Generation of quantum coherence in two-qubit cavity system: qubit-dipole coupling and decoherence effects

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Abstract

The intrinsic decoherence effect for two qubits interacting with a coherent field, under the dipole-dipole interaction and two-photon resonance, is analytically described. We investigate numerically the population inversion and the quantum coherence. The results show that the generated mixture entropy and the entanglement negativity, can be enhanced and protected by the dipole-dipole interaction and by reducing the initial coherent field intensity. In particular, we find that, the collapses and revivals of the population inversion present high sensitivity to these physical parameters. The nonlinearity of the two-photon processes leads to a generation of a strong two-qubit entanglement. This generated entanglement depends on the initial coherent field intensity, the dipole-dipole interaction and the intrinsic decoherence.

Keywords: intrinsic decoherence, quantum coherence, dipole-dipole interaction

1. Introduction

The quantum information processing depends on the entanglement as well as on the robustness of the qubits against noise and decoherence. The decoherence leads to the inhibition of the purity or the lost of the entanglement. Entanglement has been a subject of several studies due to the fact that it is considered as one of the most popular types of non-classical correlations \([1–6]\). The effect of the decoherence on the non-classical correlations has been broadly studied \([7]\). For example, one of its associated effects in the QE is the well-known ‘entanglement sudden death’ phenomenon \([8, 9]\). Several measures of the QE as: von Neumann entropy, concurrence, the entanglement of formation and negativity, have been proposed \([10–12]\).

The Jaynes–Cummings (JC) \([13]\) and Tavis-Cummings (TC) models \([14]\) are well established descriptions for the qubit-field interaction. The TC-model can be experimentally realized via superconducting circuits \([15]\) and quantum wells \([16]\). The two models were extended to study the multi-photon and multi-mode interactions \([17]\), as well as to the dipole-dipole interaction \([18]\).

The non-degenerate two-photon JC model \([19, 20]\) opens the door for the realization of the two-mode two-photon laser. Two-photon systems have revealed several nonclassical properties of multi-photon transitions. They have been experimentally implemented for an atom-micromaser system \([21]\).
Recently, the two photon-interactions have been studied and proposed in trap ions [22] and superconducting circuits [23, 24]. It is also suggested in the complete Bell measurement [25], which is crucial to the implementation of quantum information protocols.

The dipole-dipole interaction of two qubits is usually generated by the coupling of their electric or magnetic dipole moments. If the relative distance between the qubits is small, the dipole-dipole interaction becomes important and cannot be neglected. It has several practical applications in quantum information processing, such as generating quantum correlations [26–28] and implementing the quantum C-Not gate [29]. The coupling of the qubits, via the dipole-dipole interaction, improved various real quantum systems such as superconducting qubit systems [27, 30, 31]. The dynamics of the mixture and entanglement in TC-systems has been studied in [32]. Further, entanglement dynamics between two atoms interacting with single-mode field in a Kerr medium has been explored [33].

There are several approaches to describe the decoherence. One of them is intrinsic decoherence (ID), that can explain why the quantum coherences are deteriorated. It is based on the hypothesis that closed quantum systems do not evolve unitarily according to the Schrödinger equation, but are governed by more generalized equations that include intrinsic decoherence. Therefore, The dynamics of the system with ID is governed by the Milburn equation. The quantum coherence is automatically destroyed as the system evolves [34, 35]. The intrinsic decoherence is generated without the interaction of the system with a reservoir, and therefore without the usual energy dissipation associated with the decay. In the ID models, the off-diagonal elements of the density matrix are intrinsically suppressed in the energy eigenstate basis, consequently, ID is realized without the dissipation. Most of the quantum effects originate from the coherence [34]. Therefore, it is of great interest to investigate the influence of the ID on the quantum coherence. QE was explored previously under the ID effect for qubit systems interacting initially with the vacuum or Fock cavity field [36–40].

Motivated by this, the quantum coherence, for two-qubit system interacting with a coherent field, will be here investigated.

