(Pseudo) Dirac neutrino masses in Supergravity

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ABSTRACT
In this talk, we discuss the idea of how Dirac or Pseudo-Dirac neutrino masses arise naturally with a correct size, after the breaking of local N=1 supersymmetry.

1. Introduction to the idea
This talk is based on Ref.[1] and on previous works done in references therein. In a nutshell, we show that Dirac neutrino masses can arise from the Kähler potential of supergravity and they are proportional to SUSY and electroweak breaking scales. Dirac neutrino masses of the correct size ($\lesssim 0.05$ eV) are obtained, provided that the ultraviolet cutoff, $M$, of the MSSM is between the GUT and the heterotic string scale. The above is guaranteed with the implementation of an R-symmetry and the assumption of Lepton number conservation. Breaking of Lepton number results in Pseudo-Dirac neutrinos [2].

In order to obtain small Dirac neutrino masses we use, instead of the “unnatural” Dirac term

$$W \supset L H_u N^c,$$

in the superpotential, the Kähler potential term

$$K \supset \frac{L H_u N^c}{M} + \frac{L H_d N^c}{M} + H.c,$$

where $M$ is the ultraviolet cutoff of the theory. After the supersymmetry and (radiative) electroweak symmetry breaking, we find Dirac neutrino masses with magnitude

$$m_\nu \simeq \frac{m_{3/2}}{M} \left[ \langle H_u \rangle + \langle H_d \rangle \right] \sim 10^{-4} \text{ eV},$$

with $M = M_p = 2.44 \times 10^{18}$ GeV, $\langle H_u \rangle = m_{\text{top}} = 175$ GeV and $m_{3/2} \simeq 1$ TeV. The neutrino masses obtained from Eq.(3) are smaller than those implied from atmospheric neutrino oscillation experiments, $m_\nu \simeq 0.05$ eV. Turning the argument around, and using Eq.(3), right size neutrino masses imply a scale $M = 5 \times 10^{15}$ GeV, just below the GUT scale. However, with a more careful inspection, in the context of full Supergravity we obtain

$$M = (4 \times 10^{16} - 5 \times 10^{17}) \text{ GeV},$$

as explicitly shown below. The idea of obtaining neutrino masses directly from the Kähler potential has been discussed in the literature [3,4,5,6,7,8,9] mainly within the context of
global supersymmetry. Dirac neutrino masses can also be derived in the U(1) extended MSSM as described in Ref. [10].

2. Matter fermion mass terms in Supergravity

To study the idea just sketched, we need to consider contributions to fermion masses in a full supergravity set-up. To this end, consider chiral superfields \( S_i \) of some hidden sector which are responsible for spontaneous breaking of Supergravity by acquiring v.e.v’s, \( S_i = M \sigma_i \), and visible sector chiral superfields \( y_\alpha \) of the observable sector such as the leptons \( L, E \), the Higgs \( H_u, H_d \), and the right handed neutrino \( N^c \) superfield. The general formula for the fermion mass matrix \( m_{\alpha \beta} \), for an N=1 supersymmetric theory coupled to gravity, can be found in standard textbooks [12,13,14]. Denoting the gravitino mass as \( m_3/2 = \langle \frac{W(h)}{M_P^2} \rangle \exp(K(h)/2M_P^2) \), and taking the flat limit \( M_P \to \infty \), this formula reads

\[
m_{\alpha \beta} = \frac{1}{2} \left\{ \frac{\partial^2 W^{(o)}}{\partial y^\alpha \partial y^\beta} - g^{\gamma \delta} \frac{\partial^3 K^{(o)}}{\partial y^\alpha \partial y^\beta \partial y^\delta} \frac{\partial W^{(o)}}{\partial y^\gamma} - \frac{1}{M^2} \left[ g^{ij} \frac{\partial^3 K^{(o)}}{\partial y^\alpha \partial y^\beta \partial \sigma^j} \frac{\partial W^{(h)}}{\partial y^\gamma} \right] \right\} \]

\[
+ \frac{m_{3/2}}{2} \left\{ \frac{\partial^2 K^{(o)}}{\partial y^\alpha \partial y^\beta} - g^{\gamma \delta} \frac{\partial^3 K^{(o)}}{\partial y^\alpha \partial y^\beta \partial y^\delta} \frac{\partial K^{(o)}}{\partial y^\gamma} - \frac{1}{M^2} \left[ g^{ij} \frac{\partial^3 K^{(o)}}{\partial y^\alpha \partial y^\beta \partial \sigma^j} \frac{\partial K^{(h)}}{\partial \sigma^i} \right] \right\},
\]

where the “\( \cdots \)” stand for terms involving the hidden-visible mixed metric \( g^{\gamma \delta} \). In what follows we shall also assume that the metric \( g^{ij} = g^{\alpha \beta} \) is diagonal. Equation (5) is our master formula. It is devided into two parts : the first one (1st-line) exists in global supersymmetry while the second one (2nd-line) is induced from the N=1 SUSY coupled to gravity. The latter is proportional to the gravitino mass \( m_{3/2} \) and is the same as the first term with the replacement \( W^{(h,o)} \to m_{3/2}K^{(h,o)} \). The first term in first line is the well known fermion superpotential mass term. If an R-symmetry prohibits a bilinear term, it can be replaced by the first term in the second line but multiplied by \( m_{3/2} \). This is a well known mechanism [15,11]. A generalization of this mechanism is employed below in deriving small (Pseudo) Dirac neutrino masses.

