Transport through capacitively coupled embedded and T-shape quantum dots in the Kondo range

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Strong electron correlations and interference effects are discussed in capacitively coupled side attached and embedded quantum dots. The finite - $\mathcal{U}$ mean field slave boson approach is used to study many-body effects. In the linear range the many-body resonances exhibit SU(4) Kondo or Kondo-Fano like character and their properties in the corresponding arms are close to the properties of embedded or T- shape double dot systems respectively. Breaking of the spin symmetry in one of the arms or in both allows for the formation of many-body resonances of SU(3) or SU(2) symmetries in the linear range.

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I. INTRODUCTION

The advances in nanofabrication techniques opened a new path in studying correlation effects. Recently there is an increasing interest in the interplay of strong correlations and interference in multiply connected geometries [1-8], where due to tunability of couplings one can test different transport regimes. In this paper we study two capacitively coupled quantum dots differently coupled to the leads. One of the dots is tunnel coupled to a pair of electrodes (embedded dot - ED) and the second (TD) is side attached to the wire via the open dot (EDTD geometry - inset of Fig. 1). Gate voltage applied to an open dot allows a control of interference conditions. We compare the conductance of strongly coupled EDTD system with conductances of analogous symmetric systems with one type of coupling: TD, where both dots are connected in T geometry or EDED, where dots are directly connected to the leads (the schematic drawings of these systems are shown in Fig. 2a). For the weak symmetric couplings and equal values of intra and interdot Coulomb interactions SU(4) Kondo resonance is formed in EDED at low temperatures [9-11] and Kondo-Fano SU(4) resonance in TDTD [12]. The latter phenomena is the combined effect of interference and cotunneling induced fluctuations. The purpose of the present paper is to analyze the similar many-body effects in the hybrid structure EDTD. When occupancies of both interacting dots are equal cotunneling processes in the both arms together with interference occurring in this case only in one of them leads to a formation of many-body state, which in the upper embedded arm resembles Kondo resonance, while in the lower T- shape arm resembles Kondo-Fano resonance. Due to the asymmetry of the lower and upper arms strictly SU(4) resonance cannot be formed and the many body state is characterized by two characteristic temperatures, separately specifying the related processes in different arms. Interestingly for deep dot levels, the linear conductances of the arms are equal to the conductances of the corresponding symmetric systems and therefore we call this behavior zero bias SU(4) Kondo - Kondo-Fano effect. It can be considered that in the linear regime capacitively coupled EDTD system in the range of equal occupancies simultaneously demonstrates the behavior of both SU(4) TDTD and SU(4) EDED systems. For the deep dot levels and in the case of symmetric Kondo-Fano resonances also finite frequency transmissions of the interacting dots in both arms become also almost identical and then the resonance is in good approximation characterized by only one characteristic temperature. Our present discussion applied to this simplest arrangement of the dots with different types of links with electrodes is firstly aimed at the problem to what extent do subsets maintain the features of their symmetrical counterparts i.e. the systems with identical connections to each of the dots, and secondly highlights the new possibilities opened up by the diversity of the connections. We also extend the discussion by the analysis of the impact of spin polarization of electrodes. Apart from paying attention to interesting spin filtering properties of EDTD we also show that a connection of one of the dots to the fully polarized electrodes allows realization of SU(3) symmetry in EDTD system and in the case when both arms are polarized a crossover from SU(4) to SU(2) zero bias Kondo - Kondo-Fano effect results.

II. MODEL AND FORMALISM

EDTD system is modeled by an extended two-site Anderson Hamiltonian $\mathcal{H}^{EDTD}$:

$$\mathcal{H}^{EDTD} = \sum_{kappa, lambda} E_{kappa lambda} n_{kappa lambda} + \sum_{i, lambda} E_d n_{i lambda} + \sum_{sigma} E_0 n_{0 sigma} + \sum_{kappa, lambda} [V_{2, kappa lambda} d_{2 lambda} + V_{1, kappa lambda} d_{0 lambda} + t d_{0 lambda} d_{1 lambda} + \text{h.c.}]$$

$$+ \mathcal{U} \sum_i n_{i lambda} n_{i lambda'} + \mathcal{U} \sum_{sigma, sigma'} n_{1 lambda} n_{1 lambda'}$$

(1)

