Global Lepton-Quark Neutral Current Constraint on Low Scale Gravity Model

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Abstract

We perform a global analysis of the lepton-quark neutral current data on the low scale gravity model, which arises from the extra dimensions. The global data include HERA neutral current deep-inelastic scattering, Drell-yan production at the Tevatron, and fermion pair production at LEP II. The Drell-yan production, due to the large invariant mass data, provides the strongest constraint. Combining all data, the effective Planck scale must be larger than about 1.12 TeV for \( n = 3 \) and 0.94 TeV for \( n = 4 \) at 95\% CL.

1. Introduction

A solution to the gauge hierarchy, based on extra spacial dimensions, was recently proposed by Arkani-Hamed, Dimopoulos and Dvali [1]. They assumed the space-time is \( 4 + n \) dimensional, with the standard model (SM) particles living on a brane. While the electromagnetic, strong, and weak forces are confined to this brane, gravity can propagate in the extra dimensions. To solve the gauge hierarchy problem they proposed the “new” Planck scale \( M_S \) is of the order of TeV with very large extra dimensions. The size \( R \) of these extra dimensions can be as large as 1 mm, which corresponds to a compactification scale \( R^{-1} \) as low as \( 10^{-4} \) eV. The usual Planck scale \( M_G = 1/\sqrt{G_N} \sim 1.22 \times 10^{19} \) GeV is related to this effective Planck scale \( M_S \) using the Gauss’s law:

\[
R^n M_S^{n+2} \sim M_G^2, \tag{1}
\]

where \( n \) is the number of extra dimensions. For \( n = 1 \) it gives a large value for \( R \), which is already ruled out by gravitational experiments. On the other hand, \( n = 2 \) gives \( R \lesssim 1 \) mm, which is in the margin beyond the reach of present gravitational experiments.

The graviton including its excitations in the extra dimensions can couple to the SM particles on the brane with an effective strength of \( 1/M_S \) (instead of \( 1/M_G \)) after summing the effect of all excitations collectively, and thus the gravitation interaction becomes comparable in strength to weak interaction at TeV scale. Hence, it can give rise to a number of phenomenological activities testable at existing and future colliders [2]. So far, studies show that there are two categories of signals: direct and indirect. The indirect signal refers to exchanges of gravitons in the intermediate states, while direct refers to production or associated production...
of gravitons in the final state. Indirect signals include fermion pair, gauge boson pair production, correction to precision variables, etc. There are also other astrophysical and cosmological signatures and constraints. Among the constraints the cooling of supernovae by radiating gravitons places the strongest limit on the effective Planck scale \( M_S \) of order 50 TeV for \( n = 2 \), which renders collider signatures for \( n = 2 \) uninteresting. Thus, we concentrate on \( n \geq 3 \).

In this work, we perform a global analysis of the lepton-quark neutral current data on the low scale gravity model. The global data include HERA neutral current deep-inelastic scattering, Drell-yan production at the Tevatron, and total hadronic, \( b\bar{b} \) and \( c\bar{c} \) pair cross sections at LEP II. In addition, we also include the leptonic cross sections \( e^+e^- \rightarrow \mu^+\mu^- , \tau^+\tau^- \) at LEP II in our analysis. The \( \nu-N \) scattering data from CCFR and NuTeV have been shown by Rizzo [2] to be very insignificant in constraining the low scale gravity, and so we shall not include this data set in our analysis. We shall see that the Drell-yan production, due to the large invariant mass data, provides the strongest constraint among the global data. By combining all data, the effective Planck scale \( M_S \) must be larger than about 1.12 TeV for \( n = 3 \) and 0.94 TeV for \( n = 4 \) at 95%CL. This is our main result.

The organization of the paper is as follows. In the next section, we shall describe each set of data used in our global analysis and derive the effect of the low scale gravity. In Sec. III, we give the numerical results and interpretations for our fits, from which we can draw the limits on the effective Planck scale.

