QUARKS AND GLUONS IN NUCLEON POLARIZED STRUCTURE FUNCTIONS

C. BOURRELY\textsuperscript{a}, F. BUCELLA\textsuperscript{b,c}, O. PISANTI\textsuperscript{b,c}, P. SANTORELLI\textsuperscript{b,c}
and
J. SOFFER\textsuperscript{a}

\textsuperscript{a}Centre de Physique Théorique–CNRS Luminy, Case 907
F-13288 Marseille Cedex 9, France
\textsuperscript{b}Dipartimento di Scienze Fisiche, Università “Federico II”,
Pad. 19 Mostra d’Oltremare, 00195 Napoli, Italy
\textsuperscript{c}INFN, Sezione di Napoli,
Pad. 20 Mostra d’Oltremare, 00195 Napoli, Italy

Abstract

We study the available data in polarized $e - p$ deep inelastic scattering to test two different solutions to the so called spin crisis: one of them based on the axial gluon anomaly and consistent with the Bjorken sum rule and another one, where the defects in the spin sum rules and in the Gottfried sum rule are related. In this case a defect is also expected for the Bjorken sum rule. Experimental data, especially the very recent SLAC E154, favour the first solution and demand a gluon polarization $\Delta G = 2.25 \pm 1.39$.

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1 Introduction

The earlier EMC CERN experiment [1] and the importance of testing the Bjorken sum rule [2] have stimulated a considerable experimental activity in measuring $g_1^p$, $g_1^n$ ($g_1^{He^3}$) and $g_1^d$. The result found from EMC for the first moment,

$$\Gamma_1^p = \int_0^1 g_1^p(x) \, dx = 0.126 \pm 0.010 \pm 0.015,$$

has been confirmed by the SMC CERN experiment [3] (at $<Q^2> = 10 GeV^2$, which is almost the same as for EMC) and SLAC [4] (at $<Q^2> = 3 GeV^2$), giving respectively

$$\Gamma_1^p = 0.136 \pm 0.011 \pm 0.011 \quad (SMC),$$

$$\Gamma_1^p = 0.127 \pm 0.004 \pm 0.010 \quad (E143).$$

These experiments have also measured the deuteron structure function [5, 6] and from $\Gamma_1^d$ by subtracting $\Gamma_1^p$, one gets

$$\Gamma_1^n = -0.063 \pm 0.024 \pm 0.013 \quad (SMC),$$

$$\Gamma_1^n = -0.037 \pm 0.008 \pm 0.011 \quad (E143).$$

At SLAC with polarized $He^3$ targets they also obtained [7]

$$\Gamma_1^n = -0.031 \pm 0.006 \pm 0.009 \quad (E142, Q^2 = 2GeV^2),$$

and the preliminary result [8]

$$\int_{0.014}^{0.7} g_1^n(x)dx = -0.037 \pm 0.004 \pm 0.010 \quad (E154, Q^2 = 5GeV^2).$$

The preliminary result from Hermes [9] ($\Gamma_1^p = -0.032 \pm 0.013 \pm 0.017$ at $<Q^2> = 3 GeV^2$) is consistent with SLAC data.

The main issue of this experimental work is to test the validity of the Bjorken sum rule, which up to $O(\alpha_s^3)$, for $n_f = 3$, is given by [10]

$$\Gamma_1^p - \Gamma_1^n = \frac{1}{6} \frac{G_A}{G_V} \left[ 1 - \frac{\alpha_s(Q^2)}{\pi} - 3.58 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 - 20.22 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^3 \right].$$

Indeed, an interpretation of the defect in the Ellis and Jaffe sum rule [11] for $\Gamma_1^p$ implied by Eqs. (1) and (2) has been given in terms of a negative contribution coming from a large
positive polarization of the gluons $\Delta G$ [12], which is the same for proton and neutron, such that it does not affect the Bjorken sum rule.

An analysis of the existing data, excluding the most recent and very precise SLAC E154 data, has been performed in a framework consistent with the Bjorken sum rule and including next to leading order (NLO) effects in the evolution equations to relate data at different $Q^2$, and it provides a fair description of the experimental results [13].

All the existing data do not exhibit a clear evidence of $Q^2$ evolution, i.e. within error bars they are compatible with scaling behaviour in the entire $Q^2$ range accessible by all experiments. Although a complete NLO analysis of the $g_1$ data is the correct procedure without a doubt, we are aiming to demonstrate that the most accurate data provided by the SLAC experiment is really telling us something important about the gluon contribution, even at a level of a less sophisticated leading order (LO) analysis. In spite of the fact that the SMC CERN experiment can reach higher $Q^2$ and smaller $x$, the actual precision achieved cannot provide a reliable test for the NLO theoretical analysis.