The first term describes electrons in the electrodes ($i = 1, 2$, $alpha = L, R$), the second one - interacting dots with gate dependent energies, the third represents open dot,
V₁ and V₂ parameterize electrode-dot couplings for embedded dot and open dot respectively, t characterizes tunneling between open and interacting dots and the last two terms account for intra (U) and interdot (U') Coulomb interactions. The analogous, but symmetrical circuits to which we refer in our further discussion are represented by similar Hamiltonians (HEDED and HEEDTD), the only difference is that for symmetric systems the connections to both dots are identical. In the case of EDED V₂ parameterizes couplings to the upper and lower dots and for TDTD both dots are coupled to the leads indirectly via the open dots and V₁ describes coupling to the leads and t is hopping parameter between open and interacting dot. For brevity we do not give the forms of HEDED and HEEDTD. To discuss correlation effects, we use finite U slave boson mean field approach (SBMFA) of Kotlar and Rubenstein (K-R) [13] and introduce a set of boson operators, which act as projectors onto empty state e, single occupied state p₁σ, doubly occupied d, triple occupied t₁σ and fully occupied f. The single occupation projectors p₁σ are labeled by dot and spin indices, the operators of triple occupancy are also labeled by only a pair of indices and they indicate the unoccupied state (occupation of a hole), and six parts of interacting dots Green’s functions. For polarized electrodes the coupling strengths between the QDs and the leads are spin dependent due to the spin dependence of the density of states. One can express coupling strengths for the spin-majority (spin-minority) electron bands introducing polarization parameter P as Γ₁σ = Γ₁(1+P). The polarization of conductance is given by PC = (Γ₁ - Γ₂)/Γ₁. The numerical results discussed below are presented with the use of energy unit defined by its relation to the bandwidth (2D = 100).

### III. RESULTS AND DISCUSSION

Let us first discuss the case of infinite Coulomb interactions (U, U' → ∞). Fig. 1a is a map of the difference between the occupations of the lower and upper interacting dots \( \Delta N(t, E_d) \) (\( \Delta N = N_1 - N_2 \)) presented for the weak coupling to the leads drawn for symmetric transmission line i.e. Fano interference parameter q = E₀/Γ₁ = 0. For \( q = 0 \) conductance is solely determined by imaginary parts of interacting dots Green’s functions. For \( t = 0 \) QD1 is decoupled (\( N_1 = 0, \Delta N = -1 \)) and linear conductance through the lower wire is unperturbed by the presence of the dots and reaches the limit of \( 2e²/h \). Occupation of QD2 in this case is \( N_2 = 1 \) and for deep dot level spin SU(2) Kondo effect occurs in the upper arm with unitary conductance (Kondo type transmission of QD2 - left inset of Fig. 1b). Gradual increase of t turns on interdot charge fluctuations. Already at very small values of t they are revealed in corresponding finite energy transmissions by the occurrence of delta-like structure (QD1) or a dip (OQD1) (left inset of Fig. 1b).
FIG. 1: (Color online) Capacitively coupled side attached and embedded dots (EDTD) in the infinite Coulomb interaction limit ($U, U' \rightarrow +\infty$). 

a) Difference of the occupations of the dots $\Delta N = N_1 - N_2$ as a function of dot energies and interdot coupling $t$ in the T shape arm. Inset presents schematic drawing of EDTD. 

b) Transmissions of the dots in the case of the occurrence of linear Kondo-Kondo-Fano SU(4) effect ($N_1 = N_2 \sim 1/2$, $E_d = -1.5$, $t = 0.25$). Left inset: transmissions for the case of weakly perturbed SU(2) Kondo effect (QD2) occurring for weak OQD1-QD1 coupling $t (t = 0.02, E_d = -1)$, right inset: transmissions for weakly perturbed SU(2) Kondo-Fano effect (QD1) occurring in the strong OQD1-QD1 coupling limit ($t = 0.9$, $E_d = -4$). 

c) Comparison of EDTD characteristic resonance temperatures $T_{K1}$ of embedded dot (dashed red) and $T_{K2}$ of the dot in T-shape (dashed blue) with SU(4) Kondo temperature of EDED (red line) and SU(4) Kondo-Fano of TDTD (solid blue). Inset shows the relative difference of characteristic temperatures in EDTD system $\delta T_K = (T_{K1} - T_{K2})/(T_{K1} + T_{K2})$. 

d) Temperature dependencies of total and partial conductances for $q = 0$ ($E_d = -1$, $t = 0.31$). 

e) Temperature dependencies of total and partial conductances for $q = -12$ ($E_d = -1$, $t = 1.52$).

