Estimation of mass spectra and size of heavy quarkonium using $N$-dimensional Schrödinger equation with Killingbeck potential model via Laplace transform approach

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Abstract

The second order $N$-dimensional Schrödinger equation with Killingbeck potential, namely

$$V(r) = ar^2 + br - \frac{c}{r}, a > 0$$

is examined via Laplace transform approach. The stationary states are determined by good behavior of eigenfunctions at the origin and at infinity. Energy eigenvalue equation has been used to determine the mass spectra of heavy quarkonium system, specially for lower dimension $N = 3$. The eigenfunctions are used to determine the radii of the bound states. The present results suggest that the bound states $1P, 2S, 3S$ of $(\Upsilon(b\bar{b}), \psi(c\bar{c}))$ are more tightly bound than they are supposed in non relativistic hadronic study.

Keywords: Killingbeck potential, Laplace transformation approach (LTA), Schrödinger equation (SE), Quarkonium

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I. Introduction

The exact bound state solutions of the non relativistic radial Schrödinger equation with spherically symmetric potentials play an important role in atomic, molecular and hadronic spectroscopy. Over the past years, theoretical physicists have shown a great deal of interest in solving multidimensional Schrödinger equation for various physically accepted spherically symmetric potentials \([1-9]\). Besides the conventional series solution method\([10-11]\), several techniques like Nikiforov-Uvarov method \([12-14]\), Fourier transform method \([15-17]\), Shifted \(\frac{1}{N}\) expansion method \([18-19]\), Asymptotic iteration method (AIM)\([20-22]\), Super-symmetric approach (SUSY) \([23]\), Quasi-linearization method (QLM)\([24]\), Point canonical transformation (PCT) \([25]\), Hill determinant method \([26]\), Numerical method \([27-29]\) have been used to solve Schrödinger equation for lower as well as for arbitrary dimensions.

There is another very easy and efficient method commonly known as Laplace transformation approach (LTA). This was first used by Schrödinger to derive the radial eigenfunctions of the Hydrogen atom \([30]\). Later Englefield used LTA to solve the coulomb, oscillator, exponential and Yamaguchi potentials \([31]\). The success of this approach lies in reduction of the second order differential equation into first order equation, which is easy to solve by standard methods. Some recent works on LTA can be found in the reference list \([32-37]\).

The main goal of this paper is to review the bound state eigenfunctions and energy eigenvalues of \(N\)-dimensional hyperradial Schrödinger equation with Killingbeck potential\([38-40]\) via Laplace transform approach and also to determine the mass spectra and binding radii of heavy quarkonium system \((\Upsilon(\bar{b}b), \psi(\bar{c}c))\). Quarkonia are bound state of heavy quark and its antiquark with narrow decay width into open charm or bottom via strong interaction channel. They have been served as one of the most important probes to study the properties of the quark-gluon plasma for a long time. Killingbeck potential, also known as Cornell plus harmonic potential, is supposed to be responsible for the interaction between quark and antiquark. Solution of this kind of potential is quite necessary to study the electromagnetic behavior of mesons as well as the binding and dissociation of \((\Upsilon(\bar{b}b), \psi(\bar{c}c))\).
II. Overview of Laplace Transform

The Laplace transform $\phi(s)$ or $\mathcal{L}$ of a function $f(t)$ is defined by [41-42]

$$\phi(s) = \mathcal{L} \{ f(t) \} = \int_0^\infty e^{-st} f(t) dt . \quad (1)$$

If there is some constant $\sigma \in \mathbb{R}$ such that $|e^{-\sigma t} f(t)| \leq M$ for sufficiently large $t$, the integral in Eq.(1) will exist for $\text{Re } s > \sigma$. The Laplace transform may fail to exist because of a sufficiently strong singularity in the function $f(t)$ as $t \to 0$. In particular

$$\mathcal{L} \left[ \frac{t^\alpha}{\Gamma(\alpha + 1)} \right] = \frac{1}{s^{\alpha + 1}} , \alpha > -1 . \quad (2)$$

