Fast quantum information transfer with superconducting flux qubits coupled to a cavity

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Abstract
We present a way to realize quantum information transfer with superconducting flux qubits coupled to a cavity. Because only resonant qubit–cavity and qubit–pulse interactions are applied, the information transfer can be performed much faster when compared with the previous proposals. This proposal does not require adjustment of the qubit level spacings during the operation. Moreover, neither uniformity in the device parameters nor exact placement of qubits in the cavity are needed.

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(Some figures may appear in colour only in the online journal)

1. Introduction
The physical system, composed of circuit cavities and superconducting qubits (such as charge, phase and flux qubits), has appeared as one of the most promising candidates for realizing scalable quantum information processing. Superconducting qubits and microwave cavities can be fabricated with modern integrated circuit technology; a superconducting qubit has relatively long decoherence time [1], and a superconducting microwave cavity or resonator acts as a ‘quantum bus’, which can mediate long-range and fast interaction between distant superconducting qubits [3–5]. Moreover, the strong coupling between the cavity field and superconducting qubits, which is difficult to achieve with atoms in a microwave cavity, was predicted earlier by theory [6, 7] and has been demonstrated experimentally [8, 9].

In addition, much attention has been paid to quantum information transfer (QIT). One example to illustrate the importance of QIT is as follows. When performing quantum information processing in a practical system, one needs to transfer the state of the operation qubit to the memory qubit for storage after a step of processing is completed; and one needs...
to transfer the state from the memory qubit back to the operation qubit when a further step of processing is needed. Within the cavity QED technique, QIT has been experimentally demonstrated with superconducting phase qubits and transmon qubits in cavity QED [4, 10]. However, to the best of our knowledge, no experimental demonstration of QIT with superconducting flux qubits in cavity QED has been reported.

Theoretical methods for implementing QIT [3, 6, 11–14] have been presented with flux qubits (e.g., SQUID qubits) or charge–flux qubits based on the cavity QED technique. However, these methods have some drawbacks. For instance: (i) the method presented in [3] requires adjustment of the level spacings of the devices during the operation; (ii) the methods proposed in [6, 11–13] require slowly changing of the Rabi frequencies to satisfy the adiabatic passage; and (iii) the approach introduced in [14] requires a second-order detuning to achieve an off-resonant Raman coupling between two relevant levels. Note that the adjustment of the level spacings during the operation is undesirable and also may cause extra decoherence. In addition, when the adiabatic passage or a second-order detuning is applied, the operation becomes slow (the operation time required for the information transfer is of the order of 1 μs to a few microseconds [6, 14]).

In this paper, we propose an alternative method for realizing QIT with four-level superconducting flux qubits coupled to a cavity or a resonator. This proposal has the following advantages: (i) because only resonant interactions are applied, the speed of the operation is increased by two orders of magnitude (as shown below, the operation time is on the order of ∼1 ns), when compared with the previous proposals [6, 11–14] employing a second-order large detuning or adiabatic passage; (ii) the method does not need adjustment of the qubit level spacings during the operation, and thus decoherence caused due to the adjustment of the qubit level spacings is avoided; and (iii) the qubit–cavity coupling constants are not required to be identical for each qubit; therefore, superconducting devices, which often have considerable parameter nonuniformity, can be used and exact placement of qubits in the cavity is not required.

This paper is organized as follows. In section 2, we briefly review the basic theory of resonant qubit–cavity and qubit–pulse interactions. In section 3, we show how to realize QIT with superconducting flux qubits coupled to a cavity or a resonator. In section 4, we briefly discuss possible experimental implementation with superconducting flux qubits coupled to a one-dimensional transmission line resonator. A concluding summary is presented in section 5.

2. Basic theory

The flux qubits throughout this paper have four levels |0⟩, |1⟩, |2⟩ and |3⟩, as depicted in figure 1. In general, there exists the transition between the two lowest levels |0⟩ and |1⟩, which, however, can be made to be weak by increasing the potential barrier between the two levels |0⟩ and |1⟩ [1, 15, 16]. The qubits with this four-level structure could be a radio-frequency superconducting quantum interference device (rf SQUID) consisting of one Josephson junction enclosed by a superconducting loop or a superconducting device with three Josephson junctions enclosed by a superconducting loop. For flux qubits, the two logic states of a qubit are represented by the two lowest levels |0⟩ and |1⟩.

