Influences of sinusoidal density modulation on stimulated Raman scattering in inhomogeneous plasmas

Y Chen¹², C Y Zheng²³, Z J Liu²³, L H Cao²³, Q S Feng²³, Y G Chen¹, Z M Huang¹ and C Z Xiao¹⁴*

¹ Key Laboratory for Micro-/Nano-Optoelectronic Devices of Ministry of Education, School of Physics and Electronics, Hunan University, Changsha 410082, People’s Republic of China
² Institute of Applied Physics and Computational Mathematics, Beijing 100094, People’s Republic of China
³ HEDPS, Center for Applied Physics and Technology, Peking University, Beijing 100871, People’s Republic of China
⁴ Collaborative Innovation Center of IFSA (CICIFSA), Shanghai Jiao Tong University, Shanghai 200240, People’s Republic of China

E-mail: xiaocz@hnu.edu.cn

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Abstract
The influence of sinusoidal density modulation on stimulated Raman scattering (SRS) reflectivity in inhomogeneous plasmas is studied using three-wave coupling equations, fully kinetic Vlasov simulations, and particle-in-cell (PIC) simulations. Through a numerical solution of the three-wave coupling equations, we find that the sinusoidal density modulation is capable of inducing absolute SRS even though the Rosenbluth gain is smaller than $\pi$, and we describe a region of modulational wavelength and amplitude in which absolute SRS can be induced, which agrees with earlier studies. The average reflectivity obtained by the Vlasov simulations has the same trend as the growth rate of absolute SRS obtained from the three-wave equations. Instead of causing absolute instability, a modulation wavelength shorter than the basic gain length can suppress the inflation of SRS through harmonic waves. In addition, the PIC simulations qualitatively agree with our Vlasov simulations. Our results offer an alternative explanation for high reflectivity in experimental underdense plasmas, which is due to long-wavelength modulation, and a potential method to suppress SRS using short-wavelength modulation.

Keywords: stimulated Raman scattering, inhomogeneous plasma, sinusoidal density modulation

(Some figures may appear in colour only in the online journal)

1. Introduction

Stimulated Raman scattering (SRS) is an important physics process in laser plasma interactions, in which the pump laser decays to a reflected wave and a Langmuir wave. In inertial confinement fusion (ICF) [1–7], this instability can take away part of the energy of the pump laser and generate hot electrons [8], which are detrimental to fusion. The physics of SRS in homogeneous plasmas is well understood [8–14], and the behavior described above corresponds to the indirect-drive scheme [15]. On the other hand, studies of SRS in inhomogeneous plasmas are more common than studies of other ICF schemes such as the direct-drive scheme [16], the hybrid-drive scheme [17] and shock ignition [18]. In the early 1970s, Rosenbluth studied the parametric instabilities in inhomogeneous plasmas using the Wentzel–Kramers–Brillouin (WKB) approximation and described the well-known Rosenbluth gain [19, 20]. However, Rosenbluth’s results are based on ideal laser and plasma conditions. In subsequent studies, researchers...
found that some disturbances in the laser or plasma could destabilize the Rosenbluth convective instability and turn it into absolute instability. Laval et al. showed that imperfect pump lasers contributed to the development of absolute instabilities [21]. Picard and Johnston showed that absolute instability could also be induced by a sufficiently strong pump laser [22]. Li et al. showed that long-wave sinusoidal density modulation was able to induce absolute two-plasmon-decay and SRS near the $n_c/4$ surface using theory and fluid simulations [23], where $n_c$ is the critical density of the pump laser. The reflectivity of SRS saturates at a certain level when there is no density modulation. However, long-wavelength modulation can introduce phase incoherence and then eliminate the convective saturation so that absolute SRS is obtained. This process is very similar to the absolute SRS induced by turbulence [24]. In addition, absolute SRS can also be induced by long-wavelength modulation in other density profiles, such as parabolic and exponential profiles.

More importantly, Picard and Johnston showed that sinusoidal density modulation was able to induce absolute instability and gave two kinds of threshold of absolute instability; one is the exact numerical solution and the other is a quadratic-fit formula [25]. In Picard’s exact numerical solution, the threshold of absolute instability has a clear cutoff in the short modulation wavelength region; however, the quadratic-fit formula does not have this cutoff, and we note that the quadratic-fit formula agrees with the exact solution in the long-wavelength region. In addition, Nicholson pointed out that this kind of absolute instability is sensitive to the wavelength of the sinusoidal density modulation [26]. The growth rate of the absolute instability falls off rapidly when the modulation wavelength is far from $2\pi L_0$, where $L_0$ is the basic gain length of the absolute instability in a finite-length system (defined fully later). In fact, we have found by simulations that the modulation wavelength should be at least higher than $1.3L_0$ in order to trigger absolute instability. This finding explains Nicholson’s simulation results [26], and is consistent with Picard’s exact numerical solution [25] and other studies that use imperfect laser and plasma conditions [21–24, 26–28]. In addition, earlier research only considered the influence of density modulation on instability using three-wave simulations and theoretical predictions. Therefore, they only considered instability in the linear regime; the physics in the nonlinear regime is still unknown, and the influence of short-wavelength density modulation in the nonlinear regime was not considered. In this paper, we will fill these gaps.

