The Higgs Mechanism in $N = 2$ Superspace

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Abstract

We describe the Higgs mechanism for general $N = 2$ super Yang-Mills theories in a manifestly supersymmetric form based on the harmonic superspace.

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1 Introduction

During the last two years, \( N = 2, D = 4 \) supersymmetric field theories have attracted considerable interest kindled by the papers of Seiberg and Witten [1] on the exact determination of the low energy effective action in \( N = 2 \) supersymmetric \( SU(2) \) gauge models with spontaneously broken gauge symmetry (see Refs. [2, 3, 4] for a pedagogical introduction).

The \( N = 2 \) supersymmetric gauge multiplet and its action were constructed by Grimm, Sohnius and Wess [6, 7] in \( N = 2 \) superspace \( \mathbb{R}^{4|8} \) in terms of constrained superfields. In the harmonic superspace \( \mathbb{R}^{4|8} \times S^2 \) [8, 9, 10] one can formulate the \( N = 2 \) supersymmetric gauge models in terms of unconstrained (so-called analytic) superfields and study quantum corrections in a manifestly supersymmetric fashion [11, 12].

In this paper we investigate the spontaneous breakdown of gauge symmetry for general \( N = 2 \) supersymmetric gauge models in harmonic superspace (see Ref. [13] for a review of spontaneous symmetry breakdown in \( N = 1 \) supersymmetric theories). Such models admit two types of classical ground states: the ones which are invariant under \( SU(2)_A \), the (sub-) group of outer automorphisms of the \( N = 2 \) supersymmetry algebra, and the others which break \( SU(2)_A \). We analyze in detail the first type of Higgs vacua. In this case only vacuum expectation values of the scalar fields of the gauge multiplet and of matter \( \omega \)-hypermultiplets [14] can occur. The theories possess three different physical phases which were described by Fayet [15] in the framework of \( N = 2 \) supersymmetric grand unified theories.

The paper is organized as follows. In section 2 we review the geometry of \( N = 2 \) supersymmetric gauge models both in the standard \( N = 2 \) superspace and in its harmonic extension. Section 3 is devoted to the analysis of the spontaneous breakdown of gauge symmetry for \( N = 2 \) supersymmetric gauge models with matter. In section 4 we discuss the quantum equations of motion for the \( N = 2 \) supersymmetric gauge multiplet. The three appendices contain technical details.

2 \( N = 2 \) super Yang-Mills geometry

\( N = 2 \) supersymmetric gauge models are constructed in \( N = 2 \) superspace with coordinates

\[
z^M = (x^m, \theta^i, \bar{\theta}^\dot{i}) \quad \bar{\theta}^\dot{i} = \theta^{i\dot{i}} \quad i = 1, 2
\] (2.1)
in terms of gauge covariant derivatives

\[ \mathcal{D}_M \equiv (\mathcal{D}_m, \mathcal{D}_a^i, \bar{\mathcal{D}}_{\dot{a}}^i) = D_M - A_M \quad \mathcal{A}_M = A_M^a(z)\delta_a \quad [\delta_a, D_M] = 0 \quad (2.2) \]

which are restricted by \[6, 7\]

\[
\begin{align*}
\{\mathcal{D}_a^i, \bar{\mathcal{D}}_{\dot{a}}^j\} &= -2i\delta_j^i \mathcal{D}_{a\dot{a}} \\
\{\mathcal{D}_a^i, \mathcal{D}_b^j\} &= -2\varepsilon_{\alpha\beta} \varepsilon_{ij} \mathcal{W} \\
[\mathcal{D}_{a\dot{a}}, \mathcal{D}_{\dot{a}j}] &= i\varepsilon_{\alpha\beta}(\bar{\mathcal{D}}_{\dot{a}}^i \mathcal{W}) \\
[\mathcal{D}_{a\dot{a}}, \bar{\mathcal{D}}_{\dot{a}j}] &= i\varepsilon_{\dot{a}\dot{b}}(\mathcal{D}_{a\dot{b}} \mathcal{W}) .
\end{align*}
\]

Here \(\mathcal{W}\) is a complex linear combination of generators \(\delta_a\) of a real Lie algebra

\[
\mathcal{W} = W^a(z)\delta_a \quad \bar{\mathcal{W}} = \bar{W}^a(z)\delta_a \\
[\delta_a, \delta_b] = f_{ab}^c \delta_c \\
[\delta_a, D_M] = 0 .
\]

If one applies \(\delta_a\) to tensors \(\phi\) one obtains \(\delta_a\phi = -iT_a\phi\) with matrices \(T_a\) which are typically taken to be hermitean, \((T_a)^\dagger = T_a\), such that \(iT_a\) generate a unitary representation of the gauge group and satisfy the same Lie algebra as \(\delta_a\). It is, however, worthwhile to allow for more general \(\delta_a\) even though we will not pursue such a generalization here. Below we will also make use of the notation

\[
A_M = A_M^aT_a \quad W = W^aT_a \quad \bar{W} = \bar{W}^aT_a = W^\dagger \quad (2.6)
\]

for unitary representations of the gauge group.

The super field strengths \(W^a\) satisfy the Bianchi identities

\[
\bar{\mathcal{D}}_{\alpha} \mathcal{W} = 0 \quad \mathcal{D}^{\alpha(i} \mathcal{D}^{j)} \mathcal{W} = \bar{\mathcal{D}}^{\dot{a}(i} \bar{\mathcal{D}}^{j)}_{\dot{a}} \bar{\mathcal{W}} .
\]

The transformation laws of \(\mathcal{D}_M\) and of matter multiplets \(U(z)\) read

\[
\mathcal{D}_M' = e^{\tau} \mathcal{D}_M e^{-\tau} \quad U' = e^{\tau} U \quad \tau = \tau^a(z)\delta_a 
\]

where the gauge parameters \(\tau^a\) are unconstrained real superfields.

