NON–CONVENTIONAL STATISTICAL EFFECTS IN RELATIVISTIC HEAVY-IONS COLLISIONS

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We show that non–conventional statistical effects (due to the presence of long range forces, memory effects, correlations and fluctuations) can be very relevant in the interpretation of the experimental observables in relativistic heavy-ions collisions. Transverse mass spectrum, transverse momentum fluctuations and rapidity spectra are analysed in the framework of the non-extensive statistical mechanics.

1 Introduction and motivations

Most of the theoretical analyses related to observables in relativistic heavy-ion collisions involve (implicitly or explicitly) the validity of the standard Boltzmann-Gibbs statistical mechanics. In particular, if the thermal equilibrium is achieved, the Maxwell-Boltzmann (MB) distribution (Fermi-Dirac or Bose-Einstein distribution if quantum statistical effects are not negligible) is assumed to hold. When the system approaches equilibrium, the phase–space distribution should be derived as a stationary state of the dynamical kinetic evolution equation. It is well known that in the absence of non–Markovian memory effects, long–range interactions and local correlations, the MB distribution is obtained as a steady state solution of the kinetic Boltzmann equation. However, it is a rather common opinion that, because of the extreme conditions of density and temperature in ultrarelativistic heavy ion collisions, memory effects and long–range color interactions give rise to the presence of non–Markovian processes in the kinetic equation affecting the thermalization process toward equilibrium as well as the standard equilibrium distribution \[\ldots\].

The aim of the present contribution is to explore, from a phenomenological point of view, the relevance of the above mentioned statistical effects that can influence the dynamical evolution of the generated fireball toward the freeze-out stage and, as a consequence, the physical observables.
2 Generalized non–extensive statistics

A quite interesting generalization of the conventional Boltzmann–Gibbs statistics has been recently proposed by Tsallis and proves to be able to overcome the shortcomings of the conventional statistical mechanics in many physical problems, where the presence of long–range interactions, long–range microscopic memory, or fractal space–time constraints hinders the usual statistical assumptions.

The Tsallis generalized thermostatistics is based upon the following generalization of the entropy:

\[ S_q = \frac{1}{q-1} \sum_{i=1}^{W} p_i (1 - p_i^{q-1}) , \] (1)

where \( p_i \) is the probability of a given microstate among \( W \) different ones and \( q \) is a fixed real parameter. The new entropy has the usual properties of positivity, equiprobability, concavity and irreversibility, preserves the whole mathematical structure of thermodynamics and reduces to the conventional Boltzmann–Gibbs entropy \( S = -\sum p_i \log p_i \) in the limit \( q \to 1 \).

The single particle distribution function is obtained through the usual procedure of maximizing the Tsallis entropy under the constraints of keeping constant the average internal energy and the average number of particles. For a dilute gas of particles and/or for \( q \approx 1 \) values, the average occupational number can be written in a simple analytical form:

\[ \langle n_i \rangle_q = \frac{1}{[1 + (q-1)\beta(E_i - \mu)]^{1/(q-1)} \pm 1} , \] (2)

where the + sign is for fermions, the − for bosons and \( \beta = 1/T \). In the limit \( q \to 1 \) (extensive statistics), one recovers the conventional Fermi–Dirac and Bose–Einstein distribution. Under the same conditions, but in the classical limit, one has the following generalized Maxwell–Boltzmann distribution:

\[ \langle n_i \rangle_q = [1 + (q-1)\beta(E_i - \mu)]^{1/(1-q)} . \] (3)

When the entropic \( q \) parameter is smaller than 1, the distributions (2) and (3) have a natural high energy cut–off: \( E_i \leq 1/\beta(1 - q) + \mu \), which implies that the energy tail is depleted; when \( q \) is greater than 1, the cut–off is absent and the energy tail of the particle distribution (for fermions and bosons) is enhanced. Hence the nonextensive statistics entails a sensible difference of the particle distribution shape in the high energy region with respect to the standard statistics. This property plays an important rôle in the interpretation of the physical observables, as it will be shown in the following.
3 Transverse mass spectrum and momentum fluctuations

Let us consider the transverse momentum distribution of particles produced, e.g., in relativistic heavy ion collisions: it depends on the phase-space distribution and usually an exponential shape is employed to fit the experimental data. This shape is obtained by assuming a purely thermal source with a MB distribution. High energy deviations from the exponential shape are taken into account by introducing a dynamical effect due to collective transverse flow, also called blue-shift.