The Laplace transform has the derivative properties

$$\mathcal{L} \{ f^{(n)}(t) \} = s^n \mathcal{L} \{ f(t) \} - \sum_{k=0}^{n-1} s^{n-1-k} f^{(k)}(0) , \quad (3)$$

$$\mathcal{L} \{ t^n f(t) \} = (-1)^n \phi^{(n)}(s) , \quad (4)$$

where the superscript $(n)$ denotes the $n$-th derivative with respect to $t$ for $f^{(n)}(t)$, and with respect to $s$ for $\phi^{(n)}(s)$. If $s_0$ is the singular point, the Laplace transformation behaves as for $s \to s_0$

$$\phi(s) = \frac{1}{(s - s_0)^v} , \quad (5)$$

then for $t \to \infty$

$$f(t) = \frac{1}{\Gamma(v)} t^{v-1} e^{s_0 t} , \quad (6)$$

where $\Gamma(v)$ is the gamma function. On the other hand, if near origin $f(t)$ behaves like $t^\alpha$, with $\alpha > -1$, then $\phi(s)$ behaves near $s \to \infty$ as

$$\phi(s) = \frac{\Gamma(\alpha + 1)}{s^{\alpha + 1}} . \quad (7)$$

III. Bound State Spectrum

The time independent Schrödinger equation for a particle of mass $M$ in $N$-dimensional space has the form [10,36,43]

$$- \frac{\hbar^2}{2M} \nabla_N^2 \psi + V \psi = E \psi , \quad (8)$$
where \( \nabla^2_N \) is the Laplacian operator in the polar coordinates \((r, \theta_1, \theta_2, \ldots, \theta_{N-2}, \varphi)\) of \( \mathbb{R}^N \). Here \( r \) is the hyperradius and \( \theta_1, \theta_2, \ldots, \theta_{N-2}, \varphi \) are the hyperangles. The form of \( \nabla^2_N \) is given by

\[
\nabla^2_N = r^{1-N} \frac{\partial}{\partial r} \left( r^{N-1} \frac{\partial}{\partial r} \right) + \frac{\Lambda^2_N(\Omega)}{r^2},
\]

where \( \Lambda^2_N(\Omega) \) is the hyperangular momentum operator \([36,37,43]\) given by,

\[
\Lambda^2_N = -\sum_{i,j=1, i>j}^N \Lambda^2_{ij}, \quad \Lambda_{ij} = x_i \frac{\partial}{\partial x_j} - x_j \frac{\partial}{\partial x_i},
\]

for all Cartesian components \( x_i \) of the \( N \)-dimensional vector \((x_1, x_2, \ldots, x_N)\). Applying the separation variable method in Eq. (8) and taking the separation constant as \( \ell(\ell + N - 2)|_{N>1} \) \([44]\) with \( \ell = 0, 1, 2, \ldots \), the \( N \)-dimensional hyperradial or in short the “radial” Schrödinger equation becomes

\[
\left[ \frac{d^2}{dr^2} + \frac{N-1}{r} \frac{d}{dr} - \frac{\ell(\ell + N - 2)}{r^2} + 2\mu \left[ E - V(r) \right] \right] R(r) = 0, \tag{10}
\]

where \( E \) is the energy eigenvalue and \( \ell \) is the orbital angular momentum quantum number.

The above equation is equally applicable for two-particle system if \( M \) is replaced by the reduced mass \( \mu = \frac{m_1 m_2}{m_1 + m_2} \) where \( m_1 \) and \( m_2 \) are masses of the particles. Selecting natural unit \( \hbar = c = 1 \)

Eq.(10) can be rewritten as

\[
\left[ \frac{d^2}{dr^2} + \frac{N-1}{r} \frac{d}{dr} - \frac{\ell(\ell + N - 2)}{r^2} + 2\mu[E - V(r)] \right] R(r) = 0. \tag{11}
\]

Inserting the Killingbeck potential having the form

\[
V(r) = ar^2 + br - \frac{c}{r}, \quad \text{with} \ a > 0, \tag{12}
\]

Eq.(11) becomes

\[
\left[ \frac{d^2}{dr^2} + \frac{N-1}{r} \frac{d}{dr} - \frac{\ell(\ell + N - 2)}{r^2} + 2\mu[E - (ar^2 + br - \frac{c}{r})] \right] R(r) = 0. \tag{13}
\]

