Transformer hot spot temperature prediction based on basic operator information

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A B S T R A C T

A power transformer is an important component in power trains and electrical distribution networks. Predicting its life time is desirable, especially, if a zero downtime policy is applied. However, end customers often have to deal with a lack of information and cannot always use the established methods for life time prediction. Therefore, the present paper provides an alternative way to calculate the hot spot temperature and thus, the life time of power transformers based on limited information, i.e. transformer rating information and rms current and voltage measurements (including phase angles). The transformer hot spot temperature is derived from the transformer losses and a virtual twin. Therefore, the paper provides methods i) to evaluate the separate transformer losses, i.e. core, winding and stray losses, ii) to create a simple virtual transformer twin and iii) to calculate the temperature distribution in the transformer windings and thus, the hot spot temperature. The methods are applied to one phase of a 154 kV, 15MVA power transformer. It is shown that the calculated losses and hot spot temperature matches with winding measurements available in literature.

1. Introduction

Surveys of failures in wind turbine systems evaluated during the last decades have shown that power transformers have non-negligible failure rates and downtimes [1]. Although these failure rates and downtimes are relatively low in comparison to other power train components, the recent trend to a zero downtime policy also requires the consideration of the power transformer. To achieve maintenance with zero or, at least, close to zero downtime maintenance prediction becomes important. This means that end customers (transformer owners and operators) or consultancy companies seek for methods to evaluate the remaining useful life time based on actual operating conditions (see e.g. [2;3] for a similar approach for bearings). However, these two stakeholders normally deal with the problem that only limited information about the component and actual operating conditions are available to them. Therefore, methods are needed that can predict the remaining useful life time of components based on limited information, for example, based on component ratings information and simple measurements only.

To predict the transformer life time models provided by IEEE Std C57.91 [4], IEEE Std C57.100 [5] and IEC 60,354 [6] are used [7]. Alternative and improved models are proposed by [8,9,10,11]. These models calculate the insulation’s aging rate as a function of its temperature [7], since the temperature is the basic factor affecting the thermal loss of life [7]. This means that the highest temperature (also called hot spot temperature) to which the insulation is exposed is used to predict the transformer life time. The relation of insulation deterioration to changes in time and temperature is specified by the ageing acceleration factor $F_{AA}$ [7]:

$$F_{AA} = \left[ \frac{\theta_{HS,ref} + 273}{15000} \right] \left[ \frac{15000}{\theta_{HS} + 273} \right]$$

where $\theta_{HS,ref}$ is a hot spot temperature of 110 °C.

The actual transformer life time ($L$) is then obtained by dividing the normal insulation life $L_n$ (at 110 °C) by $F_{AA}$:

$$L = \frac{L_n}{F_{AA}}$$

Alternatively, following [7], the Loss of Life ($LoL$) in a certain period of operation time ($t_{op}$) yields the fraction of the normal insulation life that is consumed in $t_{op}$. This allows to calculate the remaining useful life of the transformer for scenarios in which the hot spot temperature is not constant over time.

$$LoL = \frac{F_{AA} t_{op}}{L_n}$$

Hence, to quantify the life time, the hot spot temperature $\theta_{HS}$ (°C) in
the transformer windings must be calculated. It can be estimated from (oil) temperature measurements using IEEE / IEC loading guides [4,5,6] as follows:

$$\theta_{HS} = \theta_{TO} + K \cdot \Delta \theta_{HR} \left( \frac{I}{I_R} \right)^{2m}$$

(4)

where $\theta_{TO}$ (°C) is the oil temperature measured at the top of the transformer and $\Delta \theta_{HR}$ (°C) is the temperature difference between the hot spot and the top oil temperature as measured during a heat run test at the rated power. The winding exponent $m$ is an empirical parameter recommended by the loading guide. Then, based on these measurements, the actual load current $I$ and the rated current $I_R$, the hot spot temperature is interpolated (cp. [12,13]). In addition to the interpolation, a correction factor $K$ is used which depends on the transformer design [13]. A modified approach is proposed by [8] and [14] taking into account heat convection respectively environmental variables. However, calculating the hot spot temperature according to IEEE / IEC loading guides always requires information about the correction factor (transformer design) and measured temperature drops (e.g. from winding to oil) for rated load conditions. An end customer or consultancy company may not have access to this kind of information, which means that they cannot apply this method to evaluate the transformer hot spot temperature and remaining life. Furthermore, the winding exponent is an empirical factor and the design dependent correction factor value given in the standard tends to be too small as is shown by actual winding temperature measurements [15].

Further, the transformer temperature depends on the actual losses generated in the transformer. This means that for determining the winding temperature the separate transformer core and winding losses are needed. The transformer losses can be estimated by the standard IEEE C57.120 [16]. However, to evaluate the actual winding temperature the actual losses are needed, i.e. the losses must be evaluated based on measurements. Arri et al. [17] provide an online loss measurement of transformers requiring an extensive measurement effort. An improved real time monitoring method of the transformer losses is developed by Lin and Fuchs [18] distinguishing between iron core and copper losses. They combine eddy current and hysteresis losses to quantify the iron core losses and use ohmic winding and stray losses to determine the copper losses. However, a further separation between ohmic winding and stray losses is necessary because the stray losses occur in different transformer parts [19] and thus, have only minor effect on the winding temperature. The temperature is therefore mainly affected by ohmic winding losses.

It can thus be concluded that scientific literature i) evaluating separately transformer core, stray and ohmic winding losses based on voltage and current measurements and ii) calculating the winding hot spot temperature based on these transformer losses, i.e. without design dependent correction factors and measured temperature drops seems to be very limited. Simple online monitoring methods for specifying the different transformer losses and estimating the winding hot spot temperature based on the actual losses could not be found at all. Therefore, the main objective of this paper is to propose methods to evaluate i) the actual separate transformer losses, i.e. core, stray and ohmic winding losses and ii) the actual winding hot spot temperature. The proposed methods use measured oil temperature and measured (input and output) rms voltage and current, including their phase angles, as this information is readily available, also to owners and service providers. Further, a simple virtual twin of the transformer is utilized to estimate the temperature distribution in the transformer windings and thus, to determine the winding hot spot temperature. The virtual twin is created based on only transformer nameplate information which is typically available on the transformer nameplate and therefore available for any operator. Fig. 1 provides an overview of the transformer loss and winding hot spot temperature calculation. In section 2 the calculation of the losses and the estimation of the transformer dimension for the simple virtual twin are discussed. Then, the winding temperature distribution is determined in section 3. Finally, in section 4 the loss and temperature calculations are applied to a case study and validated with measurements available in literature as well as with the hot spot temperature evaluated by the IEEE / IEC loading guides.
2. Evaluation of transformer losses and creation of the virtual twin

In this section, the calculations of the different transformer losses, i.e. ohmic winding, core and stray losses, are derived and discussed in details. To simplify the calculations one transformer phase is considered. A general transformer model including losses is analyzed in section 2.1. The calculation of ohmic resistances as well as the core hysteresis needed for the loss calculation are discussed in section 2.2. Then, the calculation procedure to evaluate the transformer losses is explained in section 2.3. Finally, the creation of the virtual transformer twin, i.e. the estimation of the transformer dimensions, is presented in section 2.4.

2.1. Transformer model

First, to simplify the theoretical considerations and calculations the common transformer phase where the primary (high voltage) winding is placed around the secondary (lower voltage) winding (cp. Fig. 2a) is approximated by a simple transformer with separate windings around the two sides of the transformer core (cp. Fig. 2b).

Then, to evaluate ohmic winding, core and stray losses, the simple transformer is considered as a real transformer. This means that the transformer shown in Fig. 2b must include the following characteristics of a real transformer [20]:

1. The permeability of the magnetic circuit is non-infinite, i.e. \( \mu_{Fe} \neq \infty \) and also is a function of the magnetic field strength \( H \), i.e. \( \mu_{Fe} = f(H) \). This means that the relation between the magnetic field \( B \) and magnetic field strength \( H \) is not linear anymore, i.e. a hysteresis and thus core losses occur as will be discussed in more detail in section 2.2. Further, considering the eddy current \( i_{ed} \) induced in an iron core (cp. Fig. 2b), the magnetomotive force \( \theta \) is given with the core length element \( ds \) and total core length \( l \) as follows [20]:

\[
\theta = \oint H ds = Hl = i_p N_p + i_s N_s + i_{ed} \neq 0
\]  

(5)

Note that the magnetomotive force \( \theta \) is the cause of the magnetic flux in the magnetic circuit [21]. It is determined by the product of the number of turns \( N \) and the current \( i \) through the circuit.

