Determining the unsaturated hydraulic conductivity of remoulded loess with filter paper method and soil column seepage test

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Abstract
Loess is very widely distributed, and the unsaturated hydraulic conductivity of loess is related to many engineering issues. To determine the unsaturated hydraulic conductivity of remoulded loess more conveniently and at a lower cost, filter paper test and soil column seepage test were carried out. The results indicate that in the one-dimensional soil column seepage process, the unsaturated hydraulic conductivity of loess increases with the increase of the volumetric water content, and as the seepage time continues, the unsaturated hydraulic conductivity of loess from the top to the depth of 40 cm gradually becomes uniform. The changes in the microstructure indicate that the collapsible settlement will occur during the seepage process, which will change the pore structure of loess, thereby reducing the unsaturated hydraulic conductivity of the underlying loess. Compared with the experimental results, the soil hydraulic conductivity curve (SHCC) obtained by the van Genuchten–Mualem model (VG–M model) underestimates the magnitude of unsaturated hydraulic conductivity in the part with a low volumetric water content (< 20%), and the Childs and Collis-George model (CCG model) has more consistent results with the experimental results, because it is based on more segments of the soil–water characteristic curve (SWCC).

Keywords Unsaturated hydraulic conductivity · Remoulded loess · Filter paper method · Instantaneous profile method · Statistical models

Introduction
The unsaturated hydraulic conductivity (also termed as unsaturated permeability coefficient) of soil is one of the primary soil parameters, which is directly related to many engineering issues, such as landslides and foundation settlement caused by rainfall infiltration, design of irrigation and drainage system, and environmental risk assessment (Fredlund and Rahardjo 1993; Gribb et al. 2004). The soil in nature (such as loess) is mostly in an unsaturated state, and the hydraulic conductivity of unsaturated soil is not a constant, but a function of suction or water content (Rahimi et al. 2010). The relationship between unsaturated hydraulic conductivity and suction or water content is usually expressed in the form of $K$ function expression or soil hydraulic conductivity curve (SHCC).

There are many methods to determine unsaturated hydraulic conductivity, but generally it can be summarized into direct and indirect methods. The direct method refers to direct measurement in laboratory or field test, such as instantaneous profile methods (Richards and Weeks 1953), constant head method (Klute 1972), constant flow method (Olsen et al. 1994), centrifuge method (Nimmo et al. 1987), and outflow method (Gardner 1956). The indirect method refers to obtaining unsaturated hydraulic conductivity through empirical models and statistical models, which are mainly based on saturated hydraulic conductivity (Gardner 1958), pore size distribution of the soil (Kunze et al. 1968; Rosas et al. 2015), and soil–water characteristic curve (van Genuchten 1980).
Genuchten 1980; Ye et al. 2014). And the commonly used models of the indirect method are summarized in Table 1. Equations (1) and (2) are the most commonly used SWCC models. Equations (3) and (4) are empirical models of unsaturated hydraulic conductivity in the form of linear and power functions, respectively. Equations (5) and (6) are more commonly used statistical models of unsaturated hydraulic conductivity. The specific expressions and references are listed in Table 1.

The instantaneous profile method (IPM) is a more commonly used direct method, first proposed by Richards and Weeks (1953). The technique refers to the fact that profile of suction is obtained by several tensiometers arranged along a soil column, independent measurement with conservation of mass in different intervals, and then the hydraulic conductivity in different intervals is calculated. Based on the principle of the IPM, many measurement techniques have been developed. Choo and Yanful (2000) calculated the hydraulic conductivity of the multilayer soils under evaporative conditions with tensiometer, TDR probe and finite element models. By monitoring the soil moisture content, suction, and wetting front advancing velocity during the seepage process, Li et al. (2009) proposed the wetting front advancing method to calculate the unsaturated hydraulic conductivity of five soils. Ng and Leung (2012) developed a stress-controllable soil column device, and obtained the unsaturated hydraulic conductivity by controlling the stress in the soil column Wang et al. (2014) conducted a seepage test on a one-dimensional loess column, combined with the SWCC of the soil sample independently obtained with a tensiometer, and calculated the SHCC of the loess sample. Leung et al. (2016) modified the boundary flux of IPM, and the modified model also has good accuracy. Li et al. (2020) combined a series of paleosol samples with filter paper into a soil column, and obtained the suction and water content of soil samples by the filter paper method, thereby calculating the unsaturated hydraulic conductivity of the paleosol samples.

The methods described in the above have their own advantages. However, some direct methods have certain requirements for the experimental equipment, which increases the cost and is not convenient enough, and if only the indirect method is used for calculation, the prediction result is not accurate enough due to the difference of soil properties. Therefore, a low-cost and easy-to-implement method to obtain the unsaturated hydraulic conductivity of the soil is necessary.

