Stability of Floating Wind Turbine Wakes

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Abstract.

Floating offshore wind turbines (FOWTs) are the next frontier in offshore wind energy, allowing exploration of deep-water regions previously unavailable to fixed-foundation turbines. Since offshore turbines operate in lower turbulence levels, the intrinsic hydrodynamic unstable modes of the tip vortices can have even more relevance than in onshore turbines. For floating turbines, platform motion induced by wind and wave loads can trigger vortex instabilities, modifying the wake structure, possibly influencing the flow reaching downstream wind turbines. In the present paper, we study those effects by the means of numerical simulations and their comparison with analytical studies. In our simulations, the wind turbine blades are modeled as actuator lines in the incompressible Navier–Stokes equations. Heave motion with different amplitudes and frequencies are studied. The effect of increasing amplitude is to advance the onset of vortex interaction. For the lower frequency of heave motion, several vortices coalesce to form a large flow structure. High amplitude of oscillations in the streamwise velocity were observed due to these flow structures, which may increase fatigue or induce high amplitude motion on downstream turbines. The number of vortices that interact, as other qualitative phenomena of the numerical simulation, were well predicted by a simple stability model of two-dimensional row of vortices. The disturbances imposed by the heave motion were also compared to the eigenvectors resulting from linear stability theory for helical vortices and the predicted growth rates for the wavenumbers resulting from this comparison were consistent with the model of a row of vortices. These results motivate further studies to understand the impact of the larger flow structures on downstream turbines.

1. Introduction

Offshore wind energy has been receiving an increasing amount of attention in the past few years. The reason for this are the several advantages it has in comparison with onshore wind turbines, such as: stronger and more consistent wind conditions, the possibility to install larger turbines, the reduced impact of the noise generated by the turbines and the mitigated harm it causes to wildlife. In order to make this kind of energy accessible and more affordable, a few technological issues need to be overcome. For example, for deep waters (typically above 50 meters), the bottom-mounted configuration is no longer feasible and Floating Offshore Wind Turbines (FOWT) have to be considered, where, inspired by naval architecture solutions, the turbine is usually mounted on a floating platform. Since this platform is not bottom-mounted,
it is allowed (under the constraint of the mooring system) to move under the action of the wind, sea waves and sea currents. This movement may produce modifications on the structure of the wake of the turbine, possibly having important performance impacts in other downstream turbines. Some studies have been carried out in the past in that direction. For example, the performance of the wind turbine (such as power and thrust) was evaluated using imposed pitch \([1, 2]\) and surge \([3]\) motion. Later, other studies \([4, 5]\) performed numerical simulations where the platform motion was not imposed, but computed. However, all those studies have paid little attention to the far-wake generated by the turbine, focusing only on near-turbine phenomena affecting the performance of that turbine. The predominant numerical model employed in those works was URANS (Unsteady Reynolds-Averaged Navier-Stokes), in which the flow close to blades was resolved.

Recently, a few studies \([6, 7]\) have payed more attention to the issue of the far wake when platform movement occurs. To avoid the dissipation of traditional low-order computational fluid dynamics (CFD) methods, the wake was modelled by Lagrangian-based vortical particles \([6]\) or vortex rings \([7]\). With these vortex-based models, they were able to provide evidence to possible patterns for the tip-vortices interactions when the platform undergoes motion on each of its six solid degrees of freedom.

Alternatively, several numerical studies have been carried out focusing on the stability of tip vortices of fixed wind turbines. Those studies are important for understanding how the wakes of several wind turbines interact with each other in a wind farm and how it affects its global performance. Earlier studies \([8, 9]\) developed a theoretical framework to study the stability of helical vortices. The results of the stability theory have been confirmed quantitatively both experimentally \([10, 11, 12]\) and numerically \([13, 14]\). Later, those instabilities were studied numerically on a flow under different configurations such as shear \([15]\) and yaw misalignment \([16]\). More recently the interaction of vortices of two in-line turbines was studied \([17]\). These recent numerical studies, focused on fixed wind turbines, relied on high-order numerical simulations of the Navier-Stokes equations for its superior resolution of the vortex structures. The perturbations were imposed as body forces to the blade tips, and the goal was to investigate the evolution and growth of instabilities. However, since the focus was not the flow around the airfoil blades, the blades were not resolved but modeled with the Actuator Line Method \([18]\).

