Micromechanical analysis on anisotropy of structured magneto-rheological elastomer

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Abstract. This paper investigates the equivalent elastic modulus of structured magneto-rheological elastomer (MRE) in the absence of magnetic field. We assume that both matrix and ferromagnetic particles are linear elastic materials, and ferromagnetic particles are embedded in matrix with layer-like structure. The structured composite could be divided into matrix layer and reinforced layer, in which the reinforced layer is composed of matrix and the homogeneously distributed ferromagnetic particles in matrix. The equivalent elastic modulus of reinforced layer is analysed by the Mori-Tanaka method. Finite Element Method (FEM) is also carried out to illustrate the relationship between the elastic modulus and the volume fraction of ferromagnetic particles. The results show that the anisotropy of elastic modulus becomes noticeable, as the volume fraction of particles increases.

1. Introduction
Magneto-rheological elastomer is comprised of polymer matrix and the micron-scale ferromagnetic particles inside it. The ferromagnetic particles are magnetized in the presence of magnetic field, and the MRE presents macroscopic behavior that the elastic modulus alters with the magnetic field. Compared with the magnetorheological fluids, the merits of MREs are their controllable stiffness, chemical and physical stability, and no sealing problems that make them attractive in semi-active vibration control [1, 2].

The elastic modulus of MRE can contribute to the magneto-induced modulus and zero-field modulus in the absence of magnetic field. The relations among magneto-induced modulus, ferromagnetic particle fraction, and magnetic field strength, have been investigated extensively [3]. In comparison with MRE whose particles are uniformly distributed, the MRE with aligned particles has a higher magneto-induced modulus [4]. Hence, it is very common to prepare the MREs where the magnetic particles are aligned orderly using magnetic field during curing process, resulting in anisotropic MREs.

However the research on the zero-field modulus is still inadequate, since only the empirical equation is utilized to evaluate the zero-field modulus of MRE in previous work [5, 6], and the particles distribution is merely taken in to consideration, which leads to an untrustworthy result. Many models are proposed in these years to help us understand the equivalent material properties, such as...
the method of upper and lower boundary from Voigt and Reuss [7, 8], Eshelby model [9] and Mori-Tanaka method [10]. Although those methods are helpful to evaluate the properties of composite like MRE, some errors worsen the theoretical result compared with empirical one. For example, the interaction between reinforced phases of composite is uncertain. Therefore, representative volume unit (RVU) [6] is set up for evaluation with the help of FEM software, and the results from FEM are proved to agree well with the experimental results, which can predict the properties of composites.

Considering the anisotropic property of the structured MREs, this paper assumes that magnetic particles are organized in layer in the presence of magnetic field, and MREs are a kind of anisotropic materials that stack the reinforced layer and matrix layer together. To investigate the zero-field modulus of the anisotropic MRE with different volume fraction of ferromagnetic particles, Mori-Tanaka method is carried out to build the theoretical model and RVU is founded and meshed to analyze the zero-field modulus of MRE by finite element theory.

2. Theoretical model

As shown in figures 1(a) and 1(b), the magnetic particles in MRE are orderly distributed in the presence of magnetic field, which make it become a kind of anisotropic material like fiber-reinforced composites.

![Figure 1. Microscopic structure of MRE. (a) Uniformed (b) Structured [1].](image)

![Figure 2. Layer-like unit cell. (a) Overall view (b) Cross-sectional view.](image)

It is assumed that both matrix and ferromagnetic particles are isotropic linear elastic materials, and ferromagnetic particles are embedded in matrix with layer-like structure. Figure 2 shows that the composite is divided into two parts, i.e. reinforced layer and matrix layer. The reinforced layer is composed of matrix and the homogenously distributed particles inside it.

In figure 2, \(l\) and \(w\) represent the thickness of matrix layer and reinforced layer respectively. \(R_0\) stands for the radius of ferromagnetic particles. In the following sections, we firstly analyze the equivalent elastic modulus of the reinforced layer. Then we obtain the equivalent elastic modulus of the MREs changed with particle volume fractions, at last we discuss the anisotropic property of the structured MREs in different deformation directions.

2.1. Equivalent modulus of reinforced layer
Given that particles in reinforced layer are seamlessly embedded in the matrix, and they are uniformly dispersed, the reinforced layer can be regarded as two phase composites. According to the relationship between stress and strain, the arithmetic mean symbol is defined as
\[
\langle \cdot \rangle = \frac{1}{V} \int_V \cdot \, dV.
\]
Inside the reinforced layer, there are two phases: matrix phase and inclusion phase, where the average stress tensor \( \langle \sigma_{ij} \rangle \) and the average strain tensor \( \langle \varepsilon_{ij} \rangle \) of each unit cell satisfy the following equations:
\[
\langle \sigma_{ij} \rangle = C_{ijkl} \langle \varepsilon_{ij} \rangle, \\
\langle \sigma_{ij} \rangle_r = C_{ijkl}^r \langle \varepsilon_{ij} \rangle_r.
\]
Where \( C_{ijkl} \) is the equivalent elastic tensor of a unit cell, \( r \) stands for the \( r \) phase of material.

