The relaxation processes calculation of fish dehydrated surface layer during drying and smoking

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Abstract. In this work the methods of study of relaxation processes that are applicable during fish dehydration based on the influence of the chemical composition, geometric size of the processing object and regime parameters of the dehydration processes are developed. The numerical methods of calculation on the basis of solution of the second-order differential equation with boundary conditions of the third type allow with sufficient for engineering practice accuracy to model the various conditions of the dehydration process conducting with periodic restoration of hydraulic conductivity properties of dehydrating objects. The proposed calculation methods applicable to dehydration processes consisting of a continuous initial period and the following combined periods of drying fish and surface layer relaxation of the object of dehydration. The importance of applying the relaxation is determined by the firming the upper layers which have lost part of the moisture during dehydration. The size of capillaries for the moisture pathway through the upper surfaces reduce. Thus, near the upper layer a free of major moisture mass zone appears, which also has low diffusion properties. As a result, the dehydration process of the entire object slows down. Applying the relaxation enables the hydraulic conductivity features of fish upper layer. During the relaxation the supply of electric power to the heating elements stops. The rate of circulation of the drying agent slows down. The drying installation is supplied with air of lower temperature and higher relative humidity than the drying agent. In the drier such conditions are created that constrain external mass transfer and enables moisture redistribution in the thickness of the fish. During the relaxation the moisture is gradually shifting from the central layers where dehydration has not yet come to the dehydrated surface layers.

1. Introduction
The intensity of mass transfer depends on regime parameters, geometric size and chemical composition of the object. Researchers have observed [1] that the speed of drying agent more than two meters per second does not influence the dehydration rate after reaching the first critical moisture content on the kinetics curves of dewatering. This could be explained by the fact that the process of moisture evaporation from the surface to the drying environment has higher speed than moisture transfer from the center to the fish surface. As the dehydration occurs the fish flesh thickens. At the same time the pore size of the capillaries shrinks by a factor of six – ten. It is more obvious in the surface layer, which verges on the drying agent. In general, the solution of differential equations of heat and mass transfer has been accomplished with boundary conditions of the first type [2, 3], although usually the thermic treatment of fish raw material occurs with boundary conditions of the third type. Many researchers and manufacturers have tried to recover the hydraulic conductivity
features in the fish surface layer by periodic termination the impact on the object of drying agent, by dampening fish surface with the little dispersed water particles or by monitoring the process under the constant coefficient value of water activity (soft drying conditions). The rest recommend to deal with a process by using the harsh drying regimes. According to our opinion, the most perspective one is the hydraulic conductivity features recovery by periodic termination the dehydration process for relaxation the fish surface layer in combination with relatively harsh drying regimes.

2. The main part
2.1. Purpose and objectives of the research
Up to the present moment the relaxation process of dehydrated fish surface has been underexplored. There are no identified dependencies which facilitate reduction in the duration and costs of manufacturing fish products. Having regard to the above, the purpose of this work is to study and define the main dependencies of inner mass transfer of moisture within the processes, which run at small value of Biot number.

In the furtherance of this purpose the following objectives should be achieved:
1. To develop the method of relaxation process studying.
2. To determine the main dependencies, which run at the moisture redistribution inside the object of dehydration.
3. To develop energy efficient methods of fish dehydration in combination with the impact on the object of drying agent and its periodic hydraulic conductivity features recovery.

2.2. The description of relaxation process
The amount of moisture \( dM_1 \), which is transferred from the inner layers of the body to the surface for the time \( d\tau \), could be written down with the help of the Fourier's Law. The moisture transfer from the center of the body to its surface equals:

\[
\frac{dM_1}{dt} = -\lambda_{m1} F \left( \frac{\partial w}{\partial n} \right) dt,
\]

where \( \lambda_{m1} \) – relative coefficient of mass transfer, kg/ (m % sec); \( F \) – the object’s surface, \( m^2 \); \( w \) – moisture content in the body, %; \( \tau \) – duration, sec; \( \left( \frac{\partial w}{\partial n} \right) \) – the derivative in the direction perpendicular to the surface \( F \) (the gradient of moisture), %/m.

Within the process of dehydration the surface layers thicken. That is why the thickened layers offer resistance to the moisture transfer more than other parts of the fish. The moisture transfer there can be described by the following differential equation:

\[
\frac{dM_2}{dt} = -\lambda_{m2} F \left( \frac{\partial w}{\partial n} \right) dt,
\]

Where \( \lambda_{m2} \) – relative coefficient of mass transfer of the dry layer; other symbols are known.

