Rumor spreading in gaming social networks

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So far, the focus on rumor spreading are mainly on some simple backgrounds, in other words, only taking consideration of the overall topological influences on its dynamical behavior. However, in the prospect of the individuality, personal strategies in the social networks play a more non-trivial role in the real social networks. To fill this gap, we will investigate the rumor spreading in gaming social networks. Our analysis is supported by the results of numerical simulations. We observe that the original rumor is still the most well known edition in case that the content is modified by the defectors. However, the portion decays with the stimulus generally. For the case that defectors keep silence in the spreading process, the scale of spreading decays with stimulus generally, suggesting the rumor can hardly spread in a community of defectors. This highlights the key role that stimulus plays in rumor spreading and the necessity to study information spreading in competitive circumstances.

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I. INTRODUCTION

Understanding the emergence of rumor is a fundamental problem in social science. In the past decade, to further understand the general principles in rumor spreading \cite{1,2} on social networks, researchers investigate the rumor spreading on many typical models, e.g., small-world networks \cite{7,8}, and scale-free networks \cite{2,9,11}. In the small world model, they reported a threshold \( P_c \) of spreading in the rewiring probability. When enhancing the probability beyond \( P_c \), i.e., shortening the path length between nodes rumors can be disseminated globally. On the other hand, in the scale-free networks, the discussions are mainly focused on the efficiency of spreading and the final infected ratio, which has been analytical solved by a mean-field approximation \cite{10}. Most recently, a model describing two propagating rumors with different probabilities of acceptance is also worth a mention \cite{12}.

Simultaneously, the modifications on the dynamical model itself are also pretty heuristic \cite{13,14}. Concretely, some researchers take consideration of accumulation effect of rumor in the process of persuading the innocent \cite{13} and others set the whole dynamics on a spatial system \cite{14}, both of which provide valuable insight on the dynamics in question.

However, humans society is full of competitions. Such competitions influence evidently to a large extent various behaviors of individuals. In this view, the rumor spreading as a classic behavior of individual can hardly avoid such influences as well. What’s more, with the advent of modern transmission technology, its fundamental influence on the information networks and social networks are increasingly emerging.

Thus, whether rumor can be spread in the real networks (social networks, technological networks and information networks) depends not only on the existence of connections among individuals but also on their individual strategy for surviving in the networks. At this point, social dilemma games, as general metaphors for studying personal strategy in social networks, happen to provide such a feasible platform. In this respect, there is a need to introduce the traditional iteration games into the rumor theory as a new access.

In the well known games, the Prisoner’s Dilemma game (PDG) \cite{16,17} and Snowdrift game (SG) \cite{18,19} are the most popular two models, which have been investigated on many classic networks, e.g., small-world and scale-free networks \cite{19}. In this paper, we thus propose a rumor model in the social networks where people are gaming. We aim to explore the spreading process influenced by two global updating personal strategies, which are modifying the content of spreading information and keeping silence. We adopt PDG and SG as typical paradigms on different network structures, including small-world and scale-free networks.

Because the recent rapid advance of experimental investigations of rumor spreading was in large part driven by social behaviors, we shall discuss the modelling approaches before reviewing the experiments. In what follows, we will investigate two strategies of defectors in the stable state of game. One delivers the rumor with a revision on purpose (see Fig. 1), while the other doesn’t believe any news and suggest any neighbor.

Considering the first strategy, as is known, the content
of information spreading in a real social network have been changing all the time, which is also called ‘Chinese Whispers’ in some countries. Sometimes, although the original information is the truth, the delivery process may turn it into a rumor as well. Usually, these changes are on purpose, when and how the rumors will be modified become a natural question. Here, we model the process running in a community playing games. The members who adopt cooperation strategy learn and disseminate a rumor directly while the defectors play tricks before spreading it.

As discussed in the original rumor model (DK model) [1], the members, no matter which strategy they adopt, can be divided into three species: innocents, spreaders, and stiflers, whose densities are denoted by \( i(t) \), \( s(t) \), and \( r(t) \), respectively, as a function of time.