FIG. 2: (Color online) a) Effect of interference on Kondo temperature: EDED (dotted red line), TDTD (dotted blue), and characteristic temperatures of EDTD $T_{K1}$ (solid blue) and $T_{K2}$ (solid red) for $E_d = -2$ and infinite Coulomb interaction. 

b) Relative difference of resonance temperatures of EDTD - $\delta T_K$ and relative differences of characteristic temperatures of resonance on embedded dot in EDTD and Kondo temperature of EDED $\delta T_{K1}$ (blue) and similarly $\delta T_{K2}$ - difference for dot in the T- shape arm of EDTD and Kondo-Fano temperature of TDTD (red) plotted as a function of Fano parameter $q$ ($E_d = -2$ solid lines, $E_d = -3$ dotted lines). 

c,d) Densities of states at QD1 in EDTD (red line) compared with DOS of TDTD (blue dashed) for $q = \pm 12$. Insets present similar comparison for QD2 (red) with DOS of EDED.

For large values of $t$ the dots change the roles, effective coupling to the leads of QD1 overcomes in this case the coupling of QD2 and Kondo SU(2) Kondo like peak is observed at QD1 and consequently also a dip structure at the open dot (SU(2) Kondo-Fano effect in the lower T-shape branch) (right inset of Fig. 1b). Linear transport through QD2 is strongly suppressed in this case, but a narrow transmission peak is visible for finite energies. In the range of intermediate values of $t$ increases the role of many-body processes which involve both spin and interdot charge fluctuations (fluctuations of charge isospin). Along the line $N_1 = N_2 = 1/2$ shown on the map (Fig. 1a) Kondo - Kondo-Fano like resonance is formed. The phase shifts are $\delta = \pi/4$ and transmissions of the interacting dots are correspondingly shifted from the Fermi energy (half transmission). Similarly transmission of the open dot, which exhibits a deep is also shifted from $E_F$ (half transmission-half reflection Fig. 1b). These shifts reflect the fact, that together with the dot - electrode hoppings also interdot fluctuations participate on equal foot in formation of these many-body resonances. Cotunneling processes in the lower arm
are directly disturbed by interference (spin and charge isospin Kondo-Fano like resonance), whereas the upper dot experiences interference only indirectly via Coulomb interaction (weakly perturbed Kondo resonance). Since the linear conductances in both arms for the deep dot levels are equal $G_1 = G_2 = e^2/h$ we call this resonance linear Kondo - Kondo-Fano resonance. The difference in cotunneling processes occurring in the upper and lower branches reflects in the difference of densities of states at the interacting dots or in the difference of finite energy transmissions (Fig. 1c). We again focus on the examination of the interference conditions modifies the many-body physics of EDTD. We again look how the change of interference conditions modifies the many-body physics of EDTD. We again focus on the examination of the region of equal occupancies of interacting dots $N_1 = N_2$. For the deep dot levels $N_1 + N_2 = 1$, for shallow levels $N$ is slightly smaller due to the increasing role of fluctuations into the state with unoccupied sites ($e \neq 0$). For finite $q$ linear conductances in different arms differ, they again get the same values in the limit $q \rightarrow \pm \infty$ (Fig. 3a, linear SU(4) Kondo effect). It is no surprise since in this limit linear conductance corresponds to the conductance of the embedded dot $16$. Fig. 1e presents the temperature dependence of conductances for $q = -12$. As opposed to the temperature dependence shown in Figure 1d (linear Kondo - Kondo-Fano effect), for large values of $q$ Kondo like character of both resonances characterized by different temperatures manifests in the two step increase of total conductance with lowering the temperature. For $q = -1$ destructive interference leads to a complete reflection and for $q = 1$ constructive interference results in the full transmission. For the deep dot levels ($E_d \rightarrow -\infty$) conductances of T-shape arm of EDTD and conductance of one of the arms in TDDTD system converge to the same value and similarly conductances of QD2 in EDTD and EDED systems (see the crossings of transmissions at $E = 0$, Figs. 2c,d). This fact is a consequence of staying on the $N_1 = N_2$ line (for $E_d \rightarrow -\infty$, $N_1 = N_2 = 1/2$) and the property that that linear conductances are specified solely by $q$ and occupations (see Appendix B). For finite frequencies however, the differences between resonances in EDTD and the resonances in systems with the single type of coupling increase with the increase of $|q|$ what is illustrated on Figs. 2a,b by presentation of characteristic temperatures and on Fig. 2c,d by showing corresponding densities of states. The corresponding relative temperature differences $\delta T_{K1} = (T_{K1}^{EDTD} - T_{K1}^{TDDTD})/(T_{K1}^{EDTD} + T_{K1}^{TDDTD})$ and $\delta T_{K2} = (T_{K2}^{EDTD} - T_{K2}^{EDED})/(T_{K2}^{EDTD} + T_{K2}^{EDED})$ are asymmetric with respect to $q = 0$ (see also resonance densities of states for $q = \pm 12$ on Figs. 2c,d). Around $q = 0$ characteristic temperatures for QD1 and QD2 are close to each other $T_{K1} > T_{K2}$, for $|q| > q_C = 2.5$ this
