Theoretical Research of Roller Enveloping the Side-worm Gear Pairs Drive

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ABSTRACT: Roller enveloping the side-worm gear pairs drive is proposed basing on roller enveloping hourglass worm drive. Roller enveloping the side-worm gear pairs drive means to assembling the roller on the end surface of the worm wheel equally. The hourglass worm tooth surface equation can be deduced by applying spatial meshing theory and differential geometry theory. The hourglass worm numerical models can be concluded by applying Matlab software to finish numerical simulation. At the same time, this theory also focus on analyzing the roller radius $R$, orifice coefficient $K_1$, circumferential angle $\alpha$ of worm gear for the meshing performance of this drive. According to the analysis, it can be concluded as that the roller radius is $5\sim20$mm.

KEYWORDS: Roller enveloping; Side-worm gear drive; Hourglass worm; Meshing performance.

1 INTRODUCTION

Roller enveloping the side-worm gear pairs drive is proposed as an new kind of practical drive pair basing on roller enveloping hourglass worm drive. Roller enveloping the side-worm gear pairs drive means to assembling the roller on the end surface of the worm wheel equally. It not only enjoys the advantages of traditional side-worm drive pairs such as high transmission ratio, compact structure and stable work status, but also includes the advantages of unique high transmission efficiency, continence to be assembled, low cost and etc.

For now, the models expanded through the side-worm gear pairs drive includes: Japanese roller pin worm gear drive and steel cylindrical worm gear with high precision-pc-surface worm pairs of Chongqing machine-tool[9].For the two kinds of drive pairs, the previous one is testing analyze about roller pin worm gear drive efficiency. According the analyze, the efficiency is above 70%. The second one only comes up with an assembly way of roller. It not emphasis the theory research about roller enveloping the side-worm gears pair drive.

Based on the spatial meshing theory and differential geometry theory, this article will deduce the hourglass worm tooth surface equation of roller enveloping the side-worm gear pairs drive, and analyze the meshing performance of this drive. This article can provide theory reference for the drive to be practical.

2 ROLLER ENVELOPING THE SIDE-WORM GEAR PAIRS DRIVE COORDINATE TRANSFORMATION

2.1 Location of coordination system

To establishing coordination system of worm-gear pair like Figure 1. Assuming the worm’s fixed and flexible coordination system as $S_1(O_1,i_1,j_1,k_1^1).S_2(O_1,i_1,j_1,k_1)$ respectively; assuming the worm-gear’s fixed and flexible coordination system as $S_3(O_2,i_2,j_2,k_2).S_4(O_2,i_2,j_2,k_2)$ respectively; $k_1^1$ represents worm’s axis of rotation and $k_2$ means worm gear’s axis of rotation. Then to establishing a coordination system $S_0(O_0,i_0,j_0,k_0)$ which rigidly connected to worm gear, $k_0$ is the direction of axis of rotation of the roller. The point $O_0$’s coordinate is $(a_2,b_2,c_2)$ in coordination system $S_4(O_2,i_2,j_2,k_2); A$ represents center distance of transmission pair, $\alpha$ is circumferential angle of worm gear, $\varphi_1$ is rotating angle of the worm, $\varphi_2$ is rotating angle of the worm wheel, $i_2l$ is the transmission ratio. When $\varphi_1$ and $\varphi_2$ is zero, the fixed and flexible coordinate system of worm are coincident, and the flexible and fixed coordinate system of worm gear are also coincident.

2.2 Transformation of coordinate system

The transformation relationship from coordination system $S_0(O_0,i_0,j_0,k_0)$ which rigidly connected to worm gear to worm’s flexible coordination system
$$S_i(O_{i1},i_{11:j1})$$ can be shown as below:

$$M_{10} = \begin{bmatrix}
D_{11} & D_{12} & D_{13} & D_{14} \\
D_{21} & D_{22} & D_{23} & D_{24} \\
D_{31} & D_{32} & D_{33} & D_{34} \\
D_{41} & D_{42} & D_{43} & D_{44}
\end{bmatrix}$$

(1)

Of which:

$$\begin{align*}
D_{11} &= -\cos(\varphi_1)\cos(\varphi_2)\sin(\alpha) + \cos(\varphi_1)\sin(\varphi_2)\cos(\alpha) \\
D_{12} &= \sin(\varphi_1)\cos(\varphi_2)\sin(\alpha) - \sin(\varphi_1)\sin(\varphi_2)\cos(\alpha) \\
D_{13} &= -\cos(\varphi_1)\cos(\varphi_2) - \sin(\varphi_1)\sin(\varphi_2) \\
D_{14} &= A\cos(\varphi_1) - a_2\cos(\varphi_2)\cos(\varphi_2) + b_2\cos(\varphi_1)\sin(\varphi_2) \\
D_{21} &= \sin(\varphi_1)\cos(\varphi_2)\cos(\alpha) + \sin(\varphi_1)\sin(\varphi_2)\sin(\alpha) \\
D_{22} &= -\sin(\varphi_1)\sin(\varphi_2)\cos(\alpha) - \sin(\varphi_1)\sin(\varphi_2)\sin(\alpha) \\
D_{23} &= -a_2\cos(\varphi_2)\sin(\varphi_2) - A\cos(\varphi_1) - b_2\sin(\varphi_2)\sin(\varphi_2) \\
D_{24} &= D_{12} = D_{33} = D_{34} = 0 \\
D_{44} &= 1
\end{align*}$$

