Abstract—In this paper, we analyze the uplink goodput (bits/sec/Hz successfully decoded) and per-user packet outage in a cellular network using multi-user detection with successive interference cancellation (MUD-SIC). We consider non-ergodic fading channels where microscopic fading channel information is not available at the transmitters. As a result, packet outage occurs whenever the data rate of packet transmissions exceeds the instantaneous mutual information even if powerful channel coding is applied for protection. We are interested to study the role of macro-diversity (MDiv) between multiple base stations on the MUD-SIC performance where the effect of potential error-propagation during the SIC processing is taken into account. While the jointly optimal power and decoding order in the MUD-SIC are NP hard problem, we derive a simple on/off power control and asymptotically optimal decoding order with respect to the transmit power. Based on the information theoretical framework, we derive the closed-form expressions on the total system goodput as well as the per-user packet outage probability. We show that the system goodput does not scale with SNR due to mutual interference in the SIC process and macro-diversity (MDiv) could alleviate the problem and benefit to the system goodput.

Index Terms—Successive interference cancellation, error propagation, macro-diversity, optimal decoding order, order statistics.

I. INTRODUCTION

Here are two important technologies that could substantially enhance the uplink performance of cellular systems, namely the multi-user detection (MUD) and the macro-diversity (MDiv). The MUD is effective to mitigate intra-cell interference while the MDiv is effective to exploit of inter-cell interference from adjacent base stations. It is well-known that jointly maximum likelihood multi-user detection (ML MUD) is optimal but with exponential order of complexity with respect to (w.r.t.) the number of users in the system. There are a lot of research works on low complexity MUD such as the linear MUD [1], [2] and the successive interference cancellation (SIC) [3], [4]. In [5], [6], the authors analyzed the system goodput (bit/s/Hz successfully delivered to mobile user) for multi-access channels with minimum mean square error (MMSE) detector. However, the MMSE MUD cannot achieve Pareto optimality in the capacity region. On the other hand, MUD-SIC is a promising technology at the base station to mitigate intra-cell interference at reasonably low complexity. In this paper, we study the uplink performance analysis of an outage-limited multi-cell system with both MUD-SIC at each base station and MDiv between adjacent base stations. While there are quite a number of works studying the MUD design and performance analysis on single cell systems [7], [8], there are still a number of open technical challenges to apply MUD-SIC in multi-cell systems with MDiv. They are elaborated in the following:

• Per-user Outage and Error Propagation in MUD-SIC

Conventional performance analysis of multi-access fading channel is usually based on the ergodic capacity [9], [10]. Uplink power adaptation for multi-access channel is addressed in [11], [12], [13] where the transmit power of mobile users are optimized with respect to a system objective function of user capacities. However, in all these works, they did not take into account of the potential packet errors (and the error propagation effects in the SIC process) due to channel outage. When error-propagation effect of the MUD-SIC is considered, the packet error events between the K users are coupled together and the outage event cannot be determined by whether the rate vector is inside the capacity region or not.1

• Power and Decoding Order Optimization

One of the consequence of the per-user outage and error propagation effects is that the system goodput cannot scale with SNR due to potential mutual interference between users. To alleviate this issue, optimization of transmit power and decoding order in MUD-SIC is needed. Yet, such optimization problem (taking into account of error propagation) is extremely complicated and has not been addressed in the literature.

• Macro-Diversity

In multi-cell systems, macro-diversity (MDiv) enhances signal detection by exploiting the intercell interference [14], [15]. For instance, packet detection is terminated at each base station locally and the decoded packets from the base stations (in the active set) are delivered to a base station controller where packet selection is performed. Macro-diversity is a well studied technique in CDMA systems with single-user detection at the base station. However, it is not clear how the MDiv could alleviate the error propagation effects in the multi-cell network with MUD-SIC.

In this paper, we attempt to address the above issues. We consider an uplink of a multi-cell system with nB base

1For example, whether the 2nd decoded packet is successful depends not only on the channel condition of that user but also on whether the 1st decoded packet is successful. Furthermore, even if a rate vector is outside the multi-access capacity region, some user(s) may still be able to decode the packet successfully. This substantially complicated the analysis.
stations (each has MUD-SIC) and $K$ mobile users. We derive the closed-form expressions on the average system goodput as well as the per-user packet outage probability of the MUD-SIC detection under macro-diversity and potential error-propagation in the SIC process. While joint power and decoding order optimization is a $NP$ hard problem, we derive a simple on/off power control and decoding ordering which is asymptotically optimal w.r.t. the transmit power. Based on the results, we found that power adaptation, decoding order and MDiv are important to enhance the system goodput of MUD-SIC in multi-cell network.

The paper is organized as follows. Section II outlines the multi-cell system and the base station MUD-SIC processing. Section III provides the analysis of the network goodput of the multi-cell system with MUD-SIC and MDiv. Section IV presents numerical results on the performance and verify with the analytical expression. Section V concludes with a summary of results.

II. SYSTEM MODEL

A. Notation

Upper and lower case letters represent random variables and realizations of the variables, respectively. $E[X]$ denotes the expectation of the random variable $X$. $X_{1:n}$ represents the $k$-th order statistic ($X_{1:n} < X_{2:n}, \ldots, < X_{n:n}$) of $n$ ordered random variables. Matrix $\Pi$ contains vectors $\{\pi_1, \pi_2, \ldots, \pi_n\}$, where $\pi_b$ represents a particular decoding order for base station $b$, $\pi_b(i)$ gives the user index of users $k$ in the $i$-th decoding iteration at the $b$-th base station and $\pi_b^{-1}(k)$ returns the decoding iteration index of user $k$ at the $b$-th base station.

