A theory of mass and gravity in 4-dimensional optics

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Abstract

The paper deals with the concepts of mass and gravity in the formalism of 4-dimensional optics, previously introduced by the author. It is shown that elementary particles can be associated with 4-dimensional standing wave patterns with the Compton wavelength and both inertial and gravitational mass are derived from this concept and shown to be attributable to an waveguide laid along the particle’s worldline; the same formalism is shown to accommodate also mass due to binding energy within compact bodies. Momentum exchange with accelerated bodies through gravitons is discussed and shown similar to mode exchange in optical fibers. Reported anomalies on the behaviour of the Foucault pendulum, both periodic and exceptional on the occasion of solar eclipses, are explained not only qualitatively but also on order of magnitude, resorting to graviton exchanges between Earth and the Sun or the Moon. It is argued that these effects provide experimental evidence of gravitons.

1 Introduction

The paper’s purpose is to show that mass and gravity can be derived from geometrical properties of space alone. Two points of departure are used in order to reach the same end, those being mass scaling of coordinates introduced in a previous paper \cite{1} and Compton wavelength for elementary particles.

In the work mentioned above, corrected and complemented in a later paper \cite{2}, mass scaling of coordinates was introduced to allow massive bodies to follow metric geodesics of equation

\[ 2L = g_{\alpha\beta} x^\alpha \dot{x}^\beta = 1, \]

where \( L \) is the movement Lagrangian and ”dot” indicates time derivative. In this formulation \( g_{\alpha\beta} \) incorporates a factor \( m^2 \), equal to the square of the moving body’s mass, which scales local coordinates. An alternative method would be to

\footnote{Greek letters are used for indices taking values between 0 and 3 and roman letters for indices with values between 1 and 3. Use is also made of indices that refer to a specific coordinate, like \( r \), \( \theta \) and \( \varphi \) with spherical coordinates.}
scale arc length as time divided by the moving body’s mass, but this approach breaks the nice symmetry of the Universal variational principle given by $dt^2 = g_{\alpha\beta}dx^\alpha dx^\beta$ and was rejected in favor of coordinate scaling.

The same papers linked Compton wavelength of elementary particles to waves of angular frequency $\omega = mc^2/\hbar$ associated to those particles and propagating along their worldline. In the previous expression $c$ is the speed of light in vacuum and $\hbar$ is Planck’s constant divided by $2\pi$. Using non-dimensional units mass becomes exactly equal to the frequency associated with the particle, so it is expected that mass and frequency are just two different views of the same reality. Further along the expression Compton frequency will sometimes be used to refer to the angular frequency associated with an elementary particle; this will be considered the same as the particle’s mass expressed in different units.

This paper develops the theory further, establishing a close connection between matter, gravity and periodic oscillations of space. The theory is based on the formation of 4-dimensional wave patterns as a result of localized resonance modes in elementary particles, something that other authors have already suggested. Both inertia and gravity will be derived from this simple concept and will assume that massive bodies act as superposition of elementary particle vibrational modes, together with modes due to orbital and vibrational frequencies within the body. This paper is concerned solely with gravity, but the author believes that other interactions, namely electrodynamics and chromodynamics, will eventually be included in the theory under a unified approach.

2 Equivalence between Compton frequency and inertial mass

Previous papers introduced a 4-dimensional space with signature 4 which in many circumstances of interest can become Euclidean. These circumstances include movement under the gravitational fields due to stationary bodies, which are generally described by the metric $g_{\alpha\beta} = m^2n^2\delta_{\alpha\beta}$, with $m$ the inertial mass of a moving body and $n$ the space curvature due to gravity. What this metric tells is that in order to use time interval to measure geodesic arc length one must use a local scale factor $n$ associated with an inertia scale factor $m$. It is possible to use unscaled coordinates if the geodesic arc length is no longer directly associated with time but rather $ds = dt/mn$. The local scale factor does not represent intrinsic curvature, but is rather a convenience that one can choose to use or not. There are situations, naturally, when space cannot be flattened by a simple change of coordinates; these include those cases when the field source is not stationary, namely when it is accelerated. The works

\footnote{Non-dimensional units are obtained dividing length, time and mass by the factors $\sqrt{G\hbar/c^5}$, $\sqrt{G\hbar/c^5}$ and $\sqrt{\hbar c/G}$, respectively; $G$ is the gravitational constant. Electric charge is normalized by the charge of the electron but this normalization will not be needed in the present work.}
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mentioned above discuss the electromagnetic field due to a moving charge, as one situation where a change of coordinates would not flatten space.

