Bridging the gap: effective interactions from microscopic dynamics

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Abstract. I discuss the derivation of an effective interaction, obtained from a realistic nuclear Hamiltonian using the formalism of Correlated Basis Functions (CBF) and the cluster expansion technique. Unlike the bare nucleon-nucleon potential, this effective interaction is well behaved, and suitable for carrying out perturbative calculations of a variety of equilibrium and non-equilibrium properties of nuclear matter using the basis of eigenstates of the non interacting system. The robustness and accuracy of the proposed approach are thoroughly examined, comparing the results of its application to the Fermi hard-sphere system to the predictions of low-density expansions. Selected properties of nuclear matter, at both zero and nonzero temperatures, are reported and discussed.

1. Introduction

Approaches based on effective interactions are widely used to study the properties of strongly interacting many-body systems, when the bare interaction between the constituents cannot be treated in perturbation theory using the basis states describing non interacting particles [1, 2].

Effective interactions specifically designed to reproduce the bulk properties of nuclear matter (see, e.g., Refs. [3, 4]), while being remarkably successful in a number of instances, fail to provide a quantitative account of nucleon-nucleon scattering—both in free space and in the nuclear medium—whose understanding is needed for the description of important non-equilibrium properties [5, 6]. For example, the results reported in Ref. [5] clearly show that the determination of the shear viscosity and thermal conductivity of pure neutron matter, relevant to many astrophysical applications [7, 8], requires effective interactions derived from ab initio microscopic approaches, capable of explaining the observed nucleon-nucleon scattering data in the zero-density limit [5].

The authors of Refs. [9, 10] have developed a procedure to determine the effective interaction in nuclear matter using the Correlated Basis Function (CBF) formalism and the cluster expansion technique. While this scheme has been thoroughly tested through comparison between its results and those obtained from G-matrix perturbation theory in pure neutron matter [5], the analysis of a somewhat simpler many-body system, several properties of which can be accurately calculated and expressed in analytic form, provides further insight into the validity and robustness of the underlying assumptions.

The results of studies of the Fermi hard-sphere system strongly suggest that the CBF effective interaction approach, provides accurate estimates of fundamental quantities other than the ground-state energy, such as the self-energy determining the two-point Green’s function. In this
context, the capability to carry out perturbative calculations beyond the lowest order—without incurring in the severe difficulties arising from the use of a non orthogonal basis—has been shown to play a critical role [11, 12].

Nuclear applications also cover a number of quantities of great astrophysical interest, such as the ground state energy and the quasiparticle spectrum of uniform matter at fixed baryon density and large neutron excess, at both zero and nonzero temperatures.

In Section 2, the derivation of the CBF effective interaction is outlined exploiting the pedagogical example of point-like fermions interacting through the hard-sphere potential, while Sections 3 and 4 are devoted to the results of numerical calculations of a variety of properties of the hard-sphere system and uniform nuclear matter, respectively. Finally, in Section 5 I summarize the present status of the CBF effective interaction approach, and outline the prospects for future developments.

2. Formalism

This section introduces the main elements entering the derivation of the CBF effective interactions, in the simple case in which the bare potential only depends on the separation distance between interacting particles.

2.1. Definition of the CBF effective interaction

The nucleon-nucleon interaction is known to be strongly repulsive at short distances, as clearly indicated by the saturation of the charge-density distributions measured by elastic electron-nucleus scattering [13]. As a consequence, the Fermi hard-sphere system, i.e. a uniform system of point-like spin one-half particles interacting through the potential

\[ v(r) = \begin{cases} \infty & r < a \\ 0 & r > a \end{cases} \]

has been long recognised as a valuable model for investigating concepts and approximations employed to study the properties of nuclear systems. Setting the degeneracy of the momentum eigenstates to \( \nu \) = 2 and 4, the hard-sphere system is meant to model pure neutron matter (PNM) and isospin-symmetric nuclear matter (SNM), respectively. Note that in both instances the system can be described in terms of a single parameter \( c = k_F a \), the Fermi momentum \( k_F \) being simply related to the density \( \rho \) through \( k_F = (6\pi^2 \rho / \nu)^{1/3} \).