In this study, the SWCC of the soil sample independently measured by the filter paper method, and a series of water content reflectometers were arranged in a soil column to measure the water content profile during the seepage process of the one-dimensional soil column. The suction profile is calculated by the water content profile and SWCC, and then the unsaturated hydraulic conductivity of the soil sample is obtained based on the principle of IPM. The test results are compared with the results of different prediction models to explore their effectiveness.

### Materials and methods

#### Experimental materials

The soil sample is a typical loess that deposited during Late Pleistocene period (i.e. Malan loess), taken from

| Table 1 SWCC models and unsaturated hydraulic conductivity models |
|----------------|----------------|----------------|
| Type | Equations | Reference |
| SWCC model | $\theta = (\theta_s - \theta_i) \left[ \frac{1 + (\psi/\psi_c)}{1 + (\psi/\psi_c)^{-m}} \right]^{m} + \theta_i$ | van Genuchten (1980) |
| | $\theta = \left[1 - \ln \left(\frac{1 + (\psi/\psi_c)}{1 + (\psi/\psi_c)^{-m}}\right) \theta_i \left(\ln \left(\frac{1}{\psi/\psi_c}\right)\right)^{m} \right]$ | Fredlund and Xing (1994) |
| Unsaturated hydraulic conductivity model | $k(\psi) = a\psi + b$ | Richards (1931) |
| | $k(\psi) = \frac{1}{1 + \psi}$ | Gardner (1958) |
| | $k(\theta) = k_s \left(\frac{\theta - \theta_i}{\theta_s - \theta_i}\right)^{1/2} \left[1 - \left(\frac{\theta - \theta_i}{\theta_s - \theta_i}\right)^{n-1}\right]^{1/(n-1)}$ | van Genuchten (1980) |
| | $k(\theta) = \frac{k_s}{\theta_s - \theta_i} \sum_{j=1}^{M} \left[2j + 1 - 2j(\psi/\psi_c)^{-1}\right]^{1/2}$ | Kunze et al. (1968) |

where $\theta$ is volumetric water content, $\theta_s$ is saturated volumetric water content, $\theta_i$ is residual volumetric water content, $a$, $n$, and $M$ are fitting parameters (where $m$ in Eq. (1) is typically set equal to $1 - 1/n$), $\psi$ is suction head (kPa), $\psi_c$ is the suction corresponding to the residual volumetric water content $\theta_i$ (kPa), $k_s$ is saturated hydraulic conductivity (cm/s), $k_w$ is calculated saturated conductivity (cm/s), $T_i$ is the surface tension of water (N/m), $\rho_w$ and $\mu$ are the density and absolute viscosity of water, respectively, and $g$ is gravitational acceleration (m/s²), $j$ and $i$ are summation indices. $M$ is the total number of pore size intervals between the saturated water content $\theta_s$ and the lowest water content $\theta_i$. The index $N$ is the number of intervals between the saturated water content and zero water content, i.e. $N = M \left[\theta_s/(\theta_s - \theta_i)\right]$, e is the natural logarithm.
Lanzhou city, China (West side of Loess Plateau of China, Fig. 1), and this kind of loess widely distributes in the Chinese Loess Plateau, so it was used in this study. The basic physical parameters of the loess were tested according to ASTM standards D854, D2487, and D4318, and the saturated hydraulic conductivity is determined with reference to the falling-head test in the Standard for Soil Test Method. The specific equipment used in the falling-head test is a Permeameter (TST-55, manufactured by Nanjing Soil Instrument Factory, China). The chemical composition of the loess is determined by an X-ray fluorescence (XRF) analyzer (PW2403, manufactured by Panalytical company, Netherlands). The basic physical parameters and the chemical composition of Malan loess are shown in Table 2. The Grain size distribution of the loess sample is shown in Fig. 2.

**Experimental method and steps**

**Experimental method**

The experimental soil used in the study was made into two different types of specimens for soil column seepage test and filter paper test, respectively. The soil column seepage test is used to obtain the variation of the water content with the seepage depth during the seepage process, that is, the profile of water content. The filter paper test was performed independently to obtain the SWCC of the experimental soil. Combining the profile of water content obtained from the soil column seepage test and the SWCC obtained from the filter paper test to obtain the profile of suction in the seepage process, and then calculate the unsaturated hydraulic conductivity according to the principle of the IPM. All tests are carried out at the ambient temperature of the laboratory at 20 °C. The flowchart of the experimental method is shown in Fig. 3.

