Effective Field Theory for Low-Energy np Systems\textsuperscript{a}

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The properties of low-energy neutron-proton systems are studied in an effective field theory where only nucleons figure as relevant degrees of freedom. With a finite momentum cut-off regularization scheme, we show that the large scattering lengths of the np systems do not spoil the convergence of the effective field theory, which turns out to be extremely successful in reproducing, with little cut-off dependence, the deuteron properties, the np $^1S_0$ scattering amplitude and most significantly, the $M1$ transition amplitude entering into the radiative np capture process.

1 Introduction

In this talk I would like to report on a recent work on the application of effective field theory to two-nucleon systems carried out in collaboration with Kuniharu Kubodera, Dong-Pil Min and Mannque Rho. Effective field theories (EFTs) have long proven to be a powerful tool in various areas of physics\textsuperscript{1,2}, so it is no surprise that they are equally powerful also in nuclear physics. Indeed we have recently had lots of successful applications of EFTs in low-energy nuclear dynamics\textsuperscript{3,4,5,6,7,8,9}. One of the spectacular cases was the chiral perturbation theory (ChPT) calculation of the np $\rightarrow d\gamma$ process at threshold\textsuperscript{4} with a perfect agreement with experiment. In Ref.\textsuperscript{4}, however, only the meson-exchange current correction relative to the one-body $M1$ transition amplitude was calculated, borrowing the latter from the phenomenological wave function of the accurate Argonne $v_{18}$ potential\textsuperscript{10}. While there is nothing wrong there and in fact it is consistent with the strategy of ChPT\textsuperscript{3} to calculate only the irreducible diagrams, there remained a “missing link” to a complete calculation, that is, to calculating everything within a given EFT. The motivation of this study is to do such a “first-principle” calculation\textsuperscript{11}. In so doing we will compute the static properties of the bound np state (the deuteron) and the np scattering amplitude in the $^1S_0$ channel.

Limiting ourselves to the processes whose typical energy-momentum scale is much smaller than the pion mass, we keep only the nucleon matter field\textsuperscript{1}.

\begin{itemize}
  \item[a] Invited talk at the APCTP Workshop on “Astro-Hadron Physics”, Seoul, Korea, October 1997
  \item[b] Present address: Fisica Teorica, Facultad de Ciencias, Edificio de Fisicas, Universidad de Salamanca, 37008 Salamanca, Spain.
  \item[c] The anti-nucleon field is suppressed due to the largeness of the nucleon mass, and is also
\end{itemize}
as an explicit degree of freedom, integrating out all massive fields as well as the pion field. The resulting EFT is a non-relativistic quantum mechanics, with all the interactions appearing as a nucleon-nucleon potential in Lippman-Schwinger (LS) equation. The potential of the EFT is zero-range or contact interactions and their derivatives, because all the meson fields which mediate the nucleon-nucleon interactions are integrated out. This innocent looking situation contains however many subtleties. First of all, even the leading order contact interactions are too singular to be solved by LS equation in three-dimensional space, thus we need to introduce a regulator and a renormalization scheme to handle the singularity. While the appearance of such a singularity is quite common in quantum field theories, the real subtlety comes from the fact that the np states in nature are all very close to the threshold: they are either weakly bound ($^3S_1$) or almost bound ($^1S_0$). Those states near threshold cannot be treated by perturbation expansion at all, which means that all the “reducible diagrams” up to infinite order should be summed by solving LS (or Schrödinger) equation. Furthermore, those states have huge scattering lengths $a$, and the appearance of extremely small mass scales $a^{-1}$ makes the convergence of EFTs by no means trivial. Indeed, using the dimensional regularization which has proven to be a very convenient and successful tool in handling singularities in most cases, Kaplan, Savage and Wise and Luke and Manohar have shown that the EFT breaks down at a very small scale, $p_{\text{crit}} = \sqrt{\frac{2}{ar_e}}$ for large scattering length $a$, where $r_e$ is the effective range and that this problem cannot be ameliorated by introducing the pionic degree of freedom. It was followed by the observation by Beane et al that, for non-perturbative cases such as ours, the physical results after the renormalization procedure may still depend on the regularization scheme. As pointed out by Beane et al and Lepage, the problem can however be resolved if one uses a cut-off regularization. In EFTs, the cut-off has a physical meaning and hence should be set by the mass of the lightest degree of freedom which is integrated out, namely the pion in our case. If one chooses too low a cut-off, the valid region of EFTs unnecessarily shrinks down, while if one chooses too high a cut-off, one introduces irrelevant degrees of freedom and hence makes the theory unnecessarily complicated. We find that the optimal cut-off in our case is $\Lambda \sim 200$ MeV as one can see from the results in Table 2 and Figures 1 and 2.
2 Renormalization and Phase shift

