Improving CO₂ emission assessment of diesel-based powertrains in dynamic driving cycles by data fusion techniques

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Abstract
This article proposes a method based on the Kalman filter to improve the accuracy of the CO₂ measurement in driving cycles such as worldwide harmonized light vehicles test cycles or real driving cycles which are inherently subject to a loss in accuracy due to the dynamic limitations of the CO₂ analysers. The information from the analyser is combined with the electronic control unit estimation of the fuel injection. The characteristics of diesel engines and, in particular, the high efficiency of the combustion process and the diesel oxidation catalyst allows to compute the CO₂ emissions from the fuel consumption estimation of the electronic control unit by applying the carbon balance method assuming negligible HC and CO emissions. Then, the assessment of the CO₂ analyser response time and accuracy allows to pose an estimation problem that can be solved by a Kalman filter. The application of the method to different driving cycles shows that analyser dynamic limitations may lead to an overestimation of the CO₂ figures that can reach 4% in highly dynamic tests such as the worldwide harmonized light vehicles test cycles. The technique thus has further potential application to replicating real driving cycles on the chassis dynamometer for real driving emission testing.

Keywords
CO₂ emissions, data fusion, worldwide harmonized light vehicles test cycles, real driving emissions

Introduction
Internal combustion engines (ICE) are still leading the powertrain technology market, despite an increase in the share of new technologies. In addition, ICEs still remain present at the core of other powertrain technologies such as hybrid electric vehicles (HEVs). In this sense, most part of the transport energy comes ultimately from petroleum, leading to energy consumption, sustainability and greenhouse gas (GHG) issues. Under this scope, there is a rising importance of light-duty vehicles CO₂ figures due to regulations and society concern about GHG. In addition, discrepancies between declared and actual performance of the vehicle in terms of fuel consumption and emissions (including CO₂)¹ have led to significant changes in vehicle certification. The traditional New European Driving Cycle (NEDC), with low dynamic content, is a poor representation of on-road driving characteristics. This cycle has been replaced by the more dynamic and aggressive worldwide harmonized light-duty vehicle test cycle (WLTC) or even real driving emission (RDE) for type approval tests in Europe. The higher dynamics of the new regulation tests may make traditional methods to measure emissions obsolete because of reduced accuracy due to their limited dynamic response.

In general, CO₂ figures are computed from exhaust CO₂ concentration and exhaust mass flow measurements. Both engine test bench and portable emissions measurement systems (PEMS) use CO₂ concentration analysers that are based on the non-dispersive infrared (NDIR) method.² Despite their high accuracy in steady state, they suffer from a slow time response (usually with a time constant between 2 and 5 s) and a delay.
that, depending on the length of the pipe connecting the exhaust with the analyser, can exceed 5–10 s.

Regarding the required exhaust mass flow to compute the CO₂ emissions from the concentration provided by the analyser, accurate measurement is a challenging task due to the high temperature of the gas and its particulate and moisture content. Complex methods can be used to measure the exhaust mass flow, for example, the smooth approach orifice (SAO) method, most of them rely on the computation of the exhaust mass flow from the ratio between the CO₂ concentrations of the exhaust flow and a diluted flow whose mass flow is known. Air mass flow can be measured with high accuracy using lab sensors or from electronic control unit (ECU) readings (if a flow metre is available). In this way, exhaust mass flow can be estimated from air mass flow measurement and fuel consumption neglecting transport from intake to exhaust or applying a correction factor. It should be noted that using the ECU signals for exhaust mass flow estimation is accepted by European Union (EU) regulation concerning the emission testing procedure for light-duty vehicles. Vu et al. report excellent correlations between engine or even injection test rigs. As commercially available methods for fuel measuring, in this way, exhaust mass flow can be estimated from ECU sensors and high accuracy SAO measurements, concluding that this is a viable method.

