Determination of the sign of the decay width difference in the $B_s^0$ system

The LHCb collaboration

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The interference between the \(K^+K^-\) S-wave and P-wave amplitudes in \(B_s^0 \to J/\psi K^+K^-\) decays with the \(K^+K^-\) pairs in the region around the \(\phi(1020)\) resonance is used to determine the variation of the difference of the strong phase between these amplitudes as a function of \(K^+K^-\) invariant mass. Combined with the results from our CP asymmetry measurement in \(B_s^0 \to J/\psi \phi\) decays, we conclude that the \(B_s^0\) mass eigenstate that is almost \(CP = +1\) is lighter and decays faster than the mass eigenstate that is almost \(CP = -1\). This determines the sign of the decay width difference \(\Delta\Gamma_s \equiv \Gamma_L - \Gamma_H\) to be positive. Our result also resolves the ambiguity in the past measurements of the \(CP\) violating phase \(\phi_s\) to be close to zero rather than \(\pi\). These conclusions are in agreement with the Standard Model expectations.

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The decay time distributions of \(B_s^0\) mesons decaying into the \(J/\psi\phi\) final state have been used to measure the parameters \(\phi_s\) and \(\Delta\Gamma_s \equiv \Gamma_L - \Gamma_H\) of the \(B_s^0\) system [1-3]. Here \(\phi_s\) is the \(CP\) violating phase equal to the phase difference between the amplitude for the direct decay and the amplitude for the decay after oscillation. \(\Gamma_L\) and \(\Gamma_H\) are the decay widths of the light and heavy \(B_s^0\) mass eigenstates, respectively. The most precise results, presented recently by the LHCb experiment [2],

\[
\phi_s = 0.15 \pm 0.18 \text{ (stat) } \pm 0.06 \text{ (syst) rad} \\
\Delta\Gamma_s = 0.123 \pm 0.029 \text{ (stat) } \pm 0.011 \text{ (syst) ps}^{-1}
\]

show no evidence of \(CP\) violation yet, indicating that \(CP\) violation is rather small in the \(B_s^0\) system. There is clear evidence for the decay width difference \(\Delta\Gamma_s\) being non-zero. It must be noted that there exists another solution

\[
\phi_s = 2.99 \pm 0.18 \text{ (stat) } \pm 0.06 \text{ (syst) rad} \\
\Delta\Gamma_s = -0.123 \pm 0.029 \text{ (stat) } \pm 0.011 \text{ (syst) ps}^{-1}
\]

arising from the fact that the time dependent differential decay rates are invariant under the transformation \((\phi_s, \Delta\Gamma_s) \leftrightarrow (\pi - \phi_s, -\Delta\Gamma_s)\) together with an appropriate transformation for the strong phases. In the absence of \(CP\) violation, \(sin\phi_s = 0\), i.e. \(\phi_s = 0\) or \(\phi_s = \pi\), the two mass eigenstates become also \(CP\) eigenstates with \(CP = +1\) and \(CP = -1\). They can be identified by the decays into final states which are \(CP\) eigenstates. In \(B_s^0 \to J/\psi K^+K^-\) decays, the final state is a superposition of \(CP = +1\) and \(CP = -1\) for the \(K^+K^-\) pair in the P-wave configuration, and \(CP = -1\) for the \(K^+K^-\) pair in the S-wave configuration. Higher order partial waves are neglected. These decays have different angular distributions of the final state particles and are distinguishable.

Solution I is close to the case \(\phi_s = 0\) and leads to the light (heavy) mass eigenstate being almost aligned with the \(CP = +1\) (\(CP = -1\)) state. Similarly, solution II is close to the case \(\phi_s = \pi\) and leads to the heavy (light) mass eigenstate being almost aligned with the \(CP = +1\) (\(CP = -1\)) state. A fit to the observed decay time distribution shows that it can be well described by a superposition of two exponential functions corresponding to \(CP = +1\) and \(CP = -1\), compatible with no \(CP\) violation [3]. In this fit the decay width to the \(CP = +1\) final state is found to be larger than that to \(CP = -1\). Thus the mass eigenstate that is predominantly \(CP\) even decays faster than the \(CP\) odd state. For solution I, we find \(\Delta\Gamma_s > 0\), i.e. \(\Gamma_L > \Gamma_H\), and for solution II, \(\Delta\Gamma_s < 0\), i.e. \(\Gamma_L < \Gamma_H\). In order to determine if the decay width difference \(\Delta\Gamma_s\) is positive or negative it is necessary to resolve the ambiguity between the two solutions.

