Neutrino Theory: Some Recent Developments

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Selected topics in neutrino theory are discussed, including tribimaximal mixing and its A₄ realization, variations of the seesaw mechanism allowing for observable deviations from unitarity of the neutrino mixing matrix, and the radiative generation of neutrino mass from dark matter.

I. INTRODUCTION

In this talk, I will discuss a number of developments in neutrino theory over the past three or four years. I will not have time to cover many interesting topics which are being pursued, but I will mention some in passing. In the following, I will give a brief account of the history of neutrino tribimaximal mixing and the tetrahedral symmetry A₄ which explains it. I will then discuss variations of the seesaw mechanism, with focus on the possibility that it may be testable at the TeV scale through the nonunitarity of the neutrino mixing matrix. The possible connection between the radiative seesaw mechanism and dark matter, i.e. scotogenic neutrino mass, is then explored with three examples.

II. BRIEF HISTORY OF NEUTRINO TRIBIMAXIMAL MIXING

In 1978, soon after the appearance of hints for the existence of the third family of quarks and leptons, it was conjectured independently by Cabibbo [1] and Wolfenstein [2] that the mixing matrix linking charged leptons to neutrinos could be given by

\[ U_{lv}^{CW} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \]

where \( \omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2. \) In the conventional notation, this corresponds to

\[ \theta_{23} = \theta_{12} = 45^\circ, \quad \theta_{13} = 35.3^\circ, \quad \delta_{CP} = 90^\circ. \] (2)

It should dispel the myth that everybody expected small mixing angles in the lepton sector as in the quark sector.

In 2002, after neutrino oscillations have been established, Harrison, Perkins, and Scott [3] proposed the tribimaximal mixing matrix, i.e.

\[ U_{lv}^{HPS} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \sim (\eta_8, \eta_1, \pi^0), \] (3)

where the columns are evocative of members of the meson nonet in terms of quark-antiquark bound states.

In 2004, I discovered [4] the simple connection:

\[ U_{lv}^{HPS} = (U_{lv}^{CW})^\dagger \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & i & 0 \end{pmatrix}, \] (4)

where the last matrix merely redefines \( \nu_{1,2,3} \) to agree with the usual convention. This shows that \( U_{lv}^{HPS} \) is obtained from the mismatch of \( U_{lv}^{CW} \) and a simple 45° rotation in the 2 – 3 sector. It means that if the charged-lepton mass matrix is given by

\[ M_l = U_{lv}^{CW} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} (U_R^l)^\dagger, \] (5)

and the Majorana neutrino mass matrix has 2 – 3 reflection symmetry, with zero 1 – 2 and 1 – 3 mixing, i.e.

\[ M_\nu = \begin{pmatrix} a + 2b & 0 & 0 \\ 0 & a - b & d \\ 0 & d & a - b \end{pmatrix}, \] (6)

\( U_{lv}^{HPS} \) will be obtained, but how? Tribimaximal mixing means:

\[ \theta_{13} = 0, \quad \sin^2 2\theta_{23} = 1, \quad \tan^2 \theta_{12} = 1/2. \] (7)

In 2002, when HPS proposed it, world data were not precise enough to test this hypothesis. In 2004, when I derived it, SNO data implied \( \tan^2 \theta_{12} = 0.40 \pm 0.05, \) which was not so encouraging. Then in 2005, revised SNO data obtained \( \tan^2 \theta_{12} = 0.45 \pm 0.05, \) and tribimaximal became a household word, unleashing a glut of papers. At present, the central value of \( \tan^2 \theta_{12} \) is more like 0.47, making it even closer to 1/2.

III. TETRAHEDRAL SYMMETRY A₄

A. WHAT IS A₄?

For three families, we should look for a group with an irreducible 3 representation, the simplest of which is \( A_4, \) the group of the even permutation of four objects. It has 12 elements and 4 equivalence classes with 3 inequivalent one-dimensional representations and 1 three-dimensional one. As such, it is tailor-made for describing 3 families. Its character table is given below.
TABLE I: Character Table of $A_4$.

| class | $a$ | $b$ | $\chi_1$ | $\chi_{1'}$ | $\chi_{1''}$ | $\chi_3$ |
|-------|-----|-----|--------|--------|--------|-----|
| $C_1$ | 1   | 1   | 1      | 1      | 1      | 3   |
| $C_2$ | 4   | 3   | 1      | 1      | $\omega$ | $\omega^2$ | 0   |
| $C_3$ | 4   | 3   | 1      | 1      | $\omega$ | $\omega^2$ | 0   |
| $C_4$ | 3   | 2   | 1      | 1      | 1      | 1   |

