A 96 GeV scalar tagged to dark matter models

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July 31, 2019

Abstract

Recently, the CMS Collaboration observed the hint of a resonance decaying to two photons at about 96 GeV with a local significance of 2.8σ. While it is too early to say whether this will stand the test of time, such a resonance can easily be accommodated in several extensions of the Standard Model (SM). Assuming that the new resonance is a scalar, we try to see its effect on several simplistic models that also contain a cold dark matter candidate. As expected, we find that with such a scalar, the parameter space gets more constrained and hence, more tractable. In this paper, we explore the consequences of such SM extensions that provide a 96 GeV scalar and one or more cold dark matter candidates. We show how significant constraints can be placed on the parameter space, not only from direct dark matter searches or LHC data but also from theoretical considerations like scattering unitarity or stability of the potential.

1 Introduction

The existence of cold dark matter (CDM) is perhaps the biggest motivation to search for physics beyond the Standard Model (SM). The SM does not contain any suitable CDM candidate, something that is massive, and singlet under $U(1)_{em}$ and $SU(3)_c$. Any extension of the SM with a $Z_2$ symmetry for which all the SM fields are $Z_2$-even can potentially come to the rescue, with the CDM candidate being the lightest $Z_2$-odd particle. The lightest neutralino of the R-parity conserving supersymmetric models, and the lightest Kaluza-Klein particle of the Universal Extra Dimension models, fall in this category. Gauge singlet objects like massive right-handed neutrinos, or additional scalars, can also fit the bill. In our subsequent discussion, we will assume a thermalised non-baryonic CDM with the density given by [1]

$$\Omega_{CDM}h^2 = 0.1186 \pm 0.0020,$$

where $h = H_0/100$ is the reduced Hubble constant. The existing literature is abound with the possible models of CDM, their detection strategies, and other phenomenological aspects, and let us refer the reader, in general, to some review articles [2], and in particular, to Ref. [3] for the analysis of several Higgs portal CDM models.

It is always an interesting exercise to link the CDM with some other beyond-SM signals or even motivations, like the generation of neutrino masses, or the flavour anomalies. In this paper, we will study some CDM models in conjunction with the recently observed hint of a (possibly) scalar resonance at about 96 GeV from the CMS Collaboration [4], decaying to a diphoton final state. Although there has been neither confirmation nor denial from the ATLAS Collaboration, this hint has given rise to a lot of interesting theoretical speculations, and the physics implication of such a resonance in the context of beyond-SM scenarios has been discussed in Refs. [5–8].
How does this new scalar, which we will call $\chi$, help in the analysis? While $\chi$—assuming that it exists—can never be a CDM candidate, it can very well be one component of a multiplet with the other component(s) constituting the dark matter, or it can be a mediator for the CDM-SM interaction. A fairly straightforward example, which we will discuss in detail, is to add a complex singlet to the SM; one of its components is the DM, while the other leads to $\chi$. The interesting part is that such a scenario reduces the number of free parameters in the model, and hence the allowed parameter space for the CDM becomes more tractable. This is, in a sense, the rationale of this paper.

In this paper, we consider three models that accommodate both $\chi$ and a prospective CDM candidate. They are, respectively,

1. Complex Scalar (CS): This consists of the SM augmented by a complex scalar singlet $S$. One of the components of $S$ mixes with the SM doublet, and these two states appear as $\chi$ and $h$, the 125 GeV Higgs boson.

2. Real Scalar with Fermion (RSF): Here one has a real gauge singlet scalar (which mixes with the SM doublet), and a vectorial gauge singlet fermion $\psi$ that plays the role of the CDM candidate.

3. Complex Scalar with Fermion (CSF): This is, in a sense, an amalgamation of the previous two models. We take a complex singlet scalar $S$ with the singlet fermion $\psi$, and consider the possibility where we have two CDM candidates, one scalar and the other fermionic.

We will concentrate on the allowed parameter space for all these models, taking the theoretical and experimental constraints into account. One major constraint is the stability of the potential; when the couplings evolve with energy, the potential should neither be unbounded from below, nor should the couplings blow up at the Landau pole (a more conservative statement is that they should remain perturbative). Up to what scale the potential should be well-behaved? Ideally, it is the Planck scale $M_{\text{Pl}}$, but that makes the parameter space too restrictive; even the SM may not be stable up to $M_{\text{Pl}}$. In fact, this scale can be anything beyond the reach of the LHC, where some new degrees of freedom appear and cure the bad behaviour of the potential. We take this scale to be 100 TeV for all our subsequent discussion. Pushing up the scale further only squeezes the parameter space, but no qualitative change takes place.

The paper is organised as follows. In Section 2, we give a brief outline of these models, including the theoretical and experimental constraints. The final parameter space for all the three models, taking into account the aforementioned constraints plus the direct CDM search results, are shown in Section 3, where we also enlist a few novel observations. Section 4 summarises and concludes the paper.

## 2 The Models

As mentioned in the Introduction, we will discuss three models in this Section. The common characteristics of these three models are: (i) they have one (or more) potential CDM candidate(s); (ii) apart from the 125 GeV scalar resonance $h$, which is dominantly the SM doublet Higgs boson, every model is constructed in such a way as to further accommodate another scalar $\chi$ at 96 GeV. Even if the 96 GeV bump does not stand the test of time, the results, on the allowed parameter space of the CDM mass and couplings, will still be more or less valid.

