\( \varepsilon'_K / \varepsilon_K \) AT NEXT-TO-LEADING IN \( 1/N_C \) AND TO LOWEST ORDER CHPT

JOHAN BIJNENS

Department of Theoretical Physics 2, Lund University
Sölvegatan 14A, S 22362 Lund, Sweden

JOAQUIM PRADES

Departamento de Física Teórica y del Cosmos, Universidad de Granada
Campus de Fuente Nueva, E-18002 Granada, Spain

We report on a calculation of \( \varepsilon'_K / \varepsilon_K \) at next-to-leading order in the \( 1/N_C \) expansion and to lowest order in Chiral Perturbation Theory. We also discuss the chiral corrections to our results and give the result of including the two known chiral corrections.

1 Introduction

Recently, direct CP violation in the Kaon system has been unambiguously established by KTeV at Fermilab and by NA48 at CERN. The present world average is

\[
\text{Re} \left( \frac{\varepsilon'_K}{\varepsilon_K} \right) = (19.3 \pm 2.4) \cdot 10^{-4}. \tag{1}
\]

Recent reviews and predictions for this quantity in the Standard Model and earlier references are in [5]. Here, we report on a calculation of this quantity in the chiral limit and next-to-leading (NLO) order in \( 1/N_C \). We also discuss the changes when the known chiral corrections - final state interactions (FSI) and \( \pi_0 - \eta \) mixing - are included.

Direct CP-violation in the \( K \to \pi \pi \) decay amplitudes is parameterized by

\[
\frac{\varepsilon'_K}{\varepsilon_K} = \frac{1}{\sqrt{2}} \left\{ \frac{A[K_L \to (\pi\pi)_{I=2}]}{A[K_L \to (\pi\pi)_{I=0}]} - \frac{A[K_S \to (\pi\pi)_{I=2}]}{A[K_S \to (\pi\pi)_{I=0}]} \right\}. \tag{2}
\]

\( K \to \pi \pi \) amplitudes can be decomposed into definite isospin amplitudes as

\[
i A[K^0 \to \pi^0 \pi^0] = \frac{a_0}{\sqrt{3}} e^{i \delta_0} - \frac{2}{\sqrt{3}} a_2 e^{i \delta_2},
\]

\[
i A[K^0 \to \pi^+ \pi^-] = \frac{a_0}{\sqrt{3}} e^{i \delta_0} + \frac{a_2}{\sqrt{6}} e^{i \delta_2}. \tag{3}
\]

with \( \delta_0 \) and \( \delta_2 \) the FSI phases.

We want to predict \( a_0 \) and \( a_2 \) to NLO order in \( 1/N_C \) and to lowest order in CHPT.

2 Short-Distance Scheme and Scale Dependence

The procedure to obtain the Standard Model effective action \( \Gamma_{\Delta S=1} \) below the \( W \)-boson mass has become standard and explicit calculations have been performed to two-loops [3]. The full process implies choices of short-distance scheme, regulators, and operator basis. Of course, physical matrix elements cannot depend on these choices.

The Standard Model \( \Gamma_{\Delta S=1} \) effective action at scales \( \nu \) somewhat below the charm quark mass, takes the form

\[
\Gamma_{\Delta S=1} \sim \sum_{i=1}^{10} C_i(\nu) \int d^4x Q_i(x) + \text{h.c.} \tag{4}
\]

where \( Q_i(x) \) are four-quark operators and \( C_i = z_i + \tau y_i \) are Wilson coefficients. In the presence of CP-violation, \( \tau \equiv -V_{td}V_{ts}^\ast /V_{ud}V_{us}^\ast \) gets an imaginary part.

At low energies, it is more convenient to use an effective action \( \Gamma_{\Delta S=1}^{\text{effective}} \) which uses different degrees of freedom. Different regulators and/or operator basis can be more practical too. The effective action \( \Gamma_{\Delta S=1}^{\text{effective}} \) depends on all these choices and in particular on the scale \( \mu_c \) introduced to regulate the divergences, analogous to \( \nu \) in (4) and on effective couplings \( y_i \), which are the equivalent of the Wilson coefficients in (4). Matching conditions between the effective field theories
of $\Pi$ and $\Gamma^L_{\Delta S=1}$ are obtained by requiring that S-matrix elements of asymptotic states are the same at some perturbative scale.

$$\left< 2 \mid \Gamma^L_{\Delta S=1} \mid 1 \right> = \left< 2 \mid \Gamma_{\Delta S=1} \mid 1 \right>.$$  \hspace{1cm} (5)

The matching conditions fix analytically the short-distance behavior of the couplings $g_i$

$$g_i(\mu_c, \cdots) = \mathcal{F}(C_i(\nu), \alpha_s(\nu), \cdots).$$  \hspace{1cm} (6)

This was done explicitly in (3) for $\Delta S = 2$ transitions and used in (4) for $\Delta S = 1$ transitions.

### 2.1 The Heavy $X$-Boson Method

For energies below the charm quark mass, we use an effective field theory of heavy color-singlet $X$-bosons coupled to QCD currents and densities. \hspace{1cm} (7)

For instance, the effective action reproducing

$$Q_1(x) = [\bar{\pi} \gamma^\mu (1 - \gamma_5) d] [\bar{\pi} \gamma^\mu (1 - \gamma_5) u] (x)$$

is

$$\Gamma_X \equiv g_1(\mu_c, \cdots) \int d^4y \; \Pi^\mu_Y \{[\bar{\pi} \gamma^\mu (1 - \gamma_5) d] (x)$$

$$+ [\bar{u} \gamma^\mu (1 - \gamma_5) u] (x) \}. \hspace{1cm} (7)$$

Here the degrees of freedom of quarks and gluons above the scale $\mu_c$ have been integrated out. The advantage of this method is that two-quark currents are unambiguously identified and that QCD densities are much easier to match than four-quark operators.

