Radiative natural supersymmetry
with mixed axion/higgsino cold dark matter

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Models of natural supersymmetry seek to solve the little hierarchy problem by positing a spectrum of light higgsinos $\sim 200$ GeV and light top squarks $\sim 500$ GeV along with very heavy squarks and TeV-scale gluinos. Such models have low electroweak finetuning and are safe from LHC searches. However, in the context of the MSSM, they predict too low a value of $m_h$ and the relic density of thermally produced higgsino-like WIMPs falls well below dark matter (DM) measurements. Allowing for high scale soft SUSY breaking Higgs mass $m_{H_u} > m_0$ leads to natural cancellations during RG running, and to radiatively induced low finetuning at the electroweak scale. This model of radiative natural SUSY (RNS), with large mixing in the top squark sector, allows for finetuning at the 5-10% level with TeV-scale top squarks and a 125 GeV light Higgs scalar $h$. If the strong CP problem is solved via the PQ mechanism, then we expect an axion-higgsino admixture of dark matter, where either or both the DM particles might be directly detected.

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I. INTRODUCTION

The recent fabulous discovery by Atlas and CMS of a Higgs-like resonance at 125 GeV$^{[1,2]}$ adds credence to supersymmetric models (SUSY) of particle physics in that the mass value falls squarely within the narrow predicted MSSM window: $m_h \sim 115 - 135$ GeV$^3$. At the same time, a lack of a SUSY signal at LHC7 and LHC8 implies squarks and gluinos beyond the 1 TeV range$^4,5$, exacerbating the little hierarchy problem (LHP):

- how do multi-TeV values of SUSY model parameters conspire to yield a $Z$-boson mass of just 91.2 GeV?

Models of natural supersymmetry$^6$ address the LHP by positing a spectrum of light higgsinos $\sim 200$ GeV and light top squarks $\sim 500$ GeV along with very heavy first/second generation squarks and TeV-scale gluinos$^7-9$. Such a spectrum allows for low electroweak finetuning while at the same time keeping sparticles safely beyond LHC search limits. In these models, the radiative corrections to $m_h$, which increase with $m_t^2$, are somewhat suppressed and have great difficulty in generating a light SUSY Higgs scalar with mass $m_h \sim 125$ GeV$^{10}$. Thus, we are faced with a new conundrum: how do we reconcile low electroweak finetuning with such a large value of $m_h$? In addition, the light higgsino-like WIMP particles predicted by models of natural SUSY lead to a thermally-generated relic density which is typically a factor 10-15 below$^9,12$ the WMAP measured value of $\Omega_{CDECM}\hbar^2 \approx 0.11$.

One solution to the finetuning/Higgs problem is to add extra matter to the theory, thus moving beyond the MSSM$^{11}$. For example, adding an extra singlet as in the NMSSM adds further quartic terms to the higgs potential thus allowing for increased values of $m_h$.$^{13}$ One may also add extra vector-like matter to increase $m_h$ while maintaining light top squarks$^{14}$. In the former case of the NMSSM, adding extra gauge singlets may lead to re-introduction of destabilizing divergences into the theory.$^{13}$ In the latter case, one might wonder about the ad-hoc introduction of extra weak scale matter multiplets and how they might have avoided detection A third possibility, which is presented below, is to re-examine EWFT and to ascertain if there do exist sparticle spectra within the MSSM that lead to $m_h \sim 125$ GeV while maintaining modest levels of electroweak finetuning.

II. ELECTROWEAK FINETUNING

One way to evaluate EWFT in SUSY models is to examine the minimization condition on the Higgs sector scalar potential which determines the $Z$ boson mass. (Equivalently, one may examine the mass formula for $m_h$ and draw similar conclusions.) One obtains the well-known tree-level expression

$$m_Z^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta - \mu^2}{\tan^2 \beta - 1}. \quad (1)$$

