Conserved higher spin supercurrents for arbitrary spin massless supermultiplets and higher spin superfield cubic interactions

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ABSTRACT

We give an explicit superspace construction of higher spin conserved supercurrents built out of 4D, \( \mathcal{N} = 1 \) massless supermultiplets of arbitrary spin. These supercurrents are gauge invariant and generate a large class of cubic interactions between a massless supermultiplet with superspin \( Y_1 = s_1 + 1/2 \) and two massless supermultiplets of arbitrary superspin \( Y_2 \). These interactions are possible only for \( s_1 \geq 2Y_2 \). At the equality, the supercurrent acquires its simplest form and defines the supersymmetric, higher spin extension of the linearized Bel-Robinson tensor.

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1 Introduction

In previous works [1–6] various conserved, higher spin, supercurrent multiplets of supersymmetric matter theories have been constructed. These were used to generate first order interactions among matter theories and higher spin supermultiplets with the lowest possible number of derivatives. In this note, we go beyond matter theories and consider first order interactions among higher spin supermultiplets.

Finding interactions involving higher spins is non-trivial as one has to satisfy the set of consistency conditions imposed by the gauge invariance of the free higher spin action. For non-supersymmetric theories the search for higher spin interactions is extensive and in many cases interaction terms were successfully constructed for flat spacetime at first order in coupling constant $g$ by using a variety of techniques, such as light-cone approach [7–18], Noether’s procedure [19–23] and BRST [29–36]. Interestingly, most of these results have been obtained by analysing tree level amplitudes of (super)strings [37–39], thus enhancing the connection between string theory and higher spin fields. For (A)dS backgrounds similar results have been obtained [40–44] which eventually led to the fully interacting equations of motion for higher spin fields [45] (see [46] for a review).

Most of these constructions, focus on interaction vertices with the lowest possible number of derivatives, which corresponds to a minimal coupling scenario. However we can have non-minimal type of interactions which in most cases will lead to lagrangians with higher derivatives. This has been done in [21,24] using gauge invariant field strengths. An interesting, distinctive aspect of these interactions is their uniqueness up to trivial redefinitions.

In this paper, we generalize these results to supersymmetric theories using the manifest 4D, $\mathcal{N} = 1$ standard superspace formulation. Specifically, we give the explicit form of gauge invariant, higher spin, conserved supercurrents for all massless, 4D Minkowski, $\mathcal{N} = 1$ higher spin supermultiplets. A particular example for the supergravity supercurrents of low spins $j \leq 1$ was given in [48]. Our results extends it in two directions: (i) we consider higher spin supermultiplets and (ii) we construct higher spin supercurrents.

Our construction is focused on the description of cubic interactions $Y_1 - Y_2 - Y_2$ between a massless higher spin supermultiplet with arbitrary half-integer superspin $Y_1 = s_1 + 1/2$ (which describes the propagation of massless spins $j = s_1 + 1$ and $j = s_1 + 1/2$) and two massless, higher spin supermultiplets with arbitrary superspin $Y_2$. We also assume that this cubic interaction can be written in the form gauge superfield times supercurrent. The supercurrent must be quadratic in the $Y_2$ supermultiplet and the gauge transformation of the $Y_1$ supermultiplet will impose on it an on-shell conservation equation. Additionally, we demand the supercurrent to be gauge invariant with respect to the gauge transformation of the $Y_2$ supermultiplet. The $Y_2$-gauge invariance fixes the supercurrent to be quadratic in the derivatives of the $Y_2$ superfield strength and the conservation equation gives a unique solution of this type. Furthermore, we derive a restriction on the allowed values of superspin, $s_1 \geq 2Y_2$, which provides a complete classification for this class of interactions. This is in agreement with the known constraints for the spin values of higher spin interactions of non-supersymmetric theories [21,16] as well as the Weinberg-Witten theorem [49]. Right at the boundary, when $s_1 = 2Y_2$, the structure of the supercurrent simplifies drastically. Its dependence on the superfield strength becomes algebraic and defines the supersymmetric, higher spin extension of the Bel-Robinson tensor [50] (and reference therein).

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4Some of these results were later generalized in [24–27]. For a review see [28].

5We use the conventions of Superspace [47].