CO₂ emissions can also be estimated from a fuel consumption measurement and the stoichiometry assuming a complete combustion and known fuel composition. Once again, the problem is to find a fuel consumption sensor with high accuracy but also good dynamic response. The reader is referred to Brace et al. and references within for a detailed survey and description of different methods to measure fuel consumption. Similarly, Gandhi et al. make a comparison of commercially available methods for fuel measuring, in this case with focus on engine or even injection test rigs. As a summary, the following can be considered:

- Gravimetric balances show good accuracy at steady-state conditions but are subject to dynamic issues due to the pipe length and dilatation coming from temperature gradients.
- Coriolis flow metres, in general, have limited accuracy and are intrusive since require the modification of the fuel circuit.
- Universal exhaust gas oxygen (UEGO) or wideband λ sensors can be also used with this aim, but their dynamic response is still limited and their accuracy is strongly reduced as the composition is moved away from stoichiometric conditions.
- The fuel estimation from the ECU has limited accuracy, since it is based on tabulated values obtained from steady-state measurements during the calibration process and does not take into account possible disturbances, engine-to-engine dispersion or ageing. However, the fuel estimation from the ECU does not suffer from the slow response of sensors. In Varella et al., compare CO₂ estimations from the ECU and from an exhaust gas analysed in different driving cycles finding higher values of 4.5% and 2% on average for compression ignition (CI) and spark ignition (SI) vehicles.

This article will discuss the impact of the dynamic limitations of standard NDIR CO₂ analysers on the accuracy of their measurements in dynamic driving cycles. Then, this article proposes the combination of the NDIR CO₂ analyser with a model of fuel consumption from the ECU in order to make up for the analyser limitations in dynamic conditions. The objective is to combine both information sources to provide a CO₂ estimation that improves both model and sensor readings. Particularly, a data fusion approach based on the Kalman filter (KF) is proposed. Assuming that the noises in both the analyser and model signals are Gaussian, the KF will provide the optimal estimation of the CO₂ in the sense of minimum square error. Then, the proposed method will improve the accuracy of the CO₂ analyser in transient conditions while keeping the CO₂ analyser accuracy in steady state. Finally, this article proposes a method to obtain the KF parameters from the analysis of the NDIR analyser and the cycle characteristics.

KF s have been previously used in the framework of exhaust gas concentration estimation. In particular, Hsieh and Wang propose to use an extended Kalman filter (EKF) to estimate the NOₓ emissions and Alberer and Del Re propose a KF to correct exhaust oxygen concentration measurements from a UEGO sensor. Guardiola et al. also propose KF-based methods to improve sensor and model estimation for NOₓ and fuel-to-air ratio estimation. A complete review on observation techniques for exhaust gas concentrations can be found in Blanco-Rodriguez. Previous works are motivated to provide real-time estimations of the required variables for control purposes. They, therefore, have a strong focus on the real-time applicability and computation burden of the methods. In the present work, the focus is on the accuracy of the method since there are no real-time requirements when the objective is to provide a realistic measure of the CO₂ emission during a driving cycle in a testing environment.

This article is structured as follows: Section “Experimental set-up” introduces the experimental set-up employed in the present work. The third section is aimed to describe the problem to be addressed, that is, provide an estimation of the CO₂ emission improving the accuracy of the state-of-the-art analyser employed in a real (and therefore dynamic) driving cycle. With this aim, analyser and model limitations will be discussed. The fourth section deals with the proposed approach, and both the method and its calibration will be described. The obtained results will be discussed in
section “Results” and finally section “Conclusion” will underline the paper contributions.

**Experimental set-up**

Tests have been carried out on a 2014 Euro5b compliant 2.0l diesel light-duty vehicle installed in a chassis dyno test cell at University of Bath. The vehicle in question, although being Euro5b compliant, was an early adopter of NOx aftertreatment and is fitted with a selective catalytic reduction (SCR) system. The AVL chassis dynamometer was operated in road load simulation mode, using representative coastdown data and the vehicle was driven by a Stahle robot driver based on a target vehicle speed trace. The environment temperature in the test cell was controlled to a set point of 23°C. A detailed description of the test cell can be found in Galindo et al.\textsuperscript{17} To maintain the repeatability of the testing, the factors identified in Brace et al.\textsuperscript{4} and Chappell et al.\textsuperscript{18} were controlled during testing.

