Fermi edge singularities in X-ray spectra of strongly correlated fermions

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Abstract

We discuss the problem of the X-ray absorption in a system of interacting fermions and, in particular, those features in the X-ray spectra that can be used to discriminate between conventional Fermi-liquids and novel ”strange metals”. Focusing on the case of purely forward scattering off the core-hole potential, we account for the relevant interactions in the conduction band by means of the bosonization technique. We find that the X-ray Fermi edge singularities can still be present, although modified, even if the density of states vanishes at the Fermi energy, and that, in general, the relationship between the two appears to be quite subtle.
Since its foundation in the late fifties, the theory of Fermi liquids has come a long way exploring the limits of its own applicability. However, the quest for possible departures from the conventional Fermi liquid behavior still remains one of the mainstays of the modern condensed matter theory.

It is well known that Fermi liquids can be more easily destroyed in low dimensions. As a common example, in one spatial dimension (1D) the Fermi liquid theory utterly fails even for a seemingly innocuous arbitrary weak short-range repulsion. The corresponding non-Fermi-liquid (NFL) metallic state is characterized by power-law decaying correlation functions and is referred to as the Luttinger liquid (LL) which can be thought of as a marginal deformation of the Fermi liquid.

Furthermore, the unscreened Coulomb interactions are known to drive 1D fermions into yet another, distinctly NFL, regime where correlations decay faster than any power law.

It is generally believed that in higher \((D > 1)\) spatial dimensions a NFL behavior can stem from sufficiently long-ranged and/or retarded ("singular") interactions. However, the necessary criteria remain unknown, which leaves room for a NFL regime to occur even as a result of non-singular, yet sufficiently strong, repulsive interactions.

In theory, a NFL behavior is commonly expected to manifest itself in single electron spectroscopy and, in particular, in angular resolved photoemission at low energies of incident photons. However, even for such a widely recognized candidate for a "strange metal" as the normal state of the high \(T_c\) cuprates, direct deduction of any ultimate evidence of the NFL behavior (e.g., electron self-energy) from the photoemission data has proven to be a tedious task \[1\].

In the present paper, we discuss a different kind of features that have already been extensively studied in a variety of conventional metals: the Fermi-edge singularities (FES) in the high-energy X-ray absorption. First predicted theoretically for weakly interacting (Fermi-liquid-like) metals back in 1967 \[2,3\], the FES became one of the hallmarks of many-body Fermi systems. However, the effect of interactions in the conduction band on the FES has not drawn much attention, except for the case of the 1D LLs studied by a number of
In retrospect, the FES provided the first example of a much more generic phenomenon of the "orthogonality catastrophe" (OC). The latter implies that in the presence of a sudden perturbation the ground states of an infinite Fermi system before and after the perturbation was switched on appear to be strictly orthogonal to each other. In the problem of the X-ray-induced photoemission such a step-like time-dependent perturbation $V(t) = V\Theta(t)$ corresponds to the attractive potential of a deep core level stripped off its electron by an incident X-ray photon.

Following the instantaneous shakeup, the initial ground state $|0>\,$ of the Fermi system tends to readjust and evolves into the final one $|V>\,$ by virtue of creating coherent multiple particle-hole pairs. These bosonic excitations dominate in the action $S_{oc}(t)$ describing the process of relaxation of the initial ground state and lead to the suppression of the time-dependent overlap between the two ground states: $<0|V>|\sim \exp(-S_{oc}(t)) \,$ at times $t \to \infty$.

Being closely related to the Green function of a localized core hole described by the operator $d(t)$ [3], the overlap factor controls the shape of the photoemission peak corresponding to the absorption of hard X-ray photons that knock core electrons out of the system

$$P(\omega) \propto \text{Im} \int_0^\infty dt e^{i\omega t} <Td(t)d^\dagger(0)>$$

Here $\omega$ is the photon energy measured from the threshold equal to the sum of the binding energy of the core level and the exit work function for the runaway electron.

