The RIT binary black hole simulations catalog

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Abstract
The RIT numerical relativity group is releasing a public catalog of black-hole-binary waveforms. The initial release of the catalog consists of 126 recent simulations that include precessing and nonprecessing systems with mass ratios $q = m_1/m_2$ in the range $1/6 \leq q \leq 1$. The catalog contains information about the initial data of the simulation, the waveforms extrapolated to infinity, as well as information about the peak luminosity and final remnant black hole properties. These waveforms can be used to independently interpret gravitational wave signals from laser interferometric detectors and the remnant properties to model the merger of black-hole binaries from initial configurations.

Keywords: binary black holes, numerical general relativity, simulations

(Some figures may appear in colour only in the online journal)

1. Introduction

The breakthroughs [1–3] in numerical relativity allowed numerical relativists to make detailed predictions for the gravitational waves from the late inspiral, plunge, merger and ringdown of black-hole-binary systems (BHB). Numerical relativity predictions were confirmed by the first direct detection [4] of gravitational waves from such binary systems [5–8] and by its comparison to targeted runs [9, 10]. The observed gravitational waves were remarkably consistent with the predictions of general relativity, thereby supporting the notion that general relativity is an accurate theory of gravity [7, 11].

Numerical relativity techniques have been used to explore the late dynamics of spinning black-hole binaries, beyond the post-Newtonian regime for many years. Indeed, the first
comparison of full numerical waveforms and post-Newtonian predictions were performed soon after the breakthroughs in [12–16]. The first generic, long-term precessing black-hole binary evolutions (i.e. without any symmetry) were performed in [17], where a detailed comparison with post-Newtonian $\ell = 2, 3$ waveforms was made. More recently, the longest such comparison using a 350 orbit full numerical simulation was performed in [18].

Other important studies include the exploration of the hangup effect, i.e. the role individual black-hole spins play to delay or accelerate their merger [16, 19, 20], the determination of the magnitude and direction of the recoil velocity of the final merged black hole [21–27], and the flip-flop of individual spins during the orbital phase [28–30], as well as precession dynamics [31–33] and the inclusion of those dynamics to construct surrogate models for gravitational waveforms [34].

Other numerical simulations have also explored the corners of parameter space, these include near extremal $\chi = 0.994^2$ spinning black-hole binaries in [35], as well as mass ratios as small as $q = 1/100$ in [36, 37], and larger initial separations $R = 100M$ in [38]. Similarly, high energy collision of black holes were studied in [39] and hyperbolic black-hole encounters in [40].

There have been several significant efforts to coordinate numerical relativity simulations to support gravitational wave observations. These include the numerical injection analysis (NINJA) project [41–44], the numerical relativity and analytical relativity (NRAR) collaboration [45], and the waveform catalogs released by the SXS collaboration [46–48] and Georgia Tech. [49].

For recent reviews of the field, see [50–53]. In this paper we describe a new waveform catalog by the RIT numerical relativity that is now available for public use. The catalog includes all $\ell = 2$ and $\ell = 3$ modes of $\psi_4$ and the strain $h$ (both extrapolated to null-infinity). The catalog can be downloaded from http://ccrg.rit.edu/~RITCatalog.

This paper is organized as follows. Section 2 describes the methods and criteria for producing the numerical simulations. We next describe in section 4 the use and content of the data in the public catalog. We conclude with a discussion in section 5 of the use and potential extensions to this work to precessing binaries.

2. Full numerical evolutions

The runs in the RIT Catalog were evolved using the LazEv [54] implementation of the moving puncture approach [2] with the conformal function $W = \exp(-2\phi)$ suggested by [55]. In all cases we use the BSSNOK (Baumgarte–Shapiro–Shibata–Nakamura–Oohara–Kojima) family of evolutions systems [56–58]. For the runs in the catalog, we used a variety of finite-difference orders, Kreiss–Oliger dissipation orders, and Courant factors [59–61]. Note that we do not upwind the advection terms. All of these are given in the metadata included in the catalog and the references associated with each run (where detailed studies have been performed).

