Nonlinear SUSY General Relativity Theory and Significances

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Abstract. We show some interesting consequences of the nonlinear supersymmetric general relativity theory (NLSUSYGR) on particle physics, cosmology and their relations. They may give new insights into the SUSY breaking mechanism, dark energy, dark matter and the low energy superpartner particles which are compatible with the recent LHC data.

1. Introduction
SUSY and its spontaneous breakdown are profound notions essentially related to the space-time symmetry. Therefore it is natural to study them in the framework describing the particle physics and the cosmology (gravitation) as well. $\text{SO}(N)$ super-Poincaré (sP) symmetry gives a natural framework. We have found by the group theoretical arguments that among all $\text{SO}(N)$ sP groups only $\text{SO}(10)$ sP shows at the low energy the standard model (SM) with just three generations of quarks and leptons in the single irreducible representation, where we have adopted the decomposition $10_{\text{SO}(10)} = 5_{\text{SU}(5)} + 5^*_{\text{SU}(5)}$ corresponding to $\text{SO}(10) \supset \text{SU}(5)$ and assigned to $5_{\text{SU}(5)}$ the same quantum numbers as those of $5$ of SU(5) GUT. Therefore it is an interesting problem to construct $N = 10$ SUSY theory in curved spacetime. For this purpose we must overcome the so called no-go theorem of the S-matrix arguments in the local field theory for $N > 8$ SUSY. We attempt to circumvent the no-go theorem by the degeneracy of the vacuum (flat space-time). We consider new (unstable) space-time whose tangent space is specified by the coordinates $\text{SO}(1,3)$ and $\text{SL}(2,\mathbb{C})$ suggested by NLSUSY. Extending the geometric arguments of GR on ordinary Riemann space-time to new space-time we construct Einstein-Hilbert type action (NLSUSYGR) which is invariant under NLSUSY transformation. We discuss in this article some basic ideas and some consequences for the low energy particle physics and the cosmology of NLSUSY GR and the subsequent (space-time) phase transitions. As the physical and the simplest case we consider $N = 2$ case and show that the lepton sector of the standard model (SM) with U(1) gauge symmetry emerges as the composites of the fundamental Nambu-Goldstone (NG) fermion in the true vacuum of flat space-time, i.e. the true vacuum of N=2 NLSUSY GR is achieved by the compositeness of particles of the SM.

2. Nonlinear supersymmetric general relativity (NLSUSY GR)
Nonlinear supersymmetric general relativity theory (NLSUSY GR) [3] is based upon the general relativity (GR) principle and the nonlinear (NL) representation [4] of supersymmetry (SUSY) [5]. In NLSUSY GR, 4 dimensional new space-time [2], as ultimate shape of nature, is introduced,
where tangent flat space-time has the NLSUSY structure, i.e. tangent space-time is specified by not only the $SO(1,3)$ Minkowski coordinates $x_a$ but also $SL(2,C)$ Grassmann coordinates $\psi^i_{\alpha}$ ($i=1,2,\cdots,N$) for NLSUSY. The Grassmann coordinates in new space-time are cost space coordinates of \( \frac{GL(4,R)}{GL(4,R)} \), which can be interpreted as the NG-fermions(superon) for NLSUSY associated with the spontaneous breaking of super-\( GL(4,R) \) down to \( GL(4,R) \). By extending the geometrical arguments of Einstein GR in ordinary Riemann space-time to new space-time, we obtain the fundamental action(NLSUSY GR theory) of the Einstein-Hilbert (EH) form. The fundamental EH-type action of NLSUSY GR is given by [3]

\[
L_{\text{NLSUSYGR}}(w) = -\frac{c^4}{16\pi G} |w| \{|\Omega(w) + \Lambda, \},
\]

where $G$ is the Newton gravitational constant, $\Lambda$ is a (small) cosmological constant related to the NLSUSY structure in Riemann flat $e^a_\mu$ $\rightarrow$ $\delta^a_\mu$ space-time, $\Omega(w)$ is the unified Ricci scalar curvature of new space-time in terms of the unified vierbein $w^a_\mu(x)$ (and the inverse $w_a^\mu$) defined by

\[
w^a_\mu = e^a_\mu + t^a_\mu(\psi), \quad t^a_\mu(\psi) = \frac{\kappa}{2\kappa} (\bar{\psi}^j \gamma^a \partial^\mu \psi^j - \partial^\mu \bar{\psi}^j \gamma^a \psi^j),
\]

and $|w| = \det w^a_\mu$. In Eq.(2), $e^a_\mu$ is the ordinary vierbein of GR for the local $SO(3,1)$, $t^a_\mu(\psi)$ is the mimic vierbein for the local $SL(2,C)$ $\psi^i(x)$ which is subsequently recasted as the stress-energy-momentum tensor of the NG fermion $\psi^i(x)$ created by the subsequent spontaneous SUSY breaking and $\kappa$ is an arbitrary constant of NLSUSY with the dimemsion (mass)$^{-2}$. Note that $e^a_\mu$ and $t^a_\mu(\psi)$ contribute equally to the curvature of space-time, which realizes the Mach’s principle in ultimate space-time.