An important feature of the \(N = 2\) supersymmetric gauge multiplet, in contrast to the \(N = 1\) case, is that one can have a covariantly constant super field strength

\[
\mathcal{D}_a^i \mathcal{W} = 0 \quad \Rightarrow \quad [\bar{\mathcal{W}}, \mathcal{W}] = 0 \quad \mathcal{D}_m \mathcal{W} = 0 .
\]

With the gauge transformations (2.8) one can cast the background value of the covariant derivatives into the form \[8\]

\[
\begin{align*}
\mathcal{D}_a^i &= D_a^i - \theta_0^a \bar{\mathcal{W}}_0 \\
\bar{\mathcal{D}}_{\dot{a}i} &= \bar{D}_{\dot{a}i} + \bar{\theta}_{0\dot{a}} \mathcal{W}_0 \\
\mathcal{D}_m &= \partial_m
\end{align*}
\]
with
\[
[W_0, W_0] = 0 \quad W_0 = W_0^\alpha \delta_\alpha \quad W_0^\alpha = \text{const}. \tag{2.11}
\]
Such a gauge fixing is super Poincaré covariant provided every supersymmetry transformation
\[
\delta U = (\epsilon_i^\alpha Q^i_\alpha + \bar{\epsilon}_i^\dot{\alpha} \bar{Q}^i_\dot{\alpha}) U \tag{2.12}
\]
is accompanied by the \(\epsilon\)-dependent gauge transformation
\[
\delta U = \tau U \quad \tau = \epsilon_i^\alpha \theta^i_\alpha W_0 + \bar{\epsilon}_i^\dot{\alpha} \bar{\theta}^i_\dot{\alpha} W_0. \tag{2.13}
\]
As a result, eq. (2.12) turns into
\[
\delta U = (\epsilon Q + \bar{\epsilon} \bar{Q}) U \tag{2.14}
\]
where
\[
Q^i_\alpha = \frac{\partial}{\partial \theta^i_\alpha} - i \bar{\theta}^\dot{\alpha} \partial_{\alpha \dot{\alpha}} + \theta^i_\alpha W_0 \quad Q_{\dot{\alpha} i} = - \frac{\partial}{\partial \theta^i_\alpha} + i \theta^\alpha \partial_{\alpha \dot{\alpha}} - \bar{\theta}^i_\dot{\alpha} W_0. \tag{2.15}
\]
These generators form the \(N = 2\) supersymmetry algebra with central charges \(W_0\) and \(\bar{W}_0\)
\[
\{Q^i_\alpha, Q^j_\beta\} = 2i \delta^i_j \partial_{\alpha \dot{\alpha}} \quad \{Q^i_\alpha, \bar{Q}^j_\dot{\beta}\} = 2 \varepsilon_{i j} \varepsilon_{\dot{\alpha} \dot{\beta}} W_0 \quad \{Q_{\dot{\alpha} i}, \bar{Q}_{\dot{\beta} j}\} = 2 \varepsilon_{\dot{\alpha} \dot{\beta}} \varepsilon_{i j} W_0 \tag{2.16}
\]
and eq. (2.11) defines the corresponding covariant derivatives, \(\{Q^i_\alpha, D^j_\beta\} = \{Q^i_\alpha, \bar{D}^j_\dot{\beta}\} = 0\).

In the harmonic superspace \(\mathbb{R}^{4|8} \times S^2\) one can solve \([8, 9, 10]\) the constraints (2.3) and (2.7) in terms of unconstrained analytic superfields and derive manifestly supersymmetric Feynman rules.

It is useful to parameterize the two-sphere \(S^2 = SU(2)/U(1)\) by harmonics, i.e. group elements
\[
(u^-_i, u^+_i) \in SU(2) \quad u^+_i = \varepsilon_{ij} u^{+j} \quad u^{+i} = u^-_i \quad u^{+i} u^-_i = 1. \tag{2.17}
\]

Then tensor fields over \(S^2\) are in a one-to-one correspondence with functions over \(SU(2)\) of definite \(U(1)\)-charges. A function \(\Psi^{(p)}(u)\) is said to have \(U(1)\)-charge \(p\) if
\[
\Psi^{(p)}(e^{i\alpha} u^+, e^{-i\alpha} u^-) = e^{i\alpha p} \Psi^{(p)}(u^+, u^-) \quad |e^{i\alpha}| = 1.
\]
The operators

\[ D_{\pm\pm} = u^{\pm i} \frac{\partial}{\partial u^{\pm i}} \quad D^0 = u^{+ i} \frac{\partial}{\partial u^{+ i}} - u^{- i} \frac{\partial}{\partial u^{- i}} \]

\[ [D^0, D_{\pm\pm}] = \pm 2D_{\pm\pm} \quad [D^{++}, D^{--}] = D^0 \]  \tag{2.18}

are left-invariant vector fields on \( SU(2) \) and \( D^0 \) is the \( U(1) \)-charge operator.

With use of the harmonics one can convert the spinor covariant derivatives into \( SU(2) \)-invariant operators on \( \mathbb{R}^4|_8 \times S^2 \)

\[ D_{\alpha}^\pm = D^i_{\alpha} u^\pm_i \quad \bar{D}_{\dot{\alpha}}^\pm = \bar{D}^i_{\dot{\alpha}} u^\pm_i. \]  \tag{2.19}

Then it follows from (2.3)

\[ \{D_{\alpha}^+, D_{\beta}^+\} = \{\bar{D}_{\dot{\alpha}}^+, \bar{D}_{\dot{\beta}}^+\} = \{D_{\alpha}^+, \bar{D}_{\dot{\alpha}}^+\} = 0 \]  \tag{2.20}

which can be solved by

\[ D_{\alpha}^+ = e^G D_{\alpha}^+ e^{-G} \quad \bar{D}_{\dot{\alpha}}^+ = e^G \bar{D}_{\dot{\alpha}}^+ e^{-G} \quad G = G^a(z, u) \delta_a. \]  \tag{2.21}

Here the superfields \( G^a \) have vanishing \( U(1) \)-charge and are real, \( \bar{G}^a = G^a \), with respect to the analyticity preserving conjugation \( \bar{\cdot} \equiv \bar{\cdot}^\ast \) [8], where the operation \( \bar{\cdot}^\ast \) is defined by \( (u^+_i)^* = u^-_i, (u^-_i)^* = -u^+_i \), hence \( (u^\pm_i)^{**} = -u^\pm_i \). Eq. (2.21) partially solves the constraints (2.3). An obvious consequence of the relations (2.20) and (2.21) is that the harmonic superspace allows one to introduce new superfield types, i.e. covariantly analytic superfields constrained by

\[ D_{\alpha}^+ \phi^{(p)} = D_{\alpha}^+ \phi^{(p)} = 0 \]  \tag{2.22}

and hence

\[ \phi^{(p)} = e^G \phi^{(p)} \quad D_{\alpha}^+ \phi^{(p)} = \bar{D}_{\dot{\alpha}}^+ \phi^{(p)} = 0. \]  \tag{2.23}

