Paschos–Wolfenstein Relationship for Nuclei and the NuTeV $\sin^2 \theta_W$ Measurement

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Abstract

We discuss nuclear effects in the Paschos–Wolfenstein relationship in the context of extraction of the weak mixing angle. We point out that the neutron excess correction to the Paschos–Wolfenstein relationship for a neutron-rich target is negative and large on the scale of experimental errors of a recent NuTeV measurement. We found a larger neutron excess correction to the Paschos–Wolfenstein relationship for total cross sections than that discussed by the NuTeV collaboration. Phenomenological applications of this observation are discussed in the context of the NuTeV deviation. Uncertainties in the neutron excess correction are estimated. Effects due to Fermi motion, nuclear binding, and nuclear shadowing are also discussed in the context of total cross sections.

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The NuTeV collaboration recently reported the results of the measurement of the weak mixing angle in deep inelastic neutrino and antineutrino scattering from a heavy target [1]. The NuTeV value of $\sin^2 \theta_W = 0.2277 \pm 0.0013 \text{(stat.)} \pm 0.0009 \text{(syst.)}$ turned out to be significantly larger than that derived from a global standard model fit to other electroweak measurements, $0.2227 \pm 0.0004$ [2]. The discussion of possible uncertainties and physics behind this discrepancy can be found in ref. [3]. However, before speculating on possible new physics, one should first worry about “standard” effects and uncertainties.

A useful tool, employed by the NuTeV collaboration to derive $\sin^2 \theta_W$, is the Paschos–Wolfenstein relationship (PW) [4]

$$R^- = \frac{\sigma^\nu_{\text{NC}} - \sigma^\nu_{\text{NC}}}{\sigma^\nu_{\text{CC}} - \sigma^\nu_{\text{CC}}} = \frac{1}{2} - \sin^2 \theta_W,$$

where $\sigma^\nu_{\text{NC}}$ and $\sigma^\nu_{\text{CC}}$ are the deep inelastic neutrino cross sections for neutral current (NC) and charged-current (CC) interactions, and $\sigma^\bar{\nu}_{\text{NC}}$ and $\sigma^\bar{\nu}_{\text{CC}}$ are corresponding antineutrino cross sections [19].

In the world without heavy quarks and with exact isospin symmetry, Eq.(1) is an exact relation for the isospin zero target for both the total and differential cross sections. The validity of this relationship is solely based on the isospin symmetry [4]. Therefore, Eq.(1) also holds for a nuclear target, provided that the nucleus is in the isoscalar state. In particular, this means that various strong interaction effects, including nuclear effects, must cancel out in the ratio (1). However, in the real world Eq.(1) must be corrected for the $s$- and $c$-quark effects. Furthermore, the targets used in neutrino experiments are usually non-isoscalar nuclei, and Eq.(1) must also be corrected for non-isoscalarity effects, as well as for other nuclear effects such as Fermi motion, binding, and nuclear shadowing.

Nuclear effects in the context of $\sin^2 \theta_W$ were recently discussed in refs.[7, 8, 9]. It was suggested in ref.[7] that a difference between the nuclear shadowing in NC and CC interactions may account for the discrepancy in NuTeV’s measurement of $\sin^2 \theta_W$, although no specific calculation of this effect was given. The impact of nuclear effects on the extraction of $\sin^2 \theta_W$ from data was discussed in ref.[8] in terms of a phenomenological parameterization of nuclear effects in parton distributions derived from charged lepton deep inelastic scattering. Phenomenological studies of the difference between nuclear effects in the $u$- and $d$-quark distributions in the context of the PW relationship were presented in ref.[9].

In this paper we address nuclear effects in the PW ratio for total cross sections. We start from the discussion of the non-isoscalarity correction to the PW ratio, since there appears to be some confusion about the sign and the magnitude of this effect in the literature. Then we discuss Fermi motion, nuclear binding, and nuclear shadowing corrections to the PW ratio.