2.3 Angular velocity and relative velocity

Setting the contact point $$P$$ of the worm tooth surface and the roller tooth surface as dynamic coordination $$S_{p}(O_{p},e_{1},e_{2},n)$$, as shown in Figure 2. Then the vector equation of the roller tooth surface in the coordinate system $$S_{p}(O_{p},i_{0},j_{0},k_{0})$$ is as below:

$$\begin{align*}
\mathbf{r}_n &= x_n\mathbf{i} + y_n\mathbf{j} + z_n\mathbf{k} \\
x_n &= R\cos\theta \\
y_n &= R\sin\theta \\
z_n &= u
\end{align*}$$

(2)

In above vector equation, $$R$$ means roller radius , $$\theta$$ and $$\mu$$ are roller surface parameters.

According to the differential geometry theory, the relative velocity vector $$V_{12}$$ of contact point $$P$$ of worm tooth surface and the roller surface in dynamic coordinate is shown as below:

$$\begin{align*}
V_{12} &= V_{12}^e + V_{12}^n \\
V_{12}^e &= -E_1(\cos\theta\sin\alpha + \cos\theta\cos\alpha) + E_2(\cos\theta\sin\alpha - \cos\alpha\sin\theta) \\
V_{12}^n &= E_1 \\
V_{12}^n &= E_2(\cos\theta\sin\alpha - \cos\alpha\sin\theta) + E_3(\cos\theta\sin\alpha + \cos\alpha\cos\theta) \\
E_1 &= z_n\cos(\varphi_1) + i_{12}(x_n\cos\varphi_2 + y_n\sin\alpha + b_2) \\
E_2 &= z_n\sin(\varphi_1) - i_{12}(x_n\sin\varphi_2 - y_n\cos\alpha + b_2) \\
E_3 &= \cos(\varphi_1)(x_n\sin\varphi_2 - y_n\cos\alpha + a_1) - \\
&\quad \sin(\varphi_1)(x_n\cos\varphi_2 + y_n\sin\alpha + b_2)
\end{align*}$$

In above formula: $$V_{12}^e$$, $$V_{12}^n$$, $$V_{12}$$ are casing shadows of elative velocity vector in dynamic coordinate $$S_{p}(O_{p},e_{1},e_{2},n)$$.

According to the meshing theory, the meshing function of the drive pairs can be shown as follow:

$$\begin{align*}
\Phi &= V_{12}^e = B_1\cos(\varphi_1) + B_2\sin(\varphi_1) + B_3 \\
B_1 &= -z_n(\cos\theta\sin\alpha - \cos\alpha\sin\theta) \\
B_2 &= z_n(\cos\alpha\cos\theta + \sin\alpha\sin\theta) \\
B_3 &= i_{12}\left[x_n\cos\varphi_2 + y_n\sin\alpha + b_2\right] - [\cos(\varphi_1)(x_n\sin\varphi_2 - y_n\cos\alpha + a_1) - \\
&\quad \sin(\varphi_1)(x_n\cos\varphi_2 + y_n\sin\alpha + b_2)]
\end{align*}$$

When $$V_{12}^n = 0$$, the meshing equation of worm drive should be:

$$\Phi = V_{12}^n = B_1\cos(\varphi_1) + B_2\sin(\varphi_1) + B_3 = 0$$

(5)

The contact line equation can be calculated by (2), (4), (5) as below:

$$\begin{align*}
r_n &= x_n\mathbf{i} + y_n\mathbf{j} + z_n\mathbf{k} \\
u &= f(\theta,\varphi_1) = \frac{M_{12}}{M_{11}} \\
M_{11} &= i_{12}\left[u_n(\cos\alpha\cos\theta + \sin\alpha\sin\theta) - b_1(\cos\theta\sin\alpha - \cos\alpha\sin\theta)\right] \\
M_{12} &= \cos(\varphi_1)[\cos\alpha\cos\theta + \sin\alpha\sin\theta] - \\
&\quad \cos(\varphi_1)[\cos\alpha\cos\theta - \cos\alpha\sin\theta] \\
\varphi_1 &= \text{const}
\end{align*}$$

The tooth surface equation of hourglass worm can be concluded through transferring the contact line equation to worm flexible coordinate system $$S_{2}(O_{i1},i_{11:j1},k_{1})$$ as be below:
2.4 Numerical simulation of hourglass worm

