Novel Calibration and Lens Distortion Correction of 3D Reconstruction Systems

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Abstract. A novel calibration and lens distortion correction of 3D reconstruction systems is introduced in this paper. Assuming zero distortion at the center of image, the parameter calibration and lens distortion correction of CCD camera and DLP projector is separated with the points in center and all the rest points. Furthermore, identification and correspondence of the interest points is done automatically because of the designed target with standard circles and multicolor code of pseudo-random array during the calibration. The experiments show that it is a fast and convenient method to carry out the calibration for lens distortion.

1. Introduction

Camera calibration is a crucial problem for computer vision where many tasks require the computation of accurate metric information from images. Calibrating a camera consists in determining the transformation which maps 3D points of a certain scene or object into their corresponding 2D projections onto the image plane of the camera. The precision of 3D reconstruction will be influenced by the veracity and reliability of camera and projector in the system. Therefore, the lens distortion of camera and projector should be considered during calibration. Many techniques and some studies concerning calibration for lens distortion have been presented in the last few years.

The first one is a simple adaptation of the Faugeras linear method with the aim of including radial lens distortion [1]. The widely used method proposed by Tsai, which is based on a two-step technique modeling only radial lens distortion [2]. Finally, the complete model of Weng, which was proposed in 1992, including three different types of lens distortion [3]. At the same time, plenty of methods to correct lens distortion have been presented, such as the method based on the invariability of cross ratio[4], the method based on slope [5] and the method based on invariability of projection [6]. In this study, camera and DLP projector of the vision system are calibrated based on two-stage method and the lens distortion have been considered and corrected.

2. Vision system of 3D reconstruction

The vision system of 3D reconstruction is shown in Figure 1. The interest point PPU on coding plane is projected to 3D scene and intersected the object at the point PW, which is corresponding to the point PIU in image plane collected by CCD camera in vision system of 3D reconstruction. The whole process narrated above is based on the supposition that CCD camera and DLP projector can be modeled as ideal optical components without lens distortion. In fact, there’re various types of geometric and chromatic aberrations, such as radial, decentering and thin prism distortion. The details about calibration and correction will be introduced as follows.
3. CCD camera calibration

3.1. Camera modeling

A camera model is a mathematical formulation which approximates the behavior of CCD camera by using a set of mathematical equations and the objective is to find the relationship between 3D scene points and 2D image points. Camera modeling depends on two sets of parameters: one is the extrinsic (or external) parameters which define the camera’s pose, i.e. the position and orientation of the camera in the scene with respect to a given reference system; and the other is the intrinsic (or internal) parameters which describe the internal geometry of the camera, i.e. the effective focal length of the camera, the lens distortion coefficients and the coordinates of the principal point [7].

As shown in Figure 1, the target coordinates is regarded as the metric world coordinates system {W}, in which the original position is regarded as origin OW. ZW is the direction of 1D precision translation; XW and YW are parallel to the level and vertical axis of the target respectively. In camera coordinates system {C}, the origin OC is optical center of camera, XC and YC are parallel to the level and vertical axis of image plane respectively. Camera modeling is usually broken down into 4 steps, the detail is as following:

At first, changing the world coordinates system to the camera coordinates system is carried out with a translation vector and a rotation matrix as shown in the equation (1). Given a 3D point PW, expressed with respect to the metric world coordinates system (i.e. WPW), the corresponding point in the camera coordinates is expressed as CPW = (CXW, CYW, CZW).

\[
\begin{bmatrix}
  cX_W \\
  cY_W \\
  cZ_W 
\end{bmatrix} =
\begin{bmatrix}
  cR_W & cT_W
\end{bmatrix}
\begin{bmatrix}
  wX_W \\
  wY_W \\
  wZ_W
\end{bmatrix}
\]

(1)

Where cR_W and cT_W express the orientation and position of the world coordinates system {W} with respect to the axis of the camera coordinates system {C}.

Next, it is necessary to carry out the projection of point CP_W on the image plane obtaining point CP_U via a projective transformation. Considering any optical sensor can be modeled as a pinhole camera, the image plane is located at a distance f from the optical center OC and is parallel to the plane defined by the coordinates axis XC and YC. Moreover, given an object point CP_W related to the camera coordinates system, if it is projected through the focal point OC, the optical ray intercepts the image plane at the 2D image point CP_U. This relation is shown in the equation (2).

\[
\begin{align*}
  cX_{IU} &= f_C \frac{cX_W}{cZ_W} \\
  cY_{IU} &= f_C \frac{cY_W}{cY_W}
\end{align*}
\]

(2)
Then, model the lens distortion according to the difference between the ideal and the practical
projection. Assuming \( \sigma_x \) and \( \sigma_y \) expresses the distortion in \( x \) and \( y \) direction respectively, the relation
between the ideal point \( C_{PIU} \) and the practical point \( C_{PID} \) is shown in the equation (3):

\[
C_{X_{ID}} = C_{X_{ID}}^{\mu} + \sigma_x \quad C_{Y_{ID}} = C_{Y_{ID}}^{\mu} + \sigma_y
\]

Assuming no distortion, \( C_{PIU} \) and \( C_{PID} \) will be the same point. Actually, the nonlinear distortion
does exist, which mainly contains radial, decentering and thin prism distortion, due to the imperfection
in lens design and manufacturing as well as camera assembly.