To obtain a natural value of $M_Z$ on the left-hand-side, one would like each term $C_i$ (with $i = H_d, H_u$ and $\mu$) on the right-hand-side to be of order $m_Z^2/2$. This leads to a finetuning parameter definition

$$\Delta \equiv \text{max}_{i} (C_i) / (m_Z^2/2) \quad (2)$$

where $C_{H_u} = | - m_{H_u}^2 \tan^2 \beta / (\tan^2 \beta - 1)|$, $C_{H_d} = | m_{H_d}^2 / (\tan^2 \beta - 1)|$ and $C_{\mu} = | - \mu^2 |$. Since $C_{H_d}$ is suppressed by $\tan^2 \beta - 1$, for even moderate $\tan \beta$ values this expression reduces approximately to

$$m_Z^2 \simeq - m_{H_u}^2 - \mu^2. \quad (3)$$
The question then arises: what is the model, what are the input parameters, and how do we interpret Eq’s [1] and [3]

Suppose we have a model with input parameters defined at some high scale \( \Lambda \gg m_{\text{SUSY}} \), where \( m_{\text{SUSY}} \) is the SUSY breaking scale \( \sim 1 \) TeV. Then

\[
m_{H_u}^2 (m_{\text{SUSY}}) = m_{H_u}^2 (\Lambda) + \delta m_{H_u}^2
\]

where

\[
\delta m_{H_u}^2 \simeq - \frac{3 f^2}{8 \pi^2} (m_{Q_3}^2 + m_{U_3}^2 + A_1^2) \ln \left( \frac{\Lambda}{m_{\text{SUSY}}} \right).
\]

The usual lore is that in a model defined at energy scale \( \Lambda \), then both \( m_{H_u}^2 (\Lambda) \) and \( \delta m_{H_u}^2 \) must be of order \( m_Z^2 / 2 \) in order to avoid finetuning. In fact, requiring \( \delta m_{H_u}^2 \lesssim m_Z^2 / 2 \) has been used to argue for a sparticle mass spectra of natural SUSY. Taking \( \Delta = 10 \) corresponds to

- \( |\mu| \lesssim 200 \text{ GeV} \),
- \( m_{l_i}, m_{b_i} \lesssim 500 \text{ GeV} \),
- \( m_{\tilde{g}} \lesssim 1.5 \text{ TeV} \).

Since first/second generation squarks and sleptons hardly enter into Eq. [1] these can be much heavier: beyond LHC reach and also possibly providing a (partial) decoupling solution to the SUSY flavor and CP problems:

- \( m_{\tilde{q}, \tilde{g}} \sim 10 - 50 \text{ TeV} \).

The natural SUSY solution reconciles lack of a SUSY signal at LHC with allowing for electroweak naturalness. It also predicts that the \( \tilde{t}_{1,2} \) and \( \tilde{b}_1 \) may soon be accessible to LHC searches. New limits from direct top and bottom squark pair production searches, interpreted within the context of simplified models, are biting into the NS parameter space [16]. Of course, if \( m_{\tilde{t}_{1,2}}, m_{\tilde{b}_1} \simeq m_Z \), then the visible decay products from stop and sbottom production will be soft and difficult to see at LHC. A more worrisome problem is that, with such light top squarks, the radiative corrections to \( m_t \) are not large enough to yield \( m_t \simeq 125 \text{ GeV} \). This problem has been used to argue that additional multiplets beyond those of the MSSM must be present in order to raise up \( m_t \) while maintaining very light third generation squarks [11]. A third issue is that the relic abundance of higgsino-like WIMPs, calculated in the standard MSSM-only cosmology, is typically a factor 10-15 below measured values. These issues have led some people to grow increasingly skeptical of weak scale SUSY, even as occurs in the natural SUSY incarnation.

One resolution to the above finetuning problem is to merely invoke a SUSY particle spectrum at the weak scale, as in the pMSSM model. Here, \( \Lambda \sim m_{\text{SUSY}} \) so \( \delta m_{H_u}^2 \sim 0 \) and we may select parameters \( m_{H_u}^2 \sim \mu^2 \sim m_Z^2 \). While a logical possibility, this solution avoids the many attractive features of a model which is valid up to a high scale such as \( \Lambda \sim m_{\text{GUT}} \), with gauge coupling unification and radiative electroweak symmetry breaking driven by a large top quark mass.

Another resolution is to impose Eq. [1] as a condition on high scale models, but using only weak scale parameters. In this case, we will differentiate the finetuning measure as \( \Delta_{EW} \), while the finetuning measure calculated using high scale input parameters we will refer to as \( \Delta_{HS} \).

