Natural convection in a liquid metal locally heated from above

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Abstract. A convective flow of liquid sodium generated nearby a hot round in the upper solid end face of a vertical cylinder has been studied experimentally and numerically. A developed turbulent flow is observed in the upper part of the cylinder. Strong velocity pulsations penetrate in the bulk of the metal up to a distance of about the diameter of the cylinder. Mean velocity fields reveal a toroidal vortex, which is localized in a narrow upper zone. Numerical simulations were done for two types of thermal boundary conditions (BCs): fixed temperature and fixed homogeneous heat flux on both heat exchangers. Experimental values of time-averaged velocity and temperature in the vortex are in good agreement with numerical data. The size and the intensity of the vortex weakly depend on BCs. The whole bulk of the metal is not involved in the motion. The temperature field depends much more on the BCs. Under fixed heat fluxes the temperature pulsations become much stronger and penetrate essentially deeper in the liquid metal, though the flow is slightly stronger under fixed boundary temperature. The considered flow is supposed to be a simplified model of the liquid magnesium flow in a reactor of metallothermic titanium reduction.

1. Introduction
Homogeneous heating of fluid from above provides a stable stratification and does not cause convective flows. Local heating on the free surface of fluid activates nearby the surface vigorous motion due to both surface-tension-driven (thermocapillary) and buoyancy-driven (thermo-gravitational) convection. To isolate the latter mechanism one consider a covered surface (solid upper boundary). Some exact solutions are known for simplest statement, e.g. for laminar flow near a linear heat source disposed on the surface of liquid [1]. In some technological apparatus, intense turbulent convective flows are produced by strong local heating from above. This is for example the case of flows generated in the reactor of metallothermic titanium reduction [2].

The reactor for titanium reduction is a cylindrical vessel with a diameter up to 2 m and a height up to 4 m filled by liquid magnesium at a temperature of about 850°C. The chemical reaction of titanium reduction starts when titanium tetrachloride $TiCl_4$ is supplied to the magnesium surface. The reaction is highly heat-producing and runs predominantly on the surface producing spongy titanium and magnesium dichloride $MgCl_2$ that sink to the retort bottom. The strong release of heat (1700 kJ per 1 kg of titanium tetrachloride), compensated by cooling at the sidewall of the reactor produces a radial temperature gradient which can reach hundreds of degrees per meter. However, it is unknown how strong can be the generated convective flow and how deep penetrate the velocity and temperature pulsations. The large mass...
and dimensions of the vessel and very high temperatures complicate the direct experimental measurements inside the reactor, which is one of the motivations for numerical study of the process [2, 3]. Adequate simulations of turbulent convective flows of liquid metals under high governing parameters, typical for industrial systems, become possible not long ago [4, 5] and numerical codes require verification using reliable experimental data.

Liquid metals are characterized by high thermal diffusivity (low Prandtl number), which reduces the contribution of convection in heat transfer. It means that intense convective flow can be provided by a strong heat injection only. The experimental studies of liquid metal convection is also made difficult by complexity of thermal and dynamical measurements in liquid metals. There are few experimental works on convective flows, made on mercury or sodium (e.g., [6, 7, 8]).

A convective flow in GaInZn alloys heated locally from above and affected by magnetic field has been studied in context of the problem of electron beam evaporation of liquid metals [9].

In this work we consider a flow of liquid metal inside a vertical cylinder. The central part of the upper end face is heated, while the circumference of the same face is cooled. In this statement, the problem can be considered as a simple hydrodynamic model of a real reactor and allows both experimental and numerical study of the convective flow. Experimentally we study the flow of liquid sodium in a cylinder of diameter $D = 200$ mm and height $H = 700$ mm, which can be considered as infinite deep because the flow is concentrated in the very top of the cylinder. Numerically we study the convective flow in a three-dimensional non-stationary formulation, which allows us to analyze the instantaneous and average characteristics of the flow and the velocity and temperature pulsation fields. We aim to validate our numeric code using experimental data and to find out the structure of the convective vortex flow, analyzing how deep the velocity and temperature pulsations penetrate in the liquid metal bulk.