Let us consider a different point of view and argue that if long tail time memory and long–range interactions are present, the MB distribution must be replaced by the generalized distribution (3). Limiting ourselves to consider here only small deviations from standard statistics \((q - 1 \approx 0)\); then at first order in \((q - 1)\) the transverse mass spectrum can be written as

\[
\frac{dN}{d m_{\perp}} = C m_{\perp} \left\{ K_1(z) + \frac{(q-1)}{8} z^2 \left[ 3 K_1(z) + K_3(z) \right] \right\}, \tag{4}
\]

where \(K_i\) are the modified Bessel function at the \(i\)-order.

The above equation is able to reproduce very well the transverse momentum distribution of hadrons produced in S+S collisions (NA35 data [6]) providing we take \(q = 1.038\). Furthermore, it is easy to see that at first order in \((q - 1)\) from Eq. (3), the generalized slope parameter takes the following form:

\[
T_q = T + (q - 1) m_{\perp}. \tag{5}
\]

Hence nonextensive statistics predicts, in a purely thermal source, a generalized \(q\)–blue shift factor at high \(m_{\perp}\); moreover this shift factor is not constant but increases (if \(q > 1\)) with \(m_{\perp} = \sqrt{m^2 + p_{\perp}^2}\), where \(m\) is the mass of the detected particle. Such a behavior has been observed in the experimental NA44 results [8].

Another observable very sensitive to non–conventional statistical effects are the transverse momentum fluctuations. In fact, in the framework of non–extensive statistics, the particle fluctuation \(\langle \Delta n^2 \rangle_q = \langle n^2 \rangle_q - \langle n \rangle_q^2\) is deformed, with respect to the standard expression, as follows

\[
\langle \Delta n^2 \rangle_q = \frac{1}{\beta} \frac{\partial \langle n \rangle_q}{\partial \mu} = \frac{\langle n \rangle_q}{1 + (q - 1)\beta(E - \mu)} (1 \mp \langle n \rangle_q), \tag{6}
\]

where \(E\) is the relativistic energy \(E = \sqrt{m^2 + p^2}\).

The fluctuations of an ideal gas of fermions (bosons), expressed by Eq. (3), are still suppressed (enhanced) by the factor \(1 \mp \langle n \rangle_q\) (as in the standard case)
but this effect is modulated by the factor \([1 + (q - 1)\beta (E - \mu)]^{-1}\). Therefore the fluctuations turn out to be increased for \(q < 1\) and are decreased for \(q > 1\). Very good agreement with the experimental NA49 analysis \(^9\) is obtained by taking \(q = 1.038\) \(^4\), notably the same value used in transverse momentum spectra.

4 Anomalous diffusion in rapidity spectra

An important observable in relativistic heavy-ion collisions is the rapidity distribution of the detected particles. In particular, there is experimental and theoretical evidence that the broad rapidity distribution of net proton yield \((p - \bar{p})\) in central heavy-ion collisions at SPS energies could be a signal of non-equilibrium properties of the system. We want to show now that the broad rapidity shape can be well reproduced in the framework of a non-linear relativistic Fokker-Planck dynamics which incorporates non-extensive statistics and anomalous diffusion.

A class of anomalous diffusions are currently described through the non-linear Fokker-Planck equation (NLFPE)

\[
\frac{\partial}{\partial t} [f(y, t)]^\mu = \frac{\partial}{\partial y} \left[ J(y) [f(y, t)]^\mu + D \frac{\partial}{\partial y} [f(y, t)]^\nu \right],
\]

where \(D\) and \(J\) are the diffusion and drift coefficients, respectively. Tsallis and Bukman \(^10\) have shown that, for linear drift, the time dependent solution of the above equation is a Tsallis-like distribution with \(q = 1 + \mu - \nu\). The norm of the distribution is conserved at all times only if \(\mu = 1\), therefore we will limit the discussion to the case \(\nu = 2 - q\).