In this paper, through numerical solution of the three-wave coupling equations, we first find that the reflectivity of SRS will show an absolute feature and that the threshold of absolute SRS agrees with Picard’s theoretical formula [25] when the modulation wavelength is close to $2\pi L_0$. However, the absolute SRS disappears when the modulation wavelength is less than $1.3L_0$, which is in good agreement with Picard’s exact solution. We also obtain a region of modulational wavelength and amplitude in which density modulation can cause absolute SRS. Second, we have carried out Vlasov simulations [29–31] to study the influence of density modulation in the linear and nonlinear regimes. In the linear regime, the Vlasov simulations show good agreement with the theoretical SRS analysis. The reflectivity of SRS increases greatly when absolute instability is induced and short-wavelength density modulation has no influence on the reflectivity. Conversely, in the nonlinear regime, in particular, when the inflation of SRS is induced by particle trapping [32–34], short-wavelength modulation has a significant effect in suppressing the inflation of SRS, because short-wavelength modulation increases the Landau damping of the Langmuir wave, and the downward harmonic wave can remove the energy of the Langmuir wave. Therefore, short-wavelength sinusoidal density modulation can be applied in the high-intensity and inhomogeneous ignition schemes [16–18, 35–37] to suppress SRS. Finally, we use particle-in-cell (PIC) simulations to prove our conclusions.

This paper is structured in the following way. First, in section 2, we describe the three-wave coupling equation analysis of SRS in an inhomogeneous plasma. Second, Vlasov and PIC simulations are performed to verify our theoretical results in sections 3 and 4. Finally, the conclusions and a discussion of sinusoidal density modulation are presented in section 5.

2. Theoretical model

2.1. Numerical solution of the three-wave coupling equations

The three-wave coupling equations in one dimension are frequently used to study parametric instabilities in inhomogeneous plasmas [19, 20, 24, 26]

\[
\begin{align*}
\left( \partial_t + \nu_0 + V_0 \partial_x \right) a_0(x,t) &= -\gamma_0 a_1(x,t) a_2(x,t), \\
\left( \partial_t + \nu_1 + V_1 \partial_x \right) a_1(x,t) &= \gamma_0 a_2^* x a_0(x,t), \\
\left( \partial_t + \nu_2 + V_2 \partial_x + iV_2 \kappa \right) a_2(x,t) &= \gamma_0 a_0(x,t) a_1^*(x,t),
\end{align*}
\]

where $a_0(x,t)$, $a_1(x,t)$ and $a_2(x,t)$ are slowly varying amplitudes of three waves: the pump wave, the back-reflected wave and the plasma wave, respectively. Here, $\nu_0$, $\nu_1$, and $\nu_2$ are the corresponding damping rates, $V_0$, $V_1$, and $V_2$ are the group velocities of the three waves, $\gamma_0 = \gamma_{\text{SRS}}/a_0(0,0)$ is the coefficient, $\gamma_{\text{SRS}}$ is the temporal growth rate of the instability in a homogeneous plasma without damping, and $\kappa$ is the wave number mismatch of parametric instabilities in an inhomogeneous plasma.

When a sinusoidal density modulation is added to the system, the plasma density profile becomes $n_e(x) = n_0 + [1 + (x - x_r)/L + \varepsilon \sin(k_x x)] \frac{L}{\varepsilon}$, which, in equation (1), is depicted by the wave number mismatch $\kappa = \kappa' + \varepsilon \sin(k_x x)$, where $L$ is the density-scale length, $\kappa' \approx \omega_p^2/6k_L^2k_{\text{De}}$, is the coefficient of spatial wave number detuning, $k_{\text{De}} \sin(k_x x)$ is the wave number mismatch caused by density modulation, and $x_r$ is the phase matching point for the instability. In the modulation, $k_x \equiv 1/L_0$ indicates the modulation’s wave number, corresponding to a wavelength of $\lambda_x = 2\pi L_0$, and $\varepsilon = \delta n_0/n_0$ is the modulation’s amplitude, which is related to $\kappa_m$ through [26]

\[
\kappa_m = \frac{k_{\text{De}} \varepsilon}{6(k_0 \lambda_{\text{De}})^2},
\]

where $k_{\text{De}}$ is the wave number of the plasma wave, $\lambda_{\text{De}} = \nu_e/\omega_p$ is the Debye length of the electron at the
phase matching point, $v_e$ is the electron thermal velocity, and $\omega_{pe} = \sqrt{4\pi n_e e^2/m_e}$ is the electron plasma frequency. Without density modulation, equation (1) has analytic solutions [20]; however, there is no analytic solution when sinusoidal density modulation is taken into consideration.

