Can we avoid dark energy?

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The idea that we live near the centre of a large, nonlinear void has attracted attention recently as an alternative to dark energy or modified gravity. We show that an appropriate void profile can fit both the latest cosmic microwave background and supernova data. However, this requires either a fine-tuned primordial spectrum or a Hubble rate so low as to rule these models out. We also show that measurements of the radial baryon acoustic scale can provide very strong constraints. Our results present a serious challenge to void models of acceleration.

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Introduction.—The last decade has seen the solidification of the Standard Model of Cosmology (SMC; see, e.g., [1]), which contains about 75% dark energy driving the acceleration of a flat, homogeneous and isotropic Friedmann-Lemaitre-Robertson-Walker (FLRW) background. A very broad range of observations are consistent with the SMC, including cosmic microwave background (CMB) (see, e.g., [2]), Type Ia supernovae (SNe Ia) (e.g. [3]), baryon acoustic oscillations (BAO) (e.g. [4]), integrated Sachs-Wolfe (ISW) effect correlations (e.g. [5]), weak lensing studies (e.g. [6]), etc.

Given this impressive convergence of observations, it would be very surprising if the data still fit an alternative model, which lacked two of the main planks of the SMC. Yet just such a radical proposal has attracted considerable attention recently. The idea is to drop the dark energy and the Copernican principle, and instead suppose that we are near the centre of a large, nonlinearly underdense, nearly spherical void surrounded by a flat, matter dominated Einstein-de Sitter (EdS) spacetime ([7]; see [8] for a brief review). By tuning the radial void profile, it is possible to match the luminosity distance-redshift relation of the SMC [9], so if these models are to be ruled out, we need more than just the SN data. Recently, constraints from the CMB acoustic scale, BAO scale, and Hubble rate have been placed on void models, although, remarkably, they have not yet been ruled out [10, 11].

We must stress that void models contain two elements which appear extremely unlikely within the standard cosmological framework, and for which no viable explanations have been proposed. Voids of the size required to fit the SN data (hundreds of Mpc to Gpc scales) correspond to fluctuations of very many σ in standard structure formation scenarios [12]. Also, we must be very close to the void centre to avoid a large CMB dipole [13].

However, as overwhelming as these difficulties may appear, they are essentially philosophical in nature, and so it would be important to rule these models out on the basis of observations. With this goal in mind we confront void models with several sets of current data, providing three main advances over previous studies. Firstly, we allow for a wide range of void profiles, employing a spline parameterization. Secondly, we carefully calculate the CMB anisotropy spectrum that a void observer would see and confront the full spectrum with recent data sets. Finally, we show that the radial BAO scale is a powerful discriminator between void and standard models.

Specifying the void model.—We model the contents of the Universe at late times as pressureless matter, with density ρ. Observations are consistent with isotropy, so we place the observer at the centre of spherical symmetry. The exact solution to Einstein’s equations in this case is known as the Lemaitre-Tolman-Bondi (LTB) spacetime [14]. It is described by two free radial functions, which correspond to the growing and decaying modes in the limit of small perturbations about FLRW [15, 16]. As stressed in [16], it is crucial to consider only voids with vanishing decaying mode (i.e. uniform “bang time”), if we are to be able to specify the initial conditions (ICs) for perturbations at early times. This is because in this case the void itself will be a small perturbation from FLRW at early times, and standard inflationary ICs can be specified on top of the void. Since our analysis will include the BAO perturbation scale (which is set before recombination) evaluated inside the void, we must assume vanishing decaying mode in our work. Thus only one radial function is required to specify the void model.

The LTB spacetime can be described by the metric

\[ ds^2 = -dt^2 + \frac{Y''}{1 - Y'} dr^2 + Y^2 d\Omega^2. \]  

(1)

Here Y(t, r) and K(r) are determined by the exact LTB solution (see, e.g., [16]) once the single free radial function is specified, and Y' \equiv \partial Y/\partial r, where r is a comoving radial coordinate. Errors due to ignoring radiation are inevitable with the LTB solution, but we estimate our results are accurate to the percent level or better [17].

