Scale-dependent enstrophy production rates in a turbulent boundary layer

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Received: 27 February 2019; Revised: 29 April 2019; Accepted: 2 June 2019

Abstract
Coarse-graining is indispensable for extracting a hierarchy of vortices in fully developed turbulent flow with multiscale nature. In the present study, for a high-Reynolds-number turbulent boundary layer, we employ two simple coarse-graining methods in real space; namely, a Gaussian filter and the combination of the Gaussian filters at two scales. The former corresponds to a low-pass filter of Fourier modes, while the latter corresponds to a band-pass filter. We also examine two different filter widths for the band-pass filter. Then, we show difference in the statistics of the three filtered fields. Since the velocity gradients in turbulence are mainly determined by the smallest-scale motions, vortical structures identified by the second invariant $Q$ of the velocity gradient coarse-grained by the filters are similar. However, there is difference between low- and band-pass filtered fields in the contribution to the enstrophy production rates. This is because the production rates are determined not only by the magnitude of the strain rates but also by the alignment between the vorticity and the stretching direction. In addition, since vortices are not created in the entire system, the conditional sampling by the value of $Q$ is essential to understand the generation mechanism of the hierarchy of vortices. The conditional analysis of the band-pass filtered fields demonstrates that small-scale vortices in the log layer are stretched by twice-larger vortices, whereas they weaken the twice-larger vortices. This observation is consistent with the picture of the energy cascade. We also show that when using the band-pass filter, these conclusions are robust irrespective of the choice of the filter width.

Keywords: Turbulent boundary layer, Coarse-graining methods, Multiscale vortices, Vortex dynamics, Energy cascade

1. Introduction
Developed turbulence is composed of a hierarchy of multiscale vortices. Although properties of large-scale vortices depend on the information of the boundary condition and/or the external force, statistics of small-scale vortices are independent of them. This small-scale universality predicted by Kolmogorov (1941) is explained in terms of the energy cascade; namely, through the energy transfer from large-scale eddies to small-scale eddies, small-scale statistics become independent of the large-scale characteristics. In the past decade, the physical mechanism of the energy cascade was investigated by means of the direct numerical simulations (DNS) of high-Reynolds-number turbulence in a periodic cube (for example, Goto, 2008, 2012; Goto et al., 2017). The main conclusion of these previous studies is that the energy is transferred from large to small scales through the events where anti-parallel vortex pairs stretch and create smaller-scale vortices around them.

These studies on homogeneous isotropic turbulence imply that the key to understanding the sustaining mechanism of developed turbulence is the multiscale coherent structures. The hierarchy of coherent structures in wall-bounded turbulence is, however, more complicated. The self-similar structure of the low- and high-speed streaks has been observed in many experimental and numerical studies (see the reviews by McKeon, 2017; Jiménez, 2018). Since the velocity is mainly determined by large-scale flow, it is relatively easy to identify the hierarchy of the streak structures by directly using simulated or measured velocity fields. However, to understand the dynamics of the interactions among different
scales, it is also necessary to extract the hierarchy of coherent vortices. In the present study, we deduce the hierarchy of multiscale coherent vortices in a turbulent boundary layer. For this purpose, coarse-graining is inevitable. For periodic turbulence, which is a model of turbulence away from solid walls, we can use a low-pass or band-pass filter of the Fourier modes. On the other hand, for wall-bounded turbulence, a Gaussian filter in real space was frequently used, which is one of the simplest coarse-graining methods. For example, many authors (Lee and Sung, 2011; Lee et al., 2014; Hwang et al., 2016; Lee et al., 2017) used the filter to highlight the large-scale motions, and Lozano-Durán et al. (2016) analyzed the dynamics of the filtered velocity gradient in the log layer in turbulent channel flow. We also used the Gaussian filter to reveal the scale-dependent contribution to the enstrophy production rates in a turbulent boundary layer (Motoori and Goto, 2019). However, we must note that the Gaussian-filtered turbulent velocity fields, which correspond to the low-pass filtered fields, possess the information of all scales larger than the filter scale.

