Laser assisted proton collision on light nuclei at moderate energies

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We present analytic angular differential cross section model for laser assisted proton nucleon scattering on a Woods-Saxon optical potential where the nth-order photon absorption is taken into account simultaneously. As a physical example we calculate cross sections for proton - 12C collision at 49 MeV in the laboratory frame where the laser intensity is in the range of $10^7 - 10^{21}$ W/cm² at optical frequencies. The upper intensity limit is slightly below the relativistic regime.

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I. INTRODUCTION

Nowadays optical laser intensities exceeded the $10^{22}$ W/cm² limit where radiation effects dominate the electron dynamics. In the field of laser-matter interaction a large number of non-linear response of atoms, molecules and plasmas can be investigated both theoretically and experimentally. Such interesting high-field phenomena are high harmonic generations, or plasma-based laser-electron acceleration. This field intensities open the door to high field quantum electrodynamics phenomena like vacuum-polarization effects of pair production \cite{1}. In most of the presented studies the dynamics of the participating electrons are investigated. Numerous surveys on laser assisted electron collisions are available as well \cite{2}. However, there are only few nuclear photo-excitation investigations done where some low-lying first excited states of medium of heavy elements are populated with the help of x-ray free electron laser pulses \cite{3}. Nuclear excitation by atomic electron re-scattering in a laser field was investigated as well \cite{4}. Various additional concepts are under consideration for photo-nuclear reactions by laser-driven gamma beams \cite{5}.

To our knowledge there are no publications available where laser assisted proton nucleus collisions (or radiative proton-nucleus scattering) were investigated. This is the goal of our recent paper. We consider the full optical potential of Woods-Saxon(WS) type \cite{7} with the proper parametrization for moderate energy proton - 12C collision \cite{8}. The optical potential formalism has been a very successful method to study the single particle spectra of nucleus in the last four decades. Detailed description of the validity of this formalism can be found in a nuclear physics textbooks or in monographs like \cite{9, 10}.

The nuclear physics community recently managed to evaluate the closed analytic form of the Fourier transformed WS interaction \cite{11} which is a great success. We incorporate these results into a first Born approximation scattering cross section formula where the initial and final proton wave functions are Volkov waves and the induced photon emission and absorption processes are taken into account up to arbitrary orders \cite{12-18}. We end our study with a physical example where 49 MeV protons are scattered on 12C nuclei in the field of a Titan sapphire laser.

II. THEORY

In the following we summarize our applied non-relativistic quantum mechanical description. The laser field is handled in the classical way via the minimal coupling. The laser beam is taken to be linearly polarized and the dipole approximation is used. If the dimensionless intensity parameter (or the normalised vector potential) $a_0 = 8.55 \cdot 10^{-10} \sqrt{I(W/cm^2)/\lambda(\mu m)}$ of the laser field is smaller than unity the non-relativistic description in dipole approximation is valid. For 800 nm laser wavelength this means a critical intensity of $I = 2.13 \cdot 10^{18}$ W/cm². In case of protons $a_0$ is replaced by $a_p = ((m_p/m_e)^{-1})a_0$, where the proton to electron mass ratio is $(m_p/m_e)=1836$ \cite{19}. Accordingly, for 800 nm wavelength the critical intensity for protons is $I_{crit} = 3.91 \cdot 10^{21}$ W/cm². Additionally, we consider moderate proton kinetic energy, not so much above the Coulomb barrier and neglect the interchange term between the proton projectile and the target carbon protons. This proton exchange effect could be included in the presented model with the help of Woods-Saxon potentials of non-local type \cite{20} but not in the scope of the recent study.

