SU(3) symmetry breaking in charmed baryon decays

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Abstract We explore the breaking effects of the SU(3) flavor symmetry in the singly Cabibbo-suppressed anti-triplet charmed baryon decays of \( B_c \to B_{cM} \), with \( B_c = (\Xi_c^0, \Xi_c^+, \Lambda_c^+) \) and \( B_{cM} (M) \) the baryon (pseudo-scalar) octets. We find that these breaking effects can be used to account for the experimental data on the decay branching ratios of \( B(\Lambda_c^+ \to \Sigma^0 K^+, \Lambda^0 K^+) \) and \( R'_{K/\pi} = B(\Xi_c^0 \to \Sigma^- K^+)/B(\Xi_c^0 \to \Sigma^- \pi^+) \). In addition, we obtain that \( B(\Xi_c^0 \to \Sigma^- K^+, \Sigma^+ \pi^-) = (4.6 \pm 1.7, 12.8 \pm 3.1) \times 10^{-4} \), \( B(\Xi_c^0 \to \Sigma^0 K^+, \Sigma^+ \pi^-) = (3.0 \pm 1.0, 5.2 \pm 1.6) \times 10^{-4} \) and \( B(\Xi_c^0 \to \Sigma^0(\pi^+\pi^0)) = (10.3 \pm 1.7) \times 10^{-4} \), which all receive significant contributions from the breaking effects, and can be tested by the BESIII and LHCb experiments.

1 Introduction

It is known that the theoretical approach based on the factorization and quantum chromodynamics (QCD) barely explains the charmed hadron decays [1]. This is due to the fact that the mass of the charm quark, \( m_c \approx 1.5 \) GeV, is not as heavy as that of the bottom one, \( m_b \approx 4.8 \) GeV, resulting in an underestimated correction to the heavy quark expansion, such that the alternative models have to take place for this correction [2–8]. On the other hand, the SU(3) flavor (SU(3)\( _f \)) symmetry that works in the b-hadron decays [9–13] can be well applied to \( D \to M M \) and \( B_c \to B_{cM} \) [14–25], where \( B_c = (\Xi_c^0, -\Xi_c^+, \Lambda_c^+) \) are the lowest-lying anti-triplet charmed baryon states, while \( B_{cM} \) and \( M \) represent baryon and pseudoscalar meson states, respectively. Particularly, the SU(3)\( _f \) symmetry has been extended to investigate the singly charmed baryon sextet states as well as the doubly and triply charmed baryon ones [23,24]. For \( D \to M M \) decays, the measurements produce [26]

\[ R_{D^0(K^0)} \equiv \frac{B(D^0 \to K^+ K^-)}{B(D^0 \to \pi^+ \pi^-)} = 2.82 \pm 0.07, \]

in comparison with \((R_{D^0(K^0)}), B_{D^0(K^0)} \) given by the theoretical calculations based on the SU(3)\( _f \) symmetry. The disagreements between the theory and experiment imply that the breaking effects of the SU(3)\( _f \) symmetry cannot be ignored in the singly Cabibbo-suppressed (SCS) processes. We note that, in the literature, the SU(3)\( _f \) breaking effects were used to relate \( R'_{K/\pi} \) to the possible large difference of the \( CP \) violating asymmetries of \( \Delta A_{CP} \equiv A_{CP}(D^0 \to K^+ K^-) - A_{CP}(D^0 \to \pi^+ \pi^-) \) [15,27,28], which is recently measured to be \((0.10 \pm 0.08 \pm 0.03)\% \) by LHCb [29].

For the two-body \( B_c \to B_{cM} \) decays, both Cabibbo flavored (CF) and SCS decays are not well explained. In particular, the experimental measurements show that

\[ B_{pK^0} \equiv B(\Lambda_c^+ \to p K^0) < 3 \times 10^{-4} \text{ (90\% C.L.)} [30,31], \]

\[ R'_{K/\pi} \equiv \frac{B(\Xi_c^0 \to \Sigma^- K^+)}{B(\Xi_c^0 \to \Sigma^- \pi^+)} = 0.028 \pm 0.006 \approx 0.6 \pm 0.2 \% \] \[ \times \frac{1}{s_c^2} [26], \]

\[ B(\Lambda_c^+ \to \Lambda^0 K^+) = (6.1 \pm 1.2) \times 10^{-4} [26], \]

\[ B(\Lambda_c^+ \to \Sigma^0 K^+) = (5.2 \pm 0.8) \times 10^{-4} [26], \]