2. The physical model

We study here, two qubits interacting with a coherent field. The dipole-dipole interaction and two-photon transitions are considered. The Hamiltonian of this system can be written as:

\[
\hat{H} = \omega \hat{a}^\dagger \hat{a} + \frac{\omega_0}{2}(\hat{\sigma}_1^+ \hat{\sigma}_2^- + \hat{\sigma}_1^- \hat{\sigma}_2^+) + \sum_{i=1}^{2}(\hat{a}^2 + \hat{a}^2\hat{\sigma}_i^+ \hat{\sigma}_i^- + \hat{\sigma}_i^+ \hat{\sigma}_i^-).
\]

where \(\hat{a}(\hat{a}^\dagger)\) represents the annihilation (creation) operator of the single-mode field with the frequency \(\omega\). The raising and lowering operators of the qubit \(i\) are \(\hat{\sigma}_i^+\) and \(\hat{\sigma}_i^-\) \((i = A, B)\), respectively, with the frequency \(\omega_\lambda\). \(\Omega\) designs the qubit dipole-dipole coupling constant and \(\lambda\) is the coupling constant between qubits and the cavity. Here, we focus on the case of the two-photon resonance \((\omega_0 = 2\omega_i)\), and the rotating-wave approximation (RWA), where the counter-rotating terms: \(\hat{a}^2\hat{\sigma}_i^+ + \hat{\sigma}_i^+ \hat{\sigma}_i^2\) (do not preserve the total number of excitations) are neglected. The rotating wave approximation is working well around the resonance. However, it is known that RWA fails a far-off resonance strong ultrashort pulse [41], which is not the case that we consider here. In this case, the Hamiltonian of equation (1) can be written as

\[
\hat{H} = \omega \hat{a}^\dagger \hat{a} + \omega (\hat{\sigma}_1^+ \hat{\sigma}_2^- + \hat{\sigma}_1^- \hat{\sigma}_2^+) + \sum_{i=1}^{2}(\hat{a}^2 \hat{\sigma}_i^+ \hat{\sigma}_i^- + \hat{\sigma}_i^+ \hat{\sigma}_i^-) + \Omega (\hat{\sigma}_1^+ \hat{\sigma}_2^- + \hat{\sigma}_1^- \hat{\sigma}_2^+).
\]

Here, only the intrinsic decoherence model of the Milburn equation, that postulating that on sufficiently short time steps the system does not evolve continuously under unitary evolution but rather in stochastic sequences of identical unitary transformations. It is worth mentioning that are others approaches, considering irreversible effects [42–44] as: dissipation (the energy of the system is not conserved) and decoherence/phase damping (the energy of the system is conserved). Various types of master equations can be employed to analyze the quantum dynamics of the systems. Usually, Master equations describe the system+reservoir interactions, that lead naturally to decoherence as the information is immediately lost after evolution.

In the ID model, the dynamics of the system is governed by the Milburn equation [34, 35]

\[
\frac{d\hat{\rho}(t)}{dt} = -i[\hat{H}, \hat{\rho}(t)] - \frac{\gamma}{2}[\hat{H}, [\hat{H}, \hat{\rho}(t)]].
\]

\(\gamma\) represents the ID rate.

The formal solution of the equation (3) is given by [45, 46],

\[
\hat{\rho}(t) = \sum_{k=0}^{\infty} \frac{\gamma^k}{k!} M^k(t) \hat{\rho}(0) M^{-k}(t),
\]

where \(M^k(t) = \hat{H}^k e^{-i\hat{H}t} e^{-\gamma t \hat{H}}\), and \(\hat{\rho}(0)\) is the density operator of the initial system. In the space states \(|1\rangle = |e_A e_B, n\rangle, |2\rangle = |e_A g_B, n + 2\rangle, |3\rangle = |g_A e_B, n + 2\rangle, |4\rangle = |g_A g_B, n + 4\rangle\), the used eigenstates \(|\Psi_0\rangle\) of the Hamiltonian (2) are given by:

\[
|\Psi_0\rangle = C_{1}\left|1\rightangle + C_{4}\left|4\rightangle,
|\Psi_1\rangle = C_{2}\left|2\rightangle + C_{3}\left|3\rightangle,
|\Psi_2\rangle = C_{1}\left|1\rightangle + C_{2}\left|2\right\rangle - C_{3}\left|3\right\rangle + C_{4}\left|4\right\rangle,
|\Psi_3\rangle = C_{1}\left|1\right\rangle + C_{4}\left|4\right\rangle + C_{3}\left|3\right\rangle + C_{4}\left|4\right\rangle,
\]

\(C_{ij}\) verify the condition of eigenvalue-problem: \(\hat{H}|\Psi_m\rangle = \Omega_m|\Psi_m\rangle\), where, \(\Omega_m\) are the corresponding eigenvalues

\[
E_1 = \omega(n + 2), \quad E_2 = \omega(n + 2) - \Omega,
E_{3(43)} = \omega(n + 2) + \frac{\Omega}{2} \pm \frac{1}{2} \sqrt{(\Omega^2 + 8(x^2 + y^2))},
\]
where \( x = \lambda \sqrt{(n + 1)(n + 2)} \) and \( y = \lambda \sqrt{(n + 3)(n + 4)} \).