We chose to define the radial profile in terms of the comoving density perturbation, δρ(t, r) = ρ(t, r)/ρ(t), specified at the early time \( t_i \). To be confident that no important regions of “profile space” are missed, we fit a three-point cubic spline to the initial density fluctuation \( \delta \rho_j \equiv \delta \rho(t_i, r_j) \), where \( j = 1, 2, 3 \). We fix \( r_1 = 0 \) and enforce the void to be smooth at the origin and to smoothly match
to EdS at large $r$ by setting $\delta \rho'(r_1) = \delta \rho'(r_3) = \delta \rho_3 = 0$. This leaves a total of four free parameters: the density at the origin, $\delta \rho_1$, the density and radius at the midpoint, $\delta \rho_2, r_2$, and the radius $r_3$ at which we match to EdS. We have checked that additional spline points do not significantly improve the fit to current data. We consider two classes of profiles: “constrained,” for which we impose $\int \delta \rho(t_i)r^2dr \leq 0$, and “unconstrained,” which are free.

A void model is completely specified by the radial profile, the Hubble rate at the void centre today, $H_0$, the radiation density today, which is fixed by the CMB mean temperature, $T_0 = 2.725$ K [18], and the baryon fraction $f_b \equiv \rho_b/\rho_m$. Outside the void we asymptote to EdS.

In Fig. 1, we plot several void profiles on the observer’s past light cone, in terms of the local density parameter $\Omega_{loc}^m = 24\pi G\rho/\theta^2$, where $\theta$ is the comoving expansion parameter (this definition reduces to the standard density parameter in the FLRW case). The profiles are sampled from a Markov-Chain-Monte-Carlo (MCMC) process, fitting to SN+CMB data as described below. In these plots, values $\Omega_{loc}^m < 1$ correspond to the central void, while $\Omega_{loc}^m > 1$ indicates an overdense shell region. The constrained voids tend to be essentially “compensated,” while the unconstrained tend to be strongly “overcompensated.”

![Image of void profile](image_url)

**FIG. 1:** Local density parameter $\Omega_{loc}^m$ versus redshift $z$ for constrained (left) and unconstrained (right) voids. The profiles are sampled from MCMC chains and the grayscale level indicates the relative likelihood.

**Cosmic microwave background.** —The CMB anisotropy power spectrum, $C_\ell$, contains much information, and so can potentially provide strong constraints on void models. Two main factors determine the detailed shape of the $C_\ell$ spectrum: the primordial perturbation spectrum, which, in the simplest models of inflation, is close to scale invariant and essentially featureless; and the local physics during recombination, which is determined by the composition of the Universe at that time. The angular scale at which features of fixed physical scale appear is determined by the physical circumference of the observer’s last scattering surface (LSS), which is affected by the geometry of the Universe and the time of observation [19].

Once the void model has been specified, it is straightforward to calculate the $C_\ell$ spectrum at all but the largest scales. The anisotropies generated at the LSS cannot be affected by the void for an on-centre observer, because of the isotropy of the LTB background [20]. Our procedure to calculate the $C_\ell$’s is simply to find the parameters for an effective EdS model with the same physics at recombination and LSS physical circumference as the void model. Then those effective parameters can be fed into public CMB anisotropy codes—we used CAMB [21].

Explicitly, we numerically integrate along the past light cone from the void centre today ($r = z = 0$) to a redshift $z_m$ to find the corresponding coordinates $(r_m, t_m) = ((t(z_m), r(z_m))$ using the exact LTB relations

$$\frac{dr}{dz} = \sqrt{1 - K/(1 + z)Y'}, \quad \frac{dt}{dz} = -Y'/Y.$$  \hspace{1cm} (2)

We choose $z_m$ large enough so that $r_m$ is sufficiently far outside the void that spatial curvature (and shear) is negligible there, but not so large that radiation is important at $t_m$ at background level. In practice, values $z_m \approx 100$ meet these criteria. We also evaluate the Hubble rate, $H_m$, at $z_m$ using the exact LTB solution. Finally, we set the spatial curvature to precisely zero, and integrate back up the light cone into EdS to comoving coordinate $r_{EdS} = 0$, using the FLRW relation $dz_{EdS}/dr_{EdS} = H_{EdS}(z)$. The correct LSS circumference is ensured by using the relation $Y(t_m, r_m) = a(t_{EdS})r_{EdS}$ for FLRW scale factor $a$, to set the IC for this integration.

