1. INTRODUCTION

Since the discovery of 51 Pegasi b (Mayor & Queloz 1995), more than 10⁵ planets orbiting stars other than the Sun have been detected and confirmed. Exoplanet observations implicate that nearly 15%–20% of solar-type stars harbor at least one gas giant planet with mass (\(M_p\)) comparable to or larger than that of Saturn (Marcy et al. 2008; Cumming et al. 2008). Despite the observational bias against the detection of additional gas giant planets, it is known that nearly 15%–20% of solar-type stars harbor at least one gas giant planet with mass (\(M_p\)) in the range of a few Earth masses (\(M_⊕\)) that migrate toward a transition radius between the inner viscously heated and outer irradiated regions of their natal disk. Their limiting isolation mass increases with the planetesimal surface density. When \(M_p > 10 M_⊕\), embryos efficiently accrete gas and evolve into cores of gas giants. We use a numerical simulation to show that despite stream line interference, convergent embryos essentially retain the strength of non-interacting embryos’ Lindblad and corotation torques by their natal disks. In disks with modest surface density (or equivalently accretion rates), embryos capture each other in their mutual mean motion resonances and form a convoy of super-Earths. In more massive disks, they could overcome these resonant barriers to undergo repeated close encounters, including cohesive collisions that enable the formation of massive cores.

Key words: planet–disk interactions – planets and satellites: formation

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The magnitude of \(M_{iso} \propto \Sigma_p^{3/2}\), where \(\Sigma_p\) is the surface density of the building block planetesimals. The growth barrier can be bypassed with either an initial \(\Sigma_p\) a few times larger than that of the MMN model or new supplies of grains, planetesimals, or embryos to replenish the feeding zones. Potential transport mechanisms include hydrodynamic drag of grains (Weidenschilling 2003) and vortice-trapping of pebbles (Johansen & Lacerda 2010). In this paper, we focus our discussion on the type I migration of embryos as the dominant mechanism to redistribute embryos’ building block materials (Alibert et al. 2005).

Embryos excite density waves in the disk, which carries flux of angular momentum (Goldreich & Tremaine 1982). When these waves are dissipated, they induce both differential Lindblad and corotation torques. Linear calculations indicate that although Earth-mass embryos do not significantly modify the disk structure, their angular momentum exchange with the disk can lead to rapid type I migration (Ward 1997; Tanaka et al. 2002; Kley & Nelson 2012; Baruteau et al. 2013). At 5 AU in an MMN (where Jupiter resides today), differential Lindblad torque alone leads to the inward migration of critical mass cores over a timescale of \(t_{mig} < t_{dep}\). Such a rapid migration rate would not only reduce the retention efficiency of cores, but would also suppress the formation of gas giants at a few AU (Papaloizou & Nelson 2005b).

Several mechanisms to reduce the rate of type I migration have been suggested. They include stochastic migration in turbulent disks (Papaloizou & Nelson 2005a), “planet-trapping” by the disk region with a positive surface density gradient (Masset et al. 2006; Morbidelli et al. 2008) or by vortices generated from Rossby wave instabilities (Koller et al. 2003; Li et al. 2009), and stalling migration at the inner disk edge (Terquem & Papaloizou 2007; Pierens & Nelson 2008).
The most promising mechanism to quench type I migration is contribution by a corotation torque due to an embryo’s interaction with nearby gas, which follows a horseshoe pattern in its corotating frame. The magnitude and sign of the corotation torque are determined by either the gas surface density ($\Sigma_g$) or entropy ($S_g$) gradient in the horseshoe region (Paardekooper & Mellema 2006; Kley & Crida 2008). In the viscously heated inner and irradiated outer unperturbed regions of the disk, embryos’ corotation torque is more intense than their differential Lindblad torque and has, respectively, positive and negative signs with the potential to induce their outward/inward migration (Kretke & Lin 2012).