Within the CBF approach, the correlated states of the hard-sphere system are obtained from the non interacting Fermi gas (FG) states through the transformation

\[ |n\rangle = \frac{F|n_{FG}\rangle}{\langle n_{FG}|F|n_{FG}\rangle^{1/2}} \]

where the operator \( F \), embodying the correlation structure induced by the interaction potential, is written in the form

\[ F = \prod_{j>i} f(r_{ij}) \]

with

\[ f(r_{ij} \le a) = 0 \quad \lim_{r_{ij} \to \infty} f(r_{ij}) = 1 \]

\( r_{ij} = |\mathbf{r}_i - \mathbf{r}_j| \) being the interparticle distance.

In principle, the shape of the two-body correlation function, \( f(r_{ij}) \), at \( r_{ij} > a \) can be determined from functional minimisation of the expectation value of the hamiltonian

\[ H = T + V = \sum_i \frac{k_i^2}{2m} + \sum_{j>i} v(r_{ij}) \]
in the correlated ground state. In the above equation, \( k_i = |k_i| \), \( m \) denotes the particle mass and \( v \) is the potential of Eq. (1).

The effective interaction

\[
V_{\text{eff}} = \sum_{j>i} v_{\text{eff}}(r_{ij}) ,
\]

is defined by the relation

\[
\frac{1}{N} \langle H \rangle = \frac{1}{N} \langle 0 | H | 0 \rangle = \frac{3k_F^2}{10m} + \frac{1}{N} \langle 0_{FG} | V_{\text{eff}} | 0_{FG} \rangle ,
\]

where \( N \) denotes the number of particles. Note that the above equation shows that, unlike the \( G \)-matrix, the CBF effective interaction is defined not in operator form, but rather in terms of its expectation value in the Fermi gas ground state.

The calculations discussed in the following Sections are largely based on the assumption—that will be tested comparing the results to those obtained from different many-body approaches—that perturbative calculations involving matrix elements of \( V_{\text{eff}} \) between Fermi gas states provide accurate estimates of all properties of the Fermi hard-sphere system.

2.2. Cluster expansion technique

The calculation of matrix elements of any many-body operator between correlated states involves largely irreducible \( 3N \)-dimensional integrations. This problem, that becomes quickly intractable with increasing \( N \), can be circumvented expanding the matrix element in a series, the terms of which represent the contributions of subsystems (clusters) involving an increasing number of particles [14]. Since correlations are short ranged, at not too high density the cluster expansion is expected to be rapidly convergent.

The effective interaction of Ref. [10] is derived expanding the left hand side of Eq. (7), and keeping the two-body cluster contribution only. The resulting expression is

\[
\frac{1}{N} \langle H \rangle = \frac{3k_F^2}{10m} + (\Delta E)_2 ,
\]

with

\[
(\Delta E)_2 = \frac{\rho}{2} \int d^3r \frac{1}{m} [\nabla f(r)]^2 \left[ 1 - \frac{1}{\nu} \ell^2(k_F r) \right] ,
\]

where the Fermi gas density-matrix, referred to as the Slater function, is defined as \( \ell(x) = 3 \sin x - x \cos x \) \( /x^3 \). The form of \( v_{\text{eff}} \) follows immediately from Eqs. (6)-(9), implying

\[
v_{\text{eff}}(r) = \frac{1}{m} [\nabla f(r)]^2 .
\]

As pointed out above, the shape of \( f(r) \) can be obtained from the functional minimisation of the Hamiltonian expectation value in the correlated ground state. Within the two-body cluster approximation, this procedure yields the Euler-Lagrange equation

\[
g''(r) - g(r) \left[ \frac{\Phi''(r)}{\Phi(r)} + m\lambda \right] = 0 ,
\]

where

\[
g(r) = f(r)\Phi(r) , \quad \Phi(r) \equiv r \sqrt{1 - \frac{1}{\nu} \ell^2(k_F r)} .
\]
Equation (11) is solved with the boundary conditions $f(a) = 0$ and $f(d) = 1$, the additional constraint $f'(d) = 0$ being fulfilled through the introduction of the Lagrange multiplier $\lambda$. The correlation range is determined requiring that the ground-state energy per particle obtained at first order in the effective interaction be equal to the corresponding result obtained from accurate many-body calculations, carried out within the Fermi Hyper-Netted Chain (FHNC) approach $[15, 16]$ or using Quantum Monte Carlo techniques $[17]$ using the bare Hamiltonian.