It is natural to address the simple cases first and defer the complications to later discussions. This paper proceeds in that line, starting with the field and associated metric due to a stationary elementary particle and generalizing the conclusions in successive steps. The premises are the Universal measurement of geodesic arc length provided by time intervals, if scaled coordinates are used, and the association of a Compton frequency with every elementary particle.

According to the theory developed in Ref. [1], the 4-dimensional worldline of a particle with mass \( m \) is given by Eq. (1). If the particle is under the influence of a stationary gravitational field the metric takes the form

\[
m^2 n^2 \delta_{\alpha \beta},
\]

with \( n \) a function of the spatial coordinates \( x^i, (i = 1, 2, 3) \). Considerations made in the next section justify the allowance that is made here for the gravitational field \( n \) of a stationary body to be a function of all the 4 coordinates without changing the diagonal form of the metric. Defining the conjugate momentum \( k_\alpha = \partial L / \partial \dot{x}^\alpha \) it is

\[
k_\alpha = g_{\alpha \beta} \dot{x}^\beta = m^2 n^2 \delta_{\alpha \beta} \dot{x}^\beta, \tag{2}
\]

which allows the geodesic equation to re-written in terms of momentum components

\[
\delta^{\alpha \beta} k_\alpha k_\beta = m^2 n^2. \tag{3}
\]

This equation remains unchanged if both sides are multiplied by the harmonic wave function \( \psi = \exp(j k_\alpha x^\alpha) \). This wave function represents a pattern of standing plane waves in 4-dimensional space but it is a truly propagating wave in 3D; in fact spatial dependence can be separated from the dependence on \( x^0 \) as \( \psi = \exp[j (k_0 x^0 + k_i x^i)] \), to highlight that when \( x^0 \) is fixed the wave exhibits a sinusoidal variation along the direction defined by \( k_i \). The 4-dimensional momentum \( k_\alpha \) functions as wave vector for the stationary wave pattern.

If both sides of Eq. (3) are multiplied by \( \psi \), noting that \( \partial_\alpha \psi \partial_\beta \psi = \dot{\psi} \partial_\alpha \partial_\beta \psi \), one gets the harmonic wave equation

\[
\delta^{\alpha \beta} \partial_\alpha \partial_\beta \psi = -m^2 n^2 \psi, \tag{4}
\]

indicating that the wave pattern has a spatial frequency \( mn \) along the wavefront normal.

The equation can have a different interpretation if a new wave function is introduced as \( \Psi = m \psi \). Then \( m^2 = \Psi \Psi^* \), with * standing for complex conjugate, and he new equation is

\[
\frac{\delta^{\alpha \beta} \partial_\alpha \partial_\beta \Psi}{\Psi \Psi^* n^2} = -\Psi. \tag{5}
\]

It is possible to say that the spatial frequency is always unity if coordinates are scaled by mass \( \sqrt{\Psi \Psi^*} \) and field \( n \). While the first interpretation is consistent with the definition of Compton wavelength, the latter is more in line with the author’s previous work.
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It is now convenient to consider a more general situation where the metric is allowed to have an arbitrary form. It is always possible to make \( g_{\alpha \beta} = m^2 n_{\alpha \beta} \) and replace Eq. (3) by

\[
n^\alpha_k k_\beta = m^2, \tag{6}
\]

with \( n^{\alpha \beta} = (n_{\alpha \beta})^{-1} \). Introducing the wave function \( \Psi \) as before,

\[
\frac{n^{\alpha \beta} \partial_\alpha \Psi \partial_\beta \Psi}{\Psi \Psi^*} = -\Psi, \tag{7}
\]

which is consistent with the definition of a Lagrangean density by

\[
\mathcal{L} = \frac{n^{\alpha \beta} \partial_\alpha \Psi \partial_\beta \Psi}{\Psi \Psi^*} - \Psi^2. \tag{8}
\]

The corresponding Euler-Lagrange equation is

\[
\partial_\alpha \left( \frac{n^{\alpha \beta} \partial_\alpha \Psi \partial_\beta \Psi}{\Psi \Psi^*} \right) = \partial_\alpha \partial^\alpha \Psi = -\Psi. \tag{9}
\]

This is the general form of wave equation (5).