relation reverses. For finite $q$ different finite temperature or finite bias characteristics are expected for the dots placed in mixed system and in the systems with one type of coupling with the leads. Apart from EDED system, which from obvious reason is independent, characteristic temperatures for other systems decrease by changing $q$ from negative to positive values (Figs. 2a,b). The $q$-dependence is the result of interference occurring in the T-shape arm of EDTD system. Interference influences directly the many-body resonances of the dots in the arm where it occurs, but it also has an indirect impact on the dot in the second arm via interdot Coulomb interaction. QD2 is subjected only to indirect interference processes, QD1 to both direct and indirect, but the latter is weak because it is only response to indirect effect on QD2. In the case of TDTD both interacting dots are influenced by both direct and indirect interference effects and as it is seen from Fig. 2a, the effect of the change of interference conditions is stronger in this case than for TDED. In Appendix A we give the analytic expressions for characteristic temperatures derived from SBMFA minimization equations. Resonance temperatures are determined by effective renormalized dot level and effective broadening. For TDTD only the former is $q$-dependent, for EDTD both. So far we have discussed the case of equal occupancies in all channels, which meant that all spin and orbital fluctuations have been involved in the many-body processes. Breaking spin symmetry opens the possibility of forming resonances of lower symmetries SU(3) or SU(2). Fig. 3 compares conductances of EDTD with non-magnetic electrodes (Fig. 3a) with the cases when completely spin-polarized electrodes are attached either to the T-shape arm (Fig. 3b) or to the arm with embedded dot (Fig. 3c) or to both arms (Fig. 3d). As described earlier for the nonmagnetic case conductance of the upper arm is $q$ - independent, conductance of T-shape arm depends on $q$ and both conductances take the same value $G_1 = G_2 = e^2/h$ for $q = 0$ or for $q \to \pm \infty$, where linear Kondo - Kondo-Fano or Kondo - Kondo resonances are created. Fig. 3b corresponds to the occupation line $N_{1\uparrow} = N_{2\sigma}$ where for $q = 0$ linear SU(3) Kondo-Fano effect is observed with $G_1 = 1/4(e^2/h)$ and $G_{2\sigma} = 3/4(e^2/h)$ for the deep dot levels (see also transmissions in the inset). For $q \to \pm \infty$ linear SU(3) Kondo effect occurs with conductances $G_1 = G_2 = 3/4(e^2/h)$. We do not present transmissions in this limit, on both interacting and open dots they have the peak structure, similarly to the case presented on Fig. 2b. For the case presented on Fig. 3c ($N_{2\uparrow} = N_{1\sigma}$) conductance of the upper dot is fully spin polarized $G_{2\uparrow} = 3/4(e^2/h)$ and $G_{1\sigma}$ take the values $1/4(e^2/h)$ for $q = 0$ (linear SU(3) Kondo - Fano effect) and $3/4(e^2/h)$ for $q \to \pm \infty$ (linear SU(3) Kondo effect). Fig. 3d illustrates reaching of SU(2) resonances for $q = 0$ and $q \to \pm \infty$ in the case of fully polarized electrodes attached to both dots. The plots correspond to the line $N_{1\uparrow} = N_{2\uparrow}$. Again for $q = 0$ Kondo-Fano resonance with characteristic dip in transmission of OQD1 at the Fermi level is observed and orbital (charge) SU(2) Kondo for $q \to \pm \infty$. Fig. 4 presents one of the possible spintronic application of EDTD, here we show an example of spin filtering for the system with spin polarized electrodes attached to the lower arm. As it is seen polarization of conductance of the lower arm (T-shape geometry) can be switched from negative onto positive by the change of $q$. For the deep dot level almost the same $q$-dependence of polarization is observed in TDTD system, what convinces us that also in the region perturbed by polarization the dots in EDTD partially take over the functions of the dots from the symmetric systems (in the case presented function of TDTD). Fig. 5 shows results for finite $U$. In this case double occupation of a given dot is also allowed. The gate dependence of linear conductance through OQD1 for different values of $q$ is drawn on Fig. 5a. For the case $q = 0$ plateaus in $N = 1$ and $N = 3$ range reaching conductance values ($G_1 \approx e^2/h$) reflect the occurrence of linear SU(4) Kondo - Kondo-Fano effect appearing from cotunneling induced fluctuations between four single-electron states ($N = 1$) or single-hole states ($N = 3$). The corresponding Kondo peaks at the interacting dots are shifted from the Fermi level in this case and half-reflection occurs at OQD1 and QD2 ($\pi/4$ phase shift, see also the plots of conductances in the inset of Fig. 5b and transmissions in the inset of Fig. 5c). For $N = 2$ range six two electron states are involved in cotunneling processes and in this case Kondo resonances are centered at $E_F$, what results in total suppression of conductance through the open dot (Kondo antiresonance at OQD1, transmission dip at $E_F$). For $q \neq 0$ the asymmetric Fano modifications of the resonance peaks introduce the e-h asymmetry (asymmetry vs. $E_d = U/2 - \langle U' \rangle$). Changing the sign of $q$ results in reversing of the conductance and $N = 1$ and $N = 3$ regions change the role (compare conductances...
for \( q = 4 \) and \( q = -4 \) on Fig. 5a). Fig. 5e shows \( \mathcal{G}_1(q) \) for \( N = 1, 2 \) and 3 respectively, single electron and single hole curves are the mirror reflections with respect to \( q = 0 \). In the limit of large values of \( q \) linear conductance through open dot resembles the conductances of interacting dots (Fig. 5b), but the pictures of the corresponding transmissions, which give account also of finite energy processes convince us about different roles played by open dot and interacting dot QD1 (examples of transmissions for \( N = 3 \) are presented on Fig. 5c). Around the Fermi level the corresponding transmissions overlap, but they differ moving away from \( E_F \). Transmission through the embedded dot QED2 at \( E_F \) is equal to the rest two transmissions, but the corresponding line is broader and further shifted towards negative energies than transmis-

