Fermi-liquid effects in the Fulde-Ferrell-Larkin-Ovchinnikov state of two-dimensional \(d\)-wave superconductors

Anton B. Vorontsov

Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803, USA

Matthias J. Graf

Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

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We study the effects of Fermi-liquid interactions on quasi-two-dimensional \(d\)-wave superconductors in a magnetic field. The phase diagram of the superconducting state, including the periodic Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state in high magnetic fields, is discussed for different strengths of quasiparticle many-body interactions within Landau’s theory of Fermi liquids. Decreasing the Fermi-liquid parameter \(F_0^a\) causes the magnetic spin susceptibility of itinerant electrons to increase, which in turn leads to a reduction of the FFLO phase. It is shown that a negative \(F_0^a\) results in a first-order phase transition from the normal to the uniform superconducting state in a finite temperature interval. Finally, we discuss the thermodynamic implications of a first-order phase transition for CeCoIn\(_3\).

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I. INTRODUCTION

The coexistence of magnetism and superconductivity is generally thought to be mutually exclusive. Recent discoveries of exotic superconductors, which exhibit coexistence with ferromagnetism or antiferromagnetism, have challenged this long standing belief.\(^1\)\(^2\)\(^3\) However, it has been known for some time that under certain circumstances conventional superconducting order can coexist with ferromagnetism or antiferromagnetism, have been known for some time that under certain circum-

stances conventional superconducting order can coexist with paramagnetic order in high magnetic fields. Since the 1960s this state of coexistence has become known as Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state.\(^4\)\(^5\) More recently, it has been thought to be observed in the heavy-fermion superconductor CeCoIn\(_3\).\(^6\)\(^7\)\(^8\)

Here, we study the Pauli paramagnetic depairing effect of a magnetic field in a \(d\)-wave superconductor. We use the weak-coupling Bardeen-Cooper-Schrieffer theory of superconductivity with quasiparticle interactions as described by Landau’s theory of Fermi liquids. The FFLO state of a spin-singlet superconductor in high magnetic fields appears as a result of the intricate interplay of two effects: (1) loss of superconducting condensation energy in a magnetic field when Cooper pairs (with antiparallel spins) are depaired, and (2) gain of magnetic energy due to the Zeeman effect by spin-polarizing quasiparticles. As a result, in high fields a spatially nonuniform superconducting state coexists with pockets of spin-polarized quasiparticles localized at the zeros of the oscillating order parameter in real space.

We address the modifications of the FFLO state by Fermi-liquid (FL) interactions. Our analysis is for quasi-two-dimensional (quasi-2D) fermionic systems with a cylindrical Fermi surface and the magnetic field applied perpendicular to the axis of the cylinder. In this geometry we can neglect any orbital effects on the superconducting condensate due to magnetic field. To further simplify our analysis, we restrict our study to FFLO states with 1D spatial modulations of the order parameter and neglect any low-temperature transition to a FFLO state with 2D modulations.\(^9\)

The effects of FL interactions in quasi-2D \(s\)-wave superconductors were first studied by Burkhardt and Rainer within quasiclassical theory.\(^10\) They reported a considerable change of the standard FFLO phase diagram when tuning the FL parameter \(F_0^a\). Since the underlying physics of Pauli depairing is the same for all spin-singlet superconductors, we expect similar new phenomena to occur for \(d\)-wave pairing states. However, we anticipate additional effects due to gap nodes and the asociated spin-polarized nodal quasiparticles. Here, we extend our earlier work\(^11\) by including many-body interactions in the form of FL effects, which enter the quasiclassical Eilenberger equation\(^12\)\(^13\)

\[
[i \varepsilon_m \tilde{\tau}_3 - \tilde{\nu}_Z - \tilde{\sigma}_{\nu L} - \tilde{\Delta}, \tilde{g}] + i \nu_f \cdot \nabla \tilde{g} = 0 ,
\]

with Matsubara frequency \(\varepsilon_m\) and quasiclassical Green’s function \(\tilde{g}\), through the FL dressed Zeeman term, \(\tilde{\nu}_Z\), and the FL self-energy, \(\tilde{\sigma}_{\nu L}\).\(^14\)\(^15\)

\[
\tilde{\nu}_Z + \tilde{\sigma}_{\nu L} = \begin{pmatrix} b \cdot \sigma & 0 \\ 0 & b \cdot \sigma^* \end{pmatrix}, \quad b = \mu_0 B_0/(1 + F_0^a) + \nu.
\]

