D0-branes in Gepner models and $N = 2$ black holes

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Abstract

In this paper D-brane boundary states constructed in Gepner models are used to analyze some aspects of the dynamics of D0-branes in Calabi-Yau compactifications of type II theories to four dimensions. It is shown that the boundary states correspond to BPS objects carrying dyonic charges. By analyzing the couplings to closed string fields a correspondence between the D0-branes and extremal charged black holes in $N = 2$ supergravity is found.

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1. Introduction

Dirichlet branes \cite{1} provide a remarkably simple way to introduce extended objects carrying Ramond-Ramond (RR) charges in string perturbation theory. One way to describe D-branes is given by the boundary state formalism \cite{2} \cite{3}. Here the open string boundary conditions are imposed on the closed string fields. The boundary state formalism is particularly useful in determining the coupling of the D-brane to the various closed string modes.

The compactification of type II string theories on Calabi-Yau threefolds (CY) yields four-dimensional theories with $N = 2$ spacetime supersymmetry. This is an interesting field of study since these theories are not as strongly constrained as theories with more supersymmetry. In addition they exhibit many of the phenomena which are central to the recent developments in non-perturbative string theories such as mirror symmetry, F-theory and heterotic duals (see \cite{4} for a review).

The wrapping of $D_p$-branes on $p$-dimensional supersymmetric cycles leads to BPS saturated non-perturbative objects. From the point of view of the four-dimensional non-compact space these objects are $D_0$-branes, i.e. particle like states. For type IIB on a CY, $D_3$-branes can wrap 3-cycles and for type IIA on a CY, $D_0, D_2, D_4$ and $D_6$ branes can wrap $0, 2, 4$ and 6 cycles respectively. Note that the NS five-brane which is present in both IIA and IIB cannot give any particle like states in four dimensions. The NS five-brane will however lead to non-perturbative corrections to the hypermultiplet geometry via euclidean wrapping on the whole CY \cite{5} \cite{6}.

As we shall see the $D_0$-branes are BPS objects which are charged with respect to the RR-vector fields. One aim of this paper is to show that they carry the same charges as extremal charged black holes in the $N = 2$ supergravity theory defined by the low energy limit of the type II string theory compactified on the CY. In the context of compactifications on $T^6 / Z_3$ orbifolds this correspondence has been discussed in \cite{7}.

Gepner models \cite{8} are exactly solvable models for strings compactified on CY manifolds, where the internal $c = 9$ SCFT is constructed as an orbifold of a tensor product of $N = 2$ minimal models. Note that the geometrical description in sigma-model language and the Landau-Ginzburg phase in which the Gepner model resides can be connected via a linear sigma-model \cite{9}. It would be interesting to study wrapped D-branes in the context of the linear sigma-model.

The construction of boundary conformal field theories from bulk conformal field theories has been discussed by various people \cite{10} \cite{11}. Boundary CFT has important applications for impurity problems in condensed matter physics \cite{12}. The construction of consistent (rational) boundary conformal field theories has been discussed by Cardy \cite{13}. Additional sewing constraints for such theories have been discussed in \cite{14} \cite{15}. Cardy's
boundary states have been used in [16] to construct boundary states in the Gepner model and some aspects of these boundary states were discussed further in [17].

In this paper we will use boundary states in Gepner models to discuss the dynamics of D0-branes in four dimensions. This analysis will be done by applying the well known techniques developed for D-brane boundary states in ten-dimensional Minkowski space to the Gepner model compactifications.

2. Boundary states in Gepner models

The construction of boundary states for free bosons and fermions satisfying Neumann or Dirichlet boundary conditions uses coherent states [2]. This construction was generalized for rational CFT’s by Ishibashi [10]. When the conformal field theory forms an extended algebra, boundary conditions have to be specified on the left- and right-moving currents of the extended algebra $W, \bar{W}$ [10]. To construct boundary states for rational conformal field theories one first defines Ishibashi states $|i\rangle\rangle$ for every primary field defining an irreducible highest weight representation $\mathcal{H}_i$ of the algebra which satisfy

$$\left(W_n - (-1)^{hw} \bar{W}_{-n}\right) |i\rangle\rangle = 0.$$  \hspace{1cm} (2.1)

In [10] it was shown that an Ishibashi state can be constructed using an anti-unitary operator $U$ which acts on the modes of the right-moving current $\bar{W}$ in the following way, $UW_nU^{-1} = (-1)^{hw} \bar{W}_n$. Such an operator $U$ is closely related to the chiral CPT operator. Explicit form of the Ishibashi state is given by

$$|i\rangle\rangle = \sum_N |i, N\rangle \otimes U|i, \bar{N}\rangle,$$ \hspace{1cm} (2.2)

where $N$ denotes the sum over the basis of $\mathcal{H}_i$. In the second step a boundary state can be constructed from a complete set of Ishibashi states,

$$|a\rangle = \sum_i B_i^a |i\rangle\rangle.$$ \hspace{1cm} (2.3)

The form of possible boundary states $|a\rangle$ are constrained by the fact that the cylinder amplitude involving two boundary states is mapped into a one-loop open string partition function by a modular transformation. Cardy [13] gives a solution in terms of the modular S-matrix $B_i^a = S_i^a / \sqrt{S_0}$.

\footnote{Gepner models in type I theory context were discussed in [18].}
In Ref. [19][20], it was shown that two different boundary conditions for \( U(1) \) current \( J \) and the superconformal generators \( G^\pm \) are consistent with \( N = 1 \) superconformal invariance. The two cases are called A and B boundary conditions, referring to the two possible topological twists of the \( N = 2 \) theory [21]. The A boundary conditions are defined by

\[
(J_n - \bar{J}_{-n}) \mid B \rangle = 0, \quad (G_r^+ + i\eta \bar{G}_r^\pm) \mid B \rangle = 0, \quad (2.4)
\]

whereas the B boundary conditions are defined by

\[
(J_n + \bar{J}_{-n}) \mid B \rangle = 0, \quad (G_r^\pm + i\eta \bar{G}_r^\pm) \mid B \rangle = 0. \quad (2.5)
\]

The choice of \( \eta = \pm 1 \) corresponds to a choice of spin structure. Due to the property of the operator \( U \) the states (2.2) satisfy B-type boundary conditions. In order to implement A-type boundary conditions \( U \) has to be replaced by \( U\Omega \) [19][16], where \( \Omega \) is the mirror automorphism of the \( N = 2 \) algebra.

For four-dimensional compactifications preserving \( N = 2 \) spacetime supersymmetry the internal degrees of freedom form a \( c = 9 \) \( N = 2 \) SCFT. In the light-cone gauge the external physical degrees of freedom are given by a \( c = 3 \) \( N = 2 \) SCFT consisting of a free complex boson and fermion. The boundary conditions in the light-cone gauge have been discussed in [17]. The results we need here are: Firstly D0-brane boundary conditions correspond to A-type boundary conditions for the external CFT and secondly the GSO projection relates the internal and external boundary conditions where the consistent internal boundary conditions for D0-branes are A-type for type IIB and B-type for type IIA.

Gepner models are exactly soluble string backgrounds in which the internal \( c = 9 \) SCFT is an orbifold of a tensor product of \( N = 2 \) minimal models. Appendix A gives a brief overview on the most important facts needed in this paper. In [16] boundary states in Gepner models corresponding to total A and B boundary conditions were constructed by applying Cardy’s construction [13] for each factor of the tensor product of \( n \) minimal \( N = 2 \) theories.

The Ishibashi states are labelled by the quantum numbers for the external part and the internal \( N = 2 \) primaries, which we denote by \( \mu = (s_0; m_1, \cdots, m_n; s_1, \cdots, s_n) \) and \( \lambda = (l_1, \cdots, l_n) \). For A-type boundary conditions for each minimal model, the Ishibashi states have the form \( \mid \lambda, \mu; \lambda, \mu \rangle \). Whereas for B-type boundary conditions the Ishibashi states have the form \( \mid \lambda, \mu; \lambda, -\mu \rangle \).

In the following we will focus on the A-type boundary conditions for the internal \( c = 9 \) SCFT, which corresponds to D0-branes in type IIB.

A boundary state is then labeled by a vector \( \alpha = (\lambda', \mu') \) where \( \lambda' = (l'_1, \cdots, l'_n) \) and \( \mu' = (s'_0; m'_1, \cdots, s'_n) \), and given by

\[
\mid \alpha \rangle = \frac{1}{\kappa_\alpha} \sum_{\lambda, \mu} B^\alpha_{\lambda, \mu} \mid \lambda, \mu \rangle \rangle. \quad (2.6)
\]
Although every boundary state in (2.6) satisfies A boundary conditions, different $\alpha$ gives different boundary conditions, e.g., for the supercharges. The normalization constant $1/\kappa_\alpha$ can be determined by Cardy’s condition \[16\]. The factor $B^\alpha_{\lambda,\mu}$ is the product of $B^\alpha_i$ using the modular $S$-matrix for the $N = 2$ minimal models \[16\];

$$B^\alpha_{\lambda,\mu} = e^{i\pi s_0^2/2} e^{-i\pi s_0'/2} \prod_{j=1}^{N} \frac{\sin \left( \frac{\pi \left( l_j+1\right) \left( l'_j+1\right)}{k_j+2} \right)}{\sin \frac{1}{2} \left( \frac{\pi \left( l_j+1\right)}{k_j+2} \right)} e^{i\pi m_j m'_j/2} e^{-i\pi s_j s'_j/2}. \quad (2.7)$$

The boundary states (2.6) are constructed using two types of data, firstly the ‘gluing automorphism’ \[16\] (i.e. $U$ and $U\Omega$), and secondly the set of $B^\alpha_{\lambda,\mu}$. It would be interesting to explore the possible generalizations of these boundary states. One possibility involves more general automorphisms of the chiral fusion algebra (a very simple example are permutations of minimal models of the same level $k$). Another possibility is to construct more general $B^\alpha_{\lambda,\mu}$ than Cardy’s. In particular, in the correspondence of D-brane boundary states in Gepner models and CY-compactifications, the boundary conditions in (2.4) and (2.5) are only necessary for the currents of the $c = 9$ conformal field theory, not for the currents associated with each minimal model factor in the tensor product. Relaxing this condition makes the theory non-rational and more general boundary states are possible.\footnote{We are grateful to A. Recknagel and V. Schomerus for correspondence on this subject.} A general solution to this problem seems rather difficult and we will not pursue this question further in this paper (see however \[22\]).

