Vortex Formation in a $U(1) \times U(1)' - N = 2 - D = 3$ Supersymmetric Gauge Model

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In this work, we study a $U(1) \times U(1)'$ model that results from a dimensional reduction of the $N=1$-$D=4$ supersymmetric version of the Cremer-Scherk-Kalb-Ramond model with non-minimal coupling to matter. Field truncations are not carried out, two Abelian symmetries coexist and three vector fields are present; two of them are gauge bosons. Then, by considering the full $N = 2 - D = 3$ supersymmetric model, we study the mechanism for magnetic vortex formation by means of the Bogomol’nyi relations, the magnetic flux and the topological charge in the presence of the two gauge potentials. A short discussion on the relation between our $N = 2 - D = 3$ supersymmetric model and vortices in superfluid films is also presented.

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I. INTRODUCTION

The study of symmetries in Physics is of crucial importance as a tool for the understanding and the description of the Elementary Particles and their process. The wide symmetry behind a Grand-Unified Theory (GUT) accommodates a large variety of phenomena in a single model. For instance, we have the Standard Model of the Elementary Particles (SM), where $SU(3) \otimes SU(2) \otimes U(1)$ describes three types of interactions. Other important invariances appear in connection with the Standard Model for Particle Physics. These symmetries are the Lorentz, CPT[1] and Supersymmetry (SUSY) [2] invariances. Despite the success of this model, there are already some important questions for which the SM has not provided a satisfactory explanation. One of these problems is the mass generation mechanism given by the Higgs boson, still lacking experimental evidences. Nowadays, this is of great motivation for detection in the Large Hadron Collider (LHC). The mass generation mechanism is based on a spontaneous symmetry breaking. Sometimes, this breaking is realised by a non-trivial vacuum configuration, so important in the formation of topological defects. In this non-trivial vacuum configuration, there may appear vortex configurations. The study of vortex configurations in a supersymmetric context is important due to the fact that supersymmetry (SUSY) is considered a fundamental symmetry in the early universe, where the vortex configuration[3–5] appears together with the symmetry breaking in GUT scenarios [4].

SUSY is a key ingredient in Superstring Theory, the Minimal Supersymmetric Standard Model and in connection with Neutrino Physics too. On the other hand, p-form potentials appear in many supersymmetric models. A 2-form field is referred to as the Kalb-Ramond field (KR) [6, 7]. In 4D, this 2-gauge form can be related with the real scalar field and can be important for the study the mediators particles of zero spin[8]. The relation between a scalar particle and a 2-form gauge potential is important to understand the spontaneous symmetry breaking carried out by KR field in a Goldstone Model. KR fields are also important in the study of the vortex superfluids[9, 10, 18]. Other aspect of the KR field is the study of its non-Abelian generalization that has not yet found a good formulation but can be associated with a non-linear chiral sigma-model [11] important to study the interaction between extended objects.

In this work, we wish to investigate the complete $N = 2 - D = 3$ gauge model with a $U(1) \times U(1)'$ symmetry. Similar models have recently been studied considering a non-Abelian Chern-Simons term and generic gauge groups [12], but without a KR field. In a previous work [14], the truncated $N = 2 - D = 3$ model including the KR field has been considered and the vortex configurations have been worked out. The truncation consisted in identifying fields that appear from the dimensional reduction of an $N = 1 - D = 4$ model, as studied in [13]. Here, we reconsider this model and discuss the full reduced model with two families of gauge potentials with a mixed Chern-Simons term and we focus on the analysis of vortex-type solutions in the presence of the second family of gauge fields. The outline of this paper is as follows: in Section 2, we present some considerations about the Cremer-Scherk-Kalb-Ramond (CSKR) model in the supersymmetric $N=2$-$D=3$ scenario. In Section 3, we devote our attention to showing the ingredients of the vortex magnetic configuration, we study the bosonic part of our SUSY model, the equations of motion and the critical coupling. In Section 4, we study the Bogomol’nyi equations and the minimal energy configuration of the vortex. Then, in Section 5, the relation between our $N = 2 - D = 3$ supersymmetric model with vor-
II. THE \( N = 2 - D = 3 \) SUSY MODEL WITHOUT TRUNCATION

In this section, we briefly review the \( N = 2 - D = 3 \) model that results from the dimensional reduction of the four-dimensional CSKR model\[14\]. This model descends from the \( N = 1 - D = 4 \) action that describes QED in the supersymmetric version coupled to the Kalb-Ramond field in a non-minimal way. This non-minimal coupling is unique. To see this, consider the pure Kalb-Ramond field in a non-minimal way. This non-minimal coupling from the 4-dimensional CSKR model\[14\]. This model descends as:

\[
S_{K-R} = \int d^3x \left\{ -\frac{1}{6} G_{\mu\nu\kappa} G^{\mu\nu\kappa} + J^{\mu\nu} B_{\mu\nu} \right\},
\]

where \( G_{\mu\nu\kappa} \) is the field-strength 3-form.

