Altarelli-Parisi Equation in Non-Equilibrium QCD

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Abstract—The $Q^2$ evolution of fragmentation function in non-equilibrium QCD by using DGLAP evolution equation may be necessary to study hadron formation from quark-gluon plasma at RHIC and LHC. In this paper we study splitting functions in non-equilibrium QCD by using Schwinger-Keldysh closed-time path integral formalism. For quarks and gluons with arbitrary non-equilibrium distribution functions $f_q(p_T)$ and $f_g(p_T)$, we derive expressions for quark and gluon splitting functions in non-equilibrium QCD at leading order in $\alpha_s$. We make a comparison of these splitting functions with that obtained by Altarelli and Parisi in vacuum.

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1. INTRODUCTION

RHIC and LHC heavy-ion colliders are the best facilities to study quark-gluon plasma in the laboratory. Since two nuclei travel almost at speed of light, the QCD matter formed at RHIC and LHC may be in non-equilibrium. In order to make meaningful comparison of the theory with the experimental data on hadron production, it may be necessary to study non-equilibrium–nonperturbative QCD at RHIC and LHC. This, however, is a difficult problem.

Non-equilibrium quantum field theory can be studied by using Schwinger-Keldysh closed-time path (CTP) formalism [1, 2]. However, implementing CTP in non-equilibrium at RHIC and LHC is a very difficult problem, especially due to the presence of gluons in non-equilibrium and hadronization etc. Recently, one-loop resumed gluon propagator in non-equilibrium in covariant gauge is derived in [3, 4].

High $p_T$ hadron production at high energy $e^+e^-$, $ep$ and $pp$ colliders is studied by using Collins-Soper fragmentation function [5, 6]. For a high $p_T$ parton fragmenting to hadron, Collins-Soper derived an expression for the fragmentation function based on field theory and factorization properties in QCD at high energy [7]. This fragmentation function is universal in the sense that, once its value is determined from one experiment it explains the data at other experiments.

Recently we have derived parton to hadron fragmentation function in non-equilibrium QCD by using Schwinger-Keldysh closed-time path integral formalism [8]. This can be relevant at RHIC and LHC heavy-ion colliders to study hadron production from quark-gluon plasma. We have considered a high $p_T$ parton in medium at initial time $\tau_0$ with arbitrary non-equilibrium (non-isotropic) distribution function $f(p_T)$ fragmenting to hadron. The special case $f(p_T) = 1/ep_0/T \pm 1$ corresponds to the finite temperature QCD in equilibrium.

We have found the following definition of the parton to hadron fragmentation function in non-equilibrium QCD by using closed-time path integral formalism [8]. For a quark ($q$) with arbitrary non-equilibrium distribution function $f_q(p_T)$ at initial time, the quark to hadron fragmentation function is given by

$$D_{H/q}(z, P_T) = \frac{1}{2z[1 + f_q(k)]} \int d^2x e^{ik \cdot x + iP_T \cdot x/z}$$

$$\times \frac{1}{2} Tr_{Dirac} \left[ \gamma \cdot \langle in|\psi(x^-, T)\Phi[x^-, x_T]\Phi_T|0\rangle \right]$$

where $z = (p_T/k^+)$ is the longitudinal momentum fraction of the hadron with respect to the parton and $P_T$ is the transverse momentum of the hadron. For a gluon ($g$) with arbitrary non-equilibrium distribution function $f_g(p_T)$ at initial time, the gluon to hadron fragmentation function is given by

$$D_{H/g}(z, P_T) = \frac{1}{2zk^+ \left[ 1 + f_g(k) \right]} \int d^2x e^{ik \cdot x + iP_T \cdot x/z}$$

$$\times \frac{1}{2} \sum_{a=1}^8 \left[ \langle in|F_{aH}^{+\mu}(x^-, x_T)\Phi[x^-, x_T]\Phi_T|0\rangle \right]$$

$$\times a_H^+(P_T, 0) a_H(P_T, 0) \Phi(0) [F_a(0)|in] \right].$$

1 The article is published in the original.
In the above equations $|m\rangle$ is the initial state of the non-equilibrium quark (gluon) medium. The path ordered exponential

$$
\Phi[x^n] = \mathcal{P} \exp \left[ ig \int_{-\infty}^{0} d\lambda n \cdot A^\alpha (x^n + n^n \lambda) T^\alpha \right]
$$

is the Wilson line $[5,7,9]$. Eqs. (1) and (2) can be compared with the following definition of Collins-Soper fragmentation function in vacuum $[5]$

