Parameters Estimation and Evaluation for the Probability Density Function of Structural Fatigue Stress of a Container Vessel

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Abstract: Fatigue stress measurement has been playing a significant role in the ship structural health monitoring and ship structural safety assessment. The PDF (probability density function) of the measured stress is one of the essentials for the further study in this domain. This paper, based on the strain-stress data collected from a container ship, focuses on the spectrum feature of the ship structural fatigue stress. A general analysis procedure for ship hull health estimation was firstly demonstrated. With the guidance of this procedure, the estimation and test of the parameters for the PDF of the stress were conducted, which showed that the stress spectrums fit well with the Weibull distribution. To review the fatigue state, the PDF and distribution function of fatigue damage increment were further developed and examined. The structural healthy assessment of this vessel shown the daily relative fatigue damage increment obeys log-normal or Weibull distribution and the increment of the fatigue damage on steel box girders of the ship hull was very low. Finally, the analyzing results yielded that the girder structure of the ship hull had a very low failure probability, matching well with the actual relative low working load of the ship.

Key words: PDF, stress, fatigue damage, container ship.

1. Introduction

Frequent and continuous wave induced load is one of the fundamental factors causing fatigue damage to ship structures, especially for those with high stress concentrations for instance connections and details. Although the damage may not be so serious that will immediately lead to the total loss of the ship, it is the main root of costly repairs and renewing of ship hull structures, influencing the service ability and operational economy. Meanwhile, the continuous accumulation of the fatigue damage on certain parts of the hull will also lead to parts in effective, causing safety and marine environment pollution problems. Special attentions have been focusing on how to reducing fatigue damage on the ship hull. These include the regulations and methods laid down by classification societies for ship hull structure strength calculation and assessment, researches on global and local stress distribution on the hull, and researches on the response of the hull structures to wave induced loads. Currently, many researchers have turned to use the stress data from stress sensor system on board ship to reveal and predicate the health state of ship hull [1-3]. However, there are few researches on the PDF of fatigue stress used for statistical purposes. This paper thus gave focus on this point based on the analysis of stress data collected from a stress monitoring system (Container Ship Structure Monitoring and Analyzing System: CSSMAS) mounted on the M/V YU FENG (IMO No. 8822038).

2. Researches on Ship Hull’s Strength and Fatigue Rate

The remained lifetime estimation and evaluation of ship hull structure work as safe guard for the safe operation of a ship. In order to precisely estimate the fatigue state and the remained lifetime of a container
vessel, the accuracy and reliability of stress data used in this purpose should be first ensured. Traditionally, the stress data are acquired through the calculation of the cargo weight and its loading distribution. In this way, the longitudinal distribution of bending and shearing forces were thus produced as an important reference for safety and ship hull fatigue assessment [4]. However, this approach cannot realistically review the complex and dynamic stress loading conditions in real time. Given to the development of ship structural healthy monitoring technology, those bending and shearing data based on estimation and calculation could be substituted by time-serial data collected by stress sensors system. The stress data from these systems could show the real stress on the ship structure more realistically and comprehensively than that produced by estimation and calculation according to the loading condition [5]. This data collecting system thus stands for a great step forward in the application of the precise evaluation of ship fatigue, which ensures the accuracy and reliability of the evaluation.

The container vessel YU FENG owned by Shanghai Maritime University is a training ship. Besides its training purpose, this ship is also engaged in the commercial shipping operating a weekly liner route of Nanjing (China)-Pusan (South Korea)-Kuangyang (South Korea)-Zhangjiagang (China)-Nanjing (China). The research focus of this study is on the spectrum features of the fatigue stress by using the collected stress data. The resultant analysis will be further used to estimate and test the parameters of the PDF of the fatigue stress. This paper could service as a fundamental research for further studying and analyzing the characteristics of fatigue stress.

