Backward Raman scattering in relativistic electron beam and intense THz light

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A new type of the THz laser is proposed. A coherent tera-hertz light is emitted through the backward Raman scattering between a visible light laser and a relativistic electron beam. The threshold conditions for the laser intensity and the electron beam density are identified. This scheme may lead to one of the most intense tera-hertz coherent light sources.

PACS numbers: 42.55.Vc, 42.65.Ky, 52.38.-r, 52.35.Hr

An intense and compact THz light source is critical component in the biomedical image, the tomography, the molecular spectroscopy, the telecommunication and many others \[1^\text{-}7\]. Many THz light sources have been invented \[8^\text{-}22\], but even the most advanced ones are neither practical nor intense enough: The free electron laser \[17^\text{-}19\] needs expensive magnets and accelerators, the quantum cascade laser \[15^\text{-}16\] cannot produce intense THz light neither be operated in the room temperature and the gyrotron suffers the scale problem unless the magnetic field is ultra intense \[11^\text{-}14\]. The current inability to produce the power (intensity) high enough for various applications aforementioned is referred as “THz gap”.

In this paper, the author proposes a new process of the THz light generation by amplifying the interaction between an intense visible-light laser and a relativistic electron beam via the backward Raman scattering (BRS). To my knowledge, it is the first scheme, in which a visible light wave is shifted down into THz light via the backward Raman scattering (BRS). The scheme may lead to one of the most intense tera-hertz coherent light sources.

To begin with, consider a relativistic electron beam and a co-traveling laser (pump laser). Let us denote the beam density in the laboratory frame by \(n_0\) and the beam relativistic factor by \(\gamma_0 = (1 - v_0^2/c^2)^{-1/2}\), where \(v_0\) is the velocity of the electron beam. In the co-traveling frame, the electron density becomes \(n_1 = n_0/\gamma_0\) due to the length dilation. The BRS between the laser (the pump pulse) and the electron beam could emit the THz light (the seed pulse). The 1-D BRS three-wave interaction in the co-traveling frame is \[23\]:

\[
\begin{align*}
\left( \frac{\partial}{\partial t} + v_{p} \frac{\partial}{\partial x} + \nu_{1} \right) A_{p} &= -ic_{p}A_{s}A_{3}, \\
\left( \frac{\partial}{\partial t} + v_{s} \frac{\partial}{\partial x} + \nu_{2} \right) A_{s} &= -ic_{s}A_{p}A_{s}^{*}, \\
\left( \frac{\partial}{\partial t} + v_{3} \frac{\partial}{\partial x} + \nu_{3} \right) A_{3} &= -ic_{3}A_{p}A_{s}^{*},
\end{align*}
\]

where \(A_{i} = eE_{i1}/m_{e}\omega_{i1}c\) is the ratio of the electron quiver velocity of the pump pulse \((i = p)\) and the seed pulse \((i = s)\) relative to the speed of light \(c\), \(E_{i1}\) is the electric field of the seed (pump) pulse, \(A_{3} = \delta n_{1}/n_{1}\) is the Langmuir wave amplitude, \(\nu_{1} (\nu_{2})\) is the inverse bremsstrahlung rate of the pump (seed), \(\nu_{3}\) is the plasmon decay rate, \(c_{i} = \omega_{i}^{2}/2\omega_{i1}\) for \(i = p, s, c_{3} = (ek_{3})^{2}/2\omega_{3}\), \((\omega_{3})\) is the frequency of the pump (seed) laser and \(\omega_{3} = \omega_{pc}/\sqrt{\gamma_{0}}\) is the plasmon fre-
frequency. In the co-traveling frame, the wave vector (frequency) of a photon satisfies the usual dispersion relation, \( \omega^2 = \omega_{pe}^2/\gamma_0 + c^2 k_1^2 \), where \( \omega_1 \) and \( k_1 \) are the photon wave frequency and the corresponding vector, and \( \omega_{pe}^2 = 4\pi n_0 e^2/m_e \) is the plasmon frequency. Denote the wave vector and the frequency of the pump laser (seed pulse or THz light) in the co-traveling frame as \( k_{p1} \) and \( \omega_{p1} \) (\( k_{s1} \) and \( \omega_{s1} \)) and their laboratory frame counterparts as \( k_{p0} \) and \( \omega_{p0} \) (\( k_{s0} \) and \( \omega_{s0} \)). The Lorentz transformation leads to the following relationship:

\[
\begin{align*}
\omega_{p0} &= \gamma_0 \left[ \frac{\omega_{pe}^2}{\gamma_0} + c^2 k_{s1}^2 + vk_{p1} \right], \\
k_{p0} &= \gamma_0 \left[ k_{p1} + \frac{\omega_{p1} v_0}{c} \right], \\
\omega_{s0} &= \gamma_0 \left[ \frac{\omega_{pe}^2}{\gamma_0} + c^2 k_{s1}^2 - vk_{s1} \right], \\
k_{s0} &= \gamma_0 \left[ k_{s1} - \frac{\omega_{s1} v_0}{c} \right].
\end{align*}
\]  

