Is QCD asymptotically free?

I.M. Dremin

Lebedev Physical Institute, Moscow 119991, Russia

Abstract

The asymptotical behaviour of the QCD coupling strength is considered. Its upper bound is found from the present experimental data. The assumption about the finite asymptotical value of the coupling strength leads to the energy scale where the perturbative predictions would fail. Possible tests of this assumption are considered.

The property of the asymptotical freedom of the QCD coupling strength $\alpha_S$ is one of the most important ingredients which justifies the perturbative approach at high momentum transfer. The running $\alpha_S$ has been predicted by the perturbative calculations and confirmed by experiments at presently available energies. However at higher energies the dynamics and, consequently, the coupling strength behaviour can change. If our ideas about the grand unification theory (GUT) are valid, the coupling strengths of strong, electroweak and gravitation interactions should tend to a common asymptotic limit.

The fine structure constant of the electromagnetic forces is supposed to increase with the transferred momentum. The standard perturbative QED statement implies that it will become infinite in asymptotics if its present value $1/137$ is taken into account. Correspondingly, if the asymptotic limit is finite, then the perturbative arguments tell us that the electromagnetic interaction vanishes (the so-called "Moscow zero" effect). At higher momentum transfers the interacting partners penetrate deeper and the screening effect of the polarization cloud becomes less pronounced. The observed value of the charge should be smaller due to the partial screening by the virtual pairs. Nevertheless, there were proposals [1] to consider the variant with the finite limit for the "bare" charge.

Thus, what concerns the limiting value $\alpha_\infty$ of all coupling strengths, it must be, at least, larger than $1/137$ and lower than the $\alpha_S$ values measured at LEP (about 0.118) because the strong coupling should decrease with the momentum transfer increase. If this tendency persists asymptotically and

\[ 1\text{Email: dremin@lpi.ru} \]
there is a common limit with no crossover in the behaviour of various coupling strengths, then $\alpha_S$ should tend to this limit from above. This would imply that QCD is not asymptotically free in a rigorous sense. Let us consider such a possibility.

The traditional lowest order expression for the running QCD coupling strength is written as

$$\alpha_S(s) = \frac{4\pi}{\beta_0 \ln \frac{s}{Q_0^2}},$$

(1)

where $s$ is the squared momentum transfer (or energy for $e^+e^-$-processes), $\beta_0 = 11 - \frac{2}{3}n_f$ ($n_f$ is the number of active flavours). $Q_0$ is a cut-off parameter. It can be chosen (see, e.g., [2]) as $Q_0 = 0.65\Lambda_{\overline{MS}}$ where $\Lambda_{\overline{MS}}$ is the cut-off of the so-called $\overline{MS}$-renormalization scheme. With such a parameter the formula (1) actually reproduces quite well the higher order expression in $\overline{MS}$-scheme for $\alpha_S$. In what follows we use (1).

The value of $Q_0$ can be determined from (1) once the mass of $Z^0$-boson $M_Z \approx 91.2$ GeV and $\alpha_S(M_Z) \approx 0.118$ are known. One gets

$$Q_0 = M_Z \exp \left( -\frac{2\pi}{\beta_0 \alpha_S(M_Z)} \right) = 0.246 \text{ GeV } (0.153; 0.088).$$

(2)

Here and everywhere below the initial number corresponds to $n_f = 3$ and the numbers in brackets to $n_f = 4$ and 5.

The formula (1) describes the asymptotically free behaviour of the coupling strength $\alpha_S(s) \to 0$ at $s \to \infty$. This would imply the crossover with the electrodynamical fine structure constant. To avoid such crossover and provide the finite asymptotical limit for $\alpha_S(s)$ the following natural generalization of the expression (1) is proposed:

$$\alpha_S^{(m)}(s) = \frac{4\pi}{\beta_0 \ln \frac{s}{(s+s_0)Q_0^2}}.$$  

(3)

The modified coupling strength $\alpha_S^{(m)}(s)$ coincides with the running coupling strength (1) at low energies and tends asymptotically to the finite limit

$$\alpha_{\infty} = \frac{4\pi}{\beta_0 \ln \frac{s_0}{Q_0^2}}.$$  

(4)
It should be the common limit for all interactions in GUT. For this generalization one has to pay by the new energy scale parameter $s_0$:

$$s_0^{1/2} = Q_0 \exp \left( \frac{2\pi}{\beta_0 \alpha_\infty} \right).$$

(5)

