, q_5$:

$$T^{(I)}_{ik} = R T^{(YM)}_{ik} - \frac{1}{2} R_{ik} F^{(a)}_{mn} F^{(a)}_{mn} + \frac{1}{2} \left[ \hat{D}_i \hat{D}_k - g_{ik} \hat{D}^l \hat{D}_l \right] \left[ F^{(a)}_{mn} F^{(a)}_{mn} \right], \quad (24)$$

$$T^{(II)}_{ik} = -\frac{1}{2} g_{ik} \left[ \hat{D}_m \hat{D}_l \left( F^{(a)}_{mn} F^{(a)}_{ln} \right) - R_{lm} F^{(a)}_{mn} F^{(a)}_{ln} \right] - F^{(a)}_{n(a)} \left( R_{ik} F_{kn(a)} + R_{ki} F_{in(a)} \right) - R^{mn} F^{(a)}_{im} F^{(a)}_{kn} - \frac{1}{2} \hat{D}^m \hat{D}_m \left( F^{(a)}_{in} F^{(a)}_{kn} \right)$$

$$+ \frac{1}{2} \hat{D}_l \left[ \hat{D}_i \left( F^{(a)}_{in} F^{(a)}_{kn} \right) + \hat{D}_k \left( F^{(a)}_{in} F^{(a)}_{kn} \right) \right], \quad (25)$$

$$T^{(III)}_{ik} = \frac{1}{4} g_{ik} R^{mn} F^{(a)}_{mn} F^{(a)}_{is} - \frac{3}{4} F^{(a)}_{is} \left( F^{(a)}_{i(a)} R_{knls} + F^{(a)}_{i(a)} R_{k(aln)ls} \right)$$

$$- \frac{1}{2} \hat{D}_m \hat{D}_n \left[ F^{(a)}_{i(a)} F^{(a)}_{k(a)} + F^{(a)}_{i(a)} F^{(a)}_{k(a)} \right], \quad (26)$$

$$T^{(IV)}_{ik} = \left( R_{ik} - \frac{1}{2} R g_{ik} \right) \hat{D}_m \Phi^{(a)} \hat{D}^m \Phi^{(a)} + R \hat{D}_i \Phi^{(a)} \hat{D}_k \Phi^{(a)}$$

$$+ \left( g_{ik} \hat{D}_n \hat{D}^n - \hat{D}_i \hat{D}_k \right) \left[ \hat{D}_m \Phi^{(a)} \hat{D}^m \Phi^{(a)} \right], \quad (27)$$

$$T^{(V)}_{ik} = \hat{D}_m \Phi^{(a)} \left[ R^{mn} \hat{D}_k \Phi^{(a)} + R^{mn} \hat{D}_i \Phi^{(a)} \right] - \frac{1}{2} R_{ik} \hat{D}_m \Phi^{(a)} \hat{D}^m \Phi^{(a)}$$

$$+ \frac{1}{2} g_{ik} \hat{D}_m \hat{D}_n \left[ \hat{D}^m \Phi^{(a)} \hat{D}^n \Phi^{(a)} \right] - \frac{1}{2} \hat{D}^m \left( \hat{D}_i \left[ \hat{D}_m \Phi^{(a)} \hat{D}_k \Phi^{(a)} \right] \right)$$

$$+ \hat{D}_k \left[ \hat{D}_m \Phi^{(a)} \hat{D}_i \Phi^{(a)} \right] - \hat{D}_m \left[ \hat{D}_i \Phi^{(a)} \hat{D}_k \Phi^{(a)} \right]. \quad (28)$$
2.3.4. Bianchi identities

The Einstein tensor $R_{ik} - \frac{1}{2}g_{ik}R$ is the divergence-free one, thus, the tensor $T^{(NMYMH)}_{ik}$ in the right-hand-side of (20) has to satisfy the differential condition

$$\nabla^k \left\{ \frac{\kappa \xi \left( \hat{D}_i \hat{D}_k - g_{ik} \hat{D}_m \hat{D}^m \right) \Phi^2 + kT^{(NMYMH)}_{ik}}{(1 + \kappa \xi \Phi^2)} \right\} = 0.$$  \hspace{1cm} (29)

One can prove that it is valid automatically, when $F^{(a)}_{ik}$ is a solution of the Yang-Mills equations (16), and $\Phi^{(a)}$ satisfy the Higgs equations (18). In order to check this fact directly, one has to use the Bianchi identities and the properties of the Riemann tensor:

$$\nabla_i R_{klmn} + \nabla_l R_{inkm} + \nabla_k R_{limn} = 0, \quad R_{klmn} + R_{mkln} + R_{lmkn} = 0,$$  \hspace{1cm} (30)

as well as the rules for the commutation of covariant derivatives

$$\left( \nabla_i \nabla_k - \nabla_k \nabla_i \right) A^l = \mathcal{A}^m R^i_{mlk},$$  \hspace{1cm} (31)

(this rule is written here for the vector only). The procedure of checking is analogous to one, described in Ref. [35] and we omit it.

