Structural Breaks, Inflation and Interest Rates: Evidence for the G7 countries.

Jesús Clemente, Antonio Montañés† and Marcelo Reyes
University of Zaragoza, Gran Vía 2, 50005 Zaragoza (Spain)
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Abstract

This paper challenges the commonly used unit root/cointegration approach for testing the Fisher effect for the economies of the G7 countries. We first prove that nominal interest and inflation rate can be better represented as being broken trend stationary variables. Later, we use the Bai-Perron procedure to show the existence of structural changes in the Fisher equation. When these characteristics are taken into account the Fisher hypothesis we can only offer evidence in favor of this hypothesis for the US, the French and the Japanese economies.

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†Corresponding author. email: amontane@posta.unizar.es
1 Introduction

One of most important results of classical economic theory is that the movements of nominal variables have no impact on real economic variables. This result, which can be verified by testing the long-run neutrality proposition, implies that a permanent movement in the inflation rate has no effect on the equilibrium real interest rate. The traditional way to represent this phenomenon is to decompose nominal interest rates into two separate components that reflect the expected inflation, on the one hand, and the ”real” interest rate, on the other. Following the very influential work of Fisher (1930), this relationship can be stated through the well-known Fisher equation:

\[ R_t = \pi_t^e + r_t \]  

where \( R \) represents the nominal interest rate, \( \pi^e \) is the expected rate of inflation and \( r \) is the (ex-ante) real interest rate. In simple economic models, this last variable is determined by deep structural parameters, such as investor preferences or the marginal efficiency of capital, and is often assumed to be constant over long horizons. According to (1), money lenders need a nominal interest rate that compensates them for the loss of purchasing power during the duration of the loan, with this compensation being proxied by the expected inflation. Thus, if we admit that there is no money illusion, then a change in the expected inflation rate should be fully transmitted to the nominal interest rate in order for the real interest rate to remain constant.

The information that (1) provides is quite useful both for theoretical researcher, as well as for taking economic policy decisions. For example, if the Fisher effect holds, then the expected inflation is a good predictor of the nominal interest rate. Further, evidence in favor of the superneutrality of money hypothesis is found. Consequently, it comes as no surprise that a huge volume of literature has directed its efforts towards the analysis of the relationship between nominal interest rates and inflation or, more exactly, towards whether the so-called Fisher effect holds. The most common approach starts by estimating the following equation:

\[ R_t = \alpha + \beta \pi_{t+1} + e_t \]  

where the presence of perfect rational expectations (\( \pi_{t+1} = \pi_t^e \)) is implicitly assumed. It is clear that whenever the value of the parameter \( \beta \), often referred to as the Fisher parameter, is equal to 1, this equation is equivalent to (1) and, therefore, we should conclude that the Fisher effect holds. At first sight, the analysis of this effect would appear to be quite straightforward, in the sense that it only requires the estimation of model (2) and, subsequently, the
testing of the null hypothesis $H_0$: $\beta = 1$. However, the literature confirms that there are several points which should be taken into account in order to accurately estimate this parameter and test for this hypothesis. Here, we are thinking in terms of the appropriate treatment of the time series properties of the variables, as well as the possible presence of changes in the values of the parameters $\alpha$ and, $\beta$, and, finally, the inclusion of dynamic effects. In this paper, we consider the relevance of all these points.

With the respect of the first point, there seems to be an almost unanimous opinion in the literature that favors the existence of unit roots in both the nominal interest and the inflation rates. Therefore, ”standard” econometric models are not longer valid; rather, the cointegration approach should be employed. We can cite several examples of the use of this unit root/cointegration approach, beginning with the seminal papers of Rose (1988) or Mishkin (1992), whose methodology has subsequently been applied in the more recent works of Crowder and Wohar (1999), Koustas and Serletis (1999), Rappach (2002) or Laatschs and Klein (2002), amongst many others. Nevertheless, other recent contributions, such as those of Malliaropoulos (2000), Lanne (2001), Olekalns (2001), Gil-Alaña (2002) or Atkins and Coe (2002), have questioned the use of such an approach. These latter authors consider that neglecting the possible presence of structural breaks in the evolution of both the nominal interest rate and the inflation rate might bias the result of unit root tests towards the failure of rejection the unit root null hypothesis. Thus, the use of the cointegration approach is nowadays being seriously questioned.

The second point concerns the constancy of the parameters. In this regard, a simple review of the literature leads us to conclude that most of the studies consider the parameters of the model (2) to be constant. This assumption is somewhat naive, in the sense that it does not correspond to what occurs empirically, especially if we take into account that these studies use sample sizes which cover the period running from the 1970’s to the present day. We need only reflect on the different monetary policies applied during this very lengthy period of time in order to realize that the validity of the constant parameters hypothesis is dubious. By contrast, we would argue that it is more sensible to advance the hypothesis that the Fisher relationship may be affected by the presence of some structural breaks. Their presence can be easily understood if we take into account that, for example, the real interest rate is the consequence of the interaction between savings and investment, in such a way that this rate may change when savings owners modify their behavior. In this regard, and as Chadha and Dimsdale (1999) point out, demographic change, technological progress, fiscal incentives, changes in the taxation of profits, the size of the public debt, the investors’ perception of risk and the degree of regulation or deregulation of capital markets could
alter the constant and the inflation parameter. Another source of the possible variation of the parameters of model (2) comes from the fact that the influence of inflation on the nominal interest rate can also vary. In this line, more robust inflation targeting and a more active monetary policy, see Söderlind (2001) and Olekalns (2001), or constraints on capital markets could be important determinants of the final value.

The third and final point concerns the presence of dynamic effects where, it should be noted, the influence of the inflation rate on the nominal interest rate may not simply be a contemporary phenomenon. Rather, the existence of such dynamic effects, which act on the generation of expected inflation or on the existence of persistence in the evolution of the nominal interest rate, should also be considered. This is the reason why a number of papers, such as Fahmy and Kandill (2002) or Atkins and Coe (2002), analyze the Fisher effect from a dynamic perspective.

Against this background, the goal of this paper is to show that most of the previous studies dedicated to analyzing the Fisher effect have not in fact done so in an appropriate manner, given that they have failed to properly reflect one or all of the three criticisms. More particularly, we demonstrate that the methodologies previously employed are not capable of providing us with useful results in order to better understand the relationship between the nominal interest rates and the inflation rates of the G7 countries. In order to prove this starting hypothesis, we should begin by appropriately testing the time series properties of these different nominal interest and inflation rates. In our view, the use of those unit root tests which allow for the presence of some structural breaks is crucial. Thus, if we can prove that these variables are better characterized as being (broken) trend stationary, then we should not use the cointegration approach. Moreover, and using similar arguments to those employed in Malliaropoulos (2000), we could also show that this approach may lead us to spuriously accept the Fisher effect. Following this strategy, and in order to reflect the second of the above criticisms, we should allow for the presence of some breaks in the relationship between the nominal interest rates and inflation rates. In a stationary scenario, we can apply the procedure proposed in Bai and Perron (1998, 2003) to test for the stability of the Fisher effect equation. This method also has the advantage of being able to provide us with consistent estimations of both the number of breaks and the periods when these occur. Finally, we can use the results obtained from applying these techniques to estimate the Fisher relationship when the structural breaks and the dynamic effects are also incorporated.

