Strategies that utilise the interference effects within $B \to DK$ decays hold great potential for improving our sensitivity to the CKM angle $\gamma$. However, in order to exploit fully this potential, knowledge of parameters associated with the $D$ decay, such as strong-phase differences, are required. This essential information can be obtained from the unique quantum-correlated $\psi(3770)$ datasets at CLEO-c. Results of such analyses involving the decay modes $D \to K\pi, K\pi\pi^0, K\pi\pi\pi$ and $K_S^0\pi\pi$ will be presented.
1. Introduction

A theoretically clean method to extract the CKM-angle $\gamma$ is to exploit the interference present in $B^\pm \to DK^\pm$, where the $D$ is a $D^0$ or $\bar{D}^0$ decaying to a common final state, $f$. Decay rates in these channels are sensitive to the following amplitude ratios

$$\frac{A(B^- \to D^0K^-)}{A(B^- \to D^0K^-)} = r_B e^{i(\delta_B - \gamma)}, \quad \frac{A(B^+ \to D^0K^+)}{A(B^+ \to D^0K^+)} = r_B e^{i(\delta_B + \gamma)}. \quad (1.1)$$

which are functions of three parameters: the ratio of the absolute magnitudes of the amplitudes, $r_B$; a $CP$-invariant strong-phase difference, $\delta_B$; and the weak phase $\gamma$. A variety of $\gamma$ extraction strategies have been suggested depending on the $D$ final state considered. For example, established final states include: two-body modes such as $K^+K^-\pi^+\pi^- [1, 2], K^{\pm}\pi^{\mp}[3]$, as well as multi-body final states such as $K^0\pi^+\pi^- [4, 5]$ and $K^{\pm}\pi^{\mp}\pi^0/K^{\pm}\pi^{\mp}\pi^+\pi^- [6].$ In all cases, the measurement of $\gamma$ is affected by properties of the $D$ decay amplitude. In order to exploit fully the sensitivity to the $B$-specific parameters ($r_B, \delta_B$ and $\gamma$) it is, therefore, highly advantageous to have prior knowledge of the parameters associated with the $D$ decay. This is where CLEO-c plays a crucial role.

These proceedings describe three sets of measurements performed by CLEO-c of $D$-specific parameters relevant to the measurement of $\gamma$. Sec. 2 introduces the $D$ parameters of interest in the context of the $B$ decay rates. Sec. 3 then explains how one can exploit quantum-correlations at the $\psi(3770)$ in order to probe these $D$ parameters. Sec. 4 describes the CLEO-c experiment and data sets used for the analyses. Secs. 5, 6 and 7 describe the experimental procedure and results.

2. $D$ Parameters Associated with the ADS Method

In the case of the so-called ADS method [3], where $f = K^\pm\pi^\mp$, $D$-specific parameters contribute to the suppressed $B^\pm$ decay-rates as follows:

$$\Gamma(B^\pm \to (K^\pm\pi^\mp)_D K^\pm) \propto r_D^{2} + (r_D^{K\pi})^{2} + 2 r_B r_D^{K\pi} \cos(\delta_B + \delta_D^{K\pi} \pm \gamma), \quad (2.1)$$

where $r_D^{K\pi}$ and $\delta_D^{K\pi}$ are analogous to the $B^\pm$ parameters $r_B$ and $\delta_B$; $r_D^{K\pi}$ is the absolute ratio of the doubly Cabibbo suppressed (DCS) to Cabibbo favoured (CF) amplitudes and $\delta_D^{K\pi}$ is the corresponding $D$ strong-phase difference. Furthermore, the extended method [6], which considers multi-body ADS modes i.e. $f = \{K^\pm\pi^\mp\pi^0, \ K^\pm\pi^\mp\pi^+\pi^-\}$, introduces an additional $D$ parameter:

$$\Gamma(B^\pm \to (f)_D K^-) \propto r_D^{2} + (r_D^{f})^{2} + 2 r_B r_D^{f} R_f \cos(\delta_B + \delta_D^{f} \pm \gamma), \quad (2.2)$$

where $R_f$ is the coherence factor, and satisfies the condition $\{R_f \in \mathbb{R} \mid 0 \leq R_f \leq 1\}$. This dilution term results from accounting for the resonant sub-structure of the multi-body mode. For modes whose intermediate resonances interfere constructively, $R_f$ tends to unity, however if the resonances interfere destructively, then $R_f$ tends to zero.

