Perturbations in a cosmological model with a hybrid potential

Srijita Sinha* and Narayan Banerjee†
IISER Kolkata, Mohanpur Campus, Mohanpur, Nadia 741246, India

Abstract

In the era of precision cosmology, the cosmological constant $\Lambda$ gives quite an accurate description of the evolution of the Universe, but it is still plagued with the fine-tuning problem and the cosmic coincidence problem. In this work, we provide a scalar field model that will provide the recent acceleration very much like the cosmological constant and will have dark energy (DE) density comparable to dark matter (DM) energy density at the recent epoch starting from arbitrary conditions. The perturbations show that this model, though it keeps the virtues of a $\Lambda$CDM model, has a distinctive quantitative feature, particularly it reduces the amplitude of the matter power spectrum on a scale of $8h^{-1}$ Mpc, $\sigma_8$ at the present epoch.

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1 Introduction

The recent cosmological observations using various independent observational data like the Type Ia supernovae (SNe Ia) measurements\[1,2,3,4\], cosmic microwave background (CMB) \[5,6\], Particle Data Group\[7\], large scale structure (LSS) \[8,9,10\] show that the Universe is expanding with acceleration for the past several Giga years. An exotic component called ‘dark energy’ (DE), in the Universe, can help overcome the attractive nature of gravity and make matter move away from each other at a faster rate. To drive the acceleration of the Universe, the pressure ($p$) of the DE must be sufficiently negative, making its ratio with the energy density ($\rho$) at least less than $-\frac{1}{3}$ ($p/\rho = w < -1/3$). A non-zero cosmological constant $\Lambda$ is undoubtedly the most preferred one\[11,12,13,14,15,16,6\]. A scalar field with potential \[17,18,19,20,21,22,23,24,25,26\] is the next popular choice. Other well-known options include Holographic Dark Energy\[27,28,29\], Chaplygin gas\[30,31,32\], phantom field\[33,34,35\], quintom model\[36,37\] (where $w$ evolve to mimic the phantom fluid) among many others. There are excellent reviews \[38,39,40,41\] that summarise the merits and problems of these candidates.

The cosmological constant with cold dark matter (LCDM) model is plagued with problems like the fine-tuning problem\[11,38\], the coincidence problem \[42,43\]. The fine-tuning problem is that the initial conditions are needed to be set to an exact value so that the cosmological constant term dominates at the current epoch. The coincidence problem is related to the question why the energy densities of dark matter and dark energy are of the same order of magnitude at the present epoch. These problems in the LCDM model has forced us to look for other candidates that can drive the acceleration. A scalar field rolling down a slowly varying potential not only gives rise to acceleration but also alleviates the cosmological coincidence problem. Such a scalar field dubbed ‘quintessence’ has been studied extensively in the literature\[44,45,46,47,48,49,50,51,52,53\]. The scaling quintessence model\[45,48\] where the energy density of the scalar field maintains a constant ratio with the energy density of the dominant component of the background can solve the initial condition problem but cannot produce enough negative pressure to drive the acceleration. Sahni and Wang\[54\] have put forward the cosp potential, and Barreiro et al.\[55\] have put forward the double exponential potential such that the energy density of the scalar field changes at late time and can produce an accelerated expansion. Albrecht and Skordis\[56\] developed an interesting model from string theory where the scalar field enters a regime of damped oscillations with $w \to -1$ leading to an acceleration. On the other hand, the tracking quintessence models\[57\] can give rise to the acceleration with a higher value of $w$, such as $-0.6[57]$ or $-0.8[58]$.

To construct a model without the problem of fixing the initial condition, the scaling potential or the tracking potential is a natural choice. As already stated, the scaling solution does not give acceleration, whereas the best-known tracking potentials cannot have $w \simeq -1$. The motivation of this work is to have a dark energy model that, though evolving, will have acceleration similar to the LCDM model. For that, we have considered a scalar field model with a potential such that it will have an accelerated expansion with $w = -1$ as well as have current energy density comparable to that of dark matter independent of the initial conditions. We engineered the model such that the scalar field $\phi$ will be subdominant.

