Modelling the income distribution in the European Union: An application for the initial analysis of the recent worldwide financial crisis

Maciej Jagielski · Ryszard Kutner

Abstract By using methods of statistical physics, we focus on the quantitative analysis of the economic income data descending from different databases. To explain our approach, we introduce the necessary theoretical background, the extended Yakovenko et al. (EY) model. This model gives an analytical description of the annual household incomes of all society classes in the European Union (i.e., the low-, medium-, and high-income ones) by a single unified formula based on unified formalism. We show that the EY model is very useful for the analyses of various income datasets, in particular, in the case of a smooth matching of two different datasets. The completed database which we have constructed using this matching emphasises the significance of the high-income society class in the analysis of all household incomes. For instance, the Pareto exponent, which characterises this class, defines the Zipf law having an exponent much lower than the one characterising the medium-income society class. This result makes it possible to clearly distinguish between medium- and high-income society classes. By using our approach, we found that the high-income society class almost disappeared in 2009, which defines this year as the most difficult for the EU. To our surprise, this is a contrast with 2008, considered the first year of a worldwide financial crisis, when the status of the high-income society class was similar to that of 2010. This, perhaps, emphasises that the crisis in the EU was postponed by about one year in comparison with the United States.

Keywords Income distribution · Yakovenko model · Econophysics

JEL Classification A12 · C46 · C81 · D63
1 Introduction

In order to describe the complexity of the world around us, contemporary scientific research quite often combines methodologies and methods that have up to now been used in different, even far-away fields of science and not only those offered separately by various scientific disciplines. A prominent example of such research provides the econophysics. This emerging branch of science applies methods and models used most often in statistical physics and condensed matter physics to describe some economic and financial processes. The term econophysics was first used by the physicist H. Eugene Stanley in Ref. (Stanley et al, 1996) – this paper is a kind of manifesto of econophysics.

The term econophysics is a neologism – a combination of two words: economics and physics, as in the case of astrophysics, geophysics, and biophysics, which use methods of physics to describe the phenomena studied within astronomy, geology, and biology, respectively. It should be emphasised that econophysics does not apply the laws of physics literally to describe the economic behaviour of different types of entities, such as investors, individuals, or households. Most often it uses reinterpreted and properly modified methods developed in statistical physics to analyse the statistical properties of complex systems (consisting of a large number of the aforementioned entities) (Yakovenko and Rosser, 2009). Shortly speaking, the econophysics focuses on the quantitative analysis of economic and financial data by the mathematical and physical modelling of a large number of interacting economic entities (also called agents) – it also has much in common with research in econometrics and multi-agent modelling (called agent-based modelling) (Yakovenko and Rosser, 2009).

One of the major trends in econophysics is the study of income and wealth redistribution in society and the analysis of social inequalities. Vilfredo Pareto, Italian economist and sociologist, is a pioneer of this research. At the fall of the nineteenth century, Pareto was the first to provide an analytical description of the distribution of wealth in society represented by the annual income of individuals. One of his most significant findings was the fact that the distribution of income of individuals from different countries is universal and has a small variability in "space" and time. In addition, Pareto stated that these distributions do not resemble the shape of the distribution that one would obtain if the accumulation of wealth was a random process. He also recognized the stability of these distributions. That is, even if one excludes the richest and the poorest from the process of gaining income, after a period of time, the distribution of income will resemble the shape of the initial distribution (Pareto, 1897; Mandelbrot, 1960; Richmond et al, 2006; Jagielski, 2009). The striking result of Pareto’s study was that the distribution of income in countries with a stable economy is described by a universal power-law known nowadays as the Pareto law. As a possible origin of this law, Pareto pointed to a hierarchical, self-similar structure of societies. Pareto’s findings inspired many researchers to continue attempting analytical descriptions of the income of societies.
The income of societies has also been analysed by French economist Robert Gibrat. He pointed out that the Pareto law is not able to describe the distribution of income over the entire range. He proposed a complementary approach known as the Rule of Proportionate Growth. Using the stochastic process to describe the dynamics of income or wealth of a single person or household, he found out that the theoretical probability distribution function described incomes belonging to the low-income society class (Gibrat, 1931; Kalecki, 1945; Armatte, 1995; Sutton, 1997; Richmond et al, 2006; Jagielski, 2009).

