Infrared-finite factorization and renormalization scheme for exclusive processes. Application to pion form factors

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Abstract

We develop and discuss an infrared-finite factorization and optimized renormalization scheme for calculating exclusive processes which enables the inclusion of transverse degrees of freedom without entailing suppression of calculated observables, like form factors. This is achieved by employing an analytic, i.e., infrared stable, effective coupling $\alpha_s(Q^2)$ which removes the Landau singularity at $Q^2 = \Lambda_{\text{QCD}}^2$ by a power-behaved correction. The ensuing contributions to the cusp anomalous dimension, related to the Sudakov form factor, and to the quark anomalous dimension, which controls evolution, lead to enhancement of the hard part of exclusive amplitudes, calculated in perturbative QCD. The phenomenological implications of this framework are analyzed by applying it to the pion’s electromagnetic form factor and the pion-photon transition.

11.10.Hi, 12.38.Bx, 12.38.Cy, 13.40.Hq, 13.40.Gp
1. Introduction

In the last few years, several works \([1,2,3,4,5]\) (among many others) addressed the possibility of power corrections to the strong running coupling, beyond the operator product expansion. Such corrections, which are subleading in the ultraviolet (UV) region, correspond to nonanalytical contributions to the $\beta$-function.

The existence of these power corrections, if proven true, would greatly affect our understanding of nonperturbative QCD effects. For instance, a power correction to $\alpha_s$ gives rise to a linear term in the interquark static potential \([6]\). On a more speculative level, one may argue \([4]\) that the source of such terms are small-size fluctuations in the nonperturbative QCD vacuum, perhaps related to magnetic monopoles in dual QCD or nonlocal condensates. Besides, and in practice, a power-behaved contribution at low scales can be used to remove the Landau singularity, present in perturbation theory, supplying in this way an infrared (IR) stable effective coupling \([2,3,5]\).

The aim of the present work is to develop in detail a factorization and renormalization scheme, which self-consistently incorporates such a nonperturbative power correction, and then use it to assess and explore exclusive processes. We do not, however, propose to involve ourselves in the discussion of whether or not such power corrections have a fundamental justification. We consider the ambiguity in removing the Landau pole as resembling the ambiguity in adopting a particular (non-IR-finite) renormalization scheme in perturbative QCD. The justification for such a procedure will be supplied a posteriori by the self-consistent incorporation of higher-order perturbative corrections and by removing any IR-sensitivity of calculated hadronic observables.

A key ingredient of our approach is that the modified effective coupling will be taken into account not only in the factorized short-distance part, i.e., through the fixed-order perturbation expansion, but also in the resummed perturbative expression for soft-gluon emission and in the renormalization-group controlled evolution of the factorized parts.

To this end, we adopt as a concrete power-corrected effective coupling, an analytic model for $\alpha_s$, proposed by Shirkov and Solovtsov \([3]\), which yields an IR-finite and universal effective (running) coupling. This model combines Lehmann analyticity with the renormalization group to remove the Landau singularity at $Q^2 = \Lambda^2_{\text{QCD}}$, without employing adjustable parameters, just by modifying the logarithmic behavior of $\alpha_s$ by a simple (nonperturbative) power term (minimality of the model).

At the present stage of evidence, it would be, however, premature to exclude other parametrizations, since neither the sign of the power correction nor the size of its exponent are established, and one could introduce further modifications \([3,7]\).

Continuing our previous exploratory study \([8]\), we further extend and test our theoretical framework by also including into the calculation of the pion form factor the next-to-leading order (NLO) perturbative contribution to the hard scattering amplitude (see, e.g., \([9]\) and earlier references cited therein).

The ultimate goal of the present analysis is not to obtain results in perfect agreement with the data, but to expose and discuss the conceptual advantages of our scheme relative to previous conventional approaches \([10,11,12]\).

The major advantage of such a theoretical framework, the object of this paper, is that it enables the inclusion of transverse degrees of freedom, primordial (i.e., intrinsic) \([1]\) and
those originating from (soft) gluonic radiative corrections \[10\], without entailing suppression of perturbatively calculated observables, viz., the pion form factor. This enhancement is due to power-term generated contributions to the anomalous dimensions of the cusped Wilson line, related to the Sudakov form factor, and such to the quark wave function which governs evolution.

Although most of our considerations refer to the pion as a case study for the proposed framework, the reasoning can be extended to describe three-quark systems as well. This will be reported elsewhere.

The outline of the paper is as follows. In the next section we briefly discuss the essential features of the IR-finite effective coupling. In sect. 3 we develop and present our theoretical scheme. Sect. 4 extends the method to the next-to-leading order contribution to the hard-scattering amplitude. In sect. 5 we discuss the numerical analysis of the electromagnetic pion form factor, revolving around the appropriate kinematic cuts in the evaluation of the IR-modified Sudakov form factor which comprises additional nonleading contributions. We also provide arguments for the appropriate choice of the renormalization scale. In this section we also show the theoretical prediction for the pion-gamma transition form factor, derived with our theoretical framework, as an independent justification of the approach. Finally, in sect. 6 we summarize our results and draw our conclusions.

2. Model for QCD effective coupling

The key element of the analytic approach of Shirkov and Solovtsov is that it combines a dispersion-relation approach, based on local duality, with the renormalization group (RG) to bridge the regions of small and large momenta, providing universality at low scales. The approach is an extension to QCD of a method originally formulated for QED \[13\].

At the one-loop level, the ghost singularity is removed by a simple power correction and the effective coupling reads

\[
\tilde{\alpha}_s^{(1)}(Q^2) = \frac{4\pi}{\beta_0} \left[ \frac{1}{\ln(Q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - Q^2} \right],
\]

where \(\Lambda \equiv \Lambda_{QCD}\) is the QCD scale parameter.

This model has the following interesting properties. It provides a nonperturbative regularization at low scales and leads to a universal value of the coupling constant at zero momentum \(\tilde{\alpha}_s^{(1)}(Q^2 = 0) = 4\pi/\beta_0 \approx 1.396\) (for three flavors), defined only by group constants. No adjustable parameters are involved and no implicit “freezing”, i.e., no saturation hypothesis of the coupling constant is invoked.

Note that this limiting value (i) does not depend on the scale parameter \(\Lambda\) – this being a consequence of RG invariance – and (ii) extends to the two-loop order, i.e., \(\tilde{\alpha}_s^{(2)}(Q^2 = 0) = \tilde{\alpha}_s^{(1)}(Q^2 = 0) \equiv \tilde{\alpha}_s(Q^2 = 0)\). (In the following the bar is dropped.) Hence, in contrast to standard perturbation theory, the IR limit of the coupling constant is stable, i.e., does not depend on higher-order corrections and is therefore universal. As a result, the running coupling constant also shows IR stability. This is tightly connected to the nonperturbative contribution \(\propto \exp(-4\pi/\alpha\beta_0)\) which ensures analytic behavior in the IR domain by eliminating the ghost pole at \(Q^2 = \Lambda^2\), and extends to higher loop orders. Besides,
the stability in the UV domain is not changed relative to the conventional approach, and UV perturbation theory is preserved.

