Quantum radiation force on a moving mirror with Dirichlet and Neumann boundary conditions at vacuum, finite temperature and coherent states.

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We consider a real massless scalar field in a two-dimensional spacetime, satisfying Dirichlet or Neumann boundary condition at the instantaneous position of a moving boundary. For a relativistic law of motion, we show that Dirichlet and Neumann boundary conditions yield the same radiation force on a moving mirror when the initial field state is invariant under time translations. We obtain the exact formulas for the energy density of the field and the radiation force on the boundary for vacuum, thermal and coherent state. In the nonrelativistic limit, our results coincide with those found in the literature.

In the 70s decade the first works investigating the quantum problem of the radiation emitted by moving mirrors in vacuum were published, motivated by the investigation of particle creation in the nonstationary universe (see Refs. [1, 2, 3, 4, 5, 6, 7]). Fulling and Davies [3] studied the moving mirror radiation problem in the context of a real scalar field in a two dimensional spacetime. They obtained an exact formula for the finite physical part of the expected value of the energy-momentum tensor, assuming the initial state as the vacuum. Their results revealed that the radiation is originated at the mirror and propagates away from it. Ford and Vilenkin [8] developed a perturbative method which can be applied to mirrors moving in small displacements and with nonrelativistic velocities. In this approximation, they obtained, for a real scalar field in a two dimensional spacetime, that the radiation force is proportional to the third time derivative of the mirror law of motion, which is the nonrelativistic limit of the result obtained in Ref. [3]. Ford and Vilenkin also applied their method to a scalar field in four-dimensional spacetime, obtaining a force proportional to the fifth time derivative of the displacement of the mirror. Davies and Fulling [3] (see also Ref. [9]) found a class of hyperbolic trajectories of a mirror moving in a 1+1 Minkowski spacetime, for which the emitted radiation corresponds to a thermal spectrum, that could be registered by an inertial detector. Moreover, several works have focused on the problem of moving mirrors placed in a thermal bath (considered as the “in” state, instead of the vacuum). Jackel and Reynaud [10] obtained for the scalar field in 1 + 1 dimensions the thermal contribution to the dissipative force proportional to the velocity of the mirror, valid in the nonrelativistic limit. Thermal effects have also been considered in Refs. [11, 12, 13, 14, 15, 16]. The coherent state is another initial field state which has been considered [1, 17, 18], as well as the superposition of coherent states used to study decoherence via the dynamical Casimir effect [19, 20]. Furthermore, attention has been given to the role that boundary conditions play on the dynamical Casimir effect. In the static Casimir effect, different boundary conditions can change the sign of the Casimir force [21, 22]. Applications of this change, for instance, in the building of nanoelectromechanical systems have been discussed [23].

The role of Dirichlet and Neumann conditions on the static Casimir force has been investigated in Refs. [24, 25, 26]. The exact method of Fulling-Davies [3] is frequently considered as less flexible than the perturbative methods because it is based on conformal transformations and applicable only for the two-dimensional spacetime [14, 15], whereas the perturbative approach can be extended to higher dimensions. On the other hand, the Fulling-Davies method enables to obtain exact results for the scalar field in the two-dimensional spacetime. In the present work, instead of following the approximate approach considered in Ref. [17], we use the exact approach proposed by Fulling and Davies [3], and show that, also for relativistic laws of motion, Dirichlet and Neumann boundary conditions yield the same radiation force on a moving mirror when the initial field state is symmetrical under time translations. In this context, we calculate for a real massless scalar field in a two-dimensional spacetime the exact (relativistic) formulas for the energy density of the field and for the force on the mirror, for Dirichlet and Neumann conditions, at vacuum, finite temperature and coherent initial states of the field. In the nonrelativistic limit, these exact formulas lead to the approximate results found in Refs. [10, 17].

We start by reviewing the exact field solution for Dirichlet and Neumann dynamical boundary conditions [2, 3, 4]. Let us consider the field satisfying the Klein-Gordon equation (we assume throughout this paper $\hbar = c = 1$): $(\partial^2_t - \partial^2_x) \phi(t, x) = 0$, and obeying Dirichlet ($\phi(t', x')_{|_{boundary}} = 0$) or Neumann ($\partial_x \phi(t', x')_{|_{boundary}} = 0$) condition, taken in the instantaneously co-moving Lorentz frame, at the moving boundary position $x = z(t)$. We
C space translations. The term form a complete set of positive-frequency solutions, and consider the averages \( \langle \ldots \rangle \)...

