The funnel annihilations of light dark matter and the Higgs invisible decay

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(Dated: March 4, 2020)

The semi-constrained NMSSM (scNMSSM), or NMSSM with non-universal Higgs masses, can naturally predict a light dark matter under current constraints including Higgs data, sparticles mass bounds, dark matter search, and muon g-2, etc. In this work, we take this scenario of scNMSSM as an example to study the funnel-annihilation mechanism of light dark matter (1\textasciitilde 62 GeV) and Higgs invisible decay. In this scenario we found that: (i) There can be four funnel-annihilation mechanisms for the LSP $\chi_1^0$, which are the $h_2$, Z, $h_1$ and $a_1$ funnel. (ii) For the $h_1$ and $a_1$ funnel with right relic density, the $\chi_1^0$ mass is lighter than 12 GeV, and the Higgs invisible decay can be 2\% at most. (iii) For the $h_2$ and Z funnel with right relic density, the Higgs invisible decay can be about 0.4\% and 1\% respectively at most. (iv) If Higgs invisible decay was discovered at the HL-LHC, the four funnel-annihilation mechanisms of light dark matter may be all excluded with $\chi_1^0$ as the only dark matter source.

I. INTRODUCTION

Dark Matter (DM) is one of the most fascinating mysteries in the universe. Particle dark matter is strongly supported by overwhelming evidence in astrophysical observations [1–3]. Among many dark matter candidates suggested by theorists, the weakly interacting massive particle (WIMP), also known as the thermal dark matter, is an influential one, whose mass can be $O(1)$ MeV $\sim O(100)$ TeV [4–7] and with which the DM relic density is naturally interpreted through the so-called freeze-out mechanism [8, 9], and also the history of the Big Band Nucleosynthesis (BBN) and recombination in the early universe are successfully interpreted. However, present negative DM search results, especially these from direct detection experiments, seriously erode the surviving space of WIMP at the electroweak scale. Thus, it motivates us to focus on lighter DM, especially its annihilation mechanisms that can not be checked by direct searches.

Correlations between Higgs and DM have been widely studied for several tens of years [10–24], especially after the first hint of the 125 GeV Higgs at the end of 2011 [25–73]. If Higgs has interaction with DM and DM mass is lighter than half of the Higgs mass, Higgs invisible decay offers another way to detect DM indirectly. Recently according to the run I and run II data at the LHC, the upper limit of Higgs invisible decay reaches 26\% by ATLAS [74], and 19\% by CMS [75]. While the future accuracy for that can reach to 5.6\%, 0.5\%, 0.24\% and 0.26\% according to the future detections HL-LHC [76], FCC [77], CEPC [78, 79] and ILC [80] respectively. So there is much space to study the nature of light DM from Higgs invisible decay, which we discuss in this work.

Supersymmetry (SUSY) can both predict a natural SM-like Higgs and a light DM candidate, and can also ensure the unification of three gauge interactions, thus has attracted much attention. There are various concrete models and scenarios in the framework of SUSY, among which a scenario in the semi-constrained NMSSM (scNMSSM) can survive with the above advantages under current constraints including Higgs data, sparticles mass bounds, DM search, and muon g-2, etc [81]. In this work, we take this scenario in the scNMSSM as an example, to discuss the annihilation mechanism of light DM from Higgs invisible decay.

The remainder of this paper is organized as follows. In Sec. II we introduce the scNMSSM and relevant analytic calculations briefly. In Sec. III we present the numerical calculations and discussions. Finally, we draw our main conclusions in Sec. IV.