At very low-momentum values, say, below 1 GeV, \( \Lambda_{\text{QCD}} \) in this model deviates from that used in minimal subtraction schemes. However, since we are primarily interested in a region of momenta which is much larger than this scale, the role of this renormalization-scheme dependence is only marginal. In our investigation we use \( \Lambda_{\text{QCD}}^{\text{an}(n_f=3)} = 242 \text{ MeV} \) which corresponds to \( \Lambda_{\text{QCD}}^{\overline{\text{MS}}(n_f=3)} = 200 \text{ MeV} \).

The extension of the model to two-loop level is possible, though the corresponding expression is too complicated to be given explicitly [3]. An approximated formula with an inaccuracy less than 0.5% in the region \( 2.5 \Lambda < Q < 3.5 \Lambda \), and practically coinciding with the exact result for larger values of momenta, is provided by [3]

\[
\alpha_s^{(2)}(Q^2) = \frac{4\pi}{\beta_0} \left[ \frac{1}{\ln \frac{Q^2}{\Lambda^2} + \frac{\beta_1}{\beta_0} \ln \left( 1 + \frac{\beta_2}{\beta_1} \ln \frac{Q^2}{\Lambda^2} \right)} + \frac{1}{2} \frac{1}{1 - \frac{Q^2}{\Lambda^2}} - \frac{\Lambda^2}{Q^2} D_1 \right],
\]

where \( \beta_0 = 11 - \frac{2}{3} n_f = 9 \), \( \beta_1 = 102 - \frac{38}{3} n_f = 64 \), and \( D_1 = 0.035 \) for \( n_f = 3 \).

With experimental data at relatively low momentum-transfer values for most exclusive processes, reliable theoretical predictions based on perturbation theory are difficult to obtain. Both the unphysical Landau pole of \( \alpha_s \) and IR instability of the factorized short-distance part are affecting such calculations. It is precisely for these two reasons that the Shirkov-Solovtsov analytic approach to the QCD effective coupling can be profitably used for computing amplitudes describing exclusive processes, like form factors, [14,15,16]. The improvements are then: (i) First and foremost, the nonperturbatively generated power correction modifies the Sudakov form factor [17,18,19,20,21] via the cusp anomalous dimension [22], and changes also the evolution behavior of the soft and hard parts through the modified anomalous dimension of the quark wave function. This additional contribution to the cusp anomalous dimension is the source of the observed IR enhancement and helps taking into account nonperturbative corrections in the perturbative domain, thus improving the quality of the predictions. (ii) Factorization is ensured without invoking the additional assumption of “freezing” the coupling strength in the IR regime by introducing, for example, an (external) effective gluon mass to saturate color forces at large distances. (iii) The Sudakov form factor does not have to serve as an IR protector against \( \alpha_s \) singularities. Hence the extra constraint of using the maximum between the longitudinal and the transverse scale, as argument of \( \alpha_s \), proposed in [10] and used in subsequent works, becomes superfluous. (iv) The factorization and renormalization scheme we propose on that basis enables the optimization of the (arbitrary) constants which define the factorization and renormalization scales [17,18,23,24]. This becomes important when including higher-order perturbative corrections.

3. Infrared-finite factorization and renormalization scheme

Application of perturbative QCD is based on factorization, i.e., how a short-distance part can be isolated from the large-distance physics. But in order that observables calculated with perturbation theory are reliable, one must deal with basic problems, like the resummation of “soft” logarithms, IR sensitivity, and the factorization and renormalization scheme dependence.
It is one of the purposes of the present work to give a general and thorough investigation of such questions.

The object of our study is the electromagnetic pion’s form factor in the spacelike region, which can be expressed as the overlap of the corresponding wave functions between the initial and final pion states: \[ F_\pi (Q^2) = e_q \int_0^1 dx d^2k_\perp \psi_\pi^{\text{out}} (x, l_\perp) \psi_\pi^{\text{in}} (x, k_\perp) , \] (3)

where we have assumed dominance of the valence quark-antiquark state, with \( e_q \) denoting the charge of the struck quark, and where 

\[
l_\perp = \begin{cases} k_\perp + (1 - x)q_\perp, & \text{struck quark} \\ k_\perp - xq_\perp, & \text{spectators} \end{cases} \] (4)

The wave function \( \psi_\pi (x, k_\perp) \) is the amplitude for finding a parton in the valence state with longitudinal momentum fraction \( x \) and transverse momentum \( k_\perp \).

In order to apply a hard-scattering analysis, we dissect the pion wave function into a soft and a hard part with respect to a factorization scale \( \mu_F \), separating the perturbative from the nonperturbative regime, and write (in the light-cone gauge \( A^+ = 0 \))

\[
\psi_\pi (x, k_\perp) = \psi_\pi^{\text{soft}} (x, k_\perp) \theta \left( \mu_F^2 - k_\perp^2 \right) + \psi_\pi^{\text{hard}} (x, k_\perp) \theta \left( k_\perp^2 - \mu_F^2 \right) .
\] (5)

Then the large \( k_\perp \) tail can be extracted from the soft wave function via a single-gluon exchange kernel, encoded in the hard scattering amplitude \( T_H \), so that

\[
\psi_{\text{hard}} (x, k_\perp) = \int_0^1 dy \int d^2l_\perp T_H (x, y, l_\perp^2; \alpha_s (l_\perp^2)) \psi_{\text{soft}} (y, l_\perp) . \] (6)

As a result, the pion form factor in Eq. (3) is expressed in the factorized form

\[
F_\pi (Q^2) = \psi_\pi^{\text{out}} \otimes \psi_\pi^{\text{in}} + \psi_\pi^{\text{out}} \otimes [T_H \otimes \psi_\pi^{\text{in}}] + \left[ \psi_\pi^{\text{out}} \otimes T_H \right] \otimes \psi_\pi^{\text{in}} + \ldots,
\] (7)

where the symbol \( \otimes \) denotes convolution defined by Eq. (6). The first term in this expansion is the soft contribution to the form factor, with support in the low-momentum domain, that is not computable with perturbative methods. The second term represents the leading order (LO) hard contribution due to one-gluon exchange, whereas the last one gives the NLO correction. We will not attempt to calculate the first term here, but adopt for simplicity the result obtained by Kroll and coworkers in [26]. For other, more sophisticated, attempts to model the soft contribution to \( F_\pi (Q^2) \), we refer to [27,28].

We now employ a modified factorization prescription [10,11], which explicitly retains transverse degrees of freedom, and define (see for illustration fig. 4)

\[
\psi_\pi^{\text{hard}} = \psi_\pi^{\text{soft}} \left( k_\perp^2 \leq \frac{C_2^2}{b^2} \right) \exp \left[ -S \left( \frac{C_2^2}{b^2} \leq k_\perp^2 \leq C_2^2 \xi^2 Q^2 \right) \right] T_{\text{Hard}} \left( Q^2 \geq k_\perp^2 \geq \frac{C_2^2}{b^2} \right), \] (8)

with \( b \), the variable conjugate to \( k_\perp \), being the transverse distance between the quark and the antiquark in the pion valence Fock state. The Sudakov-type form factor \( \exp (-S) \) comprises
leading and next-to-leading logarithmic corrections, arising from soft and collinear gluons, and resums all large logarithms in the region where $k_\perp^2 \ll Q^2 \left[23,24,29\right]$. The presence of these logarithms results from the incomplete cancellation between soft-gluon bremsstrahlung and radiative corrections. It goes without saying that the function $\mathcal{S}$ includes anomalous-dimension contributions to match the change in the running coupling in a commensurate way with the changes of the renormalization scale (see below for more details).