3. The Fermi hard-spheres system

In this section, several equilibrium and non-equilibrium properties of the Fermi hard-sphere systems obtained using the CBF effective interaction are compared to the predictions of different many-body approaches. All calculations have been carried out setting the hard-sphere radius to $a = 1$ fm and the particle mass to $m = 1$ fm$^{-1}$.

3.1. Equilibrium properties

The expansion of the ground-state energy of the quantum mechanical hard-sphere system in powers of the dimensionless parameter $c$, defined in Section 2.1, was first discussed in the 1950s by Huang and Yang $[18]$, who were able to derive its terms up to order $c^2$.

More recent calculations have been carried out exploiting the formalism developed to describe the scattering process involving two particles interacting through a strongly repulsive potential $[19]$. The main element of this approach is the replacement of the bare interaction with the $t$-matrix, which amounts to including the contribution of the infinite series of ladder diagrams. This technique, which in general allows to achieve a fast convergence of perturbative calculations, becomes essential when dealing with the hard-core interaction of Eq. (1). The resulting expressions of the ground-state energy per particle are

$$E_0 = \frac{k_F^2}{2m} \left[ \frac{3}{5} + \frac{2}{\pi} c + \frac{12}{35\pi^2} (11 - 2\ln 2) c^2 + 0.78c^3 + \frac{32}{9\pi^3} \left(4\pi - 3\sqrt{3}\right) c^4 \ln c \right] , \quad (13)$$

for degeneracy $\nu = 4$ and

$$E_0 = \frac{k_F^2}{2m} \left[ \frac{3}{5} + \frac{2}{3\pi} c + \frac{4}{35\pi^2} (11 - 2\log 2) c^2 + 0.230c^3 \right] , \quad (14)$$

for $\nu = 2$.

The ground-state energy is conveniently parametrized by the dimensionless quantity $\zeta$, defined through the equation

$$E_0 = \frac{3k_F^2}{10m} (1 + \zeta) . \quad (15)$$

Figures 1 and 2, taken from Ref. $[12]$, show the $c$-dependence of $\zeta$—providing a measure of the deviation of $E_0$ from the energy of the non interacting Fermi gas—corresponding to $\nu = 2$ and $\nu = 4$, respectively. The results of Diffusion Monte Carlo (DMC) calculations, used as baseline for the determination of the effective interaction, are compared those obtained from the Variational Monte Carlo (VMC) and FHNC approaches, the latter being only available at $c \geq 0.5$. For reference, the predictions of the low-density expansions of Ref. $[19]$ are also displayed, by the solid lines.

It clearly appears that the VMC and FHNC results are very close to one another, thus showing that at $c \geq 0.5$ the FHNC approximation does provide an upper bound to the ground-state energy. The accuracy of the variational result is measured by the difference between the VMC—or, equivalently, FHNC—values of $\zeta$ and those obtained from DMC. In the case
of degeneracy $\nu=2$, illustrated in Fig. 1, this difference ranges between $\sim 2\%$ and $\sim 9\%$ at $0.2 \leq c \leq 1$. Note that a $9\%$ difference in $\zeta$ translates in a difference of less than $3\%$ in the ground-state energy $E_0$. The low-density expansion turns out to be also quite accurate, its predictions being within $5\%$ of the DMC results at $c < 0.5$. The results corresponding to $\nu = 4$, shown in Figure 2, exhibit the same pattern.

Figure 1. $c$-dependence of the quantity $\zeta$, defined by Eq. (15), for degeneracy $\nu=2$. The solid and dashed lines show the results obtained from the low-density expansion and the variational FHNC approach, respectively. The VMC and DMC results are represented by diamonds and squares.

Figure 2. Same as in Fig. 1, but for degeneracy $\nu = 4$.

The effective interaction obtained from the procedure described in Section 2 has been employed to carry out a second-order perturbative calculation of the self-energy $\Sigma(k, E)$, which has been in turn used as an input to obtain the quantities describing single-particle properties. Figure 3 shows the energy spectrum of the hard-sphere system with $\nu = 2$, defined as [11]

$$e(k) = \frac{k^2}{2m} + \text{Re} \Sigma[k, e(k)],$$

obtained at first- and second-order in the effective interaction. The former provides the energy-independent correction corresponding to the Hartree-Fock approximation, while the latter brings about an explicit energy dependence. The momentum-dependence of $e(k)$ is illustrated for two values of density, corresponding to $c = 0.3$ and 0.6. The $c$-dependence of the corresponding effective mass, defined as

