CP violation in $\tau$ decays

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CP violation of the charged lepton sector has not been found yet. $\tau$ is a unique charged lepton which can decay into hadrons. In this talk, we argue what kind of new physics can be studied with CP violation of $\tau$ lepton decays. We also discuss how to handle hadronic final state interactions for a prediction of the direct CP violation.

1. Introduction

CP violation of the lepton sector has not been found yet. CP violation of the neutrino sector will be explored by measuring the asymmetry of the oscillation probabilities: $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$. The CP violation requires the flavor violation and non-degeneracy of the masses for the charged leptons and neutrinos. Yet another type of CP violation of the lepton sector can be explored using the $\tau$ lepton which can be produced abundantly in B factories and Super flavor factories.

First let us summarize the present status of CP and T violation of the charged lepton sector. The second example of CP violation is in semileptonic decays. There is "known" CP violation of the semileptonic $\tau$ decays even within the standard model $^{1,2}$.

$$\Gamma[\tau^- \to K_\pi^{-0} \nu] = \frac{1}{1 + A_L} \frac{|q|^2 \Gamma[\tau^- \to K^0 \pi^- \nu]}{\Gamma[\tau^+ \to K^+ \pi^+ \nu]} = 1 - A_L$$

where in the second line, we assume $\Gamma[\tau^- \to K^0 \pi^- \nu] = 1$ which is valid within the tree level approximation of the standard model. $A_L$ is CP violation of the charge asymmetry of $K_L \to \pi^{-0} \nu$ and $K_L \to \pi^{-0} \bar{\nu}$ with $l = e, \mu$

$$A_L = \frac{\Gamma[K_L \to \pi^{-0} \nu] - \Gamma[K_L \to \pi^{-0} \bar{\nu}]}{\Gamma[K_L \to \pi^{-0} \nu] + \Gamma[K_L \to \pi^{-0} \bar{\nu}]} = \frac{\Delta A}{\Gamma[\pi^- \nu]} = (3.32 \pm 0.06) \times 10^{-3},$$

where $p$ and $q$ are the mixing amplitudes of $K^0$ and $\bar{K}^0$ in the mass eigenstates $K_{S,L}$

$$|K_{S,L}| = \frac{p}{|K^0(\pi d)|} + q|K^0(\bar{s}d)|, \quad \frac{|K_{S,L}|}{\pi} = \frac{p}{|K^0_q(\pi d)|} - q|K^0(\bar{s}d)|.$$ (7)

The first limit of CP violation of $\tau$ decay is set by CLEO $^3$. Assuming the parametrization for the decay $\tau^- \to K^0 \nu$

$$A(\tau^- \to K^0 \nu) = \bar{u}_\mu(1 - \gamma_5)u_\tau \times f_V(-q^\mu + \frac{\Delta K_\tau}{Q^2}Q^\mu) + \Lambda u(1 - \gamma_5)u_\tau f_s M.$$ (8)

where $q = p_K - p_\tau, Q = p_K + p_\tau$, and $M = 1$(GeV). The bound on the parameter of CP violation is obtained as $^3$

$$-0.172 < \text{Im} A < 0.067.$$ (9)

In the present talk, we give our predictions for the CP violation of $\tau \to K \pi \nu$ decays in a two Higgs doublet model. The results including the $K\eta$ and $K\eta'$ and the improved form factors are given in $^3$.

