Interpretation of the newly observed $\Sigma_b(6097)^+$ and $\Xi_b(6227)^-$ states as the $P$-wave bottom bottom baryons

Kai-Lei Wang¹, Qi-Fang Lü¹ ²  *, Xian-Hui Zhong¹ ²  †

1) Department of Physics, Hunan Normal University, and Key Laboratory of Low-Dimensional Quantum Structures and Quantum Control of Ministry of Education, Changsha 410081, China
2) Synergetic Innovation Center for Quantum Effects and Applications (SICQEA), Hunan Normal University, Changsha 410081, China

The strong decays of the $P$-wave singly bottom heavy baryons belonging to $6_F$ are investigated with a constituent quark model in the $j$-$j$ coupling scheme. The results show that the newly observed $\Sigma_b(6097)$ and $\Xi_b(6227)$ states by the LHCb collaboration can be assigned as the $\lambda$-mode $P$-wave bottom baryons belonging to $6_F$. Given the heavy quark symmetry, both the $\Sigma_b(6097)$ and $\Xi_b(6227)$ states favor the light spin $j = 2$ states with spin-parity numbers $J^P = 3/2^-$ or $J^P = 5/2^-$. More $P$-wave baryons in the $\Sigma_b$, $\Xi_b^*$, and $\Omega_b$ families are most likely to be observed in future experiments for their relatively narrow width.

PACS numbers:

I. INTRODUCTION

Recently, the LHCb collaboration announced the observation of a new bottom baryon $\Xi_b(6227)^-$ in both $\Lambda_b^0 K^-\pi^-$ decay modes [1]. Just several weeks ago, the LHCb collaboration found two new resonances $\Sigma_b(6097)^+$ in the $\Lambda_b^0\pi^0$ channels [2]. The measured masses and widths of the $\Xi_b(6227)^-$ and $\Sigma_b(6097)^+$ are presented as follows,

\begin{equation}
\begin{aligned}
m[\Xi_b(6227)^-] &= 6226.9 \pm 2.0 \pm 0.3 \pm 0.2 \text{ MeV}, \\
\Gamma[\Xi_b(6227)^-] &= 18.1 \pm 5.4 \pm 1.8 \text{ MeV},
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
m[\Sigma_b(6097)^+] &= 6098.0 \pm 1.7 \pm 0.5 \text{ MeV}, \\
\Gamma[\Sigma_b(6097)^+] &= 28.9 \pm 4.2 \pm 0.9 \text{ MeV},
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
m[\Sigma_b(6097)^+] &= 6095.8 \pm 1.7 \pm 0.4 \text{ MeV}, \\
\Gamma[\Sigma_b(6097)^+] &= 31.0 \pm 5.5 \pm 0.7 \text{ MeV}.
\end{aligned}
\end{equation}

Compared with the mass spectrum predicted in various models [3–13], it is found that the $\Xi_b(6227)^-$ and $\Sigma_b(6097)^+$ are good candidates of the $\lambda$-mode $P$-wave bottom baryons belonging to $6_F$.

To explain the nature of $\Xi_b(6227)^-$ and $\Sigma_b(6097)^+$, recently the strong decays of the excited singly bottom states were studied with the $^3P_0$ model in Ref. [14–15]. The results indicate that $\Xi_b(6227)^-$ and $\Sigma_b(6097)^+$ may be good candidates of the $P$-wave states with $J^P = 3/2^-$ or $J^P = 5/2^-$ in the $j$-$j$ coupling scheme. In Ref. [16], we studied the strong decays of the singly bottom baryons within the chiral quark model in the $L$-$S$ coupling scheme. Comparing the predictions with the observations, one can find that both $\Xi_b(6227)$ and $\Sigma_b(6097)$ could be assigned as the $P$-wave states with $J^P = 3/2^-$ or $J^P = 5/2^-$ as well. However, the partial width ratios and decay modes of some $P$-wave $\Xi_b^*$ states predicted within the $j$-$j$ coupling scheme are different from those predicted within the $L$-$S$ coupling scheme.

We have noted that for the singly heavy baryons there may exist some effects of heavy quark symmetry, which was not carefully considered in our previous work [16]. To approximately preserve the heavy quark symmetry with the finite heavy quark masses, the physical states may be closer to the $j$-$j$ coupling basis. In this coupling scheme, the states are mixed states between the states of the $L$-$S$ coupling scheme with the same spin-parity ($J^P$) numbers. It is interesting to find that the newly observed state $\Omega_b(3000)$ at LHCb [17] may be explained as a $J^P = 1/2^-$ resonance via a $[^3P_{1/2}][^1P_{1/2}]$ mixing [18]. This may be evidence that the physical states of singly heavy baryons more favor the $j$-$j$ coupling scheme. In another word, the configuration mixing effects between the singly heavy baryon states with the same $J^P$ should be considered if one adopts the $L$-$S$ coupling scheme.

In this work, considering the requirements of the heavy quark symmetry we study the strong decay behaviors of the $\lambda$-mode $P$-wave bottom baryons $\Sigma_b$, $\Xi_b^*$, and $\Omega_b$ in the $j$-$j$ coupling scheme with the chiral quark model. In this coupling scheme, the states with light spin $j = 0$ and $j = 1$ cannot decay into some special final states, which shows selection rules. The decay modes together with the predicted total widths suggest that both the $\Xi_b(6227)^-$ and $\Sigma_b(6097)^+$ are good candidates of light spin $j = 2$ states, while the $J^P = 3/2^-$ and $J^P = 5/2^-$ cannot be distinguished due to lack of data. The predicted narrow widths of other flavor symmetric partners can provide helpful information for future experiments.

This paper is organized as follows. The spectrum and notations are presented in Sec. II. The chiral quark model is briefly introduced in Sec. III. The strong decays of the $\lambda$-mode $P$-wave $\Sigma_b$, $\Xi_b^*$, and $\Omega_b$ are estimated in Sec. IV. A short summary is presented in the last section.

II. SPECTROSCOPY

The heavy baryon containing a heavy quark violates the SU(4) symmetry. However, the SU(3) symmetry between the other two light quarks ($u$, $d$, or $s$) is approximately kept. Ac-
cording to the symmetry, the heavy baryons containing a single
ge heavy quark belong to two different SU(3) flavor representa-
tions: the symmetric sextet 6F and antisymmetric antitriplet
3F. For example, the Σc 0, 2+ and Ωc form a 6F representa-
tion, while Λb and Ξb 0− form a 3F representation.