3. Covariant external part of the boundary state

The construction reviewed in the previous section is purely algebraic in terms of $N = 2$ SCFT. However, the external part consists of usual free bosons and fermions. Thus one can split the boundary states into the internal and external parts and express the external part by coherent-type boundary states. In such an expression, the degrees of freedom from the boson zero-modes become explicit.

In the following we will construct the external part of the boundary state in a covariant formalism following \[23\]. In the light-cone gauge all longitudinal degrees of freedom are expressed in terms of transverse ones. It is possible to formulate D-brane boundary states in the light-cone gauge \[24\] (see appendix B). There are however some aspects of the boundary states which are best described in the covariant Ramond-Neveu-Schwarz formalism. In the covariant formalism, in addition to the bosonic and fermionic world sheet fields and internal CFT fields, there are anti-commuting $b, c$ ghosts and commuting $\beta, \gamma$ ghosts. The boundary conditions for the ghosts are determined by demanding BRST invariance of the boundary state. The main feature of the ghost dependent part is that it cancels...
the contribution of two bosonic and fermionic oscillators in the annulus partition function, leading to the same result as the light-cone boundary states. There are however subtleties arising in the RR sector due to the choice of a picture for the superghosts, which will be addressed later.

A D0-brane imposes Neumann boundary conditions on $X^0$ and Dirichlet boundary conditions on $X^i$, $i = 1, 2, 3$. The boundary conditions for the bosonic oscillators are given by

$$\left( a^\mu_n - M^\mu_\nu \bar{a}^\nu_{-n} \right) \left| B \right> \bigg| X = 0 ,$$

where $M^\mu_\nu = \text{diag}(-1, +1, +1, +1)$. The D0-brane will be localized in space at $y^i$, $i = 1, 2, 3$. The boundary state is then given by

$$\left| B \right> \bigg| X = T_0 \frac{d^3p}{(2\pi)^3} e^{-ip_i y^i} \exp \left( - \sum_{n>0} \frac{1}{n} M^\mu_\nu a^{\mu}_{-n} \bar{a}^{\nu}_{-n} \right) \left| p \right> ,$$

with $T_0$ a normalization constant. This is determined so that the boundary state reproduces the cylinder amplitudes from the Coleman-Weinberg formula [1]. For Dp-branes in $D$-dimensions, it is given by [23]

$$T_p = \frac{\sqrt{\pi}}{2(D-10)^{1/4}} (4\pi^2 \alpha')^{(D-2p-4)/4} .$$

The D0-brane tension is then given by $T_0/\kappa$ where $\kappa^2 = 8\pi G_N$ with $G_N$ Newton’s constant.

The $b, c$ part of the boundary state will not be important in the following and will not be displayed here. In the NSNS sector the world-sheet fermions are half integral modes and the boundary condition on the fermions and $\beta, \gamma$ ghosts are given, e.g., by

$$\left( \psi^\mu_r - i\eta M^\mu_\nu \bar{\psi}^\nu_{-r} \right) \left| B, \eta \right> \psi = 0 ,$$

with

$$\left| B, \eta \right> \psi = \exp \left( i\eta \sum_{r>0} (M^\mu_\nu \bar{\psi}^\nu_{-r} \psi^\mu_{-r} + \gamma_{-r} \bar{\beta}_{-r} - \beta_{-r} \bar{\gamma}_{-r}) \right) \left| 0 \right>_{(-1, -1)} .$$

Here the subscript $(-1, -1)$ indicates that in the NSNS sector the ghost number of the vacuum is chosen to be $(-1, -1)$. The boundary state (3.5) are GSO projected, which means that the total U(1) charge of the left- as well as the right-moving sector is odd. The boundary state (3.3) has neither even nor odd fermion number. There are two possible combinations of definite fermion number,

$$\left| B, s_0 = 0 \right>_{NSNS} = \frac{1}{2} \left( \left| B, +1 \right> \psi + \left| B, -1 \right> \psi \right)$$

$$= \cos \left( \sum_r (M^\mu_\nu \bar{\psi}^\nu_{-r} \psi^\mu_{-r} + \gamma_{-r} \bar{\beta}_{-r} - \beta_{-r} \bar{\gamma}_{-r}) \right) \left| 0 \right>_{(-1, -1)} ,$$

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\begin{align}
| B, s_0 = 2 \rangle_{NSNS} &= \frac{1}{2} \left( | B, +1 \rangle_{\psi} - | B, -1 \rangle_{\psi} \right) \\
&= \imath \sin \left( \sum_r (M_{\mu\nu} \psi_{-r,\mu}^\dagger \bar{\psi}_{-r,\nu} + \gamma_{-r} \bar{\beta}_{-r} - \beta_{-r} \bar{\gamma}_{-r}) \right) | 0 \rangle_{(-1, -1)} . \tag{3.7}
\end{align}

Here $| B, s_0 = 0 \rangle_{NSNS}$ has even fermion number and has to be tensored with a state with odd internal $U(1)$ charge. On the other hand $| B, s_0 = 2 \rangle_{NSNS}$ has odd fermion number and has to be tensored with a state with even internal $U(1)$ charge.

In the RR sector the implementation of the GSO projection is more subtle. The ground states in the RR sector are tensor products of spin fields. Due to the superghost number anomaly on the disk the boundary state is not defined in a left-right symmetric picture but in the $(-3/2, -1/2)$ picture. There are two components labeled by $s_0 = \pm 1$ which have external $U(1)$ charges. In analogy with the ten-dimensional case described in \cite{26} we can construct the RR part of the boundary state as a product

\begin{align}
| B, s_0 = +1 \rangle_{RR} &= \cos \left( \gamma_0 \bar{\beta}_0 + \sum_{n>0} M_{\mu\nu} \psi_{n,\mu}^\dagger \bar{\psi}_{n,\nu} + \gamma_{n} \bar{\beta}_{n} - \beta_{n} \bar{\gamma}_{n} \right) \sigma^0_{ab} \ | a \rangle_{-\frac{3}{2}} | b \rangle_{-\frac{1}{2}} \\
&+ \imath \sin \left( \gamma_0 \bar{\beta}_0 + \sum_{n>0} M_{\mu\nu} \psi_{n,\mu}^\dagger \bar{\psi}_{n,\nu} + \gamma_{n} \bar{\beta}_{n} - \beta_{n} \bar{\gamma}_{n} \right) \sigma^0_{ab} \ | \dot{a} \rangle_{-\frac{3}{2}} | \dot{b} \rangle_{-\frac{1}{2}} ,
\end{align}

and

\begin{align}
| B, s_0 = -1 \rangle_{RR} &= \cos \left( \gamma_0 \bar{\beta}_0 + \sum_{n>0} M_{\mu\nu} \psi_{n,\mu}^\dagger \bar{\psi}_{n,\nu} + \gamma_{n} \bar{\beta}_{n} - \beta_{n} \bar{\gamma}_{n} \right) \sigma^0_{ab} \ | a \rangle_{-\frac{3}{2}} | b \rangle_{-\frac{1}{2}} \\
&+ \imath \sin \left( \gamma_0 \bar{\beta}_0 + \sum_{n>0} M_{\mu\nu} \psi_{n,\mu}^\dagger \bar{\psi}_{n,\nu} + \gamma_{n} \bar{\beta}_{n} - \beta_{n} \bar{\gamma}_{n} \right) \sigma^0_{ab} \ | \dot{a} \rangle_{-\frac{3}{2}} | \dot{b} \rangle_{-\frac{1}{2}} .
\end{align}

Here $| a \rangle_s, | \dot{a} \rangle_s$ denote four-dimensional spinor fields in the $s$-picture (see appendix C for details on the covariant description of the RR fields in this context).

The GSO projection implies that the boundary state $| B, s_0 = -1 \rangle_{RR}$ is tensored with states in the internal sector $| q, q \rangle$ with $q = 2m + 1/2, m \in \mathbb{Z}$ whereas $| B, s_0 = +1 \rangle_{RR}$ is tensored with states in the internal sector $| q, q \rangle$ with $q = 2m - 1/2, m \in \mathbb{Z}$.

We can now calculate the external contribution to the cylinder amplitude. Denoting the contributions from the boson zero-modes and the boson and fermion oscillators by $\chi_0$ and $\chi_{s_0}$ respectively, it is given by

\begin{align}
\langle B, s_0, y_1 | q \frac{1}{2} (L_0 + \bar{L}_0 - c/12) | B, s_0, y_2 \rangle &= \chi_0 \chi_{s_0} , \\
\chi_0 &= V_0 \left( \frac{T_0}{2} \right)^{2 \frac{t-3/2}{(2\pi)^3}} e^{-\left(y_1 - y_2\right)^2/(4\pi t)} , \\
\chi_{s_0} &= \eta^{-2} \left\{ \begin{array}{ll}
\frac{1}{2} \left[ \frac{\theta_3}{\eta} + \left( -1 \right)^{s_0/2} \frac{\theta_4}{\eta} \right] & \text{for } s_0 = 0, 2 \\
\frac{1}{2} \frac{\theta_2}{\eta} & \text{for } s_0 = \pm 1
\end{array} \right.
\end{align}

with $q = e^{2\pi i \tau} = e^{-2\pi t}$ and $V_0$ the world-line volume. The combinations of theta functions are nothing but the $SO(2)$ characters $\chi_0, \chi_v, \chi_{s/c}$ for $s_0 = 0, 2, \pm 1$ respectively.
4. Open string partition function

A distinct advantage of knowing the exact conformal field theory and the boundary states is that the closed string cylinder amplitude has a dual interpretation as the open string partition function. The two are related by world-sheet duality transformation which mixes all the massless and the massive modes.

