ADIABATIC TRANSPORT OF BOSE-EINSTEIN CONDENSATE IN DOUBLE-
AND TRIPLE-WELL TRAPS

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By using a close similarity between multi-photon and tunneling population transfer schemes, we propose robust adiabatic methods for the transport of Bose-Einstein condensate (BEC) in double- and triple-well traps. The calculations within the mean-field approximation (Gross-Pitaevskii equation) show that irreversible and complete transport takes place even in the presence of the nonlinear effects caused by interaction between BEC atoms. The transfer is driven by adiabatic time-dependent monitoring the barriers and well depths. The proposed methods are universal and can be applied to a variety of systems and scenarios.

The trapped Bose-Einstein condensate (BEC) is now widely recognized as a source of new fascinating physics and interesting cross-over with other areas [1, 2]. In particular, a large attention is paid to dynamics of weakly bound condensates. Here two main systems are usually considered: multi-level BEC in a single potential well (see [3] and references therein) and a single BEC in a multi-well trap [4, 5, 6, 7, 8, 9]. In the former case, BEC components are formed by atoms in different hyperfine levels coupled by the resonant laser irradiation. In the second case, all BEC atoms are in the same state but the trap is separated by barriers into a set of potential wells and BEC dynamics reads as tunneling between the wells. This case is realized in multi-well traps [3], arrays of selectively addressable traps [10] and optical lattices [8, 11].

The multi-component and multi-well systems are quite similar and can be treated on the same footing as a BEC with (weakly) coupled fractions. In the former, the fractions coincide with BEC components while in the latter with populations of the wells. Even for the constant coupling, dynamics of multi-fractional BEC is very reach and varies from different kinds of Josephson oscillations (in analogy with internal and external Josephson effects in weakly bound superconductors, see discussion in [5]) to Mott insulator transitions and other transfer regimes pertinent to electrons in a crystal lattice [11]. The dynamics strictly depends on the interplay between the interaction of BEC atoms and the coupling [1, 3, 11].

For a constant coupling, BEC dynamics is mainly reduced to oscillating fluxes of atoms between the fractions. That was a subject of intense investigation during the last decade [3, 4, 5, 6, 7, 8, 9]. At the same time, still a little was done for exploration of irreversible transport of BEC between its fractions when their populations do not oscillate but evolve by irreversible and controlled way. Specifically, BEC transport means that condensate, being initially in one of the fractions, is then completely transferred to another (target) fraction and kept there. Being realized, BEC transport could open intriguing perspectives for investigation of topological states [12, 13], generation and exploration of geometric phases (which may serve as promising information carriers [14]), various implementations in quantum computing (e.g. to build analogs of Josephson qubits in superconductors [15]), etc.

Nowadays adiabatic population transfer methods seem to be most promising to carry out the transport of BEC. A variety of suitable adiabatic methods was already developed for the aims of atomic and molecular spectroscopy [16], between them Stark Chirped Rapid Adiabatic Passage (SCRAP) [17] and Stimulated Rapid Adiabatic Passage (STIRAP) [18]. These methods exploit two-photon population transfer schemes and intend to excite non-dipole states in electronic systems. However, the methods are quite general and can be upgraded to other systems and scenarios, see e.g. [19, 20, 21].

The aim of the present paper is to adapt the SCRAP and STIRAP ideas for BEC transport and develop the relevant population transfer schemes. For this aim the proper time-dependent protocols of the system parameters (well depths and barrier penetrabilities) will be proposed. We will concentrate on double- and triple-well traps where the necessary control of the parameters can be easily realized, e.g. the barriers can be monitored by varying depths and separations of the wells [4, 10].

By our knowledge, the SCRAP has not still employed in BEC transport. Quite recently an alternative technique, Rabi switch, was suggested, where time-dependent Rabi coupling of a fixed duration was used to transfer solitons and vortices [2]. Here we propose in principle another scheme where both Rabi frequency and detuning are adiabatically controlled. Such SCRAP-like scheme has the advantage to be rather insensitive to the process parameters, e.g. the coupling duration [10, 17].

