Multi–facility placement on lines with forbidden zones and routing of communications

G G Zabudsky¹, N S Veremchuk²
¹ Sobolev Institute of Mathematics, Novosibirsk, Russia
² Siberian State Automobile and Highway University, Omsk, Russia
E-mail: zabudsky@ofim.oscsbras.ru, n-veremchuk@rambler.ru

Abstract. This article is about placement of interconnected facilities (equipment) in a plane on parallel lines with forbidden zones. Placement of the facilities inside of the forbidden zones is not possible. The facilities are connected to each other and with the zones by some communications. Communications (routing of communications) between the facilities and between the facilities and zones which are placed on different lines (adjacent lines) pass through a fixed vertical component (viaduct). It is need to place the facilities on the lines such a way that the total cost of communications between the facilities and zones will be minimal. There are many practical applications of this problem in science and techniques, for example, when designing engineering devices. Some properties of the problem are formulated. A mathematical model of integer nonlinear programming of the problem is constructed. It is shown that algorithms for solving of a problem with rectangular metric without the viaduct can be used for solving of the problem under consideration.

1. Introduction
One of the most famous problems of operations research is placement problem. These problems have many applications in different directions of activity and engineering, for example, placement of server stations, elements of electronic devices, cell towers, service points, process equipment in workshops, and so on. The variety statements of such problems are determined by the need to take into account various factors: the geometric form and size of facilities, the structure of the placement area, restrictions on placement, type of criterion (criterions) and others. A classification of the problems by various characteristics is given in [5, 6, 10]. Studies of such problems require a construction of new mathematical models and a development of effective algorithms for their solution. This article examines one of the problems of optimal placement of interconnected facilities (facilities are connected by some communications) on lines with forbidden zones and special way of laying the communications.

The problems of optimal placement of interconnected facilities are an important class of placement problems. The concept of "facility" can have different meanings depending on the content of the problem statement. In addition, depending on the relationship of the placement area and the size of the facilities, you can consider a facility as a point or some dimensional facility. Often, the dimensional facilities are approximated by rectangles. One of the main characteristics of the class of facility placement being examined is the presence of the connections between the facilities. In different settings, connections have different meanings: pipelines; product flows; conductors. This type of problem is formulated as follows. An area has a set of
fixed facilities. A set of new facilities that need to be placed in this area in such a way that the specified restrictions are met and some indicator (indicators) of the quality of placement is optimal. Currently, the problem of placing interconnected facilities with fixed facilities is often called the Weber problem [2, 3, 7, 9].

Weber problems have a wide range of applications in practice. They are the basis of mathematical models for many application problems when automating the design of the placement of interconnected facilities in a given area. For example, when designing General plans of enterprises, it is necessary to place workshops. When designing workshops it is necessary to place technological equipment. When designing electronic devices it is necessary to place elements of printed installation. In the literature, the classical Weber problem on a plane (metric space) for point facilities without restrictions on their placement is well studied [3, 12]. If rectangular metric is used the problem is decomposed into two independent subproblems. For each subproblem we can build equivalent linear programming problem [3]. In addition, a number of streaming algorithms are proposed to solve this problem.

For efficient use of production space and existing process equipment, an important problem is to design its placement in production shops. One of the features of the problems is the need to take into account various restrictions. For example, in the development of plans workshops where equipment needs to be placed in some regular way, often along ”red” lines that are parallel to one side of the workshop. This allows for zoning of the territory, to allocate direct passages for convenient operation and maintenance of equipment. In optimal regular placement models various criteria are considered, such as minimizing the following indicators: an area occupied by the equipment, total cost of communications, cost of equipment operation. Additionally, the solutions of the problems of the regular placement of the facilities can be compared according to several criterions. Therefore, to solve such problems, you need to build multi-criterion mathematical models. For example, you must place rectangular facilities on parallel lines so that length and width of the rectangular area occupied by the facilities are minimal.

One of the examples in which at the stage of design and technological preparation of production it is necessary to solve the optimization problem of regular placement of rectangles in a multi-criterion setting is the development of plans for workshops of light industry enterprises. First, flexible production modules are formed. The main problem when placing jobs in modules is to ensure the continuity of the trajectory of movement of semi-finished products. Then the modules are replaced by rectangles and then the problem of ”regular” placement of the rectangles is solved. Regularity consists in placing modules along parallel lines. They are placed on the center lines parallel to one of the sides of the shop. Similar problems of facility placement should be solved when designing robotic systems, flexible machining systems, petrochemical industry enterprises, etc.