$$
D_{H/q}(z, P_T) = \frac{1}{2z} \int dx^d - \frac{x_T}{2} e^{i k T \cdot x_T + i P_T \cdot x_T/z} \times \frac{1}{2} \text{tr}_{Dirac} \left[ \frac{1}{3} \text{tr}_{color} \langle \gamma^+ \rangle \psi^\dagger(x_T, x_I) \Phi[x_T, x_I] \psi(x_T, x_I) \right] d_H^+(z, P_T)
$$

and

$$
D_{A/q}(z, P_T) = \frac{1}{2z} \int dx^d - \frac{x_T}{2} e^{i k T \cdot x_T + i P_T \cdot x_T/z} \times \frac{1}{8} \sum_{a=1}^{8} \left[ \langle 0 | a_T^{\dagger} \psi(x_T, x_I) \Phi[x_T, x_I] \right] \times a_H^+(P_T, 0, P_T) \Phi[0] \langle 0 | \psi(0)|0 \rangle
$$

Since the fragmentation function is a non-perturbative quantity, we do not have theoretical tools in QCD to calculate it yet. The normal procedure at high energy $pp$, $ep$ and $e^+e^-$ colliders is to extract it at some initial momentum scale $\mu_0$ and then evolve it to another scale $\mu$ by using the DGLAP evolution equation $[10-12]$

$$
\mu \frac{\partial}{\partial \mu} D_{i \rightarrow j/q}(z) = \sum_{j} \int \frac{dy}{y} P_{qj}(z, \mu) D_{j \rightarrow i/q}(y).
$$

In the above equation $P_{qj}(z)$ is the splitting function of a parton $j$ into a parton $i$ which is related to the probability of a parton $j$ emitting a parton $i$ with longitudinal momentum fraction $z$. The quark and gluon splitting functions $P_{qj}(z)$ in vacuum is evaluated by Altarelli and Parisi $[11]$ at the leading order in coupling constant $\alpha_s$.

We find the following expressions for the quark and gluon splitting functions in non-equilibrium QCD at leading order in coupling constant $\alpha_s$

$$
P_{q\bar{q}}(z) = C_A(R) \left[ 1 + f_\bar{q}(k_T, z) \right]^2 \times \left[ \frac{1}{1 - z} \right] 

P_{qg}(z) = C_A(R) \left[ 1 + f_\bar{q}(k_T, z) \right]^2 \times \left[ \frac{1}{1 - z} \right] 

P_{gg}(z) = 2C_A(R) \left[ 1 + f_\bar{q}(k_T, z) \right]^2 \times \left[ \frac{1}{1 - z} \right]
$$

where $k$ is the momentum of initial parton (which is assumed to be along longitudinal direction), $k_T$ is the transverse momentum of the emitted parton and $z$ is the longitudinal momentum fraction of the initial parton carried by the emitted parton.

Eq. (7) can be compared with the following expressions for the splitting functions in vacuum obtained by Altarelli and Parisi $[11]$ at the leading order in coupling constant $\alpha_s$

$$
P_{q\bar{q}}(z) = C_A(R) \left[ 1 + f_\bar{q}(k_T, z) \right]^2 \times \left[ \frac{1}{1 - z} \right] 

P_{qg}(z) = C_A(R) \left[ 1 + f_\bar{q}(k_T, z) \right]^2 \times \left[ \frac{1}{1 - z} \right] 

P_{gg}(z) = 2C_A \left[ \frac{1}{1 - z} \right]
$$

We will present derivation of eq. (7) in this paper.

The paper is organized as follows. In Section 2 we briefly review the derivation of quark and gluon splitting functions in vacuum. In Section 3 we describe Schwinger-Keldysh closed-time path integral formalism in non-equilibrium QCD relevant to our calculation. In Section 4 we derive quark and gluon splitting functions $P_{ij}$ in non-equilibrium QCD by using closed-time path integral formalism. Section 5 contains conclusions.
2. QUARK AND GLUON SPLITTING FUNCTIONS IN VACUUM

In this section we briefly review the derivation of quark and gluon splitting functions \( P_s \) in vacuum. We will present our calculation in the \( S \)-matrix approach. Hence, our derivation is slightly different from [11].