The weaker condition of allowing only for \( \Delta_{EW} \lesssim 20 \) allows for possible cancellations in \( m_{H_u}^2 (m_{\text{SUSY}}) \). This is precisely what happens in what is known as the hyperbolic branch or focus point region of mSUGRA [17]:

\[
m_{H_u}^2 (\Lambda) \sim - \delta m_{H_u}^2 \sim m_Z^2
\]

and consequently a value of \( \mu^2 \sim m_Z^2 \) is chosen to enforce the measured value of \( m_Z \) from Eq. [1]. The HB/FP region of mSUGRA occurs for small values of trilinear soft parameter \( A_0 \). Small \( A_0 \) leads to small \( \delta m_{H_u}^2 \) at the weak scale, which leads to \( m_h \sim 115 - 120 \text{ GeV} \), well below the Atlas/CMS measured value of \( m_h \sim 125 \text{ GeV} \). Scans over parameter space show the HB/FP region is nearly excluded if one requires both low \( |\mu| \sim m_Z \) and \( m_h \sim 123 - 127 \text{ GeV} \) [19, 20].

The cancellation mechanism can also be seen from an approximate analytic solution of the EWSB minimation by Kane et al. [21]:

\[
m_Z^2 \simeq -1.8 \mu^2 + 5.9 M_3^2 - 0.4 M_2^2 - 1.2 m_{H_u}^2 + 0.9 m_{Q_3}^2 + 0.7 m_{U_3}^2 - 0.6 A_t M_3 + 0.4 M_2 M_3 + \cdots
\]

(which adopts \( \tan \beta = 5 \) although similar expressions may be gained for other \( \tan \beta \) values). All parameters on the RHS are GUT scale parameters. We see one solution for obtaining \( m_Z \) on the left-hand side is to have all GUT scale parameters of order \( m_Z \) (this is now excluded by recent LHC limits). The other possibility— if some terms are large (like \( M_3 \simeq 0.4 \text{ TeV} \) in accord with recent LHC limits)— is to have large cancellations. The simplest possibility— using \( M_3 \simeq 0.4 \text{ TeV} \)— is to raise up \( m_{H_u}^2 \), beyond \( m_Q \) such that there is a large cancellation. This possibility is allowed in the non-universal Higgs models [22, 23].

In mSUGRA, the condition that \( m_{H_u}^2 = m_{H_d}^2 = m_{H}^2 = m_{\tilde{q}}^2 = m_{\tilde{g}}^2 = m_Z^2 = m_h^2 \) at the high scale is anyways hard to accept. One might expect that all matter scalars in each generation are nearly degenerate since the known matter in each generation fills out complete 16-dimensional representations of \( SO(10) \). However, the distinguishing feature of the Higgs multiplets is that they would live in 16-dimensional representations, and we then would not expect \( m_{10} = m_{16} \) at \( m_{\text{GUT}} \). A more likely choice would be to move to the non-universal Higgs model, which comes in several varieties. It was shown in Ref. [23] that by

\[1\] This may also occur in other varied models such as mixed moduli-AMSB [18].
adopting a simple one-parameter extension of mSUGRA—known as the one extra parameter non-universal Higgs model (NUHM1), with parameter space

$$m^2_{H_u} = m^2_{H_d} \equiv m^2_\phi, \ m_0, \ m_{1/2}, \ A_0, \ \tan \beta, \ \text{sign}(\mu) \quad (7)$$

– then for any spectra one may raise $m_\phi$ up beyond $m_0$ until at some point $m^2_{H_u}(m_{SUSY})$ drops in magnitude to $\sim m^2_Z$. The EWSB minimization condition then also forces $|\mu| \sim m_Z$. The worst of the EWFT is eliminated due to a large cancellation between $m^2_{H_u}(\Lambda = m_{GUT})$ and $\delta_{H_u}^2$ leading to low $\Delta_{EW}$ and a model which enjoys electroweak naturalness.