The rest of the paper is organized as follows. In Section 2 we describe the tests we employ. When applied to the nominal interest and inflation rates of the economies of the G7 countries, we find that they allow us to
robustly reject the unit root null hypothesis and accept the stationarity null hypothesis. This is the main finding of the paper, in that it invalidates any result obtained from the application of the cointegration approach to the analysis of the Fisher effect. Thus, an alternative methodology is clearly required and, in response, we propose a strategy based on the use of trend stationary variables and on the existence of some breaks in the relationship between the nominal interest and the inflation rate. Section 3 is devoted to a discussion of this proposed strategy, as well as to a consideration of the results obtained when it is applied to the economies of the G7 countries. Section 4 closes the paper with a review of the most important conclusions.

2 Nominal interest rates, inflation rates: unit roots versus trend stationarity

Following the seminal paper of Nelson and Plosser (1982), most of the empirical analyses based on the use of variables measured as time series begin studying the time properties of the variables. If these variables are better characterized as being integrated, then cointegration techniques are used. If, by contrast, they are considered as being stationary, then “standard” econometric techniques can be employed. The study of the Fisher effect is a scenario where we can clearly appreciate the application of this strategy and, ever since the appearance of the classic paper of Mishkin (1992), most of this literature has followed this pattern.

However, some much more recent papers have cast a number of serious doubts on the appropriateness of the unit root model when seeking to accurately describe the evolution of both inflation and nominal interest rates. In this regard, we can cite Malliaropoulos (2000) or Baum et al. (1999), where it is shown that US nominal interest and inflation rates can be better represented by way of broken trend stationary models. This finding is very important in the sense that, at least for the US data, it invalidates the use of the cointegration approach as an appropriate way to test for the Fisher effect. By way of illustration, under this approach a very commonly applied method is to test whether the real interest rate is integrated: if we can conclude that this real interest rate is stationary, this should be interpreted as evidence in favor of the Fisher effect. However, this method is only valid whenever the nominal interest and the expected inflation rate are integrated and, in other circumstances, it is not accurate. To better appreciate this, let us consider that expected inflation ($\pi$) and the nominal interest rate ($R$) can be considered as (broken) trend stationary variables. Any combination of these
variables, say $R - \beta \pi$, will also be a trend break variable and, therefore, we should observe that the real interest rate is also stationary. However, this does not imply that the Fisher effect holds, in that it only does so when the parameter $\beta$ is 1. Thus, under these circumstances, to admit that the real interest rate is stationary does not necessarily imply that the Fisher effect holds.

Such a finding requires a careful analysis of the time properties of the nominal interest and inflation rates, which the aim of the next subsection.

2.1 Analysis of the Time Properties of the Nominal Interest and Inflation rates

As we have mentioned earlier, the analysis of the time properties of the nominal interest and inflation rates should be treated carefully, and should certainly not be regarded as a mere prior step to the use of cointegration techniques. We dispose of a great range of statistics devoted to this issue. For example, most relevant papers base their analysis on the use of the Augmented Dickey-Fuller tests (Dickey and Fuller, 1979, and Said and Dickey, 1984) or those presented in Phillips and Perron (1988). In this regard, we should particularly note the recent paper of Ng and Perron (2001), which compares the performance of a wide range of unit root statistics. From amongst a number of available statistics, these authors propose the ADF$_{GLS}$, which is based on the very popular ADF test. This can be obtained from the estimating the following model:

$$y_t = \mu + \gamma t + \rho y_{t-1} + \sum_{i=1}^{k} \phi_i \Delta y_{t-i} + \varepsilon_t \quad (3)$$

and subsequently calculating the pseudo t-ratio for testing whether the parameter $\rho$ is 1. The differences between this and the simple ADF lie in the fact that ADF$_{GLS}$ is based on the use of GLS estimation methods, instead of OLS estimators, and on the determination of the value of the lag truncation parameter via the use of an information criterion, called MIC, also proposed in Ng and Perron (2001).

In some cases the use of this statistic may not be appropriate: for example, if we can admit that the variable being studied may present some structural breaks that affect to the deterministic elements. In this case, the distortions caused by the omission of these breaks on the unit root inference has been

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$^1$Ng and Perron (2001) also consider alternative tests, based on modifications of the Phillips-Perron statistics. However, their use does not modify the conclusions that we have obtained with the ADF$_{GLS}$ and, therefore, we have chose to omit these results.
very well documented in, amongst others, Perron (1989) or in Montañés and Reyes (1998). Given that the nominal interest and inflation rates may exhibit this kind of behavior, these breaks should clearly be included in the model specification. To that end, we should first take into account that we can find several types of breaks. For example, we can admit the possibility that these breaks only affect the intercept of the trend polynomial, or only the parameters associated to the trend or, the most common case, both the intercept and the slope. Secondly, we should consider that the presence of a single break cannot be enough to capture the evolution of the variables being studied. Thus, it seems to be advisable to consider the presence of more than one break. Here, in order to take into account the possible presence of these breaks, we could extend the equation (3) by including some dummy variables that can capture the effect of these changes on the deterministic elements. In fact, this approach is followed by Perron (1989), when the period when the break appears is exogenously determined, and by Perron and Vogelsang (1998), Zivot and Andrews (1992) or Lumsdaine and Papell (1997), when this period is endogenously determined by the model.

As an alternative to this à la Perron methodology, Lee and Strazicich (2002a, 2002b) have recently proposed a somewhat different approach that is based on the LM (score) principle. Following the paper of Schmidt and Phillips (1992), these statistics can be obtained by estimating the following model:

\[ \Delta y_t = \delta^t \Delta Z_t + \phi \tilde{S}_{t-1} + u_t \tag{4} \]

where \( Z_t \) reflects the deterministic components, \( \tilde{S}_t = y_t - \tilde{\psi}_x - Z_t \delta, t = 2, 3, ..., T \), are coefficients in the regression of \( \Delta y_t \) on \( \Delta Z_t \), \( \tilde{\psi}_x \) is given by \( y_1 - Z_1 \delta \) (see Schmidt and Phillips, 1992, in this regard), and \( y_1 \) and \( Z_1 \) denote the first observations of \( y_t \) and \( Z_t \), respectively. The unit root null hypothesis is described by \( \phi = 0 \) and can be tested by way of a pseudo t-ratio statistic that we will denote as \( \tilde{\tau} \). When \( Z_t = \{1, t\} \), then we have the statistic proposed in Schmidt and Phillips (1992). If we want to account for some structural breaks, we should simply reflect them in \( Z_t \). Thus, for example, we can consider the case where the breaks may affect both the intercept and the slope of the trend by simply assuming that \( Z_t = \{1, t, D_1, ..., D_n, DT_1, ..., DT_n\} \), where \( D_i = 1 \) if \( t > TB_i \) and 0 otherwise, whilst \( DT_i = t D_i \), with \( TB_i = \lambda_i T \) being the period of time where the \( i-th \) break appears and \( i = 1, 2, ..., n \). We will denote this statistic as \( \tilde{\tau}^{CC}_n \), where the sub-index \( n \) reflects the number of breaks considered and the super-index \( CC \) indicates that we are allowing for a change in the intercept and in the slope.

