Shape and Content *
Incorporating Domain Knowledge into Shape Analysis

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Abstract. The verification community has studied dynamic data structures primarily in a bottom-up way by analyzing pointers and the shapes induced by them. Recent work in fields such as separation logic has made significant progress in extracting shapes from program source code. Many real world programs however manipulate complex data whose structure and content is most naturally described by formalisms from object oriented programming and databases. In this paper, we look at the verification of programs with dynamic data structures from the perspective of content representation. Our approach is based on description logic, a widely used knowledge representation paradigm which gives a logical underpinning for diverse modeling frameworks such as UML and ER. Technically, we assume that we have separation logic shape invariants obtained from a shape analysis tool, and requirements on the program data in terms of description logic. We show that the two-variable fragment of first order logic with counting and trees can be used as a joint framework to embed suitable fragments of description logic and separation logic.

1 Introduction

The manipulation and storage of complex information in imperative programming languages is often achieved by dynamic data structures. The verification of programs with dynamic data structures, however, is notoriously difficult, and is a highly active area of current research. While much progress has been made recently in analyzing and verifying the shape of dynamic data structures, most notably by separation logic (SL) [23,17], the content of dynamic data structures has not received the same attention.

In contrast, disciplines as databases, modeling and knowledge representation have developed highly-successful theories for content representation and verification. These research communities typically model reality by classes and binary relationships between these classes. For example, the database community uses

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entity-relationship (ER) diagrams, and UML diagrams have been studied in requirements engineering. Content representation in the form of UML and ER has become a central pillar of industrial software engineering. In complex software projects, the source code is usually accompanied by design documents which provide extensive documentation and models of data structure content. This documentation is both an opportunity and a challenge for program verification. Recent hardware verification papers have demonstrated how design diagrams can be integrated into an industrial verification workflow [18].

In this paper, we propose the use of Description Logics (DLs) for the formulation of content specifications. DLs are a well established and highly popular family of logics for representing knowledge in artificial intelligence [3]. In particular, DLs allow to precisely model and reason about UML and ER diagrams [6,2]. DLs are mature and well understood, they have good algorithmic properties and have efficient reasoners. DLs are very readable and form a natural base for developing specification languages. For example, they are the logical backbone of the Web Ontology Language (OWL) for the Semantic Web [21]. DLs vary in expressivity and complexity, and are usually selected according to the expressivity needed to formalize the given target domain.

Unfortunately, the existing content representation technology cannot be applied directly for the verification of content specifications of pointer-manipulating programs. This is due to the strict separation between high-level content descriptions such as UML/ER and the way data is actually stored. For example, query languages such as SQL and Datalog provide a convenient abstraction layer for formulating data queries while ignoring how the database is stored on the disk. In contrast, programs with dynamic data structures manipulate their data structures directly. Moreover, database schemes are usually static while a program may change the content of its data structures over time.

The main goal of this paper is to develop a verification methodology that allows to employ DLs for formulating and verifying content specifications of pointer-manipulating programs. We propose a two-step Hoare-style verification methodology: First, existing shape-analysis techniques are used to derive shape invariants. Second, the user strengthens the derived shape invariants with content annotations; the resulting verification conditions are then checked automatically. Technically, we employ a very expressive DL (henceforth called $\mathcal{L}$), based on the so called $\text{ALCHOIF}$, which we specifically tailor to better support reasoning about complex pointer structures. For shape analysis we rely on the SL fragment from [7]. In order to reason automatically about the verification conditions involving DL as well as SL formulae, we identify a powerful decidable logic $CT^2$ which incorporates both logics [10]. We believe that our main contribution is conceptual, integrating these different formalisms for the first time. While the current approach is semi-manual, our long term goal is to increase the automatization of the method.
Overview and Contributions:

– In Section 2, we introduce our formalism. In particular, we formally define *memory structures* for representing the heap and we study the DL $\mathcal{L}$ as a formalism for expressing *content properties* of memory structures.
– In Section 2, we further present the building blocks for our verification methodology: We give an embedding of $\mathcal{L}$ and an embedding of a fragment of the SL from [7] into $CT^2$ (Lemmas 2 and 3). Moreover, we give a complexity-preserving reduction of satisfiability of $CT^2$ over memory structures to finite satisfiability of $CT^2$ (Lemma 1).
– In Section 3, we describe a program model for sequential imperative heap-manipulating programs without procedures. Our main contribution is a Hoare-style proof system for verifying content properties on top of (already verified) shape properties stated in SL.
– Our main technical result is a precise backward-translation of content properties along loop-less code (Lemma 5). This backward-translation allows us to reduce the inductiveness of the Hoare-annotations to satisfiability in $CT^2$. Theorem 1 states the soundness and completeness of this reduction.

1.1 Running Example: Information System of a Company

Our running example will be a simple information system for a company with the following UML diagram: The UML gives the relationships between entities in the information system, but says nothing regarding the implementations of the data structures that hold the data. We focus mostly on projects, and on the employees and managers which work on them. Here is an informal description of the programmers’ intention. The employees and projects are stored in two lists, both using the `next` pointer. The heads of the two lists are $pHd$ and $eHd$ respectively. Here are some properties of our information system. (i)-(iii) extends the UML somewhat. (iv)-(vi) do not appear in the UML, but can be expressed in DL:

(i) Each employee in the list of employees has a pointer `wrkFor` to a project on the list of projects, indicating the project that the employee is working on (or to null, in case no project is assigned to that employee).
(ii) Each project in the list has a pointer `mngBy` to the employee list, indicating the manager of the project (or to null, if the project doesn’t have one).
(iii) Employees have a Boolean field `isMngr` marking them as managers, and only they can manage projects.
(iv) The manager of a project works for the project.
(v) At least 10 employees work on each large project.
(vi) The contact person for a large-scale project is a manager.
We will refer to these properties as the *system invariants*.

The programmer has written a program $S$ (stated below) for verification. The programmer has the following intuition about her program: The code $S$ adds a new project $proj$ to the project list, and assigns to it all employees in the employee list which are not assigned to any project.

The programmer wants to verify that the system invariants are true after the execution of $S$, if they were true in the beginning (1). Note that during the execution of the code, they might not be true! Additionally the programmer wants to verify that after executing $S$, the project list has been extended by $proj$, the employee list still contains the same employees and indeed all employees who did not work for a project before now work for project $proj$ (2). We will formally prove the correctness of $S$ following our verification methodology discussed in the introduction. In Section 2.3 we describe how our DL can be used for specifying the verification goals (1) and (2). In Section 3.4 we state verification conditions that allow to conclude the correctness of (1) and (2) for $S$.

## 2 Logics for Invariant Specification

### 2.1 Memory Structures

We use ordinary first order structures to represent memory in a precise way. A *structure* (or, *interpretation*) is a tuple $M = (M, \tau, \cdot)$, where (i) $M$ is an infinite set (the universe), (ii) $\tau$ is a set of constants and relation symbols with an associated non-negative arity, and (iii) $\cdot$ is an interpretation function, which assigns to each constant $c \in \tau$ an element $c^M \in M$, and to each $n$-ary relation symbol $R \in \tau$ an $n$-ary relation $R^M$ over $M$. Each relation is either unary or binary (i.e. $n \in \{1, 2\}$). Given $A \subseteq M$, a binary $R^M$, $R^M$ and $e \in A^M$, we may use the notation $R^M(e)$ if $R^M$ is known to be a function over $A^M$.

A *memory structure* describes a snapshot of the heap and the local variables. We assume sets $\tau_{\text{var}} \subseteq \tau$ of constants $\tau_{\text{fields}} \subseteq \tau$ of binary relation symbols. We will later employ these symbols for variables and fields in programs. A *memory structure* is a structure $M = (M, \tau, \cdot)$ that satisfies the following conditions:

1. $\tau$ includes the constants $o_{\text{null}}$, $o_{\text{T}}$, $o_{\text{F}}$.
2. $\tau$ has the unary relations $\text{Addresses}$, $\text{Alloc}$, $\text{PossibleTargets}$, $\text{MemPool}$, and $\text{Aux}$.
3. $\text{Aux}^M = \{o_{\text{null}}^M, o_{\text{T}}^M, o_{\text{F}}^M\}$ and $|\text{Aux}^M| = 3$.
4. $\text{Addresses}^M \cap \text{Aux}^M = \emptyset$ and $\text{Addresses}^M \cup \text{Aux}^M = M$.
5. $\text{Alloc}^M$, $\text{PossibleTargets}^M$ and $\text{MemPool}^M$ form a partition of $\text{Addresses}^M$.
6. $c^M \in M \backslash \text{MemPool}^M$ for every constant $c$ of $\tau$.
7. For all $f \in \tau_{\text{fields}}$, $f^M$ is a function from $\text{Addresses}^M$ to $M \backslash \text{MemPool}^M$.
8. If $e \in \text{MemPool}^M$, then $f^M(e) \in \{o_{\text{null}}^M, o_{\text{F}}^M\}$.
We explain the intuition behind memory structures. Variables in programs will either have a Boolean value or be pointers. Thus, to represent null and the Boolean values $T$ and $F$, we employ the auxiliary relation $Aux^M$ storing 3 elements corresponding to the 3 values. $Addresses^M$ represents the memory cells. The relation $Alloc^M$ is the set of allocated cells, $PossibleTargets^M$ contains all cells which are not allocated, but are pointed to by allocated cells (for technical reasons it possibly contains some other unallocated cells). $MemPool^M$ contains the cells which are not allocated, do not have any field values other than null and $F$, are not pointed to by any field, do not participate in any other relation and do not interpret any constant (see (6-9)). The memory cells in $MemPool$ are the candidates for allocation during the run of a program. Since the allocated memory should by finite at any point of the execution of a program, we require that $Alloc^M$ and $PossibleTargets^M$ are finite (see (10)), while the available memory $Addresses^M$ and the memory pool $MemPool^M$ are infinite. Finally, each cell is seen as a record with the fields of $\tau_{\text{fields}}$.

2.2 The Description Logic $\mathcal{L}$

$\mathcal{L}$ is defined w.r.t. a vocabulary $\tau$ consisting of relation and constant symbols.  

