HADRONS IN THE NUCLEAR MEDIUM- ROLE OF LIGHT FRONT NUCLEAR THEORY

GERALD A. MILLER
Department of Physics, University of Washington
Seattle, WA 98195-1560
E-mail: miller@phys.washington.edu

The problem of understanding the nuclear effects observed in lepton-nucleus deep-inelastic-scattering (the EMC effect) is still with us. Standard nuclear models (those using only hadronic degrees of freedom) are not able to account for the EMC effect. Thus it is necessary to understand how the nuclear medium modifies quark wave functions in the nucleus. Possibilities for such modifications, represented by the quark meson coupling model, and the suppression of point-like-configurations are discussed, and methods to experimentally choose between these are reviewed.

1 Introduction

When the organizers asked me to give a talk entitled “Hadrons in the Nuclear Medium” I thought about what the title might mean. Since the nuclear mass $M_A \approx (NM_n + ZM_p)(1 - 0.01)$, and nucleons are hadrons, maybe the title should be “Hadrons are the Nuclear Medium!” On the other hand, the modern paradigm for the strong interaction is QCD and QCD is a theory of quarks and gluons. Maybe the title should be “Are Hadrons the Nuclear Medium?”. We have known, since the discovery of the EMC effect in 1982, that the structure functions measured in deep inelastic scattering from nuclear targets are not those of free nucleons. So one theme of this talk is to try to understand, interpret and use the EMC effect. Despite the age of this effect, no consensus has been reached regarding its interpretation, importance and implications.

The second theme of this talk arises from the kinematic variables used to describe the data. The Bjorken $x$ variable: $x = Q^2 / 2M\nu$ is, in the parton model, a ratio of quark to target momenta $p^+_q / P^+_A$, where the superscript + refers the plus-component of the four-momentum vector. This in turn can be written as: $(p^+_q / P^+_N) (p^+_N / P^+_A)$, so that one needs to know how often a nucleon has a given value of plus-momentum, $p^+_N$. Conventional nuclear wave functions are not expressed in terms of this variable, so one needs to derive nuclear wave functions which are expressed in terms of plus-momenta. Therefore, I assert that light front nuclear theory is needed.
2 Outline

I turn towards a more detailed outline. The first part of the talk is concerned with what I call the “Return of the EMC effect”. This is the statement that conventional nuclear physics does not explain the EMC effect. The physics here is subtle, so I believe that some formal development involving the construction of nuclear wave functions using light front nuclear theory is needed. I try to answer the simple queries: “light front theory- what is it? why use it?” A partial answer is reviewed in Ref. [1]. The saturation of infinite nuclear matter using the mean field approximation is discussed here. The result is that there is no binding effect which explains the EMC effect [2,3]. This statement is a natural consequence of the light version of the Hugenholtz-Van Hove Theorem [4]: the pressure of a stable system vanishes. If one goes beyond the mean-field calculations and includes correlations by using a Hamiltonian which involves only nucleon degrees of freedom, there is again no binding effect. Furthermore, I’ll argue that using mesons along with nucleons probably won’t allow a description of all the relevant data.

It is therefore reasonable, proper and necessary to examine the subject of how the internal structure of a nucleon is modified by the nucleus to which it is bound. To see how the medium modifies the wave function of nucleon, a particular model of the free proton wave function [5] is used. This allows the examination of two different and complementary ideas: the quark meson coupling model [6] and the suppression of point-like-configurations [7].

A quick summary is that the goal is to explain EMC effect and then predict new experimental consequences. This goal is not attained but is within reach.

3 Return of the EMC effect

It is necessary to use nuclear wave functions in which one of the variables is the plus-momentum of a nucleon, $p^+_N$. Thus the use of light front quantization, or light-front dynamics, which I now try to explain, is necessary.

3.1 Light Front Quantization Lite

Light-front dynamics is a relativistic many-body dynamics in which fields are quantized at a “time” $\tau = x^0 + x^3 \equiv x^+$. The $\tau$-development operator is then given by $P^0 - P^3 \equiv P^-$. These equations show the notation that a four-vector $A^\mu$ is expressed in terms of its $\pm$ components $A^\pm \equiv A^0 \pm A^3$. One quantizes at $x^+ = 0$ which is a light-front, hence the name “light front dynamics”. The
canonical spatial variable must be orthogonal to the time variable, and this is given by \( x^-=x^0-x^3 \). The canonical momentum is then \( P^+ = P^0 + P^3 \). The other coordinates are as usual \( x_\perp \) and \( P_\perp \).