2. The electrical conductivity of the magnetic circuit is non-zero, i.e. \( \kappa_{Fe} \neq 0 \). So eddy currents appear in the transformer core causing further losses. The magnitude of these losses are given by the eddy current \( i_{ed} \) and the resistance \( R_{ed} \) placed in the core (cp. Fig. 2b). Moreover, the magnetic flux of the transformer core \( (\phi_{core}) \) is obtained from the law of induction and Eq. (5),

\[
\phi_{core} = \int Bdal = \int \mu_{Fe}(H)Hdal = \frac{A}{4\pi} \mu_{Fe}(H)(i_p N_p + i_s N_s + i_{ed})
\]  

(6)

3. The permeability of the surrounding fluid (e.g. oil) is non-zero, i.e. \( \mu_{fluid} \neq 0 \). Hence, a field outside the core (stray filed) occurs causing the flux leakage \( (\phi_{leak}) \). It is assumed here that the considered transformer phase has separable stray fields (cp. Fig. 2b). This means that the flux leakage of the primary and secondary side does not affect each other and that the magnetic flux of the primary and secondary side is given by:

\[
\phi_p = \phi_{core} + \phi_{ps}
\]  

(7)

\[
\phi_s = \phi_{core} + \phi_{ps}
\]  

(8)

Note that a separation of the stray fields is normally not applicable for real transformers [20]. However, the assumption (of separable stray fields) is acceptable for the simple transformer shown in Fig. 2b and provides some calculation benefits as it will be shown in section 2.3.

- The electrical conductivity of the windings is not infinite, i.e. \( \kappa \neq \infty \). Thus, the resistance of the primary and secondary side winding has to be taken into account, i.e. resistance losses occur. Then, the primary \( (u_p) \) and secondary \( (u_s) \) side voltages are obtained from the current \( i \), the resistance \( R \), the magnetic flux linkage \( \psi \) respectively the number of turns \( N \) and magnetic flux \( \phi \).

\[
u_p = R_p i_{ps} + \frac{d\psi_p}{dt} = R_p i_{ps} + N_p \left( \frac{d\phi_{core}}{dt} + \frac{d\phi_{ps}}{dt} \right)
\]  

(9)

\[
u_s = R_s i_s + \frac{d\psi_s}{dt} = R_s i_s + N_s \left( \frac{d\phi_{core}}{dt} + \frac{d\phi_{ps}}{dt} \right)
\]  

(10)

In the same way, a voltage \( u_{ed} \) caused by eddy currents can be defined. The voltage \( u_{ed} \) is equal to zero because the eddy current is short-circuited in the transformer core (cp. also section 2.2). As the eddy currents are caused by the changing magnetic flux \( \phi \) over time this implies [20]:

\[
u_{ed} = 0 = R_{ed} i_{ed} + \frac{d\phi_{core}}{dt}
\]  

(11)

Note that there is a difference between the primary side current \( i_p \) and the transformer input current \( i_{ps} \) in Eq. (9). The input current \( i_{ps} \) contains both the primary side current \( i_p \), and the current \( i_{in} \) needed to magnetize the transformer core. The distinction between \( i_{ps} \) and \( i_p \) will be elaborated in section 2.3.

Further, based on Eqs. (5) and (9) to (11) the vector diagram\(^1\) of the (real) transformer (with separable stray fields) can be created. This is needed later for the loss calculations (in section 2.3) and is visualized in Fig. 3 for the transformer in Fig. 2b. Fig. 3a shows the voltages (Eqs. (9) to (11)) and Fig. 3b the currents (Eq. (5)). The voltages \( u_{core,p} \) and \( u_{core,s} \) coincide with the real axis and the core flux \( \phi_{core} \) with the negative imaginary axis. This is equivalent to the ideal transformer where the primary and secondary side voltages are i) parallel and ii) perpendicular to the magnetic flux in the vector diagram [20]. Hence, the core voltages \( u_{core,p} \) and \( u_{core,s} \) are defined (with \( \omega \) the fundamental angular frequency) as follows:

\[
u_{core,p} = N_p \frac{d\phi_{core}}{dt} = j\omega N_p \phi_{core}
\]  

(12)

\(^1\) Alternating quantities like currents and voltages can be represented by a (rotating) arrow in a vector diagram. The length of the arrow specifies the magnitude (i.e. the the rms value) of the alternating quantity. The phase (difference) is represented by the relative position of a vector with respect to another vector, i.e. by the angular position.
The voltage drops due to ohmic winding ($u_{R,p}$ and $u_{R,s}$) and stray ($u_{σ,p}$ and $u_{σ,s}$) losses are specified as:

$$u_{R,p} = R_p i_{R,p}$$

(14)

$$u_{R,s} = R_s i_{R,s}$$

(15)

$$u_{σ,p} = N_p \frac{dΦ_{σ,p}}{dt}$$

(16)

$$u_{σ,s} = N_s \frac{dΦ_{σ,s}}{dt}$$

(17)

By adding these losses to the (ideal) core voltages in the vector diagram, the resulting $u_p$ and $u_s$ are obtained (see Fig. 3b).

Then, the core losses are determined by the eddy currents and the hysteresis effect [20]. This is shown in Fig. 3b. In an ideal transformer the magnetomotive force $Θ$ is equal to zero and thus, the primary and secondary side currents are i) opposed to each other and ii) parallel in the vector diagram [20]. Therefore, the core losses are governed by the magnetomotive force $Θ$ and the eddy currents $i_{ed}$ causing a current drop and a phase shift between primary and secondary side current, $i_p N_p = -i_s N_s$ (cp. Fig. 3b). Note that additional losses in the transformer are caused, for example, by current displacement phenomena in the windings [20]. These losses are not explicitly considered here. On the other hand, they are already included in the winding, core and stray losses because the input and output voltages and currents are used for the loss calculation.

In addition, winding losses are calculated in a straightforward manner because they are given by the (measured) input ($i_{m,p}$) and output ($i_m,s$) current and by the ohmic resistances $R_p$ and $R_s$ (cp. Fig. 2b). The ohmic winding resistances can either be measured or taken from data sheets or estimated by calculations. However, the evaluation of the stray and core losses require some more effort. To specify the eddy current, the hysteresis effect and the stray voltages ($u_{σ,p}$ and $u_{σ,s}$) in the vector diagram shown in Fig. 3 must be solved. This will be demonstrated in section 2.3. But first, some fundamental considerations which are needed to solve the vector diagram are discussed in section 2.2.

### 2.2. Ohmic resistances, core hysteresis and stray field

In this section, the ohmic resistance ($R_p$, $R_s$) calculation, the hysteresis effect, i.e. the conversion of the magnetomotive force $Θ$ to the magnetic flux $Φ_{core}$ as well as the power losses are discussed. Simple analytical considerations and equations are used to determine them. The equations are developed for the simple transformer shown in Fig. 2b.

First, an ohmic resistance $R = \rho l / A$ is defined by the specific resistance $\rho$ [$Ω \cdot mm^2/m$], the length $l$ and the cross section $A$ of the ohmic conductor [22], i.e.:

$$R = \rho \frac{l}{A}$$

(18)

The specific resistance $\rho$ is determined by the conductor material (e.g. copper $\rho_{Cu} = 1.68 \times 10^{-8}  Ω \cdot m$ [23]). The length $l_{o}$ is given by the number of turns $N$ and the average winding circumferences, i.e. with the inner ($C_{in}$) and outer ($C_{out}$) winding circumferences:

$$l_{o} = \frac{C_{in} + C_{out}}{2} N$$

(19)

In the case of a squared ($C = 4h$) or circular ($C = 2\pi R$) winding, the length $l_{o}$ is defined with the edge length $h$ respectively radius $R$ as:

$$l_{o, square} = 2(h_{in} + h_{out}) N$$

(20)

$$l_{o, round} = \pi (R_{in} + R_{out}) N$$

(21)

The conductor cross section $A_{o}$ is obtained from the ratio between the rms current $I_{rms}$ and rms current density $J_{rms}$ (in $A/mm^2$) at rated operation.

$$A_{o} = \left( \frac{I_{o}}{J_{o}} \right)_{rms}$$

(22)
Then, the ohmic power losses are obtained from the (measured) transformer input current $i_{p,m}$ and output current $i_i$:

\[ P_{L,p} = R_{L,p}i_{p,m}^2 \]  

\[ P_{L,i} = R_{L,i}i_i^2 \]  

Second, before discussing the core hysteresis, some essential comments must be made first. A (laminated) transformer core with a magnetic field $B$ is considered (cp. Fig. 4a and 4b). Due to the magnetic field $B$ eddy currents occur in the transformer core respectively core laminates.

This means that eddy currents can be included in the hysteresis effect, i.e. the core hysteresis is modified so that the calculated hysteresis losses represent the total core losses. This is proposed by Müller and Ponick [20] and applied in this paper. To do so the effective magnetomotive force $\Theta^+$ is defined based on Eq. (5) as follows:

\[ \Theta^+ = \Theta - i_d = i_pN_p + i_iN_i \]  

(25)

This is shown in the vector diagram in Fig. 5 which is an extension of Fig. 3b, i.e. the magnetomotive force $\Theta^+$ is included. Further, Fig. 5 shows the magnetization current $i_{m,\text{mag}}$ hysteresis current $i_{hyst}$ and eddy current $i_{ed}$. The current $i_{hyst}$ could be used to describe the hysteresis losses. However, this is not further considered here.