In momentum space, the field \( B_{\mu\nu} (k) \equiv \tilde{B}_{\mu\nu} \) can be expanded as follows:

\[
\tilde{B}_{\mu\nu} = \alpha k^\mu k^\nu + \beta I k^\mu e^\nu_I + \gamma I k^I e^\nu_I + \delta J_{IJ} e^\nu_I \equiv \tilde{B}_{\mu\nu},
\]

where the basis vectors are taken as below:

\[
k^\mu = \left( k^0, \bar{k} \right); \quad \bar{k}^\mu = \left( k^0, -\bar{k} \right);
\]

\[
e_I^\mu = (0, \bar{e}_I^\mu); \quad \bar{e}_I^\mu \cdot \bar{k} = 0, \text{com} I = 1, 2.
\]

With the help of the gauge symmetry for \( B_{\mu\nu} \), \( \tilde{B}_{\mu\nu} \) can be shown to acquire the form:

\[
\tilde{B}_{\mu\nu} = \delta_{IJ} e^I_J e^\nu_I.
\]

So, the equations of motion in momentum space read as:

\[
k^2 \delta_{IJ} e^I_J e^\nu_I = \tilde{J}^\nu_I, \quad n\epsilon^{ijk}k_k = \tilde{J}^\nu_i, \quad \text{where} \quad n = k^2 \delta_{IJ}.
\]

Equation (6) ensures that the current coupled to the Kalb-Ramond field is actually a topological current with the form:

\[
J^{\mu\nu} = \varepsilon^{\mu\nu\kappa\lambda} \partial_\kappa J_\lambda.
\]

This result denies the possibility of writing down a symmetry group associated with the conservation of \( J_{\mu\nu} \). In other words, the Yang-Mills version of the Kalb-Ramond model is not possible and this is actually shown as a no-go result in the work of [15].

Back to the \( N = 2 - D = 3 \) model \[16\], we write down the gauge-field sector of the bosonic action in components as:

\[
S_{\text{gauge}} = \int d^3x \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + 2m \varepsilon^{\mu\nu\alpha} A_\mu \partial_\nu B_\alpha - \frac{1}{2} G_{\mu\nu} G^{\mu\nu} \right\},
\]

where the index \( \mu = 0, 1, 2 \), with \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) being the electromagnetic field-strength. \( B_\mu \) is the vector given by the reduction of the 4-dimensional Kalb-Ramond field, \( B^{\mu\nu} \), with a corresponding field-strength \( G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \). Another vector field, the dual of \( B^{\mu\nu} \) in 3D, comes out which is defined by \( B^{\mu\nu} = \varepsilon^{\mu\nu\rho} Z_\rho \). Having in mind that, in \((1 + 2)D\), the Kalb-Ramond field-strength may be written as a scalar,

\[
G_{\mu\nu} = S \varepsilon_{\mu\nu\kappa},
\]

then

\[
\partial_\mu Z^\mu = \frac{1}{2} \varepsilon_{\mu\nu\kappa} \partial_\mu B_{\nu\kappa} = S.
\]

However, from the free field equations and the gauge transformation \( Z'_\mu = Z_\mu + \varepsilon_{\mu\nu\kappa} \delta^{\nu\rho} \xi^\rho \), \( S \) is shown to be a constant, so that \( B_{\mu\nu} \) does not correspond to a physical degree of freedom, unless it interacts with other fields.

