Conditional Eulerian and Lagrangian velocity increment statistics of fully developed turbulent flow

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Conditional statistics of homogeneous isotropic turbulent flow is investigated by means of high-Reynolds number direct numerical simulations performed with 2048\textsuperscript{3} collocation points. Eulerian as well as Lagrangian velocity increment statistics under several conditions are analyzed and compared. In agreement with experimental data longitudinal probability density functions $P(\delta_l^u | \epsilon_i)$ conditioned on a scale-averaged energy dissipation rate are close to Gaussian distributions over all scales within the inertial range of scales. Also transverse increments conditioned on either the dissipation rate or the square of the vorticity have quasi-Gaussian probability distribution functions (PDFs). Concerning Lagrangian statistics we found that conditioning on a trajectory averaged energy-dissipation rate $\epsilon$, significantly reduces the scale dependence of the increment PDFs $P(\delta_t^u | \epsilon_t)$. By means of dimensional arguments we propose a novel condition for Lagrangian increments which is shown to reduce even more the flatness of the corresponding PDFs and thus intermittency in the inertial range of scales. The conditioned Lagrangian PDF corresponding to the smallest increment considered is reasonably well described by the K41-prediction of the PDF of acceleration. Conditioned structure functions show approximately K41-scaling with a larger scaling range than the unconditioned ones.

Keywords: Homogeneous isotropic turbulence, conditional statistics, intermittency

I. INTRODUCTION

The problem of anomalous scaling can be seen as one of the great unsolved problems in turbulence research. The scaling laws of velocity structure functions $S_p(l) = \langle (\delta u)^p \rangle \sim l^{\zeta_p}$ within the inertial range of scales of fully developed turbulence has inspired a lot of publications over the last decades\textsuperscript{1,2}. The velocity increment under consideration is usually either the longitudinal $\delta_l^u = u(x + l) - u(x)$ or the transverse one $\delta_t^u = (u(x + l) - u(x)) \times l$. Both are so called Eulerian increments because the velocity differences are taken over spatial separations at a the same instant of time. Kolmogorov’s K41-theory\textsuperscript{2} implies a linear scaling law $\zeta_p = p/3$ not distinguishing between the two different types of increments mentioned before. However, direct numerical simulations (DNS) and experiments show a deviation of the form\textsuperscript{2,3}

$$\zeta_p = p/3 - \mu_p, \quad (1)$$

with a positive $\mu_p$. The question whether longitudinal and transverse statistics possesses two different sets of scaling exponents $\zeta_l^p$, $\zeta_t^p$ respectively, is still under discussion. Experimental observations\textsuperscript{4,5,6} as well as DNS\textsuperscript{5,6} found slightly smaller scaling exponents for the high-order transverse than for the longitudinal structure functions. It is not yet clear whether these findings are finite-Reynolds number and/or anisotropy effects. In the case of election-MHD turbulence\textsuperscript{7} the differences between longitudinal and transverse scaling exponents were found to decrease with Reynolds-number. One has also to be very careful in the determination of these scaling exponent\textsuperscript{1} as longitudinal and transverse structure functions possess differing scaling ranges.

Guided by Kolmogorov’s refined self similarity hypothesis\textsuperscript{2} (RSH)

$$\delta_l^u = \beta_1 (\epsilon l)^{\zeta_l^u} \quad (2)$$

which states a relation between the local energy dissipation rate

$$\epsilon(x) = \nu \sum_{i,j} [\partial_i u_i(x) + \partial_j u_j(x)]^2 \quad (3)$$

averaged over a scale $l$

$$\epsilon_l = \frac{1}{l} \int_0^l \epsilon(x + s \hat{l}) ds \quad (4)$$

($l$ indicating the same line appearing in $\delta_l^u$ and $\delta_t^u$) and the velocity fluctuation $\delta_l^u$ over that scale Gagne et Ac\textsuperscript{5,6} experimentally measured conditional velocity increment statistics. They found the probability density functions (PDFs) $P(\delta_l^u | \epsilon_l)$ to be nearly Gaussian from the dissipation- up to the integral-scale which implies a linear scaling law with $\mu_p = 0$ in (1). It is believed that anomalous scaling ($\mu_p \neq 0$) in turbulent flows has its origin in small-scale intermittency of the local energy dissipation rate. This point of view is supported by this result, namely that the statistics of increments become

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Gaussian once they are conditioned on a scale-averaged energy dissipation rate.

