Supersymmetric Q-balls: A Numerical Study

L. Campanelli\textsuperscript{1,}\textsuperscript{2} and M. Ruggieri\textsuperscript{3,}\textsuperscript{4}

\textsuperscript{1}Dipartimento di Fisica, Universit\`{a} di Ferrara, I-44100 Ferrara, Italy
\textsuperscript{2}INFN - Sezione di Ferrara, I-44100 Ferrara, Italy
\textsuperscript{3}Dipartimento di Fisica, Universit\`{a} di Bari, I-70126 Bari, Italy and
\textsuperscript{4}INFN - Sezione di Bari, I-70126 Bari, Italy

(Dated: October, 2007)

We study numerically a class of non-topological solitons, the Q-balls, arising in supersymmetric extension of the Standard Model with low-energy, gauge-mediated symmetry breaking. Taking into account the exact form of the supersymmetric potential giving rise to Q-balls, we find that there is a lower limit on the value of the charge $Q$ in order to make them classically stable: $Q \gtrsim 5 \times 10^3 Q_{cr}$, where $Q_{cr}$ is constant depending on the parameters defining the potential and can be in the range $1 \lesssim Q_{cr} \lesssim 10^{5.16}$. If $Q$ is the baryon number, stability with respect to the decay into protons requires $Q \gtrsim 10^{17} Q_{cr}$, while if the gravitino mass is greater then $m_{3/2} \gtrsim 61$ MeV, no stable gauge-mediation supersymmetric Q-balls exist. Finally, we find that energy and radius of Q-balls can be parameterized as $E \sim \xi E Q^{3/4}$ and $R \sim \xi R Q^{1/4}$, where $\xi_E$ and $\xi_R$ are slowly varying functions of the charge.

PACS numbers: 05.45.Yv, 95.35.+d, 98.80.Cq

I. INTRODUCTION

Cosmological and astrophysical observations indicate, almost undoubtedly, that our Universe is pervaded by a not-yet-known pressureless component, called “Dark Matter”\textsuperscript{1}. Its energy density amount to about the 30\% of the entire energy density of the universe.

Promising non-baryonic, dark-matter candidates have been proposed since the discover of this mysterious component, such as axion or neutralino \textsuperscript{1}.

Recently enough \textsuperscript{2}, a particular class of non-topological solitons arising in supersymmetric extensions of the Standard Model, know as supersymmetric Q-balls \textsuperscript{3}, have been proposed as possible solution to the dark matter problem.

In particular, Q-balls admitted in supersymmetric models with low-energy, gauge-mediated symmetry breaking \textsuperscript{3}, are a plausible candidate for baryonic dark matter \textsuperscript{3}.

Q-balls \textsuperscript{4} are lumps of matter, precisely a coherent state of a complex scalar field, carrying a conserved global charge. In the context of supersymmetric extensions of the Standard Model, the charge $Q$ is some combination of baryon and lepton numbers, while the the scalar field is a gauge-singlet combination of squarks and sleptons corresponding to some flat direction of the supersymmetric potential \textsuperscript{3}. In this class of models, supersymmetry is spontaneously broken at the scale $\Lambda_{\text{SUSY}} \sim 10^3 \text{GeV}$ \textsuperscript{3}; as a result, an effective potential for the flat directions arises \textsuperscript{9} which, in turn, admits Q-balls as the non-perturbative ground state of the theory.

A deep investigation of general properties \textsuperscript{8} and astrophysical implications \textsuperscript{9} of Q-balls has been carried out in the last two decades.

Up to now, however, the main properties of gauge-mediation supersymmetric Q-balls have been analyzed using an approximate expression of the potential giving rise to Q-balls.

The aim of this paper is to study such a kind of Q-balls taking into account the exact form of the supersymmetric scalar potential quoted in Ref. \textsuperscript{7}. We find that the expressions for energy and radius of Q-balls, which fully characterize their properties from a cosmological and astrophysical viewpoint, can differ from the approximate case of about an order of magnitude.

