Abstract

A minimal extension of the Standard Model is proposed, where the observed left-handed neutrinos obtain naturally small Majorana masses from a one-loop radiative seesaw mechanism. This model has two candidates (one bosonic and one fermionic) for the dark matter of the Universe. It has a very simple structure and should be verifiable in forthcoming experiments at the Large Hadron Collider.
In the well-known canonical seesaw mechanism [1], three heavy singlet Majorana neutrinos $N_i$ ($i = 1, 2, 3$) are added to the Standard Model (SM) of elementary particles, so that

$$\mathcal{M}_{\nu}^{(e, \mu, \tau)} = -M_D M_N^{-1} M_D^T, \quad (1)$$

where $M_D$ is the $3 \times 3$ Dirac mass matrix linking the observed neutrinos $\nu_\alpha$ ($\alpha = e, \mu, \tau$) to $N_i$, and $M_N$ is the Majorana mass matrix of $N_i$. More generally [2], $M_\nu$ comes from the unique dimension-five operator

$$\mathcal{L}_\Lambda = \frac{f_{ij}}{\Lambda} (\nu_i \phi^0 - l_i \phi^+) (\nu_j \phi^0 - l_j \phi^+) + H.c., \quad (2)$$

where $(\nu_i, l_i)$ are the usual left-handed lepton doublets transforming as $(2, -1/2)$ under the standard electroweak $SU(2)_L \times U(1)_Y$ gauge group and $(\phi^+, \phi^0) \sim (2, 1/2)$ is the usual Higgs doublet of the SM. There are three and only three tree-level realizations [3] of this operator, one of which is of course the canonical seesaw mechanism. There are also three generic mechanisms for obtaining this operator in one loop [3]. Whereas the new particles required in the three tree-level realizations are most likely too heavy to be observed experimentally in the near future, those involved in the one-loop realizations may in fact be light enough to be detected, in forthcoming experiments at the Large Hadron Collider (LHC) for example.

Consider the following minimal extension of the SM. Under $SU(2)_L \times U(1)_Y \times Z_2$, the particle content is given by

$$(\nu_i, l_i) \sim (2, -1/2; +), \quad l_i^c \sim (1, 1; +), \quad N_i \sim (1, 0; -), \quad (3)$$

$$(\phi^+, \phi^0) \sim (2, 1/2; +), \quad (\eta^+, \eta^0) \sim (2, 1/2; -). \quad (4)$$

Note that the new particles, i.e. $N_i$ and the scalar doublet $(\eta^+, \eta^0)$, are odd under $Z_2$. A previously proposed model [4] of neutrino mass shares the same particle content of this model, but the extra symmetry assumed there is global lepton number, which is broken explicitly but softly by the unique bilinear term $\mu^2 \Phi^\dagger \eta + H.c.$ in the Higgs potential. Here, $Z_2$ is an
exact symmetry, in analogy with the well-known $R$–parity of the Minimal Supersymmetric Standard Model (MSSM), hence this term is strictly forbidden. As a result, $\eta^0$ has zero vacuum expectation value and there is no Dirac mass linking $\nu_i$ with $N_j$. Neutrinos remain massless at tree level as in the SM.

The Yukawa interactions of this model are given by

$$\mathcal{L}_Y = f_{ij}(\phi^- \nu_i + \bar{\phi}^0 l_i)l_j^c + h_{ij}(\nu_i \eta^0 - l_j \eta^+)N_j + H.c. \quad (5)$$

In addition, the Majorana mass term

$$\frac{1}{2} M_i \nu_i \nu_i + H.c.$$ 

and the quartic scalar term

$$\frac{1}{2} \lambda_5 (\Phi^\dagger \eta)^2 + H.c.$$ 

are allowed. Hence the one-loop radiative generation of $M_\nu$ is possible, as depicted in Fig. 1. This diagram was discussed in Ref. [3], but without recognizing the crucial role of the exact $Z_2$ symmetry being considered here.

![Figure 1: One-loop generation of neutrino mass.](image)

The immediate consequence of the exact $Z_2$ symmetry of this model is the appearance of a lightest stable particle (LSP). This can be either bosonic, i.e. the lighter of the two mass eigenstates of $Re\eta^0$ and $Im\eta^0$, or fermionic, i.e. the lightest mass eigenstate of $N_{1,2,3}$. 

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The latter possibility was first proposed in a different model [5], where neutrino masses are radiatively generated in three loops with the addition of two charged scalar singlets.

