MICROLENSING OF COLLIMATED GAMMA-RAY BURST AFTERGLOWS

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ABSTRACT

We investigate the stellar microlensing of collimated gamma-ray burst afterglows. A spherical afterglow appears on the sky as a superluminally expanding thin ring (a “ringlike” image), which is maximally amplified as it crosses a lens. We find that the image of the collimated afterglow becomes quite uniform (a “dislike” image) after the jet-break time (after the Lorentz factor of the jet drops below the inverse of the jet opening angle). Consequently, the amplification peak in the light curve after the break time is lower and broader. Therefore, detailed monitoring of the amplification history will be able to test whether afterglows are jets or not, i.e., dislike or not, if lensing occurs after the break time. We also show that some proper motion and polarization is expected, peaking around the maximum amplification. The simultaneous detection of proper motion and polarization will indicate that the brightening of the light curve is due to microlensing.

Subject headings: gamma rays: bursts — gravitational lensing

1. INTRODUCTION

Whether gamma-ray bursts (GRBs) are collimated or not is one of the most important questions regarding GRBs, since the jet configuration is crucial for almost all aspects of GRBs, such as energetics, statistics, and central engine models. There are several suggestions for the collimation of GRBs. When the Lorentz factor of the ejecta drops below the inverse of the opening angle, the ejecta begin to spread sideways, and we expect a break in the light curve of the afterglow (Rhoads 1999). Such breaks have been observed in several GRBs, such as GRB 990510 (Stanek et al. 1999; Harrison et al. 1999) and GRB 991216 (Halpern et al. 2000). A rapid decline rate (e.g., GRB 980519) or the unusually slow decline rate of GRB 980425 also suggest the collimation of GRBs (Halpern et al. 1999; Sari, Piran, & Halpern 1999; Nakamura 1999). However, in order to establish that the afterglows are jets, other observations are indispensable, such as early-afterglow observations at radio frequencies (Frail et al. 2000). Microlensing of the afterglows has the potential to be one such type of observation. The emission region of an afterglow seen by an external observer occupies an area of \( \sim 10^{17} \) cm on the sky, which is comparable to the lensing zone of a solar mass lens located at a typical cosmological distance (Loeb & Perna 1998). Therefore, microlensing can resolve afterglows, and hence we can expect that some features of the jets can be obtained by observation.

On the other hand, afterglows will be useful for constraining the cosmological density parameter of stellar mass objects, \( \Omega_{*} \), through microlensing. As a result of relativistic motion, the emission region appears to expand superluminally, so that the microlensing time is only \( \sim 1 \) day (Loeb & Perna 1998). This is much shorter than that of common sources, \( \gtrsim 10 \) yr (Gould 1995). The microlensing probability of a source at redshift \( z_{s} \sim 3 \) is \( \sim 0.3 \Omega_{*} \) (Press & Gunn 1973; Gould 1995; Koopmans & Wambsganss 2001). Since the known luminous stars amount to \( \Omega_{*} \gtrsim 4 \times 10^{-3} \) (Fukugita, Hogan, & Peebles 1998), roughly 1 in \( \sim 10^{3} \) GRBs should be within the lensing zone. If some fraction of dark matter is made of massive compact halo objects (MACHOs), as Galactic microlensing searches suggest (Alcock et al. 2000), we can expect a higher probability. Remarkably, a microlensed afterglow may have already been observed in the light curve of GRB 000301C as an achromatic bump (Garnavich, Loeb, & Stanek 2000; Gaudi, Granot, & Loeb 2001; but see also Panaitescu 2001). Since new GRB satellites such as the High Energy Transient Explorer 2 (HETE-2) and Swift will provide many opportunities to monitor afterglows frequently, it is important to study the effects of jets on the microlensing of GRB afterglows.

In §2 and 3, we calculate the equal–arrival-time surfaces and the surface brightness distribution of the afterglow image, respectively, taking jet effects into account. In §4, we calculate the microlensed light curve. In §5, we investigate the proper motion and the polarization induced by microlensing. Section 6 is devoted to a summary and discussions.

