Modeling electromagnetically driven free-surface flows motivated by the Ribbon Growth on Substrate (RGS) process

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Abstract. The Ribbon Growth on Substrate (RGS) technology is a crystallization technique that allows direct casting of silicon wafers and sheets of advanced metal-silicide compounds. With the potential of reaching high crystallization rates, it promises a very efficient approach for future photo-voltaic silicon wafer production compared to well-established processes in industry. However, a number of remaining problems, like process stability and controllability, need to be addressed for the RGS technology to eventually become a competitor in the near future. In this regard, it is very desirable to gain detailed insights into the characteristic process dynamics. To comply with this demand, we have developed a new numerical tool based on OpenFOAM (foam-extend), capable of simulating the free-surface dynamics of the melt flow under the influence of an applied alternating magnetic field. Our corresponding model thereby resolves the interaction of hydrodynamic and magnetodynamic effects in three-dimensional space. Although we currently focus on the RGS process, the modeling itself has been formulated in a more general form, which may be used for the investigation of similar problems, too. Here we provide a brief overview of these developments.

1. Introduction

Nowadays, photo-voltaic silicon wafers are mostly manufactured through a two-step process of growing and sawing large ingots. The ingots are either produced by directional solidification of multi-crystalline silicon or with the help of the Czochralski method in case of single crystals. For such a discontinuous procedure, unavoidable sawing losses are typically in the range of 40 to 50% of the feedstock. On the contrary, the Ribbon Growth on Substrate (RGS) technology, which was suggested and developed during the last decades [1, 2, 3], circumvents such a deficit in terms of energy and material almost completely.

Compared to the concept of growing and cutting ingots, the fundamental difference of the RGS process lies in a continuous and direct casting of silicon in near-net-shape. This is realized by means of feeding molten silicon into a casting frame without bottom, from where a solidified silicon foil is extracted side-wise on a sub-cooled moving substrate underneath. It results in both a close to perfect material yield and a low energy consumption due to the continuous nature of the processing. Concurrently, this also entails the crucial benefit of fully decoupled solidification and casting velocities.
Fig. 1 shows a schematic of the process principle. It shows one half of the core process assembly, where the foreground corresponds to a central sectional plane in the process direction.

![Scheme of the RGS process](image)

**Figure 1.** Scheme of the RGS process [4, 2]: one half with central cut in the process direction. A narrow slit region is situated between the casting frame and the substrate.

The silicon in the casting frame is melted by means of an induction heater with an alternating current (AC) of certain frequency. Within the RGS process there are still some effects that can negatively influence the properties and quality of the wafer. Two of them are, for example, the occurrence of flow instabilities and meniscus oscillations at the open slit, where the moving substrate enters and leaves the casting frame. The AC magnetic field, in first instance applied here for heating the silicon melt, is able to stabilize the melt at the slit since it behaves like a magnetic valve. It prevents leakage in the slit region and reduces oscillations at the extraction site of the silicon foil through electromagnetic forces. This Lorentz force field acts to counter the gravitational forces on the melt. A similar case of electromagnetic retention for liquid silicon was reported in [5]. The electromagnetic force field, however, typically gives rise to a dome-shaping of the melt in the casting frame, which represents a free surface problem complicating the numerical simulation of the system. There is only very limited literature about numerical simulations of the silicon melt flow for the RGS process (e.g., [6, 7]). Moreover, almost all related simulations were mainly focused on the crystallization process and solidification front shape of silicon ribbons. In contrast, we have performed numerical investigations in order to study the influence of the involved AC electromagnetic fields on the silicon melt during the RGS process.

