Abstract
The study presents a methodology for the optimal operation of district heating networks with a circular conduit system. The authors discuss this topic in an absolutely general form. By a special statement and solution of Kirchhoff Laws, node equations and loop equations, the hydraulic end point of the circle is determined, including the supply ratio of the consumer located at the hydraulic end point. Two objective functions are stated by the authors; one of them for the minimum of flow work, and the other for the minimum of power supplied. It is demonstrated that the objective functions yield different results. Theoretically more economical operation is ensured by the flow pattern resulting from the minimization of the power supplied.

Keywords
looped district heating network, dissipated energy, minimum of energy input, hydraulic analysis, minimum of dissipated energy

1 Introduction
As regards energetics in Europe, district heating plays a prominent role in the heat supply of large cities. District heating systems have extensive conduit networks. The hydraulic examination of conduit networks, adjustment of a hydraulic optimum, of the optimal pump operating point represent key issues in network operation. Some part of conduit networks is radial, another part is of loop topography. The operation of loop networks is more complicated; the flow pattern produced is generally unstable due to changes in consumer demands in Ref. [2-4]. The method of Krope et al. is based on nonlinear optimization in Ref. [5-8]. In Ref. [9] the optimal operation is based on fast fluid-dynamic simulation. In Ref. [10] a fluid dynamic model of the network based on conservation was built and a genetic algorithm used in order to minimize the energy required by the system. A technical-economical optimization with the aim of minimizing both the pumping energy consumption and the thermal energy losses while maximizing the yearly annual revenue is performed in Ref. [11]. A method for district heating network dimensioning, based on the probabilistic determination of the flow rate for hot water heating was carried out in Ref. [12]. In Ref. [13] a multi-objective optimization model is performed for the best network design considering both initial investment for pipes and pumping cost for water distribution. In our research, methods for the hydraulic analysis of loop and radial networks were examined. It is a widely held view that loop networks are hydraulically more advantageous than radial networks. In the course of our research, the explicitness of this theorem was disproved. This study presents only the part of our research discussing a hydraulic analysis method for district heating networks containing one loop. Objective functions are stated for this type of network and the flow pattern is determined by minimizing the objective function. One of the objective functions states the minimization of flow work. The other objective function aims for the minimum of power supplied. The flow patterns yielded by the solution of the two objective functions differ from each other. Our study discusses the solution of both objective functions.
2 Objective functions and hydraulic equations

Figure 1 shows the topological model (graph) of the pipeline system of a district heating network with a circular conduit system. Consumer hot water flow demands are given and known: \( V_0, V_1, V_2, \ldots, V_n \). An optimal flow pattern is sought for, with a minimum dissipated energy and/or energy input or pump work. Thus, the problem is examined by solving two objective functions, using two models.

\[ \sum_{i=1}^{n} R_i \dot{V}_i^2 \rightarrow \min! \]  
(1)

Conditional equations:

a) node equation:
\[ \sum_{j=1}^{n} V_{ij} = 0 \forall i \]
(2)

b) loop equation:
\[
R_{m1} \left( k \dot{V}_0^2 + R_{m2} \left( k \dot{V}_0 + \dot{V}_1 \right)^2 + \cdots \right. \\
+ R_{m3} \left( k \dot{V}_0 + \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_{m-1} \right)^2 \\
\left. + R_{m4} \left( k \dot{V}_0 + \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n \right)^2 \right)
\]

A quadratic equation is yielded for \( k \) by rearrangement of the loop equation.