To describe the scattering process of a proton on a nucleus in a spherically symmetric field the following Schrödinger
absorption and emission processes is

\[ \frac{1}{2m} \left( p - \frac{e}{c} A \right)^2 + U(r) \]

\[ \Psi = i \hbar \frac{\partial \Psi}{\partial t}, \]  

(1)

where \( p, \Psi \) are the momentum operator and the wave function of the proton, \( A(t) = A_0 \epsilon \cos(\omega t) \) is the vector potential of the external laser field with unit polarisation vector \( \epsilon \) and \( U(r) \) is the scattering potential. We search the solution in the following form of

\[ \Psi = \varphi_p + \Psi_c, \]  

(2)

where

\[ \varphi_p(r, t) = \frac{1}{(2\pi\hbar)^{3/2}} e^{i (pr - Et)} \exp \left[ -\frac{i p^2 t}{\hbar 2m} - \int_0^t dt' \left( \frac{e p A(t')}{mc} + \frac{1}{2m} \left( \frac{e A(t')}{c} \right)^2 \right) \right] \]  

(3)

is the Volkov wave function of the scattering proton. We note at this point that the \( A^2 \) term drops out from the transition amplitude, because it depends only on time (and does not depend on the proton’s momentum). The correction term is

\[ \Psi_c = \int d^3p a_p(t) \varphi_p, \]  

(4)

where \( a_p(t) \) is the scattering amplitude. Calculating the transition matrix element between the final and the initial states in the first Born approximation for \( a_p(t) \) at \( t \to \infty \) we get

\[ T_{fi} = \langle \varphi_f | U | \Psi_i \rangle = -\frac{U(q)}{(2\pi\hbar)^3} \sum_{n=-\infty}^{\infty} 2\pi i \delta \left( \frac{p_f^2 - p_i^2}{2m} + n\hbar \omega \right) J_n(z), \]  

(5)

where \( U(q) \) is the Fourier transformed of the scattering potential with the momentum transfer of \( q \equiv p_f - p_i \) where \( p_i \) is the initial and \( p_f \) is the final proton momenta. The absolute value is \( q = \sqrt{p_i^2 + p_f^2 - 2p_ip_f\cos(\theta_{p_i,p_f})} \). In our case, for 49 MeV energy protons absorbing optical photons the following approximation is valid \( q \approx 2p_i|\sin(\theta/2)| \).

The Dirac delta describes photon absorptions \( (n < 0) \) and emissions \( (n > 0) \). \( J_n(z) \) is the Bessel function with the argument of

\[ z = \frac{m_e}{m_p} a_0 (\hat{\epsilon}c) \frac{2p_i}{\hbar \omega_0} |\sin(\theta/2)| \]  

(6)

where \( m_e, m_p \) are the electron and proton masses, \( a_0 \) is the dimensionless intensity parameter, (given above), \( \hat{\epsilon} \) and \( c \) are the unit vectors of the momentum transfer and the laser polarisation direction. It can be shown with geometrical means that for low energy photons where \( (E_{ph} < E_{p^+}) \) the angle in the scalar product of \( \hat{\epsilon}c \equiv \cos(\chi) = \frac{\pi}{2} - \theta/2 \) where \( \theta \) is the scattering angle of the proton varying from 0 to \( \pi \).

From \( \frac{E_p}{\hbar \omega_0} = \sqrt{\frac{m_e}{m_p} \frac{2m_e^2 E_p}{\hbar^2 \omega_0^2}} \) collecting the constants together the final formula for \( z \) reads

\[ z = \frac{1.4166 \times 10^{-3}}{\hbar \omega_0} \sqrt{\frac{E_p}{1836} \sqrt{T \times \cos(\chi) \times |\sin(\theta/2)|}}, \]  

(7)

where the laser energy \( \hbar \omega_0 \) is measured in eV, the proton energy \( E_p \) in MeV and the laser intensity \( I \) in W/cm².

The final differential cross section formula for the laser associated collision with simultaneous nth-order photon absorption and emission processes is

\[ \frac{d\sigma^{(n)}}{d\Omega} = \frac{p_f}{p_i} J_n^2(z) \frac{d\sigma_B}{d\Omega}, \]  

(8)

The \( d\sigma_B = (\frac{m_e}{2\pi\hbar})^2 |U(q)|^2 \) is the usual Born cross section for the scattering on the potential \( U(r) \) alone (without the laser field). The expression Eq. 3 was calculated with different authors using different methods [12, 13].
The scattering interaction $U(r)$ is the sum of the Coulomb potential of a uniform charged sphere and a short range optical Woods-Saxon potential [7]

$$U(r) = V_c(r) + V_{ws}(r) + i[W(r) + W_s(r)] + V_{ls}(r)1 \cdot \sigma$$

where the Coulomb term is

$$V_c = \frac{Z_p Z_i e^2}{2R_0} \left( 3 - \frac{r^2}{R_c^2} \right) \quad r < R_c$$

$$V_c = \frac{Z_p Z_i e^2}{r} \quad r \geq R_c$$

where $R_c = r_0 A_1^{1/3}$ is the target radius calculated from the mass number of the target with $r_0 = 1.25 fm$. $Z_p, Z_i$ are the charge of the projectile and the target and $e$ is the elementary charge. This kind of regularised Coulomb potential helps us to avoid singular cross sections and is routinely used in nuclear physics.

