A Low Cost Approach to the Design of Autopilot for Hypersonic Glider

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Abstract. This paper proposes a novel integrated guidance and control (IGC) approach to improve the autopilot design with low cost for hypersonic glider in dive and pull-up phase. The main objective is robust and adaptive tracking of flight path angle (FPA) under severe flight scenarios. Firstly, the nonlinear IGC model is developed with a second order actuator dynamics. Then the adaptive command filtered back-stepping control is implemented to deal with the large aerodynamics coefficient uncertainties, control surface uncertainties and unmatched time-varying disturbances. For the autopilot, a back-stepping sliding mode control is designed to track the control surface deflection, and a nonlinear differentiator is used to avoid direct differentiating the control input. Through a series of 6-DOF numerical simulations, it’s shown that the proposed scheme successfully cancels out the large uncertainties and disturbances in tracking different kinds of FPA trajectory. The contribution of this paper lies in the application and determination of nonlinear integrated design of guidance and control system for hypersonic glider.

1. Introduction

Hypersonic flight has the characteristic of great fast velocity and long distance, thus it has attracted great interests of researchers in recent years. However, hypersonic flight dynamics is associated with coupled nonlinearities and aerodynamics uncertainties, and the guidance and control system requires both good transient performance and high steady accuracy, so its design is a challenging task.

Compared to the conventional design of guidance and control system, the integrated method benefits a lot for its low cost and high accuracy [1, 2]. The rationale of integrated method is to develop the guidance and control system based on the overall nonlinear system dynamics. This paper has further consideration of actuator dynamics based on the integrated guidance and control scheme, because large chattering and peak phenomenon always appears in the control input when the system dynamics contains large nonlinearity and uncertainty. Basically, state feedback back-stepping technique is a powerful tool in the controller design of high order nonlinear system [3]. The essence of back-stepping control is recursively design of appropriate state as pseudo control input for lower dimension subsystems. However, back-stepping control suffers from repeated differentiations of the virtual control inputs, so-called “explosion of complexity” [4]. As a result, the complexity of the controller grows drastically when the order of system dynamics is high. In [5, 6], differentiators are employed to construct the derivatives of the virtual control inputs and it solves the problem. In order to deal with unmatched disturbances in back-stepping design, the disturbance observer is introduced to improve the robustness in [7].

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To achieve the adaptive design of integrated guidance and autopilot system, this paper combines back-stepping technique and $L_1$ adaptive control. $L_1$ adaptive control can adapt to varying system dynamics, cancel out uncertainties and nonlinearities. In [8], $L_1$ adaptive control architecture is introduced, which has guaranteed transient and steady performance bounds for system’s input and output signals in the presence of fast adaptation without any gain scheduling in the controller parameters, and without high gain feedback. It is a powerful and low-cost controller in the presence of large uncertainties, un-modelled dynamics, and disturbances. The features have been verified in a large number of flight tests in recent papers [9-11]. In [9], output feedback $L_1$ adaptive control is designed and applied to missile guidance and control system. It demonstrates robustness to significant changes in the missile dynamics up to $\pm 50\%$ change in the missile longitude model parameters. In [10], the augmented base-line controller is used to improve angle of attack (AOA) control of an unstable aircraft with multiple control surfaces. In [11], it mainly addresses the application of $L_1$ adaptive control in IGC design for hypersonic glider. A dynamics pole placement controller is implemented as the baseline controller, and it is augmented with an $L1$ adaptive controller to cancel out the matched and unmatched uncertainties. In [12], control input constraint is considered in $L1$ adaptive controller with a simple form.

This paper is organized as follows: In Section 2, the nonlinear integrated guidance and autopilot model is built with uncertain aerodynamics coefficient parameters and time-varying disturbances, and a second order actuator dynamics is considered. In Section 3, the hybrid back-stepping controller is designed, first and third order nonlinear differentiators are used to avoid the direct differential of virtual control input. In Section 4, numerical simulations based on CAV-L are made to verify the proposed scheme. Two scenarios with aerodynamics coefficients uncertainties are set to track two kinds of FPA trajectories in hypersonic glide phase. In Section 5, the paper is concluded.

