Transportation modelling and solving Travelling Salesman problem

K Stoilova 1,2,* and T Stoilov1,2

1 Institute of Information and Communication Technologies – Bulgarian Academy of Sciences, Sofia, Bulgaria
2 Nikola Vaptsarov Naval Academy, Varna
e-mail: k.stoilova@hsi.iccs.bas.bg

Abstract. This research addresses the problem of solving the Traveling Salesman Problem (TSP) with wide distributed software suit Excel. The TSP problem has long history and it belongs to a class of optimization problems, which is under continuous attempt for finding appropriate numerical algorithms for its solution. This research does not make attempts for new solution of this problem. The added value of the research is that it gives example how this problem can be solved with wide available software suit. A real problem of transportation in Sofia town is defined. The problem belongs to the class of TSP. It is illustrated how this problem is solved in Excel environment. Appropriate programming in Excel’s sheet and in software application Solver is given. This work provides pragmatically example for the definition and solution of TSP transportation problem.

Keywords: optimization, transportation modelling, the shortest path problem, travelling salesman problem

1. Introduction

The transportation problem usually concerns the schedule of transportation of set of products from several source points to number of destinations. The formal definition of such problem in general is given as linear programming optimization problem. Such kind of formalization has wide usage also for problems of assignments and/or scheduling plans. These types of problems in general targets minimization of transportation costs or maximization of some values of profit.

The transportation problems initially have been used practically during the Second World War. Their primary application was the transport of troops from their training places to different battle positions in Europe and Asia. It has been seen the pragmatic and positive outcome of the implementation of the solutions of such problems.

A particular form of the transportation problem is the so-called Travelling Salesman Problem (TSP). This problem is intensively studied due to its internal complexity. The solution of this problem is widely used in many practical applications. This research is oriented to a practical usage of this problem. It illustrates the transportation rules in Sofia by means to minimize transportation
costs/distances. The research illustrates the solution of this problem by wide popular software tool like Excel, which is a prerequisite for practical implementation of the results of this work.

2. Classical Travelling Salesman problem

The Traveling Salesman Problem has long history since 18th century. Details about the definition, complexity, classifications, applications one can find in [9]. This paper uses only descriptions about the analytical definition of TSP. Algorithms for solving this problem are not presented here. The goal of the paper is to give pragmatic solution for solving this problem with popular software tool. An illustration how to solve this complex problem is given.

The TSP is a combinatorial optimization problem which makes its solution hard to be obtained. It is mainly used in traffic management for optimization the roots of transport units by means to speed up transportation services and/or increase the efficiency of operation. The content of the problem is that a Salesman has to visit only once a set of destination points by means to minimize the overall length of this trip. At the end of the trip he has to return to his starting point. The distances between the destinations are given. As a mathematical problem it has been defined in 1930 and it continues to be one of the intensively studied optimization problems. Following [4, 7] the TSP is used for analyzing the structure of crystals, for handling materials in warehouses, for clustering of data arrays, for scheduling with aggregate deadline, for orienteering problems, for applications in vehicle routing problems.

The wide way to give formal definition of the TSP is to give a description as integer linear programming model [8]. There are cases of symmetrical and asymmetrical definition of the TSP. The analytical definition of the TSP for the symmetrical case can be written as following. A graph $G = (n, A)$ is given where $n$ is a number of nodes, $A$ is a set of arcs between the nodes. Each arc has assigned value of cost (or distance) $c_{ij}, i, j \in 1, ..., n$. The TSP, written as linear integer problem, has the form

$$\min \sum_{i,j} c_{ij} x_{ij}$$

$$\sum_i x_{ij} = 1, \ \forall i \in 1, ..., n$$

$$\sum_j x_{ij} = 1, \ \forall j \in 1, ..., n$$

$$x_{ij} \text{ – integer, } \forall i, j \in 1, ..., n$$

$$x_{ij} = \begin{cases} 1 & \text{if arc } (i,j) \text{ is in the tour} \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ij} \in Z, \text{ “sub-tour breaking constraints”}.$$

The notation $Z$ defines that there are not subtours for the travelling salesman except this one, which is the solution of the TSP problem. The analytical description of the set $Z$ has several forms. Following [5, 6]

$$u_i - u_j + nx_{ij} \leq n - 1.$$ 

The variables $u_i, i = 1, n$ are dummy ones. Their meaning is the number of visited cities between the city 1 and city $i$ in the optimal tour. These compact relations are named Miller-Tucker-Zemlin formulation of the TSP.
Another well exploited formulation for the TSP is this one, given in [2]. Its form of the breaking sub-tour constraint is given with a set of many additional constraints of the form

\[ \sum_{i \in R} \sum_{j \in R} x_{ij} \leq R, \quad R \subseteq [1, ..., n], \quad 2 \leq |R| \leq n - 1. \]

Thus, the solution contains single big tour and not a set of smaller tours.