Eqs. (6) and (9) represent two different views of the same phenomenon; the first one describes a particle’s worldline, while the latter describes an equivalent wave pattern and can be seen as the analogous to Klein-Gordon equation in 4-dimensional optics. This is not unlike ray and wave descriptions of optics, which are equivalent as long as the dimensions involved remain large compared to the wavelength. Similarly in 4-dimensional optics one is allowed to deal with particle’s worldlines as long as all the dimensions involved are large compared to their Compton wavelengths.

It is possible to conclude that an elementary particle with known momentum can be associated with a 4-dimensional stationary wave with spatial angular frequency equal to the particle’s mass multiplied by the local gravitational field and wave vector equal to its momentum.

3 Vacuum, gravitons and photons

General relativity accepts that space is curved by gravity and moving bodies are affected by space curvature. The assumption of 4-dimensional optics is that not only the gravitational field but also the inertial mass of a moving body determine curvature, the latter through coordinate scaling. It has been shown before that electromagnetic fields can be assigned to space curvature, which is then determined also by the electric charge of the moving particle. The present paper is concerned mainly with gravity and so considerations about electromagnetic fields will not be extended; it is important to understand, though, that ultimately all gravitational and electromagnetic fields result from the superposition of electrostatic and gravitational fields due to elementary particles. The cited works showed that Lorentz force can effectively be deduced from the
electrostatic field of a moving charge. Accordingly the effect of a moving electrically charged elementary particle on empty space can be examined and it can be accepted that the latter must be filled with a superposition of similar effects.

In Ref. [2] the author established the fields due to both gravity and electric charge of a body of mass $M$ and charge $Q$ as $n = \exp(M/r)$ and $v = \exp(Q/r)$, respectively. Further along in this paper discusses how the gravitational field is generated, while the electrostatic field will be the subject of future work; for now it is useful to accept the expressions above just as a result of compatibility with Newtonian and electrostatic forces. If another body with mass $m$ and electric charge $q$ is under the influence of those fields, its movement follows the geodesic of the space defined by the metric

$$g_{\alpha\beta} = m^2 \begin{bmatrix} e^{2(mM+qQ)/mr} & 0 & 0 & 0 \\ 0 & e^{2M/r} & 0 & 0 \\ 0 & 0 & e^{2M/r} & 0 \\ 0 & 0 & 0 & e^{2M/r} \end{bmatrix}.$$  

It is convenient to decompose the metric into four components as $g_{\alpha\beta} = m^2 n_{\alpha\beta} (v_{\alpha\beta})^q/m$, where $n_{\alpha\beta}$ designates the gravitational field, $v_{\alpha\beta}$ the electric field, $m$ is the inertial mass and $q$ the electric charge.

There are some important consequences of the equation above. First of all notice that the metric due to electrostatic and gravitational fields is diagonal and can have an anisotropy on the 0th element if the electrostatic field is present. A stationary body could be the source of an electromagnetic field but this is never the case with an elementary particle. Notice also that the electric charge of the body that suffers the influence of the electrostatic field is equally responsible for the anisotropy, while its mass influences the whole metric. In fact the metric ceases to exist if there is no inertial mass. This is the result of the concept of metric linked to the movement and not to space itself. A further point that needs to be raised is that the fields don’t die away as distance increases, but rather tend exponentially to unity, leading to a concept of a field filled vacuum, entirely compatible with the uncertainty principle and postulates by other authors [3, 5].

The fact that inertial mass is essential for the existence of a movement metric is shown in the wave equation (9) by its collapse when $\Psi$ has zero amplitude. The question that must be addressed is the possibility of existence of some type of wave solutions in vacuum which don’t require mass. One can have a particle approach similar to what was done for photons in Refs. [1, 2] or a wave approach which is done below.

