Depth profiling of thermophysical parameters of curved solids using photothermal technique

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Abstract. The effect of curved surface of both cylindrical and spherical samples with continuous gradient structure (e.g., case hardened surface) on the photothermal radiometric signal (PTR) are investigated. Using an appropriate signal processing, it is found theoretically that the curvature effect of both cylindrical and spherical samples can be, or partially can be suppressed under certain conditions and the PTR signal from the curved composite sample can be equivalent to that of a flat surface with the same structure.

1. Introduction
Photothermal Radiometry (PTR) is a powerful tool for the thermophysical characterization and the nondestructive evaluation (NDE) of a wide variety of materials. For decades, researches in photothermal techniques have been restricted to samples with flat surfaces. With the increasing applications of PTR to the characterization of materials with curved surface, studies on curved surface samples (e.g. cylindrical or spherical samples) have been motivated in recent years[1-6]. In this paper, the frequency characteristics of in-depth inhomogeneous cylindrical and spherical solid samples are studied based on the quadruple method[2], in which the frequency dependence of PTR amplitude and phase on the sample curvature and depth profile of thermophysical properties are investigated. The relationship between an inhomogeneous cylindrical or spherical sample and an inhomogeneous flat sample is established. It is found that the curvature effect of both cylindrical and spherical samples can be eliminated or partially eliminated under certain conditions, which open a new way to characterize the curved sample with a simplified algorithm.

2. Theoretical treatment
We first introduce the theoretical models of cylindrical and spherical surfaces with a continuous gradient structure[3,4,5]. The cylindrical and spherical samples are illuminated by a uniform light beam impinging on part of their surface subtending a sector of angle $2\alpha$, the beam is modulated at a frequency of $f$. We then introduce the theoretical model of a flat-surface sample with the same structure as that of the multilayer cylindrical or spherical model. Fig. 1 shows the geometries and the coordinates of the respective cylindrical, spherical and flat surfaces. Briefly, the illumination schemes and the cross-sectional geometry of a gradient cylinder is shown in Fig. 1(a), in which the gradient cylinder consisting of $N$ layers with different materials. The thermophysical properties of layer $i$ are labelled by sub-index $i$ and its outer and inner radii by $a_i$ and $a_{i1}$, respectively. Fig. 1(b) shows the cross-sectional geometry of a gradient sphere. The structure and thermophysical properties of each layer of the sphere in Fig. 1(b) are the same as that of the cylinder in Fig. 1(a). Fig. 1(c) shows the
same structure of a flat surface as those in Fig. 1(a) and (b). The thermal conductivity and thermal diffusivity of region \( i \) are denoted with \( k_i \) and \( \alpha_i \) as shown in Fig.1(c). The thermophysical properties of region \( i \) are equal to that of layer \( i \) in the cylinder (or sphere). The thickness of each layer is: \( L_i = a(i) - a(i+1), (i=1, ..., N-1) \).

![Diagram of geometries and coordinates of a cylindrical, spherical, and flat surface.](image)

**Fig. 1** The geometries and coordinates of a cylindrical (a), spherical (b) and flat (c) surface. Three samples have the same structure which contains \( N \) layers with different materials.

The detailed theoretical thermal-wave fields for the cylindrical, spherical and flat surfaces with multi-layer structure are described in Ref. [3-5], respectively. For a cylindrical sample, the thermal-wave field can be illustrated by:

\[
T(a_1, \beta) = \frac{I_0}{2} \sum_{m=-\infty}^{\infty} g_m(\alpha) \times \frac{A_m}{C_m} e^{\frac{j2\pi}{L} (a_1 - a)}
\]

where \( I_0 \) is light intensity; \( g_m(\alpha) = (-i)^m \times \frac{m \sin(m\alpha) \cos(\alpha) - \sin(\alpha) \cos(m\alpha)}{\pi(m^2 - 1)} \), and the coefficients \( A_m, C_m \) are obtained by a recurrence relationship[3]. For a spherical sample, the thermal-wave field is represented by:

\[
T(a_1, \beta) = \frac{I_0}{2} \sum_{n=0}^{\infty} g_n(\alpha) \times \frac{A_n}{C_n} P_n(\cos \theta)
\]

where \( P_n \) is the Legendre polynomial, \( g_n(\alpha) = \frac{2n+1}{2} \int_0^\pi P_n(\cos \lambda) \cos \lambda \sin \lambda d\lambda \), and the coefficients \( A_n, C_n \) are given in Ref. [4]. For a flat surface, the thermal-wave field is represented by:

\[
T_i(r, z = 0, \omega) = \int_0^r \frac{Q_S(0)}{k_0 \beta} e^{-\frac{\gamma_1 b_i}{\beta}} \frac{1 + g_1 e^{-2\Delta_j \beta_i}}{1 + g_1 \gamma_1 b_i e^{-2\Delta_j \beta_i}} e^{-\frac{j b_i^2}{4}} J_0(\lambda r) \lambda d\lambda
\]

where \( Q_S(0) = \frac{A_s(1 - R_s)}{\pi \sigma b_i^2} e^{-r_i / b_i} \), \( R_s, A_s \) represent surface reflection and absorption coefficient of the sample. The coefficients \( g_1, \gamma_1, b_i \) in Eq. (3) is computed using a recurrent relation in Ref.[5]. The PTR signals from different geometries of the same structures will then serve as a theoretical base in the signal processing for the elimination of the sample curvature.
3. Numerical calculations and discussions

