Vortex states in high-$T_c$ superconductors and superconductivity in modern nano-science and engineering

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Abstract

A brief review is given on the recent development of the vortex engineering research in our group with special emphasis on the geometrical confinement of vortices into micron-size of samples. We show that the static as well as dynamical nature of vortices is strongly affected by the geometrical constraints, which can be manipulated by the recent nano-technology engineering artificially. One of the spectacular phenomena in this context is a formation of self-organized vortex structures occurring both in static and dynamic manners. The static behaviors have been studied by the scanning SQUID microscope technique and the dynamical behaviors are mostly done in transport measurements. Some technical development of these methods is also discussed. We stress that a common generic concept here is the confinement in nano-scale superconducting systems by the existing surfaces (boundaries), which plays an essential role for the phenomena. Some typical examples are shown, such as results on the vortex arrangement in micron size superconducting disks, a formation of the giant vortex state in such a particular case and the dynamical effects of vortices, especially in the case of Josephson vortices confined in an intrinsic Josephson junction in micron size single crystal $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8+\delta$, where the dynamical effect of the confinement provides rich linear and non-linear phenomena due to the excitation of Josephson plasma. Making use of Josephson plasma excitations, a generator operating at $T$ (Hz) frequencies is proposed in this intrinsic Josephson junctions. These phenomena are expected to be all beneficial to potential applications.

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1. Introduction

One of the most striking phenomena in superconductivity is the sudden appearance of quantum phase coherence in a macroscopic scale over the sample [1]. In such a system in
a magnetic field above the lower critical field, $H_{c1}$, quantized magnetic flux characterized by two characteristic length scales $\lambda$ and $\xi$ appear as ‘vortices’ (corresponding to the ‘rotons’ in superfluids), fundamental excitations of the coherent phase, where $\lambda$ and $\xi$ stand for the magnetic penetration depth and the coherence length, respectively.

When the size of the superconductor is large enough (and is also pure enough as well so that any pinning effects of vortices may be neglected), the position of each vortex with respect to the edge of the sample is not important because of the translational symmetry of the system. It is only the interaction between vortices themselves that are most important and relevant for determining the vortex arrangement of the system. The Abrikosov triangular vortex lattice is known to be stabilized in this case [2].

On the other hand, when the size of the superconductor is reduced down to the comparable size of $\lambda$ or $\xi$ or somewhat larger than those length scales, vortices are obliged to accommodate themselves into the narrow geometrical space. As a result, vortices inevitably feel the surface (or boundary) of the system, inducing strong competition between vortex–vortex interaction energy and surface energy. This results in the perturbation of the vortex state, and sometimes it even overcomes the vortex–vortex interactions, bringing the system into an entire new vortex state. In other words, the free energy in such a small (mesoscopic) system is strongly modified by the number of vortices, the relative position of the vortex as well as the geometrical shape of the system. In general, these all phenomena can be viewed as the phenomenon essentially similar to what is known as the Little–Parks effect [3].

In the following, we show several examples of this sort of phenomena indicating the interesting competing effects between number of vortices, positions of them and geometrical sizes and shapes of the sample. Direct images of vortices with respect to the geometrical shape will be demonstrated. The giant vortex state and the phase transition from giant vortex state to multi-vortex state, or vice versa, are evidenced by the direct measurement of the multi-tunnel junction experiment [4] as expected previously [5–7].

The similar effects can also be observed in the dynamical cases where vortices are moving in a restricted geometry in the case of high temperature superconductor $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8 + \delta$, single crystal [8]. Recently, we have also observed an oscillating behavior of the dynamic impedance of the intrinsic mesoscopic Josephson junction as functions of the current and the intensity of the applied magnetic field [9]. The phenomena can be ascribed by the Fiske steps in the multi-stacked intrinsic Josephson junctions, whereas in the static case as a corresponding phenomenon the critical currents in Josephson junctions oscillate as a function of the total penetrating magnetic flux exhibiting the Fraunhofer interference effect [10–14]. This strongly suggests that the dynamical motion of Josephson vortices in a Josephson junction may generate coherent Josephson plasma waves in the junction as suggested by Tachiki [15]. We point out here that the half quantum flux oscillatory behavior of the dynamical impedance of the junction as a function of magnetic field observed there is purely a manifestation of the dynamical effect of the Josephson vortices [12,13,16], which cannot be explained by the static model previously used for the explanation [8].

As an experimental technique for static observation, a scanning SQUID (Superconducting QUantum Interference Devices) microscope (SSM) is used [17]. This is a powerful new tool to study individual vortices with high spatial resolution ($\sim 1 \mu m$) and high magnetic flux sensitivity. For dynamical measurements in small intrinsic junctions, conventional transport measurements have been carried out. We especially use the Corbino disk arrangement for the resistivity measurement in order to exclude the surface pinning effect [18,19] as well as possibly the geometrical barrier effect [20], which causes non-linear resistivity in the vortex liquid state [21–24].

2. Vortex phase diagram in high $T_c$ superconductors

Before proceeding to the central subject, it is worthwhile mentioning the vortex phase diagram in bulk high temperature superconductors. This has been a main research subject in our group for more than 17 years after the discovery of high temperature superconductivity. As a consequence, there has been a tremendous amount of data available to properly understand the vortex phase diagram in high $T_c$ superconductors, which is very different form the one in conventional superconductors. Although we do not intend to explain all the features of the vortex phase diagram, one must be aware of the gist of the problems. This will help us better-understand the vortex state, especially in the case of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8 + \delta$ we are now going to present.

