Considerations on stress triaxiality variation for 2P armor steel

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Abstract. Stress triaxiality is considered an invariant of stress, defined as the ratio of hydrostatic stress (hydrostatic pressure by other authors) and the equivalent stress (usually calculated using von Mises criterion). If the values of the main three stresses have comparable sizes, stress triaxiality can be also calculated using the first invariant of the stress tensor. Despite that the stress triaxiality is an invariant, the authors have determined experimentally and analytically its variation with the force at the tensile test, but also with the radius of notches caused in the specimen. 2P armor steel being used in lightweight armor, these notches occur after shocks with foreign objects. Furthermore, the authors have revealed the stress triaxiality variation function of the test type. The tests were performed on tensile specimens loaded for tensile test, pure torsion test, 25% tensile - 75% torsion test, 50% tensile - 50% torsion test, 75% tensile - 25% torsion test. The mathematical model used was designed by Xue.

1. Introduction

2P armor steel is used mainly to achieve coatings for armored military vehicles. There are also some special uses, such as fuel pipes for those vehicles [1]. For this reason, a study for 2P armor steel plate and for a limited length tube, in terms of its behavior in plastic domain has to be done.

1.1. Bridgman's criterion

Bridgman provides a simple analytical solution to calculate the equivalent stress, using von Mises methodology for the minimum section of a revolution specimen [2].

Bridgman's analysis is applied to revolution specimens (with or without the initial unloading) made from a homogeneous isotropic material. The material is considered a plastic one (complying Von Mises plasticity criterion, isotropic hardening and Prandtl-Reuss equations). The specimen is tested to a tensile force in the direction of its ez axis (Figure 1).

Bridgman's specific hypothesis reflects the variation of the curvature radius ρ along the ez axis. The proposed mathematical relation has the general form:

\[ ρ = f \left( \frac{r}{a}, \frac{R}{a} \right) \]  

(1)
Bridgman had also proposed the maximum triaxiality ratio of stress \( \tau \) (the center of the minimum sample section), defined by the equation:

\[
\tau = \frac{\sigma_H}{\sigma_{eq,VM}} = \frac{1}{3} + \ln \left( 1 + \frac{a_0}{2R_0} \right)
\]  

(2)

where

- \( \sigma_H \) - hydrostatic stress;
- \( \sigma_{eq,VM} \) - equivalent von Mises stress;
- \( a_0 \) - the length of the original ligament;
- \( R_0 \) - initial notch radius

1.2. Barsoum – Faleskog’s model

Barsoum and Faleskog have used a tube with slots in their experiments, as it can be seen in Figure 2 [4]. Their results are reported through the Lode parameter, in reference to \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) main stress as a ratio of the breaking plane, as shown in Figure 3, where Lode’s parameter is defined as:

\[
\mu = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3}
\]  

(3)

1.3. Xue – Wierzbicki criterion

Xue și Wierzbicki have shown that the ductile fracture is the result of the accumulation of plastic strain and the damage depends both of the Lode’s angle and hydrostatic pressure. The model used in analytical calculations is represented by a function of the three tensors of the invariant stress:

- \( J_1 \) – shows the sensitivity of hydrostatic pressure;
- \( J_2 \) – shows the equivalent von Mises stress;
- \( J_3 \) – shows the Lode’s angle.

Xue – Wierzbicki criterion suppose that the material is isotope and bears von Mises plasticity criterion, also that the failure is characterized by a deterioration factor formally expressed as (4):
\[ \sigma_{eq,VM} = w(D)\sigma_M \] (4)

where: \( \sigma_{eq,VM} \) : equivalent von Mises stress ;
\( w(D) \) : the deterioration factor, \( D = [0,1] \).

1.4. Lode’s angle sensitivity
Lode’s angle is represented in a deviator plane, as the angle between the \( OP \) and \( O'A \) directions, where \( P \) is the stress state and \( A \) is the condition of pure shear (Figure 4) [5].

Lode’s angle is expressed and closely related to the third stress invariant \( J_3 \). Azimuthal dependence is a function on the Lode’s angle:
1.5. Hydrostatic pressure sensitivity

Stress triaxiality $\beta$ is defined as the ratio between the hydrostatic stress and the equivalent stress:

$$\beta = \frac{\sigma_m}{\sigma_{eq,VM}}$$

(6)

where $\sigma_{eq,VM}$ is the equivalent von-Mises stress expressed by:

$$\sigma_{eq,VM} = \sqrt{\frac{2}{3}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

(7)

and $\sigma_m$ is the hydrostatic stress:

$$\sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{F_1}{3}$$

(8)

Stress triaxiality (Figure 5) is not constant in the nearest notch, the maximum value is determined behind the crack. Critical effective stress triaxiality is defined as the average value from the stress triaxiality distribution on the effective length.

$$\beta_{ef,c} = \frac{1}{X_{ef,c}} \int_0^{X_{ef,c}} \beta(x) \, dx$$

(9)

This model involves identifying six material constants: $P_{lim}$ - material dependent constant; $q$ - pressure dependent constant; $\gamma$ - Lode’s angle dependent constant; $\xi_{ref}$ - crack reference strain; $m$ - damage exponent of accumulation; $\beta$ - thinning effect.

2. Experimental procedure

A tensile test allows measuring the tensile strength of any material. This test involves placing a bar of any material between the jaws of a tensile testing machine to pull the bar until it breaks, according to NF EN ISO 6892-1: 2009 [7]. As a result of the applied axial force upon a specimen by the original length ($L_0$), a reduction in cross-sectional area is caused, from $A_0$ to $A$ until the occurrence of breakage. Stress and length variation between two fixed points is recorded and used to determine the stress-strain state.

