Whistler precursor and intrinsic variability of quasi-perpendicular shocks

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The structure of whistler precursor in a quasi-perpendicular shock is studied within two-fluid approach in one-dimensional case. The complete set of equations is reduced to the KdV equation, if no dissipation is included. With a phenomenological resistive dissipation the structure is described with the KdV-Burgers equation. The shock profile is intrinsically time dependent. For sufficiently strong dissipation, temporal evolution of a steepening profile results in generation of a stationary decaying whistler ahead of the shock front. With the decrease of the dissipation parameter whistler wavetrains begin to detach and propagate toward upstream and the ramp is weakly time dependent. In the weakly dissipative regime the shock front experiences reformation.

1. Introduction

Collisionless shocks, even at low-Mach numbers, are complicated structures. The majority of the shocks, encountered in the heliosphere, are fast magnetized shocks, where the shock transition is mainly determined by the rapid increase of the magnetic field (see, e.g. Kennel \textit{et al.} 1985, for a review). While the magnetic jump in shocks must be accompanied by the drop of the fluid speed from super-magnetosonic to sub-magnetosonic, by the jump in the ion and electron density, and by the substantial increase of temperatures of both species, it is the magnetic field which is a) measured with the highest precision, and b) serves as the main signature of the shock. Knowledge of the structure of the magnetic profile is of utmost importance, since the macroscopic magnetic field is the most important factor affecting the particle motion throughout the shock. The most important parameters defining the shock behavior are thought to be the Mach number \(M\) (defined below), the angle \(\theta\) between the shock normal and the upstream magnetic field, and the ratio of the upstream kinetic-to-magnetic pressure \(\beta = \frac{8\pi p_u}{B_u^2}\). Here \(p_u = n_u T_u\) is the upstream plasma pressure and \(B_u\) is the upstream magnetic field. Hereafter the subscript \(u\) refers to the upstream values and \(d\) refers to the downstream parameters. Low-Mach number low-\(\beta\) shocks are believed to be laminar, that is, to possess an almost monotonic increase of the magnetic field with nearly no magnetic oscillations (Greenstadt \textit{et al.} 1975; Mellott \& Greenstadt 1984; Farris \textit{et al.} 1993). Recently, it was shown that even in very low-Mach number shocks downstream magnetic oscillations are caused by ion gyration (Balikhin \textit{et al.} 2008; Russell \textit{et al.} 2009; Ofman \textit{et al.} 2009; Kajdič \textit{et al.} 2012; Ofman \& Gedalin 2013; Gedalin \textit{et al.} 2015a). Low-Mach number shocks also often exhibit magnetic oscillations upstream of the ramp (Balikhin \textit{et al.} 2003; Blanco-Cano \textit{et al.} 2006; Wilson \textit{et al.} 2007, 2009; Ramírez Vélez \textit{et al.} 2012; Blanco-Cano \textit{et al.} 2013, 2016; Wilson III \textit{et al.} 2017). The latter are typically identified as low-frequency electromagnetic whistlers (Ramírez Vélez \textit{et al.} 2012). Whistler generation is quite typical for supercritical shocks (Walker \textit{et al.} 2011).

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2. Basic equations

The analysis will be done within two-fluid plasma theory with massless electrons, since the typical spatial scales are much larger than the electron inertial length and the typical temporal scales are much larger than the electron gyroperiod or inverse electron plasma frequency. For the same reason quasineutrality, $n_e = n_i = n$, is maintained with high precision. The speeds are well below the speed-of-light, so that the displacement current is negligible. We also assume that the system is one-dimensional, that is, all plasma variables and fields depend only on the coordinate $x$ along the shock normal and on time.
t. With these approximations the two-fluid equations take the form

\[ \partial_t n + \partial_x (n v_x) = 0 \quad (2.1) \]

\[ n m_i (\partial_t v_x + v_x \partial_x v_x) = ne (E_x + \hat{x} \cdot (v_i \times B)) / c - \partial_x p_i \quad (2.2) \]

\[ 0 = -en (E_x + \hat{x} \cdot (v_e \times B)) / c - \partial_x p_e \quad (2.3) \]

\[ n m_i (\partial_t v_{i\perp} + v_x \partial_x v_{i\perp}) = ne (E_{\perp} + v_x \hat{x} \times B_{\perp} / c + B_x v_{i\perp} \times \hat{x} / c) - n\nu (v_{i\perp} - v_{e\perp}) \quad (2.4) \]

\[ 0 = -en (E_{\perp} + v_x \hat{x} \times B_{\perp} / c + B_x v_{e\perp} \times \hat{x} / c) - n\nu (v_{e\perp} - v_{i\perp}) \quad (2.5) \]

\[ c \partial_x (\hat{x} \times B_{\perp}) = 4\pi ne (v_{i\perp} - v_{e\perp}) \quad (2.6) \]

\[ \partial_t B_{\perp} = -c \partial_x (\hat{x} \times E_{\perp}) \quad (2.7) \]

\[ B_x = \text{const}, \quad v_{ex} = v_{ix} = v_x \quad (2.8) \]

