The study of $\eta_c(1S) \to PP'$ decays

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Abstract

The $\eta_c(1S) \to PP'$ decays are the parity violation modes. These decays can be induced by the weak interactions within the standard model, and have been searched for based on the available experimental data. To meet the needs of experimental investigation, the $\eta_c(1S) \to PP'$ decays are studied with the perturbative QCD approach. It is found that branching ratios are the order of $10^{-15}$ and less, which offers a ready reference for future analyses.
Charmonium is a system containing the charmed quark and antiquark \( c\bar{c} \). Recently, the study of charmonium regained a great renewed interest due to many new discoveries from the massive dedicated investigation by BES-II, CLEO-c, BES-III, BaBar, Belle, Belle-II and LHCb [1].

The \( \eta_c(1S) \) meson is commonly referred to as \( \eta_c \). Both the total spin and orbital angular momentum of \( c \) and \( \bar{c} \) quarks in \( \eta_c \) are zero. The \( \eta_c \) particle is the paracharmonium state with the well-established quantum number of \( J^{PC} = 0^{-+} \) [1]. Its \( J^{PC} \) is different from that of photon. \( \eta_c \) cannot be directly produced at the \( e^+e^- \) collisions. However, \( \eta_c \) can be produced via the magnetic dipole transition process \( J/\psi \to \gamma \eta_c \) with the branching ratio of \( \mathcal{B}(J/\psi \to \gamma \eta_c) = (1.7\pm0.4)\% \) [1]. Up to now, there is over \( 10^{10} \) \( J/\psi \) data samples with BESIII detector [2], the largest amount of available statistics, and corresponding to more than \( 10^8 \eta_c \). Given the large \( J/\psi \) production cross section \( \sigma \sim 3400 \) nb [3], it is expected that more than \( 10^{13} \) \( J/\psi \), corresponding to more than \( 10^{11} \eta_c \), could be accumulated at the Super Tau Charm Facility (STCF) with 3 \( ab^{-1} \) on-resonance dataset in the future. This provides a good opportunity for studying the properties of \( \eta_c \) particle.

Although there is a large amount of data, the experimental study of \( \eta_c \) decays is comparatively limited. So far, only 33 exclusive \( \eta_c \) decay modes have been reported with concrete numerical value. The sum of the 33 branching ratios is about 63\%, and most of measurements have very large uncertainties [1]. The mass of \( \eta_c \) particle, \( m_{\eta_c} = 2983.9\pm0.5 \) MeV [1], is minimal among charmonium, and lies below the open charm threshold. So the \( \eta_c \) decay into hadronic states through the strong interactions is severely hindered by the phenomenological Okubo-Zweig-Iizuka (OZI) rule [4–6]. The \( c\bar{c} \) quark pair in the \( \eta_c \) state can annihilate into two gluons and two photons with branching ratio of \( \mathcal{B}(\eta_c \to \gamma\gamma) = (1.58\pm0.11)\times10^{-4} \) [1]. Among the nonleptonic \( \eta_c \) decays, the simplest hadronic final states are two pseudoscalar mesons. However, it should be pointed out that the \( \eta_c \to PP' \) decays are the parity violating modes, so they should be induced by the weak interactions rather than the strong and electromagnetic ones. The \( \eta_c \to PP' \) decays were experimentally studied at BES-II and BES-III, but no significant signals are observed and only the upper limits on branching ratios are obtained by now [1, 7, 8]. As far as we know, there is no theoretical investigation on the \( \eta_c \to PP' \) decays yet. In this paper, according to the future experimental prospects, we will study the \( \eta_c \to PP' \) decays within the standard model (SM) of the elementary particles in order to offer a ready reference for future analysis.
Within SM, the $\eta_c \rightarrow PP'$ decays are induced by the $W^\pm$ exchange interaction. At the quark level, based on the operator product expansion and renormalization group (RG) method, the effective Hamiltonian in charge of the $\eta_c \rightarrow PP'$ decays is written as [9],