\[
\begin{align*}
&\left( R_{m1} + R_{m2} + \cdots + R_{m4} \right) \left( k \dot{V}_0^2 \right. \\
&\quad + \cdots + kR_{m3} \dot{V}_0 \left( \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_{m-1} \right) \\
&\quad + R_{m5} \dot{V}_1^2 + \cdots + R_{m6} \dot{V}_1 \left( \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n \right) \left( \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n \right) \\
&\quad + \cdots + R_{m6} \dot{V}_n \left( \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n \right) \left( \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n \right) \\
= &\left( R_{m0} + R_{m1} + \cdots + R_{m6} \right) \left( 1 - k \right) \dot{V}_0^2 \\
&+ 2 \left[ R_{m5} \left( 1 - k \right) \dot{V}_1 \dot{V}_0^2 + \left( 1 - k \right) \left( R_{m5} \dot{V}_1 \left( \dot{V}_1 + \dot{V}_2 \right) \right) \right. \\
&\left. + \cdots + R_{m6} \left( 1 - k \right) \dot{V}_n \dot{V}_0^2 + \right. \\
&\left. + \cdots + R_{m6} \left( 1 - k \right) \dot{V}_n \left( \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n \right) \left( \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n \right) \right] \\
&+ R_{m5} \dot{V}_1^2 + \cdots + R_{m6} \dot{V}_n^2 \left( \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n \right) \left( \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n \right). \\
\end{align*}
\]
(4)

By further rearrangement:
\[
\begin{align*}
&k^2 \cdot \left( R_{m0} + R_{m1} + \cdots + R_{m6} \right) \dot{V}_0^4 + 2k \cdot \\
&\left[ R_{m1} \dot{V}_0 \dot{V}_1 + R_{m2} \dot{V}_0 \left( \dot{V}_1 + \dot{V}_2 \right) + \cdots + R_{m6} \dot{V}_0 \left( \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n \right) \right] \left( \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n \right) \\
&+ \left[ R_{m5} \dot{V}_1^2 + \cdots + R_{m6} \dot{V}_1 \left( \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n \right) \right] \left( \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n \right) \left( \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n \right) \\
= &\left( 1 - k \right)^2 \left( R_{m0} + R_{m1} + \cdots + R_{m6} \right) \dot{V}_0^4 \\
&+ 2 \left( 1 - k \right) \left[ R_{m1} \dot{V}_0 \dot{V}_1 + \cdots + R_{m6} \dot{V}_0 \left( \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n \right) \right] \left( \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n \right) \\
&+ \left[ R_{m5} \dot{V}_1^2 + \cdots + R_{m6} \dot{V}_1 \left( \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n \right) \right] \left( \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n \right) \left( \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n \right). \\
\end{align*}
\]
(5)
Signs introduced: 
\[ A = (R_{n0} + R_{1n} + \cdots + R_{nn}) \], 
\[ B = \left[ R_{i1} V_{i1}^2 + R_{i2} V_{i2}^2 + \cdots + R_{n1} V_{n1}^2 \right] \], 
\[ C = \left[ R_{i1} V_{i1}^2 + R_{i2} V_{i2}^2 + \cdots + R_{n1} V_{n1}^2 \right] \]. 
\[ A' = (R_{n0} + R_{2n} + \cdots + R_{nn}) \], 
\[ B' = \left[ R_{i1} V_{i1}^2 + R_{i2} V_{i2}^2 + \cdots + R_{n1} V_{n1}^2 \right] \], 
\[ C' = \left[ R_{i1} V_{i1}^2 + R_{i2} V_{i2}^2 + \cdots + R_{n1} V_{n1}^2 \right] \].

Constants stated in a closed form:
\[ A = \sum_{j=0}^{n} R_{j(j+1)}^* \]
\[ B = \left[ \sum_{j=0}^{n-1} R_{j(j+1),j+2} \cdot \sum_{i=1}^{n} V_{i}^2 \right] V_i \]
\[ C = \sum_{j=0}^{n-1} R_{j(j+1),j+2} \cdot \sum_{i=1}^{n} V_{i}^2 \]
\[ A' = \sum_{j=0}^{n} R_{j(j+1),j+2} \]
\[ B' = \left[ \sum_{j=0}^{n-1} R_{j(j+1),j+2} \cdot \sum_{i=1}^{n} V_{i}^2 \right] V_i \]
\[ C' = \sum_{j=0}^{n-1} R_{j(j+1),j+2} \cdot \sum_{i=1}^{n} V_{i}^2 \]

Quadratic equation using the signs:
\[ k^2 A V_o^2 + 2kB + C = (1-k^2) A' V_o^2 + 2(1-k) B' + C' \] (7)

