Rational Group Decision Making

A random field Ising model at $T = 0$

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Abstract

A modified version of a finite random field Ising ferromagnetic model in an external magnetic field at zero temperature is presented to describe group decision making. Fields may have a non-zero average. A postulate of minimum inter-individual conflicts is assumed. Interactions then produce a group polarization along one very choice which is however randomly selected. A small external social pressure is shown to have a drastic effect on the polarization. Individual bias related to personal backgrounds, cultural values and past experiences are introduced via quenched local competing fields. They are shown to be instrumental in generating a larger spectrum of collective new choices beyond initial ones. In particular, compromise is found to result from the existence of individual competing bias. Conflict is shown to weaken group polarization. The model yields new psycho-sociological insights about consensus and compromise in groups.
1 Physical approach to social behavior

Some attempts to use Statistical Physics to describe various aspects of social behavior started long ago [1], for instance to study strike process [2]. They are getting more numerous in recent years. Among others, we can cite political organisation [3], group power dynamics [4], social impact [5], outbreak of cooperation [6], stock market [7], the DNA analyse [8], evolution theory [9] and ageing [10].

Social systems often involve cooperative behavior of some small or large numbers of people. The main difficulties in studying these systems is believed to result from the rich variety of existing individual features [11, 12]. The complexity of a group could thus be expected to increase with its size. However crowds, which contain large numbers of persons, behave in some aspects like one collective individual making some behavior even simpler than in the case of one individual [13].

The theory of critical phenomena can indeed shed light on above paradox. In particular, the two basic concepts of universality and irrelevant variables [14] which mean physical characteristics like the form of microscopic interactions and their physical nature have no effect on the universal properties of the collective behavior, are of importance to tackle social behavior.

Along these two concepts, it makes sense to suppose there exists in social systems too, on the one hand, many properties associated with purely individual characteristics, and on the other, some few properties which produce the collective social state [15]. More precisely, the hypothesis behind the present approach is that micro-macro relationships are universal and hold true beyond the nature of the various entities involved.

In this paper we study group decision making, in particular conditions which lead to either a polarization or a compromise of the group [15, 16]. Here
“polarization” means that most of the people move in one direction [17].

To keep the presentation simple, we used a model in which a group of N persons has to make a decision restricted to two options. Each person can choose between only “yes” and “no”. The model is articulated around a Postulate of minimum pair individual conflict. Competing interactions are also introduced via local quenched biases.

Formally we are using a modified version of the random field Ising ferromagnetic model in an external magnetic field at zero temperature. However here the system is finite in size. Moreover “random fields” may have a non-zero configurational average. Results may depend on the field configuration.

Our model does not aim to novelty in Statistical Physics but instead sheds a new light on various aspects of human behavior. Moreover it provides ground to a class of possible social experiments. Thus we tried to write the paper such that it can be understood by non-physicists.

Section 2 considers the simplest situation introducing the symmetrical individual. It is then extended to the symmetrical group in section 3. Interactions are introduced as well as a measure of the group conflict. The concept of a symmetry breaking choice is defined in Section 4. Section 5 deals with the group emergence from isolated individuals to integrated group members. The group formation process is analysed using individual anticipating. In particular we study mechanisms by which either a compromise or a polarization of the group is produced. Social surrounding pressure and individual differences are included in Section 6 with the bias individual. This bias accounts for individual representations [18] which are well established in social sciences. It results from cultural values, beliefs and personal experiences. The model is illustrated through a few examples in Section 7. We present respectively cases of balanced equal representations, balanced unequal representations, minorities
and leaders. The last Section contains concluding remarks about extension of the model to non-rational behavior, i. e., non-zero temperature.

2 The symmetric individual

We start from the simplest situation in which one individual has to make a choice between two answers either yes or no.

Such cases are indeed numerous in the social world. Moreover cases with a larger spectrum of answers can usually be mapped at some approximative level into a two-answer case [17]. The individual choice can then be represented by a two-valued variable $c$ with $c = 1$ associated to answer yes and $c = -1$ to answer no.

For a collection of $N$ persons each individual choice is represented by a variable $c_i$, where $i = 1, 2, ..., N$ and $c_i = \pm 1$.

