Dynamic positioning of a sea vessel under the influence of a multi-harmonic external disturbance

A. O. Vediakova¹, A. A. Vedyakov², V. S. Boev

¹ Department of Computer Applications and Systems, St.Petersburg State University, 7/9 Universitetskaya nab., St. Petersburg, 199034, Russia.
² Department of Control Systems and Informatics, ITMO University, Kronverkskiy pr., 49, St. Petersburg, 197101, Russia.
e-mail: vediakova@gmail.com, vediakov@gmail.com, boev.viacheslav@gmail.com

Abstract. The paper considers the problem of dynamic positioning nonlinear model of a vessel under the multi-harmonic disturbances. The multipurpose control law makes it possible to tune the nonlinear observer, the feedback control, the dynamic corrector, and the estimator of external disturbances frequencies independently from each other. We achieve the economical ship control mode by tuning the dynamic filter to the primary external perturbation frequency. We apply a finite time approach to construct estimates of multi-harmonic external disturbance frequencies.

1. Introduction

One of the significant control theory problems is the development of dynamic positioning (DP) systems. The various practical applications widely use DP systems in hydrography, marine construction, sunken ship investigation, and submarine cable laying [1-4]. The relevant papers related to the DP system design are [1–3, 5]. The papers [1, 3, 5] describe the DP control law structure based on the nonlinear asymptotic observer, which allow independent tuning of the controller components. In the paper [1], multipurpose control theory [6-7] expands this approach and obtains ship steering's economical mode.

This paper aims to synthesize the multipurpose control law that solves the DP problem under the influence of a multi-harmonic external disturbance and provides an economical mode of the control action. For the economical ship control mode, the dynamic filter, included in the controller, is tuned to the primary external perturbation frequency. Unlike [1], which assumes the value of primary external perturbation frequency is known, in this paper, it is estimated at the finite time using the method of the Dynamic Regressor Extension and Mixing (DREM) [8, 9]. The external disturbance is represented as a multi-harmonic signal with unknown parameters. The modified approach [10] obtains the external disturbance parameter estimates at the finite time.

The paper is organized as follows. Section 2 presents the equations of the motion of DP vessels and poses the problem of optimal filtering correction. Section 3 presents the special control law structure and introduces the computational procedure employed to implement filter tuning onboard. Section 4 presents an algorithm to estimate frequencies and amplitudes of an external disturbance in a finite time. The estimator has a cascade structure. In the first step, we construct estimates of multi-harmonic signal frequencies. In the second step, we build amplitude estimates based on the obtained frequency.
estimations. Section 5 illustrates the proposed approach using a practical example of filtering corrector synthesis.

2. Problem statement

Let us consider the 3-degrees of freedom nonlinear model of DP control plant [3]:

\[
\begin{align*}
M \ddot{v}(t) &= -D \dot{v}(t) + \tau(t) + d(t), \\
\dot{\eta}(t) &= R(\eta) \nu(t), \\
\xi(t) &= \eta(t) + \eta_\omega(t),
\end{align*}
\]

(1)

where \(\nu = [u \ v \ \tau]^T \in \mathbb{R}^3\) is the generalised velocity vector defined in a vessel-fixed frame \(Ox_yy_z_y\) that includes linear velocities \(u, v\) and an angular velocity \(\tau; \eta = [x \ y \ \psi]^T \in \mathbb{R}^3\) is the joint vector relative to an earth-fixed frame \(Oxyz\) that includes position \((x, y)\) parameters and heading angle \(\psi; \tau \in \mathbb{R}^3\) is a control action generated by the propulsion system; \(\xi \in \mathbb{R}^3\) is a vector of measured variables; \(d, \eta_\omega \in \mathbb{R}^3\) are disturbances; \(M_{3 \times 3}, D_{3 \times 3}\) are positive definite matrices with constant elements, \(M = M^T; R(\eta)\) is the orthogonal rotation matrix:

\[
R(\eta) = R(\psi) = \begin{bmatrix}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}.
\]

The vessel (1) is under the action of the bias disturbance \(d(t)\) and wave-induced disturbance \(\eta_\omega(t)\).

The objective is to synthesize the control law \(\tau(t)\), which provides the closed-loop system the following design requirements.