In the present study, to extract a hierarchy of vortices in a turbulent boundary layer, we use two coarse-graining methods. One is the Gaussian filter, and the other is the difference of the Gaussian filters at two length scales, which corresponds to a band-pass filter of the Fourier modes. The objectives of the present paper are to investigate the effect of the difference of the filters on the statistics of scale-dependent enstrophy production rates and to further clarify the generation mechanism of multiscale vortices in the log layer of a turbulent boundary layer.

2. Numerical methods

2.1. Direct numerical simulation

We numerically investigate a turbulent boundary layer with zero pressure gradient over a flat plate, which is the same field previously reported by Motoori and Goto (2019). The Navier-Stokes equations for an incompressible fluid are integrated using the simplified marker and cell (SMAC) method (Amsden and Harlow, 1970). The temporal integration of the viscous and the convective terms is made by the second-order Crank-Nicolson method and the second-order Adams-Bashforth method, respectively. We use a second-order finite difference scheme for spatial discretization with the grid spacing being uniform in the streamwise (x) and spanwise (z) directions and non-uniform in the wall-normal (y) direction. For the inlet condition, we use the time-series data obtained by another DNS (Lee et al., 2013, 2017). This has enabled us to simulate a high-Reynolds-number turbulent boundary layer with relatively small numbers (6112 × 616 × 768) of grid points. In the following analyses, to evaluate the ensemble average (i.e. average over time and the spanwise direction), we store 1000 snapshots in the duration of 92δ/Us, where Us is the free-stream velocity and δ is the boundary layer thickness.

In this paper, we report the results on the turbulence with the friction-velocity Reynolds number being Reτ ≈ 1000 and the momentum-thickness Reynolds number being Res ≈ 3200. The turbulence in the log layer is fully developed in the sense that the premultiplied spanwise energy spectrum of the streamwise velocity component has the second peak (see figure 2c in Motoori and Goto, 2019).

2.2. Coarse-graining methods

Although turbulence composes of multiscale vortices, when we visualize the isosurfaces of the enstrophy or the second invariant Q of the velocity gradient tensor (see Fig. 1), only the smallest-scale structures are seen. In fully developed turbulence, the energy spectrum is proportional to −5/3 power of the wavenumber. Then, the average magnitude of the velocity difference between two points separated by ℓ is proportional to ℓ1/3, and the mean velocity gradient at the scale ℓ is proportional to ℓ−2/3. In other words, the velocity gradient is predominantly determined by the smallest scale. This is the reason why, by visualizing quantities in terms of the velocity gradient, we can only observe the smallest-scale vortices. Therefore, to capture a hierarchy of vortices by using the velocity gradient, it is imperative to coarse-grain turbulent velocity fields.

In the present paper, we examine two coarse-graining methods. One is the isotropic three-dimensional Gaussian filter. We apply it to the fluctuation velocity u′i, i.e. the deviation from the temporal average of the velocity field, as

\[ u'_i(\sigma) = C(\gamma) \int_{V(\gamma)} u'_i(z') \exp \left( -\frac{2}{\sigma^2} (x - x')^2 \right) dz', \]

where \( \sigma \) is the filter scale and \( C(\gamma) \) is a coefficient to ensure that the summation of the kernel is unity. The filtered velocity \( u'_i(\sigma) \) denotes the fluctuation velocity coarse-grained at scale \( \sigma \). The Gaussian filter is simplest but the filtered velocity includes information of all scales larger than \( \sigma \). In this sense, the Gaussian filter corresponds to the low-pass filter of the
Fourier modes. The other filter is the one corresponding to a band-pass filter of the Fourier modes, which is defined by the difference of the two Gaussian-filtered velocity fields:

\[ u_i^{(\sigma)}(x) = u_i^{(2\sigma)}(x) - u_i^{(\sigma)}(x). \]