The energy and momentum conservation of Eq. (1) are given as

\[
\begin{align*}
\omega_{p1} &= \omega_{s1} + \omega_3, \\
k_{p1} &= k_{s1} + k_3,
\end{align*}
\]  

where \( k_3 \) is the wave vector of the plasmon. With a given pump frequency \( \omega_{p0}, k_{p1} (\omega_{p1}) \) is determined from Eq. (2), \( k_{s1} (\omega_{s1}) \) is determined from Eq. (3), and, finally, \( k_{s0} (\omega_{s0}) \) is determined from Eqs. (4) and (5). In the limiting case \( c k_{s1} \gg \omega_3, \omega_{s0} \approx (1/2\gamma_0)(\omega_{p1} - \omega_3) \) or

\[
\omega_{s0} \approx \frac{1}{4\gamma_0} \left[ \omega_{p0} - 2\omega_{pe}\sqrt{\gamma_0} \right],
\]  

where \( \omega_{p1} \approx \omega_{p0}/2\gamma_0 \) and \( \omega_3 \approx \omega_{pe}/\sqrt{\gamma_0} \). Eq. (7) describes the frequency down-shift of the visible-light pump laser into the THz light via the relativistic Doppler effect. For instance, if \( \gamma_0 = 3 \), the down-shifted frequency would be 0.8 THz for the CO2 laser whose frequency is 30 THz.

The BRS growth rate can be obtained from Eq. (1). When \( |A_p| \gg |A_s| \) and \( |A_p| \gg |A_3| \), the linearization of Eq. (1) in the expansion of \( A_{s,3}(t) = A_{s,3}(\omega) \exp(\imath \omega t) \), leads to

\[
\omega^2 + (\nu_3 + \nu_2)\omega + (\nu_3 \nu_2 - c_3 c_3 |A_p|^2) = 0,
\]  

If \( \nu_2 = \imath \omega_{p0}/2\gamma_0 \), the growth rate, the imaginary part of the solution of Eq. (8), is \( \Gamma_1 \approx \sqrt{c_3 c_3 |A_p|^2} \). In the limiting case \( c k_{p0} \gg \omega_3 \), the Lorentz transformation prescribes \( E_{p1}/E_{p0} \approx 1/2\gamma_0 \) and \( A_p = e E_{p1}/m_e \omega_{p1} \) \( c \approx e E_{p0}/m_e \omega_{p0} c \), resulting in

\[
\Gamma_1 \approx \sqrt{\omega_{pe}^2 \omega_{p0}/2\gamma_0^3/2} |A_p|.
\]  

Denoting the electron beam length as \( L_b \) and the laser length as \( L_l \), the beam length in the co-traveling frame increases to \( \gamma_0 L_b \) and the laser length increases to \( 2\gamma_0 L_l \) so that the interaction time between the beam and the laser is \( \tau \approx \min(\gamma_0 L_b, 2\gamma_0 L_l)/c \). The gain-per-length \( g \) is therefore estimated as

\[
\begin{align*}
g &= \Gamma_1 \tau/L_b = \gamma_0 \Gamma_1/c \quad \text{if} \ L_b < 2L_l, \\
g &= \Gamma_1 \tau/L_l = 2\gamma_0 \Gamma_1/c \quad \text{otherwise}. \quad (10)
\end{align*}
\]

An estimation of the energy conversion efficiency of the pump to the seed pulse is as follows. Practical applications of the BRS compression of the visible-light lasers have demonstrated that a significant portion of the pump energy can be converted to the seed pulse. \( [23, 26, 30] \). Denote the total energy of the pump laser by \( E_{p0} \), which becomes \( E_{p1} = E_{p0}/2\gamma_0 \) in the co-traveling frame, and denote the conversion efficiency in the co-traveling frame by \( \epsilon_1 \). Then, the energy transferred from the pump to the seed is \( E_{s1} = \epsilon_1 E_{p0}/2\gamma_0 \), which is \( E_{s0} = \epsilon_1 E_{p0}/4\gamma_0^2 \) in the laboratory frame. Therefore, the conversion efficiency in the laboratory frame is

\[
\epsilon_0 \approx \epsilon_1/4\gamma_0^2.
\]  

| Type | Freq | \( \gamma_0 \) | \( A_1 \) | \( \Gamma_{13} \) | \( g_1 \) | \( g_2 \) | \( n_e \) |
|------|------|--------------|--------------|--------------|--------------|--------------|--------------|
| N    | 3.0  | 1.00 5.0 0.020 0.0245 40.87 81.748 2.56 |
| N    | 3.0  | 10.5 5.0 0.6324 0.775 1292 2585 2.56 |
| N    | 3.0  | 0.01 5.0 0.0006 0.0008 1.29 2.58 2.56 |
| N    | 8.3  | 1.0 3.0 0.02 0.0359 35.97 71.95 7.11 |
| N    | 8.3  | 10.5 3.0 0.63 1.14 1137 2275 7.1 |
| C    | 8.3  | 0.01 3.0 0.002 0.0036 3.597 7.19 7.1 |
| C    | 0.83 | 1.0 3.0 0.2 0.113 113.75 2275 0.007 |
| C    | 0.83 | 30.3 3.0 1.0 0.62 623 1246 0.007 |
| C    | 1.88 | 0.01 3.0 0.2 0.15 103 206 0.016 |
| C    | 1.88 | 0.01 2.0 0.02 0.015 10.28 20.56 0.016 |

