Visualization of amplitude-frequency characteristics of EEG of pathological and cognitive functions of the brain from a position of nonlinear dynamics

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Abstract. In recent years, medical imaging methods have been intensively developed to provide comprehensive and extensive data for studying the work of the brain, which makes the present study relevant. The purpose of this work is to develop a new approach for analyzing EEG signals based on nonlinear dynamics methods. To achieve this goal, it is necessary to adapt the methods of nonlinear dynamics to solving EEG analysis tasks. In this paper, the following methods are used: the principal component method for noise clearance, the wavelet transform, the Wolf, Kanz, Rosenstein, neural networks methods to calculate the spectrum of the Lyapunov exponent (LCEs). Studies have shown that the chaos of the schizophrenic’s EEG signals is higher than for healthy people. It has been revealed that the best visualization of the amplitude-frequency characteristics of the brain’s EEG pathological functions is provided by the Morlet and Gauss 32 wavelets.

1. Introduction
The medical imaging methods allow to get comprehensive and extensive data to study the work of the brain, thoroughly study the structure of the brain and find the connection between the structure of the brain and its function. Therefore, the analysis of brain activity visualization data (biomedical signals) in medical and biological studies has been intensively used to detect links between neuropsychiatric disorders and structural changes in the brain.

Biomedical signals, which are essentially chaotic signals, carry information about the physical manifestations of the physiological processes (events) of organism. This information can be measured and presented in a form suitable for processing using methods of nonlinear dynamics.

Processing of biomedical signals is carried out in order to highlight informative features or determine diagnostic indicators. The first step in the study of biological systems is the use of special medical equipment to convert the phenomena under study into electrical signals that can be measured. This includes an electroencephalogram (EEG), magnetic resonance imaging (MRI), positron emission tomography (PET), single photon emission computed tomography (SPECT), magnetic encephalography (MEG) and a number of other methods [1]. The main sources of information about the brain are the results of an electroencephalogram (EEG) [2]. The EEG signal has a complex composition with a number of characteristic rhythms and patterns that are of interest to researchers both in studying pathologies (for example, epilepsy and schizophrenia) and in analyzing cognitive processes [3]. Biomedical signals reflecting changes in the functional processes in the human body, including the brain, are continuous rather than discrete, so if such signals are converted using analog-
digital conversion into time series, then the operations performed in the second and third steps of the study of biological systems can be performed using the theory and methods of nonlinear dynamics. The second step is filtering and eliminating distortion of signals (artifacts). In most cases, the study of EEG signals is complicated by the presence of parasitic patterns - noise and artifacts, which are caused by both external sources of signals and processes in the body, for example, eye movement, heart rate, facial and neck muscles, etc. [4]. Most of the artifacts on the EEG have a significant amplitude and overlap three important low-frequency EEG ranges — delta, theta, and alpha [5]. The presence of artifacts and their variability greatly complicate the analysis of EEG signals, which makes preprocessing and filtering an important step in any EEG studies. For filtering EEG from artifacts, a number of different methods are used: based on visual search of artifacts [6], analysis of independent components [7], regression analysis [8], etc. Most methods cause distortion of the EEG signal or require joint analysis of EEG with other signals, which may not always be recorded during the experiment. An important task is the development of methods for filtering EEG signals that do not distort their structure and do not require recording of additional physiological signals.

The third step is the detection of events in biomedical signals and the analysis of their information characteristics, the modeling of processes and systems generating biomedical signals. In nonlinear dynamics, a number of methods for analyzing nonstationary signals have been developed, for example, the Fourier transform [9], wavelet analysis [10], which are very effective in analyzing EEG [11, 12]. The preliminary stage of multichannel classification of EEG, MRI, PET, SPECT and MEG signals includes its segmentation using main component analysis and noise suppression [13]. This stage is fundamental for further data processing, both for one-dimensional and multidimensional signals. Extracting features of the signal segments forms the next stage of signal segmentation, which allows combining the signal characteristics in the time and frequency domains. The commonly used spectral representation of a signal based on a discrete Fourier transform provides the same frequency resolution throughout the window function, and, accordingly, poor resolution on a time scale. To provide a different resolution of frequencies, the wavelet transform is used [14, 15] which, like a microscope, allows us to investigate the change in the spectrum over time.

One of the most important applications of nonlinear dynamics methods for medicine at the present time is their use for the localization of epileptogenic foci, while the precise localization of the epileptic foci is often a difficult task. Since there is no single method that could reliably localize the area that needs to be treated (best of all, by surgery), the preoperative assessment is based on a multimodal approach [16], which includes EEG, MRI, PET, SPECT and MEG. To identify information on the location and size of the focus, additional methods are needed that use post-processing and signal analysis, for which the authors of the article suggest using nonlinear dynamics methods.