Going over to the transverse configuration space, the pion form factor reads \cite{10}

$$F_\pi(Q^2) = \int_0^1 dx dy \int_{-\infty}^{\infty} \frac{d^2 b_\perp}{(4\pi)^2} \mathcal{P}_{\pi}^{\text{out}}(y, b, P'; C_1, C_2, C_4) T_H(x, y, b, Q; C_3, C_4) \times \mathcal{P}_{\pi}^{\text{in}}(x, b, P; C_1, C_2, C_4),$$

where the modified pion wave function is defined in terms of matrix elements, viz.,

$$\mathcal{P}_{\pi}(x, b, P, \mu) = \int |k_\perp| < \mu d^2 k_\perp e^{-i k_\perp \cdot b_\perp} \tilde{\mathcal{P}}_{\pi}(x, k_\perp, P)$$

$$= \int \frac{dz^-}{2\pi} e^{-i z^- P^+} \left< 0 \left| T \left( \bar{q}(0) \gamma^+ \gamma_5 q \left( 0, z^-, b_\perp \right) \right) \right| \pi(P) \right>_{A^+=0}$$

with $P^+ = Q/\sqrt{2} = P'^-, Q^2 = -(P' - P)^2$, whereas the dependence on the renormalization scale $\mu$ on the rhs of Eq. \cite{10} enters through the normalization scale of the current operator evaluated on the light cone. (Note that we set all light quark masses equal to zero and work in the chiral limit, i.e., $M_\pi = 0$.)

A few comments on the scales involved:

- The scale $C_3/b$ serves to separate perturbative from nonperturbative transverse distances (lower factorization scale). We assume that there exist some characteristic scale $b_{\text{nonp}}^{-1} \approx \langle k_\perp^2 \rangle^{1/2}/x(1 - x) \approx 0.5 \text{ GeV}$, related to the typical virtuality (off-shellness) of vacuum quarks. This scale should also provide the natural starting point for the evolution of the pion wave function. In the following, we match the nonperturbative scale $C_3/b$ with the scale $C_1/b$, where the resummation of soft gluons starts, i.e., we set $C_1 = C_3$. The lower boundary of the scale $C_1/b$ is set by $\Lambda_{\text{QCD}}$, though the results are not very sensitive to using a somewhat larger momentum scale, as we shall see later.
• The resummation range in the Sudakov form factor is limited from above by the scale $C_2 \xi Q$ (upper factorization scale). (Note that the constant $C_2$ here differs in notation by a factor of $\sqrt{2}$ relative to that used by Collins, Soper, and Sterman [23], i.e., $C_{2}^{\text{CSS}} = \sqrt{2}C_2$.) This scale may be thought of as being an UV-cutoff for the (soft) Sudakov form factor, and enables this way a RG-controlled scale dependence governed by appropriate anomalous dimensions within this subsector of the full theory.

• Analogously to these factorization scales, characterized by the constants $C_1, C_2,$ and $C_3,$ we have introduced an additional arbitrary constant $C_4$ to define the renormalization scale $C_4 f(x,y)Q = \mu_R$, which appears in the argument of the effective coupling $\alpha_s^{\text{an}}$. The effective coupling plays a dual role: it describes the strength of the interaction at short distances, and controls via the anomalous quark dimension the lower and upper boundaries, respectively, for the evolution of $T_H,$ and $P_\pi$ to the renormalization scale.

The appropriate choice of the unphysical and arbitrary constants $C_i$ will be discussed in our numerical analysis in sect. 5.

The ambiguities parametrized by the scheme constants $C_i$ emerge from the truncation of the perturbative series and would be absent if one would be able to derive all-order expressions in the coupling constant. In fact, the calculated (pion) form factor depends implicitly on both scales: the adopted renormalization scale via $\alpha_s$, and the particular factorization scheme through the anomalous dimensions. Since the latter also depend on $\alpha_s$, the factorization-scheme and the renormalization-scheme dependences are correlated. On the other hand, the physical form factor is independent of such artificial scales, and satisfies

$$\mu^d F_{\pi, \text{phys}}(Q^2) = 0,$$

for $\mu$ being any internal scale. Obviously, both scheme dependences should be treated simultaneously and be minimized in order to improve the self-consistency of the perturbative treatment. In order to render the perturbative prediction reliable, the parameters $C_i$ should be adjusted in such a way, as to minimize the influence of higher-order corrections, thus resolving the scheme ambiguity. However, in the present investigation we are not going to explicitly match the fixed-order NLO contributions with the corresponding terms in the resummed expression for the “soft” logarithms. This check will be included in future work.

In Eq. (9), $T_H$ is the amplitude for a quark and an antiquark to scatter collinearly via a series of hard-gluon exchanges, and, in LO perturbative expansion, it is given by

$$T_H(x, y, b, Q; \mu_R) = 8C_F \alpha_s^{\text{an}}(\mu_R^2)K_0(\sqrt{xy} bQ).$$

This result is related to the more familiar momentum-space expression

$$T_H(x, y, \mathbf{k}_\perp, \mathbf{l}_\perp, Q, \mu_R) = \frac{16\pi C_F \alpha_s(\mu_R^2)}{xyQ^2 + (\mathbf{k}_\perp + \mathbf{l}_\perp)^2},$$

via the Fourier transformation

$$T_H(x, y, \mathbf{k}_\perp, \mathbf{l}_\perp, Q, \mu_R) = \int_{-\infty}^\infty d^2 \mathbf{b}_\perp T_H(x, y, b, Q, \mu_R) \exp[i\mathbf{b}_\perp \cdot (\mathbf{k}_\perp + \mathbf{l}_\perp)],$$

where use of the symmetry of $\psi_\pi$ under $x \leftrightarrow 1 - x \equiv \bar{x}$ has been made, and where $C_F = (N_c^2 - 1)/2N_c = 4/3$ for $SU(3).$
The amplitude

\[ P_\pi (x, b, P \simeq Q, C_1, C_2, \mu) = \exp \left[ -s (x, b, Q, C_1, C_2) - s (\bar{x}, b, Q, C_1, C_2) \right. \\
\left. -2 \int_{C_1/b}^{\mu} \frac{d\mu}{\mu} \gamma_q (\alpha_s^{\text{an}} (\mu)) \right] P_\pi (x, b, C_1/b) \]  

(14)

describes the distribution of longitudinal momentum fractions of the q\bar{q} pair, taking into account the intrinsic transverse size of the pion state \[\pi^0\], and comprising corrections due to soft real and virtual gluons \[\pi^0\], including evolution to the renormalization point.