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*Email: ss13gp012@iiserkol.ac.in
†Email: narayan@iiserkol.ac.in
as tracking dark energy at early times and start to dominate as a cosmological constant in the recent past driving the acceleration. The presence of a scalar field from early times will have its impacts on the growth of perturbations and hence on the large scale structures of the Universe. The scalar field will evolve throughout the history of the Universe, and unlike ΛCDM, will have fluctuations similar to the other matter components. These fluctuations will affect the formation of structures[59] and can also cluster on their own [60, 61]. Thus, structure formation will help break the degeneracy between the ΛCDM model and our scalar field model (φCDM). It must be mentioned that the motivation of this work is not to unify inflation and dark energy and we will consider the evolution of the φCDM long after the completion of inflation.

The paper is organised as follows. Section 2 discusses the scalar field model, section 3 deals with the relevant equations of the scalar field perturbation. The evolution of the density contrast is discussed in section 3 along with the CMB temperature fluctuation, matter power spectrum, linear growth rate and fσ₈. Lastly, in section 4 we summarise the results.

2 The scalar field model

We consider a homogeneous and isotropic Universe with spatially flat constant time hypersurface, described by the well-known Friedmann-Lemaître-Robertson-Walker (FLRW) metric as,

$$ds^2 = a^2(\tau)(-d\tau^2 + \delta_{ij}dx^i dx^j),$$  \hspace{1cm} (1)

where a(τ) is the scale factor and the conformal time τ is related to the cosmic time t as a₀dτ² = dt². The Universe is filled with non-interacting fluids, namely photons (γ), massless neutrinos (ν), baryons (b), cold dark matter (c) and a scalar field (φ) with a potential V(φ) acting as dark energy. The Friedmann equations are given as

$$3\mathcal{H}^2 = -a^2\kappa \sum_i \rho_i,$$  \hspace{1cm} (2)

$$\mathcal{H}^2 + 2\mathcal{H}' = a^2\kappa \sum_i p_i,$$  \hspace{1cm} (3)

where \(\kappa = 8\pi G_N\) (\(G_N\) being the Newtonian Gravitational constant), \(\mathcal{H}(\tau) = \frac{\dot{a}}{a}\) is the conformal Hubble parameter and prime (') denotes the derivative with respect to the conformal time. The energy density and pressure of each component are respectively \(\rho_i\) and \(p_i\), where \(i = \gamma, \nu, b, c, \phi\). The equation of state (EoS) parameter is given as \(w_i = \frac{p_i}{\rho_i}\). For the photons and neutrinos, \(w_\gamma = w_\nu = 1/3\), for baryons and CDM, \(w_b = w_c = 0\). For the scalar field, \(\rho_\phi = \frac{1}{2\kappa} \phi'^2 + V(\phi)\) and \(p_\phi = \frac{1}{2\kappa} \phi'^2 - V(\phi)\) and the EoS parameter is given by

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{1}{2\kappa} \frac{\phi'^2 - V(\phi)}{\phi'^2 + V(\phi)} = 1 - \frac{2V(\phi)}{\rho_\phi}.$$  \hspace{1cm} (4)

The Klein-Gordon equation can be obtained as a consequence of Bianchi identities as

$$\phi'' + 2\mathcal{H} \phi' + a^2 \frac{dV}{d\phi} = 0.$$  \hspace{1cm} (5)

It is clear from the expression (4) that \(w_\phi\) has an evolutionary history and ranges between \(-1 \leq w_\phi \leq 1\) for a real scalar field and a positive definite \(V(\phi)\). When the kinetic energy \(E_K = \frac{\phi'^2}{2\kappa}\) is dominant with a negligible potential energy \(E_P = V(\phi)\), the scalar field behaves as a stiff fluid with \(w_\phi = 1\), and when \(E_P\) dominates with a negligible \(E_K\), it gives rise to a cosmological constant with \(w_\phi = -1\). Thus, the behaviour of the scalar field and hence the evolution of the Universe depends on the form of the potential. For the recent accelerated expansion of the Universe, the scalar field at late time should roll sufficiently slowly along the potential such that \(E_K \ll E_P\).