where \( s_c \equiv \sin \theta_c = 0.2248 [26] \) with \( \theta_c \) the well-known Cabibbo angle. However, theoretical evaluations based on the SU(3)\( _f \) symmetry lead to \( B_{pK^0} = (5.7 \pm 1.5) \times 10^{-4} \) and \( R'_{K/\pi} \approx 1.0 s_c^2 [21] \), and those in the factorization approach give \( B_{pK^0} = f_2^2/(2f_K^2) s_c^2 B(\Lambda_c^+ \to p K^0) = (5.5 \pm 0.3) \times 10^{-4} \) and \( R'_{K/\pi} = (f_K/f_\pi)^2 s_c^2 \approx 1.4 s_c^2 \), where we have used the data of \( B(\Lambda_c^+ \to p K^0) = (3.16 \pm 0.16) \times 10^{-2} [26] \). In addition, the fitted results of \( B(\Lambda_c^+ \to \Lambda^0 K^+, \Sigma^0 K^+) = (4.6 \pm 0.9, 4.0 \pm 0.8) \times 10^{-4} [22] \) are \((1.3 - 1.6)\% \) away from the data in Eq. (2). In this study, we will consider the breaking effects of the SU(3)\( _f \) symmetry due to the fact of

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where the notations of \(i, j, k\) are quark indices, to be connected to the initial and final states in the amplitudes. Note that \(H_{23}(6)\) and \(H_{32}(6)\) are derived from \(O_{6}^{l}\) and \(O_{6}^{d}\), respectively. The lowest-lying charmed baryon states \(B_{c}\) are an anti-triplet of \(\bar{3}\) to consist of \((d\bar{s} - s\bar{d})c\), \((u\bar{s} - su)c\) and \((ud - du)c\), presented as

\[
B_{c} = (\Xi_{c}^{0}, -\Xi_{c}^{+}, \Lambda_{c}^{+})
\]

together with the baryon and meson octets, given by

\[
B_{n} = \begin{pmatrix}
\frac{1}{\sqrt{6}}\Lambda_{c}^{0} + \frac{1}{\sqrt{2}}\Sigma_{c}^{0} & \frac{1}{\sqrt{6}}\Lambda_{c}^{0} - \frac{1}{\sqrt{2}}\Sigma_{c}^{0} & p \\
\Sigma_{c}^{-} & -\frac{1}{\sqrt{2}}\Lambda_{c}^{0} & -\sqrt{\frac{2}{3}}\Sigma_{c}^{0} \\
\Sigma_{c}^{+} & \frac{1}{\sqrt{2}}\Lambda_{c}^{0} & -\sqrt{\frac{2}{3}}\Sigma_{c}^{0}
\end{pmatrix},
\]

(9)

where we have removed the octet \(\eta_{8}\) and singlet \(\eta_{1}\) meson states to simplify our discussions. Subsequently, the amplitudes of \(B_{c} \rightarrow B_{n} M\) can be derived as

\[
A(B_{c} \rightarrow B_{n} M) = (B_{n} M | H_{\text{eff}} | B_{c})
\]

(10)

together with the baryon and meson octets, given by

\[
B_{n} = \begin{pmatrix}
\frac{1}{\sqrt{6}}\Lambda_{c}^{0} + \frac{1}{\sqrt{2}}\Sigma_{c}^{0} & \frac{1}{\sqrt{6}}\Lambda_{c}^{0} - \frac{1}{\sqrt{2}}\Sigma_{c}^{0} & p \\
\Sigma_{c}^{-} & -\frac{1}{\sqrt{2}}\Lambda_{c}^{0} & -\sqrt{\frac{2}{3}}\Sigma_{c}^{0} \\
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\Sigma_{c}^{-} & -\frac{1}{\sqrt{2}}\Lambda_{c}^{0} & -\sqrt{\frac{2}{3}}\Sigma_{c}^{0} \\
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\Sigma_{c}^{-} & -\frac{1}{\sqrt{2}}\Lambda_{c}^{0} & -\sqrt{\frac{2}{3}}\Sigma_{c}^{0} \\
\Sigma_{c}^{+} & \frac{1}{\sqrt{2}}\Lambda_{c}^{0} & -\sqrt{\frac{2}{3}}\Sigma_{c}^{0}
\end{pmatrix},
\]