Consider a quark with momentum \( p_A \) emitting a gluon with momentum \( p_B \) in the process \( q(p_A) \rightarrow g(p_B) + q(p_C) \). The \( S \)-matrix element for this process is given by

\[
S^{(1)} = ig \int d^4x N[\bar{\psi}(x)A^\mu(x)T^a\psi(x)]
\]

where (the normalization is from [15])

\[
\psi(x) = \psi^+(x) + \psi^-(x) = \sum_{\text{spin}} \sum_p \frac{m}{\sqrt{VE_p}} \left[ a_g(p)u(p)e^{-ip\cdot x} + a_g^+(p)v(p)e^{ip\cdot x} \right]
\]

\[
\bar{\psi}(x) = \bar{\psi}^+(x) + \bar{\psi}^-(x) = \sum_{\text{spin}} \sum_p \frac{m}{\sqrt{VE_p}} \left[ a_g(p)v(p)e^{-ip\cdot x} + a_g^+(p)\bar{u}(p)e^{ip\cdot x} \right]
\]

\[
A^\mu(x) = A_\mu^+(x) + A_\mu^-(x) = \sum_{\text{spin}} \sum_p \frac{m}{\sqrt{2VE_p}} \left[ a_g(p)e^{-ip\cdot x} + a_g^+(p)e^{ip\cdot x} \right].
\]

In the above equation \( a_g(p) \), \( a_g^+(p) \) and \( a_g^+(p) \) are annihilation operators for quark, antiquark and gluons respectively. In eq. (10) the suppression of color indices are understood. The initial and final states are

\[
|\bar{\rho}\rangle = |q(p_A)\rangle = a^+_g(p_A)|\rangle_0,
\]

\[
|\rho\rangle = |q(p_C), g(p_B)\rangle = a^+_g(p_C)a^+_g(p_B)|\rangle_0.
\]

where \( p_C = p_A - p_B \). Hence we find

\[
|\langle \bar{\rho}|S^{(1)}|\rho\rangle|^2 = \left[ \frac{V}{(E_C + E_B - E_A)} \right]^2 \frac{m}{VE_CVE_A2VE_B} \sum_{\text{spin}} |M|^2.
\]

where

\[
M = ig\bar{u}(p_C)\gamma^\mu u(p_A)e^{im(p_B)t^a}.\]

For massless quarks we find

\[
W_{gq} = |\langle \rho|S^{(1)}|\bar{\rho}\rangle|^2 \frac{Vd^3p_C}{(2\pi)^3} = C_2(R) \frac{g^2d^3p_C}{(2\pi)^3} \frac{1}{E_B} \frac{1}{2E_AE_B(E_C + E_B - E_A)^2} \times \left[ \delta^{ij} - \frac{p_{gF}^i p_{gF}^j}{p_{gF}^2} \right] \]

which gives the quark to gluon splitting function

\[
P_{gq}(z) = C_2(R) \frac{1 + (1 - z)^2}{z}.
\]

Similarly we find the quark to quark splitting function

\[
P_{qq}(z) = C_2(R) \frac{1 + z^2}{1 - z}
\]

and the gluon to gluon splitting function

\[
P_{gg}(z) = 2C_4 \left[ \frac{1 - z}{z} + \frac{z}{1 - z} + z(1 - z) \right].
\]

3. NON-EQUILIBRIUM QCD USING CLOSED-TIME PATH FORMALISM

Unlike \( pp \) collisions, the ground state at RHIC and LHC heavy-ion collisions (due to the presence of a QCD medium at initial time \( t = t_m \) (say \( t_m = 0 \)) is not a vacuum state \( |\rho\rangle \rangle > \rangle \) any more. We denote \( |in\rangle \rangle \) as the initial state of the non-equilibrium QCD medium at tin. The non-equilibrium distribution function \( f(\mathbf{k}) \) of a parton (quark or gluon), corresponding to such initial state is given by

\[
\langle a^+_g(\mathbf{k})a(\mathbf{k}') \rangle = \langle in|a^+_g(\mathbf{k})a(\mathbf{k}')|in\rangle = f(\mathbf{k})\delta^{(3)}(\mathbf{k} - \mathbf{k}'),
\]

where we have assumed space translational invariance at initial time.

Finite temperature field theory formulation is a special case of this when \( f(\mathbf{k}) = 1/ek_0/ T \pm 1 \).