Furthermore, David Champernowne proposed one of the first stochastic models, which reproduces the Pareto law (Champernowne, 1953; Jagielski, 2009). Benoît Mandelbrot described the fundamental properties of random variables from the Pareto distribution (Mandelbrot, 1963; Jagielski, 2009). The analytical description of the distribution functions of income made by Gibrat, Champernowne and Mandelbrot led to the disclosure of many significant properties of these distributions, however, it did not give an answer to the crucial question concerning the microscopic (microeconomic) mechanism determining empirical complementary distribution functions. Recently, several models have been proposed which partially explain the microscopic mechanisms (for income dynamics of individuals or households) standing behind the observed empirical complementary distribution functions of incomes.

In principle, the above mentioned models can be classified into two different groups. The first group is based on Boltzmann’s kinetic theory of collision in gases. Analogously to the collision of two particles, if kinetic energy is exchanged between them, it is assumed in models – called collision models – that two randomly selected individuals or households exchange money according to the corresponding rules (Angle, 1986, 1992, 1993, 1996; Ispolatov et al, 1998; Chakraborti and Chakrabarti, 2000; Drăgulescu and Yakovenko, 2000; Angle, 2002, 2006). By using more complex money exchange rules we obtain the basic collision model proposed by Drăgulescu and Yakovenko (Drăgulescu and Yakovenko, 2000; Chatterjee and Chakrabarti, 2007), which leads to the Boltzmann-Gibbs distribution. This group also contains: (i) models of collisions allowing a negative income (or debt) (Drăgulescu and Yakovenko, 2000; Fischer and Braun, 2003; Xi et al, 2005; Cockshott and Cottrell, 2008), (ii) models of collisions with the same saving propensity for all individuals (Angle, 1986, 1992, 1993, 1996; Ispolatov et al, 1998; Angle, 2002; Patriarca et al, 2004b; Chatterjee and Chakrabarti, 2007), and (iii) models of collisions with varying saving propensity (Chakrabarti and Chakrabarti, 2000; Angle, 2002; Patriarca et al, 2004a; Chatterjee et al, 2004; Ferrero, 2004; Scafetta et al, 2004a, b; Patriarca et al, 2005; Repetrovic et al, 2005; Chakrabarti, 2005; Chatterjee et al, 2005; Bhattacharyya et al, 2005; Angle, 2006; Chatterjee and Chakrabarti, 2007). The second group of models treats the income of an individual or household as a random variable. To describe the dynamics of this variable, the nonlinear stochastic Langevin equation and the corresponding Fokker–Planck equation are used. Depending on the specific assumptions concerning the dynamics of income, one can obtain the following models: (i) the Boltzmann–Gibbs law (Richmond et al, 2006; Yakovenko and Rosser,
2009; Banerjee and Yakovenko, 2010), (ii) the Pareto law (Richmond et al.
2006; Yakovenko and Rosser, 2009; Banerjee and Yakovenko, 2010), (iii) the
Rule of Proportionate Growth (Gibrat, 1931; Kalecki, 1945; Armatte, 1995;
Sutton, 1997; Richmond et al., 2006; Yakovenko and Rosser, 2009), (iv) the
Generalised Lotka–Volterra model (Solomon and Richmond, 2001; Solomon and
Yakovenko, 2009; Banerjee and Yakovenko, 2010; Richmond et al., 2006;
Huang, 2004; Richmond et al., 2006; Yakovenko and Rosser, 2009), and (v) the
Yakovenko et al. model (Yakovenko and Rosser, 2009; Banerjee and Yakovenko,
Remarkably, both in the case of the Boltzmann kinetic theory and in the case of Langevin
stochastic dynamics (which lead to distributions of income of individuals or
households), econophysics paves the way for new trends of research, comple-
mentary to those developed in economic and social sciences (Kiyotaki and
Wright, 1993; Molico, 2006).

