Tuning of few-electron states and optical absorption anisotropy in GaAs quantum rings

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The electronic and optical properties of a GaAs quantum ring (QR) with few electrons in the presence of the Rashba spin-orbit interaction (RSOI) and the Dresselhaus spin-orbit interaction (DSOI) have been investigated theoretically. Configuration interaction (CI) method is employed to calculate the eigenvalues and eigenstates of the multiple-electron QR accurately. Our numerical results demonstrate that the symmetry breaking induced by the RSOI and DSOI leads to an anisotropic distribution of multi-electron states. The Coulomb interaction offers additional modulation of the electron distribution and thus the optical absorption indices in the quantum rings. By tuning the magnetic/electric fields and/or electron numbers in a quantum ring, one can change its optical property significantly. Our theory provides a new way to control multi-electron states and optical properties of QR by hybrid modulations or by electrical means only.

I. INTRODUCTION

Recently all-electrical control of spin states have attracted extensive research interest in the quest for spintronics and quantum information processing with high flexibility and controllability.$^1$–$^5$ The Rashba and Dresselhaus spin-orbit interactions$^6$–$^8$ (SOI) provides us with an efficient way to control spin which has become one of the most influential concepts in semiconductor spintronics since they are electrically tunable.$^9$–$^{10}$ In the past few years, semiconductor quantum rings (QRs) have attracted intensive interests due to their unique topological geometry and energy spectrum,$^{11}$–$^{15}$ which opens new opportunities for realizing novel nano-scaled photonic detectors and sources, charge/spin memory cell, et.al. Especially, The QRs behave like giant artificial atoms in a dielectric cage and show a multi-energy-level system in the presence of magnetic fields and/or SOI, thus presenting a potential source of qubit in quantum computing. Higher spin stability in QRs than in quantum dots, another candidate for qubit, make the relaxation and decoherence processes take place in the time scale that is sufficient long for spin manipulations.$^{16}$–$^{18}$ This is very attractive for the realization of spin qubits and throughout understanding of the multi-electron spin states in QRs is required. State-of-the-art growth, etching and gate techniques have made it possible to fabricate high-quality QRs, and control the number of electrons in a QR exactly.$^{19}$–$^{22}$ The ring-shaped charge distributions can also be realized in gate-all-around nanowire or core-shell nanowire arising from potential or geometrical confinement respectively.$^{23}$–$^{27}$ The interplay between the RSOI and DSOI results in a periodic potential in an isolated QR that breaks the rotational symmetry, produces gaps in the energy spectrum and suppresses the persistent currents.$^{28}$–$^{31}$ However, in these studies, one-electron models with effective mass approximation were used, without taking into account the many-body effects. In few-electron quantum confined system, the many-body effects and Coulomb interactions affect the electronic states significantly, which have attracted considerable interests.$^{32}$–$^{37}$ So far the hybrid impacts of the RSOI, DSOI and the Coulomb interactions on the electronic and optical properties in a QR with few electrons have not been thoroughly investigated. Especially the competition between the spin-orbit interactions and the Coulomb interactions with increasing electron number in a QR from one to a relatively large number need to be discussed systematically.

In this study, we investigate theoretically the multi-electron states in GaAs QRs in the presence of the SOIs and perpendicular magnetic/electric fields. Beyond the effective one electron models on similar systems, we employ the configuration interaction (CI) method, which is numerically exact in this complex few electron system. The QR in the presence of the RSOI and DSOI behaves like two quantum dots coupled laterally along specific crystallographic direction, i.e., [110] or [1T0]. The interdot coupling can be tuned by changing the

\[ \text{FIG. 1: (Color online) Schematic diagram of a considered multi-electron QR in (001)-oriented GaAs/AlGaAs quantum well. The x (y) axis is directed along the [100] ([010]) crystallographic orientation respectively. In this calculation, we consider an ideal one-dimensional QR ignoring the finite-width impacts and the coupling to the extended reservoirs.} \]
strengths of the SOIs. Interestingly, this anisotropic electron distribution, which can be rotate from [110] to [1̅10] direction by reversing the direction of the perpendicular electric field, results in anisotropic optical properties that provide us with a possible way to detect it experimentally.

