The quantum origin of cosmic structure: Theory and observations

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Abstract. The particle production process is reviewed, through which cosmic inflation can produce a scale invariant superhorizon spectrum of perturbations of suitable fields starting from their quantum fluctuations. Afterwards, in the context of the inflationary paradigm, a number of mechanisms (e.g. curvaton, inhomogeneous reheating etc.) through which such perturbations can source the curvature perturbation in the Universe and explain the formation of structures such as galaxies are briefly described. Finally, the possibility that cosmic vector fields also contribute to the curvature perturbation (e.g. through the vector curvaton mechanism) is considered and its distinct observational signatures are discussed, such as correlated statistical anisotropy in the spectrum and bispectrum of the curvature perturbation.

1. Introduction
The standard model of cosmology at present is comprised by Hot Big Bang Cosmology and Cosmic Inflation. The cosmology of the Hot big Bang accounts for the Hubble expansion of the Universe, the observed Cosmic Microwave Background (CMB) radiation, the primordial abundance of light elements (formed in the Early Universe through the process of Big Bang Nucleosynthesis) and the age of the Universe, which agrees well with the astrophysical estimates of the ages of the oldest globular clusters. In turn inflation overcomes or at least ameliorates certain fine-tuning problems regarding the initial conditions of the Hot Big Bang, namely the so-called horizon and flatness problems [1] [2].

What is cosmic inflation? In a nutshell, inflation is a brief period of superluminal expansion of space in the Early Universe. What inflation does is that it makes the observable Universe large and uniform. However, if the Universe were perfectly uniform there would be no structures like galaxies or galactic clusters, no stars with planets orbiting around them, no ... us. It is imperative, therefore, that there is a deviation from perfect uniformity, which can give rise to these structures. Indeed, we need a Primordial Density Perturbation (PDP) for structure formation to occur. It so happens that evidence for such a PDP exists as the latter reflects itself on the CMB through the so-called Sachs-Wolfe effect [3] which states that CMB light is redshifted when crossing growing overdensities. This effect directly connects the fractional amplitude of the PDP with the fractional perturbation of the temperature of the CMB:

$$\frac{\delta T}{T}_{\text{CMB}} = \frac{1}{2} \frac{\delta \rho}{\rho}_{H} \approx 10^{-5}. \quad (1)$$

Even though the PDP appears to be very small, numerical simulations of structure formation...
have shown that it is enough to account for the observed structure in the Universe. What is the origin of this PDP? Well, it turns out that this too can be accounted for by Cosmic Inflation.

2. Particle production during cosmic inflation

To have an idea of how Cosmic Inflation is achieved consider the so-called Friedman equation, which is the temporal component of the Einstein equations for a homogeneous and isotropic spacetime. In flat space this equation reads \( H^2 = \frac{\rho}{m_P^2} \), where \( m_P^2 = (8\pi G)^{-1} \) is the reduced Planck mass\(^1\) and \( H \equiv \dot{a}/a \) is the Hubble parameter corresponding to the rate of the expansion of the Universe, with \( a = a(t) \) being the scale factor of the Universe, parameterising the Universe expansion, and the dot denotes derivative with respect to the cosmic time \( t \). Suppose now that, at some period in its early history, the Universe was dominated by an effective cosmological constant \( \Lambda_{\text{eff}} \). Then, \( \rho \approx m_P^2 \Lambda_{\text{eff}} = \text{constant} \), which means that the Hubble parameter \( H = \Lambda_{\text{eff}}/3 \) is constant and, therefore, \( a \propto e^{Ht} \), i.e. space expands exponentially in time. Thus, inflation occurs when the Universe is dominated by an effective vacuum density. When inflation ends this density has to be transferred in the density of the thermal bath of the Hot Big Bang. This, in effect, amounts to a change of vacuum\(^2\). Therefore, vacuum states during inflation are not necessarily vacuum states after inflation, but instead they can become populated.

This process is called Particle Production and it is similar to the production of particles (in the form of Hawking radiation) on the event horizon of Black Holes \([4]\). Indeed, the cosmological horizon during inflation is an event horizon and can be viewed as an “inverted” (i.e. inside-out) black hole in the sense that nothing can escape being “sucked out” by the superluminal expansion. Virtual particle pairs, corresponding to quantum fluctuations, are broken up by the expansion and are pulled away to superhorizon distances, where they can no more find eachother and annihilate, becoming thereby real particles, giving rise to classical perturbations of the corresponding fields. The amplitude of these perturbations is determined by the Hawking temperature \( \delta \phi \sim T_H \), which for de Sitter space is \( T_H = H/2\pi \) \([5]\). Let us review now the particle production process in a more rigorous manner.

The standard paradigm considers a real, minimally coupled, scalar field \( \phi \) of mass \( m \) with Lagrangian density

\[
\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{2} m^2 \phi^2. \tag{2}
\]

Using the above one obtains the equation of motion of the scalar field. The field is expected to become homogenised by the inflationary expansion as any inhomogeneities are inflated away. In this case the equation of motion becomes

\[
\ddot{\phi} + 3H \dot{\phi} + m^2 \phi = 0, \tag{3}
\]

where \( \phi = \phi(t) \). Deviations from homogeneity are introduced originally only from its vacuum fluctuations. To follow their evolution we perturb the field from its homogeneous value as \( \phi = \phi(t) + \delta \phi(\vec{x}, t) \). Then we obtain the equation of motion of the Fourier components of the field \( \delta \phi_k(\vec{k}) \equiv \int \delta \phi(\vec{x}) e^{-ik\cdot\vec{x}} d\vec{x} \). This equation reads

\[
\left[ \partial_t^2 + 3H \partial_t + m^2 + \left( \frac{k}{a} \right)^2 \right] \delta \phi_k = 0. \tag{4}
\]

The next step is to promote the perturbations of the field to quantum operators defined as

\[
\delta \hat{\phi}(\vec{x}, t) = \int \frac{dk}{(2\pi)^3} \left[ \hat{a}(\vec{k}) \delta \varphi_k(k, t) e^{ik\cdot\vec{x}} + \hat{a}^\dagger(\vec{k}) \delta \varphi^\star_k(k, t) e^{-ik\cdot\vec{x}} \right], \tag{5}
\]

\(^1\) \( G \) is Newton’s gravitational constant and we consider natural units where \( c = \hbar = k_B = 1 \).

\(^2\) from the false vacuum of inflation, corresponding to \( \Lambda_{\text{eff}} \), to the true vacuum of the present, with \( \Lambda \simeq 0 \).
where $\hat{a}(\vec{k})$ and $\hat{a}^\dagger(\vec{k})$ are creation and annihilation operators respectively and we consider canonical quantisation with $[\hat{a}(\vec{k}),\hat{a}^\dagger(\vec{k}')] = (2\pi)^3 \delta(\vec{k} - \vec{k}')$. The mode functions $\delta \varphi_k(k,t)$ satisfy the same equation of motion as the Fourier components of the field perturbations $\delta \phi_k(k,t)$ because this equation is linear. We can solve this equation considering that in the subhorizon limit ($k/aH \to +\infty$) the solution matches the so-called Bunch-Davies vacuum [6] which corresponds to the quantum fluctuations of a free scalar field in Minkowski spacetime. Thus, the boundary condition reads

$$
\delta \varphi_k \xrightarrow{k_{\text{dec}} \to \infty} e^{-ik\tau} \frac{e^{ik/aH}}{\sqrt{2k}} = e^{ik/aH} \delta \varphi_k(k,t) 
$$

where $\tau = -1/aH$ is the conformal time, which factors out the expansion of the Universe. Using the above, the solution of Eq. (4) for the mode functions is

$$
\delta \varphi_k = a^{-3/2} \sqrt{\frac{\pi}{1 - e^{2\pi\nu}}} \left[ J_\nu(k/aH) - e^{i\pi\nu} J_{-\nu}(k/aH) \right],
$$

where $J_\nu$ denotes Bessel functions of the first kind and $\nu \equiv \sqrt{\frac{9}{4} - \frac{\pi^2}{4}}$. To investigate particle production we evaluate the above solution in the superhorizon limit ($k/aH \to 0$). We find

$$
\delta \varphi_k \xrightarrow{k_{\text{dec}} \to 0} a^{-3/2} \sqrt{\frac{\pi}{1 - e^{2\pi\nu}}} \left( \frac{\sin(\pi\nu)}{\Gamma(1 - \nu)} \right) \left( \frac{aH}{k} \right)^\nu.
$$