We use a 4-dimensional Euclidean cut-off $\mu_c$ to regulate UV divergences. We can now calculate $\Delta S = 1$ Green’s functions with the $X$-boson effective theory consistently.

### 3 Long-Distance–Short-Distance Matching

Let’s study the $\Delta S = 1$ two-point function

$$\Pi(q^2) \equiv i \int d^4x \; e^{iq \cdot x} \langle 0 \mid T(p^0 \Gamma_1^\rho(0) P^1_j(x) e^{ip \cdot x} \mid 0 \rangle.$$  \hspace{1cm} (8)

The $P_i$ are pseudoscalar sources with quantum numbers describing $K \rightarrow \pi$ amplitudes.

Taylor expanding the off-shell amplitudes $K \rightarrow \pi$ obtained from these Green’s functions, in external momentum and $\pi$, and $K$ masses, one can obtain the couplings of the CHPT Lagrangian. These predict $K \rightarrow \pi\pi$ at a given order. This is unambiguous.

At leading order in the $1/N_c$ expansion the contribution to the Green function $\Pi(q^2)$ is factorizable. This only involves strong two-point functions and is model independent.

The non-factorizable contribution, is NLO in the $1/N_c$ expansion. It involves the integration of strong four-point functions $\Pi_{P, P, J_a, J_b}$ over the momentum Euclidean $r_E$ that flows through the currents/densities $J_a$ and $J_b$ from $0$ to $\infty$, schematically written as

$$\Pi(q^2) \sim \int \frac{d^4r_E}{(2\pi)^4} \Pi_{P, P, J_a, J_b}(q_E, r_E). \hspace{1cm} (8)$$

We separate long- from short-distance physics with a cut-off $\mu$ in $r_E$. The short-distance part can be treated within OPE QCD.

Recently, it was emphasized that dimension eight operators may be numerically important for low values of the cut-off scale. This issue can be studied straightforwardly in our approach.

There is no model dependence in our evaluation of $K \rightarrow \pi$ amplitudes at NLO in $1/N_c$ within QCD up to now. The long distance part from $0$ up to $\mu$ remains. For very small values of $\mu$ one can use CHPT but it starts to be insufficient already at relatively small values of $\mu$. Too small to match with the short-distance part. The first step to enlarge the CHPT domain is to use a good hadronic model for intermediate energies. We used the ENJL model. \hspace{1cm} (9)

It has several good features - it includes CHPT to order $p^4$, for instance and also some drawbacks as explained in (3). Work is in progress to implement the large $N_c$ constraints on three- and four-point functions along the lines of (3).

### 4 $\epsilon_K$ in the Chiral Limit

To a very good approximation,

$$|\epsilon_K| \simeq \frac{1}{\sqrt{2}} \frac{\text{Re\hspace{0.1cm}a}_2}{\text{Re\hspace{0.1cm}a}_0} \left\{ - \frac{\text{Im\hspace{0.1cm}a}_0}{\text{Re\hspace{0.1cm}a}_0} + \frac{\text{Im\hspace{0.1cm}a}_2}{\text{Re\hspace{0.1cm}a}_2} \right\}.$$  \hspace{1cm} (9)
The lowest order CHPT values for \(\text{Re} a_0\) and \(\text{Re} a_2\) are obtained from a fit to \(K \to \pi\pi\) and \(K \to \pi\pi\pi\) amplitudes to order \(p^4\). Our results reproduce the \(\Delta I = 1/2\) enhancement within 40\%. We use the experimental lowest order CHPT values for \(\text{Re} a_I\) to predict \(\varepsilon_K'/\varepsilon_K\) as shown in Figure 1.

For the two dominant operators and \(\varepsilon_K'/\varepsilon_K\) we obtain at NLO in \(1/N_c\) and in the chiral limit

\[
\begin{align*}
B_{6X}^{(1/2)NDR}(2 \text{ GeV}) &= 2.5 \pm 0.4 \\
B_{8X}^{(3/2)NDR}(2 \text{ GeV}) &= 1.35 \pm 0.20 \\
\left| \frac{\varepsilon_K'}{\varepsilon_K} \right| &= (60 \pm 30) \cdot 10^{-4}. \quad (10)
\end{align*}
\]

5 Higher Order CHPT Corrections

The rôle of FSI in the standard predictions of \(\varepsilon_K'/\varepsilon_K\) has been recently studied. We took a different strategy. The ratio \(\text{Im} a_I/\text{Re} a_I\) has no FSI to all orders, thus FSI only affects the ratio \(\text{Re} a_2/\text{Re} a_0\).

Among the isospin breaking effects only \(\pi^0-\eta\) mixing is under control and is known to order \(p^4\). Other real \(p^4\) and higher electromagnetic corrections are mostly unknown.

Including \(\pi^0-\eta\) mixing and FSI our result (10) becomes

\[
\left| \frac{\varepsilon_K'}{\varepsilon_K} \right| = (34 \pm 18) \cdot 10^{-4}. \quad (11)
\]

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