Critical properties in the dynamical charge correlation function for the one-dimensional Mott insulator are studied. By properly taking into account the final-state interaction between the charge and spin degrees of freedom, we find that the edge singularity in the charge correlation function is governed by massless spinon excitations, although it is naively expected that spinons do not directly contribute to the charge excitation over the Hubbard gap. We obtain the momentum-dependent anomalous critical exponent by applying the finite-size scaling analysis to the Bethe ansatz solution of the half-filled Hubbard model.

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I. INTRODUCTION

Recently strongly correlated electron systems in one dimension (1D) have attracted much interest. In particular, charge and spin excitations for the Mott insulator have been intensively studied. Various experimental methods, by which the dynamical correlation functions are directly observed, have revealed striking properties in the Mott insulator. For example, the recent photoemission experiments [1–3] for the 1D compounds SrCuO$_2$, Sr$_2$CuO$_3$ and NaV$_2$O$_5$ have clarified dynamical properties of a single hole doped in the 1D Mott insulator. These experiments have indeed stimulated extensive theoretical studies on the one-particle Green function for 1D correlated systems, which have been done numerically [4,12] and analytically. [13,19] The overall feature of the spin-charge separation found in the photoemission experiments has been explained rather well. [8] In this connection, it has been claimed that the final-state interaction between the charge and spin degrees of freedom plays the crucial role to determine the edge singularity in the spectrum, based on the conformal field theory (CFT) analysis. [16,19]

The dynamical charge correlation function is another key quantity to explore the characteristics of the Mott insulator, which is particularly important to analyze optical experiments. This quantity has also been studied intensively so far for 1D correlated electron systems. [20,23] For example, the recent theoretical treatment of a metallic system close to the Mott transition [22] has clarified that the change of the weight in the dynamical charge correlation function clearly describes the characteristics of the metal-insulator transition.

Motivated by the above investigations, in this paper we study critical properties in the dynamical charge correlation function for the 1D Mott insulator. For this purpose, we consider the 1D half-filled Hubbard model as a Mott insulator and use the exact solution of the Bethe ansatz method. [23] Applying CFT techniques, we analyze the exact finite-size spectrum to obtain the critical exponents for the dynamical charge correlation function exactly. We clarify how the edge singularity in the massive charge excitation spectrum over the Hubbard gap is controlled by massless spinon excitations by properly taking into account the final-state interaction between the charge and spin degrees of freedom. In particular, we point out that a particle-hole charge excitation is considered to act as two mobile impurities in massless spinons, and the resulting scattering phase shifts are the essential quantities to determine the edge singularity.

This paper is organized as follows. In § 2, based on the exact solution of the 1D Hubbard model, [26] we present the basic formulation following refs. 27 and 28 to investigate a particle-hole excitation over the Hubbard gap. Then in § 3 we calculate the exact finite-size spectrum, and derive the edge singularity in the charge correlation function by employing the finite-size scaling idea in CFT. We discuss the anomalous critical behavior in the singularity by evaluating the momentum-dependent critical exponents. Brief summary is given in § 4.

II. CHARGE EXCITATIONS IN THE MOTT INSULATOR

Let us start with the ordinary 1D repulsive Hubbard model,

$$\mathcal{H} = -i \sum_i \sum_{\sigma} (c_{i+1,\sigma}^\dagger c_{i,\sigma} + c_{i,\sigma}^\dagger c_{i+1,\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow} - \frac{h}{2} \sum_i (n_{i\uparrow} - n_{i\downarrow}),$$

(1)

where $c_{i,\sigma}^\dagger$ is the creation operator for electrons and $h$ is a magnetic field. We henceforth set $t = 1$ for simplicity.