To do the calculation algebraically, we choose the following form of regularization appropriate to a separable potential given by the local Lagrangian:

\[ \langle p' | \hat{V} | p \rangle = S_{\Lambda}(p'^2)V(p' - p)S_{\Lambda}(p^2) \]  

(1)

where \( S_{\Lambda}(p^2) \) is a regulator which suppresses the contributions from \( |p| \gtrsim \Lambda \), \( \lim_{|p| \leq \Lambda} S_{\Lambda}(p^2) = 1 \) and \( \lim_{|p| \gg \Lambda} S_{\Lambda}(p^2) = 0 \), and \( V(q) \) is a finite-order polynomial in \( q \). We shall do the calculation to the next-to-leading order (NLO), and so the most general form of \( V(q) \) is

\[ V(q) = \frac{4\pi}{M} \left( C_0 + (C_2 \delta^{ij} + D_2 \sigma^{ij})q^i q^j \right) \]  

(2)

where \( M \) is the nucleon mass and \( \sigma^{ij} \) is the rank-two tensor that is effective only in the spin-triplet channel,

\[ \sigma^{ij} = \frac{3}{\sqrt{8}} \left( \sigma_1^i \sigma_2^j + \sigma_1^j \sigma_2^i - \frac{\delta^{ij}}{3} \sigma_1 \cdot \sigma_2 \right) \]  

(3)

Note that the coefficients \( C_{0,2} \) are (spin) channel-dependent, and that \( D_2 \) is effective only in spin-triplet channel. Thus we have five parameters; two in \(^1S_0\) and three in \(^3S_1\) channel, which will be fixed from experiments. Since the explicit form of the regulator should not matter, we shall choose the Gaussian form,

\[ S_{\Lambda}(p^2) = \exp \left( -\frac{p^2}{2\Lambda^2} \right) \]  

(4)

where \( \Lambda \) is the cut-off. The LS equation for the wavefunction \( |\psi\rangle = |\varphi\rangle + \hat{G}_0 \hat{V} |\psi\rangle \) where \( |\varphi\rangle \) is the free wavefunction and \( \hat{G}_0 \) is the free two-nucleon propagator depending on the total energy \( E \), \( \langle p' | \hat{G}_0 | p \rangle = \frac{\langle p' | p \rangle}{E - \frac{p^2}{2M} + i\delta} \) leads to the \( S \)-wave function (for the potential (2)) of the form

\[ \psi(r) = \varphi(r) + \frac{S_{\Lambda}(ME) C_E}{1 - \Gamma_E C_E} \left[ 1 - \frac{\sqrt{Z} C_2}{C_E} (\nabla^2 + ME) \right. \right. \]

\[ - \frac{\sqrt{Z} D_2 S_{12}(\hat{r})}{C_E} \frac{1}{\sqrt{8}} \left. \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} \right] \hat{\Gamma}_\Lambda(r) \]  

(5)

where \( S_{12}(\hat{r}) = 3\sigma_1 \cdot \hat{r} \sigma_2 \cdot \hat{r} - \sigma_1 \cdot \sigma_2 \),

\[ \Gamma_E = 4\pi \int \frac{d^3p}{(2\pi)^3} \frac{S_{\Lambda}(p^2)^2}{ME - p^2 + i0^+} \]  

(6)

