NORMAlIZATION OF THE MATTER POWER SPECTRUM VIA HIGHER ORDER ANGULAR CORRELATIONS OF LUMINOUS RED GALAXIES

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Received 2007 September 20; accepted 2008 April 22

ABSTRACT

We present a novel technique with which to measure σ8. It relies on measuring the dependence of the second-order bias of a density field on σ8, using two separate techniques. Each technique employs area-averaged angular correlation functions (ω2), one relying on the shape of ω2, the other relying on the amplitude of s1 (σ8 = ω2/ω2^2). We confirm the validity of this method by testing it on a mock catalog drawn from Millennium Simulation data and finding a value of σ8 = σ8^true = -0.002 ± 0.062. We create a catalog of photometrically selected LRGs from SDSS DR5 and separate it into three distinct data sets by photometric redshift, with median redshifts of 0.47, 0.53, and 0.61. Measurements of C2 and s8 are made for each data set, with the assumption of a flat geometry and WMAP3 best-fit priors on Ω_m, h, and Ω. We find, with increasing redshift, that C2 = 0.09 ± 0.04, 0.09 ± 0.05, and 0.09 ± 0.03, and s8 = 0.78 ± 0.08, 0.80 ± 0.09, and 0.80 ± 0.09. We combine these three consistent σ8 measurements to produce σ8 = 0.79 ± 0.05. Allowing the parameters Ω_m, h, and Ω to vary within their WMAP3 σ error, we find that the best-fit value of σ8 does not change by more than 8%, and we are thus confident that our measurement is accurate to within 10%. We anticipate that future surveys, such as Pan-STARRS, DES, and LSST, will be able to employ this method in order to measure σ8 to great precision, and this will serve as an important check, complementarily, on the values determined via more established methods.

Subject headings: cosmology: observations — large-scale structure of universe

Online material: color figures

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1. INTRODUCTION

The normalization of the matter power spectrum is parameterized as the rms mass fluctuation within a top-hat radius of 8 h^{-1} Mpc and is denoted σ8. Measuring the amplitudes of the matter power spectrum, and thus σ8, is complicated by the fact that most of the matter in the universe is dark and that we must therefore rely on “tracers” of the matter: in most cases, galaxies. There is no guarantee, of course, that galaxies will cluster in the same manner as dark matter. The relationship between the clustering of galaxies and dark matter is known as the “bias” (see, e.g., Kaiser 1984). The bias essentially shifts the amplitudes of the galaxy power spectrum relative to the matter power spectrum, and there is thus a strong degeneracy between the bias and σ8. Therefore, precisely determining σ8 is important, as until this is accomplished, the full form of the relationship between the clustering of dark matter and the clustering of galaxies will remain ambiguous.

Measurements of σ8 made using cluster-counting techniques find mixed results. Using the X-ray temperature and luminosity functions and fully marginalizing over the cluster scaling relation, Pierpaoli et al. (2003) found a value of σ8 = 0.77 ± 0.05, while Henry (2004) found a value of σ8 = 0.66 ± 0.16. Using a self-calibration technique and the red sequence to optically identify clusters in the red-sequence cluster survey, Gladders et al. (2007) found a value of σ8 = 0.67 ± 0.13, while Rozo et al. (2007) found a value of σ8 = 0.92 ± 0.10 using the Sloan Digital Sky Survey (SDSS) maxBCG (Koester et al. 2007) catalog. Clearly, there is large variation in the measurements of σ8 determined via cluster abundances.

Measurements made using data from the Wilkinson Microwave Anisotropy Probe (WMAP) place some of the best constraints on the value of σ8, yet still allow it to take a wide range of values. The first-year WMAP (WMAP1) results found a value of σ8 = 0.92 ± 0.10 (Spergel et al. 2003), while the third-year (WMAP3) results determined a value of 0.744±0.025 (Spergel et al. 2007). The best-fit WMAP3 results vary significantly, depending on the adopted constraints and priors. In the currently accepted cosmological paradigm, inflation is a key ingredient. When WMAP3 is constrained by a variety of inflationary models, the best-fit values of σ8 are found to be as low as 0.702 ± 0.062 (Spergel et al. 2007). If one instead combines the WMAP3 results with the SDSS galaxy power spectrum (Tegmark et al. 2004), σ8 = 0.772±0.041 when we consider the importance of inflation to our understanding of the universe, the constraints placed by WMAP3 are quite loose. Analysis of the WMAP five-year data (WMAP5; made public during the revision process of this work) yielded a best-fit five-year mean value of 0.796 ± 0.036 (Komatsu et al. 2008). Despite the precision of this value, if one considers the high and low values measured by WMAP1 and WMAP3 and the range in values determined via cluster-counting techniques, the true value of σ8 remains unclear.

In this paper, we present a novel technique with which to measure σ8. Our technique relies on the fact that the bias relationship may be nonlinear, and if the bias is nonlinear, then both the amplitude and the shape of the correlation function (the Fourier transform of the power spectrum) are affected. As the value of σ8 affects only the amplitude of the correlation function measurement, the nonlinear bias is not degenerate with σ8. Therefore, different techniques for measuring the extent of the nonlinear bias can constrain not only the nonlinear bias, but σ8 as well.

