Investigating 10th grade students’ understanding of the structure of deductive proof

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Abstract. Miyazaki, Fujita, and Jones define the structure of deductive proofs as the relational network via deductive reasoning that combines singular and universal propositions. Related to the theory, this research aims to investigate students understanding of the structure of deductive proofs. Researchers conducted a qualitative research to 30 students of 10th grade. The data was collected from geometrical proof test that focused on Sine and Cosine rule. The result of this research showing the lack of students understanding of the structure of deductive proofs. For the future research, researchers wonder whether to apply a flow-chart proof or a table proof to teach mathematical proving can improve students understanding of the structure of deductive proofs.

1. Introduction

The need to understand and apply mathematics in everyday life and in the workplace will always increase[1]. Someone who understands and applies mathematics in his life will have the opportunity to shape his future[1]. Furthermore, mathematical competence opens the door to a productive future. However, the lack of mathematical ability will keep its closed[1].

To be able to open the door to a productive future, mathematics is known in a mathematical ability known as Higher Order Thinking Skill (HOTS). HOTS or high-level mathematical hard skills is a mathematical ability that requires the ability to link, connect, analyse and synthesize mathematical concepts that already held to form or discover new concepts, principles and or mathematical rules[2]. HOTS is an ability that is integrated by the Indonesian Minister of Education and Culture in the 2013 Curriculum revised in 2017[3].

One of the abilities that classified as HOTS is construct proof with direct or indirect proof or with mathematical induction. There are several important aspects of evidence and proof[2]. One of which is the understanding of evidence as a structural object[4]. Duval proposes that in order to construct the evidence, students are urgently needed to set up premises, conclusions, and theorems[5]. Specifically, Duval argues that one of the awareness that students need to develop is the understanding of the difference between valid reasoning and invalid reasoning[5]. Therefore, Miyazaki, Fujita, and Jones reveal that mathematical proof must be supported by valid reasoning[5].

Heinze, Cheng, Ufer, and Reiss argue that to build evidence with valid reasoning requires a "bridging process"[6]. Bridging process involves: an understanding of the information provided; recognize important elements such as premises, conclusions, and arguments; establishing intermediate conditions
for the next deduction step; and coordinate the entire process into an acceptable sequence [6]. To carry out bridging process minimal use two types of deductive reasoning, namely universal instantiation and hypothetical syllogism [5].

In general, these two types of reasoning are needed to draw conclusions from the given premise [5]. However, not only that, Miyazaki, et.al. uses both of these reasoning to relate single propositions and universal propositions [5]. The relationship between a single proposition and a universal proposition associated with universal instantiation and hypothetical syllogism is defined as the structure of mathematical proof [5].

Furthermore, Miyazaki, et al. proposed a theoretical framework of three levels of understanding of mathematical proof structures. The three levels are Pre-Structural, Partial-Structural, and Holistic-Structural [5]. Pre-Structural is the ability to complete the proof without knowing the premise and conclusion in the mathematical statement to be proved. Partial-Structural is divided into two sub-levels, namely Elemental and Rational. Partial-Structural Elemental is the ability to identify premises, conclusions, and warrants that can be implicitly used in the proofing process. Partial-Structural Rational is the ability to make connections between warrants, premises, and conclusions for logical reasons; apply one of two types of reasoning, namely universal instantiation and hypothetical syllogism. Holistic-Structural is the ability to coordinate the premises, conclusions and warrants through two types of interrelated reasoning into an acceptable proof.

2. Experimental method
This research aims to investigate students understanding of the structure of deductive proofs. Researchers conducted a qualitative research to 30 students of 10th grade. The data was collected from geometrical proof test that focused on Sine and Cosine Rule. For the test, showed on Fig.1, there were three items and all items were adapted from [7]. Students were asked to construct unfamiliar proof. The students’ response was categorized into correct, incomplete, improper, and intuitive proof. Furthermore, researchers compared this to the levels of understanding of the structure deductive proofs.

