Verification of Finite Element Computational Model for Biaxial Buckling of Stiffened Plates

Guilherme Ribeiro Baumgardt$^{1,a,*}$, Thiago da Silveira$^{1,b}$, João Paulo Lima$^{2,c}$, Luiz Alberto Oliveira Rocha$^{3,d}$, Elizaldo Domingues dos Santos$^{1,e}$, and Liércio André Isoldi$^{1,f}$

$^{1}$Graduate Program in Computational Modeling (PPGMC), Federal University of Rio Grande (FURG), km 8 Itália Ave., Rio Grande, RS, Brazil
$^{2}$Federal University of Goiás (UFG), 920 Mucuri St., Aparecida de Goiânia, Go, Brazil
$^{3}$University of Vale do Rio dos Sinos (Unisinos), 950 Unisinos Ave., São Leopoldo, RS, Brazil

E-mail: guilhermebaumgardt@gmail.com

Abstract. This study presents the verification of computational models for the analysis of biaxial buckling in plates. The verification was performed by comparing the numerical results obtained in this study with results found in scientific references published by other authors. The computational models were developed in the ANSYS® software, which is based on the Finite Element Method (FEM), and the SHELL281 element is used. The mesh convergence tests demonstrate the convergent behavior of the simulations performed for the analyzed examples, which are: a square plate under biaxial elastic buckling, and a rectangular plate with stiffeners under biaxial elasto-plastic buckling. From the converged mesh, maximum differences of 0.84% and 3.89% were achieved when compared with references, respectively, for the elastic and elasto-plastic biaxial buckling. These results indicate that the proposed computational models were adequately verified.

1. Introduction

Thin plates are structural components widely used in the aircraft industry, the shipbuilding industry and civil construction. The relatively low compressive load that a thin plate supports can be increased through the use of stiffeners. The stiffeners are beams fixed transversely and/or longitudinally in the plane of the plate, increasing its structural rigidity [1]. This increase in stiffness can be proven through the results presented in [2] where, keeping the volume of material constant, significant reductions for the central deflection of a rectangular steel plate subjected to bending were achieved due the use of stiffeners.

However, the inclusion of stiffeners in the plate generates geometric discontinuities, which makes its analysis complex or even impossible through analytical methods [3]; then the numerical methods are the most used approach for the analysis of its mechanical behavior.

In addition, it is well known that thin plates when subjected to compressive forces in the plane, can suffer buckling. But, unlike columns, the critical buckling load does not configure its collapse, since they have a stable post-buckling behavior [4].
In this sense, the behavior of plate buckling has been numerically investigated by several authors. Xu and Soares [5,6] proposed an experimental and numerical study via Finite Element Method (FEM), of uniaxial plate buckling with stiffeners, to assess the ultimate stress in the intermediate gaps of the plate, applying different configurations and boundary conditions. Through the comparison between the two studies, it was observed that the stress distributions in the reinforced panels obtained by the numerical model reached slightly higher magnitudes than the experimental tests, probably due to the residual stresses of the welding and details of the plate's manufacture. Lima et al. [7] performed the numerical study of the mechanical behavior of rectangular stiffened steel plates subjected to elastic and elasto-plastic buckling, under uniaxial compressive loading, using the Constructal Design Method associated with the Exhaustive Search technique and the FEM (by ANSYS® software) in a study of geometric optimization, evaluating the influence of the geometric variation of the rectangular stiffeners. Piscopo [8] studied analytically and numerically the uniaxial and biaxial elastic buckling of plates with no stiffeners. The analytical analysis used Shimpi’s theory instead of the energies method, determining the corrected theoretical Euler buckling stresses. Its results were compared with the data obtained in the numerical analysis via FEM performed using the ANSYS® software. It was found that Shimpi's theory always provides, in relation to the energy method, results closer to the values obtained by the FEM analysis. Paik and Seo [9] performed the non-linear numerical analysis via FEM, to evaluate the ultimate stress of plates reinforced with longitudinal and transverse stiffeners subjected to biaxial compression, and also for combined biaxial compression of lateral pressure. It was possible to define which lateral pressure actions reduce the ultimate stress of the plate under predominantly compressive actions.

Therefore, the present work seeks to verify computational models for the application of FEM, using the ANSYS® software, with the purpose of evaluating the behavior of plates biaxially requested by compressive loads. The first numerical model will be verified based on the work presented in [8], where a thin plate without stiffeners is subjected to biaxial elastic buckling. The second model will be verified from the case presented by [9], where a thin plate with stiffeners is subjected to elasto-plastic buckling under biaxial compression.