The part of the \( N = 2 - D = 3 \) action involving the scalars is written as follows:

\[
S_{\text{scalar}} = \int d^3x \left\{ e^{-2gM} \nabla_\mu \varphi (\nabla^\mu \varphi)^* + P(\varphi) \partial_\mu M \partial^\mu M + \frac{1}{2} \partial_\mu N \partial^\mu N + 2mN (\partial_\mu Z^\mu) - g^2 (\partial_\mu Z^\mu)^2 |\varphi|^2 e^{-2gM} + (\partial_\mu Z^\mu)^2 \right\},
\]

where \( P(\varphi) = 1 - g^2 |\varphi|^2 e^{-2gM} \). The covariant derivative, \( \nabla_\mu \), is given by

\[
\nabla_\mu \varphi = (\partial_\mu + ihA_\mu + i\mathcal{G}_\mu) \varphi.
\]

\( M \) and \( N \) are real scalars. The dual fields, \( F_\mu \) and \( G_\mu \), are given by:

\[
F_\mu = \frac{1}{2} \varepsilon_{\mu\nu\kappa} F^{\nu\kappa}; \quad G_\mu = \frac{1}{2} \varepsilon_{\mu\nu\kappa} G^{\nu\kappa}.
\]

Adopting the parametrisations \( \phi = e^{-gM} \varphi \) and \( \partial_\mu Z^\mu = S \), we write down the remaining piece of the bosonic action, where the auxiliary field, \( \Delta \), is present and from which we can extract the potential of the model. We denote it by \( S_U \) and it is given by

\[
S_U = \int d^3x \left\{ hN|\phi|^2 + 2h\Delta|\phi|^2 + 2\Delta^2 - 4mM\Delta + \eta\Delta \right\}.
\]
The equation of motion for the auxiliary field yields
\[
\Delta = m M - \frac{h}{2} |\phi|^2 - \frac{\eta}{4}. \tag{15}
\]

Once it is eliminated, the potential for the physical scalars takes the form below:
\[
U = \frac{h^2}{2} \left( |\phi|^2 - \frac{2m}{h} M - v^2 \right)^2 + \left( h^2 N^2 + g^2 S^2 \right) |\phi|^2 - 2SmN - S^2, \tag{16}
\]
where \( v^2 = \frac{2m}{h} \). Once this potential has been built up, we are ready to discuss the symmetry-breaking pattern that yields the vortex formation.

III. CRITICAL COUPLING AND FIELD EQUATIONS

The equations of motion for the fields involved in our Lagrangian density are given below:
\[
\begin{align*}
\partial_\nu \left[ (1 - g^2 |\phi|^2) \left( \partial_\mu Z^\nu \right) + (m + gh |\phi|^2) N \right] &= 0, \quad \tag{17} \\
\left( \square + 2h^2 |\phi|^2 \right) N - 2 \partial_\mu Z^\mu (m + gh |\phi|^2) &= 0 \tag{18} \\
\partial_\mu F^{\mu\nu} + 2m G^\nu &= J^\nu, \quad \tag{19} \\
\partial_\mu G^{\mu\nu} + \frac{m}{2} F^{\mu\nu} &= \frac{g}{2h} e^{\mu\nu\rho} \partial_\rho J_\nu, \quad \tag{20}
\end{align*}
\]
where the current is \( J_\mu = ih (\phi^* \nabla_\mu \phi - \phi (\nabla_\mu \phi^*) \). We have three vector fields, two of them coupled by a Chern-Simons term, and the other one coupled to a scalar field. Despite this complicated mixing, Bogomoln’yi equations will be helpful to us understanding the role of each field in vortex formation.

Decoupling the Eqs. (19) and (20) from one another, we obtain
\[
\begin{align*}
\left( \square + m^2 \right) F^{\nu} &= e^{\mu\nu\rho} \partial_\rho J_\nu \left( \frac{gm}{h} + 1 \right) \tag{21} \\
\left( \square + m^2 \right) G^{\nu} &= \left( \square - \frac{hm}{g} \right) \left( -\frac{g}{2h} \right) J^\nu. \tag{22}
\end{align*}
\]
Using the critical coupling, \( g = -\frac{h}{m} \), in the previous two equations yields:
\[
\begin{align*}
\left( \square + m^2 \right) F^{\nu} &= 0, \quad \tag{23} \\
G^{\nu} &= \frac{1}{2m} J^\nu. \tag{24}
\end{align*}
\]
From Eqs. (23) and (24), we see that the \( A_\mu \)-field decouples from the scalar field \( \phi \). However, the \( G^\mu \)-field gives us
\[
e^{\nu\alpha\beta} \partial_\alpha B_\beta = \frac{1}{2m} J^\nu. \tag{25}
\]
The value of the critical coupling, \( g = -\frac{h}{m} \), reveals the purely topological character of the current, which shall be a relevant information in our analysis of the asymptotic behaviour of the field configurations.