2. Modeling
The casting region in the real RGS prototype machine [2] is complex. Especially the difference in scale between, e.g., the casting frame of width 156 mm in relation to the magnetic valve at the wafer exit side of less than 1 mm represents a challenge for numerical grid definition and computing capacity. Therefore, a simplified set of modeling parameters and casting environment geometry was used [8]. Within this study we have provided an overview of the magnetohydrodynamic (MHD) effects in the RGS process at its characteristic process parameters. It has been found that the Lorentz-force is dominating the melt-flow and that the deformation of the free-surface shape may not be neglected. A retention effect was demonstrated. The hydrostatic pressure at the bottom of the casting frame can be compensated by the magnetic pressure in the slit region. A parameter study revealed the total system inductivity as a function of the melt level, which might be usable for a fill-level sensing of the silicon melt. In a subsequent publication [9], we have conducted numerical simulations of the dome-shaping effect for the RGS process by means of a two-dimensional model, where we used COMSOL Multiphysics as tool.
for the magnetodynamic part. However, it turned out that the combination of OpenFOAM [10] and COMSOL Multiphysics is not suitable for large three-dimensional simulations. Conducting simulations of such (and similar) fully coupled MHD systems with a free surface is part of a broad field of active research to investigate industrial applications, where induction melting or levitation of liquid metals is involved. Recently, first three-dimensional simulations have been performed based on coupling ANSYS products [11], [12] and [13]. Coupling solvers or methods (e.g., Spectral, Finite Volume or Finite Element) may generally bring the need of interpolation. Thus, additional overhead is produced, which means increased computational costs.

To overcome all issues with computational overhead and external coupling of different numerical tools, we have concentrated on the development of a new solver concept in the finite volume framework of OpenFOAM (foam-extend) [14], where the magnetodynamics is modeled within the same three-dimensional finite-volume framework as the free-surface movement and the flow calculation. Based on this new solver, we were able to investigate the three-dimensional behavior of a free-surface flow under the influence of magnetic fields in the RGS process with the help of the \( k-\omega \)-SST turbulence model [15]. The governing equations for the magnetic field and the flow dynamics will be presented in detail in the next section. Parts of our implementation in foam-extend were developed in close relation with [16], specially regarding the flexible and efficient application of Biot-Savart’s law (c.f. chapter 3) and a multi-mesh concept to address problems with several coupled physical effects, which are each valid on different (possibly overlapping) domains.

In order to perform a more detailed Large-Eddy-Simulation (LES) of the RGS process, it is essential to be able to distribute the work load over several processors [17]. To achieve an acceptable load balance in combination with our multi-mesh technique, we have extended foam-extend to use the multi-constraint graph-partitioning feature of the software METIS [18]. With METIS, it is now possible to decompose mesh parts individually, such that each processor holds overlapping areas of all corresponding sub-meshes. No bulk communication is necessary for this to work.

On the way of realizing coupled simulations of the RGS process, we have developed a novel approach for the numerical solution of three-dimensional eddy-current problems using the finite volume framework of foam-extend [19]. To the best of our knowledge, this is the first time that such time-dependent (quasi-steady) electromagnetic problems are addressed with the help of OpenFOAM (foam-extend). Our implementation relies on a semi-coupled solution of the magnetic vector potential in a global region and the electric scalar potential in the conducting region. The method itself is memory efficient and due to the usage of block-coupled matrices numerically robust also for higher frequencies of the alternating inductor current. Even the modeling involves a non-conducting domain around the region of interest, the former is only used for the solution of the magnetic vector potential. By means of edge-based inductor models in combination with splitting the magnetic vector potential into an impressed and a reduced part, both the size of the non-conducting region and the computational costs of Biot-Savart’s law can be minimized. In contrast to e.g. [20], our method is suitable also for very large simulations. Additionally, the basic concept of the so called Ghost-Fluid-Method (GFM) in analogy to [21] has been applied to account for discontinuities in the electrical conductivity of the modeled conductor. This development makes it possible to avoid several conducting sub-regions and allows for a simple setup. An extensive validation for all aspects of the numerical approach was accomplished and demonstrated in [19]. The method may be readily coupled to a variety of models of other physical phenomena within the library of foam-extend.