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2 The Higgs portal of real scalar singlet CDM is already ruled out, except either in a narrow resonance region around $m_h/2$, or for large CDM mass.
The following constraints on the potential need to be considered:

- Existence of a definite ground state, which means that the potential cannot be unbounded from below along any direction in the field space. As mentioned before, we allow the possibility of some new physics to take over at a scale $\Lambda$, so all we need is a well-defined ground state of the potential up to $\Lambda$, beyond which the new physics may cure any possible malady and make the potential well-behaved. We take $\Lambda = 100$ TeV.

- Partial wave unitarity, which essentially leads to the conservation of probability.

- Triviality, or the constraint that none of the couplings hit the Landau pole below $\Lambda = 100$ TeV. As we will see later, these two conditions turn out to be almost equivalent.

- The invisible decay width of the Higgs when light scalars and/or fermions are present in the model should be less than 19% [9].

- The dominantly doublet nature of $h$, which means that the mixing angle with the singlet scalar must be small.

- The constraints coming from the CDM direct detection experiments, interpreted in terms of a thermalised dark matter, as well as those coming from the relic density of the CDM, $i.e.,$ the universe must not be overclosed [10].

- Bounds on the oblique electroweak parameters, which, however, have a completely negligible effect if the singlet-doublet mixing is tiny, and/or the new fermion is a gauge singlet with no mixing to the SM fermions.

Before we go into the three models, let us have a brief recapitulation of the Real Singlet (RS) model, which is SM plus a real gauge singlet scalar $S$. The scalar potential can be written as

$$V(\Phi, S)_{RS} = -\frac{m^2}{2}\Phi^\dagger\Phi + \frac{b_2}{4}S^2 + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2 + \frac{\delta_2}{4}\Phi^\dagger\Phi S^2 + \frac{d_2}{16}S^4,$$

(2)

where $S$-odd terms are banished from the potential by a $Z_2$ symmetry: $S \rightarrow -S$. We will denote the CP-even neutral component of $\Phi$ by $\phi$, with

$$\langle \phi \rangle \equiv v = 246 \text{ GeV}.$$

(3)

In the absence of $\Phi$-$S$ mixing, $\phi = h$, where $h$ is the physical 125 GeV scalar resonance.

If $S$ does not get any vacuum expectation value (VEV), it can act as a potential CDM candidate. However, as has been shown in Refs. [11–14], the allowed region for $m_S < m_h$ is rather fine-tuned; the CDM solution survives only in a narrow resonance region around $m_S \approx m_h / 2$. There are other allowed regions for large values of $m_S$, like $m_S > 1$ TeV, where $S$ can be a viable CDM candidate. If we allow $S$ to be a component of CDM, and not the only constituent, the allowed region starts from $m_h \approx m_S$. All these situations have been extensively explored. We will not discuss this model any further, but there is a point one may note.

Leaving the dark matter solution aside, one can easily explain the 96 GeV resonance. If $\langle S \rangle \neq 0$, $S$ and the CP-even component of $\Phi$, namely, $\phi$, will mix to give rise to two states, $h$ with $m_h = 125$ GeV, which is dominantly the doublet $\phi$, and $\chi$, with $m_\chi = 96$ GeV, which is dominantly a singlet$^3$.

The decay $\chi \rightarrow \gamma\gamma$ proceeds through the top quark and gauge loops, albeit suppressed by the mixing

$^3$This mixing angle is constrained from the LHC data on Higgs, which shows it to be dominantly the doublet.
angle; whether such suppression is strong enough to prevent the \( \chi \rightarrow \gamma \gamma \) signal is another issue that we will not take up here. This scenario also leads to fermionic decay modes of \( \chi \), notably \( \chi \rightarrow b \bar{b} \), but it is hard to detect them so close to the \( Z \) resonance.

The parameter space is constrained assuming the CDM candidate(s) satisfying the relic density limit, \textit{i.e.}, there are no other particles that contribute to the relic density. To compute the relic density one solves the Boltzmann equation \cite{11} for \( Y = n/s \), where \( n \) is the number density and \( s \) is the entropy density, and calculates the thermal averaged cross-section. Assuming a scalar CDM \( S \) and a coupling of the form \( (\delta_2/4)\Phi^\dagger \Phi S^2 \), the spin-independent CDM-nucleon cross-section is given by

\[
\sigma_{SI} = \frac{\delta_2^2 m_N f^2}{16 \pi m_S^2 m_h^4},
\]

where \( m_N \) is the nucleon mass, and \( f \) is the nuclear form factor. In this paper, we take the CDM to be thermalised.

\[ \text{2.1 The Complex Scalar Model (CS)} \]