The explicit expression of the density matrix is:

\[
\rho(t) = \sum_{m,n=0}^\infty e^{-\frac{\lambda^2}{2}t} |m\rangle \langle n| \rho_{mn} |n\rangle \langle m|,
\]

where, \( \rho(0) = |e_g e_g \rangle \otimes |\alpha\rangle \langle \alpha| \) is the initial state of the entire system, i.e., the single-mode cavity field initially prepared in the coherent state,

\[
|\alpha\rangle = \sum_{n=0}^\infty e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}} |n\rangle,
\]

while the two qubits are initially in the pure excited state, \( \rho^{AB}(0) = |e_g e_g \rangle \langle e_g e_g| \). For the initial state \( \rho(0) \) and using equation (6), the final time-dependent density matrix is

\[
\rho(t) = \sum_{m,n=0}^\infty \eta_m \eta_n \left[ F_{11}|\Psi_1^m\rangle \langle \Psi_1^n| + F_{12}|\Psi_1^m\rangle \langle \Psi_2^n| + F_{13}|\Psi_2^m\rangle \langle \Psi_1^n| + F_{14}|\Psi_2^m\rangle \langle \Psi_2^n| \\
+ F_{21}|\Psi_1^m\rangle \langle \Psi_2^n| + F_{22}|\Psi_2^m\rangle \langle \Psi_2^n| + F_{23}|\Psi_1^m\rangle \langle \Psi_1^n| + F_{24}|\Psi_1^m\rangle \langle \Psi_2^n| \\
+ F_{31}|\Psi_3^m\rangle \langle \Psi_1^n| + F_{32}|\Psi_3^m\rangle \langle \Psi_2^n| + F_{33}|\Psi_3^m\rangle \langle \Psi_3^n| + F_{34}|\Psi_3^m\rangle \langle \Psi_4^n| \\
+ F_{41}|\Psi_4^m\rangle \langle \Psi_1^n| + F_{42}|\Psi_4^m\rangle \langle \Psi_2^n| + F_{43}|\Psi_4^m\rangle \langle \Psi_3^n| + F_{44}|\Psi_4^m\rangle \langle \Psi_4^n| \right],
\]

(8)

\( F_{ij} \) are given by

\[
F_{ij} = C_{ij} C_{kl} I(t) e^{-\frac{\lambda^2}{2}t} e^{-E_i^2 - E_j^2} |n|,
\]

(9)

\( I(t) = e^{-\gamma t} |e_g\rangle \langle e_g| + |e_e\rangle \langle e_e| \) is the ID term and \( C_{ij} \) are the coefficients of the \( |\Psi_n^m\rangle \) of equation (5). We will use this initial state to study the evolution of some measures of mixture and entanglement of two dipole coupled qubits in interaction under the intrinsic decoherence.

3. Quantum information quantifiers

In this work, we shall use different quantum quantifiers to investigate the nonclassical phenomena via the entropy and two types of the negativity. The sources of the quantum decoherence in the present qubits-cavity system are: (i) The unitary interaction \( \gamma = 0 \), in this case the entropy is a proper measure for the entanglement and mixture phenomena. (ii) The intrinsic decoherence \( \gamma \neq 0 \), the entanglement and the mixture are in this case different, the entropy is no more a good proper measure for the entanglement. Here, we study the intrinsic decoherence effect on the quantum coherence via different quantifiers:

3.1. Entropy

(1) The entropy of the qubits-cavity system is calculated by:

\[
S(t) = -\sum_{i=1}^{\infty} \lambda^i \ln(\lambda^i),
\]

where \( \lambda^i \) represent the eigenvalues of the state \( \rho(t) \).

(2) We use the partial entropies to measure the quantum coherence of the different partitions. Also, they are used to measure the QE between qubits and cavity states [10, 47]. If the entire qubits-cavity system has only two partitions, say \( Q \) of the two qubits and \( C \) of the cavity field, the partial entropies of i-partition, \( \rho(i) \), are given by

\[
S_i(t) = -\sum_{n=1}^{\infty} \pi_n^i \ln(\pi_n^i),
\]

where \( \pi_n^i \) represent the reduced-density matrix eigenvalues.