In order to obtain numerical solutions, we have developed software that numerically solves equation (1) using Lax–Wendroff scheme [38, 39], which we refer to as three-wave simulation below. For simplicity, the damping rates of the three waves are ignored in our three-wave simulations. We assume that SRS occurs when $\omega_0 = 0.1\omega_e$, $L = 100 \mu m$, and the electron temperature is $T_e = 1$ keV. The frequencies and group velocities of the three waves can then be calculated by dispersion relations, 

$$\begin{align*}
\omega_{d1} &= \omega_{pe}^2 + k_{d1}^2 v_e^2, \\
\omega_{d2} &= \omega_{pe}^2 + k_{d2}^2 v_e^2, \\
\omega_{d3} &= \omega_{pe}^2 + 3k_{d2}^2 v_e^2, \\
\omega_0 &= \omega_{d1} + \omega_{d2} - \omega_{d3}, \\
V_0 &= k_{d1}^2 \omega_{d1} + \omega_1 = -k_{d1}^2 \omega_1 \omega_{d2} / \omega_{d3}, \\
V_1 &= 0.9487c, \\
\omega_1 &= 0.3460c, \\
V_2 &= 0.0267c, \\
\omega_2 &= 0.0267c.
\end{align*}$$ (3)

These give $\omega_{0c} = \omega_0$, $V_{0c} = 0.9487c$, $\omega_{1c} = 0.6660c$, $V_1 = -0.8787c$, $\omega_{2c} = 0.3460c$, and $V_2 = 0.0267c$, where $\omega_0$ is the frequency of the pump laser whose wavelength is $\lambda_0 = 0.351 \mu m$, and c is the speed of light in vacuum. Here, $k_{d1}$, $k_{d2}$, and $k_0$ are the wave numbers of the pump laser, the reflected laser, and the Langmuir wave, respectively.

In the three-wave simulations, we split space and time with $\Delta x = 0.05 c/\omega_0$ and $\Delta t = 0.05 \omega_0^{-1}$. The simulation box is $x = [-2000, 2000] c/\omega_0$, the resonant point, $n_e = 0.1 n_{e0}$, is located at $x = 0$, and the simulation time is $t = 20000 \omega_0^{-1} = 3.5$ ps. The intensity of the pump laser is $4 \times 10^{15}$ W cm$^{-2}$, and the growth rate of the SRS can be calculated to be $\gamma_{SRS} = \frac{k_{d1}v_{os}}{4} \left[ \frac{\omega_{d1}^2}{\omega_0^2 - \omega_{d1}^2} \right]^{1/2} = 0.00486 \omega_0$, where $v_{os}$ is the oscillation velocity of electrons in the pump laser. The initial conditions of the three waves are $\rho_0(x, t = 0) = 1$, $a_1(x, t = 0) = 1 \times 10^{-4}$, and $a_2(x, t = 0) = 0$. Based on Rosenbluth’s analytical results [20], the maximum reflectivity of convective SRS in an inhomogeneous plasma with a linear density profile is

$$R = a_1^2 \exp(2GR).$$ (4)

In our conditions, the Rosenbluth gain is $G_R = \frac{\pi \gamma_0}{\kappa \sqrt{V_1V_2}} = 1.021$.

### 2.2. Three-wave simulation results

Figure 1 shows our three-wave simulation results, where the green solid line represents the initial seed of $a_1$, and the green dashed line represents the reflectivity of this system obtained by Rosenbluth’s theory. First, in figure 1(a), we have carried out the three-wave simulation without density modulation, as represented by the black line, which closely agrees with the reflectivity calculated by the Rosenbluth gain [19, 20].

In [23, 25, 26], absolute instability appeared when a sinusoidal density modulation was present in the system. Since

![Figure 1](image-url)

**Figure 1.** (a) Rosenbluth reflectivity in an inhomogeneous plasma (green dashed line), the reflectivity that is obtained by the three-wave simulation without density modulation (black line), and the reflectivity that is obtained by the three-wave simulation with density modulation $k_t = 0.03 k_0$, $\kappa_m = 1.0 \omega_0/c$ (red line). (b) The reflectivity that is obtained by the three-wave simulation with density modulation $k_t = 2 k_0$, $\kappa_m = 1.0 \omega_0/c$ (blue line).

there are two parameters relevant to density modulation, the wave number $k_t$ and the amplitude $\kappa_m$, the temporal growth rate of the SRS is related to these two parameters, and the absolute instability threshold given by Picard’s [25] theoretical formula is

$$\kappa_m > 2\frac{\kappa}{k_t} |\kappa_t|^{-1/2} \exp\left(-2 \frac{k_t}{\kappa} \frac{\kappa_t}{L} - 1\right).$$ (5)

Once again, we note that this formula suits long-wavelength modulation; the exact solution can be seen in figure 10 of [25]. Similarly to earlier studies, we study the influence of density modulation on SRS. We consider two different kinds of density modulation, one where the wavelength of the density modulation is close to $2\pi L_0$, and the other where the wavelength of the density modulation is much smaller than $2\pi L_0$, where $L_0 = |V_1V_2|^{1/2}/\gamma_0 = 33.3 \omega_0/c$. We note that the WKB approximation is still valid when short-wavelength density modulation is considered. Because the modulation amplitude is small, it does not affect the WKB approximation [25].

