Using Phenomenological Formulae, Deducing the Masses and Flavors of Quarks, Baryons and Mesons from Two Elementary Quarks

Jiao-Lin Xu

The Center for Simulational Physics, The Department of Physics and Astronomy
University of Georgia, Athens, GA 30602, USA

e-mail: jxu@hal.physast.uga.edu

Abstract

Using phenomenological formulae, we deduce the masses and quantum numbers of the quarks from two elementary quarks ($\epsilon_u$ and $\epsilon_d$) first. Then using the sum laws and a binding energy formula, in terms of the qqq baryon model and SU(4), we deduce the masses and quantum numbers of the important baryons from the deduced quarks. At the same time, using the sum laws and a binding energy formula, in terms of the quark-antiquark bound state meson model, we deduce the masses and quantum numbers of the mesons from the deduced quarks. The deduced masses of the baryons and mesons are 98% consistent with experimental results. The deduced quantum numbers of the baryons and mesons match with the experimental results exactly. This paper improves upon the Quark Model, making it more powerful and more reasonable. It predicts some baryons also. PACS: 12.39.-x; 14.65.-q; 14.20.-c

keywords: phenomenology, elementary quark, mass, SU(4), baryon, meson
1. Introduction

Using SU(3)$_f$, SU(4)$_f$ and SU(5)$_f$, the Quark Model [1] has successfully deduced the quantum numbers of baryons and mesons. It also has found some masses of new baryons or mesons, from the masses of the old discovered baryons or mesons, using mass relations of Gell-Mann–Okubo mass formulae [2]. Thus the Quark Model has led to many discoveries of baryons and mesons. These works are some of the greatest works in particle physics. There are, however, the important problems that need to solve:

(1) The quark’s intrinsic quantum numbers cannot deduce and have to put in “by hand” [3]. How do we deduce the flavor of the flavored quarks? Why do all quarks have the same spin ($s = \frac{1}{2}$) and baryon number ($B = \frac{1}{3}$)?

(2) Are the quarks really all elementary particles? Why the quark with large mass always automatically decays into a smaller mass quark in a very short time?

(3) Why the different flavored quarks have the flavor symmetry SU(3), SU(4) and SU(5)? What is the physical foundation of the flavor symmetry?

(4) How do we deduce the mass spectrum of the quarks? the quark masses, as arbitrary parameters, led to that the standard model has too many arbitrary parameters (nineteen) and thus the standard model is incomplete [4].

(5) How do we deduce the mass spectrums of the baryons and the mesons?

These problems are concerned with the physical foundation of the standard model. We can not dodge these problems. There may be some clues about new theory in the solutions of these problems. Using phenomenological formulae, this paper deduces the masses and quantum numbers (including the flavor numbers S, C and B) of the current quarks from the two kinds of elementary quarks $\xi_u$ and $\xi_d$. This paper shows that all current quarks inside hadrons are the excited states of $\xi_u$ or $\xi_d$. Since the two elementary quarks ($\epsilon_u$ and $\epsilon_d$) have SU(2) symmetry, thus the flavor symmetry (SU(3)$_f$,
SU(4)$_f$ and SU(5)$_f$ are the natural extensions of SU(2)$_f$. Then this paper deduces the important baryons, from the deduced quarks, using the qqq baryon model and a phenomenological binding energy formula as well as SU(4). At the same time, it also deduces the important mesons, from the deduced quarks, using the $qar{q}$ meson model and a phenomenological binding energy formula. The deduced masses of the baryons and mesons are 98% consistent with experimental results. The deduced quantum numbers of the baryons and mesons match with the experimental results exactly.

The current quarks u, c and t have $Q = +\frac{2}{3}$; and the current quarks d, s and b have $Q = -\frac{1}{3}$. This case seems to indicate that there are two kinds of the quarks. One of them has $Q = +\frac{2}{3}$, and the other one has $Q = -\frac{1}{3}$. The Quark Model implicitly assumes that all quarks are elementary particles, but a quark with a larger mass always automatically decays into a smaller mass quark in a very short time, and the smaller mass quark always decays into the u-quark or d-quark. This situation might show that the quarks are not all elementary particles. The quark pairs $(u - \bar{u})$, $(d - \bar{d})$, $(s - \bar{s})$, $(c - \bar{c})$ and $(b - \bar{b})$ can be excited from the vacuum. At the same time, these quark pairs can annihilate back to the vacuum. These facts might imply that the quarks are from the vacuum. Thus we infer that there might be only two kinds of elementary quarks $\epsilon_u$ and $\epsilon_d$ in the vacuum state essentially and all quarks (including u and d) are their excited states from the vacuum state. The quarks u and c are the excited states of $\epsilon_u$; and the quarks d, s and b are the excited states of $\epsilon_d$. The following sections will show that our inferences might be correct. Since the mass of the top quark (t) is much larger than other quark masses (about 185 time proton mass), we cannot deduce the mass of the top quark using the phenomenological formulae. How to deduce the top mass is an open problem of this paper.

In order to deduce baryons and mesons from quarks, we have to deduce quarks first. We are lucky, because there is not any baryon or meson that contains the top quark, we do not need to deduce the mass of the top quark (t) in this paper.
2. Using Phenomenological Formulae, Deducing the Masses and Flavors of Quarks from Two Elementary Quarks $\epsilon_u$ and $\epsilon_d$

In this section, we assume that there are only two kinds of unflavored elementary quarks $\epsilon_u$ and $\epsilon_d$ in the vacuum state (there is not any other kind of quark in the vacuum state). Using phenomenological formulae, we deduce the masses and quantum numbers of the current quarks from the two kinds of elementary quarks $\epsilon_u$ and $\epsilon_d$.

2.1. The two kinds of elementary quarks $\epsilon_u$ and $\epsilon_d$ in the vacuum state

We assume that there is only one kind of unflavored (S = C = B = 0) elementary quark family $\epsilon$ with a color (or red or blue or yellow), a baryon number $B = \frac{1}{3}$, a spin $s = \frac{1}{2}$, an isospin $I = \frac{1}{2}$ and two isospin states ($\epsilon_u$ has $I_z = \frac{1}{2}$ and $Q = +\frac{2}{3}$, while $\epsilon_d$ has $I_z = -\frac{1}{2}$ and $Q = -\frac{1}{3}$). For $\epsilon_u$ (or $\epsilon_d$), there are three different colored (or red or blue or yellow) members. Thus there are six kinds of elementary quarks in the $\epsilon$-elementary quark family. The elementary quarks $\epsilon_u$ and $\epsilon_d$ have flavor SU(2)$_f$ symmetry.

They are essentially in the vacuum state. When they are in the vacuum state, their baryon numbers, electric charges, spins, isospins and masses ($B = Q = I = I_z = s = m_{\epsilon_u} = m_{\epsilon_d} = 0$) cannot be seen. Although we cannot see them, as physical vacuum background, they really exist in the vacuum state. Once they obtain enough energies, they may be excited from the vacuum state to form observable baryons or mesons.