In order to guess the eigenfunction at the special limit \( r \to \infty \) we have

\[
\frac{d^2 R}{dr^2} - 4\alpha^2 r^2 R = 0, \tag{14}
\]
where the fact \( ar^2 >> br \) for large \( r \) has been assumed. The term \( 4\alpha^2 \) is for mathematical convenience with
\[
\alpha = \sqrt{\frac{\mu a}{2}}. \tag{15}
\]
The solution of the Eq.(14) is easy. Remembering the fact \( r^2 >> 1 \), we can take the solution as
\[
R(r) \propto e^{-\alpha r^2}. \tag{16}
\]
Now we are in a position to guess a complete solution of Eq.(13). The solution should be bounded in the origin. Let us try a solution of type
\[
R(r) = r^k e^{-\alpha r^2} f(r), \quad k > 0. \tag{17}
\]
The term \( r^k \) assures that, the solution at \( r = 0 \) is bounded. The function \( f(r) \) yet to be determined. Simple derivatives of Eq.(17) we get
\[
\frac{dR(r)}{dr} = r^k e^{-\alpha r^2} \left\{ \frac{k}{r} f(r) + \frac{df(r)}{dr} - 2\alpha r f(r) \right\},
\]
\[
\frac{d^2R(r)}{d^2r} = r^k e^{-\alpha r^2} \left\{ \frac{d^2f(r)}{d^2r} + \left( \frac{2k}{r} - 4\alpha r \right) \frac{df(r)}{dr} + \left( \frac{k(k-1)}{r^2} - 4\alpha k - 2\alpha + 4\alpha^2 r^2 \right) f(r) \right\}.
\]
Taking them into Eq.(13) and using Eq.(15) together, we have
\[
rf''(r) + \left\{ \frac{k(k-1) + k(N-1) - \ell(\ell + N - 2)}{r} - 2\mu br^2 + 2\mu c + r(2\mu E - 2\alpha N - 4\alpha \ell) \right\} f(r)
\]
\[
+ \left\{ 2\mu \alpha + 2\mu br^2 + r(2\mu E - 2\alpha N - 4\alpha \ell) \right\} f(r) = 0, \tag{18}
\]
were the prime over \( f(r) \)denotes the derivative with respect to \( r \).To get the Laplace transform of the above differential equation we must choose the parametric condition
\[
k(k-1) + k(N-1) - \ell(\ell + N - 2) = 0. \tag{19}
\]
This gives \( k_+ = \ell \) and \( k_- = -(\ell + N - 2) \). The acceptable physical value of \( k \) is \( k_+ = \ell \). Finally we have
\[
rf''(r) + \left\{ (2\ell + N - 1) - 4\alpha r^2 \right\} f'(r)
\]
\[
+ \left\{ 2\mu c - 2\mu br^2 + r(2\mu E - 2\alpha N - 4\alpha \ell) \right\} f(r) = 0. \tag{20}
\]
Now we are in position to get a Laplace transform of the above differential equation. Introducing the Laplace transform $\phi(s) = \mathcal{L}\{f(r)\}$ and taking the boundary condition $f(0) = 0$, the derivative properties of Laplace transform Eq.(3) and Eq.(4) give

$$(s + \beta)\frac{d^2\phi(s)}{ds^2} + \left(\frac{1}{4\alpha}s^2 + \lambda\right)\frac{d\phi(s)}{ds} + (\gamma s - \frac{\mu c}{2\alpha})\phi(s) = 0,$$

where the following abbreviations are used

$$\beta = \frac{\mu b}{2\alpha} ; \lambda = \frac{\mu E}{2\alpha} - \left(\ell + \frac{N}{2} - 2\right) ; \gamma = \frac{3 - 2\ell - N}{4\alpha}.$$

$s = -\beta$ is the singular point of the Eq.(21). Using the fact of Eq.(5) the solution of Eq.(21) can be taken

$$\phi(s) = \frac{C_{n\ell N}}{(s + \beta)^{n+1}}, n = 0, 1, 2, 3, \ldots$$

Inserting the Eq.(23) in Eq.(21) we get the following three identity relations,

$$\gamma = \frac{n + 1}{4\alpha},$$

$$\gamma\beta = \frac{\mu c}{2\alpha},$$

$$(n + 1)(n + 2) - (n + 1)\lambda - \frac{\mu c}{2\alpha}\beta = 0.$$

Using the set of Eq.(24-26) and Eq.(22), the energy eigenvalues becomes

$$E_{n\ell N} = \sqrt{\frac{a}{2\mu}}(2n + 2\ell + N) - \frac{b^2}{4a}.$$  

This result is very much similar to the ones obtained in [11, 22, 26, 49] before. The undermined function $f(r)$ can be found from the inverse Laplace transform such that $f(r) = \mathcal{L}^{-1}\{\phi(s)\}$ [41, 42]. Using the Eq.(6) we get

