Proton Decay and Realistic Models of Quark and Lepton Masses

K.S. Babu and S.M. Barr

Bartol Research Institute
University of Delaware
Newark, Delaware 19716

Abstract

It is shown that in realistic SUSY GUT models of quark and lepton masses both the proton decay rate and branching ratios differ in general from those predicted in the minimal $SU(5)$ supersymmetric model. The observation of proton decay, and in particular the branching ratio $B[p \to \pi^+\nu]/(p \to K^+\nu)$, would thus allow decisive tests of these fermion mass schemes. It is shown that the charged lepton decay modes $p \to K^0\mu^+, p \to K^0e^+$ arising through gluino dressing diagrams are significant and comparable to the neutrino modes in large tan$\beta$ models. Moreover, it is found that in certain classes of models the Higgsino-mediated proton decay amplitudes are proportional to a model-dependent group-theoretical factor which in some cases can be quite small. There is thus a natural suppression mechanism which can explain without adjustment of parameters why in the context of SUSY GUTs proton decay has not yet been seen. The most interesting such class consists of $SO(10)$ models in which the dominant flavor-symmetric contribution to the up-quark mass matrix comes from an effective operator of the form $16_1^{16}16^{10}H45_H$, where $\langle 45_H \rangle$ points approximately in the $I_{3R}$ direction. This class includes a recent model of quark and lepton masses proposed by the authors.

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2 Address starting September 1995: School of Natural Sciences, Institute for Advanced Study, Olden Lane, Princeton, NJ 08540
1 Introduction

It is well-known that in supersymmetric grand unified theories (SUSY GUTs) the dominant contribution to proton decay is through dimension-five operators which arise from the exchange of superheavy, colored Higgsinos.\textsuperscript{1,2,3} In this paper we shall take a closer look at these decays. It will be shown that two simplifications that are usually introduced in the analysis of this effect are in certain interesting cases not justified, and that in going beyond these simplifications some important features emerge.

The first simplification usually made is to analyze Higgsino-mediated proton decay in the context of the minimal SUSY $SU(5)$ model. In that model there is a very simple relation between the proton-decay amplitudes and the mass matrices of the quarks and leptons that allows one, in fact, to write these amplitudes in terms of the quark masses and the Kobayashi-Maskawa matrix elements. This relation comes from the fact that the Yukawa couplings of the colored Higgsinos which mediate proton decay are equal, because of $SU(5)$ invariance, to the Yukawa couplings of the ordinary light Higgs doublets, $H$ and $H'$, that give rise to light fermion masses. But this cannot be taken seriously precisely because the minimal SUSY $SU(5)$ model gives a wrong account of the light fermion masses. In particular, minimal $SU(5)$ predicts that $m_e^0 = m_d^0$ and $m_\mu^0 = m_s^0$ (superscript zeros refer to quantities evaluated at the GUT scale), and thus $m_e/m_\mu = m_d/m_s$, which is off by an order of magnitude.

The second simplification is to neglect gluino loops in dressing the effective, dimension-five, $\Delta B \neq 0$ operators to make four-fermion operators. This implies that only W-ino loops need be considered, which in turn leads to the conclusion that the only significant modes are those that involve neutral leptons ($p \rightarrow K^+\nu$ and $p \rightarrow \pi^+\nu$). It will be explained why for models with large $\tan\beta$ the gluino loop diagrams cannot be neglected and decay modes with charged leptons ($p \rightarrow K^0\mu^+$, $p \rightarrow K^0e^+$) can become comparable to the neutrino modes.\textsuperscript{4} (Since the Higgsino-mediated proton-decay rate goes
as $\tan^2 \beta$ it might be thought that the case of large $\tan \beta$ is in conflict with present limits. While this may be true in minimal SUSY $SU(5)$ model, there are several likely suppression mechanisms which have been proposed in the literature\textsuperscript{5} in going beyond minimal $SU(5)$ and one such group theoretical suppression mechanism is found in section 4 of this paper. Large $\tan \beta$ is actually a natural feature of many grand unified models, especially those based on $SO(10)$ or larger groups where it often is predicted to be $m_t/m_b$.

Realistic models of quark and lepton masses in the context of SUSY GUTs require that the “bad” $SU(5)$ or $SO(10)$ mass relations be broken. This means that the quark and lepton masses receive contributions from additional operators not present in the minimal $SU(5)$ model, operators which either involve Higgs fields in representations larger than the fundamental or are of higher dimension. What this implies is that the proton-decay amplitudes are related in a less direct, model-dependent way to the quark and lepton mass matrices. This allows one in principle to distinguish different theoretical models of quark and lepton masses by their proton-decay branching ratios, as will be seen. It will also be seen that an interesting group-theoretical mechanism exists by which the higgsino-mediated proton decay rate may be suppressed to levels near but consistent with present limits without any special adjustment of parameters. Interestingly, the group structure required for this suppression is precisely that suggested in a recent model of quark and lepton masses.\textsuperscript{6}

2 Review of p Decay in Minimal SU(5)

(a) W-ino loops:

In order to set the stage for the later analysis it is convenient to review briefly the standard analysis of Higgsino-mediated proton decay in the minimal SUSY $SU(5)$ model. In that model the Yukawa couplings of the quarks and leptons come from the following terms in the superpotential.
\[ W_{\text{Yukawa}} = \frac{1}{2} \sum_{i,j=1}^{3} U_{ij} \begin{bmatrix} 10, & 10 \end{bmatrix} \begin{bmatrix} 5 \end{bmatrix}_H / v + \sum_{i,j=1}^{3} D_{ij} \begin{bmatrix} 3, & 10 \end{bmatrix} \begin{bmatrix} 5 \end{bmatrix}_H / v', \quad (1) \]

where \( U_{ij} \) and \( D_{ij} \) are the mass matrices of the charge-\( \frac{2}{3} \) and charge-\( \frac{1}{3} \) quarks. In terms of multiplets of the standard model gauge group this gives

\[ W_{\text{Yukawa}}(H, H') = \sum_{ij} U_{ij} [u_i^c Q_j] H / v + \sum_{ij} D_{ij} [d_i^c L_j^+] H' / v', \quad (2) \]

and

\[ W_{\text{Yukawa}}(H_C, H'_C) = \sum_{ij} U_{ij} \left[ \frac{1}{2} Q_i Q_j + u_i^c l_j^+ \right] H_C / v + \sum_{ij} D_{ij} \left[ L_i Q_j + d_i^c u_j^c \right] H'_C / v'. \quad (3) \]

From the diagrams of Fig. 1 there arise two types of B-violating quartic terms involving the interactions in eq. 3. These are given by

\[ W_{\Delta B \neq 0} = \frac{1}{M_{C} v v'} \sum_{ij} \sum_{kl} D_{ij} U_{kl} \left[ \frac{1}{2} (L_i Q_j)(Q_k Q_l) + (d_i^c u_j^c)(u_k^c l_l^+) \right], \quad (4) \]

where \( M_C \) is the mass of the superheavy color-triplet Higgsino. Here the contractions of \( SU(2)_L \) and \( SU(3)_c \) indices are as follows:

\[ (L_i Q_j)(Q_k Q_l) = \epsilon_{\alpha \beta \gamma} (\nu_i d_j^\alpha - \epsilon_i u_j^\alpha)(u_k^\beta d_l^\gamma - d_k^\beta u_l^\gamma) \]

\[ (d_i^c u_j^c)(u_k^c l_l^+) = \epsilon_{\alpha \beta \gamma} (d_i^c u_j^\alpha u_k^\beta l_l^\gamma). \]

In terms of component fields the operators in eq. 4 are of dimension five and contain two fermions and two scalars (squarks and/or sleptons). Since squarks and sleptons are heavier than the proton they must be converted into quarks and leptons by a gluino, W-ino or neutralino. This means that the diagram in Fig. 1 must be dressed by a vertex loop.

It is generally argued that the gluino loops can be neglected. This argument can be stated in the following way. The products of superfields \( Q_j Q_k Q_l \)
and \( d_i^c u_j^c u_k^c \) appearing in eq. 4 are totally antisymmetric in color. Since they are bosonic, they must also be totally antisymmetric in flavor. (Note: flavor, not family.) Now suppose that one neglects all flavor-dependence introduced by the gluino loop. Then, in the resulting four-fermion operators, the products of quark fields, \( \psi_{Qj} \psi_{Qk} \psi_{Qi} \) and \( \psi_{d_i^c} \psi_{u_j^c} \psi_{u_k^c} \), will have the original flavor structure of eq. 4, namely they will be totally antisymmetric in flavor. But since these are now products of fermions their antisymmetry in color and flavor implies that they must also be totally antisymmetric in spin, which is clearly impossible for three spin-\( \frac{1}{2} \) objects.

The gluino loops, therefore, are only important to the extent that they introduce some additional flavor dependence. This can happen through the nondegeneracy of the different flavors of squark. But that gives a GIM-like suppression factor which is known to be very small, especially for the squarks of the first two families, because of the limits on squark non-degeneracy from the neutral Kaon system. Mixing in the squark/slepton mass–squared matrix is another source of flavor dependence. (In addition, there is a suppression factor of \( m_u \) that comes from the fact that the four-fermion operator must involve only \( u \) rather than \( c \) or \( t \) quarks. But this is not an extra suppression, because including flavor mixing in the squark masses, this \( m_u \) factor will be replaced by \( m_c \) or \( m_t \).) What shall be seen in section 2(b), however, is that if \( \tan \beta \) is large flavor-change arises in the squark mass matrix which, though very small, is nevertheless enough to make the gluino-loop contributions to proton decay significant and comparable to the W-ino contribution.

Proceeding with the standard analysis, one concludes that the dominant contribution to proton decay comes from dressing the effective, dimension-five operators coming from eq. 4 with W-ino loops. (The photino and zino loops suffer the same problems as the gluino loops, with an additional suppression of \( \alpha/\alpha_s \). The Higgsino loops have suppressions of small Yukawa couplings.) Because the second operator in the brackets in eq. 4 contains only \( SU(2)_L \)-singlet fields it can therefore be neglected.

