$Qq\bar{Q}q'$ States in Chiral SU(3) Quark Model

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Abstract

We study the masses of $Qq\bar{Q}q'$ states with $J^{PC} = 0^{++}, 1^{++}, 1^{+-}$ and $2^{++}$ in the chiral SU(3) quark model, where $Q$ is the heavy quark ($c$ or $b$) and $q$ ($q'$) is the light quark ($u$, $d$ or $s$). According to our numerical results, it is improbable to make the interpretation of $[cn\bar{c}n]_{1^{++}}$ and $[cn\bar{c}n]_{2^{++}}$ ($n = u, d$) states as $X(3872)$ and $Y(3940)$, respectively. However, it is interesting to find the tetraquarks in the $bq\bar{b}q'$ system.

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In the past three years, the discovery of few charmonium-like states $X(3872)^{[1]}$, $Y(3940)^{[2]}$ and $Y(4260)^{[3]}$ in experiments has excited great interests of physicists. Three different frameworks have been suggested to accommodate these states with their unusual characteristics: (1) $D - D^*$ molecules,$^{[4]}$ (2) $car{c}g$ hybrids,$^{[5]}$ and (3) diquark-antidiquark or four-quark (4q) states for short.$^{[6]}$ Maiani et al.$^{[6]}$ supposed that $X(3872)$ is a bound diquark-antidiquark state with $J^{PC} = 1^{++}$. With the spin-spin interactions and $X(3872)$ mass as input, they predicted the existence of $2^{++}$ state at 3952 MeV that could be identified with $Y(3940)$. Using the relativistic quark model, Ebert et al.$^{[6]}$ indicated that $X(3872)$ could be the tetraquark state with hidden charm and the masses of ground state tetraquarks with hidden bottom are below the open bottom threshold. Assuming that $X(3872)$ is a $qcar{q}ar{c}$ tetraquark and using its mass as input, Cui et al.$^{[6]}$ concluded that $0^+$ states will also exist if $X(3872)$ is really a $1^+$ tetraquark. In this Letter, we study the $Qqar{Q}ar{q}'$ states in the chiral SU(3) quark (CSQ) model, where heavy quark $Q = c, b$ and light quark $q, q' = u, d$ or $s$.

Although the $QQar{q}ar{q}'$ states were studied in the constituent quark model many years ago, no corresponding particles were found in experiments. The discovery of such charmonium-like states inspirits us to study the 4q states with content $Qqar{Q}ar{q}'$, moreover there is a lack of dynamics model calculation. In our pervious work, we concluded that $D_s(2317)$ and $D_s(2460)$ were not the pure $cnar{n}ar{s}$ 4q state in the CSQ model,$^{[7]}$ which is consistent with the general opinions about them. Thus it is interesting to study the $cqar{c}ar{q}'$ states again by using the same model parameters. Some authors$^{[8]}$ have reported that systems with a large mass difference among their components are more easily bound. Therefore, it is necessary to study the $bqar{b}ar{q}'$ states in the 4q-state picture.

The CSQ model is based on the constituent quark model of the light quark systems,
in which the constituent mass appears because of the vacuum spontaneous breaking, and at the same time the coupling between Goldstone bosons and quarks is automatically introduced for restoring the chiral symmetry and these boson exchanges are essential to obtain a correct description of the $NN$ phenomenology and the light baryon spectrum. However, for the heavy quark systems, their constituent part is very small and the vacuum spontaneous breaking effect is not important. The one-gluon-exchange (OGE) interaction between quarks and the confinement potential are enough to describe the main properties of the heavy quark systems. This basic framework is consistent with the QCD inspire and it is the usual treatment in many works. For heavy-light quark systems, since the masses of $D, D_s, B$ and $B_s$ mesons are quite large, which locate inside the chiral symmetry scale, at least as the first step, it is unnecessary to consider the Goldstone boson exchanges between light and heavy quarks. Therefore, in this work, for the light quark pairs, the interactions include confinement potential, OGE potential and Goldstone boson exchanges; and for the heavy-heavy and heavy-light quark interactions, the last one is not considered. Explicit expressions of the interacting potential derived from the nonrelativistic reduction of the Lagrangian on the static approximation and a more detailed discussion of the model can be found in Refs. [7,9].