2.1. Qubit–cavity resonant interaction

Consider a flux qubit with four levels, as shown in figure 1. Suppose that the transition between the two levels |2⟩ and |3⟩ is resonant with the cavity mode. In the interaction picture and under
Figure 1. Level diagram of a four-level flux qubit, with forbidden or weak transition between the two lowest levels. The cavity mode is resonant with the transition between the top two levels. g is the coupling constant between the cavity mode and the $|2\rangle \leftrightarrow |3\rangle$ transition.

the rotating-wave approximation, the interaction Hamiltonian of the qubit and the cavity mode is given by

$$H = \hbar g(a^\dagger \sigma_{23}^- + \text{H.c.}),$$

where $a^\dagger$ and $a$ are the photon creation and annihilation operators of the cavity mode, $g$ is the coupling constant between the cavity mode and the $|2\rangle \leftrightarrow |3\rangle$ transition of the qubit, and $\sigma_{23}^- = |2\rangle \langle 3|$. Based on Hamiltonian (1), it can easily be found that the initial states $|3\rangle_c|0\rangle_c$ and $|2\rangle_c|1\rangle_c$ of the qubit and the cavity mode evolve as follows:

$$|3\rangle_c|0\rangle_c \rightarrow -i \sin(g\tau)|2\rangle_c|1\rangle_c + \cos(g\tau)|3\rangle_c|0\rangle_c,$$

$$|2\rangle_c|1\rangle_c \rightarrow \cos(g\tau)|2\rangle_c|1\rangle_c - i \sin(g\tau)|3\rangle_c|0\rangle_c. \quad (2)$$

However, the state $|0\rangle_c|0\rangle_c$ remains unchanged under Hamiltonian (1).

The coupling strength $g$ may vary with different qubits due to non-uniform device parameters and/or non-exact placement of qubits in the cavity. Therefore, in the operation below, $g$ will be replaced by $g_1$ and $g_2$ for qubits 1 and 2, respectively.

2.2. Qubit–pulse resonant interaction

Consider a flux qubit with four levels, as depicted in figure 1, driven by a classical pulse. Suppose that the cavity mode is resonant with the transition between the two levels $|i\rangle$ and $|j\rangle$ of the qubit. Here, the level $|i\rangle$ is the lower energy level. In the interaction picture and under the rotating-wave approximation, the interaction Hamiltonian is given by

$$H_I = \hbar (\Omega_{ij} e^{i\phi}|i\rangle \langle j| + \text{H.c.}),$$

where $\Omega_{ij}$ is the Rabi frequency of the pulse and $\phi$ are the initial phases of the pulse. Based on Hamiltonian (3), it is straightforward to show that a pulse of duration $\tau$ results in the following state transformation:

$$|i\rangle \rightarrow \cos(\Omega_{ij}\tau)|i\rangle - ie^{-i\phi} \sin(\Omega_{ij}\tau)|j\rangle,$$

$$|j\rangle \rightarrow \cos(\Omega_{ij}\tau)|j\rangle - ie^{i\phi} \sin(\Omega_{ij}\tau)|i\rangle,$$

which can be completed within a very short time by increasing the pulse Rabi frequency $\Omega_{ij}$ (i.e. by increasing the intensity of the pulse).
Above we have given a discussion on the qubit-cavity resonant interaction and the qubit-pulse resonant interaction. Results (2) and (4) presented above will be used for the QIT implementation below.