Regarding the vehicle, the main engine operating variables were obtained through the vehicle CAN bus by means of an Influx Technology Rebel CT data recorder. In particular, values for the air mass flow \( \dot{m}_{\text{air}} \) and fuel mass flow estimation \( \dot{m}_{\text{fuel}} \) are obtained through this system. In addition, exhaust emissions and particularly CO2 were measured by means of a Horiba MEXA ONE system with the sampling point located downstream of the diesel oxidation catalyst (DOC) and diesel particulate filter (DPF). The gas transport due to the sampling point is corrected aligning the CO2 signal with the fuel injection \( \dot{m}_{\text{fuel}} \). The CO2 is measured with an NDIR analyser whose main characteristics are listed in Table 1.

![Table 1. Main characteristics of the CO2 analyser.](image)

| Variable                     | Value                        |
|------------------------------|------------------------------|
| Steady-state accuracy \( \sigma_{\text{SS}} \) | 0.1% (0.5% of the full scale) |
| Response time \( t_{\text{analyser}} \)          | 0.65 s \( (f_0 = 1.5 - 2.3 t_{\text{analyser}}) \) |

**Problem description**

Figure 1 shows the main issue of exhaust CO2 analysers. In particular, it shows different measured signals in the first acceleration of the NEDC as an example. The upper plot shows the vehicle speed evolution, where the driver tries to follow a predefined driving profile that in this part of the cycle consists on an acceleration from 0 to 15 km/h and back again to 0. Due to the engine start and stop system, the engine starts 3 s after the beginning of the test. At this point, a sudden increase in the fuel mass flow estimation from the ECU and measured air mass flow can be observed in the second and third plots. If the focus is put on the CO2 concentration measured by the test bench analyser (see the fourth plot), the main issue of exhaust gas analysers becomes apparent. The exhaust gas analysers have long response times in comparison with the other engine transducers, meaning that the measured emission concentrations are delayed and filtered with respect to the...
other engine parameters. In particular, in Figure 1, a smooth increase in the CO2 concentration after the engine start can be observed. It should be noted that this signal has been phased using a constant time offset with the fuel injection provided in the second plot, since the exhaust gas analyser exhibits not only a limited dynamic response but a significant delay because of the exhaust and sampling line transport delay. One can clearly identify an excessive filtered response in the measured CO2 concentration since a sudden increase in the exhaust CO2 concentration to 0% would be expected when the engine is turned off after the vehicle stop.

CO2 emissions \( (W_{CO2}) \) are computed from the concentration and mass flow measurements as

\[
W_{CO2} = \frac{MW_{CO2}}{MW_{exh}} CO2 W_{exh}
\]

where \( MW_{CO2} \) and \( MW_{exh} \) represent the molecular weights of CO2 and exhaust mass flow used to transform measurements in molecular fraction to mass concentration. Note that this ratio depends on the exhaust gas composition; however, it is usually considered constant in regulations emissions testing of light duty vehicles, imposed as 1.517 for diesel engines.\(^{19}\) CO2 is the concentration provided by the analyser and \( W_{exh} \) is the exhaust mass flow. When CO2 emissions are computed from the concentration and mass flow measurements, an error is expected due to the lack of dynamic response of the gas analyser, even when phasing the signals is the best possible. In this sense, the evolution in the CO2 mass flow shown in the lower plot of Figure 1 is subject to some filtering due to the CO2 analyser performance.

A relation between the fuel consumption \( W_f \) and the CO2 emission \( (W_{CO2}) \) can be obtained through a carbon balance, provided a known composition of the fuel \((C, H, O)\) as\(^{20}\)

\[
W_f = W_{HC} + \frac{MW_C + \frac{1}{2} MW_H + \frac{3}{2} MW_O}{MW_{CO2}} \left(W_{CO2} + \frac{MW_C}{MW_{CO2}} W_{CO2} \right)
\]

where \( MW \) represents the molecular weight of the different species involved (denoted as subscripts C, H, O for carbon, hydrogen and oxygen, respectively), while H/C and O/C are hydrogen to carbon and oxygen to carbon ratios of the specific fuel. Note that HC and CO emissions also play a role in the carbon balance, so from a strict point of view, they should be known to convert fuel to CO2 emissions or vice versa. However, in the case of diesel engines, the HC and CO emissions are negligible compared to the CO2 emissions as can be checked in Figure 2. It is observed that CO2 represents more than the 99.5% of overall carbon-based emissions during most part of the cycle, and only during the starting process and first phase of the warm up (first 200 s), the ratio drops to values around 98%.

