Potential-driven Galileon inflation

Junko Ohashi\(^1\) and Shinji Tsujikawa\(^1\)

\(^1\)Department of Physics, Faculty of Science, Tokyo University of Science, 1-3, Kagurazaka, Shinjuku-ku, Tokyo 162-8601, Japan

(Dated: May 5, 2014)

For the models of inflation driven by the potential energy of an inflaton field \(\phi\), the covariant Galileon Lagrangian \((\partial \phi)^2 \Box \phi\) generally works to slow down the evolution of the field. On the other hand, if the Galileon self-interaction is dominant relative to the standard kinetic term, we show that there is no oscillatory regime of inflaton after the end of inflation. This is typically accompanied by the appearance of the negative propagation speed squared \(c_s^2\) of a scalar mode, which leads to the instability of small-scale perturbations. For chaotic inflation and natural inflation we clarify the parameter space in which inflaton oscillates coherently during reheating. Using the WMAP constraints of the scalar spectral index and the tensor-to-scalar ratio as well, we find that the self coupling \(\lambda\) of the potential \(V(\phi) = \lambda \phi^4 / 4\) is constrained to be very much smaller than 1 and that the symmetry breaking scale \(f\) of natural inflation cannot be less than the reduced Planck mass \(M_{\text{pl}}\).

We also show that, in the presence of other covariant Galileon Lagrangians, there are some cases in which inflaton oscillates coherently even for the self coupling \(\lambda\) of the order of 0.1, but still the instability associated with negative \(c_s^2\) is generally present.

I. INTRODUCTION

The idea of inflation was originally proposed to address a number of cosmological problems plagued in standard Big Bang cosmology \(^1\). Moreover inflation provides a causal mechanism for the generation of large-scale density perturbations from the quantum fluctuation of a scalar field (“inflaton”) \(^2\). The resulting power spectra of scalar and tensor perturbations are nearly scale-invariant, whose prediction is consistent with the Cosmic Microwave Background (CMB) temperature anisotropies observed by COBE \(^3\) and WMAP \(^4\).

Most models of inflation are based on a canonical scalar field \(\phi\) with a slowly varying potential \(V(\phi)\) (see \(^3\) for reviews). For example, the simple power-law potential \(V(\phi) = \lambda \phi^n / n\) \((n \text{ and } \lambda \text{ are positive constants})\) leads to chaotic inflation for the field value larger than the reduced Planck mass \(M_{\text{pl}} = 2.435 \times 10^{18}\) GeV \(^6\). The quartic potential \(V(\phi) = \lambda \phi^4 / 4\) is in tension with the WMAP constraints of the scalar spectral index \(n_s\) and the tensor-to-scalar ratio \(r\) \(^7\). Moreover the self coupling is constrained to be \(\lambda \approx 10^{-13}\) from the WMAP normalization, which is much smaller than the typical coupling scale appearing in particle physics \((\text{e.g., } \lambda \approx 0.1\) for the Higgs boson \(^8\)).

There are several different ways to reconcile the quartic potential \(V(\phi) = \lambda \phi^4 / 4\) with observations\(^4\). One of them is to introduce a non-minimal field coupling \(\xi R \phi^2 / 2\) to the Ricci scalar \(R\) \(^12\). In the limit \(\xi \gg 1\) the tensor-to-scalar ratio can be as small as \(r \approx 10^{-3}\) with \(n_s \approx 0.96\) \(^14\), which is well inside the 1σ observational contour \(^15\). Moreover the self coupling is of the order of \(\lambda \approx 10^{-10} \xi^2\) for \(\xi \gg 1\) from the WMAP normalization. If the field \(\phi\) is a Higgs boson, however, this model is plagued by the problem of unitary violation around the energy scale of inflation \(^16\). Moreover the non-minimal coupling \(\xi R \phi^2 / 2\) does not necessarily help other inflaton potentials to be compatible with observations \(^14\) \(^15\).

The second way is to use a non-minimal field derivative coupling to gravity in the form \(G^{\mu \nu} \partial_\mu \phi \partial_\nu \phi / (2M^2)\) \(^17\), where \(\text{C}^{\mu \nu\nu}\) is the Einstein tensor and \(M\) is a mass scale (see also Ref. \(^18\) for the original work). In the regime where the Hubble parameter \(H\) is larger than \(M\), the evolution of the field slows down due to a gravitationally enhanced friction. In this case the potential \(V(\phi) = \lambda \phi^4 / 4\) is compatible with the WMAP constraints of \(n_s\) and \(r\) with \(\lambda \approx 5.9 \times 10^{-32}(M_{\text{pl}} / M)^4\) \(^19\). Moreover the mechanism of slowing down the field (“slopeon” \(^20\)) works for general steep potentials. For example this mechanism was applied to the potential \(V(\phi) = \Lambda^4 [1 + \cos(\phi / f)]\) of natural inflation, where \(\Lambda\) and \(f\) are mass parameters \(^21\). In conventional natural inflation \(^22\), the symmetry breaking scale \(f\) needs to be larger than \(3.5M_{\text{pl}}\) for the consistency with the WMAP constraints \(^23\), but in this regime standard quantum field theory is unlikely to be trustable \(^24\). In the presence of the field derivative coupling, natural inflation can be compatible with the WMAP bounds even for \(f\) smaller than \(M_{\text{pl}}\) \(^19\) \(^25\).

For the potential-driven inflation the field self-interaction of the form \((\partial \phi)^2 \Box \phi\) \(^26\) also leads to the slow evolution of inflaton along the potential \(^29\) (see Refs. \(^30\) \(^32\) for the kinetically driven case). The field equations of motion following from the Lagrangian \((\partial \phi)^2 \Box \phi\) respects the the Galilean symmetry \(\partial_\mu \phi \rightarrow \partial_\mu \phi + b_\mu\) in the limit of

---

\(^1\) In addition to a number of scenarios mentioned in Introduction, there is another way of realizing large \(\lambda\) by using non-standard kinetic terms \(^6\) \(^11\).
Minkowski space-time \( M^4 \). In a manifold having integrable (covariantly constant) Killing vectors \( \xi^i \), such a “Galileon” Lagrangian is invariant under the curved-space Galilean transformation \( \phi(x) \rightarrow \phi(x) + c + c_a \int_{x_0}^{x} \xi^a \), where \( c, c_a, x_0 \) are constants and \( x \) is a space-time coordinate \( 21 \) (whose property also holds for the derivative coupling \( G^\mu\nu \partial_\mu \phi \partial_\nu \phi \)). The presence of such a symmetry has an advantage that the theory can be quantum mechanically under control \( 33 \).

For the potential \( V(\phi) = \lambda \phi^4/4 \) the Galileon term \( (\partial \phi)^2 \Box \phi \) not only leads to the suppression of the tensor-to-scalar ratio compatible with recent observations, but also it gives rise to the coupling \( \lambda \) of the order of 0.1 consistent with the WMAP normalization \( 10, 21, 34 \). Meanwhile it is not clear whether the presence of such a non-linear field self-interaction does not disturb the oscillation of inflaton during reheating. The absence of oscillations means that the standard mechanism of reheating (decays of inflaton to other particles and the thermalization of the Universe) does not work. Moreover we need to check whether the conditions for the avoidance of ghosts and Laplacian instabilities can be avoided after inflation. Since such conditions were recently derived in Refs. \( 35, 38 \) for the most general scalar-tensor theories having second-order equations of motion \( 39, 41 \), those results can be applied to potential-driven inflation with the Galileon Lagrangian.

In this paper we study the dynamics of inflation and the subsequent reheating for the potentials \( V(\phi) = \lambda \phi^4/4 \) and \( V(\phi) = \Lambda^4[1 + \cos(\phi/f)] \) in the presence of the Galileon Lagrangian \( (\partial \phi)^2 \Box \phi \). If the Galileon self-interaction dominates over the standard kinetic term after inflation, the oscillatory regime of inflaton tends to disappear for both potentials. This is usually accompanied by a negative propagation speed squared \( c_4^2 \) of the scalar mode, which leads to the instability of scalar perturbations on smaller scales. The model parameters of the potentials can be constrained to have the coherent oscillation of inflaton as well as to match with the observational data. For the quartic potential \( V(\phi) = \lambda \phi^4/4 \), for example, the self coupling \( \lambda \) is bounded to be very much smaller than 1. In natural inflation we show that it is difficult to realize the regime where the symmetry breaking scale \( f \) is smaller than \( M_{\text{pl}} \). We also study the effect of other covariant Galileon terms \( 27 \) on the dynamics of inflation and reheating for the potentials \( V(\phi) = \lambda \phi^4/n \). It is possible to find some cases in which the self coupling of the potential \( V(\phi) = \lambda \phi^4/4 \) is of the order of 0.1, but the violent instability associated with negative \( c_4^2 \) is usually unavoidable.

This paper is organized as follows. In Sec. \( \text{II} \) we present the background and perturbation equations for potential-driven inflation in the presence of (generalized) Galileon Lagrangians. The spectra of scalar and tensor perturbations are given by using slow-roll parameters. In Sec. \( \text{III} \) we study the models of chaotic inflation as well as natural inflation in the presence of the term \( (\partial \phi)^2 \Box \phi \) alone. We clarify the viable parameter space in which the coherent oscillation of inflaton occurs during reheating. We also place observational constraints on the inflaton potentials from the information of the scalar spectral index \( n_s \), and the tensor-to-scalar ratio \( r \). In Secs. \( \text{IV} \) and \( \text{V} \) we provide similar constraints on the parameter space of chaotic inflation in the presence of other covariant Galileon terms. Sec. \( \text{VI} \) is devoted to conclusions.

\section{II. GENERAL FIELD EQUATIONS FOR THE BACKGROUND AND PERTURBATIONS}

We start with the following action

\[
S = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{pl}}^2}{2} R + P(\phi, X) - G_3(\phi, X) \Box \phi + \mathcal{L}_4 + \mathcal{L}_5 \right],
\]

where \( g \) is a determinant of the metric \( g_{\mu\nu} \), \( R \) is a scalar curvature, and

\[
\mathcal{L}_4 = G_4(\phi, X) R + G_{4,X} \left[ (\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi)(\nabla^\mu \nabla^\nu \phi) \right],
\]

\[
\mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu}(\nabla^\mu \nabla^\nu \phi) - \frac{1}{6} G_{5,X} \left[ (\Box \phi)^3 - 3(\Box \phi)(\nabla_\mu \nabla_\nu \phi)(\nabla^\mu \nabla^\nu \phi) + 2(\nabla_\mu \nabla_\alpha \phi)(\nabla^\alpha \nabla^\beta \phi)(\nabla^\beta \nabla_\mu \phi) \right].
\]

Here \( P \) and \( G_i \) (\( i = 3, 4, 5 \)) are functions in terms of \( \phi \) and \( X = -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi / 2 \) with the partial derivatives \( G_{i,X} = \partial G_i/\partial X \), and \( G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} R/2 \) is the Einstein tensor (\( R_{\mu\nu} \) is the Ricci tensor). The action \( \text{II} \) corresponds to the most general scalar-tensor theories having second-order equations of motion\(^2 \) \( 35, 40, 41 \). This was first discovered by Horndeski in a different form \( 39 \).

