21 cm signal from cosmic dawn: Imprints of spin temperature fluctuations and peculiar velocities

Raghunath Ghara\textsuperscript{1*}, T. Roy Choudhury\textsuperscript{1†} and Kanan K. Datta\textsuperscript{1‡}

\textsuperscript{1} National Centre for Radio Astrophysics, TIFR, Post Bag 3, Ganeshkhind, Pune 411007, INDIA

ABSTRACT

Observations of fluctuations in the 21 cm brightness temperature $\delta T_b$ from reionization promise to provide information on the physical processes during that epoch. We present a formalism for generating the distribution of $\delta T_b$ using dark matter $N$-body simulations and an one-dimensional radiative transfer code. The spectral energy distribution of the radiation sources is assumed to consist of a stellar-like and a mini-quasar like component. Our analysis is able to take into account the fluctuations in the spin temperature $T_S$ of neutral hydrogen arising from inhomogeneous X-ray heating and Ly\textalpha coupling during cosmic dawn. In agreement with other similar studies, we find that the power spectrum at large scales ($k \sim 0.1\, \text{Mpc}^{-1}$), when plotted as a function of redshift, shows three peaks. The middle peak, arising from fluctuations in the kinetic temperature of the gas, has the largest amplitude and occurs when $\sim 10\%$ of the gas (by volume) is heated above the CMB temperature, irrespective of the X-ray source properties. The power spectrum when plotted against $k$ shows a "bump"-like feature during cosmic dawn and its location measures the typical sizes of heated regions. Since the observations would measure the signal only in redshift space, we also account for peculiar velocity effects while simulating the signal. We find that the effect of peculiar velocities on the power spectrum is negligible at large scales ($k \lesssim 0.4\, \text{Mpc}^{-1}$) throughout the reionization history. During early stages (i.e., when the volume averaged ionization fraction $\lesssim 0.2$) this is a consequence of the fact that the signal is dominated by fluctuations in $T_S$. The peculiar velocity effects are prominent only at smaller scales where patchiness in the neutral hydrogen density dominates the signal. The conclusions are unaffected by changes in the amplitude or steepness in the X-ray spectra of the sources. We also discuss observational implications of our results.

Key words: cosmology: theory – intergalactic medium – X-rays: diffuse background

1 INTRODUCTION

The birth of first stars, galaxies, quasars is one of the landmark events in the history of the Universe. It is believed that radiation produced by these sources spread through the intergalactic medium (IGM) and changed its thermal and ionization state completely. The onset of first sources of light which marked the end of the 'dark ages' and changed the IGM thermal state is often termed as 'cosmic dawn'. Subsequent period when the neutral hydrogen (HI) was ionized is popularly known as the epoch of reionization (EoR).

Unfortunately we have a very little knowledge about this event. Many important questions such as the exact timing, thermal and ionization state of the IGM, properties of the first sources, sinks, feedback effect, impact on the structure formation of the Universe etc. are largely unknown. This is because the signal coming from this epoch is so faint that even the most sensitive instrument existing could only provide limited information of the event. Nevertheless, observations of redshift $z \geq 6$ quasars absorption spectra \textsuperscript{166}Gunn & Peterson 1965, Becker et al. 2001, Fan et al. 2003, 2006, Goto et al. 2011, and the cosmic microwave background radiation (CMB) \textsuperscript{167}Komatsu et al. 2011, Planck Collaboration et al. 2013 suggest that the event probably took place around $15 < z < 6$ \textsuperscript{171}Fan et al. 2006, Malhotra & Rhoads 2003, Choudhury & Ferrara 2003, Mitra et al. 2011, 2012.

Observations of the redshifted HI 21-cm signal from the IGM is a powerful probe and is believed to provide us with enormous amount of information about
the epoch \cite{Furlanetto2006, Morales2010, Pritchard2012}. A huge effort is in place to measure the redshifted 21 cm signal from the cosmic dawn and EoR. First generation of low frequency radio telescopes like Low Frequency Array (LOFAR) \cite{vanHaarlem2013}, Murchison Widefield Array (MWA) \cite{Bowman2013}, Tingay et al. 2013, Giant Metrewave Radio Telescope (GMRT) \cite{Ghosh2012, Parcigna2013}, Precision Array for Probing the Epoch of Reionization (PAPER) \cite{Parsons2013}, 21 Centimetre Array (21CMA) have started observing with the aim to detect the signal from the EoR. Due to lack of lower frequency band these instruments might just miss the era of cosmic dawn when the very first sources were formed and changed the thermal state of the IGM. The extremely sensitive next generation telescope, the Square Kilometer Array (SKA) is expected to measure the signal at even lower frequencies which, in addition to the EoR, should also be able to probe the cosmic dawn \cite{Mellena2014}.

The HI 21 cm signal will be observed against the CMB radiation. The signal will be observed in either emission or absorption depending on whether the HI spin temperature is higher or lower than the background CMB temperature. It is often assumed that the spin temperature is highly coupled with the IGM kinetic temperature and much higher compared to the CMB temperature right from the birth of first light sources i.e., when reionization process starts \cite{Furlanetto2006, McQuinn2006, Mesinger2007, Choudhury2007, Datta2012, Battaglia2013, Iliev2014}. This leads to a situation in which the 21 cm signal is independent of the exact value of the spin temperature. However, these assumptions may not hold during the initial stages of reionization or the cosmic dawn. Although a small amount of Ly\(\alpha\) photons is enough for establishing the coupling, it is possible that even that small number of Ly\(\alpha\) photons is not available at every location in the IGM. The heating of the IGM is also highly dependent on the nature, total number and spectra of X-ray sources at cosmic dawn which are all unknown. In a situation where the collisions (between HI atoms or HI atoms and free electrons) become inefficient and Ly\(\alpha\) photons are only confined near the sources, the coupling between the IGM kinetic temperature and HI spin temperature becomes inhomogeneous. Even if the coupling is strong and complete, the heating could be incomplete and inhomogeneous due to lack of X-ray photons. This would lead to fluctuations in the spin temperature and consequently additional features in the 21 cm signal. Recently there has been several attempts in order to understand the effect of spin temperature fluctuations on the observed HI 21 cm signal during the cosmic dawn \cite{Bararkna2005, Chuzhoy2007, Semelin2005, Santos2008, Baek2009, Thomas2014, McQuinn2013}. Models with complete Ly\(\alpha\) coupling and heating predict the variance of HI brightness temperature fluctuations to be a few 10 mK², while the same quantity for inhomogeneous Ly\(\alpha\) coupling and heating model could exceed few 100 mK².

Efforts are also in place to understand the heating from different kind of X-ray sources such as miniquasars \cite{Thomas2011}, X-ray binaries \cite{Fialkov2014, Ahn2014}, and thermal emission from hot interstellar medium \cite{Pacucci2014}. Because of their large mean free path, the hard X-ray photons can penetrate a few tens of Mpc in the IGM \cite{Shull1985}, while soft X-ray photons will be absorbed within a smaller distance from the sources and as a result the heating will be very patchy. The heating pattern will be very different in the case of high mass X-ray binaries when compared to miniquasars or hot interstellar medium as they do not contain very large amount of soft X-rays \cite{Fialkov2014, Pacucci2014}. It has also proposed that measurements of HI 21 cm power spectrum from the cosmic dawn will reveal the amount of X-ray background \cite{Christian2013} and the spectral energy distribution of X-ray sources \cite{Pacucci2014}.

In this paper we study the effects of inhomogeneous Ly\(\alpha\) coupling and IGM heating on the HI 21 cm signal from the reionization epoch and cosmic dawn using a semi-numerical code which is primarily based on the algorithm presented in \cite{Thomas2008} and \cite{Thomas2009}. We use slightly different methods for the Ly\(\alpha\) coupling and heating calculation. We consider miniquasar like objects as X-ray heating sources and focus on statistical quantities such as the variance and power spectrum of the HI brightness temperature fluctuations which are among the primary goals of instruments like LOFAR and SKA.

Our major effort, in this paper, has gone into understanding the effect of the peculiar velocity on the HI 21-cm signal from the cosmic dawn. The peculiar velocity has a significant impact on the reionization and pre-reionization HI 21-cm signal \cite{Bharadwaj2003}. This also makes 21-cm power spectrum anisotropic \cite{Bararkna2005, Majumdar2013}. Recently, it has been shown that the peculiar velocity can boost the HI power spectrum by a factor of \(\sim 5\) at large scales during the initial stages of reionization when \(z_{\text{HI}} \lesssim 0.2\) \cite{Mao2012, Jensen2013}. During the same period power spectrum becomes highly anisotropic which is detectable with LOFAR 2000 hrs of observations. It was also suggested that such observations could tell us whether reionization occurred inside-out or outside-in \cite{Jensen2013, Majumdar2013}. However, all results described above is based on the assumption that the spin temperature is much higher than the CMB temperature. Here we investigate how the above results change once the heating and Ly\(\alpha\) coupling is calculated self consistently. As we see later, implementing the peculiar velocity effect on the signal during this epoch is slightly different compared to the case when the spin temperature is much higher than the CMB temperature. Apart from that, we investigate how peaks in the power spectrum can be used to extract information about the ionization state and size of the ‘heated bubbles’.

The plan of this paper is as follows: In section 2 we have briefly described the properties of the redshifted 21 cm signal from HI. We then discuss the detailed methodology for modelling the signal, including the \(N\)-body simulation used.
for present study (section 2.1), the source model (section 2.2), radiative transfer code for generating the maps around isolated sources (section 2.3), the generation of global maps (section 2.4) and modelling the redshift space distortion (section 2.5). In section 3 we discuss the main results of our analyses. The globally averaged ionization and heating properties of our models are discussed in section 3.1 while the fluctuations in the 21 cm signal is described in section 3.2. The main results of our work, i.e., the effects of the peculiar velocities on the 21 cm signal are discussed in section 3.3. In section 3.4 we see whether our conclusions are unchanged when the X-ray properties of the sources are varied, and we check the robustness of our results with respect to the resolution of the simulation box in section 3.5. We summarize and discuss our main results in section 4. Throughout the paper, we have used the cosmological parameters Ωm = 0.32, ΩΛ = 0.68, Ωb = 0.049, h = 0.67, ns = 0.96, and σ8 = 0.83 which are consistent with the recent results of Planck mission (Planck Collaboration et al. 2013).