According to the previous paragraph, equation (3) may be simplified with little loss of accuracy to

\[
W_f = k_{CO2-f} W_{CO2}
\]

where \( k_{CO2-f} \) is a calibration constant aimed to consider any uncertainty in the fuel composition and the contribution of potential CO and HC emissions to the carbon balance. With a proper identification of constant \( k_{CO2-f} \) (in the present work 0.431), equation (3) can be used to estimate the CO2 emissions from the ECU value for fuel consumption. In order to compute the CO2 concentration at the exhaust, equation (1) can be applied.

Figure 3 shows an example where the calculated CO2 concentration from the ECU fuel consumption exhibits a very similar pattern to that provided by the exhaust gas analyser. In fact, the CO2 emissions from the ECU estimation of fuel consumption are used in this work to put the analyser raw measurement in phase with the rest of test bench signals by cross-correlation maximisation.\(^{21}\) In addition, Figure 3 shows discrepancies between raw analyser measurements and the application of equation (3) at both steady and transient conditions. In the case of steady-state operation (e.g. 20 s), some differences appear due to the inaccuracy of ECU fuel estimation and simplifications done in equation (3). Concerning transient conditions (e.g. 5–12 s), the method based on the ECU estimation is not affected by the filtering associated with sensors. This is most notable during periods of fuel cut-off (e.g. 20 s) when the engine is turned off after the vehicle stop.
The problem described in the previous section can be solved using data fusion, where among a wide set of techniques,22 the KF23 is one of the most widely used algorithms. A complete description of the algorithm is outside of the scope of the present paper and only the main equations are recalled for the sake of readability. Consider the generic discrete time invariant system

\[
\begin{align*}
    x_k &= Ax_{k-1} + Bu_k + W_k \\
    y_k &= Cx_k + v_k
\end{align*}
\]

where \( x_k, u_k \) and \( y_k \) are the state, input and output vectors of the system at instant \( k \) and \( W_k \) and \( v_k \) are the process and measurement noise, respectively, both assumed to be independent and following Gaussian distributions with 0 mean. However, \( A, B \) and \( C \) are matrixes that weight the impact of inputs and previous state in the current state and output values.

The KF23 provides the optimal solution for the estimation problem of the previously described system (i.e. estimating the state of the system \( x_k \) from the evolution of \( y_k \) and \( u_k \)) by applying an iterative process consisting of two steps:

1. **Prediction.** The model of the system is used to calculate a priori estimations of the state (\( \hat{x}_{k|k-1} \)) and the error covariance (\( P_{k|k-1} \)) as

\[
\begin{align*}
    \hat{x}_{k|k-1} &= A\hat{x}_{k-1} + Bu_k \\
    P_{k|k-1} &= AP_{k-1}A^T + Q_k
\end{align*}
\]

2. **Update.** The previous a priori estimations are improved by applying the Kalman gain (\( K_k \)) as

\[
\begin{align*}
    K_k &= \frac{P_{k|k-1}C^T}{CP_{k|k-1}C + R} \\
    \hat{x}_k &= \hat{x}_{k|k-1} + K_k(y_k - CX_{k|k-1}) \\
    P_k &= (1 - K_kC)P_{k|k-1}
\end{align*}
\]

where \( Q_k \) and \( R_k \) are the covariances of the process and measurement noises (\( \sigma_w^2 \) and \( \sigma_v^2 \), respectively) that are time-variant.

The problem addressed in the present work can be identified as a particular case of the general problem described by equations (4)–(6), where

- The state (\( x_k \)) is the CO₂ concentration.
- The input (\( u_k \)) is the CO₂ estimation from equation (3) applied to the fuel provided by the ECU.
- The measurement (\( y_k \)) is the CO₂ concentration signal from the exhaust gas analyser.