In Fig. 1, we show a generic vortex phase diagram in high $T_c$ superconductors such as highly two-dimensional layered superconductor $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8 + \delta$, when magnetic field is

![Fig. 1. The schematic vortex phase diagram in an ideal case of high $T_c$ superconductors without any pinning (a), and in a realistic case (b). In the left hand sides for both cases (a) and (b), the effect of pinning is schematically shown. No pining case for (a), while a considerable pinning effect for (b). In both (a) and (b), the lower critical field, $H_{c1}$, is omitted from the figure. Although the $H_{c2}$ line does not exist in high-$T_c$ superconductors (see text), it is added for comparison with the case for conventional superconductors.](image)
applied to the c-axis, perpendicular to the CuO2 plane. In spite of large differences in magnetic field and temperature scales, the main features observed here in Bi2Sr2CaCu2O8 +δ resemble those observed in optimally doped YBa2Cu3O7 [25]. This is not surprising, because the difference between both systems lies only in the anisotropy parameter γ, which is about eight in YBa2Cu3O7 and about 200 in Bi2Sr2CaCu2O8 +δ.

The most striking features which differ distinctly from the conventional type II superconductors are listed below [26,27];

1. disappearance of the upper critical field $H_{c2}$ line
2. instead, the first-order vortex-lattice melting transition (VLMT) line, $H_M$, comes in as a true phase transition line [28–30]
3. The vortex liquid phase occupies a large area of the $H−T$ phase diagram above the $H_M$ line, where the Lorentz-force free resistive dissipation is found [31].
4. The vortex liquid phase is continuously connected to the normal state without crossing any phase transitions.
5. The first order $H_M$ line is very sensitive to any kinds of disorders, which give rise to pinning. It easily changes its character from the first order to the continuous (second order) transition at the $H_{irr}$ or $H_{G}$ line [32].
6. The vortex lattice (Bragg glass) state transforms to the crossing lattice state [33] from the tilted lattice state at the angle of about 17° from the c-axis [34–37].
7. The angular dependence of the vortex lattice melting transition does not obey the scaling law [38], indicating that the layered structure with a periodic inhomogeneous superconducting order parameter plays an essential role in the vortex phase transition [36,39–41].
8. In slightly disordered systems including good single crystals so far available, a new line, $H_F$, emerges, dividing the liquid state into a pinned and an unpinned liquid [42,43].
9. At low temperatures, vortex glass phase is established in real samples in wide field range above about 1 kOe, where the critical current, $I_c$, of the order of $10^8$ A/cm² is realized. This $I_c$ is lost at the $H_G$ line [27].

These new features of the vortex state solely originate from much shorter coherence length, $\xi_c$, than the interlayer distance, $\sim 15$ Å, across the double CuO2 plane and high transition temperature, $T_c \approx 90$ K [44]. This is a direct consequence of the special electronic structure of this material, which consists of stacks of highly two-dimensional layered structure with different electrical properties; CuO2 layers are known to be good conducting layers which eventually undergo superconductivity at low temperatures, while Bi2O3 layers are known to be insulating layers. A simple estimate of the critical phase transition region according to the Gaussian fluctuation theory of phase transition results in $\Delta T \approx 10–20$ K in zero magnetic field and $\approx 30–40$ K in 5 T magnetic field [45]. This enormous width of the critical fluctuation region is the cause of the disappearance of $H_{c2}$ [26,31].

In an ideal case, the phase diagram is rather simple as shown in Fig. 1(a), consisting of a single phase transition line, the VLMT line, $H_M$. The topology looks the same as the one in conventional type II superconductors, except for replacement of $H_{c2} \rightarrow H_M$.

It becomes more complicated, however, when the pinning effect is involved. This is shown in Fig. 1(b), which is often seen in real single crystals in the case of Bi2Sr2CaCu2O8 +δ. In reality, the pinning effect cannot be eliminated even in the purest sample so far available, so that the phase diagram shown in Fig. 1(a) is constructed by extrapolating from the results obtained in the samples with different amount of pinning densities.

There are two new lines in Fig. 1(b) in addition to in Fig. 1(a) [27,43]. The $H_F$ line, which divides the liquid phase into two ($L_1$ and $L_2$ phases), emerges from the critical point, where the first order phase transition $H_M$ terminates, in the case of the weakest pinning. The emerging point quickly moves to the higher temperatures along $H_M$. Although the distinction between $L_1$ and $L_2$ phases are very subtle, the pinning effect is appreciable in the $L_2$ phase but not in the $L_1$ phase. Therefore, the viscosity of the liquid changes at the $H_F$ line.

The first order VLMT line, $H_M$, terminates at the critical point, then becomes the irreversibility line, $H_{irr}$, or the glass transition line, $H_G$, which seems to be the second order phase transition line [29,30,46]. From the critical point another first order line has been claimed to extend towards low temperature almost horizontally, dividing the vortex solid phase into the Bragg glass phase and the vortex glass phase [47]. There has been a serious question about whether the vortex glass phase is true phase or not. Experimentally in a practical sense, the sufficient strong supercurrent flows for a sufficiently long period of time at low temperatures even though large creep phenomena have been observed. The quantum mechanical tunneling is evidenced from the observation of large supercurrent relaxation even below 1 K. It must be noted that the first order line below the critical point is a disorder-induced transition, whereas $H_M$ is purely thermal-induced transition, irrespective of existing pinning [47].