A collective choice of the $N$ person group can be defined as the simple sum of each individual choice,

$$C = \sum_{i=1}^{N} c_i. \quad (1)$$

From Eq. (1) it is seen that aggregation enlarges drastically the spectrum of possible choices. It actually increases from 2 at the individual level up to $2^N$ at the group level. Nevertheless to materialize this spectrum a structure to collect individual answers, to sum them up and to display the net result is necessary.

Moreover, to go beyond initial twofold answer requires some complex internal transformation in order to associate a meaning to each one of the $2^N$ answers. However the use of some rules, for instance a majority rule, can bring the collective choice back to the individual one with only two answers.
3 The symmetrical group

At this stage yes and no are equiprobable. The probability distribution function is therefore,

\[ p(c_i) = \frac{1}{2}\{\delta(c_i - 1) + \delta(c_i + 1)\}, \tag{2} \]

where \( \delta \) is the Kronecker function.

Considering \( N \) isolated individuals, each one makes its choice without any interaction with others. The probability distribution for the collection of \( N \) persons is thus,

\[ p(C) = \prod_{i=1}^{N} p(c_i), \tag{3} \]

where \( p(c_i) \) is given by Eq. (2).

From Eqs. (2) and (3) the collective choice \( C \) is indeed zero on average, with fluctuations of order \( \frac{1}{\sqrt{N}} \). The result \( C = 0 \) creates a new qualitative choice which did not exist at the individual level.

\( C = 0 \) can be understood as the perfect compromise choice. Since the macroscopic quantity \( C \) is zero, the aggregation process turns out to have no effect at the macro-level making the associated group symmetrical. Perfect compromise means no group existence as such, it is a neutral group with no link among group members.

We now proceed to introduce interactions. Given a pair of persons \( i \) and \( j \), only four different choice configurations can be produced. These are, (1) \( c_i = c_j = +1 \), (2) \( c_i = c_j = -1 \), (3) \( c_i = -c_j = +1 \), and (4) \( c_i = -c_j = -1 \). In configurations 1 and 2 the two persons \( i \) and \( j \) are making the same choice. They disagree in configurations 3 and 4 with opposite choices, they
are at conflict. However agreement or conflict materializes only if \( i \) and \( j \) are both aware of the other’s choice, in other words, only once they are somehow interacting.

We can thus naturally identify a state of conflict (configurations 3 and 4) or agreement (configurations 1 and 2) using the product \( c_i c_j \). It is equal to +1 in agreement and to −1 in conflict. From now on, agreement is defined as a positive conflict. Both cases do not differentiate which choice is actually made, in accordance with the group symmetrical nature.

However prior to the decision itself, both individuals may, for instance, argue for a long time, or exchange written information. On the other hand, they may decide without any discussion. We thus need to introduce a quantity to measure this choice involvement. Let us call \( I \) the exchange amplitude. This parameter can be incorporated into the configurational choice labelling using the product \( Ic_i c_j \) instead of \( c_i c_j \). An agreement state is associated with \((+I)\) while \((-I)\) corresponds to a conflict state. \( I \) measures the conflict amplitude.

Restricting interaction to pairs the overall group conflict is measured by the function,

\[
G_I \equiv I \sum_{<i,j>} c_i c_j , \quad (4)
\]

where we have assumed that the exchange amplitude \( I \) is constant for all interacting \((i,j)\) pairs. We call \( G_I \) the group internal conflict function and \(<i,j>\) represents all interacting pairs.

4 The symmetry breaking choice

The internal conflict function \( G_I \) measures the conflict amplitude in a group for each one of the \(2^N\) decisonal configurations. It discriminates among various
possible choices, but does not indicate which one is chosen by the group. For the group decision dynamics to operate, it is necessary to invoke a criterion to select which among possible states is favored by the group.

Along the minimum energy principle, we introduce a Postulate to determine the group dynamics direction. It reads,

"Each individual favors the choice which minimizes its own conflict".

A given person will thus select its choice according to a minimum conflict principle. Justification of this Postulate is beyond the scope of the present work. It will be motivated a posteriori by the results obtained from the model. Minimum conflict means maximum agreement.