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\[ \tilde{\Gamma}_\Lambda(r) = 4\pi \int \frac{d^3p}{(2\pi)^3} \frac{S_\Lambda(p^2)}{ME - p^2 + i0^+} e^{ip \cdot r}, \]
\[ \frac{1}{\sqrt{Z}} = 1 - C_2 I_2, \]
\[ C_E = a_\Lambda \left( 1 + \frac{1}{2} a_\Lambda r_\Lambda ME \right) + \left( \sqrt{Z} D_2 ME \right)^2 \Gamma_E, \]
with
\[ a_\Lambda \equiv Z \left[ C_0 + (C_2^2 + \delta_{S,1} D_2^2) I_4 \right], \]
\[ r_\Lambda \equiv \frac{2Z}{a_\Lambda} \left[ -2C_2 - (C_2^2 - \delta_{S,1} D_2^2) I_2 \right] \]
where \( I_n \ (n = 2, 4) \) are defined by
\[ I_n \equiv -\frac{\Lambda^{n+1}}{\pi} \int_{-\infty}^{\infty} dx x^n S_\Lambda^2(x^2 \Lambda^2). \]

With the regulator (4), the integrals come out to be \( I_2 = -\frac{1}{4\sqrt{\pi}} \Lambda^3 \) and \( I_4 = -\frac{3}{4\sqrt{\pi}} \Lambda^5 \).

The phase shifts can be calculated by looking at the large-\( r \) behavior of the wavefunction. To do this, it is convenient to separate the pole contributions of the integrals Eqs.(6, 7) as
\[ \Gamma_E = -i\sqrt{ME} S_\Lambda^2(ME) + I_\Lambda(E), \]
\[ \tilde{\Gamma}_\Lambda(r) = -\frac{S_\Lambda(ME)}{r} \left[ e^{i\sqrt{ME} r} - H(\Lambda r, ME/\Lambda^2) \right], \]
which define the functions \( I_\Lambda(E) = \Lambda I\left(\frac{ME}{\Lambda^2}\right) \) and \( H(\Lambda r, ME/\Lambda^2) \), both of which are real. Note that \( H(0, \varepsilon) = 1 \) which makes \( \Gamma_\Lambda(0) \) finite, and that \( \lim_{x \gg 1} H(x, \varepsilon) = 0 \). The \( ^1S_0 \) phase shift \( \delta(^1S_0) \) takes the form
\[ p \cot \delta(^1S_0) = \frac{1}{S_\Lambda^2(ME)} \left[ I_\Lambda(E) - \frac{1}{a_\Lambda(1 + \frac{1}{2} a_\Lambda r_\Lambda ME)} \right]. \]

The “effective” low-energy constants, \( a_\Lambda \) and \( r_\Lambda \), are fixed by comparing (15) to the effective-range expansion
\[ p \cot \delta = \frac{1}{a} + \frac{1}{2} r \varepsilon p^2 + \cdots, \]
\[
\frac{1}{a_\Lambda} = \frac{1}{a} + \Lambda I(0) = \frac{1}{a} - \frac{\Lambda}{\sqrt{\pi}},
\]
(17)

\[
r_\Lambda = r_e - \frac{2I'(0)}{\Lambda} - \frac{4}{a} \left[ \frac{\partial}{\partial p^2} S_\Lambda(p^2) \right]_{p^2=0} = r_e - \frac{4}{\sqrt{\pi}a_\Lambda} + \frac{2}{a\Lambda^2}.
\]
(18)

They then give us the “renormalization conditions” of the \(C_0\) and \(C_2\), with a given value of \(\Lambda\). Two important observations to make here: (a) We note that there is an upper bound of \(\Lambda\), \(\Lambda_{\text{Max}}\), if one requires that \(Z\) be positive and that \(C_2\) be real. That is, for \(\Lambda > \Lambda_{\text{Max}}\), the potential of the EFT becomes non-Hermitian. With \(a = -23.732\) fm and \(r_e = 2.697\) fm for the \(^1S_0\) channel taken from the Argonne \(v_{18}\) potential [10] (which we take to be “experimental”), we find that \(\Lambda_{\text{Max}} \simeq 348.0\) MeV; (b) the value \(\Lambda_{Z=1}\) defined such that \(Z = 1\) when \(\Lambda = \Lambda_{Z=1} \simeq 172.2\) MeV is quite special. At this point, we have \(r_\Lambda = 0\) and \(C_2 = 0\), that is, the NLO contribution is identically zero. This corresponds to the leading-order calculation with the \(\Lambda\) chosen to fit the experimental value of the effective range \(r_e\). A similar observation was made by Beane et al. using a square-well potential in coordinate space with a radius \(R\), with \(R^{-1}\) playing the role of \(\Lambda\).