2. Methods

2.1. Experiment

The experimental setup designed to study the convection in liquid sodium with local heating from above consists of a cylindrical channel 1 (figure 1a), a heat exchanger 2, a pouring system 3, an expansion vessel 4 and a frame on which the installation is fixed. The channel 1 is made from stainless steel. The wall thickness is 8 mm, the channel length is 700 mm, the inner diameter is 200 mm. The channel is installed on the frame vertically. The bottom of the channel is connected to the pouring system. On the side wall of the channel there are ten locks 5, forming two vertical rows, opposite each other. Through these locks, the thermocouples of diameter 1 mm are inserted inside the channel. Five thermocouples are inserted to a depth of 100 mm (D1–D5), and the other five to a depth of 10 mm (N1–N5). The distance from the upper thermocouple to the end face is 40 mm. The distance between two adjacent thermocouples is 30 mm. To determine the temperature distribution along the entire cylinder, two sets of additional thermocouples (External 1 and 2 on figure 1a) with a step of 75 mm in each set are installed on channel’s wall from the outside. The angle between the sets is 90°. One more additional thermocouple (Bottom) measured the temperature at the bottom of the cylinder. At the upper boundary of the cylindrical channel, a flange is mounted to connect the heat exchanger. From the outside the channel is covered by mineral wool insulation with a thickness of about 50 mm, and additionally covered with aluminum foil. The internal volume of the unit can be connected with vacuum and argon systems.

The heat exchanger at the upper boundary of the channel provides heating of sodium in the central region and cooling in the peripheral region (figure 1b). Heating is carried out by spiral wire 6, which is wrapped around the copper cylinder 7 of diameter 40 mm. The lower end face of this cylinder contacts the liquid sodium, and the upper end face is connected to the expansion vessel 4. There is a small hole through the cylinder axis for passing the sodium to the expansion vessel. Additionally three thermocouples (H1–H3) are placed in the hole. These thermocouples
measure the temperature at the interface between the copper cylinder and liquid sodium. The thermocouples are going out through the locks 8 installed on the cover of the expansion vessel.

The main element of the cooling system is a 20 mm thickness copper plate 9. In the copper plate, five thermocouples are installed through special locks. Three locks are placed on the line along the channel radius with 18 mm pitch (C1–C3). Two other thermocouples (C4, C5) are placed in azimuth with an angle of 120° at the radius of the middle thermocouple C2. These thermocouples measure the sodium temperature at the boundary of the cooling region. The cooling system is placed into the box-type casing of the external air-flux system 10.

Neighboring thermocouples in the rows were used to estimate the mean velocity by cross-correlation analysis. The correlation method is useful to estimate the velocity in turbulent nonisothermal flows in which the temperature perturbations move with the fluid [10, 8]. In this method, the average fluid velocity in the volume between two thermocouples are determined using the position of the maximum of the cross-correlation function, which gives the mean transfer time, and the distance between the thermocouples.

2.2. Mathematical model

The computational domain is a cylinder with the height $H = 700$ mm and the diameter $D = 200$ mm (figure 1c). The convective parameters of the fluid, corresponding to liquid sodium at a temperature 125°C, are: Prandtl number $Pr = 1.01 \cdot 10^{-2}$, kinematic viscosity $\nu = 6.72 \cdot 10^{-7}$ m$^2$/s, and thermal expansion coefficient $\beta = 2.195 \cdot 10^{-4}$ K$^{-1}$. The numerical code solves the Oberbeck-Boussinesq equations of thermogravitational convection. The problem was solved in a three-dimensional nonstationary formulation. Simulations were done using a numerical mesh with a total of 2.4 million grid points. The mesh was non-uniform with a higher density of points near the boundaries in order to resolve the boundary layers. The Large Eddy Simulation (LES) approach is used for modelling turbulence, namely the Smagorinsky-Lilly model [11] with the Smagorinsky constant $C_s = 0.14$ and turbulent Prandtl number $Pr_t = 0.9$. All the simulations were run using the free and open source finite volume code OpenFOAM 4.0. The PISO (Pressure Implicit with Splitting of Operators) algorithm based on the pressure correction procedure is used to solve the derived system of equations. The terms with time
derivatives are discretised using an implicit Euler scheme. The convective terms are calculated by the TVD (Total Variation Diminishing) scheme [12]. The Courant number in our computations does not exceed 0.5. The numerical simulations were performed using Uran supercomputer of IMM UB RAS.