Lee and Strazicich (2002a, 2002b) have derived the asymptotic distribu-
tion of this pseudo t-ratio for different \( n = 1, 2 \), as well as for different types of breaks under the assumption that the innovations of the model hold the strong-mixing regularity conditions reported in, amongst others, Phillips and Perron (1988) or Schmidt and Phillips (1992). Furthermore, instead of considering that the periods where the breaks appear are a priori unknown, these authors proposed the use of the minimum value of the statistic \( \tau^C_n \) for all the combinations of possible break periods.

Alternatively, we may consider it of interest to test for the stationarity hypothesis in order to confirm the results obtained when testing for the unit root null hypothesis. In this regard, we can use the statistic proposed in Kwiatkowski et al. (1992) and commonly known as the KPSS test. To obtain this statistic, let us consider the following model:

\[
\begin{align*}
y_t &= f(t) + r_t + u_t \\
r_t &= r_{t-1} + e_t
\end{align*}
\]

(5)

where \( u_t \sim iid(0, \sigma^2_u) \), the sequence of innovation \( \{e_t\} \) is assumed to satisfy the strong-mixing regularity conditions previously mentioned and \( f(t) \) includes all the deterministic elements. Under the null hypothesis, the value of the parameter \( \sigma^2_u \) is 0, whilst it tends towards infinite when the variable \( y_t \) is I(1). Under these assumptions, the KPSS test is defined as follows:

\[
\eta = T^{-2} \sum_{t=1}^{T} S_t^2 / \hat{\sigma}^2
\]

where \( S_t = \sum_{i=1}^{t} e_i \), with \( e \) denoting the OLS residuals of the regression of \( y_t \) on \( f(t) \) and \( \hat{\sigma}^2 \) is an appropriate estimation of the long-run variance. Similarly to what we find in the case of unit root tests, the omission of some structural breaks may lead this statistic to spuriously reject the stationarity null hypothesis\(^2\). In order to solve this problem, we can follow a similar strategy to that employed for the unit root tests and include some dummy variables that should capture the effect of these structural breaks. This is the procedure applied in Lee and Strazicich (2001) and Kurozumi (2002), which consider the case of a single break, or in Carrión et al. (2003), for the existence of two breaks. However, we should note that these authors define somewhat different types of breaks. In particular, they assume that the break in the slope of the trend is better captured by the dummy variable

\(^2\)See Lee et al. (1997) in this regard.
Thus, following these authors, we can define $f(t) = \{1, t, DT_{1t}, ..., DT_{nt}\}$, which allows us to use $\eta_{n}^{B}$, where the subindex $n$ is again denoting the number of breaks admitted in the specification and $B$ is showing that a break in the slope is being considered. The asymptotic distribution of this statistic for $n = 1, 2$ is derived in the above-mentioned papers under the assumption that the periods where the breaks appear are a priori known. If we take a much more general approach, and consider them to be unknown, then we should estimate the periods where the breaks appear. To that end, we can imitate the procedure followed when testing for the unit root null hypothesis and estimate these periods by minimizing the value of the $\eta_{n}^{B}$ test. This is the procedure followed in Lee and Strazicich (2001). An interesting alternative route is that employed in Kurozumi (2002) and Carrión et al. (2003) who, following the results of Bai (1997) and Bai and Perron (1998), determine these periods by way of the minimization of the sum of the squared residuals that come from the regression of $y_{t}$ on $f(t)$. The critical values for the distributions of the min $\eta_{n}^{B}$ are also provided in these papers.

2.2 Empirical evidence for the G7 countries

As we have mentioned earlier, the methodology that should be employed in order to analyze the Fisher effect depends on the time properties of the variables that are necessary to study it, namely the nominal interest and inflation rates. Thus, we should be careful when determining the integration order of these variables. To that end, we have applied the statistics presented in the previous Section to the integration order of the nominal interest and inflation rates of the G7 countries. We have taken two different measures of the nominal interest rates. On the one hand, we have selected a short-run variable, measured by the 3-month Treasury-Bill rate (or equivalent). On the other, as a measure of the long-run behavior of the nominal interest rates, we have taken the 10-year Government bond (or equivalent). For its part, the inflation rate has been obtained from the Consumer Price Index (CPI), using the non-linear conversion $\pi_{t} = 100 \times \left[\left(\frac{CPI_{t} - CPI_{t-1}}{CPI_{t-1}}\right)^{4} - 1\right]$. All the data has been obtained from OECD Main Economic Indicators. Finally, the quarterly data, where possible, covers the sample size 1960:1-2001:4. The results of applying the previously considered unit root statistics to our database are set out in Table 1.

We can first appreciate that the use of the ADF$^{GLS}$ unit root test leads us to accept the unit root null hypothesis for all the variables under consideration, a result which coincides with the conclusion drawn in most of the papers.
devoted to this issue. A similar conclusion is also drawn when the KPSS test is employed. Thus, these initial results would lead us to analyze the Fisher effect in the commonly used unit root/cointegration scenario. However, we should note that when we consider as, we argue, should be done, the presence of some breaks, this leads us to change our conclusion in such a way that, broadly speaking, the evidence in favor of the unit root null hypothesis now becomes very limited, or even null, for the variables being studied, whilst the number of rejections of the stationarity null hypothesis clearly reduces.

Apart from this general analysis, we can study the results obtained for each group of variables in greater detail. For example, we can see that the evidence against the presence of a unit root in the long-run nominal interest rates is very robust, rejecting the unit root null hypothesis for Canada, France, Germany, the UK and the USA when a 5% significance level is used, and for Japan when a more liberal 10% significance level is applied, whilst there is no evidence against this null hypothesis when the Italian case is considered. Although the evidence against this unit root null hypothesis is less robust for the short-run interest rates, we can nevertheless reject it for Canada, the UK and the USA when the 5% significance level is used. For Germany and Italy, the evidence is even more limited, whilst we cannot reject it for France and Japan. Finally, when the inflation rate is considered, the evidence against the unit root null hypothesis is extremely robust, in that we can reject it for all the countries included in our sample at the 5% significance level.

Thus, we can appreciate that unit root tests essentially reject the presence of a unit root in the nominal interest and inflation rates whenever the presence of some breaks is properly allowed for. This evidence is even stronger when the statistics that test for the stationarity null hypothesis are employed, in that they simply do not offer any evidence against this null hypothesis, even for those cases where unit root tests failed to reject the unit root null hypothesis. Therefore, the combination of these two types of statistics leads us to consider that all the variables included in our study are better characterized as being broken trend stationary than difference stationary. This confirms our initial intuition/suspicion and, additionally, invalidates the use of the results obtained from the cointegration approach when the Fisher effect is studied for the economies of the G7 countries.

