PROMPT WAYWARDNESS: The Curious Case of Discretized Interpretation of Continuous Prompts

Daniel Khashabi\textsuperscript{1}  Shane Lyu\textsuperscript{2}  Sewon Min\textsuperscript{2}  
Lianhui Qin\textsuperscript{2}  Kyle Richardson\textsuperscript{1}  Sameer Singh\textsuperscript{1,3}  Sean Welleck\textsuperscript{1,2}  
Hannaneh Hajishirzi\textsuperscript{1,2}  Tushar Khot\textsuperscript{1}  Ashish Sabharwal\textsuperscript{1}  Yejin Choi\textsuperscript{1,2}

\textsuperscript{1}Allen Institute for AI  \textsuperscript{2}University of Washington  \textsuperscript{3}University of California-Irvine

Abstract

Fine-tuning continuous prompts for target tasks has recently emerged as a compact alternative to full model fine-tuning. Motivated by these promising results, we investigate the feasibility of extracting a discrete (textual) interpretation of continuous prompts that is faithful to the problem they solve. In practice, we observe a "wayward" behavior between the task solved by continuous prompts and their nearest neighbor discrete projections: We can find continuous prompts that solve a task while being projected to an arbitrary text (e.g., definition of a different or even a contradictory task), while being within a very small (2\%) margin of the best continuous prompt of the same size for the task. We provide intuitions behind this odd and surprising behavior, as well as extensive empirical analyses quantifying the effect of various parameters. For instance, for larger model sizes we observe higher waywardness, i.e., we can find prompts that more closely map to any arbitrary text with a smaller drop in accuracy. These findings have important implications relating to the difficulty of faithfully interpreting continuous prompts and their generalization across models and tasks, providing guidance for future progress in prompting language models.

1 Introduction

Recent work has shown the surprising power of continuous prompts to language models (LMs) for controlled generation and for solving a wide range of tasks (Li and Liang, 2021; Lester et al., 2021; Min et al., 2021). Despite these successes, the resulting continuous prompts are not easy to interpret (Shin et al., 2020). Is it possible to come up with meaningful discrete (textual) interpretations of continuous prompts, especially ones that provide a faithful explanation of the prompt’s behavior?

Towards addressing this question, we propose and investigate the Prompt Waywardness hypothesis (§3.2), a surprising disconnect between the intended behavior of continuous prompts and their nearest-neighbor discrete (language) representations.\textsuperscript{1} In particular, we show that one can find continuous prompts that perform a desired task while, at the same time, projecting to any arbitrary target text. This indicates that there is little correspondence between continuous prompts and their discrete interpretation. For instance, a continuous prompt that effectively solves the sentiment classification task in Fig.1, when projected onto discrete space, might appear as the definition of a different task ("flip the sentiment").

We conduct extensive analysis showing Waywardness on five classification datasets (§4). Empirically, we find the existence of wayward prompts — prompts that solve each of these tasks while having arbitrary natural language projections. To study a variety of projected text, we experiment with 60+ sentences, either a discrete prompt of another task (from Mishra et al. 2021b) or a random sentence

\textsuperscript{1}Nearest-neighbor projection via dot product has been previously used to study properties of continuous word embeddings (Mikolov et al., 2013; Hashimoto et al., 2016) and is commonly performed in the final layer of modern generative LMs (Radford et al., 2019; Raffel et al., 2020).
from a large text corpus. We observe that it is possible to find prompts that project to a given discrete prompt (token overlap 94% F1) while scoring within 2% accuracy of the best continuous prompts-based solution for the task. Further analysis shows that the effect of Waywardness gets worse for larger models and longer prompts. We explain this surprising behavior by relating it to several structural properties of large language models (§5).

We further provide a detailed discussion on the social and research implications of prompt waywardness to guide future research (§6). First and foremost, despite many promising attributes of continuous prompts, interpreting them is non-trivial and will require further research. In fact, careless interpretation of continuous prompts can result in vulnerabilities against malicious attacks concealed under the guise of benign discrete representation. Last but not least, the loose correspondence between continuous and discrete prompts poses a challenge for future research in differentiable interpretable-prompt optimization—optimization in search of human readable discrete prompts through the continuous space. Consequently, continuous and discrete prompts, despite their seeming similarity, are quite different and the results from one may not always transfer to the other. We hope these findings motivate further innovations in the prompting literature for NLP models.

2 Related Work

Continuous prompts. There is a line of work focused on tuning continuous prompts (Li and Liang, 2021; Lester et al., 2021; Zhong et al., 2021; Qin and Eisner, 2021; Zhou et al., 2021). A recurring theme in this line of work is the strength of continuous prompt in results in strong, yet compact models—compared to conventional architecture fine-tuning approaches. Motivated by the success of continuous prompt tuning, this paper investigates the feasibility of interpreting a learned continuous prompt and its connection to discrete prompts.

Discrete prompts. The release of GPT-3 (Brown et al., 2020) sparked a lot of excitement about the emergent ability of LMs in following discrete natural language prompts. Consequently, countless follow-up studies have used manually-designed discrete prompts for probing LMs (Petroni et al., 2019; Jiang et al., 2020), improving LMs few-shot ability (Schick and Schütze, 2021; Gao et al., 2021; Le Scao and Rush, 2021), and their zero-shot ability as well as transferability (Mishra et al., 2021a; Reynolds and McDonell, 2021). While discrete prompts have clear advantages, in addition to being human-readable and thus easily interpretable, we do not have efficient and algorithmic ways of reconstructing them. For example, Shin et al. (2020)’s algorithm discovers discrete prompts, alas the results are not human readable. Prior work also finds that model performance is highly sensitive to small changes in wordings (Mishra et al., 2021a), and optimization over the discrete space is non-trivial and often highly unstable. Our findings here about the disconnect between continuous prompts and their discrete interpretation provides another perspective on the difficulty of discovering discrete prompts via continuous optimizations algorithms that (directly or indirectly) leverage the continuous space (more discussion in §6).

