Effects of $^8$B size on the low-energy $^7$Be$(p, \gamma)^8$B cross section

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(March 13, 2018)

Abstract

We calculate several “size-like” $^8$B observables within the microscopic three-cluster model and study their potential constraints on the zero-energy astrophysical $S_{17}(0)$ factor of the $^7$Be$(p, \gamma)^8$B reaction. We find within our three-cluster model that a simultaneous reproduction of the experimental data for the $^8$B radius and quadrupole moment and of the $^8$B-$^8$Li Coulomb displacement energy implies $S_{17}(0) = (23 - 25) \text{ eV b.}$

PACS: 25.40.Lw, 26.65.+t, 21.60.Gx, 27.20.+n

Keywords: $^7$Be$(p, \gamma)^8$B; Radiative capture; Solar neutrinos; Astrophysical $S$ factor

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I. MOTIVATION

The $^7$Be($p, \gamma$)$^8$B reaction is currently considered to be one of the astrophysically most important nuclear reactions, as its low-energy cross section determines the high-energy solar neutrino flux [1]. Recently there has been a great deal of experimental and theoretical activities investigating this process. The low-energy cross section has been studied directly by using a radioactive $^7$Be target and a proton beam [2], in inverse kinematics by using a $^7$Be beam and a proton target [3], indirectly from the Coulomb dissociation of $^8$B [4], and by extracting the $^7$Be + $p$ nuclear vertex constant from the $^7$Be($d, n$)$^8$B reaction [5]. On the theoretical side some effects of $^7$Be deformations [6] and three-body dynamics [7] have been studied, and efforts to understand the nuclear vertex constant have been made [8]. In Ref. [9] we have shown that the zero-energy cross section of the $^7$Be($p, \gamma$)$^8$B reaction scales linearly with the, unfortunately yet unknown, quadrupole moment of $^7$Be. In the present paper we extend this study and investigate the relation between the zero-energy cross section and several $^8$B “size” properties.

In our approach we study the ground state properties of $^7$Be and $^8$B as well as the $^7$Be($p, \gamma$)$^8$B reaction cross section consistently within the microscopic eight-body $^4$He + $^3$He + $p$ cluster model. As this model has been discussed before we refer the reader to Refs. [10,9] for details of the theoretical background. As customary in nuclear astrophysics we define the cross sections in terms of the astrophysical $S$ factor

$$S(E) = \sigma(E)E \exp\left[2\pi\eta(E)\right], \quad \eta(E) = \frac{Z_1Z_2e^2}{hv}, \quad (1)$$

where $Z_1, Z_2$ are the charges of the two colliding nuclei, and $v$ is their relative velocity.

At low, and in particular, at solar energies the $^7$Be($p, \gamma$)$^8$B reaction is highly peripheral, which means that only the external parts of the bound- and scattering wave functions contribute to the radiative capture cross section [11]. The external wave functions are known with the exception of the asymptotic normalization constant, $\bar{c}$, of the $^8$B bound state [12]. Consequently the energy dependence of the low-energy $S_{17}(E)$ factor is well-known (e.g. Refs. [13,11,14]). Its absolute value, however, depends on $\bar{c}$ and has thus to be determined experimentally. Nevertheless theoretical constraints on the asymptotic normalization constant might be quite useful. We note that $\bar{c}$ depends mainly on the effective $^7$Be–$p$ interaction radius. A larger radius results in a lower Coulomb barrier, which leads to a higher tunneling probability into the external region, and hence to a higher cross section. A possible way to constrain the interaction radius is to study some key properties of the $A = 7$ and 8 nuclei [15]. The observables that are most sensitive to the interaction radius are “size-like” properties, for example, quadrupole moment, radius, Coulomb displacement energy [16], etc. These are the quantities which we will calculate in our microscopic cluster model and then study their effect on the astrophysical $S$ factor.

