Chiral Ball and Its Omnidirectional Circularly-Polarized Radiation

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Chiral structures have reported radiation of circular polarized electromagnetic waves (CPs) in a specific direction. Here we report a class of torus knot radiators that is not only chiral but also three-dimensional (3-D) rotational symmetric along X, Y and Z axes. Because of this exotic chirality and symmetry, the knot radiator presented is able to demonstrate omnidirectional circular polarized radiation, which has never been reported in any known structures.

1. INTRODUCTION

Symmetry is one of the universal and possible the most important properties of nature, art and science. In physics, geometrical symmetry is associated with physical conservation, which is one of the most powerful tools of theoretical physics, as it has become evident that the laws of nature originate from symmetries [1,2]. Chirality is a class of asymmetry with the absence of reflection symmetry. It widely exists in nature ranging from chemical molecules [3] to biological organisms [4]. Some two-dimensional (2-D) chiral patterns, for example, a shape like the letter “S”, possess not only chirality, but also rotational symmetry. However, their three-dimensional (3-D) counterparts, namely “chiral balls”, that possess both chirality and 3-D rotational symmetry along every axis of a Cartesian coordinate, has not yet been observed. One application of structures that possess this unique symmetry is polarization control. Chiral structures have been reported to radiate circular polarized waves (CPs). However, most chiral structures cannot generate CPs for every direction. For example, helix structures have no rotational symmetry, their chiroptical responses are dependent on their orientation, as shown in Fig. 1(A). If we could design a structure that is not only chiral but also 3-D rotational symmetric, it may generate omnidirectional circular polarized radiation, as depicted in Fig. 1 (B).

To generate omnidirectional CP radiation knotted wire structures can be a potential candidate for its combination of chirality and rotational symmetry. A (p,q) torus knot lies on the surface of a circular or elliptical torus [5,6]. Scaling a torus knot in either coordinate axis will not alter its symmetry.

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\begin{align*}
  x &= S_x \cdot (R + r \cdot \cos(q \cdot t)) \cdot \cos(p \cdot t) \\
  y &= S_y \cdot (R + r \cdot \cos(q \cdot t)) \cdot \sin(p \cdot t) \\
  z &= S_z \cdot r \cdot \sin(q \cdot t)
\end{align*}
\]

Fig. 1. The comparison of the scattering field of (a) a segment of helix; and (b) the proposed chiral ball. A helix radiates CPs in only one direction, while a chiral ball radiates CPs in almost every direction.

A torus knot is chiral, as well as q-fold rotationally symmetric on the azimuth plane. In [7], a (2,5) knot antenna generated conical CPs only at $\theta = 30^\circ$. A knot meta-molecule was found capable to rotate the polarization angle of a linear polarized wave, independent of the incident polarization direction [8]. Those works demonstrated that a torus knot can show $\phi -$ independent radiation ($\theta = 30^\circ$ in [7], and $\theta = 0^\circ$ in [8]). But in order to find an omnidirectional circular polarized radiator that is independent with not only $\phi$ but also $\theta$, a structure with higher order symmetry has to be introduced.
Namely, its symmetry should not only be on the azimuth plane \( (\varphi \text{ independent}) \), but also on the elevation plane \( (\theta \text{ independent}) \).

To be rotationally symmetric also about the \( X \) - and \( Y \) - axes (on the elevation plane), the parameter \( q \) must be even. Therefore, we reveal that the simplest torus knot with the desired symmetry is a \((3, 2)\) knot. Fig. 2 (a) depicts the perspective view of the knot chiral ball. Fig. 2 (b), (c) and (d) are the three views from \( X \), \( Y \), and \( Z \) axis, respectively. The knot is evidently chiral for its absence of reflection symmetry. Moreover, it is two-fold rotational symmetric around every axis in a Cartesian coordinate.

For experimental validation, we excite the chiral ball using an EM source. But the introduction of source will break the unique symmetry of chiral ball. To minimize the effect of source towards symmetry, we apply it on the non-radiant direction \((-Z)\) of the chiral ball. A \( S \)-shaped feeding network was designed to excite the working mode 1 at 580 MHz (S2). The chiral ball was fabricated in aluminum by employing a Selective Laser Melting (SLM) technique. The antenna was mounted on an automatic turret and measured in a chamber, as shown in Fig. 4 (a). Fig. 4 (b) and (c) depict the LCP/RCP radiation pattern of the chiral ball antenna in position on \( X-Y \), and \( Y-Z \) planes, respectively. As can be seen, the antenna has omnidirectional RCP radiation with radiation nulls in \( \pm Z \) direction, which matches well with the characteristic mode analysis. From Fig. 4 (b), we can observe its RCP radiation feature is \( \varphi \) - independent at \( \theta = 90^\circ \). Fig. 4 (b) plots the reflection of the antenna. Then we rotate the antenna to study its \( \theta \) - dependency. Fig. 4 (e) shows the axial ratio plot at the ultrawide elevation angles (100° out of 180°) of the chiral ball antenna. It cannot be 360° partly because of the theoretical limit of radiation nulls, and partly because the excitation breaks the symmetry of the chiral ball. But even though, the chiral ball structure demonstrates an omnidirectional CP radiation.
Knots have been put forward to explain various physical phenomena because of their topological stability. Nevertheless, few works have reported on the exotic symmetry that knots exhibit. Here we discover an extraordinary form of symmetry for a type of knot that is both chiral and 3-D rotationally symmetric around every axis of a Cartesian coordinate, namely a chiral ball. To shed some light on the unusual physical characteristics of this structure, we study the electromagnetic features of the conductive chiral ball. With its unique symmetry, the chiral ball demonstrates omnidirectional CP radiation, which has never been observed in any known electromagnetic structures. This work suggests that further investigation into the symmetry of knots is merited, as well as its potential relationship with various fundamental physical phenomena.

References
1. T.-D. Lee and C.-N. Yang, "Question of parity conservation in weak interactions," Physical Review 104, 254 (1956).
2. C.-S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, "Experimental test of parity conservation in beta decay," Physical review 105, 1413 (1957).
3. M. Y. B. Zion, X. He, C. C. Maass, R. Sha, N. C. Seeman, and P. M. Chaikin, "Self-assembled three-dimensional chiral colloidal architecture," Science 358, 633–636 (2017).
4. S. Armon, E. Efrati, R. Kupferman, and E. Sharon, "Geometry and Mechanics in the Opening of Chiral Seed Pods," Science 333, 1726–1730 (2011).
5. D. H. Werner, "Radiation and scattering from thin toroidally knotted wires," IEEE Transactions on Antennas and Propagation 47, 1351–1363 (1999).
6. D. H. Werner and R. Mittra, Frontiers in Electromagnetics (Wiley-IEEE Press, 2000), Vol. 2.
7. S. V. Kumar and A. R. Harish, "Generation of Circularly Polarized Conical Beam Pattern Using Torus Knot Antenna," IEEE Transactions on Antennas and Propagation 65, 5740–5746 (2017).
8. W. Mai, L. Kang, R. Jenkins, D. Zhu, C. Mao, P. L. Werner, Y. Chen, and D. H. Werner, "A knotted metamolecule with axisymmetric strong optical activity," Advanced Optical Materials 8, 2000948 (2020).
9. W. Scott and K. S. Hoo, "A theorem on the polarization of null-free antennas," IEEE Trans. Antennas Propagat. 14, 587–590 (1966).