However, relatively massive embryos significantly perturb the disk stream lines, reduce their advective transport of angular momentum, and trap them within the horseshoe region (with a width $\Delta a_{\text{hr}}$). On a liberation timescale ($\tau_{\text{lib}}$), mixing of the gas reduces the $\Sigma_g$ or $S_g$ gradients and weakens (saturates) the corotation torque (Balmforth & Korycansky 2001). Nevertheless, turbulence in the disk also induces an intrinsic outward transport of angular momentum, which enables gas to (1) flow through the horseshoe region of embryos with modest masses, (2) preserve the $\Sigma_g$ and $S_g$ distribution, and (3) retain the corotation torque (Paardekooper et al. 2010, 2011; Baruteau & Lin 2010). Around low-mass embryos, the timescale for the disk gas to diffuse through $\Delta a_{\text{hr}}$ is much smaller than $\tau_{\text{lib}}$, such that a large fraction of the gas diffuses through their horseshoe regions without being perturbed by their gravity. In the high-mass limit, the vortensity and entropy related part of corotation torque is saturated. In the low-mass limit, the magnitude of corotation torque approaches the linear corotation torque, which is below without being perturbed by their gravity. In the high-mass limit, the vortensity and entropy related part of corotation torque is saturated. In the low-mass limit, the magnitude of corotation torque approaches the linear corotation torque, which is below that of differential Lindblad torque. The embryos would undergo inward migration throughout the disk.

Within a certain mass range (approximately a few $M_\oplus$), embryos undergo convergent type I migration toward transitional radii ($r_{\text{trans}}$) where the net (Lindblad plus corotation) torque changes sign with a negative radial gradient. It has been suggested this process may lead to the accumulation of building block material and enhance the growth of embryos (Nelson 2005; Lyra et al. 2010; Paardekooper et al. 2011; Horn et al. 2012; Hellary & Nelson 2012). Pierens et al. (2013) investigated the effects of initial number of embryos and the stochastic force on the frequency of gas giant formation. They suggested that the resonant chain can be broken by increasing the initial number of embryos or by including a moderate stochastic force due to the disk turbulence. They also discussed briefly about the dependency of zero-torque radius on the protoplanetary disk mass, but the effects of different disk mass on convergent migration of embryos were not discussed. In this paper, we emphasize on this potential mechanism and explore conditions under which the critical mass cores ($\sim 10 M_\oplus$) may form and be retained during the main course of disk evolution. After we submitted our paper, a paper by Coleman & Nelson (2014) was posted on arxiv.org. They investigated the correlation between initial disk mass and the formation and survival of gas giants and pointed out that in order to form and retain gas giants, it is necessary for planetary cores to accrete gas and open gaps at large radii and they must do it in a sufficiently late epoch to prevent migration into the central star. However, during this advanced stage, much of the initial disk mass is depleted from its initial values so that the critical condition for cores retention by their natal disks is not yet explicitly determined. Although we obtain some similar results on the condition for multiple embryos to overcome resonant barriers (as found by Coleman & Nelson 2014), our consideration of more general boundary conditions in this and subsequent papers bypass some retention issues associated with their corotation saturation. Following previous work, we use in the paper a two-dimensional (2D) Fast Advection in Rotating Gaseous Objects (FARGO) code to carry out a hydrodynamic simulation of tidal interaction between multiple embryos and their natal disks. In Section 2, we briefly recapitulate the numerical method and model parameters.

Following the detailed analysis by Paardekooper et al. (2010, 2011), we verify in Section 3 that in disks with composite $\Sigma_g$ distribution, isolated embryos undergo the type I migration to $r_{\text{trans}}$. These simulations are extended to multiple embryos in Section 4. Since contribution from corotation torque determines the direction of type I migration, we introduce an idealized model to examine whether it may be affected by overlapping horseshoe regions between two nearby embryos. In this model, we neglect mutual gravitational interaction between embryos and trace diffusion across the corotation zone with passive contaminants.

As they approach each other, embryos perturb each other through secular and resonant interaction. These effects are restored in the models presented in Section 5. We show that in disks with modest $\Sigma_g$, convergent embryos are trapped into their mutual mean motion resonance. However, embryos’ rate of type I migration increases with $\Sigma_g$ and in relatively massive disks, they bypass the resonant barrier.

