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Journal of Advanced Concrete Technology, volume 15 (2017), pp. 314-327

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Scientific paper

Time-Dependent Capacity of Large Scale Deep Beams under Sustained Loads

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Received 10 March 2017, accepted 7 July 2017 doi:10.3151/jact.15.314

Abstract

The aim of this study is to investigate the effect of sustained loads on the capacity of large-scale deep beams as a part of underground reinforced concrete, and to conduct parametric studies by numerical methods. The reduced shear capacity under long-term sustained loads is investigated to be associated with the mode of failure, and the higher time-dependency is numerically suggested for deep-beams irrespective of their gain in the short-term static capacity. The main source of the possible decay of shear capacity is analyzed to originate from the creep damage of diagonal compression strut over the web concrete, and a simple expression of the strut-tie analogy is extended to the creep shear rupture of deep beams. Finally, it is concluded that the design safety factor of 1.2~1.3 for shear is appropriate for avoiding the risk of creep failure against long-term sustained loads of about 100 years.

1. Introduction

Sustained loads are generally critical for underground reinforced concrete (RC) such as cut-and-cover ducts, tunnels and retaining walls rather than varying live loads. Then, structural dimensions and detailing are normally determined by permanent service loads for long duration of life. Reported was an excessive deflection of more than 10 times greater than that expected at design stage of underground utility ducts (Chijiwa et al. 2015), and uneven consolidation of surrounding soil foundation was found to produce over-loaded soil pressure to the ducts. It was also reported that in such a static soil-structure interaction, rigid corner connections of slabs and side walls may fail in out-of-plane shear but very gradual in nature, termed as delayed or creep shear failure.

In fact, high levels of sustained loads for long durations may bring about a decay of the shear capacity (Chijiwa et al. 2015; Safiullah et al. 2016). The delayed shear failure has been further investigated for non-slender RC, and gradual shear-failure at a load level below the static capacity was experimentally observed and numerically simulated (Maekawa et al. 2016) as summarized in Fig. 1.

In the past decades, significant studies have been carried out on varying strain-rate and associated time dependent deformations (Rusch 1960; Bazant et al. 1983; Higgins et al. 2013). Rusch (1960) tested the laboratory-scaled RC prisms under sustained loads and varying strain rate conditions, and reported the impact of creep strains on the capacity of the prisms. Afterwards, intensive studies have been conducted for structural members and its mechanism of creep failure has been discussed (Sarkhosh et al. 2013; Tasevski and Muttoni 2016; Safiullah et al. 2016). Sarkhosh et al. (2013) and Sarkhosh (2014) conducted experiments of slender beams (a/d>2.5) and concluded that their shear capacity is not affected by the sustained loading. Contrary, Tasevski and Muttoni (2016) reported that nonlinear creep may affect the capacity of RC members and Safiullah et al. (2016) experimentally testified that creep-strain may and may not affect the capacity depending upon the availability of time for structure to respond.

As these studies mainly target on-ground superstructures, slender beams (a/d >2.5) have been extensively studied. Then in this study, the authors mainly focus on the creep shear failure of short or deep beams (a/d <2), whose mechanism is supposed to be close to that of corner connections of underground RC ducts, tanks and tunnels (Fig. 2).

The nominal short-term shear strength of RC beams is known to decrease as the member size increases (size-effect) of wide ranges of effective depth for rather slender members (Kani 1967; Taylor 1972; Swamy and Queshi 1971; Nojiri and Akiyama 1985; An et al. 1997). Furthermore, valuable studies for a short and deep beam of about 4m effective depth were conducted by Collins et al. (2015).

Although the long-term deformation and the creep capacity of RC were reported (Chijiwa et al. 2015; Maekawa et al. 2016) for comparatively smaller depth of RC members, large-scale RC is under way in practice. Underground LNG storage tanks, whose base slabs can be of the order of 4~10 m depth and is under sustained uplift underground water pressure from foundation, is such an example. For clarifying the reliability of design, it is required to re-evaluate the design safety factor.
whether it is appropriate or not for avoiding the risk of delayed creep failure in future.

Thus, the authors direct their attention to the combined size and long-term sustained load’s effect as explained in Fig. 2. In fact, there is some limitation of experimental approach to the large-scale RC under long-term sustained loading over several years. Then, numerical models, whose applicability to both time-dependency and size effect has been already examined, are used to investigate its mechanism and key factors affecting the delayed shear failure. For summarizing key factors, the design format of the strut-tie analogy is re-
ferred for generalizing knowledge earned by simulation in practice.

