Emission Origin for the Wave of Quanta

Sanjay M Wagh

Astrophysics & Cosmology Research Unit,
University of KwaZulu-Natal, Private Bag X54001
Durban 4000, South Africa

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Abstract

We argue that certain assumptions about the process of the emission of the quanta by their (oscillating) emitter provide for their changing (oscillatory) flux at any location. This mechanism underlying (such) wave phenomena is not based, both, on the newtonian notion of force and the field concept (of Faraday, Maxwell, Lorentz and Einstein). When applied to the case of thermal radiation, this emission origin for the wave of quanta is shown here to be consistent with the laws of the black body radiation. We conclude therefore also that a conceptual framework, which is not rooted in the notion of force and in the field concept, may provide a deterministic basis underlying the probabilistic methods of the quantum theory.

Keywords: Peculiarities of emission of quanta - Changing or Wavy flux of quanta - Black Body Radiation - Planck’s law - Statistical nature of Quantum Theory

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I. INTRODUCTION

In contrast to Maxwell’s theory of the monochromatic radiation as being a transverse and propagating wave of electromagnetic nature, Planck assumed\(^\text{[1]}\) radiation as quantal during interactions with matter. In 1900, he also assumed that the quantal energy \(\epsilon\) is related to the frequency \(\nu\) of (the electromagnetic wave of) monochromatic radiation as \(\epsilon = h\nu\). Planck’s law for the spectral energy density of radiation in thermal equilibrium agreed with experiments, then. Maxwell’s theory could explain only the Rayleigh-Jeans part of Planck’s spectral energy density function, however.

In extending Planck’s reasoning, Einstein assumed that monochromatic radiation behaves like a discrete medium consisting of quanta carrying energy \(\epsilon = h\nu\) also in transits. Einstein thence explained\(^\text{[2]}\) the photoelectric effect in 1905. He had further envisaged\(^\text{[3], p.404}\), in 1909, that the next phase in the development of theoretical physics will bring us a theory of light that can be interpreted as a kind of fusion of the wave and the emission theory \(\cdots\) [The] wave structure and [the] quantum structure are not to be considered as mutually incompatible \(\cdots\)

Theory of spectra was proposed by Bohr\(^\text{[4]}\) in 1913. Then, in 1916, Einstein had considered\(^\text{[5], [6], [7]}\) an atomic gas in thermal equilibrium with radiation. Following Bohr, he assumed that atoms of gas make transitions from one to another of their energy levels by emitting or absorbing the quanta of monochromatic radiation. In addition to processes of induced emission and absorption of quanta, Einstein was led to postulate the process also of their spontaneous emission so that Planck’s law is obtained for the spectral energy density of radiation in this situation.

In his works of 1916, Einstein’s treatment of the spontaneous emission of quanta of radiation was statistical. He then pointed to the similarity of this statistical theory with Rutherford’s theory for the radioactivity\(^\text{[8]}\), which also involves spontaneous emission of particles, or quanta of matter, by a single atom.

Many physicists, including Einstein, expected a deterministic framework to be the basis of statistical theories. However, no one could show how the statistical treatment could be supported with a deterministic framework for the process of the spontaneous emission of quanta, either of radiation or of matter.
Consequently, Einstein remained unsatisfied with the statistical nature of his theory of spontaneous emission of quanta of light. In 1951, he expressed this dissatisfaction by saying that: *All these fifty years of pondering have not brought me any closer to answering the question, What are light quanta?*

Today, the word “photon” (coined, firstly, in [9]) is synonymous with a quantum of light of vanishing rest-mass, momentum $h\nu/c$, and unit intrinsic spin.

Now, Maxwell’s theory indicates that the quanta of radiation be inertia-less. Then, just like their inertia is, their momentum must be vanishing, if defined as a product of inertia and velocity. But, Compton-like effects [10] [11] can be explained by assuming that the inertia-less quanta of radiation possess non-vanishing momentum $p = h\nu/c$. Such considerations appear to be mutually contradictory, however.

Notably, the non-vanishing of the photon momentum $p = h\nu/c$ appears to arise out of Planck’s relation $\epsilon = h\nu$ alone. Grasping the nature of Planck’s relation should thus be pivotal to our understanding of the quantum phenomena.

