Reconstruction of the spectrum of cascades generated by VHE muons in the IceCube

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Abstract. One of the best ways for the investigations of the VHE muons spectrum is measuring the spectrum of stochastic energy losses (cascades). IceCube is the detector capable of measuring the cascade spectrum in the energy region between tens of TeV and one PeV where manifestation of the prompt muon production mechanisms is predicted. In events with muon bundles, the longitudinal energy deposit profile reconstructed by means of a maximum likelihood method is analyzed. Cascade energies are estimated for the events in which the highest local energy deposit is much greater than the median energy deposit. The technique of cascade spectrum reconstruction has been tested with MC-simulated events. Criteria of the event selection, cascade parameters estimation accuracy and the efficiency of the spectrum reconstruction are discussed.

1. Introduction

Very high energy (VHE) muons with energies above 100 TeV are of a special interest, since changes of their energy spectrum can be caused by different physical reasons. The softening of the primary cosmic ray energy spectrum above the ‘knee’ should lead to softening of the spectrum of muons, while an inclusion of any additional rapid processes of muon generation (decays of charmed and other short-lived massive particles or appearance of a new state of matter) may manifest itself as a hardening of the muon spectrum.

To obtain the spectrum by measuring the energies of single muons in individual events is an extremely difficult task. Magnetic muon spectrometers (MUTRON and DEIS, for example) are limited due to their finite resolution and cannot measure momenta above 20-30 TeV [1, 2]. Studying muons by means of the pair-meter method [3,4] is not energy restricted but a calorimeter with a thickness of several hundred radiation lengths and large number of detector layers is necessary.

Another way for investigations of the VHE muon spectrum is the measurement of the spectrum of stochastic energy losses (cascades) [5]. Very large volume neutrino telescopes such as IceCube [6] are very well suited for such an investigation [7,8]. IceCube is a cubic-kilometer neutrino detector installed in the ice at the geographic South Pole between depths of 1450 m and 2450 m. The detector registers cosmic ray-induced atmospheric muon bundles at a rate of several thousand events per second [9].
2. Search for cascades and reconstruction of their energy

The cascade spectrum reconstruction technique has been tested with MC-simulated events. A five-component ($p$, $^4$He, $^{14}$N, $^{27}$Al and $^{56}$Fe) simulation with CORSIKA [10] above the ice surface was used to create the simulated events. The energies of the primary nuclei are distributed with a differential spectrum following a power-law with index $\gamma = -2$ in the energy range between $10^{14}$ and $10^{20}$ eV. The processes of the passage of muon bundles through the ice and the response of the detector were simulated using the lepton propagator PROPOSAL [11] and software developed in the IceCube collaboration [12].

For the subsequent analysis, events were selected that produced more than 1000 photoelectrons in the detector. The length of the event axis in a cylinder with a radius of 600 meters and a height of 1 km around the IceCube detector center was required to be at least 500 m.

The idea of reconstructing the cascade spectrum is based on the analysis of the longitudinal energy deposit profile reconstructed by means of the Millipede algorithm [13]. The algorithm uses a maximum likelihood method, taking into account the arrival time and charge measured at each optical module, to fit the energies of a set of cascades $\{E_i\}$, positioned every 10 meters along the axis of the event.

An example of a simulated event is shown in figure 1 (left). The primary particle was a proton with an energy of 8.05 PeV and zenith angle of 30.4°; the leading muon had an energy of 2.27 PeV on the ice surface; the muon generated a cascade with an energy of $E_{\text{sim}} = 344$ TeV at a depth of 2390 m.

The reconstructed longitudinal energy deposit profile of this event is shown in figure 1 (right). The profile is shown as a function of the distance to the surface, calculated along the axis of the event. At the slant depth of ~ 2800 m a pronounced peak is observed in the profile. For the reconstructed energy of the most energetic cascade, we take the maximum sum of the energies of three neighboring points of the profile: $E_{\text{reco}} = \max(E_{i-1} + E_i + E_{i+1})$, and we define the peak/median-ratio [8]:

$$\text{Ratio} = \frac{E_{\text{reco}}}{\text{median}\{E_i\}}.$$

In the example, the median energy deposit is 1.3 TeV, the reconstructed energy of the cascade is $E_{\text{reco}} = 293$ TeV; the peak/median-ratio is $\text{Ratio} = 75$. The reconstructed cascade vertex is 11.1 m from the simulated vertex.

![Figure 1](image.png)

**Figure 1.** An example of a simulated IceCube event (left) and its longitudinal energy deposit profile reconstructed using the Millipede algorithm (right).

The peak/median-ratio of the energy depositions is the most important parameter in the reconstruction. The distributions the peak/median-ratio for events of different energies, showing simulated (left) and reconstructed (right) energies respectively, are presented in figure 2. As can be seen, the distributions show increased maxima at small values of the $\text{Ratio}$ parameter. Cascades with low energies can be erroneously reconstructed as cascades with high energies. Therefore, events with $\text{Ratio} < 12$ have been excluded from this analysis.
Figure 2. Distributions of events in the peak/median-ratio for different energies of the simulated (left) and reconstructed (right) cascades.

