A Unified Method of Detecting Core-Periphery Structure and Community Structure in Networks

Bing-Bing Xiang\textsuperscript{a}, Zhong-Kui Bao\textsuperscript{a}, Chuang Ma\textsuperscript{a}, Xing-Yi Zhang\textsuperscript{b}, Han-Shuang Chen\textsuperscript{c}, Hai-Feng Zhang\textsuperscript{a,d,e,*}

\textsuperscript{a}School of Mathematical Science, Anhui University, Hefei 230601, China
\textsuperscript{b}Institute of Bio-inspired Intelligence and Mining Knowledge, School of Computer Science and Technology, Anhui University, Hefei 230601, China
\textsuperscript{c}School of Physics and Material Science, Anhui University, Hefei 230601, China
\textsuperscript{d}Center of Information Support & Assurance Technology, Anhui University, Hefei, 230601, China
\textsuperscript{e}Department of Communication Engineering, North University of China, Taiyuan, Shan’xi 030051, China

Abstract

Core-periphery structure and community structure are two typical meso-scale structures in complex networks. Though the community detection has been extensively investigated from different perspectives, the definition and the detection of core-periphery structure have not been attracted enough attention. Furthermore, the detection problem of the core-periphery and community structure was separately investigated previously. In this paper, we develop a unified framework to simultaneously detect core-periphery structure and community structure in complex networks. Moreover, there are several extra advantages of our algorithm: our method can detect not only single but also multiple core-periphery structures; the overlapping nodes belonging to different communities can be identified; by adjusting the size of core, different scales of core-periphery structures can be detected. The good performance of the method has been validated on synthetic and real complex networks. So we provide a basic framework
to detect the two typical meso-scale structures: core-periphery structure and community structure.

**Keywords:** Complex networks, core-periphery structure, community structure, region density curve, overlapping/active nodes.

1. Introduction

Many real-world systems in the field of communication, social, transportation, information, biology and so on, can be described as networks [25]. Meso-scale structures are very important for understanding of network properties and dynamics. Considerable investigations have focused on the study of a particular type of meso-scale structure known as community structure which has some cohesive groups called “communities”, nodes in the same community are connected densely to each other, whereas nodes in different communities are connected sparsely [16, 28]. In an ideal community structure, nodes in the same community are full connected, and there is no connection between nodes in different communities. The adjacency matrix of block model representing ideal community structure is given in Fig.1(a). Various techniques have been developed to detect community structure [15, 18], e.g., the algorithms were based on modularity [27], random walks [32], spectral clustering [39], hierarchical clustering [23, 38], nonnegative matrix factorization approach [40] and so on. There are also many algorithms for overlapping community detection based on label propagation [17, 19], link partition [2, 41], clique percolation theory [30], etc.

Another meso-scale structure known as core-periphery (abbreviated as CP) structure has not received enough attention, but it has been examined in the networks of society [37], scientific citation [13], international trade [24, 35] and other fields [12, 5, 20]. For instance, maintaining a party or an organization often needs an elite group to organize and manage it. The elite group is composed of highly influential and powerful individuals who have more connections with each other. And other people involved, namely the periphery part with few connections inside are less organized and less dominant [35]. Biological networks, such as that of the human brain, where a group of densely connected network nodes provides long-term functionality and robustness (core), while another group of sparsely connected nodes is responsible for adaptation on short time
Figure 1: Adjacency matrices of idealized block models. (a) community structure, (b) single CP structure, (c) multiple CP structure.

scales to changing conditions in the environment (periphery) [29]. All of these phenomena are the expressions of CP structure, indicating the significance of studying the CP structure [6, 31, 8].

Although the two meso-scale structures were studied more or less, the relationship between them is not well clarified. In fact, there are some intuitive relationships between the two typical meso-scale structures: on the one hand, a network with an ideal single CP structure (the adjacency matrix of ideal single CP structure is shown in Fig. 1(b)) implies the absence of community structure; on the other hand, a network with multiple CP structure (the adjacency matrix of ideal multiple CP structure is illustrated in Fig. 1(c)) often implies the existence of community structure. However, community structure does not imply the existence of multiple CP structure, if there are no closely connected core nodes and more loosely connected periphery nodes in each community, the multiple CP structure does not appear. Correctly understand the relationship can help us distinguish the two meso-scale structures and design effective methods to detect both of them.

Up to now, the definition of CP structure is unclear even though it widely exists in many real systems. A more formal and rather popular definition was proposed by Borgatti and Everett [6], in which a node belongs to a core if and only if it is well connected both to other core nodes and to periphery nodes, and periphery nodes do not connect with other periphery nodes. The adjacency matrices of block models with respect to the ideal single CP structure is illustrated in Fig. 1(b) [31]. Since the definition of ideal CP structure is too strict to meet, a looser definition of CP
structure is that core nodes are highly interconnected and periphery nodes frays into a tree [36]. However, it is a descriptive definition, the strict definition from mathematics has not been well proposed. The existing CP detection methods were mainly implemented by checking how well a network approximates the ideal case. The methods are often criticized due to several shortcomings: first, the methods need to give the size of core in advance; second, the methods rudely divide a network into a single CP structure, however, many real networks present multiple CP structure. More importantly, multiple CP structure and community structure may coexist in many real networks, the methods do not provide a unified framework to detect the both meso-scale structures. Inspired by these reasons, the goal of this paper is to propose a unified method to detect CP structure and community structure. In doing so, we firstly rank all nodes according to a connection density indicator, then we can judge whether the network exhibits a single CP structure, multiple CP structure or community structure based on the defined region density curves. Moreover, the multi-scale of CP structure and the overlapping nodes can be detected too.