Parameter selection of this dive pair: \( Z_1 = 1, Z_2 = 40, A = 160, R = 7.5, K_1 = 0.4 \). Through numerical simulation of hourglass worm by applying Matlab software, the left tooth surface, right tooth surface and cylindrical spiral worm can be shown as Figure 3:

3 ANALYSIS OF MESHING PERFORMANCE

3.1 Induced principal curvature

According to document, the induced principal curvature of the drive gear is as below:

\[
\psi = \psi_1 - \psi_2 + \psi_3 - \psi_4
\]

\[
k_{\sigma}^{12} = -k_{\sigma}^{31} = \left( -\frac{\alpha_2^{12} + V_1^{12} / R^2 + (\alpha_1^{12})^2}{2} \right)
\]

\( \Psi \) is function of undercutting limit of gear dive. According to document, the conclusion can be shown:

\[
\Psi = \Phi_1 + \alpha_2^{12} V_1^{12} - \alpha_1^{12} V_2^{12} - (V_1^{12})^2 / R
\]

The induced principal curvature is one key element for the gear drive’s transmission performance. Next analysis will focus on the different impact of roller radius \( R \), circumferential angle \( \alpha \), orifice coefficient \( K_1 \) on induced principal curvature \( k_{\sigma}^{12} \) when worm running under different angle. Result can be shown as Figure 4:
According to analyzing the curve in the Figure. 4, the conclusion can be that: the roller radius \( R \) should less than 20mm and it has little impact on the induced principal curvature within 20mm. The circumferential angle \( \alpha \) also has little impact on induced principal curvature, and the whole trend is that the induced principal curvature will become bigger complying with the circumferential angle \( \alpha \) to be bigger. The orifice coefficient \( K_1 \) is smaller, the induced principal curvature is bigger. And overall, the induced principal curvature is very small, it shows that the meshing performance is very well.

3.2 Lubrication angel

According to document, the lubrication angel of this drive should be shown as below:

\[
\mu = \arcsin \frac{V_1^{(2)} (V_2^{(2)} - \alpha R^{(2)}) + \alpha V_1^{(2)}}{\sqrt{(V_1^{(2)} - \alpha R^{(2)})^2 + (\alpha V_1^{(2)})^2} \sqrt{(V_1^{(2)})^2 + (V_2^{(2)})^2}}
\]

(10)

The Lubrication angel is one important indicate to balance the quality lubrication of worm dive. Next analyze will focus on the different impact of roller radius \( R \), circumferential angle \( \alpha \), orifice coefficient \( K_1 \), on lubrication angel \( u \) when worm running under different angel. Result can be shown as Figure .5:

By analyzing the curve in Figure. 5, the conclusion can be that: The roller radius \( R \) should bigger than 5mm. When the roller radius \( R \) is bigger than 5mm, the lubrication angel is big. When \( \varphi_2 \) is positive , the circumferential angle \( \alpha \) become bigger , the lubrication angel will become bigger; When \( \varphi_2 \) is negative, the circumferential angle \( \alpha \) become bigger, the lubrication angel will become smaller. Impact of orifice coefficient \( K_1 \) can be concluded as that: the lubrication angel will become bigger at first then become smaller during the process of orifice coefficient \( K_1 \) to become bigger.

3.3 Angel of rotation

According to the document, the rotation angel of this drive should be shown as below:
The angel of rotation is one vital indicator to measuring the rotation performance. Continues analyze will focus on the different impact of roller radius $R$, circumferential angle $\alpha$, orifice coefficient $K_1$ to angel of rotation $z_0$. Result can be shown as Figure. 6:

$$z_0 = \arccos \frac{|v_z^2|}{\sqrt{(V_{12}^1)^2 + (V_{12}^2)^2}}$$ (11)

By analyzing the curves in Figure. 6, the conclusion can be that: the angel of rotation will become bigger when the roller radius $R$ become bigger, but the process is slowly. The circumferential angle $\alpha$ has small impact on angel of rotation. The angel of rotation will become smaller gradually within the process of orifice coefficient $K_1$ to become bigger. During the whole analyze in the picture, we can know that angel of rotation of the roller in this kind of model is very small and it’s about 15 degree.

4 CONCLUSION

The tooth surface equation of hourglass worm can be deduced by applying spatial meshing theory and differential geometry theory. The numerical models of hourglass worm can be concluded by applying Matalb software to finish numerical simulation. Result of numerical simulation can reflect real situation of hourglass worm’s contact line and spiral worm. At last, to analyze meshing performance of this drive based on the tooth surface equation of hourglass worm. After comprehensive analyze on the result, the conclusion can be that: the meshing performance and the lubrication performance of this worm dive are very well. The angel of rotation of the roller is very small and it’s about 15 degree. The best roller radius $R$ of the worm drive should within 6-20mm, and the orifice coefficient $K_1$ should large than 0.25. The circumferential angle $\alpha$ has little impact on the meshing performance of this worm drive.

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