After rearrangement:
\[ k^2 \left( A - A' \right) V_o^2 + 2k \left( B + A' V_o^2 + B' \right) + \left( C - C' - A' V_o^2 - 2B' \right) \] (8)

The equation yields two solutions for proportional factor k; for the most part, only one of them has real physical content, this latter yielding flows \( V_{o1} \) and \( V_{o2} \) and finally, flows \( V_o \) and \( V_o' \). In view of these flows, pressure losses on conduit sections can be calculated and the pressure pattern of the network can be set up, simultaneously yielding feed differential pressure and pump delivery head figures. Application of the loop rule also results in the fact that the aggregate of pressure loss figures is equal along routes 0-n and 0-n*, and pumps operate at identical delivery heads.

### 2.2 Flow pattern in the minimization of input power.

**Separated pumping at the feed location**

Let us examine if the quantity of power supplied can be decreased by separated pumping, by opening the loop at the location of pumping.

Objective function to express the minimum of pumping work:

\[ C = (\sum V + k \cdot V_o') \cdot \sum \Delta p + (\sum V' + (1-k) \cdot V_o) \cdot \sum \Delta p' \rightarrow \min \]

(9)

This objective function is of different structure compared to the function to express the search for minimum dissipated energy.

The minimum is placed where \( \frac{dC}{dk} = 0 \), and

\[ \frac{dC}{dk} = V_o \cdot \sum \Delta p + (\sum V + k \cdot V_o) \cdot \frac{d}{dk} \sum \Delta p - V_o' \cdot \sum \Delta p' \] (10)

Where, using the earlier signs,

\[ \sum \Delta p = k^2 \cdot A \cdot V_o^2 + 2k \cdot B + C. \]

\[ \frac{d}{dk} \sum \Delta p = 2k \cdot A \cdot V_o^2 + 2B \cdot A' V_o^2 + 2B' \cdot A' V_o + C'. \]

\[ \frac{dC}{dk} = 2k \cdot A \cdot V_o^2 + 2A' \cdot V_o^2 - 2A' \cdot V_o^2 - 2B'. \] (11)

**Equation (8) with the signs introduced:**

\[ V_o \cdot \left( k^2 \cdot A \cdot V_o^2 + 2k \cdot B + C \right) + \left( \sum V + k \cdot V_o \right) \cdot \left( 2k \cdot A \cdot V_o^2 + 2B \right) \]
\[ -V_o' \left( (1-k)^2 \cdot A' \cdot V_o^2 + 2(1-k) \cdot B' + C' \right) \]
\[ + \left( \sum V' + (1-k) \cdot V_o' \right) \cdot \left( 2k \cdot A' \cdot V_o^2 - 2A' \cdot V_o^2 - 2B' \right) = 0 \] (12)

After rearrangement, a quadratic equation is yielded for k:

\[ k^2 \left( A - A' \right) \cdot 3 \cdot V_o^3 + k \left[ 2A \cdot V_o^2 \cdot \sum V + 4B \cdot V_o \right] \]
\[ + A' \left( 2 \cdot V_o^2 \cdot \sum V' + 6 \cdot V_o^3 \right) + 4B' \cdot V_o' \] (13)

\[ + \frac{1}{2} B \sum V + C \cdot V_o - A' \left( 3 \cdot V_o^3 + 2 \cdot V_o^2 \cdot \sum V' \right) \]
\[ - B' \cdot \left( 4 \cdot V_o + 2 \sum V' \right) - C' \cdot V_o' = 0. \]

The factor k expressing the division ratio of volumetric flow \( V_o \) can be determined by the root formula.

It can be observed that the expression to determine proportional factor k is not identical with expression (8), therefore results are also different. It is important to decide whether it is economical to invest in two pumps, that is, whether operating cost savings represent real yields in view of investment costs.

### 3 Example

Let us perform the hydraulic analysis of the looped district heating network shown in Fig. 3. Let us define the hydraulic end point of the network. Let us open the loop network into a radial network with two feed points. Our example presents calculations for the minimization of both dissipation energy and feed outputs.
### 3.1 Determination of the flow pattern with the minimum of energy dissipation

![Fig. 3](image_url)

**Fig. 3** Model of loop district heating network for calculations to minimize dissipation energy

Figure 4 illustrates the transformation of the network shown in Fig. 3 into a radial network with two input points.