B. Multi-user Multi-cell Channel Model

We consider a wireless communication system which consists of $n_B$ base stations, $K$ mobile users, and a centralized controller as shown in Figure 1. The base stations and mobile terminals all have single antenna. The signal received by the $b$-th base station is given by

$$Y_b = \sum_{i=1}^{K} \sqrt{P_i g_{i,b}} H_{i,b} X_i + Z_b,$$

where $X_i$ is the transmitted signal from the $i$-th mobile station, $P_i$ is the transmitted power of the $i$-th mobile station which has range $[0, P_{max}]$, and $Z_b$ is complex Gaussian noise with zero mean and unit variance at the $b$-th base station, i.e., $CN(0,1)$. The path loss and shadowing effect, i.e., $g_{i,b}$, between the $b$-th base station and the $i$-th mobile station can be expressed as

$$g_{i,b}(dB) = PL_{b}(d_0) + 10\psi_b \log_{10}\left(\frac{d_i}{d_o}\right) + \omega_{\sigma},$$

where $PL_{b}(d_0)$ is the average path loss at the reference point $d_o$ meters away from the $b$-th base station, $\psi_b$ is the path loss exponent in the $b$-th cell, $d_i$ is the distance in meters away the $b$-th BS, and $\omega_{\sigma}$ denotes the shadowing effect which is modeled as a zero mean Gaussian distributed random variable with standard deviation $\sigma$. In order words, $g_{i,b}$ is log-normal distributed (in dB) with mean $PL_{b}(d_0)$ and standard deviation $\sigma$ dB. We model the channel coefficient $H_{i,b}$ between the $i$-th mobile station and the $b$-th base station as circularly complex Gaussian random variable with zero mean and unit variance.

In general, power and rate adaptation can be performed w.r.t. the product of multipath fading, average path loss and shadowing variables. However, adaptation w.r.t. microscopic fading is challenging especially for fast moving mobiles because the corresponding channel state information need to be updated at the base stations in a frequent manner. These updates increase the signalling overhead significantly and the computational complexity at the base stations. As a result, in this paper we assume that the power and data rates of the $K$ users are adaptive w.r.t. long-term fading (path loss and shadowing).

C. Centralized Controller Processing

The centralized controller is responsible for determining a user assignment set of each base station and a set of users who need MDiv to enhance the performance. The $b$-th base station should pass the estimated macroscopic fading coefficients (average path loss and shadowing) from all $K$ users to the centralized controller. After collecting all the macroscopic fading information from the $n_B$ base stations, the centralized controller compares the differences of average path loss and shadowing effect, i.e., $g_{i,b}$, between each mobile user and all the base stations with a predefined threshold $\Delta_{threshold}$ and then sends out the MDiv users list to all base stations. Furthermore, for those mobile users who require MDiv, the decoded messages are passed to the centralized controller from the corresponding base stations. Then the controller selects a

\footnote{The proposed system model is a generalized model of CDMA, since the constant spreading factor/processing gain can be treated as a multiplicative factor and absorbed in the path loss variables.}

\footnote{Mapping of the $K$ users w.r.t. $n_B$ base station is not the focus of this paper in which users are assumed to be associated with the strongest base station. For a discussion on mapping algorithm, please refer to [13].}
successfully decoded packet based on the Cyclic Redundancy Check (CRC) field. Since multiple base stations are decoding the same message for a user who demands MDiv and only the correct decoding messages are selected, a form of selection diversity protection is achieved.

D. MUD-SIC Processing and Per-User Packet Error Model

In this paper, we assume that the base stations are equipped with synchronous multi-user detector with successive interference cancellation. Furthermore, we assume that the base stations have knowledge of the channel statistic of multipath fading, average path loss and shadowing for all mobile users by long term measurement. On the other hand, the mobile stations do not have channel state information (CSI) and power allocation in the uplink are calculated at the base station and fed forward to the mobile stations. The received signal at the $b$-th base station is given by

$$Y_b = \sum_{i \in A_b} P_i g_{i,b} h_{i,b} x_i + \sum_{i \notin A_b} P_i g_{i,b} h_{i,b} x_i + Z_b,$$

where $A_b$ is a user set (including the users which perform MDiv) that are associated with the $b$-th base station.

The instantaneous channel capacity between the $b$-th base station and the $k$-th user is given by the maximum mutual information between the channel input $X$ and channel output $Y$. Hence, for a given decoding order $\pi_b = \{\pi_b(1), \pi_b(2), \ldots, \pi_b(\tilde{n}_b)\}$ and user assignment set $A_b$ with cardinality $\tilde{n}_b$, the instantaneous channel capacity between the $b$-th base station and the user $j$ in the $\pi_b^{-1}(j)$-th decoding iteration is $C_b(H,G,\pi_b,j) = \log_2 \left(1 + \frac{P_j |H_{\pi_b^{-1}(j),b}|^2}{1 + W_{\pi_b^{-1}(j)}} + \Phi_b(H,G,\pi_b,j) + \Omega_b(H,G)\right)$ where $H$ is the channel state information at the receiver (CSR) matrix, $G$ is the average path loss and shadowing matrix, $\Phi_b(H,G,\pi_b,j) = \sum_{i \in A_b} P_i |H_{\pi_b(i),b}|^2 g_{\pi_b(i),b}$ is the undetected signal, $\Omega_b(H,G) = \sum_{i \notin A_b} P_i g_{i,b} h_{i,b}$ is the inter-cell interference, and $W_{\pi_b^{-1}(j)}$ denotes the accumulated undecodable interference after $\pi_b^{-1}(j) - 1$ decoding iterations.