In this work, we propose to employ a previously used high-order numerical method \([15, 16, 17]\) to study the stability of the wake created by a FOWT. Instead of imposing perturbations of body forces, the wake will be perturbed by the motion of the turbine itself. We will consider only the heave motion, but vary its amplitude and frequency, providing new extra information and understanding on how tip vortices interact in those configurations.

2. Methods

2.1. Numerical Solver, Domain and Boundary Conditions

Nek5000, a spectral element method (SEM) code, was used to solve the three-dimensional Navier-Stokes equations in a fixed frame of reference. The spectral element method exhibits low dispersion and dissipation \([19]\), which is relevant for stability calculations. In each spectral element seventh-order Lagrange polynomials on Gauss-Lobatto-Legendre quadrature points are used for spatial discretization and a third-order implicit/explicit scheme is applied for temporal discretization. To stabilize numerical instabilities, filtering is applied \([16]\).

All quantities are non-dimensionalized by the turbine radius \(R\) and the free-stream velocity \(W_\infty\). The turbine was positioned at the origin of a cylindrical domain of radius \(R_{rad} = 5\). The distances of the inlet and the outlet to the turbine were \(z_{in} = 5\) and \(z_{out} = 11.78\), respectively (figure 1(a)). Around the center of the domain, the elementwise discretization in the streamwise direction was uniform between \(-0.6 < z < 8.025\), with constant spacing \(\Delta z = 0.075\). As can be
seen in figures 1(b) and (c), the discretization was coarser in regions farther from the turbine, in order to reduce computational cost. Homogeneous Dirichlet boundary conditions with constant velocity \( W_\infty = 1 \) were imposed on the inflow and lateral boundaries. Sheared and turbulent inflow were not considered in this study. At the outlet, the natural outflow boundary condition was imposed in conjunction with a sponge region, with a width \( \Delta z = 2.5 \), that forces the \( x \) and \( y \) components of velocity to zero. Due to the nature of the outflow boundary condition, no forcing of the \( z \)-component is needed.

**Figure 1.** (a) Schematic view of the computational domain. The center of the turbine is located at \((x_{t0}, y_{t0}, z_{t0}) = (0, 0, 0)\). \( R_{rad} = 5 \), \( z_{in} = 5 \) and \( z_{out} = 11.78 \). All distances are non-dimensionalized by the radius. (b) and (c) Mesh at the center of the domain, showing the spectral elements.

### 2.2. Wind Turbine Modelling

The wind turbines were modelled using the actuator line method [18]. In this method, the blades are represented by body forces calculated from airfoil data and local velocity. Considering the local velocity and the chord and twist distributions, the normal and tangential forces are calculated for \( N_{ACL} \) points \((x_n)\) along the blades. The discrete two-dimensional force vector \((F_{2D})\) is projected on the domain \((x)\) using the convolution of the force with a three-dimensional Gaussian kernel:

\[
F(x, t) = F_{2D}(x_n, t) * \eta(|x - x_n|), \quad \eta(|x - x_n|) = \frac{1}{\varepsilon^3 \pi^{3/2}} \exp \left[ -\left( \frac{|x - x_n|}{\varepsilon} \right)^2 \right], \quad (1)
\]

where \( \varepsilon \) is a smearing parameter. Following parametric studies [16], \( \varepsilon \approx 3.5 \Delta r \) (where \( \Delta r \) is the averaged grid spacing) and each blade was discretized with 100 points.