A formulation a two-criterion problem of placing rectangles on parallel lines for an unfixed line arrangement in a plane is considered in [14]. Width and length of the rectangles, length and width of the placement area are given. In addition, a minimum technical distance from the end or side area in accordance with the technical requirements and the required width of the passage between the lines are given. You must place the rectangles on lines with the minimum length and width of the area they occupy (rectangular shell). Two-criteria model nonlinear integer programming with Boolean variables for optimal facility placement on lines in a rectangular area is discussed in [14]. A set of valid values for Boolean variables defines only the distribution of rectangles along lines and does not determine their order on lines. The problem is NP-hard. To find Pareto-optimal solutions, the method of concessions is used, in which the leading criterion is to minimize the total length of equipment on the lines. Concessions are made by the width of the area with the specified step. In this case, a series of single-criterion integer linear programming problems is solved.

Note that in practice, the most interesting solution is the one with the minimum length,
since in this case the equipment is located "evenly" on the lines. A solution of the problem to be a compact placement of the facilities on parallel lines, for which the covering rectangle has a minimum length and width. A method for finding a compact placement variant by solving two integer linear programming problems is proposed in [14]. The search for a solution that is close to a compact one is based on dynamic programming. Its idea is that the equipment is sequentially placed on lines so that its total length on the next line is the maximum, but does not exceed the specified number.

Despite many papers devoted to research and solving problems of optimal placement of facilities, problems with forbidden zones (areas where it is forbidden to place facilities) for dimensional facilities with conditions of regularity of their placement have not been studied enough. Basically, problems for only one line without taking into account any restrictions on placement are considered [1, 4, 8, 11, 13]. For example, one of the well–known problems of optimal placement of interconnected facilities on line with criterion minimal total cost of communications is the One–Dimensional Space Allocation Problem (ODSAP). Combinatorial models and integer programming models are proposed for the problem. Algorithms of branch and bounds, dynamic programming, and others have been developed to solve ODSAP [8, 11, 13]. A problem of optimal placement of rectangular interconnected facilities on one line with forbidden zones is considered in [15, 16, 18, 19]. The original continuous problem is decomposed into a series of discrete subproblems of smaller dimension. The branch and bounds algorithm is used to solve subproblems accurately [17]. The approximate solution is constructed using a polynomial heuristic procedure [16]. Review of models and methods for solving of the problem using combinatorial methods and integer programming are presented [18].

In [15, 19] a generalization of the ODSAP with forbidden zones is considered. The structure of connections between objects is defined using the directed acyclic graph. A polynomial–time algorithm for finding the local optimum when the graph of connections between objects is a composition of rooted trees and parallel–serial directed graphs is proposed. An overview of models and methods to solve of the generalization of the ODSAP is provided in [18]. Algorithms for finding an approximate solution and branch and bounds are described. Results of computational experiments are reported.

In this paper, we study the problem of placement of facilities (equipment) on the parallel lines, taking into account forbidden zones. Facilities are rectangles whose centers are connected to each other and with the zones by communications. The forbidden zones also are the rectangles in which placement of facilities is not possible. It is considered that the positions of the lines are fixed and direct passes are made between the lines for easy maintenance of the facilities. Adjacent lines are connected by a fixed vertical component (viaduct) for routing of communications between facilities and zones which are placed on different lines. Some properties of the problem are provided. For the problem a mathematical model of nonlinear programming with Boolean variables is built. The initial continuous problem is reduced to a series of discrete subproblems of lower dimension with identical structure. It is shown that algorithms for solving of a problem without the viaduct with rectangular metric can be used for solving of the problem under consideration.

2. Problem statement
Let's write a mathematical model of the problem. There are some segments parallel to the axis OX and have the length $LS$. These segments contain some fixed rectangular areas (forbidden zones). Further, the segments will be called lines. We believe that placement in forbidden zones is not possible. We can assume that the left border of each line is at the origin. In addition, the lines are placed at such a distance from each other that direct passages between them are made for the convenience of facilities maintenance. There are new rectangular facilities. Centers of the facilities are related to each other and with the centers of forbidden zones by
some communications. Moreover, routing of communications the facilities among themselves and with zones which are placed on adjacent lines passes through a fixed vertical component (viaduct).