2 SIMULATING THE 21 CM SIGNAL

Usually the redshifted 21 cm signal from neutral hydrogen is measured in terms of the deviation of 21 cm brightness temperature from the brightness temperature of background CMB radiation along a line of sight. The differential brightness temperature observed at a frequency νobs along a direction ⃗n is given by (Madau et al. 1997; Furlanetto et al. 2004)

$$\delta T_b(ν_{obs}, \vec{n}) \equiv \delta T_b(x) = 27 x_H(z, x)[1 + \delta_B(z, x)] \left(\frac{Ω_m h^2}{0.023}\right) \times \left[1 - \frac{3}{10} \left(\frac{Ω_m h^2}{Ω_b h^2}\right)^{1/2} \frac{T_{CMB}(z)}{T_s(z, x)}\right] \text{mK},$$

(1)

where $x = r_s \vec{n}$ and $1 + z = 1420 \text{MHz/}ν_{obs}$, with $r_s$ being the comoving radial distance to redshift $z$. The quantities $x_H(z, x)$ and $δ_B(z, x)$ denote the neutral hydrogen fraction and the density contrast in baryons respectively at point $x$ at a redshift $z$. The CMB temperature at a redshift $z$ is denoted by $T_{CMB}(z) = 2.73 \times (1 + z)$ and $T_s$ is the spin temperature of neutral hydrogen. We have omitted the effect of line of sight peculiar velocities (Bharadwaj & Ali 2004; Barkana & Loeb 2005a) in the above expression which essentially maps the point $x$ in real space to a point $s$ in the redshift-space, the mapping being determined by the line of sight peculiar velocity field. We will discuss how to account for this effect in Section 2.6.

The primary goal of the first generation radio telescopes is to measure the fluctuations in $δT_b$ using the spherically averaged power spectrum $P(k)$ which is defined as

$$\langle \delta T_b(k)\delta T_{b∗}(k') \rangle = (2π)^3 δ_D(k - k') P(k),$$

(2)

where $δT_b(k)$ is the Fourier transform of $δT_b(x)$ defined in equation (1). The dimensionless power spectrum is defined as $A^2(k) = k^3 P(k)/2π^2$ which also represents the power per unit logarithmic interval in $k$.

The spin temperature $T_s$ is determined by the coupling of neutral hydrogen gas with CMB photons by Thomson scattering, Lyα coupling and collisional coupling. Considering all these coupling effects, the spin temperature can be written in the following form (Field 1958; Furlanetto & Oh 2002)

$$T_s^{-1} = T_{CMB}^{-1} + x_a T_a^{-1} + x_e T_e^{-1} \left(1 + x_e + x_a\right),$$

(3)

where $T_K$ is the kinetic temperature of the gas and $T_α$ is the color temperature of the Lyα photons which, in most cases of interest, is coupled to $T_K$ by recoil during repeated scattering. The quantities $x_e$ and $x_a$ are the coupling coefficients due to collisions and Lyα scattering, respectively. The collision efficiency $x_e$ includes both the collisions between neutral hydrogen atoms (H – H) and hydrogen atom with free electrons in the medium (H – e) and can be written as (Hirata & Sigurdson 2004),

$$x_e = \frac{4T_e}{3A_{10} T_{CMB}} \left(\kappa^H_{HH}(T_K) n_H + \kappa^H_{eH}(T_K) n_e\right),$$

(4)

where $A_{10} = 2.85 \times 10^{-15} \text{ s}^{-1}$ is the spontaneous Einstein $B$-coefficient, $T_α = hν_{21cm}/k = 0.0681K$, $n_H$ and $n_e$ be the local number densities of neutral hydrogen and electrons respectively. The table of $κ_H^H$ as a function of $T_K$ is taken from Furlanetto & Furlanetto (2004) and table of $κ^H_{eH}$ is taken from Zygelman (2003); Allison & Dalgarno (1964).

The most important process which couples $T_K$ to $T_s$ during reionization is the Wouthysen-Field effect (Wouthuysen 1952; Field 1958; Madau et al. 1997; Hirata & Sigurdson 2004). The Lyα coupling coefficient in this case is given by

$$x_α = \frac{16π^2 T_α e^2 f_α}{27 A_{10} T_{CMB} m_e c} J_α,$$

(5)

where $J_α$ is the Lyα flux density, $f_α = 0.4162$ is the oscillator strength for the Lyα transition, $m_e$ and $e$ are the mass and charge of the electron respectively. In many studies of reionization, particularly those dealing with later stages (Furlanetto et al. 2004; McQuinn et al. 2006; Mesinger & Furlanetto 2007; Choudhury et al. 2009; Datta et al. 2012; Battaglia et al. 2013; Iliev et al. 2014), one assumes the Lyα coupling to be highly efficient and uses the approximation $T_s ≈ T_K$. In addition, if the IGM is assumed to be heated substantially compared to the CMB ($T_K ≫ T_{CMB}$), then the term $(T_s - T_{CMB})/T_s → 1$ in equation (1) and hence one obtains the simple expression where $δT_b$ tracks the neutral hydrogen distribution $x_H(1 + δ_H)$. These assumptions, however, have been shown not to hold in early stages of reionization where the Lyα coupling may not be uniformly strong in all locations and not all regions of the IGM will be heated uniformly (Santos et al. 2003; Back et al. 2010). This would lead to variations in $T_s$ and hence would affect the fluctuations in the 21 cm signal. We take all these effects into account in this paper using a combination of N-body simulations and an one-dimensional radiative transfer code.

2.1 Numerical simulations

The method we have used for simulating the 21 cm brightness temperature signal is essentially based on (i) obtaining the dark matter density field and the distribution of collapsed haloes from a N-body simulation, (ii) assigning
luminosities to these dark matter haloes and (iii) using a one-dimensional radiative transfer code to obtain the neutral hydrogen and spin temperature maps.

We have performed dark matter \( N \)-body simulations using the publicly available \textsc{cubePM}\footnote{http://wiki.cita.utoronto.ca/mediawiki/index.php/CubePM} \cite{Harnois-Deraps2012} which is essentially a massively parallel particle-particle-particle-mesh (P\(^3\)M) code. Initialization of the particle positions and velocities at redshift \( z = 200 \) was done by using \textsc{camb} transfer function\footnote{http://camb.info/} \cite{Lewis2000} and employing Zeldovich approximation. The fiducial simulation used in this work contains 768\(^3\) particles in a box of size 100 \( h^{-1} \) cMpc with 1536\(^3\) grid points. The mass resolution of the dark matter particles in the simulation is \( 1.945 \times 10^{6} \, h^{-1} \, M_{\odot} \). The simulation generates snapshots at \( 25 \geq z \geq 6 \) in equal time gap of \( 10^7 \) years. The output at each snapshot consists of the density and velocity fields in a grid which is 8 times coarser than the simulation grid. The code is also equipped with a run time halo finder which identifies halos within the simulation volume using spherical over density algorithm. We assume that the smallest halo at least contains 20 dark matter particles. As far as the baryonic density field is concerned, we simply assume that the baryons trace the dark matter, i.e., each dark matter particle is accompanied by a baryonic particle of mass \( (\Omega_{b}/\Omega_{m}) \times M_{\text{part.}} \). Though this assumption is not valid in very small scales, i.e., scales comparable to or smaller than the local jeans scale, it probably works fine at large scales which are of our interest.

The main difficulty with our fiducial simulation box is that it does have not smaller haloes \( (M_{\text{halo}} \sim 10^{6} \, M_{\odot}) \) which are believed to be driving the reionization at early stages. Resolving such small haloes require simulations of very high dynamic range which are beyond the computing power we have access to. Hence, to address this difficulty, we run an additional dark matter simulation of size 30 \( h^{-1} \) cMpc with 768\(^3\) particles. The mass resolution achieved in this case is \( 5.254 \times 10^{6} \, h^{-1} \, M_{\odot} \), thus giving a minimum halo mass of \( 1.05 \times 10^{6} \, h^{-1} \, M_{\odot} \). This box is helpful in probing the effects of small mass haloes at early stages, however, one should note that this box is too small for studying the large scales which are expected to be accessible to the first generation radio telescopes. Hence, we present most of our results using the fiducial box, and discuss whether missing out the small haloes make any difference to our conclusions in a separate section.

### 2.2 Source selection

The stars residing in the galaxies are believed to be the major source of ionizing photons that completes the hydrogen reionization process of the universe. The dark matter haloes are the most suitable place to form galaxies but not all of them will contain luminous sources. For galaxy formation to proceed, the gas is required to be cooled below their virial temperature by atomic (or molecular) cooling which may not be possible in the smallest mass haloes. Typically, the minimum mass of haloes which can cool via atomic hydrogen is \( \sim 10^{7} \, M_{\odot} \) while the same value can be much smaller \( \sim 10^{6} \, M_{\odot} \) in presence of hydrogen molecules. In addition, there could be further suppression of star formation in haloes lighter than \( \sim 10^{9} \, M_{\odot} \) which are residing in ionized regions because of radiative feedback. Since the smallest halo in our fiducial simulation has mass \( 3.89 \times 10^{6} \, h^{-1} \, M_{\odot} \), we are unable to tackle all these complications self-consistently; hence we assume all haloes in our simulation box to form stars at all redshifts. We will later (Section\footnote{http://www2.iap.fr/pegase/} III) discuss the simulation where haloes having masses as small as \( \sim 10^{6} \, h^{-1} \, M_{\odot} \) are resolved.

The relation between the dark matter halo mass and the galaxy luminosity of these early galaxies are all very uncertain. The fraction of baryons \( f_{\star} \) residing within the stars in a galaxy depends on the metallicity and mass of the galaxy. There is no well known relation between the stellar mass and total mass of the galaxies at very high redshifts. For simplicity here we have assumed \( f_{\star} \) to be constant throughout the reionization epoch and its value is chosen such that the resulting reionization history is consistent with the constraints obtained from CMB polarization measurements. The stellar mass of a galaxy corresponding to a dark matter halo of mass \( M_{\text{halo}} \) is

\[
M_{\star} = f_{\star} \left( \frac{\Omega_{b}}{\Omega_{m}} \right) M_{\text{halo}}. \tag{6}
\]

Given \( M_{\star} \), one can calculate the spectral energy distribution (SED) of stellar sources in a galaxy using stellar population synthesis codes. However, the SED will depend upon the initial metallicity and the stellar IMF, both of which evolve with time. For example, stars in very first galaxies are expected to be metal poor \cite{Lai2007,Enkelstein2009} and short-lived \cite{Mevnet2005}. Eventually they enrich the ISM with metals which changes the nature of subsequent star formation. Since tracking the evolution of metallicity self-consistently is not straightforward, we have taken the best fit mass metallicity relation from \cite{Daval2009} and \cite{Daval2010}.

\[
\frac{Z}{Z_{\odot}} = (0.25 - 0.05 \Delta z) \log_{10} \left( \frac{M_{\star}}{M_{\odot}} \right) - (2.0 - 0.3 \Delta z), \tag{7}
\]

where \( \Delta z = (z - 5.7) \). The evolution of stellar IMF of high redshift galaxies too is not well constrained. For our study we assume that the stars follow a Salpeter IMF with mass range \( 1 \) to \( 100 \, M_{\odot} \).