The bias relationship can be expressed as a Taylor expansion, with the parameters b1 and b2 representing the first- and second-order contributions (and b2 thus being a measure of the nonlinearity; see, e.g., Gaztañaga 1992). In general, it is convenient to express the second-order contribution as c2 = b2/b1. The average overdensity increases as the scale becomes smaller; therefore,
The median redshift of SDSS LRGs is measuring correlation functions in the weakly nonlinear regime. To test the weakly nonlinear regime, as the effects of order statistics using photometrically selected LRGs, and thus the of SDSS DR5, which significantly improves our ability to calculate the redshift-space three-point correlation function of spectroscopically selected LRGs. Padmanabhan et al. (2007) and Blake et al. (2007) selected LRGs from SDSS imaging data and studied their clustering. These efforts measured the two-point correlation function and power spectrum, respectively, at different redshifts in order to place tight constraints on the matter and baryon densities of the universe (but not \( \sigma_8 \)). Nor is this the first study to measure the second order, is

\[
\bar{\omega} = \frac{\bar{n} - \bar{n}_i}{\bar{n}}.
\]

(1)

where \( \bar{n} \) is the average number of galaxies in a cell and \( n_i \) is the number of galaxies in cell \( i \). The remaining details and equations required to determine \( \bar{\omega}_N(\theta) \) can be found in Ross et al. (2006). The hierarchical amplitudes are defined as

\[
s_N = \frac{\bar{\omega}_N}{\bar{\omega}_N^2 - 1}.
\]

(2)

2. METHODS

2.1. Angular Correlation Functions

We estimate N-point area-averaged angular correlation functions, \( \bar{\omega}_N(\theta) \), using a counts-in-cells technique identical to that used in R07. This involves calculating the statistical moments of the overdensities contained in equal-area cells. We create the cells using a modified version of the SDSSPix pixelization scheme originally developed by M. Tegmark, Y. Xu, and R. Scranton.

The overdensity for cell \( i \) is defined as

\[
\delta_i = \frac{\bar{n}_i - \bar{n}}{\bar{n}},
\]

where \( \bar{n} \) is the average number of galaxies in a cell and \( n_i \) is the number of galaxies in cell \( i \). The remaining details and equations required to determine \( \bar{\omega}_N(\theta) \) can be found in Ross et al. (2006). The hierarchical amplitudes are defined as

\[
s_N = \frac{\bar{\omega}_N}{\bar{\omega}_N^2 - 1}.
\]

2.2. Bias and \( \sigma_8 \)

In order to determine the relationship between the bias parameter \( c_2 \) and \( \sigma_8 \), measurements of \( \bar{\omega}_2 \) and \( s_3 \) are compared to theoretical models constructed via matter power spectra. In each case, the \( z = 0 \) value of \( \sigma_8 \) is input to the model, which allows us to calculate \( c_2(\sigma_8) \).

Model values of \( \bar{\omega}_2 \) are produced using power spectra calculated using the Smith et al. (2003) fitting formulae, as described in § 6 of R07. By using a modified version of Limber’s equation (Limber 1954), one can use the redshift distribution of our LRG catalog to invert \( P(k) \) and obtain \( \bar{\omega}_2(\theta) \):

\[
\bar{\omega}_2(\theta) = \frac{H_0 \pi}{c} \int \int \left( \frac{dn}{dk} \right)^2 \left[ \Omega_m(1+z)^3 + \Omega_b \right] P_2 \chi(z) \theta k dz dk,
\]

(3)

where \( P_2 = 2/[J_1(x)/x] \) is the top-hat two-dimensional window function, \( \chi(z) \) is the comoving distance to redshift \( z \), \( P(k, z) \) is the matter power spectrum, \( k \) is the spectral index, and \( J_1 \) is the first-order Bessel function of the first kind, and this equation requires \( \Omega_{\text{total}} = 1 \) (see, e.g., Bernard et al. 2002). This equation is then integrated using the assumed cosmology and the desired value of \( \sigma_8 \).

Both of the methods employed to measure \( c_2 \) are dependent on the product of \( b_1 \) and \( \sigma_8 \). In order to account for this, we determine the first-order bias using the \( \bar{\omega}_2 \) measurement and the model value of \( \bar{\omega}_2 \) at \( \sigma_8 = 0.8 \) and denote it as \( b_{1,0.8} \). This is calculated for scales where linear theory is a good approximation (>10 \( h^{-1} \) Mpc). Therefore, a valid expression for the first-order bias is given by

\[
b_{1,0.8} = (0.8/\bar{\omega}_2) b_{1,0.8}.
\]

The second-order bias can be determined by manipulating the overdensities in each cell used in the calculation of \( \bar{\omega}_2 \). The overdensity of the LRGs can be related to the overdensity of dark matter and bias terms via a Taylor expansion (to second order):

\[
\delta_{\text{LRG}} = b_{1,\text{DM}} \delta_{\text{DM}} + 0.5 b_2 \delta_{\text{DM}}^2.
\]

(4)

In order to apply a second-order bias term to the LRG correlation measurement, one must solve equation (4) for \( \delta_{\text{DM}} \), which, to second order, is

\[
\delta_{\text{DM}} = \frac{\delta_{\text{LRG}} - 2 b_2 \delta_{\text{LRG}}^2}{b_1}.
\]

3 See http://lahmu.phyast.pitt.edu/~scranton/SDSSPix/.
Thus, in order to determine the first- and second-order bias of the LRGs, we use equation (5) to apply a value of $b_1$ and a value of $b_2$ to each overdensity used in the measurement of $\tilde{\omega}_2$ and match these altered measurements to the model value of $\tilde{\omega}_2$. To find the best-fit $b_2$ for a given $\sigma_8$, one must simply calculate $\tilde{\omega}_2$ for a sampling of $b_2$-values, calculate the $\chi^2$ for each, and minimize $\chi^2$ via iteration (fully accounting for covariance as noted in § 4). This must then be repeated for all of the $\sigma_8$-values one wishes to test (which requires the determination of the model value of $\tilde{\omega}_2$ for each value of $\sigma_8$). This process requires the correlation functions to be calculated thousands of times, but is highly parallel. In practice, we determine the $\chi^2$-values for selected $\sigma_8$- and $b_2$-values on a grid with an initial spacing of 0.02 in $\sigma_8$ and 0.005 in $b_2$. This grid is then refined in areas of rapidly changing $\chi^2$-values. The $\chi^2$-value for any $\sigma_8$-$b_2$ pair is then found by using a two-dimensional spline fitted to the grid. This method will hereafter be referred to as the “shape” method.