The NLSUSY GR action (1) possesses promising large symmetries isomorphic to $SO(N)$ ($SO(10))$ sP group [6]; $L_{\text{NLSUSYGR}}(w)$ is invariant under

\[\text{[new NLSUSY]} \otimes \text{[local GL(4,R)]} \otimes \text{[local Lorentz]} \otimes \text{[local spinor translation]} \otimes \text{[local SO(N)]} \otimes \text{[local U(1)^N]} \otimes \text{[Chiral]}.\]

Note that the no-go theorem is overcome (circumvented) in a sense that the nontrivial $N$-extended SUSY gravity theory with $N > 8$ has been constructed in the NLSUSY invariant way, i.e. by the degenerate vacuum (flat space-time).

3. Particle astrophysics of NLSUSY GR

New empty (besides the constant energy density $\Lambda$) space-time for everything described by the (matter free) EH-type NLSUSY GR action (1) is unstable due to NLSUSY structure of tangent space-time, i.e. the positive potential minimum $V_{P,E} = \Lambda > 0$. It decays (called Big Decoy) spontaneously to ordinary Riemann space-time with the NG fermion (superon matter) described by the ordinary EH action with the cosmological term, the NLSUSY action for the N NG fermions and their gravitational interactions, called SGM action for superon-graviton(SG) space-time. By expanding (1) around $e^a_\mu$ the SGM action is given as follows;

\[
L_{\text{SGM}}(e, \psi) = -\frac{c^4}{16\pi G} e |w_{\text{NLA}}| \{|R(e) + \Lambda + T(e, \psi)\}
\]

where $R(e)$ is the Ricci scalar curvature of ordinary EH action, $T(e, \psi)$ represents highly nonlinear gravitational interaction terms of $\psi^i$, and $|w_{\text{NLA}}| = \det w^a_b = \det(\delta^a_b + t^a_b)$ is the NLSUSY invariant volume[4]. We can easily see that the cosmological term in $L_{\text{NLSUSYGR}}(w)$ of Eq.(1) (i.e. the constant energy density of ultimate space-time) mediated to the second term in
SGM action (3) reduces to the NLSUSY action [4], \[ L_{\text{NLSUSY}}(\psi) = -\frac{1}{2\kappa^2} |w| \] in Riemann-flat space-time, i.e. the arbitrary constant \( \kappa \) of NLSUSY is now fixed to

\[ \kappa^{-2} = \frac{c^4 A}{8\pi G}. \] (4)

The Big Decay is the phase transition of space-time, which besides the fermion of the spontaneous breakdown of SUSY(SSB) produces a fundamental mass scale depending on the \( \Lambda \) and \( G \) through the relation (4). We will show that the effect of Big Decay is mediated to the (low energy) particle physics content which is tractable and SUSY is also the case. We will investigate the low energy physics of supermultiplet \( \psi^i \) (i = 1, 2) is written in \( d = 2 \) as follows;

\[ L_{N=2} = \frac{1}{2} (\bar{\psi} \gamma^a \psi) \bar{D}^a \psi - \frac{1}{2} \kappa (\bar{\psi} \gamma^a \psi) \bar{D}^a \psi - \bar{\psi} \gamma^a \bar{D}^a \psi \] (6)

where \( \kappa \) is a constant with the dimension (mass)\(^{-1} \), which satisfies the relation (4).