Consider a body following its worldline where it is possible evaluate the derivatives $\ddot{x}^i = dx^i/dx^0$. When this body is stationary it is the source of a field which is here restricted to gravity and designated $n_{\alpha\beta} = n^2 \delta_{\alpha\beta}$; the evaluation of the field when the body is moving involves the consideration of the tensor

$$\Lambda_{\alpha\beta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\ddot{x}^1 & 1 & 0 & 0 \\ -\ddot{x}^2 & 0 & 1 & 0 \\ -\ddot{x}^3 & 0 & 0 & 1 \end{bmatrix}.$$  

(11)
where the ”bar” over an index indicates coordinates of the moving frame. The moving frame is taken to be the moving body’s frame and the field on this frame is designated by \( n_{\bar{\mu}\bar{\nu}} \); the field on the stationary frame is given by

\[
n_{\alpha\beta} = \Lambda_{\alpha}^{\bar{\mu}} \Lambda_{\beta}^{\bar{\nu}} n_{\bar{\mu}\bar{\nu}};
\]

(12)

making the substitutions one gets

\[
n_{\alpha\beta} = n^2 \begin{bmatrix}
1 - \vec{x}^1 & -\vec{x}^2 & -\vec{x}^3 \\
-\vec{x}^1 & 1 + (\vec{x}^1)^2 & \vec{x}^1 \vec{x}^2 \\
-\vec{x}^2 & \vec{x}^1 \vec{x}^2 & 1 + (\vec{x}^2)^2 \\
-\vec{x}^3 & \vec{x}^1 \vec{x}^3 & \vec{x}^2 \vec{x}^3
\end{bmatrix}.
\]

(13)

Evaluating \( n^{\alpha\beta} \):

\[
n^{\alpha\beta} = \frac{1}{n^2} \begin{bmatrix}
1 + \delta_{ij} \vec{x}^i \vec{x}^j & \vec{x}^1 & \vec{x}^2 & \vec{x}^3 \\
\vec{x}^1 & 1 & 0 & 0 \\
\vec{x}^2 & 0 & 1 & 0 \\
\vec{x}^3 & 0 & 0 & 1
\end{bmatrix};
\]

(14)

recalling Eq. (11) and passing \( \Psi \Psi^* \) to the second member the wave equation of a zero mass field is obtained

\[
\partial_{\alpha} (n^{\alpha\beta} \partial_{\beta} \phi) = 0.
\]

(15)

A gravitational field \( n^{\alpha\beta} \) is of the form given by Eq. (14), even if it is the result of a superposition of many individual gravitational fields; naturally in the latter case the \( \vec{x}^i \) must be replaced by something that results from the combined movements of all the field sources. Eq. (15) can be expanded as

\[
\partial_0 (n^{0\beta} \partial_j \phi) + \partial_{\alpha} (n^{\alpha0} \partial_0 \phi) + \frac{1}{n^2} \delta^{ij} \partial_{ij} \phi = 0.
\]

(16)

It can be shown that the equation does not hold any solutions of interest when the field is stationary, i.e. when the field is created by a body in uniform motion. For an accelerated body it is possible to try a solution on 3-space, by setting \( x^0 = 0 \); if one is interested on a 3-space solution it is possible also to try a tangent function \( \Phi \), which does not depend on \( x^0 \) but has the same dependence on the spatial coordinates as \( \phi \). The resulting equation is

\[
\partial_0 n^{0j} \partial_j \phi + \frac{1}{n^2} \delta^{ij} \partial_{ij} \Phi = 0.
\]

(17)

A plane wave type solution requires \( \Phi = \exp(jk_i x^i) \) and leads to

\[
j \partial_0 n^{0j} k_j - \frac{\delta^{ij} k_i k_j}{n^2} = 0.
\]

(18)

For simplicity it is assumed that the body follows a circular orbit with radius \( r \) and proper time angular speed \( \omega \) around the \( x^3 \) axis. Accordingly \( x^1 = \cos(\omega x^0), x^2 = \sin(\omega x^0), x^3 = 0 \); in dealing with waves it is convenient to write \( x^1 = R[\exp(j \omega x^0)] \) and \( x^2 = I[\exp(j \omega x^0)] \), with \( R() \) and \( I() \) meaning real part
and imaginary part, respectively. With this notation it is \( \tilde{x}^1 = j r \omega \cos(\omega x^0) \)
and \( \tilde{x}^2 = j r \omega \sin(\omega x^0) \). Making the replacements the equation becomes

\[
- r \omega \left[ k_1 \cos(\omega x^0) + k_2 \sin(\omega x^0) \right] - \delta^{ij} k_i k_j = 0,
\]
which has a solution

\[
\begin{align*}
k_1 &= -r \omega \cos(\omega x^0), \\
k_2 &= -r \omega \sin(\omega x^0), \\
k_3 &= 0.
\end{align*}
\]

