Elastic $K\pi$ amplitude: a simple model

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Abstract

We present a chiral model for the $J = 0$, $I = 1/2$, elastic $K\pi$ amplitude, suited to be employed in $D^+ \to K^-\pi^+\pi^+$ data analyses and valid between threshold and 1.5 GeV. Although not as precise as other versions available in the literature, it is rather simple and incorporates the essential physics in this energy domain. In the case of the $K$-matrix approximation, the model allows the pole structure of the $K\pi$ amplitude to be understood by solving a quadratic equation in $s$. We show that the solutions to this equation can be well approximated by polynomials of masses and coupling constants. This analytic structure allows a clear understanding why, depending on the values of one of the coupling constants, one may have one or two physical poles. The model yields a pole, associated with the $\kappa$, at $\sqrt{s} = (0.75 - i 0.24)$ GeV.

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I. INTRODUCTION

Since the E791 experiment[1], heavy-meson decays have systematically produced solid evidence in favour of low-energy scalar resonances[2–4]. The quality of these results renewed interest in the problem and motivated an effort aimed at determining the positions of their poles in the complex energy plane. In the case of $\pi\pi$ resonances, the pole position of the $\sigma$ can be determined from scattering data by means of the Roy equation, and the rather precise value $\sqrt{s_\sigma} = (0.441^{+16}_{-8} - i 0.272^{+9}_{-12.5})$ MeV is available[5].

The situation of scalar resonances in the $K\pi$ system is much less certain, since our knowledge of the $S$-wave $I = 1/2$ amplitude is based on just two experiments[6, 7], which include phase shifts only up to $\sim 80^0$. These data sets have been carefully analyzed in recent years, and determinations of both pole positions[8] and scattering lengths[9] became available. Nevertheless, the need of more empirical knowledge is still urgent and, in principle, information from the decay $D^+ \rightarrow K^-\pi^+\pi^+$ could be useful in either constraining or complementing $K\pi$ scattering data.

Analyses of $D^+$ decays normally rely on trial functions written in terms of Breit-Wigner expressions, which are at odds with chiral symmetry. As the symmetry is very important at low-energies, this kind of procedure has been criticized and alternatives were proposed[10, 11]. In the work by Oller[11], data were refitted with the help of a chiral amplitude, with a remarkable decrease in $\chi^2$. In a recent paper[12], we have discussed the main features of the decay $D^+ \rightarrow K^-\pi^+\pi^+$ at low energies, including explicit descriptions of both the primary weak vertex and final state interactions, based on a unitarized $K\pi$ amplitude. This elastic amplitude was obtained by the iteration of a kernel by means of a simplified Bethe-Salpeter equation[13]. The $(J,I) = (0,1/2)$ component of the kernel, which interests us here, was derived from effective lagrangians and contains a leading $O(q^2)$ contact term[14], supplemented by an explicit resonance exchange[15], corresponding to $O(q^4)$ corrections. This model is illustrated in fig.1.

Our ultimate goal is to produce a tool to be directly employed in analyses of raw data and therefore we try to avoid, as much as possible, the use of long and involved expressions. We thus neglect contributions from resonance exchanges in $t-$ and $u-$ channels, since they just give rise to small backgrounds[12]. In the present work we follow this approach and show that simplifications are possible in the chiral kernel which preserve its essential physical
II. $K\pi$ AMPLITUDE

Our unitarized $(J, I) = (0, 1/2)$ amplitude for the process $\pi K \rightarrow \pi K$ has been discussed in detail in ref.[12] and here we just summarize its main features. It is written as

$$T_{1/2}(s) = \gamma^2(s)/D(s) ,$$

$$D(s) = [m_R^2 - s + \gamma^2(s) \bar{R}_{1/2}(s)] - i \left[ \frac{\gamma^2(s) \rho(s)}{16\pi} \right] ,$$

where:
- $s$ is the usual Mandelstam variable and $\rho(s) = \sqrt{1 - 2(M_K^2 + M_\pi^2)/s + (M_K^2 - M_\pi^2)^2/s^2}$;
- $m_R$ is the parameter present in the chiral lagrangian, called nominal resonance mass;
- $\bar{R}_{1/2}(s)$ is the function describing off-shell effects in the two-meson propagator, given by

$$\bar{R}_{1/2}(s) = -\Re \left[ L(s) - L(m_R^2) \right]/16\pi^2 ,$$

$$\Re L(s) = \rho(s) \log \left[ \frac{1 - \sigma}{1 + \sigma} \right] - 2$$

$$+ \left[ \frac{(M_K^2 - M_\pi^2)/s}{s - (M_K + M_\pi)^2} \right] \log(M_K/M_\pi) ,$$

