Analysis of Multivariate Scoring Functions for Automatic Unbiased Learning to Rank

Tao Yang  
Shikai Fang  
Shibo Li  
Yulan Wang  
Qingyao Ai

University of Utah  
University of Utah  
University of Utah  
University of Utah  
University of Utah

Salt Lake City, Utah  
Salt Lake City, Utah  
Salt Lake City, Utah  
Salt Lake City, Utah  
Salt Lake City, Utah

ABSTRACT
Leveraging biased click data for optimizing learning to rank systems has been a popular approach in information retrieval. Because click data is often noisy and biased, a variety of methods have been proposed to construct unbiased learning to rank (ULTR) algorithms for the learning of unbiased ranking models. Among them, automatic unbiased learning to rank (AutoULTR) algorithms that jointly learn user bias models (i.e., propensity models) with unbiased rankers have received a lot of attention due to their superior performance and low deployment cost in practice. Despite their differences in theories and algorithm design, existing studies on ULTR usually use uni-variate ranking functions to score each document or result independently. On the other hand, recent advances in context-aware learning-to-rank models have shown that multivariate scoring functions, which read multiple documents together and predict their ranking scores jointly, are more powerful than uni-variate ranking functions in ranking tasks with human-annotated relevance labels. Whether such superior performance would hold in ULTR with noisy data, however, is mostly unknown. In this paper, we investigate existing multivariate scoring functions and AutoULTR algorithms in theory and prove that permutation invariance is a crucial factor that determines whether a context-aware learning-to-rank model could be applied to existing AutoULTR framework. Our experiments with synthetic clicks on two large-scale benchmark datasets show that AutoULTR models with permutation-invariant multivariate scoring functions significantly outperform those with uni-variate scoring functions and permutation-variant multivariate scoring functions.

CCS CONCEPTS
• Information systems → Learning to rank.

KEYWORDS
Unbiased learning to rank; Multivariate Scoring Function

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1 INTRODUCTION
The study of learning to rank with implicit user feedback such as click data has been extensively studied in both academia and industry. Usually, learning to rank directly with implicit user feedback would suffer from the intrinsic noise and propensity in user interaction (e.g., position bias). In recent years, many algorithms have been proposed to find the best model that ranks query-document pairs according to their intrinsic relevance. Among them, unbiased learning to rank (ULTR) algorithms that automatically estimate click bias and construct unbiased ranking models, namely the AutoULTR algorithms, have drawn a lot of attention [2, 7]. Because they do not need to conduct separate user studies or online experiments to estimate user bias (i.e., the propensity models), most AutoULTR algorithms can easily be deployed on existing IR systems without hurting user experiences.

Despite their differences in background theories and algorithm design, previous studies on ULTR usually use uni-variate learning-to-rank models, which score each document independently, for experiments and theoretical analysis. Recently, multivariate scoring functions, which take multiple documents as input and jointly predict their ranking scores, have been proved to be more effective than uni-variate scoring function in many learning-to-rank problems [1, 6]. By modeling and comparing multiple documents together, multivariate scoring functions naturally capture the local context information and produce state-of-the-art performances on many learning-to-rank benchmarks with human-annotated relevance labels. Whether the superior performance of multivariate scoring functions would hold in unbiased learning to rank is still unknown.

To leverage the full power of click data and multivariate scoring functions, we explore the potential of multivariate scoring functions in AutoULTR. Specifically, we investigate the compatibility
### 2 RELATED WORK

AutoULTR, which has the advantage of estimating propensity and relevance simultaneously, has drawn much attention recently. For example, Ai et al. [2] proposed Dual Learning Algorithm (DLA) based on counterfactual learning, and Wang et al. [7] proposed a regression-based EM algorithm. On the other hand, for multivariate scoring functions, there are several ways to take multiple documents as input. For example, DLCM [1] employs a RNN model to give score, SetRank [6] utilizes self-attention mechanism. To the best of our knowledge, however, research on multivariate scoring functions in AutoULTR has not been fully explored, which is exactly the focus of this paper.

### 3 PROBLEM FORMULATION

In this section, we investigate existing AutoULTR algorithms in theory and prove that permutation invariance is a sufficient and necessary condition that determines whether multivariate scoring functions could be applied to existing AutoULTR algorithms. A summary of the notations used in this paper is shown in Table 1.