FIG. 1: Moving mirror trajectory. The dashed lines are null-lines separating the regions I from II, and III from IV.

examine the particular set of mirror trajectories for which \( z(t < 0) = 0 \), as shown in Fig. 1. Using the appropriate Lorentz transformation, Dirichlet and Neumann conditions can be written in terms of quantities in the laboratory inertial frame as follows: \( \phi[t, z(t)] = 0 \) and \( \{ [\dot{z}(t) \partial_t + \partial_z] \phi(t, x) \} |_{z=0} = 0 \), respectively. The field solution can be obtained by exploiting the conformal invariance of the Klein-Gordon equation. Considering the conformal coordinate transformations \( t-x = f(w-s) \) and \( t+x = g(w+s) \), the scalar wave equation in 1+1 dimensions remains invariant:

\[
(\partial^2_w - \partial^2_s) \phi(w, s) = 0.
\]

Functions \( f \) and \( g \) can be set up, for a large class of laws of motion \( z(t) \), so that the curve \( s = 0 \) coincides with the boundary trajectory \( x = z(t): [t, z(t)] \rightarrow (w, 0) \). Then: \( \phi[t, z(t)] = 0 \rightarrow \phi(w, 0) = 0 \) and \( \{ [\dot{z}(t) \partial_t + \partial_z] \phi(t, x) \} |_{s=0} = 0 \). The mode solution of the wave equation with static Dirichlet or Neumann boundary conditions in \( (w, s) \) space are well known so that, coming back to \( (t, x) \) coordinates, we get:

\[
\hat{\phi}(t, x) = \int_{0}^{\infty} d\omega \left[ \hat{a}_\omega \phi_\omega + \hat{a}^\dagger_\omega \phi^* \right],
\]

form a complete set of positive-frequency solutions, and \( u = t-x, v = t+x \). In Eq. (1), we introduce a notation which enables us to put into a single formula the solutions for Dirichlet and Neumann boundary conditions, and also the solutions for the right hand side (regions I and II in Fig. 1) and the left side (regions III and IV) of the mirror. In this sense, for \( \gamma = 1 \), Eq. (1) gives the Neumann solution, whereas for \( \gamma = i \) we have the solution for Dirichlet boundary condition. For the regions I and II showed in Fig. 1, \( r(v) = v \) and \( 2\tau u - u = f^{-1}(u) \equiv p(u) \), where \( \tau(u) \) can be obtained from \( \tau(u) - z[\tau(u)] = u \); for the regions III and IV: \( p(u) = u \) and \( 2\tau v - v = g^{-1}(u) \equiv r(v) \), where \( \tau(v) + z[\tau(v)] = v \). As causality requires, the field in the regions I and IV, represented by \( \phi_0 \), is not affected by the boundary motion \( z(t) \), so that \( p \) and \( r \) are also chosen to be identity functions in these static regions. Hereafter we consider the averages \( \langle \ldots \rangle \) taken over an arbitrary initial field state (regions I and IV) assumed here, for simplicity, as being the same one for both sides of the mirror. We continue our analysis writing the correlator function \( C = C_{\text{vac}} + C_{(\hat{a}^\dagger \hat{a})} + C_{(\hat{a} \hat{a}^\dagger)} \), where

\[
C_{\text{vac}} = \int_{0}^{\infty} d\omega F_1(\omega, \omega', |\gamma|, |\gamma|)|_{\omega=\omega'},
\]

\[
C_{(\hat{a}^\dagger \hat{a})} = \int_{0}^{\infty} \int_{0}^{\infty} d\omega d\omega' \langle \hat{a}^\dagger_{\omega} \hat{a}_{\omega'} \rangle F_1(\omega, \omega', |\gamma|, |\gamma|) + \text{c.c.},
\]

\[
C_{(\hat{a} \hat{a}^\dagger)} = \int_{0}^{\infty} \int_{0}^{\infty} d\omega d\omega' \langle \hat{a}_{\omega} \hat{a}^\dagger_{\omega} \rangle F_1(\omega, -\omega', |\gamma|, |\gamma|) + \text{c.c.}
\]

and

\[
F_1(\omega, \omega', \rho, \lambda) = e^{-i(\omega t - \omega' t')} / (4\pi \sqrt{\omega \omega'}) \left[ \rho^2 e^{-i(\omega x - \omega' x')} + \lambda^2 e^{-i(\omega x + \omega' x')} + \text{c.c.} \right].
\]