![Fig. 4](image_url)

**Fig. 4** Opened loop

Figure 4 shows the location of the 11 consumers along the mains conduit with two input points yielded after cutting the loop. The data required for performing the hydraulic analysis – name of section, length of section, caloric output, mass flow and standard pipe diameter – are shown in Table 1. Starting from the presumed end point towards the two inputs produced by cutting, end point consumer demand is supplied in a proportion $kV_0$ from one direction, and in a proportion $(1-k)V_0$ from the opposing direction. The presumed hydraulic end point is consumer 5. For reasons of expance, results for consumers 4 and 6 as presumed hydraulic end points are not included herein. They represent figures higher than one and lower than zero, respectively, as expected.

Select consumer caloric center 5 as a hydraulic end point.

$$V_5 = V_0 = 0.0117 \left( \frac{\text{m}^2}{\text{s}} \right)$$

![Fig. 5](image_url)

**Fig. 5** Illustration of the presumed hydraulic end point

By substituting the constants yielded in the quadratic correlation as described:

$$k^2 \left( A - A' \right) V_0^2 + 2k \left( B + A'V_0^2 + B' \right) + \left( C - C' + A'V_0^2 - 2B' \right)$$

Constant values:

$A = 3211.2380,$
$B = 0.7401,$
$C = 2.2177,$
$A' = 36642.7233,$
$B' = 2.2222,$
$C' = 2.5400.$

So the factor after solution will be $k = 0.7937 \left[ \right].$

This result already corresponds to the expected solution; however, it cannot be stated clearly that this should be the end point of the system before verification thereof by calculations performed for further points.

Table 2 shows pressure values at each node and at the feed point. It can be observed that the feed pressure figures required, as calculated in the two directions, are in agreement.

| L | Q | m | D |
|---|---|---|---|
| B | 1018 | 7454 | 58.79 | 200 |
| C | 552 | 5894 | 46.48 | 150 |
| D | 194 | 3494 | 27.56 | 150 |
| E | 562 | 1934 | 15.25 | 150 |
| F | 154 | 504 | 3.98 | 125 |
| G | 468 | 64 | 0.51 | 80 |
| H | 112 | 466 | 3.67 | 80 |
| I | 160 | 906 | 7.14 | 100 |
| J | 336 | 1606 | 12.67 | 200 |
| K | 162 | 11446 | 90.27 | 250 |
| L | 240 | 20596 | 162.43 | 250 |
| M | 32 | 21426 | 168.97 | 250 |
| 1 | 324 | 830 | 6.55 | 100 |
| 2 | 790 | 1560 | 12.30 | 100 |
| C | 3 | 36 | 2400 | 18.93 | 125 |
| O | 4 | 50 | 1560 | 12.30 | 100 |
| N | 5 | 250 | 1430 | 11.28 | 100 |
| S | 6 | 42 | 440 | 3.47 | 65 |
| U | 7 | 94 | 530 | 4.18 | 65 |
| E | 8 | 38 | 440 | 3.47 | 65 |
| R | 9 | 220 | 700 | 5.52 | 65 |
| S | 10 | 9840 | 77.60 | 25 |
| 11 | 9150 | 72.16 | 25 |
Table 2 Pressure figures of nodes and feed points

| Δp [bar] | Δp [bar] |
|----------|----------|
| IV. 1.9202 | IV. 1.9202 |
| III. 2.0257 | V. 1.9257 |
| II. 2.2233 | VI. 2.8904 |
| I. 4.2035 | VII. 3.5651 |
| SZ1 5.5920 | VIII. 4.1081 |
| IX. 4.1702 | |
| X. 4.4022 | |
| XI. 5.4424 | |
| SZ2 5.5920 | |

It can be observed that the aggregate of the differential pressure figures calculated from the two directions – that is, pump delivery head figures – agree. \( \Delta p_{pump} = 5.5920 \) bar.