3 Prompt Waywardness

3.1 Preliminaries: Setup and Terminology

Here we define the setup of our study and the notation. We start by defining the space of discrete and continuous prompts (Fig 2). Let \( p_d \in \{0, 1\}^{L \times V} \) denote a discrete prompt represented as an \( L \times V \) matrix, where \( L \) is the length of the sequence of one-hot vectors over a lexicon of size \( V \) (corners of a hyper-cube). Similarly, let \( p_c \in \mathbb{R}^{L \times d} \) denote a continuous prompt, represented as a \( L \)-length sequence of \( d \)-dimensional real vectors.

Projection operators. We define operators that project these two spaces to one another. Define the \( c \)-projection as one that maps discrete inputs to a continuous space by multiplying with a fixed (often pre-trained) embedding matrix\(^2\) \( E \in \mathbb{R}^{V \times d} \):

\[
c-proj(p_d) = p_d E \in \mathbb{R}^{L \times d}.
\]

The \( d \)-projection maps the continuous inputs to nearest neighbor discrete elements, where for each position \( l \) (\( 1 \leq l \leq L \)), one of the possible (and perhaps most straightforward) methods for interpreting a continuous prompt is defined as a projection onto nearest neighbor representations (Mikolov et al., 2013; Hashimoto et al., 2016):

\[
d-proj(p_c) = [\delta_1 \cdots \delta_l \cdots \delta_L] \in \{0, 1\}^{L \times V},
\]

\(^2\)In our experiments we use the embedding matrix of the GPT2 family (Radford et al., 2019) which is used for both mapping input text to their embeddings as well as generating text in the output layer.
where $\delta_t$ is a one-hot vector corresponding to the word with the closest (highest dot product) embedding to the $t$-th position of continuous prompt $p_c$.

These projections are used in the first and last layer of virtually all the modern LMs such as GPT2 (Radford et al., 2019).

### Solving tasks with continuous prompts.

Consider any machine learning model $M$ (typically a pre-trained model) that takes textual input $x$ and produces output $y$. Normally, the parameters of $M$ are learned so as to optimize behavior on a task with a dataset $D = \{(x, y)\}$ of input/output pairs. In prompt tuning (Lester et al., 2021), one freezes the parameters of $M$ and instead optimizes for a prompt $p$ that, when fed in conjunction with $x$, makes $M$ produce the desired output $y$. Thus, $p$ represents the only learnable parameters in this method. When $p$ is a discrete prompt with $k$ tokens, it can be simply concatenated with $x$, denoted $p + x$. In our study, $p$ will be a continuous prompt (of length equal to the embedding of $k$ tokens). We will concatenate it with the embedding of the input $x$. For simplicity, with some abuse of notation, we use $p + x$ to denote concatenation in this continuous case as well.

One can quantify the amount of loss incurred when using a continuous prompt $p$ as follows:

$$\ell(p; D) = \mathbb{E}_{x,y \sim D} [\text{loss}(M(p + x), y)], \quad (3)$$

Minimizing this loss function (empirical risk minimization) over $p$ recovers a minimum risk continuous prompt for this dataset:

$$p^*_c = \arg \min_{p_c \in \mathbb{R}^{L \times d}} \ell(p_c; D^{\text{train}}) \quad (4)$$

Given this prompt, its generalization to the test data can be measured in terms of the loss incurred on the test set: $\ell(p^*_c; D^{\text{test}})$.

### 3.2 The Waywardness Hypothesis

How should one interpret the resultant continuous prompt $\tilde{p}_c$? Empirically, one can easily verify that such continuous prompts are not unique (e.g., random initializations lead to different outcomes). Additionally, the resultant prompts get projected to seemingly irrelevant discrete elements. Taking this to an extreme, we hypothesize that next to the continuous projection $\text{c-proj}(p_d)$ of any discrete prompt $p_d$, there exists a variety of continuous prompts $p_c$ that trigger responses from model $M$ that are orthogonal to the intentions described by the discrete prompt $p_d$. We formalize this idea as the following hypothesis:

**Hypothesis 1 (Prompt Waywardness)** Let $L \in \mathbb{N}$, $M$ be a prompt-based model, and $D$ be a dataset for a desired task. There exists a small $\Delta = \Delta(L, M, D)$ such that for any discrete target prompt $p_d$ with length $L$ and a continuous prompt $\tilde{p}_c \in \mathbb{R}^{L \times d}$: (1) $\tilde{p}_c$ is nearly as effective at making $M$ solve the task as the optimal continuous prompt (Eq. 4), i.e., $|\ell(\tilde{p}_c; D^{\text{test}}) - \ell(p^*_c; D^{\text{test}})| < \Delta$, yet (2) $\tilde{p}_c$ projects to the discrete target prompt $p_d$, i.e., $d\text{-proj}(\tilde{p}_c) = p_d$.

In this statement, $\Delta$ (prompt performance gap relative to the optimal prompt $p^*_c$) is a function of the prompt length $L$, the model $M$ (e.g., its embedding size and depth when $M$ is transformer based), and inherent properties of the target dataset $D$. The analysis in §4.3 will provide an empirical estimate of this gap $\Delta$ as a function of various parameters like model size and prompt length.

It is worth emphasizing that the hypothesis is stated for any task and any set of discrete prompts, even if they are irrelevant or contradictory.\(^3\)

### 3.3 Finding Wayward Prompts

While the above hypothesis promises the existence of $\tilde{p}_c$, it does not say how to discover them. We now discuss a practical method for their discovery.