The resulting phase shift with \(\Lambda = \Lambda_{Z=1}\) is plotted in Fig. 1. We see that the agreement with the result taken from the Argonne \(v_{18}\) potential [10] is perfect up to \(p \sim m_\pi/2\). Beyond that, we should expect corrections from the next-to-next-order and higher-order terms. In Fig. 2, we show how the phase-shift for a fixed center-of-mass momentum, \(p = 68.5\) MeV varies as the cut-off is changed. The solid curve is our NLO result, the dotted one the LO result (with \(C_2 = 0\)), and the horizontal dashed line the result taken from the \(v_{18}\) potential (“experimental”). We find that our NLO result is remarkably insensitive to the value of \(\Lambda\) for \(\Lambda \gtrsim m_\pi\). It demonstrates that, going to the higher-order calculations in EFTs, we have not only more accurate agreement with the data but also less dependence on the cut-off.

As for the \(^3S_1\) coupled channel, the phase shift \(^3S_1\) and the mixing angle \(\epsilon_1\) are given as

\[
p \cot \delta^{(3S_1)} = \frac{1}{S_{\Lambda}^2(ME)} \left[ I_\Lambda(E) - \frac{1 - \eta^2(E)}{a_\Lambda(1 + \frac{1}{2}a_\Lambda r_\Lambda ME)} \right],
\]
(19)

\[
\frac{\eta(E)}{1 - \eta^2(E)} = \frac{\sqrt{Z} D_2 ME}{a_\Lambda (1 + \frac{1}{2}a_\Lambda r_\Lambda ME)^4},
\]
(20)

where \(\eta(E) \equiv -\tan \epsilon_1\) and we have used the eigenphase parametrization [13]. The \(D_2\) can be fixed by the deuteron \(D/S\) ratio \(\eta_d \simeq 0.025\) at \(E = -B_d\).
Figure 1: \( np^1S_0 \) phase shift (degrees) vs. the center-of-mass (CM) momentum \( p \). Our theory with \( \Lambda = \Lambda_{Z=1} \simeq 172 \text{ MeV} \) is given by the solid line, and the results from the Argonne \( v_{18} \) potential ("experiments") by the solid dots.

Figure 2: \( np^1S_0 \) phase shift (degrees) vs. the cut-off \( \Lambda \) for a fixed CM momentum \( p = 68.5 \text{ MeV} \). The solid curve represents the NLO result, the dotted curve the LO result and the horizontal dashed line the result from the \( v_{18} \) potential.
Table 1: $C_0$, $C_2$ and $D_2$ for various values of $\Lambda$. The unit of $C_0$ is GeV$^{-1}$ while that of $C_2$ and $D_2$ is GeV$^{-3}$. $D_2$ in $^1S_0$ channel are identically zero.

| $\Lambda$ (MeV) | $^1S_0$ | $^3S_1$ |
|-----------------|---------|---------|
| $C_0$           | $C_2$   | $C_0$   | $C_2$   | $D_2$   |
| 150.0           | -9.877  | -56.4   | -15.503 | -206.6  | 186.9   |
| 172.2           | -9.482  | 0       | -12.051 | -78.0   | 169.7   |
| 200.0           | -10.185 | 37.9    | -9.925  | 2.5     | 156.3   |
| 216.1           | -11.377 | 51.8    | -9.730  | 31.7    | 152.7   |
| 250.0           | -17.167 | 74.2    | -14.342 | 83.4    | 158.1   |

with $B_d$ the binding energy of the deuteron,

$$\sqrt{Z}D_2 = \eta_d \frac{a_A}{1-\eta_d} \left[ 1 - \frac{1}{2}a_A r_A M B_d \right].$$  \hspace{1cm} (21)