The cube root of unity, i.e., $\omega$, appears again, and you can catch a glimpse of $U_{CW}^l$ in the above. The key to how this leads to Eq. (5) is the multiplication rule:

$$3 \times 3 = \frac{1}{2}(11 + 22 + 33)$$
$$+ \frac{1}{2}(11 + \omega^2 22 + \omega 33) + \frac{1}{2}(11 + \omega 22 + \omega^2 33)$$
$$+ \frac{3}{2}(23, 31, 12) + \frac{3}{2}(32, 13, 21).$$  

Note that $3 \times 3 \times 3 = 1$ is possible in $A_4$, i.e. $a_1 b_2 c_3 +$ permutations.

The non-Abelian finite group $A_4$ is also the symmetry group of the regular tetrahedron, one of five perfect three-dimensional geometric solids, as shown below.

TABLE II: Perfect Three-Dimensional Geometric Solids.

| solid       | faces | vertices | Plato | group |
|-------------|-------|----------|-------|-------|
| tetrahedron | 4     | 4        | fire  | $A_4$ |
| octahedron  | 8     | 6        | air   | $S_4$ |
| cube        | 6     | 8        | earth | $S_4$ |
| icosahedron | 20    | 12       | water | $A_5$ |
| dodecahedron| 12    | 20       | quintessence | $A_5$ |

Most models of neutrino tribimaximal mixing are based on $A_4$. Some use $S_4$ and recently, $A_5$ has also been proposed. It is amusing to note the parallel case of five string theories in ten dimensions: Type I is dual to Heterotic SO(32), Type IIA is dual to Heterotic $E_8 \times E_8$, and Type IIB is self-dual.

B. Two Steps to Tribimaximal Mixing

To realize neutrino tribimaximal mixing using $A_4$, there are two steps. The first is to obtain Eq. (5). Here there are two options. (A) Let $(\nu_i, l_i) \sim 3$, $l_i^c \sim 1$, $l_i^c \sim 1'$, $l_i^c \sim 1''$, and $(\phi^0_1, \phi^+_1) \sim 3$ with $v_1 = v_2 = v_3$. This was the original proposal of Ma and Rajasekaran.

(B) Let $(\nu_i, l_i) l_i^c \sim 3$, and $(\phi^0_1, \phi^+_1) \sim 1$, $3$ with $v_1 = v_2 = v_3$. This choice enables $A_4$ to be compatible with grand unified theories where quarks and leptons belong to a single multiplet, such as the 16 of SO(10).

In either case, the diagonalization of $M_l$ yields $U_{CW}^l$, automatically, with arbitrary $m_e$, $m_\mu$, $m_\tau$. This is the crucial reason for the success of $A_4$. Note that $A_4$ breaks to the residual symmetry $Z_3$ which maintains the condition $v_1 = v_2 = v_3$.

The second step is to obtain Eq. (6). The most straightforward way is to use Higgs triplets $(\xi^+_1, \xi^0_1, \xi^-_1) \sim 1\frac{1}{2}$ with $u_2 = u_3 = 0$, resulting in

$$M_\nu = \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix}.$$  

This is the simplest realization of tribimaximal mixing, with neutrino mass eigenvalues $a + d$, $a$, $-a + d$, allowing only normal hierarchy.

To summarize, the keys to neutrino tribimaximal mixing are (1) the choice of symmetry, i.e. $A_4$ or $S_4$, etc., (2) the choice of lepton and Higgs representations, (3) residual symmetries $Z_4$ and $Z_2$ in different sectors. The last condition is the most technically challenging. In a complete model, a large number of extra particles and auxiliary symmetries are often required, sometimes also with nonrenormalizable operators and perhaps even in the context of extra space dimensions. The above is an example of how group theory alone could determine mixing angles, but leaves all masses free. Applying this idea to the quark sector, it has been shown that the Cabibbo angle $= \pi/14$ could come from $D_7$ breaking to $Z_2$.