Recently, new experimental techniques have provoked a renewed interest in Lagrangian statistics\textsuperscript{11–13}. Here, velocity increments are taken along trajectories of fluid elements (tracers). Lagrangian velocity increments are defined by

\[ \delta \tau v_i = v_i(\tau) - v_i(0) = u_i(X(x_0, \tau), \tau) - u_i(X(x_0, 0), 0), \]

where \( X(x_0, \tau) \) denotes the trajectory of a tracer which started at the position \( x_0 \) at time \( t = 0 \). Although one might expect a scaling law of the corresponding structure functions \( S^l_p(l) = \langle |\delta \tau v|^p \rangle \sim \tau^\zeta(l) \) within the temporal inertial range of scales it has not yet been clearly observed\textsuperscript{14,15}. The origin of the strong measured intermittency, the very existence of an inertial range of scales, the corresponding scaling exponents, and their relation to Eulerian intermittency are still open issues\textsuperscript{16–18}.

In this paper we measure conditional velocity statistics both in the Eulerian as well as in the Lagrangian frame of reference by means of high-Reynolds number DNS. We approve the experimental results obtained by Gagne et al\textsuperscript{19} and complement their findings by a detailed scale by scale analysis and an investigation of the statistics of conditioned transverse velocity increments. Furthermore we analyze conditioned Lagrangian increment statistics.

In the next section we briefly present the numerical method. Section \( \text{III} \) presents the results in the Eulerian frame and section \( \text{IV} \) those in the Lagrangian frame of reference. Conclusion are summarized in section \( \text{V} \).

II. NUMERICS

The numerical simulations were performed by solving the incompressible Navier-Stokes equations

\[ \partial_t u + (u \cdot \nabla) u = f - \nabla p + \nu \Delta u \]
\[ \nabla \cdot u = 0, \]

in a periodic cube with a pseudo-spectral method using a high-order exponential cut-off\textsuperscript{20,21}.

We parallelize the computations via a pencil geometry by means of the San Diego P3D-FFT\textsuperscript{22} and explore the BlueGene/P-architecture (the 2048 simulation was performed on 32k processors). The time integration of the velocity field is done by means of a strongly stable Runge-Kutta third order scheme\textsuperscript{23}. In order to maintain a statistically stationary flow a forcing \( f \) is applied which keeps constant the modes of the two lowest Fourier-shells. Averages are taken over several statistically independent realizations of the velocity field.

Once a stationary state has been reached 10 Million tracers are seeded into the flow and integrated according to

\[ \hat{X}(x_0, t) = u(X(x_0, t), t), \]

where \( u(X, t) \) is the velocity obtained from \( \text{II} \).

In order to obtain the velocity at the particle position from the grid values we use a tri-cubic interpolation\textsuperscript{24}. All relevant quantities such as the gradient of velocity are stored in intervals of 1/7th of the dissipation timescale. The main parameters of all simulations are given in Table \( \text{I} \).

III. EULERIAN CONDITIONAL STATISTICS

In this section we present the result on conditioned Eulerian increments statistics. We are going to start with longitudinal increments, followed by transverse increments in the subsequent section.

A. Longitudinal increments

Following the experimental results obtained by Gagne et al\textsuperscript{10} as well as Naert et al\textsuperscript{19}, we split our simulation domain in subsets \( \Omega_l \) of fixed rate of energy dissipation \( \epsilon_l \) on a line \( l \) defined by \( [1] \). On these subsets we consider longitudinal velocity increments \( \delta^l_i \) in order to obtain conditional PDFs \( P(\delta^l_i | u(\epsilon_l)) \). The standard (unconditioned) PDFs can be recovered by integrating this PDF over all \( \epsilon_l \). In agreement with the experimental results we find for a separation \( l \) within the inertial range of scales nearly Gaussian statistics for different dissipation rates (see Fig. \( \text{I} \)). Also for different scales \( l \) we recover Gaussianity (see Fig. \( \text{II} \)). The unconditioned PDFs have clearly flatter tails.