Using the above we can calculate the power spectrum $P_{\delta \phi} \equiv \frac{k^3}{2\pi^2} |\delta \varphi|^2$ in the superhorizon limit. We find

$$
P_{\delta \phi} = \frac{8\pi |\Gamma(1 - \nu)|^2}{1 - \cos(2\pi\nu)} \left( \frac{H}{2\pi} \right)^2 \left( \frac{k}{2aH} \right)^{3-2\nu}. \tag{9}
$$

Thus, we see that the scale dependence of the field perturbations is only included in the last term of the above. Considering a light field, i.e. $m \ll H \Rightarrow \nu \to \frac{3}{2}$, we see that the scale dependent term is eliminated and the spectrum becomes scale invariant. Indeed, for a light field the above spectrum can be written as

$$
P_{\delta \phi} = \left( \frac{H}{2\pi} \right)^2 \left( \frac{k}{2aH} \right)^{2\eta}, \tag{10}
$$

where $\eta = \frac{1}{3} \left( \frac{m}{H} \right)^2$ and we considered $|\eta| \ll 1$. Thus, a light minimally coupled scalar field obtains a scale invariant spectrum of perturbations when they exit the horizon. The typical value of the field perturbations is $\delta \phi \approx \sqrt{P_{\delta \phi}} \approx H/2\pi$, i.e. it is determined by the Hawking temperature as expected.

What is the fate of these perturbations after horizon exit? The perturbations can now be treated classically and their evolution is determined by the equivalent of Eq. (3)

$$
(\ddot{\delta \phi}) + 3H(\dot{\delta \phi}) + m^2(\delta \phi) = 0. \tag{11}
$$

Considering that $H \approx$ constant the solution of the above for a light field is

$$
\delta \phi = \frac{H}{2\pi} \left[ e^{-\frac{1}{2} \left( \frac{m}{H} \right)^2 H \Delta t} - \frac{1}{9} \left( \frac{m}{H} \right)^2 e^{-3H \Delta t} \right] \approx \frac{H}{2\pi}. \tag{12}
$$

Thus, we see that the perturbations of a light scalar field freeze-out after horizon exit. This guarantees that scale invariance is maintained since, regardless of the time of horizon exit (i.e.
Figure 1. Schematic log-log plot of the evolution of the perturbations of a scalar field (enveloped by red solid lines) which follow the Universe expansion compared with the cosmological horizon (solid green lines). During inflation the cosmological horizon is an event horizon of constant physical radius. The perturbations of the scalar field start off as quantum fluctuations at subhorizon size, which are stretched to superhorizon scales by the (quasi)exponential expansion. After inflation, the size of the superhorizon perturbations continues to grow following the expansion of the Universe (i.e. proportional to $a(t)$) but the cosmological horizon, which is now a particle horizon) grows faster, with the speed of light. As a result, the perturbations reenter the horizon some time after the end of inflation. In the graph two different perturbations are depicted. They exit the horizon at different times, corresponding to the horizontal dashed lines. As a result they are inflated to different sizes so one is much larger that the other when they reenter the horizon. The size of the perturbations at horizon reentry is depicted by the vertical dashed lines. However, both the perturbations have the approximately the same amplitude, determined by the Hawking temperature during inflation $\delta \phi = \frac{H}{2\pi} = T_H$, which remains approximately constant. The fact that the scalar field perturbations retain the same amplitude even though they attain different sizes during their superhorizon evolution is the reason behind the scale-invariance of the perturbation spectrum.

regardless of how large the size of the perturbations becomes by inflation), the perturbations have the same amplitude because $H \approx$ constant until the end of inflation (see Fig. 1).

How are such field perturbations related with the PDP? The PDP arises because of the generation of a corresponding curvature perturbation $\zeta$, as is discussed below. If the curvature perturbation is due to the perturbations of a light scalar field then their power spectra are proportional, i.e. $P_\zeta \propto P_{\delta \phi}$. This means that they have the same scale dependence. The latter can be parametrised in the form of a power-law: $P_\zeta \propto k^{n_s-1}$, so that

$$n_s(k) - 1 \equiv \frac{d \ln P_\zeta}{d \ln k}.$$  \hspace{1cm} (13)

$^a$ Locally spacetime is flat. The subhorizon limit is well within the radius of curvature of spacetime during inflation so this curvature can be ignored. Similarly, in this limit the momentum of the virtual particles is much larger than their mass $k/a \gg m$ so that the field can be considered effectively massless.
Assuming that $\zeta$ is due to a light scalar field one obtains (cf. Eq. (10))

$$n_s = 1 + 2\eta + \mathcal{O}(\varepsilon),$$

(14)

where $\varepsilon \equiv -H/H^2 \ll 1$ quantifies the deviation from pure de Sitter expansion which we have ignored so far. Since, during inflation $|\eta| \ll 1$ for a light scalar field, we find that, if the curvature perturbation is due to this field then $n_s \approx 1$ and the scale dependence of $P_\zeta$ is eliminated. Thus, the PDP in this case would be approximately scale-invariant. Indeed, the latest WMAP observations suggest [7]

$$n_s = 0.963 \pm 0.012,$$ (15)

which agrees with the predictions of inflation with the quantum fluctuations of a light scalar field as the source of the PDP. Note that the observations deviate from exact scale invariance ($n_s = 1$) at 1-$\sigma$. This reveals some dynamics during inflation and agrees with the expectations of realistic inflation models.

3. The curvature perturbation

In general relativity the curvature of spacetime and the energy density of its content are interchangeable quantities, through the Einstein equations, depending on the choice of foliation of spacetime. Therefore, for the curvature perturbations it is useful to define a quantity $\zeta$ which is independent of such foliation (gauge invariant). This can be written as [8]

$$\zeta \equiv -\psi - H \frac{\delta \rho}{\rho},$$ (16)

where the first term on the right-hand-side is the curvature perturbation in uniform density slices of spacetime and the second term is the density perturbation in flat slices of spacetime. We call $\zeta$ the (gauge invariant) curvature perturbation from now on.

One can also define the power spectrum of the curvature perturbation $\langle \zeta^2(\vec{x}) \rangle = \int_0^\infty d(k)P_\zeta(k)$ which is given by the two-point correlator as

$$\langle \zeta(\vec{k})\zeta(\vec{k'}) \rangle = (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{k'}) \frac{(2\pi)^3}{4\pi k^3} P_\zeta(\vec{k}).$$ (17)

The latest observations of WMAP give [7]

$$\sqrt{P_\zeta(k_0)} = (4.94 \pm 0.09) \times 10^{-5},$$ (18)

where $k_0 = 0.002$ Mpc$^{-1}$ is the pivot scale. The corresponding density perturbation (at horizon re-entry) is given by

$$\left(\frac{\delta \rho}{\rho}\right)_H = \frac{2}{5} \zeta_{LS} = (1.98 \pm 0.04) \times 10^{-5},$$ (19)

which, as mentioned, is the observed measurement of the PDP.\(^\dagger\)

Another useful quantity is the so-called bispectrum $B_\zeta$ of the curvature perturbation, defined by the three-point correlator of $\zeta$ as follows

$$\langle \zeta(\vec{k})\zeta(\vec{k'})\zeta(\vec{k''}) \rangle = (2\pi)^3 \delta^{(3)}(\hat{k} + \hat{k'} + \hat{k''}) B_\zeta(\vec{k}, \vec{k'}, \vec{k''}).$$ (20)

The bispectrum can be related with the power spectrum in the following manner

$$B_\zeta(\vec{k}_1, \vec{k}_2, \vec{k}_3) = -\frac{6}{5} f_{NL}[P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_1)],$$ (21)

\(^\dagger\) The subscript ‘LS’ refers to the last-scattering surface, where the CMB is emitted.
where \( P_2(k) \equiv \frac{m^2}{m^2} \mathcal{P}_\zeta(k) \).