2. Problem Formation
In this section, the vertical flight dynamics is developed in the presence of uncertain aerodynamic parameters. The planar geometry of the hypersonic glider is depicted in figure 1. The unpowered glide phase can be divided into dive phase, pull-up phase and quasi-glide phase in sequence based on the characteristic of FPA trajectory. Thus different kinds of FPA trajectory are set in the numerical simulation. In figure 1, $x_s$ is the symmetrical body axis. The coordinate systems are built without considering the earth rotation. $\gamma$, $\alpha$, $\omega_c$ are respectively the FPA, AOA and pitch angle rate, $\delta_e$ is the control surface deflection. The angle and angle rate directions shown in figure 1 are positive. $V$ is constant glide velocity, $m$, $g$ are constant mass and gravity acceleration

![Figure 1. Planar geometry in glide phase.](image)

2.1. Longitudinal Dynamics
The nonlinear system dynamics is built as equation (1).
\[\dot{y} = \ell_o + \ell_
abla \alpha - \frac{g}{V} \cos \gamma \]
\[\dot{\alpha} = \omega_o - \ell_o - \ell_
abla \alpha + \frac{g}{V} \cos \gamma \]
\[\dot{\omega}_\varepsilon = \bar{M}_\varepsilon + \bar{M}_\delta \delta_\varepsilon \]

When the flight velocity is high, the lift force coefficient can be expressed as \[C_L = C_L^\alpha + C_L^\omega,\] so the lift consists of two parts:
\[L = \frac{1}{2} \rho V^2 C_L = L_o \alpha + L_a\] (2a)

where \[C_L^\alpha\] is the lift coefficient derivative with respect to AOA, and \[L_o = \frac{1}{2} \rho V^2 C_L^\omega\] is the other factors contributing to lift. The two parts of lift are unified by momentum as following:
\[\ell_o = \frac{L_o}{mV} = \frac{1}{2} \rho V C_L^\alpha, \quad \ell_a = \frac{L_a}{mV} = \frac{1}{2} \rho V C_L^\omega\] (2b)

Also, the aerodynamics moment is described as \[M_t = M_\delta \delta_\varepsilon + M_a,\] is the moment coefficient derivative with respect to \[\delta_\varepsilon,\] and \[M_a\] is the other factors contribution to moment. Both are unified by the moment of inertia with respect to z-axis \[I_z:\]
\[\bar{M}_a = \frac{M_a}{I_z}, \quad \bar{M}_\delta = \frac{M_\delta}{I_z}\] (3)

### 2.2. Integrated Guidance and Autopilot

Redefining the flight states as \[x_1 = \gamma, \quad x_2 = \alpha, \quad x_3 = \omega,\] and \[x_4 = \delta_\varepsilon.\] Then equation (1) is reshaped as follows:
\[\dot{x}_1 = f_1(x,t) + x_2 \]
\[\dot{x}_2 = f_2(x,t) + x_3 \]
\[\dot{x}_3 = f_3(x,t) + b x_4 \]
\[\dot{x}_4 = f_4(x,t) + b x_4 \]

The nonlinear functions with unmatched time-varying disturbances \[d_i(t), \quad i = 1, 2, 3\] in equation (4) are depicted as follows:
\[f_1(x,t) = \ell_o + (\ell_o - 1) x_2 - \frac{g}{V} \cos(x_1) + d_1(t) \]
\[f_2(x,t) = -\ell_a - \ell_a x_3 + \frac{g}{V} \cos(x_1) + d_2(t) \]
\[f_3(x,t) = \bar{M}_\delta + d_3(t), \quad b = \bar{M}_a \]

We have two assumptions about the nonlinear functions. Firstly, there exists \[B > 0, \quad \text{such that } |f_i(t,0)| \leq B, \quad i = 1, 2, 3, \text{ holds for all } t \geq 0.\] Secondly, assuming that \[f_i(x,t)\] is uniform bounded and the partial derivatives are semi-global uniform bounded. Then the following upper bound is yielded:
\[\|f_i(x,t)\|_{\infty} \leq L_{\alpha_i} \|x\|_{\infty} + B \]

where \[L_{\alpha_i}\] is the bound of partial derivatives of the nonlinear functions \[f_i(x,t).\] Besides \[b \epsilon [b_l, b_u], \quad 0 < b_l < b_u\] are known conservative bounds.