The UV and NIR spectral energy distributions of the stellar sources in galaxies are generated using the code \textsc{pegase}\footnote{http://www2.iap.fr/pegase/} \cite{Fioc1997} which computes the galactic SED using standard star formation scenarios for different initial metallicities, IMF and star formation history at different epochs. The lifetime of the stars in the galaxy may vary with metallicity and mass \cite{Mevnet2004}. For convenience, we set the stellar lifetime to be \( 10^{7} \) years which is the time difference between two simulation snapshots.

The dotted blue curve in Figure\footnote{http://camb.info/} II shows the intrinsic SED of the stellar component in a galaxy with stellar mass \( 10^{9} \, M_{\odot} \) with metallicity 0.1 \( Z_{\odot} \). The SED peaks around hydrogen ionization wavelength and falls sharply for higher energies (\( \gtrsim 50 \) eV). Thus, in absence of any other processes,
to UV luminosity from the sources
\[
f_x = \frac{\int_{10 \text{ keV}}^{100 \text{ eV}} I(E) dE}{\int_{10.2 \text{ eV}}^{100 \text{ eV}} I(E) dE},
\] (8)

where \(I(E)\) is the SED of the galaxy. Let us denote the SED of the stellar component by \(I_*(E)\), which in our case is computed using the stellar population synthesis code PEGASE2. We have seen from Figure 1 that this component does not contribute significantly to the X-rays, i.e., to the numerator of the above equation. The SED of the miniquasar-like component is assumed to have a power law form
\[
I_q(E) = A E^{-\alpha},
\] (9)

where \(A\) is a normalisation constant to be determined in terms of \(f_x\) and \(\alpha\). Let \(L_x\) be the luminosity of the stellar component in the UV band, i.e., \(L_x = \int_{100 \text{ eV}}^{100 \text{ eV}} I_*(E) dE\). Then, it is straightforward to show that the constant \(A\) is determined by the relation
\[
A = \frac{f_x L_x}{\left( \int_{10 \text{ keV}}^{100 \text{ eV}} I(E) dE - f_x \int_{10.2 \text{ eV}}^{100 \text{ eV}} E^{-\alpha} dE \right)}. \] (10)

We have varied the X-ray properties (i.e., the parameters \(f_x\) and \(\alpha\)) of the sources to study the effects on various quantities of interest. However, while choosing the values for \(f_x\) and \(\alpha\) it should be kept in mind that the normalisation coefficient \(A\) must be positive.

### 2.3 Radiative transfer around an isolated source

As the first sources of light appear in the universe, they start to ionize the surrounding IGM and create ionized bubbles around them. In addition, the UV and X-ray radiation from these sources would heat up the medium. In order to simulate the ionization and heating patterns around ionizing sources, we have developed a code which closely follows the treatment of [Thomass & Zaroubi 2008, 2011]. Essentially, the global ionization and temperature distributions are simulated in two steps: in the first step, we generate the ionization and temperature patterns as a function of time and distance around isolated sources surrounded by a uniform density field, and in the second step, we account for possible overlaps between these individual patterns around all the sources in the simulation box. We refer the readers to the original papers of [Thomass & Zaroubi 2008, 2011] for the details of the algorithm. Here we highlight some of the important features of the method and the modifications we have made to the method.

- Let us consider a galaxy of stellar mass \(M_*\), surrounded by a uniform IGM. Following the procedures outlined in the previous section, we can estimate the photons production rate at every energy band of interest. We then assume that the ionization and heating pattern around this isolated galaxy would be spherical in nature, and hence a one-dimensional radiative transfer setup is sufficient to solve the problem. The motivation for this assumption comes from the fact that at early stages of reionization the number density of the sources are low and the bubbles are believed

---

10 It turns out that the impact of X-rays from the hot interstellar medium is similar to those from the miniquasar like sources, as we will see later in this paper. This is because of the fact that the hot interstellar medium produces significant number of soft X-ray photons similar to miniquasar like sources. The HMXBs, on the other hand, show very different signatures since they do not contain any significant amount of soft X-rays.

**Figure 1.** The SED of a galaxy of mass \(10^8 \, M_\odot\) and metallicity 0.1 \(Z_\odot\). The dotted blue curve shows the stellar-like component, while the solid red curve shows the SED of the miniquasar like component. We have assumed the miniquasar to emit 5% of the UV energy in X-ray band with a power law SED (in this case the power law index \(\alpha\) is 1.5). The vertical green, black and orange lines correspond to energies 10.2 eV, 13.6 eV and 100 eV respectively.
to be separated from each other. Radiative transfer simulations studying the impact of the sources on the surrounding IGM indicate that the ionized bubbles around galaxies were probably almost spherical in nature before the overlap started (Kuhlen & Madau 2003; Thomas & Zaroubi 2008; Alvarez et al. 2010).

- The number densities of HI and HII along with those of HeI, HeII and HeIII are calculated as a function of distance $d$ from the source by solving the relevant rate equations. The most relevant processes in calculating the fraction of different species are photoionization by UV radiation, secondary ionization arising from high-energy free electrons and recombination.

- These rate equations are supplemented by the evolution equation for $T_K$. The most prominent heating processes are the photoheating due to UV and X-ray flux. The UV-heating is most prominent near the boundary of the ionized and neutral regions, while X-ray heating is substantial within the partially ionized and neutral regions because of large mean free path of high energy photons. In addition, X-rays can increase the kinetic temperature by sufficient Compton scattering with the free electrons in the medium. Among the various cooling processes described in Thomas & Zaroubi (2008), the main cooling of the IGM comes from the expansion of the universe.

- The UV and X-ray radiation flux from the central source is calculated self-consistently accounting for the $d^{-2}$ dilution and also the effects of optical depth along the photon path.

- In addition to the ionizing flux, we also keep track of the Lyα flux $J_{\alpha}$ around the source. The Lyα flux is assumed to decrease as $d^{-2}$ from the source. In addition to the continuum Lyα photons from the stellar sources, another source of Lyα photons is the X-rays from the source. In this case the fraction of energy from the primary electrons is spent in exciting the HI, which then generate Lyα photons on relaxation. Since the Lyα photons travel large distances without being absorbed, we account for the redshift of photons having energies larger than the Lyα frequency produced in the source. We have also restricted the Lyα photons to propagate with speed of light for a time equal to the age of the hosting halo.

- We store profiles of the fraction of different species, the kinetic temperature and the Lyα flux for a wide range of galaxy stellar masses, redshifts, density contrast, X-ray to UV luminosity ratio and X-ray SED spectral index. These catalogues are to be used later while constructing global maps of the 21 cm brightness temperature.

Before proceeding to the construction of global maps, we validate our code by studying the behaviour of some basic quantities for an isolated source surrounded by a uniform IGM. Figure 2 shows the ionization, heating and Lyα coupling patterns around a source which is taken to be a galaxy with stellar mass $10^8 M_\odot$ at $z = 10$. The results are shown at a time $10^7$ years after the sources began to radiate. The uniform IGM around the source is assumed to have a density equal to the mean density of the universe. The different curves in each panel represent different X-ray properties for the galaxy.

The top left panel of the figure shows the ionization fraction of hydrogen along the radial direction for different $f_X$ values: 0.05 (short dashed blue), 0.1 (dotted magenta curve), while the power law spectral index $\alpha$ is kept fixed (1.5). The three curves in second column represent three different values of $\alpha$ (0.5, 1.0, 1.5 respectively) while $f_X = 0.05$ is kept fixed.

![Figure 2](image_url)
values of $f_X$. One can see that the size of the HII region created around the galaxy is as large as $\sim 150$ pkpc. In absence of X-rays ($f_X = 0$) the ionization front shows a sharp transition, while in presence of X-rays the transition between ionized to neutral medium is relatively smooth. The reason behind this is the presence of high energy photons in the spectrum, which have longer mean free paths and are efficient in partially ionizing the gas. Note that the increase in the X-ray luminosity does not increase the size of the fully ionized HII region, but creates a partially ionized region of larger size beyond the HII region. The top right panel of Fig 2 shows the HII fraction for different values of the X-ray spectral index $\alpha$. A larger $\alpha$ implies a steeper spectrum, which in turn implies an increase in the number of soft X-ray photons for a fixed $f_X$. As a result, the size of the partially ionized region is larger for a higher value of $\alpha$.

Panels in the second row of Figure 2 show the kinetic temperature $T_K$ pattern around the isolated source. It is clear from the left panel that the presence of X-rays affects the temperature profile quite drastically. Firstly, there is rise in $T_K$ in the ionized region when $f_X$ is increased; this is mainly due to the Compton scattering of X-ray photons with free electrons. The size of the heated region is larger than the HII region in the presence of X-rays, with the size increasing with increasing values of $f_X$. Because of the larger mean free paths of high energy photons, $T_K$ shows a very smooth transition from the central highly heated ($\sim 10^4$ K) region to far away cold region ($\sim$ few K) in the presence of X-rays, while the transition is sharp in the absence of X-rays. Even when ratio of X-ray to UV luminosity is as small as 5%, regions as far as few hundreds of pkpc from the source have temperatures larger than the CMB temperature. With sufficient Ly$\alpha$ coupling, this heated region will show the signal in emission. An increase in $\alpha$ results in a larger number of soft X-ray photons, hence the temperature of the ionized region increases and so does the size of the heated regions as is seen from right panel of the second row.

The pattern of the spin temperature $T_S$ as calculated using equation 3 is shown in the third row from top. It is clear from the plots that $T_S$ closely follows $T_K$ at distances close to the source, while it tends to follow $T_{CMB}$ at larger distances. This behaviour can be explained by the $d^{-2}$ decline of the Ly$\alpha$ flux $J_\alpha$ from the source, which makes the Ly$\alpha$ coupling stronger closer to the source and weaker at larger distances.

In order to show the differences between $T_S$ and $T_K$, we plot the function $T_S - T_K$ as a function of distance from the source in the fourth row from top. As expected, $T_S = T_K$ close to the source because of strong Ly$\alpha$ coupling, and this relation holds for distances $\sim 300 - 400$ pkpc which are much larger than the HII region. If we look at larger distances, the Ly$\alpha$ coupling weakens and $T_S$ gradually tends towards $T_{CMB}$. Hence one cannot work with the simple assumption that $T_S$ follows $T_K$ at early stages of reionization. Since the value of $T_S - T_K$ is essentially determined by the Ly$\alpha$ coupling, which in turn is mainly determined by $J_\alpha$ from the central source, we find that the curves are almost independent of the X-ray properties of the source.