The superfield \( \phi^{(p)} \) turns out to be unconstrained over an analytic subspace of the harmonic superspace parameterized by \( x^m_A = x^m, \theta^{+\alpha}, \bar{\theta}^{+\dot{\alpha}} \) and \( u^\pm_i \), where [8]

\[ x^m_A = x^m - 2i \theta^{(i}(\sigma^m \bar{\theta})^{j)} u^+_i u^-_j \quad \theta^{\pm}_{\alpha} = u^\pm_i \theta^i_{\alpha} \quad \bar{\theta}^{\pm}_{\dot{\alpha}} = u^\pm_i \bar{\theta}^i_{\dot{\alpha}}. \]

Another crucial consequence of (2.20) and (2.21) is that the gauge group for the prepotential \( G \) is larger than the original \( \tau \)-group (2.8):

\[ e^{G^\prime} = e^{\tau} e^G e^{-\lambda} \quad \lambda = \lambda^a(\zeta_A, u) \delta_a \quad D_{\alpha}^+ \lambda^a = \bar{D}_{\dot{\alpha}}^+ \lambda^a = 0. \]  \tag{2.24}
Here the unconstrained analytic gauge parameters $\lambda^a$ have vanishing $U(1)$-charge. They are real, $\tilde{\lambda}^a = \lambda^a$, with respect to the analyticity preserving conjugation. The set of all $\lambda$-transformations is called the $\lambda$-group. The $\tau$-group acts on $\Phi^{(p)}$ and leaves $\phi^{(p)}$ unchanged; the $\lambda$-group acts on $\phi^{(p)}$ by

$$\phi'^{(p)} = e^{\lambda} \phi^{(p)}$$

and leaves $\Phi^{(p)}$ unchanged. The $\Phi^{(p)}$ and $\phi^{(p)}$ describe the covariantly analytic superfield in $\tau$- and $\lambda$-frame respectively.

In the $\tau$-frame, the complete set of gauge-covariant derivatives reads

$$D_M \equiv (D_M, D^{++}, D^{--}, D^0) \quad D^{\pm\pm} = D^{\pm\pm} \quad D^0 = D^0$$

and their transformation law is the same as that of $D_M$ given by (2.8). In the $\lambda$-frame, the covariant derivatives

$$\nabla_M = e^{-G} D_M e^G$$

transform by the rule

$$\nabla'_M = e^\lambda \nabla_M e^{-\lambda}$$

and their algebra reads

$$\{\nabla^+_a, \nabla^-_\bar{a}\} = -\{\nabla^+_a, \nabla^-_\bar{a}\} = 2i\nabla_{a\bar{a}}$$

$$\{\nabla^+_a, \nabla^-_\beta\} = 2\varepsilon_{a\beta}\bar{W}_\tau$$

$$\{\nabla^{\pm\pm}_a, \nabla^+_\bar{a}\} = \nabla^{\pm\pm}_a$$

$$\{\nabla^{\pm\pm}_a, \nabla^-_\bar{a}\} = \nabla^{\pm\pm}_a$$

$$[\nabla^0, \nabla^{\pm\pm}] = \pm 2\nabla^{\pm\pm}$$

$$[\nabla^{++}, \nabla^{--}] = \nabla^0$$

where

$$\bar{W}_\lambda = e^{-G} \bar{W} e^G \quad \bar{W}_\lambda = e^{-G} \bar{W} e^G.$$  

The other (anti-)commutators vanish except those involving vector covariant derivatives, the latter can be readily obtained from the relations given.

In the $\lambda$-frame, we have

$$\nabla^+_a = D^+_a \quad \nabla^+_\bar{a} = D^+_\bar{a} \quad \nabla^0 = D^0 \quad \nabla^{\pm\pm} = e^{-G} D^{\pm\pm} e^G = D^{\pm\pm} - \nabla^{\pm\pm} \quad \nabla^{\pm\pm} = V^{\pm\pm} \delta_a.$$  

Since $[\nabla^{++}, \nabla^{++}_a] = [\nabla^{++}, \nabla^{++}_{\bar{a}}] = 0$, the connection components $V^{+++}$ prove to be analytic real superfields, $D^+_a V^{+++} = D^+_\bar{a} V^{+++} = 0$, $V^{+++} = V^{+++}$, with the transformation law

$$V'^{+++} = e^{\lambda} V^{+++} e^{-\lambda} - e^\lambda D^{++} e^{-\lambda}.$$  

(2.35)
Using the (anti-)commutation relations for the covariant derivatives, especially eq. (2.31), as well as the explicit form (2.34) of $\nabla_\alpha^+$ and $\bar{\nabla}_\dot{\alpha}^+$, one easily expresses the connections associated with $\nabla_M$ in terms of $V^{-\pm}$. In particular, the super field strengths read

$$W_\lambda = \frac{1}{4}(\bar{D}^+)^2V^{-\pm} \quad \bar{W}_\lambda = \frac{1}{4}(D^+)^2V^{-\pm}.$$  \hspace{1cm} (2.36)

This implies

$$(\nabla^+)W_\lambda = (\bar{\nabla}^+)^2\bar{W}_\lambda.$$  \hspace{1cm} (2.37)

The remaining Bianchi identities

$$(\nabla^-\nabla^+ + \nabla^+\nabla^-)W_\lambda = (\bar{\nabla}^-\bar{\nabla}^+ + \bar{\nabla}^+\bar{\nabla}^-)\bar{W}_\lambda$$
$$\quad (\nabla^-)^2W_\lambda = (\bar{\nabla}^-)^2\bar{W}_\lambda$$  \hspace{1cm} (2.38)

are trivial consequences of the covariant $u$-independence of $W_\lambda$ and $\bar{W}_\lambda$ ($\nabla^{\pm\pm}W_\lambda = \nabla^{\pm\pm}\bar{W}_\lambda = 0$) and of the identities

$$[\nabla^-, (\nabla^+)^2] = \nabla^-\nabla^+ + \nabla^+\nabla^-$$
$$[\nabla^-, [\nabla^-, (\nabla^+)^2]] = 2(\nabla^-)^2$$

and their analogs with $\nabla^+$'s replaced by $\bar{\nabla}$'s. Further, the relation $[\nabla^+, \nabla^-] = D^0$ can be treated as an equation uniquely determining $V^{-\pm}$ in terms of $V^{++}$. It is given by

$$V^{-\pm}(z, u) = -\sum_{n=1}^{\infty} \int du_1 du_2 \ldots du_n \frac{V^{++}(z, u_1)\cdots V^{++}(z, u_n)}{(u^+u^-_1)(u^+_2u^-_3)\ldots(u^+_nu^+)}$$  \hspace{1cm} (2.39)

where the integration over $SU(2)$ is defined by

$$\int du \ = 1 \quad \int du^+_1 \ldots u^+_{i_1} u^-_{j_1} \ldots u^-_{j_m} = 0 \quad n + m > 0$$

and the properties of harmonic distributions are described in [8]. As a result, all geometric objects are expressed in terms of the single unconstrained analytic real prepotential $V^{++}$.

