A template-free approach for detecting a gravitational wave stochastic background with LISA

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The Laser Interferometer Space Antenna will be the first Gravitational Wave observatory in space developed by the European Space Agency. It is scheduled to fly in the early 2030’s. LISA design predicts sensitivity levels that enable the detection a Stochastic Gravitational Wave Background signal. This stochastic type of signal is a superposition of signatures from sources that cannot be resolved individually and which are of various types, each one contributing with a different spectral shape. In this work we present a fast methodology to obtain the posterior probability to measure this signal in a set of frequency bins, combining information from all the available data channels. We use this methodology in order to obtain an efficient and computationally cheap first assessment of the stochastic gravitational wave backgrounds, but also generate sensible prior densities for more computationally expensive template-based searches. We also derive useful detectability bounds as a function of frequency and prior knowledge on the instrumental noise spectrum.

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1. INTRODUCTION

The Laser Interferometer Space Antenna (LISA) is a space-borne Gravitational Wave (GW) observatory accepted by the European Space Agency (ESA) to be launched around 2034 [1]. LISA will be comprised of a constellation of three spacecraft forming an equilateral triangle with sides of 2.5 million kilometers. Each spacecraft will host two cubic test-masses maintained in free-fall conditions. The relative distance between the test-masses aboard different spacecraft will be monitored by laser interferometry. LISA aims to directly measure GWs in the spectral range 0.1 to 100 mHz. The predicted sensitivity of LISA opens up the window for the detection of a Stochastic Gravitational Wave Background (SGWB). The SGWB is a superposition of stochastic signals emitted from astrophysical and cosmological sources. The first type of source is the non-stationary, anisotropic, and partially unresolvable GW signal that is emitted by compact galactic binaries [2]. This contribution is guaranteed to be detected by LISA, and will manifest itself as a confusion noise foreground that needs to be carefully handled in a data analysis pipeline. We also expect an extra component from Stellar Origin Black Hole Binaries, as detected by LIGO-VIRGO observations [3]. This population of binaries is expected to contribute with a power law spectrum to the overall stochastic signal. There is also the possibility to detect signatures originating from cosmological sources [4], and that would bring information on the properties of the primordial Universe and the physical processes describing its evolution. The main mechanisms behind the emission of cosmological stochastic signals are attributed to high-energy processes such as phase transitions, cosmic strings, and primordial black holes. The detection of the cosmological background is of great importance as it will make it possible to distinguish between competing cosmological models.

Given the diversity of sources contributing to the stochastic background, we expect different contributions with different spectral shapes to form the overall SGWB measurement. A first approach to characterize it would be to perform template-based searches on the data. The results of a template-based search, which are of course dependent on the choice of the template, are in general computationally expensive, especially for large data sets. In addition, lack of prior information on the underlying signal adds more complexity to the analysis. In this work we focus on defining a template-free first level characterization of the stationary stochastic type of signal present in the LISA data which can be utilised as prior information for computationally expensive template-based pipelines. The technique developed in the present work is based on the assumption that we can analytically model the power excess caused by the stochastic signal for each frequency bin in the power spectrum and in all data channels. In particular, we expand on the technique employed in the data analysis of the LISA Pathfinder (LPF) mission [5], aimed at characterizing a noise contribution of unknown origin in the lower part of the differential acceleration spectrum [6].

Finally, it is worth mentioning that with the present method we focus only on the treatment of the stationary and isotropic part of the signal. Nevertheless, it is straightforward to use it in a piecewise analysis scheme, where in each segment the signal can be assumed stationary, or applied in combination with a different analysis strategy that takes into account non-stationary contributions.

In section [7] we discuss the theoretical approach used to model the power excess in the signal in each frequency bin. In section [8] we apply our approach to the Radler [7] LISA Data Challenge (LDC) data set. We then discuss
the detectability of a stochastic background as a function of its amplitude and the uncertainty on the noise spectrum in section III] by deriving an analytic expression for the Bayes factor that depends on the two aforementioned quantities. Finally, in section III, we discuss our main results and elaborate on the possible applications of our technique.