In the standard Fermi liquid case the action $S_{oc}(t)$ diverges logarithmically, which gives rise to the asymmetrical power-law singularity of the absorption peak [2,3]

$$P(\omega) \propto \Theta(\omega)|\omega|^{-1+2\sum_l \delta_l^2/\pi^2}$$

that would have been absent in an insulating state where at zero temperature the peak remains sharp: $P(\omega) \propto \delta(\omega)$.

The partial phase shifts $\delta_l$ in Eq.(2) can be expressed in terms of the angular momentum harmonics of the Fourier transformed core hole potential: $\int d\mathbf{r} V(\mathbf{r}) e^{i\mathbf{r} \cdot (\mathbf{k}-\mathbf{k}')} = \sum_l Y_l(\hat{\mathbf{k}}) -$
\( \hat{\Omega}' \hat{V}_l(k_F) \) (throughout this paper we include all the angular momentum quantum numbers into the definition of "\( l' \)" and the density of states (DOS) in the conduction band \( \nu(\omega) = \frac{1}{\pi} \int d\mathbf{k} \text{Im} G(\mathbf{k}, \omega) \) taken at the Fermi energy \( \nu_F = \nu(\omega = 0) \):

\[
\tan \delta_l = -V_l \nu_F \tag{3}
\]

At first sight, Eq. (3) seems to imply that DOS must remain finite for the divergence (2) to occur. In order to find a possible caveat in this seemingly unavoidable conclusion we consider the simplest, short-range isotropic, core hole potential \( V(\mathbf{r}) = V \delta(\mathbf{r}) \) and, following the original work by Nozieres and de Dominicis [6], introduce a transient Green function \( G(t, t'|V) \) which allows the OC action to be cast in the form: \( S_{\text{oc}}(t) = \int_0^V dV' \int_0^t dt' G(t, t'|V) \).

For free fermions this Green function obeys the equation

\[
G(t, t'|V) = G(0, t - t') + V \int_0^t dt'' G(t, t''|V) G(0, t'' - t') \tag{4}
\]

which relates it to the time dependence of the propagator \( G(\mathbf{r}, t) \) of the conduction band fermions in the absence of the core hole potential.

The above, naive, conclusion would then imply that even with the interaction effects taken into account the local value of the propagator \( G(\mathbf{0}, t) \) would have to retain its Fermi liquid-like \( \propto 1/t \) behavior, the exponent in the power-law divergence (2) being determined by the corresponding prefactor.

Below we comment on the case of fermions interacting via \( U(\mathbf{r}) \propto 1/r^{2-\eta} \) pairwise potential which demonstrates that the above condition may be somewhat more relaxed than the stringent requirement for a system to remain a Fermi liquid. However, even in this "mildly NFL" case Eq.(4) ceases to be valid, because it misses important vertex corrections that describe the effect of the electron interactions on the coupling to the core hole potential \( V(\mathbf{r}) \).

In the extensively studied case of the 1D LL, for instance, crucial relevance of the vertex corrections can be seen from the fact that although the faster-than-1/t decay of the fermion propagator \( G(0, t) \propto t^{-(2+g+g^{-1})/4} \), where \( g \) is the Luttinger parameter (0 < \( g < 1 \) for short-
range repulsive interactions), does imply a vanishing DOS, the FES retain their algebraic behavior with the \( g \)-dependent exponents \[4\].

In the 1D case of purely forward scattering off the core hole potential, one can relate the robustness of the FES to the fact that the density of particle-hole pairs with small momenta given by the integral of the density correlation function \( \chi(\omega, q) \) maintains its non-interacting functional form: 
\[
\int_0^{Q<k_F} Im\chi(\omega, q) dq \propto \omega \[4\].
\]

This property is due to the known asymptotically exact cancellation between the self-energy and vertex interaction corrections to the density correlation function at small momenta. In order to account for both types of corrections the previous studies \[4\] resorted to the standard method of 1D bosonization.

Furthermore, the bosonization approach allows one to treat an even more intricate 1D backward scattering problem. Unlike the case of pure forward scattering, here the exponents controlling the power-law FES assume universal values which turn out to coincide with those corresponding to the unitary s-wave scattering \((\delta_0 = \pi/2)\) in a Fermi liquid \((g = 1)\) \[5\].