Further interesting insights are provided by analyzing the periods where the breaks appear. If we again begin with the case of the long-run interest rates, we can see that most of the breaks can be associated with the movements in the monetary regimes during the first part of the 1980s or the late 1970s. This pattern of behavior is followed by Canada, France and the USA. Japan shows a single break at the end of the 1970s, whilst the behavior of
Germany and the UK is a little different, with both of them exhibiting a break in the 1990s.

However, the analysis of the short-run interest rates allows us to conclude that the breaks are different from those reported for the long-run interest rates. Thus, whilst we can admit that these series show a break around 1980, the presence of a second break related to the relaxation of the monetary regime in mid 1980s is not so evident. By contrast, we can observe an almost generalized presence of a break at the beginning of the 1990’s.

If we now consider the case of the inflation rates for the sample countries, we can see that the presence of two breaks seems to be an accurate hypothesis. For the USA, both of these breaks are associated with the changes in its monetary policy at the end of the 1970s and in the mid 1980s. For the rest of the countries, we can observe the existence of a first break related to the increase in the inflation rate at the beginning of the 1970s, clearly reflecting the impact of the first oil crisis. The second break mostly appears at the beginning of the 1980s, although slightly earlier for Germany and Japan. The case of Italy is somewhat different, in that the breaks appear in the mid 1970’s and late 1980’s.

Finally, if we compare the estimation of the breaks for, on the one hand, the inflation rates and, on the other, the nominal interest rates, we can obtain some additional insights. First, we can see that the estimation of the breaks for the long-run interest rates does not always coincide with the corresponding estimations for the short-run case, although they do appear to be somewhat related. To appreciate this, we can point to the case of the USA, where we can see that both long-run and short-run nominal interest rates exhibit a break at the end of the 1970’s. However, the period when the second break appears does not coincide (1985:2 and 1993:2, respectively).

Secondly, we can also see that there is no direct correspondence between the estimations of the times of the breaks for the nominal interest and inflation rates. A possible explanation for this lack of coincidence could be related to the restriction that only two breaks can affect the variable. However, the absence of statistics for testing the unit root null hypothesis under the presence of 3 or more breaks in the trend function makes it impossible to explore this alternative. In any event, for the purpose of this Section, it suffices to show the stationarity of the variables, in that we will explore the possible presence of multiple breaks in the next Section where the relationship between nominal interest rates and expected inflation rate is studied.
3 Testing the Fisher effect with stationary variables

As we have seen, the presence of a unit root in the variables being studied is not supported by our data and thus the unit root/cointegration methodology cannot be employed in such a scenario; rather, we should test for the Fisher effect by considering that the variables are not integrated, but can show some dynamic component. In this regard, we can cite the recent alternative suggested in Malliaropoulos (2000). This very appealing method is based on the analysis of the impulse-response functions calculated from a VAR composed by the cyclical components of both the interest and inflation rates. This author first suggests extracting the cyclical component of both the nominal interest and the inflation rate by estimating the following equation:

\[ y_t = g(t) + u_t \]  \hspace{1cm} (6)

where \( y \) can represent either the nominal interest rate or the inflation rate. Thus, if the (possibly broken) trend function, denoted by \( g(t) \), is appropriately defined, the residuals of the estimation of this model are an appropriate measure of the cyclical component. We could follow this method here, using the results of the preceding Section in order to define these (broken) trend functions for each of the variables. However, we should note that this method is only adequate for studying the Fisher effect whenever the nominal interest and the rate share the same trend function. Otherwise, this is not totally accurate, given that the Fisher effect is not exclusively related to the cyclical behavior of the variables.

Furthermore, a second source of criticism comes from the fact that we should take into account that the presence of breaks may affect not only the evolution of the variables, but also the structural equation. To verify this possibility, and given that the interest and inflation rates can be better characterized as being trend stationary variables, we can use the procedure recently suggested in Bai and Perron (1998, 2003). This method allows us to detect the presence of an unknown number of breaks, as well as to estimate the relationship. In our case, this method is based on the estimation of the following model where up to \( m \) breaks may appear:

\[ R_t = m_j + \beta_j \pi_{t+1} + u_t, \quad t = TB_{j-1}, ..., TB_j \quad i = 1, 2, ... m + 1 \]  \hspace{1cm} (7)

with \( TB_j \) representing the period where the break appears. Then, the Bai-Perron procedure implies the estimation of the above equation considering that the break may appear in any period of the sample size. A Chow-type
tests is then defined in order to determine the existence of a first break, which coincides with that period where this Chow-type statistic attains its maximum value. The existence of multiple breaks is subsequently analyzed by applying this procedure in a sequentially way, combined with the repartition method described in Bai (1997). To determine the existence of breaks, we can use the UD\text{max} and WD\text{max} statistics which test the null hypothesis of no structural breaks versus the presence of an unknown number of breaks. Note that we have considered a maximum number of 5 breaks and that we have used the quadratic spectral kernel in order to account for the presence of possible autocorrelation and heterogeneity in the residuals, combined with the Andrews (1991) automatic bandwidth selection with AR(1) approximation.

Alternative models can also be used. In the previous equation we assume that none of the parameters is constant. Nevertheless, we can find the case where some of them are. In these circumstances, it is more appropriate to estimate either of the following two models:

\begin{align}
R_t &= m_j + \beta \pi_{t+1} + u_t, \quad t = TB_{j-1}, \ldots, TB_j \quad i = 1, 2, \ldots, m + 1 \quad (8)
\end{align}

\begin{align}
R_t &= m + \beta_j \pi_{t+1} + u_t, \quad t = TB_{j-1}, \ldots, TB_j \quad i = 1, 2, \ldots, m + 1 \quad (9)
\end{align}

The literature does not offer any recommendations regarding the selection of the most appropriate model. In its absence, we have opted for a selection based on a number of statistics, such as the SBIC information criterion, the $\bar{R}^2$ and the analysis of the significance of the variables. The results that have emerged are presented in Table 2.

These results clearly confirm our suspicion of the presence of breaks in the structural relationships between the interest and the inflation rates. We can verify this by observing the values of both the UD\text{max} and WD\text{max} which clearly reject the null hypothesis of no breaks for almost any level of significance. Thus, it becomes necessary to us to consider the presence of these breaks in order to appropriately reflect this relationship.

A first analysis of the results of Table 2 also shows that it is enough to introduce some changes in the intercept so as to reflect the presence of these breaks. The exceptions are the cases of France and, especially, the USA, where the parameter associated to the inflation rate cannot be considered as constant across the available sample size. Therefore, for these two countries, the Fisher effect should be studied separately for the different subsamples.