II. Q-BALLS: GENERAL PROPERTIES

In this Section, we briefly review the Q-ball solution of a scalar theory with a global $U(1)$ symmetry \textsuperscript{6}. We consider a charged scalar field $\phi$ whose lagrangian density is given by

$$\mathcal{L} = (\partial_{\mu} \phi^*) (\partial^\mu \phi) - U(|\phi|^2).$$

In the next Section, we will identify $\phi$ with one of the flat directions in supersymmetric extensions of the standard model, and will specify the form of the potential $U(|\phi|)$ which is not relevant in the present discussion. For the moment, we simply require its invariance under a global $U(1)$ symmetry. The corresponding conserved Noether charge $q$ is normalized as

$$q = \frac{1}{2i} \int d^4x (\phi^\ast \dot{\phi} - \phi \dot{\phi}^*) ,$$

where a dot indicates a derivative with respect to time. (Throughout this paper, we follow the conventions of Ref. \textsuperscript{10}). For a given field configuration $\phi(t, r)$, the total energy is given by

$$E = \int d^3x \left[ \frac{1}{2} |\dot{\phi}|^2 + \frac{1}{2} |\nabla \phi|^2 + U(|\phi|^2) \right].$$
We are interested to solutions of the field equations that correspond to a fixed value of the charge, namely \( Q \), in Eq. (2). This is properly achieved by the introduction of the Lagrange multiplier \( \omega \) associated to \( q \), and by the requirement that the physical configuration makes the functional

\[
\mathcal{E}_\omega \equiv E + \omega \left[ Q - \frac{1}{2v} \int d^3x \left( \varphi^* \varphi - \varphi^2 \right) \right]
\]

stationary with respect to independent variations of \( \varphi \) and \( \omega \). The requirement of time-independence of the total energy \( E \) implies the choice \([6, 10]\)

\[
\varphi(t, r) = e^{i\omega t} \phi(r),
\]

with \( \phi(r) \) real. Consequently, the functional \( \mathcal{E}_\omega \) reduces to

\[
\mathcal{E}_\omega = \int d^3x \left[ \frac{1}{2} |\nabla \phi|^2 + U(\phi) - \frac{1}{2} \omega^2 \phi^2 \right] + \omega Q.
\]

The physical solutions have to satisfy the constraints

\[
\delta \mathcal{E}_\omega \over \delta \phi = 0, \quad \delta \mathcal{E}_\omega \over \delta \omega = 0.
\]

The first constraint leads to the equation of motion of the field \( \phi(r) \),

\[
d^2 \phi \over dr^2 + 2 \frac{d \phi}{d r} + \omega^2 \phi = \frac{\partial U}{\partial \phi}.
\]

where, for simplicity, we assumed isotropy: \( \phi = \phi(r) \) with \( r \equiv |r| \).

The second constraint is equivalent to the requirement that the charge corresponding to the solution of the equation of motion is equal to \( Q \).

A Q-ball is defined as the solution \( \phi(r) \) of Eq. (8) satisfying, at fixed charge \( Q \), the boundary conditions \( \phi(r \to \infty) = 0 \) and \( d \phi / d r(r = 0) = 0 \) \([4]\).

In the next Section, we will analyze Q-ball configurations arising in a supersymmetric model where supersymmetry is broken via low-energy gauge mediation \([4]\).

### III. SUPERSYMMETRIC Q-BALLS

We consider a supersymmetric model in which supersymmetry is broken by low-energy gauge mediation \([4]\). In this kind of model the coupling of the massive vector-like messenger fields to the gauge multiplets, with coupling constant \( g \sim 10^{-2} \), leads to the breaking of supersymmetry \([4]\). The coupling itself gives rise to an effective potential for the flat direction \( \phi \) whose lowest order (two-loop) contribution has been calculated in Ref. \([7]\):

\[
U(z) = \Lambda \int_0^1 dx \frac{z^2 - x(1 - x) + x(1 - x) \ln[x(1 - x)z^2]}{z^2 - x(1 - x)^2},
\]

where \( z \equiv \phi / M \) and \( M \equiv M_S / (2g) \), with \( M_S \) the messenger mass scale. The value of the mass parameter \( \Lambda^{1/4} \) is constrained as (see, e.g., Ref. \([11]\)):

\[
10^9 \text{GeV} \lesssim \Lambda^{1/4} \lesssim (g^{1/2}/4\pi) \sqrt{m_{3/2} M_P}, \nonumber
\]

where \( M_P \sim 2.4 \times 10^{18} \text{GeV} \) is the reduced Planck mass and \( m_{3/2} \), the gravitino mass, is in the range \( 100 \text{keV} \lesssim m_{3/2} \lesssim 1 \text{GeV} \) \([7, 11]\).