Synthesizing three kinds of distortion, the lens distortion model is obtained as follows:

\[
\sigma_x = k_1 C_{X_{ID}} (C_{X_{ID}}^2 + C_{Y_{ID}}^2) + (p_1 (3C_{X_{ID}}^2 + C_{Y_{ID}}^2) + 2p_2 C_{X_{ID}} C_{Y_{ID}}) + s_1 (C_{X_{ID}}^2 + C_{Y_{ID}}^2)
\]

\[
\sigma_y = k_2 C_{X_{ID}} (C_{X_{ID}}^2 + C_{Y_{ID}}^2) + (p_2 (3C_{X_{ID}}^2 + C_{Y_{ID}}^2) + 2p_2 C_{X_{ID}} C_{Y_{ID}}) + s_2 (C_{X_{ID}}^2 + C_{Y_{ID}}^2)
\]

Where \( k_1, k_2 \) is radial distortion coefficient; \( p_1, p_2 \) is decentering distortion coefficient; \( s_1, s_2 \) is thin
prism distortion coefficient. Generally, the radial distortion coefficient \( k_1 \) will be satisfied in
measurement.

Finally, changing the point \( C_{PID} \) in the camera image to the point \( P \) in the computer image
coordinates system is carried out in the equation (5):

\[
U = -K_x C_{X_{ID}} + U_0 \quad V = -K_y C_{Y_{ID}} + V_0
\]

Where \( K_x = 1/dx \), \( K_y = 1/dy \) are the parameters that transform from metric measures with respect to the
camera coordinates system to pixels with respect to the computer image coordinates system; \( dx \) and \( dy \)
is the interval between adjacent pixel in the \( x \) and \( y \) direction; \( (U_0, V_0) \) are the components that define
the projection of the focal point in the plane image in pixels, i.e. the principal point.

### 3.2. Experiment for camera calibration

The CCD camera is calibrated according to the camera modeling step introduced above [8]. As
shown in Figure 2, the camera system is made up of a CCD camera with the resolution of 1280×960, a
target and a precision stage. The target with array of 11×17 standard circles (diameter is four, interval
is five) is fixed on the precision stage vertically. The camera captures three images at three different
positions along with \( Z \) axis of the stage. Taking the central coordinates of circles around at the center
both in the world coordinates and the image coordinates into the equation (1)-(5), the parameters are
obtained via optimization and the lens distortion is shown in Figure 4.

\[
C_{R_w} = \begin{bmatrix}
0.9896 & -0.0352 & 0.1392 \\
0.0581 & 0.9848 & -0.1635 \\
-0.1313 & 0.1699 & 0.9767
\end{bmatrix} \quad C_{T_w} = \begin{bmatrix}
-71.5 \\
-83.3 \\
1811.1
\end{bmatrix}
\]

\( fc = 508, \quad (u_0,v_0)=(645.4, 478.6), \quad k_1=k_2=0.15, \quad p_1=p_2=0.001, \quad s_1=s_2=0.0005. \)
4. DLP projector calibration

4.1. Projector modeling

Projector calibration is different from camera calibration, because the coding interest points are fixed and the system will not be affected by the rest points. Since projector can be seen as reversal camera, the ideal camera model \([9]\) can be used in projector calibration to find the relationship between 3-D scene points \(^W P_W = (^W X_W, ^W Y_W, ^W Z_W)\) and 2-D points \(P = (U, V)\) on the coding plane as follows:

\[
\begin{bmatrix}
U \\
V
\end{bmatrix} = \begin{bmatrix}
\alpha_{PU} & s_P & u_{P0} \\
0 & \alpha_{PV} & v_{P0} \\
1 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
^P R_W & ^P T_W
\end{bmatrix} \begin{bmatrix}
^w X_W \\
^w Y_W \\
^w Z_W
\end{bmatrix} = \begin{bmatrix}
\alpha_{PU} & s_P & u_{P0} \\
0 & \alpha_{PV} & v_{P0} \\
1 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
^w X_W \\
^w Y_W \\
^w Z_W
\end{bmatrix} = \begin{bmatrix}
\alpha_{PU} & s_P & u_{P0} \\
0 & \alpha_{PV} & v_{P0} \\
1 & 0 & 0 & 1
\end{bmatrix}
\]

Where \(k\) is nonzero coefficient; \(A_P\) is intrinsic parameter matrix; \(a_{PU} = f_P/dx\) and \(a_{PV} = f_P/dy\) is the scale factor on level and vertical direction of coding plane; \(f_P\) is the effective local length; \(dx\) and \(dy\) is the interval between adjacent points on the \(x\) and \(y\) direction; \(S_P\) is the leaning scale factor; \((U_{P0}, V_{P0})\) are principle points that define the projection of the focal point in the coding plane; \(^3R_W\) and \(^3T_W\) express the orientation and position of the world coordinates system \(\{W\}\) with respect to the axis of the projector coordinates system \(\{P\}\).