### 2. Global Data

Before we come to each data set, let us first give a general expression for the square of amplitude for \( e^-e^+ \rightarrow q\bar{q} \):

\[
\sum |M|^2 = 4u^2 (|M_{LL}|^2 + |M_{RR}|^2) + 4t^2 (|M_{LR}|^2 + |M_{RL}|^2) \\
+ 2\pi^2 \left( \frac{F}{M_S^2} \right)^2 (t^4 - 6t^3u + 18t^2u^2 - 6tu^3 + u^4) \\
+ 8\pi e^2 Q_f Q_q \frac{F}{M_S^2} \frac{(u-t)^3}{s} \\
+ \frac{8\pi e^2}{\sin^2 \theta_w \cos^2 \theta_w} \frac{F}{M_S^2} \frac{1}{s-M_Z^2} \left[ g_a^e g_a^f (t^3 - 3t^2u - 3tu^2 + u^3) + g_v^e g_v^f (u-t)^3 \right], \tag{2}
\]

where \( s, t, u \) are the usual Mandelstam variables, \( e = \sqrt{4\pi\alpha} \) is the electromagnetic coupling constant, \( Q_f \) is the electric charge of the fermion \( f \), \( \theta_w \) is the Weinberg mixing angle, \( g_a^f \) and \( g_v^f \) are, respectively, the axial-vector and the vector Z couplings to the fermion \( f \). The reduced amplitudes \( M_{\alpha\beta} \) \( (\alpha, \beta = L, R) \) are

\[
M_{\alpha\beta} = \frac{e^2 Q_f Q_q}{s} \frac{g_a^e g_a^f}{\sin^2 \theta_w \cos^2 \theta_w} \frac{g_v^e g_v^f}{s-M_Z^2} \tag{3}
\]

where

\[
g_L^f = T_{3f} - Q_f \sin^2 \theta_w , \quad g_R^f = -Q_f \sin^2 \theta_w \\
g_v^f = (g_L^f + g_R^f)/2 , \quad g_a^f = (g_L^f - g_R^f)/2 .
\]
In the derivation, we have followed the notation in Han et al. and we have taken \( M_S^2 \gg s, |t|, |u| \) and in this case the propagator factor \( D(s) = D(|t|) = D(|u|) \), which is given by

\[
\kappa^2 |D(s)| = \frac{16\pi}{M_S^4} \times \mathcal{F},
\]

where the factor \( \mathcal{F} \) is given by

\[
\mathcal{F} = \begin{cases} 
\log \left( \frac{M_S^2}{s} \right) & \text{for } n = 2, \\
\frac{2}{n-2} & \text{for } n > 2.
\end{cases}
\]

For \( n \geq 3 \) \( \mathcal{F} \) is always positive. The amplitude square for the crossed channels \( e^\pm q \to e^\pm q^- \) can be obtained using the crossing symmetry.

### 2.a HERA neutral-current data

Both H1 and ZEUS have measured the neutral-current deep-inelastic scattering cross sections at high-\( Q^2 \) region. Despite the excess in cross section reported in early 1997, the 1997 data alone agreed well with the SM. The double differential cross section for \( e^+p \to e^+X \) is given by

\[
\frac{d^2\sigma}{dydQ^2} = \sum_q \frac{f_q(x)}{16\pi} \frac{Q^2}{sy^2} \left\{ (1-y)^2(|M_{LL}|^2 + |M_{RR}|^2) + |M_{LR}|^2 + |M_{RL}|^2 \right. \\
+ \frac{\pi^2}{2} \left( \frac{Q^2}{y} \right)^2 \left( \frac{\mathcal{F}}{M_S^2} \right)^2 (32 - 64y + 42y^2 - 10y^3 + y^4) \\
+ 2\pi e^2 Q_e Q_q \left( \frac{\mathcal{F}}{M_S^2} \right)^2 \left( 2 - y \right)^3 \left( 1 - y \right)^2 \\
+ \frac{\pi e^2}{2} \sum_{\alpha} f_{\alpha}(x) \frac{Q^2}{sy^2} \left( \frac{\mathcal{F}}{M_S^2} \right)^2 \left( \frac{Q^2}{y} \right)^2 \left( 1 - y \right)^2 \\
\left. + \frac{\pi}{2} f_{\gamma}(x) \frac{Q^2}{sy^2} \left( \frac{\mathcal{F}}{M_S^2} \right)^2 \left( \frac{Q^2}{y} \right)^2 \left( 1 - y \right)^2 \right\},
\]