sions of the lower arm \( (T_{K2} > T_{K1}) \). Fig. 5d illustrates evolution of conductance with the increase of interdot interaction for \( q = 0 \) and \( E_d = -2 \) (single electron range). Starting from \( U' = 0 \) case, the upper and lower subsystems are decoupled, the upper is in the SU(2) Kondo state with conductance \( \mathcal{G}_2 = 1(e^2/h) \) \((N_2 = 1)\) and the conductance of the lower wire reflects Kondo-Fano resonance with the dip of transmission at \( E_F \) of the open dot at \( E_F \), \( (\mathcal{G}_1 = 0, N_1 = 1) \). Increase of interdot interaction results in an inclusion of the effective interdot charge fluctuations into the many-body processes. For \( U = U' \) these charge isospin fluctuations participate together with the spin and mixed spin-interdot charge fluctuations, all perturbed by interference effects contribute to the formation of SU(4) Kondo - Kondo-Fano like resonance.

Summarizing the main objective of the present work was to analyze transport properties of the simplest system of the dots with different types of the links with the electrodes - EDTD in order to examine to what extent the properties of homogenous symmetric sets of embedded or T-shape dots are conserved in the system with mixed links. It is shown that in the linear range for equal occupancies of the dots the corresponding subsets, embedded or side attached dots perform the similar role as in the respective homogenous systems. We also point on spintronic applications of EDTD presenting as an example its spin filtering properties. In the case of fully polarized electrodes connected to both of the dots charge SU(2) Kondo state is formed and when fully polarized electrodes are attached to only one of the dots more peculiar, spin polarized Kondo or Kondo-Fano states of SU(3) symmetry are formed. This property with a similar assumption of equal numbers of electrons on the dots will also apply to other capacitively coupled systems with mixed types of links with electrodes and different number of dots.