II. THE SIZE OF $^8$B AND ITS EFFECT ON $S_{17}$

In Ref. [9] we demonstrated that there is a linear correlation between the zero-energy astrophysical $S$ factor, $S_{17}(0)$, of the $^7$Be($p, \gamma$)$^8$B reaction and the quadrupole moment of $^7$Be,
The $Q_7$ quadrupole moment has not been measured yet, but in Ref. [9] we have predicted it to be between $-6$ efm$^2$ and $-7$ efm$^2$. The absolute scale of the $S_{17}(0) - Q_7$ correlation, however, depends on the applied effective nucleon-nucleon ($N-N$) interaction. For our preferred MN interaction this resulted in $S_{17}(0) = 25 - 26.5$ eV b [9]. Other interactions gave slightly larger or smaller $S_{17}(0)$ values, but these interactions were found to be inferior to the MN force for other observables. Now we will turn to the other “size-like” observables and their potential constraints on $S_{17}(0)$. For this purpose we have repeated the microscopic calculations described in Ref. [9] varying the size parameter of the $^3$He and $^4$He clusters while keeping other important ingredients of the calculation fixed. Again as in Ref. [9] we have performed these calculations for several interactions (MN force [17], V2 interaction [18], and MHN interaction [19]). Our calculation thus reproduces the $S_{17}(0) - Q_7$ plot shown in Fig. 1 of Ref. [9].

In the present Fig. 1 we extend the study of Ref. [3] and investigate the relation of $S_{17}(0)$ to several size-like properties of $^8$B: (a) the $^8$B radius $r(^8$B), (b) the difference between the $^7$Be and $^8$B radii quantified by $r^2(^8$B) − $r^2(^7$Be), (c) the $^8$B quadrupole moment, and (d) the $E(^8$Li) − $E(^8$B) Coulomb displacement energy. These calculations have been performed for the same model spaces and interactions as in Ref. [9]. Importantly all four indicators scale linearly with $S_{17}(0)$. This is caused by the ‘halo’ structure of the $^8$B ground state [20] and reflects that the $^7$Be($p,\gamma)^8$B reaction at low energies is an external capture process. In the following we will discuss the four indicators in turn and will try to derive at possible constraints on $S_{17}(0)$. First, the comparison is performed for the consistent eight-body $^4$He+$^3$He+p calculation using the MN force; the dependence of our results on the model space and the interaction employed will be discussed below.

The size property that is most sensitive to the effective $^7$Be − $p$ interaction radius is $r^2(^8$B) − $r^2(^7$Be). However, a precise experimental determination of this quantity is very difficult. In fact, during the course of the work reported in Ref. [9] it seemed hopeless that the $^7$Be or $^8$B radius could be measured with relatively high precision. The radii of nuclei far from stability are usually extracted from interaction cross section measurements by using Glauber-type models with uniform density distribution for the nuclei [21]. Recently, a new and more precise method has been introduced [22] which considers the few-body structure of the nuclei involved (like $^7$Be + $p$ for $^8$B), while extracting the radius from the measured interaction cross sections. For $^8$B the resulting point-nucleon radius is $r(^8$B) = 2.50±0.04 fm, and hence $r^2(^8$B) − $r^2(^7$Be) ≈ 0.9 fm$^2$. We note, that for $^7$Be the model of Ref. [22] still uses the Glauber estimate. Our calculation reported in Ref. [9] gives $r(^8$B) = 2.73 fm and thus overestimates the experimental value. As expected we observe that $S_{17}(0)$ decreases with decreasing $^8$B radius. Using the linear relationship between $r(^8$B) and $S_{17}(0)$ and the experimental value for the $^8$B radius places the cross section into the range $S_{17}(0) = 23.2 - 24.2$ eV b. Ref. [9] found $r^2(^8$B) − $r^2(^7$Be) = 0.8 fm$^2$, which appears to be a reasonable value. However, due to the uncertainties in the phenomenological value the derivation of a constraint on $S_{17}(0)$ from $r^2(^8$B) − $r^2(^7$Be) is currently not possible.

The experimental value of the $^8$B quadrupole moment is $Q_8 = (6.83 \pm 0.21)$ efm$^2$ [23]. In Ref. [9] we calculated a slightly larger value, $Q_8 = 7.45$ fm$^2$. Like in the case of the $^8$B radius, $S_{17}(0)$ increases linearly with the $^8$B quadrupole moment. From a comparison to the experimental data we find the constraint $S_{17}(0) = 23.7 - 24.8$ eV b (Fig. 1c).