The pion distribution amplitude evaluated at the factorization scale is approximately given by

\[ P_\pi (x, b, C_1/b) \simeq \phi_\pi (x, C_1/b) \Sigma (x, b). \]  

(15)

In the present work, we follow Jakob and Kroll \[\pi^0\] and parametrize the distribution in the transverse momentum \[k_\perp\] (or equivalently the interquark transverse distance \[b\]) in the form of a nonfactorizing Gaussian function which is normalized to unity,

\[ \Sigma (x, b) = 4 \pi \exp \left[ -\frac{b^2}{4g(x)\beta^2} \right], \]  

(16)

where \( g(x) = 1/x\bar{x} \), with an appropriate width \( \beta \) to be specified below.

Neglecting transverse momenta in Eq. (12) (collinear approximation), the only dependence on \( k_\perp \) resides in the wave function. Limiting the maximum value of \( k_\perp \), these degrees of freedom can be integrated out independently for the initial and final pion states to give way to the corresponding pion distribution amplitudes, which depend only implicitly on the cutoff momentum:

\[ \frac{f_\pi}{2\sqrt{2}N_c} \phi_\pi (x, \mu^2) = \int \frac{d^2k_\perp}{16\pi^2} \frac{\Psi_\pi (x, k_\perp)}{\Psi_\pi (x, k_\perp)} , \]  

(17)

where \( f_\pi = 130.7 \text{ MeV} \) and \( N_c = 3 \). Integrating on both sides of this equation over \( x \) normalizes \( \phi_\pi \) to unity, i.e., \( \int_0^1 dx \phi_\pi (x, \mu^2) = 1 \) because the rhs is fixed to \( \frac{f_\pi}{2\sqrt{2}N_c} \) by the leptonic decay \( \pi \rightarrow \mu^+\nu_\mu \) for any factorization scale.

Hence, the full (model) wave function for the pion takes finally the form \[\pi^0\]

\[ \Psi_\pi (x, k_\perp) = \frac{16\pi^2 f_\pi}{2\sqrt{2}N_c} \phi(x) \beta^2 g(x) \exp \left[ -g(x)\beta^2k_\perp^2 \right] . \]  

(18)

Let us now return to Eq. (14). The Sudakov form factor \( F_S (\xi, b, Q, C_1, C_2) \), i.e., the exponential factor in front of the wave function, will be expressed as the expectation value of an open Wilson line along a contour of finite extent, \( C \), which follows the bent quark line in the hard-scattering process from the segment with direction \( P \) to that with direction \( P' \) after being abruptly derailed by the hard interaction which creates a “cusp” in \( C \), and is to be evaluated within the range of momenta termed “soft”, confined within the range limited
by \( C_1/b \) and \( C_2 \xi Q \) (where \( \xi = x, \bar{x}, y, \bar{y} \)). (This means that the region of hard interaction of the Wilson line with the off-shell photon is factorized out.) Thus we have \([17, 19, 21, 30]\)

\[
F_\xi (W(C)) = \left\langle P \exp \left( ig \int_C dz \cdot t^a A^a(z) \right) \right\rangle_{\text{soft}}, \tag{19}
\]

where \( P \) denotes path ordering along the integration contour, and where \( \langle ... \rangle_A \) denotes functional averaging in the gauge field sector with whatever this entails (ghosts, gauge choice prescription, Dirac determinant, etc.). Having isolated a subsector of the full theory, where only gluons with virtualities between \( C_1/b \) and \( C_2 \xi Q \) are active degrees of freedom, quark propagation and gluon emission can be described by eikonal techniques, using either Feynman diagrams \([23, 18]\) or by employing a worldline casting of QCD which reverts the fermion functional integral into a path integral \([21]\).

Then the Sudakov functions, entering Eq. (14), can be expressed in terms of the momentum-dependent cusp anomalous dimension of the bent contour to read

\[
s (\xi, b, Q, C_1, C_2) = \frac{1}{2} \int_{C_1/b}^{C_2 \xi Q} \frac{d\mu}{\mu} \Gamma_{\text{cusp}} (\gamma, \alpha_s^{an}(\mu)) \tag{20}
\]

with the anomalous dimension of the cusp given by

\[
\Gamma_{\text{cusp}} (\gamma, \alpha_s^{an}(\mu)) = 2 \ln \left( \frac{C_2 \xi Q}{\mu} \right) A (\alpha_s^{an}(\mu)) + B (\alpha_s^{an}(\mu)), \tag{21}
\]

\( \gamma = \ln \left( \frac{C_2 \xi Q}{\mu} \right) \) being the cusp angle, i.e., the emission angle of a soft gluon and the bent quark line after the external (large) momentum \( Q \) has been injected at the cusp point by the off-mass-shell photon. The functions \( A \) and \( B \) are known at two-loop order:

\[
A (\alpha_s^{an}(\mu)) = \frac{1}{2} \left[ \gamma_K (\alpha_s^{an}(\mu)) + \beta(g) \frac{\partial}{\partial g} K (C_1, \alpha_s^{an}(\mu)) \right] = C_F \frac{\alpha_s^{an}(g(\mu))}{\pi} + \frac{1}{2} K (C_1) C_F \left( \frac{\alpha_s^{an}(g(\mu))}{\pi} \right)^2, \tag{22}
\]

and

\[
B (\alpha_s^{an}(\mu)) = -\frac{1}{2} \left[ K (C_1, \alpha_s^{an}(\mu)) + \mathcal{G} (\xi, C_2, \alpha_s^{an}(\mu)) \right] = \frac{2}{3} \frac{\alpha_s^{an}(g(\mu))}{\pi} \ln \left( \frac{C_2^2 e^{2\gamma_E} - 1}{C_2^2} \right), \tag{23}
\]

respectively. The K-factor in the \( \overline{\text{MS}} \) scheme to two-loop order is given by \([17, 13, 23, 24, 31]\)

\[
K (C_1) = \left( \frac{67}{18} - \frac{\pi^2}{6} \right) C_A - \frac{10}{9} n_f T_F + \beta_0 \ln (C_1 e^{\gamma_E} / 2). \tag{24}
\]

with \( C_A = N_C = 3, n_f = 3, T_F = 1/2, \gamma_E \) being the Euler-Mascheroni constant.

The quantities \( K, \mathcal{G} \) are calculable using the nonabelian extension to QCD \([23]\) of the Grammer-Yennie method \([32]\) for QED. Alternatively, one can calculate the cusp anomalous
In this latter approach (see, e.g., [21]), the IR behavior of the cusped Wilson line is expressed in terms of an effective fermion vertex function whose variance with the momentum scale is governed by the anomalous dimension of the cusp within the isolated effective subsector. Since this scale dependence is strictly restricted within the low-energy sector of the full theory, IR scales are locally coupled and the soft form factor depends only on the cusp angle which varies with the interquark transverse distance $b$ between $C_1/b$ and $C_2 \xi Q$.

The corresponding anomalous dimensions are linked to each other (for a nice discussion, see, [17]) through the relation $2 \Gamma_{\text{cusp}}(\alpha_s^{\text{an}}(\mu)) = \gamma_K(\alpha_s^{\text{an}}(\mu))$ with $\Gamma_{\text{cusp}}(\alpha_s^{\text{an}}(\mu)) = C_F \alpha_s^{\text{an}}(\mu^2)/\pi$, which shows that $\frac{1}{2} \gamma_K = A(\alpha_s^{\text{an}}(\mu))$. (Note that $\gamma_g = -\gamma_K$.)

