We investigate the incidence of several systematic effects on the $Z'$ discovery limits of the NLC. These include the initial state radiation and the systematic errors due to the imperfect polarization measurement, the finite detector angular resolution and the uncertainty on the integrated luminosity. We focus on three reactions involving leptonic couplings: muon pair production, Bhabha scattering and Möller scattering.

I. INTRODUCTION

An important feature of the NLC is the high degree of polarization which can be obtained for the electron beams. Beam polarizations exceeding 80% are by now routinely obtained at SLAC and are steadily improving. At the NLC a 90% electron polarization seems a quite sensible assumption. For the positron beam, although there are reasonable hopes that some practicable technology may be available by the time the NLC is operating, at present no scheme for polarizing positrons has been proven to be implementable.

Another fascinating feature of the NLC, is the possibility to run it in its four different modes: $e^+e^-$, $e^-\gamma$, $\gamma\gamma$ and $e^-e^-$ [1]. In particular the latter collider mode has been shown to have a very high resolving power in searches for a heavy $Z'$ [2] and studies of its couplings to leptons [3]. This is mainly due to the huge events rates of Möller scattering and the possibility of polarizing both beams. Similar studies have also been performed for the more “bread-and-butter” muon pair-production reaction of the $e^+e^-$ mode of the NLC [4]. To the best of our knowledge, the prospects of Bhabha scattering have never been considered in the context of $Z'$ searches.

We consider here the following three reactions involving the exchange of a $Z'$:

$$e^+e^- \rightarrow \mu^+\mu^-$$  
$$e^+e^- \rightarrow e^+e^-$$  
$$e^-e^- \rightarrow e^-e^-.$$  

In the absence of a $Z'$, their unpolarized cross sections are plotted as a function of the center of mass energy in Fig. 1. These curves are obtained for a polar acceptance angle of ca $10^\circ$. The Bhabha and Möller cross sections exceed those of muon pair-production by more than two orders of magnitude and are thus well suited for precision measurements [5]. As we will demonstrate at hand of this $Z'$ analysis, they are also well adapted to searches for new physics.

There is of course a peak in the $e^+e^-$ reactions if the center of mass energy is sufficient to produce the $Z'$ on mass shell.

II. LAGRANGIAN

There are many extensions of the standard model which predict the existence of extra neutral vector bosons $Z'$. While most searches are performed within the framework of a particular model [6], it was advocated that it is important to perform a model-independent analysis [7]. Otherwise we may miss some unexpected kind of new physics lurking beyond the standard model. We also adopt this point of view here.

For each of the reactions (1–3) we analyze here the discovery prospects of a heavy $Z'$ and study the the influence of four different systematic effects:

- the polarization error
- the angular resolution
- the initial state radiation
- the luminosity error.

Figure 1: Unpolarized standard model cross sections of the three processes (1–3) considered in the text. It is easy to disentangle which curve corresponds to which reaction!

Obviously, close to this pole the $e^-e^-$ mode cannot compete with the $e^+e^-$ mode. But if the $Z'$ mass exceeds the NLC center of mass energy by as little as 20%, the discovery potential of the $e^-e^-$ mode becomes competitive or even superior [3]. We focus here on the situation where the $Z'$ is far off-shell.

For each of the reactions (1–3) we analyze here the discovery prospects of a heavy $Z'$ and study the the influence of four different systematic effects:
charged leptons $\ell = e, \mu, \tau$ can be written

\[
L = e \bar{\psi}_\ell \gamma^\mu (v_{Z'} + a_{Z'} \gamma_5) \psi_\ell Z'_\mu, \tag{4}
\]

where $v_{Z'}$ and $a_{Z'}$ are the vector and axial couplings normalized to the charge of the electron $e$. This interaction mediates both $e^+e^-\rightarrow e^+e^-$ annihilation and $e^-e^-\rightarrow e^-e^-$ scattering.