**Preparation of soil specimens**

The loess sample obtained is naturally air-dried, and the water content after natural drying is about 5%. The dry density of undisturbed loess generally ranges from 1.25 to

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**Table 2** Basic physical parameters and the chemical composition of the soil specimen

| Physical parameters          | Standard                                      |
|-----------------------------|----------------------------------------------|
| Nature water content (%)    | 8.22                                         |
| Specific gravity            | 2.72                                         |
| Liquid limit (%)            | 29.70                                        |
| Plastic limit (%)           | 19.20                                        |
| Plastic index               | 10.50                                        |
| Maximum dry density (g/cm³)| 1.75                                         |
| Optimum water content (%)   | 15.2                                         |
| Classification              | Lean clay (CL)                               |
| Saturated hydraulic conduc-
  tivity (m/s)                | 1.90×10⁻⁶                                    |
| Chemical composition (%)    |                                              |
| SiO₂                        | 54.77                                        |
| Al₂O₃                       | 11.75                                        |
| Fe₂O₃                       | 4.53                                         |
| MgO                         | 2.78                                         |
| CaO                         | 9.61                                         |
| Na₂O                        | 2.28                                         |
| K₂O                         | 2.46                                         |
| LOI                         | 10.54                                        |

*LOI* loss of ignition

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Fig. 1 The location of the Lanzhou City
1.65 g/cm$^3$ (Yao et al. 2012), therefore, the median value of 1.45 g/cm$^3$ is taken as the dry density of the remoulded sample. Two types of remoulded loess samples are prepared. The loess used for the SWCC test was first placed in an oven (105 °C) for 12 h to dry, and then a quantitative amount of distilled water was added to prepare different moisture content samples, and stored in a sealed bag for 24 h to ensure uniform moisture diffusion. Then, the loess sample was placed into a cylindrical mould with an inner diameter of 4 cm and a height of 1 cm to form a specimen with dry density of 1.45 g/cm$^3$. The loess used for the soil column seepage test was directly placed in the designed cylindrical acrylic mould (inner diameter 47.8 cm, height 100 cm) by natural accumulation method in the natural air-drying state. After adding a certain amount of soil each time, gently ram the top of soil column to the calculated height (approximately 10 cm rise for every 26 kg added) to ensure that the dry density of the soil column is 1.45 g/cm$^3$ evenly. A total of sixteen replicate specimens were made for the SWCC test, and one loess column was used for the soil column seepage test.

**SWCC test**

The SWCC of soil specimen is determined by the filter paper method (ASTM D5298-16 2016, Houston et al. 1994; Likos and Lu 2002). SWCC test is carried out with No.203 filter paper (produced by Hangzhou Xinhua Co., Ltd.), which has been proved to have high accuracy in filter paper test (Chen et al. 2018; Wang et al. 2003). Before SWCC test, the filter paper was calibrated by determining the relationship between water content and suction. To better calibrate the filter paper in the whole suction range, a combination of salt solution method and pressure plate method was used for calibration. The salt solution method refers to the use of Kelvin’s equation to calculate the relationship between suction and moisture content of filter paper after it is balanced in the vapor pressure environment formed by different concentrations of sodium chloride solution. And the specific test steps are carried out according to the method of Likos and Lu (2002). The pressure method is carried out with a pressure plate extractor (1500F2, manufactured by soil moisture equipment company, U.S.). The filter paper calibration curve drawn according to the calibration results is shown in Fig. 4.

According to the calibration curve, the relationship between suction and the water content of the filter paper can be obtained, as shown in the following equation:

![Fig. 4 Filter paper calibration curve](image-url)
where $\psi$ is suction, (kPa), $w_{fp}$ is filter paper water content, (%).

After calibration, to facilitate specimen preparation and shorten the equilibration time, the water content of the soil specimen is set from 1 to 25%, and each gradient is increased by 2–3%. The specimens were wrapped as shown in Fig. 5. After 20 days of equilibration time, open the sealed box and weigh the filter paper to calculate its water content, and retest the water content of the soil specimens. The filter paper test equilibration process requires a constant ambient temperature, and the filter paper weighing process should not exceed 30 s (Wang et al. 2003). The SWCC of the soil specimen is obtained by the water content of the filter paper after equilibrium and the calibration curve.

### Soil column seepage test

To make the added water infiltrate evenly, a 10 cm thick quartz sand cover layer is set on the top of the soil column. Similarly, to facilitate the collection of exuded water, a 10 cm quartz sand cushion layer is designed at the bottom. Under natural rainfall conditions, the moisture content of the loess at a depth to 40 cm changes significantly (Li et al. 2013; Lin et al. 2019). Therefore, the part from the top of the soil column to 40 cm is used for monitoring and calculation. The probe used in the test is a soil moisture and temperature sensor (ECH2O-5TE, manufactured by Decagon, U.S.). The basic parameters of the probe are shown in Table 3. Before the test, calibrate the probe by preparing soil samples with different moisture content gradients, and then the probes were buried in the center of the soil column. The arrangement of the probes is shown in Fig. 6.

