Testing chemical composition of highest energy cosmic rays

DALIBOR NOSER¹, JAKUB VICHA², JANA NOSKOVA³, JAN EBR².

¹ Institute of Particle and Nuclear Physics, Faculty of Mathematics and Physics, Charles University, Prague, Czech Republic
² Institute of Physics, Academy of Sciences of the Czech Republic, Prague, Czech Republic
³ Department of Mathematics, Faculty of Civil Engineering, Czech Technical University, Prague, Czech Republic

nosker@ipnp.troja.mff.cuni.cz

Abstract: We study basic characteristics of distributions of the depths of shower maximum in air showers caused by cosmic rays with the highest energies. The consistency between their average values and widths, and their energy dependences are discussed within a simple phenomenological model of shower development independently of assumptions about detailed features of high–energy interactions. It is shown that reliable information on primary species can be derived within a partition method. We present examples demonstrating implications for the changes in mass composition of primary cosmic rays.

Keywords: ultra–high energy cosmic rays, chemical composition.

1 Introduction

Knowledge of the mass distribution of cosmic rays (CR) and of its energy evolution can provide useful information about CR acceleration mechanisms and propagation through the galactic and extragalactic space. Measurements and subsequent analysis of the mass composition of ultra–high energy cosmic ray (UHECR) primaries are of particular importance. Corresponding observables can help to understand their typical spectral features, the ankle at about 4 EeV and the steep flux suppression at energies above 30 EeV. In addition, their knowledge makes it possible to examine the mass composition we utilize the particles of the mass A initiated by primaries of the mass \( A \) on a reference energy of \( E_0 \).

\[
\langle X_{\text{max}} | A \rangle = C + D \log \left( \frac{E}{E_0^A} \right),
\]

where \( C = \langle x_{\text{max}} \rangle (E_0) \) is a constant proton mean shower maximum, and \( D = \frac{d\langle x_{\text{max}} \rangle}{d\log E} \) is the proton elongation rate. \( \langle x_{\text{max}} \rangle = \langle X_{\text{max}} | A = 1 \rangle \) is the proton mean depth of shower maximum, and \( C = \langle x_{\text{max}} \rangle (E_0) \) is the proton constant proton mean depth of maximum at a reference energy of \( E_0 \). In the same line, the conditional variance of the depth of maximum is

\[
\sigma_{\text{max}}^2 \langle X_{\text{max}} | A \rangle = \sigma_{\text{fr}}^2 + \sigma_{\text{sh}}^2, \tag{2}
\]

where \( \sigma_{\text{fr}}^2 = \sigma_{\text{fr}}^2(A,E) \) is the variance of the depth where the first interaction of the CR primary takes place and \( \sigma_{\text{sh}}^2 = \sigma_{\text{sh}}^2(A,E) \) assigns the variance of the depth of shower maximum associated with its subsequent development.

Then, the total mean and total variance of the depth of shower maximum at a given energy \( E \) that are to be confronted with measurements, are respectively

\[
\langle X_{\text{max}} \rangle = \langle X_{\text{max}} | A \rangle = \langle x_{\text{max}} \rangle - d \langle \ln A \rangle, \tag{3}
\]

and

\[
\sigma_{\text{max}}^2 = \sigma_{\text{fr}}^2 + \langle \sigma_{\text{sh}}^2 \rangle + d^2 \sigma_{\ln A}^2, \tag{4}
\]

where \( D = d \ln 10 \) was inserted and the law of total variance was used, i.e. \( \sigma_{\text{max}}^2 = \langle \sigma_{\text{max}}^2(X_{\text{max}} | A) \rangle \) and \( \sigma_{\text{max}}^2(x_{\text{max}} | A) \).

2 Air shower model

Let us assume that a CR shower maximum \( X_{\text{max}} \) is measured when a UHECR particle with a mass \( A \) hits the upper part of the Earth atmosphere. In the following we will treat these two quantities as dependent random variables, \( X_{\text{max}} = X_{\text{max}}(A) \). Adopting superposition assumptions, the mean depth of shower maximum provided air showers are initiated by primaries of the mass \( A \) depends on the shower energy \( E \) as

The mean penetration depth in the atmosphere at which the shower of secondary particles reaches its maximum number, \( X_{\text{max}}(A) \), and \( \sigma_{\text{max}} = \sigma(X_{\text{max}}) \), the square root of its variance, are widely used. Recent results presented by the Auger collaboration indicate a transition from lighter to heavier primaries at the ankle region. The HiRes collaboration achieved a different conclusion. Its analysis based on the truncated fluctuation widths speaks in favor of very light primaries at the highest energies.