Since each solution corresponds to a different set of strong phases, one may attempt to resolve the ambiguity by using the strong phases either as predicted by factorisation, or as measured in \(B^0 \to J/\psi K^{*0}\) decays. Unfortunately these two possibilities lead to opposite answers [4]. A direct experimental resolution of the ambiguity is therefore desirable.

In this Letter, we resolve this ambiguity using the decay \(B_s^0 \to J/\psi K^+K^-\) with \(J/\psi \to \mu^+\mu^-\). The total decay amplitude is a coherent sum of S-wave and P-wave contributions. The phase of the P-wave amplitude, which can be described by a spin-1 Breit-Wigner function of the invariant mass of the \(K^+K^-\) pair, denoted by \(m_{KK}\), rises rapidly through the \(\phi(1020)\) mass region. On the other hand the phase of the S-wave amplitude should vary relatively slowly for either an \(f_0(980)\) contribution or a non-resonant contribution. As a result, the phase difference between the S-wave and P-wave amplitudes falls rapidly with increasing \(m_{KK}\). By measuring this phase difference as a function of \(m_{KK}\), and taking the solution with a decreasing trend around the \(\phi(1020)\) mass as the physical solution, the sign of \(\Delta\Gamma_s\) is determined and the ambiguity in \(\phi_s\) is resolved [5]. This is similar to the way the BaBar collaboration measured the sign of \(cos 2\beta\) using the decay \(B^0 \to J/\psi K^{*0}\) [6], where \(2\beta\) is the weak phase characterizing mixing-induced \(CP\) asymmetry in this decay.

The analysis is based on the same data sample as used in Ref. [2], which corresponds to an integrated luminosity of 0.37 fb\(^{-1}\) of \(pp\) collisions collected by the LHCb experiment at the Large Hadron Collider at the centre of mass energy of \(\sqrt{s} = 7\) TeV. The LHCb detector is a forward spectrometer and is described in detail in Ref. [7]. The trigger, event selection criteria and analysis method are very similar to those in Ref. [3], and here we discuss only the differences. The fraction of \(K^+K^-\) S-wave
contribution measured within ±12 MeV of the nominal \( \phi(1020) \) mass is 0.042 ± 0.015 ± 0.018 [3]. (We adopt units such that \( c = 1 \) and \( \hbar = 1 \).) The S-wave fraction depends on the mass range taken around the \( \phi(1020) \). The result of Ref. [3] is consistent with the CDF limit on the S-wave fraction of less than 6% at 95% CL (in the range 1009–1028 MeV) [2], smaller than the DØ result of (12 ± 3)% (in 1010–1030 MeV) [8], and consistent with phenomenological expectations [9]. In order to apply the ambiguity resolution method described above, the range of \( m_{KK} \) is extended to 988–1050 MeV. Figure 1 shows the \( \mu^+\mu^-K^+K^- \) mass distribution where the mass of the \( \mu^+\mu^- \) pair is constrained to the nominal \( J/\psi \) mass. We perform an unbinned maximum likelihood fit to the invariant mass distribution of the selected \( B_s^0 \) candidates.

The probability density function (PDF) for the signal \( B_s^0 \) invariant mass \( m_{J/\psi KK} \) is modelled by two Gaussian functions with a common mean. The fraction of the wide Gaussian and its width relative to that of the narrow Gaussian are fixed to values obtained from simulated events. A linear function describes the invariant mass distribution where the mass of the \( \phi \) depends on the mass range taken around the nominal \( J/\psi \) mass. We perform an unbinned maximum likelihood fit to the invariant mass distribution of the selected \( B_s^0 \) candidates. The probability density function (PDF) for the signal \( B_s^0 \) invariant mass \( m_{J/\psi KK} \) is modelled by two Gaussian functions with a common mean. The fraction of the wide Gaussian and its width relative to that of the narrow Gaussian are fixed to values obtained from simulated events. A linear function describes the invariant mass distribution where the mass of the \( \phi \) depends on the mass range taken around the nominal \( J/\psi \) mass. We perform an unbinned maximum likelihood fit to the invariant mass distribution of the selected \( B_s^0 \) candidates. The probability density function (PDF) for the signal \( B_s^0 \) invariant mass \( m_{J/\psi KK} \) is modelled by two Gaussian functions with a common mean. The fraction of the wide Gaussian and its width relative to that of the narrow Gaussian are fixed to values obtained from simulated events. A linear function describes the invariant mass distribution where the mass of the \( \phi \) depends on the mass range taken around the nominal \( J/\psi \) mass. We perform an unbinned maximum likelihood fit to the invariant mass distribution of the selected \( B_s^0 \) candidates.