The rainfall in Lanzhou City is mainly short-term heavy rainfall in summer. According to the data in the relevant literature (Hu et al. 2017), the precipitation of a heavy rainfall is taken as a reference (28 mm), and the required water volume is calculated according to the cross-sectional area of the soil column to be about 5 L, and the water is added within 1 h to simulate the environment of short-term heavy rainfall. To avoid ponding on the top of the soil column, a sprinkler was used to evenly add water to the top of the column. And cover the top of the column immediately after adding water to prevent evaporation. Record the changes of water content at intervals of 3 days, 6 days, and 12 days.

### Calculation step

Due to the velocity of the seepage is very slow, ignore the influence of velocity head. The total head at any cross-section is shown in Eq. (8).

$$h = \frac{\psi}{\rho_w g} + z,$$

where $h$ is total head, $z$ is elevation head, $\psi$ is suction head, $\rho_w$ is the density of water, and $g$ is gravitational acceleration.

Record the heads at $Z_1$ and $Z_2$ as $h_1$ and $h_2$, respectively, and take the $(h_1 + h_2)/2$ layer as the reference layer, then the amount of water $V$ passing through the plane is shown in Eq. (9).

$$V = \int_0^{(h_2-h_1)/2} \theta(z) A dz,$$

where $\theta(Z)$ is the function between volumetric water content and depth, which can be obtained from the profile of water content. $A$ is the cross-sectional area of the soil column.

![Fig. 5 Sketch (a) and picture (b) of filter paper test](image-url)
amount of water change from time \( t_1 \) to time \( t_2 \) is shown in the following equation:

\[
\Delta V = \int_0^{h_2-h_1} \theta(z) \, dz - \int_0^{h_2-h_1} \theta(z) \, dz,
\]

and seepage velocity \( v \) is shown in Eq. (11).

\[
v = \frac{\Delta V}{A \Delta t}.
\]

Linearize the hydraulic gradient \( i \), which is shown in Eq. (12).

\[
i = \frac{1}{2} \left( \frac{h_{1-1} - h_{1-2}}{z_2 - z_1} + \frac{h_{2-1} - h_{2-2}}{z_2 - z_1} \right).
\]

where the first digit in the subscript indicates time, and the second digit indicates depth, that is, \( h_{1-2} \) indicates the head with time \( t_1 \) and depth \( Z_1 \).

According to Darcy's law, the hydraulic conductivity is the ratio of \( v \) and \( i \).

**Results and discussion**

**SWCC and curve fitting**

The results obtained by the filter paper method are calibrated by Eq. (7) and plotted in the suction-water content relationship diagram. To obtain a more continuous SWCC, Eqs. (1) and (2) (i.e., the VG model and FX model) are used to fit the obtained data, and the results are shown in Fig. 7. Table 4 is fitting parameters.

From the fitting results, when describing the SWCC of the remoulded loess measured by the filter paper method, the Adj.R\(^2\) of both models is above 0.93. But as far as the test results are concerned, the FX model is more accurate for fitting the data of the high water content (> 30%) part and the low water content (< 15%) part. This may be due to the correction factor included in the FX model, which makes the change of the entire curve shape more convex and concave, thus fitting the actual data more accurately. In addition, \( m \) in the VG model is set equal to 1–1/\( n \), which makes the VG model essentially a two-parameter model, so its fitting
of simulated rainfall infiltration, the water content of the surface loess changed significantly, but there was almost no change at a depth of 40 cm. With the continuous infiltration of rainfall every day, the distribution of moisture in the soil column gradually becomes uniform. After 1 month, the moisture content is mainly concentrated at 15–20%.

**Calculation results**

Tables 5, 6, 7 and 8 show the calculation results based on profile of water content and SWCC, of which Tables 5 and 6 are the results based on the VG model fitting curve, and Tables 7 and 8 are the results based on the FX model fitting curve. The first digit of the serial number in the table represents the time period, I to IV represent the four time intervals of 1–4 days, 4–10 days, 10–22 days, 22–34 days, and the second digit is the corresponding probe depth, 0–4 correspond to 0 to 40 cm

**Variation of unsaturated hydraulic conductivity with seepage**

To analyze the migration and change law of the water in the soil column during the seepage process, the calculation results of the IPM based on VG model fitting curve are plotted in Fig. 9 as an example. From Fig. 9a, it can be seen that the variation of unsaturated hydraulic conductivity with depth of loess column has similar characteristics to the distribution of water content with depth in Fig. 8a. All of them gradually uniformed with the duration of seepage, that is, the unsaturated hydraulic conductivity values of the loess in each part gradually approached. Figure 9b reflects the same law. The variation of depth below 20 cm shows that as the loess changes from dry to wet, that is, the process of increasing the water content, the hydraulic conductivity of the loess gradually increases. The water content of the surface loess (0–10 cm) gradually decreases with the continuous seepage process, and its hydraulic conductivity also gradually decreases.