The analysis of the periods where the breaks appear also provides some rich insights. We can see that the number of breaks is greater when using
the long-run rather than the short-run nominal interest rate. Nevertheless, it is not easy to draw any clear conclusions from this first result, in that the sample sizes are always larger for the long-run nominal interest rate. We can also observe that there is no significant coincidence with regards to the estimation of the periods where the breaks appear. The US case is a good example: the break appears in 1981:2 when the long-run nominal interest rate is employed, whilst it appears in 1979:4 for the short-run nominal interest rate. Moreover, we can observe that the short-run relationship presents the breaks earlier than in the long-run case. Additionally, we can see that the most common breaks for the long-run relationships appear in late 1970’s, beginning of the 1980’s and in the mid 1990’s, whilst for the short-run relationship, the presence of a break in the late 1970’s is clear, as is the existence of a second break either around 1992 or around 1995.

If we compare the break points of the long-run relationship with the break points in the behavior of the inflation and the nominal interest rates, we can conclude that the observed change in the constant in the cases of Canada, France, Germany, Italy and Japan is not associated to changes in the nominal variables; that is to say, we can conclude that there was a change in the transmission of the inflation to the interest rate. A possible cause of this phenomenon could be the creation of the European Monetary System, in that the three eurozone countries in question are affected. Another interesting result is that the changes in the early or mid 1970s are associated to breaks in the inflation rate and, as Rapach and Wohar (2003) point out, this is a monetary phenomenon. However, the rest of the breaks hide other important elements and cannot be explained using simply monetary approaches.

Once having offered robust evidence in favor of the presence of some breaks in the relationship between the nominal interest and the inflation rates, it is clearly necessary to take it into account in order to test for the Fisher effect. In this regard, we should recall that the Bai-Perron procedure allows for the presence of some dynamics in this relationship. A sensible strategy which reflects both the presence of breaks and the dynamics is based on the estimation of the following model:

\[ y_t = \mu + \delta \text{dummies} + \sum_{i=1}^{n} \phi_i y_{t-i} + \sum_{i=0}^{n} \gamma_i \pi_{t+1-i} + \varepsilon_t \]  \hspace{1cm} (10)

where dummies reflects the presence of the breaks reported in Table 2 and where we have used \( n_{\text{max}} = 5 \) in order to account for the possible presence of a dynamic component. Then, we have applied a general-to-particular strategy where the less significant variables have been removed from the model in each step, in such a way that the final model contains all significant
variables (using a 5% significance level) and where no autocorrelation pattern is presented in the residuals. As a consequence, we could study the Fisher effect by analyzing the value of the estimation of the parameter $\gamma_{o}$, which could give us an instantaneous perspective of this effect. However, most of the precedent studies have failed to find evidence in favor of this instantaneous Fisher effect. Rather, they focus on the analysis of the long-run Fisher effect, which can be studied by analyzing the following ratio:

$$\beta = \frac{\sum_{i=0}^{n} \gamma_i}{1 - \sum_{i=1}^{n} \phi_i} = 1$$  \hspace{1cm} (11)$$

where the parameter $\beta$ represents the long-run effect of the inflation on the nominal interest rate. Given that the results that we have obtained also suggest that we should discard this instantaneous effect, we have chosen to omit them, focusing exclusively on the long-run analysis by studying the hypothesis stated in (11). Finally, we should recall that we have conclude that the breaks affect the parameter associated to the expected inflation for the cases of the USA and France and, thus, we should study the long-run Fisher effect in some sub-samples for these variables. Table 3 reports the results obtained from the estimation of equation (10) for each country.

The first results that emerges from an inspection of this Table is that the evidence in favor of the Fisher effect is very limited, especially when long-run nominal interest rates are considered. We can also see that the estimation of the effect of inflation on nominal interest rates is smaller when we use the long-run nominal interest rate than when use the short-run one.

In spite of this negative picture against the validity of the Fisher effect, we should note that we the hypothesis can nevertheless be accepted for some countries. This is the case with the USA, for example, where this effect holds during the period prior to 1981. Furthermore, we can accept it for both the long-run and the short-run nominal interest rates employed in our study. However, after the change in monetary policy that occurred at the beginning of the 1980’s, the long-run response of the nominal interest rate to changes in the expected inflation is not longer equal to 1; rather, it takes values that are clearly greater than 1. Previous studies have obtained this same result and can be explained either by reference to the influence of taxes, as in Crowder and Wohar (1999), or by way of the liquidity effect, in the sense that a rise

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3 We have used the OLS estimation method, except for the case of the Canadian short-run interest rate where, given that the Hausman statistic leads us to reject the exogeneity null hypothesis, IV estimation has instead been used. The US and UK short-run nominal interest rates have been employed as the instruments.

4 See, in this regard, Lee et al. (1998) and Both and Ciner (2001).
in expected inflation increases the cost of holding cash and loanable funds and the interest rate decreases, as in Fahmy and Kandil (2002).

The case of France provides additional evidence in favor of the Fisher effect, albeit similarly limited to some periods of time. Thus, when the short-run nominal interest rate is considered, the period when this effect holds corresponds to post-1981. By contrast, for the long-run nominal interest rate, this evidence is limited to the 1981-1996 period. The result obtained for the post-1996 period is somewhat contradictory in that, on the one hand, the value of the long-run Fisher parameter takes the value 0.23, which is far from 1. However, the corresponding F-ratio takes a value that implies the rejection of the long-run Fisher hypothesis only for a liberal 10%.

The third country that offers evidence in favor of the Fisher effect is Japan, but in this case only when the short-run nominal interest rate is employed. When we consider the long-run nominal interest rate, the F ratio clearly rejects the this effect. However, when the long-run nominal interest rate is used, the results are dramatically different and we can observe that the value of the long-run parameter \( \beta \) takes a value very close to 0.

Italy also offers some evidence for the short-run nominal interest rate, in that we can reject that the long-run Fisher parameter is equal to 1 only for a liberal 10% significance level. When the long-run rate is used, the value of this parameter reduces to some 0.40 and, therefore, we can easily reject the hypothesis stated in (11).

As regards the remaining countries, we find no evidence in favor of the Fisher effect, although we can also observe that the value of the parameter \( \beta \) takes values around 0.5. This is the case for Germany and, especially, for Canada. In this latter case, the value of the parameter \( \beta \) is 0.62, when the short-run nominal interest rate is employed. From this point of view, although the effect of inflation is not totally translated to the nominal interest rate, we can nevertheless affirm that it allows us to predict the future value of that rate.

Finally, the case of the UK presents the lowest values of this parameter. For example, this is equal to 0.32 when the short-run nominal interest rate is employed, with this value decreasing to some 0.01 when the long-run nominal interest rate is considered. Moreover, in this latter case, we cannot reject the null hypothesis that the value of the parameter \( \beta \) is 0. As a result, in this case, we are unable even to predict the future values of the nominal interest rates by way of the expected inflation.
4 Conclusions

In contrast to the findings of the majority of previous papers devoted to analyzing the so-called Fisher effect, we have proved that the nominal interest and inflation rates of the G7 group of countries are better characterized as being (broken-trend) stationary variables than as being integrated variables. This admittedly robust conclusion has been obtained from the appropriate use of unit root tests, combined with statistics that test the null hypothesis of stationarity, when we allow for the presence of some changes in the trend function. This central result implies that it is clearly unadvisable to use techniques based on cointegration when analyzing for the Fisher effect. Given that ever since the seminal work of Mishkin (1982), the great majority of papers devoted to studying this effect have used methods based precisely on this cointegration analysis, the importance of our results would seem to be clear, in that it qualifies, if not invalidates, most of the previous literature on the relationship between nominal interest and inflation rates.

