Mixed Scheduling Strategy for High Frequency Bus Routes With Common Stops

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ABSTRACT Bus routes overlapping would lead to more than one bus entering the stop simultaneously, which may trigger bus bunching. Focusing on high frequency routes with common stops, this paper proposes a mixed scheduling method combining the all-stop service and the stop-skipping service. The method optimizes scheduling strategies for multiple routes by minimizing total passenger travel time. The optimization variables are binary variables reflecting whether the stops in the overlapping area are skipped. Three exciting bus routes are employed for case study. Results show that the proposed method reduces total passenger travel time by 21.4% compared with the current scheduling strategy.

INDEX TERMS Mixed bus scheduling, common stops, high frequency routes, passenger travel time.

I. INTRODUCTION

A. BACKGROUND

Developing public transit is an effective way to alleviate traffic congestion and reduce fuel consumption. As an important component of urban transportation system, public transit has been drawing sustained attention. Public transit in densely populated cities is characterized by abundant routes and frequent departures. These two features enable the improvement of passenger travel efficiency and public transit attractiveness, as well as the increase of public transit coverage and capacity. Taking China as an example, there are more than 30 cities that respectively runs over 5000 buses and over 150 routes. Among them, Beijing has the largest urban public transportation system in China, with 31000 buses and 1221 routes. Bus route overlapping, i.e. multiple routes share common stops, is a widespread phenomenon in high-passenger-density areas, particularly on arterial roads.

Overlapping of high-frequency routes would lead to the following problems: (i) Buses from different routes enter the stop at the same time, triggering traffic blockage and increasing bus dwell times. (ii) Facing many options of bus routes, passengers tend to take the bus that arrives earlier among all the buses could take them to their final destinations, which implies that there are fewer passengers taking the succeeding alternative buses. This would cause large discrepancy of the crowdedness among buses, dwell time fluctuation and even bus bunching. (iii) In the long run, it would undermine the bus operation reliability and public transit attractiveness [1]–[3].

Optimal bus scheduling strategy would increase bus service punctuality and reduce passenger travel time. In the last decade, a large number of researchers have investigated the bus scheduling, bringing about plentiful achievement [4]–[6]. However, few studies have taken into account the impact of route overlapping on bus scheduling. Therefore, it is of theoretical and practical significance to study the impact of route overlapping on bus operations and passenger travel time, and to propose a holistic method for optimizing scheduling strategy of overlapping routes.

Many researchers have extensively studied the bus scheduling problem, focusing on departure frequency optimization considering the impacts of passenger demand and bus fleet size [7]–[11]. Routine scheduling, which requires buses stopping at every stop, is a widely used scheduling approach [12]. Salzborn proposed a continuum method for optimizing bus departure frequency, minimizing passenger waiting time and bus fleet size [13]. Ceder suggested departure frequency to be determined by passenger count.
Under the assumption of uniform headways, Ceder et al. presented a cooperative method for overlapping bus routes by calculating the weighted sum of the deviation from the scheduled arrival interval [30]. Schmöcker et al. presented a passenger queue balancing model with the considerations of bus bunching, passenger boarding behavior and bus overtaking outside stops. Allowing bus overtaking was found to benefit the operation of multi-route transit network [31]. Sun and Schmöcker studied bus bunching and passenger choice behavior at the stop served by multiple bus routes. The results showed that when two buses arrive immediately choosing to board the one after is beneficial to the operation of transit network; in addition, bus overtaking could help prevent the bus ahead from being over crowded [32]. Antoine et al. studied substitution strategy in the context of multiple bus lines under either time-independent or time varying settings. They modeled the agency’s substitution decisions and retired bus repositioning decisions as a stochastic dynamic program so as to obtain the optimal policy that minimizing the system-wide costs [33].

Appropriate bus scheduling is crucial for increasing transit operational efficiency and reducing passenger travel costs. However, some issues do exist in these bus scheduling studies:

1. Conventional bus scheduling methods are useful for low-frequency (i.e. large departure interval) or less overlapping routes; for high-frequency and highly overlapping routes, bus stopping at every stop in the overlapping area would easily lead to the simultaneous arrivals of multiple buses and then aggravate the stop congestion.

2. As to bus bunching, some have proposed the dynamic scheduling method based on overtaking strategy. However, the dynamic scheduling method becomes less effective for the overlapping bus routes with large fluctuations of ridership and bus operations.

3. Static bus scheduling strategy, including dispatching headways and timetables, is one of the most important parts of bus operation management, and plays a significant role in passenger travel and subsequent dynamic scheduling method. However, few studies on static bus scheduling considering route overlapping have been conducted.

All-stop (buses dwell at all the stops) [34]–[37], stop-skipping (some stops are skipped) [38]–[41] and short-turn (buses dwell at all stops in the specific interval) [42]–[45] are three most used bus scheduling methods. Appropriate combination of different methods in static bus scheduling can help reduce overall bus travel time, save carrying capacity and increase the operational efficiency of transit network. For high-frequency overlapping bus routes, the simultaneous arrivals of multiple buses would result in non-uniform bus headways, unstable passenger waiting time and crowded carriages. In addition, passengers in the overlapping area have shorter waiting times due to more than one alternative route. The mixed scheduling method, i.e. the all-stop service used outside the overlapping area and the stop-skipping service
used inside, could be utilized in this case to avoid the simultaneous arrivals of multiple buses and generate less impact on passengers in the overlapping area. Hence, this study proposes a mixed scheduling model that minimizes overall passenger travel time with the considerations of bus route overlapping, to optimize static scheduling strategies for the routes.

B. OBJECTIVES AND CONTRIBUTIONS

Appropriate bus scheduling scheme can improve bus operations in the transit network and level of service of public transport system. This study proposes a scheduling strategy optimization method for the high frequency overlapping bus routes based on the mixed scheduling strategy. The contributions of this study include: (1) a mixed scheduling strategy that combines the all-stop service and stop-skipping service is proposed for the overlapping bus routes, which can avoid the simultaneous arrivals of multiple buses at the stops; (2) the calculation methods for passenger waiting time, in-vehicle travel time and the increased walking time due to stop skipping are developed. A scheduling strategy optimization model for the overlapping routes is established by minimizing total passenger travel time.

