An introduction to spatial dispersion: some basic concepts

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I describe three ways that the spatial properties of a wave propagation medium can cause dispersion, and propose that they should form the basics for correctly understanding and “naming” phenomena described as “spatial dispersion”. In particular, I emphasise the specific spatial properties which generate the resulting dispersive – i.e. spatially dispersive – behaviour. The mechanisms are geometry, structure, and dynamics.

1. INTRODUCTION

Spatial dispersion is the non-local dependence of material properties on both direction and wavelength, and is of particular interest in electromagnetic systems and other fields where artificial functional materials (AFMs) are finding technological uses, such as in acoustic and elastic metamaterials. It has been long observable in crystals [1], but is also of particular importance in AFMs where the wavelength of radiation becomes comparable to the lattice parameters (see e.g. [2]). Further, it is also significant in the region of the material resonances utilized in metamaterial unit cells, where constitutive parameters such as the permittivity and permeability – and likewise their acoustic or elastic counterparts – are maximised or have near-zero local values [3] or where the response is non-reciprocal [4]. In some materials its presence is revealed by the shape of the equi-frequency surfaces of the dispersion relations – e.g. when they are non-symmetric, not elliptical or hyperbolic, or having multiple modes with the same direction and polarisation.

Spatial dispersion usually appears as either spatially non-local effect that produces a wavevector dependence of the material parameters, or as a non-trivial wavevector dependence for the dispersion relations. Although spatial dispersion can be added in an ad hoc manner to suit intuition or an empirical fit to some data, it is preferable to be more systematic.

However, the term “spatial dispersion” is often used rather loosely, and so sometimes the origin of the specific phenomenon being discussed is unclear. In an attempt to rectify this, here I describe three physically distinct ways that spatial dispersion can arise, briefly describing examples of each. Each of these mechanisms has (a) no time dependence beyond that required to support a wave, and (b) a specification of time-independent material properties. The lack of any (non-trivial) time dependence ensures that the phenomena treated here are entirely unrelated to the more commonly considered topic of temporal dispersion. The first two spatial dispersion mechanisms depend entirely on spatial properties, whereas the third derives from a coupling to a dynamic medium, leading to a changed wave velocity and/or an “effective mass” term.

The categorization here shows that instances of spatial dispersion can be caused by the geometric, structural, or dynamic properties of a system. This categorization is a result not based on existing (and often somewhat ad hoc) naming conventions or justifications for spatial dispersion, but is instead grounded in the following assertion:

Spatial dispersion refers to any dispersive behaviour that occurs (solely) as a result of spatial properties of the system.

This viewpoint is distinct from the widely used one (see e.g. [5,6]) where ‘weak’ spatial dispersion is that where the spatial effects have been approximated as a local effect and ‘strong’ being where nonlocal effects remain important. It is to be noted that so-called weak spatial dispersion is not necessarily small in any practical sense, despite the implications of ‘weak’; and likewise so-called strong spatial dispersion need not be particularly dominant.

In particular, I do not:

1. describe the example systems considered below in a detailed manner – only the minimum features necessary to make the point are presented, as they are worked through both exhaustively and frequently in other sources;
2. discuss how homogenization schemes might reduce the description of a complicated medium into a simple one, (e.g.) perhaps one based on a series expansion in terms of the wavevector \( k \);
3. consider dispersion relations that have been artificially constructed without reference to some specific physical system with spatial properties; e.g. putative dispersion based on suggested or “what-if” polynomials in frequency \( \omega \) and wavevector \( k \), or some other specification of an \( \omega, k \) interdependence.

The main focus here is on spatial dispersion in electromagnetic systems or artificial functional media such as metamate-

[1] In my view, terms such as ‘weak’ or ‘strong’ should be only used to indicate the strength or significance of a phenomenon, not its origin or type, or whether it might be effectively local or non-local.
In materials, but the distinctions apply equally well to waves in acoustic or elastic materials, or indeed potentially any material system which supports wave propagation of some kind. In what follows (Sec. II) I will first describe what I mean by dispersion (i.e., specifically non-trivial dispersion) by considering the relationships between the frequency $\omega$ of a wave and the wavevector $k$ associated with that frequency. Following that the next three sections address each of geometric (III), structural (IV), and dynamic (V) spatial dispersions in turn.