We learn a continuous prompt $p_c$ using a modification of the prompt tuning objective of Lester et al. (2021). Our modification jointly minimizes

\(^3\)While our focus is on the use of continuous prompts for solving datasets (one prompt shared among many instances), one can imagine applications of the same conjecture to special use cases such as controlled generation (Dathathri et al., 2019) with one prompt per instance.
the standard downstream task cross-entropy loss \( \ell(.) \) for the task (Eq.3) and a distance measure \( \text{dist}(.) \) between \( p_c \) and the discrete target prompt \( p_d \in \{0,1\}^{L \times V} \):

\[
\ell'(p_c; D, \gamma) = \ell(p_c; D) + \gamma \text{dist}(p_c, p_d)
\]

\[
\hat{p}_c = \arg \min_{p_c \in \mathbb{R}^{L \times d}} \ell'(p_c; D, \gamma),
\]

where \( p_c \) is the only learnable parameter, and \( \gamma \) is a hyperparamter.

When \( \gamma = 0 \), the modified objective is reduced to the standard objective (Eq.4), \( \ell'(.) = \ell(.) \). We refer to this case and its resulting prompt \( p'_c \) as the ‘unconstrained’ setting. A large value of \( \gamma \) will make \( p_c \) even closer (possibly identical) to \( c\text{-proj}(p_d) \) but lead to poor accuracy on a target dataset. Most of the experiments below are conducted via a range of \( \gamma \) values to better understand the trade-off between the two terms in the objective function. In practice, we find \( \gamma = 0.01 \) to give a reasonable trade-off regardless of the target dataset and the choice of \( p_d \).

There are at least two natural ways to define the distance measure \( \text{dist}(p_c, p_d) \) between a continuous prompt \( p_c \) and a discrete target prompt \( p_d \), by converting one so that both are in the same space:

\[
c\text{-dist}(p_c, p_d) = \| p_c - c\text{-proj}(p_d) \|_2^2
\]

\[
d\text{-dist}(p_c, p_d) = \text{F1}(d\text{-proj}(p_c), p_d)
\]

The first of these places both \( p_c \) and \( p_d \) in the continuous space and computes the squared-L2 norm, normalized by the prompt length. This is used in our training loss (Eq.5) implementation. The second places both in discrete space (text) and computes the standard word-level token overlap F1 score.\(^4\) This is used during our evaluation.

\begin{center}
\begin{tabular}{l|l|c|c}
\hline
Dataset & Task & |C| & Acc \\
\hline
SST-2 & Sentiment analysis (movie) & 2 & 92.4 \\
SST-5 & Sentiment analysis (movie) & 5 & 50.3 \\
AGNews & News classification (topic) & 4 & 88.1 \\
Subj & Subjectivity classification & 2 & 90.5 \\
TREC & Answer type classification & 6 & 88.0 \\
\hline
\end{tabular}
\end{center}

Table 1: The collection of downstream tasks used in the experiments (§4.1). |C| indicates the output size (number of classes); Acc indicates the unconstrained accuracy of a prompt tuning method (Lester et al., 2021) using GPT2 Large, as a reference point.

\(^4\)Ignoring punctuation marks and articles, and applying lemmatization.

4 Empirical Support of Waywardness

We empirically investigate the Prompt Waywardness hypothesis (§3.2) using our modification (§3.3) of the prompt tuning method from Lester et al. (2021). We show that given an arbitrary and irrelevant discrete prompt \( p_d \), it is possible to learn a continuous prompt that is mapped to \( p_d \) while retaining its accuracy on a given dataset.

4.1 Setup

\textbf{Target tasks.} Following the setup of Min et al. (2021), we select a diverse set of 5 classification datasets: SST-2 (Socher et al., 2013), SST-5 (Socher et al., 2013), AGNews (Zhang et al., 2015), Subj (Pang and Lee, 2004) and TREC (Voorhees and Tice, 2000). Statistics and the unconstrained accuracy of each dataset are provided in Table 1.

\textbf{Discrete Target Projections.} We compile two sets of discrete target prompts: (1) 32 discrete target prompts for solving tasks from Natural-Instructions\(^5\) dataset (Mishra et al., 2021b) that are distinct from and intentionally orthogonal to the end tasks solved here. These were chosen by excluding discrete target prompts that have high lexical overlap with other discrete prompts; this is because we found lexically similar prompts are often semantically similar even when they are for different subtasks. (2) 30 random sentences from The PILE,\(^6\) a large-scale, diverse text corpus that is used to pretrain GPT-J, the largest public causal language model (Wang and Komatsuzaki, 2021). We sampled sentences are drawn from the Poisson distribution with \( \lambda = 14 \), which makes the average length of the sentence to be consistent to those in Natural-Instructions. It is worth emphasizing that these selected discrete texts have almost none with the target prompts. See Table 3 for a few examples.

\textbf{Evaluation metrics.} For all experiments, we report two metrics: (1) the task accuracy\(^7\) as well as (2) prompt F1, the word-level token overlap F1 score computed as in Eq.8, since it easy to interpret and is commonly used for evaluating the textual output of models (Rajpurkar et al., 2016).

\textbf{Models.} For evaluation, we use GPT2 (Radford et al., 2019) an auto-regressive LM which has ex-

\(^5\)https://instructions.apps.allenai.org

\(^6\)https://pile.eleuther.ai

\(^7\)We did not consider alternatives like Macro-F1 because all datasets are roughly balanced across different classes.
which solve task $T$ we also train unconstrained prompts (both $\hat{\Delta}$ and $\hat{p}_c$) to ensure that they have the same size and little accuracy drop. Table 2 reports a summary of the results.