2. EQUAL–ARRIVAL–TIME SURFACE

Let \( E, \rho, \theta_{i}, \) and \( c_{b} \) be the burst energy, the density of the ambient gas, the initial opening angle, and the sound speed of the jet, respectively. The Lorentz factor of the shock front evolves approximately as

\[
\Gamma = \begin{cases} \Gamma_{0}(r/r_{b})^{-3/2} & r < r_{b}, \\ \Gamma_{s} \exp \left[ -(r-r_{b})/r_{c} \right] & r > r_{b}, \end{cases}
\]

where \( r_{b} = (75E/8\pi\rho c_{b}^{2})^{1/3} = 3.9 \times 10^{18} E_{52}^{2/3} n_{1}^{-1/3} (\sqrt{3}/c_{s}/c)^{-2/3} \) cm, \( r_{c} = (8/75)^{1/3} r_{b} \), and \( \Gamma_{s} = 2c_{b}/5c_{s} \theta_{i} \) (Rhoads 1999). Here \( E_{52} \) is the burst energy in units of \( 10^{52} \) ergs, and \( n_{1} \) is the ambient gas density in \( \text{cm}^{-3} \). We assume that the material is uniformly distributed across the jet at any \( r \), that the emission comes just behind the shock front, and that \( \Gamma \gg 1 \). Such approximations are sufficient and convenient for understanding key features before performing more detailed calculations (Rhoads 1999; Panaitescu & Mészáros 1999; Moderski, Sikora, & Bulik 2000; Huang, Dai, & Lu 2000), although future detailed analyses may demand a realistic model (Granot, Piran, & Sari 1999; Granot & Loeb 2001; Gaudi et al. 2001).

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The arrival time $T$ of a photon emitted at an angle $\theta$ from the line of sight is given by $T = t - r \mu/c$, where $t = \int dr/\gamma c(1 - \Gamma^{-2})^{1/2}$ and $\mu = \cos \theta$. This relation, combined with equation (1), can be written as

$$\frac{1 - \mu}{1/T_b} = \begin{cases} \frac{1}{81^2 \Gamma_b^5} \left( \frac{r}{r_b} \right)^{-1} \left( \frac{T}{T_b} - \frac{r}{r_b} \right)^{-1} & r < r_b, \\ \frac{1}{81^2 \Gamma_b^5} \left( \frac{T}{T_b} - 1 - 2 \frac{r}{r_b} \left[ \exp \left( \frac{2(r - r_b)}{r_b} \right) - 1 \right] \right) & r > r_b, \end{cases}$$

where $T_b = r_b \sqrt{8c^2 \Gamma_b^2} = 0.35 \Gamma_b^{1/3} n_1^{-1/3} (3c^2/c - 8/3)(\theta/0.01)^2$ days. Equation (2) gives the equal-arrival-time surface at $T$, and some examples are shown in Figure 1. In the power-law regime, $T \lesssim T_b$, the apparent size $r_{1,\text{max}}$ of the jet is determined by the equal-arrival-time surface (Sari 1998; Panaitescu & Mészáros 1998). The distance of the equal-arrival-time surface from the line of sight is given by $r_1 = r(1 - \mu^2)^{1/2} \approx r[2(1 - \mu)^{1/2}]$, with equation (2), which takes a maximum value of $r_{1,\text{max}} = (T/5T_b)^{3/5} \Gamma_b = 1.2 \\ \times 10^1 E_5^2 n_1^{-1/3}(\theta/0.01)^{-1/4}(T/1 \text{day})^{1/8} \text{ cm at } r/r_b = (T/5T_b)^{1/4}$. On the other hand, in the exponential regime, $T \gtrsim T_b$, the apparent size of the jet is determined by its opening angle, $\theta_b = \theta_0 + \gamma_c \cos \theta_0 r_b$, where $\gamma_c = \int dr/\gamma c$ is the comoving time in the jet frame. With equation (1), we find

$$\frac{\theta_b}{1/T_b} = \begin{cases} \frac{2 \gamma_c}{5 c} \left[ 1 + \left( \frac{r}{r_b} \right)^{3/2} \right] & r < r_b, \\ \frac{2 \gamma_c}{5 c} \left[ 1 + \left( \frac{r}{r_b} \right)^{-1} \left( 1 + 5 \frac{r_b}{r} \left[ \exp \left( \frac{r - r_b}{r_b} \right) - 1 \right] \right) \right] & r > r_b. \end{cases}$$

If the direction of the jet axis is $(\theta, \phi) = (\theta_0, 0)$, with $\theta = 0$ being the line of sight, the edge of the jet is given by $\theta^2 + \theta_0^2 - 2 \theta \theta_0 \cos \phi = \theta_b^2$. In Figure 1, the trajectories of the jet edges for $\theta_0 = 0$ and $\theta_0$ on the $\phi = 0$ plane are plotted. We can see that there is an intermediate regime in which the evolution of the Lorentz factor goes as a power-law, while the edge of the jet determines the apparent size (Rhoads 1999; Panaitescu & Mészáros 1999).