The bispectrum is useful because it is exactly zero if the curvature perturbation is Gaussian, i.e. if it obeys Gaussian statistics. One expects that the perturbations of the fields which are generated during inflation from their quantum fluctuations are indeed Gaussian since the quantum fluctuations are random. Any non-Gaussian signal therefore would arise by the process which translates the perturbations of such fields, e.g. \( \delta \phi \), into the curvature perturbation \( \zeta \). This process may be highly non-linear in which case \( \zeta \) would feature non-Gaussian statistics. The level of such non-Gaussianity is quantified in the bispectrum of \( \zeta \) by the so-called non-linearity parameter \( f_{\text{NL}} \). The latter also depends on the configuration of the three \( \vec{k} \)-vectors used to determine the bispectrum.

The two most popular configurations used to determine \( f_{\text{NL}} \) are 1) the equilateral configuration, where \( k_1 = k_2 = k_3 \) and 2) the squeezed configuration, where \( k_1 = k_2 \gg k_3 \). The WMAP findings for the values of \( f_{\text{NL}} \) in these configurations are [7]

\[
\begin{align*}
 f_{\text{NL}}^{\text{eq}} & = 26 \pm 140 \\
 f_{\text{NL}}^{\text{sq}} & = 32 \pm 21.
\end{align*}
\]  

Note that there is a hint (at 1-\( \sigma \)) of non-Gaussianity in the squeezed configuration. It is likely that non-Gaussianity may be detected in the near future by the observations of the Planck satellite, which are expected to improve the precision of the measurement of \( f_{\text{NL}} \) by about an order of magnitude. It is important to stress here that the PDP is highly Gaussian. Indeed, remembering that \( \mathcal{P}_\zeta(k_0) \sim 10^{-9} \) and also \( B_\zeta \propto f_{\text{NL}} \mathcal{P}_\zeta^2 \) we see that, even if the observational upper bounds on \( f_{\text{NL}} \) are saturated, non-Gaussianity in the PDP is tiny. This also agrees with the conjecture that the PDP is due to perturbations of suitable fields (e.g. light scalar fields) arising from their quantum fluctuations during a period of inflation.

4. The inflationary paradigm

Before discussing the mechanisms through which the quantum fluctuations of suitable fields can generate the curvature perturbation from inflation, it is necessary to briefly present the so-called inflationary paradigm. This is the typical manner in which inflation is modelled in particle cosmology. According to the inflationary paradigm the Universe undergoes inflation when dominated by the potential density of a scalar field, which is called the inflaton field.

We return in the Lagrangian of Eq. (2) but this time instead of the mass term we consider a generic function \( V(\phi) \), which corresponds to the potential density of the scalar field. In this case, Eq. (3) has the form

\[
\ddot{\phi} + 3H \dot{\phi} + V'(\phi) = 0,
\]  

where the prime denotes derivative with respect to the scalar field \( \phi \). The above equation is similar to the equation of motion of a body sliding down the potential \( V \) and subject to a friction term determined by the rate of the Universe expansion \( H \). Based on this analogy we will consider the field as rolling down the potential in field space.

Now, according to the inflationary paradigm, for inflation to take place the Universe must be dominated by the potential density of the inflation field, i.e. the kinetic density \( \rho_{\text{kin}}(\phi) \) of the field needs to be subdominant:

\[
V(\phi) \gg \rho_{\text{kin}}(\phi) \equiv \frac{1}{2} \dot{\phi}^2.
\]  

This means that the field hardly moves and, therefore, its potential density remains roughly constant, providing thereby the effective cosmological constant \( \Lambda_{\text{eff}} \equiv V/m^2_P \) needed for inflationary expansion. When the condition \( \rho_{\text{kin}} \ll V \) is valid it is said that the field undergoes slow-roll and its variation (motion in field space) is overdamped by the friction term in Eq. (23). In this case one can ignore the acceleration term in Eq. (23) which is then recast as

\[
3H \dot{\phi} \simeq -V'(\phi).
\]
Figure 2. Schematic representation of the inflaton potential \( V(\phi) \). During inflation the inflaton field \( \phi \) (represented by a circle) rolls down its flat potential in field space until it reaches a critical value \( \phi_{\text{end}} \) (corresponding to \( V_{\text{end}} = V(\phi_{\text{end}}) \)) when slow-roll is terminated and inflation ends. Afterwards the field undergoes rapid oscillations around its VEV, corresponding to the minimum of the potential. The field decays while oscillating reheating thereby the Universe.

Because the roll of the field is overdamped, the value of \( \dot{\phi} \) is very small, which means that the slope of the potential needs to be very small as well, according to the slow-roll equation above. Hence, the inflaton field corresponds to a flat direction in field space.

Inflation ends when the slow-roll condition is violated, i.e. at some critical value \( \phi_{\text{end}} \) of the inflaton field, the potential becomes steep and curved such that \( \rho_{\text{kin}} \sim V \). Afterwards, the inflaton condensate oscillates around its vacuum expectation value (VEV). Such coherent oscillations correspond to massive particles (inflatons)\(^5\), which eventually decay into the standard model particles that comprise the thermal bath of the Hot big Bang (see Fig. 2). This process is called Reheating and, if it occurs in a perturbative manner, it is usually modelled by adding a phenomenological decay term in the field equation

\[
\ddot{\phi} + 3H\dot{\phi} + \Gamma \phi + V'(\phi) = 0,
\]

where \( \Gamma \) stands for the decay rate of the inflaton field. Reheating occurs when the density of inflation is transferred to the newly formed thermal bath, whose temperature is \( T_{\text{rh}} \sim \sqrt{\Gamma m_P} \), called the reheating temperature.

5. Mechanisms for the formation of the curvature perturbation

We are ready now to discuss some ways that the perturbations of a light scalar field can give rise to the curvature perturbation in the Universe, which corresponds to the PDP that is responsible for structure formation.

5.1. The inflaton hypothesis

The traditional mechanism through which the curvature perturbation is generated employs the inflaton field itself for the job. This, so-called inflaton hypothesis, simply assumes that the field

\(^5\) with zero momentum since the field is homogeneous
Figure 3. Schematic representation of the way that the curvature perturbation is generated in the inflaton hypothesis. The inflaton field perturbations vary its expectation value typically by $\delta \phi = H/2\pi$. This means that, while the inflaton field (represented by a circle) rolls down its potential in field space, its value is different throughout coordinate space, represented by the $x$-labelled axis. Thus, in some locations the rolling inflaton is ahead compared to others, which means that, in these locations, it will reach the critical value $\phi_{\text{end}}$ that terminates inflation, somewhat earlier. At a given time the inflaton value is represented by the wiggly line which crosses the $\phi = \phi_{\text{end}}$ line in several places. Therefore, the Universe inflates more in some places than in others, because, at the given time that corresponds to the wiggly line shown, in some locations inflation has ended while in others it still continues.

responsible for the formation of the curvature perturbation is the same field which determines the dynamics of inflation, i.e. the inflaton field.

As mentioned in the previous section, to undergo slow-roll the inflaton must correspond to a flat direction in field space and therefore is characterised by $V'' \ll H^2$, i.e. it is a light scalar field. This means that, through the particle production process, it obtains a scale-invariant superhorizon spectrum of perturbations. As a result, the perturbed inflaton field will reach the critical value $\phi_{\text{end}}$ which terminates inflation at slightly different times at different points in space. Thus, inflation will continue a little bit more in some locations that in others (see Fig. 3). This is the cause of the generation of the curvature perturbation $\zeta$. Indeed, it can be shown that the latter is determined by the difference in logarithmic expansion between the uniform density and the spatially flat slices, i.e. $\zeta = \delta (\ln a)$ [9].