As mentioned above, biomedical signals have a complex structure and, in fact, are chaotic signals. By definition, chaos given by D. Gulick [17] chaos exists when either there is a substantial dependence on the initial conditions or the function has a positive Lyapunov exponents at each point of its definition domain and therefore is not ultimately periodic.

2. Methods
2.1. The Principal Component Analysis (PCA)
Currently, there are many ways to purify signals from noise, for example, wavelet transform, Fourier transform, Hubert-Huang transform, empirical mode decomposition, and others. This paper uses the principal component analysis (PCA), which is widely used in the compression of images and video to reduce the spatial redundancy of pixels when encoding images and video when linear transformations of blocks of pixels are used. Noise reduction is that when removing noise from a signal, it is necessary to present data in multidimensional space, apply the principal component method to it and leave only the first components of the transformation. It is assumed that the first components contain basic useful information, the remaining components contain unnecessary noise. The advantage of this method is that the method does not require the specified accuracy from the user. The target approach to estimating the number of principal components by the required fraction of the variance explained is formally always applicable. However, where there is no separation between “signal” and “noise”, any predetermined accuracy makes sense. Thus, the “signal” suggests a relatively small dimension with a
relatively large amplitude. "Noise" implies a large dimension with a relatively small amplitude. From this point of view, the principal component method works as a filter: the signal is contained mainly in the projection on the first main components, while in the other components the proportion of noise is much higher. The method of principal components solves the following problems: 1. Clearing the signal from noise. 2. Localization of the main frequencies of the signal.

In this work, the principal component analysis (PCA) was used to purify EEG signals from noise.

2.2. Wavelet transform

After clearing the noise, the signal was examined using wavelet analysis and Lyapunov parameters. The wavelet transform is a decomposition of a signal in a basis constructed from maternal functions. As which can be used the functions of the Morlet wavelet, Gauss of various orders (8, 16, 32), Mexican hat. One of the objectives of the study is to determine the maternal wavelet, which allows the most accurate localization of the EEG signal frequency. The use of wavelet transform greatly simplifies the search for differences between the EEG of a healthy and sick person.

2.3. The Lyapunov characteristic exponent (LCEs) for dynamical system

One of the basic information in understanding the behavior of a dynamical system is the knowledge of the spectrum of its Lyapunov Characteristic Exponents (LCEs). The LCEs are asymptotic measures characterizing the average rate of growth (or shrinking) of small perturbations to the solutions of a dynamical system. Positive Lyapunov exponent is one of the chaos indicators. The authors of the article have a lot of experience of application LCEs for the study of nonlinear dynamics of distributed structures [18]. In this paper we propose the use of LCEs for the analysis of human Electroencephalogram (EEG).

To exponent of largest Lyapunov exponent currently there are a number of approaches. For the numerical computation of the LCEs spectrum using the Benettin approach. [19]. This method can only be used if the evolution equations of the system are known. But for experimental data, these equations are usually unknown. In [20], Alan Wolf and his co-authors proposed an algorithm that which allows you to evaluate non-negative Lyapunov exponent based on time series. The idea of the method is to compute the highest Lyapunov exponent from a sample of a single coordinate. This approach can be used when the evolution equations of the system are unknown, and it is impossible measure all its phase coordinates. Long-term growth of elements is monitored. small volume in the attractor. The Rosenstein method [21] is simple to implement and shows a good computation speed. However, the result of his work is not a numerical value of $\lambda_1$, but a certain function of time. The Kantz algorithm [22] computes the largest Lyapunov exponent by searching all the neighbors in the neighborhood of the reference trajectory and calculates the average distance between the neighbors and the reference trajectory as a function of time (or the relative time multiplied by the data sampling rate). A new modification of the neural network method was developed in [23, 24].

For numerical implementation of this algorithm, the neural network was built (Figure 1). It can be classified using the following criteria:

- input network is analog (information is presented in the form of real numbers)
- according to the form of training network is self-organizing (forms output space solutions based only on input impacts)
- by the nature of connections network is a network direct distribution (all communications are strictly directed from input neurons to weekends)
- by character signal settings network is a network with dynamic connections (in the process of learning the synaptic
connections are tuned \( (dw/dt \neq 0) \), where \( W \) — is the weight network coefficients.