The soft amplitude $P_\pi(x, b, C_1/b, \mu)$ and the hard-scattering amplitude $T_{\Pi}(x, y, b, Q, \mu)$ satisfy independent RG equations to account for the dynamical factorization (recall that both $b$ and $\xi$ are integration variables) with solutions controlled by the modified “evolution time” (see, e.g., [29] and earlier references cited therein):

$$
\tau \left( \frac{C_1}{b}, \mu \right) = \int_{C_1^2/b^2}^{\mu^2} \frac{dk^2}{k^2} \alpha_s^{\text{an}(1)}(k^2) \frac{\alpha_s^{\text{an}(2)}(k^2)}{4\pi} = \frac{1}{\beta_0} \ln \left( \frac{\mu^2/\Lambda^2}{\alpha_s^{\text{an}}(\mu^2/\Lambda^2)} \right) + \frac{1}{\beta_0} \left[ \ln \left( \frac{\mu^2}{(C_1/b)^2} \right) - \ln \left( \frac{\mu^2 - \Lambda^2}{C_1/b} \right) \right]
$$

from the factorization scale $C_1/b$ to the observation scale $\mu$, with $\Lambda$ denoting $\Lambda_{\text{QCD}}$ as before. The evolution time is directly related to the quark anomalous dimension, viz., $\gamma_q(\alpha_s^{\text{an}}(\mu)) = -\alpha_s^{\text{an}}(\mu^2)/\pi$. One appreciates that the second term in (25) stems from the power-generated correction of the effective coupling and is absent in the conventional approach. At moderate values of $\mu^2$ it is “slowing down” the rate of evolution.

The leading contribution to the IR-modified Sudakov functions $s(\xi, b, Q, C_1, C_2)$ (where $\xi = x, \bar{x}, y, \bar{y}$) is obtained by expanding the functions $A$ and $B$ in a power series in $\alpha_s^{\text{an}}$, and collecting together all large logarithms $\left( \frac{\alpha_s^{\text{an}}}{\pi} \right)^n \ln \left( \frac{C_1^{\xi b Q}}{\Gamma_{\text{cusp}}^{\xi}} \right)$, which can be transformed back into large logarithms $\ln \left( \frac{Q_s^2}{k_1^2} \right)$ in transverse momentum space. Employing equations (1) and (2), the leading contribution results from the expression

$$
s(\xi, b, Q, C_1, C_2) = \frac{1}{2} \int_{C_1/b}^{C_2 \xi Q} \frac{d\mu}{\mu} \left( 2 \ln \left[ \frac{C_2 \xi Q}{\mu} \right] - \frac{\alpha_s^{\text{an}(2)}(\mu)}{\pi} A^{(1)} + \frac{\alpha_s^{\text{an}(1)}(\mu)}{\pi} A^{(2)}(C_1) \right)
$$

$$
+ \frac{\alpha_s^{\text{an}(1)}(\mu)}{\pi} B^{(1)}(C_1, C_2) + O \left( \frac{\alpha_s^{\text{an}}}{\pi} \right)^3,
$$

where Eq. (2) is to be used in front of $A^{(1)}$, whereas the other two terms are to be evaluated with Eq. (1). The specific values of the coefficients $A^{(i)}, B^{(i)}$ are

$$
A^{(1)} = C_F
$$

$$
A^{(2)}(C_1) = \frac{1}{2} C_F K(C_1)
$$

$$
B^{(1)}(C_1, C_2) = \frac{2}{3} \ln \left( \frac{C_1^2 e^{2\gamma_E-1}}{C_2^2} \right)
$$

(27)
As now the power-correction term in \( \alpha_s \) gives rise to polylogarithms, a formal analytic expression for the Sudakov form factor is too complicated for being presented. In the calculations to follow, Eq. (26) is evaluated numerically with appropriate kinematic bounds. Note that, neglecting the power-generated logarithms, we obtain an equation for the ordinary Sudakov function, which we write as an expansion in inverse powers of \( \beta_0 \) to read

\[
s(\xi, b, Q, C_1, C_2) = \frac{1}{\beta_0} \left[ \left( 2A^{(1)} \hat{Q} + B^{(1)} \right) \ln \frac{\hat{Q}}{\hat{b}} - 2A^{(1)} \left( \hat{Q} - \hat{b} \right) \right] - \frac{4}{\beta_0^2} A^{(2)} \left( \ln \frac{\hat{Q}}{\hat{b}} - \frac{\hat{Q} - \hat{b}}{\hat{b}} \right) + \frac{\beta_1}{\beta_0^3} A^{(1)} \left\{ \ln \frac{\hat{Q}}{\hat{b}} - \frac{\hat{Q} - \hat{b}}{\hat{b}} \left[ 1 + \ln \left( 2\hat{b} \right) \right] + \frac{1}{2} \left[ \ln^2 \left( 2\hat{Q} \right) - \ln^2 \left( 2\hat{b} \right) \right] \right\}, \tag{28}
\]

where the abbreviations \( \hat{Q} \equiv \ln \frac{C_2 q Q}{\Lambda} \) and \( \hat{b} \equiv \ln \frac{C_1 b}{\Lambda} \) have been used.

This quantity differs from the original result given by Li and Sterman in \[10\], and, though it almost coincides numerically with the formula derived by J. Bolz \[33\], it differs from that algebraically.

All told, the final expression for the electromagnetic pion form factor at leading perturbative order in \( T_H \) and next-to-leading logarithmic order in the Sudakov form factor has the form

\[
F_\pi(Q^2) = \frac{2}{3} \pi C_F f_\pi^2 \int_0^1 dx \int_0^1 dy \int_0^\infty db \alpha_s^{\text{an}(1)}(\mu_R) \phi_{\text{as}}(x) \phi_{\text{as}}(y) \exp \left[ -\frac{b^2 (x \bar{x} + y \bar{y})}{4\beta_0^2} \right] K_0(\sqrt{xy} Q b) \exp \left[ -S(x, y, b, Q, C_1, C_2, C_4) \right], \tag{29}
\]

where

\[
S(x, y, b, Q, C_1, C_2, C_4) \equiv s(x, b, Q, C_1, C_2) + s(\bar{x}, b, Q, C_1, C_2) + (x \leftrightarrow y) - 8 \tau \left( \frac{C_1}{b}, \mu_R \right) \tag{30}
\]

with \( \tau \left( \frac{C_1}{b}, \mu_R \right) \) given by Eq. (23) and \( \mu_R = C_4 f(x, y) Q \).

Before we go beyond the leading order in the perturbative expansion of the hard-scattering amplitude, \( T_H \), let us pause for a moment to comment on the pion wave function. We have proactively indicated in Eq. (29) that its asymptotic form \( \phi_{\text{as}}(x) = 6x\bar{x} \) will be used.

A few words about this choice are now in order.