II. DISPERSION

The definition of dispersion used here is based on three criteria relating to how the frequency ($f$ or $\omega$) of a wave is related to its wavelength or wavevector $k$. Note that in discrete systems, analytic solutions typically depend on an integer index $n$ (and possibly more indices as well). Since these indices usually indicate the number or spatial frequency of the oscillation in a given solution, they play the same role as the wavevector $k$, and contain essentially the same physical content. In simple cases, the wavevector $k$ can be a straightforward multiple of $n$; in any case here the wavevector associated with an index $n$ is $k_n$, and likewise the related angular frequency is $\omega_n$.

The criteria are:

- **Linear** – the relationship between $\omega$ and $k$ (or between $\omega$ and $n$ or $k_n$) is linear, i.e., it lies along a single straight line.
- **Origin** – if $\omega = 0$ then $k = 0$ (or $n = 0$), and vice versa.
- **Continuous** – the relationship is continuous, i.e., is in one piece, and has no gaps or jumps.

If all these criteria hold, then the wave is **dispersionless**, and the phase and group velocities are the same. If any of the criteria do not hold, then the wave is dispersive, and e.g., an initial pulse shape will change over time as its different spectral components evolve at different rates. In general, this behaviour might be due to either the time-response of the medium (giving rise to temporal dispersion), or to the spatial properties of the medium (giving rise to spatial dispersion), or possibly both.

Here I focus exclusively on dispersion that results solely from spatial properties, although it is worth keeping in mind that typically the structural elements relied on to create those properties in a physical device may in reality also be temporally dispersive – notably, reflecting walls will not be ideal at all frequencies, and different refractive indices exist only because of temporal dispersion. I assume below that such complications can be ignored, and/or that they are irrelevant to the point being made.

A dispersionless wave travelling along the $z$-axis with propagation speed $c$ follows the wave equation

$$\partial_t^2 E - \partial_z c^2 \partial_t E = 0. \quad (1)$$

This equation can be Fourier transformed in both time and space, and the field strength terms cancelled, giving the “dispersionless” dispersion relation

$$\omega^2 - c^2 k^2 = 0. \quad (2)$$

A depiction of this relationship between $k$ and $\omega$ can be seen on fig. 1.

Note that this is quadratic in both $\omega$ and $k$, so that either might have negative values and still help satisfy the dispersion relation. Any physical meaning(s) that is attributable to such negative spectral quantities is comprehensively discussed elsewhere [7], so for simplicity we will only consider positive values here. Typically the sign choices correspond to the different direction in which propagating waveforms will evolve [8–10].

III. GEOMETRIC SPATIAL DISPERSION

The geometry of a region in which the wave of interest is supported is perhaps the most primitive possible source of a spatial effects like dispersion.

For the purposes here, I consider a (purely) geometric system to be one where the properties of the propagation medium are homogeneous, isotropic, and are not temporally dispersive. This medium is present everywhere except at any boundaries, and to guarantee complete confinement within the propagation medium should be assumed to be perfectly reflective. Note that this category also includes systems without boundaries, such as those confined for some reason (e.g.) to a torus, or to a spherical surface.

In such systems, especially if there is sufficient symmetry, we can often find analytic solutions for its frequency eigenmodes. In such a case, we have an index $n$ rather than a Fourier-transform based wavevector, but the role is the same: $n$ indicates the number of spatial oscillations over some relevant distance interval; typically it is closely related to the number of nodes in its eigenfunction.

![dispersion relation diagram](http://www.kinsler.org/physics/)
An introduction to spatial dispersion

A. Cavity

Although any shape of wave-confining, empty cavity would a candidate for supporting spatial dispersion, the simplest would be a 1D perfectly reflective cavity with a length $L$. In such a case, where the wave amplitude is zero on the boundary, the dispersion relation is

$$\omega = \frac{n + 1}{2} \frac{2\pi c}{L},$$

where $n$ is a non-negative integer, and the equivalent wavevector is $k_n = \pi (n + 1)/L$. The dispersion relation for this system is depicted schematically on fig. 2.