Besides the construction of analytical models describing the distribution
functions of income, their verifications were intensively conducted recently, as
extensive empirical databases became, in principle, a public domain. These
verifications were carried out, among others, for the United States (Levy and
Solomon, 1997; Drăgulescu and Yakovenko, 2001a; Reed, 2003; Rawlings
et al., 2004; Scafetta et al., 2004a; Lukasiewicz and Orłowski, 2004; Yakovenko
and Silva, 2005; Silva and Yakovenko, 2005; Clementi and Gallegati, 2005a;
Niri and Souma, 2007; Clementi et al., 2008, 2009; Yakovenko and Rosser,
2009; Banerjee and Yakovenko, 2010), the United Kingdom (Pareto, 1897;
Drăgulescu and Yakovenko, 2001b; Scafetta et al., 2004b; Ferrero, 2004, 2005;
Clementi and Gallegati, 2005b; Richmond et al., 2006; Clementi et al., 2007).
Germany (Pareto, 1897; Clementi and Gallegati, 2005a; Clementi et al., 2007),
Italy (Pareto, 1897; Clementi and Gallegati, 2005b; Clementi et al., 2006, 2007),
France (Pareto, 1897), Switzerland (Pareto, 1897), Japan (Aoyama et al., 2000;
Souma, 2001; Fujiwara et al., 2003; Aoyama et al., 2003; Ferrero, 2004, 2005;
Souma and Niri, 2007), Australia (Matteo et al., 2004; Clementi et al., 2006;
Banerjee et al., 2006; Clementi et al., 2008), Canada (Reed, 2003), the Czech
Republic (Reed, 2003), New Zealand (Ferrero, 2004, 2005), India (Sinha, 2006),
Sri Lanka (Reed, 2003), Argentina (Ferrero, 2005), Peru (Pareto, 1897), South
Korea (Kim and Yoon, 2004), and Romania (Derzay et al., 2012).

However, none of the models that have been developed so far (to the best
of our knowledge) give an analytical description of the annual household in-
comes of all society classes (i.e. the low-, medium-, and high-income society
classes) by a single unified formula based on unified formalism. In our recent
papers (Jagielski and Kutner, 2013a, b) we developed the Extended Yakovenko
model, which provided such a powerful formula. This formula (with the low
number of free parameters) reproduces the empirical complementary cumu-
lative distribution function in the entire range of the income. In the present
paper we give a short review of the Extended Yakovenko model and show that
this model is valid for various income datasets, especially when matching two
different datasets.
It should be noted that the subject of this paper relates, at least partially, to the problems analysed in sociophysics. Sociophysics, in contrast to econophysics, does not focus solely on the research of economic activity of individuals but, by using the methods of physics, it studies the social mainstream subjects such as the analysis of political preferences, social networks, formed coalitions, terrorism as well as the dynamics of public opinions and emotions (Yakovenko and Rosser 2009; Galam 2012).

2 Matched dataset

We exploit the empirical data from Eurostat’s Survey on Income and Living Conditions (EU–SILC) (Eurostat, 2013, 2005, 2006, 2007, 2008, 2009, 2010) for the years 2005–2010. This database contains information on the demographic characteristics of households in the European Union (EU), their living conditions, and their income and economic activity. For our analysis we chose the *Total household gross income* variable. According to Eurostat, the definition of the annual total gross household income (we quote) “... is the total monetary and non-monetary income of a household over a period of one year, before deducting taxes on income or wealth or social security contributions by employers and employees but after including inter-household transfers received” (Eurostat 2013).

Eurostat’s EU–SILC database contains only a few records concerning the income of households belonging to the high-income society class. That is, these households cannot be subject to any statistical description. Therefore, in order to consider the high-income society class, we have additionally analysed the effective income of billionaires in the EU by using the Forbes ranking “The World’s Billionaires” (Forbes 2013). This ranking contains individuals whose value of wealth in a given year exceeds one billion US dollars. From this ranking, we selected only those who reside in the European Union. Hence, we were able to make the ranking of the richest Europeans for the years 2004–2010.

Next, by using the EU–SILC database and the ranking of the richest Europeans, we considered incomes of three society classes thanks to the following procedure (roughly described in Refs. (Jagielski and Kutner 2013a,b)):

(i) Firstly, we calculated their incomes for the years 2005–2010. This calculation was possible because we assumed that the billionaires incomes were proportional to the corresponding differences between their wealth for the pairs of successive years (here from 2004 to 2010). Notably, we took into account only the billionaires who gained effective incomes.

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1 The term ‘billionaire’ used herein is equivalent (as in US terminology) to the term ‘multimillionaire’ used in European terminology. Since we consider the wealth and income of billionaires in euros, we recalculated the US dollars to euros by using the average exchange rate on the day the Forbes list “The World’s Billionaires” was constructed.