**Microstructure**

The microstructure of soil samples after soil column seepage test at different depths was investigated (Take three replicate samples every 10 cm from 10 to 40 cm, because they have similar rules, take one set as an example to illustrate). Moreover, the obtained micrographs were binarized to reflect the

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**Table 4** Fitting parameters of SWCC models

|        | VG model |                | FX model |                |
|--------|----------|----------------|----------|----------------|
|        | a    | n    | m   | Adj. $R^2$ | a    | n    | m   | Adj. $R^2$ |
|        | 0.044 | 1.343 | 0.255 | 0.935       | 35.028 | 3.101 | 0.407 | 0.997       |

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Fig. 8 Profile of water content (a) and suction (b). Profile of suction is based on the VG model fitting curve.
Porosity changes more intuitively, as shown in Fig. 10, and the initial porosity of the sample (i.e. the soil column before the seepage test) can be calculated by Eq. (13).

\[ n = \left(1 - \frac{\rho_d}{G_s\rho_w}\right) \times 100\% \]  

(13)

where \( \rho_d \) is the dry density, \( G_s \) is the specific gravity.

According to Eq. (13), the initial porosity of the entire soil column can be calculated to be 46.58% (\( \rho_d = 1.45 \) g/cm\(^3\), \( G_s = 2.72 \), \( \rho_w = 0.998 \) g/cm\(^3\)), and through analyses with ImageJ, the micrographs after binarization show that the porosity of the soil samples from 10 to 40 cm are 41.13%, 33.36%, 25.23%, and 18.85%, respectively. Although there is a certain error between the result of binarization and the actual situation, the change in porosity reflects a trend, that is, as the penetration depth increases, the large pores in the deep soil are filled, and the soil becomes denser. This phenomenon is caused by two reasons, one is that the soil at the bottom is denser due to its own weight, and the other is the collapsibility of loess. Lei (1987) think that the interparticle pores in

| Serial number | \( t_1 \) | Volumetric water content (%) | Suction (kPa) | Total head (cm) | \( t_2 \) | Volumetric water content (%) | Suction (kPa) | Total head (cm) |
|---------------|----------|-----------------------------|---------------|----------------|----------|-----------------------------|---------------|----------------|
| I-0           | 31.50    | 28.55                        | -291.25       |                | 25.53    | 79.04                        | -806.20       |
| I-1           | 28.03    | 52.31                        | -543.60       |                | 23.86    | 104.70                       | -1077.95      |
| I-2           | 11.05    | 2301.04                      | -23,490.57    |                | 17.74    | 343.66                       | -3525.32      |
| I-3           | 8.02     | 18,058.14                    | -184,223.05   |                | 8.83     | 9669.41                      | -98,657.94    |
| II-0          | 25.53    | 79.04                        | -806.20       |                | 21.90    | 147.98                       | -1509.44      |
| II-1          | 23.86    | 104.70                       | -1077.95      |                | 20.98    | 175.63                       | -1801.38      |
| II-2          | 17.74    | 343.66                       | -3525.32      |                | 19.12    | 254.28                       | -2613.63      |
| II-3          | 8.83     | 9669.41                      | -98,657.94    |                | 12.07    | 1822.98                      | -18,624.38    |
| III-0         | 21.90    | 147.98                       | -1509.44      |                | 19.41    | 239.43                       | -2442.23      |
| III-1         | 20.98    | 175.63                       | -1801.38      |                | 18.95    | 263.53                       | -2697.97      |
| III-2         | 19.12    | 254.28                       | -2613.63      |                | 18.04    | 321.14                       | -3295.63      |
| III-3         | 12.07    | 1822.98                      | -18,624.38    |                | 15.91    | 537.31                       | -5510.58      |
| IV-0          | 19.41    | 239.43                       | -2442.23      |                | 17.71    | 346.02                       | -3529.43      |
| IV-1          | 18.95    | 263.53                       | -2697.97      |                | 17.49    | 364.02                       | -3722.97      |
| IV-2          | 18.04    | 321.14                       | -3295.63      |                | 17.07    | 401.90                       | -4119.37      |
| IV-3          | 15.91    | 537.31                       | -5510.58      |                | 16.32    | 483.42                       | -4960.91      |
| IV-4          | 8.85     | 9536.40                      | -97,311.25    |                | 14.71    | 748.64                       | -7676.18      |