Extending the scalar sector by a complex singlet \( S \equiv (S_1 + iS_2)/\sqrt{2} \), the most general renormalisable potential is of the form \cite{15}

\[
V(\Phi, S) = -\frac{m^2}{2} \Phi^\dagger \Phi + \frac{\lambda}{4} (\Phi^\dagger \Phi)^2 + \left( \frac{\delta_1}{4} e^{i\theta_1} \Phi^\dagger \Phi S + \text{c.c.} \right) + \frac{\delta_2}{2} \Phi^\dagger \Phi |S|^2 + \left( \frac{\delta_3}{4} e^{i\theta_3} \Phi^\dagger \Phi S^2 + \text{h.c.} \right) + \left( \frac{a_1}{4} e^{i\theta_1} S + \text{h.c.} \right) + \left( \frac{b_1}{4} e^{i\theta_1} S^2 + \text{h.c.} \right) + \left( \frac{c_1}{6} e^{i\theta_1} S^3 + \text{h.c.} \right) + \left( \frac{c_2}{6} e^{i\theta_2} S^2 |S|^2 + \text{h.c.} \right) + \left( \frac{d_3}{8} e^{i\theta_3} S^2 |S|^2 + \text{h.c.} \right) + \frac{d_2}{4} |S|^4,
\]

where the couplings are taken to be real apart from an explicit phase factor, generically written as \( \exp(i\theta) \), with \( 0 \leq \theta \leq \pi \). We will first try to confine ourselves to a simpler case, by reducing the number of independent parameters through a global \( U(1) \) symmetry on \( S \): \( S \rightarrow S \exp(i\zeta) \). This simplifies the potential to

\[
V(\Phi, S) = -\frac{m^2}{2} \Phi^\dagger \Phi + \frac{\lambda}{4} (\Phi^\dagger \Phi)^2 + \frac{\delta_2}{2} \Phi^\dagger \Phi |S|^2 + \frac{b_2}{2} |S|^2 + \frac{d_2}{4} |S|^4,
\]

but leads to a massless Goldstone boson in the spectrum when the symmetry breaks spontaneously to give \( S \) a VEV. To avoid this, we introduce a soft \( U(1) \) breaking term (but still keeping the \( \mathbb{Z}_2 \) of \( S \rightarrow -S \) intact) in the potential:

\[
V_{CS}(\Phi, S) = -\frac{m^2}{2} \Phi^\dagger \Phi + \frac{\lambda}{4} (\Phi^\dagger \Phi)^2 + \frac{\delta_2}{2} \Phi^\dagger \Phi |S|^2 + \frac{b_2}{2} |S|^2 + \frac{d_2}{4} |S|^4 + \left( \frac{b_1}{4} e^{i\theta} S^2 + \text{h.c.} \right).
\]

The phase \( \theta \) helps in realising the vacuum alignment condition. We take \( \theta = \pi \) and let \( S_1 \) get the VEV:

\[
\langle S \rangle = \frac{1}{\sqrt{2}} \langle S_1 \rangle = s_1/\sqrt{2}.
\]

In terms of the component fields, the potential is

\[
V_{CS}(\phi, S_1, S_2) = -\frac{m^2}{4} \phi^2 + \frac{\lambda}{16} \phi^4 + \frac{\delta_2}{8} \phi^2 (S_1^2 + S_2^2) + \frac{b_2}{4} (S_1^2 + S_2^2) + \frac{d_2}{16} (S_1^2 + S_2^2)^2 - \frac{b_1}{4} (S_1^2 - S_2^2).
\]
with the extremisation conditions
\[
\begin{align*}
\frac{\partial V}{\partial \phi} &= -\frac{m^2}{2} \phi + \frac{\lambda \phi^3}{4} + \frac{\delta_2}{4} \phi (S_1^2 + S_2^2) = 0, \\
\frac{\partial V}{\partial S_1} &= \frac{S_1}{2} \left( b_2 - b_1 + \frac{\delta_2 \phi^2}{2} + \frac{d_2}{2} (S_1^2 + S_2^2) \right) = 0, \\
\frac{\partial V}{\partial S_2} &= \frac{S_2}{2} \left( b_2 + b_1 + \frac{\delta_2 \phi^2}{2} + \frac{d_2}{2} (S_1^2 + S_2^2) \right) = 0.
\end{align*}
\] (10)

Note that the terms of the RS and the CS potentials have been written in such a way as to ensure identical CDM couplings with the Higgs as well as its self-coupling.

Obviously, the solution with \( \langle \phi \rangle = v = 0 \) is not physical. For \( v \neq 0 \), there can be two possible cases that will give rise to a CDM candidate: (i) \( s_1 = s_2 = 0 \), and \( s_1 \neq 0, s_2 = 0 \). In the first case, there is no mixing between \( \phi \) and \( S_{1,2} \), and hence no \( \chi \), so we will drop that from our focus. Only the second case is interesting, so let us treat that in more detail.

The conditions \( v \neq 0, s_1 \neq 0, s_2 = 0 \) lead to
\[
\begin{align*}
s_1^2 &= \frac{2\lambda (b_2 - b_1) + 2\delta_2 m^2}{\delta_2^2 - \lambda d_2}, \\
v^2 &= \frac{2\delta_2 (b_2 - b_1) + 2d_2 m^2}{\lambda d_2 - \delta_2^2},
\end{align*}
\] (11)
thus ensuring that the four minima at \( (\pm \sqrt{v^2}, \pm \sqrt{s_1^2}) \) are of equal depth, which is a consequence of the still-intact \( \mathbb{Z}_2 \) symmetry.