The other method we employ is detailed in R07 and takes advantage of the relationship (Fry & Gaztañaga 1993)

$$s_3 = b_1^{-1}(s_{3,DM} + 3c_2),$$

(6)

where $s_3$ is the measured amplitude and $s_{3,DM}$ is the theoretical amplitude. The $s_{3,DM}$-values are determined at scales greater than 8 $h^{-1}$ Mpc. This is accomplished by calculating $\tilde{\omega}_3$ via the integration of linear power spectra and redshift distributions given by Bernardeau (1995),

$$\tilde{\omega}_{3,DM} = 6\left(\frac{H_0 \pi}{c}\right)^2 \int \left(\frac{dn}{dz}\right)^3 \left[\Omega_m(1+z)^3 + \Omega_{\Lambda}\right] dz \times \left\{\frac{6}{7} \int kP(k)W_{2D}(D\theta k) dk + \int kP(k)W_{2D}(D\theta k) dk \times \int k^2 D\theta P(k)W_{2D}(D\theta k)W_{2D}(D\theta k) dk \right\},$$

(7)

where $D$ is the comoving distance to the median redshift. We calculate $\tilde{\omega}_3$ using equation (3) and linear power spectra and then use $s_3 = \tilde{\omega}_3/\tilde{\omega}_2$. By altering $\sigma_8$ and using the relationship described in § 7.1, equation (14), of R07, we can determine the 1 $\sigma$ allowed region of $c_2$ and $\sigma_8$. We hereafter refer to this approach as the “R07” method.

2.3. Testing via Mock Catalogs

To test our new method, we took galaxies with $M_r < -23$ from the Blaizot all-sky mock catalog created using the methods described in Blaizot et al. (2005) and Millennium Simulation data (Springel et al. 2005). In order to select red galaxies, we constrained the absolute magnitudes of our mock catalog to have $B - R > 1.4$, as we discovered that the color of the simulated galaxies was bimodal about this value. This yielded a sample of nearly 300,000 simulated LRGs. These simulated LRGs had a median redshift of 0.2; this is significantly smaller than those of the LRGs to be used in our measurements, but nonetheless quite sufficient to test our measurement techniques.

Using our mock catalog, the values of $\tilde{\omega}_N$ and $s_N$ were calculated using the methods described in § 2.1. In order to determine the bias of the simulated LRGs, we employed the methods described in § 2 to calculate model values of $\tilde{\omega}_N$ and $s_N$. This was done using $\sigma_8 = 0.8$ and the assumed cosmology of the Millennium Simulation [relevant parameters being $(\Omega_m, h, \Gamma) = (0.25, 0.73, 0.14)$; Springel et al. 2005]. This allowed us to find

$b_{1,0.8} = 2.04 \pm 0.02$, fitted at scales of greater than 8.2 $h^{-1}$ Mpc (>0.8 $h^{-1}$). With this value in hand, we could then use the shape method to find the $\chi^2$-values in the $b_2$-$\sigma_8$ parameter space. This allowed us to find the best-fit $c_2$-value as a function of $\sigma_8$. These $1 \sigma$ bounds, which represent $\Delta \chi^2 = 1$ from the minimum at that value of $\sigma_8$, are displayed in Figure 1 by solid lines. For $\sigma_8 = 0.8$, we found a value of $c_2 = 0.186 \pm 0.026$.

Using the R07 method and setting $\sigma_8 = 0.8$, we found a value of $c_2 = 0.41 \pm 0.09$. The large disagreement with the value of $c_2$ determined by the shape and R07 methods is expected for $\sigma_8 = 0.8$, as the two methods should agree only for the value of $\sigma_8$ used to create the mock catalog (0.9). To find where the methods agreed, we calculated the values of $\chi^2$ for the entire $c_2$-$\sigma_8$ parameter space using the R07 method. The resulting 1 $\sigma$ allowed region, which represents $\Delta \chi^2 = 1$, is bounded by the dashed lines in Figure 1. Combining the $\chi^2$ distributions of the two methods produced the 1 $\sigma$ ($\Delta \chi^2 = 2.3$ from the overall minimum) shaded region in Figure 1. From these measurements, we determine that $\sigma_8 = 0.898 \pm 0.062$ and $c_2 = 0.146 \pm 0.037$. This measured value of $\sigma_8$ is entirely consistent with the input value of $\sigma_8 = 0.9$ of the Millennium Simulation. This confirms that our method can indeed be used to measure the value of $\sigma_8$ both precisely and accurately.

3. DATA

We take data from the fifth data release (DR5) of the Sloan Digital Sky Survey. To create a catalog of LRGs with photometric redshifts, we applied the techniques described by Collister et al. (2007, hereafter C07) to objects in the DR5 PhotoPrimary view. Employing the color and magnitude cuts described by C07 produces a sample of just over 1.7 million objects. As in C07, we found photometric redshifts by using the ANNz software (Firth et al. 2003), with the Two-Degree Field–SDSS LRG and QSO (2SLAQ) spectroscopic LRG catalog (Cannon et al. 2006), with
stars removed, as training data. In order to separate stars, we again used the ANN software and trained it on the 2SLAQ LRG target catalog. In this case, we included the targeted objects determined to be stars and gave them a classification value of 0, while galaxies were given a classification value of 1. This same method was employed by C07 to eliminate stars from their catalog.