Here \( n, v_x, \) and \( v_{\perp} \) are the ion density and velocity, \( B_x \) and \( B_{\perp} \) are the magnetic field components, while \( E_x \) and \( E_{\perp} \) are the electric field components. The subscript \( \perp \) denotes components perpendicular to the shock normal: \( v_{\perp} = (v_y, v_z), B_{\perp} = (B_y, B_z), E_{\perp} = (E_y, E_z). \) Here \( \perp \) refers to the direction \( \hat{x}. \) We shall assume that \( p_e \) and \( p_i \) are functions of the density only. The last terms in (2.4) and (2.5) describe the friction (momentum exchange) between the two species which conserves the total momentum. Indeed, the following conservation laws can be easily derived:

\[ \partial_t n + \partial_x (n v_x) = 0 \quad (2.9) \]

\[ \partial_t (n m_i v_x) + \partial_x (n m_i v_x^2 + p + B_i^2 / 8\pi) = 0 \quad (2.10) \]

\[ \partial_t (n m_i v_{i\perp}) + \partial_x (n m_i v_{x\perp} v_{i\perp} - B_x B_{\perp} / 4\pi) = 0 \quad (2.11) \]

which are the particle number and the momentum conservation and \( p = p_i + p_e. \) Eqs. (2.5) and (2.6) give

\[ c (\hat{x} \times E_{\perp}) = v_x B_{\perp} - B_x v_{i\perp} + \frac{c B_x}{4\pi ne} \partial_x (\hat{x} \times B_{\perp}) - \frac{c^2 \nu}{4\pi ne^2} \partial_x B_{\perp} \quad (2.12) \]

so that (2.7) takes the form

\[ \partial_t B_{\perp} + \partial_x (v_x B_{\perp} - B_x v_{i\perp}) + \partial_x \left( \frac{c B_x}{4\pi ne} \partial_x (\hat{x} \times B_{\perp}) - \frac{c^2 \nu}{4\pi ne^2} \partial_x B_{\perp} \right) = 0 \quad (2.13) \]

Let us choose some point as a reference point. It is always possible to switch to a reference frame such that in the reference point \( v_{i\perp} = 0. \) It is always possible to rotate to a coordinate system around axis \( x \) in such a way so that in the reference point \( B_y = 0. \) We shall denote the other variables in the reference point as follows: \( n = n_0, B_x = B_0 \cos \theta, B_z = B_0 \sin \theta, \) and \( p = p_0. \)

We further define

\[ v_A^2 = \frac{B_0^2}{4\pi n_0 m_i}, \quad \omega_{pi}^2 = \frac{4\pi n_0 e^2}{m_i}, \quad \Omega = \frac{e B_0}{m_i c}, \quad \beta = \frac{8\pi p_0}{B_0^2}, \quad M = \frac{v_0}{v_A} \quad (2.14) \]

It is convenient to introduce the normalized variables as follows:

\[ T = \Omega t, \quad X = \frac{\omega_{pi} x}{c}, \quad N = \frac{n}{n_0}, \quad V = \frac{v}{v_A}, \quad (2.15) \]

\[ u = \frac{v_{i\perp}}{v_A}, \quad b = \frac{B_{\perp}}{B_0} \quad (2.16) \]
We shall also assume the polytropic state equation \( p/n^\gamma = \text{const.} \). In the normalized form the equations are written as follows:

\[
\begin{align*}
\partial_T N + \partial_X (Nv) &= 0 \\
\partial_T (Nv) + \partial_X \left( Nv^2 + \frac{\beta N^\gamma + b^2}{2} \right) &= 0 \\
\partial_T (N\mathbf{u}) + \partial_X (N\mathbf{u} - \cos \theta \mathbf{b}) &= 0 \\
\partial_T \mathbf{b} + \partial_X (v\mathbf{b} - \cos \theta \mathbf{u}) \\
&\quad + \cos \theta \partial_X \left( N^{-1} \partial_X (\hat{x} \times \mathbf{b}) \right) - \mu \partial_X (N^{-1} \partial_X \mathbf{b}) = 0
\end{align*}
\]