The problem is to place the facilities on the lines outside of forbidden zones, so that they do not intersect each other and, in addition, the total cost of connections (communications) between facilities and between facilities and zones was minimal. Let’s enter notations and write a mathematical model for two lines.

Facilities and zones are denoted by \( X_i \) and \( F_j \), where \( i \in I = \{1, \ldots, n\} \) and \( j \in J = \{1, \ldots, m\} \). Let \((x_i, y_i)\) and \((b_{1j}, b_{2j})\) denote the centers of \( X_i \) and \( F_j \), \( i \in I, \ j \in J \), respectively. The lengths of facilities and zones are denoted by \( p_j \), \( i \in I, \ j \in J \). While \( w_{ij} \geq 0 \), \( u_{ik} \geq 0 \) are the costs of communications between \( X_i \) and \( F_j \), \( X_i \) and \( X_k \) for \( i, k \in I, \ j \in J, \) and \( i < k \). Let the left border of each line be a point \((0, Ly_t)\), where \( t = 1, 2 \). Note that the set \( J \) can be represented as a union \( J = JL_1 \cup JL_2 \), where \( JL_t \) is the set of zone’s numbers on the line with number \( t, \ t = 1, 2 \). If \( F_j \) is placed on the line with number \( t \), then \( b_{2j} = Ly_t \). The purpose is to place the facilities \( X_1, \ldots, X_n \) on the lines outside zones \( F_1, \ldots, F_m \) and so that they do not intersect with each other and the total cost of the communications between the facilities and between facilities and zones was minimal.

To formulate the conditions of facility’s placement on the line, we introduce the Boolean variables: \( z_{it} \) for \( i \in I, \ t = 1, 2 \), so that \( z_{it} = 1 \) if \( X_i \) is on the line with the number \( t \), otherwise \( z_{it} = 0 \).

Let communications between the facilities and zones which are placed on different lines pass through the viaduct \( V \). Taking into account the entered designations, the height of the viaduct \( V \) will be \( \delta y = Ly_2 - Ly_1 \) and it is a constant value. Let the abscissa of the viaduct placement is denoted by \( x_0 \). An example of the problem for two facilities and four zones is shown in Figure 1. Where forbidden zones are represented as shaded rectangles, and communications between \( X_1 \) and other facilities and zones are shown as segments.

Taking into account the entered designations, we have a mathematical model of nonlinear programming with Boolean variables as shown below:

\[
G(x) = \sum_{t=1}^{2} \sum_{i \in I} \sum_{j \in JL_t} w_{ij} \left( z_{it} |x_i - b_{1j}| + (1 - z_{it})(|x_i - x_0| + |b_{1j} - x_0| + \delta y) \right) +
\]
\[
+ \sum_{t=1}^{2} \sum_{i \in I} \sum_{k \in I, i < k} u_{ik} \left( z_{it} z_{kt} |x_i - x_k| + z_{it}(1 - z_{kt})(|x_i - x_0| + |x_k - x_0| + \delta y) \right) \rightarrow \min,
\]

under constraints

\[
|x_i - b_{1j}| \geq z_{it} \frac{L_i + p_j}{2}, \ i \in I, \ j \in JL_t, \ t = 1, 2;
\]

\[
|x_i - x_k| \geq (z_{it} + z_{kt} - 1) \frac{L_i + L_k}{2}, \ i, k \in I, \ i < k, \ t = 1, 2;
\]

\[
\frac{l_i}{2} \leq x_i \leq L_S - \frac{l_i}{2}, \ i \in I;
\]

\[
y_i = \sum_{t=1}^{2} z_{it} Ly_t, \ i \in I;
\]

\[
\sum_{t=1}^{2} z_{it} = 1, \ i \in I;
\]

\[
z_{it} \in \{0, 1\}, \ i \in I, \ t = 1, 2.
\]
In (1) the first component determines the total cost of communications between the facilities and zones. The second part in (1) determines the total cost of communications between the facilities themselves. Since the positions of the lines are fixed, then the value $y$ is the constant. So, the goal function $G(x)$ depends only on the $x$ coordinates.

If facilities $X_i$ and $X_k$ are placed on one line, then the expression $u_{ik}(|x_i - x_k|)$ is the contribution of these facilities to the goal function $G(x)$; otherwise the expression $u_{ik}(|x_i - x_0| + |x_k - x_0| + \delta y)$ is the contribution to the goal function. For the facility $X_i$ and the zone $F_j$ the contribution to the goal function $G(x)$ is defined analogously. The disjoint conditions between facilities and zones and between facilities themselves are expressions (2) and (3). The conditions for placing of facilities on the lines are (4) and (5). Moreover, the condition for placing a facility only on one line is (6).

