Benchmarking the \textit{ab initio} hydrogen equation of state for the interior structure of Jupiter

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ABSTRACT

Context. As Juno is presently measuring Jupiter’s gravitational moments to unprecedented accuracy, models for the interior structure of the planet are putted to the test. While equations of state based on first principles or \textit{ab initio} simulations have been available and used for the two most abundant elements constituting the envelope, hydrogen and helium, significant discrepancies remain regarding the predictions of the inner structure of Jupiter. The differences are severe enough to clutter the analysis of Juno’s data and even cast doubts on the usefulness of these computationally expensive EOSs for the modeling of the interior of Jupiter and exoplanets at large.

Aims. Using our newly developed equations of state for hydrogen and helium, we assess the \textit{ab initio} equations of state currently available and establish their efficiency at predicting the interior structure of Jupiter in a two-layers model. We paid particular attention to the calculation of the total entropy for hydrogen that is required to calculate the convective H-He envelope but is a derived quantity from \textit{ab initio} simulations.

Methods. The \textit{ab initio} equations of state used in this work are based on a parameterization of the \textit{ab initio} simulation points using a functional form of the Helmholtz free energy. It extends on our previous work recently published. Compared to previous \textit{ab initio} equations of state available, this latter approach provides an independent mean of calculating the entropy that was recently pointed out as deficient in some \textit{ab initio} results.

Results. By adjusting our free energy parameterization to reproduce previous \textit{ab initio} EOS behavior, we identify the source of the disagreement previously reported for the interior structure of Jupiter. We further point to area where care should be taken when building EOS for the modeling of giant planets. This concerns the interpolation between the \textit{ab initio} results and the physical models used to cover the low density range as well as the interpolation of the \textit{ab initio} simulation results at high densities. This sensitivity falls well within the uncertainties of the \textit{ab initio} simulations. This suggests that hydrogen EOS should be carefully benchmarked using a simple planetary model before being used in the more advanced planetary models needed to interpret the Juno data. We finally provide an updated version of our \textit{ab initio} hydrogen EOS recently published.

Key words. equation of state, hydrogen, helium, Jupiter, planetary interiors, giant planets, exoplanets

1. Introduction

With the Juno spacecraft currently orbiting Jupiter, there is now a unique opportunity to constrain the planet inner structure using high-precision measurements of the gravitational field (Folkner et al. 2017, Bolton et al. 2017). With the first orbits now completed and analyzed, several remarkable results are already reported. This includes the magnetic field properties of the planet, the depth of the atmospheric jet-stream estimated to extend 3000 km below the surface (Guillot et al. 2018), and the suggestion that the core of the planet may be eroded and probably extending significantly outward in the envelope (Wahl et al. 2017). This latter result is potentially important to validate the core accretion scenario (Pollack et al. 1996), the formation model, and the time evolution of giant planets at large. It remains, however, cluttered by significant uncertainties regarding the predictions of the planet inner structure using various equations of state (EOS).

From the mid-90’s, interior models of Jupiter mostly relied on the benchmark EOSs of Saumon et al. (1995) (SCVH). These EOSs provided the first comprehensive description of hydrogen and helium properties in the entire regime relevant to giant planets (Guillot 1999). These equations of state rest on a chemical description of the dense plasma and are obtained by minimizing the Helmholtz free energy of the system represented as a collection of atoms, molecules, ions, and electrons. This physically based model provides an estimation of the gradual dissociation and ionization of the hydrogen molecules taking place as the density increases along Jupiter’s interior. Among its most notable features, it predicted that dissociation of hydrogen is a first order transition that occurs within Jupiter. This phase transition justifies a three layers model for Jupiter’s interior consisting of a H-He envelope where hydrogen is neutral, an inner envelope where hydrogen turns metallic, and a solid core made of a mixture of water and silicates (Stevenson & Salpeter 1977, Stevenson 1982). This core is assumed to correspond to the primordial planetary embryo around which hydrogen and helium
were accreted in the planetary nebula during the formation of the planet (Guillot 1999; Pollack et al. 1996). This understanding of the interior of Jupiter was completed by considering de-mixing of the hydrogen-helium mixture in the metallic envelope to account for the measurement of a sub-solar abundance of helium in the atmosphere measured by the Galileo probe (von Zahn et al. 1998; Guillot 1999). Demixing is also a convincing explanation of the strong depletion in neon observed by Galileo, as neon is more soluble in helium than in hydrogen (Wilson & Militzer 2010). As mentioned earlier, the recent measurements by Juno support the existence of a diffuse core, blurring somewhat the three-layer pictures (Wahl et al. 2017). This is, however, beyond the scope of the present paper that mainly focuses on describing the H-He envelope and specifically on benchmarking the hydrogen EOS used.