In the L-S coupling scheme, the quark model states are con-
built by
\[ |2S+1Lj⟩ = |(ℓpℓJ) S(pJQ)⟩, \] (7)
where ℓp and ℓJ correspond to the eigenvalues of the orbital
angular momenta ℓp and ℓJ for the ρ and L oscillators, re-
spectively, and L stands for the eigenvalue of the total orbital
angular momentum L = ℓp + ℓJ. The parity of a state is
determined by P = (−1)ℓp+ℓJ. Quantum numbers sρ and sQ cor-
tain to the eigenvalues of the total spin of the two
light quarks sρ and the spin of the heavy quark sQ, respec-
tively, while S stands for the eigenvalue of the total spin an-
gular momentum S = sρ + sQ. According to the quark model
classification, there are five 1-wave states belong-
ing to 6F. These states and their corresponding quantum
numbers have been collected in Table I.

| 2S+1Lj | Jp | ℓp | ℓJ | L | sρ | sQ | S |
|--------|----|-----|-----|---|----|----|---|
| 2P1/2 | 1/2 | 0 | 1 | 1 | 1/2 | 1/2 | 1 |
| 2P3/2 | 3/2 | 0 | 1 | 1 | 3/2 | 3/2 | 1 |
| 1P1/2 | 1/2 | 0 | 1 | 1 | 1/2 | 1/2 | 1 |
| 1P3/2 | 3/2 | 0 | 1 | 1 | 3/2 | 3/2 | 1 |
| 1P5/2 | 5/2 | 0 | 1 | 1 | 5/2 | 5/2 | 1 |

TABLE II: The classifications of the low-lying 1-wave states belong-
ing to 6F in the L-S coupling scheme. The states in the L-S coupling
scheme are denoted by |2S+1Lj⟩.

The five 1-wave states belonging to 6F in the j-j coupling scheme
and their corresponding quantum numbers have been collected in Table II.

The states of the j-j coupling scheme are linear combina-
tions of the states of the L-S coupling scheme. The precise
relationship is [8]
\[ |{(ℓpℓJ) S(pJQ)}⟩ = (−1)L+J+4J ′ 2S+1 \sum S \sqrt{2S+1} \]
\[ \begin{pmatrix} \ell & m & 1 \rho & m \nu & 1 \sigma & m \lambda \end{pmatrix}(|{(ℓpℓJ) S(pJQ)}⟩). \] (9)

The heavy-quark symmetry may tell us that there are configura-
tion mixing between the singly heavy baryon states with the
same Jp numbers in the L-S coupling scheme. In the heavy
quark limit, the mixing angles are determined by Eq. (4).

III. STRONG DECAY

To study the strong decays of the low-lying 1-wave bottom
baryons, we apply the chiral quark model [19]. In this model,
the light pseudoscalar mesons, i.e., π, K and η, are treated
as point-like Goldstone boson. This method has been suc-
sessfully applied to the strong decays of heavy-light mesons,
charmed and strange baryons [18, 21–30]. The nonrelativistic
form of the effective quark-pseudoscalar-meson interactions
might be described by [18, 21, 27]:
\[ H_{m}^{π} = \sum_{j} \left[ G\mathbf{σ}_j \cdot \mathbf{q} + h\mathbf{σ}_j \cdot \mathbf{p}_j \right] e^{-i\mathbf{q} \cdot \mathbf{r}}, \] (10)
with G ≡ −\(1 + \frac{\omega_m}{E_f + M_j}\), h ≡ \(\frac{\omega_m}{4\mu_q}\). In the above equa-
tion, \(\mathbf{σ}_j\) and \(\mathbf{p}_j\) stand for the Pauli spin vector and internal
momentum operator for the jth quark of the initial hadron; \(\omega_m\)
and \(\mathbf{q}\) stand for the energy and three momenta of the emit-
ted light meson, respectively; \(E_f\) and \(M_f\) are the energy and
mass of the final heavy baryon; \(\mu_q\) is a reduced mass given by
1/\(\mu_q = 1/m_j + 1/m_f\), with \(m_j\) and \(m_f\) for the masses of the jth
quark in the initial and final hadrons, respectively; and \(L\) is the
isospin operator associated with the pseudoscalar mesons,
which have been defined in Refs. [22, 25]. It should be men-
tioned that the nonrelativistic form of quark-pseudoscalar-
meson interactions expressed in Eq. (10) is similar to that in
Refs. [21, 23, 24, 25], except that the factors \(G\) and \(h\) in this work
have explicit dependence on the energies of final hadrons.

For a light pseudoscalar meson emission in a strong decay
process, the partial decay width can be calculated with [22,
25]
\[ \Gamma_m = \left( \frac{\delta}{f_m} \right)^2 \frac{(E_f + M_f)|\mathbf{q}|}{4\pi M_f(2J_f + 1)} \sum_{J_{1c},J_{2c}} |M_{J_{1c},J_{2c}}|^2, \] (11)
where \(M_{J_{1c},J_{2c}}\) corresponds to the strong amplitudes.

The quantum numbers \(J_{1c}\) and \(J_{2c}\) stand for the third components
of the total angular momenta of the initial and final heavy
baryons, respectively. \(M_f\) is the mass of the initial heavy
quark limit, the mixing angles are determined by Eq. (4).

III. STRONG DECAY

To study the strong decays of the low-lying 1-wave bottom
baryons, we apply the chiral quark model [20]. In this model,
the light pseudoscalar mesons, i.e., π, K and η, are treated
as point-like Goldstone boson. This method has been suc-
sessfully applied to the strong decays of heavy-light mesons,
charmed and strange baryons [18, 21–30]. The nonrelativistic
form of the effective quark-pseudoscalar-meson interactions
might be described by [18, 21, 27]:
\[ H_{m}^{π} = \sum_{j} \left[ G\mathbf{σ}_j \cdot \mathbf{q} + h\mathbf{σ}_j \cdot \mathbf{p}_j \right] e^{-i\mathbf{q} \cdot \mathbf{r}}, \] (10)
with G ≡ −\(1 + \frac{\omega_m}{E_f + M_j}\), h ≡ \(\frac{\omega_m}{4\mu_q}\). In the above equa-
tion, \(\mathbf{σ}_j\) and \(\mathbf{p}_j\) stand for the Pauli spin vector and internal
momentum operator for the jth quark of the initial hadron; \(\omega_m\)
and \(\mathbf{q}\) stand for the energy and three momenta of the emit-
ted light meson, respectively; \(E_f\) and \(M_f\) are the energy and
mass of the final heavy baryon; \(\mu_q\) is a reduced mass given by
1/\(\mu_q = 1/m_j + 1/m_f\), with \(m_j\) and \(m_f\) for the masses of the jth
quark in the initial and final hadrons, respectively; and \(L\) is the
isospin operator associated with the pseudoscalar mesons,
which have been defined in Refs. [22, 25]. It should be men-
tioned that the nonrelativistic form of quark-pseudoscalar-
meson interactions expressed in Eq. (10) is similar to that in
Refs. [21, 23, 24, 25], except that the factors \(G\) and \(h\) in this work
have explicit dependence on the energies of final hadrons.

