Comparison of effectiveness of models for solving fuzzy assignment problem

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Abstract. The task of assigning particular task to a particular person can be solved using assignment problem. If the problem is considered under fuzzy environment, then uncertainty can be minimized which is usually involved in human complex problem. There are various models for solving fuzzy assignment problem. In this paper, the triangular fuzzy assignment problem is formulated. It is then solved by the fuzzy Hungarian method, Hungarian and direct method. The results are obtained and comparison of optimal solution among these methods is done for the effectiveness of the method. The method is illustrated by numerical example based on real life data.

1. Introduction

An assignment problem is a special type of linear programming problem whose main purpose is to provide optimum assignment for allocating given number of tasks to equal number of resources. There are many applications of assignment problem in various sectors. They can be successfully used to solve various human resource problems like a problem of assigning a particular job to particular person or allocating sale region to particular salesperson and many such more. These decision making problems are often solved on the basis of vague information. Therefore, it becomes essential to solve these assignment problems using fuzzy concept for more reliable solution.

The concept of fuzzy theory was first introduced by Zadeh (Zadeh, 1965). A new method was proposed by (Kumar, Gupta, & Kaur, 2009) for providing solution to triangular fuzzy assignment problem which is represented using triangular fuzzy number. The results are discussed with advantages of using the method in real life situations. Kadhivel K and Balamurugan K (Kadhivel & Balamurugan, 2012) proposed fuzzy jobs and workers by Hungarian assignment problem. The problem is solved by using Hungarian and Robust ranking method for triangular and trapezoidal fuzzy number. They concluded that fuzzy assignment problem can be well managed by fuzzy ranking method.

A systematic procedure of Hungarian ones assignment is applied to find the optimal solution to fuzzy assignment problem by (Kalairasi, Sindhu, & Arunadevi, 2014). They suggested that the results obtained are optimal and the method can be applied to all type of assignment problem. Jatinder Pal Singh and Neha Ishesh Thakur (Singh & Thakur, 2015) solved the fuzzy assignment problem by
fuzzy Hungarian method without defuzzification process. D. Anuradha [Anuradha, 2015] described plane point method for finding the solution to fuzzy solid assignment problem with Robust ranking method.

A Lexisearch algorithm is discussed by Yadaiah & Haragopal, 2016) for solving unbalanced assignment problem. They showed that the method outperforms as compared to existing methods. S. Narayananamoorthy and P. Vidhya [Narayananamoorthy & Vidhya, 2017] used branch and bound technique for fuzzy assignment problem represented by trapezoidal fuzzy number and defuzzification is done by Robust ranking method. The new approach of subtract row and add one is given by Humayra Dil Afroz and Mohammad Anwar Hossen [Afroz & Hossen, 2017]. The optimal solution of the proposed method is compared with optimal solutions obtained by Hungarian and MOA method. S. Muruganandam and K. Hema [Muruganandam & Hema, 2017] suggested Fourier elimination method for getting optimal assignment. The defuzzification is done using graded mean integration representation method. The method is also illustrated with numerical example.

The fuzzy assignment problem is solved by the Hungarian algorithm where centroid ranking method is used as a defuzzification technique [Mary & Selvi, 2018]. A fuzzy based amalgamated technique was proposed by Anju Khandewal [Khandelwal, 2019] where task allocation problem is examined and solved with fuzzy performance time and fuzzy communication time.

This paper is organized as follows: In section 2, some basic definitions and operations are given. In section 3, the methodology is given for the methods, where formulation and solution to the fuzzy assignment problem is provided. This will be followed by results and discussion. Lastly, section 5 contains conclusion which is then followed by references.