It is legitimate to say that an orbiting point mass is the source of a wave of frequency \( \omega \) and amplitude \( r \omega \) that propagates towards the center of the orbit. A massive body is composed of many elementary particles and orbital movements so, in general, it is expected that a massive body is the source of waves, or gravitons when quantization is introduced, with a spectral distribution that results from the masses of the individual elementary particles. These waves propagate in all directions of space, interfering with other bodies which are the source of the gravitational field that determines the first body’s worldline. In the particular case of two bodies orbiting each other in circular orbits, they generate waves of equal frequency and opposing phase which cancel each other or, to put it in terms of gravitons, they interchange gravitons with equal total momentum.

4 Inertial mass of orbital systems

It is now appropriate to consider an orbiting elementary particle and search for the mass field solution of the wave equation (19) which yields the inertial mass of this particle as part of an orbital system instead of searching for massless solutions as was done in the previous section. Without loss of generality one can use a frame fixed to the center of the orbital system, so as to set the whole system stationary. In this frame the elementary particle is described by a wave \( \Psi = m \exp(j k_\alpha x^\alpha) \), which obeys Eq. (14) with the field given by Eq. (14). The general solution of the equation is rather complex but there is special interest on a tangential solution valid on the origin, so only the 0th component is examined. It is legitimate to do this if the particle is part of an orbiting system that cancels out all 3-space components. The wave equation on the origin is then

\[
\partial_\alpha (n^{\alpha 0} \partial_0 \Psi) = -m^2 \Psi.
\]

If the body is in circular motion and setting \( \tilde{x}^i = j \omega x^i \), as before, one gets

\[
(1 - r^2 \omega^2) (k_0)^2 = n^2 m^2.
\]

The value of \( k_0 \) can be obtained from the particle’s worldline equation \( M^2 n^2 \delta_{\alpha \beta} \dot{x}^\alpha \dot{x}^\beta \), with \( M \) the particle’s mass. This equation can be set in spherical coordinates for a circular orbit as

\[
M^2 n^2 \left[ (\dot{\varphi})^2 + \dot{r}^2 + r^2 \dot{\varphi}^2 \right] = 1,
\]
where $\dot{\phi} = \omega \dot{x}^0$. Replacing and solving for $\dot{x}^0$

$$\dot{x}^0 = \frac{1}{M n \sqrt{1 + r^2 \omega^2}}; \quad (24)$$

inserting into Eq. (22)

$$m = M \sqrt{\frac{1 - r^2 \omega^2}{1 + r^2 \omega^2}}. \quad (25)$$

It is important to compare the result of Eq. (25) with the predictions of general relativity; in order to do this one takes the first two terms of the series expansion, whereby the inertial mass can be seen to be approximately $M(1 - r^2 \omega^2)$. Note that $\omega$ is a derivative with respect to $x^0$ and so the linear speed is $v = r \omega \dot{x}^0$ and taking $\dot{x}^0 = 1/\sqrt{n^2 - v^2}$ from Eq. (1) with the necessary substitutions, it is finally $m = M(1 - v/\sqrt{n^2 - v^2})$. In first approximation, the mass is equivalent to the particle’s mass reduced by an amount equal to the sum of potential and kinetic energies.

As conclusion to the present section one can say that the inertial mass of a compact body is the sum of all its elementary constituents masses minus a contribution of masses resulting from all the orbital frequencies within the body. On the wave picture, a complex body acts as a complex wave pattern resulting from the superposition of many harmonic wave patterns.

5 Gravity as an evanescent field

In section 2 the inertial mass of an elementary particle was associated to its Compton frequency, through a field $\Psi$ whose nature was unspecified and in section 4 it was shown that a compact body’s mass can be viewed as a superposition of many harmonic waves due to each of the elementary particles’ masses and to the multitude of orbital frequencies within the body. In section 3 it was shown also that an orbiting body must exchange momentum with the metric in order to follow a circular orbit and necessarily this conclusion could be extended to any accelerated movement. All the previous discussions were centered on inertial mass and it has not yet been established how a particle or a body affects the metric in order to allow this momentum exchange.