2.2. The normally excited quarks $u$ and $d$

Generally, when a particle normally excited from the vacuum state, it will keep
its intrinsic quantum numbers unchanged and get a continuous energy spectrum from the lowest energy (the rest mass) to infinite. As a colored (or red or blue or yellow) elementary quark $\epsilon_u$ is normally excited from the vacuum state, it will excite into the observable state inside a hadron with the color (or red or blue or yellow), the electric charge $Q = \frac{2}{3}$, the spin $s = \text{the isospin } I = \text{the isospin } z\text{-component} = \frac{1}{2}$ and the $S = C = B = 0$; and it will get an energy of a continuous energy spectrum from the lowest energy (the rest mass $m_u$) to infinite. Comparing its quantum numbers with the current quark quantum numbers $[5]$, we recognize that it is the current u-quark. Similarly, as a colored (or red or blue or yellow) elementary quark $\epsilon_d$ is normally excited from the vacuum state, it will excite into the observable state inside a hadron with $B = \frac{1}{3}$, $Q = -\frac{1}{3}$, $I = s = \frac{1}{2}$, $I_z = -\frac{1}{2}$, $S = C = B = 0$ and it will get an energy of a continuous energy spectrum from the lowest energy (the rest mass $m_d$) to infinite. Comparing its quantum numbers with the current quark quantum numbers $[5]$, we recognize that it is the current d-quark. Since $\epsilon_u$ and $\epsilon_d$ have the flavor SU(2)$_f$ symmetry, the normally excited quarks u and d have the flavor SU(2)$_f$ symmetry also. Thus we have

\[
\begin{align*}
\text{the u-quark: } & B = \frac{1}{3}, \quad Q = \frac{2}{3}, \quad I = I_z = s = \frac{1}{2}, \quad S = C = B = 0, \quad m_u; \\
\text{the d-quark: } & B = \frac{1}{3}, \quad Q = -\frac{1}{3}, \quad I = -I_z = s = \frac{1}{2}, \quad S = C = B = 0, \quad m_d.
\end{align*}
\]

Let us find the rest masses $m_u$ of the u-quark and the rest mass $m_d$ of the d-quark now.

According to the Quark Model $[1]$, a proton is composed of three quarks (uud) and a neutron is composed of three quarks (udd) also. Thus the proton mass $M_p$ and the neutron mass $M_n$ generally will be

\[
\begin{align*}
M_p &= m_u + m_u + m_d - |E_{\text{Bind}}(p)|, \\
M_n &= m_u + m_d + m_d - |E_{\text{Bind}}(n)|,
\end{align*}
\]

where $|E_{\text{Bind}}(p)|$ and $|E_{\text{Bind}}(n)|$ are the binding energy of the three quarks inside p and n. Since the mass difference between p(938) and n(940) is very small, we omit the differences of the masses and binding energies between p(938) and n(940). Taking an average mass (939) of p and n masses, from $[2]$, we get
m_u + m_u + m_d - |E_{Bind}(p)| = m_u + m_d + m_d - |E_{Bind}(n)| = 939

|E_{Bind}(p)| ≈ |E_{Bind}(n)|.

Then we have approximately:

m_u ≈ m_d, \hspace{1cm} (3)

|E_{Bind}(p)| ≈ |E_{Bind}(n)| = |E_{Bind}| is an unknown complex function. As a phenomenological approximation, we assume that |E_{Bind}| = 3\Delta (\Delta is an unknown constant). Because any high energy particle has not destroyed a proton or a neutron to get a free quark, we assume that \Delta is an unknown huge constant (\Delta \gg m_P = 938 \text{ Mev}). From |E_{Bind}| = 3\Delta, \hspace{1cm} (2) \hspace{1cm} and \hspace{1cm} (3), we find m_u = m_d = 313 + \Delta:

m_u = m_d = 313 + \Delta
\Delta = \frac{1}{3} |E_{Bind}| \gg m_P = 938 \text{ Mev}. \hspace{1cm} (4)

Now we have two unflavored quarks u(313+\Delta) and d(313+\Delta). They are the most important quarks. They compose the most important baryons (p and n) and mesons (\pi^\pm and \pi^0). How do the flavored quarks (s, c and b) come out?

2.3. Using phenomenological formulae, deducing the flavored quarks s, c and b from the elementary quarks \epsilon_u or \epsilon_d

The flavored quarks (s, c and b) are also the excited states of the elementary quarks \epsilon_u or \epsilon_d; they all have the baryon number \mathbb{B} = \frac{1}{3} and the spin s = the isosin = \frac{1}{2} because \epsilon has \mathbb{B} = \frac{1}{3} and s = \frac{1}{2}. The excited quarks of \epsilon_u have the electric charge Q = +\frac{2}{3} since \epsilon_u has Q = +\frac{2}{3}. The excited quarks of \epsilon_d have the electric charge Q = -\frac{1}{3} since \epsilon_d has Q = -\frac{1}{3}. Thus the current flavored quark c with Q = +\frac{2}{3} is the excited quark of \epsilon_u;
the current flavored quarks s and b with $Q = -\frac{1}{3}$ are the excited quarks of $\epsilon_d$. Although the flavored quarks are also the excited states of the elementary quarks, there are large differences from the normally excited states (u and d) of $\epsilon_u$ and $\epsilon_d$:

1. A flavored quark has a flavor that is different from the normally excited quarks u and d that are unflavored quarks.

2. None of the flavored quarks can compose any stable baryon or meson; this case means that they are short lifetime quarks.

3. All flavored quarks are single isospin ($I = 0$) particles that are different from the unflavored quarks u and d with $I = \frac{1}{2}$.

4. They have quantized energies (masses) that are different from the continuous energy spectrum of u and d.

In order to deduce the quantizing energies (masses) of the flavored quarks, let us recall Planck’s and Bohr’s quantizations. Using the quantization condition ($\varepsilon = n\hbar\nu$, $n = 0, 1, 2, ...$), Planck [6] selected correct energies from a classic continuous energy spectrum. Using the quantization condition ($L = \frac{n\hbar}{2\pi}$, $n = 1, 2, 3, ...$), Bohr [7] selects correct orbits (energy levels) from the classic infinite orbits (continuous energy spectrum). Similarly, extending the Planck-Bohr quantization from the linear function of $n$ to a quadric function of $n$, we get a quantization condition (a phenomenological quark mass spectrum which depends on the electric charge of the possible flavored quark). If an excited state with a quantized energy $E_Q$ (mass):

$$E_Q = m_0 + 360(1+\tilde{Q})[(2n+\frac{1}{2}\tilde{Q})^2 - \frac{1}{4}\text{Sign}(Q)n], \quad n+\tilde{Q}= 1, 2, 3, ... , \quad (5)$$

it is a possible flavored quark with a flavored value:

$$\text{Flavor value} = \text{Sign}(Q) \times 1, \quad n+\tilde{Q}= 1, 2, 3, ... . \quad (6)$$

Where $Q$ is the electric charge of the excited state; $m_0 = m_u = m_d = (313+\Delta)$ [5]; $\tilde{Q} \equiv |Q-\frac{2}{3}|$, for $Q = +\frac{2}{3}$, $\tilde{Q} = 0$, for $Q = -\frac{1}{3}$, $\tilde{Q} = 1$. The Sign$(Q)$ is signum of the $Q$; for
the flavored excited states of \( \epsilon_u, Q = +\frac{2}{3} \) and \( \text{Sign}(Q) = "+" \); for the flavored excited state of \( \epsilon_d, Q = -\frac{1}{3} \) and \( \text{Sign}(Q) = "-" \). This phenomenological mass formula has only one arbitrary parameter \( m_0 \) that is determined by proton mass and neutron mass \( \text{(4)} \).