2. Some Basic Definitions and Operations

2.1. Fuzzy Set

The set which is defined by a following characteristic function is fuzzy set and it is stated as,

\[ \mu_A : X \rightarrow [0,1] \]

\[ \mu_A(x) = \begin{cases} 
1, & \text{if } x \text{ is totally in } A \\
0, & \text{if } x \text{ is not in } A \\
(0,1), & \text{if } x \text{ is partially in } A 
\end{cases} \]

2.2. Triangular Fuzzy Number

A fuzzy number \( \tilde{A} = (a_1, a_2, a_3) \) on \( R \) defined by membership function (2.1) is considered as triangular fuzzy number and its membership function is,

\[ \mu_\tilde{A}(x) = \begin{cases} 
\frac{x - a_1}{a_2 - a_1}, & \text{if } a_1 \leq x \leq a_2 \\
1, & \text{if } x = a_2 \\
\frac{a_3 - x}{a_3 - a_2}, & \text{if } a_2 \leq x \leq a_3 \\
0, & \text{Otherwise} 
\end{cases} \] (2.1)

where, \( a_1, a_2, a_3 \in R \).

Here, \( a_2 \) represents the middle value or midpoint, \((a_3 - a_1), (a_3 - a_2)\) represent the left, right spread of triangular fuzzy number \( \tilde{A} = (a_1, a_2, a_3) \).

2.3. Some Basic Operations on Triangular Fuzzy Number
If \( \tilde{A} = (a_1, a_2, a_3) \) and \( \tilde{B} = (b_1, b_2, b_3) \) are two positive triangular fuzzy numbers and \( k \) is a positive real number, then various arithmetic operations defined on these triangular numbers are given as follows:

\[
\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3) \tag{2.2}
\]
\[
\tilde{B} - \tilde{A} = (-b_1, b_2 - a_2, b_3 - a_3) \tag{2.3}
\]
\[
\tilde{A} \times \tilde{B} = \{\min(a_1 b_1, a_2 b_2, a_3 b_3), a_2 b_2, \max(a_1 b_1, a_2 b_2, a_3 b_3)\} \tag{2.4}
\]
\[
\tilde{A} \div \tilde{B} = \{\min(a_1 / b_1, a_2 / b_2, a_3 / b_3), a_2 / b_2, \max(a_1 / b_1, a_2 / b_2, a_3 / b_3)\} \tag{2.5}
\]

2.4. Defuzzification

The process of finding singleton value as the average value of triangular fuzzy number is defuzzification. The magnitude ranking method is taken as a defuzzification process. Because of simplicity and accuracy, the method of magnitude of ranking is convenient to use ([Selvi, Mary, & Velemmal, 2017]). The magnitude of triangular fuzzy number \( (a_1, a_2, a_3) = (a_0 + a_+, a_-) \) is given by equation (2.8) as

\[
\text{Mag}(a_1, a_2, a_3) = \frac{1}{2} \left( (a_+^* + 3a_0 - a_-) f(r) dr \right) \tag{2.8}
\]

Here, for a convenience, non-negative and increasing function \( f(r) \) on \([0,1]\) is taken as \( r \).

3. Methodology

In this section, fuzzy Hungarian method, Hungarian method and direct methods are used for solving the formulated triangular fuzzy assignment problem.

3.1. Formulation of the problem

The allocation of persons to different tasks based on their performances against these tasks is done using formulated maximal fuzzy assignment. The data is collected from one of the reputed organizations. Four persons are taken as P1, P2, P3 and P4 along rows and four tasks are taken as T1, T2, T3 and T4 along columns. The maximal fuzzy assignment problem is given as

\[
\begin{bmatrix}
(20, 26, 32) & (14, 20, 26) & (16, 22, 28) & (14, 20, 26) \\
(18, 24, 30) & (20, 26, 32) & (16, 22, 28) & (18, 24, 30) \\
(10, 16, 22) & (14, 20, 26) & (14, 20, 26) & (10, 16, 22) \\
(16, 22, 28) & (12, 18, 24) & (16, 22, 28) & (10, 16, 22)
\end{bmatrix} \tag{3.1}
\]

The fuzzy problem is converted into crisp problem with magnitude ranking method. We calculate magnitude of all triangular fuzzy numbers using equation (2.8). From equation (2.8), we write,

\[
\text{Mag}(20, 26, 32) = \frac{1}{2} \left( (32 + 3(20) - 26) r \right) dr = 16.5 \tag{3.2}
\]
In this way, we calculate all the magnitudes of remaining fuzzy numbers and we get crisp assignment problem as,

$$\text{Mag}(14, 20, 26) = \frac{1}{2} \int (26 + 3(14) - 20) r \, dr = 12 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.3)$$

3.2. Solution by Hungarian method

The steps of Hungarian method ([Sharma & Sharma]) is given as follows:

Step 1: Consider the balanced assignment problem of order n. If it is unbalanced then convert it into balanced problem. Subtract smallest element of each row from each element of that respective row. This will result in creation of at least one zero in each row.