Across all datasets, we find that it is possible to learn a continuous prompt $p_c$ whose discrete projection is very close to $p_d$ and mostly retains the task accuracy. There is a trade-off between the task accuracy and prompt F1, which can be controlled by the choice of $\gamma$ (more extensive ablations in the forthcoming paragraphs (§4.3)). Overall, with $\gamma = 0.01$, it is possible to achieve $\geq 94\%$ prompt F1 with under 2\% relative drop in task accuracy. The only outlier is the TREC dataset where we achieved a prompt F1 score of 86\% for a $\Delta = 2.3\%$ relative drop in accuracy. This is likely because learning an effective continuous prompt on TREC is relatively hard due to its inherent differences with language modeling, as discussed by Min et al. (2021).

Example prompts with varying values of prompt F1 scores are shown in Table 3. Generally, $\geq 94\%$ prompt F1 indicates no semantically meaningful difference; $\geq 90\%$ prompt F1 indicates around 1 word flipped.

### 4.3 Further Analysis

**Effect of Gamma.** Fig. 3 shows the trade-off between task accuracy and the prompt F1 when varying $\gamma$ from 0 to 0.03. As $\gamma$ increases, the task accuracy goes down while the prompt F1 increases. The drop in task accuracy is relatively minor—it is possible to learn a continuous prompt for which prompt F1 is near 1.00 and the accuracy drop relative to the unconstrained accuracy is less than 1\%.

**Effect of Prompt Length ($L$).** We randomly sample sentences from The PILE with a constraint that its length must be $L$ (chosen from $\{4, 7, 14, 28, 56\}$). The left and the middle parts of Fig. 4 illustrate the results. We find that when $L$ is very small (e.g., 4) it is relatively difficult to learn a continuous prompt $p_c$ that is close to $p_d$ (F1 $\leq 60\%$) while retaining the task accuracy. This is likely because the prompt being too short significantly hurts the expressivity of the prompt. Nonetheless, when $L$ is reasonably larger, e.g., 14 (the average length of in Natural Instructions) or longer, all cases lead to a continuous prompt with near 1.00 prompt F1 and little accuracy drop.

### Table 2: Main Results

| Data | $p_d$ Source | Task Accuracy (%) | Prompt $\Delta$ $(\text{Acc}(\hat{p}_c) \rightarrow \text{Acc}(\hat{\Delta}))$ | Prompt F1 (%) |
|------|-------------|-----------------|---------------------------------|---------------|
| SST-2 | NI | 0.7 | (92.4 → 91.8) | 99.0 |
|       | PILE | 0.5 | (92.5 → 92.0) | 97.1 |
|       | Avg | 0.6 | (92.4 → 91.9) | 98.1 |
| SST-5 | NI | 3.3 | (50.2 → 48.5) | 95.9 |
|       | PILE | 0.7 | (50.5 → 50.2) | 92.4 |
|       | Avg | 2.0 | (50.3 → 49.3) | 94.2 |
| AGNews | NI | 1.6 | (88.0 → 86.6) | 97.4 |
|        | PILE | -0.1 | (88.1 → 88.2) | 97.5 |
|        | Avg | 0.8 | (88.1 → 87.3) | 97.4 |
| Subj | NI | 2.0 | (91.3 → 89.5) | 97.3 |
|       | PILE | 0.9 | (89.6 → 88.8) | 94.4 |
|       | Avg | 1.5 | (90.5 → 89.2) | 95.9 |
| TREC | NI | 3.3 | (87.5 → 84.7) | 86.5 |
|       | PILE | 1.2 | (88.5 → 87.5) | 85.6 |
|       | Avg | 2.3 | (88.0 → 86.0) | 86.1 |

Table 2: Main Results: Accuracy of solving five classification datasets, in an unconstrained setting ($p_c^*$) vs. constrained by the projection to various irrelevant pieces of text ($\hat{p}_c$). Optimization is done using $\gamma = 0.01$ in the objective function (Eq. 5). $\Delta$ indicates the relative accuracy drop (in %) from unconstrained accuracy. Each reported score (Accuracy and Prompt F1) are the average over 62 discrete target prompts and 3 random seeds. Overall, it is possible to achieve $\geq 94\%$ prompt F1 with under 2\% drop in accuracy.

---

8While this is different from prior work (Lester et al., 2021; Min et al., 2021) that uses a random subset of the top-5000 vocabs, we find no meaningful differences in an unconstrained accuracy between two initialization methods.
Table 3: Examples of the target prompts \( p_t \) and their reconstructions via \( \delta\text{-proj}(p_c) \) for different ranges of prompt F1 scores. The first \( p_d \) is from Natural-Instructions; the rest two are sampled from The PILE. The mismatches with the original prompt are color-coded.

| Task: AGNews | \( p_d \) | \( \delta\text{-proj}(p_c) \) |
|--------------|----------|-------------------------|
| \text{Write down the conclusion you can reach by combining the given Fact 1 and Fact 2.} | 89.2 | 89.7 |
| \text{Write down the conclusion you can reach by combining the given Fact 1 and Fact 2.} | 88.1 | 88.8 |
| \text{Write down the conclusion you can reach by combining the given Fact 1 Category Fact 2.} | 89.0 | |
| \text{Write Messi in conclusion you can reach by combining the given Fact 1 and Fact 2.} | 88.1 | |

Effect of Model Size. We vary the size of the GPT2 models—small, medium, large, and XL—with 124M, 355M, 774M, and 1.5B parameters, respectively. Figure 5(right) reports the result on SST-2. We find that (1) across different sizes of the LM, our findings in learning continuous prompts with the prompt F1 of near 1.0 and little drop in the accuracy generally hold, and (2) in particular, the drop in accuracy is more negligible with larger LMs (0.2% with XL, 0.5–0.7% with medium and large, 1.2% with small).

