Mathematical modeling of the process electrostatic fumigation of grain legumes in a drum type unit

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Abstract. Intensification of the processes of post-harvest processing of leguminous crops is an urgent task for enterprises of the agro-industrial complex. The improvement of the known methods of ensuring reliable protection of leguminous crops from the effects of pathogenic microflora, quarantine insect pests is of substantive interest for research. The most common way of disinfecting food is fumigation - the elimination of insect pests and pathogens with a gaseous or vaporous toxic chemical. The article discusses the process of fumigation of leguminous crops with an ionized flow of an aerodispersed mixture in a drum-type installation, and also presents the results of mathematical modeling of this process. The mathematical description of the proposed model of the fumigation process was carried out using the applied package "Mathcad". An analytical study of the obtained mathematical model made it possible to establish a number of dependencies and patterns characterizing the course of the process of electrostatic fumigation of leguminous crops in a constant stirring mode.

1. Introduction
Effective neutralization of agricultural products is carried out in special devices for carrying out the fumigation process, in which the critical parameters reach the required level throughout the mass of products [5, 10].

Currently, the procedure for fumigation of products is carried out by fumigation with a gaseous (vaporous) poisonous substance - fumigant [2, 9]. Quite often, fumigation of cereals or leguminous crops occurs already in warehouses, and the products, as a rule, are in the mode of a motionless layer [8, 11]. When the process is carried out under such conditions, the effect of uneven distribution of the active components of the fumigant throughout the entire mass of the product arises. Only the surface layers are subject to fumigation, while the seeds located in the deep layers do not experience proper interaction with the fumigant, due to a significant increase in the hydrodynamic resistance of the seed layer. Intensification of the process of fumigation of grain and leguminous crops can be achieved through constant mixing in an ionized environment, the use of which allows for an additional destructive effect on pathogenic microflora, quarantine insect pests [1, 3].

2. Purpose of the study
The aim of the study is to improve the process of fumigation of seeds of leguminous crops (hereinafter referred to as seeds) with an ionized flow of an aerodispersed mixture (powder fumigant). To achieve this goal, the design of the installation was modeled, which allows carrying out this process (figure 1).
Figure 1. General view of the designed model of the drum-type electrostatic fumigation plant for leguminous crops.

A schematic diagram of the process of electrostatic fumigation of leguminous crops in a drum-type installation is shown in figure 2.

Figure 2. Schematic diagram of a drum-type installation for carrying out the electrostatic fumigation process: 1 - loading station; 2 - unloading station; 3 - fumigation chamber; 4 - perforated drum; 5 - partition; 6 - fitting for removing the aerodispersed mixture; 7 - seed layer; 8 - conical perforated insert; 9 - branch pipe for supplying the aerodispersed mixture to section 2; 10 - branch pipe for supplying an aerodispersed mixture to section 1.

The considered method of carrying out the process is as follows: the initial product (for example, a flow of seeds) enters the loading station 1, after which it passes along the horizontal axis of the installation and is discharged through the unloading station 2. At the same time, the aerodispersed mixture with pre-charged in the ionization chamber (on not shown in the diagram) suspended particles are injected into the zone of section 1 (figure 2) of the fumigation chamber 3 through the inlet pipe 10, as well as into the zone of section 2 through the pipe 9. The zones are separated by a partition 5.

Ionized components of the flow of the aerodispersed mixture penetrate the moving mass of seeds in section 1 towards the positively charged part of the conical perforated insert 8. Then, the active components of the flow settle on the surface of the seeds, and the remaining components of the aerodispersed mixture that have not come into contact are removed from the seed layer. While getting to section 2, where they are mixed with the incoming aerodispersed mixture through the inlet pipe 9. The resulting air suspension of flows penetrates into the overflowing layer of seeds 7 in the 2nd section, passes through it towards the positively charged part of the perforated drum 4 and is removed after the limits of the fumigation chamber by means of the fitting 6.

Simulating the electrostatic fumigation of seeds in a drum-type installation, it is possible to obtain the values of the rational operating parameters of this process.

3. Mathematical model
When developing a mathematical model, some assumptions were made:
• The overflowing seed layer is electrically neutral.
• The electric charge is evenly distributed between the suspended particles of the aerodispersed mixture.
• The rotation of the drum provides a constant concentration of ionized components of the aerodispersed mixture for any of its sections, changing only along the horizontal axis.
• The movement of the aerodispersed mixture through the seed layer occurs in the radial direction. The influence of the axial component is negligible.
• The porosity of the seed layer is constant in its volume.
• The mass of seeds changes negligibly due to interaction with active ionized components of the aerodispersed mixture.
• The change in the height of the seed layer $H$ does not occur along the horizontal axis.