In the next sections, we will restrict ourselves by the study of unitary matrix representations of the gauge group and make use of the notation

$$G = G^a T_a \quad V^{\pm\pm} = V^{\pm\pm a} T_a.$$  \hspace{1cm} (2.40)

The gauge freedom (2.35) can be used to choose the Wess-Zumino gauge [8]

$$V^{++}(x_A, \theta^+, \bar{\theta}^+, u) = \theta^+\bar{\theta}^+ N(x_A) + \bar{\theta}^+\theta^+ \bar{N}(x_A)$$
$$-2i\theta^+\sigma^m\bar{\theta}^+ V_m(x_A) + \bar{\theta}^+\theta^+ \bar{\theta}^+\theta^+ \Psi^i(x_A)u_i^- + \theta^+\theta^+ \bar{\Psi}^{i\alpha}(x_A)u_i^-$$
$$+ i \theta^+\theta^+\bar{\theta}^+\bar{\theta}^+ D^{(ij)}(x_A)u_i^- u_j^-$$  \hspace{1cm} (2.41)
where the triplet $D_{ij}$ satisfies the reality condition (A.4). Thus we stay with the field multiplet of $N = 2$ supersymmetric gauge theory [5]. The residual gauge freedom is given by $\lambda^a = \xi^a(x_A)$ describing the standard Yang-Mills transformations.

In the case of constant curvature, the prepotentials $G$ and $V^{\pm\pm}$ read

$$G_0 = \theta^{-\alpha} \tilde{\theta}^+ \bar{W}_0 + \tilde{\theta}^{-\tilde{\alpha}} \tilde{W}_0 \quad V^{\pm\pm}_0 = -\theta^{\pm\alpha} \tilde{\theta}^+ \bar{W}_0 - \tilde{\theta}^{-\tilde{\alpha}} \tilde{\theta}^\pm \bar{W}_0 . \quad (2.42)$$

### 3 Spontaneous breakdown of gauge symmetry

We consider a general $N = 2$ supersymmetric gauge theory with matter hypermultiplets being described by unconstrained analytic superfields $\{q^+(\zeta_A, u), \bar{q}^+ (\zeta_A, u)\}$ ($q$-hypermultiplet) and $\omega (\zeta_A, u), \tilde{\omega} = \omega$ (real $\omega$-hypermultiplet) [8] in some representations of the gauge group. The gauge-invariant action is given by

$$S[V^{++}, q^+, \omega] = S_{SYM}[V^{++}] + S_{MAT}[V^{++}, q^+, \omega] \quad (3.1)$$

where the pure $N = 2$ supersymmetric gauge action has the form [3, 8, 10]

$$S_{SYM}[V^{++}] = \frac{1}{2g^2} \text{tr} \int d^4xd^4\theta W^2 = \frac{1}{2g^2} \text{tr} \int d^4xd^4\bar{\theta} \bar{W}^2 \quad (3.2)$$

$$= \frac{1}{g^2} \text{tr} \int d^{12}z \sum_{n=2}^{\infty} \frac{(-i)^n}{n} \int d\zeta du_1 du_2 \cdots du_n \frac{V^{++}(z, u_1)V^{++}(z, u_2) \cdots V^{++}(z, u_n)}{(u_1^+ u_2^+)(u_2^+ u_3^+) \cdots (u_n^+ u_1^+)}$$

with $\text{tr} (T_a T_a) = \delta_{ab}$. The matter action reads [8]

$$S_{MAT}[V^{++}, q^+, \omega] = -\int d\zeta^{(-4)} du \bar{q}^+ \nabla^{++} q^+ - \frac{1}{2} \int d\zeta^{(-4)} du \nabla^{++} \omega^T \nabla^{++} \omega \quad (3.3)$$

with $d\zeta^{(-4)} = d^4x_A d^2\theta d^2\bar{\theta}$. It is not difficult to derive the dynamical equation for $V^{++}$ (see Appendix C):

$$\frac{1}{4g^2} (\nabla^+)^2 W_\lambda^\alpha - i q^+ T_a q^+ + i \omega^T T_a \nabla^{++} \omega = 0 . \quad (3.4)$$

Therefore, the theory possesses $SU(2)_A$-invariant solutions of the form

$$D^a_\alpha W = 0 \quad q^+ = 0 \quad \nabla^{++} \omega = 0 \quad (3.5)$$

which correspond to the possible $SU(2)_A$-invariant Higgs vacua. The importance of $\omega$-hypermultiplets for realizing $SU(2)_A$-invariant Higgs vacua was first recognized by Delamotte, Delduc and Fayet [4].
More generally, there may exist vacuum solutions with broken $SU(2)_A$. Such solutions are described by the requirements

\[
\begin{align*}
\mathcal{D}^i_\alpha W &= 0, \\
\nabla^{++} q^+ &= 0, \\
(\nabla^{++})^2 \omega &= 0.
\end{align*}
\]

In what follows, we restrict ourselves by the study of the $SU(2)_A$-invariant Higgs vacua.