From \( \text{(5)} \) and \( \text{(6)} \), for the excited states of \( \epsilon_u, Q = +\frac{2}{3}, \tilde{Q} = 0, \text{Sign}(Q) = +1 \) and \( n = 1, 2, 3, ... \); for the excited states of \( \epsilon_d, Q = -\frac{1}{3}, \tilde{Q} = 1, \text{Sign}(Q) = -1 \) and \( n = 0, 1, 2, ... \). Thus we have the masses and flavor values of the possible flavored excited quarks of \( \epsilon_u \) and \( \epsilon_d \):

for \( \epsilon_u, m_{\epsilon_u} = m_0 + 360[(2n)^2 - \frac{1}{4} n] \) and flavor = +1 \( n = 1, 2, 3, ... \);
for \( \epsilon_d, m_{\epsilon_d} = m_0 + 720[(2n+\frac{1}{2})^2 + \frac{1}{4} n] \) and flavor = -1 \( n = 0, 1, 2, ... \). \( \text{(7)} \)

Putting the \( m_0 = 313 + \Delta \) and \( n \) values into \( \text{(7)} \), we find the masses and flavor values of the flavored excited states (possible flavored quarks) shown in Table 1. We have already deduced two unflavored quarks \( u \) and \( d \) \( \text{(1)} \) and \( \text{(4)} \) shown in Table 1 also:

| \( n \) | \( 0 \) | \( 1 \) | \( 2 \) | \( 3 \) |
|---|---|---|---|---|
| Excited states of \( \epsilon_u \) | \( \text{u}(\Delta+313) \) | \( \text{c}(\Delta+1663) \) | \( \text{c}^*(\Delta+5893) \) | \( \text{c}^{2*}(\Delta+13003) \) |
| \( S=C=B=0 \) | \( C = 1 \) | \( C^* = 1 \) | \( \text{C}^{2*} = 1 \) |
| \( \text{*********} \) | \( \text{*********} \) | \( \text{*********} \) | \( \text{*********} \) |
| \( n \) | \( -1 \) | \( 0 \) | \( 1 \) | \( 2 \) |
| Excited states of \( \epsilon_d \) | \( \text{d}(\Delta+313) \) | \( \text{s}(\Delta+493) \) | \( \text{b}(\Delta+4993) \) | \( \text{b}^*(\Delta+15253) \) |
| \( S=C=B=0 \) | \( S = -1 \) | \( B = -1 \) | \( \text{B}^* = -1 \) |

In Table 1, we use the quark names to show their quantum numbers as usual.

Table 1 shows that there are three kinds of possible quarks: the first kind for \( n < 1 \) light quarks \( u(313+\Delta), d(313+\Delta) \) and \( s(493+\Delta) \); the second kind for \( n = 1 \), heavy quarks \( c(1663+\Delta) \) and \( b(4993+\Delta) \); the third kind for \( n > 1 \) the flavored excited states \( c^*(5893+\Delta), c^{2*}(13003+\Delta) \) and \( b^*(15253+\Delta) \). Today’s experiments show that for \( n < 1 \) light quarks, we have found almost all possible baryons and mesons made by the light quarks; for \( n = 1 \), the heavy quarks \( c(1663+\Delta) \) and \( b(4993+\Delta) \), we have found many baryons and mesons made by the heavy quarks; for \( n > 1 \), the higher energy flavored
excited states \( c^*(5893+\Delta) \), \( c^{2*}(13003+\Delta) \) and \( b^*(15253+\Delta) \), there is not any confirmed baryon or meson that is composed by these excited states. From today’s experimental result, we cannot think these excited states are quarks that can compose baryons and mesons. Thus, we conclude that the ground unflavored excited state \( u(313+\Delta) \) with \( Q = +\frac{2}{3} \), the ground unflavored excited state \( d(313+\Delta) \) with \( Q = -\frac{1}{3} \), the ground excited strange state \( s(493+\Delta) \) with \( S = -1 \), the ground charmed excited state \( c(1663+\Delta) \) with \( C = +1 \) and the ground bottom excited state \( b(4993+\Delta) \) with \( B = -1 \) are the quarks that can compose baryons and mesons, shown in Table 2; but the flavored excited states \( c^*(5893+\Delta) \), \( c^{2*}(13003+\Delta) \) and \( b^*(15253+\Delta) \) are not quarks. The foundation of this conclusion is on today’s experimental results. As new experimental technology and equipment become available, in future, many new baryons and mesons might be discovered. Some new quark might be discovered also.

### Table 2: The quarks and their quantum numbers and masses

| Quark | \( u(313) \) | \( d(313) \) | \( s(493) \) | \( c(1663) \) | \( b(4993) \) |
|-------|----------------|---------------|---------------|----------------|---------------|
| \( B \) - Baryon number | \( \frac{1}{3} \) | \( \frac{1}{3} \) | \( \frac{1}{3} \) | \( \frac{1}{3} \) | \( \frac{1}{3} \) |
| \( s \) - Spin | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) |
| \( I \) - Isospin | \( \frac{1}{2} \) | \( \frac{1}{2} \) | 0 | 0 | 0 |
| \( I_z \) - Isospin z-component | \( \frac{1}{2} \) | \( \frac{1}{2} \) | 0 | 0 | 0 |
| \( S \) - Strangeness | 0 | 0 | -1 | 0 | 0 |
| \( C \) - Charm | 0 | 0 | 0 | +1 | 0 |
| \( B \) - Bottomness | 0 | 0 | 0 | 0 | -1 |
| \( Q \) - Electric Charge | +\( \frac{2}{3} \) | -\( \frac{1}{3} \) | -\( \frac{1}{3} \) | +\( \frac{2}{3} \) | -\( \frac{1}{3} \) |
| \( m \) - mass (Mev) | 313+\( \Delta \) | 313+\( \Delta \) | 493+\( \Delta \) | 1663+\( \Delta \) | 4993+\( \Delta \) |

\( u(313) \) and \( d(313) \) are the two component states \( I_z = \frac{1}{2}(u(313)) \) and \( I_z = -\frac{1}{2}(d(313)) \) of one isospin quark \( q_N(313) \) with \( I = \frac{1}{2} \).

When the quark pairs \( (u - \bar{u}) \), \( (d - \bar{d}) \), \( (s - \bar{s}) \), \( (c - \bar{c}) \) and \( (b - \bar{b}) \) annihilate back to the vacuum state, the \( u \)-quark and the \( c \)-quark will decay back to the elementary quark \( \epsilon_u \) with unobservable color, electric charge, spin, isospin, mass and \( S = C = B = 0 \) in
the vacuum state; and the d-quark, the s-quark and the b-quark will decay back to the elementary quark $\epsilon_d$ with unobservable color, electric charge, spin, isospin, mass and $S = C = B = 0$ in the vacuum state.

2.4 The extensions of SU(3)$_f$, SU(4)$_f$ and SU(5)$_f$ from SU(2)$_f$

Since the elementary quarks $\epsilon_u$ and $\epsilon_d$ have the flavor SU(2)$_f$ symmetry, their normally excited quarks (u and d) have the flavor SU(2)$_f$ symmetry also. Because the five quarks (u, d, s, c and b) are all the excited states of the elementary quarks $\epsilon_u$ or $\epsilon_d$, SU(3)$_f$ (basic quarks u, d and s), SU(4)$_f$ (basic quarks u, d, s and c) and SU(5)$_f$ (basic quarks u, d, s, c and b) are the natural extensions of SU(2)$_f$. Since $\Delta \gg m_p = 938$ Mev, from Table 2, $m_u (=\Delta+313) \approx m_d (=\Delta+313) \approx m_s (=\Delta+493) \approx m_c (=\Delta+1663) \approx m_b (=\Delta+4993)$. Thus SU(4)$_f$ and SU(5)$_f$ are not badly broken by the quark masses. We will use SU(4) and the qqq baryon model to deduce baryons from the deduced quarks in Table 2.