To summarize, the main contributions of the work are as follows.

1) We propose a unified method to detect CP structure and community structure simultaneously.
2) Multiple CP structure can also be detected. Moreover, each periphery node is assigned a class value denoting the closeness with its core group.
3) The overlapping and active nodes taking part in different communities can be detected, furthermore, multi-scale of CP structure can be found by adjusting the size of core [33].

The rest of this paper is organized as follows. In Section 2, we briefly recall the related work on CP detection algorithms and overlapping community detection. The detailed descriptions of our method are described in Section 3. Empirical results on several typical networks are presented in Section 4. Finally, conclusions are given in Section 6.

2. Related Work

2.1. Algorithms of core-periphery structure

The detection of CP structure has not been well studied, this problem is formally considered in a brief review written by Borgatti and Everett [6].
In this review, authors formalized the notion of a CP structure and also proposed several intuitive algorithms to detect this structure. A discrete-version algorithms aims to find a vector $C$ of length $N$ whose entries can be either 1 or 0. The $i$th entry $C_i$ equals 1 if the corresponding node is assigned to the core, otherwise, it equals 0 if the corresponding node is assigned to the periphery. And define a pattern matrix with the element $C_{ij} = 1$ if $C_i = 1$ or $C_j = 1$, and let $C_{ij} = 0$ otherwise. At last, a core quality $\rho = \sum_{i,j} A_{ij} C_{ij}$ is defined to measure how well a network approximates the ideal case, where $A_{ij}$ is the element of adjacency matrix $A$. They also presented a continuous version in which each node is assigned a continue “coreness” value between 0 and 1, and the element of the pattern matrix $C_{ij} = C_i C_j$. The continue version is further developed in Ref. [31], where the “coreness” of each node is defined in different ways. Moreover, some heuristic methods, such as based on MINERS were proposed to maximize the core quality $\rho$ [7]. All of them are to make sure the core quality $\rho$ as large as possible, which is sensitively dependent on the ideal block adjacency matrix. Therefore, the size of core is fixed in advance. Also, the complexity of algorithms are high.

Some methods based on centrality indices, including network capacity [10], geodesic paths [9], knotty centrality and so forth [34], were proposed to divide nodes with high centrality values as the core. However, hereafter, we will address that nodes with high centrality value are not always the cores. In addition, Rossa et al. proposed a detection method by deleting the nodes with the smallest persistence probability $\alpha_s$, which is calculated by implementing the behaviour of a random walker in networks [11]. Zhang et al. also proposed a method for the detection of CP structure by fitting a stochastic block model to observed network data using a maximum likelihood method [43]. However, These methods fail in detecting multiple CP structures, or cannot detect community structure simultaneously, or hold a high computational cost.

The performance of different methods cannot be well compared owing to the following two reasons: on the one hand, there is no a recognized and standard indictor to measure which one method is better; on the other hand, previous methods need to fix the size of core group in advance, which restricts the comparison of different methods.
2.2. Algorithms of overlapping community

How to detect overlapping communities has been recognized to be one of the hot areas in the last few years. The most well-known algorithm is the clique percolation method (CPM) proposed by Palla et al. CPM builds up communities from k-cliques, where two k-cliques are said to be adjacent if they share \( k - 1 \) nodes. Communities are the connected components of the clique adjacency matrix. Since the high computational complexity of CPM, some methods were proposed. Such as, Kumpula et al. have proposed a fast sequential clique percolation algorithm, the main idea of the algorithm is to decrement the number of comparisons between cliques when determining whether the found cliques can be merged into larger communities [21].

A link community detection algorithm considering hierarchical and overlapping relationships between nodes simultaneously was proposed by Ahn et al., the idea of it is to partition links instead of nodes to discover community structure [2]. Similar idea has also been considered by Evans et al. [14]. One shortcoming of link community method is that too many overlapping nodes are detected.

Another type of overlapping community detection is based on local expansion or optimization, where a community is detected by maximizing a local fitness function from a seed in the network to be detected. For example, Lancichinetti et al. proposed a local expansion and optimization algorithm (LFM), where a fitness function for detecting communities is defined, and a node is randomly selected to expand its group members by that function [23]. Bandyopadhyay et al. also developed a fast overlapping community search (FOCS) method, which is also based on the idea of local expansion and optimization [4].

Overlapping community structure can be also found by extending the traditional label propagation algorithm by allowing a node in the network to have multilabels during the process of propagating the labels [17]. All of these overlapping community detection methods just focus on how to detect overlapping structure but do not concern the CP structure.

3. The Proposed Algorithm

In this section we describe the method used for detecting both CP structure and community structure. Consider an undirected and unweighted network, the general framework of our method is presented in Algorithm
1, which consists of three main steps: 1) we re-rank all nodes in a new sequence; 2) the region density of each node is calculated, and the region density curve is plotted; 3) we detect main meso-scale structures of the network based on the region density curves.