The intrinsic decoherence

\[
\rho(0) = Tr_{Q} \{ \hat{\rho}(t) \}.
\]

The partial entropies as well as the total entropy of the system satisfy the Araki-Lieb inequality [48]:

\[
|S_Q(t) - S_C(t)| \leq S(t) \leq S_Q(t) + S_C(t).
\]

3.2. The population inversion

Due to the indistinguishability of the qubits, we can study the population inversion for any one of them, let us consider for the qubit A. The reduced density matrix of the qubit A is given by:

\[
\hat{\rho}^A(t) = Tr_B \{ \hat{\rho}(t) \} = \begin{pmatrix}
\rho_{11}^A(t) + \rho_{22}^A(t) \\
\rho_{33}^A(t) + \rho_{44}^A(t)
\end{pmatrix}
\]

(13)

Where \( \rho_{ij}^A(t) \) are the elements of the reduced density matrix of the two-qubit \( \hat{\rho}(t) \). The population inversion \( W(t) \) can be expressed as

\[
W(t) = \langle e_A | \rho^A(t) | e_A \rangle - \langle g_A | \rho^A(t) | g_A \rangle.
\]

(14)

Numerical results are displayed in figures 1-4, where we plot the time evolution of the population inversion \( W(t) \) (solid plots), the entropy functions of the two qubits \( S_{Q(g)} \) (dotted plots) and the cavity-field \( S_C(t) \) (dashed plots) for different values of the intrinsic decoherence and the initial entanglement field intensity. The effect of the dipole-dipole interaction is examined with two different values, \( \Omega = 0 \) and \( \Omega = 25\lambda \).

In figure 1(a), solid curve shows that the population inversion, equation(12), has regular oscillations around the zero-value between \(-1 \) and \( 1 \). The phenomena of the revivals and collapses appear. The revivals appear at the revival time \( T_R = n\pi \) \( n = 0, 1, 2, \ldots \). In the collapse intervals, the expectation values of the upper \( |e_e\rangle \) and the lower \( |g_g\rangle \) A-qubit states are equal, i.e., the amount of the energy transfers from the upper two-qubit state \( |e_e e_g\rangle \) \( \rho_{11}^A(0) = 1, \rho_{22}^A(0) = 0 \) to the lower states \( |g_g e_g\rangle + |g_g g_g\rangle \), and the system is in the mixture state. Therefore, we deduce that the population inversion can be used as indicator for the mixture.

Dotted curve of the qubit-cavity entropy \( S(t) \) shows that, for a closed system \( \gamma = 0 \), the purity of the total system
state is not affected by the unitary qubit-cavity interaction. The entire entropy is time-independent $S(t) = 0$. While the sub-entropies grow with the same regular oscillatory behavior $S_Q = S_C$ with $n\pi$-period ($n = 0, 1, 2, ...$). So, one of them can be considered as measure for the entanglement between the qubit and cavity-field states. The qubit-cavity states are entangled during the collapse intervals, i.e., these intervals are indicators of the generation of the quantum coherence (entanglement and mixture).

In figures 1(b), (c), the effect of the intrinsic decoherence, is shown for $\gamma = 0.001\lambda$ and $\gamma = 0.1\lambda$. Due to the ID term: $I(t) = e^{-\gamma(t\hat{E}_0^2 - \hat{E}_1^2)}$, the revival phenomenon disappears and the oscillation amplitude of the population inversion deteriorates completely with the increase of the decoherence. The collapse intervals are much larger compared to the case $\gamma = 0$, i.e., appearance of more generated mixture with $\gamma > 0$.

For the case $\gamma > 0$, we note that:

1. As the $\gamma/\lambda$ increases, $S_C(t)$ splits further from $S_Q(t)$, and their oscillations disappear. The increase of the intrinsic decoherence leads to fact that the two-qubit and cavity-field states reach their stationary mixture state.

2. Due to intrinsic decoherence, the entropy $S(t)$ indicates that the purity of the entire qubit-cavity system loses its initial value and reaches a stationary mixture.

3. For $\gamma = 0.1\lambda$, the entropies reach quickly their stationary values, with more deterioration in their oscillatory behavior. This is due to the fact that the entropy functions depend on the off-diagonal elements of their matrices which are affected by the intrinsic decoherence.