As for STIRAP, it was already proposed to transfer individual atoms [21] and BEC [5]. In BEC the nonlinearity due to the interatomic interaction $U$ was shown to be detrimental for the adiabatic transfer (like dynamical Stark shifts in multi-level system) [5]. As a remedy, a large detuning $\Delta$ of the well depths (trap asymme-
try) was proposed. We will show that, unlike conclusio

1 both large non-linearity and asymmetry are det
mental for STIRAP but the transfer is robust at the
modest values. Instead, in SCRAP-like scheme a lar
time-dependent detuning $\Delta(t)$ is useful and even crui

Altogether, we will develop and justify simple and ef
fective adiabatic schemes for BEC transport in double-
triple- and multi-well traps.

Our calculation have been performed in the mean-fi
approximation by using the non-linear Schrödinger,
Gross-Pitaevskii equation \(^ (2)\) for BEC:

$$i\hbar \dot{\Psi}(\vec{r},t) = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{ext}}(\vec{r},t) + g_0 |\Psi(\vec{r},t)|^2 \right] \Psi(\vec{r},t)$$

where the dot means time derivative, $\Psi(\vec{r},t)$ is the classi
cal order parameter of the system, $V_{\text{ext}}(\vec{r},t)$ is the external trap potential involving both (generally tin
dependent) confinement and coupling, $g_0 = 4\pi a/m$ the pa
meter of interaction between BEC atoms, $a$ the scattering length and $m$ is the atomic mass.

For BEC with $M$ fractions, the order parameter can ex
panded as \(^ (3)\)

$$\Psi(\vec{r},t) = \sqrt{N} \sum_{k=1}^{M} \psi_k(t) \Phi_k(\vec{r})$$

where $\Phi_k(\vec{r})$ is the static ground state solution of \(^ (1)\) for the isolated (without coupling) $k$-th well \(^ (23)\) and $\psi_k(t) = \sqrt{N_k(t)} e^{i\phi_k(t)}$ is the amplitude related with the relative population of the $k$-th well $N_k(t)$ and corresponding phase $\phi_k(t)$. The total number of atoms $N$ is fixed:

$$\int d\vec{r} |\psi_k(t)|^2 / N = \sum_{k=1}^{M} N_k(t) = 1$$

Being mainly interested in evolution of populations $N_k(t)$, we dispose by integration of the spatial distribu
ions $\Phi_k(\vec{r})$ and finally get \(^ (4)\)

$$i\dot{\psi}_k = \left[ E_k(t) + UN |\psi_k|^2 \right] \psi_k - \sum_{j \neq k}^{M} \Omega_{kj}(t) \psi_j$$

where

$$\hbar \Omega_{kj}(t) = -\left[ \frac{\hbar^2}{2m} \nabla \Phi_k^* \cdot \nabla \Phi_j + \Phi_k^* V_{\text{ext}}(t) \Phi_j \right]$$

is the coupling between BEC fractions,

$$\hbar E_k(t) = \int d\vec{r} \left[ \frac{\hbar^2}{2m} |\nabla \Phi_k^*|^2 + \Phi_k^* V_{\text{ext}}(t) \Phi_k \right]$$

is the potential depth, and

$$\hbar U = g_0 \int d\vec{r} |\Phi_k|^4$$

labels the interaction between BEC atoms. The values $\Omega_{kj}(t)$, $E_k(t)$, and $U$ have frequency dimension.

![FIG. 1: Schemes of the adiabatic population transfer in double-level a)-b) and double-well c)-d) systems. a) Double-
level scheme with off-resonant pump laser pulse $\Omega_p(t)$ and detuning $\Delta(t)$. b) Time evolution of pump $\Omega_p(t)$ and Stark
oscillations $\Omega_{\text{ext}}(t)$ laser pulses. c) Double-well scheme with barrier coupling $\Omega(t)$ and detuning $\Delta(t)$. The condensate (dot at the
left well) should be transferred to the right well, as indicated by the long arrow. d) Time evolution of the coupling and detuning.]