where \( Q^2 = sy = \sqrt{s}y = 300 \text{ GeV} \), and \( f_{q/g}(x) \) are the parton distribution functions. The reduced amplitudes \( M_{\alpha\beta} \) are given in Eq. with \( s \) replaced by \(-Q^2\). The last term in the above equation comes from the additional channel \( e^+q \to e^+g \) allowed by the low scale gravity interactions. The channels \( \sum_q e^+\bar{q} \to e^+\bar{q} \) are also included. The \( Q^2 \) data of ZEUS, including systematics, are recently published while the H1 data used in our analysis are shown in Table.

### 2.b Drell-yan at the Tevatron

CDF measured the double differential cross section \( d^2\sigma/dM_\ell\ell dy (\ell = e, \mu) \) for the Drell-yan production, where \( M_\ell\ell \) and \( y \) are, respectively, the invariant mass and the rapidity of the lepton pair. Essentially, CDF measured the cross section in the range \(-1 < y < 1\) and then average over \( y \) to obtain the double differential cross section. The double differential cross section for \( pp \to \ell^+\ell^- \) is given by

\[
\frac{d^2\sigma}{dM_\ell\ell dy} = K \sum_q \left[ \frac{M_\ell^3}{72\pi s} f_q(x_1) f_{\bar{q}}(x_2) \right] \left( |M_{LL}|^2 + |M_{LR}|^2 + |M_{RL}|^2 + |M_{RR}|^2 \right)
\]
\[ + \frac{\pi M_{\ell\ell}^2}{120s} f_q(x_1)f_{\bar{q}}(x_2) \left( \frac{F}{M_S^2} \right)^2 + \frac{\pi M_{\ell\ell}^2}{80s} f_g(x_1)f_g(x_2) \left( \frac{F}{M_S^2} \right)^2, \]  

(7)

where \( x_{1,2} = M_{\ell\ell}e^{\pm y}/\sqrt{s} \), \( \sqrt{s} = 1800 \text{ GeV} \), \( f_{q,g}(x_i) \) are the parton distribution functions, and the sum over all possible \( q\bar{q} \) pairs is understood. Note that after integrating the scattering angle \( \cos \theta^* \) over the whole range in the center-of-mass frame, the interference terms proportional to \( (F/M_S^4) \) vanish. The reduced amplitudes \( M_{\alpha\beta} \) (\( \alpha, \beta = L, R \)) are given in Eq. (3) with \( s \) replaced by \( \hat{s} = M_{\ell\ell}^2 \). The QCD \( K \) factor is given by \( K = 1 + \frac{\alpha_s}{\pi} + 1.409(\alpha_s/\pi)^2 - 12.77(\alpha_s/\pi)^3 \). The leptonic cross sections \( \sigma(e^+e^- \rightarrow q\bar{q}) \) do not have this \( K \) factor. The LEPII data \( \text{[1]} \) we used in our analysis are given in Tables 3 and 4.

