Wavelet packet transform-based time of arrival estimation method for orthogonal frequency division multiplexing ultra-wideband signal

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Abstract: This study discusses the estimation of time of arrival (TOA) used for ranging and positioning applications in orthogonal frequency division multiplexing ultra-wideband- (OFDM-UWB) based system. The correlation-based method applied for impulse radio-UWB (IR-UWB) is not suitable for OFDM-UWB because OFDM-UWB signal is much longer than IR-UWB in time domain. This study proposes a new method for TOA estimation of OFDM-UWB signal based on wavelet packet transform (WPT). The author’s method estimates TOA of OFDM-UWB signal through the time information of the frequency band of UWB, which is one of the branches of WPT decomposition of the oversampled received baseband signal. It does not need the whole OFDM signal to estimate TOA of UWB signal because every segment of OFDM-UWB signal in time domain contains all information of UWB frequency band; therefore the resolution of TOA is improved. Signal-to-noise ratio can also be improved because WPT decomposes the power of additive white Gaussian noise equally in all wavelet packet branches, and false alarm rate can be controlled. The author’s method is robust in that it can counter narrow band noise and impulse noise effectively. Numerical results show our method is effective for accurate estimation of TOA for OFDM-UWB signal.

1 Introduction

Ultra-wideband (UWB), as a promising technology in short-range high-speed wireless communication, has many good features including low-power consumption and capability of ranging. UWB is proposed as wireless sensor network physical layer standard because it can combine low-power communication and accurate positioning [1], the latter relying on precise ranging. So far, most UWB ranging schemes realise ranging by estimating the time of arrival (TOA) of UWB pulse because UWB pulse is very narrow, about nanosecond scale, in time domain and suitable for correlation-based method to realise precise estimation of TOA [2–4].

However, for orthogonal frequency division multiplexing ultra-wideband (OFDM-UWB) system, it is not possible to acquire the same resolution of TOA as impulse radio-UWB (IR-UWB) system by using the same method since the symbol of OFDM-UWB is much longer than IR-UWB pulse in time domain. For OFDM-based UWB systems, positioning is mainly based on received signal strength (RSS) method [5, 6]. Ranging of the RSS method must be based on a channel model, which describes the RSS with the propagating distance. The exact relation between distance and signal energy in a practical wireless environment is quite complicated because of propagation mechanisms such as reflection, scattering and diffraction, especially for complex indoor environment of UWB. Commonly, the RSS technique cannot provide accurate range estimates because of its heavy dependence on the channel parameters [1].

Moreover, because multipath effect is very severe in indoor environment, the first path signal may not be the strongest signal. Accordingly, improving the signal-to-noise ratio (SNR) is very important to find the first path signal and reduce false alarm rate.

This paper proposes a method based on wavelet packet transform (WPT). First, the received baseband signal is oversampled. Then the discrete signal is decomposed into several frequency bands with WPT. Frequency band of UWB signal lies in the first wavelet packet branch. TOA estimation is made according to the time information of UWB frequency band. This method can improve the accuracy, which is dependent on the time resolution of WPT coefficients. This paper shows that the resolution is the reciprocal of bandwidth of UWB signal. This method can improve the SNR, and hence reduce the false alarm rate, because the power of noise distributes equally in every branch. This method can control the false alarm rate with reference to non-UWB branches, which obey Gaussian distribution. In addition, another advantage of this method is the robustness of resistance against noise. This method can eliminate the effect of the narrow band noise and impulse noise (wide band noise).

The paper is organised as follows. Section 2 describes a system model. Section 3 presents our method. In Section 4,
numerical simulation results are presented and analysed for IEEE indoor channel model. Finally, Section 5 concludes the paper.

