First Study for the Pentaquark Potential in SU(3) Lattice QCD

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The static penta-quark (5Q) potential $V_{5Q}$ is studied in SU(3) lattice QCD with $16^3 \times 32$ and $\beta=6.0$ at the quenched level. From the 5Q Wilson loop, $V_{5Q}$ is calculated in a gauge-invariant manner, with the smearing method to enhance the ground-state component. $V_{5Q}$ is well described by the OGE plus multi-Y Ansatz: a sum of the OGE Coulomb term and the multi-Y-type linear term proportional to the minimal total length of the flux-tube linking the five quarks. Comparing with QQ and 3Q potentials, we find a universality of the string tension, $\sigma_{QQ} \simeq \sigma_{3Q} \simeq \sigma_{5Q}$, and the OGE result for Coulomb coefficients.

In this paper, motivated by the recent discovery of the exotic anti-strange baryon $\Theta^+(1540)$ at SPring-8 (LEPS), and was confirmed by ITEP(DIANA), JLab(CLAS) and ELSA(SAPHIR). The $\Theta^+(1540)$ was theoretically predicted in the Skyrme model, and is regarded as a penta-quark (5Q) baryon of $u^2d^2s$ in the valence-quark picture. Another 5Q baryon $\Xi^-(1862)$ was found by CERN(NA49), and also an anti-charmed 5Q baryon $\Theta_c(3099)$ was found by HERA(H1).

Accordingly, many theoretical studies have been done for the 5Q baryon using various approaches such as lattice QCD, the constituent quark model, the diquark model, the QCD sum rule, the string theory, and so on. However, there are several puzzling problems on the $\Theta^+(1540)$: its mass seems to be rather small and its decay width is extremely narrow. To solve them, one encounters the many-body problem of quarks, and therefore it is quite desired to clarify the inter-quark force in the multi-quark system based on QCD.

In this paper, motivated by the recent discovery of the penta-quark baryons, we perform the first study of the static penta-quark (5Q) potential $V_{5Q}$, i.e., the inter-quark force in the 5Q system, in SU(3) lattice QCD with $\beta=6.0$ and $16^3 \times 32$ at the quenched level. Note that the lattice QCD result of $V_{5Q}$ presents a key information in modeling the multi-quark system based on QCD.

For the penta-quark system, we investigate the QQ-\bar{Q}-QQ type configuration with the two “QQ clusters” belonging to the $3^*$ representation of the SU(3) color as shown in Fig.1, since this type of the 5Q configuration is expected to have a small energy and seems to be natural as a realistic candidate of the $\Theta^+(1540)$. Indeed, in the perturbative sense, an attractive (repulsive) force acts between two quarks, when their total SU(3) color belongs to $3^*$ ($6$). Therefore, the nearest QQ cluster tends to form $3^*$ rather than $6$ in the low-lying 5Q system, which leads to the $3^*$-diquark model.

![FIG. 1: The QQ-\bar{Q}-QQ type configuration for the pentaquark system. The two QQ clusters belong to the $3^*$ representation of the color SU(3).](image-url)

Similar to the derivation of the Q-\bar{Q} (3Q) potential from the (3Q) Wilson loop, the 5Q static potential $V_{5Q}$ can be calculated with the 5Q Wilson loop $W_{5Q}$, which is defined in a gauge-invariant manner as shown in Fig.2.

We define the 5Q Wilson loop $W_{5Q}$ as

$$W_{5Q} \equiv \frac{1}{3!} \epsilon^{abc} \epsilon^{a'b'c'} M^{ab'} \langle \bar{L}_3 \bar{L}_{12} \bar{L}_4 \rangle^{bb'} \langle \bar{R}_3 \bar{R}_{12} \bar{R}_4 \rangle^{cc'} ,$$

where $\bar{M}, \bar{L}_i, \bar{R}_i (i=1, 2, 3, 4)$ are given by

$$\bar{M}, \bar{L}_i, \bar{R}_i \equiv P \exp \{ ig \int_{M, L_i, R_i} dx A_\mu (x) \} \in SU(3)_c. \quad (2)$$

As shown in Fig.2, $\bar{M}, \bar{L}_i, \bar{R}_i (i=3, 4)$ are line-like variables and $\bar{L}_i, \bar{R}_i (i=1, 2)$ are staple-like variables. Here,
in the Q- ¯Q potential heat-bath algorithm. The lattice spacing for the 5Q potential $V_{5Q}$ configurations using SU(3) action with $\beta$ lead to a large enhancement of the ground-state color current.)

obtained from the 5Q Wilson loop five quarks (4Q- ¯Q) being spatially fixed in which lead to a large enhancement of the ground-state component of the 5Q state in the 5Q Wilson loop.