#### 3.1 Univariate and Multivariate Scoring

In a standard learning-to-rank system, given a specific query $q$ and its associated retrieved document set $D = \{d_1, d_2, \ldots, d_N\}$, a vector $x_i \in \mathbb{R}^H$ can be extracted and used as the feature representation for $d_i$. Let $\pi_q$ be the ranked list for query $q$. Then there will be a feature matrix for a ranked list $\pi_q$:

$$X = [x_1, x_2, \ldots, x_N]^T, \quad x_i \in \mathbb{R}^{N \times H}$$

Then, univariate ranking function $f_q(x)$ and multivariate scoring function $G_{\phi}(X)$ for a ranked list can be defined as:

$$f_q(x) = [f_q(x_1), f_q(x_2), \ldots, f_q(x_N)]$$

$$G_{\phi}(X) = [G_{\phi}^1(X), G_{\phi}^2(X), \ldots, G_{\phi}^N(X)]$$

where $\phi$ is the parameter. The main difference is that multivariate scoring functions take the whole list as input, while univariate scoring functions only score one document a time.

#### 3.2 AutoULTR Framework

In this paper, we adopt DLA [2], an AutoULTR framework which treats the problem of learning a propensity model from click data (i.e., the estimation of bias in clicks) as a dual problem of constructing an unbiased learning-to-rank model. Formally, let $a_q$, $r_q$ and $c_q$ be the sets of Bernoulli variables that represent whether a document in $\pi_q$ is observed, perceived as relevant, and clicked by a user, respectively. In DLA, an unbiased ranking system $S$ and a propensity model $E$ can be jointly learned by optimizing the local AttRank losses [1] as

$$l(E, q) = - \sum_{i=1}^{[\pi_q]} a_q^i \times \log G_{\phi}^i(\pi_q)$$

$$l(S, q) = - \sum_{i=1}^{[\pi_q]} r_q^i \times \log F_{\theta}^i(X_q)$$

$$\sum_{i=1}^{[\pi_q]} G_{\phi}^i(\pi_q) = 1, \quad \sum_{i=1}^{[\pi_q]} F_{\theta}^i(X_q) = 1$$

where $G_{\phi}$ and $F_{\theta}$, parameterized by $\phi$ and $\theta$, compute the propensity scores and relevance scores of each document in the ranked list with a softmax function constrained by Eq (2). To compute $l(S, q)$ and $l(E, q)$, we need to know the actual relevance (i.e. $r_q^i$) and observation (i.e., $a_q^i$) information of each document. However, in practice, the only data we can get is click (i.e. $c_q$), and $r_q$ and $a_q$ are unknown latent variables. In order to deal with clicks, a common assumption used by most studies is

$$P(c_q^i = 1) = P(a_q^i = 1)P(r_q^i = 1)$$

which means that users click a search result ($c_q^i = 1$) only when it is both observed ($a_q^i = 1$) and perceived as relevant ($r_q^i = 1$), and $a_q$ and $r_q$ are independent to each other. With this assumption, unbiased estimation of $l(S, q)$ and $l(E, q)$ can be achieved through inverse propensity weighting (IPW) [4] and inverse relevance weighting (IRW) [2] as

$$\tilde{l}_{\text{IRW}}(E, q) = - \sum_{i=1}^{[\pi_q]} F_{\theta}^i(X_q) \times \log G_{\phi}^i(\pi_q)$$

$$\tilde{l}_{\text{IPW}}(S, q) = - \sum_{i=1}^{[\pi_q]} G_{\phi}^i(\pi_q) \times \log F_{\theta}^i(X_q)$$

$$\mathbb{E}_{q}[\tilde{l}_{\text{IRW}}(E, q)] \Delta l(E, q), \mathbb{E}_{q}[\tilde{l}_{\text{IPW}}(S, q)] \Delta l(S, q)$$

where $\Delta$ means equal or linearly correlated with a positive constant factor. Then the final optimization losses in DLA are

$$\tilde{L}(S) = \sum_{q \in Q} \tilde{l}_{\text{IPW}}(S, q), \quad \tilde{L}(E) = \sum_{q \in Q} \tilde{l}_{\text{IRW}}(E, q)$$

where $Q$ is the set of all possible queries. During training, we update $\theta$ and $\phi$ with the derivatives of $\tilde{l}_{\text{IPW}}(S, q)$ and $\tilde{l}_{\text{IRW}}(E, q)$ respectively and repeat the process until the algorithm converges.

#### 3.3 Convergence Analysis

In this section, we analyze the compatibility of multivariate scoring functions and DLA in theory. Firstly, we give definition of permutation invariance as:

### Table 1: A summary of notations.

| $Q$, $q$ | All possible query $Q$ and a query instance $q \sim P(q)$. |
|----------|----------------------------------------------------------|
| $S$, $F$, $\theta$, $E$, $G$, $\phi$ | A multivariate relevance estimation function $F$ parameterized by $\theta$ for ranking system $S$ and a propensity estimation function $G$ parameterized by $\phi$ for propensity model $E$. |
| $l$, $I$, $\mathcal{L}$ | $l$ is local loss, while $I$ is unbiased estimation of $l$ and $\mathcal{L}$ is the unbiased estimation of global loss. |
| $\pi_q$, $d_i$, $i$, $x_i$, $X$, $y$ | A ranked list $\pi_q$ produced by $S$ for $q$, a document $d_i$ with features $x_i$ on the $i$-th position in $\pi_q$ and its relevance $y$. $X$ is the feature matrix for whole $\pi_q$. |
| $a_q$, $r_q$, $c_q$ | Bernoulli variables that represent whether a document is observed ($a_q$), perceived as relevant ($r_q$) and clicked ($c_q$). |