2 System model

Fig. 1 illustrates the scheme of our method, in which the TOA of OFDM-UWB signal is the parameter to be acquired for ranging applications. OFDM-UWB signal experiences multipath arriving at reference node with additive white Gaussian noise (AWGN). Then, the baseband of received signal is oversampled. After oversampling, the signal is decomposed into multiple frequency sub-band by WPT, which can provide high resolution in both time domain and frequency domain. TOA of UWB signal is estimated by the time information of UWB frequency band, which is one of the branches of WPT. Here, only AWGN is taken into consideration. Discussion in Section 5 shows that our method works well when there are narrow band noise and impulse noise.

In OFDM systems, $N$ data symbols $X_k$, $k = -(N/2)$, $-(N/2) + 1$,

\ldots, $(N/2) - 1$ are modulated on a set of $N$ orthogonal subcarriers, and the time-domain signal $x(t)$ may be written as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=-(N/2)}^{(N/2)-1} X_k e^{j2\pi kt/T}, \quad 0 \leq t \leq T \quad (1)$$

where $N$ is the number of subcarriers, and $T$ is the OFDM symbol period. Samples of $x(t)$ are efficiently computed via an inverse discrete Fourier transform (IDFT), that is

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=-(N/2)}^{(N/2)-1} X_k e^{j2\pi nk/N} \quad (2)$$

$x(t)$ is obtained from $x_n$ via digital-to-analogue conversion [7]. If the bandwidth of the UWB signal, BW, is 1 GHz, $T$ will be $N$/BW, that is, $N$ nanoseconds. If the number of subcarriers is 128, the length of OFDM-UWB signal will be 128 ns, much larger than 1 ns, which is not suitable for the correlation-based TOA estimation as IR-UWB.

The UWB channel model adopted in this paper was developed for wireless personal area network by IEEE 802.15.3a task group. The model includes path loss model and multipath model to reflect the characteristics of indoor environment, the former based on free space path loss model and the latter based on modified Saleh-Valenzuela (S-V) model [8].

The received signal at reference node is

$$r(t) = h(t) \ast x(t) + n(t) \quad (3)$$

where $h(t)$ is the channel response, $x(t)$ is OFDM-UWB signal and $n(t)$ is zero-mean white Gaussian noise. Mathematically, the impulse response of IEEE 802.15.3a is described as

$$h(t) = X \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_{nk} \delta(t - T_n - \tau_{nk}) \quad (4)$$

where $X$ is the log-normal shadowing, $N$ is the number of clusters, $K$ is the number of multipaths in cluster $n$, $\alpha_{nk}$ is the multipath gain coefficient, $T_n$ is the delay of the $n$th cluster and $\tau_{nk}$ is the delay of the $k$th multipath relative to the $n$th cluster. The arriving of cluster and multipaths in each cluster are Poisson processes with different arriving rates. Power of clusters decays exponentially with cluster delay, and power of multipaths in each cluster also decays exponentially with delay. The distribution of power obeys a double exponential distribution.

3 OFDM-UWB signal detection via WPT

From (2), time-domain signal at any point contains all frequency information in UWB band. Wavelet analysis is a kind of time–frequency analysis that can provide the time information of specific frequency band. WPT can provide good resolution both in frequency and time domain. Our method estimates TOA of UWB signal by using the time information of UWB frequency band.

3.1 Generation of OFDM-UWB signal for detection

The WPT coefficients vary with time because at different timepoints the power of OFDM signal is different. To illustrate this point, a brief introduction to discrete wavelet transform is given.

The wavelet transform of $x(t)$ is

$$c_{jk} = \int_{-\infty}^{+\infty} x(t) \psi_{jk}(t) dt = x, \psi_{jk} \quad (5)$$

where $\psi_{jk}(t)$ is $a_0^{(j/2)} \phi(t - k_0 b_0) / \sqrt{d_0}$, $d_0 = a_0^{(j/2)} \phi(a_0^{(j/2)} t - k_0 b_0)$, $j = 1, 2, 3, \ldots$; $a_0 \in \mathbb{R}^+$ and $b_0 \in \mathbb{R}$ are, respectively, the scaling and translation parameters. The functions $\psi_{j,k}$ generated from $\phi(t)$ by the operation of dilation and translation, form a basis in $L^2(\mathbb{R})$. Here, $\phi(t)$ is called a mother wavelet which satisfies the following admissibility condition [9]