To grasp part of the decision making dynamics we can start randomly from one person who made at random a choice of either yes or no. Afterwards it does not change its choice. Then all persons interacting with this particular person will make the same choice, to minimize their own conflicts. Within a sequence process all people interacting with them will again make the same choice to favor agreement. In so doing the whole group will end up making the same initial choice the first person did select.

The net result of these dynamics is an extreme polarization of collective choice with $C = \pm N$. The sign, i. e., the polarization direction, is determined by the initial individual choice which was made randomly. This polarization phenomenon holds whoever is chosen to be the initial person. Only the direction (yes or no) will change from one initial person to another one.

In real life situations, the above process starts simultaneously from several persons. The dynamics of choice spreading is a rather complex phenomenon. Monte Carlo simulations on zero-temperature dynamics on the Ising model showed indeed non trivial behavior at all dimensions [19]. However, the group succeeds somehow to select only one initial choice at long distance al-
lowing everyone to minimize its own conflicts. Therefore we conclude,

Symmetrical groups polarize themselves towards an extreme choice. The direction of that choice however is arbitrary. Each extreme is equiprobable.

From the definition of the group internal conflict function $G_I$ (Eq. 4) and above polarization result, the Postulate of individual minimum conflict turns out to be identical to maximize $G_I$.

The polarization effect which results from group member interactions is identical to the Spontaneous Symmetry Breaking phenomenon well known in the physics of collective phenomena [14]. Individual local interactions make the group to behave as one super-person [13]. That super-person chooses between two possible choices with equiprobability likewise the isolated individual.

In parallel the individual within the group has lost its freedom of choice. It must now make a choice identical to people it interacts with. Individual freedom has been given up in favor to group freedom.

Here perfect compromise has disappeared. Simultaneously the group decision produces an effect on its surrounding somehow proportional to $N$ since $C = \pm N$. Without interactions $c_i = \pm 1$ individual were overall self-neutralised macroscopically. Interactions have produced strong individual correlations associated with a Symmetry Breaking.

Our finding sheds new light on results obtained from group decision making experiments conducted in social psychology. The polarization effect was clearly evident in data reported in [17]. However, until now, most theoretical explanations have been unconvincing in connecting choices at respectively the individual level [12] and the group level [16]. Our proposal is that polarization effect arises quite naturally from first principles, i.e., from interactions.
5 Anticipating effect

At this stage we need to formalize the internal group dynamics which proceeds from initial individual choices towards the final collective choice. The exchange term must be modified to account for the emergent group reality. We first rewrite $G_I$ as,

$$G_I = \frac{I}{2} \sum_{i=1}^{N} \left\{ \sum_{k=1}^{n} c_{j(k)} \right\} c_i .$$

(5)

where $n$ is the number of persons one individual interacts with. To keep the presentation simple this number is assumed equal for everyone. In case everyone interacts with everyone $n = N$.

Now we modify Eq. (5) to account for the process of group formation. People do anticipate the emergence of a collective choice. Each individual $i$ will thus try to project through its partner’s choices $c_j$ (the people $i$ discusses with), its expectation of the overall final group decision.

Individual $i$ then extrapolates the $j$’s choice $c_j$ to the expected collective choice the group will eventually make without its own participation. Within this process, individual $i$ perceives the $j$’s choice as given by the transformation,

$$c_j \rightarrow \frac{1}{N-1} (C - c_i) ,$$

(6)

where $C$ is the collective choice defined as before. Once this process is completed, Eq. (5) becomes,

$$G_I^a = \frac{I}{2} \sum_{i=1}^{N} \left\{ \sum_{j=1}^{n} \frac{1}{N-1} (C - c_i) \right\} c_i ,$$