For A boundary conditions the cylinder amplitude for two boundary states implementing the same boundary conditions $\alpha$ is given by

$$Z_{\alpha\alpha}(q) = \langle \alpha | q^{\frac{c}{24}}(L_0+\bar{L}_0-c/24) | \alpha \rangle = \frac{1}{\kappa^2} \sum_{\lambda,\mu}^{\beta} B_{\lambda,\mu}^{\alpha} B_{\lambda,-\mu}^{\alpha} \chi_\lambda^{\alpha}(q).$$

(4.1)

Here we have suppressed the contribution from the boson zero-modes $\chi_0$. A modular transformation into the open string channel gives

$$Z_{\alpha\alpha}(\tilde{q}) = \frac{1}{\kappa^2} \sum_{\lambda,\mu}^{\beta} \sum_{\lambda,\bar{\mu}}^{ev} B_{\lambda,\mu}^{\alpha} B_{\lambda,-\mu}^{\alpha} S_{\lambda,\mu}^{\lambda,\bar{\mu}} \chi_\lambda^{\alpha}(\tilde{q}),$$

(4.2)

where $\tilde{q} = e^{-2\pi/t}$ and $\sum_{\lambda,\bar{\mu}}^{ev}$ stands for the constraints $l_i + m_i + s_i = 2Z$. This expression can be evaluated using the explicit form of $B_{\lambda,\mu}^{\alpha}$ (2.7) and the modular matrix $S_{\lambda,\mu}^{\lambda,\bar{\mu}}$ for the Gepner models. This calculation was done in [16]. Here we only need the result with the same boundary conditions on both ends of the cylinder,

$$Z_{\alpha\alpha}(\tilde{q}) = \sum_{\lambda,\bar{\mu}}^{ev} \sum_{v_0=0}^{K-1} \sum_{v_1,\cdots,v_{n-1},=0} \hspace{-1cm} \prod_{j=1}^{n} (\cdots (-1)^{s_0} \delta^{(4)}_{s_0,2+v_0+2} \sum_{v_i} \sum_{l_j=0}^{l_j} N_{l_j,\mu}^{l_j,\nu} \delta^{(4)}_{l_j,\nu,2+2v_j} \chi_\lambda^{\alpha}(\tilde{q}).$$

(4.3)

Here the condition that the characters $\chi_\lambda^{\alpha}$ in (4.3) appear only in integer multiplicities determines the normalization $1/\kappa^2$ up to an overall integer factor. $\delta^{(4)}_{m,n}$ are non-zero for $m = n$ (mod $k$). $N_{l_j,\mu}^{l_j,\nu}$ is the matrix appearing in the fusion rules among the primaries with spin $l_{1,2}/2$ in the $SU(2)_k$ WZW model; $\phi_{l_{1}/2} \times \phi_{l_{1}/2} \sim \sum_{l_2} N_{l_1}^{l_2} \phi_{l_2}/2$. Namely, $N_{l_1}^{l_2} = 1$ for $0 \leq l_2 \leq \min(2l_1, 2k - 2l_1)$ and otherwise vanishing. Note that the open string partition function $Z_{\alpha\alpha}$ for two identical D-branes only depends on $\lambda = (l'_1, \cdots, l'_m)$ in $\alpha = (\lambda', \mu')$. It is easy to see that the open string spectrum of (4.3) is GSO projected, i.e. the all the states appearing in $Z_{\alpha\alpha}(\tilde{q})$ have odd integer $U(1)$ charges. However the set of labels $(\lambda, \mu)$ for fields entering in the open string partition function (4.3) is very different from the fields entering the closed string boundary state (4.1), since all $m_i = v_0$ in (4.3) take the same value.

The primary fields appearing in (4.3) define a set of boundary operators $\psi^{(\alpha\alpha)}_{\lambda,\mu}$. Here the label $(\alpha\alpha)$ stands for operators which do not change the boundary condition $\alpha$. In general there are also ‘boundary condition changing’ operators $\psi^{(\alpha\beta)}$, which inserted at a
point on the boundary will change the boundary condition from $\alpha$ to $\beta$. The spectrum of such operators can be determined considering $Z_{\alpha\beta}(q) = \langle \alpha | \Delta | \beta \rangle$ instead of (4.1). For boundary conditions $\alpha, \beta$ which are not mutually supersymmetric there will be an open string tachyon in the spectrum. The ‘massless’ open string modes for D0-branes in Gepner models correspond to primaries with $\tilde{q}^0$ in the partition function.

As an example, we consider the quintic hypersurface which corresponds to the $(k = 3)^5$ Gepner model. There is a ‘universal’ $\psi_{\lambda^g,\mu^g}$, with labels $s_0 = 2, (l_i, m_i, s_i) = (0, 0, 0)^5$. This corresponds to the excitations $\psi^\mu_{-1/2} | 0 \rangle, \mu = 0, \cdots, 3$. In spacetime these modes correspond to the position of the D0-brane and a gauge field $A_0$ living on the world-line.

If $l_j' = 0, 3$ the $N_{l_j'}$ in (1.3) is only non-zero for $\bar{L}_j = 0$. It is then easy to see using the formulas given in Appendix A that the fields given above are the only massless fields if there exits $l_j' = 0, 3$. These fields and their fermionic partners are a $d = 4 \ N = 1$ vector multiplet reduced to $0 + 1$ dimensions. Introducing $U(n)$ Chan-Paton factors the resulting quantum mechanics describing $n$ D0-branes will be given by the dimensional reduction of $N = 1 \ U(n)$ Super Yang-Mills to $(0 + 1)$ dimensions.

If all $l_j' = 1, 2$ there are additional massless fields denoted by $s_0 = 0, (l_i, m_i, s_i) = (2, 4, 2)^5$ which by field redefinition is mapped into $s_0 = 0, (l_i, m_i, s_i) = (1, -1, 0)^5$ and $s_0 = 0, (l_i, m_i, s_i) = (2, 6, 2)^5$ which is mapped into $s_0 = 0, (l_i, m_i, s_i) = (1, 1, 0)^5$. For the quintic one can check that these two are the only massless states (together with their fermionic partners) and they correspond to scalars (fermions) in the external sector. Hence depending on the Gepner model and the boundary conditions $\alpha$ there are additional massless modes which are given by the dimensional reduction of a chiral $N = 1$ multiplet to $(0 + 1)$ dimensions. For $n$ D0-branes this multiplet would transform in the adjoint of $U(n)$. In principle the couplings of these fields are determined by calculating correlation functions on the disk with $\psi^{\alpha\alpha}$ operators inserted on the boundary. The simplest such quantity would be given by the three-point function of two fermionic open string states and one bosonic one, which is connected to the superpotential of the chiral fields.

When there is more than one type of D0-brane present there can also be additional massless (and tachyonic) modes coming from open strings stretched between the two branes. Such open string states are given by boundary changing operators $\psi^{\alpha\beta}$.

It is an interesting and open question whether the quantum mechanics of D0-branes in compactified theories can be used to give a Matrix theory definition of M-theory compactified on CY [27]. In this context, D0-branes in type IIA are relevant and they are described by the B-type boundary states. In this case, the expression of the cylinder amplitude is slightly different from (1.3) [10]. Then one finds that if all $l_j' = 1, 2$ the characters including massless states are the same in the closed and open channels, which is similar to the ten-dimensional case.
5. Coupling to closed string fields

Boundary states for Dp-branes can be efficiently used \[23\] to calculate the coupling (tadpoles) of massless closed string modes to the D-brane. Such couplings were then compared to the large distance behavior of fields around a black p-brane of supergravity \[28\].

In the case of D0-branes in Gepner model compactifications the boundary state will be used to read off the coupling to the gravitational field and the vector multiplets.

The massless states in the NSNS sector corresponding to the universal fields are of the form

\[ | \xi_{\mu\nu} \rangle = \xi_{\mu\nu} \psi_\mu \frac{1}{2} \bar{\psi}_\nu \frac{1}{2} | k \rangle, \]

where the four-dimensional polarization tensor contains the graviton, NSNS anti-symmetric tensor and dilaton. The dilaton polarization tensor is given by

\[ \xi_{\phi \mu\nu} = \frac{1}{\sqrt{2}} (\eta_{\mu\nu} - n_{\mu} k_{\nu} - n_{\nu} k_{\mu}) \]

where the vector \( n_{\mu} \) satisfies \( n \cdot k = 1, n^2 = 0 \). The coupling of the dilaton to the D0-brane vanishes for D0-brane boundary conditions defined in (3.1) since \( \xi_{\phi \mu\nu} M^{\mu\nu} = 0 \). This result is expected since there is no direct coupling of the D0-brane to the hypermultiplet moduli. The coupling of the anti-symmetric tensor also vanishes. Using the traceless-ness of the graviton the coupling of the graviton to the D0-brane boundary state is given by

\[ \langle \xi_{\mu\nu} | B \rangle = M^{\mu\nu} \xi_{\mu\nu} N B_{\lambda_g,\mu_g} = \xi_{00} N B_{\lambda_g,\mu_g}^\alpha. \]

The scalar fields in the vector multiplets are found in the NSNS sector. Each scalar field corresponds to a primary \( \Phi_{\lambda,\mu,\lambda,\mu} \) with \( h = \bar{h} = 1/2 \). For type IIB, which we are interested in, we have \( \lambda = \pm 1, \bar{\lambda} = \pm 1 \) whereas for type IIA we have \( \lambda = \pm 1, \bar{\lambda} = \mp 1 \). We denote by \( \phi^A \) a particular scalar associated with a primary \( \Phi_{\mu,\bar{\mu}}^{\lambda,\lambda} \) with \( q = 1 \), and by \( \bar{\phi}^A \) a conjugate field with primary \( \Phi_{\mu,\bar{\mu}}^{\lambda,\lambda} \) and \( q = -1 \). The one-point function for the two scalars is then given by

\[ \langle \lambda, \mu, \lambda, \mu | B \rangle = N B_{\lambda,\mu}^\alpha, \quad \langle \lambda, -\mu, \lambda, -\mu | B \rangle = N B_{\lambda,-\mu}^\alpha = N^* B_{\lambda,\mu}^\alpha, \]

where the last equality follows from the definition of \( B_{\lambda,\mu}^\alpha \) (2.7).