- There is the only equilibrium point satisfying the equalities \(\nu(t) \equiv 0, \eta(t) \equiv \eta^*, \) if \(d(t) \equiv 0, \eta_\omega(t) \equiv 0\), where \(\eta^* = [x^* \ y^* \ \psi^*]^T \in \mathbb{R}^3\) is the desired constant position vector.
- The equilibrium point must be globally asymptotically stable.
- The controller \(\tau(t)\) must provide an integral action to the bias vector \(b(t)\), and a filtering action to the high frequencies wave generated components of the external disturbances \(\eta_\omega(t)\).

3. Dynamic controller structure

Let us construct the non-linear dynamic control law in the form [1]:

\[
\begin{align*}
M \ddot{v}(t) &= -D \dot{v}(t) + \tau(t) + R^T(y)K_2(y(t) - \hat{\eta}(t)), \\
\dot{\hat{\eta}}(t) &= R(y) \hat{\nu}(t) + K_2(y(t) - \hat{\eta}(t)), \\
\tau_{dp}(t) &= -K_d \hat{v}(t) - R^T(y)K_p(\hat{\eta}(t) - \eta^*), \\
\dot{p}(t) &= \alpha \dot{p}(t) + \beta \hat{\eta}(t), \\
\tau_f(t) &= \gamma \dot{p}(t) + \mu \hat{\eta}(t), \\
\tau(t) &= \tau_{dp}(t) + \tau_f(t),
\end{align*}
\]

(2)

where \(\hat{v}, \hat{\eta} \in \mathbb{R}^3\) are estimates of state vectors \(v\) and \(\eta\) of the model (1); \(p \in \mathbb{R}^l\) is the state vector of the dynamic controller; \(\tau_{dp}\) is the part of the control law, which guarantees the global asymptotic stability of the desired equilibrium position of the closed system; \(\tau_f\) is another part of the control law, which suppresses external disturbances; \(K_1, K_2, K_p, K_d, \alpha, \beta, \gamma, \mu\) are constant matrices of corresponding dimensions, \(a_{1 \times 1}\) is the Hurwitz matrix.

If the matrices \(K_1, K_2, K_p, K_d\) are chosen according to papers [3, 5] and there are no external disturbances, then the control law \(\tau(t) = \tau_{dp}(t)\) provides estimation errors \(\hat{v}(t): = v(t) - \hat{v}(t), \hat{\eta}(t): = \eta(t) - \hat{\eta}(t)\) asymptotic convergence to zero and global asymptotic stability of the equilibrium position \(v(t) \equiv 0, \eta(t) \equiv \eta^*\) of the closed system (1)–(2).

In the presence of the external disturbances \(d(t), \eta_\omega(t)\) the transfer matrix \(F(s)\) of dynamic controller (2) should satisfy the following conditions:
\[ F(0) = -(D + K_d)R^T(\psi^*)K_2 - R^T(\psi^*)(K_p + K_1), \]
\[ F(j\omega_0) = -P_2^{-1}(j\omega_0)P_1(j\omega_0), \]
\[ P(s) = C(\eta)(E_{6\times6} - A(\eta))^{-1}B(\eta) + D = [P_1(s) \ P_2(s)], \]
\[ A(\eta) = \begin{bmatrix} -M^{-1}(D + K_d) & -M^{-1}R^T(\eta) & K_p \\ 0 & 3x3 \end{bmatrix}, \]
\[ B(\eta) = \begin{bmatrix} -M^{-1}R^T(\eta)K_1 \\ M^{-1} \end{bmatrix}, \]
\[ C(\eta) = \begin{bmatrix} -K_d & -R^T(\eta)K_p \end{bmatrix}, \]
where \( P_1(s)_{3\times3}, P_2(s)_{3\times3} \) are blocks of the matrix \( P(s)_{3\times6} = [P_1(s) \ P_2(s)] \) and \( \text{det}P_2(j\omega_0) \neq 0; \)
\( \mathbb{O}_{3\times3}, E_{6\times6} \) are zero and identity matrices of corresponding dimensions. Constructed dynamic controller \( \tau_f(t) \) with the transfer matrix \( F(s) \) from (3) provides filtering action for the continual and the slowly varying disturbance \( d(t) \) and the central frequency \( \omega_0 \) of sea waves \( \eta_\omega(t) \).

4. The external disturbance parameters estimation

In this section, we build the estimation \( \hat{\omega}_0(t) \) of the central external disturbance frequency \( \omega_0 \) to tune the controller.