Here we want to compare the current interpretation of the defect in the Ellis and Jaffe sum rule in terms of a large flavour singlet contribution to the nucleon polarization coming from the gluons, with another one [14], where one relates this defect to the one in the Gottfried sum rule [15] and to the role that Pauli principle seems to play [16]. This is done by relating the first moments and the shapes of the parton distributions, as first proposed in Ref. [14]. Indeed, if one assumes the validity of the Adler sum rule [17],

$$u - d = [(u - \bar{u}) - (d - \bar{d})] + \bar{u} - \bar{d} = 1 + \bar{u} - \bar{d},$$

(7)

the defect in the Gottfried sum rule implies

$$u - d < 1.$$

(8)

If one thinks that the Pauli principle is responsible for the inequality (8), it is reasonable to assume that it is $u^+$, the most abundant valence parton, which receives less contribution from the sea, so that we have [14]

$$\Delta u = u^+ - u^- \simeq \Delta u_{val} + \bar{u} - \bar{d},$$

(9)

producing a defect in the Ellis and Jaffe sum rule for the proton

$$\Delta \Gamma_1^p = \frac{2}{9} (\bar{u} - \bar{d}) = \frac{2}{9} (-0.15 \pm 0.04) \simeq -0.033 \pm 0.009,$$

(10)
in fair agreement with the experiments.

An empirical test for the two interpretations might be given using the experimental information on the $x$-dependence of the polarized structure functions. The higher precision of SLAC data (especially the ones of E154 for $g_1^p$) and the agreement between E142 ($<Q^2>=2 GeV^2$) and E154 ($<Q^2>=5 GeV^2$) data, suggest to describe them together with the E143 measurements ($<Q^2>=3 GeV^2$) on proton and deuteron in terms of the same parton distributions, and we consider for these distributions two options corresponding to the two different solutions to the spin crisis.

We shall neglect higher-twist terms, supported by more recent theoretical evaluations of the contribution of these terms [18] which lead to results smaller in modulus and sometimes opposite in sign than the previous one, consistent with an experimental determination of these terms by the SLAC group [19].

The paper is organized as follows. In the forthcoming section we shall describe the SLAC data, with proton and deuteron targets at $<Q^2>=3 GeV^2$ and $He^3$ target at $<Q^2>=2 GeV^2$ (E142) and at $<Q^2>=5 GeV^2$ (E154), with the two different options. Then we shall present the method we used to solve the Altarelli-Parisi evolution equations and to find the parton distributions at $Q^2=10 GeV^2$ which we shall compare with CERN data. Finally, we shall give our conclusions.

2 Description of SLAC data

We describe the proton and neutron polarized structure functions at $Q_0^2=3 GeV^2$, in terms of the valence quark and gluon polarized distributions only, using a simplified version of the functional forms used in Ref. [20] (in our case we take $\gamma_q(q=u,d,G)=0$), namely

$$x\Delta u_v(x,Q_0^2) = \eta_u A_u x^{a_u} (1-x)^{b_u},$$
$$x\Delta d_v(x,Q_0^2) = \eta_d A_d x^{a_d} (1-x)^{b_d},$$
$$x\Delta G(x,Q_0^2) = \eta_G A_G x^{a_G} (1-x)^{b_G},$$

where $\eta_q$ $(q=u,d,G)$ are the first moments of the distributions and $A_q = A_q(a_q,b_q)$,

$$A_q^{-1} = \int_0^1 dx x^{a_q-1} (1-x)^{b_q} = \frac{\Gamma(a_q)\Gamma(b_q+1)}{\Gamma(a_q+b_q+1)},$$

in such a way that

$$\int_0^1 dx A_q x^{a_q-1} (1-x)^{b_q} = 1.$$
As pointed out by several authors [21], to avoid the inclusion of soft contributions into the coefficient functions one has to choose a factorization scheme in which the gluon polarization contributes to the first moments of $g_1^p$ and $g_1^n$ (for $n_f = 3$):

$$
\Gamma_1^{p(n)}(Q^2) = \frac{2}{9} \left( \frac{1}{18} \right) \eta_u(Q^2) + \frac{1}{18} \left( \frac{2}{9} \right) \eta_d(Q^2) - \frac{\alpha_s(Q^2)}{6\pi} \eta_G(Q^2).
$$

(14)

The gluonic term appears to be a higher order correction but is not, because $\eta_G(Q^2)$ rises logarithmically with $Q^2$ and, if the gluons had a positive polarization, it could, in principle, be large enough to explain the defect in the Ellis and Jaffe sum rule.