The turbine was modelled after the turbine of the Blind Test [20]. The chosen operation condition was the tip-speed ratio \( \lambda = \Omega R/W_\infty = 6 \), which corresponds to the optimum performance for this turbine. Hub and tower were not included, since the focus of the work is the stability of the tip vortices. The code was extensively validated for this turbine, showing good agreement between numerical simulations and experimental results [21, 22, 16] and was previously used in vortex stability studies [15, 16, 17] with the same turbine, and similar discretization and actuator line parameters.

The lift and drag coefficients of the 14\% thick NREL S826 airfoil were obtained by [23]. For the computational simulation, a Reynolds number based on the radius of \( Re_R = U_\infty R/\nu = 50,000 \) was used (where \( \nu \) is the dynamic viscosity). However, since the Reynolds number of the lookup table can be independent of the CFD Reynolds number, a constant factor of 5.64 is applied to the local Reynolds number used to interpolate the airfoil lift and drag coefficients,
so that the tip local Reynolds number (based on the chord at the tip, \( c_t \)) is in the order of 
\[
Re_c^{tip} = U_\infty \lambda c_t / \nu \approx 10^5,
\]
matching the value from the experiments [20].

2.3. Heave motion

When a wind turbine is mounted on a floating platform, it is subjected to motions that result
from the loads imposed by the wind on the turbine, the waves and sea currents on the platform,
and the reaction and constraints of the mooring lines. Wind and waves are usually aligned,
and the yaw control system of the turbine guarantees that the rotor faces the wind directly.
The floating platforms that are usually employed for floating turbines, namely spar-buoy, semi-
submersible platform, and barge, have high degree of spatial symmetry with respect to the
vertical axis. All these aspects make the movement of the turbine to occur mostly at the
vertical plane parallel to the wind. In other words, the most relevant motions of a floating
offshore wind turbine are heave, surge and pitch.

In this paper, we consider heave motion only. Heave is directly caused by the local oscillation
of the sea level due to the passage of sea waves. To assess its amplitude and its frequency,
we performed simulations using the software FAST v8 [24], for the NREL 5MW turbine [25]
mounted on the OC4 semi-submersible platform [26], subject to a 10.3 m/s wind speed and
waves corresponding to a JONSWAP wave spectrum with significant height (Hs) of 1.5 m peak
spectral period (Tp) of 8s. These environmental conditions are typical of the Southeast Brazilian
continental shelf. Simulations were run considering water depths from 500 m to 1000 m, with
catenary mooring lines designed specifically for each of the depths. We observed amplitudes
of approximately 1% of the radius (2% peak-to-peak), and frequency of about a half of the
rotation of the blades. So we employed these values as starting points in the current study, and
investigated how variations around this set point affect the wakes produced.

The heave motion was implemented as movement of the center of rotation of the actuator
lines:
\[
(x_t, y_t, z_t) = (x_{t0}, y_{t0}, z_{t0}) + (0, A \sin \omega t, 0)
\]
where \( \omega \) is the angular frequency. We define the \( \omega^* = \omega / \Omega \) as the normalized angular frequency.

3. Results

3.1. Direct numerical simulations

Five cases with different heave motion parameters were simulated, according to table 1. The
reference case has amplitude \( A = 1\% \) and angular frequency \( \omega = 0.5\Omega \), according to estimates
detailed in section 2.3. A higher amplitude of 2\%, still within the same order of magnitude of the
reference case, and a lower amplitude of 0.1\% were considered. The effect of the amplitude can
be seen in figure 2(a)-(c), for the reference frequency. The effect of the increase in amplitude is
to advance the onset of the instability, bringing it closer to the turbine. It is possible to note that
the effect of displacing the tip vortices follows the results from Ivanell et al. [13], who showed
the effect of the amplitude of a body force in disturbing the tip vortices, a property confirmed
by later studies and used to develop a model for the stable wake length based on linear stability
theory [14, 15].

