Applying deep neural networks to the detection and space parameter estimation of compact binary coalescence with a network of gravitational wave detectors

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In this paper, we study an application of deep learning to the advanced laser interferometer gravitational wave observatory (LIGO) and advanced Virgo coincident detection of gravitational waves (GWs) from compact binary star mergers. This deep learning method is an extension of the Deep Filtering method used by George and Huerta (2017) for multi-inputs of network detectors. Simulated coincident time series data sets in advanced LIGO and advanced Virgo detectors are analyzed for estimating source luminosity distance and sky location. As a classifier, our deep neural network (DNN) can effectively recognize the presence of GW signals when the optimal signal-to-noise ratio (SNR) of network detectors ≥ 9. As a predictor, it can also effectively estimate the corresponding source space parameters, including the luminosity distance D, right ascension α, and declination δ of the compact binary star mergers. When the SNR of the network detectors is greater than 8, their relative errors are all less than 23%. Our results demonstrate that Deep Filtering can process coincident GW time series inputs and perform effective classification and multiple space parameter estimation. Furthermore, we compare the results obtained from one, two, and three network detectors; these results reveal that a larger number of network detectors results in a better source location.

deep neural networks, advanced LIGO and advanced Virgo coincident detection of gravitational waves, multiple space parameter estimation

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1 Introduction

As the authority of gravitational wave (GW) detection, last year, the advanced Laser Interferometer Gravitational-Wave Observatory (advanced LIGO) and advanced Virgo performed the first three-detector detection of a GW signal from a binary neutron star coalescence [1]. This is the fifth direct detection of GWs from compact binary coalescences by advanced LIGO [2-6]. With the continuous improvement of advanced LIGO and advanced Virgo [7-10], numerous GW triggers are expected to appear in the time series. Since, the current data analysis of advanced LIGO and advanced Virgo is computationally expensive for detecting the matched GW signals in noisy data and for distinguishing signals from glitches, new methodologies for signal processing are required that can process data rapidly and with high ac-

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In recent years, the application of machine learning to gravitational-wave astronomy has been widely proposed [11-20]. Machine learning enables a computer to use specific learning algorithms to program itself based on large supplies of data to solve particular problems [21]. Among them, a deep learning architecture that is usually in the form of deep neural networks (DNNs) simulates the learning process of a human brain. By processing input information and sharing it with relevant neurons with sufficient neural connections, deep learning enables autonomous learning from data, yielding simpler data analysis [22, 23]. Most importantly, its ability to directly process raw noisy time series has been demonstrated, which can enable real-time GW observation [12]. Deep learning is expected to be a popular approach for processing big data and analyzing problems in the field of astrophysics.

Traditional NNs composed of three layers—the input, hidden, and output layers—have a limited ability to solve problems. However, researchers have discovered that if the hidden layer is extended with additional layers, its solving ability can be greatly enhanced [24]. NNs with long, interconnected layers between the input and output layers are called deep neural networks (DNN). Depending on the properties of the task, many categories of DNNs can be used, such as a recurrent neural network (RNN) [12, 24, 25] for exhibiting temporal behavior in data with variable input lengths, and a convolutional neural network (CNN) [26-28] for extracting features from data. Because a CNN is much easier to train due to its local receptive fields and weight sharing, and its effectiveness in automatically recognizing input, we have selected CNN as an integral part of our DNN system. Comparing the DNN to the matched filtering, widely used in the advanced LIGO’s and advanced Virgo’s pipelines, indicates that the DNN can significantly quicken GW searches [12].

There are a variety of GW sources in the advanced LIGO’s and advanced Virgo’s frequency band, such as the mergers of compact binary stars, which have been researched intensively. In particular, binary neutron stars have proven to be associated with abundant electromagnetic counterparts [29, 30]. As in ref. [12], we first train the DNN as a classifier to distinguish signal from noise. Second, the DNN structure is adjusted and trained as a predictor to estimate a source’s space parameters, which can constrain the locations of compact binary stars and provide insight for future multi-messenger observations [31-35].

In this paper, we take into account the coincident detection of three detectors, H, L, and V (HLV) referring to the advanced LIGO-Hanford, advanced LIGO-Livingston, and advanced Virgo detectors, respectively, and perform multiple space parameters estimation directly from noisy time series inputs after our DNN is trained by simulated signals. Gabbard et al. [19] has shown that a deep convolutional neural network can reproduce the sensitivity of a matched-filtering search for simulated binary black hole gravitational-wave signals. In the first application of the Deep Filtering method to advanced LIGO and advanced Virgo GW detection, George and Huerta [13] have detected GW150914 from noisy time series in one detector and effectively measured their masses. We, then, study the DNN as a predictor to estimate the luminosity distance and sky location of binary black holes.