3. The General Pattern of Strain-Stress on the Structural Beam of YU FENG

The sensors of the CSSMAS of M/V YUFENG consist of tress-strain gages, accelerometers, GPS and video cameras. Fig. 1 illustrates the main types and locations of these sensors. In this figure, S stands for stress variable, V is accelerated speed of the ship’s vertical vibration motion, X is transverse stress, Z is longitudinal stress, and y is a stress variable in 45 degree against X. M/V YU FENG has a total length of 139.8 m and a largest width of 21 m. Stress-strain gages are mainly used for the measurement of fatigue stress, which are mounted on the inner top surface of the deck, the hull wall and top surface of double bottom tanks in an optimistic way. The accelerometers are to measure the ship’s vibration motion, providing another approach for the analyzing of fatigue stress. The cameras system is used to record the wave conditions around the vessel which could provide not only the direct appreciation of wave load the relatively

![Sea wave camera](image1)

![Vibration sensors](image2)

**Fig. 1** Sensors arrangement of the CSSSMAS.
more precise measurement of the external wave load. The GPS provides location data, allowing the plotting of the ship position on the ECDIS for more detailed analyzing of ship’s fatigue in combination with the external environment conditions.

Figs. 2 and 3 are typical data used to verify the effectiveness of the system. The curves in Figs. 2 and 3 are the stress measured at point 6 and 7, which varied against time with a time span of 25 hours. The section 1 of the curve stands for the data measured when the ship was operating in the Yangzi River voyage leg. During this voyage leg, the river surface was calm and the wave load was week, reflecting on a relative flat and low curve. The section 2 was measured on the sea legs of the voyage when the vessel move from the Yangzi River onto open sea. A relative high and rough curve therefore was produced. This character was induced by the swells and waves on the sea surface. Normally, the average stress and the amplitude of stress are two typical indicators to show features of a strain process [6]. The average stress is influenced by those factors mainly including the weight of cargo loaded, temperature and stowage of the cargo along the vessel [7]. While, the stress amplitude is usually affected by the wave load surround the ship. The affecting factors like wave load, temperature and cargo weight at the point 6 and point 7 are almost the same at a given time; therefore the trend of the two curves also keeps the same. These two data examples could be an apparent verification that the monitoring system could work effectively and be used to reflect the stress condition sensitively and correctly.

In this study, the approach of ship hull fatigue stress and fatigue rate assessment consists of five steps:

Step 1: to calculate the sequential stress by using the strain data under the assumption that all the components of the ship hull structure working generally in elastic state [8].

Step 2: to calculate the stress spectrum and the nominal stress amplitude spectrum at the sensor site by using the Rain-flow counting method [9].

Step 3: to study the detailed welding characteristics of the nearest welding site and get the stress concentration parameters. Use the parameters to correct the stress spectrums at the site prone to be fatigued.

Step 4: to calculate the equivalent stress amplitude and identify the significant locations for further study according to the values of the equivalent stress amplitude. The greater the value, the danger the site is.

Step 5: to evaluate the fatigue state and ship hull reliability state by various fatigue damage accumulation models.

Since M/V YUFENG is still at its middle age, majority of the stress amplitudes from different sensors are under their amplitude thresholds; therefore, more attention will be paid to factors reviewing the fatigue accumulation, for instance, the characteristics of the spectrums of the fatigue stress and its corresponding PDFs.

![Fig. 2 Stress of S-6.](image)

![Fig. 3 Stress of S-7.](image)
4. Parameters Estimation and the PDF Verification

4.1 Parameters Estimation and Testing for the PDF of Fatigue Stress

The actual stress amplitude of ship hull structure normally satisfies certain types of distribution [10]. According to the statistics of data measured from relatively similar navigational conditions, these data samples confirm a certain distribution. Study [11] reviews that the PDF of the stress amplitude complies with Weibull distribution. In this paper, a maximum likelihood method [12] was employed to estimate the parameters of the Weibull distribution.