2. Scheme of time-dependent simulation

The framework for time-dependent models of structural concrete is illustrated in Fig. 3. This system used in this study can handle transient volumetric deformation related to pore water in motion through micro pores, creep arising due to mixed ambient drying and multidirectional cracking in hardened concrete. The time-dependent simulation models used have been verified under various experiments (Asamoto et al. 2006; Mabrouk et al. 2004; Maekawa et al. 2008). This simulation scheme have been also utilized to investigate time-dependent post-cracking structural concrete (El-Kashif and Maekawa 2004; Alca et al. 1997; Jansen and Shah 1997; Jansen et al. 1995; Maekawa et al. 2008).

For higher stress levels in compression, creep rupture is numerically simulated as the combination of fracturing and plastic evolution of concrete and examined by long-term deformations of RC (Mozer et al. 1970; Chong et al. 2004).

Although this computational framework coded as DuCOM-COM3 (Maekawa et al. 2008) is capable of simulating structural concrete from early ages to the matured, the linkage of thermo-hygral modeling with solid mechanics can be disconnected optionally if mechanical properties of concrete are assumed to be constant over the life regardless of ambient conditions. For deep beams of large sizes as a focus of this study, compressive forces are dominant and hence the time-dependent deformation associated with compressive stress is a major contributor towards the long-term time-dependent performance. In order to highlight this pure impact of compressive time-dependency and to intentionally avoid other factors related to ambient actions and cement hydration, the stand-alone of mechanical simulation is applied as an underlined point of interest as shown in Fig. 3.

Thus, the time-dependent mechanics simulated by the stand-alone version as stated above is rooted in the post-cracking time-dependent model (El-Kashif and Maekawa 2004; Maekawa et al. 2006, 2008), which have been developed for constant environmental input of 20°C temperature and 60% relative humidity. As a matter of fact, the full coupling of thermo-hygral simulation is quite necessary for the case of middle or smaller sized structural members as shown in Fig. 1.

The simulation scheme has been verified for size effect of large-scale RC in shear as indicated in Fig. 2, but its applicability to large-scale short or deep beams with and without shear reinforcement has been less examined although advanced studies have been conducted with experiments (An et al. 1997; Tan and Cheng 2006; Walraven 2007). As a valuable experiment for the large short beam was recently reported by Collins et al. (2015), the simulation was conducted for experimental verification as shown in Fig. 4. The capacity predicted is about 8% higher than the experiment and thought to be within the tolerance in consideration of reproducibility of shear tests. The crack pattern predicted is similar to that observed in the experiment. As the short-term capacity of small and large-scale beams and the long-

![Fig. 3 Scheme of time-dependent modeling.](image-url)
term one of small sized short or deep beams are encompassed by the scheme, the authors deem it to satisfy the requirement to challenge the creep capacity of large-scale short or deep beams as an alternative to experimental approaches.

3. Numerical simulation

The finite element model is selected in consideration of large-scale underground RC tanks and ducts in reference to the shear span of the experiment conducted by Collins et al. (2015) as shown in Fig. 4, which fits the objective of this paper as stated in Section 2. The schematic of the FE mesh for sensitivity analysis is shown in Fig. 5(a). Reinforcement is smeared over the volume of RC elements. To maintain the consistency with the verified case (Collins et al. 2015), the compressive and tensile strengths of concrete for all further simulations are set forth as $f'_c=40$ MPa and $f_t=2.69$ MPa, and the initial stiffness is $28.0$ GPa. For more information on material properties specified, we can refer to Appendix A. For simplification, all the analyses are assumed to start at 28 days and corresponding material properties are provided as an initial input. Hence, $\text{Time}_{\text{absolute}} = \text{Time}_{\text{analysis}} + 28(\text{days})$. The yield strength of the bottom and top reinforcement is 573 and 522 MPa, and the corresponding reinforcement ratio is 0.094 and 0.66%, respectively. Mesh size is kept as uniform as possible in all directions. Shear-lock-free enhanced strain formulation of finite elements (Kasper and Taylor 2000; Taylor et al. 1976) is adopted for avoiding numerical ill-conditions, especially for rather slender finite elements.