(7)
and,

\[ G_i^g = \frac{In}{2(N-1)} \left\{ C \sum_{i=1}^{N} c_i - \sum_{i=1}^{N} c_i^2 \right\}, \tag{8} \]

where superscript \( g \) denotes active anticipating process. Using the collective choice definition \( C = \sum_{i=1}^{N} c_i \) and the property \( c_i^2 = 1 \) we get,

\[ G_i^g = \gamma \frac{C}{N} \sum_{i=1}^{N} c_i - \gamma, \tag{9} \]

where \( \gamma \equiv \frac{nIN}{2(N-1)} \) is a constant independent of the group choice. As such it is irrelevant to the collective choice. \( C \) is not yet the final decision. Rather it is the expected final collective choice. We can rewrite Eq. (9) in the form,

\[ G_i^g = S_g \sum_{i=1}^{N} c_i - \gamma. \tag{10} \]

where

\[ S_g \equiv \gamma \frac{C}{N}, \tag{11} \]

acts as a group field which couples with each individual choice. The field notion is a natural way to account for some pressure towards a definite choice. Within our convention of minimum conflict the product \( S_g c_i \) measures that influence. A positive field \( S_g \) favors a positive choice +1, while −1 is associated to a negative field. The conflict amplitude is given by \( S_g \).

We have indeed a self-consistent expression since on one hand, individual \( i \) wants to go along the virtual field \( S_g \), and on the other hand it contributes directly to this virtual field through its dependance on the collective choice \( C \).
Rewriting Eq. (10) as

\[ G_i^2 = \gamma \frac{C_i^2}{N} - \gamma \]  \hspace{1cm} (12)

shows that maximizing \( G_i^2 \) results in maximizing \( C_i^2 \) which is obtained by \( C_i^2 = N^2 \). It is an extreme polarization with either \( C_i = +N \) or \( C_i = -N \). Again individual minimum conflicts appear clearly identical to the maximum of the group internal conflict function.

At this stage of the model our “group formation process” is formally identical to the mean field theory of phase transitions. While in physics, it is an approximation, here it embodies the social mechanism of anticipation.

6 The bias individual

We are now in position to overcome two simplifying assumptions made earlier. First, most choices an individual and a group have to make are not independent of the surroundings as assumed above. We now will account for pressure applied to the group from the outside. Second, assuming identical individuals, with no apriori individual differences in preferences about the issue, does not hold in most cases. Individual differences will now be included.

6.1 Social pressure

The existence of external pressure on group members means the equiprobability hypothesis (Eq. 2) no longer holds. It is achieved introducing a quantity which differentiates the two possible choices. We call this quantity the social field \( S \). It turns the symmetrical individual into a social one with now \( p(c_i = 1) \neq p(c_i = -1) \).

Similarly with above group field \( S_g \), each person’s conflict with \( S \) is represented by the product \( S c_i \). Agreement is associated with \( S c_i > 0 \), i. e.,
the choice is made along the field with \( S \) and \( c_i \) having the same sign. In contrast \( Sc_i < 0 \) represents a conflict between the individual and the social pressure, since \( S \) and \( c_i \) have opposite signs. The surrounding group conflict measure is,

\[
G_S \equiv \sum_{i=1}^{N} Sc_i .
\]  

(13)

Applying the Postulate to the sum \( G_I + G_S \) still results in an extreme polarization but now its direction is no longer random. The group choice is \( C = +N \) for \( S > 0 \), and \( C = -N \) with \( S < 0 \). Under external pressure, even extremely weak, the group and the individual behave identically. They both follow the pressure induced by the external pressure. The Symmetry Breaking choice is no longer random.

Here the super-person represented by the whole group is identical to the individual person. They are both aligned along the field. This result is at contrast with the symmetrical state, where the individual loses its freedom of choice in favor of the group choice freedom.

6.2 The Representational state

To get closer to experimental situations, individual differences in preferences about some issues are now introduced. Following social literature they are called “individual representations” [18]. A representation varies in both, direction and amplitude, from one person to another. It depends upon cultural values, past experiences, ethics and beliefs. It is attached to a person.

The representational effect can be included within our formalism by introducing an additional field. We call \( S_i \) the internal social field attached to individual \( i \). Its properties are similar to those of a social field \( S \). The difference being that the social field applies uniformly to each group member while
an internal social field acts only on one person.