For the hypersonic glider, the control objective is to design a robust controller in the presence of uncertainties and disturbances to ensure the system output \[y = x_1 = \gamma\] to track the reference signal under \[x_4 = \delta_\varepsilon.\] Additionally, the dynamics of actuator is integrated with the vertical flight. This paper considers a second order actuator dynamics given by the following transfer function:
By defining $x_s = \delta_s$, $u = \delta_u$, the state space model of actuator dynamics is built as follows with uncertainty and disturbance.

\[
\dot{x}_d = x_s \\
\dot{x}_s = f_s(x_s, x_d) + \Delta f_s + b_s u(t) + d_s(t)
\]

where

\[
f_s(x_s, x_d) = 2\omega_s \dot{x}_s - \omega_s x_s, \quad b_s = K_s
\]

In equation (8), $\Delta f_s$ is the uncertainty and $d_s(t)$ is the time-varying disturbance. They are bounded under positive constant $F$, $D$.

To get a strict feedback form of the state space, RBF neural network (RBFNN) is implemented to make $f_s(x_s, t)$ only relate to $(x_s, t)$. RBFNN is one of the most widely used networks models in nonlinear control systems. Theoretically, as long as a sufficient number of neurons are employed, RBFNN can approximate any continuous function to an arbitrary accuracy on any compact set.

\[
f_s(x_s, t) = \Phi(x_s) - \frac{g}{V} \cos(x_s) + d_s(t)
\]

In equation (11), $\Phi(\cdot)$ is depicted as follows:

\[
\Phi(x_s) = \theta^T \xi(x_s) + \Delta
\]

where $\Delta$ is neural network functional approximation error, $x_s$ is the input of RBFNN, $\theta = [\theta_1, \theta_2, \ldots, \theta_n]^T \in \mathbb{R}^n$ is the weight vector to be determined, and $n$ is the node number of RBFNN. $\xi(x_s)$ is a vector of Gaussian functions of equation (13).

\[
\xi_j(x_s) = \exp\left(-\frac{||x_s - \xi_j||^2}{2\beta^2}\right), \quad j = 1, 2, 3, \ldots, n
\]

$\xi = [\xi_1, \xi_2, \ldots, \xi_n]^T$ is the central vector of the Gaussian function, and $\beta$ is the width. In order to realize small reconstruction errors, it is necessary to use high-order networks. The weighing vector updates as the following adaptive law:

\[
\dot{\theta} = -\lambda (e_{of} \xi + \sigma \theta)
\]

Where $\lambda$, $\sigma$ is positive constant, $e_{of}$ is the estimate error. Assuming there has an ideal estimate of $\Phi_d(x_s)$, then the optimal weighing vector is given as follows:

\[
\theta^* = \arg\min\left[\sup|\Phi_d - \Phi|\right]
\]

3. Controller Design

In this section, back-stepping $L_1$ adaptive controller is designed for IGC system depicted as equation (5). This state feedback controller mainly compensates the uncertainties of the nonlinear plant with the help of low-pass command filters $C(s)$. The filters ensure the controller remains in low frequency range in the presence of fast adaptation and large reference input. Here this adaptive control scheme is chosen for the following reasons: 1) fast adaption without high gain; 2) great robustness to large system uncertainties and different reference signals; 3) reduced cost for the validation and verification process. Then sliding mode controller with nonlinear differentiator is implemented for the autopilot to track the control surface deflection.

3.1. Command Filters
Let

\[ C(s) = \begin{bmatrix} C_1(s) \\ C_2(s) \\ C_3(s) \end{bmatrix} \]  

(16)

In this case, the design of L₁ adaptive controller involves three stable low-pass filters. Technically, any transfer function can be used as long as the filters keep the stability of the state feedback controller. Simple first and second order linear transfer function can often result in good performance. Here \( C_1(s) \), \( C_2(s) \) is verified by relative degree of at least 2 within unity DC gain, i.e. \( C_1(0) = 1 \), \( C_2(0) = 1 \). \( C_3(s) \) have positive feedback gains \( k_2 \), \( k_3 \) and strictly proper transfer functions \( D_2(s) \), \( D_3(s) \).

\[ C_1(s) = \frac{k_1 D_1(s)}{1 + k_1 D_1(s)} \]
\[ C_2(s) = \frac{k_2 D_2(s)}{1 + k_2 D_2(s)} \]
\[ C_3(s) = \frac{b_k D_3(s)}{1 + b_k D_3(s)} \]  

(17)

\( C_i(s) \) is also within unity DC gain.

