The Gaussian cell two-point “energy-like” equation: Application to large scale galaxy redshift and peculiar motion surveys

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ABSTRACT
We introduce a simple linear equation relating the line-of-sight peculiar velocity and density contrast correlation functions. The relation, which we call the Gaussian cell two-point “energy-like” equation, is valid in the distant-observer-limit and requires Gaussian smoothed fields. In the variance case, \textit{i.e.}, at zero lag, the equation is similar in its mathematical form to the Layzer-Irvine cosmic energy equation. Estimation with this equation from the 6dF redshift galaxy survey and the SEcat catalogue of peculiar velocities is carried out, returning a value of $\beta \approx 0.34 \pm 0.08$. The applicability of the method for the 6dF galaxy redshift and peculiar motions survey is demonstrated with mock data where it is shown that beta could be determined with \%5 accuracy. The prospects for constraining the dark energy equation of state with this method from the kinematic and thermal Sunyaev-Zel’dovich cluster surveys are discussed. The equation is also used to construct a nonparametric mass density power spectrum estimator from peculiar velocity data.

Key words: galaxies: clusters: general – galaxies: distances and redshifts – cosmology: theory – large-scale structure of Universe – cosmological parameters

1 INTRODUCTION
In the linear regime of the gravitational instability scenario the underlying mass distribution is directly traced by the galaxy peculiar velocities. Measurement of the radial component of the galaxy peculiar velocities, the only component that one can easily observe, is carried out with one of many available techniques, the most common among which are the Tully-Fisher-like methods \textit{e.g.}, Tully & Fisher 1977, Faber & Jackson 1976). Normally, these methods exploit a well defined intrinsic relation between two or more of the galaxy\textsuperscript{1} observed properties that facilitates establishing its actual distance from the observer. The estimated distance is then used together with the measured galaxy redshift, to determine the galaxy radial peculiar velocity. Assuming an irrotational flow on large scales and the knowledge of $n$ (the cosmological mass density parameter), it is straightforward to use the measured radial peculiar velocities to recover the full underlying mass overdensity (Bertschinger & Dekel 1989, Dekel, Bertschinger & Faber 1990, Zaroubi 2002, Zaroubi \textit{et al.} 2002). In addition, the same mass-density could be probed by galaxy redshift catalogues assuming a simple linear biasing scheme that connects it to the spatial galaxy distribution (Kaiser 1984, Bardeen \textit{et al.} 1986). To date, galaxy redshift and peculiar motion surveys are the main tools with which astronomers explore the distribution of matter in the nearby universe.

Since the two types of data, galaxy peculiar velocity catalogues and galaxy redshift surveys, probe the underlying mass distribution comparing the two provides a simple and powerful test on the paradigm of gravitational instability and gives a model independent measurement of $\beta$, the ratio between the linear growth factor, $f (n)$ ($\frac{0.8}{n}$) and the linear biasing factor of the galaxy population. In most cases the comparison is either performed by deriving galaxy peculiar velocities from the galaxy density field and confront them with the measured velocities point-by-point, an approach usually called “velocity-velocity” comparison \textit{e.g.}, Davis, Nusser & Willick 1996, Willick & Strauss 1998, Zaroubi 2002, Zaroubi \textit{et al.} 2002). Or by adopting the so called “density-density” approach in which the velocity data is used to infer the full mass density field (Bertschinger & Dekel 1989, Zaroubi 2002) and compare it with the galaxy distribution \textit{e.g.}, Sigad \textit{et al.} 1998, Zaroubi \textit{et al.} 2002). With the exception of the POTENT algorithm \textit{see e.g.}, Sigad \textit{et al.} 1998), all the comparison methods yield a low value of consistent with $n$ $0.3$ and bias factor of $\beta$.