4.2. Experiment for projector calibration

Projector calibration is on the basis of the camera calibration, as shown in figure 3, in which the designed multicolor code of pseudo-random array project to 3D scene via projector, and then acquired by CCD camera. The camera acquires three images of the target at three different positions along with \(Z\) axis of the stage. The 3D interest point coordinates which multicolor code of pseudo-random array projects to the target are gotten with the corrected CCD image. Taking the points in world coordinates and in coding plane into the equation (7), the parameters of projector have been obtained as follows:

\[
A_P = \begin{bmatrix}
2190.4 & 8.7 & 239.3 \\
0 & 2026 & 368.5 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
^P R_W = \begin{bmatrix}
0.994 & -0.0282 & 0.0217 \\
0.0303 & 0.9943 & -0.1023 \\
-0.0187 & 0.1029 & 0.9945
\end{bmatrix}
\]

\[
^P T_W = \begin{bmatrix}
-88.6 \\
-182.1 \\
1071.6
\end{bmatrix}
\]

Compared with the correction of camera lens distortion, the project’s is more convenient. Since the position of interest points on multicolor code of pseudo-random array are fixed, only the distortion of the interest points need to be taken into account. According to the difference between the original points on coding plane and the points gained via ideal model, the program of multicolor code of pseudo-random array should be modified. The position of the interest points has been moved in the
opposite direction of the distortion trend in order to reduce or even eliminate the influence of lens distortion of DLP projector, as shown in Figure 5.

![a. the version before compensation](image1) ![b. the version after compensation](image2)

**Figure 5.** Compensation for lens distortion.

5. Conclusion

In this study, the interest points of the designed target and multicolor code of pseudo-random array can be identified and matched automatically. The parameter calibration and lens distortion correction of CCD camera and DLP projector is separated with the points in center and all the rest points assuming little distortion the center of lens, and equation solving and optimization is carried out in Matlab. In this way, the influence of lens distortion will be decreased, and it will be applicable in computer vision. This allows for a major computational advantage with respect to the techniques that estimate these parameters numerically. The high accuracy of our feature extractor and the ease of implementation of our multi-view rectification algorithm make up a powerful tool for the complete, reliable, and high precision calibration of digital cameras affected by any kind of lens distortion, making it very attractive for a wide range of applications both in computer vision and in photogrammetry.

References

[1] J. Salvi, J. Battle and E. Mouaddib 1998 A robust-coded pattern projection for dynamic 3D scene measurement *Int. J.Pattern Recognition Lett.* **19** 1055-65

[2] R.Y. Tsai 1987 A versatile camera calibration technique for high-accuracy 3D machine vision metrology using off-the shelf TV cameras and lenses *IEEE Int. J. Robot. Automat.* **RA-3** 323-344

[3] Weng, P. Cohen, M. Herniou 1992 Camera Calibration with Distortion Models and Accuracy Evaluation *IEEE Trans. Pattern Anal Mach Intell.* **14** 965-980

[4] He Junji, Zhang Guangjun and Yang Xianmin 2004 Approach for Calibration of Lens Distortion Based on Cross Ratio Invariability *Chinese Journal of Scientific Instrument* **25** 577-599

[5] ZHANG Yan-zhen, OU Zong-ying and XUE Bin-dan 2002 Error Correction Method Based on Slope for Camera Radial Distortion *Mini-Micro Systems* **23** 625-627

[6] QIU Zhiqiang, LU Hongwei and YU Qifeng 2003 A Correction Method of Fish eye Lens Distortion Using Projective Invariability *Journal of Applied Optics* **24** 36-38

[7] Joaquim Salvi, Xavier Armangue and Joan Battle 2002 A comparative review of camera calibrating methods with accuracy evaluation *Pattern Recognition* **35** 1617-35

[8] MAO Jianfei, ZOUXiyong and ZHUJing 2004 Improved Two stages Camera Calibration from Plane *Journal of Image and Graphics* **9** 846-852

[9] Luca Lucchese 2005 Geometric calibration of digital cameras through multi-view rectification. *Image and Vision Computing* **23** 517-539