Bedford, Eric; Kim, Kyounghee
Periodicities in linear fractional recurrences: degree growth of birational surface maps. (English) Zbl 1171.37023
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The authors study periodicity of linear fractional recurrences and degree growth of birational surface mappings. They start with the family of birational transformations of the plane defined in affine coordinates by

\[ f(x, y) = \left( \frac{a_0 + a_1 x + a_2 y}{b_0 + b_1 x + b_2 y} \right), \quad a = (a_0, a_1, a_2), \quad b = (b_0, b_1, b_2), \]

or its canonically induced map \( f_{a, b} : \mathbb{P}^2 \to \mathbb{P}^2 \). The authors study the degree growth of the iterates \( f_{a, b} \).

For a generic choice of parameters \( a \) and \( b \), \( f_{a, b} \) is not birationally conjugate to an automorphism. The remaining possibilities are thoroughly classified.

This family of mappings are usually used to construct examples with interesting dynamics.

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MSC:
37F10 Dynamics of complex polynomials, rational maps, entire and meromorphic functions; Fatou and Julia sets
32H50 Iteration of holomorphic maps, fixed points of holomorphic maps and related problems for several complex variables
37B40 Topological entropy
37C25 Fixed points and periodic points of dynamical systems; fixed-point index theory; local dynamics

Keywords:
natural mapping; degree growth; birational morphism; periodic mapping

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