2. Material and methods

For the developed computational models, this work used the ANSYS® software, based on FEM, which can be used in the most diverse types of engineering problems. For a structural analysis in ANSYS®, displacements and rotations (nodal degrees of freedom) are calculated numerically, and deformations and stresses are determined from these values [10].

For the development of the computational models of the present work, the finite element SHELL281 was adopted, being suitable for analyzing thin to moderately thick shell structures. The element has eight nodes with six degrees of freedom in each node: translations on the $x$, $y$ and $z$ axes and rotations on the $x$, $y$ and $z$ axes, being its formulation based on the first-order shear-strain theory (Reissner-Mindlin Theory) [11].

Based on El-Sawy and Nazmy [12], Helbig et al. [13], and Lima et al. [7] the computational models for biaxial elasto-plastic buckling of plates with stiffeners proposed in the present work was developed. For that, it is necessary to assume an imperfect initial configuration for the plate, which is obtained from its first elastic buckling mode. Therefore, a computational model for biaxial elastic plate buckling was preliminarily developed. For the numerical simulation of biaxial elastic buckling, the eigenvalue analysis was adopted. The finite element equilibrium equations for this type of analysis involve the solution of homogeneous algebraic equations, in which the lowest eigenvalue corresponds to the critical buckling load and the associated eigenvector represents the first buckling mode [14]. The formulation used includes both linear and non-linear terms, with the total stiffness matrix $K$ obtained by adding the conventional stiffness matrix for small deformations, $K_E$, with the geometric stiffness matrix $K_G$. The latter, $K_G$, does not depend only on the geometry, since the internal load $P_0$, existing at the beginning of the loading must be considered. Thus, the total plate stiffness matrix for a load level $P_0$, is given by [15]:
\[ [K] = [K_E] + [K_G]. \] (1)

In case the load reaches a level of \( \{P\} = \lambda\{P_0\} \), where \( \lambda \) is a scalar, the stiffness matrix is written as:

\[ [K] = [K_E] + \lambda[K_G]. \] (2)

Then, the equilibrium equations, governing for the plate, can be written as:

\[ ([K_E] + \lambda[K_G])\{U\} = \lambda\{P_0\}, \] (3)

where \( \{U\} \) is the total displacement vector, which can be determined by:

\[ \{U\} = \left( [K_E] + \lambda[K_G] \right) - \lambda\{P_0\}. \] (4)

When \( \lambda \) reaches a critical value, the plate is subjected to a large increase in displacements without the load increasing. By mathematical definition it is possible to obtain the inverse matrix as the adjunct matrix divided by the coefficient determinant. So the displacements \( \{U\} \) tend to infinity when:

\[ \text{det} \left( [K_E] + \lambda[K_G] \right) = 0, \] (5)

being \( \text{det} \) the determinant of the matrix.

The Eq. 5 represents an eigenvalue problem which, in which the smallest eigenvalue \( \lambda_1 \), corresponds to the critical buckling load given by:

\[ \{P_{Cr}\} = \lambda_1\{P_0\}, \] (6)

where \( P_{Cr} \) is the limit load that defines the biaxial elastic buckling of the plate, while the associated displacement vector \( \{U\} \) defines the first buckling mode. Finally, it is worth noting that the eigenvalue problem was solved in ANSYS® using the Lanczos numerical method [16].

Then, for the numerical simulation of the biaxial elastoplastic buckling of plates, an elastic-perfectly plastic behavior (with no strain hardening) was assumed. In addition, as previously mentioned, a plate with an imperfect initial geometric configuration needs to be considered. For this, the indication made by El-Sawy and Nazmy [12] was adopted, which, from the first elastic buckling mode, defines the displacement as the maximum value of the initial imperfection, given by:

\[ w_0 = b/2000, \] (7)

where \( b \) is the width of the plate.

Then, the ultimate biaxial buckling load acting on the plate is determined by gradually applying a reference load on the four sides of the plate, defined as:

\[ P_y = \sigma_y t, \] (8)

where \( \sigma_y \) is the yield stress of the material. For each load increase, the Newton-Raphson method is applied to determine the displacements that correspond to the equilibrium configuration of the plate. At
the beginning of the loading step \( i + 1 \), there is an out-of-balance load vector \( \psi \), equal to the load increment \( \Delta P \), between the vector of external loads, \( P_{i+1} \), and the vector of nonlinear internal forces \( F_{NL} \), that is equal to the previous external load vector \( P_i \), then:

\[
\psi = \Delta P = P_{i+1} - F_{NL} = P_{i+1} - P_i.
\] (9)

Therefore, the Newton-Raphson method reduce iteratively the out-of-balance load vector \( \psi \) bellow a prescribed tolerance, by using the next equations:

\[
\psi_{r+1} = P_{i+1} - F_{NLr},
\] (10)

\[
[K_t]_r \{\Delta U\}_{r+1} = \psi_{r+1},
\] (11)

\[
\{U\}_{r+1} = \{U\}_r + \{\Delta U\}_{r+1},
\] (12)

where \( \psi_{r+1} \) is the updated out-of-balance load vector; \( F_{NLr} \) is the nonlinear internal forces vector at the iteration \( r \); \( [K_t]_r \) is the tangent stiffness matrix calculated as a function of the displacement vector \( \{U\}_r \); \( \{\Delta U\}_{r+1} \) is the updated displacement increment vector; and \( \{U\}_{r+1} \) is the updated displacement vector.