This analysis uses the sWeight technique [10] for background subtraction. The signal weight, denoted by \( W_s(m_{J/\psi KK}) \), is obtained using \( m_{J/\psi KK} \) as the discriminating variable. The correlations between \( m_{J/\psi KK} \) and other variables used in the analysis, including \( m_{KK} \), decay time \( t \) and the angular variables \( \Omega \) defined in Ref. [3], are found to be negligible for both the signal and background components in the data. Figure 2 shows the \( m_{KK} \) distribution where the background is subtracted statistically using the sWeight technique. The range of \( m_{KK} \) is divided into four intervals: 988–1008 MeV, 1008–1020 MeV, 1020–1032 MeV and 1032–1050 MeV. Table I gives the number of \( B_s^0 \) signal and background candidates in each interval.

| \( k \) | \( m_{KK} \) interval (MeV) | \( N_{sig,k} \) | \( N_{bkg,k} \) | \( W_{p,k} \) |
|---|---|---|---|---|
| 1 | 988–1008 | 251 ± 21 | 1675 ± 43 | 0.700 |
| 2 | 1008–1020 | 4569 ± 70 | 2002 ± 49 | 0.952 |
| 3 | 1020–1032 | 3952 ± 66 | 2244 ± 51 | 0.938 |
| 4 | 1032–1050 | 726 ± 34 | 5442 ± 62 | 0.764 |

In this analysis we perform an unbinned maximum likelihood fit to the data using the sFit method [11], an extension of the sWeight technique, that simplifies fitting in the presence of background. In this method it is only necessary to model the signal PDF, as background is cancelled statistically using the signal weights.

The parameters of the \( B_s^0 \to J/\psi K^+K^- \) decay time distribution are estimated from a simultaneous fit to the four intervals of \( m_{KK} \) by maximizing the log-likelihood function

\[
\ln L(\Theta_P, \Theta_S) = \sum_{k=1}^{4} \ln \prod_{i=1}^{N_k} W_s(m_{J/\psi KK;i}) \times P_{sig}(t_i, \Omega_i, q_i, \omega_i; \Theta_P, \Theta_S)
\]

where \( N_k = N_{sig;k} + N_{bkg;k} \). \( \Theta_P \) represents the physics parameters independent of \( m_{KK} \), including \( \phi_s, \Delta \Gamma_s \) and the magnitudes and phases of the P-wave amplitudes. Note that the P-wave amplitudes for different polarizations share the same dependence on \( m_{KK} \). \( \Theta_S \) denotes the values of the \( m_{KK}\)-dependent parameters averaged over each interval, namely the average fraction of S-wave contribution for the \( k \)-th interval, \( F_{S;k} \), and the average phase difference between the S-wave amplitude and

![FIG. 1. Invariant mass distribution for \( B_s^0 \to \mu^+\mu^-K^+K^- \) candidates with the mass of the \( \mu^+\mu^- \) pair constrained to the nominal \( J/\psi \) mass. The result of the fit is shown with signal (dashed curve) and combinatorial background (dotted curve) components and their sum (solid curve).](image1)

![FIG. 2. Background subtracted \( K^+K^- \) invariant mass distribution for \( B_s^0 \to J/\psi K^+K^- \) candidates. The vertical dotted lines separate the four intervals.](image2)
the perpendicular P-wave amplitude for the k-th interval, δS⊥,k. Psig is the signal PDF of the decay time t, angular variables Ω, initial flavour tag q and the mistag probability ω. It is based on the theoretical differential decay rates [3] and includes experimental effects such as decay time resolution and acceptance, angular acceptance and imperfect identification of the initial flavour of the B_d^0 particle, as described in Ref. [3]. The factors W_p,k account for loss of statistical precision in parameter estimation due to background dilution. Their values are given in Table I.