The Pauli matrices \(\sigma_i\) describe the coupling of quasiparticle spins to the FL dressed external field \(\text{B}_0/(1 + F_0^a)\) and the internal exchange field \(\nu\). The latter satisfies the self-consistency condition given by the spin part of the diagonal component of \(\tilde{g}\),

\[
\nu(R) = A_0^a T \sum_{\varepsilon_m} \int dp' g(R, p'; \varepsilon_m) .
\]

\(A_0^a\) is the isotropic channel of the antisymmetric part of the Landau interaction \(A^a(p, p')\). The Landau parameter \(A_0^a\) is related to the quasiparticle FL parameter \(F_0^a\)
the magnetic moment), calculating the jump in the spin magnetization (density of states per spin at the Fermi energy. It follows that the free parameter $g$ is a free electron.

II. RESULTS AND DISCUSSION

In Figs. 1 and 2 we show the computed phase diagrams of a 2D $d$-wave superconductor for three different strengths of the FL parameter $F_0^a$ ranging from negative to positive. The evolution of the $d$-wave phase diagram with $F_0^a$ is similar to the s-wave case.

We determine the order of a phase transition by calculating the jump in the spin magnetization (density of the magnetic moment $\mu_B \mathbf{M}$) across the transition line. Simultaneously, we check it by directly evaluating the free energy. A discontinuity of the magnetization, $\mathbf{M} = \partial (F/V)/\partial \mathbf{B}$, defines a first-order phase transition, while a kink in $\mathbf{M}$ (discontinuity in the susceptibility) defines a second-order transition.

We adopt the following notation for drawing transition lines: second-order transitions have solid lines, while first-order transitions have dashed lines. For comparison, the unphysical part of the normal (N) to uniform superconducting (USC) state transition (Pauli limited) is shown by a thin dot-dashed line inside the Larkin-Ovchinnikov (LO) phase, as well as the corresponding second-order phase transition between USC and LO phases (solid magenta line).

a. Second-order transition line at $B_{c2}$: First, we consider the second-order instability line of the upper critical field $B_{c2}$ from the N state into the FFLO state with an order parameter $\Delta(R)$ similar to the result by Clogston. At $T = 0$, the free energy density can be expressed in a very intuitive way, $\Delta F/V = -\frac{1}{2} N_p (|\Delta(p)|^2)_{FS} + \frac{1}{2} |\Delta M| \cdot B_0$, similar to the result by Clogston, where $\Delta M = M_N - M$ and $N_p$ is the density of states per spin at the Fermi energy. It follows that a gain in condensation energy happens at the expense of the magnetic energy of the spin-polarized quasiparticles, which is proportional to the difference of the spin magnetization between the N and USC state.