These predictions of Jupiter’s inner structure based on the SCVH EOS (Saumon et al. 1995) were putted into question in the early-2000s by shock measurements of hydrogen up to a few Mbars. These measurements indicated that along the principal Hugoniot, the dissociation and metalization of hydrogen do not coincide with a first order transition in the regime relevant to giant planet interior but happen rather continuously as pressure and temperature increase (Collins et al. 1998; Knudson et al. 2001). It further showed that hydrogen is not as compressible as predicted by the SCVH EOS. These findings immediately cast shadows on the resulting model of Jupiter’s interior structure and triggered intense activities on both the experimental and theoretical sides, mostly focusing on the rate of dissociation of molecular hydrogen at planetary conditions (Saumon & Guillot 2005).

This lead to a new generation of EOS for hydrogen and helium based on density functional theory (DFT) (Lenosky et al. 1997; Militzer & Ceperley 2000; Desjarlais 2003; Holst et al. 2008; Caillabet et al. 2011; Hu et al. 2011; Becker et al. 2014; Militzer & Hubbard 2013; Miguel et al. 2016; Chabrier et al. 2019). As demonstrated by their success at describing all the experimental data obtained so far, these ab initio EOSs provide a description of the ionization and dissociation processes occurring along the principal shock Hugoniot and Jupiter’s adiabat without adjustable parameters. Despite this now undisputed ability at providing an improved description of hydrogen and helium properties at planetary conditions (Knudson et al. 2018), the situation for the interior of Jupiter still remains cluttered. At the moment two different ab initio EOSs published in the past ten years for the hydrogen-helium mixture are leading to significantly different predictions for the interior of Jupiter. The situation was recently summarized by Miguel et al. (2016). These two ab initio EOSs are leading to drastically different predictions regarding the size of the core, the distribution of metallic elements within the envelope as well as the temperature profile within the planet (Nettelmann et al. 2012; Militzer & Hubbard 2013; Miguel et al. 2016). These differences are significant enough to currently challenge our ability to correctly interpret the data from the Juno mission and, ultimately, to use this improved knowledge to validate formation models of giant planets.

To resolve this issue, we recently developed equations of state for hydrogen, helium, and the associated H-He mixture based on density functional molecular dynamics simulations (Chabrier et al. 2019; Soubrirat 2012). Compared to previous ab initio EOSs developed by Militzer & Hubbard (2013) and Becker et al. (2014), these latest EOSs rest on an independent mean of evaluating the Helmholtz free energy. This allows us to deduce the total entropy needed to model convective envelopes, that is in agreement with both the high-pressure melting properties as well as Monte Carlo simulations (Caillabet et al. 2011). Using these EOSs, we critically compare our results with previous predictions for both the entropy and the interior structure obtained for Jupiter. Following the work of Miguel et al. (2016, 2018), we paid particular attention to the evaluation of the entropy for the case of hydrogen. To detangle the discrepancies reported so far, we adjust our free energy model to reproduce previous ab initio simulation results. Lastly, we update our initial release of the hydrogen EOS (Chabrier et al. 2019) to provide a more accurate fit of the ab initio data in the thermodynamical regime relevant to the modeling of the interior structure of Jupiter.

