1. Evolution of the hBN and 2D line with magnetic field

2D line

In contrast to the G line, the 2D line is based on a double resonant Raman process\textsuperscript{S1} where two large wave vector phonons connect the electronic bands at the K and K' points at an
excitation energy of $\pm E_{\text{Laser}}/2 \approx \pm 1.16 \text{ eV}$ away from the Fermi energy of undoped graphene. Thanks to the linear electronic dispersion of graphene, the 2D mode phonons cannot connect electronic states near the charge neutrality point since there is a missing momentum, i.e. a missing wave vector of around $\Delta q \approx 2 \times 0.167 \text{ Å}^{-1}$. This leads to a complete suppression of Landau damping for the 2D mode (see e.g. Figures 2e and 2g in Ref. [S2]), in contrast to the G mode. In other words, at low doping there is no intrinsic coupling of the phonons relevant for the 2D process to the electronic system. Even at high magnetic fields of up to 9 T, where wave vectors are getting mixed on the scale of the inverse magnetic length, $1/l_B = \sqrt{eB/\hbar} \approx 0.017 \text{ Å}^{-1}$ (at $B = 9$ T) the Kohn anomaly at K (at zero doping) remain inaccessible for the 2D mode phonons ($\Delta q \gg 1/l_B$). It is important to note that typical doping values in graphene are far below 1.16 eV such that this also completely suppresses any potential coupling between 2D mode phonons and filled and unfilled Landau levels (at these high energies). Consequently, there is no strong $B$ field dependence expected for the 2D mode.

In Figure S1 we show both the 2D peak position (Figure S1a) and FWHM (Figure S1b) as a function of $B$ field. In contrast to the G line there is only a weak dependence on magnetic field, which is most likely due to the finite curvature of the trajectories of the photo-excited electrons and holes in the magnetic field.

**hBN line**

Due to the band gap of around 6.4 eV in hBN, there are - apart from unwanted trap states - no electronic states to which the hBN $E_{2g}$ phonons can couple. This makes the hBN mode nearly $B$ field independent as confirmed by Figure S2, where both the hBN peak position (Figure S2a) and FWHM (Figure S2b) are plotted as a function of magnetic field.
Figure S1: Evolution of the 2D line frequency (a) and FWHM (b) with $B$ field for one spot on the hBN-SLG-hBN part.

Figure S2: Evolution of the hBN line frequency (a) and FWHM (b) with $B$ field for one spot on the hBN-SLG-hBN part.
2. Substrate dependence of the magneto-Raman spectrum

In Figure S3 we show the $B$ field evolution of the FWHM (Figure S3b) of the G line and the peak intensity $I(G)$ (Figure S3a) as obtained from single Lorentzian fits to the G peak data displayed in Figures 2c and 2d of the manuscript. For hBN-SLG-hBN (blue data points), MPRs can be identified as regions with increased FWHM($G$). At the same time, $I(G)$ drops at resonance points. Indeed, there is an inverse relation between FWHM($G$) and $I(G)$ (see Figure. S3c), indicating that the area of the Raman G line remains mostly constant with magnetic field (Figure S3d), as long as the phonon mode is not in resonance. It is important to note that right at the MPRs at 2.1 T and 3.7 T we observe a clear increase of the integrated intensity (area) recorded on hBN-SLG-hBN (blue data). For SiO$_2$-SLG-hBN (red data points), no distinct MPRs can be identified.

Figure S3: Evolution of the G line intensity $I(G)$ (a) and FWHM($G$) (b) with $B$ field for one spot on the hBN-SLG-hBN part (blue data points) and one spot on the SiO$_2$-SLG-hBN part (red data points). (c) Dependence of $I(G)$ on FWHM($G$)$^{-1}$ for both substrates. (d) Area of the G peak as a function of $B$ field for both substrates.
3. Doping dependence of the strength of the MPRs

In Figure S4 we show the calculated dependence of the maximum broadening of the $T_1$-MPR on the charge carrier density. For carrier densities above $n_{el} \approx 8 \times 10^{11} \text{ cm}^{-2}$, the $T_1$-resonance is fully blocked due to the Pauli principle.