\footnote{For a review of all these methods, and a summary of current and future $B^\pm \to DK^\pm \gamma$ measurements, see Refs. [3] and [6].}
3. Quantum Correlations at the $\psi(3770)$

Determination of strong-phase differences and coherence factors can be made from analysis of quantum-correlated $D^0\bar{D}^0$ pairs. Such an entangled state, with $C = -1$, is produced in $e^+e^-$ collisions at the $\psi(3770)$ resonance. To conserve this charge-conjugation state, the final state of the $D^0\bar{D}^0$ pair must obey certain selection rules. For example, both $D^0$ and $\bar{D}^0$ cannot decay to $CP$-eigenstates with the same eigenvalue. However, decays to $CP$-eigenstates of opposite eigenvalue are enhanced by a factor of two. More generally, final states that are accessible by both $D^0$ and $\bar{D}^0$ (such as $K^-\pi^+$) are subject to similar interference effects. Consequently, by considering time-integrated decay rates of double tag (DT) events, where both the $D^0$ and $\bar{D}^0$ are reconstructed, one is sensitive to interference dependent parameters such as strong-phases and coherence factors. Furthermore, these decay rates are also sensitive to charm mixing. Charm mixing is described by two dimensionless parameters: $x \equiv (M_1 - M_2)/\Gamma$ and $y \equiv (\Gamma_1 - \Gamma_2)/2\Gamma$, where $M_{1,2}$ and $\Gamma_{1,2}$ are the masses and widths, respectively, of the neutral $D$ meson $CP$-eigenstates. The explicit dependence on the mixing parameters can be seen by considering the generalised, time-integrated, DT rate. That is, for a $D^0\bar{D}^0$ pair decaying to the final state $(f, g)$:

$$
\Gamma(f|g) = Q_M |A_f \tilde{A}_g - \tilde{A}_f A_g|^2 + R_M |A_f A_g - \tilde{A}_f \tilde{A}_g|^2,
$$

where $A_i \equiv \langle i|D^0 \rangle$, $\tilde{A}_i \equiv \langle i|\bar{D}^0 \rangle$. The coefficients $Q_M$ and $R_M$ posses the dependence on the mixing parameters, where $Q_M \equiv 1 - (x^2 - y^2)/2$ and $R_M \equiv (x^2 + y^2)/2$.

3.1 Probing strong-phases and coherence factors

Letting $f$ represent the signal $D$ decay of interest, it is possible to obtain access to strong-phases and coherence factors by considering specific states of the ‘tag’, $g$. As an example, we demonstrate here how sensitivity to strong-phases can be obtained by considering $g$ to be in a $CP$-eigenstate with eigenvalue $\lambda_{CP}$. For the purpose of this discussion, we simplify the problem by ignoring $D$-mixing effects, i.e. $x, y \rightarrow 0$. In this scenario, $Q_M \rightarrow 1$, $R_M \rightarrow 0$. Consequently, for $f = K^-\pi^+$, Eqn.(3.1) reduces to:

$$
\Gamma(K^-\pi^+|CP) \propto |A_{K\pi} A_{CP} - \tilde{A}_{K\pi} A_{CP}|^2 = |A_{K\pi}|^2 |A_{CP}|^2 \left(1 + (r_D^{K\pi})^2 - 2\lambda_{CP} r_D^{K\pi} \cos(\delta_{K\pi}^{D})\right). \tag{3.2}
$$

Therefore, with a knowledge of $|A_{K\pi}|$, $|A_{CP}|$ and $r_D^{K\pi}$, the observed asymmetry between the rates for $\lambda_{CP} = +1$ and $\lambda_{CP} = -1$ provides direct sensitivity to $\cos(\delta_{K\pi}^{D})$. When a multi-body signal mode is considered, such as $f = \{K^+\pi^-\pi^0, K^+\pi^- \pi^+ \pi^\mp\}$, the amplitude $A_f$ must be integrated over all phase-space. This has the effect of modifying Eqn. (3.2) through the transformation $\cos(\delta_{K\pi}^{D}) \rightarrow R_f \cos(\delta_{K\pi}^{D})$. Therefore, for $f = K^-\pi^+$:

$$
\Gamma(K^-\pi^+\pi^0|CP) = |A_{K\pi\pi^0}|^2 |A_{CP}|^2 \left(1 + (r_D^{K\pi\pi^0})^2 - 2\lambda_{CP} r_D^{K\pi\pi^0} R_{K\pi\pi^0} \cos(\delta_{K\pi\pi^0}^{D})\right). \tag{3.3}
$$