Despite the simplicity in the formulation and the performance in practical application of the KF, it is rather complex to find a system that strictly fulfils its requirements, not only because most of the real systems are not linear but also because noises rarely follow a Gaussian distribution and their covariances are not usually a priori known.22 In addition, the variances of process noise (\( Q \)) and measurement noise (\( R \)) have a key impact on the performance of the KF.24

**KF with augmented model for drift correction**

Taking into account that the ECU-based estimation contains the high-frequency information of the actual CO₂ emissions but has some drift, the analyser signal (slow but accurate in steady state) can be used to compensate this drift as proposed in Blanco-Rodriguez16 for the NOx emissions and \( \lambda \) estimation. In particular, the proposed method is based on observing the bias \( \theta \) between the measurement \( y \) and the estimation \( u \), considering the measurement dynamics. The augmented state leads to considering the following state vector

\[
    x_k = [\theta_k, \text{CO}_2\text{actual,filter}]
\]

where the state vector \( x_k \in \mathbb{R}^2 \) consists of the bias \( \theta \) and the actual CO₂ concentration filtered with the analyser model (\( \text{CO}_2\text{actual,filter} \)), that in the case at hand is a first-order model with the time constant provided in

![Figure 3. Example of the CO₂ signal phasing. Blue: CO₂ estimation from ECU signals. Dashed black: raw CO₂ measurement from the exhaust gas analyser. Red: CO₂ signal from the exhaust gas analyser phased with the ECU estimation.](image-url)
the analyser characteristics (see Table 1). The state-space matrices $A$, $B$ and $C$ can be identified as

$$
A = \begin{bmatrix} 1 & 0 \\ 1 - a & a \end{bmatrix}
$$

$$
B = \begin{bmatrix} 0 \\ 1 - a \end{bmatrix}
$$

$$
C = [0 \ 1]
$$

(8)

where $a$ is the discrete time version of the system time constant that can be obtained from the analyser response time (see Table 1) and the sample time to convert model from continuous to discrete time. The process noise ($W_k \in \mathbb{R}^2$) can be defined as

$$
W_k = [w_k \ 0]^T
$$

(9)

and the output noise ($v_k \in \mathbb{R}$) leads to the following covariance matrices

$$
Q = \begin{bmatrix} \sigma_w^2 & 0 \\ 0 & 0 \end{bmatrix}
$$

$$
R = \sigma_v^2
$$

(10)

Note that from the structure of matrix $A$, it can be observed that

- The model implicitly involves a slow evolution in the drift since $\theta$ only varies due to the consideration of the process noise.
- The process noise is applied exclusively to the model bias $\theta$ and is transmitted to $CO_{\text{actual}}$ by means of the state equation (matrix $A$). Accordingly, the actual $CO_2$ can be computed as follows

$$
CO_{\text{actual}}^{\text{measured}} = CO_{\text{actual}}^{\text{expected}} + \theta
$$

(11)

In addition, provided that the system is linear time invariant (LTI), if the covariances are considered constant, the Kalman gain ($K$) rapidly tends to a constant value. This value depends on the time response of the analyser ($a$) and the ratio between process and output covariances ($\sigma_w^2/\sigma_v^2$). This approach with constant covariances is widely covered in literature for observing exhaust gas properties such as NOx concentration or temperature.\cite{Guardiola et al.} Figure 4 shows the result of applying this approach to estimate $CO_2$ emissions in part of the NEDC. In particular, it can be observed how the solution obtained from the KF with augmented model tends to the analyser signal when $\sigma_w^2/\sigma_v^2$ tends to 1 and conversely tends to the model estimation as $\sigma_w^2/\sigma_v^2$ approaches 0.

Figure 5 shows the ratio between $CO_2$ emissions obtained from the extended model KF approach and the analyser in three different driving cycles. One can observe how as the $\sigma_w^2/\sigma_v^2$ ratio is reduced (the noise in the analyser increases respect to the model noise) the result diverges from the value provided by the analyser. It can be also noticed how differences can rise up to levels in the order of 8%–10% depending on the $\sigma_w^2/\sigma_v^2$ ratio and the dynamics of the cycle. The synthetic driving cycle representing urban conditions in green consists mainly of constant speed segments linked with constant acceleration profiles. In this sense, the weight of transitory phases in this cycle is lower than in the NEDC and especially in the WLTC, so the differences between the analyser and the proposed method is reduced to less than 4%.

As discussed above, the selection of the proper $\sigma_w^2/\sigma_v^2$ is critical to the final performance of the KF,
while the method itself does not provide any clue on this decision.