For a light pseudoscalar meson emission in a strong decay
process, the partial decay width can be calculated with [22,
25]
\[ \Gamma_m = \left( \frac{\delta}{f_m} \right)^2 \frac{(E_f + M_f)|\mathbf{q}|}{4\pi M_f(2J_f + 1)} \sum_{J_{1c},J_{2c}} |M_{J_{1c},J_{2c}}|^2, \] (11)
where \(M_{J_{1c},J_{2c}}\) corresponds to the strong amplitudes.

The quantum numbers \(J_{1c}\) and \(J_{2c}\) stand for the third components
of the total angular momenta of the initial and final heavy
baryons, respectively. \(M \) is the mass of the initial heavy
baryon. $\delta$ as a global parameter accounts for the strength of the quark-meson couplings. It has been determined in our previous study of the strong decays of the charmed baryons and heavy-light mesons. Here, we fix its value the same as that in Refs. \[22, 25\], i.e. $\delta = 0.557$.

In the calculation, we adopt the same quark model parameter set as that in Refs. \[16, 30\]. The masses of the well-established hadrons used in the calculations are taken from the Particle Data Group (PDG) \[33\], and the masses of the undiscovered initial states adopted from the predictions in Ref. \[4\].

![Graph](image)

**FIG. 1:** Partial and total strong decay widths of the the $J^P = 1/2^-$ and $J^P = 3/2^-$ states with $M = 6097$ MeV in the $\Sigma_0$ families as a function of mixing angle $\phi$. The solid curves stand for the total widths.

IV. RESULTS AND DISCUSSIONS

A. $\Sigma_0(6097)\bar{\Sigma}_0$ as the P-wave $\Sigma_0$ states

The measured mass of the $\Sigma_0(6097)$ indicates that it is a good candidate of the $\lambda$-mode P-wave excitations. In the L-S coupling scheme, the strong decay properties of the P wave $\Sigma_0$ baryons have been studied in Ref. \[16\] (see Table. III). From the decay behaviors, it is found that $\Sigma_0(6097)$ favors the $J^P = 3/2^-$ $[\Sigma_0(2P_{3/2})]$ state or the $J^P = 5/2^-$ $[\Sigma_0(4P_{5/2})]$ state. Moreover, the possibilities of $\Sigma_0(6097)$ as a candidate of the other 1P wave $\Sigma_0$ states cannot be excluded, because their predicted total widths are close to that of $\Sigma_0(6097)$, and the $\Lambda_0\pi$ decay mode is allowed.

1. $J^P = 1/2^-$ states

In the $j-j$ coupling scheme, according to Eq. (9), we can obtain two $J^P = 1/2^-$ states as mixed states between $|2P_{1/2}\rangle$ and $|4P_{1/2}\rangle$:

$$\left| J^P = \frac{1}{2}, 0 \right\rangle = -\sqrt{\frac{1}{3}}|2P_{1/2}\rangle + \sqrt{\frac{2}{3}}|4P_{1/2}\rangle,$$

$$\left| J^P = \frac{1}{2}, 1 \right\rangle = \sqrt{\frac{2}{3}}|2P_{1/2}\rangle + \sqrt{\frac{1}{3}}|4P_{1/2}\rangle.$$  \(12\)

If let $\cos \phi = \sqrt{\frac{2}{3}}$ and $\sin \phi = \sqrt{\frac{1}{3}}$, we get the mixing angle $\phi = 35^\circ$. The strong decay properties of $|J^P = \frac{1}{2}, 0\rangle$ and $|J^P = \frac{1}{2}, 1\rangle$ are calculated within the chiral quark model, our results are presented in Table. III. It is seen that the $|J^P = \frac{1}{2}, 0\rangle$ is a narrow state with a width of a few MeV, and mainly decays into $\Lambda_0\pi$; while the $|J^P = \frac{1}{2}, 1\rangle$ state has a width of $\sim 30$ MeV, and dominantly decays into $\Sigma_0\pi$. For a comparison, in Table. III we also list the strong decay properties of the $J^P = 1/2^-$ states $|^2P_{1/2}\rangle$ and $|^4P_{1/2}\rangle$ obtained from the L-S coupling scheme in Ref. \[16\]. It is found that the decay properties of $|J^P = \frac{1}{2}, 1\rangle$ in the $j-j$ coupling scheme are similar to those of $|^2P_{1/2}\rangle$ in the L-S coupling scheme, for the strong decays of $|J^P = \frac{1}{2}, 1\rangle$ are governed by the component of $|^2P_{1/2}\rangle$. However, the strong decay properties of $|J^P = \frac{1}{2}, 0\rangle$ are very different from those in the L-S coupling scheme. The decay properties of the $J^P = 1/2^-$ states are inconsistent with the observations of $\Sigma_0(6097)$, thus, the $\Sigma_0(6097)$ as the $J^P = 1/2^-$ states should be excluded.

It is interesting to note that the $\Sigma_0(3000)$ resonance may be explained as a mixed state $|P = \frac{1}{2}, 1\rangle_1 = \cos \phi |2P_{1/2}\rangle + \sin \phi |4P_{1/2}\rangle$ with the mixing angle $\phi \approx 24^\circ$ \[18\], which is close to $\phi \approx 35^\circ$ determined by the heavy-quark symmetry in the $j-j$ coupling scheme, but not actually equal to that determined by the heavy-quark symmetry. This indicates that on the one hand the heavy-quark symmetry requires the physical states might be mixed states between the states with the same $J^P$ in the L-S coupling scheme, on the other hand because the heavy-quark symmetry is only an approximation, the physical mixing angle should be slightly different from that determined within the $j-j$ coupling scheme. Thus, the physical P-wave singly-heavy baryon states might be mixed states between $|^2P_{1/2}\rangle$ and $|^4P_{1/2}\rangle$ in the L-S coupling scheme, i.e.,

$$\left( \begin{array}{c} |P J^P_1\rangle \\ |P J^P_2\rangle \end{array} \right) = \left( \begin{array}{cc} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{array} \right) \left( \begin{array}{c} |2P_{1/2}\rangle \\ |4P_{1/2}\rangle \end{array} \right), \quad (14)$$

and the mixing angle $\phi$ may range from the value of the L-S coupling scheme ($\phi = 0^\circ$) to that of the $j-j$ coupling scheme. Specially, for the P-wave states with $J^P = 1/2^-$ the mixing angle $\phi \in (0, 35^\circ)$, while for the P-wave states with $J^P = 3/2^-$ the mixing angle $\phi \in (0, 24^\circ)$.