(9)
is due to the fact that the contributions to the branching rates from $H(15)$ and $H(6)$ lead to a small ratio of $R(15/6) = c_2^2/c_2^2 \simeq 17\%$ with $(c_+, c_-) = (0.76, 1.78)$ calculated at the scale $\mu = 1$ GeV in the NDR scheme [33, 34]. There remain two measurements to be explained. In Eq. (2), the prediction for $B_{pr0}$ has the $2\sigma$ gap to reach the edge of the experimental upper bound. However, with $R(15/6)$ to be small, it is nearly impossible that, by restoring $a_{4,5,6,7}$ that have been ignored in the literature [20–23, 25], one can accommodate the data of $B_{pr0}$ but without having impacts on the other decay modes, which are correlated with the same sets of parameters. Moreover, as seen from Eq. (12), there is no room for $R_{K^*}/\pi$ as it is fixed to be $(1.0)^{2}_{-2}$. On the other hand, the results for $D \to MM$ decays in Eq. (1) suggest some possible corrections from the breaking effects of the $SU(3)_f$ symmetry in the SCS processes. In the charm baryon decays, we consider the similar effects. Due to $m_s \gg m_{ud, d}$, we present the matrix of $M_s = \epsilon(\lambda_s)\gamma_j^{(1)}$ [14] to break $SU(3)_f$, where $\epsilon \sim 0.2 - 0.3$ and $\lambda_s$ is given by [14, 15, 18]

$$\lambda_s = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix},$$

which transforms as an octet of 8, such that its coupling to $H(6)$ is in the form of $8 \times 6 = \bar{3} + 6 + 6 + 15 + 24$, and $\bar{3}$ is the simplest break effects to be confined in the SCS processes [18, 35]. Note that from $\frac{1}{8}(\delta_1(\lambda_s)\gamma_j^{(1)}H(6))_{kn} - \delta_1(\lambda_s)\gamma_j^{(1)}H(6)_{kn} + \delta_1(\lambda_s)\gamma_j^{(1)}H(6)_{kn} - \delta_1(\lambda_s)\gamma_j^{(1)}H(6)_{kn}$ and the nonzero entry of $H(\bar{3})_l = s_l$ from the coupling of $H(6)_{23}$ and $H(6)_{12}$ [14], one can trace back to the break effect between SCS $c \to u\bar{s}s$ and $c \to u\bar{d}d$ transitions. As a result, the $SU(3)_f$ symmetry breaking gives rise to the new $T$-amplitudes, given by

$$T(H_3) = v_1(B_c)H(\bar{3})^{(1)}(B_u^{(1)})^{(1)}(M)_{kn}^{(1)} + v_2(B_c)H(\bar{3})^{(1)}(B_u^{(1)})^{(1)}(M)_{kn}^{(1)} + v_3(B_c)H(\bar{3})^{(1)}(B_u^{(1)})^{(1)}(M)_{kn}^{(1)},$$

where $v_{1,2,3}$ are the parameters related to the $SU(3)_f$ breakings. It is interesting to note that the $v_i$ terms associated with $(B_c)H(\bar{3})^{(1)}$ in Eq. (14) occur in some of the $\Xi_c^{0,+}$ decays, but disappear in all $\Lambda_c^{*5}$ modes. By adding $T(H_3)$ to $T(B_c \to B_u M)$ in Eq. (10), the full expansions of $T(B_c \to B_u M)$ are given in Table 1, to be used to calculate the decay widths, given by [26]

$$\Gamma(B_c \to B_u M) = \frac{|\tilde{p}_{cm}|}{8\pi m_{B_c}} |A(B_c \to B_u M)|^2,$$

where $|\tilde{p}_{cm}| = \sqrt{[(m_{B_c} - (m_{B_u} + m_M))^2][(m_{B_c} - (m_{B_u} - m_M))^2]}/(2m_{B_c}).$

3 Numerical analysis

In the numerical analysis, we examine $B(\Lambda_c^{+} \to \Lambda^0 K^+, \Sigma^0 K^+, \pi\pi^0)$ and $R_{K^*/\pi}$ by including the breaking effects of the $SU(3)_f$ symmetry to see if one can explain their data in Eq. (2). The theoretical inputs for the CKM matrix elements are given by [26]

$$(V_{cs}, V_{ud}, V_{us}, V_{cd}) = (1 - \lambda^2/2, 1 - \lambda^2/2, \lambda, -\lambda),$$

with $\lambda = 0.2248$ in the Wolfenstein parameterization. We perform the minimum $\chi^2$ fit, in terms of the equation of [22]