After their orbits cross, embryos undergo close encounters. We show in Section 5 that although some embryos scatter each other to locations outside the corotation zone, they resume their convergent type I migration. These embryos are entrained near $r_{\text{trans}}$ by persistent type I migration and undergo repeated close encounters until they collide with each other. In the 2D simulations, physical collisions occur within a few hundred orbital periods. However, if the embryos have a nearly isotropic velocity dispersion rather than mono-layer distribution, their collision timescale is two to three orders of magnitude longer, i.e., a significant fraction of the (megayear) disk evolution timescale. It is impractical to carry out a high-resolution hydrodynamic simulation of embryos’ close interaction during various stages of disk evolution. In a subsequent paper, we will utilize a prescription for embryo–disk interaction to construct a Hermit-embryo scheme. This prescription was constructed by Paardekooper et al. (2010, 2011) based on the results of a comprehensive series of hydrodynamic simulations of the embryo–disk interaction. This approximation is justified under the assumption that embryos do not strongly modify the disk’s intrinsic structure. Based on the results here, we assume this prescription remains valid for individual embryos in multiple systems. Finally, in Section 6, we summarize our results and discuss their implications.

## 2. HYDRODYNAMIC SIMULATION OF EMBRYO–DISK INTERACTION

Here, we briefly recapitulate the numerical method and model parameters.

Following previous investigations, we utilize a publicly available FARGO (Masset 2000) scheme to simulate the interaction between multiple embryos with their natal disks. FARGO is a 2D hydrodynamical polar grid code centered on the star, based on the van Leer upwind algorithm on a staggered mesh. It solves the Navier–Stokes and continuity equations for a Keplerian disk subject to the gravity of the central object and that
of embedded protoplanets as well as the energy equation in its more recent version (Baruteau & Masset 2008). The energy equation implemented in FARGO is

$$\frac{\partial e}{\partial t} + \nabla \cdot (e \mathbf{v}) = -P \nabla \cdot \mathbf{v} + Q_+ - Q_-$,$$

(1)

where $e$ is the thermal energy density (thermal energy per unit area), $\mathbf{v}$ is the flow velocity, $P$ is the vertically integrated pressure, and $Q_+ (Q_-)$ denote heating (cooling) source terms, assumed to be positive quantities. The cooling source is defined by a cooling time, and the disk will go back to the initial energy within the given cooling time. Here we choose the cooling time to be about five orbital periods of the planet at $r = 1$.

There are several boundary conditions available in the FARGO code. Here we choose the EVANESCENT boundary condition that is described in de Val-Borro et al. (2006), used for the EU test comparison problem. It aims at implementing wave killing zones at each edge of the grid. It allows the disk values (surface density, velocities, and thermal energy density) to damp toward the instantaneous axisymmetric disk conditions. The damping regions are located in the radial ranges $[r_{\text{min}}, 1.25r_{\text{min}}]$ and $[0.8r_{\text{max}}, r_{\text{max}}]$, where $r_{\text{min}}$/$r_{\text{max}}$ denotes the inner (outer) edge radius of the grid.

The basic equations are solved in a cylindrical coordinates from $r = 0.3$ to $r = 1.7$ and full $2\pi$ in azimuth. The typical resolution is $\delta r/r_p \sim 0.004$ and $\delta \phi = 0.01$, which gives about five grids in the radial direction within the Hill’s radius of a $10M_\odot$ planet.

For illustration, we adopt a simple $\alpha$ disk model with an effective viscosity. We assume power-law distributions:

$$\frac{\partial \ln \Sigma}{\partial \ln r} = p, \quad \frac{\partial \ln T}{\partial \ln r} = q, \quad \text{and} \quad \frac{\partial \ln \alpha}{\partial \ln r} = \xi.$$

(2)

In a steady state, the accretion rate $M = 3\pi \Sigma_0 \alpha C_s^2 \Omega$ is independent of $r$ so that $p + q + \xi = -1.5$.

We adopt a disk model based on the assumption that the inner region of the disk is heated by viscous dissipation, whereas the outer region is heated by stellar luminosity (Garaud & Lin 2007). We set the transit region at $r_t$, the disk inside $r_t$ has $p = 0.0$ and $q = -1.5$ and outside has $p = -0.5$ and $q = 1.0$. In the simulations, these model parameters follow a continuous transition at $r_t$. This composite power law $\Sigma_0$ and $T$ distribution model is an extension of the single power-law simulations pioneered by Paardekooper et al. (2010, 2011).

We simulate the planetary migration in disks of both high and low accretion rates to investigate the dependency of orbital structure on the convergent migration rate. According to the disk model structured by Garaud & Lin (2007), $h/r \propto M^{0.25}$ and $r_t \propto M^{0.72}$. For normalization, we specify $h/r = 0.079$ at $r_t$ for a disk with accretion rate as high as $10^{-7} M_\odot$ yr$^{-1}$, and $h/r = 0.05$ at $r_t$ for $1.2 \times 10^{-9} M_\odot$ yr$^{-1}$ (model 5). To reduce the simulation time for planetary migration in low $M$ disks, we set $r_t = 7.1$ AU for all disk models. The viscosity $\alpha$ is 0.001 as constant and the smoothing length is $b/h = 0.4$.