IV. BP-STATES AND ASYMPTOTIC BEHAVIOUR

The explicit form of BPS-states can be worked out in a supersymmetric context. This result is explained in ref. [17]. Based on that work, we could define new supersymmetric generators as follows,
\[
Q_\pm = Q_\theta \mp i \gamma^0 Q_\tau, \tag{26}
\]
where \( Q_\theta \) e \( Q_\tau \) are the initially generators for the \( N = 2 \) supersymmetry. The generators (26) render manifest one of the results of Houkse and Spector [17],
\[
\{ Q_+, \overline{Q}_+ \} = 4 \gamma^0 (P_0 + Z); \quad \{ Q_-, \overline{Q}_- \} = 4 \gamma^0 (P_0 - Z), \tag{27}
\]
where \( Z \) it is a central charge of the extended supersymmetry.

Using these generators and setting to zero all fermionic variations, we can obtain BPS-states; however, here, we shall present another approach (more heuristic) that can be used also in the case of non supersymmetric models. To do so, we begin with the energy density of our model
\[
E = \int d^4x \left\{ \frac{1}{2} \left( \overline{F}^2 + B^2 \right) + P \left( \overline{G}^2 + B^2 \right) + \right. \\
+ \left. PS^2 + e^{-2gM} (D_0 \varphi)^* (D_0 \varphi) + e^{-2gM} (D_1 \varphi)^* (D_1 \varphi) + P (\partial_0 M)^2 + \right. \\
+ \left. P (\partial_1 M)^2 + \frac{1}{2} (\partial_0 N)^2 + \frac{1}{2} (\partial_1 N)^2 + \right. \\
+ \left. \left( 2mN + 2gh |\phi|^2 \right) S + U \right\}, \tag{28}
\]
where, contrary to the work of ref. [14], the second family of gauge potentials is not truncated. This is one of ours proposals: to understand the role of the \( U (1) \) factor and its corresponding gauge potential, \( B^\mu \), in the process of vortex formation.

Upon completion of squares,
\[
E = \int d^4x \left\{ \frac{1}{2} \left[ B \mp h \left( \frac{2m}{h} M - |\phi|^2 + v^2 \right) \right]^2 + \right. \\
+ \left. \frac{1}{4} (E_i \pm \partial_i N)^2 + P (G_0 \pm S)^2 + \right. \\
+ \left. P (G_i \pm \partial_i M)^2 + e^{-2gM} |(D_0 \pm ihN) \varphi|^2 + \right. \\
+ \left. e^{-2gM} |(D_1 \pm iD_2) \varphi|^2 \right. \pm \left. hB \left( \frac{2m}{h} M - |\phi|^2 + v^2 \right) \right. \pm \left. E_i \partial_i N + 2PG_0 S \mp 2PG_0 \partial_i M \mp 2e^{-2gM} NH_0 + \right. \\
+ \left. e^{-2gM} \left( \frac{1}{h} \varepsilon_{ij} \partial_i H_j + hB |\varphi|^2 \right) \right. + \left. \left( 2mN + 2gh |\phi|^2 \right) S + U \right\}, \tag{29}
\]
with
\[
H_\mu = \frac{ih}{2} \left( \varphi^* D_\mu \varphi - \varphi (D_\mu \varphi)^* \right). \tag{30}
\]
Now, we drop all quadratic terms as we are interested in the minimum energy configuration. Then, we obtain the BPS-equations:

\[ B \equiv h \left( \frac{2m}{h} M - |\phi|^2 + v^2 \right) = 0; \quad (31) \]
\[ \partial_{\mu} Z^\mu = S = \pm G_0; \quad E_i = \partial_i N = 0; \quad (32) \]
\[ G_i \pm \partial_i M = 0; \quad (1/\sqrt{2}) (\nabla_1 \pm i \nabla_2) \phi = 0. \quad (33) \]

Introducing Eq. (32) in (17) and (18), we recover Eqs. (19) and (20), showing that BPS-states agree with the results from the equations of motion, as expected. It is worthy to mention that the field-strength for the Kalb-Ramond potential becomes the topological charge.