For this work, the potential is constructed to be a hybrid one, as the sum of an exponential potential and a constant potential, shown in figure (1). The exponential potential will rescue from fine-tuning of the initial condition while the constant one will provide the slow-roll condition and drive the recent acceleration. The potential is written as,

$$V(\phi) = V_0 e^{-\lambda \phi} \theta(-\phi) + V_0 \theta(\phi),$$  \hspace{1cm} (6)

where \(\theta(\phi)\) is the Heaviside theta defined as

$$\theta(\phi) = \begin{cases} 0 & \phi < 0, \\ 1 & \phi \geq 0. \end{cases}$$  \hspace{1cm} (7)

Here \(V_0\) is a constant. The exponential potential has been studied extensively in the literature[44, 45, 46, 47]. As shown by Copeland et al.[48], the exponential potential has two attractor solutions,
The evolution of the energy density parameters, \( \Omega \), of radiation \((r \equiv \gamma + \nu)\), matter \((m \equiv b + c)\) and scalar field \((\phi)\) with the scale factor, \(a\), in logarithmic scale are shown in figure (2a) and that of the deceleration parameter \( q = -\frac{\dddot{a}/a}{\frac{\dot{a}^2}{a^2}} - 1 \) with \(a\) in figure (2b) for \( \lambda = 14.0 \). Figure (2) shows that though \( w_\phi = -1 \) at the present epoch, the evolution dynamics of the Universe is different from the \( \Lambda \)CDM model. The scalar field model, henceforth called \( \phi \)CDM, gives accelerated expansion at a little higher value of \( a \) compared to the \( \Lambda \)CDM model.
adiabatic sound speed \([69, 62]\) for the scalar field reads as in the Fourier space with wavenumber \(k\).

For an adiabatically expanding Universe, the sound speed is \(c_s^2 = p'/\rho'\). Using the Klein-Gordon equation (5), the adiabatic sound speed \([69, 62]\) for the scalar field reads as

\[
c_s^2 = \frac{1}{3} - \frac{2\phi''}{3 \mathcal{H} \phi'} = 1 + \frac{2\alpha^2}{3 \mathcal{H} \phi'} \frac{dV}{d\phi}.
\]
Figure 3: (a) Plot of the matter density contrast \( \delta_m \) against \( a \). Both the axes are in logarithmic scale. (b) Plot of scalar field density contrast \( \delta_\phi \) against \( a \) in logarithmic scale. The solid line represents \( k = 1.0 \, h \, \text{Mpc}^{-1} \), dashed line represents \( k = 0.1 \, h \, \text{Mpc}^{-1} \) and dashed-dot line represents \( k = 0.01 \, h \, \text{Mpc}^{-1} \) with \( \lambda = 14.0 \).

Figure 4: Plot of the matter density contrast \( \frac{\delta_m}{\delta_{m,0,\Lambda CDM}} \) against \( a \) in logarithmic scale for \( \varphi \text{CDM} \) with \( \lambda = 13.6 \) (solid line with solid circles) and \( \lambda = 14.0 \) (solid line) and \( \Lambda \text{CDM} \) (dashed-dot line) for \( k = 0.1 \, h \, \text{Mpc}^{-1} \). The difference in the growth of \( \delta_m \) for \( \varphi \text{CDM} \) and \( \Lambda \text{CDM} \) is prominent in the recent past, hence \( a \) is from \( 10^{-3} \).

In order to solve the perturbation equation (12), the second derivative of the potential is written in terms of the sound speed \( c_{\varphi, \phi}^2 \) as

\[
\frac{d^2 V}{d\varphi^2} = \frac{3}{2} \frac{\mathcal{H}^2}{a^2} \left[ c_{\varphi, \phi}^2 - \frac{1}{2} (c_{\varphi, \phi}^2 - 1) (3c_{\varphi, \phi}^2 + 5) + \frac{\mathcal{H}'}{\mathcal{H}} (c_{\varphi, \phi}^2 - 1) \right].
\]  

(17)

The sound speed, \( c_{\varphi, \phi}^2 \), is constant in the different phases of evolution, like in the scaling regime \( c_{\varphi, \phi}^2 = w_\phi = w_D \) and in the slow-roll regime \( c_{\varphi, \phi}^2 = 1 \). We shall henceforth take it to be described by equation (16) but neglect its derivative, \( c_{\varphi, \phi}^2 \) in equation (17). The perturbation equations (10) and (11) are solved along with equations (12), (13) and (14) with adiabatic initial conditions and \( k = [1.0, 0.1, 0.01] \, h \, \text{Mpc}^{-1} \) using CAMB.