The result of integration, $r_{EdS}^{\text{EdS}}$, allows us to calculate the effective EdS mean temperature and Hubble rate via

$$T_0^{\text{EdS}} = T_0 (1 + z_{m}^{\text{EdS}}), \quad H_0^{\text{EdS}} = H_m \left(1 + z_{m}^{\text{EdS}}ight)^{3/2}. \hspace{1cm} (3)$$

The parameters $T_0^{\text{EdS}}, H_0^{\text{EdS}}, \Omega_m = 1, \Omega_A = \Omega_K = 0, and f_b$ are then fed into CAMB to calculate the $C_\ell$ spectrum which would be observed in the specified void. Note that the effective parameters $T_0^{\text{EdS}}$ and $H_0^{\text{EdS}}$ will generally differ from the true temperature and Hubble rate observed at the void centre today, $T_0 = 2.725$ K and $H_0$.

Importantly, we find that the effective set $T_0^{\text{EdS}} = 3.372$ K, $H_0^{\text{EdS}} = 51.0$ km s$^{-1}$ Mpc$^{-1}$, and $f_b = 0.165$ produce a $C_\ell$ spectrum that matches that of the Wilkinson Microwave Anisotropy Probe (WMAP) best-fit Λ model [2] at all but the largest scales. A void model must have effective parameters close to these if it is to fit the CMB.

**Data fitting and results.** —We used COSMOMC [22] to create MCMC chains to estimate confidence limits on the parameters. Along with the four void spline quantities, the basic cosmological parameters used in the fit are $f_b, H_0$, and the amplitude $A_s$ and spectral index $n_s$ of adiabatic initial perturbations [20].

SNe Ia provide important evidence for acceleration within the standard FLRW framework, and hence we must ensure that our voids fit these observations. We used the recent Union compilation [3], consisting of 307 SNe with $z = 0.015–1.55$ [23]. We also used CMB data, namely the 5-year WMAP results [2] and four experiments at higher resolution: ACBAR [24], Boomerang
[25], CBI [26], and QUaD [27]. We also applied a conservative prior that $\Omega_{m}^{\text{loc}}>0.1$ at the void centre.

In Fig. 2 we show a selection of 2D likelihood surfaces for the constrained and unconstrained void models. The parameters are very different in each case. For constrained voids, the effective temperature, $T_{0}^{\text{EdS}}=2.760\pm0.008$ K, is similar to $T_{0}$. This $T_{0}^{\text{EdS}}$ is far too low to provide a good fit to the observed CMB spectrum—we find $\Delta\chi^{2}=162$ between the constrained void and $\Lambda$ model for CMB+SN data, with almost all of this difference coming from the CMB. This poor fit also leads to very low $f_{b}=0.100\pm0.001$ and $n_{s}=0.88\pm0.01$.

For the unconstrained voids, however, much higher effective temperatures are possible, due to the geometrical effect of the overdense shell. These temperatures are sufficiently high to fit the CMB well—the fit to CMB+SN is actually slightly better than $\Lambda$, with $\Delta\chi^{2}=-1.4$. However, this requires an unusually low local Hubble rate of $H_{0}=44\pm2$ km s$^{-1}$ Mpc$^{-1}$, as Fig. 2 shows. Recent local estimates range between 57 and 79 km s$^{-1}$ Mpc$^{-1}$ at 1$\sigma$ [28], and so this class of void model is ruled out at high confidence.

In Fig. 3 we present the residuals of the void $C_{l}$'s from the best-fit $\Lambda$ model. The best-fit void model is shown by the black curve, along with 100 other spectra sampled randomly from the MCMC chains. The grayscale level indicates the relative likelihoods. It is clear that in the constrained case one cannot fit the CMB data without introducing fine-tuned features to the primordial spectrum, since the physics at recombination is wrong.

**Baryon acoustic scale.**—The physics before recombination imprints a fixed comoving scale into the matter power spectrum in the form of the BAO scale, carrying much useful information about the geometry and expansion history assuming an FLRW background (see, e.g., [29]). It is therefore important to assess the usefulness of BAO data in constraining void models for acceleration.