We first simulate the migration of a single isolated planet in a disk of composite $\Sigma_0$ or $S_\phi$ distributions to confirm that the planets would undergo convergent migration in this kind of disk model (models 1a–1f). We then compare the results with the migration of a single planet in a disk disturbed by other planets (model 1g). At last, we simulate the migration of multiple planet systems in disks with different accretion rates (models 5 and 6). In all models, the planet has initial circular orbit. The model parameters are listed in Table 1.

### Table 1

| Model | $N_p$ | $M_p$ ($M_\odot$) | $M$ ($M_\odot$ yr$^{-1}$) | Feel Others | Feel Disk |
|-------|-------|-------------------|-----------------------------|-------------|-----------|
| 1a-1c | 1     | 10                | 1e-7                        | No          | Yes       |
| 1d    | 1     | 20                | 1e-7                        | No          | Yes       |
| 1e    | 1     | 30                | 1e-7                        | No          | Yes       |
| 1f    | 1     | 40                | 1e-7                        | No          | Yes       |
| 1g    | 3     | 10                | 1e-7                        | No          | Yes       |
| 5     | 4     | 10                | 1.2e-8                      | Yes         | Yes       |
| 6a    | 4     | 10                | 1e-7                        | Yes         | Yes       |
| 6b    | 4     | 10                | 2e-7                        | Yes         | Yes       |

### 3. MIGRATION OF SINGLE ISOLATED PLANETS

The planet with $M_p = 10 M_\odot$ migrates outward when released inside the transit location (model 1a) and it migrates inward when starting outside (model 1c; Figure 1). The sign and magnitude of the initial torque agree well with that obtained for a planet in a disk with a single power-law $\Sigma$ distribution. In both cases, the migration is slowed down and stalled when the planet approaches $r_t$. Although the strength of one-side torque is preserved, the net Lindblad torque is suppressed by the cancelation from the two regions of the disk. The reduction of the differential Lindblad torque enables it to balance the corotation torque. These models indicate that in disks with a bimodel ($p, q$) distribution, the net torque can indeed be summarized as a linear combination of that from two separate regions of the disk.

In models 1d–1f, we place a single isolated planet at initial location $r = 0.8$ with $M_p = 20 M_\odot, 30 M_\odot$ and $40 M_\odot$, respectively. The planet migrated outward with stalling at $r_t$ in model 1d and migrated inward in models 1e and 1f (Figure 1). For this $M_p$, perturbation on $\Sigma_0$ remains relatively small. These results are in general agreement with those obtained by Paardekooper et al. (2010, 2011) and they indicate that the corotation torque of massive planets is saturated (weakened) by the suppression of diffusion of disk gas across the horseshoe stream lines.

### 4. MIGRATION OF MULTIPLE PLANETS

When two or more planets converge near $r_t$, the location of their Lindblad resonances and horseshoe regions may overlap with each other. They also directly interact with each other. We assess the relative contribution from different effects with idealized simulations of the concurrent evolution of multiple planets.

We first examine the extent of resonant interference by releasing three planets at the same location as the previous single-planet models. In an artificial, idealized model, we neglect the mutual interaction between the planets (model 1g). Initially, the disk response is a linear combination perturbation of three widely separated planets. These planets independently evolve along paths similar to those of the analogous individual planets (Figure 2). As they approach $r_t$, their combined perturbation amplitude on the disk is locally enhanced, albeit changes in $\Sigma_0$ remain small and linear (Figure 2).