2. What kind of CP violation might manifest itself in $\tau$ decay?

We first give an example of the beyond the standard model which gives rise to the CP violation in $\tau$ hadronic decays $^3, 4, 5$. The direct CP violation of
τ decay may appear when there are two or more interfering amplitudes with different weak phases and the strong phases. In $\tau^- \to K^- \pi^0 \nu$ and its CP conjugate process $\tau^+ \to K^+ \pi^0 \bar{\nu}$, within the standard model, the charged current interaction leads to the decays as,

$$\begin{align*}
\tau^- &\to \nu_\tau W^{-*} \to \nu_\tau \bar{u}s \to \nu_\tau K^- \pi^0, \\
\tau^+ &\to \bar{\nu}_\tau W^{+*} \to \bar{\nu}_\tau \bar{u}u \to \bar{\nu}_\tau K^+ \pi^0.
\end{align*}$$

In the two Higgs doublet model, the charged Higgs interaction may also generate the amplitude,

$$\tau_R^- \to \nu_i H^{-*} \to \nu_i S_R u_L \to \nu_i K^- \pi^0,$$  

where $i = e, \mu$ and $\tau$. When the flavor of the neutrino is $\tau$–flavored, the charged Higgs contribution may interfere with the contribution of the standard model. The charged Higgs contributes to the angular momentum $L = 0$ state (s wave) of the hadronic ($K \pi$) system in the hadronic rest frame and the charged current due to $W$ boson interaction contributes to $L = 1, 0$ state. The strong phase due to the final state interaction for the s wave of $K$ and $\pi$ is different from the phase shift of the p wave. Moreover, the CP phase of the charged Higgs contribution may be also different from the weak phase $V_{us}$ of the standard model contribution. Therefore one may expect the direct CP violation in the interference of the amplitudes of the p wave and the s wave of $K$ and $\pi$ system. On the flavor of neutrino in Eq.(11), by ignoring the mass term of the neutrino compared with the $\tau$ lepton mass, only in the case that the flavor of the neutrino is $\tau$ flavored, the interference occurs. For the other flavor case which corresponds to $\nu_\tau$ and $\nu_\mu$, the interference term in the charged current contribution is absent. Therefore, the direct CP violation may not be expected from the flavor changing case in Eq.(11).

3. CP violation of $\tau$ decay, edm and flavor changing neutral current (FCNC).

In Fig.(1), it is shown how the direct CP violation of $\tau$ hadronic decay and edm of the $\tau$ lepton is related to each other in the two Higgs doublet model. We also show, in the same model, how the flavor changing interaction in charged current mediated by the charged Higgs boson exchange is related to the flavor changing neutral current (FCNC) mediated by the neutral Higgs boson. Since the charged Higgs boson forms the SU(2) doublet with the neutral Higgs boson, we can expect the CP violation and the flavor changing in Eq.(11) also contributes to the CP violation and flavor changing in the neutral Higgs sector, namely,

$$\tau_R \to H^- \nu_L \leftrightarrow \tau_R \to H^0 l_i (L).$$

where $H^0$ is a linear combination of the three mass eigenstates of the neutral Higgs bosons. For $i = \tau$, the flavor diagonal coupling of $\tau$ lepton and neutral/charged Higgs boson denoted by $y_{33}$ is CP violating and it contributes to the $\tau$ electric dipole moment as shown in Fig. 1. For the flavor changing coupling $i = \mu$ denoted by $y_{32}$ in Fig. 1, it also leads to the flavor changing neutral current process in the charged lepton such as $\tau \to \mu \bar{\nu}_\mu$ mediated by the neutral Higgs boson exchange. Therefore, one may constrain the flavor changing coupling of Eq. 11 from the charged lepton FCNC process such as $\tau \to l_i l_j^\pm l_j^-$ ($l_i, l_j = \mu, e$) while CP violation of the flavor diagonal coupling can be constrained from edm of $\tau$ lepton.