II. THE MODEL AND ANALYTIC CALCULATIONS

The NMSSM extends the MSSM by a singlet superfield $\hat{S}$, which provides an effective $\mu$ term when $\hat{S}$ gets a VEV. The superpotential of the $Z_3$-invariant NMSSM is

$$W_{\text{NMSSM}} = W_Y + \lambda \hat{S}^2 H_u H_d + \frac{\kappa}{3} \hat{S}^3 \quad (1)$$

where the Yukawa term $W_Y$ is the same as that of MSSM, the $H_{u,d}$ denote the doublet superfields, and the $\lambda$, $\kappa$ are the dimensionless couplings. After electroweak symmetry breaking, the doublet and singlet scalar fields mix to generate 3 CP-even Higgses $h_{1,2,3}$ and 2 CP-odd Higgses $a_{1,2}$ respectively, and their superpartners $\tilde{H}_{u,d}$ (higgsino) and $\tilde{S}$ (singlino) mix with gauginos $\tilde{W}$ and $\tilde{B}$, generating 5 neutralinos $\tilde{\chi}_i^0$ ($i = 1, 2, 3, 4, 5$). Hereafter we use $\chi$ to denote the lightest neutralino (LSP) $\chi_1^0$ for convenience.

The scNMSSM is also called the NMSSM with non-universal Higgs masses (NUHM), for it assumes the unification of gauginos and sfermions respectively at the GUT scale, while not that of the Higgs sector [82–87]. In the scNMSSM, the gauginos are constrained to be very heavy, for the high mass bound of gluino and the unification of gauginos at the GUT scale. And because of the large interactions with the SM sector, the higgsino-dominated LSP usually predicts very small relic density. Thus the lightest neutralino $\chi$ predicting right relic density is usually singlino-dominated in the sc-NMSSM. If singlino is lighter than the SM-like Higgs, the...
singlet-dominated scalar is usually also lighter than 125 GeV, thus $h_1$, $a_1$ are usually singlet-dominated scalars, and $h_2$ is usually the SM-like Higgs with $m_{h_2} \approx 125$ GeV. Considering $\chi$ as mixed only by singlino and higgsinos, we can write the couplings between Higgs and $\chi$ as [88]

$$C_{h_0\chi\chi} = \sqrt{2}\lambda(S_{a_0}N_{12}N_{13} + S_{a_1}N_{13}N_{13} + S_{a_2}N_{11}N_{13} + S_{a_3}N_{11}N_{12}) - \sqrt{2}\kappa S_{a_3}N_{13}N_{13},$$

(2)

$$C_{a_0\chi\chi} = i\left[\sqrt{2}\lambda(P_{a_1}N_{12}N_{13} + P_{a_2}N_{11}N_{13} + P_{a_3}N_{11}N_{12}) - \sqrt{2}\kappa P_{a_3}N_{13}N_{13}\right],$$

(3)

where $N_{1i}$ ($i = 1, 2, 3$) are the coefficients of $\tilde{H}_u$, $\tilde{H}_d$ and $\tilde{S}$ in $\chi$ respectively. Similarly, $S_{a_i}$ and $P_{a_i}$ are the coefficients of $\tilde{H}_u$, $\tilde{H}_d$ and $\tilde{S}$ in $h_2$ and $a_2$ respectively. When $\chi$ is singlino-dominated, $h_2$ is SM-like, $h_1$ and $a_1$ are singlet-dominated,

$$C_{h_0\chi\chi} \approx -\sqrt{2}\kappa, \quad C_{a_0\chi\chi} \approx -i\sqrt{2}\kappa,$n

$$C_{h_2\chi\chi} \approx \sqrt{2}\lambda N_{11} - \sqrt{2}\kappa S_{23}.$$  

(4)

When $2m_\chi < m_\phi$, with $\phi = h_2, h_1, a_1$, the invisible width of the SM-like Higgs $h_2$ can be calculated as

$$\Gamma_{\phi \rightarrow \chi\chi} = \frac{C_{\phi\chi\chi}^2 m_\phi}{16\pi} \left(1 - \frac{4m_\chi^2}{m_\phi^2}\right)^{3/2}.$$  

(5)