Our final catalog comprises only objects with classification values greater than 0.8. On the basis of the training data, cutting on this value should reduce stellar contamination to less than 2% while keeping 99.9% of the LRGs. This results in a catalog of 1,662,390 LRGs with a median photometric redshift of 0.52. Our redshift distribution is nearly identical to the distribution found by C07. These LRGs are then processed through the same imaging, reddening, and seeing masks as in R07, leaving 1,168,702 objects.

We split these LRGs into three distinct photometric redshift ranges with similar numbers of objects: 0.4 < z < 0.5 (444,175 LRGs), 0.5 < z < 0.57 (398,250 LRGs), and 0.57 < z < 0.7 (326,277 LRGs). These data sets will hereafter be referred to as Z_{0.47}, Z_{0.53}, and Z_{0.61}, since they have median redshifts of 0.47, 0.53, and 0.61, respectively. This gives us three distinct data sets with which to test the consistency of our measurements and that can be combined to increase the precision of our final σ_b measurement.

4. measurements

We calculate the area-averaged angular correlation functions (w_θ) and hierarchical amplitudes (s_N) for photometrically classified SDSS DR5 LRGs using the methods described in § 2.1. For every measurement, errors and covariance matrices are calculated using a jackknife method (e.g., Scanton et al. 2002), with inverse-variance weighting for both the errors (e.g., Myers et al. 2005, 2006) and the covariance (e.g., Myers et al. 2007), identical to the one described in § 3.4 of R07. This allows us to minimize χ^2, fully accounting for covariance via

$$\chi^2 = \sum_{i,j} [\tilde{w}_i(\theta_i) - \tilde{w}_m(\theta_i)]C_{i,j}^{-1} [\tilde{w}_i(\theta_i) - \tilde{w}_m(\theta_i)],$$

where C is the covariance matrix and i and j refer to the ith and jth jackknife subsample.

Focusing first on Z_{0.53}, we fit \tilde{w}_2 for measurements made between 0.4' and 1.6' (10.2'–40.2 h^{-1} Mpc). We determine a value of b_{1.08} = 1.63 ± 0.02. The measured w_2 values are well fitted by a single bias parameter in this range, as can be seen from the values of χ^2 = 1.7 and P(<χ^2) = 0.89. Thus, we measure b_{1, LRG} = (0.8/σ_b)(1.63 ± 0.02). The w_2 measurement, divided by 1.63^2 (to account for b_{1, 0.8}), is presented in Figure 2, along with the model value of w_2 at σ_b = 0.8. At scales of less than 0.3', the measurement grows larger with the model, which is indicative of positive second-order bias. The measurement also grows larger at scales of greater than ~2' (50.2 h^{-1} Mpc), but at these scales the errors begin to grow larger, and systematics due to reddening and projection effects also increase. On the basis of the results of Simon (2007), our theoretical curve, which employs a modified version of Limber’s equation, should not be accurate to better than 10% at scales of greater than ~2' for any of the redshift ranges we use. Our measurement at 2' differs from the model by 8.5%; thus, the disagreement is no greater than would be predicted by Simon (2007). We thus fit no measurements to scales that are greater than 1.6'.

Altering the w_2 measurement using the shape method and fitting the measurements at scales between 0.1' and 0.7' (equivalent to ~2.5–17.6 h^{-1} Mpc; there are eight measurements in this range and thus 7 degrees of freedom), we measure a value of b_2 = 0.15 ± 0.05 for σ_b = 0.8. The fit is acceptable [χ^2 = 0.60, P(<χ^2) = 0.999]. Attempting to fit the data with a single bias parameter model, we find that χ^2 = 12.18 and P(<χ^2) = 0.09. For other redshift ranges, we find similar results. Again for σ_b = 0.8, we measure a value of b_2, Z_{0.47} = 0.150 ± 0.040 [χ^2 = 0.49, P(<χ^2) = 0.999]. A single bias parameter model is rejected at 87%. Finally, we find a value of b_2, Z_{0.61} = 0.165 ± 0.025 [χ^2 = 1.91, P(<χ^2) = 0.96]. A single bias parameter is rejected at >99% for this redshift range. On the basis of the marginal rejections of a single-parameter model, a two-parameter model is needed to fit the measurements for each redshift range. For the two lower redshift ranges, the minimum χ^2-values are quite small, which implies that perhaps our error bars are overestimated for these redshift ranges, which further implies that our quoted errors on b_2 are overestimated.

In the bottom right panel of Figure 3, the w_2 measurement, corrected for b_1 = 1.63 and b_2 = 0.15, is displayed along with the theoretical values of w_2 at σ_b = 0.8. The model curve clearly fits the data. The other panels of this figure display the measured values of s_3 (triangles) for Z_{0.47}, Z_{0.53}, and Z_{0.61} (top left, top right, and bottom left, respectively), corrected for the best-fit values of b_1 and c_2 in accordance with equation (6). Each panel also includes a solid line displaying the model values of s_3. The Z_{0.53} measurement appears to be extremely consistent with the model, while the curves defined by the other two measurements do not share the same shape as the model. Despite this fact, the size of the error bars allows the Z_{0.47} and Z_{0.61} measurements to appear consistent with the model.

