Platform for controllable Majorana zero modes using superconductor/ferromagnet heterostructures

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Abstract. We propose a novel platform for the creation and manipulation of Majorana zero modes consisting of a ferromagnetic metallic wire placed between conventional superconductors which are in proximity to ferromagnetic insulators. Our device relies on the interplay of applied supercurrents and exchange fields emerging from the ferromagnetic insulators. We assert that the proposed superconductor/ferromagnet heterostructures exhibit enhanced controllability, since topological superconductivity can be tuned apart from gate voltages applied on the ferromagnetic wire, also by manipulating the applied supercurrents and/or the magnetisation of the ferromagnetic insulators.

1 Introduction

Topological quantum computation is the most promising answer to the quantum decoherence problem hindering the development of quantum computers [1]. Topologically protected qubits formed by non-Abelian anyons would allow to store and process quantum information in decoherence-resilient schemes. Majorana zero modes (MZMs) are arguably the most natural realization of such non-Abelian anyons. Indeed, MZMs are expected to emerge at the edges of 1D [2] and at the vortices of 2D [3] topological p-wave superconductors but unfortunately p-wave superconductivity appears to be scarce in nature. However, a topological p-wave superconducting state has been demonstrated to emerge in heterostructures comprising topological insulators and conventional superconductors [4], where proximity induced singlet superconductivity pairs the helical states confined at the boundaries of the topological insulator realising an effectively spinless superconducting state. Following this seminal work numerous proposals have been put forward involving superconductor-semiconductor heterostructures [5–7], non-centrosymmetric superconductors [8], surfaces of heavy metals [9], the quantum anomalous Hall state [10] and 2D electron gases [11]. Key elements for realising effectively spinless superconductivity in all the aforementioned proposals, is spin momentum locking and a Zeeman energy term, thereby many schemes for realising MZMs rely on effective spin orbit coupling emerging from the application of laser beams in ultracold atom superfluids [12], nanomagnets in superconducting wires [13] or carbon nanoribbons [14], magnetic tunnel junctions in proximized 2D electron gases [15], from the helical ordering of magnetic adatoms embedded in conventional superconductors [16–20] as well as from the curved geometry of carbon nanotubes [21] and semiconducting wires [22] and from the spatially modulated width of planar Josephson junctions [23]. Moreover, in most of the above setups for engineering MZMs, the required Zeeman energy is provided by the application of an external magnetic field. Superconductor/ferromagnet heterostructures (SC-FM) support the emergence of topological superconductivity without the requirement of an external magnetic field, as has been already demonstrated for ferromagnetic metal/superconductor heterostructures with spin active interfaces [24,25] and ferromagnetically aligned magnetic adatoms chains embedded in conventional superconductors with spin orbit coupling [26]. However, the particular ferromagnet-superconductor heterostructures exhibit limited controllability of the emergent MZMs.

Here we propose a platform for engineering controllable TSC and MZMs based on a superconductor/ferromagnet heterostructure and the application of supercurrents and gate voltages. The heterostructure consists of a ferromagnetic metallic wire (FM) placed among conventional superconductors (SCs) which are in proximity to ferromagnetic insulators (FIs) (see Fig. 1). The general underlying mechanism has already been presented in our previous publication [27] and consists of two stages. In the first stage, triplet p-wave correlations are induced in the SCs, due to the coexistence of conventional superconductivity with applied supercurrents and exchange fields emerging from the FIs.
As we show below, the d-vector [28] of the induced p-wave correlations is parallel to the polarization of the exchange field of the FIs. In the second stage, these p-wave correlations mediate into the FM where an analogous to Kitaev’s chain spinless superconducting state is realised, provided the magnetisation of the wire and the d-vector of the p-wave correlations are misaligned. We demonstrate that our platform exhibits enhanced controllability through the externally applied supercurrents and the manipulation of the exchange fields resulting from the FIs, which can be achieved by the use of currents [29] or applied gate voltages [30,31]. We remark that although supercurrents were involved in other proposals for engineering MZMs [32–37], in our remark that although supercurrents were involved in [27] these诱导 p-wave superconductivity

2 Induced p-wave superconductivity

Initially, we describe how the coexistence of conventional superconductivity with supercurrent and an exchange field, induces triplet p-wave correlations with a d-vector parallel to the polarization of the exchange field. As we have already pointed out in [27] these four fields or order parameters constitute a quartet [38] meaning that the presence of any three of them induces the missing fourth one. To this end we employ the following Hamiltonian representing a 1D SC,

\[ H = \int \Psi^\dagger \left( \frac{\partial^2}{2m} + \mu \right) \tau_z + \tau_z \mathbf{h} \cdot \mathbf{\sigma} + \Delta \tau_y \sigma_y \Psi \]  