3. SURFACE BRIGHTNESS DISTRIBUTION

Let us consider a system that has an isotropic luminosity $dL_v$ in its rest frame and is moving with Lorentz factor $\gamma$ in a direction $(\theta, \phi)$ with a solid angle $d\Omega = d\theta d\phi$. An emitted photon with frequency $v$ is blueshifted to frequency $v/\gamma(1 - \beta \mu)$, where $\beta = (1 - \gamma^{-2})^{1/2}$. Using the Lorentz transformation, the observed flux at a distance $D$ and frequency $v$ is given by $dF_v = dL_v(1 - \beta \mu)/4\pi D^2 v^2(1 - \beta \mu)^2$. A jet, with a luminosity $L_v$ in its local frame, is a collection of such systems, so that we have $dL_v = L_v d\Omega/\gamma^4 \mu^2$, with equation (3). Now we assume that the luminosity in the local frame is $L_v \propto \Gamma^4 \mu^2$. Then, the flux per unit area, i.e., the surface brightness as a function of the distance from the center at times $T/T_b = 10^{-2}, 10^{-1}, 1, 10, 10^{-2}$, shown for frequencies above and below the typical synchrotron frequency by solid and dotted lines, respectively. Each surface brightness is normalized by the average surface brightness. The relative brightness at the center increases with time $T$. The image is ringlike at $T \lesssim T_b$ and disklke at $T \gtrsim T_b$. The divergence of the brightness at $r_1 = r_{1,\text{max}}$ is an artifact of the assumption that the radiation comes from a two-dimensional shell.
brightness $S(r_\ell, \phi; T) \equiv d^2F r_\ell dr_\ell d\phi$, can be obtained by
\[ S(r_\ell, \phi; T) \propto \Gamma^{3+b-3}(1-\beta \mu)^{b-3}\theta_0^{-2} \frac{d\mu/dr}{dr^2/d\mu}. \] (4)
where we use $d\Omega/r_\ell dr_\ell d\phi = 2d\mu/d\mu$ and the fact that the Lorentz factor of the shock front ($\Gamma$) is higher than that of the material behind it ($\gamma$) by a factor of $\sqrt{2}$ (Blandford & McKee 1976). With equations (1), (2), and (3), we find that equation (4) is a function of $r_\ell$, noting that $r_\ell \approx r(2(1 - \mu)^1/2$ and $1 - \beta \mu \approx 1 - \beta + 1 - \mu \approx 1/2^2 + 1 - \mu$. Since we can obtain $r$ for a given $r_\ell$ by $r_\ell \approx r(2(1 - \mu)^1/2$ with equation (2), the surface brightness can be calculated as a function of $r_\ell$. Note that there are two solutions of $r$ for each $r_\ell$, one from the front of the equal–arrival-time surface and the other from the back. As time goes on, the emission from the back does not contribute because of the jet geometry, as we can see from Figure 1.

The assumption $L_\ell \propto \Gamma^{3+b}$ is valid for frequencies far from the typical synchrotron frequency $\nu_m$ (Sari, Piran, & Narayan 1998; Sari et al. 1999). For simplicity, we neglect scattering, self-absorption, and electron cooling. The luminosity is then proportional to the total number of swept-up electrons, $N_e \propto \Gamma^{-2}$ (Rhoads 1999), times the radiation power from each electron, $P \propto \gamma^2 B^2 \propto \Gamma^4$. Since the typical frequency is $\nu_m \propto B^2 \propto \Gamma^2$, the luminosity at the typical frequency is $L_m \propto \Gamma^{-1}$. At $\nu \ll \nu_m$, we have $L_\ell = L_{n\ell}(\nu/\nu_m)^{1/2} \Gamma^{-1} \nu^{1/3}$. At $\nu \gg \nu_m$, if the electrons are accelerated to a power-law distribution with index $p$, we have $L_\ell = L_{n\ell}(\nu/\nu_m)^{p-1/2} \Gamma^{p-5/2} \nu^{-p/2}$. Therefore, we find $a = -2$, $b = 1/2$ at $\nu \ll \nu_m$, and $a = (3p - 5)/2$, $b = -(p - 1)/2$ at $\nu \gg \nu_m$. We adopt $p = 2.5$.