![Figure 1. Design test based on Sine and Cosine Rule [8]](image)

|   |   |
|---|---|
| 1. | Pada \( \Delta ABC \) diketahui \( D \) adalah titik tengah \( AC \). Jika \( BC = a, AC = b, AB = c \), dan \( BD = d \). Buktikan bahwa \( d^2 = \frac{1}{2}a^2 - \frac{1}{4}b^2 + \frac{1}{2}c^2 \). |
| 2. | Diagonal persegi \( ABCD \) berpotongan di titik \( S \) dengan panjang sisi-sisinya adalah \( 4a \). Tunjukkan bahwa jika \( T \) titik tengah ruas garis \( SC \) maka \( \sin \angle TBS = \frac{1}{5} \). |
| 3. | Panjang sisi-sisi \( \Delta ABC \) berturut-turut adalah \( AB = 4cm, BC = 6cm \), dan \( AC = 5cm \) sedangkan \( \angle BAC = \alpha, \angle ABC = \beta \), dan \( \angle BCA = \theta \). Tunjukkan bahwa \( \sin \alpha : \sin \beta : \sin \theta = 6 : 5 : 4 \). |

3. Results and discussion
Table 1 shows the results of student written test. The columns Q.1, Q.2, and Q.3 mean the results of the first, the second, and the third question, respectively. The data shows that only less than 27% of students are able to construct a valid deductive proof. Between 10% until 40% of students answer the question using incorrect geometric properties and using properties inappropriately, and in all questions, most (20% - 73%) students makes a visual judgment or just gives an intuitive response. It is reasonable because Indonesian students just beginning to learn proof at 10th grade. Students who make intuitive solution is divided into two types of student. First, using visual judgment or intuitive response and knowing the information of the question, these students are at Partial Structural Elemental level. Then, students solve the question without knowing the information, they only reach Pre-Structural level. To illustrate the results shown in Table 1, below researchers address for the case of the first and the second question.
Table 1. Results of written test.

| Solution       | Q.1 (%) | Q.2 (%) | Q.3 (%) |
|----------------|---------|---------|---------|
| Correct*       | 2 (7)   | 0 (0)   | 8 (26)  |
| Incompleteb    | 6 (20)  | 5 (17)  | 5 (17)  |
| Improperc      | 12 (40)| 3 (10)  | 11 (37) |
| Intuitived     | 10 (33)| 22 (73)| 6 (20)  |

*Correct solution means that the students judge correctly and then construct a solution which would get the best score in school tests

b Incomplete solution means that the students judge correctly and understand the crucial elements for a solution, but there is a lack of some process or detailed explanation

c Improper solution means that the students using incorrect geometric properties, and using properties inappropriately

d Intuitive solution means that the students makes a visual judgment or just gives an intuitive response

For the first question, students can prove it with Cosine Rule. There are two or more ways to solve this question and the left part of Figure 2 present one of them. Only two students can construct a valid deductive proof. Figure 2 shows student written work for the first question. Even if there is no formal proof, the left part of Figure 2 shows a correct answer and reasoning, and it is very good for student who just beginning to learn proof, while the right part of Figure 2 shows an improper argumentation. An improper argumentation lies on the fifth line equation. Point $D$ is a mid-point of $AC$ but it is not conduce that $DB \perp AC$, so students can not apply the comparisons of right triangle on this case.

Figure 2. Representative examples of student written work for the first question.

Student on the left part of Figure 2 can construct a valid deductive proof. With a correct answer and reasoning, based on Miyazaki et.al. theory, this student is at Holistical-Structural level. Meanwhile, student on the right part of Figure 2 only reach Partial-Elemental-Structural level.
Table 3 shows the student written work for the second question. Both the left and the right part of Figure 3 are an incomplete proof. Students does not explain the detail of each steps. The left part of Figure 3 solve this question by Pythagorean’s theorem and Sine rule, while the right part of Figure 3 use Cosine and Sine rule. Based on Miyazaki et.al. theory, students on the Figure 3 are at Partial-Elemental-Structural level.

4. Conclusion
From results described in the previous section, we draw the following conclusions. The result showing the lack of students’ understanding of the structure of deductive proof. As investigated through this Sine and Cosine rule problem, teachers must encourage to improve the content of mathematics courses and to emphasize the use of deductive reasoning through doing mathematical proving. Teachers do not need to directly teach how proving in formal proof but they can use a flow-chart proof or a table proof to help them teaching. There are many student applying the comparison of right triangle to an arbitrary triangle. This fundamental error are often found in the class, so teacher might notice and clarify about this to their students. For the future research, researcher wonder whether to apply a flow-chart proof or a table proof to teach mathematical proving can improve the students’ understanding of the structure of deductive proof.

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