Now, the hysteresis effect is elaborated. The magnetization and demagnetization of a steel transformer core follows a hysteresis (cp. Fig. 4a and 4b). Due to the magnetic field $B$ demagnetization of a steel transformer core follows a hysteresis (cp. Fig. 4a and 4b). Due to the magnetic field $B$ demagnetization of a steel transformer core follows a hysteresis (cp. Fig. 4a and 4b). Due to the magnetic field $B$ demagnetization of a steel transformer core follows a hysteresis (cp. Fig. 4a and 4b). Due to the magnetic field $B$ demagnetization of a steel transformer core follows a hysteresis (cp. Fig. 4a and 4b). Due to the magnetic field $B$ demagnetization of a steel transformer core follows a hysteresis (cp. Fig. 4a and 4b).

As the permeability $\mu_{Fe}$ and, on the other hand, by the permeability $\mu_{Fe}$ which is a function of the magnetic field strength $H$, i.e. $\mu_{Fe} = f(H)$ and, on the other hand, by the residual magnetism of the core material. As the permeability $\mu_{Fe}$ depends on the magnetic field strength $H$, the relation between the magnetic field $B$ and the magnetic field strength $H$ is nonlinear, i.e.:

\[ B = \mu_{Fe}(H)H \]  

(26)

This leads to the flattening of the hysteresis, i.e. magnetic core saturation, for higher values of $H$ (cp. Fig. 6). The residual magnetism causes a remaining magnetic field ($B$) at a magnetic field strength ($H$) equal to zero. This means that the magnetization and demagnetization curve of the transformer core differ and the hysteresis occur. It also means that the residual magnetism influences the enlargement of the hysteresis. Further, with Eq. (5) and (6) the hysteresis ($B$-$H$ diagram) can also be shown in a $\Phi$-$\Theta$ (magnetic flux - magnetomotive force) diagram (cp. Fig. 6).

As the magnetization and demagnetization curve of the transformer do not coincide (hysteresis effect), energy is dissipated in the transformer core removing the residual magnetism (every hysteresis cycle). This energy is equivalent to the area $A_{hyst}$ in Fig. 6. The dissipated energy of the hysteresis is calculated for the one (hysteresis) cycle based on Eq. (5) and (6) as follows [20]:

\[ E_{hyst} = A_l \int B dH = \int \Phi d\Theta \]  

(27)

Taking into account Eq. (25) the total energy lost in the transformer core during one cycle is given by:

\[ E_{L} = \int \Phi d\Theta^+ \]  

(28)

Then, with the number of cycles per second given by the angular frequency $\omega$ the total power losses in the core are:

\[ P_L = \frac{\omega}{2\pi}E_L = \frac{\omega}{2\pi} \int \Phi d\Theta^+ \]  

(29)

It can be seen that the magnetic flux $\Phi_{\text{core}}$ and magnetomotive force $\Theta^+$ must be known to calculate the total core losses. This means that Eq. (9), (10) and (25) must be solved. However, solving these equations is not a straightforward procedure. Its solution is shown in section 2.3.

Third, the stray fields occur in the transformer because the permeability of the materials and fluids surrounding the transformer core is non-zero (cp. section 2.1). This means that the stray fields are generated in different transformer components [19]. It also means that the stray losses are distributed over several parts in the transformer and that they rather influence the temperature of the entire transformer than the temperature of one particular transformer component. Consequently, the stray losses cannot be assigned to one specific transformer component like the windings. Therefore, in this paper the stray losses are not considered in the calculation of the winding hot spot temperature because i) the portion of stray losses occurring in the windings cannot be evaluated by simple methods and ii) the stray losses are significantly smaller than the ohmic winding losses [15,19] and thus are non-dominant for the winding hot spot temperature. Nevertheless, for the sake of completeness, the stray loss calculation is briefly shown here.

From Fig. 3a it can be seen that the stray fields cause a voltage drop given by the stray voltages $u_{dp}$ and $u_{ds}$. Due to the assumption of separable stray fields, the power losses are computable individually for the primary and secondary side. The power losses are defined by the product of stray voltage and current, i.e.:

\[ P_{L,p} = u_{p,d}i_p \]  

(30)

\[ P_{L,s} = u_{s,d}i_s \]  

(31)

Then, the total transformer losses are given with Eq. (23), 24 and Eq. (29) to (31):

\[ P_{loss} = P_{L,p} + P_{L,i} + P_{L,p} + P_{L,s} \]  

(32)

To verify the loss calculation result, i.e. Eq. (23), (24) and Eq. (29) to (31), the total transformer losses according to Eq. (32) must coincide with losses $P_{loss,m}$ evaluated by the measured input and output currents and voltages [18], i.e.:

\[ P_{loss,m} = i_{p,m}u_p - i_iu_i \]  

(33)

This will be demonstrated for a literature case in section 4.

2.3. Loss calculation procedure

Before explaining the calculation procedure used to evaluate the magnetic flux $\Phi_{\text{core}}$, magnetomotive force $\Theta^+$ and thus the total transformer losses, the data required for the calculation, i.e. the calculation inputs, are specified. To solve the voltage and current vector diagram (cp. Fig. 3), at least, the rms values of currents, voltages and phase angles are needed (cp. Fig. 7). This means that the following transformer input and output measurements are required:

- primary ($u_p$) and secondary ($u_s$) side voltages
- input current ($i_{p,m}$) and secondary (output) current ($i_s$)
- phase angle $\phi_i$ between $u_p$ and $i_p$
- phase angle $\phi_i$ between $i_{p,m}$ and $i_i$
- phase angle $\phi_i$ between $i_{p,m}$ and $i_i$
- phase angle $\phi_i$ between $i_{p,m}$ and $i_i$ (determined by $\phi_i$, $\phi_p$ and $\phi_s$)

An overview of the calculation procedure is provided in Fig. 8. The
magnetic flux $\Phi_{\text{core}}$ is calculated based on the vector diagram of voltages (cp. Fig. 3a) and the magnetomotive force $\Theta^+$ based on the vector diagram of currents (cp. Fig. 3b or 5). (Note that the definition of the angles $\phi$, $\chi$ and $\psi$ is given while presenting the calculation procedure for $\Theta^+$.)

The procedure sketched in Fig. 8 is as follows. First, the magnetic flux $\Phi_{\text{core}}$ is considered. The time derivative of the magnetic flux $d\Phi_{\text{core}}/dt$ must comply with both Eqs. (9) and (10). This means that the vector diagram of voltages visible in Fig. 9 is determinable (cp. also Fig. 3a). The voltages $u_p$ and $u_s$ as well as the phase angles $\phi_{u,p}$ (sum of $\phi_{u,p}$ and $\phi_{u,s}$), $\phi_p$ and $\phi_s$ are directly measured. The voltages $u_{R,p}$ and $u_{R,s}$ are specified by the measured current $i_{p,m}$ and $i_s$ and ohmic resistances $R_p$ and $R_s$ (cp. also Eqs. (14) and (15)). Further, for a linearized core permeability, i.e. $\mu_{Fe} = \text{const.}$ and sinusoidal currents with the fundamental angular frequency $\omega$, i.e.

$$i(t) = \hat{I} \sin(\omega t)$$

$$\frac{d\hat{I}}{dt} = \omega \hat{I} \cos(\omega t)$$

the time derivative of the (stray) flux is with Eq. (6) [20]:

$$\frac{d\Phi_{\text{s}}}{dt} = \frac{d\hat{I}}{dt} j\omega \hat{I}$$

This means that in the vector diagram the stray voltages $u_{s,p}$ and $u_{s,s}$ are perpendicular (phase shift of $\pi/2$ due to $j\omega$) to the currents $i_p$ and $i_s$ (cp. Fig. 9). Note that this is valid for ideal conditions and thus, implies that an approximation is made for the real transformer.

Further, to solve the voltage vector diagrams directly, it is assumed that the phase angle $\phi_{s,p}$ is equal to zero ($\phi_{s,p} = 0$). This means that the voltages $u_p$ and $u_{\text{core},p}$ are parallel and that the phase angle $\phi_{s,s}$ is equal to the measured voltage phase angle $\phi_s$. It also means that the minimum stray voltage $u_{s,p}$ and the maximum core voltage $u_{\text{core},p}$ are calculated. With the measured voltage $u_p$ current $i_{p,m}$ phase angle $\phi_p$ and ohmic resistance $R_p$ this yields:

$$u_{s,p} = u_{s,p} \tan(\phi_p) = R_{p,m} n \tan(\phi_p)$$

$$u_{\text{core},p} = u_p - u_{R,s} = u_p - \sqrt{u_{k,p}^2 + u_{s,p}^2} = u_p - R_p i_{p,m} \sqrt{1 + (\tan(\phi_p))^2}$$

Moreover, the core voltages $u_{\text{core},p}$ and $u_{\text{core},s}$ are equivalent to the primary and secondary side voltages of an ideal transformer which are perpendicular to the magnetic flux $\Phi_{\text{core}}$ [20]. This means that the ratio of the core voltages $u_{\text{core},s}$ is given by the voltage $u_{\text{core},p}$ and the transformer turns ratio $n = N_s / N_p$.