(2)

Here, we reemphasize that, whereas \( u_i^{(g)} \) (coarse-grained by the Gaussian filter) has contributions from the scales larger then \( \sigma \), \( u_i^{(\sigma)} \) (coarse-grained by the combination of two Gaussian filters) has those from only around \( \sigma \). In order to investigate the effect of the filter width, we also examine another band-pass filter with a narrower width,

\[ u_i^{(\sigma)}(x) = u_i^{(2\sigma)}(x) - u_i^{(\sigma)}(x). \]

(3)

Note that the filter width for (2) is \( \sigma \), whereas that for (3) is \((2^{1/4} - 1)\sigma \approx 0.19\sigma\).

In this study, we examine these three filters. Note that \( \langle \sigma \rangle \), \( [\sigma] \) and \( [\sigma] \) denote quantities evaluated from the velocity fields \( u_i^{(\sigma)} \), \( u_i^{(\sigma)} \) and \( u_i^{(\sigma)} \), respectively.

3. Results and discussions

3.1. Visualization of the hierarchy of vortices

Figure 1 shows the isosurfaces of \( Q \) evaluated directly from simulated velocity fields without coarse-graining. Comparing the visualized structures with the grid width (200 wall units) drawn on the wall, we see that the size of the tubular vortices is approximately 10 wall units in the visualized region, where \( Re_\tau \approx 1000 \). As mentioned in the previous section, vortices identified by velocity gradients are in the smallest scale. Figure 1 is the direct evidence of the fact. To observe arbitrary-scale vortices, we use the second invariant \( Q^{(\sigma)} \), \( Q^{(\sigma)} \) or \( Q^{(\sigma)} \) of the fluctuation velocity gradients coarse-grained by the three filters. Figure 2 shows the isosurfaces of \( (a, d, g) Q^{(\sigma)} \), \( (b, e, h) Q^{(\sigma)} \) and \( (c, f, i) Q^{(\sigma)} \) coarse-grained at \( (a, b, c) \sigma^\theta = 20 \), \( (d, e, f) 40 \) and \( (g, h, i) 80 \). Here, \( \sigma^\theta \) denotes the wall units defined in terms of the friction velocity \( u_\tau \) on the exit plane of the visualized domain and the kinematic viscosity \( \nu \), whereas \( \langle \sigma \rangle \) denotes wall units at each streamwise location. We can observe the hierarchy of multiscale vortices in this figure. The thresholds (Table 1) are set in an objective way which will be explained in the next subsection. The reason why the thresholds for the low-pass filter are larger than those for the band-pass filter is that the velocity gradient with the former filter includes the contributions from the larger scales than \( \sigma \). However, once we appropriately choose the threshold, regardless of the coarse-graining methods and the band-pass filter widths, the radius of the tubular vortices is comparable with the filter scale. It is further important to note that their axial length is much longer than the filter scale. This means that the isotropic filters can capture the anisotropic structures. This feature of isotropic filters can be shown if we assume that turbulence is composed of vortex tubes because the velocity changes its sign in the radial direction but not in the axial direction; see also the results (Goto et al., 2017) that tubular, i.e. anisotropic, vortices are captured by the isotropic band-pass filters in periodic turbulence. Incidentally, as expected, the green vortices coarse-grained at \( \sigma^\theta = 20 \) (Fig. 2a, b, c) look like those identified without the filtering (Fig. 1).