3. Governing equations
The magnetic fields are computed using the \( A - V \) – formulation of the quasi-static Maxwell equations in the magnetohydrodynamic approximation [22, 23]. It can be shown that due to
the very small magnetic Reynolds number this approximation is valid for modeling the RGS process with sufficient quality. The magnetic fields can be described as follows with the applied Coulomb gauge \( \nabla \cdot A = 0 \):

\[
\nabla^2 A = \mu_0 \sigma (\partial_t A + \nabla V) - \mu_0 j_0 (1)
\]

\[
\nabla \cdot (\sigma \nabla V) = -\partial_t A \cdot \sigma (2)
\]

Here the magnetic vector potential is denoted by \( A \), the electric scalar potential by \( V \), the time by \( t \), the electric conductivity by \( \sigma \) and the vacuum permeability by \( \mu_0 \). The total current density was divided into two terms \( j = j_0 + j_I \), where \( j_0 \) is the contribution from the external sources and represent the effect of the excitation coils. The field \( j_I \) represents the induced current density and can be computed using the Ohm’s law: \( j_I = -\sigma (\partial_t A + \nabla V) \). To avoid meshing of the inductor it is possible to split the magnetic vector potential into an impressed and a reduced part [24], i.e. \( A = A_0 + A' \). We can easily compute the impressed magnetic vector potential in an algebraic way be means of Biot-Savart’s law:

\[
A_0(x) = \frac{\mu_0}{4\pi} \int_{\Omega} \frac{j_0(r)}{|x-r|} d\Omega(r) (3)
\]

where the integration has to be carried out only over the region of the excitation coil. This can be done by an exact integration of a parameterized geometry [19].

The induced current \( j_I \) in the electrically conducting region results in an electromagnetic force density (Lorentz force)

\[
F = j_I \times (\nabla \times A) (4)
\]

In most industrial applications, the source current \( j_0 \) originates from an excitation coil with a time-harmonic alternating current at frequency \( f \). If this is the case, a complex, quasi-steady formulation of equations 1 and 2 is more convenient, where all time derivatives are replaced with a complex-valued angular frequency \( \partial_t = i \omega \), \( \omega = 2\pi f \). For a given frequency \( f \), alternating magnetic fields appear in shape of a harmonically oscillating magnetic field \( B = \nabla \times A \) and the induced current density \( j_I \). Both together cause a time dependent Lorentz-force \( F = j_I \times (\nabla \times A) \). Due to the product of two harmonic functions, this force can be split in a steady part \( F_L = \langle F \rangle \) and a superposed part \( F' \) oscillating at twice the frequency. The constant part \( F_L \) is balanced mainly by friction forces, and the oscillating part is in equilibrium with inertial forces. The ratio of the corresponding velocities \( U(\langle F_L \rangle)/U(F') \) scales with \( (\omega L^2)/\nu \). For liquid metals, this ratio is typically in the order of \( 10^6 \) to \( 10^8 \), which means that the oscillating part can be neglected. A more detailed explanation can be found in [25].

The flow dynamics in the electrically conducting melt is modeled by solving the incompressible Navier-Stokes equations

\[
\rho (\partial_t u + (u \cdot \nabla) u) = \nabla \cdot \tau + F_L; \quad \nabla \cdot u = 0 (5)
\]

The equation contains the fluid velocity \( u \), the density \( \rho \) and the stress tensor \( \tau = \eta (\nabla U + (\nabla U)^T) - p I \) based on a modified diagonal fluid pressure \( p' = p - \rho (g \cdot x) \) with hydrostatic part according to the spatial coordinate \( x \) and the gravitational vector \( g \). The parameter \( \eta \) denotes the dynamic viscosity of the fluid.