Note, however, that we can only conclude against the presence of a unit root in these variables when some breaks are included in the specification. The presence of these breaks not only modifies the results on the integration order of the variables, but also opens the door to the possible presence of breaks in the Fisher equation, a question that, in our view, has not received sufficient attention on the part of the literature. The use the procedure recently proposed in Bai and Perron (1998, 2003) would appear to confirm our hypothesis, proving the existence of different regimes in the relationship between nominal interest and inflation rates.

On the basis of these findings, an appropriate method for testing for the Fisher effect should take into account, on the one hand, that the variables are broken-trend stationary and, on the other hand, the presence of breaks in the structural model. So as to properly reflect these aspects, we have chosen to test for the Fisher effect by estimating an ADL polynomial, combined with the inclusion of some dummy variables that account for the breaks in the structural relationship. The results based on this method show that the evidence in favor of the Fisher effect is in fact very limited. Indeed, of all the G7 countries, we have found that it clearly holds only for the cases of the USA and France, whilst we have detected only limited evidence in its favor for the cases of Japan and Italy. As regards the remaining G7 countries, our results cast considerable doubts on whether the Fisher effect is present.
References

[1] Andrews, D.W.K. (1991): Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation, *Econometrica* 59, 817-858.

[2] Atkins, F.J. and Coe, P.J. (2002): An ARDL Bounds Test of the Long-term Fisher Effect in the United States and Canada. *Journal of Macroeconomics*, 24, 255-266.

[3] Bai, J. (1997): Estimation of a Change Point in Multiple Regression Models. *Review of Economics and Statistics*, 551-563.

[4] Bai, J. and Perron, P. (1998): Estimating and Testing Linear Models with Multiple Structural Changes. *Econometrica*, 66:47-78.

[5] Bai, J. and Perron, P. (2003): Computation and analysis of multiple structural-change models. *Journal of Applied Econometrics*, 18, 1-22.

[6] Baum, C., Barkoulas, J. and Caglayan, M. (1999): Persistence in International Inflation Rates, *Journal of Southern Economic Journal*. 65(4), 900-913

[7] Both, G., and Ciner ,C.(2001): The relationship between Nominal Interest Rate and Inflation: International Evidence. *Journal of Multinational Financial Management*, 11, 269-280.

[8] Carrión, J.L., Sansó, A. and Artís, M. (2003): The KPSS Test with Two Structural Breaks, Working Paper, U. Central de Barcelona.

[9] Crowder, W.J. and Wohar, M.E. (1999): Are Tax Effects Important in the Long-Run Fisher Relationship? Evidence from the Municipal Bond Market. *Journal of Finance*, LIV(1), 307-317.

[10] Chadha, J.S. and Dimsdale, N.H. (1999): A Long Review of Real Rates. *Oxford Review of Economic Policy*, 15(2), 17-45.

[11] Dickey, D. and Fuller, W. (1979): Distribution of the Estimators for Autoregressive Time Series With a Unit Root. *Journal of the American Statistical Association*, 74, 427-431.

[12] Fahmy, Y.F. and Kandil, M. (2002): The Fisher Effect: New Evidence and Implications. *International Review of Economics and Finance*, 178.

[13] Fisher, I. (1930): The Theory of Interest. MacMillan Ed., New York.
[14] Gil-Alaña, L.A. (2002): A Mean Breaks in the US Interest Rate. *Economics Letters*, 77, 357-362.

[15] Koutras, Z. and Serletis, A. (1999): On the Fisher Effect. *Journal of Monetary Economics*, 44, 105-130.

[16] Kurozumi, E. (2002): Testing for Stationarity with a Break. *Journal of Econometrics*, 108, 63-99.

[17] Kwiatkowski, D., Phillips, P. C. B., Schmidt, P. J. and Shin Y. (1992), Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root: How Sure are We that Economic Time Series Have a Unit Root ?. *Journal of Econometrics*, 54, 159-178.

[18] Laatsch, F. and Klein, D.P. (2002): Nominal Interest Rate and Expected Inflation: Results from a Study of US Treasury Inflation-Protected Securities. *Quarterly Review of Economics and Finance*, 187, 1-13.

[19] Lanne, M. (2001): Near Unit Root and the Relationship between Inflation and Interest Rate: A Reexamination of the Fisher Effect. *Empirical Economics*, 26, 357-366.

[20] Lee, J., Clark, C, and Ahn, S. (1998): Long- and Short-Run Fisher Effect: New Tests and New Results, *Applied Economics*, 30, 223-124.

[21] Lee, J., Huang, C. J. and Shin, Y. (1997): On Stationary Tests in the Presence of Structural Breaks. *Economics Letters*, 55, 165-172.

[22] Lee, J., and Strazicich , M. (2001): Testing the Null of Stationarity in the Presence of One Structural Break. *Applied Economics Letters*, 8, 377-382.

[23] Lee, J., and Strazicich , M. (2002a): Minimum LM Unit Root Test, *Working Paper*.

[24] Lee, J., and Strazicich, M. (2002b): Minimum LM Unit Root Tests with Two Structural Breaks. *forthcoming in Review of Economics and Statistics*.

[25] Lumsdaine, R. L. and Papell, D. H. (1997): Multiple Trends and the Unit Root Hypothesis. *Review of Economic and Statistics*, 79, 212-218.

[26] Malliaropulos, D. (2000): A Note on Nonstationarity, Structural Breaks and the Fisher Effect. *Journal of Banking and Finance*, 24, 695-707.
[27] Mishkin, F., (1992): Is the Fisher effect for real: A Reexamination of the Relationship between Inflation and Interest Rates. *Journal of Monetary Economics, 95*-215.

[28] Montañés, A., Reyes, M. (1998): The asymptotic behaviour of the Dickey-Fuller unit root tests under a shift in the trend function. *Econometric Theory, 14*, 355-363.

[29] Ng, Serena and Perron, P. (1995): Unit Root Tests in ARMA Models with Data-Dependent Methods for the Selection of the Truncation Lag. *Journal of the American Statistical Association, 90:429*, 268-281.

[30] Ng, Serena and Perron, P. (2001): Lag Length Selection and the Construction of Unit Root Tests With Good Size and Power. *Econometrica, 69*, 1519-1554.

[31] Olekalns, N. (2001): An Empirical Investigation of the Structural Breaks in the Ex Ante Fisher Effect. *Research Paper number 786*, Department of Economics, University of Melbourne, Australia.

[32] Perron, P. (1989): The Great Crash, the Oil Price Shock and the Unit Root Hypothesis. *Econometrica, 57*, 1361-1401.

[33] Perron, P. (1997): Further Evidence on Breaking Trend Functions in Macroeconomic Variables. *Journal of Econometrics, 80:2*, 355-385.

