Interaction between elements of the track ground contacting area with the soil at curvilinear motion of the timber harvesting machine

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Abstract. This article describes interaction between the tracks of the timber harvesting machine and the soil. It concerns the research of interaction of the elements of the ground contacting area of the tracks with the soil at curvilinear motion of the tracked timber harvesting machine. The expressions was found to determine relationship between the value of the longitudinal and lateral travel of an element of the track ground contacting area and the traveltime parameters of motion of the tracked timber harvesting machine and in turn characterize kinematics of the elements of the track ground contacting area at curvilinear motion of the machine.

1. Introduction
It is impossible to accomplish the task set by the development strategy of the timber complex of the Russian Federation until 2020 to increase timber harvesting up to 294 mln. cu.m. without providing the timber harvesting enterprises with a complex of machines and equipment ensuring efficient timber harvesting, including in the severe weather conditions which prevail in most forest areas (weak soil, heavily waterlogged area, deep snow, etc.).

It is advisable to use the tracked machines on the waterlogged cutting areas, steep slopes, in deep snow, multi-storeyed wood, cross-country full of obstacles in the form of stubs, stones and fallen trees, where the performance and traction power of the wheeled vehicles are insufficient. The advantage of the tracked running gear is even more evident on the ice surface [1, 2, 3, 4].

In this article a model of interaction between elements of the track ground contacting area with the soil is described. The kinematics of the elements of the ground contacting areas of the tracked timber harvesting machine at its curvilinear motion is examined

2. Model of interaction between elements of the track ground contacting area with the soil
The analytical description of the track-soil interaction involves considerable difficulties, though the physical and mechanical properties of soils have been well studied today. However, application of the soil mechanics laws to describe the track-soil interaction is not acceptable due to dynamic loading of the track ground contacting area elements on the soil.
Due to the above, experimental data which allows to find out some common patterns are used to describe the process of the track-soil interaction [5, 6, 7]. Let us examine interaction between the elements of the track ground contacting area with the soil using those patterns.

To evaluate this interaction let us take a track-related triangular coordinate system which zero coincides with the track centre-of-mass (Fig. 1).

During the curvilinear motion of the tracked timber harvesting machine, each element of the track ground contacting area carries out a complex motion, gets displaced on the soil in longitudinal and lateral direction relative to the machine rolling axis [8, 9]. While examining the track-soil interaction, we assume that the track travel on the soil at a specific time coincides with the direction of the absolute velocity of the track centre-of-mass travel. We can approximately accept that the track centre-of-mass coincides with the geometric centre of its surface.

![Figure 1. Coordinate system to analyze the track-soil interaction](image)

Let us suppose the track to be loaded with force $Q$ (normal track-on-soil load) which is normal to its surface. When the track travels relative to the soil, this causes a response force that we mark as $R$. This force is proportional to the normal load:

$$R = \mu Q.$$  \hfill (1)

The proportionality factor $\mu$ is called a factor of the track-soil interaction.

Let us assume that the force of the track-soil interaction is opposite in direction to the instantaneous sliding velocity of the centre-of-mass of the track relative to the soil $V$ (Fig. 1).

The track-soil interaction force $R$ in general depends on the value and direction of travel of the track relative to the soil because the track ground contacting area has a complex shape and the forces resisting its travel in longitudinal and lateral direction are different.

Therefore, with the same value of track travel relative to the soil $\sigma$ and constant value of the normal track-on-soil load $Q$, the reaction force $R$ will be different depending on the track sliding direction (there will be different value of coefficient $\mu$).

Let us call the hodograph of force $R$ at $Q=1$ the hodograph of the track-soil interaction coefficient $\mu$.

The results of experiments showed that this hodograph practically has the shape of ellipsis and may be described in the following equation:

$$\mu = \frac{\mu_x \cdot \mu_y}{\sqrt{\mu_x^2 \cdot \sin^2 \alpha + \mu_y^2 \cdot \cos^2 \alpha}},$$ \hfill (2)
where $\mu_x$, $\mu_y$ is a value of the coefficient of the track-soil interaction at (separate) travel of the track by value $\sigma$ in longitudinal and lateral directions; $\alpha$ is an angle between direction of the track displacement and longitudinal axis of the tracked timber harvesting machine.

Unequal soil resistance to the track displacement in longitudinal and lateral directions is taken into consideration with use of the anisotropy coefficient $\lambda$:

$$\lambda = \frac{\mu_y}{\mu_x}. \quad (3)$$

3. Kinematics of the elements of the track ground contacting areas at curvilinear motion of the tracked timber harvesting machine

Let us examine the kinematics of the elements of the ground contacting areas of the tracked timber harvesting machine at its curvilinear motion (Fig. 2).

