Mode Frequencies from 17, 15 and 2 Years of GONG, MDI, and HMI Data

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Abstract. We present results from fitting all the available data to date from the three major helioseismic instruments: MDI, GONG and HMI. These data were fitted using an innovative and independent methodology devised a few years ago. The mode fitting was carried out on time series of varying lengths (1×, 2×, 4×, 8×, 16×, 32×, 64×72 day-long), using co-eval epochs for all three data sets. By fitting time series of varying lengths, one trades off some temporal resolution for a better precision. We present a comparison of these results, and discuss the potential sources of the residual small discrepancies. We also present inferences from these results on the determination of the solar internal rotation and changes with epoch and thus activity levels.

1. Introduction

We present results from fitting all the available data sets acquired to date by the GONG, MDI and HMI instruments, namely: 171×36 days of GONG, 76×72 days of MDI and 10×72 days of HMI data¹. The time series of spherical harmonic coefficients for all three instruments were fitted using the same procedure, described in Korzennik [3, 4]. This method fits all the individual singlets (n,ℓ,m), for a given multiplet (n,ℓ). It uses an optimal sine-multitaper spectral estimator, the complete leakage matrix, and fits an asymmetric mode profile. The spherical harmonic coefficients for MDI we used are the ones computed using an improved spatial decomposition, i.e., the one that includes our best knowledge of both components of the image distortion. The spherical harmonic coefficients for HMI we used were still preliminary values.

As a trade-off between precision and temporal resolution, we fitted time series of different lengths instead of the canonical 72 or 108 days used by either the MDI or GONG projects, respectively. Instead, we fitted 1×, 2×, 4×, 8×, 16×, 32×, and 64×72 day long time series. For all but the 1×72 day-long ones, the epochs we fitted overlap by half their length. Figure 1 illustrates the temporal coverage of the available data, the length of the fitted segments and the corresponding mean solar activity level.

To investigate the influence of the leakage matrix on the results, we also fitted some time series using different leakage matrices: one computed by Schou for \( B_o = 0 \), one computed by Korzennik also for \( B_o = 0 \), and one computed by Korzennik for \( B_o = \bar{B}_o \) (i.e., the mean heliographic latitude at disk center, \( B_o \), over the time series of each epoch).

¹ Two 72 day-long MDI epochs are missing due to the loss of contact with the SOHO spacecraft.
This paper presents comparisons of the resulting fitted frequencies and the solar internal rotation rate inferred from the frequency splittings. These are comparisons of results obtained with the same fitting method and for identical time intervals, by contrast to comparing table of frequencies produced by the respective projects where neither the fitting method nor the time interval are similar.

2. Comparisons

Figure 2 compares GONG results to MDI’s, complemented by HMI's, in terms of scaled changes of the multiplet frequencies, \( \delta \nu / \sigma \nu \), as a function of time and for epochs of different lengths. The frequency changes with activity, as measured by both sets of observations, are quite similar, but not identical.

Figure 3 compares scaled multiplet differences between GONG and MDI results, as a function of epoch and for epochs of different lengths. The upgrade of the GONG instrument, during the summer of 2001, from a 256 \( \times \) 242 to a 1024 \( \times \) 1024 pixels CCD camera, is visible in the frequency differences. Frequency comparison, also in terms of scaled multiplet differences, for the 360 days of available overlap between MDI and HMI data is shown in Fig. 4. This comparison shows no systematics, but differences at the 1 to 2 \( \sigma \) level.

The effects of the leakage matrix are illustrated in Figs. 5 and 6. the scaled differences for the singlets are plotted, as well as the corresponding distribution. The comparison between two independent leakage matrix calculations shows almost no bias, but systematic differences for some ridges, as large as 2\( \sigma \). The inclusion of the mean heliographic latitude at disk center is barely affecting the low and intermediate degrees results (\( i.e., \) remaining mostly at or below the 0.1\( \sigma \) level).
Figure 2. Scaled changes of multiplet frequencies, \((\text{i.e., } \delta \nu / \sigma_{\nu})\), as a function of time, and for fitting epochs of different lengths (1, 2, 4, and 8 × 72 day-long, top to bottom), between fitting GONG data (left) and MDI augmented by HMI data (right). The scaled differences are computed with respect to the first fitted epoch, and averaged over bins equispaced in \(\log(\nu/L)\).
Figure 3. Scaled differences of multiplet frequencies, \( \delta \nu / \sigma_\nu \), as a function of time, and for fitting epochs of different lengths (1, 2, 4, 8 \times 72 day-long), using GONG data or MDI data. The horizontal dash line corresponds to the summer of 2001, when the GONG instrument was upgraded from a 256 \times 242 to a 1024 \times 1024 pixels CCD camera. As for Fig. 2, The scaled differences are averaged over bins equispaced in \( \log(\nu/L) \).