The couplings can be further constrained from the existence of a well-defined ground state:
\[
\lambda > 0, \quad d_2 > 0, \quad \lambda d_2 > \delta_2^2.
\] (12)

The mass matrix for the neutral scalars is
\[
M^2 = \frac{1}{2} \begin{pmatrix}
\lambda v^2 & \delta_2 v s_1 & 0 \\
\delta_2 v s_1 & d_2 s_1^2 & 0 \\
0 & 0 & 2b_1
\end{pmatrix}.
\] (13)

with the mass eigenvalues
\[
\begin{align*}
m_{h,\chi}^2 &= \frac{\lambda v^2}{4} + \frac{d_2 s_1^2}{4} \pm \sqrt{\left( \frac{\lambda v^2}{4} - \frac{d_2 s_1^2}{4} \right)^2 + \frac{\delta_2^2 v^2 s_1^2}{4}}, \\
m_{S_2}^2 &= b_1.
\end{align*}
\] (14)

\( S_2 \) is a stationary state itself; there is no mixing with the other scalars as \( s_2 = 0 \).

The \( h-\chi \) mixing angle \( \alpha \) is given by
\[
\tan (2\alpha) = \frac{2\delta_2 v s_1}{\lambda v^2 - d_2 s_1^2}.
\] (15)

For the analysis, it is helpful to express the parameters of the potential, namely, \( \lambda, d_2, \delta_2, m, b_2, \) and \( b_1 \), in terms of the VEVs \( v \) and \( s_1 \), the mixing angle \( \alpha \), and the three masses \( m_h, m_\chi \) and \( m_{S_2} \).
Among these, $m_h$, $m_\chi$, and $v$ are known, so there are only three free parameters\(^4\). The transformation equations between the two bases are

\[
\begin{align*}
\lambda &= \frac{2}{v^2} \left( m_h^2 \cos^2 \alpha + m_\chi^2 \sin^2 \alpha \right), \\
d_2 &= \frac{2}{s_1^2} \left( m_h^2 \sin^2 \alpha + m_\chi^2 \cos^2 \alpha \right), \\
\delta_2 &= \frac{(m_h^2 - m_\chi^2) \sin(2\alpha)}{v s_1}, \\
m^2 &= \frac{1}{2} \left( \delta_2 s_1^2 + \lambda v^2 \right), \\
b_1 - b_2 &= \frac{1}{2} \left( d_2 s_1^2 + \delta_2 v^2 \right), \\
b_1 &= m^2 S_2, \quad (16)
\end{align*}
\]

We would also like to see how the dimensionless couplings evolve with energy. The one-loop $\beta$-functions are [16]

\[
\begin{align*}
16\pi^2 \beta_{d_2} &= 5d_2^2 + 2\delta_2^2, \\
16\pi^2 \delta_2 &= \delta_2 \left( 2d_2 + 2\delta_2 - 3\frac{g_1^2}{2} g_2^2 - 9\frac{g_1^2}{2} + 3\lambda + 6g_t^2 \right), \\
16\pi^2 \beta_\lambda &= \left( \delta_2^2 + 3\frac{g_1^4}{2} + 3g_1^2 g_2^2 + 9\frac{g_1^4}{2} g_2^2 - 3g_1^2 \lambda - 9g_2^2 \lambda + 6\lambda^2 + 12\lambda g_t^2 - 24g_t^4 \right), \quad (17)
\end{align*}
\]

with $\beta_x = dx/dt$, and $t \equiv \ln(q/q_0)$, where $q_0$ is some reference scale to set the boundary conditions on the couplings, and $q$ is the relevant energy scale where they are measured. Higher orders do not affect our results in any appreciable way. We must ensure that nowhere in the parameter space the triviality bound is reached below 100 TeV.

The parameter space becomes more complicated with the introduction of several new couplings if we allow the breaking of the $Z_2$ symmetry. However, qualitatively it does not add much over the parameter space of the CS model that we will discuss in the next Section. One important modification is that the vacua are no longer degenerate, and one has to put the additional constraint that either the SM vacuum is the deepest one, or it is a false vacuum with the tunnelling time larger than the age of the universe. We will not discuss the $Z_2$ breaking case any further.

### 2.2 Real singlet scalar with vectorial fermion (RSF)

Let us add one vector singlet fermion $\psi$ to the real singlet model of Eq. (2). This leads to the potential

\[
V(\Phi, S, \psi) = -\frac{m^2}{2} \Phi^\dagger \Phi + \frac{\lambda}{4} \left( \Phi^\dagger \Phi \right)^2 + \frac{\delta_2}{4} \Phi^\dagger \Phi S^2 + \frac{b_2}{4} S^2 + \frac{d_2}{16} S^4 + m_{\psi} \tilde{\psi} \psi + \left( Y/\sqrt{2} \right) \tilde{\psi} \psi S, \quad (18)
\]

with $Y$ being the Yukawa coupling, whose presence already breaks the $Z_2$ symmetry of $S \rightarrow -S$.