**Definition 1 (Syntax of $\mathcal{L}$).** The sets of roles and concepts of $\mathcal{L}$ is defined inductively: (1) every unary relation symbol is a concept (atomic concept); (2) every constant symbol is a concept; (3) every binary relation symbol is a role (atomic role); (4) if $r, s$ are roles, then $r \cup s$, $r \cap s$, $r\backslash s$ and $r^{-}$ are roles; (5) if $C, D$ are concepts, then so are $C \cap D$, $C \cup D$, and $\neg C$; (6) if $r$ is a role and $C$ is a concept, then $\exists r.C$ is also a concept; (7) if $C, D$ are concepts, then $C \times D$ is a role (product role).

The set of formulae of $\mathcal{L}$ is the closure under $\land, \lor, \neg, \rightarrow$ of the atomic formulae: $C \subseteq D$ (concept inclusion), where $C, D$ are concepts; $r \subseteq s$ (role inclusion), where $r, s$ are roles; and $\text{func}(r)$ (functionality assertion), where $r$ is a role.

**Definition 2 (Semantics of $\mathcal{L}$).** The semantics is given in terms of structures $\mathcal{M} = (M, \tau, \cdot)$. The extension of $\mathcal{M}$ from the atomic relations and constants in $\mathcal{M}$ and the satisfaction relation $\models$ are given below. If $\mathcal{M} \models \varphi$, then $\mathcal{M}$ is a

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3 Here $n \in \{1, 2\}$.
4 In DL terms, $\mathcal{L}$ corresponds to Boolean $\text{ALCHOIF}$ knowledge bases with the additional support for role intersection, role union, role difference and product roles.
model of \( \varphi \). We write \( \psi \models \varphi \) if every model of \( \psi \) is also a model of \( \varphi \).

\[
\begin{align*}
(C \cap D)^M &= C^M \cap D^M \\
(C \cup D)^M &= C^M \cup D^M \\
(-C)^M &= M \setminus C^M \\
(C \times D)^M &= C^M \times D^M \\
(\exists r.C)^M &= \{ e \mid \exists e' : (e,e') \in r^M \} \\
M \models C \subseteq D &\text{ if } C^M \subseteq D^M \\
M \models r \subseteq s &\text{ if } r^M \subseteq s^M \\
\end{align*}
\]

The closure of \( \models \) under \( \land, \lor, \neg, \to \) is defined in the natural way. We abbreviate: \( \top = C \lor \neg C \), where \( C \) is an arbitrary atomic concept and \( \bot = \neg \top \); \( \alpha \equiv \beta \) for the formula \( \alpha \subseteq \beta \land \beta \subseteq \alpha \); and \( \exists r \) for the concept \( \exists r ; (o, o') \) for the role \( o \times o' \). Note that \( \top^M = M \) and \( \bot^M = \emptyset \) for any structure \( M = (M, r, \cdot) \).

### 2.3 Running Example: Content Invariants in \( \mathcal{L} \)

Now we make the example from Section 1.1 more precise. The concepts \( ELst \) and \( PLst \) are interpreted as the sets of elements in the employee list resp. the project list. \( mngBy, isMngr \) and \( wrkFor \) are roles. \( o_{Hd} \) and \( o_{Pld} \) are the constants which correspond to the heads of the two lists. The invariants of the systems are:

- The employee and project lists are allocated: \( PLst \cup ELst \subseteq Alloc \)
- Projects and employees are distinct: \( PLst \cap ELst \subseteq \bot \)
- \( wrkFor \) is set to null for projects: \( PLst \subseteq \exists wrkFor.o_{null} \)
- \( mngBy \) is set to null for employees: \( ELst \subseteq \exists mngBy.o_{null} \)
- \( wrkFor \) of employees in the list point to projects in the list or to null: \( \exists wrkFor^-.ELst \subseteq PLst \cup o_{null} \)
- \( isMngr \) is a Boolean field: \( \exists isMngr^-.ELst \subseteq Boolean \)
- \( mngBy \) of projects point to managers or null: \( \exists mngBy^-.PLst \subseteq (ELst \cap \exists isMngr.o_{Hd}) \cup o_{null} \)
- The manager of a project must work for the project: \( mngBy \land (\top \times ELst) \subseteq wrkFor^- \)

Let the conjunction of the invariants be given by \( \varphi_{\text{invariants}} \).

Consider \( S \) from Section 1. The states of the heap before and after the execution of \( S \) can be related by the following \( \mathcal{L} \) formulae. \( \varphi_{\text{lists-updt}} \) and \( \varphi_{\text{p-assgn}} \).

\( \varphi_{\text{lists-updt}} \) states that the employee list at the end of the program \( (ELst) \) is equal to the employee list at the beginning of the program \( (ELst_{gho}) \), and that the project list at the end of the program \( (PLst) \) is the same as the project list at the beginning of the program \( (PLst_{gho}) \), except that \( PLst \) also contains the new project \( o_{proj} \). \( ELst_{gho} \) and \( wrkFor_{gho} \) are ghost relation symbols, whose interpretations hold the corresponding values at the beginning of \( S \).

\[
\begin{align*}
\varphi_{\text{lists-updt}} &= ELst_{gho} \equiv ELst \land PLst_{gho} \cup o_{proj} \equiv PLst \\
\varphi_{\text{p-assgn}} &= ELst_{gho} \cap \exists wrkFor_{gho}.o_{null} \equiv ELst \cap \exists wrkFor.o_{proj}
\end{align*}
\]
Ghost symbols As discussed in Section 2.3, in order to allow invariants of the form \( \varphi_{\text{lists-updt}} = E \text{Lst}_{gho} \equiv E \text{Lst} \land P \text{Lst}_{gho} \cup o_{\text{proj}} \equiv P \text{Lst} \) we need ghost symbols. We assume \( \tau \) contains, for every symbol e.g. \( s \in \tau \), the symbol \( s_{gho} \). Therefore, memory structures actually contain two snapshots of the memory: one is the current snapshot, on which the program operates, and the other is a ghost snapshot, which is a snapshot of the memory at the beginning of the program, and which the program does not change or interact with. We denote the two underlying memory structures of \( \mathcal{M} \) by \( \mathcal{M}_{\text{cur}} \) and \( \mathcal{M}_{\text{gho}} \). Since the interpretations of ghost symbols should not change throughout the run of a program, they will sometime require special treatment.

2.4 The Separation Logic Fragment SLs

The SL that we use is denoted \( \text{SLs} \), and is the logic from [7] with lists and multiple pointer fields, but without trees. It can express that the heap is partitioned into lists and individual cells. For example, to express that the heap contains only the two lists \( E \text{Lst} \) and \( P \text{Lst} \) we can write the \( \text{SLs} \) formula \( \text{ls}(pHd, \text{null}) \ast \text{ls}(eHd, \text{null}) \).

We denote by \( \text{var}_i \in \text{Var} \) and \( f_i \in \text{Fields} \) the sets of variables respectively fields to be used in \( \text{SLs} \)-formulae. \( \text{var}_i \) are constant symbols. \( f_i \) are binary relation symbols always interpreted as functions. An \( \text{SLs} \)-formula \( \Pi \vdash \Sigma \) is the conjunction of a pure part \( \Pi \) and a spatial part \( \Sigma \). \( \Pi \) is a conjunction of equalities and inequalities of variables and \( o_{\text{null}} \). \( \Sigma \) is a spatial conjunction \( \Sigma = \beta_1 \ast \ldots \ast \beta_r \) of formulae of the form \( \text{ls}(E_1, E_2) \) and \( \text{var} \mapsto [f_1 : E_1, \ldots, f_k : E_k] \), where each \( E_i \) is a variable or \( o_{\text{null}} \). Additionally, \( \Sigma \) can be \( \text{emp} \) and \( \Pi \) can be \( \text{T} \). When \( \Pi = \text{T} \) we write \( \Pi \vdash \Sigma \) simply as \( \Sigma \).

The memory model of [7] is very similar to ours. We give the semantics of \( \text{SLs} \) in memory structures directly due to space constraints. See the appendix for a discussion of the standard semantics of \( \text{SLs} \). \( \Pi \) is interpreted in the natural way. \( \Sigma \) indicates that \( \text{Alloc}^\mathcal{M} \) is the disjoint union of \( r \) parts \( P_1^\mathcal{M}, \ldots, P_r^\mathcal{M} \). If \( \beta_i \) is of the form \( \text{var} \mapsto [f_1 : E_1, \ldots, f_k : E_k] \) then \( |P_i^\mathcal{M}| = 1 \) and, denoting \( v \in P_i^\mathcal{M} \), \( f_i^\mathcal{M}(v) = E_j^\mathcal{M} \). If \( \beta_i \) is of the form \( \text{ls}(E_1, E_2) \), then \( |P_i^\mathcal{M}| \) is a list from \( E_1^\mathcal{M} \) to \( E_2^\mathcal{M} \). \( E_2^\mathcal{M} \) might not belong to \( P_i^\mathcal{M} \). If \( \Sigma = \text{emp} \) then \( \text{Alloc}^\mathcal{M} = \emptyset \).

2.5 The Two-variable Fragment with Counting and Trees \( C^{T^2} \)

\( C^2 \) is the subset of first-order logic whose formulae contain at most two variables, extended with counting quantifiers \( \exists^k \), \( \exists^k \) and \( \exists^k \) for all \( k \in \mathbb{N} \). W. Charatonik and P. Witkowski [10] recently studied an extension of \( C^2 \) which trees which, as we will see, contains both our DL and our SL. \( C^{T^2} \) is the subset of second-order logic of the form \( \exists F_1 \varphi(F_1) \land \varphi_{\text{forest}}(F_1) \) where \( \varphi \in C^2 \) and \( \varphi_{\text{forest}}(F_1) \) says that \( F_1 \) is a forest. Note that \( C^{T^2} \) is not closed under negation, conjunction or disjunction. However, \( C^{T^2} \) is closed under conjunction or disjunction with \( C^2 \)-formulae.