In Figure 2, we show the surface brightness distribution as a function of distance from the center for $\theta_0 = 0$ (the jet axis coincides with the line of sight). In the power-law regime, $T/T_b \ll 1$, the surface brightness is brighter near the edge and dimmer at the center, i.e., “ringlike” (Waxman 1997; Sari 1998; Panaitescu & Mészáros 1998). However, in the exponential regime, $T/T_b \gg 1$, the surface brightness becomes nearly constant, i.e., “disklike.” This qualitative feature does not depend on $\theta_0$. In the intermediate regime, a part of the ring will be missing because of the jet edge when $\theta_0 \neq 0$.

4. MICROLENSED LIGHT CURVE

We now consider a point-mass lens of mass $M$ that is located at $L$ from the source center on the sky (see Mao & Loeb 2001 for binary lenses). The Einstein radius on the source plane is given by $R_\ell = [(4GM/c^2)(D_sD_l/D_\ell)]^{1/2}$, with $D_s$, $D_l$, and $D_\ell$ being the angular diameter distance to the lens, distance to the source, and distance from the lens to the source, respectively (e.g., Schneider, Ehlers, & Falco 1992). A point lens is described by three parameters, the Einstein radius, $r_\ell \equiv R_\ell / (r_\ell/T_b) = 0.45(M/M_\odot)^{1/2}[(D_sD_l/D_\ell)/10^{28} \text{ cm}]^{1/2}E_{53}^{1/3}n_{11}^{-1}(3c/\epsilon)^{3/2}(\theta_0/0.01)^{-1}$, the impact parameter, $l \equiv L/(r_\ell/T_b)$, and the azimuthal angle of the lens position, $\phi_\ell$, with respect to the source center on the sky. Here we measure the distance from the line of sight in units of $r_\ell/T_b$.

The observed flux can be calculated numerically by $F_\ell = \int A(r_\ell, \phi)S(r_\ell, \phi; T) r_\ell dr_\ell d\phi$, where $A(r_\ell, \phi) = [u_\ell(d\ell^2 + 4)]/[u_\ell d\ell^2 + 4])$, and $u_\ell = [L^2 + r_\ell^2 - 2Lr_\ell \cos (\phi - \phi_\ell)]^{1/2}/R_\ell$. The unlensed flux can be obtained by setting $A = 1$. In Figure 3, the lensed flux of an afterglow, the unlensed flux, and the lensed flux while always retaining the initial surface brightness as a function of the time $T$, plotted by solid, dashed, and dotted lines, respectively. The origin of the vertical axis has no meaning. We set $r_\ell=1$, $l=1$, $\theta_0=0$, and $c_\ell = c/\sqrt{3}$. The top panel shows the flux at frequencies above the typical synchrotron frequency, and the bottom panel is at frequencies below the typical synchrotron frequency. In the top panel, the boxed region is expanded.

In the top panel of Figure 3, we consider the case $\nu \gg \nu_m$. At the maximum amplification, an amplification peak occurs around when the edge of the image crosses the lens, $r_\ell,\text{max} \sim L = r_\ell/T_b$. Here we set $l = 1$ so that the maximum amplification occurs at $T \sim T_b$, with the image being disklike.

In the top panel of Figure 3, we consider the case $\nu \gg \nu_m$. At the maximum amplification, an amplification peak would appear if the image were ringlike, as in the power-law regime$^3$ (Loeb & Perna 1998). However, since the image is disklike, the amplification peak becomes lower and broader, and apparently disappears for $\nu \gg \nu_m$, while for $\nu \ll \nu_m$, as shown in the bottom panel of Figure 3, the low amplification peak does exist in the light curve, even if the image is disklike. This is because the decline rate of the unlensed flux at $\nu \gg \nu_m$ is too steep ($-p = -2.5$) for the time derivative of the flux to be positive, but the slope at $\nu < \nu_m$ is very mild ($-\tfrac{1}{3})$. Note that the amplification factor is nearly the same in both cases. In Figure 4, the amplification factor is plotted by solid lines, and the amplification factor retaining the initial surface brightness is also plotted by dotted lines. The amplification factor for a uniform ring with a fractional width of 10%, which reproduces the bump in the light curve of GRB 000301C (Garnavich et al. 2000), is also plotted by dashed lines. Error bars of magnitude

$^3$ This statement may depend on a more detailed profile of the ring (Granot et al. 1999).
F. 4.—Amplification factor as a function of the time $T$ (solid lines), and while retaining the initial surface brightness (dotted lines). The amplification factor for a uniform ring with a fractional width of 10% is also plotted (dashed lines). Error bars of $\pm 0.1$ mag are also shown for reference. We set $r_E = 1, l = 1, \theta_0 = 0$, and $c_s = c/\sqrt{3}$. The top panel shows the flux at frequencies above the typical synchrotron frequency, and the bottom panel is at frequencies below the typical synchrotron frequency.