For the scalar field \( \phi^A \) or the graviton with quantum numbers \( (\lambda, \mu) \), there is a vector field in the Ramond sector associated with \( s_0 = +1 \) and it has the quantum numbers \( (\lambda, \mu - \beta_0) \). Then the charge conjugated component with \( s_0 = -1 \) has quantum numbers \( (\lambda, -\mu + \beta_0) \). Because the RR boundary states (3.8) and (3.9) are in the \((-3/2, -1/2)\) picture the relevant tadpoles use the vector vertex operators \( W^\pm \) which are discussed in
Appendix C. In particular, it turns out that only the components $W^\pm_0(A)$ couple to the boundary states. The relevant part of the boundary state $|B\rangle$ is given by

$$|B\rangle = \cdots + B^\alpha_\lambda,\mu-\beta_0 |\lambda,\mu-\beta_0\rangle + B^\alpha_\lambda,-\mu+\beta_0 |\lambda,-\mu+\beta_0\rangle,$$  \hfill (5.4)

where the external part in the above is given by (3.8) or (3.9) depending on the value of $s_0 = \pm 1$. As discussed in Appendix C, the couplings of the boundary states to $W^\pm_0(A)$ are electric and magnetic, respectively. They are determined by using the explicit forms of the external boundary state (3.8) and (3.9) and the vertices $W^\pm$,

$$\langle W^+_0(A) | B \rangle = \mathcal{N}(B^\alpha_\lambda,\mu-\beta_0 + B^\alpha_\lambda,-\mu+\beta_0) A^c_0, \quad \langle W^-_0(A) | B \rangle = \mathcal{N}(B^\alpha_\lambda,\mu-\beta_0 - B^\alpha_\lambda,-\mu+\beta_0) A^m_0.$$  \hfill (5.5)

Hence the boundary states are dyonic and the electric and magnetic charges are given by

$$q^i_e = \mathcal{N} |B^\alpha_\lambda,\mu-\beta_0| \cos \theta, \quad q^i_m = \mathcal{N} |B^\alpha_\lambda,\mu-\beta_0| \sin \theta,$$  \hfill (5.6)

where the angle $\theta$ depends on the quantum numbers of the vector field and the boundary condition $\alpha$. The graviphoton field has quantum numbers $\lambda = \lambda_g$ and $\mu = \mu_g$. Thus using the formula for $B^\alpha_\lambda,\mu$ (2.7), the angle $\theta_0$ is given by

$$\theta_0 = 2\pi \mu' \cdot \beta_0 = -\pi s_0' + \pi \sum_{i=1}^r \left( \frac{m_i'}{k_i} + 2 - \frac{s_i'}{2} \right) = \pi Q_\alpha,$$  \hfill (5.7)

where $Q_\alpha$ defines a $U(1)$ charge associated with the boundary condition $\alpha$ (see Appendix A). Note that the condition for mutually supersymmetric branes is given by $Q_\alpha = Q_\alpha + 2n, n \in \mathbb{Z}$. For the vector field associated with a scalar $\phi^A$ with $(\lambda, \mu)$, the angle $\theta_A$ is given by

$$\theta_A = -\pi s_0 s_0' + \pi \sum_{i=1}^r \left( \frac{m_i m_i'}{k_i} + 2 - \frac{s_i s_i'}{2} \right) - \pi Q_\alpha.$$  \hfill (5.8)

Alternatively for a given boundary condition $\alpha$ one might introduce a phase in the definition of the vertex operators $W^\pm_0$ and make the coupling to the boundary state purely electric or magnetic. For different boundary conditions $\bar{\alpha}$ the coupling will in general be dyonic.\footnote{These results are obtained also by a simple argument in the light-cone gauge. In this case, it is easy to calculate $\langle A_1 \pm iA_2 | B \rangle$. After an analytic continuation (see Appendix B), these give the couplings of $A_0 \pm iA_2$. However, since the gauge fields are supposed to live on the world-line, the field strength coming from $A_2$ should be interpreted as a magnetic dual of a field strength, namely, $F_{\mu\nu}(A_2) = * F_{\mu\nu}(A'_\mu)$ where $A'_\mu$ has only $A'_0$ component. Then one finds the same dyonic coupling as in (5.6).}
Note that a connection between \((l_i, m_i, s_i)\) and the geometrical picture of the boundary states has been discussed in [17]. In this sense, the ratio of the electric and magnetic charges may be determined by geometrical data similarly to the orbifold limit case [7].

In ten dimensions D-brane charges automatically satisfy Dirac-Zwanziger quantization conditions [1]. In the case of CY compactifications there are \(n_V + 1\) different vector fields and charges, and for two different boundary conditions \(\alpha\) and \(\tilde{\alpha}\) we denote the charges by \(q\) and \(\tilde{q}\) respectively. The quantization condition should then read

\[
q^0_e \tilde{q}^0_e - q^0_m \tilde{q}^0_m + \sum_{A=1}^{n_V} \left( q^A_e \tilde{q}^A_e - q^A_m \tilde{q}^A_m \right) = 2\pi n, \quad n \in \mathbb{Z}.
\]  

(5.9)

Hence we would like to interpret the boundary state as a D0-brane which carries electric and magnetic charges \(q_e, q_m\) and has a following coupling to the world-line to the lowest order in the bulk fields:

\[
S = \int d\tau (1 + c^*_A \phi^A + c_A \phi^{A*}) \sqrt{g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} + \left( q^0_e A^0_\mu + q^0_m A^0_\mu + q^A_e A^A_\mu + q^A_m A^A_\mu \right) \dot{x}^\mu.
\]  

(5.10)

Here the values for \(c_i\) are determined by (5.3) and the charges \(q^i\) are determined by (5.6).

6. Supersymmetry and vanishing of cylinder amplitude

The BPS nature of D0-branes defined by the boundary states (2.6) ensures that the static force between two of them vanishes. This force can be calculated to lowest order in string perturbation theory by the cylinder diagram.

In the limit of large separations only the exchange of massless closed string states is important. The vanishing of the cylinder amplitude implies the BPS cancelation between the massless NSNS and RR fields. We will see that this occurs in each multiplet. In the universal sector the cancelation between the graviton and the graviphoton exchange determines the BPS relation between the mass and the central charge. For the vector multiplets the cancelation relates the scalar and the vector coupling.

The cancelation between the NSNS and RR fields at every mass level follows from two facts. Firstly the contribution of a state to the cylinder amplitude is proportional to

\[
C_{\lambda, s_0}^{\lambda'} = B^\alpha_{\lambda, \mu} B^{\alpha}_{\lambda, -\mu} = (-1)^{s_0} \prod_{j=1}^n \sin^2 \left[ \pi \frac{(l_j + 1)(l'_j + 1)}{k_j + 2} \right] / \sin \left[ \pi \frac{l_j + 1}{k_j + 2} \right].
\]  

(6.1)

Secondly, the supersymmetry ensures that for every state in the NSNS sector there is an RR state in the same mass level. The supersymmetry connecting the NSNS and RR states
is essentially the spectral flow and shifts $\mu$ in (6.1) by an odd number of $\beta_0$. Since $\beta_0$ is
independent of $\lambda, \lambda'$, the contributions from the NSNS and RR states have just an opposite
sign,

$$C_{\lambda, s_0 = \pm 1}^{\lambda'} = - C_{\lambda, s_0 = 0, 2}^{\lambda'}.$$  \hspace{1cm} (6.2)

This gives the cancelation in each multiplet at each mass level.

Alternatively, we can apply an argument in the light-cone gauge given by Gepner in [29]. The spacetime supersymmetry current can be bosonized defining (see Appendix B for details on the notation)

$$\phi = \frac{1}{2} \phi_{ext} + \frac{\sqrt{3}}{2} H.$$  \hspace{1cm} (6.3)

The exponentials by $\psi = e^{i\phi}$ and $\psi^* = e^{-i\phi}$ (omitting a cocycle factor) are free fermions. The fact that all fields have odd $U(1)$ charge implies that the fermion $\psi$ is in the Ramond sector (this is not to be confused with the NSNS and RR sector described above).

The supersymmetry charge is simply the zero-mode of these free complex fermions $Q = \psi_0, Q^\dagger = \psi_0^*$. The $c = 12$ SCFT splits into a $c = 1$ free complex fermion and a $c = 11$ CFT, which is neutral under the $U(1)$ charge. Acting on a state with $Q$ will shift $\mu \rightarrow \mu + \beta_0$.

Similarly for the right-mover we have a free complex fermion $\bar{\psi}$ associated with the supercharges $\bar{Q}, \bar{Q}^\dagger$. In [17] it was shown that the boundary state preserves a supersymmetry $(Q + i\bar{Q}^\dagger) | B \rangle = 0$. This implies that the boundary state must be the product of a $c = 11$ part times

$$| B \rangle_{c=1} \sim | - - \rangle + i\psi_0^* \bar{\psi}_0^* | - - \rangle,$$

where $| - - \rangle$ is annihilated by $\psi_0$ and $\bar{\psi}_0$. It is clear from (6.1) that a shift by $\beta_0$ introduces a minus sign for each pair $\psi_0^* \bar{\psi}_0^*$ inserted. Hence the cylinder amplitude will vanish since

$$\langle - - | - - \rangle - \langle ++ | ++ \rangle = 1 - 1 = 0 \text{ with } | ++ \rangle \text{ a state annihilated by } \psi_0^* \text{ and } \bar{\psi}_0^*.$$ 

After modular transformation to the open string channel, the same argument basically implies that the free fermion part of the partition function is give by $\text{tr}(-1)^F q^{L_0} = 0$, which vanishes because of the presence of fermionic zero modes for the free fermion $Q$ in the odd spin structure. This is reminiscent of the GS formalism where the Green-Schwarz fermions become world-sheet fermions in the light-cone gauge, and cancelations due to supersymmetry are manifest in the odd spin structure.
7. Velocity dependent cylinder amplitude

The boundary state formalism can be used to describe D-branes moving with a constant velocity \([30]\). Such boundary states are simply constructed by applying a boost operator on the boundary states at zero velocity. Hence the boosted boundary state is defined by

\[
|B, v\rangle \equiv e^{ivK_{01}} |B, 0\rangle, \tag{7.1}
\]

where \(\tanh v\) is the velocity and the boost operator \(K_{01} = K_X^{01} + K_{\psi}^{01}\) is given by

\[
K_X^{01} = x^0 p^1 - x^1 p^0 - i \sum_{n>0} \frac{1}{n} (a_n^0 a_n^1 - a_{-n}^1 a_{-n}^0 + \tilde{a}_{-n}^1 \tilde{a}_n^0 - \tilde{a}_n^1 \tilde{a}_{-n}^0), \tag{7.2}
\]

and the fermionic part in the NSNS and RR sector respectively

\[
K_{\psi}^{01} = -i \sum_{r>0} (\psi_r^0 \psi_r^1 - \psi_{-r}^1 \psi_{-r}^0 + \tilde{\psi}_{-r}^1 \tilde{\psi}_{-r}^0 - \tilde{\psi}_r^0 \tilde{\psi}_r^1) \quad \text{(NSNS)},
\]

\[
K_{\psi}^{01} = -\frac{i}{2} ([\psi_0^0, \psi_0^1] + [\tilde{\psi}_0^1, \tilde{\psi}_0^0]) - i \sum_{n>0} (\psi_{-n}^0 \psi_n^1 - \psi_{-n}^1 \psi_n^0 + \tilde{\psi}_{-n}^0 \tilde{\psi}_n^1 - \tilde{\psi}_{-n}^1 \tilde{\psi}_n^0) \quad \text{(RR)}. \tag{7.3}
\]

It is easy to see that as far as the cylinder amplitude is concerned the boost is just acting as a twist on the bosonic and fermionic oscillators. After careful consideration of the zero-mode part the changes in the cylinder amplitude can be separated into the bosonic and fermionic part and we get

\[
\begin{align*}
\chi_0 &\quad \rightarrow \left(\frac{T_0}{2}\right)^2 \frac{t^{-1}}{(2\pi)^2} e^{-b^2/(4\pi t)}, \\
\chi_{s_0=0,2} &\quad \rightarrow \frac{1}{i\theta_1 (iv/\pi |it)} \left[ \theta_3 (iv/\pi |it) + (-1)^{s_0}/2 \theta_4 (iv/\pi |it) \right], \\
\chi_{s_0=\pm 1} &\quad \rightarrow \frac{1}{i\theta_1 (iv/\pi |it)} \theta_2 (iv/\pi |it),
\end{align*}
\tag{7.4}
\]

where \(b^2 = \sum_{i=2,3} (y_i^1 - y_i^2)^2\).