Based on widely accepted empirical characteristics of the energy evolution of the mean depth of shower maximum and its variance, we present a method in which reasonable inferences about the partition of the primary CR mass are naturally achieved. Utilizing a generalized Heitler model, two illustrative examples are presented. We make use of the recently measured value of the \( p – \text{air cross section} \) and try intentionally to account for the details of EAS development independently of assumptions about detailed features of hadronic interactions.

3 Partition problem

To examine the mass composition we utilize the partition method described briefly in Appendix. To this end, we use two \( A \)-dependent constrains, \( F_1(A) = d \ln A \) and \( F_2(A) = d^2 \ln^2 A + \sigma_{\text{fr}}^2 + \sigma_{\text{sh}}^2 \) respectively. Their average values are given by the available experimental information.
We successfully applied the maximum–entropy method where and variance at given energy, principal as described in Appendix A. Knowing the total mean shower maximum connected with the shower development, characteristics resembling their measured energy for a number of artificially chosen examples with average energy. The widths of colored bands correspond to aforementioned uncertainties in parameters and to the total sample mean, contained in the measurements. They are directly connected to the total sample mean, \( \langle X_{\text{max}} \rangle \), and to the total sample variance of \( X_{\text{max}} \), \( \sigma_{\text{max}}^2 \), measured at given energy.

The aforementioned constrains are written as

\[
\begin{align*}
( F_1 ) &= d \langle \ln A \rangle = Q_{\text{max}}, \\
( F_2 ) &= d^2 \langle \ln^2 A \rangle + ( \sigma_Q^2 ) = \sigma_{\text{max}}^2 + Q_{\text{max}}^2,
\end{align*}
\]

where \( Q_{\text{max}} = \langle X_{\text{max}} \rangle - \langle X_{\text{max}} \rangle \).

In the partition method, the probability distribution of the mass number is dictated by the maximum–entropy principal as described in Appendix A. Knowing the total mean and variance at given energy, \( \langle X_{\text{max}} \rangle \) and \( \sigma_{\text{max}}^2 \), the form of this distribution is given by Eq. (10) with two Lagrangian multipliers deduced numerically in such a way that the two constrains written in Eqs. (5) and (6) are satisfied.

In this study, the proton mean depth of shower maximum \( C = \langle X_{\text{max}} \rangle (E_0) \) at a reference energy of \( E_0 \) and the energy independent proton elongation rate \( D = d \ln 10 \) are only two free parameters.

The \( A \)–dependence of the depth of shower maximum is given by the Heitler conjecture, see Eq. (1). For other mass dependent terms we use simple phenomenological arguments described in Appendix B. The variance of the depth of the first interaction, \( \sigma_{E_1}^2 = \sigma_Q^2 (A, E) \), is deduced from the measured \( p–\)air–cross section and its extrapolated energy dependence. The variance of the depth of shower maximum connected with the shower development, \( \sigma_{E_2}^2 = \sigma_Q^2 (A, E) \), is inferred from basic characteristics of underlying interaction processes. Let us stress that other parametrizations of the EAS \( A \)–dependent terms, different from that ones introduced in Appendix B can be adopted in our treatment.

### 4 Illustrative examples

We successfully applied the maximum–entropy method for a number of artificially chosen examples with average shower characteristics resembling their measured energy evolution. Within the partition method, we decomposed these observables into different sets of primary masses assuming different parametrization of the mean depth of shower maximum and its variance. In the following, we present results of two of these hypothetical examples.

In the first example, we used the mean depth of shower maximum with a constant elongation rate and a logarithmically increasing square root of the depth variance with energy. These shower statistics, displayed in Figs. 1 and 2 as black empty points, were parametrized by

\[
\frac{\langle X_{\text{max}} \rangle (E) - X_0}{D_0} = \frac{\sigma_{\text{max}} (E) - \sigma_0}{s_0} = \log \left( \frac{E}{E_0} \right),
\]

where \( X_0 = 673 \ \text{gcm}^{-2} \) and \( \sigma_0 = 36 \ \text{gcm}^{-2} \) are shower statistics at a reference energy of \( E_0 = 1 \ \text{EeV} \), and parameters \( D_0 = 80 \ \text{gcm}^{-2} \) and \( s_0 = 10 \ \text{gcm}^{-2} \). An energy interval \( \text{Log}(E/\text{EeV}) \in (17.1, 19.7) \) with 14 equidistant values \( \Delta \text{Log}(E/\text{EeV}) = 0.2 \) was assumed.