$$\chi^2 = \sum_i \left( \frac{B_{th}^i - B_{ex}^i}{\sigma_{th}^i} \right)^2 + \sum_j \left( \frac{R_{thj} - R_{exj}}{\sigma_{thj}} \right)^2,$$

with $B = B(\Lambda_c^{+} \to \Lambda_c K^+)$ and $R = B(\Sigma_c^{0} \to \Sigma_c K^+)$, where the subscripts $th$ and $ex$ are denoted as the theoretical inputs from the amplitudes in Table 1 and the experimental data points in Table 2, respectively, while $\sigma_{th}$ correspond to the $1\sigma$ errors. By following Refs. [21–23], we extract the parameters, which are in fact complex numbers, given by

$$a_1, a_2 e^{i\delta_{a2}}, a_3 e^{i\delta_{a3}}, v_1 e^{i\delta_{v1}}, v_2 e^{i\delta_{v2}}, v_3 e^{i\delta_{v3}},$$

where $a_{4,5,...}$ have been ignored as discussed in Sect. 3. Since only the relative phases contribute to the branching ratios, $a_1$ is set to be real without losing generality. However, we take $v_j$ to be real numbers in order to fit 8 parameters with the 9 data points in Table 2. In the calculation, $\delta_{a_i}$ ($i=2,3$) from $a_i e^{i\delta_{a_i}}$ is a fitting parameter, which can absorb the phase of $\delta_{v_j}$ from the interference in the data fitting. Note that $\delta_{v_j}$ ($i=2,3$) have been fitted with the imaginary parts [22]. As a result, we may set $\delta_{v_{2,3}}$ along with the overall phase of $\delta_{a_i}$ to be zero for the estimations of the decay branching ratios due to the $SU(3)$ breaking effects. We will follow Ref. [25] to test our assumption, where a similar global fit in the approach of the $SU(3)_f$ symmetry has been done to extract $a_2 e^{i\delta_{a2}}$ by freely rotating the angle of $\delta_{a2}$ from $-180^\circ$ to $180^\circ$ to estimate the uncertainties of the branching ratios. Subsequently, the fit with the breaking effects in the $SU(3)_f$ symmetry yields

$$T(\Lambda_c^{+} \to \Lambda^0 K^+),$$

$$T(\Sigma_c^{0} \to \Sigma_c K^+),$$

$$(v_1, v_2, v_3) = (0.090 \pm 0.032, -0.037 \pm 0.013, 0.025 \pm 0.012) \text{ GeV}^2,$$

with $\chi^2/d.o.f = 3.0/1$, where $d.o.f$ represents the degree of freedom. Note that $a_{1,2,3}$ and their phases are nearly the same as those without the breaking of $SU(3)_f$ [22]. With the parameters in Eq. (19), we obtain the branching ratios of the CF and SCS $B_c \to B_u M$ decays, shown in Table 3.
Table 1: Amplitudes of $T(B_c \rightarrow B_n M)$, where $T$-amps refers to $T(B_c \rightarrow B_n M)$ and CF (SCS) represents Cabibbo favored (singly Cabibbo-suppressed).