2.c LEPII fermion pair Cross sections

LEPII has measured the leptonic cross sections, hadronic cross sections, and heavy flavor (\( b \) and \( c \)) production. Since the low scale gravity interactions are naturally flavor-blind, we shall include the effects on all flavors here. For a massless \( q \) the expression for \( \sigma(e^+e^- \rightarrow q\bar{q}) \) is given by

\[ \sigma(e^+e^- \rightarrow q\bar{q}) = K \left\{ \frac{s}{16\pi} \left( |M_{LL}|^2 + |M_{LR}|^2 + |M_{RL}|^2 + |M_{RR}|^2 \right) + \frac{3\pi s^3}{80} \left( \frac{F}{M_S^4} \right)^2 \right\}, \]  

(8)

where the reduced amplitudes \( M_{\alpha\beta} \) (\( \alpha, \beta = L, R \)) are given in Eq. (3), and the QCD \( K \) factor is \( K = 1 + \alpha_s/\pi + 1.409(\alpha_s/\pi)^2 - 12.77(\alpha_s/\pi)^3 \). The leptonic cross sections \( \sigma(e^+e^- \rightarrow \mu^+\mu^-, \tau^+\tau^-) \) do not have this \( K \) factor. The LEPII data \( \text{[1]} \) we used in our analysis are given in Tables 3 and 4.

2.d \( \nu\)-N Scattering

The recent measurements by CCFR and NuTeV \( \text{[7]} \) are the most precise ones on the neutrino-quark couplings. Since the gravitons also couple to neutrinos, the measurements should, in principle, place a constraint on the scale \( M_S \). However, the analysis by Rizzo in Ref. \( \text{[2]} \) showed that the constraint coming from these low energy \( \nu\)-N scattering is very weak. This is because the center-of-mass energies of these collisions are only of order of tens of GeV’s, even though the neutrino beam energy is as high as a few hundred GeV. In addition, the effective operators induced by the low scale gravity interactions are at least of dimension eight. This is in contrast to the traditional four-fermion contact interactions, which are only of dimension six. Therefore, at such low center-of-mass energies in these collisions, the effect of low scale gravity is extremely minimal. We are not going to include these data in our global fit.
3. Fits

For the fitting we follow the procedures in Refs. [8], where the four-fermion contact interactions are analyzed with a similar but larger global data set. The difference is that Refs. [8] include also the data sets from the low energy e-N scattering and the atomic parity violation, which constrain parity violating interactions. But the gravity is certainly parity conserving and thus it receives no restriction from these parity-violating experiments.

The interference effect between the SM amplitude and the low scale gravity scales as $F/M_S^4$, while the pure low scale gravity scales as $(F/M_S^4)^2$. To linearize the fitting we use the parameter $\eta = F/M_S^4$. We use MINUIT to minimize the $\chi^2$. The best estimates of $\eta$ for each individual data set and the combined set are shown in Table 5. The SM fit gives a $\chi^2 = 94.10$ for 118 d.o.f., while the fit with non-zero $\eta$ gives a $\chi^2 = 94.05$ for 117 d.o.f. Since the fits do not show any evidence for new physics, we can then place limits on the $\eta = F/M_S^4$. The physical region of $\eta$ is $\eta \geq 0$, and so we define the 95%CL upper limit for $\eta$ as

$$0.95 = \int_{0}^{\eta_{+}} \mathcal{P}(\eta) \, d\eta \int_{\eta_{+}}^{\infty} \mathcal{P}(\eta) \, d\eta,$$

where

$$\mathcal{P}(\eta) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left(-\frac{\chi^2(\eta) - \chi^2_{\text{min}}}{2}\right).$$

From $\eta_{+}$ we find the limits as $M_S/F^{1/4} = \eta_{+}^{-1/4}$. The limits on $M_S/F^{1/4}$ for each data set and the combined are also shown in Table 5. The combined limit is $M_S/F^{1/4} > 939$ GeV, or

$$M_S > \begin{cases} 1120 \text{ GeV} & \text{for } n = 3 \\ 939 \text{ GeV} & \text{for } n = 4 \end{cases}$$

at 95%CL.

It is the Drell-yan production at the Tevatron that gives the strongest constraint among all data. This is easy to understand because the $s$ of the Drell-yan process is the largest of all. The new HERA data are now consistent with the SM and only cause a very slight pull of the fit, about one third of a $\sigma$ from zero; see Table 5. Both the “ALL w/o HERA” and the “ALL” fits agree well with the SM. We show in Fig. 1 the Drell-yan production for the SM and for the low scale gravity model with $M_S$ and $n$ given in Eq. (11), together with the D0 data.