The exact solution of this model [26] is given by a set of the Bethe equations for the charge rapidities $k_j$ and the spin rapidities $\lambda_\beta$.

$$N k_j = 2\pi l_j - \sum_{\beta=1}^M 2 \tan^{-1} \left[ \frac{\sin k_j - \lambda_\beta}{c} \right]$$
where we set \( c = U/4 \) (> 0). \( N_e \) and \( M \) represent the number of electrons and down spins, and \( N \) is the number of the lattice sites. For the repulsive case \( U > 0 \), the rapidities are real numbers for the ground state as well as for excited states within the lower Hubbard band, which are classified by the set of the quantum numbers \( \mathcal{I}_j \) and \( \mathcal{J}_\alpha \) for the charge and spin sectors.

A. Particle-hole excitations at half-filling

For the half-filled case \( (N_e = N) \) the available quantum numbers \( \mathcal{I}_j \) for the charge sector are completely filled, giving rise to the incompressible state of the Mott insulator. So, charge excitations over the Hubbard gap, which are necessary for the evaluation of the dynamical charge correlation function, cannot be described by real rapidities.

It is known that the charge excitations of particle-hole type are specified by complex charge rapidities in bound pairs. Following the methods of Woynarovich, we briefly summarize how to treat such complex rapidities. As a simple charge excitation at half-filling, we here introduce a pair of complex charge rapidities, which are denoted as \( k^\pm = \kappa \pm i\chi \) with the real quantities \( \kappa \) and \( \chi \). In the thermodynamic limit, these charge rapidities are coupled with the corresponding spin rapidity \( \Lambda \) to yield a bound pair.

\[
\sin(\kappa \pm i\chi) = \Lambda \mp ic. \tag{2}
\]

Since this pair adds two particles to the system, we need to introduce two holes, labeled by the real charge rapidities \( k_l \) and \( k_m \), in order to keep the particle number unchanged. We thus obtain the Bethe equations for a particle-hole excitation over the Hubbard gap,

\[
Nk_j = 2\pi\mathcal{I}_j - \sum_{\beta=1}^{M-1} 2\tan^{-1}\left[\frac{\sin k_j - \lambda_\beta}{c}\right] - 2\tan^{-1}\left[\frac{\sin k_j - \Lambda}{c}\right] = 2\pi\mathcal{J}_\alpha + \sum_{\beta=1}^{M-1} 2\tan^{-1}\left[\frac{\lambda_\alpha - \lambda_\beta}{2c}\right]. \tag{3}
\]

These equations are subject to the constraints for the momentum conservation,

\[
N\kappa = 2\pi\mathcal{I} - \frac{\pi}{2} \sum_{\beta=1}^{M-1} \text{sign}(\Lambda - \lambda_\beta) - \sum_{\beta=1}^{M-1} \tan^{-1}\left[\frac{\Lambda - \lambda_\beta}{2c}\right] \\
\sum_{j\neq l,m} 2\tan^{-1}\left[\frac{\Lambda - \sin k_j}{c}\right] = 2\pi\mathcal{J} + \sum_{\beta=1}^{M-1} 2\tan^{-1}\left[\frac{\lambda_\alpha - \lambda_\beta}{2c}\right], \tag{4}
\]

which are recast into more convenient form,

\[
\Lambda = \frac{1}{2}(\sin k_l + \sin k_m). \tag{5}
\]

Note that the unknown parameters for excitations are now reduced to two rapidities \( k_l \) and \( k_m \) thanks to the above constraints, which naturally reproduces the fact that a particle-hole excitation is classified by two independent momentums. Eqs. (3) and (4) are our starting equations to analyze the finite-size corrections to a particle-hole excitation over the Hubbard gap and to get further information on the charge correlation function.

B. Dispersion relations in magnetic fields

Before computing the finite-size corrections, we first summarize the results on the dispersion relation by slightly extending the method of ref. 28 to include the effect of magnetic fields. Following a standard way, let us introduce the distribution functions for the charge and spin sectors, which are denoted as \( \rho(k) \) and \( \sigma(\lambda) \), respectively.