![Figure 2. Kinematics of the elements of the track ground contacting areas at curvilinear motion of the tracked timber harvesting machine](image)

Assume that point $O$ is an instantaneous center-of-turn. Select $i$-nd element on the ground contacting area of the leading track. It will participate in two motions: translational (around the instantaneous center-of-turn) at a rate $V_i^t$ and relative (track return relative to the body) at a rate $V_{0i}$. (Further all parameters related to the leading track will be marked with subindex "2", and to the retreating track - with subindex "1".) Let us break down translational velocity $V_i^t$ into longitudinal $V_{ix}^t$ and lateral $V_{iy}^t$ components. The difference between relative velocity $V_{i2}$ and longitudinal component $V_{ix}^t$ will give slipping velocity $V_{ix}^s$ of the leading track relative to the soil. The vector sum of velocities $V_{iy}^t$ and $V_{ix}^s$ will give an absolute velocity of element displacement $V_{i\text{abs}}$ which the direction of the soil absolute response $dR_i$ depends on. Let us assume the following: vector of soil absolute response coincides with the vector of absolute displacement of the $i$-nd element of the track ground contacting area relative to the soil but is opposite in direction. Considering this assumption, the soil absolute response $dR_i$ may be broken down into two components: lateral - relative to the axis of the tracked timber harvesting machine.
(elementary turn resistance force $dS_i$) and longitudinal one which is directed along this axis (elementary traction power $dP_i$).

In view of the above, the soil response acting on each element of the track ground contacting area may be expressed with a reasonable degree of accuracy through longitudinal and lateral displacement of the element of the ground contacting area relative to the soil.

To determine the values of longitudinal and lateral travel of each element of the track ground contacting area over the soil, let us examine their kinematics at the curvilinear motion of the tracked timber harvesting machine.

At each point of time of the plane curvilinear motion of the tracked timber harvesting machine, its travel as a solid object can be replaced with rotating motion relative to the instantaneous center-of-turn $O$ (Fig. 3).

**Figure 3.** Kinematics of the curvilinear motion of the tracked timber harvesting machine relative to the instantaneous center-of-turn

Direction of axis $x$ coincides with the direction of the machine longitudinal axis.

Take $\omega$ as an angular rate of rotation of the tracked timber harvesting machine relative to the instantaneous center-of-turn $O$, record the expression of motion velocity of its center-of-mass:

$$V_C = \omega \cdot \rho_C \cdot \cos \beta$$

(4)

Break down the center-of-mass velocity of the tracked timber harvesting machine $V_C$ into longitudinal $V_{CX}$ and lateral $V_{CY}$ components:

$$V_{CX} = V_C \cdot \cos \beta = \omega \cdot \rho_C \cdot \cos \beta = \omega \cdot \rho_\phi = \omega \cdot \sqrt{\rho_C^2 - \chi^2};$$

(5)

$$V_{CY} = V_C \cdot \sin \beta = \omega \cdot \rho_C \cdot \sin \beta = \omega \cdot \chi,$$

(6)

where $\rho_C$ is a radius of curvature of trajectory of the machine center-of-mass;

$\chi$ is a projection of instantaneous center-of-turn to axis $x$;

$\rho_\phi$ is an actual turn radius;
\( \beta \) is an angle between the tangent to the required motion trajectory and longitudinal axis of the tracked timber harvesting machine.

It is evident that

\[
\beta = \arcsin(\frac{\chi}{\rho_C}).
\]  

(7)

The projection of instantaneous velocity of the body arbitrary point \( A \) with coordinates \( x \) and \( y \) similarly with velocities \( \dot{V}_x \) and \( \dot{V}_y \) shall be determined with the following expressions:

\[
\dot{V}_x = \omega \cdot (\sqrt{\rho_C^2 - \chi^2} + y);
\]

\[
\dot{V}_y = \omega \cdot (\chi - x).
\]

(8)

The tracks are not rigidly connected with the body of the tracked timber harvesting machine in longitudinal direction in relative motion and have rigid connection in the lateral direction. For this the lateral velocity of travel of the elements of the ground contacting areas shall be determined with the following expression:

\[
V_y(x) = V_y(x) = \omega \cdot (\chi - x),
\]

(9)

where \( V_y(x) \) and \( V_y(x) \) are the lateral velocities of travel of the point with coordinate \( x \) belonging to the ground contacting area of the leading and retreating tracks, respectively.

