A low scale left-right symmetric mirror model

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Abstract

An effective left-right symmetric mirror model restoring parity at a high scale in a unique way is proposed. One of the remarkable features is that mirror fermions and mirror gauge sector simultaneously can exist at TeV scale. We also provide a simple ultraviolet completion of the model with vector-like fermions. The model has implications for recently observed flavour anomalies in the quark-level $b \to s l \bar{l}$ transitions.

Keywords: Parity, gauge extension of the standard model

There are two classes of parity restoring models in general. In the first class, the right-handed degrees of freedom of the standard model (SM) and three right-handed neutrinos are accommodated in the fundamental representation of the gauge group $SU(2)_R \{1, 2, 3, 4, 5, 6\}$. This class may further be divided into two sub-classes, minimal $\{2, 3, 4\}$ and non-minimal $\{5, 6\}$. The second class of models have mirror fermions in the fundamental representation of the gauge group $SU(2)_R$ instead of the right-handed degrees of freedom of the SM $\{7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$. In these models, the right-handed degrees of freedom of the SM and three right-handed neutrinos are treated as singlets under the gauge group $SU(2)_R$. The latest phenomenological status of these models can be found in Refs. $\{11, 12, 13, 14, 17, 18, 19\}$.

It is quite disappointing that mirror gauge sector of the models having mirror fermions and mirror symmetries turns out to be extremely heavy. The reason lies in the fact that parity invariance makes the Yukawa couplings of the SM and mirror sector identical. For instance, the Yukawa Lagrangian in

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Ref. [11] with gauge symmetry $SU(2)_L \otimes SU(2)_R \otimes U(1)_Y$ is

$$\mathcal{L}_Y = \Gamma \left( \bar{\psi}_L \varphi_L \psi_R + \bar{\psi}'_R \varphi_R \psi'_L \right) + \text{H.c.},$$  

(1)

where $\psi'$ fermions are mirror counter-parts of the SM $\psi$ fermions. They are singlet under the SM gauge group $SU(2)_L$ and charged under the mirror gauge group $SU(2)_R$. The scalar Higgs field $\varphi_L$ is doublet under the SM gauge group $SU(2)_L$, and singlet under the mirror gauge group $SU(2)_R$. Similarly, scalar Higgs field $\varphi_R$ is doublet under the gauge group $SU(2)_R$, and singlet under the SM gauge group $SU(2)_L$. $\Gamma$ is a $3 \times 3$ matrix in family space.

We have not seen these mirror fermions at the Large Hadron Collider (LHC) around TeV scale yet. The mass of the lightest charged lepton is given as $m_{e'} = m_e \langle \varphi_R \rangle / \langle \varphi_L \rangle$ where $m_e$ is mass of the electron, $\langle \varphi_L \rangle = 246$ GeV is the vacuum expectation value (VEV) of the SM Higgs field, and $\langle \varphi_R \rangle$ is VEV of the Higgs field $\varphi_R$. Now, for instance, $m_e = 0.511$ MeV and $\langle \varphi_R \rangle = 5 \times 10^8$ GeV, the mass of the lightest charged lepton is $m'_{e} = 1038.65$ GeV that could be looked for at the LHC.

Hence, for sufficiently heavy mirror fermions to search at the LHC, we need an unusual parity breaking scale around $10^8$ GeV. Hence, mirror gauge bosons corresponding to the gauge group $SU(2)_R$ have masses of order $10^8$ GeV [11, 12]. Thus, restoring parity in this way creates another unusual disparity of scale in the mirror gauge sector of these models. Moreover, requirement of small neutrino masses further increases scale of parity breaking. Furthermore, these mirror gauge bosons are out of the reach of the LHC, and being so heavy may not be able to search in near future.

This problem is solved in the low scale left-right-right-left (LSLRRL) model where gauge symmetry was enlarged to accommodate mirror fermions while the right-handed degrees of freedom of the standard model (SM) and three right-handed neutrinos are kept in the fundamental representation of the gauge group $SU(2)_R$ [6].