3. Isotropic cosmological model associated with six-parameter nonminimal EYMH theory

Generally, the application of the EYMH model to cosmological problems requires the spacetime to be considered as anisotropic one. Clearly, when the spacetime is isotropic, the Einstein tensor in the left-hand-side of (20) is diagonal, while the tensor $T^{(NMYMH)}_{ik}$ in the right-hand-side is generally non-diagonal. This can be also motivated by the analogy with Einstein-Maxwell theory: it is well-known, for instance, that the minimal models with magnetic field are inevitably anisotropic and can be properly described in terms of Bianchi models. Nevertheless, as it was shown in Ref. [54] the nonminimal extension of the Einstein-Maxwell theory admits the models in which the spacetime is isotropic while the magnetic field is non-vanishing. Below we discuss the first example of analogous problem in the framework of nonminimal EYMH theory. Our goal is to present explicitly an exact solution to the equations of spatially isotropic EYMH model. When the Yang-Mills field is non-vanishing, the stress-energy tensor (21) is non-diagonal, as in the case of Einstein-Maxwell theory, thus, the symmetry of equations for the gravitational field is, generally, broken. Nevertheless, we will indicate a special choice of the coupling parameters $q_1, q_2, \ldots, q_5, \xi$, for which one can guarantee, that these equations become self-consistent. Since the de Sitter model is associated with the spacetime of constant curvature, we consider a number of properties of desired solution without solving the master equations.
3.1. Constant curvature spacetime and restrictions on the Yang-Mills-Higgs fields

We consider isotropic cosmological models with constant curvature $K$. For these spacetimes the Riemann tensor takes the form

$$R_{ikmn} = -K (g_{im}g_{kn} - g_{in}g_{km})$$

and the Ricci tensor, the Ricci scalar are

$$R_{ik} = -3K g_{ik}, \quad R = -12K.$$  

The tensors $\mathcal{R}_{ikmn}$ and $\mathcal{R}_{ik}$, introduced phenomenologically, can be transformed into

$$\mathcal{R}_{ikmn} = -K (6q_1 + 3q_2 + q_3) (g_{im}g_{kn} - g_{in}g_{km}), \quad \mathcal{R}_{ik} = -3K (4q_4 + q_5) g_{ik}.$$  

Then, the $H^{ik}$ tensor and the $\Psi^m$ vector simplify significantly, and the equations (17) and (19) convert, respectively, into

$$[1 - 2K (6q_1 + 3q_2 + q_3)] \hat{D}_k F_{ik}^{(a)} = -\mathcal{G} [1 - 3K (4q_4 + q_5)] \hat{D}^i \Phi^{(b)} \Phi^{(c)} \Phi^{(a)},$$

$$[1 - 3K (4q_4 + q_5)] \hat{D}_m \hat{D}^m \Phi^{(a)} = \left[12K - V'(\Phi^2)\right] \Phi^{(a)}.$$  

We focus on the case, when the equation for the Yang-Mills field turns into identity for arbitrary (non-vanishing) $F_{ik}^{(a)}$. It is not possible, when the EYMH theory is minimal one. Nevertheless, in the framework of nonminimal EYMH theory with non-vanishing Higgs field, $\Phi^{(a)} \neq 0$, the Yang-Mills equations admit an arbitrary non-vanishing solution, when

$$2(6q_1 + 3q_2 + q_3) = \frac{1}{K}, \quad 3(4q_4 + q_5) = \frac{1}{K}.$$  

If (37) is valid, the Higgs equations are self-consistent, when

$$[12K - V'(\Phi^2)] \Phi^{(a)} = 0.$$  

In its turn, it is possible in two cases: first, when $V(\Phi^2)$ is a linear function of its argument,

$$V(\Phi^2) = \frac{2\Lambda}{\kappa} + 12K \xi \Phi^2,$$  

$\Phi^{(a)}$ being arbitrary, second, when $\Phi^2$ is constant satisfying the equation (38).