3.1 Key factors for parametric study

As diagonal compression stress clearly arises around the web of short or deep beams, so called strut-tie analogy has been successfully applied to these members’ design. Thus, this analogy is expected to be suggestive for identifying the key factors for a parametric study. Strut-tie analogy was first proposed by Ritter (1899), Mörsch (1909), Mörsch and Goodrich (1910) and have been generalized by Collins and Mitchell (1986), and Schlaich et al. (1987). The strength of strut is found to be affected by the shear-span to depth ratio (Arabzadeh
et al. 2009), the longitudinal restrain at support (Vecchio and Collins 1986) and the shape and size of the compressive strut. It is logical to study these parameters affecting strength of compressive strut for the time-dependent performance as well. Then, the authors pay their attention to the development of compressive strut and the structural size on the delayed shear capacity, and Table 1 summarizes the parameters to be discussed.

Reduced depth of beams may generally bring changes of failure mode from shear to flexure. Then, the sensitivity analysis on the beam depth will be advisable for creep effect on both shear and flexural failure. For investigating the effect of longitudinal restrain at support (Vecchio and Collins 1986), boundary conditions are set up to be simply supported as well as statically indeterminate one (Fig. 5(c)). Here, the longitudinal reinforcement ratio in both top and bottom is kept constant as 0.66%.

Shear reinforcement is hypothetically changed to 0.1, 0.5 and 2 times of the referential case (minimum shear reinforcement, same as Collins et al. 2015) by keeping spacing of reinforcing bars as constant. The referential discretized mesh is shown in Fig. 5(b). To analyze the size effect on delayed shear capacity, the depth of beams is changed while keeping the width and the a/d ratio as constant.

### 3. 2 Application of sustained loads
First, the short-term static capacity is obtained by the numerical simulation of a constant loading rate of 0.03 mm/min (nodal displacement control). Next, the constant load of 95-70% of the static capacity is applied to the member until the delayed failure, which is specified when the rate of deflection at the span center exceeds 1mm/sec under the sustained nodal forces. The failure point was confirmed with computed visuals of crack patterns as we do in laboratory experiments. Owing to this less restrictive failure criterion, this sensitivity analysis gets qualitative in nature.

### 4. Numerical Simulation

#### 4. 1 Size of the member
First, let us start the sensitivity analysis on the basis of Fig. 5(a) and change the depth of the beam from the referential value of 4m to 3m (a/d from 1.82 to 2.4) keeping the width of the beam and the shear span length unchanged. Next, the span length is changed from the referential value of 7 m to 4 m (a/d=1) keeping the depth constant. The life-time to failure and the decayed capacity are normalized by the static capacity and corresponding time (see Appendix B) as shown in Fig. 6(a). To improve comprehensibility, time-zones corresponding to 1 year and 100 years are placed as a finite strip in the figure as an easy indicator.

The computed decay of shear capacity under higher sustained loads results from the time-dependency of concrete compression parallel to cracks and tension normal to cracks. Reduced tension stiffness may lead to increase in the flexural deflection, which is also associated with the extension of shear cracking. The nonlinear creep and following rupture along cracks may also have substantial impact to the creep failure, especially for deep beams where clear diagonal stress flow of compression develops.

The same issue can be also confirmed from the failure pattern of the beam (a/d=1.81) as shown in Fig. 6. The crack pattern is illustrated not at the load application but just before the failure after the long-term sustained loading. For the load level of 85%, localized flexure cracks appear quickly. When the same high load level is sustained, so-called bond shear failure occurs accompanying excessive cracking along longitudinal reinforcement which leads to tension shear failure. When the medium-level of loads such as 70% of the capacity is sustained, the initial flexural cracks appear around the mid-span similarly. However, there is enough time left for inclined cracks from flexure to start participating in diagonal compressive flows, which may lead to a diagonal shear-compression failure. In other words, lower sustained load-levels allow for greater interactions of flexural cracks with diagonal struts. Such an interaction may influence on the position and propagation of other cracks.

Although the shear transfer is decayed under the high cycle repetition of loads, no time-dependency of shear transfer is modeled (Maekawa et al. 2008). Then, the computed life-time to failure on the shear capacity of deep beams is expected to be more than the slender ones as discussed in later section. For all cases as shown in Fig. 6, 20~30% safety margin of capacity seems to be sufficient for avoiding creep failure for 100 years’ design period in practice.