A \( C^{T^2} \)-formula \( \varphi \) is satisfiable in a memory structure if there is a memory structure \( \mathcal{M} \) such that \( \mathcal{M} \models \varphi \). We write \( \psi \models_m \varphi \) if \( \mathcal{M} \models \psi \) implies \( \mathcal{M} \models \varphi \) for
Appendix D discusses the translation of cyclic data structures.

Lemma 1. Satisfiability of $CT^2$ by memory structures is in NEXPTIME.

2.6 Embedding $\mathcal{L}$ and SLs in $CT^2$

$\mathcal{L}$ has a fairly standard reduction (see e.g. [8]) to $C^2$: \(\alpha\).

Lemma 2. For every vocabulary, there exists $tr : \mathcal{L}(\tau) \rightarrow C^2(\tau)$ such that for every $\varphi \in \mathcal{L}(\tau)$, $\varphi$ and $tr(\varphi)$ agree on the truth value of all $\tau$-structures.

E.g., $tr(C_1 \sqsubseteq C_2) = \forall x. C_1(x) \rightarrow C_2(x)$. The details of $tr$ are given in Table 1.

The translation of SLs requires more work. Later we need the following related translations: $\alpha : SL_{ls} \rightarrow \mathcal{L}$ extracts from the SLs properties whatever can be expressed in $\mathcal{L}$, $\beta : SL_{ls} \rightarrow CT^2$ captures SLs precisely.

Given a structure $M$, $L^M$ is a singly linked list from $o_{var_1}^M$ to $o_{var_2}^M$ w.r.t. the field $next^M$ if $M$ satisfies the following five conditions, or it is empty. Except for (5), the conditions are expressed fully in $\mathcal{L}$ below:

1. $o_{var_1}^M$ belongs to $L^M$;
2. $o_{var_2}^M$ is pointed to by an $L^M$ element;
3. $o_{var_2}^M$ does not belong to $L^M$;
4. Every $L^M$ element is pointed to from an $L^M$ element, except possibly for $o_{var_1}^M$;
5. All elements of $L^M$ are reachable from $o_{var_1}^M$ via $next^M$.

Let

\[
\begin{align*}
\alpha^1(ls) &= (o_{var_2} \sqsubseteq L) \\
\alpha^2(ls) &= (o_{var_2} \sqsubseteq \exists next^- L) \\
\alpha_{emp\text{-}ls}(ls) &= (L \sqsubseteq \bot) \land (o_{var_1} = o_{var_2}) \\
\alpha(ls) &= \alpha^1(ls) \land \cdots \land \alpha^4(ls) \lor \alpha_{emp\text{-}ls}(ls)
\end{align*}
\]

In memory structures $M$ satisfying $\alpha(ls)$, if $L^M$ is not empty, then it contains a list segment from $o_{var_1}^M$ to $o_{var_2}^M$, but additionally $L^M$ may contain additional simple $next^M$-cycles, which are disjoint from the list segment. Here we use the finiteness of $Alloc^M$ (which contains $L^M$) and the functionality of $next^M$. A connectivity condition is all that is lacking to express $ls$ precisely. $\alpha(ls)$ can be extended to $\alpha : SL_{ls} \rightarrow \mathcal{L}$ in a natural way (see Appendix C) such that:

Lemma 3. For every $\varphi \in SL_{ls}$, $\varphi$ implies $\alpha(\varphi)$ over memory structures.

To rule out the superfluous cycles we turn to $CT^2$. Let

\[
\beta^5(ls) = \forall x \forall y \left[(L(x) \land L(y)) \rightarrow (F_1(x, y) \leftrightarrow next(x, y))\right] \land
\forall x \left[(L(x) \land L(y) \rightarrow \neg F_1(y, x)) \rightarrow (x \approx o_{var_1})\right]
\]

$\beta^5(ls)$ states that the forest $F_1$ coincides with $next$ inside $L$ and that the forest induced by $F_1$ on $L$ is a tree. Let $\beta(ls) = \exists F_1 \ tr(\alpha(ls)) \land \beta^5(ls) \land \varphi_{forest}(F_1)$.

In fact [10] allows existential quantification over two forests, but will only need one.
Lemma 4. For every \( \varphi \in \text{SLis} \): \( \varphi \) and \( \beta(\varphi) \) agree on all memory structures.

\( CT^2 \)'s flexibility allows to easily express variations of singly-linked lists, such as doubly-linked lists, or lists in which every element points to a special head element via a pointer head, and analogue variants of trees.

2.7 Running Example: Shape Invariants

At the loop header of the program \( S \) from the introduction, the memory contains two distinct lists, namely \( \text{PLst} \) and \( \text{ELst} \). \( \text{ELst} \) is partitioned into two parts: the employees who have been visited in the loop so far, and those that have not. This can be expressed in \( \text{SLis} \) by the formula: \( \varphi_{\ell_1} = T : \text{ls}(eHd,e) * \text{ls}(e,nil) * \text{ls}(pHd,nil) \). The translation \( \alpha(\varphi_{\ell_1}) \) is given by

\[
\begin{align*}
P_1 \cup P_2 \cup P_3 & \equiv \text{Alloc} \land \alpha(\text{ls}(eHd,e,\text{next},P_1)) \land \\
\alpha(\text{ls}(e,\text{null},\text{next},P_2)) & \land \alpha(\text{ls}(pHd,\text{null},\text{next},P_3)) \land \\
P_1 \cap P_2 & \equiv \bot \land P_1 \cap P_3 \equiv \bot \land P_2 \cap P_3 \equiv \bot \land \alpha_T
\end{align*}
\]

The translation from \( \text{SL} \) assigns concepts \( P_i \) to each of the lists. \( \alpha_T \) which occurs in \( \alpha(\varphi_{\ell_1}) \) is the translation of \( II = T \) in \( \varphi_{\ell_1} \). In order to clarify the meaning of \( \alpha(\varphi_{\ell_1}) \) we relate the \( P_i \) to the concept names from Section 2.3 and simplify the formula somewhat. Let \( \psi_i = P_1 \cup P_2 = \text{ELst} \land P_3 = \text{PLst} \). \( P_1 \) contains the elements of \( \text{ELst} \) visited in the loop so far. \( \alpha(\varphi_{\ell_1}) \) is equivalent to:

\[
\begin{align*}
\alpha'(\varphi_{\ell_1}) & = \psi_1 \land \text{ELst} \lor \text{PLst} \equiv \text{Alloc} \land \text{ELst} \lor \text{PLst} \equiv \bot \land \alpha(\text{ls}(eHd,e,\text{next},P_1)) \land \\
\land \alpha(\text{ls}(e,\text{null},\text{next},\text{ELst} \lor \neg P_1)) & \land \alpha(\text{ls}(pHd,\text{null},\text{next},\text{PLst})) \\
\beta^5(\Sigma) & = \beta^5(\text{ls}(eHd,e,\text{next},P_1)) \land \beta^5(\text{ls}(e,\text{null},\text{next},\text{ELst} \lor \neg P_1)) \land \\
\beta(\varphi_{\ell_1}) & = \exists F_1 \ \text{tr}(\alpha(\varphi) \land \beta^5(\Sigma) \land \varphi_{\text{forest}}(F_1))
\end{align*}
\]

3 Content Analysis

3.1 Syntax and Semantics of the Programming Language

Loopless Programs are generated by the following syntax:

\[
\begin{align*}
\text{e} & : = \text{var} \cdot f \mid \text{var} \mid \text{null} \quad (f \in \tau_{\text{fields}}, \text{o}_{\text{var}} \in \tau_{\text{var}}) \\
\text{b} & : = (e_1 = e_2) \mid \sim b \mid (b_1 \text{ and } b_2) \mid (b_1 \text{ or } b_2) \mid \text{T} \mid \text{F} \\
\text{S} & : = \text{var} := e_2 \mid \text{var} := f :: \text{skip} \mid S_1; S_2 \mid \text{var} := \text{new} \mid \text{dispose(\text{var})} \mid \\
& \quad \text{if } b \text{ then } S_1 \text{ fi } \mid \text{if } b \text{ then } S_1 \text{ else } S_2 \text{ fi } \mid \text{assume(\text{b})}
\end{align*}
\]

Let \( \text{Exp} \) denote the set of expressions \( e \) and \( \text{Bool} \) denote the set of Boolean expressions \( b \). To define the semantics of pointer and Boolean expressions, we extend \( f^M \) by \( f^M(\text{err}) = \text{err} \) for every \( f \in \tau_{\text{fields}} \). We define \( \mathcal{E}_e(M) : \text{Exp} \rightarrow \text{Addresses}^M \cup \{ \text{null}, \text{err} \} \) and \( \mathcal{B}_b(M) : \text{Bool} \rightarrow \{ o_T, o_F, \text{err} \} \) (with \( \text{err} \notin M \)):

\[
\begin{align*}
\mathcal{E}_{\text{var}}(M) & = o_{\text{var}}^M, \text{ if } o_{\text{var}}^M \in \text{Alloc}^M \quad \mathcal{B}_{e_1 = e_2}(M) = \text{err} \text{ if } \mathcal{E}_{e_1}(M) = \text{err}, i \in \{1, 2\} \\
\mathcal{E}_{\text{var}}(M) & = \text{err}, \text{ if } o_{\text{var}}^M \notin \text{Alloc}^M \quad \mathcal{B}_{e_1 = e_2}(M) = o_T, \text{ if } \mathcal{E}_{e_1}(M) = \mathcal{E}_{e_2}(M) \\
\mathcal{E}_{\text{var},f}(M) & = f^M(\mathcal{E}_{\text{var}}(M)) \\
\mathcal{B}_{e_1 = e_2}(M) & = o_F, \text{ if } \mathcal{E}_{e_1}(M) \neq \mathcal{E}_{e_2}(M)
\end{align*}
\]
Definition 3 (Program). A program is $G = (V, E, \ell_{init}, shp, cnt, \lambda)$ such that $G = (V, E)$ is a directed graph with no multiple edge but possibly containing self-loops, $\ell_{init} \in V$ has in-degree 0, $shp : V \to SLis$, $cnt : V \to \mathcal{L}(\tau)$ are functions, and $\lambda$ is a function from $E$ to the set of loopless programs.