6The Bel-Robinson tensor is known in the context of General Relativity and is a spin 4, gauge invariant and conserved tensor constructed out massless spin 2 fields. It is the generalization of the energy-momentum tensor.
The organization of the paper is as follows. In section 2, we review the description of free, massless, 4D, $\mathcal{N} = 1$ higher spin supermultiplets and their corresponding superfield strengths. In section 3, we construct the gauge invariant, higher spin, conserved supercurrent which generates the cubic interactions. In section 4, we include a component discussion for the higher spin currents that can be extracted from the higher spin supercurrent superfield. The last section summarizes our results.

## 2 Free higher spin supermultiplets and their superfield strengths

The massless, higher spin irreducible representations of the 4D, $\mathcal{N} = 1$, super-Poincaré were first described in [51]. Later a superfield formulation was introduced in [52–54] and further developments can be found in [55–57]. A quick synopsis of the description of higher spin supermultiplets is the following:

1. The integer superspin $Y = s$ ($s \geq 1$) supermultiplets ($s+1/2$, $s^7$) are described by a pair of superfields $\Psi_{\alpha(s)\dot{\alpha}(s-1)}$ and $V_{\alpha(s-1)\dot{\alpha}(s-1)}$ (is real) with the following zero order gauge transformations

   \begin{align}
   \delta_0 \Psi_{\alpha(s)\dot{\alpha}(s-1)} = & -D^2 L_{\alpha(s)\dot{\alpha}(s-1)} + \frac{1}{(s-1)!} \bar{D}(\dot{\alpha}_{s-1} L_{\alpha(s)\dot{\alpha}(s-2)}) , \\
   \delta_0 V_{\alpha(s-1)\dot{\alpha}(s-1)} = & D^{\dot{s}} L_{\alpha(s-1)\dot{\alpha}(s-1)} + \bar{D}^{\dot{s}} L_{\alpha(s-1)\dot{\alpha}(s-1)} .
   \end{align}

   Off-shell, this supermultiplet carries $8s^2 + 8s + 4$ bosonic and equal number of fermionic degrees of freedom.

2. The half-integer superspin $Y = s + 1/2$ supermultiplets ($s + 1$, $s + 1/2$) have two descriptions. The first is called the transverse formulation ($s \geq 1$) and it uses the pair of superfields $H_{\alpha(s)\dot{\alpha}(s)}$ (is real) and $\chi_{\alpha(s-1)\dot{\alpha}(s-1)}$ with the following zero order gauge transformations

   \begin{align}
   \delta_0 H_{\alpha(s)\dot{\alpha}(s)} = & \frac{1}{s} D_{\alpha(s)\dot{\alpha}(s-1)} + \frac{1}{s} \bar{D}(\dot{\alpha}_{s} L_{\alpha(s)\dot{\alpha}(s-1)}) , \\
   \delta_0 \chi_{\alpha(s-1)\dot{\alpha}(s-1)} = & \bar{D}^2 L_{\alpha(s-1)\dot{\alpha}(s-1)} + \bar{D}^{\dot{s}} L_{\alpha(s-1)\dot{\alpha}(s-1)} .
   \end{align}

   This supermultiplet, off-shell describes $8s^2 + 8s + 4$ bosonic and equal fermionic degrees of freedom. The second formulation is called the longitudinal ($s \geq 2$) and it includes the superfields $H_{\alpha(s)\dot{\alpha}(s)}$ (is real) and $\chi_{\alpha(s-1)\dot{\alpha}(s-2)}$ with the following zero order gauge transformations

   \begin{align}
   \delta_0 H_{\alpha(s)\dot{\alpha}(s)} = & \frac{1}{s} D_{\alpha(s)\dot{\alpha}(s-1)} + \frac{1}{s} \bar{D}(\dot{\alpha}_{s} L_{\alpha(s)\dot{\alpha}(s-1)}) , \\
   \delta_0 \chi_{\alpha(s-1)\dot{\alpha}(s-2)} = & \bar{D}^{\dot{s}-1} D^{\alpha(s-1)} L_{\alpha(s-1)\dot{\alpha}(s-1)} + \frac{s-1}{s} D^{\dot{s}} \bar{D} L_{\alpha(s-1)\dot{\alpha}(s-1)} + \frac{1}{(s-2)!} \bar{D}(\dot{\alpha}_{s-2} \beta L_{\alpha(s-1)\dot{\alpha}(s-3)}) .
   \end{align}

   This supermultiplet carries $8s^2 + 4$ off-shell bosonic and equal number of fermionic degrees of freedom. However one can show that it is dual to the first, transverse, formulation [52, 6].