It is useful to write down the above equations using the variables \( J = Nv \), and \( W = nu \):

\[
\begin{align*}
\partial_T N + \partial_X J &= 0 \\
\partial_T J + \partial_X \left( J^2 \frac{N}{2} + \frac{\beta N^\gamma + b^2}{2} \right) &= 0 \\
\partial_T W + \partial_X \left( W \frac{N}{2} - \cos \theta \mathbf{b} \right) &= 0
\end{align*}
\]

3. Stationary states

If there is no time dependence we arrive at the following conservation laws

\[
\begin{align*}
J &= M \\
\frac{M^2}{N} \frac{N}{2} + \frac{\beta N^\gamma + b_y^2 + b_z^2}{2} &= M^2 + \frac{\beta + \sin^2 \theta}{2} \\
\frac{W_y}{N} - \cos \theta b_y &= 0 \\
\frac{W_z}{N} - \cos \theta b_z &= -\cos \theta \sin \theta
\end{align*}
\]

while the remaining equations for \( b_y \) and \( b_z \) take the form

\[
\begin{align*}
Mb_y - \cos \theta W_y - \cos \theta \partial_X b_z - \mu \partial_X b_y &= C_y \\
Mb_z - \cos \theta W_z + \cos \theta \partial_X b_y - \mu \partial_X b_z &= C_z
\end{align*}
\]

where the constants \( C_y \) and \( C_z \) should be determined by additional conditions in the reference point. Since we are particularly interested in solutions which asymptotically converge to an uniform state, we take the reference point at \( x \to \pm \infty \) and assume that all derivatives vanish there, that is, \( C_y = 0 \) and \( C_z = \sin \theta \). Eventually, one has

\[
\frac{1}{N} + \frac{\beta N^\gamma}{2M^2} + \frac{b^2}{2M^2} = 1 + \frac{\beta + \sin^2 \theta}{2M^2} \\
\frac{b_y}{M^2} (M^2 - N \cos^2 \theta) - \cos \theta \partial_X b_z - \mu \partial_X b_y &= 0 \\
b_z (M^2 - N \cos^2 \theta) + \cos \theta \partial_X b_y - \mu \partial_X b_z = N \sin \theta (M^2 - \cos^2 \theta)
\]\n
We start with the analysis of the stationary point by linearizing the equations for \( N = 1 + \delta N \), \( b_y = \sin \theta \pm \delta b_x \), \( b_y = \delta b_y \) and searching for solutions \( \propto \exp(kx) \) (see, e.g. Gedalin 1998). One has

\[
(M^2 - \cos^2 \theta - k\mu)(M^2 - \cos^2 \theta - k\mu)(M^2 - \beta \gamma/2) \\
- M^2 \sin^2 \theta (M^2 - \cos^2 \theta) + k^2 \cos^2 \theta (M^2 - \beta \gamma) = 0
\]
It is easy to recognize the MHD speeds (normalized on the Alfven speed):

\[ v_I^2 = \cos^2 \theta, \quad v_s^2 = \beta \gamma / 2, \quad v_{F,SL}^2 = \frac{1}{2}[(1 + v_s^2) \pm \sqrt{(1 + v_s^2)^2 - 4v_s^2 \cos^2 \theta}] \]  

(3.11)

\[ v_F^2 > v_I^2 > v_{SL}^2, \quad v_F^2 > v_s^2 > v_{SL}^2 \]  

(3.12)

then (3.10) takes the form

\[ (\cos^2 \theta + \mu^2) k^2 - k \mu L_1 + L_2 = 0 \]  

(3.13)

\[ L_1 = \frac{(M^2 - v_f^2)(M^2 - v_s^2) + (M^2 - v_F^2)(M^2 - v_{SL}^2)}{M^2 - v_s^2} \]  

(3.14)

\[ L_2 = \frac{(M^2 - v_f^2)(M^2 - v_{SL}^2)(M^2 - v_F^2)}{M^2 - v_s^2} \]  

(3.15)

\[ k_{1,2} = \pm \frac{\mu L_1 \pm \sqrt{\mu^2 L_1^2 - 4(\cos^2 \theta + \mu^2)L_2}}{2(\cos^2 \theta + \mu^2)} \]  

(3.16)

