Quark and Lepton Flavour Physics in the Littlest Higgs Model with T-Parity

Andrzej J. Buras and Cecilia Tarantino
Technische Universität München, D-85748 Garching, Germany

The Littlest Higgs model with T-parity (LHT) contains new sources of flavour and CP violation both in the quark and the lepton sector. They originate from the interactions of ordinary fermions with mirror fermions mediated by new gauge bosons: $W_H^\pm, Z_H$ and $A_H^\mp$. The most spectacular departures from the Standard Model are found in $K_L \to \pi^0 \nu\bar{\nu}$, $K^+ \to \pi^+ \nu\bar{\nu}$, in the CP-asymmetry $S_{\text{CP}}$ and in lepton flavour violating decays. In particular, the latter decays offer a clear distinction between the LHT model and the MSSM. We summarize the most interesting results of three extensive analyses of flavour physics in the LHT model.

I. THE LHT MODEL

One of the most attractive solutions to the so-called little hierarchy problem that affects the Standard Model (SM) is provided by Little Higgs models. They are perturbatively computable up to $\sim 10$ TeV and have a rather small number of parameters, although their predictivity can be weakened by a certain sensitivity to the unknown ultraviolet (UV) completion of the theory. In these models, in contrast to supersymmetry, the problematically quadratic divergences to the Higgs mass are cancelled by loop contributions of new particles with the same spin-statistics of the SM ones and with masses around 1 TeV.

The basic idea of Little Higgs models is that the Higgs is naturally light as it is identified with a Nambu-Goldstone boson (NGB) of a spontaneously broken global symmetry. An exact NGB, however, would have only derivative interactions. Gauge and Yukawa interactions of the Higgs have to be incorporated. This can be done without generating quadratically divergent one-loop contributions to the Higgs mass, through the so-called collective symmetry breaking. Collective symmetry breaking (SB) has the peculiarity of generating the Higgs mass only when two or more couplings in the Lagrangian are non-vanishing, thus avoiding one-loop quadratic divergences.

The most economical, in matter content, Little Higgs model is the Littlest Higgs (LH) model, where the global group $SU(5)$ is spontaneously broken into $SO(5)$ at the scale $f \approx O(1$ TeV) and the electroweak (ew) sector of the SM is embedded in an $SU(5)/SO(5)$ nonlinear sigma model. Gauge and Yukawa Higgs interactions are introduced by gauging the subgroup of $SU(5)$: $[SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2$. In the LH model, the new particles appearing at the TeV scales are the heavy gauge bosons ($W_H^\pm, Z_H, A_H$), the heavy top ($T$) and the scalar triplet $\Phi$.

In the LH model, significant corrections to ew observables come from tree-level heavy gauge boson contributions and the triplet vacuum expectation value (vev) which breaks the custodial $SU(2)$ symmetry. Consequently, ew precision tests are satisfied only for quite large values of the New Physics (NP) scale $f \geq 2 - 3$ TeV$^{[3, 4]}$, unable to solve the little hierarchy problem. Motivated by reconciling the LH model with ew precision tests, Cheng and Low proposed to enlarge the symmetry structure of the theory by introducing a discrete symmetry called T-parity. T-parity forbids the tree-level contributions of heavy gauge bosons and the interactions that induced the triplet vev. The custodial $SU(2)$ symmetry is restored and the compatibility with ew precision data is obtained already for smaller values of the NP scale, $f \geq 500$ GeV$^{[5]}$. Another important consequence is that particle fields are T-even or T-odd under T-parity. The SM particles and the heavy top $T_+$ are T-even, while the heavy gauge bosons $W_H^\pm, Z_H, A_H$ and the scalar triplet $\Phi$ are T-odd. Additional T-odd particles are required by T-parity: the odd heavy top $T_-$ and the so-called mirror fermions, i.e., fermions corresponding to the SM ones but with opposite T-parity and $O(1$ TeV$)$ masses. Mirror fermions are characterized by new flavour interactions with SM fermions and heavy gauge bosons, which involve in the quark sector two new unitary mixing matrices analogous to the CKM matrix. They are $V_{Hd}$ and $V_{Hu}$, respectively involved when the SM quark is of down- or up-type, and satisfying $V_{Hd}^\dagger V_{Hd} = V_{\text{CKM}}^\dagger V_{\text{CKM}}$. Similarly, two new mixing matrices, $V_{Ht}$ and $V_{H\nu}$, appear in the lepton sector, respectively involved when the SM lepton is charged or a neutrino and related to the PMNS matrix$^{[8]}$ through $V_{Ht}^\dagger V_{H\nu} = V_{\text{PMNS}}^\dagger V_{\text{PMNS}}$. Both $V_{Hd}$ and $V_{Ht}$ contain 3 angles, like $V_{\text{CKM}}$ and $V_{\text{PMNS}}$, but 3 (non-Majorana) phases$^{[10]}$, i.e., two additional phases relative to the SM matrices, that cannot be rotated away in this case.