By the criteria demanded in Sec. II, this is spatially dispersive since although the relationship between $\omega$ and $k$ is linear, with $\omega = ck$, there is no supported wave with $\omega = 0$ and $k = 0$, and only a discrete spectrum of waves exist.

The primary effect of spatial dispersion here is simply the imposition of a discrete spectrum, although the removal of any $(\omega, k) = (0, 0)$ solution is also important. Other cavity shapes, such as cylindrical or spherical, are widely covered in undergraduate textbooks on electromagnetism so I do not present them here, but they also will (at least) have a discrete spectrum and no $(0, 0)$ solution.

B. Topology: torus

A wave-supporting space without boundaries is a candidate for supporting spatial dispersion (only) if it is also finite, so that it’s size provides an intrinsic length scale. The simplest situation is probably a toroidal space, which is essentially the same as the case of an infinite and periodic lattice, or indeed of periodic boundary conditions. In 1D, the torus is a simple loop, with the length $L$ giving the periodicity scale and setting an effective maximum wavevector $k_{\text{max}} = \frac{2\pi}{L}$. In a dispersion plot, this gives rise to band folding [11] – i.e. although a non-periodic dispersion relation would normally extend to both high wavevector and high frequency, those parts of the dispersion at too-high wavevector are “folded” back to lower wavevector; so that the periodic case is restricted to a finite $k$ range, as can be seen on fig. 3. This means that at low wavevectors and lowest frequencies, the system appears dispersionless, but once band folding occurs for the higher frequencies, even though the dispersion might be linear and continuous, the folded dispersion can no longer be extrapolated from every point to pass through $(0, 0)$.

C. Topology: sphere

A more interesting case than those above is a resonator where the waves are confined on a spherical surface, which is equivalent to a Maxwell's fisheye lens [12]. The supported modes or this are derived from the Legendre polynomials $P_n(\xi)$. These mode functions $P_n$ provide a complete and countable orthonormal basis set for all possible radial field patterns in the resonator, with the argument $\xi = (r^2 - 1)/(r^2 + 1)$ being derived from the radial displacement $r$ from some choice of preferred origin. Of course there can also be an angular dependence to field patterns, which can easily be included using the usual angular mode functions [13], but I omit those details here in the interests of brevity. Each discrete mode has a frequency $f_n$ determining its physical properties as determined from its index $n$,

$$\omega_n \propto \sqrt{n(n+1)}$$

with modes of larger $n$ (or $\omega_n$) having more spatial oscillations. A depiction of the spatial dispersion properties on a sphere can be seen on fig. 4.

Here the effect of spatial dispersion is significant, because two of the criteria are violated – the relationship is not linear, and the spectrum is discrete.

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2 See [http://dlmf.nist.gov/14](http://dlmf.nist.gov/14)
IV. STRUCTURAL SPATIAL DISPERSION

Structural spatial dispersion is distinct from the geometric spatial dispersion above, in that it is due to material inhomogeneity. This type is the origin of most instances of spatial dispersion that are considered. As we see in the slab waveguide example below, even very simple types of inhomogeneity can generate spatial dispersion; however, the necessary calculations are usually non-trivial.

A. Slab waveguide

An electromagnetic slab waveguide is one of the simplest structures that might be considered when looking for an example of structural spatial dispersion in a non-periodic system. It consists of a planar slab of one type of material sandwiched on either side by half-infinite regions of an alternative material, where the difference in material properties allows modes to exist that are localised in a way centred on the slab.

Electromagnetic slab waveguides are treated in a wide range of textbooks (see e.g. [14], or the abbreviated summary in [9] with a discussion of dispersion handling). Here the waveguide is taken to have thickness \( d \) in the perpendicular \( x \) direction, propagation in the \( z \) direction, and with core and cladding permittivities \( \varepsilon_1 \) and \( \varepsilon_2 \). To solve this system, we take Maxwell’s equations in component form, assume plane-wave like behaviour in orientations parallel to the slab, and, at the boundaries, match the \( \sin() \) or \( \cos() \) functional form in the core with decaying exponentials in the cladding. Even for this simple slab design, the boundary conditions give the dispersion relation a non trivial form. Notably, for the bound modes it is given by the solution to a transcendental equation [14]. Typically, this is written in a way implying we want to calculate \( k_x \) and \( \beta = k_z \), from a specified \( \omega \), although a compelling argument can be made [9] that it is better to calculate \( \omega \) from a provided \( k_z \).