$$\sigma = \sqrt{|s - (M_K + M_\pi)^2|/\left|s - (M_K - M_\pi)^2\right|} ;$$

- $\bar{R}_{1/2}(m_R^2) = 0$ by construction and therefore the phase shift is $\pi/2$ at $s = m_R^2$;
- $\gamma^2(s)$ is the function which incorporates chiral dynamics, given by

$$\gamma^2(s) = \left\{ \left(1/F^2\right) \left[ (1 - 3 \rho^2(s)/8) s - (M_\pi^2 + M_K^2) (m_R^2 - s) \right] \right\}_L$$

$$+ \left\{ (3/F^4) \left[ c_d \left( s - M_\pi^2 - M_K^2 \right) + c_m \left( 4 M_K^2 + 5 M_\pi^2 \right) /6 \right] \right\}_R ;$$
- the labels $L$ and $R$ in the curly brackets denote contributions from the leading $\mathcal{O}(q^2)$ contact term and the $\mathcal{O}(q^4)$ resonance correction;

- $F$ is the pseudoscalar decay constant and the parameters $c_d$ and $c_m$ are the resonance couplings defined in ref.[15].

This model for the $K\pi$ amplitude depends on six parameters, three of which are well known and given by $M_\pi = 0.1396$ GeV, $M_K = 0.4937$ GeV and $F = \sqrt{F_\pi F_K} = 0.103$ GeV. The other three, namely $m_R$, $c_d$ and $c_m$ need to be taken from experiment. In ref.[15] one finds $(|c_d|, |c_m|) = (0.032, 0.042)$ GeV, obtained from the process $a_0 \rightarrow \eta \pi$. We adopt these values provisionally and, at the end, suggest our own choice.

The influence of the various dynamical mechanisms over $|T_{1/2}|$ is displayed in Fig.2. The full curve is based on eq.(3), whereas the dashed ones describe partial contributions: $L$ corresponds to $c_d = c_m = 0$ and those labeled $c_d$ and $c_m$ arise from just the terms proportional to these parameters. The curve $c_m$ has a typical Breit-Wigner shape, but that labeled $c_d$ is much wider. Nevertheless, both of them are constrained to be small at threshold, since
the resonance corresponds to a $\mathcal{O}(q^4)$ correction. The leading $\mathcal{O}(q^2)$ contact term clearly dominates at low-energies and, as expected, full and resonance curves coincide at $s = m_R^2$, since the resonance nominal mass is independent of coupling parameters. By construction, $T_{1/2}$ is purely imaginary at this point.

### III. POLES

The poles of $T_{1/2}$, determined by the condition $D(s) = 0$, can be easily found out by numerical methods. Before doing this, however, it is instructive to discuss the mathematical structure of the problem, by means of a simplified version of the amplitude, in which:
- the function $\bar{R}_{1/2}$ is neglected, which corresponds to the $K$-matrix approximation;
- the pion mass is neglected, which corresponds to the $SU(2)$ limit.

The poles can then be found by solving the complex quartic equation

$$\left[\frac{5}{8} - \frac{3c_d^2}{F^2}\right] s^4 + \left[-(5m_R^2 + 7M_K^2)/8\right] s^3 + \left[(7m_R^2 - M_K^2)\frac{M_K^2}{8}\right] s^2 + \left[-(c_d - 2c_m/3)(9c_d - 4c_m)\frac{M_K^4}{F^2} + i 16\pi F^2m_R^2\right] s + \left[-3m_R^2M_K^6/8\right] = 0 . \quad (4)$$

Around physical poles, the quantity $M_K^2/|s|^2$ is small and one obtains the quadratic equation

$$A \ s^2 + B \ s + C = 0 , \quad (5)$$

$$A = \left[5/8 - 3c_d^2/F^2\right] ,$$
$$B = \left[-(5m_R^2 + 7M_K^2)/8 + c_d(9c_d - 4c_m)\frac{M_K^2}{F^2} + i 16\pi F^2\right] ,$$
$$C = \left[7M_K^2/8 - i 16\pi F^2\right]m_R^2 .$$

The parameter $A$ plays an important role in this problem and receives contributions from both the leading contact term and the resonance. In particular, the condition $A = 0$ yields $c_d = F\sqrt{5/24} = 0.047$ GeV, which is just 50% larger than accepted empirical values. In the sequence, we discuss the behavior of the solutions of this equation as functions of $A$, in the interval $5/8 \leq A \leq 0$. Exact solutions are available in its two extremes:
- $A = 5/8 \rightarrow c_d = 0$: in this case, the resonance $R$ corresponds to a bound state in the real axis, which does not couple with the $K\pi$ system, and one has

$$s_+(5/8) = m_R^2 \quad \text{and} \quad s_-(5/8) = \left[7M_K^2/5 - i \ 128\pi F^2/5\right];$$ (6)