The asymptotic expressions of \( U(z) \), for small and large \( z \) are \([7]\):

\[
\frac{U(z)}{\Lambda} \simeq \begin{cases} 
 z^2, & \text{if } z \ll 1, \\
 (\ln z^2)^2 - 2 \ln z^2 + \frac{1}{4}, & \text{if } z \gg 1,
\end{cases}
\]

where \( m = \sqrt{2} \Lambda / M \) is the soft breaking mass and is of order 1 TeV \([11]\). In Fig. 1, we plot the potential \( U(z) \) with its asymptotic expansions \([10]\).

Defining the critical charge \( Q_{\text{cr}} \equiv \Lambda / m^4 \) (whose meaning will be clear in the following), the constraint on \( \Lambda \) can be translated to a constraint on \( Q_{\text{cr}} \), namely:

\[
\left( \frac{\text{TeV}}{m} \right)^4 \simeq Q_{\text{cr}} \lesssim 10^8 \left( \frac{g}{10^{-2}} \right)^2 \left( \frac{\text{TeV}}{m} \right)^4 \left( \frac{m_{3/2}}{100 \text{keV}} \right)^2.
\]

A widely used approximation consists in replacing the full potential \( U(z) \) with its asymptotic expansions \([10]\) in which a plateau plays the role of the logarithmic rise for large values of \( z \).

Within this approximation, it has been shown that the potential \( U(z) \) allows Q-balls solutions as the non perturbative ground state of the model \([3, 12]\). Such states are known as supersymmetric Q-balls. In particular, for large charges, \( Q \gg Q_{\text{cr}} \), one can deduce analytically the most important characteristics of Q-ball solutions. In more detail, the Q-ball profile is given by \([3, 12]\):

\[
\phi(r) \simeq \phi_0 \sin(\omega r) / (\omega r) \quad \text{for } r \leq R , \quad \text{and zero for } r \geq R , \nonumber
\]

where \( R = \pi / \omega \) is the radius of the Q-ball, and \( \phi_0 \equiv \phi(0) \).
Moreover, one has [3, 12]

\[
\frac{\omega}{m} \approx \sqrt{2\pi} \left( \frac{Q}{Q_{\text{cr}}} \right)^{-1/4},
\]

(12)

\[
\frac{E}{mQ_{\text{cr}}} \approx \frac{4\sqrt{2\pi}}{3} \left( \frac{Q}{Q_{\text{cr}}} \right)^{3/4},
\]

(13)

\[
\frac{R}{m^{-1}} \approx \frac{1}{\sqrt{2}} \left( \frac{Q}{Q_{\text{cr}}} \right)^{1/4},
\]

(14)

\[
\frac{\phi_0}{M} \approx \frac{1}{\sqrt{2}} \left( \frac{Q}{Q_{\text{cr}}} \right)^{1/4}.
\]

(15)

Although the previous (simplified) analysis reveals the major properties of supersymmetric Q-balls, which are widely used in the literature in a cosmological and astrophysical context, we wish to study them by taking into account the full potential [9]. Since the form of the potential is involved, we need to solve the problem numerically.

The computational procedure has been depicted in the previous Section. From a numerical viewpoint, it is simpler to fix \( \omega \) rather than the total charge \( q \). Then, once \( \omega \) is fixed, we solve the equation of motion \([5]\) with the condition \( d\phi/dr(r = 0) = 0 \). We look for the value of \( \phi_0 \) such that the Q-ball solution exists; once this is achieved we insert \( \phi(r) \) in Eqs. (2) and (3), obtaining the values of the charge and energy. Finally, we define the “radius” of the Q-ball, \( R \), such that \( \phi(R)/\phi_0 = 0.1 \).

In Fig. 2, we plot the quantity \( E/mQ \) as a function of the charge. If the energy \( E \) of the Q-ball at fixed charge \( Q \) is less than \( mQ \), the soliton decays into \( Q \) quanta of the field (the perturbative spectrum of the theory), each of them with mass \( m \). Instead, if \( E < mQ \) the Q-ball is said to be classically stable, and then represents the ground state of the theory.

Numerically, we find classical stability, \( E/mQ < 1 \), for \( Q > Q_{\text{min}} \), with \( Q_{\text{min}} \simeq 504Q_{\text{cr}} \).

In Fig. 3, we plot \( \omega \), the energy, the radius \( R \), and \( \phi_0 \) as a function of the charge in the interval \( Q \in [Q_{\text{min}}, 7.2 \times 10^{37}Q_{\text{cr}}] \).