The most important consequence of this is that the relation between energy and momentum of a free particle is given by:

\[
p_\mu p^\mu = m^2 = p^+ p^- - p_\perp^2 \rightarrow p^- = \frac{p_\perp^2 + m^2}{p^+},
\]

a relativistic formula for the kinetic energy which does not contain a square root operator. This feature allows the separation of center of mass and relative coordinates, so that the computed wave functions are frame independent.

The philosophy, for the beginning of this talk, is to use a Lagrangian density, \( \mathcal{L} \) which is converted into an energy-momentum tensor \( T^{\mu\nu} \). The total-four-momentum operator is defined formally as

\[
P^\mu = \frac{1}{2} \int d^2 x_\perp dx^- T^{+\mu},
\]

with \( P^+ \) as the “momentum” operator and \( P^- \) as the “energy” operator. We then need to express \( T^{+\mu} \) in terms of independent variables. In particular, the nucleon, usually described as a four-component spinor, is a spin 1/2 particle and therefore there are only two independent degrees of freedom.

I’ll start with the well-known Walecka model in which \( \mathcal{L}(\phi, V^\mu, N) \) is expressed in terms of nucleon \( N \), vector meson \( V^\mu \), and scalar meson \( \phi \) degrees of freedom. The plan is to first carry out calculations using the mean field approximation, and then include the effects of \( NN \) correlations using other Lagrangians.

### 3.2 Light Front Quantization

The mode equation for nucleons in infinite nuclear matter nuclear matter is given by

\[
(\not{k} - g_v \not{V} - (M + g_s \phi)) \psi = 0,
\]

within the mean field approximation of the Walecka model. The quantity of relevance for understanding deep inelastic scattering is the nuclear plus component of momentum given by

\[
P^+_A = \langle A | \psi_+^{\dagger} (i\partial^+ + g_v V^+) \psi_+ | A \rangle,
\]

where \( \psi_+ = \frac{1}{2} \gamma^0 \gamma^+ \psi \), the independent component of the nucleon field. Furthermore

\[
P^-_A = \langle A | \psi_+^{\dagger} \left( \frac{-\partial^2 + (M + g_s \phi)^2}{i\partial^+} \right) \psi_+ | A \rangle + m_s^2 \psi_+^2.
\]
In the rest frame we must have $P_A^+ = M_A$, $P_A^- = M_A = E_A$, a result not obvious from the above equations. However, if we minimize $P_A^-$ subject to the constraint that the expectation value of $(P_A^+ - P_A^-)$ vanish, we indeed get the same $E_A$ as Walecka and more! The more refers to information about the plus momenta.

The result $P_A^+ = P_A^-$ means $P_A^3 = 0$, which is a statement that pressure vanishes for a stable system. According to a venerable 1958 theorem by Hugenholtz & Van Hove, a vanishing pressure, plus the definition that the nucleon Fermi energy $E_F \equiv \langle \frac{dE}{dA} \rangle_{\text{volume}}$, gives $E_F = \frac{E_A}{M_A} = \frac{M_A}{M}$. This has important consequences now because we may express the result (4) as

$$P_A^+ = M_A = A \int dk^+ f_N(k^+) k^+. \quad (6)$$

Next we use a dimensionless variable $y \equiv \frac{k^+}{M}$ to find that

$$\int f_N(y) \, y = 1, \quad (7)$$

which means that nucleons carry all of the plus momentum.

The relevance of this can be seen by calculating the effects of nucleons in deep inelastic lepton nucleus scattering using a manifestly covariant calculation of the handbag diagram. One finds

$$\frac{F_{2A}(x_A)}{A} = \int_x^{\infty} df_N(y) F_{2N}(x_A/y), \quad (8)$$

where

$$f_N(y) = \int \frac{d^4k}{(2\pi)^4} \delta(y - \frac{k^0 + k^3}{M}) Tr \left[ \frac{\gamma^+}{A} \chi(k, P) \right], \quad x_A = xM/M. \quad (9)$$