Inspired by the success of 1D bosonization, there has been a strong recent effort toward extending this method to higher dimensions \[7,8\]. In what follows we employ the “tomographic” version of this technique (see Refs. \[7\]) which is capable of yielding asymptotically exact results provided that fermions undergo predominantly forward scattering due to the interactions in the conduction band and the core hole potential.

Under these conditions the Lagrangian of the problem can be expressed solely in terms of the bosonic partial densities \( \rho^\sigma_{\hat{\Omega}}(r, t) \) associated with different points on the Fermi surface parameterized by the unit vector \( \hat{\Omega} \). These degrees of freedom obey the infinite algebra
\[
[\rho^\sigma_{\hat{\Omega}}(r, t), \rho^{\sigma'}_{\hat{\Omega}'}(r', t)] = K_{\sigma\sigma'}\delta(\hat{\Omega} - \hat{\Omega}')(\nabla \hat{\Omega})\delta(r - r')
\] (5)

Non-commutativity of the density operators at the same point of the Fermi surface can be interpreted as the "chiral anomaly" which reflects, in compliance with the Luttinger theorem, conservation of the total number of states enclosed by the nominal Fermi surface in the presence of interactions between fermions.
In the case of sufficiently weak spin-independent interactions the $K$-matrix reduces to the unit: $K_{\sigma\sigma'} = \delta_{\sigma\sigma'}$. By introducing the $K$-matrix into the theory one can incorporate strong correlations between fermions of opposite spins which modify the very kinematics of the system and cannot be treated as regular quadratic Landau-type terms in the effective Lagrangian.

Although the conduction band fermions $\psi_\sigma(r, t)$ couple to the forward scattering core hole potential only via the charge density operator

$$\int d^3 r \sum_\sigma \psi_\sigma^\dagger(r)V(r)\psi_\sigma(r) \approx V \sum_\Omega \sum_\sigma \rho_\sigma^\dagger,$$

to incorporate the abovementioned exclusion principle-type spin-dependent correlations in the conduction band. Then the relevant part of the Lagrangian reads as

$$L = \sum_\Omega \int d^3 q \rho_\Omega^\dagger K_{\sigma\sigma'}^{-1}(q) \frac{i\partial_t - v_F \hat{\Omega} q}{\hat{\Omega} q} \rho_{\sigma'}^\dagger(-q) - \frac{1}{2} \sum_{\Omega, \Omega'} \int d^3 q \rho_\Omega^\dagger(q) F_{\Omega, \Omega'}(q) \rho_{\Omega'}^\dagger(-q) +$$

$$+ d^\dagger(i\partial_t - E_d)d - (Vd^\dagger d - E_F) \sum_\Omega \int d^3 q \rho_\Omega^\dagger(q)$$

where we introduced a generalized (momentum and/or frequency-dependent) Landau function $F_{\Omega, \Omega'}(q)$ to describe the spin-independent ("residual") part of the forward scattering between the conduction band fermions.

The quantity of primary physical interest is the X-ray absorption/emission intensity in the processes of electron transfer from the core level to the conduction band and vice versa

$$I(\omega) \propto \text{Im} \int_0^\infty dt e^{i(\omega + E_F - E_d)t} D(t), \quad D(t - t') =< Td(t)\psi_\sigma(0, t)\psi_\sigma^\dagger(0, t')d^\dagger(t') >$$

The amplitude (7) can be conveniently expressed as a gaussian average that has to be taken with respect to the Lagrangian (6)

$$D(t) \propto < \exp[i \int d^3 q \int d\omega \left( J_\Omega(t, \omega, q)\phi_\Omega^\dagger(-q) + V j(t, \omega)\rho_\Omega^\dagger(-q) \right) + V j(t, \omega)\rho_\Omega^\dagger(-q) ] >$$

where the source densities

$$J_\Omega(t, \omega, q) = \frac{1 - e^{i\omega t}}{\omega - v_F \hat{\Omega} q}, \quad j(t, \omega) = \frac{1 - e^{i\omega t}}{\omega}$$