2 The billionaires who gained effective incomes are billionaires whose incomes are greater than zero.
(ii) Secondly, having calculated the incomes for the high-income society class, we simply matched them with the EU–SILC dataset. Then, by using the dataset completed in this way, we constructed the initial empirical complementary cumulative distribution function for the years 2005–2010 separately. For that, we used the well known Weibull recipe (see below for details) (Huang, 2004; Chow et al, 1998). However, this direct but too simplified approach shows a wide gap of incomes among the high-income society class resulting in a horizontal line of the complementary cumulative distribution function. The reason for the gap is that the first segment of high-income society class consists in all the data points derived from the EU–SILC dataset, whereas the other segment of high-income society class, comprised of the remaining data points, has been taken from the Forbes dataset.

(iii) In the final step, we eliminated this gap by adopting the assumption that the empirical complementary cumulative distribution function (concerning the whole society) has no horizontal segments. That is, we assumed that the statistics of income is a continuous function of income (i.e. there is no disruption). Hence, we were forced to multiply the billionaire incomes from the Forbes dataset by the properly chosen common proportionality factor. This factor is not an arbitrary one – it is equal to $1.0 \times 10^{-2}$, since the obvious requirement of a full overlap of the first (above mentioned) segment by the subsequent (second) segment was assumed. Hence, this approach leads to a unique solution (up to some negligible statistical error) for this proportionality factor. Furthermore, we found that this factor was only a slowly-varying function of time (or years).

Thus, we obtained the matched dataset (MDS) already containing the sufficient data points covering all society classes, i.e. containing also the high-income society class.

In order to analyse the presented empirical data in a more stable form, we used an empirical complementary cumulative distribution function. We calculated it according to the commonly used two-step procedure. Firstly, the income empirical data was ordered according to its rank, i.e. from the richest household incomes to the poorest. Next, in accordance with the Weibull formula (Chow et al, 1998; Haneberg, 2004), we calculated the ratio $\frac{l}{n+1}$, where $l$ is the position of the household in the rank and $n$ is the size of the empirical data record. This ratio directly determines the required fraction of households of an income higher than that related to a given household position $l$ in the rank. The complementary cumulative distribution function obtained this way is sufficiently persistent. Furthermore, it does not reduce the size of the output in comparison to that of the original empirical data record.
3 Extended Yakovenko et al. model

We present the necessary theoretical background that is extended by us, the Yakovenko et al. model. A detailed description of the model can be found in our earlier paper (Jagielski and Kutner, 2013b).

Let $m$ be an influx of income per unit of time for a given household. We treat $m$ as a variable obeying the stochastic dynamics. Then, we can describe the time evolution of the income probability distribution function by using the so called second diffusion equation (Yakovenko and Rosser, 2009; Banerjee and Yakovenko 2010; van Kampen, 2011)

$$\frac{\partial}{\partial t} P(m,t) = \frac{\partial}{\partial m} [A(m)P(m,t)] + \frac{\partial^2}{\partial m^2} [B(m)P(m,t)].$$

(2)

where, $B(m) = C^2(m)/2$ and $P(m,t)$ is the temporal income distribution function. In general, functions $A(m)$ and $B(m)$ can additionally be determined by the first and second moments of the income change per unit of time, respectively, only if these moments exist. Subsequently, the equilibrium solution of Eq. (2), $P_{eq}$ takes the form (van Kampen, 2011):

$$P_{eq}(m) = \frac{const}{B(m)} \exp \left( - \int_{m_{init}}^{m} \frac{A(m')}{B(m')} dm' \right).$$

(3)

where $m_{init}$ is the lowest household income and const is a normalisation factor.

Using Eq. (3) we derive such a distribution function which covers all three ranges of the empirical data records, i.e. the low-, medium-, and high-income society classes (including also two short intermediate regions between them). Therefore, we provide function $A(m)$ in a threshold form (Jagielski and Kutner, 2013a,b):

$$A(m) = \begin{cases} 
A^< (m) = A_0 + a m & \text{if } m < m_1 \\
A^> (m) = A'_0 + a' m & \text{if } m \geq m_1
\end{cases},$$

$$B(m) = B_0 + b m^2 = b (m_0^2 + m^2),$$

(4)

where parameters used in the above equation are defined and considered below.