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q_1,q_2} V_{cq_1} V_{cq_2}^* \left\{ C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu) \right\} + \text{h.c.}, \quad (1)$$

where $G_F \simeq 1.166 \times 10^{-5} \text{GeV}^{-2}$ [1] is the Fermi coupling constant, and $q_{1,2} = d$ and $s$. The averaged values of the Cabibbo-Kobayashi-Maskawa (CKM) elements are $|V_{cs}| = 0.987(11)$ and $|V_{cd}| = 0.221(4)$ [1]. The parameter $\mu$ is a factorization scale, which divides the physical contributions into two parts, the short- and long-distance contributions. The Wilson coefficients $C_{1,2}$ summarize the short-distance physical contributions above the scales of $\mu$. They are computable with the RG-improved perturbation theory at the scale of the mass of gauge $W$ boson, $m_W$, and then evolved to a characteristic scale of $\mu$ for $c$ quark decay.

$$\tilde{C}(\mu) = U_4(\mu, m_b) U_5(m_b, m_W) \tilde{C}(m_W), \quad (2)$$

where the explicit expression of $U_f(\mu_f, \mu_i)$ can be found in Ref. [9]. The operators describing the local interactions among four quarks are defined as,

$$O_1 = \left[ \bar{c}_\alpha \gamma_\mu (1 - \gamma_5) q_{1,\alpha} \right] \left[ \bar{q}_{2,\beta} \gamma_\mu (1 - \gamma_5) c_\beta \right], \quad (3)$$

$$O_2 = \left[ \bar{c}_\alpha \gamma_\mu (1 - \gamma_5) q_{1,\alpha} \right] \left[ \bar{q}_{2,\beta} \gamma_\mu (1 - \gamma_5) c_\beta \right], \quad (4)$$

where $\alpha$ and $\beta$ are color indices. The contributions of penguin operators are neglected because of the strong suppression from the CKM factors, $(V_{uq_1} V_{uq_2}^* + V_{cq_1} V_{cq_2}^*)/(V_{cq_1} V_{cq_2}^*) = -(V_{tq_1} V_{tq_2}^*)/(V_{cq_1} V_{cq_2}^*) = O(\lambda^4)$ with the Wolfenstein parameter $\lambda \approx 0.2$.

The decay amplitudes can be written as,

$$\mathcal{A}(\eta_c \rightarrow PP') = \langle PP' | \mathcal{H}_{\text{eff}} | \eta_c \rangle = \frac{G_F}{\sqrt{2}} \sum_{q_1,q_2} V_{cq_1} V_{cq_2}^* \sum_{i=1}^2 C_i(\mu) \langle PP'| O_i(\mu) | \eta_c \rangle. \quad (5)$$

In Eq.(5), the Fermi constant $G_F$, the CKM elements $|V_{cs}|$ and $|V_{cd}|$ have been pretty well determined from data, and the Wilson coefficients $C_{1,2}$ could be reliably computed. So the remaining theoretical work is the evaluations of hadronic matrix elements (HMEs) $\langle O_i \rangle = \langle PP'| O_i(\mu) | \eta_c \rangle$. HMEs describe the complex transformation between quarks and hadrons, and contain the perturbative and nonperturbative contributions.

Recently, some QCD-inspired phenomenological models, such as the QCD factorization (QCDF) approach [10–15] and the perturbative QCD (pQCD) approach [16–22], have been
technically proposed and widely applied to evaluate HMEs. With these phenomenological models, HMEs are generally written as the convolution of scattering amplitudes and the hadronic wave functions (WFs). The scattering amplitudes and WFs reflect the contributions at the quark and hadron levels, respectively. The scattering amplitudes arising from hard gluon exchanges among quarks are calculable with the perturbative theory. WFs including the momentum distributions of hadronic compositions are universal, and could be obtained by nonperturbative methods or from data. In the practical theoretical calculation, the transverse momentum and Sudakov factors are proposed by the pQCD approach to provide an effective cutoff for the endpoint singularities from the collinear approximation. In this paper, we will investigate the $\eta_c \rightarrow PP'$ decays with the pQCD approach, where the decay amplitudes are expressed as the convolution integral of three parts: the Wilson coefficients $C_i$, scattering amplitudes $H$ and hadronic WFs $\Phi$.

$$A_i = \int dx_1 dx_2 dx_3 db_1 db_2 db_3 C_i(t_i) H_i(x_1, x_2, x_3, b_1, b_2, b_3) \Phi_{\eta_c}(x_1, b_1) e^{-S_{\eta_c}} \Phi_P(x_2, b_2) e^{-S_P} \Phi_{P'}(x_3, b_3) e^{-S_{P'}},$$

where $x_i$ is the longitudinal momentum fraction of the valence quark, $b_i$ is the conjugate variable of the transverse momentum, and $e^{-S_i}$ is the Sudakov factor.