![Figure S4](image)

Figure S4: Doping dependence of the $T_1$-MPR strength. Maximum broadening of the $T_1$-MPR as a function of charge carrier density. The curve was calculated with the theory presented in the main text, with the filling factor dependence taken from Ref. [S5]. The parameters were fixed to $\omega_{ph} = 1586 \text{ cm}^{-1}$, $\gamma_{ph} = 5.5 \text{ cm}^{-1}$, $\gamma_{el} = 160 \text{ cm}^{-1}$, $v_F = 1.17 \times 10^6 \text{ m/s}$, $\lambda = 4 \times 10^{-3}$, and $\lambda_L = 0.015 \lambda$.

4. Estimation of the width $n_{el}^*$ of the minimum conductance

In Figure S5 we show the method used to estimate the width $n_{el}^*$ of the minimum conductance in our contacted hBN-SLG-hBN sample, following Ref. [S6]. The data shown is taken from the black trace (Figure S5a) and the red trace (Figure S5b) in Figure 5a of the main text.
Figure S5: Estimation of the charge carrier density fluctuation. Double logarithmic plot of charge carrier density versus conductivity (red trace). The charge carrier density fluctuation $n_{el}^*$ can be estimated from the intersection of the two black lines. The back gate voltage has been set to $V_g = 0$ V (a) and $V_g = -20$ V (b) prior to illuminating the sample with the laser light. From the traces we obtain values of $n_{el}^* = 6.6 \times 10^{10}$ cm$^{-2}$ (a) and $n_{el}^* = 5.6 \times 10^{10}$ cm$^{-2}$ (b).

5. Theoretical description of magneto-phonon resonances in bilayer graphene

A theoretical description of magneto-phonon resonances in bilayer graphene can be obtained by calculating the self-energy of the G mode, which can then be used to obtain a prediction for the expected shift of the G line position ($\omega_G$) and the G line width ($\Gamma_G$). Here we follow Ando$^{57}$ and use the simplest possible model with the two graphene layers being coupled by a single inter-layer hopping term $\Delta = \gamma_1$ with $\gamma_1 \approx 0.39$ eV being the largest inter-layer tight-binding parameter. Like in single-layer graphene, electrons condense into Landau levels when a perpendicular magnetic field is applied. They are labeled by integer numbers $n \in \mathbb{Z}$ and $p = -1, +1$, with the latter labeling the upper ($p = +1$) or lower subband ($p = -1$) of bilayer graphene. Note that we omit the subband index in the main text as we only refer to the states in the lower subband. For $n = 0$, there is only one state in the lower subband with energy $\varepsilon_{n=0,p=-1} = 0$. For $|n| = 1$ one finds three states, one in the lower subband also with energy $\varepsilon_{n=1,p=-1} = 0$ and two states in the upper subband with energies $\varepsilon_{n=\pm1,p=+1} = \pm \sqrt{\Delta^2 + \varepsilon_B^2}$, where $\varepsilon_B$ is given by $\varepsilon_B = \hbar \omega_B = v_F \sqrt{2eB\hbar}$, with $v_F$ being the Fermi velocity. For the case $|n| \geq 2$, there are two states in each subband with their energies described by$^{58}$
\[\varepsilon_{n,p} = \text{sgn}(n) \left[ \frac{\Delta^2 + (2|n| - 1)\varepsilon_B^2}{2} + \frac{p}{2} \sqrt{\Delta^4 + 2(2|n| - 1)\Delta^2\varepsilon_B^2 + \varepsilon_B^4} \right]^{1/2}. \]  \hspace{1cm} (S1)