The simulations were done with two types of boundary conditions (BC) for temperature on the top surface. BC I – first-type (Dirichlet) thermal BC, fixed temperature on the heater equal to 350°C and fixed temperature on the cooler equal to 125°C. BC II – second-type (Neumann) BC, fixed homogeneous heat flux on the heater and on the cooler. The total flux \( Q = 1 \text{ kW} \), thereby the heat flux density on the heater is \( q_h = 795.8 \text{ kW/m}^2 \), the heat flux density on the cooler is \( q_c = -37.9 \text{ kW/m}^2 \). The ring with inner diameter 40 mm and with outer diameter 80 mm, which is placed between the heater and cooler, is heat insulated. The sidewalls are heat insulated. The no-slip velocity conditions are applied at all boundaries.

3. Results
Experimental setup allowed us to set the power of the heater and the air flow rate in the cooler. Setting these parameters we waited for a saturated regime, which established in some hours after the beginning of heating. Then the record of signals from all thermocouples was performed during few hours more. Figure 2 shows time-averaged temperature distribution along the cylinder (a) and the power spectrum of temperature pulsations for the uppermost thermocouple, set on the channel axis, D5 (b). The form of the power spectrum of temperature pulsations for this thermocouple indicates a developed turbulent flow in this area. The spectra of other thermocouples show a transient flow regime, and the thermocouple N1 (figure 3b) displays very weak pulsations of temperature (if a flow exists in its vicinity, than it is a laminar flow).

Thus, the turbulent convective flow is recorded only by the upper probe under the hot spot of the heat exchanger. The slope of “−5/3” is shown on the graphs as the reference point and not for discussing the existence of the inertial interval in the spectrum, which can be expected at higher frequencies. In the range of distances, covered by thermocouples, the mean temperature gradually decreases to the cylinder bottom, following a similar law inside the cylinder and on the outer side of the cylinder wall (figure 2a).

To estimate the mean vertical velocity along both lines of thermocouples, we have calculated the cross-correlation function for the signals of neighbouring thermocouples. The location of the maximum of cross-correlation and the distance between corresponding thermocouples allow to estimate the vertical velocity averaged over the region in the between of thermocouples and under the time. The accuracy of this estimation in our case is low because the pulsations are stronger than the mean values. The estimations done gave values of average velocity in the range 5 – 15 mm/s.

In numerical simulations we tried to reproduce the conditions of the experiment as close as possible, but the temperature distribution on the faces in the experiments is non-uniform and it is difficult to measure the actual temperature field on the heater and cooler. In the experimental setup the total power of the heater was fixed, but neither the distribution of heat flux density, nor the distribution of the temperature on the upper boundary could be fixed or controlled, because the thermal diffusivity of the copper is only a little higher than the thermal diffusivity of sodium. Moreover, the position of the heater, which was installed inside the cooler, made impossible to avoid some direct heat flux from the heater to cooler. Therefore, the numerical study was performed for two idealized boundary conditions, fixed temperature on the surface of the heater and cooler (BC I), and fixed homogeneous heat flux on both heat exchangers (BC II).

Figure 4 shows the temperature fields obtained in numerical simulations with BC II for horizontal cross-sections on the distance 20 mm and 40 mm from the top surface. The temperature distribution near the top surface in both instant and time-averaged fields (figure 4a,b) has a region with pronounced temperature maximum right below the heater. The
Figure 2. Time-averaged temperature distribution along the cylinder (a) and power spectrum of temperature pulsations for the thermocouple D5 (b).

Figure 3. Power spectra of temperature pulsations for thermocouples D1 (a), N1 (b) and N5 (c).

Field of temperature standard deviation (figure 4c) shows that the most intense pulsations occur in the ring region which separates the heater and cooler, and in the region close to the sidewalls. On the distance 40 mm from the top, where upper thermocouples were placed in the experiment, the temperature distributions (figure 4d-f) are very different from the previous case: there is no temperature maximum in the center and the amplitude of temperature variations is much lower.