To give a more concrete overview, expressions from evaluating Eqn. (3.1) are listed in Table I for various tag modes against $f = K^-\pi^+$. As is demonstrated in Ref.[8], while $|A_{K\pi}|^2$ has direct correspondence to the CF branching fraction $(B_{K\pi}^{CF})$, $|\tilde{A}_{K\pi}|^2$ and $|A_{CP}|^2$ possess dependence on the mixing parameters $x$ and $y$, i.e. $|\tilde{A}_{K\pi}|^2 = B_{K\pi}^{DCS}(1 + g(x,y))$. Consequently, a linear dependence on $x$ and $y$ is observed in some of the quantum correlated branching fractions quoted in Table I.
This information is gathered by averaging results of single-tagged yields at the 5. Measurement of the strong-phase difference in described in Ref. [12]. The results of the fit are given in Table 2. The analysis finds a result of are normalised to the multiple of the uncorrelated branching fractions. Some rates show dependence to the wrong-sign rate ratio, the method described in Ref. [11], this analysis has performed the first measurements of the mixing parameters are used as constraints. All correlations amongst the inputs are accounted for.

| Mode                   | Relative Correlated Branching Fraction |
|------------------------|----------------------------------------|
| $K^{-}\pi^+$ vs. $K^{-}\pi^+$ | $R_M$                                   |
| $K^{-}\pi^+$ vs. $K^{+}\pi^-$  | $(1 + R_W)^2 - 4r \cos \delta_D^{K\pi}(r \cos \delta_D^{K\pi} + y)$ |
| $K^{-}\pi^+$ vs. $CP\pm$       | $1 + R_W \pm 2r \cos \delta_D^{K\pi} + y$ |
| $K^{-}\pi^+$ vs. $e^-$         | $1 - r \cos \delta_D^{K\pi} - r x \sin \delta_D^{K\pi}$ |
| $CP\pm$ vs. $CP\pm$           | 0                                      |
| $CP+ vs. CP-$                | 4                                      |
| $CP\pm vs. e^-$              | $1 \pm y$                              |

Table 1: Correlated $(C = -1)$ effective $D^0\bar{D}^0$ branching fractions to leading order in $x$, $y$ and $r^2$. The rates are normalised to the multiple of the uncorrelated branching fractions. Some rates show dependence to the wrong-sign rate ratio, $R_W = r^2 + ry' + R_M$, where $y' = (y \cos \delta_D^{K\pi} - x \sin \delta_D^{K\pi})$.

4. CLEO-c

All measurements presented are made with $e^+e^- \rightarrow \psi(3770)$ data accumulated at the Cornell Electron Storage Ring (CESR). The CLEO-c detector was used to collect these data. Details of the experiment can be found elsewhere [10]. The total integrated luminosity of the data is 818 pb$^{-1}$, however, only 281 pb$^{-1}$ have been used so far for the measurement of $\delta_D^{K\pi}$ presented in Sec. 5.

5. Measurement of the strong-phase difference in $D \rightarrow K^{-}\pi^+$

The first analysis presented is that of the strong-phase difference in $D \rightarrow K^{-}\pi^+$. Implementing the method described in Ref. [11], this analysis has performed the first measurements of $y$ and $\cos(\delta_D^{K\pi})$ in quantum-correlated $\psi(3770)$ data. By comparing the correlated event yields, whose rates are listed in Table 1, with the uncorrelated expectations, we are able to extract $r^2$, $r \cos(\delta_D^{K\pi})$, $y$ and $x^2$. To achieve this, a knowledge of the relevant uncorrelated branching-ratios are needed. This information is gathered by averaging results of single-tagged yields at the $\psi(3770)$ with external measurements using incoherently-produced $D^0$ mesons. In addition, to extract $\cos(\delta_D^{K\pi})$ from $r \cos(\delta_D^{K\pi})$, knowledge of $r$ is required. This necessary information is obtained by including $R_W$ and $R_M$ as external inputs to the least-squares fit. Furthermore, external measurements of the mixing parameters are used as constraints. All correlations amongst the inputs are accounted for.