2 Obtaining training and test data

The GW signal from a compact binary coalescence can be divided into three phases: the inspiral, merger, and ringdown of the final object. The waveform \( h_{+}, h_{\times} \) of all the phases has been successfully simulated using the effective-one-body model. Herein, we adopt the “EOBNRv4” to generate training and test data sets through the PyCBC software package (https://ligo-cbc.github.io). The GW strain \( h(t) \) of an interferometer is a linear combination of \( h_{+} \) and \( h_{\times} \):

\[
h(t) = F^{+}(t)h_{+}(t) + F^{\times}(t)h_{\times}(t),
\]

where, the antenna functions \( F^{+}(t) \) and \( F^{\times}(t) \) for a specific detector geometry are mainly related to the angles describing GW polarization \( \psi \) and source location at a celestial sphere frame coordinate (right ascension \( \alpha \), declination \( \delta \)), and the positions and orientations of the detectors. The detailed expressions of these functions, as well as the specific latitude of the detector’s location along with the orientation of each of the detector’s arms with respect to the local geographical directions, can be found in refs. [36, 37].

GW signals are submerged in Gaussian noise with the advanced LIGO’s and advanced Virgo’s noise power spectral density (PSD) at designated sensitivities, respectively [38, 39]. Figure 1 shows one sample of a coincident GW signal in the three detectors. The prior distributions on \( \alpha, \delta, \psi \) and the orbital inclination \( \iota \) in \( h_{+}, h_{\times} \) should be uniformly distributed in their respective ranges. The priors on \( m_{1}, m_{2} \) are assumed to be uniform in the range from 10 to 40\( M_{\odot} \), and the distribution of luminosity distance \( D \) are assumed to be uniform on the sky spherical surface from 10 to 4000 Mpc (see Figure 2). In each time series, all of our simulated binary black hole mergers are included in a 2-s window with cutoff frequency \( f_{\text{cutoff}} = 40 \) Hz, and the arrival time of the GW at the Earth’s center is selected from the uniform distribution on the interval [1.7, 1.75]s. For each detector, the time strain is

\[
s_{i}(t_{0} + \tau_{i} + t) = F^{+}_{i}(t)\hs_{+}(t) + F^{\times}_{i}(t)\hs_{\times}(t) + n(t_{0} + \tau_{i} + t),
\]

\[0 < t < T,
\]

\[\text{for } i = H, L, V,
\]
where, \( t_0 \) is the time at which the GW reaches the origin of any fixed coordinate, \( \tau_i \) is the time at which the GW propagates from the origin to the \( i \)th detector, and \( T \) is the duration of the GW. The time delay and antenna functions for each detector are based on the space locations of the advanced LIGO and the advanced Virgo (cf. Table 1 of ref. [40]). In this paper, we adopt the optimal SNR \( \rho = 2 \left[ \int_0^{\infty} \left| \tilde{h}(f) \right|^2 df / S_h(f) \right]^{1/2} \) to describe the SNR [41], where, \( S_h(f) \) is the noise PSD. In the following calculations, all of the SNRs are calculated from network detectors.

Splitting data into separate sets for training and testing is necessary for supervised learning. We randomly generate 3000 training sets and 2000 test sets with a given SNR (see Figure 2(b)). In addition to these, 2000 validation sets are separately generated for each SNR in order to adjust the hyperparameters of the DNN. We, then, standardize all of the sets. Herein, we select a sampling frequency of 2048Hz (cf. Figure 6 of ref. [42]). For coincident observation, the data from H, L, and V should be inputted together, leading to a 3×4096 dimensional tensor for each input data set. In total, we generate 100000 sets for training by adding different noise to 10000 templates, and 2000 sets for testing with each specific SNR.

3 DNN for GW detection and space parameter estimation

Structure of a DNN: An NN composed of many neurons can simulate the learning process of a human brain and is one of the most popular methods in machine learning. The standard structure of an NN includes an input layer, a hidden layer, and an output layer (see Figure 3(a)). Information enters the input layer and is transferred to neurons in the next layer (i.e., the hidden layer) via synapses. Using vector \( x \) as an example, herein, the input information is expressed by linear combina-
In the training process, the NN gets input $x$ and $Y$ and produces output $y$. (b) Illustration of a DNN structure with convolution layers and fully connected layers, where $*$ is a convolution operator.