The fit results for φ, ΔΓ, and the fractions of S-wave contribution and phase differences between the S-wave and P-wave amplitudes for the four m_{KK} intervals are given in Table II. Figure 3 shows the estimated K^+K^- S-wave and P-wave contributions in the four m_{KK} intervals. The shape of the measured P-wave m_{KK} distribution is in good agreement with that of B_d^0 → J/ψφ events simulated using a spin-1 relativistic Breit-Wigner function for the φ(1020) amplitude. In Fig. 4, the phase difference between the S-wave and the perpendicular P-wave amplitude is plotted in four m_{KK} intervals for solution I and solution II. Figure 4 shows a clear decreasing trend of the phase difference between the S-wave and the perpendicular P-wave amplitude as a linear function of the average m_{KK} value in the k-th interval. This leads to a slope of −0.050^{+0.013}_{−0.020} rad/MeV for solution I and +0.050^{+0.013}_{−0.020} rad/MeV for solution II, where the uncertainties are statistical only.

![Fig. 3](image1.png)

**FIG. 3.** Distribution of (a) K^+K^- S-wave signal events, and (b) K^+K^- P-wave signal events, both in four invariant mass intervals. In (b) the distribution of simulated B_d^0 → J/ψφ events in the four intervals assuming the same total number of P-wave events is also shown (dashed). Note the interference between the K^+K^- S-wave and P-wave amplitudes integrated over the angular variables has vanishing contribution in these distributions.

![Fig. 4](image2.png)

**FIG. 4.** Measured phase differences between S-wave and perpendicular P-wave amplitudes in four intervals of m_{KK} for solution I (blue full circles) and solution II (black full squares). The asymmetric error bars correspond to Δ ln L = −0.5 (solid) and Δ ln L = −2 (dotted).

| Parameter | Solution I | Solution II |
|-----------|------------|-------------|
| φ_k (rad) | 0.167 ± 0.175 | 2.975 ± 0.175 |
| ΔΓ / (ps^{-1}) | 0.120 ± 0.028 | −0.120 ± 0.028 |
| F_{S;1} | 0.283 ± 0.113 | 0.283 ± 0.113 |
| F_{S;2} | 0.061 ± 0.022 | 0.061 ± 0.022 |
| F_{S;3} | 0.044 ± 0.022 | 0.044 ± 0.022 |
| F_{S;4} | 0.269 ± 0.067 | 0.269 ± 0.067 |
| δ_{S⊥,1} (rad) | −0.46 ± 0.35 | −2.68 ± 0.42 |
| δ_{S⊥,2} (rad) | −2.92 ± 0.15 | −0.22 ± 0.13 |
| δ_{S⊥,3} (rad) | −3.25 ± 0.16 | 0.11 ± 0.18 |
| δ_{S⊥,4} (rad) | −4.11 ± 0.43 | 0.97 ± 0.28 |

**TABLE II.** Results from a simultaneous fit of the four intervals of m_{KK}, where the uncertainties are statistical only. Only parameters which are needed for the ambiguity resolution are shown.
The difference of the ln$L$ value between this fit and a fit in which the slope is fixed to be zero is 11.0. Hence the negative trend of solution I has a significance of 4.7 standard deviations. Therefore, we conclude that solution I, which has $\Delta \Gamma_{\perp} > 0$, is the physical solution. The trend of solution I is also qualitatively consistent with that of the phase difference between the $K^+K^-$ S-wave and P-wave amplitudes versus $m_{KK}$ measured in the decay $D_s^+ \to K^+K^-$ $\pi^+$ by the BaBar collaboration [12].