Imposing the validity of the Einstein relation for Brownian particles, we can generalize to the relativistic case the standard expressions of diffusion and drift coefficients as follows

\[
D = \alpha T, \quad J(y) = \alpha m_\perp \sinh(y) \equiv \alpha p_\parallel,
\]

where \(p_\parallel\) is the longitudinal momentum and \(\alpha\) is a common constant. It is easy to see that the above coefficients give us the Boltzmann stationary distribution in the linear Fokker-Planck equation \((q = \nu = 1)\) (such a result cannot be obtained if one assumes a linear drift coefficient as in Ref. \(^{11}\)) while the stationary solution of the NLFPE \(^4\) with \(\nu = 2 - q\) is a Tsallis-like distribution with the relativistic energy \(E = m_\perp \cosh(y)\):

\[
f_q(y, m_\perp) = \left\{ 1 - (1 - q) \beta m_\perp \cosh(y) \right\}^{1/(1-q)}.
\]
Basic assumption of our analysis is that the rapidity distribution is not appreciably influenced by transverse dynamics, which is considered in thermal equilibrium. This hypothesis is well confirmed by the experimental data and adopted in many theoretical works. Therefore, the time dependent rapidity distribution can be obtained, first, by means of numerical integration of \( \frac{dN}{dy} \) with initial \( \delta \)-function condition depending on the value of the experimental projectile rapidities and, second, by integrating such a result over the transverse mass \( m_\perp \) (or transverse momentum) as follows

\[
\frac{dN}{dy}(y, t) = c \int_0^\infty m_\perp^2 \cosh(y) f_q(y, m_\perp, t) \, dm_\perp,
\]

where \( c \) is the normalization constant fixed by the total number of the particles. The calculated rapidity spectra will ultimately depend on the nonextensive parameter \( q \) only, since there exists only one “interaction time” \( \tau = \alpha t \) which reproduces the experimental distribution.

Figure 1. Rapidity spectra for net proton production \((p - \bar{p})\) in central Pb+Pb collisions at 158\(A\) GeV/c (grey circles are data reflected about \(y_{cm} = 0\)). Full line corresponds to our results by using a non-linear evolution equation \((q = 1.25)\), dashed line corresponds to the linear case \((q = 1)\).

In Fig.1 we show the calculated rapidity spectra of net proton compared with the experimental NA49 data from central Pb+Pb collisions at 158 GeV/c. The obtained spectra are normalized to 164 protons and the beam rapidity is fixed to \(y_{cm} = 2.9\) (in the c.m. frame). The full line corresponds to the NLFPE solution at \(\tau = 0.82\) and \(q = 1.25\); the dashed line corresponds to
the solution of the linear case \((q = 1)\) at \(\tau = 1.2\). Only in the non-linear case \((q \neq 1)\) there exists a (finite) time for which the obtained rapidity spectra well reproduces the broad experimental shape. A value of \(q \neq 1\) implies anomalous superdiffusion in the rapidity space, i.e., \([y(t) - y_M(t)]^2\) scales like \(t^\alpha\) with \(\alpha > 1\). 

5 Conclusions

The nonextensive statistics appears suitable to evaluate physical observables recently measured in heavy ion collision experiments. The physical motivation for the validity of a non-conventional statistical behavior can be related to the presence of memory effects and long range interactions at the early stage of the collisions, even if a microscopic justification of these effects is still lacking. A rigorous determination of the conditions that produce a nonextensive statistical regime should be based on microscopic calculations relative to the parton plasma originated during the high energy collisions. Non–perturbative QCD effects in the proximity of hadronic deconfinement could play a crucial role in the determination of the quantum kinetic evolution of the system toward the equilibrium.

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