JET QUENCHING OF MASSIVE QUARKS IN A NUCLEAR MEDIUM

BEN-WEI ZHANG
Institute of Particle Physics, Huazhong Normal University, Wuhan 430079, China
E-mail: bwzhang@iopp.ccnu.edu.cn

Utilizing the generalized factorization of twist-4 processes, we derive the modified heavy quark fragmentation function after considering the gluon radiation induced by multiple scattering in DIS. It is found that the mass effects of heavy quark may reduce the gluon formation time and change the medium size dependence of heavy quark energy loss. The radiative energy loss is also significantly suppressed relative to a light quark due to the dead-cone effect.

In relativistic heavy-ion collisions, when a fast parton propagating in a dense medium, it may experience multiple scattering with other partons in nucleus and lose a large amount of energy loss via induced gluon radiation\(^1\). Such kind of jet quenching phenomenon has aroused a lot of attention recently\(^2,3,4,5\), and it is found that the total energy loss of a massless parton (light quark or gluon) has a quadratic dependence on the medium size due to non-Abelian Landau-Pomeranchuk-Migdal (LPM) interference effect. In order to get a complete understanding of the mechanism of jet quenching it is important to investigate the heavy quark energy loss by multiple scattering in nuclei with uniform methods. Currently, though the framework of the light quark energy loss induced by gluon radiation has been well established, there are only few studies about the energy loss of a heavy quark in literature\(^6,7,8,9\). Here I would like to report our group’s study on jet quenching of massive quark in nuclei with the twist expansion approach\(^8\) by utilizing the generalized factorization in pQCD\(^8,10\).

To separate the complication of heavy quark production and propagation, we consider a simple process of charm quark production via the charge-current interaction in DIS off a large nucleus. The results can be easily extended to heavy quark propagation in other dense media.

To the leading-twist in collinear approximation, the semi-inclusive cross section factorizes into the product of quark distribution \(f_{s\mu}^A(x_B + x_M)\), the heavy quark fragmentation function \(D_{Q\rightarrow H}(z_H)\) \((z_H = \ell_H/\ell_Q)\) and the hard partonic part \(H_{\mu\nu}^{(0)}(k, q, M)\)

\[
\frac{dW^S_{\mu\nu}}{dz_h} = \sum_q \int dx f_{q\mu}^A(x, Q_1^2) H_{\mu\nu}^{(0)}(x, p, q, M) \times D_{Q\rightarrow h}(z_h, Q_2^2).
\]

As illustrated in Fig. 1, in medium, the propagating heavy quark in DIS will suffer additional scattering with other partons from the nucleus, which will induce gluon radiation and cause the leading quark to lose energy. Such induced gluon radiations will effectively give rise to additional terms in the evolution equation leading to the modification of the heavy quark fragmentation functions in nuclei. There are total 23 cut diagrams that contribute to these double scattering processes.

Similar to the case of light quark propagation in nuclear medium\(^10\), the generalized factorization of twist-4 processes will be employed to evaluate these contributions. The dominant contribution gives the semi-inclusive tensor for heavy quark fragmentation from double quark-gluon scattering,

\[
\frac{W^D_{\mu\nu}}{dz_h} = \sum \int dz H_{\mu\nu}^{(0)}(\ell_T) \times D_{Q\rightarrow H}(z_H) \times C_A^2 \frac{1 + z^2}{2\pi} \times T_{A,C}(x, x_L)
\]
which can be approximately factorized as
\[ f_{\bar{q}q}(x) = \left[ \frac{\ell_T^2}{\ell_T^2 + z^2 M^2} \right]^4 \left[ 1 + \frac{\theta^2}{\ell_T^2} \right]^{-4}, \tag{2} \]

where \( \theta = M/q^- \) and \( \theta = \ell_T/q^- z \). This is often referred to as the “dead-cone” phenomenon that suppresses small angle gluon radiation and therefore reduces radiative energy loss of a heavy quark.