Using the same notation as for the CS model, we get the scalar mass eigenstates as

\[
m_{h,\chi}^2 = \frac{1}{4} \left[ \lambda v^2 + d_2 s_1^2 \pm \sqrt{ \left( \lambda v^2 - d_2 s_1^2 \right)^2 + 4\delta_2^2 v^2 s_1^2 } \right], \quad (19)
\]

\(^4\alpha\) is also known to be small.
The mixing angle

$$\tan (2\alpha) = \frac{2\delta_2 s_1}{\lambda v^2 - d_2 s_1^2}.$$  \hfill (20)

The fermion $\psi$ is the CDM candidate, whose mass is $m_{\text{CDM}} = m_\psi + Y s_1 / \sqrt{2}$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{The dark matter annihilation cross-section as a function of the singlet scalar VEV, $s_1$, in the RSF model. The cross-section dips at $s_1 \sim 50$ GeV, due to destructive interference of the $s$- and $t$-channel amplitudes. For more details, see text.}
\end{figure}

The inclusion of $\psi$ helps in stabilising the potential: The negative contribution in the $\beta$-function keeps $\delta_2$ and $d_2$ in check, which in turn affects the running of $\lambda$. The one loop $\beta$-functions are given by

\begin{align*}
16\pi^2 \beta_{d_2} &= \frac{9}{2} d_2^2 + 2\delta_2^2 + 4d_2 Y^2 - 8Y^4, \\
16\pi^2 \beta_{\delta_2} &= \delta_2 \left( \frac{3}{2} d_2^2 + 2\delta_2 - \frac{3}{2} g_1^2 - \frac{9}{2} g_2^2 + 2Y^2 + 3\lambda + 6g_1^2 \right), \\
16\pi^2 \beta_{\lambda} &= \frac{\delta_2^2}{2} + \frac{3}{2} g_1^4 + 3g_1^2 g_2^2 + \frac{9}{2} g_2^4 - (3g_1^2 + 9g_2^2) \lambda + 6\lambda^2 + 12\lambda g_1^2 - 24g_1^4, \\
16\pi^2 \beta_Y &= \frac{5}{2} Y^3. \hfill (21)
\end{align*}

With more scalars added to the SM, the scalar quartic couplings tend to blow up, hitting the Landau pole much below the Planck scale. One has to choose points in the parameter space such that not only Eq. (1) is satisfied, but the couplings also remain perturbative up to $\Lambda = 100$ TeV. As we will see later, this puts some nontrivial constraints on the parameter space.

In Fig. 1, we show the annihilation cross-section of the CDM, $\psi \bar{\psi} \to X$, where $X$ is any allowed SM particle-antiparticle pair, or even $\chi \bar{\chi}$. The $s$-channel amplitude is mediated by the singlet scalar $S$ (and hence by $h$ and $\chi$ after mixing), while the $t$-channel process is simply $\psi \bar{\psi} \to SS$. The amplitudes depend on the Yukawa coupling $Y$. For this plot, the CDM mass is kept fixed at 100 GeV, so the cross-section, in turn, depends on the VEV $s_1$. We find that there is a dip in the cross-section near $s_1 \approx 50$ GeV, as the $s$- and $t$-channel amplitudes interfere destructively. The position of the dip, obviously, will depend on the chosen value of $m_\psi$ and $Y$. This fact will be relevant when we study the allowed parameter space.
2.3 Complex singlet scalar with vectorial fermion (CSF)

Analogous to RSF, the particle content is that of the CS model plus the singlet vectorial fermion $\psi$. The potential looks like

$$V_{\text{CSF}}(\Phi, S, \psi) = V_{\text{CS}} + m_{\psi} \bar{\psi} \psi + \frac{Y}{\sqrt{2}} \left( \bar{\psi} \psi S_1 + i \bar{\psi} \gamma_5 \psi S_2 \right),$$

(22)

where $V_{\text{CS}}$ is given by Eq. (7). This can lead to two distinct possibilities:

(i) $\langle \Phi \rangle = v/\sqrt{2}$, $\langle S_1 \rangle = s_1$, $\langle S_2 \rangle = 0$: The scalar sector will be completely identical to that of the CS model, and there are two possible CDM candidates, namely, $\psi$ and $S_2$. Based on the number of CDM candidates, we will call this model CSF-2.

(ii) $\langle S_2 \rangle = s_2 \neq 0$. This leads to $\phi$-$S_1$-$S_2$ mixing. In other words, the field $S$ gets a complex VEV $s = s_1 + is_2$. The only CDM candidate is $\psi$. This will, therefore, be called the CSF-1 model.

The phenomenology of CSF-1 is more or less the same as that of RSF, so we will concentrate on the CSF-2 model. The scalar mass eigenstates are given by Eq. (19), as well as $m^2_{S_2} = b_1$, while the mass of $\psi$ is, as before, $m_{\text{CDM}} = m_{\psi} + Y s_1/\sqrt{2}$. The $h$-$\chi$ mixing angle $\alpha$ is the same as shown in Eq. (20). The one loop $\beta$-functions are:

$$16\pi^2 \beta_d = 5d_2^2 + 2\delta_2^2 + 4d_2 Y^2 - 8Y^4,$$
$$16\pi^2 \beta_d = \delta_2 \left( 2d_2 + 2\delta_2 - \frac{3}{2} g_1^2 - \frac{9}{2} g_2^2 + 2Y^2 + 3\lambda + 6g_1^2 \right),$$
$$16\pi^2 \beta_\lambda = \delta_2^2 + \frac{3}{2} g_1^4 + 3g_1^2 g_2^2 + \frac{9}{2} g_2^4 - (3g_1^2 + 9g_2^2) \lambda + 6\lambda^2 + 12\lambda g_1^2 - 24g_1^4,$$
$$16\pi^2 \beta_Y = 2Y^3.$$