**Proposed noise tuning method**

In particular, the output variance is usually obtained from the analyser accuracy data, generally represented using a standard deviation. In the case of the CO₂ analyser, the data sheet of the equipment provides an uncertainty value for steady-state measurement ($\sigma_{r,s}$), but higher uncertainty can be expected during transient evolutions as shown in Figures 1 and 3. Even though it is not possible to know the actual value of the analyser error ($j$), defined as the absolute value of the difference between the actual and sensor CO₂ concentrations, will evolve according to

$$\sigma_{r,k} = \sigma_{r,s} + \sigma_{r,dyn,k}$$ \hspace{1cm} (12)

where $\sigma_{r,s}$ is assumed to be constant (see Table 1) and the term $\sigma_{r,dyn,k}$ represents a correction for dynamic conditions that evolves with time. One may expect this term ($\sigma_{r,dyn,k}$) to be strongly related to both the analyser response time and the dynamics of the actual CO₂ concentration to be measured.

The general assumption that exhaust gas sensors follow a first-order response leads in the case at hand to

$$\tau_{analyser} \frac{dCO_2^{analyser}}{dt} + CO_2^{analyser} = CO_2^{actual}$$ \hspace{1cm} (13)

where $\tau_{analyser}$ is the time constant of the analyser (see Table 1). If the analyser described by equation (13) is tested with CO₂ variations at different rates (ramp response), the analyser error ($\hat{\xi}$), defined as the absolute value of the difference between the actual and sensor CO₂ concentrations, will evolve according to

$$\hat{\xi} = |CO_2^{actual} - CO_2^{analyser}| \%$$

In Figure 6, equation (13) is represented showing the sensor response of the analyser. The proposed method consists of assuming $\sigma_{r,dyn,k} = \hat{\xi}_k$, so equation (12) becomes

$$\sigma_{r,k} = \sigma_{r,s} + \hat{\xi}_k$$ \hspace{1cm} (16)

Note that $\sigma_{r,k}$ contains both the measurement noise and the error due to the assumption of a first-order response of the analyser.

Regarding the estimation of $\hat{\xi}_k$, $\tau_{analyser}$ is obtained from the analyser data or by simple identification during step tests. The variation in the actual CO₂ concentration is estimated from the variation in the CO₂ estimation from the ECU signals, since that signal contains most of the dynamic information. Note that for highly dynamic variations in CO₂, $dCO_2^{analyser}/dt$ tends to $dCO_2^{actual}/dt$. In this sense, equation (15) leads to

$$\hat{\xi} = \frac{dCO_2^{actual}}{dt} \tau_{analyser}$$ \hspace{1cm} (17)

With respect to the process variance ($\sigma_{r,k}$), it will be assumed that the noise is related to the input term $u$, considering $\tau_{analyser} = 2\,s$ to variations in the actual CO₂ concentration (dashed lines) at different rates. $\xi_1$ and $\xi_{0.5}$ represent the considered error for the CO₂ evolutions with $1\%\,s^{-1}$ and $0.5\%\,s^{-1}$ rates.

**Figure 6.** Example theoretical sensor response (continuous lines) considering $\tau_{analyser} = 2\,s$ to variations in the actual CO₂ concentration (dashed lines) at different rates. $\xi_1$ and $\xi_{0.5}$ represent the considered error for the CO₂ evolutions with $1\%\,s^{-1}$ and $0.5\%\,s^{-1}$ rates.

**Figure 7.** Considered sensor error due to dynamics ($\hat{\xi}$) as a function of the sensor time constant ($\tau_{analyser}$) and rate of variation in the actual CO₂ concentration $dCO_2/dt$.

$$\hat{\xi}(t) = \frac{dCO_2^{actual}}{dt} \tau_{analyser} \left(1 - e^{-t/\tau_{analyser}}\right)$$ \hspace{1cm} (14)

Figure 6 shows an example of the response of the sensor described by equation (13) for two different ramp rates ($1\%\,s^{-1}$ and $0.5\%\,s^{-1}$). In both cases with $\tau_{analyser} = 2\,s$.