II. PROBABILITY OF POWER EXCESS

Let us suppose that we have a series of \( k \) data channels \( d_k(t) \), which in our case are the time series after applying the Time Delayed Interferometer (TDI) algorithms (see section III for details). For this exercise, the overall measured noise is going to be considered equal for all channels, \( S_{n,k} (f) = S_n(f) \). The power spectrum of the noise can depend on a set of parameters, \( \vec{\theta}_n \), but for now we consider it completely known. Then, and if we assume Gaussian and zero mean noise sources \([9, 10]\), the noise can depend on a set of parameters, \( \vec{\theta}_n \), and its spectral properties. In this work, we follow the approach of \([12]\) to produce the series of frequency power spectra \( S_n(i) \) for each frequency bin \( i \). Marginalizing over \( S_n \), the resulting PDF for each frequency bin \( i \) is

\[
p(D_k | \vec{\theta}_n, S_n, S_o) \propto \int_{S_o - \epsilon}^{S_o + \epsilon} e^{-\frac{1}{2} \frac{n \tau_k}{\epsilon}} \frac{S_{o}^{N-1}}{S_o} dS_n, \quad (4)
\]

After a change of variable, we can use the incomplete gamma function \( \Gamma(x) = \int_0^\infty y^{x-1}e^y \, dy \) in eq. (4). Then, the posterior PDF for the signal power \( S_n[i] \) for each frequency bin can be expressed as

\[
p(S_o | D_k, S_n) \propto (\Gamma_{N-1} (A^+) - \Gamma_{N-1} (A^-)), \quad (5)
\]

with

\[
A^\pm = \frac{N D_k}{S_n + S_o + \epsilon}, \quad (6)
\]

where again the \([i]\) indices have been dropped for the sake of clarity.

Let us now reintroduce the dependence of \( S_n \) on the parameters \( \vec{\theta}_n \). In this study, our simple noise model is comprised of the test mass position and acceleration noise levels, that for the sake of convenience are considered equal for all test masses. Prior information on these parameters from instrumental studies exist and we can assume that their prior densities follow \( \theta_n \sim U[\theta_n^{\min}, \theta_n^{\max}] \). Consequently, the aforementioned limits generate lower and upper bounds on the overall power spectral density of the instrumental noise such that

\[
S_n^{\min} = S_n[i, \vec{\theta}_n^{\min}] \leq S_n[i, \vec{\theta}_n] \leq S_n[i, \vec{\theta}_n^{\max}] = S_n^{\max}. \quad (7)
\]

Without loss of generality and for the sake of simplicity, we can just choose prior ranges for \( \vec{\theta}_n \), so that \( S_n = S_n[i, \vec{\theta}_n] \). Thus, if we substitute eq. (4) into (3) and marginalize over \( S_n \), the resulting PDF for each frequency bin \( i \) is reduces again to eq. (4).

So far we have defined the statistical framework to be applied directly to the data set \( d_k(t) \) in order to infer the signal \( S_n[i] \). But before proceeding, we must carefully perform the averages over the data series and estimate the errors on the binned averaged spectra \( \bar{D}_k \). Following \([12]\) and \([13]\), we compute the power spectra of the time series on a logarithmic frequency axis, by adjusting \( N \). In essence, short data segments are chosen for higher frequencies, while longer data segments are chosen at lower frequencies, and the number of averages is in fact \( N[i] \). In \([9, 13, 15]\), the approach of \([12]\) was extended in order to take into account the correlations between Fourier coefficients by carefully choosing an appropriate \( N[i] \) in a procedure depending on the choice of windowing function and its spectral properties. In this work, we follow this methodology to produce the series of \( k \) averaged in frequency power spectra \( \bar{D}_k \).

The above results can directly be applied to the detection of stationary and isotropic stochastic types of signals. In the end, in order to construct the posteriors of
Figure 1: The reconstruction of the SGWB signal for the Radler LDC data set. The grey curve represents the numerically computed power spectrum $S_{\text{num}}(f)$ of the TDI X channel of the Radler data set, while the actual signal present in the data $S_{h}(f)$, is depicted in red. The shaded area represents the normalized log-posterior probability densities for each of the spectral coefficient assumed in the analysis, while the black dots represent the maximum a-posteriori $S_{o,\text{MAP}}$, together with the associated 2-σ error bars (see text for details). The maximum posterior noise estimate $S_{n}(f, \hat{\theta}_n, \text{MAP})$, is shown in black and follows the grey spectrum. Finally, the grey dots denote the maximum a-posteriori $S_{o,\text{MAP}}$ for the case where there is no SGWB signal in the data (see text for details).

