Influence of mass affecting tractor's rear axle and rigidity of tires on the control coefficient

G D Voropaev¹, V N Sidorov¹,⁴, K Yu Maksimovich², V A Sokolova³, A S Krivonogova¹, Yu L Pushkov³ and G S Taradin³

¹Kaluga Branch Russian State Agrarian University - Moscow Timiryazev Agricultural Academy, 27 Vishnevskogo str., Kaluga, 248007, Russian Federation
²Novosibirsk State Agrarian University, 160 Dobrolyubova st., Novosibirsk, 630039, Russian Federation
³Saint Petersburg State Forest Technical University named after S.M. Kirov, Institutskiy per., 5, St. Petersburg, 194021, Russian Federation
⁴E-mail: sidorov-kaluga@yandex.ru

Abstract. To increase the versatility of a class 1.4 tractor in a small farm, a universal hydraulic mounted system has been proposed, which allows the tractor to be used without changeover, both for agricultural technological operations, and for loading and construction works. The article discusses the use of mathematical modeling in the MathLab, Simulink environment and the steep ascent method to identify the optimal values of the mass on the rear axle and the stiffness of the tractor tires when driving on a horizontal supporting surface. When studying the dependence of the change in the control coefficient from two factors of variation: the mass on the rear axle and the stiffness of the tractor tires, the response function was obtained. By the method of steep ascent, the optimal design parameters of a tractor with attachments were determined, which make it possible to ensure sufficient controllability when driving on country roads.

1. Introduction
On modern tractors, the rear hydraulic mounted system is widely used, with the help of which various agricultural machines and tools are attached to the tractor [1,2]. To increase the versatility of a class 1.4 tractor in a small farm, we have developed a universal hydraulic mounted system that allows us to use the tractor without changeover, both for agricultural technological operations, and for loading and construction works (figure 1).

Figure 1. Diagram of a tractor with designed mounted attachments.
The use of the developed universal hydraulic mounted system leads to the redistribution of the tractor mass along the axles. This affects the controllability of the tractor, therefore, the search for the permissible weight of the attachment in combination with the stiffness of the tires is an urgent task.

Objective of the paper:

- to determine the optimal values of the mass of the attachment and mass distribution of the tractor along the axles in combination with the stiffness of the tires to achieve sufficient tractor controllability.

2. Research methods

To obtain the calculated, optimal values of the mass on the rear axle and the stiffness of the tractor tires when driving on a horizontal support surface, we used the MatLab automation system for mathematical calculations (software package). The structural modeling of dynamic systems Simulink application [3,4] is used. To create a simulation model in Simulink we used the Simulink and SimEvent standard library.

In the study of controllability and distribution of mass along the axes, the scheme shown in figure 2 was adopted.

![Figure 2](image)

**Figure 2.** Design scheme for determining the steering coefficient and movements of the front axle from the movements of the rear axle of the tractor.

The control coefficient is calculated by the formula [1,5]:

$$\lambda_1 = \frac{G_1}{G_{total}},$$  \hspace{1cm} (1)

where $G_1$ – weight on the front axle, $G_{total}$ – total weight of the tractor with attachments.

Movements of the front axle of the tractor along the vertical axis $Z$:

$$Z_1 = \frac{Z_2 + L_1}{L_2},$$  \hspace{1cm} (2)

where $L_1$ – distance from the center of mass to the front axle, $L_2$ – distance from the center of mass to the rear axle, $Z_1$ and $Z_2$ - movement values of the front and rear axles along the vertical $Z$ axis.

The design scheme for researching the movement of the rear axle of the tractor is shown in figure 3.
Equations of movement of the rear axle of the tractor [4]:

\[ m\ddot{z} = P - mg, \]  

where \( m \) — mass on the rear axle.

Here

\[ P = P_e + P_d, \]  

where \( P_e \) — elastic force, \( P_d \) — damping force of the tire.