With this aim, we assume certain aspects of the emission of quanta by their emitter. Propagating spherical fronts of quanta emitted at different instances are then centered about different locations of their (oscillating) emitter. This leads to the changing (wavy) nature of the quantal flux at any location, then. Emission aspects thus constitute the *emission origin* for the changing (wavy) nature of the flux of quanta.

From the perspective of this emission origin for the wavy flux of quanta, we may then focus on the situation of radiation contained within a cavity and in thermal equilibrium with the walls of that cavity. In this situation of the thermal equilibrium of radiation with a cavity, or that of the black body radiation, we must obtain Planck’s law for the spectral energy density of radiation. In other words, if the wave phenomena of radiation arise indeed due to aspects of emission of quanta, then such an origin must be consistent with Planck’s law.

If the above is indeed the case, then we could expect Planck’s relation $\epsilon = h\nu$ to arise from these considerations. This is the subject of the present study, which attempts to comprehend Planck’s $\epsilon = h\nu$ relation in this manner.
II. STATISTICAL CONSIDERATIONS

In what follows, we recall the statistical basis of Planck’s law for the spectral energy density of the black body radiation. (See [12] for concerned details.) However, we will not assume Planck’s relation $\epsilon = h\nu$ to begin with.

For us, noteworthy here is S N Bose’s derivation [13] of Planck’s formula for the spectral energy density of the black body radiation. Bose considered a gas of (inertia-less) particles of (non-kinetic) energy $\epsilon = h\nu$. He then distributed these particles in energy boxes, and computed their most probable distribution.

(The term “box” will refer to the momentum part of the phase space of the system, and will not be used to represent its spatial dimensions.)

Bose did not explain how a corpuscular or quantal property $\epsilon$ can be so related to a wave property $\nu$ as in the relation $\epsilon = h\nu$. Nevertheless, assumptions underlying his derivation of Planck’s formula are of our interest here.

Bose considered indistinguishable objects, each with an individuality of its own. (For an interesting discussion related to these notions, see [14].) Number $N$ of these objects are to be distributed in number $m$ of boxes, with the boxes being labeled by energy as $E_i = (i-1)\epsilon$. Here, $\epsilon$ is a constant; and the index $i$ takes integer values as $i = 1, 2, \cdots, m$. Then, a specific state of this system of $N$ objects has $n_i$ objects in the $i$-th box with $\sum_{i=1}^{m} n_i = N$. The number $N$ is not necessarily a constant. Both $N$ and $m$ are assumed to be very large integers.

Bose had also assumed additional conditions leading to the statistical independence of objects while getting distributed in energy boxes. He assumed also that objects do not prefer any particular box. We also know that, for the case of radiation, it is necessary to allow a single box to be populated by more than one object.

Then, as can be easily shown using the canonical distribution, in the most probable state of the system, the mean occupation number of the $i$-th box is

$$\bar{n}_i = 1/ (e^{\epsilon/\Theta} - 1)$$

Then, in the most probable state of the system, the mean energy of the $i$-th box is

$$\bar{E}_i = \epsilon / (e^{\epsilon/\Theta} - 1)$$
Here $\Theta \equiv kT$ is called as the *modulus of the distribution*, with $T$ as the temperature of the system and $k$ as Boltzmann’s constant.

As can also be easily shown, when in the most probable state, the system displays the mean square fluctuations in number given by

$$\overline{n_i^2} = \pi_i \left[ 1 + \pi_i \right]$$

When the system is in the most probable state, the mean square fluctuations in energy are then given by

$$\overline{E_i^2} = \overline{E_i} \left[ \epsilon + \overline{E_i} \right]$$

Now, these fluctuations, when the system is in the most probable state, are *entirely statistical results*. These are to be “interpreted” in terms of the physical motions of objects (whether the inertia of these objects is vanishing or not).

The most probable state of the system is now taken to be its equilibrium state. For the physical problem of the radiation enclosed in a cavity, the most probable state is then the state of thermal equilibrium of radiation.

Consider the first term of the equilibrium fluctuations in energy (or in the number of quanta) of radiation. Quanta have been assumed to possess an individuality of their own. Then, we expect that the fluctuations in energy, as in the first term, arise due to collisions of quanta. Thus, this term is known as the *particle term*.

For thermal radiation in a cavity, the second term of these equilibrium fluctuations can be interpreted as being due to the wave character of radiation as per Maxwell’s theory. It is therefore called as the *wave term*.

Wavy motions of objects may lead to the second term of equilibrium fluctuations, also. This would require appropriate (Hooke’s law) forces acting on objects. But, no physical mechanism producing such forces exists within this situation.