In Fig. 2, a typical example of the real vortex phase diagram is schematically shown in reduced scale of both axes [27].

All arguments so far are given for the case of magnetic field applied parallel to the c-axis (perpendicular to the superconducting CuO2 plane). When the magnetic field is applied to the ab-plane, the field scale becomes extremely high due to the huge anisotropy parameter $\gamma \sim 150–200$ in optimally doped Bi2Sr2CaCu2O8 +δ, so that the measurable quantities easily go beyond the available field range of 6 T in our facility. Furthermore, it is extremely difficult to meet the exact experimental condition for the field being parallel to the ab-plane, because the allowance of the angle to orient the magnetic field parallel to the ab-plane is theoretically very narrow in macroscopic samples, of the order of $4 \times 10^{-5}$ or less. Practically, this condition is formidable to realize. However, the recent resistivity measurements by the Corbino disk arrangement surprisingly shows a clear lock-in behavior within 0.04° even in the bulk single crystal sample with a few millimeter dimensions [36,41,48]. We know from the X-ray parallel beam double crystal diffraction method [49] that
the rocking curve width of the single crystal is approximately 0.03° [50], which coincides well with the width of the lock-in transition found by the Corbino method. Therefore, the lock-in angle is perhaps much larger even in the bulk sample, although experiments are limited by the parallelism of the crystal plane. It is interesting to note that the bulk lock-in behavior is compared with the recent results of the micrometer size samples, where the lock-in angle becomes several degrees, three orders of magnitude bigger than the bulk sample [16,51].

The obtained phase diagram for $H$ parallel to the $ab$-plane is shown in Fig. 3. The line $H_{SL}$ is the boundary, which seems to be a continuous transition (may be the second order transition) and merges to the $H_{SS}$ line just at CP (critical point) in a few degree K below $T_c$. The line $H_{SS}$ above CP is likely to be the first order melting transition line, which corresponds essentially to the $H_M$ line for the field parallel to the $c$-axis. The fact that the $H_{SL}$ line is almost vertical and has a strong current dependent behavior in the resistivity measurement, this is attributed to the smectic-liquid transition, below which a smectic Josephson vortex state may be realized. This smectic phase is bounded by the solid phase through the second order transition line $H_{SS}$ below the temperature of CP.

In magnetic fields in-between $c$-axis and $ab$-plane, two tilted lattice phases and a crossing lattice state are expected to occur theoretically [52]. This is in fact observed in recent work as shown in Fig. 4 [36,37,39–41,48,53–56]. Here, we note that one tilted lattice of pancake vortices occurs in a region of $0° < \theta < 17°$ (not seen in Fig. 4) [37], whereas another tilted phase may be in a higher angle region of $\sim 86° < \theta < 89.90°$ [36,41]. A large portion of angle in $\sim 17° < \theta < \sim 86°$ is well described by the crossing lattice of the pancake vortex system and the Josephson vortex system [33], and a narrow angular region of $89.9° < \theta < 90°$ is certainly attributed to be as the lock-in state of the Josephson vortices [54]. Although the field scale in both axes increases with decreasing temperature, the critical angles described above may change only little.

It should be mentioned that the generic feature of the vortex phase diagram in whole angle region is rather similar topologically, in spite of some dissimilarities due to modification originated from the nature of large anisotropy parameter with the intrinsic inhomogeneity of the superconducting order parameter.

3. Experimental techniques

In this section, we summarize the techniques we have been using for the present study.
3.1. Scanning SQUID Microscope (SSM)

3.1.1. General description

The main part of the scanning SQUID microscope (model SQM2000) was purchased from SEIKO Instruments Inc [17]. The block diagram of the scanning SQUID microscope was shown in Fig. 5. The double washer type of DC-SQUID is made by thin film technology using pure Nb on Si chip with the size of $3 \times 3$ mm at the position where is at a distance of about 1 mm from the SQUID pick-up coil as shown in Fig. 5(A). The pick-up coil is located at the corner of the square of the Si chip, which works as a cantilever mounted at the bottom of the cold head. This stage is kept as cold as possible ($\sim 2$–$3$ K), while the temperature of the sample stage can be controlled by the heater up to 100 K.

The principal specification of the model SQM2000 is as follows [57]:

1. detection pick-up coil diameter: 10 $\mu$m
2. flux sensitivity: $5\mu\Phi_0/\sqrt{Hz}$
3. maximum field to be allowed to apply: 10 mT
4. maximum scan range $(x \times y)$: $10 \times 10$ mm
5. spatial resolution of the $xy$ stage: 50 nm
6. scan speed: 12.5 $\mu$m/s
7. variable range of temperature: 2.2 – 100 K
8. temperature stability: $\pm 0.5$ K
9. consumption of liquid Helium: 2.9 l/h

According to the specification of SQM2000 the spatial resolution is several $\mu$m, which is similar to the one developed by the IBM group [58], but not sufficient for the imaging of the individual vortex, although the flux sensitivity is good enough for the purpose. This situation can be compared with the scanning Hall probe microscope (SHPM), where the spatial resolution is better, but the flux sensitivity is three to four orders of magnitude worse [59]. The spatial resolution can be improved by reducing the size of the pick-up coil to a few micrometers as described below.