The above solutions are restricted by some consistency conditions. First of all, the requirement of $W$ to be covariantly constant implies $[\bar{W}, W] = 0$. Another consistency condition follows from the fact that for negative $p$ the equation $D^{++} f(p)(u) = 0$ has the unique solution $f(p) = 0$. Therefore, if we have a scalar superfield $\phi(z, u)$ constrained by $D^{++} \phi = 0$, then $D^{--} \phi = 0$ also holds ($D^{--}D^{++} \phi = D^{++}D^{--} \phi = 0$), and hence $\phi$ is $u$-independent, $\phi = \phi(z)$. If, in addition, $\phi$ is an analytic superfield by construction, then we automatically have $\phi = \text{const}$ since the analyticity requirements $D^i_{\alpha} \phi = 0$ and $\bar{D}^i_{\dot{\alpha}} \phi = 0$ are now equivalent to $D^i_{\alpha} \phi = 0$ and $\bar{D}^i_{\dot{\alpha}} \phi = 0$. Keeping all this in mind, we analyze the last equation in (3.5). By definition

\[
\nabla^{++} \omega = e^{iG} D^{++} e^{-iG} \omega = 0 \quad \Rightarrow \quad \omega = e^{iG} \omega_\tau(z).
\]

Therefore, $\omega$ is $u$-independent in the $\tau$-frame. Then, however, the analyticity requirements imply

\[
\mathcal{D}^i_{\alpha} \omega_\tau = 0, \quad \bar{\mathcal{D}}^i_{\dot{\alpha}} \omega_\tau = 0. \quad (3.8)
\]

These are consistent only if

\[
\bar{W} \omega_\tau = 0, \quad W \omega_\tau = 0 \quad (3.9)
\]

and then $\omega_\tau \equiv \omega_0 = \text{const}$. Let us choose the gauge in which $W = W_0 = \text{const}$. The explicit form of $G_0$ (2.42) and eq. (3.7) tell us that $\omega = \omega_0$. In summary, the admissible $SU(2)$-invariant Higgs vacua are parameterized by the expectation values $W_0$, $\bar{W}_0$ and $\omega_0$ constrained by

\[
[W_0, \bar{W}_0] = 0, \quad W_0 \omega_0 = \bar{W}_0 \omega_0 = 0. \quad (3.10)
\]

Physically, the three choices (i) $W_0 \neq 0$, $\omega_0 = 0$; (ii) $W_0 \neq 0$, $\omega_0 \neq 0$; (iii) $W_0 = 0$, $\omega_0 \neq 0$ describe different phases of the theory.

Let us first examine the case $W_0 \neq 0$, $\omega_0 = 0$. We choose the supersymmetric gauge in which the vacuum covariant derivatives look like in eq. (2.10). It is supersymmetric since the Higgs vacuum conditions are invariant under the gauge transformation with parameter
Then we still have an unbroken gauge group generated by the subalgebra \( \mathcal{Y} \) of elements of the Lie algebra \( \mathcal{G} \) which commute with \((\text{Re} \, W^a_0) T_a \) and \((\text{Im} \, W^a_0) T_a \). As is obvious, \( \mathcal{Y} \) includes the abelian subalgebra \( \mathcal{H} \) spanned by \((\text{Re} \, W^a_0) T_a \) and \((\text{Im} \, W^a_0) T_a \).

But now, however, we deal with supersymmetry with central charges, and there appear massive superfields: not only several \( q \)- and \( \omega \)-hypermultiplets, but also the components of the gauge multiplet \( \{V^{++a}\} \) which belong to the orthogonal complement \( \mathcal{K} \) to \( \mathcal{Y} \) in the Lie algebra \( \mathcal{G} \) of the gauge group, \( \mathcal{G} = \mathcal{Y} \oplus \mathcal{K} \). Herewith all the massive superfields, not only the massive hypermultiplets, describe short representations of the \( N = 2 \) supersymmetry with central charges, since the mass matrix turns out to look like

\[
M^2 = W_0 \bar{W}_0 = \bar{W}_0 W_0
\]

and, hence, the values of the mass and central charge coincide, as a consequence of (2.16).

The simplest way to prove the above assertion is to analyze the gauge structure upon the spontaneous breakdown. Let us represent the gauge superfield in the form

\[
V^{++} = V^{++}_0 + \mathcal{V}^{++}
\]

where \( V^{++}_0 \), given by (2.42), corresponds to the Higgs vacuum and \( \mathcal{V}^{++} \) describe deviations from the ground state. The gauge transformation (2.35) turns into

\[
\delta \mathcal{V}^{++} = D^{++} \lambda + i[V^{++}_0, \lambda] + i[\mathcal{V}^{++}, \lambda] = D^{++} \lambda + i[\mathcal{V}^{++}, \lambda] \quad \lambda = \lambda^a T_a .
\]

As in the case of unbroken gauge symmetry we can impose the Wess-Zumino gauge. We can use the residual gauge transformations with real parameters \( \xi^a \)

\[
\delta N = i[N, \xi] - i[W_0, \xi] \quad \xi = \xi^a T_a
\]

to gauge away a half of the complex \( N \)'s which correspond to the broken symmetries. The spin content of the multiplets obtained is \( 2 (0 \oplus \frac{1}{2} \oplus \frac{1}{2} \oplus 1) \) and the doubling of fields is caused by the central charges \( W_0 \) and \( \bar{W}_0 \) in the supersymmetry algebra. This corresponds to the short massive multiplet with highest spin one [16].

To analyze the mass spectrum of the theory, we insert (3.12) in the action functional (3.1). This gives for the supersymmetric gauge action

\[
S_{SYM} = \frac{1}{g^2} \text{tr} \int d^{12} z \sum_{n=2}^{\infty} \frac{(-i)^n}{n} \int du_1 du_2 \cdots du_n \frac{V^{++}_0(z, u_1) \cdots V^{++}_0(z, u_{n-1}) V^{++}_0(z, u_n)}{(u_1^+ u_2^+) \cdots (u_{n-1}^+ u_n^+)}
\]

(3.15)
where
\[ \mathcal{V}^{++} = e^{-iG_0} \mathcal{V}^{++} e^{iG_0}. \] (3.16)

The matter action takes the form
\[ S_{\text{MAT}} = -\int d\zeta^{(-4)} du \tilde{q}^+ (D^{++} + i\mathcal{V}^{++}) q^+ + \frac{1}{2} \int d\zeta^{(-4)} \omega^T(D^{++} + i\mathcal{V}^{++})^2 \omega. \] (3.17)