IV. SEESAW VARIATIONS AND THE NONUNITARITY OF THE NEUTRINO MIXING MATRIX

A. Canonical Seesaw

With one doublet neutrino $\nu$ and one singlet neutrino $N$, their $2 \times 2$ mass matrix is the well-known

$$M_{\nu N} = \begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix},$$  

resulting in the famous seesaw formula $m_\nu \simeq -m_D^2/m_N$. Hence $\nu - N$ mixing $\simeq m_D/m_D \approx \sqrt{m_\nu/m_N} < 10^{-6}$, for $m_\nu < 1$ eV and $m_N > 1$ TeV. This means that even if $N$ is light enough to be kinematically accessible, it cannot be produced at the Large Hadron Collider (LHC) because its interaction with other particles is too weak. I assume here that $m_N$ is not much below the electroweak symmetry breaking scale of $10^2$ GeV. If this assumption is relaxed, there could be interesting phenomenological implications.
B. Inverse Seesaw

Consider next one $\nu$ and two singlets $N_{1,2}$. Their $3 \times 3$ mass matrix is then

$$M_{\nu N} = \begin{pmatrix} 0 & m_D & 0 \\ m_D & m_1 & m_N \\ 0 & m_N & m_2 \end{pmatrix}, \quad (11)$$
resulting in $m_\nu \approx m_D^2 m_N / m_2^2$. Since the limit $m_1 = m_2 = 0$ corresponds to lepton number conservation ($L = 1$ for $\nu$ and $N_2$, $L = -1$ for $N_1$), their smallness is natural. Thus $m_\nu$ is small, not because $m_N$ is big, but rather that $m_2$ is small. This is the inverse seesaw mechanism \[13,16,17\]. Here $\nu - N_1$ mixing remains small as in the previous case, but $\nu - N_2$ mixing $\approx m_D / m_N$ may now be big, say of order $10^{-2}$. Thus $U_{\nu N}$ may deviate significantly from being unitary and the effect may be decipherable \[18\] at the LHC. If all three neutrinos are considered, this will also be a new source of lepton flavor violation, which may also be experimentally observable.

C. Seesaw Textures

For two families, consider $\nu_{1,2}$ and $N_{1,2}$, with the special choice \[19\]

$$M_{\nu N} = \begin{pmatrix} 0 & 0 & a_1 b_1 & a_1 b_2 \\ 0 & 0 & a_2 b_1 & a_2 b_2 \\ a_1 b_1 & a_2 b_1 & M_1^2 & 0 \\ a_1 b_2 & a_2 b_2 & 0 & M_2^2 \end{pmatrix}. \quad (12)$$
In that case, the arbitrary imposed condition

$$b_1^2 / M_1^2 + b_2^2 / M_2^2 = 0 \quad (13)$$
renders all two light neutrinos massless, and yet $\nu - N$ mixing may be large, because $a, b$ need not be small. Small deviations from this texture will allow small neutrino masses to appear, but retain the large $\nu - N$ mixing. This is an active topic of study with many recent papers, including Refs. \[20,21\].

To understand the mechanism and symmetry of the texture hypothesis, change the neutrino basis to \[22\]

$$M_{\nu N} = \begin{pmatrix} 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_2 \\ m_1 & 0 & M_1 & M_2 \\ 0 & m_2 & M_3 & M_3 \end{pmatrix}. \quad (14)$$

Then Eq. (13) implies $m_1 = M_1 = 0$, so that $\nu_1$ and $\nu_2 = (M_2 \nu_2 - m_2 N_1) / \sqrt{M_2^2 + m_2^2}$ are massless, showing how large mixing actually occurs, i.e. through the inverse seesaw mechanism. However, lepton number conservation would not only forbid $M_1$ but also $M_2$, which is in fact arbitrary here. Where is the symmetry which does this?

Let $\nu_{1,2}, N_{1,2}$ have $L = 1, 1, 3, -1$. Add the usual Higgs doublet $(\phi^0, \phi^0)$ with $L = 0$ and the Higgs singlet $\chi_2$ with $L = 2$. Then $m_2$ comes from $(\phi^0, M_2$ from $(\chi_2)$, and $M_3$ from $(\chi_2^\pm)$ The absence of a Higgs doublet with $L = -4$ and a singlet with $L = \pm 6$ means that $m_1 = M_1 = 0$ at tree level. However, $M_1$ is induced in one loop, from the coupling of $N_1$ to $N_2$ through $\chi_2$. This effect is proportional to $M_2$ and finite, because the breaking of $L$ by $\chi_2$ results in $\text{Re}(\chi_2)$ and $\text{Im}(\chi_2)$ having different masses, the latter being zero if $L$ is broken spontaneously and nonzero if it is also broken explicitly, by a soft term such as $\chi_2^2$ for example. Note that $(-)^L$ is still conserved. Thus $\nu_2$ acquires an inverse seesaw mass $M_1 m_2^2 / M_2^2$. Once $\nu_2$ is massive, $\nu_1$ also gets a two-loop radiative mass \[23\] from the exchange of two $W$ bosons.