The analytical formulation and explicit description in bi-linear integer form is possible to be written only for very small networks, \( n < 4 \). The number of the constraints in the problem is very high, formalizing all possible paths, which can connect the \( n \) nodes without making small tours. The peculiarities of the TSP do not allow it to be solved by linear programs. The algorithms, derived for the solution of the TSP have different formal nature: ant colony algorithms [12], evolutionary algorithms [3], heuristics, which decrease the computational workload to polynomial level [1, 10]. For an extensive analysis of the nature of TSP, solving algorithms and application areas one can refer to [11]. According to the accuracy of solving the TSP the algorithms are classified on two sets: exact algorithms and heuristic algorithms [5]. The exact algorithms find optimal solution but in exponential set of calculations. These algorithms are quite complex and need considerable computer power. That is why heuristic algorithms are derived, which give approximate, suboptimal solution but for reasonable computer workload.

This research does not derive new formulation neither algorithms for solving this combinatorial problem. Here it is derived an appropriate application of the TSP and it is illustrated how this problem can be solved with Excel. The peculiarities, applied in this research are the usage of built-in functions in Excel suit, which makes the TSP problem non-analytical with procedural definition. This specific form of the problem later is included in the optimization function Solver. Thus, by making non-analytical constraints and integrating them in optimization function, it is achieved a practical for implementation approach for solving the TSP. This manner of solving the complex TSP has been applied for planning the logistic operation and transportation of goods in the region of Sofia.

3. Case study of the TSP on the example of transportation system in Sofia

The transportation system consists of five stock markets (SM), situated in Sofia, Figure 1. The urban distances between the SMs (in km) are given in Table 1. These stock markets have to be visited by travelling salesman. Particularly, he/it could be boss of a company, supplying vehicle, service company, postman, advertising agent, etc. Each of these nodes has to be visited only once for a round trip. The problem’s target is to minimize the travelled path among the five SMs.

The problem will be solved by popular software tool like Excel. At first, the model is designed in Excel and after that the problem is solved by the optimization tool Solver, which is application software of Excel. Table 1 is modeled in the Excel environment in the massive B3:G7, Figure 2.
Figure 1. The transportation scheme in Sofia

Table 1. The distance between the SMs

|     | SM1 | SM2 | SM3 | SM4 | SM5 |
|-----|-----|-----|-----|-----|-----|
| SM1 |     | 12.5| 18.7| 12.3| 9.6 |
| SM2 | 12.5|     | 8.3 | 12.5| 19.5|
| SM3 | 18.7| 8.3 |     | 10.2|16.3 |
| SM4 | 12.3| 12.5|10.2 |     |20.2 |
| SM5 |  9.6| 19.5|16.3 |20.2 |     |

The model for solving the problem continues in the area B10:E14. The unknown variables are the sequence of the SMs, each of which has to be visited by the salesman. Column B (B10:B14) represents the description of the sequence of the visited SMs. The unknown variables are in cells C10:C14. At the beginning we put arbitrary variables, for instance from the first to the fifth SM, C10:C14. The third column D corresponds to the SMs in the row in correspondence with the variables, D10:D14. These values are obtained by the embedded in Excel function “INDEX”. It is a statistical function which finds a cell in massive. This function has three arguments: the area of the values in which we are looking for a cell (in our model this is the massive $B$3:$B$7); the second argument is a cell, which content determines the row of the specified massive; the third argument specifies the column of the massive. For instance, the content of cell D10 is =INDEX($B$3:$B$7,$C10,1). In E10:E14 is the distance from the previous SM. Each of these cells contains again the function “INDEX” as it is shown in Figure 2. The solution of the problem gives the order of the sequentially visiting nodes of the network, C10:C14. These values are the solutions of the optimization problem. The goal function is in cell E16. We have to find the minimum travelling distance. The goal function is formalized in Excel by the function SUM(E10:E15), which minimum has to be estimated.
The TSP is formalized in Solver according to Figure 3. As a tool for solving optimization problems, the Solver consists of three main fields, which have to be determined: Target cell or objective function, in our case E16, which minimum has to be obtained. It can be reached by changing variables C10:C14. The third main field is the determination of the problem’s constraint – C10:C14 have to be different values because the Travelling Salesman has to visit only once the network’s node/SM. The solver has to use nonlinear optimization solving method. That’s why in this research the “Evolutionary” solving method is chosen.

The TSP solution is given in Figure 4. The minimum value is 57.9 km and the sequence of travelling is SM5-SM1-SM4-SM2-SM3.

The illustration for the usage and solution of the TSP problem in EXCEL environment can be easily extended for larger set of points for the salesman. One has to take into consideration that the solution of the problem takes time because of its complexity. For this research the TSP solution is found after nearly 3 minutes of computer calculations. If the number of nodes of the problem increases, the solution time is significantly long.
4. Conclusions

This research presents useful example for definition and solution of practical transportation problem. The transportation problem is defined for the logistics management of a set of nodes/stores in urban
environment. A real case for town of Sofia is under consideration. The transportation problem was defined in the class of TSP. Nevertheless the difficulties for solving such kind of problems, the paper illustrates a practical case for its solution. It has been used the software suit Excel, which is widely available. The manner of programming the main sheet, the set of parameters, given by the application Solver, and the usage of specific Excel functions give a useful procedure for the definition and solution of the TSP transportation problem. This research can be used as an approach for solution of hard problems like TSP.

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