Link Scheduling in Amplify-and-Forward Cooperative Wireless Networks

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Abstract—In this letter we are concerned with the problem of link scheduling for throughput maximization in wireless networks that employ a cooperative amplify and forward (AF) protocol. To address this problem first we define the signal-to-interference plus noise ratio (SINR) expression for the complete cooperative AF-based transmission. Next, we formulate the problem of link scheduling as a mixed integer linear program (MILP) that uses as a constraint the developed SINR expression. The proposed formulation is motivated by the observation that the aggregate interference that affects a single cooperative transmission can be decomposed into two separate SINR constraints. Results for the optimal solution and a polynomial time approximation algorithm are also presented.

Index Terms—Link scheduling, cooperative systems, amplify-and-forward, wireless networks, relay network, power allocation, mixed integer linear program.

I. INTRODUCTION

Link scheduling in a wireless network consists of the activation of point-to-point links between source-destination pairs at specific time instants [1]. A schedule that is optimized will allow more network nodes to transmit concurrently by minimizing interference to each other. Existing link scheduling algorithms that model the transmission of wireless signals with the interference model [2], perceive the network as a collection of single transmitter-single receiver point-to-point links.

However, when node cooperation is employed through relays [3], the concept of a communication link is altered since a third node is involved. In this case link scheduling becomes a tougher problem to address. The two aspects of the same fundamental problem that arises in this case can be explained with the network topology depicted in Fig. 1(a).

When source node $S_1$ selects node $R_1$ to be the cooperative relay that amplifies and forwards (AF) its signal destined to $D_1$, the interference generated by this "composite" transmission consists of the interference generated from both $S_1$ and $R_1$ (although in different time slots). The second problem is that this cooperative transmission that originates from $S_1$ is affected by the aggregate interference that is accumulated at node $D_1$ and also at relay $R_1$ (another node $S_2$ interferes in this case). The same type of problems can also occur in networks that employ a centralized architecture (Fig. 1(b)).

Link scheduling that considers point-to-point wireless links has been studied extensively in the literature. However, the impact of relay nodes that AF signals has not attracted significant attention. Hong et. al in [4], [5] considered relays as a mechanism to aid in multi-hop communication but without using them as a means to improve the PHY performance. The problem is casted as a minimization of the total interference across multiple hops. Recently, Goussevskaia and Wattenhofer analyzed the complexity of scheduling wireless links under the physical interference model with a single relay [6]. The authors considered a canonical network with a specific line structure and bidirectional traffic flow. Xue et. al in [7] considered opportunistic scheduling for two-way physical layer network coding in line networks with bidirectional traffic.

In this letter we investigate link scheduling for a wireless network that employs relays that use an AF protocol. To address the problem of link scheduling in this setting, first we define an extended physical interference model for AF-based cooperative transmissions. Next, we decouple the signal-to-interference plus noise ratio (SINR) expression of the cooperative transmission and we proceed with the formulation of the link scheduling throughput maximization problem as a mixed integer linear program (MILP).

II. SYSTEM MODEL

The network model we define in this letter consists of a set $S$ of $N$ source nodes that want to communicate with a group of $D$ destination nodes denoted as the set $D$. Communication can be accomplished with the assistance of $M$ relays that belong in the set $R$. Each destination may also have multiple incoming sources. Thus, the network model is generic and can reflect either distributed or centralized network topologies (Fig. 1). To accommodate the previous design choice we denote the existence of a link from a source $i$ to a destination $j$ as $l_{ij}$, while all these links are contained in the set $L$. Since a particular relay
might serve any of the valid source/destination links \( l_{ij} \), there is a need to define the extended group of point-to-point relay links. This group \( R^i \) consists of the links between all the combinations of source/relay/destination links.

During the forwarding phase, interference will be generated from the relays that forward their signals. This can be expressed for the particular link \( l_{ij} \) as:

\[
I_{j,b} = \sum_{l_{mk} \in S \setminus \{l_{ij}\}} \gamma_{mj} P_{m}^t,
\]

where \( l_{mk} \) is an auxiliary variable that counts all the links that are active in slot \( t \). \( P_{m}^t \) corresponds to the transmitter power of a source \( m \) during slot \( t \). Also \( \gamma_{mj} = 1/d_{mj}^2 \), where \( d_{mj} \) is the distance between source \( m \) and destination \( j \) while \( a \) is usually set to a value between 3 and 4. During the same broadcasting phase, interference will also be present at relay nodes. The aggregate power in this case that is denoted as \( I_{r,b} \) and is captured with the same formula given in (1).

During forwarding, interference will be generated from the relays that forward their signals. This can be expressed for destination \( j \) as:

\[
I_{j,f} = \sum_{l_{mk} \in R_{irj} \setminus \{l_{ij}\}} \gamma_{mj} P_{m}^t
\]

Now we can define the SINR of the cooperative transmission that occurs in two orthogonal time slots as:

\[
SINR_{i,j,r} = \frac{P_{t}^i \gamma_{ij}}{I_{j,b} + \sigma^2} + \frac{P_{t}^i \gamma_{ij} g_r^2}{\sigma_a^2 + I_{j,f} + (\sigma^2 + I_{r,b}) \gamma_{ij} g_r^2}.
\]

In the above expression the received signal scales the received signal by a factor \( g_r \) so as to maintain the power constraint [8].