To evaluate the fitness of a distribution function with the real distribution of the stress amplitudes, daily-based data simples measured by the CSSMAS need to be examined continually. Hereby, the Kolmogorov-Smirnov testing method [13] was applied in order to evaluate the fitting effectively. Under this approach, the theoretic frequency and the corresponding cumulative probability $F(X_i)$ of the sample was first calculated. Then, the actual frequency $n_i$ and actual cumulative probability $F_n(X_i)$ of each group of the sample were identified respectively. Based on the theoretical results and actual statistical data, the comparison between $F(X_i)$ and $F_n(X_i)$ was made by using $\Delta = |F(X_i) - F_n(X_i)|$ to reflect the difference between the two. The value of $D_n = \sup \Delta = \sup |F(X_i) - F_n(X_i)|$ for each section of the data sample was then calculated and the critical value of $D_n$ was found. Finally, conclusion was made based on the comparison between results from the theoretical distribution function and the sample cumulative frequency distribution function of the same data point.

4.2 The PDF of the Stress Amplitude Evaluation

Since the CSSMAS of M/V YU FENG came into operation in 2006, huge amount of strain data has been collected over the past ten years. The data sampling period of this system is 1/8 second. Literature review shows that the former researches were usually based upon the daily data as a research object or the data collected in a specific time period by investigators [11]. This research finds that the daily or monthly sequential strains of the hull structure varied significantly. Therefore, it is not sufficient or comprehensive to use data of a specific day or week to represent the total strain features of a ship during a particular length of days. Hence, this research, based upon large amount of data, did statistics by using the Rain-flow Method to get the distribution of the cycle numbers of all scales of stress amplitudes. The statistics results were further used to fit the PDF of the structural stress amplitudes.

An amount of one year’s data from point 7 was selected as a research object. The initial data processes include the transferring from sequential strain data to sequential stress data, and the counting of the amplitude of the stress by the rain-flow counting method to produce the everyday stress spectrum. Fig. 4 gives the statistical results of stress amplitudes with the same scale level, which reviews the daily distribution features of the various stress amplitudes under the same loading condition. Then, fit the Weibull distribution with the statistical resultant data to get the cumulative Weibull distribution function. Fig. 5 shows the comparison of two functions between the typical cumulative Weibull distribution and fitted cumulative Weibull distribution. The comparison yields that the two functions fit each other well, which
proves that the Weibull method could be applied to analyze the distribution of the cycles of the stress amplitudes with the same rate levels. Through maximum likelihood estimation, the fitted PDF of the Weibull distribution is as follow:

\[ p = F(x|a, b) = \int_0^x abt^{b-1}e^{-at^{b}} dt \]

After the above parameters estimation, the hypothesis test based on Kolmogorov-Smirnov method [12] was conducted. Because the observed probability density \( p \) is 0.1733, which is far greater than the significance level of 0.05. Therefore, the null hypothesis that the numerical value obeys the Weibull distribution can be accepted.

Extreme sea working conditions have a significant negative influence on the structure health of the ship. Reflected on the stress curves, these extreme working conditions usually correspond to large value of stress [14]. Therefore, the distribution of these large values of stress could be applied to reflect the significance of extreme sea working conditions. Find the daily average cycle number of different stress amplitudes rates by summing of all the cycling times of various levels of stress amplitudes, and then averaging them. The result is shown in Fig. 6. Taking out those stress amplitudes with values greater than 10 MPa from Fig. 6 and examining their total cycle times during a half year the statistic shows that the maximum stress amplitude is less that 20 MPa and those higher than 10 MPa appears not more than 8 times. This statistic means that the high sea waves in this sea region were not frequent and the ship normally had proper stowage plans. Based on the statistic, it fits the cycle times with the Weibull distribution to produce the fitted cumulative Weibull distribution function. The comparison between the typical and the fitted cumulative Weibull distribution functions is shown in Fig. 7. The result shown that the fitted Weibull function has a slight deviation with the
actual data curve; Therefore, the fitted function can be used to represent the distribution of cycle numbers. By using maximum likelihood estimation, the fitted PDF of the Weibull distribution is as follows:

\[
p = F(x|a, b) = \int_{0}^{x} abt^{b-1}e^{-ax^{b}} dt
\]

\[
= \int_{0}^{x} 0.6532 \times 1.3436 t^{1.3436-1}e^{-0.6532x^{1.3436}} dt
\]

\[
= \int_{0}^{x} 0.8776 t^{0.3436}e^{-0.6532x^{1.3436}} dt \quad (2)
\]