At the higher load level (95-80%), the gradient of the shear versus life-time (S-L) curve as shown in Fig. 6(a) falls as the a/d ratio increases from 1.00 to 2.41. Especially, there is a significant difference in the life-time to failure between 1.00 and 1.82 of the a/d ratio, that is to say, creep decay of the shear capacity is comparatively large for deep beams although its absolute static capacit-
ity goes up. In other words, slender RC beams of a/d>2.5 is estimated to possess almost time-independent capacity. Accordingly, it is considered that the arch action is more influenced by the long-term sustained compression rather than so called truss mechanism.

4.2 Bearing boundary of loads
As the size of loading plates is increased, the compressive strut prism becomes larger with increased capacity (JSCE 2012). Figure 7 illustrates the varying compressive strut as the size of the loading plate is changed. Here, Fig. 6(b) exhibits the sensitivity of the load bearing size on the delayed creep capacity. It is evident from the analysis that the creep rupture of the wider strut may gain an important position, because the higher level of sustained compression develops over the greater volume of web concrete. Of course, the absolute shear capacity is great for wide struts even though the time-dependency gets predominant compared to the slender ones. Thus, the safety factor to avoid the risk of creep shear failure shall be specified in considering both plus and minus effects of these factors.

Here, the authors highlight a point as observed in Fig. 6(a) and Fig. 6(b) for a/d=1 and 800 mm cases, respectively. The failure time for these cases does not follow a continuous smooth trend. To gain more insights into this results, additional 800mm Scenerio1 was further examined numerically. The perturbation of just 2-3% from the originally sustained load level brings about substantial fluctuations of life-time as shown in Fig. 6(b), especially in the zone near the original discontinuity (95-85% sustained load range).

Here, be reminded that the parallel processed computation is executed for this study, and allocation of massive CPUs to parallel jobs of finite elements may automatically bring about small perturbation, which may...
successfully introduce the broken symmetry of failure (Geabreyouhannes and Maekawa 2016). Then, it is known that there is no need to introduce the fluctuation of material properties of 3D extent for obtaining the realistic broken symmetry of failure modes.

As the same fluctuation definitely arises in nature as well, overall behaviors shall be understood with the possible variation of loads versus life-time relation. Then, the authors illustrate the highlighted green-belt zone as shown in Fig. 6(b). To provide more insights, failure pattern at different sustained load levels is also indicated in Fig. 6. The possibility of impact of failure-pattern on shape of the load-failure time curve is investigated again in later sections.

4. 3 Support boundary

When support conditions are changed, the longitudinal restrain at support provided to members (Vecchio and Collins 1986) is also changed. Pinned supports without translational degree of freedom provide additional restrain to the member compared to the one of roller supported. Similarly, for continuous beam as shown in Fig. 5, additional moment from middle support increases the restrain effect even further. In this case, two extreme supports are modeled as a roller without sliding, and the mid support as well. For the continuous beam, two-point loading is provided at the middle of each shear span.

The longitudinal restrain at support causes the higher static shear capacity, but as shown in Fig. 6(c), it is evident that the life-time to failure is clearly shortened when the same magnitude of shear force normalized by the static one is continuously applied. We have high static capacity of statically indeterminate RC, but their long-term capacity might be possibly influenced by boundary conditions. This will be a point when the safety factor for serviceability is specified in an engineering viewpoint.

4. 4 Shear reinforcement

Even minimum shear reinforcement specified by codes can significantly improve the shear capacity (Collins et al. 2015). The detail of shear reinforcement for the referential case (Fig. 5(b)) is 1500mm spacing of 20mm diameter bars, and the yield strength of the bars is 520 MPa.

To meet the challenge of sensitivity analysis, different amount of shear reinforcement is placed from that of the referential case, while maintaining the same spacing between bars.

It is evident from Fig. 6(d) that the time-dependency on shear failure gets sensitive with more shear reinforcement. For the case of 50%, the normalized lifetime to failure is reduced by two order from that of the no-reinforcement case under the same normalized sustained loads. With the presence of shear reinforcement, the localization of shear cracking is suppressed and more uniform stress field develops. Thus, members can have higher capacities, but it is possibly suggested that their performance is more affected by sustained loads.