In fact, DNNs, which combine fully connected layers with convolutional and modification layers, have been proposed to be more effective for both signal detection and parameter estimation from noisy time series data [12]. Figure 3(b) provides an illustration of the DNN structure. In the convolution layers, output from the previous layer, $x_k$, is filtered by neurons through convolution operations and transferred to the output of this layer, $x_{k+1}$, i.e., $x_{k+1} = g(w \ast x_k + b)$, where $\ast$ is the convolution operator. Therefore, we plan to use this type of DNN to learn how to recognize GW signals from raw time series data sets and estimate the corresponding parameters of the GW sources. Determining the optimal hyperparameters of the DNN, including the number of convolutional and fully connected layers, size of pooling layers, number of neurons in each layer, learning rate, iterations, and activation functions, remains a challenging problem [24]. Currently, several approaches for determining optimal hyperparameters have been proposed, such as the randomized trial-and-error-based methods adopted in this paper (http://neuralnetworksanddeeplearning.com), Bayesian optimization [47], and genetic algorithms [48].

**Training process:** Once the structure of a DNN is determined, then, for each attempted hyperparameter, the DNN should be trained. As in human learning, the training process is the most important step for DNN learning, and the learning algorithm plays a key role in this process. The goal of training is to determine the optimal weights and bias of the DNN. Initially, the computer provides random weights, $w$, and bias, $b$, to the DNN, which then undergoes several rounds of training using the training input data and obtains output $y$. Finally, the optimal $w$ and $b$ are determined when the loss function $l(w, b)$, which describes the difference between the DNN result $y$ and the real answer $Y$, reaches the minimum value. The loss function differs for classification and parameter estimation, which is discussed further in the following subsections. Applying each trained DNN to the validation sets, the optimal hyperparameters can be determined according to accuracy. Our optimal hyperparameters are illustrated...
in Table 1.

In this paper, we adopt the ADAM method, which uses stochastic gradient descent with an adaptive rate, to train the DNN [49]. Gradient descent with an adaptive rate is invariant to the diagonal rescaling of the gradients of the loss function. When the local minimum is reached, the first-order partial differential equations equals to zero: \( \frac{\partial l}{\partial w} = \frac{\partial l}{\partial b} = 0 \). This applies only to neurons from the input layer to the output layer; for adjusting the weights of neurons in the preceding layers, a back-propagation algorithm should be used [50]. From the updated weights in the output layer and by differentiating the activation function, back-propagation can be used to determine the updated weights in all preceding layers [51]. On the Mathematica 11.1 platform, we use the NetChain command to design the DNN, whose hyperparameters are selected by random trials. Then, for each designed DNN, we apply the NetInitialize command to initialize the network with a random distribution of weights and bias before training it.

### 3.1 DNN as a classifier

The main task of the DNN as a classifier is to detect GW signals in numerous time series data sets. The aim of the classifier is to correctly distinguish the class corresponding to x with maximum likelihood; therefore, the loss function is defined as the negative mean log-likelihood:

\[
I(N, Y, x) = -\frac{1}{N} \sum_{i=1}^{N} \log(P(Y = y|x_i)),
\]

where, \( x_i \) represents the \( i^{th} \) training input data set and \( N \) is the size of the training data sets, which in our case is 100000. \( Y \) and \( y \) are the real and predicted classes for \( x_i \), respectively. \( \log(P(Y = y|x_i)) \) represents the log probability of the predicted class of the DNN, \( y \), is the real class \( Y \) for a given \( x_i \).

The input of the classifier comprises several 3×4096 time series from three detectors. Each sample is recorded as three time series, which are simultaneously measured by three separate detectors, H, L, and V. Based on the structure in ref. [12], we have made some modifications. First, the three time series inputs from H, L, and V are joined into a 1×12288 matrix using a reshape layer, since the dimension of each neuron can then be simplified without any loss in accuracy in feature extraction. Second, specific values for kernel size, stride, dilation, and other DNN hyperparameters are set. Third, the DNN is trained. Then, validation sets are used to check the effectiveness of the trained DNN. After selecting a wide variety of hyperparameters for the DNN, the structure of our classifier is finally determined (shown in Table 1) based on the accuracy and operation time of the hyperparameters. Later on, we use NVIDIA GPU (Gtx1080ti, 11G ram) to implement the deep learning process.

After training for nearly nine hours, the classifier obtains 98.70% sensitivity of detection for SNR = 8 and 100% sensitivity for SNR ≥9, with a false alarm rate of 0.4%. The confusion matrices are shown in the left and middle panels of Figure 4. The right panel of Figure 4 illustrates the sensitivity varying with the SNR in the three network detectors, where, sensitivity increases for a higher SNR.

### 3.2 DNN as a predictor

With a structure similar to that of our classifier, the predictor uses a ReLU function instead of the final softmax layer [12] and a different output size. When trained with a template bank of expected signals, a DNN is able to estimate multiple parameters directly from noisy time series data [12]. Herein, we focus on the parameters \( D, \alpha \), and \( \delta \) for multiple parameter estimation, which directly provide the location of sources. With respect to the DNN as a predictor, we also compare its performance on one, two, and three network detectors, which can reveal meaningful insights into the coincident observation of GWs.