- $A = 0$: the dynamical equation is no longer quadratic and its single solution reads

$$s_-(0) = \frac{\left[7M_K^2/5 - i \ 128\pi F^2/5\right]}{1 + \left[\frac{7M_K^2}{5} - 8c_d(9c_d - 4c_m)^M_K^2/5 - i \ 128\pi F^2/5\right]/m_R^2}.$$ (7)

We note that these two solutions already show a prominent feature of the problem, namely the stability of the solution $s_-(A)$ in the whole interval considered. As $m_R^2$ is a large parameter, $s_-(0) = s_-(5/8) + \mathcal{O}(1/m_R^2)$.

In the general case, the solutions of eq.(5) will have the form

$$s_\pm = \left[-B \pm \sqrt{D}\right]/2A \quad \leftrightarrow \quad D = B^2 - 4AC.$$ (8)

The square root prevents algebraic simplification of results. However, at the point $A = -\Re B/2m_R^2 \sim 5/16$, one has $3D = 0$ and an approximate solution can be obtained for $\sqrt{D}$. By imposing a quadratic polynomial in $A$ to interpolate this function at $A = 5/8$, $-\Re B/2m_R^2$ and 0, one finds

$$s_+ = \frac{1}{A} \left\{ \left[\frac{5}{8} \ M_K^2 - \frac{c_d}{F} \left(\frac{24c_d}{5F} - \frac{4c_m}{F}\right) \right] \ M_K^2 \\
- \frac{3c_d^2}{m_R^2F^2} \left(1 - \frac{24c_d^2}{5F^2} \left(\frac{128\pi F^2}{5}\right)^2\right) \right\}$$

$$- \frac{i}{F} \left[\frac{c_d}{F} \left(\frac{3c_d}{5F} - \frac{4c_m}{F}\right) \frac{M_K^2}{m_R^2} \right] \left[1 - \frac{24c_d^2}{5F^2} \left(\frac{128\pi F^2}{5}\right)^2 \right] \frac{128\pi F^2}{5},$$ (9)

$$s_- = \left\{ \left[\frac{7}{5} \ M_K^2 + \frac{24m_R^2c_d^2}{5F^2} \left(\frac{128\pi F^2}{5m_R^2}\right)^2\right] \right\}$$

$$- \frac{i}{5F^2} \left[1 - \frac{24c_d^2}{5F^2} \left(\frac{128\pi F^2}{5m_R^2}\right)^2 \right] \frac{128\pi F^2}{5}.$$ (10)

In tables I and II we display figures derived from eqs.(4), (5) and (9-10), for a wide sample of values for $c_d/F$. We recall that empirical estimates of this quantity lie in a narrow band around $c_d/F \sim 0.3$ and the condition $A = 0$ corresponds to $c_d/F = 0.456$. As far as accuracy
is concerned, one learns that predictions from the quartic and quadratic equations deviate very little. The approximate algebraic solutions are less precise but, nevertheless, describe well the qualitative features of the results in the whole range considered and are reasonably precise in the region of empirical interest. The accuracy of eqs.(9-10) could be improved by keeping more terms in series expansions, but this would just yield larger expressions, without further conceptual gains.

| $c_d/F$ | quartic eq.(4) | quadratic eq.(5) | analytic eq.(9) | simplified eq.(11) |
|---------|----------------|------------------|---------------|-------------------|
| 0       | 1.2000         | 1.2000           | 1.2000        | 1.2000            |
| 0.1     | 1.2351 $-i$ 0.0269 | 1.2357 $-i$ 0.0272 | 1.2394 $-i$ 0.0269 | 1.2300 $-i$ 0.0175 |
| 0.2     | 1.3135 $-i$ 0.0898 | 1.3155 $-i$ 0.0918 | 1.3257 $-i$ 0.0874 | 1.3371 $-i$ 0.0758 |
| 0.3     | 1.5346 $-i$ 0.2501 | 1.5346 $-i$ 0.2500 | 1.5513 $-i$ 0.2245 | 1.6050 $-i$ 0.2022 |
| 0.4     | 2.4694 $-i$ 0.6287 | 2.4644 $-i$ 0.6327 | 2.4566 $-i$ 0.6408 | 2.5521 $-i$ 0.5534 |
| 0.5     | 0.8660 $+i$ 2.7361 | 0.8586 $+i$ 2.7414 | 1.0882 $+i$ 2.8707 | 0.9040 $+i$ 2.8315 |