The figures of volumetric flow delivered by the pumps correspond to the volumetric flow figures of sections B and M, respectively. \( \dot{V}_{sz1} = \dot{V}_b = 0.0547 \) bar, \( \dot{V}_{sz2} = \dot{V}_m = 0.1826 \) bar.

The power absorbed by each pump:

\[
P_{p1} = \dot{V}_{sz1} \cdot \Delta p_{sz1} = 5.5920 \cdot 0.0547 = 30.57 kW, \\
P_{p2} = \dot{V}_{sz2} \cdot \Delta p_{sz2} = 5.5920 \cdot 0.1826 = 102.1 kW.
\]

The total power absorbed:

\[
P_{total} = P_{p1} + P_{p2} = 30.57 + 102.1 = 132.67 kW.
\]

The pressure pattern developed in the network is shown in Fig. 6.

![Fig. 6 Illustration of the presumed hydraulic end point](image)

3.2 Determination of the flow pattern with the minimum of energy input

The calculation principle of the basic data (heat power, mass flow, volumetric flow) coincides with the one presented for flow work minimization. In the calculation based on minimum energy input, the hydraulic end point came to be consumer 7. The data used in the calculation are shown in Table 3.

Table 3 Hydraulics data of each section

| L | Q | m | D |
|---|---|---|---|
| B | 1018 | 7454 | 58.79 | 200 |
| C | 552 | 5894 | 46.48 | 150 |
| D | 194 | 3494 | 15.25 | 125 |
| E | 562 | 1934 | 64 | 80 |
| F | 154 | 504 | 3.98 | 125 |
| G | 468 | 102 | 0.51 | 80 |
| H | 112 | 466 | 3.67 | 80 |
| I | 160 | 906 | 7.34 | 100 |
| J | 240 | 20596 | 162.43 | 250 |
| K | 32 | 21426 | 168.97 | 250 |
| L | 324 | 830 | 6.55 | 100 |
| M | 790 | 1560 | 12.30 | 100 |
| N | 36 | 2400 | 18.93 | 125 |
| O | 50 | 1560 | 12.30 | 100 |
| P | 250 | 1430 | 11.28 | 80 |
| Q | 42 | 440 | 3.47 | 65 |
| R | 74 | 530 | 4.18 | 65 |
| S | 38 | 440 | 3.47 | 65 |
| T | 220 | 700 | 5.52 | 65 |
| U | 9840 | 77.60 | 72.16 |

When selecting consumer caloric center 7 as a hydraulic end point, the volumetric flow is \( \dot{V}_0 = 0.0044 \) bar.

The figures of volumetric flows, resistance factors, and differential pressures developed along each section are included in Table 4. The correlations used correspond to those described earlier.

Calculated constants are as follows in this case:

\[
A = 36779.8, \\
B = 0.4998, \\
C = 4.903, \\
A' = 9490.96, \\
B' = 0.0877, \\
C' = 1.2923. 
\]
On the basis thereof, the value of $k$ can be calculated. 

$$k = 0.1208$$

In case of other nodes, figures lower than 0 or higher than 1 are yielded for $k$ on the basis of this calculation principle as well, therefore only consumer 7 can be the hydraulic end point. Table 5 contains the values of $k$ in case of different presumed hydraulic end points.

As regards the calculation principle, it is allowable to infringe Kirchhoff Laws, meaning that the aggregate figures of differential pressures calculated in the two directions are not required to be identical.

Table 5 Values of $k$

| Hydraulic end point | Value of $k$ |
|---------------------|-------------|
| Consumer 4          | 2.192       |
| Consumer 5          | 1.326       |
| Consumer 6          | 1.080       |
| Consumer 7          | 0.121       |
| Consumer 8          | -1.120      |

Figures of design pressure at each node as well as pump feed differential pressure are included in Table 6.