The ultimate load of the plate is obtained if, in a specific loading step, convergence cannot be achieved in the iterative process. In this case, a finite increment in the displacement vector cannot be defined so that the out-of-balance load vector is nullified. This happens because no matter how large the strains and displacements, the internal dimensions and stresses cannot increase, as it requires necessary to balance the external loads, which indicates that the material of the plate reached its final strength [13,14,17].

From this, firstly it was verified the biaxial elastic buckling model, through a thin plate without stiffeners, like the one presented in Piscopo [8]. It is a simply-supported square plate, with length \( a \) and width \( b \) of 1000 mm, and thickness \( t \) of 10 mm, subjected to a biaxial compressive loading with the same intensity in both axes. The properties of the material used are: Young’s modulus of 206 GPa and Poisson's ratio of 0.3. Fig. 1 shows the schematic representation of this plate.

![Figure 1. Plate simply supported on the edges, subjected to biaxial compression.](image)
rectangular plate with T-shaped stiffeners was performed, with the following dimensions: $a = 4300$ mm, $b = 16300$ mm, $b_1 = 815$ mm, $t = 17.8$ mm, $h_w = 463$ mm, $t_w = 8$ mm, $b_f = 172$ mm and $t_f = 17$ mm, with compressive biaxial loading of different intensities in each two axes ($\sigma_x : \sigma_y = 0.4 : 0.6$) [9]. The properties of the material used are: Young’s modulus of 205.8 GPa, yield stress of 315 MPa and Poisson’s ratio of 0.3. Fig. 2 shows the schematic representation of the plate and its stiffeners used in this verification.

![Figure 2](image-url)

**Figure 2.** Simply-supported stiffened plate subjected to biaxial compression: (a) geometric configuration (Adapted from [9]), (b) computational domain.

3. Results and discussions

As described in item 2, the verification of the numerical model for biaxial elastic buckling was developed, since its result is used to determine the initial imperfect geometric configuration for the biaxial elasto-plastic buckling numerical model. In sequence, the verification of the computational model of biaxial elastoplastic buckling of a plate with stiffeners was performed.

3.1. Verification of the computational model for biaxial elastic plate buckling.

The case presented in Fig. 1 was analyzed by Piscopo [8], who obtained analytically a critical buckling load of $P_{Cr} = 372$ kN/m; and numerically a value of $P_{Cr} = 370$ kN/m. Thus, through the computational model proposed in this work, it was possible to numerically simulate the case of Fig. 1. For this, a mesh convergence test, varying the size of finite element from 100 to 10 mm. Therefore, it was obtained the variation of the critical load as the increase of mesh refinement, as can be seen in Fig. 3.
Inspecting the obtained results, it can be seen that from the mesh with 2500 elements, the numerical solution does not vary with the increase in the number of elements in the mesh, maintaining the critical load value of $P_{cr} = 368.88$ kN/m. Therefore, the result obtained for the proposed model presents a difference of 0.84% and 0.30%, in relation to the results obtained by Piscopo [8] in his analytical and numerical models, respectively.

3.2. Verification of the computational model for biaxial elastoplastic buckling of plates with stiffeners.

In Paik and Seo [9], see Fig. 2, it was obtained numerically the ultimate stress in $x$ direction of 72.10 MPa, and the ultimate stress in $y$ direction of 105.34 MPa. Then, by means the computational model developed in this work, the mesh convergence test was carried out, varying the size of each elements from 1000 to 100 mm, obtaining the variation of the ultimate buckling load, as can be seen in Fig. 4.

Comparing the results obtained in this work with those obtained by Paik and Seo [9], it was achieved a difference of 3.89% in relation to the $x$ direction, and a difference of 1.32% in relation to the $y$ direction.

Conclusions
The present work proposed computational models for biaxial elastic and elasto-plastic plate buckling, developed using the Finite Element Method (FEM) in ANSYS® software. The biaxial elastic buckling model is used to determine the imperfect initial geometric configuration of the plate, which is used as an initial condition in the biaxial elasto-plastic buckling model.

