Application of wavelet analysis of X-Ray computed tomography histogram for phase segmentation

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Abstract. With the development of unconventional resources, the standard task of evaluation the porosity and oil saturation needs a new approach. Heavy oil and bitumen can act as the cement in many sand reservoirs, which excludes the use of standard measuring techniques. A method for phase segmentation and estimating the porosity and oil saturation of heavy oil reservoirs, based on wavelet analysis of X-ray computed tomography histogram was proposed. Its advantages are reducing human influence on phase thresholding and the ability to work with loose sand reservoirs, for which it is not possible to use standard assessment methods. The main problem found for this method is the need for high detailing of segmentation objects since a large amount of noise and transition voxels adversely affect class recognition and quantitative interpretation. The application of wavelet analysis approach for naturally heavy oil saturated samples requires high-resolution microCT (less than 5 µm) and high oil saturation of the samples.

1. Introduction

X-Ray computed tomography (CT) is an established and rapidly evolving technology of proven value for geological investigations [1]. The advantage of tomography over other porous media characterization techniques is direct imaging of microstructure, from which we can evaluate statistical and geometrical parameters such as porosity or porous media surface area. Tomography can then be used to determine the three dimensional microstructure of the porous samples, from which petrophysical parameters as porosity, accessible pore space, permeability, etc. can be derive [2].

Each three-dimensional cell of tomography model (voxel) has the grayscale value of X-Ray attenuation coefficient of material. Standard histogram of X-Ray computed tomography of clean reservoir sample has a form of a double-peaked curve, where the x-ray attenuation coefficient is plotted along the x-axis and the number of voxels with a given grayscale value – along the y-axis. The minimum value of histogram between two peaks usually is close to a threshold between rock matrixes and air in porous media and around the sample (fig. 1a). After cutting of the air around the sample, the histogram changes form to one-peaked (fig. 1b).
Figure 1. X-Ray computed tomography histograms of sandstone (resolution 11µm): a – original sample with air outside, b – cropped cube sample (350x350x350 voxels)

To date, a large number of tomographic image segmentation methods have been developed [3–6], but the most popular methods are still interactive thresholding by operator. The big problem of this method is high subjectivism in threshold choice, which can lead to a significant difference in the reservoir properties even when the same operator is re-segments the same sample. In addition, tomography artifacts, noise, low resolution can aggravate the situation. Also that some transitional voxels on the border of phases could include characteristics both of them. These transitional cells have the name “mixels” in the article by Kobayashi Y [7].

One of interesting thresholding method is based on fitting Gaussians to a histogram. This approach was briefly described in several papers [8–10]. The main idea of our work is to evaluate the possibility of applying a modified Gaussian algorithm called “wavelet analysis” for the phase segmentation of 3D reservoir image.
2. Methodology

A wavelet is a mathematical function used to divide a given function into different scale components. Like Fourier analysis, wavelet analysis deals with expansion or functions in terms of a set of basis functions. Unlike Fourier analysis, wavelet analysis expands functions not in terms of trigonometric polynomials but in terms of wavelets, which are generated in the form of translations and dilations of a fixed function called the “mother wavelet”. The objective of wavelet analysis is to define these powerful wavelet basis functions and find an efficient method for their computation [11].

In this work the superposition of Gaussian function with three indeterminate parameters (maximum position, outline width and amplitude) is taken as a mathematical model of a density function of relative X-ray attenuation coefficient:

\[ f(x) = \sum_n A_n g_0(x, m_n, \sigma_n) = \sum_n A_n e^{-\frac{(x-m_n)^2}{2\sigma_n^2}} \]  

(1)

The parameterization of the mathematical model of the histogram comes down to searching for indeterminate parameters of the Gaussian curves \( \{A_n, m_n, \sigma_n\} \) from (1). The method based on continuous wavelet transformations with specially selected basis is provided to solve this inverse problem [12].

Since the density function in the model (1) is a superposition of Gaussian functions, the most suitable basis functions for continuous wavelet transformations are functions from the derivatives set of the same Gaussian function:

\[ g_m(x) = (-1)^m \frac{\partial^m}{\partial x^m} \exp\left(-\frac{x^2}{2}\right) \]  

(2)

In this case, the form peculiarities of the basis function have an insufficient effect on the wavelet spectrum [13]. The wavelet of the second order is optimal from the set (2):

\[ g_2(x) = \left(1-x^2\right) e^{-\frac{x^2}{2}} \]  

(3)

Using the function (3) as a basis wavelet and the normalization constant \( a^{1.5} \), the wavelet spectrum of the Gaussian function with parameters \( \{A_0, m_0, \sigma_0\} \) is written as:

\[ W(a,b) = a^{1.5} A_0 \int_{-\infty}^{\infty} g_0(x, m_0, \sigma_0) g_2\left(\frac{x-a}{b}\right) dx = A_0 \frac{2\pi a}{2\pi a^2 + \sigma_0^2} \left(1 - \frac{(b-m_0)^2}{2(a^2 + \sigma_0^2)}\right) e^{-\frac{(b-m_0)^2}{2(a^2 + \sigma_0^2)}} \]  