[34] Perron, P. and Vogelsang, T. (1992): Testing for a Unit Root in Time Series with a Changing Mean: Corrections and Extensions. *Journal of Business and Economic Statistics, 10:4*, 467-470.

[35] Phillips, P., and Perron, P. (1988): Testing for a Unit Root in Time Series Regression, *Biometrika 75:2*, 335-346.

[36] Rapach, D. E. (2002): The Log-run Relationship between Inflation and Real Stock Price. *Journal of Macroeconomics, 24*, 331-351.

[37] Rapach, D.E. and Wohar, M. (2003): Regime Changes in International Interest Rates: Are They a Monetary Phenomenon?, *Journal of Money, Credit and Banking*, forthcoming.

[38] Rose, A.K. (1988): Is the Real Interest Rate Stable?. *Journal of Finance, 43*, 1095-1112.
[39] Said, S. E. and Dickey, D. (1984): Testing for Unit Roots in Autoregressive-Moving Average Models of Unknown Order. *Biometrika*, 71, 599-607

[40] Schmidt, P. and Phillips, P. (1992): LM Tests for a Unit Root in the Presence of Deterministic Trends. *Oxford Bulletin of Economics and Statistics*, 54:3 (1992), 257-287.

[41] Söderlind, P. (2001): Monetary Policy and the Fisher Effect. *Journal of Policy Modelling*, 23, 491-495.

[42] Vogelsang, T. and Perron (1998): Additional Tests for a Unit Root Allowing for a Break in the Trend Function at an Unknown Time. *International Economic Review*, 39(4): 1073-1100

[43] Zivot, E. and Andrews, D. (1992): Further Evidence on the Great Crash, the Oil-Price Shock and the Unit Root Hypothesis. *Journal of Business and Economic Statistics*, 10:3 (1992), 251-270.
Table 1. Testing for unit roots

| Sample        | DFGLS | Unit Root Tests | Stationary Tests |
|---------------|-------|-----------------|------------------|
|               |       | $\hat{\tau}_1^C$ | $TB_1$ $\hat{\tau}_2^C$ | $TB_1$ $TB_2$ $\eta$ $\eta_1^B$ $TB_1$ $\eta_2^B$ $TB_1$ $TB_2$ |
| **Panel A. Long-run interest rates** | | | |
| Canada 60:1-01:4 | -1.16 | -5.15 | a  | -6.43 | a  | 80:3 | 80:3 | 84:4 | 1.38 | c  | 0.04 | 79:4 | 0.02 | 79:4 | 89:1 |
| France 60:1-01:4 | -1.01 | -4.12 | a  | -7.76 | a  | 80:2 | 80:2 | 85:3 | 1.44 | c  | 0.08 | b   | 79:1 | 0.02 | 74:3 | 84:3 |
| Germany 60:1-01:4 | -2.27 | -4.50 | c  | 72:2 | -5.42 | b  | 72:2 | 96:1 | 0.79 | c  | 0.11 | a  | 78:2 | 0.03 | 74:1 | 94:3 |
| Italy 60:1-01:4 | -1.27 | -3.99 | a  | -4.83 | a  | 80:3 | 80:3 | 85:4 | 1.29 | c  | 0.15 | a  | 79:2 | 0.04 | 70:4 | 79:4 |
| Japan 66:1-01:4 | -1.67 | -4.32 | c  | 80:4 | -4.95 | b  | 80:4 | 90:1 | 0.91 | c  | 0.11 | b  | 77:4 | 0.02 | 80:4 | 88:2 |
| UK 60:1-01:4 | -1.25 | -5.04 | b  | 80:2 | -5.96 | a  | 80:2 | 84:4 | 0.79 | c  | 0.11 | a  | 78:4 | 0.03 | 67:1 | 78:3 |
| USA 60:1-01:4 | -1.14 | -4.02 | c  | 80:3 | -7.16 | a  | 79:2 | 85:3 | 1.31 | c  | 0.07 | b  | 79:2 | 0.02 | 79:4 | 85:1 |
| **Panel B. Short-run interest rates** | | | |
| Canada 61:1-01:4 | -1.44 | -5.04 | b  | 80:2 | -5.96 | a  | 80:2 | 83:4 | 1.07 | c  | 0.05 | 78:2 | 0.02 | 78:3 | 93:5 |
| France 69:1-01:4 | -1.80 | -3.93 | a  | -4.66 | a  | 82:1 | 82:1 | 89:3 | 0.71 | c  | 0.07 | c  | 79:1 | 0.03 | 81:1 | 88:3 |
| Germany 60:1-01:4 | -2.15 | -4.18 | c  | 69:2 | -4.61 | c  | 74:3 | 90:1 | 0.39 | c  | 0.10 | a  | 77:4 | 0.03 | 70:4 | 90:4 |
| Italy 71:1-01:4 | -1.73 | -4.38 | c  | 85:4 | -5.10 | b  | 85:4 | 91:4 | 0.33 | c  | 0.13 | b  | 82:1 | 0.03 | 84:1 | 90:3 |
| Japan 79:1-01:4 | -1.55 | -3.96 | b  | 93:1 | -4.56 | c  | 86:3 | 91:3 | 0.18 | c  | 0.07 | b  | 94:1 | 0.02 | 89:3 | 94:4 |
| UK 69:1-01:4 | -2.51 | -5.88 | b  | 84:4 | -5.44 | b  | 78:2 | 92:4 | 0.61 | c  | 0.07 | b  | 83:4 | 0.02 | 83:1 | 92:2 |
| USA 69:1-01:4 | -2.25 | -5.62 | b  | 78:4 | -5.62 | b  | 79:2 | 93:2 | 0.54 | c  | 0.09 | a  | 85:1 | 0.03 | 78:3 | 94:3 |
| **Panel C. Inflation** | | | |
| Canada 60:1-01:3 | -1.70 | -5.13 | a  | -6.27 | a  | 72:2 | 72:2 | 82:3 | 1.00 | c  | 0.050 | 82:4 | 0.03 | 72:1 | 83:3 |
| France 60:1-01:3 | -1.34 | -4.33 | c  | 82:1 | -7.10 | a  | 72:1 | 83:2 | 1.13 | c  | 0.08 | b  | 83:2 | 0.03 | 72:2 | 83:4 |
| Germany 60:1-01:3 | -2.17 | -4.67 | c  | 69:4 | -5.37 | b  | 71:4 | 78:1 | 0.36 | c  | 0.07 | c  | 77:2 | 0.03 | 70:4 | 89:2 |
| Italy 60:1-01:3 | -1.09 | -4.30 | c  | 76:2 | -5.38 | b  | 76:2 | 89:4 | 1.01 | c  | 0.12 | a  | 83:2 | 0.02 | 72:3 | 84:1 |
| Japan 66:1-01:3 | -2.08 | -5.07 | b  | 77:2 | -8.30 | a  | 73:4 | 79:2 | 0.19 | c  | 0.04 | b  | 76:3 | 0.02 | 75:3 | 87:2 |
| UK 60:1-01:3 | -1.68 | -5.48 | a  | 80:1 | -6.53 | a  | 73:4 | 82:2 | 0.77 | c  | 0.05 | c  | 80:2 | 0.02 | 73:3 | 87:1 |
| USA 60:1-01:3 | -1.78 | -5.23 | a  | 77:1 | -6.99 | a  | 78:1 | 86:1 | 0.88 | c  | 0.03 | 82:1 | 0.02 | 82:3 | 91:4 |

DFGLS reflects the value of the Ng-Perron modification of the ADF statistic when an intercept and a deterministic trend are included in the specification. $\hat{\tau}_1^C$ and $\hat{\tau}_2^C$ report the results of the Lee-Strazicich statistics for 1 and 2 breaks, respectively. In both cases, the breaks affect both the intercept and the slope of the trend.