Interactions of quanta of radiation with the “matter of the walls” of the cavity could be considered as the subject of *the emission theory* that Einstein referred to in 1909. But, *forces causing wavy motions of quanta all over the space within the cavity cannot arise even from such interactions.*
In summary, objects of Bose’s considerations have an *individuality* of their own, *ie*, they are particulate. The second term of equilibrium fluctuations cannot then be due to their being waves, or due to their wavy motions. Then, what is the origin of these fluctuations? In what follows, we address this issue for radiation, all whose quanta are assumed to move with the same speed.

III. EMISSION AND THE FORMATION OF WAVE OF QUANTA

Now, we show that certain assumptions about the nature of the emission of quanta explain the second term of their equilibrium fluctuations. Wavy changes in the flux of quanta arise out of these assumptions about the nature of their emission. However, forces are not invoked to act on these quanta. We expect all the wave characteristics of quantal propagation to emerge from these emission aspects.

To this end, let the emission of quanta not occur unless the kinetic energy of their emitter changes. Change in the kinetic energy of the emitter is then assumed to be distributed among the emitted quanta as their (non-kinetic) energy.

Now, as per Maxwell’s theory, an oscillating charge produces an electromagnetic wave propagating in all directions away from that charge. This wave emission has the property that any wavefront is spherically symmetric about the instantaneous position of the charge at the instant of its emission. To be in conformity with this and, thus, with many experiments, we assume that the propagating front formed by the simultaneously emitted quanta is spherically symmetric about the instantaneous position of the source at their emission. (Notably, this assumption is also not inconsistent with the laws of radioactivity, if the quanta are those of matter.)

In what follows, we use these assumed peculiarities of the emission of quanta to show that Planck’s law obtains for their spectral energy density, when in thermal equilibrium with the walls of the cavity containing them.

In the above context of the emission of light-quanta by their emitter, the following questions then arise. How many quanta get emitted at an instant? How does the kinetic energy of the emitter get distributed among quanta? What are the laws underlying the process of the emission of quanta? Issues arising out of such questions are related to
aspects of Maxwell’s theory dealing with the power radiated by an accelerated charge. These will be the subject of a later communication, however.

We are not requiring, at this stage here, the details of the laws of emission of quanta as well as those of how the accelerated motion of the emitter is arising. Presently, we only need to know that the emitters of quanta undergo oscillatory motions at the cavity walls. This is so for the following reasons.

We assumed that quanta are emitted only when the kinetic energy of their emitter changes. For the change $\Delta E$ in the kinetic energy of the emitter, let it emit number $n$ of light-quanta at that instant. (When the energy $\Delta E$ is distributed equally among the emitted $n$ quanta, each quantum has energy $\epsilon = \Delta E/n$.)

After one emission of quanta, the second emission takes place when the kinetic energy of the emitter changes next. But, this second spherical front of quanta is now centered at a different location of their emitter. Then, passing any specified or given location, the flux of quanta due to the first emission front would be different than that due to the second emission front.

The changing flux of quanta at a given location can now arise due to the following two reasons. Firstly, the number of quanta emitted at the instances of the emission of two different spherical fronts could be different. Secondly, for the given location of the unit area, its angle subtended at the location of the emitter would be different, spherical fronts being centered at different locations.

Clearly, for oscillatory changes in the location of the emitter, oscillatory changes would be seen in the flux of quanta at any location as well. Then, the frequency of oscillations of the number of quanta at any arbitrary location is also the frequency of oscillations of their emitter.

Assumed peculiarities of the emission of light-quanta are responsible for the wavy behavior of their flux passing any location, therefore.

No light-quantum is undergoing wavy motion. We do not therefore require (Hooke’s type) forces causing the wavy motions of quanta. But, the wavy behavior of the flux of quanta results. It is due to the oscillatory motion of their emitter. Such wave aspects of quantal propagation thus originate in their emission aspects.
Now, for the case of radiation in thermal equilibrium with the walls of a cavity, an emitter of quanta is necessarily required to be a linear oscillator. If this were not the situation, then we would expect the density of radiation within the cavity to be varying with time. This contradicts our assumption of the thermal equilibrium of radiation with the walls of the cavity containing it.

Planck had assumed linear oscillators for matter at the walls of the cavity. We too have linear oscillators at the walls of the cavity in thermal equilibrium with radiation, then. Planck’s law for the cavity radiation should thus be obtainable within the premise of the present considerations.