\[
\rho(k) = \frac{1}{2\pi} + \int_{-\lambda_0}^{\lambda_0} \frac{d\lambda}{2\pi} \cos kK_{12}(\sin k - \lambda)\sigma(\lambda) + \frac{1}{2\pi} \cos kK_{12}(\sin k - \Lambda) \int_{-\pi}^{\pi} \frac{dk}{2\pi}K_{21}(\lambda - \sin k)\rho(k) - \frac{1}{2\pi} \sum_{\alpha=l,m} K_{21}(\lambda - \sin k_\alpha) \sigma(\lambda) = \sigma(\lambda) + \int_{-\lambda_0}^{\lambda_0} \frac{d\lambda}{2\pi} K_{22}(\lambda - \lambda')\sigma(\lambda') \tag{6}
\]

where \( K_{12}(x) = K_{21}(x) = 2c/(c^2 + x^2), K_{22}(x) = 4c/(4c^2 + x^2) \). Note that a particle-hole excitation is incorporated in the integral equations through the terms of order of \( 1/N \) including \( k_l \) and \( k_m \).

The energy and momentum for this excited state are given by,

\[
E = \sum_{j\neq l,m} (h_c - 2\cos k_j) + \sum_{\alpha=1}^{M} h_s + (h_c - 2\cos k_+ + h_c - 2\cos k_-), \tag{7}
\]

\[
P = \sum_{j\neq l,m} k_j + (k_+ + k_-), \tag{8}
\]

where \( h_c = -h/2 \) and \( h_s = h \). Using (8), we recast them to simple formulas,

\[
\omega_c \equiv E - E_0 = U - \varepsilon_c(k_l) - \varepsilon_c(k_m), \tag{9}
\]

\[
q_c \equiv P - P_0 = -p(k_l) - p(k_m), \tag{10}
\]
where $E_0$ and $P_0$ is the energy and momentum of the ground state. Here we have introduced the dressed energy $\varepsilon_c(k)$ and the dressed momentum $p(k)$ for the simplest charge excitation (often referred to as holon), which are given by

$$
\varepsilon_c(k) = -2 \cos k - 2 \int_0^\infty \frac{e^{-\omega}}{\omega} \cos k J_1(\omega) \cos(\omega \sin k) d\omega \\
- \int_{|\lambda| > \lambda_0} \frac{1}{4c} \cosh^{-1} \frac{\pi}{2c(\sin k - \lambda)} \varepsilon_s(\lambda) d\lambda,
$$

$$
p(k) = k + \int_0^\infty \frac{e^{-\omega}}{\omega} \cos k J_0(\omega) \sin(\omega \sin k) d\omega \\
- \int_{|\lambda| > \lambda_0} G(\sin k - \lambda) \sin \lambda d\lambda,
$$

where $J_n(\omega)$ is the $n$-th order Bessel function. This charge excitation is coupled with spin excitation (spinon), whose energy $\varepsilon_s$ is given by the solution to the following integral equation,

$$
\varepsilon_s(\lambda) = \varepsilon_s^0(\lambda) - \int_{-\lambda_0}^{\lambda_0} \frac{d\lambda'}{2\pi} K_{22}(\lambda - \lambda') \varepsilon_s(\lambda')
$$

where,

$$
\varepsilon_s^0(\lambda) = h - 2 \int_{-\lambda_0}^{\lambda_0} \frac{dk}{2\pi} \cos^2 k K_{12}(\sin k - \lambda),
$$

$$
G(\lambda) = \int_0^\infty \frac{\sin \omega \lambda}{\omega \cosh \omega} d\omega.
$$

The dispersion for the particle-hole excitation spectrum is described by $\omega_\mu(q)$, which includes the effects of magnetic fields.

In Fig. 1 we show the charge excitation spectrum ($q = q_c$) for a given $U$ in several choices of magnetic fields. The charge excitation spectrum distributions in the continuum over the Hubbard gap. It is seen from the figure that the lower edge of the spectrum changes its character at the critical momentum $q = \pi \pm q_L \hbar$. For $\pi - q_L < q < \pi + q_L$, the lower edge is given by changing a rapidity $k_m$ with the other being fixed as $k_l = -\pi$. This excitation corresponds to a particle-hole excitation from the top of the lower Hubbard band to an excited state in the upper Hubbard band. On the other hand, for $0 < q < \pi - q_L$ and $\pi + q_L < q < 2\pi$, the lower edge is featured by the excitation with $k_l = k_m$, which makes a particle (hole) with the momentum $q/2 (-q/2)$ in the upper (lower) Hubbard band. So, the corresponding critical behavior of the edge singularity is different between these two regimes, which will be explicitly studied in the next section.