From the form of the first term in eq. 4 it is readily seen that the W-ino
loop leads to the following effective four-fermion operators:

\[ \mathcal{L} = \frac{1}{M_C \pi} \frac{\alpha^2}{4 \pi} D_{ij} U_{kl} \epsilon_{\alpha \beta \gamma} \]

\[ \times [(u^\alpha_k d^\beta_l)(d^\gamma_i \nu^\gamma_j)(f(d_k, u_l) + f(d_k, u_l))] \]

\[ + (d^\alpha_k u^\beta_l)(u^\gamma_i l^\gamma_j)(f(u_k, d_l) + f(u_k, d_l)) \]

\[ + (d^\alpha_k u^\beta_l)(d^\gamma_i \nu^\gamma_j)(f(u_k, d_l) + f(u_k, d_l)) \]

\[ + (u^\alpha_k d^\beta_l)(u^\gamma_i l^\gamma_j)(f(d_k, u_l) + f(d_k, u_l))]. \]

(5)

Here the color indices \((\alpha, \beta, \gamma)\) and flavor indices \((i, j, k, l)\) are understood to be summed over, and the fermion fields paired together in parentheses are spin-contracted to singlets. \(f\) is a loop integral with dimension of \(M^{-1}\) defined by \(f(a, b) \equiv \frac{m_W}{m_a - m_b} \left( \frac{m_b^2}{m_a - m_b} \ln \frac{m_a^2}{m_W^2} - [a \to b]\right)\). For \(m_a \simeq m_b \gg m_W\), \(f(a, b) \simeq \frac{m_b}{m_a}\) and for \(m_a \simeq m_b \ll m_W\), \(f(a, b) \simeq \frac{1}{m_W} \ln \frac{m_W^2}{m_a^2}\). While deriving eq. (5), it has been assumed that flavor mixing in the squark sector is negligible, which should hold to a good approximation in supergravity models. We have verified that the effect of flavor mixing that occurs in large \(\tan \beta\) models (see discussion in section 2.(b)) is negligible in so far as the W-ino dressing graphs are concerned.

If one neglects contributions proportional to \(m_u\), then it is easily seen that only the third term in the brackets in eq. 5 contributes to proton decay. One can rewrite that term in the physical basis of the charge-\(\frac{2}{3}\) quarks, where \(U_{ij}\) is diagonal, as follows

\[ \mathcal{L} \propto \sum_{j=1}^{3} \sum_{k=2}^{3} D_{jj} U_{kk} \epsilon_{\alpha \beta \gamma} (d^\alpha_k u^\beta_l)(d^\gamma_k \nu^\gamma_j), \]

(6)

where we have taken the \(f\) factors to be flavor independent (which should be a very good approximation since the squarks (and sleptons) must be nearly degenerate.) Here \(d_j^i = \sum_j (V_{KM})_{ij} d_j\). The important conclusion is immediate that only the neutrino modes of proton decay are important.
Note the (paradoxical) result that the ratio of $d$ to $s$ quarks, which determines the branching ratio of $\pi$ to $K$ modes, is controlled by the elements of the matrix $U_{ij}$. In particular, the ratio of the coefficients of the $(du)(d\nu)$ and $[(su)(d\nu)+(du)(s\nu)]$ terms is (neglecting terms proportional to $m_u$) given by

$$\frac{C_{dd}}{C_{ds}} \approx \frac{m_c \sin^2 \theta_c + A_s m_t V_{td}^2}{-m_c \sin \theta_c - A_s m_t V_{td} V_{cb}}.$$  

(7)

Or:

$$\frac{C_{dd}}{C_{ds}} \approx -\sin \theta_c \left( \frac{1 + a^2 b}{1 - ab} \right).$$  

(8)

where $a \equiv \frac{V_{td}}{V_{ts} \sin \theta_c}$, and $b \equiv \frac{V_{td}^2 m_t A_s}{m_c}$ are quantities that can be measured in low-energy experiments. (Except for the phase of $b$. $m_c/m_t$ has in general a non-trivial phase which cannot be measured in low energy experiments, but which can be predicted in specific models of quark and lepton masses.) $A_s$ is a short distance renormalization factor, proportional to the Yukawa couplings. If $\tan \beta$ is small ($\tan \beta \leq 10$), $A_s = (1 - Y_t/Y_f)^{1/12}$ where $Y_t = h_t^2$ is the top-quark Yukawa coupling–squared and $Y_f$ is its ‘true’ fixed point values, i.e., the value of $Y_t$ at the weak scale if $Y_t$ were infinite at the GUT scale. ($Y_f \simeq 1.29$ corresponding to $\alpha_s(M_Z) = 0.12$). For $\tan \beta = m_t/m_b$, $A_s = 1$. If $\tan \beta$ takes intermediate values, $A_s$ is to be evaluated numerically. The parameters $a$ and $b$ are of order unity, present experiments allow a factor of 2 uncertainty in their numerical values.