From these equations, we see that, in the presence of the boundaries, \( C_{\text{vac}}, C_{(\hat{a}^\dagger \hat{a})} \) and \( C_{(\hat{a} \hat{a}^\dagger)} \) are not symmetric under space translations. The term \( C_{\text{vac}} \) is symmetric under time translation, whereas \( C_{(\hat{a}^\dagger \hat{a})} \) is symmetric if \( \langle \hat{a}_{\omega}^\dagger \hat{a}_\omega \rangle \propto \delta(\omega' - \omega) \).
On the other hand, the part related to time translations. In the context of the perturbative approach \[8\], the radiation force can be given in terms of correlation functions depending on the unperturbed field operator \(\phi_0\): \(\langle \partial_x \partial_x C \rangle_{x=0}^{x=0} + [C]_{x=x}^{x=0}\) for Dirichlet and Neumann boundary conditions respectively \[8, 17\]. As shown in Ref. \[17\], the parts of the force related to \(C_{\text{vac}}\) and \(C_{\text{corr}}\) are the same for Dirichlet and Neumann boundary conditions. On the other hand, the part related to \(C_{\text{corr}}\) is different by a sign. In other words, in Ref. \[17\] it is shown that the difference between the force acting on Dirichlet and Neumann emerges, in the context of the non-relativistic mirror motion with small amplitude, from the part of the non-perturbed correlator \(C\) which is non-invariant under time translations.

Now we will investigate this problem in the context of an exact approach, starting from the field solution \((1)\) and calculating the exact formulas for the expected value of the energy density operator \(T\), the net force acting on the moving boundary defined by (since \(T_{00} = T_{11}\) in this model): \(F(t) = T(t, z(t))^{(–)} - T\left[ t, z(t) \right]^{(+)}, \) where the superscript “+” indicates the regions I and II, whereas “-” indicates the regions III and IV in Fig. 1. Let us split \(T\), writing \(T = T_{\text{vac}} + T_{(a^+ a)} + T_{(a^– a)}\), where:

\[
T_{\text{vac}} = 1/2 \int_0^\infty d\omega F_2(\omega, \omega', |\gamma|, |\gamma|)\text{d}\omega = \int_0^\infty d\omega d\omega' \langle \tilde{a}_\omega, \tilde{a}_{\omega'} \rangle F_2(\omega, \omega', |\gamma|, |\gamma|),
\]

\[
T_{(a^+ a)} = -1/2 \int_0^\infty d\omega d\omega' \langle \tilde{a}_\omega, \tilde{a}_{\omega'} \rangle F_2(\omega, \omega', |\gamma|, |\gamma|) + \text{c.c.},
\]

and

\[
F_2(\omega, \omega', \rho, \lambda) = \frac{\sqrt{\omega \omega'}}{2\pi} \left\{ \rho^2 |p'(v)|^2 e^{-i(\omega - \omega')r(v)} + \lambda^2 |p'(u)|^2 e^{-i(\omega - \omega')p(u)} \right\}.
\]

The first term \(T_{\text{vac}}\) is the local energy density related to the vacuum state, which is divergent. After regularization and renormalization (see Ref. \[3\]), \(T_{\text{vac}}\) can be redefined as the renormalized local energy density:

\[
T_{\text{vac}} = -\frac{|\gamma|^2}{24\pi} \left[ p'''(u)/p'(u) - (3/2)p''(u)^2/p'(u)^2 \right] + p(u) \to r(v),
\]

where the object appearing in the brackets is known as the Schwarzian derivative. We remark that, in our notation, for the right side of the mirror we have \(r(v) = v\), so that this equation gives back the formula found in Ref. \[3\] for Dirichlet, and in addition we verify that the same formula is valid for the Neumann boundary condition. Now we investigate the net radiation force \(F\) acting on the moving mirror. Let us consider \(F = F_{\text{vac}} + F_{(a^+ a)} + F_{(a^– a)}\), where:

\[
F_{\text{vac}} = |\gamma|^2 \left( 1 + \frac{\dot{z}}{z} \right) \times \left[ \left( \frac{\dot{z}^2}{2\pi} \right)^2 \frac{1}{1 - \dot{z}^2} \right] + |\gamma| + \dot{z}^2 / 24\pi - (3/2)p''(u)^2/p'(u)^2 \left[ p'(u) \right] + \text{c.c.},
\]

\[
F_{(a^+ a)} = 2 \int_0^\infty d\omega d\omega' \langle \tilde{a}_\omega, \tilde{a}_{\omega'} \rangle \cdot F_3(\omega, \omega', |\gamma|, |\gamma|) + \text{c.c.},
\]