Step 2: Get the first modified matrix by subtracting the smallest element of each column from that column.

Step 3: We find now the possibility of optimal assignment. Cover all the zeros in the matrix obtained after step 2 by drawing minimum number of horizontal and vertical lines. We have 2 cases:

(i) Make a zero assignment, if the number of straight lines is equal to the number of rows (or columns) of the matrix

(ii) Go to next step if number of straight lines is less than number of rows of the matrix.

Step 4: Subtract the smallest element of the matrix which is not covered by above lines from all elements of the matrix and add this smallest element to the intersection of horizontal and vertical lines.

Step 5: Now, repeating the step 3 and 4 until optimal assignment is obtained.

Step 6: Make the assignment by examining the row which has exactly one zero and marking that zero with a box. Simultaneously, crossing off all the zeros in the column of the mark zero. Do this until all the rows are done and repeat the same process for all the columns.

Step 7: If we find more than one of the unmarked zeros in any row or column, then mark one of the unmarked zeros arbitrarily with box and mark cross in the cells of remaining zeros in its row and column. Repeat the process, until no unmarked zero is left in the matrix.

We use this Hungarian algorithm to get optimal assignment to the problem (3.2). We first convert maximal assignment problem to minimization by subtracting each number of the matrix from the highest number of the matrix, we get from (3.2),

$$\begin{bmatrix}
16.5 & 12 & 13.5 & 12 \\
15 & 16.5 & 13.5 & 15 \\
9 & 12 & 12 & 9 \\
13.5 & 10.5 & 13.5 & 9
\end{bmatrix} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.4)$$

After applying the Hungarian algorithm to above matrix, we get matrix....
Here, fuzzy optimal assignment by Hungarian method is given as:

\[ P_1 - T_1, P_2 - T_4, P_3 - T_2, P_4 - T_3 \]

Fuzzy optimal Value = \( (20, 26, 32) + (18, 24, 30) + (14, 20, 26) + (16, 22, 28) = (68, 92, 116) \)

By magnitude ranking method,

\[ Mag(68, 92, 116) = 57 \]

### 3.3 Solution by Direct Method

The direct method [(Seethalakshmy & Srinivasan, 2016)] steps are:

1. Consider assignment problem which is balanced. Subtract each row from maximum element of that row to get new matrix.
2. Now, we identify for each column, the zero position of (i,j)th place and make assignment where zero has unique position. Here, the corresponding row and column will be deleted simultaneously. Do this, till we get the final optimal assignment.
3. Find the value of nest successor of zero is some rows have same column and do allocation to the row where we find successor with maximum value. If again the tie is observed for these maximum values, then get next to next successor of zero and assignment is made for value which is maximum.
4. Subtract row minimum from that particular row in the reduced matrix if each row is not having at least one zero.
5. Get the optimal assignment by going repeatedly through step 3 to 5.

We can directly use this technique to the maximal assignment problem. It doesn’t require any conversion to minimization problem. After applying this direct method to the above constructed assignment problem (3.2), we get same optimal assignment as,

\[ P_1 - T_1, P_2 - T_4, P_3 - T_2, P_4 - T_3 \]

Fuzzy optimal Value = \( (20, 26, 32) + (18, 24, 30) + (14, 20, 26) + (16, 22, 28) = (68, 92, 116) \)

By magnitude ranking method,

\[ Mag(68, 92, 116) = 57 \]

### 3.4 Solution by Fuzzy Hungarian Method

The steps of Fuzzy Hungarian method [(Singh & Thakur, 2015)] is given as:

1. Consider the balanced fuzzy assignment problem of order \( n \). If it is unbalanced then convert it into balanced fuzzy assignment problem by adding dummy source or dummy destination with entries as fuzzy zero. Then subtract row minimum from that respective row.
2. Next, subtract each column minimum from that respective column. With this step, we find at least one fuzzy zero in each row and in each column.
3. Make the assignment, where row has single fuzzy zero and crossing all other fuzzy zeros in its column. Do this for all rows successively and then for all columns until final optimal assignment is obtained. If a row and/ or a column have two or more fuzzy zeros, assign arbitrary any one of these arbitrary fuzzy zeros and cross off all the other fuzzy zeros of that row or column.
4. If the order of the matrix is equal to number of assignments then optimal assignment is done, else go to step 5.
Step 5: Cover all the fuzzy zeros in reduced matrix by drawing minimum number of horizontal and vertical lines.