3.1.2. Pick-up sensing coil and design

We have made a great effort to improve the spatial resolution down to a few $\mu$m by reducing the size of the pick-up loop as shown Fig. 6. The SQUID loop was originally designed for the effective diameter of 10 $\mu$m of the pick-up coil, the sensitivity is somewhat decreased by reducing the size of the pick-up coil down to 2 $\mu$m [60]. It turns out that this size is almost the smallest limit technically possible without modifying the rest of the SQUID system. The reduction of sensitivity is inevitable due to impedance mismatching when the inductance of the rest of the SQUID coil is unchanged. Furthermore, for the thin film processes to fabricate pick-up coil smaller than a few $\mu$m, it is necessary to have $\sim 100$ nm level accuracy of the processing, which becomes very difficult and makes it unreliable. Considering these results, it is highly desirable to have an entirely different concept of the pick-up coil design in order to further improve and increase the spatial resolution of the SQUID microscope.

There are more difficult problems to overcome in order to make better spatial resolution down to the range of several 100 nm, although the SQUID itself is expected to work even in such a small size. The difficulty is to eliminate deformation of the image originating from the incomplete closure of the pick-up coil at the neck. This can be improved by changing the design of the pick-up coil. In Fig. 6(c), we developed a new pick-up coil with the effective diameter of 10 $\mu$m using a three dimensional lithography processing technology in combination with FIB (Focused Ion Beam) processes and succeeded in improving the ghost tail effect greatly.

Another important issue is to control the distance between the sensing pick-up coil and the object. This issue becomes more serious when the size of the pick-up coil is reduced. As shown in Fig. 5(A), the pick-up coil is located at the corner of the Si chip with $\sim 50$ $\mu$m distance from the corner edge. This Si chip is gently pressed on the surface of the sample at an angle of 3° while operation. This makes about 3–4 $\mu$m from the surface to the centre of the pick-up coil. Because of this the magnetic flux from a vortex spreads out at the point where the pick-up coil is measuring the magnetic flux, the image is blurred and faded. In order to make the pick-up coil as closer as possible to the object, the corner edge is grounded off by the micro-grinder. By doing so the distance can be made as close as 1 $\mu$m to the object.

In general from our experience it is required to bring down the pick-up coil closer than the radius of the pick-up coil in order to obtain the reasonable quality of the vortex image. The more quantitative analysis will be shown in Section 4.

All SSM measurements shown here have been done in the field cooling process in order to avoid the surface current effects.
3.2. Corbino disk method

In our previous resistivity studies of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ with four terminal method, it has been known that the results of the measurement are strongly sample dependent. It turns out later that this was not due to the sample quality or experimental set-up but due to the surface barrier effect [61], which short-cuts the current path via the sample edges [19,21,22,62]. In order to avoid this effect, the Corbino disk method is introduced [23,24,46,63–66].

In Fig. 7, we show the schematic view of the Corbino disk and the electrode arrangement for $\rho_{ab}$. Two independent voltage taps were made in order to check asymmetrical effects, in particular, when the magnetic field is tilted from the symmetric direction such as the c- or ab-direction. Although the two resistances differ by about several to 10%, no appreciable difference after scaling of the value of resistance was detected within our experimental error. This means that even in a tilted magnetic field, which may break symmetry of the vortex flow with respect to the magnetic field direction, causing asymmetric inhomogeneous vortex flow resistivity in the sample, the Corbino method gives the correct resistivity value irrespective of the position of the voltage tap.

Although this method needs larger single crystal with uniform and high quality, and requires complicated preparation processes of samples, another spurious effect coming from the geometrical barrier [20] may also be eliminated, providing that the Corbino disk is made near the centre of the sample (sufficiently far from the edges) [63]. Another disadvantage of the Corbino disk measurement exists in the non-uniform current distribution, because the current flows to the radial direction of the disk. This may introduce additional complications of the data interpretation.

In spite of these disadvantages the dramatic improvement can clearly be seen in the quality of the data as shown in Fig. 8, where a set of in-plane resistivity data of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ measured by the Corbino disk method is shown. These data are compared with Fig. 9, in which an example of the same resistivity measured by the conventional four terminal measurement method is shown [41,46,48].

First of all, the sharpness of the first order transition is remarkable, and is greatly improved with large jump in
resistivity, at least one order of magnitude larger at the onset of resistivity relative to the normal state resistivity just above \( T_c \).

A large current dependent non-linear resistivity observed in most of the region in the data obtained by the conventional four terminal measurement as seen in Fig. 9 cannot be found at all in the Corbino disk measurement. Furthermore, it is obvious that the results of the resistivity measurement shown in Fig. 9 are something wrong, because the observed resistance in the smaller sample measured after cutting the larger sample gives even smaller values, contradicting common knowledge of electrical conduction. The non-linear behavior is even larger in the smaller sample. All these facts strongly indicate that the observed unconventional behaviors originate from the surface barrier effect, and perhaps as well as the geometrical barrier, which contribute to the pinning, giving rise to the lower resistance portion at the edge of the sample. Since, most of experimental data so far available in the literature are obtained by the strip geometry with four terminal method, the resistivity of Bi₂Sr₂CaCu₂O₈+\( d \) deserves reconsideration. One should be careful to read experimental data so far available, because even data obtained by the Corbino disk are not always consistent [23,24,46,63,64].

4. Direct observation of vortices

In this section, we will demonstrate some examples of the vortex images in various systems observed by the SSM technique.