Another approach to the comparison that doesn’t fit into the two general classes outlined earlier, is the one proposed by Juszkiewicz \textit{et al.} (1999) who start from the pair conservation equation (Peebles 1980) and evolve it further to the quasi-linear regime of gravitational instability. In the pair conservation approach, which yields a value of $\beta$ that is consistent with the one derived from velocity-velocity analyses (Ferreira \textit{et al.} 1999, Feldman \textit{et al.} 2003), a relation between the mean pairwise velocity at a certain separation and the density correlation function is derived. The com-
2 THEORETICAL DERIVATIONS

In this section, we first derive the main theoretical relation (subsection 2.1) and show how it could be used to estimate the matter power spectrum from peculiar velocity data (subsection 2.3). Then in subsection 2.4.1 we derive the variance component of the main relation is used in order to estimate the variance of the measurement noise and sparseness of the sample to a bare minimum. The paper concentrates on the relation between the variance (two-point correlation at zero distance) of the two fields. This relation basically reduces the comparison between the catalogues to a couple numbers allowing robust extraction of the parameters. The proposed equation is especially suited to future peculiar velocity data sets like, the 6dF galaxy survey and the kinematic and isotropy.

2.1 The basic relation

Consider a radial peculiar velocity field $v_{\text{los}}(r)$ measured in a very distant patch of the sky smoothed with a Gaussian window function with scale $R_s$, $W_{\text{los}}(r) = (2\pi R_s^2)^{3/2} \exp (-r^2/2R_s^2)$. In the limit of $R_s \rightarrow R$, where $R_s$ is the correlation radius of peculiar velocities and $R$ is the distance of the patch from the observer. A smoothed radial field within a given observed volume can be written as,

$$ v_{\text{los}}^2(\mathbf{r}) = \frac{(H_0/2)^3}{(2\pi)^3} \frac{P_k}{k^2} \mathbf{W}_{\text{los}}(k) \exp (-ik \cdot r) d^3k; \quad (1) $$

where the superscript $S$ refers to values smoothed with a Gaussian kernel of radius $R_s$, $W_{\text{los}}$ is a unit vector along the line-of-sight, and $\mathbf{W}_{\text{los}}(k)$ is Fourier transform of the smoothing kernel.

The two-point correlation function of the Gaussian smoothed line-of-sight galaxy peculiar velocity is:

$$ h_\nu^2(r_{\text{los}}) i = \frac{(H_0^2/2)^{3}}{(2\pi)^3} \frac{P_k}{k^2} W_{\text{los}}^2 x \mathbf{x} \mathbf{k} \cdot \mathbf{r} d^3k; \quad (2) $$

Where $P_k$ is the mass density power spectrum and $r$ is the radial vector separating between any two points. Since there are two independent directions that appear in eq. one can’t invoke symmetry arguments in order to proceed. However, since in the distant observer limit the line-of-sight direction is approximately constant across the observed volume and independent of the direction of $r$, one can average over all possible directions of $r$ relative to $k$ (arbitrary direction) by integrating equation with $\mathbf{r}$, where is the cosine angle between the two vectors $r$ and $k$:

$$ h_\nu^2(r_{\text{los}}) i = \frac{(H_0^2/2)^{3}}{(2\pi)^3} \frac{P_k}{k^2} W_{\text{los}}^2 x \mathbf{x} \mathbf{k} \cdot \mathbf{r} d^3k; \quad (3) $$

Where $\mathbf{r}$ is an average over statistical ensemble and over $\mathbf{k}$, is the zero order Spherical Bessel function and $x = \mathbf{x} \cdot \mathbf{k}$

Assuming statistical isotropy for the velocity field, i.e., symmetry between the line of sight and the other two orthogonal directions one obtains the following equation:

$$ h_\nu^2(r_{\text{los}}) i = \frac{(H_0^2/2)^{3}}{(2\pi)^3} \frac{P_k}{k^2} W_{\text{los}}^2 x \mathbf{x} \mathbf{k} \cdot \mathbf{r} d^3k; \quad (4) $$