III. APPLICATION TO LISA DATA

We can now apply the method described in Section II to LISA simulated data. For our purposes, we choose to work with the Radler LISA Data Challenge data set [7] that contains only signals originating from stationary and isotropic stochastic sources. This is akin to working on data residuals, after the subtraction of high signal-to-noise ratio spurious signals, and either after the subtraction of the anisotropic and non-stationary stochastic signals due to compact galactic binaries [16], or in combination with a methodology to simultaneously deal with the latter. Finally, each of the data channels use the same noise parameters, which means that the displacement and acceleration noises for all test masses in all spacecraft are equal. In particular, the level of the acceleration noise for each test mass is $S_a = 3 \times 10^{-15} \text{m}/\text{sec}^2/\text{Hz}$, while the displacement noise level is $S_i = 15 \times 10^{-12} \text{m}^2/\text{Hz}$. We can use a simple analytical formula for the noise curve for all channels, that can be written as

$$S_K(f) = 16 \sin(\beta)^2 \left[ S_i + 2S_a (1 + \cos(\beta)^2) \right],$$

with $K = \{X, Y, Z\}$, and $\beta = 2\pi L f/c$, $L$ being the LISA arm length [17]. The SGWB signal of Radler is a simple power law with amplitude $\log_{10}(\Omega_0) = -8.445$ and spectral index $\alpha = 2/3$, where

$$S_h = \frac{3H_0^2\Omega_{gw}(f)}{4\pi^2f^3}, \text{ with}$$

$$\Omega_{gw}(f) = \Omega_0 \left( \frac{f}{25\text{Hz}} \right)^\alpha.$$
We begin the complete analysis starting from the TDI X, Y and Z channels \[8\], where the data are windowed, averaged in frequency, and binned. Together with the spectral coefficients that characterize the SGWB signal we also sample the extra noise parameters \( S_a \) and \( S_i \). We follow a conservative approach on the prior for the total noise power at each frequency \( f_i \), choosing \( \epsilon[i] = 5\% S_n[i] \) in all frequency bins.

The methodology described in the previous section \[11\] yields the results presented in figure \[1\]. The contour in this figure illustrates the logarithm of the posterior probability densities for each frequency bin, while the data points indicate the maxima of these probabilities. The two sigma error estimate is shown in the corresponding vertical bars. It is quite evident that we can accurately reconstruct the excessive signal due to the SGWB and separate it from the noise of the instrument. The reconstruction of the signal is also poorer, as expected, for lower signal-to-noise ratio areas of the spectrum. For the sake of comparison, we also include in this plot, as a null test, the maximum a posteriori estimates (grey dots) for a signal excess on a data set in which there is no other noise sources other than the LISA instrument noise. The noise parameters are marginally estimated to be within the 2-\( \sigma \) limit, an expected result due to the differences between the simple analytical model of eq. \[9\] and the simulator noise \[18\]. Finally, the parameters of eq. \[9\] and eq. \[10\] can be extracted by fitting a line in log-space on the extracted data points, deconvolved by the LISA instrument response function \( R(f) \). \( R(f) \) is the response of LISA to the generic observed GW signal, and it basically depends on the orbits of the constellation. Given a LISA configuration, one can analytically approximate the response function \[19, 20\], but here we directly use the LISA simulator \[15\] to estimate it numerically. For the data presented in figure \[4\] we find \( \log(\Omega_e) = -8.25 \pm 0.6 \) and \( \alpha = 0.72 \pm 0.2 \). To put this result into perspective, we compare with a template-based analysis (as in \[21, 22\]) on the same data set. For the ideal case of a known noise power spectrum, we recover \( \log(\Omega_e) = -8.55 \pm 0.13 \) and \( \alpha = 0.64 \pm 0.03 \), which is in good agreement with our previous result. At this point it is worth mentioning that in this application, we demonstrate the performance of this methodology in an ideal scenario: the data is Gaussian and presents no artefacts such as gaps, or spurious noise signals. We also assume that we are working on perfect residuals, meaning that all “loud” sources have been perfectly subtracted.