The bottom row shows the 21 cm signal $J_{21}$ profile around the source. As is already known, (Thomas & Zaroubi 2003, Alvarez et al. 2010, Yajima & Li 2013), there are four main regions in the profile starting from the centre going outward: (i) the signal vanishes completely in the HII region, (ii) there is an emission region just beyond the HII region arising from X-ray heating in the partially ionized medium, (iii) the signal decreases and turns into an absorption feature because of the decrease in the value of $T_K$ and finally (iv) the absorption signal decreases and gradually vanishes as the Ly$\alpha$ coupling becomes weaker. When $f_X = 0$, the heating due to X-rays is absent. Hence the second region with emission signal is obviously absent, and the absorption signal is much stronger. As the value of $f_X$ is increased, the size of the region with emission signal increases, however the amplitude of the signal remains almost the same. The amplitude of the absorption signal in the third region decreases with increasing $f_X$. Similarly, increasing the value of $\alpha$ shows a similar effect on the emission and absorption signal.

2.4 Global maps

Having discussed the generation of ionization and temperature maps around an isolated galaxy, we now discuss the method for generating such maps in the full simulation box. Our treatment closely follows that of (Thomas et al. 2009) to account for overlap between the individual patterns, though we have introduced some modifications in the method which are discussed below.

The method starts by listing the location and mass of all the sources in the box. One then determines the radius $R_{HII}$ of the HII region, defined as the distance at which $x_{HII}$ falls to $x_{HII}^0 = 0.5$, for each source. Since the ionization profile shows a sharp drop from $x_{HII} = 1$ to $x_{HII} \sim 0.1$ for the values of $f_X$ and $\alpha$ considered in this paper, varying the threshold value in the range $0.1 < x_{HII}^0 < 1$ should not affect our results. For a given source the average HI number density is measured within a sphere of radius $R_0$ around the source. Initially $R_0$ is taken to have a small value. Having found the overdensity within $R_0$, we find out the radius of the ionization front $R_1$ for the source from the 1-D catalog as obtained in Section 2.3. If $R_0$ is taken to be sufficiently small, we usually end up with $R_1 > R_0$. Since the sources form preferentially at high-density peaks, we expect the average density within the sphere to decrease with increasing radius. Hence we take $R_1$ to be the next guess for $R_0$ and iterate the process till $R_1 \approx R_0$. We then assign the radius of the HII region to be $R_{HII} = R_1$. Let us denote the radius of the HII region and the mean hydrogen number density within the HII region for $i^{th}$ source to be $R_{HII}^i$ and $n_{H_{-1D}}^i$, respectively.

Having found the quantities $R_{HII}^i$ and $n_{H_{-1D}}^i$, the correct ionization and heating profile around the source can be selected from the previously generated catalogue of 1-D profiles. While assigning the ionization profile within the HII region is straightforward, one needs to be slightly careful while assigning the ionization fraction in the partially ionized regions. If one assigns the same profile corresponding to $n_{H_{-1D}}^i$ in the partially ionized region, as is done by (Thomas et al. 2009), then there will be an underestimation in the value of ionization fraction. This is due to the fact that the average density decreases with distance from the source. In order to account for this effect, we assume that the number of photons from the $i^{th}$ source at a point $x$ in the partially ionized region is given by $x_{HII_{-1D}}^i(x) \times n_{H_{-1D}}^i$, where $x_{HII_{-1D}}^i(x)$ is the ionization fraction as obtained from our catalogue of 1-D
profiles. If the number density of hydrogen in that particular pixel is \( n_{\text{H}}(x) \), then the ionization fraction will be modified to \( x_{\text{HIH}1-D}(x) \times n_{\text{HIH}1-D}/n_{\text{H}}(x) \). Note that we do not account for the fact that the number of recombinations will also be less for lower densities as that would involve modelling of sub-grid physics.

When the individual HII regions overlap there will be excess photons residing at the overlapped regions which need to be accounted for. Following Thomas et al. (2009) we use an iterative process to estimate the total number of excess photons in the overlapping regions and distribute them among the contributing ionizing sources. The main difference between our approach and that of Thomas et al. (2009) arises while assigning the ionization profile in the partially ionized regions. In the presence of X-rays, these regions are larger than the HII region and hence their overlap begins much earlier. The ionization fraction in overlapping partially ionized regions is given by

\[
x_{\text{HI}}(x) = \sum_i x_{\text{HI}1-D}(x) \times n_{\text{HI}1-D}/n_{\text{H}}(x).
\]

The heated regions too extend well beyond the HII regions, and hence they start overlapping very early during reionization. Let a point be heated up by photons from \( n \) sources. Let \( \{T_1, T_2, ..., T_n\} \) be the set of temperatures at that point obtained from the catalogue of 1-D profiles for these sources. In such a situation, a possible approach could be to assign the temperature in the overlap region by invoking the conservation of energy Thomas et al. (2009). In our study, however, we have used a different approach which is more straightforward to implement.

Our approach is based on the correlation between \( T_K \) and \( x_{\text{HII}} \) in the heated regions for isolated sources. Figure 3 shows the plot of \( T_K \) as a function of \( x_{\text{HII}} \) for different types of source properties. The top-left panel shows the plot for different stellar masses in the galaxy, the bottom-left panel shows the same for different values of the density contrast, the top-right is for different values of \( f_X \) and the bottom-right panel shows the plots for different values of \( \alpha \). Clearly a strong correlation exists between \( T_K \) and \( x_{\text{HII}} \) for \( x_{\text{HII}} < 0.1 \) and this correlation is completely independent of the source properties and densities for \( f_X > 0 \). Thus we can use this correlation for determining temperature in the overlapped regions given that we already know how to estimate the ionized fraction. This method is expected to be inaccurate for \( 0.1 < x_{\text{HII}} < 1 \), however, the fraction of such points is negligible as can be seen from the top panel of Figure 2. In addition these points are highly heated and have strong Ly\( \alpha \) coupling, thus making the signal almost independent of \( T_K \).

The correlation is somewhat different when \( f_X = 0 \). In such cases, however, we find that the temperature falls sharply beyond the HII regions and hence the IGM can be treated as a two-phase medium characterized by the complete ionized and neutral regions. Thus we can still estimate the \( T_K \) from the correlation without introducing any significant error.

Finally, we discuss how to deal with points which receive Ly\( \alpha \) radiation from more than one source. In this case, accounting for overlap is almost trivial as \( J_n \) essentially measures the number of Ly\( \alpha \) photons and hence one just has to add the fluxes from different sources (Thomas & Zaroubi 2011).

### 2.5 Redshift space distortion

Observations of the redshifted 21-cm radiation at a specific frequency can be mapped to a redshift and thus to a position along the line of sight. In absence of the peculiar velocities of HI gas, 21 cm radiation is redshifted only due to the expansion of the universe. The redshift will be modified in presence of the peculiar velocities of HI gas. As gas tends to move toward over dense regions, over/under dense regions will appear more over/under dense at large scales. This changes the strength of the observed 21 cm power spectrum and makes it anisotropic (see Bharadwaj & Ali 2005; Barkana & Loeb 2005; Mao et al. 2012; for detailed reviews of the theory).

The method for implementing the effect of peculiar velocities on the reionization 21 cm signal during the ‘emission’ phase (i.e. \( T_h \gg T_{\text{CMB}} \)) is reasonably well studied (Mellema et al. 2006; Mao et al. 2012; Jensen et al. 2013; Majumdar et al. 2013). The situation is different in our case because at high redshift there are regions where Ly\( \alpha \) coupling is not sufficiently strong and regions which may not be highly heated. We follow a method outlined in Mao et al. 2012 with certain modifications to account for the fluctuations in the spin temperature. We know the positions and peculiar velocities of all the dark matter particles at a certain redshift in our simulation box. Let the \( i^{th} \) dark matter particle have position \( (x_i, y_i, z_i) \) and velocity \( (v_{x_i}, v_{y_i}, v_{z_i}) \) and have an associated hydrogen mass \( M_{\text{HI}} \). We also know the neutral fraction of hydrogen and spin temperature at the grid point that contains the particle. For the scenario where \( T_S \gg T_{\text{CMB}} \), the mass of the neutral hydrogen associated

\[ T_{\text{CMB}} \] is the temperature of the cosmic microwave background radiation.

\[ T_{\text{CMB}} \] is the temperature of the cosmic microwave background radiation.
with the $i^{th}$ particle can be written as
\[ M_{\text{HI}} = M_{\text{HI}}^i z_{\text{HI}}^i \] (12)

where $z_{\text{HI}}^i$ is the neutral fraction of hydrogen associated to the particle. This is sufficient in case the only fluctuations in $\delta T_b$ arise from the neutral hydrogen field. If we want to account for the spin temperature fluctuations, we need to modify the above relation suitably. Since the fluctuations in $\delta T_b$ arises from the combination $x_{\text{HI}} (1 - T_{\text{CMB}}/T_s)$, we define an effective HI mass associated with the particle as
\[ M_{\text{HI}}^i = M_{\text{HI}} x_{\text{HI}}^i \left(1 - \frac{T_{\text{CMB}}}{T_s}\right). \] (13)

If the line of sight is taken to be along the $x$-axis, the position of the particle $s$ in redshift space coordinate will be given by,
\[ s^i = x^i + \frac{v^i(1 + z_{\text{abs}})}{H(z_{\text{obs}})} \] (14)

where $z_{\text{obs}} = (1 + z_{\text{cos}})(1 - v_i/c)^{-1}$ is the observed redshift and $z_{\text{cos}}$ is the cosmological redshift. Once this mapping from $x \rightarrow s$ is established, we interpolate the HI contributions $M_{\text{HI}}^i$ of each particle to an uniform grid in the redshift-space. The resultant $\delta T_b$ map will contain the effects of redshift space distortions which would correspond to the observed signal.

\section{RESULTS}

We present the results of our analysis of the 21 cm signal in this section. As mentioned in section\textsuperscript{2}, the signal depends on the neutral hydrogen fraction, kinetic temperature, the Ly$\alpha$ coupling and the line of sight velocity of the neutral gas. The model we have used accounts for all these effects. However, it is important to note that there exist studies which simply assume $T_s \gg T_{\text{CMB}}$, which is obtained when the medium is highly heated ($T_K \gg T_{\text{CMB}}$) and Ly$\alpha$ coupling is very strong ($T_s = T_K$). In such cases, the effect of fluctuations in $T_s$ on the signal can be ignored. Similarly, many studies ignore the effect of redshift-space distortion. While presenting our results using the model where all the effects are accounted for, we will also look into the effects of not accounting for the fluctuations in $T_s$.