The rest of this paper is organized as follows: Section II introduces the mixed scheduling method for overlapping routes, including parameter selection, bus operation analysis, optimization model development and solution algorithm. Section III uses three exciting bus routes in Harbin city, China as an example to validate the proposed method. Section IV concludes the findings from the case study in Section III.

II. METHODOLOGY

This paper proposes the mixed scheduling strategy that combines the all-stop service used outside the overlapping area and the stop-skipping service used inside the overlapping area to address the problems induced by route overlapping. The mixed scheduling model that minimizes overall passenger travel time is established with the consideration of bus carrying capacity. The framework of methodology is shown as Figure 1.

A. MODEL PARAMETER AND VARIABLES

The transit network studied in this paper is illustrated in Figure 2, where there are 1 bus routes. Route \( i (i = 1, 2, \ldots, I) \) has \( K_i \) buses, \( J_i \) stops in total, \( B_i \) stops outside the overlapping area and \( G \) stops inside the overlapping area.

A binary variable \( x_{i,j,k} \) equaling to either 1 or 0 is used to indicate the dwelling state of bus \( k \) at stop \( j \) of route \( i \), where \( k = 1, 2, \ldots, K_i \) and \( j = 1, 2, \ldots, J_i \). \( x_{i,j,k} = 1 \) represents that bus \( k \) stops at stop \( j \); \( x_{i,j,k} = 0 \) indicates that bus \( k \) skips stop \( j \). This study adopts the mixed scheduling scheme that combines the all-stop service and the stop-skipping service. Stops can be skipped only within the overlapping area.

Passengers are divided into two types. The first type is that both the boarding and alighting stops are located in the overlapping area; the second type is that at least one of the boarding and alighting stops is outside the overlapping area. Passengers are assumed to arrive at all stops randomly, and their arrival rate is subject to Poisson distribution.

The parameters and variables in the proposed model are summarized in Table 1.

B. BUS OPERATION ANALYSIS UNDER ROUTINE SCHEDULING STRATEGY

1) BUS OPERATION ANALYSIS UNDER THE ALL-STOP STRATEGY

Due to the limitation of bus carrying capacity, the number of boarding passengers is the minimum value between the number of waiting passengers and the available bus carrying capacity when arriving at the stop. The calculation method is expressed by (1):

\[
b_{i,j,k} = \min\left(W_{i,j,k}, C_{i,k} - Z_{i,j,k} + m_{i,j,k}\right)
\]  

(1)

When a bus arrives at stop \( j \), the number of passengers on-board equal to the sum of the difference between the numbers of boarding and alighting passengers from stop 1 to \( j - 1 \). Thus, \( Z_{i,j,k} \) can be formulated as:

\[
Z_{i,j,k} = \sum_{j=1}^{j-1} \max(b_{i,j,k} - m_{i,j,k}, 0)
\]

(2)

Due to the limit of passenger capacity of buses, the number of passengers failing to board the bus \( k \) at stop \( j \) can be
### (a). Parameters list in the proposed model

| Parameters | Description |
|------------|-------------|
| $t_o$      | bus door opening time, min |
| $t_c$      | bus door closing time, min |
| $t_b$      | average boarding time of each passenger, s |
| $t_a$      | average alighting time of each passenger, s |
| $q_{i,j-1,k}$ | acceleration time of bus $k$ on route $i$ when departing from stop $j-1$, min |
| $u_{i,j,k}$ | deceleration time of bus $k$ on route $i$ when arriving at stop $j$, min |
| $L$        | distance needed for buses to decelerate or accelerate at stops, m |
| $v_{i,j,k}$ | average speed of bus $k$ on route $i$ traveling from stop $j-1$ to stop $j$, m/s |
| $h_{i_{\text{min}}}$ | minimum bus departure interval of route $i$, min |
| $h_{i_{\text{max}}}$ | maximum bus departure interval of route $i$, min |
| $e_{i,j,k}$ | time required for bus $k$ of route $i$ at stop $j$ travel across the acceleration or deceleration area, min |
| $v_f$      | average passenger walking speed, m/s |
| $h_{i,k}$  | scheduled departure interval between bus $k$ and bus $k-1$ on route $i$, min |
| $a_{i,j,k}$ | acceleration or deceleration of bus $k$ on route $i$ at stop $j$, m/s² |
| $K_i$      | total number of buses for route $i$ scheduling |
| $C_{i,k}$  | bus carrying capacity for bus $k$ of route $i$, passengers/veh |

### (b). Variables list in the proposed model

| Variables | Description |
|-----------|-------------|
| $b_{i,j,k}$ | number of passengers that can board bus $k$ of route $i$ at stop $j$ |
| $W_{i,j,k}$ | number of passengers waiting for bus $k$ of route $i$ at stop $j$ |
| $Z_{i,j,k}$ | number of passengers on-board of bus $k$ of route $i$ upon arriving at stop $j$ |
| $m_{i,j,k}$ | number of alighting passengers of bus $k$ of route $i$ at stop $j$ |
| $r_{i,j,k}$ | number of passengers failing to board bus $k$ of route $i$ at stop $j$ |
| $d_{i,j,k}$ | dwell time of bus $k$ of route $i$ at stop $j$, min |
| $T_{i,j,k}$ | departure time of bus $k$ on route $i$ at stop $j$ |
| $S_{i,j,k}$ | travel time from stop $j-1$ to stop $j$ for bus $k$ of route $i$, min |
| $F_{i,j,1}$ | departure time of the first bus of route $i$ at the first stop |
| $H_{i,j,k}$ | headway between bus $k$ and bus $k-1$ on route $i$ at stop $j$, min |
| $\lambda_{i,j}$ | passenger average arrival rate for route $i$ at stop $j$, passenger/min |
| $\theta_{i,j}$ | passenger alighting rate at stop $j$ of route $i$, passenger/min |
| $n_{i,j,k}$ | number of alighting passengers of bus $k$ of route $i$ at stop $j$ under the stop-skipping service |
| $t$        | total travel time of all passengers, min |
| $t_1$      | waiting time of the first-type passengers at stops, min |
| $t_2$      | waiting time of the second-type passengers at stops, min |
| $t_3$      | total in-vehicle travel time of passengers, min |
| $t_4$      | walking time, min |