It can be observed that according to equation (14), the error increases up to a maximum ($\hat{\xi}$) when $t \gg \tau_{analyser}$, then when the ramp is finished, the error is progressively reduced up to 0. The maximum error $\hat{\xi}$ can be deducted from the limit of equation (14) for large times ($t$) leading to

$$\hat{\xi} = \frac{dCO_2^{actual}}{dt} \tau_{analyser}$$ \hspace{1cm} (15)

In Figure 7, equation (15) is represented showing the analyser error ($\hat{\xi}$, with its particular definition) direct dependence with its time constant and the dynamics of the signal to be measured.
that is, the ECU estimation. In reality, the accuracy of the ECU fuel estimation will depend on the operating conditions and the engine state (ageing, unit to unit dispersion), but for simplicity here it is considered constant. As an example, Figure 8 shows the histogram of the CO$_2^\text{ecu}$ estimation error at steady state for two different engines over its complete engine operating map (engine speed vs load). Both are Euro 6 Direct Injection Turbocharged Diesel engines with 1.6 L (black line) and 2.0 L (grey line) of displacement. The standard deviation obtained in both cases is around 0.4%, which is the value used in the present work as $\sigma_w$. In any case, if an accurate method to determine the actual CO$_2$ emissions is available, $\sigma_w$ can be used as calibrating factor that will make the KF estimation swing between the value provided by the ECU estimation and the analyser.

Results

Figures 9–11 show the results obtained at particular segments of three different driving cycles. In the upper plot, the vehicle speed profile can be observed. The three cases show accelerations in the urban range, up to 50 km/h at different rates. In the bottom plot, the evolution of the CO$_2$ concentration can be tracked. It can be clearly observed how, independent of the test, the CO$_2$ estimation provided by the proposed method (black line) tends to the analyser measurement (dashed line) as the rate of change in the CO$_2$ signal is reduced and replicates it in steady-state phases. On the contrary, the CO$_2$ estimation provided by the proposed method tends to the estimation based on the ECU signals (grey line) during dynamic phases. It can be also noted that during step decelerations where there is not fuel injection, unlike the analyser, the proposed estimation leads to null CO$_2$ concentration.

In Figure 12, the differences between the accumulated CO$_2$ emissions obtained from the proposed method and the raw analyser signal are tracked during the three tested cycles. Independent of the driving cycle, in the phases with constant velocity, there are no
The differences between the values provided by both methods (the line showing the difference between accumulated emissions remains constant), while in the zones with sharp changes in velocity, differences appear to correct the lack of dynamic response of the analyser.

In this way, the proposed approach is able to maintain the steady-state accuracy of the CO₂ analyser while keeping the dynamic response of the ECU estimation.

Figure 12. Difference between accumulated CO₂ emissions estimated with the proposed method and with the analyser in three driving cycles: NEDC (top), WLTC (medium), and synthetic driving cycle representing urban conditions (bottom). The grey shades represent the velocity profile.

To underline this idea, Figure 13 shows the CO₂ signals from the three assessed methods during the WLTC in the frequency domain. One may observe how at low frequencies (below 0.2 Hz), the analyser and the proposed method behave similarly, while the ECU estimation slightly underpredicts the CO₂ emissions. For higher frequencies (above 0.2 Hz), the dynamic response of the analyser is not high enough to capture the CO₂ variations, while the proposed method is able to keep the dynamic response of the ECU estimation.

In summary, Figure 14 shows the overall CO₂ from the three methods, normalised with the analyser results. Discrepancies between the ECU estimation and analyser results range from 4% in the test with lower dynamics (urban) to 14% in the more dynamic WLTC.

Regarding the proposed method, differences with the analyser method are below 2% for tests where steady-state phases prevail (like the NEDC or the synthetic urban driving cycle used in this work), but differences up to 4% may appear in driving cycles where dynamics play an important role and then showing that the lack of dynamic response of the analyser has a noticeable impact on the CO₂ figures.

Conclusion

This article has discussed the impact of the dynamic limitations of standard CO₂ analysers used for engine and vehicle testing over highly transient driving cycles. In those situations, typical analyser response times and transport delays in the sample lines (5–10 s) may lead to noticeable measurement errors. The assessment of the CO₂ analyser and ECU-based methods response time and accuracy allows to pose an estimation problem that can be solved by a KF. In particular, the proposed approach is based on the following:

- A model augmentation to consider, inside the Kalman filter, the model bias and the sensor dynamics.
- Time-dependent covariance $\sigma_t$ that depends on the analyser response and CO₂ dynamics.