[Raw text content is not shown, as it is not essential to the translation process.]
Definition 3.1. Let $S_n$ be the set of all permutations of indices $\{1, \ldots, n\}$. A function $f: X^n \rightarrow Y^n$ is permutation invariant [5] if for any permutation function $\Pi$, $f(\Pi(X)) = \Pi(f(X))$, i.e.,

$$f(x_{\Pi(1)}, \ldots, x_{\Pi(n)}) = [f^{\Pi(1)}(X), \ldots, f^{\Pi(n)}(X)]$$

where $X = [x_1, \ldots, x_n]$, $f^{\Pi(i)}(X)$ is the $i$-th dimension of $f(X)$.

For simplicity, we consider position bias [3] as the only bias in click data. Then we have $G^\phi_{o} = G^\phi_{o}(\pi_q)$, which means propensity is independent of query. Let $\Pi(i) = j$ mean putting $x_i$ on $i$-th position of permutated matrix $\Pi(X)$. In theory, DLA will converge when

$$\frac{\partial \tilde{L}(E)}{\partial G^\phi_{o}} = 0 \implies \frac{\partial^2}{\partial G^\phi_{o}^2} = \frac{\mathbb{E}[c_q^1]}{\mathbb{E}[c_q^2]} = \frac{\mathbb{E}[r_q^1]}{\mathbb{E}[r_q^2]} = \frac{\mathbb{E}[o^1]}{\mathbb{E}[o^2]}$$

(6)

Considering any permutation function $\Pi$, the original $i$-th document in $\pi_q$ is in the $\Pi^{-1}(i)$-th document in the permuted list, where $\Pi^{-1}$ is the inverse function of $\Pi$. Note that in the following analysis, the default ranking of documents is original ranking $\pi_q$ shown to users if not explicitly pointed out. For a permutated ranking, we have

$$G^\Pi_{o}^{-1}(i) = \frac{\mathbb{E}[\Pi^{-1}(\pi_{\Pi(i)})]}{\mathbb{E}[\Pi^{-1}(\pi_q)]} \mathbb{E}[o^1],$$

(7)

(3.3.1) Necessary Condition. When DLA converges and propensity is correctly estimated, we have

$$\forall \Pi, \forall i, \frac{G^\Pi_{\phi}^{-1}(i)}{G^\Pi_{\phi}^{-1}(i)} = \frac{\mathbb{E}[o^1]}{\mathbb{E}[o^2]},$$

(8)

Assuming that the relevance of a document $d_i$ would not change after moving to a different position, then we have

$$\Pi(r_q^{-1}(\Pi^{-1}(i)) = r_q^i$$

(9)

Considering Equations (2) and (6) to (9), we can get

$$\frac{\mathcal{F}_{\phi}^1(X)}{\mathcal{F}_{\phi}^j(X)} = \frac{\mathcal{F}_{\phi}^{\Pi^{-1}(i)}(\Pi(X))}{\mathcal{F}_{\phi}^{\Pi^{-1}(j)}(\Pi(X))} \implies \mathcal{F}_{\phi}^j(X) = \mathcal{F}_{\phi}^{\Pi^{-1}(i)}(\Pi(X))$$

(10)

Then, we insert $i = \Pi(j)$ in Eq.10, and we have

$$\forall \Pi, \forall j, \mathcal{F}_{\phi}^j(\Pi(X)) = \mathcal{F}_{\phi}^{\Pi^{-1}(j)}(X)$$

(11)

which means that $\mathcal{F}_{\phi}$ is permutation invariant according to Definition 3.1. This indicates that permutation invariance is a necessary condition for the convergence of DLA.

3.3.2 Sufficient Condition. Suppose that $\mathcal{F}_{\phi}$ is permutation invariant, then the estimated relevance score for a document from $\mathcal{F}_{\phi}$ is independent of its position. Because $G_{\phi}$ only takes the positions as input, $\mathcal{F}_{\phi}$ and $G_{\phi}$ are independent to each other and can separately estimate the relevance and propensity during training. As proven by Ai et al. [2], DLA is guaranteed to converge in this case, which means that permutation invariance can be a sufficient condition for the convergence of DLA.

