On topological properties of vacuum defects in lattice Yang-Mills theories

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Abstract

We study correlations between low-lying modes of the overlap Dirac operator and vacuum defects, center vortices and three-dimensional volumes, in lattice SU(2) gluodynamics. The low-lying modes are apparently sensitive to topological properties of the underlying gluon field configurations while the vacuum defects are crucial for the confinement. We find distinct positive correlation in both cases. In case of vortices the correlation is stronger.
1 Introduction

We study properties of the low-lying modes of the Dirac operator

$$D_\mu \gamma_\mu \psi_n = \lambda_n \psi_n ,$$

where the covariant derivative is constructed on the lattice vacuum gluonic field configurations $\{A_\mu^a(x)\}$. Low-lying modes play a special role in understanding topological properties of the gluonic field configurations. Namely, the difference between number of zero modes with positive and negative chirality is related to the total topological charge of the lattice volume:

$$n_+ - n_- = Q_{\text{top}} .$$

Near-zero mode modes determine the value of the quark condensate via the Banks-Casher relation:

$$< \bar{q} q > = - \pi \rho(\lambda_n \to 0) ,$$

where $\lambda_n \to 0$ with the total volume tending to infinity. Properties of the low-lying fermionic modes were studied in great detail for cooled gluon-filed configurations and these studies confirmed the quasiclassical picture, at least in its gross features, for review see, e.g., [1].

Recently it turned possible to investigate the topological fermionic modes working with the original fields $\{A_\mu^a(x)\}$. The use of the overlap operator [2] is crucial for this purpose. Measurement brought some unexpected results [3]. Namely the volume occupied by low-lying modes apparently tends to zero in the continuum limit of vanishing lattice spacing, $a \to 0$,

$$\lim_{a \to 0} V_{\text{loc}} \sim a^\alpha \to 0 ,$$

where $\alpha$ is a positive number of order unit (for details see original papers [3]) and the localization volume [2] is defined in terms of the Inverse Participation Ratio (IPR).

Observation [2] implies that there exists short-distance description of the topological properties of the vacuum. In particular, one can ask, where the modes shrink to.

Independently of the measurements on topological fermionic modes, there accumulated evidence that lower-dimensional vacuum defects are crucial for the confinement. We have in mind center vortices and 3d volumes. The role of the center vortices has been discussed since long, for review see [4]. These defects are defined as 2d surfaces. However, commonly one was assuming that these, by definition thin vortices are void of physical meaning and only mark, approximately ‘thick’ vortices. Measurements reported in Ref [5] indicate, however, that thin vortices possess remarkable gauge invariant properties, for review see, e.g., [6]. The role of three-dimensional defects [7] is highlighted by observation [8] that by changing a part of lattice in a special way (defined in terms of projected fields) one loses confinement and nullifies the quark condensate. This part of the lattice turns to be, up to gauge transformations, a three-dimensional volume [7].

In this note we report results of measurements of correlation between low-lying fermionic modes and vacuum defects.
2 Measurements

2.1 Definitions

We use standard definitions of the center vortices in the Direct Maximal Center Projection [9] which is defined in SU(2) lattice gauge theory by the maximization of the functional

\[ F(U) = \sum_{n,\mu} (\text{Tr} U_{n,\mu})^2, \]

with respect to gauge transformations, \( U_{n,\mu} \) is the lattice gauge field. The maximization of (3) fixes the gauge up to \( Z(2) \) gauge transformations and the corresponding \( Z(2) \) gauge field is defined as: \( Z_{n,\mu} = \text{signTr} U_{n,\mu} \). The plaquettes \( Z_{n,\mu\nu} \) constructed as product of links \( Z_{n,\mu} \) along the border of the plaquette have values \( \pm 1 \). The P-vortices (forming closed surfaces in 4D space) are made from the plaquettes, dual to plaquettes with \( Z_{n,\mu\nu} = -1 \).

We have computed the eigenvalues \( \lambda_{\text{lat}} \) and the eigenfunctions \( \psi_{\lambda}(x) \) of the overlap Dirac operator on the lattice with \( \beta = 2.45 \) and \( L_s = L_t = 14 \) by solving an eigenvalue problem

\[ D_{ov} \psi_{\lambda}(x) = \lambda_{\text{lat}} \psi_{\lambda}(x). \]

To make a connection with continuum physical eigenvalues \( \lambda \) we have stereographically project \( \lambda_{\text{lat}} \) onto the imaginary axis and divided by lattice spacing [10],

\[ \lambda = \text{Im} \left( \frac{\lambda_{\text{lat}}}{1 - \lambda_{\text{lat}}/2} \right) / a. \]

The scalar density of eigenmode is defined by

\[ \rho_{\lambda}(x) = \psi_{\lambda}^\dagger(x) \psi_{\lambda}(x), \quad \sum_x \rho_{\lambda}(x) = 1. \]

2.2 Correlation between vortices and fermionic modes

To clarify the role of the vortices in the topological structure of the vacuum we measure the correlator between intensity of fermionic modes and of center vortices. The correlator depends on the eigenvalue and on the local geometry of the vortex. The vortex lives on the dual lattice and we consider the correlator of the points on the dual lattice, \( P_i \), which belong to center vortex with the value of \( \rho(x) \) averaged over the vertices of the 4d hypercube, \( H \), dual to \( P_i \). Thus we consider the correlator:

\[ C_{\lambda} = \frac{\sum_{P_i} \sum_{x \in H} (V \rho_{\lambda}(x) - \langle V \rho_{\lambda}(x) \rangle)}{\sum_{P_i} \sum_{x \in H} 1}. \]

We find out that this correlator strongly depends on the number of the vortex plaquettes, \( N_{\text{vort}} \) attached to point \( P_i \). Numerical results for \( C_{\lambda} \) are shown on Fig. 1. We see that the more plaquettes are attached to given point, the larger correlator is. Also, the larger eigenvalue \( \lambda \) is, the smaller is the correlator. These results are in agreement with the general picture that vortices are related to chiral symmetry breaking, which is due to the low lying eigenmodes.
Figure 1: Dependence of $C_\lambda$ on $N_{\text{vort}}$ for $\beta = 2.45$, $L^4 = 14^4$.

2.3 Correlation between three-dimensional volumes and Dirac eigenmodes

By gauge transformation we can minimize the number of negative links in $Z(2)$ projection from which we construct center vortices. This gauge is called $Z(2)$ Landau gauge. These links are dual to 3d cubes on the dual lattice, these cubes form 3d volumes, which scale in physical units [7]. Thus these negative links (or corresponding 3d volumes) play the role in the confinement. Removing these negative links leads to gauge field configurations with zero string tension. We calculated in $Z(2)$ Landau gauge the correlator

$$C_{3d} = \frac{\left( \sum_s V_{\rho\lambda} - \langle V_{\rho\lambda} \rangle \right)}{\sum_s 1}.$$  

where the sums are over sites, $s$, which are endpoints of negative links. This correlator can be considered as the correlator of dual 3d volumes and Dirac eigenmode with the energy $\lambda$. The result is shown on Fig. The larger eigenvalue is, the smaller correlation we observe, from general grounds it is clear that the high energy eigenmodes should not be related to topology and confinement.

3 Conclusions

We have found strong evidence for correlation between local vortices density and density of low-lying eigenmodes of the overlap Dirac operator. The correlation of fermionic modes with density of three-dimensional volume is also positive, although
weaker. These observations support strongly the idea that there exists picture of confining fields and of topological structures in the vacuum state of YM theories in terms of short distances.

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