We set $l \leq 1$, and the top panel shows the flux $r_E l = 0.1, h_0 = 0, c_s = c/\sqrt{3}$. at frequencies above the typical synchrotron frequency, and the bottom panel is at frequencies below the typical synchrotron frequency.

Even when the multiple images formed by microlensing cannot be resolved, the centroid of the combined image is expected to move (Hosokawa et al. 1993; Hög, Novikov, & Polnarev 1995; Walker 1995; Miyamoto & Yoshii 1995; Paczyński 1998; Mao & Witt 1998). The order of the maximum displacement is estimated by $R_E/D_s \sim 10^{-11}(M/M_\odot)^{1/2} \sim 2(M/M_\odot)^{1/2}$ cm and $D_s \sim 10^{28}$ cm. This proper motion might be measured by upcoming missions, such as the Space Interferometry Mission (SIM), with positional accuracy down to $\sim 1$ $\mu$as (Paczyński 1998). To neglect the proper motion due to the jet-edge effect (Sari 1999), we set $\theta_0 = 0$. We can calculate the light centroid numerically by weighting the image positions with brightness (Walker 1995; Mao & Witt 1998). The top panel of Figure 5 shows $\pm 0.1$ are also shown, for reference. As analyzed by Gaudi & Loeb (2001) and Gaudi et al. (2001), a future monitoring campaign of a lensed afterglow will be able to reconstruct the radial structure of the afterglow image, although the quality of the observational data for GRB 000301C (about $\sim 0.1$ mag) may not be sufficient for this purpose. If the reconstructed image is disklike after the break time, it will indicate that the afterglows are jets.

In the intermediate regime, where the evolution of the Lorentz factor is a power law but the edge of the jet determines the image size, the amplification depends on the lens position when the ring is cut off by the jet edge, the amplification peak is small, while if the lens is on the other side, the peak is relatively large.

5. PROPER MOTION AND POLARIZATION

Even when the multiple images formed by microlensing cannot be resolved, the centroid of the combined image is expected to move (Hosokawa et al. 1993; Hög, Novikov, & Polnarev 1995; Walker 1995; Miyamoto & Yoshii 1995; Paczyński 1998; Mao & Witt 1998). The order of the maximum displacement is estimated by $R_E/D_s \sim 10^{-11}(M/M_\odot)^{1/2} \sim 2(M/M_\odot)^{1/2}$ $\mu$as with $R_E \sim 10^{17}(M/M_\odot)^{1/2}$ cm and $D_s \sim 10^{28}$ cm. This proper motion might be measured by upcoming missions, such as the Space Interferometry Mission (SIM), with positional accuracy down to $\sim 1$ $\mu$as (Paczyński 1998). To neglect the proper motion due to the jet-edge effect (Sari 1999), we set $\theta_0 = 0$. We can calculate the light centroid numerically by weighting the image positions with brightness (Walker 1995; Mao & Witt 1998). The top panel of Figure 5 shows $\pm 0.1$ are also shown, for reference.

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FIG. 5.—Top: Light centroid position of a GRB afterglow on the sky in units of the ratio of the Einstein radius $R_E$ to the source distance $D_s$ as a function of the time $T$, shown for three cases: $(r_E, l) = (0.1, 0.03), (1, 0.2), and (1, 1)$. The horizontal axis has the same meaning as in the bottom panel. Bottom: Polarization of a GRB afterglow as a function of the observed time $T$, shown for $(r_E, l) = (0.1, 0.03), (1, 0.2), and (1, 1)$. The polarization scales linearly with the polarization at each point of the image $\Pi_0$, and we assume $\Pi_0 = (p + 1)/(p + 7/3) \approx 72\%$. In both panels, frequencies are above the typical synchrotron frequency.

FIG. 6.—Top: Maximum magnitude of the light centroid displacement as a function of the normalized impact parameter $1/r_E$, shown for uniform surface brightness (solid line), initial surface brightness (dotted line), and a uniform ring with a fractional width of 10% (dashed line). The horizontal axis has the same meaning as in the bottom panel. Bottom: Maximum polarization as a function of the normalized impact parameter $1/r_E$, shown for uniform surface brightness (solid line), initial surface brightness (dotted line), and a uniform ring with a fractional width of 10% (dashed line). We assume $\Pi_0 = (p + 1)/(p + 7/3) \approx 72\%$. In both panels, the surface brightness is retained, so that the displacement and the polarization depend only on $1/r_E$.