The physical and propagating degrees of freedom of the two massless integer and half-integer superspin theories above are described by corresponding superfield strengths $W_{\alpha(2s)}$ and $W_{\alpha(2s+1)}$ respectively. They are defined by:

$$Y = s + 1/2 : \quad W_{\alpha(2s+1)} \sim \bar{D}^2 D_{\alpha_{2s+1}} \partial_{\alpha_{2s}} \dot{\alpha}_{s} \partial_{\alpha_{2s-1}} \dot{\alpha}_{s-1} \cdots \partial_{\alpha_{s+1}} \dot{\alpha}_{1} H_{\alpha(s)} \dot{\alpha}(s)$$

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7On-shell they describe the propagation of degrees of freedom with helicity $\pm (s + 1/2)$ and $\pm s$

8The notation $\alpha(k)$ is a shorthand for $k$ undotted symmetric indices $\alpha_1 \alpha_2 \ldots \alpha_k$. The same notation is used for the dotted indices

9A detailed counting of the off-shell degrees of freedom can be found in [55, 56].
\[ Y = s : \quad W_{\alpha(2s)} \sim \bar{D}^2 D_{(\alpha_{2s}} \partial_{\alpha_{2s-1}} \dot{\alpha}_{s-1} \partial_{\alpha_{2s-2}} \dot{\alpha}_{s-2} \ldots \partial_{\alpha_{s+1}} \ddot{\alpha}_1 \Psi_{(s)} \ddot{\alpha}_{(s-1)} \]  

These are superfields which respect the gauge symmetries mentioned above and at the component level they include the bosonic and fermionic higher spin field strengths. Moreover, they have two interesting characteristics. Firstly, they are both chiral superfields

\[ \bar{D}_\beta W_{\alpha(2s+1)} = 0 \quad , \quad \bar{D}_\beta W_{\alpha(2s)} = 0 \]  

Secondly, they have a special index structure. Specifically, they have indices of only one type (the undotted ones) and in both cases the number of indices is \(2Y\). Furthermore, on-shell they satisfy the following equations of motions:

\[ D^\beta W_{\beta\alpha(2s)} = 0 \quad , \quad D^\beta W_{\beta\alpha(2s-1)} = 0 . \]  

### 3 Gauge invariant, conserved, higher spin supercurrent

Now let us consider the cubic interaction \( Y_1 - Y_2 - \bar{Y}_2 \) between a massless superspin \( Y_1 \) supermultiplet and two massless superspin \( Y_2 \) supermultiplets. We select \( Y_1 \) to be half-integer \( Y_1 = s_1 + 1/2 \) for arbitrary positive integer \( s_1 \) and \( Y_2 \) can be either integer, \( Y_2 = s_2 \), or half-integer, \( Y_2 = s_2 + 1/2 \), for arbitrary positive integer \( s_2 \). Moreover, we assume that that the first order interaction vertices can be written in a lagrangian form which is generated by a higher spin supercurrent \( J \) and a higher spin supertrace \( T \) as follows\(^\text{10}\):

\[ S_I = g \int d^8z \left\{ H^{\alpha(s_1)} \dot{\alpha}(s_1) J_{\alpha(s_1)} \dot{\alpha}(s_1) + \chi^{\alpha(s_1)} \dot{\alpha}(s_1-1) T_{\alpha(s_1)} \dot{\alpha}(s_1-1) \right\} . \]  

In this case, because the \( Y_2 \) supermultiplets are not interacting the second term can be ignored because there are improvement terms that can eliminate the supertrace \((T_{\alpha(s_1)} \dot{\alpha}(s_1-1) = 0)\). Hence the only non-trivial object to focus in the supercurrent \( J_{\alpha(s_1)} \dot{\alpha}(s_1) \):

It is straightforward to show that, due to the gauge transformation (2a) the higher spin supercurrent must satisfy on-shell (up to terms that depend on the equation of motion) the following conservation equation

\[ \bar{D}^{\dot{\alpha}_{s_1}} J_{\alpha(s_1)} \dot{\alpha}(s_1) = 0 . \]  

Also due to the reality of \( H^{\alpha(s_1)} \dot{\alpha}(s_1) \), \( J_{\alpha(s_1)} \dot{\alpha}(s_1) \) must be real:

\[ J_{\alpha(s_1)} \dot{\alpha}(s_1) = \bar{J}_{\alpha(s_1)} \dot{\alpha}(s_1) . \]  