In the calculation, it is convenient to use the light-cone vectors to define the kinematic variables. In the rest frame of the $\eta_c$ meson, one has

$$p_{\eta_c} = p_1 = \frac{m_{\eta_c}}{\sqrt{2}}(1, 1, 0),$$

$$p_P = p_2 = \frac{m_{\eta_c}}{\sqrt{2}}(1, 0, 0),$$

$$p_{P'} = p_3 = \frac{m_{\eta_c}}{\sqrt{2}}(0, 1, 0),$$

$$k_1 = \frac{m_{\eta_c}}{\sqrt{2}}(x_1, x_1, \vec{k}_{1T}),$$

$$k_2 = \frac{m_{\eta_c}}{\sqrt{2}}(x_2, 0, \vec{k}_{2T}),$$

$$k_3 = \frac{m_{\eta_c}}{\sqrt{2}}(0, x_3, \vec{k}_{3T}),$$

where $k_i$, $x_i$ and $\vec{k}_{iT}$ are respectively the momentum, longitudinal momentum fraction and transverse momentum, as shown in Fig. 1 (a).
FIG. 1. The Feynman diagrams for the $\eta_c \to K^- K^+$ decay with the pQCD approach, where (a,b) are factorizable diagrams, and (c,d) are nonfactorizable diagrams. The dots denote appropriate interactions, and the dashed circles denote scattering amplitudes.

With the convention of Refs. [23–26], the WFs and distribution amplitudes (DAs) are defined as follows.

$$\langle 0 | \bar{c}_\alpha(0)c_\beta(z) | \eta_c(p_1) \rangle = -\frac{i}{4} f_{\eta_c} \int_0^1 dx_1 e^{-i k_{1z}} \left\{ \left[ \phi_{\eta_c}^a + m_{\eta_c} \phi_{\eta_c}^p \right] \gamma_5 \right\}_{\beta\alpha},$$  (13)

$$\langle P(p_2) | \bar{q}_\alpha(0) q_1\beta(z) | 0 \rangle = -\frac{i f_P}{4} \int_0^1 dx_1 e^{+i k_{2z}} \left\{ \gamma_5 \left[ \phi_P^a \phi_P^p + \mu_P \phi_P^p - \mu_P \left( \gamma \gamma_5 \phi_P^p \right) \right] \right\}_{\beta\alpha},$$  (14)

$$\langle P'(p_3) | \bar{q}_{2\alpha}(0) q_2\beta(z) | 0 \rangle = -\frac{i f_{P'}}{4} \int_0^1 dx_1 e^{+i k_{3z}} \left\{ \gamma_5 \left[ \phi_{2P}^a \phi_{2P}^p + \mu_{P'} \phi_{2P}^p - \mu_{P'} \left( \gamma \gamma_5 \phi_{2P}^p \right) \right] \right\}_{\beta\alpha},$$  (15)

where $f_{\eta_c}$ and $f_{P,P'}$ are decay constants. $\mu_{P,P'} = 1.6\pm0.2$ GeV [25] is the chiral mass. $n_+ = (1,0,0)$ and $n_- = (0,1,0)$ are the null vectors. The explicit expressions of $\phi_{\eta_c}^{a,p}$ and $\phi_P^{a,p,t}$ can be found in Ref. [23] and Refs. [24, 25], respectively. We collect and display these WFs and DAs as follows.