\subsection{Global ionization and heating history}

Before carrying out any analysis, we need to fix the value of the star-forming efficiency $f_s$. As mentioned earlier, the value of $f_s$ would determine the global reionization history. Hence, we fix its value by demanding that it matches the electron scattering optical depth $\tau$ as observed by the CMB observations. We find that, in the absence of X-rays ($f_X = 0$), the choice $f_s = 0.03$ gives $\tau = 0.08$ which is consistent with the observed constraints \cite{Hinshaw et al. 2009}. Adding X-ray photons to the luminosity increases the value of $\tau$, however, the effect is quite small. For example, using $f_X = 0.1$ with $\alpha = 1.5$ increases $\tau$ to 0.089 which still is consistent with the CMB constraints. Hence, we fix the value $f_s = 0.03$ and concentrate on studying the effects of changing the values of $f_X$ and $\alpha$. The evolution of the ionized hydrogen fraction $x_{\text{HI}}$ is shown in Figure\textsuperscript{3}. Most of the reionization process occurs between redshifts 16 and 8 during which the ionization fraction grows from 0.01 to 1. Since our simulation boxes are not equipped for treating small mass haloes, the reionization occurs faster than what would be allowed by quasar spectra constraints at $z \approx 6$ \cite{Gunn & Peterson 1965; Fan et al. 2003; 2006; Malhotra & Rhoads 2006; Choudhury & Ferrara 2006; Bolton & Haeckel 2007}. However, since the main purpose of this work is to study the effects of peculiar velocities on the 21 cm signal, we did not concern ourselves too much on reproducing the reionization constraints. As mentioned earlier, we have run a simulation of smaller box to check if ignoring the small mass haloes at early stages has any effect on our conclusions, which we will discuss later in Section\textsuperscript{4.3}.

We can see from Figure\textsuperscript{4} that increasing the value of $f_X$ causes reionization to occur earlier, though the effect is not that drastic (at least for $f_X < 0.1$, the highest value of reionization considered in this paper). However, the heating pattern is very sensitive to the value of $f_X$. As can be seen from the figure, the volume fraction in which the kinetic temperature is above the CMB temperature reaches unity only at $z \approx 8$ (at the same time when reionization is complete) for $f_X = 0$, while the same happens much earlier, at $z \approx 12(14)$ for $f_X = 0.05(0.1)$. We have considered a number of models in this paper which depending on the how various effects have been accounted for. These models are summarised in Table\textsuperscript{1} For model A, we have assumed the IGM to be uniformly heated and the Ly$\alpha$ coupling to be highly efficient, thus making the signal independent of $T_s$. Model B accounts for the fact that the IGM may not be uniformly heated, i.e., the pattern of $T_K$ is calculated self-consistently, however the Ly$\alpha$ coupling is still taken to be highly efficient, thus making $T_s = T_K$. In model C, the Ly$\alpha$ coupling too is calculated self-consistently accounting for the inhomogeneities in the Ly$\alpha$ flux $J_{\alpha}$. These models are similar to those considered by, e.g., \cite{Baek et al. 2009}.
Table 1. Different kinds of models considered in this paper. The terms ‘coupled’ and ‘heated’ represent the scenarios $x_\alpha \gg 1$ and $T_k \gg T_{\text{CMB}}$ respectively.

| Model | Ly$\alpha$ coupling | Heating |
|-------|----------------------|---------|
| A     | coupled              | heated  |
| B     | coupled              | self-consistent |
| C     | self-consistent      | self-consistent |

Figure 5. Evolution of volume weighted brightness temperature for model A (solid red curve), B (long dashed green curve) and C (short dashed blue) with redshift. The X-ray properties are chosen such that $f_X = 0.05$ and $\alpha = 1.5$. The dashed dot cyan curve represents model C with a higher value of $f_X = 0.1$.

The evolution of the volume averaged brightness temperature $\delta T_b$ is shown in Figure 5. For model A, the IGM is assumed to be heated and Ly$\alpha$ coupled, thus the brightness temperature is always positive and essentially traces the neutral hydrogen distribution. Once the non-uniform heating is accounted for (model B), the signal shows absorption at earlier times. This represents colder regions in the IGM where the X-ray flux may not have percolated as yet. As the fraction of regions with X-ray heating increases, the signal shows up in emission and follows model A at later stages. Since the Ly$\alpha$ coupling is assumed to be strong, $T_\delta = T_k$, and hence the signal is always non-zero (except for the point where $T_k = T_{\text{CMB}}$). When the effects of Ly$\alpha$ coupling are treated self-consistently (model C), the $T_\delta$ can depart from $T_k$ towards $T_{\text{CMB}}$ at earlier stages when the coupling is not efficient enough, and hence $\delta T_b$ tends to vanish. As soon as the ionized fraction $x_{\text{HH}} \sim 0.05$, the Ly$\alpha$ coupling seems to be sufficiently strong so that model C and B becomes identical. We also show in the figure the $\delta T_b$ evolution when the fraction of X-rays is increased $f_X = 0.1$ in model C. Clearly, the effects of heating are visible relatively early in reionization history. It is interesting to note that all the models A, B and C are identical when $x_{\text{HH}} \gtrsim 0.2$ and the assumption of $T_\delta = T_k \gg T_{\text{CMB}}$ works extremely well in these stages. It is only at very early stages that we need to account for the fluctuations in the spin temperature as the Ly$\alpha$ radiation is accounted for (model B), the signal shows absorption. At this stage, the maps have both emission and absorption features.

3.2 Fluctuations in the brightness temperature

Since the main target of the radio interferometers is to measure the fluctuations in the 21 cm signal, we discuss how different effects impact the fluctuations. The maps of $\delta T_b$ obtained from a random slice through our simulation box is shown in Figure 6. The left panels show the maps for model A for three different redshifts $z = 19.2, 15.4, 9.6$ respectively. Since this model corresponds to the case $T_\delta \approx T_k \gg T_{\text{CMB}}$, the brightness temperature essentially follows the neutral hydrogen distribution. In the absence of any significant sources at $z = 19.2$, the IGM is neutral and the fluctuations are mainly those corresponding to underlying dark matter density field. Once can see spherical ionized bubbles appearing in the slice at $z = 15.4$, where the signal drops to zero. These bubbles percolate in the IGM and gives the patchy reionization map at $z = 9.6$ as is expected.

The middle panels of Figure 6 show the brightness temperature maps for model B where it is assumed that $T_\delta \approx T_k$ but the temperature $T_k$ is estimated self-consistently. At early stages $z = 19.2$, the map for model B is very different from that for model A because the effect of X-ray heating is not that strong. In fact, most of the IGM shows up in absorption because it is colder than the CMB. Once the sources form, the signal shows a number of features. One can identify the spherically-shaped regions around the sources where the signal vanishes as expected. However, there is a “ring” of emission which corresponds to the regions which are heated by X-ray from the sources. As the effect of X-rays decreases away from the sources, the emission signal drops, changes sign and shows up in absorption. At this stage, the maps have both emission and absorption features.

As we come to lower redshifts $z = 9.6$, the X-ray heating is dominant all over the IGM and model B is almost identical to model A. The right panels of Figure 6 show the maps of $\delta T_b$ for the model C. The main difference between this model and B is at very early stages $z = 19.2$. Since the Ly$\alpha$ coupling is calculated self-consistently using the value of $J_\alpha$, the map shows effects of inefficient coupling when sources are sparse. As a result, the magnitude of the absorption signal is less than that in model B. However, since one requires only a small amount of Ly$\alpha$ radiation for efficient coupling, we find that the differences between model B and C go away by $z = 15.4$.

The same conclusions can be drawn if we plot the power spectrum of $\delta T_b$ for the three models, which is done in Figure 7. It is clear that at relatively later stages of reionization $z = 9.6$, all the models overlap with each other when the signal traces the HI distribution. At $z = 15.4$, model B and C differ from A because of the inhomogeneities in the X-ray heating which is not accounted for in A. Since there are regions showing strong absorption signals in models B and C, it leads to a stronger contrast in the maps, and hence the amplitude of the power spectrum is much larger (almost $\sim 30$ times). Interestingly, the power spectra for these models show a “bump” or a peak around $k_{\text{peak}} \approx 0.2$ Mpc$^{-1}$. This scale corresponds to the typical sizes of regions which are

© ? RAS, MNRAS 000.
heated by the sources, i.e., the sizes of the heated bubbles. In fact, one can estimate the typical radius of heated regions as $R_{\text{heat}} = 2.46/k_{\text{peak}}$ \cite{Friedrich2011}, which in our case turns out to be $\sim 12$ Mpc. The amplitude of this peak is determined by the contrast in the signal between heated (i.e., emission) and colder (i.e., absorption) regions. This feature was noted in simulations of \cite{Baek2009} too. Since the peak has a relatively high amplitude ($\sim 5000$ mK$^2$) it can easily be detected by an instrument like the SKA. The size of a typical heated region is an important parameter which can help in constraining the nature of X-ray sources such as the SED and the total X-ray flux. At the earliest stages ($z = 19.2$), the effects of inhomogeneous Ly$\alpha$ coupling makes model B different from C. As is clear, because of inefficient Ly$\alpha$ coupling in regions away from sources, the brightness temperature amplitude becomes smaller in model C compared to B. This leads to a decrease in the power spectrum amplitude.

The evolution of the power spectra for different models for a typical scale $k = 0.1$ Mpc$^{-1}$ accessible to first generation low-frequency telescopes is shown in Figure 8. For model A, the amplitude decreases with increase in ionization fraction during initial stages of reionization. However as the characteristic size of the ionized bubbles increase with time, $\Delta^2$ starts to increase. This leads to a prominent trough-like feature at $z \approx 14.2$ when the ionization fraction is $\sim 0.05$. The rise in $\Delta^2$ at $z < 14$ is halted when the bubbles start overlapping and the patchiness in the ionization fraction de-
there is another peak at \( z \approx 19.2 \) arises because of ray heating. Since the heating is not efficient at high redshifts, the signal is in absorption and hence larger than that in model A. As sources heat up the IGM, the amplitude of the signal decreases and tends towards model A. Once sufficient heating is completed in the IGM, the signal follows model A. Model C is quite similar to B except that there is another peak in the amplitude at \( z \sim 19 \). This peak corresponds to inhomogeneities in Lyα coupling which is not accounted for in the other models. Once the Lyα coupling becomes efficient at \( z \sim 16 \), the signal for model C follows B.

### 3.3 Effects of redshift-space distortion

As is well known, the effects of including the peculiar velocities in the brightness temperature calculation is two-fold: one is to introduce anisotropies in the signal, and the other is to modify the amplitude of the spherically averaged power spectrum. In this work, we will mostly study the second effect, i.e., the change in the amplitude because of redshift-space effects. We will also comment on the anisotropies wherever appropriate.

