Mathematical modeling of the temperature distribution under the cathode spot of the vacuum arc

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Abstract. We present a solution to the problem of the temperature distribution under the cathode spot of taking into account melting and spare deposits of metal, brought to boiling temperature on the surface of the cathode spot. The process of heat transfer in the metal is described by the unsteady three dimensional heat conduction equation in Cartesian coordinate system. Similarly, we present a solution to the problem of the temperature distribution in the presence of the pores in the surface layer of the metal. To solve this task we used a numerical method to finite differences and variable directions. We present the calculated data on the distribution of temperature under the cathode spot for different values of spot diameters and speeds its movement.

1. Introduction
Vacuum-arc treatment of surface of materials or surface treatment of the cathode spots of vacuum arcs – the direction of scientific research and practical applications, which in recent years actively developing. In addition to the traditional applications of vacuum-arc treatment to clean the surface of metals and alloys from scale, rust, oil films, technological sediments and other contaminants with the purpose of flaw detection in metal or for the subsequent application of coatings, and new applications. Hardening of the surface layer of metals, the change of physic-chemical characteristics of the surface layer of structural materials, restoration of the plastically deformed surface shape of products, cleaning of surfaces prior to their welding to obtain qualitative weld, the creation of a given surface forms, for example, to retain oil films on the surface, creating on the surface passivation films for corrosion protection – is not a complete list of modern technology using vacuum-arc surface treatment [1–4].

One of the main parameters determining the efficiency of technological processes to the surface by the cathode spot of vacuum-arc discharge is the temperature near the cathode spot and the average temperature of the surface during its movement.

2. Mathematical modeling of temperature distribution
When solving the problem about the temperature distribution under the cathode spot of taking into account melting and evaporation of metal, brought to boiling temperature on the surface of the cathode spot, take into account that the movement of the cathode spot is a intermittent nature. Some time, the spot remains in one place and then jumps to the new plot surface, separated from the initial one or two radius of the spot. Numerically was solved the problem of heat conduction in Cartesian coordinate system for the computational domain presented in figure 1.
The process of heat transfer in the metal is described by the unsteady three-dimensional heat conduction equation in Cartesian coordinate system as:

\[ \rho c_p \frac{\partial T}{\partial t} = \lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right), \]  

(1)

where \( \rho \) is the material density; \( \lambda \) is thermal conductivity coefficient; \( c_p \) is the heat capacity; \( T \) is the temperature; \( x, y, z \) – coordinates.

Since the task has a plane of symmetry, the solution was carried out in half the expected volume. The cathode spot is moved along the plane B (figure 1). The plane A was chosen as the plane of symmetry and on it was set the boundary conditions of symmetry, i.e. no flow through the surface:

\[ \lambda \frac{\partial T}{\partial s} \bigg|_A = 0. \]  

(2)

Figure 1. An example of the calculated region.

The surface B faces towards the vacuum and it was set to a boundary condition of radiation:

\[ \lambda \frac{\partial T}{\partial s} \bigg|_B = \sigma \varepsilon \left( T_{\infty}^4 - T^4 \right), \]  

(3)

where \( T_{\infty} \) – ambient temperature; \( \sigma \) – Stefan–Boltzmann constant; \( \varepsilon \) – is the integral coefficient of radiation.

The effect of cathode spots was recorded as heat flow in the metal surface on the surface area B. The heat flow is proportional to the voltage \( U \) on the vacuum arc, to the current intensity \( I \) and to the power cathode coefficient \( h_k \), then:

\[ \lambda \frac{\partial T}{\partial s} \bigg|_S = \sigma \varepsilon \left( T_{\infty}^4 - T^4 \right) + \frac{UIh_k}{S}, \]  

(4)

where \( S \) – is the area of the cathode spot.

Considering that the size of the computational domain much larger than the size of cathode spots and the surface C (figure 1) sufficiently far from the heat source, then \( T_{\infty} = T_0 \), where \( T_0 \) – the initial temperature of the metal.

When solving the problem was taken into account melting and evaporation of metals under the action of a cathode spot. To account of these phenomena consider the phase transition from the solid to the liquid state. The transition from liquid to gaseous phase is carried out similarly. The transition into the liquid phase is carried out when the temperature of metal the melting temperature \( T_{pl} \). In addition, for the transition it is necessary to spend certain amount of thermal energy \( Q_{pl} \). The value of the specific heat of melting is defined as \( \lambda_{pl} = Q_{pl}/m \), where \( m \) – is the mass of the consumable substance.
The solution of equations was performed numerically. The computational domain was divided into a differential three-dimensional grid in the directions $x$, $y$, $z$. To solve these tasks, the numerical method of finite differences and variable directions was used.

Figure 2 presents a picture of the thermal field surrounding the crater of the cathode spot, at a current arc $I = 100$ A, the cathode spot radius $r = 5 \cdot 10^{-5}$ m, the speed of movement of the cathode spots of $V = 1$ m/s.