A conclusion drawn from Fig. 2 is that there are only small differences in the visualizations of the isosurfaces of the three filtered fields. This implies that the difference of the coarse-graining methods and the band-pass widths does not much affect the identification of vortices on the basis of the magnitude of velocity gradients. This is again consistent with the fact that the magnitude of the velocity gradients is predominantly determined by the smallest scale in the filtered fields.
3.2. Enstrophy production rates

In this subsection, we further examine the effect of the coarse-graining methods by investigating the generation mechanism of the hierarchy of vortices observed in Fig. 2. For this purpose, we focus on the vortex-stretching term \( \omega_i S_{ij} \omega_j \), which plays a role in the amplification of the vorticity. Here, \( \omega_i \) is the vorticity and \( S_{ij} \) is the rate-of-strain tensor. As emphasized in the previous section, \( \omega_i \) and \( S_{ij} \) predominantly reflect the smallest scale. Therefore, we need to decompose them into different scales so that we can examine the vortex generation mechanism at a given scale. More concretely, we evaluate the coarse-grained fluctuation vorticity \( \omega_i^{(r)} \) and rate-of-strain tensor \( S_{ij}^{(r)} \) and we introduce a quantity defined by

\[
G^{(r)} = \frac{\omega_i^{(r)}}{S_{ij}^{(r)}} \frac{\omega_j^{(r)}}{\omega_i^{(r)}}.
\]

This quantity indicates the contribution of the strain-rates field coarse-grained at \( \sigma^r \) to the stretching of the vorticity coarse-grained at \( \sigma_w \). If \( G \) is positively large, the vortices at \( \sigma_w \) are stretched by strain rates at \( \sigma^r \), whereas they are compressed when \( G \) is negatively large. As was demonstrated in our previous study (Motoori and Goto, 2019), the generation mechanism of vortices depends on the distance from the wall and the scale of vortices. In the log layer, while vortices larger than approximately one-fifth of the height are stretched and amplified directly by the mean flow, smaller

![Fig. 2 Isosurfaces of the second invariant Q of the velocity gradient tensor coarse-grained at the scale (a, b, c) \( \sigma^r = 20 \), (d, e, f) 40 and (g, h, i) 80. The coarse-graining methods are (a, d, g) the low-pass filter, (b, e, h) the band-pass filter and (c, f, i) the narrower band-pass filter. The thresholds \( Q_{th} \) are set as in Table 1. The instance and location are the same as in Fig. 1.](image)

| Filter scale (\( \sigma^r \)) | Low-pass \((a)\) | Band-pass \((b)\) | Narrower band-pass \((c)\) |
|-----------------------------|-----------------|-----------------|-----------------|
| 20                          | \(1.4 \times 10^{-3}\) | \(1.8 \times 10^{-4}\) | \(9.8 \times 10^{-6}\) |
| 40                          | \(4.9 \times 10^{-4}\) | \(1.3 \times 10^{-4}\) | \(1.0 \times 10^{-5}\) |
| 80                          | \(8.3 \times 10^{-5}\) | \(3.1 \times 10^{-5}\) | \(3.4 \times 10^{-6}\) |
Golds are chosen on the basis of the time-scale of the self-contribution with the filter width \( \sigma \). Thus, we can evaluate the contribution to the vortex stretching from the thresholds (Table 1) of the isosurfaces shown in Fig. 2 were set as the same and the prefactor 30 was chosen so that 

This means that the twice-larger strain-rate fields contribute most for the stretching of the small-scale vortices in the log layer. It is worth mentioning that this result is consistent with the observation in high-Reynolds-number turbulence in a periodic cube (Goto et al., 2017). On the other hand, looking at the gray lines in Fig. 3(a) which are the results with the low-pass filter, we cannot observe that larger scales contribute significantly to stretching. This is because the value of \( y^+ \) is larger than \( 150 \) for \( Re_\tau = 1030 \), we show results at a position \( y^+ = 150 \) in the log layer.

We plot the ensemble average \( G \) of \( g \) with gray lines in Fig. 3. First, looking at the results with the band-pass filter with the filter width \( \sigma \) (Fig. 3b), we see that \( G^{[2\sigma - \sigma_w]} \) is most significant for smaller-scale vorticity \( (\sigma_w = 20 \) and \( 40 )\). This means that the twice-larger strain-rate fields contribute most for the stretching of the small-scale vortices in the log layer. It is worth mentioning that this result is consistent with the observation in high-Reynolds-number turbulence in a periodic cube (Goto et al., 2017). On the other hand, looking at the gray lines in Fig. 3(a) which are the results with the low-pass filter, we cannot observe that larger scales contribute significantly to stretching. This is because the value of \( G^{[\sigma_w - \sigma_w]} \) contains the contribution of the band-pass filtered fields at all scales larger than \( \sigma_w \).