We take

$$
g_1^{p(n)}(x, Q^2) = \frac{2}{9} \left( \frac{1}{18} \right) \Delta u_v(x, Q^2) + \frac{1}{18} \left( \frac{2}{9} \right) \Delta d_v(x, Q^2) - \frac{\alpha_s(Q^2)}{6\pi} (\Delta \sigma \otimes \Delta G)(x, Q^2),
$$

$$
g_1^d(x, Q^2) = \frac{1}{2} \left( 1 - \frac{3}{2} \omega_D \right) (g_1^p(x, Q^2) + g_1^n(x, Q^2)),
$$

(15)

where $\omega_D = 0.058$ [22] takes into account the small D-wave component in the deuteron ground state. In Eq. (15) the QCD corrections in the quark sector are included in the $\tilde{F}$ and $\tilde{D}$ values entering in the expressions of the first moments of the quark distributions ($F = 0.46 \pm 0.01$, $D = 0.79 \pm 0.01$ [23]):

$$
\tilde{F}(Q^2) = \frac{1}{5} \left[ 5 F \left( 1 - \frac{\alpha_s(Q^2)}{\pi} \right) - (10.46 F + 2.48 D) \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 
$$

$$
- 20.22 (2 F + D) \left( \frac{\alpha_s(Q^2)}{\pi} \right)^3 \right],
$$

$$
\tilde{D}(Q^2) = \frac{1}{5} \left[ 5 D \left( 1 - \frac{\alpha_s(Q^2)}{\pi} \right) - (7.44 F + 15.42 D) \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 
$$

$$
- 20.22 (3 F + 4 D) \left( \frac{\alpha_s(Q^2)}{\pi} \right)^3 \right],
$$

(16)

and the gluon contribution appears as a convolution [24],

$$
(\Delta \sigma \otimes \Delta G)(x, Q^2) = \int_x^1 \frac{dz}{z} (1 - 2 z) \left( \ln \frac{1 - z}{z} - 1 \right) \Delta G \left( \frac{x}{z}, Q^2 \right).
$$

(17)

We fix ($\alpha_s(3 GeV^2) = 0.35 \pm 0.05$)

$$
\eta_d(Q_0^2) = \tilde{F}(Q_0^2) - \tilde{D}(Q_0^2) = -0.26 \pm 0.02,
$$

(18)
and we explore the two options A and B, the first one with

$$\eta_u(Q_0^2) = 2\tilde{F}(Q_0^2) = 0.76 \pm 0.04,$$

(19)

and $\eta_G$ free, the second one with $\eta_u$ free and $\eta_G = 0$. Options A and B correspond respectively to the interpretation of the defect in the Ellis and Jaffe sum rule for $\Gamma^0$ in terms of the anomaly, assuming that the Bjorken sum rule is obeyed, and to the case of a smaller $\Delta u$ resulting from the Pauli principle.

Since we know that $u^\uparrow$ dominates at high $x$ and that the gluons dominate in the small $x$ region, we restrict, as in Ref.[25], the values of the parameters in Eqs. (11), to be consistent with the information we already have for the parton distributions, by the following limitations

$$b_u > 1, \quad b_d > 3, \quad b_G > 5,$$

(20)

and we assume

$$a_u = a_d.$$  

(21)

Indeed, especially for option A, where one describes two functions, $g_1^p(x)$ and $g_1^n(x)$, in terms of three distributions, Eqs. (11), one has to make sure to exclude some choices of the parameters describing well the data, but not consistent with the information one has from the unpolarized data, that is, e.g. about 1/2 of the proton momentum (in the $P_z = \infty$ frame) is carried by the gluons and that the partons $u^\uparrow$ are dominating the high $x$ region.

The parameters corresponding to the best fit of the SLAC proton and deuteron data for options A and B are given in Table [4], while in Figs. 1, 2 and 3 one compares the two resulting curves with SLAC data.

All the data are well described with the two options, except for the ones by E154, which are better described by the option A, the one with gluon contribution and consistent with the Bjorken sum rule. Option A implies a large value of $\Delta G = 2.25 \pm 1.39$.