FIG. 1. Charge excitation spectrum for $U = 6$ in several choices of magnetic fields: (a), (b) and (c) correspond to the case of the magnetization $m = 0, 0.5$ and 1. The excitations are allowed for the dashed region. The lower edge of spectrum changes its character at the critical momentum $\pi \pm q_L$. In the case of $U \rightarrow \infty$ or $m \rightarrow 1$, $q_L$ equals to $0$ in $0 < q < 2\pi$. On the other hand, $q_L$ becomes $\pi$ at $U = 0$.

III. CRITICAL BEHAVIOR IN DYNAMICAL CHARGE CORRELATION FUNCTION

We now consider the dynamical charge correlation function for the Mott insulator and show that the anomalous power-law behavior appears in the spectrum, which is controlled by massless spinon excitations subject to the two phase shifts due to a particle-hole excitation.

Let us start with the charge correlation function,

$$
D(x, t) = \langle \rho(x, t), n(0, 0) \rangle \\
= \sum_\mu \langle 0| n(0) \mu |\rho_i \Delta \rho_x, x \mu \Delta E \rangle \\
\simeq \sum_{\{k, k_m\}} e^{i\omega_x x - i\omega_t t} D_s(x, t),
$$

where $n = \sum_\alpha n_\alpha$ is the charge density operator. In the third line of the equation, we have approximately substituted $\omega_c(k_l, k_m)$ and $q_c(k_l, k_m)$ given by [8] for the energy and the momentum of excited states. Since the action of the charge density operator $n$ creates not only particle-hole charge excitations, but also induces low-energy spin excitations, the contribution from the spin sector to $\langle n(x, t) n(0, 0) \rangle$ is denoted by the spin correlator $D_s(x, t)$. In the following we will mainly deal with the Fourier transform of the correlation function,

$$
D(q, \omega) \simeq \sum_{\{k_l, k_m\}} \int dx \int dt \ e^{-i(q - q_c)x + i(\omega - \omega_c)t} D_s(x, t),
$$

for which the main spectrum is featured by the massive charge excitation $\omega_c(q)$ while its critical behavior is essentially determined by the spin correlator $D_s(x, t)$. It is now clear that the long-time or large-distance behavior of $D_s(x, t)$ is important to discuss the critical behavior of the dynamical charge correlation function. Therefore our remaining task is to study low-energy properties of $D_s(x, t)$ exactly when a particle-hole excitation is created.

A. Finite-size spectrum and conformal properties

In order to apply the methods developed in CFT [24] to the evaluation of $D_s(x, t)$, let us first compute the finite-size corrections to the excitation energy. [25] We recall here that in the calculation of the spectrum in the previous section, we have not taken into account the fact that
massless spinons are scattered by two dressed particles created for the charge part (holons), and have discarded the resulting scattering phase shifts in the thermodynamic limit. However, it turns out that this coupling between the charge and spin degrees of freedom is essential to determine anomalous critical properties in the charge correlation function. Since we are now considering the situation that a particle-hole pair excitation is suddenly created, this interaction may be regarded as the so-called final-state interaction. We show explicitly the above fact by correctly evaluating the final-state interaction.