On top of these requirements, we demand that the higher spin supercurrent is gauge invariant with respect the gauge transformation of the \( Y_2 \) supermultiplets. This can be achieved if the supercurrent explicitly depends on the superfield strength \( W_{\alpha(2Y_2)} \) of the \( Y_2 \) supermultiplet. Furthermore, due to the cubic nature of the interaction the supercurrent must be quadratic in the \( Y_2 \) information. However because of the special index structure of \( W_{\alpha(2Y_2)} \), we conclude that the supercurrent must explicitly depend on both \( W_{\alpha(2Y_2)} \) and \( \bar{W}_{\dot{\alpha}(2Y_2)} \). The most general ansatz one can write is:

\[ J_{\alpha(s_1)} \dot{\alpha}(s_1) = \sum_{p=0}^{s_1-2Y_2} \alpha_p \partial^{(p)} W_{\alpha(2Y_2)} \partial^{(s_1-2Y_2-p)} \bar{W}_{\dot{\alpha}(2Y_2)} \]

\[ + \sum_{p=0}^{s_1-2Y_2-1} \beta_p \partial^{(p)} D W_{\alpha(2Y_2)} \partial^{(s_1-2Y_2-1-p)} D \bar{W}_{\dot{\alpha}(2Y_2)} \]

\(^\text{10}\)Due to the duality between the transverse and longitudinal formulations of the half-integer superspin, we only have to consider one of them. Our choice is the transverse description.
for some coefficients $\alpha_p$ and $\beta_p$. We have intentionally not explicitly written the external indices of the spacetime or spinorial derivatives in order to simplify the expression. Also one must not forget the independent symmetrization of all the dotted and undotted indices of the right hand side together with appropriate symmetrization factors that we also omit. The symbol $\partial^{(k)}$ is a replacement for a string of $k$ spacetime derivatives.

An immediate consequence of all the above is that there is a constraint in the $Y_1$ and $Y_2$ values. A supercurrent of this type can exist only if:

$$s_1 \geq 2Y_2 \Rightarrow Y_1 \geq 2Y_2 + 1/2.$$  \hspace{1cm} (12)

In other words, if $Y_2 > s_1/2$ there is not enough room for derivatives in order to have a gauge invariant supercurrent. Hence the conclusion is that if a supersymmetric theory allows the construction of a gauge invariant, conserved (as in (9)), real supercurrent of rank $s_1$ then its spectrum can include massless supermultiplets with superspin $Y > s_1/2$. This is a supersymmetric, higher spin extension of the Weinberg-Witten theorem [49]. Also at the component level the constrain (12) is consistent with the restrictions found in [21, 16].

Now we have to actually check, whether a solution of (9) and (10) of the form (11) exist. The conservation equation (9) relates the $\beta_p$ and $\alpha_p$ coefficients:

$$\beta_p = -i (-1)^{2Y_2} \alpha_{p+1} \frac{p+1}{s_1-p} \ , \ p = 0, 1, \ldots, s_1 - 2Y_2 - 1$$  \hspace{1cm} (13)

whereas the reality of the supercurrent gives the following constraints:

$$\alpha_p = \alpha_{s_1-2Y_2-p}^* \ , \ p = 0, 1, \ldots, s_1 - 2Y_2 \ ,$$  \hspace{1cm} (14)

$$\beta_p = \beta_{s_1-2Y_2-1-p}^* \ , \ p = 0, 1, \ldots, s_1 - 2Y_2 - 1 .$$  \hspace{1cm} (15)

The above three constraints fix uniquely the coefficients $\alpha_p$ and $\beta_p$

$$\alpha_p = c \ (i)^{s_1-2Y_2} (-1)^p \frac{(s_1-2Y_2)(p)}{1}\frac{(2Y_2+p)}{2Y_2} \ , \ p = 0, 1, \ldots, s_1 - 2Y_2 \ ,$$  \hspace{1cm} (16)

$$\beta_p = c \ (i)^{s_1-2Y_2+1} (-1)^{p+2Y_2} \frac{(s_1-2Y_2)(p)}{1}\frac{(s_1-2Y_2-p)(2Y_2+p)}{2Y_2 + 1 + p} \ , \ p = 0, 1, \ldots, s_1 - 2Y_2 - 1$$  \hspace{1cm} (17)

up to an overall real constant $c$. Hence not only is there a solution but it is unique. The higher spin supercurrent we find is