On the other hand, in Eq.(5), the \( N = 2 \) LSUSY QED action \( L_{N=2 \text{SUSYQED}}(V, \Phi) \) is constructed from a \( N = 2 \) minimal off-shell vector supermultiplet and a \( N = 2 \) off-shell scalar supermultiplet denoted \( V \) and \( \Phi \) respectively. Indeed, the most general \( L_{N=2 \text{SUSYQED}}(V, \Phi) \) in \( d = 2 \) with a Fayet-Iliopoulos (FI) \( D \) term and Yukawa interactions, is given in the explicit component form as follows for the massless case;

\[
L_{N=2 \text{SUSYQED}}(V, \Phi) = -\frac{1}{4} (F_{ab})^2 + \frac{i}{2} \bar{\lambda}^i \delta \lambda^i + \frac{1}{2} (\partial_a A)^2 + \frac{1}{2} (\partial_a \phi)^2 + \frac{1}{2} D^2 - \frac{\xi}{\kappa} D + \frac{i}{2} \chi \bar{\lambda}^i \lambda^i + \frac{1}{2} (\partial_a B^i)^2 + \frac{i}{2} \bar{\nu} \delta \nu + \frac{1}{2} (F^i)^2 + f(A \bar{\lambda}^i \lambda^i + \bar{\nu} \lambda^i \phi) - A^2 D + \phi^2 D + e^{ab} A \phi F_{ab} + i \nu \bar{\lambda}^i \nu - e^{ij} v^a B^i \partial_a B^j + \bar{\lambda}^i \lambda^i + e^{ij} \bar{\lambda}^i \nu B^j
\]

\[
-\frac{1}{2} D(B^i)^2 + \frac{1}{2} A(\bar{\lambda} \lambda + \bar{\nu} \nu) - \phi \bar{\lambda}^i \nu \]

\[+ \frac{1}{2} e^2 (v_a^2 - A^2 - \phi^2)(B^i)^2. \] (7)

where \((v^a, \lambda^i, A, \phi, D) \) \( (F_{ab} = \partial_a v_b - \partial_b v_a) \) are the staffs of the minimal off-shell vector supermultiplet \( V \) representing \( v^a \) for a \( U(1) \) vector field, \( \lambda^i \) for doublet (Majorana) fermions,
relations for $\Phi$ found one of the vacua where $\xi$ we obtain the particle (mass) spectra of the linearized theory in SGM scenario [12, 13]. The relation (5) are shown explicitly (and systematically) by $\psi$ fermion self-interaction terms (i.e. the condensation of $SO(2)$) overall parameters in the SUSY invariant relations for $\Phi$ and $(\psi^i)^2 = \psi^i\psi^i, e^{jk}\psi^i\gamma^j\psi^k, e^{jk}\psi^j\gamma^i\psi^k$, etc.. For example, some of SUSY invariant relations for $V$ are

\begin{align*}
v^a &= -\frac{i}{2}\xi\kappa\epsilon^i\bar{\psi}^i\gamma^a\psi^i|w|, \\
\lambda^i &= \xi \left[ \psi^i|w| - \frac{1}{2}\kappa^2\partial_a\{\gamma^a\psi_i\bar{\psi}^j(1-i\kappa^2\bar{\psi}^k\psi^k)\} \right], \\
A &= \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^i|w|, \\
\ldots
\end{align*}

which are promising features for the SGM scenario. The explicit form [12] of the SUSY invariant relations (8) are obtained systematically in the superfield formulation (for example, see Refs. [8, 10, 13]). The familiar LSUSY transformations among the component fields of the LSUSY supermultiplet $(V, \Phi)$ are reproduced among the composite LSUSY supermultiplet by taking the NLSUSY transformations of the constituents $\psi^i$. We just mention that four-NG fermion self-interaction terms (i.e. the condensation of $\psi^i$) appearing only in the auxiliary fields $F_i$ of the scalar supermultiplet $\Phi$ is crucial for the local $U(1)$ gauge symmetry of LSUSY theory in SGM scenario [12, 13]. The relation (5) are shown explicitly (and systematically) by substituting Eq. (8) into the LSUSY QED action (7) [12, 13].

Now we briefly show the (physical) true vacuum structure of $N = 2$ LSUSY QED action (7) related (equivalent) to the $N = 2$ NLSUSY action (6) [14]. The vacuum is determined by the minimum of the potential $V_{P.E.}(A, \phi, B^i, D)$ in the action (7). The potential is given by using the equation of motion for the auxiliary field $D$ as

\begin{align*}
V_{P.E.}(A, \phi, B^i) = \frac{1}{2}f^2 \left\{ A^2 - \phi^2 + \frac{e}{2f}(B^i)^2 + \frac{\xi}{f\kappa} \right\}^2 + \frac{1}{2}e^2(A^2 + \phi^2)(B^i)^2 \geq 0, \tag{9}
\end{align*}

The configurations of fields corresponding to true vacua $V_{P.E.}(A, \phi, B^i) = 0$ in $(A, \phi, B^i)$-space in the potential (9) are classified according to the signatures of the parameters $e, f, \xi, \kappa$.

