1. Introduction

In modern development process of rail vehicle computer aided simulations are employed. In this way costly experiments and prototypes can be reduced. Production of a rail vehicle is composed of several phases. There is a design phase, a development and optimisation phase, production of a rail vehicle, verification and validation of a rail vehicle and, in the end, commissioning of a rail vehicle. At this time computer software allows to perform complex simulations. Thus, shorter development periods and rising requirements like durability, efficiency or mass reduction demand precise simulations, which intensify the usage of lightweight structures.

2. Principles of rail vehicle multibody system with a flexible body

The need for more accurate models of a rail vehicle to describe the complex behaviour of flexible systems experiencing large motion while undergoing small elastic deformations motivated the development of many powerful analysis techniques. The most popular formulations use time-variant mass matrices to describe the inertia coupling between the rigid body motion and the elastic deformation [1].

To describe the dynamic behaviour of a rail vehicle mechanical system which undergoes large nonlinear working motions the multibody system (MBS) approach is often most useful [2]. A classic MBS of a rail vehicle consists of rigid elements which are connected by ideal joints, coupling elements [3], contact elements [4 and 5] and force elements. The phenomena of the wheel/rail contact [6] significantly influence the rail vehicle properties and wheel/rail contact stress evaluation [7 and 8].

For applications in the field of rail vehicle analysis where the deformation of the bodies cannot be neglected, the method of flexible multibody systems has to be applied. In the flexible multibody system of a rail vehicle the approach is extended by flexible bodies.

2.1 Foundations of flexible multibody dynamics

For the kinematic description of the motion of flexible bodies that are subjected to large displacements several methods are used. Among these methods are, for example, the floating frame of reference, convected coordinate system, finite segment method and large rotation vector. Large deformation problems in flexible

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multibody system can be efficiently solved using the absolute nodal coordinate formulation [10].

In the absolute nodal coordinate formulation, neither infinitesimal nor finite rotations are used as the element coordinates. The locations and deformations of the material points on the finite element are defined in the global coordinate system using the element shape function and nodal coordinates (Fig. 1).

$$u_R^t + u_P^t(j)$$

$$\Phi(R) \cdot q_r(t), \Psi(R) \cdot q_r(t)$$. (2)
where $q_e(t)$ refers to the nodal displacements of a finite element model and $\Phi(R)$ and $\Psi(R)$ are the elastic shape functions [9]. This results in the equation of motion of a flexible body

$$M_e \cdot \ddot{q}_e(t) + K_e \cdot q_e(t) = h_e,$$

(3)

as formulated in [9]. The matrices $M_e$ and $K_e$ are the mass and stiffness matrices of the flexible structure and have the following characteristics if the system is constrained sufficiently to avoid rigid body motion

$$M_e = M_e^r > 0, \quad K_e > 0,$$

(4)

The generalised surface and volume forces are summarised in the force vector $h_e$. To consider the dissipative effects an additional damping matrix $D_e$ is often introduced and can be approximated, e.g. by viscous damping (Rayleigh damping):

$$D_e = \alpha M_e + \beta K_e,$$

(5)

with the proportional factors $0 \leq \alpha, \beta \in \mathbb{R}$. The need for high precision and complex geometries often leads to a fine spatial discretisation. Mathematically the flexible bodies are described by a large set of linear ordinary differential equations, whose solution increases the computational effort of the simulation. Linear model reduction is a decisive component to efficient simulation. To get a representation used for some model reduction techniques the forces acting on the finite element structure are described by the time dependent excitation $u_e(t)$ and the input or control matrix $B_e \in \mathbb{R}^{m \times p}$. This matrix captures the spatial distribution of the boundary and coupling conditions. Further on, the output or observation matrix $C_e \in \mathbb{R}^{n \times q}$ is introduced for the calculation of the interesting displacements $y(t)$. In this case, equations of motion of a single flexible body can be formulated as a linear time-invariant second order multi input multi output system:

$$M_e \cdot \ddot{q}_e(t) + D_e \cdot \dot{q}_e(t) + K_e \cdot q_e(t) = B_e \cdot u_e(t),$$

(6)

$$y(t) = C_e \cdot q_e(t).$$

(7)

Due to increasing demands on the characteristics of technical products and their simulation, the requirements on the calculation accuracy and the calculation time are often extremely high. On the bottom line this trend means that the dimension of the equation of motion rises and the time to run the simulation should be as quick as possible. Such problems particularly require an adequate model order reduction to decrease the number of equations and keep the significant characteristics of the system. Using the floating frame of reference formulation and the linear model order reduction via projection of equation of motion (3), (5), (6) (see [9 and 11]) can be used. Therefore, the large number of degrees of freedom of the flexible coordinates $q_e \in \mathbb{R}^{n \times 1}$ are approximated in a subspace $\mathcal{V}_s$ of smaller dimension $n < N$ by the reduced displacement vector $\tilde{q}_e \in \mathbb{R}^{n \times 1}$

$$q_e \approx V \cdot \tilde{q}_e.$$

(8)