To derive a phenomenological value for the Coulomb displacement energy to be compared
with our calculated values, we have to consider that the physics that accounts for the Nolen-Schiffer effect \[25\] is not present in our model. This effect is estimated to cause a \( \approx 130 \) keV shift in the \( E(^8\text{Li}) - E(^8\text{B}) \) Coulomb displacement \[10\]. So we should compare our results to a phenomenological value of \( \Delta = 3.54 - 0.13 = 3.41 \) MeV. Ref. \[7\] found a too small Coulomb displacement energy, \( \Delta = 3.2 \) MeV. From Fig. 1d we observe that \( S_{17}(0) \) decreases linearly with \( \Delta \); thus the experimental value for the Coulomb displacement energy corresponds to \( S_{17}(0) = 24.3 \) eV b.

We can summarize the results for the complete \(^4\text{He} + ^3\text{He} + p\) cluster study (with the MN force) of Ref. \[9\] as follows: This calculation \[9\] gives consistently values for those indicators, for which reliable experimental data exist, which point to the use of a too large \(^7\text{Be} + p\) interaction radius. This implies that the value for \( S_{17}(0) \) (26.1 eV b) predicted in \[9\] is too large. We note that, by slightly varying the interaction radius, all three experimentally determined “size-like” parameters (the \(^8\text{B} \) radius and quadrupole moment and the Coulomb displacement energy) can consistently be reproduced (see Table 1); the corresponding value for the \(^7\text{Be}(p,\gamma)^8\text{B}\) reaction cross section then is \( S_{17}(0) = (23 - 25) \) eV b.

How much do our results depend on the chosen model space and the adopted interaction? To answer these questions we have at first performed a series of restricted calculations involving only \((^3\text{He} + ^4\text{He}) + p\) configurations \((^7\text{Be} + p\) like configurations) rather than all possible arrangements of the three clusters. (As in all calculations reported in this paper the experimental value of the \(^8\text{B}\) binding energy relative to the \(^7\text{Be} + p\) threshold has been reproduced by a slight modification in the \(N - N\) interaction, see Ref. \[7\].) Again we find that \( S_{17}(0) \) scales linearly with all indicators. From Fig. 1 we also observe that, for a fixed value of \( S_{17}(0) \), the extension of the model space (going from the restricted space to the full three-cluster model) reduces the \(^8\text{B} \) radius slightly, but increases the \(^8\text{B} \) quadrupole moment and the Coulomb displacement energy. The reason why the radius \( (a) \) and the quadrupole moment \( (c) \) of \(^8\text{B}\) change in opposite direction if the model space is enlarged is that the addition of the \(^5\text{Li} + ^3\text{He}\) and \(^4\text{He} + ^4\text{Li}\) channels brings in large charge polarization which increases the quadrupole moment even if \( r(^8\text{B}) \) is reduced. Even if we allow the variation of the \(^7\text{Be} + p\) interaction radius, the restricted model space calculation does not simultaneously reproduce the experimental data for our indicators (see Table 1). While the \(^8\text{B}\) radius and the Coulomb displacement energy puts \( S_{17}(0) \) at around 23 - 24 eV b, the \(^8\text{B}\) quadrupole moment favors a larger value of \( S_{17}(0) = 25.3 - 27.5 \) eV b. In fact, the \(^8\text{B}\) quadrupole moment is the quantity which is clearly most sensitive to the model space. Note that enlargening the model space does not only reduce \( S_{17}(0) \) for fixed value of \( Q_8 \), it also changes the slope of \( S_{17}(0) - Q_8\) scaling. Obviously the reproduction of the \(^8\text{B}\) quadrupole moment requires a 3-body approach and is sensitive to the internal structure of the clusters.

In Ref. \[9\] it has been observed that the \( S_{17}(0) - Q_7\) scaling depends on the adopted NN interaction. We have therefore repeated the \(^7\text{Be} + p\)-type model calculations with two other interactions (V2 and MHN). We observe that, for fixed values of the \(^8\text{B}\) radius, of the \( r^2(^8\text{B}) - r^2(^7\text{Be})\) difference and of the Coulomb displacement, the rather repulsive V2 interaction gives larger \( S_{17}(0) \) values than the MN interaction, while the MHN interaction gives smaller values. Assuming a linear relation between \( S_{17}(0) \) and our indicators, constraints on the zero-energy S-factor can be derived; the corresponding values are listed in Table 1. For the MHN interaction we find that the \(^8\text{B}\) quadrupole moment and the other indicators are not simultaneously reproduced in the restricted model space. For the V2 interaction the
The 8B quadrupole moment and radius and the Coulomb displacement energy are reproduced for a 7Be+p interaction radius corresponding to \( S_{17}(0) \approx 26 \text{ eV b} \). However, for this value of \( S_{17}(0) \) the \( r^2(8B) - r^2(7Be) \) difference becomes unreasonably large.