The interaction parameters include the OGE coupling constant $g_i$, the confinement strengths $(a_{ij}, a_{ij}^0)$, and the chiral coupling constant $g_{ch}$. The parameters for the light quark pairs are taken from our previous works,[7,9] which gave a satisfactory description for the light baryon spectrum and the binding energy of the deuteron. The parameters for $cq$ or $cc$ quark pairs are taken from Ref.[7], which fitted the masses of $D, D^*, D_s, D_s^*, \eta_c, J/\Psi$ and $h_1(1p)$. Followed the same method, the model parameters for $bq$ and $bb$ quark pairs can be fixed by the masses of $B, B^*, B_s, B_s^*, \eta_b$ and $\Upsilon(1s)$, which are listed
TABLE I: Model parameters for the $bq$ and $bb$ quark pairs.

| Parameter | Value 1 | Value 2 |
|-----------|---------|---------|
| $m_b$(MeV) | 4717 | 0.52 |
| $(\text{MeV/fm}) a_{ba}$ | 275 | 275 |
| $(\text{MeV}) a^0_{ba}$ | 275 | 275 |
| $(\text{MeV/fm}) a_{bs}$ | -141.1 | -112 |
| $(\text{MeV}) a^0_{bs}$ | -141.1 | -112 |
| $(\text{MeV/fm}) a_{bb}$ | 275 | 275 |
| $(\text{MeV}) a^0_{bb}$ | 275 | 275 |

TABLE II: Masses (MeV) of $Q\bar{q}$ and $Q\bar{Q}$ mesons. Experimental data are taken from PDG.

| Mesons | $D$ | $D^*$ | $D_s$ | $D^*_s$ |
|--------|-----|------|------|--------|
| Exp.   | 1867.7 | 2008.9 | 1968.5 | 2112.4 |
| Theor. | 1888 | 2099 | 1969 | 2130 |

| Mesons | $B$ | $B^*$ | $B_s$ | $B^*_s$ |
|--------|-----|------|------|--------|
| Exp.   | 5279.2 | 5325 | 5369.6 | 5416.6 |
| Theor. | 5288 | 5320 | 5371 | 5412 |

| Mesons | $\eta_c$ | $J/\Psi$ | $h_c(1p)$ | $\eta_b$ | $\Upsilon(1s)$ |
|--------|----------|----------|-----------|---------|-------|
| Exp.   | 2979.6 | 3096.9 | 3526.2 | 9300 | 9460.3 |
| Theor. | 2990 | 3098 | 3568 | 9404 | 9460 |

in Table 1, and the theoretical results for the masses of $Q\bar{q}$ and $Q\bar{Q}$ mesons are shown in Table 2. It is seen that the theoretical masses of mesons are reasonably consistent with their experimental values.
For the wave function of a \( 4q \) state, let us describe it in the form

\[
\Psi(4q) = \Psi_{4q}(0s^4) \left[ (q_1q_2)^{I_1}_{S_1,C_1} (\bar{q}_3\bar{q}_4)^{I_2}_{S_2,C_2} \right]^{I}_{S,C},
\]

where \( \Psi_{4q}(0s^4) \) is the orbital part and \( \left[ (q_1q_2)^{I_1}_{S_1,C_1} (\bar{q}_3\bar{q}_4)^{I_2}_{S_2,C_2} \right]^{I}_{S,C} \) is the flavor-spin-color part. As the first step, let us take the spacial wave function of such four quarks in \( S \) wave state. Since there is no need to antisymmetrize the wave function under the interchange of the coordinates of a heavy and a light quark, the possible configurations of \( Qq\bar{Q}\bar{q}' \) states with \( (J^P; I) = (1^+; 0) \) read

