Magnetic Rapid Quench Measurement in U(1) and SU(2) superconductors

H. Nobukane, S. Tsuchiya, K. Konno, K. Inagaki, and S. Tanda
Department of Applied Physics, Hokkaido University, Sapporo 060-8628, Japan

I. Kawasaki, H. Amitsuka
Department of Physics, Hokkaido University, Sapporo 060-810, Japan

K. Tenya
Department of Education, Shinsyu University, Nagano 380-8544, Japan

Y. Maeno
Department of Physics, Kyoto University, Kyoto 606-8502, Japan
E-mail: nobukane@eng.hokudai.ac.jp

Abstract. We observed the power law and the critical exponent of quantum phase transitions by measuring a transient response in an Al/AlO\(_x\)/Al junction of U(1) superconductors and Sr\(_2\)RuO\(_4\) single crystals of SU(2) superconductors, when magnetic field decreased rapidly from more than the critical field to zero magnetic field below \(T = 0.1\) K. In the Al/AlO\(_x\)/Al junction, the result shows that the domain length \(\xi\) is a power function of the quench time \(\tau_Q\). The critical exponent is 0.30 \(\pm\) 0.01. Moreover, in the Sr\(_2\)RuO\(_4\) crystal, the result shows \(\xi\) is a power function of \(\tau_Q\) and the critical exponent is 0.40 \(\pm\) 0.01. From the results of U(1) and SU(2) superconductors, we consider the critical exponents observed by the quenching are derived by a mean-field value in charged systems. Using typical U(1) and SU(2) superconductors, we find that the magnetic quench is efficient method to study formation processes of topological defects for a quantum phase transition near \(T = 0\).

1. Introduction
Phase transitions have occurred in the early universe and condensed matter systems. In cosmology, topological defects such as monopoles, cosmic strings and textures are considered to be important for understanding the unified theory of particle physics [1]. Zurek pointed out that analogues of cosmological phase transitions can be studied in condensed matter systems of the laboratory [2], which is the so-called Kibble-Zurek mechanism (KZM). In fact, the formation of topological defects have been studied in thermal phase transitions using superfluid helium [3, 4], liquid crystals [5, 6] and superconductors [7, 8]. Recently, theoretical approaches have been discussed in a quantum phase transition rather than a thermal one [9]. However the experimental study in a quantum phase transition have not been carried out yet. Here we focus
on a magnetic quench measurement of superconductors, which is the rapid sweeping magnetic fields from more than the critical field to zero magnetic field.

Superconductors are a promising candidate for embodying the early universe because they have a local gauge invariant and also provide $U(1)$ and $SU(2)$ order parameter fields as in the universe. What kind of influence do internal degrees of freedom of the order parameter have in the KZM? Here we adopted $Al/AlO_x/Al$ junctions of $U(1)$ superconductors and $Sr_2RuO_4$ single crystals of $SU(2)$ superconductors [10].

In this paper, we observed the power law and the critical exponent of the KZM using a magnetic quench measurement in an $Al/AlO_x/Al$ junction and a $Sr_2RuO_4$ crystal. Using the $Al/AlO_x/Al$ junction, we show the magnetic quench measurement is efficient method in order to determine the critical exponent of $z$ and $\nu$ in quantum phase transitions. The magnetic quench measurement of the $Sr_2RuO_4$ have the critical exponent of the KZM given by internal degrees of freedom of order parameter. From the results, we suggest the critical exponents of the $U(1)$ and $SU(2)$ superconductors are derived by a mean-field value in charged systems of $z = 1$.

2. Experiments

We fabricated an annular $Al/AlO_x/Al$ junction of diameter $d = 100 \mu m$ and width $w = 2 \mu m$ using standard overlay electron beam lithography. We measured the transient response of the voltage when magnetic field decreased rapidly from $H = 1000 \text{ G}$ to zero magnetic field parallel to the junction plane below 0.1 K. The sample voltage was amplified 100-fold and measured with a signal analyzer (35670A, Agilent). The magnetic quenching rate was determined by monitoring the current of the magnet. In the magnetic quench method, the rate of temperature rise was suppressed by up to 4.4 % of $T_c$. Using the same setup, we measured the transient response of the voltage in $ab$ plane of a $Sr_2RuO_4$ single crystal using the magnetic quench measurement parallel to $c$ axis. The sample of the $Sr_2RuO_4$ crystal have an electrode spacing of 150 $\mu m$.