To calculate the relic density of the LSP $\chi$ with velocity $v$, one has to solve first the number density $n$ from the Boltzmann equation

$$\frac{dn}{dt} = -3Hn - \langle v\sigma_{\text{eff}} \rangle (n^2 - n_{\text{eq}}^2),$$

where $H$ is the Hubble rate, $n_{\text{eq}}$ is the density in thermal equilibrium, $\sigma_{\text{eff}}$ is the total cross section of dark matter annihilation or co-annihilation, and $\langle v\sigma_{\text{eff}} \rangle$ is the thermal average of $v\sigma_{\text{eff}}$. For the $s$-channel annihilation processes, two dark matter particles annihilate to a medium boson $\phi$, and $\phi$ decay promptly into two SM particles, which is also called $\phi$-funnel annihilation [1]. According to perturbation theory, the cross section of $\phi$-funnel annihilation to a pair of fermions $f\bar{f}$ at leading order (LO) can be written as [53]

$$\sigma_{\phi\rightarrow f\bar{f}}^{\text{eff}} = \frac{1}{2} C_{\phi\chi\chi}^2 \frac{s\Gamma_{\phi \rightarrow f\bar{f}} m_\phi}{(s - m_\phi^2)^2 + m_\phi^2 \Gamma_\phi^2} \sqrt{1 - \frac{4m_\chi^2}{s}}.$$  

(7)

where $s$ is the squared total center-of-mass energy, $\Gamma_{\phi}$ and $\Gamma_{\phi \rightarrow f\bar{f}}$ are the total width and partial $f\bar{f}$ width of $\phi$ decay respectively. The total annihilation cross section $\sigma_{\text{eff}}$ is the sum to all channels including different medium particles and different final states. For cold dark matter, whose average velocity is non-relativistic, one can expand $v\sigma_{\text{eff}}$ in powers of $v^2$, obtaining

$$v\sigma_{\text{eff}} \equiv a + bv^2 + \mathcal{O}(v^4)$$

(8)

Then the relic density can be expressed as [89]

$$\Omega_\chi h^2 \equiv \frac{m_\chi n_0 h^2}{\rho_c} = 0.847 \left(\frac{m_\chi}{T_i}\right)^{2/3} \left(\frac{g_*}{100}ight) \frac{10^{-27}\text{cm}^3\text{s}^{-1}}{(v\sigma_{\text{eff}}^{\text{eff}})},$$

(9)

where $n_0$ is the present number density, $g_*$ is the total effective degrees of freedom, $T_i$ is the freezing-out temperature, and $\rho_c = 3H_0^2/(8\pi G_N)$ is the critical density of the universe.

The calculation of the DM relic density in SUSY models are performed detailed in several public codes, e.g., the micrOMEGAs [90–92], where all relevant annihilation and co-annihilation processes are implemented.

III. NUMERICAL CALCULATIONS AND DISCUSSIONS

In this work, we use the result we got in our former work on scNMSSM [81], where we used NMSSMTools-5.5.2 [88, 93, 94] to generate the particle spectrum, using micrOMEGAs-5.0 [90–92] there to calculate the DM relic density and $\chi$-nucleon cross section, imposing the corresponding constrains there, and also used HiggsBounds-5.5.0 [95–98] to constraint the extra Higgses, used SModelS-v1.2.2 [99–105] with official 1.2.2 database to constraint the sparticle sectors. Thus eventually the surviving samples we are using here satisfy the constraints including Higgs data, muon g-2, B physics, sparticle searches, DM relic density and direct searches, etc. As shown in Ref.[81], the particle spectrum of this scenario is characterized by

- In the colored sectors, the gluinos and the first two generation squarks are heavier than 2 TeV, thus $M_{3/2}$ at the GUT scale, or $M_{3/2}/2.4 \approx M_{3/2}/0.8 \approx M_{1/2}/0.4$ at the $M_{\text{SUSY}}$ scale due to RGE runnings, is larger than about 1 TeV. And due to the RGE runnings including $M_3$, even the lightest squark, $\tilde{t}_1$, is also over 700 GeV.

- In the slepton sector, due to the RGE runnings including $M_2$ and $M_1$, the charged sleptons in the first two generations and all sneutrinos are heavier than about 300 GeV, while only $\tilde{\tau}_1$ can be lighter and to about 100 GeV.

- In the chargino sector, $\tilde{\chi}_2^\pm$ are wino-like and heavier than about 800 GeV, while $\tilde{\chi}_1^\pm$ are higgsino-like at 100–200 GeV.