According to the definition of arithmetic mean symbol and the structure of reinforced layer, the average stress tensor of a unit cell can be also described as equation (3), which is the sum of average stress of each phase:
\[
\langle \sigma_{ij} \rangle = \frac{1}{V} \int_V \sigma_{ij} \, dV = \sum_{r=0}^{N-1} V_r \frac{1}{V_r} \int_{V_r} \sigma_{ij} \, dV = \sum_{r=0}^{N-1} c_r \langle \sigma_{ij} \rangle_r.
\]
Where \( V \) is the total volume, \( V_r \) is the relative volume of phase \( r \), \( c_r = V_r / V \) is the volume fraction that phase \( r \) accounts for. The local relation between each phase is:
\[
\langle \varepsilon_{ij} \rangle = B_{ijkl}^r \varepsilon_{kl}.
\]

According to Mori–Tanaka method, given that the strain around the certain inclusion is the average strain of matrix, and the average strain of ferromagnetic particles is expressed as:
\[
\langle \varepsilon_{ij} \rangle = \frac{1}{V} \int_{\Omega} \varepsilon_{ij} \, dV + \frac{1}{V} \int_{\Omega} \varepsilon_{ij} \, dV.
\]
Where \( \Omega \) is the fourth-rank tensor, \( \delta \mathbf{C} = \mathbf{C}_1 - \mathbf{C}_0 \) is the difference between elastic modulus of inclusions \( \mathbf{C}_1 \) and elastic modulus of matrix \( \mathbf{C}_0 \). Tensor \( \mathbf{P} \) is related with the inclusion shape. When the inclusions are chosen as spherical shape, equation (7) is derived as:
\[
P_{ijkl} = \frac{1}{3G_0(4G_0 + 3K_0)} \delta_{ij} \delta_{kl} + \frac{3(2G_0 + K_0)}{10G_0(4G_0 + 3K_0)} \left( \delta_{ij} \delta_{kl} + \delta_{ij} \delta_{kl} - \frac{2}{3} \delta_{ij} \delta_{kl} \right)
\]
Where \( G_0, K_0 \) are shear modulus and volume modulus of matrix phase, respectively. \( \delta_{ij} \) is Kronecker tensor. By considering equations (6) and (7), the equivalent modulus of reinforced layer satisfies the following equation:
\[
\mathbf{C} = \mathbf{C}_0 + c_1 [(\mathbf{C}_1 - \mathbf{C}_0) + c_0 \mathbf{P}]^{-1}
\]

Where \( \mathbf{P} \) is the \( r \) phase's \( \mathbf{P} \) tensor, by substituting equation (7) into equation (8), it can be concluded that reinforced layer is isotropic, while the equivalent volume modulus and equivalent shear modulus are expressed as:
\[
\bar{G} = G_0 + \frac{10G_0(4G_0 + 3K_0)(G_1 - G_0)}{10G_0(4G_0 + 3K_0) + 12G_0(3G_0 + K_0)(G_1 - G_0)} c_1,
\]
\[
\bar{K} = K_0 + \frac{3G_0(4G_0 + 3K_0)(K_1 - K_0)}{3G_0(4G_0 + 3K_0) + 9G_0(K_1 - K_0)} c_1.
\]

2.2. Equivalent modulus of MRE
The reinforced layer and matrix layer are made up of isotropic materials. Assuming that MRE is transverse isotropic, the five independent transverse elastic constants of elastic modulus are described
as equations (12)-(16). Along the $x$ or $z$ axis stretching the MRE uniaxially, the displacement is $\Delta l$. Therefore, the resultant force generated by two layers is as:

$$P = A\sigma_{11} = A_p\sigma_p + A_m\sigma_m,$$

(11)

Where $A_p$ is the cross-sectional area of reinforced layer, $A_m$ is the cross-sectional area of matrix. $\sigma_p$ is the normal stress of reinforced layer, $\sigma_m$ is the normal stress of matrix layer. Considering that the two layers have the same normal strain $\varepsilon_{11} = \Delta l/l$, according to the equation (11), the vertical Young's modulus of MRE $E_1, E_2$ are expressed as

$$E_1 = E_2 = E_p c_p + E_m (1 - c_p) = E_m (1 - c_p) + \frac{\sigma_p c_p K}{3K + G},$$

(12)

Where $c_p = A_p/(A_{pm} + A_p)$ represents the volume fraction of reinforced layer. Similarly, along the $y$ axis stretches the MRE, assuming that both layers share the same normal stress, so the horizontal Young's modulus $E_3$ can be obtained:

$$E_3 = \frac{1}{(1-c_p) + c_p (E_m/E_p)}.$$

(13)

