Characterizing Wake Turbulence with Staring Lidar Measurements

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Abstract.
Lidar measurements in the German offshore wind farm Alpha Ventus were performed to investigate the turbulence characteristics of wind turbine wakes. In particular, we compare measurements of the free flow in the surroundings of the wind turbines with measurements in the inner region of a wake flow behind one turbine. Our results indicate that wind turbines modulate the turbulent structures of the flow on a wide range of scales. For the data of the wake flow, the power spectrum as well as the multifractal intermittency coefficient reveal features of homogeneous isotropic turbulence. Thus, we conjecture that on scales of the rotor a new turbulent cascade is initiated, which determines the features of the turbulent wake flow quite independently from the more complex wind flow in the surroundings of the turbine.

1. Introduction
It is a well-known fact that the power output and loads of wind turbines are strongly influenced by wakes of neighboring turbines [1, 2]. These wakes are highly turbulent flows which even show an increased turbulence intensity compared to the atmospheric inflow [3, 1]. Since the response times of wind turbines are typically in the order of seconds the small scale properties of turbulent flows have a strong impact on the turbines [4, 5, 6, 7, 8]. Hence, to understand the impact of wakes on neighboring turbines, the turbulence present in a wake flow needs to be investigated in detail.

Recently, laboratory experiments with model wind turbines have been performed [9, 10, 11] showing that the turbulent statistics in the wake flow differ strongly from the inflow characteristics.

Since laboratory experiments cannot fully emulate the atmospheric boundary layer turbulence and since they are performed for relatively small Reynolds numbers, field measurements play an essential role for the investigation of wake flows. Measurements performed on met masts offer only a limited number of measurement locations and are therefore not sufficient to characterize flows in wind farms. Thus, the use of remote sensing techniques, such as sodars or Doppler lidars, has increased rapidly in the last years [12, 13, 14, 15, 16, 17]. However, field measurements focusing on turbulent features of wakes[16, 17, 18] are still rare and have to the authors knowledge...
not yet been performed in an offshore environment. Furthermore, they mostly do not investigate higher order statistics. Therefore, this study aims for a characterization of wake turbulence using lidar measurements in an offshore wind farm. By comparing the statistics of the wind field in a wake region with the statistics of a region with approximately free atmospheric flow, we investigate the modulation of the turbulent structures through the wind turbine.

We start with a concise description of the measurements in Sec. 2. The statistical analysis is performed in Sec. 3 including a description of power spectral densities and higher order statistics of temporal increments. In Sec. 4 the results of our analysis are discussed and conclusions are drawn in Sec. 5.

2. Measurements

The measurements analyzed in this work stem from a measurement campaign in the offshore wind farm “Alpha Ventus” (Fig. 1) located in the German North Sea. In this campaign, three scanning long range Doppler lidar systems of type Leosphere Windcube200S were used for a detailed analysis of the flow in the wind farm. One lidar was positioned on the wind farm’s sub station while two others were positioned on the nearby research platform and meteorological mast “Fino 1”.

Figure 1. Illustration of the measurement scenario in the wind park “Alpha Ventus”. The blue color represents the wakes of the different turbines in the farm. Note that the wakes do also exist in larger distances to the turbine than indicated by the blue color. The black cross represents the position of the research platform “Fino 1”. The red line represents the lidar beam. The black lines through the lidar beam show approximate positions of the center of the measurement regions, defined in Sec. 3.1.

In this work, we analyze data from a measurement scenario using only the lidar on the substation platform and an ultrasonic anemometer located at a height of $z = 60$ m on the nearby met mast Fino 1. Measurements were taken for approx. 5 days but only 10 hours of this data are selected for the analysis presented in this work. In the selected time interval the wind speed and the wind direction indicate a relatively stationary atmospheric velocity field, as illustrated by the sonic measurements shown in Fig. 2. The average wind speed, noted as $U$
in the following, is given by approx. 9.9 m/s and the average wind direction by approx. 213°. The approximate stationarity for this relatively long period is essential for a reliable statistical analysis.

![Ultrasonic Anemometer measurements](image)

**Figure 2.** Ultrasonic Anemometer measurements on “Fino 1” (z = 60 m). Left: Absolute value of the wind speed measured with a sampling rate of 10 Hz (black line) and its ten minute average (red line). Right: Ten minute average of the wind direction α.