A shock profile can exist if the upstream point has an unstable solution and the downstream point has a stable solution, that is, if there exist \( \text{Re} \ k > 0 \) for \( x \to -\infty \) (upstream) and \( \text{Re} \ k < 0 \) for \( x \to \infty \) (downstream). For a fast shock, in the upstream point one has \( M^2 > v_F^2 \) and, therefore, \( L_1 > 0 \), \( L_2 > 0 \). As a result, \( \text{Re} \ k_{1,2} > 0 \), which means that the upstream point is unstable. If \( \mu^2 L_1^2 - 4(\cos^2 \theta + \mu^2)L_2 > 0 \) this point is an unstable node, otherwise it is an unstable focus (spiral point). In the downstream asymptotic point of a fast shock one has \( v_F^2 > M^2 > v_s^2 \). If \( M^2 < v_s^2 \) then again \( L_1 > 0 \) and \( L_2 > 0 \), so that \( \text{Re} \ k_{1,2} > 0 \), which means that no solution can converge to the downstream point for \( M^2 < v_s^2 \). If \( M^2 > v_s^2 \), then \( L_2 < 0 \) and \( \text{Re} \ k_1 > 0 \), \( \text{Re} \ k_2 < 0 \). The downstream point becomes a saddle in this case, having exactly one direction along which the solution is converging to the asymptotic state.

In the MHD approximation \( k^2 \) is neglected and one has \( k \mu = L_2 / L_1 \), which gives an unstable upstream and stable downstream for \( M^2 > v_s^2 \), corresponding to the classical first sub- to super-critical transition (see, e.g. Kennel et al. 1985). From the above analysis it is clear that the MHD description loses the whistler mode and forces linear polarization throughout.

The stationary point analysis does not, however, provide all the information about the topology of the solutions, since \( b_{max}^2 \) limits the magnetic field magnitude from above:

\[ b_{max}^2 = 2M^2(1 - N_c^{-1}) + \beta(1 - N_c^\gamma) + \sin^2 \theta \]  

(3.17)

\[ N_c = \left( \frac{2M^2}{\beta \gamma} \right)^{1/(\gamma+1)} \]  

(3.18)

4. Weakly nonlinear weakly time-dependent solutions

Let us now extend our analysis onto the weakly time-dependent regime. In the weakly nonlinear case the time-derivative term, the second- and third-order spatial derivative terms, and the nonlinearity due to \( N \) are all small in \( 2.19, 2.20 \). Thus, it is sufficient
to express \( J \), and \( N \), and \( W \) from the stationary equations:

\[
J = M = \text{const} \tag{4.1}
\]

\[
\frac{M^2}{N} + \frac{\beta N \gamma + b^2}{2} = \text{const} \tag{4.2}
\]

\[
W_y = \cos \theta Nb_y \tag{4.3}
\]

\[
W_z = \cos \theta N (b_z - \sin \theta) \tag{4.4}
\]

\[
\partial_T b_y + \partial_X \left( \frac{M^2 b_y}{N} - \cos^2 \theta b_y \right) - \cos \theta \partial_X^2 b_y - \mu \partial_X^2 b_y = 0 \tag{4.5}
\]

\[
\partial_T b_z + \partial_X \left( \frac{M^2 b_z}{N} - \cos^2 \theta b_z \right) + \cos \theta \partial_X^2 b_y - \mu \partial_X^2 b_z = 0 \tag{4.6}
\]

Defining \( V = 1/N \) and expanding further one gets

\[
\partial_T b_z + \partial_X \left( V M b_z - \cos^2 \theta b_z \right) + \cos \theta \partial_X^2 b_y - \mu \partial_X^2 b_z = 0 \tag{4.7}
\]

\[
b_y = \left( \frac{\cos \theta}{VM^2 - \cos^2 \theta} \right) \partial_X b_z \tag{4.8}
\]

\[
b_y \approx \left( \frac{\cos \theta}{M - \cos^2 \theta} \right) \partial_X b_z \tag{4.9}
\]

\[
\partial_T b_z + \partial_X \left( V M^2 b_z - \cos^2 \theta b_z \right) + \left( \frac{\cos^2 \theta}{M - \cos^2 \theta} \right) \partial_X^2 b_z - \mu \partial_X^2 b_z = 0 \tag{4.10}
\]

\[
(M^2 - v_s^2)(V - 1) = -\sin \theta (b_z - \sin \theta) \tag{4.11}
\]