II. MODEL AND FORMULISM

A. SINGLE-ELECTRON HAMILTONIAN

We consider a GaAs QR which can be fabricated in (001)-oriented symmetrical GaAs/AlGaAs quantum well as shown in Fig. 1. With coordinate axes directed along cubic crystallographic axes, i.e., x||[100] direction and y||[010] direction, the single electron Hamiltonian with both RSOI and DSOI terms in a finite-width QR under a perpendicular magnetic field is given by,38-41

\[ H = \frac{\hbar^2 k^2}{2m^*} + \alpha \left( \sigma_x k_y - \sigma_y k_x \right) + \beta \left( \sigma_y k_z - \sigma_z k_y \right) + \frac{1}{2} g' \mu_B B r \sigma_z + V(r), \]

where \( \vec{k} = -i \nabla + e\vec{A}/\hbar \). \( \vec{A}(r) = B/2(\gamma, x, 0) \) is the vector potential. \( m^* \) is the electron effective mass. The fourth term describes the Zeeman splitting with Bohr magnetron \( \mu_B = e\hbar/2m_0 \) and the effective g factor \( g' \). \( \sigma_x, \sigma_y, \sigma_z \) are the Pauli matrices. \( \alpha(\beta) \) specify the RSOI (DSOI) strengths. To include the RSOI and DSOI, an additional up-asymmetry need to be present by, e.g., electric field applied normally to QR plane. Consequently the rotation symmetry of the system is reduced to \( C_{2v} \). \( V(r) \) is the radial confining potential. We adopt hard-wall boundary conditions which can be guaranteed by the strong confinement potential,

\[ V(r) = \begin{cases} 0, & r = R \\ \infty, & \text{otherwise} \end{cases} \]

where \( R \) is radii of the quantum ring. In the presence of both RSOI and DSOI, the single-electron dimensionless 1D Hamiltonian is written as,39,40

\[ H_e = -\frac{i}{\hbar} \frac{\partial}{\partial \varphi} + \frac{\Phi}{\Phi_0} + \frac{\sigma_x}{2} \sin \varphi + \frac{\sigma_y \sin \varphi \sigma_z}{4} + \frac{\sigma_y}{2} \sin 2\varphi + \frac{1}{2} g' \mu_B \sigma_z, \]

where \( \sigma_x, \sigma_y, \sigma_z \) are Pauli matrices, \( \Phi = B R^2 \) is the magnetic flux threading the ring, \( \Phi_0 = \hbar/e \) is the flux unit, \( b = e\hbar B/m^* E_0 = 4\Phi/\Phi_0 \) is the dimensionless magnetic field, \( \sigma_y \mu_B = \alpha(\beta)/E_0 R \) specifies the dimensionless RSOI (DSOI) strength, \( E_0 = \hbar^2/2m^* R^2 \) with the ring radius \( R \), and \( g' = g^* m^*/2m_0 \) is the dimensionless g factor.

B. FEW-ELECTRON HAMILTONIAN

The total Hamiltonian of the multi-electron QR can be rewritten in second-quantization

\[ H = \sum_i E_n a_i^+ a_i + \frac{1}{2} g \sum_{i,j} \langle ij | U | ij \rangle a_i^+ a_j^+ a_i a_j, \]

where \( | ij \rangle = a_i^+ a_j^+ | 0 \rangle \), \( a_i^+ (a_i) \) is the electron creation (annihilation) operator of the states. \( i \) and \( j \) denoting the ith and jth single-electron energy states, respectively, \( E_i \) is the energy of the \( i \)-th single electron level, which can be obtained numerically by solving the single electron Schrödinger equation shown in Eq. 3. The parameter \( g = \epsilon^2 / 4 \pi \alpha_0 \epsilon_0 \) and \( U = 1/r \), where \( r \) is the electron-electron distance.