D. Seesaw Extensions

Type I seesaw, i.e. using the singlet fermion $N$, is generically difficult to verify, even if $m_N$ is at the TeV scale. If large $\nu - N$ mixing occurs, as discussed in the above, then there is a chance. On the other hand, the Higgs triplet $(\xi^+, \xi^0, \xi^0)$ of Type II seesaw and the fermion triplet $(\Sigma^+, \Sigma^0, \Sigma^-)$ of Type III seesaw have accompanying charged particles as well as electroweak gauge interactions which are much easier to find at the LHC. These are also active topics of study with many recent papers, including Ref. \[21\]. Another attractive possibility is to gauge lepton number or some related quantity, such as $B - L$. This would predict a $Z'$ boson which couples to $N$ and allows the latter to be produced, resulting thus in many interesting phenomenological consequences \[25,26,27,28,29,31\]. However, $U(1)_{B-L}$ is not orthogonal to $U(1)_Y$. It is theoretically much better to use $U(1)_\chi$ instead \[32,33\], because it comes from the breaking of $SO(10) \rightarrow SU(5) \times U(1)_\chi \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\chi$ through the vacuum expectation value of the Higgs \[45\] along the $(24, 0)$ component. Here $Q_\chi = 5 (B - L) - 4 Y$.

V. RADIATIVE SEESAW AND DARK MATTER: SCOTOGENIC NEUTRINO MASS

A. One-Loop Prototype Model

Neutrino mass may be radiative in origin. This is a very old idea, but if the particles in the loop are all odd under an exactly conserved $Z_2$ symmetry whereas the ordinary particles are even, then a connection with dark matter may be established. The simplest such model \[34\] assigns $N_{1,2,3}$ and a second scalar doublet \[35\] $(\eta^+, \eta^0)$ to be odd under $Z_2$. Hence $\nu N \eta^0$ is forbidden and $\nu N \eta^0$ is allowed, but $(\eta^0) = 0$, so that
N is not the Dirac mass partner of \( \nu \). A finite neutrino mass is generated in one loop because \( \text{Re}(\eta^0) \) and \( \text{Im}(\eta^0) \) have different masses from the allowed quartic scalar interaction term \( (\lambda_5/2)(\Phi/2)^2 + H.c. \). This mass splitting also enables \( \text{Re}(\eta^0) \) or \( \text{Im}(\eta^0) \) to be a realistic dark-matter candidate, as studied in some detail over the last two months by Barbieri, Hall, and Rychkov [30]. They call \( \eta \) the inert Higgs doublet, but it is neither inert because it has gauge interactions, nor a “Higgs” because it has no vacuum expectation value. I call it the dark scalar doublet.

The one-loop radiative seesaw neutrino mass is easily calculated:

\[
(M_\nu)^{\alpha\beta} = \sum_i \frac{h_{\alpha i} h_{\beta i} M_i}{16\pi^2} \left[ f \left( \frac{m_i^2}{M_i^2} \right) - f \left( \frac{m_R^2}{M_i^2} \right) \right],
\]

where

\[
f(x) = -\ln x/(1 - x).
\]

Let \( m_R^2 - m_i^2 = 2\lambda_5 v^2 \leq m_0^2 = (m_R^2 + m_i^2)/2 \), then

\[
(M_\nu)^{\alpha\beta} = \sum_i \frac{h_{\alpha i} h_{\beta i} M_i}{16\pi^2} \left( \frac{m_0^2}{M_i^2} \right),
\]

where

\[
I(x) = -\frac{\lambda_5 v^2}{8\pi^2} \left( \frac{1}{1 - x} \right) \left[ 1 + \frac{\ln x}{1 - x} \right].
\]

For \( x_i \ll 1 \), i.e. \( N_i \) very heavy, this reduces to

\[
(M_\nu)^{\alpha\beta} = -\frac{\lambda_5 v^2}{8\pi^2} \sum_i \frac{h_{\alpha i} h_{\beta i}}{M_i} (1 + \ln x_i).
\]

instead of the canonical seesaw expression of \( v^2 \sum_i h_{\alpha i} h_{\beta i}/M_i \). In the context of leptogenesis, if this mechanism is extended to include supersymmetry, the Davidson-Ibarra lower bound [38] of about 10^9 GeV for the lightest \( N_i \) may then be evaded, avoiding thus a potential conflict of gravitino overproduction and thermal leptogenesis.