2. Equation of state

The study reported here is based on a new set of EOSs for hydrogen and helium covering the complete thermodynamical range relevant to astrophysical modeling (Chabrier et al. 2019; Soubrirat 2012). This density-temperature range extends significantly beyond the regime relevant to giant planets of the solar system to include hot exoplanets and brown dwarfs several times the size of Jupiter. This set of EOSs follows on the previous work of Caillabet et al. (2011) and complete ab initio simulations data using results obtained from physical models for low and extreme densities as well as for high temperatures. Figure 1 shows how the various methods are used to build a complete hydrogen EOS covering the complete thermodynamical range of interest for astrophysical applications. For hydrogen, density functional theory based molecular dynamics simulations (DFT-MD) can be used for densities down to 0.1-0.2 g/cm³. Below this density range, the method becomes less efficient numerically. Standard DFT functionals, such as the commonly used PBE functional (Perdew et al. 1997), also become less reliable. These functionals do not account for the van der Wall interactions that start to become relevant as the density decreases. The high-temperature limit of the method is more a practical one that steams from the number of Kohn-Sham orbitals that can be included in the simulation while keeping the overall simulation time tractable.

In the DFT-MD region, we used the parameterization of the ab initio results provided by Caillabet et al. (2011). This consists in adjusting two physical models on the ab initio results:
Fig. 2. Comparison between the experimental data and the various theoretical predictions for the hydrogen principal Hugoniot. The initial state is taken as $p_0=0.085$ GPa and $T = 20$ K. EOS-0.2 and EOS-0.8 are principal Hugoniot obtained with an interpolation range between the SCVH and ab initio data extending from 0.05 g/cm$^3$ to, respectively, 0.2 g/cm$^3$ and 0.8 g/cm$^3$. The former corresponds to the CHABRIER et al. (2019) EOS and is thus labeled CMS19-EOS-0.2 in the figure.

The experimental data extending from 0 to 100 K vs. the temperature are available in most of the recent EOSs is clearly visible for both the pressure and internal energy at densities above 0.1 g/cm$^3$. This has been previously documented by several authors.

This revision of the experimental data has a noticeable impact for the modeling of the inner structure of Jupiter. The principal Hugoniot represents the density-temperature conditions at the edge of the interpolation region between the ab initio data and the SCVH results. Reducing the error bars on the Hugoniot measurements adds a stronger constraining on the interpolation procedure in a density region where the two data sets do not coincide for either the internal energy or the entropy. The original shock data (Knudson et al. 2004) allowed for a looser interpolation extending on a density region between $\rho = 0.05$ and 0.8 g/cm$^3$. Figure 2 shows that this leads to a principal Hugoniot, EOS-0.8, with a maximum compressibility reduced and compatible with the original shock data, the predictions of Hu et al. (2011), and Kerley (2013) (not shown in figure 2). The re-analyzed shock data lead to a reduction of the density range over which the interpolation between the SCVH and the ab initio data is performed, between $\rho = 0.05$ and 0.2 g/cm$^3$ (labeled as CMS-19-EOS-0.2 in figure 2).

In figure 3, we compare the pressure and internal energy for hydrogen as given by our recently published EOS, CMS-EOS-0.2, with the latest EOS of Becker et al. (2014), the standard SCVH EOS (Saumon et al. 1995) and the Militzer & Hubbard (2013) EOS as calculated by Miguel et al. (2016), MH-SCVH. We see that differences between the SCVH result and all the ab initio results are clearly visible for both the pressure and internal energy at densities above 0.1 g/cm$^3$. This has been previously documented by several authors.

In figure 3, we also compare the pressure data for various principal EOSs, showing a close agreement between the latest analysis of the shock Hugoniot data by Knudson et al. (2018) and the experimental data obtained by several authors.
and speculated that it propagates to the evaluation of the entropy. We confirm this result here and confirm that it may explain some of the differences in the temperature profile of Jupiter obtained using various ab initio EOSs.