In this paper, we assume packet errors are contributed by channel outage which is a systematic error and cannot be avoided even when a capacity achieving coding is applied to protect the packet. As a result, traditional system performance measure using ergodic capacity may not be a good choice in this situation since it fails to account for the penalty of packet errors. In order to model the effect of packet errors, we consider the performance in terms of the system goodput (bit/s/Hz successfully received).

We model the undecodable interference and per-user goodput as follows. The undecodable interference at the $b$-th base station of user $j$ in the $\pi_b^{-1}(j)$-th decoding iteration is

$$W_{\pi_b^{-1}(j)} = \sum_{i=1}^{\pi_b^{-1}(j)-1} P_i |H_{\pi_b^{-1}(i),b}|^2 g_{\pi_b^{-1}(i),b} \times I\left\{r_{\pi_b^{-1}(i)} > C_b(H,G,\pi_b,\pi_b^{-1}(i))\right\}.\tag{5}$$

For the per-user goodput of user $k$, let $B_k$ denotes the MDiv base station assignment list and the instantaneous goodput of a packet transmission (bit/s/Hz successfully delivered) to the $b$-th base station is given by

$$\rho_b = r_b \times \left[1 - \prod_{b \in B_k} I\left\{r_b > C_b(H,G,\pi_b,k)\right\}\right],\tag{6}$$

where $r_b$ is the transmitted data rate of user $k$, which is a function of the average path loss and shadowing realization only. $I\{\cdot\}$ is an indicator function that evaluates to 1 when the event is true and 0 otherwise. In [6], we can see that the goodput of user $k$ depends on a set of base stations $B_k$ if the user is performing MDiv, otherwise the goodput of this user only depends on one base station. If strong error correction code is applied to the packet, the conditional average packet error rate (PER) of the user $k$ (conditioned on the path loss and shadowing realization) can be expressed as

$$\text{PER}_k(r_k, P_k; G) \approx \frac{P_{out}(r_k, P_k; G)}{\prod_{b \in B_k} \{\text{Pr}(r_k > C_b(H,G,\pi_b,k) | \pi_b) \} \text{Pr}(\pi_b)} \tag{7}$$

where the first summation accounts for all the possible combinations of decoding order in $|B_k|$ number of MDiv stations. Therefore, the average system goodput (conditioned on the path loss and shadowing matrix $G$) is given by

$$U_{gp}(P, R, \Pi; G) = E_H \left[\sum_{k=1}^{K} \rho_k |G|\right] = E_H \left[\sum_{k=1}^{K} r_k \left(1 - \frac{P_{out}(r_k, P_k; G)}{|G|}\right) \right].\tag{8}$$

Note that the average system goodput and PER are both functions of the transmission power of users and the decoding order. In the next section, we shall derive the optimal transmit power of each user and the asymptotically optimal decoding order w.r.t. the transmit power.

III. PERFORMANCE ANALYSIS

In this section, we shall analyzes the average system goodput and per-user outage probability of the MUD-SIC system taking into account of transmission power, potential error propagation and macro-diversity.

\footnote{The maximum mutual information can be achieved if we assume Gaussian random codebook is used.}
A. Optimal Power Transmission Level with MUD-SIC under Macro-Diversity

Traditionally, power control is employed to eliminate the near/far problem by maintaining equal received SINR among all mobile users when base stations are configured to perform single user detection [19]. On the other hand, for ML detection at the base station, the optimal power control (under peak power constraint) to maximize the ergodic sum capacity is simply for each user to transmit at its maximum power [20]. Yet, in our case of outage-limited MUD-SIC with potential error-propagation, it is not obvious if all the users should transmit at their maximum power due to potential interference in the SIC process. In the following lemma, we prove that a simple on/off power control is asymptotically optimal with respect to high transmit power in the outage limited case.

Lemma 1 (Optimal Power Allocation): With the same peak power constraint \(0 \leq P_k \leq P_{\text{max}}\) for all users, the optimal power allocation that maximizes the instantaneous mutual information in the outage-limited MUD-SIC system (with potential error propagation) is given by the simple on/off rule:

\[
P_k = \{0, P_{\text{max}}\}, \quad \forall k
\]

(9)

This lemma suggests that a user either transmits at full power or does not transmit at all.

Proof: Please refer to Appendix A.