The cancellation obviously can also be implemented in the 2-extra-parameter model NUHM2 where both $m^2_{H_u}(m_{\text{GUT}})$ and $m^2_{H_d}(m_{\text{GUT}})$ may be taken as free parameters (as in an $SU(5)$ SUSY GUT) or using the EWSB minimization conditions—these may be traded for weak scale values of $\mu$ and $m_A$ as alternative inputs. A third possibility that will allow for an improved decoupling solution to the SUSY flavor and CP problems would occur if we allow for split generations $m_0(3)$ and $m_0(1) \simeq m_0(2) \equiv m_0(1, 2)$ (SGNUHM). The latter condition need not require exact degeneracy, since with $m_0(1) \sim m_0(2) \sim 10 \sim 20 \text{ TeV}$ we obtain only a partial decoupling solution to the flavor problem. Taking $m_0(1, 2) \sim 10 \sim 20 \text{ TeV}$ solves the SUSY CP problem[24].

III. RADIATIVE NATURAL SUSY

Motivated by the possibility of cancellations occurring in $m^2_{H_u}(m_{SUSY})$, we go back to the EWSB minimization condition and augment it with radiative corrections $\Sigma^u_d$ to $\Sigma^u_d$ such that since if $m^2_{H_u}$ and $\mu^2$ are suppressed, then these may dominate:

$$m^2_Z = \frac{(m^2_{H_u} + \Sigma^u_d) - (m^2_{H_u} + \Sigma^u_d)}{2} \tan^2 \beta - \mu^2. \quad (8)$$

Here the $\Sigma^u_d$ terms arise from derivatives of the radiatively corrected Higgs potential evaluated at the potential minimum. At the one-loop level, $\Sigma^u_d$ contains the contributions $\Sigma^u_d(\tilde{f}_1, \tilde{f}_2, \tilde{b}_1, \tilde{b}_2, \tilde{t}_1, \tilde{t}_2, \tilde{W}_1, \tilde{Z}_1, \tilde{b}, \tilde{H}, \tilde{Z}_2, \tilde{H}^\pm, \tilde{W}^\pm, \tilde{Z})$, $\Sigma^u_d(H)$, $\Sigma^u_d(t)$, $\Sigma^u_d(t)$. $\Sigma^u_d(\tilde{f}_1, \tilde{f}_2)$ contains similar terms along with $\Sigma^u_d(\tilde{b}, \tilde{f}_1, \tilde{f}_2)$ and $\Sigma^u_d(\tilde{t}, \tilde{t})$ and $\Sigma^u_d(\tilde{t}) = 0$. There are also contributions from D-term contributions to first/second generation squarks and sleptons which nearly cancel amongst themselves (due to sum of weak isospins/hypercharges equaling zero). Once we are in parameter space where $m^2_{H_u}(m_{SUSY}) \sim \mu^2 \sim m^2_Z$, then the radiative corrections $\Sigma^u_d$ may give the largest contribution to $\Delta_{EW}$.

The largest of the $\Sigma^u_d$ almost always come from top squarks, where we find

$$\Sigma^u_d(\tilde{f}_1, \tilde{f}_2) = \frac{3}{16\pi} F(m^2_{\tilde{f}_1, \tilde{f}_2}) \times \left[ f^2 - g^2_Z + \frac{f^2_A t^2 - 8 g^2_Z (\frac{1}{4} - \frac{3}{2} x_W) \Delta t}{m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1}} \right] \quad (9)$$

where $\Delta t = (m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2})/2 + m^2_Z \cos 2\beta (\frac{1}{4} - \frac{3}{2} x_W), \ g^2_Z = (g^2 + g'^2)/8, \ x_W \equiv \sin^2 \theta_W \quad (\text{and} \quad F(m^2) = m^2 / (\log(m^2/Q^2) - 1)).$ In Ref. 27, it is shown that for the case of the $\tilde{t}_1$ contribution, as $|A_t|$ gets large there is a suppression of $\Sigma^u_d(\tilde{f}_1)$ due to a cancellation between terms in the square brackets of Eq. 9. For the $\tilde{t}_2$ contribution, a large splitting between $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ yields a large cancellation within $F(m^2_{\tilde{t}_2})$ 

$$\left( \log(m^2_{\tilde{t}_2}/Q^2) \log(m_{\tilde{t}_2}/m_{\tilde{t}_1}) \rightarrow 1 \right) \text{ for } Q^2 = m_{\tilde{t}_1} m_{\tilde{t}_2},$$

leading also to suppression. So while large $|A_t|$ values suppress both top squark contributions to $\Sigma^u_d$, at the same time they also lift up the value of $m_h$, which is near maximal for large, negative $A_t$. Combining all effects, one sees that the same mechanism responsible for boosting the value of $m_h$ into accord with LHC measurements also suppress the $\Sigma^u_d$ contributions to EWFT, leading to a model with electroweak naturalness.