3. Properties of the problem

Note some properties of problem (1)–(7). Let’s denote by $B$ a feasible area of the problem. Note that $B$ is disconnected and it consists of the set of $r$ disjoint segments $B_k, k = 1, \ldots, r$, $B = \bigcup_{k=1}^{r} B_k$. Let’s denote the lengths of these segments $B_k$ by $L_k$, $k = 1, \ldots, r$. We will place facilities $X_i, i \in I,$ in them. These segments we will call as blocks. Such blocks clearly are defined as forbidden zones. Problem (1)–(7) is NP–hard because the problem for one line is NP–hard. Really a feasible solution to the problem can be found by construction a one–dimensional packing in containers. When facilities with lengths $l_i, i \in I,$ are packed in containers with dimensions $L_k, k = 1, \ldots, r$. If there are no zones then the problem for each line is a one–dimensional space allocation problem. It is NP–hard for an arbitrary set of relationships between the facilities [13] and it is polynomial–solvable for a rooted tree [11].

Let a feasible placement is given. Then a remainder in the block $B_k$ is a segment. Let $x = (x_1, \ldots, x_n)$ be a feasible solution of the problem. Denote by $I_k(x)$ the set of the facility’s numbers in $B_k$, $I = \bigcup_{k=1}^{r} I_k(x)$. The set of remainders in $B_k$ for $x$ is denoted by $H_k(x)$. Denote by $n_k$ the capacity of set $I_k(x)$, then $|H_k(x)| \leq n_k + 1$. A feasible solution $x$ can be represented as $x = (x^1, \ldots, x^r)$, in which $x^k$ is the coordinates of the centers of facilities that are placed in $B_k$ with numbers from $I_k(x)$.

In [16] it is proved that for a given feasible solution $x$ of the problem for one line, we can find a feasible solution $x'$ such that $|H_k(x')| \leq 1, k = 1, \ldots, r$. In this case, the value of the goal
function for $x'$ will be no worse than for $x$, so $G(x') \leq G(x)$. This way we can consider no more than one remainder in each block $B_k$. The original continuous problem is reduced to a discrete problem. Next, we show that this property is true for formulated problem (1)–(7).

Consider an arbitrary block $B_k$ and let $LB_k$ and $RB_k$ denote the coordinates of the left and right borders of $B_k$. Let a partition of facilities into blocks is fixed. Designate by $J_L(B_k)$ and $J_R(B_k)$ the sets of zones to the left and to the right from the block $B_k$; $I_L(B_k)$ and $I_R(B_k)$ denote the set of facilities to the left and to the right from the block $B_k$ respectively.

Note that, taking into account the specifics of the formulated problem, many of its properties are based on determining the relative placement of facilities and zones in relation to the block under consideration. This is the main idea for formulating the following properties. Consider the cases of the viaduct placement, depending on which we define sets $J_L(B_k)$, $J_R(B_k)$ and $I_L(B_k)$, $I_R(B_k)$. Let’s assume that the viaduct does not pass through arbitrary block. The following cases are possible.

Case a). Viaduct $V$ is placed to the right from the block $B_k$ (see Fig. 2), i.e., $x_0 > RB_k$. Since the problem condition requires that connections between facilities and zones placed on different lines pass through the viaduct, then the following assumption is possible. We will assume that numbers of zones and facilities to the left from $B_k$ on the same line as $B_k$ form the sets $J_L(B_k)$, $I_L(B_k)$, respectively. All other zones and facilities, including those which are placed on different lines, form sets $J_R(B_k)$, $I_R(B_k)$, respectively. Thus, $J_R(B_k) = J \setminus J_L(B_k)$, $I_R(B_k) = I \setminus I_L(B_k)$.

In order not to clutter the Figure 2, only blocks in square brackets are shown on it. Zones and facilities are not shown.

![Figure 2](image-url) 

**Figure 2.** The placement of the viaduct to the right from the block $B_k$.

Case b). Viaduct $V$ is placed to the left from $B_k$ (see Fig. 3), i.e., $x_0 < LB_k$. Then similar we assume that numbers of zones and facilities to the right from $B_k$ on the same line as $B_k$ form the sets $J_R(B_k)$, $I_R(B_k)$, respectively. All other zones and facilities, including those which are placed on different lines, form sets $J_L(B_k)$, $I_L(B_k)$, respectively.