B. Asymptotically Optimal Decoding Order with MUD-SIC under Macro-diversity

In the existing literature, the decoding order of successive interference cancellation is usually designed to either minimize the transmit power subject to performance requirement constraints or to maximize system capacity with power constraint. In [21], the authors show that solving for optimal decoding order is \(\mathcal{NP}\)-hard when the decoding order is jointly optimized with power allocation, but can be approximated by means of the discrete stochastic approximation (DSA) algorithm. In [22], the authors show that for any point on the boundary of the capacity region, the optimal decoding policy is successive decoding with the same decoding order of users for all channel, when the mobile station has perfect CSIT. However, these results failed to account for the packet errors in slow fading channels. Furthermore, due to the mutual coupling of the outage events in the MUD-SIC processing, the optimal decoding order, which is given by \(\mathbf{\Pi}^* = \arg \max \Pi_{\text{goodput}}(P, R, \mathbf{\Pi}; \mathbf{G})\), is very complicated and requires exhaustive search in general. Yet, we shall show in Lemma 2 that a simple decoding ordering would be asymptotically optimal for large transmit power.

Lemma 2 (Asymptotically Optimal Decoding Order): For a given path loss realization \(\mathbf{G}\), let \(A_k(\mathbf{G}) = \{1, 2, \ldots, \mu_b\}\) be the set of active users (users with non-zero transmit power). Suppose all the users have the same conditional average PER requirement, i.e., \(\text{PER}_k(r_k, P_k; \mathbf{G}) = \epsilon\), then the following decoding order is asymptotically optimal for sufficiently large \(P_{\text{max}}\).

\[
\pi_k^*(j) = \arg \max \left\{k \in [1,K] \setminus \{\pi_k(1), \pi_k(2), \ldots, \pi_k(j-1)\} \right\} \gamma_k
\]

(10)

where \(\gamma_k = P_{\text{max}} |H_{k,b}|^2 g_{k,b}\) is the instantaneous receive SNR of all active users.

Proof: Please refer to Appendix B.

While the decoding rule in (10) is only asymptotically optimal, we show in Figure 2 that the decoding rule in (10) achieves close-to-optimal performance even in moderate SNR.

In order to characterize the per-user outage probability, let’s define \(S_i = \{0, 1\}\) as the \(i\)-th stage iteration decoding event with \(S_i = 1\) denotes successful decoding and \(S_i = 0\) denotes decoding failure. Given the asymptotically optimal decoding order policy in (10), we assume that for user \(\pi_k^*(i)\) fails in the \(i\)-th decoding iteration, then we can declare packet error for all the remaining users in the same base station. This assumption cause a neglectable sub-optimality to the system performance which can be verified by numerical simulation, however, it can provide a tractable analysis expression and provide some important insights regarding the system performance.

Next, we define the event \(S_i\) which is given by

\[
S_i = \mathcal{I}\left\{r_{\pi_k^*(i)} < \log_2 \left(1 + \text{SINR}_{\pi_k^*(i)}\right)\right\} = \{0, 1\},
\]

(11)

where \(\text{SINR}_{\pi_k^*(i)} = \frac{\gamma_{\pi_k^*(i)}}{1 + \sum_{j<i} \gamma_{\pi_k^*(j)} (1 - S_j) + \sum_{j>i} \gamma_{\pi_k^*(j)}}\) is the signal-to-interference plus noise ratio and \(\mathcal{I}(\cdot)\) is the indicator function which is 1 when the event is true and 0 otherwise. Since we assume that any packet error before the \(i\)-th stage will result decoding error in the remaining stages, we can define the following event

\[
O_i = \mathcal{I}\left\{r_{\pi_k^*(i)} < \log_2 \left(1 + \frac{\gamma_{\pi_k^*(i)} }{1 + \sum_{j>i} \gamma_{\pi_k^*(j)}}\right)\right\}.
\]

(12)

Based on the above assumption and event definition, we can ducue that:

\[
S_i = 0 \Rightarrow O_1 \cup O_2 \cup \ldots \cup O_i.
\]

(13)
Therefore, the packet outage probability of user $k$ is given by
\[
\Pr_{\text{out}}(r_k, P; k; G) = \sum_{\pi^*_k \in B_k} \prod_{b \in B_k} \sum_{i=1}^{\pi^*_k-1(k)} \Pr \left[ O_i \cup O_2 \cup \ldots \cup O_i = 0 | \pi^*_k \right] \Pr(\pi^*_k)
\]
\[
\leq \sum_{\pi^*_k \in B_k} \prod_{b \in B_k} \sum_{i=1}^{\pi^*_k-1(k)} \Pr \left[ O_j = 0 | \pi^*_k \right] \Pr(\pi^*_k) \tag{14}
\]
By substituting (14) into (8), the average system goodput under the asymptotically optimal decoding order is given by
\[
U_{\text{gp}}(P, R, \Pi^*; G) = \sum_{k=1}^{K} r_k (1 - \Pr_{\text{out}}(r_k, P; k; G))
\]
\[
\geq \sum_{k=1}^{K} r_k (1 - \prod_{\pi^*_k \in B_k} \sum_{i=1}^{\pi^*_k-1(k)} \Pr \left[ O_j = 0 | \pi^*_k \right] \Pr(\pi^*_k)) \tag{15}
\]
C. Per-user PER and Average System Goodput