The CO2 laser has the wavelength of 10 \( \mu m \) and the Nd:YAG laser has the wavelength of 1 \( \mu m \). From Eq. (7), for a fixed \( \omega_{p0} \), the required relativistic factor for the CO2 laser should be lower than the Nd:YAG laser by a factor of \( \sqrt{10} \) and thus the conversion efficiency \( \epsilon_0 \) for the CO2 laser will be larger than the Nd:YAG laser by a factor 10 for the same \( \epsilon_1 \). For \( \gamma \approx 3 \), the conversion efficiency for the CO2 laser can be as high as a few percents, assuming that \( \epsilon_1 \) is a few tens of percents. In order for the current scheme to work, there exist a few necessary conditions for the laser and the electron
beam. One necessary condition is $\Gamma_1 \tau > 1$ for a sufficient amplification or

$$|A_p| > \frac{21/2}{\gamma_0^{3/4}} \frac{c}{\text{min}(L_1, L_2)} \left( \frac{1}{\omega_{p0}\omega_{p0}} \right)^{1/2},$$

which is readily satisfied by currently available intense visible-light laser $[31-34]$. Another necessary condition for the electron beam density is $n_0/\gamma_0 > k_3^2/(2\pi)^3$, as only the plasmons with $n_0/\gamma_0 \gg k_3^3$ are collective waves. In the limiting case $k_{p1} \gg \omega_3$, the condition is

$$n_0 > \gamma_0^{-2} \left( \frac{k_{p0}^3}{8\pi^3} \right) \approx 64 \times \gamma_0^4 \left( \frac{k_{p0}^3}{8\pi^3} \right),$$

where $k_{p1} \approx k_{s1} \approx k_{s0}/2\gamma_0$ and $k_{s0} \approx k_{p0}/4\sqrt{\gamma_0}$. In addition, the wave vector of the Langmuir wave should be larger than the Debye length

$$k_3 < \lambda_{de}^{-1}$$

where $\lambda_{de}^{-2} = 4\pi n_0 e^2/\gamma_0 m_e$. The condition given by Eqs. (13) and (14) estimates the minimum electron beam density for the BRS compression for a given electron temperature. Note that much higher density can be achieved through the current electron accelerator or dense electron beams $[24, 32-38]$ while the electron temperature is low enough to satisfy Eq. (14).

The estimations for various electron beams and lasers are provided in the Table I. The Table suggests that the proposed scheme is plausible for a wide range of frequencies, the growth rate (the gain-per-length) can be as high as $10^{13}/\text{sec} (10^{13}/\text{cm})$ and the requirement of the laser intensity, $I > 10^{12} \text{W/cm}^2$, is moderate. If $J_{15} \geq 10^3$ for the case of the Nd:YAG laser as in Table I the electron quiver velocity becomes marginally relativistic and the full relativistic treatment is necessary $[39]$, which is ignored in this paper.

In Fig. (2), a 1-D simulation of Eq. (1) is performed, where the pump laser is the CO2 laser with $I = 6 \times 10^{12} \text{W/cm}^2 (A_p = 0.02)$. The pump laser (the seed pulse) moves from the left (right) to the right (left) and the THz pulse extracts the energy from the pump laser via the BRS, resulting the energy gain of the THz pulse by a factor of 1000. In this example, the pump laser has the intensity of $I = 1.5 \times 10^{12} \text{W/cm}^2 (I = 6 \times 10^{12} \text{W/cm}^2)$ in the co-moving frame (the laboratory frame); the intensity of the pump laser (THz pulse) is higher (lower) by 36 times in the laboratory frame than in the co-moving frame due to the Doppler’s effect. In the simulation shown in Fig. (2), the final peak intensity of the THz pulse in the co-moving frame is comparable to the intensity of the pump laser; the attained peak intensity of the THz pulse is $I = 1.1 \times 10^{10} \text{W/cm}^2$ in the laboratory frame, which is 0.1 percent of the laser pump intensity. Various simulation suggests that the attained peak intensity of the THz pulse in the laboratory frame can be as high as the the laser pump intensity.
peak THz intensity can be comparable the pump laser intensity. In addition, because it does not need expensive magnets or high quality electron beam, the disclosed light source has advantages in the compactness, the mobility and the operating (construction) cost. While some researches have attempted to shift the visible light laser up to the XUV regime via the BRS [39–41], the new idea for the scheme presented here is that the visible-light laser is shifted down to the THz regime via the BRS.

There are technical issues to consider in the realization of the proposed scheme. The performance of the proposed scheme relies on the quality of the electron beam such as the uniformity and the time duration in which the beam maintains its quality. Also, a rather intense seed THz pulse would be needed in order to extract the significant fraction of the pump pulse energy. However, even if the difficulties mentioned compromises the efficiency of the scheme proposed to some degree, its prospect as a powerful THz light source remains high.

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