4. CP violation measurement of the unpolarized $\tau$ decay

Now we turn to the measurement of the CP violation. The issue has been discussed in [2]. In order to extract the interference term from Eq.(10) of [5] by replacing $\beta \sin^2 \theta$ in Eq.(29) with $\theta$ where $\cos \theta = n_\tau \cdot n_k$. Here, $n_k$ and $n_\tau$ are the directions of the kaon and $\tau$ lepton in the hadronic rest frame respectively as shown in Fig. 2,

$$\frac{d^2\Gamma}{d\sqrt{s}d\cos \theta} = \frac{G_F^2|V_{us}|^2}{2\pi^3} \frac{(m_\tau^2 - s)^2}{m_\tau^4} l(s) \left( \frac{m_\tau^2}{s} \cos^2 \theta + \sin^2 \theta |l(s)|^2 |F(s)|^2 + \frac{m_\tau^2}{4} |F_\pi(s)|^2 \right) - \frac{m_\tau^2}{\sqrt{s}} l(s) \cos \theta \text{Re}(FF_\pi^*) \right)$$

(13)
where \( l(s) \) is the three momentum of kaon in the hadronic rest frame. \( \sqrt{s} \) is the invariant mass for hadrons. \( F \) and \( F_S \) are the vector and the scalar form factors defined below.

\[
(K^+(p_K)P(p_P)|\bar{u}\gamma_\mu s|0) = F(Q^2)q^\mu + (F_S(Q^2) - \frac{\Delta_{K^0}}{Q^2} F(Q^2))^\mu, \quad (14)
\]

with \( Q^\mu = (p_K + p_\pi)^\mu \) and \( \Delta_{K^0} = m^2_K - m^2_\pi \). From the angular distribution, one can define the forward and the backward asymmetry, \( A_{FB}(s) \),

\[
A_{FB}(s) = \frac{dT|_{\cos\theta > 0} - dT|_{\cos\theta < 0}}{dT|_{\cos\theta < 0}},
\]

\[
-\frac{G^2_F|V_{us}|^2 (m^2_\pi^2 - s)^3 m^2_\pi}{8\sqrt{s}} \psi(s)^2 \text{Re.}(FF^*_S). (15)
\]

As we can see from Eq. \((15)\), the forward and backward asymmetry defined at the hadronic rest frame is the difference of the numbers of the events for the kaon scattered into the forward direction and the backward direction with respect to the incoming \( \tau \). Note that the asymmetry is proportional to the interference term of the vector and the scalar form factors.

**5. The scalar and vector form factors from the chiral Lagrangian**

To predict the forward and the backward asymmetry, we must estimate the form factors. We have used an effective chiral Lagrangian including the vector \( [9] \) and the scalar \( [10] \) resonances.

\[
\mathcal{L} = \frac{f^2}{4} \text{Tr} D^\mu D_{\mu} U + B \text{Tr} M(U + U^t)
\]

where \( U \) is the chiral field defined by,

\[
U = \xi^2, \quad \xi = \exp(\frac{i\pi}{2}),
\]

\[
\pi = \begin{pmatrix}
\frac{\pi^-}{\sqrt{2}} & \frac{\pi^0}{\sqrt{2}} & \frac{\pi^+}{\sqrt{2}} \\
\frac{\pi^-}{\sqrt{2}} & \frac{\pi^0}{\sqrt{2}} & \frac{\pi^+}{\sqrt{2}} \\
\frac{\pi^-}{\sqrt{2}} & \frac{\pi^0}{\sqrt{2}} & \frac{\pi^+}{\sqrt{2}} \\
\end{pmatrix}
\]

\[
M \text{ is the chiral breaking term,}
\]

\[
M = \begin{pmatrix}
m_u & 0 & 0 \\
0 & m_d & 0 \\
0 & 0 & m_s \\
\end{pmatrix}
\]

The scalar and the vector fields are given as,

\[
S = \begin{pmatrix}
\frac{\delta^0}{\sqrt{2}} & \frac{\delta^+}{\sqrt{2}} & \frac{\kappa^+}{\sqrt{2}} \\
\frac{\delta^0}{\sqrt{2}} & \frac{\delta^+}{\sqrt{2}} & \frac{\kappa^+}{\sqrt{2}} \\
\frac{\delta^0}{\sqrt{2}} & \frac{\delta^+}{\sqrt{2}} & \frac{\kappa^+}{\sqrt{2}} \\
\end{pmatrix}
+ S_0
\]