Let us consider what corrections to the ordinary $\alpha_S$ are introduced by this parameter. From (3) and (1) one gets

$$\alpha^{(m)}_S(s) = \alpha_S(s) \left[ 1 - \frac{\ln(1 + \frac{s}{s_0})}{\ln \frac{s}{Q_0}} \right]^{-1}.$$  

(6)

In the low energy region $Q_0^2 < s \ll s_0$ it reduces to

$$\alpha^{(m)}_S(s) \approx \alpha_S(s) \left[ 1 - \frac{s}{s_0 \ln \frac{s}{Q_0}} \right]^{-1}.$$  

(7)

Thus the power-like correction to (1) is provided by the scale $s_0$. At very high energies $s \gg s_0$ the coupling strength tends to its asymptotical limit also in a power-like manner:

$$\alpha^{(m)}_S(s) \approx \alpha_\infty \left[ 1 + \frac{s_0}{s \ln \frac{s}{Q_0^2}} \right].$$  

(8)

The upper bound on the constant $\alpha_\infty$ is imposed by the accuracy of the present experimental data on $\alpha_S(M_Z)$:

$$\alpha_\infty \leq \frac{\alpha_S(M_Z)}{1 - x \ln(e^{\frac{\delta}{1}} - 1)},$$  

(9)

where

$$x = \frac{\beta_0 \alpha_S(M_Z)}{4\pi}$$  

(10)

and $\delta$ denotes the relative error in the determination of $\alpha_S(M_Z)$. For $\delta \ll x$ the good estimate is provided by the approximate expression:

$$\alpha_\infty \leq \frac{\alpha_S(M_Z)}{1 - \frac{\delta}{2} + x \ln \frac{s}{\delta}}.$$  

(11)
With present accuracy [3] about 2% (i.e., \( \delta = 0.02 \)) the following upper bounds are imposed on \( \alpha_\infty \):

\[
\alpha_\infty \leq 0.1061 \quad (0.1076; 0.1090). \tag{12}
\]

It would be necessary to improve the accuracy of the determination of \( \alpha_S(M_Z) \) to the level about 0.03%, to reach the upper bound on \( \alpha_\infty \) as low as 0.08. Unfortunately, such accuracy is hardly achievable at present.

This number is important because there are various arguments [4, 5] in favour of a definite limiting value of \( \alpha_\infty = 1/4\pi \approx 0.0796 \). Actually, these arguments are suggested for the asymptotical value of the "bare" electromagnetic charge \( e_0 = \sqrt{\hbar c} \) but the same limit is valid for strong interactions in GUT. The strong coupling strength is the closest one to this value (0.118 at \( Z^0 \)) at present. Therefore, it is most reasonable to try to get some knowledge about \( \alpha_\infty \) from QCD-governed processes.

If the limiting value \( \alpha_\infty = 1/4\pi \) is really preferred by nature, it becomes possible to estimate the characteristic energy scale \( s_0 \) from the formula (5):

\[
\sqrt{s_0} \approx 1.6 \text{ TeV} \quad (2; 2.6). \tag{13}
\]

It is seen that this scale is high but, surprisingly enough, much lower than Planck scales. It lies in the reach of the next generation of colliders. Nevertheless, to confirm or disprove these scales in experiment is very difficult if not problematic. To demonstrate this statement, we consider two methods of \( \alpha_S \) determination in \( e^+e^- \) experiments.

At the beginning, let us estimate the ratio \( \alpha_S^{(m)}(s)/\alpha_S(s) \) at various energies. At the \( Z^0 \) peak they differ by 0.03 - 0.01 per cents only. At \( \sqrt{s} = 1 \) TeV this difference increases to 2 - 3%. For energies \( \sqrt{s} = 3\sqrt{s_0} \) it can be as large as 10 -15%. Thus, in principle, the experiments in the high energy region can distinguish between the two possibilities.