The PDP in this case is given by

$$\left. \frac{\delta \rho}{\rho} \right|_H = \frac{H^2}{5\pi \phi} \bigg|_* ,$$

where the asterisk denotes the epoch of horizon exit of the inflaton perturbations. Since in the inflaton hypothesis a single degree of freedom (the inflaton) determines both the dynamics of inflation and the PDP, the latter can be written in terms only of the characteristics of the inflaton potential. Indeed, Eq. (27) can be recast as [10]

$$\left. \frac{\delta \rho}{\rho} \right|_H = \frac{1}{5\sqrt{3}\pi} \left. \frac{V^{3/2}}{m_p^2 |V'|} \right|_* ,$$

(28)
which again is to be evaluated at horizon exit. Observational constraints on the amplitude of the PDP (cf. Eq. (19)) suggest that, if $\varepsilon$ is not extremely small, the energy scale of inflation is comparable to that of grand unification, i.e., $V^{1/4} \sim 10^{15-16}$ GeV. This seems a natural scale to introduce new physics but it turns out that, for single field models, it requires fine-tuning of model parameters.

The spectral index in the inflaton hypothesis is [10]

$$n_s = 1 + 2\eta_\phi - 6\varepsilon,$$

where $\eta_\phi \equiv m_\phi^2 V''/V$ corresponds to the curvature of the potential along the direction of the inflaton field. If $V(\phi) = \frac{1}{2}m_\phi^2 \phi^2$ as in Eq. (2), then $\eta_\phi = \frac{1}{4}(\frac{m_\phi^4}{H})^2$ as discussed after Eq. (10).\(^6\)

Under the inflaton hypothesis the generated non-Gaussianity is expected to be negligible with $f_{NL} \ll 1$, i.e. below the limit of observability. This means that if non-Gaussianity is indeed observed in the PDP then all the single field inflation models are going to be falsified.

The beauty and the curse of the inflaton hypothesis is that it relies on a single degree of freedom, namely the inflaton field, to account for all the problems that inflation aims to address, namely provide the period of accelerated expansion that deals with the horizon and flatness problems and also generate the appropriate spectrum of curvature perturbations, which agrees with observational constraints on its amplitude and spectral index. As a result, models of single field inflation are typically overconstrained and suffer from substantial fine-tuning. This is why alternative hypotheses have been put forward for the generation of the PDP from inflation.

5.2. The curvaton hypothesis

This hypothesis assumes that the field responsible for the formation of the curvature perturbation has nothing to do with the dynamics of inflation, i.e., it is other than the inflaton field. This scalar field is called curvaton $\sigma$ [11] (see also Refs. [12][13][14][15]). In order for the curvaton to play this role it needs to obtain a superhorizon spectrum of perturbations during inflation. Thus, it needs to be a light field during inflation so that it can undergo particle production.

It is evident that by introducing another degree of freedom, the fine-tuning problems of inflation model-building are substantially improved [16][17]. However, it must be stressed that the curvaton is not necessarily a new ad hoc addition to the theory, which is introduced by hand. Indeed, the fact that the curvaton is not connected to the inflaton sector allows it to correspond to physics at energy scales much smaller than that of inflation (e.g. the TeV scale which is accessible by LHC). Consequently, there exist a number of candidates for the curvaton which correspond to realistic fields already present in simple extensions of the Standard Model. Prominent examples are a right-handed neutrino [18], an MSSM flat direction [19][20] or the so-called orthogonal axion in supersymmetric realisations of the Pece-Quinn symmetry [21][22].

Under the curvaton hypothesis the curvature perturbation is given by [23]

$$\zeta = \hat{\Omega}_\sigma \zeta_\sigma,$$

where $\hat{\Omega}_\sigma \equiv H\frac{3\Omega_\sigma}{\rho_\sigma} \simeq \Omega_\sigma$, where $\Omega_\sigma \equiv (\rho_\sigma/\rho_{dec})$ is the density parameter of the curvaton field at the time of its decay after inflation\(^7\). In the above $\zeta_\sigma$ is the curvature perturbation attributed to the curvaton field, which is determined by the fractional perturbation of the field itself:\(^8\)

$$\zeta_\sigma \equiv -H \frac{\delta \rho_\sigma}{\rho_\sigma} = \frac{1}{3} \frac{\delta \rho_\sigma}{\rho_\sigma} = \frac{2}{3} \frac{\delta \sigma}{\sigma} \simeq \frac{2}{3} \frac{\delta \sigma}{\sigma} |_{s = \frac{H_s}{3\pi\sigma}}.$$

\(^6\) we considered also the Friedmann equation with $\rho \simeq V$ since we have potential domination during inflation.

\(^7\) $\hat{\Omega}_\sigma$ is also denoted as $r$ in much of the literature.

\(^8\) in spatially flat hypersurfaces
Figure 4. Log-log plot of the evolution of the inflaton energy density which decays into radiation \( \rho_\gamma \) (purple line) at the end of inflation (denoted by ‘end’) and the curvaton energy density \( \rho_\sigma \) (green line) (prompt reheating is assumed). During inflation, the curvaton density is negligible. After inflation \( \rho_\gamma \propto a^{-4} \). In contrast, \( \rho_\sigma \) remains constant (the curvaton is frozen at some value \( \sigma_* \)) until \( m \sim H(t) \), when the curvaton unfreezes and begins oscillating, after which time (denoted ‘osc’) \( \rho_\sigma \propto a^{-3} \). At some moment (denoted ‘dom’) the curvaton density dominates the Universe (\( \bar{\Omega}_\sigma = \bar{\Omega}_\gamma = 1 \)) until, some time later (denoted ‘dec’) when it decays into the thermal bath of the Hot Big Bang. The dashed slanted line depicts the possibility that the curvaton decays before domination (\( \bar{\Omega}_\sigma = \bar{\Omega}_\gamma = 1 \)), when substantial non-Gaussianity can be generated.

where we used that \( \delta \sigma_* = H_*/2\pi \) and we assumed that near its decay the curvaton density is \( \rho_\sigma \approx m_\sigma^2 \sigma^2 \), where \( m_\sigma \ll H_* \) is the mass of the curvaton field. The spectral index of the PDP in this case is [23]

\[
\eta_s = 1 + 2\eta_\sigma - 2\varepsilon,
\]

where \( \eta_\sigma \) corresponds to the curvature of the potential along the curvaton direction.

Since by definition the curvaton should not affect the dynamics of inflation, during inflation we expect \( \rho_\sigma \ll \rho \). Thus the density parameter of the curvaton is extremely small and its contribution to the overall curvature perturbation (cf. Eq. (30)) is also small. For the curvaton to significantly contribute to \( \zeta \) we need to consider the period after inflation, when its contribution to the density can increase.

In the simplest case when \( V(\sigma) \approx \frac{1}{2} m_\sigma^2 \sigma^2 \) the equation of motion for the curvaton is of the same form as Eq. (3). Since during inflation the field is light and \( m_\sigma \ll H \), the curvaton is overdamped and remains frozen in some value \( \sigma_* \). After inflation, however, the Hubble parameter decreases in time \( H(t) \propto 1/t \). As a result, there will be a moment when \( m_\sigma \sim H(t) \), at which time the curvaton condensate will unfreeze and will begin coherent oscillations around its VEV. These oscillations correspond to massive particles (curvatons) whose density is diluted as \( \rho_\sigma \propto a^{-3} \), which is less drastic that the density of the radiation background \( \rho_\gamma \propto a^{-4} \). Thus, the oscillating curvaton has a chance to dominate (or nearly dominate) the Universe before its decay.

\[ \text{This is the radiation that was generated by the decay of the inflaton field after the end of inflation.} \]
(see Fig. 4). Thus, at curvaton decay $\hat{\Omega}_\sigma$ can be as large as unity and the curvaton contribution to $\zeta$ can be substantial. Consequently, the curvaton imposes its curvature perturbation onto the Universe at (or near) its domination.

Because of its spectrum of perturbations $\delta \sigma$ the amplitude of the curvaton oscillations is also perturbed, i.e. it is larger in some places than in others. This means that the density of the oscillating field is perturbed too and the field will dominate the Universe at different times at different locations. This is what generates the curvature perturbation (cf. Eq. (31)).

In contrast to the inflaton hypothesis, under the curvaton hypothesis non-Gaussianity can be substantial. Indeed, in this case [23]

$$f_{NL} = \frac{5}{4\Omega_\sigma},$$

which can be large if the decay of the curvaton happens before domination when $\Omega_\sigma \ll 1$. In fact, the WMAP observations in Eq. (22) set the lower bound $\Omega_\sigma \gtrsim 0.01$.