$$m^*(k) = \left(\frac{1}{k} \frac{d e}{d k}\right)^{-1},$$

with

$$\frac{d e}{d k} = \left[ \frac{k}{m} + \frac{\partial}{\partial k} \text{Re} \Sigma(k, E) \right] \times \left[ 1 - \frac{\partial}{\partial E} \text{Re} \Sigma(k, E) \right]^{-1}_{E=e(k)},$$

evaluated at $|k| = k_F$, is displayed in Fig. 4. From Figs. 3 and 4, it clearly appears that the appearance of the energy dependence of the self-energy, while resulting in a small modifications
of the spectrum, dramatically affects both magnitude and density dependence of $m^*(k_F)$. For comparison, Fig. 4 also shows the effective mass predicted by the low-density expansion

$$\frac{m^*(k_F)}{m} = 1 + \frac{8}{15\pi^2}(7\ln 2 - 1)c^2.$$  

obtained by Galitskii back in the 1950s [20].

Figure 3. Single-particle energy spectrum of the fermion hard-sphere system of degeneracy $\nu=2$ at $c=0.3$ [panel (B)] and 0.6 [panel(A)]. The dashed and solid lines correspond to the first order (i.e. Hartree-Fock) and second order perturbative calculation of the self-energy $\Sigma(k, E)$, respectively. For comparison, the dot-dash lines show the kinetic energy spectrum.

The calculated self-energy can be also used to obtain the two-point Green’s function, defined as

$$G(k, E) = \frac{Z_k}{E - e(k) + i\Gamma_k},$$  

with $e(k)$ given by Eq. (16). The spectroscopic factor $Z_k$, yielding the normalization of the quasiparticle state of momentum $k$, and the corresponding width $\Gamma_k$ can be obtained from the self-energy using the relations

$$Z_k = \left[1 - \frac{\partial}{\partial E} \text{Re}\Sigma(k, E)\right]^{-1}_{E=e(k)}, \quad \Gamma_k = Z_k \text{Im}\Sigma(k, E). \tag{21}$$

The momentum distribution

$$n(k) = \frac{1}{2\pi i} \int_C d\omega \ G(k, \omega),$$  

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure4}
\caption{c-dependence of the ratio $m^*(k_F)/m$ for the hard-sphere system of degeneracy $\nu=2$. The dot-dash and solid lines represent the results of calculations carried out using the first- and second-order approximations to the self-energy $\Sigma(k, E)$. For comparison, the dashed line shows the results computed using the low-density expansion of Eq. (19).}
\end{figure}
where the integration on the complex variable $\omega$ is carried out along a closed contour in the upper half-plane $\text{Im}\;\omega > 0$, exhibits a discontinuity at $k = k_F$, given by

$$n(k_F - \eta) - n(k_F + \eta) = Z = Z_{k_F},$$

(23)

with $\eta = 0^+$. At second order in the effective interaction, $n(k)$ can be conveniently written in the form

$$n(k) = n_< (k) + n_> (k),$$

(24)

with $n_< (k > k_F) = n_> (k < k_F) = 0$, and

$$n_< (k < k_F) = 1 + \left[ \frac{\partial}{\partial E} \text{Re} \Sigma_p (k, E) \right]_{E=k^2/2m},$$

(25)

$$n_> (k > k_F) = - \left[ \frac{\partial}{\partial E} \text{Re} \Sigma_c (k, E) \right]_{E=k^2/2m},$$

(26)

where $\Sigma_p (k, E)$ and $\Sigma_c (k, E)$ denote the polarization and correlation contributions to the second order self-energy, respectively (see, e.g., Ref. [21]). Note that the above equations imply that within the Hartree-Fock approximation $n(k) = \theta(k_F - k)$, and $Z = 1$.

Figure 5 shows a comparison between the momentum distribution of the Fermi hard-sphere system with degeneracy $\nu = 4$, evaluated at $c = 0.55$, and the momentum distribution of SNM at equilibrium density, corresponding to Fermi momentum $k_F = 1.33$ fm$^{-1}$, obtained by the authors of Ref. [22] using non-orthogonal CBF perturbation theory at second order. The emerging picture suggests that nucleons in nuclear matter behave like hard spheres of radius $a = 0.55/1.33 \approx 0.4$ fm.