First, we add long-wavelength density modulation to the system. The modulation wave number is $k_t = 0.03 k_0$, i.e., $L_0$ is equal to $L_0$, and the amplitude is $\kappa_m = 0.1 \omega_0/c$. The amplitude in real space is $\epsilon = 0.018$, which is obtained using equation (2). Since the density modulation used here is above the threshold of Picard’s formula [25], absolute SRS can be induced. As expected, in figure 1(a), the red line increases exponentially over time, which is a clear sign of absolute SRS, and when $t > 2.1$ ps, the reflectivity becomes flat (because the pump depletion occurs). Thus, the average reflectivity of the red line is much larger than the average reflectivity obtained by Rosenbluth gain.

In addition, the temporal growth rate of absolute SRS caused by density modulation can also be obtained by three-wave simulations. As shown in figure 2(a), the time history of $a_1$ is collected at the leftmost edge of the simulation box, and we take a linear fitting of $\log(a_{1f}/a_{1seed})$, where $a_{1seed} = 1 \times 10^{-4}$. The growth rate of absolute SRS in figure 2(a) is $\gamma_{abs} = 0.07545^{+5}_{-5}$ SRS. Using a stationary modulational wave number, $k_t = 0.03 k_0$, the $\kappa_m$ dependence of the absolute SRS growth rate is shown in figure 2(b). Based


understand the reason for the existence of the growth rate peak of point. In our case, the wave number dependence of the growth rate of absolute SRS and the maximum growth rate is $\kappa_0$ with a fixed amplitude $m_0$. The blue diamonds in figure 3. Diagram of the plasma profile without density modulation (black line) and with density modulation $k_x = 0.03k_0$, $\varepsilon = 0.03$ (blue line).

(a) The linear fitting of log($\alpha_{SL}/\alpha_{SL,max}$) to obtain the growth rate of absolute SRS with density modulation $k_x = 0.03k_0$, $\kappa_m = 0.1\omega_0/c$. (b) The dependence of the growth rate on the wave number $k_x$ with $\kappa_m = 0.1\omega_0/c$. (c) The dependence of growth rate on the amplitude $m_0$ with $\kappa_m = 0.1\omega_0/c$. (d) The threshold of absolute SRS in the two-dimensional plane of $k_x$ and $\kappa_m$ with equation (5) (red line); the grey shaded area is the region where absolute SRS can occur, together with the threshold obtained by the three-wave simulations (blue diamonds).

on our three-wave simulations, the threshold of absolute SRS for $k_x = 0.03k_0$ is $\kappa_m \geq 0.06\omega_0/c$ ($\varepsilon = 0.0108$). The growth rate reaches a maximum when $\kappa_m = 0.14\omega_0/c$ ($\varepsilon = 0.0252$), and the maximum growth rate is $\gamma_{abs} = 0.11979\gamma_{SRS}$. Also, the wave number dependence of the growth rate of absolute SRS with a fixed amplitude $\kappa_m = 0.1\omega_0/c$ is shown in figure 2(c). The threshold of absolute SRS is $k_x \geq 0.02k_0$ when $\kappa_m = 0.1\omega_0/c$. The peak growth rate is also shown to be around $k_x = 0.04k_0$ and the maximum growth rate is $\gamma_{abs} = 0.10547\gamma_{SRS}$.

Peaks of the growth rate appear in both figures 2(b) and (c). This is due to the fact that the derivative of the wave number mismatch vanishes at the growth rate’s peak point, i.e. $d\gamma_{abs}/dk_x = k_x\kappa_m\cos(k_x) = 0$ [26]. The vanishing condition is $k_x \approx k_x\kappa_m$, in figure 2(b), at the peak point, $k_x\kappa_m = 0.0042$, and in figure 2(c), $k_x\kappa_m = 0.004$ at the peak point. In our case, $k_x' \approx 0.0031$; this is close to the product of $k_x$ and $\kappa_m$ at the peak points of the growth rate. Thus, we understand the reason for the existence of the growth rate peak points.

Furthermore, as shown in figure 2(d), the red line is the threshold of absolute SRS in the two-dimensional plane of $k_x$ and $\kappa_m$ using equation (5), and the grey shaded area represents the region where absolute SRS can be induced by sinusoidal density modulation. The blue diamonds in figure 2(d) are the threshold obtained by three-wave simulations, which agrees with Picard’s quadratic fit formula in a specific wave number region ($k_x \leq 0.145k_0$). The large wave number region will be discussed below.

We now consider the second situation and find that a system with short-wavelength density modulation ($k_x = 2k_0$, $\kappa_m = 0.1\omega_0/c$) cannot cause absolute SRS. As shown in figure 1(b), the reflectivity (blue line) remains unchanged compared to that without density modulation (black line) in figure 1(a) and that obtained by Rosenbluth gain represented by the green dashed line. In figure 2(d), a sharp cutoff of the short-wavelength modulation is plotted using three-wave simulations. The cutoff is seen as $k_x \leq 0.145k_0$, equally, $\lambda_0 = 2\pi/k_x \geq 1.3L_0$, which also appears in Picard’s exact solution [25] and Li’s three wave simulations [23]. This result is also consistent with the requirement for absolute instability under imperfect laser and plasma conditions, as studied by many authors [21, 22, 24, 26–28]. Since the absolute SRS needs a basic length $L_0$ to grow, the modulation length needs to be comparable to $L_0$ to induce absolute SRS.