Several possible sources of systematic uncertainty on the phase variation versus $m_{KK}$ have been considered. A possible background from decays with similar final states such as $B^0 \to J/\psi K^0$ could have a small effect. From simulation, the contamination to the signal from such decays is estimated to be 1.1% in the $m_{KK}$ range of 988–1050 MeV. We add a 2.2% contribution of simulated $B^0 \to J/\psi K^0$ events to the data and repeat the analysis. The largest observed change is a shift of $\delta_{\perp,4}$ by 0.06 rad, which is only 20% of its statistical uncertainty and has negligible effect on the slope of $\delta_{\perp}$ versus $m_{KK}$. The effect of neglecting the variation of the values of $F_{S}$ and $\delta_{\perp}$ in each $m_{KK}$ interval is determined to change the significance of the negative trend of solution I by less than 0.1 standard deviations. We also repeat the analysis for different $m_{KK}$ ranges, different ways of dividing the $m_{KK}$ range or different shapes of the signal and background $m_{J/\psi KK}$ distributions. The significance of the negative trend of solution I is not affected. To measure precisely the S-wave lineshape and determine its resonance structure more data are needed. However, the results presented here do not depend on such detailed knowledge.

In conclusion the analysis of the strong interaction phase shift resolves the ambiguity between solution I and solution II. Values of $\phi_s$ close to zero and positive $\Delta \Gamma_{\perp}$ are preferred. It follows that in the $B_s^0$ system, the mass eigenstate that is almost CP even is lighter and decays faster than the state that is almost CP odd. This is in agreement with the Standard Model expectations [13]. It is also interesting to note that this situation is similar to that in the neutral kaon system.

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[1] DØ collaboration, V. M. Abazov et al., Measurement of the CP violating phase $\phi_s^{J/\psi\phi}$ using the flavor-tagged decay $B_s^0 \to J/\psi\phi$ in 8 fb$^{-1}$ of pp collisions, arXiv:1109.3166.
[2] CDF collaboration, T. Aaltonen et al., Measurement of the CP violating phase $\beta_s$ in $B_s^0 \to J/\psi\phi$ decays with the CDF II Detector, arXiv:1112.1726.
[3] LHCb collaboration, R. Aaij et al., Measurement of the CP-violating phase $\phi_s$ in the decay $B_s^0 \to J/\psi\phi$, arXiv:1112.3183. Accepted by Phys. Rev. Lett..
[4] S. Nandi and U. Nierste, Resolving the sign ambiguity in $\Delta \Gamma_{\perp}$ with $B_s^0 \to D_sK$, Phys. Rev. D77 (2008) 054010, arXiv:0801.0143.
[5] Y. Xie, P. Clarke, G. Cowan, and F. Muheim, Determination of $2\beta_s$ in $B_s^0 \to J/\psi K^+K^-$ decays in the presence of a $K^+K^-$ S-wave contribution, JHEP 09 (2009) 074, arXiv:0908.3627.
[6] BaBar collaboration, B. Aubert et al., Ambiguity-free measurement of $\cos 2\beta$: time-integrated and time-dependent angular analyses of $B \to J/\psi K\pi$, Phys. Rev. D71 (2005) 032005, arXiv:hep-ex/0411016.
[7] LHCb collaboration, J. Alves, A. Augusto et al., The LHCb Detector at the LHC, JINST 3 (2008) P08005.
[8] DØ collaboration, V. M. Abazov et al., Measurement of the relative branching ratio of $B_s^0 \to J/\psi f_0$ to $B_s^0 \to J/\psi\phi$, Phys. Rev. D85 (2012) 011103, arXiv:1110.4272.
[9] S. Stone and L. Zhang, S-waves and the measurement of CP violating phases in $B_s^0$ decays, Phys. Rev. D79 (2009) 074024, arXiv:0812.2832.
[10] M. Pivk and F. R. Le Diberder, sPlot: a statistical tool to unfold data distributions, Nucl. Instrum. Meth. A555 (2005) 356–369, arXiv:physics/0402083.
[11] Y. Xie, sFit: a method for background subtraction in maximum likelihood fit, arXiv:0905.0724.
[12] BaBar collaboration, P. del Amo Sanchez et al., Dalitz plot analysis of $D_s^0 \to K^+K^\mp \pi^\pm$, Phys. Rev. D83 (2011) 052001, arXiv:1011.4190.
[13] A. Lenz et al., Anatomy of new physics in $B \to B$ mixing, Phys. Rev. D83 (2011) 036004, arXiv:1008.1593.