| Interval   | \( \Delta V/A \) (cm) | \( v \) (cm/s) | Hydraulic gradient | Hydraulic conductivity (cm/s) | Average volumetric water content (%) |
|------------|------------------------|----------------|--------------------|------------------------------|-------------------------------------|
| I-0–I-1   | 27.60                  | 1.06E−04       | 26.20              | 4.06E−06                     | 27.23                               |
| I-1–I-2   | 58.54                  | 2.26E−04       | 1269.72            | 1.78E−07                     | 20.28                               |
| I-1–I-3   | 13.12                  | 5.06E−05       | 12,793.25          | 3.96E−09                     | 11.52                               |
| II-0–II-1 | 17.21                  | 3.32E−05       | 28.18              | 1.18E−06                     | 23.07                               |
| II-1–II-2 | 41.63                  | 8.03E−05       | 162.98             | 4.93E−07                     | 20.43                               |
| II-2–II-3 | 30.82                  | 5.95E−05       | 5557.17            | 1.07E−08                     | 14.44                               |
| III-0–III-1 | 11.87              | 1.14E−05       | 27.38              | 4.18E−07                     | 20.31                               |
| III-1–III-2 | 31.56              | 3.04E−05       | 70.49              | 4.32E−07                     | 19.27                               |
| III-2–III-3 | 37.40              | 3.61E−05       | 911.29             | 3.96E−08                     | 16.29                               |
| IV-0–IV-1 | 8.20                   | 7.91E−06       | 22.46              | 3.52E−07                     | 18.39                               |
| IV-1–IV-2 | 22.49                  | 2.17E−05       | 49.70              | 4.36E−07                     | 17.89                               |
| IV-2–IV-3 | 31.08                  | 3.00E−05       | 152.82             | 1.96E−07                     | 16.84                               |
| IV-3–IV-4 | 21.89                  | 2.11E−05       | 4725.80            | 4.47E−09                     | 13.95                               |
loess are the main pore types that cause loess collapsibility. Although the interparticle pore structure of remoulded loess may be different from that of undisturbed loess, the micro-mechanism of collapsibility is similar (Wang et al. 2017; Shao et al. 2018). That is: when moisture penetrates into the soil, the connection force between particles is rapidly decreased under its own weight or a certain pressure, and the particles around the pores sink into the pores, and the particles rearrange and become compact, resulting in collapsibility. The decrease in the number of interparticle pores in the microphotographs of soil samples from top to bottom of the soil column also confirms this view. This process is also accompanied by the occurrence of collapsible settlement. The change in the collapsible settlement of the soil column can be observed through the scale engraved on the acrylic cylinder. Figure 11 shows the relationship between settlement displacement of the soil column and seepage time. It can be seen that with the passage of seepage time, the wetting front continues to advance downward, and the settlement of the soil continues to increase. However, due to the remodeling of the soil changes the original structure, the settlement displacement is not large, the maximum is only 11 mm. Compared with remoulded loess, there are some root holes and wormholes in the undisturbed loess and a smaller dry density of the undisturbed loess, so there will be more

### Table 7 Values of volumetric water content, suction and head of loess at different times and depths (based on FX model fitting curve)

| Serial number | $t_1$ | $t_2$ |
|---------------|-------|-------|
|               | Volumetric water content (%) | Suction (kPa) | Total head (cm) | Volumetric water content (%) | Suction (kPa) | Total head (cm) |
| I-0           | 31.50 | 44.47 | −453.58       | 25.53 | 76.81 | −783.42       |
| I-1           | 28.03 | 60.76 | −629.78       | 23.86 | 92.50 | −953.53       |
| I-2           | 11.05 | 10,722.71 | −109,391.61   | 17.74 | 287.30 | −2950.49       |
| II-0          | 25.53 | 76.81 | −783.42       | 21.90 | 120.78 | −1231.93       |
| II-1          | 23.86 | 92.50 | −953.53       | 20.98 | 138.61 | −1423.79       |
| II-2          | 17.74 | 287.30 | −2950.49       | 19.12 | 201.15 | −2071.68       |
| II-3          | 8.83  | 48,262.67 | −492,309.28   | 12.07 | 6936.10 | −70,778.21     |
| III-0         | 21.90 | 120.78 | −1231.93      | 19.41 | 186.93 | −1906.70       |
| III-1         | 20.98 | 138.61 | −1423.79      | 18.95 | 209.28 | −2144.67       |
| III-2         | 19.12 | 201.15 | −2071.68      | 18.04 | 262.69 | −2699.47       |
| III-3         | 12.07 | 6936.10 | −70,778.21   | 15.91 | 557.88 | −5720.42       |
| IV-0          | 19.41 | 186.93 | −1906.70      | 17.71 | 291.98 | −2978.15       |
| IV-1          | 18.95 | 209.28 | −2144.67      | 17.49 | 311.44 | −3186.69       |
| IV-2          | 18.04 | 262.69 | −2699.47      | 17.07 | 358.23 | −3637.91       |
| IV-3          | 15.91 | 557.88 | −5720.42      | 16.32 | 469.08 | −4814.63       |
| IV-4          | 8.85  | 47,971.08 | −489,345.00  | 14.71 | 1024.96 | −10,494.58     |