**Appendix A: SBMFA SU(4) Kondo temperatures for the systems of capacitively coupled dots**

Here we give the formulas for SU(4) Kondo temperatures for EDED and TDTD systems and characteristic resonance temperatures for EDTD obtained within SBMFA formalism in the \( U \rightarrow \infty \) limit. The equation of minimization of energy with respect to slave boson operator \( p \left( p_{1\sigma} = p_{2\sigma} = p \right) \) for EDED takes the form:

\[
\frac{\partial H_\text{EDED}}{\partial p} = \sum_{k\alpha i\sigma} \left( \frac{1}{p} \frac{\partial^2}{\partial p} - \frac{1}{e^2} \frac{\partial}{\partial e} \right) (V/z) n_{k\alpha i\sigma,\sigma} - \lambda = 0,
\]

where correlation function of conduction electrons with electrons of the dot can be expressed through the lesser Green’s functions \( \left( \sum_{k\alpha} (V/z) n_{k\alpha i\sigma,\sigma} = \sum_{k\alpha} \int \frac{(V/z G^<_{k\alpha i\sigma,\sigma} dE)}{2\pi} \right) \), which finally takes the form

\[
\sum_{k\alpha} (V/z) G^<_{k\alpha i\sigma,\sigma} = \frac{(f_L + f_p)(E - E_d)f_q}{(E - E_d)^2 + f_q^2},
\]

where \( E_d = E_d + \)
λ is the renormalized energy and \( \bar{\Gamma} \) is renormalized coupling strength. Kondo temperature is defined by these quantities: \( T_K^{EDTD} = \sqrt{\bar{E}_d^2 + \bar{\Gamma}_2^2} \approx 2\bar{\Gamma} \). Putting it into (A.1) one gets the equation for \( T_K^{EDTD} \):

\[
\frac{\partial H^{EDTD}}{\partial p} = \frac{1 - 8p^2}{p^2 - p^2} \left( \frac{\Gamma_2}{\pi} \right) \ln \left( \frac{D}{T_K^{EDTD}} \right)
\]

\[
\lambda = 0,
\]

(A2)

Taking into account that for the deep dot levels \( p \rightarrow 1/2 \) for SU(4) symmetry and \( \lambda \rightarrow |E_d + \bar{\Gamma}| \), the below exponential dependence of Kondo temperature on the system parameters is obtained \( T_K^{EDTD} = D e^{-(\bar{E}_d + 2\bar{\Gamma}_2)}/\pi \).

Evaluation of Kondo temperature for TDTD systems proceeds in a similar way and minimization equation with respect to \( p \) reads:

\[
\frac{\partial H^{TDTD}}{\partial p} = \sum_{i\sigma} \left( \frac{1}{p^2} \frac{\partial z}{\partial p} - \frac{1}{e^1} \frac{\partial E}{\partial e} \right) (t/z)n_{0\sigma,i\sigma} - \lambda = 0
\]

(A3)

where the electron open dot - interacting dot correlation function replaces here direct correlation function of the leads and the dot in (A.1), \( (t/z)n_{0\sigma,i\sigma} = \int t(z)G(z,t)e^{iEz}G(z,E)\partial E \) Green’s functions \( (t/z)G(z,t) = \frac{\tau q(E - Ed)\Gamma}{(t^2 + (E - Ed)(E - Ed) + (E - Ed)\Gamma_q)^2} \) depend not only on \( E_d \), but also on \( q \) (on \( E_d \)). Equation (A.3) takes the form:

\[
\frac{\partial H^{TDTD}}{\partial p} = \frac{1 - 8p^2}{p^2 - p^2} \left[ \frac{\Gamma_1}{2} \ln \left( \frac{E_0 + \Gamma_1^2}{2(E_0 + \Gamma_1^2)} \right) \right] - \lambda = 0
\]