This subspace $\mathcal{V}_s$ is described by the projection matrix $V \in \mathbb{R}^{N \times n}$. The use of this relation in FEM equations of motion (5) and (6) leads to an over-determined system and leaves a residuum because the exact solution $q_e$ is generally not an element of the subspace $\mathcal{V}_s$. To obtain a unique solution the residual should be orthogonal on a second subspace $\mathcal{W}_s$ represented by $W \in \mathbb{R}^{n \times s}$. The orthogonality conditions or Petro-Galerkin conditions result in the reduced FE equations

$$\bar{M}_e \cdot \bar{\ddot{q}}_e + \bar{D}_e \cdot \bar{\dot{q}}_e + \bar{K}_e \cdot \bar{q}_e = \bar{B}_e \cdot u_e,$$

(9)

with the reduced matrices $\bar{M}_e := W^\top \cdot M_e \cdot V$, $\bar{D}_e := W^\top \cdot D_e \cdot V$, $\bar{K}_e := W^\top \cdot K_e \cdot V \in \mathbb{R}^{n \times n}$ and $\bar{B}_e := W^\top \cdot B_e \in \mathbb{R}^{n \times p}$, $\bar{C}_e := C_e \cdot V \in \mathbb{R}^{n \times q}$.

The projection is called orthogonal if the subspaces are identical $V = W$ and oblique otherwise. This procedure leads to the reduced equations of motions of one flexible body:

$$\begin{bmatrix} M_e & M_e \cdot V \\ W^\top \cdot M_e & \bar{M}_e \end{bmatrix} \begin{bmatrix} \tilde{q}_e \\ \tilde{\dot{q}}_e \end{bmatrix} + \begin{bmatrix} \bar{K}_e \cdot \tilde{q}_e + \bar{D}_e \cdot \tilde{\dot{q}}_e \end{bmatrix} = \begin{bmatrix} h_e \\ W^\top \cdot h_e \end{bmatrix}.$$

(10)

A task of different reduction techniques is to find the projection matrices $V$ and $W$.

4. Approach for the reduction of rail vehicle parts

For the simulation of the rail vehicle multibody system with flexible bodies some preprocessing steps for obtaining a reduced flexible body are necessary. It is possible to make in FEM software, for example in Ansys [14 and 15]. Ansys software allows engineers to construct computer models or structures, machine components or system, apply operating loads and other design criteria and study physical responses [16 and 17]. This software also allows to reduce flexible bodies for import into MBS software.

The general process to integrate flexible bodies into the rail vehicle multibody system consists of several operations:

- setting up the finite element model,
- integrating the finite element model into the MBS software,
- setting up the MBS model of the rail vehicle.

It is needed to reduce the size (number of freedom) of FEM model before working with the MBS interface. For this it is needed to perform several operations:

- define the interface nodes. The MBS interacts with the FEM superelement on these nodes.
- connect the interface nodes with structure. In Ansys it is recommended to use the following elements types:
Once the FEM model of a part of the rail vehicle is reduced, the input files generation for MBS software is required. The file with the flexible body input data is necessary for including flexible bodies in the MBS software. After loading the file with the FEM input data into the MBS software it is possible to define the interaction between flexible body and MBS system by using joints, constraints or force elements which apply loads to the flexible body. The flexible body deformation is caused by these loads [15].

For the needs of a rail vehicle simulation the FEM model of a bogie frame of a freight wagon was created. This is the most commonly used bogie in the Central and Eastern Europe – the Y25 bogie [19, 20 and 21].

4.1 Reduction of the bogie frame

In this section the procedure of preparation of the flexible model of the bogie frame is described.

The CAD model of the bogie frame was imported into the FEM software. For the preparation of the FEM data the software

- **rigid body element** - interface nodes have independent DOFs, coupling nodes on the FE structure have dependent degrees of freedom, dependent nodes perform rigid body motion only and independent node (interface node) defines this rigid body motion. Element types of rigid body elements for Ansys FEM code are CE, CERIG, MPC184 and RBE 2.

- **force distributing constraints** - interface node has dependent DOFs, coupling nodes on the FE structure have independent DOFs, motion at the interface node is the weighted average of the motion of the coupling nodes, forces and moments at the reference node are distributed either as forces or moments at the coupling nodes. Element types of force distributing constraints for Ansys FEM code are TARGE 170+CONTA173, TARGE 170+CONTA174.

- define the coupling nodes as retained nodes,
- define the retained DOFs. This step is important in the reduction process for yielding accurate superelement matrices [18].

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*Fig. 3 FEM model of the bogie frame with interface nodes (INode)*

*Fig. 4 Interface node (INode) on the friction surface (left) and constraint equations (right)*
simulation allows better optimization of rail vehicles design as well as prevention of potential problems during their long-term operation.

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