The dwell time of bus $k$ of route $i$ at stop $j$ is denoted by $d_{i,j,k}$, which includes the door opening time, the door closing time, maximum time between passengers boarding and alighting time. $d_{i,j,k}$ can be calculated by (4):

$$d_{i,j,k} = t_o + t_c + max(t_b \cdot b_{i,j,k}, t_a \cdot m_{i,j,k})/60 \quad (4)$$

The departure time of a bus at the stop is determined by the departure time at the previous stop, inner-stop travel time, acceleration time, deceleration time and dwell time, which...
can be described by (5).

\[ T_{i,j,k} = T_{i,j-1,k} + q_{i,j-1,k} + S_{i,j,k} + u_{i,j,k} + d_{i,j,k} \]  

(5)

When \( j = 1 \), the departure time of a bus at stop 1 is determined by the departure time of the first bus on each route, the dwell time of the bus at stop 1 and the departure interval. For example, the departure time of bus \( k \) at stop 1 equals to:

\[ T_{i,1,k} = F_{i,1,1} + d_{i,1,k} + \sum_{k=2}^{k} h_{i,k} \]  

(6)

As shown in Figure 3, assuming that the distance needed for buses to decelerate or accelerate is \( L \) (m), and the average inner-stop travel speed between stop \( j - 1 \) and stop \( j \) is \( v_{i,j,k} \) (m/s). The acceleration and deceleration of a bus entering or leaving a bus stop is assumed to be constant, with exactly the same absolute value. This means that the acceleration time of the bus at stop \( j \) equals to the slowing-down time at stop \( j \). The following equations can be obtained.

\[ q_{i,j,k} = u_{i,j,k} = v_{i,j,k} / \left| a_{i,j,k} \right| \]  

(7)

\[ L = \frac{(v_{i,j,k})^2}{2 \left| a_{i,j,k} \right|} \]  

(8)

By indicating the bus departure times at each stop, the headways of adjacent buses on the same route at each stop can be further obtained. The headway between bus \( k \) and bus \( k - 1 \) at stop \( j \) is expressed by (9).

\[ H_{i,j,k} = T_{i,j,k} - T_{i,j,k-1} \]  

(9)

Passengers waiting at a stop can be classified into two parts: one part is the remaining passengers failing to board the previous bus; the other part is the passengers arrive after the departure of the previous bus. The number of passengers waiting at stop \( j \) for bus \( k \) of route \( i \) is denoted by \( W_{i,j,k} \) and can be calculated as (10):

\[ W_{i,j,k} = r_{i,j,k-1} + \lambda_{i,j} \cdot H_{i,j,k} \]  

(10)

Under the all-stop service strategy, the number of alighting passengers of bus \( k \) of route \( i \) at stop \( j \) is denoted by \( m_{i,j,k} \) and can be calculated by (11):

\[ m_{i,j,k} = \theta_{i,j} \cdot H_{i,j,k} \]  

(11)

2) BUS OPERATION ANALYSIS UNDER THE STOP-SKIPPING STRATEGY

Considering the stop-skipping strategy and passenger capacity of a bus, the number of passengers that can board the bus at each stop is measured by (12):

\[ b_{i,j,k} = x_{i,j,k} \cdot \min \left( C_{i,k} - Z_{i,j,k} + m_{i,j,k} \right) \]  

(12)

where \( x_{i,j,k} \) is 0—1 variable, when \( x_{i,j,k} = 0 \), bus \( k \) of route \( i \) does not dwell at stop \( j \); when \( x_{i,j,k} = 1 \), bus \( k \) of route \( i \) dwells at stop \( j \).

\( n_{i,j,k} \) is the number of alighting passengers of bus \( k \) of route \( i \) at stop \( j \) under the stop-skipping service strategy. Due to the existence of the stop-skipping strategy, when calculating \( n_{i,j,k} \), it is necessary to consider whether the stop is skipped.

\[ n_{i,j,k} = x_{i,j,k} \cdot \left[ m_{i,j,k} + (1 - x_{i,j+1,k}) \cdot m_{i,j+1,k} \right] \]  

(13)

In this study, it is stipulated that if the destination stop of a passenger is skipped, the passenger needs to alight at the last stop of the destination stop and walk to the destination stop. Thus, in (13), \( x_{i,j+1,k} \) is considered when calculating \( n_{i,j,k} \). In addition, in this study a bus is not allowed to skip two consecutive stops.

The dwell time of bus \( k \) on route \( i \) at stop \( j \) is expressed by (14):

\[ d_{i,j,k} = x_{i,j,k} \cdot \left[ t_o + t_c + \max(t_b \cdot b_{i,j,k}, t_d \cdot n_{i,j,k}) \right] / 60 \]  

(14)

Considering the bus stop-skipping strategy, the departure time of bus \( k \) from route \( i \) at stop \( j \) is given by (15):

\[ T_{i,j,k} = T_{i,j-1,k} + x_{i,j-1,k} \cdot q_{i,j-1,k} + (1 - x_{i,j-1,k}) \cdot e_{i,j-1,k} + S_{i,j,k} + x_{i,j,k} \cdot \left( (m_{i,j,k} + d_{i,j,k}) + (1 - x_{i,j,k}) \cdot e_{i,j,k} \right) / 60 \]  

(15)

The lengths of acceleration area and deceleration area equal to \( L \) meters. Thus,

\[ e_{i,j-1,k} = \frac{L}{60v_{i,j,k}} \]  

(16)

When \( j = 1 \), the departure time of bus \( k \) at stop \( j - 1 \) is denoted by \( T_{i,1,k} \) and it can be obtained by (6).