To further compare the various ab initio EOSs available for the modeling of Jupiter, we show in figure 2 the high-density behavior along three isotherms particularly relevant for the interior structure of Jupiter. This density-temperature regime corresponds to the planet deep envelope and close to the core-envelope boundary where significant differences have been reported between the various ab initio predictions. Figure 3 shows some differences at high densities between the three EOSs for all the temperatures investigated. When comparing to the raw ab initio data points, we see that the EOSs remain within 5% of the simulation result. For a temperature of $T = 10000$ K and a density of $\rho = 4$ g/cm$^3$, the original Caillabet et al. (2011) fit is 2% lower than the ab initio data while both the Miguel et al. (2016) and Becker et al. (2014) are 4% higher. This difference worsened as temperature increases. At $T = 20000$ K and for the same density range, the difference in pressure between the Becker et al. (2014), Miguel et al. (2016), and Caillabet et al. (2011) reaches 7%. While this stays within the prescribed boundary for a fit adjusted over a broad density-temperature range, this difference is significant enough to require attention when considering the interior structure of Jupiter. We recall here that the Debras & Chabrier (2019) EOS is based on the Caillabet et al. (2011) parameterization.

We further point out that the ab initio simulation points themselves have some uncertainty associated to them. Various simulation parameters such as the number of particles used in the simulation cell, the plane wave cut-off, the functional used, as well as the fluctuation naturally occurring within the simulation all bring a combined uncertainty of a few percents in the final pressure. In this context, a difference of a few percents between the different EOS based on ab initio results can be expected. While this likely explains the difference between our EOS (Chabrier et al. 2019) and the result of Becker et al. (2014), we also mention that the hydrogen EOS extracted by Miguel et al. (2016) from the H-He of Militzer & Hubbard (2013) adds an additional uncertainty by using the SCVH He EOS and by neglecting the non-ideal mixing contribution accounted for in the Militzer & Hubbard (2013) EOS. The former probably explains the difference we see here for the pressure between the hydrogen EOS extracted by Miguel et al. (2016) and the ab initio simulation points.

To test how this uncertainty propagate for the inner structure of Jupiter, we adjusted the parameterization provided by Caillabet et al. (2011) and used in Chabrier et al. (2019) to reproduce precisely the ab initio results and the pressures obtained by the two other ab initio EOSs. This is obtained by writing $d_e(\rho) = d_0 \rho$ for $\rho \geq 0.419$ and varying the $d_0$ parameter to a value of respectively, $d_0 = 0.0326858$, $0.123$, and $0.223$. The first value corresponds to the initial parameterization of Caillabet et al. (2011) used in Chabrier et al. (2019) and Debras & Chabrier (2019), the second matches exactly the ab initio results, while the latest value reproduces the Becker et al. (2014) behavior at high densities.

To address the issue of the evaluation of the entropy pointed out by Miguel et al. (2016), we show in figure 5 the various ab initio predictions for isotherms representative of the interior of Jupiter. The labeling follows the one given by Miguel et al. (2016) where the entropy deduced from the EOS of Becker et al. (2014) and corrected for the energy are labeled respectively eos3b and eos3c. The entropy deduced from the H-He EOS of

![Figure 3](image1.png)  
**Fig. 3.** Comparison between the (a) pressure normalized to the density and (b) internal energy as given by various EOS for hydrogen. Same legend as in figure 2. MH-SCVH stands for the Militzer & Hubbard (2013) EOS as calculated by Miguel et al. (2016).

![Figure 4](image2.png)  
**Fig. 4.** Comparison between the initial ab initio simulation points of Caillabet et al. (2011) and the various ab initio EOS available at high-density.
ties between \( \rho \) is considered as a benchmark result for the entropy. As expected, the figure 5-a shows that the predictions are overall rather consistent and the high pressure behavior, EOS-0.2 \( d_0 = 0.132 \), and with the SCVH predictions and another well used physically based EOS (Kerley 2013).

Militzer & Hubbard (2013) is labeled MH-SCVH. We see in figure 5-a that the predictions are overall rather consistent and in rather good agreement with the coupled electron-ion Monte Carlo calculations of Morales et al. (2010). The latter is considered as a benchmark result for the entropy. As expected, the largest differences are seen in the interpolation region for densities between \( \rho = 0.1 \) g/cm\(^3\) and \( \rho = 0.8 \) g/cm\(^3\). These differences reflect the variation that was observed for the principal Hugoniot predictions shown in figure 2. We also note that once corrected (Miguel et al. 2018), the high-temperature behavior initially noticed for the entropy deduced from the EOS of Becker et al. (2014) using thermodynamics relations tends to disappear. The results shown in figures 5 suggest that the estimation of the entropy is rather consistent between the various models. The evaluation of the entropy itself is not likely the direct cause of the differences that are reported in the predictions for the interior of Jupiter.