The certain amount of moisture is transferred from the surface to the environment:

\[
\frac{dM_3}{dt} = \beta_m (w_e - w_n) dF dt,
\]

where \( \beta_m \) – relative coefficient of mass transfer from the surface to the environment kg/ (m % sec); \( F \) – the object’s surface, \( m^2 \); \( w_e \) – moisture content in the body, %; \( w_n \) – moisture content in the drying agent, %.

With the absence of drying agent movement in the relaxation period the coefficient \( \beta_m \) essentially decreases, tending to zero. Then \( dM_3 = 0 \). Taking into account that \( dM_1 = dM_2 \), we receive the following equation:

\[
\frac{dM_1}{dt} = \frac{dM_2}{dt} \approx -\lambda_{m2} F \left( \frac{\partial w}{\partial n} \right) dt
\]

Divide the both parts of the equation in \( dF dt \),

\[
-\lambda_{m1} \left( \frac{\partial w}{\partial n} \right) \approx -\lambda_{m2} \left( \frac{\partial w}{\partial n} \right)
\]

As a matter of convenience we pass on to the one spatial coordinate and will not consider the case of unlimited disk in thickness \( 2l \). From the equation (5) it follows that during the process of moisture redistribution the relaxation curves have the common point with a single moisture level, where they intersect. This common point with constant moisture after deep dehydration and before the relaxation
is located near the inner boundary of the “dry layer”. The relative coefficient of moisture transfer \( \lambda m_2 \), as the surface layer absorbs the moisture, increases more remarkably than \( \lambda m_1 \) decreases. In this case the common point will gradually move up and to the right on the curves of dehydration dynamics: from the point A to the point A1 (figure 1). The difference in the dynamics of \( \lambda m_1 \) and \( \lambda m_2 \) change can be explained by the fact that the volume of the dry layer is significantly smaller than the volume of layer with high hydraulic conductivity features. Rotating around the common point of equal moisture level, the left part of dynamics curves of relaxation goes up, while the right part moves down. Further we will call the point of equal moisture level as a rotation point. After a rather long time both of the parts will reach the medium-volume humidity.

Figure 1. The curves of the relaxation of fish dehydrated surface layer: 1 – at the beginning of the process; 2 – at the time of 0.5 hours, 3 – at the time of 1 hour; 4 – at the time of 1.5 hours; 5 – at the time 2 hours; 6 – average humidity level

2.3. The characteristics of the relaxation processes calculation

Mathematical modelling techniques are successfully used in science and in many industries [4-7]. In the earlier works we have developed the method of dehydration processes and moisture relaxation calculation with numerical methods, taking into the account the chemical composition, geometric size of the processing object and regime parameters of the dehydration processes [8, 9]. According to this method, during the relaxation period the calculation of the object’s moisture level is conducted from the center to the surface. Thus, the task of the calculation is to determine the surface’s moisture and to trace the curve of dehydration dynamics. The surface’s moisture calculates with the following equation[6]:

\[
w_s = w - \sum_{i=1}^{n} \frac{w_i}{n},
\]

where \( w_s \) – the humidity of the object’s surface; \( w_i \) – the moisture in the mesh points, apart from the moisture on the surface; \( n \) – amount of intervals, into which the half of object’s thickness is divided; \( w \) – the medium-volume humidity, which is influenced by the following parameters:
where \( N \) – the dehydration speed in the first drying period; \( X_p \) – the hardness of drying regimes; \( w_0 \) – the initial moisture level of the drying object, \( s/m \) – relative surface of the drying object.

One of the conditions of the relaxation process calculation is the presence of the dehydration dynamics curve in the moment of the stopping delivery of the drying agent. Although the dehydration dynamics curves of the drying object, that are traced with the calculation Finite difference method, near-surface layer have a convex downward. This shows that the moisture level on the surface grows faster than in the near-surface layer, so, in this case, the supposition \( dM_3 \) = 0 causes erroneous results when calculating the relaxation process using the numerical methods.

As the near-surface layer moistens from the direction of moisture supply from the central layers, hydraulic conductivity features partly recover and at the same time some moisture evaporates from the surface. That is why the curve of relaxation dynamics should be a concave upward. In order to reduce the moisture measurements errors on the surface of relaxation object it is preferably to use the data calculated by the numerical methods only for tracing the relaxation process using the numerical methods.