In numerical experiments, we have investigated the temperature distribution under the cathode spot at the moment when the cathode spot moves to a different place. This point corresponds to the maximum temperature under the cathode spot, which is approximately equal to the boiling temperature of the metal. As variable parameters, we took the radius of the cathode spot, the velocity of the spot, and the current of the vacuum arc.

Table 1 presents the calculated values of the residence time of the cathode spot on one place on its radius and speed.

| $V$, m/s | $r$, m   |
|----------|----------|
| $5 \cdot 10^{-5}$ | $1 \cdot 10^{-5}$ | $3 \cdot 10^{-6}$ |
| $1$      | $5 \cdot 10^{-5}$ | $1 \cdot 10^{-5}$ | $3 \cdot 10^{-6}$ |
| $10$     | $2.5 \cdot 10^{-6}$ | $5 \cdot 10^{-7}$ | $1.5 \cdot 10^{-7}$ |
| $100$    | $1 \cdot 10^{-6}$ | $2 \cdot 10^{-7}$ | $6 \cdot 10^{-8}$ |

The calculated temperature distribution from the surface under the cathode spot is presented in figure 3. Surface in this case is considered to be the deepest point of the molten hole.

The cathode spot of vacuum-arc discharge moving along the surface of the cathode, it affects how a localized, highly concentrated source of energy. The temperature of the material at the surface in the spot area, as a rule, exceeds the temperature boil. If the speed of tens meters per second the heat affected zone in the metal around the cathode spot is small. Calculations show that the temperature distribution from the surface under the cathode spot is plummeting conductive and at a distance equal to the radius of the spot, the temperature approaches to the temperature of the bulk metal. In the presence on the cathode surface of thick oxide films in the form of, for example, scale or rust, when the cathode spot is used to clean these films, the speed of movement of the spots decreases significantly and may reach units of centimeters per second. The zone of thermal action on the metal is greatly increased. In the presence of near surface layers of metal cavities in the form of pores, cracks, etc. heat sink from the zone of the cathode spot to a mass of metal is deteriorating, the cathode spot can thin melted surface wall and open surface cavity. Such situations often occur in a vacuum-arc cleaning the surface of various products, for example, if a poor quality casting (figure 4).
This figure represents a photograph of the interior surface of the housing axle box truck of a railroad car, which are made with the help of technology of the “ground” cast. Cleaning was carried out on the installation in which the axle box housing, having a cylinder form, which acted as vacuum chamber. Vacuum arc cleaning has been used as an effective method of flaw detection of the metal.

Consider the amount of metal with sides A, B, C, within which cavity are D (figure 5). The cathode spot moves on the surface B, which is in a vacuum.

Figure 3. The temperature vs. distance from the surface under the cathode spot in different speeds of motion of cathode spots ($I = 300$ A, $r = 10^{-4}$ m): (a) – $V = 10$ m/s; (b) – $V = 1$ m/s.

Figure 4. Uncovered near the surface the pore as a result of vacuum-arc cleaning of axle box housing for railway car.

Figure 5. An example of the calculated region with a pore.

For the computational domain (figure 5) was numerically solved the problem of heat conduction. The power fed from the discharge to the cathode (the surface) is proportionality to current of vacuum arc $I$, the voltage drop across the discharge $U$ and the coefficient of cathode power $h_k$. The process of heat transfer in the metal is described by three-dimensional non-stationary heat conduction equation in Cartesian coordinate system. From surface power is given also by radiation to the side of the vacuum volume. On the surface of the cavity D was defined the condition of radiation:

$$
\lambda \frac{\partial T}{\partial s} \bigg|_{id} = \sigma \epsilon \left(T_{av}^4 - T^4 \right),
$$

(5)

where $T_{av}$ – the average temperature of the cavity surface.
Were taken into account melting and evaporation of metals under the action of a cathode spot. The solution of equations was performed numerically. Was studied, in that the distance $h$ a cavity inside the metal may be situated, so the cathode spot could have melted through the metal, separated the surface from the cavity, during its stay on one place. Figure 6 shows an example of calculation of fields of temperature and depth of melting of the metal under the cathode spot.

![Figure 6](image)

Table 2 presents the results of calculations of maximum depth of metal, which the cathode spot is melts, while over at cavity, from speed of movement of the spot and its radius.

| V, m/s | $r$, m | $5 \times 10^{-5}$ | $1 \times 10^{-5}$ | $3 \times 10^{-6}$ |
|-------|--------|------------------|------------------|------------------|
| 1     | $1 \times 10^{-4}$ | $7 \times 10^{-5}$ | $4,5 \times 10^{-5}$ |
| 20    | $3,2 \times 10^{-5}$ | $2,4 \times 10^{-5}$ | $1,7 \times 10^{-5}$ |
| 50    | $2 \times 10^{-5}$ | $1,8 \times 10^{-5}$ | $1,3 \times 10^{-5}$ |

3. Conclusions
The results of the calculations were used to assess the efficiency of some technological processes in the design of the equipment.

References
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