As a measure of the deviation from Gaussianity we present in Fig. \( \text{III} \) the flatness \( \langle (\delta^l_i u)^4 \rangle / \langle (\delta^l_i u)^2 \rangle \). The figure includes the logarithmic derivative of the third-order structure function \( S_3^l(t) \) in order to illustrate the inertial

\[ \begin{align*}
\text{FIG. 1.} & \quad \text{Conditioned PDFs} \quad P(\delta^l_i | u(\epsilon_l)) \quad \text{for different space-averaged dissipation rates} \quad \epsilon_l \quad \text{for} \quad l = 93\eta \quad \text{in comparison to} \quad \epsilon_l = 1 \quad \text{corresponds to the most probable energy dissipation rate, the others of multiples of this rate.} \quad \text{All PDFs are normalized to unit variance.} \end{align*} \]
TABLE I. Parameters of the numerical simulations. $\mathcal{R}_\lambda = \sqrt{15V\ell_L/\nu}$: Taylor-Reynolds number, $u_{\text{rms}}$: root-mean-square velocity, $\epsilon_\lambda$: mean kinetic energy dissipation rate, $\nu$: kinematic viscosity, $dx$: grid-spacing, $\eta = (\nu^3/\epsilon_\lambda)^{1/4}$: Kolmogorov dissipation length scale, $\tau_\eta = (\nu/\epsilon_\lambda)^{1/2}$: Kolmogorov time scale, $L = (2/3E)^{1/2}/\epsilon_k$: integral scale, $T_L = L/u_{\text{rms}}$: large-eddy turnover time, $N^3$: number of collocation points, $N_p$: number of tracer particles.

\begin{tabular}{cccccccccc}
$\mathcal{R}_\lambda$ & $u_{\text{rms}}$ & $\epsilon_\lambda$ & $\nu$ & $dx$ & $\eta$ & $\tau_\eta$ & $L$ & $T_L$ & $N^3$ & $N_p$ \\
\hline
460 & 0.189 & 3.6 $\cdot$ $10^{-3}$ & 2.5 $\cdot$ $10^{-5}$ & 3.07 $\cdot$ $10^{-3}$ & 1.45 $\cdot$ $10^{-3}$ & 0.083 & 1.85 & 9.9 & 2048 & 10$^7$
\end{tabular}

For comparison we conditioned the velocity increments on other quantities composed of velocity-gradient tensor elements, namely the vorticity $\omega = \nabla \times u$ and the longitudinal gradient $\hat{l} \cdot \nabla u$. As for the energy dissipation rate

we consider spatial averages of the square of vorticity

$$\Omega_l = \frac{1}{l} \int_0^l ds \nu |\omega(x + s \hat{l})|^2. \quad (9)$$

and the square of the longitudinal gradient

$$\Delta_{ll}^l = \frac{1}{l} \int_0^l ds \nu |\hat{l} \cdot \nabla u(x + s \hat{l})|^2. \quad (10)$$

From Fig. 3 one recognizes that the flatness of $P(\delta_{ll}^l)(\epsilon_l)$ is closer to a Gaussian distribution over all scales than $P(\delta_{ll}^l)(\Omega_l)$ or $P(\delta_{ll}^l)(\Delta_l)$. It is interesting to remark that the integral of the longitudinal gradient over $l$ is the longitudinal increment. That the energy dissipation rate $\epsilon_l$ nevertheless works better than this longitudinal gradient implies that correlations of the form $\partial_j u_i \partial_i u_j$ with $i \neq j$ are essential in the condition (3) and in the RSH (2).

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energy dissipation rate than to the vorticity. This is important for diverse models such as the She-Lévéque model\(^25\), where physical reasoning is based on the one hand on the energy dissipation rate and the RSH and on the other hand on the dimensionality of the coherent structures of vorticity. This question is also closely related to the issue of different scaling laws for longitudinal and transverse structure functions as we will explain in the next section.

We conclude this section on longitudinal increments by an examination of the corresponding conditioned structure functions \(S_{p,\epsilon_l}\). From the scale-invariant PDFs in Fig. 3 we expect them to follow the linear K41-scaling law \(p/3\) within the inertial range. Indeed, as shown in Fig. 5, the conditioned structure functions follow Kolmogorov’s prediction while the unconditioned higher-order functions exhibit lower plateaus expressed by a non-zero \(\mu_p\) in (1).