From Figure 2, we can find that when the stop-skipping strategy is implemented, the distance traveled by the bus for skipping the stop is 2\( L \). Average operation speed is \( v_{i,j,k} \), then the time needed for bus \( k \) to skip the distance is expressed by (17):

\[ e_{i,j,k} = \left( \frac{2L}{v_{i,j,k}} \right) / 60 = \frac{L}{30v_{i,j,k}} \]  

(17)

Under the stop-skipping service, the calculation methods for \( H_{i,j,k} \) and \( W_{i,j,k} \) are the same as that under the all-stop service. Whereas \( m_{i,j,k} \) can be expressed as (18):

\[ m_{i,j,k} = x_{i,j,k} \cdot \theta_{i,j} \cdot H_{i,j,k} \]  

(18)

C. BUS OPERATION ANALYSIS UNDER MIXED SCHEDULING STRATEGY

The objective of the proposed mixed scheduling optimization strategy is minimizing the total travel time of all passengers, including stop waiting time \( (t_1 + t_2) \), in-vehicle travel time \( t_3 \) and walking time \( t_4 \). The independent variables are binary
variables that representing whether stops in the overlapping area are skipped, i.e. the value of $x_{i,j,k}$. The objective function is displayed below.

$$\min \ t = t_1 + t_2 + t_3 + t_4$$  \hfill (19)

1) STOP WAITING TIME

a: STOP WAITING TIME OF THE FIRST-TYPE PASSENGERS

In section 2.1, passengers are divided into two types. The first type of passengers only travel within the overlapping area, in which they have multiple choices of routes for their rides. The stop $g$ in the overlapping area in Figure 1 will be taken as an example to explain the calculation process. First, the bus departure times at stop $g$ of all routes are sorted ascendingly. In the analysis period, there will be $Y$ buses depart from stop $g$ and $Y = \Sigma_{i=1}^{l} K_i$. Bus $y-1$ departs earlier from stop $g$ than bus $y$, $1 < y \leq Y$, and $1 \leq g \leq G$. Please note that bus $y$ and bus $y-1$ may belong to different routes. Let $H'_{g,y}$ denote the headway between bus $y$ and bus $y-1$, and it can be calculated by (20).

$$H'_{g,y} = T'_{g,y} - T'_{g,y-1}$$  \hfill (20)

where $T'_{g,y}$ is the departure time of bus $y$ at stop $g$; $T'_{g,y1}$ is the departure time of bus $y-1$ at stop $g$.

The average waiting time of the first-type passengers at stop $j$ consists of two parts. The first part is the average waiting time of passengers arriving within the arrival times between bus $y-1$ and bus $y$. If the headway is $H'_{g,y}$, the average waiting time is computed as $H'_{g,y} - (1 - e^{-\lambda_{g,y}t}) / \lambda_{g,y}$. The first part is calculated as the product of the number of passengers and the average waiting time.

The second part is the waiting time of passengers failing to board bus $y-1$ and waiting for bus $y$. In the calculation of the waiting time for bus $y-1$, the average waiting time of the passengers left has been covered. Therefore, in the calculation of the waiting time for bus $y$, the waiting time of the passengers left is the sum of average waiting time $H'_{g,y} - (1 - e^{-\lambda_{g,y}t}) / \lambda_{g,y}$ and headway $H'_{g,y}$. The second part is calculated as the product of the number of passengers left and the headway.

The average waiting time of the first-type passengers at stop $g$ is denoted by $t_1$:

where $\lambda_{g,y}$ is the boarding rate of bus $y$ at stop $g$; $p_{g,y}$ is the proportion of the first-type passengers that take bus $y$ at stop $g$; $r_{g,y-1}$ is the number of passengers failing to board bus $y-1$ at stop $g$. $r_{g,y-1} = 0$ when $y = 1$.

In (21), as shown at the bottom of this page, the first part is the waiting time of passengers arriving within the headway of two consecutive buses; the second part is the waiting time of passengers failing to board the previous bus.

b: STOP WAITING TIME OF THE SECOND-TYPE PASSENGERS

The second-type passengers can only choose one route to travel at a stop, even the stop is in the overlapping area. The waiting time for the second-type passengers is denoted by $t_2$ and it is given in (22), as shown at the bottom of this page, where $\zeta_{i,j,k}$ is the boarding rate for bus $k$ of route $i$ at stop $j$; $p_{i,j,k}$ is the proportion of the first-type passengers that take bus $k$ of route $i$ at stop $j$; $(1 - p_{i,j,k})$ is the proportion of the second-type passengers that take bus $k$ of route $i$ at stop $j$.

2) IN-VEHICLE TRAVEL TIME

The definition of a passenger’ total in-vehicle travel time is the time from s/he boarding at the origin stops to arriving at the destination stops, which is the sum of the inner-stop travel time and the dwell times at stops. The difference of bus departure times between origin stop and destination stop is used to calculate the total travel time. Therefore, the passenger’s total in-vehicle travel time $t_3$ is shown in (23).

$$t_3 = \sum_{i=1}^{l} \sum_{j=1}^{J_i} \sum_{k=1}^{K_i} Z_{i,j,k} \cdot (T_{i,j,k} - T_{i,j-1,k})$$  \hfill (23)

where $(T_{i,j,k} - T_{i,j-1,k})$ denotes the travel time of bus $k$ on route $i$ between stop $j-1$ and stop $j$; $T_{i,j-1,k} = 0$ when $j = 1$.