This is confirmed in figure 5-b, where we compare the predictions of our EOS (Chabrier et al. 2019) with the results given by the SCVH and Kerley (Kerley 2013) EOSs. The latter is a refined version of the SCVH EOS that takes into account the experimental Z-pinch data of Knudson et al. (2004). In the interpolation region, the difference is surprisingly only noticeable at the lowest and highest temperatures displayed. We also show in figure 5-b the effect of varying the interpolation region between the SCVH and the \textit{ab initio} results from the density range extending from 0.05 to 0.2 g/cm\(^3\) to 0.05 to 0.8 g/cm\(^3\) while keeping \( d_0 = 0.0326858 \). These EOS are labeled as, respectively, CMS19-EOS-0.2 and EOS-0.8. At \( T = 2000 \) K, we see that a larger interpolation range provides a smoother transition between the SCVH and \textit{ab initio} data sets. The EOS-0.8-\( d_0 = 0.0326858 \) result is almost indistinguishable from the SCVH result. We recall from the discussion of figure 2 that this occurs at the expense of not reproducing the re-analyzed shock data. As this also corresponds to the differences in the internal energy noticed between the various models, this tends to further suggest that the different predictions for the internal energy only impact the entropy deduced in the low-temperature region of the isentrope. It thus is not likely the origin of the difference noted for the whole adiabat of Jupiter.

We also show in figure 5-b how the entropy changes when the EOS is adjusted to reproduce the initial \textit{ab initio} calculations to less than 1%; noted as EOS-0.2-\( d_0 = 0.123 \). We see that adjusting the pressure at high densities reproduces the variations noticed between the various \textit{ab initio} predictions at high densities. This suggests that the evaluation of the entropy itself, which varies in the various methods, is probably not a cause for concern. The variation between the various predictions is mostly a propagation of the difference noticed for the energy and pressure in the interpolation between the \textit{ab initio} and SCVH results and, at higher densities, in the interpolation of the \textit{ab initio} results.

3. Jupiter Inner Structure

We now turn to the predictions of the interior profile of Jupiter obtained using the various EOSs discussed in the previous section. We show in figure 6 the hydrogen-helium adiabat obtained using our recent EOS (Chabrier et al. 2019) with a helium concentration \( Y_{He} = 0.245 \). This profile is obtained by considering the planet as constituted of an isentropic hydrogen-helium envelope with an homogeneous He concentration and a central core made of heavier elements. We used the temperature measured at 1 bar by the Galileo probe (von Zahn et al. 1998), \( T_{1 \text{ bar}} = 167 \) K, to fixed the entropy of the isentrope using the SCVH EOS. This model assumes that the envelope is fully convective, neglects a potentially radiative outer layer (Guillot 1999), the effect of demixing and of the multiple molecular species detected in the atmosphere by the Galileo probe (Hubbard & Militzer 2016). With these effects neglected, the Jupiter’s interior profiles calculated thus corresponds to the H-He isentrope.