Figure 2 shows the effect of the dipole-dipole interaction between two qubits on the population inversion $W(t)$, the entropy functions, $S_Q(t)$, $S_C(t)$ and $S(t)$, are plotted as
From equation (8), we find that the dynamic of the qubit-cavity state depends on the coefficients $F_{jk}$, which is a function of $\frac{\Omega}{2} \pm \lambda \sqrt{\left(\frac{k}{\sqrt{\pi}}\right)^2 + 4n^2 + 20n + 28}$. This clear explain the dependence of the quantum effects on the dipole-dipole interaction parameter $\Omega$. In figure 2(a), without the intrinsic decoherence $\gamma = 0$, we observe a decay on the amplitude of the revival of the population inversion and an increase of the its period of the collapse with more oscillations. The amount of the energy transfers from the upper two-qubit state $|\epsilon_a \epsilon_b\rangle + |\epsilon_a \gamma_b\rangle$ to the lower states $|\gamma_b^a \epsilon_b\rangle + |\gamma_b^a \gamma_b\rangle$ reduce and the two-qubit state evolves to the state $|\epsilon_a \gamma_b\rangle$. The lower values of the oscillations of the two-qubit and the cavity-field entropies increase. The dipole-dipole interaction enhances the generated entanglement between the qubits and the cavity-field, which tends to a maximally entangled state more than the case of the $\Omega = 0$.

Figure 2(b), shows that the dipole-dipole interaction works as additional damping in the presence of the intrinsic decoherence $\gamma = 0.001\lambda$. The quantum coherence quantifiers grow and reach their stationary values quickly, due to the dipole-dipole interaction, compared to the case $\gamma = 0$.

Figures 3 and 4 show the effect of the initial coherent field intensity on the entropy functions. For the case $\gamma = 0$, the oscillations of the population inversion are shifted up to around the value 0.3 (see figure 3). The collapse intervals disappears completely and the energy transfers reduce, i.e., the two-qubit state evolve to the upper state. The increase of the decoherence parameter $\gamma$ leads to the annihilation of the quantum coherence and an increase of the mixture. The stationary values of the entropy functions, $S(t)$, $S_C(t)$ and $S_Q(t)$ depend on the mean photon number $|\alpha|^2$.

In figure 4, for the dipole-dipole interaction $\Omega = 25\lambda$, the population inversion and the entropy functions have more irregular oscillations, and their oscillation frequencies speed up. We also observe that the effect of the intrinsic decoherence is reduced. The shifted up population inversion is an indictor of more generated quantum coherence (entanglement and mixture).

### 3.3. Cavity-qubit entanglement

Here, we use the negativity to investigate the generated entanglement between the cavity-field and the two-qubit states by using the density matrix $\rho(t)$. The negativity is defined by [12]

$$N(t) = \frac{1}{2}(\|\rho(t)^T\| - 1),$$

where the matrix $\rho(t)^T$ is the partial transpose of the matrix $\rho(t)$. The cavity-qubits state is maximally entangled for $N(t) = 1$ while it is disentangled state for $N(t) = 0$. 

Figure 3. The same as figures 1(a), (b), but with $\alpha = \sqrt{2}$.

Figure 4. As figures 3, but for $\Omega = 25\lambda$. 

![Figure 3](image3.png)

![Figure 4](image4.png)
Now to quantify the cavity-qubit entanglement, the negativity function $N(t)$ for $\rho(t)$ is plotted in figure 5 for $|\alpha|^2 = 16$ with different values of the intrinsic decoherence: $\gamma = 0.0$ in figure 5(a), $\gamma = 0.001 \lambda$ in figure 5(b) and $\gamma = 0.1 \lambda$ in figure 5(c). The solid curves are for the case $\Omega = 0$ while dashed curves for the case $\Omega = 25 \lambda$.

Without the dipole-dipole interaction and the intrinsic decoherence, $\Omega = 0$, the entanglement negativity has regular oscillations. Strong QE is generated between the cavity-field and the two-qubit states as expected, and the cavity-qubit states are disentangled at revival times $T_R = n \pi$, $n = 0, 1, 2, \ldots$ (see solid curve of figure 5(a)). Dashed curve of figure 5(a) shows that the dipole-dipole interaction $\Omega = 25 \lambda$ leads to irregular oscillations of the negativity. We can deduce that the cavity-qubit entanglement can be enhanced in the presence of the dipole-dipole interaction.