### III. MILP for Throughput Maximization in Wireless Relay Networks

The objective of the CLS (cooperative link scheduling) problem formulation is to maximize the throughput in an arbitrary wireless network by allowing the dynamic activation of the links that originate from sources and relays. To model this link scheduling problem, in the formulation we introduce binary optimization variables that indicate whether a link is active or not. First, we define variable \( x_{ij}^l \) that denotes whether source \( i \) transmits to its destination \( j \) during slot \( t \). Next, we define variable \( y_{ij}^{l,r} \) that indicates whether the point-to-point links from source \( i \) to relay \( r \) and from relay \( r \) to destination \( j \) are activated or not. We also introduce the continuous optimization variable \( P_{i}^t \) that corresponds to the transmission power of source \( i \). Thus, the optimization problem constitutes a mixed integer linear program. We also denote with \( T \) the maximum number of slots for which the problem is solved, and finally \( \beta \) is the SINR decoding threshold at the destination.

The most important feature of the problem formulation is that it separates the SINR expression of the cooperative transmission so that the interference constraints maintain a linear form. The key observation that allows the above approach is that the precise SINR values of the broadcasting and forwarding transmissions for a specific packet and a specific destination do not matter as long as their sum is higher or equal to \( \beta \), i.e. if the packet is decodable at the final destination after forwarding. To implement the previous insight in practice we separate the cooperative SINR expression shown in (3) into the direct and AF transmission parts by introducing the auxiliary packet decoding thresholds \( \beta_1 \) and \( \beta_2 \). The detailed problem formulation is explained below.

The objective is to maximize the throughput by increasing the number of scheduled valid source-destination pairs from the set \( \mathcal{L} \). The first constraint, C1 refers to the integer optimization variables. Next, C2 ensures that each source \( i \) is scheduled at least \( B_i \) slots which may be an optimal constraint. C3 refers to the SINR of the direct transmission that must be higher than the required threshold \( \beta_1 \). \( \Delta \) is a high value constant that essentially disables this constraint when the link is not scheduled (\( x_{ij}^l = 0 \)). With our formulation a direct transmission can be scheduled if the resulting SINR is higher than \( \beta_1 \) which is not necessarily the same with \( \beta \). This means that the scheduling condition for link \( l_{ij} \) can be less restrictive. C4 that is slightly more complicated, and it corresponds to the SINR of the AF transmission that must be at least equal to the second threshold \( \beta_2 \). In C4 the interference summations that are included are first the collective interference during the relay forwarding phase, and also the interference from the sources that were activated when source \( i \) was broadcasting. For the packet to be decoded, the value of the aggregate SINR must be higher than \( \beta \) and so we must set \( \beta_1 + \beta_2 \) equal to \( \beta \). C5 ensures that a source \( i \) that transmits in slot \( t \) will either use one relay or not. C6 is formulated in a very simple form but it is critical since it ensures that a relay link cannot be active before the corresponding source link was activated. With this constraint it is possible that a source was activated in slot \( t \) and the relay was not activated in that slot, but not the other.
max. $\frac{1}{T} \sum_{t=1}^{T} \sum_{i,j \in E} x^t_{ij}$

s. t. $x_{ij}, y_{irj} \in \{0, 1\}$ (C1), $\sum_{t=1}^{T} x^t_{ij} \geq B_t, \forall i,j \in E$ (C2), $P^t_{i}\gamma_{ij} + (1 - x^t_{ij})\Delta \geq \beta_1 + (1 - y_{irj})\beta_2, \forall i,j \in E, \forall t \in T$ (C3)

$$\frac{P^t_{i}\gamma_{ij} + (1 - x^t_{ij})\Delta \sigma^2 + \sum_{k \in R_i - \{r\}} P^t_{k}\gamma_{kj} (1 - y_{irj}) \sigma^2}{\sigma^2 + \sum_{k \in R_i - \{r\}} P^t_{k}\gamma_{kj} (1 - y_{irj}) \sigma^2} \geq \beta_2, \forall i,j \in E, \forall t \in T$$ (C4), $\sum_{t=1}^{T} P^t_{i} \leq P^t_{i,\max}$ (C7), $x^t_{ij} P^t_{i,\min} \leq P^t_{i} \leq x^t_{ij} P^t_{i,\max}$ (C8).

randomized_rounding_contraint_check()

