Intrinsic parton transverse momentum in next-to-leading-order pion production

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We study pion production in proton-proton collisions within a pQCD-improved parton model in next-to-leading order augmented by intrinsic transverse momentum \( k_\perp \) of the partons. We find the introduction of intrinsic transverse momentum necessary to reproduce the experimental data in the CERN SPS to RHIC energy range, and we study its influence on the so-called \( K \) factor, the ratio of the NLO cross section to the Born term. A strong \( p_T \) dependence is seen, especially in the 3–6 GeV transverse momentum region of the outgoing pion, where nuclear effects (e.g. the Cronin effect) play an important role.

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I. INTRODUCTION

Today’s collider facilities raise the interest in testing perturbative QCD (pQCD) at work, and in searching for phenomena beyond its capability. However – at least for the experimentally available transverse momentum region – pQCD parton models underestimate the production of mesons in proton-proton \((pp)\) collisions \[1,2\], even at next-to-leading order (NLO) \[3\]. In order to restore the consistency with the data, two methods were proposed at leading order (LO): inclusion of the intrinsic transverse momentum of partons \[1,2\], and/or an effective correction factor (\( K \) factor), accounting for higher order contributions \[4,5\]. The physical background is to account for both, the missing higher order perturbative corrections and radiation effects. The latter is not present in electron-proton processes, however, it becomes important in \( pp \) collisions \[6\].

In this paper we present the first results on pion production in \( pp \) collisions applying a NLO pQCD parton model with intrinsic transverse momentum, taken from a Gaussian distribution with width \( \langle k_\perp^2 \rangle \). In Section II the intrinsic transverse momentum is introduced into the formalism, and the appropriate NLO expressions are presented. The necessity of such an extension is demonstrated at \( \sqrt{s} = 27.4 \) GeV in Section III. Next, we display the best fit values of the width of the intrinsic transverse momentum distribution at NLO level for available \( pp \to \pi + X \) experiments, in the energy range \( 20 \text{ GeV} \lesssim \sqrt{s} \lesssim 200 \text{ GeV} \). Similarly to Ref. \[1\], we study the pion production in the 2 GeV < \( p_T < 6 \) GeV transverse momentum region, where nuclear effects are considered to be important.

Finally, we extract the ratio of the NLO cross section to the Born term (\( K \) factor) at different energies and transverse momenta, and study its dependence on the amount of the included intrinsic transverse momentum. This way we provide a numerical foundation to the correction factors used in LO calculation \[7\]. We demonstrate, that a leading order calculation with a fitted, c.m. energy, transverse momentum and scale dependent \( K \) factor and additional intrinsic transverse momentum reproduces well the full NLO results at higher energies and momenta, and can be used as a fast method to get a reasonable estimate of a full NLO calculation.

II. MODEL

In order to extend the applicability of the original, infinite momentum frame parton model \[8\] to smaller transverse momenta, we introduce the intrinsic transverse momentum of the partons \[9\]. We write the four-momenta of the interacting partons \((a, b)\) as

\[
p_a = \left(x_a \frac{\sqrt{s}}{2} + \frac{k_{\perp,a}^2}{2x_a\sqrt{s}} \right) \vec{k}_{\perp,a}, \quad n_a = \frac{\sqrt{s}}{2} - \frac{k_{\perp,a}^2}{2x_a\sqrt{s}},
\]

\[
p_b = \left(x_b \frac{\sqrt{s}}{2} + \frac{k_{\perp,b}^2}{2x_b\sqrt{s}} \right) \vec{k}_{\perp,b}, \quad n_b = -x_b \frac{\sqrt{s}}{2} + \frac{k_{\perp,b}^2}{2x_b\sqrt{s}}.
\]

In this notation \( x \), the momentum fraction carried by the parton, becomes a parameter. The apparent fraction is \( x - k_{\perp}^2/\langle x_s \rangle \), however, for practical applications at high energy \( \langle p_T \rangle \gtrsim 3 \text{ GeV}; \sqrt{s} \gtrsim 40 \text{ GeV} \); \( \langle k_{\perp}^2 \rangle \lesssim 2 \text{ GeV}^2 \), the distinction has a negligible (\( \lesssim 5\% \)) effect. Furthermore, we require that the longitudinal direction of the partons does not change sign due to the transverse momentum, i.e. \( x > k_{\perp}/\sqrt{s} \).

