Research on Nonlinear Opto-Mechanical System for Improving Detection Performance

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Abstract. In this paper, the use of optical parametric amplifier to effectively improve the detection performance of nonlinear opto-mechanical system is studied theoretically. At the same time, it is found that by adjusting the nonlinear gain of the optical parametric amplifier and the effective detuning between the input light field and the cavity field, the quantum noise of about 14dB in the squeezed quadrature can be reduced, and the sensitivity of weak force detection can be increased by more than 2 times. On the other hand, the phenomenon of reaction noise cancellation caused by nonlinear cavity combined with homodyne measurement is analyzed and the method of quantum non-destructive measurement is realized, which provides technical reference for weak force detection of nonlinear opto-mechanical system.

Keywords: Nonlinear optics, opto-mechanical system, optical parametric amplifier, weak force detection.

1. Introduction

Optical resonator and mechanical resonator are coupled together through radiation pressure to form cavity opto-mechanical system [1-2]. With the development of nanotechnology, cavity optical mechanical systems are now widely used, such as compressed light [3], quantum ground state cooling [4], photo force-induced transparency [5], etc. A very important application that is most relevant to the study in this paper is weak force detection based on cavity opto-mechanical system [6]. Weak force detection based on cavity optical-mechanical system requires a mechanical resonator with high quality factor as the detection mass. The weak force to be measured on the mechanical resonator is converted into the time-dependent displacement of the resonator. The displacement of the resonator is monitored by interference technology, so the weak force can be detected by optical signal. Light radiation pressure coupling between the force model, however, will produce the photon shot noise and quantum effect on noise, shot noise decreases with the increase of input power, and quantum effect on noise as the input power increases, so the photon shot noise and noise combined reaction and there is a minimum, the minimum value is the standard quantum limits in the detection of weak light mechanical...
systems. Braginsky first proposed that the avoidance of reaction noise through quantum non-destructive measurement could break through the standard quantum limit [7-8].

Recently, it has been proposed that high precision displacement measurement can be achieved by using optical parametric amplifier to realize in-field compression [9]. This study proves that the nonlinear cavity as a phase-sensitive parametric amplifier can amplify the amplitude quadrature components of the input field. At the same time, the phase quadrature component containing all mechanical information is attenuated. Although the signal in the phase quadrature component is attenuated, the quantum noise in the quadrature component is attenuated to a greater extent, thus improving the signal-to-noise ratio (SNR). On the other hand, the dissipative coupled cavity opto-mechanical system assisted by OPA can also improve the sensitivity of weak force detection. OPA nonlinear cavity opto-mechanical system can also be used to improve the cooling and compression of mechanical modes. Enhanced optical-mechanical coupling and adjustable normal mode splitting.

In this paper, the sensitive performance of degenerate OPA nonlinear cavity opto-mechanical system for weak force detection is analyzed. Compared with the previous displacement measurement under mechanical resonance frequency (detection frequency $\omega$ equals mechanical resonance frequency $\omega_m$), this paper will discuss the weak force sensing performance of the system in a wider frequency range ($\omega$ resonance $\omega_m$). The text will be divided into four parts: the model of this paper. Calculate the dynamic evolution process of the system, numerical simulation analysis, discuss the influence of effective detuning and nonlinear gain on weak force measurement, and combine nonlinear system with variable output measurement scheme. a quantum non-destructive measurement method which can realize the cancellation of reaction noise is analyzed theoretically.

2. Brief introduction of the model

In this paper, considering the dispersion force type optical resonator and degenerate optical parametric amplifier composite system. One side of the system and the spring connected to the mobile cavity mirror can be used as an effective mass $m$ mechanical resonator, its resonant frequency and mechanical damping are $\omega$ respectively, while the other side of the cavity mirror fixed $\omega_m, y_m$ .The dimensionless forms of displacement $x$ and momentum $P$ in the mechanical model are respectively expressed as:

$$X = x/x_{zp}, P = p/p_{zp} \quad (1)$$

And satisfies the Bose commutation relation $[X, P] = I$. In the formula, $x_{zp} = /m\omega_m, p_{zp} = /m\omega_m$ represents the zero fluctuation of the displacement and momentum operators, respectively. Using the input light field $a_{in}$ to drive the optical cavity with a resonant frequency $\omega_a$, the cavity field and the mechanical resonator are coupled by radiation pressure interaction, and the coupling intensity $g_0 = x_{zp}x_{a}/L L$. Where $L$ is the length of the cavity field. The degenerate OPA with a nonlinear gain of $G$ is usually used to generate squeeze. Due to the interaction between the pump field and the second-order nonlinear optical crystal, the output frequency is equal to the $1_{\text{max}}$ of the pump frequency $\omega_p$. According to the phase shift produced by the output field, the weak force $F_{\text{ex}}$ applied on the mechanical resonator can be detected by the system.