(b) Gluino loops, large $\tan \beta$ and charged lepton modes:

The coefficient of the effective dimension-five operator, $L_i Q_j Q_k Q_l$, in eq. 4 depends on the mass matrix of the charge-$\frac{2}{3}$ quarks, $U_{ij}$. Thus, a gluino loop which dressed this operator would be correspondingly enhanced by containing a virtual $t$ squark. Such a diagram is shown in Fig. 2. Since, however, the external charge-$\frac{2}{3}$ quarks must be $u$’s, for this to happen a flavor change must
occur in the gluino loop, such as is indicated by the $\tilde{t}^* \tilde{u}$ squark mass insertion shown in Fig. 2. This gives a compensating suppression; the question is by how much. If large enough flavor change can occur in the squark mass matrix, then the gluino loops can be important. As will now be explained, this is indeed the case for large $\tan \beta$.

If one neglects the Yukawa couplings of the lighter two generations then there are only two Yukawa terms left. These can be written

$$L_{\text{Yukawa}} = \frac{m_b}{v'} b^c \left( \frac{t'}{b} \right) \times \left( \frac{H^0}{H^{i'}} \right) + \frac{m_t}{v} t^c \left( \frac{t}{b'} \right) \times \left( \frac{H^+}{H^0} \right).$$

(9)

Here and throughout, the unprimed quark fields denote the mass eigenstates, while $t'$ denotes the $SU(2)_L$ partner of $b_L$, and $b'$ denotes the $SU(2)_L$ partner of $t_L$. Thus $t' = V_t^* t + V_{ts} s + V_{td} d$ and $b' = V_{tb} b + V_{ts} s + V_{td} d$.

Corresponding to eq. 9 there are soft SUSY-breaking trilinear terms

$$L_{\text{soft}} = A \frac{m_b}{v'} b^c \left( \frac{\tilde{t}'}{\tilde{b}} \right) \times \left( \frac{H^0}{H^{i'}} \right) + A \frac{m_t}{v} \tilde{t}^c \left( \frac{\tilde{t}}{\tilde{b}'*} \right) \times \left( \frac{H^+}{H^0} \right).$$

(10)

Here and throughout, the squark fields $\tilde{t}$, $\tilde{b}$, $\tilde{c}$ etc. are not the squark-mass eigenstates but the superpartners of the quark-mass eigenstates $t$, $b$, $c$ etc. Similarly, $\tilde{t}'$ and $\tilde{b}'$ are the superpartners of $t'$ and $b'$.

The interactions given in eq. 10 give rise to the diagrams shown in Fig. 3. There are also related diagrams in which the vertices are the hard, SUSY-invariant Yukawa couplings and the SUSY breaking comes from the non-equality of the squark and quark masses. Altogether, these give effective low-energy terms of the form

$$\Delta \mathcal{L} \approx m_0^2 (3 + A^2) (16\pi^2)^{-1} \ln \left( \frac{M_{\text{GUT}}}{M_{\text{SUSY}}} \right)^2 \left[ c_1 \left( \frac{m_b}{v'} \right)^2 \tilde{t}'^* \tilde{t}' + c_2 \left( \frac{m_t}{v} \right)^2 \tilde{b}'^* \tilde{b}' \right].$$

(11)
where $c_1$ and $c_2$ are numbers of order one that depend on the precise spectrum of sparticles. Eq. (11) with $c_1 = c_2 = 1$ is what one would obtain with supergravity boundary conditions at the GUT scale for the squark flavor mixing, if one ignores the running of all the relevant parameters. This is, of course, very naive, but this should still tell us the correct results to within a factor of 3 or so. $c_1$ and $c_2$ parametrizes the effect of such running.

Substituting $\tilde{t}' = V_{ib}^* \tilde{t} + V_{cb}^* \tilde{c} + V_{ub}^* \tilde{u}$, one sees that, for example, a $\tilde{u}\tilde{t}$ mixing of order $(m_b v')^2 V_{ub}$ is produced. For large $\tan \beta$, $m_b v' \sim 1$ and such flavor mixing becomes significant.

Armed with this result one can go back and evaluate the graph shown in Fig. 2. One obtains (in the minimal $SU(5)$ model)

$$\mathcal{L}_{\text{gluino}}^{\text{eff}} \sim \frac{\alpha_s}{4\pi} \left( \frac{m_b}{v'} \right)^2 \frac{1}{M_C v v'} \sum_{ij} D_{i1} U_{33} V_{ub}^* V_{ij} \left[ (l_i^- u - \nu_i d^')(d_j u) \right].$$

This is to be compared to the dominant W-ino graph contribution which gives

$$\mathcal{L}_{\text{Wino}}^{\text{eff}} \sim \frac{\alpha_2}{4\pi} \frac{1}{M_C v v'} \sum_i D_{i1} U_{22} \left[ (s' u)(s' \nu_i) \right].$$

Since $U_{33}/U_{22} = m_t^0/m_c^0$ one sees that for $m_b/v' \sim 1$ (that is, for $\tan \beta \sim m_t/m_b$) the gluino and W-ino contributions are comparable, and therefore the charged lepton and neutrino modes are comparable.