5.3. Other mechanisms

There are numerous other ways to generate the PDP from a superhorizon spectrum of scalar field perturbations. This section briefly reviews two of them, namely the inhomogeneous reheating and the end of inflation mechanisms. As with the curvaton mechanism, these mechanisms assume that the contribution to the curvature perturbation from the inflaton field itself is negligible.

5.3.1. Inhomogeneous Reheating

This mechanism generates the PDP by assuming that the inflaton decay rate $\Gamma$ is modulated by another scalar field $\sigma$ [24][25]. This scalar field is the one which undergoes particle production during inflation and which obtains thereby a superhorizon spectrum of perturbations.

In this scenario we can ignore the perturbations of the inflaton field. Thus, we can consider that inflation is terminated at the same time throughout space\textsuperscript{10}. The inflaton then begins its coherent oscillations around its VEV until it decays when $\Gamma \simeq H(t)$. However, since in this case $\Gamma = \Gamma(\sigma)$ and $\sigma$ is perturbed in space, the decay rate is different at different locations so that the Hot Big Bang begins at different times. This is what generates the curvature perturbation which corresponds to the PDP. Roughly speaking we have

$$\frac{\delta \rho}{\rho} \sim \frac{\delta \Gamma}{\Gamma} \sim \frac{\delta \sigma}{\sigma}.$$ (34)

In this scenario the spectral index is again given by Eq. (32). Non-Gaussianity in this model is quantified as [26]

$$f_{NL} = 5 \left( \frac{\Gamma''}{\Gamma^2} - 1 \right),$$ (35)

where now the prime denotes derivative with respect to $\sigma$. If $\Gamma$ has a power-law dependence on $\sigma$ then $f_{NL} = O(1)$, which is marginally observable.

5.3.2. End of inflation

This mechanism applies to a particular type of inflation model, the so-called hybrid inflation. Therefore, before discussing the mechanism a brief summary of hybrid inflation is necessary.

Hybrid inflation couples the inflaton field $\phi$ to another scalar field $\psi$, called the waterfall field, in such a way that inflation is terminated by a phase transition which sends the waterfall to its VEV [27]. Since inflation is likely to be at the scale of grand unification, in many cases

\textsuperscript{10} at least in the observable Universe.
the waterfall field is assumed to be the Higgs field of a Grand Unified Theory (GUT). Thus, inflation is terminated by the breaking of grand unification, i.e. the GUT phase transition.

The scalar potential for hybrid inflation has the following form

$$V(\phi, \psi) = \frac{1}{4} \lambda (\psi^2 - M^2)^2 + \frac{1}{2} g \phi^2 \psi^2 + V_\phi(\phi)$$  \hspace{1cm} (36)$$

where $M$ is the GUT energy scale, $\lambda$ is the self-coupling of the waterfall field and $g$ is the interaction coupling between the waterfall and the inflaton. The potential $V_\phi(\phi)$ is responsible for the slow-roll of the inflaton and its precise form is not relevant to this discussion\textsuperscript{11}. From the above potential it is evident that the effective mass-squared of the waterfall is

$$m_\psi^2 = g \phi^2 - \lambda M^2 + \lambda \phi^2.$$  \hspace{1cm} (37)$$

The above implies that there is a critical value of the inflaton

$$\phi_c = \sqrt{\frac{\lambda M}{g}}$$ \hspace{1cm} (38)$$

such that if $\phi \gg \phi_c$ then $m_\psi^2 > 0$ and $\psi \to 0$. In this case, Eq. (36) becomes $V = \frac{1}{4} \lambda M^4 + V_\phi$. The constant contribution to the scalar potential provides the effective cosmological constant for (quasi)de Sitter inflation: $\Lambda_{\text{eff}} \sim \lambda M^4 / m_\psi^2$. If however, $\phi < \phi_c$ then $m_\psi^2 < 0$ and a phase transition occurs which results in $\psi \to M$, which gives the inflaton a large mass (through the interaction term) and so $\phi \to 0$. Assuming $V_\phi(0) = 0$, after the phase transition $V \to 0$ and inflation ends.

The end of inflation mechanism for the production of the PDP introduces an extra coupling between the waterfall field and another scalar field $\sigma$ [30][31]. The scalar potential now becomes

$$V(\phi, \psi) = \frac{1}{4} \lambda (\psi^2 - M^2)^2 + \frac{1}{2} g \phi^2 \psi^2 + V_\phi(\phi) + \frac{1}{2} h \sigma^2 \psi^2$$  \hspace{1cm} (39)$$

where $h$ parametrises the strength of the interaction between $\sigma$ and the waterfall field $\psi$. With this addition the effective mass-squared of the waterfall becomes

$$m_\psi^2 = g \phi^2 - \lambda M^2 + \lambda \phi^2 + h \sigma^2$$  \hspace{1cm} (40)$$

and the critical value which triggers the phase transition that ends inflation is now

$$\phi_c(\sigma) = \left( \frac{\lambda M^2 - \frac{h}{g} \sigma^2}{g} \right)^{1/2},$$ \hspace{1cm} (41)$$

i.e. it is modulated by the value of $\sigma$. Therefore, if $\sigma$ is light during inflation, then it undergoes particle production and obtains a superhorizon spectrum of perturbations with typical magnitude $\delta \sigma = H/2\pi$. This means that the value of $\phi_c$ is also perturbed. Hence, the phase transition which terminates inflation occurs earlier in some parts of the Universe than in others depending on when the inflaton reaches the critical value $\phi_c$. Consequently, the Universe inflates more in some locations than in others and this generates the curvature perturbation. As in the curvaton scenario, the spectral index in this case is given by Eq. (32) [32].

\textsuperscript{11}In supersymmetric versions of hybrid inflation the slow-roll potential is provided by radiative corrections and it is of the form $V_\phi \propto \ln \phi$ [28][29].
6. Cosmic vector fields and the curvature perturbation

Tantalising evidence exists of a preferred direction in the CMB temperature perturbations. In particular, the low multipoles in the CMB appear to be aligned [33][34] at a level which is statistically rather improbable [35][36]. A preferred direction in the CMB cannot be accounted for if inflation and the generation of the PDP is due to scalar fields only. Moreover, despite their abundance in theories beyond the Standard Model, scalar fields have not been observed as yet. If the Higgs field is not found in the LHC, the credibility of the ubiquitous usage of scalar fields in cosmology will be shaken.

Until recently only scalar fields have been considered both as responsible for the dynamics of inflation and also for the generation of the observed PDP. However, in the pioneering work of Ref. [37] the possibility that a vector field contributes in the generation of the PDP was first considered. Since then, a number of attempts have been made to investigate the role and the implications of cosmic vector fields in the generation of the PDP (see Ref. [38] and references therein).

Why vector fields were not considered originally for the generation of the PDP? After all, these are fields which are similar to the massive gauge bosons, which have indeed been observed in LEP. The main difficulty had to do with the inherent anisotropic nature of vector fields. Inflation would homogenise a vector field condensate, and a homogeneous vector field picks up a preferred direction in space. Thus, if this vector field condensate were to dominate the density of the Universe (so that it can affect the expansion and generate the PDP) it was felt that the anisotropic stress generated would lead to strongly anisotropic expansion which would be impossible to reconcile with the predominant isotropy of the CMB. However, as is discussed below, this difficulty can be circumvented in the vector field remains subdominant during inflation. Still, if (in analogy to scalar fields) light vector fields were needed for a scale invariant spectrum then a more subtle problem arises. Massless Abelian vector fields are conformally invariant and they cannot undergo particle production during inflation\(^{12}\). Thus, for light vector fields particle production is expected to be suppressed. Nevertheless, there are numerous mechanisms which break the conformality of vector fields so that particle production can be efficient. These mechanisms are model dependent which means that they can have distinct observational signatures as is discussed below.

7. Particle production of vector fields during inflation

Suppose that there is a suitable theory which breaks the vector field conformality during inflation. How are we to study particle production of the vector field? We follow the same recipe as in Sec. 2 assuming that the inflationary expansion remains isotropic.