Under the asymptotically optimal decoding order in Lemma 2 the average system goodput and per-user outage probability can be expressed in terms of the conditional outage probability. In order to solve the per-user outage probability and average system goodput, we should obtain the closed form expression of the conditional outage probability. For a given asymptotically optimal decoding order $\pi^*_k$, the conditional outage probability of user $k$ in the $j$-th iteration can be expressed as:
\[
\Pr \left[ O_j = 0 | \pi^*_k \right] = \Pr(\gamma_j > C_{\pi^*_k}(H, G, \pi^*_k, k) | \pi^*_k)
\]
\[
= \Pr \left\{ \gamma_{\pi^*_k(j)} - \vartheta_{\pi^*_k(j)} \sum_{l=j+1}^{\mu_k} \gamma_{\pi^*_k(l)} < \vartheta_k \right\} \tag{16}
\]
where $\vartheta_k = 2^{r_k} - 1$. In general, the conditional outage probability involve $\mu_k$ dimensions nested integration which is complicated and non-traceable when the dimension of integration grows. However, by taking the advantage of the additive Markov chain property from the exponential random variable order statistics, the conditional outage probability can be calculated by a one dimensional integration. We first introduce the following lemma.

Lemma 3 (Closed-form Expression of Conditional PER): The conditional outage probability of user $k$ in the $j$-th iteration in (16) can be written in a summation of exponential functions which is given by
\[
\Pr \left[ O_j = 0 | \pi^*_k \right] = 1 - \sum_{l=j+1}^{\mu_k} \Psi_l \beta_l \exp(-\vartheta_l \beta_l / v_l) \tag{17}
\]
where $\Psi_l = \prod_{j=l+1}^{\mu_k} \frac{v_l}{v_l - \vartheta_l}, \beta_l = \frac{1}{v_l - \vartheta_l}$, and $\vartheta_k = 2^{r_k} - 1$.

Proof: Please refer to Appendix C.

After obtaining the closed-form of the conditional outage probability, we need to calculate the probability of a particular decoding order which is summarized in the following:

Lemma 4 (Probability of a Decoding Order Policy $\pi_k$): Consider a set of independent non-identical distributed (i.i.d.) exponential random variables $X_1, X_2, X_3, \ldots, X_{\mu_k}$ which has a p.d.f. as defined in (11). From (23), the probability of a particular order $X_{1:1}, X_{2:2}, \ldots, X_{\mu_k:n_{\pi_k}}$ is given by
\[
\Pr(X_{1:1}, X_{2:2}, \ldots, X_{\mu_k:n_{\pi_k}}) = \frac{1}{\prod_{j=1}^{\mu_k} \vartheta_j^{\vartheta_j} \prod_{j=1}^{\mu_k} \vartheta_j^{\vartheta_j}}
\]
As a result, the per-user outage probability is given by the following lemma.

Lemma 5 (Per-User Conditional PER with Macro-diversity): The average packet error probability of user $k$ under the asymptotically optimal decoding order policy $\Pi^*$ is given by:
\[
\Pr_{\text{out}}(r_k, P_{\text{max}}; G) \leq \sum_{\pi^*_k \in B_k} \prod_{b \in B_k} \sum_{i=1}^{\pi^*_k-1(k)} \sum_{j=1}^{\mu_k} \Pr \left[ \sum_{l=j}^{\mu_k} \beta_l \exp(-\vartheta_l \beta_l / v_l) \right] \Pr(\pi^*_k) \tag{19}
\]
where $\Pr(\pi^*_k)$ is given in equation (19) and $\vartheta_k = 2^{r_k} - 1$. Therefore, the average system goodput can be summarized by the following theorem:

Theorem 1 (Lower Bound for the Average System Goodput): The data rate $r_k(G)$ can be determined by solving the per-user conditional packet error requirement $\Pr_{\text{out}}(r_k, P_{\text{max}}; G) = \epsilon$.

IV. RESULTS AND DISCUSSIONS

In this section, we evaluate the theoretical results in the preceding section using simulations. We consider a multicell system with 2 base stations. Every cell has radius of 1 km and path loss exponent 3.6. Assume that the minimum
distance from a mobile station to the home base station is 30 m, the average path loss of a particular user in the cell has a dynamic range up from -48 dB to -103 dB. The noise power level is equal to -105 dBm. The log-normal shadowing is assumed to have a standard derivation 8 dB. There are $K$ active users uniformly distributed in the cells and the distance the mobile and $b$-th base station and the $k$-th mobile user is $d_{k,b}$. All the channel fading coefficients $\{H_1, H_2, ..., H_K\}$ are generated as i.i.d. complex Gaussian random realizations with zero mean and unit variance. Average system goodput is obtained by counting the number of packets which are successfully decoded by the base station for all users and average the result over both macroscopic and microscopic fading. In the simulation, each point is obtained by averaging 100000 macroscopic and microscopic realizations.

A. Average System Goodput

Figure 3 illustrates the average system goodput versus the transmit power (dBm) of mobile user for $K = 10$ with asymptotical optimal decoding order. Each curve in the graph represents different type of power control with same target outage probability for all user (5% or 10%).

The optimal data rate of each user is obtained by numerical method such as Newton method in solving equation (20) for $\mathcal{P}_{\text{out}}(k; P_{\max} ; G) = \epsilon$, $\forall k$. We compare the performance of the proposed design with a conventional baseline 1 CDMA power control algorithm in which the transmit powers of all users are adjusted such that the received SINR of them are the same at base station. For the baseline 1, the system
depends on the number of transmitters and receivers. Therefore, the system capacity for MUD-SIC receiver is obtained by counting the number of packets which are successfully decoded by the base station for all users and average the result over both macroscopic and microscopic fading.