$$u_{\text{core},s} = n u_{\text{core},p} = \frac{N_s}{N_p} u_{\text{core},p}$$

Then, with the measured secondary side voltage $u_s$, current $i_s$ and phase angle $\phi_s$, the stray voltage $u_{s,s}$ of the secondary side can be evaluated. With the law of cosines applied to Fig. 9b and $\phi_{s,s} = \phi_s$, this yields:

$$u_{R,s} = \frac{\sqrt{u_{R,s}^2 + u_{\text{core},s}^2 - 2 u_{R,s} u_{\text{core},s} \cos(\phi_s)}}{u_{\text{core},s}}$$

$$u_{s,s} = \frac{\sqrt{u_{R,s}^2 + u_{\text{core},s}^2 - u_{\text{core},s}^2 - 2 u_{R,s} u_{\text{core},s} \cos(\phi_s) - (R_l i_s)^2}}{u_{\text{core},s}}$$

Finally, after calculating the core and stray voltages, the time derivative of the magnetic flux $d\Phi_{\text{core}}/dt$ and the magnetic flux $\Phi_{\text{core}}$ must...
still be evaluated. With the fundamental angular frequency $\omega$ of the vector diagram it follows (cp. Eqs. (12) and (13)) that:

$$\frac{d\Phi_{\text{core}}}{dt} = \frac{1}{N_p} \Phi_{\text{core},p} = \frac{1}{N_s} \Phi_{\text{core},s}$$  \hspace{1cm} (42)

$$\Phi_{\text{core}} = \frac{1}{\omega} \frac{d\Phi_{\text{core}}}{dt} \hspace{1cm} (43)$$

From this, it can be seen that the calculation of the core voltages $u_{\text{core},p}$ and $u_{\text{core},s}$ based on transformer input and output measurements provides the actual magnet flux $\Phi_{\text{core}}$ in the transformer core. Knowing the actual magnet flux $\Phi_{\text{core}}$ is the first of two pieces which are needed to calculate the transformer core losses.

Second, the magnetomotive force $\Theta^+$ is evaluated. To do so the primary side current $i_p$ has to be calculated based on the measured transformer input current $i_{p,m}$ and the secondary side current $i_s$. Remember that in the previous section it was stated that the input current $i_{p,m}$ contains the magnetization current $i_{mag}$, hysteresis current $i_{hyst}$ and eddy current $i_{ed}$ while the primary side $i_p$ current does not. This will become clear when the transformer is considered at idling operation where $i_p = i_s = 0$ and $i_{p,m} \neq 0$. In other words, at idling operation an input current is measured which is needed for the core magnetization and losses, i.e. for $i_{mag}$, $i_{hyst}$ and $i_{ed}$ [20]. Therefore, the current $i_p$ and $i_{p,m}$ can be distinguished as it is shown in the current vector diagram visible in Fig. 10a.

This means that the primary side current $i_p$ can be calculated from the current triangle determined by $i_p$, $N_p i_{p,m}$, $N_s i_s$ and $i_{p,m}$ as demonstrated in Fig. 10b. By applying the law of cosines to this triangle, the current $i_p$ is given with the measured current phase angle $\phi_i$ as follows:

$$i_p = \frac{1}{N_p} \sqrt{i_{p,m}^2 + N_s^2 i_s^2 - 2N_p i_{p,m} i_s \cos(\chi)}$$ \hspace{1cm} (44)

with

$$\chi = 2\pi - \phi_i$$ \hspace{1cm} (45)

Using further the law of sines for this current triangle, the angle $\chi_{th}$ between the currents $i_p$ and $i_{p,m}$ is evaluated with the calculated current $i_p$ and measured current $i_{p,m}$ (cp. Fig. 10b).

$$\sin(\chi) = \frac{N_i}{N_p + 1} \frac{i_p}{i_{p,m}} \sin(\chi_{th})$$ \hspace{1cm} (46)

Then, the magnetomotive force $\Theta^+$ can be determined based on Fig. 10c. Again, by applying the law of cosines to the current triangle $N_{p,i_{p,m}}$, $N_{s,i_s}$ and $\Theta^+$, the magnetomotive force $\Theta^+$ is specified.

$$\Theta^+ = \sqrt{N_{p,i_{p,m}}^2 + N_{s,i_s}^2 - 2N_{p,i_{p,m}} N_{s,i_s} i_p \cos(\chi_{th})}$$ \hspace{1cm} (47)

with (cp. also Fig. 10b and c)

$$\chi_{th} = \phi_i - \phi_s - \pi$$ \hspace{1cm} (48)

Furthermore, with the law of sines the angle $\psi_{th}$ between the current $i_p$ and the magnetomotive force $\Theta^+$ is given and thus, also the angle $\phi_{th}$ (cp. Figs. 9a and 10c).

---

**Fig. 8.** Calculation procedure for transformer losses.

**Fig. 9.** Vector diagram of voltages at the a) primary and b) secondary side for $\phi_i > \pi$ and $\phi_s > \pi$. 

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The numbers of turns \( N_p \) and \( N_r \) are necessary to solve the current vector diagram shown in Fig. 10. Further, the number of turns and the transformer dimension are needed to calculate the temperature distribution in the windings and thus to evaluate the hot spot temperature as it will be shown in section 3.

The transformer dimensions and number of turns can be determined from a few fundamental considerations and equations which must be fulfilled by any transformer and which can be derived from the theory discussed in section 2.1. First, the number of turns \( N_p \) of the primary side is related to the primary side voltage. With Eq. (6) and neglecting the voltage drops due to ohmic resistance \( R_p \) and stray flux \( \Phi_{sp} \) in Eq. (9) (cp. ideal transformer [20]), the magnetic flux \( \Phi(t) \) as a function of the primary side voltage, is given by:

\[
\Phi(t) = \int B(t) dA = \frac{1}{N_p} \int u(t) dt
\]  

(55)

Based on Eqs. (54) and (55) the number of turns \( N_p \) is obtained with the primary side voltage \( u(t)p \), the core cross section \( A_c \), the magnetic field \( B \) and fundamental angular frequency \( \omega \). Using the rms value for the primary side voltage \( U_{rms} \) at rated operation, which is available on the transformer nameplate, and assuming the rms value of the magnetic field \( B_{rms} \) at rated operation, the number of turns \( N_p \) is calculated as follows:

\[
N_p = \frac{1}{\omega A_c} \frac{U_{rms}}{B_{rms}}
\]  

(56)

Second, a similar equation is obtained for calculating the number of turns \( N_r \) with the ratio of winding window area \( A_w \) multiplied by the fill factor \( k_f \) and the wire cross section \( A_w \) (cp. Fig. 12). The wire cross section area \( A_{w} \) is obtained from the rms current \( I_{rms} \) and rms current density \( J_{rms} \) (in [A/mm²]) at rated operation (cp. Eq. (22)). The winding window is specified by the winding length \( l_w \) and height difference \( \Delta h \) (cp. Fig. 12). The fill factor \( k_f \) describes the ratio of wire cross section area \( A_{f,w} \) to the provided winding space in the window area \( A_w \). For example, the ratio of a circular area (round wire) and a square (winding space) leads to a fill factor \( k_f = \pi/4 \). Then, the number of turns \( N_r \) is:

\[
N_r = \frac{k_f A_w}{A_{f,w}} = k_f l_w \Delta h \left( \frac{J_{rms}}{l_{rms}} \right)_{rated}
\]  

(57)

Third, combining Eq. (56) and (57) provides a simple relation for the product of the transformer core cross section \( A_c \) and winding
window area \( A_w \), i.e.:
\[
A_w = \frac{1}{\alpha k_f} \left( \frac{U_{rms_{1m}}}{B_{rms_{1m}}} \right)_{rad}
\]  
(58)

Further, if it is assumed that the ratio of the transformer lengths \( b \) and \( h \) (cp. Fig. 12) is fixed, e.g. \( b/h = 3 \), the main transformer dimensions, i.e. the virtual twin, are determined with \( A = h^2 \) and \( A_w = (b-2h)b \):
\[
b = \sqrt{\frac{2}{\alpha k_f} \left( \frac{1 - \frac{2h}{b}}{1 - \frac{h}{b}} \right)^{\frac{1}{2}} \left( \frac{U_{rms_{1m}}}{B_{rms_{1m}}} \right)_{rad}}
\approx \frac{2}{\alpha} \sqrt{1 - \frac{2h}{b}} \left( \frac{U_{rms_{1m}}}{B_{rms_{1m}}} \right)_{rad}
\]  
(59)

Note that i) the current density \( J_{rms} \) (approx. 1.5–2 A/mm²) and the magnetic field \( B_{rms} \) (approx. \( I \) \( T \)) are estimated and that ii) the fourth root of the fill factor \( k_f \) is approx. one. Therefore, the fill factor is neglected in Eq. (59).

Fourth, after calculating the core cross section area \( A \) based on Eq. (59) and then, the number of turns \( N_p \) based on Eq. (57), the number of turns \( N_s \) (secondary side) is determined by:
\[
N_s = nN_p
\]  
(60)

Note that both numbers of turns \( N_p \) and \( N_s \) must be integers. This means that \( N_p \) must be rounded downwards appropriately. The transformer ratio \( n \) is obtained from the rated voltages of primary and secondary side which, again, are available on the transformer nameplate.