3.2. State Predictor

Under the assumptions of the nonlinear functions, the IGC model can be approximated as follows:

\[ x_i = \theta_i \|x\|_e + \sigma_1 + x_2 \]
\[ x_2 = \theta_2 \|x\|_e + \sigma_2 + x_3 \]
\[ x_3 = \theta_3 \|x\|_e + \sigma_3 + bx_4 \]  

(18)

then consider the following state predictor:

\[ \dot{x}_1 = -a_1 \dot{x}_1 + \dot{\theta}_1 \|x\|_e + \dot{\sigma}_1 + x_2 \]
\[ \dot{x}_2 = -a_2 \dot{x}_2 + \dot{\theta}_2 \|x\|_e + \dot{\sigma}_2 + x_3 \]
\[ \dot{x}_3 = -a_3 \dot{x}_3 + \dot{\theta}_3 \|x\|_e + \dot{\sigma}_3 + bx_4 \]  

(19)

where \( \dot{x}_1, \dot{x}_2, \dot{x}_3 \) are the predictor states, \( \dot{x}_4 = \dot{x}_1 - x_1, \dot{x}_2 = \dot{x}_2 - x_2, \dot{x}_3 = \dot{x}_3 - x_3 \) are the prediction error, \( \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3, \dot{\sigma}_1, \dot{\sigma}_2, \dot{\sigma}_3, b \) are the adaptive estimates.

3.3. Adaptation

The adaptive estimates are governed by

\[ \dot{\theta}_1 = \Gamma \text{proj} \left( \dot{\theta}_1, -x^T P \begin{bmatrix} 1,0,0 \end{bmatrix}^T \|x\|_e \right) \]
\[ \dot{\theta}_2 = \Gamma \text{proj} \left( \dot{\theta}_2, -x^T P \begin{bmatrix} 0,1,0 \end{bmatrix}^T \|x\|_e \right) \]
\[ \dot{\theta}_3 = \Gamma \text{proj} \left( \dot{\theta}_3, -x^T P \begin{bmatrix} 0,0,1 \end{bmatrix}^T \|x\|_e \right) \]
\[ \dot{\sigma}_1 = \Gamma \text{proj} \left( \dot{\sigma}_1, -x^T P \begin{bmatrix} 1,0,0 \end{bmatrix}^T \right) \]
\[ \dot{\sigma}_2 = \Gamma \text{proj} \left( \dot{\sigma}_2, -x^T P \begin{bmatrix} 0,1,0 \end{bmatrix}^T \right) \]
\[ \dot{\sigma}_3 = \Gamma \text{proj} \left( \dot{\sigma}_3, -x^T P \begin{bmatrix} 0,0,1 \end{bmatrix}^T \right) \]
\[ \dot{b} = \Gamma \text{proj} \left( \dot{b}, -x^T P \begin{bmatrix} 0,0,1 \end{bmatrix}^T u \right) \]  

(20)

Let \( A_a \) be defined as:

\[ A_a = \text{diag} \begin{bmatrix} -a_1 & -a_2 & -a_3 \end{bmatrix} \]  

(21)

where \( a_1, a_2, a_3 > 0 \) are positive constants specifying the desired closed-loop dynamics. \( \Gamma \in \mathbb{R}^+ \) is the adaptation gain, and the symmetric positive definite matrix \( P = P^T > 0 \) solves the Lyapunov equation \( A_a^T P + PA_a = -Q \) for arbitrary \( Q = Q^T > 0 \). The projection operator \( \text{proj}(\bullet, \bullet) \) ensures that \( \| \dot{\theta}_i(t) \| \leq \theta_a, \| \dot{\sigma}_i \| \leq \Delta, i = 1, 2, 3 \) and \( \| \dot{b}(t) \| \leq \Omega \).
\[
A_g = \begin{bmatrix}
-a_1 & 1 \\
-a_2 & 1 \\
-a_3 & 1
\end{bmatrix}
\]  \hspace{1cm} (22)

and \( G(s) = H(s)(I-C(s)) \), \( H(s) = (sI-A_g)^{-1} \), the selection of \( C(s) \) need to ensure the following \( L_1 \)-norm upper bound can be verified.

\[ \|G(s)\|_{L_1} < \frac{\rho - P}{K_1 \rho + K_2} \]  \hspace{1cm} (23)

where \( K_1, K_2, P \) are constants related to \( \rho \) and \( B \).