The analysis has considered a total of seven $CP$-eigenstates reconstructed against the $K^+\pi^+$ signal mode: $K^+ K^-$, $\pi^+\pi^-$, $K^0\bar{\pi}^0$, $K^0\eta$, $K^0\bar{\pi}^0\pi^0$, $K^0\bar{\pi}^0\eta$ and $K^0\bar{\pi}^0\pi^0$. In those DTs without a $K^0\bar{\pi}^0$, the signal is identified using two kinematic variables: the beam-constrained mass, $M \equiv \sqrt{E^2_{\text{Beam}} - p_D^2}$, and $\Delta E \equiv E_D - E_{\text{Beam}}$, where $E_{\text{Beam}}$ is the beam energy, $p_D$ and $E_D$ are the $D^0$ candidate momentum and energy, respectively. The reconstruction of $K^0\bar{\pi}^0$ events utilises the missing-mass technique described in Ref. [12]. The results of the fit are given in Table 2. The analysis finds a result of $\delta_D^{K\pi} = (22^{+11}_{-12} \pm 9)^o$, which is the first direct determination of this phase [13]. An updated result following analysis of the full 818 pb$^{-1}$ dataset is in preparation.
6. Measurement of the coherence factor and average strong-phase difference in
\( D \to K^{\pm} \pi^{\mp} \pi^0 \) and \( D \to K^{\pm} \pi^{\mp} \pi^+ \pi^- \)

Determination of the average strong-phase difference and associated coherence factors for the modes \( f = \{K\pi\pi^0, K3\pi\} \) have been made using an analogous technique to that described in Sec. 5 [14]. As shown in Eqn.(5.13), CP-tagged multi-body rates provide sensitivity to the product \( R_f \cos(\delta'_f) \). A means of decoupling these parameters fortunately comes from considering the rate \( \Gamma(f|f) \). Evaluating Eqn.(3.1) for \( g = f \), one obtains:

\[
\Gamma(f|f) = Q_M |A_f|^2 |\bar{A}_f|^2 \left( 1 - (R_f)^2 \right) + |A_f|^4 R_M \left( 1 - 2(R_f)^2 + (R_f)^4 \right).
\]

(6.1)

In the case of the two-body mode, \( f = K^{\pm} \pi^\mp, R_f = 1 \) and Eqn.(6.1) reduces to \( |A_f|^4 R_M \) as quoted in Table 1. However, for multi-body final states, one observes that \( (1 - R_f^2) \) is the leading term in Eqn.(6.1). Consequently, the rate \( \Gamma(f|f) \) provides direct sensitivity to \( R_f \) and allows for a decoupling of the parameters. All the CP-tags listed in Sec.5 are employed in this analysis, as well as \( K_3^0 \phi, K_3^0 \eta' \) and \( K_3^0 \omega \).

As was done in the \( K^{\pm} \pi^\mp \) analysis, a least-squares fit has been used to extract both mixing and strong-phase parameters. Likelihood contours in \( R_f, \delta_f' \) parameter space are shown in Fig. 1(a) for \( f = K\pi\pi^0 \), and Fig. 1(b) for \( f = K3\pi \). The best-fit values of the coherence factors and average strong-phases are \( R_{K\pi\pi^0} = 0.84 \pm 0.07, \delta_{K\pi\pi^0}^{K\pi\pi^0} = (227^{+14}_{-11})^\circ, R_{K3\pi} = 0.33^{+0.20}_{-0.23} \) and \( \delta_{K3\pi}^{K3\pi} = (114^{+28}_{-23})^\circ \). The uncertainties quoted are a combination of statistical and systematic errors.

![Figure 1](image1.png)

**Figure 1:** The limits determined on (a) \( (R_{K\pi\pi^0}, \delta_{K\pi\pi^0}^{K\pi\pi^0}) \) and (b) \( (R_{K3\pi}, \delta_{K3\pi}^{K3\pi}) \) at the 1, 2 and 3σ levels.

The results show significant coherence for \( D^0 \to K\pi\pi^0 \), but much less so for \( D^0 \to K\pi\pi\pi \). These results will improve the measurement of \( \gamma \) and the amplitude ratio \( r_B \) in \( B^\pm \to DK^\pm \), where the \( D \) decays to \( K\pi\pi^0 \) and \( K\pi\pi\pi \). Earlier preliminary results of \( R_{K3\pi} \) and \( \delta_{K3\pi}^{K3\pi} \) [15] combined with CLEO-c’s measurement of \( \delta_{K3\pi}^{K3\pi} \) were shown to improve the expected sensitivity to \( \gamma \) at LHCb in a combined ADS analysis of \( K\pi \) and \( K\pi\pi\pi \) final states by up to 40% [16].