We focus on the models in which inflation is mainly driven by a field potential \( V(\phi) \), i.e.,

\[
P(\phi, X) = X - V(\phi).
\]

\(^2\) Note that the cosmological dynamics in the presence of the general Lagrangian \( G_3(\phi, X) \Box \phi \) was studied in Ref. \( 21 \) in the context of dark energy. In Refs. \( 31 \) the authors chose some particular forms of the function \( G_3(\phi, X) \) to discuss the dynamics of dark energy.
For the functions $G_i$ ($i = 3, 4, 5$) we take
\[ G_3(\phi, X) = f_3(\phi)X, \quad G_4(\phi, X) = f_4(\phi)X^2, \quad G_5(\phi, X) = f_5(\phi)X^2, \]
(5)
where $f_i(\phi)$ depend on $\phi$ alone. The covariant Galileon [27] corresponds to the choice [42]
\[ f_3 = \frac{c_3}{M^2}, \quad f_4 = -\frac{c_4}{M^6}, \quad f_5 = \frac{3c_5}{M^9}, \]
(6)
where $c_3$, $c_4$, $c_5$ are dimensionless constants, and $M$ is a constant having a dimension of mass. We derive the background and perturbation equations for the general functions (5) in order to cover both the covariant Galileon and the Galileon-like self-interactions on the dynamics of inflation and reheating. After Sec. III we mainly focus on the more general forms (like the Horndeski’s action 39), but our interest in this paper is to understand the effect of the Galileon-like self-interactions on the dynamics of inflation and reheating. After Sec. III we mainly focus on the covariant Galileon.

A. Background equations

On the flat Friedmann-Lemaître-Robertson-Walker (FLRW) space-time with the scale factor $a(t)$ (where $t$ is cosmic time) the background equations for the theories described by the functions (4) and (5) are given by [28, 51]
\[ E_1 \equiv 3M^2_H^2H^2 - X - V - 6Hf_3\dot{\phi}X + 2(f_3,\phi - 45H^2f_4)X^2 + 2H(15f_4,\phi - 14H^2f_5)\dot{\phi}X^2 + 42H^2f_5,\phi X^3 = 0, \]
(7)
\[ E_2 \equiv 3M^2_H^2H^2 + X - V + 2\left(M^2_H - 6f_4X^2 - 4Hf_5\dot{\phi}X^2 + 2f_5,\phi X^3\right)\dot{H} \]
\[ -2\left[f_3 + 12Hf_3\dot{\phi} - (5f_4,\phi - 10H^2f_5)X - 6Hf_5,\phi\dot{X}\right]X\dot{\phi} \]
\[ -2(f_3,\phi + 9H^2f_4)X^2 - 4(3f_4,\phi + 2H^2f_5)\dot{\phi}X^2 + 2(2f_4,\phi - H^2f_5,\phi)X^3 + 4Hf_5,\phi\dot{\phi}X^3 = 0, \]
(8)
\[ E_3 \equiv 3H^2 + V_\phi + 18H^2f_3X + 108H^2f_4\dot{\phi}X - 2(f_3,\phi + 18H^2f_4,\phi - 30H^4f_5)X^2 \]
\[ + \left[1 + 6Hf_3^2 - 4(f_3,\phi - 27H^2f_4)X - 20(3f_4,\phi - 2H^2f_5)H\dot{\phi}X - 90H^2f_5,\phi X^2\right]\dot{\phi} \]
\[ + 2\left[3f_3 + 36Hf_3\dot{\phi} - (15f_4,\phi - 30H^2f_5)X - 18Hf_5,\phi\dot{X}\right]X\dot{\phi} \]
\[ - 2\left(12f_4,\phi + 19H^2f_5,\phi\right)H\dot{\phi}X^2 - 30H^2f_5,\phi X^3 = 0, \]
(9)
where $H \equiv \dot{a}/a$ is the Hubble parameter, and a dot denotes a derivative with respect to $t$.

Let us consider the covariant Galileon Lagrangian where the functions $f_i$ ($i = 3, 4, 5$) are given by Eq. (5). In this case it is convenient to introduce the following dimensionless quantities
\[ x = \frac{\phi}{M_{pl}}, \quad y = \frac{\dot{\phi}}{MM_{pl}}, \quad z = \frac{H}{M}, \]
(10)
and
\[ \tau = Mt, \quad U(x) = \frac{V}{M^2M_{pl}^2}, \quad U,\phi(x) = \frac{V,\phi}{M^2M_{pl}^2}, \quad \alpha = \frac{M_{pl}}{M}. \]
(11)
The constraint equation (7) can be written as
\[ 6z^2 - y^2 - 2U(x) - 6c_3\alpha y^3 z + 45c_4\alpha^2 y^4 z^2 - 42c_5\alpha^3 y^5 z^3 = 0. \]
(12)
Combining Eqs. (7) and (11) to eliminate $V$ and then using Eq. (10) to solve for $\ddot{\phi}$ and $\dot{H}$, it follows that
\begin{align}
\frac{dx}{d\tau} &= y, \\
\frac{dy}{d\tau} &= [9c_3^2\alpha^2 y^5 z + 3c_3\alpha y^2 (78c_5\alpha^3 y^5 z^3 - 63c_4\alpha^2 y^4 z^2 + y^2 - 6z^2) + 810c_2^4\alpha^4 y^7 z^3 - 3c_4\alpha^2 y^3 (603c_5\alpha^3 y^5 z^4 \\
+ 15y^2 z + U,\phi(x)y - 36z^3) + 945c_2^2\alpha y^6 z^5 + 3c_5\alpha^3 y^4 z (21y^2 z - 30z^2 + 2U,\phi(x)y) - 2U,\phi(x) - 6yz]/\Delta, (14)
\end{align}
\begin{align}
\frac{dz}{d\tau} &= -[27c_3^2\alpha^2 y^2 z^2 + c_3\alpha (450c_5\alpha^3 y^4 z^3 - 432c_4\alpha^2 y^3 z^3 + 12yz + U,\phi(x)) + 1620c_2^4\alpha^4 y^4 z^4 - 12c_4\alpha^2 yz (9yz \\
+ 270c_5\alpha^3 y^4 z^4 + U,\phi(x)) + 1 + 1575c_2^4\alpha^6 y^6 z^6 + 15c_5\alpha^3 y^2 z^2 (8yz + U,\phi(x))]y^2/\Delta, \]
(15)
where
\[ \Delta = 2 + 3c_3 \alpha y(4z + c_3 \alpha y^3) - 3c_4 \alpha^2 y^2(36z^2 - y^2 + 18c_3 \alpha y^3 z - 90c_4 \alpha^2 y^4 z^2) + 3c_5 \alpha^3 y^3 z(40z^2 - 2y^2 + 18c_3 \alpha y^3 z - 192c_4 \alpha^2 y^4 z^2 + 105c_5 \alpha^3 y^5 z^3). \] (16)

Numerically it is usually more stable to solve Eqs. (13) and (14) with the constraint equation (12) rather than solving Eqs. (12)–(14).

B. The spectra of density perturbations

The spectra of scalar and tensor perturbations generated in the theories given by the action (1) were derived in Refs. [35–37]. Here, we briefly review their formulas in order to apply them to concrete inflaton potentials.

The perturbed line element about the flat FLRW background is given by
\[ ds^2 = -(1 + 2A)dt^2 + 2\partial_t B dt dx^i + a^2(t)(1 + 2\mathcal{R})\delta_{ij} + h_{ij} \] dx^i dx^j, \] (17)
where \( A, B, \mathcal{R} \) are scalar metric perturbations, and \( h_{ij} \) are tensor perturbations which are transverse and traceless. The inflaton field is decomposed into the background and inhomogeneous parts, as \( \phi = \phi_0(t) + \delta \phi(t, x) \). We choose the uniform-field gauge characterized by \( \delta \phi = 0 \), which fixes the time-component of a gauge-transformation vector \( \xi^\mu \). The scalar perturbation \( \mathcal{E} \), which appears as the form \( E_{ij} \) in the last term of (17), is gauged away, so that the spatial part of \( \xi^\mu \) is fixed. Vector perturbations decay during inflation, so that their contribution is negligibly small.

We expand the action (11) up to second-order in perturbations by using the Hamiltonian and momentum constraints. For the theories given by Eqs. (1) and (3), the second-order action for scalar perturbations reduces to [35–37]
\[ S_s^{(2)} = \int dt d^3x a^3 Q_s \left[ \dot{\mathcal{R}}^2 - \frac{c_s^2}{a^2} (\partial \mathcal{R})^2 \right], \] (18)
where
\[ Q_s = \frac{w_1(4w_1w_3 + 9w_2^2)}{3w_2^2}, \quad c_s^2 = \frac{3(2w_1^2w_2H - w_2^3w_4 + 4w_1w_1w_2 - 2w_2^2w_2)}{w_1(4w_1w_3 + 9w_2^2)}, \] (19)
and
\[ w_1 = M_{pl}^2 - 2 \left( 3f_4 + 2H f_5 \phi \right) X^2 + 2f_{5,\phi} X^3, \] (20)
\[ w_2 = 2M_{pl}^2 H - 2f_3 \phi X - 2 \left( 30H f_4 - 5f_{4,\phi} + 14H^2 f_5 \phi \right) X^2 + 28f_{5,\phi} X^3, \] (21)
\[ w_3 = -9M_{pl}^2 H^2 + 3 \left( 1 + 12H f_3 \phi \right) X + 6 \left( 135H^2 f_4 - 2f_{3,\phi} - 45H f_{4,\phi} + 56H^3 f_5 \phi \right) X^2 - 504H^2 f_{5,\phi} X^3, \] (22)
\[ w_4 = M_{pl}^2 + 2 \left( f_4 - 2f_{5,\phi} \right) X^2 - 2f_{5,\phi} X^3. \] (23)
The conditions for the avoidance of ghosts and Laplacian instabilities correspond to \( Q_s > 0 \) and \( c_s^2 > 0 \), respectively. The two-point correlation function of the curvature perturbation \( \mathcal{R} \) can be derived by employing the standard method of quantizing the fields on a quasi de Sitter background [44]. Using the solution for \( \mathcal{R} \) obtained under the slow-roll approximation, the power spectrum of the curvature perturbation is
\[ P_s = \frac{H^2}{8\pi^2 Q_s c_s^2}, \] (24)
which is evaluated at \( c_s k = aH \) (where \( k \) is a comoving wavenumber).

We decompose the intrinsic tensor perturbation \( h_{ij} \) into two independent polarization modes, as \( h_{ij} = h_+ \epsilon_{ij}^+ + h_\times \epsilon_{ij}^\times \). Then the second-order action for tensor perturbations is given by
\[ S_t^{(2)} = \sum_p \int dt d^3x a^3 Q_t \left[ h_p^2 - \frac{c_t^2}{a^2} (\partial h_p)^2 \right], \] (25)
where \( p = +, \times, \) and
\[ Q_t = \frac{w_1}{4}, \quad c_t^2 = \frac{w_4}{w_1}. \] (26)
We require that $Q_t > 0$ and $c_t^2 > 0$ to avoid ghosts and Laplacian instabilities. The tensor power spectrum is
\[ P_t = \frac{H^2}{2\pi^2 Q_t c_t^3}, \] (27)
which is evaluated at $c_t k = aH$.