Including the model bias and analyser dynamics in the KF framework allows a compact formulation that
can cope with the system dynamics, while the values of \( \sigma_2^2 \) and \( \sigma_3^2 \) are obtained from the analysis of the CO\(_2\) sensor characteristics. In addition to the method itself, the main conclusion of this article is that after applying the previous method to different driving cycles, it can be stated that while analyser dynamic limitations did involve accuracy penalties below 2% in the former European regulation cycle (NEDC), they may lead to a substantial overestimation of the CO\(_2\) figures that can reach 4% in highly dynamic tests such as the current regulation cycle WLTC. In this sense, this article not only shows a problem in the measurement procedure for the current and more realistic driving cycles used for research and certification (such as WLTC or RDE) but also proposes different solutions to minimise this issue.

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Appendix 1

Notation

| Symbol     | Description                                      |
|------------|--------------------------------------------------|
| $a$        | equivalent time constant with sampling time 0.1 s|
| $A$        | system state matrix                             |
| $B$        | system input matrix                             |
| $C$        | carbon                                           |
| $C_{\text{actual, filter}}$ | system output matrix                             |
| $CO_{2, k}^\text{actual, filter}$ | CO$_2$ concentration filtered with the analyser model at instant $k$ |
| $CO_{2, k}^\text{actual}$ | actual CO$_2$ concentration at instant $k$ |
| $C_{\text{O$_2$ cu, k}}$ | CO$_2$ concentration estimation from the ECU at instant $k$ |
| $C$        | system output matrix                             |
| $D$        | system feedforward matrix                        |
| $DOC$      | diesel oxidation catalyst                        |
| $DPF$      | diesel particulate filter                        |
| $ECU$      | electronic control unit                          |
| $GHG$      | green house gases                                |
| $H$        | hydrogen                                         |
| $HEV$      | hybrid electric vehicle                          |
| $ICE$      | internal combustion engine                       |
| $k_{\text{CO$_2$-f}}$ | conversion factor from CO$_2$ to fuel |
| $KF$       | Kalman filter                                    |
| $K_k$      | Kalman gain at instant $k$                       |
| $LTI$      | linear time invariant system                     |
| $MW$       | molecular weight                                 |
| $NDIR$     | non-dispersive infrared                          |
| $NEDC$     | New European Driving Cycle                       |
| $O$        | oxygen                                           |
| $P_k$      | error covariance matrix at instant $k$           |
| $Q_k$      | covariance matrix of the process noise at instant $k$ |
| $R_k$      | covariance matrix of the measurement noise at instant $k$ |
| $RDE$      | read driving emission                             |
| $t_{90}$   | time taken by the analyser to reach 90% of the applied CO$_2$ concentration |
| $u_k$      | system input at instant $k$                      |
| $UEGO$     | universal exhaust gas oxygen                     |
| $v_k$      | measurement noise at instant $k$                 |
| $W$        | mass flow                                        |
| $W_{\text{mass flow}}$ | measured air mass flow |
| $W_{\text{exh}}$ | exhaust mass flow |
| $W_f$      | fuel mass flow                                    |
| $W_{\text{fuel flow}}$ | fuel mass flow estimated by the ECU |
| $WLTC$     | worldwide harmonised light-duty vehicles test cycle |
| $x_k$      | system state at instant $k$                      |
| $\hat{x}_k$ | estimated state of the system at instant $k$    |
| $y_k$      | system output at instant $k$                     |
| $\theta_k$ | measurement bias at instant $k$                 |
| $\xi$      | analyser error                                   |
| $\xi$      | maximum analyser error                           |
| $\sigma^2_w$ | variance of the process noise                    |
| $\sigma^2_v$ | variance of the measurement noise                |
| $\sigma_{x,k}$ | standard deviation of the measurement at instant $k$ |
| $\sigma_{x,\text{dyn},k}$ | dynamic correction for the standard deviation of the measurement at instant $k$ |
| $\sigma_{x,ss}$ | steady-state standard deviation of the measurement at instant $k$ |
| $\tau_{\text{analyser}}$ | time constant of the analyser |