### Table 8 Change of unsaturated hydraulic conductivity with calculation interval (based on FX model fitting curve)

| Interval     | $\Delta V/A$ (cm) | $\nu$ (cm/s) | Hydraulic gradient | Hydraulic conductivity (cm/s) | Average volumetric water content (%) |
|--------------|-------------------|--------------|--------------------|-----------------------------|-------------------------------------|
| I-0–I-1     | 27.60             | 1.06E−04     | 17.32              | 6.15E−06                    | 27.23                               |
| I-1–I-2     | 58.54             | 2.26E−04     | 5537.94            | 4.08E−08                    | 20.28                               |
| II-0–II-1   | 17.21             | 3.32E−05     | 18.10              | 1.83E−06                    | 23.07                               |
| II-1–II-2   | 41.63             | 8.03E−05     | 132.24             | 6.07E−07                    | 20.43                               |
| II-2–II-3   | 30.82             | 5.95E−05     | 27,903.27          | 2.13E−09                    | 14.44                               |
| III-0–III-1 | 11.87             | 1.14E−05     | 21.49              | 5.33E−07                    | 20.31                               |
| III-1–III-2 | 31.56             | 3.04E−05     | 3586.37            | 1.01E−08                    | 16.29                               |
| III-2–III-3 | 37.40             | 3.61E−05     | 60.13              | 5.06E−07                    | 19.27                               |
| IV-0–IV-1   | 8.20              | 7.91E−06     | 22.33              | 3.54E−07                    | 18.39                               |
| IV-1–IV-2   | 22.49             | 2.17E−05     | 52.10              | 4.16E−07                    | 17.89                               |
| IV-2–IV-3   | 31.08             | 3.00E−05     | 208.08             | 1.44E−07                    | 16.84                               |
| IV-3–IV-4   | 21.89             | 2.11E−05     | 24,465.23          | 8.63E−10                    | 13.95                               |
intense collapsibility. The penetration of water in the soil is mainly promoted in the form of capillary wetting fronts, and capillary water mainly exists in pores with a diameter of 0.002–0.5 mm. Collapsible settlement leads to closure and reduction of capillary pores, making capillary water difficult to conduct. Since the IPM is based on the conservation of mass to calculate the unsaturated hydraulic conductivity between different sections of the soil column, the unsaturated hydraulic conductivity of the high water content part is calculated from the upper cross-section of the soil column, and the unsaturated hydraulic conductivity of the low water content part is obtained from the lower cross-section of the soil column. This leads to the fact that when calculating with IPM, with the decrease of water content, the decreasing trend of the unsaturated hydraulic conductivity in the low water content part is faster than

Fig. 9 Variation of hydraulic conductivity with depth (a) and time (b). The hydraulic conductivity is calculated based on the VG model fitting curve

Fig. 10 Microstructure photos and binarization photos of soil samples at different depths. a 10 cm. b 20 cm. c 30 cm. d 40 cm. Yellow circles indicate interparticle pores

Fig. 11 The relationship between settlement displacement and seepage time
that in the high water content part. That is, the slope of SHCC in the low water content section is higher than that in the high water content section.

Comparison with statistical models results and related test results

Indirect use of statistical model calculation is also a common method to obtain unsaturated hydraulic conductivity (Rahimi et al. 2015; Nan et al. 2021). van Genuchten–Mualem model (VG–M model) and Childs and Collis-George model (CCG model) are the two most commonly used prediction models.

**VG–M model**

The parameters in the VG-M model (i.e., Eq. 5) are determined by the SWCC (i.e., Fig. 7), and it were determined according to the method described by van Genuchten (1980). The results are shown in Table 9.

| SWCC fitting model | \(a\)   | \(n\)   | \(m\)   |
|-------------------|---------|---------|---------|
| VG                | 0.002   | 0.370   | 1.588   |
| FX                | 0.001   | 0.435   | 1.770   |

![Figure 12](image) IPM data and VG–M model calculation results

**CCG model**

The expression of the CCG model is shown in Eq. (6). The obtained SWCC is divided into 20 equal parts along the horizontal axis, that is, \( M = 20 \). The rest of the parameter values take the values of water at a test temperature of 20 °C. The calculated results are shown in Table 10. A comparison of IPM data and calculated results of CCG model is shown in Fig. 13.

**Comparison with related test results**

The test results are compared with the results obtained by high-cost direct measurement methods in related literature (Yao et al. 2012). The comparison of IPM data and related test results is shown in Fig. 14. It can be obviously seen that when the volumetric water content of the soil sample is between 18 and 38%, the results of this study have a very high degree of fit with the results obtained by the high-cost direct methods. This also proves that the more conveniently and at a lower cost method adopted in this study is very effective for testing the unsaturated hydraulic conductivity of remoulded loess.