$$\text{(9)}$$
represent a conduction band fermion and an immobile core hole, respectively, while $\phi_\Omega(q)$ is an auxiliary Hubbard-Stratonovich variable conjugate to $\rho_\Omega^\sigma(q)$

$$\int D\rho_\Omega^\sigma \exp\left[ -\frac{1}{2} \rho_\Omega^\sigma K_{\sigma\sigma'} q_\Omega \rho_\Omega^\sigma' \right] = \int D\phi_\Omega^\sigma \exp\left[ -\frac{1}{2} \phi_\Omega^\sigma K_{\sigma\sigma'} i\dot{q} - v_F \Omega q \phi_\Omega^\sigma + \phi_\Omega^\sigma \rho_\Omega^\sigma' \right]$$  \hspace{1cm} (10)

In order to perform the averaging explicitly one first has to compute the kernel of the angular decomposition for the density correlator $\chi(\omega, q) = \sum_{\Omega,\Omega'} \sum_{\sigma\sigma'} \chi_{\Omega,\Omega'}^{\sigma\sigma'}(\omega, q)$. In practice, this amounts to inverting the operator

$$\chi_{\Omega,\Omega'}^{-1}(\omega, q) = \delta(\Omega - \Omega') K_{\omega} - v_F(\Omega q)$$  \hspace{1cm} (11)

Having carried out the gaussian functional averaging one arrives at the result

$$D(t) \propto \exp(-S(t)), \hspace{0.5cm} S = S_{\text{dos}} + S_{\text{exc}} + S_{\text{oc}}$$  \hspace{1cm} (12)

where the three different contributions to the total action $S(t)$ can be identified with, respectively, time dependence of the local conduction band propagator $G(0, t)$ which is directly related to DOS:

$$S_{\text{dos}}(t) = \int d\omega (1 - \cos \omega t) \text{Im} \int dq \chi_{\Omega,\Omega'}(\omega, q)$$  \hspace{1cm} (13)

the "excitonic" term due to the interaction between the conduction band and the core hole in the final state:

$$S_{\text{ex}}(t) = V \int d\omega \frac{1 - \cos \omega t}{\omega} \text{Im} \sum_{\Omega'} \int dq \chi_{\Omega,\Omega'}(\omega, q)$$  \hspace{1cm} (14)

and the OC term perse:

$$S_{\text{oc}}(t) = V^2 \int d\omega \frac{1 - \cos \omega t}{\omega^2} \text{Im} \sum_{\Omega,\Omega'} \int dq \chi_{\Omega,\Omega'}(\omega, q)$$  \hspace{1cm} (15)

We first comment on the use of Eqs.(13-15) in the free limit where one is supposed to recover the known results of Refs. [2,3] by simply putting the Landau function equal to zero.

Although for a sufficiently weak attractive core hole potential ($0 < \delta_l << 1$) we do reproduce the expected results for $S_{\text{dos}} = \log t$ and $S_{\text{ex}} = (2V\nu / \pi) \log t$, the expression for
\(S_{oc}\) turns out to be formally divergent. This non-physical divergence is solely due to our assumption of a strictly forward scattering off the core hole, for in this extreme limit all the angular harmonics of the core hole potential become equal: \(V_l = V\), and therefore the sum
\[
\sum_{\hat{\Omega}} V^2 = \sum_{l} V_l^2 \tag{16}
\]
diverges. As a result, the vanishing OC factor \(\exp(-S_{oc})\) in \(I(\omega)\) outpowers any divergence that may result from the excitonic DOS enhancement at the location of the core hole, and, as a result, all the FES are gone.

However, for any realistic core hole potential \(V(\mathbf{r})\) the sum (16) is finite, and the overall energy dependence of the X-ray absorption near the threshold assumes its classical form:
\[
I(\omega) \propto \Theta(\omega)|\omega|^{-2\delta_0/\pi + 2\sum_l \delta_l^2/\pi^2} \tag{2,3}.
\]

It is worthwhile mentioning that the opposite case of an isotropic core hole potential (\(\delta_l = 0\) for all \(l \neq 0\)) allows one to construct an alternate bosonization scheme, thanks to the effectively 1D character of the radial motion of non-interacting fermions in the \(s\)-wave orbital channel [3].