4 There is information on the SIM at http://sim.jpl.nasa.gov.
the proper motion as a function of time at $v \gg v_m$. Initially, since the source is pointlike, the initial displacement is a function of $l/r_E$ (see Walker 1995, Fig. 1). As the source expands, the centroid moves toward the lens because of the finite source effect (Mao & Witt 1998). The centroid goes through the origin in coincidence with the maximum amplification. The top panel of Figure 6 shows the maximum magnitude of the displacement as a function of the normalized impact parameter $l/r_E$ for uniform surface brightness ($\text{thick line}$), initial surface brightness (dotted line), and a uniform ring with a fractional width of 10% (dashed line). Since the surface brightness is retained, the displacement depends only on $l/r_E$. The displacement is smaller for more uniform surface brightness but is always larger than the initial displacement.

We also expect some polarization in the microlensed afterglows (Loeb & Perna 1998). Here, following Sari (1999), we consider the optimal conditions in which the polarization at each point in the image is $\Pi_0 = (p + 1)/(p + 7/3) \approx 72\%$ toward the source center. To neglect the polarization due to the jet-edge effect (Ghisellini & Lazzati 1999; Sari 1999), we set $\theta_0 = 0$. The net polarization can be calculated by averaging with the jet-edge effect (Ghisellini & Lazzati 1999; Sari 1999), we set $\theta_0 = 0$. The net polarization can be calculated by averaging with the jet-edge effect (Ghisellini & Lazzati 1999; Sari 1999), we set $\theta_0 = 0$. The net polarization can be calculated by averaging with The bottom panel of Figure 5 shows the net polarization as a function of time at $v \gg v_m$. The polarization has a maximum toward the lens in coincidence with the maximum amplification. The bottom panel of Figure 6 shows the maximum polarization as a function of the normalized impact parameter $l/r_E$ for uniform surface brightness (thick line), initial surface brightness (dotted line), and a uniform ring with a fractional width of 10% (dashed line). Even in the exponential regime, the polarization due to microlensing is comparable to the polarization that is expected around the jet-break time, when $\theta_0 \neq 0$ (see Sari 1999, Fig. 4).

6. SUMMARY AND DISCUSSIONS

We have investigated the stellar microlensing of collimated GRB afterglows, taking jet effects into account. Using analytical expressions for the evolution of the jet, we have first calculated the surface brightness distribution of the jet. We find that the image is disklike after the jet-break time, rather than ringlike, as before the break time. We have further analyzed the microlensing signal in the light curve of the afterglow observed at frequencies far below and far above the typical synchrotron frequency $v_m$, where the radio is below $v_m$ for about 1 month, and the optical is above $v_m$ after about 1 day. If the edge of the image crosses the lens before the break time, the microlensing signal appears as an achromatic amplification peak in the light curve (Loeb & Perna 1998). The peak seen in GRB 000301C is believed to be a microlensing event occurring before the break time, in the ringlike regime (Garnavich et al. 2000; Gaudi et al. 2001; but see also Panaitescu 2001). We find that after the break time, the amplification peak becomes lower and broader because of the disklike image. Since detailed monitoring of the amplification history will be able to reconstruct the afterglow image (Gaudi & Loeb 2001; Gaudi et al. 2001), it could be tested whether the afterglows are jets or not, i.e., whether the reconstructed image is disklike after the break time or not. We should take care, since the amplification peak apparently disappears for $v \gg v_m$. At the break time, the peak amplitude depends on the lens position.

Microlensing also induces proper motion and polarization of the afterglow in coincidence with the maximum amplification. The magnitude of the proper motion is of the order of the Einstein radius of the lens, which might be detected by upcoming missions, such as SIM, with positional accuracy down to $\approx 1$ mas. From the proper motion, we can estimate the apparent size of the afterglow explicitly. The maximum polarization due to microlensing is comparable to the polarization that is expected around the jet-break time (Sari 1999), even if the image is disklike. The simultaneous detection of proper motion and polarization would indicate that the brightening of the light curve is due to microlensing, although an initial constant positive offset in the amplification history (Loeb & Perna 1998) and a specific level of chromaticity of the amplification history (Granot & Loeb 2001) are unique features of lensed afterglows.

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5 The radio scintillation is suppressed at $v \gtrsim 10$ GHz (Goodman 1997).

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