$\eta$ reflects the KPSS statistic when an intercept and a deterministic trend are included in the specification. $\eta_n^B$ (n=1,2) reports the value of the minimum value of this KPSS statistic when n changes in the slope of the trend are permitted.

TB_1 and TB_2 means the estimations of the periods when the breaks appears.

$^{a, b, c}$ signify the rejection of the unit root null hypothesis for a given 1%, 5% and 10% significance level, respectively.
Table 2. Bai-Perron procedure

| Country | $\text{UD}_{max}$ | $\text{WD}^{0.05}_{max}$ | $TB_1$ | $TB_2$ | $TB_3$ | $TB_4$ | model |
|---------|-------------------|--------------------------|--------|--------|--------|--------|-------|
| Canada  | 291.7             | 640.0                    | 68:3   | 79:3   | 85:4   | 95:2   | 1     |
| France  | 294.5             | 646.2                    | 81:1   |        |        |        | 2     |
| Germany | 20.7              | 24.6                     | 66:1   | 95:3   |        |        | 1     |
| Italy   | 41.3              | 90.3                     | 76:1   | 95:2   |        |        | 1     |
| Japan   | 38.9              | 46.4                     | 85:2   | 95:1   |        |        | 1     |
| UK      | 47.9              | 84.1                     | 66:1   | 73:2   | 82:3   | 92:4   | 1     |
| USA     | 111.4             | 132.3                    | 81:2   |        |        |        | 2     |

Panel A. Long-run case

Panel B. Short-run case

| Country | $\text{UD}_{max}$ | $\text{WD}^{0.05}_{max}$ | $TB_1$ | $TB_2$ | $TB_3$ | $TB_4$ | model |
|---------|-------------------|--------------------------|--------|--------|--------|--------|-------|
| Canada  | 34.4              | 50.9                     | 78:2   | 92:2   |        |        | 1     |
| France  | 99.9              | 132.6                    | 81:1   | 96:1   |        |        | 2     |
| Germany | 13.3              | 14.6                     | 95:1   |        |        |        | 1     |
| Italy   | 61.5              | 119.8                    | 96:4   |        |        |        | 1     |
| Japan   | 324.0             | 710.9                    | 92:4   |        |        |        | 1     |
| UK      | 59.8              | 85.8                     | 79:3   | 92:3   |        |        | 1     |
| USA     | 100.4             | 172.6                    | 79:4   |        |        |        | 2     |

$\text{UD}_{max}$ and $\text{WD}^{0.05}_{max}$ test for the non-structural break null hypothesis. Both statistics reject this null hypothesis for a given 5% significance level for all the reported cases. $TB_i$ ($i=1, 2, 3, 4$) reflects the estimation of the periods when the breaks appear. Model 1 implies that only the intercept varies, whilst Model 2 means that the break only affects the parameter associated to the inflation rate.
|                | R² | μ₁  | μ₂  | μ₃  | μ₄  | μ₅  | \( \sum \gamma_{1,i} \) | \( \beta_1 \) | \( \sum \gamma_{2,i} \) | \( \beta_2 \) | \( \sum \gamma_{3,i} \) | \( \beta_3 \) |
|----------------|----|-----|-----|-----|-----|-----|----------------|-------------|----------------|-------------|----------------|-------------|
| **Panel A. Long-run nominal interest rate** |    |     |     |     |     |     |                         |             |                   |             |                   |             |
| Canada         | 0.97| 4.36| 5.20| 9.23| 7.79| 5.36| 0.13                      | 0.46         | 0.06             | 0.39         | 0.05            | 0.13         |
| France         | 0.98| 4.56|     |     |     |     | 0.07                      | 0.40         | 0.01             | 0.07         | 0.00            |             |
| Germany        | 0.95| 5.32| 6.29| 4.55|     |     | 0.05                      | 15.22        | 0.01             | 18.21        | 0.00            |             |
| Italy          | 0.99| 4.14| 9.88| 4.17|     |     | 0.01                      | 17.51        | 0.01             |             | 0.01            |             |
| Japan          | 0.97| 7.09| 5.04| 3.89|     |     | 0.07                      | 15.59        | 0.00             |             | 0.00            |             |
| UK             | 0.96| 5.93| 8.25| 13.44| 9.96| 6.24| 0.00                      | 0.01         | 1.81             | 1.45         | 7.35            |             |
| USA            | 0.97| 2.55|     |     |     |     | 0.07                      | 0.75         | 0.13             | 1.45         | 7.35            |             |
| **Panel B. Short-run nominal interest rate** |    |     |     |     |     |     |                         |             |                   |             |                   |             |
| Canada         | 0.93| 3.02| 6.87| 4.01|     |     | 0.16                      | 0.62         | 0.06             | 0.39         | 2.61            | 0.23         |
| France         | 0.94| 3.83|     |     |     |     | 0.13                      | 0.59         | 0.29             | 1.36         | 0.05            | 0.23         |
| Germany        | 0.92| 4.70| 2.72|     |     |     | 0.09                      | 10.15        | 0.45             | 2.81         | 2.61            | 2.78         |
| Italy          | 0.97| 6.57| 2.34|     |     |     | 0.11                      | 0.69         | 0.11             | 1.53         | 1.36            |             |
| Japan          | 0.98| 4.33| 1.47| 3.06|     |     | 0.21                      | 0.32         | 0.32             | 1.82         | 35.05           |             |
| UK             | 0.91| 5.52| 9.77| 1.05|     |     | 0.14                      |             | 0.22             |             | 0.03            |             |
| USA            | 0.92| 0.00|     |     |     |     | 0.13                      | 1.05         | 0.22             |             | 1.82            |             |

\( \sum \gamma_{1,i} \) reflects the sum of the estimation of the \( \gamma \) parameters of either model (8) or (9). For those cases where model (9) has been selected, the subindex \( I \) means that we are analyzing the Fisher sample in the first sub-sample, with the same applying to \( II \) and \( III \). The value of \( \beta \) is obtained from (11). \( F_i (i = I, II, III) \) reports the value of the F-statistic for testing the \( \beta = 1 \) null hypothesis.

\( a, b \) and \( c \) signify the rejection of the unit root null hypothesis for a given 1%, 5% and 10% significance level, respectively.