The fact that an orbiting body’s mass is reflected in the center as a different mass, when applied to an elementary particle is indicative that containment or localization of a particle determines its effective mass. How an elementary particle’s mass is generated is not known but it is possible to assume that it is the result of some containment of yet another wave of different frequency and so one speculates that all mass probably results from some sort of containment of harmonic waves. Eventually one may find that all mass is the result of containment of a single frequency which would then deserve to be designated by Higgs frequency. Containment can be generated by a local change of the scale factor $n$, acting as a 4-dimensional refractive index, but it can also result from more
complex metric changes. In this framework a body or even an elementary particle is seen as a 4-dimensional waveguide extended along the body or particle’s worldline.

In a similar way to 3-dimensional optical waveguides, namely optical fibers, the guided field originates an external evanescent wave with the same spatial frequency along the waveguide direction as exists inside. The following paragraphs set the equations for this evanescent wave in the case of an elementary particle and it will not be difficult to extend the conclusions to more general situations. The present analysis deals solely with the radially symmetric component of the evanescent field, although a guided wave is expected to produce an evanescent field due to the circular component of the wave vector. The latter evanescent field is probably connected with spin and electric charge and is thus outside the scope of the present paper.

The radially symmetric component of the evanescent wave due to a stationary particle of mass $m$ must exhibit a frequency $m$ along the $0$th direction and has an equation

$$v^2 \delta^{ij} \partial_{ij} \psi = -m^2 \psi,$$

where the letter $v$ on the equation designates a propagation speed of wavefronts defined generally as the derivative $ds/dt$, with $ds$ the arc length of the wavefront normal in flat Euclidean space. Notice that this equation could also be applied to the mass of an orbiting particle given by Eq. (25).

Naturally the resulting field must have spherical symmetry, which implies that the wave equation will have a more manageable form in spherical coordinates. Furthermore, because $\psi$ is a function of $r$ and $x^0$ alone, it is possible to express $v$ as

$$v = \frac{\partial_r \psi}{\partial_r \psi}.$$ 

(27)

The operator $\delta^{ij} \partial_{ij}$ is a Laplacian; considering spherical symmetry one can make the replacement $\delta^{ij} \partial_{ij} = \partial_{rr} + 2 \partial_r/r$. Re-writing Eq. (26) in spherical coordinates and inserting Eq. (27) one gets upon simplification

$$\psi \left( \partial_{rr} \psi + 2 \frac{\partial_r \psi}{r} \right) = (\partial_r \psi)^2,$$

(28)

which has the general solution

$$\psi = C_1 e^{(C_2/r \pm j mx^0)},$$

(29)

where $j = \sqrt{-1}$.

So far no comments were about the nature of the field $\psi$ but this question must be addressed if in order to understand its relation to gravity. It is postulated that $\psi$ is the local coordinate scale factor, by which it is meant that space is corrugated with the Compton frequency on the particle’s worldline and that this corrugation is extended to infinity on the form of an evanescent field. There must be a transition from the field on the worldline to the evanescent
field but so far there are no means to choose among the many possibilities. In any case a particle will always act as a 4-dimensional waveguide for the field $\psi$, which will allow the extrapolation of many effects known in their 3-dimensional counterparts.

The field $\psi$ defines the local scale factor or alternatively it defines how the geodesic arc length should be measured; accordingly in Eq. (2) one makes the assignment $n = \sqrt{\psi \psi^*}$. In the absence of mass it is expected that the scale factor will be unity and so constant $C_1$ in the equation above can be made unity; constant $C_2$ must become zero for zero mass. The field does not die away completely but an oscillation with unit amplitude is extended to infinity; this is seen as one possible source of quantum vacuum fluctuations required by the uncertainty principle or the zero point field as proposed as early as 1916 [3, 5]. The actual value for constant $C_2$ is easy to establish resorting to compatibility with Newton mechanics. If this path is taken constant $C_2$ can be made equal to the mass, in a similar way to what was used in Refs. [2, 6]. This argument will be used in the present work and an independent derivation of this constant’s value will be deferred until there is better understanding of the waveguiding process. Consequently the gravitational field due to a stationary elementary particle will be written as

$$\psi = e^{(m/r \pm jma^0)}.$$  

(30)

It is now easy to understand the mechanism of momentum exchange by accelerated particles discussed previously, through a parallel with 3-dimensional waveguides. It is well known that some guided modes in an index-difference waveguide, such as an optical fiber, can be lost when the waveguide is bent [7]. The reverse is also true, although less common; a bent waveguide can gain energy from the outside, which becomes guided energy if further along the waveguide is straightened up. Similarly, elementary particles are seen as 4-dimensional waveguides where similar processes can occur. Accelerated particles have curved worldlines which correspond to bent waveguides and are able to exchange momentum with other particles these being the ultimate field sources.