**IV. DETECTABILITY ASSESSMENT**

Having a relatively easily integrable expression for the posterior distribution can be very useful to study the level of detectability of a signal in different scenarios. Working in a Bayesian framework, the detectability can be assessed by computing the ratio of the marginalized posterior distributions, or evidences, between the competing models. In the case at hand, we can directly calculate the Bayes Factor \( B_{10} \) for the two models of interest. The first model, \( M_1 \), corresponds to the model that includes the presence of GW signal in the data, while the second, \( M_0 \) corresponds to the instrumental noise only hypothesis. The Bayes factor will then take values greater than unity in regions of the parameter space where detection of a signal is more probable. We can write the evidence \( M_1 \) as the double integral of eq. \[2\] over the excess signal \( S_o \) and noise \( S_n \) for each frequency bin as

\[
p(\tilde{D} | M_1) = C \int_{\tilde{S}_n}^{\tilde{S}_n + \epsilon} \int_{0}^{\tilde{D} - S_n} e^{-\frac{N}{S_o + S_n}} (S_o + S_n)^N dS_o dS_n, \tag{11}
\]

with \( C \) a constant. Similarly, for \( M_0 \) we get

\[
P(\tilde{D} | M_0) = C' \int_{\tilde{S}_n - \epsilon}^{\tilde{S}_n + \epsilon} e^{-\frac{2D}{S_n}} S_n^N dS_n. \tag{12}
\]

Taking their ratio yields a Bayes factor that depends on the uncertainty \( \epsilon \) of the power spectral density of the instrument noise \( S_n \), and the quantities \( \tilde{D} \) and \( N \) that are constant, and depend on the spectral preprocessing.
of the time series data:

\[ B_{10}(\epsilon) = \frac{P(\bar{D}|M_1)}{P(D|M_0)} = \frac{\bar{D}N(\Gamma^+-\Gamma^-)}{(N-2)!D-S_n(\Gamma^-\Gamma^+)} + \frac{\bar{S}_n + \epsilon}{D-S_n} \]

with \( \Gamma^\pm = \Gamma_{N-2} \left( (N\bar{D}) / (\bar{S}_n \pm \epsilon) \right) \). The expression in eq. (13) can thus be used to assess the detectability of a stochastic gravitational wave signal as a function of the level of uncertainty in the noise spectrum.

We apply this expression directly to the Radler data set. The result is shown in figure 2, where the logarithm of \( B_{10} \) is plotted for each frequency bin as a function of the value of \( \epsilon \). As expected, for frequencies lower than 1 mHz where the signal-to-noise ratio is low, the \( B_{10} \) takes values smaller than unity, supporting the noise-only model \( M_0 \). It is also quite evident that for this idealized data set, the detection of a SGWB with \( \log_{10}(\Omega_0) = -8.445 \) and \( \alpha = 2/3 \) is becoming more challenging when our knowledge of the instrumental noise reaches the relative limit of 20% for each of the available data channels.

Reversing this argument, we can start by considering a fixed level of noise uncertainty for all frequencies, and predict the gravitational wave signal amplitudes that would yield greater than unity \( B_{10} \), that would in-turn indicate higher probability levels for \( M_1 \). To do that, we assume a measurement with the same properties as the one of the Radler LDC data set (zero-mean Gaussian noise, idealized and an uninterrupted data stream for a duration of two years). We then substitute \( D \) with \( S_n + S_o \) and compute the logarithm of \( B_{10} \) given by eq. (13), for different values of the SGWB signal level \( S_o \). This enables us to infer the signal \( S_o \) levels that would yield positive detection levels, given a particular instrument configuration, observation duration, and confidence in the instrument noise power spectrum for each frequency bin. The results of this computation are shown in figure 3. The dashed blue lines correspond to the contour levels for \( B_{10} = 10 \), while the solid blue lines correspond to values of Bayes Factors of \( B_{10} = 100 \). These two levels of Bayes Factors correspond, respectively, to positive and very strong evidence of a SGWB signal present in the data. The black solid line represents the corresponding noise level. This figure has been produced for a data set similar to Radler, i.e. assuming an uninterrupted data series of length two years for all TDI channels (see text for details). The frequency bins of this analysis are represented by the minor grind on this plot.

V. DISCUSSION

We have presented an efficient methodology to extract the characteristics of the underlying isotropic and stationary stochastic Gravitational-Wave Background from the LISA TDI measurements. A variation of this technique has already been employed to analyze the LPF data [6]. For the case of LPF, this analysis technique was utilized to identify an excess of noise power of unknown origin in the lower frequency part of the differential acceleration spectra, measured throughout the mission. Here, we start from the assumption that we can employ an arbitrary number of channels that measure the same quantity simultaneously, while we allow different noise spectral shapes in those channels. In this work, we focus on the Radler LISA simulated data set, but this methodology can be straightforwardly adapted in numerous potential applications related to LISA data analysis or otherwise.