\[
\begin{align*}
\Psi_A &= \Psi_{Qq\bar{Q}\bar{q}'}(0s^4) \left[ (Qq)^{\frac{1}{2}}_{1,3c} (\bar{Q}\bar{q}')^{\frac{1}{2}}_{0,3c} \right]^{0}_{1,1c}, \\
\Psi_B &= \Psi_{Qq\bar{Q}\bar{q}'}(0s^4) \left[ (Qq)^{\frac{1}{2}}_{1,6c} (\bar{Q}\bar{q}')^{\frac{1}{2}}_{0,6c} \right]^{0}_{1,1c}, \\
\Psi_C &= \Psi_{Qq\bar{Q}\bar{q}'}(0s^4) \left[ (Qq)^{\frac{1}{2}}_{0,3c} (\bar{Q}\bar{q}')^{\frac{1}{2}}_{1,3c} \right]^{0}_{1,1c}, \\
\Psi_D &= \Psi_{Qq\bar{Q}\bar{q}'}(0s^4) \left[ (Qq)^{\frac{1}{2}}_{0,6c} (\bar{Q}\bar{q}')^{\frac{1}{2}}_{1,6c} \right]^{0}_{1,1c}, \\
\Psi_E &= \Psi_{Qq\bar{Q}\bar{q}'}(0s^4) \left[ (Qq)^{\frac{1}{2}}_{1,3c} (\bar{Q}\bar{q}')^{\frac{1}{2}}_{1,3c} \right]^{0}_{1,1c}, \\
\Psi_F &= \Psi_{Qq\bar{Q}\bar{q}'}(0s^4) \left[ (Qq)^{\frac{1}{2}}_{1,6c} (\bar{Q}\bar{q}')^{\frac{1}{2}}_{1,6c} \right]^{0}_{1,1c}.
\end{align*}
\]

Note that both the configurations \([ (Qq)_{3c} (\bar{Q}\bar{q}')_{3c} ]_{1c} \) and \([ (Qq)_{6c} (\bar{Q}\bar{q}')_{6c} ]_{1c} \) are taken into account in wave functions. Under charge conjugation, \( \Psi_A (\Psi_B) \) and \( \Psi_C (\Psi_D) \) interchange while \( \Psi_E (\Psi_F) \) is odd. Thus, \( J^P = 1^+ \) complex contains two \( C \)-even and four \( C \)-odd states:

\[
\begin{align*}
| J^{PC}, I \rangle &= |1^{++}, 0 \rangle_1 = \frac{1}{\sqrt{2}} [ \Psi_C + \Psi_A ], \\
|1^{++}, 0 \rangle_2 &= \frac{1}{\sqrt{2}} [ \Psi_D + \Psi_B ],
\end{align*}
\]
$|1^{+-}, 0\rangle_1 = \frac{1}{\sqrt{2}}[\Psi_C - \Psi_A]$,  
$|1^{+-}, 0\rangle_2 = \frac{1}{\sqrt{2}}[\Psi_D - \Psi_B]$,  
$|1^{+-}, 0\rangle_3 = \Psi_E$,  
$|1^{+-}, 0\rangle_4 = \Psi_F$,  

where $|1^{++}, 0\rangle_1$ in Eq. (1) and $|1^{+-}, 0\rangle_1$ and $|1^{+-}, 0\rangle_3$ in Eq. (2) are the so-called diquark-antidiquark states in Ref.[6]. In order to analyse the $C$ values of Eqs. (1) and (2), we recouple the color basis from $\{|3_{Qq}, 3_{Qq'}, 6_{Qq}, 6_{Qq'}\}$ to $\{|1_{QQ}, 1_{qq'}, 8_{Qq}, 8_{q'q}\}$. Take $|1^{++}, 0\rangle_1$ for example,

$$
|1^{++}, 0\rangle_1 = \frac{1}{\sqrt{3}}\psi_{QqQq'}(0s^4)[(Q\bar{Q})^0_{1,1^c}(q\bar{q'})^0_{1,1^c}]^0_1^1,
$$

or

$$
= \frac{1}{\sqrt{6}}\psi_{QqQq}(0s^4)[(Q\bar{Q})^{\frac{3}{2}}_{0,1^c}(q\bar{q'})^{\frac{3}{2}}_{1,1^c}]^0_1^1.
$$

From Eq. (3), the only one with $C = +$ is that both spins of $Q\bar{Q}$ and $q\bar{q'}$ pairs equal to 1, namely, $S_{Q\bar{Q}} = S_{q\bar{q'}} = 1$. Similarly, the states with $C = -$, $S_{Q\bar{Q}}$ and $S_{q\bar{q'}}$ should be 0 and 1, or 1 and 0, respectively. After a similar deducing, we can confirm that Eq. (2) are $C$-odd states. For $Qq\bar{Q}q'$ states with $(J^{PC}; I) = (2^{++}; 0)$, the possible configurations read

$$
|2^{++}, 0\rangle_1 = \psi_{QqQq'}(0s^4) [(Qq)^{\frac{3}{2}}_{1,3^c}(Q\bar{Q})^{\frac{3}{2}}_{1,3^c}]^0_{2,1^c},
$$

$$
|2^{++}, 0\rangle_2 = \psi_{QqQq'}(0s^4) [(Qq)^{\frac{3}{2}}_{1,6^c}(Q\bar{Q})^{\frac{3}{2}}_{1,6^c}]^0_{2,1^c}.
$$