### 3.4. Control law

The adaptive controller for the integrated guidance and autopilot system made up by equation (5) and (8) is designed step-by-step. Consider the first subsystem:

\[ \dot{x}_1 = \theta_1 \|x\|_n + \sigma_1 + x_2 \]  \hspace{1cm} (24)

In equation (24), \( x_2 \) is the virtual control input, it’s depicted as follows:

\[ x_2 = -a_4 (x_1 - \hat{\eta}_c(t) + \dot{r}) \]  \hspace{1cm} (25a)

and

\[ \hat{\eta}_c(s) = C_1(s) \hat{\eta}(s) \]  \hspace{1cm} (25b)

where

\[ \hat{\eta}(t) = \hat{\eta}_1(t) \|x(t)\|_n + \sigma_i \]  \hspace{1cm} (26)

\( \hat{\eta}(s) \) is the Laplace transform of \( \hat{\eta}(t) \).

Consider the second subsystem:

\[ \dot{x}_2 = \theta_2 \|x\|_n + \sigma_2 + x_3 \]  \hspace{1cm} (27)

where

\[ x_3 = -k_2 D_s(s) \hat{\eta}_3(s) \]  \hspace{1cm} (28)

In equation (28), \( \hat{\eta}_3(s) \) is the Laplace transform of the following equation (29a).

\[ \hat{\eta}_3(t) = \hat{\eta}_3(t) + a_4 (x_3(t) - \hat{x}_3(t)) - \tilde{x}_3(t) \]  \hspace{1cm} (29a)

and

\[ \hat{\eta}_3(t) = \sigma_1(t) + \tilde{\eta}_3(t) \|x(t)\|_n + \hat{\sigma}_3(t) \]  \hspace{1cm} (29b)

The third subsystem is depicted as

\[ \dot{x}_3 = \theta_3 \|x\|_n + \sigma_3 + b x_4 \]  \hspace{1cm} (30)

The control input is depicted as follows:

\[ \tilde{x}_4(s) = -k_1 D_s(s) \hat{\eta}_4(s) \]  \hspace{1cm} (31)

where \( \hat{\eta}_4(s) \) is the Laplace transform of the signal in equation (32a).

\[ \hat{\eta}_4(t) = \hat{\eta}_4(t) + a_4 (x_4(t) - \hat{x}_4(t)) - \tilde{x}_4(t) \]  \hspace{1cm} (32a)

and

\[ \hat{\eta}_4(t) = \dot{b}(t) u(t) + \tilde{\eta}_4(t) \|x(t)\|_n + \hat{\sigma}_4(t) \]  \hspace{1cm} (32b)

To get the second order derivatives of \( \tilde{x}_4 \), a third order differentiator is implemented:

\[ \dot{x}_{\text{shod}1} = x_{\text{shod}2} \]

\[ \dot{x}_{\text{shod}2} = x_{\text{shod}3} \]

\[ \dot{x}_{\text{shod}3} = \frac{4}{\varepsilon^3} \left[ 2^{-7} \|x\|_n^7 \operatorname{sgn}(\nu) - \varepsilon^2 x_{\text{shod}3} \|x\|_n^7 \operatorname{sgn}(\varepsilon^2 x_{\text{shod}3}) \right] \]  \hspace{1cm} (33)

\[ \nu = x_{\text{shod}1} - x_{\text{shod}3} + \frac{1}{\varepsilon^3} x_{\text{shod}2} \|x\|_n^7 \operatorname{sgn}(\varepsilon x_{\text{shod}2}) \]  \hspace{1cm} (34)

\[ x_{\text{shod}1}(0) = \tilde{x}_4(0) \]

\[ x_{\text{shod}2}(0) = 0 \]

\[ x_{\text{shod}3}(0) = 0 \]
The error between \( x_{\text{shod}} \) and \( \bar{x}_i \) can be enough small through choosing suitable \( \bar{e} \). Then \( x_{\text{shod}} \) and \( x_{\text{shod}} \) are respectively the first and second order derivatives of \( \bar{x}_i \).