We can perform the calculations of the finite-size spectrum by extending those previously done for the photoemission spectrum [19]. In order to deal with the finite-size effects, let us rewrite the coupled integral equations of distribution functions [4],

\[ \rho(k) = \frac{1}{2\pi} + \int_{\lambda^-}^{\lambda^+} \frac{d\lambda}{2\pi} \cos k K_{12}(\sin k - \lambda) \sigma(\lambda) \]
\[ + \frac{1}{2\pi N} \cos k K_{12}(\sin k - \Lambda), \]
\[ \int_{-\pi}^{\pi} \frac{dk}{2\pi} K_{21}(\lambda - \sin k) \rho(k) - \frac{1}{2\pi N} \sum_{\alpha=l,m} K_{21}(\lambda - \sin k_{\alpha}) \]
\[ = \sigma(\lambda) + \int_{\lambda^-}^{\lambda^+} \frac{d\lambda'}{2\pi} K_{22}(\lambda - \lambda') \sigma(\lambda'), \]

where we have introduced the asymmetric cutoffs for spin rapidities, \( \lambda^\pm \), which are essential to include the final-state interaction. Namely, two massive holons created give rise to the phase shifts in massless spinons, which naturally induce the change in cutoffs in the order of \( 1/N \) (compare them with (3)). The importance of this effect has been previously noticed for the photoemission spectra in the Mott insulator [16,19]. In this way, although it is naively expected that the massless spinons do not directly coupled with the charge excitation, the edge singularity in the charge correlation function is affected by spinons via the final-state interaction.

Let us now exactly analyze the excitation energy, including the contribution from low-energy massless spinon excitations. By applying the computational techniques presented in [19] to eqs. (4), we straightforwardly end up with the following formulas,

\[ \Delta E = \Delta \epsilon_c + \Delta \epsilon_s \]
\[ = U - \epsilon_c(k_l) - \epsilon_c(k_m) + \frac{2\pi v_s}{N} (x + N_+ + N_-), \]

where \( v_s \) is the velocity of massless spinons,

\[ v_s = \frac{\epsilon_s'(\lambda)}{2\pi \sigma(\lambda)} \bigg|_{\lambda=\lambda_0}. \]

Note that the first term of the order of unity in (15) represents the energy \( \omega_c \) for the massive charge excitation, which is referred to as the surface energy in boundary CFT. Now, according to the finite-size scaling idea in CFT [20,21] the scaling dimension \( x \) for the spin sector is read from the \( 1/N \) corrections to the excitation energy, which is obtained as,

\[ x = \frac{1}{4\pi^2} (-1 + n_c(k_l) + n_c(k_m))^2 + \xi^2_x(d_c(k_l) + d_c(k_m))^2. \]

(17)

This formula for the scaling dimension is typical for Tomonaga-Luttinger liquids classified by \( c = 1 \) CFT. Here, the quantity \( \xi_x = \xi_x(\lambda_0) \) (referred to as the dressed charge),

\[ \xi_x(\lambda) = 1 - \int_{-\lambda_0}^{\lambda_0} \frac{d\lambda'}{2\pi} K_{22}(\lambda - \lambda') \xi_x(\lambda'), \]

(18)

features the U(1) critical line of \( c = 1 \) CFT when we change the strength of interaction or the magnetic field. A remarkable point in (17) is that there exist two kinds of phase shifts \( n_c \) and \( d_c \), which are caused by the final-state interaction between the charge and spin degrees of freedom. These phase shifts are explicitly obtained as

\[ n_c(k_{\alpha}) = \int_{-\lambda_0}^{\lambda_0} \sigma_\alpha(\lambda), \]
\[ d_c(k_{\alpha}) = -\frac{1}{2} \left( \int_{-\infty}^{-\lambda_0} \sigma_\alpha(\lambda) - \int_{-\lambda_0}^{\infty} \sigma_\alpha(\lambda) \right), \]

(19)

where

\[ \sigma_\alpha(\lambda) = \frac{1}{2\pi} K_{22}(\lambda - \sin k_{\alpha}) - \int_{-\lambda_0}^{\lambda_0} \frac{d\lambda'}{2\pi} K_{22}(\lambda - \lambda') \sigma_\alpha(\lambda'). \]

(20)

It is thus seen that although the scaling dimension \( x \) for massless spinons is typical for \( c = 1 \) CFT, [22,23] the massive charge sector also contributes to \( x \) via the phase shifts \( n_c \) and \( d_c \). In this sense, (17) is classified as shifted \( c = 1 \) CFT, whose fixed point is different even from that of the static impurity problem, as pointed out by Sorella and Parola [24]. Since in the present case, these phase shifts are considered to be caused by the two holons created, the fixed point belongs to Tomonaga-Luttinger liquid with two mobile impurities [24]. It will be shown that these phase shifts are the key quantities to control the anomalous low-energy properties in the correlation function.