In Figures 5(b), (c), we observe that as the intrinsic decoherence $\gamma$ increases, the QE gradually decreases and tends to its stationary value (see figure 5(c)). From dashed curves, we can conclude that the dipole-dipole interaction, leads to the increase of the degree of entanglement between the cavity-field and the two qubits in the presence of the weak intrinsic decoherence $\gamma = 0.001 \lambda$. However, for the case of the strong intrinsic decoherence $\gamma = 0.1 \lambda$, the stationary cavity-qubit entanglement is not affected the dipole-dipole interaction.

### 3.4. Two-qubit entanglement

We study in this section, the dynamics of entanglement between the pair of coupled qubits with the intrinsic decoherence, showing the entanglement sudden birth, sudden death and revival. Here, the logarithmic negativity (LN) is used to investigate the QE between the two qubits in the state $\rho^{AB}(t)$. LN is quantified by [12]:

$$N_q(t) = \log[1 + 2n_p],$$  

where $n_p$ is the negativity function of $\rho^{AB}(t)$. $N_q(t) = 0$ for a separable state, and if $N_q(t) > 0$ the two-qubit states $\rho^{AB}(t)$ have partial entanglement.

Figure 6(a) shows how the nonlinearity of the unitary interaction between the two qubit and the cavity field through the two-photon processes leads the generation of strong entanglement between the two qubits, without the dipole-dipole interaction. The logarithmic negativity, $N_q(t)$, is plotted for $|\alpha|^2 = 16$ and $\Omega = 0$ with different values of $\gamma$: $\gamma = 0$, $\gamma = 0.001 \lambda$ and $\gamma = 0.1 \lambda$. Solid curves of figure 6(a) shows the dynamical behavior of the LN Function. We observe that the initially disentangled two-qubit state evolves periodically to partial entangled states. The generated qubit-qubit entanglement has regular fluctuations between its maximum and minimum with a $\pi$ - periodical dynamics. Due to the ID term: $f(t) = e^{-\gamma (t^n - t^n)}$, the LN function vanishes for a short time (see the next figure), then LN suddenly grows. The phenomena of the sudden death and birth entanglement is observed for several time windows [8, 9]. Dashed curves of figure 6(a), shows that the intrinsic decoherence leads to the decreasing of the LN amplitudes.

Figure 6(b) illustrates the dependence of the two-qubit entanglement on the dipole-dipole interaction. We observe that LN has higher frequency of irregular oscillations. The dipole-dipole interaction accelerates the accomplishment of the two-qubit stationary entanglement.

In figure 7(a), the LN is plotted for the small value of the mean photon number $|\alpha|^2 = \sqrt{2}$. We note that the results are different, where the amplitudes of LN decrease with higher oscillation frequencies and more additional fluctuations. The LN shows sudden death and birth two-qubit entanglement.

In figures 7(b), with the dipole-dipole interaction parameter $\Omega = 25 \lambda$, we observe more speeding and more irregular oscillations. The average value of the two-qubit
entanglement gradually increases. In the case of a small value of the initial coherent field intensity, the curves of figure 7 (a), (b) prove that the dipole-dipole interaction can enhance the generated entanglement. This generated entanglement is more robust against the intrinsic decoherence.

4. Conclusions

In this manuscript, we have explored dipole-dipole coupled two qubits in interaction with a coherent cavity-field. An analytical study of the Milburn equation is used to explore the population inversion and the generation of the quantum coherence via the entropy, partial entropies and the negativity in the presence of the intrinsic decoherence. Without the intrinsic decoherence, the entropy and the negativity show the same quantum coherence. The population inversion presents periodic revivals and collapses. During the collapse intervals, the system is in mixture state due to the transfer of population from upper to lower states. The dynamics of the generated entanglement in the qubit-cavity and the qubit-qubit systems, due to the unitary interaction, are sensitive to the dipole-dipole interaction as well as to the initial coherent field intensity. When the intrinsic decoherence is considered, the dynamics of the entropy and the negativity are different, and the mixtures of the entropy grow, while the entanglement negativity degrades. The dipole-dipole interaction accelerates the accomplishment of the stationarity. The growth of the entropy and the degradation of the entanglement negativity, due to the intrinsic decoherence, can be protected by increasing the dipole-dipole interaction and by reducing the initial coherent field intensity. For a small initial coherent field intensity, the dipole-dipole interaction not only enhances the generated entanglement but also empowers it to be more robust against the intrinsic decoherence.

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