If \( Q \) is the baryon number, stability with respect to the decay into protons requires the energy of a Q-ball to be less than \( E < m_pQ \), where \( m_p \simeq 1\text{GeV} \) is the proton mass. We find numerically that this is attained.
TABLE I: Nonlinear fit of the functions ξ’s defined in Eqs. (16)-(19) using a power-function of the type ξ(x) = (a + bx^p)^q. The parameters ε1 and ε2 represent the maximum percentage error of the functions ξ’s with respect to their numerical values in the range Q ∈ [Q_{min}, 7.2 \times 10^{37}Q_{cr}] and Q ∈ [10^{17}Q_{cr}, 7.2 \times 10^{37}Q_{cr}] respectively.

|   | a    | b    | p    | q    | ε1  | ε2  |
|---|------|------|------|------|-----|-----|
| ω | -9.567 | 9.686 | 0.381 | 1 | 1.32% | 0.16% |
| E | -17.438 | 15.559 | 0.352 | 1 | 2.08% | 0.16% |
| R | -7.162 | 4.300 | 0.560 | -0.712 | 2.76% | 0.24% |
| φ_0 | -2.324 | 1.292 | 0.616 | 0.668 | 2.60% | 0.23% |

for charges larger then Q \gtrsim 4.1 \times 10^{17}Q_{cr}, where we assumed m = 1\text{TeV}.

We now wish to compare our numerical results to those obtained in the flat-potential approximation. Inspired by Eqs. (12)-(15), we write the quantities characterizing the Q-ball solution in the following way:

\[
\frac{\omega}{m} = \xi_\omega \left( \log_{10} \frac{Q}{Q_{cr}} \right) \left( \frac{Q}{Q_{cr}} \right)^{-1/4},
\]

\[
\frac{E}{mQ_{cr}} = \xi_E \left( \log_{10} \frac{Q}{Q_{cr}} \right) \left( \frac{Q}{Q_{cr}} \right)^{3/4},
\]

\[
\frac{R}{m^{-1}} = \xi_R \left( \log_{10} \frac{Q}{Q_{cr}} \right) \left( \frac{Q}{Q_{cr}} \right)^{1/4},
\]

\[
\frac{\phi_0}{M} = \xi_\phi \left( \log_{10} \frac{Q}{Q_{cr}} \right) \left( \frac{Q}{Q_{cr}} \right)^{1/4}.
\]

The functions ξ’s (which depend only logarithmically on the charge Q) parameterize the deviation from the simple power-laws (12)-(15), and are shown in Fig. 4. We fit their numerical values by the power-function

\[
\xi(x) = (a + bx^p)^q.
\]

In Table I, we report the values of the coefficients a, b, p, and q found by least-squaring the numerical data. We also show the maximum percentage error of the functions ξ’s with respect to their numerical values. In particular, ε1 and ε2 refer to the maximum percentage errors in the ranges Q ∈ [Q_{min}, 7.2 \times 10^{37}Q_{cr}] and Q ∈ [10^{17}Q_{cr}, 7.2 \times 10^{37}Q_{cr}], respectively.

In is worth noting that, in the flat-potential approximation, the functions ξ’s are constants whose values differ from the numerical results of about an order of magnitude in the limit of large charges (say Q \gg 10^{17}Q_{cr}).

In Fig. 5, we show the Q-ball profile for three different values of the charge. We observe that to the charges Q = Q_{min} \approx 504Q_{cr} and Q \approx 10^6Q_{cr} there corresponds the same value of radius, namely R \approx 11m^{-1} (see also the third panel in Fig. 3). However, looking at the shapes of the corresponding profiles, we see that in the first case the profile is spreader than the second one, indicating a larger “wall-thickness” of the Q-ball. Indeed, the larger the charge, the smaller is the wall-thickness, so that, as the charge increases the Q-ball approaches to the so-called “thin-wall” regime [3, 6] (see continuous line in Fig. 5).

As pointed out in Ref. [7], for large values of the field φ, supergravity effects become important and give a con-
tribution to the scalar potential of the form $U_{\text{gravity}}(\phi) \simeq m_3^2/2\phi^2$, approximatively. When this contribution dominates, $U_{\text{gravity}}(\phi) \gg U(\phi)$, Q-ball properties change drastically. Indeed, a different type of stable Q-balls are generated, the so-called “New-type Q-balls” [13]. The energy-charge relation is, in this case, $E_{\text{new-type}} \sim m_3/2Q$ (if $Q$ is the baryon number, being $m_p > m_3/2$, the Q-ball is also stable with respect to the decay into protons).