I. THE PW RELATIONSHIP FOR GENERIC TARGET

We discuss (anti)neutrino deep inelastic scattering in the leading twist QCD approximation assuming that the four-momentum transfer $Q$ is large enough. In this approximation the NC and CC structure functions are given by well-known expressions in terms of quark and antiquark distribution functions [10]. In order to simplify discussion of isospin effects, we consider the isoscalar, $q_0(x) = u(x) + d(x)$, and isovector, $q_1(x) = u(x) - d(x)$, quark distributions (for simplicity, we suppress the explicit notation for the $Q^2$ dependence of parton
distributions). The calculation of the NC and CC cross sections, and the PW ratio in the QCD parton model is straightforward. A QCD radiative correction to the PW ratio for the total cross sections was calculated in ref.[3]. The result can be written as

\[ R^\pm = \frac{1}{2} - s^2_W + \left[ 1 - \frac{7}{3} s^2_W + \frac{4\alpha_s}{9\pi} \left( \frac{1}{2} - s^2_W \right) \right] \left( \frac{x^-}{x_0} - \frac{x^+}{x_0} + \frac{x^-}{x_0} \right), \tag{2} \]

where \( s^2_W = \sin^2 \theta_W \), \( \alpha_s \) is the strong coupling, and \( x^- = \int dx \, x(q_a - \bar{q}_a) \), with \( q_a \) and \( \bar{q}_a \) the distribution functions of quarks and antiquarks of type \( a \) [21]. The subscripts 0 and 1 refer to the isoscalar \( q_0 \) and isovector \( q_1 \) quark distributions, respectively. In the derivation of Eq.(2) we expanded in \( x^-/x_0 \) and \( x^+/x_0 \) and retained only linear corrections. Eq.(2) applies to any (not necessarily isoscalar) nuclear target. We observe that the PW relationship (1) is corrected by the C-odd parts of the isovector component in the target \( (x^-) \) and the strange and charm components of the target’s sea.

The isovector quark distribution \( q_1 \) vanishes in an isoscalar target, provided that the isospin symmetry is exact. However, a correction due to the quark–antiquark asymmetry in the nucleon (or nuclear) strange sea is possible. In particular, a positive \( x^-_s \) would move \( s^2_W \) towards a standard model value [3]. However, available phenomenological estimates are controversial even in the sign of the effect: a shift in the \( s^2_W \) relationship (1) is corrected by the \( q_0/p \) and \( q_1/p \) distribution functions of quarks and antiquarks. Assuming exact isospin symmetry, we have \( q_A = Z q_{A/p} + N q_{A/n} = A/2 (q_{A/p} + q_{A/n}) + Z - N/2 (q_{A/p} - q_{A/n}) \), \tag{3} where \( A = Z + N \). We now apply this relation to the isoscalar and the isovector quark distributions. Assuming exact isospin symmetry, we have \( q_{1/p} = q_{0/n} \) and \( q_{1/p} = -q_{1/n} \). For nuclear parton distributions per one nucleon we then obtain

\[ A^{-1} q_{0/A} = q_{0/p}, \tag{4a} \]
\[ A^{-1} q_{1/A} = -\delta N q_{1/p}, \tag{4b} \]

where we introduced a fractional excess of neutrons \( \delta N = (N - Z)/A \). Similar equations can readily be written for antiquark distributions.

Eqs.(4) suggest a negative neutron excess correction to \( R^- \) in a neutron-rich target. Indeed, using Eqs.(4), we find from Eq.(2) that the correction is

\[ \delta R^- = -\delta N \frac{x^-}{x_0} \left[ \frac{7}{3} s^2_W + \frac{4\alpha_s}{9\pi} \left( \frac{1}{2} - s^2_W \right) \right], \tag{5} \]
where \( x_1^- \) and \( x_0^- \) are taken for the proton.