Hadron wave functions are clearly the essential variables needed to model and describe the properties of an intact hadron. In the past, most attempts to improve the theoretical predictions for the hard contribution to the pion form factor have consisted of using end-point concentrated wave functions. In this analysis, we refrain from using such wave functions of the Chernyak-Zhitnitsky (CZ) type \[14\], referring for a compilation of objections and references to \[8\] (see also \[35\]), and present instead evidence for an alternative source of enhancement due to the nonperturbative power correction in the effective coupling. This effect is found \[8\] to be quite significant, even for the asymptotic pion wave function which has its maximum at \( x = 1/2 \). Indeed, the IR-enhanced hard contribution can account
already at leading perturbative order for a sizable part of the measured magnitude of the electromagnetic pion form factor, though agreement with the data calls for the inclusion of the soft contribution (cf. Eq. (7)) \[36,37,38,26\] – even if the NLO correction is taken into account (see sect. 5). Nevertheless, the true pion distribution amplitude may well be a “hybrid” of the type \( \Phi^{\text{true}}_\pi = 90\% \Phi^{\text{as}}_\pi + 9\% \Phi^{CZ}_\pi + 1\% C^{(3/2)}_4 \), where the mixing ensures a broader shape with the fourth-order, “Mexican hat”-like, Gegenbauer polynomial \( C^{(3/2)}_4 \), being added in order to cancel the dip of \( \phi^{\text{CZ}}_\pi \) at \( x = 1/2 \).

First tasks from instanton-based approaches show that the extracted pion distribution amplitudes are very close to the asymptotic form \[39,40\]. Similar results are also obtained using nonlocal condensates \[41\]. The discussion of non-asymptotic pion distribution amplitudes will be conducted in a separate publication.

4. Pion form factor to order \((\alpha_s^{\text{an}}(Q^2))^2\)

Next, we generalize our calculation of the hard contribution to the pion form factor by taking into account the perturbative correction to \( T_H \) of order \( \alpha_s^2 \), using the results obtained in \([12,13,14,9]\), in combination with our analytical scheme.

To be precise, we only include the NLO corrections to \( T_H \) leaving NLO corrections to the evolution of the pion distribution amplitude aside. The reason is that for the asymptotic distribution amplitude, at issue here, these corrections are tiny \[15\]. For subasymptotic distribution amplitudes, however, NLO evolutional corrections \[15\] have to be taken into account. The calculation below does not claim to be strictly correct, the reason being that the transverse degrees of freedom in the NLO terms to \( T_H \) have been neglected, albeit the intrinsic ones in the wave functions are taken into account. Hence our prediction should be regarded rather as an upper limit for the size of the hard contribution to the pion form factor than as an exact result. Taking into account the \( k_\perp \)-dependence of \( T_H \) at NLO, this result will be reduced, though we expect that due to IR enhancement this reduction should be rather small. Thus, we have

\[
F_\pi(Q^2) = 16\pi C_F \left( \frac{f_\pi}{2\sqrt{N_C}} \right)^2 \int_0^1 dx \int_0^1 dy \int_0^\infty b db \alpha_s^{\text{an}}(\mu_R^2) \\
\times K(\sqrt{xy}Qb) \phi_{\text{as}}(x) \phi_{\text{as}}(y) \\
\times \exp \left( -\frac{b^2 (x\bar{x} + y\bar{y})}{4\beta_\text{as}^2} \right) \exp (-S(x, y, b, Q, C_1, C_2, C_4)) \\
\times \left[ 1 + \frac{\alpha_s^{\text{an}}}{\pi} \left( f_{\text{UV}}(x, y, Q^2/\mu_R^2) + f_{\text{IR}}(x, y, Q^2/\mu_F^2) + f_C(x, y) \right) \right], \tag{31}
\]

where the Sudakov form factor, including evolution, is given by Eq. (30), \( \mu_F = C_1/b \), and the functions \( f_i \) are taken from \[9\]. They are given by

\[1\] We wish to thank Dr. B. Melić for sending us the corrected version of this paper prior to publication.
\[ f_{\text{UV}}(x, y, Q^2/\mu_R^2) = \frac{\beta_0}{4} \left( \frac{5}{3} - \ln(\bar{x}\bar{y}) + \ln \frac{\mu_R^2}{Q^2} \right) \]
\[ f_{\text{IR}}(x, y, Q^2/\mu_F^2) = \frac{2}{3} \left( 3 + \ln(\bar{x}\bar{y}) \right) \left( \frac{1}{2} \ln(\bar{x}\bar{y}) - \ln \frac{\mu_F^2}{Q^2} \right) \]
\[ f_C(x, y) = \frac{1}{12} \left[ 34 - 12 \ln(\bar{x}\bar{y}) + \ln x \ln y \right. \\
+ \ln \bar{x} \ln \bar{y} - \ln x \ln \bar{y} - \ln \bar{x} \ln y \\
\left. + (1 - x - y)H(x, y) + R(x, y) \right] \]

and are related to UV and IR poles, as indicated by corresponding subscripts, that have been removed by dimensional regularization along with the associated constants \( \ln(4\pi) - \gamma_E \), whereas the last term is scale-independent. In evaluating expression \( f_C \) in (32), we found it particularly convenient to use the representation of the function \( H(x, y) \) given by Braaten and Tse [44],

\[ H(x, y) = \frac{1}{1 - x - y} \left[ \text{Li}_2(x) + \text{Li}_2(y) - \text{Li}_2(x) - \text{Li}_2(y) + \ln x \ln y - \ln \bar{x} \ln \bar{y} \right] , \] (33)

where \( \text{Li}_2 \) denotes the Spence function. For the function \( R(x, y) \) we have used the expression derived by Field et al. [42], except at point \( x \approx y \), where we employed the Taylor expansion displayed below:

\[ R(x, y) = \frac{1}{3} \frac{y^2}{(-1 + y)^2} \left[ (-1 + 33y - 45y^2 + 13y^3) \ln \bar{y} \right. \\
+ y \left. (-1 + y + (9 - 13y) y \ln y) \right] \\
+ \frac{x - y}{3(-1 + y)^2 y^3} \left[ (-1 + y)^2 (-1 + 16y) \ln \bar{y} \right. \\
\left. + y \left( -1 + 13y - 12y^2 + 2y^2 \ln y \right) \right] \\
+ \frac{(x - y)^2}{30(-1 + y)^2 y^4} \left[ (-1 + y)^3 \left( 9 - 148y + 9y^2 \right) \ln \bar{y} \right. \\
\left. - y \left( 9 - 148y + 328y^2 - 189y^3 + y^3 (5 + 9y) \ln y \right) \right] . \] (34)

Note that this expression does not reproduce its counterpart in [42].

Having developed in detail the theoretical apparatus, let us now turn to the concrete calculation of the pion form factor up to NLO.

5. Validity of the analysis

This numerical analysis updates and generalizes our previous investigation in [8]. In order to set up a reliable algorithm for the numerical evaluation of the expressions presented above, we have to ensure that this is done in kinematic regions where use of fixed-order or resummed perturbation theory is legal. Further, restrictions have to be imposed to
avoid double counting of gluon corrections by carefully defining the validity domain of each contribution to the pion form factor. These kinematic constraints are compiled below.

**Kinematic cuts**

1. $C_1/b > \Lambda_{\text{QCD}}$; otherwise the whole Sudakov exponent $\exp(-S)$ (cf. Eq. (30)) is continued to zero because this large-$b$ region is properly taken into account in the wave functions. This condition excludes from the resummed perturbation theory soft gluons with wavelengths larger than $C_1/\Lambda$, which should be treated nonperturbatively.