The elementary move of the point belonging to the track ground contacting area shall be defined by the following dependence:

\[
d\sigma_y = d\sigma_y = \omega \cdot (\chi - x) \cdot dt_k,
\]

where \( k \) is the time of contact of this point of the ground contacting area with the soil.

For the leading track:

\[
t_{k2} = [(l_1 + t_{f2} / 2) - x] / V_{02}.
\]

(11)

Similar for the retreating track:

\[
t_{k1} = [(l_1 + t_{f1} / 2) - x] / V_{01}.
\]

(12)

where \( V_{02}, V_{01} \) are respective velocities of the leading and retreating tracks' turn relative to the tracked timber harvesting machine body.

It is evident that

\[
(l_n - t_{f2} / 2) \leq x \leq (l_n + t_{f2} / 2),
\]

(13)

where \( l_n \) is a distance between the center-of-mass of the tracked timber harvesting machine to the last support roller.

The fill displacement of the element of the track ground contacting area over the soil in lateral (relative to the longitudinal axis of the tracked timber harvesting machine) direction shall be determined by the following expressions:

\[
\sigma_y(x) = \int_{0}^{t_{k2}} \omega(t) \cdot (\chi - x) \cdot dt_{k2};
\]

\[
\sigma_y(x) = \int_{0}^{t_{k1}} \omega(t) \cdot (\chi - x) \cdot dt_{k1};
\]

at \( \omega = \text{const} \)

\[
\sigma_y(x) = \omega \int_{0}^{t_{k2}} (\chi - x) \cdot dt_{k2};
\]

(15)
\[ \sigma_{y1}(x) = \omega \int_{0}^{t_k} (\chi - x) \cdot dt_k. \]

In formulas (15) let us go to variable \( x \) instead of \( t_k \). In accordance with expressions (11) and (12) we arrive at:
\[ dt_{k1} = -dx/V_{01}; \]
\[ dt_{k2} = -dx/V_{02}. \]

Consequently, expressions (15) may be recorded in the following form:
\[ \sigma_{y2}(x) = (\omega/V_{02}) \cdot \int_{x}^{l_1 + t_{k2}/2} (\chi - x) \cdot dx; \]
\[ \sigma_{y1}(x) = (\omega/V_{01}) \cdot \int_{x}^{l_1 + t_{k1}/2} (\chi - x) \cdot dx. \]

Having calculated integrals in expressions (16), we arrive at:
\[ \sigma_{y2}(x) = [\omega/(2 \cdot V_{02})] \cdot [(\chi - x)^2 - (\chi - l_1 - t_\omega/2)^2]; \]
\[ \sigma_{y1}(x) = [\omega/(2 \cdot V_{01})] \cdot [(\chi - x)^2 - (\chi - l_1 - t_\omega/2)^2]. \]

Let us determine longitudinal travel of the elements of the ground contacting areas over the soil:
\[ \sigma_{x2} = V_{x2} \cdot t_{k2}; \]
\[ \sigma_{x1} = V_{x1} \cdot t_{k1}, \]
where \( V_{x2} \) is a slipping velocity of the leading track; \( V_{x1} \) is a slipping velocity of the retreating track. But for \( V_{x2} \) and \( V_{x1} \) we can record:
\[ V_{x2} = V_{x2}^n - V_{02}; \]
\[ V_{x1} = V_{x1}^n - V_{01}, \]
where \( V_{x2}^n \) and \( V_{x1}^n \) are translational velocities of the center-of-mass of the track line that may be determined as follows:
\[ V_{x2}^n = \omega \cdot (\sqrt{\rho_C^2 - \chi^2} + B/2) = V_{CX} + \frac{\omega \cdot B}{2}; \]
\[ V_{x1}^n = \omega \cdot (\sqrt{\rho_C^2 - \chi^2} - B/2) = V_{CX} - \frac{\omega \cdot B}{2}. \]

Having substitute expressions (11), (12), (20), (21) в (18) and (19), we arrive at:
\[ \sigma_{x2}(x) = \left( \omega \cdot (\sqrt{\rho_C^2 - \chi^2} + B/2) - V_{02} \right) \cdot (l_1 + t_\Gamma/2 - x)/V_{02}; \]
\[ \sigma_{x1}(x) = \left( \omega \cdot (\sqrt{\rho_C^2 - \chi^2} - B/2) - V_{01} \right) \cdot (l_1 + t_\Gamma/2 - x)/V_{01}. \]

4. Conclusion

Thus the expressions determined relationship between the value of the longitudinal and lateral travels of an element of the track ground contacting area and the travel time parameters of motion of the tracked timber harvesting machine and in its turn characterize kinematics of the elements of the track ground contacting area at curvilinear motion of the machine was found.
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