So the facilities and zones to the left from the block $B_k$ and to the right from the block $B_k$ are defined. Next for each facility $X_i$ in $B_k$, we define the total cost of communications $Lw_i$ and $Rw_i$ as follows:

$$Lw_i = \sum_{j \in J_L(B_k)} w_{ij} + \sum_{k \in I_L(B_k)} u_{ik}, \quad Rw_i = \sum_{j \in J_R(B_k)} w_{ij} + \sum_{k \in I_R(B_k)} u_{ik}.$$
Taking into account the entered designations, the following theorem can be formulated.

**Theorem 1** For an arbitrary feasible solution of problem (1)–(7), we can constructed another feasible solution \( x' \) such that \( |H_k(x')| \leq 1, k = 1, \ldots, r \) and \( G(x') \leq G(x) \) when the partition of \( X_1, \ldots, X_n \) into blocks is fixed.

Using the total cost of communications \( Lw_i \) and \( Rw_i \) defined above, the proof of the theorem 1 is similar to the proof given in [16].

Note that when we fixed a partition of the facilities into blocks, the goal function \( G(x) \) can be represented as follows:

\[
G(x) = \sum_{k=1}^{r} G_k(x^k) + \overline{C},
\]

where

\[
G_k(x^k) = \sum_{s \in I_k(x)} \sum_{t \in I_s(x), t > s} u_{st} |x_s - x_t| + \sum_{s \in I_k(x)} |x_s - LB_k| \left( \sum_{j \in J_s(B_k)} w_{sj} \right) + \sum_{i \in I_L(B_k)} u_{si} + \sum_{t \in I_k(x)} |x_t - RB_k| \left( \sum_{j \in J_t(B_k)} w_{tj} + \sum_{i \in I_R(B_k)} u_{ti} \right),
\]

\( \overline{C} \) is some constant.

The sum of costs of communications between the facilities in \( B_k \) is the first component in \( G_k(x^k) \). The total cost of communications between the facilities from \( B_k \) and \( LB_k \) is the second component in \( G_k(x^k) \). The total cost of communications between the facilities from \( B_k \) and \( RB_k \) is the third component in \( G_k(x^k) \).

When a partition of \( X_1, \ldots, X_n \) into blocks is fixed, you can consider a placement problem with \( n_k + 2 \) facilities in each block. In this case, \( n_k \) facilities are placed in a segment with two fixed facilities which are placed at the ends of the segment. Taking into account the entered designates, this problem is written in following way

\[
G_k(x^k) = \sum_{s \in I_k(x)} \sum_{t \in I_k(x), t > s} u_{st} |x_s - x_t| + \sum_{s \in I_k(x)} Lw_s |x_s - LB_k| + \]
\[ + \sum_{t \in I_k(x)} Rw_t |x_t - RB_k| \rightarrow \min, \]  
\[ |x_i - x_k| \geq \frac{l_i + l_k}{2}, \quad i, k \in I_k(x), \quad i < k, \]  
\[ LB_k + \frac{l_i}{2} \leq x_i \leq RB_k - \frac{l_i}{2}, \quad i \in I_k(x). \]

An admissible solution \( x \) of problem (1)–(7) we will call a local minimum if \( G(x) \leq G(x') \) for all \( x' : I_k(x) = I_k(x'), \ k = 1, \ldots, r. \)

From all of the above, the validity of the following theorem follows

**Theorem 2** To find a local optimum of problem (1)–(7), it is sufficient to solve \( r \) subproblems of the form (8)–(10).

Accounting for these properties allows us to offer the following scheme for solving problem (1)–(7). At the first step, we find a feasible partition of the facilities into blocks using, for example, an algorithm from [16]. Then, for the resulting partition, we solve a series of subproblems of smaller dimension (8)–(10).

In [16] the algorithm of an approximate solution of problem (8)–(10) is offered. The main idea of which is that, when a partition of facilities into blocks is fixed, the facilities in the blocks are ordered for the minimization of communications. Namely, the facility \( X_i \) is placed to the left border of the block \( B_k \) if \( Lw_i > Rw_i \), otherwise \( X_i \) is placed to the right border of \( B_k \) and so for all \( i \in I_k(x) \).

You can find exact solution of this problem using a branch and bound method from work [17]. In which facilities in a certain order are placed sequentially either to the left border of the block or to the right. The placement order is determined based on the calculated lower estimates of the goal function.