To facilitate the investigation of short time scales, the lidar was set to a staring mode with a fixed direction of the laser beam, as illustrated in Fig. 1. In the following, the distance along the laser beam will be noted as r and the time as t. The angle between the average wind vector and the lidar beam is approx. 51° and the vertical position of the laser beam is shown in Fig. 3. The lidar measures the velocity component along the laser beam, which will be called line of sight (los) velocity in the following and be noted as $u_{\text{los}}$. Further technical settings for the lidar can be found in table 1. With these settings $u_{\text{los}}$ could be measured instantaneously at different positions with a relatively high temporal resolution, as shown in Fig. 4. Measurements with a carrier to noise ratio (CNR) lower than $-22.5$ dB were omitted from the analysis which were less than 4% of the data in the analyzed measurement regions, defined in Sec. 3.1. The measurements were not taken at equidistant times and the average time step was given by approximately 1.5 s corresponding to an average sampling rate of approx. 0.6 Hz. Since in the following 2-point statistics, such as moments of temporal increments are investigated, a resampling procedure was applied. Linear interpolation was used to obtain values at equidistant time steps of 1.5 s. Values stemming from an interpolation over a time of more than 2 s were omitted.

**Table 1.** Lidar settings for the measurement scenario described in Sec. 2.

| Setting                              | Value          |
|--------------------------------------|----------------|
| Azimuth angle                        | -17.62°        |
| Elevation angle                      | 1.80°          |
| Pulse length (Full Width Half Maximum)| 200 ns         |
| Range gate size                      | 64 points      |
| Sampling rate of the photodiode      | 250 Mhz        |
| Accumulation time                    | 0.60 s         |
| Physical resolution                  | 30 m – 40 m    |
| Range gate distance                  | 4 m            |
3. Results

3.1. Defining Regions of Interest

To compare the statistics of the lidar measurements in a wake and a free flow situation, we choose two different regions for our analysis. The first region is centered at $r = 460$ m (lower black line crossing the laser beam in Fig. 1 and lower dashed line in Fig. 4) and contains all range gates with $r \in [430, 490]$ m. For the wind directions occurring in the selected measurement period this region is assumed to be only weakly influenced by the presence of the wind farm (see Fig. 1). Thus, it will be referred to as the free flow region the following.

The second region corresponds to the wake of AV 5 (Fig. 1 and upper dashed line in Fig. 4) and contains all range gates with $r \in [r_c(t) - 30 \text{ m}, r_c(t) + 30 \text{ m}]$, as illustrated in Fig. 6. $r_c(t)$ is an approximation of the position of the wake center given by the maxima of the 10-min averages of the measured los velocities (Fig. 6). Note that these maxima are the minima of the absolute wind speed due to $u_{los} < 0$. The defined region is approximately $5D$ away from the turbine, where $D = 126$ m is the rotor diameter. Due to the described tracking procedure we expect that almost all measurements in this region correspond to the inner region of the wake and we will refer to it as the wake region or inner wake region in the following. Note that we do not aim to analyze the outer region of the wake in this work.

For both defined regions, statistical averages will be taken over all enclosed range gates and the complete time domain.

Additionally, we compare the lidar measurements to the ultrasonic anemometer measurements done on the platform “Fino 1” with a sampling frequency of 10 Hz. To do this, the three dimensional sonic measurements are projected on the line of sight direction of the lidar. After this procedure, we see that, both, the lidar measurements for the free flow case and the projected sonic data, show the same large scale dynamics (Fig. 5). The obvious offset is likely to be caused by the height difference between sonic and the center of the free flow region. In Sec. 3.2 the sonic data is used for a simplified correction of the power spectra of the lidar measurements.
3.2. Spectral Analysis and Correlations

As expected, we find a reduction of the absolute mean los velocity in the wake region to 3.3 m s\(^{-1}\) compared to 5.1 m s\(^{-1}\) in the free flow case. The turbulent kinetic energy (variance) in the los direction is increased in the wake situation to 1.15 m\(^2\) s\(^{-2}\) from 0.62 m\(^2\) s\(^{-2}\) in the free flow, corresponding to a turbulence intensity of \(u_{\text{los}}\) of 34% and 12% respectively. This indicates that the mean kinetic energy of the free flow gets partially converted into turbulent kinetic energy.

To investigate the scale dependence of this conversion process we now compare the spectral characteristics in the inner wake region and the free flow region. To estimate the power spectral density (PSD) the averaging of the power spectrum is done over 5–min windows using every range gate in the respective region, as defined in Sec. 3.1. In the wake case, the 10–min windows defined by the wake tracking (Fig. 6) are split in two while the same wake center position is used for both windows.