To be more precise, we shall present below the effective Lagrangian arising through gluino dressing to lowest order in the flavor mixing parameters $\Delta_{ij}^{u,d} \equiv (m_{u,d}^2)_{ij}/[(m_{u,d}^2)_{ii} - (m_{u,d}^2)_{jj}]$ for $i \neq j$ for the case of minimal SUSY $SU(5)$:

$$\mathcal{L}_{\text{eff}}^{\text{gluino}} = -\frac{4}{3} \frac{\alpha_s}{4\pi} U_i V_{ki}^* D_l \times \left[ (u_i^\alpha d_i^\beta) (u_k^e e_l) \Delta_{ij}^{u,i} (f(u_i, d_i^') - f(u_j, d_j^')) + (u_i^\alpha d_i^\beta) (u_k^e e_l) \Delta_{ij}^{d,i} (f(u_i, d_i^') - f(u_j, d_j^')) + (d_i^\alpha u_i^\beta) (u_k^e e_l) \Delta_{ij}^{u,i} (f(u_k, d_i^') - f(u_j, d_j^')) \right]$$

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In the above, we have used a basis where the up–quark mass matrix is diagonal and the down-quark mass matrix is given by $V_{KM}D$, $D$ being the diagonal down mass matrix. It should be noted that in the approximation of keeping only the third family Yukawa couplings, the mixing parameter $\Delta_{ij}^d$ in this basis is nonvanishing only for $i = j = 3$ because the only mixing is the down quark sector is of the form $\tilde{b}\bar{\nu}\tilde{b}'$. It is clear from eq. (14) that the dominant contribution for proton decay via gluino exchange arises from the first and the fifth terms, all other terms being proportional to the small up–quark mass. eq. (14) justifies the qualitative discussion preceding it.

In particular, if the SUSY spectrum is such that the gluino is lighter than the squarks, since $m_{\tilde{g}} \simeq \frac{m_{\tilde{W}}}{\alpha_s/\alpha_2}$, the gluino dressing effect has a factor of $4/3(\alpha_s/\alpha_2)^2 \sim 18$ in the amplitude relative to W-ino dressing. Using the naive estimate of $c_1 = 1$ in eq. (11) and $A = 1$, we find that the two amplitudes are about the same.

It should be noted that, just as for the W-ino loops, the second operator in eq. 4, namely the $\tilde{d}^c u^c u^c t^+$ operator, can be neglected in the gluino loops. This is because there is no flavor-changing in the right-handed squark masses coming from diagrams analogous to Fig. 3. If one interchanges $\tilde{b}$ and $\tilde{b}^c$ in Fig. 3a, for example, one merely gets the effective flavor-conserving operator $\tilde{b}\bar{\nu}\tilde{b}'$. 
3 Models With Realistic Fermion Masses

So far we have been considering the unrealistic case where the quark and lepton Yukawa couplings are those of minimal SUSY SU(5). As we have seen, what controls proton decay in the general case (for both W-ino and gluino loops) is the Yukawa coupling of the colored Higgsino, $H_C$, to the quark bilinear $Q_i Q_j$ and of the $H'_C$ to $L_i Q_j$. For minimal SUSY SU(5) the former is given by the up quark mass matrix, $U_{ij}$, as shown in eq. 3, but in general it will be some different matrix, which we will denote $U'_{ij}$:

$$W_{\text{Yukawa}}(H_C) = \sum_{ij} U'_{ij} [Q_i Q_j] H_C / v \ldots.$$ 

Similarly, the coupling of $H'_C$ is given by some matrix $D'_{ij}$ which is not in general the same as $D_{ij}$. Let us now examine the relationship between $U'_{ij}$ and $U_{ij}$ in some simple cases.

The matrix $U_{ij}$ can be written as the sum of a symmetric and an antisymmetric piece:

$$U_{ij} = f_{(ij)} + g_{[ij]}.$$ 

(15)

Let us assume for simplicity that there is only a single term that contributes to the symmetric piece. Then the matrix $U'_{ij}$ will have the form

$$U'_{ij} = r \cdot f_{(ij)}.$$ 

(16)

It is easily seen that the antisymmetric piece $g_{[ij]}$ does not contribute to $U'_{ij}$, for $\epsilon_{\alpha\beta\gamma}Q^\alpha_i Q^\beta_j H^\gamma_C$ being antisymmetric under $SU(2)_L$ and $SU(3)_c$ has to be symmetric in flavor. The factor $r$ is group-theoretical and gives the ratio of the coupling of the color-triplet Higgs(ino) to the doublet Higgs(ino). This factor will be of great interest to us later.

