Comment on “Collective Excitations of a Bose-Einstein Condensate in a Magnetic Trap”

In a recent Letter, Mewes et al. [1] experimentally investigated the collective excitations for a Bose-Einstein condensate. For a nearly pure condensate they observed a damping time of 250(40)ms for the collective excitations at 30Hz. They argued that for a nearly pure condensate (T ≈ 0) the damping due to thermal contributions should be negligible and that so far there is no theoretical prediction for the damping of collective excitations of a trapped condensate.

In this Comment, we shall calculate the damping of collective excitations for their experiments by using the existing theories [2–4]. Let us consider a dilute gas model of N weakly interacting bosons at finite temperature with interaction $v(\mathbf{x} - \mathbf{x}') = \frac{4\pi a^2}{m}(\mathbf{x} - \mathbf{x}')$ with a the s-wave scattering length. It is well known that the long-wavelength excitations ($k \to 0$) are phonons with the sound velocity $c \equiv \sqrt{\frac{\pi}{m}}$ where the sound velocity $c \equiv \frac{h}{\sqrt{4\pi a m}}$. The damping rate is given by Hohenberg and Martin [2] and Popov [3] for the low temperature and by Szepfalusy and Kondor [4] for the intermediate temperature,

$$\gamma = \begin{cases} \frac{3\hbar k^5}{640\pi mn_0 (k_BT)ak} + \frac{3\pi^3(k_BT)^4k}{40mn_0c^4} & (T \ll T^*) \\ \frac{1}{\hbar}\epsilon^3T^4(T^* \ll T \ll T_c) & (T^* \ll T \ll T_c) \end{cases}$$

where $T^* \equiv \frac{4\pi a^2n_0}{mk_B}$. Next, we shall estimate the damping rate of Ref. [1] using Eq. (1).

First, we checked the temperature-independent part of the damping that arises from the interaction between quasi-particles. We found that for typical parameters (see below) $\gamma_0 \equiv \gamma(T = 0) \sim 10^{-18}s^{-1}$ at an excitation frequency $\omega = 2\pi \times 30$Hz, which yields a decay time in the order of $10^3$s much longer than 250ms found in Ref. [1]. Hereafter, $\gamma_0$ shall be ignored when discussing the finite temperature case.

We plot $\gamma$ against $T$ using [1] in Fig. 1, where the following numbers are used [1,2]: $a = 65a_{Bhar}$, $n_0$ in the order of $10^{14}$cm$^{-3}$, the number of atoms in condensate $N_0 = 5 \times 10^6$, the trap frequencies of 250Hz (radially) and 19Hz (axially), and the sodium atom $m = 23 \times 1.66 \times 10^{-27}$kg. The temperature axis has been scaled against $T_c$ defined by $T_c(N) \equiv \frac{\hbar \omega_c(N/1.202)^{1/3}}{k_B}$ with $\omega_c$ the geometric mean of the harmonic trap frequencies [1,3]. Ketterle [5] pointed out that a “nearly pure condensate” meant that the condensate fraction of atoms was greater than or about 90%. This infers that the total number of atoms $N \approx 5.5 \times 10^6$, leading to $T_c \approx 0.84\mu$K. The temperature can be implied in the experiment from the condensate fraction according to $N_0/N = 1 - (T/T_c)^3$ such that $T \approx 0.5T_c$. With this temperature, we find from Fig. 1 $\gamma \approx 5.1s^{-1}$ and $\gamma \approx 3.6s^{-1}$ for $n_0 = 1.5 \times 10^{14}$cm$^{-3}$ and $n_0 = 3.0 \times 10^{14}$cm$^{-3}$, respectively. In other words, the theoretical value of the decay time of collective excitations at frequency 30Hz ranges from 190ms to 280ms as $n_0$ is from $1.5 \times 10^{14}$cm$^{-3}$ to $3.0 \times 10^{14}$cm$^{-3}$, consistent with the experimental result 250(40)ms.

We therefore consider that the damping is caused by the interaction between the collective excitation and the thermal cloud rather than the interaction between collective modes. A more complete microscopic theory taking into account the inhomogeneity and the presence of the harmonic trap shall be published elsewhere.

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FIG. 1. The interpolation of the damping rate for different density of the condensate. The dotted lines mean that $T$ is near $T^*$ and simple analytical solutions are not yet available. a. $n_0 = 0.5 \times 10^{14} \text{cm}^{-3}$; b. $n_0 = 1.5 \times 10^{14} \text{cm}^{-3}$; c. $n_0 = 3.0 \times 10^{14} \text{cm}^{-3}$; and d. $n_0 = 4.5 \times 10^{14} \text{cm}^{-3}$. 