The starting point of our calculation is factorization: the hadronic cross sections up to a power correction may be written as a convolution over hard partonic (pQCD)
processes,
\[ \frac{d\sigma}{dyd^2p_T} = \sum_{abc} \int dx_a dx_b d^2k_{\perp,a} d^2k_{\perp,b} \frac{dz_c}{\pi z_c^2} \] (2)
\[ f_{a/p}(x_a, Q; k_{\perp,a}) f_{b/p}(x_b, Q; k_{\perp,b}) \frac{d\sigma}{df} D_{\pi/c}(z_c, Q_f) , \]
where \( d\sigma/d\hat{f} \) is the partonic cross section of the reaction
\( a + b \rightarrow c + d \) (LO with condition \( \delta(1 + (\hat{t} + \hat{u})/\hat{s}) \)), or
\( a + b \rightarrow c + d + e \) (NLO with fixed \( z_c \)) and (at fixed scales)
\( \hat{Q} \) on three scales, as proposed in Ref. [8]. However, in our case we found
\( \hat{Q} \) is the function of the partonic Mandelstam variables only.
In order to avoid singularities due to the intrinsic transverse momentum we use regularization
\[ s \rightarrow \hat{s} + M^2, \quad \hat{t} \rightarrow \hat{t} - M^2/2, \quad \hat{u} \rightarrow \hat{u} - M^2/2 , \] (3)
with \( M = 1.8 \) GeV. The factorization is done at the factorization scale \( Q \), where the parton content of the initial proton is determined by the parton distribution function
\( f_{a/p} \) (PDF). For simplicity, we assume a Gaussian dependence of the PDFs on the intrinsic transverse momentum, with a width \( \langle k^2_\perp \rangle \),
\[ f(x, Q, k^2_\perp) = f(x, Q) \frac{1}{\pi \langle k^2_\perp \rangle} e^{-k^2_\perp/\langle k^2_\perp \rangle} . \] (4)
We note, that such a separation may be viewed as a first approximation to the unweight PDF, where recent studies indeed found a shape close to Gaussian [11].
Finally, the hadrons are created collinearly with the outgoing parton \( c \) with momentum fraction \( z_c \) at fragmentation scale \( Q_f \). The partonic cross section explicitly depends on three scales, \( Q, Q_f \) and the renormalization scale \( Q_r \).
In principle, the scales can be determined such that the final result has the minimum sensitivity to them [3], and usually are set to be equal. However, in this paper we use the results of a previous study [11, 12] of pp and \( pA \) data, where the reproduction the Cronin effect in \( pA \) reactions [12] imposed such scales, that the corresponding \( \langle k^2_\perp \rangle \approx 2 \) GeV².
We note that there is some ambiguity in the choice of scales.
In the literature typical scales are fixed to hadronic or partonic variables, \( \kappa p_T / z_c \) and \( \kappa/\kappa_T / z_c \), respectively, where \( \kappa \) is an \( \mathcal{O}(1) \) number. Other choices are also possible, e.g., an invariant scale, \( Q^2 = \kappa^2 \hat{s} \hat{u}/(\hat{s}^2 + \hat{t}^2 + \hat{u}^2) \) as proposed in Ref. [8]. However, in our case we found that this choice is equivalent to \( \kappa p_T / z_c \).
At NLO level and no intrinsic transverse momentum Eq. (2) is usually rewritten with variable change \( (x_a, x_b, z_c) \rightarrow (\hat{v}, \hat{w}, \hat{z}_c) \), where \( \hat{t} = -(1 - \hat{v}) \hat{s} \) and \( \hat{u} = -\hat{v} \hat{w} \hat{s} \), as
\[ \frac{d\sigma}{dyd^2p_T} = \frac{1}{\hat{s}} \sum_{abc} \int d\hat{v} d\hat{w} d\hat{z}_c \frac{dz_c}{\pi z_c^2} J \] (5)
\[ d^2k_{\perp,a} d^2k_{\perp,b} f_{a/p} f_{b/p} D_{\pi/c}(z_c, Q_f) \frac{d\sigma^{NLO}}{d\hat{f}} , \]
with the proper kinematical boundaries, \( J \) being the Jacobian of the transformation \( (1/J = \hat{v}(1 - \hat{v})/\hat{w} \) for

![FIG. 1: Comparison of experimental data to the NLO pQCD parton model result in \( p + p \rightarrow \pi^+ + X \) reaction at \( E_{lab} = 400 \) GeV, with and without intrinsic transverse momentum.](image)

### III. RESULTS

#### A. Comparison to data

First, we demonstrate the importance of the intrinsic transverse momentum to reproduce experimental data in the transverse momentum region \( 3 \) GeV < \( \sqrt{s} < 6 \) GeV. In this part, we use the scales proposed in Ref. [3] to make a direct comparison to the 400 GeV FNAL experiment [17]. Fig. 1 shows, that at \( \sqrt{s} = 27.4 \) GeV the NLO calculations of pion production in \( pp \) collision underpredict the experimental data by a factor of 2 using
the scale parameters \( Q = Q_r = Q_f = p_T/2 \) and neglecting intrinsic transverse momentum (as in [3]). Decreasing the scales (but still keeping them hard for the applicability of pQCD) the agreement can be improved, however, without intrinsic transverse momentum, even if lower scales are chosen, the data are still underestimated.