Firstly we perturb the vector field around its homogenised value \( A_\mu = A_\mu (t) + \delta A_\mu (\vec{x}, t) \). Then we Fourier transform the perturbations \( \delta A_\mu (\vec{k}) = \int \delta A_\mu (\vec{x}) e^{-i\vec{k}\cdot\vec{x}} d^3\vec{x} \) and we obtain the equations of motion for the Fourier components in the theory which we are considering\(^{13}\). Then we promote the perturbations of the vector field into quantum operators

\[
\delta \hat{A}_\mu (\vec{x}, t) = \int \frac{d^3k}{(2\pi)^3} \sum_\lambda \left[ \hat{e}_\lambda^\lambda (\vec{k}) \delta \hat{A}_\lambda (k, t) e^{i\vec{k}\cdot\vec{x}} + \hat{e}_\lambda^\dagger (\vec{k}) \delta \hat{A}_\lambda^\dagger (k, t) e^{-i\vec{k}\cdot\vec{x}} \right],
\]

where \( \delta \hat{A}_\lambda \) are the mode functions and we consider again canonical quantisation \([\hat{a}_\lambda (\vec{k}), \hat{a}_\lambda^\dagger (\vec{k}')] = (2\pi)^3 \delta (\vec{k} - \vec{k}') \delta_{\lambda\lambda'}\). In the above we have introduced the polarisation vectors

\[
\epsilon^L \equiv \frac{1}{\sqrt{2}} (1, i, 0), \quad \epsilon^R \equiv \frac{1}{\sqrt{2}} (1, -i, 0), \quad \epsilon^\parallel \equiv (0, 0, 1),
\]

\(^{12}\) They view the Universe expansion as a conformal transformation to which they are insensitive.

\(^{13}\) We focus on the spatial components of the cosmic vector field, as they are the ones which would generate anisotropy. Also, the temporal component of a homogeneous massive Abelian vector field is zero.
where $L, R$ denote the left and right transverse polarisations and $\parallel$ denotes the longitudinal polarisation\textsuperscript{14}.

As with the scalar field case, the mode functions of the above expansion are expected to satisfy the equations of motion for the Fourier components of the vector field perturbations. The form of these equations depends on the theory which breaks the conformality of the vector field. To solve them we again consider vacuum boundary conditions in the subhorizon limit $(k/aH \rightarrow \infty)$. They are [40]:

\[
\delta A_k^{L,R} \rightarrow \frac{e^{ik/aH}}{\sqrt{2k}} \quad \text{and} \quad \delta A_k^\parallel \rightarrow \gamma \frac{e^{ik/aH}}{\sqrt{2k}}. \quad (44)
\]

We see that the boundary condition is again the Bunch-Davies vacuum but the longitudinal component it is multiplied by the Lorentz boost factor $\gamma \equiv \frac{E}{m} = \sqrt{\left(\frac{k}{aH}\right)^2 + 1}$, which takes us from the frame with $\vec{k} = 0$ where all components are equivalent, to the frame of momentum $\vec{k}$\textsuperscript{15}.

After solving the equations of motion we evaluate the solutions in the superhorizon limit $(k/aH \rightarrow 0)$. Then we can obtain the power spectrum for each polarisation as $P_\chi(k) \equiv \frac{k^3}{2\pi^2} |\delta A_k^\lambda|^2$, where $\delta A_k^\lambda$ is evaluated in the superhorizon limit. Because we have three degrees of freedom there are three possibilities:

Case A: $P_\parallel \neq P_L \neq P_R \neq P_\parallel$
Case B: $P_L = P_R \neq P_\parallel$
Case C: $P_L = P_R = P_\parallel$

Even though Case A appears to be the most generic, in practice it is Case B that is the most common because usually the theories which break the vector field conformality are parity conserving. We have isotropic particle production only in the special Case C.

What would happen if the spectrum of the produced perturbations of the vector field affected the curvature perturbation? Since particle production is in general anisotropic we expect an anisotropic contribution to $\zeta$. This contribution can be parametrised as follows. For the power spectrum we can write [39]

\[
P_\zeta(\vec{k}) = P_\zeta^{\text{iso}}(k) \left[1 + g(\vec{A} \cdot \vec{k})^2 \cdots \right], \quad (45)
\]

where $\vec{A} \equiv \vec{A}/A$ and $\vec{k} \equiv \vec{k}/k$ with $A \equiv |\vec{A}|$ and the ellipsis denotes higher order contributions. The anisotropic part of the spectrum corresponds to a new observable, namely \textit{Statistical Anisotropy}, which amounts to direction dependent patterns in the CMB [40] (see Fig. 5). Statistical anisotropy is quantified by the so-called anisotropy parameter $g$ in Eq. (45). The WMAP observations set a surprisingly weak upper bound on the anisotropy parameter: $g \lesssim 0.3$ [41]\textsuperscript{16}. Thus we see that, at present, observations allow up to 30% statistical anisotropy in the spectrum of the PDP. The forthcoming data of the Planck satellite will improve precision by an order of magnitude and are likely to detect statistical anisotropy. If this will be so then a cosmic vector field will have to be involved in the generation of the PDP during inflation.

In a similar manner one can parametrise the statistically anisotropic contribution from a vector field in the bispectrum. Indeed, we can write [44][45]

\[
f_{NL} = f_{NL}^{10} \left(1 + gA_1^2 \cdots \right), \quad (46)
\]

\textsuperscript{14} The index $\lambda$ in Eq. (42) runs over these values.

\textsuperscript{15} Note that for a massless vector field the longitudinal component is unphysical.

\textsuperscript{16} In Ref. [42] statistical anisotropy with $g = 0.29 \pm 0.03$ is claimed to be detected at a level of 9-$\sigma$ but the authors acknowledge that, because the direction is suspiciously near the ecliptic plane, this is probably due to an unknown systematic error. Hence we treat this number as an upper bound.
Figure 5. Patterns in the CMB temperature perturbations which may arise from statistical anisotropy in the CMB spectrum. The left panel shows an isotropic signal, while the middle and right panels show patterns due to statistical anisotropy along the vertical and horizontal direction respectively. The figure is taken from Ref. [43].

where $f_{NL}^{\text{iso}}$ denotes the isotropic part and $\hat{A}_\perp$ is the projection of $\hat{A}$ onto the plane of the three $k$-vectors which are used to define the bispectrum. Note that, as non-Gaussianity has not been observed yet, there are no bounds on $\mathcal{G}$. If $\mathcal{G} > 1$ this means that non-Gaussianity is predominantly anisotropic (even though $\mathcal{P}_\zeta$ is not). If non-Gaussianity is indeed observed (e.g. by the Planck satellite) and no angular modulation of $f_{NL}$ is found on the microwave sky then all models which predict $\mathcal{G} > 1$ will be falsified. It is important also to stress that the directions of statistical anisotropy in the spectrum and the bispectrum are correlated (determined by the direction of $\hat{A}$), which is a smoking gun for the contribution of a vector field to the PDP [44].

In the Cases A and B the spectra of the superhorizon perturbations of the vector field are predominantly anisotropic. Since $\mathcal{P}_\zeta$ is isotropic at least at the level of 70% the contribution of the vector field to the PDP in these two cases has to be subdominant and its significance is only the possibly observable statistical anisotropy in the spectrum and bispectrum. Thus, in the Cases A and B we still require some other, isotropic source (such as a scalar field) to provide the dominant contribution to the PDP. On the other hand, in Case C the perturbation spectra of the vector field are isotropic. In this case, the vector field alone can generate $\zeta$ and no input from any scalar field is necessary. Note that, in this case, the vector field may also produce statistical anisotropy if the $\mathcal{P}_\lambda$ are not exactly the same but differ slightly, albeit by no more that 30%.

8. Models for particle production of cosmic vector fields

In this section we discuss a couple of proposals for the generation of a flat superhorizon spectrum of vector field perturbations.

8.1. Non-minimal coupling to gravity

Consider the following theory

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (m^2 + \alpha R) A_\mu A^\mu,$$

(47)

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of the Abelian vector field $A_\mu$, $m$ is its bare mass, $\alpha$ is constant and $R$ is the Ricci scalar. From the above we see that the vector field has effective
mass-squared $m^2_a = m^2 + \alpha R$.