To conclude, we have used the global lepton-quark neutral current data plus the leptonic cross sections at LEPII to constrain the low scale gravity interactions arising from the extra dimensions. The limit that we place on the effective Planck scale $M_S$ is about 1.12 TeV for $n = 3$ and 0.94 TeV for $n = 4$ at 95%CL.

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Figure 1: The differential distribution $d\sigma/dM_{ee}$ for Drell-yan production at the Tevatron for the low scale gravity model and for the SM. The data shown are from D0.
Table 1: The measured $N_{\text{obs}}$ and expected $N_{\text{exp}}$ number of events as a function of $Q^{2}_{\text{min}}$ for H1 at HERA.

| $Q^{2}_{\text{min}}$ (GeV$^{2}$) | $N_{\text{obs}}$ | $N_{\text{exp}}$ |
|-------------------------------|----------------|-----------------|
| 2500                          | 1297           | 1276±98         |
| 5000                          | 322            | 336±29.6        |
| 10000                         | 51             | 55.0±6.42       |
| 15000                         | 22             | 14.8±2.13       |
| 20000                         | 10             | 4.39±0.73       |
| 25000                         | 6              | 1.58±0.29       |
Table 2: The Drell-Yan data by CDF and D0. $N_{\text{obs}}$ is the observed and $N_{\text{exp}}$ is the expected number of events.

| $M_{\ell\ell}$ (GeV) | $e^+e^-$ | $\mu^+\mu^-$ |
|----------------------|----------|---------------|
|                      | $N_{\text{obs}}$ | $N_{\text{exp}}$ | $N_{\text{obs}}$ | $N_{\text{exp}}$ |
| 50–150               | 2581      | 2581          | 2533            | 2533            |
| 150–200              | 8         | 10.8          | 9               | 9.7             |
| 200–250              | 5         | 3.5           | 4               | 3.2             |
| 250–300              | 2         | 1.4           | 2               | 1.3             |
| 300–400              | 1         | 0.97          | 1               | 0.94            |
| 400–500              | 1         | 0.25          | 0               | 0.27            |
| 500–600              | 0         | 0.069         | 0               | 0.087           |

| $M_{\ell\ell}$ (GeV) | $\sigma$ (pb) |
|----------------------|---------------|
| 120–160              | 1.93 $^{+0.43}_{-0.44}$ |
| 160–200              | 0.49 $^{+0.16}_{-0.18}$  |
| 200–240              | 0.28 $^{+0.09}_{-0.10}$  |
| 240–290              | 0.066 $^{+0.052}_{-0.058}$ |
| 290–340              | 0.033 $^{+0.032}_{-0.030}$  |
| 340–400              | 0.057 $^{+0.042}_{-0.047}$  |
| 400–500              | < 0.063 (0.039) |
| 500–600              | < 0.060 (0.037) |
| 600–1000             | < 0.058 (0.035) |
Table 3: The LEPII hadronic cross sections and the ratio $R_b$ and $R_c$. Note that the cross sections given by ALEPH have an angular cut of $|\cos \theta| < 0.95$. The two sets of DELPHI data for $R_b, R_c$ are given at $\sqrt{s} = 130 - 136$ and 161 – 172 GeV, respectively. $\sigma_{\text{had}}^{\text{SM}}$ and $R_{b,c}^{\text{SM}}$ are the SM expectations.