In view of the structure of PTR signal from either a cylindrical or a spherical sample, it is seen that there are some common geometrical factors, e.g., angular dependence(θ, ϕ) in the PTR signal for the same shape of samples with different layer structures, for example, a homogeneous solid cylindrical sample and a multilayer cylindrical sample with the same diameters[1]. The common geometrical factors can be eliminated, or partially eliminated by a self-normalizing process. The signal after normalization will be solely or mainly from the contribution of the thermophysical properties of the material, which reduces the complexity of the curved-surface PTR to that of a flat surface structure. The self-normalizing procedure is performed as follows: first, the surface temperature of a cylinder (or sphere) with gradient structure is normalized to a homogenous cylinder (or sphere) with the same total radii and the same material as the inner core of the multi-layer gradient structure; second, the surface temperature of a flat surface with the same multilayer structure as the cylinder (or sphere) is normalized to a homogenous flat surface with semi-infinite thickness and the same material as the substrate of the multilayer flat sample. The amplitude and phase of a semi-infinite flat sample are calculated by the well-known 1-D model in Ref [7].

To show the effect of the normalization process on the PTR signal of the curved and inhomogeneous gradient structures, we assume that the cylindrical, spherical and flat samples have exactly the same arbitrary depth profile of thermal conductivity/diffusivity. The assumed formula is[5]:

$$k(z) = k_0 \left(1 + \frac{\Delta e^{-zc}}{1 + \Delta}ight)^2,$$

and

$$\Delta = \frac{1 - \sqrt{k_i/k_0}}{\sqrt{k_i/k_0} - e^{-zc}}.$$  \hspace{1cm} (4)

where $k_0$ and $k_i$ represent the values of thermal conductivity at two boundary surfaces $z=0$ (surface) and $L_0$ (inhomogeneous depth inside the sample), respectively. $L_0$ is the total thickness of the inhomogeneous surface layer (i.e., $a_1-a_N$, in Fig. 1(a) and (b), or $L_1+L_2+...+L_{N-1}$ in Fig. 1(c)). The inner core (or substrate of the flat surface) of the sample is assumed to be AISI 1018 steel ($k_N=51.9W/mK, \alpha_N=13.57 \times 10^{-6} m^2/s$) as shown in Fig. 1. The thermal conductivity of the inhomogeneous layer (and thermal diffusivity, which is linked by $\alpha(z)=k(z)/\rho c$, $\rho c$ is constant) is continuously increased from $k_1=36.05W/mK$ at surface to saturated $k_N=51.9W/mK$ at $L_0$ inside the material. $q=4500(1/mm)$ is assumed. The thickness of the inhomogeneous layer $L_0$ is varied to show the effect of thickness on the PTR signal. The diameter of cylindrical and spherical outer surface are assumed to be $D_1=2a_1=2, 4, 8$ and $16$mm, respectively. A ratio factor $\chi$ is defined to measure the relative thickness of the inhomogeneous layer $L_0$ (=a_1-a_N) and the radius of curvature ($a_1$) of a cylinder and a sphere: $\chi=(a_1-a_N)/a_1$.

Fig. 2(a) and (b) shows the normalized amplitude and phase of the surface temperature of the cylindrical and spherical samples with a ratio of $\chi=6\%$, respectively. The thicknesses of the inhomogeneous layer $L_0$ are 0.06, 0.12, 0.24, and 0.48mm for $D_1=2, 4, 8$ and $16$mm, respectively. The normalized amplitude and phase from the corresponding flat surfaces with the same structure as the curved ones, i.e., the inhomogeneous layer $L_0=0.06, 0.12, 0.24$ and 0.48mm, respectively, are compared with the cylindrical and spherical cases with the solid line plotted in Fig. 2. In the simulation, the beam size is assumed to be large enough so as to cover the entire upper part of the cylindrical and spherical samples and also meet 1-D condition in the case of flat surface. The temperature at north pole of cylinder and sphere is calculated. It is seen that all the normalized amplitude and phase of the cylindrical and spherical sample are well overlapped with the amplitude and phase of the corresponding flat surface of the same structure (same inhomogeneous layer), especially at the high frequency range. The featured peaks in the phase channel of the cylindrical and spherical samples are all approximately coincident with those of the flat surface, with a discrepancy in the frequency position at which the phase peak reaches maxima being <4% when $\chi=6\%$. With the increase of $\chi$, the discrepancy between the normalized phase of the curved surfaces and the flat
surface increases accordingly, from which the validation range of the technique can thus be deduced based on the acceptable tolerance of mismatched frequency position of the phase peak. The relative large deviations seen in samples of smaller diameters and at the lower frequency range is due to the competition of the sample radius and thermal diffusion length ($\frac{\alpha}{\pi f}$), in which a relative large sample radius and a small thermal diffusion length gives a better suppression of the curvature effect. With the fixed ratio $\chi$, samples with different diameters have different thicknesses of the inhomogeneous layer, the featured peak in the phase channel is shifting to the lower frequencies when the inhomogeneous layer is thicker, which is understandable due to the longer thermal diffusion length at lower frequencies.

4. Conclusions
In this work, an appropriate signal processing is present to eliminate the effect of curved surface on the PTR signals. By using the self-normalizing process, a curved surface PTR can be treated as a flat-surface PTR without the need of the precise dimension of cylindrical and/or spherical samples when the ratio $\chi$ is within a limited range. This technique opens a new way for extending PTR technique to the characterization of curved surface samples.

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