For the transverse electric (TE) field modes, the traditional presentation shows that in a slab waveguide we have

\[
T(k_x,d) = k_x^{-1} \sqrt{\omega^2 \mu_0 (\varepsilon_1 - \varepsilon_2) - k_z^2},
\]

where \( \beta^2(\omega) = \omega^2 \mu_0 \varepsilon_1 - k_z^2 \),

and \( T(k_x,d) \) is either \( \tan(k_xd) \) or \( -\cot(k_xd) \).

Fig. 5 indicates the appearance of the dispersion relations for the first few bound modes of such a waveguide. This slab waveguide case is a somewhat similar problem to finding the modes of an optical fibre, although the cylindrical symmetry of a fibre means that the solutions involve matching Bessel functions across the slab boundaries.

As a passing note, the spatial dispersion due to the structural configuration of an optical fibre is typically treated as instead as if it were temporal in origin. This is because a majority of optical pulse simulations are propagated along a spatial axis, and so the dispersion computations are more convenient in \( \omega \) than they would be in \( k \).

B. Bragg mirror

A Bragg mirror is composed of multiple thin layers of dielectric material, with the layers designed so that the device is highly reflective at some wavelength, or in some wavelength range. Simple versions consist of stacks of alternating high and low refractive material, with thicknesses chosen so that the path-length differences for internal reflections are integer multiples of the design wavelength. Here, a unit cell for the stack consists of just the two contrasting layers.

In the model calculation of Horsley et al. [15], an infinite periodic stack is considered. The analysis shows that the relationship between structure and eigenvalue – i.e. its dispersion relation is controlled by the expression

\[
\lambda = z(\omega) \pm \sqrt{z(\omega)^2 - 1} = \exp[i \kappa (a + b)].
\]

Here \( \kappa \) is the unit-cell Bloch wavevector, and the layer thicknesses are \( a \) and \( b \). The effect of the material refractive indices...
The maximum and minimum $k$-mode structure gives it similarities with the torus case of fig. 3, but in the maximum and minimum $k$ regions, the two branches bend apart to create bandgaps, where certain $(\omega, k)$ combinations do not propagate. The dispersion relation is not linear, does include the origin, and is not continuous.

$n_a$ and $n_b$ are subsumed into the real valued $z(\omega)$, which is calculated from the unit-cell’s transfer matrix.

For our purposes it is sufficient to note that in cases where $|z| > 1$, $\kappa$ becomes complex and $|\lambda|$ is no longer unity. These represent diverging (non-periodic) solutions which are physically prohibited and do not form part of the dispersion relation. The resulting dispersion relation is nonlinear, and is discontinuous across the (reflective) bandgap, although continuous elsewhere; it is depicted in fig. 6.

FIG. 6: Structural spatial dispersion in a Bragg mirror. The periodic structure gives it similarities with the torus case of fig. 3, but in the maximum and minimum $k$ regions, the two branches bend apart to create bandgaps, where certain $(\omega, k)$ combinations do not propagate. The dispersion relation is not linear, does include the origin, and is not continuous.

C. Wire media

Wire media are a class of metamaterials consisting of a regular (rectangular) array of parallel wires or rods. Thus the system is periodic along both the transverse axes, and (usually) uniform along the longitudinal one. This is a hard problem to solve in the general case, but results can be found when the wires have a radius is small compared to their spacing [16–18]. Conveniently, it turns out that under approximation, such media have a quadratic spatio-temporal dispersion relation. This behaviour for the light in such a structure mimics that of existing usage; it is depicted in fig. 6.

\[ \omega^2 - \beta^2 k^2 = c^2 K^2, \]  

which has a cut-off frequency $c^2 K^2$. The resulting dispersion curve is depicted on fig. 7.