The LazEv code uses the EinstEintoolkit [62, 63] / Cactus [64] / Carpet [65] infrastructure. The Carpet mesh refinement driver provides a ‘moving boxes’ style of mesh refinement. In this approach, refined grids of fixed size are arranged about the coordinate centers of both holes. The Carpet code then moves these fine grids about the computational domain by following the trajectories of the two BHs.

We use AhfinderDirect [66] to locate apparent horizons. We measure the magnitude of the horizon spin using the isolated horizon (IH) algorithm detailed in [67] (as implemented in

$^2\chi$ denotes the spin angular momentum of a black hole in units of the square of its mass. The maximum possible spin is $\chi = 1$.}
The IH formalism requires that one finds an approximate Killing vector $\omega^i$ (rotational symmetry) on the horizon. The horizon spin angular momentum, $S_H$, is then given by

$$S_{[\omega]} = \frac{1}{8\pi} \int_{AH} (\omega^a R^b K_{ab}) d^2V,$$

(1)

where $K_{ab}$ is the extrinsic curvature of the 3D-slice, $d^2V$ is the natural volume element intrinsic to the horizon, and $R^a$ is the outward pointing unit vector normal to the horizon on the 3D-slice. The approximate Killing vector $\omega^i$ is calculated using the procedure described in Dreyer et al section III [67]. Briefly, $\omega^i$ is obtained by considering the propagation of 1-form $\omega_a$ and 2-form $L_{ab}$ intrinsic to the 2-sphere of the horizon along an arbitrary closed-curve $\gamma$ from an arbitrary point $p$ to that same point. The Killing propagation equations then induce a linear map from the space of 1 and 2-forms at a point $p$ to itself. One then looks for eigenvectors of this transformation with (near) unit eigenvalue. In order to increase accuracy, 2-sphere of the horizon is covered with spherical coordinates aligned (or nearly aligned) with the spin direction. The curve $\gamma$ is then chosen to be the equator.

Note that once we have the horizon spin, we can calculate the horizon mass via the Christodoulou formula $m_H = \sqrt{m_{\text{irr}}^2 + S_H^2 / (4m_{\text{irr}}^2)}$, where $m_{\text{irr}} = \sqrt{A / (16\pi)}$ and $A$ is the surface area of the horizon.

To compute the numerical initial data, we use the puncture approach [69] along with the TwoPunctures [70] thorn. To compute initial low eccentricity orbital parameters, we use the post-Newtonian techniques described in [71].

### 3. Error assessments

We use the variation in the measured horizon irreducible mass and spin during the simulation as a measure of the error in computing these quantities, since the levels of gravitational wave energy and momentum absorbed by the holes is 4–5 orders of magnitude smaller [72]. We measure radiated energy, linear momentum, and angular momentum, in terms of the radiative Weyl Scalar $\psi_4$, using the formulas provided in [73, 74]. However, rather than using the full $\psi_4$, we decompose it into $\ell$ and $m$ modes and solve for the radiated energy-momentum, dropping terms with $\ell > 6$. The formulas in [73, 74] are valid at $r = \infty$. We extract the radiated energy-momentum at finite radius and extrapolate to $r = \infty$ using a least squares fit to a polynomial in $1/r$. We take the differences between a linear and quadratic extrapolation as an estimate for the uncertainty.

We also compared the horizon quantities with the radiative counterparts and found that they converge towards the horizon values (see figures 24 and 25 and tables VIII and IX of [75] and figures 8–11 and tables XIV–XVII in [61]).

To calculate $\psi_4$, we use the Antenna thorn [76, 77]. Here, we use the standard coordinate tetrad which is based on unit timelike vector normal to the slice and the unit (coordinate) radial vector in the slice. This tetrad is asymptotically (at $r \to \infty$) equivalent to a quasi-Kinnersley tetrad.