\[
F_{(a^– a)} = -1 \int_0^\infty d\omega d\omega' \langle \tilde{a}_\omega, \tilde{a}_{\omega'} \rangle \cdot F_3(\omega, \omega', |\gamma|, |\gamma|) + \text{c.c.},
\]

and

\[
F_3(\omega, \omega', \rho, \lambda) = \frac{\sqrt{\omega \omega'}}{4\pi} \left\{ -\rho^2 \left( 1 + \frac{\dot{z}}{z} \right)^2 e^{-i(\omega - \omega')p[t-z(t)]} + \rho^2 e^{-i(\omega - \omega'[t-z(t)] - [(z, \dot{z}, p, \rho) \to (\dot{z}, -\dot{z}, r, \lambda)]} \right\}.
\]
We see that $F_{\text{vac}}$ and $F_{(\hat{a}^\dagger \hat{a})}$ depend on $|\gamma|^2$, which has the same value for Dirichlet and Neumann conditions. On the other hand $F_{(\hat{a}\hat{a})}$ depends on $\gamma^2$ (or $\gamma^2$), which differs by a sign in Dirichlet and Neumann cases. Noting that the term $C_{(\hat{a}\hat{a})}$ ($\langle \hat{a}\hat{a} \rangle \neq 0$) is non-symmetric under time translations, we can generalize the result found in Ref. [17], concluding that for a general (relativistic) law of motion, Dirichlet and Neumann boundary conditions yield the same radiation force on a moving mirror when the initial field state is symmetric under time translations. In the non-relativistic limit, we recover $F_{\text{vac}}(t) \approx |\gamma|^2 z^2/6\pi$, found in Ref. [8] for Dirichlet, and in Ref. [17] for Neumann condition.

Let us examine the radiation force on moving mirrors when there are real particles in the initial state of the field. We start with the thermal bath with temperature $T$, which is an example of invariant field state under time translations. For this state we need to take into account that $\langle \hat{a}\hat{a} \rangle = \langle \hat{a}\hat{a} \rangle \delta (\omega - \omega')$ where $\ddot{n}(\omega) = 1/(e^{\hbar\omega/T} - 1)$, with the Boltzmann constant equal to 1. We get $T_{(\hat{a}\hat{a})} = 0$ and $T_{(\hat{a}^\dagger \hat{a})}$, renamed as the energy density $T^{(T)}_{(\hat{a}^\dagger \hat{a})}$, is given by: $T^{(T)}_{(\hat{a}^\dagger \hat{a})} = |\gamma|^2 \pi T^2/12 \left[ p'(v)^2 + p'(u)^2 \right]$. The force $F_{(\hat{a}\hat{a})} = 0$, whereas $F_{(\hat{a}^\dagger \hat{a})}$, renamed as the net thermal force $F^{(T)}$, is given by:

$$F^{(T)} = -\sigma_T \left[ z \frac{(1 + z^2)}{(1 - z^2)^2} \right] = -\sigma_T \sum_{n=0}^{\infty} (2n + 1) z^{2n+1},$$

where $\sigma_T = 2|\gamma|^2 \pi T^2/3$ is the viscosity coefficient. This exact formula is a generalization of the one obtained in Ref. [10]. For non-relativistic velocities related, for instance, to mechanical motions of the mirror ($\dot{z} \sim 10^{-8}$) or to the simulation of the mirror motion by changing the reflectivity of a semiconductor by irradiation from laser ($\dot{z} \sim 10^{-3}$) [24], the series can be truncated in $n = 0$, leading to the approximate formula: $F^{(T)} \approx F^{(T)}_{(0)} = -2|\gamma|^2 \pi T^2 \ddot{z}/3$, in agreement with Ref. [10] (for Dirichlet), and also with Ref. [17] (for Neumann boundary condition). In the nonrelativistic context, this approximate formula is in good agreement with the exact value. For relativistically moving mirrors (for instance, relativistic electron beam may be an embodiment of a relativistic mirror [25]), corrections to the approximate formula can become necessary, as it can be seen in Fig. 2.