3) WALKING TIME

Due to the stop-skipping service, some stops are skipped during operation. Passengers destined for a skipped stop need to alight at adjacent stops, and then walk to the destination stops. It is assumed that passengers will alight at the previous stop of the original destination, and the passenger total walking time is given by (24).

$$t_4 = \sum_{i=1}^{l} \sum_{j=1}^{J_i} \sum_{k=1}^{K_i} x_{i,j,k} \cdot (1 - x_{i,j+1,k}) \cdot m_{i,j+1,k} \cdot t_{ij+1,k}'$$  \hfill (24)

where $t_{ij+1,k}'$ is the walking time from stop $j$ to stop $j+1$ for alighting passengers of bus $k$ of route $i$. 

\begin{align*}
\frac{t_1}{\text{sum}} &= \sum_{g=1}^{G} \sum_{y=1}^{Y} \left[ \left( \frac{H'_{g,y} - (1 - e^{-\lambda_{g,y}t})}{\lambda_{g,y}} \right) p_{g,y} \cdot \left( H'_{g,y-1} - (1 - e^{-\lambda_{g,y-1}t})/\lambda_{g,y-1} + H'_{g,y} \right) \right] \\
\text{sum} &= \sum_{g=1}^{G} \sum_{y=1}^{Y} \left[ \left( \frac{H'_{g,y} - (1 - e^{-\lambda_{g,y}t})}{\lambda_{g,y}} \right) p_{g,y} \cdot \left( H'_{g,y-1} - (1 - e^{-\lambda_{g,y-1}t})/\lambda_{g,y-1} + H'_{g,y} \right) \right]
\end{align*}  \hfill (21)

\begin{align*}
\frac{t_2}{\text{sum}} &= \sum_{i=1}^{l} \sum_{j=1}^{J_i} \sum_{k=1}^{K_i} \left[ \left( \frac{H'_{i,j,k} - (1 - p_{i,j,k})}{\lambda_{i,j}} \right) \cdot (H_{i,j,k-1} - (1 - e^{-\lambda_{i,j}t})/\lambda_{i,j} + H_{i,j,k}) \right] \\
\text{sum} &= \sum_{i=1}^{l} \sum_{j=1}^{J_i} \sum_{k=1}^{K_i} \left[ \left( \frac{H'_{i,j,k} - (1 - p_{i,j,k})}{\lambda_{i,j}} \right) \cdot (H_{i,j,k-1} - (1 - e^{-\lambda_{i,j}t})/\lambda_{i,j} + H_{i,j,k}) \right]
\end{align*}  \hfill (22)
TABLE 2. Inner-stop distances for route 63, 10 and 11 (unit: km).

| Stop ID |  1  |  2  |  3  |  4  |  5  |  6  |  7  |  8  |  9  | 10  | 11  | 12  |
|---------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Route 63 | 0.00 | 0.61 | 0.56 | 0.26 | 0.34 | 0.30 | 0.70 | 0.58 | 0.90 | 0.37 | 0.45 | 0.92 |
| Route 10 | 0.00 | 1.80 | 0.53 | 0.75 | 0.91 | 0.82 | 0.92 | 0.75 | 0.95 | 0.40 | 0.65 | 0.37 |
| Route 11 | 0.00 | 0.50 | 0.80 | 0.35 | 0.46 | 0.93 | 0.42 | 0.33 | 1.00 | 0.57 | 0.53 | 0.47 |

If the bus dwells at stop $j+1$, the alighting passengers of the bus do not need to walk to the destination stop, then
$$1 - x_{i,j+1,k} = 0.$$ If the bus dwells at stop $j$ and doesn’t dwell at stop $j+1$, the number of passengers who alight at stop $j$ and walking to stop $j+1$ is
$$\left(1 - x_{i,j+1,k}\right) \cdot m_{i,j+1,k} \cdot t_{i,j,k}^f \quad \text{can be obtained by (25)}.$$

$$t_{i,j,k}^f = \left(\frac{S_{i,j,k} \cdot 60 \cdot v_{i,j,k} + 2L}{v_f}\right)/60 \quad (25)$$

4) MIXED SCHEDULING MODEL CONSTRAINTS

Some passengers would experience an increase in travel time because of the stop-skipping service. To avoid overlong waiting time and walking time, the proposed mixed scheduling model needs to satisfy the following constraints.

(i) Every two consecutive buses of all routes cannot skip the same stop in the overlapping area:
$$x_{g,y} + x_{g,y-1} \geq 1, \quad 1 \leq g \leq G, \quad 1 < y \leq Y \quad (26)$$

(ii) A bus cannot skip two consecutive stops in the overlapping area:
$$x_{g,y} + x_{g+1,y} \geq 1, \quad 1 \leq g \leq G - 1, \quad 1 \leq y \leq Y \quad (27)$$

D. SOLUTION ALGORITHM

The mixed scheduling model proposed in this paper is a nonlinear model with linear constraints and integer variables. The addressed problem is clearly a complex NP-hard one, which is difficult to be efficiently solved when the scale of the problem increases. Hence, genetic algorithm (GA) is used to efficiently solve the model. The objective function shown in (19) is used as a fitness function. An analogy for a feasible scheduling strategy is a chromosome. The variables (i.e., $x_{g,y}, x_{g+1,y}, x_{g+1,y+1}$) correspond to $(Y \times G)$ genes form a chromosome. Different values of these variables generate different chromosomes, which then constitute an initial group, that is, a set of model solutions. The chromosomes of a group go through crossover, mutation, and selection, eventually producing a chromosome with the highest fitness. The scheduling strategy associated with the highest-fitness chromosome is the optimal model solution.

III. CASE STUDY

A. DATA COLLECTION

Bus routes 63, 10 and 11 in Harbin city, China are selected for the case study. These three routes are overlapped with...
TABLE 3. Passenger volumes and inter-stop travel time of three bus routes.