According to the mixing scheme defined in Eq. (14), the strong decay widths of the $J^P = 1/2^-$ states in the $\Sigma_0$ family
as a function of the mixing angle $\phi$ are shown in Fig. 1. It is found that the mixed state $|\Sigma_b P_{\frac{3}{2}}^{-}\rangle_1$ has a width of $\sim 20 - 30$ MeV, its decays are dominated by the $\Sigma_b\pi\pi$ channel, the decay properties are less sensitive to the mixing angle. If the mixing angle is less than the angle $\phi \approx 35^\circ$ obtained in the heavy quark limit, there is a small decay rate into the $\Lambda\pi\pi$ channel. The other mixed state $|\Sigma_b P_{\frac{3}{2}}^{-}\rangle_2$ has a relatively narrower width, which is sensitive to the mixing angle. If we take the same mixing angle $\phi \approx 24^\circ$ as that of $\Omega_b(3000)^{13}$, except for the main decay mode $\Lambda_b\pi\pi$, the $\Sigma_b\pi\pi$ may play an obvious role in its strong decays. The decay properties of $|\Sigma_b P_{\frac{3}{2}}^{-}\rangle_1$ and $|\Sigma_b P_{\frac{3}{2}}^{-}\rangle_2$ are inconsistent with the observations of $\Sigma_b(6097)$.

If let $\cos \phi = \sqrt{\frac{5}{6}}$ and $\sin \phi = \sqrt{\frac{1}{6}}$, we get the mixing angle $\phi \approx 24^\circ$. The decay properties of $|J^P = \frac{3}{2}^+, 1\rangle$ and $|J^P = \frac{3}{2}^+, 2\rangle$ in the $\Sigma_b$ family are calculated within the chiral quark model. Our results are listed in Table III as well. From the Table III, it is found that the $|J^P = \frac{3}{2}^+, 1\rangle$ state has a width of $\sim 28$ MeV, and dominantly decays into the $\Sigma_b\pi\pi$ channel; while the $|J^P = \frac{3}{2}^-, 2\rangle$ state has a relatively broad width of $\sim 39$ MeV, and dominantly decays into the $\Lambda_b\pi\pi\pi$ channel. Both the decay mode and width indicate that $\Sigma_b(6097)$ is a good candidate of the light spin $j = 2$ state ($|J^P = \frac{3}{2}^-, 2\rangle$). The partial width ratio between $\Lambda_b\pi\pi$ and $\Sigma_b\pi\pi$ for $|J^P = \frac{3}{2}^-, 2\rangle$ is predicted to be

$$\frac{\Gamma[\Lambda_b\pi\pi]}{\Gamma[\Sigma_b\pi\pi]} \approx 12,$$  \hspace{1cm} (17)

which can be tested in future experiments. It should be mentioned that the main decay properties, such as the total decay width and dominant decay modes, of the $J^P = 3/2^-$ states $|J^P = \frac{3}{2}^-\rangle_1$ and $|J^P = \frac{3}{2}^-\rangle_2$ in the $j$-$j$ coupling scheme are similar to those of $|\psi_{P_{3/2}}\rangle_2$ and $|\psi_{P_{3/2}}\rangle_3$ in the $L$-$S$ coupling scheme, respectively (see Table IV).

Considering the mixing angle of the physical states with $J^P = 3/2^-$ may have some deviations from the $j$-$j$ couplings, adopting the mixing scheme defined in Eq. (14), we also plot the strong decay width as a function of mixing angle $\phi$ in Fig. III for a reference. It is shown that when $\phi$ varies in $0 - 24^\circ$, the total widths of both $|\Sigma_b P_{\frac{3}{2}}^{-}\rangle_1$ and $|\Sigma_b P_{\frac{3}{2}}^{-}\rangle_2$ are less sensitive to the mixing angle, however, the partial width

| State | $|J^P, J\rangle$ | Channel | $\Gamma$ (MeV) | $B_t$ | $|\psi_{P_{3/2}}\rangle_2$ | $\Gamma$ (MeV) | $B_t$ |
|-------|----------------|--------|-------------|--------|----------------|-------------|--------|
| $\Sigma_b(6101)$ | $|\frac{1}{2}^+, 1\rangle$ | $\Lambda_b\pi\pi$ | $\cdots$ | $\cdots$ | $|\psi_{P_{3/2}}\rangle_2$ | $\Lambda_b\pi\pi$ | $1.74$ | $7.68\%$
| | | $\Sigma_b\pi\pi$ | $28.89$ | $92.04\%$ | | $\Sigma_b\pi\pi$ | $19.26$ | $85.00\%$
| | | $\Sigma^*\pi\pi$ | $2.50$ | $7.96\%$ | | $\Sigma^*\pi\pi$ | $1.66$ | $7.33\%$
| | | total | $31.39$ | | | total | $22.66$ | |
| $\Sigma_b(6095)$ | $|\frac{1}{2}^-, 0\rangle$ | $\Lambda_b\pi\pi$ | $6.00$ | $100.00\%$ | | $\Lambda_b\pi\pi$ | $4.00$ | $28.15\%$
| | | $\Sigma_b\pi\pi$ | $3.25$ | $8.32\%$ | | $\Sigma_b\pi\pi$ | $9.50$ | $66.85\%$
| | | $\Sigma^*\pi\pi$ | $0.71$ | $5.0\%$ | | $\Sigma^*\pi\pi$ | $0.71$ | $5.0\%$
| | | total | $6.00$ | | | total | $14.21$ | |
| $\Sigma_b(6096)$ | $|\frac{1}{2}^+, 2\rangle$ | $\Lambda_b\pi\pi$ | $35.18$ | $90.07\%$ | | $\Lambda_b\pi\pi$ | $29.31$ | $74.60\%$
| | | $\Sigma_b\pi\pi$ | $3.25$ | $8.32\%$ | | $\Sigma_b\pi\pi$ | $4.81$ | $12.24\%$
| | | $\Sigma^*\pi\pi$ | $0.63$ | $1.61\%$ | | $\Sigma^*\pi\pi$ | $0.57$ | $13.16\%$
| | | total | $39.06$ | | | total | $39.29$ | |
| $\Sigma_b(6087)$ | $|\frac{1}{2}^-, 1\rangle$ | $\Lambda_b\pi\pi$ | $1.47$ | $5.28\%$ | | $\Lambda_b\pi\pi$ | $5.38$ | $20.46\%$
| | | $\Sigma_b\pi\pi$ | $26.39$ | $94.72\%$ | | $\Sigma_b\pi\pi$ | $20.71$ | $78.78\%$
| | | total | $27.86$ | | | total | $26.29$ | |
| $\Sigma_b(6084)$ | $|\frac{1}{2}^-, 2\rangle$ | $\Lambda_b\pi\pi$ | $31.38$ | $81.85\%$ | | $\Lambda_b\pi\pi$ | $31.38$ | $81.85\%$
| | | $\Sigma_b\pi\pi$ | $1.09$ | $2.84\%$ | | $\Sigma_b\pi\pi$ | $1.09$ | $2.84\%$
| | | $\Sigma^*\pi\pi$ | $5.77$ | $15.05\%$ | | $\Sigma^*\pi\pi$ | $5.77$ | $15.05\%$
| | | total | $38.34$ | | | total | $38.34$ | |
ratio of $\Gamma(\Lambda_b\pi)/\Gamma(\Sigma_0\pi)$ is very sensitive to the mixing angle. The measurements of the ratio $\Gamma(\Lambda_b\pi)/\Gamma(\Sigma_0\pi)$ might be helpful to determine the mixing angle.