For these types of 5Q configurations, we calculate the 5Q potential $V_{5Q}$ from the 5Q Wilson loop ($W_{5Q}$) using the smearing method. Owing to the smearing, the ground-state component is largely enhanced, and therefore the 5Q Wilson loop ($W_{5Q}$) composed with the smeared link variable exhibits a single-exponential behavior as ($W_{5Q}$) $\simeq e^{-V_{5Q}T}$ even for a small value of $T$. Then, for each 5Q configuration, we extract $V_{5Q}$ from the least squares fit with the single-exponential form

$$\langle W_{5Q} \rangle = \tilde{C}e^{-V_{5Q}T} \quad (5)$$

in the range of $T_{\text{min}} \leq T \leq T_{\text{max}}$ listed in Table I. The prefactor $\tilde{C}$ physically means the ground-state overlap, and $\tilde{C}$ corresponds to the quasi-ground-state. Here, we choose the fit range of $T$ such that the stability of the “effective mass” $V(T) \equiv \ln(\langle W_{5Q}(T) \rangle / \langle W_{5Q}(T+1) \rangle)$ is observed in the range of $T_{\text{min}} \leq T \leq T_{\text{max}} - 1$. For the lattice calculation of ($W_{5Q}$), we use the translational and the rotational symmetries on lattices.

For 56 different patterns of the 5Q configurations as shown in Figs.3 and 4, we present the lattice QCD data for the 5Q potential $V_{5Q}$ together with the ground-state
TABLE I: Lattice QCD results for the penta-quark potential $V_{5Q}$ for the planar 5Q configuration labeled by $(d, h_1, h_2)$ as shown in Fig.3. We list also the ground-state overlap $\tilde{C}$, the fit range of $T$ and the theoretical form $V_{5Q}^{\text{theor}}$ of the OGE plus multi-Y Ansatz $\mathcal{B}$ with $(A_{5Q}, \sigma_{5Q})$ fixed to be $(A_{3Q}, \sigma_{3Q})$ in $V_{3Q}$ in Ref.[2]. All the data are measured in the lattice unit.