With the factor 3 coming from the symmetry argument. Now to the last step in the calculation, for a Gaussian smoothing kernel, i.e., $W_{\text{los}}(k) = \exp (-kR_s^2/2)$, the derivative of the line-of-sight velocity two point correlation function with respect to $R_s$ yields:

$$ \frac{\partial h_\nu^2(r_{\text{los}})}{\partial R_s} i = \frac{(H_0^2/2)^{3}}{(2\pi)^3} \frac{P_k}{k^2} W_{\text{los}}^2 x \mathbf{x} \mathbf{k} \cdot \mathbf{r} d^3k; \quad (5) $$

Here $(x; R_s)$ is the two point correlation function of the smoothed densities. Notice that eq. can only be obtained when $\partial x / \partial R_s = \partial W_{\text{los}} / k^2 W_{\text{los}}(k)$, strictly valid only with Gaussian smoothing kernel.

Obviously, for a given 3 dimensional peculiar velocity field the two point correlation function of the Gaussian smoothed velocity, $v^2$, is related to its density counterpart through the equation,

$$ \frac{\partial v^2(r_{\text{los}})}{\partial R_s} i = \frac{(H_0^2/2)^{3}}{(2\pi)^3} \frac{P_k}{k^2} W_{\text{los}}^2 x \mathbf{x} \mathbf{k} \cdot \mathbf{r} d^3k; \quad (6) $$

2 There are other functions that satisfy this relation but they do not satisfy the requirements of smoothing kernels.
The Gaussian cell two-point “energy-like” equation

\[ = 2 \frac{\delta^2}{3} R_s \frac{\delta}{R_s} \text{ (r}^2 \text{r}_s) : \]

which is similar to equation (9) without the factor of 3 and with no need for averaging over .

It might be easier to interpret equation (9) in its integral form, where the integral is performed over the smoothing radius. Let \( R_1 \) and \( R_2 \) the two smoothing radii that bound our integral, therefore,

\[ \frac{1}{2} \frac{\delta}{ \langle \delta \rangle } (\tau R_s) \frac{d^2}{R_s} \frac{d}{R_s} = \frac{2}{3} \frac{\delta}{ \langle \delta \rangle } \frac{d^2}{R_s} \frac{d}{R_s} : \]

In the \( r = 0 \) limit, the left-hand side of equation (6) describes the mean change in the kinetic energy associated with the smoothed velocity due to the variation of the smoothing radius, whereas the right-hand side depicts the 3-dimensional integral of the density variance of the smoothed field over the smoothing scale. The right-hand side term is very similar to the normal potential energy except that \( R_s \) does not represent a proper distance between points. In other words, the variation in the kinetic-like energy comes from the “potential-energy-like” behavior of the modes corresponding to the scales between \( R_1 \) and \( R_2 \), with exponentially decreasing contributions from larger and smaller scale modes. If \( r \neq 0 \) then the interpretation is not as simple as but still the left- and right-hand sides correspond to a sort of a two-point kinetic-energy and potential-energy, respectively, with the same scales contributing to the modification as before. Therefore, equation (6) is an “energy equation” of sorts as it describes the two point “potential-like” and “kinetic-like” energy partition within a Gaussian window function. Dimensional arguments significantly restrict the mathematical form of the relation between the velocity and the density 2 points correlation functions. Therefore, it is not surprising that the theoretical relation shown by equation (6) is very similar to the Irvine-Layzer cosmic energy equation which describes how the energy of the Universe is partitioned between kinetic and potential energy (Irvine 1961, 1965; Layzer 1963, 1966; see also Mo, Jing & Börner 1997).

2.2 The mass density power spectrum

The main approach currently used to measure the mass-density power spectrum from peculiar velocity data is the likelihood method introduced by Zaroubi et al. (1997; see also, Freudling et al. 1999 and Zaroubi et al. 2001) in which a theoretical power spectrum with few free parameters and a noise model are assumed. Since in this method the data is forced to fit a specific power spectrum shape, an inaccurate description of the noise model could propagate to large scales and contaminate the measured power spectrum. Indeed the results obtained with this method have been yielding unrealistically high amplitude of the mass-density fluctuations power spectrum (consistent with \( n > 0 \)). Therefore, direct nonparametric methods for power-spectrum estimation from peculiar velocity data are needed.