In order to further investigate the interference of planets on each other’s migration, we introduce a tracer (as a passive contaminant) to highlight the diffusion of fluid elements around the planets when they are very close to each other. For comparison, the fluid elements’ tracer is examined under four situations: (model 0) an unperturbed disk (without any embedded planet), (model 2a) a disk that bears a single planet with a fixed position...
Figure 1. Left: the three solid lines indicate the semi-axis evolution of a 10 $M_\oplus$ planet released from $r = 0.8$, $r = 1.0$, and $r = 1.2$, respectively (models 1a–1c). The three dot–dashed lines indicate the semi-axis evolution of a planet from $r = 0.8$ with masses of 20, 30, and 40 $M_\oplus$, respectively (models 1d–1f). The dashed line indicates the location of $r_\text{c}$. Right: surface density profile scaled with unperturbed $\Sigma(r_\text{c})$. The dashed line indicates the profile of the unperturbed disk. The solid and three dot–dashed curves, with values at $r_\text{c}$ from highest to lowest, indicate that disturbed by a planet with masses of 10, 20, 30, and 40 $M_\oplus$, respectively (models 1a, 1d–1f) after 200 orbits.

Figure 2. Left: the semi-axis evolution of three planets with equal masses of 10 $M_\oplus$ (model 1g). The planets are released from $r = 0.8$ (dark blue solid line), $r = 1.0$ (red solid line), and $r = 1.2$ (light blue solid line). During their migrations, planets’ direct gravitational perturbations on each other are neglected. However, the disk is perturbed by all three planets. The dot–dashed color lines represent the migration with single planets from previous comparisons (models 1a–1c). The dashed black line indicates the location of $r_\text{c}$. Right: the dashed solid and black lines represent the surface density profile at $t = 0$ and $t = 2000$ orbits in the case of multiple planets’ migrations, respectively. The color lines represent the surface density profile at $t = 2000$ orbits of the previous three separate cases of isolated planets’ migrations. (A color version of this figure is available in the online journal.)

at $R = 1.0$, (model 3a) disk with two planets fixed at $R = 1.0$ and $R = 1.1$, and (model 4a) disk with two planets at $R = 1.0$ and $R = 0.9$. Figure 3 shows the diffusion pattern of tracer elements under these four cases with $M_p = 10 M_\oplus$. Models 1a–4a are presented from top to the bottom rows. The tracer distribution at times 0, 400, and 800 orbital periods are plotted in left to right columns.

Comparisons between models 0 and 2a highlight the horse-shoe stream lines near the corotation region around an embedded planet. The width of the corotation zone around the
The corotation region around a 2 $M_\oplus$ planet (model 2c) is ~0.02. The corotation regions around the coexisting planets in models 3c and 4c are well separated. There is no evidence of horseshoe stream line interference to enhance the diffusion of tracer elements.

In contrast, model 2b shows that around a 20 $M_\oplus$ planet, the width of the corotation region is ~0.07 such that two planets with 0.1 separation have overlapping corotation regions. The horseshoe stream lines around two planets are clearly intertwined in models 3b and 4b. Gravitational perturbation from
the exterior planet enhances outward diffusion, whereas that from the interior planet promotes more rapid inward migration.

In a previous analysis, Paardekooper et al. (2010, 2011) showed that the preservation of the $\Sigma_g$ profile near the embedded planets and the maintenance of their corotation torque require gas diffusion through the corotation region. For relatively large planets, the diffusion is quenched by the horseshoe stream lines and saturates the corotation torque. The elevated diffusion of tracer elements in disks with closely packed multiple planets (models 2c and 2d) limits the modification of the unperturbed $\Sigma_g$ distribution and unsaturates the corotation torque in comparison with that between the disk and isolated planets (model 2b).

Our simulated models suggest that (1) the migration direction, pace, and the net torque of individual planets are not affected by other planets in well-separated multiple systems and (2) closely packed multiple planets also retain their unsaturated corotation torque. Thus, resonant interference enhances rather than suppresses convergent migration. Based on these results, in subsequent papers of this series, we approximate the torque exerted by individual planets in multiple systems with the prescription obtained from simulations with single planets (Paardekooper et al. 2010, 2011).

5. GRAVITATIONAL INTERACTION AND MERGER THROUGH PHYSICAL COLLISIONS

We now incorporate the gravitational interaction between planets. We place four equal-mass (10 $M_\oplus$) planets into the disk (with the same $p, q$ distribution as previous models) at $r = 0.7, 0.9, 1.1, 1.3$, respectively.