As for the other fields, conflict with the internal social field is accounted for in the product $S_i c_i$. It is positive for a choice made along the representation (internal agreement with personal values), and negative otherwise (internal conflict with personal values). The group representation conflict measure is given by,

$$G_R \equiv \sum_{i=1}^{N} S_i c_i .$$

(14)

The distribution of individual representations is required to determined the group collective choice. The representation effect is enhanced in the isolated-person case where both exchange amplitude and social external field are zero. There, from the Postulate, final decision is found to result from every individual following its own representation. It gives,

$$C = \sum_{i=1}^{N} \frac{S_i |S_i|}{|S_i|} ,$$

(15)

where the $|...|$ denotes absolute value.

This equation illustrates the qualitative change driven by the existence of representations. Actual $C$ value can now vary over the whole spectrum of values $-N, ..., 0, ..., +N$. Compromise $C = 0$ can again be an outcome. Individual representations are thus instrumental for making the whole model relevant to real situations in which collective choices are far richer than $C = \pm N$.

In others words, prior to group formation, individuals have their own representations which determine their apriori answers to the initial question. All these representations result in either yes or no. Then, in the process of
group formation, people start to interact through the yes and no distribution in the group.

However to reach a collective choice, due to the existence of opposite representations, people must construct new answers in addition to the initial yes and no. Answers are thus enriched during group formation, due to driving representations. On the other hand, within the neutral state groups do not produce new answers.

Once the final decision is reached, each group member identifies with the collective choice triggering its new individual choice to \( d = \frac{c}{N} \) which may differ from the initial \( c_i \). In the neutral state \( d = \pm 1 \).

Thus, in the process of building a new answer \( d \), a new representation has been produced by the group. This new representation is integrated by each individual to yield the \( d \) choice. Group formation has qualitatively modified individual representations.

This process shows that while individuals resist adopting a representation opposed to their own, via the group transformation, they will join a new common representation which accounts for the overall balance of initial representations. The preceding takes place around a new answer which did not exist prior to the group forming.

Note our qualitative departure from usual Statistical Physics. Here we are not considering an average individual position, but a well defined and fixed individual position. This position results from the group forming. In most cases \( d \) is different from \( \pm 1 \). We are thus passing from a class of Ising variables \( c_i = \pm 1 \) to one continuous variable \( \frac{-1}{N} \leq d \leq \frac{1}{N} \).
6.3 The frustrated individual

Adding together all the effects introduced until now results in an extended group internal conflict function \( G = G^g_I + G^s + G_R \) which is,

\[
G = I \sum_{i,j} c_i c_j + S \sum_{i=1}^{N} c_i + \sum_{i=1}^{N} S_i c_i .
\]  

(16)

The extended form of Eq. (16) makes maximizing \( G \) a more difficult task since competing effects are active. A given individual wants now to minimize its overall own conflict. There exist three contributions,

- Interacting group members: the individual wants to come up with the same final decision as preferred by interaction partners.
- External social field: the individual wants to comply to the external pressure from immediate surroundings.
- Internal social field: the individual wants to comply to the internal pressure from its personal representations.

These three elements are not necessarily satisfied simultaneously. From the Postulate, the individual wants to minimize overall personal conflict. It could result in simultaneous agreement with some of above items, and conflict with others. It is clearer to write Eq. (16) as,

\[
G = \sum_{i=1}^{N} S^r_{g,i} c_i - \gamma ,
\]  

(17)

where,

\[
S^r_{g,i} = S_g + S + S_i ,
\]  

(18)
is the resulting field applied to individual $i$ in the group formation process. Maximum $G$ and minimum individual conflicts are achieved when each individual follows his resulting field sign. If $S_{g,i}^r > 0$, then $c_i = 1$ and $c_i = -1$ for $S_{g,i}^r < 0$.

The case $S_{g,i}^r = 0$ results in an undetermination of the $i$ choice as in the isolated neutral case. Satisfying $S_{g,i}^r$ sign does imply satisfying simultaneously $S$, $S_i$, and $S_g$ signs. This competing effect is the signature of the psychological complexity involved in the decision making process. Each person first follows its resulting field $S_{g,i}^r$ to produce a collective choice $C$. Then this collective choice is individually integrated back with $c_i \rightarrow d = \frac{C}{N}$.