- In the the neutralino sector, $\tilde{\chi}_1^0$ is wino-like and heavier than 800 GeV, $\tilde{\chi}_2^0$ is higgsino-like and heavier than 400 GeV, the two higgsino-dominated neutralinos are $100 \sim 200$ GeV, while the singlino-dominated neutralino can be 1–400 GeV.

- In the Higgs sector, $h_2$ is the SM-like Higgs with mass at $123 \sim 127$ GeV, $h_1$ is singlet-dominated and lighter than 123 GeV, while the light CP-odd Higgs $a_1$
FIG. 1. The surviving samples projected in the $m_{a_1}$ versus $m_\chi$ planes, with colors indicate the relic density $\Omega h^2$ (left), the singlino component $|N_{13}|^2$ of the LSP $\chi$ (middle) and Higgs invisible branching ratio $Br(h_2 \rightarrow \chi \chi)$ (right) respectively. The dashed line is $|\kappa/\lambda| = 125/400 \approx 0.31$. The $\chi$ denotes the lightest neutralino $\tilde{\chi}_1^0$ in this paper.

In Fig.2 we project the surviving samples in the $m_{a_1}$ versus $m_\chi$ planes, with colors indicate the relic density $\Omega h^2$ (left and right) and the invisible branching ratio $Br(a_1 \rightarrow \chi \chi)$ (middle) respectively. The dashed, dash-dotted and dotted lines denote $m_\chi = m_{a_1}, m_{h_1}$ and 125 GeV respectively. In the right panel, only samples with $m_\chi \leq 15$ GeV are projected, and the three axis are $m_{a_1}, m_{h_1}$ and $m_\chi$ respectively. The $\chi$ denotes the lightest neutralino $\tilde{\chi}_1^0$ in this paper.

is also singlet-dominated but can be lighter or heavier than $m_{h_2}$. The Higgs invisible decay caused by $m_\chi \lesssim 62$ GeV is lower than 19% as constrained by the LHC result [74, 75], the Higgs exotic decays caused by $m_{h_1,a_1} \lesssim 62$ GeV is also lower than about 20% for $h_2$ is SM-like.

In the following, we focus on the lightest neutralino (LSP) $\chi$ with mass at $1 \sim 62$ GeV in the scN MSSM: its annihilation mechanisms and relic density, and the Higgs invisible decay into them.

In Fig.1 we project the surviving samples in the $\kappa$ versus $\lambda$ planes, with colors indicate the relic density $\Omega h^2$ and singlino component $|N_{13}|^2$ of the LSP $\chi$ respectively. From Fig.1, we can see that

- For samples with the right relic density, $\lambda \gtrsim 0.3$, $|\kappa/\lambda| \lesssim 0.3$, $\chi$ is highly singlino-dominated, and Higgs invisible branching ratio $Br(h_2 \rightarrow \chi \chi) \lesssim 0.1$.

- For samples with $\lambda \gtrsim 0.3$, there can be large mixing between higgsino and singlino in $\chi$, and the singlino component is inversely proportional to $|\kappa/\lambda|$, while that of higgsino is opposite.

- For samples with $\lambda \gtrsim 0.3$, the relic density of $\chi$ is smaller than 0.02, for the large interaction between singlino and SM-like Higgs caused by large $\lambda$ and/or sizable higgsino component. And also for the same reason, most samples have sizable invisible branching ratio $Br(h_2 \rightarrow \chi \chi) \lesssim 0.1$. Besides, we checked that for most of the samples, the singlet component in $h_2 |S_{23}|^2 \ll 0.1$.

In Fig.2 we project the surviving samples in the $m_{a_1}$ versus $m_\chi$ planes, with colors indicate the relic density $\Omega h^2$ and the invisible branching ratio $Br(a_1 \rightarrow \chi \chi)$ respectively. According to this figure, we can sort the surviving samples into four cases, and combine with Fig.1 we can see that:
• **Case I:** $2m_\chi \approx m_{h_2}$, thus the annihilation mechanism of $\chi$ is $h_2$ funnel, or $h_{SM}$ funnel. This case mainly locates in the $0.05 \lesssim \lambda \lesssim 0.3$ and $|\kappa/\lambda| < 0.3$ region.