Next, we evaluate the ensemble average \( G_Q \) conditioned by the criterion that \( Q_{[\sigma_w]} > Q_{[\sigma_w]}^{[\sigma_w]} \). Here, the positive thresholds are chosen on the basis of the time-scale of the self-contribution \( G^{[\sigma_w - \sigma_w]} \) as \( Q_{[\sigma_w]}^{[\sigma_w]} = 30 (G^{[\sigma_w - \sigma_w]})^2 \). Incidentally, the thresholds (Table 1) of the isosurfaces shown in Fig. 2 were set as the same and the prefactor 30 was chosen so that the vortices in the log layer were visualized. Thus, we can evaluate the contribution \( G_Q \) to the stretching of the vortices. 

Looking at the black lines in Fig. 3(b), which is obtained with the band-pass filter, we see that \( G^{[2\sigma_w - \sigma_w]} \) is largest for each of \( \sigma_w \). This implies again that the twice-larger strain fields contribute most to the vortex stretching. As for the low-pass filtered fields (black lines in Fig. 3a), we can see the same trend that twice-larger scale contributes most. Incidentally, this result is consistent with the observation in turbulent channel flow on the contribution to the vortex stretching from the scale-dependent strain rates (Lozano-Durán et al., 2016). However, the trend is less clear than the black lines in Fig. 3(b).

This is also because the results with the low-pass filters contains contributions from larger scales. Moreover, comparing \( G \) (gray lines) and \( G_Q \) (black lines) in each panel of Fig. 3, we see that \( G_Q \) is larger than \( G \) for \( \sigma_S/\sigma_\omega \geq 2 \). This means that vortices are more significantly stretched by larger scales. On the other hand, \( G_Q \) is smaller than \( G \) and it takes negative value for \( \sigma_S/\sigma_\omega \leq 0.5 \). This means that vortices are more likely to be compressed by smaller-scale strain-rate fields.

We have shown (Fig. 3) that the difference between the results of the two coarse-graining methods is remarkable. In contrast, the filter width of the band-pass filter does not alter the conclusion. The evidence is shown in Fig. 4, where we
Fig. 5  PDF of the cosine of the angle between the vorticity coarse-grained at the fixed scale $\sigma_0^2 = 20$ and the first eigenvector of the rate-of-strain tensor coarse-grained at $\sigma_{ij}^2 = 10$ (blue dashed line), 20 (blue solid line), 40 (light-gray), 80 (gray) and 160 (black) at $y^+ = 150$ for $Re_T = 1030$. The results are evaluated from (a) the low- and (b) band-pass filtered fields.

Fig. 6  Conditional PDF of the same quantities as in Fig. 5.

show the same quantities (gray lines, $G$ and black lines, $G_Q$) evaluated for the narrower band-pass filtered fields with the filter width 0.19. We see that the curves in Fig. 4 are smoother (because there are more bands than for the broader filter), but they are similar to those in Fig. 3(b). For example, looking at the black open triangles ($\sigma_{ij}^2 = 40 = 14\eta^+$, where $\eta$ is the Kolmogorov scale), the small-scale vortices are stretched predominantly by the twice-larger-scale strain rates. This is the same observation as in Fig. 3(b) for the broader filter width. For the smaller (black open squares, $\sigma_{ij}^2 = 20 \approx 7\eta^+$) and larger (black closed circles, $\sigma_{ij}^2 = 80 \approx 0.53\eta^+$) scales, which are outside of the inertial range, the conclusions drawn from Fig. 4 and Fig. 3(b) are also the same, though, more precisely, Fig. 4 shows that the peaks are at $\sigma_{ij}/\sigma_{ij} = 2^{5/4} \approx 2.4$ and $2^{3/4} \approx 1.7$ rather than 2. In summary, the conclusion that the twice-larger strain rate contributions most to the enstrophy production is robust irrespective of the choice of the filter width.