By adopting the simple parametrization $(\rho, \theta, \phi, \omega)$ for the vacuum configuration of $(A, \phi, B^i)$-space and by expanding the fields $(A, \phi, B^i)$ around the vacua, e.g.

\begin{align*}
A &= -(k + \rho) \cos \theta \cos \varphi \cosh \omega, \\
\phi &= (k + \rho) \sinh \omega, \\
B^1 &= (k + \rho) \sin \theta \cosh \omega, \\
B^2 &= (k + \rho) \cos \theta \sin \varphi \cosh \omega.
\end{align*}

we obtain the particle (mass) spectra of the linearized theory $N = 2$ LSUSY QED. We have found one of the vacua $V_{P.E.}(A, \phi, B^i) = 0$ describes $N = 2$ LSUSY QED containing
one charged Dirac fermion ($\psi^{D^c} \sim \chi + i\nu$),
one neutral (Dirac) fermion ($\lambda_D^0 \sim \lambda^1 - i\lambda^2$),
one massless vector (a photon) ($v_\mu$),
one charged scalar ($\phi^c \sim \theta + i\varphi$), one neutral complex scalar ($\phi^0 \sim \rho(+i\omega)$),

with masses $m_{\phi^c}^2 = m_{\lambda_D^0}^2 = 4f^2k^2 = -\frac{4e^2}{\kappa}$, $m_{\psi^{D^c}}^2 = m_{\phi^c}^2 = e^2k^2 = -\frac{4e^2}{\kappa}$, $m_{v_\mu} = 0$, which are the composites of NG-fermions superon and the vacuum breaks SUSY alone spontaneously (The local $U(1)$ is not broken. For further details, see [14][15]). For large $N$ case we can anticipate the large (broken) SU(N) LSUSY with different large mass scales.

Remarkably these arguments show that the true vacuum of (asymptotic flat space-time of) $L_{N=2\text{SGM}(e,\psi)}$ is achieved by the compositeness(eigenstates) of fields of the supermultiplet of global $N = 2$ LSUSY QED. This phenomena may be regarded as the relativistic second order phase transition of massless superon-graviton system, which is dictated by the symmetry of space-time (analogous to the superconducting states achieved by the Cooper pair). Here we should notice that R-parity (for $N \geq 2$) may not be a good quantum number in the true vacuum of SGM scenario as seen from the particle spectra (without superpartners) mentioned above. These situations are very favourable in constructing the consistent model with the recent LHC data which exclude the TeV scale SUSY breaking. As for the cosmological significances of $N = 2$ SUSY QED in the SGM scenario, the (physical) vacuum for the above model explains (predicts) simply the observed mysterious (numerical) relation between the (dark) energy density of the universe $\rho_D \sim \frac{\xi^4\Lambda}{8\pi G}$ and the neutrino mass $m_\nu$,

$$\rho_D^{\text{obs}} \sim (10^{-12}\text{GeV})^4 \sim (m_\nu)^4 \sim \frac{\Lambda}{G} \sim (g_{\text{sv}})^2,$$

(10)

provided $-\xi f \sim O(1)$ and $\lambda_D^0$ is identified with the neutrino, which gives a new insight into the origin of (small) mass [7, 14] and produce the mass hierarchy by the factor $\xi(\sim O(m_\nu/m_\psi)$ in case of $\psi^{D^c}$ as electron!).

Furthermore, the neutral scalar field $\phi^0(\sim \rho)$ with mass $O(m_\nu)$ of the radial mode in the vacuum configuration may be a candidate of the dark matter, for $N = 2$ LSUSY QED structure and the radial mode in the vacuum are preserved in the realistic large $N$ SUSY GUT model. (Note that $\omega$ in the model is a NG boson and disappears provided the corresponding local gauge symmetry is introduced as in the standard model.) These arguments show the potential of the SGM scenario which gives unified understandings for particle physics and cosmology. The no-go theorem for $N > 8$ SUSY may be overcome(irrelevant) in a sense that the linearized (equivalent) $N > 8$ LSUSY theory would be massive theory with SSB.