The free-surface boundary is represented by means of a surface-tracking method on the basis of the Arbitrary Lagrangian-Eulerian (ALE) technique [26]. Boundary conditions for \( u \) and \( p \) can be derived from the force balance at the fluid interface. While the normal force balance results in a curvature and surface tension dependent Dirichlet-type condition for the pressure, the tangential force balance leads to a Neumann-type boundary condition for the gradient of the velocity. For more details about the conditions at the free surface, the reader is referred to [27] and [17].

4
4. Results

As mentioned before, an efficient solution algorithm with a solver named eddyCurrentFoam has been developed to solve electromagnetic (eddy-current) problems in OpenFOAM.

![Graph](image.png)

**Figure 2.** Results for the amplitude of the induced current density $j'$ from simulations with jump discontinuity in the electrical conductivity ($\sigma_{left} = 12 \sigma_{right}$) for two different frequencies.

For example both contour plots in Fig. 2 show numerical results of the induced current density from test simulations with a jump discontinuity in the electrical conductivity ($\sigma_{left} = 12 \sigma_{right}$) at two different frequencies of 1 kHz and 10 kHz. For the case of the lower frequency, a mesh consisting of $\approx 1.8 \times 10^5$ cells was used and the calculation converged within $\approx 80$ s to a normalized residual of $1 \times 10^{-5}$. The calculation for the higher value of the frequency is based on a mesh of $\approx 1.15 \times 10^6$ cells, while convergence was achieved after $\approx 840$ s. The interface separating the two regions with different values of the electrical conductivity is inclined with an angle of 45 degrees with respect to the vertical direction. Details of the used setup can be found in [19]. With the application of our embedded discretization presented in [19], the flux continuity across the material boundary is preserved, while the jump in the tangential part of the current density is sharply resolved.

We have computed the induced Lorentz force density distribution in a circular cylinder caused by both a rotating (RMF) and a traveling magnetic field (TMF) [30]. In [19], the simulated results for the RMF case were validated with analytical expressions and numerical simulations with the commercial code Opera 3D [28].

Fig. 3 shows a comparison of results from a numerical simulations of the TMF Lorentz force density distribution: our implementation in foam-extend (solid) vs. results from the commercial software Cobham Opera 3D [28] (dashed) showing a very good agreement. The results from foam-extend are related to a computational grid of $\approx 7 \times 10^5$ cells and convergence at a residual of $1 \times 10^{-8}$ was reached after $\approx 500$ s with the help one processor. The right part of the figure includes a comparison with the in the literature often used analytical expression $F_L = \sigma \omega k_{TMF} B_0^2 r^2 e_z/8$ (dotted), where $k_{TMF} = 2\pi/\lambda_{TMF}$ denotes the TMF wave number [29, 30]. The TMF was realized by means of an axial array of multiple inductor coils supplied with alternating current and with different phase shifts. We have performed numerical simulations for a fluid region with radius $R$ and height equal 30 mm, an AC angular frequency of $\omega = 2\pi 50$ Hz and an magnetic induction amplitude of $B_0 = 2$ mT in the center of the coil system. The electrical conductivity of the modeled liquid metal alloy Gallium-Indium-Tin (GaInSn) at room temperature was $\sigma = 3.289 \times 10^6$ S/m. The TMF wave length $\lambda_{TMF} = 446.6$ mm and all the other parameters were chosen in agreement with [31].

The solution procedure for the fully coupled system of electrodynamics, flow motion and free surface deformation is separated according to physical effects. The coupling is addressed by means of iterative bulk data exchange and recurring updates of the mesh topology.
Figure 3. Comparison of results from a numerical simulation of the TMF Lorentz force density distribution: our implementation in foam-extend (solid) vs. results from the commercial software Cobham Opera 3D [28] (dashed, barely visible behind the solid lines due to the very good agreement between both methods). The dotted lines at the right part corresponds to an analytical expression linear in the radial coordinate [29, 30].

Fluid flow, surface-tracking and fluid mesh motion is numerically solved on the basis of a modified *interTrackFoam* solver algorithm which was derived from the originally published and implemented version by Tukovic and Jasak (2012) [27].