In the following calculations, we tried to decompose the mass composition represented by the shower statistics, \( \langle X_{\text{max}} \rangle (E) \) and \( \sigma_{\text{max}} (E) \), into four pieces corresponding to primary species generating underlying CR showers. Namely, we assumed proton primaries \( (A = 1) \), and helium \((A = 4) \), nitrogen \((A = 14) \) and iron \((A = 56) \) nuclei.

In the first step, we solved the partition problem numerically treating the two unknown quantities, \( C \) and \( D \) introduced in Eq. (11), as free parameters. This way, we obtained a two–dimensional domain where maximum–entropy solutions exist. In the second step, we performed the partition analysis with parameters \( C = (730 - 740) \ \text{gcm}^{-2} \) and \( D = (56 - 60) \ \text{gcm}^{-2} \) that provided us the best solutions of the partition problem.

Our results are summarized in top panels in Figs. 3 and 4. In the top panel in Fig. 4 decomposition probabilities of hypothetical shower statistics are depicted as functions of energy. The widths of colored bands correspond to aforementioned uncertainties in parameters \( C \) and \( D \). The mean and variance of logarithmic mass are depicted in the top panel in Fig. 4. Both characteristics give the expected trends with
steeply falling \((\ln A)\) and growing up variance \(\sigma_{\ln A}^2\) with the increasing energy. Large uncertainties of heavier primaries at energies where small values of \(\langle x_{\text{max}} \rangle\) and \(\sigma_{\text{max}}\) were chosen are salient features of our treatment.

In the second example, we tried to analyze hypothetical shower characteristics for the constant elongation rate (top panel) and the elongation rate with a break (bottom panel) as shown in Figs.1 and 2. Red, green, gray and blue bands are for proton, helium, nitrogen and iron primaries. Their widths correspond to uncertainties of parameters

\[
C = \langle x_{\text{max}} \rangle |E_0| \quad \text{and} \quad D.
\]

Also in this hypothetical example we obtained reasonable solutions. The chosen breaks in shower statistics are well visible in the energy evolution of the resultant partition probabilities. The lightest component, driven up to the chosen break dies out rapidly after it reaches its maximum value near the break at \(\log(E/eV) = 18.4\). Interestingly, the proton–iron mixture is not able to explain the chosen energy evolution of shower statistics. For a reasonable description intermediate mass nuclei are necessary.

5 Conclusions

We used the well justified maximum–entropy method to deduce the partition of the mass of CR primaries from the hypothetical characteristics of the EAS development that they initiated. This method combines simple properties of the generalized Heitler model, multiplication characteristics of air showers and the measured \(p–\text{air cross section. It is independent of details on hadronic interactions.}\n
The partition method enables us to establish a reasonable connection between the mean value of the logarithmic primary mass number, its variance and other observ-
ables as well. The resultant decomposition of the mass distribution describes what we know from experiment as effectively as possible provided the selected model of the shower evolution holds. Let us finally stress that the consistency of deduced quantities, as interpreted in the Heitler reasoning, is emphasized rather than questioned within the partition method.

A Partition formalism
Let us assume that the quantity A is capable to take n discrete values $A = 1, \ldots, n$. Corresponding probabilities $p_A$ are not known, however. Only a set of r expectation values of the functions $F_i(A)$, $i = 1, \ldots, r$, $r < n$, is measured. For setting up a probability distribution which satisfies the given data, the least bias possible estimate on the basis of partial knowledge is used. This method, known as the maximum–entropy principle, is widely used in statistical mechanics [8], for its statistical background see e.g. [7].

In our method, the depth of shower maximum caused by a primary proton at the reference energy of $E_0 = 1$ EeV, $s_{fr0} \approx 46$ g cm$^{-2}$, is deduced from the measured $p$–air cross section as well as a function $\xi(E)$ of $E$. The A–dependent term in Eq.[13] accounts for details of the first interaction given by individual nucleon–nucleon interactions and subsequent nuclear fragmentation [5]. A statistical treatment assuming a subset of interacting nucleons supplemented by simple geometrical arguments gives approximately $\sigma_{fr0} \approx 0.3 – 3.0$ yielding slightly different results that were negligible if uncertainties of other parameters were taken into account.