| $c_d/F$ | quartic eq.(4) | quadratic eq.(5) | analytic eq.(10) |
|---------|----------------|------------------|-----------------|
| 0       | 0.7908 $-i$ 0.5294 | 0.7908 $-i$ 0.5294 | 0.7908 $-i$ 0.5294 |
| 0.1     | 0.8064 $-i$ 0.5109 | 0.8014 $-i$ 0.5173 | 0.8001 $-i$ 0.5242 |
| 0.2     | 0.8458 $-i$ 0.4819 | 0.8390 $-i$ 0.4879 | 0.8206 $-i$ 0.4848 |
| 0.3     | 0.8904 $-i$ 0.4085 | 0.8908 $-i$ 0.4126 | 0.8586 $-i$ 0.4215 |
| 0.4     | 0.8756 $-i$ 0.3228 | 0.8837 $-i$ 0.3165 | 0.9189 $-i$ 0.3391 |
| 0.5     | 0.8440 $-i$ 0.2736 | 0.8576 $-i$ 0.2575 | 1.0041 $-i$ 0.2459 |

Conceptually, the behaviours of $\sqrt{s_+}$ and $\sqrt{s_-}$ shown in the tables are strikingly different. The latter is a rather slow-varying function, whereas the former changes rapidly and even has the sign of the imaginary part reversed in the last row. In order to understand this
behavior of $\sqrt{s_+}$, we keep just the leading term in eq.(9) and find

$$s_+ = \frac{1}{A} \left\{ \frac{5}{8} m_R^2 - i \frac{384\pi}{5} c_d^2 \right\}.$$  

(11)

In spite of its simplicity, this result yields quite reasonable predictions, as indicated in the last column of table I. Moreover, it shows clearly that the major features of $s_+$ are determined by the factor $A$ in the denominator. In particular, it explains the change in the sign of the imaginary part of $\sqrt{s_+}$ observed in the bottom line of the table, since $A$ vanishes at $c_d/F = 0.456$. As this factor does not occur in $s_-$, table II is much more monotonic.

The following scenario is supported by eqs.(9-10):

- in case the resonance $R$ is absent, one has just the pole $\sqrt{s_-}$, which is due to the leading contact interaction;
- if the resonance is present, but its couplings to mesons are not turned on ($c_d = c_m = 0$), one has the pole $\sqrt{s_-}$ and a bound state in the real axis at $s = m_R^2[26]$;
- when the parameters $c_d$ and $c_m$ are turned on and the resonance $R$ couples to $\sqrt{s_-}$, both the mass and width of $\sqrt{s_+}$ increase monotonically, driven by the factor $A$ in the denominator;
- as a consequence, for realistic values of $c_d$, one has necessarily $\Re\sqrt{s_+} > m_R$ and the pole on the complex plane has to stand on the right of the point at which the phase shift passes through $\pi/2$;
- the pole $\sqrt{s_+}$ blows up at the critical value $c_d/F = \sqrt{5/24}$ and, beyond this point, just $\sqrt{s_-}$ is present;

Although the couplings of the resonance $R$ to mesons is implemented by both $c_d$ and $c_m$, the former is much more important than the latter, for it is incorporated into the parameter $A$. The term proportional to $c_m$ is less relevant, because it contains meson masses and vanishes in the chiral limit. For instance, if one chooses $c_d/F = 0.3$ and sets $c_m = 0$, the solutions of the quadratic equation become $\sqrt{s_+} = 1.3631 - i 0.1973$ GeV and $\sqrt{s_-} = 0.9971 - i 0.4835$ GeV. This means that gentle variations of $c_m$ around empirical averages would have little influence over pole positions.

In figs. 3 and 4, predictions from the full model for $E = \sqrt{s}$, given by eq.(1), are compared with results from K-matrix and the quadratic approximations. Quartic and quadratic approximations cannot be distinguished visually and the former is not shown. Inspecting these figures, one learns that the inclusion of the pion mass is not numerically important, but off-shell effects in the two-meson propagator do influence the positions of the poles and
tend to decrease both masses and widths. Nevertheless, it does not alter the qualitative features of the scenario discussed above.

![Graph](image)

FIG. 3: (Color online) Real (full lines) and imaginary (dashed lines) components of the function $E_\pm = \sqrt{s_{\pm}}$; the full curve comes for eq(1), whereas those labelled $K$ and $q$ correspond to the $K$-matrix and quadratic approximation.