The interaction of vortices occurs further upstream also for the higher frequency, maintaining
the amplitude constant (figure 2(b), (d) and (e)). The change in frequency of the heave motion
highlights a more interesting effect. For heave motion with lower frequency, larger flow structures
are created inside the wake. Observing the vortex interaction in figure 2, it is possible to notice
that these flow structures are created by the merging of the vortices. The formation of similar
structures for lower frequencies can also be noted in the simulations reported in [6]. However, in
their case, the lower frequency was accompanied by a modification in the nature of the motion,
\[
\omega^* = \frac{\omega}{\Omega}
\]

Angular Frequency | Amplitude
---|---
0.5 | 0.1%, 1% and 2%
1 | 1%
1.5 | 1%

Table 1. Parameters of heave motion for the cases simulated.

...from translation (heave, sway and surge) to rotation (yaw, pitch and roll), making it difficult to isolate the effects.

The streamwise flow inside the wake experiences higher fluctuations due to these flow structures. These fluctuations might affect downstream turbines, increasing unsteady loads that may worsen the fatigue. In the case of a downstream FOWT, it may increase the amplitude of its motion. It is even possible to imagine an extreme case, where the unsteady flow and the sea waves add to each other with the same frequency, creating very high amplitudes. It is specially worth investigating because the frequency of the flow structures is the same frequency of the upstream heave motion, caused by sea waves.

In order to estimate the effect of the unsteadiness of the streamwise velocity on a downstream turbine, the parameter \( F^* \) was defined. It is the streamwise dynamic pressure integrated over the area of a disk, normalized by the equivalent value at infinity:

\[
F^* = \frac{\int_S \frac{1}{2} \rho w^2 ds}{\frac{1}{2} \rho W_*^2 S}
\]

where \( S \) is the area of a disk with the same radius of the turbine (\( R = 1 \)) centered at \((x_c, y_c, z_c)\) and \(\rho\) is the fluid density. Two positions for the disk were chosen for this calculation, one completely inside the wake, \((x_c, y_c, z_c) = (0, 0, 6)\), and another which is impinged by the vortical structures from one side of the upstream turbine, \((x_c, y_c, z_c) = (1, 0, 6)\). As can be seen in table 2, the root mean square (RMS) of the integral of the dynamic pressure is considerably higher for angular frequency \( \omega^* = 0.5 \). This indicate that this frequency might be more dangerous for downstream turbines. It should be noted, however, that our domain does not extend to a position where downstream FOWTs are expected to be installed. To confirm this effect, these flow structures would have to be present beyond 12 or 14 radii. Turbulent mechanisms, either intrinsic of the wake or from the atmospheric boundary layer, might dissipate the larger flow structures by that point.

| \( \omega/\Omega \) | Disk centered at (0,0,6) | Disk centered at (1,0,6) |
|---|---|---|
| Mean | RMS | RMS/Mean | Mean | RMS | RMS/Mean |
| 0.5 | 0.322 | 0.101 | 31.3% | 0.674 | 0.147 | 21.9% |
| 1 | 0.316 | 0.046 | 14.7% | 0.662 | 0.0409 | 6.2% |
| 1.5 | 0.316 | 0.021 | 6.7% | 0.662 | 0.051 | 7.7% |

Table 2. Mean and RMS values of normalized integral of the dynamic pressure, \( F^* \).

3.2. Stability analysis

In this section, a first order approximation of the disturbance of the tip vortices introduced by the heave motion is presented. The determination of the wavenumber of the disturbance allows...
Figure 2. Instantaneous streamwise velocity along the wake in the $yz$-plane (left) and $xz$-plane (right). Three dimensional iso-surfaces of vorticity magnitude ($|\xi| = 10$) are shown in grey for the figures in the left. (a) $A=0.1\%, \omega^* = 0.5$. (b) $A=1\%, \omega^* = 0.5$. (c) $A=2\%, \omega^* = 0.5$. (d) $A=1\%, \omega^* = 1.0$. (e) $A=1\%, \omega^* = 1.5$. 
results from simpler analytical stability models, such as a 2-d row of vortices [27] and perfect helical vortices [8, 9], to be used to interpret the numerical results.