Projection onto true task definitions. In all our results so far, the target projected text was orthogonal to the tasks being solved. One might naturally wonder whether there is any benefit in projecting continuous prompts to the texts that truly describe the task being solved, i.e., a “true” prompt for the task. To this end, we manually authored 5 “true” prompts for each of the tasks. We then follow the exact same setup used earlier for Table 2 to fine-tune continuous prompts \( \tilde{p}_c \) for the task while projecting onto these true task definitions. As before, we also fine-tune unconstrained prompts \( p_c^* \) of the same length, without any projection requirement.

By design, \( \tilde{p}_c \) can be no more effective at solving the task than the unconstrained prompt \( p_c^* \) (barring suboptimal search issues), which is what we find in practice. For completeness, we report detailed results for “true” target prompts (analogous to Table 2) in Table 4.

More interestingly, as shown in Table 5, continuous prompts that project to “true” target prompts are no more effective at solving the task than continuous prompts that project to the 62 irrelevant target prompts considered earlier (Table 2). Specifically, the average performance gap \( \Delta \) (relative to unconstrained prompts of the same length) is about the same (around 1.5) for continuous prompts that map to true task definitions.
Figure 4: The effect of the length of the prompt \((L)\) on AGNews. Each point computed via the average over 32 discrete target prompts from Natural Instructions and 3 random seeds \((\gamma = 0.01)\) used. The corresponding prompt F1 is reported as a orange line. The accuracy of \(p^*_c\) and \(\tilde{p}_c\) increase as a function of prompt length, however, the gap between them tends to decrease. The relative accuracy drop is marginal when \(L\) is not too small (e.g. 7 or larger).

Figure 5: The effect of the size of the model—small, medium, large, and XL—on SST-2. Each point in the experiment is computed by averaging over 30 experiments each with a different discrete target prompt from PILE and 3 random seeds. We vary \(\gamma = \{0.01, 0.005, 0.003\}\) and choose the one for which prompt F1 is larger than 0.98. The relative accuracy drop (gap between the two trends) decreases as models become larger.

Table 4: Accuracy of solving five classification datasets, unconstrained setting \((p^*_c)\) vs. constrained by the projection to the true definition of tasks \((\tilde{p}_c)\) using \(\gamma = 0.01\) in the objective function (Eq.5). Subscript \(T\) in \(\Delta_T\) denotes this being the case for true task definitions. The mapping between continuous and discrete space is not one-to-one. The mapping between the space of discrete input and that of word embeddings (Fig.2) is not a bijection. While a discrete target prompt is mapped to exactly one continuous prompt (via its embedding, Eq.1), the reverse is not true: except for some unnatural or rare cases (as formalized in the following propositions) there are (infinitely) many continuous prompts that project back to a fixed discrete target prompt (Eq.2).

Nearest-neighbor projections are arguably natural, computationally efficient, and useful in practice. Although we have considered them in the Euclidean space so far, they can be defined for an arbitrary distance metric\(^9\) \(m\) on \(\mathbb{R}^d\). As before, consider an embedding of a lexicon of size \(V\) into \(\mathbb{R}^d\) and the corresponding one-hot vectors in \(\{0, 1\}^V\). We call \(d\)-proj a nearest-neighbor projection operator w.r.t. \(m\) if it maps each \(x \in \mathbb{R}^d\) to the one-hot vector in \(\{0, 1\}^V\) that corresponds to

\(^9\)https://en.wikipedia.org/wiki/Metric_(mathematics)

Table 5: Task accuracy gap comparison between unconstrained prompts and those fine-tuned to project to a true task definition \((\Delta_T)\) as reported in Table 4. For comparison, we also show the corresponding performance gaps with irrelevant \((\Delta)\) from Table 2. The average performance gaps are about the same (around 1.5) for true and irrelevant target prompts—further evidence that continuous prompts don’t relate to the task being solved.

5 Explaining Waywardness

Here we provide intuitions behind the factors that enable Prompt Waywardness.

The mapping between continuous and discrete space is not one-to-one. The mapping between
The remaining question is, how is this region able to respond to a fixed discrete representation (Fig. 6). A whole region of continuous prompts that correspond to a fixed discrete token.

A proof is included in the Appendix. In effect, the projection operators induce a clustering of the space of continuous prompts $\mathbb{R}^{d \times E}$ into regions that have the same discrete projection (Fig. 6).

The infinite-to-one mapping aspect is not limited to the class of nearest-neighbor projection operators. It is rather an inherent property of the interaction between continuous and discrete spaces, and holds for a broader family consisting of all but a negligible portion of possible projection operators:

**Proposition 1** Every nearest-neighbor projection operator, under any metric, maps infinitely many elements of $\mathbb{R}^d$, forming one or more continuous subspaces, to every one-hot vector in $\{0,1\}^V$.

The above propositions indicate that there is a whole region of continuous prompts that correspond to a fixed discrete representation (Fig. 6). The remaining question is, how is this region able to have a diverse set of prompts that can solve a variety of tasks?

**Deep models give a lot of expressivity power to the earlier layers.** The deeper a network is, the more expressivity it has with respect to its inputs (Telgarsky, 2016; Raghu et al., 2017). Since continuous prompts reside just before the first layer, they enjoy a lot of expressivity. Therefore, no matter how narrow the regions corresponding to individual tokens are (Fig. 6), they are extremely powerful in solving a variety of tasks. Previously in §4.2 we provide an empirical analysis showing evidence that the effect of Waywardness is stronger in deeper models.

## 6 Implications of Prompt Waywardness

We discuss the implications of these findings on several inter-related lines of research. Note that all the following statements are valid within the boundaries of the existing architectures. Moving beyond these barriers likely requires major innovations in terms of LM architectures or how continuous prompts are optimized.