For the other quantum numbers, such as $J^{PC} = 0^{++}$ or isospin $I = 1$, the wave functions can be written with the same rule. For saving space, we do not show them here.
TABLE III: Masses (MeV) of $cq\bar{c}q'$ and $b\bar{b}q'$ states with different $(J^{PC}, I)$.

| 4q states $cn\bar{c}n'$ $cs\bar{s}$ $bn\bar{b}n'$ $bs\bar{s}$ |
|------------------|------------------|------------------|------------------|
| $(0^{++};0)$    | 3956 4177 10347 10608 |
| $(0^{++};1)$    | 3862 — 10260 — |
| $(1^{++};0)$    | 4047 4444 10395 10799 |
| $(1^{++};1)$    | 4123 — 10464 — |
| $(1^{-+};0)$    | 4003 4271 10366 10639 |
| $(1^{-+};1)$    | 3926 — 10285 — |
| $(2^{++};0)$    | 4047 4444 10395 10799 |
| $(2^{++};1)$    | 4123 — 10464 — |

By using the variation method, the energies of these states can be obtained. For the 4q states with the same $(J^{PC}, I)$, the configuration mixture should be considered.

For the recently observed $X(3872)$ and $X(3940)$ resonances the $S$ wave, $cn\bar{c}n$ assignment has been suggested with $J^P = 1^{++}$ and $2^{++}$, respectively.\[^{[10]}\] Thus, we focus our calculation on $J^{PC} = 0^{++}, 1^{++}$ and $2^{++}$. At the same time, the $b\bar{b}q'$ states with the same quantum numbers are studied. The masses of $cq\bar{c}q'$ and $b\bar{b}q'$ states are calculated and the numerical results are presented in Table 3.

In principle, the allowed decay modes depend on the relationship between the tetraquark mass and the sum of the masses of the possible decay products. In the case of $[cn\bar{c}n]_{(1^{++},0)}$ states, according to Eqs. (1), (3), and (4), the possible decay products are $J/\Psi + \omega(n\bar{n})$ and $D + D^*$. Similarly, for the $[cn\bar{c}n]_{(1^{++},1)}$ states, they are $J/\Psi + \rho$ and $D + D^*$. From Table 3, the masses of $[cn\bar{c}n']_{1^{++}}$ states are 4047 MeV and 4123 MeV for
isospin equal to 0 and 1, respectively. They are all above the threshold of $DD^*$ (3897 MeV). Therefore, $[cn\bar{c}n']_{1^{++}}$ is broad and not easy to be detected experimentally, and this result is not consistent with the experiments of $X(3872)$. Thus, we think that the 4q-state picture cannot be suitable to $X(3872)$ in our calculation. For $X(3940)$, the same conclusion can be obtained, which is different from the conclusion of Maiani et al.$^6$ and Ebert et al.$^6$ who indicated that $Y(3940)$ is a $2^{++}$ tetraquark state. On the other hand, in our model the lightest scalar $0^{++}$ $cn\bar{c}n'$ states are above the threshold of $DD$ (3776 MeV) and thus are broad, which is consistent with the conclusion of Ebert et al.$^6$, but different from the result of Maiani et al.$^6$ and Cui et al.$^6$. In short, the $cq\bar{c}q'$ states with $J^{PC} = 0^{++}$, $1^{++}$ and $2^{++}$ may not be tetraquark states in our present calculation.

How about the $bq\bar{b}q'$ states? Do they have any chance to exist as tetraquark states? For states $[bn\bar{b}n']_{1^{++}}$, from Table 3, the masses are 10395 MeV and 10464 MeV for isospin $I = 0$ and 1, respectively, which is below the threshold of $BB^*$ (10608 MeV). Thus, $[bn\bar{b}n]_{(1^{++},0)}$ can only decay to $\Upsilon(1s) + \omega(n\bar{n})$, and $[bn\bar{b}n]_{(1^{++},1)}$ can go to $\Upsilon(1s) + \rho$. Although $[bn\bar{b}n']_{1^{++}}$ is not a bound state here, its mass is below the threshold of $BB^*$. Therefore, we think that it may be a 4q state and can be predicted to be narrow. Such state is worth being found in experiment. Additionally, our numerical results show that other $bq\bar{b}q'$ states we have studied may be 4q states, for instance, $[bn\bar{b}n]_{0^{++}}$ and $[bq\bar{b}q']_{1^{+-}}$. Although the masses of such states are high, about 10 GeV, we also expect them to be found in the future experiments.