Following the procedure outlined in the previous section, we obtain the following solution for the mode functions of the transverse perturbations of the physical vector field $[46]^{17}$

$$\delta A_k^{L,R} = a^{-3/2} \sqrt{\frac{\pi}{H} e^{i\frac{\nu}{2} \frac{k}{aH}} [J_\nu(k/aH) - e^{i\nu} J_{-\nu}(k/aH)],}$$ (48)

The above is identical to Eq. (7) with the crucial difference that $\nu \equiv \sqrt{\frac{1}{4} - \left(\frac{m}{H}\right)^2}$. Using this solution we obtain the following expression for the power spectrum of the transverse perturbations in the superhorizon limit

$$\mathcal{P}_{L,R} = \frac{8\pi |\Gamma(1 - \nu)|^2}{1 - \cos(2\pi \nu)} \left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{2\alpha H}\right)^{3-2\nu},$$ (49)

which again is identical to Eq. (9) but for the different value of $\nu$. From the above it is evident that a scale invariant spectrum is attained for $\nu \approx \frac{3}{2}$. This translates into a requirement for both $\alpha$ and $m$ as follows.

In a spatially flat, homogeneous and isotropic spacetime, the scalar curvature is $R = 3(3w - 1)H^2$, where $w$ is the barotropic parameter of the content of spacetime. During inflation, $w \approx -1$ so that $R \approx -12H^2$. This means that we can attain a scale invariant transverse spectrum if 1) $\alpha = \frac{1}{R}$ and 2) $m \ll H$, i.e. the vector field is a light field with a negative effective mass-squared $[46]^{18}$. Under these conditions the transverse spectra are indeed scale-invariant and are given by $\mathcal{P}_{R,L} = \left(\frac{H}{2\pi}\right)^2$, exactly as a massless scalar field.

Employing the same conditions for $\alpha$ and $m$ one obtains the following solution for the longitudinal component of the physical vector field $[40]$:  

$$\delta A_k^\parallel = \frac{1}{\sqrt{2}} \left[ \left(\frac{k}{aH}\right)^2 - 2 \frac{aH}{k} + 2i \right] \frac{\delta k/aH}{\sqrt{2k}}.$$ (50)

Clearly the above is totally different from Eq. (48). In the superhorizon limit the power spectrum of the longitudinal component is $\mathcal{P}^\parallel = 2(\frac{H}{2\pi})^2 = 2\mathcal{P}_{R,L}$. This means that we are in Case B since $\mathcal{P}^\parallel > \mathcal{P}_{L,R}$ and $\mathcal{P}_L = \mathcal{P}_R$ because the theory is parity invariant. Thus, particle production in this theory is anisotropic at a level of 100%. Therefore, the vector field cannot alone be responsible for the PPD. Its contribution to $\zeta$ has to be subdominant and it can only be the source of statistical anisotropy. Before concluding, we should point out that this model has been criticised for suffering from instabilities (ghosts) $[47][48]$ (see however Ref. [49]).

8.2. Varying kinetic function and mass

Now, consider the theory

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu,$$ (51)

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $f = f(t)$ is the kinetic function and we also consider $m = m(t) > 0$ during inflation. The Maxwell-type kinetic term in combination with the positive mass-squared guarantees the stability of the model $[50]$ and therefore is motivated even if the vector field is

$^{17}$In a flat homogeneous and isotropic Universe the spatial components of the physical (in contrast to comoving) vector field are $A_\mu / a$.

$^{18}$The non-minimal coupling to gravity $\alpha = \frac{1}{R}$ has a special property. It turns the conformally invariant massless Abelian vector field equivalent (in the sense that its field equations are the same) to a (set of two) minimally coupled massless scalar field(s). If applied to a scalar field it renders it conformally invariant. In this respect it appears to reflect a deeper symmetry and therefore it may be a natural value for $\alpha$.  

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not a gauge boson. Note also that a massive Abelian vector field is renormalisable even if it is not a gauge field [51].

The solutions for the mode functions of the field perturbations are too complicated to reproduce here. It suffices to say that scale invariance in the transverse components requires the kinetic function to scale with the expansion as $f \propto a^{-1/3}$ and also the physical vector field to be light at horizon exit $M \ll H$, where $M \equiv m/\sqrt{f}$. Scale invariance for the longitudinal component additionally requires $m \propto a^{-1/3}$ [38][52].

If the vector field is a gauge boson then $f$ is the gauge kinetic function, which is related to the gauge coupling as $f \sim 1/e^2$. This means that only the case when $f \propto a^{-4}$ is acceptable because only then the gauge field remains weakly coupled during inflation. Note that the model is naturally realisable in supergravity theories where $f$ is a holomorphic function of the scalar fields of the theory $f(\phi_i)$ [53]. In general, Kähler corrections to the scalar potential result in masses of order $H$ for the scalar fields$^{19}$, which are therefore expected to be fast-rolling down the potential slopes during inflation, causing significant variation to $f$. Indeed, for a power-law dependence of the gauge kinetic function to the scalar fields, it is easy to show that $\dot{f}/f \sim H$ [38], i.e. $f$ has a power-law dependence on $a$ as assumed in this model. Now, if $f$ is modulated by the inflaton field itself, then it can be shown that, under fairly general conditions, the backreaction to the inflaton’s roll renders the scaling $f \propto a^{-4}$ an attractor solution [54]. Similarly, for a gauge boson, $m$ can be modulated by a fast-rolling Higgs field with tachyonic mass $m_H = 2H$ [38].

Given the conditions $f \propto a^{-1/3}$ and $m \propto a$ the power spectra for the transverse and longitudinal components depend on whether the vector field remains light until the end of inflation or not (note that $M = M(t)$). In particular we find [38][52]

$$M \ll H : \quad \mathcal{P}_{L,R} = \left( \frac{H}{2\pi} \right)^2 \quad \text{and} \quad \mathcal{P}_\parallel = \left( \frac{H}{2\pi} \right)^2 \left( \frac{3H}{M} \right)^2$$

$$M \gg H : \quad \mathcal{P}_{L,R} = \mathcal{P}_\parallel = \frac{1}{2} \left( \frac{H}{2\pi} \right)^2 \left( \frac{3H}{M} \right)^2 .$$

From the above we see that, if the vector field remains light until the end of inflation, $\mathcal{P}_\parallel \gg \mathcal{P}_{L,R}$, i.e. we are in Case B. Therefore, particle production is strongly anisotropic and the vector field contribution to the PDP has to be subdominant but it can still give rise to substantial statistical anisotropy. In contrast, if the field becomes heavy by the end of inflation then particle production is isotropic and the vector field can be solely responsible for the generation of the PDP. Note that, because we need the field to be light when the cosmological scales exit the horizon, for it to become heavy we need $M > 0$ during inflation, which is possible only in the case when $f \propto a^{-4}$. If $f \propto a^2$ then $M = \text{constant}$ and we have $M \ll H$ throughout inflation. But if $f \propto a^{-4}$ then we have $M \propto a^3$ so that we may end up having $M \gg H$ at the end of inflation even though we started of with a light field at horizon exit. In the latter case the vector field begins coherent oscillations a few exponential expansions (e-folds) before the end of inflation. Assuming that at the end of inflation $f \rightarrow 1$ and the field becomes canonically normalised, we find that there is ample parameter space for Case C to be realised, namely: $1 < m/H < 10^6$ [38][52].

9. The vector curvaton paradigm

Through the examples in the previous section it is evident that a scale invariant spectrum of perturbations (isotropic or not) of a vector field can indeed be generated if one assumes some theory which appropriately breaks the conformality of the vector field. However, in order for these perturbations to affect or even generate the PDP the vector field needs to affect the Universe expansion, i.e. its density should become dominant (or nearly dominant) at some point.

$^{19}$This is the source of the famous $\eta$-problem of inflation, since slow-roll requires $|\eta| = \frac{1}{3}(\frac{\dot{f}}{f})^2 \ll 1$ for the inflaton.
But, even if particle production is isotropic the homogeneous zero-mode condensate is not. How can we avoid excessive anisotropc stress when the vector field dominates the expansion? It turns out that this is possible if the vector field plays the role of the curvaton.