Turning now to applications of the general formalism, we first consider the limit of an exclusively "intra-tomograph" spin-independent interaction that can only couple fermions with the same direction of the momenta: \(K = 1\) and \(F_{\hat{\Omega}\hat{\Omega}'}(q) \propto \delta(\hat{\Omega} - \hat{\Omega}')\). A straightforward analysis shows that, similarly to the chiral 1D case, the only effect of such an interaction is a multiplicative renormalization of the Fermi velocity \(v_F\) and, accordingly, the phase shifts: \(\delta_l \rightarrow \delta_l/(1 + F/v_F)\) that determine the, otherwise unaltered, FES exponents.

We note, in passing, that in order for a quasi-1D LL-like behavior to occur in \(D > 1\)-dimensions, the spin-independent fermion interaction must couple each tomograph \(\hat{\Omega}\) to its antipodal one at \(-\hat{\Omega}\): \(F_{\hat{\Omega}\hat{\Omega}'}(q) \propto \delta(\hat{\Omega} - \hat{\Omega}') + \delta(\hat{\Omega} + \hat{\Omega}')\).

In this situation the effective Luttinger parameter is given by the ratio \(g = v_F/c\) between the Fermi velocity and the speed of zero sound that can be read off from the low-\(q\) limit of the density correlator: \(\chi(\omega, q) = \sum_{\hat{\Omega}}(\hat{\Omega}q)^2/(\omega^2 - c^2(\hat{\Omega}q)^2)\).

Again, in a close analogy with the situation in the 1D LL, coupling between the opposite
points on the Fermi surface gives rise to, both, vanishing DOS \( \nu(\omega) \sim |\omega|^{(g+1/g-2)/4} \) and the modified, yet non-zero, FES exponents

\[
P(\omega) \propto |\omega|^{-1+2g^3 \sum_l \delta_l^2/\pi^2}, \quad I(\omega) \propto \nu(\omega)|\omega|^{-2g\delta_0/\pi + 2g^3 \sum_l \delta_l^2/\pi^2}
\]

where the phase shifts are given by Eq.3 with a finite \( \nu_F \) corresponding to the non-interacting case \( g = 1 \).

As follows from Eq.(17), repulsive interactions weaken the OC exponent via the reduced compressibility, thereby enhancing the overlap \(< 0|V >\) between the initial and the final ground states.

In principal, the asymmetrical profile of the photoemission peak \( P(\omega) \) can be measured in absorption of hard X-rays that knock core electrons out of the system, whereas measuring \( I(\omega) \) requires soft X-rays that can only promote core electrons up to the conduction band. In practice, however, for the exponents in question to be reliably deduced from a real X-ray spectrum, all the absorption lines and thresholds have to be well separated from one another. Apart from that, one has to carry out a deconvolution of the measured intensity onto the FES factor and the extra factors describing recombination of the core hole as well as other mechanisms of the line broadening.

Furthermore, if the core hole potential can be approximated by the first two orbital harmonics: \( V(\hat{\Omega} - \hat{\Omega}') \approx V_0 + V_1 Y_1(\hat{\Omega} - \hat{\Omega}') \), then by measuring the two exponents in (17) and invoking the Friedel sum rule \( (2 \sum_l \delta_l = \pi) \) one might be able to extract the numerical values of, both, the relevant phase shifts \( \delta_{0,1} \) and the Luttinger parameter \( g \) that serves as a measure of the strength of the LL-like correlations in the conduction band.

Yet another evidence in favor of the above quasi-1D behavior would be the presence of the FES in the X-ray spectra even if the core hole has a finite, as opposed to infinite, mass. Such an observation would be in a marked contrast with the situation in the \( D > 1 \) Fermi liquids, where the hole recoil is known to reduce the number of density excitations, thereby smearing the FES out [10].