C. Slow-roll analysis

For the covariant Galileon theory (6) we employ the slow-roll approximation to estimate the physical quantities introduced in previous subsections. Eliminating the term $V$ from Eqs. (7) and (8), we obtain the equation for $\epsilon \equiv -\dot{H}/H^2$ expressed in terms of the slow-roll parameters
\[
\delta_X = \frac{X}{M_{pl}^2 H^2}, \quad \delta_3 = \frac{c_3 \dot{\phi} X}{M_{pl}^2 M^3 H}, \quad \delta_4 = -\frac{2c_4 X^2}{M_{pl}^2 M^6}, \quad \delta_5 = \frac{6c_5 \dot{\phi} X^2}{M_{pl}^2 M^9}, \quad \delta_\phi = \frac{\ddot{\phi}}{H \dot{\phi}},
\] (28)
which are much smaller than unity during inflation. It then follows that
\[ \epsilon = \frac{\delta_X + 3 \delta_3 + 18 \delta_4 + 5 \delta_5 - \delta_\phi (\delta_3 + 12 \delta_4 + 5 \delta_5)}{1 - 3 \delta_4 - 2 \delta_5} \approx \delta_X + 3 \delta_3 + 18 \delta_4 + 5 \delta_5. \] (29)
In the second approximate equality we neglected the terms at second-order in slow-roll.

Under the slow-roll approximation the field equations (7) and (9) reduce to
\[
3M_{pl}^2 H^2 = V, \quad 3H\dot{\phi}(1 + A) + V,\phi \approx 0,
\] (30)
(31)
where
\[
A = 3c_3 \frac{H \dot{\phi}}{M^3} - 18c_4 \left( \frac{H \dot{\phi}}{M^3} \right)^2 + 15c_5 \left( \frac{H \dot{\phi}}{M^3} \right)^3 = \frac{3\delta_3 + 18 \delta_4 + 5 \delta_5}{\delta_X}. \] (32)
Using Eqs. (30) and (31), the parameter $\delta_X$ can be estimated as
\[
\delta_X \approx \frac{\epsilon_\phi}{(1 + A)^2}, \quad \epsilon_\phi = \frac{M_{pl}^2}{2} \left( \frac{V,\phi}{V} \right)^2. \] (33)
(34)
From Eqs. (29) and (32) it follows that
\[ \epsilon \approx (1 + A) \delta_X \approx \frac{\epsilon_\phi}{1 + A}. \] (35)
The conventional slow-roll inflation corresponds to the limit $A \to 0$, in which case $\epsilon \approx \epsilon_\phi \approx \delta_X$. In the regime where $|A|$ is much larger than 1 the evolution of the field slows down relative to that in standard inflation.

We define the number of e-foldings from the time $t$ to the time $t_f$ at the end of inflation, as $N = \int_t^{t_f} H(t) \, dt$. From Eqs. (30) and (31) we have
\[ N \approx \int_{\phi_f}^{\phi} \frac{1}{M_{pl}^2} \frac{1}{(1 + A)} \frac{V}{V,\phi} \, d\phi. \] (36)
The field value $\phi_f$ at the end of inflation is known by solving $\epsilon(\phi_f) = 1$, that is
\[ \epsilon_\phi(\phi_f) = 1 + A(\phi_f). \] (37)
Since the factor $A$ in Eq. (32) involves the field velocity, we need to express $\dot{\phi}$ in terms of $\phi$ according to Eq. (31) for the evaluation of $\dot{\phi}_f$ and $N$.

Under the slow-roll approximation the quantities $Q_s$ and $c_s^2$ read

\[
Q_s \simeq M_{\text{pl}}^2 (\delta_X + 6\delta_3 + 54\delta_4 + 20\delta_5),
\]

\[
c_s^2 \simeq \frac{\delta_X + 4\delta_3 + 26\delta_4 + 8\delta_5}{\delta_X + 6\delta_3 + 5\delta_4 + 20\delta_5}.
\]

(38)

(39)

In the regime where the Galileon self-interactions dominate over the standard kinetic term we have $\{ |\delta_3|, |\delta_4|, |\delta_5| \} \gg \delta_X$. In order to avoid that $Q_s$ becomes negative we demand the following conditions

\[c_3\dot{\phi} > 0, \quad c_4 < 0, \quad c_5\dot{\phi} > 0.\]

(40)

If $\delta_X$ is much larger than $|\delta_3|, |\delta_4|$, and $|\delta_5|$, then the scalar propagation speed squared is close to 1. If either of $\delta_i$ ($i = 1, 2, 3$) is the dominant contribution in Eq. (39), we have

\[
c_s^2 \simeq 2/3 \quad (\delta_3 \text{ dominant}),
\]

\[
c_s^2 \simeq 13/27 \quad (\delta_4 \text{ dominant}),
\]

\[
c_s^2 \simeq 2/5 \quad (\delta_5 \text{ dominant}).
\]

(41)

(42)

(43)

This shows that the Laplacian instability of scalar perturbations is absent during slow-roll inflation.

The quantities $Q_t$ and $c_t^2$ are approximately given by

\[
Q_t \simeq \frac{M_{\text{pl}}^2}{4} (1 - 3\delta_4 - 2\delta_5), \quad c_t^2 \simeq 1 + 4\delta_4 + 2\delta_5,
\]

(44)

which are both positive. Since we require that $\delta_4 > 0$ and $\delta_5 > 0$ to avoid scalar ghosts [see Eqs. (28) and (40)], the tensor propagation speed squared is slightly superluminal in the presence of the couplings $G_4$ and $G_5$.

Under the slow-roll approximation the power spectra of scalar and tensor perturbations are given, respectively, by

\[
\mathcal{P}_s \simeq \frac{H^2}{8\pi^2 M_{\text{pl}}^2} \frac{1}{c_s \epsilon_s} \simeq \frac{V}{24\pi^2 M_{\text{pl}}^4} (\delta_X + 6\delta_3 + 54\delta_4 + 20\delta_5)^{1/2},
\]

\[
\mathcal{P}_t \simeq \frac{2H^2}{\pi^2 M_{\text{pl}}^2} \simeq \frac{2V}{3\pi^2 M_{\text{pl}}^4},
\]

(45)

(46)

where

\[
\epsilon_s = \frac{Q_s c_s^2}{M_{\text{pl}}^2} \simeq \delta_X + 4\delta_3 + 26\delta_4 + 8\delta_5.
\]

(47)

The tensor-to-scalar ratio is

\[
r = \frac{\mathcal{P}_t}{\mathcal{P}_s} = 16c_s \epsilon_s = 16 \frac{(\delta_X + 4\delta_3 + 26\delta_4 + 8\delta_5)^{3/2}}{(\delta_X + 6\delta_3 + 54\delta_4 + 20\delta_5)^{3/2}}.
\]

(48)

Defining the spectral indices as $n_s - 1 = d \ln \mathcal{P}_s / d \ln k|_{k = aH}$ and $n_t = d \ln \mathcal{P}_t / d \ln k|_{k = aH}$, it follows that

\[
n_s - 1 = -2\epsilon - \eta_s - s,
\]

\[
n_t = -2\epsilon,
\]

(49)

(50)

where $\epsilon$ is given in Eq. (29), and

\[
\eta_s = \frac{\epsilon_s}{H \epsilon_s}, \quad s = \frac{c_s}{H \epsilon_s}.
\]

(51)

The consistency relation between $r$ and $n_t$ is

\[
r = -8c_s (n_t - 2\delta_3 - 16\delta_4 - 6\delta_5).
\]

(52)

In the regime $\delta_X \gg |\delta_i|$ ($i = 1, 2, 3$) the standard consistency relation $r = -8n_t$ holds. If either of the terms $|\delta_i|$ ($i = 1, 2, 3$) dominates over other terms, it follows that

\[
r = -8.71n_t \quad (\delta_3 \text{ dominant}),
\]

\[
r = -8.02n_t \quad (\delta_4 \text{ dominant}),
\]

\[
r = -8.10n_t \quad (\delta_5 \text{ dominant}).
\]

(53)

(54)

(55)

Since the ratio $r/n_t$ is close to $-8$ in all cases, the observational bounds on $n_s$ and $r$ are similar to those derived by using the consistency relation $r = -8n_t$. 

III. THEORIES WITH $G_3 \neq 0, G_4 = 0, G_5 = 0$

We first study the covariant Galileon theory in which only the term $- (c_3/M^3) X \Box \phi$ is present in the action $\mathcal{L}$, i.e.,
\[ c_3 \neq 0, \quad c_4 = 0, \quad c_5 = 0. \] (56)

Solving the slow-roll equation (31) for $\dot{\phi}$, it follows that
\[ \dot{\phi} = \frac{M^3}{6c_3 H} \left( \sqrt{1 - \frac{4c_3 V_{\phi}}{M^3}} - 1 \right), \quad \text{and} \quad \mathcal{A}(\phi) = \frac{1}{2} \left( \sqrt{1 - \frac{4c_3 V_{\phi}}{M^3}} - 1 \right). \] (57)

For $c_3 > 0$ one has $\dot{\phi} > 0$ and $V_{\phi} < 0$ from Eqs. (33) and (34). If $c_3 < 0$, then $\dot{\phi} < 0$ and $V_{\phi} > 0$. In the former and latter cases we choose the coefficients $c_3 = 1$ and $c_3 = -1$, respectively, without loss of generality. The transition from Galileon inflation to standard inflation can be quantified by the condition $\mathcal{A}(\phi_G) = 1$, which translates into
\[ c_3 V_{\phi}(\phi_G) = -2M^3. \] (58)

The field value $\phi_f$ at the end of inflation is known from Eq. (37), i.e.,
\[ \epsilon_\phi(\phi_f) = \frac{1}{2} \left[ 1 + \sqrt{1 - \frac{4c_3 V_{\phi}(\phi_f)}{M^3}} \right]. \] (59)

For the scalar potential with $V_{\phi} > 0$ the transition from the regime $\delta_3 \gg \delta_X$ to the regime $\delta_3 \ll \delta_X$ occurs during inflation provided that $|\phi_G| > |\phi_f|$. On the other hand, if $|\phi_G| < |\phi_f|$, the Galileon self-interaction dominates over the standard kinetic term during the whole stage of inflation.

Since $c_3 V_{\phi}/M^3 = -\mathcal{A}(1 + \mathcal{A})$, the number of e-foldings (56) reads
\[ N = -\frac{c_3}{M^3 M_p^2} \int_{\phi_f}^{\phi} \frac{V(\phi)}{\mathcal{A}(\phi)} d\phi. \] (60)

Using Eqs. (33), (34), and the relation $\delta_3 = (\mathcal{A}/3)\delta_X$, the scalar power spectrum (65) reduces to
\[ \mathcal{P}_s = \frac{V^3}{12\pi^2 M_p^6 V_{\phi}^2} \left( \frac{1 + \mathcal{A}}{1 + 4\mathcal{A}/3} \right)^{1/2}. \] (61)

For a given inflaton potential and a mass scale $M$, the field value $\phi_f$ is known by solving Eq. (39). Then the number of e-foldings can be evaluated from Eq. (60) to find the value of $\phi$ at $N = 60$ (for which we denote $\phi_{60}$). The WMAP normalization of the scalar power spectrum is $\mathcal{P}_s(\phi_{60}) = 2.4 \times 10^{-9} [7]$, by which the mass $M$ can be related to model parameters of the potential.