3.1. Quarks in Non-Equilibrium

The non-equilibrium (massless) quark propagator at initial time \( t = t_m \) is given by (suppression of color indices are understood)

\[
G(\mathbf{k})_{ij} = \left[ \begin{array}{cc} \frac{1}{k^2 + i\epsilon} + 2\pi\delta(k^2)f_q(\mathbf{k}) & -2\pi\delta(k^2)\theta(-k_0) + 2\pi\delta(k^2)f_q(\mathbf{k}) \\ -2\pi\delta(k^2)\theta(k_0) + 2\pi\delta(k^2)f_q(\mathbf{k}) & \frac{1}{k^2 + i\epsilon} + 2\pi\delta(k^2)f_q(\mathbf{k}) \end{array} \right]
\]
where where \( i, j = +, - \) and \( f_{\bar{s}}(\hat{k}) \) is the arbitrary non-equilibrium distribution function of quark.

### 3.2. Gluons in Non-Equilibrium

We work in the frozen ghost formalism \([3, 4]\) where the non-equilibrium gluon propagator at initial time \( t = t_{in} \) is given by (the suppression of color indices are understood)

\[
G^{\mu
\nu}(k)_{ij} = -i\left[g^{\mu
\nu} + (\alpha - 1)\frac{k^{\mu}k^{\nu}}{k^2} - \frac{(k^{\mu}k^{\nu})}{(k\cdot u)^2 - k^2} \right]
\]

where \( i, j = +, - \). The transverse tensor is given by

\[
\Pi^{\mu
\nu}(k) = g^{\mu
\nu} - \frac{(k\cdot u)(u^{\mu}k^{\nu} + u^{\nu}k^{\mu}) - (k^{\mu}k^{\nu}) - k^2u^{\mu}u^{\nu}}{(k\cdot u)^2 - k^2},
\]

with the flow velocity of the medium \( u^{\mu} \). \( G_{ij}^{\mu
\nu}(k) \) are the usual vacuum propagators of the gluon

\[
G_{ij}^{\mu
\nu}(k) = \begin{pmatrix}
\frac{1}{k^2 + i\epsilon} & -2\pi\delta(k^2)\theta(\epsilon)k_0 \\
-2\pi\delta(k^2)\theta(k_0) & \frac{1}{k^2 + i\epsilon}
\end{pmatrix}
\]

and the medium part of the propagators are given by

\[
G_{ij}^{\mu
\nu}(k) = 2\pi\delta(k^2)f_{\bar{s}}(\hat{k})\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}
\]

### 3.3. Ratio of Characteristic Relaxation Time of the Non-Equilibrium State to the QCD Evolution Time

The typical relaxation time in the non-equilibrium QCD plasma can be written as \([16–18]\)

\[
\tau_c = \frac{1}{n\hat{\sigma}_{tr}}
\]

where

\[
n = \int \hat{d}\hat{k}f(\hat{k}),
\]

is the parton number density in terms of the non-equilibrium parton distribution function \( f(\hat{k}) \) and \( \hat{\sigma}_{tr} \) is the typical transport cross section of the partonic collisions in the non-equilibrium QCD plasma which depends on the non-equilibrium parton distribution function \( f(\hat{k}) \).

Consider for example, the \( gg \rightarrow gg \) scattering. The leading order partonic differential cross section in vacuum is given by

\[
\frac{d\hat{\sigma}}{dt} = \frac{9\pi\alpha_s^2}{2\hat{s}^2} \left[ 3 - \frac{u^\nu}{\hat{s}^2} \frac{\hat{u}^\nu}{\hat{u}^2} - \frac{\hat{u}^\nu}{\hat{s}^2} \right]
\]

where in the infrared limit \( t \rightarrow 0 \) diverges

\[
\frac{d\hat{\sigma}}{dt} = \frac{9\pi\alpha_s^2}{2t^2}.
\]

However, in the medium, the medium modified resumed gluon propagator removes this infrared divergence and the typical finite differential cross section becomes \([19]\)

\[
\frac{d\hat{\sigma}}{dt} = \frac{9\pi\alpha_s^2}{8} \left[ \frac{1}{(\Pi_L - i)(\Pi_L - i)} + \frac{1}{(\Pi_L - i)(\Pi_L - i)} \right]
\]

\[
+ \frac{1}{(\Pi_L - i)(\Pi_L - i)} + \frac{1}{(\Pi_L - i)(\Pi_L - i)}
\]

where \( \Pi_L \) and \( \Pi_T \) are the longitudinal and transverse component of the gluon self energy which depend on the non-equilibrium distribution function \( f(\hat{k}) \). One can see that even at the one-loop level of the self energy the magnetic screening mass is non-zero \([3, 4, 20, 21]\) as long as the non-equilibrium distribution function \( f(\hat{k}) \) is non-isotropic i.e., it depends on the direction of \( \hat{k} \) of the parton, which is the case at early stage of the heavy-ion collisions at RHIC and LHC. The expressions of the medium modified resumed gluon propagator at the one-loop level of self energy in non-equilibrium in covariant gauge is recently derived in \([3, 4]\).