The lower line in Fig. 1. presents the NLO calculation with a partonic intrinsic transverse momentum distribution of width \( \langle k_2^2 \rangle = 1 \) GeV, and shows a nice agreement with the data. We note, however, that there is a delicate interplay between the choice of the scales and the intrinsic transverse momentum \( \langle k_2^2 \rangle \), needed to reproduce the data [18]. Typically, increasing the scales increases the value of \( \langle k_2^2 \rangle \).

B. Intrinsic transverse momentum

In this section we summarize the results on the intrinsic transverse momentum width of the partons in the nucleon fitting the NLO pQCD calculations to the data, generalizing the LO scale choice of [11] to \( Q = Q_r = \kappa p_T/z_c, Q_f = \kappa p_T \), where \( \kappa \) is a \( O(1) \) number. In this work, we fix \( \kappa = 2/3 \) at NLO level. Our previous study [11] also showed, that the reproduction of the Cronin peak in \( pA \) collision requires the width of the intrinsic transverse momentum distribution to be on the order of \( \langle k_2^2 \rangle \approx 2 \) GeV\(^2\) at energies \( \sqrt{s} \sim 30 \) GeV, which is achieved by the above choice of scales. A similar value of \( \langle k_2^2 \rangle \) was extracted from the experimental analysis of jet-angle distribution [18].

Analyzing the \( pp \rightarrow \pi + X \) experimental data [17, 19, 20], we deduced the best fit value \( \langle k_2^2 \rangle \) for each experiment, similarly to what is shown in Fig. 1, minimizing the \( \chi^2/(D/T - 1) \) (data over theory) ratio in the range \( 3 - 6 \) GeV. The result is presented in Fig. 2, separately indicating the runs from different experiments and shows a need for a considerable amount of partonic transverse momentum. We also checked, that similarly to the LO results [11], the extracted width does not depend on the charge of the pion.

Usually, changing the order of a calculation requires changing of the underlying scales. Comparing NLO result to the previous LO ones [11] one notices that in order to keep the average transverse momentum width, we had to increase the scales, dictated by the experimentally obtained Cronin peak [11]. Keeping LO scales (\( \kappa = 1/2 \)) would lead to a substantial reduction of the width, originating in the mechanism of NLO graphs to automatically generate transverse momenta. It is remarkable, that fitting to the nuclear reaction data requires the same width independent of the order of the calculation used!

Recent \( dAu \) data at RHIC [21] also indicate that at \( \sqrt{s} = 200 \) GeV more transverse momentum is necessary, than \( \langle k_2^2 \rangle \approx 0 - 0.5 \) GeV\(^2\), indicated in Fig. 2. This shows, that the scale parameter \( \kappa \) possibly may also depend on the energy, and a higher, \( \kappa = 4/3 \) scale with \( \langle k_2^2 \rangle = 2.5 \) GeV\(^2\) reproduces well both the \( pp \) and the \( dAu \) data without jet quenching effects [22]. The dependence of the scale parameter on the c.m. energy and its consequences will be studied in a separate paper.
FIG. 4: Ratio of the pionic $K$ factor at $\langle k_T^2 \rangle = 1$ GeV$^2$ (left) and 2 GeV$^2$ (right) to $K$ factor at $\langle k_T^2 \rangle = 0$ at energies from $\sqrt{s} = 20$ GeV to 1800 GeV.

IV. CONCLUSIONS

In this paper we introduced initial transverse momentum distributions into the parton-based description of hadron production in NLO level pQCD calculations and demonstrated, that such an extension is necessary even in NLO to reproduce the $pp$ experimental data. From the analysis of most experiments in the $3 \text{ GeV} \lesssim p_T \lesssim 6$ window a width of intrinsic transverse momentum distribution on the order of $\langle k_T^2 \rangle \approx 2$ GeV$^2$ was fitted. The obtained precision of the description of pion production in $pp$ collisions is high enough to find possible collective effects in nuclear collisions.

We presented the pionic $K$-factor (full NLO cross section to the Born term) for several energies and transverse momentum values, and found a pronounced dependence on $p_T$ in the $3 \text{ GeV} \lesssim p_T \lesssim 6$ window, where nuclear effects show up most prominently. Furthermore, we investigated in detail the modifications of the Born and higher order terms due to the presence of an intrinsic transverse momentum and concluded, that above certain energy in the above mentioned $p_T$ window the higher order contribution raises faster, than the Born term and hence the $K$-factor increases compared to its value without intrinsic transverse momentum. For $p_T \gtrsim 4$ GeV this enhancement may be neglected, justifying $\langle k_T^2 \rangle$ independent $K$-factor calculations, however, at smaller transverse momenta the correction goes up to 20%, just in the order of the measured nuclear enhancement. The detailed study of the nuclear enhancement, and its role in fixing the scales of an NLO calculation will be carried out separately.
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