Distributed Optimization Strategy for Multi Area Economic Dispatch Based on Electro Search Optimization Algorithm

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Abstract—A new adopted evolutionary algorithm is presented in this paper to solve the non-smooth, non-convex and non-linear multi-area economic dispatch (MAED). MAED includes some areas which contains its own power generation and loads. By transmitting the power from the area with lower cost to the area with higher cost, the total cost function can be minimized greatly. The tie line capacity, multi-fuel generator and the prohibited operating zones are satisfied in this study. In addition, a new algorithm based on electro search optimization algorithm (ESOA) is proposed to solve the MAED optimization problem with considering all the constraints. In ESOA algorithm all probable moving states for individuals to get away from or move towards the worst or best solution needs to be considered. To evaluate the performance of the ESOA algorithm, the algorithm is applied to both the original economic dispatch with 40 generator systems and the multi-area economic dispatch with 3 different systems such as: 6 generators in 2 areas; and 40 generators in 4 areas. It can be concluded that, ESOA algorithm is more accurate and robust in comparison with other methods.

Index Terms—Multi Area Economic Dispatch, Optimization, MJAYA Algorithm, Prohibited Operating Zone, Tie Line Capacity, Deregulation System.

Nomenclature

Indices

\( i, j \) generating unit indices
\( k \) Iteration index
\( l \) Prohibited operating zone index
\( t \) Candidate solution index
\( M \) Number of area

Constants

\( a_{ij}, b_{ij}, c_{ij}, e_{ij}, f_{ij} \) cost coefficients of \( j \)th generator in the \( i \)th area

\( B_{qi} \) loss coefficient relating the productions of \( q \)th and \( j \)th generators in area \( i \)
\( B_{ij} \) loss coefficient associated with the production of \( j \)th generator in area \( i \)
\( B_{i0} \) loss coefficient parameter (MW) in area \( i \)
\( L_{i} \) number of POZs for \( i \)th generator
\( N_{g} \) number of generating units
\( P_{gi \text{min}} \) lowest output power of \( i \)th generator (MW)
\( P_{gi \text{max}} \) highest output power of \( i \)th generator (MW)
\( P_{gi \text{Low}}, P_{gi \text{Up}} \) minimum/maximum boundary of the \( i \)th POZ for \( i \)th generator, respectively
\( \text{rand}(1, n) \) vector consists of random numbers in the range \([0,1]\)

Variables

\( F(p_{g}) \) generating unit cost function
\( H(X) \) objective function
\( P_{i} \) power output of generating unit \( i \) (MW)
\( T_{i,j} \) Transmission power between area \( i \) and \( j \) areas

I. INTRODUCTION

Economic dispatch is highly concerned when optimization and power system are the topic of discussion. Allocating the required power among the committed generators and minimizing the cost function is the main aim of the economic dispatch. Moreover, it is demanded to satisfy all the physical and operational constraints at the mean-time [1]. The original economic dispatch problem is a

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second order polynomial problem. However, for modeling the valve point loading effect a sinusoidal term should be added [2].

Based on the literature, the economic dispatch problem is solved by different mathematical technique such as Lambda Iteration [3], Gradient Method [4] and linear programming [5]. However, using mathematical methods are not so suitable because of the discontinuity and nonlinearity of the fuel cost which is resulted from the valve point effect [6]. Although in some researches, dynamic programming was utilized for economic dispatch [2], this method is not recommended due to the curse of dimensionality [2]. Moreover, some meta-heuristics methods such as genetic algorithm (GA) [7,8], particle swarm optimization (PSO) [9-11], tabu search (TS) [12], simulated annealing (SA) [13], quasi-oppositional group search optimization (QOGSO) [14], chaotic global best artificial bee colony (CGABC) [15], Firefly algorithm (FA) [16], continuous quick group search optimizer (CQGSO) [17], fuzzy adaptive chaotic ant swarm optimization (FCASO) [18], augmented Lagrange hopfield network (ALHN) [19] were employed. It should be noted that, some of the mentioned methods do not assure the best and optimal solution.