Defining a scalar \( e = x_i - \bar{x}_i \) and a vector \( E = [e, \dot{e}]^T \), then the following nonlinear sliding mode surface is designed.

\[
\sigma = CE - CP
\]  

In equation (34), \( C = [c_1, c_2] \) is a constant vector, and \( P = [p, \dot{p}]^T \) is determined by the following nonlinear function:

\[
p(t) = \begin{cases} 
e + \frac{1}{2} \dot{\bar{e}} \bar{e}^2 &= \left( -\frac{10}{T^3} \bar{e} + \frac{6}{T} \bar{e} + \frac{3}{2T} \dot{\bar{e}} \right) t^3 + \left( \frac{15}{T^3} \bar{e} + \frac{8}{T^3} \dot{\bar{e}} + \frac{3}{2T} \ddot{\bar{e}} \right) t \bigg|_{t=0}^{t=T} \\ 0, t > T & 0 \leq t \leq T \end{cases}
\]  

(35)

It can be seen that \( T \) is the convergence time of this nonlinear sliding mode controller. Finally, the control input of integrated guidance and autopilot is designed as follows:

\[
u = -\frac{1}{b} \left( f_1(x) - x_{\text{shod}} - \dot{p} + \frac{c_1}{c_2} (\dot{e} - \ddot{p}) \right) - \frac{1}{b} \operatorname{sgn}(c_2 \sigma)(F + D)
\]  

(36)

4. Numerical Simulation

To verify numerically the performance of the proposed method, two simulation scenarios with aerodynamics force and moment coefficient uncertainties are performed. The aerodynamics coefficients are chosen based on CAV-L in Appendix A. Two kinds of FPA trajectory are set as follows:

\[
r(t) = 10 \sin \left( \frac{2\pi}{15} t \right) + 3
\]  

(37a)

\[
r(t) = \begin{cases} 2t, t \geq 15 & 0 \leq t \\ -2t + 60, t < 15 & \end{cases}
\]  

(37b)

Scenario 1:

\[
\begin{align*}
\bar{L}_p &= -1.25, \quad \bar{L}_q = 1.1 \\
f_1(x, t) &= \Phi(x_1) - 0.1 \cos(x_1) - 0.2 \sin(0.1 r) \\
f_2(x, x, t) &= -0.25 - 1.1 x_2 + 0.1 \cos(x_1) - 0.2 \sin(0.1 r) \\
f_3(x, t) &= 1 \\
b &= 0.8
\end{align*}
\]

Scenario 2:

\[
\begin{align*}
\bar{L}_p &= -1.25, \quad \bar{L}_q = 1.1 + 0.1 x_2 \\
f_1(x, t) &= -1.25 + \Phi(x_2) - 0.1 \cos(x_1) - 0.2 \sin(0.1 r) \\
f_2(x, x, t) &= -1.25 - 0.1 x_2^2 - 1.1 x_2 + 0.1 \cos(x_1) - 0.2 \sin(0.1 r) \\
f_3(x, t) &= 0.01 x_1 + 0.01 x_2 + 0.01 x_3 + 0.05 \sin(t) \\
b &= 0.8
\end{align*}
\]

(39)

Comparing with Scenario 1, the lift force coefficient in Scenario 2 has factor of AOA and the moment is related to flight states. The second order actuator dynamics is set with \( \omega_n = 30, \xi = 0.3 \), and is saturated with magnitude maximum 30deg. However, a single \( L_1 \) is designed for the two Scenarios with the following control parameters:

The state predictor is given by \( a_1 = a_2 = a_3 = 2 \). The adaptation is determined by \( Q = \operatorname{diag} [150 \quad 1 \quad 1] \), and the projection bound is set as \( \Omega = [0.1, 3] \), the adaptation gain is \( \Gamma = 10000 \).


