Introducing two Mathematical Models using Data Functional Approximation to Determine the Shadow Angles and Lengths in North of Baghdad City

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Abstract. In this study, two models are presented to estimate a shadow angle and length of an object located outdoor during daytime. A stick, with 50 cm length and diameter 1 cm, used to measure the shadow angle and length from 9 am to 3 pm for 1st, 11th, 12th, 21st, 22nd, and 31st of August, 1st, 11th, 12th, 25th, 26th, and 27th of September, 11th, 12th, 25th, 26th, and 31st of October, 8th, 16th, 17th, 25th, 26th, and 30th of November, 1st, 16th, 17th, 22nd, and 23rd December 2019. The introduced model's results, for shadow angles and shadow lengths, show good agreement with the real data. The Normalize mean square error (NMSE) criterion used to measure the quality matching between the real and modeled data. The highest value of NMSE between the real and result shadow angles and shadow length from model is 0.006% and 0.003% respectively.

1. Introduction
Shadow is a natural phenomenon that occurs when sun light is blocked by any object [1]. Shadows are two dimensions that are produced from three-dimensional objects [2, 3]. The direction and length of any shadow changes due to sun position in the sky and object length [4, 5] shadow angle is the azimuth angle located between the position of the sun and the direction of the object considered (the position of the sun in the sky can be described by the elevation angle and the azimuth angle)[6,7]. Earth moves in the orbit around the sun, so the sun's rays differ in reaching earth’s surface[8,9]. Therefore, the shadow length of any object is a longest at the sunrise and sunset time that approaches to infinity, and shorter at midday and changing depends on the latitude and time of the day in that year[10,11]. At midday, the shadow length is zero at the equator and increases as we move away from the equator towards the north or the south. It changes through the course of a day just as the year progressed, for instance, an afternoon shadow is a lot shorter in summer than in winter [12,13].

Many models have been presented to estimate objects shadow on earth surface [14]. Technically, shadow detection methods can be categorized based on geographic location (object case geometry and
environment variables), and physical information of the sun's location in the sky as a function of the day time in the year and the state of weather. However, it is extremely difficult to obtain the exact model of an arbitrary scene because the environment is complex and the light sources vary from time to time and from place to place[15]. Several studies have been presented to estimate shadow model parameters. In 2010[16], Khalil Ethar, Naji Yarob and Jasim Ayad M. designed a mechatronics system to track sun position using a solar cell and shadow. The suggested design was based on a sensory system checking a database such as sunrise, sunset, and midday time. In 2015 [17], Akram A. Abood studied specific case to analyze the pattern of solar angles and the solar path then compute the best solar angles as a function of daytime to get high solar energy. Yi-Kai Wang, in 2017[18], determined the shadow length of a straight rod at a fixed time and location by trigonometric function relationship. The relationship of shadow length changed over time can be obtained. While in 2018 [3], Emad Ali Al-Helaly and Israa J. Muhsinin used a high accurate empirical Gaussian equation to find the percentage of shadow for any building in the study region along the year. In 2020 Nedaa. M. Salman, et.al. [19]introduced models to estimated day time and shadow length of object with length 10 cm by tracking the shadow object for the north of Baghdad city. The results of their models have a good matching with real data and can be used globally.

In this study, two models are proposed to estimate the shadow parameters (angle and length). The angles of the shadow have been estimated for different days from 1st August until 23rd December in the first model. While in the second model, the shadow lengths for the same period time is estimated. These models proved to be very effective in estimating the shadow angles and lengths of outdoor object with known length for each day hours from 9am to 3pm.

2. System overview

In this study, a home-made system shown in figure 1 used to track a stick’s shadow. This system consists of the stick with length 50 cm and diameter 1 cm, a horizontal flat wood board, scale for leveling the ground horizontally, a compass, colored chalk, colored pins to mark shadow length and path, and a ruler to measured shadow lengths. The studied position details are located at (44.37, 33.66), 7 meters above ground level, and 40 meters above sea level.

The shadow length measured manually as well as the shadow angles for the object (stick) with length 50 cm which marked on the wood board for the dates 1st, 11th, 12th, 21st, 22nd, 31st of August; 1st, 11th, 12th, 21st, 22nd, 30th of September; 11th, 12th, 25th, 26th, 31st of October, 8th, 16th, 17th, 25th, 26th 30th of November, 1st, 16th, 17th, 22nd, 23rd December 2019 form 9 am to 3 pm in the sunny day time, see figure 2.