With the calculation of the left part of the curve of relaxation dynamics \( R = S \), where \( S \) is the thickness of relaxation zone. For the right part of the curve of relaxation dynamics from the points \( A \) and \( A_l \) the value \( R = l \), where \( l \) is the half of the object’s thickness (figure 1). When \( Fo \geq 0,5 \) it is enough to use only one root \( \mu_i \) of the equation (8). The usage of the equation (8) for the left part calculation from point \( A \) during the relaxation process:

\[
\theta_{(x,i)} = \sum_{j=1}^{n} A e^{-n^2 Fo} \cdot \cos(\mu_i x/R)
\]  

where \( n \) – the amount of evenly-spaced points of curve of relaxation dynamics (\( n = 4 \sim 7 \)); \( A_i \) – the constant coefficient, different for each term of series (which also does not depend on coordinates or time \( \tau \)); \( Fo \) – Fourier number, \( Fo = D \tau / R^2 \), where \( D \) – the coefficient of moisture diffusion; \( \mu_i \) – roots of standard equation:

\[
\csc \mu = \frac{\mu}{Bi}
\]  

Taking into account that the third type boundary conditions are initially included when the numerical methods are used, it is important to include them in the further calculations by trying in the solution of the second-order differential equation the coefficients of relaxation dynamics traced with the numerical methods would match the one with \( \tau_p = \tau_i \), traced on the basis of equations described below.

With the calculation of the left part of the curve of relaxation dynamics \( R = S \), where \( S \) is the thickness of relaxation zone. For the right part of the curve of relaxation dynamics from the points \( A \) and \( A_l \) the value \( R = l \), where \( l \) is the half of the object’s thickness (figure 1). When \( Fo \geq 0,5 \) it is enough to use only one root \( \mu_i \) of the equation (8). The usage of the equation (8) for the left part calculation from point \( A \) during the relaxation process:

\[
\theta_{(x,i)} = \frac{w_0' - w_0}{w_0' - w_{(w)})}, \quad \theta_{(x,i)} = \frac{w_1' - w_0}{w_1' - w_{(w)})}, \quad \ldots, \quad \theta_{(x,i)} = \frac{w_n' - w_{n-1}}{w_n' - w_{(w)})}
\]  

where \( w_0', w_1', w_n' \) – the values of moisture in the rotation points for the initial, first and following groups of relaxation curves (figure 1), which intersect in one point common for the group of curves: \( B_1 \), \( B_2 \), \( B_3 \); \( w_{(w), \tau} \) – desired value of the moisture level in any of its points within time interval equaled \( \tau = \tau_p \) (The common points of intersection of relaxation curves \( B_2, B_3, \ldots, B_n \) are not shown on the fig. 1. They characterize the relaxation processes after relatively deep fish dehydration); \( w_{(w)} \) – the moisture of the object in the corresponding point of the previous relaxation curve.

For the right part from point \( A \) (figure 1) of the curve of relaxation dynamics the desired constituent value in dimensionless form reads as follows:

\[
\theta_{(x,i)} = \frac{w_0' - w_0}{w_{(w)} - w_0'}, \quad \theta_{(x,i)} = \frac{w_1' - w_0}{w_{(w)} - w_1'}, \quad \ldots, \quad \theta_{(x,i)} = \frac{w_n' - w_{n-1}}{w_{(w)} - w_n'}
\]
It follows from the equation (8) that for the relaxation processes of the moisture distribution through-thickness of the “dry layer” for the different time τ the symmetrical curves which increase monotonically from the outer surface of the object’s “dry layer” to its inner layer are typical (figure 1). The tangents sketched towards the curves of relaxation dynamics go through the same point B. If these tangents intersect, then the curves must meet in any point A.

The initial data for calculation were the left and right parts of the curves 1 and 2 (figure 1). These curves have been traced by calculation with Finite difference method. The points on a curve are initial data for finding \( A, \) and \( \mu, \) in the equation (8).

For the relaxation calculation of surface “dry layer” (to the left of point A) the value \( S \) has been used as the determinant of size. The criteria \( Fo \) has been calculated for the left and right parts of the relaxation curves from the equation: \( Fo = D\tau S^2. \) The identification of the potential conductivity coefficients \( D \) (the coefficient of moisture diffusion) has been calculated by the method developed earlier [8, 9].

The value of \( D \) has been identified for the surface “dry layer”. By trying the values of \( A, \) and \( \mu, \) we achieved the close value match, received from the equation (8) as on the first relaxation curve 2 (figure 1) when \( \tau = 0.5 \) hour. Here the third type boundary conditions, which have been taken into account during the process of dehydration calculation with the numerical methods, remain the same in the further calculations. The relaxation duration \( \tau = 0.5 \) hour is the minimum possible time within real production systems in terms of sustainable management system of automatic support of predetermined process conditions. When the dehydration takes longer the curves intersection 1 and 2 (point A) is located within the inner boundary of near-surface “dry layer”.

Then the tangent was sketched to the left part of the curve 1. On the tangent the point B has been marked where the tangent and vertical line (which passes through the point A) intersect. Tangents sketched to two – three relaxation curves intersect in point B, while the curves 1, 2 and 3 pass through the point A (the rotation point of relaxation curves). Tangents sketched to the curves 4 and 5 have a different intersection point B1 and, as a result the curves themselves intersect in the point A1, which is located above the point A.