This definition of $A(m)$ and $B(m)$ allows for the coexistence of additive and multiplicative stochastic processes since we assume that household income consists of two components. The first, a pure deterministic component of income, arises from the fact that household income is determined by wages and salaries. The second, already an indeterministic component, expresses profits which go to households mainly through investments and capital gains.

Physicists call the second diffusion equation the Fokker–Planck equation. This equation is equivalent to the Langevin equation

$$\frac{d m}{dt} = -A(m) + C(m) \eta(t).$$

(1)

Here, $A(m)$ is a drift term and $\eta(t)$ is a white noise, where the coefficient $C(m)$ is its $m$-dependent amplitude.
The threshold parameter $m_0$ is the crossover income between the low- and medium-income society classes, while parameter $m_1$ is interpreted as a crossover income between the medium- and high-income society classes.

Subsequently, by substituting Eq. (4) into Eq. (3), we finally get

$$P_{eq}(m) = \begin{cases} 
  c' \exp\left(-\frac{(m_0/T) \arctan(m/m_0)}{\alpha_1 + 1}\right) & \text{if } m < m_1 \\
  c'' \exp\left(-\frac{(m_0/T_1) \arctan(m/m_0)}{\alpha_1 + 1}\right) & \text{if } m \geq m_1
\end{cases}$$

where exponents $\alpha = 1 + a/b$, $\alpha_1 = 1 + a'/b$, and income temperatures $T = B_0/A_0$, $T_1 = B_0/A'_0$. Parameter $T$ can be interpreted in this case as the average income per household for low- and medium-income society classes. Parameter $T_1$ has the same interpretation but for a high-income society class. Apparently, the number of free (effective) parameters driving the two-branch distribution function, given by Eq. (5), is reduced because this function depends only on the ratios of the corresponding parameters defining the nonlinear Langevin dynamics given by Eq. (1).

For $m \ll m_0$, the distribution function, given by the first expression in Eq. (5), becomes the Boltzmann–Gibbs law. For $m_0 \ll m < m_1$ it gives the weak Pareto law with exponent $\alpha$. For $m \gg m_1$ we obtain, from the second expression in Eq. (5), also the weak Pareto law but with the exponent $\alpha_1 < \alpha$.

4 Results and concluding remarks

In this Section we compared the theoretical complementary cumulative distribution functions, based on Eq. (5), with the corresponding empirical ones. The latter are constructed from data coming from two sources – the EU–SILC dataset and the MDS.

The corresponding plots of the theoretical and empirical complementary cumulative distribution functions are compared, in a log-log scale, in Figs. 1-3 for the most intriguing years 2008–2010, respectively. Besides, for further comparisons, Tables 1-3 provide estimates of the parameters of the EY formula for the years 2005–2010 for both above mentioned datasets.

Notably, the high-income society class almost disappeared in 2009, which can define this year as the economic crash for the EU. This is in contrast to 2008, which was defined as a worldwide financial crisis, when the status of the high-income society class was very similar to that of 2010. This, perhaps, emphasises that the crisis in the EU was postponed by about one year in comparison with that of the United States. This is the striking utilitarian result of the paper.

Apparently, the predictions of the Extended Yakovenko et al. Formula (5), agree with the corresponding empirical cumulative distribution functions of the annual total gross incomes of households in the European Union. We confirmed that the value of parameter $m_0$ can be considered a crossover income between low- and medium-income society classes. Similarly, the value of parameter $m_1$ can be considered as a (subsequent) crossover income between medium- and
Fig. 1 Two plots of complementary cumulative distribution functions obtained from the extended Yakovenko et al. formula \[ \mathcal{F}(x) \] (dashed and solid curves) and income empirical datasets (open squares and full circles; cf. Eurostat 2008, Forbes 2013 for details). The lower plot (dashed curve and open squares) concerns the EU–SILC database. The upper plot (solid curve and full circles) concerns MDS – both for the year 2008. The dotted vertical line denotes the value \( m_0 \) common for both plots. The dashed and solid vertical lines denote two different values of \( m_1 \) for both plots, respectively.