| Stop ID | Route 63 |       | Route 10 |       | Route 11 |       |
|---------|----------|-------|----------|-------|----------|-------|
|         | BPV      | APV   | ISTT     |       | BPV      | APV   | ISTT     |       | BPV      | APV   | ISTT     |
| 1       | 201      | 0     | 0.00     |       | 216      | 0     | 0.00     |       | 180      | 0     | 0.00     |
| 2       | 132      | 0     | 2.23     |       | 240      | 0     | 1.58     |       | 30       | 0     | 2.00     |
| 3       | 75       | 0     | 2.58     |       | 160      | 45    | 1.22     |       | 6        | 0     | 1.50     |
| 4       | 60       | 12    | 2.08     |       | 103      | 30    | 1.38     |       | 60       | 12    | 3.15     |
| 5       | 42       | 0     | 2.23     |       | 65       | 30    | 1.83     |       | 114      | 0     | 2.10     |
| 6       | 147      | 12    | 1.50     |       | 60       | 114   | 2.00     |       | 186      | 6     | 4.09     |
| 7       | 54       | 24    | 2.77     |       | 105      | 120   | 1.50     |       | 114      | 12    | 1.78     |
| 8       | 69       | 132   | 2.00     |       | 45       | 45    | 3.15     |       | 66       | 78    | 2.91     |
| 9       | 68       | 60    | 1.50     |       | 60       | 53    | 2.10     |       | 78       | 42    | 6.33     |
| 10      | 21       | 66    | 3.15     |       | 95       | 120   | 4.09     |       | 66       | 24    | 1.43     |
| 11      | 30       | 54    | 2.10     |       | 79       | 75    | 1.78     |       | 90       | 36    | 6.24     |
| 12      | 39       | 108   | 4.09     |       | 23       | 59    | 1.18     |       | 138      | 102   | 5.70     |
| 13      | 27       | 72    | 1.78     |       | 74       | 75    | 1.30     |       | 102      | 36    | 2.93     |
| 14      | 27       | 60    | 4.00     |       | 45       | 150   | 0.95     |       | 192      | 48    | 1.47     |
| 15      | 63       | 48    | 1.55     |       | 60       | 66    | 1.02     |       | 156      | 102   | 3.08     |
| 16      | 21       | 12    | 1.92     |       | 45       | 48    | 1.53     |       | 114      | 324   | 2.52     |
| 17      | 45       | 108   | 2.73     |       | 30       | 60    | 7.30     |       | 96       | 222   | 5.10     |
| 18      | 15       | 36    | 1.45     |       | 13       | 75    | 0.95     |       | 72       | 210   | 3.87     |
| 19      | 3        | 72    | 4.43     |       | 6        | 46    | 2.50     |       | 18       | 192   | 2.45     |
| 20      | 0        | 198   | 1.78     |       | 15       | 67    | 1.15     |       | 0        | 96    | 1.38     |
| 21      | 0        | 96    | 1.72     |       | 9        | 90    | 1.48     |       | 12       | 78    | 1.17     |
| 22      | —        | —     | —        |       | 0        | 45    | 1.07     |       | 0        | 150   | 9.11     |
| 23      | —        | —     | —        |       | 0       | 135   | 1.93     |       | 0       | 120   | 3.27     |

NOTE: BPV—boarding passenger volume, passengers/h; APV—alighting passenger volume, passengers/h; ISTT—Inter-stop travel time, m.

TABLE 4. Timetables of the three bus routes under existing scheduling strategies (Unit: min).

| Stop ID | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---------|---|---|---|---|---|---|---|---|---|----|----|----|
| Route 63 | 0 | 3 | 6 | 7 | 9 | 11| 14| 17| 21| 23 | 25 | 29 |
| Route 10 | 0 | 6 | 8 | 11| 14| 17| 20| 23| 27 | 29 | 31 | 32 |
| Route 11 | 0 | 2 | 5 | 6 | 8 | 12| 14| 15| 19 | 22 | 24 | 26 |

| Stop ID | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | |
|---------|----|----|----|----|----|----|----|----|----|----|----|---|
| Route 63 | 31 | 32 | 34 | 37 | 40 | 42 | 45 | 46 | 48 | -  | -  | -  |
| Route 10 | 34 | 36 | 38 | 39 | 43 | 45 | 48 | 50 | 52 | 54 | 55 | -  |
| Route 11 | 29 | 30 | 34 | 36 | 39 | 41 | 43 | 45 | 47 | 49 | 50 | -  |

TABLE 5. Parameter values of mixed scheduling mode.

| Parameter | $t_a$ | $t_z$ | $t_y$ | $t_y$ | $q_{i,j,k}$ | $h_{i,j,k}$ | $L$ | $v_{i,j,k}$ |
|-----------|-------|-------|-------|-------|-------------|-------------|-----|-------------|
| Value     | 1/30  | 1/30  | 2.5   | 2.0   | 7/60        | 7/60        | 35  | 10          |
| Unit      | min   | min   | s     | s     | min         | min         | m   | m/s         |

| Parameter | $h_{i,max}$ | $h_{i,max}$ | $e_{i,j,k}$ | $v_y$ | $h_{i,k}$ | $a_{i,j,k}$ | $K_y$ | $C_{i,k}$ |
|-----------|-------------|-------------|-------------|------|-----------|-------------|------|----------|
| Value     | 2           | 10          | 3.5         | 1.2  | 5         | 1.4         | 12   | 80       |
| Unit      | min         | min         | m/s         | m/s2 | min       | passengers/veh |

The figures for route 11 are 12.2 km and 23 respectively. Table 2 shows the inner-stop distances of three bus routes.

The layout of the three routes in Harbin’s transportation network is shown in Figure 4. The three routes share six

each other, passing through business districts and large residential areas. At terminal stops, their departure headways are uniform. The full fare for each route is 1 RMB. The length of route 63 is 10.1 km and the total number of bus stops is 21. Route 10 is 15 km long and has 23 stops.
common stops. All of the overlapping stops are located on an important arterial road in Harbin with large bus passenger demand.

Data survey was implemented to obtain the bus travel time and boarding and alighting passenger volumes at each stop. The data was collected during the evening peak hour (17:00-19:00) from December 19th to 23rd, 2017. In-vehicle survey method was used to obtain the required data. Two investigators were assigned to each bus. One sat near the front door and recorded the bus arrival time, the number of boarding passengers, the average boarding time, and the number of bus seats. The other sat near the back door and recorded the number of alighting passengers, the average alighting time, and the bus departure time.

Departure intervals of the three bus routes during the evening peak period are all 5 min. Average passenger boarding and alighting volumes in an hour at each stop and inter-stop bus travel time are shown in Table 3. Units for the boarding and alighting volume are passenger per hour and that for inter-stop travel time is minute.