Three unknown parameters are involved here: the $Z'$ mass $m_{Z'}$, and its vector $v_{Z'}$ and axial $a_{Z'}$ couplings. If the $Z'$ is heavy compared to the collider energy, the couplings and the mass are correlated in such a way that it is difficult to disentangle a large mass from small couplings and vice versa. Asymptotically, for $m_{Z'}^2 \gg s$, the following scaling law applies

\[
v_{Z'}, a_{Z'} \propto m_{Z'} (sL)^{\frac{1}{2}}. \tag{5}
\]

Having determined bounds on the couplings for a given energy, luminosity and $Z'$ mass, it is trivial to use this scaling law to determine the corresponding bounds on the couplings for different energies, luminosities and $Z'$ masses.

In the limit we consider here of a heavy $Z'$, there are thus only two independent parameters we need to deal with. We therefore express the discovery limits to be set by the three reactions (6) in terms of $95\%$ confidence exclusion contours in the $(v_{Z'}, a_{Z'})$ plane, for a fixed $Z'$ mass.

### III. CROSS SECTIONS

For completeness, we give here the differential cross sections of the three processes (6-8) under consideration. As we consider the situation where all intermediate bosons are far off-shell $m_{Z'}^2 \ll s < m_{Z'}^2$, we may neglect the width of the $Z^0$ and $Z'$. In terms of the fine structure constant $\alpha$ and the Mandelstam variables $s, t, u$ we have

\[
\frac{d\sigma(e^+e^- \rightarrow \mu^+\mu^-)}{dt} = \frac{4\pi\alpha^2}{s^2} \times \left[ [RR] \left( \sum_i R_i^2 \frac{u}{s-m_i^2} \right)^2 + \left( \sum_i L_i R_i \frac{t}{s-m_i^2} \right)^2 \right] + [LR] \left( \sum_i L_i R_i \frac{t}{s-m_i^2} \right)^2 \tag{6}
\]

\[
\frac{d\sigma(e^+e^- \rightarrow e^+e^-)}{dt} = \frac{4\pi\alpha^2}{s^2} \times \left[ [RR] \left( \sum_i R_i^2 \left( \frac{u}{s-m_i^2} + \frac{u}{t-m_i^2} \right) \right)^2 + \left( \sum_i L_i R_i \frac{t}{s-m_i^2} \right)^2 \right] + [LR] \left( \sum_i L_i R_i \frac{t}{s-m_i^2} \right)^2 \tag{7}
\]

\[
\frac{d\sigma(e^+e^- \rightarrow e^-e^-)}{dt} = \frac{2\pi\alpha^2}{s^2} \times \left[ [RR] \left( \sum_i R_i^2 \left( \frac{s}{t-m_i^2} + \frac{s}{u-m_i^2} \right) \right)^2 + \left( \sum_i L_i R_i \frac{t}{s-m_i^2} \right)^2 \right] + [LR] \left( \sum_i L_i R_i \frac{t}{s-m_i^2} \right)^2 \tag{8}
\]

The summations are performed over the three intermediate bosons $i = \gamma, Z^0, Z'$. Their chiral couplings are given by

\[
R_\gamma = 1 \quad L_\gamma = 1 \tag{9}
\]

\[
R_{Z^0} = -\frac{\sin\theta_w}{\cos\theta_w} \quad L_{Z^0} = \frac{1-2\sin^2\theta_w}{2\sin\theta_w\cos\theta_w} \tag{10}
\]

\[
R_{Z'} = v_{Z'}, a_{Z'} \quad L_{Z'} = v_{Z'} - a_{Z'}. \tag{11}
\]

The polarization coefficients are defined in terms of the average beam polarizations $P_1$ and $P_2$ by

\[
[RR] = \frac{1+P_1+P_2+P_1P_2}{4} \tag{12}
\]

\[
[LL] = \frac{1-P_1-P_2+P_1P_2}{4} \tag{13}
\]

\[
[LR] = \frac{1-P_1P_2}{2}, \tag{14}
\]

where $P_1$ and $P_2$ run between -1 and 1 for left to right polarized beams.

### IV. METHOD

As the Bhabha (6) and Möller (3) cross sections are large, the systematic errors can easily become far dominant and spoil the advantage provided by the large statistics. The most dangerous systematic error stems from the luminosity measurement. However, the dependence on the luminosity cancels out in the angular distributions, i.e. the differential cross sections normalized to one. We therefore choose for the processes (6-8) to use normalized numbers of events as observables.