The transport cross section \([16, 19]\)

\[
\hat{\sigma}_{tr} = \int \hat{d}\hat{k}f(\hat{k})\sin^2\theta_{cm} = \int \hat{d}\hat{k}4\hat{u}^{\nu}\hat{s}^{\mu}\hat{\hat{s}}^{\nu}/\hat{s}^2.
\]

for this process can be obtained by using eq. (28). Since gluons are dominate part of the total parton production at the early stage of the heavy-ion collisions at RHIC and LHC one can get an estimate of the relaxation time of the non-equilibrium state from eqs. (24), (25), (28) and (29). For example, the typical value of the maximum relaxation time in non-equilibrium state found in \([16]\) is \( \sim 1.5 \) fm at RHIC and LHC heavy-ion colliders.

The typical QCD evolution time associated with the DGLAP evolution equation of the fragmentation function is given by

\[
t = \ln Q
\]
where

\[ Q = \mu \]  

(31)

is the energy scale determined by the hard process probing fragmentation function [22].

4. QUARK AND GLUON SPLITTING FUNCTIONS IN NON-EQUILIBRIUM QCD

In this section we evaluate quark and gluon splitting functions \( P_g \) in non-equilibrium QCD. Similar to the vacuum case in [11] (see eq. (11)) we define the state \(| i \rangle \) and \(| f \rangle \) in non-equilibrium QCD as follows

\[ | i \rangle = | q(p_A) \rangle = a_q^\dagger(p_A)| in \rangle, \]

\[ | f \rangle = | q(p_c), g(p_b) \rangle = a_q^\dagger(p_c)a_g^\dagger(p_b)| in \rangle, \]

(32)

where \(| in \rangle \) is the initial state of the non-equilibrium QCD medium. It has to be remembered that for evaluating the Feynman diagrams and \( S \)–matrix we work in the interaction picture where the fields \( \psi(x) \) and \( A_q(x) \) obey the free field equations in terms of creation and annihilation operators as given by eq. (10). From eqs. (32) and (9) we find

\[ \langle f | S^{(1)} | i \rangle = ig \int d^4x \langle in | a_q(p_c)a_g(p_b)N \]

\[ \times [\bar{\psi}(x)\Psi^\dagger(x)T^a\psi(x)]a_q^\dagger(p_A)| in \rangle, \]

(33)

which gives by using eq. (10)

\[ \langle f | S^{(1)} | i \rangle = ig \int d^4x \sum_{spin, q, g} \frac{V}{VE_p} \frac{m}{VE_p} \frac{1}{2VE_p} \]

\[ \times \langle in | a_q(p_c)a_g(p_b)[a_q^\dagger(p)c\bar{u}(p)e^{ip\cdot z}][\gamma_\mu a_g^\dagger(p)\epsilon^{\mu\nu}(p')T^a] \]

\[ \times e^{i(p'-z)T^a}[a_q(p''\mu)u(p'')e^{-i(p'\cdot z)}]a_q^\dagger(p_A)| in \rangle. \]

Performing \( x \)–integration we find

\[ \langle f | S^{(1)} | i \rangle \]

\[ = ig \sum_{spin, q, g} \frac{V}{E_p} \frac{m}{VE_p} \frac{1}{2VE_p} \]

\[ \times \langle in | a_q(p_c)a_g(p_b)[a_q^\dagger(p)c\bar{u}(p)][\gamma_\mu a_g^\dagger(p)\epsilon^{\mu\nu}(p')T^a] \]

\[ \times [a_q(p''\mu)u(p'')a_q^\dagger(p_A)| in \rangle. \]

(35)

In the interaction picture the commutation relations are same as that for free field operators

\[ [a(p), a^\dagger(p')] = \delta_{pp'} \]

\[ [a(p), a(p')] = [a^\dagger(p), a^\dagger(p')] = 0. \]