(1)

where \( \Psi^\dagger \) = \( (\psi^\dagger_1, \psi^\dagger_2, \psi_1, \psi_2)^\dagger \) the extended Nambu spinor and \( \tau \) and \( \mathbf{\sigma} \) = \( (\sigma_x, \tau_\sigma, \sigma_y, \sigma_z) \) are the Pauli matrices acting on particle-hole and spin space, respectively, \( \mathbf{h} \) the exchange field and \( \Delta = |\Delta| e^{iJx} \tau_z \) the singlet superconducting field which acquires a phase gradient \( J \) proportional to the applied supercurrent density. Under the gauge transformation \( \Psi(x) \rightarrow e^{-iJx/2}\tau_z \Psi(x) \), kinetic term \( \frac{\partial^2}{2m} \) transforms to \( \frac{\partial^2}{2m} + i \frac{J}{2m} \partial_x - \frac{J^2}{8m} \tau_z \) where \( i \frac{J}{2m} \partial_x \) is the velocity corresponding to the supercurrent density and \(- \frac{J^2}{8m} \tau_z \) can be absorbed in the chemical potential \( \mu \). Thus, Hamiltonian Eq. 1 takes the form

\[ H = \int dx \Psi^\dagger \left( \frac{\partial^2}{2m} + \mu \right) \tau_z + \tau_z \mathbf{h} \cdot \mathbf{\sigma} + \Delta \tau_y \sigma_y \Psi \]

For periodic boundary conditions, a Fourier transformation to momentum \( k \) space leads to the energy bands of the wire \( E_{s,\pm}(k) = -sh + \frac{J^2}{2m} k \pm \sqrt{\frac{J^2}{2m} - \mu} + \Delta^2 \), where \( s = \pm \), with + and - corresponding to different spin configurations depending on the orientation of the \( h \).

For translationally invariant systems, p-wave triplet correlations derive from equation

\[ < \Delta_p > \propto \frac{1}{\beta} \Sigma \omega_n Tr \{ k\tau_z d \cdot \mathbf{\sigma} i \sigma_y \hat{G}(k, i\omega_n) \} \]  

(2)

where \( \hat{G}(k, i\omega_n) \) the Matsubara Green’s function with \( \omega_n = (2n + 1)\pi/\beta \) the fermion frequencies and \( \beta \propto 1/T \) the inverse of temperature. The above expression simplifies to

\[ < \Delta_p > \propto \sum_k \sum_m n_F(E_m) [U(k)^\dagger k\tau_z d \cdot \mathbf{\sigma} i \sigma_y U(k)]_{mm} \]  

(3)

where \( m \) is the energy band index, \( U(k) \) the transformation matrix which diagonalises the Hamiltonian matrix \( H(k) \) while \( [\cdot]_{mm} \) denotes the \( m \) diagonal term of the corresponding matrix. Based on the above analysis it is straightforward to derive the expression

\[ < \Delta_{p,d||h} > \propto \frac{\Delta}{\sqrt{\Delta^2 + (h^2 - \mu)^2}} \times \left[ n_F(E_{s,-}) - n_F(E_{s,+,+}) - [n_F(E_{s,-}) - n_F(E_{s,+,+})] \right] \]  

(4)

for the triplet correlations with \( d \parallel h \). Since for \( h = 0 \), \( n_F(E_{s,\pm}) = n_F(E_{s,\pm}) \), triplet correlations vanish, \( < \Delta_{p,d||h} > = 0 \). Similarly, for \( J = 0 \) we get \( < \Delta_{p,d||h} > = 0 \), since the expression \( k n_F(E_{s,-}) + n_F(E_{s,+,+}) - [n_F(E_{s,-}) + n_F(E_{s,+,+})] \) is odd in momenta \( k \). However, when both \( h \) and \( J \) are finite, both spin and spatial inversion symmetries break, leading to a finite sum for triplet correlations \( < \Delta_{p,d||h} > \). Regarding correlations \( < \Delta_{p,d\pm h} > \), matrix \( U(k)^\dagger k\tau_z d \pm h \cdot \mathbf{\sigma} i \sigma_y U(k) \) has no diagonal elements, therefore they vanish. Thus, only when \( h \) and \( J \) are both finite in a conventional superconductor, \( < \Delta_{p,d||h} > \) correlations are induced. From Eq. 4 it becomes apparent that \( < \Delta_{p,d||h} > \) correlations emerge from the imbalance among the two configurations \( \psi^\dagger_{k,s} \psi^\dagger_{-k,s} \) and \( \psi^\dagger_{k,s} \psi^\dagger_{-k,s} \) of the interband singlet pairing. (See Sup. Mat.). Moreover, since \( J \rightarrow -J \) leads to \( E_{s,\pm}(k) \rightarrow E_{s,\pm}(-k) \) we get \( < \Delta_{p,d||h} > \rightarrow - < \Delta_{p,d||h} > \) or equivalently the phase of the triplet order parameter changes by \( \pi \). Most importantly, there are no threshold values for \( h \) and \( J \) in order for \( < \Delta_{p,d||h} > \) correlations to be induced. Therefore the particular mechanism is valid even for small values of these parameters which leave conventional superconductivity unaffected.