In contrast, muon pair-production (6) does not have too large cross sections. Moreover, as this reaction proceeds solely via $s$-channel exchanges, it is very sensitive to a possible $Z'$ tail. It is therefore important to keep the information about absolute rates.
For these reasons, we use in our analysis the following two observables:

\[ e^+e^- \rightarrow \mu^+\mu^- : \quad A_i = n_i \quad (15) \]
\[ e^-e^- \rightarrow e^-e^- , \quad e^+e^- \rightarrow e^+e^- : \quad B_i = n_i/N \quad (16) \]

where \( N \) is the total number of events and \( n_i \) is the number of events within one angular bin, labeled by the index \( i \). These numbers of events are obtained by integrating the differential cross sections

\[ n_i = \int_{\theta_1}^{\theta_2} dy_i \Psi(y_i, \sqrt{s}) \int_{\theta_1}^{\theta_2} dy_2 \Psi(y_2, \sqrt{s}) \]

\[ \int_{\theta_1}^{\theta_2} d\theta \frac{d\sigma(y_1y_2s)}{d\theta} . \quad (17) \]

The integrated luminosity is given by \( \mathcal{L} \).

At this stage we only consider the effect of initial state radiation for the energy spectrum \( \Psi \) of the initial electrons. This means that we ignore the effects of beamstrahlung and the beam energy spread after bunch compression. Even though the initial state radiation should be the dominant contribution to the energy spread, this is by no means sufficient. However, the savings in calculation time are substantial and this should provide a feeling for the importance of these effects. To implement the initial state radiation we use for \( \Psi \) the Kuraev-Fadin spectrum [8].

In the linear approximation the error contours corresponding to 95% confidence levels are given by the quadratic form in \( v_{Z^{'}}^2 \) and \( a_{Z^{'}}^2 \),

\[ \left( v_{Z^{'}}^2 , a_{Z^{'}}^2 \right) \mathbf{W}^{-1} \left( v_{Z^{'}}^2 , a_{Z^{'}}^2 \right) = 6 , \quad (18) \]

where the inverse covariance matrix \( \mathbf{W}^{-1} \) is given by

\[ \mathbf{W}^{-1} = \sum_{\text{polarizations}} \sum_{a=1}^{\text{bins}} \frac{1}{\Delta O_i^2} \left( \frac{\partial O_i}{\partial \epsilon_a} \right) \left( \frac{\partial O_i}{\partial \epsilon_b} \right) \quad (19) \]

and

\[ O_i = A_i, B_i . \quad (20) \]

For each observable [15] the squared errors in the denominator of Eq. (19) are given by the quadratic sum of the statistical and systematic errors

\[ \Delta A_i^2 = n_i + \sum_a \left( \frac{\partial A_i}{\partial \eta_a} \Delta \eta_a \right)^2 \quad (21) \]
\[ \Delta B_i^2 = n_i/N^2 (1 - n_i/N) + \sum_a \left( \frac{\partial B_i}{\partial \eta_a} \Delta \eta_a \right)^2 \quad (22) \]

where the first terms on the right-hand-side are the statistical error. The second terms include the systematic errors on the bin edges \( \theta_1, \theta_2 \), the polarizations \( P_1, P_2 \) and the integrated luminosity \( \mathcal{L} \), all added in quadrature. The systematic errors tend to cancel out for the ratios \( B_i / A_i \). In particular the error on the luminosity cancels out exactly, of course.

It is not a good approximation here, to assume a linear dependence of the observables on the parameters, in this case \( v_{Z^{'}}^2 \) and \( a_{Z^{'}}^2 \). But this simplification considerably accelerates the time needed for computing the error contours. Moreover, our prime interest here is to evaluate the relative importance of the different systematic effects, which should not be significantly influenced by this approximation.

V. RESULTS

We concentrate our numerical analysis on the first stage of the NLC, with a center of mass energy

\[ \sqrt{s} = 500 \text{ GeV} . \quad (23) \]

For the luminosities and their errors we take

\[ \mathcal{L}_{e^+e^-} = 50 \text{ fb}^{-1} \]
\[ \mathcal{L}_{e^-e^-} = 25 \text{ fb}^{-1} \quad (24) \]
\[ \frac{\Delta \mathcal{L}}{\mathcal{L}} = 0.5\% . \]

Since in the LEP1 experiments the errors on the luminosity measurements are approximately 0.3% and are dominated by the theoretical error, this choice is quite conservative for the NLC. The lower luminosity of the \( e^-e^- \) mode is due to the anti-pinching at the interaction point [9].