Finally, assuming a squared wire cross section with the edge length \( h_{\Omega} \), i.e. a fill factor \( k_\Omega \) of approx. one, the number of turns \( N_p,h \) along the length \( L_p \) and \( N_s,h \) along the height difference \( \Delta h \) can be estimated as follows (cp. Fig. 12):
\[
N_{p,h} = \frac{l_p}{h_{\Omega}} = \frac{l_p}{\sqrt{l_{ms}}}
\]  
(61)
\[
N_{s,h} = \frac{\Delta h}{h_{\Omega}} = \frac{\Delta h}{\sqrt{l_{ms}}}
\]  
(62)

The latter two equations also apply for the secondary side winding by using the secondary side rms current.

These considerations and equations show that the transformer main dimension and the number of turns of the primary and secondary side winding can be estimated, i.e. a simple virtual twin can be created based on the transformer rating information (rated power and voltages) only. Due to several assumptions and simplifications, the actual transformer dimensions and number of turns might not be evaluated correctly. However, based on the simple virtual twin the current vector diagram (cp. Fig. 10) and the winding temperature can be calculated. The latter is shown in the next section.

3. Calculation of transformer temperature

For the life time prediction of the transformer it is necessary to know the temperature in the transformer windings. Therefore, it is important to describe the conversion of the dissipated power (winding and core losses) to an actual temperature. In order to do so, winding and core losses are assumed to be evenly distributed over the (copper) winding \( (V_{p,Cu}, V_{s,Cu}) \) respectively core volume \( (V_c) \). With Eq. (23), (24), (29) and Fig. 12 this leads to the following definition of the volumetric heat generation in the primary \( (q_{v,p}) \) and secondary \( (q_{v,s}) \) side windings as well as core \( (q_{v,c}) \):
\[
q_{v,p} = \frac{Q_p}{V_{p,Cu}} = \frac{R_{i_{rms}^2} k_{f} \Delta h}{(4h + 3\Delta h)A_w, \Omega k_f}
\]  
(63)
\[
q_{v,s} = \frac{Q_s}{V_{s,Cu}} = \frac{R_{i_{rms}^2} \Delta h}{4(h + \Delta h)A_w, \Omega k_f}
\]  
(64)
\[
q_{v,c} = \frac{Q_c}{V_c} = \frac{2\pi c_d}{\alpha \lambda} \int \Phi d\Theta^\times
\]  
(65)

Note that the volumetric heat generation \( q_v \) is in [W/m³] and the heat flow \( Q \) in [W].

Then, to simplify the temperature calculation a few assumptions are made here:

- The windings and core have square cross sections, i.e. \( A = h^2 \).
- The circumferences of the squared winding \( (4h) \) is described by an equivalent circular circumferences \( (2\pi r) \) as shown in Fig. 13, i.e.
\[
R = 2h/\pi\text{orr} = 4x/\pi
\]  
(66)

- The heat flow is uniform and only occurs in radial direction. This means that 1D considerations are sufficient.
- The transformer operates under quasi-stationary conditions, i.e. the change of the volumetric heat generation \( q_v \) and winding temperature \( T \) over time does not have a significant effect on the spatial temperature distribution in the windings. This means that the general 1D (radial direction) heat equation in cylinder coordinates [24]
\[
\frac{1}{r} d \left( r d\frac{dT}{dr} \right) + \varphi_T = \varphi_p dT dT
\]  
(67)
with the density \( \rho \), heat capacity \( c_p \) and thermal conductivity \( k \) can be simplified to

---

**Fig. 12.** Virtual twin (dimensions) of common transformer phase.

**Fig. 13.** Cross section approximation.
\[
\frac{1}{r} \frac{d}{dr} \left( kr \frac{dT}{dr} \right) + q_r = 0
\]  
\tag{68}
\]

as the temporal variation in temperature is assumed to be approx. zero, i.e. \( \frac{dT}{dr} = 0 \), steady state. It also means that the winding temperature \( T \) apart from the amount of heat \( q_r \) generated, will only depend on the winding radius \( r \) (cp. Eq. (68)).

- The transformer windings are assumed to consist of consecutive circumferential layers of copper and insulation (including oil) and the magnitude of \( q_r \) depends on their different thermal conductivities in the radial heat flow (cp. Fig. 14). This means that the \( N_{ip} \) (or \( N_{op} \)) turns along the winding length \( L_w \) merge to one solid copper layer with the following thickness (primary side winding):

\[
\Delta r_{Cu} = \Delta h_{Cu} \frac{k_f}{N_{p,AR}}
\]  
\tag{69}
\]

Then the insulation layer thickness is:

\[
\Delta r_{ins} = \Delta h \frac{1 - k_f}{N_{p,AR}}
\]  
\tag{70}
\]

Note that the number of turns \( N_{p,AR} \) and \( N_{s,AR} \) along the radius difference \( \Delta r \) are equal to the number of turns \( N_{p,sh} \) and \( N_{s,sh} \) along the height difference \( \Delta h \) (cp. Figs. 12 and 14). The approximated transformer windings (cp. Fig. 14) have \( N_{p,AR} \) respectively \( N_{s,AR} \) layers of both copper and insulation in radial direction. Further, the number of layers will have to be rounded to an integer, if the number of turns \( N_{p,AR} \) respectively \( N_{s,AR} \) are evaluated based on Eqs. (61) and (62). In addition, as the thermal conductivities of insulation material, e.g. Enamel \( (k_{en} = 0.26-0.54 \text{ W/mK} \ [25]) \) and oil \( (k_{oil} = 0.3714 \text{ W/mK} \ [26]) \) are similar, they are merged into one insulation layer.

- The ambient oil temperature \( (T_{oil,amb}) \) of the windings is available, i.e. it is measured during operation (cp. Fig. 14). It is used as boundary condition to approximate the windings temperature, i.e. copper layers temperature.

Now, the radial temperature distribution for the different winding layers is defined. The layers can be categorized in two types: copper and insulation layers. Although, these layers have different radial heat flow gradients, the radial temperature calculation can be developed simultaneously for both type of layers because the gradient \( dQ/dr = 0 \) (insulation layer) is a special case of the gradient \( dQ/dr = \text{const.} \) (copper layer).

In general, the heat flow \( Q \) in the layers is expressed by the product of the specific radial heat flow \( q_r \) in [W/m\(^2\)] and the surface area \( A_{nr} \), i.e. for a cylindrical layer with the winding length \( L_w \):

\[
Q = q_r A_r = q_r 2\pi r L_w
\]  
\tag{71}
\]

The specific heat flow in radial direction is given by the Fourier’s law of conduction, i.e. with thermal conductivity \( k \).

\[
q_r = -k \frac{dT}{dr}
\]  
\tag{72}
\]

Further, rearranging and integrating Eq. (68) yields:

\[
dT = -\frac{q_r}{2k} rdT + C_1 \frac{dr}{r}
\]  
\tag{73}
\]

\[
T(r) = -\frac{q_r}{4k} r^2 + C_1 \ln(r) + C_2
\]  
\tag{74}
\]

The constants \( C_1 \) and \( C_2 \) are determined by boundary conditions, i.e. by the temperatures at the inner \( (T_{in}) \) and outer \( (T_{out}) \) side of the layers. Hence, the constants \( C_1 \) and \( C_2 \) are specified with the inner \( (r_{in}) \) and outer \( (r_{out}) \) layer radii (cp. Fig. 15) by:

\[
T_{in} = -\frac{q_r}{4k} r_{in}^2 + C_1 \ln(r_{in}) + C_2
\]  
\tag{75}
\]

\[
T_{out} = -\frac{q_r}{4k} r_{out}^2 + C_1 \ln(r_{out}) + C_2
\]  
\tag{76}
\]

Moreover, with Eqs. (71) to (73) this yields:

\[
Q^* = -2\pi r L_w k \left( \frac{dT}{dr} \right) = 2\pi r L_w k \left( \frac{T_{out} - T_{in}}{\ln(r_{out}/r_{in})} \right)
\]  
\tag{77}
\]

with the modified radial heat flow \( Q^* \) specified by the radial heat flow \( Q \) and volumetric heat generation \( q_r \).

\[
Q^* = Q - q_r r w L_w
\]  
\tag{78}
\]

As the ambient oil temperature is measured, i.e. \( T_{oil} \) is known and the modified heat flow \( Q^* \) through the winding is calculated based on the transformer losses (cp. previous section), the inner layer temperature \( T_{in} \) can be evaluated as follows:

\[
T_{in} = T_{out} + \frac{Q^*}{2\pi r L_w k} \ln\left( \frac{r_{out}}{r_{in}} \right) - \frac{q_r}{4k} (r_{out}^2 - r_{in}^2)
\]  
\tag{79}
\]

Note that the winding temperature is evaluated from the winding outside to its inside, i.e. from the outermost to the innermost winding layers. This means that the outer temperature of the \( i \) th copper layer is equal to the inner temperature of the \( i \) th insulation layer (cp. Fig. 15). Therefore, the outer layer temperature is known and the inner layer temperature must be determined. Further, note that Eqs. (78) and (79) apply to copper layers for \( q_r > 0 \) and to insulation layers for \( q_r = 0 \).