Table 4 shows existing timetables of the three bus routes. Buses on each route are dispatched uniformly and the headway is 5 min.

### B. OPTIMAL MIXED SCHEDULING STRATEGIES

Parameters of the mixed scheduling model are firstly calibrated based on the field survey data. The values of parameters are shown in Table 5. The evening peak

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**TABLE 6.** (a) Stop service strategies of 12 buses on route 63 in overlapping area. (b) Stop service strategies of 12 buses on route 10 in overlapping area. (c) Stop service strategies of 12 buses on route 11 in overlapping area.

| (a). Stop service strategies of 12 buses on route 63 in overlapping area |
|-----------------|-----|-----|-----|-----|-----|-----|-----|
| Stop ID | 9   | 10  | 11  | 12  | 13  | 14  |
| Bus 1   | 1   | 1   | 1   | 1   | 1   | 1   |
| Bus 2   | 1   | 1   | 1   | 1   | 1   | 1   |
| Bus 3   | 1   | 1   | 1   | 1   | 1   | 1   |
| Bus 4   | 1   | 1   | 1   | 0   | 1   | 1   |
| Bus 5   | 1   | 0   | 1   | 1   | 1   | 1   |
| Bus 6   | 1   | 1   | 1   | 1   | 0   | 1   |
| Bus 7   | 1   | 1   | 0   | 1   | 1   | 1   |
| Bus 8   | 1   | 0   | 1   | 1   | 1   | 1   |
| Bus 9   | 1   | 1   | 1   | 0   | 1   | 1   |
| Bus 10  | 1   | 1   | 1   | 1   | 1   | 1   |
| Bus 11  | 1   | 1   | 1   | 1   | 1   | 1   |
| Bus 12  | 1   | 0   | 1   | 1   | 1   | 1   |

| (b). Stop service strategies of 12 buses on route 10 in overlapping area |
|-----------------|-----|-----|-----|-----|-----|-----|
| Stop ID | 6   | 7   | 8   | 9   | 10  | 11  |
| Bus 1   | 1   | 1   | 1   | 1   | 1   | 1   |
| Bus 2   | 1   | 1   | 1   | 1   | 1   | 1   |
| Bus 3   | 1   | 1   | 1   | 1   | 1   | 1   |
| Bus 4   | 1   | 0   | 1   | 1   | 0   | 1   |
| Bus 5   | 1   | 1   | 1   | 0   | 1   | 1   |
| Bus 6   | 1   | 1   | 0   | 1   | 1   | 1   |
| Bus 7   | 1   | 1   | 1   | 1   | 1   | 1   |
| Bus 8   | 1   | 1   | 1   | 1   | 0   | 1   |
| Bus 9   | 1   | 1   | 1   | 1   | 1   | 1   |
| Bus 10  | 1   | 1   | 1   | 1   | 1   | 1   |
| Bus 11  | 1   | 1   | 1   | 1   | 1   | 1   |
| Bus 12  | 1   | 1   | 1   | 1   | 1   | 1   |

| (c). Stop service strategies of 12 buses on route 11 in overlapping area |
|-----------------|-----|-----|-----|-----|-----|-----|
| Stop ID | 2   | 3   | 4   | 5   | 6   | 7   |
| Bus 1   | 1   | 1   | 0   | 1   | 1   | 1   |
| Bus 2   | 1   | 0   | 1   | 1   | 0   | 1   |
| Bus 3   | 1   | 1   | 0   | 1   | 1   | 1   |
| Bus 4   | 1   | 1   | 1   | 1   | 1   | 1   |
| Bus 5   | 1   | 1   | 1   | 1   | 1   | 1   |
| Bus 6   | 1   | 1   | 1   | 1   | 1   | 1   |
| Bus 7   | 1   | 1   | 0   | 1   | 1   | 1   |
| Bus 8   | 1   | 0   | 1   | 1   | 0   | 1   |
| Bus 9   | 1   | 1   | 1   | 1   | 1   | 1   |
| Bus 10  | 1   | 0   | 1   | 1   | 0   | 1   |
| Bus 11  | 1   | 1   | 0   | 1   | 1   | 1   |
| Bus 12  | 1   | 1   | 1   | 1   | 0   | 1   |
TABLE 7. (a) Optimized timetables for buses on route 63. (b) Optimized timetables for buses on route 10. (c) Optimized timetables for buses on route 11.

(a) Optimized timetables for buses on route 63

| Stop | Bus 1 | Bus 2 | Bus 3 | Bus 4 | Bus 5 | Bus 6 | Bus 7 | Bus 8 | Bus 9 | Bus 10 | Bus 11 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|--------|
| 1    | 17:00 | 17:05 | 17:10 | 17:15 | 17:20 | 17:25 | 17:30 | 17:35 | 17:40 | 17:45  | 17:50  |
| 4    |       |       |       |       |       |       |       |       |       |        |        |
| 5    |       |       |       |       |       |       |       |       |       |        |        |
| 6    |       |       |       |       |       |       |       |       |       |        |        |
| 7    |       |       |       |       |       |       |       |       |       |        |        |
| 8    |       |       |       |       |       |       |       |       |       |        |        |
| 9    |       |       |       |       |       |       |       |       |       |        |        |
| 10   |       |       |       |       |       |       |       |       |       |        |        |
| 11   |       |       |       |       |       |       |       |       |       |        |        |

(b) Optimized timetables for buses on route 10

| Stop | Bus 1 | Bus 2 | Bus 3 | Bus 4 | Bus 5 | Bus 6 | Bus 7 | Bus 8 | Bus 9 | Bus 10 | Bus 11 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|--------|
| 1    | 17:00 | 17:05 | 17:10 | 17:15 | 17:20 | 17:25 | 17:30 | 17:35 | 17:40 | 17:45  | 17:50  |
| 4    |       |       |       |       |       |       |       |       |       |        |        |
| 5    |       |       |       |       |       |       |       |       |       |        |        |
| 6    |       |       |       |       |       |       |       |       |       |        |        |
| 7    |       |       |       |       |       |       |       |       |       |        |        |
| 8    |       |       |       |       |       |       |       |       |       |        |        |
| 9    |       |       |       |       |       |       |       |       |       |        |        |
| 10   |       |       |       |       |       |       |       |       |       |        |        |
| 11   |       |       |       |       |       |       |       |       |       |        |        |