Step 6: Identify the smallest fuzzy element of the matrix which is not covered by these lines and subtract it from all the uncovered elements of the fuzzy matrix and also subtract it from itself. At the same time, add it to the intersection of horizontal and vertical lines.

Step 7: Get the optimal assignment by repeating step 3 to 6.

We use this method to solve the problem (3.1). After converting the problem (3.1) to minimization problem, we get

\[
\begin{bmatrix}
0,0,0 & 6,6,6 & 4,4,4 & 6,6,6 \\
2,2,2 & 0,0,0 & 4,4,4 & 2,2,2 \\
10,10,10 & 6,6,6 & 6,6,6 & 10,10,10 \\
4,4,4 & 8,8,8 & 4,4,4 & 10,10,10 \\
\end{bmatrix}
\] (3.7)

After applying the Fuzzy Hungarian algorithm to above matrix, we get matrix

\[
\begin{bmatrix}
[0,0,0] & 6,6,6 & 4,4,4 & 4,4,4 \\
2,2,2 & 0,0,0 & 4,4,4 & [0,0,0] \\
4,4,4 & [0,0,0] & 0,0,0 & 2,2,2 \\
0,0,0 & 4,4,4 & [0,0,0] & 4,4,4 \\
\end{bmatrix}
\] (3.8)

Here, fuzzy optimal assignment by Fuzzy Hungarian method is given as

P₁-T₁, P₂-T₄, P₃-T₂, P₄-T₃

Fuzzy optimal Value= (20,26,32) + (18,24,30) + (14,20,26) + (16,22,28) = (68,92,116)

By magnitude ranking method,

\[\text{Mag}(68,92,116) = 57\]

4. Results and Discussion

In this paper, the modification of the paper [(Singh & Thakur, 2015)] is done. They have applied and compared two techniques of Fuzzy Hungarian method and Hungarian Matrix ones assignment method for the fuzzy assignment problem. They concluded that fuzzy Hungarian method outperforms the other method. We have done here the comparison of three models of solving fuzzy assignment problem. Four tasks T₁, T₂, T₃, T₄ are allocated to four persons P₁, P₂, P₃, P₄ in an optimum manner using fuzzy Hungarian method, direct method and Hungarian method.

For Hungarian and direct method, the fuzzy assignment problem is defuzzified using magnitude ranking method and then solved by respective algorithms. By all three methods, optimal assignment is obtained as same. The person P₁ is assigned task T₁, person P₂ is assigned task T₄, person P₃ is assigned task T₂ and person P₄ is assigned task T₃ with an optimal value of 57. It is also observed that, Hungarian and fuzzy Hungarian method requires conversion of the given maximal fuzzy assignment problem into minimization assignment problem, whereas direct method can be directly applied to the maximal assignment problem. Therefore, it can be said that the direct method is more convenient to use as compared to rest of the two methods.

5. Conclusion

In this paper, the maximal fuzzy assignment problem is used for the allocation of four tasks to four different persons. The task allocation problem using three models of Hungarian method, Fuzzy Hungarian method and direct method is solved with success. The comparison of the results of these three methods is done. Magnitude ranking method is used for the defuzzification process. The same
optimal assignment is obtained by all the methods and right task is allocated to right person. The results are also confirmed with the experts of the field. It is our observation that the direct method doesn’t require any conversion of maximal assignment problem into minimization assignment problem as in case of Hungarian and fuzzy Hungarian method. So, direct method is more convenient to use. The fuzzy assignment problem offers more relevant solution to the decision making problem by considering vague information.

6. References

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