Figure (3a) shows the variation of the density contrast, \( \delta_m = \delta \rho_m / \rho_m \) for the cold dark matter (c) together with the baryonic matter (b) and figure (3b) shows the variation of the density contrast \( \delta_\phi = \delta \rho_\phi / \rho_\phi \) of the scalar field against \( a \) in logarithmic scale for \( k = [1.0, 0.1, 0.01] \, h \, \text{Mpc}^{-1} \). In the matter dominated era, the modes of \( \delta_m \) grow in a very similar fashion. The modes of \( \delta_\phi \) oscillate rapidly with decreasing amplitude after entering the horizon. Figure (4) shows the evolution of the matter density contrast \( \delta_m \), for \( \varphi \text{CDM} \) and \( \Lambda \text{CDM} \). For a better comparison, \( \delta_m \) for both the models have been scaled by \( \delta_{m,0} = \delta_m (a = 1) \) of \( \Lambda \text{CDM} \). It can be seen that there is a difference in the growth of \( \delta_m \) in the two models (\( \varphi \text{CDM} \) and \( \Lambda \text{CDM} \)) and it is almost independent of \( \lambda \) for \( \varphi \text{CDM} \).

3.2 Effect on CMB temperature, matter power spectra and \( f_8 \sigma_8 \)

For more insight into the effect of the scalar field \( \varphi \) on different physical quantities, we look at the CMB temperature spectrum, matter power spectrum and \( f_8 \sigma_8 \). The CMB temperature power spectrum is given as

\[
C_l^{TT} = \frac{2}{k} \int k^2 dk P_g(k) \Delta T_l^2(k),
\]

(18)
where $P_\ell(k)$ is the primordial power spectrum, $\Delta T_T(k)$ is the temperature transfer function and $l$ is the multipole index. For the detail calculation of the CMB spectrum we refer to [70, 71]. The matter power spectrum is given as

$$P(k, a) = A_\ell k^n T^2(k) D^2(a),$$

(19)

where $A_\ell$ is the normalizing constant, $n_\ell$ is the spectral index, $T$ is the matter transfer function and $D(a) = \frac{\delta_\ell(a)}{\delta_\ell(a = 1)}$ is the normalized density contrast. For the detail calculation we refer to [72]. The $C^TT$ and $P(k, a)$ are computed numerically using CAMB. The values $A_\ell = 2.100549 \times 10^{-9}$ and $n_\ell = 0.9660499$ are taken from Planck 2018 data[6]. Figure 5a shows that the CMB temperature power spectra, $C^TT$ are almost independent of the values of the model parameter $\lambda$. For clarity of the plots only two values of $\lambda$ are given. The presence of the scalar field $\varphi$ decreases the matter content of the Universe slightly during matter domination making the amplitude of first two peaks of the CMB spectra marginally higher than that in the $\Lambda$CDM model. The scalar field also lowers the low-l CMB spectra through the integrated Sachs-Wolfe (ISW) effect. A lesser amount of matter leads to a marginally lower matter power spectra at small scales (figure 5b). Both these figures are for the present epoch.

To differentiate the $\varphi$CDM and $\Lambda$CDM decisively, we have studied the linear growth rate,

$$f(a) = \frac{d \ln \delta_m}{d \ln a} = \frac{a}{\delta_m(a)} \frac{d \delta_m}{da}.$$

(20)

Observationally the growth rate is measured using the perturbation of the galaxy density $\delta_g$, which is related to the matter density perturbations $\delta_m$ as $\delta_g = b \delta_m$, where $b \in [1, 3]$ is the bias parameter. The estimate of the growth rate $f$ is sensitive to the bias parameter, and thus not very reliable. A more dependable observational quantity is the product $f(a) \delta_m(a)$ [73], where $\delta_m(a)$ is the root-mean-square (rms) fluctuations of the linear density field within the sphere of radius $R = 8h^{-1}$ Mpc. The rms mass fluctuation can be written as $\delta_m(a) = \delta_m(1) \frac{\delta_m(a)}{\delta_m(1)}$, where $\delta_m(1)$ is the value at $a = 1$ (table 2), calculated by integrating the matter power spectrum over all the wavenumber $k$ using CAMB. Thus, the combination becomes