**Faithful interpretation of continuous prompts is difficult.** Given the intuitions behind and empirical support for the Waywardness hypothesis (§5), faithful discrete interpretations of continuous prompts via common discrete projections (like nearest-neighbor projection) are unlikely to be robust, based on current approaches. It is an open question whether there is a better way of interpreting continuous prompts with human language, or whether explaining continuous prompts with human language is inherently impossible because they lie in completely different spaces. Future work may investigate more on this topic in order to improve the interpretability of prompt-based language models.

**Risk of interpreting continuous prompts: concealed adversarial attacks.** It is not difficult to imagine a future where proprietary model development is driven by fine-tuned continuous prompts. In such a world, not addressing the challenges involved in discrete interpretation of continuous prompts can lead to harmful (and potentially, adversarial) consequences (Slack et al., 2020; Wallace et al., 2021), as discussed below.

We consider the following scenario: a model designer comes up with a set of continuous prompts that solve a target task (e.g., ranking resumes according to each applicant’s qualifications and merits). Whether intentionally or not, such prompts may maliciously target, for example, a minority group. To assure their customers, the model designer uses the projection of the prompt that expresses a benign definition for the task, which does not reveal the true nature of the egregious behavior.
The customers might even evaluate the prompt on a few instances but not notice this harmful behavior, e.g., when it effects a minority group not in the evaluation set. In a way, the benign discrete projections may provide a false sense of security.

**Continuous differentiable optimization in search of discrete human-readable prompts can lead to degenerate solutions.** Manually-written discrete prompts have many nice properties (Schick and Schütze, 2021; Le Scao and Rush, 2021; Mishra et al., 2021a), but we don’t yet know how to automatically and efficiently find them. Is there a way to use continuous and differentiable optimization on top of auto-regressive LMs like GPT (Radford et al., 2019) to find human readable prompts? We hypothesize that Waywardness poses a challenge toward this goal.

A differentiable prompt optimization can be described as an optimization in the space of embeddings $p_c \in \mathbb{R}^d$ such that:

$$
\max_{p_c \in \mathbb{R}^d} \text{utility}(D|p_c) \times \text{readability}(d\text{-proj}(p_c)),
$$

which is a joint optimization on a utility objective (the extent to which it can solve a dataset) and a human readability objective. According to the Waywardness hypothesis, there are many $p_c$’s that assign high mass to the utility (high $\mathbb{P}(D|p_c)$) while also mapping to human interpretable text (high $\mathbb{P}(d\text{-proj}(p_c))$) that is irrelevant (or even contradictory) to the task solved by the prompt (hence, degenerate solutions).

We hypothesize that this challenge holds for many reformulations of this objective. For example, consider an optimization that, instead of continuous prompts, is done in the space of word probabilities (e.g., see Kumar et al. (2021, Sec 2.2)). Searching in the space of word probabilities is analogous to a search in a word embedding space.\textsuperscript{10}

In summary, Waywardness poses a clear challenge for mapping an embedding space into meaningful and readable textual representation that is faithful to its output behavior. Nonetheless, it may be possible to use additional signals to alleviate these challenges. For example, several existing works have tackled variants of this problem (abductive generation for language inference (Qin et al., 2020) and for sub-question generation (Khot et al., 2021)) by employing a variety of constraints as additional, domain-specific signals. We hope to see more design innovations for further progress.

**Gradients alone are insufficient to reverse engineer a model.** Suppose we are given a fixed (fine-tuned or otherwise) model $M$ (e.g., an open question-answering model) and an expected output $y$ from this model (e.g., $y$ = “Joe Biden”). Can we use gradients alone to generate a semantically meaningful input question that makes the model $M$ generate this given answer? (without any additional assumptions on the input). More formally, if $q \in [0, 1]^{L \times V}$ is a probability distribution over all questions of length $L$, are gradients with respect to the question input, $\partial M(c\text{-proj}(q))=y\otimes q$, alone informative enough to move us towards the best human readable input that is faithful to the task being solved by $M$?

Our findings and the earlier argument about continuous differentiable optimization suggests this may not be feasible with current methods. To see the correspondence to Prompt Waywardness, we can replace $D$ in Eq.9 with the desired outcome $y$ and run the optimization over word distributions (cf. Footnote 10). While gradients can help guide us towards some input that makes $M$ produce $y$, such input is quite likely to not be faithful to the task being solved by $M$. In the context of the above example ($M$ being a QA system), gradients might lead to inputs that are perhaps linguistically fluent but are neither proper queries nor semantically descriptive of “Joe Biden”.

Nevertheless, as noted earlier, gradients are still useful when they are applied using domain-specific constraints. For example, one can find local (word-level) perturbations that lead to a certain adversarial outcome, if the perturbations are restricted to well-

\textsuperscript{10} A distribution over words $p \in [0, 1]^V$ corresponds to a continuous prompt $p_c = c\text{-proj}(p)$ which is a linear combination of $V$-many basis vectors $\sum_{i=1}^V p[i]E[i]$ (weighted combination of word embeddings) that span $\mathbb{R}^d$. 

Figure 7: Waywardness implies that continuous prompts can be mapped to seemingly innocuous descriptions while the underlying model is acting maliciously.
defined semantic categories (e.g., “blue” can be perturbed to any other color name) (Guo et al., 2021; Yuan et al., 2021).

**Continuous prompt tuning does not necessitate task-specific initialization.** Several recent works on continuous prompt-tuning have shown the effectiveness of initialization from embeddings of random popular words (Lester et al., 2021; Min et al., 2021). What is surprising is that many of these random words are irrelevant to the task solved by these prompts. This, however, makes sense given the observations made in this work regarding the existence of effective prompts around word embedding subspaces.

### 7 Conclusion

The prompting literature has seen many recent developments both in terms of continuous and discrete prompts. This work introduces the Prompt Waywardness hypothesis, which expresses a surprising disconnect between continuous and discrete prompts: for *any* discrete target prompt $p_d$, there exists a continuous prompt that projects to $p_d$ while achieving strong performance on a given downstream task. We provided empirical evidence for this hypothesis, studied various parameters around it, and ended with several implications of this hypothesis.