b. First-order transition line at $B_{c1}$: For first-order transitions, we must solve the general expressions (1) - (3), which are nonlinear in the mean fields, and calculate the corresponding Green’s functions and free energy. However, the calculation of the Pauli-limited transition line from the normal into the uniform state is straightforward, since we know already the general form of $\Delta$ for a uniform superconductor. At $T = 0$, the free energy density can be expressed in a very intuitive way, $\Delta F/V = -\frac{1}{2} N_p (|\Delta(p)|^2)_{FS} + \frac{1}{2} |\Delta M| \cdot B_0$, similar to the result by Clogston, where $\Delta M = M_N - M$ and $N_p$ is the density of states per spin at the Fermi energy. It follows that a gain in condensation energy happens at the expense of the magnetic energy of the spin-polarized quasiparticles, which is proportional to the difference of the spin magnetization between the N and USC state. In the s-wave superconductor in the absence of the Meissner effect, the electron magnetization $\mathbf{M}$ vanishes at $T = 0$, and the
Pauli limited field is $\mu B_p = \sqrt{\frac{1}{2}(1 + F_0^a)/(|\Delta(\mathbf{p}; B_p)|^2)}$. For $d$-wave, $\mathbf{M}(T = 0)$ is nonzero due to nodal quasiparticles, but is reduced from the normal-state magnetization by a fraction $p = |\Delta M|/|\mathbf{M}_N|$. Then the right-hand side of the previous equation needs to be divided by $\sqrt{p}$.

If many-body interactions, like FL effects, are considered in the $N$ state, then the spin magnetization of an isotropic Pauli paramagnet is given by $\mathbf{M}_N = \chi_N \mathbf{B}_0 = 2\mu^2 N_p B_0/(1 + F_0^a)$. Thus, for positive FL parameters $F_0^a$ the normal-state susceptibility $\chi_N$ is suppressed compared to a noninteracting Fermi gas, while for negative $F_0^a$ it is enhanced. The FFLO state, as well as the USC state, become stable in a wider range of magnetic fields for positive $F_0^a$ (Fig. 1), and in a smaller region for negative $F_0^a$ (Fig. 2). This happens because pockets of polarized electrons do not gain as much in magnetic energy for $F_0^a > 0$ as for $F_0^a = 0$, and the opposite happens for $F_0^a < 0$. The FFLO state exists for any $F_0^a > 0$, but disappears at some critical negative value, when the upper critical field $B_{c2}$ of the FFLO state, $B_{c2}(T; F_0^a) = B_{c2}(T; 0)(1 + F_0^a)$, drops below the Pauli-limited field, $B_P(T; F_0^a) \approx B_P(T; 0)/(1 + F_0^a)$. At that point the FFLO state becomes unstable against the USC state for any field and temperature. The numerically determined critical value is $F_0^a \approx -0.765$ for a $d$-wave superconductor, which is lower than that for an $s$-wave superconductor, $F_0^a = -0.5$. 

We see that the FFLO state in a 2D $d$-wave superconductor is more stable against FL effects compared to the $s$-wave case discussed in detail by Burkhardt and Rainer. Again, this is not completely unexpected, because spin-polarized nodal quasiparticles can gain magnetic energy without the breaking of Cooper pairs, which is unavoidable in a fully gapped superconductor.

Further, negative values of $F_0^a$ make part of the LO-N transition, $T_s < T < T_{FFLO}$, to be first order (Fig. 2). This is in qualitative agreement with the $s$-wave case. However, in contrast to the $s$-wave case we failed to detect any first-order transition inside the LO phase between phases with different periods $\mathbf{q}$.

c. Second-order transition line at $B_{c1}$: Fig. 2 also shows details of the lower critical transition $B_{c1}$ from the USC state to the periodic LO state. We calculate the free energy as a function of the field for four different types of order parameters. We find successive transitions from the USC solution to the single-domain wall (SDW) and then to the double-domain wall (DDW) and next to the periodic solution (PER) in a thin but finite wedge of magnetic fields. For $T/T_c = 0.15$, we sketch in the inset of Fig. 2 the different energetically favorable solutions with their respective transitions in magnetic field. Although the transition from the USC to SDW state is continuous, there is a sequence of transitions, as domain walls enter the bulk one by one with increasing field.

d. The USC-N transition: Finally, in Fig. 3 we show the normalized Pauli limited transition between the normal and uniform superconducting state for several Fermi-liquid parameters. The break in line from solid to dashed indicates, as before, the change from a second to first-order transition. On the right side of Fig. 3 we show the normalized magnetization in the USC state as a function of field. Note that the magnetization jump, when crossing into the normal state, is larger for negative Fermi-liquid parameters.