(4)

The maximum \( \{a^*, b^*\} \) of the function can be reached at the point with coordinates

\[ a^* = m_0, \quad b^* = \sigma_0 \]  

(5)
and the value of the wavelet coefficient at the point is

$$W(a^*, b^*) = A_0 \sqrt{\frac{\pi}{4\sigma_0}}. \quad (6)$$

Consequently, the position and the maximum of the wavelet spectrum (4) determine the parameters \(\{A_0, m_0, \sigma_0\}\) of the Gaussian function precisely and unambiguously. The graph of the Gaussian function with the parameters \(A_0 = 1, m_0 = 3, \sigma_0 = 1\) is given as an example on the fig. 2a and its wavelet spectrum with basis (3) are depicted on the fig. 2b.

If the superposition is given for several Gaussian functions, the formulas (4)-(6) can not be applied to identify them, as the extreme points do not show an accurate position of sources. The iteration algorithm is established for these cases. To explain this algorithm, we take the simplest interference of the two Gaussian functions. If parameters of the one function are known, due to the flatness of the wavelet transformations, when the wavelet spectrum (formula (4)) is subtracted from the general spectrum, the remaining values of the wavelet coefficients are a spectrum of the second function. Even if the accurate parameters of the Gaussian curve are not given, the determination of the approximate parameters of the first function provides the specification of the parameters of the other function. Repetition of this procedure provides an accurate solution step by step. This approach is correct for the case of the compound superposition of Gaussian curves. The parameters recovery result of three interfering Gaussian functions is given as an example on the fig. 3.

The procedure for wavelet analysis of CT histogram consists of several steps. The first step is to make X-ray computed tomography of the heavy oil sandstone sample from one oil field of Russia and to cut the surrounding air and artefact areas at the edges of the 3D image. We used micro and nanofocus X-ray control system for computed tomography General Electric V|tome|X S 240 and Avizo 7.1 software for these objectives.

Figure 2. Wavelet spectrum of the Gaussian function with the parameters \(m_0 = 3, \sigma_0 = 1\). A: Gaussian function graph; B: its wavelet spectrum. White zones of the wavelet spectrum are the function maximum. The point with the coordinates \(a = m_0, b = \sigma_0\) is circled in black.
Figure 3. An example of the determination of Gaussian functions parameters. A: graphs of the given superposition of the Gaussian functions (fine solid line) and the recovered superposition of the Gaussian functions (bold dot line). B: Circles - given coordinates of the Gaussian functions parameters, circumferences - positions of the found parameters of the Gaussian functions; substructure - wavelet spectrum of the given superposition for the Gaussian functions. Size of circles and circumferences are proportionate to the amplitudes of the Gaussian components. This mathematical tool helps to identify sources - the Gaussian distribution curves, which form the initial histogram [12].

Next step is to make a histogram of the cut sample. In our case, the CT histogram is located on a grayscale from 0 to 65535. After that, it is possible to start the wavelet analysis of the obtained curve. We used GraviMagic software that was developed for the wavelet analysis of gravimetric data [12,14].

The important question is how many sources (wavelets) set to search in the mother wavelet. This number is related from number of main phases in the sample. Because often this information is absent, we recommend setting from 4 to 8 sources for searching in the similar sandstone reservoirs. The search for fewer sources than general phases in the sample, the threshold between some phases maybe not founded. The setting of more sources than phases, some threshold can be inside 1 phase, but these results are possible to work with, subject to the exclusion of this threshold.

The received sources are assigned numbers from left to right, where the leftmost corresponds to the phase with the smallest value of the grayscale, and the rightmost - with the largest. The criterion of three sigma was used to exclude gross errors from the measurement results. Next, the wavelets are integrated and the proportion of the wavelet area to the area of the mother wavelet is determined. To identify the threshold values, the last grayscale value of histogram is sequentially divided into the received percentages from the first wavelet to the last. For example, the results of the wavelet analysis of histogram from fig. 1b are shown in the fig. 4.
3. Results and Discussions

As a result of the wavelet analysis for each source, we obtained the percentage ratio of their areas relative to the parent wavelet and the threshold values (table 1). The sources with close threshold values were combined into one phase.

| Source | Grayscale threshold | Phase percentage, % | Conversion to the mineral phase, % | Estimated phase         |
|-------|---------------------|----------------------|------------------------------------|-------------------------|
| 1     | 6120                | 2.37                 | 0                                  | Air in the pores        |
| 2     | 10785               | 27.99                | 0                                  | Heavy oil in pores      |
| 4     | 14467               | 64.01                | 91.92                              | Albite – microcline – quartz |
| 6     | 29444               | 5.63                 | 8.08                               | Calcite                 |

In order to identify each mineral phase, we checked the researched sample of sandstone by powder diffraction method. (fig. 5.). The main reservoir minerals were albite, quartz and microcline, with a small amount of calcite and clay minerals. We used MUCALC tool to compare the attenuation coefficients of received minerals [15,16]. The attenuation coefficients of albite, quartz and microcline were very close to each other, so we merged them into one phase. The attenuation coefficient of calcite was higher than previous for phase, so it could be classified on CT data. In addition to them, the phase composition of the 3D image includes the air in the pores and heavy oil. The results of phase classification of heavy oil-saturated sandstone based on wavelet analysis presented on fig. 6.
Figure 5. Results of powder diffraction analysis of the sandstone sample. General minerals are albite, quartz, microcline and calcite.