Table 6 Pressure values at each node and feed point

| $\Delta p$ | $\Delta p$ |
|------------|------------|
| bar        | bar        |
| VI.        | 0.3000     | VI.        | 0.3000     |
| V.         | 0.3091     | VII.       | 0.3945     |
| IV.        | 1.9202     | VIII.      | 1.2160     |
| III.       | 2.2184     | IX.        | 2.4210     |
| II.        | 2.5482     | X.         | 2.6239     |
| I.         | 5.1920     | XI.        | 3.5884     |
| SZ1        | 6.9298     | SZ2        | 3.7275     |

When minimizing energy input, the two delivery head values do not agree: in this case, there are two different pumps. The operating points established are as follows:

$$\Delta p_{\text{SZ1}} = 6.9298 \text{bar},$$

$$\Delta p_{\text{SZ2}} = 2.4210 \text{bar},$$

$$v'_{\text{SZ1}} = v'_{\text{SZ2}} = 0.0612 \frac{\text{m}}{\text{s}}.$$
\[\Delta p_{S2} = 3.7275 \text{bar},\]
\[\bar{V}_{S2} = \bar{V}_M = 0.1760 \text{ m}^3/s.\]

On the basis thereof, the power absorbed by each pump can be determined. Thus, the total power absorbed will be:
\[P_{S21} = P_{S21} \cdot \bar{V}_{S21} = 6.9298 \cdot 0.0612 = 0.4243 \text{bar} \cdot \text{m}^3/s = 42.43 \text{kW},\]
\[P_{S22} = P_{S22} \cdot \bar{V}_{S22} = 3.7275 \cdot 0.1760 = 0.6561 \text{bar} \cdot \text{m}^3/s = 65.61 \text{kW},\]
\[P_{\text{total}} = P_{S21} + P_{S22} = 42.43 + 65.61 = 108.04 \text{kW}.\]

By breaking up the loop at the pump, the network was converted into a two-feed radial network. In case of an appropriate design, after the correct selection of the hydraulic end point, the pressure pattern would be characterized by the fact that progressing along the mains, the pressure drop of the incoming consumer branch is always smaller at each node, meaning that damping is required on these branches. In this case, however, the design differential pressure at nodes IV, VIII and IX is the pressure drop of the consumer branch. Figure 7 illustrates the differential pressures available along the mains and their values required at critical nodes. The pressure pattern drawn on the mains had to be modified in accordance therewith, meaning that the feed differential pressure had to be increased. In addition to those listed above, critical nodes also include node x because a greater one would be necessary than what is available on the mains. It follows from this that the network was designed improperly. Pipe diameters are not large enough, therefore very high flow rates are produced. Consequently, pressure drops are too large on consumer branches.

In the diagram, light green lines indicate the differential pressures required at critical nodes. Thin red and blue lines show values available between the forward and return sections of the mains, while thick lines show the actual state established.

The results yielded by the two calculation methods can be compared with the power absorbed by the pumps.
\[P_{\text{dissipated}} = 132.67 \text{kW},\]
\[P_{\text{input}} = 108.04 \text{kW}.\]

Output reduction by the minimization of energy input can be calculated as compared to the one calculated for the minimum of dissipated energy.
\[\Delta P = \frac{P_{\text{dissipated}} - P_{\text{input}}}{P_{\text{dissipated}}} \cdot 100 = \frac{132.67 - 108.04}{132.67} \cdot 100 = 18.56\%.\]

It can be shown that a much more cost-effective operating state can be achieved by minimizing work input than by minimizing flow work. So it is worthwhile to break up the loop at the feed point and to apply two pumps instead of one. Obviously, these savings must be compared to the additional investment cost of the installation of two pumps.

**4 Conclusion**

Our study presented a hydraulic analysis method for district heating networks of a circular conduit system with given consumer volumetric flow demands, for both the dissipated energy minimum and the input energy minimum. After stating and arranging a loop equation and the node equations (Kirchhoff Laws I and II), the result is a quadratic equation for the distribution of consumer volumetric flow at the hydraulic end point. This is indicated by the so-called k factor, which is a figure lower than 1. Adoption of the procedure was presented in an example for each of the minimum of dissipated energy and pump work. By comparison of the results yielded, it was demonstrated that circular conduit operation by separated pumping is energetically more advantageous. Obviously, the issue to be examined is whether the installation of two pumping stations represents a better solution in respect of investment costs.
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