To illustrate these ideas, we adopt a simple benchmark point from the 2-parameter non-universal Higgs mass SUSY model NUHM2[22], but with split generations, where $m_0(3) < m_0(1, 2)$. In Fig. 1, we take $m_0(3) = 5 \text{ TeV}, m_0(1, 2) = 10 \text{ TeV}, m_{1/2} = 700 \text{ GeV}, \ tan \beta = 10$ with $\mu = 150 \text{ GeV}, m_A = 1000 \text{ GeV}$ and $m_t = 173.2 \text{ GeV}$. We allow the GUT scale parameter $A_0$ to vary, and calculate the particle mass spectrum using Isajet 7.83[28], which includes the new EWFT measure. In frame a), we plot the value of $m_h$ versus $A_0$. While for $A_0 \sim 0$ the value of $m_h \sim 120 \text{ GeV}$, as $A_0$ moves towards $-2m_0(3)$, the top squark radiative contributions to $m_h$ increase, pushing its value up to $125 \text{ GeV}$. (There is an expected theory error of $\pm 2 \text{ GeV}$ in our RGE-improved effective potential calculation of $m_h$, which includes leading 2-loop effects[29].) At the same time, in frame b), we see the values of $m_{\tilde{t}_1, \tilde{t}_2}$ versus $A_0$. In this case, large values of $A_0$ suppress the soft terms $m^2_{\tilde{t}_1, \tilde{t}_2}$ and $m^2_{\tilde{t}_1, \tilde{t}_2}$ via RGE running. But also large weak scale values of $A_t$ provide large mixing in the top squark mass matrix which suppresses $m_{\tilde{t}_1, \tilde{t}_2}$ and leads to an increased splitting between the two mass eigenstates which suppresses the top squark radiative corrections $\Sigma^u_d(\tilde{t}_2)$. The EWFT measure $\Delta = \Delta_{EW}$ is shown in frame c), where we see that while $\Delta \sim 50$ for $A_0 = 0$, when $A_0$ becomes large, then $\Delta$ drops to 10, or $\Delta^{-1} = 10\% \text{ EWFT}$. In frame d), we show the weak scale value of $A_t$ versus $A_0$ variation. While the EWFT is quite low— in the range expected for natural SUSY models we note that the top squark masses remain above the TeV level, and in particular $m_{\tilde{t}_1, \tilde{t}_2} \sim 3.5 \text{ TeV}$, in contrast to previous natural SUSY expectations.
m_{(3)=5 TeV, m_{(1,2)=10 TeV, m_{2}=0.7 TeV, tan\beta=10, \mu=150 GeV, m_A=1 TeV}

FIG. 1: Plot of a) \( m_t \), b) \( m_{\tilde{t}_1, \tilde{t}_2} \), c) \( \Delta \) and d) \( A_t \) versus variation in \( A_0 \) for a model with \( m_0(1,2) = 10 \) TeV, \( m_0(3) = 5 \) TeV, \( m_{1/2} = 700 \) GeV, \( \tan \beta = 10 \) and \( \mu = 150 \) GeV and \( m_A = 1 \) TeV.

IV. SPARTICLE SPECTRUM

The sparticle mass spectrum for this radiative NS benchmark point (RNS1) is shown in Table I for \( A_0 = -7300 \) GeV. The heavier spectrum of top and bottom squarks seem likely outside of any near-term LHC reach, although in this case gluino and possibly heavy electroweak-ino pair production may be accessible to LHC14. The \( A_0 \) parameter up to \( -8 \) TeV allows for \( m_h = 125.2 \) GeV but increases EWFT to \( \Delta = 29.5 \), or 3.4\% fine-tuning. Alternatively, pushing \( m_t \) up to 174.4 GeV increases \( m_h \) to 124.5 GeV with 6.2\% fine-tuning; increasing \( \tan \beta \) to 20 increases \( m_h \) to 124.6 GeV with 3.3\% fine-tuning. We show a second point RNS2 with \( m_0(1,2) = m_0(3) = 7.0 \) TeV and \( \Delta = 11.5 \) with \( m_t = 125 \) GeV; note the common fermion mass parameter at the high scale. For comparison, we also show in Table I the NS2 benchmark from Ref. [9]; in this case, a more conventional light spectra of top squarks is generated leading to \( m_h = 121.1 \) GeV, but the model– with \( \Delta = 23.7 \)– has higher EWFT than RNS1 or RNS2.