In model 5, we specify a low-mass disk with $\dot{M} = 1.2 \times 10^{-8} M_\odot$ yr$^{-1}$. In this case, the embryos rapidly capture each other on their 6:5 or 5:4 mean motion resonances (Figure 5). This resonant configuration is maintained with a fluctuating eccentricity $e \sim R_R/a \sim 0.02$ while the convoy’s inward migration slows down and comes to a halt. In this compact configuration, the periastrons of some embryos cross the apoapses of other embryos that are slightly closer to their host stars. After 1300 orbits, two middle embryos’ orbits overlapped and they exchange their orbits to re-establish the resonant configuration. In general, the embryos preserve their integrity and may undergo further migration together as the disk is depleted. We suggest that these convoys are the progenitors of multiple super-Earths.

We also introduce models 6a and 6b with similar embryo masses and initial locations as model 5. However, the gas accretion in the disk is specified to be $10^{-7}$ and $2 \times 10^{-7} M_\odot$ yr$^{-1}$, respectively. Similar to the results of model 5 and our previous simulation (Zhang et al. 2014), embryos in model 6a first capture each other into lower-order mean motion resonances. During their subsequent collective migration, their separation is reduced to about half of their initial spacing. Embryos’ $e$ fluctuates mostly with an average amplitude $<0.02$, as it is excited by the embryos’ resonant interaction with each other and damped by the tidal torque between them and the gas. Since their separation is $\sim 0.1 r_p$, their orbits do not generally cross each other. However, on some occasions (e.g., at $\sim 300$ orbits), pairs of closest embryos may become dynamically unstable and undergo orbit crossing. Close encounters lead to the exchange of semimajor axes and eccentricity excitation ($\sim R_R/a$ up to $\sim 0.05$; Figure 6). After this brief episode of close encounters and intense interaction, embryo eccentricities are quickly damped by the tidally induced gas drag from the disk gas.
A convoy of embryos settles near but not precisely around \( r_t \). This slight asymmetry is due to the embryos’ torque balance between the inner and outer regions of the disk. After \( \sim 700 \) orbits, the two innermost embryos again cross each other’s orbits and exchange semimajor axes. Models 5 and 6a indicate the possibility of repeated orbit crossing. Since embryos would undergo orbit crossing when their separation is smaller than the width of feeding zone (Zhou et al. 2007), under some circumstances, the embedded embryos may converge into regions where they undergo repeated close encounters. Some encounters may be sufficiently close that the participating planets physically collide and merge with each other. In this series of papers, we will explore the possibility that these mergers may attain critical mass for the onset of efficient gas accretion. In order to simulate this possibility with FARGO, we remove the gravitational softening parameter in the calculation of the force between the participating embryos and assume they would merge with each other, with the conservation of mass and momentum, if their impact parameter is smaller than a critical merger size \( R_c \approx 4 R_p \).

In a previous analysis (Zhang et al. 2014), we showed that the embryos’ convergent type I migration is halted with an interplanetary spacing \( \Delta a \), which is a decreasing function of \( M \). In a steady disk with a constant \( \alpha \), \( \Sigma_g \) generally increases with the accretion rate \( M \). The strength of both the differential Lindblad and corotation torques as well as the embryos’ type I migration speeds increase with \( \Sigma_g \). We introduce model 6b to show that in disks with relatively large \( \Sigma_g \) (or equivalently \( M = 2 \times 10^{-7} M_\odot \text{yr}^{-1} \)), the embryos undergo similar paths as those in the previous models.

We find that the faster convergent migration rate leads to the merge of the middle two embryos shortly after release. Later, they converged into a resonant state (Figure 6). If we use the embryos’ actual physical size \( R_p \) with a comparable density as the Earth, the estimated collision timescale would be \( \tau_c \sim (a \Delta a / N R_p^2 \Theta) P \), which is several orders of magnitude longer than their orbital periods \( P \), even after they converge into a region with a radial width \( \Delta a \) comparable to their Roche radius or the extent of their corotation region. However, under the 2D disk simulations, the collision frequency is much higher and the orbital crossing timescale could be reduced to several hundred orbital periods. Realistic collisions require protracted simulations, which is beyond the current computational constraint.
The total mass of the embryos (40 $M_\oplus$) in models 5, 6a, and 6b exceeds the $M_p (= 20 M_\oplus$) in models 1d–1f (see Section 3). However, this convoy of resonant embryos is retained near $r_t$, whereas the more massive isolated embryo migrated toward the inner boundary of the disk. These results indicate that as separate entities, lower-mass embryos preserve their corotation torques (in model 6a). If they merge into sufficiently massive isolated embryos, their corotation torques would be saturated and they may no longer be trapped near $r_t$ (model 6b). After the merger event, the massive embryo continues to migrate inward. Its perturbation on the disk structure is enhanced. In isolation, the merger’s mass is sufficiently large for it to confine the horseshoe stream lines and saturate the corotation torque. As we have shown in Section 4, the interference by other nearby embryos induces mixing of stream lines and restores diffusion across their corotation regions.