Because of these new mixing matrices, the LHT model does not belong to the Minimal Flavour Violation (MFV) class of models and significant effects in flavour observables are possible, without adding new operators to the SM ones. Finally, it is important to recall that Little Higgs models are low energy non-linear sigma models, whose unknown UV-completion introduces a theoretical uncertainty, as discussed in detail in$^{[13, 14]}$.

II. LHT FLAVOUR TRIOLOGY

Several studies of flavour physics in the LH model without T-parity have been performed in the last four years...

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years [13, 14]. Without T-parity, mirror fermions and new sources of flavour and CP-violation are absent, the LH model is a MFV model and NP contributions result to be very small.

More recently, flavour physics analyses have also been performed in the LHT model, for both quark [8, 14, 16] and lepton sectors [17, 18]. In this model, new mirror fermion interactions can yield large NP effects, mainly in $K$ and $B$ rare and CP-violating decays and in lepton flavour violating decays.

Below, we summarize the main results found in our trilogy on FCNC processes in the LHT model [14, 16, 18].

A. $\Delta F = 2$ Processes in the Quark Sector [16]

The short distance structure of $\Delta F = 2$ processes in the LHT model is fully encoded in three perturbatively calculable functions

$$S_K \equiv |S_K| e^{i2\varphi_K}, S_{B_d} \equiv |S_{B_d}| e^{i2\varphi_{B_d}}, S_{B_s} \equiv |S_{B_s}| e^{i2\varphi_{B_s}},$$

relevant for $K^0 - \bar{K}^0$, $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixings, respectively.

In the SM they all reduce to a real single function $S_{SM} = S_0(x_i)$ that is dominated by box diagrams with top quark exchanges. In the LHT model, where the new mixing matrix $V_{Hz}$ is present, the inclusion of box diagrams with internal mirror quarks and heavy gauge bosons $(W^z_H, Z_H, A_H)$ makes the one-loop functions in (1) complex quantities. Moreover, the universality between $K^0$, $B_d$ and $B_s$ systems, valid in the SM is broken so that the magnitudes $|S_i|$ and the phases $\varphi_i$ depend on $i = K, B_d, B_s$. We recall that in constrained MFV models [11, 14] the short distance functions are as in the SM real and universal, although different from $S_{SM}$.

The size of $\varphi_i$ depends on the structure of the matrix $V_{Hz}$, which is so far only weakly constrained by the existing data on $\Delta M_K$, $\Delta M_d$, $\Delta M_s$, $\sin 2\beta$ and $\varepsilon_K$, mainly due to significant hadronic uncertainties in $\Delta M_{d,s}$ and $\varepsilon_K$. This allows to obtain interesting departures from the SM, in particular in the $B^0_s - \bar{B}^0_s$ system, but also in connection with the slight discrepancy [19, 20, 21], existing within the SM, between the values of $|V_{ub}|$ obtained from tree level decays and the value of $\sin 2\beta$ measured from the CP-asymmetry $S_{\psi_K}$.

The main messages from [16] are as follows:

- The presence of a non-vanishing phase $\varphi_{B_d}$ implies that $S_{\psi_K} = \sin(2\beta + 2\varphi_{B_d})$, so that with $\varphi_{B_d} \approx -5^\circ$ the possible discrepancy between $|V_{ub}|$ and $S_{\psi_K}$ can be cured.
- The presence of a non-vanishing phase $\varphi_{B_s}$ allows to enhance the CP-asymmetry $S_{\psi_{K^0}}$ from the SM prediction 0.04 to 0.30 with an analogous enhancement of the semileptonic asymmetry $A_{SL}^d$ and a smaller but sizable enhancement of $A_{SL}^s$.

- A non-vanishing $\varphi_{B_s}$ allows also to slightly suppress $\Delta M_s$ below its SM value, thus further improving the agreement with the CDF measurement [22].