Before proceeding into presenting our results, let us briefly discuss a quasi-linear model (Mao et al. 2012) for studying the effects of redshift space distortion. In order to see this, let us write equation (1) as

\[
\delta T_b(z, \mathbf{x}) = \tilde{\delta T_b}(z)\tilde{\eta}(z)[1 + \delta_{\text{HI}}(z, \mathbf{x})][1 + \delta(\mathbf{x})],
\]

where

\[
\tilde{\delta T_b}(z) = 27 \bar{x}_{\text{HI}}(z) \left( \frac{\Omega_m h^2}{0.023} \right) \left( \frac{0.15 + z}{10} \right)^{1/2} \text{mK},
\]

is the average brightness temperature at \( z \) and

\[
\eta(z, \mathbf{x}) = 1 - \frac{T_{\text{CMB}}(z)}{T_b(z, \mathbf{x})}.
\]

The average value of \( \eta \) is denoted as \( \bar{\eta} \) while the corresponding contrast is given by \( \delta_\eta = \eta / \bar{\eta} - 1 \). The power spectrum in the redshift space can be decomposed as

\[
P^\prime(k) = P_0(k) + P_2(k) \mu^2 + P_4(k) \mu^4,
\]
where $\mu = k_\parallel / |k|$, $k_\parallel$ being the component of $k$ along the line of sight. In the quasi-linear approximation, the different components appearing in the above equation are given in terms of the real-space power spectra as

$$P_0(k) = \left(\delta T_b \eta\right)^2 \left[P_{\delta_{\rho HI} \delta_{HI}}(k) + P_{\delta_{\eta} \delta_{\eta}}(k) + 2P_{\delta_{\rho HI} \delta_{\eta}}(k)\right],$$

$$P_2(k) = 2 \left(\delta T_b \eta\right)^2 \left[P_{\delta_{\rho HI} \delta_{HI}}(k) + P_{\delta_{\rho HI} \delta_{\eta}}(k)\right],$$

$$P_4(k) = \left(\delta T_b \eta\right)^2 P_{\delta_{\rho HI} \delta_{HI}}(k),$$

where $\delta_x = x/x - 1$ is the contrast of the quantity $x$. The quantities of the form $P_{\delta_{\rho HI} \delta_{\eta}}$ denote the real-space auto-power spectrum of the quantity $x$, while $P_{\delta_{\rho HI} \delta_{\eta}}$ are the real-space cross-power spectrum between fields $x$ and $y$. The spherically-averaged power spectrum in redshift-space is simply given by

$$P_{\text{ave}}^2(k) = P_0(k) + \frac{1}{3} P_2(k) + \frac{1}{8} P_4(k).$$

We should mention here that we have not used this quasi-linear model to calculate any of our results, nor have we verified its validity for our work. The model is introduced so as to help understand the various properties of the redshift-space power spectrum in a simple way.

We now present our results incorporating the effects of line-of-sight peculiar velocities on the $\delta T_b$ fluctuations. The resulting power spectra for the three models are shown in Figure 9. The left panel shows the results for model A for three different redshifts and for the cases with and without the redshift space distortion. At higher redshifts, the effect of including the redshift space effects is to increase the amplitude of the power spectra, without affecting the shape significantly. For example, the increase in amplitude is about a factor of 1.87 at $z = 19.2$ and 15.4 at scales probed by our simulation when the redshift space distortion is accounted for. This is expected from the quasi-linear model when there are no fluctuations in $T_b$, i.e., $\delta_\eta = 0$ and the HI density follows the baryonic density field $\delta_{\rho HI} = \delta_{\rho B}$. The effect decreases as the neutral hydrogen fraction decreases further and we find that the effect is negligible for model A at $z = 9.6$. Since the difference between real-space and redshift-space power spectra for model A is essentially determined by the cross term $P_{\delta_{\rho HI} \delta_{\eta}}(k)$, it is obvious that the correlation between the underlying baryonic field and the HI field vanishes when the ionization fronts propagate into the low-density regions at later stages of reionization.

The situation is clearly different for models B and C at higher redshifts. We find that the redshift space power spectra are higher than the real-space ones by a factor $\sim 1.87$ at smaller scales $k \gtrsim 0.4$ Mpc$^{-1}$ at $z = 19.2, 15.4$, implying that the fluctuations in $T_b$ are negligible at these scales. At larger scales however, the $T_b$-fluctuations dominate the brightness-temperature power spectrum, and in addition, these fluctuations are not correlated with the baryonic density field. This makes the effect of redshift-space distortion negligible at scales $k \lesssim 0.4$ Mpc$^{-1}$. The models B and C are identical to A at $z = 9.6$, and hence the effect of redshift space is not visible.

In order to bring more clarity to these arguments, we plot the various terms which contribute to the redshift-space power spectrum in Figure 10. We show the plots only for model A. At $z = 19.2$ and 15.4, the power spectra will be dominated by the terms $P_{\delta_{\rho HI} \delta_{\eta}}$, $P_{\delta_{\rho HI} \delta_{\rho HI}}$, and $P_{\delta_{\rho HI} \delta_{\rho HI}}$ at smaller scales, all of which are almost identical because of very little ionization. At larger scales, the fluctuations are dominated by the $P_{\delta_{\rho HI} \delta_{\eta}}$ term. Clearly the spin temperature fluctuations $\delta_\eta$ are not correlated with the other fields as shown by the smallness of the terms $P_{\delta_{\rho HI} \delta_{\eta}}$ and $P_{\delta_{\rho HI} \delta_{\rho HI}}$, hence we find that the difference between redshift-space and real-space power spectra are negligible. The fluctuations in $\eta$ decrease as the IGM is heated, however, because the ionization fronts have propagated into low-density regions by then, the effects of redshift-space distortions are negligible. Hence overall we find that the effects of redshift-space dis-
Figure 10. The auto and cross-power spectra of baryonic density contrast \( \delta_B \), neutral hydrogen density contrast \( \delta_{\text{HI}} \) and the contrast \( \eta \) in the term \( \eta = 1 - T_{\text{CMB}}/T_\beta \) at three different redshifts. The solid parts in the cross-power spectra curves represent positive values, while the other parts represent negative values.

Figure 11. Effects of redshift space distortion on the evolution of \( \delta T_k \) power spectra for models A, B and C at a scale \( k = 0.1 \) Mpc\(^{-1} \).

tortion are negligible at large scales \( k \lesssim 0.4 \) Mpc\(^{-1} \), though the reasons differ over the history of reionization. At smaller scales however, the redshift space power spectrum is \( \sim 2 \) times higher in amplitude that the real space counterpart at early stages of reionization, mainly because the signal is dominated by the HI fluctuations at these scales. These fluctuations, in turn, are strongly correlated with the density field because of formation of sources at preferentially high density peaks.

The evolution of the power spectra in real and redshift-space for a typical scale \( k = 0.1 \) Mpc\(^{-1} \) accessible to first generation low-frequency telescopes is shown in Figure 11. One obvious conclusion is that the effects of redshift-space distortion is only visible for model A and that too at early stages of reionization \( \bar{x}_{\text{HI}} \lesssim 0.1 \). Unfortunately, this model is not appropriate at early stages of reionization because it does not account for fluctuations in \( T_\beta \). Once these fluctuations are taken into account, the power at large scales are dominated by \( \delta_B \) contribution which do not correlate with the underlying density field. Hence the effects of peculiar velocities is negligible at large scales throughout the reionization history.

It has been suggested that, for scenarios similar to our model A, the anisotropies in the power spectrum produced by the redshift space distortion effect is detectable in 2000 hrs of observations with LOFAR \citep{Jensenetal2013}. It was also suggested that such observations could tell us whether reionization occurred inside-out or outside-in \citep{Jensenetal2013, Majumdaretal2013}. However, redshift space distortion effect at large scales \( k \lesssim 0.4 \) Mpc\(^{-1} \) seems to be quite small in a scenario where the spin temperature fluctuations dominate the power spectrum, and hence it would be much more difficult to detect any signatures of anisotropy using LOFAR. It could be interesting to explore the possibility of detecting this anisotropy in intermediate scales \( k \sim 1 \) Mpc\(^{-1} \) where the effects of redshift space distortion are much more significant.

The root mean square (RMS) of HI brightness temperature fluctuations \( \delta T_k \) is another major statistical quantity that will also be targeted by the first generation instruments \citep{Mellemaetal2003, Jelicetal2008, BittnerLoecl2011, Patiletal2014}. It is defined as,

\[
\text{RMS}(\delta T_k) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\delta T^i_k - \bar{T}_k)^2},
\]

where \( N \) is the number of \( \delta T_k \) data cube pixels after smoothing over a scale of 15 cMpc and \( \bar{T}_k \) is the average value of \( \delta T_k \). The plot of RMS deviation as a function of \( z \) is shown in Figure 12. The behaviour of this quantity, as expected, is very similar to the evolution of \( \Delta^2(k) \) at large scales (Figure
It shows three peaks at different locations corresponding to different physical processes discussed earlier. Recently, it was proposed that a function having a single peak (e.g., $\propto (z/z_R)^{\beta} (1 + \text{tanh} ([z - z_R]/\Delta z))$, where $\beta$ is the power law index, $z_R$ is the location and $\Delta z$ is the width of the peak) can be fitted to the measured RMS vs. $z$ data to reliably extract information about the redshift of reionization $\sim z_R$ and the duration of reionization $\sim \Delta z$ (Patil et al. 2014). The conclusion was based on an reionization model similar to A where the RMS vs. $z$ curve has just one peak around $x_{HI} \sim 0.5$. However in Figure 12 we find that there could be two additional peaks at large scales, due to inhomogeneous Ly$\alpha$ coupling and heating respectively. The presence of these two additional peaks makes the interpretation of the measured RMS vs. $z$ slightly more complicated. In particular, the peak corresponding to the inhomogeneous heating can have a very large amplitude and can affect the detection of the low-redshift peak in a noisy data. Fitting a single peak function over the entire range of redshifts could thus possibly result in misinterpretation of the observations. One should thus either restrict the analysis within lower redshifts (e.g., $8 < z < 11$ for model C) or choose a function having three peaks. One should also keep in mind that the influence of the spin temperature fluctuations in the RMS (and also in the power spectrum) remains for a while even after the entire IGM is heated above the CMB temperature.

As expected, we find that the effects of redshift space on the RMS are negligible when $T_s$-fluctuations are accounted for in the model (model C). This implies that the modelling of the 21 cm signal at large scales is possible without accounting for peculiar velocities of the gas, however, it is critical that one accounts for the fluctuations in $T_s$.

### 3.4 Effect of different X-ray source properties on 21 cm power spectrum

It is clear from the previous section that the redshift-space effects are negligible when the X-ray heating and Ly$\alpha$ coupling are accounted for self-consistently. It is necessary to check whether this result is independent of the parameters related to the X-ray emissivities, which we do in this section. We will essentially consider model C where all the physical effects are accounted for self-consistently, and vary the parameters $f_X$ and $\alpha$ to see the effects on the real and redshift-space power spectrum. In particular, we would concentrate on the evolution of the power spectrum amplitude at a scale $k = 0.1$ Mpc$^{-1}$.