$$\phi_{\eta_c}^{a}(x,b) = N^a x \bar{x} \exp\left\{ -\frac{m_c}{\omega} x \bar{x} \left[ \left( \frac{x - \bar{x}}{2 x \bar{x}} \right)^2 + \omega^2 b^2 \right] \right\},$$  (16)

$$\phi_{\eta_c}^{p}(x,b) = N^p \exp\left\{ -\frac{m_c}{\omega} x \bar{x} \left[ \left( \frac{x - \bar{x}}{2 x \bar{x}} \right)^2 + \omega^2 b^2 \right] \right\},$$  (17)

$$\phi_{\eta_c}^{t}(x,b) = 6 x \bar{x} \left\{ 1 + a_1^P C_1^{3/2}(\xi) + a_2^P C_2^{3/2}(\xi) \right\},$$  (18)

$$\phi_{\eta_c}^{P}(x) = 1 + 3 \rho_+^P - 9 \rho_-^P a_1^P + 18 \rho_+^P a_2^P$$

$$+ \frac{3}{2} (\rho_+^P + \rho_-^P) (1 - 3 a_1^P + 6 a_2^P) \ln(x)$$

$$+ \frac{3}{2} (\rho_+^P - \rho_-^P) (1 + 3 a_1^P + 6 a_2^P) \ln(\bar{x})$$

$$+ \frac{3}{2} (\rho_+^P + \rho_-^P) (1 - 3 a_1^P + 6 a_2^P) \ln(x)$$

$$+ \frac{3}{2} (\rho_+^P - \rho_-^P) (1 + 3 a_1^P + 6 a_2^P) \ln(\bar{x})$$
\[
- \left( \frac{3}{2} \rho^P - \frac{27}{2} \rho^P a_1^P + 27 \rho^P a_2^P \right) C_1^{1/2}(\xi) \\
+ (30 \eta_P - 3 \rho^P a_1^P + 15 \rho^P a_2^P) C_2^{1/2}(\xi),
\]

(19)

\[
\phi^p_p(x) = \frac{3}{2} \left( \rho^P - 3 \rho^P a_1^P + 6 \rho^P a_2^P \right) \\
- C_1^{1/2}(\xi) \{1 + 3 \rho^P - 12 \rho^P a_1^P + 24 \rho^P a_2^P \\
+ \frac{3}{2} (\rho^P + \rho^\prime) (1 - 3 a_1^P + 6 a_2^P) \ln(x) \\
+ \frac{3}{2} (\rho^P - \rho^\prime) (1 + 3 a_1^P + 6 a_2^P) \ln(\bar{x}) \} \\
- 3 \left( 3 \rho^P a_1^P - \frac{15}{2} \rho^P a_2^P \right) C_2^{1/2}(\xi),
\]

(20)

where \( \bar{x} = 1 - x \) and \( \xi = x - \bar{x} = 2x - 1 \). \( \omega = m_c \alpha_s(m_c) \) is the shape parameter. The parameters \( N_{\alpha,P} \) is determined by the normalization conditions,

\[
\int \phi_{n_c}^{a,P}(x, 0) \, dx = 1.
\]

(21)

The meaning and definition of other parameters can refer to Refs. [24, 25].

From Fig. 1, it can be clearly seen that there are only annihilation amplitudes for the \( \eta_c \to PP' \) decays in SM, because the valence quarks of the final states are entirely different from those of the initial state. The annihilation contributions are necessary and important in nonleptonic two-body \( B \) meson decays [27–40]. The \( \eta_c \to PP' \) decays offer another processes to investigate the annihilation contributions within the factorization approaches, besides the \( B_d \to K^+K^- \) and \( B_s \to \pi\pi \) decays. The decay amplitudes are written as follows.

\[
\mathcal{A}(\eta_c \to K^+K^-) = \frac{G_F}{\sqrt{2}} V_{cs} V_{cs}^* \{ a_2 \mathcal{A}_{ab}(\bar{K}, K) + C_1 \mathcal{A}_{cd}(\bar{K}, K) \},
\]

(22)

\[
\mathcal{A}(\eta_c \to K^0\bar{K}^0) = \frac{G_F}{\sqrt{2}} \{ V_{cs} V_{cs}^* [ a_2 \mathcal{A}_{ab}(\bar{K}, K) + C_1 \mathcal{A}_{cd}(\bar{K}, K) ] \\
+ V_{cd} V_{cd}^* [ a_2 \mathcal{A}_{ab}(K, \bar{K}) + C_1 \mathcal{A}_{cd}(K, \bar{K}) ] \},
\]