7.1. potential between two moving D0-branes

For non-zero \(v\), the cylinder amplitude does not vanish and one obtains non-vanishing potential between two moving D0-branes. The calculation is straightforward and, by the standard procedure, we get

\[
V(r) = -\frac{1}{\kappa_\alpha^2} \int_0^\infty \frac{dt}{t^{1/2}} \chi_0 \bigg|_{b \rightarrow r} \sum_{\lambda,\mu}^B C_{\lambda, s_0}^{s_0} \chi_\mu^{s_0}(q), \tag{7.5}
\]
where \( r \) is the separation of the two D0-branes and \( \chi_{s_0} \) in \( \chi_\mu^\lambda \) are given by (7.4).

The contributions from the massless closed string states are extracted using the asymptotic forms of the characters as \( q = e^{-2\pi t} \to 0 \). At low velocity, the leading contributions are given by

\[
V(r) = -\frac{N^2}{2\pi r} \left[ C_{s_0}^{\lambda',2}(\cosh 2v - \cosh v) + \sum_c C_{s_0}^{\lambda',0}(1 - \cosh v) \right].
\]

(7.6)

Here \( \sum_c \) indicates the summation over the chiral primaries and includes the contributions from \( n_V \) vector multiplets whereas the first term is the contribution from the gravity multiplet. We see that the cancelation at \( v = 0 \) occurs in each multiplet.

We also see that the potential starts from \( v^2 \) and hence the moduli space has a non-trivial metric. This is as expected because the boundary states preserve only \( N = 1 \) spacetime supersymmetry. Moreover, since theta functions \( \theta_i(iv/\pi|it) \) can be expanded in \( v \) by Eisenstein functions, it is possible to get the exact coefficients for \( v^n \) terms.

7.2. absorptive part

At non-zero \( v \), the cylinder amplitude \( A = (\pi/2) \int_0^\infty dt \ Z(q) \) gets imaginary part because \( \theta_1^{-1} \) has simple poles in the open string channel. The imaginary part is interpreted as indicating pair creation of open strings as discussed in the ten-dimensional case [31]. Although the amplitudes in Gepner models are more complicated, the argument here is almost the same as in the ten-dimensional case since we know the modular transformation between the closed and open string channels.

Following the procedure in [31], the imaginary part is given by

\[
\text{Im}A = \frac{N^2}{4\pi} \sum_{k=1}^{\infty} \frac{1}{k} e^{-kb^2/(4v)} \chi\left(\frac{k\pi}{v}\right).
\]

(7.7)

Here \( \chi(s) = (i\theta_1/2\eta^3)Z_{\alpha\alpha}(\hat{q}) \) with \( \hat{q} = e^{-2\pi s} \) and \( Z_{\alpha\alpha}(\hat{q}) \) is given by (1.3) and \( \chi_{s_0} \) in (7.4) with \( \theta_i(isv/\pi|is) \) instead of \( \theta_i(iv/\pi|it) \).

At low velocity \( v \ll 1 \), the leading contribution comes from the massless open string states and it is vanishingly small for \( b^2 \gg v \) as \( e^{-b^2/v} \). On the other hand, at high velocity \( v \gg 1 \), the dominant contribution comes from the graviton in the closed string channel and the internal minimal model part does not matter. By a modular transformation of \( \chi(s) \), one finds

\[
\text{Im}A \sim \frac{N^2}{4} v^{-1} e^{-b^2/(4v) + v} C_{s_0}^{\lambda',2}.
\]

(7.8)

This shows the behavior of fundamental strings at high energy and black absorptive disks with area \( \ln(s/M_0^2) \) where \( s \) and \( M_0 \) are the center of mass energy squared and the D0-brane mass, respectively.
8. Comparison to black holes in $N = 2$ supergravity

D-branes in the ten-dimensional theory have corresponding black $p$-brane solutions in
supergravity [28]. They are regarded as different representations of the same BPS objects.
Since D-branes have boundary state description, it should be possible to relate them to
the black $p$-branes. In fact, a precise correspondence has been found in this case [23] [24].
Furthermore, a four-dimensional Reissner-Nordstrom black hole is obtained by wrapping
a D3-brane on a Calabi-Yau threefold and the connection to the corresponding boundary
state has been shown in the orbifold limit $T^6/Z^3$. In this section, we show that our
boundary states correspond to some extremal black holes in $D = 4$ $N = 2$ supergravity.

Our boundary states couple to the bosonic sector of 1 gravity multiplet and
$n_V = h^{2,1}$
vector multiplets, where $h^{2,1}$ is a Hodge number of the corresponding CY. Note that there
are no couplings to the hypermultiplets (including the dilaton) and the anti-symmetric
tensor. The boundary states also satisfy the BPS condition.

In $D = 4$ $N = 2$ supergravity, there exist the BPS black hole solutions
which have the same couplings. To see this, let us first recall the low energy action for the bosonic
part of the gravity and vector multiplets

$$S = \int \sqrt{-g} \, d^4 x \left[ \frac{1}{2} R - g_{A\bar{B}} \partial^\mu z^A \partial_\mu \bar{z}^\bar{B} - \mathcal{F}_{\mu \nu} (\ast \mathcal{G}^{\mu \nu}) \right], \quad (8.1)$$

with

$$\mathcal{G}_{I \mu \nu} = \text{Re} \mathcal{N}_{I J} \mathcal{F}_{\mu \nu}^J - \text{Im} \mathcal{N}_{I J} * \mathcal{F}_{\mu \nu}^J. \quad (8.2)$$

Here the complex scalars $z^A$ ($A = 1, ..., n_V$) parametrize a special Kähler manifold with
the metric $g_{A\bar{B}} = \partial_A \partial_{\bar{B}} K(z, \bar{z})$ where $K$ is the Kähler potential. The symplectic period
matrix $\mathcal{N}_{I J}$ ($I, J = 0, ..., n_V$) and the Kähler potential are expressed by the holomorphic
potential $F(X)$ :

$$K = -\log \left[ i (X^I F_I - \bar{X}^I \bar{F}_I) \right],$$

$$\mathcal{N}_{I J} = \bar{F}_{I J} + 2i \left( \frac{\text{Im} F_{I K} X^K \text{Im} F_{J L} X^L}{X^M (\text{Im} F_{M N}) X^N} \right), \quad (8.3)$$

with $F_I = \frac{\partial F}{\partial X^I}$ and $F_{I J} = \frac{\partial^2 F}{\partial X^I \partial X^J}$. $X^I$ are homogeneous coordinates and satisfy the
constraint $L^I \text{Im} F_{I J} \bar{L}^J = -1/2$ with $L^I = e^{K/2} X^I$. In Calabi-Yau compactifications,
$F(X)$ is given by

$$F(X) = d_{ABC} \frac{X^A X^B X^C}{X^0}, \quad (8.4)$$

where $d_{ABC}$ are the intersection numbers of the CY. The physical field strengths of the
graviphoton and the other vector fields are given by

$$T_{\mu \nu}^- = 2i \text{Im} \mathcal{N}_{I J} L^I \mathcal{F}^J_{\mu \nu}, \quad G_{\mu \nu}^{-A} = -g_{A\bar{B}} \bar{F}^I_{\bar{B} \mu \nu} \text{Im} \mathcal{N}_{I J} \mathcal{F}^J_{\mu \nu}, \quad (8.5)$$

$^6$ For a review of $D = 4$ $N = 2$ supergravity, see, e.g., [32].
where \( f_A^I = (\partial_A + \frac{1}{2} \partial_A K) L^I \) and \(-\) stands for the antiself-dual part such as \( \frac{1}{2} (T_{\mu\nu} - i T_{\mu\nu}) \).

The electric and magnetic charges are defined by

\[
q_I = \frac{1}{4\pi} \int_{S^2_\infty} G_{\mu\nu} dx^\mu \wedge dx^\nu, \quad p^I = \frac{1}{4\pi} \int_{S^2_\infty} F_{\mu\nu} dx^\mu \wedge dx^\nu. \tag{8.6}
\]

The complex charges of \( F^{-I}_{\mu\nu} \), which are defined similarly to (8.6), are related to \( q_I \) and \( p^I \) by

\[
t^I = \frac{1}{2} \left[ p^I + i(\text{Im} N^{-1})^{IJ} (\text{Re} N_{JK} p^K - q_J) \right] \bigg|_\infty. \tag{8.7}
\]

If we define the electric and magnetic charges by the asymptotic behavior

\[
F^I_{0i} \sim \frac{e^{I}}{r^3} x^i, \quad \ast F^I_{0i} \sim \frac{g^{I}}{r^3} x^i, \tag{8.8}
\]

with \( r^2 = x^i x_i (i = 1, 2, 3) \), they are related to \( t^I \) by

\[
t^I = \frac{1}{2} (g^I - ie^I). \tag{8.9}
\]

Here we consider the spherically symmetric solutions whose metric can be written in the form

\[
d s^2 = -e^{2U} (r) d t^2 + e^{-2U} (r) \left( d r^2 + r^2 d\Omega_2^2 \right). \tag{8.10}
\]

The BPS solutions without non-trivial fermionic fields are obtained from the condition of vanishing supersymmetry transformations for the gravitino and the gauginos, \( \delta \psi_{\alpha \mu} = \delta \lambda^{A\alpha} = 0 \). For a particular choice of the supersymmetry parameter, these conditions give the first order differential equations [33],

\[
\frac{dU}{dr} = -2i \text{Im} N_{IJ} t^I e^U, \quad \frac{d z^A}{d r} = -2i g^{AB} F^I_{B\ast} \text{Im} N_{IJ} t^I e^U. \tag{8.11}
\]