Both \( S_{\text{SYM}} \) and \( S_{\text{MAT}} \) are manifestly invariant under the supersymmetry transformations (2.14) generated by (2.15). They also are invariant under the gauge transformations (3.9) supplemented by those of the matter superfields (2.25). The gauge freedom can be fixed by imposing the gauge condition
\[ D^{++} \mathcal{V}^{++} = 0 \] (3.18)
or, equivalently, by adding to \( S_{\text{SYM}} \) the following gauge-fixing term (see Refs. [9, 12] for more details)
\[ S_{\text{GF}}[\mathcal{V}^{++}] = \frac{1}{2g^2\alpha} \text{tr} \int d^{12}z du_1 du_2 \left( \frac{u_1^- u_2^-}{(u_1^+ u_2^+)^2} \right) (D_2^{++} \mathcal{V}^{++}_\tau(1)) (D_2^{++} \mathcal{V}^{++}_\tau(2)) \]
\[ = \frac{1}{2g^2\alpha} \text{tr} \int d^{12}z du_1 du_2 \frac{\mathcal{V}^{++}_\tau(1) \mathcal{V}^{++}_\tau(2)}{(u_1^+ u_2^+)^2} - \frac{1}{4g^2\alpha} \text{tr} \int d^{12}z du \mathcal{V}^{++}_\tau (D^{--})^2 \mathcal{V}^{++}_\tau. \] (3.19)

The equations of motion corresponding to \( S_{\text{SYM}} + S_{\text{GF}} \) should be supplemented by the gauge condition (3.18).

For the special choice \( \alpha = -1 \) we obtain
\[ S_{\text{SYM}} + S_{\text{GF}} = -\frac{1}{2g^2} \text{tr} \int d\zeta^{(-4)} du \mathcal{V}^{++} \bar{\nabla} \mathcal{V}^{++} \]
\[ + \frac{1}{g^2} \text{tr} \int d^{12}z du_1 \ldots du_n \sum_{n=3}^{\infty} \frac{(-i)^n \mathcal{V}^{++}_\tau(z, u_1) \cdots \mathcal{V}^{++}_\tau(z, u_n)}{(u_1^+ u_2^+) \cdots (u_n^+ u_1^+)} . \] (3.20)

Here we have used the relation\footnote{We use the notation \((D^+)^4 = \frac{1}{16}(D^+)^2(D^+)^2\), \((D^\pm)^2 = D^\pm D_\alpha^\pm\), \((D^\pm)^2 = D_\alpha^\pm D^\pm_\alpha\) and similar notation for the gauge-covariant derivatives.}
\[ \frac{1}{2} \text{tr} \int d^{12}z du \mathcal{V}^{++}_\tau (D^{--})^2 \mathcal{V}^{++}_\tau = \frac{1}{2} \text{tr} \int d^{12}z du \mathcal{V}^{++}_\tau (D^{--})^2 \mathcal{V}^{++}_\tau = -\text{tr} \int d\zeta^{(-4)} du \mathcal{V}^{++} \bar{\nabla} \mathcal{V}^{++} \] (3.21)

where
\[ \tilde{\nabla} = -\frac{1}{2} (D^+)^4 (D^{--})^2 \quad \bar{\nabla} \phi^{(p)} = (\nabla + \bar{W}_0 W_0) \phi^{(p)} \] (3.22)
for any analytic superfield $\phi^{(p)}$ in arbitrary representation of the gauge group.

From eqs. (3.15) and (3.17) we can single out the part which is quadratic in the superfields

$$S_{(2)} = \int d\zeta (-4) du \left\{ -\frac{1}{2g^2} \text{tr} V^{++} \bar{\nabla} V^{++} - \bar{q}^+ D^{++} q^+ + \frac{1}{2} \omega^T (D^{++})^2 \omega \right\}.$$

Because of the identity $(D^{--})^2 D^{++} q^+ = D^{++} (D^{--})^2 q^+$, the dynamical equation $D^{++} q^+ = 0$ implies $(D^{--})^2 q^+ = 0$. Therefore, we have $(D^+)^4 (D^{--})^2 q^+ = 0$ on the mass shell. Because of the identity $(D^{--})^2 (D^{++})^2 \omega = (D^{++})^2 (D^{--})^2 \omega$, the free dynamical equation $(D^{++})^2 \omega = 0$ implies $(D^+)^4 (D^{--})^2 \omega = 0$. Therefore, the on-shell superfields satisfy the equations

$$\Box V^{++} + [\bar{W}_0, [W_0, V^{++}]] = 0 \quad (3.24)$$
$$\Box q^+ + \bar{W}_0 W_0 q^+ = 0 \quad (3.25)$$
$$\Box \omega + \bar{W}_0 W_0 \omega = 0 \quad (3.26)$$

which determine the masses of the superfields. Let us notice again that $V^{++}$ is also restricted by the requirement (3.18).

Now, we turn to the analysis of the case $W_0 \neq 0$ and $\omega_0 \neq 0$. Here we split $\omega = \omega_0 + \omega_{\text{dynamical}}$ and skip the subscript “dynamical” for readability. The matter gauge transformation (2.25) takes the form

$$\delta \omega = -i \lambda \omega_0 - i \lambda \omega \quad \lambda = \lambda^a T^a. \quad (3.27)$$

This situation exactly corresponds to the standard Higgs mechanism where some scalar fields can be gauged away due to the presence of non-vanishing vacuum expectation values. In our case we can completely gauge away several $\omega$-hypermultiplets. But then we stay with a number of massive $V^{++}$-superfields whose masses no longer satisfy eq. (3.11) and are greater than the central charge values. Therefore, the massive gauge superfields now describe the long massive vector multiplets [16].

Upon the splitting $V^{++} \rightarrow V_0^{++} + V^{++}$ and $\omega \rightarrow \omega_0 + \omega$, the classical action takes the form

$$S[V_0^{++} + V^{++}, q^+, \omega_0 + \omega] = S_{(2)}[V^{++}, q^+, \omega] + S_{\text{int}}[V^{++}, q^+, \omega] \quad (3.28)$$

where

$$S_{(2)} = \frac{1}{2g^2} \text{tr} \int d^4 z du_1 du_2 \frac{V^{++}(1) V^{++}(2)}{(u_1^+ u_2^+)^2} - \frac{1}{2} \int d\zeta (-4) du \omega_0^T (V^{++})^2 \omega_0$$
$$+ \int d\zeta (-4) du \left\{ -q^+ D^{++} q^+ + \frac{1}{2} \omega^T (D^{++})^2 \omega + i \omega_0^T V^{++} D^{++} \omega \right\} \quad (3.29)$$

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and $S_{\text{int}}$ includes the third- and higher-orders in the dynamical superfields. The $S_{\text{lin}}$ is invariant under the linearized gauge transformations

$$
\delta \mathcal{V}^{++} = D^{++} \lambda \quad \delta q^+ = 0 \quad \delta \omega = -i \lambda \omega_0 \quad \lambda = \lambda^a T_a
$$

which can be used to impose some sort of unitary gauge on $\omega$ just to eliminate the mixed $\mathcal{V}^{-}\omega$ term in the action.