Here is the code $S$ from the introduction:

\[ V = \{\ell_b, \ell_1, \ell_e\} \]
\[ E = \{(\ell_b, \ell_1), (\ell_1, \ell_1), (\ell_1, \ell_e)\} \]
\[ \lambda(\ell_b, \ell_1) = S_b \]
\[ \lambda(\ell_1, \ell_1) = \text{assume}(\sim (e = \text{null})); S_{l_1} \]
\[ \lambda(\ell_1, \ell_e) = \text{assume}(e = \text{null}); S_e \]
\[ \ell_{init} = \ell_b \]  

$S_b$, $S_{l_1}$, and $S_e$ denote the three loopless code blocks which are respectively the code block before the loop, inside the loop and after the loop. The annotations $shp$ and $cnt$ are described in Section 3.4.
The semantics of programs derive from the semantics of loopless programs and is given in terms of program paths. Given a program $G$, a path in $G$ is a finite sequence of directed edges $e_1, \ldots, e_t$ such that for all $1 \leq i \leq t - 1$, the tail of $e_i$ is the head of $e_{i+1}$. A path may contain cycles.

**Definition 4 (leadsto*) for paths.** Given a program $G$, a path $P$ in $G$, and memory structures $M_1$ and $M_2$ we define whether $\langle P, M_1 \rangle \leadsto M_2$ holds inductively.

- If $P$ is empty, then $\langle P, M_1 \rangle \leadsto M_2$ iff $M_1 = M_2$.
- If $e_t$ is the last edge of $P$, then $\langle P, M_1 \rangle \leadsto M_2$ iff there is $M_3$ such that $\langle P\{e_t\}, M_1 \rangle \leadsto M_3$ and $\langle \lambda(e_t), M_1 \rangle \leadsto M_3$. $P\{e_t\}$ denotes the path obtained from $P$ by removing the last edge $e_t$.

### 3.2 Hoare-style Proof System

Now we are ready to state our two-step verification methodology that we formulated in Section 1 precisely. Our methodology assumes a program $P$ as in Definition 3 as input (ignoring the $shp$ and $cnt$ functions for the moment).

**I. Shape Analysis.** The user annotates the program locations with SL formulae from $\text{SL}_{ls}$ (stored in the $shp$ function of $P$). Then the user proves the validity of the $\text{SL}_{ls}$ annotations, for example, by using techniques from [7].

**II. Content Analysis.** The user annotates the program locations with $\mathcal{L}$-formulae that she wants to verify (stored in the $cnt$ function of $P$). We point out that an annotation $\text{cnt}(\ell)$ can use the concepts occurring in $\alpha(shp(\ell))$ (recall that $\alpha : \text{SL}_{ls} \rightarrow \mathcal{L}$ maps SL formulae to $\mathcal{L}$-formulae).

In the rest of the paper we discuss how to verify the $\text{cnt}$ annotations. In Section 3.3 we describe how to derive a verification condition for every program edge. The verification conditions rely on the backwards propagation function $\Theta$ for $\mathcal{L}$-formulae which we introduce in Section 3.5. The key point of our methodology is that the validity of the verification conditions can be discharged automatically by a satisfiability solver for $\text{CT}^2$-formulae. We show that all the verification conditions are valid if and only if $\text{cnt}$ is inductive. Intuitively, $\text{cnt}$ being inductive ensures that the annotations $\text{cnt}$ can be used in an inductive proof to show that all reachable memory structures indeed satisfy the annotations $\text{cnt}(\ell)$ at every program location $\ell$ (see Definition 6 below).

### 3.3 Content Verification

We want to prove that, for every initial memory structure $M_1$ from which the computation satisfies $shp$ and which satisfies the content pre-condition $\text{cnt}(\ell_{\text{init}})$, the computation satisfies $\text{cnt}$. Here are the corresponding verification conditions, which annotate the vertices of $G$:

**Definition 5 (Verification conditions).** Given a program $G$, $VC$ is the function from $E$ to $\mathcal{L}$ given for $e = (\ell_0, \ell)$ by

$$VC(e) = \neg[\beta(shp(\ell_0)) \land \text{tr}(cnt(\ell_0)) \land \text{tr}(\Theta_{\lambda(e)}(\alpha(shp(\ell)) \land \neg\text{cnt}(\ell)))]$$

$VC(e)$ holds if $VC(e)$ is a tautology over memory structures ($\top \models_m VC(e)$).
is discussed in Section 3.5. As we will see, $VC(\ell_0, \ell)$ expresses that when running the loopless program $\lambda(e)$ when the memory satisfies the the annotations of $\ell_0$, and when the shape annotation of $\ell$ is at least partly true (i.e., when $\alpha(shp(\ell))$), the content annotation of $\ell$ holds.

Let $J$ be a set of memory structures. For a formula in $CT^2$ or $L$, we write $J \models \varphi$ if, for every $M \in J$, $M \models \varphi$. Let $Init$ be a set of memory structures.

**Definition 6 (Inductive program annotation).** Let $f : V \to CT^2$. We say $f$ is inductive for $Init$ if (i) $Init \models f(\ell_{init})$, and (ii) for every edge $e = (\ell_1, \ell_2) \in E$ and memory structures $M_1$ and $M_2$ such that $M_1 \models f(\ell_1)$ and $(\lambda(e), M_1) \leadsto M_2$, we have $M_2 \models f(\ell_2)$. We say $shp$ is inductive for $Init$ if the composition $shp \circ \beta : V \to CT^2$ is inductive for $Init$. We say $cnt$ is inductive for $Init$ relative to $shp$ if $shp$ is inductive for $Init$ and $g : V \to CT^2$ is inductive for $Init$, where $g(\ell) = tr(cnt(\ell)) \land \beta(shp(\ell))$.

**Theorem 1 (Soundness and Completeness of the Verification Conditions).** Let $G$ be a program such that $shp$ is inductive for $Init$ and $Init \models cnt(\ell_{init})$. The following statements are equivalent:

(i) For all $e \in E$, $VC(e)$ holds.  
(ii) $cnt$ is inductive for $Init$ relative to $shp$.

We make the notion of a computation satisfying the verification conditions precise using the following definition:

**Definition 7 (Reach($\ell$)).** Given a program $G$, a node $\ell \in V$, and a set $Init$ of memory structures, $Reach(\ell)$ is the set of memory structures $M$ for which there is $M_{init} \in Init$ and a path $P$ in $G$ starting at $\ell_{init}$ such that $(P, M_{init}) \leadsto^* M$.

In particular, $Reach(\ell_{init}) = Init$. The proof of Theorem 1 and its consequence Theorem 2 below are given in in Appendix F.

**Theorem 2 (Soundness of the Verification Methodology).** Let $G$ be a program such that $shp$ is inductive for $Init$ and $Init \models cnt(\ell_{init})$. If for all $e \in E$, $VC(e)$ holds, then for $\ell \in V$, $Reach(\ell) \models cnt(\ell)$.

### 3.4 Running Example: General Methodology

To verify the correctness of the code $S$, the $shp$ and $cnt$ annotations must be provided. The shape annotations of program $S$ are:

$$shp(\ell_0) = ls(eHd, null) * ls(pHd, null)$$

$$shp(\ell_1) = \varphi_{\ell_1}$$

$$shp(\ell_2) = (\text{proj} = pHd) \uplus ls(eHd, null) * ls(pHd, null)$$

$\varphi_{\ell_1} = ls(eHd, null) * ls(e, null) * ls(pHd, e)$ was considered in Section 2.7.

The three content annotations require that the system invariants $\varphi_{\text{invariants}}$ from Section 2.3 hold. The post-condition additionally requires that $\varphi_{\text{p-assgn}}$ and $\varphi_{\text{lists-updts}}$ hold. Recall $\varphi_{\text{p-assgn}}$ states that every employee which was not
assigned a project, is assigned to \( o_{proj} \). \( \varphi_{lists-updts} \) states that the content of the two lists remain unchanged, except that the project \( o_{proj} \) is inserted to \( PLst \).

In order to interact with the translations \( \alpha(shp(\cdot \cdot \cdot)) \) of the shape annotations, we need to relate the \( P_i \) to the concepts \( ELst \) and \( PLst \). In Section 2.7 we defined \( \psi_1 \), which relates the \( P_i \) generated by \( \alpha \) on \( shp(\ell_i) \).

\[
\begin{align*}
\psi_{t_b} &= \psi_{t_c} = P_1 \equiv ELst \wedge P_2 \equiv PLst \\
\text{cnt}(\ell_b) &= \psi_{t_b} \wedge \varphi_{\text{invariants}} \\
\text{cnt}(\ell_c) &= \psi_{t_c} \wedge \varphi_{\text{invariants}} \wedge \varphi_{\text{lists-updt}} \wedge \varphi_{\text{p-as-\ell_i}} \\
\varphi_{\text{p-as-\ell_i}} &= P_1 \cap \exists wrkFor_{gho-o_{null}} \equiv P_1 \cap \exists wrkFor_{o_{proj}} \\
\varphi_{p-as-\ell_i} &\text{ makes no demands on elements of } ELst \text{ which have not been reach in the loop so far. The verification conditions of } G \text{ are, for each } (\ell_1, \ell_2) \in E,
\end{align*}
\]

The verification conditions \( VC(e) \) express that the loopless programs on the edges \( e \) of \( G \) satisfy their annotations. To prove the correctness of \( G \) w.r.t. \( VC(e) \) using Theorem 2, we prove that \( VC(e), e \in E, \) hold, in order to get:

**Conclusion 1** Reach(\( \ell \)) \( \models \text{cnt}(\ell) \), for all \( \ell \in V \).

### 3.5 Backwards Propagation and the Running Example

Here we shortly discuss the backwards propagation of a formula along a loopless program \( S \). Let \( (S, M_1) \sim M_2 \) where \( M_1 \) and \( M_2 \) are memory structures over the same vocabulary \( \tau \). E.g., in our running example, for \( i = 1, 2, \) \( M_i \) is

\[
\langle M, ELst_{M_i}, next_{M_i}, mngBy_{M_i}, \ldots, ELst_{gho}, next_{gho}, \ldots, Alloc_{M_i}, Aux_{M_i}, \ldots \rangle
\]