Let us now consider the coherent state, as an example of a non invariant state under time translations. The coherent state of amplitude $\alpha$ is defined as an eigenstate of the annihilation operator: $\hat{a}_\omega |\alpha\rangle = \alpha \delta (\omega - \omega_0) |\alpha\rangle$, where $\alpha = |\alpha| \exp(i\theta)$ and $\omega_0$ is the frequency of the excited mode. For this case, we have $T_{(\hat{a}^\dagger \hat{a})}$ and $T_{(\hat{a}\hat{a})}$ relabeled as $T^{(\alpha)}_{(\hat{a}^\dagger \hat{a})}$ and $T^{(\alpha)}_{(\hat{a}\hat{a})}$ respectively, and given by:

$$T^{(\alpha)}_{(\hat{a}^\dagger \hat{a})} = |\gamma|^2/(2\pi)\omega_0 \left\{ \alpha^2 \left[ |v'|^2 + p'(u)^2 \right] \right\},$$

$$T^{(\alpha)}_{(\hat{a}\hat{a})} = -\frac{\omega_0}{4\pi} \left\{ \alpha^2 \left[ |v'|^2 e^{-2i\omega_0 r(v)} + |p'(u)|^2 e^{-2i\omega_0 p(u)} \right] + \text{c.c.} \right\}.$$
The exact forces $F_{(\hat{a}^{\dagger}\hat{a})}$ and $F_{(\hat{a}\hat{a})}$, relabeled as the coherent forces $F^{(\alpha)}_{(\hat{a}^{\dagger}\hat{a})}$ and $F^{(\alpha)}_{(\hat{a}\hat{a})}$ respectively, are given by:

$$F^{(\alpha)}_{(\hat{a}^{\dagger}\hat{a})} = -\frac{4|\gamma|^2}{\pi} \omega_0 |\alpha|^2 \frac{1}{(1+\frac{\dot{z}}{\ddot{z}^2})^2},$$

$$F^{(\alpha)}_{(\hat{a}\hat{a})} = -\frac{\omega_0}{4\pi} |\alpha|^2 e^{-2i(\omega_0 t - \theta)}$$

$$\times \left\{ \gamma^2 \left[ e^{2i\omega_0 z(t)} \left( \frac{1}{1+\frac{\dot{z}}{\ddot{z}^2}} - e^{-2i\omega_0 z(t)} \right)^2 \right] \right. $$

$$\left. -\gamma^2 \left[ \left( \frac{1+\frac{\dot{z}}{\ddot{z}^2}}{1+\frac{\ddot{z}}{\dot{z}^2}} \right)^2 e^{-2i\omega_0 z(t)} - e^{2i\omega_0 z(t)} \right] \right\} + \text{c.c.}$$

If we consider simultaneously nonrelativistic velocities and small displacements (in the sense considered in Ref. [17]), according to what is required by Ford-Vilenkin approach [3], the force $F^{(\alpha)} = F^{(\alpha)}_{(\hat{a}^{\dagger}\hat{a})} + F^{(\alpha)}_{(\hat{a}\hat{a})}$ can be approximated as:

$$F^{(\alpha)} \approx -\frac{2\omega_0}{\pi} |\alpha|^2 \left\{ 2|\gamma|^2 \dot{z}(t) - (\gamma^2 + \gamma^*\gamma) \right\} \times \cos (2\omega_0 t - 2\theta) \dot{z}(t) - \sin (2\omega_0 t - 2\theta) \omega_0 z(t)).$$

In Fig. 3 we plot the exact coherent force as a function of time for the Dirichlet boundary condition and different values of the mirror velocity. Assuming the mirror moving with uniform velocity toward the negative direction of the x-axis, the Fig. 3 shows the force oscillating $F^{(\alpha)}$ and the graph shifting to the positive region of the vertical axis as the mirror velocity grows, becoming the force more intense and opposite to the motion. For the Neumann boundary condition, the force oscillates in a different manner, but exhibits analogous shift for relativistic velocities.

In summary, focusing on the advantages of the Fulling-Davies approach for the case of a massless scalar field in 1 + 1 dimensions, in the present paper we showed the exact dynamical Casimir force acting on a moving boundary under Neumann condition, with the vacuum as the initial state, generalizing the non-relativistic result found in Ref. [17]. For the thermal field, considering both Dirichlet and Neumann conditions, we wrote the exact formula for the thermal force, generalizing the approximate formula found in Ref. [10] and also, for instance, in Refs. [11, 12, 13, 16, 17]. For the coherent initial state, we found exact formulas for the radiation force, which are different if we consider Dirichlet or Neumann condition, generalizing the perturbative formulas found in Ref. [17]. Finally, we extended to a general (relativistic) law of motion the conclusion found in the literature [17] that Dirichlet and Neumann boundary conditions yield the same radiation force on a moving mirror when the initial field state is invariant under time translations.