B. $\Delta F = 1$ Processes in the Quark Sector [14]

The short distance structure of the LHT model, that is relevant for rare $K$ and $B$ decays, is fully encoded in nine perturbatively calculable functions $(i = K, d, s)$

$$X_i = |X_i| e^{i\theta_i}, \quad Y_i = |Y_i| e^{i\theta_i}, \quad Z_i = |Z_i| e^{i\theta_i},$$

that result from the SM box and penguin diagrams and analogous diagrams with new particle exchanges. In the case of the radiative decay $B \to X_s\gamma$, also the function $D'$ has to be considered. In the SM and in models with constrained MFV all these functions are real and independent of $i$, with consequent strong correlations between various observables in $K$, $B_d$ and $B_s$ systems.

In the LHT model, the non-vanishing $\theta$'s originate from the new phases of the $V_{Hz}$ matrix. The additional dependence on $i$ and the possibility of large $\theta^i_{K,y,z}$ imply a pattern of FCNC processes that differs significantly from the SM and the constrained MFV one.

The main messages from [14] are as follows:

- The most evident departures from the SM predictions are found in $K \to \pi\nu\bar{\nu}$ decays (Fig. 1). $Br(K_L \to \pi^0\nu\bar{\nu})$ can be enhanced even by an order of magnitude and $Br(K^+ \to \pi^+\nu\bar{\nu})$ by a factor 5. Moreover, $Br(K_L \to \pi^0\nu\bar{\nu})$ can be larger than $Br(K^+ \to \pi^+\nu\bar{\nu})$, which is not possible in MFV models.
- $Br(K_L \to \pi^0\epsilon^+\epsilon^-)$ and $Br(K_L \to \pi^0\mu^+\mu^-)$ can be both enhanced by a factor $2 - 3$ and are strongly correlated, as shown in Fig. 2.
- A strong correlation between $Br(K_L \to \pi^0\ell^+\ell^-)$ and $Br(K_L \to \pi^0\nu\bar{\nu})$ exists, as shown in Fig. 3.

![FIG. 1: $Br(K_L \to \pi^0\nu\bar{\nu})$ vs. $Br(K^+ \to \pi^+\nu\bar{\nu})$. The shaded area represents the experimental 1σ-range for $Br(K^+ \to \pi^+\nu\bar{\nu})$. The model-independent Grossman-Nir bound [23] is displayed by the dotted line, while the solid line separates the two areas where $Br(K_L \to \pi^0\nu\bar{\nu})$ is larger or smaller than $Br(K^+ \to \pi^+\nu\bar{\nu})$.](image-url)
The branching ratios for $B_{s,d} \to \mu^+\mu^-$ and $B \to X_{s,d} \gamma\mu\bar{\nu}$, instead, are modified by at most 50% and 35%, respectively, and the effects of new electroweak penguins in $B \to \pi K$ are small, in agreement with the recent data. The new physics effects in $B \to X_{s,d} \gamma$ and $B \to X_{s,d} \ell^+\ell^-$ turn out to be below 5% and 15%, respectively, so that agreement with the data can easily be obtained.

The universality of new physics effects, characteristic for MFV models, can be largely broken, in particular between $K$ and $B_{s,d}$ systems. NP effects, in fact, are typically larger in $K$ system where the SM contribution is CKM-suppressed. In particular, sizable departures from MFV relations between $\Delta M_{s,d}$ and $Br(B_{s,d} \to \mu^+\mu^-)$ and between $S_{\ell K}$ and the $K \to \pi\nu\bar{\nu}$ decay rates are possible.

**C. Lepton Flavour Violation**

In contrast to rare $K$ and $B$ decays, where the SM contributions play an important and often dominant role, in the LHT model the smallness of ordinary neutrino masses assures that mirror fermion contributions to lepton flavour violating (LFV) processes are far the dominant effects. Moreover, the absence of QCD corrections and hadronic matrix elements allows in most cases to make predictions entirely within perturbation theory.

**FIG. 2:** $Br(K_L \to \pi^0\mu^+\mu^-)$ vs. $Br(K_L \to \pi^0 e^+e^-)$.

**FIG. 3:** $Br(K_L \to \pi^0 e^+e^-)$ (upper curve) and $Br(K_L \to \pi^0\mu^+\mu^-)$ (lower curve) as functions of $Br(K_L \to \pi^0\nu\bar{\nu})$. The corresponding SM predictions are represented by dark points.

**FIG. 4:** Correlation between the branching ratios for $\mu \to e\gamma$ and $\mu^- \to e^-\mu^-e^-$. The shaded area represents present experimental upper bounds.