The power spectra \(S\) multiplied by the frequency \(f\) in Fig. 7 reveal an enhanced energy in the wake for all investigated frequencies (0.05 \(f\) < \(f\) < 0.07 \(f\)). The strongest energy enhancement is found at around 0.35 \(f\) in the wake region corresponds to a sharp bend in the double logarithmic plot of the PSD (Fig. 8). This bend is followed by an approximate power law behavior with an exponent of approx. –2.3. In free flow case no
clear peak or bend can be found but a power law behavior for higher frequencies \((f > 2.5 U_D)\) is indicated. The exponent is approx. \(-2.1\). These rather unusual exponents could be related to the volume average effect of the lidar \([19, 20, 21]\). Therefore, we additionally investigate the PSD of the sonic anemometer measurements which shows a common \(f^{-5/3}\) scaling for higher frequencies \((f > 2.5 U_D)\). Assuming that the sonic anemometer shows the correct spectrum we obtain a simplified correction of the PSD of the lidar in the wake case by multiplying it with the ratio

\[
\frac{S_{\text{sonic}}(f)}{S_{\text{lidar\ (free flow)}}(f)}
\]

This procedure reveals an approx. \(f^{-5/3}\) scaling almost directly after the bend in the wake spectrum and thus for much lower frequencies than for the sonic measurements representing the free flow case.

In addition to the spectral analysis, we investigate the corresponding autocorrelation functions defined as

\[
c(\tau) := \frac{\langle (u_{\text{los}}(t + \tau) - \langle u_{\text{los}} \rangle) \cdot (u_{\text{los}}(t) - \langle u_{\text{los}} \rangle) \rangle}{\langle (u_{\text{los}}(t) - \langle u_{\text{los}} \rangle)^2 \rangle}
\]

Due to our wake tracking procedure, first the autocorrelation function \(c(\tau)\) is calculated for every 10-min window instead of using a complete time series. The mean of the autocorrelation functions of all windows is then used as a final estimate for \(c(\tau)\). As shown in Fig.9, the correlations decrease much faster in the wake region than in the free flow case. Even though large-scale correlations are partially suppressed since we calculated the autocorrelation functions of 10-min windows only, these results indicate a smaller integral time scale in the wake region in the order of \(\tau \approx 12 \text{ s} \approx \frac{D}{U}\).

**Figure 7.** Frequency multiplied with power spectral densities of the lidar measurements in the wake region and the free flow. The dashed line marks the maximum of wake spectrum which also corresponds to the highest enhancement through the turbine. The frequency is normalized by \(\frac{U}{D} \approx 0.08 \text{ Hz}\).

**Figure 8.** Power Spectra of lidar measurements in the wake, of lidar measurements in the free flow and of ultrasonic measurements. Additionally, a corrected wake spectrum, as described in Sec. 3.2, is plotted. The frequency is normalized by \(\frac{U}{D} \approx 0.08 \text{ Hz}\).
3.3. Higher Order Statistics

To investigate higher order two point statistics, we analyze the statistics of velocity increments, defined as

$$\Delta \tau u_{los} := u_{los}(t + \tau) - u_{los}(t)$$

for a certain time lag $\tau$. The power spectral density and the autocorrelation function, presented in the last subsection, are related to the variance of such increments. Additional information about the turbulent structures can be found in higher order cumulants of increments [22, 23]. Therefore, the normalized increment pdfs of the lidar data corresponding to different time scales $\tau$ are shown in Fig. 10. For smaller scales (3 s), the pdfs in the wake and free flow situation show heavy-tails. While the pdf in the wake case becomes Gaussian between 6 s and 12 s the free flow pdf looses its heavy tails only for large time scales (higher than 30 s). These results are consistent with the idea of a reduced integral time scale in the wake in the order of $\frac{D}{U} \approx 12$ s.

One possibility to quantify the structural change of the pdfs and their heavy tailed structure is to determine the flatness, defined as

$$F(\tau) := \frac{\langle \Delta \tau u_{los}^4 \rangle}{\langle \Delta \tau u_{los}^2 \rangle^2}. \tag{2}$$