| $(d, h_1, h_2)$ | $V_{5Q}$ | $\tilde{C}$ | $T_{\text{min}}$-$T_{\text{max}}$ | $V_{5Q}^{\text{theor}}$ |
|-----------------|----------|-----------|-----------------|---------------------|
| $(1,1,1)$       | 1.4452(11) | 0.9539(21) | 2-7              | 1.4433              |
| $(1,1,2)$       | 1.5409(13) | 0.9506(25) | 2-6              | 1.5414              |
| $(1,1,3)$       | 1.6177(19) | 0.9512(33) | 2-7              | 1.6146              |
| $(1,1,4)$       | 1.6793(20) | 0.9431(35) | 2-7              | 1.6767              |
| $(1,1,5)$       | 1.7381(23) | 0.9394(40) | 2-6              | 1.7332              |
| $(1,1,6)$       | 1.7918(28) | 0.9311(49) | 2-6              | 1.7866              |
| $(1,1,7)$       | 1.8441(31) | 0.9232(56) | 2-6              | 1.8380              |
| $(1,2,2)$       | 1.6314(17) | 0.9503(29) | 2-6              | 1.6322              |
| $(1,2,3)$       | 1.7011(20) | 0.9427(34) | 2-5              | 1.7021              |
| $(1,2,4)$       | 1.7808(25) | 0.9478(43) | 2-6              | 1.7623              |
| $(1,2,5)$       | 1.8190(29) | 0.9297(50) | 2-6              | 1.8177              |
| $(1,2,6)$       | 1.8717(33) | 0.9205(57) | 2-5              | 1.8704              |
| $(1,3,3)$       | 1.7723(24) | 0.9405(39) | 2-5              | 1.7702              |
| $(1,3,4)$       | 1.8336(28) | 0.9351(49) | 2-7              | 1.8293              |
| $(1,3,5)$       | 1.8913(33) | 0.9320(62) | 2-5              | 1.8839              |
| $(1,4,4)$       | 1.8939(31) | 0.9293(53) | 2-4              | 1.8877              |
| $(2,1,1)$       | 1.7531(23) | 0.9393(40) | 2-5              | 1.7515              |
| $(2,2,2)$       | 1.8803(31) | 0.9292(54) | 2-6              | 1.8887              |
| $(2,3,3)$       | 2.0030(37) | 0.9284(64) | 2-5              | 2.0098              |
| $(2,4,4)$       | 2.1122(49) | 0.9116(91) | 2-5              | 2.1211              |
| $(3,1,1)$       | 1.9734(37) | 0.9138(66) | 2-5              | 1.9850              |
| $(3,2,2)$       | 2.0811(45) | 0.9070(75) | 2-5              | 2.0942              |
| $(3,3,3)$       | 2.1886(53) | 0.9003(92) | 2-4              | 2.2047              |
| $(3,4,4)$       | 2.3043(68) | 0.9084(113) | 2-5 | 2.3105 |
| $(4,1,1)$       | 2.1697(60) | 0.8948(100) | 2-5 | 2.1958 |
| $(4,2,2)$       | 2.2734(60) | 0.8890(100) | 2-5 | 2.2829 |
| $(4,3,3)$       | 2.3657(73) | 0.8606(120) | 2-4 | 2.3864 |
| $(4,4,4)$       | 2.4706(104) | 0.8534(164) | 2-5 | 2.4884 |

The statistical errors are estimated with the jackknife method. We find a large ground-state overlap as $\tilde{C} > 0.85$ for almost all 5Q configurations.

Next, we consider the theoretical form of the 5Q potential $V_{5Q}$. The lattice QCD studies at the quenched level show that the Q-$Q$ potential $V_{QQ}$ takes a form of

$$V_{QQ}(r) = -\frac{A_{QQ}}{r} + \sigma_{QQ}r + C_{QQ},$$

and the 3Q potential $V_{3Q}$ takes a form of

$$V_{3Q} = -A_{3Q} \sum_{i<j} \frac{1}{|r_i - r_j|} + \sigma_{3Q}L_{\text{min}} + C_{3Q}.$$  

where $L_{\text{min}}$ denotes the minimal value of total length of color flux tubes linking the three quarks. In fact, both $V_{QQ}$ and $V_{3Q}$ are described by a sum of the short-distance one-gluon-exchange (OGE) result and the long-distance flux-tube result.

For the static penta-quark (5Q) system, we find that the lattice QCD results are well described by the OGE plus multi-Y Ansatz: a sum of the OGE Coulomb term and the multi-Y type linear term $\mathcal{B}$,

$$V_{5Q} = \frac{g^2}{4\pi} \sum_{i<j} \frac{T_i T_j}{|r_i - r_j|} + \sigma_{5Q}L_{\text{min}} + C_{5Q}$$

$$= -A_{5Q} \left\{ \left( \frac{1}{r_{12}} + \frac{1}{r_{34}} \right) + \frac{1}{2} \left( \frac{1}{r_{15}} + \frac{1}{r_{25}} + \frac{1}{r_{35}} + \frac{1}{r_{45}} \right) + \frac{1}{4} \left( \frac{1}{r_{13}} + \frac{1}{r_{14}} + \frac{1}{r_{23}} + \frac{1}{r_{24}} + \frac{1}{r_{34}} + \frac{1}{r_{35}} + \frac{1}{r_{45}} \right) \right\} + \sigma_{5Q}L_{\text{min}} + C_{5Q}$$

with $r_{ij} \equiv |r_i - r_j|$ and $i$th quark location $r_i$ in Fig.1. Here, $L_{\text{min}}$ is the minimal length of the flux-tube linking five quarks as shown in Fig.1. (For the extreme case, e.g., $d > \sqrt{3}h_1$, we here assume that the flux-tube is formed...
as the two straight lines on Q1Q5 and Q2Q4, considering the color combination, although there may appear several possibilities as the “flip-flop”.