From the sensitivity analysis as mentioned in previous sections, the stronger strut action helps the members to achieve the higher shear-capacity but on the contrary, it may violate the stability more or less in terms of creep shear capacity as summarized in Fig. 8.

| Key parameters          | Strut (arch) action | Absolute Capacity | Stability under sustained loads |
|-------------------------|---------------------|-------------------|---------------------------------|
| a/d ratio               |                     |                   |                                 |
| Support condition       |                     |                   |                                 |
| Plate width             |                     |                   |                                 |
| Shear reinforcement     |                     |                   |                                 |

Fig. 8 Summary of strut-action and time-dependency.
4.5 Characteristic S-L curve

Sarkhosh et al. (2013) reported the load-life curve from their experiments of notched short-beams as shown in Fig. 9. This advanced research indicates the simple linear relation of load versus the life-time to failure by logarithmic scale. This is analogous to the fatigue S-N diagram. However, in this study, the shear versus lifetime (S-L) curve does not show simple linear relation but rather distorted as shown in Fig. 6. It will attribute to the combination of some mixed mechanisms. Then, further investigation will be required.

For the case of varying shear span to depth ratio (a/d=3.5-1.75), S-L curves are almost linear in the load level up to 80%. The similar trend was experimentally reported by Sarkhosh et al. (2013). However, it is not followed for the load level of 70%. Normalized life-time for all the cases converges at a load level of 70%. For the depth of 0.8 m (a/d=9), the life-time follows a log-linear relation with all load levels.

For the beams of a/d=3.5-1.75, the principle failure mode under the static condition is so called shear-tension. Under sustained load conditions, long-term cracking pattern similar to the static one is computationally produced up to the load level of 85%. However, the time-dependent failure turns to somehow flexural dominated shear-compression of diagonal tension around the medium load level of 80-70%. For the depth of d=0.8 m (a/d=9), flexural cracks appear soon after the loading. The depth of the cracks gradually increases as the load level increases. On further increment in the load level, flexural cracks penetrate the compression zone and the top layer concrete is crushed near ultimate capacity. Under sustained loading conditions, however, the failure mode is flexure dominated for all load levels.

Similar shift of failure patterns can be seen in other cases. When the bearing size of load is changed, failure mode for 800mm case shifts from shear-tension to shear-compression around 90% load level. Failure mode for the pinned support remains shear-tension throughout the load range. Hence, its S-L curve follows a log-linear for the whole range of parameters. Thus, it can be concluded that the characteristic S-L curve is a combination of both shear and flexural modes of failure. The same can be confirmed from the various failure patterns and corresponding S-L curves as shown in Fig. 6. Under such considerations, special attention should be given when the sustained load is in the medium range of (70-80%) as the failure could occur much faster than what is expected to only shear-failure mechanism. Figure 9 explains the characteristic trend of S-L curves and its dependence on failure-mode.

The Green-broken line as shown in Fig. 9 is fictitious load-life relation. This line cannot be logically obtained by both analysis and experiment, but is imaginary in nature if the diagonal compression of concrete would have no time-dependency but just the tension time dependency might be substantially mechanistic. Then, the green-broken line conceptually corresponds to the mode of so called tension-shear failure of brittleness, which mainly takes place for slender beams. As a matter of fact, this is similar to the time-independence of slender beams’ capacity (Sarkhosh 2014).

The red-broken line corresponds to flexure or bending-tension failure. The time dependency of tension-stiffness of concrete is taken into account (Maekawa et al. 2006, 2008) in the frame of simulation as shown in Fig. 3. As the tension-stiffness represents the space-averaged bond mechanism, its time-dependency is asso-
associated with local bond shear failure along main reinforcement. As the magnitude of tension time-dependency is much less than that of concrete compression, the structural effect may come up at so long-term ranges as the red-broken lines indicate. Then, the load versus life-time relation of large-scale deep beams, which is indicated by the black solid line, may lie in between the two broken lines of different failure modes and may be surrounded by them as shown in Fig. 9.

4. 6 Time-dependent size effect

For the combined time dependency and the size effect on shear capacity, FE models are conducted as 1/2, 1/3, 1/5, 1/10 scales of the referential case (d=4 m) but excluding shear-reinforcement, while keeping the shear span to depth ratio as constant. The computed S-L curve is shown in Fig. 10. The size effect of the nominal shear strength is also observed according to the life-time to failure. It is clear that the remaining life at a certain load level does not vary significantly regardless of the scale. For 1/10 scale, however, the failure time is greatly shortened (1 order difference) as compared to the others (see Fig. 10).