The scalar spectral index $n_s$ is known from Eq. (31) according to $n_s - 1 = \hat{P}_s/(HP_s)$. Taking the time derivative of the quantity $\mathcal{A}$ given in Eq. (57) and using Eq. (50), it follows that $\dot{\mathcal{A}}/H = -\eta_\phi \mathcal{A}/(1 + 2\mathcal{A})$, where
\[ \eta_\phi = M_p^2 V_{\phi}/V. \] (62)

Then we obtain
\[ n_s - 1 = -\frac{6\epsilon_\phi}{1 + \mathcal{A}} + \frac{2\eta_\phi}{1 + 4\mathcal{A}/3} \left[ 1 - \frac{\mathcal{A}}{6(1 + 2\mathcal{A})^2} \right], \] (63)

where $\epsilon_\phi$ is defined in Eq. (33). For $\mathcal{A} \rightarrow 0$ the formula (63) recovers the result $n_s - 1 \simeq -6\epsilon_\phi + 2\eta_\phi$ in conventional slow-roll inflation. In the limit $\mathcal{A} \gg 1$ we have that $n_s - 1 \simeq -6\epsilon_\phi/\mathcal{A} + 3\eta_\phi/(2\mathcal{A})$. From Eq. (48) the tensor-to-scalar ratio reads
\[ r = 16\epsilon_\phi \left( \frac{1 + 4\mathcal{A}/3}{1 + \mathcal{A}} \right)^{3/2}/(1 + 2\mathcal{A})^{1/2}. \] (64)

In the limit $\mathcal{A} \rightarrow 0$ this reproduces the standard relation $r \simeq 16\epsilon_\phi$, but for $\mathcal{A} \gg 1$ we have $r \simeq 64\sqrt{6} \epsilon_\phi/(9\mathcal{A})$. The observables (63) and (64) are functions of $\phi(N)$, so that they can be evaluated for a given inflaton potential.
If the Galileon term is dominant even after the end of inflation, this affects the oscillation of inflaton during reheating. In order to avoid that the $1 + 6H f_3 \dot{\phi}$ term in front of $\dot{\phi}$ in Eq. (61) become negative, we require

$$1 + 6H c_3 \dot{\phi}/M^3 > 0.$$  \hfill (65)

The field velocity $\dot{\phi}$ changes its sign during the oscillating stage of inflaton. This means that the condition (65) can be violated depending on the model parameters. Note that the determinant (16) is approximately given by $\Delta \simeq 2(1 + 6H c_3 \dot{\phi}/M^3)$, so that the violation of the condition (65) leads to the divergence of Eqs. (14) and (15)\(^3\). In the regime $1 + 6H c_3 \dot{\phi}/M^3 < 0$ the field climbs up the potential like a phantom field, so successful reheating cannot be realized.

For smaller values of $M$ the condition (65) tends to be violated. In this case we also show that $c_s^2$ can be negative due to the dominance of the Galileon term. For the potentials (a) $V(\phi) = \lambda \phi^n/n$ (chaotic inflation) and (b) $V(\phi) = \Lambda^4[1 + \cos(\phi/f)]$ (natural inflation), we clarify the parameter space in which inflaton oscillates coherently and $c_s^2$ remains positive. We also place observational bounds on each model from the information of the scalar spectral index $n_s$ and the tensor-to-scalar ratio $r$.\(^4\)

### A. Chaotic inflation

First we study the case of chaotic inflation characterized by the potential

$$V(\phi) = \frac{\lambda}{n} \phi^n,$$  \hfill (66)

where $n$ and $\lambda$ are positive constants. The initial value of the field $\phi$ is assumed to be positive, so that $\dot{\phi} < 0$ and hence $c_3 = -1$. From Eq. (58) the field value at the transition is given by

$$\phi_G = \left(2M^4/\lambda\right)^{1/(n-1)}.$$  \hfill (67)

If the slow-roll parameter (35) at $t = t_G$ is smaller than 1, the transition from Galileon inflation to standard inflation occurs during inflation. The condition $\epsilon(\phi_G) < 1$ translates into

$$M > 2^{-n/3} n^{-1/3} M^2_{\text{pl}(n-1)/3} \lambda^{1/3} \equiv M_c.$$  \hfill (68)

For the potential (66) the function $A(\phi)$ in Eq. (54) is given by

$$A(\phi) = \frac{1}{2} \left(\sqrt{1 + \frac{4\lambda \phi^{n-1}}{M^3}} - 1\right),$$  \hfill (69)

in which case, apart from the case $n = 2$, the number of $e$-folding $N$ is not integrated analytically. Moreover, in order to find the field value $\phi_f$, we need to solve Eq. (59) numerically. In the limit $A \gg 1$, however, it is possible to derive the analytic expression of $\phi$ in terms of $N$. Since $A(\phi) \simeq \sqrt{\lambda \phi^{n-1}/M^3}$ in this limit, we have $\phi_f^{(n+3)/2} \simeq n^2 M^2_{\text{pl}} M^{3/2}/(2\sqrt{\lambda})$ from Eq. (59). Then, integration of Eq. (60) gives

$$\phi_f^{(n+3)/2} \simeq \frac{n M^2_{\text{pl}} M^{3/2}}{2\sqrt{\lambda}} [(n+3)N + n].$$  \hfill (70)

Substituting this solution into Eq. (61) and using the WMAP normalization $\mathcal{P}_s = 2.4 \times 10^{-9}$ at $N = 60$, we find

$$\lambda \frac{M^n}{M^4_{\text{pl}}} = 8^{(n+1)/3} \frac{(3.1 \times 10^{-7})^{(n+3)/3}}{(61n + 180)^{n+1}}.$$  \hfill (71)

For smaller $M$, $\lambda$ tends to be larger. The scalar spectral index (63) and the tensor-to-scalar ratio (64) reduce to

$$n_s = 1 - \frac{3(n + 1)}{(n + 3)N + n}, \quad r = \frac{64\sqrt{6}}{9} \frac{n}{(n + 3)N + n}. $$  \hfill (72)

---

\(^3\) Note that a similar determinant singularity appears in the context of anisotropic string cosmology [13].

\(^4\) Note that we also place observational bounds on each model from the information of the scalar spectral index $n_s$ and the tensor-to-scalar ratio $r$.  

---
Figure 1: The mass parameters $M$ and $m$ satisfying the WMAP normalization $P_s = 2.4 \times 10^{-9}$ at $N = 60$ for the quadratic potential $V(\phi) = m^2 \phi^2/2$ with the term $G_3 = -X/M^3$. The solid line represents the region in which the coherent oscillation occurs during reheating.

which agree with those given in Ref. [29]. For $n = 4$ and $N = 60$, for example, we have $n_s = 0.965$ and $r = 0.164$.

In the limit $A \ll 1$ we have $\phi_f = nM_{pl}/\sqrt{2}$ and $\phi^2 = 2nM_{pl}^2(N + n/4)$. The WMAP normalization at $N = 60$ gives

$$\lambda = 2.8 \times 10^{-7} n^3 (n(120 + n/2))^{-(n+2)/2} M_{pl}^{4-n}.$$  \hspace{1cm} (73)

The scalar spectral index and the tensor-to-scalar ratio are

$$n_s = 1 - \frac{2(n+2)}{4N+n}, \quad r = \frac{16n}{4N+n},$$ \hspace{1cm} (74)

which correspond to those for standard chaotic inflation. In the regime between $A \gg 1$ and $A \ll 1$ we need to evaluate $n_s$ and $r$ numerically. For $n = 4$ and $N = 60$, for example, we have $n_s = 0.951$ and $r = 0.262$.

I. $V(\phi) = m^2 \phi^2/2$

Let us consider the case of the quadratic potential $V(\phi) = m^2 \phi^2/2$, i.e., $\lambda = m^2$ and $n = 2$. In the limit $A \gg 1$ the WMAP normalization (71) leads to the following relation

$$\frac{m}{M_{pl}} \frac{M}{M_{pl}} \approx 4.1 \times 10^{-9}.$$ \hspace{1cm} (75)

In another limit $A \ll 1$, we have $m \simeq 6.2 \times 10^{-6} M_{pl}$ from Eq. (73). In Fig. 1 we plot $m$ versus $M$ constrained by the normalization $P_s = 2.4 \times 10^{-9}$ at $N = 60$. For small $M$ satisfying $M/M_{pl} \ll 10^{-3}$ the numerical result in Fig. 1 is in good agreement with the analytic estimation (75). In the regime $M/M_{pl} \gg 10^{-3}$ the mass $m$ approaches the constant value $m \simeq 6.2 \times 10^{-6} M_{pl}$. Under the condition (68), i.e., $M > 2^{-1/3} M_{pl}^{1/3} m^{2/3}$, the transition from the regime $A > 1$ to the regime $A < 1$ occurs during inflation. Combining this condition with the constraints on $M$ and $m$ shown in Fig. 1 it follows that $M > 4.0 \times 10^{-4} M_{pl}$. If $M < 4.0 \times 10^{-4} M_{pl}$, the Galileon self-interaction dominates over the standard kinetic term during inflation.

In order to see the effect of the Galileon term during inflation and reheating, we numerically solve the background equations (12)-(14) with the initial conditions determined by the slow-roll analysis. We confirm that the slow-roll approximation is accurate enough to reproduce the numerical values of $N$ with the difference less than a few percent.
Figure 2: Evolution of the field $\phi$ (left) and the scalar propagation speed squared $c_s^2$ (right) for the quadratic potential $V(\phi) = m^2 \phi^2/2$ with the term $G_3 = -X/M^3$ in three different cases: (a) $M = 3.0 \times 10^{-4} M_{\text{pl}}$, $m = 1.45 \times 10^{-5} M_{\text{pl}}$, (b) $M = 4.2 \times 10^{-4} M_{\text{pl}}$, $m = 1.1 \times 10^{-5} M_{\text{pl}}$, and (c) $M = 1.0 \times 10^{-3} M_{\text{pl}}$, $m = 6.9 \times 10^{-6} M_{\text{pl}}$. We choose the initial conditions at $N = 60$ determined by the slow-roll analysis, i.e., (a) $x_i = 6.28$, $y_i = -5.25 \times 10^{-3}$, $z_i = 1.24 \times 10^{-1}$, (b) $x_i = 8.32$, $y_i = -5.07 \times 10^{-3}$, $z_i = 8.89 \times 10^{-2}$, (c) $x_i = 13.55$, $y_i = -3.90 \times 10^{-3}$, $z_i = 3.82 \times 10^{-2}$, respectively. In the case (a) the system enters the region with negative values of $c_s^2$, whereas in the case (c) $c_s^2$ is always positive. The case (b) is the marginal one in which the minimum value of $c_s^2$ is 0.

Figure 3: Observational constraints on the quadratic potential $V(\phi) = m^2 \phi^2/2$ with the term $G_3 = -X/M^3$ in the $(n_s, r)$ plane for the numbers of e-foldings $N = 50, 60, 70$. The thin solid curves show the 1$\sigma$ (inside) and 2$\sigma$ (outside) observational contours constrained by the joint data analysis of WMAP7, BAO, and HST. For smaller values of $M$ the tensor-to-scalar ratio $r$ gets smaller, whereas the scalar spectral index $n_s$ increases.
In Fig. 2 we plot the evolution of $\phi$ and $c_s^2$ for three different mass parameters $M$ and $m$ constrained by the WMAP normalization. The case (a) corresponds to the mass $M = 3.0 \times 10^{-4} M_{\text{pl}}$, which is smaller than the critical mass $M_c = 4.0 \times 10^{-4} M_{\text{pl}}$. Hence the Galileon self-interaction dominates over the standard kinetic term by the end of inflation. As we see in the right panel of Fig. 2, the solutions enter the regime in which $c^2_s$ is negative. For $M$ smaller than $3.0 \times 10^{-4} M_{\text{pl}}$ the period in which $c^2_s$ is negative tends to be longer with $|c^2_s|$ much larger than 1. Since scalar perturbations grow very rapidly in such cases, the Universe becomes inhomogeneous at the level of destroying the homogenous background. The case (b) shown in Fig. 2 corresponds to the marginal one in which the minimum value of $c^2_s$ is 0. In the case (c) the transition from the regime $A > 1$ to the regime $A < 1$ occurs during inflation and $c^2_s$ always remains positive. For the range of masses $M$ used in the numerical simulations of Fig. 2 the inflaton oscillates coherently as long as the backreaction of created particles is neglected.