In this paper, we propose a low scale effective left-right symmetric mirror model (LRSSM) which restore parity in a way such that mirror gauge sector and mirror fermions can exist simultaneously at TeV scale. In contrast to LSLRRL, the LRSSM is a minimal model. A simple ultraviolet (UV) completion of the LRSSM with vector-like fermions is also discussed. A similar model having spontaneous break down of $CP$ is discussed in Ref. [21]. Finally we provide phenomenological signatures and consequences of the LRSSM.
The LRSMM can be described by the following field transformations under $SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y'}$:

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \sim (1, 2, 1, -1), \quad e_R \sim (1, 1, 1, -2);$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \sim (3, 2, 1, \frac{1}{3}), \quad u_R \sim (3, 1, 1, \frac{4}{3}), \quad d_R \sim (3, 1, 1, -\frac{2}{3});$$

$$l'_R = \begin{pmatrix} \nu' \\ e' \end{pmatrix}_R \sim (1, 1, 2, -1), \quad e'_L \sim (1, 1, 1, -2);$$

$$q'_R = \begin{pmatrix} u' \\ d' \end{pmatrix}_R \sim (3, 1, 2, \frac{1}{3}), \quad u'_L \sim (3, 1, 1, \frac{4}{3}), \quad d'_L \sim (1, 1, 1, -\frac{2}{3});$$

where $l_L, q_L$ are the SM doublets of leptons and quarks, and $e_R, \nu_e R, d_R$ and $u_R$ are the SM singlets. $l'_R, q'_R, e'_L, \nu'_{eL}, d'_L$ and $u'_L$ denote mirror fermions and their quantum numbers. The field transformations for second and third families are identical to the first family.

The fermionic and gauge fields under parity transform as:

$$\psi_L \longleftrightarrow \psi'_R, \quad \psi_R \longleftrightarrow \psi'_L, \quad \mathcal{W}_L \longleftrightarrow \mathcal{W}_R, \quad \mathcal{B}_\mu \longleftrightarrow \mathcal{B}_\mu, \quad \mathcal{G}_{\mu\nu} \longleftrightarrow \mathcal{G}_{\mu\nu},$$

where $\psi_L$ is a doublet of the gauge groups $SU(2)_L$, and $\psi'_R$ is a doublet of the gauge group $SU(2)_R$. $\psi_R$ and $\psi'_L$ are singlets under either of them. $\mathcal{W}_L$ is the gauge field corresponding to the gauge group $SU(2)_L$, and $\mathcal{W}_R$ is the gauge field of the gauge symmetry $SU(2)_R$. $\mathcal{B}_\mu$ represents the gauge field corresponding to the gauge symmetry $U(1)_{Y'}$. $\mathcal{G}_{\mu\nu}$ denotes the gluon field strength tensor.

The spontaneous symmetry breaking (SSB) occurs in the following way:

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y'} \rightarrow SU(2)_L \otimes U(1)_{Y} \rightarrow U(1)_{EM}. \quad (4)$$

The above symmetry breaking pattern is achieved by introducing two Higgs doublets which transform in the following way under $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y'}$:

$$\varphi_L = \begin{pmatrix} \varphi^+ \\ \varphi_0 \end{pmatrix}_L \sim (1, 2, 1, 1), \quad \varphi_R = \begin{pmatrix} \varphi^+ \\ \varphi_0 \end{pmatrix}_R \sim (1, 1, 2, 1). \quad (5)$$
and behave under parity as follows:

$$\varphi_L \leftrightarrow \varphi_R$$  \hspace{1cm} (6)

Besides doublets, two real scalar singlets are also needed to provide masses to fermions as discussed later. Singlets scalar fields have following quantum numbers under $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)'$:

$$\chi : (1, 1, 1, 0), \quad \chi' : (1, 1, 1, 0),$$  \hspace{1cm} (7)

and their behaviour under parity is described as,

$$\chi \leftrightarrow \chi'.$$  \hspace{1cm} (8)

| Fields | $Z_2$ | $Z'_2$ |
|--------|-------|-------|
| $\psi_R$ | +     | -     |
| $\chi$   | +     | -     |
| $\psi'_L$ | -     | +     |
| $\chi'$  | -     | +     |

Table 1: The charges of fermionic and singlet scalar fields under $Z_2$ and $Z'_2$ symmetries.