The NuTeV collaboration takes into account the non-isoscalarity correction in the analysis and discusses this effect on \( R^- \) \cite{5, 6}. However, Eq.(5) is different from the corresponding NuTeV equation (Eq.(7) in ref.\cite{5} and Eq.(9) in ref.\cite{6}). First, it must be noted that the NuTeV equation has the wrong sign of the \( \delta N \) term. Nevertheless, the NuTeV non-isoscalarity correction is eventually negative \cite{11}. Therefore, in the following discussion we assume a correct sign in Eq.(7) in ref.\cite{5}. Second, Eq.(5) involves only \( C \)-odd terms, hence the factor \( x_1^- / x_0^- \). The corresponding factor in refs.\cite{5, 6} is \((x_u - x_d)/(x_u + x_d)\), where \( x_a = \int dx xq_a(x) \) for the quark distribution \( q_a \) in the proton. This factor has a mixed \( C \)-parity and for this reason is not allowed in Eq.(\ref{eq:5}). Furthermore, Eq.(\ref{eq:5}) includes the non-isoscalarity correction and compute \( \delta N \) of ref.\cite{14}, we have the estimation on the uncertainty in the ratio of cross sections. The NuTeV collaboration also reported the neutron excess correction to be \( -2251 \), which is now about 1\( \sigma \) away from the standard model value. The corresponding correction computed using Eq.(7) in ref.\cite{6} is \(-0.0094\). This correction is smaller in magnitude because the factor \((x_u - x_d)/(x_u + x_d)\) is smaller than the factor \( x_1^- / x_0^- \) by about 25\%. If we simply apply the difference, \(-0.0026\), to the NuTeV central value of \( \sin^2 \theta_W \), we get a new value, 0.2251, which is now about 1.5\( \sigma \) away from the standard model value. The NuTeV collaboration also reported the neutron excess correction to be \(-0.0080\) \cite{22}. In this case the additional shift in the NuTeV \( \sin^2 \theta_W \) is \(-0.004\), and the discrepancy between the NuTeV and the standard model value of \( \sin^2 \theta_W \) practically disappears. However, it must be noted that although the non-isoscalarity correction of \(-0.008\) was discussed in ref.\cite{5}, in the context of \( R^- \), it does not directly apply to the ratio of cross sections. The NuTeV collaboration explains that this is the shift in the NuTeV \( \sin^2 \theta_W \) that takes into account experimental background and the cuts \cite{11}.

The \( \alpha_s \) correction to \( R^- \) is also negative, but small compared to the leading term. Using the NLO \( \alpha_s = 0.2161 \) at \( Q^2 = 20 \text{ GeV}^2 \), we find the \( \alpha_s \) correction to \( R^- \) about \(-0.0002\).

Let us now discuss possible uncertainties in the non-isoscalarity correction. According to NuTeV \cite{11}, the uncertainty in this correction to \( \sin^2 \theta_W \) is 0.00005. Apparently, this is due to the uncertainty in the neutron excess in the target \cite{22}. However, this is not the only source of uncertainties in the \( \delta N \) correction. Indeed, the ratio \( x_1^- / x_0^- \) is subject to theoretical and experimental uncertainties. This ratio depends on the set of parton distributions as well as on the order of perturbation theory to which the analysis is performed \cite{23}. For example, the ratio \( x_1^- / x_0^- \) calculated with the MRST parton distributions \cite{13} at the same value of \( Q^2 \) is very similar to that of CTEQ5. However, \( x_1^- / x_0^- = 0.445 \) for Alekhin’s parton distributions \cite{14}. Furthermore, the parton distributions are known with some error. This error was estimated in a recent analysis in ref.\cite{14}. Using the parton distributions of ref.\cite{14}, we have the estimation on the uncertainty in the ratio \( \delta(x_1^- / x_0^-) \approx 0.04 \). This results in the uncertainty in \( R^- \) about 0.001, which is the order of magnitude larger than the uncertainty in the neutron excess in the target. As an illustration, we treat this uncertainty as an independent theoretical uncertainty in \( \sin^2 \theta_W \) and add it in quadrature to the NuTeV error. Then we find some 1.2\( \sigma \) distance between the standard model value and the corrected value of NuTeV \( \sin^2 \theta_W \) discussed above. We also comment that the variations of the parton distributions with \( Q^2 \) introduce additional uncertainty, which is hard to access in the present
III. FERMI MOTION AND NUCLEAR BINDING CORRECTIONS