In addition, to solve Eq. (79) for every layer, the inner \( (r_{in,i}) \) and outer \( (r_{out,i}) \) radii and the heat flow \( (Q) \) are needed for every \( i \)th copper and insulation layer. These quantities are determined as follows for \( i \in \{1, N_{p,AR}\} \) and with Figs. 14 and 15 (primary side winding):

\[
r_{in,Cu,i} = R_{max} - i \frac{\Delta R}{N_{p,AR}}
\]  
\tag{80}
\]

\[
r_{out,Cu,i} = r_{in,Cu,i} + k_f \frac{\Delta R}{N_{p,AR}}
\]  
\tag{81}
\]

\[
r_{in,ins,i} = r_{in,Cu,i} + \frac{\Delta R}{N_{s,AR}}
\]  
\tag{82}
\]

\[
r_{out,ins,i} = r_{out,Cu,i} - (1 - k_f) \frac{\Delta R}{N_{s,AR}}
\]  
\tag{83}
\]

\[
Q_i = q_r \pi r_{in}^2 L_w
\]  
\tag{84}
\]

\[
Q_p = R_{p,em}^2 \pi L_w
\]  
\tag{85}
\]

\[
Q_i = Q_p + \sum_{i=1}^{N_{p,AR}} q_{v,p} m_r (r_{out,Cu,i} - r_{in,Cu,i})
\]  
\tag{86}
\]

\[
Q^* = Q_p + \sum_{i=1}^{N_{p,AR}} q_{v,p} m_r (2r_{out,Cu,i} - r_{in,Cu,i})
\]  
\tag{87}
\]
Finally, from these considerations, it can be seen that based on the main transformer dimensions (virtual twin) and winding and core losses the temperature distribution in the transformer windings can be evaluated. The hot spot temperature is the highest winding temperature. It is normally the winding temperature closest to the core, i.e. at the innermost copper layer. Further, if the primary side winding is placed around the secondary side winding (cp. Fig. 12), then the heat flow through the primary side winding does not only contain the heat flow from the core but also from the secondary side winding.

4. Case study and validation

In this section, the losses and winding hot spot temperature are calculated for a single phase of a 154 kV, 15MVA power transformer. Kwean et al. [15] provide loss and winding temperature measurements as well as the hot spot temperature calculated according to the IEEE / IEC loading guide for this transformer. The available transformer information are shown in Table 1 [15].

First, the main transformer dimensions of the virtual twin are estimated based on Table 1 and section 2.4. Table 2 shows the properties of the virtual twin. Note that the turn ratio \( n = \frac{N_s}{N_p} \) is estimated based on Eq. (18) and (19) (cp. section 2.2) and the ratio of the measured winding resistances (cp. Table 1). For three parameters in Table 2 values need to assumed, i.e. \( B_{rms}, J_{rms} \) and \( b/h \). The magnetic field \( B_{rms} \) is estimated based on the magnetic field saturation of the transformer core, which is typically \( B_s = 1.5 – 1.7 \ T \) [27]. This means that a magnetic field of \( B_{rms} = 1 \ T \) of one phase at nominal transformer operation leads to a magnetic field peak of \( B_{peak} = 1.4 \ T \). In other words, the magnetic field is chosen such that the transformer core almost reaches saturation at nominal operation. Then, the current density \( J \) is typically in the range of 2–5 A/mm\(^2\) [28]. Due to the compact transformer windings, for safety reasons lower values of the current density are chosen. Therefore, a current density peak for one phase at nominal operation of \( J_{peak} = 2.5 \ A/mm^2 \), i.e. \( J_{rms} = 1.75 \ A/mm^2 \), is assumed here. In addition, the dimension ratio \( \frac{b}{h} = 3 \) is obtained by minimizing the length \( b \) using Eq. (59). This might not yield the optimal transformer dimensions where the sum of core and winding losses is minimized. However, as the core losses are proportional to the core volume, i.e. the product of core cross section \( A \) and length \( l \) (cp. Fig. 12), and the winding losses are a function of the window area \( A_w \) (cp. Eq. (18) and (57)), i.e. the dimensions \( h \) and \( b \), the optimization of the transformer volume provides a rough estimation of the dimension ratio \( \frac{b}{h} \).

By comparing the winding resistances in Table 1 and 2, it can be seen that the calculated resistances are between the cold and hot measured resistances. This means that the virtual twin is an adequate approximation of the actual transformer because the winding resistances (lengths, cp. Eq. (18)) are also a function of the transformer dimensions, i.e. winding radii.

Second, to calculate the transformer losses, rms currents and voltages including phase angles are needed. This is shown in Table 3.

The transformer loss calculations are executed with the calculated winding resistances available in Table 2. Further, as the phase angles were not available, they are estimated here such that the calculated transformer losses match with the measurements provided by [15], assuming an inductive load impedance, i.e. \( \varphi_i > \pi \), connected to the transformer output. The stray losses determine the phase angle \( \varphi_s \) (cp. Eqs. (30) and (31)) as well as Eq. (40) and (41)), while the core losses determine the phase angles \( \varphi_c \) and \( \varphi_p \) (cp. Eqs. (45), 48 and 50). Finally, the phase angle \( \varphi_i \) is derived from the phase angles \( \varphi_s \), \( \varphi_c \) and \( \varphi_p \) (cp. Fig. 7). Note that this means that the comparison of the measured and calculated losses (cp. Table 4) demonstrates the feasibility of the loss calculation with the proposed method, but it does not quantify the calculation errors.

On the other hand, the calculated losses shown in Table 4 depend differently on the estimated phase angles. The stray losses are sensitive to the phase angle \( \varphi_p \) because it is used to calculate the secondary side stray voltage in Eq. (41). Doubling the phase angle \( \varphi_p \) from \( 3.470 \times 10^\pi \) to \( 6.940 \times 10^\pi \) increases the stray losses from 16,180 W to 39,594 W. The core losses are much less sensitive to the phase angles (cp. Eq. (44) and the followings). Reducing the phase angle \( \varphi_i \) from 1.46 to 1.36 decreases the core losses from 11,023 W to 8,931 W. Then, the winding losses do not depend at all on the phase angles. Rather, they are determined by the calculated winding resistances, i.e. they depend on the estimation of the virtual transformer twin and its dimensions. This means that estimating the phase angles is non-critical here because the stray losses will not be considered in the winding hot spot temperature calculation. As already mentioned in the section introduction, the stray losses occur in different transformer parts and thus, only a minor portion of them can be assigned to the heat flow through the windings. This portion is difficult to evaluate and therefore, it is not considered in the winding hot spot temperature calculation.

Furthermore, the core losses do neither flow entirely through the windings. The part of the transformer core that is surrounded by the windings generates the heat which must flow through the windings (cp. Figs. 12 and 14). In addition, the part of transformer core which is outside the windings affects the temperature distribution, too (cp. Fig. 16) because the heat is not directly dissipated (in radial direction) to the surrounding fluid (oil). Rather, before it is absorbed by the oil, it has to flow, at least to a certain extent, through the transformer core causing a higher winding temperature. This is demonstrated by the heat flow path A and path B in Fig. 16 which shows the schematically the top view of the transformer used by Kwean et al. [15]. On path A the heat is directly dissipated to the surrounding fluid, while on path B the heat must flow through the outer transformer part which also generates additional heat (core losses). To consider the latter in the winding temperature calculation, a fictive shell around the windings can be assumed through which the heat must flow (cp. Fig. 16). Note that this only applies for path B and is a rough approximation describing the worst case scenario.

Moreover, from Table 4 it can be seen that the core losses are significantly lower than the winding losses. Using in addition to that only a part of the core losses means that small deviations in the core losses calculations have an insignificant influences on the winding hot spot.

| parameter | value | unit | description |
|-----------|-------|------|-------------|
| S         | 15    | [MVA] | rated power |
| \( U_{rms} \) | 77,798 | [V] | rated voltage (measured) |
| \( I_{rms} \) | 206.1 | [A] | rated current |
| \( R_{s,cold} \) | 0.79277 | [Ω] | HV winding resistance (cold measured) |
| \( R_{s,hot} \) | 0.89897 | [Ω] | HV winding resistance (hot measured) |
| \( R_{p,hot} \) | 0.017228 | [Ω] | LV winding resistance (hot measured) |
| \( R_{p,cold} \) | 0.021456 | [Ω] | LV winding resistance (hot measured) |
The calculations are executed for a measured oil temperature of 73.7 °C. B. The calculations are executed for a measured oil temperature of 73.7 °C.

On the other hand, as the winding losses are the main losses, they are crucial for the winding temperature. Consequently, a proper evaluation of the winding resistances is important for the hot spot temperature calculation.