The question we pose here is: can equation (7) be used to directly estimate the mass power spectrum from peculiar velocity data? In the ideal case in which the uncertainties in the measurement are neglected and the data extends to infinity and samples the universe very accurately, the answer is clearly yes.

A good point from which to start the derivation of the power spectrum estimator is the relation between the power spectrum and the smoothed density two point correlation function,

\[(\tau R_s) = \frac{1}{2} \int _{\mathbb{R}^3} \frac{d^3}{d^3k} \frac{\delta^2}{\delta k^2} \frac{\delta}{k^2} \frac{d}{d^2 k^2} : \]

Substituting this into equation (8) and integrating over the variable \( r^2 \) from zero to 1 after multiplying with \( \frac{6}{\delta} (k \tau)^2 \), one obtains the following relation,

\[ \frac{6}{\delta} \int _{\mathbb{R}^3} \frac{d^3}{d^3k} \frac{\delta}{k^2} \frac{d}{d^2 k^2} : \]

Where the orthonormality of the spherical Bessel functions is used (e.g., Arfken & Weber 2002). The left-hand side of eq. (12) is a quantity that can be directly measured from the velocity data; whereas, the right-hand side shows the estimated quantity. Since we use smoothed velocity data, the power spectrum is determined up to a factor of \( \tau (n) \) and with a resolution that cannot exceed the scale imposed by the Gaussian smoothing.

For a real application, the discrete and noisy nature of the data should be taken into account, i.e., some of the steps leading to eq. (12) has to be modified. For example, to maintain the orthogonality of the spherical Bessel functions on a finite spherical volume one has to impose appropriate boundary conditions (see Fisher et al. 1995 & Zaroubi et al. 1995 for examples). However, the main hurdle for this direct approach to power spectrum estimation is the noise contribution; this issue is deferred to a future work.

2.3 Estimation of

2.3.1 Estimator

Eq. (7) is the most general relation derived in this paper. However, in order to use it to estimate the value of \( \delta \), it is simpler to restrict ourselves to the relation between the density and velocity variances, namely, apply the equation in the limiting case of \( r = 0 \) to yield:

\[ \frac{d}{dR_s} = 2 \frac{\delta}{\langle \delta \rangle} \frac{d^2}{R_s} \frac{d}{R_s} : \]

Where \( \frac{\delta}{\langle \delta \rangle} \) and \( \frac{d}{dR_s} \) are the peculiar velocity and density-contrast variances, respectively.

The numerical calculation of \( \frac{d}{dR_s} = \frac{d}{dR_s} \times \frac{d}{dR_s} \) is straightforward. One has to smooth the measured velocity field with a Gaussian window, calculate its variance and obtain its derivative by finite differencing (see subsection 2.3.3 for a similar explicit calculation). The right-hand-side of equation (13) is obtained from the galaxy redshift catalogue by taking the variance of the smoothed real-space density field.

The proposed estimator requires no heavy data manipulation and is easy to calculate. Due to the smoothing involved, the estimator is robust with regard to instabilities caused by the large random noise. In addition, to avoid the cosmic variance contribution to the error analysis, the comparison between the two types of data sets is performed within the same region of space. Both features, simplicity and stability, render the estimator very appealing to use.

2.3.2 Noise

The contribution of the measurement error to the estimator in equation (13) is readily calculated with the following discrete approach. Let \( \tau (\omega) \) be the noise associated with particle \( \omega \), then the smoothed noise is,

\[ \tau (\omega) = \tau (\tau) \tau (\tau) W (\omega) (\tau) \]

Subsequently, the expectation value of the noise two point correlation is,

\[ \int _{\mathbb{R}^3} \frac{d}{dR_s} \frac{d}{dR_s} : \]

(15)
The last equation assumes that the measured errors are statistically uncorrelated.