We briefly discuss other nuclear effects on $R^-$. In order to sort out different effects, in the present discussion we do not consider explicit violations of the isospin symmetry. We recall that the isospin symmetry requires nuclear effects to cancel out in the $R^-$ ratio for the isoscalar target. For a non-isoscalar nucleus, Eq. (5) was derived neglecting the effects of Fermi motion and nuclear binding. However, it is rather obvious that these effects and the non-isoscalarity correction must be considered in a unified approach. In such an approach Eq. (4) must be replaced by a convolution of quark distributions with nuclear distributions [15]. Note that in the approximation in which the proton and neutron distributions are identical, the Fermi motion and nuclear binding effects cancel out in the ratio $x^-_1/x^-_0$ [24]. Therefore, a possible correction comes through the difference between the proton and neutron distribution functions and is likely to be small. One source of the correction is a nonlinear dependence of the nucleon distribution functions on the number of bound particles. In heavy nuclei with $Z \neq N$ this effect results in different nucleon distribution functions in the isovector and isoscalar channels even if no violation of the isospin symmetry is admitted [25]. Because of this effect the ratio $x^-_1/x^-_0$ receives a correction factor of $1 - \frac{2}{9}T/M$, where $T$ is the average kinetic energy of the bound nucleon and $M$ the nucleon mass [16]. For the iron nucleus this factor is 0.993 that gives only a small correction to $R^-$. 

IV. NUCLEAR SHADOWING EFFECT

We now discuss nuclear shadowing effect in the context of the isoscalar and isovector quark distributions. The nuclear shadowing corrections to the quark distributions can be written as

$$A^{-1} q_0/A = q_0/p + \delta_{sh} q_0,$$

$$A^{-1} q_1/A = -\delta Nq_1/p + \delta_{sh} q_1.$$ 

Similar equations can be written for antiquark distributions. The nuclear shadowing effect in the isoscalar $C$-even ($q^+_0 = q_0 + \bar{q}_0$) and $C$-odd ($q^-_0 = q_0 - \bar{q}_0$) quark distributions was studied in ref. [17], where a different magnitude of the shadowing effect in $q^+_0$ and in $q^-_0$ was observed. In particular, it was argued that the relative shadowing effect in the $C$-odd quark distribution is enhanced compared to that in the $C$-even distribution (see also [18]). In order to estimate the shadowing effect in the total cross sections, we apply the approach of ref. [17] and calculate the shadowing corrections to the average quark light-cone momentum, $\delta_{sh} x_0$, in the $C$-even and $C$-odd distributions. For the iron nucleus the result is $\delta_{sh} x^+_0/x^+_0 = -0.01$ and $\delta_{sh} x^-_0/x^-_0 = -0.002$. We observe that the shadowing effect in the total cross sections is significantly bigger in the $C$-even channel, in spite of the enhancement of the relative nuclear shadowing effect in the $C$-odd quark distributions. This, paradoxical from the first glance, result can be understood by observing that the $C$-odd cross section is saturated by the valence region at large $x$, where the nuclear shadowing effect is small. On the other hand, a large part of the $C$-even cross section comes from the sea region, where the nuclear shadowing effect is essential.
Since the isovector distribution must vanish in the isoscalar nucleus, the shadowing effect in the isovector channel appears at least in the order $\delta N$. This effect has not been quantitatively studied yet. In our estimates we neglect the shadowing correction to $x_1^-$, leaving this interesting question to further studies. Then, the nuclear shadowing correction reduces to the renormalization of the ratio $x_1^-/x_0^-$ by the factor of 1.002 for the iron nucleus.