![Figure 1: The suggested homemade system tools to track the stick’s shadow.](image1)

![Figure 2: The system output showing the shadow angle with shadow length during one-day for a stick length 50cm.](image2)
The recorded data are fitted using Origin software 2105[20, 21], the fitting method depends on the adjusted R^2 criterion used to test the results [22, 23] where the best fitting function determine the best models for the shadow angle and length for stick’s lengths. The normalized mean square error (NMSE) used in this study to measure the quality between the real and resulting data according the following formula [24]:

\[
NMSE = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{(f_i - g_i)^2}{g_i^2} \right) 
\]

(1)

where \( N \) is the number of based data, \( f_i \) is the values-based data, \( g_i \) is the tested data. The local day time (\( t_n \)) in military format (24h) convert to the term of midday time (\( t \)) using:

\[
t = t_n - t_{mid}
\]

(2)

where \( t_n = 9, 10, 11, \) midday, 13, 14, and 15, i.e. 9 am to 3 pm. Note: the midday time (\( t_{mid} \)) collected online [25].

3. The introduced models

Two mathematical models introduced to determine the shadow angles and lengths in day hour time from August to December as a function of the day indices in the year.

3.1 Determined the shadow angle model (Model1)

The recorded shadow angles data are fitted, using Origin software, as shown in figure 3 with day indices for the period from August to December.

![Figure 3](image)

**Figure 3:** Experiment data with the fitting curve of stick 50 cm during year days at (a) 9 am, (b) 10 am, (c) 11 am, (d) 1 pm, (e) 2 pm, and (f) 3 pm for August to December.
The value of shadow angles is largest at sunrise and sunset, but it starts to decrease until it reaches to zero at midday time. The sign of the shadow angle ($\theta_{sh}$) before midday time is negative while after midday is positive, which results to recognized between them. The fitted function that describes the recorded data in figure 3 can be written as:

$$\theta_{sh\text{-}hour} = S + U d + H d^2 + K d^3$$ (3)

where, $S$, $U$, $H$, and $K$ represent the fitted coefficients.

We assumed the 1st day in the year is the 21st March. So, the day index for 1st August equal to 134. Equation (3) is a general form describing fitting shadow angles with the day indices depending on Adj-$R^2$. This formula represents the first introduced model (Model1), the shadow angles for any day hour from August to December can be estimated. The fitting coefficients of this equation listed in table 1.

If the coefficients in table 1 substitute in equation (3), it is easy to estimate the shadow angle of any day time for the period 1st August to 23rd December.

3.2. Determined the shadow length model (Model2)

The second attempt is to find an optimal mathematical equation to estimate the shadow length that can be used globally. This achieved by experimen data of a stick with 50 cm length from 9 am to 3 pm in period days from August to December 2019. Figure 4 represents the fitted shadow length data as a function of day. The data is fitted nicely, however, there are some points shift from the red curve because of some errors occur due to light shift and scatter or cloudy days.

The estimated model to extract the shadow length ($L_{sh\text{-}hour}$), for the period from August to December in 2019 at 9 am, 10 am, 11 am, Midday time, 1pm, 2pm, and 3pm, can be written as:

$$L_{sh\text{-}hour} = E + F.d + G.d^2 + P.d^3 + Q.d^4$$ (4)

where, the $E$, $F$, $G$, $P$, and $Q$ represent the fitting coefficients and their values listed in table 2.
Figure 4: Fitted shadow length data of stick 50 cm with day indices for (a) 9 am, (b) 10 am, (c) 11 am, (d) Midday time, (e) 1 pm, (f) 2 pm, and (g) 3 pm from August to December 2019.
Table 2. The fitted coefficients of equation (4) according to day hours for 1st Aug. to 23rd Dec.