To extrapolate the waveforms to infinity, we use a different procedure which proved to be very robust. In [78] the Teukolsky equation is used to derive expressions for $r^{-1} \psi_4$ at infinity based on its values on a finite sphere. The expressions there contain the corrections of order $O(1/r)$ and $O(1/r^2)$ to $r \psi_4$ and are of the form
\[ r \psi_4^{\text{em}}(r, t) = \left( 1 - \frac{2M}{r} \right) \left( \psi_4^{\text{NR}}(r, t) - \frac{(\ell - 1)(\ell + 2)}{2} \int dr |\nabla_r \psi_4^{\text{NR}}(r, t)| \right), \]  

where \( r \) is the areal radius and \( \psi_4^{\text{NR}} \) indicates \( \psi_4 \) as measured from the numerical simulation at a given coordinate radius. As shown in [78], this extrapolation is consistent with both the waveform and the radiated energy-momentum extrapolated using ordinary polynomial extrapolation. It has advantages over the latter in that an accurate waveform can be obtained at a smaller radius, and the noise amplification observed with polynomial extrapolation is suppressed. Additionally, the \( O(1/r) \) perturbative corrections were shown to be consistent with a Cauchy-Characteristic extraction for an equal-mass binary in [79].

Representative runs of the catalog have been studied in detail in previous papers. In appendix A of [75], which includes 36 of the waveforms provided in this catalog, we performed a detailed error analysis of prototypical configuration with equal mass and spins aligned/antialigned with respect to the orbital angular momentum. We varied the initial separation of the binary, the resolutions, grid structure, waveform extraction radii, and the number of \( \ell \) modes used in the construction of the radiative quantities. While in appendix B of [61], which includes 71 waveforms in this catalog, we calculated the finite observer location errors and performed convergence studies for typical runs with mass ratio \( q = 1, 3/4, 1/2, 1/3 \), at three different resolutions.

In both studies we conclude that the waveforms at the resolutions provided in this catalog are well into the convergence regime (roughly converging at 4th-order with resolution), that the remnant final mass and spins have errors of the order of 0.1%, and that the recoil velocities and peak luminosities are evaluated at a typical 5% error.

Finally, in Reference [10] we showed that for the parameter estimated for GW150914, the simulations we provided (also available in this catalog) with mass ratio \( q = m_1/m_2 = 0.82 \) and spins for the small/large holes of \( \chi_1 = -0.44 \) and \( \chi_2 = +0.33 \), have an excellent matching of 0.999 with those waveforms produced completely independently by the SXS collaboration. This shows, that for this kind of gravitational wave events, the typical resolution used by our simulations, and provided in this catalog, is accurate enough to provide an appropriate representation of the theoretical match, solution to the field equations of the general theory of relativity. In this study, we also investigated the phase and amplitude error of our waveforms at different resolutions (figure 2 of [10]) and found between N110 and N120 resolutions, a phase difference \( \delta \phi \) and amplitude difference \( |\delta A|/A \) on the order of \( 10^{-4} \) during the inspiral, and growing to \( 10^{-3} \) and \( 10^{-2} \) at merger for the amplitude and phase, respectively. Assuming 4th order convergence, the difference between an infinitely resolved waveform and the N120 resolution would be about 2.4 times larger, or around \( 2.4 \times 10^{-2} \) for the phase difference at merger, and 0.2% for the amplitude difference at merger.

4. Catalog

The RIT Catalog can be found at http://ccrg.rit.edu/~RITCatalog. Figure 1 shows the distribution of non-precessing runs in the catalog in terms of \( \chi_{1,2} \) and \( q \) (where \( \chi_i \) is the component of the dimensionless spins of BH \( i \) along the direction of the orbital angular momentum). The information currently in the catalog consists of the metadata describing the runs and all the \( (\ell = 2, m = 0, \pm 1, \pm 2) \) and \( (\ell = 3, m = 0, \pm 1, \pm 2, \pm 3) \) modes of \( M \psi_4 \). The extrapolations of \( M \psi_4 \) to \( r = \infty \) were performed using the perturbative approach of [78] (note that we normalize our data such that the sum of the two initial horizon masses is 1M). The associated metadata include the initial orbital frequencies, ADM masses, initial waveform frequencies,
black hole masses, momenta, spins, separations, and eccentricities, as well the black-hole masses and spins once the initial burst of radiation has left the region around the binary. These relaxed quantities (at $t_{\text{relax}} = 150M$) are more accurate for modeling purposes than the initial masses and spins. In addition, we also include the final remnant masses and spins.