The value of tire deformation can be calculated by the formula:

\[ f = -z + s + f_{st}. \]  

Let us define a function describing the irregularities of the support surface in the form:

\[ s = \frac{H}{2}\sin x. \]  

where

\[ x = V \cdot t. \]  

The research of the mathematical model of the rear axle of the tractor will be carried out using the MatLAB/SimuLink program (figure 4).
Let us review the blocks of the model. Massa block is a subsystem in which the mass on the rear axle of the tractor is calculated.

![Massa block](image5)

**Figure 5.** Massa block.

The Oporna poverhnost block is a subsystem in which the parameters of the surface are set on which the tractor with attachments will move (figure 6).

![Oporna poverhnost subsystem](image6)

**Figure 6.** Oporna poverhnost subsystem.

Kolesa block is a subsystem in which the stiffness and damping properties of the tire are set and the elastic force and damping force of the tire are determined (figure 7).

![Kolesa block](image7)
To search for the permissible mass of the attachment in combination with the stiffness of the tires while maintaining tractor controllability, we will use the planning of the experiment and the method of steep ascent. With two factors, the model of the response function has the following form [6-8]:

\[ y = b_0 + b_1 \cdot x_1 + b_2 \cdot x_2 \]

3. Research results
The limits of variation of the factors are given in table 1 and the matrix plan of experiments is in table 2.

| Name and designation of factors | Variation levels | Variation intervals |
|---------------------------------|------------------|---------------------|
| Tire stiffness                  | -1 0 +1          | 780 820 840         |
| Rear axle mass                  | -500 1000 1500   | 500                 |

| Experiment No. | Natural values | Factors | Response |
|----------------|----------------|---------|----------|
|                | \( x_1 \) | \( x_2 \) | \( x_1 \) | \( x_2 \) |     |
| 1              | 1500 | 840   | 1     | 1     | 0.62 |
| 2              | 500  | 840   | -1    | 1     | 0.77 |
| 3              | 1500 | 780   | 1     | -1    | 0.541|
| 4              | 500  | 780   | -1    | -1    | 0.7  |

With two factors, a model of the response function was obtained in the form of a regression equation:

\[ y = -0.215 - 0.003 \cdot x_1 + 1.226 \cdot x_2 \] (8)

The adequacy of the mathematical model is verified by Fisher's criterion [9].

\[ F_{\text{calc}} = \frac{s^2_{\text{res}}}{s^2_{(e)}} = \frac{0.01}{0.0025} = 0.59 \] (9)

The value of \( F \) – Fisher's criterion at a confidence coefficient of 0.95 is \( F_{\text{tabl}} = 0.62 \). The calculated value of the Fisher criterion is less than the table value \( F_{\text{calc}} \leq F_{\text{tabl}} \), therefore, the model is adequate. The obtained relation (1) shows the relationship between the steering coefficient of the tractor and such factors as the stiffness of the tire \( x_1 \) and mass \( x_2 \), affecting the rear axle of the tractor [10-14].

As a result of the search for the optimum by the steep ascent method, it was possible to determine at what values of the factors of the load on the rear axle and the stiffness of the tire, the value of the control
coefficient will not be lower than the maximum permissible. This value is achieved with a rear axle mass of 1000 kg and a rear tire stiffness of 800 N*s/m and equals 0.62.

4. Conclusion
The developed mathematical model for calculating the control coefficient of a tractor makes it possible to study the influence on it of such parameters as the mass on the rear axle and the stiffness of the tire of the rear wheels of the tractor.

Based on the planning of the experiment and the method of steep ascent, the optimal values of the load on the rear axle and the tire stiffness of the tractor rear wheels were identified. The calculation was carried out for the conditions of movement on a horizontal surface. The following optimal values of the mass per rear axle of 1000 kg and the stiffness of the rear wheel tire of 800 N*s/m were obtained.

The tests were carried out on the equipment of the Shared Use Center "Materials Research Center".

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