4. Conclusion

We consider the placement of interconnected facilities in a plane on parallel lines taking into account the forbidden zones. Routing communications between the facilities and between the facilities and zones which are placed on different lines pass through a fixed vertical viaduct. The mathematical model of integer nonlinear programming of the problem is constructed. Some properties of the problem are proposed. According obtained properties approximate and exact algorithms for solving of the problem without the viaduct with rectangular metric can be used for solving of the considered problem with the viaduct. There are many practical applications of this problem in science and techniques, for example, when designing engineering devices. The offered model and decision’s algorithms can be used in the computer–aided engineering systems.

**Acknowledgments**

The work was supported by the program of fundamental scientific research of the SB RAS No. I.5.1., project No. 0314-2019-0019.

**References**

[1] Adolfson, D., Hu, T.C.: Optimal linear ordering. SIAM J. Appl. Math. 25(3), 403–423 (1973)
[2] Bischoff, M., Klamroth, K.: An efficient solution method for Weber problems with barriers based on genetic algorithms. Eur. J. Oper. Res. 177, 22–41 (2007)
[3] Cabot, A.V., Francis, R.L., Stary, M.A.: A network flow solution to a rectilinear distance facility location problem. AIIE Transactions, 2(2), 132–141 (1970)
[4] Chan, A.W., Francis, R.L.: Some layout problem on the line with interdistance constraints and costs. Oper. Res., 27(5), 952–971 (1979)
[5] Farahani, R.Z., Hekmatfar, M.: Facility location: Concepts, models, algorithms and case studies. (Heidelberg: Physica-Verlag, (2009))
[6] Klamroth, K.: Single–facility location problems with barriers. (Springer Series in Operations Research, (2002))
[7] Kuhn, H.W.: A note on Fermat’s problem. Math. Programming, 4, 98–107 (1973)
[8] Love, R.F., Wong, J.Y.: On solving a One–Dimensional Space Allocation Problem with Integer Programming. INFOR 14(2), 139–143 (1976)
[9] McGarvey, R.G., Cavalier, T.M.: Constrained location of competitive facilities in the plane. Comput. Oper. Res., 32, 539–578 (2005)
[10] Nickel, S., Puerto, J.: Location theory. A unified approach. (Berlin: Springer, (2005))
[11] Picard, J.C., Queyranne, M.: On the one–dimensional space allocation problem. Oper. Res. 29(2), 371–391 (1981)
[12] Picard, J.C., Ratliff, D.H.: A cut approach to the rectilinear distance facility location problem. Oper. Res. 26(3), 422–433 (1978)
[13] Simmons, D.M.: One–dimensional space allocation: an ordering algorithm. Oper. Res. 17(5), 812–826 (1969)
[14] Zabudskii, G.G., Amzin, I.V.: Algorithms of compact location for technological equipment on parallel lines (in Russian). Sib. Zh. Ind. Mat. 16(3), 86–94 (2013)
[15] Zabudsky, G.G., Veremchuk, N.S.: About One–Dimensional Space Allocation Problem with forbidden zones. IOP Conf. Series: Journal of Physics: Conf. Series, 1260 (2019), 082006, doi:10.1088/1742-6596/1260/8/082006
[16] Zabudsky, G.G., Veremchuk, N.S.: An algorithm for finding an approximate solution to the Weber problem on a line with forbidden gaps. Diskret. Anal. Issled. Oper., 23(1), 82–96 (2016) [J. Appl. Ind. Math., 10(1), 136–144 (2016)].
[17] Zabudsky, G.G., Veremchuk, N.S.: Branch and Bound Method for the Weber Problem with Rectangular Facilities on Lines in the Presence of Forbidden Gaps. Springer International Publishing AG, part of Springer Nature 2018 A. Eremeev et al. (Eds.): OPTA 2018, CCIS 871, 29–41 (2018). https://doi.org/10.1007/97833199380043
[18] Zabudsksy, G.G., Veremchuk, N.S.: Models and methods for One–Dimensional Space Allocation Problem with forbidden zones. IOP Conf. Series: Journal of Physics: Conf. Series, 1441 (2020), 012177, doi:10.1088/1742-6596/1441/1/012177
[19] Zabudsky, G.G., Veremchuk, N.S.: On the One–Dimensional Space Allocation Problem with partial order and forbidden zones. CCIS, 1090, 131–143 (2019), doi: 10.1007/978303033394211