The equation that defines the position of the tip of the blades is

\[
(x_b, y_b, z_b) = (R \cos(\Omega t + \phi_n), R \sin(\Omega t + \phi_n) + A \sin \omega t, 0),
\]

where \(\phi_n = 2\pi n/N_b\) is the initial azimuthal position of each blade \(n (n = 0 \to N_b - 1, \text{where } N_b = 3 \text{ is the number of blades})\). Previous studies identified that the tip vortices are mainly convected by the flow in the streamwise direction [15]. Considering a convection velocity \(w_c\), the position of the tip vortices would be

\[
(x_v, y_v, z_v) = (R \cos(\Omega t + \phi_n), R \sin(\Omega t + \phi_n) + A \sin \omega t, w_c t).
\]

Then, the perturbation directly induced by the heave motion is \((\delta x_v, \delta y_v, \delta z_v) = (0, A \sin(\omega t), 0)\), in cartesian coordinates. For \(A \ll R\), a first order approximation of the disturbance in the radial direction is

\[
\delta r_v = \sqrt{x_v^2 + y_v^2} - R \approx \sqrt{R^2 + 2RA \sin(\Omega t + \phi_n) \sin(\omega t)} - R \approx A \sin(\omega t) \sin(\Omega t + \phi_n)
\]

The disturbance in the azimuthal direction is not usually significant for stability of helical vortices, so it is neglected. Defining \(\theta = \Omega t + \phi_n\), the disturbance \(\delta r_v = A \sin(\omega^*(\theta - \phi_n)) \sin \theta\) is defined for the helices in function of the azimuthal angle. This initial disturbance induced by the heave motion grows in time and in space, as the vortices move downstream. The growth rate can be estimated using analytical stability models.

### 3.2.1. Stability of a 2-d Row of Vortices

Several studies made the connection between the stability of helical vortices and a two dimensional row of vortices [10, 14, 11, 12], showing that the scaled growth rate of instabilities collapse to the theoretical value for an infinite row of vortices calculated by Lamb [27] (see also [28]). The growth rate \(\sigma\), when scaled using the distance of neighboring vortices, \(h\), and the circulation \(\Gamma\), \(\tilde{\sigma} = \sigma(2h^2/\Gamma)\), has a maximum value of \(\pi/2\). Kleusberg et al.[15, 16] showed that the growth rate collapse to \(\pi/2\) even for non-uniform helices created by turbines under sheared inflow or yawed, if scaled by local properties. Applying the same idea to the present configuration, we take values of \(\theta = \theta_0 + 2\pi j\), where \(j\) is an integer and \(\theta_0\) is chosen to define a cross-section that contains the row of vortices \((0 \leq \theta_0 < 2\pi)\). The displacement of the vortices is

\[
\delta r_{v2d} = A \sin (\omega^*\theta_0 + \omega^*(2\pi j - \phi_n)) \sin \theta_0 = A \sin \left(\omega^*\theta_0 + \frac{\omega^*}{N_b} 2\pi (N_b j - n)\right) \sin \theta_0.
\]

Since \(N_b j - n\) can be any integer \(m\) and can interpreted as the index of each 2-d vortex,

\[
\delta r_{v2d} = (A \sin \theta_0) \sin \left(\omega^*\theta_0 + \frac{\omega^*}{N_b} 2\pi m\right)
\]

Comparing equation 8 with the disturbance imposed in [28] in a 2-d row of vortices, we see that the term \(\sin \theta_0\) is only a scaling factor in the amplitude, the term \(\omega^*\theta_0\) is a phase shift, and, most importantly, \(\omega^*/N_b\) is the dimensionless subharmonic wavenumber denoted \(p\) by Saffman [28]. Using this scaling, the theoretical growth rate for a row of vortices is shown in figure 3(a). The number of vortices that interact with each other can be calculated from the maximum of \(1/p\) and \(1/(1 - p)\). The values of table 3 agree well with figure 2. For example, for \(\omega^* = 0.5\), six vortices interact with each other and coalesce to form the large flow structure discussed in section 3.1. It is unclear if this agreement would also be observed for higher frequencies, since the high wavenumbers involved might not allow the formation of larger structures.
\[
\omega^* = \frac{\omega^*}{N_b} \frac{1}{p} \frac{1}{1-p}
\]