3. Realizing QIT with flux qubits in cavity QED

Consider two flux qubits 1 and 2. Each qubit has a four-level configuration, as depicted in figure 1. To begin with, it should be mentioned that during the operations below, the following conditions are required: (i) the cavity mode is resonant with the $|2\rangle \leftrightarrow |3\rangle$ transition of each qubit; (ii) the cavity mode is highly detuned (decoupled) from the transition between any other two levels; and (iii) the pulse is resonant with the transition between two relevant levels of each qubit but highly detuned (decoupled) from the transition between any two irrelevant levels of each qubit. The first condition can be achieved by setting the level spacing between the two levels $|2\rangle$ and $|3\rangle$ to be the same for each qubit. Note that for superconducting qubits, by designing the qubits appropriately, one can easily make the level spacing between certain two levels (the two levels $|2\rangle$ and $|3\rangle$ here) to be identical [17], though it is hard to have the level spacing between any two levels be identical for each qubit due to non-uniformity of the device parameters. In addition, the second and third conditions can be achieved via prior adjustment of the qubit level spacings before the operation. For superconducting flux qubits, the level spacings can be readily adjusted by changing the external flux applied to the superconducting loop [1, 15, 16, 18, 19]). With these in mind, we now give a detailed discussion on how to realize the QIT.

The cavity mode is initially in the vacuum state $|0\rangle_c$. The procedure for realizing QIT is listed below.

Step 1. (a) Apply a pulse (with a frequency $\omega = \omega_{31}$, a phase $\phi = \pi$) and a pulse (with a frequency $\omega = \omega_{20}$, a phase $\phi = -\frac{\pi}{2}$) to qubit 1 (figure 2(a)); the duration of each pulse is $t_{1,a} = \frac{\pi}{\omega_{31}}$; according to equation (4), the first pulse leads to $|1\rangle_1 \rightarrow i|3\rangle_1$ while the second pulse results in $|0\rangle_1 \rightarrow |2\rangle_1$. (b) Wait for a time $t_{1,b} = \frac{\pi}{2\omega_{20}}$ to have the cavity mode resonantly interacting with the $|2\rangle \leftrightarrow |3\rangle$ transition of qubit 1 (figure 2(a')), resulting in
The cavity mode returns to its original vacuum state, while the following transformation is obtained:

\[
\begin{align*}
&|0\rangle_1 |0\rangle_c \otimes |0\rangle_2 &\rightarrow & |2\rangle_1 |0\rangle_c \otimes |0\rangle_2 &\rightarrow & |2\rangle_1 |0\rangle_c \otimes |0\rangle_2 \\
&|1\rangle_1 |0\rangle_c \otimes |0\rangle_2 &\rightarrow & i|3\rangle_1 |0\rangle_c \otimes |0\rangle_2 &\rightarrow & |2\rangle_1 |1\rangle_c \otimes |0\rangle_2 &\rightarrow & |0\rangle_1 |1\rangle_c \otimes |0\rangle_2.
\end{align*}
\]

(5)

Step 2. (a) Apply a pulse (with a frequency \(\omega = \omega_{20}\), a phase \(\phi = -\frac{\pi}{2}\), and a duration \(t_{2,a} = \frac{\pi}{\Delta_1}\)) to qubit 2 (figure 2(a)), resulting in |2\rangle_1 \rightarrow |0\rangle_1. It can be seen that the operation of this step results in the following transformation:

\[
|0\rangle_1 |0\rangle_c \otimes |0\rangle_2 \rightarrow |0\rangle_1 |0\rangle_c \otimes |0\rangle_2 \rightarrow |2\rangle_1 |0\rangle_c \otimes |0\rangle_2 \rightarrow |0\rangle_1 |0\rangle_c \otimes |0\rangle_2 \rightarrow |0\rangle_1 |0\rangle_c \otimes |0\rangle_2.
\]

Equation (5) shows that during the operations of step 1 on qubit 1 and the cavity, the states |0\rangle_2 and |1\rangle_2 of qubit 2 do not change. In addition, equation (6) shows that during the operation of step 2 on qubit 2 and the cavity, the states |0\rangle_1 and |1\rangle_1 of qubit 1 remain unchanged. This is because the cavity mode was initially assumed to be resonant with the [2] ↔ [3] transition but highly detuned (decoupled) from the transition between any other two levels of each qubit.