### Algorithm 1: General framework of our method

**Input:** Network $G(V, E)$, threshold $\beta$

**Output:** Core set $Cset$, Core-periphery set $CPset$

1. $U \leftarrow \text{RerankNodes}(G)$
2. $\alpha \leftarrow \lfloor < k > \rfloor$
3. for each $u_i \in U$
4. $RD(u_i) \leftarrow \text{Compute region density of node } u_i \text{ by Eq. (3)}$
5. end for
6. $Cset \leftarrow \text{FindCoreSet}(G, U, \beta)$
7. $CPset \leftarrow \text{FindCPSet}(G, Cset, \text{NumC})$

### 3.1. Re-rank nodes in a new sequence

Consider a network $G(V, E)$, where $V$ is the set of nodes and $E$ is the set of links. Given that the links between core nodes are denser, so we want to re-rank all nodes such that nodes with more common connections approach each other in the new sequence. In doing so, we define a set $U$ and a set $V' = V \setminus U$ to store re-ranked nodes and the remaining nodes, respectively. Initially, $U = \emptyset$ and $V' = V$. To start the sorting process, we need to choose one node as the starting node. We can choose a node with the highest centrality value since the node is more likely to be core node. We here choose the node with the highest closeness centrality as the first node in set $U$, and renumber it as $u_1$, i.e., $U = \{u_1\}$ (we found that the results are not dependent on different centrality indices, such as degree, betweenness, eigenvector, and so on [25]). Now we need to choose a node from $V'$ and put it into $U$ to make sure the new added node has the most connections with the nodes in $U$. If more than one nodes are found, we choose the node with the maximum degree and put it into $U$. Namely, choose the node with the largest value of $P$ and put it into set $U$, which is defined as:

$$P(i) = \sum_{j \in U} A_{ij} + k(i)/k_{max}$$  \hspace{1cm} (1)
The framework of re-ranking process is given in Algorithm 2:

### Algorithm 2: RerankNodes(G)

**Input:** Network \( G(V, E) \)

**Output:** Node set \( U \)

1. \( U \leftarrow \emptyset, V' \leftarrow V \)
2. /*Select the node with the highest closeness centrality as the first node*/
   3. \( u \leftarrow \max(V.\text{closeness}) \)
   4. Add \( u \) into \( U \)
   5. Remove \( u \) from \( V' \)
6. /*Select the node with the most connections with the nodes in \( U \), if more than one node were found, we choose the node with the maximum degree*/
   7. while \( V' \neq \emptyset \) do
      8. for each \( i \in \text{neighbor}_U \) do
         9. \( P(i) \leftarrow \text{Compute priority of node } i \text{ by Eq. (1)} \)
      10. end for
      11. \( u \leftarrow \max_i(P(i)) \)
      12. Add \( u \) into \( U \)
      13. Remove \( u \) from \( V' \)
   14. end while
15. return \( U \)

Take a simple illustration in Fig. 2 as an example, there are four red nodes in \( U \), now one node in \( V' \) (outside of the circle) is going to be added into \( U \) (see Fig. 2(a)). However, nodes \( a \) and \( b \) have the same connections with \( U \), we choose node \( a \) and put it into \( U \) since node \( a \) has larger degree value (see Fig. 2(b)). If their degree values are also the same, one of them is randomly chosen. In this way the nodes who have more common connections are able to close each other in the new sequence.

According to the above method, all nodes are sorted in \( U \) as: \( U = \{u_1, u_2, \ldots, u_N\} \). For instance, the nodes in the karate club network can be re-ranked in this way. The network consists of 34 nodes that represent club members and 78 links that represent friendships among members [16]. The club was split into two social groups because of a conflict of the club president and the instructor, as shown in Fig. 3(a). According
to our defined re-ranking way, node 1 is firstly put into the set $U$ owing to its the highest closeness value (i.e., $u_1 = 1$). Though there are a lot of nodes in $V'$ connecting to node 1, node 3 is secondly added into $U$ since it has the maximal degree value (i.e., $u_2 = 3$). At this time, nodes 1 and 3 are included in set $U$. Next, nodes 2, 4, 14, 8 and 9 in $V'$ connecting to all of nodes in $U$, however, node 2 is added into $U$ since its degree value is the largest. Repeat the above steps until $V' = \emptyset$, then all nodes are renumbered (the re-ranked numbers are shown in the curve of Fig. 3(b)).

3.2. Plot region density curve

We first define a local indicator—connection density (CD) to characterize the density of connections in a subgraph $S$, which is given as:

$$ CD(S) = \frac{2m'}{n'(n' - 1)}. \quad (2) $$

where $n'$ is the number of nodes in $S$ and $m'$ is the number of existing connections in $S$.