However, it is beyond the aim of this paper to study the numerical properties of such kind of Q-balls, and then we demand that the gauge-mediated potential dominates over the gravity-mediation one.

Defining $\phi_{\text{eq}}$ such that $U'(\phi_{\text{eq}}) = U'_{\text{gravity}}(\phi_{\text{eq}})$, it results $U'(\phi) \geq U'_{\text{gravity}}(\phi)$ for $\phi \leq \phi_{\text{eq}}$, where a prime indicates differentiation with respect to $\phi$. Here, following Ref. [7], we have compared the derivatives of gauge- and gravity-mediation potentials in order to determine their relative importance, since these are the quantity entering into the equation of motion [5].

It is useful to introduce the “maximum charge” $Q_{\text{max}}$ such that, if $Q \leq Q_{\text{max}}$ then $\phi_0 \leq \phi_{\text{eq}}$. Since $\phi_0 = \max \phi$ for a Q-ball configuration, the gravity effects can be neglected when $Q < Q_{\text{max}}$. In Fig. 6, we plot $Q_{\text{max}}$ as a function of the gravitino mass. It is easy to see that, neglecting logarithmic terms in the gauge-mediation potential, one finds $U(\phi) \geq U_{\text{gravity}}(\phi)$ for $Q \leq Q_{\text{max}} \simeq (m/m_3/2)^4$ [where we used Eq. [15]]. Therefore, it is convenient to write the maximum charge as

$$Q_{\text{max}} \simeq \frac{m}{Q_{\text{cr}}} \left( \log_{10} \frac{m}{3/2} \right)^4 \left( \frac{m}{m_3/2} \right)^4,$$  \hspace{1cm} (21)$$

where $\xi_Q(x)$ takes on the same form as in Eq. [20]. By least-squaring the numerical data, we find $a \simeq 3.260$, $b \simeq 5.750$, $p \simeq -1.311$, and $q \simeq 1.209$, with a maximum percentage error on $Q_{\text{max}}$ of about 0.06%. Moreover, we find $Q_{\text{max}} \simeq 4.9 \times 10^{28}$ and $Q_{\text{max}} \simeq 6.4 \times 10^{12}$ for $m_3/2 = 100 \text{keV}$ and $m_3/2 = 1 \text{GeV}$, respectively.

If $Q$ is the baryon number, it is interesting to observe that, depending on the value of the gravitino mass, it can be $Q_{\text{max}} \lesssim 4.1 \times 10^{17} Q_{\text{cr}}$, indicating that no stable Q-ball solution exists. Numerically, we find this the happens for gravitino masses above $m_3/2 \gtrsim 60.8 \text{MeV}$.

### IV. CONCLUSIONS

In this paper, we have studied Q-balls-type solutions admitted in supersymmetric particle-physics models with low-energy, gauge-mediated supersymmetry breaking. Taking into account the exact form of the supersymmetric potential, we have analyzed classical stability of Q-balls. We have found, numerically, that only Q-balls with charge $Q \gtrsim 5 \times 10^2 Q_{\text{cr}}$ are stable against the decay into quanta constituting the perturbative spectrum of the theory. Here, the “critical charge” $Q_{\text{cr}}$ is a model-dependent parameter given in Eq. [11]. Moreover, if the conserved charge $Q$ is the baryon number, stability with respect to the decay into protons (the lightest baryonic particle) requires $Q \gtrsim 10^{17} Q_{\text{cr}}$.

Although no analytical expressions for the quantity characterizing Q-ball solutions can be found, we were able to approximate the numerical results by suitable functions: we have found, indeed, that energy and radius of Q-balls, which fully characterize their properties from a cosmological and astrophysical viewpoint, can be parameterized as $E \sim \xi_E Q_3^{3/4}$ and $R \sim \xi_R Q_3^{1/4}$, where $\xi_E$ and $\xi_R$ are slowly varying functions of the charge [see Eqs. [17]-[15], Eq. [20], and Table I].

In the (approximate) case of exactly flat potential considered in the literature, the functions $\xi$’s are constants whose values can differ from our results of about an order of magnitude.