3. $J^P = 5/2^-$ state

For the $J^P = 5/2^-$ state, the $L-S$ coupling scheme is equal to the $j-j$ coupling scheme, i.e.,

$$|J^P = \frac{5}{2}, 2\rangle = |P_{5/2}\rangle.$$  (18)

From Table II it is seen that both the total decay width and the dominant $\Lambda_b\pi$ decay mode of $|J^P = \frac{5}{2}, 2\rangle$ also favor the observations of $\Sigma_0(6097)$. The decay rate of the $J^P = 5/2^-$ state into $\Sigma_0\pi$ is sizable as well. The partial width ratio between the two main decay channels $\Lambda_b\pi$ and $\Sigma_0\pi$ is predicted to be

$$\frac{\Gamma(\Lambda_b\pi)}{\Gamma(\Sigma_0\pi)} \approx 5.$$  (19)

If the $\Sigma_0(6097)$ resonance is the $J^P = 5/2^-$ state indeed, it should be observed in the $\Sigma_0\pi$ channel as well.

As a whole, $\Sigma_0(6097)$ seems to favor the $j = 2$ states with $J^P = 3/2^-$ or $J^P = 5/2^-$. To distinguish these two states, more observations in the $\Sigma_0\pi$ and $\Sigma_0\pi$ channels are needed in future experiments. If $\Sigma_0(6097)$ corresponds to $|J^P = \frac{5}{2}, 2\rangle$, it should be observed in the $\Sigma_0\pi$ channel. On the other hand, if $\Sigma_0(6097)$ is the $J^P = 5/2^-$ state, it should be observed in the $\Sigma_0\pi$ channel.

B. $\Xi_8^b(6227)^-$ as the $P$-wave $\Xi_8^b$ states

The measured mass of $\Xi_8^b(6227)$ indicates that it is a good candidate of the $\lambda$-mode $P$-wave excitations [14]. Within the $L-S$ coupling scheme, the strong decay behaviors of the $P$ wave $\Xi_8^b$ states have been studied in Ref. [16]. It is found that $\Xi_8^b(6227)$ favors the $J^P = 3/2^- |\Xi_8^b, P_{3/2}\rangle$ state or $J^P = 5/2^- |\Xi_8^b, P_{5/2}\rangle$ state. Moreover, due to the theoretical uncertainties and lack of the branching ratio information, the possibilities of the $\Xi_8^b(6227)$ as other $P$ wave $\Xi_8^b$ states cannot be excluded.

In this work, we study the strong decays of the $\lambda$-mode $P$-wave $\Xi_8^b$ state in the $j-j$ coupling scheme, and our results are listed in Table IV.

1. $J^P = 1/2^-$ states

It is found that the $J^P = 1/2^-$ state $|J^P = \frac{1}{2}^-, 0\rangle$ has a width of 51 MeV, this state mainly decays into $\Lambda_bK$ and $\Xi_8\pi$ channels. The decay width of $|J^P = \frac{1}{2}^-, 0\rangle$ is about a factor 3 larger than that of $\Xi_8^b(6227)$. While the other $J^P = 1/2^-$ state $|J^P = \frac{1}{2}^-, 1\rangle$ has a width of 15 MeV, and dominantly decays into $\Xi_8^b\pi$ channel. Although the decay width of $|J^P = \frac{1}{2}^-, 1\rangle$ is close to that of $\Xi_8^b(6227)$, the decay modes are inconsistent with the observations. For a comparison, in Table IV we also list the strong decay properties of the $J^P = 1/2^-$ states $|P_{1/2}\rangle$ and $|P_{1/2}\rangle$ which are calculated within the $L-S$ coupling scheme in Ref. [16]. The predictions show obvious differences between the $j-j$ and $L-S$ coupling schemes (see Table IV).

Considering the mixing angle between $|P_{1/2}\rangle$ and $|P_{1/2}\rangle$ for the physical states with $J^P = 1/2^-$ may have some deviations from the $j-j$ coupling scheme, the strong decay widths $|\Xi_8^b, P_{3/2}\rangle_1$ and $|\Xi_8^b, P_{5/2}\rangle_2$ as a function of mixing angle $\phi$ are shown in Fig. 2. It is found that the decay properties of $|\Xi_8^b, P_{3/2}\rangle_2$ are less sensitive to the mixing angle $\phi$. However, the decay properties of $|\Xi_8^b, P_{5/2}\rangle_1$ shows some sensitivities to the mixing angle. For example, if one adopts the mixing angle $\phi = 35^\circ$ in the heavy quark limit the $\Lambda_bK$ mode for $|\Xi_8^b, P_{5/2}\rangle_1$ is forbidden, while if $\phi = 15^\circ$ the decay rate into $\Lambda_bK$ is sizable. From the decay properties shown in Fig. 2 one can find that the $\Xi_8^b(6227)$ do not favor the mixed states $|\Xi_8^b, P_{3/2}\rangle_1$ and $|\Xi_8^b, P_{5/2}\rangle_2$.