Finally, in the third variant $W_0 = 0$, $\omega_0 \neq 0$ we deal with $N = 2$ supersymmetry without central charges in the spontaneously broken phase. All the hypermultiplets remains massless, but there appear massive gauge superfields which realize the first type of $N = 2$ massive vector multiplets [16, 15, 8]. This picture has been described in [14].

Up to now we have discussed only mass generation by vacuum expectation values of scalar fields and have not considered explicit mass terms. This does not restrict the validity of our discussion because mass terms for hypermultiplets can be written as vacuum expectation values of scalar fields. A mass term for hypermultiplets is equivalent to their coupling to a background abelian gauge superfield $\Gamma^{++}_0$ with constant strength [19]

$$
\Gamma^{++}_0 = -(\theta^\pm)^2 \bar{w}_0 - (\bar{\theta}^\pm)^2 w_0 \quad w_0 = \frac{1}{4}(\bar{D}^+)^2 \Gamma_0^{-}.
$$

Here $w_0$ is a fixed constant of mass dimension, and all information about the masses of the hypermultiplets is encoded in the $U(1)$ generator $\mathcal{M}$, to which $\Gamma^{++}_0$ is associated and which should commute with the gauge group, $[\mathcal{M}, T_a] = 0$. Then, the matter action reads

$$
S_{\text{MAT}}[V^{++}, q^+, \omega] = -\int d\zeta (-4) du \tilde{q}^+(\nabla^{++} + i\Gamma^{++}_0 \mathcal{M})q^+ + \frac{1}{2} \int d\zeta (-4) du \omega^T(\nabla^{++} + i\Gamma^{++}_0 \mathcal{M})^2 \omega.
$$

By construction, this theory possesses $N = 2$ supersymmetry with central charges. In eqs. (3.4) and (3.5) $\nabla^{++}$ is shifted to $\nabla^{++} + i\Gamma^{++}_0 \mathcal{M}$. The analog of eq. (3.10) reads

$$
[\bar{W}_0, W_0] = 0 \quad (W_0 + w_0 \mathcal{M}) \omega_0 = (\bar{W}_0 + \bar{w}_0 \mathcal{M}) \omega_0 = 0.
$$

4 Effective equations of motion

Long ago, West [20] showed that the perturbative quantum corrections can not remove the degeneracy in the classical vacuum solutions for the supersymmetric theories with unbroken supersymmetry. Here we extend this result to account for non-perturbative
quantum corrections in the pure $N = 2$ supersymmetric gauge theory. The general form of low-energy effective action $\Gamma[V^{++}]$, including non-perturbative quantum corrections, in the pure $N = 2$ supersymmetric gauge theory reads

$$\Gamma[V^{++}] = \text{tr} \int d^4x d^4\theta F(W) + \text{tr} \int d^4x d^4\bar{\theta} F(\bar{W}) + \text{tr} \int d^4x d^4\theta d^4\bar{\theta} H(W, \bar{W}) \quad (4.1)$$

with holomorphic $F(W)$ and Hermitian $H(W, \bar{W})$ functions of the super field strengths. In the framework of perturbation theory, the holomorphic part of the effective action was found by Seiberg [21] by integrating the anomaly of $R$-symmetry. The result was rederived in terms of $N = 1$ superfields [23, 24] and $N = 2$ superfields [11, 12]. The non-perturbative holomorphic effective action was found by Seiberg and Witten [1]. It has also been shown, using the $N = 1$ supergraph technique [23, 24] and the $N = 2$ harmonic superspace approach [11], that the effective potential gets perturbative non-holomorphic corrections to $H(W, \bar{W})$.

It is an instructive exercise to obtain the effective equations of motion

$$\frac{\delta \Gamma[V^{++}]}{\delta V^{++}} = 0 \quad (4.2)$$

As is shown in Appendix C, the variational derivative of $\Gamma[V^{++}]$ is

$$\frac{\delta \Gamma[V^{++}]}{\delta V^{++}} = (\mathcal{D}^+)^2 F'(W) + (\bar{\mathcal{D}}^+)^2 \bar{F}'(\bar{W}) + \frac{1}{16} (\mathcal{D}^+)^2 (\bar{\mathcal{D}}^+)^2 \left\{ (\mathcal{D}^-)^2 \frac{\partial H(W, \bar{W})}{\partial W} + (\bar{\mathcal{D}}^-)^2 \frac{\partial H(W, \bar{W})}{\partial \bar{W}} \right\}. \quad (4.3)$$

Each classical vacuum solution (2.9) of the pure $N = 2$ supersymmetric gauge theory satisfies the effective equations of motion (4.2), so the vacuum expectation values are not changed by quantum corrections.

**Acknowledgements.** We are grateful to Friedemann Brandt for fruitful discussions. This work was supported by the RFBR-DFG project No 96-02-00180, the RFBR project No 96-02-16017 and by the Alexander von Humboldt Foundation.

**A Conventions**

We use the Lorentz and two-component spinor notations and conventions adopted in [13]. The $SU(2)_A$ indices are raised and lowered by $\varepsilon^{ij}$ and $\varepsilon_{ij}$, $\varepsilon^{12} = \varepsilon_{21} = 1$, in the standard
The SU(2)-invariant matrices \( (\tau^I)_{ij} \equiv \sigma^I \), where \( I = 1, 2, 3 \), and their descendants

\[
(\tau^I)^{ij} \equiv \varepsilon^{jk} (\tau^I)_{ki} = (\tau^I)^{ij} \\
(\tau^I)^{ij}(\tau^I)_{kl} = -(\delta_i^k \delta_j^l + \delta_i^l \delta_j^k)
\]

are used to convert a real triplet \( D^I \) into the symmetric isotensor

\[
D^{ij} = (\tau^I)^{ij} D^I \\
D^I = -\frac{1}{2} (\tau^I)_{ij} D^{ij}
\]

which satisfies the reality condition

\[
\overline{D^{ij}} = -D_{ij}.
\]