For having mirror gauge sector and mirror fermions simultaneously at TeV scale, we note that fermion-scalar interactions are not governed by any symmetry in the SM and put into the SM by hand. Their description through the Yukawa operator is a simple selection. However, it is possible that these interactions are actually described by dimensional-5 operator.

Hence, for this purpose, a pair of discrete symmetries $Z_2$ and $Z'_2$ is imposed on the fermionic fields $\psi_R$, $\psi'_L$ and scalar singlets $\chi$, $\chi'$, keeping all other fields even under $Z_2$ and $Z'_2$. This is shown in Table I.

The discrete symmetries $Z_2$ and $Z'_2$ forbid the Yukawa operator, and mass term, for instance, for first family leptons is given by dimensional-5 operator,

$$\mathcal{L}_{mass} = \frac{1}{\Lambda} \left[ l_L (\Gamma_1 \varphi_L \chi + \Gamma_2 \tilde{\varphi}_L \chi) e_R + \tilde{l}_R (\Gamma'_1 \varphi_R \chi' + \Gamma'_2 \tilde{\varphi}_R \chi') e'_L \right]$$  \hspace{1cm} (9)

$$+ \frac{1}{\Lambda} \left[ \rho_1 \tilde{l}_L \varphi_L \varphi^\dagger_R l'_R + \rho_2 \tilde{l}_L \tilde{\varphi}_L \varphi^\dagger_R l'_R + \sigma_1 e'_L \chi \chi' e_R \right] + \text{H.c.,}$$

where $\Gamma_i = \Gamma'_i$ ($i = 1, 2$) due to parity and, $\Gamma, \rho, \sigma$ are $3 \times 3$ matrices in family space. $\tilde{\varphi} = i \sigma_2 \varphi^*$ is charge conjugated Higgs field, and $\sigma_2$ is the second Pauli matrix. We can write a similar Lagrangian for other fermions.
We need the pattern of the SSB such that \( \langle \chi' \rangle \gg \langle \varphi_R \rangle \gg \langle \varphi_L \rangle \) and \( \langle \chi' \rangle \gg \langle \chi \rangle \). This will result in a mirror gauge and fermionic sector at TeV scale simultaneously. Now, mass of the mirror counter-part of the electron is 

\[
m_{e'} \approx m_e \frac{\langle \varphi_R \rangle \langle \chi' \rangle}{\langle \varphi_L \rangle \langle \chi \rangle}.
\]

The LHC has searched for mirror fermions, and has excluded them up to 690 GeV\(^2\). However, these searches are model dependent. The CMS collaboration has searched for an extra \( W \) boson and has excluded it up to 4.1 TeV\(^2\). Hence, being conservative, we can assume \( \langle \varphi_R \rangle \geq 4 \) TeV. The SM extended by a real singlet scalar field is studied in Ref. \(^2\), and range of \( \langle \chi \rangle \) is given between 2.5 GeV to 3075 GeV. Using these numbers, we provide a rough bound between approximately 500 GeV to 3400 GeV on the mass of the lightest charged mirror lepton in Fig.\text{\ref{fig:mass_bound}} where we have chosen \( \langle \chi' \rangle = 10^8 \) GeV. However, this bound depends on the chosen value of \( \langle \chi' \rangle \). Thus, we observe that remarkably new mirror gauge and fermionic sector could exist simultaneously around TeV scale.

The Majorana mass term for neutrinos is provided by the following equation:

\[
\mathcal{L}_{\text{Majorana}}^\nu = \frac{c}{\Lambda} \left[ \bar{L}_L \gamma^\nu \gamma^5 L_L + \bar{L}_R \gamma^\nu \gamma^5 R_R \right] + \text{H.c.} \tag{10}
\]
The most general scalar potential of the model reads,

\[
V = -\mu_L^2 \phi_L^\dagger \phi_L - \mu_R^2 \phi_R^\dagger \phi_R - \mu_\chi^2 \chi^2 - \mu_\chi' \chi'^2 + \lambda_1 \left((\phi_L^\dagger \phi_L)^2 + (\phi_R^\dagger \phi_R)^2\right) + \lambda_2 \phi_L^\dagger \phi_L \phi_R^\dagger \phi_R + \lambda_3 \left(\chi^4 + \chi'^4\right) + \lambda_4 \chi^2 \chi'^2 + \lambda_5 \phi_L^\dagger \phi_L \chi^2 + \phi_R^\dagger \phi_R \chi'^2 + \lambda_6 \phi_L^\dagger \phi_L \chi^2 + \phi_R^\dagger \phi_R \chi'^2.
\] (11)