3 Topological superconductivity in superconductor-ferromagnet heterostructures

Evidently, topological superconductivity cannot be realised simply by applying a supercurrent in a superconducting wire under the presence of an exchange field,
Fig. 1 a Device for engineering controllable MZMs based on superconductor-ferromagnet heterostructures. A ferromagnetic metallic wire (FM) is placed between two conventional superconductors (SCs) where supercurrents (blue arrows) are applied in an antiparallel configuration. Exchange fields are induced in the SCs due to proximity with the ferromagnetic insulators (FIs). The optimal configuration is when the magnetisations of the FIs (black arrows) are perpendicular to that of the FM (white arrow) and antiparallel with each other. The application of supercurrent \( J \) and exchange field \( h_x \) in the SC induces an equal spin triplet pairing component (blue dots) which mediates in the FM resulting in the emergence of MZMs \( \gamma_a = (\psi^\dagger \sigma^x \psi) \) (yellow dot) and \( \gamma_b = i(\psi^\dagger - \psi) \) (green dot) at the edges of the wire. b The energy bands of a system with an applied current \( J \) and a \( h_x \) exchange field. The singlet superconducting field \( \Delta_p \) pairs electron from different bands. The application of \( J \) and \( h_x \) results in an imbalance of singlet pairing between the two spin configurations, and therefore an equal spin triplet pairing component is induced. However, the induced triplet superconducting field still pairs electrons from different bands. c The induced equal spin triplet superconducting correlations \( \Delta_p \) mediate in the FM where an exchange field \( h_x \) is present. In the spin configuration of the energy bands of FM, \( \Delta_p \) pairs electrons of the same band, i.e., intraband pairing, realising an effectively spinless superconducting state

since the emergent triplet pairing correlations are parallel to the polarization of the exchange field and therefore connect different energy bands of the wire, i.e. we get interband triplet pairing. To realise MZMs we need intraband superconductivity. In our device intraband superconductivity emerges in the FM where the triplet correlations induced in the conventional SCs mediate by means of the proximity effect. Necessary condition for the emergence of intraband superconductivity is that the exchange field \( h \) of the wire and the \( d \) vector of the induced triplet correlations are misaligned. Therefore, a topological superconducting state is realised in a FM separating two conventional SCs when supercurrents are applied in an antiparallel configuration and the exchange fields are perpendicular to the magnetisation of the wire and antiparallel to each other (see Fig. 1). The antiparallel configuration guarantees that the supercurrent and the exchange field vanish into the wire and therefore no \(< \Delta_{p,d}/\mu_{FM} >\) correlations are induced there. Although this is the optimal configuration, we show that TSC is realised even for small misalignment angles among the magnetisations of the FM and the FIs and even for not exactly anti-parallel supercurrents and exchange fields (see Fig. 2).

To illustrate numerically the above arguments we employ a lattice Bogoliubov de Gennes equation, \( \mathcal{H} = \sum_{\alpha,j} \psi^\dagger_i \hat{H}_{\alpha,j} \psi_j \) \( \psi_j \), which describes the SC-FM-SC presented in Fig. 1. The lattice Hamiltonian consists of three parts which describe each region of the heterostructure separately and a corresponding coupling Hamiltonian. In particular, we have

\[
\mathcal{H} = [H_{l,SC} + H_{l,tc} + H_{FM} + H_{r,tc} + H_{r,SC}],
\]

where

\[
H_{a,SC} = \sum_{i,j} \psi^\dagger_{a,SC} \left[ (tf_{i,j} + \mu_{SC}) \tau_z + h_{a,SC} \tau_z \sigma_x \right.
+ \left. \Delta \tau_y \sigma_y + J_{a,SC} \hat{g}_{i,j}^z \right] \psi_{a,j,SC}
\]

the Hamiltonian of the superconductor, with \( a = l, r \) for the left and right SC respectively, and \( h_{l,SC} = -h_{r,SC}, J_{l,SC} = -J_{r,SC} \), while