The moisture values \( w'_{10} \) and \( w'_{11} \) in the points B and B1 (figure 1) are the initial values when calculating the dimensionless value \( \theta. \) When the process of relaxation with duration of 2,5-3 hours is calculated, it is possible that the amount of points “B” would increase up till three or four. In that case the calculating of \( \theta \) the smaller value of \( w'_{10} \) in the point B is substituted by the larger value in points B1 or B2, or even B3. Thus, the rotation point moves up and to the right to the points \( A2, A3 \) (which are not shown on the fig. 1).

In the reality the turning point A smoothly goes up and to the right as the relaxation goes on (from the point A till point A1). The less ratio \( w_{cr}/w_2 \) (a ratio of average moisture level to the moisture of the second critical point) or \( w_{cr}/w_0 \) (a ratio of average moisture level to the initial moisture of fish), smaller is difference between the value of ordinates of \( AI \) and \( A \) point and vice-versa.

At high values \( w_{cr}/w_2 \) or \( w_{cr}/w_0 \) the significant receding of point A1 from A could be observed. This could be explained by the fact that with relatively high humidity while relaxation hydraulic conductivity features of the near-surface layer recover faster, so the relaxation process occurs under the smaller values of \( \Delta w = w'_{10} - w'_{1} \), gradually achieving the state of equilibrium in both right and left parts of the curve of relaxation dynamics away from point A.

With an infinitely long relaxation time, the curves of relaxation dynamics reach the average moisture content of the drying object. The tangent of the angle \( \alpha \) of tangential slope (figure 1) is

\[
\tan \alpha = \frac{BIC}{FC} = \frac{\delta_{\theta} \theta}{\delta \theta_{\theta}}
\]

On the other hand, by continuing the tangent to the left of the point F, it is obvious that \( \tan \alpha = \tan \varphi \), \( \tan \varphi = (w_f - w_0) / S \), where \( w_f \) is the water content on the fish surface (figure 1).
For the case when all the moisture passing from the center to the surface completely evaporates into the environment, the basic equation of mass transfer in a differential form looks as follows:

\[-\lambda_m (\partial w / \partial x) = b_m (w_j - w_u) \quad \text{or} \quad \lambda (\partial w / \partial x) = \nabla \phi = (w_j - w_u) / S = b_m / \lambda_m (w_j - w_u).\]

It means that

\[S = \beta_m / \lambda_m.\]

Thus,

\[-(\partial w) / \partial x \bigg|_{x=1} = (w_j - w_u) / (\lambda_m / \beta_m) = (w_j - w_u) / Bi / t\]

The last expression is allowed to find the Biot criteria by calculation for both the processes of dehydration and the processes of relaxation of moisture. Consider an option in which most of the moisture remains on the left side of the sample to restore the moisture-conducting properties of the latter, and some of it evaporates into the environment. Define the proportion of moisture evaporated from the surface: \(a = dM_3 / dM_2\). In this case, the basic equation of mass transfer in differential form takes the following form:

\[\alpha \lambda_m (\partial w / \partial x) = b_m (w_j - w_u)\]

After small transformations, we get the following expression:

\[-(\partial w) / \partial x \bigg|_{x=1} = (w_j - w_u) / (\alpha \lambda_n / \beta_n) = (w_j - w_u) / Bi / (a \cdot t)\]

The last expression can be used to calculate the criterion \(Bi\), when the value of the coefficient of moisture exchange \(\beta_m > 0\). Figure 2 shows the calculation of the relaxation process in accordance with the methodical approach outlined above. The results of calculation using numerical methods almost coincide with the analytical solution of a second-order differential equation with third-kind boundary conditions. The nature of the relaxation dynamics curves constructed using an analytical solution of a second-order differential equation corresponds to that found by constructing curves on the basis of an experiment.

![Figure 2. Dynamics curves of dehydration and relaxation of the flounder-ruff (Hippoglossoides platessoides): 1 - the curve at the beginning of the relaxation; 2 - curve at the time of relaxation 0.5 hours; 3 - curve at the time of relaxation 1 hour; 4 - curve at the time of relaxation 1.5 hours; 5 - curve at the time of relaxation 2 hours; 6 - curve at the time of relaxation 0.5 hours, built by the method of nets; 7 - average moisture content in a sample of fish.

3. Conclusion
The theoretical framework of fish near-surface layer relaxation mechanism has been laid by analyzing dynamics of the moisture distribution curves calculated by numerical methods and analytic solution of the second-order differential equation for the infinite plate with the third type boundary conditions.

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