| $\sqrt{s}$ (GeV) | $\sigma_{\text{had}}$ (pb) | $\sigma_{\text{had}}^{\text{SM}}$ (pb) | $R_b$  | $R_b^{\text{SM}}$ | $R_c$  | $R_c^{\text{SM}}$ |
|------------------|-----------------------------|---------------------------------|-------|------------------|-------|------------------|
| ALEPH            |                             |                                 |       |                  |       |                  |
| 130              | 71.6 ± 3.96                 | 70.7                            | 0.176 ± 0.044 | 0.190 | -                | -                |
| 136              | 58.8 ± 3.61                 | 57.3                            | 0.214 ± 0.050 | 0.186 | -                | -                |
| 161              | 29.94 ± 1.84                | 30.7                            | 0.159 ± 0.035 | 0.175 | -                | -                |
| 172              | 26.4 ± 1.75                 | 25.1                            | 0.134 ± 0.037 | 0.173 | -                | -                |
| 183              | 21.71 ± 0.74                | 21.1                            | 0.176 ± 0.020 | 0.171 | -                | -                |
| 189              | 19.59 ± 0.46                | 19.3                            | -      | -                | -      | -                |
| DELPHI           |                             |                                 |       |                  |       |                  |
| 130.2            | 82.1 ± 5.2                  | 83.1                            | 0.174 ± 0.028 | 0.182 | 0.199±0.073      | 0.225            |
| 136.2            | 65.1 ± 4.7                  | 67.0                            | 0.142 ± 0.024 | 0.165 | 0.314±0.055      | 0.250            |
| 161.3            | 40.9 ± 2.1                  | 34.8                            | -      | -                | -      | -                |
| 172.1            | 30.3 ± 1.9                  | 28.9                            | -      | -                | -      | -                |
| L3               |                             |                                 |       |                  |       |                  |
| 130.3            | 81.8 ± 6.4                  | 78                              | -      | -                | -      | -                |
| 136.3            | 70.5 ± 6.2                  | 63                              | -      | -                | -      | -                |
| 140.2            | 67 ± 47                     | 56                              | -      | -                | -      | -                |
| 161.3            | 37.3 ± 2.2                  | 34.9                            | -      | -                | -      | -                |
| 170.3            | 39.5 ± 7.5                  | 29.8                            | -      | -                | -      | -                |
| 172.3            | 28.2 ± 2.2                  | 28.9                            | -      | -                | -      | -                |
| OPAL             |                             |                                 |       |                  |       |                  |
| 130.25           | 64.3 ± 5.1                  | 77.6                            | -      | -                | -      | -                |
| 133.17           | -                           | -                               | 0.195 ± 0.041 | 0.182 | -                | -                |
| 136.22           | 63.8 ± 5.2                  | 62.9                            | -      | -                | -      | -                |
| 161.34           | 35.5 ± 2.2                  | 33.7                            | 0.162 ± 0.041 | 0.169 | -                | -                |
| 172.12           | 27.0 ± 1.9                  | 27.6                            | 0.131 ± 0.047 | 0.165 | -                | -                |
| 183              | 23.7 ± 0.81                 | 24.3                            | 0.190 ± 0.022 | 0.163 | -                | -                |
| 189              | 22.1 ± 0.64                 | 22.3                            | 0.167 ± 0.014 | 0.162 | -                | -                |
Table 4: The LEPII leptonic cross sections for $e^+e^- \rightarrow \mu^+\mu^-, \tau^+\tau^-$. Note that the cross sections given by ALEPH have an angular cut of $|\cos \theta| < 0.95$. $\sigma^{\text{SM}}$ are the SM expectations.