Therefore, one indirect manifestation of the "tomographic" picture of fermion scattering...
would be an observation of the FES in valence band photoemission and a related algebraic decay (as opposed to the delta-function peak) of the spectral function of the mobile valence band hole [11].

An effective 1D LL-like behavior was conjectured in the very beginning of the high $T_c$ saga on the basis of such suggestive experimental findings about the normal state of the high $T_c$ cuprates as the anomalous power-law decay of the optical conductivity $\sigma(\omega) \propto \omega^{-1+2\alpha}$ with $\alpha = 0.15 \pm 0.05$, the $\propto 1/T^2$ temperature dependence of the low-frequency limit of the magnetic susceptibility at momentum $\mathbf{q} = (\pi, \pi)$, the $\propto \omega^{-1/2}$ tail of the electron spectral function $\text{Im}G(\mathbf{k}, \omega)$, the linear $c$-axis tunneling density of states $dI/dV \sim V$, and others [12].

Thus far, traditional perturbative analyses did not turn in supporting evidence that the above quasi-1D regime may indeed occur in a low-density weakly interacting system of fermions with a spherical Fermi surface. Nonetheless, one can expect the situation to be different in paramagnetic lattice systems close to half-filling where the Hubbard-like no-double-occupancy constraint gives rise to the off-diagonal matrix elements $K_{\sigma,-\sigma} = \phi/\pi$ in Eq.(5) which are proportional to the on-shell phase shift $-\pi/2 \leq \phi \leq 0$ for a pair of fermions with opposite spins [7].

A simple analysis shows that even in the absence of any coupling between different "tomographs" the modified commutation relations (5) alone give rise to the anticipated spin-charge separation and change the OC exponent entering both $P(\omega)$ and $I(\omega)$ from $2 \sum \delta^2_l/\pi^2$ to $(1 + 2\phi/\pi + \sqrt{1 + (2\phi/\pi)^2}) \sum \delta^2_l/\pi^2$ while leaving DOS unaltered (of course, the latter gets affected, too, once coupling between different $\hat{\Omega}$’s is introduced).

As a more traditional alternative to the kinematical constraint-type interactions, it is believed that even a continuous (low-density) spinless fermion system may develop a NFL behavior provided that the pairwise fermion interaction potential $U(\mathbf{r})$ is sufficiently long-ranged. In terms of the Fourier transform $U(\mathbf{q}) = \lambda/q^n$ the sufficient condition reads as $\eta \geq 2(D-1)$ [13]. In Ref. [13], the 2D fermion distribution function was found to exhibit a non-analytical singularity instead of the finite step at the Fermi surface: $\Delta n(k) \sim |k - k_F|^\eta$
with the exponent $\beta \propto \lambda^{1/2}$, which prompted speculations that all the other characteristics should exhibit a marked NFL behavior as well.

Surprisingly enough, our subsequent analysis of the physical 2D case reveals that Fermi surface DOS may not vanish even in the presence of the singular $U(q) \sim 1/q^2$ interactions. Likewise, the FES remain largely intact.

We arrived at these conclusions by considering the situation when the Landau function is independent of the location on the Fermi surface, although it may well be singular as a function of the transferred momentum. This, in turn, demands the solution of Eq.(11) to have the RPA form:

$$F_{\Omega, \Omega'}(q) = F(q) = \frac{U(q)}{1 + U(q)\Pi(\omega, q)}, \quad \Pi(\omega, q) = \sum_{\Omega} \frac{(\Omega q)}{(\omega - v_F(\Omega q))}$$

(18)

and, accordingly, the density correlator to be given by the RPA formula $\chi = \Pi/(1 + U\Pi)$.

Elaborating on Eq.(13) that represents the effect of the fermion interaction on DOS, we obtain

$$\nu(\omega) = \nu_F \int_0^\infty dt e^{i\omega t} \exp(-Im \int d\omega (1 - \cos \omega t) \int dq \frac{F(\omega, q)}{(\omega - v_F(\Omega q))^2})$$

(19)

Our analysis of the momentum integral in Eq.(19) shows that the most important correction to DOS originates from the momenta $q << \omega/v_F$. In this regime the RPA Landau function is given by the expression

$$F(\omega, q) = \frac{\lambda \omega^2}{q^n \omega^2 - \lambda q^2}$$

(20)

where the pole reveals a sublinear plasmon dispersion $\omega \sim q^{1-\eta/2}$.