Note that there appear three kinds of Coulomb coefficients \( A_{3Q}, \frac{1}{2} A_{5Q}, \frac{1}{3} A_{5Q} \) in the penta-quark system, while only one Coulomb coefficient, \( A_{3Q} \) or \( A_{5Q} \), appears in the QQ or the 3Q system. Here, the Coulomb coefficient \( A_{5Q} \) in Eq. (8) corresponds to \( A_{3Q} \) or \( \frac{1}{2} A_{QQ} \) in terms of the OGE result.

We add in Table I and II the theoretical form \( V_{5Q}^{\text{theor}} \) of the OGE plus multi-Y Ansatz with \((A_{3Q},\sigma_{3Q})\) fixed to be \((A_{5Q},\sigma_{5Q})\) in the 3Q potential \( V_{3Q} \) obtained in Ref. [1], i.e., \( A_{5Q} = A_{3Q} \simeq 0.1366 \), \( \sigma_{5Q} = \sigma_{3Q} \simeq 0.046a^{-2} \) and \( C_{5Q} \simeq 1.58a^{-1} \). (Note that there is no adjustable parameter for the theoretical form \( V_{3Q}^{\text{theor}} \) besides an irrelevant constant \( C_{5Q} \), since \( A_{5Q} \) and \( \sigma_{5Q} \) are fixed to be \( A_{3Q} \) and \( \sigma_{3Q} \), respectively.) Thus, the 5Q potential \( V_{5Q} \) is found to be well described by the OGE Coulomb plus multi-Y-type linear potential.

We show in Fig. 5 typical examples of the lattice QCD data for the penta-quark potential \( V_{5Q} \). The symbols denote the lattice data, and the curves denote the theoretical form of the OGE plus multi-Y Ansatz with \((A_{3Q},\sigma_{3Q})\) fixed to be \((A_{5Q},\sigma_{5Q})\). One finds a good agreement between the lattice QCD data and the theoretical curves.

![Figure 5: Lattice QCD results of the penta-quark potential \( V_{5Q} \) for the planar 5Q configuration with \( h_1 = h_2 \equiv h \) in Fig. 3 in the lattice unit. Each 5Q system is labeled by \( d \) and \( h \). The symbols denote the lattice data, and the curves the theoretical form of the OGE plus multi-Y Ansatz.](image)

Note that the planar and the twisted 5Q configurations with the same \((d,h_1,h_2)\) are almost degenerate, although the energy of the planar one is slightly smaller. In terms of the OGE plus multi-Y Ansatz, the only energy difference between the two states originates from a small difference of the Coulomb interaction between \( Q_i (i = 1, 2) \) and \( Q_j (j = 3, 4) \), where the Coulomb coefficient is reduced as \( \frac{1}{3} A_{5Q} \simeq \frac{1}{3} A_{5Q} \). Then, no special configuration is favored in the 5Q system in terms of the energy. This fact also indicates that the 5Q system is unstable against the twisted motion of the two QQ clusters as shown in Fig. 4. In fact, general 5Q systems tend to take a three-dimensional configuration \[12\] in terms of the entropy.

From the comparison with the QQ and the 3Q potentials, the universality of the string tension and the OGE result are found among QQ, 3Q and 5Q systems as

\[ \sigma_{QQ} \simeq \sigma_{3Q} \simeq \sigma_{5Q}, \quad \frac{1}{2} A_{QQ} \simeq A_{3Q} \simeq A_{5Q}. \]  

This result supports the flux-tube picture on the confinement mechanism even for the multi-quark system \[5\].

To conclude, we have performed the first study of the penta-quark potential in lattice QCD, and have found that the 5Q potential is well reproduced by the OGE Coulomb plus multi-Y-type linear potential.

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