| CF mode | $T$-amp | SCS mode | $T$-amp |
|---------|---------|-----------|---------|
| $\Sigma_0^+ \rightarrow \Sigma^+ K^-$ | $2 \left( a_2 + \frac{a_3 + a_4}{2} \right)$ | $\Sigma_0^+ \rightarrow \Sigma^+ \pi^-$ | $2 \left( a_2 + v_1 + v_3 + \frac{a_3 + a_4}{2} \right) S_c$ |
| $\Sigma_0^+ \rightarrow \Sigma^0 K^0$ | $-\sqrt{2} \left( a_2 + a_3 - \frac{a_4}{2} \right)$ | $\Sigma_0^+ \rightarrow \Sigma^- K^0$ | $2 \left( a_1 + v_1 + v_2 + \frac{a_3 + a_4}{2} \right) S_c$ |
| $\Sigma_0^+ \rightarrow \Sigma^0 \pi^0$ | $-\sqrt{2} \left( a_1 - a_3 - \frac{a_4}{2} \right)$ | $\Sigma_0^+ \rightarrow \Sigma^0 \pi^0$ | $2 \left( a_2 + a_3 - 2v_1 - v_2 - v_3 - \frac{a_3 + a_4 + a_5}{2} \right) S_c$ |
| $\Sigma_0^+ \rightarrow \Sigma^- \pi^+$ | $2 \left( a_1 + \frac{a_3 + a_4}{2} \right)$ | | |
| $\Sigma_0^+ \rightarrow \Lambda^0 K^0$ | $-\sqrt{2} \left( 2a_1 - a_2 - a_3 + \frac{2a_3 + a_4}{2} \right)$ | $\Sigma_0^+ \rightarrow \Lambda^0 \pi^0$ | $2 \left( a_1 - a_3 - v_1 + \frac{a_3 + a_4}{2} \right) S_c$ |
| $\Sigma^+ \rightarrow \Sigma^+ K^0$ | $-6 \left( a_3 - \frac{a_4 + a_5}{2} \right)$ | $\Sigma^+ \rightarrow \Sigma^+ \pi^+$ | $-\sqrt{2} \left( a_3 - a_2 + v_2 - v_3 + \frac{a_3 + a_4 + a_5 + a_7}{2} \right) S_c$ |
| $\Sigma^+ \rightarrow \Sigma^0 \pi^+$ | $2 \left( a_1 + \frac{a_3 + a_4}{2} \right)$ | $\Sigma^+ \rightarrow \Sigma^0 \pi^0$ | $\sqrt{2} \left( a_1 - a_2 + v_2 - v_3 - \frac{a_3 + a_4 + a_5 + a_7}{2} \right) S_c$ |
| $\Sigma^+ \rightarrow \Sigma^+ K^+$ | $2 \left( a_2 + a_3 + \frac{a_4 + a_5}{2} \right)$ | $\Sigma^+ \rightarrow \Sigma^+ K^+$ | $2 \left( a_2 + a_3 + v_2 + \frac{a_4 + a_5}{2} \right) S_c$ |
| $\Sigma^+ \rightarrow \Sigma^+ \pi^+$ | $2 \left( a_2 - a_3 - \frac{a_4}{2} \right)$ | $\Sigma^+ \rightarrow \Lambda^0 \pi^+$ | $\sqrt{2} \left( a_1 + a_2 + 2a_3 + v_2 - v_3 - \frac{3a_3 + a_4 + a_5 + a_7}{2} \right) S_c$ |
| $\Lambda^+_c \rightarrow \Sigma^0 \pi^+$ | $-\sqrt{2} \left( a_1 - a_2 + a_3 - \frac{a_4 + a_5}{2} \right)$ | $\Lambda^+_c \rightarrow \Sigma^+ K^0$ | $2 \left( a_1 - a_3 + v_3 - \frac{a_3 + a_5}{2} \right) S_c$ |
| $\Lambda^+_c \rightarrow \Sigma^+ \pi^+$ | $\sqrt{2} \left( a_1 - a_2 - a_3 - \frac{a_4}{2} \right)$ | $\Lambda^+_c \rightarrow \Sigma^0 K^+$ | $\sqrt{2} \left( a_1 - a_3 + v_3 - \frac{a_3 + a_5}{2} \right) S_c$ |
| $\Lambda^+_c \rightarrow \Sigma^0 K^+$ | $-2 \left( a_2 - \frac{a_3 + a_4}{2} \right)$ | $\Lambda^+_c \rightarrow \Sigma^0 K^+$ | $\sqrt{2} \left( a_2 + a_3 + v_2 - \frac{a_3 + a_5}{2} \right) S_c$ |
| $\Lambda^+_c \rightarrow \Sigma^+ K^+$ | $2 \left( a_1 - a_3 + \frac{a_4 + a_5}{2} \right)$ | $\Lambda^+_c \rightarrow \Sigma^+ \pi^+$ | $2 \left( a_2 + a_3 + v_2 + \frac{a_3 + a_5}{2} \right) S_c$ |
| $\Lambda^+_c \rightarrow \Lambda^0 \pi^+$ | $-\frac{3}{2} \left( a_1 + a_2 + a_3 - \frac{a_4 + a_5 + a_7}{2} \right)$ | $\Lambda^+_c \rightarrow \Lambda^0 K^+$ | $\sqrt{2} \left( a_1 + a_2 + a_3 + v_2 + v_3 - \frac{3a_3 + a_4 + a_5 + 2a_7}{2} \right) S_c$ |

4 Discussions and conclusions

As seen from Tables 2 and 3, the breaking effects associated with $v_2$ and $v_3$ on the branching ratios of the SCS $\Lambda^+_c \rightarrow B_n M$ decays are at most around 30%, which is close to the naive estimation of $(f_K/f_\pi)^2 \simeq 40\%$. In particular, we get $B(\Lambda^+_c \rightarrow \Lambda^0 K^+, \Sigma^0 K^+ \rightarrow (6.1 \pm 0.9, 5.2 \pm 0.7) \times 10^{-4}$, which explain the data in Eq. (2) well and alleviate the $(1.3 - 1.6)\sigma$ deviations by the fit with the exact $SU(3)_f$ symmetry [22]. Meanwhile, the branching ratios for the CF modes are fitted to be the same as those without the breaking except $B(\Lambda^+_c \rightarrow \Sigma^0 K^+)$, which is slightly different in order to account for the recent observational value given by BESIII [36].