Multi area economic dispatch (MAED) is an extension of economic dispatch problem [1]. The economic dispatch problem needs to be solved in each area and power exchange among the areas need be determined whereas all constraints are satisfied and the cost function is minimized at the meantime. Furthermore, the transmission tie line capacity should be highlighted as one of the most significant distinctive features constraint in MAED. MAED has also been solved by using some mathematical methods such as linear programming [20], Dantzig–Wolfe decomposition principle [21] and decomposition approach using expert systems [22]. The mathematical problem will be more complex by increasing the problem’s decision variables, the non-linearity because of the valve point effect, the discontinuity because of the prohibited operating zones and etc. Hence, some meta-heuristics methods such as particle swarm optimization (PSO) with Reserve-constrained multi-area environmental/economic dispatch [23], a new nonlinear optimization neural network approach [24], artificial bee colony optimization [25], teaching-learning based optimization (TLBO) in [1] and chaotic global best artificial bee colony [15] are proposed.

In this study, a new modified electro search optimization algorithm (ESOA) is utilized for solving the MAED is proposed. ESOA is proposed in [21] in power system operation and planning in which individual trying to approaches the best solution and far from the worst solution to get the best solution. This algorithm is simple, but, by increasing the dimensions of the problem, the performance of the algorithm reduces and defining the optimal solution is not guaranteed. Therefore, by considering all the possible states (such as individuals motion towards the worst solution or the individuals get away from the best solution or others states). In addition, a mutation operator is considered to increase the speed on convergence. Finally, the proposed method is applied for the multi-area economic dispatch problem under different conditions. Meta-heuristics methods are one of the successful tool to solve problem which is used in many areas and fields [26-29].

II. Multi Area Economic Dispatch

A. Original Economic Dispatch

Economic dispatch is considered as one of the most important optimization problems in power system field. The main goal of this problem is to define the generation level so that minimizes the cost while all constrained are satisfied. In order to model the economic dispatch, a quadratic function is utilized. However, in the huge generators the valve point effect and prohibited operating zones can lead to non-linearity and non-convexity of the cost function. Thus, for modeling the huge generators, a sinusoidal term needs to be added to the cost function as the valve point effect:

\[
\text{Min } H(X) = \sum_{i=1}^{N_g} F_i(p_{gi})
\]

\[
F_i(p_{gi}) = a_i \times p_{gi}^2 + b_i \times p_{gi} + c_i + d_i \times \sin(f_i \times (p_{min} - p_{gi}))
\]

Where,

\[
Z = \{p_{g1}, p_{g2}, p_{g3}, ..., p_{gN_g}\}
\]

Figure 1 demonstrates the power generator fuel curve with and without the valve point effect.

![Fuel Cost vs Power Output](image)

**Fig. 1.** The cost function with and without the valve point effect

B. Multi-area Economic Dispatch

Multi area economic dispatch (MAED) is an advanced economic dispatch problem. The aim of MAED is to determine the power generation and the power transmission among all areas so that the cost function would be minimized while the load demand and constraints are still satisfied [30]-[36]. Mathematical modeling of multi area economic dispatch problem, decision variables, objective function and constraints are detailed in the following expression:

\[
\text{Min } H(X) = \sum_{i=1}^{M} \sum_{j=1}^{N_{gi}} F_{ij}(P_{gij})
\]

where
\[
\begin{align*}
F_g(p_{gi}) &= a_{gi} \times p_{gi}^2 + b_{gi} \times p_{gi} + c_{gi} \\
&+ |e_{gi} \times \sin(f_{gi} \times (p_{gi} \text{ min} - p_{gi}))| \\
\text{(4)}
\end{align*}
\]