The most general BPS black hole solutions to these equations are given by [34]

\[
z^A = X^A / X^0, \quad e^{-2U} = e^{-K} = i (\bar{X}^I F_I - X^I \bar{F}_I), \quad F^I_{ij} = \frac{1}{2} \epsilon_{ijk} \partial_k H^I, \quad G_{IJ} = \frac{1}{2} \epsilon_{ijk} \partial_k N_I \quad (i, j, k = 1, 2, 3), \tag{8.12}
\]

\[
i (X^I - \bar{X}^I) = H^I, \quad i (F_I - \bar{F}_I) = N_I,
\]

where \( H^I (r) \) and \( N_I (r) \) are the harmonic functions

\[
H^I = h^I + \frac{p^I}{r}, \quad N_I = n_I + \frac{q_I}{r}. \tag{8.13}
\]
In addition, we have the constraints

\[ e^{-K}|_{r=\infty} = 1, \quad h^I q_I - n_I p^I = 0. \]  

(8.14)

The electric components of the gauge fields are obtained from (8.2). We see that these black holes have the same couplings to the massless fields as the boundary states. Moreover, in a generic case, \( F_{\mu\nu} \) are dyonic and hence so are the physical fields \( G_{\mu\nu}^A \). This is in agreement with the dyonic gauge couplings discussed in section 5. Without introducing an additional phase in the definition of \( T_{\mu\nu}^- \), it follows from (8.5) and (8.11) that the charge of \( T_{\mu\nu}^- \) is real. Hence \( T_{\mu\nu} \) is always magnetic. This is again consistent with the discussion in section 5; by proper definition of the vertex operator, one can always make the graviphoton magnetic as long as the mutually supersymmetric boundary states, which satisfy \( Q_\alpha = Q^{\tilde{\alpha}} \) (mod 2), are considered.

One can find more precise correspondence using the potential from the massless particle exchange between two moving black holes. The coupling constant (charge) for the graviton is given by the black hole mass \( M \), which is read off from the asymptotic behavior \(-e^{2U} + 1 \sim 2G_N M/r\). Similarly, the charges for the complex scalars are obtained from \( z^A \sim w^A/r + \text{constant} \). For the gauge fields, the electric and magnetic charges are obtained similarly to (8.8),(8.9) and we denote by \( u^0 \) and \( u^A \) the complex charges for \( T_{\mu\nu}^- \) and \( G_{\mu\nu}^- \), respectively. Then the potential is given by [35]

\[ V = -\frac{1}{2\pi r} \left( \hat{M}^2 \cosh 2v - |\hat{w}^0|^2 \cosh v + \sum_{A=1}^{n_V} |\hat{w}^A|^2 - \sum_{A=1}^{n_V} |\hat{u}^A|^2 \cosh v \right), \]  

(8.15)

where \( r \) is the distance between the two black holes and \( \tanh v \) is the relative velocity. A hatted quantity is equal to the corresponding un-hatted one up to a constant coming from the proper normalization of the physical field.

Because of supersymmetry, there are constraints among charges. First, from the first equations of (8.5) and (8.11), one finds that \( M \sim u^0 \). This is nothing but the BPS mass formula for \( N = 2 \) black holes [32][36]. Furthermore, the second equations of (8.5) and (8.11) give the relation \( w^A \sim iu^A \). Consequently, the potential is brought into the form

\[ V = -\frac{1}{2\pi r} \left[ \hat{M}^2 (\cosh 2v - \cosh v) + \sum_{i=1}^{n_V} |\hat{u}^A|^2 (1 - \cosh v) \right]. \]  

(8.16)

We find that this is the same form as (7.6) from Gepner models. The boundary state then corresponds to the black hole with

\[ \hat{M} = \hat{w}^0 = |q^0_e + iq^0_m|, \quad |\hat{w}^A| = |\hat{u}^A| = |q^A_e + iq^A_m|. \]  

(8.17)
Furthermore, one can make precise comparison also by using the tadpoles discussed in section 5 [23]. The couplings between the boundary states and the massless states are regarded as the sources for the massless fields. Thus overlaps calculated in section 5 correspond to the charges of the massless fields on the supergravity side which are read off from the asymptotic behavior of the fields as \( r \to \infty \). However, the massless contribution to the potential takes the form \( V \sim r^{-1} \sum_{i \text{massless}} \langle B, v \mid i \rangle \langle i \mid B, 0 \rangle \). Hence the matching of the potential implies the correspondence of the overlaps and the massless field charges. In fact, this is nothing but (8.17). The advantage of using tadpoles is that one can see the tensor structure as in (5.2) and the dyonic couplings as in (5.5) [23][7]. (Though we have summed the components of the polarization tensor \( \xi_{\mu\nu} \) in (5.2), it is possible to extract a specific component, of course.)

In addition, the boundary states can be purely magnetic (or electric). In particular, by the proper definition of the physical fields, we can always make a boundary state magnetic as long as only one boundary state is considered. It is easy to find the corresponding magnetic black hole solution on the supergravity side. It is given by the ‘magnetic’ solution discussed in [36][37], i.e., the solution with \( H^0 = N_A = 0 \). In this case, the metric and the homogeneous coordinates are given by

\[
X^0 = \frac{1}{2} \sqrt{d_{ABC} H^A H^B H^C / N_0}, \quad X^A = \frac{i}{2} H^A, \quad e^{-2U} = 2 \sqrt{N_0 d_{ABC} H^A H^B H^C}.
\] (8.18)

We then find that the scalars \( z^A \) are purely imaginary and so are their charges \( w^A \). By (8.7) and (8.11), this means that \( u^A \) are real and hence \( G^A_{\mu\nu} \) are magnetic.

**9. Scattering involving open string states**

String scattering amplitudes on the disk with D0-brane boundary conditions give the lowest order contributions to scattering processes involving the D0-brane. Such amplitudes are given by inserting \( n \) boundary operators (open string vertices) \( \psi^{\alpha \alpha}_{\lambda, \mu} \) on the boundary of the disk and \( m \) closed string vertices \( \Phi^{\lambda, \bar{\lambda}}_{\mu, \bar{\mu}} \) in the interior of the disk.

In general the boundary states which define the boundary CFT are determined in terms of the quantities \( B^i_{\alpha} \), where the label \( i \) runs over all the chiral primary fields of the bulk CFT. The \( B^i_{\alpha} \) are determined via the one-point functions of primary fields \( \Phi^{\lambda, \bar{\lambda}}_{\mu, \bar{\mu}} \) on the disk where \( \langle \Phi^{\lambda, \bar{\lambda}}_{\mu, \bar{\mu}} \mid \alpha \rangle = B^i_{\alpha} \). Here \( \Omega \) denotes the ‘gluing automorphism’ \[\Omega\] which determines which fields can couple consistently to the boundary. The primary fields in the bulk live in \( \mathcal{H} \otimes \bar{\mathcal{H}} \). The so called ‘doubling trick’ turns the anti-holomorphic field living on the upper half plane into a holomorphic field on the lower half plane,

\[
\bar{\phi}_{\Omega(i)}(\bar{z}) \rightarrow R^i_{\alpha} \phi_{(i+)}(\bar{z}), \quad (9.1)
\]
where $i^+$ denotes the conjugate field which is determined by the chiral OPE $\phi_i(z)\phi_{i^+}(0) = z^{-2h}1 + \cdots$. The reflection coefficients $R^\alpha_i$ can be determined through the connection of the bulk to boundary operator product

$$
\lim_{\text{Im}(z) \to 0} \Phi_{i,\Omega_i}(z, \bar{z}) = \sum_k (z - \bar{z})^{-2h_i + h_k} C_{i}^{\alpha} k \psi_k(x).
$$

(9.2)

The OPE coefficient for the identity operator $C_{i}^{\alpha\,0}$ can be related to the insertion of $\Phi_{i,\Omega(i)}$ on the disk. The calculation of this quantity using the boundary states determines $C_{i}^{\alpha\,0} = B_i^\alpha / B_0^\alpha$ [38]. One can then show that the reflection coefficient for the boundary condition labeled by $\alpha$ is given by

$$
R^\alpha_i = \frac{B_i^\alpha}{B_0^\alpha}.
$$

(9.3)

As an example we will consider the two-point function of two massless closed string fields given by

$$
\langle V_{\Omega(j)} e^{ik_1 X(z_1)} V_{\Omega(l)} e^{ik_2 X(z_2)} \rangle_H = |z_1 - z_2|^{k_1 k_2} |\bar{z}_1 - \bar{z}_2|^{k_1 M k_2}
\times |z_1 - \bar{z}_1|^{k_1 M k_1} |z_2 - \bar{z}_2|^{k_2 M k_2} R^\alpha_i R^\alpha_j \langle \Phi_i(z_1) \Phi_j^+(\bar{z}_1) \Phi_k(z_2) \Phi_{i^+}(\bar{z}_2) \rangle_S,
$$

(9.4)

where $M$ is the matrix in (3.1), and $\langle \cdots \rangle_S$ denotes the correlation function for the chiral fields on the plane. After fixing the conformal Killing symmetry and integrating over the position of the vertex operators we have a very similar result to [39]. The detailed properties of the amplitude are hidden in the four-point function of the Gepner model fields. Correlators of $N = 2$ minimal models can be expressed in terms of free bosonic and parafermionic correlators [40][41]. The amplitude will display an infinite set of s-channel poles in the region where $z_1 \to z_2$, and the bulk OPE

$$
\Phi_i(z, \bar{z}) \Phi_{j^+}(0) \sim \sum_k z^{h_k - h_i - h_j} \bar{z}^{h_k - h_i - h_j} C_{i j}^{k} \Phi_k
$$

(9.5)

will determine the couplings. On the other hand for the t-channel poles (when one vertex operator approaches the boundary) the bulk to boundary OPE (9.2) determines the couplings. The fact that these two-point functions have an infinite set of poles in the open string channel indicates that the D0-branes has many internal excitations determined by open string spectrum, which is not surprising for a black hole.

Correlation functions with only boundary field (open string vertex operators) inserted can be used to calculate terms appearing in the effective quantum mechanics governing the open strings stretched between branes.