Poisson coefficient $\nu$ affects the axial deformation when the material suffers from a uniaxial stretch. Assuming that the material is stretched along the direction which is perpendicular to the $y$ axis, the total deformation of the elementary cell comes from the sum of vertical deformation of two elementary layers. Hence, the Poisson’s ratio is derived as:

$$\nu_{21} = \nu_{23} = \nu_3 = (1 - c_p)\nu_m + c_p\nu_p,$$

(14)

When determining the shear modulus, given that a shear is along the $x$-$y$, and the shear stress of reinforced layer and matrix layer are the same, the $x$-$y$ shear modulus is derived as:

$$\bar{G}_{12} = \frac{G_m\sigma_p}{(1-c_p)\sigma_p + G_m c_p}.$$ $$

(15)

When a shear is along the opposite side, the reinforced layer and matrix layer have the same shear strain. So, the $x$-$z$ shear modulus is derived as:

$$\bar{G}_{13} = \bar{G}_{23} = G_m (1 - c_p) + G_p c_p.$$ $$

(16)

3. FEM simulation

To further evaluate the theoretical model, the FEM is employed to obtain the elastic modulus of structured MRE. The model of a unit cell is shown as figure 3. When the composite suffers from the stretch along the $i$ direction, a fixed constraint is applied on $S_{efgh}$, $S_{dfgc}$ and $S_{adef}$, and the boundary condition on $S_{abcd}$ is a prescribed displacement in the $i$ direction, then we have:

$$\bar{E}_i = \frac{\bar{\varepsilon}_i}{\varepsilon_i} = \frac{\int \sigma_i d\nu}{\int \varepsilon_i d\nu},$$

$$\bar{\nu}_{ij} = \frac{\bar{\varepsilon}_{ij}}{\varepsilon_{ij}} = \frac{\int \varepsilon_{ij} d\nu}{\int \varepsilon_{ij} d\nu},$$

$$\bar{G}_{ij} = \frac{\bar{\sigma}_{ij}}{2\bar{\varepsilon}_{ij}} = \frac{\int \sigma_{ij} d\nu}{2\int \varepsilon_{ij} d\nu},$$

(17)-(19)

When the composite suffers a shear force on the $i$-$j$ plane, the equivalent shear modulus of MRE satisfies equation (19). In summary, by varying the volume fraction and layer distribution in the finite element model of composite element, the stress and strain of composite element model in different working conditions are acquired. Hence the equivalent elastic modulus is derived by introducing equations (17)-(19).
Figure 3. Finite element model of composite cell (w/l = 1), where the particles are aligned in layer-like form and cubic model represents a micro part of the material, and thus derives the equivalent elastic modulus of the MRE (a) Model (b) Von misses stress of model’s cross section.

4. Results
The relation between particles proportion and anisotropy of MRE in layer-like structure is studied, with the results from FEM and theoretical calculation. Table 1 reveals the simulation parameters while figure 4 demonstrates the results of FEM simulation and model calculations.

Figure 4. Comparison between Model and FEM results (a) Relative elastic modulus (b) Relative shear modulus (c) Relative difference of elastic modulus in x and y directions.
Table 1. Simulation parameter.

| Parameter                  | Value | Parameter                  | Value (μm) |
|----------------------------|-------|----------------------------|------------|
| Particle Young's modulus   | 200 (Gpa) | Reinforced layer width     | 1.5        |
| $\nu_p$                    | 0.29  | Matrix layer width $w$     | [0-1.5]    |
| Matrix Young's modulus     | 1 (Mpa) | Spheres radius $R_0$     | [0.3-1.3]  |
| $\nu_m$                    | 0.49  | Particle spacing $l_0$     | 1.5        |

Figure 4(a) shows that $E/E_m$ varies with the particles volume fraction. Both of $E_1$, $E_2$ increase with the particles volume fraction. Due to the present theoretical model does not consider the interaction among neighboring particles, the theoretical solution for equivalent Young's modulus is lower than the FEM simulation results. However, both methods show the equivalent Young's modulus changing with particle volume fraction in the same trend. Figure 4(b) describes the relationship between shear modulus and particles volume fraction. The shear modulus of $G_{12}$, $G_{13}$ has the same increasing trend with elastic modulus. In the shearing condition, the space between particles are fixed, which reduces the interaction among particles. Hence, the theoretical solutions for shear modulus well agree with the FEM solution.

Figure 4(c) illustrates that the difference between the Young's modulus $E_1$ in the $x$ direction and the Young's modulus $E_2$ in the $y$ direction. The result indicates that the elastic modulus along the reinforced direction increases significantly, as the particles proportion increases.

5. Conclusions

- In the same particle volume fraction, structured MRE has an enhanced Young's modulus along the direction of axis $x$, and the Young's modulus decreases along the $y$ axis.
- As the particle percentage increases, the anisotropy of MRE becomes more noticeable.
- Solutions of equivalent shear modulus match well with the finite element solutions. Both solutions of equivalent Young's modulus have the same trend, but the errors still exist and the theoretical solutions are slightly lower.

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