(23)

Figure 2: Renormalisation group flow of the scalar quartic couplings. Left and Centre plots are for the CS model, while the right one is for the CSF model. For the CS model, the low-energy values of $\delta_2$ and $d_2$ are, (a): (0.065, 2.9), (b) (0.06, 2.45), (c) (0.056, 2.2) respectively, and they hit the Landau pole at $5.4 \times 10^7$ GeV, $4 \times 10^8$ GeV, and $1.7 \times 10^9$ GeV respectively. For the CSF model (right plot), we use $\delta_2 = 0.035$ and $d_2 = 0.82$, with $Y = 0$ (a), 0.8 (b), and 0.84 (c).

Once the new fermion is added, the negative pull of the $Y^4$ term coming from the box diagram tends to delay the onset of Landau pole, and if $Y$ is large enough, can even make the potential unbounded from below. In Fig. 2 we show these trends; the first two plots are for the CS model, while the last one is for the CSF model. The plots are explained in the figure caption, and the values were taken in such a way that they satisfy $\Omega_{\text{CDM}}h^2 \leq 0.12$. 


3 Constraints on the parameter space

The major constraints on the parameter space have been enlisted in Section 2. We assume a thermalised dark matter distribution and use micrOMEGAs v5.0.8 [17] to obtain the relic density as well as the CDM-nucleon scattering cross-section. The one-loop RG equations were solved through SARAH v4.14.1 [18], and the unitarity constraints were found from Ref. [19]. The rest of the constraints were dealt with analytically.

3.1 The CS and the RSF models

Let us first explain the strategy for the CS model. As $m_\chi$ is fixed at 96 GeV, only two relevant free parameters remain: $\alpha$ and $s_1$. From Eq. (16), one can trade them for the scalar quartics $d_2$ and $\delta_2$. The third free parameter, $b_1$, is nothing but the mass squared of the CDM, i.e., $S_2$.

![Figure 3](image1.png)

Figure 3: The allowed parameter space for the CS model with $\alpha = 0.01$. The purple region is excluded from overclosure ($\Omega h^2 \geq 0.12$), the region above the dashed black line is excluded from the invisible decay of $h$ (BR[$h \rightarrow \text{invis}$] $< 0.19$), the region above the dashed orange line is excluded from the direct detection experiments, the region above the long-dashed blue line is excluded from the scattering unitarity constraints, and the region above the solid red line is excluded from potential stability (no Landau pole before 100 TeV). The light green region is allowed from all theoretical and experimental constraints.

![Figure 4](image2.png)

Figure 4: The CS parameter space plot as in Fig. 3, but for $\alpha = 0.1$. Legends remain the same.
The relic density, therefore, is a function of \( d_2, \delta_2, \) and \( m_{S_2} \). The mixing angle \( \alpha \) has to be small enough to maintain the dominantly doublet nature of \( h \). Thus, a good strategy is to fix \( \alpha \) and see what values of \( s_1 \) produce the correct relic density for different CDM masses \( m_{S_2} \). Alternatively, one can show the constraints taking \( d_2 \) or \( \delta_2 \) as the free parameter.

![Graph](image)

**Figure 5:** The CDM-nucleon cross-section versus the CDM mass for the CS model, for \( \alpha = 0.01 \) (left) and \( \alpha = 0.1 \) (right). Legends remain the same as in Fig. 3. Only the green shaded region is allowed.

A look at Fig. 3 should tell the reader how the constraints work. The left plot displays the allowed region for \( d_2 \), and the right one for \( \delta_2 \). The green shaded region is what ultimately remains allowed, after all the theoretical and experimental constraints are imposed. The salient features of these plots are:

- The mixed quartic \( \delta_2 \) is much more tightly constrained from the pure singlet quartic \( d_2 \). This is because of its role in the scalar mass matrix as well as the RG equations. However, the nature of the two plots is very similar.

- For low values of \( \alpha \), the parameter space ruled out by the direct detection limit and that ruled out from the invisible Higgs decay are competitive. However, stability of the potential and scattering unitarity (which essentially put some upper bounds on the couplings) rule out a significant amount of parameter space that is still allowed by direct detection. They rule out a large chunk of the parameter space, particularly for \( m_{S_2} > 200 \) GeV. Again, this depends on the scale \( \Lambda \) where the CS model is taken over by some ultraviolet-complete theory. The plots were drawn with \( \Lambda = 100 \) TeV; if it is higher, the green region will be even more squeezed.

- The single narrow resonance region for the RS model now expands to two closely lying such regions. The lower mass one comes from the apparent stability of \( \chi \), and the higher mass one from that of \( h \).