Recently, by taking the more general auxiliary-field structure for the general off-shell vector supermultiplet [16] and discussing the NL/L SUSY relation describing the phase transition to the true vacuum we have shown that the magnitude of the bare (dimensionless) gauge coupling constant $e$ (i.e. the fine structure constant $\alpha = \frac{e^2}{\pi}$) is expressed (determined) in terms of vacuum values of auxiliary-fields [16]:

$$e_C = \frac{\ln\left(\frac{e^2}{\pi}\right)}{4\xi_C},$$

(11)

where $e_C$ is the bare gauge coupling constant, $\xi$, $\xi^\xi$ and $\xi_C$ are the vacuum-value scale parameters (the relative magnitudes in the NL/L SUSY relation) of auxiliary-fields of the general off-shell supermultiplet in $d = 2$. This mechanism is natural and very favourable for SGM scenario as a theory for everything.
4. Conclusions

We have proposed a new paradigm for describing the unity of nature, where the ultimate shape of nature is new unstable ($V_{P,E} > 0$) space-time described by the NLSUSY GR action $L_{\text{NLSUSYGR}}(w)$ in the form of the free EH action for empty space-time with the constant energy density. Big Decay of new space-time $L_{\text{NLSUSYGR}}(w)$ creates ordinary Riemann space-time with massless spin-$\frac{1}{2}$ superon described by the SGM action $L_{\text{SGM}}(e, \psi)$ with ($V_{P,E} > 0$) and ignites Big Bang of space-time and matter accompanying the dark energy (cosmological constant).

Interestingly on Riemann-flat tangent space (in the local frame), the familiar renormalizable LSUSY theory emerges on the true vacuum ($V_{P,E} = 0$) of SGM $L_{\text{SGM}}(e, \psi)$ as composite-eigenstates of superon. We can anticipate in the true vacuum the larger gauge symmetry and the consequent different mass scales for the NL/L SUSY relation for the lager $N$. We have seen that the physics before/of the Big Bang may play crucial roles for understanding unsolved problems of the universe and the particle physics.

In fact, we have shown explicitly that $N = 2$ LSUSY QED theory as the realistic $U(1)$ gauge theory emerges in the physical field configurations on the true vacuum of $N = 2$ NLSUSY theory on Minkowski tangent space-time, which gives new insights into the origin of mass and the cosmological problems. The cosmological implications of the composite SGM scenario seems promising but deserve further studies.

Remarkably the physical particle states of $N = 2$ LSUSY as a whole look the similar structure to the lepton sector of ordinary SM with the local $U(1)$ and the implicit global $SU(2)$ [11] disregarding the R-parity, i.e. without the trivial superpartner. Such SUSY breaking mechanism may allow the model construction which is compatible with the recent LHC data. (Note that the scalar mode $\omega$ is a NG boson and disappears provided the corresponding local gauge symmetry is introduced.) We anticipate that the physical consequences obtained in $d = 2$ hold in $d = 4$ as well, for the both have the similar structures of on-shell helicity states of $N = 2$ supermultiplet though scalar fields and off-shell (auxiliary field) structures are modified (extended). However, the similar investigations in $d = 4$ are urgent for the realistic model building based upon SUSY.

The extension to large $N$, especially to $N = 5$ is important for superon quintet hypothesis of SGM scenario with $N = 10 = 5 + 5^*$ for equipping the $SU(5)$ GUT structure [2] and to $N = 4$ may shed new light on the mathematical structures of the anomaly free non-trivial $d = 4$ field theory. ($N = 10$ SGM predicts double-charge heavy lepton state $E^{2+}$ and new one neutral singlet massive vector state[1]). Further investigations on the spontaneous symmetry breaking for $N \geq 2$ SUSY remains to be studied. It may be helpful to point out the formal similarity between superconductivity or superfluidity and SGM scenario.

Linearizing SGM action $L_{\text{SGM}}(e, \psi)$ on curved space-time, which elucidates the topological structure of space-time [19], is a challenge. The corresponding NL/L SUSY relation will give the supergravity (SUGRA) [17, 18] analogue with the vacuum which breaks SUSY spontaneously.

Locally homomorphic non-compact groups $SO(1,3)$ and $SL(2,C)$ for space-time degrees of freedom are analogues of compact groups $SO(3)$ and $SU(2)$ for gauge degrees of freedom of ’t Hooft-Polyakov monopole and are special, because they are unique homomorphic groups among $SO(1,D-1)$ and $SL(d,C)$, i.e. $\frac{D(D-1)}{2} = 2(d^2 - 1)$ holds for $D = 4, d = 2$. NLSUSYGR/SGM scenario predicts four dimensional space-time.

Finally we just mention that NLSUSY GR and the subsequent SGM scenario for the spin-$\frac{3}{2}$ NG fermion [6, 20] is in the same scope. Our discussion shows that considering the physics before/of Big Bang may be significant for cosmology and the (low energy) particle physics as well.
5. References

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