It would be interesting to determine whether in some cases the massless states in the open string spectrum found in section 4 correspond to exactly marginal boundary perturbations [12]. A condensate of such operators would provide a continuous deformation of the boundary state and correspond to a modulus associated with the wrapped brane.
10. Conclusions

In this paper we have used the D-brane boundary states constructed in Gepner models to analyze some aspects of the dynamics of D0-branes in Calabi-Yau compactifications of type II theories to four dimensions. The advantage of working with an exactly soluble conformal field theory lies in the fact that the spectrum and in principle all the correlation functions are exactly known. In particular this implies that the relation between the cylinder and annulus allows to find the spectrum of stretched open strings, which is not possible if one only knows the massless part of the boundary state.

We have found a correspondence between the D0-brane boundary states and a class of $D = 4 \ N = 2$ black holes. They are both the BPS objects for the same $N = 2$ supersymmetry. It would be interesting to find more precise correspondence between the D0-branes and the black holes, e.g., the matching of the black hole entropy and the Hawking radiation. Gepner models and the CY compactification models are at different points of the moduli space and the classical black holes have macroscopic charges. Thus the issues of the moduli and quantum corrections might be important to further investigations.

Note that the boundary states (2.6) carries the minimal amount of charge because it satisfies the Cardy’s constraint with minimal multiplicities for the open string characters. We can in principle multiply the boundary state by an integer factor. This corresponds to introducing $N$ Chan Paton factors. A disk diagram with no open string vertices inserted simply picks up a trace which is equal to $N$. Note that by looking at a single boundary state, we are restricted to a regime where $N$ is finite and $g$ is small. In the ten-dimensional case, the Dp-branes are constructed in flat Minkowski space, but represent curved black p-branes of supergravity. The D-branes in the Gepner model are constructed using the asymptotic vacuum defined by a Gepner model compactification without any D-branes. On the other hand we have seen that the BPS-character and the charges that they carry makes it possible to identify D0-branes with extremal black holes in $N = 2$ supergravity.

The extreme $N = 2$ black holes have $AdS_2 \times S_2$ near-horizon geometries and exhibit the attractor mechanism [43]. Considering a spherical world-sheet with large number of disks cut out, i.e. a large number of boundary states on the disk, corresponds to a planar large $N$ limit. In this limit the insertions of boundary states should be equivalent to a new string theory on a world-sheet with no boundaries but living in the near-horizon geometry where the internal Calabi-Yau manifold is determined by the fixed point moduli of the attractor equations. It would be very interesting to explore whether it is possible to connect the attractor equations with a flow in the CFT induced by the insertion of boundaries.

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**Note added**

After this work is completed, a paper appeared which discusses related issues [44].

**Appendix A. Gepner models**

In this appendix we will give a brief review of some aspects of Gepner models. Gepner models are exactly soluble supersymmetric compactifications of type II and heterotic strings which use tensor products of $N = 2$ minimal models to construct the internal SCFT. The $N = 2$ minimal models are unitary representations of the $N = 2$ SCFT which are labeled by an integer $k = 1, 2, \cdots$, where the central charge is given by

$$c = \frac{3k}{k + 2}.$$  \hspace{1cm} (A.1)

Primary fields $\Phi_{l,m,s}^i$ are labeled by three integers $l, m, s$ with the ranges

$$l = 0, 1, \cdots, k, \quad m = -(k + 1), \cdots, k + 2, \quad s = 0, 2, \pm 1,$$  \hspace{1cm} (A.2)

together with constraint $l + m + s \in 2\mathbb{Z}$. The field identifications $(l, m, s) \sim (l, m, s + 4)$ and $(l, m, s) \sim (l, m + 2(k + 2), s)$ imply that $m$ is defined modulo $2(k + 2)$ and $s$ is defined modulo 4. The labels $(l, m, s)$ can be brought into the ‘standard range’ by another field identification $(l, m, s) \sim (k - l, m + k + 2, s + 2)$. The conformal dimension $h$ and $U(1)$ charge $q$ of the primary fields (with $(l, m, s)$ in the standard range) are given by

$$h = \frac{l(l + 2) - m^2}{4(k + 2)} + \frac{s^2}{8}, \quad q = \frac{m}{k + 2} - \frac{s}{2}.$$  \hspace{1cm} (A.3)

A Gepner model is constructed by tensoring $n$ minimal models with $k_i, i = 1, \cdots, n$ such that the sum of the central charges of the $n$ minimal models is equal to

$$\sum_{i=1}^{n} \frac{3k_i}{k_i + 2} = c_{\text{int}}.$$  \hspace{1cm} (A.4)

\footnote{Note that states with $s = 2$ are really descendants. Nevertheless splitting each module into subsets with $s = 0$ and $s = 2$ is a very useful bookkeeping device.}
The total currents $T, G^\pm, J$ of the tensor product are given by the sum of the currents of each minimal model. For $c_{int} = 9$, the external theory is given by one complex boson and a level one $SO(2)$ current algebra. The primary fields can be labeled by two vectors

$$\lambda = (l_1, \cdots, l_n), \quad \mu = (s_0; m_1, \cdots, m_n; s_1, \cdots, s_n). \quad (A.5)$$

Here $s_0 = 0, 2, +1, -1$ labels the four characters corresponding to $o, v, s, c$ conjugancy classes of the $SO(2)$ current algebra. Gepner constructed a supersymmetric partition function for the tensor product by using ‘$\beta$-projections’ (generalizing the GSO projection) and adding twisted sectors to achieve modular invariance. This ‘$\beta$-method’ uses the $(2n+1)$-dimensional vectors: $\beta_0$ which has 1 everywhere and $\beta_i, i = 1, \cdots, n$ which has 2 in the first and $n + 1 + j$ entry and is zero everywhere else. An inner product of two $(2n+1)$-dimensional vectors is defined by

$$\mu \cdot \tilde{\mu} = -\frac{1}{4}s_0\tilde{s}_0 - \sum_{j=1}^{n} \frac{s_j\tilde{s}_j}{4} + \sum_{j=1}^{n} \frac{m_j\tilde{m}_j}{2(k_j + 2)}. \quad (A.6)$$

Note that with the help of this inner product the total $U(1)$ charge of a primary field is given by $q_\mu = 2\beta_0 \bullet \mu$. The GSO projection is then implemented by projecting onto states with an odd integer charge $q_\mu$. In order to preserve the $N = 1$ superconformal invariance all fields in the tensor product have to be in the same sector (R or NS). This can be achieved by projecting onto states which satisfy $\beta_j \cdot \mu \in Z$ for $j = 1, \cdots, n$. Gepner constructed a modular invariant partition function by including twisted sectors,

$$Z = \frac{1}{2n}(\text{Im } \tau)^{-2} \sum_{b_0, b_j}^{\beta} \sum_{\lambda, \mu} (-1)^{b_0} \chi_\lambda^\mu(q)\chi_\lambda^\mu + \sum_{j=1}^{n} \sum_{\beta_j} (\bar{q}). \quad (A.7)$$

Here $b_j = 0, 1; b_0 = 0, 1, \cdots, K - 1; K = \text{lcm}(4, 2(k_j + 2))$ and $q = e^{2\pi i \tau}$. $\chi_\lambda^\mu$ are the characters corresponding to the primaries $\Phi_\lambda^\mu$. The contribution from the external boson oscillators is included in $\chi_{s_0}$ in $\chi_\lambda^\mu$. In (A.7) the diagonal affine $SU(2)$ invariant is used which exists for all levels $k_j$. Other choices according to the ADE classification of affine $SU(2)$ invariants are possible and lead to different models [45]. The notation $\sum^{\beta}$ indicates the summation over the $\beta$-projected range $(\lambda, \mu)$ and the $(-1)^{b_0}$ imposes the connection between spin and statistics. Note that the supersymmetries have a very simple action on the characters $\chi_\lambda^\mu$: acting with $Q$ corresponds to $\mu \rightarrow \mu + \beta_0$ and acting with $Q^\dagger$ corresponds to $\mu \rightarrow \mu - \beta_0$. 

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Appendix B. Boundary states in the light-cone gauge

In [24], a construction of D-brane boundary states in the light-cone gauge was given. A peculiar feature of these states is that they describe \((p + 1)\) D-instantons instead of \(D_p\)-branes since the light-cone coordinates \(X^\pm\) satisfy Dirichlet boundary conditions. One can use these boundary states simply as a calculational tool since they are related to the \(D_p\)-brane boundary states by an analytic continuation (double Wick rotation). In fact, the calculation of cylinder amplitudes is much easier in this gauge.

A ‘\(D_p\)-brane’ boundary condition is then given by

\[
(\partial X^i \pm \bar{\partial} X^i) | B_p \rangle \chi = 0, \quad (\psi^i \pm i\eta \bar{\psi}^i) | B_p, \eta \rangle \psi = 0, \quad (B.1)
\]

where \(i = 1, 2\), and plus sign for \(i = 1, \ldots, 1+p\) and minus sign for \(i = 2+p, \ldots, 2\) \((p = \pm 1, 0)\). The complex combinations of these free bosons and fermions form the simplest example of a \(c = 3\ N = 2\) SCFT. Except for the RR zero-mode part, the boundary state is obtained by removing the ghost and light-cone oscillator parts of the corresponding boundary state in the covariant case.

The construction of the RR zero-mode part is similar to the ten-dimensional case [46]. We start from a state \(| B_1, + \rangle^0_{RR}\) satisfying (B.1) for the zero-modes with \(p = 1\) and \(\eta = +1\). It is useful to define \(\psi_i^\prime \equiv (\psi_0^i \pm i\bar{\psi}_0^i)/\sqrt{2}\) and \(\bar{\Gamma}_3 = 2i\bar{\psi}_0^1\psi_0^2, \Gamma_3 = 2i\psi_0^1\bar{\psi}_0^2\). Then the other zero-mode states are given by

\[
| B_1, - \rangle^0_{RR} = \bar{\Gamma}_3 | B_1, + \rangle^0_{RR}, \quad | B_p, \pm \rangle^0_{RR} = \prod_{i=2+p}^2 \psi_i^\prime \pm | B_1, \pm \rangle^0_{RR}. \quad (B.2)
\]

The GSO projection enforces that the left and right \(U(1)\) charges take odd integer values. This relates the boundary conditions of the external and internal parts. Using free bosons, the left \(U(1)\) current is expressed by \(J_{tot} = J_{ext} + J_{int} = i\partial \phi + i\sqrt{3}\partial H\) whereas the right \(U(1)\) current is

\[
\text{IIB} : \bar{J}_{tot} = i\bar{\partial} \bar{\phi} + i\sqrt{3}\bar{\partial} \bar{H}, \quad \text{IIA} : \bar{J}_{tot} = -i\bar{\partial} \bar{\phi} + i\sqrt{3}\bar{\partial} \bar{H}. \quad (B.3)
\]

The difference between IIB and IIA is the reversal of the sign of \(\bar{\phi}\) (or equivalently the sign of \(\bar{H}\)). This can be traced back to the fact that IIB in ten dimensions is chiral whereas IIA is not. Because of (B.3), the external chirality operators are given by

\[
\Gamma = \Gamma_3(-1)^F, \quad \bar{\Gamma} = \epsilon \Gamma_3(-1)^F, \quad (B.4)
\]

with \(\epsilon = +1\) for type IIB and \(-1\) for type IIA.