After being plugged into Eq.(19), the Landau function (20) yields

$$S_{dos} \propto \beta \int_1^{E_F} d\omega \omega^{2(D-2)/(2-\eta)}.$$  Thus, for $\eta = 2(D - 1)$ and $1 \leq D < 2$ we obtain $S_{dos} \propto \log t$ which does imply a power-law DOS:

$$\nu(\omega) = \int_0^\infty dt/t e^{i\omega t - S_{dos}} \propto |\omega|^{\beta/(2-D)}.$$  

However, despite this suggestive behavior, in the physical dimension $D = 2$ the plasmon mode develops a gap and fails to generate any singular corrections to DOS. Nonetheless, this does not necessarily contradict the NFL-like behavior of the distribution function reported
in Ref. [13], since a non-linear plasmon dispersion breaks the symmetry between energies and momenta.

Hence, in 2D the X-ray absorption intensity \( I(\omega) \propto \nu_F \int_0^\infty e^{i\omega t - S_{oc+ex}} dt/t \) can only acquire a non-trivial energy dependence via the combined effect of the OC and "excitonic" terms in the total action (12)

\[
S_{oc+ex} = \int d\omega \frac{1 - \cos \omega t}{\omega^2} Im \sum_{\mathbf{q}'} \int d\mathbf{q}(1 + F(\omega, \mathbf{q})\Pi(\omega, \mathbf{q}))
\]

\[
= (2\delta_0 - \sum_{l} \delta_l^2) \int \frac{d\omega}{\omega}(1 - \cos \omega t) \int \frac{d^{d-1}q_{\perp}}{1 + \lambda/q^\eta}
\]  

which, in contrast to the momentum integral in (19), receives its main contribution from the momenta \( q > \omega/\nu_F \) while the contribution of the momenta \( q < \omega/\nu_F \) can only become important for a 1D short-range interaction (\( \eta = 0 \)).

At this point, a word of caution is in order. After having obtained Eq.(21), one may question the use of bosonization employed for its derivation. Indeed, unlike Eq.(19), the region of momenta contributing to the integral over \( q_{\parallel} \) does not scale down to zero in the asymptotic limit \( \omega \to 0 \). In fact, one has to set the integration limit to be of order the inverse Fermi surface curvature for the classical results of Refs. [2,3] to be recovered for \( \lambda = 0 \).

Nonetheless, we believe that, being formulated solely in terms of the density correlator (or, more precisely, its angular decomposition \( \chi_{\hat{\Omega},\hat{\Omega}'}(\omega, \mathbf{q}) \)), the general Eqs.(13-15) remain qualitatively valid, thereby indicating that, although the singular repulsive interactions in the conduction band do affect the FES exponents via the reduced compressibility, they do not eliminate the FES altogether.

In a pursuit of a genuine \( D > 1 \) NFL behavior, one can proceed further by incorporating a finite inelastic scattering time \( \tau(\omega) \) into the theory. The latter is proportional to the optical conductivity \( \sigma(\omega) \) at frequencies \( \omega >> 1/\tau(T) \), while in the opposite limit it saturates at a finite value \( \tau(T) \).
As a result, the density modes acquire an overdamped character even in the absence of any real disorder (elastic scattering), and the pole of the density correlator \( \chi(\omega, q) = q^2/\left(\omega^2 - q^2 + i\omega/\tau(\omega)\right) \) moves to the imaginary axis.

In order to gain a better insight into the overdamped regime, we consider a generic Landau function \( F(\omega, q) = q^m/(i\omega + q^n) \) whose functional form is inspired by the studies of the compressible Quantum Hall states at even-denominator filling fractions \(^{14}\) as well as quantum critical points in itinerant magnets and Mott insulators \(^{15}\).