To see the branching ratios in proton decay we want to go to the physical basis of the charge-$1/3$ quarks, as in the discussion leading to eq. 6. This is done by some transformation under which $Q_i \rightarrow W_{ij} Q_j$, $u_i^c \rightarrow W_{ij} u_j^c$, and $U \rightarrow W^T U W$, so that $U'$ gets transformed as $U' \rightarrow W^T U' W$. It is interesting to see what happens if there is a hierarchy among the elements of $U$, as is
usually the case in models of quark and lepton masses. Assume, therefore that $f_{33} > f_{23} \sim g_{23} > f_{22}$, and that the elements of the first row and column can be neglected. Then one has, approximately, that

$$U \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & f_{22} - \frac{f_{23}^2 - g_{23}^2}{f_{33}} & 0 \\ 0 & 0 & f_{33} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix},$$

(17)

while

$$U' \rightarrow r \cdot \begin{pmatrix} 0 & 0 & 0 \\ 0 & f_{22} - \frac{f_{23}^2 - g_{23}^2}{f_{33}} & g_{23} \\ 0 & g_{23} & f_{33} \end{pmatrix} = r \cdot \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_c & g_{23} \\ 0 & g_{23} & m_t \end{pmatrix}.$$  

(18)

One sees that the relative contributions to proton decay from the (2,2) and (3,3) elements are the same as in the minimal SUSY $SU(5)$ model. But now there is also the contribution from the (2,3) element, which is obviously model-dependent. In particular models of quark and lepton masses $g_{23}$ would in general be known. If $\tan \beta \ll m_t/m_b$ then the W-ino loops dominate and proton decay is controlled by the terms in eq. 6 with the matrices $D$ and $U$ replaced by $D'$ and $U'$. One can then generalize eqs. 7 and 8 using eq. 18.

$$\frac{C_{dd}}{C_{ds}} \approx - \sin \theta_c \left( \frac{1 + a^2 b - 2ab \frac{1}{2} g_{23}}{1 - ab - (a - 1)b \frac{1}{2} g_{23}} \right),$$

(19)

where $g_{23} \equiv g_{23}/(m_c m_t)^{1/2}$ has been normalized so that it is typically a number of order unity (if non–zero). Thus

$$\frac{\Gamma(p \rightarrow \pi \overline{\nu})/\Gamma(p \rightarrow K \overline{\nu})}{\Gamma(p \rightarrow \pi \overline{\nu})/\Gamma(p \rightarrow K \overline{\nu})_{\text{min SU(5)}}} = \left| 1 - 2 \left( \frac{ab}{1+ab} \right) g_{23} \right|^2.$$  

(20)

One can straightforwardly derive the analogous expression when the first row and column of $U'$ cannot be neglected.
What one sees is that by measuring the $\pi^+\nu$ to $K^+\nu$ ratio in proton decay one could distinguish different models of quark and lepton masses, since in different models $|\mathcal{F}_{23}|$ would take different — and for predictive models, computable — values.

If $\tan \beta$ is large, then these neutrino modes can get comparable contributions from both the W-ino and gluino loops. This would make the expression for the branching ratio $\Gamma(p \to \pi^+\nu)/\Gamma(p \to K^+\nu)$ much more complicated (unless $\tan \beta$ is large and the spectrum is such that the gluino loop dominated). One can tell the relative importance of the W-ino loops and gluino loops by the relative importance of the neutrino and charged lepton modes, of course.

A further test of models comes, if $\tan \beta$ is large, from examining the ratio of $e^+$ to $\mu^+$. This probes the value of $D'_{11}/D'_{21}$, as can be seen from eq. 12, where $D$ and $U$ should be replaced in the general case by $D'$ and $U'$. In minimal $SU(5)$, $D'_{ij} = D_{ij}$ and $\Gamma(p \to K^0 e^+)/\Gamma(p \to K^0 \mu^+) \approx |D_{11}/D_{21}|^2 \approx |m_d/m_s \sin \theta_c|^2 \approx 0.05$. (Recall that we are working in the basis where $U_{ij}$ is diagonal, so that $D_{ij}$ is nondiagonal.) In a realistic model of quark and lepton masses this branching ratio would be different. For example, in the Georgi-Jarlskog model it comes out to be about 0.02.

Finally, for large $\tan \beta$, the branching ratios $\Gamma(p \to K^0 l^+)/\Gamma(p \to \pi^0 l^+)$ give independent information about the elements of $U'_{ij}$. (Cf. eq. 12.)

## 4 Group-theoretical Suppression of p Decay

Where eq. 16 applies one sees that the proton decay amplitudes coming from either W-ino or gluino loops are proportional to the group-theoretical factor $r$. This factor is given for various simple cases in Table I.

In discussing the value of $r$ it should be kept in mind that the form given in eq. 16, in which $r$ appears, is only valid for cases in which the symmetric contribution to $U_{ij}$ comes from a single term (or at least from several terms which all have the same gauge structure). The simplest possibility in $SU(5)$
is that the symmetric contribution to $U_{ij}$ comes from a 5 of Higgs, as in the minimal $SU(5)$ model. In that case the couplings of doublet and triplet Higgs are trivially the same and $r = 1$. The other $SU(5)$ multiplet that can couple flavor-symmetrically to the charge-$2\over 3$ quarks is the $50_H$. But, while this contains a color-triplet it does not contain the usual type of Higgs doublet, so that if only a $50_H$ were present then $U_{ij}$ would vanish and $r$ would be infinite, which is clearly unrealistic. If both a $5_H$ and a $50_H$ contribute then eq. 16 does not apply.