We will show how to translate a formula for \( M_2 \) to a formula for an extended \( M_1 \). Fields and variables in \( M_2 \) will be translated by the backwards propagation into expressions involving elements of \( M_1 \). For ghost symbols \( s_{gho} \), \( s_{gho}^{M_1} \) will be used instead of \( s_{gho}^{M_2} \) since they do not change during the run of the program. Let \( \tau_{\text{rem}} \subseteq \tau \) be the set of the remaining symbols, i.e. the symbols of \( \tau \setminus (\{ \text{PossibleTargets, MemPool} \} \cup \text{fields}) \) which are not ghost symbols, for example \( ELst \), but not \( ELst_{gho}, next \) or \( mngBy \). We need the result of the backwards propagation to refer to the interpretations of symbols in \( \tau_{\text{rem}} \) from \( M_2 \) rather than \( M_1 \). Therefore, these interpretations are copied as they are from \( M_2 \) and added to \( M_1 \) as follows. For every \( R \in \tau_{\text{rem}} \), we add a symbol \( R_{\text{ext}}^{M_1} \) the tuple
\[(\langle R^\text{ext} \rangle \mathcal{M}_1 : (R^\text{ext})\mathcal{M}_1 = R^\mathcal{M}_2 \text{ and } R \in \tau^\text{rem}) \] Let \(\tau^\text{ext}\) extend \(\tau\) with \(R^\text{ext}\) for each \(R \in \tau^\text{rem}\). The backwards propagation updates the fields and variables according to the loopless code. Afterwards, we substitute the symbols \(R \in \tau^\text{rem}\) in \(\varphi\) with the corresponding \(R^\text{ext}\). We present here a somewhat simplified version of the backwards propagation lemma. The precise version is similar in spirit and is given in Appendix E.

**Lemma 5 (Simplified).** Let \(S\) be a loopless program, let \(\mathcal{M}_1\) and \(\mathcal{M}_2\) be memory structures, and \(\varphi\) be an \(L\)-formula over \(\tau\).

1. If \(\langle S, \mathcal{M}_1 \rangle \sim \mathcal{M}_2\), then: \(\mathcal{M}_2 \models \varphi\) if \(\varphi\) iff \(\mathcal{M}_1, (\langle R^\text{ext} \rangle \mathcal{M}_1) \models \Theta_S(\varphi)\).
2. If \(\langle S, \mathcal{M}_1 \rangle \sim\text{abort}\), then \(\mathcal{M}_1, (\langle R^\text{ext} \rangle \mathcal{M}_1) \not\models \Theta_S(\varphi)\).

As an example of the backwards propagation process, we consider a formula from Section 3.4, which is part of the content annotation of \(\ell I\) and perform the backwards propagation on the loopless program inside the loop:

\[
\varphi_{p-as-\ell I} = P_1 \land \exists \text{wrkFor}_{gho-o\text{null}} \equiv P_1 \land \exists \text{wrkFor}_{o\text{proj}}.
\]

Since \texttt{next} does not occur in \(\varphi_{p-as-\ell I}\), backwards propagation of \(\varphi_{p-as-\ell I}\) over \(e := e\texttt{next}\) does not change the formula (however \(o(\text{shp}(\ell I))\) by this command).

The backwards propagation of the \texttt{if} command gives

\[
\Psi_{S_{\ell I}}(\varphi_{p-as-\ell I}) = (\neg(\exists \text{wrkFor}^- e_o \equiv o\text{null}) \land \varphi_{p-as-\ell I}) \lor (\exists \text{wrkFor}^- e_o \equiv o\text{null} \land \Psi_e \text{wrkFor}:\text{proj}(\varphi_{p-as-\ell I}))\]

\[
\Psi_e \text{wrkFor}:\text{proj}(\varphi_{p-as-\ell I}) = P_1 \land \exists \text{wrkFor}_{gho-o\text{null}} \equiv P_1 \land \exists (\text{wrkFor}(o_e \times T)) \lor (o_e, o\text{proj}), o\text{proj}.
\]

\(\Psi_e \text{wrkFor}:\text{proj}(\varphi_{p-as-\ell I})\) is obtained from \(\varphi_{p-as-\ell I}\) by substituting the \text{wrkFor} role with the correction \((\text{wrkFor}(o_e \times T)) \lor (o_e, o\text{proj})\) which updates the value of \(o_e\) in \text{wrkFor} to \text{proj}. \(\Phi_{S_{\ell I}}(\varphi_{p-as-\ell I})\) is obtained from \(\Psi_{S_{\ell I}}(\varphi_{p-as-\ell I})\) by substituting \(P_1\) with \(P_1^\text{ext}\). \(\Theta\) is differs from \(\Phi\) from technical reasons related to aborting computations (see Appendix E).

## 4 Related Work

**Shape Analysis** attracted considerable attention in the literature. The classical introductory paper to SL [23] presents an expressive SL which turned out to be undecidable. We have restricted our attention to the better behaved fragment in [7]. The work on SL focuses mostly on shape rather than content in our sense. SL has been extended to object oriented languages, cf. e.g. [22,11], where shape properties similar to those studied in the non objected oriented case are the focus, and the main goal is to overcome difficulties introduced by the additional features of OO languages. Other shape analyses could be potential candidates for integration in our methodology. [24] use 3-valued logic to perform shape analysis. Regional logic is used to check correctness of program with shared
dynamica memory areas [5]. [16] uses nested tree automata to represent the heap. [20] combines monadic second order logic with SMT solvers.

**Description Logics** have not been considered for verification of programs with dynamically allocated memory, with the exception of [13] whose use (mostly undecidable) DLs to express shape-type invariants, ignoring content information. In [9] the authors consider verification of loopless code (transactions) in graph databases with integrity constraints expressed in DLs. Verification of temporal properties of dynamic systems in the presence of DL knowledge bases has received significant attention (see [4,14] and their references). **Temporal Description Logics**, which combine classic DLs with classic temporal logics, have also received significant attention in the last decade (see [19] for a survey).

**Related Ideas.** Some recent papers have studied verification strategies which use information beyond the semantics of the source code. E.g., [18] is using diagrams from design documentation to support verification. [12,1] infer the intended use of program variables to guide a program analysis. Instead of starting from code and verifying its correctness, [15] explores how to declaratively specify data structures with sharing and how to automatically generate code from this specification. Given the importance of both DL as a formalism of content representation and of program verification, and given that both are widely studied, we were surprised to find little related work. However, we believe this stems from large differences between the research in the two communities, and from the interdisciplinary nature of the work involved.

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A Separation Logic

Here we expand on the treatment of separation logic in the paper. We have defined the semantics of SL using our memory structures. The memory model used in [7] is very similar to our memory structures. We give the standard semantics of SL here in terms of heaps and stacks, and relate it to memory structures. It is convenient to define first $\text{SL}$, which does not allow list segments, and then extend to $\text{SL}_{\text{ls}}$.

The allocated cells of the memory all have the same finite collection of fields (denoted $\text{Fields}$). Let $\text{Add}$ and $\text{Val}$ be disjoint sets. $\text{Add}$ is the set of addresses (locations in the terminology of [7], not to be confused with our use of “locations in a program”). $\text{Val}$ is a set of values which include $\text{nil}$. The description of the memory consists of two parts, a heap and a stack. A heap is a partial function $h: \text{Add} \to (\text{Fields} \cup \text{Val} \cup \text{Add})$ which is only defined on a finite subset of $\text{Add}$.

A stack is a function from a finite set $\text{Var}$ of local variables $s: \text{Var} \to \text{Val} \cup \text{Add}$.

The syntax of $\text{SL}$ is as follows:

$$
\begin{align*}
\text{var}_i & \in \text{Var}, i \in \mathbb{N} \quad \Pi ::= T \mid E = E \mid E \neq E \mid \Pi \land \Pi \\
\text{fields}, i \in \mathbb{N} \quad \Sigma ::= \text{emp} \mid \Sigma \ast \Sigma \\
\text{Add} & ::= \text{null} \mid \text{var}_i \\
\text{SL}-\text{formula} ::= \Pi \mid \Sigma
\end{align*}
$$

When $\Pi = T$ we write $\Pi \mid \Sigma$ simply as $\Sigma$. The semantics of $\text{SL}$ is given by a relation $s, h \models \phi$ where $s \in \text{Stacks}, h \in \text{Heaps}$. We define $[\text{var}_i]s \overset{\text{def}}{=} s(x)$ and $[\text{nil}]s \overset{\text{def}}{=} \text{nil}$, and:

$$
\begin{align*}
s, h & \models \text{true} \quad \text{always} \\
s, h & \models E_1 = E_2 \quad \text{iff } [E_1]s = [E_2]s \\
s, h & \models E_1 \neq E_2 \quad \text{iff } [E_1]s \neq [E_2]s \\
s, h & \models \Pi_0 \land \Pi_1 \quad \text{iff } s, h \models \Pi_0 \text{ and } s, h \models \Pi_1 \\
s, h & \models \text{var} \rightarrow [f_1 : E_1] \quad \text{iff } h = [\text{var}_1]s \rightarrow r \text{ where } r(f_i) = [E_i]s \\
s, h & \models \text{emp} \quad \text{iff } h = \emptyset \\
s, h & \models \Sigma_1 \ast \Sigma_2 \quad \text{iff } \exists h_0 h_1, h = h_0 \ast h_1 \text{ and } s, h_0 \models \Sigma_0 \text{ and } s, h_1 \models \Sigma_1 \\
s, h & \models \Pi \mid \Sigma \quad \text{iff } s, h \models \Pi \text{ and } s, h \models \Sigma
\end{align*}
$$

where $h_0 \ast h_1$ enforces there is no address in $\text{Add}$ on which both $h_0$ and $h_1$ are defined, and $h = h_0 \ast h_1$ denotes that $h$ is the union of $h_0$ and $h_1$. Additionally, not all fields in $\text{Fields}$ need to occur in $\text{var} \rightarrow [f_1 : E_1]$, and those that do not are assigned $\text{nil}$ implicitly.