The renormalization procedure is the same as for the $^1S_0$ channel. The only difference is that the value of $\Lambda_{Z=1}$ that makes $Z = 1$ does not coincide with $\Lambda_{r_A=0}$ that makes $r_A = 0$. Using $a = 5.419$ fm and $r_e = 1.753$ fm for the $^3S_1$ channel, we find that $\Lambda_{\text{max}} = 304.0$ MeV, $\Lambda_{Z=1} = 198.8$ MeV and $\Lambda_{r_A=0} = 216.1$ MeV. The values of $C_0$, $C_2$ and $D_2$ with respect to various values of $\Lambda$ are listed in Table 1. One can see the “naturalness” of the coefficient by making them dimensionless, for example, $m_\pi C_0$, $m_3 \pi C_2$ and $m_\pi D_2$ in spin-triplet channel with $\Lambda = 216.1$ MeV are $-1.4$, $0.1$ and $0.4$, respectively.

### 3 Results and Discussion

Given $C_0$, $C_2$ and $D_2$ for a given $\Lambda$, all other quantities are predictions. The binding energy of the deuteron is determined by the pole position,

$$\gamma S^2_{\Lambda}(-\gamma^2) + I_{\Lambda}(-\gamma^2) = \frac{1-\gamma^2}{a_\Lambda \left( 1 - \frac{1}{2} a_A r_A \gamma^2 \right)}$$

with $\gamma \equiv \sqrt{MB_d}$. The $S$- and $D$-wave radial wavefunctions of the deuteron are

$$u(r) = e^{-\gamma r} - H(\Lambda r, \frac{-\gamma^2}{\Lambda^2}) + \beta_\Lambda \frac{4\pi r}{\Lambda^2} \delta^{(3)}(r),$$

$$\omega(r) = \eta_d \frac{\gamma^2}{\gamma^2} \frac{\partial}{\partial r} \frac{1}{r} \left[ e^{-\gamma r} - H(\Lambda r, \frac{-\gamma^2}{\Lambda^2}) \right].$$

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where

$$\delta^{(3)}_{\Lambda}(r) = \int \frac{d^3 p}{(2\pi)^3} S_{\Lambda}(p^2) e^{ip \cdot r}, \quad (25)$$

$$\beta_{\Lambda} = \frac{(\sqrt{Z} - 1)\Lambda^2}{a_{\Lambda} (1 + \frac{1}{2} a_{\Lambda} ME) I_2 S_{\Lambda}(ME)}. \quad (26)$$

We now have all the machinery to calculate the deuteron properties: the wavefunction normalization factor $A_s$, the radius $r_d$, the quadrupole moment $Q_d$ and the $D$-state probability $P_D$, which are defined as

$$A_s^{-2} = \int_{0}^{\infty} dr \left[ u^2(r) + \omega^2(r) \right],$$

$$r_d^2 = \frac{A_s^2}{4} \int_{0}^{\infty} dr r^2 \left[ u^2(r) + \omega^2(r) \right],$$

$$Q_d = \frac{A_s^2}{\sqrt{50}} \int_{0}^{\infty} dr r^2 \left[ u(r) \omega(r) - \frac{\omega^2(r)}{\sqrt{8}} \right],$$

$$P_D = A_s^2 \int_{0}^{\infty} dr \omega^2(r). \quad (27)$$

The magnetic moment of the deuteron $\mu_d$ is related to the $P_D$ through

$$\mu_d = \mu_S - \frac{3}{2} \left( \mu_S - \frac{1}{2} \right) P_D \quad (28)$$

where $\mu_S \approx 0.8798$ is the isoscalar nucleon magnetic moment. Finally the one-body isovector $M1$ transition amplitude relevant for $n + p \rightarrow d + \gamma$ at threshold is

$$M_{1B} \equiv \int_{0}^{\infty} dr u(r) u_0(r) \quad (29)$$

where $u_0(r)$ is the $np \, {}^1S_0$ radial function,

$$u_0(r) = \frac{\sin(\sqrt{ME} r + \delta(1S_0))}{\sin \delta(1S_0)} - H(\Lambda r, \frac{ME}{\Lambda^2}) + \beta_{\Lambda} \frac{4\pi r}{\Lambda^2} \delta^{(3)}_{\Lambda}(r) \quad (30)$$