Let us consider the external disturbance acting as the multi-harmonic signal:
\[ \eta_\omega(t) = \sum_{i=1}^{n} A_{\omega i} \sin(\omega_i t + \phi_i), \]
where \( \omega_i \in \mathbb{R}_+ \) are the signal frequencies, \( A_{\omega i} \in \mathbb{R}_+ \) are the amplitudes, \( \phi_i \in \mathbb{R} \) are the phases, \( i = 1, n, n \) is the number of the signal harmonics \( \eta_\omega(t) \). The parameters \( \omega_i, A_{\omega i}, \phi_i \) are unknown.

Our goal is to construct the estimates \( \hat{\omega}_i(t) \) of the frequencies \( \omega_i \) and \( \hat{A}_{\omega i} \) of the amplitudes \( A_{\omega i} \) that guarantee convergence of the estimation errors \( \tilde{\omega}_i(t) = \omega_i - \hat{\omega}_i(t) \) and \( \hat{A}_{\omega i}(t) = A_{\omega i} - \hat{A}_{\omega i}(t) \) to zero in a determined finite time \( t_f > 0 \), i.e. \( |\tilde{\omega}_i(t)| = 0, |\hat{A}_{\omega i}(t)| = 0 \) for \( \forall t \geq t_f \). Let us further assume that the frequency signals (4) upper bound \( 0 < \omega_i < \bar{\omega} \) is known, \( i = 1, n \).

Then, we define the central frequency as one of the obtained estimates \( \hat{\omega}_i(t) \), which has a maximum amplitude \( \hat{A}_{\omega i}(t) \), i.e. \( \hat{A} = \hat{A}_{\omega i}(t) \), \( \forall t > t_f \).

Consider the difference between the measured signal \( y(t) = \eta(t) + \eta_\omega(t) \) of the model (1) and the state vector \( \hat{\eta}(t) \) of the nonlinear observer to allocate external disturbance:
\[ \tilde{y}(t) = y(t) - \hat{y}(t) = \hat{\eta}(t) + \eta_\omega(t) = \sum_{i=1}^{n} A_{\omega i} \sin(\omega_i t + \phi_i) + \epsilon(t), \]
where \( \epsilon(t) \) is an exponentially decreasing signal related to transition processes. Starting from a certain moment in time, influence of the exponentially decreasing term can be neglected and the signal \( \tilde{y}(t) \) can be used to construct estimates \( \hat{\omega}_i(t) \).

Remark. With a long transition process, it is recommended to take into account an exponentially decreasing term in the regression model.

4.1. Finite-time frequencies estimation

In this subsection, we aim to obtain finite-time frequencies estimation for the multi-harmonic signal, similar to (4):
\[ y(t) = \sum_{i=1}^{n} A_{\omega i} \sin(\omega_i t + \phi_i). \]

First of all, we find a linear regression model with measurable variables and a constant vector depending on the signal frequencies.

Introduce an \( h \)-second delay operator:
\[ [Z(\cdot)](t) = \begin{cases} 0, & t < h, \\ (\cdot)(t - h), & t \geq h, \end{cases} \]
where \( h \in \mathbb{R}_+ \) is a chosen delay value, \( h < \frac{\pi}{2\omega_0}. \)
Signals with multiple delays can be represented using this delay operator, as \( y(t - kh) = Z^k y(t) \), for \( k = 1, 2n \).

Let us find an expression, where the signal \( y(t) \) with \( n \) harmonics is expressed via \( 2n \) delayed signals \( y(t - h), \ldots, y(t - 2nh) \).

**Proposition.** For any \( n \in \mathbb{N} \) the following relation holds

\[
[Z^2 + 1 - 2Zc_1] \cdots [Z^2 + 1 - 2Zc_n]y^{(n)}(t) = 0, \quad \forall t \geq 2nh,
\]

where \( c_i = \cos \omega_i t \) are constants, \( i = 1, n; Z \) is the delay operator (7), the upper index in the brackets \( (n) \) denotes the number of harmonics in the signal.

Detailed proof by induction is given in the paper [11].