3. Variation of the Solar Rotation
We inverted the resulting rotational frequency splittings for all the data and all the fitted epochs, using the inversion methodology described in [1, 2]. Note that this method uses the actual frequency splittings, not the coefficients of a polynomial expansion in \( m \) of the splittings. This method optimizes the model grid used for the inversion based on the precision of the inverted data set.

Figure 7 shows the resulting variation of the solar rotation, at four depths spanning most of the convection as a function of time and latitude, derived from fitting epochs of different lengths and either GONG data or MDI augmented by HMI data. The figure also shows the difference of the inferred rotation between the two data sets.Animations of the resulting rotation changes as a function of depth or time, and for other lengths of fitted time series, are available in the supplementary material.

4. Conclusions
We have derived sets of frequencies\(^2\), using a single fitting methodology and identical epochs, that are consistent across the three instruments (GONG, MDI and HMI). Detailed comparisons show: (a) a systematic offset at the 1\( \sigma \) level (relatively high) between GONG and MDI that corresponds to the upgrade of the GONG instrument; (b) no systematic differences when comparing MDI

\(^2\) Tables of frequencies are available at \texttt{www.cfa.harvard.edu/~sylvain/research/tables/MediumL/}. 
Figure 4. Scaled differences of multiplet frequencies, \((i.e., \frac{\delta \nu}{\sigma_{\nu}})\), between MDI and HMI data, as a function of time, and for 1 & 2 × 72 day-long fitting epochs. The left panels show averaged scaled differences, averaged over bins equispaced in \(\log(\nu/L)\), as in Figs. 2 and 3. The right panels show the distributions of the differences.

Figure 5. Scaled differences of singlet frequencies, \((i.e., \frac{\delta \nu}{\sigma_{\nu}})\), when fitting a 64 × 72 day-long time series but using two independent leakage matrix calculations. The color coding corresponds to the mode’s order, \(n\). This comparison shows almost no bias, but systematic differences for some ridges, as large as 2\(\sigma\).
Figure 6. Scaled differences of singlet frequencies, \(i.e., \frac{\delta \nu}{\sigma_{\nu}}\), when fitting a 2 \(\times\) 72 day-long time series but using leakage matrix calculations that account for the mean heliographic latitude at disk center.

and HMI results, except at the 2\(\sigma\) level (again surprisingly high); and (c) that the effect of using different leaks results in small effects, at the 0.1\(\sigma\) to 2\(\sigma\) level, when comparing singlets, with almost no bias.

Our rotation inversions show how the high latitude branch becomes more visible and significant when using longer time series, while differences of the inferred rotation using different instruments illustrate the difficulty of deriving significant information on potential changes at the deeper layers.

References
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$\Omega(t, \theta) - \bar{\Omega}, \text{at}\ r/R_{\odot} = 1$

$\Omega(t, \theta) - \bar{\Omega}, \text{at}\ r/R_{\odot} = 0.98$

$\Omega(t, \theta) - \bar{\Omega}, \text{at}\ r/R_{\odot} = 0.87$

$\Omega(t, \theta) - \bar{\Omega}, \text{at}\ r/R_{\odot} = 0.71$

**Figure 7.** Variation of the solar rotation as a function of time and latitude, inferred from fitting epochs of different lengths (rows in each box: 1×, 4× & 16×72 day-long) at four depths (each box: $r/R_{\odot} = 1, 0.98, 0.87, 0.71$). The columns in each box show results from GONG, MDI augmented by HMI data, and their difference (left to right). The residual variations of the differences are an indication of the remaining uncertainties of these inferences, especially for the deeper layers.