**B. Transverse increments**

In analogy to the RSH for the longitudinal velocity increments Chen et al.\(^26\) proposed a refined self-similarity hypothesis for the transverse velocity increments (RSHT). This relation of the scale-averaged square of vorticity and the transverse velocity increments reads

\[
\delta_{l}^\perp u = \beta_2 (\Omega_l)^{\frac{4}{3}},
\]

where \(\beta_2\) is a statistical variable independent of \(l\) and \(\Omega_l\), given by (9).

Following Chen et al., it is reasonable to look in the transverse case at the statistics of velocity increments conditioned to \(\Omega_l\), namely the PDFs \(P(\delta_{l}^\perp u|\Omega_l)\). They are scale-invariant and only slightly flatter than Gaussian PDFs (see Fig. 6). As in the case of longitudinal increments one can ask how other conditions such as the energy dissipation rate perform compared to \(\epsilon_l\). The PDFs conditioned on \(\epsilon_l\) shown in Fig. 7 are indistinguishable from the PDFs conditioned on the vorticity \(\Omega_l\). From this point of view it is impossible to conclude whether \(\epsilon_l\) or \(\Omega_l\) is the better condition for the transverse fluctuations. In order to make a more precise statement on the difference between these two conditions it is helpful to look at the flatness in Fig. 8. From this we conclude that conditioning to \(\epsilon_l\) or \(\Omega_l\) yields quasi-identical results and surprisingly \(P(\delta_{l}^\perp u|\epsilon_l)\) are even slightly more Gaussian than \(P(\delta_{l}^\perp u|\Omega_l)\). The averaged transverse gradient

\[
\Delta_{l}^\perp = \frac{1}{l} \int_{0}^{l} ds \nu \times \nabla u(x + s \hat{l})^2.
\]

reduces the flatness less than the two other conditions. The transverse structure functions are shown in Fig. 9. As expected, we find that the conditioned ones follow the K41 prediction with the inertial range of scale while the high-order unconditioned functions have significantly lower plateaus.
is the correct quantity in [13]. Yu et al. [22] conditioned the velocity increments on a spatially averaged energy dissipation rate at one foot-point of the increments.

In this work we stick to trajectory-averaged conditions and propose yet another one for Lagrangian increment statistics. In order to motivate this on dimensional grounds we recall that Eulerian increments \((u_i(l \tau_j) - u_i(0))/l\) tend to spatial derivatives \(\partial_j u_i\) of the velocity field in the limit \(l \to 0\). Those derivatives appear in the local energy dissipation rate \([3]\). Instead, Lagrangian increments \((u_i(\tau_j) - u_i(0))/\tau\) tend to the fluid-particle acceleration in the limit \(\tau \to 0\) which involve a term \(u_j \partial_j u_i\). We therefore propose to replace (14) by

\[
\epsilon^{L}_\tau = \frac{1}{2} \int dt \sum_{i,j} [u_j \partial_j u_i + u_i \partial_i u_j] \Omega^{2} \tag{15}
\]

in the LRSH [13].

The calculation of \(\epsilon_\tau\), \(\Omega_\tau\), and \(c^{L}_\tau\) for a given time lag \(\tau\) is done by averaging the local quantities over all stored points along the particle trajectory. We achieved converged statistics by taking the average over 10 Million particles and several large-eddy turn-over times.

In Fig. 10 we compare the flatness of velocity increment PDFs conditioned on \(\epsilon_\tau\), \(\Omega_\tau\), and \(c^{L}_\tau\). We added the logarithmic derivative of the second order Lagrangian structure function \(S_2^{L}(l) = \langle (\delta v_i)^2 \rangle\) in order to clarify three different ranges of scales: The dissipative scales up to \(\tau \approx 1\), the inertial ones \(1 < \tau < 60\), followed by the large scales. If we restrict our attention to the inertial range we observe that the flatness is most efficiently reduced by \(c^{L}_\tau\). Also the trajectory integrated energy dissipation rate \(\epsilon_\tau\) diminishes significantly the flatness while the integrated vorticity \(\Omega_\tau\) has a negligible effect. This indicates that \(\epsilon^{L}_\tau\) might be a more appropriate condition than \(\epsilon_\tau\).