This potential is parity invariant except for mass terms of scalar fields which constitutes a soft parity breaking. This is required to break parity spontaneously in the gauge sector of the model such that \(\langle \chi' \rangle = \omega'/\sqrt{2} \gg \langle \phi_R \rangle = v_R/\sqrt{2} \gg \langle \phi_L \rangle = v_L/\sqrt{2}\) and \(\langle \chi' \rangle \gg \langle \chi \rangle = \omega/\sqrt{2}\). All couplings of the scalar potential are real, and VEVs of the scalar doublets can be made real through the gauge symmetry. Hence, the strong CP phase is zero in the model as discussed in Ref. [24].

The gauge-scalar interactions of the model are described by the following Lagrangian:

\[
\mathcal{L}_{GS} = (\mathcal{D}_{\mu,L} \phi_L)^\dagger (\mathcal{D}_{\mu,L}^\mu \phi_L) + (\mathcal{D}_{\mu,R} \phi_R)^\dagger (\mathcal{D}_{\mu,R}^\mu \phi_R),
\] (12)

where, \(\mathcal{D}_{L,R}\) are the covariant derivatives given by,

\[
\mathcal{D}_{\mu,L}(\mathcal{D}_{\mu,R}) = \partial_\mu + ig^2 \tau^a \mathcal{W}^{a}_{\mu,L}(\mathcal{W}^{a}_{\mu,R}) + ig' B_\mu,
\] (13)

where, \(\tau^a\)'s are the Pauli matrices. \(g\) is the common coupling of the gauge groups \(SU(2)_L\) and \(SU(2)_R\). The coupling constant \(g'\) corresponds to the gauge group \(U(1)_{Y'}\).

The charged gauge bosons masses after the SSB are given by,

\[
M_{W_{L}^\pm} = \frac{1}{2}gv_L, \quad M_{W_{R}^\pm} = \frac{1}{2}gv_R.
\] (14)

The neutral gauge boson mass matrix in the basis \((W_{L}^3, W_{R}^3, B)\) is given by,

\[
M = \frac{1}{4} \begin{pmatrix}
2g^2v_L^2 & 0 & -gg'v_L^2 \\
0 & 2g^2v_R^2 & -gg'v_R^2 \\
-gg'v_L^2 & -gg'v_R^2 & g^2(v_L^2 + v_R^2)
\end{pmatrix}.
\] (15)

The matrix in Eq. (15) can be diagonalized through an orthogonal transformation \(\mathcal{T}\) which transforms the weak eigenstates: \((W_{L}^3, W_{R}^3, B)\) into the
physical mass eigenstates: \((Z_L, Z_R, \gamma)\);

\[
\begin{pmatrix}
W^3_L \\
W^3_R \\
B
\end{pmatrix} = T \begin{pmatrix}
Z_L \\
Z_R \\
\gamma
\end{pmatrix}. \tag{16}
\]

The physical masses of the neutral gauge bosons are then written as,

\[
M_{Z_L}^2 = \frac{1}{4} v_L^2 g^2 + \frac{2 g'^2}{g^2 + g'^2} \left[ 1 - \frac{g'^4}{(g^2 + g'^2)^2} \epsilon \right],
\]

\[
M_{Z_R}^2 = \frac{1}{4} v_R^2 (g^2 + g'^2) \left[ 1 + \frac{g'^4}{(g^2 + g'^2)^2} \epsilon \right], \tag{17}
\]

where \(\epsilon = v_L^2/v_R^2\). Since \(v_R >> v_L\), we have ignored terms of order \(\mathcal{O}(\epsilon^2)\) in Eq.(17).