$$f(r) = \frac{C_{n\ell N}}{n!}r^ne^{-\beta r}.$$  

Finally using Eq.(17), Eq.(28) and $k_+ = \ell$ it is east to determine the corresponding eigenfunctions of the system as

$$R_{n\ell N}(r) = \frac{C_{n\ell N}}{n!}r^{\ell+n}\exp(-\sqrt{\frac{\mu a}{2}}r^2 - b\sqrt{\frac{\mu}{2a}}r).$$
The normalization constant $C_{\ell n}N$ can be obtained from the condition [11,36,37]

$$
\int_0^\infty [R_{\ell n}(r)]^2 r^{N-1} dr = 1 .
$$

(30)

A fair approximation $r + \frac{\beta}{2\alpha} \approx r$ will make the integration much more easy to evaluate. The use of integral formula

$$
\int_0^\infty x^p e^{-Ax^q} dx = \frac{1}{q^2} \frac{\Gamma(p+q)}{A^{p+q}} , \quad p, q > 0,
$$

gives

$$
C_{\ell n}N = n! \left\{ \frac{2(2\alpha)^{\ell+n+\frac{N}{2}}}{\Gamma(\ell+n+\frac{N}{2})} e^{-\frac{a^2}{2\alpha}} \right\}^{\frac{1}{2}} .
$$

(31)

In case of three dimensional isotropic harmonic oscillator $V(r) = \frac{1}{2} \mu \omega r^2 = \alpha r^2$ and $b = c = 0$. Within the frame work of natural unit, Eq.(27) gives the energy eigenvalue $E = \omega (n' + \frac{3}{2})$, which is a very common result with the definition of quantum number $n' = n + \ell$.

IV. Useful Applications Based On The Results ($N = 3$)

1. Mass spectra of heavy quarkonium systems

Here we can formulate the mass spectra of the heavy quarkonium systems consisting of a quark and antiquark of same flavor in three dimensions. The reduced mass for charmonium system ($m_1 = m_2 = m_c$) is $\mu = \frac{m_c}{2}$ and the same for bottomonium system ($m_1 = m_2 = m_b$) is $\mu = \frac{m_b}{2}$. The mass spectra can be given by the relation [45]

$$
\mathcal{M} = 2m + E_{n\ell 3} .
$$

(32)

Using Eq.(27) the mass spectra of bottomonium can be given by

$$
\mathcal{M}_b = 2m_b + \sqrt{\frac{a}{m_b}} (2n + 2\ell + 3) - \frac{b^2}{4a} .
$$

(33)

Replacing the mass $m_b$ by $m_c$ one can find the mass spectra of charmonium $\mathcal{M}_c$ also.
### TABLE I: Mass Spectra of Bottomonium

\( (m_b = 4.68\text{GeV}, a = 0.143\text{GeV}^3, b = 0.465\text{GeV}^2) \) in GeV

| States | Parameter \( c \) | \( M_b \) From Eq. (33) | Reference: [46] |
|--------|-----------------|--------------------------|-----------------|
| 1S     | 0.284           | 9.506                    | 9.460           |
| 1P     | 0.284           | 9.856                    | 9.900           |
| 2S     | 0.568           | 9.856                    | 10.023          |
| 1D     | 0.284           | 10.205                   | 10.161          |
| 2P     | 0.568           | 10.205                   | 10.260          |
| 3S     | 0.853           | 10.205                   | 10.355          |
| 4S     | 1.137           | 10.555                   | 10.580          |