(36)

which gives

\[ \langle f | S^{(1)} | i \rangle \]

\[ = ig \sum_{spin, p, p'} \frac{V}{E_p} \frac{m}{VE_p} \frac{1}{2VE_p} \]

\[ \times \langle in | a_q^\dagger(p)c\bar{u}(p)\delta_{pp'}\bar{u}(p)\gamma_\mu [a_q^\dagger(p')a_g(p_b) + \delta_{pp'a}u(p''\mu)T^a] \]

\[ \times \langle in | a_q^\dagger(p')a_g(p_b)\epsilon^{\mu\nu}(p)T^a \]

\[ \times \epsilon^{\nu\nu'}(p') [a_q(p''\mu)u(p'')e^{-i(p'\cdot z)}]a_q^\dagger(p_A)| in \rangle, \]

(37)

For our purpose of evaluating Feynman diagrams in momentum space we use eq. (18)

\[ \langle in | a_q^\dagger(p)\bar{u}(p) | in \rangle = \frac{1}{2VE_p} \]

\[ \times [1 + f_q(\hat{p}_c)]^2[1 + f_q(\hat{p}_g)]^2[1 + f_q(\hat{p}_d)] \]

\[ \sum_{spin} [M]^2 \]

(38)

where

\[ M = ig\bar{u}(p_c)\gamma_\mu u(p_a)\epsilon^{\mu\nu}(p_b)T^a. \]

(40)

\[ \hat{p}_f = \frac{p_f + zp_T}{1 + z} \]

(41)

From now onwards we can follow exactly the same steps as in the vacuum case (see Section 2, the derivations after eq. (12)) to find the probability

\[ W = C_f(R)\frac{z}{2\pi} \]

\[ \times [1 + f_q(\hat{p}_c)]^2[1 + f_q(\hat{p}_g)]^2[1 + f_q(\hat{p}_d)]\]

\[ \times \frac{1 + (1 - z)^2}{z} dzd(lnp_T^2). \]

\[ \hat{p}_f = \frac{p_f + zp_T}{1 + z} \]

(42)

which reproduces eq. (7).
4.2. Quark to Quark Splitting Function in Non-Equilibrium QCD

The quark to quark splitting function [in the process $q(p_1) \rightarrow q(p_2) + g(p_3)$] can be obtained from the quark to gluon splitting function [in the process $q(p_1) \rightarrow g(p_2) + q(p_3)$] with the replacement $z \rightarrow (1-z)$:

$$P_{qg}(z) = P_{Gq}(1-z), \quad z < 1.$$  \hfill (43)

Hence we find from eq. (42) the quark to quark splitting function in non-equilibrium QCD

$$P_{qq}(z) = C_2(R)[1 + f_q(p)]^2 \left[1 + f_g(-p_T, (1-z)p)\right]^2 \times \left[1 + f_g(p_T, zp)\right]^{1+z^2-1-z}$$ \hfill (44)

which reproduces eq. (7).

4.3. Gluon to Gluon Splitting Function in Non-Equilibrium QCD

Similarly, using three gluon vertex and carrying out the similar algebra we find gluon to gluon splitting function in non-equilibrium QCD

$$P_{gg}(z) = 2C_A[1 + f_g(p)]^2 \left[1 + f_g(p_T, zp)\right]^2 \times \left[1 + f_g(-p_T, (1-z)p)\right]^{1+z^2-1-z}$$ \hfill (45)

which reproduces eq. (7).

The splitting functions in non-equilibrium QCD as given by eqs. (42), (44) and (45) can be used to study DGLAP evolution equation of fragmentation function in non-equilibrium QCD [8, 14] to study high $p_T$ hadron production from quark-gluon plasma at RHIC and LHC.

5. CONCLUSIONS

RHIC and LHC heavy-ion colliders at RHIC and LHC one needs to prove factorization of fragmentation function in non-equilibrium QCD. Recently we have proved factorization theorem in non-equilibrium QED in [13] and in non-equilibrium QCD in [14].

In this paper we have evaluated the quark and gluon splitting functions in non-equilibrium QCD at the leading order in coupling constant $\alpha_s$ by using closed-time path integral formalism. For quarks and gluons with arbitrary non-equilibrium distribution functions $f_q(\hat{p})$ and $f_g(\hat{p})$, we have derived expressions for quark and gluon splitting functions in non-equilibrium QCD. We have found that the quark and gluon splitting functions depend on non-equilibrium distribution functions $f_q(\hat{p})$ and $f_g(\hat{p})$. We have made a comparison of these splitting functions with that obtained by Altarelli and Parisi in vacuum.

The splitting functions in non-equilibrium QCD can be used to study DGLAP evolution equation of fragmentation function in non-equilibrium QCD [8, 14] to study high $p_T$ hadron production from quark-gluon plasma [16, 23] at RHIC and LHC.

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