![Figure 5. Comparison between the momentum distribution of the Fermi hard-sphere system obtained using the CBF effective interaction (squares) and that of SNM at equilibrium density reported in Ref. [22] (solid line).](image)

3.2. Collision probability and transport properties

Within the approach based on Landau’s theory of normal Fermi liquids [23], originally developed by Abrikosov and Khalatnikov [24, 25], the shear viscosity and thermal conductivity coefficients—denoted $\eta$ and $\kappa$, respectively—are determined from the momentum and energy fluxes obtained from the kinetic equation for the distribution function

$$\frac{\partial n_k}{\partial t} + \frac{\partial n_k}{\partial r} \cdot \frac{\partial v_k}{\partial k} = \frac{\partial n_k}{\partial k} \cdot \frac{\partial v_k}{\partial r} = I [n_k],$$

(27)
where $\epsilon_k$ denotes the energy of a quasiparticle carrying momentum $k$, and $I[n_k]$ is the collision integral, the definition of which involves the in-medium scattering probability $W$.

In general, the scattering probability depends on the initial and final momenta of the particles participating in the process. In the low-temperature limit, however, the system is strongly degenerate, and only quasiparticles occupying states in the vicinity of the Fermi surface can be involved in interactions. As a consequence, the magnitudes of their momenta can be all set equal to the Fermi momentum, and $W$ reduces to a function of the two angular variables $\theta$ and $\phi$ only. The former is the angle between the initial momenta, whereas the latter is the angle between the planes specified by the initial and final momenta, respectively.

Besides the collision probability, that can be obtained in Born approximation using the CBF effective interaction, the calculation of the shear viscosity and thermal conductivity involves the effective mass, defined by Eq. (17). The results corresponding to the case of degeneracy $\nu = 2$ are reported in Fig. 6 and 7, showing the $c$-dependence of the $T$-independent quantities $\eta T^2$ and $\kappa T$, where $T$ denotes the temperature.

The most striking feature emerging from the figures is the sizable effect of the second order contributions to the effective mass, highlighted in the insets. It clearly appears that these corrections lead to sharp increase of $m^*$, which in turn implies a decrease of the shear viscosity and thermal conductivity coefficients.

4. Nuclear matter at zero and nonzero temperature

The extension of the approach based on the CBF effective interaction to describe the properties of uniform nuclear matter does not involve any conceptual problems, although the complex operatorial structure of the nuclear interaction, reflected by the correlation functions, and the occurrence of strong three-body forces entail non trivial computational difficulties.
The procedure developed by the authors of Ref. [27, 28], taking into account the contributions of three-nucleon clusters, allows to use a realistic nuclear Hamiltonian, comprising two- and three-nucleon potentials. The results reported in this paper have been obtained with the nucleon-nucleon potential referred to as Argonne $v'_6$ (AV6P) [29]—embodying the spin-isospin dependence of the nuclear forces, as well as the presence of non-spherically-symmetric interactions—and the Urbana IX (UIX) three-nucleon potential [30].

The resulting effective interaction—derived using as baseline the FHNC variational estimates of the ground state energy of PNM and SNM—can be used to evaluate the energy per nucleon of neutron reach and spin polarized matter at fixed baryon density

$$\rho = \sum_\lambda \rho_\lambda = \rho \sum_\lambda x_\lambda ,$$

where $\lambda = 1, 2, 3, 4$ labels spin-up protons, spin-down protons, spin-up neutrons and spin-down neutrons, respectively, the corresponding densities being $\rho_\lambda = x_\lambda \rho$. In SNM $x_1 = x_2 = x_3 = x_4 = 1/4$, while in PNM $x_1 = x_2 = 0$ and $x_3 = x_4 = 1/2$.

At first order in the CBF effective interaction, the energy per baryon can be written in the form

$$\frac{E}{A} = \frac{3}{5} \sum_\lambda x_\lambda \frac{k_{F\lambda}^2}{2m} + \frac{\rho}{2} \sum_{\lambda\mu} x_{\lambda\mu} \int d^3r \left[ v^{\text{eff},d}_{\lambda\mu}(r) - v^{\text{eff},e}_{\lambda\mu}(r) \ell(k_{F\lambda}r) \ell(k_{F\mu}r) \right]$$

where $v^{\text{eff},d}_{\lambda\mu}(r)$ and $v^{\text{eff},e}_{\lambda\mu}(r)$ denote the direct and exchange matrix elements of the effective interaction in spin-isospin space, $k_{F\lambda} = (6\pi^2\rho_\lambda)^{1/3}$ is the Fermi momentum of the nucleons of type $\lambda$ and

$$\rho_\lambda \ell(k_{F\lambda}r) \equiv \frac{1}{V} \sum_{k} e^{ikr} n_{\lambda}(k),$$

where $n_{\lambda}(k) = \theta(k_{F\lambda} - |k|)$ is the zero-temperature Fermi distribution and $V$ is the normalization volume.