We now require that $z_i = z_j$ and sum over all the data points. The expectation value of the noise contribution to the variance of the velocity is:

$$\frac{2}{N} \langle \mathcal{R}_s \rangle = \frac{1}{N} \sum_{i=1}^{N} h^2 \langle z_i \rangle \frac{N}{\mathcal{R}_s} \langle \mathcal{R}_s \rangle \langle z_i \rangle$$

(16)

Therefore, the noise variance that adds to the left hand side of equation (13) is readily obtained by finite differencing:

$$\frac{d}{dR_s} \left( \frac{2}{N} \langle \mathcal{R}_s \rangle \right) = \frac{2}{N} \langle \mathcal{R}_s + \mathcal{R}_s \rangle \frac{2}{N} \langle \mathcal{R}_s \rangle$$

(17)

The contribution of the noise variance to the right hand side of eq. (13) is typically small and is neglected here. However, it is straightforward to account for in the case of unusually noisy data.

3 COMPARISON BETWEEN THE SECAT AND THE PSCz CATALOGUES

In this section equation (13) is employed for comparison between the PSCz galaxy redshift catalogue (Saunders et al. 2000), Branchini et al. 2000) and the SECat galaxy peculiar velocity catalogue (Zaroubi et al. 2002) which is a combination of the two homogeneous peculiar velocity catalogues, the SFI catalogue of spiral galaxies (Giovanelli et al. 1998, Haynes et al. 1999) and the ENEAR catalogue of early-type galaxies (da Costa et al. 2000). The SECat catalogue extends to a distance of about $70 \ Mpc$ and the PSCz goes to about twice of that. Therefore, in order to avoid cosmic variance contamination of the measurement the comparison between the two is restricted to the closer distance.

Prior to applying the method to the actual data, however, one needs to address the question of whether it is realistic to expect a reliable estimation of the value of with noisy and close by catalogue such as the SECat. Hence, the next subsection is dedicated to testing with mock catalogues how robust our estimator is.

3.1 Testing with mock data

The density and peculiar velocity mock catalogues used in this section are derived from the $32 h^{-1} Mpc$ resolution reconstruction of the density field from the PSCz galaxy redshift catalogue (Branchini et al. 2000), where the peculiar velocity field is obtained using linear theory from the galaxy redshift space positions assuming a value of $= 0.2$. The mock SECat peculiar velocity catalogue has the same distances and number of points the real SECat has, but with the velocities of the PSCz reconstructed velocity field. Obviously, it would have been better to use a full nonlinear N-body simulation with which to test the method. However, since the positions of the actual measured velocities are controlled by the specific distribution of the galaxies in the nearby universe we choose to test the method with data that has the same spatial distribution as the real universe, albeit the lack of full nonlinearity. Given the heavy smoothing involved in the analysis this way of assigning velocities to mock data is satisfactory – for testing the method with full nonlinear simulation see section 4. After assigning the velocities to the noise free mock data, we generate 30 mock catalogue with the random errors added to their distance and velocity values in concordance with the observational uncertainties.

However, as a first step we wish to test whether the method works in the distant observer limit with homogeneous sampling and noise free data. In this case we have constructed the mock velocity data by sampling the PSCz catalogue on every 8-th grid point where the velocity is taken to be equal to the z component to mimic the distant observer limit. Figure 1 shows as calculated from a homogeneously sampled velocity catalogue in the distant-observer-limit. The dashed line shows beta from a homogeneously sampled velocity catalogue but with the actual volume coverage of SECat, i.e., the distant-observer-limit requirement is relaxed. The solid line is as deduced from noise free mock peculiar velocity data with the same selection effects as SECat. The dotted line shows the correct value of

![Figure 1](image_url)