A simple example is given in Fig. 10, where the vortex image is taken from a YBa₂Cu₃O₇−\( \delta \) (hereafter abbreviated as YBCO) thin film [67] under various external magnetic fields. The number of vortices, \( N \), in the area, \( S \), of the frame (500×500 \( \mu \)m) is counted as a function of the applied magnetic field, \( H_a \). Since, the demagnetization factor can be negligible in such a thin and large film, a relation \( N_\Phi = (H_a + H_{\text{earth}})S \) must be satisfied, where \( H_{\text{earth}} \) stands for the earth magnetic field. As seen in Fig. 11, a clear linear relation is observed with a small value of \( H_{\text{earth}} \approx 10.1 \) mG, which is the stray earth field in our superconducting shield.

Another interesting observation in Fig. 10 is concerning the vortex arrangement, which seems to be random without spatial correlation. This can be checked by using two-dimensional Fourier transformation of the location of the vortex. It turned out that there is indeed no appreciable spatial correlation,

![Fig. 9](image_url)

**Fig. 9.** A set of resistivity curves of single crystal Bi₂Sr₂CaCu₂O₈+\( d \) measured by the conventional four terminal method. The arrangement of the four electrodes is schematically depicted in the inset. The data with blue color are obtained from a larger sample before cutting, while the data with green color are obtained from the same sample with same four contacts but after cutting by green line.

![Fig. 10](image_url)

**Fig. 10.** A typical example of the vortex image trapped in a YBCO thin film obtained at 2.0 K. Each white spot in the picture represents an individual single vortex. The scanning area is 500×500 \( \mu \)m and the external magnetic field applied is written at the upper right corner of the picture. The scanning step for \( x \) and \( y \) directions are 2.0 \( \mu \)m.
suggesting the random individual pinning working in the YBCO thin film.

It is interesting to note that all vortices so far observed in two different films are singly quantized and no half-quantized or multi-quantized vortex has been detected [68].

Closer looking at a single vortex, a beautiful example of the magnetic field distribution image of a vortex trapped in a thin Nb film is shown in Fig. 12. Such a clear image can only be obtained by the SQUID microscope because of high field sensitivity. In addition to this excellent sensitivity, the absolute value of the integrated magnetic flux can be measured, since a SQUID sensor measures the absolute value of the flux in a unit of $\phi_0 = 2.07 \times 10^{-7}$ G cm$^2$. This is another advantage of SSM in comparison with other similar techniques such as SHPM, in which the field sensitivity (≤ $10^{-3} \phi_0$, at least three to four orders of magnitude less than SQUID [59]) and the noise condition vary strongly with temperature, especially above 30 K.

There is, however, a disadvantage of SSM, which is related with a lack of sufficient spatial resolution. As seen in Fig. 12, the field profile of the vortex in Nb is much more extended than the expected one. This is nothing but the defocusing effect due to a finite diameter of the pick-up coil, usually, 10 µm, which is much bigger than the size of the vortex $2\lambda \approx 400$ Å in Nb.

Another important factor to get better image is the distance, $z$, of the pick-up coil from the sample surface. Because of rapid spread-out of the magnetic field lines from the surface, the field intensity at the centre should decrease as $z^{-2}$ with increasing the $z$-value. Taking both factors into account, using a simple magnetic monopole model as shown schematically in Fig. 13(a), the field profile can be calculated. The results are shown in Fig. 13(b), where magnetic field profiles are displayed with different value of $z$ for the 10 µm standard pick-up coil. It is concluded from these calculations that, firstly, the width of the field profile is mostly determined by the width of the pick-up coil, when it is located at a distance being less than the diameter of the pick-up coil. This means that the spatial resolution is limited mainly by the size of the pick-up coil. Secondly, the field intensity at the centre grows sharply as $z^{-2}$ with decreasing the $z$-value. This means that in order to achieve a good field resolution, it is essential to bring down the sensor as close as possible, say, possibly less than 1 µm. The profile shown in Fig. 12 was analyzed and is shown in Fig. 13(b). For the $z$-value $z = 6.3$ µm was obtained.

5. Mesoscopic superconducting disks

5.1. Multi-vortex state

We have studied the vortex arrangement in mesoscopic size of superconducting disks with diameter from 10 µm to 50 µm and with the shape of circular disks, squares and triangles made of Nb, Pb and YBa$_2$Cu$_3$O$_{7-\delta}$ thin films [69,70]. We have also studied antidots, i.e. arrays of holes with various shapes and periods. Fig. 14 presents such an example of the images of the vortices trapped in a Nb mesoscopic disk with 50 µm in diameter in 2, 4 and 6 µT magnetic fields.

As is seen in Fig. 14, the first vortex enters into a central region of the disk. But it is not always so. Sometimes it comes at the rim and is pinned strongly, perhaps due to some defects created when the sample was made. Keeping the experimental condition the same and having been measured repeatedly, the first vortex seems to have a tendency to come into the same place in the sample, but avoiding the edge region. This means that there is a certain favorable position for the first entering vortex, indicating that pinning is playing a major role to determine the position of the first entry vortex.

When the second vortex enters into the disk the first one moves to other positions. This means that the vortex repulsive interaction is stronger than the pinning energy of the vortices. This becomes more clearly when the number of vortices is

Fig. 11. The number of vortices, $N$, as a function of applied magnetic field $H_a$. All vortices observed in this YBCO thin-film are singly quantized with $\phi_0 = 2.07 \times 10^{-7}$ G cm$^2$.