3.2. One-parameter nonminimal EYMH model

In order to obtain an analytical progress in the searching for the solution to the gravity field equations let us consider the one-parameter model, which is characterized by the following conditions:

$$q_1 = q_4 = \frac{1}{12K}, \quad q_2 = q_3 = q_5 = 0, \quad \mu = \frac{\mu}{12K}.$$  

These conditions guarantee that the Yang-Mills equations (16) and the Higgs equations (18) are the trivial identities for arbitrary $F_{(a)}$ and $\Phi^{(a)}$. The Einstein equations for this case take the form

$$3Kg_{ik}(1 + \kappa\xi\Phi^2) = \kappa\xi\left(\dot{D}_i\dot{D}_k - g_{ik}\dot{D}_p\dot{D}^p\right)\Phi^2 + \frac{\kappa}{2}V(\Phi^2)g_{ik}$$

$$+ \frac{\kappa}{8}g_{ik}\left(F^m_{mn}F^{mn}_{(a)} - 2\dot{D}_m\Phi^{(a)}\dot{D}^m\Phi^{(a)}\right)$$

$$+ \frac{\kappa}{24K}\left(D_i\dot{D}_k - g_{ik}\dot{D}_p\dot{D}^p\right)\left[F^m_{mn}F^{mn}_{(a)} - 2\dot{D}_m\Phi^{(a)}\dot{D}^m\Phi^{(a)}\right].$$

(41)

It can be reduced formally to ten equations for one scalar function

$$\nabla_i\nabla_kW = Kg_{ik}W,$$

(42)

where

$$W = F^m_{mn}F^{mn}_{(a)} - 2\dot{D}_m\Phi^{(a)}\dot{D}^m\Phi^{(a)} + 24K\xi\Phi^2 + \frac{24}{\kappa}\left(\Lambda - K\right).$$

(43)

Let us consider the integrability conditions for such system and calculate the commutator of the covariant derivatives $\tilde{K}_{ijk} \equiv [\nabla_i\nabla_j - \nabla_j\nabla_i]\nabla_kW$. On the one hand with (42) this commutator yields directly

$$\tilde{K}_{ijk} = -K\left(g_{ik}\nabla_jW - g_{jk}\nabla_iW\right).$$

(44)

On the other hand due to (31)

$$\tilde{K}_{ijk} = -R^p_{\ ijk}\nabla_pW = -K\left(g_{ik}\nabla_jW - g_{jk}\nabla_iW\right).$$

(45)

Thus, the integrability conditions are satisfied identically, and the equations (42) are completely integrable.

### 3.3. De Sitter spacetime

In order to represent the exact solution to (42) explicitly we consider the model with positive curvature, $K > 0$, and reduce the metric to the de Sitter form

$$ds^2 = dt^2 + \exp\{2\sqrt{K}t\}(\eta_{\alpha\beta}dx^\alpha dx^\beta),$$

(46)

where $\alpha, \beta = 1,2,3$ and $\eta_{\alpha\beta}$ is the spatial part of the Minkowski metric with the signature $(-, -, -)$. Then (42) splits into three subsystems

$$\partial_t^2W - KW = 0, \quad \partial_t[\partial_tW - \sqrt{K}W] = 0,$$

$$\partial_\alpha[\partial_\betaW + \eta_{\alpha\beta}\sqrt{K}\exp\{2\sqrt{K}t\}[\partial_tW - \sqrt{K}W] = 0,$$

(47)

which can be readily solved

$$W = C_1e^{\sqrt{K}t} + C_2e^{-\sqrt{K}t} + e^{\sqrt{K}t}\left[L_\alpha x^\alpha + C_2K\eta_{\alpha\beta}x^\alpha x^\beta\right].$$

(48)
Here $C_1$, $C_2$ and $L_\alpha$ are arbitrary constants. Thus, we obtain an exact solution of the total EYMH system of equations for which the Yang-Mills field $F^{(a)}_{mn}$ and the Higgs fields $\Phi^{(a)}$ are connected by unique condition

$$
F^{(a)}_{mn} F_{mn}^{(a)} - 2 \dot{D}_m \Phi^{(a)} D^m \Phi^{(a)} + 24K\xi \Phi^2 + \frac{24}{\kappa} \left( \frac{\Lambda}{3} - K \right)
$$

$$
= C_1 e^{\sqrt{K}t} + C_2 e^{-\sqrt{K}t} + e^{\sqrt{K}t} \left[ L_\alpha x^\alpha + C_2 K \eta_{\alpha \beta} x^\alpha x^\beta \right] . 
$$

Clearly, there exists a lot of various Yang-Mills-Higgs configurations, which satisfy this condition.