\[
H_{FM} = \sum_{i,j} \psi^\dagger_{i,FM} \left[ (tf_{i,j} + \mu_{FM}) \tau_z + h_{FM} \tau_z \sigma_z \right] \psi_{j,FM}
\]

the 1D Hamiltonian of the FM wire. We remark that part \( H_{a,SC} \) of the Hamiltonian Eq. 5 is the lattice counterpart of Hamiltonian Eq. 1 extended in two dimensions. We have introduced the Nambu spinors \( \psi^\dagger_{a,i,SC} = (\psi_{a,i,\uparrow,SC}, \psi_{a,i,\downarrow,SC}, \psi_{a,i,\uparrow,SC}, \psi_{a,i,\downarrow,SC}) \) and \( \psi^\dagger_{i,FM} = (\psi_{i,\uparrow,FM}, \psi_{i,\downarrow,FM}, \psi_{i,\uparrow,FM}, \psi_{i,\downarrow,FM}) \), where \( \psi_{a,i,\uparrow,SC} \) creates (destroys) an electron with spin \( s \) at lattice site \( i \) in the \( a \) SC, while \( \psi_{i,\uparrow,FM} \) creates (destroys) an electron with spin \( s \) at lattice site \( i \) in the FM wire. We have also employed the Pauli matrices \( \tau \) and \( \sigma \) acting on particle-hole and spin space respectively, and the functions \( f_{i,j} = \delta_{j,i,\pm \pm} + \delta_{j,i,\pm \pm} \) and \( g_{i,j} = \hat{g}_{i,j}^z = (\pm i \delta_{j,i,\pm \pm} \pm i \delta_{j,i,\pm \pm}) \), even and odd in spatial inversion, connecting nearest neighbours lattice points. Moreover, \( t \) is the hopping integral, \( \mu_{SC} \) and \( \mu_{FM} \) the chemical potential in the superconducting regions and the ferromagnetic wire respectively, while \( h_{a,SC} \) and \( h_{FM} \) are the corresponding exchange fields. Moreover, we include the conventional superconducting field \( \Delta \) and the supercurrent density term \( J_{a,SC} \).

The SCs are connected with the FM wire through the following coupling Hamiltonian
\[ H_{a\text{,tc}} = \sum_{i,j,s} t_c \psi_{a,i,j,s}^\dagger \psi_{i,j,s} + H.c. \] (8)

The proximity to the FIs is modelled by considering the finite exchange fields \( h_{a,SC} \) over the SC regions emerging from the interfacial exchange interaction between the localized magnetic moments of the FIs and the SCs conduction band electrons. The dependence of the emerging MZMs on the width of the conventional superconductors eludes this simplified model and will be examined in a more detailed work. However, the width of the conventional superconductors along the x-axis is not important when the FIs are deposited on top or below the SCs. MZMs are anticipated to emerge localised at the edges of the wire for \( |2t' - \sqrt{(h_{FM})^2 - \Delta'^2} - |\mu_{FM}| < |2t' + \sqrt{(h_{FM})^2 - \Delta'^2}| \) where \( \Delta' \) is the induced by proximity singlet superconducting field over the FM [27]. From Fig. 2a we conclude that the hopping term along the wire is renormalised to \( t' \approx 0.85t \), due to the coupling of the wire to the SCs [39]. Figure 2b, c and d demonstrate the significant robustness of our device against deviations from the optimal antiparallel alignment of the magnetisation in the FIs and the antiparallel configuration of the supercurrents in the SCs.