$$\mathcal{J}_{\alpha(s_1)\bar{\alpha}(s_1)} = c (i)^{s_1-2Y_2} \sum_{p=0}^{s_1-2Y_2} (-1)^p \frac{(s_1-2Y_2)(p)}{1}\frac{(2Y_2+p)}{2Y_2} \left\{ \partial^{(p)} \mathcal{W}_{\alpha(2Y_2)} \partial^{(s_1-2Y_2-p)} \bar{\mathcal{W}}_{\bar{\alpha}(2Y_2)} \right. \left. + i (-1)^{2Y_2} \frac{s_1-2Y_2-p}{2Y_2 + 1 + p} \partial^{(p)} \mathcal{D} \mathcal{W}_{\alpha(2Y_2)} \partial^{(s_1-2Y_2-1-p)} \bar{\mathcal{D}} \bar{\mathcal{W}}_{\bar{\alpha}(2Y_2)} \right\}$$  \hspace{1cm} (18)

There are two interesting cases to look at. The first one is at the boundary of (12), when $s_1 = 2Y_2$. In this case the supercurrent (18) is simplified and takes the following form:

$$\mathcal{J}_{\alpha(s_1)\bar{\alpha}(s_1)} = c \mathcal{W}_{\alpha(2Y_2)} \bar{\mathcal{W}}_{\bar{\alpha}(2Y_2)} \ .$$  \hspace{1cm} (19)
This supercurrent is the supersymmetric and higher spin extension of the Bel-Robinson tensor. That means that for $Y_2 = 3/2$ (linearized supergravity) the supercurrent $\mathcal{J}_{\alpha_2 \beta_2 \alpha_1 \beta_1}$ as defined by (19) in its components includes the Bel-Robinson tensor of General Relativity (see [50] and references therein). For higher values of $Y_2$ we get higher spin generalizations of the Bel-Robinson tensor.

The second interesting observation is in the limit $Y_2 = 0$. This case technically is not allowed because it is not a gauge theory but a matter theory, hence there is no zero order transformation and $\mathcal{W}$ is not a superfield strength. However, it is the chiral superfield of the free chiral theory and the supercurrent (18) gives the cubic interactions of the higher spin supermultiplet $Y_1$ with the chiral supermultiplet. Therefore it is allowed to take this limit. In that case the expression (18) exactly matches to the higher spin supercurrent of matter theories found in [1, 2].

4 Component structure of the higher spin supercurrent

The components of the higher spin supercurrent (18) will include bosonic and fermionic gauge invariant, conserved higher spin currents. It will be useful to extract the expressions for these currents and compare with known results for non-supersymmetric theories. For the projection of the superfield to its components, we will follow [55,2]. Due to the conservation equation (9), on-shell the higher spin supercurrent $\mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)}$ has only three independent components (2 bosonic and one fermionic)

$$
\mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} = \mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} \bigg|_{\theta=0} \quad \mathcal{J}_{\alpha(s_1+1)\dot{\alpha}(s_1)} = \frac{1}{s_1+1 !} \mathcal{D}^{(s_1+1)} \mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} \bigg|_{\theta=0} ,
$$

$$
\mathcal{J}_{\alpha(s_1+1)\dot{\alpha}(s_1+1)} = - \frac{1}{2(s_1+1) ! (s_1+1)!} \left[ \mathcal{D}^{(s_1+1)} \mathcal{D}^{(s_1+1)} \mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} \right] \bigg|_{\theta=0} .
$$

The rest of them either vanish or are derivatives of the components above. It is straight forward to show, using (10) and (9), that these components satisfy the traditional spacetime conservation equations

$$
\partial^{s_1+1} \mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} = 0, \quad \partial^{s_1+1-1} \mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} = 0, \quad \partial^{s_1+1-1} \mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} = 0
$$

hence they correspond to gauge invariant, conserved, higher spin currents. Their explicit expressions are:

$$
\mathcal{J}^{(0,0)}_{\alpha(s_1)\dot{\alpha}(s_1)} \sim (i)^{s_1-2Y_2} \sum_{p=0}^{s_1-2Y_2} (-1)^p \frac{(s_1-2Y_2)}{2Y_2+p} \left\{ \partial^{(p)} \mathcal{W}^{(0,0)}_{\alpha(2Y_2)} \partial^{(s_1-2Y_2-p)} \mathcal{W}^{(0,0)}_{\alpha(2Y_2)} \right\}
$$