The correlation of the reconstructed cascade energy and the simulated energy is shown in figure 3a. Only events where the reconstructed vertex of the cascade is at least 50 m from the edge of the detector and that have a Ratio > 12 are presented in the figure. As can be seen, for most of the cascades the energy is reconstructed correctly (the events are near the red diagonal line). However, there are events in which the error in the reconstructed energy reaches several orders of magnitude.

Figure 3. Correlation of the reconstructed energy and the simulated cascade energy: a – cascades in the whole detector volume; b – cascades in the dust layer excluded; c – cascades in the lower part of the detector only.
In the central part of IceCube, a dust layer is located. The optical properties of the dust layer are significantly worse than the properties of the ice above or below the layer. The correlation between the reconstructed and the simulated cascade energy with the exclusion of cascades within the dust layer is shown in figure 3b. The number of cascades with incorrectly restored energy has noticeably decreased.

Additionally, muon bundles of large multiplicity imitate cascades in the reconstructed longitudinal energy deposit profile in the upper part of the detector. This contamination is weaker in the lower part. The dependence for cascades generated only in the lower part of IceCube is shown in figure 3c. As can be seen, there are no events with wrong reconstructed cascade energies above 80-100 TeV.

The distributions of the ratio between the reconstructed and simulated cascade energies are presented in figure 4. The distributions are given in linear (left pictures) and logarithmic (right pictures) scales. In the top pictures the events with reconstructed energies above 31.6 TeV are shown, in the bottom pictures – events with reconstructed energies above 100 TeV. As can be seen, if we use the entire volume of the detector, the error of the reconstructed energy can reach more than four orders of magnitude (the right ‘tail’ of the black curves), exclusion of the dust layer somewhat reduces the maximum error value. If we use only the lower part of the detector (blue curves), the maximum error value does not exceed one and a half order of magnitudes for cascades with energies above 31.6 TeV and 100 TeV. In this case, the standard deviations of the distributions are σ[^\log_{10}(E_{reco}/E_{sim})] = 0.15 (for $E_{reco} > 31.6$ TeV) and 0.10 (for $E_{reco} > 100$ TeV), i.e., the energy reconstruction resolution can be estimated as 40 % and 25 %.

![Figure 4](image_url). Distributions of events in the $\log(E_{reco}/E_{sim})$ in linear and logarithm scales for $E_{reco} > 31.6$ TeV (top) and $E_{reco} > 100$ TeV (bottom).

Since the depth at which the cascade was generated is of great importance for the reconstruction of the muon spectrum via the spectrum of cascades, the accuracy of the vertex reconstruction of the cascade was investigated. The distribution of the distance between the simulated and reconstructed cascades vertices for events with $Ratio > 12$ in the lower part of detector is presented in figure 5. The
most probable distance is 5-6 meters, the width of the distribution is FWHM = 10 meters. Considering the vertical distance between optical modules of 17 meters, the obtained spatial accuracy is quite high.

Figure 5. Distribution of the distance between simulated and reconstructed cascades.

3. Reconstruction of the cascade energy spectrum

In the simulation, a differential spectrum of primary particles $dN/dE \sim E^{-2}$ was used. The spectra of primary particles in events that have passed the selection conditions (1000 photoelectrons in IceCube, etc.) are presented in figure 6 (left). To match these spectra to the real ones, the spectra index was increased to $\gamma = -3$ (figure 6, right).

Figure 6. Energy spectra of primary particles simulated with a spectral index of $\gamma = -2$ (left) and $\gamma = -3$ (right).
Figure 7. Energy spectra of simulated and reconstructed maximum cascade energies for a primary particle spectrum with a spectral index of $\gamma = -2$ (left) and $\gamma = -3$ (right).

Figure 8. Ratio of the spectra of simulated and reconstructed maximum cascade energies for primary particle spectrum with a spectral index of $\gamma = -2$ (left) and $\gamma = -3$ (right).

The spectra of simulated and reconstructed maximum cascade energies for the selected events are shown in figure 7. Ratios of reconstructed and simulated cascade spectra are presented in figure 8. It can be seen that the ratio gets constant in the energy range above 10-20 TeV, only mildly dependent on the slope of the primary particles spectrum.

4. Conclusion
A technique for the reconstruction of the spectrum of cascades generated by muons in the lower part of the IceCube detector has been developed. It can be used for effective energies of cascades $> 10$-30 TeV. The accuracy of the energy reconstruction is $\sigma[\log_{10}(E_{\text{reco}}/E_{\text{sim}})] = 0.10$ for cascades with $E_{\text{reco}} > 100$ TeV, while the accuracy of the vertex reconstruction is 5-6 meters.

The IceCube is capable of measuring the cascade spectrum in the energy region from tens of TeV to one PeV where a manifestation of prompt muons is predicted.

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