Based on results (5) and (6), we obtain the transformation below:

\[
|0\rangle_1 |0\rangle_2 \rightarrow |0\rangle_1 |0\rangle_2 \rightarrow |0\rangle_1 |0\rangle_2 \rightarrow |0\rangle_1 |0\rangle_2.
\]

(7)

From the above equation, it is easy to see that when qubit 1 is initially in the state \(|\alpha\rangle|0\rangle + |\beta\rangle|1\rangle\) (with \(|\alpha|^2 + |\beta|^2 = 1\)) and qubit 2 initially in the state |0\rangle_2 before the operation of step 1, the initial state \((|\alpha\rangle|0\rangle + |\beta\rangle|1\rangle)|0\rangle_2\) of the whole system is transformed to the state \(|\alpha\rangle|0\rangle_2 + |\beta\rangle|1\rangle_2\) after the above two-step operation. This result implies that after the above operations, the cavity mode returns to its original vacuum state; while the following transformation

\[
(|\alpha\rangle|0\rangle + |\beta\rangle|1\rangle)|0\rangle_2 \rightarrow |0\rangle_1 (|0\rangle_2 + |\beta\rangle|1\rangle_2),
\]

(8)

which describes the QIT from qubit 1 to qubit 2, is completed.

From the description above, it can be seen that the proposal presented here does not require adiabatic passage (slow variation of the pulse Rabi frequency) or a second-order large detuning \(\delta = \Delta_c - \Delta\) during the entire operation. Here, \(\Delta_c = \omega_{32} - \omega_c\) is the first-order large detuning between the cavity frequency \(\omega_c\) and the [2] ↔ [3] transition frequency \(\omega_{32}\) of the qubits, while \(\Delta = \omega_{32} - \omega\) is the first-order large detuning between the pulse frequency \(\omega\) and the [2] ↔ [3] transition frequency \(\omega_{32}\) of the qubits. In addition, one can see that the present proposal does not require a first-order large detuning \(\Delta_c\) or \(\Delta\), either. Note that only resonant qubit–cavity and qubit–pulse interactions are used in this proposal, while a second-order large detuning or adiabatic passage was employed for the previous proposals [6, 11–14]. Thus, when compared with the previous proposals [6, 11–14], the speed of QIT in the present proposal is increased by two orders of magnitude.

It can also be seen from the operations above that the present proposal does not require adjustment of the level spacings of the qubits during the entire operation, which was however
needed by the previous proposal \[3\]. Furthermore, since the qubit–cavity coupling constants \(g_1\) and \(g_2\) are not required to be identical, either nonuniformity in the qubit device parameters (resulting in nonidentical qubit level spacings) or non-exact placement of qubits in the cavity is allowed by this proposal.

Several points need to be addressed as follows.

(i) Four levels of each qubit are necessary in order to have qubit 2 (qubit 1) to be decoupled from the cavity mode during the operation of step 1 (step 2).

(ii) The decay of the level \(|1\rangle\) of each qubit can be suppressed by increasing the potential barrier between the two lowest levels \(|0\rangle\) and \(|1\rangle\) \[1, 15, 16\].

(iii) For simplicity, we considered the identical Rabi frequency \(\Omega_1\) for each pulse during the operations above. However, this requirement is unnecessary. The Rabi frequency for each pulse can be different and thus the pulse durations for each step of operations above can be adjusted accordingly.

(iv) During the operation above, to have the effect of the qubit–cavity resonant interaction during the pulse negligible, the pulse Rabi frequency \(\Omega_1\) needs to be set such that \(\Omega_1 \gg g_1, g_2\), which can be met by increasing the pulse Rabi frequency (i.e., by increasing the intensity of the pulse).

4. Possible experimental implementation

As shown above, it can be found that the total operation times \(\tau\) is given by

\[
\tau = \pi/(2g_1) + \pi/(2g_2) + 2\pi/\Omega, 
\]

which should be much shorter than the energy relaxation time \(\gamma_{3r}^{-1}\), and dephasing time \(\gamma_{3p}^{-1}\) of the level \(|3\rangle\) (note that the level \(|1\rangle\) or \(|2\rangle\) has a longer decoherence time than the level \(|3\rangle\)), such that decoherence, caused due to spontaneous decay and dephasing process of the qubits, is negligible during the operation. The \(\tau\) needs to be much shorter than the lifetime of the cavity photon, which is given by \(\kappa^{-1} = Q/2\pi v_c\), such that the decay of the cavity photon can be neglected during the operation. Here, \(Q\) is the (loaded) quality factor of the cavity and \(v_c\) is the cavity field frequency. To obtain these requirements, one can design the qubits (for solid-state qubits) to have sufficiently long energy relaxation time and dephasing time, such that \(\tau \ll \gamma_{3r}^{-1}, \gamma_{3p}^{-1}\); and choose a high-\(Q\) cavity such that \(\tau \ll \kappa^{-1}\).