A parameter $\alpha$ is defined to measure the minimal size of core. For a given value of $\alpha$, the region density of a node $u_i$ is defined as the connection density of subgraph consisting of nodes from $u_{i-\alpha+1}$ to itself, i.e.,

$$ RD(u_i) = \begin{cases} CD(\{u_1, \cdots, u_i\}), & i \leq \alpha; \\ CD(\{u_{i-\alpha+1}, \cdots, u_i\}), & i > \alpha. \end{cases} \quad (3) $$
Figure 3: Detection on karate club network. (a) original structure, where nodes with the same color are in the same community, (b) region density curve of the network, the numbers in the curve are their original numbers, (c) network shows a dual CP structure, and all nodes are assigned a class value. In addition, node 10 is an overlapping node, (d) matrix representation of the network. Here $\alpha = 4$ and $\beta = 1$. 
We mainly set $\alpha = \lfloor \langle k \rangle \rfloor$ in this work, with $\langle k \rangle$ be the average degree of network and $\lfloor \cdot \rfloor$ be the integral function (In the next context, we also address that different scales of meso-scale structures can be observed by adjusting the value of $\alpha$). We can draw a region density curve after calculating the value of RD regarding to each node, where the horizontal axis denotes the node sequence defined in Sec. 3.1, and the ordinate axis is the value of RD (see Fig. 2(b)). The framework of plotting region density curve is summarized in Algorithm 3:

**Algorithm 3: FindCoreSet($G, U, \beta$)**

**Input:** Network $G(V, E)$, Node set $U$, threshold $\beta$

**Output:** Core set Cset, NumC

1. $Cset \leftarrow \emptyset$
2. $NumC \leftarrow 1$
3. /*if the values of RD for two sequential nodes are greater than or equal to $\beta$, the two cores are merged as a single one*/
4. for $i = \alpha$ to numNode do
5.  if $RD(u_i) \geq \beta$ then
6.     $Cset(NumC) \leftarrow Cset(NumC) \cup \{u_{i-\alpha+1}, \ldots, u_i\}$
7.  else
8.      if $RD(u_{i-1}) \geq \beta$ and $i > \alpha$ then
9.         $NumC \leftarrow NumC + 1$
10.     end if
11. end if
12. end for
13. if $RD(u_{numNode}) < \beta$ then
14.     $NumC \leftarrow NumC - 1$
15. end if
16. return Cset and NumC

3.3. Detect meso-structures

We define a subgraph $S$ be a core if the $CD(S)$ is larger than a threshold value $\beta$. Larger value of $\beta$ gives rise to the stricter definition of core. Once the region density curve is presented, the meso-scale structures can be detected by comparing how many peaks are larger than or equal to $\beta$. Note that some peaks in the beginning of the curve are not valid even though
Figure 4: Region density curves of three types of ideal meso-scale structures. (a) single core-periphery structure, (b) multiple core-periphery structure, (c) community structure. The dot lines are the threshold value $\beta$, which are used to determine the number of sub-CP structures or communities.

whose $RD(u_i) \geq \beta$. For example, if we set $\alpha = 4$, the first three nodes in the region density curve can not form core even though $RD(u_2) = 1$ or $RD(u_3) = 1$, since the number of nodes is smaller than the minimal size of core $\alpha$. Moreover, if the values of RD for two sequential nodes are greater than or equal to $\beta$, the two cores or communities are merged as a single one. A region density curve of networks with CP structure has the following characteristic: the difference between maximum and minimum values is very huge, and the values of $RD$ for most nodes are very low. This is consistent with the fact that the number of periphery nodes is far greater than the number of core nodes, and the connections among core nodes are very dense but among periphery nodes are very few.

Fig. 4 schematically shows the region density curves of three types of ideal meso-scale structures. The region density curves in Fig. 4(a) and Fig. 4(b) correspond to an ideal single CP structure and ideal multiple CP structure, respectively. The region density curves of a community structure without CP structure is similar to a cosine curve, where the difference between peak and least values is not so huge, as shown in Fig. 4(c). Of course, the number of community structures in network is determined by the number of the peaks which reach the value of $\beta$. In sum, we can judge the meso-scale structure of a network according to its region density curve.

Though we can determine the core node and the number of cores based on the region density curve, some important problems should be further considered. Firstly, when multiple CP structure is detected, we should
know that the periphery nodes belong to which core and how close with their core nodes; Secondly, when community structure exists in a network, can we find the overlapping nodes who belong to different communities? To this end, we begin to expand periphery nodes from each core to form its sub-CP structure. At the beginning, each sub-CP structures only contains core nodes themselves and they are defined as class 0. The periphery nodes who have direct connections with core nodes are valued as class 1, and they are pre-allocated to the core which has the most connections with them. In this way, one or several initial sub-CP structures are formed (each sub-CP structure only contains core nodes and their periphery neighbors). Next, the neighbors of the initial sub-CP structures are defined as class 2, and they are pre-allocated to the sub-CP structure which has the most connection with them. The expanding process finishes until all periphery nodes are pre-allocated and their class values are determined. Many experimental results have demonstrated that most of real networks are “small-world”, i.e., their average length of paths are very short [3]. Thus, the above expanding process can be finished in several steps.

Now, we want to address why we need to pre-allocate the periphery nodes. The reason is that some periphery nodes may be pre-allocated to more than one sub-CP structure or communities, namely, they have the same number of connections with two or several sub-CP/community structures. These nodes are viewed as active nodes. One may intuitively think that these active nodes are the overlapping nodes because they connect different sub-CP structures or communities. However, we should address that many active nodes are not real overlapping nodes, for example, even though an important person need to frequently connect the leaders in different organizations owing to his special role, but which does not mean that the important person belongs to different organizations. Therefore, we need to extract these active nodes and re-distribute them to a sub-CP structure/community who has the most connections with the active nodes. If there are still some nodes who are assigned to more than one sub-CP structure/community, this kind of nodes are viewed as the real overlapping nodes. The framework of detecting meso-structure is demonstrated in Algorithm 4:

We still use karate club network as an example to help us understand our method. In Sec. 3.1, all nodes have been re-ranked and the region density curve is plotted in Fig. 3(b). Now we should use the number of peaks in the curve to determine the meso-scale structures. According to
Algorithm 4: \textit{FindCPSet}(G, Cset, NumC)

\textbf{Input:} Network $G(V, E)$ Core set $Cset$, NumC

\textbf{Output:} Core-periphery set CPset

1: $CPset \leftarrow Cset$
2: $class \leftarrow 0$
3: $nodeCset.class \leftarrow class$
4: $NodeP \leftarrow V - nodeCset$
5: /*Pre-allocate the periphery nodes*/
6: \textbf{while} $NodeP \neq \emptyset$ \textbf{do}
7: \hspace{1em} $neighbor\_CPset.class \leftarrow class + 1$
8: \hspace{1em} \textbf{for} each $j \in neighbor\_CPset$ \textbf{do}
9: \hspace{2em} \textbf{if} $connections(j, CPset(i))$ is max \textbf{then}
10: \hspace{3em} Add $j$ into $CPset(i)$
11: \hspace{2em} \textbf{end if}
12: \hspace{1em} \textbf{end for}
13: \hspace{1em} Remove $neighbor\_CPset$ from $NodeP$
14: \textbf{end while}
15: $NodeActive \leftarrow$ node in different $CPset(i)$
16: /*Re-distribute active nodes*/
17: \textbf{for} each $j \in NodeActive$ \textbf{do}
18: \hspace{1em} \textbf{if} $connections(j, CPset(i))$ is max \textbf{then}
19: \hspace{2em} Add $j$ into $CPset(i)$
20: \hspace{1em} \textbf{end if}
21: \textbf{end for}
22: \textbf{return} $CPset$
the assumption $\alpha = \lceil \langle k \rangle \rceil$, one has $\alpha = 4$ owing to $\langle k \rangle \approx 4.59$. Here we set core density $\beta = 1$, that is to say, all core nodes are fully connected. As shown in the region density curve, the values of $RD(4)$, $RD(14)$ and $RD(31)$ are equal to 1. Thus, the region formed by these three nodes are $\{1, 3, 2, 4\}$, $\{3, 2, 4, 14\}$ and $\{9, 34, 33, 31\}$ (though $RD(2) = 1$ and $RD(3) = 1$, neither of them is valid peak since their region is too small to form core). Because node 4 and node 14 are sequently placed in the curve, the two cores are merged into one larger core, i.e., $\{1, 3, 2, 4, 14\}$. Now there are two cores: $C_1 = \{1, 3, 2, 4, 14\}$ and $C_2 = \{9, 34, 33, 31\}$. As shown in Fig. 3(c), the red and dark blue nodes (class 0) are core nodes. According to the above expanding method, we can know that each periphery node belongs to which core and how far from its own core.

In the first allocation, nodes 10, 28 and 29 are allocated to the two sub-CP structures, so the three active nodes need to be re-allocated. In the secondary distribution, nodes 28 and 29 are allocated to the right sub-CP structure (see Fig. 3(c)). However, node 10 still has the same number of connections to the two sub-CP structures, which is viewed as the real overlapping node.

Fig. 3(d) shows the adjacency matrix of karate club network, which is very similar to the ideal multiple CP structure shown in Fig. 1(c). Although karate club network has been used as a standard benchmark network for community detection, our result indicates that the meso-scale structure of karate club network is a dual core-periphery structure, leading to a community network with two communities. We should address that even though some community detection algorithms can divide the karate club network into two groups, which do not clearly clarify each node being core node or periphery node, namely the inner structure in each group is not well answered. Our method not only divides the network into two groups, but also accurately differentiate the core nodes and periphery nodes in each group. In addition, our method can further distinguish the roles of periphery nodes, identify the active nodes and overlapping nodes simultaneously. In short, our method provides a more systematic and accurate description of the meso-scale structure and the roles of nodes in networks.
4. Experimental Results

In this section, we examine the meso-scale structures in some real-world networks and a synthetic benchmark network used in community network detection.

4.1. USA airport network

The USA airport network has 332 nodes representing airports and 2126 unweighted links describing the airlines between airports [44]. The average degree of this network is $\langle k \rangle = 12.81$, so the value of $\alpha$ is 12. By setting $\beta = 1$, as shown in Fig. 5(b), only one peak in the region density curve reaches the value of 1, the values of RD for the other nodes are very low. So this network exhibits a strong single CP structure. There are 22 core nodes (including nodes 118, 261, 255, 152, 182, 230, 166, 67, 112, 201, 147, 162, 293, 258, 248, 217, 299, 174, 109, 219, 131 and 167. The core nodes are the red nodes in Fig. 5(a), and the core nodes are also enlarged in the region density curve of Fig. 5(c)).