After some straightforward algebra, eventually we arrive at the following equation for \( \xi = b_z / \sin \theta - 1 \):

\[
\partial_T \xi + P \partial_X \xi - Q \partial_X \xi^2 + R \partial_X^2 \xi - \mu \partial_X^2 \xi = 0 \tag{4.12}
\]

\[
P = \frac{(M^2 - v_F^2)(M^2 - v_{SL}^2)}{M^2 - v_s^2} \tag{4.13}
\]

\[
Q = \frac{\sin^2 \theta}{M^2 - v_s^2} \tag{4.14}
\]

\[
R = \frac{\cos^2 \theta}{M^2 - \cos^2 \theta} \tag{4.15}
\]

In the stationary case one has

\[
P \xi - Q \xi^2 + R \partial_X^2 \xi - S \partial_X \xi = 0 \tag{4.16}
\]

Let \( g = \tanh kX \), so that \( \partial_X g = k(1 - g^2) \). Ignoring dispersion \( R \) one gets

\[
\partial_X \xi = \left( \frac{Q}{S} \right) \xi \left( \frac{P}{Q} - \xi \right) \tag{4.17}
\]

with the shock solution \( \xi(-\infty) = 0, \xi(\infty) = P/Q \):

\[
\xi = A(1 + g), \quad A = \frac{P}{2Q}, \quad k = \frac{P}{2S} \tag{4.18}
\]

provided \( P/Q > 0, P/S > 0 \).
Ignoring dissipation $S$ one gets
\[ R\partial_X^2\xi = -P\xi^2 + Q\xi^2 \] (4.19)
\[ R(\partial_X\xi)^2 + P\xi^2 - \frac{2Q\xi^3}{3} = 0 \] (4.20)
with the soliton solution $\xi(-\infty) = \xi(\infty) = 0$:
\[ \xi = A(1 - \varphi^2), \quad k = \sqrt{-\frac{P}{R}}, \quad A = \frac{-3P}{2Q} \] (4.21)
provided $P/R < 0$ and $Q/R < 0$.

In what follows we are particularly interested in the case when $M^2 > v^2_F$ so that all parameters $P, Q, R, S$ are positive. Using the scaling transformation
\[ T = \alpha T, \quad X = \beta X, \quad \xi = \gamma \xi \] (4.22)
\[ \beta = \sqrt{\frac{R}{P}} = \sqrt{\frac{\cos^2 \theta (M^2 - v^2_s)}{(M^2 - \cos^2 \theta)(M^2 - v^2_F)(M^2 - v^2_{SL})}} \] (4.23)
\[ \alpha = \frac{\beta}{P} = \sqrt{\frac{\cos^2 \theta (M^2 - v^2_s)^3}{(M^2 - \cos^2 \theta)(M^2 - v^2_F)^3(M^2 - v^2_{SL})^3}}, \] (4.24)
\[ \gamma = \frac{P}{Q} = \frac{(M^2 - v^2_F)(M^2 - v^2_{SL})}{\sin^2 \theta} \] (4.25)
equation (4.12) can be rewritten in the following one-parametric form
\[ \partial_T \xi + \partial_X \xi - \partial_X^2 \xi + \partial_X^3 \xi - S\partial_X^2 \xi = 0 \] (4.26)
where
\[ S = \frac{\mu}{\sqrt{PR}} = \frac{\mu}{\cos \theta} \sqrt{\frac{(M^2 - \cos^2 \theta)(M^2 - v^2_s)}{(M^2 - v^2_F)(M^2 - v^2_{SL})}} \] (4.27)
is the dimensionless parameter which characterizes the dissipation strength. It is significantly Mach number dependent. The parameters $\alpha$ and $\beta$ are the typical temporal and spatial scales, while $\gamma$ is the shock amplitude. In the cold case $v_s = 0$, $v_{SL} = 0$, $v_F = 1$, and for weak shocks $M^2 = 1 + \delta$, $\delta \ll 1$, one would have
\[ \beta = \cot \frac{\theta}{\delta^{1/2}}, \quad \alpha = \cot \frac{\theta}{\delta^{3/2}}, \quad \gamma = \frac{\delta}{\sin \theta}, \quad S = \frac{\mu \tan \theta}{\delta^{1/2}} \] (4.28)
In particular, these expressions show that even for small $\mu$ the effective dissipation parameter $S$ can be large for sufficiently low Mach numbers.