Now, for the radiation in thermal equilibrium within a cavity, let us note that the wave modes of radiation would be standing modes. Let us also assume two polarization states for these wave modes. (We offer no explanation for this assumption. But, it should relate to results established in [15].)

Consider now a cubical cavity of each side of length \( \ell \). Assume the wavelength \( \lambda \) of the standing wave mode to be smaller than the cavity length-scale \( \ell \). Under equilibrium, the number \( f(\nu)\delta\nu \) of standing wave modes within the cavity and within the frequency range \( \nu \) to \( \nu + \delta\nu \) is, then,

\[
f(\nu)\delta\nu = \frac{4\ell^3\nu^2}{c^3}\delta\nu
\]

with \( c \) being the wave speed. Then, the number \( \delta n_\nu \) of light-quanta involved in the wavy flux and within the frequency range \( \nu \) to \( \nu + \delta\nu \) is, simply, given by

\[
\delta n_\nu = \pi_i f(\nu)\delta\nu
\]

Then, the energy of radiation within the same frequency range is

\[
\delta E_\nu = \epsilon \delta n_\nu
\]

The spectral energy density of the cavity radiation is therefore obtained as

\[
\frac{\delta E_\nu}{\ell^3} = \frac{(4\epsilon \nu^2/c^3)\delta\nu}{e^{\epsilon/kT} - 1}
\]

For \( \epsilon = h\nu \), this expression now yields Planck’s law for the black body radiation. Planck’s law is consistent with the corresponding equilibrium fluctuations. Therefore, our considerations are consistent with those fluctuations, too.
Emission origin for the wavy nature of the number of quanta at any location in a cavity in thermal equilibrium with radiation is then consistent with the laws of the black body radiation. The second term of the statistical fluctuations of the most probable state of cavity radiation arises due to the wavy nature of the “number” of quanta at a cavity location. But, the wavy flux is due to aspects of the emission of quanta at the walls of a cavity they are in thermal equilibrium with.

We needed to resolve the question of the origin of the wavy fluctuations in the number of quanta within a cavity, without invoking “forces” to act on them. Aspects of the emission of quanta provide such a resolution of this question.

IV. CONCLUDING REMARKS

In summary, we assumed that quanta move with constant velocity till interactions. For radiation, speed is the same for all the quanta. Emitter of quanta was assumed to undergo a change $\Delta E$ in its kinetic energy for the quantal emission. The energy $\Delta E$ was assumed to be distributed (as non-kinetic energy) among the quanta emitted at the same instant. Emission of quanta was assumed to be spherically symmetric about the position of the emitter at the instant of emission.

Then, the flux of quanta at any location changes as per the motion of the emitter. If the motion of the emitter were oscillatory with frequency $\nu$, then the flux of quanta at any location would also be oscillatory with the same frequency $\nu$.

We showed here that this “emission origin” for the wavy behavior of the flux of light-quanta is consistent with the laws of the black body radiation.

However, the (non-kinetic) energy $\epsilon$ of any emitted light-quantum is not related here to the frequency $\nu$ of the material oscillator of the walls of a cavity that the radiation is assumed to be in thermal equilibrium with. Planck’s relation $\epsilon = h\nu$ appears, therefore, to be an ad-hoc assumption relating a corpuscular property $\epsilon$ to a wave property $\nu$. Consequently, Planck’s constant $h$ too has an ad-hoc status, here.

Why is Planck’s postulate $\epsilon = h\nu$ consistent with experimental results then? To comprehend this, we recall here our assumption that no quanta are emitted unless the kinetic energy of the emitter changes. Then, the quantal energy $\epsilon$ is vanishing when the
frequency \( \nu \) of the oscillator is vanishing. As entirely mathematical relationship, without any physical basis, we may postulate their proportionality. This we understand as a reason for \( \epsilon = h\nu \) being consistent with the observed laws of the black body radiation. On the basis of present considerations, we therefore “comprehend” Planck’s this relation as being an ad-hoc postulate.