Moreover, the fitted value of $\mathcal{R}_f'\mathcal{R}_f = (0.6 \pm 0.2) \pm 0.03 \pm 0.01$ explains the data very well for the first time. This leads to the prediction of $B(\Sigma^0 \rightarrow \Sigma^- K^+) = (4.6 \pm 1.7) \times 10^{-4}$, with $v_1 + v_2$ as the destructive contribution to reduce the value of $(7.6 \pm 0.4) \times 10^{-4}$ under the exact $SU(3)_f$ symmetry, whereas $B(\Sigma^0 \rightarrow \Sigma^- \pi^+) = (12.8 \pm 3.1) \times 10^{-4}$ receives the constructive contribution from $v_1 + v_2$, with $T(\Sigma^0 \rightarrow \Sigma^- K^+, \Sigma^- \pi^+) = \mp [a_1 \mp (v_1 + v_2)] S_c$. Since
Table 2  The data of the $B_c \rightarrow B_m \ M$ decays, together with the reproduction with the exact (broken) $SU(3)_f$ symmetry in the 3rd (4th) column

| (Branching) | Data [26,30,36] | Exact [22] | Broken |
|-------------|----------------|------------|--------|
| $10^3 B(A_{c}^{+} \rightarrow \Sigma^{+} K^+)$ | 5.2 ± 0.8 | 4.0 ± 0.8 | 5.2 ± 0.7 |
| $10^3 B(A_{c}^{+} \rightarrow \Lambda^{0} K^+)$ | 6.1 ± 1.2 | 4.6 ± 0.9 | 6.1 ± 0.9 |
| $R'_{K/\pi} = \frac{B(\Xi_1^0 \rightarrow \Xi^- K^+)}{B(\Xi_1^0 \rightarrow \Xi^- \pi^+)}$ | $(0.6 \pm 0.2) s_{c}$ | $(1.0) s_{c}$ | $(0.6 \pm 0.2)s_{c}$ |
| $10^2 B(\Lambda_{c}^{+} \rightarrow \Sigma^{0} \pi^+)$ | 1.29 ± 0.07 | 1.3 ± 0.2 | 1.3 ± 0.1 |
| $10^2 B(\Lambda_{c}^{+} \rightarrow \Sigma^{+} \pi^0)$ | 1.24 ± 0.10 | 1.3 ± 0.2 | 1.3 ± 0.1 |
| $10^2 B(\Lambda_{c}^{+} \rightarrow \Xi_0^0 K^+)$ | 0.59 ± 0.09 | 0.5 ± 0.1 | 0.6 ± 0.1 |
| $10^2 B(\Lambda_{c}^{+} \rightarrow \rho K^0)$ | 3.16 ± 0.16 | 3.3 ± 0.2 | 3.2 ± 0.1 |
| $10^2 B(\Lambda_{c}^{+} \rightarrow \Lambda^{0} \pi^+)$ | 1.30 ± 0.07 | 1.3 ± 0.2 | 1.3 ± 0.1 |
| $R''_{K/\pi} = \frac{B(\Xi_1^0 \rightarrow \Lambda^{0} K^0)}{B(\Xi_1^0 \rightarrow \Xi^- \pi^+)}$ | 0.42 ± 0.06 | 0.5 ± 0.1 | 0.5 ± 0.1 |