5. The Statistical Analysis of the Increment of the Relative Fatigue Damage

The increment of the relative fatigue damage can be obtained by statistical analysis of the daily equivalent stress amplitudes and the corresponding cycle times during a time period [13], which is a half year in this research. According to the detailed fatigue residence classifications specified in the ship structural profiles and the relevant welding procedure diagrams, the welds nearby the strain sensor 7 are thus classified as F2. By using these data, the relevant increment of fatigue damage is produced and shown in Fig. 8. Furthermore, the traditional experience tells that the accumulated fatigue damage satisfies the Normal or Weibull distribution [10]. To verify this experience, the normal probability plot function, normplot, and the Weibull probability plot function, wblplot, in the Matlab were used separately to produce the normal distribution diagram and the Weibull distribution diagram as shown in Figs. 9 and 10. If the data satisfied the Normal or Weibull distribution, the main section of the data plotted should be in a straight line. As can be seen from these two diagrams, the relative increment of fatigue damage of the half year does satisfy Normal or Weibull distribution.

In order to test whether the relative cumulative damage increment obeys the Normal distribution with an unknown mean and variance, the Jarque-Bera test method [11] was employed. This test is based on the skewness and kurtosis of sample D because for any normal data, its skewness is close to 0 and kurtosis to 2.5 [9]. Therefore, through checking whether the actual values of these two parameters deviate far away from their anticipate values, a judgment could be made. The testing result showed that the daily relative cumulative damage increment refused to normal distribution under the null hypothesis. However, after logarithmic transformation, the distribution of the processed data was more close to the normal distribution. At a significant level of 5%, the test statistic of D were 6.2314 and 4.6932 respectively, and

![Fig. 8 The frequency distribution of the increment damage for the ship hull.](image)

![Fig. 9 Lognormal probability plot for increment damage.](image)
the critical value for rejection of the null hypothesis was 7.3562 (5.9915). So, the values of the test statistics were less than the critical value.

Kolmogorov-Smirnov method [12] for goodness of fit was also used to explore whether the sample data D satisfied a specified theoretical distribution. The test based on this method revealed that D did not reject logarithmic normal distribution with parameters of (-12.3523, 0.4946), and the PDF was:

\[ y = f(\alpha|\mu, \sigma) = \frac{1}{x \sqrt{2\pi} \sigma} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}} = \frac{1}{x \times 0.4946 \sqrt{2\pi}} e^{-\frac{(\ln x + 12.3523)^2}{0.49}} \]  

Similarly, D does not reject a Weibull distribution with parameters of \((4.3655 \times 10^{-8}, 2.1447)\), and had a PDF of

\[ y = F(x|a, b) = \int_0^x abt^{b-1} e^{-ax^b} I_{(0, \infty)}(x) = 4.3655 \times 10^{-8} \times 2.1447 x^{1.1447} e^{-4.3655 \times 10^{-8} x^{2.1447}} I_{(0, \infty)}(x) \]  

Based on the above analysis and statistics, the PDF and distribution function of fatigue damage increment were derived. Then the critical damage increment was examined. The analyzing result revealed that the reliability indexes of the ship under different estimated lifetimes are high and the failure probability of the hull structure is low.

6. Conclusions

The analyzing of strain-stress data of M/V YUFENG shows that the increment of the fatigue damage on the girders of the hull is very low. The rain-flow counting on the sequential strain data reveals that the maximum stress amplitude is still under the threshold value. The distribution of the stress amplitudes and its cycles times satisfy the Weibull distribution. Meanwhile the daily relative fatigue damage increment obeys log-normal or Weibull distribution. Finally, the analyzing results yielded that the ship hull girder structure had a very low failure probability, which matched well with the relative low working load of the ship currently.

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