Let us consider the issue of applying the above assumptions to the study of the process of electrostatic fumigation of seeds in a drum-type installation. Let us define the outer radius of the layer through the piecewise given function $R_H = R_H(\theta)$:

$$R_H(\theta) = \begin{cases} 
\frac{1}{\cos \theta} \left( H - \frac{D_H}{2} \right) - \psi, & 0 \leq \theta \leq \psi \\
\frac{D_H}{2} - \psi, & \psi < \theta < \pi 
\end{cases} 
$$

In this way:

$$\psi = \arccos \left( \frac{2 \cdot H}{D_H} - 1 \right) 
$$

The average height of the seed layer in the section through which active ionized particles of the aerodispersed mixture pass will be determined by the ratio:

$$h = R_{H,\text{cp}} - \frac{d}{2} 
$$

$$R_{H,\text{cp}} = \frac{1}{\pi} \int_{0}^{\pi} R_{H}(\theta) d\theta 
$$
Where: $d$ - outer diameter of the inner perforated conical insert in section $z$, $R_{\text{h,exp}}$ - average value of the radius of the seed layer in the section $z$ (figure 3).

$$d(z) = d_1 - \frac{d_1 - d_2}{L} \cdot z$$  \hspace{1cm} (5)

![Figure 3. An arbitrary section $z$ in the installation.](image)

Then, together with $h = h(z)$, the rate of filtration of the aero-dispersed mixture through layer $\omega = \omega(z)$ will also functionally change.

Filtration rate at a certain pressure difference in the 2nd section $\Delta p_2$ will search using the formula:

$$\Delta p = \Delta p' + \Delta p^*,$$  \hspace{1cm} (6)

Where: $\Delta p'$ - pressure difference in the seed layer:

$$\Delta p' = \left[ 150 \cdot \frac{(1 - \varepsilon)^2}{\varepsilon^3} \cdot \frac{\mu \cdot \omega}{d_c} + 1,75 \frac{1 - \varepsilon}{\varepsilon^3} \cdot \frac{\rho_d \cdot \omega^2}{2} \right] \cdot h,$$  \hspace{1cm} (7)

$\Delta p^*$ - pressure difference in switchgear:

$$\Delta p^* = \xi_p \cdot \frac{\rho_d \cdot \omega^2}{2},$$  \hspace{1cm} (8)

Where: $\varepsilon$ - layer porosity; $\mu$ - dynamic viscosity of the aerodispersed mixture; $d_c$ - average seed diameter; $\rho_d$ - density of the aerodispersed mixture.

Using (6) and also taking into account (7) and (8), we write the equation for the velocity of the aerodispersed mixture $\omega$:

$$A(z) \cdot \omega^2 + B(z) \cdot \omega - \Delta p = 0,$$  \hspace{1cm} (9)

Where:

$$A(z) = 1,75 \cdot \frac{1 - \varepsilon}{\varepsilon^3} \cdot \frac{\rho_d \cdot h(z)}{d_c} + \xi_p \cdot \frac{\rho_d}{2},$$  \hspace{1cm} (10)

$$B(z) = 150 \cdot \frac{(1 - \varepsilon)^2}{\varepsilon^3} \cdot \frac{\mu \cdot h(z)}{d_c^2}.$$  \hspace{1cm} (11)
Solving (9), we find a non-negative root satisfying our conditions:

\[ \omega(z) = \frac{-B(z) + \sqrt{[B(z)]^2 + 4A(z) \cdot \Delta p}}{2A(z)}, \quad (12) \]

Then, through the integral over the layer element \( dz \), we find the volumetric flow rate of the aerodispersed mixture penetrating the seed layer in the 2nd section:

\[ V_2 = \int_{l_2}^{l_1} \omega(z) \cdot 2\pi \left( R_{\text{usp}} - \frac{h(z)}{2} \right) dz, \quad (13) \]

For the 1st section, the volume of the aerodispersed mixture is determined as the fraction of the aerodispersed mixture passing through the bypass pipeline, \( k_2 \):

\[ V_1 = \frac{V_2}{1 + k_2}. \quad (14) \]

Let us write down the integral equation for the pressure drop of the aerodispersed mixture in the 1st section:

\[ V_1(\Delta p) = \int_{0}^{l_1} \omega(z, \Delta p) \cdot 2\pi \cdot \left( R_{\text{usp}} - \frac{h(z)}{2} \right) dz, \quad (15) \]

Thus, to find the distribution of velocities, it is sufficient to substitute the found value \( \Delta p \) for the 1st section in (12).

Now you can calculate the degree of deposition of ionized components in the seed layer. As is known, the deposition of ionized components occurs due to inertial deposition, engagement, and electrical deposition [4, 6].