In Fig. 4, we show identical plots but for \( \alpha = 0.1 \). While the qualitative features remain similar to Fig. 3, there are two important changes. First, while the direct detection limit remains more or less the same, the stability and unitarity lines have become more relaxed for \( \delta_2 \) (and remain the same for \( d_2 \)), which can qualitatively be understood from the RG equations. As a result, the direct detection bound has started to cut in the theoretical constraints for lower values of \( m_{S_2} \), approximately \( m_{S_2} < 200 \) GeV. Second, the resonance regions have become wider, which can be explained easily from the \( \alpha \)-dependence of the couplings between physical scalars.
One can translate these bounds directly to the CDM-nucleon scattering cross-section limits, which is shown in Fig. 5, for both $\alpha = 0.01$ and $\alpha = 0.1$.

Figure 6: The allowed parameter space for the RSF model for $\alpha = 0.05$ and Yukawa coupling $Y = 0.7$. The purple region is excluded from overclosure, the portion above the red and blue horizontal lines (which are overlapping in these plots) are excluded from scattering unitarity and triviality bounds respectively, and the portion below the horizontal cyan line near the bottom is excluded from the stability of the potential (which becomes unbounded from below). The region left to the short-dashed black line is excluded from the invisible decay of the Higgs. Only the green shaded region remains allowed. The entire region is allowed from the direct detection bounds.

Figure 7: The same as Fig. 6 but with $Y = 0.85$. All other legends remain the same.

The allowed parameter space for the RSF model is shown in Figs. 6, 7, and 8. The fermion $\psi$ is the CDM candidate, and couples only to the real singlet $S$. Thus, for a large enough Yukawa coupling $Y$ between $S$ and $\psi$, the potential runs the risk of being unbounded from below at a high scale. In Figs. 6-8, we have also delineated the region where such a catastrophe takes place (the lower part of the horizontal cyan line). The instability threshold is taken, again, to be at 100 TeV or more, i.e., some other new physics comes into play at that scale to make the potential stable. While the entire region showed in these figures is allowed from the direct detection limits, one may note the power of the triviality and stability constraints, which ultimately leaves only a narrow region around 150 GeV as allowed. The allowed region expands for larger values of $Y$, as the overclosure bound starts shrinking.

For Figs. 6-8, the mixing angle $\alpha$ is fixed at 0.05. For higher values of $\alpha$, $\psi$ couples with $h$ more strongly and hence direct detection constraints tend to rule out more of the allowed parameter space.
shown here. For even smaller values of $\alpha$, resonance regions no longer exist. It is also easy to understand why the constraint from the invisible decay of the Higgs is a vertical line. The invisible decay width depends on the CDM mass, $\alpha$, and $Y$, but here $\alpha$ and $Y$ are fixed, so the only dependence is on $m_{\text{CDM}}$.

The window near $m_{\text{CDM}} \sim 150$ GeV, in Figs. 6-8, is interesting. Let us see what happens when we keep $m_{\text{CDM}}$ fixed in that region but increase $d_2$ or $\delta_2$. Such a shift decreases $s_1$, the VEV of $S$. This increases the relic density to such a point as to hit the overclosure bound, so the region becomes disallowed and the neck begins. If we continue increasing $d_2$ or $\delta_2$, the relic density starts decreasing after a point (e.g., one may look at Fig. 1 to see how the annihilation cross-section changes, moving from right to left), and the parameter space again opens up, terminating the neck.

Let us also display the interdependence of the couplings $d_2$, $\delta_2$, and $Y$ in Fig. 9. We kept the mixing angle fixed at $\alpha = 0.05$ and changed the scalar VEV $s_1$, which in turn controls $d_2$ and $\delta_2$. Apart from the shaded overclosure region, there are three lines in each plot. The vertical line at the right is the bound from direct detection; as the CDM mass is kept fixed at 100 GeV, the limit depends only on $Y$, and not on the scalar quartics. The region above the almost horizontal red line is ruled out because the couplings blow up before 100 TeV. Large values of $Y$ can delay the onset of the Landau pole; that is
why the line veers slightly upwards for large \( Y \) (although that region is ruled out from direct detection). The region below the cyan line is ruled out because the potential, at least at the tree-level, becomes unstable there. Only the island within these three lines and the overclosure region remains allowed.

### 3.2 The CSF-2 model

The CSF-1 model, with the fermion being the sole dark matter candidate, is qualitatively very similar to the RSF model, even more so if the second singlet scalar is heavy. We, thus, do not explicitly show the features, and rather concentrate on CSF-2. Between the two CDM components, for consistency, we denote the scalar mass by \( m_{S_2} \) and the fermion mass by \( m_{\text{CDM}} \).

![Figure 10](image1.png)

**Figure 10:** The composition of the CDM in the CSF-2 model, with the mass difference \( \Delta M \) (in GeV) of the CDM components plotted against the relic density. The blue line corresponds to the contribution of \( \psi \) and the red line to that of \( S_2 \), while the green line gives the combined contribution. Left: \( m_{\text{CDM}} > m_{S_2} \), with \( m_{S_2} = 100 \) GeV. Right: \( m_{S_2} > m_{\text{CDM}} \), with \( m_{\text{CDM}} = 100 \) GeV. For both the plots, the Yukawa coupling \( Y = 0 \). The relic density limit \( \Omega h^2 = 0.12 \) is also shown by the black horizontal line.