Also,
\[
Z = [ r_{g1}, r_{g2}, r_{g3}, ..., r_{gL}, T_1, T_2, ..., T_M ]
\text{(5)}
\]

\[
[ p_{g1}, p_{g2}, p_{g3}, ..., p_{gL} ] = [ P_{g11}, P_{g12}, P_{g13}, ..., P_{gL1} , P_{g21}, P_{g22}, P_{g23}, ..., P_{gL2} , P_{g31}, P_{g32}, P_{g33}, ..., P_{gL3} , ..., P_{gL,M} ]
\text{(6)}
\]

\[
[ T_1, T_2, ..., T_M ] = [ T_{i1,1}, T_{i1,2}, ..., T_{i1,M} , T_{i2,1}, T_{i2,2}, ..., T_{i2,M} , ..., T_{iL,1,M} ]
\text{(7)}
\]

In some power plants, multi types of fuel using as sources instead of one specific fuel for power generation. Thus, the coefficients of the cost function in each horizon are varied [15]. Therefore, applying the valve point effect, prohibited operating zones, and tie line capacity constraint, the multi-area economic dispatch lead to a complex, non-linear and non-convex problem. Consequently, a robust and effective optimization method is required to solve the problem [1].

C. Constraint

1. Power Generation Constraint

The power generation of each generator has a limitation as follows:
\[
P_{gi} \text{ min} \leq P_{gi} \leq P_{gi} \text{ max}
\text{(8)}
\]

2. Load Balancing Constraint

Power generators should provide the total load demand and the transmission network losses. Hence, in multi area, the load demand in each area represents as the following expression:
\[
P_{gi} = P_{Di} + P_{Li} + \sum_{j=1,j \neq i}^{N} T_{ij} \text{ i}=1,2,...,M
\text{(9)}
\]

\[P_{Li}\] is represented as the transmission network losses in the \(i^{th}\) area as following [15]:
\[
P_{Li} = \sum_{q=1}^{N} \sum_{j=1}^{L} P_{gij} B_{1}^{i} q_{j} P_{gij} + \sum_{j=1}^{L} B_{1}^{i} P_{gij} + B_{1}^{i} 0_{j}
\text{(10)}
\]

3. Prohibited Operating Zone Constraint

Because of some practical limitations which damages the plant or network instability, in some intervals each generator should not generate power. So, power generation in these intervals is prohibited. Fig (2), demonstrates the fuel curve of the power generators with two prohibited operating zones.

In addition, equation (11) shows the prohibited operating zone constraint as follows:
\[
P_{gi} \text{ min} \leq P_{gi} \leq P_{gi} \text{ max}
\text{(11)}
\]

\[P_{gi,low} \leq P_{gi} \leq P_{gi,high}
\text{(12)}
\]

4. Tie Line Capacity Constraint

The transmission tie line capacity is one of the most important distinctive features constraint in MAED. Power exchange among areas should be between minimum and maximum capacity of the transmission line as follows:
\[
-T_{i,j} \text{ max} \leq T_{i,j} \leq T_{i,j} \text{ max}
\text{(12)}
\]

III. ELECTRO SEARCH OPTIMIZATION ALGORITHM

In this paper, the Electro Search Optimization Algorithm (ESOA) is to overcome the nonlinearity of the problem. ESOA is a new meta-heuristic approach which is developed based on the theory of the electron movement around the nuclear atom. The main reason behind this approach is some significant advantages such as less mathematical calculation and no need to control parameters in comparison with other methods like particle swarm optimization (PSO), genetic algorithm (GA), simulated annealing (SA), hybrid PSO-GA algorithm (PGHA). ESOA is developed based on the following steps [24]:

1) Atom spreading phase

In the phase, the candidate solutions are spread all around the search space randomly.

2) Orbital transition

Based on the quantized energy concept, electrons around each nucleus effort to move to the greater orbit to achieve higher energy level.
\[
e_{i} = N_{i} * (2 \times \text{rand-1}) (1 - \frac{1}{n}) \times r, \quad n \in [2,5]
\text{(13)}
\]
The new electrons in the greater level of energy are considered as the best solution \( (e_{\text{best}}) \) and utilized atom relocating in the next step [37-41].