The RNS model shares some features of generic NS models, but also includes important differences. The several benchmark points shown in Table I imply that RNS is characterized by:

- a higgsino mass \( \mu \approx M_Z \approx 100 - 300 \) GeV,
- a light top squark \( m_{\tilde{t}} \approx 1 - 2 \) TeV,
- a heavier top-squark \( m_{\tilde{t}_2} \approx em_{\tilde{t}_1} \approx 2 - 4 \) TeV (here, \( e \approx 2.718 \ldots \)),
- \( m_{\tilde{\gamma}} \approx 1 - 5 \) TeV,
- first/second generation sfermions \( \approx 5 - 20 \) TeV.

While \( \mu \approx m_Z \) as in usual NS models, the heavier top squarks and gluinos implied by RNS allow for \( m_h \approx 125 \) GeV but may make this model more difficult to detect at LHC than usual NS.

To illustrate how low EWFT comes about even with rather heavy top squarks, we show in Fig. 2 the various third generation contributions to \( \Sigma_u \) for a model with \( m_0 \approx 100 \) GeV and \( m_t = 123.2 \) GeV.

![Diagram showing contributions to \( \Sigma_u \)]

TABLE I: Input parameters and masses in GeV units for two radiative natural SUSY benchmark points and one NS point with \( \mu = 150 \) GeV and \( m_t = 173.2 \) GeV.
VI. MIXED AXION-HIGGSINO COLD DARK MATTER:

In R-parity conserving SUSY models with higgsino-like WIMPs\cite{12}, the relic density is usually a factor $10 - 15$ below the WMAP measured value of $\Omega_{CDM} h^2 \simeq 0.11$. This is due to a high rate of higgsino annihilation into WW and ZZ final states in the early universe. Thus, the usual picture of thermally produced WIMP-only dark matter is inadequate for the case of models with higgsino-like WIMPs.

A variety of non-standard cosmological models have been proposed which can ameliorate this situation. For instance, at least one relatively light modulus field is expected from string theory\cite{33}, and if the scalar field decays after neutralino freeze-out with a substantial branching fraction into SUSY particles then it will typically augment the neutralino abundance\cite{36}.

Alternatively, or in addition, if the strong $CP$ problem is solved by the Peccei-Quinn mechanism in a SUSY context, then we expect the presence of axions in addition to $R$-parity odd spin $\frac{1}{2}$ axinos $\tilde{a}$ and $R$-parity even spin-$0$ saxions $s$. In this case, dark matter could consist of two particles: an axion-higgsino admixture\cite{37–39}. The neutralinos are produced thermally as usual, but are also produced via thermal production followed by cascade decays of axinos at high $T_R$. The late decay of axinos into higgsinos can cause a re-annihilation of neutralinos at temperatures below freeze-out, substantially augmenting the relic abundance. In addition, saxions can be produced both thermally at lower range of PQ breaking parameter $f_a$, and via coherent oscillations at high $f_a$, and in fact may temporarily dominate the energy density of the universe. Their decay $s \to aa$ is expected to dominate and would add to the measured $N_{eff}(\nu)$. Their decay $s \to gg$ or $aa$ would augment the neutralino abundance, while late decays into SM particles such as $gg$ would dilute all relics present at the time of decay. Exact dark matter abundances depend on the specific SUSY axion model and choices of PQMSSM parameters. It is possible one could have either axion or higgsino dominance of the relic abundance, or even a comparable mixture. In the latter case, it may be possible to directly detect both an axion and a higgsino-like WIMP.