One can also consider higher order operators involving bilinears of Higgs fields. Suppose that the flavor-symmetric contribution to $U_{ij}$ comes from the operator $[10, 10]_j 5_H 24_H$. The product $5 \times 24$ contains $5 + 45 + 70$. Of these, $70$ does not couple to $10 \times 10$, and $45$ couples antisymmetrically. Thus there is only one term that couples symmetrically in flavor and eq. 16 does apply with a value $r = -2/\sqrt{3}$, as can be straightforwardly shown.

The next simplest possibility is that the symmetric contribution to $U_{ij}$ comes from the bilinear $45_H \times 24_H$. There are two independent ways to contract this to get a flavor symmetric term, namely into a 5 or a $50$, so eq. 16 does not apply unless for some reason only a single contraction or linear combination of contractions of $45_H \times 24_H$ contributes. For example, if only the operator $[10, 10]_j (45_H 24_H)_5$ appears, where $45_H \times 24_H$ is contracted in the 5 channel, then $r = 2/\sqrt{3}$. (Such a contraction might come about by integrating out a $5 + \overline{5}$, as shown in Fig. 4a. However, in that case there is also in general the direct $10, 10, 5_H$ term, which, if it contributes significantly to $U_{ij}$ and $U'_{ij}$, invalidates eq. 16.) A different term arises from integrating out a $10 + \overline{10}$, as shown in Fig. 4b. If this gives the only symmetric contribution to $U_{ij}$ then $r = 0$. (The reason for this is that the color triplet in the $45_H$ does not have a coupling to a pair of quark doublets. See the discussion above of why $g_{ij}$ does not contribute to $U'_{ij}$.) This would be a way of suppressing Higgsino-mediated proton decay group-theoretically in $SU(5)$. However, $SU(5)$ seems to require an elaborate Higgs structure involving $75_H + 50_H + \overline{50}_H$ to solve the doublet-triplet-splitting problem,
and the mechanism being discussed here would require in addition a $45_H$. We thus consider the $SO(10)$ example of a group-theoretical suppression of proton decay discussed below to be more interesting.

Turning now to $SO(10)$, the simplest possibility is that the sole symmetric contribution to $U_{ij}$ is from a $10_H$, which, like the case of the fundamental Higgs in $SU(5)$, gives $r = 1$. On the other hand, if the sole contribution to $U_{ij}$ is from a $126_H$ then $r = \sqrt{3}$. (It should be noted that there are two color-triplets in the $126_H$, or more precisely two $(3,1,-\frac{1}{3})$ representations of $SU(3)_c \times SU(2)_L \times U(1)_Y$. $[Q_i Q_j]$ couples to one linear combination of these. $r = \sqrt{3}$ is the ratio of the strength of this coupling to the strength of the Higgs doublet’s coupling to $[u_i c Q_j]$. But the proton decay amplitude will also depend on the mixing angle which tells how much of the lightest color-triplet Higgsino is contained in the linear combination that couples to $[Q_i Q_j]$.)

The simplest Higgs bilinear to be considered in $SO(10)$ is $10_H \times 45_H$. This contains $10 + 120 + 320$. Since the $320$ does not couple to $[16,16]$, and $120$ couples antisymmetrically, there is only one symmetric contribution from this term and eq. 16 applies. The value of $r$ depends on what direction in group space $\langle 45_H \rangle$ points in. Let us call that direction, which is a linear combination of $SO(10)$ generators, $Q$. Then it is easily seen that $r = \left( \frac{2Q_{(u,d)}}{Q_{(u,d)}+Q_{\nu}} \right)$. (Note that the same expression applies to the $SU(5)$ case of $5_H \times 24_H$ with $Q = Y/2$ and gives $r = 2(\frac{1}{6})/(\frac{1}{6} - \frac{2}{3}) = -\frac{2}{3}$.) This is made small if $Q$ points approximately in the $I_{3R}$ direction, since the left-handed quark doublets are singlets under $SU(2)_R$. One can write $Q$ as a linear combination of $I_{3R}$ and $Y/2$. (There is a two-dimensional space of generators of $SO(10)$ that commute with the generators of $SU(3)_c \times SU(2)_L \times U(1)_Y$.) Since $Y/2$ is non-zero for the left-handed quark doublets, it is obvious that $r$ will be small only if $Q$ points approximately in the $I_{3R}$ direction.

This result is quite interesting, since, in a recent paper, a model of quark and lepton masses is proposed by us in which the matrix $U_{ij}$ arises from an
effective term $[16,16] \cdot 10_H \cdot 45_H$ and the $\langle 45_H \rangle$ points approximately in the $I_{3R}$ direction. Indeed, in that model, $Q \sim I_{3R}$ explains three relations among known quantities: the Georgi-Jarlskog relation $m_\mu^0/m_s^0 \approx 3$, the smallness of the second generation masses compared to those of the third generation, and the smallness of $V_{cb}$. (In the “long version” of that model $Q \sim I_{3R}$ also explains why $m_\tau^0/m_t^0 \ll m_\mu^0/m_b^0$.) What we have found here is that the same group theoretical assumption also gives rise to a (needed) suppression of Higgsino-mediated proton decay.