We conclude that in the three-wave simulations above, absolute SRS can be induced by density modulation when $L_m \sim L_0$, and the threshold we obtained from three-wave simulations agrees with the theoretical formula [25]. However, short-wavelength density modulation cannot induce absolute instability. We give the applicable range of wave numbers for equation (5), $\lambda_0 \geq 1.3L_0$. Li et al. also considered the transition of SRS at a higher densities, e.g., near $1/4n_r$, the intensity of the pump laser is lower than that of our lower-density case, and the threshold of absolute SRS at higher densities still agrees with equation (5) [23]. However, the three-wave simulations can only consider the linear stage of SRS. Next, we use Vlasov simulations to study the influence of sinusoidal density modulation on SRS in inhomogeneous plasmas in the linear and nonlinear regimes.

3. Vlasov simulations

Fully kinetic Vlasov software [29, 30] is used here to study the SRS with density modulation. The plasma density profile is shown in figure 3. The density region ranges from $0.0667n_r$ to $0.2n_r$ according to the formula $n_e(x) = n_{e0}[1 + (x - x_r)/L]$ (black line), and the blue line represents the plasma density with a sinusoidal density modulation, $n_e(x) = n_{e0}[1 + (x - x_r)/L + \varepsilon \sin(k_x)]$, where $L = 100 \mu m$. 

![Figure 2](image2.png)

![Figure 3](image3.png)
$\varepsilon = 0.03$, and $k_s = 0.03k_0$. The electron temperature is $T_e = 1$ keV. The ions are set to be immobile in the Vlasov simulation, to exclude stimulated Brillouin scattering (SBS), because we only study SRS in Vlasov simulations.

The whole simulation box is $L_{box} = 573 \lambda_0$ with a small length of vacuum space on both sides of the simulation box and we add a small length region with strong collision damping on both sides of the simulation box to avoid particle reflection. Space and time are discretized with $dx = 0.1c\omega_0$ and $dt = 0.1\omega_0^{-1}$. The total simulation time is $t_{total} = 20000\omega_0^{-1} = 3.5$ ps. The pump laser enters the simulation box on the left hand side at $\lambda_0 = 0.351 \mu m$ and an intensity $I_{pump} = 1 \times 10^{15} \ W \ cm^{-2} \times 10^{-15} \ W \ cm^{-2}$. The normalized light intensity of the pump laser is $a_0 = 0.0095 - 0.03$. The seed laser enters the plasma from the right side of the simulation box; the intensity of the seed laser $a_1 = 0.02a_0$, and its wavelength is $\lambda_1 = 0.530 \mu m$, which corresponds to SRS reflected light at $n_{10-\sigma} = 0.1n_e$.

### 3.1. Linear stage: absolute SRS

In the three-wave simulations, absolute SRS can be induced when the density modulation amplitude is above the threshold. The absolute SRS threshold is $\kappa_m = 0.06\omega_0/e$ ($\varepsilon = 0.0108$) when the wave number of the density modulation is $k_s = 0.03k_0$. In our Vlasov simulation, the wave number and amplitude of the long-wavelength case are $k_l = 0.03k_0$ and $\varepsilon = 0.03$, which are above the threshold required for absolute SRS. Therefore, as shown in figure 4, the red line shows a clear feature of absolute SRS at $t = 0.8-1.2$ ps. The reflectivity is much larger than that observed without density modulation. We note that the reflectivity of the SRS without density modulation agrees well with the convective reflectivity represented by the green dashed line using equation (4). At $t > 1.2$ ps, the absolute SRS in figure 4 comes into saturation due to the nonlinear frequency shift. We will discuss this nonlinear phenomenon in the next subsection.

Also, in figure 5, the black circles show the average reflectivity of SRS with varying density modulation amplitudes when the wave modulation number equals $0.03k_0$; the average reflectivity is obtained by $\bar{R} = \int_0^{t_{total}} R(t)dt/\int_0^{t_{total}} dt$. When the amplitude of density modulation is $\varepsilon = 0.01$, which is lower than the absolute SRS threshold, we observe that the reflectivity is still at a convective level. However, when $\varepsilon = 0.02$, which is above the threshold of absolute SRS, the average reflectivity increases to 29.75%. As we know, the average reflectivity of absolute SRS is related to the temporal growth rate of SRS; thus, the growth rate of SRS in figure 2(b) has the same trend with the average reflectivity as that of figure 5. They both have a peak point in terms of a varying $\varepsilon$ and the peak point of the growth rate located at $\varepsilon = 0.025$ is consistent with that of the average reflectivity located at $\varepsilon = 0.025$. In addition, from three-wave simulations, we find that the absolute SRS can only be induced when $\lambda_0 \geq 1.3L_0$. The blue line in figure 5 is the time history of reflectivity when the short-wavelength density modulation ($k_s = 2k_0$, $\varepsilon = 0.03$) is added to the system. The blue line is at the level of convective SRS reflectivity because in this case, $\lambda_0 = 0.094L_0 < 1.3L_0$, which is not in the region where absolute SRS can occur.