| Day hours | E     | F. d  | G. d^2 | P. d^3 10^4 | Q. d^4 10^-7 | Adj-R^2 |
|-----------|-------|-------|--------|-------------|--------------|---------|
| 9am       | - 600.842 | 14.90492 | - 0.12618 | 4.62126  | - 5.98701 | 0.99    |
| 10am      | -559.6274 | 13.45432 | - 0.11345 | 4.1695   | - 5.48267 | 0.99    |
| 11am      | -239.71488 | 6.16407 | - 0.05549 | 2.19936  | - 3.06416 | 0.99    |
| midday    | -60.45206 | 2.00974 | - 0.02206 | 1.04965  | - 1.62307 | 0.99    |
| 1pm       | -219.78821 | 5.65596 | - 0.05169 | 2.09187  | - 2.95979 | 0.97    |
| 2pm       | -763.01917 | 18.01248 | - 0.15242 | 5.64202  | - 7.50972 | 0.98    |
| 3pm       | -1300.00000 | 30.33523 | - 0.25226 | 9.11258  | - 11.8267 | 0.98    |

Table 2 is important in terms of having parameters key for the general model represented by equation (4). In this case, it is easy now to estimate shadow length at any day precisely.

4. Models results

The NMSE method showed the quality of the introduced two models results. The models data are plotted and compared with the experiment data to measure the NMSE.

4.1. Shadow angles resulted from model

The Model1 represented by equation (3) and fitting coefficients values in table 1 can be used to estimated shadow angle ($\theta_{d\text{-hour}}$) for each day time hour from August to December. The resulted shadow angles values with the real shadow angles shown in figure 5 for the period from August to December 2019 for day time hours (form 9am to 3pm).

![Figure 5: Model 1 and real data to measure shadow angles from equation (3).](image)
The NMSE values calculated to test real shadow angles with resulting shadow angles from Model1, see table 3.

| Day hours | NMSE    |
|-----------|---------|
| 9 am      | 0.0003% |
| 10 am     | 0.0060% |
| 11 am     | 0.0018% |
| 1 pm      | 0.0015% |
| 2 pm      | 0.0007% |
| 3 pm      | 0.00047%|

The NMSE values showed high matching between real and Model 1 data. This lead that Model1 is accurate in determining the object shadow angles in different day time. Shadow angles decrease gradually whenever going from summer towards the winter, i.e. its value large in August and then gradually decreases until 23rd December.

**4.2. Shadow length Results from Model 2**

The Model2 and experiment data are plotted in figure 6 to compare the reliability of equation (4) for the period from August to December 2019 for day time hours (form 9am to 3pm). The hour day split into two parts, before and after midday. The shadow length results are promising.

![Figure 6](image_url)

Figure 6: Model 2 and real data of the shadow length, (a) before midday, (b) after midday time.

The fluctuation of the data is presented in Model2 nicely as a function of day. The suggested home-made system was helpful and useful to collect the data to estimate a general model for shadow length and angle. The NMSE shows good matching between the recorded and Model2 data which shown in table 4.
Table 4. The NMSE between the real and Model 2 data for shadow lengths

| Day hours   | NMSE      |
|------------|-----------|
| 9am        | 0.00066%  |
| 10am       | 0.00054%  |
| 11am       | 0.00032%  |
| Midday-time | 0.00088%  |
| 13pm       | 0.00300%  |
| 14pm       | 0.00099%  |
| 15pm       | 0.00026%  |

The NMSE values showed high matching between real and Model2 data for shadow lengths measurement. The shadow length increases whenever going from summer towards the winter. While the length shadow at midday time stays shortest for all seasons.

5. Conclusions

The proposed two models work well in estimating the shadow angles and lengths for the outdoor object on sunny days for every day hours. NMSE measure used to check the data quality matching. The highest NMSE value in measuring shadow angles in Model1 was 0.006%. While, the highest NMSE value between the experiment shadow length data and the data from Model2 was 0.003%. It is important to record the data in a sunny day to obtain accurate results from the proposed models. The suggested system chooses to be easy, cheap, and everyone can repeat it.