The quantity $\chi(k, P)$ is the nuclear expectation value of the connected part of the nucleon Green’s function. This can easily be calculated for our light front nuclear wave functions. The result is

$$f_N(y) = 4 \int \frac{dk^+ d^2k_\perp}{(2\pi)^4 \rho_B} \theta \left( k_F^2 - k^2 \right) \left( k^+ + g_v V^+ - E_F \right)^2 \right) \delta \left( y - \frac{k^+ + g_v V^+}{M} \right) \quad (10)$$

or

$$f_N(y) = \frac{3M^3}{4 k_F^2} \theta(1 + k_F/M - y) \theta(y - (1 - k_F/M)) \left[ \frac{k_F^2}{M^2} - (1 - y)^2 \right], \quad (11)$$
which obeys the baryon $\int dy f_N(y) = 1$ and momentum $\int dy f_N(y)$ sum rules, so that nucleons carry all of the plus momentum.

This is important because $f_N(y)$ is narrowly peaked at $y = 1$, so that $F_{2A}(x_A) \approx A F_{2N}(x_A)$, and there is almost NO Binding Effect.

One can see this more directly by expanding $F_2(x/y)$ in Taylor series about $y = 1$ to find:

$$F_{2N}(x_A)/A = F_{2N}(x) + \epsilon F_{2N}'(x) + \frac{k^2}{10M} (2x_A F_{2N}'(x_A) + x_A^2 F_{2N}''(x_A)).$$

where $\epsilon = 1 - M/M_N \approx 16/940$. The resulting figure is shown as Fig. 2 of Ref. 3 but Eq. (12) shows clearly that $F_{2N}(x_A)/F_{2N}(x)$ is too large.

Similar calculations can now be done for finite nuclei 10. Although being able to do these calculations is a major technical achievement (according to me), the results also show a huge disagreement with experiment.

3.3 Beyond the Mean Field Approximation

Suppose one assumes nucleons are the only degrees of freedom in the Hamiltonian, so that

$$H = \sum_i t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk} + \cdots. \quad (13)$$

Any correct solution gives must be consistent with the Hugenholtz Van Hove Theorem $P^+ = P^- = M_A = P^+_N$, so that once more one finds

$$\int dy f_N(y) y = 1. \quad (14)$$

One may again expand make an expansion about $y = 1$ to find that

$$F_{2N}(x_A)/A = F_{2N}(x) + \langle \epsilon \rangle F_{2N}'(x) + \gamma (2x_A F_{2N}'(x_A) + x_A^2 F_{2N}''(x_A)), \quad (15)$$

where $\gamma = \int dy f_N(y)(y - 1)^2$. Again there is NO binding effect, and the results are even worse, in comparison with experiment, than before. Using a more elaborate many-body calculation with a Hamiltonian involving only nucleons can not explain the EMC effect.

3.4 Nucleons $N$ and Mesons $m$

The best version of the conventional approach is to explicitly include the effects of mesons. Then one may compute the nuclear expectation value of
\[ T^{++} \text{ as} \]
\[ P_A^+ = P_N^+ + P_m^+ = M_A, \quad \epsilon \equiv \frac{P_m^+}{M_A}, \]  
so that one would find
\[ \int dy f_N(y) = 1 - \epsilon. \]

Many authors can reproduce the EMC binding effect using \( \epsilon \approx 0.07 \), but \( \epsilon \approx 0.07 \) corresponds to a BIG enhancement of the nuclear sea. Some time ago theorists suggested that the nuclear Drell-Yan process could be used to disentangle the EMC effect, but data from E772 at Fermilab showed no enhancement, and no nuclear effects. This finding was termed a “Crisis in Nuclear Theory”.

### 3.5 Summary of Return of the EMC Effect

Conventional nuclear theory is unable to provide an explanation of the EMC effect. Mean field theory gives no EMC effect. Any theory involving using only nucleons as degrees of freedom gives no EMC effect. Including the effects of mesons explicitly (and not buried in the nucleon-nucleon potential) can provide an explanation or description of the EMC effect, but seems to cause a disagreement with the Drell-Yan data. Thus it is entirely legitimate, correct and proper to take very seriously the proposition that the structure of the nucleon is modified by its presence in the nuclear medium.