Exploiting finite-size scaling techniques in CFT, we can now write down the asymptotic form for the spin correlation function as

\[ D_s(x, t) \approx \frac{1}{(x - v_s t)^{2\Delta^+_s} (x + v_s t)^{2\Delta^-_s}}, \]

(21)

where \( \Delta^\pm_s \) are conformal dimensions for the spin sector, which are related to the scaling dimension as \( x = \Delta^+_s + \Delta^-_s \). Thus we can read the low-energy behavior in \( D_s(x, t) \) correctly from the scaling dimension (17).
B. Anomalous critical behavior

Let us now discuss the critical properties in the dynamical correlation function by substituting the above results for $D_s(x, t)$ to (13). As noted in the previous section, there appear two different lower edges in the charge spectrum depending on the momentum $q$, so we deal with two cases separately.

1. Edge singularity for $\pi - q_L < q < \pi + q_L$

We now discuss the critical properties along the lower edge of particle-hole excitation spectrum. Let us start with the case for $\pi - q_L < q < \pi + q_L$ shown in Fig. 2. As shown in the previous section, the lower edge is given by $\omega_c(q) = U - \varepsilon(-\pi) - \varepsilon(k_m)$. In this case, if we neglect the contribution from the spin sector, there does not exist the power-law edge singularity in the charge correlation function, so that the anomalous behavior around the edge may be completely determined by spinon excitations. So the Fourier transform of the charge correlation function is given by

$$D(q, \omega) \propto \theta(\omega - \omega_c(q))(\omega - \omega_c(q))^X_1(q),$$

for $\omega \simeq \omega_c(q)$. The corresponding critical exponent $X_1(q)$ directly follows from the scaling dimension for the spin sector as

$$X_1(q) = 2x - 1,$$

where $x$ is given by (11). We emphasize here that the critical exponent is dependent on the momentum $q$. Recall that this anomalous behavior for the critical exponent is due to the two phase shifts in $x$, which is caused by the final-state interaction. In Fig. 2, we show the results for $X_1(q)$ as a function of $q$ for a given magnetic field. In the limit of vanishing magnetic fields, the exponent gradually approaches the value of $-1$ because the scaling dimension $x$ tends to zero. On the other hand, exactly at $h = 0$, the power-law singularity is expected to disappear, which implies that $X_1 = 0$. We can show that there indeed exists an energy scale which characterizes the crossover between these two behaviors. Namely, only in the small region close to the edge, $(\omega - \omega_c(q))(2x - 1) < 1/\Gamma(2x)$, we can observe a power-law singularity with the exponent $X_1 = 2x - 1$ where $\Gamma$ is the gamma function. When the magnetic field is decreased, this region is gradually shrunk because $\Gamma(2x) \rightarrow \infty$, and is eventually replaced by $D(q, \omega \rightarrow \omega_c) \approx$ constant.

2. Edge singularity for $0 < q < \pi - q_L$ and $\pi + q_L < q < 2\pi$

Let us now turn to the momentum region $0 < q < \pi - q_L$ and $\pi + q_L < q < 2\pi$ in Fig. 1, in which the lower edge is given by the excitation with $k_l = k_m$. Along this edge there occurs a specific excitation where a particle (a hole) is created at symmetric points in the upper (lower) Hubbard band with the momentum $q/2 (-q/2)$ for a given $q$. So, we should be a little bit careful to deduce its critical behavior. We start by discarding the spinon contributions to clearly see the origin of the singularity. Let us first recall that for a given $q$ the charge excitation energy $\omega_c$ takes its minimum at $k_l = k_m$ when we change the values of $k_l$ and $k_m$. So, around the edge, $\omega_c$ has the following properties for a fixed momentum $q$,

$$\omega_c|_{k_m=k_l} = 0, \quad \omega_c''|_{k_m=k_l} > 0,$$

which naturally leads to the well known square-root behavior like the van Hove singularity. $$(\omega - \omega_c(q))^{-1/2}.$$ This singularity depends on neither the coulomb interaction $U$ nor the magnetic field $h$, so long as the spinon contribution is neglected.