Now, consider the ratio \( R \) of the hadronic to muon cross sections:

\[
R = R_0(1 + \frac{\alpha_S(s)}{\pi} + ...). \tag{14}
\]

The difference in \( R \) is \( \pi \) times smaller than the above differences. This implies that extremely precise measurements are needed at very high energies to discover the limiting value \( \alpha_\infty = 1/4\pi \).
Even more dramatic is the situation with the energy dependence of mean multiplicities in quark and gluon jets. It is known (for a review see, e.g., [6]) that these multiplicities increase as \( \exp c \sqrt{\ln s} \) for the running coupling strength [7] and like a power \( s^\kappa \) for the fixed coupling [8]. Surely, the power law takes over the first one at high energies. If the coupling strength stops its running at high energy, one would await this effect to show up. It really does, however, at extremely high and unreachable energies.

It is easy to estimate the corresponding exponents in the DLA expressions for the multiplicities from the abovementioned papers. The running coupling behaviour is

\[
\langle n \rangle_r = A \exp(2c \ln^{1/2} \frac{s}{Q_0^2}).
\]  

Here \( c = (4N_c/\beta_0)^{1/2} \), \( N_c = 3 \) is the number of colours, \( A \) is a common normalization constant, defined by the non-perturbative region.

The fixed coupling theory predicts the power-like growth of mean multiplicity with energy increase:

\[
\langle n \rangle_f = A \exp(c \ln \frac{s}{Q_0^2} / \ln^{1/2} \frac{s_0}{Q_0^2}).
\]  

Herefrom, the energy at which the fixed coupling regime would show up is

\[
\sqrt{s} = Q_0 \left( \frac{s_0}{Q_0^2} \right)^2 = Q_0 \exp \left( \frac{8\pi}{\beta_0 \alpha_\infty} \right).
\]  

These energies are far outside the particle energy region available in nature if \( \alpha_\infty = 1/4\pi \). They are about 10\(^{(23-25)} \) eV. These estimates show that the results of the jet calculations with the running coupling strength at present energies are safe and do not depend on its particular asymptotical value.

The only hope to get in the nearest future \( \alpha_\infty \) appears if its value is as "high" as 0.1 by some unknown reasons. Then the improvement by a factor of 2 in accuracy at \( Z^0 \) peak would already allow to check this hypothesis. At the energy 1 TeV, the modified coupling strength would be larger than the traditional perturbative value by about 15 - 20 \%. In addition, some other data (shape variables etc) could be more sensitive to the variations of the coupling strength. More complete analysis is needed to compare various upper bounds.

To conclude, the absence of the asymptotical freedom in QCD is hard to confirm or disprove in experiment in the nearest future if the limiting
value of the coupling strength is $1/4\pi$ or lower. If the asymptotical values of the coupling strength are very close to $0.1$, they can be quantitatively established when the accuracy of the data processing increases. The low values of the energy scale parameter $s_0$ are encouraging for new prospects at the next generation of accelerators. Apart from the considered data, other experimentally measured characteristics (like event shapes) could be more sensitive to these features of QCD. The complete analysis of all the data from this point of view should be done and upper bounds on $\alpha_\infty$ determined and compared to answer the question posed to the title of the paper.

This work has been supported in parts by the RFBR grants N 02-02-16779, 03-02-16134, NSH-1936.2003.2.

References

[1] M. Gell-Mann and F. Low, Phys. Rev. 95 (1954) 1300.

[2] A.V. Radyushkin, Fiz. Elem. Chast. i Atom. Yadra 14 (1983) 59 (in Russian).

[3] Particle Data Group, D.E. Groom et al., Eur. Phys J. C15 (2000) 1.

[4] A.F. Ranada, Ann. de la Fond. L. de Broglie 27 (2002) 505.

[5] V.I. Ritus, ZhETF 124 (2003) N7 (to be published).

[6] I.M. Dremin and J.W. Gary, Phys. Rep. 349 (2001) 301.

[7] Yu.L. Dokshitzer, V.A. Khoze, A.H. Mueller and S.I. Troyan, Basics of perturbative QCD ed. by J. Tran Thanh Van (Gif-sur-Yvette, Editions Frontieres, 1991).

[8] I.M. Dremin and R.C. Hwa, Phys. Lett. B324 (1994) 477; Phys. Rev. D49 (1994) 5805.