$$\int_{-\infty}^{+\infty} \frac{\Psi(\omega)}{|\omega|} d\omega = 0, \quad \int_{-\infty}^{+\infty} \frac{\Psi(\omega)}{|\omega|^2} d\omega = C_\phi < +\infty \quad (6)$$

where $\omega$ is the frequency of the signal, and $\Psi(\omega)$ is the Fourier transform of $\phi(t)$.

Fig. 2(a) is an OFDM signal of time acquired by fast Fourier transform (FFT) computation of quadrature phase shift keying (QPSK) modulated 256 bits which are generated randomly. Apparently, power at the start of the OFDM-UWB symbol is not the largest.

From (5), wavelet coefficients at specific timepoint depend on the value of $x(t)$ at that timepoint. It is desired that power at the start of OFDM-UWB is as large as possible, which needs appropriate signal design.

According to FFT theory, signals in time domain and frequency domain are all periodic. Shift in time domain just introduces phase delay into frequency domain, and does not

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**Fig. 1** Diagram of the scheme
change power spectrum density, which makes it possible for us to design the OFDM signal for detection. First, OFDM signal is generated with FFT operation as normal OFDM-UWB communication. Then, shift in time domain makes the start of OFDM signal the position of the highest power as shown in Fig. 2.

OFDM-UWB signal uses a large number of subcarriers because of its large bandwidth. The number of subcarriers has a great influence on the decrease of the side lobes. The side lobes of the complete OFDM spectrum show a steeper decay and the spectrum comes closer to a rectangular shape with increasing number of subcarriers [10]. The energy of OFDM-UWB signal can be considered concentrated in its bandwidth.

3.2 Improvements of SNR and accuracy

If the sampling frequency is $f_s$, then the period in frequency domain is $f_s$. When the number of levels of WPT is $l$, the frequency band from 0 to $f_s$ is decomposed into $2^l$ sub-bands after $l$ levels WPT. The $2^l$ frequency bands have the same bandwidth, as shown in Fig. 3. That is [9]

$$\left[\frac{(m-1)f_s}{2^l}, \frac{mf_s}{2^l}\right], \ m = 1, 2, \ldots, 2^l; \ l \in \mathbb{R} \quad (7)$$

where $f_s$ is the sampling frequency, and $m$ denotes the $m$th frequency band.

If the ratio of sampling frequency to bandwidth of OFDM-UWB signal is equal to $2^l$, OFDM-UWB signal only occupies the whole first sub-band, that is, 0 to $(f_s/2^l)$, after WPT on oversampled received signal.

Sampling frequency is decided by the number of levels of WPT and bandwidth of UWB signal, that is

$$f_s = 2^lf_u \quad (8)$$

Here, $f_u$ is the bandwidth of UWB signal. The received baseband signal is oversampled by power of 2, such as 4 and 8 after analogue-to-digital conversion. Then the discrete signal is decomposed by WPT into branches of power of 2, as shown in Fig. 3.

The addition of zero-mean Gaussian noise to the original signal has the effect of adding zero-mean Gaussian noise to the wavelet representation as well [11]. After WPT, the power of AWGN is distributed equally in $2^l$ sub-bands. Thus, SNR is improved by $2^l$ times compared with SNR in time domain through oversampling and WPT.

Fig. 4 illustrates a three-level wavelet packet tree structure. The sampling rate of WPT coefficients in different levels decreases with the level number as $f_s/2^m$, where $m$ is the level number of WPT tree.