### 3.2. Nonlinear stage: suppression of inflation

As we know, most nonlinear processes in SRS are related to the amplitude of Langmuir waves. When the amplitude of the Langmuir waves is large enough to trap an number of electrons near the phase velocity, the distribution function near the phase velocity is flattened, and then the Landau damping of the Langmuir wave is reduced, accompanied by a nonlinear frequency shift. Vu et al reported that the burst of SRS reflectivity is...
induced by particle trapping and they called it SRS inflation [32–34]. A nonlinear frequency shift also occurs, which is an SRS saturation mechanism. The inflation and the nonlinear frequency shift cause pulse-like bursts of SRS. Similarly, in our Vlasov simulations, we find that a higher intensity of the pump laser can result in the nonlinear regime of SRS.

In figure 6, the black line is the pump laser intensity’s dependent theoretical reflectivity based on Rosenbluth gain, and the red circles represent the average reflectivity without density modulation. We observe that the simulations agree with Rosenbluth theory when the intensity of the pump laser is lower than $4 \times 10^{15} \text{ W cm}^{-2}$. This is because the SRS is in the linear regime. When the intensity of the pump laser is higher than $4 \times 10^{15} \text{ W cm}^{-2}$, the reflectivity is much higher than that obtained by equation (4), which means that nonlinear processes occur.

We observe that the reflectivity has bursts during 2 ps, shown in figure 7(b), which is the frequency spectrum of the reflected light when $I_0 = 8 \times 10^{15} \text{ W cm}^{-2}$. The burst of reflected light is not observed in figure 7(a) when the intensity of the pump laser is $I_0 = 4 \times 10^{15} \text{ W cm}^{-2}$.

Figures 7(c) and (d) show the electron phase structure near the phase velocity at $t = 1.5 \text{ ps}$ for $I_0 = 4 \times 10^{15} \text{ W cm}^{-2}$ and $I_0 = 8 \times 10^{15} \text{ W cm}^{-2}$, respectively. The velocity width of the trapping structure in figure 7(d) is larger than that in figure 7(c), and the trapping structure begins to merge when $I_0 = 8 \times 10^{15} \text{ W cm}^{-2}$. Correspondingly, the electron distribution functions are shown in figures 7(e) and (f). The electron distribution of $I_0 = 4 \times 10^{15} \text{ W cm}^{-2}$ stays unchanged near the phase velocity from $t = 1.5 \text{ ps}$ to $t = 2.1 \text{ ps}$ while the electron distribution in the $I_0 = 8 \times 10^{15} \text{ W cm}^{-2}$ case is flatter and much wider near the phase velocity as time goes on. Based on the results of [32], particle trapping reduces the Landau damping of the Langmuir waves, which causes SRS inflation. Thus, we believe that the inflation of SRS happens when $I_0 = 8 \times 10^{15} \text{ W cm}^{-2}$. Also, we have changed the intensity of the pump laser from $1 \times 10^{15} \text{ W cm}^{-2}$ to $10 \times 10^{15} \text{ W cm}^{-2}$, and we find that the average reflectivity is at the convective level when $I_0$ is in the range from $1 \times 10^{15} \text{ W cm}^{-2}$ to $4 \times 10^{15} \text{ W cm}^{-2}$, and increases to 18.66% when $I_0 = 8 \times 10^{15} \text{ W cm}^{-2}$; this is the inflation feature reported by Vu et al [32–34]. Next, we discuss the influence of sinusoidal density modulation on SRS in the nonlinear regime.

Finally, we study the influence of short-wavelength density modulation on the SRS in the nonlinear regime. As shown by the blue triangles and green stars of figure 6, the reflectivity remains unchanged, as does the amplitude of the short-wavelength density modulation when $I_0 \leq 4 \times 10^{15} \text{ W cm}^{-2}$. This agrees well with our three-wave simulations. The SRS reflectivity decreases with the modulation amplitude when $I_0 \geq 6 \times 10^{15} \text{ W cm}^{-2}$. In [40, 41], the Landau damping of the Langmuir wave increases with the modulation amplitude, and the Landau damping of the Langmuir wave is sensitive to the plasma temperature when short-wavelength modulation...
is added. This is one of the reasons that SRS reflectivity decreases with the modulation amplitude in the nonlinear regime.

O’Neil [42] stated that particle trapping can reduce the Landau damping for \( \omega_{\text{bounce}} > 2\pi c \), where \( \omega_{\text{bounce}} = \sqrt{k_0^2 + \lambda_0^2} \phi / T_e \) is the potential of the Langmuir wave. Thus, reducing the amplitude of the Langmuir wave is an effective way to avoid SRS inflation.

When we add the short-wavelength modulation, the harmonic waves are induced. Figure 8(a) is the time history of the plasma wave in \( k \) space, and we can observe two clear harmonic waves [13] at \( k_0 = k_0 \) and \( k_0 = k_0 + k_h \), where \( k_h = 2k_0 \).