(23)

\[
\mathcal{A}(\eta_c \to \pi^+\pi^-) = \frac{G_F}{\sqrt{2}} V_{cd} V_{cd}^* \{ a_2 \mathcal{A}_{ab}(\bar{K}, \pi) + C_1 \mathcal{A}_{cd}(\bar{K}, \pi) \},
\]

(24)

\[
\mathcal{A}(\eta_c \to \pi^0\pi^0) = -\frac{G_F}{2} V_{cd} V_{cd}^* \{ a_2 \mathcal{A}_{ab}(\bar{K}, \pi) + C_1 \mathcal{A}_{cd}(\bar{K}, \pi) \},
\]

(25)

\[
\mathcal{A}(\eta_c \to \pi^+\pi^-) = \frac{G_F}{\sqrt{2}} V_{cd} V_{cd}^* \{ a_2 \mathcal{A}_{ab}(\bar{K}, \pi) + C_1 \mathcal{A}_{cd}(\bar{K}, \pi) \},
\]

(26)

\[
\sqrt{2} \mathcal{A}(\eta_c \to \pi^0\pi^0) = \frac{G_F}{\sqrt{2}} V_{cd} V_{cd}^* \{ a_2 \mathcal{A}_{ab}(\bar{K}, \pi) + C_1 \mathcal{A}_{cd}(\bar{K}, \pi) \},
\]

(27)
where the mixing angle is \( \phi = 39.3(1.0)^\circ \), and the flavor bases are \( \eta_q = (u \bar{u} + d \bar{d})/\sqrt{2} \) and \( \eta_s = s \bar{s} \) [41]. Here, it is assumed that (a) the components of glueball, charmonium or bottomonium are negligible, and (b) that the DAs of \( \eta_q \) and \( \eta_s \) are the same as those of \( \pi \) meson, but with different decay constants and mass [39, 41, 42],

\[
 f_q = 1.07(2) \ f_\pi, \quad (41) \\
 f_s = 1.34(6) \ f_\pi, \quad (42)
\]
\[ m_{\eta_s}^2 = m_\eta^2 \cos^2 \phi + m_{q'}^2 \sin^2 \phi - \frac{\sqrt{2} f_s}{f_q} (m_{q'}^2 - m_\eta^2) \cos \phi \sin \phi, \quad (43) \]
\[ m_{\eta_c}^2 = m_\eta^2 \sin^2 \phi + m_{q'}^2 \cos^2 \phi - \frac{f_q}{\sqrt{2} f_s} (m_{q'}^2 - m_\eta^2) \cos \phi \sin \phi, \quad (44) \]

(2) One distinctive feature is that the amplitudes for \( \eta_c \to K^+K^- \) and \( \pi^+\pi^- \) decays are respectively proportional to the module square of the CKM elements \( V_{cs} \) and \( V_{cd} \). It is well known that the magnitudes of \( V_{cs} \) and \( V_{cd} \) are extracted from leptonic and semileptonic charm decays. If these nonleptonic decay modes are accurately measured in the future, they could offer another determinations or constraints of \( |V_{cs}| \) and \( |V_{cd}| \).

**TABLE I.** The values of the input parameters, where their central values will be regarded as the default inputs unless otherwise specified. The numbers in parentheses are errors.

| mass, width and decay constants of the particles [1] |
|-----------------------------------------------------|
| \( m_{\eta^0} = 134.98 \text{ MeV} \), \( m_{K^0} = 497.61 \text{ MeV} \), \( f_\pi = 130.2(1.2) \text{ MeV} \), |
| \( m_{\eta^\pm} = 139.57 \text{ MeV} \), \( m_{K^\pm} = 493.68 \text{ MeV} \), \( f_K = 155.7(3) \text{ MeV} \), |
| \( m_\eta = 547.86 \text{ MeV} \), \( m_{\eta'} = 957.78 \text{ MeV} \), \( f_{\eta_c} = 398.1(1.0) \text{ MeV} \) [43], |
| \( m_c = 1.67(7) \text{ GeV} \), \( m_{\eta_c} = 2983.9(5) \text{ MeV} \), \( \Gamma_{\eta_c} = 32.1(8) \text{ MeV} \), |
| Gegenbauer moments at the scale of \( \mu = 1 \text{ GeV} \) [25] |
| \( a_1^\pi = 0 \), \( a_2^\pi = 0.25(15) \), \( a_1^K = 0.06(3) \), \( a_2^K = 0.25(15) \) |