The condition for the avoidance of negative values of $c^2_s$ is

$$M > 4.2 \times 10^{-4} M_{\text{pl}},$$

under which $c^2_s$ finally approaches 1 without entering the regime $c^2_s < 0$. Note that $c^2_s = 1$ in the presence of the $G_3$ term alone. Numerically we find that inflaton oscillates coherently during reheating for

$$M > 2.5 \times 10^{-4} M_{\text{pl}},$$

which is related to the condition \((65)\). During inflation in which $c_3 \dot{\phi}$ is always positive, the condition \((65)\) is always satisfied. However, after $\dot{\phi}$ changes its sign during reheating, the condition \((65)\) is violated for $M < 2.5 \times 10^{-4} M_{\text{pl}}$. The criterions \((74)\) and \((77)\) are not very different from each other. We also confirmed that the conditions $Q_s > 0$ and $Q_t > 0$ are satisfied in such cases.

The superluminal behavior of the scalar propagation speed seen in Fig. 2 is a matter of debate \([46–50]\). This behavior does not necessarily imply a violation of causality because general solutions of Galileon models break Lorentz symmetry on the FLRW background. A problem occurs if closed time-like curves (CTCs) are developed by the existence of such a superluminal mode. Hawking argued that the formation of CTCs may be generally avoided because the backreaction from the energy-momentum tensor of a quantum field becomes so large before the onset of formation of the CTC (which is called chronology protection conjecture) \([51]\). According to the acoustic analogue of the chronology protection conjecture, Refs. \([47]\) showed that CTCs do not form even in the presence of the superluminal mode in k-essence theories. In Ref. \([49]\) it was claimed that in Galileon theories the CTCs appear only when there exists some region in which higher derivative Galileon terms are larger than the 2-derivative kinetic term. On the other hand, Ref. \([50]\) showed that the CTCs do not arise because the Galileons become strongly coupled at the onset of formation of a CTC. In our work we do not put the bounds $c_s^2 \leq 1$ and $c_t^2 \leq 1$ by taking an attitude that the existence of superluminal modes does not pose a problem associated with the CTCs.

In Fig. 3 the theoretical values of $n_s$ and $r$ are plotted as a function of $M$ ranging in the region \((77)\) with three different values of $N = 50, 60, 70$. We also show the 1$\sigma$ and 2$\sigma$ observational contours constrained by the joint data analysis of WMAP7 \([7]\), Baryon Acoustic Oscillations (BAO) \([52]\), and the Hubble constant measurement using the the Hubble Space Telescope (HST) \([53]\). As we decrease the value of $M$, the two observables shift from the values in Eq. \((74)\) to those in Eq. \((72)\). For smaller $M$, $r$ gets smaller whereas $n_s$ increases, so that the quadratic potential shows better compatibility with the data. Even for the mass $M$ corresponding to the lower limit of Eq. \((77)\) the term $A$ is larger than 1 during most stage of inflation, in which case $n_s$ and $r$ are close to the asymptotic values given in Eq. \((72)\).

2. $V(\phi) = \lambda \phi^4 / 4$

We proceed to the case of the quartic potential $V(\phi) = \lambda \phi^4 / 4$. In the limits $A \gg 1$ and $A \ll 1$ the WMAP normalizations \((71)\) and \((73)\) give $\lambda(M/M_{\text{pl}})^4 \simeq 2.4 \times 10^{-25}$ and $\lambda \simeq 1.6 \times 10^{-13}$, respectively. Figure 4 shows the viable parameter space in the $(M, \lambda)$ plane satisfying the WMAP normalization at $N = 60$. Under the condition \((68)\), i.e., $M > 2^{2/3} \lambda^{1/3} M_{\text{pl}}$, the transition from the regime $A > 1$ to the regime $A < 1$ occurs during inflation. Combining this condition with the constraints on $M$ and $\lambda$ shown in Fig. 3 it follows that $M > 2.8 \times 10^{-4} M_{\text{pl}}$. If $M < 2.8 \times 10^{-4} M_{\text{pl}}$, the Galileon term dominates over the standard kinetic term during inflation.

By solving the background equations of motion \([12, 14]\), we find that $c_s^2$ remains positive for

$$M > 1.7 \times 10^{-4} M_{\text{pl}}. \tag{78}$$

The inflaton oscillation occurs during reheating provided that

$$M > 9.5 \times 10^{-5} M_{\text{pl}}. \tag{79}$$
Figure 4: The parameters $M$ and $\lambda$ satisfying the WMAP normalization $P_s = 2.4 \times 10^{-9}$ at $N = 60$ for the quartic potential $V(\phi) = \lambda \phi^4/4$ with the term $G_3 = -X/M^3$. The solid line represents the region in which the inflaton oscillation occurs during reheating and the model is within the $2\sigma$ observational contour in the $(n_s, r)$ plane.

Figure 5: Observational constraints on the quartic potential $V(\phi) = \lambda \phi^4/4$ with the term $G_3 = -X/M^3$ for three different values of $N$. The $1\sigma$ and $2\sigma$ observational contours are the same as those in Fig. 4. While the standard case ($M \to \infty$) is outside the $2\sigma$ bound, the presence of the Galileon term can make the quartic potential compatible with observations.

In Fig. 5 the theoretical values of $n_s$ and $r$ are plotted as a function of $M$ ranging in the region $79$ with $N = 50, 60, 70$. In the limit $M \to \infty$ the quartic potential is outside the $2\sigma$ observational contour for $N$ smaller than 70. In the presence of the Galileon term the model can be compatible with the current observations due to the suppressed tensor-to-scalar ratio and the larger scalar spectral index. For $N = 60$ the model is within the $2\sigma$ contour for

$$M < 7.7 \times 10^{-4} M_{\text{pl}}.$$  

(80)
In terms of the parameter $\lambda$ the conditions \ref{79} and \ref{80} translate into

\[ 3.4 \times 10^{-13} < \lambda < 3.1 \times 10^{-10}. \] \hfill (81)

Under the constraint \ref{78} the upper bound is $\lambda < 3.0 \times 10^{-11}$. The result \ref{81} shows that one cannot accommodate the self coupling $\lambda \sim 0.1$ of the Higgs boson in the presence of the coupling $G_3 = -X/M^3$.

We also studied the case of the generalized Galileon term $-G_3(\phi, X)\Box\phi$, where

\[ G_3 = \frac{c_3}{M^4}\phi X, \] \hfill (82)

which was proposed in Ref. \cite{29}. Numerically we find that the inflaton oscillation occurs for $M > 3.6 \times 10^{-4} M_{\text{pl}}$ and $\lambda < 2.7 \times 10^{-8}$. The self coupling $\lambda$ is still much smaller than the order of 0.1.

### B. Natural inflation

Natural inflation \cite{22} is characterized by the potential

\[ V(\phi) = \Lambda^4 \left[ 1 + \cos \left( \frac{\phi}{f} \right) \right], \] \hfill (83)

where $\Lambda$ and $f$ are constants having the dimension of mass. In the absence of the Galileon term this potential can be compatible with the observational data only for $f \gtrsim 3.5 M_{\text{pl}}$ \cite{28}. This is the regime in which standard quantum field theory may not be reliable. If the field $\phi$ is a string axion, $f$ is usually smaller than the order of $M_{\text{pl}}$ \cite{24, 54}. In the following we study whether this problem can be alleviated or not in the presence of the Galileon term $G_3 = c_3 X/M^3$.

We assume that inflation occurs in the region $0 < \phi/f < \pi$, in which case $\dot{\phi} > 0$. We choose $c_3 = 1$ to satisfy the condition \ref{10}. For the potential \ref{83} the function $A(\phi)$ in Eq. \ref{57} is given by

\[ A(\phi) = \frac{1}{2} \left( \sqrt{1 + \frac{4\gamma \sin(\phi/f)}{q}} - 1 \right), \] \hfill (84)

where

\[ q = \frac{f}{M_{\text{pl}}}, \quad \gamma = \frac{\Lambda^4}{M^4 M_{\text{pl}}}. \] \hfill (85)

Note that in the limit $\phi \to 0$ one has $A \to 0$. We are mainly interested in the case where the initial displacement of the field $\phi_i$ satisfies the condition $A(\phi_i) > 1$. This can be achieved for $4\gamma \gg q$ provided that $\phi_i$ is not very close to 0.

The field value $\phi_G$ at the transition ($A = 1$) is given by

\[ \sin(\phi_G/f) = 2q/\gamma. \] \hfill (86)

For the existence of $\phi_G$ we require that $2q < \gamma$, i.e., $M^3 < \Lambda^4/(2f)$. From Eq. \ref{58} the field value $\phi_f$ at the end of inflation satisfies

\[ \frac{1 - \cos(\phi_f/f)}{1 + \cos(\phi_f/f)} = q^2 \left[ 1 + \sqrt{1 + \frac{4\gamma \sin(\phi_f/f)}{q}} \right]. \] \hfill (87)

The transition from Galileon inflation to standard inflation occurs under the condition $\phi_G < \phi_f$. In the limit $\gamma \to 0$ one has $\cos(\phi_f/f) = (1 - 2q^2)/(1 + 2q^2)$, so that $\phi_f/f \to 0$ for $q \ll 1$. This implies that, in the absence of the Galileon self-interaction, it is difficult to realize sufficient amount of inflation for $f \ll M_{\text{pl}}$. If the Galileon term is present with $\gamma \to \infty$, the field value $\phi_f$ can be close to $\pi f$ even for $f \ll M_{\text{pl}}$.

The slow-roll parameters $\epsilon_\phi$ and $\eta_\phi$ are given by

\[ \epsilon_\phi = \frac{1}{2q^2} \frac{\sin^2(\phi/f)}{1 + \cos(\phi/f)^2}, \quad \eta_\phi = -\frac{1}{q^2} \frac{\cos(\phi/f)}{1 + \cos(\phi/f)}. \] \hfill (88)

In the limit $\gamma \to 0$, i.e., $A \to 0$, the scalar spectral index is $n_s \simeq 1 - 6\epsilon_\phi + 2\eta_\phi$. When $q \ll 1$ inflation occurs in the region $\phi/f \ll 1$, so that $|\eta_\phi| \simeq 1/(2q^2) \gg 1$. This case is in contradiction with observations because $n_s$ significantly
Figure 6: Observational constraints on natural inflation with $f = 0.1M_{\text{pl}}$ in the presence of the term $G_3 = X/M^3$ for three different values of $N$. The parameter range of $\gamma = \Lambda^4/(M^3M_{\text{pl}})$ corresponds to $4.0 \times 10^5 \leq \gamma \leq 1.0 \times 10^{11}$. The 1$\sigma$ and 2$\sigma$ observational contours are the same as those in Fig. 3.