3. Using the Sum Laws, SU(4) and a Phenomenological Binding Energy Formula, Deducing the Important Baryons from the Deduced Quarks

According to the Quark Model, a colorless baryon is composed of three different colored quarks. From Table 2, we can see that there is a term $\Delta$ inside the masses of each quark. $\Delta$ is an unknown huge constant. Since the masses of the three quarks in a baryon are huge (from $\Delta$) and the mass of the baryon composed by the three quarks is not, we infer that there will be a part of binding energy ($E_{Bind} = -3\Delta$) to cancel the
3Δ of the three quark masses. Thus the baryon mass will essentially be:

$$M_B = m_{q_1}^* + m_{q_2}^* + m_{q_3}^* - |E_{Bind}|$$

$$= (m_{q_1} + \Delta) + (m_{q_2} + \Delta) + (m_{q_3} + \Delta) - 3\Delta$$

$$= m_{q_1} + m_{q_2} + m_{q_3} \equiv M_{123}. \quad (8)$$

Therefore we will omit the term Δ in the three quark masses of the baryon and the term \((-3\Delta)\) in the binding energy of the baryon when we deduce the masses of baryons. The sum laws of the baryon number \(B\), the strange number \(S\), the charmed number \(C\), the bottom number \(B\), the electric charge \(Q\) and the isospin \(I_z\) component of a baryon are:

$$B_{\text{Baryon}} = B_{q_1} + B_{q_2} + B_{q_3}, \quad S_{\text{Baryon}} = S_{q_1} + S_{q_2} + S_{q_3},$$

$$C_{\text{Baryon}} = C_{q_1} + C_{q_2} + C_{q_3}, \quad B_{\text{Baryon}} = B_{q_1} + B_{q_2} + B_{q_3},$$

$$Q_{\text{Baryon}} = Q_{q_1} + Q_{q_2} + Q_{q_3}, \quad I_{z,\text{Baryon}} = I_{z,q_1} + I_{z,q_2} + I_{z,q_3}. \quad (9)$$

3.1. Using the sum laws, Deducing the most important baryons from the Deduced quarks

Using the quantum number sum laws (9) and the mass sum law (8), from the \(S\), \(C\), \(B\), \(Q\), \(I_z\) and the masses of the deduced quarks in Table 2, omitting the \(\Delta\) in the quark masses and the binding energy \((-3\Delta)\), we deduce the most important baryons, shown in Table 3:

| Table 3: The most important baryons \((J^P = \frac{1}{2}^+\) and \(B = 1)\) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Quark\(_1\)    | u(313)          | d(313)          | s(493)          | c(1663)         | b(4993)         | c\(^*\)(5893) |
| Quark\(_2\)    | u(313)          | u(313)          | u(313)          | u(313)          | u(313)          | u(313)          |
| Quark\(_3\)    | d(313)          | d(313)          | d(313)          | d(313)          | d(313)          | d(313)          |
| Deduced        | p(939)          | n(939)          | \(\Lambda\)(1119) | \(\Lambda_c\)(2289) | \(\Lambda_b\)(5619) | \(\Lambda_{c^*}\)(6519) |
| Exper.         | p(938)          | n(940)          | \(\Lambda\)(1116) | \(\Lambda_c\)(2285) | \(\Lambda_b\)(5624) | ? |
| \(\Delta M/M\) | 0.1             | 0.1             | 0.3             | 0.2             | 0.09            | ? |

\(\Lambda_{c^*}\)(6519) is not a normal baryons; it might be a quasi-baryon.
From Table 3, we see that the deduced quantum numbers of the most important baryons match the experimental results \[8\] exactly (using the same names to show the same quantum numbers between the deduced baryons and the experimental baryons); and the deduced masses of the most important baryons (p, n, Λ, Λc and Λb) are 99.7% consistent with experimental results. The baryon masses are deduced only using sum law $M_B = m_{q_1} + m_{q_2} + m_{q_3}$. These cases might show that the deduced masses of the quarks could indeed be correct. Λ∗c(6519) is not a normal baryon, it might be a quasi-baryon.

The mass of a baryon essentially is the sum $M_{123}$ of the three quark masses inside the baryon. For higher isospin (I) and higher spin (J) baryons, adding a small phenomenological adjustment binding energy ($\Delta e$), we can find the masses of the baryons with higher I and J:

$$M_{\text{Baryon}} = m_{q_1} + m_{q_2} + m_{q_3} + \Delta e = M_{123} + \Delta e,$$

$$\Delta e = 68 [\Delta I + 3 \Delta J + 2C \times I \times \delta(\Delta J)].$$

Where $\Delta I$ is the difference between the isospin of the baryon and the minimum isospin of the three quark system in the baryon, $\Delta J$ is the difference between the spin J of the baryon and the minimum J of the three quark system in the baryon (see Table 4 and Table 9). $C$ is the charm number and $I$ is the isospin number of the baryon. $\delta(\Delta J)$ is a Dirac $\delta$ function. For $\Delta J = 0$, $\delta(\Delta J) = 1$; for $\Delta J \neq 0$, $\delta(\Delta J) = 0$.

Using the sum laws (9), the mass formula (10), a flavor-spin SU(6) and the qqq baryon model of the Quark Model, we can deduce the masses and the quantum numbers of the baryons from the masses and quantum numbers of the deduced quarks in Table 2.

3.2. Deducing the baryons of Octet and Decuplet using the sum laws and a phenomenological binding energy formula from the deduced quarks

The flavor and spin of the “ordinary” quarks (u, d and s) may be combined in an
approximate flavor-spin SU(6) in which the six basic states are \( u^\uparrow, u^\downarrow, d^\uparrow, d^\downarrow, s^\uparrow \) and \( s^\downarrow \) (\( \uparrow, \downarrow = \) spin up, down). Then the baryons belong to the multiplets on the right side of

\[
6 \otimes 6 \otimes 6 = 56_S \oplus 70_M \oplus 70_M \oplus 20_A.
\]  

(11)

These SU(6) multiplets decompose into flavor SU(3) multiplets as follows:

\[
56_S = ^410 \oplus ^28, \\
70_M = ^210 \oplus ^48 \oplus ^28 \oplus ^21, \\
20_A = ^28 \oplus ^41.
\]  

(12)

where the superscript \((2J + 1)\) gives the net spin \( J \) of the baryons for each baryon in the SU(3) multiplets. \( J^P = \frac{1}{2}^+ \) Octet and \( J^P = \frac{3}{2}^+ \) Decuplet together make up the “ground-state” 56-plet in which the orbital angular momenta between the quark pairs are zero (so that the spatial part of the state function is trivially symmetric). \( 70_M \) and \( 20_A \) require some excitation of the spacial parts of the state function. For simplicity we only discuss the “ground-state” 56-plet in this paper. For the SU(6) multiplets, since there is not any charmed quark \((C = 0)\), the formula (10) is simplified into:

\[
M_{\text{Baryon}} = M_{123} + \Delta e, \\
\Delta e = 68 (\Delta I + 3\Delta J).
\]  

(13)

3.2.1. Deducing the baryons of the SU(3) Octet from the deduced quarks

The baryons \( p(938), n(940), \Lambda^0(1116), \Sigma(1193), \Xi(1318) \) belong to SU(3)\( f \) Octet with \( J^P = \frac{1}{2}^+ \). The SU(3) Octet has given the three quarks for each baryon of the Octet. Using sum laws (9) and mass formula (13), from the deduced quarks in Table 2, we deduce the strange number \( S \), the isospin \( I \), the \( I_z \), the charge \( Q \) and the mass of the baryons shown in Table 4:
Table 4: SU(3) Octet baryons with $B = C = 0$ and $J^P = \frac{1}{2}^+$

| $q_1$ | $q_2$ | $q_3$ | S | $I$ | $I_2$ | $\Delta I$ | $\Delta e$ | $M_{123}$ | Deduced | Exper. | $\frac{\Delta M}{M} \%$ |
|-------|-------|-------|---|-----|-----|----------|---------|----------|---------|-------|----------------|
| u(313) | u | d | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 | 939 | p(939) | p(938) | 0.1% |
| d(313) | u | d | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | 0 | 939 | n(939) | n(940) | 0.1% |
| s(493) | u | d | -1 | 0 | 0 | 0 | 0 | 1119 | $\Lambda^0(1119)$ | $\Lambda^0(1116)$ | 0.3% |
| s(493) | u | u | -1 | 1 | 1 | 1 | 68 | 1187 | $\Sigma^+(1187)$ | $\Sigma^+(1189)$ | 0.2% |
| s(493) | u | d | -1 | 1 | 0 | 1 | 68 | 1187 | $\Sigma^0(1187)$ | $\Sigma^0(1193)$ | 0.5% |
| s(493) | d | d | -1 | 1 | -1 | 1 | 68 | 1187 | $\Sigma^-(1187)$ | $\Sigma^-(1197)$ | 0.8% |
| s(493) | s | u | -2 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 | 1299 | $\Xi^0(1299)$ | $\Xi^0(1315)$ | 1.2% |
| s(493) | s | d | -2 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | 0 | 1299 | $\Xi^-(1299)$ | $\Xi^-(1321)$ | 1.7% |