### 4 Non-Standard Nuclear Physics

The idea that modification of nucleon properties has important experimental consequences can be termed as non-standard nuclear physics. The main goal I wish to discuss here is that of first finding some model (or models) that reproduce both the EMC effect and the nuclear Drell-Yan data, and then predict other experimental consequences. There are many ideas in the literature. We shall take up only two here. The first involves nucleon wave function modification by the nuclear mean field. This is the quark meson coupling model introduced by Guichon and applied to understanding the EMC effect by Saito and Thomas. The nucleus is bound by the effects of mesons which are exchanged between quarks in different nucleons, so that the wave function of a quark in a bound nucleon is modified.

The second idea involves the suppression of point like configurations (PLC), introduced by Frankfurt and Strikman as a consequence of the effects of color neutrality of nucleons. The idea is that the nucleon consists of an
infinite number of configurations of different sizes, each having different interactions with the nuclear medium. There are configurations with anti-quarks and gluons which are large and blob-like (BLC). These are influenced by the attractive force provided by other nucleons. There are also rarer configurations consisting of only three quarks which are close to each other. These PLC do not interact because the effects of gluons emitted by such a color-singlet configuration are canceled. The energy differences between the BLC and the rarer PLC are increased by the nuclear medium, so the probability of the PLC are decreased. This is the suppression we speak of. The validity of this idea was checked and confirmed by Frank, Jennings and Miller.

Our attitude is the quark meson coupling model and the suppression of PLC are different reasonable hypotheses for nuclear modifications of nucleon wave functions which should be taken seriously.

4.1 Light Front Model of Proton

The basic idea is to put a nucleon in the medium and study how it responds to the external forces provided by the other nucleons. We need a relativistic model, and a convenient one is that of Schlumpf, which was recently exploited by us. The model wave function is written in terms of light-cone variables and can be written schematically as

$$\Psi(p_i) = u(p_1)u(p_2)u(p_3)\psi(p_1, p_2, p_3),$$

(18)

in which \(p_i\) represent space (cm), spin and isospin variables, and in which the \(u\) are conventional Dirac spinors. The function \(\psi\) depends on spin and isospin and includes a spatially symmetric function \(\Phi(M_0^2)\). The quantity \(M_0^2\) is the square of the mass of a non-interacting system of three quarks which plays the role that the square of spatial three momentum would in an ordinary wave function. Thus:

$$M_0^2 = \sum_{i=1,3} \frac{p_{i+}^2 + m^2}{p_i^3} + \Phi(M_0^2) = \frac{N'}{(M_0^2 + \beta^2)\gamma},$$

(19)

The wave function is now specified. It is expressed in terms of relative variables and is a boost invariant light front wave function.

The first application is how the electromagnetic form factors are modified in the medium, so we need to consider the model’s version of the form factors of a free nucleon. These are obtained by sandwiching the wave functions around the current operator \(J^+ \sim \gamma^+\). The evaluation of this Dirac operator is simplified by making a unitary transformation to represent the wave function \(\Psi\) in terms of light front spinors which have the nice property:
\[ \bar{u}_L(p')\gamma^+u_L(p\lambda) = 2p^+\delta_{\lambda\lambda'}. \]

This allows us to interpret the results in an analytic fashion. The coefficients of the unitary transformation are known as the Melosh transformation, and one example is given by

\[ \bar{u}_L(p_3,\lambda_3)u(p_3,s_3) = \langle \lambda_3 \left| \frac{m + (1 - \eta)M_0 + i\sigma \cdot (\mathbf{n} \times \mathbf{p}_3)}{\sqrt{(m + (1 - \eta)M_0)^2 + p_{3\perp}^2}} \right| s_3 \rangle. \] (20)

The basic idea is that the spin flip term given by \( i\sigma \cdot (\mathbf{n} \times \mathbf{p}_3) \) is as large as the non-spin flip term given by \( m + (1 - \eta)M_0 \). For large momentum transfer, \( Q \), each of these is proportional to \( Q \). The form factor \( F_1 \) depends on the non-spin-flip term: \( F_1 \sim Q \cdots \), and \( F_2 \) depends on the spin-flip term, so \( QF_2 \sim Q \cdots \), as well. Thus the ratio \( QF_2/F_1 \) is approximately constant for sufficiently large \( Q \). The results are shown in Fig. 1 and are in good agreement with the recent exciting data\(^{17,18} \). This means we have a reasonable model nucleon to put in the nuclear medium.