3.3. Alignment of vorticity and stretching directions

In the previous subsection, we have shown that the scale-dependent contribution to the enstrophy production rates depends on the coarse-graining methods (i.e. low- and band-pass filters), whereas it does not depend on the band-pass filter width. This is explained by the fact that the low-pass filter contains all the contributions from scales larger than the filter scale. However, since, as demonstrated in Fig. 2, the magnitude of the velocity gradient is mainly determined by the vortices at the filter scale, the observation in Fig. 3 is nontrivial. In this subsection, we explain it in terms of the alignment between the vorticity and stretching direction at different scales.

For this purpose, we rewrite the enstrophy production rate as

$$\frac{\omega_j S_{ij} \omega_j}{\omega_i \omega_j} = s_i (\vec{e}_i \cdot \vec{\omega})^2,$$

where $s_i (s_1 \geq s_2 \geq s_3)$ are the eigenvalues of the rate-of-strain tensor $S_{ij}$, $\vec{e}_i$ are the corresponding eigenvectors, and $\vec{\omega}$
is the unit vector in the direction of the vorticity. Since \( s_1 + s_2 + s_3 = 0 \) (i.e. the incompressibility of the fluid), \( \mathbf{e}_1 \) and \( \mathbf{e}_3 \) always correspond to the stretching and contraction directions, respectively. Equation (5) implies that the strong vortex stretching requires the alignment of the vorticity with \( \mathbf{e}_1 \) or a large value of \( s_1 \). Recall that the average of \( s_1 \) is larger for smaller scales.

To focus on the alignment between vorticity at a given scale and the stretching direction at another scale, we evaluate the cosine

\[
\cos \theta = \mathbf{e}_1^{(\sigma_s)} \cdot \mathbf{\hat{e}}_1^{(\sigma_w)}
\]

of the angle between them. Here, \( \mathbf{e}_1^{(\sigma_s)} \) is the stretching direction of the fluctuation strain-rate coarse-grained at \( \sigma_s \) and \( \mathbf{\hat{e}}_1^{(\sigma_w)} \) is the normalized fluctuation vorticity coarse-grained at \( \sigma_w \). When \( \cos \theta \) is unity, vorticity at the scale \( \sigma_w \) aligns with the stretching direction at the scale \( \sigma_s \).

Figure 5 shows that the probability density function (PDF) of \( |\cos \theta| \) for \( \sigma_w = 20 \) and several scales \( \sigma_s \). We show the results with the broader band-pass filter in Fig. 5(b). The vorticity is more likely to align with the larger-scale (gray or black lines) stretching directions, whereas it does not align with the same or smaller scales (blue lines). More precisely, the tendency of the alignment with four-times-larger stretching direction is strongest. This is consistent with the result that vortical structures are more likely to align with the stretching direction at scales 3–5 times as large as themselves in decaying homogeneous isotropic turbulence (Leung et al., 2012). We observe that this trend is weaker in the low-pass filtered fields (Fig. 5a). This is also explained by the contamination from different scales in the low-pass filtered fields. Furthermore, we show in Fig. 6 the conditional \((Q^{(\sigma_s)} > Q_{th}^{(\sigma_s)})\) PDF of \( |\cos \theta|\). With the both filters, the preferential alignments of vorticity with larger-scale strain fields is more likely than the non-conditional cases (Fig. 5). As expected from the results in the previous subsection, we have confirmed (figure is omitted) that the preferential alignment at \( \sigma_s / \sigma_w = 4 \) is observed even if we change the filter width.