3.2.2. Deducing the baryons of SU(3) Decuplet from the deduced quarks

The baryons $\Delta(1232), \Sigma(1385), \Xi(1532)$ and $\Omega^-(1672)$ belong to SU(3) Decuplet with $J^P = \frac{3}{2}^+$. The SU(3) Decuplet has given the three quarks for each baryon of the Decuplet. Using the sum laws and the mass formula, from the deduced quarks in Table 2, we deduce the strange number S, the isospin I, the z component $I_z$ of the isospin, the charge Q and the masses of the baryons shown in Table 5:

Table 5: SU(3) Decuplet with $B = C = 0$ and $J^P = \frac{3}{2}^+ (\Delta J = 1)$

| $q_1$ | $q_2$ | $q_3$ | S | $I$ | $I_2$ | $M_{123}$ | $\Delta I$ | $\Delta e$ | Deduced | Exper. | $\frac{\Delta M}{M} \%$ |
|-------|-------|-------|---|-----|-----|----------|---------|---------|---------|-------|----------------|
| u(313) | u | u | 0 | $\frac{3}{2}$ | $\frac{3}{2}$ | 939 | 1 | 272 | $\Delta^{++}(1211)$ | $\Delta^{++}(1232)$ | 1.7% |
| u(313) | u | d | 0 | $\frac{3}{2}$ | $\frac{1}{2}$ | 939 | 1 | 272 | $\Delta^+(1211)$ | $\Delta^+(1232)$ | 1.7% |
| d(313) | u | d | 0 | $\frac{3}{2}$ | $-\frac{1}{2}$ | 939 | 1 | 272 | $\Delta^0(1211)$ | $\Delta^0(1232)$ | 1.7% |
| d(313) | d | d | 0 | $\frac{3}{2}$ | $-\frac{3}{2}$ | 939 | 1 | 272 | $\Delta^-(1211)$ | $\Delta^-(1232)$ | 1.7% |
| s(493) | u | u | -1 | 1 | 1 | 1119 | 1 | 272 | $\Sigma^+(1391)$ | $\Sigma^+(1385)$ | 0.4% |
| s(493) | u | d | -1 | 1 | 0 | 1119 | 1 | 272 | $\Sigma^0(1391)$ | $\Sigma^0(1385)$ | 0.4% |
| s(493) | d | d | -1 | 1 | -1 | 1119 | 1 | 272 | $\Sigma^-(1391)$ | $\Sigma^-(1385)$ | 0.4% |
| s(493) | s | u | -2 | $\frac{1}{2}$ | $\frac{1}{2}$ | 1299 | 0 | 204 | $\Xi^0(1503)$ | $\Xi^0(1532)$ | 1.9% |
| s(493) | s | d | -2 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 1299 | 0 | 204 | $\Xi^-(1503)$ | $\Xi^-(1535)$ | 2.1% |
| s(493) | s | s | -3 | 0 | 0 | 1479 | 0 | 204 | $\Omega^-(1683)$ | $\Omega^-(1672)$ | 0.7% |
3.3. Deducing the baryons of the SU(4) 20-plet with SU(3) Octet from the deduced quarks

The addition of the c quark to the light quarks (u, d and s) extends the flavor symmetry to SU(4) (basic quarks u, d, s and c). Fig 14.4 of [10] shows SU(4) multiplets of baryons. Fig 14.4 (a) shows the 20-plet baryon multiplet with the SU(3) Octet as its bottom level (see Table 6). All the baryons in a given SU(4) multiplet have the same spin and parity. Thus from the spin and parity of the baryons in the bottom layer, we can find the spin and parity of the 20-plet baryon multiplet:

Table 6: The baryons of 20-plet with SU(3) Octet ($\frac{1}{2}^+$)

| Top level | C = 2; $\Xi_{cc}^{++}$, $\Xi_{cc}^+$, $\Omega_{cc}^+$ |
|-----------|--------------------------------------------------|
| Middle level | C = 1; $\Lambda_c^+$, $\Sigma_c^{++}$, $\Sigma_c^+$, $\Sigma_c^0$, $2\Xi_c^+$, $2\Xi_c^0$, $\Omega_c^0$ |
| Bottom level | C = 0; $p^+$, $n^0$, $\Lambda^0$, $\Sigma^+$, $\Sigma^0$, $\Sigma^-$, $\Xi^0$, $\Xi^-$ |

We have already shown the baryons of the bottom level (the SU(3) Octet of the 20-plet) baryon multiplet with C = 0 and $J^P = \frac{1}{2}^+$ in Table 4. Using the sum laws [2] and the mass formula [10], we deduce the charmed number C, the strange number S, the isospin I, the z-component $I_z$ of the isospin, the charge Q and the masses of the baryons. We show the baryons on the top level in Table 7 and the middle level in Table 8 (the same name between deduced and experimental baryons means the same quantum numbers):

Table 7: The top level baryons of the 20-plet ($C = 2$, $J^P = \frac{1}{2}^+$)

| $q_1$ | $q_2$ | $q_3$ | S | C | B | I | $I_z$ | $M_{123}$ | $\Delta I$ | $\Delta J$ | Baryon | Exp. |
|-------|-------|-------|---|---|---|---|-------|-----------|-----------|----------|--------|------|
| u(313) | c | c | 0 | 2 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 3639 | 0 | 0 | $\Xi_{cc}^{++}(3639)$ | ? |
| d(313) | c | c | 0 | 2 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 3639 | 0 | 0 | $\Xi_{cc}^+(3639)$ | ? |
| d_s(493) | c | c | -1 | 2 | 0 | 0 | 0 | 3819 | 0 | 0 | $\Omega_{cc}^0(3819)$ | ? |
Table 8: The middle level baryons of the 20-plet (C=1, J^P = \frac{1}{2}^+, \Delta J = 0)

| q_1 | q_2 | q_3 | S | I | I_z | M_{123} | ΔI | Δe | Deduced | Exper. | ΔM/M % |
|-----|-----|-----|---|---|-----|---------|----|----|---------|--------|--------|
| c(1663) | u | d | 0 | 0 | 0 | 2289 | 0 | 0 | \Lambda^+_c(2289) | \Lambda^+_c(2285) | 0.2% |
| c(1663) | u | u | 0 | 1 | 1 | 2289 | 1 | 204 | \Sigma^{++}_c(2493) | \Sigma^{++}_c(2455) | 1.5% |
| c(1663) | u | d | 0 | 1 | 0 | 2289 | 1 | 204 | \Sigma^{++}_c(2493) | \Sigma^{++}_c(2455) | 1.5% |
| c(1663) | d | d | 0 | 1 | -1 | 2289 | 1 | 204 | \Sigma^0_c(2493) | \Sigma^0_c(2455) | 1.5% |
| c(1663) | u | s | -1 | \frac{1}{2} | \frac{1}{2} | 2469 | 0 | 68 | \Xi^+_c(2537) | \Xi^+_c(2520) # | 0.7% |
| c(1663) | d | s | -1 | \frac{1}{2} | -\frac{1}{2} | 2469 | 0 | 68 | \Xi^0_c(2537) | \Xi^0_c(2526) # | 0.5% |
| c(1663) | s | s | -2 | 0 | 0 | 2649 | 0 | 0 | \Omega^0_c(2649) | \Omega^0_c(2698) | 1.8% |