Third, based on the properties of the virtual transformer twin (Table 2) and the calculated core and winding losses (Table 4) the temperature distribution in the windings is calculated along path A and B. The calculations are executed for a measured oil temperature of 73.7 °C [15] and a thermal conductivity of copper \( k_{Cu} = 385 \, W/mK \) [25], wire insulation (Epoxy impregnation) \( k_{ins} = 1 \, W/mK \) [25] and iron \( k_{Fe} = 80 \, W/mK \) [29].

The winding temperature distribution is calculated for a fill factor \( k_f = 1 \) and \( k_f = 0.9974 \) along path A (cp. Fig. 17). A fill factor equal to one \( (k_f = 1) \) means that the wire insulation is neglected (cp. "no insulation A" in Fig. 17) and that the heat is only transmitted via the copper. A fill factor \( k_f = 0.9974 \) means that there are small layers of insulation in between the windings through which the heat must flow (cp. Figs. 14 and 15). As the insulation material has a significantly lower thermal conductivity than the copper, very thin layers of insulation cause a notable increase of the calculated winding temperature, i.e. up to 10 K (cp. "no insulation A" and "insulation A" in Fig. 17).

Further, the winding temperature distribution is calculated neglecting the core losses including the insulation (cp. "no core losses" in Fig. 17). It is visible that the maximum calculated temperature (hot spot temperature) will be approx. 2.5 K on path B, if the core losses are neglected. Moreover, due to the fictive shell around the windings the temperature distribution on path B is increased by an offset of approx. 7.5 K in comparison to path A (cp. "insulation" in Fig. 17). This increase is caused by i) the heat (core losses) generated in the fictive shell and ii) the thermal conductivity of iron which is lower than that of copper. In addition, note that the winding radius in Fig. 17 is normalized with the maximum radius of the primary side winding and that in this case study the secondary side winding is surrounded by the primary side winding (cp. also [15] and Fig. 16). The low normalized winding radii thus represent the windings that are a the inside of the transformer, and Fig. 17 shows that the temperature reaches its maximum at these windings.

From Fig. 17, it is visible that the calculated winding hot spot temperature (at the inner side of the secondary winding) considering the insulation is 93.6 °C on path A and 101.0 °C on path B. For comparison, Kwean et al. [15] measured a temperature of 98.0 °C on path A, 105.9 °C on path B and calculated a temperature of 92.6 °C based on the IEEE / IEC loading guide. So the hot spot temperature calculated with insulation approximates the measured hot spot temperatures. The hot spot temperature according to the loading guide is lower than evaluated with the method proposed in this paper (cp. path B). So values of the correction factor \( K \) (cp. Eq. (4)) provided by the IEC standard appear to be too small. This confirms the Kwean et al. [15] findings and concerns that higher correction factors should be applied in the IEEE and IEC standards. The results also confirm that the proposed method allows to quite accurately determine the winding temperatures.

Fourth, the winding temperature calculation with the proposed method also provides the evaluation of the winding exponent \( m \) (cp. Eq. (4)). To do so, the calculated differences between hot spot and oil temperature are normalized by the rated difference, i.e. the temperature drop ratio is evaluated. Then, this ratio is plotted over the winding loss ratio (cp. “calc.” Fig. 18). The winding loss ratio is defined by the ratio of the actual winding losses to rated winding losses. Note that the core losses are considered in the temperature drop ratio. Therefore, the

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**Table 2**

Properties of virtual transformer twin.

| parameter | value | unit | description |
|-----------|-------|------|-------------|
| \( B_{rms} \) | 1 | [T] | magnetic field (assumed) |
| \( I_{rms} \) | 1.75 | [A/mm\(^2\)] | current density (assumed) |
| \( f \) | 50 | [Hz] | operating frequency |
| \( b/h \) | 3 | - | dimension ratio (assumed) |
| \( d \) | 1.11 | [m] | length |
| \( R \) | 0.20875 | [m] | core radius |
| \( n \) | 5 | - | turn ratio |
| \( N_p \) | 1800 | - | number of turns (primary side) |
| \( N_{p,al} \) | 100 | - | number of turns (along length) |
| \( N_{p,al} \) | 18 | - | number of turns (along radius) |
| \( N_s \) | 360 | - | number of turns (secondary side) |
| \( N_{s,al} \) | 45 | - | number of turns (along length) |
| \( N_{s,al} \) | 8 | - | number of turns (along radius) |
| \( R_{p,calc} \) | 0.20875 | [Ω] | HV winding resistance (calculated) |
| \( R_{s,calc} \) | 0.0197352 | [Ω] | LV winding resistance (calculated) |

**Table 3**

Input data for transformer loss calculations.

| parameter | value | unit | description |
|-----------|-------|------|-------------|
| \( \phi_u \) | 77,798 | [V] | primary side rms voltage |
| \( \phi_m \) | 206.1 | [A] | primary side rms current |
| \( \phi_i \) | 15,513 | [V] | secondary side rms voltage |
| \( \phi_s \) | 960.4 | [V] | secondary side rms current |
| \( \phi_1 \) | 3.47 \times 10^{-6} | [rad] | phase angle between \( u_s \) and \( u_p \) |
| \( \phi_2 \) | 0.1π | [rad] | phase angle between \( u_s \) and \( i_s \) |
| \( \phi_3 \) | 1.36π | [rad] | phase angle between \( i_s \) and \( i_p \) |
| \( \phi_4 \) | 1.6π | [rad] | phase angle between \( u_p \) and \( i_p \) |

**Table 4**

Calculated and measured transformer losses.

| loss [W] | calculated | measured |
|----------|------------|----------|
| core | 11,023 | 11,280 |
| winding | 65,091 | 62,769 |
| stray | 16,180 | 16,277 |

**Fig. 16.** Transformer top view with different paths of radial heat flow.

calculations. On the other hand, as the winding losses are the main losses, they are crucial for the winding temperature. Consequently, a proper evaluation of the winding resistances is important for the hot spot temperature calculation.
temperature drop ratio is not zero at zero winding losses in Fig. 18. For comparison the winding loss ratio, which is equivalent to the power two of the current ratio in Eq. (4), is calculated for a winding exponent $m = 0.8$ and $m = 0.9$ (cp. Fig. 18). It is visible that a winding exponent of 0.8 fits better to the calculated temperature drop ratio. Kwean et al. [15] used based on the IEC standard recommendations a winding exponent of 0.8 for the considered transformer. This means that the calculated and empirically evaluated winding exponents coincide.

5. Discussion

Now, after the proposed methods for calculating the transformer losses and hot spot temperature has been introduced and demonstrated, some issues remain for discussion.

First, the case study and validation reveal that with the proposed method the hot spot temperature can be estimated for an effective fill factor $kf = 0.9974$, i.e. considering small layers of insulation. This means on the one hand that the winding insulation cannot be neglected and, on the other hand, that the actual fill factor, which is significantly smaller than one (e.g. $\pi/4$), does not apply. The effective fill factor is derived from the ratio of the thermal conductivity of the insulation ($k_{ins} = 1 \text{ W/mK}$) and the copper ($k_{Cu} = 385 \text{ W/mK}$) as follows:

$$k_f = 1 - \frac{k_{ins}}{k_{Cu}} = 1 - \frac{1}{385} = 0.9974$$

(88)

Now, to explain Eq. (88), two wires are considered in radial winding direction (cp. Fig. 19). It is assumed that heat is only transported through the parts of the (insulated) wires that are in contact, i.e. the gaps between the wire cross sections do not contribute to the heat transport. This assumption is motivated by the significant difference in thermal conductivity of the wires and the gaps (oil), $k_{Cu} \gg k_{oil}$.

It is assumed that the specific heat flow $q_r$ in radial direction is equal in the $i$th insulation and copper layer. Then, it follows with Eq. (72):

$$\frac{k_{ins}}{k_{Cu}} = \frac{\Delta T_{ins}}{\Delta T_{Cu}}$$

(89)

Further, the Wiedemann-Franz law applies. It defines the ratio of the thermal and electrical conductivity with the (material) temperature $T$ and the Lorenz number $L = 2.44 \times 10^8 \text{ W} \Omega/\text{K}^2$.

$$\frac{k}{g} = LT$$

(90)

with the electrical conductivity $g$ specified by (cp. Eq. (18)):

$$g = \frac{1}{R_a A_a}$$

(91)

If small temperature drops $\Delta T$ across $i$th layers are assumed, the temperature $T$ in the $i$th layers is almost constant and hence, the ratio of the thermal and electrical conductivity (cp. Eq. (90)). With a constant specific heat flow $q_r$ in the $i$th layers and with $\Delta q_r = \Delta T$ and $A_a = A_a$ (cp. Eq. (71)), this yields the following relation for the temperature drop and radial ohmic resistance of the insulation and copper layers:

$$\frac{k_{ins}}{k_{Cu}} = \frac{\Delta T_{ins}}{\Delta T_{Cu}} = \frac{\Delta A_{ins}}{A_{ins}} \cdot \frac{R_{ins}}{R_{Cu}} A_{ins} \frac{A_{ins}}{A_{ins}} = \frac{g_{ins}}{g_{Cu}}$$

$$\frac{\Delta T_{ins}}{\Delta T_{Cu}} \approx \frac{R_{ins}}{R_{Cu}}$$

(92)

This means that a high radial ohmic layer resistance $R_{ins}$ leads to a high temperature drop $\Delta T_{ins}$. Both $R_{ins}$ (cp. Eq. (91)) and $\Delta T_{ins}$ (cp. Eq. (72)) increase with the insulation layer thickness $\Delta ins$. A high radial ohmic layer resistance $R_{ins}$ is desired to minimize the radial current $i_r$ across the windings caused by the radial winding voltage drop. The latter is determined by the winding terminal voltages and thus, is given by external transformer conditions. On the other hand, a low temperature drop $\Delta T_{ins}$ is preferred to reduce the temperature increase in the windings, i.e. the hot spot temperature and eventually the insulation aging process. However, this is a dilemma and requires the optimal insulation layer thickness to satisfy both a high radial resistance $R_{ins}$ and a low temperature drop $\Delta T_{ins}$.