2. $J^P = 3/2^-$ states

For the $J^P = 3/2^-$ state $|J^P = \frac{3}{2}^-, 1\rangle$ in the $\Xi_8^b$ family, it has a width of $\sim 14$ MeV, and dominantly decays into $\Xi_8^b(5945)\pi$, the decay mode is inconsistent with the observations of $\Xi_8^b(6227)$. While the other $J^P = 3/2^-$ state $|J^P = \frac{3}{2}^-, 2\rangle$ has a width of $\sim 24$ MeV, and mainly decays into $\Lambda_bK$.
TABLE IV: Partial widths (MeV) and branching fractions for the strong decays of the 1P-wave states in the j-j coupling scheme compared with that in the L-S coupling scheme taken form Ref. [16] in the $\Xi_b^*$ family. The masses of the P-wave bottom baryons are adopted from the quark model predictions in Ref. [8].

| State   | $|J,j\rangle$ | Channel | $\Gamma_j$ (MeV) | $\mathcal{B}_j$ | $^{3S+1}L_J$ Channel | $\Gamma_j$ (MeV) | $\mathcal{B}_j$ |
|---------|---------------|---------|------------------|----------------|----------------------|------------------|----------------|
| $\Xi_b^*(6233)$ | $|J = \frac{3}{2}^+, 1\rangle$ | $\Lambda_bK$ | $\cdots$ | $\cdots$ | $\Xi_b\pi$ | $\cdots$ | $\cdots$ |
|         |               | $\Xi_b^*\pi$ | $13.85$ | $90.76\%$ | $\Xi_b(5945)\pi$ | $1.41$ | $92.4\%$ |
|         | total         |          | $15.26$ |          |          |          |          |
| $\Xi_b^*(6227)$ | $|J = \frac{3}{2}^-, 0\rangle$ | $\Lambda_bK$ | $38.07$ | $71.71\%$ | $\Xi_b\pi$ | $15.02$ | $28.29\%$ |
|         |               | $\Xi_b^*\pi$ | $\cdots$ | $\cdots$ | $\Xi_b(5945)\pi$ | $\cdots$ | $\cdots$ |
|         | total         |          | $53.09$ |          |          |          |          |
| $\Xi_b^*(6234)$ | $|J = \frac{3}{2}^+, 2\rangle$ | $\Lambda_bK$ | $7.31$ | $26.02\%$ | $\Xi_b\pi$ | $17.89$ | $63.69\%$ |
|         |               | $\Xi_b^*\pi$ | $1.60$ | $5.70\%$ | $\Xi_b(5945)\pi$ | $1.29$ | $4.59\%$ |
|         | total         |          | $28.09$ |          |          |          |          |
| $\Xi_b^*(6224)$ | $|J = \frac{3}{2}^-, 1\rangle$ | $\Lambda_bK$ | $\cdots$ | $\cdots$ | $\Xi_b\pi$ | $0.72$ | $5.02\%$ |
|         |               | $\Xi_b^*\pi$ | $\cdots$ | $\cdots$ | $\Xi_b(5945)\pi$ | $13.61$ | $94.98\%$ |
|         | total         |          | $14.33$ |          |          |          |          |
| $\Xi_b^*(6226)$ | $|J = \frac{3}{2}^-, 2\rangle$ | $\Lambda_bK$ | $4.20$ | $17.22\%$ | $\Xi_b\pi$ | $16.37$ | $67.12\%$ |
|         |               | $\Xi_b^*\pi$ | $0.60$ | $2.46\%$ | $\Xi_b(5945)\pi$ | $3.22$ | $13.20\%$ |
|         | total         |          | $24.39$ |          |          |          |          |

and $\Xi_b\pi$ final states, and the partial width ratio between these two channels is predicted to be

$$\frac{\Gamma[\Lambda_bK]}{\Gamma[\Xi_b\pi]} \approx 0.36.$$  \hspace{1cm} (20)

This ratio predicted by us is obviously smaller than the $^3 P_0$ model prediction $\frac{\Gamma[\Lambda_bK]}{\Gamma[\Xi_b\pi]} \approx 0.89$ in Ref. [14]. Both the decay modes and width of $|J^P = \frac{3}{2}^-, 2\rangle$ are consistent with the observations of $\Xi_b(6227)$. It should be mentioned that the strong decay properties of the $|J^P = 3/2^-\Xi_b^*\pi_\rho$ states $|J^P = \frac{3}{2}^-, 1\rangle$ and $|J^P = \frac{3}{2}^-, 2\rangle$ in the j-j coupling scheme are similar to those of $|^3 P_3/2\rangle$ and $|^3 P_3/2\rangle$ in the L-S coupling scheme, respectively (see Table. [IV]).

The mixing angle of the physical states with $J^P = 3/2^-$ may lie between the L-S and j-j coupling limit, we show the strong decay widths as a function of mixing angle $\phi$ from $0^\circ \sim 24^\circ$ in Fig. [2] It can be found that the deviation from the j-j coupling mixing has small influence on the strong decay behaviors, and our conclusions remain. It should mentioned that the recent QCD sum rule analysis also suggested that $\Xi_b(6227)$ may be a $J^P = 3/2^-$ state [34].

3. $J^P = 5/2^-$ state

For the $|J^P = \frac{5}{2}^-, 2\rangle$ state, the L-S coupling scheme is equal to the j-j coupling scheme. The results indicate that the $\Xi_b^*(6227)$ can be assigned as the $|J^P = \frac{5}{2}^-, 2\rangle$ state as well. The partial width ratio between $\Lambda_bK$ and $\Xi_b\pi$ is predicted to be

$$\frac{\Gamma[\Lambda_bK]}{\Gamma[\Xi_b\pi]} \approx 0.25,$$  \hspace{1cm} (21)

which is slightly smaller than that for $|J^P = \frac{3}{2}^-, 2\rangle$. The ratio predicted by us is obviously smaller than the $^3 P_0$ model prediction $\frac{\Gamma[\Lambda_bK]}{\Gamma[\Xi_b\pi]} \approx 0.94$ in Ref. [14].