The Higgs potential of this model is given by

\[ V = m_1^2 \Phi^\dagger \Phi + m_2^2 \eta^\dagger \eta + \frac{1}{2} \lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{2} \lambda_2 (\eta^\dagger \eta)^2 + \lambda_3 (\Phi^\dagger \Phi)(\eta^\dagger \eta) + \lambda_4 (\Phi^\dagger \eta)(\eta^\dagger \Phi) + \frac{1}{2} \lambda_5 ([\Phi^\dagger \eta]^2 + H.c.], \]

(6)

where \( \lambda_5 \) has been chosen real without any loss of generality. For \( m_1^2 < 0 \) and \( m_2^2 > 0 \), only \( \phi^0 \) acquires a nonzero vacuum expectation value \( v \). The masses of the resulting physical scalar bosons are given by

\[ m^2(\sqrt{2}Re\phi^0) = 2\lambda_1 v^2, \]

(7)

\[ m^2(\eta^+) = m_2^2 + \lambda_3 v^2, \]

(8)

\[ m^2(\sqrt{2}Re\eta^0) = m_2^2 + (\lambda_3 + \lambda_4 + \lambda_5) v^2, \]

(9)

\[ m^2(\sqrt{2}Im\eta^0) = m_2^2 + (\lambda_3 + \lambda_4 - \lambda_5) v^2. \]

(10)

The diagram of Fig. 1 is exactly calculable from the exchange of \( Re\eta^0 \) and \( Im\eta^0 \) and is given by

\[ (\mathcal{M}_\nu)_{ij} = \sum_k \frac{h_{ik} h_{jk} M_k}{16\pi^2} \left[ \frac{m_R^2}{m_R^2 - M_k^2} \ln \frac{m_R^2}{M_k^2} - \frac{m_I^2}{m_I^2 - M_k^2} \ln \frac{m_I^2}{M_k^2} \right], \]

(11)

where \( m_R \) and \( m_I \) are the masses of \( \sqrt{2}Re\eta^0 \) and \( \sqrt{2}Im\eta^0 \) respectively. If \( m_R^2 - m_I^2 = 2\lambda_5 v^2 \) is assumed to be small compared to \( m_0^2 = (m_R^2 + m_I^2)/2 \), then

\[ (\mathcal{M}_\nu)_{ij} = \frac{\lambda_5 v^2}{8\pi^2} \sum_k \frac{h_{ik} h_{jk} M_k}{m_0^2 - M_k^2} \left[ 1 - \frac{M_k^2}{m_0^2 - M_k^2} \ln \frac{m_0^2}{M_k^2} \right]. \]

(12)

If \( M_k^2 \gg m_0^2 \), then

\[ (\mathcal{M}_\nu)_{ij} = \frac{\lambda_5 v^2}{8\pi^2} \sum_k \frac{h_{ik} h_{jk}}{M_k} \left[ \ln \frac{M_k^2}{m_0^2} - 1 \right]. \]

(13)

If \( m_0^2 \gg M_k^2 \), then

\[ (\mathcal{M}_\nu)_{ij} = \frac{\lambda_5 v^2}{8\pi^2 m_0^2} \sum_k h_{ik} h_{jk} M_k. \]

(14)
If \( m^2_0 \simeq M^2_k \), then

\[
(M_\nu)_{ij} \simeq \frac{\lambda_5 v^2}{16\pi^2} \sum_k \frac{h_{ik} h_{jk}}{M_k}.
\]

(15)

From the above, it is clear that the seesaw scale is reduced by roughly the factor \( \lambda_5/16\pi^2 \).

Assuming \( \lambda_5 \sim h^2 \sim 10^{-4} \), the corresponding canonical seesaw scale of \( 10^9 \) GeV (with \( m_\nu \sim h^2 v^2/M \sim 1 \) eV) is then reduced to just 1 TeV, which is amenable to experimental verification in forthcoming experiments at the LHC, for example.

This radiative seesaw mechanism of neutrino mass also predicts the existence of dark matter, either in the form of \( N_1 \) (assuming \( M_1 < M_2 < M_3 \)) or \( \sqrt{2} \text{Re}\eta^0 \) (assuming that it is the lightest scalar particle odd under \( Z_2 \)). In the former case, if the \( \eta \) masses are all greater than \( M_k \), there will be observable decays

\[
\eta^\pm \rightarrow l^\pm N_{1,2,3},
\]

(16)

then

\[
N_2 \rightarrow l^\pm l^\mp N_1
\]

(17)

and

\[
N_3 \rightarrow l^\pm l^\mp N_{1,2}
\]

(18)

through \( \eta^\pm \) exchange. The Yukawa couplings \( h_{ij} \) may then be extracted and compared against the neutrino mass matrix as a means of verifying the seesaw mechanism [4].