The (parameter-free) numerical results are listed in Table 2 for various values of the cut-off $\Lambda$. We see that the agreement with the experiments (particularly for $\Lambda = 216.1$ MeV) is excellent with very little dependence on the precise value of $\Lambda$. It may be coincidental but highly remarkable that even the quadrupole moment, which (as the authors of Ref. [12] stressed) the $v_{18}$ potential fails to reproduce, comes out correctly.
Table 2: Deuteron properties and the $M_1$ transition amplitude entering into the np capture for various values of $\Lambda$.

| $\Lambda$ (MeV) | 150 | 198.8 | 216.1 | 250 | Exp. $v_{1B}$ |
|-----------------|-----|-------|-------|-----|----------------|
| $B_d$ (MeV)     | 1.799 | 2.114 | 2.211 | 2.389 | 2.225 |
| $A_s$ (fm$^{-2}$) | 0.869 | 0.877 | 0.878 | 0.878 | 0.884(8) |
| $r_d$ (fm)      | 1.951 | 1.960 | 1.963 | 1.969 | 1.966(7) |
| $Q_d$ (fm$^2$)  | 0.231 | 0.277 | 0.288 | 0.305 | 0.286 |
| $P_d$ (%)       | 2.11 | 4.61 | 5.89 | 9.09 | – |
| $\mu_d$        | 0.868 | 0.854 | 0.846 | 0.828 | 0.8574 |
| $M_{1B}$ (fm)   | 4.06 | 4.01 | 3.99 | 3.96 | – |

Let us compare our result (15) with that obtained with the dimensional regularization $^{6}$:

$$ p \cot \delta \bigg|_{\text{Dim.}} = -\frac{1}{a(1 + \frac{1}{2}ar_e ME)} $$

Expanding $p \cot \delta$ of (31) in $ME$, we find that the coefficient of the $n$-th order term is order of $a^{n-1}r_e^n$. This increases rapidly with $n$ when $a$ is large, disagreeing strongly with the fact that the low-energy scattering is well described by just two terms of the effective range expansion in (16). This observation led the authors of Ref. $^6$ to conclude that the critical momentum scale at which the EFT expansion breaks down is very small for a very large $a$:

$$ p_{\text{crit}} \bigg|_{\text{Dim.}} \sim \sqrt{\frac{2}{ar_e}}. $$

We arrive at a different conclusion. With the cut-off regularization, the scattering length $a$ is replaced by an effective one, $a_{\Lambda}$, that is order of $\Lambda^{-1}$ for large $|a|$. This agrees with the findings of Beane et al.$^8$ and Lepage.$^{12}$ Counting $r_e$ to be order of $\Lambda^{-1}$, the $n$-th order coefficient now is $\Lambda^{1-2n}$, as one would expect on a general ground. Recalling that a small (and negative) scattering length corresponds to weak (and attractive) interactions, it is also remarkable that the mechanism of the finite cut-off EFTs is quite similar to the “quasi-particle” phenomena since both convert highly non-linear systems into weakly interacting systems.

Using the finite cut-off regularization scheme which is the most faithful way to realize the principles of EFTs, we have demonstrated that the low-energy nuclear physics can be well-described by EFTs. In particular, it is satisfying that the classic np capture process can be completely understood from a “first-principle” approach. Here the cut-off regularization was found
to be highly efficient. With the dimensional regularization the $M1$ matrix element was found to be in total disagreement with the result of the Argonne $v_{18}$ potential.

There are several important and urgent extensions and applications of the finite EFTs in nuclear physics. One of the most important applications is the proton fusion process, $p + p \rightarrow d + e^+ + \nu$, at threshold which plays a crucial role in the stellar evolution. This process has been recently worked out and will appear soon. From the theoretical side, the most urgent task is to take into account the pion degree of freedom so as to extend the calculation to higher chiral order. This would enable us to study the interplay between the breakdown of an EFT and the emergence of a “new physics.”

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