1: for all links do
2: Assign $\tilde{x}_{ij}, \tilde{y}_{irj}$ according to [4]
3: if $\tilde{x}_{ij} = 1$ AND C1 = true then
4: continue; //C6 is valid anyway
5: else if $\tilde{x}_{ij} = 1$ AND C1! = true then
6: $\hat{x}_{ij} \leftarrow 0$; //C1 will become valid because of $\Delta$
7: end if
8: if $\tilde{x}_{ij} = 0$ AND $P^t_i! = 0$ then
9: set $\hat{x}_{ij} \leftarrow 0$; //So that C7 becomes valid
10: end if
11: continue //C1 will be valid anyway
12: end if
13: if $\tilde{y}_{irj} = 1$ AND $\tilde{x}_{ij} = 1$ then
14: if C2! = true then
15: $\hat{y}_{irj} \leftarrow 0$
16: end if
17: else if $\tilde{y}_{irj} = 1$ AND $\tilde{x}_{ij} = 0$ then
18: set $\hat{y}_{irj} \leftarrow 0$
19: else if $\tilde{y}_{irj} = 0$ then
20: continue;
21: end if
22: end for

Fig. 3. Pseudo-code for checking the constraints after applying randomized rounding to the LP solution.

A suboptimal and approximate approach has to be designed for performing the previously mentioned assignments. To this aim, we adopt the well known randomized rounding algorithm for creating our heuristic [9], [10]. The main idea of the approximation algorithm to assign the final binary values with a certain probability as follows. If we denote with $\tilde{x}_{ij}, \tilde{y}_{irj}, P^t_i$ the solutions of the LP, the binary values are approximated with randomized rounding as follows:

$$\hat{x}_{ij} = \begin{cases} P_r[1] = \tilde{x}_{ij} & P_r[0] = 1 - \tilde{x}_{ij} \quad \text{if } \tilde{x}_{ij} \text{ is equal to } 1 \text{ w.p. } \tilde{x}_{ij} \text{ and equal to } 0 \text{ w.p. } 1 - \tilde{x}_{ij} \\ \hat{y}_{irj} = \begin{cases} P_r[1] = \tilde{y}_{irj} & P_r[0] = 1 - \tilde{y}_{irj} \end{cases} \end{cases}$$ (4)

The above rule means that the final binary solution $\hat{x}_{ij}$ is equal to 1 w.p. $\tilde{x}_{ij}$ and equal to 0 w.p. $1 - \tilde{x}_{ij}$. A value for $\tilde{x}_{ij}$ closer to 1 increases the probability that a binary 1 is actually assigned. The above process ensures that the expectation of the cost of the MILP and LP solutions are the same [9]. Since some constraints might be invalid after the assignment in (4), they must be verified before we obtain the final result. This is accomplished with the algorithm depicted in Fig[3] Consider for example the case that $\tilde{x}_{ij} = 1$. It is possible that C3 does not hold since the solution of the LP can have an optimal value $\tilde{x}_{ij}$ lower than 1, that could lead to an arbitrarily low power level $P^t_i$. If this is the case, our approximation algorithm does not schedule the link by setting $\hat{x}_{ij} \leftarrow 0$ and $P^t_i \leftarrow 0$. For the variable $\hat{y}_{irj}$ the algorithm must only check if the corresponding source was activated during broadcasting (line 13). Similar reasoning follows for the remaining cases.

B. Impact of Packet Buffers

A note should be made here regarding the ability of the proposed approach to take into account the number of buffered packets at each node. In this case the network throughput should be maximized for multiple periods each of duration $T$. The buffered packets should be taken into account and a priority should be given to nodes that have more buffered packets. This could be accomplished by changing $B_i$ for a specific node every time the problem is solved. One potential metric would be to assign $B_i = \left[ \frac{\text{Buffer size at } j}{T} \right]$, where sources with higher buffer occupancy are prioritized. This approach could also be used for implementing different fairness policies and for adaptation to dynamic traffic conditions.
all the sources. Calculated by considering all the successful transmissions from the vertical axis the normalized throughput. Throughput is important to observe that the performance of the cooperative approach. Most of the results correspond of the cooperative link is tougher to meet through a higher $\beta_2$. This result with minor throughput variations was also obtained for even lower values of $\beta_1$ and supports our original choice of decoupling the two SINR constraints.

For a lighter traffic load of $B=4$ slots with $T=8$ in Fig. 5(a,b), we see that the performance trend is similar. The throughput increase is now lower for a smaller number of sources due to the decreased traffic load. Also the peak performance does not reach the same level as before. Nevertheless, the cooperative relays still offer performance benefits in this case of lower traffic demand. The performance of our heuristic is also very good when compared with the optimal solution, and is also consistent with the behavior of the randomized rounding approach [9].

V. CONCLUSIONS

In this paper we presented a MILP formulation for the problem of link scheduling in wireless networks that cooperate through an amplify-and-forward protocol. A simple MILP formulation is enabled by the separation of the SINR of the cooperative transmission into linear constraints that correspond to the direct and AF transmissions. The performance results indicate that as the number of sources is increased, it is more critical to employ the proposed approach that allows the scheduling algorithm to be more flexible in the allocation of individual source and relay transmissions across time.

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