Figure 7: The allowed values of $\gamma$ versus $f/M_{\text{pl}}$ in natural inflation with the term $G_3 = X/M^3$. In the region (i) the model is within the 2$\sigma$ observational contour in the $(n_s, r)$ plane for $N = 60$. In the region (ii) the coherent oscillation of inflaton occurs during reheating. There is a viable parameter space only for $f/M_{\text{pl}} > 1.7$.

deviates from 1. In another limit $\gamma \to \infty$ one has $A \to \infty$ and hence the scalar spectral index can be as close as 1 even for $q \ll 1$. In this limit, inflation occurs in the regime close to the potential minimum $(\phi/f = \pi)$. Since the potential is approximately given by $V(\phi) \simeq (\Lambda^4/2f^2)(\phi - \pi f)^2$ in this regime, $n_s$ and $r$ are the same as those given...
in Eq. (72) with \( n = 2 \). Hence, for \( \gamma \to \infty \), it follows that

\[
n_s = 1 - \frac{9}{5N + 2}, \quad r = \frac{128\sqrt{6}}{9(5N + 2)},
\]

(89)

which give \( n_s = 0.970 \) and \( r = 0.115 \) for \( N = 60 \).

In the intermediate regime characterized by \( 0 < \gamma < \infty \) we need to evaluate \( n_s \) and \( r \) numerically according to Eqs. (94) and (95). For given values of \( f, \Lambda, \) and \( M, \phi_f \) is known by solving Eq. (87). Integrating Eq. (60) numerically, we can determine the field \( \phi \) in terms of the number of e-foldings \( N \). The WMAP normalization \( P_s = 2.4 \times 10^{-9} \) at \( N = 60 \) provides one constraint between the three parameters \( f, \Lambda, \) and \( M \). In other words, for a given \( f \), the two parameters \( \Lambda \) and \( M \) are related to each other.

Let us first consider the case \( f = 0.1M_{pl} \), i.e., \( q = 0.1 \). In Fig. 6 we plot the theoretical values of \( n_s \) and \( r \) in the range \( 4.0 \times 10^5 \leq \gamma \leq 1.0 \times 10^{11} \) for three different values of \( N \), together with the \( 1\sigma \) and \( 2\sigma \) observational contours. When \( N = 60 \) the model is within the \( 2\sigma \) contour provided that \( \gamma > 4.3 \times 10^5 \). In the limit \( \gamma \to \infty \), \( n_s \) and \( r \) approach the asymptotic values given in Eq. (93). This asymptotic case is within the \( 1\sigma \) contour for \( N > 55 \).

For larger \( \gamma \), however, the inflaton oscillation during reheating tends to be disturbed by the Galileon term. After the inflaton velocity \( \dot{\phi} \) changes its sign from positive to negative, the condition (65) can be violated for \( \gamma > \gamma_{\text{c}} \). Detailed numerical simulations show that the coherent oscillations of inflaton occur provided that \( \gamma < 0.5 \). Thus there is no viable parameter space of \( \gamma \) satisfying both the WMAP bound and the successful reheating. We also note that, if \( \gamma \) is larger than 0.05, \( c_s^2 \) becomes negative. Hence, for \( \gamma > 0.5 \), the model is plagued by the reheating problem as well as the negative instability of scalar perturbations.

We also study the models of other values of \( f \) ranging in the region \( 0.1 < f/M_{pl} < 2 \). In Fig. 7 we show the two kinds of constraints on the parameter \( \gamma \) versus \( f/M_{pl} \). Above the dotted line (i) the model is within the \( 2\sigma \) observational contour in the \((n_s, r)\) plane, whereas under the solid line (ii) the coherent oscillation of inflaton occurs during reheating. For the compatibility of two constraints we require that \( f \) is bounded to be

\[
f > 1.7M_{pl}. \tag{90}
\]

Hence the problem of the super-Planckian values of \( f \) in standard natural inflation is not circumvented by the Galileon term \( G_3 = X/M^3 \).

IV. THEORIES WITH \( G_3 = 0, G_4 \neq 0, G_5 = 0 \)

We proceed to the covariant Galileon theory (6) with

\[
c_3 = 0, \quad c_4 \neq 0, \quad c_5 = 0. \tag{91}
\]

Since \( c_4 < 0 \) to avoid ghosts, we set \( c_4 = -1 \) without loss of generality. From Eq. (91) the quantity \( \mathcal{A} = 18(H\dot{\phi}/M^3)^2 \) satisfies the following relation

\[
\mathcal{A}(1 + \mathcal{A})^2 = 2V_{\phi}^2/M^6. \tag{92}
\]

The field value \( \phi_G \) at the transition from Galileon inflation to standard inflation obeys

\[
V_{\phi}^2(\phi_G) = 2M^6. \tag{93}
\]

From Eq. (92) we have \( \delta_4 = \mathcal{A}\delta_X/18 \). Using Eq. (93), the scalar power spectrum (15) can be written as

\[
P_s = \frac{V^3}{12\pi^2M_{pl}^2V_{\phi}^2}(1 + \mathcal{A})^2(1 + 3\mathcal{A})^{-1/2}. \tag{94}
\]

Taking the time derivative of Eq. (92) and making use of Eq. (31), it follows that \( \dot{\mathcal{A}}/H = -2\eta_{\phi}\mathcal{A}/(1 + 3\mathcal{A}) \). Then the scalar spectral index is given by

\[
n_s - 1 = -\frac{6\mathcal{A}}{1 + \mathcal{A}} + \frac{2\eta_{\phi}}{1 + 3\mathcal{A}/2} \left[ 1 - \frac{3\mathcal{A}(5 + 8\mathcal{A})}{2(1 + 3\mathcal{A})^2(9 + 13\mathcal{A})} \right]. \tag{95}
\]

The tensor-to-scalar ratio (38) reads

\[
r = 16\mathcal{A}\frac{(1 + 3\mathcal{A})^3}{(1 + \mathcal{A})^2(1 + 3\mathcal{A})^{1/2}}. \tag{96}
\]
In the limit $A \to \infty$ we have $n_s - 1 \simeq -6 \epsilon_\phi / A + 4 \eta_\phi / (3A)$ and $r \simeq 208 \sqrt{39} \epsilon_\phi / (81A)$. In the following we focus on the potential $V(\phi) = \lambda \phi^n / n$ of chaotic inflation. From Eq. (99) the field value at the transition is
\[
\phi_G = \left( \frac{2M^6}{\lambda^2} \right)^{1/(2(n-1))}.
\]
The condition under which the transition occurs during inflation corresponds to $\epsilon(\phi_G) < 1$, which translates into
\[
M > 2^{(1-2n)/6} n^{(n-1)/3} M_{pl}^{(n-1)/3} \lambda^{1/3}.
\]  
(98)

Let us consider the case in which the condition $A \gg 1$ is satisfied during the whole stage of inflation. Then we have $54(H\phi)^3 \simeq -M^6 V_\phi$ from Eq. (31). The end of inflation is characterized by the condition $\epsilon(\phi_f) \simeq \epsilon_\phi(\phi_f)/A = 1$, which gives $\phi_f = [n^3 M_{pl}^3 M^3/(4\lambda)]^{1/(n+2)}$. The number of e-foldings (36) is related to the field value $\phi$ during inflation, as
\[
\phi^{2(n+2)/3} \simeq \frac{n M_{pl}^2 M^2}{6(2\lambda^2)^{1/3}} [4(n+2)N + 3n] .
\]  
(99)

The WMAP normalization $P_s = 2.4 \times 10^{-9}$ at $N = 60$ provides the relation
\[
\lambda^2 M_{pl}^{3n+8} = \frac{2^{3n+2} n^{n/2+4}}{39^{n/2+1}} \frac{(1.8 \times 10^{-6})^{n+2}}{(81n + 160)(5n+4)/2} .
\]  
(100)

Substituting Eq. (99) into Eqs. (95) and (100) in the regime $A \gg 1$, it follows that
\[
n_s = 1 - \frac{2(5n + 4)}{4(n+2)N + 3n} , \quad r = \frac{208 \sqrt{39}}{27} \frac{n}{4(n+2)N + 3n} .
\]  
(101)

For $n = 4$ and $N = 60$, for example, $n_s = 0.967$ and $r = 0.133$. The tensor-to-scalar ratio is smaller than that studied in Sec. [III] in the regime $A \gg 1$.

In another limit $A \ll 1$, we have the same relations as those given in Eqs. (12) and (13). In the intermediate regime between $A \gg 1$ and $A \ll 1$ we need to solve the background equations (12)-(14) numerically in order to find the values of $n_s$ and $r$ as well as the relation between $\lambda$ and $M$ from the WMAP normalization. In Fig. 5, we show the parameter space for the two potentials $V(\phi) = m^2 \phi^2 / 2$ and $V(\phi) = \lambda \phi^4 / 4$ satisfying the WMAP normalization at
Figure 9: Evolution of the field $\phi$ for the quartic potential $V(\phi) = \lambda \phi^4/4$ with $\lambda = 0.1$ in the presence of the term $G_4 = X^2/M^6$ with $M = 7.3 \times 10^{-6} M_{\text{pl}}$. The initial conditions are chosen to be $x_i = 2.07 \times 10^{-2}$, $y_i = 4.69 \times 10^{-4}$, and $z_i = 5.49$ at $N = 60$.

Figure 10: The same as Fig. 9 but for the evolution of $c_s^2$ (left) and $c_t^2$ (right).

In the two asymptotic regimes $A \gg 1$ and $A \ll 1$, the analytic estimation given above agrees well with the numerical results.

Unlike the case of the coupling $G_3 = c_3 X/M^3$, the term $1 + 54 H^2 \dot{\phi}^2 / M^6$ in front of $\ddot{\phi}$ in Eq. (49) remains positive even if $\dot{\phi}$ changes its sign. Numerically we confirmed that the determinant $\Delta$ defined in Eq. (16) does not cross 0 even for the mass $M$ much smaller than the r.h.s. of Eq. (98). In Fig. 9 we show the field evolution during reheating for the quartic potential $V(\phi) = \lambda \phi^4/4$ with $\lambda = 0.1$ and $M = 7.3 \times 10^{-6} M_{\text{pl}}$. In fact the coherent oscillation of inflaton is not disturbed by the dominance of the term $G_4 = X^2/M^6$. In this case, however, the scalar propagation speed squared oscillates significantly between largely negative and positive values (see the left panel of Fig. 10). This leads to the strong enhancement of scalar perturbations for the modes inside the Hubble radius during reheating. While this instability does not directly affect the evolution of large-scale density perturbations relevant to CMB, the rapid growth of perturbations can invalidate the analysis without the backreaction of created particles after some stage of
reheating \[55\]. Our numerical simulations without the backreaction effect show that both \(c_s^2\) and \(c_t^2\) finally approach 1 with oscillations. The tensor propagation speed is superluminal during most stages of inflation and reheating, but it does not enter the region \(c_t^2 < 0\) (see the right panel of Fig. 10).

It remains to see how the created particles can change the evolution of \(\phi\), \(c_s^2\), and \(c_t^2\) at the late stage of reheating. This is beyond the scope of our paper, since nonlinear lattice simulations (along the line of Refs. \[56\]) are required to deal with such a problem properly.