The full configuration interaction (CI) method has been widely used to solve the many-body, non-relativistic Schrödinger equation.42-47 Rather than the ab-initio approaches.48 We adopt the CI method with employing adequate single-particle electron states, to calculate the eigenvalues and eigenstates of the above Hamiltonian. The total wavefunction can be expanded as \( | \psi \rangle = \sum C_i | i \rangle \). The two electrons configuration is \( | i \rangle = | \cdots 01,0 \cdots 0 \rangle = a_i^+ a_j^+ | 0 \rangle \) with \( i < j \), (or \( | i \rangle = | \cdots 01,0 \cdots 0 \rangle = a_i^+ a_j^+ | 0 \rangle \) with \( p < q \). Here \( | 0 \rangle \) represents the vacuum state and \( a_i^+ (a_i) \) is the electron creation (annihilation) operator of the states. \( i \) and \( j \) denoting the \( i \)-th and \( j \)-th single electron energy states, respectively. The matrix element of the total Hamiltonian for two electrons can be calculated

\[ \langle i | H | j \rangle = (E_i + E_j) \delta_{i,j} - \gamma \langle ij | U | pq \rangle - \langle ij | U | qp \rangle. \]

The N-electron configuration is \( | M \rangle = \left( a_{i_1}^+ \right)^{n_1} \left( a_{i_2}^+ \right)^{n_2} \cdots | 0 \rangle \), where \( n_1, n_2, \cdots \) are either 1 or 0, and satisfied \( \sum n_i = N \). Then the matrix element of the total Hamiltonian for \( N \) electrons is given by,

\[ \langle M | H | M' \rangle = \langle E_{i_1} + E_{i_2} + E_{i_3} + \cdots + E_{i_N} \rangle \delta_{M,i'} + \sum_{q=1}^{Q} \left( \gamma m_{dq} m_{eq} | U | m_{cq} m_{bq} \right) \]

\[ \times (-1)^{n_b+q+n_{a+q}}, \]

where \( \pi_i \) identify the position of nonzero \( n_{i_1} \), \( \delta_{M,i'} = \delta_{M,M'} \). The factor \( (-1)^{n_b+q+n_{a+q}} \) comes from moving two electrons from states \( m_{aq}, m_{bq} \) to \( m_{cg}, m_{dg} \). \( n_{a+q} = d_q - b_q + 1 \) is the number of electrons in the states \( m_a \) and \( m_b \) respectively. The number \( Q \) obeys the following rules: (1) If there are four \( m_i \) differing between \( | M \rangle \) and \( | M' \rangle \), there is only one choice, \( Q = 1 \). (2) If only two \( m_i \) differing between \( | M \rangle \) and \( | M' \rangle \), \( Q = N - 1 \). (3) If \( | M \rangle = | M' \rangle \), \( Q = N(N - 1)/2 \). Solving the above secular equation, we
can obtain the eigenenergies and the eigenstates of the multi-electron system, and thus calculate the electron distributions, and optical property of the multi-electron QRs.

C. OPTICAL ABSORPTION

The optical absorption rate is obtained within the electric-dipole approximation

$$W = \frac{2\pi}{\hbar} \sum_j |(f|\hat{H}_{ep}|i)|^2 \delta(E_f - E_i)$$

$$= \frac{2\pi}{\hbar} \sum_{j,i} \frac{eA_0}{m^*} |(f|\tilde{\epsilon}_i \cdot (\vec{p}_1 + \vec{p}_2)|i)|^2$$