For the sake of definitiveness, let us consider the experimental possibility using two identical superconducting flux qubits coupled to a one-dimensional coplanar waveguide transmission line resonator (figure 3(a)). Without loss of generality, let us consider \(g_1 \sim g_2 \sim 3.0 \times 10^9\) s\(^{-1}\), which is available at present \[20\]. By choosing \(\Omega \sim 10g_1\), we have \(\tau \sim 1\) ns, much shorter than \(\min(\gamma_{3r}^{-1}, \gamma_{3p}^{-1}) \sim 1\) \(\mu\)s \[1, 2\]. In addition, consider a resonator with frequency \(v_c \sim 3\) GHz (e.g., \[21\]) and \(Q \sim 2 \times 10^4\), we have \(\kappa^{-1} \sim 1.1\) \(\mu\)s, which is much longer than the operation time \(\tau\) here. Note that superconducting coplanar waveguide resonators with a (loaded) quality factor \(Q \sim 10^6\) have been experimentally demonstrated \[22, 23\].

It should be mentioned that the two qubits can be addressed by pulses separately, through the ac current loops placed on their own superconducting loops (figures 3(b) and (c)). Note that for superconducting qubits located in a microwave resonator, the qubits can be well separated, because the dimension of a superconducting qubit is \(10–100\) \(\mu\)m while the wavelength of the cavity mode for a microwave superconducting resonator is 1 to a few centimeters \[6, 20\]. As long as the two qubits are well separated in space (figure 3(a)), the effect of ‘loop current of one qubit’ on the other qubit and the direct coupling between the two qubits are negligible,
Figure 3. (a) Setup for two superconducting flux qubits (red dots) and a (grey) standing-wave one-dimensional coplanar waveguide resonator. \(\lambda\) is the wavelength of the resonator mode and \(L\) is the length of the resonator. The two (blue) curved lines represent the standing wave magnetic field in the \(z\)-direction. Each qubit (a red dot) could be a rf SQUID consisting of one Josephson junction enclosed by a superconducting loop as depicted in (b), or a superconducting device with three Josephson junctions enclosed by a superconducting loop as shown in (c). \(E_j\) is the Josephson junction energy \((0.6 < \alpha < 0.8)\). The qubits are placed at locations where the magnetic fields are the same to achieve an identical coupling strength for each qubit. The superconducting loop of each qubit, which is a large circle for (b) while a large square for (c), is located in the plane of the resonator between the two lateral ground planes (i.e. the \(x-y\) plane). For each qubit, the external magnetic flux \(\Phi_1\) through the superconducting loop for each qubit is created by the magnetic field threading the superconducting loop. A classical magnetic pulse is applied to each qubit through an ac flux \(\Phi_2\) threading the qubit superconducting loop, which is created by an ac current loop (i.e. the red dashed-line loop) placed on the qubit loop. The pulse frequency and intensity can be adjusted by changing the frequency and intensity of the ac loop current.

which can be reached by designing the qubits and the resonator appropriately [6, 20]. We should mention that further investigation is needed for each particular experimental setup. However, this requires a rather lengthy and complex analysis, which is beyond the scope of this theoretical work.

5. Conclusion

We have presented a way to implement quantum information transfer with four-level superconducting flux qubits in cavity QED. As shown above, this proposal avoids most problems existing in the previous proposals, and the speed of the information transfer is significantly increased when compared with the previous proposals. The method presented here is quite general, which can be applied to other physical qubit systems, such as atoms and quantum dots within cavity QED.

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