The data rate in the simulation of CDMA is set to a value such that the outage requirement is fulfilled for the weakest user. For the spread spectrum in the CDMA system, we assume the synchronized orthogonal spreading codes are used and the spreading factor is always equal to the number of users. Therefore, the orthogonal multiple access incurs no loss in total system capacity for equal rate and equal SINR users. Therefore, the system capacity for equal rate and equal SINR users.

A high disparities received power can significantly increase the system capacity for MUD-SIC receiver. Therefore, the system saturated at high SNR because strong interference from inter-cell becomes a dominate factor in the system performance. On the contrary, the average system goodput of the proposed design increase with the transmit power when MDiv is performed in the base station. The reason is that strong interference is regarded as desired user signal and it will be decoded by corresponding base station. Furthermore, the optimal power control (either full power transmission or completely silent) create a high disparities received power at the base station and strong enough interference environment for MDiv to exploit. Therefore, the system goodput grows with SNR at small SNR but quickly saturated at moderate SNR.

This is because the performance is always limited by the weakest user. On the other hand, the goodput performance of the proposed on/off power control scheme does not saturate even at high SNR regime. It can be explained that in the proposed on/off power control, strong users do not required to decrease the transmission power to maintain the same SINR as those weak users, this factor contribute significantly to the system goodput. Furthermore, we compare the proposed design with a baseline 2 (FDMA system) where each user transmits at its peak power. Although multiple access interference does not exist in the FDMA system due to orthogonal transmission, it has a very low spectral efficiency. On the contrary, the proposed design provides a substantial performance gain compared with the FDMA system in the interference limited environment.

Figure 4 shows the average system goodput versus the transmit power with different MDiv threshold ($\Delta_{\text{threshold}}$). Each user is power controlled by the on/off scheme and there is 5% outage probability requirement. We compare the performance of the proposed design with a system that does not perform MDiv in which all the inter-cell users are treated as interference. For the system without MDiv, the average system goodput saturated at high SNR because strong interference from inter-cell becomes a dominate factor in the system performance. On the contrary, the average system goodput of the proposed design increase with the transmit power when MDiv is performed in the base station. The reason is that strong interference is regarded as desired user signal and it will be decoded by corresponding base station. Furthermore, the optimal power control (either full power transmission or completely silent) create a high disparities received power at the base station and strong enough interference environment for MDiv to exploit. Therefore, the system goodput grows with SNR at small SNR but quickly saturated at moderate SNR. This is because the performance is always limited by the weakest users. On the other hand, the goodput performance of the proposed on/off power control scheme does not saturate even at high SNR regime. It can be explained that in the proposed on/off power control, strong users do not required to decrease the transmission power to maintain the same SINR as those weak users, this factor contribute significantly to the system goodput. Furthermore, we compare the proposed design with a baseline 2 (FDMA system) where each user transmits at its peak power. Although multiple access interference does not exist in the FDMA system due to orthogonal transmission, it has a very low spectral efficiency. On the contrary, the proposed design provides a substantial performance gain compared with the FDMA system in the interference limited environment.

The data rate in the simulation of CDMA is set to a value such that the outage requirement can be fulfilled for the weakest user. For the spreading in the CDMA system, we assume the synchronized orthogonal spreading codes are used and the spreading factor is always equal to the number of users. Therefore, the orthogonal multiple access incurs no loss in total system capacity for equal rate and equal SINR users.

A high disparities received power can significantly increase the system capacity for MUD-SIC receiver.
goodput has a significant gain when MDiv is performed in multi-cell environment. Furthermore, the goodput of the joint ML detection (which consider common outage) is plotted for comparison. In low SNR regime, the SIC outperforms the joint ML detection. This is because in the joint ML detection, a common outage will be declared if the rate vector lies outside the instantaneous capacity region. Hence, the outage performance of the joint ML detection is always limited by the weakest users. On the other hand, the SIC approach consider per-user outage and packets for some users may be decoded correctly even though the rate vector lies outside the capacity region. In high SNR region, the performance of SIC is limited by strong interference from both intra-cell and inter-cell interference. Nevertheless, using MDiv, the performance of the SIC scheme can be improved at high SNR regime. On the other hand, the joint ML detection does not suffer from multi-user interference and hence the performance is able to scale with SNR.

Similarly, Figure 5 shows the average system goodput versus different value of path loss exponent for K=10. Along all the curves, the same user transmit power is fixed at -10 dBmW and -3 dBmW respectively. It is very interesting that the average system goodput first increase with the value of path loss exponent and then decrease when the path loss exponent is beyond certain value. This counter intuitive result is due to the fact that when the path loss exponent increases, both desired signal and interference signal received by base station decreases. However, the attenuation of interference occurs to be larger than desired signal because interference users are usually located far away from desired home base station. As path loss exponent increases, the operating region of the system is shifting from interference limited region to noise limited region, and the desired users signal strength attenuate to a level that high data rate communication is impossible, and it results in a decreasing trend of average system goodput.