In the latter case, with \( \sqrt{2} \text{Re}\eta^0 \) as a bosonic dark-matter candidate [6], the fact that \( \sqrt{2} \text{Im}\eta^0 \) must be just slightly heavier is a natural condition for their coannihilation in the early Universe [7]. This is better than the usual supersymmetric scenario for dark matter, where coannihilation requires the accidental degeneracy of two unrelated particles.

If \( M_k \) are all greater than the \( \eta \) masses, there will be observable decays

\[
N_{1,2,3} \rightarrow l^\pm \eta^\mp,
\]

(19)
then
\[ \eta^\pm \to \eta^0 + \{W^\pm\}, \]  
(20)

where the real or virtual \( W^\pm \) becomes a quark or lepton pair. Again the Yukawa couplings \( h_{ij} \) may be extracted.

The \( \eta \) particles can be produced in pairs directly by the SM gauge bosons \( W^\pm, Z, \) or \( \gamma \). Their subsequent decays will produce \( N_i \) if kinematically allowed. In the case where \( N_{1,2,3} \) are all heavier than the \( \eta \) particles, pair production by \( e^+e^- \) annihilation through \( \eta^\pm \) exchange appears to be the only realistic possibility.

This model is also a very suitable framework for considering lepton family symmetry. It has the flexibility of having the neutrino mass matrix proportional to the inverse mass matrix of \( N_i \) as in the canonical seesaw mechanism [8], or to the mass matrix of \( N_i \) itself. For example, using the tetrahedral symmetry \( A_4 \) [9], many recent ideas [10] of implementing tribimaximal mixing [11] can be easily incorporated.

In conclusion, with a minimal addition to the Standard Model, i.e. a second scalar doublet and three heavy neutral fermion singlets transforming as \(-1\) under an exact \( Z_2 \) symmetry, realistic radiative neutrino masses can be obtained together with candidates for the dark matter of the Universe. This framework parallels that of the SM in family structure and the new particles are very likely to be observable in forthcoming experiments at the Large Hadron Collider, or at a future Linear Collider.

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References

[1] M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, edited by P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979), p. 315; T. Yanagida, in Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe, edited by O. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba, Japan, 1979), p. 95; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).

[2] S. Weinberg, Phys. Rev. Lett. 43, 1566 (1979).

[3] E. Ma, Phys. Rev. Lett. 81 1171 (1998).

[4] E. Ma, Phys. Rev. Lett. 86, 2502 (2001).

[5] L. M. Krauss, S. Nasri, and M. Trodden, Phys. Rev. D67, 085002 (2003).

[6] C. Boehm and P. Fayet, Nucl. Phys. B683, 219 (2004); M. Cirelli, N. Fornengo, and A. Strumia, hep-ph/0512090.

[7] For a recent review, see for example K. A. Olive, hep-ph/0412054, to be published in “Dark 2004”, Proceedings of 5th International Heidelberg Conference on Dark Matter in Astro and Particle Physics (Mitchell Institute, Texas A&M University), ed. H.-V. Klapdor-Kleingrothaus and R. Arnowitt.

[8] E. Ma, Phys. Rev. D71, 111301R (2005).

[9] E. Ma and G. Rajasekaran, Phys. Rev. D64, 113012 (2001); K. S. Babu, E. Ma, and J. W. F. Valle, Phys. Lett. B552, 207 (2003).

[10] See for example E. Ma, Phys. Rev. D70, 031901R (2004); Phys. Rev. D72, 037301 (2005); hep-ph/0511133 G. Altarelli and F. Feruglio, Nucl. Phys. B720, 64 (2005);
hep-ph/0512103, K. S. Babu and X.-G. He, hep-ph/0507217, I. de Medeiros Varzielas, S. F. King, and G. G. Ross, hep-ph/0512313, X.-G. He, Y.-Y. Keum, and R. R. Volkas, hep-ph/0601001.

[11] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. B530, 167 (2002). See also X.-G. He and A. Zee, Phys. Lett. B560, 87 (2003).