For parameter estimation, the loss function is generally the
mean squared error:

\[ l(N, Y, y) = \frac{1}{N} \left( \sum_{i=1}^{N} (Y_i - y_i) \right)^2, \tag{4} \]

where, \( Y_i \) and \( y_i \) are the real and predicted values of parameters \( D_i, \alpha_i, \delta_i \), respectively, corresponding to the \( i \)th training data set, and \( N \) is the number of training data sets.

Results: Our predictor is capable of estimating multiple parameters from simulated time series data in network detectors. By comparing the estimation errors for one, two, and three network detectors (Figure 5), we have found that luminosity distance errors \( \epsilon_D = y_D - Y_D \) decrease for a higher number of network detectors. In addition, the areas of angular uncertainty of celestial coordinates are constrained from several hundred deg\(^2\) for one detector to 100 deg\(^2\) for three network detectors. Figure 5(e) indicates that the gradient direction of the declination error, \( \epsilon_\delta \), on the right ascension error, \( \epsilon_\alpha \), tends to be vertical, which is in accordance with existing error ellipses of angular parameters of all-sky maps [52]. Furthermore, when SNR \( \geq 8 \), the relative errors of \( D, \alpha, \) and \( \delta \) in a multi-parameter predictor for advanced LIGO and advanced Virgo are all less than 23\% for three network detectors.

4 Conclusions

We have investigated the robustness of DNNs in coincident GW searches and multiple space parameter estimation in advanced LIGO and advanced Virgo, and have also shown that Deep Filtering can be extended to the network detection of

![Figure 4](image-url) (Color online) (a) Confusion matrix of a classifier on a test set with SNR = 8 (sensitivity is 98.70\%). (b) Confusion matrix of a classifier on a test set with SNR = 9 (sensitivity is 100\% for all signals with a higher SNR). (c) Sensitivity of detection varying with the SNR, where, the false alarm rate = 0.4\%.

![Figure 5](image-url) (Color online) (a)-(c) Distribution of errors \( \epsilon_D = y_D - Y_D \) in a multi-parameter predictor for advanced H, HL, and HLV, respectively. (d) Cumulative distribution of areas of angular parameter (\( \alpha, \delta \)) uncertainty. (e) Distribution probability of the gradient angle for \( \epsilon_\delta \) on \( \epsilon_\alpha \). Here, SNR = 8. (f) Mean relative errors of \( D, \alpha, \delta \) in the multi-parameter estimation varying with optimal SNR.
binary black hole mergers. Based on the features of a compact binary star’s GW waveform, 100000 theoretical inspiral-merger-ringdown signals are simulated to train our DNN. The training process takes approximately nine hours, after which the instant classification or prediction with a specific SNR is achieved (shown in Figure 6). From these results, we can determine the fundamental relation between the performance of the DNN and the optimal SNR of the data sets. This is meaningful for broadening the scope of current GW searches in advanced LIGO and advanced Virgo and for future real-time multi-messenger astrophysics.

For the above DNN structure, we have discussed signal-to-noise classification and space parameter estimation in one, two, and three detectors. The signal-to-noise classification indicates that the DNN can correctly classify the GW signals of compact binary stars with 100% detection sensitivity for time series with a SNR ≥ 9. From the parameter estimation, the DNN is able to estimate space parameters simultaneously. Our results also confirm that advanced LIGO and advanced Virgo can better estimate luminosity distance and sky location with a greater number of network detectors. Moreover, when a deep learning algorithm is used in advanced LIGO’s and advanced Virgo’s pipeline, there will be powerful instruments and equipment such as FPGAs for large data processing, and the corresponding sensitivity will be greatly improved. At present, our work mainly provides inspiration for deep learning in the network detection and source location of advanced LIGO and advanced Virgo.

Furthermore, the main advantage of adding a DNN to GW searches is that retraining a DNN is time-saving once the DNN is trained well at a given PSD of advanced LIGO and advanced Virgo [12]. Using a DNN facilitates real-time coincident GW detection, since a trained DNN is very efficient in performing classification and prediction. Based on the instant time detection and space parameter estimation of binary black hole mergers, the real-time observation of binary neutron star coalescence will likely be determined in the near future, which is highly important for the search for electromagnetic counterparts [53-55].

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![Figure 6](Color online) Data processing times for classification and prediction by our DNN and by other machine learning methods. Note: times refer to the processing of one test set by our DNN (using GPU: NVIDIA GPU (Gtx1080ti, 11G ram)) and the processing of one test set by other machine learning methods (using CPU: 2.7 GHz Intel Core i5).

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