### B. Supersymmetric SU(5) Completion

This model of scotogenic neutrino mass also has a straightforward SU(5) completion [33] with gauge-coupling unification. Simply add the complete superfield multiplets

\[
\tilde{5} = h(3, 1, -1/3) + (\eta_1^0, \eta_2^0)(1, 2, 1/2),
\]

\[
\tilde{5}^* = h^c(3^*, 1, 1/3) + (\eta_1^0, \eta_2^0)(1, 2, -1/2),
\]

and singlets \( N_{1\,2\,3} \) and \( \chi \). Let these be odd under \( Z_2 \) and the usual superfields be even. Furthermore, let \( N \) be odd under the usual matter parity and \( \chi \) be even. Then the usual R parity of the Minimal Supersymmetric Standard Model (MSSM) is maintained together with the new \( Z_2 \). Gauge-coupling unification of the MSSM is undisturbed if all the \( \tilde{5} \) and \( \tilde{5}^* \) particles are at the TeV scale. Proton decay is safe because \( h \) and \( h^c \) are odd under \( Z_2 \). The strong production of \( h \) and \( h^c \) at the LHC and their subsequent decays, such as \( h \rightarrow d^-\eta_2^+ \) and \( d^-\eta_2^- \), would yield same-sign dileptons plus quark jets plus missing energy, which is a possible unique signature of this model.

### C. Multipartite Dark Matter

At least two out of the following three particles are dark-matter candidates: (1) the usual lightest neutralino of the MSSM with \((R, Z_2) = (-, +)\), (2) the lightest exotic neutral particle with \((+, -)\), and (3) that with \((-,-)\). The dark matter of the Universe may not be all the same, as most people have taken for granted! For a general discussion, see Ref. [40].

### D. Three-Loop Neutrino Mass

Neutrino mass may also be obtained in three loops, with the addition of \( N \) and a charged scalar \( S_2^+ \) which are odd under \( Z_2 \). This model [41] also has a charged scalar \( S_2^- \) which is even under \( Z_2 \). It was the first proposal that \( N \) could be dark matter. However, since \( lNS_2^+ \) is the only interaction involving \( N \), it cannot be too weak to have the correct annihilation cross section for the observed dark-matter relic density. This implies generically large flavor-changing leptonic radiative decays, such as \( \mu \rightarrow e\gamma \), and requires delicate fine tuning [42] to suppress. In the one-loop scotogenic model [41], this constraint may be relaxed, because \( \text{Re}(\eta^0) \) is available for dark matter.

Another three-loop model has been proposed [43] where the singlet \( S_1^+ \) is replaced by a a second Higgs doublet and a neutral singlet \( \eta^0 \) is added with odd \( Z_2 \). In this case, \( \eta^0 \) (with a mass of 40 to 65 GeV) is a suitable dark-matter candidate. Since its interactions with the other scalar particles are unconstrained, the desired relic abundance is easily obtained without requiring the \( lNS_2^+ \) coupling to be large, thus avoiding the problem of flavor violating leptonic interactions. This model also allows for electroweak baryogenesis, coming from a first-order phase transition in the Higgs potential. At the LHC, the \( \eta^0 \) of this model is hard to produce and detect because it is a singlet, whereas the \( \text{Re}(\eta^0) \) and \( \text{Im}(\eta^0) \) of the one-loop scotogenic model are produced by the \( Z \) boson, with the subsequent decay \( \text{Im}(\eta^0) \rightarrow \text{Re}(\eta^0) l^+ l^- \) as a possible signature [14].

### VI. CONCLUDING REMARKS

- With the application of the non-Abelian discrete symmetry \( A_4 \), a plausible theoretical un-
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An understanding of the tribimaximal form of the neutrino mixing matrix has been achieved.

- Seesaw variants at the TeV scale may allow this mechanism (in its inverse or linear manifestation) to be observable through the nonunitarity of the $3 \times 3$ neutrino mixing matrix, as well as flavor changing leptonic interactions.

- Dark matter may be the origin of radiative neutrino mass. This scotogenic mechanism may be implemented in a number of different models, and be observable also at the TeV scale. A complete supersymmetric SU(5) version also exists with gauge-coupling unification.

- Other recent topics in neutrino theory, such as Type III seesaw, using a Majorana fermion triplet $(\Sigma^+, \Sigma^0, \Sigma^-)$, and small Dirac and pseudo-Dirac neutrino masses, are not covered in this talk, but are being actively pursued.