FIG. 3: (Color online) Left panel: The Pauli-limited upper critical field for Fermi-liquid parameters $F_0^a = \{0.5, 0.0, -0.5\}$. Right panel: Normalized uniform magnetization vs. field for temperatures $T/T_c = \{0.1, 0.3, 0.5, 0.7, 0.9\}$ as labeled in the bottom window.

III. THERMODYNAMIC IMPLICATIONS

So far we discussed specific results for the phase diagram of a quasi-2D Fermi liquid model in the superconducting state. As we have seen, the existence and extent of a first-order phase transition between the normal and superconducting state - USC or FFLO - at lower temperatures can be modeled by invoking a negative Fermi liquid parameter $F_0^a$. On the other hand, in three dimensional superconductors of type-II the first-order transition line the free energy is continuous.
and results in a generalized Clausius-Clapeyron equation, \( \frac{dS}{dT} = -\frac{\Delta S}{\Delta T} \), which relates the jumps in entropy, \( \Delta S = S_N - S_U \), and magnetization, \( \Delta M = M_N - M_U \), and volume \( V \). If the magnetization in the superconducting state is reduced by a fraction \( 0 < p < 1 \) from \( M_N \), then \( \Delta S/V \sim -pM_N dB_p/dT \). Consequently, the latent heat associated with this transition is \( Q = T\Delta S = pT(M_N dB_p/dT) \).

The experiments by Bianchi et al.\(^{20} \) show a value of \( \Delta S/V_{mol} \approx 200 \text{ mJ/(mol K)} \) at \( T \approx 0.5 \text{ K} \), and a measured slope of the upper critical field of \( dB_p/dT \approx -1.5 \text{ T/K} \) at \( T \approx 0.5 \text{ K} \). Whereas Tayama et al.\(^{20} \) reports a magnetization jump of \( \Delta M \approx 0.1 \text{ MN} \approx 80 \text{ mJ/(mol T)} \) at \( T \approx 0.45 \text{ K} \). It is obvious that the agreement between the ratio of the discontinuities, \( \Delta S/(\Delta V M) \sim 2.5 \text{ T/K} \), and the slope of \( B_p \) is poor at \( T \approx 0.5 \text{ K} \) (a deviation of \( \sim 70\% \)), despite good overall agreement between both phase transition lines. The origin of this inconsistency is poorly understood, but might be related to the nature of the localized f electrons and their contributions to the magnetization. Further entropy and magnetization studies are needed to resolve this open problem.

### IV. CONCLUSIONS

We studied the superconducting phase diagram of quasi-2D \( d \)-wave superconductors in the presence of Fermi-liquid effects in high magnetic fields. We found that a negative Fermi-liquid parameter \( F_0^a \) increases the gain in magnetic energy, while at the same time it reduces the available phase space of a FFLO state. The uniform superconducting and periodic FFLO state are competing and at a critical Fermi-liquid parameter \( F_0^a \approx -0.765 \) the FFLO state is completely suppressed.

We note that in order to explain the high-field phase diagram of the CeCoIn\(_5\) superconductor in terms of an FFLO state one needs to go beyond the simplistic Fermi-liquid picture considered here. While we find that the inclusion of Fermi-liquid effects considerably changes the phase diagram, the changes are not consistent with the experimental findings. For example, (1) the magnitude of the calculated critical temperature \( T_p \), where the Pauli-limited upper critical transition changes from second to first order between the uniform superconducting and normal state, and (2) the corresponding magnetization jump are much larger than those seen in experiment. We also note that the shape of the transition lines of the calculated FFLO state is qualitatively different from experiments. It is not unreasonable to expect that some of those discrepancies may be overcome by including the effects of impurity scattering\(^{20} \) antiferromagnetic spin fluctuations, local magnetic moments, or orbital effects.

Finally, our analysis of the first-order transition puts stringent constraints on thermodynamic properties and reveals a significant discrepancy between specific heat and magnetization measurements that requires further studies.

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