\[
V = \begin{pmatrix}
\frac{\rho^0}{\sqrt{2}} & \frac{\omega^0}{\sqrt{2}} & \frac{K^+}{\sqrt{2}} \\
\frac{\rho^0}{\sqrt{2}} & \frac{\omega^0}{\sqrt{2}} & \frac{K^+}{\sqrt{2}} \\
\frac{\rho^0}{\sqrt{2}} & \frac{\omega^0}{\sqrt{2}} & \frac{K^+}{\sqrt{2}} \\
\end{pmatrix},
\]

where \( S_0 \) is the vacuum expectation value of the scalar and is given as \( S_0 = \frac{g_2 M^2}{M^2} \). We note the kinetic terms of the scalar generate the mixing of the scalar (\( \kappa(0^+) \)) and the vector mesons (\( K^*(1^-) \) ) because,

\[\text{Tr} D^\mu S D^\nu S = \text{Tr} (\partial^\mu S + i g [V^\mu, S_0]) (\partial^\nu S + i g [V^\nu, S_0]).\]

We have computed the hadronic form factors relevant for the processes \( \tau \to \nu K^\pi^0 \) defined in Eq. \((15)\). The vector form factor is given as,

\[
F(Q^2) = \frac{1}{\sqrt{2}} \left( -\frac{R + R^{-1}}{2} + \frac{M^2}{2g^2 F^2 \pi} \left( 1 - \frac{M^2}{M^2_K - Q^2} \right) \right).
\]

The scalar form factor is,

\[
Q^2 F^\pi_S = -(m_s - m_u) K^+ \pi^0 |\bar{u}s|0 >
\]

\[
= \frac{1}{2\sqrt{2}} \left( \frac{Q^2}{M^2_K - Q^2}(R + R^{-1}) + \frac{Q^2}{M^2_K - Q^2} \frac{m^2_\pi}{2\Delta} (2\Delta R + (1 + \Delta) R^{-1}) \right) + \Delta \frac{Q^2}{M^2_K - Q^2} (1 - \Delta) \left( \frac{m^2_\pi}{2\Delta} R + \frac{m^2_K}{1 + \Delta} R \right).
\]
where \( R = \frac{g_{K\pi}}{F_{\pi}}, \Delta_{K\pi} = m_{K}^2 - m_{\pi}^2, \Delta = \frac{m_{s} + m_{\pi}}{2m_{s}} \sim \frac{1}{3}. \) The form factors are computed using the chiral Lagrangian of Eq. (11). The major contribution of the \( \tau \to \nu K\pi \) decay comes from the decay chain of \( \tau \to K^0\nu \to \nu K\pi. \) We also have the contribution from the scalar resonance \( \kappa \) as \( \tau \to \kappa\nu \to \nu K\pi. \) The latter contributes to the scalar form factor. The form factors given in Eq. (21) and Eq. (22) do not include the contribution of the width of the resonances. We have included the width by computing the imaginary part of the self-energy corrections for \( K^*, \kappa \) and their mixing. Then for instance, the inverse propagator for \( K^* \) is modified as,

\[
A = M_{K^*}^2 - Q^2 - iM_{K^*}\Gamma_{K^*}(Q^2)
\]

\[
\Gamma_{K^*}(Q^2) = \frac{3}{48\pi M_{K^*}} \left( \frac{\nu_{K\pi}^2}{Q^2} + \frac{\nu_{K\pi}^2}{Q^4} \left( \frac{F_s}{F_{\pi}} \right)^2 \right) g_{K^*K\pi}^2,
\]

(23)

where \( g_{K^*K\pi} = \frac{M_{K^*}^2}{g_{K\pi}} = 3.246 \) which reproduces \( \Gamma_{K^*}(M_{K^*}) = 50.8(\text{MeV}). \) The form factors including the width of the \( K^* \) and \( \kappa \) are compared with the prediction of the chiral perturbation theory in Fig. (3).