6 Gravitational shielding and Foucault’s pendulum

This section makes use of the anomalies of Foucault’s pendulum oscillation reported by Allais [8] as experimental evidence of gravitons. Some references will also be made to the later observations with a torsion pendulum [9] and to the 1990 experiment in Finland, which did not confirm the previous results [10].

The point of departure is that gravity is carried by massless gravitons, which are momentum carriers such as photons. The distinction between photons and gravitons is a question of function and essence which will be discussed in future work. Gravitons, like photons, are expected to have essentially straight trajectories that can be slightly bent by gravitational fields. The latter effect is too
small to be detected in normal experiments and will not be considered. Gravitons must interact with matter if they are to interchange momentum with it. So in the majority of cases massive bodies must be considered opaque to gravitons, although it is conceivable that in some cases they could be transparent.

It will be shown below that all the anomalies reported by Allais [8] can be explained by shielding of Solar and Lunar gravity by the Earth or the Moon. Allais reported two periodic anomalies, with 24h and 25h periods respectively, and a sporadic anomaly during the Solar eclipse of 1959. It is intended to show that the 24h period anomaly can be explained by Earth shielding of Solar gravity, a similar thing happening with the 25h period anomaly but this time due to shielding of Lunar gravity. The Solar eclipse anomaly can be explained by Lunar shielding of Solar gravity.

The main effect of Terrestrial gravity on the pendulum is precisely the oscillatory motion. Azimuth change of Foucault’s pendulum is due to minor differences in the direction of gravitational pull in neighboring points on Earth. Naturally, the fact that the Earth is rotating has the consequence that the pendulum oscillation plane must rotate in order to eliminate the “drag” caused by the passage of the differential gravity. Foucault’s effect is easier to understand if one thinks of a referential which is not subject to Earth’s rotation; this approach, however, is not the best for the comparisons made below and so the discussion will be set on a referential on Earth.

Allais dismissed Solar and Lunar effects as possible explanation for the observed anomalies as follows [8]:

These effects are so small that none of the 19th-century authors who worked on the theory of the pendulum, some of whom were excellent mathematicians, ever had a desire to compute them.

The extreme smallness of the effects computed can readily be accounted for if we allow for the fact that, in order to obtain the true gradient \( \vec{f} \) of the Moon and Sun attraction at a point on the surface of the ground, with respect to Earth, we must take the difference between the attractions at this point and at the center of the Earth, respectively. Gradient \( \vec{f} \) is of the order of \( 10^{-8} \).

Furthermore, the plane of oscillation of the pendulum can rotate, under the influence of the solar and lunar attraction, only because of the variations of the gradient about the point considered. Therefore, the difference \( \Delta \vec{f} \) between the value of \( \vec{f} \) at the mean position of the pendulum and its magnitude at a nearby point must be considered. It is of some \( 10^{-13} \).

Furthermore, nothing in the current theory of gravitation can be considered likely to account for the screen phenomenon observed during 1954.

The objective is to show that if gravity screening is allowed, solar and lunar gravitational fields have effects which are of the same order of magnitude as terrestrial ones. In the following S.I. units are used instead of non-dimensional ones, so that the values obtained in the calculations are easy to compare with everyday results.

Using spherical coordinates in a frame fixed to Earth with origin at its center, the gravitational field can be represented by the vector \( \vec{g} = g \hat{r} \), where \( g \) is the acceleration of gravity \( g = 9.8 \text{ ms}^{-2} \) and \( \hat{r} \) is the unit vector along the radial
direction. If two points on Earth are separated by an angle $d\phi$, there is a gravity difference between those two points whose magnitude is given by $dg = g d\phi$.