The orthogonal matrix \(T\) in Eq.(16) can be parametrized in terms of mixing angle \(\theta_W\) which is given by the following equation:

\[
\cos^2 \theta_W = \left( \frac{M_{W_L}^2}{M_{Z_L}^2} \right)|_{\epsilon=0} = \frac{g^2 + g'^2}{g^2 + 2g'^2}. \tag{18}
\]

and transformation matrix \(T\) can be written as,

\[
T = \begin{pmatrix}
-cos\theta_W & -\sqrt{cos^2\theta_W} tan^2\theta_W \epsilon & \sin\theta_W \\
\sin\theta_W tan\theta_W \left[ 1 + \frac{cos^2\theta_W}{cos\theta_W} \epsilon \right] & -\sqrt{cos^2\theta_W} \left[ 1 - tan^4\theta_W \epsilon \right] & \sin\theta_W \\
\sin\theta_W \sqrt{cos^2\theta_W} \left[ 1 - \frac{tan^2\theta_W}{cos\theta_W} \epsilon \right] & tan\theta_W \left[ 1 + tan^2\theta_W \frac{cos^2\theta_W}{cos^2\theta_W} \epsilon \right] & \sqrt{cos^2\theta_W}
\end{pmatrix}. \tag{19}
\]

We note that the third column of that matrix is unchanged by further terms of order \(\epsilon^2\) in the transformation matrix \(T\).

The couplings of the original symmetries and the residual symmetry are related by the following equation:

\[
g = \frac{e}{sin\theta_W}, \quad g' = \frac{e}{\sqrt{cos^2\theta_W}}, \quad \frac{1}{e^2} = \frac{2}{g^2} + \frac{1}{g'^2}. \tag{20}
\]

The masses of fermions are given by Eq.(9) and (10). For instance, the Lagrangian for the down type quark and its mirror counter-part can be writ-
Figure 2: The pair production of the mirror quarks at the LHC and their subsequent decay to the SM $Z_L$ boson and a quark.

ten as,

$$\mathcal{L}_d = \frac{\Gamma_d}{\Lambda} (\bar{q}_L \varphi_L d_R \chi + \bar{q}_R' \varphi_R d'_L \chi') + \frac{1}{\Lambda} \left[ \rho_d \bar{q}_L \varphi_L \varphi_R' q_R' + \sigma_d' \bar{q}_L' \chi' q_R \right] + \text{H.c.}$$

where the mass matrix in general is $6 \times 6$.

The mass matrices can be diagonalized through bi-unitary transformations given as $[25, 26, 27, 28]$,

$$\begin{pmatrix} u \\ u' \end{pmatrix}_{L,R} = \begin{pmatrix} X_u \\ Y_u \end{pmatrix}_{L,R} \begin{pmatrix} u \\ u' \end{pmatrix}_{L,R} \quad \text{and} \quad \begin{pmatrix} d \\ d' \end{pmatrix}_{L,R} = \begin{pmatrix} X_d \\ Y_d \end{pmatrix}_{L,R} \begin{pmatrix} d \\ d' \end{pmatrix}_{L,R},$$

(22)

where $X_{u,d}$ and $Y_{u,d}$ are $3 \times 6$, and CKM matrices are given by $V_{\text{CKM}} = X_{uL}^\dagger X_{dL}$ and $V_{\text{CKM}}' = Y_{uR}^\dagger Y_{dR}$.

We observe that due to mixing of the SM and mirror fermions, the mirror fermions can decay into a SM $W_L$ or $Z_L$ boson in association with a SM fermion. Since mirror quarks can couple to gluons, for illustration, we show the pair production of the mirror quarks in Fig.2 at the LHC via gluon-gluon and quark-antiquark initial states.

There are flavor changing neutral current interactions at tree level in the model due to mirror fermions and vector-like fermions (to be discussed later). However, these interactions are suppressed by $v_L^2/v_R^2$ or by $v_L^2/M^2$ where $M$ is the mass of vector-like fermion$[26, 27, 28]$. Hence, low energy phenomenological consequences will appear, for instance, in flavour changing neutral
current processes which are highly suppressed in the SM. For example, apart from the SM box diagrams, new diagrams, having $W_R$ in the box, contributing to the $\Delta F = 2$ transitions, as shown in Fig.3 of $K$ and $B$ mesons may place non-trivial constraints on the masses of the new gauge bosons and mirror fermions.