Consider a minimally coupled massive Abelian vector field for which
\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu, \] (54)
where \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) and \( m = \text{constant} > 0 \). It is clear that both models in the previous section eventually approach the above theory, when \( m^2 \gg R \sim H^2 \) and also after the end of inflation when \( f \to 1 \) and \( m = \text{constant} \). The vector field condensate is homogenised by inflation so that \( A_\mu = \hat{A}_\mu(t) \). In this case it can be shown that the temporal component is zero and the spatial components satisfy the following equation [37]
\[ \hat{A}_i + H \hat{A}_i + m^2 \hat{A}_i = 0, \] (55)
which is similar to Eq. (3) for the scalar field. The energy-momentum tensor of this theory can be written in the form [37]
\[ T^\mu_\nu = \text{diag}(\rho_A, -p_\perp, -p_\perp, +p_\perp), \] (56)
where
\[ \rho_A = \rho_{\text{kin}} + V \quad \rho_{\text{kin}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \]
\[ p_\perp = \rho_{\text{kin}} - V \quad V = -\frac{1}{2} m^2 A_\mu A^\mu. \] (57)

Eq. (56) is reminiscent of a perfect fluid with the crucial difference that the pressure in the longitudinal direction is of opposite sign compared to the transverse pressure. Thus, it seems that if the homogeneous vector field were to dominate the Universe it would indeed generate excessive anisotropic stress. Therefore, we assume that the vector field remains subdominant during inflation so that (quasi)de Sitter expansion is not spoilt.

After inflation the Hubble parameter decreases until \( m > H(t) \). When this happens one can ignore the friction term in Eq. (55), which therefore suggests that the vector field condensate starts rapid (quasi)harmonic oscilations. It is easy to show that, during a Hubble time, on average \( \rho_{\text{kin}} \approx 0 \), which means that, over many oscillations, the average pressure is zero: \( \bar{p}_\perp = 0 \) [37]. Thus the oscillating vector field condensate behaves as pressureless, isotropic matter. Its density, therefore, scales as \( \rho_A \propto a^{-3} \) which not as drastic as the radiation background, which scales as \( \rho_\gamma \propto a^{-4} \). Therefore, the oscillating vector field can gradually increase its density parameter and come to dominate (or nearly dominate) before its decay (see Fig. 6). Because it is isotropic, it dominates without causing any excessive anisotropic stress, so that the Universe expansion remains isotropic [37].

From the above we see that a massive Abelian vector field can follow the curvaton scenario and play the role of the curvaton without problem. Indeed, the perturbations of the vector field imply a perturbation in its local density \( \rho_A(\vec{x}) \), which means that in some locations it dominates the Universe earlier than in others. This is how it can generate the curvature perturbation according to the curvaton mechanism. It is important to note that, since \( \rho_A \) is a scalar quantity the curvature perturbation generated is scalar and not vector in nature. Also, note that the perturbations of the vector field \( \delta A_i \) follow a similar equation to Eq. (55), which means that they too are rapidly oscillating and do not introduce anisotropic stress. Thus, at domination the perturbations do not cause anisotropic expansion either, not even in a small scale. If particle
Figure 6. Log-log plot of the evolution of the inflaton energy density which decays into radiation \( \rho_r \) (purple line) at the end of inflation (denoted by ‘end’) and the vector curvaton energy density \( \rho_A \) (green line) (prompt reheating is assumed). During inflation, the vector curvaton density is negligible (its evolution depends on the model which breaks its conformality). After inflation \( \rho_r \propto a^{-4} \). Similarly, \( \rho_A \) decreases also as radiation after inflation when the vector field is light (in contrast to the scalar curvaton case where \( \rho_\theta \) remains constant when the curvaton is light, cf. Fig. 4). However, when \( m \sim H(t) \), the vector curvaton becomes heavy and begins oscillating, after which time (denoted ‘osc’) \( \rho_A \propto a^{-3} \). At some moment (denoted ‘dom’) the vector curvaton density dominates the Universe until some time later (denoted ‘dec’) when it decays into the thermal bath of the Hot Big Bang. As the vector curvaton is rapidly oscillating it does not give rise to anisotropic stress at domination. The dashed slanted line depicts the possibility that the vector curvaton decays before domination \( (\Omega_A \ll 1) \), when substantial non-Gaussianity can be generated.

production of the vector field is anisotropic then there are direction dependent patterns in the amplitude of the oscillating zero mode that lead to statistical anisotropy in the PDP [40].

In Ref. [44] statistical anisotropy in the bispectrum in the vector curvaton model was investigated. It was found that it manifests itself only quadratically, i.e. the expansion in Eq. (46) is truncated to \( f_{NL} = f_{NL}^{\Theta} \left( 1 + G \bar{A}^2 \right) \). \( G \) was evaluated in the two models discussed in the previous section. In the non-minimally coupled to gravity model \( G^{eq} = \frac{m}{\Lambda} \) in the equilateral configuration, whereas \( G^{eq} = 1 \) in the squeezed configuration. This means that the angular modulation of \( f_{NL} \) is prominent and should be detected if non-Gaussianity is found. In the varying kinetic function and mass model \( G^{eq} = \frac{1}{f}(\frac{3H}{M})^2 \gg G^{eq} = (\frac{3H}{M})^2 \gg 1 \) when \( M \ll H \), i.e. if the field remains light until the end of inflation. Here we see that non-Gaussianity is predominantly anisotropic. Should no angular modulation of \( f_{NL} \) be observed this possibility will be excluded. However, if \( M \gg H \), i.e. if the field becomes massive by the end of inflation, particle production is isotropic (Case C) and \( f_{NL} = f_{NL}^{\Theta} \) \( = \frac{5}{4\Omega_A} \), which is identical to the scalar curvaton case (cf. Eq. (33)).

\( \hat{\Omega}_A \) is defined in the same way as \( \hat{\Omega}_s \).
The vector curvaton is an elegant mechanism for vector fields to contribute of even generate the PDP since it does not need to couple the fields to the inflaton sector. However, it is by no means the only way that a vector field can affect the curvature perturbation. For example, in Ref. [55], the end of inflation mechanism is employed (see Sec. 5.3.2), where, instead of coupling the waterfall field $\phi$ of hybrid inflation to a scalar field $\sigma$, the authors considered introducing a coupling of the form $\Delta V = \frac{1}{2} h A_{\mu} A^\mu \phi^2$. This model too produces distinct observational signatures. For example, in this model $f_{NL} = f_{NL}^{2c}(1 + g A^2_{\perp} + g' A^4_{\perp})$, where the maximum value for $g'$ in the squeezed configuration is independent from the model parameters: $g'_\text{max} = \frac{1}{4}$. 

10. Conclusions
Cosmic structure originates from the growth of quantum fluctuations during a period of cosmic inflation in the Early Universe. The particle production process generates an almost scale invariant spectrum of superhorizon perturbations of suitable fields, e.g. light scalar fields. These perturbations give rise to the primordial density/curvature perturbation via a multitude of mechanisms (e.g. inflaton, curvaton, inhomogeneous reheating etc.). Observables such as the spectral index $n_s$ or the non-linearity parameter $f_{NL}$ will soon exclude whole classes of models. Indeed, the Planck satellite observations are expected to increase precision up to $f_{NL} = O(1)$.

Recently, the possibility that cosmic vector fields contribute or even generate the curvature perturbation $\zeta$ (e.g. through the vector curvaton mechanism) is being explored. If it is so then vector fields can produce distinct signatures such as correlated statistical anisotropy in the spectrum and bispectrum of $\zeta$. WMAP observations allow up to 30% statistical anisotropy in the spectrum but the Planck satellite mission is expected to reduce this bound down to 2% [56], if statistical anisotropy is not observed. Anisotropy in the non-Gaussianity can be dominant, which means that $f_{NL}$ may feature an angular modulation on the microwave sky.

The above suggest that cosmological observations allow for detailed modelling and open a window to fundamental physics complementary to Earth based experiments such as the LHC.

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