If we reduce the value of $\alpha$, we can find different sizes of meso-scale structures. When the value of $\alpha$ ranges from 6 to 11, the USA airport network exhibits still a single CP structure, just a slight change of the number of core nodes. When $\alpha = 5$, besides the largest CP structure, another three smaller meso-scale structures can be observed. Each of them has 5 core nodes. And their number of periphery nodes are 3, 14, and 1 respectively. Generally speaking, the number of periphery nodes is larger than the number of core nodes, so only secondary small meso-scale structure can be viewed as a CP structure, the other two are the small scale community structure. Namely, there are a large CP structure, a small CP structure and two small community structures in this network. Region density curve for $\alpha = 5$ is shown in Fig. 5(d), and there are four peaks reaching 1. The first peak contains more nodes, while the other three peaks contain fewer nodes. And the values of RD for most remaining nodes are very low. Figs. 5(e) and (f) show the adjacency matrix of USA airport network for $\alpha = 12$ and $\alpha = 5$, respectively. The adjacency matrix shown in Fig. 5(e) is very similar to the ideal case shown in Fig. 1(a), indicating this network exhibits a single CP structure. But when $\alpha = 5$, there are three smaller meso-scale structures shown in the lower right area of Fig. 5(f).
Figure 5: Detection on USA airport network. (a) original structure, where red nodes are core nodes, (b) fully region density curve when $\alpha = 12$, (c) region density curve of the first 25 nodes when $\alpha = 12$, (d) full region density curve when $\alpha = 5$, (e) matrix representation of the network when $\alpha = 12$, (f) matrix representation of the network when $\alpha = 5$. Here $\beta = 1$. 
4.2. Dolphin social network

The dolphin social network is an undirected social network of frequent associations between 62 dolphins in a community living off Doubtful Sound, New Zealand. The network has 62 nodes representing the dolphins and 159 links denoting the frequent associations between dolphins [28]. The community is composed of two families, as shown in Fig. 6(a), nodes with different colors belong to different families.

The value of $\alpha = 5$ in this network. As shown in Fig. 6(b), the difference between the peak and lowest value in region density curve is very huge. The values of RD for most nodes are very low. So we can judge the dolphin social network is a CP structure. If we set $\beta = 1$, there are two peaks, but the first peak is invalid since there are only three nodes (nodes 34, 38 and 41). Only the subgraph including nodes 46, 30, 22, 52 and 19 forms a core. Here we can reduce $\beta$ to 0.9 to relax the definition of core. We should emphasize that the assumption is reasonable since only one connection is missed in the core group (for a subgraph with five nodes, the possible number of connections is $\binom{5}{2} = 10$). Now, $RD(25) = 0.9$ and $RD(10) = 0.9$, that is to say, two cores emerge: $C_1 = \{46, 30, 22, 52, 19, 25\}$ and $C_2 = \{14, 42, 58, 18, 10\}$, and they are marked as red and blue colors, respectively. So we can say that the dolphin social network is a dual CP structure too. Similar to the method in the karate club network, all periphery nodes can be classified to one corresponding core group and assigned a class value denoting how far from their core group. Fig. 6(d) demonstrates the adjacency matrix of the dolphin social network.

4.3. Political blogs network

Adamic and Glance constructed a network of political blogs during the 2004 U.S. Presidential election. The nodes of this network are blogs about US politics and the edges are hyperlinks between these blogs [1]. There are 1222 blogs and 16714 connections in the network. Since this network displays a marked division into groups of conservative and liberal blogs, which has been viewed as a typical example of community structure [42].

After plotting the region density curve (see Fig. 7(b)), one can see that there are two obvious peaks in the curve. So the network is a typical dual CP structure. By using the expanding method, the periphery nodes can be divided into their core group and form a corresponding sub-CP structure (see Fig. 7(a)), where blue nodes are the periphery of yellow cores and the red nodes are the periphery of green cores, respectively. Moreover, there
Figure 6: Detection on Dolphin social network. (a) original structure, where the nodes with the same color in one community, (b) region density curve, (c) detection by our method, where the network is a dual CP structure and each node is assigned a class value, (d) matrix representation of the network. Here $\alpha = 5$ and $\beta = 0.9$. 
are 11 nodes marked by light blue color are the overlapping nodes. The adjacency matrix in Fig. 7(c) also validates that the network is a dual CP structure.

Some methods based on centrality indices assume that the nodes with higher centrality values are core nodes [10]. In this network, we pick two nodes with larger degree values (orange node in left side and the light purple node in right side of Fig. 7(a)) as examples to argue that nodes with larger centrality values are not always be the core nodes. The main reason is that these nodes have many connections with periphery nodes but few connections with core nodes.

4.4. Synthetic benchmark network

Our experimental results indicate that our method can not only find out CP structure but also the community structure. But one should note that not all community networks denote the existence of the CP structure. We construct a synthetic benchmark network based on the approach in Ref. [22] as an example. The generated network is often used as a benchmark for testing the performance of community detection algorithms. The parameters are set as follows: the average degree $\langle k \rangle = 8$, the maximum degree $k_{\text{max}} = 12$, the mixing parameter $\mu = 0.1$, and the exponents of the power law distribution of node degrees $\gamma_1$ and community size $\gamma_2$ are -2 and -1, respectively. The network has 100 nodes and 404 links, and it
is composed of 10 communities whose sizes range from 7 to 12. The formation mechanism can only guarantee emergence of community structure but cannot ensure there is no core in each community. The region density curve of the synthetic network is shown in Fig. 8(a), which is very similar to the curve shown in Fig. 4(c). So we can judge the network does not exhibit CP structure. The adjacency matrix of the network shown in Fig. 8(b) is very similar to the Fig. 1(a), which further validates that the network has no CP structure.