For larger values of \(M\), the instability associated with negative \(c_s^2\) tends to be less significant. For the power-law potential \(V(\phi) = \lambda \phi^n / n\) we find that \(c_s^2\) remains positive for

\[
M > 4.3 \times 10^{-4} M_{pl} \quad \text{(for } n = 2),
\]
\[
M > 2.3 \times 10^{-4} M_{pl} \quad \text{(for } n = 4),
\]

respectively. The regions in which these conditions are satisfied are shown as solid curves in Fig. 8. In Fig. 11 we plot the theoretical values of \(n_s\) and \(r\) for \(n = 2\) and \(n = 4\) as a function of \(M\). Even for the lower bounds of Eqs. (102) and (103), \(n_s\) and \(r\) are close to the values \[101\] corresponding to the limit \(\mathcal{A} \gg 1\). For the quadratic potential the presence of the term \(G_4 = X^2/M^6\) leads to better compatibility with the WMAP data (see the left panel of Fig. 11). In the case of the quartic potential the model is within the 2σ observational contour under the condition

\[M < 1.1 \times 10^{-3} M_{pl},\]

for \(N = 60\). From Fig. 8 this condition translates into

\[\lambda > 1.7 \times 10^{-13}.\]

If we demand the condition \[103\] for the avoidance of negative values of \(c_s^2\), the self coupling is bounded to be \(\lambda < 9.9 \times 10^{-11}\). Recall that we do not have a constraint coming from the absence of inflaton oscillations.

V. THEORIES WITH \(G_3 = 0, G_4 = 0, G_5 \neq 0\)

Finally we study the covariant Galileon theory \[6\] with

\[c_3 = 0, \quad c_4 = 0, \quad c_5 \neq 0.\]
From Eq. (31) the quantity $A = 15c_5H^3\dot{\phi}^4/M^9$ satisfies

$$A(1 + A)^3 = -5c_5V_\phi^3/(9M^9).$$

(107)

The field value $\phi_G$ at the transition from Galileon inflation to standard inflation is determined by

$$V_\phi^3(\phi_G) = -72M^9/(5c_5).$$

(108)

Since $\delta_5 = A\delta_X/5$, the scalar power spectrum (15) reduces to

$$P_s = \frac{V^3}{12\pi^2M^6V^2_{\phi}} \frac{(1 + A)^2(1 + 4A)^{1/2}}{(1 + 8A/5)^{3/2}}.$$  

(109)

On using the relation $\dot{A}/H = -3\eta_\phi A/(1 + 4A)$, the scalar spectral index is expressed as

$$n_s - 1 = - \frac{6\epsilon_\phi}{1 + 4A} + \frac{2\eta_\phi}{1 + 8A/5} \left[ 1 - \frac{9A}{5(1 + 4A)^2} \right].$$

(110)

The tensor-to-scalar ratio (18) reads

$$r = 16\epsilon_\phi \frac{(1 + 8A/5)^{3/2}}{(1 + 4A)^{3/2}}.$$  

(111)

Let us focus on the power-law potential $V(\phi) = \lambda\phi^n/n$. We assume that inflation occurs in the regime $\phi > 0$ with the coefficient $c_5 = -1$ (under which the condition $c_5\phi > 0$ is satisfied). Then the field value at the transition is

$$\phi_G = [72M^9/(5\lambda^3)]^{1/3(n-1)}. $$

(112)

The condition under which the transition occurs during inflation is

$$M > 0.94\cdot 2^{-n/3}n^{-n-1/3}M^{(n-1)/3}.$$

(113)

If $A \gg 1$ during the whole stage of inflation, the field value at the end of inflation can be estimated as $\phi_f = [(9/5)^{1/4}n^2M^2M^9/4/(2\lambda^3)^{1/4}]^{(3n+5)}/(3n+5).$ The field $\phi$ is related to the number of e-foldings $N$, as

$$\phi^{(3n+5)/4} \simeq \frac{nM^2_{pl}}{4\lambda^{3/4}} \left( \frac{9M^9}{5} \right)^{1/4} \left[ (3n+5)N + 2n \right].$$

(114)

From the WMAP normalization $P_s = 2.4 \times 10^{-9}$ at $N = 60$ it follows that

$$\lambda^5 \frac{M^{3n}}{M^{(n+5)}_{pl}} = 1.33 \times 10^{-7} \cdot (4.44 \times 10^{-7})^{n/2} \cdot (1.36 \times 10^{-5})^{3n+5} \cdot \frac{n^{2(n+5)}}{91n + 150}.$$

(115)

From Eqs. (10) and (11) the asymptotic values of $n_s$ and $r$ in the regime $A \gg 1$ are

$$n_s = 1 - \frac{7n + 5}{(3n + 5)N + 2n}, \quad r = \frac{256\sqrt{10}}{25} \frac{n}{(3n + 5)N + 2n}.$$  

(116)

If $n = 4$ and $N = 60$, for example, $n_s = 0.968$ and $r = 0.126$. The tensor-to-scalar ratio is slightly smaller than that for the coupling $G_4 = X^2/M^6$. In the regime $A \ll 1$ the relations (13) and (14) also hold for the coupling $G_5 = -3X^2/M^9$. In the intermediate regime between $A \gg 1$ and $A \ll 1$ we resort to the numerical analysis to derive the relation between $M$ and $\lambda$ from the WMAP normalization as well as to evaluate the observables $n_s$ and $r$.

In order to avoid that the term in front of $\ddot{\phi}$ in Eq. (9) becomes negative, we require that

$$1 + 120H^2Xc_5\dot{\phi}/M^9 > 0.$$  

(117)

For smaller $M$ this condition can be violated during reheating because of the sign change of $\dot{\phi}$ (as it happens for the coupling $G_3 = c_3X/M^3$). Note that this also leads to the divergence of Eqs. (13) and (15) through the crossing at $\Delta = 0$. For the quadratic and quartic potentials we find that the inflaton oscillations occur under the conditions

$$M > 2.7 \times 10^{-4} M_{pl} \quad \text{(for } n = 2),$$

$$M > 1.5 \times 10^{-4} M_{pl} \quad \text{(for } n = 4).$$

(118)

(119)
respectively. The instability associated with negative values of \( c_4^2 \) tends to be stronger for smaller \( M \). The conditions under which \( c_4^2 \) remains positive are given by

\[
M > 4.0 \times 10^{-4}M_{\text{pl}} \quad \text{(for } n = 2) ,
\]
\[
M > 2.9 \times 10^{-4}M_{\text{pl}} \quad \text{(for } n = 4) ,
\]

respectively. The similar lower bounds on \( M (\gtrsim 10^{-4}M_{\text{pl}}) \) also follow from Eq. (113).

In the presence of the coupling \( G_5 = -3X^2/M^6 \) the tensor-to-scalar ratio gets smaller relative to that in standard inflation, so that the quartic potential \( V(\phi) = m^2\phi^2/2 \) is compatible with the current observational data. For the quartic potential \( V(\phi) = \lambda \phi^4/4 \) the model is within the 2\( \sigma \) observational contour in the \((n_s, r)\) plane under the condition

\[
M < 8.6 \times 10^{-4}M_{\text{pl}} ,
\]

for \( N = 60 \). Translating the conditions (119) and (122) in terms of the parameter \( \lambda \), it follows that

\[
1.6 \times 10^{-13} < \lambda < 2.6 \times 10^{-9} .
\]

As in the case of the coupling \( G_3 = -X/M^3 \), the self coupling \( \lambda \) is required to be very much smaller than unity.

VI. CONCLUSIONS

We have studied the viability of potential-driven Galileon inflation described by the action (1). We mainly focused on the covariant Galileon theory in which the functions \( G_i \) \( (i = 3, 4, 5) \) are given by Eq. (5) with the choice (6). The Galileon self-interactions generally lead to the slow down for the evolution of the field, which allows the possibility to accommodate steep inflaton potentials. In Ref. [29], for example, it was suggested that even the Higgs potential \( V(\phi) = \lambda \phi^4/4 \) with \( \lambda \sim 0.1 \) can be consistent with the observed CMB temperature anisotropies because of the presence of the term \( G_3 = c_3X/M^3 \).

The dominance of the Galileon self-interactions relative to the standard kinetic term \( X \) can modify the dynamics of reheating after inflation. In order to clarify this issue, we numerically solved the background equations (12)-(15) for several different inflaton potentials. We found that, depending on the couplings \( G_i \) \( (i = 3, 4, 5) \) and their associated mass scales \( M \), there is no oscillatory regime of inflaton. Moreover the dominance of the Galileon terms generally gives rise to the negative scalar propagation speed squared \( c_4^2 \) during reheating, which leads to the instability of small-scale density perturbations.

For the theories where the covariant Galileon term \( G_3 = c_3X/M^3 \) is present, we found that the system does not enter the oscillatory regime of inflaton after the field velocity \( \dot{\phi} \) changes its sign around the onset of reheating. This corresponds to the violation of the condition (66), which is related to the crossing of the determinant \( \Delta \) in Eq. (16) at 0. The latter leads to the divergence of the background equations (14) and (15). For the potentials \( V(\phi) = \lambda \phi^4/4 \) the coherent oscillation of inflaton occurs for \( M > 2.5 \times 10^{-4}M_{\text{pl}} \) \( (n = 2) \) and \( M > 9.5 \times 10^{-5}M_{\text{pl}} \) \( (n = 4) \). When \( n = 4 \) this constraint translates into \( \lambda < 3.1 \times 10^{-10} \), which is much smaller than the coupling constant \( \lambda \sim 0.1 \) of the Higgs boson. In the presence of the term \( G_3 = c_3X/M^3 \) the quartic potential \( V(\phi) = \lambda \phi^4/4 \) is within the 2\( \sigma \) observational contour in the \((n_s, r)\) plane for \( M < 7.7 \times 10^{-4}M_{\text{pl}} \). Taking into account this constraint, the self coupling is bounded to be \( 3.4 \times 10^{-13} < \lambda < 3.1 \times 10^{-10} \). We also found that \( c_4^2 \) remains positive under the condition \( M > 1.7 \times 10^{-4}M_{\text{pl}} \), which provides even the stronger upper bound \( \lambda < 3.0 \times 10^{-11} \). We extended our analysis to the generalized Galileon term \( G_3 = c_3\phi X/M^4 \) with the potential \( V(\phi) = \lambda \phi^4/4 \) and derived the bound \( \lambda < 2.7 \times 10^{-8} \) for successful reheating.

In the presence of the term \( G_3 = c_3X/M^3 \) we studied the case of natural inflation described by the potential \( V(\phi) = \Lambda^4[1 + \cos(\phi/f)] \) as well. While this potential can be compatible with the observed CMB anisotropies for \( \gamma = \Lambda^4/(M^3M_{\text{pl}}) \gg 1 \) even in the regime \( f \ll M_{\text{pl}} \), there is no oscillatory regime under the condition \( \gamma \gg 1 \). For the compatibility of two constraints, we found that \( f \) needs to be larger than 1.7\( M_{\text{pl}} \). Hence the super-Planckian problem of the symmetry breaking scale in standard inflation \( (f > 3.5M_{\text{pl}}) \) is not improved significantly.

For the Galileon coupling \( G_4 = -c_4X^2/M^6 \) the scalar ghost is absent for \( c_4 < 0 \), in which case the sign change of the determinant \( \Delta \) in Eq. (16) can be avoided. In fact, we numerically confirmed that the oscillation of inflaton occurs even for small \( M \) corresponding to the large self coupling \( \lambda \sim 0.1 \) of the quartic potential \( V(\phi) = \lambda \phi^4/4 \). On the other hand, for such small values of \( M \), the scalar propagation speed squared \( c_4^2 \) heavily oscillates between largely negative and positive values (see the left panel of Fig. 11). This leads to the rapid growth of scalar perturbations for the modes inside the Hubble radius during reheating, which can invalidate the analysis without taking into account the backreaction of created particles. For the potentials \( V(\phi) = \lambda \phi^4/n \) the conditions for the avoidance of this negative instability are given by \( M > 4.3 \times 10^{-4}M_{\text{pl}} \) \( (n = 2) \) and \( M > 2.3 \times 10^{-4}M_{\text{pl}} \) \( (n = 4) \). Taking into
account this condition, the quartic potential $V(\phi) = \lambda \phi^4/4$ is compatible with the current CMB observations for $1.7 \times 10^{-13} < \lambda < 9.9 \times 10^{-11}$.