We note again that a measurement of the 7Be quadrupole moment would place some additional constraints on the consistency of our calculations. For the complete 4He+3He+p model calculation the simultaneous reproduction of the indicators predict \( Q_7 \) to be in the range \(-(5.5 - 6.0) \text{ e fm}^2\). However, this value is smaller than the one \( (Q_7 = -6.9 \text{ e fm}^2) \) obtained if we chose the cluster size parameters such to reproduce the quadrupole moment of the analog nucleus 7Li. Does this already point to the necessity of a further enlargement of the model space beyond the 4He + 3He + p three-cluster model which would then also effect our results obtained for 7Be, e.g., change the 7Be quadrupole moment? To investigate this we have performed calculations for 7Be in which we have added the 6Li+p = 4He+d+p configuration to the 4He+3He model space, adopted above. In all cases the exchange mixture parameter of the \( N - N \) interaction was fixed to reproduce the 7Be binding energy relative to the 4He + 3He threshold. We also made sure that the 6Li+p threshold was correctly reproduced. Our results show that \( |Q_7| \) is increased by 0.5 - 1 e fm\(^2\) in the coupled-channel model relative to the single-channel 4He + 3He value perhaps suggesting the need for an even larger model space than the complete 4He+3He+p model space adopted here. Which consequences such an enlargement might have on \( S_{17}(0) \) has to wait for 8B calculations performed in larger model spaces, which are beyond 4He + 3He + p.

### III. SUMMARY

In summary, we have adopted a microscopic 4He+3He+p cluster model to calculate the 8B radius and quadrupole moment, the difference in the 7Be and 8B radii, \( r^2(8B) - r^2(7Be) \), and the Coulomb displacement energy \( E(8Li) - E(8B) \) and to study their relation to the \( S_{17}(0) \) astrophysical S factor. We find that all these indicators scale linearly with the zero-energy 7Be(p,\( \gamma \))8B cross section.

Within our three-cluster model we find that the experimentally determined values for the 8B radius and quadrupole moment as well as the Coulomb displacement energy is consistently described if the internal cluster size parameters are chosen such that the values for \( S_{17}(0) \) are between \((23 - 25) \text{ eV b}\). This range is more or less compatible with the value currently used in most solar models, \( S_{17}(0) = 22.4 \pm 2.1 \text{ eV b} \) [24]. However, it is slightly inconsistent with the recently adopted new experimental value \( S_{17}(0) = 19^{+4}_{-2} \text{ eV b} \) [27].

As a note of caution we mention that this result has been derived from a linear relation between our four indicators and \( S_{17}(0) \) found in our three-cluster model. However, we found that enlarging the 7Be model space by adding a 4He + d + p configuration to the 4He + 3He configuration increased the 7Be quadrupole moment by about 10%. To investigate the effects which additional configuration might have on the 8B properties and in particular on the \( S_{17}(0) \) value, requires calculations in model spaces which are beyond 4He + 3He + p.

The work of A. C. was performed under the auspices of the U.S. Department of Energy. The work has been partly supported by the Danish Research Council and by OTKA grant F019701.
REFERENCES