4 Controlling Majorana zero modes

To demonstrate the controllability of our setup we consider a junction between two ferromagnetic wires FM1 and FM2 as depicted in Fig. 3. The conventional superconductors in this configuration are separated by an insulator in order for the supercurrents adjacent of FM1 and FM2 to be controlled independently. First, we consider a sinusoidal variation \( J = J_0 \cos(\phi) \) of the magnitude of the supercurrents applied in the SCs that are adjacent to the FM2, where \( \phi \) is just a parameter without any physical meaning. The supercurrents always flow in opposite directions. In Fig. 3 is presented the supercurrents configuration for \( \phi = \pi \) while in Fig. 4a is shown the low-energy spectrum of this junction with respect to parameter \( \phi \). The FM wires are coupled through the insulating region with a hopping term \( t_c = 0.5t \). For \(-\pi/2 < \phi < \pi/2\), the superconducting states in the two FMs acquire the same phase (0-Josephson junction) and MZMs \( \gamma_{1,a} \) localised in the right edge of FM1 and \( \gamma_{2,a} \) localised at the left edge of the FM2 interact with each other giving rise to an ordinary ferromagnetic state with finite energy. The other two MZMs \( \gamma_{1,b} \) at the left edge of FM1 and \( \gamma_{2,b} \) at the right edge of FM2 remain unpaired and therefore stick at zero energy. For \( \phi = \pm \pi/2 \) the energy gap in FM2 closes and MZMs \( \gamma_{2,a} \) and \( \gamma_{2,b} \) fuse, while \( \gamma_{1,a} \) and \( \gamma_{1,b} \) remain uncoupled. Remarkably for \( \pi/2 < \phi < 3\pi/2 \), for which the supercurrent flow is reversed on both sides of FM2, the phase of the induced p-wave superconductivity in FM2 changes by \( \pi \) leading to the formation of a p-Josephson junction among the FM wires. The reversal of the flow of the supercurrent leads to an interchange of the position of \( \gamma_{2,a} \) and \( \gamma_{2,b} \). Since in our case Majorana operators, \( \gamma_b \) and \( \gamma_a \) have opposite time-reversal T parities, \( T\gamma_a T^{-1} = \gamma_a \) while \( T\gamma_b T^{-1} = -\gamma_b \), the coupling term \( i\gamma_{1,b} \gamma_{2,a} \) is odd in time-reversal [40]. However, in our setup the two FM wires are only coupled by the even in time-reversal \( t_c \) hopping term. Thus, \( \gamma_{2,a} \) cannot couple with \( \gamma_{1,a} \) and the heterostructure hosts in this case four MZMs reflected in the low-energy quasiparticle spectrum (Fig. 4a). Note on the same figure, that the four MZMs are stabilised even for very small values of the supercurrent.

Next, we investigate the same junction by rotating the magnetisation and therefore the exchange field of the FIs adjacent to FM2, in the x-z plane, \( h = h_x \sin(\theta) + h_z \cos(\theta) \) where \( \theta \) is the angle from the x-axis. Since we require that the x component of the exchange fields of FIs adjacent to FM2, point at the opposite direction for every \( \theta \), the magnetisation in the upper FI rotates clockwise while in the lower FI rotates counter-clockwise (Fig. 3). The supercurrent in the two FMs flows along the same direction, therefore for \( \theta = 0 \)
the junction is formed by two identical copies of FMs embedded in conventional SCs. According to our mechanism topological SC emerges only when the magnetizations of the FIs and that of the FM wire are misaligned. Indeed, for \( \theta = \pi/2 \) the energy gap at the wire closes (Fig. 4b). Remarkably for \( \pi/2 < \theta < 3\pi/2 \) for which the \( x \) component of the magnetisation of the FIs adjacent to FM2 is opposite to that of the FIs adjacent to FM1, a \( \pi \)-Josephson junction is formed and four Majorana zero modes emerge localised at the edges of the two wires. In Fig. 4c and 4d we display the spectral function \( |\Psi|^2_z \) along the \( z \)-axis over the FM wires, where \( \mathcal{H}\Psi = E\Psi \), with \( \Psi \) corresponding in this context, to the wavefunction of the eigenstates of the two lowest eigenvalues for \( \theta = 0.45\pi \) and \( \theta = 0.55\pi \). We remark, that small rotations of the FIs magnetisation can be achieved by the application of gate voltages [30,31].

5 Conclusions

We present a novel platform for engineering MZMs based on SC-FM heterostructures. Key elements of the underlying mechanism are the externally applied supercurrents and the emergent exchange fields, due to proximity of the SCs with FIs. Developments in the fabrication of SC-FI heterostructures [41] and ferromagnetic metallic Co [42,43] or Fe [44] nanowires embedded in conventional SCs, enhance the experimental feasibility of our proposal. Several works have demonstrated both theoretically [45,46] and experimentally [47–49] the emergence of exchange fields in conventional SCs in proximity to FIs, e.g. in EuS/Al and EuO/Al heterostructures. Finally, our platform does not require externally applied magnetic fields, yet it facilitates the control of the emergent MZMs, either through the applied supercurrents or the manipulation of FIs magnetisation by use of currents or applied gate voltages.

In this way we circumvent the screening effects problem [50] that undermines schemes for braiding MZMs based on semiconductors [51].

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Author contributions

Dr. Giorgos Livanas has conducted the analysis and wrote the initial version of the manuscript. Nikolaos Vanas contributed to numerical calculations. Georgios Varelogiannis and Prof. Manfred Sigrist have revised and wrote the final version of the manuscript. The work was conceptualized by Dr. Giorgos Livanas and Prof. Georgios Varelogiannis.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request. Data can be reproduced using Equation 5 of the manuscript.]

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