The three filters are set as follows:

\[ C_1(s) = \frac{3}{s^2 + 1.1s + 1.3} \]
\[ C_2(s) = \frac{100}{s^2 + 15s + 100} \]
\[ C_3(s) = \frac{30}{s + 30} \]

(40a) (40b) (40c)

The RBFNN is setting as follows:

\[ j = 9, \ b = 2.5, \ \lambda = 0.15, \ \sigma = 0.5, \ \epsilon = [-1 \ -0.7 \ -0.5 \ -0.3 \ 0 \ 0.3 \ 0.5 \ 0.7 \ 1]. \]

Here the estimate error of RBFNN can be combined in the system uncertainties.

The simulation results of tracking sinusoidal FPA reference trajectory in equation (37a) are shown in figure 2 to figure 5 and that of step FPA reference trajectory in equation (37b) are shown in figure 6 to figure 9. From figure 2 and figure 6, it can be seen that the proposed IGC control method can track the reference FPA trajectory under lift and moment coefficient uncertainties and time-varying disturbances in different channel. Figure 3 shows the AOA curves under reference FPA of equation (37a), and compares the AOA curve of Scenario 1 with Scenario 2. The second order item of AOA in S2 makes the curve smoother. Figure 4 shows the pitch angle rate comparison. The control surface deflections are presented in figure 5, which shows that the nonlinearity of moment coefficients results in control surface deflection chasing in a small range.

Figure 2. The tracking result of FPA

Comparison of FPA Curves

Figure 3. The AOA comparison S1 with S2

Comparison of IGC AOA

Figure 4. The pitch angle rate comparison

Figure 5. The control surface deflection comparison

Figure 7 shows the AOA curves under reference FPA of equation (37b), and compares the AOA curve of Scenario 1 with Scenario 2. Figure 8 shows the pitch angle rate comparison. The control surface deflections are presented in Figure 9.
Motivated by the reviews and discussions on the integrated guidance and control design, adaptive command filtered control is firstly proposed for the nonlinear IGC with second order actuator dynamics in the presence of aerodynamics coefficient uncertainties and disturbances for hypersonic glider in the glide phase. Through the numerical simulations of FPA tracking in different scenarios, it can be concluded that adaptive command filtered IGC can keep fast transient response and good robustness without changing the controller setting, thus significantly reduce the amount of Monte-Carlo analysis for V&V. What’s more, the proposed IGC avoids input peaking and chasing, which means better in practical application.

### 6. Appendix A

The aerodynamics of NASA CAV-L in 1998 is performed in the following tables.

| Mach | AOA | 3.5 | 5 | 8 | 15 | 20 |
|------|-----|-----|---|---|----|----|
| 10°  | 0.3401 | 0.3264 | 0.3108 | 0.2856 | 0.2760 |
| 15°  | 0.5786 | 0.5358 | 0.4883 | 0.4991 | 0.4349 |
| 20°  | 0.7975 | 0.7291 | 0.6731 | 0.6137 | 0.5975 |
In hypersonic flight, the lift coefficient can be simplified as \( C_L = C^\alpha_L \alpha + C_{\alpha} \). The fit results of lift coefficient in Table A.1 are shown in Figure A.1, and the parameters are listed in Table A.2.

**Figure A.1** Polynomial fit results

Take \( Mach = 4, \ H = 20km \) and \( S_{ref} = 0.35m^2 \) as example to calculate the aerodynamics parameters:

\[
\mathcal{L}_a = \frac{L_a}{mV} = \frac{1}{2} \rho V S_{ref} C_{\alpha} = -1.673, \quad \mathcal{L}_e = \frac{L_e}{mV} = \frac{1}{2} \rho V S_{ref} C^\alpha_{\alpha} = 0.9149
\]

**Table A.2.** Fit curve parameters

| Mach | \( C^\alpha_{\alpha} \)  | \( C_{\alpha} \)  |
|------|----------------|----------------|
| 3.5  | 0.04574        | -0.114         |
| 5    | 0.04027        | -0.07362       |
| 8    | 0.03623        | -0.05272       |
| 15   | 0.03281        | -0.04268       |
| 20   | 0.03215        | -0.4612        |

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