Using the form factor described above, we have computed the double differential rate in Fig. (4), the angular distribution in Fig. (5), and the hadronic invariant mass spectrum \( \frac{d^2\Gamma}{d\nu d\cos\theta} \) in Fig. (6). The branching fraction for \( \tau^+ \to K^+\pi^0\nu \) is obtained by integrating the hadronic invariant mass spectrum. The theoretical prediction is given by,

\[
\text{Br}(\tau^+ \to \pi^0K^+\nu)_{\text{th}} = 0.448 \pm 0.0044\%,
\]

(24)

where we have changed the scalar mass as \( M_{\kappa} = 800 \pm 50. \) The theoretical prediction of Eq. (24) can be compared with the experimental result from Babar [12]:

\[
\text{Br} = 0.416 \pm 0.003 \pm 0.018\%.
\]

(25)

One may also use the branching fraction \( \tau^\pm \to K^\pm\pi^\pm\nu \) to estimate the \( \tau^\pm \to K^\pm\pi^0\nu \) in the isospin limit,

\[
\text{Br}(\tau^\pm \to K^\pm\pi^\pm\nu) = |p|^2 \text{Br}(\tau^+ \to K^0\pi^+\nu) + |q|^2 \text{Br}(\tau^- \to K^0\pi^-\nu),
\]

\[
= (1 + |\epsilon_{m^2}|^2)\text{Br}(\tau^- \to K^0\pi^-\nu),
\]

\[
= 2(1 + |\epsilon_{m^2}|^2)\text{Br}(\tau^- \to K^-\pi^0\nu)
\]

\[
\approx 0.90\%.
\]

(26)

where \( p = \frac{1 + m_{\pi^0}}{\sqrt{2}}, q = \frac{1 - m_{\pi^-}}{\sqrt{2}}. \) The numerical value of Eq. (26) should be compared with the Belle measurement [11] \( \text{Br}(\tau \to K^\pm\pi^\pm\nu) = 0.808 \pm 0.004 \pm 0.026\%. \) In Eq. (20), we assumed \( \frac{\text{Br}(\tau^- \to K^0\pi^-\nu)}{\text{Br}(\tau^+ \to K^0\pi^+\nu)} = 1 \) and use
the isospin relation \( \text{Br}(\tau^- \to K^0\pi^-\nu) = 2\text{Br}(\tau^- \to K^-\pi^0\nu) \).

Figure 6: The prediction of the differential branching fraction for \( \tau \to K^-\pi^0\nu \). The horizontal axis shows the hadronic invariant mass \( \sqrt{s} \) (MeV). The vertical axis shows \( \frac{d\text{Br}}{d\sqrt{s}} \). We vary the scalar meson mass as \( M_s = 750, 800, 850 \). As for comparison with experimental data, the spectrum for \( \tau^\pm \to K^\pm\pi^\mp\nu \) has been measured by Belle [11].

### 6. New Physics interaction and source of CP violation

Having explored the various distributions of the decay within the standard model, we turn to CP violation of the two Higgs doublet model. It has been known that the two Higgs doublet model with the condition of the natural flavor conservation, the charged Higgs coupling to the \( \tau \) lepton and neutrino is real as in the standard charged current interaction. Therefore within the scheme, we may not have the CP phase. Then we relax the condition of natural flavor condition as [4],

\[
-\mathcal{L} = y_{1ij} R_i H_1^\dagger l_{Lj} + y_{2ij} R_i H_2^\dagger l_{Lj} + h.c.
\]