Now let us take into account the final-state interaction between the charge and spin degrees of freedom. Then low-energy spinons subject to the phase shifts show up in the infrared singularity. By properly incorporating this contribution, we finally end up with the edge singularity for $0 < q < \pi - q_L$ and $\pi + q_L < q < 2\pi$,

$$D(q, \omega) \propto \theta(\omega - \omega_c(q))(\omega - \omega_c(q))^X_2(q),$$

where the corresponding critical exponent is

$$X_2(q) = -\frac{1}{2} + \frac{1}{2}(2x - 1).$$

The first term of $-1/2$ comes from the square-root singularity in the dispersion relation, while the latter contribution reflects low-energy spinon excitations. In Fig. 2 we show the momentum dependent critical exponent for $0 < q < \pi - q_L$ and $\pi + q_L < q < 2\pi$ for several choices of magnetic fields.

We note here again that the above exponent seems to approach $-1$ for vanishing magnetic fields, while the correct exponent should be given by $-1/2$ at $h = 0$, corresponding to the square-root singularity. As mentioned above, there is the crossover between these two behaviors. In this case, in the region $(\omega - \omega_c(q))(2x - 1)/2 < 1/\Gamma(2x)$, we can see a power-law behavior with the exponent $X_2 = -1/2 + (2x - 1)/2$, while for higher energies, we can see the square root singularity. When $h \rightarrow 0$, this region with the exponent $X_2$ is gradually shrunk, being replaced by the square-root behavior.

FIG. 2. Momentum-dependent critical exponent $X_1(q)$ and $X_2(q)$, which correspond to the lower edge of spectrum in $\pi - q_L < q < \pi + q_L$ and $0 < q < \pi - q_L$, $\pi + q_L < q < 2\pi$. The magnetization is given by $m = 0, 0.5$ and 1 with the fixed $U = 6$. 
C. Comparison with photoemission spectrum

As shown above, the edge singularity in the dynamical charge correlation function for the 1D Mott insulator is controlled by massless spinons subject to the phase shifts caused by the final-state interaction. We recall again that similar anomalous properties have been already reported in the study on the photoemission spectrum, for which one electron is removed from the system and the induced massless spinon excitations play a crucial role for the edge singularity. [16][19] Therefore, the origin of the anomalous critical behavior in both cases is essentially same in the sense that their class belongs to the Tomonaga-Luttinger liquid with mobile impurities [3]. It should be noticed, however, that in the present case for the charge correlation function two mobile impurities are created as a particle-hole excitation, while for the photoemission spectra, only a single mobile impurity is created. So, the momentum-dependence of the critical exponents are quite different between these two cases.

IV. SUMMARY

We have studied the critical behavior in the dynamical charge correlation function for the 1D Mott insulator. Anomalous critical exponents have been calculated for the edge singularity in the charge excitation spectrum for the half-filled Hubbard model, by combining the Bethe ansatz with finite-size scaling methods in CFT. It has been claimed that the final-state interaction between the charge and spin degrees of freedom plays an important role to produce the anomalous critical behavior dependent on the momentum. We have further pointed out that the universality class of the present system belongs to the Tomonaga-Luttinger liquid with two mobile impurities.

In this paper, we have studied the charge density correlation function for the half-filled Hubbard model. Similar analysis can be straightforwardly applied to the dynamical spin correlation function for spin gapped systems in a metallic phase. In such cases, the roles played by the charge and spin degrees of freedom are interchanged; namely massless holons subject to the spinon phase shifts essentially determine the anomalous critical properties in the spin excitation spectrum.

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