7 Illustration of the model

To gain a deeper insight about the meaning of Eqs. (17) and (18), we now analyse four different specific cases. Each will illustrate some basic feature of the model.

7.1 Two balanced opposite biases case

We consider an evenly divided group of $N$ persons with no external social field, i.e., $S = 0$. Half the persons have a positive representation $S_i = +S_0$, and the other half have a negative representation with the same amplitude $S_j = -S_0$. The whole group has thus no net representation. Interactions are of amplitude $I$ and each person discusses with $n$ other persons. In small, face-to-face groups, everyone usually interacts with everyone else, so $n = N$. The corresponding internal conflict function is,

$$G = -\gamma + \frac{\gamma}{N}C^2 + \frac{N}{2}(S_0c_i^+ - S_0c_j^-),$$

(19)
where \( c_i^+ \) and \( c_j^- \) are attached to persons with respectively positive and negative representation. The constant \( \gamma \equiv \frac{n I N}{2(N-1)} \) has been introduced earlier in the group formation section. The collective choice may be written as \( C = \frac{N}{2}(c_i^+ + c_j^-) \). The actual choice is the one which maximises \( G \). In this case it is easily singled out, since there exist only two different kinds of persons symbolised by \( c_i^+ \) and \( c_j^- \). Four choice configurations are possible, (a) \( c_i^+ = +1; c_j^- = +1; C = N \), (b) \( c_i^+ = -1; c_j^- = -1; C = -N \), (c) \( c_i^+ = +1; c_j^- = -1; C = 0 \), (d) \( c_i^+ = -1; c_j^- = +1; C = 0 \).

The first two (a and b) are agreement and others (c and d) are conflict. Associated internal conflict functions are, \( G(a) = G(b) = -\gamma + N\gamma \), \( G(c) = -\gamma + NS_0 \), \( G(d) = -\gamma - NS_0 \).

Clearly \( G(d) < G(c) \), reducing the choice to either (a and b) or (c). In case \( \gamma > S_0 \), we have \( G(a, b) > G(c) \), indicating that the interaction strength proportional to \( nI \) is stronger than \( S_0 \). The group then polarizes with \( C = \pm N \). The direction of the extreme choice occurs at random.

Half of the members are fully satisfied with both their representation and their partners while the other half is in conflict with its own representation. This result means in particular that the “losing” subgroup has to build a new representation which embodies some level of internal conflict. The “winning” part does not modify its initial representation. In this case, no new answer was built. We have \( c_i \rightarrow d = \pm 1 \).

On the other hand, strong representation, i.e., \( \gamma < S_0 \) favors compromise, with the collective choice \( C = 0 \). Each member \( i \) starts from a personal representation to decide eventually through weak interactions on a medium compromise, with the creation of a new answer \( d = 0 \). Again, this compromise choice did not exist prior to the group formation. It is the result of cooperation between the group level and the individual level.
Within a balanced representation group, exchange favors a compromise.

Weak exchange result in an extreme polarization along a random direction.

7.2 Two unbalanced opposite biases case

We now go back to the previous example, but consider a stronger positive representation. This is done by writing the negative representation fields as $S_j = -\alpha S_0$, with $0 < \alpha < 1$. Respective numbers of positive and negative representations are equal. Only the internal conflict function values are changed to become respectively, $G(a) = -\gamma + N\gamma + \frac{N}{2}(1 - \alpha)S_0$, $G(b) = -\gamma + N\gamma - \frac{N}{2}(1 - \alpha)S_0$, $G(c) = -\gamma + \frac{N}{2}(1 + \alpha)S_0$, $G(d) = -\gamma - \frac{N}{2}(1 + \alpha)S_0$.

Since $0 < \alpha < 1$, $G(a) < G(b)$ and $G(d) < G(c)$, always. However in the case $\gamma > S_0$ the polarization direction is determined with $C = +N$.

Before we had $\alpha = 1$ which made the direction arbitrary, but now it is the strongest initial representation which wins. The discussion process within the forming group has made the weaker-biased people align themselves with the stronger ones. Here we have $c_i \rightarrow d = 1$.