# 2520 of \Xi^+_c(2520) is the average of the mass of \Xi^+_c(2466) and the mass of \Xi^+_c(2574)

# 2526 of \Xi^0_c(2526) is the average of the mass of \Xi^0_c(2472) and the mass of \Xi^0_c(2579)

3.4. Deducing the baryons of SU(4) 20-plet with SU(3) Decuplet from the deduced quarks

Fig 14.4(b) of [10] shows SU(4) the 20-plet baryon multiplet with the SU(3) Decuplet as its bottom level (see Table 9):

Table 9: The baryons of 20-plet with Decuplet J^P = \frac{3}{2}^+

| Top level | C = 3 | \Omega^{++}_{ccc} |
|-----------|-------|-------------------|
| Middle-Up | C = 2 | \Xi^{++}_{cc}, \Xi^{++}_{cc}; \Omega^{++}_{cc} |
| Middle-Down | C = 1 | \Sigma^{++}_c, \Sigma^+_c, \Sigma^-_c, \Xi^+_c, \Xi^-_c; \Omega^0_c |
| Bottom | C = 0 | \Delta^{++}, \Delta^+, \Delta^0, \Delta^-; \Sigma^+, \Sigma^0, \Sigma^-; \Xi^0, \Xi^-; \Omega^- |

We have already shown the baryons in the bottom level (SU(3) Decuplet) in Table 5. Using sum laws [9] and mass formula [10], we deduce S, C, I, I_z, Q and the mass of the baryons from the S, C, I, I_z, Q and masses of the deduced quarks in Table 2. The baryons on the middle-down level with C = 1 are shown in Table 10 (the same name between deduced and experimental baryons means the same quantum numbers):
Table 10: The baryons on the middle-down level with $C = 1$ and $J^P = \frac{3}{2}^+$ $(\Delta J = 1)$

| $q_1$  | $q_2$ | $q_3$ | $S$ | $I$ | $I_z$ | $M_{123}$ | $\Delta I$ | $\Delta J$ | $\Delta e$ | Deduced | Experim. | $\frac{\Delta M}{M}$% |
|-------|-------|-------|-----|-----|-------|-----------|------------|-----------|-----------|---------|----------|------------------|
| c(1663) | u     | u     | 0   | 0   | 0     | 2289      | 1          | 272       | $\Sigma^+ (2561)$ | $\Sigma^+ (2519)$ | 1.7%     |
| c(1663) | u     | d     | 0   | 1   | 1     | 2269      | 1          | 272       | $\Sigma^0_c (2561)$ | $\Sigma^0_c (2515)$ | 1.8%     |
| c(1663) | d     | d     | 0   | 1   | 0     | 2289      | 1          | 272       | $\Sigma^0_c (2561)$ | $\Sigma^0_c (2518)$ | 1.7%     |
| c(1663) | u     | s     | -1  | 1   | $\frac{1}{2}$ | 2469      | 0          | 204       | $\Xi^+ (2673)$   | $\Xi^+ (2645)$   | 1.1%     |
| c(1663) | d     | s     | -1  | 1   | $\frac{1}{2}$ | 2469      | 0          | 204       | $\Xi^0 (2673)$  | $\Xi^0 (2645)$  | 1.1%     |
| c(1663) | s     | s     | -2  | 0   | 0     | 2649      | 0          | 204       | $\Omega^0_c (2853)$ | $\Omega^0_c (2853)$ | ?        |

The baryons on the middle-up level with $C = 2$ and the top level with $C = 3$ are shown in Table 11:

Table 11. The baryons on the middle-up and top level, $J^P = \frac{3}{2}^+$

| $q_1$  | $q_2$ | $q_3$ | $S$ | $I$ | $I_z$ | $M_{123}$ | $\Delta I$ | $\Delta J$ | $\Delta e$ | Deduced | Experim. |
|-------|-------|-------|-----|-----|-------|-----------|------------|-----------|-----------|---------|----------|
| u(313)  | c     | c     | 0   | $\frac{1}{2}$ | $\frac{1}{2}$ | 3639      | 0          | 1         | 204       | $\Xi^+_{ccc} (3843)$ | ?        |
| d(313)  | c     | c     | 0   | $\frac{1}{2}$ | $-\frac{1}{2}$ | 3639      | 0          | 1         | 204       | $\Xi^+_{ccc} (3843)$ | ?        |
| s(493)  | c     | c     | 0   | 0   | 0     | 3819      | 0          | 1         | 204       | $\Omega^+_{ccc} (4023)$ | ?        |
| c(1663) | c     | c     | 0   | 0   | 0     | 4989      | 0          | 1         | 204       | $\Omega^+_{ccc} (5193)$ | ?        |

Using sum laws and a phenomenological binding energy formula, in terms of $q\bar{q}$ meson model of the Quark Model, from the deduced quarks in Table 2, we can deduce the masses and the quantum numbers of the important mesons also.

4. Deducing the Important Mesons from the Deduced Quarks

Using the $q\bar{q}$ Meson Model

In the quark model, a meson is the $q_i\bar{q}_j$ bound state of a quark $q_i$ and an antiquark $\bar{q}_j$. In this short paper, we only deduce the important mesons to show the main physical idea and to check the deduced quark masses. The sum laws of the baryon number $B$, the strange number $S$, the charmed number $C$, the bottom number $B$, the electric charge $Q$
The masses of the quarks are huge from the term $\Delta (\gg M_p = 938 \text{ Mev})$ of the quark’s masses in Table 2. The masses of mesons \[11\], however, are not huge. Thus we infer that there will be $(-2\Delta)$ in the binding energy of the meson to cancel $2\Delta$ in the masses of the quark and antiquark inside the meson. We assume a phenomenological binding energy formula

\[ E_M(q_i \bar{q}_j) = -2\Delta - 338 + 100\left[\frac{\Delta m}{938}\right] + C_{ij} - |SC| - 2.5 |B| - \delta(\Delta m) - 2I_i I_j, \]

where $\Delta m = |m_{q_i} - m_{\bar{q}_j}|$, $C_{ij} = C_i - \overline{C}_j$; $|SC|$ is the absolute value of strange number $\times$ charmed number of the meson; $|B|$ is the absolute value of the bottom number of the meson. $\delta(\Delta m)$ is Dirac $\delta$ function; for $\Delta m = 0$, $\delta(\Delta m) = 1$; for $\Delta m \neq 0$, $\delta(\Delta m) = 0$. $I_i$ is the isospin of $q_i$; $I_j$ is the isospin of $q_j$. The meson mass formula is

\[ M_{\text{Meson}} = m_{q_i} + m_{\bar{q}_j} + E_M(q_i \bar{q}_j). \]

Using \[16\], \[15\] and sum laws \[14\], from the deduced quark masses in Table 2, we can deduce the masses of the important mesons as shown in the following. Since $(-2\Delta)$ in the binding energy of the meson is always cancelled by $2\Delta$ in the masses of the quark and antiquark inside the meson, we can omit the $(-2\Delta)$ inside \[15\] and the $2\Delta$ inside the masses of the quark and antiquark when we deduce masses of mesons.