To do so, it is assumed here that the radial current $i_r$ across the windings is equal in the $i$th copper and insulation layer. Further, the total radial voltage drop across the $i$th copper and insulation layer is evenly distributed, i.e. $\Delta u_{Cu} = \Delta u_{ins}$. With Eq. (92) this yields $\Delta T_{Cu} = \Delta T_{ins}$. The even distribution of both voltage and temperature drop provides the benefit that the mechanical stresses (i.e. deformation due to the voltage drop [30,31]) and thermal stresses (due to the temperature drop) are limited in the insulation layer. As the total radial winding drop is given by external conditions a minimum insulation layer thickness is needed to limit the mechanical stresses level. On the other hand, this minimum thickness determines the thermal stress level. So the conditions $\Delta u_{Cu} = \Delta u_{ins}$ and $\Delta T_{Cu} = \Delta T_{ins}$ define a balance point of mechanical and thermal stress levels in the insulation layer. Based on these considerations Eq. (89) can be simplified and Eq. (88) is obtained again assuming that only the ratio between the radial dimensions (4r) of copper and insulation determine the (effective) fill factor (and the gaps can be neglected). This also explains the high $k_f$ value.

In addition, as the calculated hot spot temperature (93.6 °C) (cp. “insulation A” in Fig. 17) is between the measured (98.0 °C) and calculated (92.6 °C) temperature based on the IEEE / IEC loading guide [15], it appears that the here assumed voltage and temperature drop distribution across the ith copper and insulation layer are appropriate. However, to limit the scope of this paper the optimal insulation layer thickness and balance point of mechanical and thermal stress levels are not further discussed. This is left for future research.
Second, the accuracy in creating the virtual transformer twin also influences the hot spot temperature. If the dimensions of the virtual twin are smaller than the actual transformer, then a higher hot spot temperature will be calculated because the winding volume affects the calculations (cp. Eq. (63) and (64)). To validate whether the dimensions of the virtual twin are estimated properly, the measured and calculated winding resistances are compared. Remember that the winding resistances depend on the winding lengths (cp. Eq. (18)), i.e. winding radii. Hence, if the measured and calculated winding resistances coincide, then it can be concluded that the dimensions of the virtual twin and actual transformer match. As this is the case here (cp. Table 1 and 2), the calculated temperature distribution (cp. Fig. 17) and hot spot temperature can be considered to be representative for the actual transformer. Furthermore, the assumption of radial heat flow leads to a deviation between the calculated and measured hot spot temperatures. Although the heat mainly flows in radial direction, multiple directional heat flows occur in reality. This means that the winding temperature is not only determined by the losses generated in the windings and in the transformer core surrounded by the windings. Rather, the losses of the entire core as well as the stray losses, which are not considered here at all, additionally affect the winding temperature distribution. Consequently, higher local temperatures occur in the transformer windings. This makes that the measured temperature is higher than the calculated hot spot temperature, which matches with the results presented in the previous section. From this perspective, the presented method leads to a non-conservative prediction.

Third, the core losses are determined by the vector diagram and the approximation of the core hysteresis by an ellipse. The latter applies only for ideal working conditions where harmonics are not present. However, as the core losses are significantly smaller than the winding losses and, in addition to that, the core losses are only partly considered in the winding temperature calculation, the proposed method can still be used with the presence of harmonics. This would then mean that the harmonics are neglected in the core loss calculation, but they are considered in the winding loss calculation. This yields a slightly less conservative temperature prediction, as the core losses have a minor influence on the winding temperature distribution (cp. Fig. 17).

Fourth, to demonstrate the significance of the hot spot temperature on the winding life time, Eq. (1) is applied for a specific temperature range. The inverse of the ageing acceleration factor (1/FAA) is shown in Fig. 20. It is visible that the higher the hot spot temperature, the lower the inverse value 1/FAA. This means for an increase of the hot spot temperature by 1 K, the winding life time is reduce by approx. 10% (Eq. (2)). Therefore, a proper determination of the hot spot temperature is essential to predict the transformer life time.

Fifth, the here proposed method uses Eq. (79) to evaluate the hot spot temperature (i.e. $T_{in}$) while the IEEE / IEC load guides apply Eq. (4). Comparing these equations yields the following:

$$Q^* = \frac{q_0 K}{2 \pi K} \ln \left( \frac{r_{in}}{r_{out}} \right) - \frac{dQ}{dK} \left( r_{in}^2 - r_{out}^2 \right) \leftrightarrow K \cdot \Delta \theta_{hil} \left( \frac{I}{I_o} \right)^{2m}$$

(93)

It can be seen that both Eqs. (4) and (79) consider the transformer dimensions. The proposed method uses the inner ($r_{in}$) and outer ($r_{out}$) winding radius as well as the winding length ($L_w$). In the IEEE / IEC load guide the correction factor K, exponent m and measured temperature drop $\Delta \theta_{hil}$ depend on the transformer type and design (dimensions). Then, the generated heat ($q_0$) as well as the heat flow (Q) through the windings is determined by the core and winding losses. As the winding losses dominate, the generated heat and the heat flow are almost proportional to the square of the winding currents (cp. ohmic winding losses in Eqs. (23) and (24)). This is equivalent to the exponent two of the current ratio used in the IEEE / IEC loading guide. Further, the proposed method use the thermal conductivity of the windings (copper) and insulation material. The IEEE / IEC loading guide includes the thermal conductivity in the measured temperature drop $\Delta \theta_{hil}$. This comparison shows that the proposed method utilizes the same elements as the IEEE / IEC loading guide and thus, is general and representative for many transformers.

Sixth, the input parameters for calculating the transformer losses rely on (phase angle) estimations. It was already discussed in the previous section that this has a minor influence on the calculated hot spot temperature because core losses are only slightly affected and winding losses are not at all affected by these estimations. However, if the proposed method is used to evaluate not only core and winding losses but also the stray losses, then further validations will be necessary as the stray losses are sensitive to these estimations. This is left for future research and is thus outside the scope of this paper.

6. Conclusion

End customers and consultancy companies often deal with a lack of information and cannot always use the established methods for life time prediction. Therefore, the present paper provides an alternative way to evaluate the life time of power transformers based on limited information. The approach is applied to one phase of a 154 kV, 15MVA power transformer and the results are compared with loss and winding temperature measurements as well as temperature calculations according to the IEEE / IEC loading guides which are available in literature.

- The transformer losses are calculated based on rms current and voltage measurements (including phase angles). Then, the temperature distribution in the transformer windings is evaluated using i) core and winding losses and ii) a virtual transformer twin. Stray losses are not considered in the winding temperature calculation because only a minor portion of them can be assigned to the windings. The virtual transformer twin is created based on the transformer rating information.
- Depending on the considered path of heat flow, the hot spot temperature calculated with the proposed methods deviates from the temperature calculated according to the IEEE / IEC loading guides. Hot spot temperature measurements provided in literature show a similar deviation: higher temperatures are measured than those calculated according to the IEEE / IEC loading guides. Therefore, it appears that values of the hot spot factor recommended in these standards are too small, leading to an underestimation of the hot spot temperature. Moreover, the proposed method yields temperature values that are closer to the measured values.
- It is shown that the consideration of the winding insulation is crucial to properly determine the hot spot temperature. The factor used to indicate the winding insulation thickness is slightly smaller than one, but significantly higher than the actual fill factor. It appears that the insulation thickness factor must be determined from the thermal conductivity of the insulation material and copper
(winding). An even distribution of the radial voltage and temperature drop over the copper and insulation layer appears to specify the optimal insulation (layer) thickness.

- It has been demonstrated that the hot spot temperature prediction of power transformers based on limited information is not only feasible but also provides a calculation accuracy equivalent to the established methods. Therefore, the proposed methods are an alternative way to calculate the transformer hot spot temperature and thus, to predict the transformer life time.

CRediT authorship contribution statement

**D.P. Rommel:** Conceptualization, Methodology, Software, Validation, Data curation, Writing - original draft. **D. Di Maio:** Writing - review & editing, Supervision. **T. Tinga:** Writing - review & editing, Supervision, Project administration, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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