The wave number of the primary Langmuir wave is \( k_{a0} = 1.53k_0 \), so the wave number of the downward harmonic wave is \( k_{h-1} = -0.47k_0 \), which means that the propagation direction of the downward harmonic wave is opposite to that of the Langmuir wave. The downward harmonic wave has a higher intensity than the upward harmonic wave because the downward harmonic wave has less Landau damping. The downward harmonic wave can take energy away from the Langmuir wave and then suppress the SRS. As depicted in figure 8(b), the green dashed line represents the phase velocity of the Langmuir wave, \( v_{\phi0} = 0.22c \), corresponding to the electron trapping velocity, and the brown dashed line is the phase velocity of the downward harmonic wave. In theory, the phase velocity should be \(-0.66c\), but in the simulation, \( v_{\phi h-1} = -0.48c \) due to the broadening of the downward harmonic wave in the \( k \) spectrum, as shown in figure 8(a). Since the \( k \) spectrum of the Langmuir wave broadens to around \( 1.3k_0 \), the real trapping velocity of the downward harmonic wave is at around \(-0.48c\), as shown in the electron distribution function at different times. The electron trapping around the primary Langmuir wave velocity stays at the same level because of the downward harmonic wave, and more electrons are trapped by the downward harmonic wave near \( v_{\phi h-1} \) from \( t \approx 1.05 \) ps to \( t \approx 2.1 \) ps. We note that there is no electron trapping in the minus \( v \) direction in the case of no density modulation in figure 7(f). Thus, the suppression of inflation is a consequence of the downward harmonic wave induced by short-wavelength density modulation.

In figure 6, the average SRS reflectivity decreases with the amplitude of the density modulation in the nonlinear regime.

This phenomenon is different from the long-wavelength modulation case shown in figure 5. The long-wavelength density modulation can easily cause the absolute SRS to increase the reflectivity of SRS. Thus, in ICF experiments, one should try to avoid long-wavelength density modulation, and one should add short-wavelength density modulation to suppress the inflation of SRS.

4. PIC simulations

To test our model in three-wave simulations and Vlasov simulations, we decided to use the PIC software EPOCH [14]. The electron density profile is a linear profile, \( n_e \in [0.0667, 0.2]n_0 \), the electron temperature is \( T_e = 1 \) keV. We consider two different situations, one is the fixed-ion case, and the other is the mobile-ion case. In the mobile-ion case, we use \( He^{2+} (M = 7344n_e, Z = 2) \) as the ions, and the ion temperature is \( T_i = 0.5 \) keV. The wavelength of the pump laser is \( \lambda_0 = 0.351 \) \( \mu m \). In the fixed ion case, a seed laser is added with \( \alpha_{\text{seed}}/\alpha_0 = 0.02 \), i.e. \( \alpha_{\text{seed}} = 1.6 \times 10^{-12} \) W cm\(^{-2}\), \( \lambda_{\text{seed}} = 0.530 \) \( \mu m \), and there is no seed laser in the mobile-ion case. The pump laser profile is a plane wave with a 50 \( T_0 \) ramp up. The boundary condition for waves is an open boundary condition, and for particles it is a thermal boundary condition, which means that when the boundary absorbs a particle moving out of the simulation box, the system creates a new particle with an initial Maxwell distribution. The total simulation time is \( t_{\text{total}} = 3.5 \) ps, and the simulation box is [0, 382] \( \lambda_0 \) with 100 cells/\( \lambda_0 \). In the fixed-ion case, we use 100, 500, and 1000 particles per cell (PPC), and PPC = 100 is used in the mobile-ion case.

First, we study the fixed-ion case and compare it with our Vlasov simulations, where the intensity of the pump laser is \( I_0 = 4 \times 10^{15} \) W cm\(^{-2}\). In figure 9(a), the black line is the reflectivity without density modulation with PPC = 500. Convective saturation is observed at \( t \approx 0.5 \) ps, which is similar to that of the Vlasov simulation in figure 4. However, after 2.5 ps, the reflectivity increases because the SRS has a higher density \( n_e \sim 0.2n_0 \). In PIC simulations, the noise level is higher than in the Vlasov simulations, especially when the PPC is small.
modulation; in figure Vlasov simulations. The SBS is hardly affected by density which is in agreement with our three-wave simulations and modulation induces absolute SRS at by long-wavelength modulation, because the long-wavelength plasma density. Figure long-wavelength density modulation induces the SRS at lower than that without density modulation. This occurs because the $t > t_0$.

The average reflectivity $R = \int_0^{t_{\text{total}}} R(t) \, dt / \int_0^{t_{\text{total}}} dt$ is 2.8%. If we calculate the average reflectivity at $[0, 2]$ ps, the average reflectivity is 0.27%, which agrees with the average reflectivity (black line) in figure 4, i.e. 0.266%. The red line in figure 9(a) is the reflectivity with long-wavelength density modulation and PPC = 500. It clearly shows the absolute SRS feature at an early stage. The average reflectivity of the red line is 43%. In the Vlasov simulation, the corresponding average reflectivity is 33%. They are comparable. The number of hot electrons in the modulation case is much higher than in the no-modulation case in figure 9(a) [2, 8].