**TABLE II.** Branching ratios for the \( \eta_c \to PP' \) decays, where the uncertainties come from \( m_c, \mu_P \) and \( a_2^P \), respectively.

| modes \( \eta_c \to K^+K^- \) | branching ratio |
|-------------------------------|----------------|
| \( \eta_c \to K^+K^- \) | \( (1.47^{+0.14+0.63+0.22}_{-0.13-0.48-0.19}) \times 10^{-15} \) |
| \( \eta_c \to K^0K^0 \) | \( (1.55^{+0.15+0.67+0.22}_{-0.13-0.51-0.26}) \times 10^{-15} \) |
| \( \eta_c \to \pi^+K^- \) | \( (5.15^{+0.41+1.61+17.00}_{-0.38-1.22-4.32}) \times 10^{-17} \) |
| \( \eta_c \to \pi^0K^0 \) | \( (2.65^{+0.21+0.81+8.55}_{-0.19-0.62-2.22}) \times 10^{-17} \) |
| \( \eta_c \to \pi^+\pi^- \) | \( (1.38^{+0.13+0.62+0.20}_{-0.12-0.46-0.18}) \times 10^{-18} \) |
| \( \eta_c \to \pi^0\pi^0 \) | \( (6.91^{+0.64+3.12+1.02}_{-0.58-2.32-0.89}) \times 10^{-19} \) |
| modes \( \eta_c \to \eta \) | branching ratio |
|-------------------------------|----------------|
| \( \eta_c \to \eta \) | \( (4.36^{+0.12+1.07+14.18}_{-0.12-0.95-1.60}) \times 10^{-17} \) |
| \( \eta_c \to \eta' \) | \( (1.84^{+0.08+0.57+1.89}_{-0.08-0.46-1.24}) \times 10^{-16} \) |
| \( \eta_c \to \pi^0\eta \) | \( (9.18^{+0.85+4.15+1.35}_{-0.77-3.07-1.18}) \times 10^{-19} \) |
| \( \eta_c \to \pi^0\eta' \) | \( (5.71^{+0.53+2.58+0.84}_{-0.48-1.91-0.73}) \times 10^{-19} \) |
| \( \eta_c \to \eta\eta \) | \( (5.08^{+0.47+2.18+1.02}_{-0.42-1.54-0.92}) \times 10^{-16} \) |
| \( \eta_c \to \eta\eta' \) | \( (1.36^{+0.12+0.55+0.26}_{-0.11-0.39-0.24}) \times 10^{-15} \) |
| \( \eta_c \to \eta'\eta' \) | \( (9.12^{+0.84+3.89+1.84}_{-0.76-2.75-1.66}) \times 10^{-16} \) |
The branching ratio is defined as follows.

\[
\mathcal{B}\!\mathcal{r} = \frac{p_{\text{cm}}}{8 \pi m_{\eta_c}^2 \Gamma_{\eta_c}} |A(\eta_c \to PP')|^2,
\]

where \( p_{\text{cm}} \) is the center-of-mass momentum of final states in the rest frame of the \( \eta_c \) meson. The numerical results of branching ratios obtained with the input parameters in Table I are listed in Table II. Our comments are listed as follows.