To summarize, the difference observed for the gray curves in Fig. 3(a) and Fig. 3(b) stems from the difference in Fig. 5(a) and Fig. 5(b). We emphasize that, regardless of the coarse-graining methods, vortices are likely to align with the stretching direction 2–8 times as large as themselves, whereas \( s_1 \) is larger for smaller scales. The balance between these two features explains the reason why the contribution from the twice-larger stretching dominates in the enstrophy production. We have also shown that, when conditioned by the value of \( Q \), the alignment is pronounced (Fig. 6). This is also interesting observation because it is consistent with the picture of the energy cascade.

4. Conclusion

To extract the hierarchical structure of multiscale vortices in fully developed turbulence, we need coarse-graining. In our previous study (Motoori and Goto, 2019), we employed the simplest choice; namely, the isotropic three-dimensional Gaussian filter. This corresponds to a low-pass filter of the Fourier modes. In the present study, we have examined the second simplest choice; namely, the combination of the Gaussian filters at two length scales, which corresponds to a band-pass filter, where we have examined two different filter widths. Then, we have studied the difference in the statistics of the coarse-grained turbulent velocity fields obtained by applying these filters to a turbulent boundary layer.

Since the magnitude of the velocity gradients is predominantly determined by the smallest scales in the filtered field, there is only small difference in the isosurfaces of \( Q \) of the velocity fields coarse-grained by these filters (Fig. 2). However, when we investigate the contribution \( G \) to the enstrophy production rate at scale \( \sigma_w \), difference of the low- and band-pass filters appears (Fig. 3). This is because \( G \) is subject not only to the magnitude of the strain rate but also to the direction of the stretching. Looking at the gray lines in Fig. 3(b), we see the positive contribution from the scales \( \sigma_s \) larger than \( \sigma_w \). More precisely, the twice-larger scale (2\( \sigma_w \)) most significantly contributes. Although the magnitude of the strain rate is smaller for larger scales, vortices tend to align with the stretching direction at the 2–8 times larger scales (Fig. 5 and 6). Consequently, the contribution \( G \) from the scale 2\( \sigma_w \) dominates. In this analysis, band-pass filters are more appropriate for the precise evaluation of the contribution, since the effect of larger scales contaminates the statistics when we use a low-pass filter. Moreover, comparing Fig. 3(b) and Fig. 4, we see that the results are independent of the choice of the filter width of the band-pass filter, though the narrower width makes it possible to examine the scale-dependence in more detail.

We emphasize that vortices are not created in the entire space. This is the reason why the analysis conditioned by the magnitude of \( Q \) reveals the clearer generation mechanism of the vortices. The black lines in Fig. 3(b), or in Fig. 4 for more detailed analyses with narrower band-pass filter, show that vortices are predominantly stretched by twice-larger
vortices, and, at the same time, they weaken the twice-larger vortices. The latter events are consistent with the fact that the energy at the twice-larger vortices transfers to the vortices (i.e. the energy cascade). Thus, the conditional sampling by the value of $Q$ is essential to investigate energy cascading events. In fact, the conclusions drawn by using the low-pass filter (black lines in Fig. 3a) and by using the band-pass filter (black lines in Fig. 3b) are the same.

In conclusion, the band-pass filter is a better choice for more precisely understanding the generation mechanism of the hierarchy of vortices in turbulence, where we may use an arbitrary filter width smaller than the filter scale. Although we encounter a practical difficulty to construct a band-pass filter in real space, we can make it by a simple way shown in (2) with the combination of the Gaussian filters at two different scales.

Acknowledgements

We are grateful and thankful to Dr. T. A. Zaki and Dr. S. Y. Jung for providing us with their turbulent inlet data. This work was partly supported by JSPS Grant-in-Aid for Scientific Research No.16H04268, and the DNS was conducted under the auspices of the NIFS Collaboration Research Programs (NIFS17KNSS101 and NIFS18KNSS108).

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