$$\times \delta(E_f - E_i - \hbar\omega),$$

where $E_i$ and $E_f$ are the energies of the initial and final states, respectively. $H_{ep} = (e/m^*) (\vec{A} \cdot \vec{p}_1 + \vec{A} \cdot \vec{p}_2)$, $\vec{p}_1, \vec{p}_2$ are the canonical momenta of the electrons and $\vec{A} = \sum_k A_0 \tilde{\epsilon}_i (a_{k\lambda} e^{i(kx + \omega t - \vec{k} \cdot \vec{r})} + a_{k\lambda}^* e^{i(kx + \omega t + \vec{k} \cdot \vec{r})})$ in which $a_{k\lambda}$, $a_{k\lambda}^*$ are photon annihilation and creation operators, respectively. $A_0$, $\omega$ and $\tilde{\epsilon}_i$ are the amplitude, frequency and polarization vector of the incident linear-polarized light. In the calculation we replace the energy delta function $\delta(E_{12} - \hbar\omega)$ with a Lorentz broadened function $(\Gamma/\pi)/((\hbar\omega - E_{12})^2 + \Gamma^2$ where $\Gamma$ is the broadening parameter describing the homogeneous broadening of the energy levels in the ring. Then we can obtain the experimentally measurable absorption index $A$, which is defined by the following light damping equation,

$$F(x) = F_0 e^{-Ax},$$

where $F_0$ is the flux at $x = 0$. The energy flux equation is $
abla \cdot \vec{F} + \frac{\partial n}{\partial t} = 0$, and the flux can also be written as $\vec{F} = \nu \vec{v} n_\perp$, here $\nu = c/\eta$ is the speed of energy flux and $\eta$ is the refraction index of the medium. Thus the absorption index $A$ can be written as,

$$A = \frac{1}{F} \frac{dn_\perp}{dt} = \frac{W}{\nu \vec{v} n_\perp}$$

$$= \frac{2\pi}{\hbar} \frac{(eA_0)}{m^*} \sum_{j,i} \left| \frac{(f|\tilde{\epsilon}_i \cdot (\vec{p}_1 + \vec{p}_2)|i)}{c/\eta} \right|^2 \delta(E_f - \hbar\omega)$$

III. RESULTS AND DISCUSSIONS

The accuracy of the calculated multi-electron energy spectrum depends on the number of possible many-particle configurations that are used which is determined by the number of electrons ($N_e = 3, 4, 5,$ and $6$ in this paper), and the number of single-particle states $N_S$. In the calculation we include 30 single-particle electron states to ensure that the lower multi-electron states are numerically accurate, for instance, the accuracy of the lowest levels can approach to $1.0 \times 10^{-4} \text{meV}$. For simplicity, all physical quantities are taken dimensionless, e.g., the length unit is the radius of the ring $R$, the energy unit is $E_0$ and the magnetic field unit is $b$. The relevant parameters for GaAs are: $\text{m}_e = 0.067 \text{m}_0$, and the dielectric constant $\varepsilon = 12.5$. For an example, for $R = 30 \text{nm}$ we find $E_0 = 0.633 \text{meV}$ and $b = 2.73$ when $B = 1 \text{T}$. In this study, we focus on the interplay between RSOI, DSOI and the Coulomb interactions with ignoring the Zeeman term.

![FIG. 2](image)

FIG. 2: (Color online)The energy spectrum for 1D GaAs rings without Coumb interaction and SOIs for (a) $N_e = 3$, (b) $N_e = 4$, (c) $N_e = 5$, and (d) $N_e = 6$, respectively.