The contribution of double scattering is proportional to a twist-4 parton correlation function
\[ T_{qg}^{A,C}(x, x_L, M^2) = \frac{1}{2} \int \frac{dy^-}{2\pi} \int dy_1^+ dy_2^+ \bar{H} \psi \left( A|\bar{q}(0) \gamma^+ F_{a^+}(y_2^-) F^{a\sigma}(y_1^-) \psi_q(y^-)|A \right) \times e^{i(x+x_L)p^+ y^-} \theta(-y^- y_2^-) \theta(y^- - y_1^-), \]

which can be approximately factorized as
\[ T_{qg}^{A,C}(x, x_L, M^2) \approx \frac{\bar{C}}{x_A} f_{\bar{q}}^A(x) \left[ 1 - e^{-x_L^2/x_A^2} a_1 + a_2 \right] \tag{3} \]
in the limit \( x_L \ll x \), where \( x_L = \ell_L^2/2p^+ q^- z(1-z), x_A \equiv 1/\alpha N C A \). The coefficients \( a_1 \) and \( a_2 \) are polynomial functions of \( M^2/\ell_T^2 \) and become \((1+z)/2\) and \( C_F(1-z)/2CA \), respectively for \( M = 0 \).

Rewriting the sum of single and double scattering contributions in a factorized form for the semi-inclusive hadronic tensor, one can define a modified effective fragmentation function \( \bar{D}_{Q \rightarrow H}(zH, \mu^2) \) as
\[ \bar{D}_{Q \rightarrow H}(zH, \mu^2) = D_{Q \rightarrow H}(zH, \mu^2) \]
\[ + \int_0^{\mu^2} \frac{d\ell_T^2}{\ell_T^2 + (1-z)^2 M^2} \times \frac{\alpha_s}{2\pi} \int_{z_h}^1 \frac{dz}{z} \Delta_{y\rightarrow qg}(z, M^2) D_{Q \rightarrow H}(\frac{zH}{z}) \]
\[ + \int_0^{\mu^2} \frac{d\ell_T^2}{\ell_T^2 + z^2 M^2} \frac{\alpha_s}{2\pi} \int_{z_h}^1 \frac{dz}{z} \times \Delta_{y\rightarrow qg}(z, M^2) D_{y \rightarrow H}(\frac{zH}{z}), \]

where \( D_{Q \rightarrow H}(zH, \mu^2) \) and \( D_{y \rightarrow H}(zH, \mu^2) \) are the leading-twist fragmentation functions of the heavy quark and gluon. The modified splitting functions are given as
\[ \Delta_{y\rightarrow qg}(z) = \left[ \frac{1 + z^2}{(1-z)^+} T_{qg}^{A,C}(x, x_L, M^2) \right. \]
\[ + \delta(1-z) \Delta T_{qg}^{A,C}(x, \ell_T^2, M^2) \]
\[ \times \frac{2\pi C_A \alpha_s \ell_T^2}{(\ell_T^2 + (1-z)^2 M^2)^3 N_c f_{\bar{q}}^A(x)} \]
\[ \Delta T_{qg}^{A,C}(x, \ell_T^2, M^2) \equiv \int_0^{1} \frac{dz}{1-z} \left[ 2T_{qg}^{A,C}(x, x_L, m^2) \right]_{z=1} \]
\[ - (1+z)T_{qg}^{A,C}(x, x_L, M^2) \tag{4} \]
and \( \Delta_{y\rightarrow qg}(z) = \Delta_{y\rightarrow qg}(zH, \mu^2) \)

The gluon formation time for radiation from a heavy quark can be read out from the phase factors in the effective twist-four matrix element as
\[ \tau_f = \frac{1}{p^+ x_L} = \frac{2x(1-z)q^-}{\ell_T^2 + (1-z)^2 M^2}, \tag{5} \]

which is shorter than that for gluon radiation from a light quark. This should have significant consequences for the effective modified
quark fragmentation function and the heavy quark energy loss.