Table 3  The branching ratios of the $B_c \rightarrow B_m \ M$ decays, where the numbers with the dagger (†) correspond to the reproductions of the experimental data input, instead of the predictions

| CF mode | Exact [22] | Broken | SCS mode | Exact [22] | Broken |
|---------|------------|--------|----------|------------|--------|
| $10^3 B(\Xi_{c}^{0} \rightarrow \Sigma^{+} K^-)$ | 3.5 ± 0.9 | 3.8 ± 0.6 | $10^4 B(\Xi_0^0 \rightarrow \Sigma^{+} \pi^-)$ | 2.0 ± 0.5 | 5.2 ± 1.6 |
| $10^3 B(\Xi_{c}^{0} \rightarrow \Sigma^{0} K^-)$ | 4.7 ± 1.2 | 5.2 ± 0.8 | $10^4 B(\Xi_0^0 \rightarrow \Sigma^{-} \pi^-)$ | 9.0 ± 0.4 | 12.8 ± 3.1 |
| $10^3 B(\Xi_{c}^{0} \rightarrow \Xi^{0} \pi^-)$ | 4.3 ± 0.09 | 4.4 ± 0.4 | $10^4 B(\Xi_0^0 \rightarrow \Sigma^{0} \pi^-)$ | 3.2 ± 0.3 | 7.7 ± 2.2 |
| $10^3 B(\Xi_{c}^{0} \rightarrow \Xi^{-} \pi^-)$ | 15.7 ± 0.7 | 15.2 ± 0.7 | $10^4 B(\Xi_0^0 \rightarrow \Xi^{-} \pi^-)$ | 7.6 ± 0.4 | 4.6 ± 1.7 |
| $10^3 B(\Xi_{c}^{0} \rightarrow \Lambda^{0} K^-)$ | 8.3 ± 0.9 | 7.8 ± 0.5 | | | |
| $10^3 B(\Xi_{c}^{+} \rightarrow \Sigma^{+} K^-)$ | 8.0 ± 3.9 | 7.8 ± 2.7 | $10^4 B(\Xi_0^0 \rightarrow \Sigma^{0} \pi^-)$ | 18.5 ± 2.2 | 10.3 ± 1.7 |
| $10^3 B(\Xi_{c}^{+} \rightarrow \Xi^{0} \pi^-)$ | 8.1 ± 4.0 | 7.9 ± 2.7 | $10^4 B(\Xi_0^0 \rightarrow \Sigma^{+} \pi^-)$ | 18.5 ± 2.2 | 10.3 ± 1.7 |
| $10^3 B(\Lambda_{c}^{+} \rightarrow \Sigma^{0} \pi^-)$ | $(1.3 \pm 0.2) t$ | $(1.3 \pm 0.1) t$ | | | |
| $10^2 B(\Xi_{c}^{0} \rightarrow \Sigma^{0} \pi^-)$ | $(1.3 \pm 0.2) t$ | $(1.3 \pm 0.1) t$ | | | |
| $10^2 B(\Lambda_{c}^{+} \rightarrow \Sigma^{0} \pi^-)$ | $(0.5 \pm 0.1) t$ | $(0.6 \pm 0.1) t$ | | | |
| $10^2 B(\Lambda_{c}^{+} \rightarrow \rho K^0)$ | $(3.3 \pm 0.2) t$ | $(3.2 \pm 0.1) t$ | | | |
| $10^2 B(\Lambda_{c}^{+} \rightarrow \Lambda^{0} \pi^+)$ | $(1.3 \pm 0.2) t$ | $(1.3 \pm 0.1) t$ | | | |

there are other similar interferences between $a_i$ and $v_i$, which come from $T(\Xi_0^0 \rightarrow \rho K^-)$, $\Sigma^{+} \pi^-$, $\Sigma^{+} \pi^0$, and $T(\Xi_1^{+} \rightarrow \Sigma^{0} \pi^+)$, $\Sigma^{+} \pi^0) = \mp \sqrt{2}((a_1 - a_2) + (v_2 - v_3)s_c)$, it is predicted that $B(\Xi_0^0 \rightarrow \rho K^-)$, $\Sigma^{+} \pi^-$) = $(3.0 \pm 1.0, 5.2 \pm 1.6) \times 10^{-4}$ and $B(\Xi_1^{+} \rightarrow \Sigma^{0} \pi^+)$, $\Sigma^{+} \pi^0)$) = $(10.3 \pm 1.7) \times 10^{-4}$. It is interesting to note that the important roles of the terms associated with $v_1$ in the $T$ amplitudes are also projected in the $\Xi_0^0$ modes, particularly, $\Xi_0^0 \rightarrow \Xi^{-} K^+$ and $\Xi_0^0 \rightarrow \Xi^{-} \pi^0$. Clearly, these SCS $\Xi_0^0$ decays all contain sizable $SU(3)_f$ breaking effects, and can be treated as golden modes to test the $SU(3)_f$ symmetry.