We use the R07 method to find c_2 for each data set. For the data sets Z_{0.47} and Z_{0.53}, we fit between 0.4' and 1.6' (equivalent to 9.0 and 35.9 h^{-1} Mpc for Z_{0.47} and 10.0 and 40.2 h^{-1} Mpc for Z_{0.53}; there are seven measurements in this range and thus 6 degrees of freedom). For Z_{0.61}, we fit between 0.3' and 1.6' (8.5–45.6 h^{-1} Mpc; again, seven measurements and 6 degrees of freedom). We find that for σ_b = 0.8, c_2, Z_{0.47} = 0.08 ± 0.12, c_2, Z_{0.53} = 0.07 ± 0.13, and c_2, Z_{0.61} = 0.14 ± 0.16. For Z_{0.53}, c_2 = 0.033, which means that P(<χ^2) = 1.0 – 1.0 × 10^{-6}. This is a remarkably small
the errors on our $\chi^2$-value, which one might expect (to a degree) on the basis of how well the measured values appear to match the model, despite the size of the error bars. This suggests that the error bars are overestimated for the $s_3$ measurements in this redshift range, and thus the errors on our $c_2$ measurements may be overestimated as well. For both the $Z_{0.47}$ and $Z_{0.61}$ ranges, $\chi^2 = 1.5$, with $P(<\chi^2) = 0.96$. These values are quite reasonable, implying that if our errors are being overestimated, it is happening only for the $Z_{0.53}$ range.

Using the shape and R07 methods (see §2.2), the $\chi^2$-values over the entire $s_8$-$c_2$ parameter space are determined for each data sample. The 1 $\sigma$ allowed regions of $c_2$ and $s_8$ determined via the R07 (solid lines; $\Delta \chi^2 = 1$ from a fixed value of $s_8$) and the shape (dashed lines; $\Delta \chi^2 = 1$ from a fixed value of $s_8$) methods are plotted in Figure 4 for $Z_{0.53}$. Fortunately, the two methods bound significantly different regions of the parameter space, allowing us to make a precise determination of $c_2$ and $s_8$. Combining the two measurements produces the 1 $\sigma$ (triangles; $\Delta \chi^2 = 2.3$ from overall minimum) allowed regions for $c_2$ and $s_8$, which are also displayed in Figure 4. To 1 $\sigma$ precision, we thus find values of $c_2, Z_{0.53} = 0.692 \pm 0.052$ and $s_8 = 0.796 \pm 0.086$. Repeating the process for $Z_{0.47}$, we find values of $c_2, Z_{0.47} = 0.088 \pm 0.041$ and $s_8 = 0.776 \pm 0.080$, and for $Z_{0.61}$, we find values of $c_2, Z_{0.61} = 0.092 \pm 0.033$ and $s_8 = 0.798 \pm 0.094$.

The best-fit values of $s_8$ for our three data sets are consistent to 0.275 $\sigma$. Combining the three measurements, we find that $s_8 = 0.789 \pm 0.050$. Adopting this value in order to determine the first-order bias, we find values of $b_1, Z_{0.47} = 1.47 \pm 0.09, b_1, Z_{0.53} = 1.65 \pm 0.09$, and $b_1, Z_{0.61} = 1.80 \pm 0.10$. These values make sense, given that the median luminosity of the galaxies increases with redshift, since our sample is not volume-limited. If we multiply each of the best-fit $c_2$ measurements by 0.789 divided by the best-fit value of $s_8$ for each respective data set (approximately correct for small changes in $s_8$, on the basis of our shape method measurements), we find that $c_2, Z_{0.47} = 0.69 \pm 0.04, c_2, Z_{0.53} = 0.09 \pm 0.05$, and $c_2, Z_{0.61} = 0.09 \pm 0.03$ for $s_8 = 0.789$. The fact that there is no significant change in $c_2$ is moderately surprising and implies...
that there are differences in the halo occupation distribution (HOD). If the HOD was not changing as a function of halo mass, $c_2$ would increase with $b_1$; see, e.g., Nishimichi et al. 2007). We will discuss the HOD more in § 5.2.

Of interest is the fact that the $s_3$ measurements for both $Z_{0.47}$ and $Z_{0.61}$ have a local minimum (Fig. 3, top left and bottom left), but there is no such minimum in the $s_3$ of $Z_{0.53}$. The minimum is at $\sim 0.6^\circ$, which is equivalent to $11.4 h^{-1}$ Mpc for the lower redshift range, while it is at $\sim 0.3^\circ$, which is equivalent to $8.5 h^{-1}$ Mpc, in the high-redshift range. In R07, it was found that early-type galaxies also displayed a minimum in their $s_3$ measurement at $\sim 10 h^{-1}$ Mpc. As in R07, the errors dominate the LRG measurement (although to a lesser extent), but it appears to be unlikely that this is a coincidence. The feature is seen at approximately the same physical scale, but at a different angular scale, due to the differences in redshift. This rules out any possibility of observational systematics such as seeing or reddening. It is unclear as to whether the feature may be due to projection effects or complicated halo dynamics that do not affect the middle redshift range.

5. DISCUSSION

We have presented a technique for measuring $\sigma_8$ using the two- and three-point angular-area-averaged correlation functions and have applied it to photometrically classified LRGs from the SDSS DR5, split into three distinct redshift ranges. Using a method that depends on the shape of $\omega_2$ and the technique described in R07, we determined two separate relationships between $\sigma_8$ and $c_2, \text{LGR}$. These relationships split the degeneracy between bias and $\sigma_8$, allowing us to make an unambiguous determination of the first- and second-order bias and $\sigma_8$. The measured values of $\sigma_8$ in the three redshift ranges are consistent and combine for a best-fit value of $\sigma_8 = 0.789 \pm 0.048$. Our determination of $\sigma_8$ is quite precise. It is thus important for us to investigate the assumptions, implicit and explicit, that we made when determining our measurements, to compare our measurements to the relevant dark matter halo/bias theory, and to investigate how consistent our measurements are with previous results.