Two application examples are presented in the following as demonstration for the capabilities of our development. The first exemplary case is a simplified three-dimensional RGS process as examined and discussed in Beckstein et al. (2015) [9] and Beckstein et al. (2017) [15]. A typical snapshot of the instantaneous fluid velocity magnitude and time-averaged Lorentz-force density is illustrated in Fig. 4.

Figure 4. Deformed bulk melt shape of a three-dimensional RGS model (right part: velocity magnitude $|u|$, left part: Lorentz-Force density magnitude $|F_L|/\rho$)
The URANS-simulation presented in Fig. 4 was conducted with a mesh of \( \approx 1.5 \times 10^6 \) cells. Within one hour of calculation time, a simulation time of \( \approx 0.05 \) s could be realized using a single CPU. A more detailed LES-simulation is already planned.

Another demonstration is given in Fig. 5. It shows the simulation of an electromagnetically levitated (EML) drop of silicon. The simulation and its parameters were chosen according to the experimental setup in [32]. In the related experiment, the levitated drop is held within a helical coil, which is modeled as five independent, toroidal loops. Due to the more complex coil geometry this simulation was realized using the reduced magnetic vector potential in combination with Biot-Savart’s law as mentioned above. In this case, we have deployed LES-modeling with a corresponding spherical shaped mesh of \( \approx 1 \times 10^6 \) cells. The simulation runs revealed that within one CPU-hour of calculation time, \( \approx 0.01 \) s of simulation time could be achieved.

\[ 0.4 \text{ m/s} \]
\[ 0 \text{ } \]
\[ 3 \times 10^5 \text{ N/m}^3 \]

**Figure 5.** Numerical simulation of the induced electromagnetic force (left), the flow (right) and the shape deformation of a levitated liquid metal drop using the complete coupled code `interTrackEddyCurrentFoam` and the setup from [32].

### 5. Conclusions

We have presented a newly developed multi-physics method for the simulation of free-surface flows under the influence of alternating magnetic fields, which has been implemented in OpenFOAM (foam-extend). However, due to the complexity of the whole modeling we could only provide an overview of it in the scope of this paper. The key novelty is that electromagnetic and hydrodynamic effects are addressed in one single framework named `interTrackEddyCurrentFoam`. With our idea of using multiple overlapping meshes to capture individual physical effects, we have successfully created a versatile tool for a broad field of multi-physical applications, which is fully parallelized. As part of our work, a new solver for eddy-current problems called `eddyCurrentFoam` on the basis of the finite volume method was developed and validated. Additionally, the existing implementation of the surface-tracking algorithm of `interTrackFoam` has been improved and extended. In future work we would like address remaining problems like mesh-quality degradation of long simulation runs, time-step restrictions and general stability issues. This should also be accompanied with an extensive validation based on numerical and experimental data.

Concerning the RGS process, it was illustrated that a retention effect is correlated with the electromagnetically driven fluid flow. A parameter study revealed the total system inductivity as a function of the melt level, which might be usable for a fill-level sensing of the silicon melt in this process. Preliminary simulations of the flow and surface deformations including all relevant effects were conducted for the model geometry of the RGS process. A detailed insight into the
flow and its surface dynamics is a key for controlling the RGS process by means of tailored magnetic fields which can be achieved by different coil geometries, power supply parameters as well as by a different electrical conductivity of the moving substrate. Such parameters studies and more detailed LES simulations with the help of our software may be used for further optimizing the RGS process.