### 3 Parton evolution equations

For the polarized parton distributions one has the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution equations (DGLAP) [26], which are, in the variable $t \equiv \ln Q^2/\Lambda_{QCD}^2$ and at LO in
\[ \alpha_s (\Delta q^i \equiv x \Delta q^i \text{ and } \Delta \tilde{g} \equiv x \Delta G), \]
\[
\frac{d \Delta \tilde{q}^i}{dt}(x, t) = \frac{\alpha_s(t)}{2\pi} \left[ \int_x^1 dz \left\{ \frac{4}{3} \left[ \frac{2}{1-z} - 1 - z + \frac{3}{2} \delta(1-z) \right] \Delta \tilde{q}^i \left( \frac{x}{z}, t \right) \right. \right. \\
+ \left. \left. \int_x^1 dz \left( z - \frac{1}{2} \right) \Delta \tilde{g} \left( \frac{x}{z}, t \right) \right] \right] , \quad (i = 1, \ldots, 2n_f) \tag{22}
\]
\[
\frac{d \Delta \tilde{g}}{dt}(x, t) = \frac{\alpha_s(t)}{2\pi} \left[ \int_x^1 dz \left\{ \frac{4}{3} (2-z) \sum_{i=1}^{2n_f} \Delta \tilde{q}^i \left( \frac{x}{z}, t \right) \right. \right. \\
+ \left. \left. \int_x^1 dz \left[ \frac{2}{(1-z)} + 2 - 4z + \left( \frac{11}{6} - \frac{n_f}{9} \right) \delta(1-z) \right] \Delta \tilde{g} \left( \frac{x}{z}, t \right) \right] . \right.
\]

We work in the fixed flavour scheme where the number of flavours in the splitting functions is \( n_f = 3 \), while for the \( Q^2 \)-evolution of \( \alpha_s \) one has
\[
\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f) \ln \frac{Q^2}{\Lambda_{QCD}^{(4)}}} , \tag{23}
\]

with \( \Lambda_{QCD}^{(4)} = 201 \text{ MeV} \) to reproduce, with \( n_f = 4 \), \( \alpha_s(3 \text{ GeV}^2) = 0.35 \pm 0.05 \).

For the solution of the DGLAP equations we have used a method \cite{27} that consists in expanding the parton distributions \( p^i \) into a truncated series of Chebyshev polynomials,
\[
p^i(x, t) \rightarrow p^i(\tau(x), t) = \frac{2}{n} \sum_{s=0}^{n-1} \sum_{l=0}^{n-1} v_l p^i(x_s, t) T_l(\tau_s) T_l(\tau(x)) , \tag{24}
\]

where \( T_l \) are the Chebyshev polynomials and
\[
\tau(x) = -\frac{2 \ln x - y_{max}}{y_{max}}, \\
y_{max} = 4 \ln 10, \\
v_l = \begin{cases} 0.5 & l = 0 \\ 1 & l > 0 \end{cases}, \\
x_s = \exp\left[ -\frac{y_{max}}{2} (\tau_s + 1) \right], \\
\tau_s = \cos \left[ \frac{2s+1}{2n} \pi \right]. \tag{25}
\]

Substituting Eq. (24) in Eqs. (22) gives rise to a system of coupled differential equations
\[
\frac{dp^i}{dt}(t) = \frac{\alpha_s(t)}{2\pi} \sum_{j=1}^7 \sum_{s=0}^{n-1} A_{ks}^{ij} p^j_s(t), \quad (i = 1, \ldots, 7; \ k = 0, \ldots, n - 1), \tag{26}
\]
in which $p^i_k(t) = p^i(x_k, t)$ are the values of the polarized distributions $\Delta \tilde{q}^i$ and $\Delta \tilde{g}$ in the $n$ points $x_k$ corresponding to the nodes $\tau_k$ of the Chebyshev polynomials.

With the initial conditions given by the results of the fits $A$ and $B$ to the SLAC data we get the evolved distributions at $Q^2 = 10 GeV^2$.

4 Comparison of the evolved distributions with experiments

The predictions for the evolved distributions at $Q^2 = 10 GeV^2$ are compared with CERN measurements at $Q^2 = 10 GeV^2$ in Figs. 4 and 5. There is a better agreement for option $B$ for the proton (total $\chi^2$ of 10.6 for 12 points to be compared with $\chi^2 = 16.2$ for option $A$) while for the deuteron the option $A$ has a slightly lower $\chi^2$ (total $\chi^2$ of 14.2 for 12 points to be compared with $\chi^2 = 16.8$ for option $B$). Note that for option $A$ one has six free parameters, but only four for option $B$.

It is interesting to remark that with both options one fails to reproduce the rise at low $x$ of $x g_1^p(x)$, which turns negative below $x = 10^{-2}$ for option $A$, while $x g_1^d(x)$ in the same region is in agreement with the trend of the data (see Fig. 5). This is due to the isoscalar nature of the anomaly contribution, which is expected to be the same for proton, neutron and deuteron (neglecting the small correction coming from the D-wave component in its ground state).