Let us first consider the effect of varying $f_X$ and take three representative values $f_X = 0, 0.05, 0.1$. The corresponding power spectra are shown in Figure 13. When $f_X = 0$, the neutral hydrogen will contain almost no photons of higher energies and hence there would be almost no heating. As a result, the neutral regions will be visible in absorption while the ionized regions will show no signal. This also means there will be no fluctuations in $T_K$ once the Ly$\alpha$ coupling is complete. This is clear from the figure which shows that the case with $f_X = 0$ contains the initial peak at $z \sim 20$ arising from inhomogeneities in the Ly$\alpha$ coupling. However, as expected the model does not have the subsequent peak corresponding to the $T_K$ fluctuations. The amplitude again peaks because of the increase in bubble sizes and then decreases as reionization is completed. The amplitude at these last stages, interestingly, is much larger compared to the X-ray heated models which is because of the fact that the IGM is cold and shows strong absorption signal. This model is essentially same as the model A discussed above but the mean brightness temperature $\delta T_K$ being replaced by the mean kinetic temperature of the IGM. When $f_X > 0$, the signal shows the prominent peak corresponding to the $T_K$ fluctuations. It is expected that the amount of heating and the size of the heated regions would increase as the X-ray intensity increases. This suggests that the IGM will be heated at an earlier redshift when $f_X$ in increased which is clearly seen from Figure 13. In fact, the peak arising from $T_K$ fluctuations appear earlier when the value of $f_X$ is increased. Hence the position of the peak as a function of redshift (or frequency of radio observations) can
be used for probing the level of total X-ray background. This is consistent with earlier results \cite{ChristianLoeb2013}.

As far as the effect of redshift-space distortion is concerned, we find that there is some difference between the redshift and real space amplitudes at $13 < z < 18$ when $f_X = 0$. This is arising because there are no fluctuations in the spin temperature at these epochs and hence the model is similar to model A. However, the moment when $f_X > 0$, the effect of redshift-space distortion vanishes and becomes independent of the actual value of $f_X$.

The conclusions remain similar when we vary the value of the spectral index $\alpha$. The effect of considering different values of $\alpha$ is shown in Fig. 14. We find that the peak related to $T_K$ fluctuations appear earlier when $\alpha$ is increased. This is because for the same value of $f_X$, increasing $\alpha$ means redistributing photons to relatively lower energies. Since lower energy photons are easier to be absorbed by HI, the amount of ionization and heating is more. The most interesting aspect to check is the effect of redshift space distortions, and we find that the effect is negligible independent of the value of $\alpha$.

It is clear that our results on redshift space distortion is independent of the exact X-ray background chosen, as long as it is non-zero. The only effect of varying $f_X$ and $\alpha$ is to change the location of the peak in the power spectrum amplitude arising from $T_K$ fluctuations. In Fig. 14, we show the evolution of the power spectrum amplitude as a function of the heated fraction $x_{\text{heated}}$ for different values of $f_X$ and $\alpha$. We should mention that the quantity $x_{\text{heated}}$ is defined as the fraction of points which have $T_K > T_{\text{CMB}}$. It is interesting that for $f_X > 0$, the location of the peak is independent of the level of X-ray background or the spectral index of the X-ray sources and appears when $x_{\text{heated}} = 0.1$. It would be interesting to discuss the possibility of detecting this peak using the next generation of radio telescopes as that would clearly establish the signature of fluctuations in the IGM heating. Additionally, the position of the peak will tell us

![Figure 14. Evolution of $\delta T_K$ power spectra with redshift at scale $k = 0.1 \, \text{Mpc}^{-1}$ for three different values of the spectral index $\alpha$ of the miniquasar SED. The results are shown for cases with and without the effects of redshift space distortion. The value of $f_X$ is kept fixed to 0.05.](image)

![Figure 15. Evolution of the $\delta T_K$ power spectra at $k = 0.1 \, \text{Mpc}^{-1}$ as a function of heated volume fraction in the IGM. Regions with $T_K > T_{\text{CMB}}$ are referred to as heated regions.](image)

about the redshift when 10% of the IGM volume was heated above the CMB temperature.

### 3.5 Effect of box size on the 21 cm power spectrum

One difficulty with the models we have considered so far is that they do not contain haloes smaller than $3.89 \times 10^8 \, h^{-1} \, M_\odot$. The initial stages of reionization are expected to be driven by sources in haloes as small as $\sim 10^6 \, M_\odot$, and hence it is necessary to verify if our results remain unchanged when such small sources are included in the analysis.

We have run another simulation in smaller box of length $30 \, h^{-1} \, \text{cMpc}$ with 768$^3$ particles. The mass resolution of the simulation is $5.254 \times 10^5 \, h^{-1} \, M_\odot$ and the corresponding minimum halo mass obtained is $1.05 \times 10^8 \, h^{-1} \, M_\odot$. Thus, this box should be able to capture the essential features at early stages of reionization. We should also mention here that since the box has a small size, it is not appropriate for studying the later stages of reionization when the sizes of the ionized regions become large.

Figure 15 show the effects of including small haloes in the analysis. The solid cyan curve represents the evolution of $\Delta^2$ at a scale $k = 0.3 \, \text{Mpc}^{-1}$ for the $100 \, h^{-1} \, \text{cMpc}$ box, while the solid red curve represents the same for the $30 \, h^{-1} \, \text{cMpc}$ box with $f_X$ kept same as that for the $100 \, h^{-1} \, \text{cMpc}$ box. For the same value of the efficiency parameter $f_X$, the presence of small haloes drive the reionization faster than what is found with the $100 \, h^{-1} \, \text{cMpc}$ box. However, the basic features, i.e., the three peaks in the power spectrum remain the almost similar in the smaller box. What is

---

11 We should mention that we have plotted the power spectra at scale $k = 0.3 \, \text{Mpc}^{-1}$ in Figure 15 rather than at $k = 0.1 \, \text{Mpc}^{-1}$ as we had done in earlier sections. This is because larger scales are not accessible in the smaller $30 \, h^{-1} \, \text{cMpc}$ box.
interesting to note is that the effect of redshift space distortion at these scales is negligible even when the small mass haloes are taken into account.

We also study the case where \( f_* \) is varied for the small box so as to recover the ionization history identical to what we studied using the 100 \( h^{-1} \) cMpc box. The solid blue curve in Figure 16 represents the results for such a scenario. We find that the plots with and without the redshift space distortion fall on top of each other. Thus the conclusion that the peculiar velocities do not affect the large scale power remains unchanged.

### 4 SUMMARY AND DISCUSSION

The main aim of this paper is to investigate the effects of peculiar velocities of the neutral gas and spin temperature fluctuations in the IGM on the spherically averaged power spectrum of 21 cm brightness temperature during the early stages of reionization, i.e., cosmic dawn. We have developed a code, based on the methods of Thomas & Zaroubi (2008), Thomas et al. (2009), to generate self-consistent brightness temperature maps. Our method consists of the following main steps: (i) We generate the dark matter density and velocity field using a \( N \)-body code CUBEP3M (Harnois-Deraps et al. 2012). The same simulation data is used for identifying locations and masses of collapsed haloes. Since the main purpose of this work is to study the effects of peculiar velocities, it is important to model the velocity fields and the correlation between density and velocity fields as accurately as possible, which is only achieved in \( N \)-body simulations. (ii) The sources of reionization are modelled assuming each source has a stellar component and a mini-quasar like component for producing photons. The stellar component is modelled using a population synthesis code PEGASE2 (Fioe & Rocca 1997), while the other component was assumed to have a power-law spectrum. (iii) The radiative transfer was implemented using a one-dimensional code for isolated sources, and we have accounted for overlaps in ionized and heated regions of different sources appropriately.

We have validated our code by comparing with various existing results and found that all the features expected in the fluctuation power spectrum at early stages of reionization are nicely reproduced in our calculations (see e.g., Pritchard & Furlanetto 2009, Santos et al. 2008, Baek et al. 2014, Mesinger et al. 2013, Christian & Loeb 2013). In particular, we have studied in some detail the effects of inhomogeneities in the gas temperature and the Lyα coupling. Each of these effects produce distinct peak-like features in the large scale power spectrum when plotted as a function of redshift. The peak which appears the latest in the history corresponds to the fluctuations in the HI field, which is targeted by the present generation of radio telescopes. The second peak corresponds to fluctuations in heating and occurs when \( \sim 10\% \) of the volume is heated above the CMB temperature. The third peak, which occurs the earliest in reionization history, arises because of inhomogeneities in the Lyα coupling.

Since the spin temperature fluctuations during the early stages of reionization introduce two additional peaks in the RMS vs. redshift plot, fitting a single peak function over the entire redshift range like the one proposed earlier (Patil et al. 2014) could possibly result in misinterpretation of the data. One can, however, restrict the analysis within lower redshift (e.g., \( 8 < z < 11 \) for model C) or choose a function with three peaks. One should also be careful that the influence of the spin temperature fluctuations in the RMS (and also in the power spectrum at large scales) remains for a while even when the entire IGM is heated above the CMB temperature.

In a situation when the spin temperature is coupled to the kinetic temperature but the gas is heated inhomogeneously, the power spectrum shows a “bump” when plotted as a function of \( k \). The bump corresponds to typical size of the heated bubbles. Thus, this peak can be used to constrain the sizes of heated bubbles which has further implications in constraining properties of X-ray sources.

Once we incorporate the effects of peculiar velocities in our model and compare with the cases when such effects are absent, we find that at large scales, i.e., \( k \lesssim 0.4 \) Mpc\(^{-1} \), the effects on the spherically averaged power spectra are negligible throughout reionization history. It is not difficult to understand the reasons for this: the effect of peculiar velocities is substantial only when the 21 cm fluctuations are correlated with the underlying baryonic density field. At late stages of reionization (\( x_{\text{HI}} \gtrsim 0.2 \)), the 21 cm fluctuations are dominated by the fluctuations in HI field. The ionization fronts at these epochs have percolated in the low-density cosmological density field and the HI field is not correlated with the density field any more. Hence one finds no effect of the peculiar velocities on the 21 cm signal. On the other hand, during early stages of reionization (\( x_{\text{HI}} \lesssim 0.2 \)), the fluctuations at large scales are dominated by the \( T_b \)-fluctuations, which too are only very mildly correlated with the density field. Interestingly, if the \( T_b \)-fluctuations are not accounted for in the model, the 21 cm fluctuations are dominated by HI fluctuations even during early stages. In the inside-out models of reionization, the HI fluctuations are highly correlated with the density field and thus one seems to find relatively stronger effects of peculiar velocities on the 21 cm fluctu-
In fact, at smaller scales $k \gtrsim 0.4$ Mpc$^{-1}$, the 21 cm fluctuations even at the early stages of reionization are dominated by patchiness in the HI field, and it is not surprising that the power spectrum in the redshift space is enhanced in the amplitude compared to the real space counterpart. It was suggested that, for a reionization scenario where the spin temperature is much higher than the CMB temperature, the redshift space distortion effect which makes the power spectrum anisotropic is detectable with 2000 hrs of observations using LOFAR. It was also suggested that such observations could tell us whether reionization occurred inside-out or outside-in [Jensen et al. 2013; Majumdar et al. 2013]. However, possibility of detecting the redshift space distortion effect in the power spectrum seems very difficult with the first generation instruments in a scenario where the spin temperature fluctuations dominate the power spectrum.