Now we construct the regression model using (8):

\[
\kappa(t) = \varphi^T(t)\theta, \quad t \geq 2nh,
\]

where \( \kappa(t) \in \mathbb{R}^1 \) is a regressand, \( \varphi^T(t) = [\varphi_1(t) \ldots \varphi_n(t)] \in \mathbb{R}^n \) is a regressor, \( \theta^T = [\theta_1 \ldots \theta_n] \in \mathbb{R}^n \) is an unknown parameter vector:

\[
\kappa(t) = [Z^2 + 1]^n y(t) = \sum_{i=0}^{n} C_i y(t - 2h(n-i)),
\]

\[
\varphi_1(t) = 2Z[Z^2 + 1]^{n-1} y(t) = 2 \sum_{i=0}^{n-1} C_{n-i} y(t - h(2(n-i) - 1)),
\]

\[
\varphi_2(t) = 2^2 Z^2[Z^2 + 1]^{n-2} y(t) = 2^2 \sum_{i=0}^{n-2} C_{n-i} y(t - 2h(n-i - 1)),
\]

\[
\varphi_n(t) = 2^n Z^n y(t) = 2^n y(t - nh),
\]

\[
\theta_1 = -c_1 - c_2 - \ldots - c_n,
\]

\[
\theta_2 = c_1 c_2 + c_1 c_3 + \ldots + c_{n-1} c_n,
\]

\[
\theta_n = (-1)^n c_1 c_2 \cdots c_n,
\]

where \( C_i = \frac{n!}{i!(n-i)!} \).

We use DREM method [8–11] to obtain \( n \) separate first order linear regression models based on (9).

Following DREM procedure we introduce new delay operator similarly to (7):

\[
[H_d(\cdot)](t) = \begin{cases} 0, & t < b, \\ (\cdot)(t - b), & t \geq b, \end{cases}
\]

where \( b \in \mathbb{R}_+ \) is the delay value.

Let us apply delay operators to the linear regression model (9):

\[
H^i \{ \kappa(t) \} = H^i \{ \varphi(t) \}^T \theta, i = 1, n,
\]

where \( H^i \{ \cdot \} = H \{ H \{ \ldots \{ H \{ \cdot \} \ldots \} \} \} \).

Next, we introduce new variables: \( \kappa_i(t) = H^i \{ \kappa(t) \} \), \( \Phi_i(t) = H^i \{ \varphi(t) \} \) and write the extended system from the expressions (9) in a matrix form:

\[
e \Psi_f(t) = e \Phi_f(t) \theta,
\]

where \( e \in \mathbb{R}_+ \) is normalization gain, \( \Psi_f(t) = [\kappa_1(t) \ldots \kappa_n(t)]^T \in \mathbb{R}^n \) is an extended regressand, \( \Phi_f(t) = [\Phi_1(t) \ldots \Phi_n(t)]^T \in \mathbb{R}^{n \times n} \) is an extended regressor.

At the mixing step of DREM procedure the regression model (16) is multiplied by the adjugate matrix \( \text{adj} \{ \Phi_f(t) \} \) and we get

\[
\Psi(t) = \Delta(t) \theta,
\]
where $\Psi(t) := \text{adj}\{ \epsilon \Phi_f(t) \} \epsilon \Psi_f(t) = [\Psi_1(t) \ldots \Psi_n(t)]^T$, $\Delta(t) := \text{det}\{ \epsilon \Phi_f(t) \}$, $\text{adj}\{ \cdot \}$ is the adjugate matrix, $\text{det}\{ \cdot \}$ is the determinant.

Rewrite the equation (17) in a component way:

$$\Psi_i(t) = \Delta(t) \theta_i, \quad i = \overline{1, n},$$

(15)

where $\Delta(t) \in \mathbb{R}$, $\Psi(t) \in \mathbb{R}$.

Parameters estimates of the first order regression model (18) can be obtained separately using the standard gradient method [12]:

$$\dot{\theta}_i(t) = \gamma_i \Delta(t) \left( \Psi_i(t) - \Delta(t) \tilde{\theta}_i(t) \right),$$

(16)

where $\tilde{\theta}_i(t) \in \mathbb{R}$ is an estimate of $\theta_i$, $\gamma_i \in \mathbb{R}^+$ is a tuning gain.