In figure 6-a, we compare our calculation for the H-He isentrope obtained for a fixed concentration of helium, \( Y_{He} = 0.245 \), with the \textit{ab initio} predictions of Militzer & Hubbard (2013) and Nettelmann et al. (2012). At low pressures, figure 6-a shows that varying the interpolation domain from 0.2 to 0.8 g/cm\(^3\), noted respectively EOS-0.2 and EOS-0.8, leads to a cooler isentrope in the 10 – 100 GPa range. It also removes the change of slope noticeable from 10 to 80 GPa. This suggests that a different interpolation scheme in this density region is likely at the origin of the difference obtained for the adiabat in the 10 – 300 GPa range. This range also corresponds to tighter constrains from the Hugoniot data. We further see that the revision of the Hugoniot data confirms a slight variation of the slope of the adiabat of Jupiter as the pressure increase and dissociation takes place.
We see that up to 100 GPa, the agreement with the prediction of Militzer & Hubbard (2013) is almost perfect while some departure is noticeable with the calculation of Nettelmann et al. (2012) and Kerley (2013). More specifically, we see that the isentrope deduced from the Becker et al. (2014) data, noted es3b, is consistently higher in temperature than the other EOSs in the 0.03 – 1.3 g/cm³ density range. It is even close to the SCVH isentrope in this density range. In contrast to Miguel et al. (2016), we attribute this difference to the interpolation region between the SCVH and ab initio data and the internal energy obtained rather than the method used to evaluate the entropy. Beyond a density of 1.5 g/cm³, we also see that the isentrope obtained reached higher temperatures when calculated using the Becker et al. (2014) EOS. The latter clips our estimation corresponding to the highest value of $d_0$. In contrast, the isentrope obtained using a modified version of Militzer & Hubbard (2013) remains consistently colder. Figure 6-b shows that the mean value $d_0$ gives an isentrope that lays almost in between the interior profiles obtained using the Becker et al. (2014) EOS or based on the Militzer & Hubbard (2013) EOS. This result suggests that the MH13-SCVH result is not completely consistent with the initial Militzer & Hubbard (2013) EOS calculated at a fixed helium concentration. We recall here that Miquel et al. (2016) extract a pure hydrogen EOS by using the SCVH EOS for helium. As this does not coincide with the ab initio results for pure helium (Soubiran 2013), it is not surprising that some differences remain for the isentrope. The comparison shown in figure 7 suggests that the error is on the order of 1000 K when using the EOS deduced by Miquel et al. (2016).

We show, in figure 7, the variation of the first two gravitational moments obtained using the two extreme values of $d_0$. These calculations are performed using the theory of figures to the third order and considering a two-layers model consisting of an H-He envelope and a core made of water. While it is now well documented that such simple model is not sufficient to calculate the gravitational moments in the Juno era (Hubbard & Militzer 2016; Nettelmann 2017), it enables us to quantify the uncertainty stemming from the EOS without having to consider the additional planetary model assumptions that vary from authors to authors. Following Kerley (2013), we propose that this model being used to benchmark future EOS intended for the modeling of Jupiter and the interpretation of the Juno data so as to decipher the uncertainties coming from the EOS from the ones arising from the planetary model. In figure 7, each curve represents an interior structure calculation at a fixed value of the helium concentration in the envelope while the size of the core varies. We assumed the core as made of pure water and used the ab initio EOS for dense water (Mazevet et al. 2019) to model

 afflictio data for, respectively, the internal energy and the entropy 

previously in the interpolation region between the SCVH and ab initio

feff (2012). The latter is consistent with the departure is noticeable with the calculation of Nettelmann et al. (2012) and Chabrier et al. (2019). (b) Comparison with the predictions of

Militzer & Hubbard (2013), Nettelmann et al. (2012), and Miguel et al. (2016); corrected by Miguel et al. (2018). The shaded area corresponds to the result obtained when $d_0$ is varied as in indicated in (a).

Fig. 6. Jupiter interior profiles obtained with various version of the EOS adjusted at low and high densities and an helium mass fraction $Y_{He} = 0.245$. (a) Comparison with previous ab initio predictions of Militzer & Hubbard (2013), Nettelmann et al. (2012), and Chabrier et al. (2019). (b) Comparison with the predictions of Miguel et al. (2016) corrected by Miguel et al. (2018). The shaded area corresponds to the result obtained when $d_0$ is varied as in indicated in (a).
various predictions regarding the size of the core and the amount of metallic elements in the envelope as obtained when we vary the two gravitational moments when using the EOS modified to match the two extreme values of $d_0$. The latter only appears in the interpolation region between the SCVH and \textit{ab initio} data and, even in this case, is a consequence of the interpolation in energy and pressure between the \textit{ab initio} predictions.