3. Results and Discussions

We observed the power law and the critical exponent in the magnetic quench measurement of the $Al/AlO_x/Al$ junction and the $Sr_2RuO_4$ crystal. Figure 1(a) shows a transient response of the voltage of the $Al/AlO_x/Al$ junction and the sweeping magnetic field, as magnetic field is swept rapidly. In what follows, we define a quench time $\tau_Q$ as $dH/dt = -H_c/\tau_Q$. In the equilibrated transition, electrical properties of the $Al/AlO_x/Al$ junction were superconducting transition temperature $T_c = 1.16 \text{ K}$ and the critical field $H_c = 682 \text{ G}$. We observed distinct steps of the voltage by the magnetic quenching. The steps exhibit the vortices are kept out of the junction. Here we focus on the second step, which denotes the second decrease of the voltage from normal state to superconducting state, as shown in the vertical dot line of Fig. 1(a), since the step arises each time even by varying the quench time $\tau_Q$. In general, the transition occurs in the critical magnetic field $H_c = 682 \text{ G}$ as the magnetic field decreases from $H = 1000 \text{ G}$ to zero magnetic field in equilibrium. On the other hand, we observed a delay of the transition by the magnetic quenching. In Fig. 1(a), the transition of the second step occurs in magnetic field of $H' = 300 \text{ G}$. Here the magnetic field $H'$ represents magnetic field of the second step each quench time $\tau_Q$. We performed the magnetic quench measurement by varying $\tau_Q$ from about 0.1 s to 4.0 s. The squares of Fig. 2 shows the inverse of $(H_c - H')/H_c$ is power function of $\tau_Q$. Thus we find the power law in the magnetic quench measurement using the $Al/AlO_x/Al$ junction of $U(1)$ superconductors. The critical exponent is $0.30 \pm 0.01$.

Figure 1(b) shows a transient response of the voltage of the $Sr_2RuO_4$ crystal and the sweeping magnetic field, as magnetic field is swept rapidly. We observed distinct steps of the voltage and a delay of the transition to superconducting state. In equilibrated transition, the properties of $Sr_2RuO_4$ were $T_c = 0.92 \text{ K}$ and $H_c = 550 \text{ G}$. Using the same analysis method, we obtain $(H_c - H')/H_c$ is a power function of $\tau_Q$ as shown in the triangles of Fig. 2. The critical
exponent of the Sr$_2$RuO$_4$ crystal is 0.40 ± 0.01, which is close to the critical exponent of U(1) superconductor.

Let us discuss about the power law and the critical exponent in the quantum phase transition. In thermal phase transitions, Zurek predicts that the domain (correlation) length $\xi$ is a power function of $\tau_Q$ [2]. Recently the critical exponent of $\mu_{U(1)} = 1/2$ in a thermal phase transition of U(1) superconductors have reported in Ref. [7]. However the critical exponent in a quantum phase transition may be different due to quantum fluctuations of the superconducting phase $\theta$ [11]. In quantum phase transitions at temperature $T = 0$, the KZM works as follows

$$\xi = \tau_Q^{\frac{\nu}{1+z\nu}}, \quad (1)$$

where $z$ is a dynamical critical exponent and $\nu$ is a static critical exponent [9]. Here $\nu/(1+z\nu)$ is represented as $\mu_{U(1)}$ and $\mu_{SU(2)}$. The results show the inverse of $(H_c - H')/H_c$ is the power function of $\tau_Q$. Thus we consider the inverse of $(H_c - H')/H_c$ is related to $\xi$. The magnetic quench measurement is adaptable to the KZM instead of thermal quench. We have $\nu = 3/7$ which assign $\mu_{U(1)} = 0.30 \pm 0.01$ and charged systems of $z = 1$ to the critical exponent $\mu_{U(1)}$ of Eq. (1). Thus the critical exponent of $\nu$ is close to a mean-field value in charged systems. Assuming $z = 1$ and a mean-field of $\nu = 1/2$, the critical exponent represents $\mu_{U(1)} = 1/3$. In this way, the magnetic quench measurement is efficient method to determine $z$ and $\nu$ of the KZM in quantum phase transitions.

In SU(2) superconductors, internal degrees of freedom of order parameter may generate unconventional vortices such as textures, while, in U(1) superconductors, the Abrikosov vortex (line vortex) is occurred by an integer winding number of phase. It is important to make a comparison of the critical exponent between U(1) and S(2) superconductors. We obtain $\mu_{SU(2)} = 0.40 \pm 0.01$ which is consistent with $z = 1$ and $\nu = 2/3$. Both $\mu_{U(1)}$ and $\mu_{SU(2)}$ have an approximate value in spite of topological defects given by internal degrees of freedom of order parameter.

In summary, we have observed a power law and a critical exponent of the KZM in a quantum phase transition using an Al/AlO$_x$/Al junction and a Sr$_2$RuO$_4$ crystal. Using U(1) superconductors, we find the magnetic quench measurement is efficient method to determine the
critical exponent $z$ and $\nu$ of quantum phase transitions. The critical exponent of $0.30 \pm 0.01$ is different from that of thermal phase transition. We have carried out the magnetic quench measurement of SU(2) superconductors. From the comparison of the results, we consider the critical exponent of $\mu_{U(1)}$ and $\mu_{SU(2)}$ is approximately equal despite internal degrees of freedom of order parameter.

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