Fig. 12. The three dimensional SSM image of a single vortex trapped in a Nb thin film obtained at 1.9 K. The scanning step was 0.5 µm.
more than three. In such cases, it may be stated that the arrangement of the vortices inside the sample is mainly determined by the interaction between vortices and between vortex and the edge of the sample. The former interaction is known to stabilize the Abrikosov triangular lattice, whereas the latter interaction depends strongly on the shape of the sample and causes a geometrical confinement of the vortex into the fixed symmetry. Since, this boundary condition gives strongest restriction to the vortices, the triangular lattice is distorted to accommodate them with the shape of the sample at the minimum cost of the free energy. This re-organization phenomenon is observed experimentally, and it is in good accordance with the recent theoretical calculations [6,7,71–82].

In square and triangle samples with the sides of 10 and 30 µm similar behaviors have been observed.

A typical and beautiful example of the image of the multi-vortex state for the purpose of display is shown in Fig. 15 for YBa$_2$Cu$_3$O$_{7-x}$ thin films, in which one vortex sits at the centre and five vortices are arranged in a circular fashion, forming an ordered structure in a thin circular YBa$_2$Cu$_3$O$_7$ disk. It is interesting to note that the magnetic field inside the disk is almost zero (the external magnetic field is almost cancelled as if the disk is in the Meissner state), if all vortices are removed. At the position of the vortex, it is also clearly observable that the magnetic field is much larger than the external field, regardless of the size of the vortex.

Fig. 13. A model calculation of the magnetic field profile of a single vortex trapped in a Nb thin film: (a) a schematic drawing of the monopole model used for the actual calculation; (b) the calculated results of magnetic field distribution with various sensor distance, $z$, assuming the diameter of the pick-up coil to be 10 µm.

Fig. 14. The examples of the entry of one (a), two (b) and three (c) vortices in the superconducting Nb disk with 50 µm in diameter in magnetic field of 2, 4 and 6 µT, respectively. The upper row shows the images of the field intensity, the middle row the 3D image of the first row, and the bottom row the field profile cut along the line indicated in the upper row.
Fig. 15. The Magnetic field profile with six vortices trapped in the YBa2Cu3O7−δ thin film with the thickness of 480 nm in applied magnetic field of 10 μT observed at 4.2 K.

5.2. Giant vortex state

In further increase of applied magnetic field it is predicted that the above mentioned multi-vortex state is enforced highly by the geometrical constraint and transforms to the giant vortex state, where the vortex quantization is more than one quantum flux with some integral number of $\phi_0$ [6,7]. Although we made much effort to directly observe the giant vortex state by the scanning SQUID microscope, we are not convinced whether or not the images obtained are the unresolved multi-vortex state, in which the vortices lie at very close positions each other because of the insufficient spatial resolution of our pick-up coil. However, our recent multi-tunnel junction experiment on an Al disk of 1.5 μm in diameter and 33 nm in thickness showed a clear evidence of the giant vortex, and the phase transition from multi-vortex state to a giant vortex state of vice versa was identified [4,83]. This result is in excellent agreement with the results of the recent theoretical calculations [83–85]. Further work on the other shape of samples such as squares is underway.

6. Observation of vortex in thin films

So far, we have considered samples with geometrical constraints on the shape, for example, such as circular disks in thin films, where the film thickness can be neglected. Now that the geometrical constraint is on the thickness $d$, which is much less than $\lambda$ in a large size of the sample, a new phenomenon is expected. Since, the superconductor is so thin that the sufficient screening current cannot be maintained within $\lambda$, the effective magnetic penetration depth $\Lambda$ is enhanced as $\Lambda \approx 2\lambda^2/d$, as pointed by Pearl [86].

According to Pearl, Pearl vortices have interesting characteristics compared with the conventional Abrikosov vortices [87]. Since, the vortex energy associated with a single Pearl vortex mostly lies in magnetic fields outside the superconductor, the interaction becomes long range with spatial variation of $V_{\text{int}}(z) \sim \Lambda/r$ for $\Lambda = r$, not like $\sim e^{-r/\Lambda}$ for conventional Abrikosov vortices, which is short range for $r \to \infty$. When $r = \Lambda$ the magnetic field intensity right above the Pearl vortex ($x = y = 0$) diverges as $1/r$, whereas it diverges as $\ln(\lambda/r)$ in Abrikosov vortices. Although such distinct differences of the characteristic field profile of the Pearl vortex from the Abrikosov vortex, no serious attention has been paid until just recently [88], perhaps, because of no experimental tools to access into that range of spatial microscopic scale.

Recently, we have studied the vortex state in a series of very thin amorphous MoGe films where the thickness is varied as $d = 30, 100, 55, 200$ and $400$ nm, which are (much) thinner than the penetration depth $\lambda \sim 670$ nm [89]. In such a relation, $\Lambda = 2\lambda^2/d$, predicted by Pearl can directly be proved by measuring the magnetic field distribution of the vortex in various thickness of samples by the scanning SQUID microscope. This measurement has indeed been carried out. The direct image of the Pearl vortex and the field profile fitted to the theory are shown in Fig. 16. As is easily seen from Fig. 16, the peak intensity is greatly reduced and the tail becomes somewhat longer as the thickness is reduced. According to the theory [86,87] it is noted that although the peak flux intensity is greatly reduced, the full width at half-maximum of the vortex images is relatively independent of $\Lambda$. Using analytical expression of the field profile, it is fitted to the experimental data as shown in Fig. 17. The effective $\Lambda$ tends to diverge with decreasing the thickness, while it converges with increasing the thickness to a constant value of $\Lambda(d \to \infty) = \lambda \approx 550$ nm, a bulk value of the magnetic penetration depth, which agrees with the previously known value of $\lambda(0) \sim 670$ nm for MoGe amorphous thin films reasonably well.