Assuming an experimental value $\sigma_{max} \approx 60$ g cm$^{-2}$ at about 1 EeV [1, 2], and predominantly proton primaries, we estimated the variance of the depth of shower maximum in the subsequent shower development by

\[ \sigma_{sh}^2 = \sigma_{fr,0}^2 + \sigma_{fr,0}^2, \]

where $\sigma_{fr0}$ is the radiation length in air, $\varepsilon \approx 84$ MeV denotes the critical energy in air, $\kappa$ is the elasticity of the first interaction and $M$ assigns its multiplicity. This relationship is well documented by physical arguments and by MC simulations as well. It can also be derived as an approximate solution of Yule birth process.

For the variance of the depth of the first interaction we have adopted the measured $p$–air cross section at a center of mass energy of $\sqrt{s} = 57$ TeV [6]. Relying upon a smooth extrapolation from accelerator measurements, and in agreement with model predictions, here we used a parametrization $\Sigma_{p-Air} \approx [000 + 50 \log(E/\text{EeV})]$ mb. Within a naive model, the variance of the depth of the first interaction is then approximately

\[ \sigma_{fr}^2 = \sigma_{fr,0}^2 (A,E) \approx A^{-\alpha} \xi(E) \sigma_{fr,0}^2, \]

where $A$ assigns the mass number of a primary CR particle and $\alpha$ is a constant index. The variance of the depth of shower maximum caused by the proton primary at the reference energy of $E_0 = 1$ EeV, $s_{fr0} \approx 46$ g cm$^{-2}$, is deduced from the measured $p$–air cross section as well as a function $\xi(E)$ of $E$. The A–dependent term in Eq.[13] accounts for details of the first interaction given by individual nucleon–nucleon interactions and subsequent nuclear fragmentation [5]. A statistical treatment assuming a subset of interacting nucleons supplemented by simple geometrical arguments gives approximately $\sigma_{fr0} \approx 0.3 – 3.0$ yielding slightly different results that were negligible if uncertainties of other parameters were taken into account.

Assuming an experimental value $\sigma_{max} \approx 60$ g cm$^{-2}$ at about 1 EeV [1, 2], and predominantly proton primaries, we estimated the variance of the depth of shower maximum in the subsequent shower development by

\[ \sigma_{sh}^2 = \sigma_{fr,0}^2 + \sigma_{fr,0}^2, \]

where $\sigma_{fr0}$ is the radiation length in air, $\varepsilon \approx 84$ MeV denotes the critical energy in air, $\kappa$ is the elasticity of the first interaction and $M$ assigns its multiplicity. This relationship is well documented by physical arguments and by MC simulations as well. It can also be derived as an approximate solution of Yule birth process.

For the variance of the depth of the first interaction we have adopted the measured $p$–air cross section at a center of mass energy of $\sqrt{s} = 57$ TeV [6]. Relying upon a smooth extrapolation from accelerator measurements, and in agreement with model predictions, here we used a parametrization $\Sigma_{p-Air} \approx [000 + 50 \log(E/\text{EeV})]$ mb. Within a naive model, the variance of the depth of the first interaction is then approximately

\[ \sigma_{fr}^2 = \sigma_{fr,0}^2 (A,E) \approx A^{-\alpha} \xi(E) \sigma_{fr,0}^2, \]

where $A$ assigns the mass number of a primary CR particle and $\alpha$ is a constant index. The variance of the depth of shower maximum caused by the proton primary at the reference energy of $E_0 = 1$ EeV, $s_{fr0} \approx 46$ g cm$^{-2}$, is deduced from the measured $p$–air cross section as well as a function $\xi(E)$ of $E$. The A–dependent term in Eq.[13] accounts for details of the first interaction given by individual nucleon–nucleon interactions and subsequent nuclear fragmentation [5]. A statistical treatment assuming a subset of interacting nucleons supplemented by simple geometrical arguments gives approximately $\sigma_{fr0} \approx 0.3 – 3.0$ yielding slightly different results that were negligible if uncertainties of other parameters were taken into account.

Assuming an experimental value $\sigma_{max} \approx 60$ g cm$^{-2}$ at about 1 EeV [1, 2], and predominantly proton primaries, we estimated the variance of the depth of shower maximum in the subsequent shower development by

\[ \sigma_{sh}^2 = \sigma_{fr,0}^2 + \sigma_{fr,0}^2, \]

where $\sigma_{fr0}$ is the radiation length in air, $\varepsilon \approx 84$ MeV denotes the critical energy in air, $\kappa$ is the elasticity of the first interaction and $M$ assigns its multiplicity. This relationship is well documented by physical arguments and by MC simulations as well. It can also be derived as an approximate solution of Yule birth process.