Figure 5 presents the velocity fields in the vertical cross-section. The instant velocity field simulated with BC II (figure 5a) reveals a developed turbulent flow in the upper part of the cylinder. The intensity of nonstationary vortices decreases with depth and vanishes at a depth of the order of the cylinder diameter. The time-averaged velocity field (figure 5b) demonstrates a ring vortex near the upper face and a very weak counter-rotating vortex under it. The standard deviation of velocity fields (figure 5c) shows that the strongest pulsations are concentrated in the area of the upper vortex and that strong pulsations are observed on the distance up to 200 mm from the top face.

The difference in the flow structure between simulations with BC II and BC I can be revealed by comparing the top panel (a)-(c) and the bottom panel (d)-(f) in figure 5. In case of fixed temperature on the top face (BC I) the height of the upper vortex is lower but its intensity is higher (figure 5e). The figure 5f shows that the vortex itself becomes more stable, however the most pronounced pulsations occur in the central part of the vessel, in the region below the heater.

Figure 6a demonstrates time-averaged profiles of vertical velocity component $\langle V_z \rangle$ along
Figure 4. Results of simulations with BC II in the $xOy$ plane: $z = 0.68$ m (a) – (c), $z = 0.66$ m (d) – (f). Instant temperature field (a), (d); time-averaged temperature field (b), (e); standard deviation of temperature field (c), (f).

Figure 5. Results of simulations with BC II (a) – (c) and with BC I (d) – (f) in the $yOz$ plane, only upper part of the fields is shown. Instant velocity field (a),(d); time-averaged velocity field (b),(e); standard deviation of velocity field (c), (f).
the cylinder axis and along the line at a distance of 10 mm from the wall. One sees again that the changing of BC II to BC I leads to change in localisation of the upper vortex: it pins more tight to the upper heat exchanger and the vertical velocity rises. Note that the maximum velocity on the axis is observed at the distance 15 mm from the top, while on the side line the maximum is at 40 mm from the top face. This is the distance where the upper thermocouples were installed in the experimental setup. In the region where lower thermocouples were placed the velocity values are much lower.

The profiles of temperature pulsations obtained in simulations and in experiment are shown in figure 6b. Results of simulations show that temperature pulsations under BC II are stronger than under BC I, especially along the line near the sidewall. The level of temperature pulsations under BC I is much closer to the experimental results.

4. Conclusions
A convective flow of liquid metal generated nearby a hot round in the upper solid end face of a vertical cylinder has been studied experimentally and numerically. The considered flow is supposed to be a simplified model of the liquid magnesium flow in a reactor of metallothermic titanium reduction though the typical Grashof number in the reactor is about three order higher as in the experiment ($10^{12}$ versus $10^9$).

Ten thermocouples installed in the experimental setup along two vertical lines (on the cylinder axis and near the sidewall) allowed one to control the temperature in range of depth from 40 to 160 mm. Two upper thermocouples demonstrated strong temperature pulsations with developed turbulent spectra, while lower thermocouples shows strong reduction of pulsations with depth. The mean temperature distribution reveal in this range of depths a weak gradient, about 20 K/m, corresponding to stable stratification. The cross-correlation analysis of temperature pulsations allowed us to estimate the mean vertical velocity along both lines of thermocouples. The highest velocity is found around the upper thermocouples and is estimated as about 0.01 m/s.

The numerical study was performed for two idealized boundary conditions, fixed temperature on the surface of the heater and cooler (BC I), and fixed homogeneous heat flux on both heat exchangers (BC II). Obtained velocity fields show that a developed turbulent flow exists in the
upper part of the cylinder. Strong velocity pulsations penetrate up to distance of about 200 mm from the upper boundary. Mean velocity fields reveals one toroidal vortex, which is localized in a narrow upper zone, mainly above the upper thermocouples (40 mm from the upper boundary). The size and the intensity of this vortex weakly depends on the boundary conditions. The experimental estimations of the value of the mean velocity nearby the upper thermocouples agree with numerical data. The whole bulk of the metal is not involved in the motion. The temperature field depends much more on the boundary conditions, what is common for the liquids with small Prandtl number. Under fixed heat fluxes (BC II) the temperature pulsations become much stronger and penetrate essentially deeper in the liquid metal, though the flow is slightly stronger under BC I.

Acknowledgement
This study was partially supported by the Government of Perm Krai, project No C-26/060 11.03.2016. We thank Andrey Vasiliev, Alexander Pavlinov and Alexander Shestakov for help in experiment preparation and performance.

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