As a whole, $\Xi_b^*(6227)$ seems to favor the $j = 2$ states with $J^P = 3/2^-$ or $J^P = 5/2^-$. Our conclusion is also consistent with the calculations of the $^3 P_0$ model and QCD sum rule [14, 35]. To further understand the nature of $\Xi_b^*(6227)$, more measurements, such as the ratio $\Gamma[\Lambda_bK]/\Gamma[\Xi_b\pi]$, are suggested to be carried out in future experiments.
TABLE V: Partial widths (MeV) and branching fractions for the strong decays of the 1P-wave states in the j-j coupling scheme compared with that in the L-S coupling scheme taken from Ref. [16] in the Ω_b family. The masses of the P-wave bottom baryons are adopted from the quark model predictions in Ref. [4].

| State | | | \(\xi_b(6339)\) | | \(\xi_b(6330)\) | | \(\xi_b(6340)\) | | \(\xi_b(6331)\) | | \(\xi_b(6334)\) |
|-------|---|---|---|---|---|---|---|---|---|
| \(J^p, j\) | \(L_{\Sigma_b} \) | \(\Gamma(\xi_b\rightarrow\Omega_b\pi)\) | \(\Gamma(\xi_b\rightarrow\Omega_b\pi)\) | \(\Gamma(\xi_b\rightarrow\Omega_b\pi)\) | \(\Gamma(\xi_b\rightarrow\Omega_b\pi)\) | \(\Gamma(\xi_b\rightarrow\Omega_b\pi)\) | \(\Gamma(\xi_b\rightarrow\Omega_b\pi)\) | \(\Gamma(\xi_b\rightarrow\Omega_b\pi)\) | \(\Gamma(\xi_b\rightarrow\Omega_b\pi)\) |
| \(\frac{1}{2}^+, 0\) | \(\frac{1}{2}^+, 1\) | 143 | 49 | 95 | 1.8 | 0.22 | 1.6 |
| \(\frac{3}{2}^-, 2\) | \(\frac{3}{2}^-, 1\) | | | | | | |
| \(\frac{3}{2}^-, 1\) | \(\frac{3}{2}^-, 2\) | 2.2 | 1.6 | | | | |

FIG. 3: Partial decay widths of the \(\xi_b(6339)\) channel for the \(J^p = 1/2^-\) and \(J^p = 3/2^-\) \(\Omega_b\) states as a function of mixing angle \(\phi\). The resonance mass is taken the quark model prediction \(M = 6330\) MeV.

C. The P-wave \(\Omega_b\) states

For the 1P-wave \(\Omega_b\) states, there are no signals from experiments. By taking the predicted masses from RQM [4], their strong decays in j-j coupling scheme are calculated and shown in Table [X]. For the \(J^p = 1/2^-\) state \(J^p = 1/2^-, 0\), a large decay width of 148 MeV is obtained, its decays are governed by the \(\xi_bK\) mode. The decay mode \(\xi_bK\) for the \(J = 1\) states \(|J^p = 1/2^-, 1\rangle\) and \(|J^p = 3/2^-, 1\rangle\) are forbidden, thus, these states should be very narrow states. The two \(J = 2\) states \(|J^p = 1/2^-, 2\rangle\) and \(|J^p = 3/2^-, 2\rangle\) are predicted to be very narrow states with a width of \(\sim 1 - 2\) MeV, which might be found in future experiments. The strong decay properties of the 1P-wave \(\Omega_b\) states were also studied in Ref. [15, 34]. Our main results are consistent with the predictions in Ref. [15]. For a comparison, the results calculated from the L-S coupling scheme [16] are also listed in Table [X]. It is found that the strong decay properties of \(J = 1/2^-, 0\) and \(J = 3/2^-, 2\) are similar to those of \(|P_{3/2}\rangle\) and \(|P_{3/2}\rangle\) in the L-S coupling scheme, respectively. However, the decay properties of \(J = 1/2^-, 1\) and \(J = 3/2^-, 1\) are very different from those in the L-S coupling scheme.

The physical states with \(J^p = 1/2^-\) as mixed states between \(|P_{1/2}\rangle\) and \(|P_{1/2}\rangle\), the mixing angle may not be the ideal angle \(\phi = 35^\circ\) predicted in the heavy quark limit. Taking the mixing scheme as defined in Eq. [14], the strong decay widths of the \(J^p = 1/2^-\) states as a function of the mixing angle \(\phi\) are presented in Fig. 3. When the mixing angle lies in \(0^\circ \sim 35^\circ\), the width of \(|\Omega_b K_{P}^\pm\rangle_1\) is relatively small, while \(|\Omega_b K_{P}^\pm\rangle_2\) is a broad state with a width of \(\sim 120 \pm 20\) MeV. For the \(J^p = 3/2^-\) states, when the mixing angle varies from \(0^\circ\) to \(24^\circ\), the \(|\Omega_b K_{P}^\pm\rangle_1\) state is a narrow state with a width of \(\sim 1\) MeV, while the partial width of \(|\Omega_b K_{P}^\pm\rangle_2\) into \(\xi_bK\) channel may be less than 0.2 MeV. The \(J^p = 3/2^-\) states might be found in the \(\xi_bK\) channel in future experiments.

V. SUMMARY

In the j-j coupling scheme, the strong decays of the low-lying P-wave singly bottom heavy baryons belonging to \(6_F\) are studied within the chiral quark model. Our results show that the newly observed resonances \(\Sigma_b(6097)^\pm\) can be assigned as the light spin \(j = 2\) states with spin-parity numbers \(J^p = 3/2^-\) or \(J^p = 5/2^-\). If \(\Sigma_b(6097)\) corresponds to \(J^p = 5/2^-\), it might be observed in the \(\Sigma_b\pi\) channel, while if \(\Sigma_b(6097)\) is the \(J^p = 5/2^-\) state, it might be observed in the \(\Sigma_b\pi\) channel.

The newly observed resonance \(\Sigma_b(6227)^+\) favors the light spin \(j = 2\) states with spin-parity numbers \(J^p = 3/2^-\) or \(J^p = 5/2^-\) in the \(\xi_b\) family. \(\Sigma_b(6227)^-\) may be the strange partner of \(\Sigma_b(6097)^-\). The \(J^p = 3/2^-\) state \(|J^p = 5/2^-\rangle\) in the \(\xi_b\) family have similar strong decay properties. In order to identify them, angular distributions of their decays in either strong decay modes or radiative transitions should be needed [16].

Considering the heavy quark symmetry, the \(J^p = 1/2^-\) P-wave \(\Omega_b\) states \(|J^p = 1/2^-, 0\rangle\) might be a broad state with a width of \(\sim 150\) MeV. The two \(J = 2\) states \(|J^p = 3/2^-, 2\rangle\) and \(|J^p = 5/2^-, 2\rangle\) are predicted to be very narrow states with a width of \(\sim 1 - 2\) MeV, which might be found in future experiments. The \(\xi_bK\) decay mode for the \(J = 1\) states \(|J^p = 1/2^-, 1\rangle\) and \(|J^p = 3/2^-, 1\rangle\) are forbidden in the heavy quark limit, thus these two states should be very narrow states. To looking for the missing P-wave \(\Omega_b\) states, the \(\xi_bK\) decay mode is worth to observing.
Acknowledgments

This work is supported, in part, by the National Natural Science Foundation of China under Grants No. 11775078, No.U1832173, and No. 11705056.