Figure 6 depicts that average number of system goodput versus the number users in a two cells system. Similarly, the transmit power of an active user is again fixed at -10dBmW and -3dBmW respectively. It can be observed that the system goodput gain due to MDiv is not significant when the number of user is small, especially users are transmitting at low power (-10dBmW). When the transmit power is low, signal strength of interference can not satisfy the MDiv threshold requirement, so there is nearly no MDiv performed in the base stations. However, when the number of users increases, it is more likely that there exists a user who locates near the cell boundary, creates large interference to neighboring cells. Therefore, base stations can take advantage of the strong interference and perform MDiv to improve the system goodput. On the other hand, there is a diminishing return in the system goodput when the number of users increases, particular in high transmit power with small MDiv threshold value (2dB). This is due to the fact that base stations do not fully utilize the benefit of strong interference by setting a small threshold value, therefore interference can not be decoded and causes the degradation in system performance.

V. CONCLUSION

In this paper, a generic multi-cell system with K client users, n_B base stations and a centralized controller is considered. Based on the asymptotic optimal decoding order with respect to the transmit power, we incorporate the mathematical tool of order statistics to obtain the closed-form solution of system performance. Numerical simulations result are obtained to verify the analytical expressions. The closed form solutions allow efficient numerical evaluations to find out how the system performance is affected by the system parameter such as number of users and path loss exponent. From the results, we see that in interference limited region (users transmit at high power), MDiv improves the system goodput significantly by introducing macro-diversity protection to alleviate the consequences of error propagation. Furthermore, system with

9Common outage is declared as rate vector is outside the instantaneous capacity region.

10A small threshold value implies a few users are satisfied with the MDiv requirement.
MDiv allows more users to be served at the same time through taking advantage of strong interference.

APPENDIX

A. Proof of Lemma 1

Note that since our power constraint is instantaneous, average system goodput maximization is the same as maximize the instantaneous goodput for each fade vector. It can be observed in equation (4) that the system goodput is contributed by the instantaneous channel capacity and transmitted data rate. In fact, the system goodput is upper bounded by the instantaneous mutual information of user $k$ is given by equation (4). Therefore, in high SNR the total instantaneous capacity $Q = \sum_{k=1}^{K} \sum_{b \in B_k^*} C_b(H,G, \pi_b, k) \approx$ 

\[
\sum_{k=1}^{K} \sum_{b \in B_k^*} \log_2 \left( \frac{P_k |H_{\pi_b^{-1}(k),b}|^2 g_{\pi_b^{-1}(k),b}}{W_{\pi_b^{-1}(k)}(k)} + \Phi_b(H,G, \pi_b, j) + \Omega_b(H,G) \right)
\]

where $B_k^*$ denotes the base station which has the maximum mutual information for user $k$. To find the optimal power allocation which maximizes the instantaneous mutual information, we consider the following optimization problem.

\[
P^* = \{P_1^*, P_2^*, \ldots, P_K^*\} = \arg \max_{\{P_1, P_2, \ldots, P_K\}} Q
\]

Differentiating the system capacity twice with respect to $P_j$, which yields $\frac{\partial^2 Q}{\partial P_j^2} = \frac{-1}{P_j \ln 2}$

\[
+ \frac{1}{\ln 2} \sum_{k \neq j}^{K} \sum_{b \in B_k^*} \frac{(|H_{\pi_b^{-1}(j),b}|^2 g_{\pi_b^{-1}(j),b})^2}{(P_j + \sum_{i \in K - \{k\}} P_i |H_{\pi_b^{-1}(i),b}|^2 g_{\pi_b^{-1}(i),b})^2} \approx -\frac{1}{P_j^2 \ln 2} \leq 0.
\]

It can be observed that the first term in the derivative is the dominating term since the other terms converge to zero much faster with respect to the transmit power of all users in high transmit power regime. Therefore, $\frac{\partial^2 Q}{\partial P_j^2}$ is non-positive and the system goodput is a concave function of $P_j$. Similarly, by differentiating the system capacity once with respect to $P_j$, we obtain $\frac{\partial Q}{\partial P_j} = \frac{1}{P_j \ln 2}$

\[
- \frac{1}{\ln 2} \sum_{k \neq j}^{K} \sum_{b \in B_k^*} \frac{|H_{\pi_b^{-1}(j),b}|^2 g_{\pi_b^{-1}(j),b}}{(P_j + \sum_{i \in K - \{k\}} P_i |H_{\pi_b^{-1}(i),b}|^2 g_{\pi_b^{-1}(i),b})} \approx \frac{1}{P_j \ln 2} \geq 0.
\]

As the first derivative is approximately non-negative and the objective function is a concave function with respect to $P_j$, we conclude that $P_j = P_{max}$ achieves the maximum system capacity.

11Since the packet selection is performed when users are involved in the MDiv and a packet can be possibly decoded by more than one base stations, we can focus on the base station which gives the maximum mutual information for user $k$.