### TABLE II: Mass Spectra of Charmonium

\( (m_c = 1.48\text{GeV}, a = 0.042\text{GeV}^3, b = 0.255\text{GeV}^2) \) in GeV

| States | Parameter \( c \) | \( M_c \) From Eq. (33) | Reference: [46] |
|--------|-----------------|--------------------------|-----------------|
| 1S     | 0.511           | 3.078                    | 3.068           |
| 1P     | 0.511           | 3.415                    | 3.525           |
| 2S     | 1.023           | 3.415                    | 3.663           |
| 1D     | 0.511           | 3.752                    | 3.770           |
| 2P     | 1.023           | 3.752                    | –               |
| 3S     | 1.534           | 3.752                    | 4.159           |
| 4S     | 2.045           | 4.089                    | 4.421           |

2. **Radius of bound state**

One of the most important feature of quarkonia is their small size. Compared with the typical hadron radius 1 fm, the radii of charmonium and bottomonium range from
The eigenfunctions play a key role in calculation of \( r.m.s \) radii of the bound states of \( b\bar{b} \) and \( c\bar{c} \) system. To determine the \( r.m.s \) radius of the bound state, we first evaluate

\[
<r^2> = \int_0^\infty r^2 [R_{n\ell 3}(r)]^2 r^2 dr. \tag{34}
\]

Using \( C_{n\ell 3} \) from Eq.(31) and the same approximation as done after Eq.(30), the \( r.m.s \) radius becomes

\[
r_{rms} = \sqrt{<r^2>} = \sqrt{\frac{\ell + n + \frac{3}{2}}{2\alpha}} \times 0.1973 \text{fm}. \tag{35}
\]

The above formula for the \( r.m.s \) radius has been used to construct the table:

### TABLE III: \( r.m.s \) radii of the \( b\bar{b} \) and \( c\bar{c} \) bound states in fm

| \( q\bar{q} \) | States         | Reference:[47] | Reference:[48] | Our Results from Eq.(35) |
|--------------|----------------|----------------|----------------|-------------------------|
| \( b\bar{b} \) | 1S(\( \Upsilon \)) | 0.2249         | 0.2211         | 0.2672                  |
|              | 2S(\( \Upsilon' \)) | 0.5040         | 0.4998         | 0.3449                  |
|              | 3S(\( \Upsilon'' \)) | 0.7336         | 0.7457         | 0.4081                  |
|              | 1P(\( \chi_b \)) | 0.4041         | 0.3982         | 0.3449                  |
| \( c\bar{c} \) | 1S(\( J/\psi \)) | 0.4490         | 0.4453         | 0.4839                  |
|              | 2S(\( \psi' \)) | 0.8655         | 0.9034         | 0.6247                  |
|              | 3S(\( \psi'' \)) | 1.2025         | 1.3765         | 0.7392                  |
|              | 1P(\( \chi_c \)) | 0.6890         | 0.7000         | 0.6247                  |
V. Conclusions

Firstly, the energy eigenvalues of the quarkonium systems and hence the mass spectra of $b\bar{b}$ and $c\bar{c}$ bound states are obtained. The obtained analytical results are in close agreement with the experimental results and the results of other studies\cite{22,45,47-49}. Being a non-relativistic model it is a fair achievement. The presence of degeneracy level in the energy states as well as in mass spectra confirm that entire phenomenon of the quarkonium system yet to be known completely. May be proper choice of quantum number or the relativistic two-body equation, like Bethe-Salpeter (BS) equation, will provide a more close satisfactory results with this same potential model.

Secondly, r.m.s radii of few bound states of $b\bar{b}$ and $c\bar{c}$ are calculated based on the eigenfunctions that we have derived. Killingbeck potential model provides the confirmation that $J/\psi$ and $\Upsilon$ states are most tightly bound states. Moreover, the present model of the potential suggests that $1P, 2S, 3S$ states of $b\bar{b}$ as well as $c\bar{c}$ are more tightly bound than they are supposed in literature \cite{47,48}. This signifies about higher critical temperature and density for them to dissociate due to the mechanism of color screening.

However, it is worth mentioning that entire present evaluations are based on non relativistic model. Though they are capable of discussing the properties of quarkonium in the quark-gluon plasma, relativistic quantum mechanical approach will surely give far more better realistic results. Generally speaking, the motion of quark and antiquark in meson is relativistic, even for charmonium. It is well known fact that Bethe-Salpeter equation (BS) is the only effective relativistic equation of two-body bound state and it is very much consistent with the quantum field theory. The study of Killingbeck potential via BS equation may give far more promising results and we will have to proceed future study for that.
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