While our experiments are done on the GPT family, we expect many of our findings to apply to a broader set of architectures that, in one way or another, use similar mechanisms for mapping discrete elements to their continuous representation (Eq.1) and vice versa. Similarly, our projection to the discrete space (Eq.2) is a popular operator in the field (cf. Footnote 1). Furthermore, the intuition explained in Propositions 1 and 2 contends that the behavior likely holds for a broad class of projection operators.

Prompt Waywardness poses a challenge for future progress on the algorithmic discovery of human readable prompts that are faithful to the task they solve. We hope the observations made in this work motivate architectural innovations that overcome such challenges and guide future steps in the prompting literature.

### Acknowledgment

The authors are thankful to Nicholas Lourie and Vered Shwartz for helpful conversations and the Beaker team at AI2 for their support with the experiments. This work was supported in part by DARPA MCS program through NIWC Pacific (N66001-19-2-4031), DARPA SemaFor program, and Google Cloud Compute.

### References

- Tom B Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, et al. 2020. Language models are few-shot learners. *arXiv preprint arXiv:2005.14165*.
- Sumanth Dathathri, Andrea Madotto, Janice Lan, Jane Hung, Eric Frank, Piero Molino, Jason Yosinski, and Rosanne Liu. 2019. Plug and play language models: A simple approach to controlled text generation. In *Proceedings of ICLR*.
- Tianyu Gao, Adam Fisch, and Danqi Chen. 2021. Making pre-trained language models better few-shot learners. In *Proceedings of ACL*.
- Chuan Guo, Alexandre Sablayrolles, Hervé Jégou, and Douwe Kiela. 2021. Gradient-based adversarial attacks against text transformers. *arXiv preprint arXiv:2104.13733*.
- Tatsunori B Hashimoto, David Alvarez-Melis, and Tommi S Jaakkola. 2016. Word embeddings as metric recovery in semantic spaces. *TACL*, 4:273–286.
- Zhengbao Jiang, Frank F Xu, Jun Araki, and Graham Neubig. 2020. How can we know what language models know? *TACL*, 8:423–438.
- Teven Le Scao and Alexander M Rush. 2021. How many data points is a prompt worth? In *Proceedings of NAACL*, pages 2627–2636.
- Brian Lester, Rami Al-Rfou, and Noah Constant. 2021. The power of scale for parameter-efficient prompt tuning. *arXiv preprint arXiv:2104.08691*.
- Xiang Lisa Li and Percy Liang. 2021. Prefix-tuning: Optimizing continuous prompts for generation. In *Proceedings of ACL*.
- Tomas Mikolov, Ilya Sutskever, Kai Chen, Greg S Corrado, and Jeff Dean. 2013. Distributed representations of words and phrases and their compositionality. In *Proceedings of NeurIPS*, pages 3111–3119.
Sewon Min, Mike Lewis, Hannaneh Hajishirzi, and Luke Zettlemoyer. 2021. Noisy channel language model prompting for few-shot text classification. *arXiv preprint arXiv:2108.04106*.

Swaroop Mishra, Daniel Khashabi, Chitta Baral, Yejin Choi, and Hannaneh Hajishirzi. 2021a. Reframing instructional prompts to gptk’s language. *arXiv preprint arXiv:2109.07830*.

Swaroop Mishra, Daniel Khashabi, Chitta Baral, and Hannaneh Hajishirzi. 2021b. Cross-task generalization via natural language crowdsourcing instructions. *arXiv preprint arXiv:2104.08773*.

Bo Pang and Lillian Lee. 2004. A sentimental education: Sentiment analysis using subjectivity summarization based on minimum cuts. In *ACL*.

Fabio Petroni, Tim Rocktäschel, Sebastian Riedel, Patrick Lewis, Anton Bakhtin, Yuxiang Wu, and Alexander Miller. 2019. Language models as knowledge bases? In *Proceedings of EMNLP*, pages 2463–2473.

Guanghui Qin and Jason Eisner. 2021. Learning how to ask: Querying lms with mixtures of soft prompts. In *Proceedings of NAACL*, pages 5203–5212.

Lianhui Qin, Vered Shwartz, Peter West, Chandra Bhagavatula, Jena D Hwang, Ronan Le Bras, Antoine Bosselut, and Yejin Choi. 2020. Backpropagation-based decoding for unsupervised counterfactual and abductive reasoning. In *EMNLP*.

Alec Radford, Jeffrey Wu, Rewon Child, David Luan, Dario Amodei, Ilya Sutskever, et al. 2019. Language models are unsupervised multitask learners. *OpenAI blog*, 1(8):9.

Colin Raffel, Noam Shazeer, Adam Roberts, Katherine Lee, Sharan Narang, Michael Matena, Yanqi Zhou, Wei Li, and Peter J Liu. 2020. Exploring the limits of transfer learning with a unified text-to-text transformer. *JMLR*, 21:1–67.

Maithra Raghu, Ben Poole, Jon Kleinberg, Surya Ganguli, and Jascha Sohl-Dickstein. 2017. On the expressive power of deep neural networks. In *Proceedings of ICML*, pages 2847–2854.

Pranav Rajpurkar, Jian Zhang, Konstantin Lopyrev, and Percy Liang. 2016. SQuAD: 100,000+ questions for machine comprehension of text. In *Proceedings of EMNLP*, pages 2383–2392.

Laria Reynolds and Kyle McDonell. 2021. Prompt programming for large language models: Beyond the few-shot paradigm. In *Proceedings of CHI*, pages 1–7.

Timo Schick and Hinrich Schütze. 2021. Exploiting cloze-questions for few-shot text classification and natural language inference. In *Proceedings of EACL*, pages 255–269.