4.5. College football network

The college football network describes games between Division I-A American college football teams in the year 2000 [26]. It has 115 nodes and 613 connections, leading to $\alpha = 10$. From the region density curve in Fig. 9(a), we can conclude that the college football network only has community structure but no CP structure. By setting $\alpha = 8$, the characteristics of region density curve is more clear and the peaks are easier to be observed (see Fig. 9(b)). For the football network, the inner connections in each community are not very dense, so is hard to achieve full connection in each community. For this purpose, we set $\beta = 0.6$ to detect the community structure. According to our method, the network can be divided into 11 communities. The adjacency matrix of the network is shown in Fig. 9(c), which is very similar to the adjacency matrix shown in Fig. 1(a).
5. The detection on overlapping community networks

The main purpose of our method is not just to propose a community detection algorithm, thus we do not redundantly compare our method with other community detection algorithms. We just use the normalized mutual information (NMI) index to compare our method with other methods on the overlapping community detection. Three classical networks whose true community structures are known are considered: karate club network, dolphin social network and college football network. We compare our method with three well-known overlapping community detection algorithms: clique percolation method (CPM) proposed by Palla et al. [30], a local expansion and optimization algorithm (LFM) proposed by Lancichinetti et al. [22], and a fast overlapping community search (FOCS) method developed by Bandyopadhyay et al. [4]. The comparison of them are summarized in table 1.

| Network          | CPM | LFM | FOCS | Our method |
|------------------|-----|-----|------|------------|
| karate           | 0.17| 0.69| 0.2249| 0.9186     |
| dolphins         | 0.254| 0.781| 0.2375| 0.8889     |
| college football | 0.697| **0.754**| 0.6650| 0.741      |

From the table 1, one can see that the performance of our method on overlapping community detection is also surprising.
6. Conclusions

In this work, we have proposed a unified method to detect CP structure and community structure in networks. This method is effective not only on single CP structure, but also on community structure and multiple CP structures, and further on finding active nodes and overlapping nodes in networks. We can also obtain precise position of each node in a network. In addition, our method does not fix the size of core in advance, where some sequential cores can form a larger core. Therefore, our method provides a tool for the identification of meso-scale structures in network, and may also provide some inspirations in identifying the influential nodes. Of course, many places are worthy of future study. In most cases, we set $\alpha = \lfloor \langle k \rangle \rfloor$ and $\beta = 1$. However, for each real network, how to determine proper values for them is a non-negligible problem.

Acknowledgments

This work is funded by the NSFC (Grant Nos. 61473001, 11331009, 61672033).

References

References

[1] Lada A Adamic and Natalie Glance. The political blogosphere and the 2004 us election: divided they blog. In Proceedings of the 3rd international workshop on Link discovery, pages 36–43. ACM, 2005.

[2] Yong-Yeol Ahn, James P Bagrow, and Sune Lehmann. Link communities reveal multiscale complexity in networks. Nature, 466(7307):761–764, 2010.

[3] Réka Albert and Albert-László Barabási. Statistical mechanics of complex networks. Reviews of Modern Physics, 74(1):47, 2002.

[4] Sanghamitra Bandyopadhyay, Garisha Chowdhary, and Debarka Sengupta. FOCS: Fast overlapped community search. IEEE Transactions on Knowledge and Data Engineering, 27(11):2974–2985, 2015.
[5] Paolo Barucca and Fabrizio Lillo. Disentangling bipartite and core-periphery structure in financial networks. *Chaos, Solitons & Fractals*, 88:244–253, 2016.

[6] Stephen P Borgatti and Martin G Everett. Models of core/periphery structures. *Social networks*, 21(4):375–395, 2000.

[7] John P Boyd, William J Fitzgerald, and Robert J Beck. Computing core/periphery structures and permutation tests for social relations data. *Social networks*, 28(2):165–178, 2006.

[8] Peter Csermely, András London, Ling-Yun Wu, and Brian Uzzi. Structure and dynamics of core/periphery networks. *Journal of Complex Networks*, 1(2):93–123, 2013.

[9] Mihai Cucuringu, M Puck Rombach, Sang Hoon Lee, and Mason A Porter. Detection of core-periphery structure in networks using spectral methods and geodesic paths. *arXiv preprint arXiv:1410.6572*, 2014.

[10] Marcio Rosa Da Silva, Hongwu Ma, and An-Ping Zeng. Centrality, network capacity, and modularity as parameters to analyze the core-periphery structure in metabolic networks. *Proceedings of the IEEE*, 96(8):1411–1420, 2008.

[11] Fabio Della Rossa, Fabio Dercole, and Carlo Piccardi. Profiling core-periphery network structure by random walkers. *Scientific Reports*, 3:1467, 2013.

[12] Susan Doolittle. The self-organizing economy. *Business Economics*, 31(2):71–73, 1996.

[13] Patrick Doreian. Structural equivalence in a psychology journal network. *Journal of the American Society for Information Science*, 36(6):411–417, 1985.

[14] TS Evans and R Lambiotte. Line graphs, link partitions, and overlapping communities. *Physical Review E*, 80(1):016105, 2009.

[15] Santo Fortunato. Community detection in graphs. *Physics Report*, 486(3):75–174, 2010.
[16] Michelle Girvan and Mark EJ Newman. Community structure in social and biological networks. Proceedings of the National Academy of Sciences, 99(12):7821–7826, 2002.