The catalog is organized using an interactive table that includes an identification number, resolution, type of run (nonspinning, aligned spins, precessing), the initial proper length of the coordinate ray joining the two BH centroids that is outside both horizons, the coordinate separation of the two centroids, the mass ratio of the two black holes, the components of the dimensionless spins of the two black holes, the starting waveform frequency, time to merger, number of gravitational wave cycles\(^3\), remnant mass, remnant spin, recoil velocity, and peak luminosity. The final column gives the appropriate bibtex keys for the relevant publications where the waveforms were first presented. The table can be sorted (ascending or descending) by any of these columns.

The proper length of the coordinate ray joining the two horizons is gauge dependent, but still gives reasonable results (see [38] for more details). Note that in the precessing case [80], using oscillations in this length to measure eccentricity will lead to incorrect results as both eccentricity and precession lead to oscillations in the proper length of this ray.

The initial waveform frequency, denoted by $M_{f_{22,\text{start}}}$ in the table gives the starting frequency (measured directly from the waveform after the initial burst has dissipated) in units of $2.03 \times 10^5 \left( \frac{M}{M_\odot} \right)$ Hz, where $M$ is the mass of the binary and $M_\odot$ is one solar mass (e.g. $M_{f_{22,\text{start}}} = 0.01$ corresponds to 34 Hz for a 60 $M_\odot$ binary). Note that $2\pi f_{22} = \omega_{22}$. The runs in the catalog span initial frequencies from 0.003 to 0.012, with a corresponding initial proper separations of 10.59M to 25.18M. Times from the start of the simulation to merger range from 556M to 19 219M, and the number of inspiral cycles in the $(\ell = 2, m = 2)$ mode of $\psi^4$ range from 8.3 to 89.9.

\(^3\)The number of cycles is measured from the cumulative phase of the tracks from the beginning of the orbital motion to the merger, this produces very close results to the use of the phase of $\psi^4$ from the beginning of the inspiral signal to the amplitude peak.
Resolutions are given in terms of the gridspacing of the refinement level where the waveform is extracted (which is typically two refinement levels below the coarsest grid) with $R_{\text{obs}} \sim 100M$. We use the notation nXY, where the gridspacing in the wavezone is given by $M/X.YY$, e.g. n120 corresponds to $\delta = M/1.2$.

For each simulation in the catalog there are two files: one contains the metadata information in ASCII format, the other is a tar.gz file containing ASCII files with the $(\ell = 2, m = 0, \pm 1, \pm 2)$ and $(\ell = 3, m = 0, \pm 1, \pm 2, \pm 3)$ modes of $Mr\psi_4$. These modes are extrapolated to $r = \infty$ using equation (2).

Note that the primary data in our catalog is the Weyl scalar $Mr\psi_4$ extrapolated to infinity rather than the strain $h$. We leave it to the user to convert $Mr\psi_4$ to strain for most modes since this is still a sensitive process and is best handled on a mode-by-mode basis. The subtleties associated with transforming $\psi_4$ to $h$ arise from the two integrations required. One of the standard techniques, developed by Reisswig and Pollney [81], performs this integration in Fourier space with a windowing function and a low-frequency cutoff. Both of these require fine-tuning of parameters. The codes to do this are open-source and publicly available from http://svn.einstein toolkit.org/pyGWAnalysis/trunk. As an example of the reconstruction of the strain from $\psi_4$, we provide the $(2, 2)$ and $(2, 1)$ modes of $h$ for the 126 simulations as displayed in figure 4.

Figure 2. Initial parameters in the $(q, \chi_1, \chi_2)$ plane for the 120 nonprecessing binaries. Note that $\chi_i$ denotes one of the component of the dimensionless spin of BH $i$ along the orbital angular momentum and $q = m_1/m_2 < 1$. 
Figure 2 shows the distribution of the 120 non-precessing runs in the catalog in terms of \( \chi_{1,2} \) and \( q \) (\( \chi_i \) is the component of the dimensionless spin of BH \( i \) along the direction of the orbital angular momentum). Those runs were motivated by two systematic studies. First to produce a set of accurate remnant formulas to represent the final mass, spin and recoil of a merged binary black hole system as a function of the parameters of the precursor binary, as reported in \([61, 75]\). The second was to provide a (coarse) grid of simulations for parameter estimation of gravitational wave signals detected by LIGO using the methods described in \([9]\). The 6 precessing runs were used in the study of unstable spin flip-flop, as reported in \([30]\).