The corresponding conditioned increment PDFs are labeled \(P(\delta v_i | \epsilon_\tau)\) and show in Fig. 11. The PDF corresponding to the shortest time-lag considered is reason-
FIG. 11. Conditioned PDFs $P(\delta_v | \epsilon)$ for different time lags $\tau$ in comparison to a Gaussian distribution and to the K41-prediction for the PDF of acceleration, normalized to unit variance

ably well described by the K41-acceleration PDF

$$P(a) = (a/b)^{-5/9} \exp[-0.5 (a/b)^{8/9}/c]$$

(16)

normalized to unit variance with $a = 0.48$ and $b = 2.72$. This PDF is the Lagrangian analogon to a Gaussian distribution for Eulerian velocity gradients.

It is important to note that contrarily to the results in Eulerian setup the conditioned Lagrangian PDFs $P(\delta_v | \epsilon_\tau)$ (see Fig. 11) are still scale-dependent. One notes a transition from stretched tails (K41-prediction) for short time-lags to Gaussian PDFs (uncorrelated statistics) for time lags of the order of the integral time scale. This implies that Lagrangian increment statistics is ‘naturally’ scale dependent.

As can be see from the unconditioned structure function in Fig. 10, Lagrangian structure functions do not show a clear scaling law at today accessible Reynolds numbers. We therefore refer to relative structure functions $S_p(S_2)$. In the computation of the conditioned structure functions we fixed one $\epsilon_\tau$ for all increments $\tau$. In Fig. 12 their logarithmic derivatives are shown which clearly change under the condition $\epsilon_\tau$. There are two major differences between the conditioned and unconditioned functions. The first concerns intermittency: The conditioned functions have larger values than the unconditioned ones. We observe a value of approximately 1.43 which is close to the K41 prediction of 1.5. This implies that intermittency is significantly reduced on subsets $\Omega_{\epsilon_\tau}$. A second feature of Lagrangian increment statistics is the so called bottleneck around a few $\tau_\eta$. It has been attributed to the characteristic trajectories (spirals) of tracers in the vicinity of coherent vortex filaments. This bottleneck in the local slop is absent once velocity increments are conditioned (see again Fig. 12), which means that their scaling range is enlarged. Its origin is supposed to be in the coexistence of two different power-laws. The first related to dissipative effects and the second to inertial range physics.

V. CONCLUSION

This work investigates the statistics of Eulerian and Lagrangian velocity increments when conditioned to different scale-averaged quantities such as the energy dissipation rate, the square of vorticity or the velocity gradient. In the case of Lagrangian increments we propose a novel condition dimensionally related to the acceleration of fluid elements.

Considering Eulerian statistics we find that longitudinal as well as transverse increment PDFs are Gaussian shaped with flatness factors close to three when conditioned to the scale-averaged energy dissipation rate. The averaged vorticity produces slightly flatter tails while the longitudinal and transverse velocity gradient perform significantly worse. Therefore, there is no preferential link of transverse increments and vorticity as of longitudinal increments and energy dissipation rate which is important for models of intermittency. Conditional structure functions show clear K41-scaling within the inertial range of scales.

Considering Lagrangian statistics we investigated velocity increments conditioned to trajectory-averaged quantities such as the energy dissipation rate, the vorticity and a novel condition. The latter is motivated by dimensional arguments. Conditioning to the dissipation rate and to the novel condition yields flatnesses of the increment PDFs much smaller than without conditioning. More precisely, the conditioned PDF of the shortest increment considered agrees reasonably well with the K41-prediction for the PDF of acceleration. Within the inertial range of scales the flatnesses of PDFs under the novel separation of dissipative and inertial scales might lead to the observed dip in the local slope of structure functions. Interestingly, this bottleneck is negligible in the case of conditioned structure functions. This implies that it is due to a mixture of statistics from different subset $\Omega_{\epsilon_\tau}$. 

FIG. 12. Logarithmic derivatives of relative Lagrangian velocity structure function. $\epsilon_\tau = 1$ corresponds to the one with the most statistics.
condition are even smaller than the flatness of PDFs conditioned on the averaged energy dissipation.

Conditioned and unconditioned Lagrangian structure functions differ significantly. First, conditioning yields quasi-K41 scaling exponents. Secondly, the characteristics bottleneck of the unconditioned functions at the onset of the inertial range disappears once conditioned.

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