[1] J. N. Bahcall, Neutrino Astrophysics (Cambridge University Press, 1989).
[2] F. Hammaache et al., Phys. Rev. Lett. 80 (1998) 928.
[3] L. Campaolada et al., Z. Phys. A 356 (1996) 107.
[4] T. Motobayashi et al., Phys. Rev. Lett. 73 (1994) 2680.
[5] W. Liu et al., Phys. Rev. Lett. 77 (1996) 611.
[6] F. M. Nunes, R. Crespo, and I. J. Thompson, Nucl. Phys. A627 (1997) 747.
[7] L. V. Grigorenko, B. V. Danilin, V. D. Efros, N. B. Shul’gina, and M. V. Zhukov, Chalmers University preprint (1998).
[8] N. K. Timofeyuk, D. Baye, and P. Descouvemont, Nucl. Phys. A 620 (1997) 29; N. K. Timofeyuk, University of Surrey preprint SCNP-97/17.
[9] A. Csótó, K. Langanke, S. E. Koonin, and T. D. Shoppa, Phys. Rev. C 52 (1995) 1130.
[10] A. Csótó, Phys. Lett. B 315 (1993) 24.
[11] K. Langanke, in Solar Modeling, ed. A. B. Balantekin and J. N. Bahcall (World Scientific, Singapore, 1995) p. 31.
[12] H. M. Xu, C. A. Gagliargi, R. E. Tribble, A. M. Mukhamedzhnov, and N. K. Timofeyuk, Phys. Rev. Lett. 73 (1994) 2027.
[13] R. A. Williams and S. E. Koonin, Phys. Rev. C 23 (1981) 2773.
[14] A. Csótó, Phys. Lett. B 394 (1997) 247.
[15] A. Csótó, Heavy Ion Physics, in press, [nucl-th/9704053; nucl-th/9712033].
[16] B. A. Brown, A. Csótó, and R. Sherr, Nucl. Phys. A 597 (1996) 66.
[17] D. R. Thompson, M. LeMere, and Y. C. Tang, Nucl. Phys. A 268 (1977) 53; I. Reichstein and Y. C. Tang, Nucl. Phys. A 158 (1970) 529.
[18] A. B. Volkov, Nucl. Phys. 74 (1965) 33.
[19] F. Tanabe, A. Tohsaki and R. Tamagaki, Prog. Theor. Phys. 53 (1975) 677.
[20] K. Riisager and A.S. Jensen, Phys. Lett. B301 (1993) 6; K. Riisager, A.S. Jensen and P. Møller, Nucl. Phys. A548 (1992) 393.
[21] I. Tanihata et al, Phys. Lett. B 206 (1988) 592.
[22] J. S. Al-Khalili and J. A. Tostevin, Phys. Rev. Lett. 76 (1996) 3903.
[23] T. Minamisono et al., Phys. Rev. Lett. 69 (1992) 2058.
[24] F. Ajzenberg-Selove, Nucl. Phys. A 490 (1988) 1.
[25] J. A. Nolen and J. P. Schiffer, Ann. Rev. Nucl. Sci. 19 (1969) 471.
[26] C. W. Johnson, E. Kolbe, S. E. Koonin and K. Langanke, Astrophys. J. 392 (1992) 320.
[27] E. G. Adelberger et al., Rev. Mod. Phys., to be published.
TABLE I. The constraints derived for $S_{17}(0)$ (in eV b) from the $^8$B radius and quadrupole moment and from the Coulomb displacement energy in our complete $^4$He+$^3$He+p 8-body calculation (full) and in the restricted $^7$Be+p model spaces for the Minnesota force (MN), the Volkov force (V2) and the Hasegawa-Nagata force (MHN).

| indicator | full       | MN         | V2         | MHN         |
|-----------|------------|------------|------------|-------------|
| r($^8$B)  | 23.2 – 24.2| 22.8 – 23.6| 25.7 – 26.6| 21.7 – 22.7 |
| $Q_8$     | 23.7 – 24.8| 25.3 – 27.5| 24.1 – 27.0| 24.3 – 27.2 |
| $\Delta$  | 24.3       | 23.8       | 26.5       | 23.0        |
FIGURES

FIG. 1. Correlation between the zero-energy astrophysical $S$ factor of the $^7$Be$(p, \gamma)^8$B reaction and (a) the $^8$B point-nucleon radius, $r(^8$B) (in fm), (b) the $r^2(^8$B) $-$ $r^2(^7$Be) value (in fm$^2$), (c) the quadrupole moment of $^8$B, $Q_8$ (in e fm$^2$), and (d) the $\Delta = E(^8$Li) $-$ $E(^8$B) Coulomb displacement energy (in MeV). The correlations have been calculated in our microscopic eight-body model, using several $N$ $-$ $N$ interactions and model spaces. For a given model space and interaction, different results are obtained by varying the cluster size parameters. For a detailed description of the model spaces and interactions, see Ref. [9]. The phenomenological values are $r(^8$B) = $2.50 \pm 0.04$ fm [22], $r^2(^8$B) $-$ $r^2(^7$Be) $\approx 0.9$ fm$^2$, $Q_8$ = $6.83 \pm 0.21$ e fm$^2$ [23], and $\Delta$ = $3.41$ MeV. In the Coulomb displacement energy the Nolen-Schiffer anomaly is removed from the value given in Ref. [24] (see the text). The phenomenological values are indicated by dashed lines in the respective figures.