[1] R. Aaij et al. [LHCb Collaboration], Observation of a new $\Sigma^*_b$ resonance, Phys. Rev. Lett. 121, 072002 (2018).
[2] R. Aaij et al. [LHCb Collaboration], Observation of two resonances in the $\Lambda^0\pi^-$ systems and precise measurement of $\Sigma^0_b$ and $\Sigma^*_{b0}$ properties, [arXiv:1805.1809 [hep-ex]].
[3] S. Capstick and W. Roberts, Quark models of baryon masses and decays, Prog. Part. Nucl. Phys. 45, S241 (2000).
[4] D. Ebert, R. N. Faustov and V. O. Galkin, Spectroscopy and Regge trajectories of heavy baryons in the relativistic quark-diquark picture, Phys. Rev. D 84, 014025 (2011).
[5] D. Ebert, R. N. Faustov and V. O. Galkin, Masses of excited heavy baryons in the relativistic quark model, Phys. Lett. B 659, 612 (2008).
[6] D. Ebert, R. N. Faustov and V. O. Galkin, Masses of heavy baryons in the relativistic quark model, Phys. Rev. D 72, 034026 (2005).
[7] T. Yoshida, E. Hiyama, A. Hosaka, M. Oka and K. Sadato, Spectrum of heavy baryons in the quark model, Phys. Rev. D 92, 114029 (2015).
[8] W. Roberts and M. Pervin, Heavy baryons in a quark model, Int. J. Mod. Phys. A 23, 2817 (2008).
[9] A. Valcarce, H. Garcilazo and J. Vijande, Towards an understanding of heavy baryon spectroscopy, Eur. Phys. J. A 37, 217 (2008).
[10] M. Karliner, B. Keren-Zur, H. J. Lipkin and J. L. Rosner, The Quark Model and $b$ Baryons, Annals Phys. 324, 2 (2009).
[11] B. Chen, K. W. Wei and A. Zhang, Assignments of $\Lambda_d$ and $\Xi^0_d$ baryons in the heavy quark-light diquark picture, Eur. Phys. J. A 51, 82 (2015).
[12] Q. Mao, H. X. Chen, W. Chen, A. Hosaka, X. Liu and S. L. Zhu, QCD sum rule calculation for P-wave bottom baryons, Phys. Rev. D 92, 114007 (2015).
[13] Z. G. Wang, Analysis of the $1/2^-$ and $3/2^-$ heavy and doubly heavy baryon states with QCD sum rules, Eur. Phys. J. A 47, 81 (2011).
[14] B. Chen, K. W. Wei, X. Liu and A. Zhang, Role of newly discovered $\Xi_b(6227)^-$ for constructing excited bottom baryon family, Phys. Rev. D 98, 031502 (2018).
[15] B. Chen and X. Liu, Assigning the newly reported $\Sigma_b(6097)$ as a P-wave excited state and predicting its partners, [arXiv:1810.00389 [hep-ph]].
[16] K. L. Wang, Y. X. Yao, X. H. Zhong and Q. Zhao, Strong and radiative decays of the low-lying $5^-$ and $P$-wave singly heavy baryons, Phys. Rev. D 96, 116016 (2017).
[17] R. Aaij et al. [LHCb Collaboration], Observation of five new narrow $\Omega^0_b$ states decaying to $\Xi^+_bK^-$, Phys. Rev. Lett. 118, 182001 (2017).
[18] K. L. Wang, L. Y. Xiao, X. H. Zhong and Q. Zhao, Understanding the newly observed $\Omega_b$ states through their decays, Phys. Rev. D 95, 116010 (2017).
[19] H. Y. Cheng, Charmed baryons circa 2015, Front. Phys. 10, 101406 (2015).
[20] A. Manohar and H. Georgi, Chiral quarks and the nonrelativistic quark Model, Nucl. Phys. B 234, 189 (1984).
[21] X. H. Zhong and Q. Zhao, Strong decays of newly observed $\Omega^0_b$ states in a constituent quark model with effective Lagrangians, Phys. Rev. D 81, 014031 (2010).
[22] X. H. Zhong and Q. Zhao, Strong decays of heavy-light mesons in a chiral quark model, Phys. Rev. D 78, 014029 (2008).
[23] X. H. Zhong, Strong decays of the newly observed $D(2550)$, $D(2600)$, $D(2750)$, and $D(2760)$, Phys. Rev. D 82, 114014 (2010).
[24] L. Y. Xiao and X. H. Zhong, Strong decays of higher excited heavy-light mesons in a chiral quark model, Phys. Rev. D 90, 074029 (2014).
[25] X. H. Zhong and Q. Zhao, Charmed baryon strong decays in a chiral quark model, Phys. Rev. D 77, 074008 (2008).
[26] L. H. Liu, L. Y. Xiao and X. H. Zhong, Charm-strange baryon strong decays in a chiral quark model, Phys. Rev. D 86, 034024 (2012).
[27] L. Y. Xiao and X. H. Zhong, $\Xi_b$ baryon strong decays in a chiral quark model, Phys. Rev. D 87, 094002 (2013).
[28] L. Y. Xiao, K. L. Wang, Q. F. Liu, X. H. Zhong and S. L. Zhu, Strong and radiative decays of the doubly charmed baryons, Phys. Rev. D 96, 094005 (2017).
[29] H. Nagahiro, S. Yasui, A. Hosaka, M. Oka and H. Noumi, Structure of charmed baryons studied by pionic decays, Phys. Rev. D 95, 014023 (2017).
[30] Y. X. Yao, K. L. Wang and X. H. Zhong, Strong and radiative decays of the low-lying $D$-wave singly heavy baryons, [arXiv:1803.03634 [hep-ph]].
[31] R. Koniuk and N. Isgur, Baryon Decays in a Quark Model with Chromodynamics, Phys. Rev. D 21, 1868 (1980) Erratum: [Phys. Rev. D 23, 818 (1981)].
[32] S. Godfrey and N. Isgur, Mesons in a Relativized Quark Model with Chromodynamics, Phys. Rev. D 32, 189 (1985).
[33] M. Tanabashi et al. [Particle Data Group], Review of Particle Physics, Phys. Rev. D 98, 030001 (2018).
[34] S. S. Agaev, K. Azizi and H. Sundu, Decay widths of the excited $\Omega^0_b$ baryons, Phys. Rev. D 96, 094011 (2017).
[35] T. M. Aliev, K. Azizi, Y. Sarac and H. Sundu, Structure of the $\Sigma_b(6227)^-$ Resonance, [arXiv:1808.08032 [hep-ph]].