2. $C_2\xi Q > C_1/b$; otherwise each Sudakov exponent $\exp[-s(\xi, b, Q, C_1, C_2)]$ in Eq. (30) is “frozen” to unity because this small-$b$ region is dominated by low orders of perturbation theory rather than by the resummed perturbation series, and consequently contributions in this region should be ascribed to higher-order corrections to $T_H$. Yet evolution is taken into account to match the scales in our “gliding” factorization scheme.

3. $C_4 f(x, y) Q > C_1/b$; otherwise evolution time $\tau(C_1/b, \mu_R)$ in Eq. (30) is contracted to zero, i.e., evolution is “frozen”. The renormalization scale should be at least equal to the factorization scale, so that the effective coupling has arguments in the range controlled by perturbation theory.

4. $C_4 f(x, y) Q > C_2\xi Q$; otherwise evolution to that scale is “frozen” because this region is appropriately accounted for by the Sudakov contribution. This helps avoiding double counting terms which belong to the resummed rather than to the fixed-order perturbation theory.

5. $C_4 f(x, y) Q > C_1/b$; otherwise the two scales $\mu_R = \mu_F = C_1/b$ are identified in the function $f_{\text{UV}}(x, y)$. If $\mu_R \leq \Lambda_{\text{QCD}}$, then $f_{\text{UV}}(x, y)$ is set equal to zero.

6. $C_1/b > \Lambda_{\text{QCD}}$; otherwise the function $f_{\text{IR}}(x, y)$ is set equal to zero.

To illustrate the difference in technology between approaches employing the conventional expression for the full Sudakov exponent [18, 19], on one hand, and our analysis, on the other hand, we show $\exp(-S)$ graphically in Fig. 2 for three different values of the momentum transfer. In contrast to Li and Sterman [10], the evolutional contribution is not cut-off at unity, whenever $C_2\xi Q < C_1/b$. The dotted curve shows the result for Eq. (28) without this cutoff. One infers from this figure that their suggestion to ignore the enhancement due to the anomalous dimension does not apply in our case because the IR-modified Sudakov form factor is not so rapidly decreasing as $b$ increases owing to the IR-finiteness of $\alpha_s^{\text{an}}$. Indeed, as $Q$ becomes smaller, $\exp(-S)$ remains constant and fixed to unity for increasing $b$, providing enhancement only in the large-$b$ region before it reaches the kinematic boundary $C_1/b = \Lambda_{\text{QCD}}$, where it is set to zero. As a result, for small $Q$-values, like $Q_1 = 2$ GeV, the enhancement due to the quark anomalous dimension does not apply in our case because the IR-modified Sudakov form factor is not so rapidly decreasing as $b$ increases owing to the IR-finiteness of $\alpha_s^{\text{an}}$. Indeed, as $Q$ becomes smaller, $\exp(-S)$ remains constant and fixed to unity for increasing $b$, providing enhancement only in the large-$b$ region before it reaches the kinematic boundary $C_1/b = \Lambda_{\text{QCD}}$, where it is set to zero. As a result, for small $Q$-values, like $Q_1 = 2$ GeV, the enhancement due to the quark anomalous dimension cannot be associated with higher-order corrections to $T_H$, since it operates at larger $b$-values, and for that reason it should be taken into account. Only for asymptotically large $Q$ values, when the IR-modified Sudakov form factor and the conventional one become indistinguishable, the evolutional enhancement becomes a small effect, strictly confined in the small-$b$ region, and can be safely ignored.
Figure 2. Behavior of the Sudakov form factor with respect to the transverse separation $b$ for three different values of the momentum transfer $Q_1 = 2$ GeV, $Q_2 = 5$ GeV, and $Q_3 = 10$ GeV with all $\xi_i = 1/2$, and where we set $C_1 = 2e^{-\gamma_E}$, $C_2 = e^{-1/2}$ and $\Lambda_{\text{QCD}} = 0.242$ GeV. The dotted curve shows the result obtained with $\alpha_s^{\overline{MS}}$ and $\Lambda_{\text{QCD}} = 0.2$ GeV for $Q_2 = 5$ GeV using the same set of $C_i$ as before. Notice that in this case, evolution is limited by the (renormalization) scale $\mu_R = t = \{\max \sqrt{\pi y Q_1}, C_1/b\}$, proposed in [10]. However, the enhancement at small $b$-values due to the quark anomalous dimension is not neglected.
Table 1. Different sets of coefficients $C_i$, and values of the $K$-factor and the quantity (cf. Eq. (23)) $\kappa = \ln \left( C_1^2 e^{2\gamma_E-1}/4C_2^2 \right)$, corresponding to different factorization and renormalization prescriptions.

| Choice | $C_1$ | $C_2 = \frac{1}{\sqrt{2}}C_2^{CSS}$ | $C_3$ | $C_4$ | $K$     | $\kappa$ |
|--------|-------|------------------------------------|-------|-------|---------|---------|
| canonical | $2\exp(-\gamma_E)$ | $1/\sqrt{2}$ | $2\exp(-\gamma_E)$ | $-$ | 4.565   | -0.307  |
| SSK    | $\exp\left[ -\frac{1}{2}(2\gamma_E - 1) \right]$ | $1/\sqrt{2}$ | $\exp\left[ -\frac{1}{2}(2\gamma_E - 1) \right]$ | $-$ | 2.827   | 0       |
| this work | $2\exp(-\gamma_E)$ | $\exp(-1/2)$ | $2\exp(-\gamma_E)$ | $\exp(-1/2)$ | 4.565   | 0       |

This behavior of the Sudakov form factor, we emphasize, shows that power-induced subleading logarithmic corrections are relevant in the range of currently probed momentum-transfer values. Then the advantage of employing such a scheme to calculate observables is that the hard contribution to the pion form factor is enhanced. This is because the range in which soft gluons build up the Sudakov form factor is enlarged and inhibition of bremsstrahlung sets in at larger $Q^2$. Let us mention in this context that power corrections could also lead to additional suppression of soft gluon emission at large transverse distance $b$. Indeed, Akhoury, Sincovics, and Sotiropoulos [46] have resummed nonperturbative power corrections in $b$ to the hadronic (meson) wave function and found Sudakov-type suppression on top of the Sudakov suppression discussed so far. The discussion of such IR-renormalon-based contributions in conjunction with our approach would be interesting, but goes beyond the scope of the present work.

Using the techniques discussed above, we obtain the theoretical predictions shown in Fig. 3. Recall that the asymptotic pion distribution amplitude $\phi_{as} = 6x(1-x)$ is employed with $\beta_{as}^2 = 0.883$ [GeV$^{-2}$] which corresponds to $(k_t^2)^{1/2} = 0.35$ GeV (see, [11]). A set of constants $C_i$, ($i = 1, 2, 3$) which eliminates artifacts of dimensional regularization, while practically preserving the matching between the resummed and the fixed-order calculation, is given in Table II in comparison with other common choices of these constants. In addition, we set $f(x, y) = \sqrt{xy}$, a choice for the renormalization scale which eliminates large kinematical corrections due to soft gluon emission. However, these favored values of the scheme constants do not constitute a strict constraint on the validity of the numerical analysis. They merely indicate the anticipated appropriate choice of the factorization and renormalization scales with respect to observables and theoretical self-consistency.