Fig. 2 Two, almost overlapping plots of complementary cumulative distribution functions obtained from the EY Formula \[ \mathcal{F}(x) \] (both dashed and solid curves) and income empirical datasets (open squares and full circles as well; cf. Eurostat 2009, Forbes 2013 for details). The slightly lower plot (dashed curve and open squares) concerns the EU–SILC database. The slightly upper plot (solid curve and full circles) concerns MDS – both for the year 2009. The dotted vertical line denotes the value \( m_0 \) common for both plots. The solid vertical line denotes the value \( m_1 \), common for both plots.

high-income society classes. The values of exponents \( \alpha \) and \( \alpha_1 \) indicate social stratifications within the medium- and high-income society classes, respectively. The lower values of \( \alpha \) and \( \alpha_1 \) mean higher social stratifications in the corresponding classes. For values of parameters \( T \) and \( T_1 \), we obtained quite reasonable quantities, which confirm their interpretation given in Sec. 3.

Furthermore, Tables 4–5 show that regardless of whether the Extended Yakovenko model et al. is compared with empirical data coming from the EU–SILC database or with empirical data coming from the matched databases
In conclusion, the completed database MDS, which we constructed, emphasises the decisive role of the high-income society class in a thorough, systematic analysis of the annual household incomes in the EU.
Table 2 Values of parameters $m_0$ and $m_1$ found by the fit of the EY Formula (5) as well:
(i) to the empirical cumulative distribution functions of the annual total gross income of households obtained from the EU–SILC database as (ii) from the EU–SILC+Forbes database – the years 2005–2010.

| Year | EU–SILC database | EU–SILC+Forbes database |
|------|------------------|------------------------|
|      | $m_0$ [EUR]      | $m_1$ [EUR]           | $m_0$ [EUR]      | $m_1$ [EUR]           |
| 2005 | 160,000 ± 20,000 | 330,000 ± 50,000       | 155,000 ± 20,000 | 430,000 ± 50,000       |
| 2006 | 150,000 ± 20,000 | 330,000 ± 50,000       | 145,000 ± 20,000 | 445,000 ± 50,000       |
| 2007 | 160,000 ± 20,000 | 325,000 ± 50,000       | 160,000 ± 20,000 | 480,000 ± 50,000       |
| 2008 | 120,000 ± 20,000 | 320,000 ± 50,000       | 120,000 ± 20,000 | 450,000 ± 50,000       |
| 2009 | 145,000 ± 20,000 | 290,000 ± 50,000       | 145,000 ± 20,000 | 290,000 ± 50,000       |
| 2010 | 140,000 ± 20,000 | 200,000 ± 50,000       | 135,000 ± 20,000 | 450,000 ± 50,000       |

Table 3 Values of exponents $\alpha$ and $\alpha_1$ found by the fit of the EY Formula (5) as well:
(i) to the empirical cumulative distribution functions of the annual total gross income of households obtained from the EU–SILC database as (ii) from the EU–SILC+Forbes database – the years 2005–2010.

| Year | EU–SILC database | EU–SILC+Forbes database |
|------|------------------|------------------------|
|      | $\alpha$         | $\alpha_1$            | $\alpha$         | $\alpha_1$            |
| 2005 | 3.216 ± 0.002    | 1.54 ± 0.02            | 2.907 ± 0.004    | 0.793 ± 0.009          |
| 2006 | 3.094 ± 0.003    | 2.15 ± 0.02            | 2.892 ± 0.004    | 0.86 ± 0.01            |
| 2007 | 3.057 ± 0.003    | 2.32 ± 0.01            | 2.735 ± 0.004    | 0.79 ± 0.01            |
| 2008 | 3.063 ± 0.0005   | 2.13 ± 0.02            | 2.965 ± 0.001    | 0.890 ± 0.007          |
| 2009 | 2.979 ± 0.001    | 2.750 ± 0.005          | 2.974 ± 0.001    | 2.608 ± 0.006          |
| 2010 | 3.329 ± 0.001    | 2.43 ± 0.01            | 3.153 ± 0.002    | 0.77 ± 0.01            |

As the extended Yakovenko et al. model describes well the income of the EU households while the (usual) Yakovenko model (Yakovenko and Rosser, 2009; Banerjee and Yakovenko, 2010) is valid for the incomes of US households, we believe it would be interesting to see the results of a comparative analysis of the incomes of the EU and the US households.

We suppose that our results give the basis for a better understanding of the mechanisms of enrichment and impoverishment of societies. It is also very likely that we can find a quite precise classification of income ranges which determine whether a given household belongs to the low-, medium- or high-income society class.

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