**Error analysis**

The root mean square error (RMSE) can measure the degree of consistency between the calculated values of different models and the experimental values. The smaller the value of RMSE, the closer the calculated value is to the experimental value, which indirectly indicates that the model is more applicable. The RMSE in this paper is calculated by Eq. (14), and the results are shown in Table 11.

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{N} (\ln x_i - \ln x_0)^2}{N}}.
\] (14)

As far as this experiment is concerned, through RMSE analyses, for the prediction of unsaturated hydraulic conductivity, the CCG model is superior to the VG–M model in accuracy and precision based on the same kind of SWCC. Compared with the FX model, the VG model
lacks a correction factor, which makes the description of the relationship between water content and suction in the low water content part (i.e. high suction part) inaccurate (Fayer and Simmons 1995). As shown in Fig. 7, there are differences in the shape of the curves of the two models. Furthermore, the parameters of the VG–M model largely depend on the slope at the midpoint of the SWCC, so that the predicted value of the VG–M model and the experimental results will produce certain errors, while the CCG model divides SWCC into \( N \) intervals, which can reduce errors caused by some intervals. The loess will collapse in the seepage process, and the porosity will change accordingly, which also leads to the error of the prediction model and the experimental results.

### Table 10  The calculated results of CCG model

| Interval i | Volumetric water content (%) | Based on VG SWCC | Based on FX SWCC |
|------------|------------------------------|------------------|------------------|
|            | Suction (kPa)                | Hydraulic conduc- |
|            |                             | tivity (cm/s)    | Suction (kPa)    |
|            |                             | Hydraulic conduc- |
|            |                             | tivity (cm/s)    |                  |
| 1          | 37.18                        | 4.27             | 2.21E-04         |
| 2          | 35.51                        | 10.67            | 7.60E-05         |
| 3          | 33.84                        | 17.38            | 3.82E-05         |
| 4          | 32.17                        | 25.07            | 2.08E-05         |
| 5          | 30.50                        | 34.31            | 1.15E-05         |
| 6          | 28.83                        | 45.77            | 6.41E-06         |
| 7          | 27.16                        | 60.38            | 3.51E-06         |
| 8          | 25.49                        | 79.53            | 1.88E-06         |
| 9          | 23.82                        | 105.36           | 9.70E-07         |
| 10         | 22.15                        | 141.30           | 4.79E-07         |
| 11         | 20.49                        | 193.13           | 2.23E-07         |
| 12         | 18.82                        | 271.09           | 9.68E-08         |
| 13         | 17.15                        | 394.53           | 3.83E-08         |
| 14         | 15.48                        | 602.93           | 1.35E-08         |
| 15         | 13.81                        | 985.50           | 4.08E-09         |
| 16         | 12.14                        | 1773.41          | 9.96E-10         |
| 17         | 10.47                        | 3694.28          | 1.79E-10         |
| 18         | 8.80                         | 9856.39          | 1.99E-11         |
| 19         | 7.13                         | 43,695.63        | 8.84E-13         |
| 20         | 5.46                         | 1,074,361.11     | 1.45E-15         |
Table 11  RMSE between calculated values of statistical models*, the results of related study and the results of this experiment

| Experimental results | RMSE  |  |  |
|----------------------|-------|-----------|-----------|
|                      | VG-M  | CCG       | Yao et al.|
| IPM data (VG SWCC)   | 1.688 | 1.210     | 0.495     |
| IPM data (FX SWCC)   | 3.209 | 1.023     | 0.613     |

*RMSE between statistical models and experimental results is performed under the same SWCC fitting model, that is, 1.688 is the RMSE value between IPM data (VG SWCC) and VG-M model (VG SWCC)

Conclusion

The unsaturated hydraulic conductivity of remoulded loess was measured based on filter paper method and soil column seepage test. The method is easy to implement and very low cost. The results indicate that in the one-dimensional soil column seepage process, the unsaturated hydraulic conductivity increases with the increase of the volumetric water content. And as the seepage time continues, the unsaturated hydraulic conductivity of the loess at different depths gradually becomes uniform. The micrograph shows that with the seepage process, the loess will collapse, resulting in the change of the pore structure, which will reduce the unsaturated hydraulic conductivity of the underlying loess to a certain extent. As far as the RMSE between the prediction model and the results of this experiment is concerned, because the CCG model has more subdivisions for SWCC, the error is relatively small. The prediction results of VG–M model will have a large error in the low volume water content (< 15%) part, and the lower the water content, the greater the error.

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Declarations

Conflict of interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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