Action Principle and Ritz Method for Yarn Dynamics in Ring Spinning

Michael Lenz1,∗ and Michael Beitelschmidt1
1 Institute of Solid Mechanics, Technische Universität Dresden, D-01062 Dresden

Ring spinning is the most relevant method of producing high quality staple yarns. The rotating yarn forms one or multiple balloon shapes governed by inertial, eyelet friction and drag forces. Traditionally, the stationary yarn path has been computed by direct integration of an equation of motion, and stability and natural oscillations have been assessed by application of Galerkin’s method to these ([1], [4], and others). Recent technological advances by using frictionless superconducting magnetic bearings allow for increased process speeds and thus motivate a renewed interest in the dynamics of ring spinning.

An integral formulation of the yarn dynamics problem is presented using Hamilton’s principle for the one-dimensional continuum and the magnetic bearing. While therefrom the known equations of motions can be deduced elegantly, here Ritz’s method is employed to achieve direct discretization of the problem, with cases to be made for both local and global shape functions.

While applicable to the classical study of stationary balloon shapes and their stability, the resulting models in particular enable the study of the instationary yarn path and natural and driven yarn oscillations, which both have received limited attention in literature, but are known to affect yarn tension and thus the process viability.

© 2021 The Authors. Proceedings in Applied Mathematics & Mechanics published by Wiley-VCH GmbH.

1 Problem definition

Spinning is the process of twisting loosely connected filaments, e.g. of cotton fibre, in the roving to produce a yarn of appreciable longitudinal strength and stiffness. In ring spinning, this is achieved by subjecting the material to circular motion about a vertical axis, as outlined in Fig. 1. The motion of the yarn in formation, transported axially at a speed \( v(t) \), is guided through the roving inlet \( O \), intersecting with the axis of rotation, and a rotating eyelet \( E \). For the system being considered here, \( E \) sits on a ring rotating at the angular velocity \( \Omega(t) \), suspended by a frictionless superconducting magnetic bearing (SMB). The movement is driven by the rotating spindle, whose radius increases as yarn is wound onto it. The yarn between the inlet and the guide \( E \), rotating at high speed, takes the characteristic balloon form. In order to achieve a yarn cob of the desired length, the balloon height \( h(t) \) is being changed perpetually, by means of a vertical movement of the frame supporting the SMB. Air resistance effects a drag force \( F_D \) on the yarn, and a drag moment \( M_{DR} \) on the SMB ring.

Fig. 1: SMB ring spinning frame [2].

![Diagram of SMB ring spinning frame](image)

A study of the yarn dynamics is of particular interest for the balloon section between the eyelets \( O \) and \( E \). The section tensile forces in the yarn are \( T_0 \) and \( T_2 \), both acting downstream from the eyelets. The position of a yarn element within is denoted as \( r(S) \), in dependence of the material coordinate. For most applications the assumption of a (quasi-) stationary state is suitable, i.e. of no or slow change of the quantities \( \Omega, h, v, m(S) \), and \( b \). For that case the decomposition \( r = \tilde{r} + u \) into a rotating stationary equilibrium configuration and small oscillatory movements \( u \) is used and the movement is described in dependence of the convective coordinate \( \sigma = S - vt \).

### Table 1: Process parameters.

| Symbol | Description                        |
|--------|-----------------------------------|
| \( a \) | spindle radius incl. cob          |
| \( b(t) \) | radial position of \( E \) on SMB ring |
| \( d_R(\Omega) \) | SMB ring air damping coefficient |
| \( D \) | yarn air drag coefficient         |
| \( J_R \) | SMB ring moment of inertia       |
| \( m(S, t) \) | yarn mass per unit length        |

* Corresponding author: e-mail michael.g.e.lenz@tu-dresden.de, phone +49 351 463 36628, fax +49 351 463 37969

This is an open access article under the terms of the Creative Commons Attribution-NonCommercial License, which permits use, distribution and reproduction in any medium, provided the original work is properly cited and is not used for commercial purposes.
Hamilton’s principle of stationary action,
\[ \int_{t_0}^{t_1} (\delta K - \delta U + \delta W) \, dt = 0, \]  
(1)
is applied, with the kinetic energy \( K \), potential energy \( U \), and virtual work of external forces \( \delta W \). With \( S = 0 \) in \( O \), \( K \) can be written in a generalized form as \( K = \int_{S=0}^{L} k(S,t) \, dS + K_D \), where \( K_D \) captures discrete terms – in this case the SMB ring energy. A particular aspect of the problem is that the balloon length \( L \) is not known beforehand, meaning that
\[ \delta K = \int_{0}^{L} \delta k(S,t) \, dS = \int_{0}^{L} \frac{\partial k}{\partial S} \, dS + k \Big|_{S=L} \delta L + \delta K_D \]  
(2)
An equivalent result is obtained for \( U \). It follows that (1) includes terms proportional to \( \delta L \), in addition to the commonly expected terms proportional to variations of \( r \) and its derivatives. These yield an additional condition for the solution.

2 Ritz method
Exemplarily, an application is shown for the stationary case \( r = \tilde{r} \). The application for natural oscillations including experimental validation is covered in [3]. The general instationary problem can be deduced analogously.

\( \tilde{r} \) is described in cylindrical coordinates \( r, \varphi, z \), for which in turn shape functions are set such that \( r = \sum R_j(\xi)q_j = R^T(\xi)q_r \), and analogously for \( \varphi \) and \( z \), with the normalized coordinate \( \xi = \sigma/L \). The use of a normalized coordinate introduces further \( \delta L \) proportional terms in (1), but is practical for the numerical evaluation of the integrals such as in (2).

After inserting into (1), the result is a system of nonlinear equations of order \( N + 1 \) for the unknowns \( q \) and \( L \), where \( N \) is the number of elements in \( q \).

Fig. 3: Stationary balloon shape at \( h = 180 \) mm, rotating at 9,000 rpm, as calculated with the Ritz method.

A preliminary solution is shown in Fig. 3, and can be compared to a validated result (Fig. 4). The solution for the radius shows a qualitatively reasonable behaviour and also acceptable quantitative accordance. The computed angular position \( \varphi \) is as of yet implausible, and even negative over most of the domain. It is suspected that the chosen global polynomial shape functions are not fully suitable to produce an accurate solution, in particular with respect to the derivative. The near constant positive tensile force requires a near constant positive derivative, which could not yet be reproduced successfully. A possible remedy could be the use of localized shape functions.

The computation of \( L \) by means of the additional equation introduced into (1) yields accurate results.

Acknowledgements This research is funded by the German Research Foundation , DFG (Project Nos. CH 174/33-2 and SCHU 1118/12-2). The authors would like to thank DFG for the financial support. Open access funding enabled and organized by Projekt DEAL.

References
[1] W. B. Fraser, Philos Trans A Math Phys Eng Sci342, 439-468 (1993).
[2] M. Hossain, C. Telke et al., Text Res J 87, 1011-1022 (2017).
[3] M. Lenz, M. Hossain et al., Appl Math Model 88, 518-528 (2020).
[4] F. Zhu, et al., J Appl Mech 64, 676-683 (1997).