Taylor Shin, Yasaman Razeghi, Robert L Logan IV, Eric Wallace, and Sameer Singh. 2020. Eliciting knowledge from language models using automatically generated prompts. In *Proceedings of EMNLP*, pages 4222–4235.

Dylan Slack, Sophie Hilgard, Emily Jia, Sameer Singh, and Himabindu Lakkaraju. 2020. Fooling lime and shap: Adversarial attacks on post hoc explanation methods. In *Proceedings of the AAAI/ACM Conference on AI, Ethics, and Society*, pages 180–186.

Richard Socher, Alex Perelygin, Jean Wu, Jason Chuang, Christopher D Manning, Andrew Y Ng, and Christopher Potts. 2013. Recursive deep models for semantic compositionality over a sentiment treebank. In *Proceedings of EMNLP*.

Matus Telgarsky. 2016. Benefits of depth in neural networks. In *Proceedings of COLT*, pages 1517–1539.

Ellen M Voorhees and Dawn M Tice. 2000. Building a question answering test collection. In *Proceedings of SIGIR*.

Eric Wallace, Tony Zhao, Shi Feng, and Sameer Singh. 2021. Concealed data poisoning attacks on NLP models. In *Proceedings of NAACL*, pages 139–150.

Ben Wang and Aran Komatsuzaki. 2021. GPT-J-6B: A 6 Billion Parameter Autoregressive Language Model. https://github.com/kingoflolz/mesh-transformer-jax.

Lifan Yuan, Yichi Zhang, Yangyi Chen, and Wei Wei. 2021. Bridge the gap between CV and NLP! a gradient-based textual adversarial attack framework. *arXiv preprint arXiv:2110.15317*.

Xiang Zhang, Junbo Zhao, and Yann LeCun. 2015. Character-level convolutional networks for text classification. In *Proceedings of NeurIPS*.

Zexuan Zhong, Dan Friedman, and Danqi Chen. 2021. Factual probing is [mask]: Learning vs. learning to recall. In *Proceedings of NAACL*, pages 5017–5033.

Kaiyang Zhou, Jingkang Yang, Chen Change Loy, and Ziwei Liu. 2021. Learning to prompt for vision-language models. *arXiv preprint arXiv:2109.01134*. 
Supplementary Material

A Proofs

Proof of Prop. 1: Let $c_i \in \mathbb{R}^d$ for $i \in \{1, \ldots, V\}$ be fixed vectors (denoting the embedding of words in a lexicon of size $V$). Let $e_i \in \{0,1\}^V$ denote the one-hot vector with 1 in the $i$-th position and 0 elsewhere. Since $d$-proj is a nearest-neighbor projection operator w.r.t. $m$, by definition it maps $x \in \mathbb{R}^d$ to $e_i$ whenever $x$ is closest to $c_i$, i.e., $i = \arg\min_j m(x, c_j)$ (breaking ties arbitrarily).

Let $S_i \subseteq \mathbb{R}^d$ denote the pre-image of $e_i$, i.e., the elements that the nearest-neighbor projection $d$-proj maps to the $i$-th one-hot vector. By definition, $c_i \in S_i$. Let $d' = \min_j m(c_i, c_j) > 0$ denote the distance of $c_i$ to the nearest $c_j$ w.r.t. the metric $m$. Consider the subspace $C_i = \{x \mid m(c_i, x) < d'/2\}$. By design, we have $C_i \subseteq S_i$. Further, moving $x$ by some small distance $\epsilon$ (w.r.t. $m$) to another point $x'$ changes its distance to $c_i$ only by at most $\epsilon$ (by the triangle inequality property of $m$). This implies that if $\epsilon$ is chosen to be small enough such that $m(c_i, x) + \epsilon < d'/2$, then $x'$ must also be in $C_i$. It follows that $C_i$ is an open subset of $\mathbb{R}^d$ and thus has infinitely many elements forming a continuous subspace. Hence $S_i$ also has infinite elements in one or more continuous subspaces. □

Proof of Prop. 2: For simplicity, assume $V = 2$. A projection operator $d$-proj $\in \mathcal{D}$ can then be fully characterized by the subset $S \subseteq \mathbb{R}^d$ that it maps to any one arbitrarily chosen one-hot vector. Choosing $d$-proj uniformly at random from $\mathcal{D}$ thus amounts to choosing the subset $S$ uniformly at random from $\mathbb{R}^d$. We show that the probability of choosing an $S$ such that $|S|$ is finite, is 0. (The same argument applies to $|\mathbb{R} \setminus S|$ being finite.)

To see this, let $\mathcal{S}_i$ denote the set of all (finite) subsets of $\mathbb{R}^d$ that have size exactly $i$. First, observe that the probability of choosing an $S$ that lies in $\mathcal{S}_0 \cup \mathcal{S}_1$ (i.e., a subset of $\mathbb{R}^d$ that has at most 1 element) is 0; this is a degenerate case in the underlying continuous probability space. Second, for any $i \geq 2$, $\mathcal{S}_i$ has the same “size” (in the measure theoretic sense) as $\mathcal{S}_1$, because one can construct an injective map from either one to the other—which follows from the fact that they both have the same cardinality as the set $\mathbb{R}$.\footnote{This can be proved using the rules of cardinal multiplication applied to $\mathcal{S}_i$, viewed as (a subset of) the Cartesian product of $\mathcal{S}_1$ with itself, $i$ times.}

Lastly, the space $\mathcal{S}$ of all finite subsets of $\mathbb{R}^d$ is the countable union $\bigcup_i \mathcal{S}_i$ of disjoint sets. Therefore, $\Pr[S \in \mathcal{S}] = \sum_i \Pr[S \in \mathcal{S}_i] = 0$. □