There is a hidden layer of neurons with hyperbolic tangent as a transfer function. The derivative of the hyperbolic tangent, as well as the logistic function, is expressed by a quadratic function. However, unlike the logistic function, the range of values of the hyperbolic tangent lies in the interval \((-1, 1)\). This gives the advantage of faster convergence, in contrast to the standard logistic function;

Scalar time series prediction \( \hat{x}_k \) taking into account the formula (1) was fulfilled.

\[
\hat{x}_k = \sum_{i=1}^{n} b_i \tanh \left( a_{i0} + \sum_{j=1}^{d} a_{ij} x_{k-j} \right)
\]

where \( n \) — is the number of neurons, \( d \) — is the number of the desired Lyapunov exponent, \( a_{ij} - n^*(d+1) \) is a matrix of coefficients, and \( b_i \) — is a vector of length \( n \). The matrix \( a_{ij} \) represents the strength of the connection to the input of the network; \( b_i \) the vector is used to control the contribution of each neuron to the output of the network. A vector \( a_{i0} \) is an offset that facilitates learning from non-zero mean data. Weights \( a \) and \( b \) are chosen in a probabilistic way, and the dimension of the searched solution is decreased in the process of learning. The associated Gaussian is chosen in a way to have initial standard distribution \( 2^j \), centered with respect to zero in order to promote the most recent time delays (small values of \( j \)) in the phase space. The coupling forces are chosen in a way to minimize the averaged one step mean square error of a forecast:

\[
e = \frac{\sum_{k=d+1}^{c} (\hat{x}_k - x_k)^2}{c - d}
\]

When the network is trained, output sensitivity to each time delay is determined by computation the partial derivatives of the output with respect to each time delay averaged by all points of the time series:

\[
\hat{S}(j) = \frac{1}{c - j} \sum_{k=j+1}^{c} \frac{\partial \hat{x}_k}{\partial x_{k-j}}
\]

Optimal attachment dimension assumed as the greatest value \( j \), for which \( \hat{S}(j) \) is of great importance, false nearest neighbour (FNN) methods. Individual values \( \hat{S}(j) \) give quantify the importance of each time delay, as members of the autocorrelation function or ARMA coefficients of a linear system model. Weight the coefficients of the trained neural network are substituted into the decision matrix. Input data used to determine the initial state. Spectrum is computed at using the generalized Benettin algorithm for the obtained system of equations.

3. Numerical experiment

In our work, we compared the EEG for schizophrenic patients and healthy people. These EEG signals are taken on the website [25]. For the study, EEG of 40 healthy people and 40 patients with schizophrenia was taken. For each EEG result given in the database, wavelet spectra were calculated based on different maternal wavelets Gauss8, Gauss16, Gauss32, Morlax, Mexican hat, Meier, in the frequency range \( \omega \in [0; 40] \) and time, and also obtained the values of the Lyapunov senior exponent based on the Kantz, Rosenstein, Wolf methods and the neural network method. For recording EEG, the recording electrodes were arranged according to the “10–20%” scheme Fig. 2.
Consider the wavelet spectra of EEG signals (point 1 - F7) obtained for a schizophrenic patient and a healthy person. Beta rhythm in the range of 14 – 40 Hz has been studied. Normally, it is associated with higher cognitive processes and attention focusing. These frequencies are present on the EEG of a person in the usual waking state, when he is observing the events, or is focused on solving any current problems. These waves dominate the conversation and learning activities. During the day, most people's brains work in beta range frequencies. Beta activity is very important for thinking process. Inadequate beta activity leads to depression, can cause various mental disorders, attention deficit hyperactivity syndrome. Wavelet analysis shows that the EEG frequencies of a healthy person are evenly distributed over a range of 14 – 40 Hz. For a schizophrenic patient, the frequencies are not constant over time, i.e. the phenomenon of frequency intermittency is observed, the power is shifted from low to higher frequencies (Table 1).

Table 1. Wavelet spectrum Gauss 32

| Wavelet spectrum Gauss 32 for EEG of a schizophrenic patient | Wavelet spectrum Gauss 32 for EEG of a healthy person |
|-------------------------------------------------------------|------------------------------------------------------|
| ![Wavelet spectrum Gauss 32 for EEG of a schizophrenic patient](image1) | ![Wavelet spectrum Gauss 32 for EEG of a healthy person](image2) |

A comparative analysis of the amplitude-frequency characteristics of the EEG in time using the wavelet transform for one healthy person shows (Table 2) that the wavelet transform based on Meier and Mexh maternal wavelets is not suitable for the study due to poor localization of frequencies over time. Increasing the order of the maternal Gauss wavelet (8, 16, 32) allows us to achieve good visualization of low frequencies. Thus, the EEG frequencies of a human Morlet and Gauss wavelets are particularly well localized.

Table 2. Wavelet spectrum

| Gauss 8 wavelet | Gauss 16 wavelet | Gauss 32 wavelet |
|----------------|-----------------|-----------------|
Conclusions
1. An approach to the study of human EEG from the perspective of nonlinear dynamics is proposed.
2. It is shown that for visualization of the amplitude-frequency characteristics of the EEG should use the wavelet transform on the basis of the maternal wavelet Gauss 32 or Morlet.
3. A comparative analysis of the wavelet spectra of a schizophrenic patient and a healthy person showed that in a schizophrenic patient, the EEG frequencies are not constant over time, i.e. the phenomenon of frequency intermittency is observed, the power is shifted from low to higher frequencies.

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