The accuracy of TOA is dependent on the time resolution of WPT coefficients. The resolution of WPT coefficients in $l$th level satisfies the following equation

$$t_l = \frac{2^l}{f_s} = \frac{f_u}{f_s} \quad (9)$$

That is the reciprocal of bandwidth of UWB signal. Taking three-level WPT of 500 MHz UWB signal as an example, the time resolution is 2 ns for WPT coefficients after eight times oversampling and three-level WPT.
3.3 Detection of UWB signal

The existence of UWB signal is judged by analysing the WPT coefficient of UWB branch, with non-UWB branches as reference because WPT of zero-mean Gaussian noise is still zero-mean Gaussian [11]. After WPT, assuming there is no UWB signal, the coefficients in UWB branch will obey Gaussian distribution with variance information provided by non-UWB branches. The occurrence probabilities of the WPT coefficients obey Gaussian distribution. If the probability is lower than a threshold, we can say the signal is Gaussian noise with a probability less than the threshold. Accordingly, the signal is probably UWB signal with a certain false alarm rate corresponding to the threshold. Different thresholds of probability correspond to different thresholds of WPT coefficients. For example, with the variance of WPT coefficients of AWGN being \( \sigma^2 \), if the absolute value of WPT coefficient is \( > 2.58 \sigma \), there exists UWB signal with false alarm rate of 1% from Gaussian distribution. The false alarm rate can be calculated and controlled by setting different thresholds according to Gaussian distribution.

\[
\text{Prob}_{d} = \text{Prob}(|u + g| \geq T_h) = \text{Prob}(u + g \geq T_h) + \text{Prob}(u + g \leq -T_h)
\]

where \( u \) is the WPT coefficients of UWB signal and \( g \) is the WPT coefficients of AWGN, \( T_h \) is the threshold of WPT coefficient for certain false alarm rate.

\[
\text{Prob}(u + g \geq T_h) + \text{Prob}(u + g \leq -T_h) = \text{Prob}(g \geq T_h - u) + \text{Prob}(g \leq -T_h - u)
\]

\[
= 1 - \text{Prob}(-T_h - u \leq g \leq T_h - u)
\]

Take \( T_h = 2.58\sigma \) as an example

\[
\text{Prob}_{d} = \text{Prob}(g \geq 2.58\sigma - u) + \text{Prob}(g \leq -2.58\sigma - u) = 1 - \text{Prob}(-2.58\sigma - u \leq g \leq 2.58\sigma - u)
\]

From (12), the probability of detection increases with \(|u|\) because \( g \) obeys Gaussian distribution. When \(|u| \to \infty\), \( \text{Prob}_{d} \to 1 \). The probability of detection of UWB under different SNR is shown in Fig. 5 when \( T_h = 2.58\sigma \) which corresponds to 1% false alarm rate.

3.4 Robustness of the method

In Section 2, the noise is assumed AWGN. In this part, we will show that this method works well for narrow band noise and impulse noise. It does not need additional complex processing.

For narrow band noise, it does not need narrow band filter to eliminate the narrow band signal. There are two possibilities: (i) the narrow band noise is out of the band of UWB signal; (ii) the narrow band is in the band of UWB signal. In the first case, it does not need any processing except that the reference branch selected does not cover the band of the narrow band noise. In the second case, it needs to change the frequency band of UWB away from the
narrow band noise, which is easy to achieve for OFDM-UWB system.

For impulse noise, there will be peaks in every branch of WPT because its bandwidth is very large, as shown in Fig. 6. The peaks are at the same position, which is the time when impulse noise occurs. If there are such patterns in WPT coefficients, our method is applicable by removing the corresponding points in every branch.