![Figure 11](image2.png)

**Figure 11:** The allowed parameter space for the CSF-2 model for \( \alpha = 0.05 \) and Yukawa coupling \( Y = 0.7 \). The legends are identical with that of Fig. 6, apart from the dashed orange line, the region above which is excluded from the direct detection experiments. For more explanation, see text.

With the two possible CDM candidates, there can be interconversions, \( \bar{\psi}\psi \leftrightarrow S_2S_2 \), and the standard annihilations to bosonic and fermionic final states. The composition of the dark matter is shown in Fig.
Figure 12: The CDM-nucleon cross-section versus the CDM mass for the CSF-2 model for $\alpha = 0.05$ and Yukawa coupling $Y = 0.7$. Legends remain the same as in Fig. 11. Only the green shaded region is allowed.

Figure 13: The allowed region for the CSF-2 model, with $\alpha = 0.05$ and $m_{\text{CDM}} = m_{S_2} + 200$ MeV. The region below the red horizontal line, above the cyan line, right to the overclosure patch, and left to the vertical line, is allowed.

10, with $Y = 0.7$ and scalar VEV $s_1 = 100$ GeV. In the left plot, we take the fermion $\psi$ to be heavier than the scalar $S_2$, and in the right plot, just the opposite. The lighter CDM mass is fixed at 100 GeV. Note that if $S_2$ is heavier than $\psi$, it quickly saturates the $\Omega h^2 = 0.12$ limit. On the other hand, if $\psi$ is heavier, its contribution slowly rises with the mass difference $\Delta M$, and hits the relic density limit at $\Delta M \approx 450$ GeV.

The allowed parameter space for the CSF-2 model is shown in Fig. 11, with $m_{\text{CDM}} = m_{S_2} + 200$ MeV, $\alpha = 0.05$ and $Y = 0.7$. One finds that the plots display traits similar to both CS and RSF models, as expected. For example, the line showing the invisible decay constraint starts out vertically for small $d_2$ or $\delta_2$, like the RSF model, and then shows a rightward shift as found for the CS model. One may note the important role the triviality and unitarity lines play; they cut out a significant portion of the otherwise allowed parameter space.

Fig. 12 shows the allowed region for the CDM-nucleon scattering cross-section, analogous to Fig. 5. Fig. 13, similarly, shows the allowed region for the parameters of the potential, analogous to Fig. 9.
4 Conclusion

In this paper, we have studied the parameter space for several extensions of the SM that provide one or more cold dark matter candidates as well as a scalar at 96 GeV. The existence of the latter was only hinted by the CMS Collaboration, but if tagged with a CDM model, this provides further constraints on the parameter space, by reducing the number of free parameters in the Lagrangian.

We analysed the parameter space of three models: (i) SM plus a complex scalar singlet (CS), with one of the scalars being the CDM, and the other giving rise to the 96 GeV resonance after mixing with the SM doublet; (ii) SM plus a real scalar singlet and a singlet vectorial fermion (RSF), with the fermion being the dark matter; and (iii) CS plus a singlet vectorial fermion (CSF), with both fermion and scalar being dark matter candidates. The theoretical constraints that were taken into account includes the stability of the potential (both triviality and boundedness), the scattering unitarity (although it hardly differs from the triviality constraints), and the oblique parameters, the effect of the latter being negligible. The experimental constraints include those coming from the direct detection of dark matter, the overclosure bound on the relic density ($\Omega h^2 < 0.12$), and the invisible decay width of the Higgs boson. We have assumed a thermalised nonrelativistic dark matter and used micrOMEGAs v5.0.8 to generate the relic density.

Several interesting features emerge from the analysis, and they have been displayed in the previous section. One may enlist them once again here:

1. The triviality/unitarity constraints play a vital role in restricting the allowed parameter space for the CS and the CSF-2 models. In fact, the parameter space gets further squeezed if we assume the possible onset of a new physics at higher than 100 TeV. The severity of their effect depends on the singlet-doublet mixing angle $\alpha$. If $\alpha$ is small enough, one finds that these constraints are always more powerful than those coming from direct detection. With larger values of $\alpha$, they become comparable.

2. The CS and the CSF-2 models allow two narrow resonance regions, at approximately $m_h/2$ and $m_{\chi}/2$, and they become wider as the mixing angle increases. Thus, one may still have a sub-100 GeV scalar dark matter, which is not overly fine-tuned.

3. The RSF model is much more tightly constrained than its CS or CSF counterparts. A large part of the parameter space allowed from the relic density bound is ruled out by triviality and unitarity limits. However, there exists a narrow window, whose position depends on the singlet scalar VEV. This window appears because of the destructive interference between the $s$- and $t$-channel annihilation amplitudes. Even then, a significant part of this window is truncated from the stability of potential.

While such a study might be interesting to the model builders as well as those looking for collider signatures of beyond-SM physics, one must be cautious in applying these bounds. The parameter space, even after allowing for the 96 GeV scalar, is complicated enough, and we have refrained from doing a complete scan over all the parameters. The allowed regions will shift for different values of the scalar VEV $s_1$, the CDM mass, or the splitting between the two CDM candidates for the CSF-2 model. However, we expect the general trends to remain qualitatively similar.

Acknowledgement: AK acknowledges the Science and Engineering Research Board, Government of India, for support through the Grant EMR/2016/001306 and the DIA fellowship.
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