5 Conclusions

We have studied the precise SLAC data on polarized structure functions with the main purpose of testing the Bjorken sum rule and the necessity of a gluon contribution to the polarized structure functions. As a result we find a better description of the data, especially of the very precise ones by the SLAC E154, with a gluon contribution and imposing the Bjorken sum rule than for the option without gluons and with the first moment of $\Delta u$ free. The best fit in the first case is obtained with a rather large value of $\Delta G = 2.25 \pm 1.39$.

The $\Delta G$ distribution appears to be more singular than the $\Delta q$’s, leading to the conclusion that $g_1^p(x)$ should also become negative at small $x$. These results are in agreement with what was found in Ref. [28]. Although we have found here $a_G = 0.44$ and $a_G = 0.13$ in [28], these two different powers lead to comparable values for $\int_{0.01}^{0.1} \Delta G(x) \, dx$, which is 1.5 in Ref. [28] and 1.0 here. Note that in Ref. [28], since the $(1-x)$ terms were omitted, one expects to
get, for gluons and for valence quarks, a slightly more singular behaviour when \( x \to 0 \) than in the present analysis.

Concerning the test of the Bjorken sum rule, the fact that one gets a better fit by allowing the presence of a contribution coming from the gluons speaks in favour of its validity. Assuming the validity of the Bjorken sum rule, this contribution was advocated to explain the defect of the Ellis and Jaffe sum rule for the proton. However, the fact that, according to our description, at low \( x \) the isovector contribution is expected to be overwhelmed by the isoscalar one, which is more singular, suggests that a precise test of the Bjorken sum rule rather requires very accurate measurements in the \( x \) region where the two contributions are comparable. Indeed, in the very small \( x \) region, where \( g_1^p(x) - g_1^n(x) \) should have a small value being the difference between two almost equal negative large quantities, normalizations uncertainties can produce substantial errors. SLAC E154 has measured \( g_1^n(x) \) with an outstanding precision and we look forward to have, in the same \( x \) range, a comparable precision for \( g_1^p \) from SLAC E155 \[29\].

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Table 1: The results of the options A and B for the values of the parameters of the fits at $Q^2 = 3 \text{GeV}^2$. The free parameters are marked with an asterisk.

| Parameter | A       | B       |
|-----------|---------|---------|
| $a_u = a_d$ | $0.79 \pm 0.05^{(*)}$ | $1.03 \pm 0.07^{(*)}$ |
| $b_u$     | $1.51 \pm 0.17^{(*)}$ | $1.88 \pm 0.23^{(*)}$ |
| $b_d$     | $3.00 \pm 0.13^{(*)}$ | $4.52 \pm 0.52^{(*)}$ |
| $a_G$     | $0.44 \pm 0.18^{(*)}$ | - |
| $b_G$     | $5.00 \pm 1.05^{(*)}$ | - |
| $\eta_u$  | $0.76 \pm 0.04$ | $0.63 \pm 0.03^{(*)}$ |
| $\eta_d$  | $-0.26 \pm 0.02$ | $-0.26 \pm 0.02$ |
| $\eta_G$  | $2.25 \pm 1.39^{(*)}$ | - |
| $\chi^2_{NDF}$ | 0.84 | 1.09 |
Figure Captions

Fig. 1 The best fit for the options A (solid line) and B (dashed line) (see text) are compared with the SLAC data on proton for $xg_1^p(x)$ at $<Q^2>=3\,GeV^2$ from Ref. [4].

Fig. 2 Same as Fig. 1 for the deuteron SLAC data for $xg_1^d(x)$ at $<Q^2>=3\,GeV^2$ from Ref. [6].

Fig. 3 Same as Fig. 1 for the neutron SLAC data for $xg_1^n(x)$ at $<Q^2>=2\,GeV^2$ from Ref. [7] (triangle) and at $5\,GeV^2$ from Ref. [8] (boxes).

Fig. 4 The data on proton for $xg_1^p(x)$ from SMC at $<Q^2>=10\,GeV^2$ from Ref. [3] are compared with the results of the options A (solid line) and B (dashed line), evolved to $Q^2=10\,GeV^2$.

Fig. 5 Same as Fig. 4 for the deuteron SMC data for $xg_1^d(x)$ from Ref. [5].
Fig. 2

$Q^2 = 3 GeV^2$
Fig. 3

\( \langle Q^2 \rangle = 2 \text{GeV}^2 \)

\( \langle Q^2 \rangle = 5 \text{GeV}^2 \)
\[ \langle Q^2 \rangle = 10 \text{GeV}^2 \]
Fig. 5

$\langle Q^2 \rangle = 10 \text{GeV}^2$