We also study the PPC dependence of the average reflectivity of the PIC simulations in figure 9(b). As shown by the black squares, without modulation, the average reflectivity decreases from 4% to 2.8% when PPC changes from 100 to 500. The average reflectivity with long-wavelength modulation is revealed by the red dots. The reflectivity level remains at around 45% because of the appearance of absolute SRS.

We now study the influence of long-wavelength modulation on the SRS when the ions are mobile. As shown in figure 10(a), the intensity of the pump laser is $I_0 = 4 \times 10^{15}$ W cm$^{-2}$, and we filter out the SRS-reflected light. The blue line represents the SRS reflectivity without density modulation, the average SRS reflectivity for the whole simulation time is 1%, and the red line represents the SRS reflectivity when the long-wavelength modulation is present, and the average SRS reflectivity is 2%. At an early stage, $t < 1.5$ ps, the SRS is dominant, and the SBS becomes the main instability when $t > 1.5$ ps. We observe that the SRS reflectivity is more intense than that without density modulation. This occurs because the long-wavelength density modulation induces the SRS at lower plasma density. Figure 11 is the corresponding reflected light spectrum; the reflected light at $\omega = 0.6\omega_0 - 0.7\omega_0$ is improved by long-wavelength modulation, because the long-wavelength modulation induces absolute SRS at $n_s = 0.075n - 0.15n_0$, which is in agreement with our three-wave simulations and Vlasov simulations. The SBS is hardly affected by density modulation; in figure 11, the position and amplitude of SBS peaks for the two cases are almost identical. Next, we study the influence of short-wavelength modulation on SRS in the nonlinear regime. In figure 10(b), we filter the SRS reflected light when $I_0 = 1 \times 10^{16}$ W cm$^{-2}$. The black line is the SRS reflectivity without density modulation, and the green line represents the SRS reflectivity with short-wavelength modulation $k_s = 2k_0$, $\epsilon = 0.03$. We observe that the SRS reflectivity decreases when short wavelengths are present; the maximum value changes from 2 to 1.6, and the average SRS reflectivity changes from 7% to 5%. This phenomenon is in qualitative agreement with our Vlasov simulations.

The SRS reflectivity in the fixed-ion case is much larger than in the mobile-ion case. Since SBS is dominant in the mobile-ion case, the ion acoustic wave in SBS could be a potential source of short-wavelength density modulation that suppresses SRS.

5. Conclusions and discussion

In this paper, we first numerically solved the three-wave equations to study the influence of density modulation on the SRS reflectivity in underdense inhomogeneous plasmas. We found that long-wavelength density modulation was able to destabilize the convective SRS and cause an absolute SRS when the modulation amplitude was above the threshold. The absolute SRS threshold had a sharp cutoff, $\lambda_s \geq 1.3L_0$, which agreed with the exact solution of Picard and Johnston [25]. Second, the Vlasov simulation was used to study the influence of density modulation in the linear and nonlinear regimes. In the linear regime, the Vlasov simulations were very consistent with the numerical solution of the three-wave equations. The average reflectivity had the same trend as the temporal growth rate, and short-wavelength density modulation, i.e., $k_s = 2k_0$ was not able to induce absolute SRS. We then found...
that the inflation of SRS as able to be induced by particle trapping when \( I_0 \gtrsim 6 \times 10^{15} \text{ W cm}^{-2} \), and a pulse-like burst of reflectivity was caused by inflation and a nonlinear frequency shift. Finally, we studied the influence of short-wavelength modulation, i.e., \( k_s = 2k_0 \) on SRS inflation. The Landau damping increased with the modulation amplitude. The downward harmonic wave was an effective way to remove the energy of the Langmuir wave; thus, the reflectivity of SRS decreased. Finally, our PIC simulations were qualitatively consistent with our Vlasov simulations.

In the early experiments, a threshold of SRS lower than the Rosenbluth threshold was observed in underdense plasmas [43–46]. Estabrook and Krue pointed out that these phenomena may be related to plasma noise using simulation [47]. In this paper, we offer an alternative explanation, which is that the intense SRS reflected light in the low-density plasmas may be caused by the long-wavelength density modulation. Based on our simulations, absolute SRS can be induced by a density modulation with a very small amplitude \( \varepsilon \sim 1\% \). The sinusoidal density modulation that we consider here is similar to the turbulent density gradient [24]; both these kinds of modulation can induce absolute SRS. In addition, our results show that short-wavelength density modulation can suppress the inflation of SRS by the downward harmonic wave. This mechanism may be applied in high-intensity and inhomogeneous ignition schemes [16–18, 35–37] to suppress SRS.

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ORCID iDs

Y Chen https://orcid.org/0000-0003-1074-5500
Q S Feng https://orcid.org/0000-0002-0757-8978
C Z Xiao https://orcid.org/0000-0002-5232-2947

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