5.1. Assumptions

The main assumption that goes into our measurements of $c_2$ is that the bias can be expressed solely as a function of the overdensity; i.e., it is not a function of both the overdensity and the smoothing scale. If the bias was a strong function of the scale, it would invalidate any measurement made using the shape method, as a changing value of $b_1$ would change the shape of $\omega_2$. Similarly, the R07 technique assumes constant values of $b_1$ and $c_2$ over the range of scales that are fitted. Further, the R07 measurement is fitted at a different range of scales than the shape measurement. Bias that is a strong function of scale between 2.5 and $40 h^{-1}$ Mpc would completely invalidate any comparison between measurements using the shape and R07 methods and thus would invalidate our $\sigma_8$ measurements.

Our assumption that the bias can be expressed solely as a function of the overdensity is validated by the goodness of the fit to our bias-corrected measurements. These measurements demonstrated that, between $0.1^\circ$ and $1.6^\circ$, our $\omega_2$ measurement is quite consistent with a two bias parameter model, as all of the model fits using the shape method were accepted to better than 96%. Further, in each redshift range, the probability that a single bias parameter fits the data is less than 9%. The simplest model that fits the data between $0.1^\circ$ and $1.6^\circ$ is thus that the bias is independent of scale and can be described by two parameters. Therefore, we believe our measurement techniques to be valid and that our comparison of those techniques is valid as well.

The error on our $\sigma_8$ measurement is quite low, in part because we held the values of $\Omega_m, h$, and $\Gamma$ fixed to their WMAP3 best-fit values. Allowing these values to change does alter our best-fit values of $c_2$ and $\sigma_8$. In order to determine the degree to which the uncertainty in these parameters should affect the uncertainty of our results, we repeated our $\sigma_8$ measurements using the $Z_{0.53}$ data set ($0.5 < z < 0.57$) and produced model values of $\omega_2$ and $s_3$ for each parameter at the $1 \sigma$ limits determined by the WMAP3-alone best-fit values (while holding the other parameters at their best-fit values).

As long as the geometry of the universe is kept flat, changes in the matter density have little effect on either the amplitude or the shape of $\omega_2$. Thus, we expected that the value of $\Omega_m$ would have little effect on our measurement of $\sigma_8$. This was indeed the case, as we found that $\sigma_8 = 0.796 \pm 0.086$ and $0.800 \pm 0.088$ for values of $\Omega_m$ equaling $0.251$ and $0.2134$, respectively, with $\Omega_m$ fixed at 1 and $(\Gamma, h) = (0.135, 0.73)$. Conversely, we expected our measurement of $\sigma_8$ to be fairly dependent on the value of $h$, as this value significantly affects the distance to the LRGs. We found that $\sigma_8 = 0.846 \pm 0.088$ and $0.748 \pm 0.085$ for values of $h = 0.7$ and 0.76, respectively, with $(\Omega_m, \Gamma, \Omega_{\Lambda}) = (0.135, 0.238)$. The percentage change in the measured value of $\sigma_8$ was approximately the same as the percentage change in $h$, suggesting a close relationship between the two (again, as expected).

We also expected the value of $\Gamma$ to have a significant effect on the measurement, as changing $\Gamma$ alters the shape of $\omega_2$. For values of $\Gamma = 0.149$ and 0.12, we found that $\sigma_8 = 0.842 \pm 0.088$ and $0.736 \pm 0.082$, with $(\Omega_m, h, \Omega_{\Lambda}) = (1, 0.73, 0.238)$. While the uncertainty in $\Gamma$ produced the largest range in $\sigma_8$-values, the percentage change in $\sigma_8$ was actually $\sim 25\%$ smaller than the percentage change in $\Gamma$. We thus determine that although the uncertainty in $\Gamma$ introduces the most uncertainty into our measurement of $\sigma_8$, our measurement technique is most sensitive to the value of $h$. Despite the changes in the value of $\sigma_8$ that we measure, our measurement of $\sigma_8$ has not changed by more than $8\%$.
suggests that our quoted uncertainty of 0.05 would increase by less than a factor of 2 when uncertainties in \( h \) and \( \Gamma \) were taken into account. We are thus confident that our measurements are accurate to within 10%.

We also assumed no error in our redshift distribution when making our measurements. Precise knowledge of the redshift distribution is necessary in order to use equations (3) and (7). In order to explicitly test our measurements’ dependence on the redshift distribution, we created two new distributions for the \( Z_{0.53} \) redshift range (see R07 for the details of how these distributions are constructed). For one, we systematically increased the error of each photometric redshift by 10% (effectively broadening the redshift distribution), and for the other we decreased the error of each photometric redshift by 10% (effectively narrowing the distribution). We then recalculated the best-fit value of \( \sigma_8 \) for each distribution, finding that it increased to 0.84 ± 0.09 for the distribution with greater redshift errors and that it decreased to 0.76 ± 0.09 for the distribution with smaller redshift errors. These differences are significant, but they are smaller than our combined 1 σ error. We thus do not believe that this issue significantly adds to our quoted uncertainty.