The particular data set contains only a power excess signal that originates from a stationary stochastic Gravitational-Wave background that follows a power law. We manage to recover the shape of the injected spectrum, with very good accuracy in the frequency bins where the signal-to-noise ratio is high (see figure 1). In addition, together with the spectral coefficients of the underlying signal, we estimate the parameters of a simple analytic model of the instrument noise spectrum. The Radler data set was generated by a LISA instrument where all test-mass acceleration and displacement noises are equal, further simplifying the calculations. However, this methodology can easily be extended to the more complicated case where each of the readout channels has different noise.
properties. The effect of adding more free parameters in this problem remains to be studied in future work, however we naturally expect that larger uncertainty about the noise power spectrum would yield worse estimates of the stochastic signals characteristics. The present analysis could be extended to use the noise orthogonal TDI variables A, E, and T, expressed as linear combinations of the X, Y and Z channels [24]. In addition, the use of TDI channel cross-spectra could also be included in the analysis. Their characteristic shapes in frequency, and their dependance on the noise parameter vector $\boldsymbol{\theta}_n$, would yield a more accurate estimation of the noise parameters and the signals. This method, however, cannot disentangle the various contributions to the overall signal and does not provide any information about the possible sources of the excess of power detected. To differentiate between the various contributions, one must perform meta-analysis on the results of the method presented here, together with the associated data and their cross-spectra, and conduct template-based model selection studies.

The approach introduced in this paper provides a template-free first level characterization pipeline, for any given underlying stationary excess of power as measured by different channels simultaneously. The posterior densities of the signal power for each frequency bin are calculated analytically, which means that the computational cost remains in acceptable levels. The resulting posterior densities for the power spectra amplitudes can be used as prior densities for a more thorough template-based search. This use case is in fact the main motivation for adapting the method to LISA data. For the particular example of the SGWB analyzed here, we can straightforwardly use the densities of figure [1] in a computationally expensive Markov Chain Monte Carlo (MCMC) algorithm that can efficiently sample through the parameter space of a more sensible (and limited in number) set of templates.

A step further has been taken, where the expression of the posterior densities for the stochastic signal has been integrated over the parameter space for two model cases: the data plus signal $M_1$, and the instrument noise only hypothesis $M_0$. The result is an analytic formula for the Bayes factor $B_{10}$ between those two models that depends on the level of uncertainty of the noise power spectral density. This expression can directly be used to assess the efficiency of LISA at detecting a SGWB, given the data and the instrument noise in each frequency bin. In the case of the Radler data set, we find that the high SNR signal in parts of the frequency range can be detected with increased confidence when the noise PSD for each data channel is less than 20%. In addition, we can reverse the argument and study the efficiency of LISA for various levels of stochastic signals, given a particular uncertainty on the instrument noise PSD and observation time. As expected, the SGWB signal that yields a positive log$_{10} B_{10}$ greatly depends on our knowledge of the noise spectrum $S_n$. Note that in this work we have used a level of noise uncertainty parametrized by a constant $\epsilon$, but one could consider a more realistic scenario, by taking into account different level of uncertainties in different parts of the frequency range. The magnitude of $\epsilon(f)$ can depend, for example, on known instrumental noise features, like noise transients, noise bursts and other non-stationarities, or on the imperfect subtraction of brighter sources, and finally from imperfect knowledge of foreground noises, such as the one produced by the compact galactic binaries.

Finally we wish to comment on recent work [25] on the same topic. The main result of our work derives from eq. (2). This expression is constructed by first following the procedure given in [11] and by further binning the data as in [6, 12]. The resulting joint conditional probability density function of eq. (2) follows $\chi^2$ statistics and yields an analytic formula for the posterior distribution eq. (5) and for the Bayes factor eq. (13). This methodology thus yields an analytic estimate of the signal amplitude’s probability distribution on a sparse and equally spaced in log-space frequency grid. It is particularly useful if one wishes to gain intuition on the efficiency of LISA in detecting SGWBs given a certain level of noise uncertainty, without the need for any simulated data. By contrast, in [25], the observed power spectra are divided in segments of width estimated by computing model selection criteria between segments of varying widths. The data model for each bin is parameterized by an amplitude, a spectral slope and a noise model. Model parameters are then determined numerically assuming a Gaussian likelihood function. This is an iterative and adaptive procedure capable of determining the spectral shape and amplitude of stochastic backgrounds without the need for a single template. Both approaches yield comparable results, and could be used in a single data analysis pipeline for cross-validation purposes.

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