Figure 1. as deduced from mock velocity data (as taken from PSCz high resolution data) designed to test various selection effects. The dotted-dashed curve shows as calculated from a homogeneously sampled velocity catalogue in the distant-observer-limit. The dashed line shows beta from a homogeneously sampled velocity catalogue but with the actual volume coverage of SECat, i.e., the distant-observer-limit requirement is relaxed. The solid line is as deduced from noise free mock peculiar velocity data with the same selection effects as SECat. The dotted line shows the correct value of
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Figure 2. as deduced from mock SEcat data where the underlying in this catalogues is $0.5$ (dotted line). The solid line shows the mean as obtained from 30 SEcat mock catalogues with the error bars reflecting the 1-uncertainty around the mean. The dashed line shows the recovered value of for the noise free case; clearly, there is an underestimation of from the noisy data at small smoothing radii.

the volume of the data set itself and one has to start to worry about sampling issues within the Gaussian kernel itself. As will be shown later, the lack of convergence is an issue, however less severe, for the real SEcat and PSCz data.

The final step in our testing is to apply the method to a “full” mock SEcat catalogue (with noise and actual sampling). The solid line in figure 2 shows the mean value of $\beta$ as a function of the smoothing radius as recovered from 30 mock SEcat catalogues with the error bars indicating the 1 scatter about the mean. is clearly well reconstructed with large smoothing radii. There is also some bias in the mean value of at the smaller smoothing radii with respect to the obtained from the noise-free data (dashed line), which is probably due to overestimation of the noise variance at smaller scales. This is not a big worry as on small scales the PSCz catalogue used to produce the velocity data has limited nonlinear evolution due to its poor resolution ($32 h^{-1} Mpc$) and its velocities are purely linear.

Please note that the error bars shown here are correlated. The uncertainty estimates made for this figure, and for the rest of the figures in the paper, are based on one of the error bars and not their combination.

3.2 from the real data

Having tested the method on mock catalogues and demonstrated that, on large scales, it gives unbiased results for a SEcat-PSCz comparison, we now apply it to the real data. Figure 3 shows the measured as a function of the smoothing radius where it has a value of $0.6$ at smoothing radius of $10 h^{-1} Mpc$ but drops down as the smoothing radius increases to $0 \pm 0.5$, the curve becomes almost flat at $R_s > 18 h^{-1} Mpc$. The error bars here are taken from the 1 uncertainties determined from the 30 mock catalogues.

Branchini et al. (1999) have used two methods to solve for the redshift distortion equation (Kaiser 1987) and reconstruct the real-space density from the PSCz galaxy redshift distribution, one is based on the Yahil et al. (1989) iterative method and the other on the Nusser & Davis (1994) spherical harmonic expansion approach. In the previous analysis we have used data obtained with the former method. However, to examine the robustness of the measured value of we perform the same comparison but with the later method. The dashed line in figure 3 shows as a function of smoothing radius deduced from the second method which is well within the 1 uncertainty level, albeit being slightly smaller.

On small smoothing scales the behavior of the curves shown in figure 3 is systematically different from those obtained from the analysis of the mock catalogues, the former drops with scale while the later increases. We attribute this difference to the fact that the PSCz catalogue has a limited resolution and its velocity field is purely linear.

4 FUTURE SURVEYS

4.1 Application to Mock 6dF catalogue

In the near future, the 6dF Galaxy Survey (Jones et al. 2004) will measure the redshifts of around 150000 galaxies, and the peculiar velocities of a 15000-member sub-sample, over almost the entire southern sky. When complete, it will be the largest redshift survey of the nearby universe, reaching out to about $z < 0.15$, and more than an order of magnitude larger than any peculiar velocity survey to date. Since the two datasets will be obtained from the same survey, the galaxy redshift and peculiar velocity catalogues will have
the valuable attribute of being subjected to the same selection effects.