First we consider three electrons in a 1D GaAs ring. The electron states are obtained by coupling the well known singlet and triplet two-electron states with a third electron. There are total of $2^3 = 8$ spin states including one set of quartet states and two sets of doublet states that involve two pairs of total spin 1/2 states built up of a singlet and an unpaired spin. The symmetries of both the spatial wavefunctions and the spin states which finally make an antisymmetric wavefunction as requested by the Pauli principle. In Fig. 2 we plot the low energy dispersion relations of a 1D GaAs ring with $N_e = 3, 4, 5, 6$ electrons without considering the SOIs or the Coulomb interactions. For a QR with three electrons but without Coumb interactions and SOIs, we plot the lowest few energy levels versus a perpendicular magnetic field in Fig. 2(a). One can find that the ground energy level lies in the degenerated doublet states in the absence of an external perpendicular magnetic field. The first exited energy level lies in the quartet states. As the magnetic field increases, the degeneracy is lifted due to the effect of the magnetic field on Landau level. The energy spectra exhibit a periodic dependence on the dimensionless magnetic field $b$ accounting for the moving of Landau levels. For a QR including more than three electrons, similar low-energy spectra are obtained following the same considerations. Interestingly we find that the parabolic and crossing or anticrossing features in energy levels of are similar for set of (1, 3, 5) electrons and the other set of (2, 4, 6) electrons alternatively (see Fig. 2 and Refs. 39, 40 for a QR with 1 or 2 electrons). The two sets of multi-electron QRs arising
from the alternatively changed basis configurations given in Table I.

The SOIs and Coulomb interactions play important roles in the multi-electron system. We also start by exploring the energy spectra with three electrons in a 1D GaAs ring. When the SOIs are induced, the eigenvalues of spin operator $S^2$ are no longer good quantum numbers, the singlet and triplet states are mixed due to spin-orbit coupling. The energy levels of the three-electron QR get lower, and the spin degenerate is lifted as shown in Fig. 3(b). Since the SOIs reduce the total energy of the three-electron system as determined by the Hamiltonian. The RSOI and DSOI with different strengths in our calculations, i.e., $\overline{\sigma} = 2.0, \overline{\beta} = 1.0$, break the symmetry of the 1D Hamiltonian. Then the spin-up and spin-down electron energy levels with the same quantum number are separated. For comparison, the Coulomb interactions are considered without inducing SOIs, the energy levels of the three-electron QR are increased significantly, since the Coulomb interactions increase the repulsive energy. Importantly the Coulomb exchange interactions give rise to the splitting of the triplet states $S_z = 0$ and $S_z = \pm 1$ and the coupling of the doublet and quartet states. We therefore observe complex crossings and anticrossings in the energy spectrum as shown in Fig. 3(c). The interplay of both Coulomb interaction and the SOIs destroy the parabolic energy dispersions and lift the spin degeneracy in the three-electron QR due to the competition between the two kinds of interactions, resulting in very complicated energy spectra as shown in Fig. 3(d). For a QR with more than three electrons, above considerations are still valid. Accounting for the configuration of singlet, triplet, and quintet states instead of doublet and quartet states, the feature of energy spectrum is modulated by adding the fourth electron in the QR as shown in Fig. 3(e-h). The oscillations length $b$ is doubled, this can be readily explained in terms of the occupation of an even or odd number of single-electron levels.

Next we investigate the electron distributions in the GaAs QR and the dominating physical mechanism. Fig. 4(a)-(d) show the exclusive or hybrid impacts of Coulomb interaction and SOIs on the electron distributions in the QR containing three, four, five and six electrons respectively. The Coulomb repulsion makes the multiple electrons tend to avoid each other and localize equally separated in the ring but at any crystallographic direction as indicated by blue lines in Fig. 4. The interplay between the RSOI and DSOI breaks the rotational symmetry and results in an azimuthal periodic potential, whose height is determined by the product of the strengths of RSOI and DSOI in Eq. 3. From this Hamiltonian, one can see clearly that the QR with RSOI and DSOI does not possess the rotational symmetry, because there are two potential wells at $\varphi = 3\pi/4$ and $\varphi = -\pi/4$. Consequently, it leads to an azimuthal anisotropic electron distribution. The electron distribution of the ground state shows a barbell-like shape along the specific crystallographic directions at $\varphi = 3\pi/4$ and $\varphi = -\pi/4$. When both Coulomb interaction and SOIs are incorporated in Eq. 3, the competition between the two type interactions gives rise to complicated electron distribution which differs when adding more electrons and/or applying a finite perpendicular magnetic field. The QR behaves like a laterally-coupled double quantum dots. In the following we illustrate