The error model for $\tilde{\theta}_i(t) = \theta_i - \tilde{\theta}_i(t)$ is expressed as

$$\tilde{\theta}_i(t) = -\gamma_i \Delta^2(t) \tilde{\theta}_i(t).$$

(17)

One can easily find the solution for (20):

$$\tilde{\theta}_i(t) = \tilde{\theta}_i(0) e^{-\gamma_i \int_0^t \Delta^2(r) dr}.$$  

(18)

As shown in [16], the algorithm (19) provides an exponential convergence of the estimation error $\tilde{\theta}_i(t)$ to zero at $i = \overline{1, n}$.

Let us replace $\tilde{\theta}_i(t)$ with $\theta_i - \tilde{\theta}_i(t)$ in equation (21):

$$\theta_i - \tilde{\theta}_i(t) = \theta_i W(t) - \tilde{\theta}_i(0) W(t),$$

(19)

where $W(t) := e^{-\gamma_i \int_0^t \Delta^2(r) dr}$.

Now from (22) parameter $\theta_i$ can be explicitly found

$$\theta_i (1 - W(t)) = \tilde{\theta}_i(t) - \tilde{\theta}_i(0) W(t),$$

(20)

and for some $t_{ft} > n(h + b)$

$$\tilde{\theta}_i(t) = \frac{1}{1 - W(t)} \left( \tilde{\theta}_i(t) - \tilde{\theta}_i(0) W(t) \right), \quad t \geq t_{ft}.$$  

(21)

where $\tilde{\theta}_i(t)$ is the finite-time estimates of the parameter $\theta_i$, $i = \overline{1, n}$ of the regression model (9).

We can reconstruct $c_i = \cos(\omega_i t)$ from Vieta's formulas (10) using parameter estimates (24):

$$\tilde{\theta}_i^{ft}(t) = -\hat{c}_i^{ft}(t) - \hat{c}_j^{ft}(t) \ldots - \hat{c}_n^{ft}(t),$$

$$\tilde{\theta}_2^{ft}(t) = \hat{c}_1^{ft}(t) \hat{c}_2^{ft}(t) + \hat{c}_1^{ft}(t) \hat{c}_3^{ft}(t) + \ldots + \hat{c}_{n-1}^{ft}(t) \hat{c}_n^{ft}(t),$$

$$\ldots$$

$$\tilde{\theta}_n^{ft}(t) = (-1)^n \hat{c}_1^{ft}(t) \hat{c}_2^{ft}(t) \ldots \hat{c}_n^{ft}(t).$$

(22)

Finally, we obtain the desired frequency estimates, using identity $c_i = \cos(\omega_i t)$, as

$$\tilde{\omega}_i^{ft}(t) = \frac{1}{n} \arccos(\hat{c}_i^{ft}(t)), \quad i = \overline{1, n},$$

(23)

at the predefined time $t_{ft}$.

The estimation algorithm (26) guarantees the convergence of frequency estimates $\tilde{\omega}_i^{ft}(t)$ to the true values $\omega_i$, $i = \overline{1, n}$ for the finite time $t_{ft}$.  


4.2. Finite-time amplitudes and phases estimation

In this subsection, we construct a new linear regression model, which depends on the measured signal \( y(t) \), the frequency estimates \( \tilde{\omega}_i^{ft}(t) \), and the vector of unknown parameters related to amplitudes \( A_i \) and phases \( \phi_i \), \( i = 1, \ldots, n \) of the signal (6).

Introduce the signal \( \tilde{y}(t) \), similar to (6), which uses the estimates \( \tilde{\omega}_i^{ft}(t) \) instead of the frequency values \( \omega_i \):

\[
\tilde{y}(t) = \sum_{i=1}^{n} A_{oi} \sin(\tilde{\omega}_i^{ft}(t) t + \phi_i).
\]  

(24)

Represent the measured signal \( y(t) \) as \( \tilde{y}(t) \) for \( t \geq t_{ft} \):

\[
y(t) = \sum_{i=1}^{n} A_{oi} \sin(\omega_i t + \phi_i) = \sum_{i=1}^{n} A_{oi} \sin(\tilde{\omega}_i^{ft}(t) t + \phi_i) = \sum_{i=1}^{n} A_{oi} \sin(\tilde{\omega}_i^{ft}(t) t + \phi_i) + \sin(\tilde{\omega}_i^{ft}(t) t + \phi_i)] =
\]

(25)

where \( \tilde{\omega}_i^{ft}(t) = \omega_i - \hat{\omega}_i^{ft}(t) \) are the frequency estimation and \( \hat{\omega}_i^{ft}(t) = 0 \) for \( \forall t \geq t_{ft}, i = 1, \ldots, n \).