[17] Steve Gregory. Finding overlapping communities in networks by label propagation. New Journal of Physics, 12(10):103018, 2010.

[18] Steve Harenberg, Gonzalo Bello, La Gjeltema, Stephen Ranshous, Jitendra Harlalka, Ramona Seay, Kanchana Padmanabhan, and Nagiza Samatova. Community detection in large-scale networks: A survey and empirical evaluation. Wiley Interdisciplinary Reviews: Computational Statistics, 6(6):426–439, 2014.

[19] Miao He, Mingwei Leng, Fan Li, Yukai Yao, and Xiaoyun Chen. A node importance based label propagation approach for community detection. In Proceedings of the Seventh International Conference on Intelligent Systems and Knowledge Engineering, pages 249–257, 2014.

[20] Petter Holme. Core-periphery organization of complex networks. Physical Review E, 72(4):046111, 2005.

[21] Jussi M Kumpula, Mikko Kivelä, Kimmo Kaski, and Jari Saramäki. Sequential algorithm for fast clique percolation. Physical Review E, 78(2):026109, 2008.

[22] Andrea Lancichinetti and Santo Fortunato. Community detection algorithms: A comparative analysis. Physical Review E, 80(5):056117, 2009.

[23] Andrea Lancichinetti, Santo Fortunato, and János Kertész. Detecting the overlapping and hierarchical community structure in complex networks. New Journal of Physics, 11(3):033015, 2009.

[24] Roger J Nemeth and David A Smith. International trade and world-system structure: A multiple network analysis. Review (Fernand Braudel Center), 8(4):517–560, 1985.

[25] Mark Newman. Networks: an introduction. Oxford university press, 2010.
[26] Mark EJ Newman. Fast algorithm for detecting community structure in networks. *Physical Review E*, 69(6):066133, 2004.

[27] Mark EJ Newman. Modularity and community structure in networks. *Proceedings of the National Academy of Sciences*, 103(23):8577–8582, 2006.

[28] Mark EJ Newman and Michelle Girvan. Finding and evaluating community structure in networks. *Physical Review E*, 69(2):026113, 2004.

[29] Mari Otokura, Kenji Leibnitz, Tetsuya Shimokawa, and Masayuki Murata. Evolutionary core-periphery structure and its application to network function virtualization. *Nonlinear Theory and Its Applications, IEICE*, 7(2):202–216, 2016.

[30] Gergely Palla, Imre Derényi, Illés Farkas, and Tamás Vicsek. Uncovering the overlapping community structure of complex networks in nature and society. *Nature*, 435(7043):814–818, 2005.

[31] M Puck Rombach, Mason A Porter, James H Fowler, and Peter J Mucha. Core-periphery structure in networks. *SIAM Journal on Applied mathematics*, 74(1):167–190, 2014.

[32] Martin Rosvall and Carl T Bergstrom. Maps of random walks on complex networks reveal community structure. *Proceedings of the National Academy of Sciences*, 105(4):1118–1123, 2008.

[33] Michael T Schaub, Renaud Lambiotte, and Mauricio Barahona. Encoding dynamics for multiscale community detection: Markov time sweeping for the map equation. *Physical Review E*, 86(2):026112, 2012.

[34] Murray Shanahan and Mark Wildie. Knotty-centrality: finding the connective core of a complex network. *PLoS One*, 7(5):e36579, 2012.

[35] David A Smith and Douglas R White. Structure and dynamics of the global economy: network analysis of international trade 1965–1980. *Social forces*, 70(4):857–893, 1992.

[36] T Verma, F Russmann, NAM Araújo, J Nagler, and HJ Herrmann. Emergence of core-peripheries in networks. *Nature communications*, 7:10441, 2016.
[37] Harrison C White, Scott A Boorman, and Ronald L Breiger. Social structure from multiple networks. i. blockmodels of roles and positions. American journal of sociology, pages 730–780, 1976.

[38] Bo Yang, Jin Di, Jiming Liu, and Dayou Liu. Hierarchical community detection with applications to real-world network analysis. Data & Knowledge Engineering, 83:20–38, 2013.

[39] Bo Yang, Jiming Liu, and Jianfeng Feng. On the spectral characterization and scalable mining of network communities. IEEE Transactions On Knowledge and Data Engineering, 24(2):326–337, 2012.

[40] Jaewon Yang and Jure Leskovec. Overlapping community detection at scale: A nonnegative matrix factorization approach. In Proceedings of the Sixth ACM International Conference on Web Search and Data Mining, pages 587–596, 2013.

[41] Le Yu, Bin Wu, and Bai Wang. Topic model-based link community detection with adjustable range of overlapping. In Proceedings of the 2013 IEEE/ACM International Conference on Advances in Social Networks Analysis and Mining, pages 1437–1438, 2013.

[42] Shihua Zhang, Xuemei Ning, and Xiang-Sun Zhang. Identification of functional modules in a PPI network by clique percolation clustering. Computational Biology and Chemistry, 30(6):445–451, 2006.

[43] Xiao Zhang, Travis Martin, and Mark EJ Newman. Identification of core-periphery structure in networks. Physical Review E, 91(3):032803, 2015.

[44] Tao Zhou, Linyuan Lü, and Yi-Cheng Zhang. Predicting missing links via local information. The European Physical Journal B, 71(4):623–630, 2009.