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[1] G.T. Moore, J Math. Phys. 11, 2679 (1970).
[2] B.S. DeWitt, Phys. Rep. 19, 295 (1975).
[3] S.A. Fulling and P.C.W. Davies, Proc. R. Soc. London, A 348, 393 (1976).
[4] P.C.W. Davies and S.A. Fulling, Proc. R. Soc. London, A 354, 59 (1977).
[5] P.C.W. Davies and S.A. Fulling, Proc. R. Soc. London, A 356, 237 (1977).
[6] P. Candelas and D.J. Raine, J. Math. Phys. 17, 2101 (1976).
[7] P. Candelas and D.J. Raine, Proc. R. Soc. London, A 354, 79 (1977).
[8] L.H. Ford and A. Vilenkin, Phys. Rev. D 25, 2569 (1982).
[9] N.D. Birrell and P.C.W. Davies, Quantum Fields in Curved Space (Cambridge University Press, Cambridge, England, 1982).
[10] M.-T. Jaekel and S. Reynaud, J. Phys. I (France) 3, 339 (1993).
[11] G. Plunien, R. Schutzhold and G.Soff, Phys. Rev. Lett. 84, 1882 (2000).
[12] M.-T. Jaekel and S. Reynaud, Phys. Lett. A 172, 319 (1993).
[13] A. Lambrecht, M.T. Jaekel, and S. Reynaud, Europhys. Lett. 43, 147 (1998); J. Hui, S. Qing-Yun, and W. Jian-Sheng, Phys. Lett. A 268, 174 (2000).
[14] L. A. S. Machado, P. A. Maia Neto and C. Farina, Phys. Rev. D 66, 105016 (2002).
[15] R. Schutzhold, G. Plunien and G. Soff, Phys. Rev. A 65, 043820 (2002).
[16] A. Lambrecht, J. Opt. B: Quantum Semiclass. Opt. 7 S3 (2005).
[17] D.T. Alves, C. Farina and P.A. Maia Neto, J. Phys. A 36, 1333 (2003).
[18] V. V. Dodonov, A. Klimov and V. I. Man’ko, Phys. Lett. A 149, 225 (1990); M. A. Andreata and V. V. Dodonov, J. Phys. A 33, 3209 (2000).
[19] V. V. Dodonov, M. A. Andreata and S. S. Mizrahi, J. Opt. B: Quantum Semiclass. Opt. 7 S468 (2005); D. A. R. Dalvit and P. A. Maia Neto, Phys. Rev. Lett 84, 798 (2000).
[20] T.H. Boyer, Phys. Rev. A 9, 2078 (1974); I. Klich, A. Mann and M. Revzen, Phys. Rev. D 65, 045005 (2002); O. Kenneth, I. Klich, A. Mann and M. Revzen, Phys. Rev. Lett. 89, 033001 (2002).
[21] E. Buks and M. L. Roukes, Nature (London) 419, 119 (2002); D. Iannuzzi and F. Capasso, Phys. Rev. Lett. 91, 029101 (2003); O. Kenneth, I. Klich, A. Mann and M. Revzen, Phys. Rev. Lett. 91, 029102 (2003).
[22] T. H. Boyer, Am. J. Phys. 71, 990 (2003); S. A. Fulling, L. Kaplan and J. H. Wilson, Phys. Rev. A 76, 012118 (2007); X. H. Zhai and X. Z. Li, Phys. Rev. D 76, 047704 (2007).
[23] M. Montazeri and M. F. Miri, Phys. Rev. A 71, 063814 (2005); D.T. Alves, C. Farina and E.R. Granhen, ibid. 73, 063818 (2006); D.A.R. Dalvit, F.D. Mazzitelli and O. Millán, J. Phys. A 39, 6261 (2006); B. Mintz, C. Farina, P.A.M. Neto and R.B. Rodrigues, ibid. 39, 6559 (2006); D.T. Alves and E.R. Granhen, Phys. Rev. A 77, 015808 (2008); J. Sarabadani and M. F. Miri, ibid. 75, 055802 (2007).
[24] C. Braggio, G. Bressi, G. Carugno, C. Del Noce, G. Galeazzi, A. Lombardi, A. Palmieri, G. Ruoso and D. Zanello, Europhys. Lett., 70, 754 (2005).
[25] V. L. Granatstein, P. Sprangle, R. K. Parker, J. Pasour, M. Herndon and S. P. Schlesinger, Phys. Rev. A 14, 1194 (1976); M. Lampe, E. Ott and J. Walker, Phys. Fluids 21, 42 (1978).