(A4)

with the solution \( T_K^{TDTD} = D e^{2\pi}|E_d + 2\bar{\Gamma}_2|/\pi \). \( \delta E_q \) denotes correlation and interference induced \( q - \) dependent energy corrections to the effective dot level \( \delta E_q = \frac{4\pi}{3} \frac{\Gamma_1}{E_0 + \Gamma_1^2} \left[ \frac{\Gamma_1}{2(E_0 + \Gamma_1^2)} \right] + \frac{8\pi}{3} \frac{E_0 \arctan \left( \frac{E_0}{E_0 + \Gamma_1} \right)}{E_0 + \Gamma_1^2} \) and \( \Gamma_q \) is the effective indirect hybridization \( \Gamma_q = \frac{\tau q(E - Ed)^2}{2(E_0 + \Gamma_1^2)} \). For EDTD system one has to consider minimization equations with respect to \( p_{i\sigma} \) (\( i = 1 \) for the lower interacting dot and \( i = 2 \) for the upper). The equations read:

\[
\frac{\partial H^{EDTD}}{\partial p_1} = \sum_{i\sigma} \left( \frac{1}{p_1} \frac{\partial z_1}{\partial p_1} - \frac{1}{e^1} \frac{\partial E}{\partial e} \right) (t/z)n_{0\sigma,i\sigma} + \sum_{k\sigma} \left( - \frac{1}{e^1} \frac{\partial z_2}{\partial e} \right) (V_2/z_2)n_{k\sigma,2\sigma} - \lambda_1 = 0
\]

(A5)

and,

\[
\frac{\partial H^{EDTD}}{\partial p_2} = \sum_{i\sigma} \left( - \frac{1}{e^1} \frac{\partial z_1}{\partial e} \right) (t/z)n_{0\sigma,i\sigma} + \sum_{k\sigma} \left( \frac{1}{p_1} \frac{\partial z_2}{\partial p_2} - \frac{1}{e^1} \frac{\partial E}{\partial e} \right) (V_2/z_2)n_{k\sigma,2\sigma} - \lambda_2 = 0
\]

(A6)

By a similar derivation as above one gets two characteristic temperatures for EDTD system \( T_K^{EDTD} = D e^{-(\bar{E}_d + 2\bar{\Gamma}_2)/\pi} \) and \( T_K^{EDTD} = D e^{-(\bar{E}_d + 2\bar{\Gamma}_2)/\pi} \).

Appendix B: Generalized Friedel sum rules

Linear conductance expresses through the imaginary part of the Green’s function at the Fermi energy \( (E_F = 0) \). For EEDT system the Green’s functions at both dots are identical and they read:

\[
\frac{G_{2\sigma}}{e^2/h} = -\Gamma_2 z_{2\sigma} \Im \left[ \frac{1}{E_0 - \lambda_2 + i\Gamma_2 z_{2\sigma}^2} \right] = \sin^2(\pi N_{2\sigma}) = \sin^2(\delta_{2\sigma}),
\]

(B1)

where we have used the expression \( \lambda_{2\sigma} = -E_d - \Gamma_2 z_{2\sigma}^2 \tan(\pi N_{2\sigma} - \frac{\pi}{2}) \) and \( N_{2\sigma} \) is the occupation number of QD2. The phase shifts are expressed solely by occupations \( (\delta_{2\sigma} = \pi N_{2\sigma}) \). For TDTD system:

\[
\frac{G_{1\sigma}}{e^2/h} = -\Gamma_1 \Im \left[ \frac{1}{-E_0 + i\Gamma_1 \left( 1 + \frac{t^2 z_{1\sigma}^2}{E_0 + i\Gamma_1} \right) - \frac{1}{E_0 + i\Gamma_1}} \right] = \left( \Gamma_1 \cos(\pi N_{1\sigma}) + E_0 \sin(\pi N_{1\sigma})^2 \right)^2, \]

(B2)

where \( \lambda_{1\sigma} = -E_d (E_0^2 + \Gamma_1^2) + E_0 - t^2 z_{1\sigma}^2 \Gamma_1 \tan(\pi N_{1\sigma} - \frac{\pi}{2}). \)

Conductance is expressed by occupations and Fano parameter \( q (E_0) \). One can easily check that the Green’s functions of the dots in EDTD system have exactly the same form as the functions of the corresponding dots in homogenous systems, (B1) for the embedded dot QD2 and (B2) for the QD1 in the T-shape arm, in general however they are specified by different occupancies. For the special case of the assumed equal occupancies of the dots in hybrid EDTD system and in homogenous TDTD or EDTD discussed by us, a conclusion on the equality of linear conductances can be found.
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