| $\sqrt{s}$ (GeV) | $\sigma_{\mu\mu}$ (pb) | $\sigma_{\mu\mu}^{\text{SM}}$ (pb) | $\sigma_{\tau\tau}$ (pb) | $\sigma_{\tau\tau}^{\text{SM}}$ (pb) |
|------------------|------------------------|-------------------------------|------------------------|-------------------------------|
| ALEPH            |
| 130              | 7.9 ± 1.2              | 7.0                           | 10.9 ± 1.8             | 7.3                           |
| 136              | 6.9 ± 1.1              | 6.1                           | 5.6 ± 1.3              | 6.3                           |
| 161              | 4.49 ± 0.70            | 3.88                          | 5.75 ± 0.99            | 4.01                          |
| 172              | 2.64 ± 0.53            | 3.32                          | 3.26 ± 0.75            | 3.43                          |
| 183              | 2.98 ± 0.25            | 2.89                          | 2.90 ± 0.30            | 2.98                          |
| 189              | 2.66 ± 0.14            | 2.69                          | 2.39 ± 0.16            | 2.78                          |
| DELPHI           |
| 130.2            | 9.7 ± 1.9              | 8.1                           | 10.2 ± 3.1             | 8.3                           |
| 136.2            | 6.6 ± 1.6              | 7.0                           | 8.8 ± 3.0              | 7.2                           |
| 161.3            | 3.6 ± 0.7              | 4.5                           | 5.1 ± 1.2              | 4.6                           |
| 172.1            | 3.6 ± 0.7              | 3.8                           | 4.5 ± 1.1              | 3.9                           |
| L3               |
| 161.3            | 4.59 ± 0.84            | 4.4                           | 4.6 ± 1.1              | 4.5                           |
| 172.1            | 3.60 ± 0.75            | 3.8                           | 4.3 ± 1.1              | 3.9                           |
| OPAL             |
| 130.25           | 9.0 ± 1.8              | 8.0                           | 6.8 ± 2.0              | 8.0                           |
| 136.22           | 11.4 ± 2.1             | 7.0                           | 7.2 ± 2.1              | 6.9                           |
| 161.34           | 4.5 ± 0.7              | 4.4                           | 6.2 ± 1.0              | 4.4                           |
| 172.12           | 3.6 ± 0.6              | 3.8                           | 3.9 ± 0.8              | 3.8                           |
| 183              | 3.46 ± 0.29            | 3.45                          | 3.31 ± 0.32            | 3.45                          |
| 189              | 3.13 ± 0.16            | 3.21                          | 3.53 ± 0.21            | 3.21                          |
Table 5: The best estimate of the $\eta = \mathcal{F}/M_S^4$, 95%CL upper limit $\eta_+$, and the 95%CL lower limit on $M_S/\mathcal{F}^{1/4}$.

| Data Set        | $\eta = \mathcal{F}/M_S^4$ (GeV$^{-4}$) | $\eta_+$ (GeV$^{-4}$) | 95%CL $M_S/\mathcal{F}^{1/4}$ |
|-----------------|----------------------------------------|------------------------|---------------------------------|
| DY-CDF          | $(0.00 \pm 0.15) \times 10^{-11}$      | $0.22 \times 10^{-11}$ | 819 GeV                         |
| DY-D0           | $(0.00 \pm 0.12) \times 10^{-11}$      | $0.15 \times 10^{-11}$ | 906 GeV                         |
| DYcombined      | $(0.00 \pm 0.10) \times 10^{-11}$      | $0.14 \times 10^{-11}$ | 925 GeV                         |
| LEPII hadronic  | $(0.45 \ ^{+0.23}_{-1.18}) \times 10^{-11}$ | $0.82 \times 10^{-11}$ | 591 GeV                         |
| LEPII leptonic  | $(-0.34 \ ^{+0.80}_{-0.68}) \times 10^{-11}$ | $1.01 \times 10^{-11}$ | 561 GeV                         |
| LEPII b,c       | $(1.63 \ ^{+1.69}_{-5.87}) \times 10^{-11}$ | $4.23 \times 10^{-11}$ | 392 GeV                         |
| LEPII           | $(-0.44 \ ^{+1.01}_{-0.29}) \times 10^{-11}$ | $0.74 \times 10^{-11}$ | 607 GeV                         |
| HERA            | $(-0.050 \ ^{+0.18}_{-0.17}) \times 10^{-11}$ | $0.47 \times 10^{-11}$ | 679 GeV                         |
| ALL             | $(-0.026 \ ^{+0.076}_{-0.01}) \times 10^{-11}$ | $0.13 \times 10^{-11}$ | 939 GeV                         |
| ALL w/o HERA    | $(-0.0055 \ ^{+0.11}_{-0.10}) \times 10^{-11}$ | $0.14 \times 10^{-11}$ | 925 GeV                         |