### 6. DISCUSSION AND SUMMARY

In the sequential accretion scenario, the formation of critical mass cores (with $M_p > M_c \sim 10 M_\oplus$) prior to the severe depletion of gas in protostellar disks is a prerequisite for the formation of gas giant planets. In this paper, we present simulations to examine the conditions for the formation of critical mass cores.

We confirm the previous hypothesis that the core formation probability is greatly enhanced by the type I migration of protoplanetary embryos. The magnitude and direction of the embryos’ migration is determined by their net differential Lindblad and corotation torques. In the outer irradiated regions of protostellar disks, these torques lead to inward migration, but in the inner, viscously heated regions of the disk, unsaturated (full strength) corotation torque is stronger than the differential Lindblad torque and it induces an outward migration. However, the corotation torque is suppressed by saturation for both massive ($> 10 M_\oplus$) and low-mass ($< 3 M_\oplus$) embryos. Nevertheless, super-Earth embryos undergo convergent migration toward the transition radius ($r_t \sim$ a few AU) that separates these two disk regions.

Here we present numerical simulations to show that the torque prescription previously constructed by Paardekooper et al. (2010, 2011) for idealized single power-law $\Sigma$, and $S_\Sigma$ distribution can be generalized to more complex disk models. This applicability is a reflection that the embryo–disk torque is mostly applied to the proximity of the planet’s orbit over length scales much shorter than those that modify the disk structure.

We show that when multiple embryos congregate near $r_t$, they interact with each other through secular and resonant perturbations. In disks with modest $\Sigma$ (or $M$), the convergent speed is relatively slow. They would capture each other on their mutual mean motion resonances if the resonant libration timescale is shorter than that for the approaching embryos to cross the resonant width. These planets form a convoy of resonant embryos. The separation between the planets decreases with $M$.

In disks with sufficiently large $M$, the embryos’ separation becomes comparable or smaller than the total width of their mean motion resonance. Interference between horseshoe stream lines around each planet enhances diffusion of gas through the corotation region and suppresses the saturation of corotation resonances. In this limit, embryos converge with overlapping orbits.

Orbit-crossing embryos undergo repeated close encounters. Although some embryos are scattered outside the corotation zones, converging type I migrations continually re-��iate them back to the proximity of $r_t$. The trapped embryos continue to scatter each other until they undergo physical collision.

The conditions for the collisional products to evolve into a super-critical mass core are (1) there is adequate time for collision to occur, (2) the physical collisions need to be mostly cohesive, and (3) the merger products must be retained. The value of $r_t$ is also an increasing function of $M$. In disks with sufficiently large $M$ for embryos to bypass the resonant barrier, $r_t$ is at least a few AU where the local Keplerian velocity is smaller than the surface escape speed of super-critical mass cores. Since the embryos’ velocity dispersions are much smaller than their Keplerian velocities, their collisions are not sufficiently energetic to cause any significant fragmentation.

The collision timescale for the embryos is expected to be much longer than the orbital period but shorter than the gas depletion timescale. We used an idealized model to simulate the consequences of a merger event on the residual embryos. Hydrodynamic simulations with a realistic collision timescale are beyond the current computational feasibility, especially for a systematic model parameter study. Based on the results presented here, we will present a newly constructed Hermit-embryo code to simulate the evolution of multiple embryos in subsequent papers of this series. We will also replace the idealized composite power-law $\Sigma$ distribution with comprehensive evolving disk models.

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**Figure 6.** Semi-axis and eccentricity evolution of four 10 $M_\oplus$ planets’ orbits released from $r = 0.7, 0.9, 1.1,$ and $1.3$, respectively, in disks with different accretion rates. Each group of curves shows the paths of $a, a \times (1 + e)$, and $a \times (1 - e)$ for the same planet. (Top: $M = 10^{-7} M_\odot$yr$^{-1}$ (model 6a), bottom: $M = 2 \times 10^{-7} M_\odot$yr$^{-1}$ (model 6b)).
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