The first step is to evaluate the physical BAO scale, $r_{BAO}^{l}$ (also called the sound horizon at the drag epoch), at some time $t_{i}$ early enough that the void background is well approximated by FLRW. To do this, we find an effective EdS model with the same physics at early times as the specified void model using the same procedure used to calculate the void $C_{l}$’s, except that it is not necessary here to match the effective and true LSS circumferences. Then we calculate $l_{BAO}$ using the fitting function from [30] applied to the effective EdS parameters. Next, $l_{BAO}$ is propagated up to redshift $z$ on the void observer’s past light cone using the LTB metric. The background shear causes the physical BAO scales in the transverse and radial directions to differ; they are given, respectively, by

$$l_{\perp} = \frac{r_{BAO}^{l}(z)}{Y(t, r(z))}, \quad l_{\parallel} = \frac{r_{BAO}^{l}(z)}{Y(t, r(z))}. \quad (4)$$

Here $Y(z)=Y(t(z), r(z))$, and similarly for $Y'(z)$. BAO observations are often expressed as a BAO length scale today, but such values necessarily depend on the assumed background. Instead, a model-independent expression of the transverse and radial BAO (RBAO) scales is in terms of the corresponding angular and redshift increments,

$$\Delta\theta_{BAO}^{l} = \frac{r_{BAO}^{l}(z)}{Y(z)}, \quad \Delta z_{BAO}^{l} = \frac{r_{BAO}^{l}(z)}{Y(z)}. \quad (5)$$

Reference [31] emphasized the importance of distinguishing radial and angular scales in this context.

Importantly, $\Delta z_{BAO}^{l}$ contains two factors that reinforce each other in the peripheral void region. The quantity $Y'(z)/Y(z)$ is the expansion rate in the radial direction, which is suppressed in the overdense periphery. This, in turn, results in a suppressed RBAO scale, $l_{\parallel}^{BAO}(z)$, in the periphery. The net effect is a strong suppression of $\Delta z_{BAO}^{l}$ in this region compared with the stan-
standard Λ case, as illustrated in Fig. 4 for a set of profiles from the same MCMC chain as we obtained above [32].

![Figure 4: RBAO scale \( \Delta z_{\text{BAO}} \) for constrained (left) and unconstrained (right) voids. Also shown are the RBAO data from [33] (error bars), best-fit Λ model (blue, dashed), and best-fit void including RBAO constraints (green, dot-dashed).](image)

Recently, RBAO measurements have been presented [33] for the observable \( \Delta z_{\text{BAO}} \) rather than the model-dependent \( \ell_{\text{BAO}} \). Figure 4 shows that the new data are in excellent agreement with standard Λ, but strongly at odds with the likeliest void models. Including the new RBAO data points worsens our best fits by \( \Delta \chi^2 = 16 \) (constrained profiles) and 9.8 (unconstrained). Such large \( \Delta \chi^2 \) values for only two extra data points show that the new RBAO technique is already a considerable obstacle to void models.

**Discussion.**—We have concentrated here on constraints from SN, CMB, and RBAO data. In future work [17] we will also consider the independent constraints from the amplitude of matter fluctuations [16]. These techniques all rely on observations confined to our past light cone. Very promising are measurements that sample the interior of the light cone, via spectral distortions [34] or the kinematic Sunyaev-Zel’dovich effect [35] (see also [36] for a distinct approach).

Nevertheless, we have already shown here that an appropriate overcompensated void profile can fit both the SN and CMB data, but at the expense of an \( H_0 \) so low that it can be ruled out. Constrained voids provide a better \( H_0 \), but a very poor fit to the CMB without fine tuning of the primordial spectrum. We stress the significance of this result: it is an extraordinary achievement of the SMC that it predicts the detailed shape of the \( C_\ell \) spectrum using only a few parameters. Losing this predictive power and requiring a fine-tuned primordial spectrum is a severe price to pay for the allure of \( \Lambda = 0 \).

We also showed that early RBAO results already impose very strong constraints on void models. In particular, RBAO poses a serious challenge to the constrained profiles which is free of any subjectivity that some may argue is inherent in the above fine-tuning argument. Future BAO surveys such as PAU [37] and BOSS [38] will provide improved precision out to greater redshifts, placing unprecedented constraints on inhomogeneity. In this era of precision cosmology we can in fact begin to test the Copernican principle.

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