$\text{SL}_{\text{ls}}$ extends $\text{SL}$ by adding list segments: the syntax is extended by

$$
\Sigma ::= \cdots \mid \text{ls}(E_1, E_2)
$$

and the semantics of $\text{ls}(E_1, E_2)$ is the least fixed point of the predicate given by

$$
(E_1 = E_2 \land \text{emp}) \lor (E_1 \neq E_2 \land \exists y. E_1 \rightarrow [n : y] \ast \text{ls}(y, E_2))
$$
Heaps and Stacks vs. Memory Structures The memory model of SL and our memory model are easily translatable. The distinction between values in $Val$ and addresses in $Add$ does not play a major role in [7], so we simplify by setting $Val = \{nil, true, false\}$.

Given $M$ we define $s_M$ and $h_M$ as follows. $Fields$ is equal to $\tau_{fields}$ and $Add = Addresses$ and we have $M = Val \cup Add$ with $nil = o^M_{null}$, $true = o^M_T$ and $false = o^M_F$. For every variable $var_i$, $s(var_i) = o^M_{var_i}$. For every $add \in Add$ we define $h(add) = h_{add}$ as follows: for every $f \in Fields$, $h_{add}(f) = f^M(add)$.

Given $s$ and $h$, we define $M_{s,h} = M$ as follows. The universe $M_{s,h}$ is $Add \cup \{o^M_{null}, o^M_T, o^M_F\}$, with $o^M_{null} = nil$, $o^M_T = true$ and $o^M_F = false$ and $Addresses^M = Add$. The set of addresses on which $h$ is defined is $Alloc^M$. For every field $f \in Fields$, $f^M = h_f$, where $h_f(add) = h(add)(f)$ is the value the field $f$ of address $add$ receives under $h$. For every variable $var_i$, $o^M_{var_i} = s(var_i)$.

The above shows how to transform the memory models into each other in a natural way. More precisely:

**Lemma 6.** If $s, h$ are a stack and a heap, and if $s_{M_{s,h}}, h_{M_{s,h}}$ are the stack and heap obtained by applying the above transformations on $s$ and $h$ in order, then $s_{M_{s,h}} = s$ and $h_{M_{s,h}} = h$.

**Remark 1.** The semantics of the standard programming language used with separation logic allows memory cells to be reallocated, while our programming language forbids this for technical simplicity.

**B CT²-Satisfiability in Memory Structures**

In this appendix we prove Lemma 1, i.e. we show that satisfiability of $CT^2$ formulae by memory structures is in NEXPTIME. We employ the fact that finite satisfiability of $CT^2$-formulae, i.e. truth in a structure with a finite domain, is in NEXPTIME:

**Theorem 3 (W. Charatonik and P. Witkowski [10]).** Finite satisfiability of $CT^2$ is NEXPTIME-complete.

To show that satisfiability of a formula $\psi \in CT^2$ in a memory structure can be decided in non deterministic exponential time, it suffices to construct in linear time a formula $\psi_m \in C^2$ such that $\psi$ is satisfiable in a memory structure iff $\psi_m \land \psi$ is finitely satisfiable. The formula $\psi_m$ is the conjunction of formulae corresponding to requirements (3)-(9) we placed on memory structures. The conjoined formulae are the translations using $tr : L \rightarrow C^2$ from Table 1 of the following formulae:

- $Aux \equiv o_{null} \sqcup o_T \sqcup o_F$,
- $Addresses \sqsubseteq \neg Aux$ and $Addresses \sqcup Aux \equiv \top$,
- $Alloc \sqsubseteq \neg MemPool$, $Alloc \sqsubseteq \neg PossibleTargets$, $MemPool \sqsubseteq \neg PossibleTargets$, and
  
  $Alloc \sqcup PossibleTargets \sqcup MemPool \sqsubseteq Addresses$,
while MemPool through the next. These cycles are not reachable from the variables of the head variable of the list.

As discussed in Section 2.6, $\alpha$ and $\beta$ are restated here:

Table 1. Translation of $\mathcal{L}$ into $\mathcal{C}^2$ by employing only two variables $x$ and $y$. In each translation rule, $z \in \{x, y\}$. Moreover, $z = y$ if $z = x$, and $z = x$ if $z = y$.

- $o \subseteq -\text{MemPool}$ for every constant symbol $o$ in $\psi$,
- $\text{func}(f)$ and $\text{Addresses} \subseteq \exists f.\neg\text{MemPool}$ for every $f$ of $\psi$ with $f \in \tau_{\text{fields}}$,
- $\text{MemPool} \subseteq \exists f.\alpha_{\text{null}} \sqcup \alpha_f$ for every $f \in \tau_{\text{fields}}$, and
- $C \subseteq \neg\text{MemPool}$ for every atomic concept $C \in \tau$ with $C \neq \text{MemPool}$.

Requirements (1) and (2) hold by the correct choice of vocabulary. To see that requirement (10) holds, namely that $\text{Alloc}^M$ and $\text{PossibleTargets}^M$ are finite while $\text{MemPool}^M$ is infinite, note that any finite model $M$ of $\psi_m \land \psi$ is almost the desired memory structure with $M \models \psi$. The desired $M$ is obtained by adding to $\text{MemPool}^M$ infinitely many fresh elements $e$ and setting $f^M(e) = \alpha_{\text{null}}$ for all $f \in \tau_{\text{fields}}$.

C Translations of $\mathcal{L}$ and SLIs into $\mathcal{C}^2$

As discussed in Section 2.6, $\alpha$ translates SLIs-formulae into $\mathcal{L}$ almost exactly, with the caveat that for every list $L$, some redundant cycles may exist in $L$. These cycles are not reachable from the variables of the head variable of the list through the next pointer. Since $\mathcal{L}$ is a fragment of first order logic, properties related to connectivity cannot be used to rule out these cycles. We use $\mathcal{C}^2$ to express the necessary connectivity property. Recall $\beta^\text{loc}(\bar{ls})$ stated that the forest $F_1$ coincides with $\text{next}$ inside $L$ and that the forest induced by $F_1$ on $L$ is a tree.

Here we define the two translations $\alpha$ and $\beta$ so that the satisfy the Lemmas 3 and 4, restated here:
Lemma 7. i. For every heap $h$ and stack $s$, if $\varphi \in \text{SL}\mathbf{is}$ then: if $s, h \models \varphi$ then $\mathcal{M}_{s,h} \models \alpha(\varphi)$.

ii. For every heap $h$ and stack $s$, if $\varphi \in \text{SL}\mathbf{is}$ then: $s, h \models \varphi$ iff $\mathcal{M}_{s,h} \models \beta(\varphi)$.

It is convenient to define the following notation: if $\varphi = \Pi_l \Sigma$, then there exist $\delta_1, \ldots, \delta_r$ such that $\Sigma = \delta_1 \cdots \delta_r$ and the $\delta_i$ are of the form $\text{var}_i \mapsto [f_i : E_i]$. We use the concepts $P_1, \ldots, P_r$ which partition the allocated memory cells according to $\Sigma$.

First we define the formula $\alpha(\varphi)$ for $\varphi \in \text{SL}$.

\[
\alpha(E_1 = E_2) = (o_{E_1} \equiv o_{E_2}) \\
\alpha(E_1 \neq E_2) = \neg(o_{E_1} \equiv o_{E_2}) \\
\alpha(\Pi_0 \land \Pi_1) = \alpha(\Pi_0) \land \alpha(\Pi_1) \\
\alpha(\text{true}) = (\top \subseteq \top) \\
\alpha(\text{emp}) = (\text{Alloc} \equiv \bot) \\
\alpha(\delta_i) = (P_i \equiv o_{\text{var}_i} \land \bigwedge_{i=1}^{k} ((o_{\text{var}_i}, o_{E_i}) \subseteq f_i), \\
\text{for } \delta_i = \text{var}_1 \mapsto [f_i : E_i : i = 1, \ldots, k] \\
\alpha(\Sigma) = \bigwedge_{1 \leq i \leq r} \alpha(\delta_i) \land \bigwedge_{1 \leq i, t \leq r} (P_i \cap P_t \equiv \bot) \\
\alpha(\Pi \mid \Sigma) = P_1 \sqcup \cdots \sqcup P_r \equiv \text{Alloc} \land \alpha(\Pi) \land \alpha(\Sigma)
\]

By construction we have for every $\varphi \in \text{SL}$, $s, h \models \varphi$ iff $\mathcal{M}_{s,h} \models \alpha(\varphi)$.

Now we turn to $\text{SL}\mathbf{is}$. Here $\Sigma = \delta_1 \cdots \delta_r$ where the $\delta_i$ are of either of the forms $\text{var}_i \mapsto [f_i : E_i]$ or $\text{ls}(E_1, E_2)$. If $\delta_i = \text{ls}(E_1, E_2)$, then $\alpha(\delta_i)$ is defined similarly to the definition of $\alpha(\text{ls})$ in Section 2.6 using $P_i$, $E_1$ and $E_2$:

\[
\begin{align*}
\alpha^1(\delta_i) &= (o_{E_1} \subseteq P_i) \\
\alpha^2(\delta_i) &= (o_{E_2} \subseteq \exists \text{next}^- P_i) \\
\alpha^3(\delta_i) &= (o_{E_2} \subseteq \neg P_i) \\
\alpha^4(\delta_i) &= (P_i \equiv o_{E_1} \sqcup \exists \text{next}^- P_i)
\end{align*}
\]

Let

\[
\begin{align*}
\alpha_{\text{emp} - \text{ls}}(\delta_i) &= (P_i \subseteq \bot) \land (o_{E_1} = o_{E_2}) \\
\alpha(\delta_i) &= \alpha^1(\delta_i) \land \cdots \land \alpha^4(\delta_i) \lor \alpha_{\text{emp} - \text{ls}}(\delta_i)
\end{align*}
\]

and $\alpha(\text{ls}(E_1, E_2)) = \alpha^1(\text{ls}(E_1, E_2)) \land \cdots \land \alpha^4(\text{ls}(E_1, E_2))$.