| \(\omega^*\) | \(p\) | \(\frac{1}{p}\) | \(\frac{1}{1-p}\) | Number of interacting vortices |
|--------------|---------|-----------------|---------------------|-----------------------------|
| 0.5          | 1/6     | 6               | 6/5                 | 6                           |
| 1.0          | 1/3     | 3               | 3/2                 | 3                           |
| 1.5          | 1/2     | 2               | 2                   | 2                           |

Table 3. Number of interacting vortices as function of angular frequency.

![Graph](image1)

![Graph](image2)

Figure 3. (a) Theoretical growth rate for a two-dimensional row of vortices. In order to make it comparable to helical vortices, the x-axis is scaled using the relationship \(\omega^* = pN_b\). (b) Theoretical growth rate for the eigenvalues in function of the wave number for a triple helix. Black filled circles correspond to modes relevant to case \(\omega^* = 1\).

3.2.2. Stability of Helical Vortices  

Going back to the three-dimensional helical vortices, as discussed in [15] and [17], for disturbances imposed as body forces at tip of the blades, the angular frequency is related to the wavenumber according to \(\omega = k\Omega\). However, as it is going to be shown, the heave motion induces other wavenumbers in the helical system. In the two-dimensional case, the disturbance in one direction, when projected onto the eigenvectors, create a disturbance in the other component with equal magnitude (see [28]). For three-dimensional helical vortices, this means that the heave motion induces a component in the streamwise direction \(\delta z_v \approx \delta r_v\):

\[
\delta z_v \approx A \sin (\omega^* (\theta - \phi_n)) \sin \theta. \quad (9)
\]

Using trigonometric identities, this can be decomposed in

\[
\delta z_v \approx \frac{A}{2} (\cos ((\omega^* - 1)\theta - \omega^* \phi_n) - \cos ((\omega^* + 1)\theta - \omega^* \phi_n)). \quad (10)
\]

Therefore, the heave motion with frequency \(\omega^*\) creates perturbations with wavenumber \(k = \omega^* - 1\) and \(k = \omega^* + 1\). The term \(\sin \theta\), which comes from the decomposition of the oscillation of \(y\) in the radial direction, is not present in the previous studies where body forces were used as disturbances at the tip, hence their conclusion that \(k = \omega^*\) [15, 17]. This latter result is also expected for a surge motion, since it does not requires the term \(\sin \theta\). Nevertheless, for all the cases discussed (heave motion, surge and disturbances at the tip), it should be noted that an angular frequency \(\omega^* = 1.5\) would excite the out-of-phase vortex pairing mechanism that has the highest growth rate \(\tilde{\sigma} \approx \pi/2\), according to the mechanism described in [14] (see figure 15(a) of that reference, for \(k = 0.5\)). A dynamic consistent with vortex pairing can be observed in figure 2 for this frequency.
Figure 4. Comparison of disturbance imposed on vortices with eigenvectors of the stability theory. Unrolled tip vortices, indicating distinct vortices by distinct colors. Dashed line: undisturbed helix. (a) Disturbance on helix for $\omega^* = 1$ (equation 9). (b) and (c) Disturbance on helix decomposed in components with $k = 0$ and $k = 2$, respectively (equation 10). (d) and (e) Eigenvectors of stability theory of helical vortices corresponding to eigenvalues indicated in figure 3, obtained using the method of [9].

The stability for this system of vortices was calculated using the method of [9] and the eigenvalues are shown in figure 3(b) (considering perfect helices, with constant radius and pitch). The eigenmodes corresponding to $\omega^* = 1$ are highlighted. In figure 4, the eigenvectors of the stability calculation for the eigenvalues indicated in figure 3(b) are compared to the two terms of equation 10. The similarity is impressive. Even though the figures were obtained with completely different methods, they look identical. The only adaptation necessary was to scale the amplitudes of the disturbances and the eigenvectors, since eigenvectors are non-dimensional. For all other cases, similar agreement was found. This indicates that the disturbances imposed by the heave motion agree almost identically to eigenvectors of the stability theory, thus, only these modes would be excited.