Before we proceed with the discussion of these results, let us first present the theoretical prediction for the pion-photon transition form factor $F_{\pi\gamma\gamma}(Q^2, q^2 = 0)$ in which one of the photons is highly off-shell and the other one is close to its mass-shell. In leading perturbative order this form factor is given by the expression (cf. [26])

$$F_{\pi\gamma}(Q^2) = \frac{1}{\sqrt{3}\pi} \int_0^1 dx \int_0^\infty db \frac{f_\pi(x)b}{2\sqrt{6}} \exp\left( -x\bar{x}b^2/4\beta_{as}^2 \right) \times \left( 4\pi K_0 \left( \sqrt{x}bQ \right) \right) e^{-S_{\pi\gamma}}, \quad (35)$$

where the Sudakov exponent, including evolution, has the form

$$S_{\pi\gamma}(x, \bar{x}, b, Q, C_1, C_2, C_4) = s(x, b, Q) + s(\bar{x}, b, Q) - 4\tau \left( \frac{C_1}{b}, \mu_R \right). \quad (36)$$
Figure 3. Spacelike pion form factor calculated with $\phi_{\text{las}}$ within our theoretical scheme in comparison with existing experimental data [16, 18]. The dotted line shows the enhanced hard contribution to the form factor, obtained in our analysis at leading perturbative order and with inclusion of the NLO correction to $T_H$ (lower solid line). The upper solid line represents the sum of the lower solid line and the dashed line, the latter curve giving the result for the soft contribution, computed in [20]. The dot-dashed line reproduces the recent calculation of Tung and Li [12] which does not include an intrinsic $k_{\perp}$-dependence and uses for $\alpha_s$ the conventional 1-loop expression.
The main difference relative to the previous case is that this form factor contains only one pion wave function, whereas the associated hard-scattering part, being purely electromagnetic at this order, does not depend on $\alpha_s$. The only dependence on the strong coupling constant enters through the anomalous dimensions in the Sudakov form factor. The result of this calculation is displayed in Fig. 4.

All constraints on kinematics set forward in the numerical evaluation of the electromagnetic pion form factor are relevant to this case too, except the requirement which deals specifically with the choice of the renormalization scale, which now is set equal to $\mu_R = C_4 x Q$. Another reasonable choice would be $\mu_R = C_4 \sqrt{x} Q$, which entails evolution to a lower scale, hence reducing evolitional enhancement through $\tau (C_1/\mu, \mu_R)$ by approximately 6%.

It is obvious from Fig. 3 that the IR-enhanced hard contribution to $F_\pi(Q^2)$ is providing a sizeable fraction of the magnitude of the form factor. This behavior is IR stable and extends from low to high $Q^2$ values, exhibiting exact scaling. In contrast to the dot-dashed line, which is calculated with the conventional $\alpha_s^{(1)}$, enhancement is not resulting from approaching the unphysical Landau singularity at $Q^2 = \Lambda^2_{QCD}$ – still appreciable at $Q^2$ values around $(6 - 8)$ GeV$^2$ – but is induced by the nonperturbative power correction.

To make these findings more transparent, we compare our results in table 2 with those obtained in other analyses.

Comparison of our values with those calculated by Jakob and Kroll [11] shows that the suppression of the hard part of the form factor due to the inclusion of transverse degrees of
Table 2. Calculated pion form factor at two values of $Q^2$. The first two columns show the results obtained in the present work, in comparison with those calculated by Jakob and Kroll (JK) [11] (third column), and by Melič et al. (MNP) [9] (last two columns).

| $Q^2$ [GeV$^2$] | LO (this work) | LO+NLO (this work) | LO (JK) | LO (MNP) | LO+NLO (MNP) |
|----------------|----------------|-------------------|---------|----------|-------------|
| 4              | 0.119          | 0.167             | 0.08    | 0.131    | 0.211       |
| 10             | 0.128          | 0.168             | 0.08    | 0.109    | 0.164       |

freedom is counteracted by the power-induced enhancement. On the other hand, comparison with the values computed by Melič et al. at leading order, by ignoring completely transverse degrees of freedom, reveals that in the $Q^2$ domain, where the influence of the Landau singularity has died out, there is enhancement. Furthermore, comparing our results with theirs at next-to-leading order, we conclude that our choice of the renormalization scale is consistent with a proper matching between gluon corrections, calculated on a term-by-term perturbation expansion, and those due to the resummed perturbative series. Moreover, the scaling behavior of the calculated perturbative contribution to the full pion form factor is also improved.

The calculation of $F_{\pi\gamma}(Q^2)$ provides an additional confirmation of our method. Fig. 4 shows that our theoretical prediction for this form factor reproduces the recent high-precision CLEO [49] and the earlier CELLO [50] data with the same numerical accuracy as the dipole interpolation formula, without using any phenomenological adjustment.

In this context, we mention, however, that other authors [51,52,53] obtain similarly good numerical agreement of $F_{\pi\gamma}$ with the experimental data, following different premises on the basis of QCD sum rules.

Another important point worth to be mentioned is that the proposed scheme exceeds the quality of the phenomenological predictions derived from schemes which involve saturation prescriptions [14], even if these employ “comensurate rescaling” to “pre-sum” into the running coupling constant NLO contributions [13]. A more detailed study of the BLM procedure [54], which helps avoid scheme ambiguities, in combination with our approach will be presented elsewhere.

6. Summary and conclusions

We have developed in detail a theoretical framework which self-consistently incorporates effects resulting from a modification of the running $\alpha_s$ by a nonperturbative power correction [3] which provides IR universality. Though a deep physical understanding of such contributions is still lacking, we have given, as a matter of practice, quantitative evidence that in this way it is possible to get a hard contribution to the electromagnetic form factor $F_{\pi}(Q^2)$, which is IR-enhanced relative to conventional approaches, using solely the asymptotic form of the pion distribution amplitude, hence avoiding end-point concentrated distribution amplitudes. The presented IR-finite factorization and renormalization scheme makes it possible to take into account transverse degrees of freedom both in the pion wave function [11] as well as in the form of Sudakov effects [10], without entailing suppression of the form-factor mag-
nitude. In addition, use of this modified form of $\alpha_s(Q^2)$ renders the theoretical predictions insensitive to its variation with $Q^2$ at small momentum values. An appropriate choice of the factorization and renormalization scales ensures matching of fixed-order with resummed perturbation theory and helps avoid double counting of higher-order corrections due to gluon radiation, ensuring this way the self-consistency of the whole perturbative treatment. The same procedure applied to $F_{\pi^0\gamma^*\gamma}$ yields a prediction which agrees with the experimental data very well, hence supporting the theoretical basis of our approach.

Taking our results for the pion form factor at face value, they seem to suggest that its magnitude may be significantly larger than predicted in previous analyses, even without employing broad pion distribution amplitudes. However, because of the restricted accuracy of the existing experimental data such a conclusion would be surely premature.

We believe that the insight gained through our analysis gives a strong argument that if power corrections to the effective coupling of QCD really exist, their implications and applications are important and revealing. In this respect, the Shirkov-Solovtsov approach may provide a convenient tool for improving theoretical predictions for exclusive observables, based on perturbation theory.

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