This flatness depending on the scale $\tau$ is presented in Fig. 11. As expected from Fig. 10, we find a higher flatness in the free flow case compared to the wake situation. The flatness in both regions obviously varies with the scale $\tau$ indicating a multifractal scaling behavior as typical for turbulent flows [22, 23]. As for homogeneous isotropic turbulence (HIT), the flatness in the wake region approximately behaves like a power law for small $\tau$ ($\tau < \frac{D}{U}$). Following K62-theory [24, 23] for HIT, we would expect:

$$F(\tau) \propto \tau^{-\frac{4}{3} \mu}, \tag{3}$$

where $\mu$ is often called intermittency factor. The dashed line in Fig. 11 corresponds to a value of $\mu \approx 0.27$, which is a realistic value for HIT (see e.g.,[25, 23]). For the free flow case, no clear power law behavior for the small scales is found.
Figure 10. Normalized increment pdfs of the lidar measurements in the wake and free flow region. For clarity, the pdfs are multiplied by different factors for different temporal increments. Time lags used from top to bottom: 3 s, 6 s, 12 s, 45 s.

Figure 11. Double logarithmic plot of the flatness of the increments dependent on the temporal lag $\tau$ for the wake and the free flow region. The dashed line corresponds to a power law as given in Eq. (3) with $\mu = 0.27$. The time scale is normalized by $D/\bar{U} \approx 12$ s.

4. Discussion

In contrast to the results for the free flow region, we find clear indications of homogeneous isotropic turbulence (HIT) in the inner far wake region of a wind turbine. The power spectra indicate that energy injection on the scales of the rotor diameter seems to be the starting point of an energy cascade. Features of HIT such as the approximate $f^{-5/3}$ scaling of the spectrum and multifractal scaling behavior with an intermittency factor in the same order of magnitude as known from HIT are found. For the undisturbed flow, the HIT-like properties are much less clearly present, may be seen only in the higher frequency part. One possible explanation of this phenomenon is that the anisotropic features of the ambient atmospheric flow become less relevant in the wake due to the mixing by the rotor. An alternative interpretation is offered by the works of [26, 27] which indicate that HIT cannot be seen in the atmospheric flow due to its non-stationary character. From this point of view, our results indicate that the wake flow might be in some sense more stationary than its surroundings.

As in our case of the wind turbine wake, properties of HIT are also found in the center of free jet flows or flows behind bluff bodies such as a cylinder (see e.g. [28]). When moving further away from the center of these flows the velocity field obviously becomes less isotropic (see e.g. [29]). Analogously, we only expect statistical behavior similar to HIT in the inner region of a wind turbine wake and not in the outer region where the wake strongly interacts with the ambient field. In contrast to most laboratory experiments concerning turbulence behind bluff bodies, the inflow for a wind turbine in the atmosphere is strongly anisotropic and furthermore the rotational direction of the turbine also breaks the rotational symmetry. This is particularly important in the near wake [30]. Therefore, our findings of statistical properties similar to HIT in the far wake are quite remarkable.

It is important to note that the wind turbine seems to modulate the turbulent structures of the atmosphere on a wide range of scales possibly even for frequencies smaller than $0.05 \frac{1}{\bar{U}}$.
also supported by the results from Singh et al.[9] for model turbines in laboratory experiments. This might be relevant when modeling the wake as a passive tracer driven by the large scale atmospheric structures [31, 32]. In agreement with our results, Singh et al.[9] also found a maximum energy enhancement for frequencies on the order of $0.4 \frac{U_D}{L}$. Since our resolution in the frequency domain was comparably coarse we could not confirm that the wake flow has a reduced energy for $0.01 \frac{U_D}{L} < f < 0.1 \frac{U_D}{L}$. As Singh et al. [9] in the lab and Wessel [18] in the atmosphere, we also observed a less heavy-tailed distribution of increments in the wake compared to the ambient flow. A homogenization in the sense of a reduced intermittency factor in the wake [9] could not be found here.

It should be noted that the free flow and wake region compared here, are located at two different heights. Thus, the statistical differences can partially stem from this height difference and should be interpreted with care. However, we do not expect that our major findings in the wake region, such as the integral length scale related to the rotor diameter together with a scaling behavior similar to HIT, could also be found in the free flow when measured in the same height as the wake.

5. Conclusions

Our results and their partial agreement with lab experiments clearly show that staring line of sight lidar measurements can be a useful tool to qualitatively investigate the turbulent flow inside a wind farm. A further and more detailed investigation of corrections due to the volume averaging effect, particularly for higher order statistics would be very useful.

Since a wind turbine operates in a complex anisotropic flow field, statistical properties similar to HIT in the inner region of a far wake are a remarkable result. It would be very interesting to investigate whether further statistical quantities also show HIT-like behavior. Multiple lidars could for example be used to measure other velocity components providing information about the turbulent momentum fluxes in the wake.

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