Figure 3 shows the distributions of the minimal total mass of the BHB systems in the RIT catalog corresponding to a starting gravitational wave frequency of 20 Hz (red) and 30 Hz (blue) in bins of 5\( M_\odot \). Below the cumulative version of the above plot also in bins of 5\( M_\odot \).

Figure 2 shows the distribution of the 120 non-precessing runs in the catalog in terms of \( \chi_{1,2} \) and \( q \) (\( \chi_i \) is the component of the dimensionless spin of BH \( i \) along the direction of the orbital angular momentum). Those runs were motivated by two systematic studies. First to produce a set of accurate remnant formulas to represent the final mass, spin and recoil of a merged binary black hole system as a function of the parameters of the precursor binary, as reported in \([61, 75]\). The second was to provide a (coarse) grid of simulations for parameter estimation of gravitational wave signals detected by LIGO using the methods described in \([9]\). The 6 precessing runs were used in the study of unstable spin flip-flop, as reported in \([30]\).

Figure 3 shows the distributions of the minimal total mass of the BHB systems in the catalog given a starting gravitational wave frequency of 20 or 30 Hz in the source frame. This provides a coverage for the current events observed by LIGO (redshift effects improve this coverage by a factor of \( 1 + z \), where \( z \) is the redshift). Coverage of lower total masses would require longer simulations or hybridization of the current waveforms.
Conclusions and discussion

The breakthroughs [1–3] in numerical relativity were instrumental in identifying the first detection of gravitational waves [4] with the merger of two black holes. The direct comparison of numerical waveforms with observations also allows one to determine the parameters of such binaries [9]. The current catalog of waveforms as displayed in figure 4 can be used to perform independent analysis by the wider gravitational wave community and serves as a platform to deliver new sets of simulations as they become available.

Aside the interest in producing waveforms for direct comparison with observation, the simulations of orbiting black-hole binaries produce information about the final remnant of the merger of the two holes. This was already the subject of early studies using the Lazarus approach [82, 83]. With the advent of the breakthroughs that allowed for longer accurate computations, numerous empirical formulas relating the initial parameters \((q, \vec{\chi}_1, \vec{\chi}_2)\) (individual masses and spins) of the binary to those of the final remnant \((m_f, \vec{\chi}_f, \vec{V}_f)\) have been proposed. These include formulas for the final mass, spin, and recoil velocity [75, 84–90], as well as algebraic properties of the final metric [91, 92]. Recently, the computation of the peak frequency [93] and peak luminosity has also been the subject of interest in relation to the observation of gravitational waves [4, 7, 61, 94]. The data in RIT catalog, along with the SXS [46–48] and Georgia Tech. [49] catalogs, can be used by other groups to develop and improve new empirical formulas for the remnant properties and approximate/phenomenological waveform models [95, 96]. This first release of the RIT waveforms is planned to be followed up by periodic updates including new precessing and nonprecessing simulations as they become suitable for public release as well as increasing search and post-processing capabilities. We expect to be double the total number of waveforms in the next release, as well as include accurate strain reconstructions for the higher modes, up to \(\ell = 6\). Another planned upgrade to the catalog is to transform the waveforms to the average rest frame of the binary. To obtain the average rest frame of the binary, one would calculate the average velocity of the center.

Figure 4. Visual display of the different lengths of the (2,2) (in blue) and (2,1) (in red) strain \(h\) waveforms in this first set of 126 simulations in the RIT Catalog. Each row of waveforms spans \(\sim 22\ 700 M\) of simulation time from edge to edge, with each tic mark denoting \(500 M\).
of mass of the binary (from $\psi_4$) over the inspiral and then boost the waveform in the opposite direction. This is done using equations (4) and (5) in [97] and equations (7) and (8) in [98]. By performing this boost, the amount of mode-mixing, where information from the dominant $\ell = 2, m = \pm 2$ modes would otherwise leak into the other modes, is reduced. Note that this would not change the physical waveforms, only how they are spread over modes. Choosing the frame in this way may also be useful for direct comparisons with other codes.

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