In the case of the Galileon coupling $G_5 = 3c_5 X^2/M^9$ the sign change of $\Delta$ can occur for small $M$, as it happens for the coupling $G_3 = c_3 X/M^3$. For the potentials $V(\phi) = \lambda \phi^n/n$ the inflaton oscillations occur for $M > 2.7 \times 10^{-4} M_{pl}$ ($n = 2$) and $M > 1.5 \times 10^{-4} M_{pl}$ ($n = 4$). Using the latter bound, the quartic potential is consistent with the CMB observations for $1.6 \times 10^{-13} < \lambda < 2.6 \times 10^{-9}$. We also found that the instability associated with negative $c_2^2$ is present for small $M$, which puts even severer upper bounds on $\lambda$.

Compared to the models of non-minimal field derivative couplings to the Einstein tensor \cite{17,19}, the allowed parameter space of potential-driven Galileon inflation is more severely constrained because of the modified dynamics of reheating. We note, however, that there are some viable parameter spaces even for the quartic potential $V(\phi) = \lambda \phi^4/4$ due to the presence of the Galileon terms. It will be of interest to see whether future observations such as PLANCK 57 can place tighter constraints on such inflationary scenarios.

ACKNOWLEDGEMENTS

J. O. and S. T. are supported by the Scientific Research Fund of the JSPS (Nos. 21111006). S. T. also thanks financial support from Scientific Research on Innovative Areas (No. 2111006).

[1] A. A. Starobinsky, Phys. Lett. B 91, 99 (1980); D. Kazanas, Astrophys. J. 241 L59 (1980); K. Sato, Mon. Not. R. Astron. Soc. 195, 467 (1981); A. H. Guth, Phys. Rev. D 23, 347 (1981).
[2] V. F. Mukhanov and G. V. Chibisov, JETP Lett. 33, 532 (1981); A. H. Guth and S. Y. Pi, Phys. Rev. Lett. 49, 1110 (1982); S. W. Hawking, Phys. Lett. B 115, 295 (1982); A. A. Starobinsky, Phys. Lett. B 117, 175 (1982).
[3] G. F. Smoot et al., Astrophys. J. 396, L1-L5 (1992).
[4] D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 148, 175 (2003).
[5] J. E. Lidsey et al., Rev. Mod. Phys. 69, 375 (1997); D. H. Lyth and A. Riotto, Phys. Rept. 314, 1 (1999); A. D. Linde, “Particle physics and inflationary cosmology,” Chur, Switzerland: Harwood (1990) 362 page (Contemporary concepts in physics, 5) [hep-th/0503203]; B. A. Bassett, S. Tsujikawa and D. Wands, Rev. Mod. Phys. 78, 537 (2006).
[6] A. D. Linde, Phys. Lett. B 129, 177 (1983).
[7] E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. 192, 18 (2011).
[8] C. Amsler et al. [Particle Data Group Collaboration], Phys. Lett. B 667, 1 (2008).
[9] K. Nakayama and F. Takahashi, JCAP 1011, 009 (2010).
[10] A. De Felice, S. Tsujikawa, J. Elliston and R. Tavakol, JCAP 1108, 021 (2011).
[11] S. Ummarino, V. Saini and A. Toporensky, [arXiv:1205.0780] [astro-ph.CO].
[12] T. Futamase and K. -i. Maeda, Phys. Rev. D 39, 399 (1989); R. Fakir and W. G. Unruh, Phys. Rev. D 41, 1783 (1990).
[13] F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B 659, 703 (2008).
[14] D. S. Salopek, J. R. Bond and J. M. Bardeen, Phys. Rev. D 40, 1753 (1989); N. Makino and M. Sasaki, Prog. Theor. Phys. 86, 103 (1991); D. I. Kaiser, Phys. Rev. D 52, 4295 (1995).
[15] E. Komatsu and T. Futamase, Phys. Rev. D 59, 064029 (1999); S. Tsujikawa and B. Gumjudpai, Phys. Rev. D 69, 123523 (2004).
[16] C. P. Burgess, H. M. Lee and M. Trott, JHEP 0909, 103 (2009); J. L. Barbon and J. R. Espinosa, Phys. Rev. D 79, 081302 (2009); C. P. Burgess, H. M. Lee and M. Trott, JHEP 1007, 007 (2010); R. N. Lerner and J. McDonald, JCAP 1004, 015 (2010).
[17] C. Germani and A. Kehagias, Phys. Rev. Lett. 105, 011302 (2010); C. Germani and A. Kehagias, JCAP 1005, 019 (2010).
[18] L. Amendola, Phys. Lett. B 301, 175 (1993).
[19] S. Tsujikawa, Phys. Rev. D 85, 083518 (2012).
[20] C. Germani, L. Martucci and P. Moyssaris, Phys. Rev. D 85, 103501 (2012).
[21] C. Germani and A. Kehagias, Phys. Rev. Lett. 106, 161302 (2011).
[22] K. Freese, J. A. Frieman and A. V. Olinto, Phys. Rev. Lett. 65, 3233 (1990); F. C. Adams, J. R. Bond, K. Freese, J. A. Frieman and A. V. Olinto, Phys. Rev. D 47, 426 (1993).
[23] C. Savage, K. Freese and W. H. Kinney, Phys. Rev. D 74, 123511 (2006).
[24] T. Banks, M. Dine, P. J. Fox and E. Gorbatov, JCAP 0306, 001 (2003); N. Barnaby and M. Peloso, Phys. Rev. Lett. 106, 181301 (2011).
[25] C. Germani and Y. Watanabe, JCAP 1107, 031 (2011).
[26] A. Nicolis, R. Rattazzi and E. Trinchieri, Phys. Rev. D 79, 064036 (2009).
[27] C. Deffayet, G. Esposo-Farese and A. Vikman, Phys. Rev. D 79, 084003 (2009); C. Deffayet, S. Deser and G. Esposo-Farese, Phys. Rev. D 80, 064015 (2009).
[28] C. de Rham and A. J. Tolley, JCAP 1005, 015 (2010); K. Van Acoleyen and J. Van Doorsselaere, Phys. Rev. D 83, 084025 (2011).
[29] K. Kamada, T. Kobayashi, M. Yamaguchi and J. "i. Yokoyama, Phys. Rev. D 83, 083515 (2011).
[30] C. Deffayet, O. Pujolas, I. Sawicki and A. Vikman, JCAP 1010, 026 (2010).
[31] P. Silva and K. Yokoyama, Phys. Rev. D 80, 121301 (2009); T. Kobayashi, H. Tashiro and D. Suzuki, Phys. Rev. D 81, 063513 (2010); A. De Felice and S. Tsujikawa, JCAP 1007, 024 (2010); R. Gannouji and M. Sami, Phys. Rev. D 82, 024011 (2010).
[32] T. Kobayashi, M. Yamaguchi and J. Yokoyama, Phys. Rev. Lett. 105, 231302 (2010); C. Burrage, C. de Rham, D. Seery and A. J. Tolley, JCAP 1101, 026 (2010); X. Gao, JCAP 1110, 026 (2011); T. Kobayashi, M. Yamaguchi and J. Yokoyama, Phys. Rev. D83, 103524 (2011); S. Renaux-Petel, Class. Quant. Grav. 28, 072001 (2011); S. Renaux-Petel, S. Mizuno and K. Koyama, JCAP 1111, 042 (2011).
[33] M. A. Luty, M. Porrati and R. Rattazzi, JHEP 0309, 029 (2003); R. Geroch, arXiv:1203.4059 [hep-ph].
[34] L. A. Popa, JCAP 1110, 025 (2011).
[35] T. Kobayashi, M. Yamaguchi and J. "i. Yokoyama, Prog. Theor. Phys. 126, 511 (2011).
[36] A. De Felice and S. Tsujikawa, Phys. Rev. D 84, 083504 (2011).
[37] A. De Felice and S. Tsujikawa, JCAP 1202, 007 (2012).
[38] A. De Felice and S. Tsujikawa, Phys. Rev. D 84, 083504 (2011).
[39] A. De Felice and S. Tsujikawa, Phys. Rev. D 84, 083504 (2011).
[40] A. De Felice and S. Tsujikawa, Phys. Rev. D 83, 043515 (2011); C. Deffayet et al., JCAP 1102, 006 (2011). A. Naruko and M. Sasaki, Class. Quant. Grav. 28, 072001 (2011); X. Gao, JCAP 1110, 026 (2011); T. Kobayashi, M. Yamaguchi and J. Yokoyama, Phys. Rev. D83, 103524 (2011); S. Renaux-Petel, Class. Quant. Grav. 28, 182001 (2011) [Erratum-ibid. 28, 249601 (2011)]; S. Renaux-Petel, S. Mizuno and K. Koyama, JCAP 1111, 042 (2011).
[41] M. A. Luty, M. Porrati and R. Rattazzi, JHEP 0309, 029 (2003); K. Hinterbichler, M. Trodden and D. Wesley, Phys. Rev. D 82, 124018 (2010); G. Ellis, R. Maartens and M. A. H. MacCallum, Gen. Rel. Grav. 39, 1651 (2007).
[42] A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis and R. Rattazzi, JHEP 0610, 014 (2006); C. Bonvin, C. Caprini and R. Durrer, Phys. Rev. Lett. 97, 081303 (2006); G. Ellis, R. Maartens and M. A. H. MacCallum, Gen. Rel. Grav. 39, 1651 (2007).
[43] E. Babichev, V. Mukhanov and A. Vikman, JHEP 0802, 101 (2008).
[44] J. P. Bruneton, Phys. Rev. D 75, 085013 (2007); R. Geroch, arXiv:1005.1614 [gr-qc].
[45] J. Evslin and T. Qiu, JHEP 1101, 032 (2011); J. Evslin, JHEP 1203, 009 (2012).
[46] C. Burrage, C. de Rham, L. Heisenberg and A. J. Tolley, JCAP 1207, 004 (2012).
[47] S. W. Hawking, Phys. Rev. D 46, 603 (1992).
[48] W. J. Percival et al. [SDSS Collaboration], Mon. Not. Roy. Astron. Soc. 401, 2148 (2010).
[49] A. G. Riess et al., Astrophys. J. 699, 539 (2009).
[50] J. E. Kim, H. P. Nilles and M. Peloso, JCAP 0501, 005 (2005); S. Dimopoulos, S. Kachru, J. McGreevy and J. G. Wacker, JCAP 0808, 003 (2008); R. Easther and L. McAllister, JCAP 0605, 018 (2006); L. McAllister, E. Silverstein and A. Westphal, Phys. Rev. D 82, 046003 (2010).
[51] L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. D 56, 3258 (1997).
[52] S. Y. Khlebnikov and I. I. Tkachev, Phys. Rev. Lett. 77, 219 (1996); S. Y. Khlebnikov and I. I. Tkachev, Phys. Rev. Lett. 79, 1607 (1997); T. Prokopec and T. G. Roos, Phys. Rev. D 55, 3768 (1997); G. N. Felder and I. Tkachev, Comput. Phys. Commun. 178, 929 (2008).
[53] [Planck Collaboration], astro-ph/0604069.