(c) Optimized timetables for buses on route 11

| Stop | Bus 1 | Bus 2 | Bus 3 | Bus 4 | Bus 5 | Bus 6 | Bus 7 | Bus 8 | Bus 9 | Bus 10 | Bus 11 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|--------|
| 1    | 17:00 | 17:05 | 17:10 | 17:15 | 17:20 | 17:25 | 17:30 | 17:35 | 17:40 | 17:45  | 17:50  |
| 4    |       |       |       |       |       |       |       |       |       |        |        |
| 5    |       |       |       |       |       |       |       |       |       |        |        |
| 6    |       |       |       |       |       |       |       |       |       |        |        |
| 7    |       |       |       |       |       |       |       |       |       |        |        |
| 8    |       |       |       |       |       |       |       |       |       |        |        |
| 9    |       |       |       |       |       |       |       |       |       |        |        |
| 10   |       |       |       |       |       |       |       |       |       |        |        |
| 11   |       |       |       |       |       |       |       |       |       |        |        |
The max generation in GA is set to 200 with the crossover probability and the mutation probability set to 0.8 and 0.05, respectively. The independent variables are binary variables representing whether stops in the overlapping area are skipped. The mixed scheduling strategies are shown in Table 6, where 1 denotes stopping at the stop and 0 indicates the stop is skipped. Table 7 displays the optimized timetables for 12 buses on routes 63, 10 and 11 respectively in the evening peak period.

C. COMPARISONS AND ANALYSIS

In this section, the proposed mixed scheduling strategy is compared with the existing scheduling strategy. Under the existing scheduling strategy, the all-stop service is employed by the three routes. The evaluation indexes are calculated and shown in Table 8.

Table 8 shows that Strategy 2 can save passenger total travel time by 21.4% compared with that of Strategy 1. Due to reasonable selection of skipped stops, passengers’ waiting time at stops and in-vehicle travel time decrease by 4.5% and 25.1%, respectively. It can be concluded that the

| Evaluation index   | Strategy 1 | Strategy 2 | Decrease percentage |
|--------------------|------------|------------|---------------------|
| Total travel time  | 107930 min | 84792 min  | 21.4%               |
| Waiting time       | 11752 min  | 11217.2 min| 4.5%                |
| In-vehicle travel time | 96178 min  | 72067 min  | 25.1%               |
| Walking time       | 0 min      | 1507.8 min | —                   |

NOTE: Strategy 1- the strategy generated by the existing scheduling method; Strategy 2- the strategy generated by the mixed scheduling method.

| Headways | 0 | (0, 1] | (1, 2] | (2, 3] | (3, 4] | (4, 5] | ≥5 |
|----------|---|--------|--------|--------|--------|--------|----|
| Strategy 1 | 5.6% | 12.2% | 7.2% | 3.9% | 20.9% | 11.7% | 38.5% |
| Strategy 2 | 0% | 1.3% | 2.9% | 18.0% | 26.3% | 31.4% | 19.8% |

NOTE: Std Dev= standard deviation
mixed scheduling strategy significantly reduces passengers’ in-vehicle travel time and enhances the travel efficiency of most passengers. Despite the increase in walking time of the passengers at the skipped stops due to the stop-skipping service, the waiting time of all passengers still reduces under the mixed scheduling strategy. This is because the skipped stops are limited. In addition, under the mixed scheduling strategy, the walking time only accounts for 1.78% of the total travel time.

Under Strategy 2, bus bunching problem caused by non-uniform headways and simultaneously bus arrival from different routes can be mitigated. The proportions of headways in different intervals under two scheduling strategies are given by Table 9. The unit of headways in the first row is minute. From the table we can find that the total percentage of headways that are smaller than 2 min under Strategy 1 is 25%. However, under Strategy 2 the percentage is only 4.2%. Thus, Strategy 2 is helpful to prevent bus bunching on the three routes.

Crowding degrees of all dispatched buses on the three routes are shown in Table 10. For example, the crowding degree of bus 1 on route 63 under Strategy 1 is 46.06%, it is equal to the average of crowding degrees when bus 1 departs from each stop on route 63. In the last row, the variances of all bus crowding degrees under the two strategies are displayed. By comparing Strategy 1 with Strategy 2, the standard deviations of route 63, route 10 and route 11 decreased by 2.67%, 1.75% and 1.44% respectively. The corresponding decrease ranges are 47.88%, 32.95% and 40.09% respectively. Hence, it can be concluded that Strategy 2 can produce more balanced bus crowding degree for each route.

The optimal mixed scheduling plans are generated by GA. We compare the GA with enumeration method to test the effectiveness of the GA. The computing times of the two solving algorithms are 3.2s and 4.6s respectively.

IV. CONCLUSION

This study has drawn the following conclusions:

(i) Compared with the existing scheduling strategy with all-stop service, the mixed scheduling strategy can significantly reduce passenger total travel time. The increased walking time due to stop-skipping service only accounts for a small percentage of total travel time.

(ii) Compared with the existing scheduling strategy, the mixed scheduling strategy can significantly reduce passengers’ in-vehicle travel time, which is the main factor leading to the significant reduction of total travel time. In addition, the waiting time is reduced slightly even with the stop-skipping service. This is because passengers in the overlapping area can select more than one bus route and the stop-skipping service would not increase their waiting time significantly under the high-frequency bus service.

(iii) Using the stop-skipping service in the overlapping area could help mitigate non-uniform bus headways and crowdedness caused by the simultaneous arrival of multiple buses, and pay less impact on passenger travel time in the overlapping area.

The scope of this study is developing a new method to optimize the static scheduling plans for multiple routes with common stops. Dynamic scheduling issues such as real-time travel time estimation are not considered. In the future, the impacts of route overlap on dynamic scheduling will be analyzed.

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