7. Confinement of Josephson vortex

Another very interesting example of the vortex phenomena in a restricted geometry is the case of the Josephson vortex in highly anisotropic superconductors such as Bi2Sr2CaCu2O8 + δ. When a magnetic field is applied parallel to the superconducting CuO2 plane, Josephson vortices are introduced in between CuO2 layers, forming a multi-stacked intrinsic Josephson junction system [10,90–92]. From the result of numerical calculations it is expected that a highly distorted isosceles triangular Josephson vortex lattice state may be realized at sufficiently high fields, $H^* \approx \sqrt{3}\phi_0/2\gamma s^2 = 3.85$ T, for $\gamma = 200$ and $s = 1.5$ nm in the case of Bi2Sr2CaCu2O8 + δ, where all block layers (in-between CuO2 layers) are occupied by the Josephson vortexes [93–96]. In the vicinity of $T_c$ (a few degree of Kelvin below $T_c$), the Josephson vortex lattice is predicted to melt with the first order transition at a field, $H^* = \phi_0/2\sqrt{3}\gamma s^2$, above which Koserlitz-Thouless transition may be realized [95,96]. These theoretical predictions are certainly very intriguing, and it provides the most challenging topics to study. Along with this line the phase diagram and
the resistivity, $\rho_{ab}$, in bulk size of samples related to this topic have briefly been discussed in Section 2.

When the dimension of the sample is reduced and becomes the same order of the size or somewhat larger than the Josephson vortex length $\lambda_J = \gamma s \sim 3000$ nm, confinement of the Josephson vortex in-between the layers becomes more stringent and significant. In Fig. 18, the actual intrinsic Josephson junction made of single crystal Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ by FIB (focused ion beam) is shown as an example. Here, the junction area, $S = w \times t = 4.4 \times 9.5$ $\mu$m, and the thickness, $t = 150$ nm, which has approximately 100 layers of Josephson junctions. In this case, the Josephson vortex system profoundly finds more favorable conditions energetically, depending on the number of Josephson vortices in the layers. This can be regarded as the matching effect to the size of the Josephson junction, similar to the Little–Parks effect [3]. Because of this matching effect, it is expected that the Josephson current flow crossing the junction may be oscillating as a function of total magnetic flux penetrating through the junction [11,12], similar to the Fraunhofer pattern in the case of the single Josephson junction [97], where the critical current, $I_c$, of the junction oscillates as a function of magnetic flux penetrated through the junction as

$$I_c(\phi) = I_0(0) \left| \frac{\sin \frac{\phi}{\phi_0}}{\phi/\phi_0} \right|,$$

where $I_0(0)$ is the critical current of the junction at zero magnetic field, $\phi = BS$ is the magnetic flux penetrated through the junction, and $\phi_0 = 2.07 \times 10^{-7}$ G cm$^2$.

It is noted that the critical current becomes zero when the magnetic flux penetrated through the junction becomes exactly the integer times $\phi_0$. Similarly, it is expected that the critical current oscillates and has a minimum at the magnetic field, which satisfies the relation of $nH_0 = \phi_0/\omega$, where $n$ is an integer and $\omega$ the width of the junction perpendicular to the magnetic field as shown in Fig. 18. Actually, we measure the flux flow resistance (impedance) $\rho_c$, not the critical current, as a function of applied magnetic field, so that a maximum of

Fig. 16. The Pearl vortex observed in an extremely thin-MoGe amorphous film of 30 nm in thickness. Left panel: a direct scanning SQUID microscope image of the pearl vortex, right panel: the magnetic field profile of the pearl vortex shown in the left panel.

Fig. 17. The effective magnetic penetration depth $\Lambda$ as a function of the film thickness $d$ measured at 4.5 K. The bulk value extrapolated to $d \rightarrow \infty$ is $\Lambda \sim 550$ nm.

Fig. 18. A photograph of the intrinsic Josephson junction of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ fabricated by FIB. The size of the junction shown here is the width $w = 4.4$ $\mu$m, the thickness $t = 0.15$ $\mu$m, and the length $\ell = 9.5$ $\mu$m. The thickness of 0.15 $\mu$m corresponds to 100 layers of CuO$_2$ plane in the junction. A magnetic field is applied along with $\ell$. 
the resistivity should satisfy the relation of \( nH_0 = \phi_0/\omega s \), which corresponds to the minimum critical current.

In Fig. 19(a), a beautiful example of the oscillating c-axis Josephson vortex flow resistance with a period of \( H_0 = 2.49 \, \text{kOe} \) is shown as the field is swept slowly as a function of field intensity. This phenomenon is similar to that previously reported [98]. The period of the oscillation can exactly be calculated by the matching condition as \( H_0 = \phi_0/\omega s \), where \( w = 5.5 \, \mu\text{m} \) and \( s = 15 \, \mu\text{m} \). This is another strong evidence that the CuO\(_2\) double layers work as real atomic scale Josephson junctions, experimentally proving existence of intrinsic Josephson junctions in Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_{8+\delta}\).