The inertial settlement coefficient is determined as follows:

\[ \eta_{\text{Stk}} = \frac{Stk^3}{Stk^3 + 1.54 \cdot Stk^2 + 1.76}, \quad (16) \]

Where: \( Stk \) - Stokes number.

\[ Stk = \frac{2 \cdot \eta \cdot \rho_A \cdot \omega}{9 \cdot \mu \cdot R}, \quad (17) \]

Where: \( R = \frac{d_c}{2} \) - average radius of seeds.

Mesh deposition rate \( \eta_R \):

\[ \eta_R \approx \frac{\eta}{R}, \quad (18) \]

And, finally, we compare the expression for the coefficient of electrical deposition of ionized components of the aerodispersed mixture:

\[ \eta_{\text{el}} = 6 \cdot \sqrt{\pi} \cdot K_{\text{el}}, \quad (19) \]

Where: \( K_{\text{el}} \) - parameter
\[ K_H = \frac{q^2}{96 \cdot \pi^2 \cdot \varepsilon_0 \cdot \mu \cdot \eta \cdot R^2 \cdot \omega \cdot \varepsilon_u + 1}, \]  
\[ (20) \]

Where: \( q \) - average charge of the ionized component of the aerodispersed mixture.

\[ q = \frac{4 \cdot I \cdot \rho_A \cdot \pi \cdot \eta^3}{3 \cdot G_A}, \]  
\[ (21) \]

Where: \( I \) - useful current in the ionization chamber; \( \varepsilon_0 \) - electrical constant \( (\varepsilon_0 = 8,85 \cdot 10^{-12} \Phi / \text{m}) \) [7]; \( \varepsilon_u \) - relative dielectric constant of seeds.

Summing up the coefficients found above, we get:

\[ \eta_{\Sigma} = \eta_{sk} + \eta_R + \eta_{sz}. \]  
\[ (22) \]

Now, using the value found above \( \eta_{\Sigma} \), we calculate the overshoot coefficient \( k_{np} \) from the expression:

\[ \lg \frac{1}{k_{np}} = 0,651 \cdot \frac{h \cdot (1 - \varepsilon)}{d_c \cdot \varepsilon} \cdot \eta_{\Sigma}, \]  
\[ (23) \]

Further, using \( k_{np} \), we will calculate the concentration of ionized components of the aero-dispersed mixture in the seed layer and in the flow of the aero-disperse mixture itself at the exit from the layer.

The mass volume concentration of ionized components in the flow at the inlet and outlet to the layer of the 1st section is given, respectively, by the expressions:

\[ c_1 = \frac{G_A}{V_2}, \]  
\[ (24) \]

\[ c_2 (z) = c_1 \cdot k_{np} (z), \]  
\[ (25) \]

Average value \( c_2(z) \) write through the integral:

\[ c_{2zp} = \frac{1}{L_1} \int_0^{L_1} c_2(z)dz, \]  
\[ (26) \]

For the 2nd section, the expressions for the mass volumetric concentrations at the inlet and outlet, respectively, look as follows:

\[ c_{1B} = \frac{c_{2zp} \cdot V_1 + c_1 \cdot V_p}{V_2}, \]  
\[ (27) \]

Where: \( V_p \) - volume after mixing with a fresh stream of ionized aerodispersed mixture.

\[ c_{2B} (z) = c_{1B} \cdot k_{np} (z), \]  
\[ (28) \]

\[ c_{2Bzp} = \frac{1}{L_2} \int_0^{L_2} c_{2B}(z)dz, \]  
\[ (29) \]
Using the material balance equations, we find the concentration of the ionized components of the aerodispersed mixture in the layer:

- for the 2nd section:
  \[
  G_c \cdot \frac{dW(z)}{dz} = -\frac{V_2}{L_2} \cdot (c_{iB} - c_{zB}(z)),
  \]
  \[
  (30)
  \]

- for the 1st section:
  \[
  G_c \cdot \frac{dW(z)}{dz} = -\frac{V_1}{L_1} \cdot (c_1 - c_2(z)),
  \]
  \[
  (31)
  \]

Where: \( W(z) \) - mass fraction of ionized components of the aerodispersed mixture in the seed layer.

The boundary conditions for (30) and (31), respectively, have the form:

\[
W(L) = 0, \quad W(L_1) = W_{iP},
\]
  \[
  (32)
  \]
  \[
  (33)
  \]

Where: \( W_{iP} \) - mass fraction of ionized components of the aerodispersed mixture in the seed layer at the border of the 1st and 2nd sections.