TABLE I: Basis electronic configurations of 2 to 6 identical electrons in a QR.

|        | $S = 0$ | $S = 1$ |
|--------|---------|---------|
| Two    | Singlet| Triplet |
| Three  | $S = 1/2$ | $S = 3/2$ |
| Four   | $S = 1$ | $S = 2$ |
| Five   | $S = 1/2$ | $S = 3/2$ | $S = 5/2$ |
| Six    | $S = 0$ | $S = 1$ | $S = 2$ |

FIG. 3: (Color online) The energy spectrum for 1D GaAs rings, (a) $N_e = 3$ electrons without Coulomb interaction and SOIs, (b) $N_e = 3$ electrons without Coulomb interaction but with SOIs, (c) $N_e = 3$ electrons with Coulomb interaction but without SOIs, (d) and $N_e = 3$ electrons with both Coulomb interaction and SOIs. (e)-(h) The same with (a)-(d) but with $N_e = 4$ electrons. $\overline{\sigma} = 2.0, \overline{\beta} = 1.0$. |
The behavior of electron distribution is very different with increasing magnetic field \( b \) for different electron numbers. This is caused by the differences of overlap between electrons of each single electron distribution. For three-electron case there are two kinds of possible single electron distribution: (i) two electrons are localized at \( \varphi = \frac{3\pi}{4} \) or \( \varphi = -\frac{\pi}{4} \), while the third electron localizes at the opposite side, i.e., \( \varphi = -\frac{\pi}{4} \) or \( \varphi = \frac{3\pi}{4} \); (ii) all three electrons localize at \( \varphi = \frac{3\pi}{4} \) or \( \varphi = -\frac{\pi}{4} \). Due to the repulsive Coulomb interaction, this configuration has higher energy, therefore the ground state of three-electron case prefers a triangular configuration. This is because the azimuthal confining potential (the third term in Hamiltonian (1)) squeezes the electron wavefunctions and forces them align along the specific crystallographic direction, i.e., \([1\bar{1}0]\) (see Fig. 5(a)), while the repulsive Coulomb interaction pushes two electrons away from this direction. Therefore one can see clearly that the peaks of electron distribution at \( \varphi = \frac{3\pi}{4} \) or \( \varphi = -\frac{\pi}{4} \) become broadening comparing with the two-electron case (see Fig. 7 in Ref. [40]). For four and five electron QR they both have three kinds of possible single electron distribution which give more complicated features shown in Figs. 5(b) and 5(c). However, the six-electron QR shows in Fig. 5(d) releases a feature very similar with the two-electron’s. This is because the repulsive Coulomb interaction makes the ground state of six-electron case prefer a bar-bell like configuration, i.e., three electrons are localized at \( \varphi = \frac{3\pi}{4} \) or \( \varphi = -\frac{\pi}{4} \), while the other three localize at the opposite side. In this case the three electrons are too close and therefore the density distribution (see the color scales of Fig. 5). We can switch the potential minima from \([110]\) to \([1\bar{1}0]\) easily by reversing the direction of the perpendicular electric field, i.e., \( \phi \to -\phi \). The orientation of the electron distribution can be switched from \([1\bar{1}0]\) to \([110]\). Besides, by tuning the external magnetic field \( b \), we can tune the shape of the electron distribution. Thus, it provides a method to control the electron state by using SOIs and the external magnetic field.