Now we turn the translation $\beta$. Similarly to $\beta^0(\text{ls})$ from Section 2.6, we define $\beta^0(\delta_i)$ for each $\delta_i = \text{ls}(E_1, E_2)$ as:

\[
\begin{align*}
\forall x \forall y \left[ (P_i(x) \land P_i(y)) \rightarrow (F_1(x, y) \iff \text{next}(x, y)) \right] \land \\
\forall x \left[ (P_i(x) \land \forall y (P_i(y) \rightarrow \neg F_1(y, x))) \rightarrow (x \approx o_{\text{var}_i}) \right]
\end{align*}
\]

$\beta^0(\delta_i)$ states that the forest $F_1$ coincides with $\text{next}$ inside $L$ and that the forest induced by $F_1$ on $P_i$ is a tree. Let $I \subseteq \{1, \ldots, r\}$ be the set of $i$ such that $\delta_i$ is of the form $\text{ls}(E_1, E_2)$. Let

\[
\begin{align*}
\beta^0(\Sigma) &= \bigwedge_{i \in I} \beta^0(\delta_i) \\
\beta(\Pi \mid \Sigma) &= \exists F_1 \text{tr}(\alpha(\Pi \mid \Sigma)) \land \beta^0(\Sigma) \land \varphi_{\text{forest}}(F_1)
\end{align*}
\]
Note that to get that $\beta^5(\Sigma)$ indeed states the connectivity condition for each of the lists we use the fact that $P_1, \ldots, P_r$ are disjoint, and therefore the trees we quantify for the different lists are disjoint.

D Cyclic Data Structures in $CT^2$

Here we want to clarify that cyclic data structures such as cyclic lists which are expressible in $SL_{ls}$ are translated correctly into $CT^2$ by $\beta$.

Consider the formula $ls(a,b) * ls(b,a)$ which defines a cyclic list with at least two elements. In the translation to $CT^2$, $P_1$ contains the elements of the list from $a$ to $b$, and $P_2$ contains the elements of the list from $b$ to $a$. Importantly, $b$ does not belong to $P_1$, and $a$ does not belong to $P_2$. This is captured by $\alpha$ in the translation of $SL_{ls}$ to $L$ in Section 2.6 or Appendix C.

The translation requires (in $\beta^5$) that the forest $F_1$ coincides with $next$ inside $P_1$ and $P_2$. However, crucially, there is no requirement on $next$ between $P_1$ and $P_2$, see the definitions of $\beta^5$ and $\beta$ in Section 2.6 and Appendix C.

As a result, in the cyclic list $ls(a,b) * ls(b,a)$, not all $next$ edges are required to belong to $F_1$. Rather, the two edges that point to $a$ and to $b$ respectively are not required to belong to $F_1$.

In more detail, if $M \models ls(a,b) * ls(b,a)$ then:

1. $P_1^M = \{a_1, \ldots, a_i\}$ such that $a_1 = a$ and the edges $(a_j, a_{j+1})$ belong both to $next^M$ and to $F_1^M$, for $1 \leq j \leq i - 1$.
2. The edge $(a_i, b)$ belongs to $next^M$ but might not belong to $F_1^M$.
3. $P_2^M = \{b_1, \ldots, b_k\}$ such that $b_1 = b$ and the edges $(b_j, b_{j+1})$ belong both to $next^M$ and to $F_1^M$, for $1 \leq j \leq k - 1$.
4. The edge $(b_k, a)$ belongs to $next^M$ but might not belong to $F_1^M$.

Additionally note that the translation of a formula of the form $var_1 \mapsto [f_1 : E_i]$ also does not require the $next$ edge from $var_1$ to belong to $F_1$.

E Backwards propagation

Here we give the exact formulation of the backwards propagation lemma and prove it.

It is convenient to consider a program $\overline{S}$, which behaves like $S$, except that it does not abort. $\overline{S}$ uses a fresh variable $abo$ to indicate whether $S$ aborts. The command $abo := F$ is added at the beginning of the code. Every command $C$ of the form $var_1 := var_2.f$, $var_2.f := var_1$ or $dispose(var_2)$ is replaced with $\overline{C} = if \ var_2 = null \ then \abo := T \ else \ C \ fi$. For if, assume and assignments of the form $var_1.f_1 := var_2.f_2$ commands the case is similar, except that there may be two evaluations of the form $var_1.f_1$, which need to be reflected in the condition in $\overline{C}$. By the construction of $\overline{S}$, $\overline{S}$ has the following properties:

1. The run of $M_1$ on $\overline{S}$ does not abort for any $M_1$. 

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Lemma 9. Let \( \tau \) be an \( \mathcal{L} \)-formula over \( \tau \).

1. If \( \langle S, M_1 \rangle \leadsto_{d_y} M_2 \), then:
   \[ M_2 \models \varphi \iff \langle M_1, (\bar{R}^{ext})M_1, \bar{d}_{\bar{Y}}, d_{abo} \rangle \models \Theta_S(\varphi). \]

2. If \( \langle S, M_1 \rangle \leadsto_{d_y} \) abort, then for every tuple \( \bar{d}_{\bar{Y}} \) of \( M \) elements, \( \langle M_1, (\bar{R}^{ext})M_1, \bar{d}_{\bar{Y}}, d_{abo} \rangle \not\models \Theta_S(\varphi). \)

The vocabulary of the structure \( \langle M_1, (\bar{R}^{ext})M_1, \bar{d}_{\bar{Y}}, d_{abo} \rangle \) is \( \tau_{\bar{Y}}^{ext} \cup \{o_{abo}\} \).

The definition of \( \Theta_S \) is:
Definition 8. \( \Theta_S(\varphi) = \Phi(\varphi \land (\sigma_{\text{nth}} \equiv \sigma_{\text{th}}))) \), \( \Phi \) is obtained from \( \Psi \) by substituting every symbol \( R \in \tau_{\text{mem}} \) in \( \varphi \) by \( R^{\text{ext}} \), and \( \Psi \) is defined as:

\[
\begin{align*}
\Psi_{\text{skip}}(\varphi) &= \varphi \\
\Psi_{\text{var} := e}(\varphi) &= \varphi[\sigma_{\text{var}}, \sigma_e], \quad e = \text{var}_2 \text{ or } e = \text{null} \\
\Psi_{\text{gvar} := \text{var}_2 \cdot \text{f} \cdot e}(\varphi) &= \varphi[\sigma_{\text{var}}, \sigma_2] \land (\exists f \cdot \sigma_{\text{var}} \equiv \sigma_f) \\
\Psi_{\text{var} := e}(\varphi) &= \varphi[f \cdot f \cdot (\sigma_{\text{var}} \times \top) \cup (\sigma_{\text{var}}, \sigma_e)], \\
&\text{where } e = \text{var}_2 \text{ or } e = \text{null} \\
\Psi_{\text{if } b \text{ then } S_1 \text{ else } S_2 \cdot \text{f} \cdot e}(\varphi) &= \varepsilon_b \land \Psi_{S_1}(\varphi) \lor \neg \varepsilon_b \land \Psi_{S_2}(\varphi) \\
\Psi_{\text{dispose} := \text{new}}(\varphi) &= \varphi[\sigma_{\text{var}}, \sigma_{\text{f}}](\text{Alloc/Alloc} \cap \sigma_e) \\
&\land \text{new} \equiv \neg \text{Alloc} \\
\Psi_{\text{dispose}(\text{var})}(\varphi) &= \Psi_{\text{dispose}(\text{var})}(\varphi[\text{Alloc/Alloc} \cap \neg \sigma_{\text{var}}]), \\
&\text{where } S_{\text{disp}} = \var.f_{k_1} := \text{null}; \\
&\ldots; \var.f_{k_w} := \text{null} \\
\Psi_{S_1 \cdot S_2}(\varphi) &= \Psi_{S_1}(\Psi_{S_2}(\varphi))
\end{align*}
\]

The notation \( [A/B] \) should be interpreted as the syntactic replacement of any occurrence of \( A \) with \( B \). We write \( e.g. \ y : \text{var} \equiv \text{new} \) to indicate that the command \( \text{var} := \text{new} \) is labeled with \( y \). \( \varepsilon_b \) is defined inductively: for \( e_1 = e_2 \) we set \( \varepsilon_b = (A_{e_1} = A_{e_2}) \), with \( A_{\text{var}} = \sigma_{\text{var}} \) and \( A_{\text{var}_2} = \exists f \cdot \sigma_{\text{var}} \); \( \varepsilon \) extends naturally to the Boolean connectives. In the definition of \( \Psi_{\text{dispose}(\text{var})} \), \( f_{k_1}, \ldots, f_{k_w} \) are the members of \( \tau_{\text{fields}} \) which occur in \( \varphi \). W.l.o.g. we assume that \( S \) does not contain commands of the form \( \text{if } b \text{ then } S_1 \cdot f \) or \( \text{var}_1 \cdot f_1 := \text{var}_2 \cdot f_2 \), since they can be expressed using the other commands.

To prove Lemma 9 we need the following lemma:

Lemma 10. Let \( S \) be a loopless program without assume commands, \( Y_S \) be the set of labels of commands in \( S \), \( Y \) be a set of labels disjoint from \( Y_S \), \( M_1 \) and \( M_2 \) be memory structures with universe \( M \) and \( d_YS \) a tuple of \( M \) elements such that \( \langle S, M_1 \rangle \sim dYS M_2 \). Let \( dY \) be a tuple of \( M \) elements and \( \varphi \) be an \( \mathcal{L} \)-formula over \( \tau \cup \{o_y : y \in Y\} \). \( \langle M_2, dY \rangle \models \varphi \) iff \( \langle M_1, (R^{\text{ext}})M_1, dY, dYS \rangle \models \Phi_S(\varphi) \).

Proof. We prove the lemma by induction.