Here, we set the normalization condition \( i(t) + s(t) + r(t) = 1 \) and \( s_{\text{sum}} = \sum x s_x \), where \( s_x \) denote the \( x \)th, \( x = 1, 2, ..., n \) version of the rumor. A cooperater in this case learns a rumor at first. Subsequently, it will turn to be a stiffer directly when encountering the rumor or the likes. Unlike, once hearing the rumor, a defector doesn’t only learn it but also revise it before disseminating to the other members. As what Napoleon Bonaparte said “Never interrupt your enemy when he is making a mistake”; no matter whether the rumor can help their competition, doing so is a wise choice to keep their advantage on the accuracy of information.

In evolutionary biology on the heterogeneous networks as BA networks [20], the set of coupled properties can be written as follow: for cooperators,

\[
P_{x \rightarrow s_x}(t,k) = f_c k N P(k) i_k(t) \langle k \rangle \frac{\langle k \rangle k' P(k') s_{k'}(t)\rangle}{k'}, \tag{1}
\]

\[
P_{s_x \rightarrow r}(t,k) = f_c k N P(k) s_k(t) \langle k \rangle \frac{\langle k \rangle k' P(k') [s_{k'}(t) + r_{k'}(t)]\rangle}{k'}. \tag{2}
\]

For defectors,

\[
P_{x \rightarrow s_x}(t,k) = (1 - f_c) k N P(k) i_k(t) \langle k \rangle \frac{\langle k \rangle k' P(k') s_{k'}(t)\rangle}{k'}, \tag{3}
\]

\[
P_{s_x \rightarrow r}(t,k) = (1 - f_c) k N P(k) s_k(t) \langle k \rangle \frac{\langle k \rangle k' P(k') [s_{k'}(t) + r_{k'}(t)]\rangle}{k'}. \tag{4}
\]

the rate equation for the revision \( R_k(t) \) can be written as:

\[
\frac{dR_k(t)}{dt} = (1 - f_c) k N P(k) i_k(t) \langle k \rangle \frac{\langle k \rangle k' P(k') s_{k'}(t)\rangle}{k'} - k N P(k) s_k(t) \langle k \rangle \frac{\langle k \rangle k' P(k') [s_{k'}(t) + r_{k'}(t)]\rangle}{k'}. \tag{5}
\]

where the evolution of the densities \( s_k(t) \) and \( r_k(t) \) satisfy the following set of coupled differential equations:

\[
\frac{ds_k(t)}{dt} = -k N P(k) i_k(t) \langle k \rangle \frac{\langle k \rangle k' P(k') s_{k'}(t)\rangle}{k'}, \tag{6}
\]

\[
\frac{dr_k(t)}{dt} = k N P(k) s_k(t) \langle k \rangle \frac{\langle k \rangle k' P(k') [s_{k'}(t) + r_{k'}(t)]\rangle}{k'}. \tag{7}
\]

In evolutionary biology on the homogeneous networks as WS networks [21], the set of coupled properties can be written as follow:

for cooperators,

\[
P_{x \rightarrow s_x}(t,k) = f_c \langle k \rangle i(t)s(t), \tag{9}
\]

\[
P_{s_x \rightarrow r}(t,k) = f_c \langle k \rangle s(t) [s(t) + r(t)]. \tag{10}
\]

For defectors,

\[
P_{x \rightarrow s_x}(t,k) = (1 - f_c) \langle k \rangle i(t)s(t), \tag{11}
\]

\[
P_{s_x \rightarrow r}(t,k) = (1 - f_c) \langle k \rangle s(t) [s(t) + r(t)]. \tag{12}
\]