**TABLE I:** Comparison between the LHT model and the MSSM without and with significant Higgs contributions.

| ratio | LHT | MSSM (dipole) | MSSM (Higgs) |
|-------|-----|---------------|--------------|
| $Br(\mu^+ \to e^+\nu\bar{\nu})$ | $0.4 \ldots 2.5$ | $\sim 6 \times 10^{-3}$ | $\sim 6 \times 10^{-3}$ |
| $Br(\mu^- \to e^-\nu\bar{\nu})$ | $0.4 \ldots 2.3$ | $\sim 1 \times 10^{-2}$ | $\sim 1 \times 10^{-2}$ |
| $Br(\tau \to e\nu\bar{\nu})$ | $0.4 \ldots 2.3$ | $\sim 2 \times 10^{-3}$ | $0.06 \ldots 0.1$ |
| $Br(\tau \to \mu\nu\bar{\nu})$ | $0.3 \ldots 1.6$ | $\sim 2 \times 10^{-3}$ | $0.02 \ldots 0.04$ |
| $Br(\tau \to e\mu\nu\bar{\nu})$ | $0.3 \ldots 1.6$ | $\sim 1 \times 10^{-2}$ | $\sim 1 \times 10^{-2}$ |
| $Br(\tau \to \mu^+\mu^-\mu^-)$ | $1.3 \ldots 1.7$ | $\sim 5$ | $0.3 \ldots 0.5$ |
| $Br(\tau \to \mu^+\mu^-\mu^+)$ | $1.2 \ldots 1.6$ | $\sim 0.2$ | $5 \ldots 10$ |
| $R(\mu \to e\gamma)$ | $10^{-2} \ldots 10^2$ | $\sim 5 \times 10^{-3}$ | $0.08 \ldots 0.15$ |

In [18] we have studied the most interesting LFV processes: $\ell_i \to \ell_j \gamma$, $\tau \to \ell P$ (with $P = \pi, \eta, \eta'$), $\mu^- \to e^-\mu^-e^-$, the six three-body decays $\tau^- \to l_i l_j l_k^-$ and the rate for $\mu \to e$ conversion in nuclei. We have also calculated the rates for $K_{L,S} \to \mu e$, $K_{L,S} \to \pi^0\mu\bar{\nu}$, $B_{d,s} \to \mu e$, $B_{d,s} \to \tau e$ and $B_{d,s} \to \tau\mu$.

The main messages from [18] are as follows:

- Several rates can reach or approach the present experimental upper bounds. In particular, in order to suppress the $\mu \to e\gamma$ and $\mu^- \to e^-\mu^-e^-$ decay rates below the experimental upper bounds (see Fig. 4), the $V_{H\ell}$ mixing matrix has to be rather hierarchical, unless mirror leptons are quasi-degenerate.

- The pattern of the LFV branching ratios in the LHT model differs significantly from the MSSM one, allowing a clear distinction of these two models. The origin of this difference is that in the MSSM the LFV rates are dominated by the dipole operator, whose role is instead negligible in the LHT model.

- These different patterns of LFV in the LHT and the MSSM can best be seen by studying certain correlations between branching ratios that have...
been previously considered in the context of the MSSM \cite{24, 25, 26, 27}. We find that the ratios in Table\ref{tab:ratios} could allow for a transparent distinction between the LHT model and the MSSM. In particular, the ratios involving $Br(\ell_i \to \ell_j \gamma)$ turn out to be of $O(1)$ and $O(\alpha)$ in the LHT model and the MSSM, respectively.

- We also note that a measurement of $\mu \to e\gamma$ at the $10^{-13}$ level would necessarily imply within the MSSM a rate for the $\mu \to e$ conversion in Ti below $10^{-15}$, while it could be much larger within the LHT model.

- Finally, we have studied the muon anomalous magnetic moment $(g-2)_\mu$, and found that LHT effects are roughly a factor 5 below the current experimental uncertainty \cite{28}, implying that the possible discrepancy between the SM prediction and the data cannot be solved in the LHT model. This represents another clear difference from the MSSM.

III. CONCLUSIONS

Our trilogy on FCNC processes in the LHT model revealed very interesting and peculiar patterns that not only differ from those found in the SM and MFV models but also in the MSSM. These differences can most clearly be seen in LFV processes. We are looking forward to the forthcoming data from Tevatron, LHC, $B - K$ and LFV dedicated experiments that will tell us whether the LHT model represents a good description of nature.

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