Attention should be paid again to the sudden drop for 1/3 scale (d=1333 mm) case in Fig. 10. Reproducibility of experimentally obtained shear-capacity was found to be about 5% around the mean values based upon the statistical survey of about 2,000 experimental specimens (Okamura and Higai 1980). As stated on the CPU parallel process-induced perturbation, variation in computed capacity can be also reproduced as the nature does (Maekawa et al. 2016; Gebreyouhannes and Maekawa 2016) owing to the broken symmetry of failure mode.

For this particular case, two predominant failure modes are highlighted in Fig. 10. There is about 5% difference in static shear capacity for the two cases (468 kN and 446 kN). This may affect the reproducibility of sustained loads, especially at much high load-levels such as 95-90% and may result in one order difference in failure-time. In the simulation, sustained load-levels are calculated by using static capacity of 468 kN as highlighted in Appendix B. Occurrence of alternative failure mode may explain the apparent drop in failure-time. The real load sustained may reach near-capacity for the alternative mode despite application of 95% level assuming original failure mode. This explains the almost instantaneous failure. The authors re-iterate the fact as stated previously that, some fluctuation never be avoided in nature for more realistic simulation as highlighted in Fig. 6(b).

Here, let us pay attention to the shear capacity having the same life-time to failure. There exists clear size effect and roughly speaking, the nominal shear strength is about 20% reduced if the life-time is about 100 years and 30% for 1000 years. Then, if the partial safety factor for shear capacity calculation is specified 1.3, we may avoid the risk of creep failure of members in shear.
5. Strut-tie analogy for creep shear capacity

For the case of deep-beams, concrete in the diagonal strut zone could be analogous to an inclined RC prism subjected to compressive sustained stresses. Thus, the time-dependency of the prism found by Rusch (1960) will be effective for the inclined strut as well. Here, the strut-tie analogy is simply assumed, and the compressive stress level, obtained through Mohr’s transformation for principal stress at any given sustained load as shown in Fig. 11, is normalized by the strut-capacity by Arabzadeh et al. (2009) as shown in Fig. 11(b). The computed normalized stress of the diagonal RC elements is related to the S-L diagram for the limit state for further verification in next part. Here, it should be noted again that the used constitutive model of concrete (Maekawa et al. 2008) for nonlinear analysis has been verified for uniaxial time-dependent strength reported by Rusch (1960) as shown in Fig. 11(a).

First verification of the strut-tie analogy for time-dependency is conducted in view of the effect of boundary conditions. As shown in Fig. 12, normalized stress level at any given load level in strut-elements for the indeterminate boundary conditions (Fig. 5(c)) is 20% higher than pinned boundary condition. The negative sign in the graphs generated through inclined prism stress represents compressive stress. The higher stress level in the indeterminate case corresponds to 100 times difference in failure time for the two cases as highlighted in S-L curve. This difference in the life-time to failure and the compressive stress-level is similar to that observed by Rusch (1960) as shown in left bottom cor-
ner of Fig. 12, confirming that the simple strut-tie analogy may be coupled with the time-dependency of the capacity irrespective to varying boundary conditions.

It was reported by experimental studies on size-effect of the deep beams that flexural cracks well develop and release much energy as the depth of the beam is increased (Zhang and Tan 2007). This implies less contribution from compression zone (weaker strut action), when the size is increased. Actually, normalized strut stress is 10% lower (Fig. 13) for 1/5 scale mode compared to 1/10 scale one. One order difference in time-dependency for a 10% stress difference confirms with the results by Rusch (1960), confirming the usefulness of the strut-tie model even under the combined effect of size and time. Hence, similar to the design formulae for the static shear capacity of deep beams, the strut-tie analogy is confirmed to be applicable to the delayed failure of RC members under sustained loads by combining the time-dependent strength model of concrete prism.

6. Conclusion

Systematically arranged parametric analyses by the numerical method were conducted to understand the time-dependent capacity of large-scale deep beams. First, the applicability of time-dependent constitutive models used in the sensitivity analyses was confirmed for deep beams as well as slender ones. The effects of variation in the depth of beams and size, impact of shear-reinforcement and boundary conditions on creep delayed shear strength were also examined. By using the element stress-data available from numerical methods, a simple strut tie analogy was used for summarizing the key factors for delayed shear failure.