1. \( S = \text{skip} \): \( Y_S = \emptyset \), and we have \( \langle M_2, dY \rangle \models \varphi \) iff \( \langle M_1, R^{\text{ext}}, dY \rangle \models \Phi_S(\varphi) \), as required.
2. \( S = \text{if } b \text{ then } S_1 \text{ else } S_2 \cdot f \) depending on whether \( \varepsilon_b \) is true or false, \( \Phi_{S_1}(\varphi) \) or \( \Phi_{S_2}(\varphi) \) should be used.
3. \( \text{var}_1 := e \), where \( e \) is a variable \( \text{var}_2 \) or \( \text{null} \): every reference to \( \text{var}_1 \) in \( \varphi \) is replaced by a reference to \( \text{var}_2 \) or \( \text{null} \), respectively.
4. \( \text{var}_1 := \text{var}_2 \cdot f \): every reference to \( \text{var}_1 \) in \( \varphi \) is replaced with a reference to \( \sigma_f \), whose interpretation is \( d_y \), in accordance with \( \sim dYS \), which requires that \( d_y \) be the result of applying \( f \) on \( \text{var}_2 \).
5. \( \text{var}_1 := e \), where \( e \) is a variable \( \text{var}_2 \) or \( \text{null} \): the function symbol \( f \) is updated by removing the current value of \( f \) on \( \text{var}_1 \) by subtracting \( (\sigma_{\text{var}_1} \times \top)M_1 \) from \( fM_1 \) and setting the new value explicitly by adding the pair \( (\sigma_{\text{var}_1}, \sigma_{\text{f}}) \) to \( fM_1 \).

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relations \( \varphi \).

Proof (Proof of Lemma 9).

Using Lemma 10 with \( Y = \emptyset \), if \( (S, M_1) \sim_{\bar{d}_Y} \) abort, then for every tuple of relations \( \bar{U} \) interpreting \( R^{ext} \) we have \( \langle M_1, \bar{U}, \bar{d}_Y, d_{abo} \rangle \not\models \Phi_S(\varphi \land (o_{abo} \equiv o_T)) \), because \( \varphi \) is set to true during the run of \( S \). If \( (S, M_1) \sim_{\bar{d}_Y} M_2, M_2 \models \varphi \) iff \( \langle M_1, \bar{d}_Y, d_{abo} \rangle \not\models \Phi_S(\varphi \land (o_{abo} \equiv o_T)) \).

Note that \( d_{abo} \) is the value of \( \varphi \) at the beginning of the run of \( S \). Since the first command of \( S \) assigns \( \varphi \) a new value, \( d_{abo} \) plays no role (it appears because, technically, \( \varphi \) still needs a value at the beginning of the run).

Also note that in Lemma 5, \( \bar{d}_Y \) strictly extends \( \bar{d}_Y \), since \( S \) extends \( S \).

However, the semantics of all of the new commands in \( S \) does not actually depend on the relevant \( \bar{d}_Y \) (since none of them of new commands or assignments of the form \( var_1.var_2.f \)). Hence, any extension of \( \bar{d}_Y \) into \( \bar{d}_Yo \) will do.

Remark 2. Revisiting the example in Section 3.5 with the detailed version of Lemma 9 in mind, we now see that \( \Theta \) is actually the backwards propagation of programs of the form \( S \). The backward propagation of \( \lambda(\ell_1, \ell_1) \) is similar to that presented in Section 3.5, with \( \Theta_{\lambda(\ell_1, \ell_1)}(\varphi_{p-as-\ell_1}) = \Phi_{\lambda(\ell_1)}(\varphi_{p-as-\ell_1} \land (o_{abo} \equiv o_T)) \).

F Soundness and Completeness Theorems

Here we restate and prove Theorems 1 and 2:

**Theorem 1.** Let \( \Gamma \) be a program such that \( \text{shp} \) is inductive for \( \text{Init} \) and \( \text{Init} \models \text{cnt}(\ell_{init}) \). The following statements are equivalent:

(i) For all \( e \in E \), \( VC(e) \) holds.

(ii) \( \text{cnt} \) is inductive for \( \text{Init} \) relative to \( \text{shp} \).

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Proof. Assume $VC(e)$ holds for every $e \in E$. Let $e = (\ell_1, \ell_2) \in E$ and memory structures $M_1$ and $M_2$ such that $M_1 \models g(\ell_1)$ and $\langle \lambda(e), M_1 \rangle \sim M_2$. Since $M_1 \models g(\ell_1)$ we have $s_{M_1}, h_{M_1} \models shp(\ell_1)$ and $M_1 \models g(\ell_1)$. Since $shp$ is inductive, $s_{M_2}, h_{M_2} \models shp(\ell_2)$. There exists a tuple $d_{Y_{\lambda(e)}}$ such that $\langle \lambda(e), M_1 \rangle \leadsto^* d_{Y_{\lambda(e)}} M_2$.

Let $(R^{ext})_M$ be the tuple of copies of $R_M$ relations from Section 3.5, i.e. $(R^{ext})_M \equiv (G_M, M)$. Let $N = \langle M_1, (R^{ext})_M, d_{Y_{\lambda(e)}} \rangle$. Since $s_{M_1}, h_{M_1} \models shp(\ell_1)$ and $M_1 \models cnt(\ell_1)$, $N \models \beta(shp(\ell_1)) \land tr(cnt(\ell_1))$.

By Lemma 9, $N \models \Theta_{\lambda(e)}(a(shp(\ell_2)) \land \neg cnt(\ell_2))$ if $M \models \beta(shp(\ell_1)) \land tr(cnt(\ell_1))$, it must be that $N \models \Theta_{\lambda(e)}(a(shp(\ell_2)) \land \neg cnt(\ell_2))$, so $M \models \beta(shp(\ell_2)) \land \neg cnt(\ell_2)$. Since $M \models \beta(shp(\ell_2))$, in particular $M \models \alpha(shp(\ell_2))$. Hence $M \models cnt(\ell_2)$. We get that $cnt$ is inductive for $Init$ relative to $shp$.

Conversely, assume $cnt$ is inductive for $Init$ relative to $shp$. Assume for contradiction that there exists $e = (\ell_1, \ell_2) \in E$ such that $VC(e)$ does not hold. Then there exists a memory structure $N$ such that

$$N \models \beta(shp(\ell_1)) \land tr(cnt(\ell_1)) \land tr(\Theta_{\lambda(e)}(a(shp(\ell_2)) \land \neg cnt(\ell_2)))$$

Let $N = \langle M_1, (R^{ext})_M, d_{Y_{\lambda(e)}}, Y_{shp} \rangle$. Then $M_1$ is also a memory structure and

$$M_1 \models \beta(shp(\ell_1)) \land tr(cnt(\ell_1)),$$

so $s_{M_1}, h_{M_1} \models shp(\ell_1)$ and $M_1 \models cnt(\ell_1)$. Since $N \models \Theta_{\lambda(e)}(\ell_1)$, $M_2$ must not be set to true in the computation of $\lambda(e)$ starting from $M_1$ with $d_{Y_{\lambda(e)}}$. Therefore, there exists $M_2$ such that $\langle \lambda(e), M_1 \rangle \leadsto^* d_{Y_{\lambda(e)}} M_2$. Since the computation of $\lambda(e)$ is not affected by the interpretations of $R \in \tau \setminus \text{Fields}$, assume w.l.o.g. that for each $R \in \tau \setminus \text{Fields}$, $R^M = (R^{ext})_M$.

Since $\langle \lambda(e), M_1 \rangle \sim M_2$ and $s_{M_1}, h_{M_1} \models shp(\ell_1)$, we get $s_{M_2}, h_{M_2} \models shp(\ell_2)$. By Lemma 9, $M_2 \models \alpha(shp(\ell_2)) \land \neg cnt(\ell_2)$. In particular, $M_2 \not\models cnt(\ell_2)$, in contradiction to $cnt$ being inductive relative to $shp$.

For every $e = (\ell_1, \ell_2) \in E$, let $Reach(e)$ be the set of memory structures $M \in Reach(\ell_2)$ for which there exist $M_{init} \in Init$ and a path $P$ in $G$ starting at $\ell_{init}$ and ending with $e$ such that $\langle P, M_{init} \rangle \leadsto^* M$.

**Theorem 2.** Let $G$ be a program such that $shp$ is inductive for $Init$ and $Init \models cnt(\ell_{init})$. If for all $e \in E$, $VC(e)$ holds, then for $\ell \in V$, $Reach(\ell) \models cnt(\ell)$.

**Proof.** By Theorem 1, $cnt$ is inductive for $Init$ relative to $shp$. Let $M \in Reach(\ell)$, and let $P$ be a path and $M_{init} \in Init$ as guaranteed for members of $Reach(\ell)$. We prove the following claim by induction on the length of $P$:

If for all $e \in E$, $VC(e)$ holds, then for $\ell \in V$, $Reach(\ell) \models cnt(\ell)$ and $Reach(\ell) \models shp(\ell)$.

If $P$ is empty, then $M \in Init$ and the claim holds.
If \( P \) is not empty, let \( e = (\ell_0, \ell) \) be the last edge of \( P \), and let \( P_0 \) be the path obtained from \( P \) by removing \( e \). Let \( M_0 \) be a memory structure such that 
\[
\langle P_0, M_{\text{init}} \rangle \overset{*}{\leadsto} M_0 \quad \text{and} \quad \langle \lambda(e), M_0 \rangle \overset{*}{\leadsto} M.
\]
Let \( \bar{d}_{\gamma_{\lambda(e)}} \) be a tuple of \( M \) elements such that 
\[
\langle \lambda(e), M_0 \rangle \overset{\bar{d}_{\gamma_{\lambda(e)}}}{\leadsto} M.
\]
By the induction hypothesis, \( s_{M_0}, h_{M_0} \models shp(\ell_0) \) and \( M_0 \models cnt(\ell_0) \). Since \( shp \) is inductive, and since \( cnt \) is inductive relative to \( shp \), \( s_M, h_M \models shp(\ell) \) and \( M \models cnt(\ell) \).

Remark 3. In the proofs in this appendix only case 1. of Lemma 9 is used. The purpose of case 2. of Lemma 9 is to make the verification conditions less strict, in the sense that they require nothing of aborted executions.
DTree

UnusedList

UsedList

DTree