- Neutrino theory marches on, but what we really need are corroborating data!

Acknowledgments

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[1] N. Cabibbo, Phys. Lett. B72, 333 (1978).
[2] L. Wolfenstein, Phys. Rev. D18, 958 (1978).
[3] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. B530, 167 (2002).
[4] E. Ma, Phys. Rev. D70, 031901 (2004).
[5] E. Ma, Mod. Phys. Lett. A17, 2361 (2002).
[6] L. L. Everett and A. J. Stuart, arXiv:0812.1057 [hep-ph].
[7] E. Ma and G. Rajasekaran, Phys. Rev. D64, 113012 (2001).
[8] E. Ma, Mod. Phys. Lett. A21, 2931 (2006).
[9] G. Altarelli and F. Feruglio, Nucl. Phys. B72, 64 (2005).
[10] E. Ma, Phys. Rev. D72, 037301 (2005).
[11] C. S. Lam, Phys. Lett. B656, 193 (2007).
[12] A. Blum, C. Hagedorn, and M. Lindner, Phys. Rev. D77, 076004 (2008).
[13] A. Atre, T. Han, S. Pascoli, and B. Zhang, JHEP 0905, 030 (2009).
[14] A. de Gouvea, W.-C. Huang, and J. Jenkins, arXiv:0906.1611 [hep-ph].
[15] D. Wyler and L. Wolfenstein, Nucl. Phys. B218, 205 (1983).
[16] R. N. Mohapatra and J. W. F. Valle, Phys. Rev. D34, 1642 (1986).
[17] E. Ma, Phys. Lett. B191, 287 (1987).
[18] E. Ma, arXiv:0904.1580 [hep-ph].
[19] W. Buchmuller and D. Wyler, Phys. Lett. B249, 458 (1990).
[20] A. Pilaftsis, Phys. Rev. Lett. 95, 081602 (2005).
[21] J. Kersten and A. Yu. Smirnov, Phys. Rev. D76, 073005 (2007).
[22] X.-G. He and E. Ma, arXiv:0907.2737 [hep-ph].
[23] K. S. Babu and E. Ma, Phys. Rev. Lett. 61, 674 (1988).
[24] F. del Aguila and J. A. Aguilar-Saavedra, Nucl. Phys. B813, 22 (2009).
[25] S. Khalil, J. Phys. G35, 055001 (2008).
[26] K. Huitu, S. Khalil, H. Okada, and S. K. Rai, Phys. Rev. Lett. 101, 181802 (2008).
[27] S. Khalil and H. Okada, Phys. Rev. D79, 083510 (2009).
[28] R. Allahverdi, B. Dutta, K. Richardson-McDaniell, and Y. Santoso, Phys. Rev. D79, 075005 (2009).
[29] Y. Kajiyama, S. Khalil, and M. Raidal, Nucl. Phys. B820, 75 (2009).
[30] L. Basso, A. Belyaev, S. Moretti, and C. H. Shepard-Themistocleous, arXiv:0812.4313 [hep-ph].
[31] K. S. Babu, Y. Meng, and Z. Tavartkiladze, arXiv:0901.1041 [hep-ph].
[32] E. Ma, Phys. Rev. D80, 013013 (2009).
[33] P. Fileviez Perez, T. Han, and T. Li, arXiv:0907.4186 (2009).
[34] E. Ma, Phys. Rev. D73, 077301 (2006).
[35] N. G. Deshpande and E. Ma, Phys. Rev. D18, 2574 (1978).
[36] R. Barbieri, L. J. Hall, and V. S. Rychkov, Phys. Rev. D74, 015007 (2006).
[37] E. Ma, Ann. Fondation de Broglie 31, 85 (2006).
[38] S. Davidson and A. Ibarra, Phys. Lett. B535, 25 (2002).
[39] E. Ma, Phys. Lett. B659, 885 (2008).
[40] Q.-H. Cao, E. Ma, J. Wudka, and C.-P. Yuan, arXiv:0711.3881 [hep-ph].
[41] L. M. Krauss, S. Nasri, and M. Trodden, Phys. Rev. D67, 085002 (2003).
[42] J. Kubo, E. Ma, and D. Suematsu, Phys. Lett. B642, 18 (2006).
[43] M. Aoki, S. Kanemura, and O. Seto, Phys. Rev. Lett. 102, 051805 (2009).
[44] Q.-H. Cao, E. Ma, and G. Rajasekaran, Phys. Rev. D76, 095011 (2007).