In order for a compromise outcome to be favored, a decrease in exchanges among group members is required. For $\gamma < S_0$, the final choice is $C = 0$ which gives $c_i \rightarrow d = 0$.

Within an unbalanced representation group, exchange favors the initially strongest representation. Only a limitation of exchange may produce a compromise.

7.3 The minority case

Most cases do not have an equal number of people in two opposite subgroups. Usually there exist a majority and a minority. Let us consider a minority number $M$ of people $(M < \frac{N}{2})$ with a positive representation $S_i = +S_0$. The majority then contains $(N - M)$ with an unequal negative representation $S_j =$
−αS₀. We chose, for instance, the case of a minority more motivated than the majority, i.e., 0 < α < 1.

Denoting $c_i^+$ and $c_j^−$ the respective minority and majority choices, the collective choice is given by $C = Mc_i^+ + (N − M)c_j^−$. The internal conflict function is $G = \frac{n}{2(N − 1)}IC^2 + (Mc_i^+ − (N − M)αc_j^−)S₀$. Associated four choice configurations are, (a) $c_i^+ = +1; c_j^− = +1; C = N$, (b) $c_i^+ = −1; c_j^− = −1; C = −N$, (c) $c_i^+ = +1; c_j^− = −1; C = 2M − N$, (d) $c_i^+ = −1; c_j^− = +1; C = −2M + N$.

The first two (a and b) are minority-majority agreement while the others (c and d) are minority-majority conflict. Associated internal conflict functions are, $G(a) = −\gamma + N\gamma + \{(1 + \alpha)M − \alphaN\}S₀$, $G(b) = −\gamma + N\gamma − \{(1 + \alpha)M − \alphaN\}S₀$, $G(c) = −\gamma + \frac{\gamma}{N}(2M − N)^2 + \{(1 − \alpha)M + \alphaN\}S₀$ $G(d) = −\gamma + \frac{\gamma}{N}(2M − N)^2 − \{(1 − \alpha)M + \alphaN\}S₀$.

Analysis of above expressions is complicated since now several parameters are involved. There are $N$, $M$, $nI$, $S₀$ and $α$. Let us comment on some cases. Again, case (d) is never selected since indeed nothing is satisfied in that case, neither interactions nor representations.

• Interaction effects are winning over representation effects: the group is polarized, i.e., $G(a)$ or $G(b) > G(c)$.

(a) The minority wins, turning the majority to its side if $G(a) > G(b)$.

It is the case if $M < (N − M)α$. Condition $G(a) > G(c)$ is ensured by $nIM > (N − 1)αS₀$.

(b) The majority wins, turning the minority to its side if $G(a) < G(b)$.

It is the case if $M > (N − M)α$. Condition $G(b) > G(c)$ is ensured by $nI(N − M) > (N − 1)S₀$. The condition does not depend on $α$ since in both cases (b) and (c) the majority follows its own representation $−αS₀$. 

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• Representation effects are winning over interaction effects: the group is balanced, i. e., \( G(a) \) and \( G(b) < G(c) \).

Condition \( G(a) < G(c) \) is ensured by \( nIM < (N - 1)\alpha S_0 \). Condition \( G(b) < G(c) \) is ensured by \( nI(N - M) < (N - 1)S_0 \). A balanced collective choice reflecting the respective strength in numbers of each group is given by case (c).

7.4 The leader case

In most groups, persons are not all equal in status. The inequality can stem from either a strong character or an institutional position, like for instance a group president who has a tie-breaking vote. To account for such situations it is enough to associate a stronger individual field to the leading person in the group. In other words the leader case is a special case of a strong minority which reduces to one person.

We can thus use the minority case equations putting \( M = 1 \). In the case of a charismatic leader we take \( \alpha \sim 0 \) with \( 0 < \alpha < 1 \) to emphasize its weak aspect. However for another kind of leader, the authoritarian for instance, an external field would account for the pressure the leader applies to all group members.