Form Table 3, we note that the masses of $[Qq\bar{Q}q']_{1^{++}}$ and $[Qq\bar{Q}q']_{2^{++}}$ with the same isospin are equal to each other. As an example, the energies of every configuration state for $[cn\bar{c}n]_{(1^{++},0)}$ and $[cn\bar{c}n]_{(2^{++},0)}$ are given in Table 4. If only the $|1^{++}, 0\rangle_1$ and $|2^{++}, 0\rangle_1$ states are taken into account, the masses of such two states are different. However, the
configuration mixture should be considered for the state with the same quantum numbers, unless the contribution of some configuration states are not important. From Table 4, we also note that the energy of $|1^{++}, 0\rangle_1$ in Eq. (1) is slightly higher than the energy of $|1^{++}, 0\rangle_2$. This result is model dependent, and the possible main cause is that the color electrostatic field (CEF) energy is considered in our calculation. Additionally, whether the heavy-light diquark is formed or not in the heavy-light 4q systems is also an open question now. After considering the configuration mixture, the masses of $|1^{++}, 0\rangle$ and $|2^{++}, 0\rangle$ are the same as shown in Table 3. The reason is as follows: Since $|1^{++}, 0\rangle$ and $|2^{++}, 0\rangle$ have the same $C$ number, both spins of $c\bar{c}$ and $q\bar{q}$ pairs in such states equal to 1.

If $E_1 > E_2$, the lower eigenvalue of $|1^{++}, 0\rangle$ is $2E_2 - E_1$, where $E_1$ and $E_2$ are the expected values of Hamiltonian on states $|1^{++}, 0\rangle_1$ and $|1^{++}, 0\rangle_2$, respectively. The corresponding eigenvector is $|1^{++}, 0\rangle = \sqrt{4/5}|1^{++}, 0\rangle_1 + \sqrt{2/3}|1^{++}, 0\rangle_2$. By calculating the contribution of the interaction potentials, we note that only the component $[(c\bar{c})^0_{1,1}; (n\bar{n})^0_{1,1}][0]_1$ contributes.

Following the same process, we can obtain $|2^{++}, 0\rangle = \sqrt{2/3}|2^{++}, 0\rangle_1 + \sqrt{3/2}|2^{++}, 0\rangle_2$, and the interesting component is $[(c\bar{c})^0_{1,1}; (n\bar{n})^0_{1,1}][0]_2$. Since a $c\bar{c}$ $(n\bar{n})$ pair in such two states have the same quantum numbers, the masses of $|1^{++}, 0\rangle$ and $|2^{++}, 0\rangle$ are equal to each other.

This result means that the molecule picture may be more suitable to describe $X(3872)$ and $Y(3940)$. Obviously, if the CEF energy is deleted in our calculation, the mass of such two states will be different. Possibly, this is the place to review whether considering the contribution of CEF or not.

In summary, we have studied the masses of $Qq\bar{Q}q'$ states in the CSQ model, and attempted to give a reasonable interpretation of $X(3892)$ and $X(3940)$. Our numerical results show that they may not be explained as the pure 4q state, while the $bnb\bar{n}'$ states with $J^{PC} = 0^{++}$, $1^{++}$ and $1^{+-}$ may be tetraquarks and they are worth being detected in
TABLE IV: Energies (MeV) of every configuration state for $cn\bar{c}\bar{n}$ states with $(J^{PC}, I) = (1^{++}, 0)$ and $(2^{++}, 0)$.

| state | $|1^{++}, 0\rangle_1$ | $|1^{++}, 0\rangle_2$ | $|2^{++}, 0\rangle_1$ | $|2^{++}, 0\rangle_2$ |
|-------|-----------------|-----------------|-----------------|-----------------|
| Energy | 4167 | 4125 | 4230 | 4151 |

experiments. In our present calculation, the orbital wave function of the four quarks are in the $S$ wave state. This is the simplest condition, and other conditions are also allowed. Since our 4q states are roughly of hadronic size, the contribution of annihilation mechanics should be taken into account. Moreover, the two quark-antiquark cluster structure is worth using in the study of $X(3872)$ and $Y(3940)$. These aspects will be studied in the next step to improve our calculations.

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