Despite the relatively large volume covered by the 6dF galaxy peculiar velocity survey the relative nature of the errors in the $D_n$ distance estimation might still diminish the information content of the data. To evaluate this effect we apply eq. 14 to mock 6dF galaxy redshift and peculiar velocity catalogues. In this experiment the catalogues are constructed from the full nonlinear N-body numerical simulation described by Cole et al. (1998), specifically, the simulation labeled L3S in their paper. The simulation assumes a CDM power spectrum of fluctuations with $n = 0.3$, $H_0 = 0.7$, rms fluctuation of the mass contained in spheres of radius $8\, h^{-1} Mpc$, $s = 1.3$ and a CDM power spectrum shape parameter, of 0.25. The simulation box side is $345 \pm 6 \, h^{-1} Mpc$ and has $192^3$ particles. The mock catalogues where produced by carving out 6 hemispheres of radius $150 \, h^{-1} Mpc$ of the simulation box. We obtain the redshift and peculiar velocity catalogues with uniform sampling of the galaxies in the simulated hemisphere in accordance with the expected sampling of the 6dF survey. The real-space distribution is presumed to have negligible errors; but the distances in the peculiar velocitycatalogues carry errors of $20\%$ of their actual values. The input linear bias factor, $b$ is one.

The star symbols in figure 4 shows the average value recovered from the 12 mock catalogues, as a function of smoothing scale. The error bars show the associated variance. The continuous line shows the average value recovered from the same 12 mock catalogues in which no errors have been added to velocities. Clearly, the recovery is close to its input value at all smoothing scales. If the error level we get is realistic the accuracy with which the 6dF galaxy survey will recover the parameter ($0.05\%$) is indeed encouraging.

The recovery down to $5\, h^{-1} Mpc$ scale is very encouraging too as it indicates that the Gaussian cell “energy-like” equation holds also for the quasilinear regime. Obviously, this point needs to be further explored with many simulations and over a wide range of point separations.

### 4.2 Kinematic and Thermal SZ Clusters

Inverse Compton scattering of cosmic microwave background (CMB) photons off thermal electrons within the hot intra-cluster medium of galaxy clusters produce two effects. First, distortion of the CMB black-body spectrum causing the cluster to appear brighter or dimmer at different frequencies and, second, an achromatic modification of its surface brightness. These effects are known, respectively, as the thermal and kinematic Sunyaev-Zeldovich effects (Sunyaev & Zeldovich 1972). The two combined with a measure of the cluster temperature give the cluster’s radial peculiar velocity component to a high degree of accuracy. Current estimates of the measured distance-independent absolute uncertainty are as low as $130 \, km \, s^{-1}$ (Holder 2004).

The thermal component of the SZ effect is now routinely measured with interferometers and major efforts are underway to survey the sky with in the thermal SZ relevant spectral range. Since the SZ effect is redshift independent, this kind of survey will provide the expected superior quality of the measured peculiar velocity data. On the other hand however, this insensitivity facilitates a very accurate measurement of the clusters biasing factor and its evolution as a function of redshift.

The expected superior quality of the measured peculiar velocity of individual clusters is hampered by their sparseness. Therefore, it is essential to analyze the data with methods that are stable with respect to this feature. The method developed here is a good candidate as it is simple, easy to apply, involves no complicated inversion schemes and the vast majority of the measured clusters will satisfy the distant-observer-limit assumed in the derivation. Initial application of the method to realistic mock catalogues shows a good success in the recovery of the thermal SZ component will provide wide angle surveys of galaxy-cluster peculiar velocities up to redshift of about 2. Such data will probe the evolution of the dark energy and galaxy-cluster bias evolution and clearly distinguish between various theoretical scenarios of cosmological evolution.