Let us consider a leader with a positive representation \( S_1 = +S_0 \). The majority figure is then \( N - 1 \) with an unequal negative representation \( S_j = -\alpha S_0 \). Denoting \( c_0^+ \) and \( c_j^- \) as the respective leader and majority choices. The collective choice is given by \( C = c_0^+ + (N-1)c_j^- \). The internal conflict function is \( G = \frac{n}{2(N-1)}IC^2 + (c_0^+ - (N-1)\alpha c_j^-)S_0 \). The associated four choice configurations are, (a) \( c_0^+ = +1; \ c_j^- = +1; \ C = N \), (b) \( c_0^+ = -1; \ c_j^- = -1; \ C = -N \), (c) \( c_0^+ = +1; \ c_j^- = -1; \ C = 2 - N \), (d) \( c_0^+ = -1; \ c_j^- = +1; \ C = -2 + N \).

The first two (a and b) are leader-majority agreement while the others (c
and d) are leader-majority conflict. Associated internal conflict functions are,
\[ G(a) = -\gamma + N\gamma + \{ (1 + \alpha) - \alpha N \} S_0, \]
\[ G(b) = -\gamma + N\gamma - \{ (1 + \alpha) - \alpha N \} S_0, \]
\[ G(c) = -\gamma + \frac{\alpha}{N}(2 - N)^2 + \{ (1 - \alpha) + \alpha N \} S_0, \]
\[ G(d) = -\gamma + \frac{\alpha}{N}(2 - N)^2 - \{ (1 - \alpha) + \alpha N \} S_0. \]

Analysis of above expressions is as complicated as in the minority case with parameters \( N, nI, S_0 \) and \( \alpha \). Let us comment on some cases. Again, case (d) is never selected since indeed nothing is satisfied in that case, neither interactions nor representations.

- **Interaction effects are winning over representation effects:** the group is polarized, i.e., \( G(a) \) or \( G(b) > G(c) \).
  
  a. The leader wins, turning the majority to its side if \( G(a) > G(b) \). It is the case if \( \frac{1}{\alpha} < N - 1 \). Condition \( G(a) > G(c) \) is ensured by \( nI > (N - 1)\alpha S_0 \).
  
  b. The majority wins, turning the minority to its side if \( G(a) < G(b) \). It is the case if \( \frac{1}{\alpha} < N - 1 \). Condition \( G(b) > G(c) \) is ensured by \( nI > S_0 \). The condition does not depend on \( \alpha \) since in both cases (b) and (c) the majority follows its own representation \(-\alpha S_0\).

- **Representation effects are winning over interaction effects:** the group is balanced, i.e., \( G(a) \) and \( G(b) < G(c) \).
  Condition \( G(a) < G(c) \) is ensured by \( nI < (N - 1)\alpha S_0 \) and condition \( G(b) < G(c) \) by \( nI < S_0 \). A balanced collective choice reflecting the respective numerical strength of each group is given by case (c).

## 8 Conclusion

A simple Ising-like model has been presented to describe group decision making. It is indeed a modified version of the random field Ising ferromagnetic
model in an external magnetic field at zero temperature. However, our system is finite in size and fields may have a non-zero configurational average. In principle, results may also depend on the field configuration. Moreover, we crossover in the group decision making process from a class of Ising variables to one continuous variable.

The hypothesis behind our approach is that group decision making obeys universal laws which are independent of the nature of the issue at stake. Our main results with respect to the qualitative properties of group decision making are,

* Exchanges among individuals do not aim to select an issue, but rather to align people along the same issue. The issue itself is random with respect to exchanges.

* Exchanges among individuals do not favor compromise about an issue. On the opposite, it produces polarization, i.e., extreme options.

* Reducing exchanges favors compromise.

* External social pressure is extremely efficient on selecting an option.

* Individual bias is a necessary ingredient to both weaken extreme options and oppose an external social pressure.

These theoretical results must be put in parallel to various data obtained from a large number of experimental studies which show groups polarize along an extreme position reflecting the dominant pole of attitudes and not around an average position as a priori expected [16, 17].

Our emphasis is on building a conceptual methodology rather than a final complete theory. In a forthcoming paper, we will introduce non-rational behavior which is a real-life basic feature. It will be analogous to temperature.
However within our model we will define a “local temperature” in a finite system.

Acknowledgments

I would like to thank Dietrich Stauffer for stimulating comments on the manuscript.

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