(1) Almost all of the decay width of the \( \eta_c \) meson should come from the strong interactions. The parity violating \( \eta_c \to PP' \) decays can only be induced from the weak interactions. For the \( \eta_c \) meson, compared with the strong decays, the occurrence probability of the weak decay is very tiny, only about \( 1/\tau_D \Gamma_{\eta_c} \sim \mathcal{O}(10^{-11}) \). By analogy with the nonleptonic \( B \) meson decays, the pure annihilation decay modes are dynamically suppressed by helicity. Branching ratios for the pure annihilation \( B_s \to \pi\pi \) decays are about 4 orders of magnitude smaller than those of \( B_s \to D_s\pi \) decay [1]. So it is not difficult to imagine that the pure annihilation \( \eta_c \to PP' \) decays should have very small branching ratios, about \( 10^{-15} \) or less.

(2) It is turned out that branching ratios for the \( \eta_c \to PP' \) decays within SM are the order of \( 10^{-15} \sim 10^{-19} \), and far beyond the measurable precision limit of BES-III and future STCF. However, these branching ratios are not as small as the order of \( 10^{-27} \) expected in Ref. [7]. More particularly, the branching ratios for the \( \eta_c \to K\bar{K} \) decays can reach up to the order of \( 10^{-15} \) even without a considerable additional contribution from new physics (NP) beyond the SM. The observation of these decays at any level in the next few decades would be a signal of parity violations from new sources and a hint of NP.

(3) The \( \eta_c \to K\bar{K} \) decays are Cabibbo-favored. The \( \eta_c \to \pi\bar{K} \) decays are singly Cabibbo-suppressed. And the \( \eta_c \to \pi\pi \) decays are doubly Cabibbo-suppressed. In addition, the decay constants \( f_K > f_\pi \). Hence, there is a clear hierarchical pattern among branching ratios,

\[
\mathcal{B}\!\mathcal{r}(\eta_c \to K\bar{K}) > \mathcal{B}\!\mathcal{r}(\eta_c \to \pi\bar{K}) > \mathcal{B}\!\mathcal{r}(\eta_c \to \pi\pi).
\]

(4) The study of the pure annihilation \( \eta_c \to PP' \) decays further confirmed that when the two final states are the particle and antiparticle pair, such as the \( K\bar{K} \) and \( \pi\pi \), the factorizable annihilation contributions from Fig. 1 (a) and (b) exactly cancel each other because of the isospin symmetry, as analyzed in Refs. [12, 37, 38]. In addition, the interferences between the
nonfactorizable annihilation amplitudes for Fig. 1 (c) and (d) are destructive for $\eta_c$ decays because of the opposite signs of the momentum of charm quark propagators. The above factors also led to the small branching ratios for the $\eta_c \to PP'$ decays.

In summary, the parity violating $\eta_c \to PP'$ decays have been investigated based on the available BES-II and BES-III data, while the corresponding theoretical study is lack of references for a long time. In this paper, considering the experimental needs and the high enthusiasms in searching for NP at the intensity frontier, the $\eta_c \to PP'$ decays are studied with the pQCD approach within SM. It is found that branching ratios for the concerned processes are the order of $10^{-15}$ and less, and beyond the current detection capability. This study offer a ready reference for future analyses.

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Appendix A: Building blocks of decay amplitudes

For the sake of convenience in writing, some shorthands are used.

\[
\phi^{a,p}_{\eta_c} = \phi^{a,p}_{\eta_c}(x_1) e^{-S_{\eta_c}},
\]

\[
\phi^{a}_{P,P'} = \phi^{a}_{P,P'}(x_i) e^{-S_{P,P'}},
\]

\[
\phi^{p,t}_{P,P'} = \frac{\mu_P}{m_{\eta_c}} \phi^{p,t}_{P,P'}(x_i) e^{-S_{P,P'}},
\]

\[
C_i A_{jk}(P', P) = i m_{\eta_c}^4 f_{\eta_c} f_P f_{P'} \pi C_F \frac{N_c}{N_c} \left\{ A_j(P', P, C_i) + A_k(P', P, C_i) \right\},
\]

The subscript $i$ of building block $A_i$ corresponds to the indices of Fig. 1. The expressions of $A_i$ are written as follows.