For the variance of the depth of the first interaction we have adopted the measured $p$–air cross section at a center of mass energy of $\sqrt{s} = 57$ TeV [6]. Relying upon a smooth extrapolation from accelerator measurements, and in agreement with model predictions, here we used a parametrization $\Sigma_{p-Air} \approx [000 + 50 \log(E/\text{EeV})]$ mb. Within a naive model, the variance of the depth of the first interaction is then approximately

\[ \sigma_{fr}^2 = \sigma_{fr,0}^2 (A,E) \approx A^{-\alpha} \xi(E) \sigma_{fr,0}^2, \]

where $A$ assigns the mass number of a primary CR particle and $\alpha$ is a constant index. The variance of the depth of shower maximum caused by the proton primary at the reference energy of $E_0 = 1$ EeV, $s_{fr0} \approx 46$ g cm$^{-2}$, is deduced from the measured $p$–air cross section as well as a function $\xi(E)$ of $E$. The A–dependent term in Eq.[13] accounts for details of the first interaction given by individual nucleon–nucleon interactions and subsequent nuclear fragmentation [5]. A statistical treatment assuming a subset of interacting nucleons supplemented by simple geometrical arguments gives approximately $\sigma_{fr0} \approx 0.3 – 3.0$ yielding slightly different results that were negligible if uncertainties of other parameters were taken into account.

Assuming an experimental value $\sigma_{max} \approx 60$ g cm$^{-2}$ at about 1 EeV [1, 2], and predominantly proton primaries, we estimated the variance of the depth of shower maximum in the subsequent shower development by

\[ \sigma_{sh}^2 = \sigma_{fr,0}^2 + \sigma_{fr,0}^2, \]

where $\sigma_{fr0}$ is the radiation length in air, $\varepsilon \approx 84$ MeV denotes the critical energy in air, $\kappa$ is the elasticity of the first interaction and $M$ assigns its multiplicity. This relationship is well documented by physical arguments and by MC simulations as well. It can also be derived as an approximate solution of Yule birth process.

For the variance of the depth of the first interaction we have adopted the measured $p$–air cross section at a center of mass energy of $\sqrt{s} = 57$ TeV [6]. Relying upon a smooth extrapolation from accelerator measurements, and in agreement with model predictions, here we used a parametrization $\Sigma_{p-Air} \approx [000 + 50 \log(E/\text{EeV})]$ mb. Within a naive model, the variance of the depth of the first interaction is then approximately

\[ \sigma_{fr}^2 = \sigma_{fr,0}^2 (A,E) \approx A^{-\alpha} \xi(E) \sigma_{fr,0}^2, \]

where $A$ assigns the mass number of a primary CR particle and $\alpha$ is a constant index. The variance of the depth of shower maximum caused by the proton primary at the reference energy of $E_0 = 1$ EeV, $s_{fr0} \approx 46$ g cm$^{-2}$, is deduced from the measured $p$–air cross section as well as a function $\xi(E)$ of $E$. The A–dependent term in Eq.[13] accounts for details of the first interaction given by individual nucleon–nucleon interactions and subsequent nuclear fragmentation [5]. A statistical treatment assuming a subset of interacting nucleons supplemented by simple geometrical arguments gives approximately $\sigma_{fr0} \approx 0.3 – 3.0$ yielding slightly different results that were negligible if uncertainties of other parameters were taken into account.

Assuming an experimental value $\sigma_{max} \approx 60$ g cm$^{-2}$ at about 1 EeV [1, 2], and predominantly proton primaries, we estimated the variance of the depth of shower maximum in the subsequent shower development by

\[ \sigma_{sh}^2 = \sigma_{fr,0}^2 + \sigma_{fr,0}^2, \]

where $\sigma_{fr0}$ is the radiation length in air, $\varepsilon \approx 84$ MeV denotes the critical energy in air, $\kappa$ is the elasticity of the first interaction and $M$ assigns its multiplicity. This relationship is well documented by physical arguments and by MC simulations as well. It can also be derived as an approximate solution of Yule birth process.

B Shower variances
In our method, the depth of shower maximum caused by a primary proton with energy $E$ is assumed to be

\[ \langle s_{max} \rangle (E) \approx \lambda (E) + X \ln \left( \frac{kE}{2Me} \right), \]

where $\lambda (E)$ is the average interaction length for inelastic $p$–air collisions, $X \approx 37$ g cm$^{-2}$ is the radiation length in air, $\varepsilon \approx 84$ MeV denotes the critical energy in air, $\kappa$ is the elasticity of the first interaction and $M$ assigns its multiplicity. This relationship is well documented by physical arguments and by MC simulations as well. It can also be derived as a...