One can then calculate the heavy quark energy loss, defined as the fractional energy carried by the radiated gluon,

\[
\langle \Delta x_q^2 \rangle (x_B, Q^2) = \frac{\alpha_s}{2\pi} \int_0^{Q^2} d\ell_T^2 \int_0^1 dz \frac{\Delta_2 \gamma_q \gamma_q (1-z)}{\ell_T^2 + z^2 M^2} z
\]

\[
= \frac{CC_A \alpha_s^2 x_B}{N_c Q^2 x_A} \int_0^1 dz \frac{1 + z^2}{z(1-z)}
\]

\[
\times \int_{\bar{x}_M}^{\bar{x}_L} d\bar{z}_L \frac{(\bar{x}_L - \bar{x}_M)^2}{x_A^2}
\]

\[
\times \left[ \left(1 - e^{-\bar{x}_L/x_A^2} \right)^2 a_1 + a_2 \right],
\]

where \( \bar{x}_M = (1-z)M^2/2zp^+q^+ \) and \( \bar{x}_L = \mu^2/2p^+q^+z(1-z) + \bar{x}_M \). Note that \( \bar{x}_L/x_A = L_A/\tau_f \) with \( L_A = R_A m_N/p^+ \) the nuclear size in the chosen frame. The LPM interference is clearly contained in the second term of the integrand that has a suppression factor \( 1 - e^{-\bar{x}_L/x_A^2} \).

Since \( \bar{x}_L/x_A \sim x_B M^2/x_A Q^2 \), there are two distinct limiting behaviors of the energy loss for different values of \( x_B/Q^2 \) relative to \( x_A/M^2 \). When \( x_B/Q^2 \gg x_A/M^2 \) for small quark energy (large \( x_B \)) or small \( Q^2 \), the formation time of gluon radiation off a heavy quark is always smaller than the nuclear size. In this case, \( 1 - \exp (-\bar{x}_L/x_A^2) \sim 1 \), so that there is no destructive LPM interference. The integral in Eq. (6) is independent of \( R_A \), and the heavy quark energy loss

\[
\langle \Delta x_q^2 \rangle \sim C_A \frac{\widetilde{\alpha}_s^2}{N_c} \frac{x_B}{x_A Q^2}
\]

(7)

is linear in nuclear size \( R_A \). In the opposite limit, \( x_B/Q^2 \ll x_A/M^2 \), for large quark energy (small \( x_B \)) or large \( Q^2 \), the quark mass becomes negligible. The gluon formation time could still be much larger than the nuclear size. The LPM suppression factor \( 1 - \exp (-\bar{x}_L/x_A^2) \) will limit the available phase space for gluon radiation. Therefore, the heavy quark energy loss

\[
\langle \Delta x_q^2 \rangle \sim C_A \frac{\widetilde{\alpha}_s^2}{N_c} \frac{x_B}{x_A Q^2}
\]

(8)

now has a quadratic dependence on the nuclear size similar to the light quark energy loss. Shown in Fig. 2 are the numerical results of the \( R_A \) dependence of charm quark energy loss, rescaled by \( C(Q^2)C_A \alpha_s^2(Q^2)/N_c \), for different values of \( x_B \) and \( Q^2 \). One can clearly see that the \( R_A \) dependence is quadratic for large values of \( Q^2 \) or small \( x_B \). The dependence becomes almost linear for small \( Q^2 \) or large \( x_B \). Here we take \( M = 1.5 \text{ GeV} \) for charm quark in the numerical calculation.

To illustrate the mass suppression of ra-
diative energy loss imposed by the “dead-cone”, we plot the ratio \( \frac{\langle \Delta z_Q \rangle(x_B, Q^2)}{\Delta z_q(x_B, Q^2)} \) of charm quark and light quark energy loss as functions of \( Q^2 \) and Bjorken variable \( x_B \) in Fig. 3 and Fig. 4 respectively. Apparently, the heavy quark energy loss induced by gluon radiation is significantly suppressed as compared to a light quark when \( x_B \) is large and when the momentum scale \( Q \), or the quark initial energy \( q^- \) is not too large as compared to the quark mass. Only in the limit \( M \ll Q, q^- \), is the mass effect negligible. Then the energy loss approaches that of a light quark.

In summary, we have calculated the energy loss of the massive quark in terms of modified heavy quark fragmentation function in nuclei with the twist expansion approach. We show that mass effects such as “dead-cone” effect gives a significant suppression to the induced heavy quark energy loss. In particular, (nuclear) medium size dependence of heavy quark energy loss is found to change from a linear to a quadratic form when the initial energy and momentum scale are increasing, which is quite different from the light quark energy loss where the total energy loss always have a quadratic dependence on medium size. The result can be easily extended to a hot and dense medium, which may be applied to study heavy quark production and suppression in heavy-ion collisions at RHIC and LHC.

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