We use the coupling of the Gauss form with a common amplitude $K$:

$$\Omega_{kj}(t) = K \bar{\Omega}_{kj}(t), \quad \bar{\Omega}_{kj}(t) = \exp\left\{ -\frac{(t_{kj} - t)^2}{\Gamma_{kj}^2} \right\}$$

where $t_{kj}$ and $\Gamma_{kj}$ are centroid and width parameters. Then, dividing \(^ (3)\) by $1/2K$, one gets

$$i\hbar \dot{\psi}_k = \left[ \bar{E}_k(t) + \Lambda |\psi_k|^2 \right] \psi_k - \frac{1}{2} \sum_{j \neq k}^{M} \Omega_{kj}(t) \psi_j$$

where

$$\bar{E}_k(t) = E_k(t)/2K, \quad \Lambda = UN/2K$$

and the time is scaled as $2Kt \rightarrow t$ and so is dimensionless. In \(^ (8)\) the key parameter $\Lambda$ is responsible for the interplay between the coupling and interaction. In principle, one may further upgrade \(^ (8)\) by canonical transformation of $N_k$ and $\phi_k$ to other unknowns, population imbalances and phase differences, and thus removing from \(^ (8)\) the integral of motion $N$ \(^ (24)\). It should be empha
sized that, unlike the previous studies of the oscillating
BEC fluxes in traps with constant parameters \(^ (3)\), \(^ (4)\), \(^ (6)\), \(^ (8)\), we will deal with irreversible BEC transport by monitoring time-dependent parameters, namely depths $E_k(t)$ and couplings $\Omega_{kj}(t)$.

In the present study, the cases with $M=2$ and $M=3$ are ex
plored. For $M=2$ the SCRAP scheme for a double-level
system \(^ (16)\) \(^ (17)\) and its counterpart proposed for BEC
transport in a double-well system are illustrated in Fig.
needs a rapid change of the detuning, which agrees with the population transfer improves with increasing the detuning. The condition $\theta(t) = 0.3(t - 47)$ has to be fulfilled, where $\theta$ is the mixing angle of the states $|1\rangle$ and $|2\rangle$ during the adiabatic evolution.

In Fig. 2 the results of our calculations for the double-well case are demonstrated. The plots a)-c) show that the population transfer improves with increasing the detuning rate $\Delta(t)$ from 0.003 to 0.3. For $\Delta(t)=0.3$ the transfer is complete. So, the robust transport of BEC needs a rapid change of the detuning, which agrees with the adiabatic condition. \[ |\theta(t)| \ll \sqrt{\Omega_p^2(t) + \Delta^2(t)} \] (10)

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Such a cyclic adiabatic transport can be used to generate the geometric Berry phase [24]. Being initially in the first well (interaction), Λ = 0, our proposals are relevant for the interaction between BEC atoms and related nonlinearity. The results are straightforwardly generalized for multi-well traps with M > 3. The schemes work in the presence of a modest nonlinearity caused by interatomic interaction. Our developments open interesting perspectives for investigation of various geometric phases [24], e.g. using the prescription [25]. There could be also various applications for Josephson qubits [15], adiabatic quantum computing [24], topological states [12, 13], etc. At zero interaction, Λ = 0, our proposals are relevant for the transport of individual atoms.

In summary, we propose simple and effective adiabatic population transfer schemes for the complete and irreversible transport of BEC in double- and triple-well traps. The results are straightforwardly generalized for multi-well traps with M > 3. The schemes work in the presence of a modest nonlinearity caused by interatomic interaction. Our developments open interesting perspectives for investigation of various geometric phases [24], e.g. using the prescription [25]. There could be also various applications for Josephson qubits [15], adiabatic quantum computing [24], topological states [12, 13], etc. At zero interaction, Λ = 0, our proposals are relevant for the transport of individual atoms.

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