Torsion test involves twisting a specimen in the absence of other loads (bending or shear) as is specified in ASTM E143: 2013 [8]. Tests were performed on smooth and notched specimens. The tensile strength was carried out by applying a load force between 10 kN and 25 kN. For different notches, the range of the radius was between 0.5 mm and 10 mm (Figure 6). The same type of specimens was used to analyze the influence and the impact of Lode’s angle and triaxiality.

To study the effects of Lode’s angle, smooth specimens have chosen for different tests (pure traction, pure torsion and combined test: tensile 75% - 25% torque, tensile 50% - 50% torque, tensile 25% - 75% torque).

Lode's effectively parameter $\xi_{ef}$ is evaluated based on actual distance in the middle of the notch. The effective distance $X_{ef}$ is given by the volumetric method [9].

Figure 6. Notch specimen
3. Results
In order to study the effects of hydrostatic stress, selected notched specimens with the radius of the notch between 0.5 mm and 10 mm were studied.

The value of Lode’s effectively parameter ($\xi_{ef}$) at pure tensile test is shown in Figure 7. The values for the radius of the notch and the effective distance ($X_{ef}$) are reported depending on the length of the notch in Tables 1, 2 and 3.

Table 1. Values for $\xi_{ef}$ and $X_{ef}$ function of notch radius at pure traction test

| $\sigma_g/\sigma_N$ > 1 ; 100% Traction | $\xi_{ef}$ | $X_{ef}$ [mm] |
|---|---|---|
| r [mm] | | |
| 0.5 | 0.971068 | 0.64 |
| 2 | 0.944137 | 0.7 |
| 4 | 0.955766 | 0.79 |
| 10 | 0.980475 | 0.88 |

Table 2. Values for $\xi_{ef}$ and $X_{ef}$ function of notch radius at pure traction test

| $\sigma_g/\sigma_N$ = 1 ; 100% Traction | $\xi_{ef}$ | $X_{ef}$ [mm] |
|---|---|---|
| r [mm] | | |
| 0.5 | 0.909574 | 0.66 |
| 2 | 0.85741 | 0.72 |
| 4 | 0.907483 | 0.86 |
| 10 | 0.966831 | 0.94 |

Table 3. Values for $\xi_{ef}$ and $X_{ef}$ function of notch radius at pure traction test

| $\sigma_g/\sigma_N$ < 1 ; 100% Traction | $\xi_{ef}$ | $X_{ef}$ [mm] |
|---|---|---|
| r [mm] | | |
| 0.5 | 0.866405 | 0.68 |
| 2 | 0.816663 | 0.74 |
| 4 | 0.854189 | 0.92 |
| 10 | 0.936259 | 0.98 |

Figure 7. Lode’s effective parameter variation at pure tensile test

The results of the tests made for combined loads conditions are presented in Figures 8, 9 and 10. As it may be observed, Lode's parameter values vary between $\theta$ and $l$. A “zero” value corresponds to pure torsion and the “one” value corresponds to pure traction. Values in the range “zero – one” are for combination of tensile and torsion load corresponding to different ratios of these two loads.

Preserving at a constant value the triaxiality (pure tensile load on a smooth specimen), Lode’s parameter can take different values (Lode’s angle varies between $0^\circ$ and $30^\circ$). This variation can cause a significant difference in deformation and also to the material damage, so the material is directly dependent of Lode’s parameter. This variation can show the influence of the load level. For a higher tensile load and a reduced torsion load Lode’s parameter is close to the value $l$, which means a small
influence of this parameter. For a higher torsion and a lower tensile load applied, Lode’s parameter is near 0 values, which means a greater influence of Lode’s parameter.

25% traction - 75% torsion test

Figure 8. Lode’s effective parameter variation at 25% tensile – 75% torsion combined test

50% tensile-50% torsion test

Figure 9. Lode’s effective parameter variation at 50% tensile – 50% torsion combined test

75% tensile -25% torsion test

Figure 10. Lode’s effective parameter variation at 75% tensile – 25% torsion combined test
Contrariwise, the dependence of the Lode’s angle and stress triaxiality (shown in Figure 11 for only one load situation) enables us to make an indirect correlation between the damage risk and the hydrostatic stress.

![Graph showing triaxiality values and Lode's angle for different notch radius in complex load state: 75% tensile - 25% torsion](image)

**Figure 11.** Triaxiality values and Lode's angle for different notch radius in complex load state: 75% tensile - 25% torsion

4. Conclusions
In this paper the authors have proposed to determine stress triaxiality variation on 2P armor steel. Therefor it was determined:
- Stress triaxiality for various types of specimens with different notch radius;
- Lode's angle, using the model Xue-Wierzbicki;
- Effective stress triaxiality and effective Lode's angle;
- The influence of stress triaxiality and the influence of Lode angle upon ductile material breakage.

From the analysis of experimental results is easy to observe that stress triaxiality (via Lode's angle) is a very sensitive indicator for material deformation before a ductile fracture. Thus, for 2P armor steel, only at low levels of stress triaxiality the ductile fracture deformation respect the model described in fracture mechanics. At high levels of stress triaxiality (small parameter values for Lode's) a breakage of the material can occur suddenly. Therefore, the authors believe that the introduction of Lode’s parameter as a new indicator in the model of ductile fracture is required. Lode's parameter can be associated with the breakage material characteristics.

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