In our calculation, we treat $v_3$ as the norm in $T(\Lambda_c^+ \rightarrow \Sigma^0 K^+) \simeq \sqrt{2}(a_1 - a_3 + v_3)s_c$ of Table I, such that $\delta v_3$ is allowed to rotate from $-90^\circ$ to $50^\circ$ without letting $B(\Lambda_c^+ \rightarrow \Sigma^0 K^+) \succ 0$ exceed the data. Since the allowed range for $\delta v_3$ is large, it is clear that its value is insensitive to the data. On the other hand, in order to explain the experimental data of $R''_{K/\pi}$ with the smallest corrections from $v_ie^{\delta v_1}$, we assume maximally destructive interferences between $a_i e^{\delta v_i}$ and $v_i e^{\delta v_i}$. Explicitly, in $T(\Xi_0^0 \rightarrow \Xi^{-} K^-)$, $\Xi_1^{+} \rightarrow \Sigma^{+} \pi^0)$, $\Sigma^{+} \pi^0)$), we can take $\delta v_1 = \delta v_2 = \delta v_1 = 0$ as an overall phase in $T(\Xi_1^{+} \rightarrow \Xi^{-} K^-)$. Consequently, we are able to assume real values for $v_i (i = 1,2,3)$ without loss of generality. Finally, we remark that, even with the breaking effects, we are still unable to fit the data of $\Lambda_c^+ \rightarrow \rho \pi^0$ in Eq. (1) as our result for its branching ratio of $(5.4 \pm 1.0) \times 10^{-4}$, which is close to $(5.5 \pm 0.3) \times 10^{-4}$ from the factorization.
approach [22], is lower than the current experimental upper bound of $3 \times 10^{-4}$ [30, 31]. However, it is possible that $H(15)$ would be non-negligible in $\Lambda_c^+ \rightarrow p\pi^0$. For example, with $T(\Lambda_c^+ \rightarrow p\pi^0) = \sqrt{2}(a_2 + a_3 + v_z - (a_6 - a_f))/2$s, the contribution from $(a_6 - a_f)$ of $H(15)$ in Table 1 might be comparable with that from $a_2 + a_3 + v_z$ of $H(6)$, while $a_{1,2,3}$ and $v_2$ of Eq. (19) are taken to be small. In particular, with $(a_6 - a_f)$ to be around 25% of $a_2 + a_3 + v_z$, $B(\Lambda_c^+ \rightarrow p\pi^0)$ can be reduced to be within the experimental upper bound due to the destructive interference. In this case, there is a corresponding constructive interference in $\Lambda_c^+ \rightarrow n\pi^+$, leading to $B(\Delta^+_c \rightarrow n\pi^+) \sim 17 \times 10^{-4}$, which breaks the relation of $A(\Lambda_c^+ \rightarrow n\pi^+) = \sqrt{2}A(\Lambda_c^+ \rightarrow p\pi^0)$ [8]. Clearly, in order to confirm the importance of $H(15)$, both experimental observations of $\Lambda_c^+ \rightarrow p\pi^0$ and $\Lambda_c^+ \rightarrow n\pi^+$ are needed.

In sum, we have studied the singly Cabibbo-suppressed charmed baryon decays. We have shown that the breaking effect of the $SU(3)_f$ symmetry can be used to understand the experimental data of $B(\Lambda_c^+ \rightarrow \Sigma^0 K^+, \Lambda^0 K^+)$ and $R'_{K/\pi} = B(\Sigma_c^0 \rightarrow \pi^0 K^+)/B(\Sigma_c^0 \rightarrow \pi^- \pi^+)$. With these effects, we have obtained that $B(\Sigma_c^0 \rightarrow \pi^0 K^+, \pi^- \pi^+) = (4.6 \pm 1.7, 12.8 \pm 3.1) \times 10^{-4}$, $B(\Sigma_c^0 \rightarrow pK^-, \pi^- \pi^+) = (3.0 \pm 1.0, 5.2 \pm 1.6) \times 10^{-4}$ and $B(\Sigma_c^0 \rightarrow \Sigma^0 \pi^0(\pi^+)) = (10.3 \pm 1.7) \times 10^{-4}$, which are quite different from those predicted by the approach with the exact $SU(3)_f$ symmetry. However, even with the breaking effects, our result for the branching ratio of $\Lambda_c^+ \rightarrow p\pi^0$ is still higher than the current experimental upper bound, which clearly requires a closer examination by a future dedicated experiment.

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