Thus, in this case, the rate equation for the revision \( R_k(t) \) can be written as:

\[
\frac{dR_k(t)}{dt} = (1 - f_c) \langle k \rangle i(t)s(t) - \langle k \rangle s(t) [s(t) + r(t)]. \tag{13}
\]

where the evolution of the densities \( s_k(t) \) and \( r_k(t) \) satisfy the following set of coupled differential equations:

\[
\frac{ds_k(t)}{dt} = -\langle k \rangle i(t)s(t). \tag{14}
\]

\[
\frac{dr_k(t)}{dt} = \langle k \rangle s(t) [i(t) - s(t) + r(t)]. \tag{15}
\]

In the infinite time limit, for WS networks, one can have a transcendental equation,

\[
r(\infty) = 1 - e^{-2r(\infty)}, \tag{17}
\]

base on the analytical approach of the reference [9, 10].

For the second strategy that defectors don’t believe and disseminate any rumors, they will turn to be silent automatically when hearing the rumor. Note that they belong to the informed although silent. For the sake of compare, the settings of the simulations are the same as the former one.

In this case, we define a new variable \( b_k(t) \) to describe the density of these defectors. At this point, the evolution of the densities for BA networks satisfies the following set of coupled differential equations:

\[
\frac{db_k(t)}{dt} = k N (1 - f_c) P(k) i_k(t) \langle k \rangle \frac{\langle k \rangle k' P(k') s_{k'}(t)\rangle}{k'}. \tag{18}
\]
\[
\frac{di(t)}{dt} = -kN P(k) i(t) \Sigma_{k'} \frac{k' P(k') s_{k'}(t)}{\langle k \rangle}.
\]
\[
\frac{ds(t)}{dt} = k N_{c} P(k) i(t) \Sigma_{k'} \frac{k' P(k') s_{k'}(t)}{\langle k \rangle} - k N P(k) s(t) \Sigma_{k'} \frac{k' P(k') s_{k'}(t) + r_{k'}(t)}{\langle k \rangle}.
\]
\[
\frac{dr(t)}{dt} = k N_{c} P(k) s(t) \Sigma_{k'} \frac{k' P(k') [s_{k'}(t) + r_{k'}(t)]}{\langle k \rangle}.
\]

For WS networks, the set of coupled differential equations can be rewritten as:
\[
\frac{dl(t)}{dt} = \langle k \rangle (1 - f_{c}) i(t) s(t).
\]
\[
\frac{di(t)}{dt} = -\langle k \rangle i(t) s(t).
\]
\[
\frac{ds(t)}{dt} = \langle k \rangle f_{c} i(t) s(t) - \langle k \rangle f_{c} s(t) [s(t) + r(t)].
\]
\[
\frac{dr(t)}{dt} = \langle k \rangle f_{c} s(t) [s(t) + r(t)].
\]

Notice that, at this point, the reliability \( q = r(\infty)+l(\infty) \).

In the infinite time limit, for WS networks, one can also have a transcendental equation,
\[
q(\infty) = 1 - e^{A q(\infty) + B}
\]

for the final reliability, where \( A = \frac{N(1-f_{c})}{1-f_{c}+N f_{c}} \) and \( B = \frac{1-f_{c}-(2N-1)f_{c}^{2}}{f_{c}(1-f_{c}+N f_{c})} + \ln \frac{N-1}{N} \). We show the result of simulation directly in Fig. 3 where both BA and WS networks are formed by 64 nodes. The average degree of BA and WS networks are \( z = 3 \) and \( z = 4 \) respectively. The initial conditions are the same as the former case.

To test the analytical prediction, we then run a number of extensive simulations on both WS and BA networks. First, we show our settings on the prisoner’s dilemma game 17 and snowdrift game. We set \( T = b > 1, R = 1 \) and \( P = S = 0 \) in the PDG, where \( b \) represents the temptation to defect, being typically constrained to the interval. For the SG, we make \( T = \beta > 1, R = \beta - \frac{1}{2}, S = \beta - 1 \) and \( P = 0 \). So that the cost-to-benefit ratio of mutual cooperation can be written as \( r = \frac{1}{2 b - 1}, \) with \( 0 \leq r \leq 1 \) 22. In each step of evolution, one node \( i \) plays the PDG and SG game with its directly connected neighbors, accumulating the payoff as \( P_{i} \). Whenever the strategy of the player \( i \) with \( k_{i} \) neighbors is to be updated, a neighbor \( j \) is randomly selected from \( i \)'s neighborhood. If \( P_{j} > P_{i} \), the chosen neighbor \( j \) spreads its strategy to the player \( i \) with probability, otherwise, the player \( i \) holds its strategy 22.