Taking a look at the data shown in Fig. 19(a) more carefully, one notices that the periodic oscillation of \( \rho_c \) with \( H_0 \) occurs only above about 4 T. Below it there is a field region, \( \sim 10 \, \text{kOe} < H < \sim 33 \, \text{kOe} \), where \( \rho_c \) oscillates with a double frequency. Furthermore, the minima occur at the field of \( nH_0/2 \), even \( n \) is even number, where the maxima are observed in \( H > \sim 2 \, \text{T} \). This means that the critical current is reversed while crossing a field region of 3.5–4 T and has maxima instead of minima at the field of \( H_0/2 \). This fact may imply that the double layers become a set of the junction as if it is the single junction, simply because the area of the effective junction area becomes two times greater, as suggested previously [12,99]. It is noted here that the crossing over region depends on the sample size as seen in Fig. 19(b), where it occurs at about 1.5 T in a smaller junction. Although this double layer composite model may be applicable to understand the double frequency of the period, one must find the deeper reason for that. Perhaps, the simplest model may be to imagine the triangular lattice model of the Josephson vortex arrangement, where the periodicity along the c-axis becomes double, as already suggested previously [12,99]. However, one still cannot explain the reversal of the peak of \( \rho_c \) as long as only the static Josephson vortex configuration as a rigid body is considered. It seems that the dynamical effect of Josephson vortices is essential to account for the full characteristics of the oscillatory behavior of \( \rho_c \) as a function of \( H \). This is also very suggestive to account for the recent result of calculation of the single Josephson junction, where the dynamical impedance shows an oscillatory behavior with half quantum flux per junction, corresponding to the \( H_0/2 \) period, especially in the lower field region [100].

Finally, we add some more characteristic features found in our experiments associated with the Josephson vortex flow phenomena. Firstly, it was found that the amplitude of the oscillation of the flow resistivity \( \rho_c \) strongly depends on the current. The amplitude of the oscillation tends to become larger as the current is reduced. This means that the dynamical impedance is highly non-linear, indicating non-linear dynamics of Josephson vortex motion is essential for understanding the phenomena, and is the characteristic feature of the Josephson junction system [9].

![Fig. 19. An example of the c-axis Josephson vortex flow resistance in Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_{8+\delta}\) as a function of magnetic field applied parallel to the CuO\(_2\) plane: (a) The oscillatory behavior with a period of \( H_0/2 \) at lower fields (\( \sim 0.7 \, \text{T} < H < \sim 1.7 \, \text{T} \)) shifts gradually to the one with a period of \( H_0 = 2.49 \, \text{kOe} \) at higher fields (\( H > \sim 2 \, \text{T} \)). Notice that the peak and dip positions are inversed in lower and higher field regions. The junction size is \( \ell = 41 \, \mu\text{m}, w = 5.5 \, \mu\text{m} \) and \( t = 2.0 \, \mu\text{m} \); (b) The similar oscillatory behavior of \( \rho_c \) with the dimensions of \( \ell = 11.7 \, \mu\text{m}, w = 1.9 \, \mu\text{m} \) and \( t = 0.5 \, \mu\text{m} \). The \( H_0 \)-value is estimated to be 7.7 kOe in this case.](image1)

![Fig. 20. The principle of a THz generator is shown schematically by dynamical motion of Josephson vortices.](image2)
Secondly, the highly non-linear current–voltage relation (I–V characteristics) shows very peculiar stepwise oscillation. This is more clearly seen in the dynamic differential impedance, which suggests the Fiske-like behavior in the multi-stacked Josephson junction [9,101]. This means that the motion of Josephson vortices excite Josephson plasma waves in a junction, as shown in Fig. 20. Here, we only show schematically the principle of operation of the THz generator. The frequency estimated from the Fiske steps is order of 0.2–0.5 THz, depending on the sample preparation conditions. Since, Josephson plasma waves are identical with the electromagnetic waves in superconductors as shown in our previous work, this provides us an extremely important evidence for the generation of THz electromagnetic waves for applications. This possibility has recently been proposed by Tachiki by using the earth simulator, an ultra-fast supercomputer in RIST [15]. As for more detailed results it will be published elsewhere separately.

Thirdly, we have not mention about the dynamical lock-in behavior of the Josephson vortices in the small junctions in Bi\textsubscript{2}Sr\textsubscript{2}CaCu\textsubscript{2}O\textsubscript{8+δ}, though we have explained the phase diagram in the case of the bulk sample in the previous chapter [51]. Our experimental results of \( \rho_c \), as a function of angle, \( \theta \), show the sharp lock-in behavior: the lock-in angle width, \( \Delta \theta \), becomes wider in shorter junctions, obeying the relations as \( \Delta \theta \propto 1/\sqrt{\xi} \) and \( \propto 1/H \), and does not depend much on \( \omega \) and \( t \) [16]. These behaviors can be ascribed reasonably well by the competing two energies involved in the phenomena; one is the energy, which confines the Josephson vortices into the layers (which is the origin of magnetic torque), and the other is the energy to create pancake vortices by making Josephson vortices penetrate through the superconducting layers. The energy cost due to the confinement is simply reduced in shorter junctions. Further detailed experimental observations and their interpretations will be given in coming publications.

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