The value of Sun’s gravity on Earth is easily evaluated using Newton’s formula $Gm/d^2$ and it averages $5.9 \times 10^{-3}$ ms$^{-2}$. Considering shielding, that value corresponds to the gravity on the illuminated portion of the planet, while on the dark side it must be zero. The transition zone from full to zero solar gravity is remarkably small and is responsible for an appreciable gradient, in spite of the comparatively small value of solar gravity. Dividing Sun’s diameter by the average distance from Sun to Earth one gets an angle of $9.3 \times 10^{-3}$ radians, which corresponds roughly to 55Km on Earth’s surface. Dividing the value of solar gravity by the transition zone angle yields an angular dependence of $0.63$ ms$^{-2}$ per radian, which is approximately 1/15 of terrestrial gravity variation and perfectly in line with Allais’ findings. This can easily account for the 24h period of the anomalies.

Similar calculations performed for the Moon lead to a gravity variation of $0.0037$ ms$^{-2}$ per radian, which is about 250 times smaller than terrestrial variation. Although the value obtained is about one order of magnitude smaller than would be necessary to account for the observed effect, the period is 25h as desired.

The eclipse anomaly was rightly attributed by Allais to a screening effect. In this case the angular dependence is considerably smaller than in the daily variation, because the transition zone is much wider than the mere 55Km of the latter. The felt effect is of the same order of magnitude or even larger because the linear speed of the transition zone is very high.

Consider now the torsion pendulum experiments [9, 10], one of which apparently confirmed Allais’ results and the other could not detect any anomalies. The second experiment was conducted by Ullakko et al. in Helsinki during the 1970 total solar eclipse. Naturally, both the size of the transition zone and its speed are latitude dependent. At high latitudes the transition zone is wider and its speed is smaller, so it is normal that the manifestation of solar gravity variation is more difficult to detect. This is true also for the 24h and 25h period anomalies as well.

The effects of solar and lunar gravity variations on Foucault’s pendulum are easier to explain than on a torsion pendulum. In fact, the period of a torsion pendulum is determined by the suspended mass and the spring constant of the suspension. The effect of a gravitational change in the period can be due both to the variation in the weight of the suspended mass and on some change of the spring constant due to extension or contraction. These effects are necessarily much smaller than azimuthal changes in Foucault’s pendulum, where a change in period is also expected but is probably too small to be detected. Saxl reports a period increase during the eclipse, which was not recovered after the eclipse was over. This can only be attributed to creep on the suspension caused by the gravitational change during the eclipse.

The question arises about what sort of experiments could be conducted on Earth to prove or disprove gravity shielding theory. Why can’t one just use any sort of screen to shield form Earth’s gravity, for instance? The reason is
that any screen at a fixed height from the surface will absorb and shed an equal amount of momentum and so gravity shielding does not have any effect. The screen actually seems transparent to gravity. Things will be different behind a free falling object or any object on a geodesic orbit around Earth, for in this case the object will absorb all the momentum that reaches it and will incorporate this momentum in its own. Objects in free fall or in geodesic orbits are expected to cast a gravitational shadow behind them and effectively shield any other objects in that shadow from Earth’s gravity.

7 Conclusions and further developments

4D optical theory was used to successfully explain inertial and gravitational mass. In this process it was shown that the worldline equation for an elementary particle could be transformed into a wave equation with the Compton spatial frequency. It was also shown that the gravitational field could be derived from an evanescent wave equation if it was assumed that the particle acted like a 4-dimensional waveguide. Particles or bodies constrained to a region of space, namely to an orbit, were found to reflect their own mass on the center of gravity, as well as a negative mass resulting from the binding energy.

The author was able to show that massless particles (gravitons and photons) have wave equations similar to elementary particles but while the latter are standing wave patterns, those of massless particles are propagating waves in 3-dimensional space supported on an oscillatory 0th coordinate of 4-space. Massless particles act as momentum carriers for accelerated particles and the momentum exchange process was shown to be similar to mode loss and gain in optical fibers.

Allais’ experimental results with a Foucault pendulum were used as evidence of graviton existence and the order of magnitude of most of his reported anomalies was explained through terrestrial and lunar shielding of gravitons origination on the Sun and the Moon.

The author expects to have shown sufficient evidence that elementary particles are indeed 4-dimensional waveguides, because this hypothesis proves entirely satisfactory for the explanation of their mass, be it inertial or gravitational mass. It is not clear what sort of wave is being guided nor what is the guidance mechanism. No comments were made about electromagnetism or weak and strong interactions, although the author has some hope that the future will allow the derivation of all the elementary particles as guided modes of one single frequency wave (the Higgs frequency) and all the interactions as evanescent fields relative to these modes. Some preliminary results have already shown that, at least qualitatively, this might be true.
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