B. Proof of Lemma 2

From equations (4), (7) and (8), it can be observed that the optimal decoding order in maximizing the system capacity is equivalent to a decoding order, which maximize the instantaneous mutual information in each decoding iteration at each base station. The total mutual information in the $b$-th station can be expressed as:

\[
C_b(H,G, \pi_b, \pi_b(1)) \approx \log_2 \left( 1 + \frac{P_{max} g_{\pi_b(1),b} |H_{\pi_b(1),b}|^2}{U_1^{\pi_b} + \Phi_b(H,G, \pi_b, \pi_b(1)) + \Omega_b(H,G)} \right)
\]

where $U_1^{\pi_b} = 0$ in the first iteration, the channel capacity of the user in the first decoding iteration at the $b$-th base station is given by:

\[
C_b(H,G, \pi_b, \pi_b(1)) = \log_2 \left( 1 + \frac{g_{\pi_b(1),b} |H_{\pi_b(1),b}|^2}{U_1^{\pi_b} + \Phi_b(H,G, \pi_b, \pi_b(1)) + \Omega_b(H,G)} \right)
\]

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\]

\[
\approx \log_2 \left( 1 + \frac{P_{max} g_{\pi_b(1),b} |H_{\pi_b(1),b}|^2}{U_1^{\pi_b} + \Phi_b(H,G, \pi_b, \pi_b(1)) + \Omega_b(H,G)} \right)
\]

Consider the second iteration of the decoding process. The accumulated undecodable interference has value $U_2^{\pi_b} \in \{0, g_{\pi_b(1),b} |H_{\pi_b(1),b}|^2 \}$. Therefore, the mutual information in the second iteration is given by $C_b(H,G, \pi_b, \pi_b(2)) = \log_2 \left( 1 + \frac{g_{\pi_b(2),b} |H_{\pi_b(2),b}|^2}{U_2^{\pi_b} + \Phi_b(H,G, \pi_b, \pi_b(2)) + \Omega_b(H,G)} \right)$. Similarly, the choice of $\pi_b(2)$ that maximizes the mutual information is given by:

\[
\pi_b^{*}(2) = \arg \max_{k \in [1, \mu_b]} g_{k,b} |H_{k,b}|^2
\]

As such, by induction, the asymptotically optimal decoding order is to decode the users sequentially in decreasing receive SNR as in (10).

C. Proof of Lemma 3

By (23) and (24), for a given ordered $\Gamma_1, \mu_b < \Gamma_2, \mu_b < \ldots < \Gamma_{\mu_b, \mu_b}$ channel gains where $\Gamma_i = |H_{i,b}|^2 g_{i,b}$, define a

12Undecodable interference is due to cancellation error in the previous decoding stage.

13Given the path loss and CSR realization, the optimal decoding order should gives the largest number of successfully decoded users, or equivalently the lowest potentially accumulated undecoded interference.
new set of random variables \( \{D_1, D_2, ..., D_{\mu_b}\} \) to denote the spacing between \( \Gamma_{l,\mu_b} \) and \( \Gamma_{l-1,\mu_b} \) as follows:

\[
\begin{align*}
D_1 &= \Gamma_{\mu_b,\mu_b}, \\
D_l &= \Gamma_{\mu_b-l+1,\mu_b} - \Gamma_{\mu_b-l,\mu_b}, \quad l=2, \ldots, \mu_b-1
\end{align*}
\]  

(29)

Then, a linear combination of the spacing is defined as:

\[
\mathcal{M}_i = i \{D_i\}
\]

(30)

where \( \mathcal{M} \) is a set of independent exponential random variables with p.d.f. given by:

\[
f_{\mathcal{M}_i}(m) = \beta_i \exp(-m\beta_i), \quad \forall m, \beta_i \geq 0
\]

where \( \beta_i \) is defined as:

\[
\beta_i = \sum_{u=1}^{i} \frac{1}{g_{\mu_u} u!}
\]

(32)

Hence, the conditional outage probability can be written as:

\[
\Pr[O_j = 0 | \pi_b] = \Pr \left\{ \sum_{l=j}^{\mu_b} \mathcal{M}_l | v_l < \vartheta_{\pi_b}(j) | \pi_b \right\}
\]

(33)

\[
= \Pr \left\{ W_l < \vartheta_{\pi_b}(j) | \pi_k \right\}
\]

where \( v_l = 1 - l \times \vartheta_{\pi_b}(j) + j \times \vartheta_{\pi_b}(j), \quad W_l = \sum_{l=j}^{\mu_b} \mathcal{M}_l | v_l, \) and \( \vartheta_{\pi_b}(j) = 2^{\pi_b} - 1 \).

Representing \( \phi_l(\omega) = \frac{\beta_l}{\vartheta_{l+1} + \vartheta_l \omega} \) as characteristic function of \( \mathcal{M}_l | v_l \), then the p.d.f of the \( W_l \) is given by the inverse Laplace transform of the following:

\[
f_{W_l}(x) = L^{-1} \left\{ \sum_{l=j}^{\mu_b} \frac{\beta_l}{\vartheta_l} \exp(-\frac{\vartheta_{\pi_b}(j)\beta_l}{\vartheta_l}) \right\}
\]

(34)

By using the partial-fraction decomposition technique [26], the conditional outage probability results in a summation of exponential function which is given by:

\[
\Pr[O_j = 0 | \pi_b] = \Pr \left\{ W_l < \vartheta_{\pi_b}(j) | \pi_b \right\}
\]

\[
= \int_{-\infty}^{\vartheta_{\pi_b}(j)} f_{W_l}(x) dx
\]

\[
= 1 - \sum_{l=j}^{\mu_b} \Psi_l \frac{\beta_l}{\vartheta_l} \exp(-\frac{\vartheta_{\pi_b}(j)\beta_l}{\vartheta_l})
\]

(35)

where \( \Psi_l = \prod_{u=j, u \neq l}^{\mu_b} \frac{u!}{(u-\pi_{u,v})!} \) are the partial fraction coefficients.
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