4.1. Deducing the mesons made of quarks and their own antiquarks from the deduced quarks
For these mesons \((i = j)\), \(\Delta m = 0\), \(\delta(\Delta m) = 1\), \(|SC| = 0\), \(|B| = 0\), the binding energy formula (15) is simplified into

\[E_M(q_i\bar{q}_i) = -438 + 100(C_{ii} - 2I_i I_i).\] (17)

Using sum laws (14), the mass formula (16) and the binding energy formula (17), we deduce the mesons made of the quarks and their own antiquarks as shown in Table 12 (the same names show the same quantum numbers of the deduced and experimental mesons):

| Table 12. The Most Important Mesons (Baryon number \(B = 0\)) |
|---------------------------------------------------------------|
| \(q_i(m_i)\overline{q}_j(m_j)\) | \(C_{ij}\) | \(E_M(q_i\overline{q}_j)\) | Deduced | Exper. | \(R\) |
|---------------------------------|----------|-----------------|--------|-------|-----|
| \(q_N(313)\overline{q}_N(313)\) | 0        | -488            | \(\pi(138)\) | \(\pi(137)\) | 0.7 |
| \(s(493)\overline{s}(493)\)   | 0        | -438            | \(\eta(548)\) | \(\eta(548)\) | 0.0 |
| \(c(1663)\overline{c}(1663)\) | 2        | -238            | \(J/\psi(3088)\) | \(J/\psi(3097)\) | 0.3 |
| \(b(4993)\overline{b}(4993)\) | 0        | -438            | \(\Upsilon(9548)\) | \(\Upsilon(9460)\) | 0.9 |
| \(c^*(5893)\overline{c}^*(5893)\) | 2       | -238            | \(\psi^*(11548)\) | ? |
| \(c^*(13003)\overline{c}^*(13003)\) | 2       | -238            | \(\psi^*(25768)\) | ? |
| \(b^*(15253)\overline{b}^*(15253)\) | 0       | -438            | \(\Upsilon^*(29135)\) | ? |

\(\psi^*(11548)\), \(\psi^*(25768)\) and \(\Upsilon^*(29135)\) might be only quasi-mesons

In Table 12, the first row \(q_N(313)\overline{q}_N(313) = \pi(138)\) is deduced using Table 13. The quark \(q_N(313)\) represents the quarks \(u(313)\) and \(d(313)\) with \(I = \frac{1}{2}\) (see Table 2). The mass \(137\) of \(\pi(137)\) is the average mass of \(\pi^+(140)\), \(\pi^-(140)\) and \(\pi^0(135)\):

| Table 13. The Deduction of \(\pi(138)\) and \(\pi(137)\) |
|----------------------------------------------------------|
| \(q_i(m_i)\overline{q}_j(m_j)\) | \(C_{ij}\) | \(100 \times 2I_i I_j\) | \(E_{bind}\) | Deduced | Exper. |
|---------------------------------|----------|------------------------|--------------|---------|-------|
| \(u(313)\overline{d}(313)\)    | 0        | -438                   | 50           | -488    | \(\pi^+(138)\) | \(\pi^+(140)\) |
| \(d(313)\overline{u}(313)\)   | 0        | -438                   | 50           | -488    | \(\pi^-(138)\) | \(\pi^-(140)\) |
| \(u(313)\overline{u}(313)\)   | 0        | -438                   | 50           | -488    | \(\pi^0(138)\) | \(\pi^0(135)\) |
| \(d(313)\overline{d}(313)\)   | 0        | -438                   | 50           | -488    | \(\pi^0(138)\) | \(\pi^0(135)\) |
| \(q_N(313)\overline{q}_N(313)\)| 0        | -438                   | 50           | -488    | \(\pi(138)\) | \(\pi(137)\) |
Table 12 shows that the deduced masses of the most important mesons are more than 99% consistent with the experimental results [11] and the deduced quantum numbers match the experimental result exactly (the same names show the same quantum numbers of the deduced and experimental mesons). These results might show that the deduced quark masses are really correct. \(\psi^*(11548)^8\), \(\psi^*(25768)^8\) and \(\Upsilon^*(29135)^8\) are not normal mesons. They might be quasi-mesons, and we are not sure that they really exist in nature.

4.2. Deducing the mesons made of quarks and other antiquarks \((J^P = 0^-)\) from the deduced quarks

For these mesons \((i \neq j)\); \(\Delta m \neq 0 \rightarrow \delta(\Delta m) = 0\); and \(i \neq j \rightarrow 2I_i I_j = 0\) since there is only one kind of quark \(q_N(313)\) with \(I = \frac{1}{2}\), and all other kinds of quarks have \(I = 0\). The binding energy formula (15) is simplified into

\[
E_M(q_i q_j) = -338 + 100[\frac{\Delta m}{938} + C_{ij} - |SC| - 2.5|B|]. \tag{18}
\]

Using the sum laws (14), the binding energy (18) and the meson mass formula (16), from the deduced quarks in Table 2, we deduce the mesons shown in Table 14.

Table 14 shows that the deduced quantum numbers (I, S, C, B and Q) of the mesons are the same as the experimental result (the same names show the same quantum numbers of the deduced and experimental mesons), and the deduced masses of the important pseudoscalar mesons are about 98% consistent with the experimental results. These results furthermore show that the deduced masses and quantum numbers might be indeed correct.
Table 14. The Pseudoscalar Mesons ($B=0, J^P=0^-, R = \frac{M_D-M_E}{M}$)

| $q_i \bar{q}_j$ | S | C | B | $\Delta m$ | $E_M$ | Deduced | Exper. | R  |
|-----------------|---|---|---|-----------|------|---------|--------|----|
| u(313)s(493)    | 1 | 0 | 0 | 1 | 19   | 0       | -319   | K$^+(487)$ | K$^+(494)$ | 1.4 |
| d(313)s(493)    | 1 | 0 | 0 | 0 | 19   | 0       | -319   | K$^0(487)$ | K$^0(498)$ | 2.2 |
| s(493)u(313)    | -1| 0 | 0 | -1| 19   | 0       | -319   | K$^-(487)$ | K$^-(494)$ | 1.4 |
| s(493)d(313)    | -1| 0 | 0 | 0 | 19   | 0       | -319   | K$^0(487)$ | K$^0(498)$ | 2.2 |
| c(1663)d(313)   | 0 | 1 | 0 | 1 | 144  | 1       | -94    | D$^+(1882)$ | D$^+(1869)$ | 0.7 |
| c(1663)u(313)   | 0 | 1 | 0 | 0 | 144  | 1       | -94    | D$^0(1882)$ | D$^0(1865)$ | 0.9 |
| c(1663)s(493)   | 1 | 1 | 0 | 1 | 125  | 1       | -213   | D$^+_s(1943)$ | D$^+_s(1968)$ | 1.3 |
| d(313)c(1663)   | 0 | -1| 0 | -1| 144  | 1       | -94    | D$^-(1882)$ | D$^-(1869)$ | 0.7 |
| u(313)c(1663)   | 0 | -1| 0 | 0 | 144  | 1       | -94    | D$^0_0(1882)$ | D$^0_0(1865)$ | 0.9 |
| s(493)c(1663)   | -1| -1| 0 | -1| 125  | 1       | -213   | D$^-_s(1943)$ | D$^-_s(1968)$ | 1.3 |
| u(313)b(4993)   | 0 | 0 | 1 | 1 | 499  | 0       | -89    | B$^+_u(5217)$ | B$^+_u(5279)$ | 1.2 |
| b(4993)u(313)   | 0 | 0 | -1| -1| 499  | 0       | -89    | B$^-_u(5217)$ | B$^-_u(5279)$ | 1.2 |
| d(313)b(4993)   | 0 | 0 | 1 | 0 | 499  | 0       | -89    | B$^+_u(5217)$ | B$^+_u(5279)$ | 1.2 |
| b(4993)d(313)   | 0 | 0 | -1| 0 | 499  | 0       | -89    | B$^0(5217)$ | B$^0(5279)$ | 1.2 |
| b(4993)s(493)   | 1 | 0 | -1| 0 | 480  | 0       | -108   | B$^0_s(5378)$ | B$^0_s(5370)$ | 0.2 |
| s(493)b(4993)   | -1| 0 | 1 | 0 | 480  | 0       | -108   | B$^0_s(5378)$ | B$^0_s(5370)$ | 0.2 |
| c(1663)b(4993)  | 0 | 1 | 1 | 1 | 355  | 1       | -133   | B$^+_c(6523)$ | B$^+_c(6400)$ | 1.9 |
| b(4993)c(1663)  | 0 | -1| -1| -1| 355  | 1       | -133   | B$^-_c(6523)$ | B$^-_c(6400)$ | 1.9 |

5. Discussion and Predictions

1). The extended Planck-Bohr quantization ($\Box$) plays a crushing role in deducing the flavored quarks. Without this quantization, from two elementary quarks $\epsilon_u$ and $\epsilon_d$ in the vacuum state, we cannot deduce the flavored excited quarks; we only can deduce the normally excited quarks $u$ and $d$. The flavored quarks ($s$, $c$ and $b$) are the results of the extended Planck-Bohr quantization.