3. (Pseudo) Dirac neutrino masses

We first need to prohibit the superpotential term in Eq. (1). We also want to generate the \( \mu \)-term with the same mechanism. For simplicity, we assume that the observable superpotential is a function only of observable fields, \( W^{(o)}(y) \). Then by imposing a discrete R-symmetry (see Table.1 in [11]) we obtain

\[
W^{(o)}(y) \supset Y_E LH_d E^c + Y_D QH_d D^c + Y_U QH_u U^c,
\]

\[
K^{(o)}(\sigma, \sigma^*, y, y^\dagger) \supset c_1(\sigma, \sigma^*)H_u H_d + \frac{c_2(\sigma, \sigma^*)}{M} L H_u N^c + \frac{c_3(\sigma, \sigma^*)}{M} L H_d^* N^c + \text{H.c.,}
\]

where \( c_i(\sigma, \sigma^*) \) are functions of the hidden sector superfields. Supergravity is spontaneously broken and, soon after that, electroweak symmetry is radiatively broken. Use of
the master formula, Eq.(5), results in the following Dirac neutrino masses

\[ m_D^{\nu} = v \left( \frac{m_3/2}{M} \right) \sin \beta \left[ c_2(\sigma, \sigma^*) - c_1(\sigma, \sigma^*)c_3(\sigma, \sigma^*) \right] \]

\[ - v \left( \frac{F_S}{M^2} \right) \sin \beta \left[ \partial_{\sigma^*}c_2(\sigma, \sigma^*) + \cot \beta \partial_{\sigma^*}c_3(\sigma, \sigma^*) \right], \] (8)

where \( F_S = \partial_s W^{(h)} + m_3/2 \partial_s K^{(h)} \). In local supersymmetry, for example, vanishing of the vacuum energy implies that \( F_S = \sqrt{3} M_p m_3/2 \). Right-size neutrino masses imply that \( M = (4 \times 10^{16} - 5 \times 10^{17}) \) GeV, for \( 100 \) GeV \( \leq m_3/2 \leq 10 \) TeV. The second term in Eq.(8) is enhanced by a factor \( M_p/M \) as compared to the first one. In addition, the gravitino can be very light. Furthermore, the non-holomorphic term proportional to \( \partial_{\sigma^*}c_3 \) is dominant if \( c_1 = c_2 = 0 \) or if cancelations take place. Another generic aspect (and maybe a problem) of this mechanism is that soft breaking masses, as well as the \( \mu \)-parameter, are expected to be of order \( \tilde{m} \sim F_S/M \sim 10 \) TeV, much larger than desired. If we relax the assumption of lepton number conservation, then Dirac neutrinos obtained from the Kähler potential can be “polluted” by the presence of Majorana neutrino masses derived from extra non-renormalizable terms

\[ W^{(o)}(y) \supset g_L(LH_u)(LH_u), \] (9)

\[ K^{(o)}(\sigma, \sigma^*, y, y^\dagger) \supset \frac{c_4(\sigma, \sigma^*)}{M^3} W^{(h)} N c^2 + \text{H.c.} \] (10)

Let us assume that only the term \( (9) \) is present. Then, using the range for \( M \) given above, we obtain, in addition to the Dirac mass \( m_D^{\nu} \simeq 0.05 \) eV, a much smaller Majorana mass \( m_L^{\nu} \simeq 10^{-5} \) eV. Assuming one generation, the mass matrix reads

\[ \begin{pmatrix} m_L^{\nu} & m_D^{\nu} \\ m_D^{\nu} & 0 \end{pmatrix}. \] (11)

Pseudo-Dirac neutrinos result, with mass difference \( \delta m^2 \simeq 2 m_D^{\nu} m_L^{\nu} \simeq 10^{-6} \) eV^2 and maximal mixing. Astrophysical techniques to distinguish between Dirac and Pseudo-Dirac neutrinos have been described in Ref. [16].

4. Questions/Conclusions

The first question one asks is: what is the source of the non-renormalizable operators i.e., the mechanism which generates the scale \( M \)? The answer may be: a) radiative corrections to the Kähler potential [17], b) effective operators in heterotic string theories [18], or c) a GUT model. We have tried the last possibility but we did not find an acceptable model that also complies with the proton decay stability. Another important question is how to account for non-trivial (maximal) neutrino mixing matrix. The answer to this question may be linked to the fact that the Kähler potential parameters are not protected by the non-renormalization theorem, and vertex corrections may induce large flavour mixing through RGE running. Another question concerning leptogenesis with Dirac neutrinos...
has been (and is currently being) addressed in Ref. [19]. Furthermore, a question concerning “fundamentals” is that, in order to put the light pseudo-Dirac neutrino mechanism to work and for the superpotential and Kähler potential to obtain the particular form given in Eqs. (6,7), one has to impose a symmetry such as the $R$-symmetry in Table 1 of Ref. [1]. Is this symmetry broken and, if so, where and how? One attractive answer is to gauge a $U(1)$ anomaly-free $R$-symmetry. It is known [20] that such a symmetry must be broken at scales $M \leq M_P$, which favours our case, with no effects from gauging the $R$-symmetry remaining at low energies.

The bottom-line of the idea presented in Ref. [1] is that neutrino masses are not filtered through unknown Majorana mass terms but carry direct information about the structure of the Kähler metric (more accurately of the Christoffel symbols of the metric). (Pseudo) Dirac neutrino with mass of the correct size can naturally arise in supergravity.

5. References

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