We allow the charged lepton to acquire mass from two Higgs \( H_1 \) and \( H_2 \). One can, in general, take the following parametrizations for Higgs fields,

\[
H_1 = e^{i\frac{\theta_{CP}}{2}} \left( \frac{v_1 + h_1 - i \sin \beta A}{\sqrt{2}}, -\sin \beta H^- \right),
\]

\[
H_2 = e^{i\frac{\theta_{CP}}{2}} \left( \frac{-\cos \beta H^+}{\sqrt{2}}, v_2 + h_2 - i \cos \beta A \right),
\]

where the relative phase of the vacuum expectation value of the two Higgs is denoted by \( \theta_{CP} \). In this case, the CP violation occurs in the charged Higgs coupling with \( \tau \) lepton and neutrino. One can parametrize the coupling [4],

\[
\mathcal{L} = H^+ \overline{\nu}_{Lj} R_j \left( \frac{Y_2^* e^{i\kappa_{CP}}}{\cos \beta} - \delta_{ij} \frac{g \tan \beta m_j}{\sqrt{2}M_W} \right)
\]

where \( Y_2 = V_R y_{2j} V_L^\dagger \) and \( V_R \) and \( V_L \) are unitary matrices which diagonalize the charged lepton mass matrix. Then one can show that the charged Higgs coupling to \( \tau \) lepton and neutrino can be complex and CP violating as shown in Fig. 7. We have introduced the following parametrization,

\[
r \exp(i\gamma) = 1 - \frac{\sqrt{2}M_W}{m_\tau} Y_{2\tau \nu} e^{i\kappa_{CP}} \frac{g \tan \beta m_\nu}{m_\tau}.
\]

In the minimal two Higgs doublet model with the natural flavor conservation, \( r = 1 \) and \( \gamma = 0 \). Note that CP phase may come from either CP violating phase \( \theta_{CP} \) of Higgs vacuum expectation value or the new Yukawa couplings \( y_2 \). The charged Higgs contribution can be easily incorporated by replacing the scalar form factor \( F_S \) as,

\[
F_S \to F_S \left( 1 - \frac{Q^2}{M_H^2} \tan^2 \beta r \exp(\pm i\gamma) \right).
\]

for \( \tau^\pm \) decay. We define the CP violation of the forward and the backward asymmetry,

\[
A_{FB}(s) - \tilde{A}_{FB}(s).
\]

In Fig. 8, we have plotted the forward and backward asymmetries and their CP violation in Eq. (32). We have shown the dependence of the CP violation of the forward and the backward asymmetry on the CP violating phase \( \gamma \) as \( \frac{\pi}{2} \), \( \pi \), and \( \pi \) in Fig. 9. We also have changed \( \kappa \) mass and studied how the CP violation depends on the resonance parameter in Fig. 10.

### 7. Summary

- We have presented a realistic calculation of the form factors of the \( \tau \) decays. We study the vector and scalar form factors including \( \kappa(800) \) and
Figure 8: The forward and backward asymmetries for $\tau^-$ decay (green) and $\tau^+$ decay (blue). CP violation of the forward and backward asymmetries defined in Eq.(32) is shown in the red solid line. We choose, $M_H = 200$ GeV, $\tan \beta = 50$, $r = 2$, $\gamma = \frac{\pi}{2}$, $M_\kappa = 800$ MeV.

Figure 9: CP phase dependence ($\gamma = \frac{\pi}{2}, \frac{\pi}{6}, \frac{\pi}{18}$) of the CP violation of the forward and the backward asymmetry.

$K^*$. The hadron invariant mass spectrum is obtained.

- We study the forward and backward asymmetry and find the large asymmetry (not CP) $\sim 60\%$ near the threshold region within the standard model using the form factors which we obtain.

Figure 10: The dependence on the mass of $\kappa$ ($M_\kappa = 700, 800, 850$ MeV) of the CP violation of the forward and backward asymmetry.

- Including the new physics contribution from charged Higgs exchange, we have seen that CP violation of the forward and backward asymmetry can be as large as 10\%. ($M_H = 200, r = 2, \tan \beta = 50, \gamma = \frac{\pi}{2}$)

- The CP violating source of the charged Higgs coupling is identified as the non-minimal ($r \neq 1, \gamma \neq 0$) two Higgs doublet model structure. We have shown the CP violation may originate from the phase of the vacuum expectation value of Higgs field.

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