2). The fact that physicists have not found any free quark shows that the binding energies of baryons and mesons are huge. The binding energy ($-3\Delta$) of baryons (or
-2\Delta of mesons) is a phenomenological approximation of the interaction energy in a baryon (or a meson). The binding energy (-3\Delta) (or -2\Delta) is always cancelled by the corresponding parts (3\Delta) (or 2\Delta) of the masses of the three quarks inside the baryon (or the quark and antiquark inside the meson). Thus we can omit the binding energy and the corresponding mass parts of the three quarks (or the quark and antiquark) when we account the mass of the baryon (or the meson). This effect makes it appear as if there is no binding energy in the baryon (or the meson). We, however, cannot always forget the huge binding energy (-3\Delta for baryon) (or -2\Delta for meson). In fact the huge binding energy is a necessary condition for the quark confinement.

3). Since the mass of the top quark is too large (about 185 times proton mass), using these phenomenological formulae, we cannot deduce it. This paper tries to deduce the masses of baryons and mesons (there are experimental results that can compare with the deduced results). Because there is not any baryon or meson that contains the top quark, we do not need the mass of the top quark in this paper. How to deduce the very large mass of the top quark is still an open problem of this paper.

4). This paper predicts some baryons shown in Table 15:

| Baryon $J^P$ | $\Xi_{cc}^{++}(3639)$ | $\Xi_{cc}^{++}(3639)$ | $\Omega_{cc}^{++}(3819)$ | $\Omega_{cc}^{++}(5193)$ |
|--------------|-----------------------|-----------------------|------------------------|------------------------|
| $\frac{1}{2}^+$ | $\Xi_{cc}^{++}(3639)$  | $\Xi_{cc}^{++}(3639)$ | $\Omega_{cc}^{++}(3819)$ |                        |

The predicted baryons have higher energies than the discovered baryons or higher flavor values such as $C = 2$ or 3; they are difficult to discover. As new experimental technology and equipments become available, these predicted baryons might be more easily discovered. At the same time, a quasi-baryon $\Lambda_{cc}^*(6519)$ [$e^+(5893)u(313)d(313)$] and the quasi-mesons $\psi^*(11548)$, $\psi^*(25768)$ and $\Upsilon^*(29135)$ might have a slight possibility of being discovered using the new experimental technology and equipments. We are not sure that they really exist in nature.
6. Conclusions

1). There might be only one kind of unflavored \((S = C = B = 0)\) elementary quark family \(\epsilon\) with three possible colors (red or blue or yellow), the baryon number \(B = \frac{1}{3}\), the spin \(s = \frac{1}{2}\), the isospin \(I = \frac{1}{2}\) and two isospin states \((\epsilon_u\) with \(I_z = \frac{1}{2}\) and \(Q = +\frac{2}{3}\), and \(\epsilon_d\) with \(I_z = -\frac{1}{2}\) and \(Q = -\frac{1}{3}\)). Thus there are six elementary quarks in the elementary quark family.

2). The elementary quarks are essentially in the vacuum state. When the elementary quarks are in the vacuum state, their colors = \(B = s = I = I_z = Q = m = 0\). Although they cannot be seen, as the physical vacuum background, they really exist in the vacuum state and there is not any other kind of quark in the vacuum state.

3). When an elementary quark gets enough energy, it may be excited into a current quark. All quarks inside baryons and mesons are the excited states of the elementary quark \(\epsilon\). The u-quark and the c-quark are the excited states of the elementary quark \(\epsilon_u\); and the d-quark, the s-quark and the b-quark are the excited states of the elementary quark \(\epsilon_d\). The masses and flavors of the quarks can deduce from the elementary quarks using (5) and (6).

4). Since all quarks are the excited states of the elementary quarks \((\epsilon_u\) and \(\epsilon_d\)) and the two elementary quarks \((\epsilon_u\) and \(\epsilon_d\)) have \(SU(2)\) symmetry, we can easily understand that \(SU(3)\), \(SU(4)\) and \(SU(5)\) are natural extensions of \(SU(2)\). The physical foundation of the extensions is that all quarks inside hadrons are the excited states of the same elementary quark family \(\epsilon\).

5). Using the sum laws and the binding energy formula of baryons, in terms of \(SU(4)\) and \(qqq\) baryon model, we have deduced the masses and quantum numbers of the important baryons from the deduced quarks in Table 2. At the same time, using
the sum laws and the binding energy formula of mesons, in terms of $q\bar{q}$ meson model, we have also deduced the masses and quantum numbers of the important mesons from the deduced quarks in Table 2. The deduced quantum numbers of the baryons and mesons match the experimental results [8] and [11] exactly, and the deduced masses of the baryons and mesons are 98% consistent with experimental results. This case might show that the deduced masses of the quarks could be indeed correct.

6). This paper not only deduce the masses and the quantum numbers (including the flavors) of the quarks, the important baryons and mesons, but also provides a physical foundation for the flavor symmetry (SU(3)$_f$, SU(4)$_f$ and SU(5)$_f$) on a phenomenological level. Since the elementary quark number decreases to two, it can decrease the number of the adjustable parameters of the Standard Model [12], and might simplify the calculations. In fact this paper improves upon the Quark Model, making it more powerful and more reasonable.

Acknowledgments

I sincerely thank Professor Robert L. Anderson for his valuable advice. I acknowledge my indebtedness to Professor D. P. Landau for his help also. I would like to express my heartfelt gratitude to Dr. Xin Yu. I sincerely thank Professor Kang-Jie Shi for his important advice.

References

[1] M. Gell-Mann, Phys. Lett. 8, 214 (1964); G. Zweig, CERN Preprint CERN-Th-401, CERN-Th-412 (1964); Particle Data Group, Phys. Lett. B592, 154 (2004).

[2] S. Okubo, Prog. Theor. Phys. 27, 949 (1962); 28, 24(1962).

[3] The Oecd Frum, Particle Physics (Head of Publications Service, OECD) 55 (1995).
[4] M. K. Gaillard, P. D. Grannis, F. J. Sciulli, Rev. Mod. Phys., 71 No 2, Centenary S96 (1999).

[5] Particle Data Group, Phys. Lett. B592, 154 (2004).

[6] R. M. Eisberg, Fundamentals of Modern Physics, (John Wiley & Sons, New York, 1961), p. 64.

[7] R. M. Eisberg, Fundamentals of Modern Physics, (John Wiley & Sons, New York, 1961) p. 114.

[8] Particle Data Group, Phys. Lett. B592, 66–78 (2004).

[9] Appendix II of Particle Data Group, Phys. Lett. 111B (1982).

[10] Particle Data Group, Phys. Lett. B592, 158 (2004).

[11] Particle Data Group, Phys. Lett. B592, 38–65 (2004).

[12] A. Pais, Rev. Mod. Phys., 71 No. 2 Centenary, S16 (1999).