Then, initially, the verification of the computational model of biaxial elastic buckling was performed, numerically simulating the problem proposed in Piscopo [8]. The numerical result for the critical load was compared with the analytical and numerical solutions of Piscopo [8], being identified a difference of 0.84% and 0.30%, respectively.

Afterwards, the verification of the computational model proposed for biaxial elastoplastic buckling was performed, comparing its results to those presented by Paik and Seo [9]. A plate with T-shaped transverse stiffeners and I-shaped longitudinal stiffeners was considered. A difference of 3.89% and 1.32% was found for the ultimate buckling stress in the $x$ and $y$ directions, respectively, comparing the numerical results obtained in the present study with those presented in Paik and Seo [9].

Therefore, it can be concluded, from the small differences found, that both proposed computational models were properly verified. In future work, we intend to use these models to carry out a study of geometric optimization of plates with stiffeners subjected to biaxial buckling.

Acknowledgments

G.R. Baumgardt thanks to the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001; and L.A.O. Rocha, E.D. dos Santos, and L.A. Isoldi thank to the Conselho Nacional de Desenvolvimento Científico e Tecnológico - Brasil (CNPq) - Processes: 307791/2019-0, 306024/2017-9, and 306012/2017-0, respectively.

References

[1] V. Birman, Solid Mechanics and Its Applications: Plate Structures, Springer, Netherlands, 2011.
[2] J.P.T.P. De Queiroz, M.L. Cunha, A. Pavlovic, L.A.O. Rocha, E.D. Dos Santos, G.S. Troina, L.A. Isoldi, Geometric Evaluation of Stiffened Steel Plates Subjected to Transverse Loading for Naval and Offshore Applications, J. Mar. Sci. Eng. 7 (2019) 7.
[3] M.L. Gambhir, Stability Analysis and Design of Structures, Springer-Verlag, Berlin, 2004.
[4] K.K. Kapur, B.J. Hartz, Stability of plates using the finite element method, Journal of the Engineering Mechanics Division. 92 (1966) 177-196.
[5] M.C. Xu, C. G. Soares, Experimental study on the collapse strength of wide stiffened panels, Mar. Struct. 30 (2013) 33–62.
[6] M.C. Xu, C. G. Soares, Comparison of calculations with experiments on ultimate strength of wide stiffened panels, Mar. Struct. 31 (2013) 82–101.
[7] J.P.S. Lima, M.L. Cunha, E.D. Dos Santos, L.A.O. Rocha, M.V. Real, L.A. Isoldi, Constructal Design for the ultimate buckling stress improvement of stiffened plates submitted to uniaxial compressive load, Engineering Structures. 203 (2020) 109883.
[8] V. Piscopo, Refined Buckling Analysis of Rectangular Plates Under Uniaxial and Biaxial Compression. World Academy of Science, Engineering and Technology International Journal of Mechanical and Mechatronics Engineering, 4 (2010) 1018-1025.
[9] J.K. Paik, J.K. Seo, Nonlinear finite element method models for ultimate strength analysis of steel stiffened-plate structures under combined biaxial compression and lateral pressure actions—Part II: Stiffened panels, Thin-Walled Structures. 47 (2009) 998–1007.
[10] Ansys. Analysis Guide. Cannonsburg: Ansys Inc, 2004.
[11] ANSYS Help Viewer. Release 18.1 - © ANSYS, Inc. All rights reserved.
[12] K.M. El-Sawy, A.S. Nazmy, M.I. Martini, Elasto-Plastic Buckling of Perforated Plates under Uniaxial Compression. Thin-Walled Structures. 42 (2004) 1083-1101.
[13] D. Helbig, C.C.C. Da Silva, M.V. Real, E.D. Dos Santos, L.A. Isoldi, L.A.O. Rocha, Study About Buckling Phenomenon in Perforated Thin Steel Plates Employing Computacional Modeling and Constructal Design Method, Latin American Journal of Solids and Structures. 13 (2016) 1912-1936.
[14] E. Madenci, I. Guven, The Finite Element Method and Applications in Engineering Using ANSYS®, second ed., Boston: Springer, 2015.
[15] J.S. Przemieniecki, Theory of Matrix Structural Analysis, Dover Publications, 1985.
[16] Ansys. User’s Manual (version 10.0), Swanson Analysis System Inc, Houston, 2005.
[17] J.P.S. Lima, L.A.O. Rocha, E.D. Dos Santos, M.V. Real, L.A. Isoldi, Constructal design and numerical modeling applied to stiffened steel plates submitted to elasto-plastic buckling, Proceedings of the Romanian Academy - Series A: Mathematics, Physics, Technical Sciences, Information Science. 19(Special Issue) (2018) 195-200.