Computing the integrals (13-15) we obtain the following asymptotics for the interaction terms that add to the bare \( \log t \) terms corresponding to the non-interacting limit

\[
\Delta S_{\text{dos}} \propto t^{3-n-m-D}/n, \quad \Delta S_{\text{oc}} \propto t^{2-(m+D)/n}, \quad \Delta S_{\text{ex}} \propto t^{1-(D-1+m)/n}
\]

(22)

For \( n = 1 \) all of the above long time asymptotics coincide and, therefore, all the three terms in (22) are equally important, whereas for \( n > 1 \) it is the OC term that dominates over the other two. Moreover, for \( D + m < 2n \) the divergence of \( \Delta S_{\text{oc}} \) is strong enough for the FES to be replaced by a rounded edge whose profile appears to be strongly non-analytical if the parameters satisfy the additional condition \( n < D + m \):

\[
I(\omega) \propto \exp\left(-|\omega|^{-(2n+D+m)/(D+m-n)}\right)
\]

(23)

The non-algebraic behavior described by Eq.(23) gives the final result as long as the core hole potential can be considered as a predominantly forward scatterer, and the bosonized theory (6) remains gaussian. The same reservation applies to the conduction band interactions that we chose to be quadratic in terms of the bosonic density modes.

For a realistic core-hole potential, however, the non-forward-scattering contribution is likely to become increasingly more and more important at \( \omega \to 0 \). It remains to be seen whether or not an algebraic behavior controlled by some universal exponent gets restored at the lowest energies (see Ref. \(^{4}\) for the analysis of the problem of 1D backscattering where Eq.(15) yields \( S_{\text{oc}} \propto t^{2(1-g)} \)).

Indeed, in the framework of the conventional cluster expansion that does not rely on any bosonic representation, Eqs.(15) appears to be merely the first significant \( (k = 2) \) term in
the infinite series $S_{oc} = \sum_k S_{oc}^{(k)}$, where $S_{oc}^{(k)} \propto V^k$ is given by a convolution of the $k$ functions $\chi(\omega, q)$.

In analogy with the 1D backscattering problem, one may expect that a universal asymptotic behavior $S_{oc} \propto \log t$ with a prefactor independent of the strength of a generic (non-forward-scattering) core-hole potential occurs in the case when each $k^{th}$ order term of the cluster expansion (unless it vanishes) has a stronger divergence than the $(k - 1)^{th}$ one at $t \to \infty$. However, regardless of whether or not the photoemission line broadening washes this regime out, the universal exponents themselves do not provide much insight into a nature of the electron correlations in the conduction band.

Putting it another way, for a generic core-hole potential a simplified picture of the conduction band (suggested by the lowest order of the cluster expansion Eq.(15)) as a "Fermi bath" characterized solely by its spectral density of particle-hole excitations $A(\omega) = \int dq \Im \chi(\omega, q)$ can only hold at some intermediate time scales, until the higher terms $S^{(k)}$ with $k > 2$ come into play.

Before concluding, we briefly comment on the effect of a finite temperature that tends to further amplify the divergence of the expressions given by Eqs.(13-15) via the occupation number of thermally excited collective modes $N(\omega) \approx T/\omega$. However, the resulting hardening of the OC contribution may not be detectable, since it will be superimposed with the smearing of the FES features stemming from the free fermion Green functions $G(0, t) \propto 1/t \to \pi T/\sinh(\pi Tt)$.

To summarize, in this paper we considered the problem of the X-ray absorption in the presence of interactions in the conduction band. By restricting our attention to the case of the forward scattering core hole potential we were able to incorporate the fermion interactions in the framework of the bosonization approach. We illustrated the general method with such examples as the $D > 1$-dimensional "tomographic" LL, fermions with a singular pairwise potential, and coupling via an overdamped collective mode. We found that, in general, neither a vanishing single-particle DOS precludes the existence of the FES, nor can
one guarantee that the FES remain algebraic at all energy scales. Altogether, these findings testify in favor of the FES features in the X-ray spectra offering an independent way to probe many-body correlations in strongly interacting Fermi systems.

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