(We would like to note here that the ad-hoc status of Planck’s celebrated constant is consistent with Einstein’s views about the fundamental role of constants of Nature in a theory [16, p. 74]: *In a sensible theory there are no [dimensionless] numbers whose values are determinable only empirically. I can, of course, not prove that · · · dimensionless constants in the laws of nature, which from a purely logical point of view can just as well have other values, should not exist.*)

Now, our problem of the radiation existing in thermal equilibrium with the walls of a cavity has two aspects, namely, the emission/absorption of radiation at the walls and the propagation of radiation inside the cavity. We established here that the wavy flux of propagating quanta of radiation originates in aspects of their emission by material oscillators at the walls of the cavity containing them.

If we focus on only one of these two aspects and neglect the other, then we would be dividing this problem into two parts - the wave part and the emission part. We may then attempt to describe them separately of each other. This would be the divide and conquer strategy of problem-solving. Within this strategy, we need to establish the mutual consistency of the separate solutions of the two parts, however.

Any emitter of quanta in thermal equilibrium with radiation has an oscillatory motion. It is due to continual emission and absorption of the quanta of radiation. Under the “divide and conquer strategy” of problem-solving, such motion could be described as being due to a force acting on it. Such a force, the Lorentz force, causing the motion of a radiating charge is the basis of Maxwell’s electrodynamics.

Under the same “divide and conquer strategy” of problem-solving, the wave part can now be described in terms of a wave equation. Then, a wave equation could be based on an appropriate (Hooke’s type) force, or on an appropriate substratum (field) providing a medium or basis for the propagation of waves.
A physical mechanism causing wavy motions of the quanta of radiation does not exist within the cavity situation. A wave equation could then be based on the concept of a field. Maxwell’s theory provided such a wave equation.

However, such a framework of concepts was unable to explain the laws of the black body radiation, the photoelectric effect, · · · Planck’s radical and revolutionary hypothesis of the quantal nature of radiation turned out to be necessary for explaining these phenomena. Faraday, Maxwell and Lorentz’s framework of concepts was then replaced by the probabilistic one of the quantum theory.

Schrödinger’s equation now describes the mechanics of a material body, of non-vanishing inertia, using the concept of a potential (arising out of that of force acting on that body). It then provides for the motion of the emitter of the quanta of radiation. Bohr’s theory of spectra arises out of these considerations.

Path to the description of the quanta of radiation in transit was, however, long. It required a mathematical formalism providing for their number, that is to say, a formalism consistent with their quantum nature. The second quantization formalism of the quantum theory is such a method to obtain the number of quanta within a given wave mode. It is applicable to quanta of matter and of radiation, both.

Quantum theory considers then only the wave aspect of the problem at hand. We then “comprehend” the formalism of the quantum theory as arising due to the “divide and conquer strategy” of problem-solving. Based on only the wave part, the quantum theory provides the probabilistic description for the quanta.

Clearly, the formalism of the quantum theory then completely misses crucial aspects of the emission of quanta. Needless to say then, but important also to point out here, the description of the physical world provided by the quantum theory is, therefore, an incomplete one in this respect.

In this context, we then note that of pivotal importance to Einstein’s thinking was Boltzmann and Gibbs’s statistical theory of many particle systems, which is incomplete in relation to Newtonian mechanics. He considered the quantum theory to be similarly an incomplete description of physical systems, as only a statistical theory that is inherently incapable of describing the dynamics of a single object of any many-body system. He therefore expected some deterministic framework to be the basis underlying the methods
of the quantum theory. Then, from Einstein’s aforementioned point of view, Heisenberg’s indeterminacy relations hold only statistically.

We emphasize that we have not used the concepts of force and field to explain the wave of quanta. Notably, in 1954, Einstein [3, p. 467] considered it quite possible that physics cannot be based on the field concept, i.e., on continuous structures.

Here, continuous structures are to represent the “objects” themselves, and not the probability of their being at some space location at some instant of time. This is the “field” concept of Faraday, Maxwell, Lorentz and Einstein.

The present results then imply that Einstein’s aforementioned opinion may have merit, because we have not used Einstein’s concept of field to represent quanta and to also explain their wave. It should be noted that we have not even specified here which mathematical structure represents objects of our statistical considerations.

In this context, spontaneous emission of quanta (of radiation or not) by one atom may have to be dealt statistically, but the computation of its probability can then have basis in deterministic ideas not based on the concepts of force and field, both.

Then, we also conclude here that a deterministic basis, which is rooted neither in the newtonian notion of force nor in the field concept, may underly the probabilistic methods of the quantum theory. This possibility was first explored by Hertz [19] with an intention to free Newtonian mechanics from the notion of the potential energy, which he considered as unsatisfactory a concept.

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