1. It is confirmed that stronger the diagonal strut-action develops on webs of members; greater time-dependency arises. Reduction in shear span to depth ratio, increase in bearing size of load and longitudinal restraint at support, and shear-reinforcement result in higher capacity for the members as previously known. However, this also leads to larger creep evolution and damaging under sustained loads. Thus, the time-dependent stability is degraded although the absolute capacity is advanced for deep beams.

2. The characteristics time-dependent capacity and shear versus remaining life-time to failure (S-L) relation can be explained by a combination of shear and flexure failure mechanisms for various load levels.

3. By substituting the time-dependent compressive strength in material level into the strut-tie analogy for structural concrete, we may have the simple estimate of RC shear capacity under sustained loads as well as short term static strength.

This study suggests there are various factors that should be considered for design of structures performing under sustained loads for long time. This study highlights the need to consider such time-dependency while designing of the structures. Safety factors for structural concrete subjected to sustained loads could somewhat be improved by using the general explanations. Within the knowledge of this study, the safety factor of 1.2–1.3 is thought to be appropriate for avoiding the risk of delayed shear failure of large-scale short and/or deep beams during the design period of about 100 years.

However, such an estimation for the safety factor in

Fig. 13 Consistency of strut-tie analogy to FE analysis in terms of size effect.
practice should be improved by considering the linkage with thermo-hygral aspects especially for middle and small sized members, which are under greater impacts of ambient actions unlike the case of large-sized ones. Furthermore, varying mode of failure at various sustained load levels, which is raised by this study as a possible scenario of structural concrete, should be further experimentally verified, examined and investigated.

**Acknowledgements**
The authors express their gratitude to Dr. Zhu of Shimizu Corporation and Dr. Nakarai of Hiroshima University for their suggestions and discussions. This research is financially supported by MEXT/JSPS KAKENHI 23226011.

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Appendix A

To maintain the consistency across computations, numerical analysis for the RC member by Collins et al. (2015) and others are conducted by using the same stand-alone computation with the same material input values of the strength and the fracture energy. The reported compressive strength of concrete used is 40 MPa at 28 days of material age. Pure-uniaxial compressive strength is used in the simulation. It is a bit smaller than the tested nominal compressive strength, because of the friction between loading plates and specimens owing to confinement (Maekawa et al. 2008). Then, 90% of the reported nominal compressive strength of concrete is used as the input parameter. Corresponding tension-softening parameter is computed based upon the principle of the fracture energy balance as proposed by An et al. (1997).

Appendix B

The normalizer load and normalizer time are provided in each analysis case. The normalizer load represents the static capacity for any given numerical model in this paper. These values are obtained under the displacement-controlled for the nodal displacement as 0.03mm/min. The normalizer time is the time corresponding to the static capacity. These values have been used to calculate appropriate the load level and the normalized time as represented in Fig. 6.

The load-deflection curves for a few cases are graphically represented in Fig. A. The case of depth = 1.33m, 0.8m and 0.4m are shown here. The failure-point was chosen as the first drop in the load carrying capacity. The normalized load is selected as the peak load corresponding to failure-point and the corresponding time is selected as normalizer time. This also explains the lower value of failure-time for the case of d=0.8m. The similar approach was taken for all the cases.

| Mechanism | Parameter | Value | Normalizer Load (kN) | Normalizer Time (days) |
|-----------|-----------|-------|-----------------------|------------------------|
| Strut-Action | Span/depth | 1.00 | 1915.99 | 0.065 |
| | | 1.81 | 1206.40 | 0.127 |
| | | 2.41 | 765.52 | 0.169 |
| | Plate Width | 200 mm | 1149.76 | 0.109 |
| | | 400 mm | 1206.40 | 0.127 |
| | | 800 mm | 1438.27 | 0.123 |
| | Boundary Conditions | Roller | 1206.40 | 0.127 |
| | | Pinned | 2091.55 | 0.101 |
| | | Indeterminate | 5231.22 | 0.106 |
| | Shear-Reinforcement | 0.1 | 1225.45 | 0.130 |
| | | 0.5 | 1583.96 | 0.333 |
| | | 1 | 1899.68 | 0.426 |
| Size-Effect | Depth Variation | 4m | 1206.40 | 0.127 |
| | | 2m | 631.67 | 0.064 |
| | | 1.33m | 468.65 | 0.051 |
| | | 0.8m | 314.99 | 0.039 |
| | | 0.4m | 214.13 | 0.049 |

Fig. A Definition of normalizers of the capacity and the corresponding deflection.