This is an example where the effect of inhomogeneity under approximation induce a structural spatial dispersion that mimics a mass-like term $\propto K^2$ in the dispersion (see e.g. [19]). We can see this directly by back Fourier transforming this relation for the electric field $E$ from $\omega, k$ into $t, x$

\[ \partial_t^2 E - \beta^2 \partial_x \partial_x E - c^2 K^2 E = 0. \]

FIG. 7: Spatial dispersion in an approximated wire medium. The cut-off $c^2 K^2$ gives a minimum allowed frequency, but in the limit as $k$ becomes large the curve approaches the line $\omega = \beta k$.

V. DYNAMIC SPATIAL DISPERSION

Dynamic spatial dispersion is a result of non-trivial propagation properties of the material, and how that affects the wave of interest that propagates through it. For the spatial dispersion induced in wire media, we have already seen above that spatial properties can alter the effective dynamics of the light propagation. Partly because of this, and partly because of existing usage, I categorise such propagation effects as spatial mechanism here because it results from excitations (spatially) moving through the medium\(^3\). The origin of the resulted dispersion is a direct result of the material properties, where the propagating wave of interests drives excitations in the medium, and those excitations then affect the propagation of the orginal wave.

One might consider quite a wide variety of wave models when treating dynamic spatial dispersion; for example, in acoustic or elastic media, we might think that any of the somewhat eclectic selection in [10] could be suitable. Indeed, if used as a priori mechanisms, they might be, but it is worth noting that such wave models result from applying approximations to an underlying, more complicated and/or microscopic description. As such, the physical effects treated are in fact more like the effects or side-effects of the structural spatial dispersion as treated in section IV rather than native dynamic spatial dispersion.

Because of such complications, here I will consider only one model of dynamic spatial dispersion, the hydrodynamic model for plasmonics (HMP) [20]. As a simple wave equation, which is (only and exactly) second order in both space and time it avoids issues of causality as long as its built-in speed parameter stays less than that of light.

\(^3\) Note that it is arguable that it should be instead regarded as a space-time effect, due to its wave-like underlying model. However, here I will retain it as a pure spatial effect for consistency with existing usage; also note that the $\omega^2$ term is not modified from that in the usual vacuum case.
A. Plasmons

The HMP has recently found widespread popularity in the field of plasmonics and electromagnetism. One of its key features is that unlike simpler plasmonics approaches based on the Drude model for the dielectric permittivity, it incorporates spatial derivatives terms which represent the dynamics of the charge distribution. Although these are often called “non-local” effects, they are more usefully called propagation effects, since they are not non-local in any sense that violates relativistic (signalling) constraints on the physics. However, although much of its usage in plasmonics is recent, the basic model itself dates back to the 1970’s [21] and has been used in a number of other contexts [22, 23].

One point regarding naming conventions that needs careful attention here is the use of “plasmon”, which can be applied in a number of other contexts [22, 23].

The equation for the HMP in a 1D case is [20]

\[ \partial_t^2 P + \gamma \partial_x P - \partial_x^2 \partial_t P = \epsilon_0 \omega_0^2 E, \]  

where the polarization field is \( P \equiv P(t, x) \) and the driving electric field is \( E \equiv E(t, x) \). The speed of disturbances in the polarization field \( E \) is \( \beta \), polarization losses are given by \( \gamma \), and \( \omega_0^2 \) is a measure of how strongly the electric field drives disturbances in the amplitude of \( P \). With respect to discussions about the utility and meaning of the HMP, we may consider the deconstructed plasmon model at [24], and also the variant current-based version as opposed to the usual polarization version as used here.

Taking the 1D case for simplicity, then after a double transform into \( \omega \) and \( k \), while also ignoring losses\(^5\) we get

\[ \omega^2 P - k^2 \beta^2 k P = \epsilon_0 \omega_0^2 E. \]  

When linked to the standard EM wave equation we find that this gives an effective permittivity of (simplified)

\[ \epsilon(\omega, k) = \epsilon_0 \left( \frac{\omega_0^2}{\omega^2 - k^2 \beta^2} \right), \]  

and a dispersion relation

\[ \omega^2 - \beta^2 k^2 = \omega_0^2. \]

Again, this (as expected) shows dynamic spatial dispersion, with the dispersion relation being non-linear and lacking a \((0, 0)\) solution.