6. References

[1] Liere R., Adriaansen T. and Zudilova-Seinstra E. eds., 2009. Trends in interactive visualization: state-of-the-art survey. Springer-Verlag London.
[2] Al-Hilaly, E.A. and Jameel, I., 2018. Examining the Efficiency of Sun Lightening and Shadow Tools in AutoCAD Program. arXiv preprint arXiv:1803.03823.
[3] Al-Hilaly, E.A. and Muhsin, I.J., 2018. Gaussian Equation to Describe the Percent of Shadow Length in Satellite Image. arXiv preprint arXiv:1803.05304.
[4] Barlow J.F. and Harrison G., 1999. Shaded by trees?. Arboricultural Advisory and Information Service.
[5] Dare, P.M., 2005. Shadow analysis in high-resolution satellite imagery of urban areas. Photogrammetric Engineering & Remote Sensing, vol. 71, no. 2, pp. 169-177.
[6] S.V. Szokolay, 2007. SOLAR GEOMETRY," Passive and Low Energy Architecture International.
[7] Wu, J.F., Wu, S. and Wu, X.J., 2016. Sun Shadow Position Based on the Sundial. Computer & Digital Engineering, vol. 44, no. 3, pp. 393-395.[8] R. Walraven, "Calculating the position of the sun," Solar energy, vol. 20, no. 5, pp. 393-397, 1978.
[8] Williams, G.E., 2000. Geological constraints on the Precambrian history of Earth’s rotation and the Moon’s orbit. Reviews of Geophysics, vol. 38, no. 1, pp.37-59.

[9] Huang, L., 2016, October. The research of the sun shadow positioning. In 2016 4th International Conference on Mechanical Materials and Manufacturing Engineering (pp. 743-745). Atlantis Press.

[10] S. Ray, "Calculation of sun position and tracking the path of sun for a particular geographical location," International Journal of Emerging Technology and Advanced Engineering, vol. 2, no. 9, pp. 81-84, 2012.

[11] Kolivand H. and Sunar, M.S., 2011. Real-time projection shadow with respect to sun’s position in virtual environments. International Journal of Computer Science Issues, vol. 8, no. 6, pp. 80-84.

[12] J. Appelbaum and J. Bany, "Shadow effect of adjacent solar collectors in large scale systems," Solar Energy, vol. 23, no. 6, pp. 497-507, 1979.

[13] "suns shadow," 10-5-2020. [Online]. Available: https://astroedu.iau.org/en/activities/1503/suns-shadow/10-5-2020.or a Rotate Target,” Journal of College of Education, no.3, 2018.

[14] Gualla F., 2015. Sun position and PV panels: a model to determine the best orientation. Student thesis series INES.

[15] Tian J., Sun J. and Tang Y., 2009. Tricolor attenuation model for shadow detection. IEEE Transactions on image processing, vol. 18, no. 10, pp. 2355-2363.

[16] Khalil E.H., Naji Y.O. and Jasim A.M., 2010. Sun Tracking System According to the Shadow.

[17] Abood, A.A., 2015. A comprehensive solar angles simulation and calculation using matlab. International Journal of Energy and Environment, vol. 6, no. 4, pp.367-376.

[18] Wang, Y.K., 2017, March. Mathematical model for solving the relationship of shadow length changing over time through solar elevation. In 2017 2nd International Conference on Automation, Mechanical Control and Computational Engineering (AMCCE 2017) (pp. 226-229). Atlantis Press.

[19] Nedaa M. Salman, Ali Alzuky, Haidar J. Mohamed and Anwar H. Al-Saleh, 2020. New Models to Estimate Daytime and Shadow Length by Tracking the Shadow, International Journal of Psychosocial Rehabilitation, vol. 24, no. 05, 2020.

[20] "TheKaleidaGraphGuide to CurveFitting," 20-4-2020. [Online]. Available: http://www.synergy.com.

[21] Chung K.S., Choe H.S., Lee J.I. and Kim J.L., 2007. A new method for the numerical analysis of thermoluminescence glow curve. Radiation measurements, vol. 42, no. 4-5, pp. 731-734.

[22] Ngo T.H.D. and La Puente C.A., 2012, April. The steps to follow in a multiple regression analysis. In SAS Global forum (Vol. 2012, pp. 1-12).

[23] Motulsky H. and Christopoulos A., 2004. Fitting models to biological data using linear and nonlinear regression: a practical guide to curve fitting. Oxford University Press.

[24] Grigsby M.R., Di J., Leroux A., Zipunnikov V., Xiao L., Crainiceanu C. and Checkley W., 2018. Novel metrics for growth model selection. Emerging themes in epidemiology, vol. 15, no. 1, pp.1-10.

[25] T. Hoffmann. "SunCalc.org." Torsten Hoffmann. https://www.suncalc.org/#/45.6175,-63.2806,3/2020.01.05/09:14:1/3.

[26] Shubham Sharma and Ahmed J. Obaid 2021 IOP Conf. Ser.: Mater. Sci. Eng. 1145 012054.