3) Nucleus relocation

In nucleus relocation phase, the position of the new nucleus \( (N_{\text{new}}) \) is related to the energy stages of the atoms. As the result, the nucleus relocation is defined as follows:

\[
D_k = (e_{\text{best}} \cdot N_{\text{best}}) + Re_k \otimes \left( \frac{1}{N_{\text{best}}} - \frac{1}{N_k} \right)
\]

\[N_{\text{new},k} = N_k + Ac_k \times D_k\] (14)

This step is continued for all nucleus and repositions of all atoms to find the global optimum point. According to the above equations, the convergence speed of the algorithm is related to the Rydberg’s energy constant \( (Re) \) and accelerator coefficient \( (Ac) \) coefficients. These numbers are selected randomly for the first iteration while they will be updated in the next iteration using the following step information.

4) Orbital-Tuner method

Orbital-Tuner technique is utilized to update the Rydberg’s energy and accelerator coefficients. This technique is proposed according to the cumulative normal density function. Therefore, instead of the center of gravity between two candidates, the center of mass is calculated.

\[
Re_{k+1} = \frac{Re_{\text{best}} + \sum_{i=1}^{n} \frac{Re_i / f_{N_i | Re}}{1/f_{N_i | Re}}}{2}
\]

\[
Ac_{k+1} = \frac{Ac_{\text{best}} + \sum_{i=1}^{n} \frac{Ac_i / f_{N_i | Ac}}{1/f_{N_i | Ac}}}{2}
\] (16) (17)

In the ESOA, the initial numbers have no influence on the efficiency of the algorithm. This is due to these coefficients are controlled by self-tuning technique.

IV. CASE STUDY AND RESULTS

In order to validate the effectiveness of the proposed algorithm, it is applied to two different cases as follows:

A. Six generators in two area
B. Forty generators in four areas

A) Six generators in two areas

The selected test system includes six generation units in two different areas. Each area includes three generation units, in which the total load demand is 1263 MW. The load demand of the first area is 758.7 MW (60%) and for the second area is 505.2 MW (40%). The optimization problem includes prohibited operating zone, losses and the valve point effect. The capacity of the transmission line between two areas is 100 MW. Table I demonstrates the superiority of the proposed algorithm in comparison with other well-known optimization algorithm such as genetic algorithm (GA), and teaching-learning based optimization (TLBO).

Convergence speed is one of the significant factor for the heuristic algorithm. Figure 3 demonstrates the high convergence speed of the proposed algorithm.

Table I

| Power Generation (MW) | ESOA | TLBO | GA |
|-----------------------|------|------|----|
| P11                   | 499.94 | 500  | 500 |
| P12                   | 200  | 200  | 200 |
| P13                   | 150  | 150  | 150 |
| P21                   | 199.87 | 204.3271 | 204.3341 |
| P22                   | 146.63 | 154.7095 | 154.7048 |
| P23                   | 75   | 67.5795 | 67.5770 |
| T12                   | 87.68 | --   | --  |
| PL                    | 13.41 | 13.61 | 13.59 |
| Cost                  | 12210.66 | 12255.39 | 12255.42 |

Based on the Fig. 3, the algorithm is optimized and reached the best answer in less than ten iterations.

A) Forty generators in four areas

This system includes forty generators in four areas. This system is very complicated due to many nonlinear and nonconvex parameters. Fig. 4 shows eleven independent run for system B.

Table II compared the distribution of power generated...
is an advance form of ESO and other methods.

One of the most important constraints in multi area economic dispatch problem is tie line capacity constraint. Fig 6, shows the iterative evolution of tie-line flows convergence in system B. According to the figure, the power transmission between areas converge very fast to the optimal value.