• **Case II:** $2m_\chi \approx m_Z$, thus the annihilation mechanism of $\chi$ is $Z$ funnel. This case also mainly locates in the $0.05 \lesssim \lambda \lesssim 0.3$ and $|\kappa/\lambda| < 0.3$ region, but with smaller $m_{\chi} \sim 2\kappa \mu/\lambda$.

• **Case III:** $2m_\chi \approx m_{a_1}$, thus the annihilation mechanism of $\chi$ is $a_1$ funnel. This case can locate in all the surviving region in Fig.1, with $m_\chi$ varying in $1 \sim 62$ GeV, while only in $1 \sim 12$ GeV the $\chi$ can provide right relic density.

• **Case IV:** $2m_\chi \approx m_{h_1}$, thus $\chi$ is $h_1$ funnel. This case can only locate in the $0.05 \lesssim \lambda \lesssim 0.3$ and $|\kappa/\lambda| \ll 0.3$ region, with $m_{\chi} \lesssim 15$ GeV.

From the middle panel, we can also see that when $2m_\chi < m_{a_1}$, the invisible branching ratio $a_1 \rightarrow \chi \chi$ is proportional to $m_\chi$, for $\chi$ and $a_1$ are both from the singlet superfield, and their interaction and $\chi$ mass are both proportional to the parameter $\kappa$.

![Graph](image.png)

**FIG. 3.** The surviving samples projected in the Higgs invisible branching ratio $Br(h_2 \rightarrow \chi \chi)$ versus $m_\chi$ planes, with colors indicate the relic density $\Omega h^2$. The dashed, dot-dashed and dotted line show the current upper limit 10% of Higgs invisible decay [75], future detection accuracy of that at HL-LHC 5.6% [79] and CEPC 0.24% [78] respectively. The $\chi$ denotes the lightest neutralino $\tilde{\chi}^0_1$ in this paper.

In Fig.3 we project the surviving samples in the Higgs invisible branching ratio $Br(h_2 \rightarrow \chi \chi)$ versus $m_\chi$ planes, with colors indicate the relic density $\Omega h^2$. From this figure, we can see that for $h_2$-funnel and $Z$-funnel annihilation of $\chi$ with right relic density, Higgs invisible decay can be about 0.4% and 1% respectively at most. While for $h_1$-funnel and $a_1$-funnel annihilation with right relic density, Higgs invisible decay can be 2% at most, and the corresponding $\chi$ mass is about 12 GeV at most. All the samples with right relic density can not be detected at the HL-LHC, but may be covered at the CEPC.

**IV. CONCLUSIONS**

In this work, we have discussed the funnel-annihilation mechanisms of light dark matter and the Higgs invisible decay in the scNMSSM, which is also called the NMSSM with non-universal Higgs masses (NUHM). We use the scenario we got in our former work on scNMSSM, where we considered the theoretical constraints of vacuum stability and Landau pole, and experimental constraints of Higgs data, muon g-2, B physics, sparticle searches, dark matter relic density and direct searches, etc. In our scenario, the bino and wino are heavy because of the high mass bound of gluino and the unification of gaugino masses at the GUT scale. Thus the lightest neutralino $\chi$, or LSP, can only be singlino-dominated or higgsino-dominated.

Finally, we come to the following conclusions regarding the funnel-annihilation mechanisms of light dark matter and Higgs invisible decay in scNMSSM:

- **There can be four funnel-annihilation mechanisms for the LSP $\chi$, which are the $h_2$, $Z$, $h_1$ and $a_1$ funnel.**

- **For the $h_1$ and $a_1$ funnel with right relic density, the $\chi$ mass is lighter than 12 GeV, and the Higgs invisible decay can be 2% at most.**

- **For the $h_2$ and $Z$ funnel with right relic density, the Higgs invisible decay can be about 0.4% and 1% respectively at most.**

- **If Higgs invisible decay was discovered at the HL-LHC, the four funnel-annihilation mechanisms of light dark matter can be excluded with $\chi$ as the only dark matter source.**

**Acknowledgements.** This work was supported by the National Natural Science Foundation of China (NNSFC) under grant Nos. 11605123.

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