\[
A_a = \int_0^1 dx_2 dx_3 \int_0^\infty db_2 db_3 \alpha_s(t_a) H_a(\alpha_g, \beta_a, b_2, b_3) C_i(t_a)
\]

\[
S_t(\vec{x}_2) \left\{ \phi^a_P \phi^{p,t}_{P', \vec{x}_2} + 2 \phi^p_{P'} [\phi^P_{P'} (1 + \vec{x}_2) + \phi^{t}_{P} x_2] \right\},
\]

\[
A_b = - \int_0^1 dx_2 dx_3 \int_0^\infty db_2 db_3 \alpha_s(t_b) H_b(\alpha_g, \beta_b, b_2, b_3) C_i(t_b)
\]
\begin{equation}
S_t(x_3) \{ \phi_p^a \phi_{P'}^a, x_3 + 2 \phi_P^p [\phi_{P'}^p (1 + x_3) - \phi_{P'}^p \bar{x}_3] \}, \quad (A6)
\end{equation}

\begin{equation}
\mathcal{A}_c = \frac{1}{N_c} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty db_1 db_2 \alpha_s(t_c) H_c(\alpha_g, \beta_c, b_1, b_2) C_i(t_c)
\end{equation}

\begin{equation}
\times \{ \phi_{\eta_c}^a \phi_{\eta_c}^a, (x_3 - x_1) + (\phi_P^p \phi_{P'}^p - \phi_{P'}^p \phi_P^p) (x_3 - \bar{x}_2) 
+ (\phi_P^p \phi_{P'}^p - \phi_{P'}^p \phi_P^p) (x_3 + \bar{x}_2 - 2x_1) \}
\end{equation}

\begin{equation}
+ \phi_{\eta_c}^a \left[ \frac{1}{2} \phi_P^p \phi_{P'}^p + 2 \phi_P^p \phi_{P'}^p \right] \} \big|_{b_2=b_3}, \quad (A7)
\end{equation}

\begin{equation}
\mathcal{A}_d = \frac{1}{N_c} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty db_1 db_2 \alpha_s(t_d) H_d(\alpha_g, \beta_d, b_1, b_2) C_i(t_d)
\end{equation}

\begin{equation}
\times \{ \phi_{\eta_c}^a \phi_{\eta_c}^a, (x_2 - x_1) + (\phi_P^p \phi_{P'}^p - \phi_{P'}^p \phi_P^p) (x_3 - \bar{x}_2) 
+ (\phi_P^p \phi_{P'}^p - \phi_{P'}^p \phi_P^p) (2 \bar{x}_1 - \bar{x}_2 - x_3) \}
\end{equation}

\begin{equation}
- \phi_{\eta_c}^a \left[ \frac{1}{2} \phi_P^p \phi_{P'}^p + 2 \phi_P^p \phi_{P'}^p \right] \} \big|_{b_2=b_3}, \quad (A8)
\end{equation}

\begin{equation}
S_{n_c} = s(x_1, p_1^+, 1/b_1) + 2 \int_{1/b_1}^t \frac{d\mu}{\mu} \gamma_q, \quad (A9)
\end{equation}

\begin{equation}
S_P = s(x_2, p_2^+, 1/b_2) + s(\bar{x}_2, p_2^-, 1/b_2) + 2 \int_{1/b_2}^t \frac{d\mu}{\mu} \gamma_q, \quad (A10)
\end{equation}

\begin{equation}
S_{P'} = s(x_3, p_3^-, 1/b_3) + s(\bar{x}_3, p_3^+, 1/b_3) + 2 \int_{1/b_3}^t \frac{d\mu}{\mu} \gamma_q, \quad (A11)
\end{equation}

\begin{equation}
\alpha_g = m_{\eta_c}^2 \bar{x}_2 x_3, \quad (A12)
\end{equation}

\begin{equation}
\beta_a = m_{\eta_c}^2 \bar{x}_2, \quad (A13)
\end{equation}

\begin{equation}
\beta_b = m_{\eta_c}^2 x_3, \quad (A14)
\end{equation}

\begin{equation}
\beta_c = \alpha_g - m_{\eta_c}^2 x_1 (\bar{x}_2 + x_3), \quad (A15)
\end{equation}

\begin{equation}
\beta_d = \alpha_g - m_{\eta_c}^2 \bar{x}_1 (\bar{x}_2 + x_3), \quad (A16)
\end{equation}

and other definitions can be found in Ref. [44].

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