$\delta_m(a)$

| Model  | $\lambda$ | $\sigma_8$ |
|--------|-----------|------------|
| $\varphi$CDM | 13.6 | 0.7548 |
| $\varphi$CDM | 14.0 | 0.7581 |
| $\varphi$CDM | 14.4 | 0.7611 |
| $\Lambda$CDM | --- | 0.8123 |

$$f \delta_m(a) \equiv f(a) \sigma_8(a) = \sigma_8(1) \frac{a}{\delta_m(1)} \frac{d \delta_m}{da}.$$

(21)

Since $f \sigma_8$ measurements provide a tighter constraint on the cosmological parameters, it will give a better insight into the growth of the density perturbations. We have studied the variation of $f$ and $f \sigma_8$ with redshift $z$ for three different values of $\lambda$. Redshift $z$ is related to the scale factor $a$ as $z = \frac{a_0}{a} - 1$, $a_0$ being the present value (taken to be unity). The linear growth rate $f$ and $f \sigma_8$ are independent of the wavenumber $k$ for low redshift. As the $f \sigma_8$ analysis is valid for $z \in [0, 2]$, the redshift from $z = 0$ to $z = 2$ are considered here.
The linear growth rate $f$ is same for all the models at low redshift (figure 6(a)). The difference in matter power spectrum is manifested in its amplitude $\sigma_8$ as given in table 2 and hence in $f\sigma_8$ as in figure 6(b). It is interesting to note that there is a substantial difference in the CMB temperature and matter power spectra. It must be noted that making $\lambda$ high enough will give higher value of $\sigma_8$ and hence $f\sigma_8$ but that will increase the age of the Universe as well, which is around 13.797 ± 0.023 Giga years according to the recent Planck 2018 data[6]. Thus, a low $\sigma_8$ can be said to be the characteristic distinguishing feature of the present model from $\Lambda$CDM.

4 Summary and discussion

The motivation of the present work is to construct a dynamical dark energy model that will alleviate the initial condition problem associated with the cosmological constant and leads to an EoS parameter $w_\phi = -1$ at the present epoch thereby retaining the virtues of the $\Lambda$CDM model. A scalar field with an exponential potential at early times and a constant potential at late times appears to serve the purpose. At early times the scalar field energy density tracks the dominant component of the background fluid and later on starts to roll sufficiently slowly to drive the accelerated expansion of the Universe. That $w_\phi = -1$ for the present epoch is independent of the choice of the model parameters, and the present dark energy density parameter $\Omega_{m0}$ is dependent only on the height of the constant potential, $V_0$.

Linearized scalar perturbations of the FLRW metric in synchronous gauge are studied using our modified CAMB. The growth of matter density contrast, $\delta_m$ is similar to the $\Lambda$CDM model. The linear growth rate $f$, which is the logarithmic derivative of $\delta_m$ with respect to $a$ is same for both the models. The presence of the scalar field slightly decreases the matter content of the Universe during the evolutionary history. This decrease in matter content is manifested in the matter power spectrum and even more clearly in the evolution of the $f\sigma_8$. Thus, $f\sigma_8$ helps in breaking the degeneracy between the present model and the standard $\Lambda$CDM. Another interesting result is that the decrease in the rate of clustering decreases the variance of the linear matter perturbation, $\sigma_8$. The $\sigma_8$ obtained here is more towards the side of the value obtained from the galaxy cluster counts using thermal Sunyaev- Zel’dovich (tSZ) signature[6], $\sigma_8 = 0.77^{+0.04}_{-0.03}$ rather than the value obtained from Planck spectrum[6], $\sigma_8 = 0.811 \pm 0.006$.

It can be said quite conclusively that this scalar field model resolves the initial condition problem, produces late-time acceleration with $w_\phi = -1$ as predicted by the recent data as well as decreases rms mass fluctuation $\sigma_8$. This model is also successful in the context of the structure formation in the Universe. Thus, the potential constructed in this work can successfully describe the evolution of the Universe. It will be interesting to check if such a potential can also be arrived at from other physical considerations.

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