4 EXPERIMENT

So far, we have proven that permutation invariance is the sufficient and necessary condition for learning-to-rank models to converge in DLA in theory. In this section, we describe our experiments on two large-scale benchmarks for further demonstrations.

4.1 Simulation Experiment Settings

4.1.1 Datasets and Click Simulation. To fully explore the performance of multivariate scoring functions in AutoULTR, we conducted experiments on Yahoo! LETOR set $^1$ and Istella-S LETOR $^2$ with derived click data. Similar to previous studies [4], we trained a SVMrank model $^3$ (which we refer to as Prod) using 1% of the training data with real relevance judgements to generate the original ranked list $\pi_q$. We then sampled clicks ($c_q^q$) on documents according to Eq. (3) with $o_q^i$, and $r_q^i$ as,

$$P(o_q^i = 1|\pi_q) = P(r_q^i = 1) = \rho_i$$

$$P(r_q^i = 1|\pi_q) = \epsilon + (1 - \epsilon) \frac{2y_{max} - 1}{2y_{max} - 1}$$

where $\rho$ is acquired through eye-tracking experiments [3], $\epsilon \in [0, 4]$ is the 5-level relevance label in both datasets where $y_{max} = 4$, and $\epsilon$ is used to model click noise so that irrelevant documents ($y = 0$) can also be clicked. For simplicity, we fixed the value of $\epsilon$ as $0.1$.

4.1.2 Models and Evaluation Measures. In this paper, we focus on two state-of-art multivariate scoring functions. The first one is DLCM$^1$, a RNN model with gated recurrent unit (GRU) that treats the final network state as context to score each document. It is permutation variant by nature. The second one is SetRank$^6$, constructed with multi-head self-attention networks, which is permutation invariant. For DLMC, we adopt three kinds of input orders, namely the original ranking of documents (i.e., DLCMinit$^i$) created by Prod., the reverse of the original ranking (i.e., DLCMrev$^r$), and a random permutation of the original ranking (i.e., DLCMrand$^r$). For comparison, we include a uni-variate scoring function based on deep neural networks (DNN) as our baseline. We also include a DNN model that directly use clicks as relevance labels, which is referred to as DNNnav$^i$. The source code can be found here.$^4$

All models were tuned and selected based on their performances on the validation set according to nDCCG@10. Each model was trained for five times and reported the mean performance. The batch size was set to be 64. Learning rate was tuned between 0.1 and 0.01, and we stopped training after 60k steps. During training, we set the size of ranked list as 10, while during validating and testing, we tested our ranking model on all documents to each query. We reported both ERR and nDCG metrics at ranks of 3 and 10 to show ranking performance. Besides, in order to show performance of propensity estimation, we computed the mean square error (MSE) between the true inverse propensity weights ($\rho_1/\rho_i$) and the estimated inverse propensity weights ($\hat{G}^1/\hat{G}^1$) as

$$\text{MSE}_{\text{propen}} = \frac{1}{|\pi_q|} \sum_{i=1}^{|\pi_q|} \left(\frac{\hat{G}^1_i}{\hat{G}^1} - \frac{\rho_i}{\rho_i}\right)^2$$

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$^1$https://webscope.sandbox.yahoo.com
$^2$http://blog.istella.it/istella-learning-to-rank-dataset/
$^3$http://www.cs.cornell.edu/people/tj/svm_light/svm_rank.html
$^4$https://github.com/Taosheng-ty/CIKM_2020_Multivariate_AutoULTR.git
4.2.3 Comparison of DLCM with different input order. We noticed that DLCM$^\text{rand}$ could get a perfect estimation of the propensity but failed to get ranking performance as good as SetRank and DNN. We think the reason might be that random input order makes permutation variant models hard to remember any pattern in order, thus empirically achieve permutation invariance. As for ranking performance, we can interpret it from the RNN structure. In DLCM, the final input has the most impact on the final network state, which is viewed as context to help score each document. Random input sequence results in a random context which would make DLCM$^\text{rand}$ hard to score documents. This could also explain the bad performance of DLCM$^\text{reverse}$ as it always takes documents in the reverse order of the original ranking produced by Prod.

5 CONCLUSION AND FUTURE WORK

In this work, we explore the potential of multivariate scoring functions in AutoULTR. Based on existing AutoULTR algorithms, we prove that permutation invariance is a crucial factor for a multivariate scoring function to be included in AutoULTR. With two existing multivariate functions and one AutoULTR algorithm, we conduct experiments based on two benchmark datasets, the results of which align with our theoretical analysis. AutoULTR models with permutation-invariant multivariate scoring functions significantly outperform those with uni-variate scoring functions and permutation-variant multivariate scoring functions. Our work represents an initial attempt to include multivariate scoring in AutoULTR. In the future, we may base on our analysis to propose novel multivariate scoring functions for AutoULTR.

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