**4. Summary**

Using our newly developed equations of state for hydrogen and helium \cite{Chabrier2019}, we investigate the long standing disagreement regarding the predictions of the amount of metallic elements in the envelope, the size of the core, and the temperature at the core-envelope boundary for Jupiter. We find the origin of the disagreement between the previous \textit{ab initio} predictions by varying the parameters in our parameterization. We confirm the prediction of \cite{Militzer2013} and point out to deficiencies in the parameterization of \cite{Becker2014} regarding the size of the core and the amount of metallic elements in the envelope. We also find that Jupiter inner structure and the associated gravitational moments are very sensitive to the evaluation of pressure and internal energy to a level that approaches the uncertainty in the \textit{ab initio} simulations. It further enters the regime where the different functionals used can have some influence \cite{Schottler2018,Mazzola2018}. This is a source of concern as neither the input \textit{ab initio} points or the fit developed for planetary modeling are brought to this level of accuracy. This result should be accounted for in more refined planetary models required to interpret the Juno data by using benchmarked equation of states carefully validated for all the quantities involved in planetary modeling, and particularly, the pressure, internal energy, and entropy. We suggest that a simple two-layers model provides a useful framework to benchmark future EOS. Application of this new EOS for hydrogen in a more refined planetary model as needed to interpret Juno data will be the object of further work. The benchmarked EOS for hydrogen, matching the initial \textit{ab initio} points of \cite{Caillabet2011} by less than 1%, and providing an updated version of the CMS19 EOS, is provided at the following address.

\begin{equation}
\text{Hubbard & Marley (1989, Figure [8]) shows that the variation in the hydrogen \textit{ab initio} EOS, embodied in the value of $d_0$, leads to a variation in the amount of metallic elements predicted that is comparable to the one found by Militzer & Hubbard (2013) and Nettelmann et al. (2012). Based on the comparison shown before for the EOS and the isentrope, we suggest that the remaining difference with the Nettelmann et al. (2012) comes from the remaining difference for the isentrope around 1 g/cm$^3$ that is not completely covered by varying the $d_0$ parameter. We further see that this result does not depend on the nature of the core as the amount of metallic elements predicted in the envelope as $d_0$ is varied is similar for a pure water or a water-silicate core.}
\end{equation}

Our preferred EOS, EOS-0.2 $d_0=0.123$, that coincides with the \textit{ab initio} simulation points to better than 1%, and corrects the original Caillabet et al. (2011) and Chabrier et al. (2019) EOSs, is in rather good agreement with the predictions of Militzer & Hubbard (2013). The detailed study performed here shows that \textit{ab initio} EOS for hydrogen leads to Jupiter model with a low amount of metallic elements in the envelope. It furthermore provides an explanation for the differences reported with the predictions of Nettelmann et al. (2012) that overestimates the amount of metallic elements in Jupiter’s envelope. We further point out that this difference is likely due to the uncertainty in the evaluation of the pressure either coming from the fit and/or the initial \textit{ab initio} data but not due to differences in the evaluation of the entropy as suggested by Miguel et al. (2016). The latter only appears in the interpolation region between the SCVH and \textit{ab initio} data and, even in this case, is a consequence of the interpolation in energy and pressure between the \textit{ab initio} and SCVH data.
https://luth.obspm.fr/~luthier/mazevet/wordpress/planets-and-exoplanets/.

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### Appendix A: EOS and references

| Name               | interpolation region in $\rho$ g/cm$^3$ | $d_0$     | ab initio data          | reference               |
|--------------------|-----------------------------------------|-----------|-------------------------|-------------------------|
| CMS19-EOS-0.2 $d_0=0.0326858$ | 0.05-0.2                               | 0.0326858 | Caillabet et al. (2011) | Chabrier et al. (2019)  |
| EOS-0.8 $d_0=0.0326858$          | 0.05-0.8                               | 0.0326858 | Caillabet et al. (2011) |                         |
| EOS-0.2 $d_0=0.123$              | 0.05-0.2                               | 0.123     | Caillabet et al. (2011) |                         |
| EOS-0.2 $d_0=0.223$              | 0.05-0.2                               | 0.223     | Caillabet et al. (2011) | This work               |
| eos3b                | -                                      | -         |                         | Becker et al. (2014)    |
| eos3c                | -                                      | -         |                         | Becker et al. (2014)    |
| MH-SCVH              | -                                      | -         |                         | Militzer & Hubbard (2014) | Miguel et al. (2016) |