We do not consider polarized positrons, but we assume for the electron beam polarizations the realistic future values

\[ P_{e^-} = \pm 0.9 \quad \Delta P_{e^-} = 1\% . \quad (25) \]

For the \( e^+e^- \) processes, there are thus only two combinations of polarizations to sum over in Eq. (19): \( e^+e^- \) and \( e^+e^- \). For Möller scattering, there are three possibilities: \( e^-_{L} e^-_{L} \), \( e^+_{R} e^+_{R} \) and \( e^-_{L} e^-_{R} \). We do not consider the third one, since it is not very sensitive to the existence of a \( Z' \) [3].

Since the NLC and LEP detectors are comparable, we may assume an angular coverage and resolution of

\[ |\cos \theta| < 0.985 \quad (26) \]
\[ \Delta \theta = 10 \text{ mrad} . \]

We subdivide this angular range into 10 equal size bins in the cosine of the polar angle. This number is only dictated by the savings in computation time. A larger number of bins would only yield better results. The formal limit of an infinite number of infinitesimal bins is equivalent to a maximum likelihood fit and would yield the Cramér-Rao bound [10].
Finally, to avoid the backgrounds from two-photon processes, for instance, we demand the total energy of the final state lepton pair to be sufficiently close to the center of mass energy

\[ E_{\text{event}} > \sqrt{s} - 10 \text{ GeV} . \]  

(27)

We plot for each reaction (1–3) in Figs 2–4 the \((v_{Z'}, a_{Z'})\) contours defined by the quartic form (18). In the absence of a signal the parameter areas outside the contours are excluded to better than 95% confidence. These results are obtained assuming a \(Z'\) mass of 2 TeV. For other choices the contours approximately scale according to Eq. (5).

Even with our quite conservative assumptions for the systematic errors, the resolving power of muon pair-production (1) remains dominated by the statistical errors. The effect of initial state radiation amounts to an increase by 15% of the lower limit on the observable \(Z'\) couplings.

Bhabha scattering (2) suffers a similar loss of resolving power due to initial state radiation. However, the finite detector angular resolution is now the major source of degradation in accuracy. This is due to the fact that this reaction is most sensitive to the presence of a \(Z'\) in the forward scattering region, where the effect of the angular error is largest. The dependence on the polarization, though, is not a matter of concern.

Møller scattering (3) is mildly sensitive to all systematic effects. In order of increasing importance we have the angular resolution, the error on the polarization and the smearing due to initial state radiation. As for muon pair-production and Bhabha scattering, the latter decreases the sensitivity by no more than 15%.

To gauge the discovery potential of the three reactions (1–3) we plot their 95% confidence discovery contours together in Fig. 5. These contours include the effects of all four systematic sources of accuracy loss we have considered here.

VI. CONCLUSIONS

Most current analysis of \(Z'\) searches at the NLC are performed for lepton or quark pair-production. We have considered here the prospects of Bhabha and Møller scattering and have compared them with muon pair-production. The three reactions...
turn out to have a similar discovery potential. Møller scattering is slightly more performant than the two other processes.

All three reactions are relatively insensitive to systematic effects. The most important degradation of the resolving power is due to the finite angular resolution for Bhabha scattering, and to the initial state radiation for Møller scattering and muon pair-production. The lower discovery limits to be set on the leptonic $Z'$ couplings worsens with respect to the case of an ideal beam and detector by about 15%, 41% and 26% for muon pair-production, Bhabha scattering and Møller scattering respectively.

To perform this study we have used a linear approximation and we have ignored the effects of the beam energy spread and of beamstrahlung. Our results are, therefore, correct only at the qualitative level. It is unlikely, though, that the full calculation will yield significant departures from these conclusions.

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Figure 5: Same as Fig. for the three reactions (1–3). All the systematic effects discussed in the text are included.