We have checked our conclusions for different X-ray properties of the sources (i.e., the amount of X-rays produced compared to the UV and the steepness of the X-ray spectra), and also on the resolution of the simulation box. The conclusions seems to be quite robust on this respect. It thus implies that the 21 cm fluctuations would be quite isotropic at large scales throughout the reionization epoch, while one expects some departures from isotropy at relatively smaller scale $k \gtrsim 0.4$ Mpc$^{-1}$.

In future, it would be interesting to study the anisotropies in the 21 cm power spectrum arising from peculiar velocities at early stages of reionization. While we expect no signatures of anisotropy at scales $k \lesssim 0.4$ Mpc$^{-1}$, there would be some anisotropic signal at smaller scales. It would be interesting to explore the possibilities of constraining early source properties using this feature. Also, one should keep in mind that we have concentrated only on high-mass X-ray binaries and verify if the conclusions remain unchanged.

ACKNOWLEDGEMENT

RG would like to thank Aritra Basu, Narendra Nath Patra and Prasun Dutta for valuable discussions and suggestions regarding numerical computations. KKD thanks the Department of Science & Technology (DST), India for the research grant SR/FTP/PS-119/2012 under the Fast Track Scheme for Young Scientist.

REFERENCES

Ahn, K., Xu, H., Norman, M. L., Alvarez, M. A., & Wise, J. H. 2014, arXiv:1405.2085
Allison A. C., Dalgarno A., 1969, ApJ, 158, 423
Alvarez, M. A., Pen, U.-L., & Chang, T.-C. 2010, ApJ, 723, L17
Baek S., Di Matteo P., Semelin B., Combes F., Revaz Y., 2009, A&A, 495, 389
Baek, S., Semelin, B., Di Matteo, P., Revaz, Y., & Combes, F. 2010, A&A, 523, A4
Barkana, R., & Loeb, A. 2005a, ApJ, 624, L65
Barkana, R., & Loeb, A. 2005b, ApJ, 626, 1
Battaglia, N., Trac, H., Cen, R., & Loeb, A. 2013, ApJ, 776, 81
Becker, R. H., Fan, X., White, R. L., et al. 2001, AJ, 122, 2850
Bharadwaj, S., & Ali, S. S. 2004, MNRAS, 352, 142
Bharadwaj, S., & Ali, S. S. 2005, MNRAS, 356, 1519
Bittner J. M., Loeb A., 2011, J. Cosmology Astropart. Phys., 4, 38
Bolton, J. S., & Haehnelt, M. G. 2007, MNRAS, 382, 325
Bowman, J. D., Cairns, I., Kaplan, D. L., et al. 2013, Pub. Astro. Soc. Australia, 30, 31
Choudhury, T. R., & Ferrara, A. 2006, Albert Einstein Century International Conference, 861, 835
Choudhury, T. R., Haehnelt, M. G., & Regan, J. 2009, MNRAS, 394, 960
Christian, P., & Loeb, A. 2013, JCAP, 9, 14
Chuzhoy L., Alvarez M. A., Shapiro P. R., 2006, ApJ, 648, L1
Chuzhoy, L., & Zheng, Z. 2007, ApJ, 670, 912
Datta, K. K., Friedrich, M. M., Mellema, G., Iliev, I. T., & Shapiro, P. R. 2012a, MNRAS, 424, 762
Datta, K. K., Mellema, G., Mao, Y., et al. 2012b, MNRAS, 424, 1877
Datta, K. K., Jensen, H., Majumdar, S., Mellema, G., Iliev, I. T., Mao, Y., Shapiro, P., Ahn, K., 2014, MNRAS, (in press)
Dayal P., Ferrara A., Saro A., Salvaterra R., Borgani S., Tornatore L., 2009, MNRAS, 400, 2000
Dayal P., Ferrara A., Saro A., 2010, MNRAS, 402, 1449
Elvis M., et al., 1994, ApJS, 95, 1
Fan, X., Strauss, M. A., Schneider, D. P., et al. 2003, AJ, 125, 1649
Fan X., et al., 2006, AJ, 131, 1203
Fialkov, A., Barkana, R., & Visbal, E. 2014, Nature, 506, 197
Field G. B., 1958, Proc. I. R. E., 46, 240
Finkelstein S. L., Rhoads J. E., Malhotra S., Grogin N., 2009, ApJ, 691, 465
Fioc M., Rocca V. B., 1997, A&A, 326, 950
Fragos, T., Lehmer, B., Tremmel, M., et al. 2013, ApJ, 764, 41
Fragos, T., Lehmer, B. D., Naoz, S., Zezas, A., & Basu-Zych, A. 2013, ApJ, 776, L31
Friedrich M. M., Mellema G., Alvarez M. A., Shapiro P. R., Iliev I. T., 2011, MNRAS, 413, 1353
Furlanetto, S. R., Zaldarriaga, M., & Hernquist, L. 2004, ApJ, 613, 1
Furlanetto S., Zaldarriaga, M., & Hernquist, L. 2004, ApJ, 613, 1
Furlanetto S., Furlanetto M., 2006, MNRAS, submitted (astro-ph/0608067)
Furlanetto S. R., Oh S. P., 2006, ApJ, 652, 849
Furlanetto S. R., Oh S. P., Briggs F. H., 2006, Phys. Rep., 433, 181
Ghosh, A., Prasad, J., Bharadwaj, S., Ali, S. S., & Chngalur, J. N. 2012, MNRAS, 426, 3295
Goto, T., Utsumi, Y., Hattori, T., Miyazaki, S., & Yamashita, C. 2011, MNRAS, 415, L1

Gunn, J. E., & Peterson, B. A. 1965, ApJ, 142, 1633

Harnois-Deraps, J., Pen, U. L., Iliev, I. T., Merz, H., Emerson, J. et al. 2012, arXiv:1208.5098

Hinshaw, G., et al. 2009, ApJS, 180, 225

Hirata C. M., Sigurdson, K., 2006, MNRAS, submitted (astro-ph/0605071)

Hirata C. M., 2006, MNRAS, 367, 259

Iliev, I. T., Mellema, G., Ahn, K., et al. 2014, MNRAS, 439, 725

Jelíč V. et al., 2008, MNRAS, 389, 1319

Jensen, H., Datta, K. K., Mellema, G., et al. 2013, MNRAS, 435, 460

Komatsu, E., Smith, K. M., Dunkley, J., et al. 2011, ApJS, 192, 18

Kuhlen M., Madau P., 2005, MNRAS, 363, 1069

Lai K., Huang J.-S., Fazio G., Cowie L. L., Hu E. M., Kakazu Y., 2007, ApJ, 655, 704

La Plante P., Battaglia N., Natarajan A., Peterson J. B., Trac H., Cen R., Loeb A., 2013, arxiv-1309.7056

Laor A., Fiore F., Elvis M., Wilkes B. J., McDowell J. C., 1997, ApJ, 477, 93

Lewis, A., Challinor, A., & Lasenby, A. 2000, ApJ, 538, 473

Madau P., Meiksin A., Rees M. J., 1997, ApJ, 475, 429

Majumdar, S; Bharadwaj, S; Choudhury, T. Roy, 2013, MNRAS, 434, 1978

Majumdar, S., Mellema, G., Datta, K. K., et al. 2014, arXiv:1403.0941

Malhotra, S., & Rhoads, J. E. 2006, ApJ, 647, L95

Mellema G., Iliev I. T., Pen U.-L., Shapiro P. R., 2006, MNRAS, 372, 679

Mellema, G., Koopmans, L. V. E., Abdalla, F. A., et al. 2013, Experimental Astronomy, 36, 235

Mao Y., et al., 2012, MNRAS, 422, 926954

McQuinn, M., Zahn, O., Zaldarriaga, M., Hernquist, L., & Furlanetto, S. R. 2006, ApJ, 653, 815

McQuinn, M. 2012, MNRAS, 426, 1349

Meynet, G., & Maeder, A. 2005, A&A, 429, 581

Mesinger, A., & Furlanetto, S. 2007, ApJ, 669, 663

Mesinger, A., Furlanetto, S., & Cen, R. 2011, MNRAS, 411, 955

Mesinger, A., Ferrara, A., & Spiegel, D. S. 2013, MNRAS, 431, 621

Mitra, S., Choudhury, T. R., & Ferrara, A. 2011, MNRAS, 413, 1569

Mitra, S., Choudhury, T. R., & Ferrara, A. 2012, MNRAS, 419, 1480

Morales, M. F., & Wyithe, J. S. B. 2010, ARA&A, 48, 127

Paciga, G., Albert, J. G., Bandura, K., et al. 2010, MNRAS, 403, 639

Pacucci, F., Mesinger, A., Mineo, S., & Ferrara, A. 2014, arXiv:1403.6125

Parsons, A. R., Liu, A., Aguirre, J. E., et al. 2013, arXiv:1304.4991

Patil, A. H., Zaroubi, S., Chapman, E., et al. 2014, arXiv:1401.4172

Planck Collaboration, Ade, P. A. R., Aghanim, N., et al. 2013, arXiv:1303.5076

Pritchard, J. R., & Furlanetto, S. R. 2007, MNRAS, 376, 1680

Pritchard, J. R., & Loeb, A. 2012, Reports on Progress in Physics, 75, 086901

Sanchez, M. G., Amblard, A., Pritchard, J., Trac, H., Cen, R., & Cooray, A. 2008, ApJ, 689, 1

Semelin, B., Combes, F., & Baek, S. 2007, A&A, 474, 365

Shull J. M., van Steenberg M. E., 1985, ApJ, 298, 268

Tingay, S. J., Goeke, R., Bowman, J. D., et al. 2013, Pub. Astro. Soc. Australia, 30, 7

Thomas R. M., Zaroubi, S., 2008, MNRAS, 384, 1080

Thomas, R. M., et al. 2009, MNRAS, 393, 32

Thomas R. M., Zaroubi, S., 2011, MNRAS, 410, 1377

van Haarlem, M. P., Wise, M. W., Gunst, A. W., et al. 2013, A&A, 556, A2

Vanden Berk D. E., et al., 2001, AJ, 122, 549

Vignali C., Brandt W. N., Schneider D. P., 2003, AJ, 125, 433

Wouthuysen S. A., 1952, AJ, 57, 31

Yajima, H., & Li, Y. 2013, arXiv:1308.0381

Zawada K., Semelin B., Vonlanthen P., Baek S., Revaz Y., 2014, MNRAS(in press), arXiv:1401.1807

Zygelman B., 2005, ApJ, 622, 1356