The obtained solutions of equations (30), (31) can be written in the form:

\[
W_2(z) = \frac{V_2}{L_2 \cdot G_c} \int_{z}^{L_2} (c_{iB} - c_{zB}(z)) dz,
\]
  \[
  (34)
  \]

\[
W_1(z) = \frac{V_1}{L_1 \cdot G_c} \int_{z}^{L_1} (c_1 - c_2(z)) dz + W_{iP},
\]
  \[
  (35)
  \]

Where: \( W_{iP} = W_2(L_1) \).

Using the equation of electric charge balance for the element of the seed layer \( dz \), we find the distribution of the linear current density along the length of the fumigation chamber:

\[
dQ = (c_1 - c_2(z)) \cdot \omega \cdot 2\pi \cdot \left( R_{\text{sep}} - \frac{h}{2} \right) \cdot \frac{q}{4 \cdot 3 \cdot \pi \cdot \eta^2 \cdot \rho_A} \cdot d\tau dz,
\]
  \[
  (36)
  \]

The linear current density is defined by the expression:

\[
i = \frac{dQ}{d\tau dz},
\]
  \[
  (37)
  \]

From (36) we get:

- for the 1st section:
  \[
i_1(z) = \frac{3}{2} \cdot (c_1 - c_2(z)) \cdot \left( R_{\text{sep}} - \frac{h(z)}{2} \right) \cdot \frac{q \cdot \omega(z)}{\eta^2 \cdot \rho_A},
\]
  \[
  (38)
  \]

- for the 2nd section:
\[ i_2(z) = \frac{3}{2} \left( \varepsilon_{1B} - c_{2B}(z) \right) \left( \frac{R_{\text{up}} - h(z)}{2} \right) \cdot \frac{q \cdot \omega(z)}{\eta^3 \cdot \rho_A}. \] (39)

4. Analysis of the mathematical model

The initial data for the calculations are presented above. In addition to these, a number of physical parameters are introduced for the aerodispersed mixture: density \( \rho_A \); operating pressure \( p \); molecular diameter \( d_M \); particle density \( \rho_T \); dynamic viscosity \( \mu \). Boltzmann constant values \( k \); electrical constant \( \varepsilon_0 \); relative dielectric constant of seeds \( \varepsilon_r \) were selected from reference literature [7].

Below are the results of modeling the process of electrostatic fumigation of seeds (figures 4-8) with the following parameter values in the Mathcad 15 environment:

- \( L_1 = 0.7 \, m \); \( L_2 = 0.4 \, m \); \( D_b = 0.5 \, m \); \( H = 0.4 \, m \); \( d_1 = 0.3 \, m \); \( d_2 = 0.2 \, m \); \( \Delta p_2 = 50 \, Pa \);
- \( \rho_A = 1,2 \, \kappa \, e / m^3 \); \( p = 10^5 \, Pa \); \( d_M = 3 \cdot 10^{-10} \, m \); \( d_c = 7 \cdot 10^{-3} \, m \); \( \xi_R = 10 \); \( k_2 = 1 \);
- \( G_A = 3 \cdot 10^{-4} \, \kappa \, e / c \); \( \rho_T = 1500 \, \kappa \, e / m^3 \); \( \mu = 1.8 \cdot 10^{-5} \, Pa \cdot c \); \( G_c = 1 \cdot 10^{-2} \, \kappa \, e / c \);
- \( k = 1.38 \cdot 10^{-23} / K \); \( T = 303K \); \( \varepsilon_0 = 8.85 \cdot 10^{-12} \, \Phi / m \); \( I = 1 \cdot 10^{-3} \, A \); \( \varepsilon = 0.5 \); \( \varepsilon_c = 20 \).

Below are graphs of various dependencies of the electrostatic fumigation of leguminous crops in a drum-type installation.

**Figure 4.** Distribution of odds deposition of ionized particles aerodispersed mixture in the seed layer.

**Figure 5.** Concentration distribution ionized particles of the aerodispersed mixture in the seed layer along the length of the drum-type installation.
Figure 6. Concentration dependence ionized particles in an aerodispersed mixture from the useful current in the ionization chamber.

Figure 7. Dependency final concentration of ionized particles on seeds from useful current in the ionization chamber.

5. Conclusion
The obtained results of mathematical modeling allow us to draw the following conclusions:

- Due to the application of the mechanism of ionization of solid suspended particles of an aerodispersed mixture (fumigant), a significant increase in the efficiency of deposition of fumigating particles on seeds is observed. In the absence of an ionization factor, the degree of deposition is close to zero.
- The installation of an additional bypass pipeline has a significant impact on the concentration distribution along the length of the fumigation chamber. In the place of its installation, there is a sharp abrupt increase in the degree of deposition of ionized components of the aerodispersed mixture (fumigant) by 2 times.
- With an increase in the value of the supplied current into the ionization chamber, the concentration of ionized particles of the aerodispersed mixture on the seeds increases.

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