Excitations between two of these states again couple to the G mode, leading to a contribution to the self-energy, which has been calculated before\textsuperscript{S7} and is given by

\[\Pi(\omega) = -\lambda \hbar \omega_B^2 \sum_{n,n'=-N_{\text{Cut}}}^{N_{\text{Cut}}} \sum_{p,p'=\pm 1} N_{n,p}^2 N_{n',p'}^2 f(n,p;n',p') \delta_{|n|,|n'|-1} \Delta \bar{\nu}_{n,p;n',p'} \frac{2\Delta \varepsilon_{n,p;n',p'}}{2\hbar \omega^2 - \Delta \varepsilon_{n,p;n',p'}^2}, \]  \hspace{1cm} (S2)

where \(\lambda\) denotes the electron-phonon coupling constant. The function \(f(n,p;n',p')\) stems from the squared matrix elements of the electron-phonon interaction Hamiltonian and can be calculated by using the wave functions of the Landau levels.\textsuperscript{S7} An explicit expression for \(f(n,p;n',p')\) is

\[f(n,p;n',p') = \left\{ \sqrt{|n|\delta^2 \bar{\varepsilon}_{n,p}} \sqrt{|n'|(|n'| - 1)} + \sqrt{|n'|} \left[ -\frac{|n|||n| - 1)}{\bar{\varepsilon}_{n,p}} + |n| \varepsilon_{n,p} \right] \left[ \bar{\varepsilon}_{n',p'}^2 - (|n'| - 1) \right] \right\}^2, \]  \hspace{1cm} (S3)

where we introduced the shorthand notations \(\delta = \Delta/\varepsilon_B\) and \(\bar{\varepsilon}_{n,p} = \varepsilon_{n,p}/\varepsilon_B\). \(N_{n,p}\) is a wave function normalization constant given by

\[N_{n,p} = \left\{ n(|n| - 1) \delta^2 + \left[ \frac{|n|||n| - 1)}{\bar{\varepsilon}_{n,p}} \right]^2 + |n| \left[ \bar{\varepsilon}_{n,p}^2 (2\delta^2 + |n| + 1) - 2|n|^2 + |n| + 1 \right] \right\}^{-\frac{1}{2}}. \]  \hspace{1cm} (S4)

We further introduced \(\Delta \bar{\nu}_{n,p;n',p'} = \bar{\nu}_{n,p} - \bar{\nu}_{n',p'}\) to denote the difference of the partial filling factors of the involved Landau levels. For bilayer graphene, we take the latter to be given by
\[ \bar{\nu}_{n,p} = \begin{cases} 
(n + 4)/8, & n = 0, 1; p = -1 \\
(n + 4 - 4n)/4, & |n| \geq 2; p = -1 \\
1, & n < -1; p = +1 \\
0, & n > +1; p = +1 
\end{cases} \] (S5)

where \( \nu = n_{el}h/(eB) \) is the usual filling factor, which depends on the charge carrier density \( n_{el} \) and magnetic field. Note that for reasonable doping values and magnetic fields, the states in the upper subband can be regarded as completely filled (valence band) or completely empty (conduction band). The first line of Eq. S5 specifically takes into account the doubly degenerate Landau level at zero energy.

Finally, we introduced the excitation energies \( \Delta \varepsilon_{n,p,n',p'} = \varepsilon_{n,p} - \varepsilon_{n',p'} \). In order to account for a finite lifetime of the excitations, we again introduce a phenomenological broadening parameter \( \gamma_{el} \) via \( |\Delta \varepsilon_{n,p,n',p'}| \to |\Delta \varepsilon_{n,p,n',p'}| - i\hbar\gamma_{el}/2 \). In contrast to the model we use for single-layer graphene, we find that one generic broadening parameter for all LL excitations is sufficient for an accurate description of the data.

As for the SLG, the electronic contribution to the self-energy in the adiabatic limit \( \omega \to 0 \) has to be subtracted from \( \Pi(\omega) \) as this contribution is already included in the definition of \( \omega_G \), i.e. we replace \( \Pi(\omega) \to \Pi(\omega) - \Pi(\omega = 0) \).

The above expression for the phonon self-energy can then be used to find the renormalized phonon frequency \( \omega_G \) and FWHM \( \Gamma_G \) by solving the equation \( \omega^2 - \omega_{ph}^2 - 2\omega_{ph}\Pi(\omega) = 0 \), where \( \omega = \omega_G - i\Gamma_G/2 \) and \( \omega_{ph} \) denotes the phonon frequency at zero magnetic field. Non-electronic broadening mechanisms are accounted for by making the replacement \( \omega_{ph} \to \omega_{ph} - i\gamma_{ph}/2 \), where \( \gamma_{ph} \) is the phonon width without electronic contributions.

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