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References
[1] Lange H and Schwirtlich I A 1990 Journal of Crystal Growth 104 108–112
[2] Schönecker A, Geerligs L and Müller A 2003 Gettering and Defect Engineering in Semiconductor Technology – X (Solid State Phenomena vol 95) (Trans Tech Publications) pp 149–158
[3] Hahn G and Schönecker A 2004 Journal of Physics - Condensed matter 16 R1615–R1648
[4] Beckstein P, Galindo V and Gerbeth G 2016 Proc. of the 10th PAMIR International Conference - Fundamental and Applied MHD (Cagliari, Italy) pp 257–261
[5] Santara F, Delannoy Y and Autruffe A 2012 Journal of Crystal Growth 340 41–46
[6] Jeong H M, Chung H S and Lee T W 2010 Journal of Crystal Growth 312 555–562
[7] Steinbach I, Apel M, Rettelbach T and Franke D 2002 Solar energy materials and solar cells 72 59–68
[8] Beckstein P, Galindo V and Gerbeth G 2015 Magnetohydrodynamics 51 385–396
[9] Beckstein P, Galindo V, Gerbeth G and Schönecker A 2015 Proc. of the 8th international conference on electromagnetic processing of materials EPM pp 167–170
[10] OpenCFD Ltd. 2015 OpenFOAM - The Open Source CFD Toolbox - User’s Guide OpenCFD Ltd. United Kingdom 3rd ed
[11] Spitans S, Jakovics A, Baake E and Nacke B 2011 Magnetohydrodynamics 47 385–397
[12] Spitans S, Jakovics A, Baake E and Nacke B 2013 Metallurgical and Materials Transactions B 44 593–605
[13] Spitans S, Baake E, Nacke B and Jakovics A 2014 International Journal of Applied Electromagnetics and Mechanics 44 171–182
[14] Wikki Ltd. 2017 The foam-extend project 3rd ed URL http://www.foam-extend.org
[15] Beckstein P, Galindo V and Gerbeth G 2017 International Journal of Applied Electromagnetics and Mechanics 53 S43–S51
[16] Weber N, Beckstein P, Herremann W, Horstmann G, Nore C, Stefani F and Weier T 2017 Physics of Fluids 29 054101
[17] Beckstein P, Galindo V and Gerbeth G 2017 11th OpenFOAM Workshop ed Nobrega J M and Jasak H (Springer) accepted
[18] Karypis G 2003 Proceedings of the 2003 ACM/IEEE Conference on Supercomputing SC ’03 (New York, NY, USA: ACM) pp 56– ISBN 1-58113-695-1 URL http://doi.acm.org/10.1145/1049335.1050206
[19] Beckstein P, Galindo V and Vukcevic V 2017 Journal of Computational Physics 344 623–646
[20] Djambazov G, Bojarevics V, Pericéous K and Croft N 2015 Applied Mathematical Modelling 39 4733–4745 ISSN 0307904X
[21] Vukcevic V, Jasak H and Gatin I 2017 Computers & Fluids 153 1–19
[22] Moreau R 1990 Magnetohydrodynamics (Springer) ISBN 978-0-7923-0937-6
[23] Binns K J, Lawrenson P J and Trowbridge C W 1992 The Analytical and Numerical Solution of Electric and Magnetic Fields (Wiley) ISBN 978-0-471-92460-9 URL
[24] Biro O and Preis K 2000 IEEE Transactions on Magnetics 36 3128–3130
[25] Davidson P A and Hunt J C R 1987 Journal of Fluid Mechanics 185 67–106
[26] Hughes T J, Liu W K and Zimmermann T K 1981 Computer Methods in Applied Mechanics and Engineering 29 329 – 349
[27] Tukovic Z and Jasak H 2012 Computers & Fluids 55 70–84
[28] Cobham plc. 2016 Opera-3D Design Software URL http://www.operafea.com
[29] Grants I and Gerbeth G 2004 Journal of Crystal Growth 269 630–638
[30] Grants I, Galindo V and Gerbeth G 2013 The European Physical Journal Special Topics 220 215–225
[31] Pal J, Cramer A, Gundrum T and Gerbeth G 2009 Flow Measurement and Instrumentation 20 241–251
[32] Gao L, Shi Z, Li D, Yang Y, Zhang G, McLean A and Chattopadhyay K 2016 Metallurgical and Materials Transactions B 47 67–75