Our results suggest that the pole $\sqrt{s_{+}}$ can be identified with the $K_0^*(1430)$. The fitting of both the mass and width of this state supplies two constraints for the resonance parameters. As $c_m$ has little influence over numerical results, we fix it at the value $c_m = 0.042$ GeV and choose $m_R$ and $c_d$ so that $\sqrt{s_{+}} = [(1.414 \pm 0.006) - i(0.145 \pm 0.010)]$ GeV [27]. We then find $c_d/F = 0.2705 \pm 0.0078$ , $m_R = 1.1865 \pm 0.079$ GeV and $\sqrt{s_{-}} = (0.7505 \pm 0.0010) - i(0.2363 \pm 0.0023)$ GeV. It is interesting to compare our results with those from the much more complete analysis by Jamin, Oller and Pich[8]. Although there are no data in the
FIG. 4: (Color online) Real (full lines) and imaginary (dashed lines) components of the function \( E_+ = \sqrt{s_+} \); the full curve comes for eq(1), whereas those labelled \( K \) and \( q \) correspond to the K-matrix and quadratic approximation.

\((J,I) = (0,1/2)\) channel for phase shifts around \( \pi/2 \), parametrizations produced in their paper suggest that \( m_R \sim 1.32 \) GeV and our value misses this point by 10%. On the other hand, by fixing the mass of the \( K_0^* \) and imposing \( m_R = 1.32 \) GeV, we find \( c_d/F = 0.1640 \), \( \sqrt{s_-} = 0.7419 - i \ 0.2541 \) GeV, and \( \Gamma_{K_0^*} = 0.1035 \) GeV, which is too small. In both cases, the values of \( c_d/F \) are below those of ref.[15]. The difficulty of fitting simultaneously \( m_R \) and \( \Gamma_{K_0^*} \) may be associated with the fact that we did not include inelasticities present in the region \( s > 1 \) GeV.

For the sake of completeness, in table III, we compare our result, \( \sqrt{s_-} = 0.751 - \)
$0.236$ GeV, with the positions of the $\kappa$ pole obtained in previous works.

### TABLE III: Pole position of the $\kappa$, in GeV.

| year | ref. | $\kappa$       |
|------|------|-----------------|
| 1986 | [16] | $0.727 - i 0.263$ |
| 1997 | [17] | $0.905_{-0.30}^{+0.65} \pm i 0.222_{-0.35}^{+0.15}$ |
| 1998 | [18] | $0.911 - i 0.158$ |
| 1999 | [19] | $0.779 - i 0.330$ |
| 2000 | [8]  | $0.708 - i 0.305$ |
| 2002 | [2]  | $0.721 \pm 19 \pm 44 - i 0.292 \pm 21 \pm 44$ |
| 2003 | [20] | $0.750_{-0.55}^{+0.30} - i 0.342 \pm 60$ |
| 2004 | [21] | $0.750 \pm 18 - i 0.226 \pm 11$ |
| 2006 | [22] | $0.694 \pm 53 - i 0.303 \pm 30$ |
| 2006 | [23] | $0.658 \pm 13 - i 0.279 \pm 12$ |
| 2006 | [24] | $0.841 \pm 23_{-0.55}^{+0.64} - i 0.306 \pm 26_{-0.44}^{+0.37}$ |
| 2008 | [25] | $0.772 - i 0.281$ |

### IV. SUMMARY

In this work we have discussed a model for the $K\pi$ amplitude in the $(J,I) = (0,1/2)$ channel, suited to energies up to 1.5 GeV. It is aimed at being used in data analyses of processes such as $D^+ \rightarrow K^-\pi^+\pi^+$ and given by

$$T_{1/2} = \frac{\gamma^2}{(m_R^2 - s) - i \gamma^2 \rho/(16\pi)}, \quad (12)$$

$$\gamma^2 = \left( s - M_K^2 \right) \left[ \frac{5}{8} s + \frac{3}{8} M_K^2 \right] \left( m_R^2 - s \right) + s \frac{c_d}{F} \left[ \frac{3}{F} c_d \left( s - M_K^2 \right) + \frac{4}{F} c_m M_K^2 \right].$$

It represents a compromise between simplicity and the essential phenomena of this channel. The factor $(s - M_K^2)$ in $\gamma^2$ is the Adler zero, which arises naturally in the framework of chiral
symmetry. It allows automatically for one complex pole, identified with the \( \kappa \). Depending on the values obtained for the free parameters \( m_R, c_d \) and \( c_m \), another pole can be present, associated with the \( K_0^* \).

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