5. **Stationary case**

Stationary solutions satisfy the time-independent equation which, after integration once, gives
\[ \xi - \xi^2 + \xi'' - S\xi' = 0 \] (5.1)
where we require the $\xi' = 0$ and $\xi'' = 0$ for $\xi = 0$. The other point where the derivatives vanish is $\xi = 1$. It was suggested (Jeffrey & Mohamad 1991) that a travelling wave solution may be sought for in the form
\[ \xi = A_1 + A_2 \tanh[k(X - UT)] + A_3 \cosh^{-2}[k(X - UT)] \] (5.2)
Figure 1. Initial (dotted line) and evolved profiles for $S = 1$.

where the parameters $U$, $A_i$, $i = 1, 2, 3$, should be found by substitution in (5.1). Since one can always switch to the frame moving with the travelling wave and re-define/re-normalize all variables, it is equivalent to finding a solution of (5.1). We, therefore, will try a solution of the form

$$\xi = \frac{1}{2}(1 + g) + A(1 - g^2) = (A + \frac{1}{2}) + \frac{g}{2} - Ag^2$$

(5.3)

where $g = \tanh kX$. Upon substitution in (5.1) one has

$$k = \frac{1}{2\sqrt{6}}, \quad A = -\frac{1}{4}, \quad S = \frac{5\sqrt{6}}{6}$$

(5.4)

It can be easily seen that $\xi' > 0$ for this solution, that is, it describes a monotonic shock transition. The condition on $S$ means that for a given Mach number $M$ the resistivity $\mu$ must have some definite value for this solution to exist.

6. Shock-like profile

In the focus of our study is the evolution of an initially smooth profile, of the kind

$$b = 1 + \left(\frac{1}{2}\right) \left(\tanh \left(\frac{x}{D}\right) + 1\right)$$

(6.1)

such that $\xi(-\infty) = b(-\infty) - 1 = 0$, $\xi(\infty) = b(\infty) - 1 = 1$, and $D$ is larger than the typical wavelength. Roughly speaking, this profile describes the transition from one asymptotic value of the magnetic field to another within the distance $D$. In what follows we present the results of the numerical integration of KdV-Burgers (4.26) for various values of the dissipation parameter $S$. The integration is done using spatial finite-difference method with the grid spacing $\Delta x = 0.1$ and temporal 4th order Runge-Kutta method. Figure 1 shows two profiles achieved at two different times as a result of steepening for $S = 1$. The initial profile is shown by the dotted line. In this case the dissipation is strong enough to ensure quick convergence to a final stationary state, which is not monotonic. This is clearly seen in the movie 1 (online supplementary materials). Figure 2 shows two profiles for two different times for $S = 0.25$. The ramp and the magnetic dip before the ramp
are weakly time-dependent. Time-dependent wavetrains are generated both upstream and downstream by the steepened profile. The variations are illustrated in the movie 2 (online supplementary materials). Finally, Figure 3 shows two profiles achieved at two different times for $S = 0.005$. The two late profiles are quite different which indicates essential non-stationarity of the shock front. The evolution of the profile in this case is shown in the movie 3 (online supplementary materials). It is worth mentioning that although downstream wavetrains are also present, they should not be taken seriously, since ion kinetics is important in this region and the assumptions of an isotropic polytropic state equation for ions are not valid in this region (Gedalin et al. 2015a).

7. Conclusions

We have shown that weak shocks with weak resistive dissipation can be described by KdV-Burgers equation. This equation is expected to possess stationary solutions. How-
ever, it is quite plausible that not any initial smooth profile with different asymptotical values of the magnetic field converges to the stationary point or that the convergence rate is extremely slow. In this case the profile will steepen until it begins to generate time-dependent wavetrains which prevent further steepening. Thus, the shock profile is intrinsically time-dependent and stationary only on average. Although the wavetrains are generated in a time-dependent manner, the time-dependence of the ramp (the main magnetic increase) may be weak for a range of dissipation parameter, so that the stationary approximation would be justified for the analysis of the ion motion across the shock front. If the dissipation is strong enough the upstream wavetrain is also nearly stationary or even reduces to a small number of phase standing oscillations adjacent to the ramp. For sufficiently weak dissipation propagating wavetrains are formed with the changing shape and the shock front experiences reformation. These findings may explain the observed variety of the low-Mach number shock profiles (Farris et al. [1993] [Wilson et al. 2007] [Russell et al. 2009] [Kajdič et al. 2012] [Wilson III et al. 2017]). At this stage, there is no good theory providing the dissipation parameters for throughout the shock, including the upstream region. Therefore, no first principle based estimates are available for the control parameter $S$.

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