In A-type boundary states, \(s_0\) takes the same sign in the left and right sectors whereas it takes the opposite sign in B-type boundary states. Thus the GSO projected states are given by

\[
| Bp, s_0 = \pm 1 \rangle = \frac{1}{2\sqrt{2}}(1 + s_0 \Gamma)(1 \pm s_0 \bar{\Gamma}) | Bp, + \rangle_{RR}, \quad (B.5)
\]

where \(+(-)\) for A(B)-type boundary states. One then finds that the projected states are non-vanishing only when (i) for type IIB, \(p = 0\) for A-type and \(p = \pm 1\) for B-type, (ii) for type IIA, \(p = \pm 1\) for A-type and \(p = 0\) for B-type. This is in agreement with the possible combinations of the internal and external boundary conditions discussed in [17]. This is also an analog of the ten-dimensional case in which the RR boundary states exist only for odd \(p\) for type IIB and even \(p\) for type IIA because of the GSO projection.
Appendix C. Vertex operators in the RR sector

The vertex operators for left- or right-moving Ramond states contain three parts (we only display the left-moving part, the right-moving fields will be barred): a bosonized superghost part, an $SO(3,1)$ spin field and a Ramond field in the internal CFT,

$$ V_s = \exp(s\phi) S^A V_{q_{int}}. $$

Here $s$ denotes the superghost picture and $q_{int}$ denotes the internal $U(1)$ charge. For the spin field $S^A$ we introduce a Weyl notation where $S^a$ is a spinor of positive helicity and $\bar{S}^\dot{a}$ is a spinor of negative helicity. In the $s = -1/2$ picture the GSO projected vertices are given by

$$ V^a = e^{-\frac{1}{2}\phi} S^a V_{q_\phi = \frac{1}{2}, \frac{-3}{2}}, \quad \bar{V}^{\dot{a}} = e^{-\frac{1}{2}\bar{\phi}} S^{\dot{a}} V_{q_{\bar{\phi}} = \frac{1}{2}, \frac{-3}{2}}, $$

whereas for in the $s = -3/2$ picture the GSO projected vertices are given by

$$ V^a_{-3/2} = e^{-\frac{3}{2}\phi} S^a V_{q_\phi = \frac{1}{2}, \frac{-3}{2}}, \quad \bar{V}^{\dot{a}}_{-3/2} = e^{-\frac{3}{2}\bar{\phi}} S^{\dot{a}} V_{q_{\bar{\phi}} = \frac{1}{2}, \frac{-3}{2}}. $$

This is because the GSO projection projects onto odd $U(1)$ charge where the $U(1)$ charge is given by $q_{GSO} = s + h + q_{int}$ with $h = 1$ for un-dotted and $h = 0$ for dotted spinors.

We will consider IIB strings compactified on a CY in the following. The standard vertex operators for the RR fields are given by tensor products of left- and right-moving RR fields in the $-1/2$ picture. These vertices contain the field strength of the appropriate RR potential. This implies that for IIB the RR vector fields in the vector multiplet are given by

$$ V(F) = e^{-\frac{1}{2}(\phi + \bar{\phi})} e^{ikX} \left( S^a \sigma_{ab} \bar{S}^b V_{q_\phi = \frac{1}{2}, \bar{q}_{\bar{\phi}} = \frac{1}{2}, \bar{q}_{\bar{\phi}} = \frac{1}{2}} F^+ + S^{\dot{a}} \tilde{\sigma}_{\dot{a}b} \bar{S}^b V_{q_{\bar{\phi}} = \frac{1}{2}, \bar{q}_\phi = \frac{1}{2}, q_{\bar{\phi}} = \frac{1}{2}} F^- \right). $$

Here $F^+$ and $F^-$ are the self-dual and antiself-dual part of the field strength, which satisfy $F^\pm = \frac{1}{2}(F \pm i*F)$ and $*F^\pm = \mp iF$. For the vector multiplets in type IIB, the $F^+$ part of the vertex (C.4) contains an RR field $V_{q_\phi = 1/2, \bar{q}_{\bar{\phi}} = 1/2}$ related to the $(a,a)$ ring by spectral flow, whereas the $F^-$ part of the vertex (C.7) contains an RR field $V_{q_{\bar{\phi}} = -1/2, \bar{q}_{\bar{\phi}} = -1/2}$ related to the $(c,c)$ ring by spectral flow. These vertices correspond to the $s_0 = \pm 1$ states in the light-cone gauge respectively. For the graviphoton vertex the internal part is replaced by $V_{q_{\phi} = -3/2, \bar{q}_{\bar{\phi}} = -3/2}$ and $V_{q_{\phi} = 3/2, \bar{q}_{\bar{\phi}} = 3/2}$ respectively which are related to the identity by spectral flow. Because of the superghost anomaly of the disk, the RR states which couple to D-branes are not the ones described above but are in an asymmetric picture. This fact makes it possible that D-branes couple minimally to the appropriate RR-fields [11][1].

The vertex operator in the $(-3/2, -1/2)$ picture is given by

$$ W_0^\pm(A) = e^{-\frac{3}{2}\phi - \frac{1}{2}\bar{\phi}} e^{ikX} \left( S^a \sigma_{ab} \bar{S}^b V_{q_\phi = \frac{1}{2}, \bar{q}_{\bar{\phi}} = \frac{1}{2}} \pm S^{\dot{a}} \tilde{\sigma}_{\dot{a}b} \bar{S}^b V_{q_{\bar{\phi}} = \frac{1}{2}, \bar{q}_\phi = \frac{1}{2}} \right) A_\mu. $$

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The sign will later be seen to correspond to an electric or magnetic vertex. The picture changing operator mapping \( V_s \to V_{s+1} \) is defined by
\[
V_{s+1}(w) = \{Q_{BRST}, \xi V_s(w)\},
\]
(C.6)
where the BRST charge is \( Q_{BRST} = Q_0 + Q_1 + Q_2 \) and the three distinct contributions to the charge are given by
\[
Q_0 = \int dz \, c(T_{ext} + T_{int} + T_{gh}), \quad Q_1 = \frac{1}{2} \int dz \, e^\phi \eta (\psi^\mu \partial X_\mu + G_{int}),
\]
Q_2 = \frac{1}{4} \int dz \, b \eta \partial \eta e^{2\phi}.
\]
(C.7)

On the vertex operators of the form (C.5) (i.e., \( s = -3/2 \)), the picture changing operation effectively reduces to
\[
V_{s+1}(w) = \lim_{z \to w} e^\phi G(z) V_s(w),
\]
(C.8)
where \( G = \psi^\mu \partial X_\mu + G_{int} \) and \( G_{int} \) is the internal \( N = 1 \) supercurrent defined by \( G = G^+ + G^- \) in terms of the two \( N = 2 \) superconformal currents. A simple observation which will be very important in the following is that since \( V_{q=\pm 1/2, \bar{q}=\pm 1/2} \) define Ramond/Ramond ground states in the internal CFT and \( V_{q=\pm 3/2, \bar{q}=\pm 3/2} \) are the internal parts of the spacetime supercharges, we have the OPE \( G(z) V_{q, \bar{q}}(w) \) which is at most \( O((z-w)^{-1/2}) \). This implies that in the picture changing operation only the external part of (C.6) will be important. Picture changing the vertex (C.3) gives
\[
\lim_{z \to w} e^\phi G(z) W^\pm_0 = e^{-\frac{1}{2}(\phi + \bar{\phi})} e^{ikX} \left( S^a_{\sigma\mu\nu} \bar{S}^b V_{q=\frac{1}{2}, \bar{q}=\frac{1}{2}} F^+_{\mu\nu} \pm S^a_{\sigma\mu\nu} \bar{S}^b V_{q=-\frac{1}{2}, \bar{q}=-\frac{1}{2}} F^-_{\mu\nu} \right),
\]
(C.9)
where the following OPE's were used
\[
\psi(z) S^a_\mu(w) \sim \frac{1}{(z-w)^{1/2}} \sigma^a_{\mu\nu} S_b(w), \quad \partial X_\mu(z) e^{ikX}(w) \sim \frac{-ik^\mu}{(z-w)} e^{ikX}(w),
\]
(C.10)
\[
e^{\phi(z)} e^{-\frac{3}{2} \phi(w)} \sim (z-w)^{3/2} e^{-\frac{1}{2} \phi(w)}.
\]

To derive (C.9) we have further to impose the Lorentz gauge condition \( k_\mu A^\mu = 0 \) on the gauge potential. Note that for the picture changed vertex operator the choice of sign in (C.5) implies that the picture changed vertex \( W^+_0 \) turns into \( V(F) \) with \( dA = F \) whereas \( W^-_0 \) turns into \( iV(*F) \) with \( dA = *F \). If we define the vertex \( W^+_0(A^e) \) with electric variables, then the vertex \( W^-_0(A^m) \) corresponds to the magnetic dual variables.

Note that the vertex (C.5) in the asymmetric picture is not BRST invariant as it stands. Due to the comment regarding the action of \( G_{int} \) above, the resolution of this fact is exactly along the lines of the ten-dimensional case discussed in [20]. The basic idea of [26] is that \( W_0 \) fails to be BRST invariant because of \( Q_1 \). A new vertex is then defined as \( W = \sum_{n=0}^\infty W^{(n)} \) such that \( [Q_1, W^{(n)}] + [Q_1, W^{(n+1)}] = 0 \) for \( n = 0, 1, \ldots \), and hence \( [Q_1 + Q_1, W] = 0 \). For this to work one has to impose the Lorentz gauge condition \( k_\mu A^\mu = 0 \). Note that under picture changing all the \( W^{(n)} \) with \( n > 0 \) vanish.
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