With the initial conditions \( i(0) = \frac{N-1}{N}, s(0) = \frac{1}{N} \), and \( r(0) = 0 \), simulations were carried out for a population with \( N = 1024 \) individuals. Initially, cooperator and defector strategies were distributed randomly among the members. Equilibrium frequencies of cooperators and defectors were obtained by a stable state after a transient time of 10000 generations. Considering the frequency of cooperators is a function of the advantage of defectors \( b \) for the PDG (the cost-to-benefit ratio \( r \) for the SG). We measure the relation between the times of revision on rumors \( R \) and \( b \) (or \( r \)) in Fig. 2. For generality, we set a random picked node with degree 4 as the first spreader. Each data point corresponds to the average of \( 10 \times N \) simulations, where \( N \) is the number of nodes with degree 4. In another word, we test every node with degree 4 as the rumor spreader on ten different realizations of the same type of network specified by the appropriate parameters (the population size \( N \) and the average connectivity \( z \)).

As shown in Fig. 2, in this case one can easily find out no matter which game on which networks the original rumor is the best known one all the time. However, the advantage is challenged by the higher “stimulus to defect” exponent \( b \) and \( r \). So that, one can observe the proportion of vertices influenced by the original rumor decays with \( b \) and \( r \) respectively. This phenomenon can be more apparently observed on WS networks for both games. As is known, in the WS networks, the transport efficiency is higher than the BA networks. Also, we find the final scope affected by the rumor is unrelated with \( b \) and \( r \), which confirms Eq. 17 at this point. But, the inducing level does influence the times of revision and its distributions. One can observe the species of rumors and the proportion of the revised rumors grow with the two parameters gradually.

For the second strategy, the relations between \( q \) of rumor and \( b \) (or \( r \)) for BA and WS networks are shown as Fig. 3. One can observe \( q \) decays with \( b \) (or \( r \)) generally. To test the relation between the size \( N \) and \( q \), we run extensive simulations in the same networks with different size in Fig. 3 where each plot corresponds to 10 runs of games on 10 different realizations of networks. One can observe the relations are relatively stable when \( N \geq 10^{3} \). Here, we only show the special cases of \( b = 1, 1.5, 2 \) and \( r = 0, 0.5, 1 \) for observation. We expect that in larger system sizes \( q \) would be more stable, where \( A(N \rightarrow \infty) = \frac{1-f_{c}}{f_{c}} \) and \( B(N \rightarrow \infty) = 2f_{c} \) in Eq. 20.

II. CONCLUSION

To sum up, rumor as a common social phenomenon has been investigated on many pure topological models. However, its owned social property make it different from other dynamics, e.g., synchronization, epidemic processes, reaction diffusion, driving the researchers to move one step further to investigate it on a more realis-
FIG. 1: (Color online) Spreading of a rumor in a group of cooperators and defectors. The smile faces denote the cooperators and the tricky faces denote the defectors. If the rumor starts from the black face, in three time steps all the people in the graph will know it, where people in black and red denote the person who knows the original and once-revised rumor respectively. Those people who are innocents or stiflers are colored white and blue respectively.

FIG. 2: (Color online) The cumulated probability of $R$ as a function of $b$ and $r$, where $1 < b < 2$ and $0 < r < 1$ respectively. Note that our simulations are running on the BA networks with $z = 3$ and the WS networks with $z = 4$, where networks are formed by 64 nodes.

FIG. 3: (Color online) Spreading of a rumor in networks with silent defectors. The PDG and SD are realized on the BA network with $z = 3$ and WS network with $z = 4$ respectively, where networks are formed by 64 nodes.