## B Bosonic action

In this appendix we consider the bosonic sector of the general \( N = 2 \) supersymmetric gauge theory \((3.1)\) in components. To pass to components, it is useful to choose the Wess-Zumino gauge \((2.41)\). It turns out that only the leading (bosonic) components of \( q^+ \) and \( \tilde{q}^+ \)

\[
q^+(x_A, \theta^+, \bar{\theta}^+, u) = u^{+i} C_i(x_A) + \cdots \\
\tilde{q}^+(x_A, \theta^+, \bar{\theta}^+, u) = -u_i^+ \tilde{C}^i(x_A) + \cdots \\
\tilde{C}^i \equiv \overline{C_i}
\]

constitute the physical fields. the remaining fields, denoted by dots, are auxiliary. Similarly, the bosonic sector of \( \omega \) reads

\[
\omega(x_A, \theta^+, \bar{\theta}^+, u) = A(x_A) + i B^{ij}(x_A) u^+_i u^-_j + \cdots \\
\tilde{A} = A \\
\overline{B^{ij}} = -B_{ij}
\]

and the fields indicated by dots have no independent dynamics.

The Lagrangian of bosonic fields looks like

\[
\mathcal{L}_{\text{BOS}} = -\frac{1}{2g^2} \text{tr} F^{mn} F_{mn} + \frac{1}{g^2} \text{tr} \partial^m \tilde{N} \partial_m N + \nabla^m \tilde{C}^i \nabla_m C_i + \nabla^m A^T \nabla_m A + (\nabla^m B^I)^T \nabla_m B^I + \tilde{D}^{Ia} \tilde{D}^{Ia} - \mathcal{P}(\varphi)
\]
where
\[ \nabla_m = \partial_m + i V_m \quad F_{mn} = \partial_m V_n - \partial_n V_m - i [V_m, V_n] \quad (B.4) \]
\[ \hat{D}^{ia} = \frac{1}{3g} D^{ia} + ig A^{T} T_b B^I - \frac{1}{2} g \tilde{C}^i (\tau^I)_i^j T_a C_j . \quad (B.5) \]
The scalar potential reads
\[ \mathcal{P}(\varphi) = \frac{1}{4g^2} \text{tr} \left( [\bar{N}, N] \right)^2 + g^2 \left( \frac{1}{2} \tilde{C}^i (\tau^I)_i^j T_a C_j - i A^{T} T_a B^I \right)^2 \]
\[ + \bar{C}^{i} \{ \bar{N}, N \} C_i + \frac{1}{2} A^{T} \{ \bar{N}, N \} A + \frac{1}{2} (B^I)^T \{ \bar{N}, N \} B^I . \quad (B.6) \]

C Derivation of the effective equations of motion

In this appendix we derive eq. (4.3). We consider the effective action as a functional of the unconstrained analytic prepotential \( V^{++} \), i.e. \( W \) and \( \bar{W} \) are given by (2.36). If we make use of the crucial relation
\[ \nabla^{++} \delta V^{--} = \nabla^{--} \delta V^{++} , \quad (C.1) \]
which follow from (2.32) and (2.34), we can determine the variation of \( V^{--} \) with respect to an arbitrary variation of \( V^{++} \). For simplicity, we will handle only the holomorphic functional
\[ \mathcal{F} = \text{tr} \int d^4 x d^4 \theta F(W) . \quad (C.2) \]
The other terms can be treated similarly.

We start with the identities
\[ \delta \mathcal{F} = \text{tr} \int d^4 x d^4 \theta \delta W F'(W) = \text{tr} \int d^4 x d^4 \theta d u \delta W F'(W) = \text{tr} \int d^4 x d^4 \theta d u \delta W_{\lambda} F'_{\lambda}(W_{\lambda}) \]
and insert here the expression for \( W_{\lambda} (2.36) \). Next, the covariant \( u \)-independence of \( W_{\lambda} \), eq. (C.1) and the identities
\[ (\bar{\nabla}^+)^2 = [\nabla^{++}, \nabla^+ \nabla^-] \quad (\bar{\nabla}^-)^2 = [\nabla^{--}, \nabla^+ \nabla^-] \quad (C.3) \]
allow us to continue as follows
\[ \delta \mathcal{F} = \frac{1}{4} \text{tr} \int d^4 x d^4 \theta d u (\bar{\nabla}^+)^2 \delta V^{--} F'(W_{\lambda}) = - \frac{1}{4} \text{tr} \int d^4 x d^4 \theta d u \nabla^+ \nabla^- \nabla^{++} \delta V^{--} F'(W_{\lambda}) \]
\[ = - \frac{1}{4} \text{tr} \int d^4 x d^4 \theta d u \nabla^+ \nabla^- \nabla^{--} \delta V^{++} F'(W_{\lambda}) = \frac{1}{4} \text{tr} \int d^4 x d^4 \theta d u (\bar{\nabla}^-)^2 \delta V^{++} F'(W_{\lambda}) . \]
The next step is to formally rewrite

\[ F'(W_\lambda) = W_\lambda \frac{F'(W_\lambda)}{W_\lambda} = \frac{1}{4}(\nabla^+)^2V^- F'(W_\lambda) \quad \text{.} \tag{C.4} \]

Then one gets

\[
\delta \mathcal{F} = \frac{1}{16} \text{tr} \int d^4x d^4\theta du \left( (\bar{\nabla}^-)^2(\nabla^+)^2 \right) \left\{ \delta V^{++} V^- F'(W_\lambda) \right\} \\
= \frac{1}{16} \text{tr} \int d^4x d^4\theta du \left( (\bar{\nabla}^-)^2(\nabla^+)^2 \right) \left\{ \delta V^{++} V^- F'(W_\lambda) \right\} \\
= \text{tr} \int d^4x d^4\theta du \left( (\bar{\nabla}^-)^2(\nabla^+)^2 \right) \left\{ \delta V^{++} V^- F'(W_\lambda) \right\} \\
= \frac{1}{16} \text{tr} \int d^4x d^4\theta du \left( (\bar{\nabla}^-)^2(\nabla^+)^2 \right) \left\{ \delta V^{++} V^- F'(W_\lambda) \right\} . \tag{C.5} \]

Since \( W \) is covariantly chiral, we finally obtain

\[ \delta \mathcal{F} = \frac{1}{4} \text{tr} \int d\zeta d^4x d^4\theta du \delta V^{++}(\nabla^+)^2F'(W_\lambda) . \tag{C.6} \]

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