Then from (28) we get

\[
y(t) = \sum_{i=1}^{n} A_{oi} [\sin(\hat{\omega}_i^{ft}(t) t) \cos \phi_i + \cos(\hat{\omega}_i^{ft}(t) t) \sin \phi_i].
\]  

(26)

Rewrite expression (29) as a linear regression model:

\[
v(t) = \xi^T(t) \eta,
\]

(27)

where \( v(t) \in \mathbb{R} \) is a regressand, \( \xi^T(t) = [\xi_1(t) \ldots \xi_{2n}(t)] \in \mathbb{R}^{2n} \) is a regressor, \( \eta^T = [\eta_1 \ldots \eta_{2n}] \in \mathbb{R}^{2n} \) is an unknown parameter:

\[
v(t) = v(t), \xi(t) = \begin{bmatrix}
\sin(\hat{\omega}_1^{ft}(t) t) \\
\cos(\hat{\omega}_1^{ft}(t) t) \\
\sin(\hat{\omega}_2^{ft}(t) t) \\
\cos(\hat{\omega}_2^{ft}(t) t) \\
\vdots \\
\sin(\hat{\omega}_n^{ft}(t) t) \\
\cos(\hat{\omega}_n^{ft}(t) t)
\end{bmatrix}, \eta = \begin{bmatrix}
A_{o11} \cos \phi_1 \\
A_{o11} \sin \phi_1 \\
A_{o22} \cos \phi_2 \\
A_{o22} \sin \phi_2 \\
\vdots \\
A_{onn} \cos \phi_n \\
A_{onn} \sin \phi_n
\end{bmatrix}.
\]

In the next step, we construct amplitudes \( A_{oi} \) and phases \( \phi_i \), \( i = 1, \ldots, n \) estimates of the signal (6) from the linear regression model (30). For this purpose, we apply the DREM method to (30) similar to the approach described in subsection 4.1. As a result, we obtain \( 2n \) first order independent linear regression models, whose unknown parameters are the components of the constant vector \( \eta \) of the initial regression model (30). As a result, we obtain the estimates \( \hat{\eta}_j^{ft}(t) \), \( j = 1, 2n \) of the unknown vector components. The estimation errors \( \tilde{\eta}_j^{ft}(t) \) converge to zero in the finite time, \( j = 1, 2n \).

Then the estimates \( \hat{A}_i^{ft}(t) \) for the amplitudes \( A_{oi} \) can be obtained from \( \hat{\eta}_i^{ft}(t) \) as follows:

\[
\hat{A}_i^{ft}(t) = \sqrt{[\tilde{\eta}_i^{ft}(t)]^2 + [\hat{\eta}_i^{ft}(t)]^2},
\]

where \( \tilde{\eta}_j^{ft}(t) \) are the estimates of the unknown vector components \( \eta_j, j = 1, 2n \).

The estimation algorithm provides convergence of the errors \( \hat{A}_i^{ft}(t) = A_{oi} - \hat{A}_i^{ft}(t) \) to zero at the predefined finite time \( \tilde{t}_{ft} > 0 \).

Now, the central frequency estimate is
\[ \hat{\omega} f^t_0(t) = \hat{\omega} f^t \arg\max_{\omega}(t), \forall t > \bar{t}_f. \]

5. Simulation results

In this section, we present simulation results that illustrate the efficiency of the proposed multipurpose control. All simulations have been performed in MATLAB Simulink.

Let us illustrate the idea of the separate filtering correction by a practical example based on the DP control system for the vessel ‘Northern Clipper’ (the length is \( L = 76,2 \) m and mass is \( m = 4,59 \cdot 10^6 \) kg). The matrices of model, observer, and regulator were taken from [1, 3].

The preferred position and external disturbance parameters are the following:

\[
\eta^* = [x^*, y^*, \psi^*]^T = [30, 30, 15]^T, \\
d(t) = [3 \cdot 10^4, -4 \cdot 10^4, -2 \cdot 10^6]^T, \\
\eta_\omega(t) = 0.01 \sin(0.455t) + 0.003 \sin(0.39t).
\]

The estimation of unknown frequencies and amplitudes of the external disturbance \( \eta_\omega(t) \) is performed from the signal (5). Fig. 1 shows the graph of the signal \( \hat{y}(t) \). As can be noticed, the influence of the term \( \varepsilon(t) \) in the signal (5) can be neglected, starting from the moment 500 s.