In the model of Ref. 6 the group-theoretical suppression of the proton decay amplitude is numerically of order $10^{-1}$. This means that, while suppressed, $p$–decay is still within the reach of Super Kamiokande.

It is worth noting that the term $[16,16] \cdot 10_H \cdot 45_H$ has another beautiful property, which is exploited in Ref. 6. For $i = j = 3$, $10_H \times 45_H$ must be in the symmetric product, and, in particular, be contracted to into a $10$, which gives equal contributions to $D_{33}$ and $L_{33}$. But for $i,j = 2,3$ or $3,2$ contractions into both $10$ and $120$ are allowed, and the latter contributes differently to $D$ and $L$. Thus a term of the form $[16,16] \cdot 10_H \cdot 45_H$ explains why $m_b^0 = m_s^0$ while $m_s^0 \neq m_\mu^0$.

5 Conclusions

We have shown that in realistic models of quark and lepton masses in the context of SUSY GUTs, the proton decay rate and branching ratios are different from the predictions of minimal SUSY $SU(5)$ model. This would enable one to test quark and lepton mass schemes in SUSY GUTs by measuring, for example, the branching ratio $B[(p \to \pi^+\nu)/(p \to K^+\nu)]$. In predictive models of fermion masses, such branching ratios are computable as they are related to other low energy observables. We have also emphasized that in large $\tan \beta$ models, as often occurs in $SO(10)$ GUT, the gluino dressing of the effective $\Delta B \neq 0$ dimension 5 operator is quite significant. This opens up the possibility that the charged lepton decay modes of the proton,
$p \to K^0\mu^+, p \to K^0e^+$, may be comparable to the neutrino modes. We give simple analytic arguments showing how this happens, especially in the case of $\tan\beta = m_t/m_b$ schemes. The branching ratio $B[(p \to K^0\mu^+)/ (p \to K^0e^+)]$ will then provide additional clue to the texture of the fermion mass matrices. The relative strengths of the neutral to charged lepton modes will tell us about the parameter $\tan\beta$ itself. While the precise values of these branching ratios depend on poorly known (presently) Kobayashi–Maskawa mixing angles ($V_{ub}, V_{td}$), that situation should change in the near future.

We have also found, as discussed in section 4, an interesting group theoretical suppression mechanism for proton decay rate. This could provide a simple answer to the question why in the context of SUSY GUTs, proton decay has not yet been observed. We find it interesting that a recent model of quark and lepton masses proposed on quite independent grounds automatically has this ingredient needed for group theoretical suppression of proton decay rate.

In closing, let us emphasize that the discovery for proton decay would not only provide evidence for the violation of baryon number symmetry near the Planck scale, but it would also provide, in the context of SUSY GUTs, important clues to the structure of the fermion mass matrices. The search for these decays should continue in earnest.
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Table I: The group-theoretical factor $r$ that enters into the proton-decay amplitude in SUSY GUT models for which the flavor-symmetric contribution to the mass matrix of the up quarks comes from a single kind of operator. The second column gives the product of Higgs fields appearing in that operator. In the case of the $45_H \times 24_H$ of $SU(5)$ more than one contraction of indices is possible.

| Gauge group | Higgs operator          | $r$          |
|-------------|-------------------------|--------------|
| $SU(5)$     | $\bar{5}_H$             | 1            |
| $SU(5)$     | $50_H$                  | $\infty$    |
| $SU(5)$     | $5_H \times 24_H$       | $-\frac{2}{3}$ |
| $SU(5)$     | $(45_H \times 24_H)_5$  | $2/\sqrt{3}$ |
| $SU(5)$     | $(45_H \times 24_H)_{\text{Fig.2b}}$ | 0            |
| $SO(10)$    | $10_H$                  | 1            |
| $SO(10)$    | $126_H$                 | $\sqrt{3}$  |
| $SO(10)$    | $10_H \times 45_H$      | $\frac{2Q_{(u,d)}}{Q_{(u,d)}+Q_{u^c}}$ |
Figure Captions

**Fig. 1:** Diagrams involving the exchange of supermassive, colored Higgsinos which give effective dimension-five $\Delta B \neq 0$ operators. The dominant contribution to $p$ decay in general SUSY GUTs comes from (a).

**Fig. 2:** A dimension-five $\Delta B \neq 0$ operator dressed by a gluino loop to produce a four-fermion interaction. By having a virtual $t$-squark this diagram is enhanced by a factor of $U_{33} = m_t$. This requires the flavor-changing squark-mass insertion indicated by the solid circle.

**Fig. 3:** A one-loop diagram giving a flavor-changing squark mass term.

**Fig. 4:** Two diagrams that would give operators of the form $10_i 10_j 45_H 24_H$. Diagram (a) gives the $45_H$ and $24_H$ contracted into a $5$. Diagram (b) gives a different contraction, for which $r = 0$. 


Fig. 1a

Fig. 1b
Fig. 3a

Fig. 3b