5.2. Testing via Halo Models

It is important to determine how our measured bias values compare to theoretical values. We calculate \( b_1 \) and \( c_2 \) using halo models and the methods prescribed by Nishimichi et al. (2007). They show that one can combine the \( N \)-th-order bias coefficient of halos as a function of mass and redshift, \( B_N(m, z) \), the number of halos of a certain mass and redshift, \( n_{\text{halo}}(m, z) \), and the mean number of galaxies occupying a halo of a certain mass, \( \langle n_g|M \rangle \), to find the bias of a population of galaxies. This can be expressed by

\[
b_N = \frac{\int dM n_{\text{halo}}(M, z)B_N(M, z)\langle n_g|M \rangle}{\int dM n_{\text{halo}}(M, z)}.
\]

We determine both \( n_{\text{halo}}(m, z) \) and \( B_N(m, z) \) by using an ellipsoidal collapse model (e.g., Sheth et al. 2001) and following the methods described in detail in Nishimichi et al. (2007). For \( \langle n_g|M \rangle \), we first model the number of central LRGs per halo as having a “soft” transition between \( n_g = 0 \) and \( n_g = 1 \) such that

\[
\langle n_\text{central}|M \rangle = 0.5\left\{1 + \text{erf}\left[\frac{\log(M/M_{\text{cut}})}{\sigma_{\text{cut}}}\right]\right\},
\]

as in Zheng et al. (2005) and Blake et al. (2008). This helps account for the fact that we are not using a volume-limited sample, and thus at smaller redshifts the mass limit is likely to be smaller than that at higher redshifts. As in Blake et al. (2008), we model the number of satellite galaxies using a simple power law. Thus,

\[
\langle n_\text{sat}|M \rangle = \left(\frac{M}{M_0}\right)^\alpha.
\]

The bias model has four free parameters: \( M_{\text{cut}}, M_0, \sigma_{\text{cut}}, \) and \( \alpha \). It is beyond the scope of this paper to fit for these parameters. It is instead our intention to determine if reasonable values for these parameters can reproduce the bias values that we measure. If this is possible, it suggests that our bias measurements are themselves reasonable. Using \( \sigma_8 = 0.793, z = 0.532 \), and halo parameters \( \log(M_{\text{cut}}/M_0) = 13.3 \), \( \sigma_{\text{cut}} = 0.6 \), \( \log(M_0/M_0) = 14.5 \), and \( \alpha = 2.0 \), we find that \( b_1 = 1.61 \) and \( c_2 = 0.09 \): results that are consistent with our best-fit values. The values of \( \log(M_0/M_0) \) and \( \alpha \) were chosen to be equal to the best-fit parameters found by Blake et al. (2008) for LRGs with \( 0.5 < z < 0.55 \). The log of \( M_{\text{cut}}/M_0 \) and \( \alpha \) parameters are slightly lower than the Blake et al. (2008) parameters, likely because our LRGs have a lower luminosity and thus a lower bias, minimum mass, and \( \alpha \) (\( \alpha \) has been seen to increase with \( b_1 \) in both Blake et al. [2008] and Zehavi et al. [2005]). This is not to suggest that we favor a steep power law to a shallow one. If we change to a value of \( \alpha = 1.4 \) (as measured by Kulkarni et al. 2007), we calculate values of \( b_1 = 1.62 \) and \( c_2 = 0.09 \) if we also reduce \( \log(M_0/M_0) \) to 14.41. This implies that, from a theoretical standpoint and in the context of relevant measurements of LRG halo properties, our measurements of \( b_1 \) and \( c_2 \) are reasonable.

The bias model can also be used to test our implicit assumption that \( \langle c_2 \rangle = \langle b_2 \rangle/\langle b_1 \rangle \). We calculate the values of \( b_1, b_2, \) and \( c_2 \) at redshifts between 0.4 and 0.7 (essentially the redshift range of our entire sample) in intervals of 0.02. These values are used to calculate theoretical values for \( \langle b_2 \rangle/\langle b_1 \rangle \) and \( \langle b_2/b_1 \rangle \). We find that the difference between the two is less than 0.1%, meaning that the systematic error introduced by assuming that \( \langle c_2 \rangle = \langle b_2 \rangle/\langle b_1 \rangle \) is insignificant for our measurements.

5.3. Comparison with Other Measurements

Our measurement of \( \sigma_8 = 0.789 ± 0.050 \) is consistent with most previous measurements. There are, however, notable exceptions. Our result is inconsistent to 1 σ with a photometric optical cluster-counting technique employing SDSS data that found \( \sigma_8 = 0.92 ± 0.10 \) (Rozo et al. 2007). Another recent result (Harker et al. 2007) found a similarly high value of \( \sigma_8 = 0.97 ± 0.06 \) using \( N \)-body simulations in combination with semianalytic galaxy formation models and the projected two-point correlation function of SDSS galaxies. Other results determine \( \sigma_8 \) to be too small to be consistent with our measurement (to 1 σ). Notable examples are the result from WMAP3 constrained by inflationary models, with a value of 0.702 ± 0.062 (Spergel et al. 2007), and the Voevodkin & Vikhlinin (2004) results that found a value of \( \sigma_8 = 0.72 ± 0.04 \) by using the cluster baryon mass function.

However, many notable results are consistent with our measurement. The best-fit WMAP3 data alone found a value of \( \sigma_8 = 0.744^{+0.053}_{-0.064} \), which is just barely consistent with our measurement to 1 σ. The WMAP3+SDSS and WMAP3+LRG best-fit values of \( \sigma_8 = 0.775^{+0.036}_{-0.046} \) and 0.781 ± 0.032 are consistent with our measurement to less than 1 σ. Large disagreement with these results would be surprising, given that we used the WMAP3 best-fit priors for the relevant input cosmological parameters. The WMAP5 results are quite similar to our results, as Komatsu et al. (2008) find a value of \( \sigma_8 = 0.796 ± 0.036 \) for the WMAP-alone five-year mean value. More significantly, our results are also consistent to 1 σ with the values of \( \sigma_8 = 0.67^{+0.14}_{-0.13} \) as derived from optical cluster-finding techniques (Gladders et al. 2007) and \( \sigma_8 = 0.66 ± 0.16 \) from X-ray cluster measurements (Henry 2004). Due to the fact that the inconsistent measurements appear to be as likely to be lower than our measurement as they are higher, we feel that these results are hinting at a potential convergence to a \( \sigma_8 \)-value that is close to 0.8.