The short range nuclear part is given via

$$V(r) = -V_p f_{ws}(r, R_0, a_0)$$

$$W(r) = -V_p f_{ws}(r, R_s, a_s)$$

$$W_s(r) = -W_s(-4a_s) f'_{ws}(r, R_s, a_s)$$

$$V_{ls}(r) = -(V_{so} + iW_{so})(-2) g_{ws}(r, R_{so}, a_{so})$$

$$f_{ws}(r, R, a) = \frac{1}{1 + e^{(r - R)/a}}$$

$$f'_{ws}(r, R, a) = \frac{d}{dr} f_{ws}(r, R, a)$$

$$g_{ws}(r, R, a) = f'_{ws}(r, R, a)/r.$$ (11)

The constants $V_p, W_p, V_{so}$ and $W_{so}$ are the strength parameters, and $a_{0,s,so}, R_{0,s,so}$ are the diffuseness and the radius parameters given for large number of nuclei. As we will see at moderate collision energies the complex terms become zero. In the last part of the present paper we will use the numerical parameters of [8] for proton carbon-collision.

According to the work of Hlophe [11] the complete analytic form of the Fourier transform of the Woods-Saxon potential is available

$$V_s(q) = \frac{V_p}{\pi^2} \left\{ -\pi a_0 e^{-\pi a_0 q} \left[ R_0 (1 - e^{-2\pi a_0 q}) \cos(q R_0) - \pi a_0 (1 + e^{-2\pi a_0 q}) \sin(q R_0) \right] - a_0^3 e^{-\frac{R_0}{a_0}} \left[ \frac{1}{(1 + a_0^2 q^2)^2} - \frac{2 e^{-\frac{R_0}{a_0}}}{(4 + a_0^2 q^2)^2} \right] \right\}. \quad (12)$$

For the $W(q)$ imaginary term, the same expression was derived with $W_v, a_s, R_s$ instead of $V_p, a_0$ and $R_0$. The surface term $W_s(r)$ gives the following formula in the momentum space:

$$W_s(q) = -4a_s W_s \left\{ -\pi a_s e^{-\pi a_s q} \left[ (\pi a_s (1 + e^{-2\pi a_s q}) - \frac{1}{q} (1 - e^{-2\pi a_s q}) \cos(q R_s) + R_s (1 - e^{-2\pi a_s q}) \sin(q R_s) \right] + a_s^2 e^{-\frac{R_s}{a_s}} \left[ \frac{1}{(1 + a_s^2 q^2)^2} - \frac{4 e^{-\frac{R_s}{a_s}}}{(4 + a_s^2 q^2)^2} \right] \right\}. \quad (13)$$

The transformed spin-orbit coupling term leads to

$$V_{ls}(q) = -\frac{a_{so}}{\pi^2} (V_{so} + iW_{so}) \left\{ \frac{2 \pi e^{-\pi a_{so} q}}{1 - e^{-2\pi a_{so} q}} \sin(q R_{so}) + e^{-\frac{R_{so}}{a_{so}}} \left( \frac{1}{1 + a_{so}^2 q^2} - \frac{2 e^{-\frac{R_{so}}{a_{so}}}}{4 + a_{so}^2 q^2} \right) \right\}. \quad (14)$$

where the momentum transfer is defined as above $q \equiv p_i - p_f$. The low energy transfer approximation formula $q \approx 2p_i |\sin(\theta/2)|$ is valid.