$$
= i (1)^{s_1-2Y_2} \sum_{p=0}^{s_1-2Y_2} (-1)^p \left\{ \partial^{(p)} \mathcal{W}^{(0,0)}_{\alpha(2Y_2)} \partial^{(s_1-2Y_2-p)} \mathcal{W}^{(0,0)}_{\alpha(2Y_2)} \right\}
$$

$$
\mathcal{J}^{(1,0)}_{\alpha(s_1+1)\dot{\alpha}(s_1)} \sim (i)^{s_1-2Y_2} \sum_{p=0}^{s_1-2Y_2} (-1)^p \frac{(s_1-2Y_2)}{2Y_2+p} \left\{ \partial^{(p)} \mathcal{W}^{(1,0)}_{\alpha(2Y_2+1)} \partial^{(s_1-2Y_2-p)} \mathcal{W}^{(0,0)}_{\alpha(2Y_2)} \right\}
$$

$$
\mathcal{J}^{(1,1)}_{\alpha(s_1+1)\dot{\alpha}(s_1+1)} \sim (i)^{s_1-2Y_2} \sum_{p=0}^{s_1-2Y_2} (-1)^p \left\{ \partial^{(p)} \mathcal{W}^{(0,0)}_{\alpha(2Y_2)} \partial^{(s_1-1-2Y_2-p)} \mathcal{W}^{(0,0)}_{\alpha(2Y_2)} \right\}
$$

\[\text{[The labels (0,0), (1,0) and (1,1) correspond to the position of the components in the } \theta \text{ and } \bar{\theta} \text{ expansion of the supercurrent.]}\]
The components $W^{(0,0)}_{\alpha(2Y_2)}$ and $W^{(1,0)}_{\alpha(2Y_2+1)}$ are the only non-trivial components of the superfield strength $W_{\alpha(2Y_2)}$

$$
W^{(0,0)}_{\alpha(2Y_2)} = W_{\alpha(2Y_2)} \bigg|_{\theta=0} , \quad W^{(1,0)}_{\alpha(2Y_2+1)} = \frac{1}{(2Y_2+1)!} D(\alpha_{2Y_2+1} W_{\alpha(2Y_2)}) \bigg|_{\theta=0}
$$

and are the higher spin field strengths for spins $j = Y_2$ and $j = Y_2 + 1/2$ respectively. These (22) higher spin currents are consistent with the results of [21, 24].

5 Discussion

We consider cubic interactions $Y_1 - Y_2 - Y_2$ among massless supermultiplets of superspin $Y_1$ and $Y_2$, where $Y_1$ is half integer $Y_1 = s_1 + 1/2$ for arbitrary positive integer $s_1$ and $Y_2$ has an arbitrary non-negative integer or half-integer value. Specifically, we consider the class of these interactions that are generated by a higher spin supercurrent $J_{\alpha(s_1)\dot{\alpha}(s_1)}$ which respects the gauge symmetry of the $Y_2$ supermultiplet. We find that:

i. There is a unique gauge invariant, conserved, real, higher spin supercurrent $J_{\alpha(s_1)\dot{\alpha}(s_1)}$ (18).

ii. The existence of this supercurrent puts constraints (12) on the allowed $Y_2$ superspin values $|s_1| \geq 2|Y_2|$. This is a supersymmetric and higher spin generalization of the Weinberg-Witten theorem and also is consistent with various spin restrictions coming from the consideration of non-supersymmetric, higher spin, cubic interactions.

iii. For the special case of $s_1 = 2Y_2$ the higher spin supercurrent simplifies a lot (19) and provides a supersymmetric, higher spin extension of the linearized Bel-Robinson tensor known in the context of General Relativity.

iv. Another special limit is $Y_2 = 0$ where we recover previously known results for the higher spin supercurrent of the free, massless chiral theory.

v. The component structure includes two bosonic and one fermionic gauge invariant, conserved, higher spin currents (22) which depend on the derivatives of higher spin field strengths for spins $j = Y_2$ and $j = Y_2 + 1/2$.

The list of the supercurrents we find refer only to non-minimal couplings (higher number of derivatives) and that is due to the explicit dependence of the supercurrent on the superfield strengths. Therefore, in spite of this list being infinite (for arbitrary values of $Y_1$ and $Y_2$ that respect (12)), it is incomplete. This is simply understood from the fact that ordinary conserved currents such as the energy momentum tensor and their higher spin extensions [58] are not included in the class of gauge invariant currents. It would be interesting to search for non gauge invariant higher spin supercurrents that generate minimal coupling (least number of derivatives) among higher spins and give gauge invariant conserved charges.

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