To confirm the correct classification of the heavy oil phase, we carried out the extraction of hydrocarbons, re-made X-ray computed tomography of the sample and performed spatial registration of two 3D images: before extraction and after extraction. As shown on the fig. 7A the histogram of the sample between two peaks in the grayscale range 6000-11000 was changed after extraction. The same changes took a place on the slices in the pore space area: here the voxels have become darker. This demonstrates the fundamental efficiency of the proposed method.
We tested the method ability to evaluate the porosity and volume oil saturation on heavy oil-saturated sandstone samples. We used as big standard samples (cylinders 30x30 mm), as little samples for microCT. Hydrocarbon extraction was done for all samples after tomography of naturally oil-saturated sandstones. After that, we re-made X-ray computed tomography and spatially registered all samples in not extracted 3D images. The operator made interactive thresholding on the saturated volumes and masked it on non-extracted ones. Then he classified manually air and heavy oil in the segmented pore space volume. The results of the comparison performed in table 2 and table 3.

4. Conclusions

Thus, a method for phase segmentation and estimation the porosity and bulk saturation based on wavelet analysis of X-ray computed tomography histogram, was proposed. Its advantages are reducing human influence on phase thresholding and the ability to work with loose sand reservoirs, for which it is not possible to use standard assessment methods. The results demonstrates that low-resolution samples are difficult to analyze by wavelet method. Low oil saturation of the sample leads to errors in searching sources and thresholds for heavy oil phase.

Figure 7. MicroCT histograms and slices of the sandstone sample before hydrocarbon extraction (on the left) and after it (on the right). A – changes on the histogram in the range 6000-11000 after hydrocarbon extraction. B – changes in gray color of voxels in pore space area after hydrocarbon extraction.
Table 2. Comparison of porosities coefficients, obtained by different methods for naturally heavy oil saturated samples

| Sample No. | Tomography resolution, µm | Standard method* | Porosity coefficient, % | Wavelet analysis (air+oil in pores) | Masking pore space on extracted volumes |
|------------|--------------------------|------------------|-------------------------|-------------------------------------|----------------------------------------|
| 42_1_big   | 34.25                    | 25.69            | 8.62                    | 6.73                                |                                        |
| 42_1_small | 10.75                    | 24.96            | 24.36                   | 16.37                               |                                        |
| 37_2_big   | 34.25                    | 30.25            | 15.43                   | 15.04                               |                                        |
| 37_2_small | 13.27                    | 31.10            | 50.35                   | 26.64                               |                                        |
| 153_1_small| 9.86                     | 23.30            | 48.39                   | 29.37                               |                                        |
| 153_2_small| 9.86                     | 16.37            | 46.79                   | 17.55                               |                                        |
| 153_3_small| 9.86                     | 22.53            | 44.67                   | 18.08                               |                                        |
| 153_3_small| 11.36                    | 17.74            | 41.30                   | 22.07                               |                                        |

* - standard method was made for standard samples (cylinders 30x30 mm)

Table 3. Comparison of bulk oil saturation, obtained by different methods for naturally heavy oil saturated samples

| Sample No. | Tomography resolution, µm | Standard method* | Volume oil saturation, % | Wavelet analysis (air+oil in pores) | Masking pore space on extracted volumes |
|------------|--------------------------|------------------|--------------------------|-------------------------------------|----------------------------------------|
| 42_1_big   | 34.25                    | 71.57            | 70.72                    | 67.07                                |                                        |
| 42_1_small | 10.75                    | 69.71            | 76.07                    | 60.38                                |                                        |
| 37_2_big   | 34.25                    | 48.96            | 92.83                    | 77.42                                |                                        |
| 37_2_small | 13.27                    | 21.70            | 92.20                    | 73.73                                |                                        |
| 153_1_small| 9.86                     | -                | 81.75                    | 57.07                                |                                        |
| 153_2_small| 9.86                     | -                | 84.25                    | 55.78                                |                                        |
| 153_3_small| 9.86                     | -                | 76.20                    | 44.41                                |                                        |
| 153_3_small| 11.36                    | -                | 76.02                    | 40.19                                |                                        |

* - standard method was made for standard samples (cylinders 30x30 mm)

The comparison of histograms of these samples before and after hydrocarbon extraction shows an almost complete lack of changes in the heavy oil phase area. The values of porosity and bulk saturation were close to the data of the standard method in the sample 42_1_small with a high oil saturation. We recommend using wavelet analysis method for evaluation the porosity and bulk saturation with high-resolution tomography (less than 5 µm) and high heavy oil-saturated samples. Specially, it can be useful for sand reservoir, where heavy oil acts as cement.

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