### 4.2 Medium modifications of nuclear form factors

Let’s start with the quark meson coupling model QMC. We approximate that the nuclear scalar \( \sigma \) and vector potentials are constant over the volume of the nucleon. Then the modification is simply expressed as \( m \rightarrow m - \sigma \), with the average scalar field experienced by a quark is given by \( \sigma \approx 40 \text{ MeV} \). Thus we simply reduce the quark mass used in the previous calculation by 40 MeV. The results are shown in Figs. 2 and 3.

These results seem to give huge effects, but one must understand that these are form factors evaluated in the nuclear ground state. The only attempts to observe such effects have involved using the \((e,e'p)\) reaction. Thus only the proton in the initial state is modified in the manner described here. Furthermore, the reaction occurs at the nuclear surface where the \( \sigma \) field is falling off towards zero. When such realities are included, the results would be similar to the theory of Ref.\(^{19} \) and similar to the \(^4\text{He} \) data of Dieterich \( et \ al.\(^{20} \) \) and the \(^{16}\text{O} \) data Malov \( et \ al.\(^{21} \) \). For \(^4\text{He} \), the actual effects are about four times smaller than shown here.

### 4.3 Suppression of PLC

The idea here is that the interaction of a bound nucleon with a nucleus depends on the distances between the quarks. The relevant operator is

\[ r^2 \equiv \sum_{i<j}(\mathbf{r}_i - \mathbf{r}_j)^2, \] (21)
so that the nuclear mean field $U$ is given by

$$U(r, R) = U_0 \rho(R) \frac{r^2}{\langle N | r^2 | N \rangle},$$

which vanishes for PLC. Then, in first-order perturbation theory, the modification of the expectation value of any operator $\mathcal{O}$ is given by

$$\delta\langle \mathcal{O} \rangle = 2 \frac{U_0 \rho(R)}{\Delta E} \left( \frac{\langle N | \mathcal{O} r^2 | N \rangle}{\langle N | r^2 | N \rangle} - \langle N | \mathcal{O} | N \rangle \right).$$

The resulting form factors are shown in Figs. 10,11 of Ref. 15. The effects of the medium are smaller here than for the QMC. But there are substantial modifications of the valence quark distribution functions, as shown in Fig. 16 of Ref. 15. These are relevant for understanding the EMC effect.
There are many ideas available to explain EMC effect. More experiments are needed to select the correct ones. Here I only have room to discuss one possibility in which one studies the reaction: $e'D \rightarrow e + N + X$ for the kinematics of deep-inelastic scattering. The ratio

$$G(x_1, x_2) = \frac{\sigma(x_1, k^+, k_\perp, Q^2)}{\sigma(x_1, k^+, k_\perp, Q^2)} = \frac{F_{2N}^{\text{mod}}(x_1/(2 - k^+/M), k_\perp, Q^2)}{F_{2N}^{\text{mod}}(x_2/(2 - k^+/M), k_\perp, Q^2)}$$

(24)

is extremely sensitive to the different models, in which models giving the same DIS have differences of more than 50%. This is shown in Fig. 6 of Ref. 22.
5 Summary

The use of the standard, conventional meson-nucleon dynamics of nuclear physics is not able to explain the nuclear deep-inelastic and Drell-Yan data. The logic behind this can be understood using the Hugenholtz-van Hove theorem, which states that the stability (vanishing of pressure) causes the energy of the single particle state at the Fermi surface to be $M_A/A \approx 0.99 M_N$. In light front language, the vanishing pressure is achieved by obtaining $P^+ = P^- = M_A$. But $P^+ = \int dk^+ f_N(k^+)k^+$. This, combined with the relatively small value of the Fermi momentum (narrow width of $f_N(k^+)$) means that the probability $f_N(k^+)$ for a nucleon to have a given value of $k^+$ must be narrowly peaked about $k^+ = 0.99 M_N \approx M_N$. Thus the effects of nuclear binding and Fermi motion play only a very limited role in the nuclear structure function, and the resulting function must very close to the one of a free nucleon unless some quark-gluon effects are included. This means that some
non-standard explanation involving quark-gluon degrees of freedom is necessary. Many contending non-standard ideas available. Testing these models, selecting the right ones, and ultimately determining the dynamical significance depends on having new high accuracy experiments at large momentum transfer and energy.

Acknowledgments

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