The solid lines of Figures 8 and 9 illustrate the density dependence of the energy per nucleon of PNM and SNM, respectively, obtained from Eq. (29) using the CBF effective interaction. The shaded regions show the FHNC results obtained using the bare Hamiltonian, with the associated theoretical uncertainty arising from the treatment of the kinetic energy [14]. For comparison, the results of a calculation carried out using the Auxiliary Field Diffusion Monte Carlo (AFDMC) technique are also displayed. It clearly appears that the FHNC variational estimates provide a very accurate upper bound to the ground state energy. Note, however, that while the empirical equilibrium density of SNM is well reproduced, the corresponding energy is underestimated.

The main advantage of the effective interaction approach is the possibility of carrying out calculations of a variety of nuclear matter properties of astrophysical interest using perturbation theory and the basis of eigenstates of the non interacting system. Of great importance, in this context, is the ground state energy of matter at fixed baryon density and arbitrary proton fraction, $x_p = x_1 + x_2$, shown in Fig. 10.

To the extent to which thermal effects do not lead to modifications of the underlying nuclear dynamics, the approach described in this paper can be readily generalized to treat nuclear matter at nonzero temperature, by replacing the $T = 0$ Fermi distribution appearing in Eq. (30) with the corresponding distribution at temperature $T > 0$

$$n_{\lambda}(k, T) = \left\{ 1 + e^{\beta [\epsilon_{\lambda}(k) - \mu_\lambda]} \right\}^{-1},$$
Figure 8. Density dependence of the energy per nucleon of PNM. The solid lines display results obtained using Eqs. (29)-(30) and the CBF effective interaction. The variational FHNC results are represented by the shaded regions, accounting for the uncertainty arising from the treatment of the kinetic energy, while the open circles correspond to the results obtained using the AFDMC technique.

Figure 9. Density dependence of the energy per nucleon of SNM. The solid lines display results obtained using Eqs. (29)-(30) and the CBF effective interaction. The variational FHNC results are represented by the shaded regions, accounting for the uncertainty arising from the treatment of the kinetic energy.

Figure 10. Energy per nucleon of uniform nuclear matter, computed as a function of baryon density, $\rho$, and proton fraction, $x_p = x_1 + x_2$, using Eqs. (29)-(30) and the CBF effective interaction.

where $e_\lambda(k)$ is the energy of a nucleon of type $\lambda$ carrying momentum $k$, and the chemical potential $\mu_\lambda$ is determined by the constraint

$$ \frac{1}{V} \sum_{k\lambda} n_{\lambda}(k, T) = \rho_\lambda . $$

(32)

As an example, Figs. 11 and 12 show the temperature- and density-dependence of the Gibbs free energy per baryon of PNM and SNM, respectively, defined as

$$ \frac{F}{N} = \frac{E_0 - TS}{N} , $$

(33)

where $S$ denotes the entropy.
5. Summary and outlook
The results reported in this paper provide convincing evidence that the effective interaction obtained from a microscopic Hamiltonian using the (CBF) formalism and the cluster expansion technique is a powerful tool to carry out consistent calculations of a variety of properties of strongly interacting many-body systems, ranging from the ground-state energy to the quasiparticle spectrum, the in-medium collision probability and the transport coefficients.

The outcome of systematic studies of the Fermi hard-sphere system is quite encouraging, and suggest that the CBF effective interaction approach can be employed to describe a variety of nuclear matter properties, exploiting the flexibility of perturbation theory in the Fermi gas basis.

Future applications to neutron star matter will include the calculations of the shear and bulk viscosity coefficients, the superfluid gaps and the neutrino emission and absorption rates.

Acknowledgements
The results reported in this paper have been obtained in collaboration with Alessandro Lovato, Angela Mecca and Artur Polls.

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