The Fourier transform of the charged sphere Coulomb field is also far from being trivial

$$V_c(q) = \frac{Z_p Z_i e^2}{2 \pi \sqrt{\pi} q^3} \left\{ -2 \cdot 3^q q \cos(2^q 3^q q) + 2^q (1 + 2 \cdot 2^q 3^q q^2) \sin(2^q 3^q q) \right\}$$

$$+ 3 Z_p Z_i e^2 \sqrt{\frac{2}{\pi}} \left\{ \frac{i \pi |q|}{2q} - Ci[2^q 3^q q] + \log(q) - \log|q| - iSi[2^q 3^q q] \right\}. \quad (15)$$

where Ci and Si are the cosine and and sine integral functions, respectively.
Figure 1 presents the angular differential cross section in the first Born approximation of the various Woods-Saxon potential terms for 49 MeV elastic proton - $^{12}$C scattering. The different lines represent the different terms (12, 13, 14). The laboratory frame is used in the calculation. For a better transparency the contributions of the regularised Coulomb term is not presented. Our calculated total cross section of the elastic scattering is 201 mbarn which is consistent with the data of [8].

In our case the laser photon energy is $\hbar \omega_0 = 1.56$ eV, which means 800 nm wavelength and the proton energy is $E_i = 49$ MeV. With these values the argument of the Bessel function Eq. (7) becomes the following

$$\begin{align*}
z &= 1.48 \times 10^{-3} \sqrt{I} \times \cos(\chi) \times |\sin(\theta/2)| = I \times \cos(\chi) \times |\sin(\theta/2)|.\end{align*}$$

(16)

Figure 2 presents the angular differential cross section for $n = 0, 1, 2, 3$ photon absorptions for $I = 1$ which means $I = 4.56 \times 10^7$ W/cm$^2$ intensity. The single, double and triple photon absorption total cross sections are 2.26, 1.1 and 0.71 mbar, respectively.

Figure 3 shows the angular differential cross section for $n = 0, 1, 2$ photon absorptions for $I = 100$ which means $I = 4.56 \times 10^{11}$ W/cm$^2$ intensity. Note, that the cross sections for single and double photon absorption are almost the same. The single photon absorption total cross section is 0.5 mbar.
FIG. 2: The calculated angular differential cross sections from Eq. (8) for $I = 4.56 \times 10^7$ W/cm$^2$ laser field intensity ($I = 1$). The thick solid, thin long dashed, thin short dashed and thin solid lines are for $n = 0, 1, 2, 3$ photon absorptions, respectively.

For large laser field intensities, which means large $z$ arguments of the Bessel functions the following asymptotic expansion can be used for a fixed index $[21]$

$$J_n(z) = \sqrt{2/(\pi z)} \cos(z - n\pi/2 - \pi/4).$$

Which means an approximate $1/\sqrt{\sin(\theta)\cos(\theta)}$ angle dependence which and has a strong decay for large scattering angles. Note, that even this function shows very rapid oscillations. Figure 4 shows the same kind of cross sections for for $z = 10000$ (which means $I = 4.56 \times 10^{15}$ W/cm$^2$ intensity) and for $z = 6.61 \times 10^6$ (which means $I = 2.0 \times 10^{21}$ W/cm$^2$ intensity), respectively. Only the $n = 1$ one photon absorption process is considered.

IV. SUMMARY

We presented a formalism which gives analytic angular differential cross section model for laser assisted proton nucleon scattering on a Woods-Saxon optical potential where the nth-order photon absorption is taken into account simultaneously. As an example the proton - $^{12}$Ca collision was investigated at moderate 49 MeV proton energies. The
FIG. 4: The calculated angular differential cross sections for $n = 0$ and photon absorption. The black thin solid line is for $I = 4.56 \times 10^{15}$ W/cm$^2$ intensity ($I = 10000$). The dashed gray line is close to the relativistic threshold with $I = 2.0 \times 10^{21}$ W/cm$^2$ laser field intensity ($I = 6.61 \times 10^6$).

laser intensities vary from from $10^7$ W/cm$^2$ to $10^{21}$ W/cm$^2$. We found that at $10^7$ W/cm$^2$ laser intensities the elastic Born total cross section is a factor of 90 times higher than the same process where a single photon is absorbed. At higher laser field intensities this ratio is even higher. We hope that this study will stimulate laser assisted nuclear scattering collisions which will be done in the Romanian ELI facility.

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