7. Measurement of strong-phase variations in \( D \to K_S^{0} \pi^{\mp} \pi^- \)

The current best constraints on \( \gamma \) come from measurements in \( B^\pm \to D(K_S^{0} \pi^- \pi^-)K^\pm \) and
related modes [17, 18] by performing likelihood fits to the \( K_S^0 \pi^+ \pi^- \) Dalitz plot [4]. These fits require models to represent the \( D^0 \to K_S^0 \pi^+ \pi^- \) resonant amplitude structure. Since these models possess certain assumptions, an inherent systematic uncertainty is associated with this technique. Current estimates predict this error to be between 5° and 9°, meaning the \( \gamma \) measurement would soon become systematically limited at the next generation of flavour-physics experiment. However, an alternative model-independent method has been proposed where events are counted in specified regions of the \( K_S^0 \pi^+ \pi^- \) Dalitz plot [4, 5], thus eliminating the model-uncertainty. This method relies on necessary strong-phase parameters having been determined at CLEO-c.

As Dalitz plot variables we use the invariant-mass squared of the \( K_S^0 \pi^- \) and \( K_S^0 \pi^+ \) pairs, which we label as \( s_- \) and \( s_+ \), respectively. The strong-phase at a given point in the \( K_S^0 \pi^+ \pi^- \) Dalitz plot is then \( \delta_D(s_-, s_+) \). For the phase difference between \( D^0 \to K_S^0 \pi^+ \pi^- \) and \( \bar{D}^0 \to K_S^0 \pi^+ \pi^- \) at the same point in the Dalitz plot, we define

\[
\Delta \delta_D \equiv \delta_D(s_-, s_+) - \delta_D(s_+, s_-).
\]

(7.1)

The quantities measured by CLEO-c that provide input to the model-independent \( \gamma \) determination are the averages of \( \cos(\Delta \delta_D) \) and \( \sin(\Delta \delta_D) \) in the \( i \)th Dalitz plot bin. We denote these terms \( c_i \) and \( s_i \), respectively. In a completely analogous manner to the analyses presented in Secs. 5 and 6, \( c_i \) can be determined from \( CP \)-tagged decay rates, while \( s_i \) is extracted from considering the double Dalitz plot of \( K_S^0 \pi^- \pi^+ \) vs. \( K_S^0 \pi^- \pi^- \). Furthermore, additional constraints on \( c_i \) and \( s_i \) are obtained through \( K_L^0 \pi^+ \pi^- \) events.

The choice of Dalitz plot binning affects the statistical precision of the analysis. It has been demonstrated in Ref. [5] that it is beneficial to choose bins such that \( \Delta \delta_D \) varies as little as possible across each bin. The binning used in this analysis, with eight-pairs of bins uniformly dividing \( \Delta \delta_D \) over the range \( [0, 2\pi] \), is shown in Fig. 2(a). The location of these bins in phase space is evaluated from referring to the BaBar isobar model given in Ref. [19].

\[\text{Figure 2: In (a), the uniform } |\Delta \delta_D| \text{ binning of the } K_S^0 \pi^+ \pi^- \text{ Dalitz plot. In (b), the comparison of the measured } c_i \text{ and } s_i \text{ (circles with error bars) to the predictions from the BaBar isobar model (stars).}\]
The values of $c_i$ and $s_i$ from the combined analysis of $K^0_S\pi^+\pi^-$ and $K^0_L\pi^+\pi^-$ tagged events are shown graphically in Fig [2(b)]. When used as input to the $\gamma$ measurement, these results are expected to replace the current model uncertainty of $5^\circ - 9^\circ$ with an uncertainty due to the statistically dominated error on $c_i$ and $s_i$ of $1.7^\circ$ [20].

8. Conclusion

The importance of CLEO-c’s quantum-correlated $\psi(3770)$ dataset in the context of measuring the CKM angle $\gamma$ has been described. Analysis of a variety of two- and multi-body $D^0$ decays with these data have provided vital measurements of $D^0$ strong-phases, and associated parameters, for model-independent $\gamma$ measurements at LHCb. In addition to the modes presented here, results are in preparation for other promising final states, such as $D^0 \rightarrow K^0_S K^+ K^-$. 

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