Combining the nuclear shadowing effect with the Fermi motion and nuclear binding corrections to $R^-$, we observe a partial cancellation between these effects. For the iron nucleus the resulting correction factor to the ratio $x_1^-/x_0^-$ is 0.995 that gives a negligible correction to $R^-$. 

V. SUMMARY

We have studied nuclear effects in the PW relationship for total cross sections. We observed that for a neutron-rich target (such as iron, used in the NuTeV experiment) the neutron excess correction to the PW relationship is negative and large on the scale of experimental errors. We found a larger neutron excess correction than that discussed by the NuTeV collaboration in the context of $R^-$. The ratio $R^-$ is rather sensitive to the value of the neutron excess correction and a consistent treatment of this correction may well explain at least a large part of the NuTeV deviation.

The uncertainties in nuclear non-isoscalarity correction were discussed. We found that the uncertainty is dominated by the uncertainty in the parton distributions leading to the variation in $R^-$ about 0.001 that should be considered as an additional source of theoretical uncertainty in $\sin^2 \theta_W$.

We discussed the effects of Fermi motion, nuclear binding and nuclear shadowing on the ratio $R^-$ for the total cross sections. We point out that nuclear effects on $R^-$ vanish for the isoscalar nucleus and appear in the first or higher order in $\delta N$. For this reason these effects are small. Furthermore, we observed a partial cancellation between Fermi motion and nuclear shadowing effects in the ratio $R^-$. However, this cancellation of nuclear effects in the ratio $R^-$ for the total cross sections does not, of course, mean that these effects identically cancel out in the ratio of differential cross sections. Since the NuTeV experiment does not measure the total cross sections, some nuclear effects which cancel out in $R^-$ remain in actual experimental observables. Therefore, in order to clarify the impact of nuclear effects on the NuTeV result, it is necessary to explicitly take them into account in the analysis.

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[14] NuTeV experiment was primarily designed to measure \( \sin^2 \theta_W \) using the Paschos–Wolfenstein relationship [5]. However, it must be noted that the NuTeV collaboration does not directly measure the cross sections and the ratio \( R^- \), but rather measures “ratios of experimental candidates within kinematic criteria and compares this to a Monte Carlo simulation” [6].

[15] Eq. (2) was derived in the tree approximation. Electroweak corrections depend on the definition of the weak mixing angle. To be specific, the weak mixing angle is defined here in the on-shell scheme, \( s^2_W = 1 - M^2_W/M^2_Z \).

[16] The total cross sections involve the integration of the structure functions over the full phase space of \( x \) and \( Q^2 \). Therefore \( \alpha_s \) and the moments \( x_i^- \) of the parton distributions are taken at some average scale \( Q^2 \), which has to be chosen according to specific experimental conditions.

[17] Using the 0.05% uncertainty in \( \delta N \) [4] and Eq. (5), we obtain the uncertainty in \( \sin^2 \theta_W \) about 0.0001.

[18] The NuTeV collaboration uses its own parton distributions obtained in the leading order fit to their CC data. We comment in this respect that, since the NuTeV collaboration aims at precise determination of the weak mixing angle, the inclusion of NLO corrections to parton distributions appears as a necessary improvement of the analysis.

[19] However, it must be emphasized that this cancellation applies to the total cross sections. For the ratio of differential cross sections (or for the quark distributions) this effect remains finite even in the approximation in which the distribution functions of bound protons and neutrons are identical.

[20] It is interesting to note that this effect may mask the effects of isospin violation in quark distribution functions. It must be also commented that possible difference between nuclear modifications of \( u \)- and \( d \)-quark distributions was discussed in ref. [5] in the context of the NuTeV deviation in terms of a phenomenological approach.