3.5 Implementation

The filter bank analysis algorithm represented in Fig. 7 can be used to compute the wavelet packet coefficients. That is WPT can be implemented by hardware. The algorithm consists of

\[
\begin{align*}
d^{(2l)}_{j+k+1}(n) &= \left[ h_0(n) * d^{(l)}_{j+k}(n) \right]_2 \\
d^{(2l+1)}_{j+k+1}(n) &= \left[ h_1(n) * d^{(l)}_{j+k}(n) \right]_2 \\
k &= 0, 1, 2, 3, \ldots, \quad l &= 0, 1, 2, 3, \ldots
\end{align*}
\]

4 Numerical simulation and analysis

Our simulations include two-level and three-level WPT for 500 MHz UWB signal under different SNRs. The number of WPT levels depends on SNR, accuracy requirement and sampling limitations. For example, if the bandwidth of UWB signal is 500 MHz, it needs 2 GHz sampling rate to realise two-level WPT and 4 GHz sampling rate to do three levels WPT. Two-level WPT improves SNR by four times, whereas three-level WPT improve SNR by eight times. The wavelet used in simulation is db3 in Matlab.

First, the variance of the WPT coefficients of Gaussian noise is computed to acquire the distribution of WPT coefficients. Assume the bandwidth of UWB signal is 500 MHz, the received signal is oversampled eight times of bandwidth of UWB signal and SNR is $-6$ dB. The SNR refers to power ratio between UWB signal and Gaussian noise in the duration of multipaths of UWB signal. The variances of WPT coefficients of branches (3, 6) and (3, 7) are computed and illustrated in Fig. 5, which shows that the variance of coefficients of Gaussian noise is almost the same in all branches of WPT tree structure. One hundred simulations show that the variances of the coefficients vary in a small range. If the number of samples of coefficient gets larger, the fluctuation of variances will be smaller and the fluctuation of variance can be reduced by averaging variances of different branches (Fig. 8).

With the variance of non-UWB branch as reference, the existence of UWB signal is judged by the probability of occurrence under the Gaussian distribution. For Gaussian noise, the occurrence possibility is < 1% if the received signal amplitude is > 2.58$\sigma$ or power is > $(2.58\sigma)^2$. Different thresholds for amplitude of received signal (WPT decomposed) correspond to different false alarm rates. In our simulation, 2.58$\sigma$ is set as threshold for 1% false alarm rate in consideration of the fluctuation of variance, that is, if the absolute value of WPT coefficient is > 2.58$\sigma$, there is UWB signal with false alarm rate being 1%

Figs. 9 and 10 show three-level WPT of received signal with $-6$ and $-3$ dB SNR, respectively. After WPT decomposition, UWB signal is located in branch (3, 0). The power of coefficients of branch (3, 6) is also provided as a reference. The first arrival of UWB signal is at the 0 ns point. If the power of WPT coefficients at 0 point is > $(2.58\sigma)^2$, the first arrival of UWB signal is detected with false alarm rate being 1%

Simulations investigate the detected ratio of UWB signal under different SNR, which are presented in Fig. 11. The detected ratio increases with SNR similar to Fig. 5. From
Fig. 11, if SNR is near 0 dB the probability of detection is about 90%. The SNR of 0 dB in time domain corresponds to SNR of 9 dB in branch (3, 0). The simulation results are very close to theoretical analysis with the SNR improvement taken into consideration. Simulations show that two-level WPT provides less SNR improvement compared with three-level WPT, which can be seen in Figs. 11 and 12.

5 Conclusion

The WPT-based method proposed in this paper can greatly improve the accuracy of TOA estimation in OFDM-based UWB system. WPT decomposes oversampled received signal into multiple sub-bands. UWB signal occupies only one of the sub-bands, whereas Gaussian noise is equally distributed in the all sub-bands. By using our method, SNR in UWB branch is improved times the factor of oversampling, compared with that in time domain, which
can increase the probability of detection of the direct path signal. The TOA resolution is the reciprocal of bandwidth of OFDM-UWB signal, which is nanosecond scale. Thus, our method can realise the same ranging resolution in OFDM-UWB-based system as in IR-UWB-based system. With the reference of non-UWB branches in WPT, the false alarm rate can be easily calculated and controlled. Moreover, this method is very robust in resistance against narrow band and wide band noise.

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7 References

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