6. CONCLUSIONS

We present and test a new method for determining the value of \( \sigma_8 \). The method and the results of our testing can be summarized as follows:

1. The technique for measuring \( \sigma_8 \) utilizes two measures of the second-order bias of a density field. The two methods have different dependencies on the value of \( \sigma_8 \) and can thus be combined to determine a best-fit value of \( \sigma_8 \) and second-order bias. One measure of the second-order bias (the R07 method) has been used many times before (e.g., R07) and depends on the amplitude of \( s_8 \).
The other method (the “shape” method) has (to our knowledge) never been used before. It relies on correcting the overdensities for the given first- and second-order bias parameters and determining the bias parameters that allow the shapes of the $\sigma_2$ measurement and model to become consistent.

2. The method was tested using a mock catalog of LRGs drawn from the Blaizot all-sky catalog (Blaizot et al. 2005) that was constructed using Millennium Simulation data (Springel et al. 2005) and its input cosmological parameters for $\Omega_m$, $h$, and $\Gamma$. The Millennium Simulation assumed a value of $\sigma_8 = 0.9$, and we measured a value of $\sigma_8 = 0.898 \pm 0.062$. This measurement proved that our method is both accurate and precise.

3. We photometrically selected LRGs from SDSS DR5, determined photometric redshifts for each LRG, and removed stars following the prescriptions of Collister et al. (2007). We divided this LRG catalog into three samples by redshift, with the separate ranges being $0.4 < z < 0.5$ ($Z_{0.47}$), $0.5 < z < 0.57$ ($Z_{0.53}$), and $0.57 < z < 0.7$ ($Z_{0.61}$).

4. We measured the value of $\sigma_8$ in each sample and found values of $\sigma_8 = 0.776 \pm 0.080$, $0.796 \pm 0.086$, and $0.798 \pm 0.094$, respectively. Combining these consistent results, we determined that $\sigma_8 = 0.789 \pm 0.050$. For each measurement, we assumed that the relevant cosmological parameters were equal to their WMAP3 best-fit values.

5. Allowing the relevant cosmological parameters to vary within their WMAP3 $1 \sigma$ errors, we found that our measurement of $\sigma_8$ changed by less than 8%. Thus, even allowing for these uncertainties, our method produces a precise measurement that we are confident is accurate to within 10%.

6. We measured values of $b_1, Z_{0.47} = 1.47 \pm 0.09$, $b_1, Z_{0.53} = 1.65 \pm 0.09$, and $b_1, Z_{0.61} = 1.80 \pm 0.10$ and $c_2, Z_{0.47} = 0.09 \pm 0.04$, $c_2, Z_{0.53} = 0.09 \pm 0.05$, and $c_2, Z_{0.61} = 0.09 \pm 0.03$. Using a halo model, we determined that the bias values for $Z_{0.53}$ were consistent with reasonable, and previously measured, HOD parameters.

The techniques described herein can easily be repeated and tested using other cosmic samples. Future surveys, such as the Panoramic Survey Telescope and Rapid Response System (Pan-STARRS), the Dark Energy Survey (DES), and the Large Synoptic Survey Telescope (LSST), will provide opportunities to measure both the two- and three-point area-averaged correlation functions to extreme precision. Repeating the techniques we have presented here will allow researchers to determine $\sigma_8$ accurately, precisely, and independently of future cluster-counting and CMB techniques, thus providing an important check on those results.

In addition to our determinations of the bias and of $\sigma_8$, we have found that a feature in the hierarchical amplitude of $s_3$ at $\sim 10 h^{-1}$ Mpc exists in two of the three data sets. Given that a feature existed at approximately the same physical location for early-type galaxies at significantly smaller redshifts (R07), the feature appears to be physical in nature. The fact that it is absent in one of our data sets hints that the possible cause may be due to projection effects, or that perhaps the feature is indicative of complicated halo occupation statistics. The feature demands further study, both observationally and theoretically, and we are currently focusing our efforts to explain this phenomenon.

A. J. R., R. J. B., and A. D. M. acknowledge support from Microsoft Research, the University of Illinois, and NASA through grant NNG06GH156. The authors made extensive use of the storage and computing facilities at the National Center for Supercomputing Applications and thank the technical staff for their assistance in enabling this work.

We thank Ani Thakar and Jan Van den Berg for help with obtaining a copy of the SDSS DR5 databases. We thank Ravi Sheth for comments that helped to improve the paper. We thank an anonymous referee whose comments helped us to significantly improve our results.

Funding for the creation and distribution of the SDSS Archive has been provided by the Alfred P. Sloan Foundation, the Participating Institutions, the National Aeronautics and Space Administration, the National Science Foundation, the U.S. Department of Energy, the Japanese Monbukagakusho, and the Max Planck Society. The SDSS Web site is http://www.sdss.org/.

The SDSS is managed by the Astrophysical Research Consortium (ARC) for the Participating Institutions. The Participating Institutions are the University of Chicago, Fermilab, the Institute for Advanced Study, the Japan Participation Group, Johns Hopkins University, the Korean Scientist Group, Los Alamos National Laboratory, the Max Planck Institute for Astronomy (MPIA), the Max Planck Institute for Astrophysics (MPA), New Mexico State University, the University of Pittsburgh, the University of Portsmouth, Princeton University, the United States Naval Observatory, and the University of Washington.

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