Like the 6dF galaxy survey, the future SZ surveys will probe the density and the peculiar velocity of the same region of space with the same objects and therefore allow a measurement of $w$ (through $D_n$). The left hand panel of figure 5 shows the evolution of $H \, \xi (z)$ as a function of redshift for different values of $z$ for a flat CDM universe normalized to the case of $z = 0$. The right hand panel of figure 5 shows the evolution of $H \, \xi (z)$ as a function of redshift for various values of the dark energy equation of state parameter, $w$, in a flat universe (Haiman, Mohr & Holder 2001). As pointed out by Lahav et al. (1991) the evolution of $\xi (z)$ partially cancels out with the evolution in the Hubble parameter. The weak dependence of the evolution of $w$ clearly shows that the equation of state is very hard to measure with peculiar velocity data. On the other hand however, this insensitivity facilitates a very accurate measurement of the clusters biasing factor and its evolution as a function of redshift.
which we applied it were at redshift zero and limited in size. When
the cluster mass cut-off exceeds $8 \times 10^3 \text{M}_\odot$ the simulation box is
left with very small number of clusters. In order to test the applica-
bility of the method properly one should apply it to very large scale
simulations that span the redshift range of $0 \leq z \leq 2$; a task that will be
deferred to the future.

5 DISCUSSION

In this paper we introduced the Gaussian cell two point “energy-
like” equation connecting the two point density and peculiar velocity
correlation functions. The interpretation of this equation is that the
change in the velocity correlation function is caused by density variation coming from scales larger than the scale set by the
Gaussian smoothing; this analytic cancellation of the small scale
power is particular to Gaussian kernels. Two practical applications of
the Gaussian cell two-point energy-like equation have been de-
veloped here, the first is direct matter power spectrum estimator
from peculiar velocity data, and the second is measurement from
comparison of galaxy peculiar velocity and redshift surveys. The later application was restricted to the velocity dispersion, i.e., the
$z = 0$, case.

In the $z = 0$ case, the relation derived here is similar in its
mathematical form to the Irvine-Layzer cosmic energy equation.
This is not surprising as each of the two relations reflect some sort
of energy balance and should, due to dimensionality arguments, be homologous.

Restricting the main formula to the variance case the relation
could be easily used to estimate the value of $\Omega_m$ from comparison
between galaxy peculiar velocity and redshift catalogues. In this paper we showed that despite their proximity the PSCz galaxy red-
shift survey and the SEdcat galaxy Peculiar velocity data could be
reliably used to derive the value of $\Omega_m$. The result is consistent with
that of previous analyses. The variance case has also been shown to
apply to the 6dF galaxy survey, despite being far from the distant-
observer-limit. Using mock 6dF catalogues we have demonstrated
that our method can be successfully used to extract cosmological
parameters from the real sample.

In the future, the eminent detectability of the kinematic
Sunyaev-Zeldovich effect will provide peculiar velocity measure-
ments for large number of galaxy clusters at redshifts extending
back to the formation epoch of cluster ($z \leq 2$). This type of data is
deal to explore with the Gaussian cell two-point energy-like equa-
tion as it satisfy all of the required assumptions and have small
measurement errors with large spatial coverage. The redshift cov-
ervation of the SZ data will allow an accurate measurement of the evo-
olution of the clusters biasing factor with redshift. The main hurdle
these data sets will pose is the limited resolution with which they
will sample the universe as the comoving rms distance between rich
galaxy-clusters is of the order of $30 \text{ h}^{-1} \text{Mpc}$.

We have also shown that the Gaussian cell two-point energy-
like equation could be used to estimate the matter power spectrum
peculiar velocity data in a non-parametric fashion. This is a very
important application since the current measurements of the mass
power spectrum from peculiar velocity employs likelihood analy-
sis with specific models that almost certainly do not properly ac-
count for the noise contribution (Zaroubi et al. 1997, 2001). A
non-parametric measurement on the other hand will allow a scale-
by-scale dissection of the various components contributing to the
measured power spectrum allowing the isolation of the noise part.

Finally, given the simplicity of equation (7) it is tantalising to
attempt to extend it to the quasi-linear regime to obtain a nonlinear
description of the evolution of the two point peculiar velocity corre-
lation function similar to the very successful quasi-linear extension of its density counterpart (Hamilton et al. 1991). Indeed, figure 4 gives an encouraging indication that the equation might hold for the quasi-linear regime. This will be further explored in a future work.

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