4. Discussions

1. The main mathematical result of the presented paper is the establishing of a new self-consistent nonminimal system of master equations for the coupled Yang-Mills, Higgs and gravity fields from the gauge-invariant nonminimal Lagrangian [13]. The obtained mathematical model contains six arbitrary parameters, and, thus, admits a wide choice of special sub-models interesting for the applications to the nonminimal cosmology (isotropic and anisotropic) and nonminimal colored spherical symmetric objects. The applications require the phenomenological coupling constants $q_1$, $q_2$, . . . , $q_5$ and $\xi$ to be interpreted adequately. Following the idea, discussed in Ref. [13], we intend not to introduce “new constants of Nature”, but to relate the phenomenological parameters with the constants well-known in the High Energy Particle Physics, on the one hand, and with the constants of cosmological origin, on the other hand. Indeed, in the specific cosmological model, established above, the sixth phenomenological parameter $\xi$ is expressed in terms of the square of the effective mass of the Higgs bosons $\mu$ and constant curvature $K$, $\xi = \frac{\mu}{12K}$. Other parameters are expressed in terms of $K$ (see [10]). Since in the de Sitter model the Hubble constant is $H = \sqrt{K}$, one can say that $q_1$, $q_2$, . . . , $q_5$ are connected with $H$. Analogously, one can consider the equality $H^2 = K = \frac{1}{3}$ and thus, one can say that they are connected with the cosmological constant $\Lambda$. In any case the parameters of nonminimal coupling $q_1$, $q_2$, . . . , $q_5$ can be expressed in terms of cosmological parameters $K$, $H$ or $\Lambda$, and define a specific radius of curvature coupling, $r_q \equiv \frac{1}{\sqrt{K}}$ and the corresponding time parameter $t_q \equiv r_q/c$.

2. The curvature coupling modifies the master equations for the Yang-Mills and Higgs fields. According to [10] a new tensor $H^{ik}_{(a)}$ appears (see [14]), which is an analog of the induction tensor in the Maxwell theory [63]. This means that the curvature coupling of the non-Abelian gauge field with gravity acts as a sort of quasi-medium with a nonminimal susceptibility tensor $R^{ikmn}$ (see [17]). As well, the curvature coupling modifies the master equations for the Higgs field, and the tensor $R^{mn}$, according to [18], can be indicated as a simplest nonminimal susceptibility tensor for the Higgs field, and the vector $\Psi^{(a)}_m$ (see [19]) can be defined as scalar induction. For the specific set of coupling constants (see [57], [40]) the
non-Abelian induction $H_{(a)}^{ik}$ and the scalar induction $\Psi_{(a)}^{m}$ can turn into zero, despite the fact that the Yang-Mills field strength $F_{(a)}^{ik}$ and the Higgs field $\Phi^{(a)}$ are non-vanishing. This means that, when $\tilde{\epsilon}$ holds, the possibility exists to satisfy the nonminimally extended Yang-Mills and Higgs equations for arbitrary $F_{(a)}^{ik}$ and $\Phi^{(a)}$. This possibility gives, in principle, a new option for modeling physical processes in Early Universe and shows very interesting analogy between this nonminimal model and resonance phenomena in plasma physics. Indeed, when we deal with plasma waves (for instance, with the longitudinal waves) one can see that electric induction $\vec{D}$ is connected with the longitudinal electric field $\vec{E}$ by the relation $\vec{D} = \tilde{\epsilon} |E||$. Here $\tilde{\epsilon}$ is the longitudinal dielectric permittivity, the simplest expression for this quantity can be obtained in the limit of long waves and gives $\tilde{\epsilon} = 1 - \frac{\Omega_p^2}{\omega^2}$, where $\Omega_p$ is the well-known plasma frequency. When $\omega = \Omega_p$, one obtains $\vec{D} = 0$ and electrodynamic equations are satisfied for arbitrary $\vec{E}$. Analogous feature can be found in the nonminimal model described above (see Eq. (35)). Indeed, the quantity $K$ with the dimensionality of squared frequency ($c = 1$) can be regarded as an analog of $\Omega_p^2$, the quantity $2(6q_1 + 3q_2 + q_3)$ can be indicated as $1/\omega^2$, then the term $1 - 2K(6q_1 + 3q_2 + q_3)$ plays a role of effective permittivity scalar $\tilde{\epsilon}$. When this effective permittivity scalar vanishes, i.e., when the constants of nonminimal coupling are connected with the constant curvature $K$ according to (37), we obtain the resonance case, for which the Yang-Mills and Higgs equations are satisfied identically for arbitrary strength field tensor $F_{(a)}^{ik}$ and Higgs multiplet $\Phi^{(a)}$, the color induction $H_{(a)}^{ik}$ being equal to zero.

3. The vector potential of the Yang-Mills field $A_{(a)}^{i}$ enters the master equations via the gauge covariant derivative $\hat{D}_k$, thus, the gauge field generates an anisotropy in the spacetime. Such an anisotropy, in general case, breaks down the symmetry of the model and produces the isotropy violation. Nevertheless, as it was shown above, the nonminimal coupling can effectively screen the anisotropy and guarantee the symmetry conservation. In the framework of this model one can speak about hidden anisotropy of the Yang-Mills field, keeping in mind that non-Abelian gauge field enters the master equations for the gravity field in the isotropic combinations only.

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