VII. CONCLUSIONS:

Models of Natural SUSY are attractive in that they enjoy low levels of EWFT, which arise from a low value of $\mu$ and possibly a sub-TeV spectrum of top squarks and $b_1$. In the context of the MSSM, such light top squarks are difficult to reconcile with the LHC Higgs boson discovery which requires $m_{h} \sim 125$ GeV. By imposing naturalness using $\Delta_{EW}$ with weak scale parameter inputs, we allow for large cancellations in $m_H^2$ as it is driven to the weak scale. In this case, for some range of $m_H^2 (m_{GUT}) > m_0$ (as in NUHM models), the weak scale value of $m_H^2$ will

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**Figure 2:** Plot of third generation contributions to $\Sigma_n$ versus $A_{\nu}$ for benchmark point RNS1 where solid curves come form the lighter mass eigenstate and dashed curves from the heavier. The black solid curve is $\Sigma_n$ which has summed over all contributions.

V. RADIATIVE NATURAL SUSY AT COLLIDERS:

What chance does LHC have of detecting RNS? Unlike previous NS models, RNS has top and bottom squarks more typically in the $m_{\tilde{t}_1} \sim 1 - 2$ TeV and $m_{\tilde{t}_2} \sim 2 - 4$ TeV range, likely beyond LHC reach. It also has light higgsinos $\tilde{W}_1^{\pm}$, $\tilde{Z}_{1,2}$. While these latter particles can have substantial production cross sections at LHC, the mass gaps $m_{\tilde{W}_1} - m_{\tilde{Z}_1}$ and $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$ are typically in the $10 - 50$ GeV range. Thus, the visible decay products from $\tilde{W}_1^{\pm} \tilde{W}_1^{-}$ and $\tilde{Z}_1^{\pm} \tilde{Z}_2$ production tend to be at rather low energies, making observability difficult. The best bet may be if gluinos lie in the lower half of their expected range $m_{\tilde{g}} \sim 1 - 5$ TeV. In this case, $\tilde{g} \to tb\tilde{W}_1$ or $tt\tilde{Z}_i$ decays occur, and one might expect an observable rate for $gg \to 4t + E_T^{miss}$ signals. A portion of these events will contain cascade decays to $\tilde{Z}_2 \to \tilde{Z}_1 \ell^+ \ell^-$ and if the OS/SF dilepton pair can be reconstructed, then its distinctive invariant mass distribution bounded by $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$ may point to the presence of light higgsinos.

The hallmark of RNS and other NS models is the presence of light higgsinos with mass $m_{\tilde{W}_1}, m_{\tilde{Z}_{1,2}} \lesssim 200 - 300$ GeV. In this case, a linear $e^+ e^- \to \tilde{Z}_1 \tilde{Z}_2$ collider operating with $\sqrt{s} \gtrsim 2m_{\tilde{W}_1}$ would be a higgsino factory in addition to a Higgs factory\cite{12,34}! The soft decay products from $W_1 \to Z_1 f f^\prime$ decay are not problematic for detection at an ILC, and will even be boosted as $\sqrt{s}$ increases well beyond threshold energy for creating charginos pairs. The reaction $e^+ e^- \to \tilde{Z}_1 \tilde{Z}_2$ will also be distinctive.
be $\sim m_Z^2$ thus generating the natural SUSY model radiatively. Models with a large negative trilinear soft-breaking parameter $A_t$ can maximize the value of $m_h$ into the 125 GeV range without recourse to adding exotic matter into the theory. The large value of $A_t$ also suppresses 1-loop top squark contributions to the scalar potential minimization condition leading to models with low EWFT and a light Higgs scalar consistent with LHC measurements. (More details on the allowable parameter space of RNS will be presented in Ref. [20]). The large negative $A_t$ parameter can arise from large negative $A_0$ at the GUT scale.

While RNS may be difficult to detect at LHC unless gluinos, third generation squarks or the heavier electroweak-inos are fortuitously light, a linear collider with $\sqrt{s} \gtrsim 2|\mu|$ would have enough energy to produce the hallmark light higgsinos which are expected in this class of models. Since the model predicts a lower abundance of higgsino-like WIMP dark matter in the standard cosmology, there is room for mixed axion-higgsino cold dark matter. The axions are necessary anyway if one solves the strong CP problem via the PQ mechanism.

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