VI. SUMMARY

Here I have addressed the basic causes of spatial dispersion, and briefly summarized examples of each. However, since the term “spatial dispersion” is often used rather loosely, I first set out to specify clearly what I meant by dispersion, and what kinds of spatial properties might generate it: notably by the geometric, structural, or dynamic properties of a system. Since my classification is not based on existing (and often somewhat ad hoc) naming conventions or justifications for phenomena called (or attributed to) “spatial dispersion”, it may be challenging to adapt to or accept. However, it is a definition derived solely based on trying to answer the question: What different types of (solely) spatial properties can result in dispersive behaviour?

Acknowledgments

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Material properties generally are a result of both spatial (structural) and temporal effects, and often these are coupled together. Even apparently simple properties such as the refractive index (which is usually simply a proxy for the material permittivity) are in fact a complicated combination of the (many) individual temporal responses of the material’s constituent atoms and molecules. In a crystal, the electronic structure of each atom gives that atom its own independent temporal response, and the arrangement of those atoms in space specifies the material’s spatial (structural) properties. The combination gives the crystal a combined spatio-temporal dispersion which potentially quite complicated behaviour.

These complications were deliberately not addressed above, because that would have obscured the goal of defining those spatial features which by themselves can induce dispersive behaviour. Of course, in many practical cases the fine details can be ignored. If we are happy with the approximate behaviour. Of course, in many practical cases the fine details can be ignored. If we are happy with the approximate behaviour of each atom then we will adopt its independent spatial properties, and the arrangement of those atoms in space specifies the material’s spatial (structural) properties. The combination gives the crystal a combined spatio-temporal dispersion which potentially quite complicated behaviour.

Appendix: Materials and Metamaterials
atoms and molecules within the glass. In such natural media it is typically the case that the scales on which microscopic and bulk properties act are distinct enough so that we can treat them additively.

In contrast to natural media, metamaterials tend to have dispersive behaviour which is harder to approximate. There are two significant reasons for this. Firstly, metamaterial design often relies on time dependent (dynamic) behavior to get strong responses, so since the impinging wave field is – by design – rather near the metamaterial resonance, we cannot easily ignore the temporal contributions to dispersion. Secondly, because metamaterials are themselves constructed of natural materials, their unit cell scales are larger with respect to the impinging wavelengths than for natural materials where unit cells are atomic of molecular in size; thus we cannot easily ignore the spatial (structural) contributions to dispersion. Indeed, if we want to engineer a magnetic response, then we cannot make a metamaterial cell size “negligible” in any useful sense [25].

An important point to emphasize is that many types of metamaterials rely on both their spatial and temporal properties to work. For example, many (such as the SRR) are based on shaped metal structures, in which electric currents can be induced by any impinging electromagnetic fields. Once induced, such currents can then follow their own shape-dependent time-domain dynamics, whilst also being driven by, and emitting into, the field. Strictly, therefore, the true response of such metamaterials is due to interlinked spatial and temporal dispersions. However, in the appropriate (or convenient) limits, the spatial properties can often be homogenized away, leaving only the temporally dispersive properties as significant.

The extra work required to reduce a lattice of metamaterial cells is significant, and has lead to a great interest in how to measure, simplify, or otherwise describe their dispersive properties – leading to the topic known as homogenization [26]. This goal of getting approximate but still sufficiently accurate dispersion relations for uniform arrays of unit-cell structures has been widely examined, but except in rare cases [6, 16] the spatial properties are either ignored (as in e.g. the F-model for split-ring resonators (SRRs) [27, 28]) or poorly resolved. It is well beyond the intended scope of this paper to attempt to present the wide variety of possible spatial dispersia generated by metamaterials, but the interested reader might perhaps look at either the Special Issue of PNFA [26], or the more recent possibilities covered by the work of Mnasri et al. [6] and Khrabustovskyi et al. [5].