| Power generation (MW) | ESOA | TLBO | GA |
|-----------------------|------|------|----|
| P11                   | 110.8215 | 110.8791 | 111.5448 |
| P12                   | 111.0259 | 112.955 | 111.7092 |
| P13                   | 97.40176 | 97.4151 | 98.2429 |
| P14                   | 179.7358 | 179.9466 | 179.8834 |
| P15                   | 87.90663 | 89.4955 | 95.95 |
| P16                   | 139.9999 | 139.8937 | 139.3533 |
| P17                   | 259.5976 | 259.7338 | 259.3395 |
| P18                   | 284.601 | 284.6387 | 285.3569 |
| P19                   | 284.597 | 284.7414 | 284.9627 |
| P20                   | 1.39 | 1.30.1151 | 1.30.2217 |
| P21                   | 94.00712 | 168.8311 | 243.6005 |
| P22                   | 94.01658 | 168.8214 | 95.389 |
| P23                   | 304.5255 | 125.0623 | 214.5171 |
| P24                   | 394.2784 | 394.2799 | 394.0808 |
| P25                   | 394.2884 | 394.2529 | 394.248 |
| P26                   | 394.2787 | 484.0429 | 394.436 |
| P27                   | 489.2967 | 489.284 | 489.9552 |
| P28                   | 489.2897 | 489.2703 | 488.8885 |
| P29                   | 534.7352 | 511.3347 | 511.4713 |
| P30                   | 511.3035 | 511.4548 | 511.4125 |
| P31                   | 523.2761 | 523.2816 | 523.2896 |
| P32                   | 523.2831 | 523.4321 | 523.295 |
| P33                   | 523.29 | 523.377 | 523.4129 |
| P34                   | 523.2834 | 523.5974 | 523.4073 |
| P35                   | 523.2865 | 523.5493 | 523.7703 |
| P36                   | 523.2919 | 523.2773 | 523.5424 |
| P37                   | 10.00002 | 10.1442 | 10.1621 |
| P38                   | 10.0037 | 10.0248 | 10.1326 |
| P39                   | 10 | 10.0862 | 10.6366 |
| P40                   | 88.10562 | 88.2354 | 88.1189 |
| P41                   | 190 | 189.919 | 161.222 |
| P42                   | 189.9999 | 189.9718 | 189.5668 |
| P43                   | 190 | 190 | 189.924 |
| P44                   | 164.7963 | 164.8927 | 165.6621 |
| P45                   | 164.8027 | 165.1343 | 165.4321 |
| P46                   | 164.8005 | 165.2322 | 164.9686 |
| P47                   | 89.14464 | 90.2758 | 109.8137 |
| P48                   | 89.12672 | 109.9813 | 109.7935 |
| P49                   | 102.5187 | 100.2019 | 100.1543 |
| P50                   | 511.2834 | 458.9376 | 459.114 |
| T1                    | 199.9866 | 185.5862 | 172.0652 |
| T2                    | -7.82561 | -23.6686 | -36.3060 |
| T3                    | -81.4729 | -47.1037 | -86.8070 |
| T4                    | -199.994 | -183.0863 | -191.1128 |
| T5                    | -99.9999 | -94.6933 | -98.8231 |
| T6                    | -100 | -97.7497 | -45.0391 |

Cost 121694.384 121760.5 121794.8

Table II Optimal Power generation for case B

Figure 5, compared the convergence curve EASO with GA as one of the well-known optimization algorithm. Based on the figure, GA algorithm is trapped in the local minimums, but EASO algorithm passed easily from the local minimums and reached the best solution.

One of the most important constraints in multi area economic dispatch problem is tie line capacity constraint. Fig 6, shows the iterative evolution of tie-line flows convergence in system B. According to the figure, the power transmission between areas converge very fast to the optimal value.

Figure 7. Iterative evolution of tie-line flows convergence in system B

Multi area economic dispatch is an advance form of economic dispatch. In multi area economic dispatch, in addition to the commonly constraint, the transmission constrains such as tie line capacity should be satisfied as well. In this paper, a new modified and effective algorithm known as the electro search optimization algorithm is proposed. In order to evaluate the ability and effectiveness of the algorithm, it is applied to MAED with different complexity. The results proved the high efficiency, performance, accuracy and robustness of the algorithm. Indeed, ESOA algorithm is simpler with less mathematical calculation, high speed, no need any control parameters and simple implementation for both constraint and unconstrained optimization problems. Moreover, by increasing the dimension and complexity of the problem, the performance of the algorithm is same.