However, the electron states and electron distributions are difficult to measure directly. We therefore propose an optical measurement method to detect the multi-electron spatial anisotropic distribution by monitoring the optical absorption in the infrared regime. The anisotropic electron distribution and the corresponding overlap factor between the ground and first excited states gives rise to anisotropic absorption. We consider a beam of linear-polarized light incident along the \( z \) axis and calculate the optical absorption at zero magnetic field. The optical absorption oscillates periodically which is consistent with the electron distributions accounting for their dependence on crystallographic angle \( \theta \) that shown in Fig. 5. As we observed before, the RSOI and DSOI and the Coulomb repulsion between these electrons make the localized electron distribution in the ring along

\[ \sum_{i<j} C_{ij} + \sum_{i<j<k} C_{ijk} \]

is caused by the interplay between the RSOI and DSOI which breaks the rotational symmetry and results in an azimuthal periodic potential. The behavior of electron distribution is very different with increasing magnetic field \( b \) for different electron numbers. This is caused by the differences of overlap between electrons of each single electron distribution. For three-electron case there are two kinds of possible single electron distribution: (i) two electrons are localized at \( \varphi = \frac{3\pi}{4} \) or \( \varphi = -\frac{\pi}{4} \), while the third electron localizes at the opposite side, i.e., \( \varphi = -\frac{\pi}{4} \) or \( \varphi = \frac{3\pi}{4} \); (ii) all three electrons localize at \( \varphi = \frac{3\pi}{4} \) or \( \varphi = -\frac{\pi}{4} \). Due to the repulsive Coulomb interaction, this configuration has higher energy, therefore the ground state of three-electron case prefers a triangular configuration. This is because the azimuthal confining potential (the third term in Hamiltonian (1)) squeezes the electron wavefunctions and forces them align along the specific crystallographic direction, i.e., \([1\bar{1}0]\) (see Fig. 5(a)), while the repulsive Coulomb interaction pushes two electrons away from this direction. Therefore one can see clearly that the peaks of electron distribution at \( \varphi = \frac{3\pi}{4} \) or \( \varphi = -\frac{\pi}{4} \) become broadening comparing with the two-electron case (see Fig. 7 in Ref. [40]). For four and five electron QR they both have three kinds of possible single electron distribution which give more complicated features shown in Figs. 5(b) and 5(c). However, the six-electron QR shows in Fig. 5(d) releases a feature very similar with the two-electron’s. This is because the repulsive Coulomb interaction makes the ground state of six-electron case prefer a bar-bell like configuration, i.e., three electrons are localized at \( \varphi = \frac{3\pi}{4} \) or \( \varphi = -\frac{\pi}{4} \), while the other three localize at the opposite side. In this case the three electrons are too close and therefore the density distribution (see the color scales of Fig. 5). We can switch the potential minima from \([110]\) to \([1\bar{1}0]\) easily by reversing the direction of the perpendicular electric field, i.e., \( \phi \to -\phi \). The orientation of the electron distribution can be switched from \([1\bar{1}0]\) to \([110]\). Besides, by tuning the external magnetic field \( b \), we can tune the shape of the electron distribution. Thus, it provides a method to control the electron state by using SOIs and the external magnetic field.

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\[ \sum_{i<j} C_{ij} + \sum_{i<j<k} C_{ijk} \]
This distinct variation provides us an efficient tool to detect the anisotropy in the electron distribution.

IV. CONCLUSIONS

In summary, we have studied multi-electron energy spectra in a GaAs QR by using the CI method. In our calculation, the Coulomb interaction, Rashba SOI and Dresselhaus SOI are explicitly included. We demonstrate theoretically the anisotropic distribution of multi-electron states in a semiconductor quantum ring. Our numerical results illustrate that the energy spectra and the charge distributions are controlled by the interplay between the Rashba SOI, Dresselhaus SOI and the Coulomb interaction in the presence of perpendicular magnetic/electric fields. We propose a possible experimental manifestation that the anisotropy in multi-electron QR can be switched by reversing the direction of the perpendicular electric field and be detected by the optical absorption measurement.

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