For $k = (\omega^* + 1) = 2$ there is more than one eigenvalue that could be chosen, since it is near the intersection of the parabola between $0 < k < 3$ and the parabola between $1 < k < 4$ of figure 3(b). The eigenvector that was observed to be identical to the disturbance is the one whose eigenvalue belongs to the parabola between $1 < k < 4$. It is worth noting that the relative position of $k = 2$ to the parabola between $1 < k < 4$ is similar to the relative position of $\omega^* = 1$ to the parabola of figure 3(a). This last observation was noted for all the cases investigated. In summary, in the case of ambiguity of which mode is selected, the relative position comparable to the parabola can be used. For example, there are three modes with $k = 1.5$, but the one excited by $\omega^* = 0.5$ would be the one belonging to the parabola between $1 < k < 4$ (with growth $\tilde{\sigma} / (\pi / 2) \approx 0.55$), since this corresponds to the relative position of $\omega^* = 0.5$ in the parabola of 2-d row of vortices (figure 3(a)). Consequently, the results of stability of the full tridimensional helical system might not be needed to estimate the growth rate. It is expected that the growth rate can be predicted from the theory of infinite row of two-dimensional vortices, within the restrictions discussed at [11] and [12]. Qualitatively, the influence of frequency on the position of the onset of vortex interaction observed in figure 2 seems to agree with this prediction.
Nevertheless, this still lacks further experimental or numerical confirmation.

4. Conclusions
The wake of a floating offshore wind turbine under heave motion was analysed by means of numerical simulations and compared with the linear stability theory. For the numerical simulations, the use of actuator lines with a high-order SEM code made the analysis of the vortex interactions possible.

Frequency and amplitude of the heave motion for the reference case were estimated using real-world parameters. The effect of amplitude and frequency of the motion was analysed by simulating a total of five cases. A higher amplitude of motion brings the onset of the vortex interaction closer to the turbine. Increasing the frequency from half to one and a half the rotational frequency has the same qualitative effect. This is consistent with the stability theory, which indicates that the most amplified mode occurs for a frequency of 1.5 times the turbine rotation.

Using a simplified model, it was noted that the perturbations induced by the heave motion have wavenumbers of \( k = \omega^* - 1 \) and \( k = \omega^* + 1 \), in contrast to the perturbations imposed by body forces, which have wavenumber \( k = \omega^* \) [15]. However, for these two cases, the growth rate is expected to be the same and compatible with a model of an infinite row of two-dimensional vortices calculated by Lamb [27]. When the stability theory of helical vortices predicted multiple modes for the same wavenumber, the eigenvectors that were identical to the components of the perturbations were those compatible with the stability theory of a row of vortices. Further studies are needed to quantitatively confirm whether the modes induced by the first order approximation of the disturbance are present and grow according to the theoretical predictions.

The stability theory of a row of two-dimensional vortices also predicted the number of vortices that interact with each other in the numerical simulations. For \( \omega^* = 1.5 \), our simulations showed the out-of-phase mechanism of two vortex pairing observed in other numerical [13, 14, 15] and experimental works [29, 11, 12]. For \( \omega^* = 0.5 \), larger structures with the interaction of six vortices were observed. These larger flow structures created higher oscillations of the streamwise velocity inside the wake. The RMS of the integrated dynamic pressure was considerably higher for this case. This could increase the fatigue loads and the amplitude of the motion of downstream FOWTs. Nevertheless, the extension of the computational domain of the present study did not allow for study of the flow and computation of loads at a position where downstream FOWTs are expected to be installed. Also, the free-stream turbulence, not considered here, or turbulent mechanisms intrinsic of the wake could dissipate or modify these flow structures. Hence, future studies are necessary to confirm if these effects are sustained further downstream.

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