FIG. 4: (Color online) The final density of the stiflers $q$ as a function of $b$ and $r$, where $1 < b < 2$ and $0 < r < 1$ respectively.
In this paper, we have proposed and investigated a rumor model on the social networks, where people are gaming. Here, we only focus on two most investigated game models, the prison dilemma game and snowdrift game. When encountering the news, defectors in game have two choices, to change its content, namely, Chinese whisper, or to keep silence. Base on the former strategy, we present numerical results for the distribution of editions people finally get. For the latter strategy, we have showed the influence of inducement of game to the scale of rumor spreading numerically. Also, we have derived analytically the solution for the final scale of the rumor spreading for both strategies. We found that with the increase of inducement, the number of editions of the rumor grows and the number of people informed by the original rumor decays slowly. But, the original rumor remains the most well-known basically. The simulation results indicate that WS networks are better at keeping the original edition of rumor than BA networks. On the other hand, for the second strategy, the scale of rumor spreading decays with the inducement. We have showed WS networks have a larger spreading scale than BA networks when the inducements are the same. To some extent, our model can thus perform well in understanding and mimicking a variety of social dynamics in competitive surrounding.

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[1] D. J. Daley and D. G. Kendall, Nature 204, 1118, (1964).
[2] P. G. Lind, L. R. da Silva, J. J. S. Andrade, and H. J. Herrmann, Phys. Rev. E 76, 036117, (2007).
[3] P. Lind, L. da Silva, J. A., Jr., and H. Herrmann, Europhys. Lett. 78, 68005, (2007).
[4] P. Blanchard, A. Krueger, T. Krueger, and P. Martin, e-print arXiv: physics/0505031, (2005).
[5] J. L. Iribarren, E. Moro, Phys. Rev. Lett. 103, 038702 (2009).
[6] A. Grabowski, N. Kruszewska, and R.A. Kosiński, Phys. Rev. E 78, 066110, (2008).
[7] D. H. Zanette, Phys. Rev. E 64, 050901, (2001).
[8] D. H. Zanette, Phys. Rev. E 65, 041908, (2002).
[9] Y. Moreno, M. Nekovee, and A. Pacheco, Phys. Rev. E 69, 066130, (2004).
[10] M. Nekovee, Y. Moreno, G. Bianconi, and M. Marsili, Physica A 374, 457, (2007).
[11] Y. Moreno, M. Nekovee, and A. Vespignani, Phys. Rev. E 69, 055101, (2004).
[12] D. Trpevski, W. K. S. Tang and L. Kocarev, Phys. Rev. E 81, 056102, (2006).
[13] P. S. Dodds, and D. J. Watts, Phys. Rev. Lett. 92, 218701, (2004).
[14] E. Agliari, R. Burioni, D. Cassi, and F. M. Neri, Phys. Rev. E 73, 046138, (2006).
[15] G. Szabó and G. Fath, Phys. Rep. 446, 97 (2007).
[16] R. Axelrod and W. D. Hamilton, Science 211, 1390, (1981).
[17] M. A. Nowak and R. M. May, Nature 359, 826, (1992).
[18] H. Gintis, Game Theory Evolving (Princeton University, Princeton, NJ, 2000).
[19] G. Szabó and C. Hauert, Phys. Rev. Lett. 89, 118101 (2002); G. Abramson and M. Kuperman, Phys. Rev. E 63, 030901(R) (2001); J. Ren, W. X. Wang, and F. Qi, Phys. Rev. E 75, 045101(R) (2007).
[20] A. L. Barabási and R. Albert, Science 286, 509, 1999.
[21] D. J. Watts and S. H. Strogatz, Nature 393, 440, (1998).
[22] J. L. Iribarren, E. Moro, Phys. Rev. Lett. 98, 108103 (2007).
[23] F. C. Santos and J. M. Pacheco, Phys. Rev. Lett. 95 098104, (2005).