Taking the input light field frequency $\omega_l$ as the rotation frame, the total Hamiltonian of the system

$$H = H_0 + H_{om} + H_{opa}$$

$$H_0 = h\Delta \alpha^* \alpha + \hbar \omega_m (X^2 + P^2)$$

$$H_{om} = h\omega_m \alpha^* \alpha X$$

$$H_{opa} = i\hbar G (\alpha^* \alpha^2 - \alpha^2) \quad (2)$$

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3. Numerical simulation analysis

\[ X_a(\omega) = P_a(\omega) \], that is the particle noise is equal to the reaction noise, the corresponding effective optical force coupling intensity for the optimized measurement intensity \( g_{opt} \), using the corresponding input power at this time can achieve the best sensitivity. The effects of adjusting the nonlinear gain and effective detuning on sensitivity after optimizing the input power will be discussed next. In order to carry out specific numerical analysis, feasible parameters in the experiment are selected, as shown in Table 1.

| Parameters | \( \omega_f/(2\pi) \) | \( \gamma_m/(2\pi) \) | \( \omega_m/(2\pi) \) | \( \kappa/(2\pi) \) | \( g_0/(2\pi) \) |
|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Numerical value (Hz) | \( 2 \times 10^{11} \) | \( 10^3 \) | \( 10^7 \) | \( 10^7 \) | \( 10^2 \) |

The function of noise power spectral density changing with detection frequency is fitted numerically, as shown in Figure 1. The solid lines in Figure 1 (a) and (b) represent the standard quantum limit in the case of a standard cavity optical-mechanical system, where the effective detuning is equal to zero and there is no compression, and the adjusted nonlinear gain coincides with the standard quantum limit fitting line.

![Figure 1](image1.png)

**Figure 1.** Noise power density spectrum fitting diagram.

In areas where the detection frequency is lower than the mechanical resonance, the additional noise can be compressed under the standard quantum limit and the sensitivity of the measurement can be improved. However, in this case, the phase rotation degree of the field becomes larger, so there will be limited mechanical motion information contained in the unmeasured output amplitude orthogonal, resulting in certain quantum information loss. In fact, the smaller the effective detuning is and the larger the nonlinear gain is, the smaller the field phase deflection is, thus the quantum information loss can be reduced.

Figure 2 shows the fitting graph of noise power spectrum function in low frequency domain, that is, the noise power spectrum is equal to the standard quantum limit. Below the horizontal line, the additional noise is compressed and breaks the standard quantum limit.
By adjusting the strength of the measurement and the nonlinear gain, it was found that large areas where sensitivity could be improved were contained within the range of $g < g_{SQL}$. In this case, the darker area of $0.2 < G/k < 0.3$ increased sensitivity to a greater extent. In the range of $S_{FF} < 0.5S_{SQL}$ within the dotted line, the sensitivity of nonlinear cavity for weak force detection can be improved by more than 2 times.

4. **Zero deviation measurement analysis**

Effective detuning results in the correlation between amplitude and phase orthogonality, and some mechanical motion information is lost. Next, based on the nonlinear system combined with the variable output, namely the zero difference measurement method, this paper theoretically analyzes a method that can break through the standard quantum limit and has no quantum information loss. Select effective first $\Delta = 0$ harmonic loss makes the amplitude and phase of orthogonal back coupling, all mechanical motion information are included in the quadrature, considering the output from the input in the orthogonal field amplitude and phase of orthogonal signal, the output field orthogonal can be expressed as:

$$
\begin{align*}
    x_{\alpha}^{out} &= K_{xx}(\omega)x_{\alpha}^{in} \\
    p_{\alpha}^{out} &= K_{px}(\omega)x_{\alpha}^{in} + K_{pp}(\omega)p_{\alpha}^{in}
\end{align*}
$$

Since the reaction noise is proportional to the input power, after eliminating the reaction noise, theoretically, increasing the input power and reducing the particle noise can improve the sensitivity infinitely, breaking the standard quantum limit. On the other hand, the noise can be further compressed by adjusting the nonlinear gain parameters. As shown in Figure 3, the simulation curve of the power spectral density changes with the detection intensity after eliminating the reactive noise is fitted. At this time, only the granular noise exists. The triangle fitting line represents the nonlinear gain equal to 0, and the circle fitting line represents the nonlinear gain $G/k = 0.1$. It is found that adjusting the nonlinear gain is not equal to zero, and the particle noise is further compressed. When the measurement intensity is small, the noise can be reduced by about 7dB, thus further improving the measurement sensitivity. In other words, compared with the standard optical force system, the reactive noise phase is realized by means of zero-difference measurement. The sensitivity of the nonlinear optical force cavity can be improved by compressing the particle noise when the measured intensity is small.
5. Conclusion
By introducing a non-degenerate optical parametric amplifier into the opto-mechanical resonator, it is found that the field compression can suppress the quantum noise and improve the sensitivity of weak force detection. Compared with the widely used input variable frequency compressed light scheme, the direct compression in the field can be immune to the influence of input loss. This paper discusses the cavity field and effective mismatch between input light field and the influence of nonlinear gain on detection sensitivity, found at the same time adjust the effective nonlinear gain can inhibit the loss of harmonic measurement output field of quantum noise, and sensitivity can be increased by more than 2 times, but as a result of the existence of effective detuning, to improve the performance of sensitive at the same time there are certain quantum information loss, decrease the detuning and increase the nonlinear gain can inhibit the loss. In addition, by combining the nonlinear cavity with the zero-difference detection, the theoretical analysis can achieve the effect of counteracting noise cancellation so as to achieve the quantum non-destructive measurement and break the standard quantum limit. Meanwhile, by further adjusting the nonlinear gain, the quantum noise compression of 7dB can be achieved when the measurement intensity is small. In the future, the weak force sensing performance of non-degenerate OPA-assisted cavity optical mechanical systems and magneton-photon coupling systems can be further studied.

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