For large values of the scalar condensate defining a supersymmetric Q-ball, supergravity effects become important and give a contribution to the scalar potential which, in turn, change drastically the Q-ball properties. Introducing the “maximum charge” $Q_{\text{max}}$ such that, if
$Q < Q_{\text{max}}$ supergravity effects are negligible, we have found that $Q_{\text{max}}$, as a function of the gravitino mass, can be well approximated by $Q_{\text{max}} \sim \xi Q_{m}^{-4/3}$, with $\xi$ a slowly varying functions of $m_{3/2}$ [see Eq. (21)]. In particular, if $Q$ is the baryon number we have found that, for gravitino masses above $m_{3/2} \gtrsim 61 \text{MeV}$, the maximum charge is less then the charge required for classical stability, indicating that no stable gauge-mediation supersymmetric Q-balls exist.

[1] For reviews on Dark Matter see: G. Jungman, M. Kamionkowski and K. Griest, Phys. Rept. 267, 195 (1996); G. Bertone, D. Hooper and J. Silk, Phys. Rept. 405, 279 (2005).
[2] A. Kusenko and M. E. Shaposhnikov, Phys. Lett. B 418, 46 (1998).
[3] G. R. Dvali, A. Kusenko and M. E. Shaposhnikov, Phys. Lett. B 417, 90 (1998).
[4] M. Dine and A. E. Nelson, Phys. Rev. D 48, 1277 (1993); M. Dine, A. E. Nelson and Y. Shirman, Phys. Rev. D 51, 1362 (1995); M. Dine, et al., Phys. Rev. D 53, 2658 (1996).
[5] For a review on Q-balls as Dark Matter see: A. Kusenko, [hep-ph/0009089] Invited talk at 3rd International Conference on Dark Matter in Astro and Particle Physics (Dark 2000), Heidelberg, Germany, 10-16 Jul 2000. Published in *Heidelberg 2000, Dark matter in astro- and particle physics* 306-315.
[6] S. R. Coleman, Nucl. Phys. B 262, 263 (1985) [Erratum-ibid. B 269, 744 (1986)].
[7] A. de Gouvea, T. Moroi and H. Murayama, Phys. Rev. D 56, 1281 (1997).
[8] A. G. Cohen, et al., Nucl. Phys. B 272, 301 (1986); K. M. Lee, et al., Phys. Rev. D 39, 1665 (1989); A. Kusenko, M. E. Shaposhnikov and P. G. Tinyakov, Pisma Zh. Eksp. Teor. Fiz. 67, 229 (1998); [JETP Lett. 67, 247 (1998)]; T. Multamaki and I. Vilja, Nucl. Phys. B 574, 130 (2000); M. Axenides, et al., Phys. Rev. D 61, 085006 (2000); S. Theodorakis, Phys. Rev. D 61, 047701 (2000); R. Battye and P. Sutcliffe, Nucl. Phys. B 590, 329 (2000); F. Paccetti Correia and M. G. Schmidt, Eur. Phys. J. C 21, 181 (2001); N. Graham, Phys. Lett. B 513, 112 (2001); T. A. Ioannidou, A. Kouiroukidou and N. D. Vlachos, J. Math. Phys. 46, 042306 (2005); S. Clark, [arXiv:0706.1429] [hep-th].
[9] K. Enqvist and J. McDonald, Phys. Lett. B 425, 309 (1998); S. Kasuya and M. Kawasaki, Phys. Rev. D 61, 043010 (2000); A. Kusenko and P. J. Steinhardt, Phys. Rev. Lett. 87, 141301 (2001); T. Multamaki and I. Vilja, Phys. Lett. B 555, 170 (2002); M. Fuji and K. Hamaguchi, Phys. Lett. B 525, 143 (2002); M. Postma, Phys. Rev. D 65, 085035 (2002); K. Enqvist, et al., Phys. Lett. B 526, 9 (2002); M. Kawasaki, F. Takahashi and M. Yamaguchi, Phys. Rev. D 66, 043516 (2002); A. Kusenko, L. Loveridge and M. Shaposhnikov, Phys. Rev. D 72, 025015 (2005); Y. Takenaga et al. [Super-Kamiokande Collaboration], Phys. Lett. B 647, 18 (2007); S. Kasuya and F. Takahashi, [arXiv:0709.2634] [hep-ph].
[10] A. Kusenko, Phys. Lett. B 404, 285 (1997).
[11] S. Kasuya and F. Takahashi, Phys. Rev. D 72, 085015 (2005).
[12] A. Kusenko, L. C. Loveridge and M. Shaposhnikov, JCAP 0508, 011 (2005).
[13] S. Kasuya and M. Kawasaki, Phys. Rev. Lett. 85, 2677 (2000); Phys. Rev. D 62, 023512 (2000); Phys. Rev. D 64, 123515 (2001).