![Fig. 1. The observation error \( \hat{y}(t) \)](image)

The parameters of the proposed parameter estimation algorithm are \( h = 2, b = 2.5, \ varepsilon = 1, \gamma_1 = \gamma_2 = 2 \cdot 10^7 \). Estimation of the external disturbance parameters by the proposed approach starts from 600 s. The behaviour of the estimators is shown in Fig. 2–3.

![Fig. 2. Frequency estimates \( \hat{\omega}_1 f^t(t) \) and \( \hat{\omega}_2 f^t(t) \) by the proposed finite time method](image)
To illustrate the control filtering effect, let us consider a stabilization process presented by the graph of the third component of the control vector $\tau(t)$ in Fig. 4 for the closed-loop system. Before the 1000th second, the controller (2) works with the astatic corrector

$$\tau_d(t) = -K_d \dot{\theta}(t) - R^T(y)K_p(\theta(t) - \theta^*), \quad t < 1000 \text{ s},$$

but then the tuned corrector (2) switches on to provide filtering features.

As can be seen in the Fig. 4, the proposed control law correction provides significant a filtering effect.

Fig. 5 shows results of vessel motion simulation for the considered closed-loop DP system, where it is possible to see the transient positioning process $\psi(t)$ under influence of the sea waves.
6. Conclusion
We have synthesized the multipurpose control law for nonlinear model of a vessel, solving the dynamic positioning problem and providing economical mode of executive mechanisms work under the influence of a multi-harmonic external disturbance. The specialized structure of nonlinear control law allows to adjust the nonlinear observer, the feedback control, the dynamic corrector, and the estimator of external disturbances frequencies independently from each other. We use primary frequency finite-time estimation method, involved in tuning of the controller. The proposed approach is focused on implementation of the control law on ship board.

References
[1] Veremey E. I. Separate filtering correction of observer-based marine positioning control laws // International journal of control. 2017. Vol. 90, no. 8.P. 1561–1575.
[2] Sørensen A. J. Marine control systems propulsion and motion control of ships and ocean structures lecture notes. — 2012.
[3] Fossen T. I., Strand J. P. Passive nonlinear observer design for ships using lyapunov methods: full-scale experiments with a supply vessel // Automatica. 1999. Vol. 35, no. 1. P. 3–16.
[4] Koschorrek P., Siebert C., Haghani A., Jeinsch T. Dynamic positioning with active roll reduction using voith schneider propeller // IFAC- PapersOnLine. 2015. Vol. 48, no. 16. P. 178–183.
[5] Loria A., Fossen T. I., Panteley E. A separation principle for dynamic positioning of ships: Theoretical and experimental results // IEEE Transactions on Control Systems Technology. 2000. Vol. 8, no. 2. P. 332–343.
[6] Veremey E. I. Optimization of filtering correctors for autopilot control laws with special structures // Optimal Control Applications and Methods. 2016. Vol. 37, no. 2. P. 323–339.
[7] Veremey E. I. Special spectral approach to solutions of siso h-optimization problems // International Journal of Automation and Computing. 2019. Vol. 16, no. 1. P. 112–128.
[8] Aranovskiy, S. V., Bobtsov, A. A., Ortega, R., & Pyrkin, A. A. (2016). Improved transients in multiple frequencies estimation via dynamic regressor extension and mixing. IFAC-PapersOnLine, 49(13), P. 99–104.
[9] Wang, J., Gritsenko, P. A., Aranovskiy, S. V., Bobtsov, A. A., Pyrkin, A. A. (2017). A method for increasing the rate of parametric convergence in the problem of identification of the sinusoidal signal parameters. Automation and Remote Control, 78(3), P. 389–396.
[10] Gromov V. S. et al. First-order frequency estimator for a pure sinusoidal signal //2017 25th Mediterranean Conference on Control and Automation (MED). IEEE, 2017. P. 7–11.
[11] A. Vediakova, A. Vedyakov, A. Pyrkin, A. Bobtsov, and V. Gromov, Frequency estimation of multi-sinusoidal signals in finite-time, 2020.
[12] P. A. Ioannou and J. Sun, Robust adaptive control. California: PTR Prentice-Hall, 1996.