If asymptotically we write \( \phi = ve^{i\theta} \), then, from equation (33), we get

\[ \frac{1}{\sqrt{2}} \phi^{-1} (\partial_1 \pm i \partial_2) \phi = -e^{\pm i\theta} \frac{N}{r}, \quad (34) \]
\[ -e^{\pm i\theta} \frac{N}{r} = e (A_1 \pm i A_2) + g (G_1 \pm i G_2). \quad (35) \]

Therefore, in the minimum energy configuration both fields, \( A_\mu \) and \( G_\mu \), participate of the vortex formation. However, for the critical coupling \( (g = -\frac{N}{m}) \), asymptotically, only the field that appears in the non-minimal coupling, \( G_\mu \), is relevant for the vortex configuration:

\[ 2m \int d^2 x b = Q_{\text{top}} = 2m \Phi_{\text{lux}}. \quad (36) \]

By analyzing the critical coupling and the asymptotic behaviour, we see that the non-minimal coupling in the covariant derivative contributes directly to the topological current, in agreement with equation (25).

**V. A POSSIBLE RELATION WITH A CONDENSED MATTER PHYSICS SYSTEM**

The relation between a global vortex in the Abelian Higgs model and vortices in a superfluid has been exploited in [19]. This work is developed in 4D and basically two problems are found when we try to identify them. The first difference has to do with the energy density that falls off like \( 1/r^2 \) in the case of the global vortex; on the other hand, vortices in a superfluid have non-zero energy density at infinity. The second main difference is related with the angular momentum, that is well-defined for vortices in a superfluid, but is zero for global vortices, when considering static configurations.

These problems have been solved when Davis and Shellar considered time dependent equations and a non-trivial background, as below:

\[ G^{ijk} = \alpha e^{ijk}. \quad (37) \]

This is clearly done in 4D; but, it is similar to the equation (9) that naturally shows up in 3D. The reason why they achieve this result is that (37) simulates a preferencial background for the superfluid and contributes with a non-zero energy at infinity. An important fact to mention is that, in order to introduce a non-trivial background in our \( N = 2 - D = 3 \) model, the SSB must also be realised by the Kalb-Ramond field. This has been done in 4D because the scalar action and the Kalb-Ramond action are simply related by a canonical transformation [20]. However in 3D the SSB cannot be realised by the Kalb-Ramond field and will be entirely described by a scalar field.

Another relation of our \( N = 2 - D = 3 \) with Condensed Matter concerns the gauge action:

\[ S_{\text{gauge}} = \int d^4 x \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + 2 m \varepsilon^{\mu\nu\rho} A_\mu \partial_\nu B_\rho \right\}. \quad (38) \]

In a lower-dimensional Condensed Matter system, the Chern-Simons-like term in equation (38) could also provide a non-trivial background. This mixing has been studied as an effective theory [21] in which a dynamical vortex is coupled with a superfluid film at zero temperature. In the \( \varepsilon^{\mu\nu\rho} A_\mu \partial_\nu B_\rho \)-term, the \( A_\mu \)-field is chosen as the responsible for the vortex formation and the \( B_\mu \)-field as the electromagnetic potential, which becomes part of the source that describes a uniform magnetic field. Also here, time-dependent equations must be considered.

Therefore, references [19] and [21] seem to be very closely related with our \( N = 2 - D = 3 \) supersymmetric model. We have pointed it out as a suggestion for future works. In both cases, the requirement of considering dynamical solutions is evident, so time dependence must be considered in equations (21 and 22).

**VI. GENERAL CONCLUSIONS**

In this work, we have shown that the Kalb-Ramond current has a topological conservation law in four dimensions. So, it seems reasonable that the coupling of the KR field to any other theory must be non-minimal. This also supports the non-existence of a non-Abelian generalization for these theories. Our result agrees with the ”no-go” theorem [15]. We however would like to point out the efforts in building up an interesting extension of the gauge approach to allow minimal couplings of the 2-form gauge potential [22], [23].

In the study of vortex formation, the KR-field strength in 1 + 2 dimensions is a simple constant and it couples to the present model as the topological charge of the vortex. This may also describe a non-trivial background. Also the non-minimal coupling of the vector field in the covariant derivative becomes directly identified with the topological current, which seems to stabilize the topological solutions for configuration of non-minimal energy.
We analyzed how BPS-states in this model reduce the number of differential equations and give us some insight on the role of each field whenever half of the supersymmetry charges become zero. We see that the mixing of the minimal and non-minimal couplings contributes for the ansatz on the scalar field, in general. However, with the critical coupling, \( g = -\frac{h}{m} \), only the non-minimal coupling is actually relevant for the vortex configuration.

Finally, our perspectives are to study the possibility of having a minimal coupling of the KR model in higher dimensions and study whether or not this coupling is allowed in presence of a gravity background. It would also be interesting to explore further the relation between our \( N = 2 - D = 3 \) and dynamical vortices in a superfluid film.

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