Now we discuss an anomaly free UV completion of the LRSMM. For this purpose, we add two vector-like isosinglet up type quark, two vector-like isosinglet down type quarks, two vector-like iso-singlet charged leptons, and one iso-singlet vector-like neutrino. They transform under $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_Y$, as follows:

$$Q = U_{L,R} : (3, 1, 1, \frac{4}{3}); \quad D_{L,R} : (3, 1, 1, -\frac{2}{3}); \quad (23)$$

$$L = E_{L,R} : (1, 1, 1, -2); \quad N_{L,R} : (1, 1, 1, 0).$$

The mass term for vector-like fermions can be written as,

$$L_V = M_U \bar{U} U_R + M_D \bar{D} D_R + M_E \bar{E} E_R + M_N \bar{N} N_R + H.c. \quad (24)$$
The interactions of vector-like fermions with the SM fermions are given by,

\[ L'_{V ff} = y' \left[ \bar{q}_L \varphi R Q_R + \bar{q}_R \varphi R Q_L \right] + c' \left[ \bar{L}_L \varphi L L_R + \bar{L}_R \varphi L L_L \right] + \text{H.c.} \quad (25) \]

The interactions of singlet SM and mirror fermions with vector-like fermions are given by,

\[ L''_{V ff} = y'' \left[ \bar{q}_R Q L \chi + \bar{q}_L Q R \chi' \right] + c'' \left[ \bar{L}_R L \chi + \bar{L}_L L \chi' \right] + \text{H.c.} \quad (26) \]

Now the Lagrangian giving masses of the SM and mirror fermions in Eq.(9) can be realized as shown in Fig.4. Vector-like fermions are extensively studied in literature, and their latest phenomenological status can be found in Refs.[29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43]

We briefly comment on recently observed anomalies in the flavor-changing neutral current transition \( b \rightarrow s l^+ l^- \) and their consequences in the LRSMM. The first deviation from the SM is in the optimised observable \( P'_5 \) measured by the LHCb of 3.7\( \sigma \) significance [44]. Another interesting observable measured by the LHCb which is hinting the lepton flavour universality (LFU) violation is the ratio \( R_K = \frac{B_{B \rightarrow K^+ \mu^-/e^-}}{B_{B \rightarrow e^+ e^-}} \) [46]. More interestingly, the LHCb has recently presented their results on the ratio \( R_{K^*} = \frac{B_{B \rightarrow K^{*+} \mu^-/e^-}}{B_{B \rightarrow K^{*+} e^+ e^-}} \) showing significant deviation from the SM lepton-flavour universality [47]. In the LRSMM, LFU violation enters through mixing of the SM and mirror fermions and a possible explanation of deviations observed in \( P'_5 \), \( R_{K,K^*} \) may be provided by a contribution due to new heavy vector gauge boson at tree level.

In addition to this, a solid evidence for an explanation of flavour anomalies in the quark-level \( b \rightarrow s l^+ l^- \) transitions comes from a recent nice paper by Botella et al [48]. In this paper, flavour anomalies in the quark-level \( b \rightarrow s l^+ l^- \) transitions are explained by the FCNC effects of vector-like quarks and a heavy neutrino. The model presented in this work is in fact a possible UV completion of the framework used by Botella et al [48] since it has naturally vector-like quarks \( U, D \) and a heavy neutrino \( N \) required in Ref.48. Hence, an explanation of flavour anomalies in the quark-level \( b \rightarrow s l^+ l^- \) transitions in the model discussed in this paper is already on concrete ground due to the analysis performed in Ref.48.

Finally we comment that the LRSMM restores parity in a way such that mirror gauge and fermionic sectors can coexist at the same scale. An interesting feature of the model is LFU violation which could be interesting keeping in mind recent observed flavour anomalies. A detailed phenomenological investigation is under progress.
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