Security region-based small signal stability analysis of power systems with FSIG based wind farm

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Abstract. Based on the Security Region approach, the impact of fixed-speed induction generator based wind farm on the small signal stability of power systems is analyzed. Firstly, the key factors of wind farm on the small signal stability of power systems are analyzed and the parameter space for small signal stability region is formed. Secondly, the small signal stability region of power systems with wind power is established. Thirdly, the corresponding relation between the boundary of SSSR and the dominant oscillation mode is further studied. Results show that the integration of fixed-speed induction generator based wind farm will cause the low frequency oscillation stability of the power system deteriorate. When the output of wind power is high, the oscillation stability of the power system is mainly concerned with the inter-area oscillation mode caused by the integration of the wind farm. Both the active power output and the capacity of reactive power compensation of the wind farm have a significant influence on the SSSR. To improve the oscillation stability of power systems with wind power, it is suggested to reasonably set the reactive power compensation capacity for the wind farm through SSSR.

1. Introduction
Increasing environmental pollution and worsening energy crisis make it a consensus to explore and utilize the sources of clean and renewable energy. As one of the most technology-matured and the most potential renewable energy, wind power has been widely developed and utilized around the world [1][2]. Compared with traditional coal-fired plants, modern wind farms widely adopt fixed-speed induction generators, doubly-fed induction wind generators and direct-drive permanent magnet generators, the static and dynamic characteristics of which are quite different from synchronous generators [3]. With the integration and long-distance transmission of large-scale wind power, the secure and stable operation of power grid is faced with more severe challenges.

Small signal stability of power systems has attracted broad attention from the electric operators at home and abroad [4]. At present, the domestic and foreign scholars have carried out extensive research around the small signal stability of power systems with wind power [5]. Generally speaking, the existing
research methods can be roughly divided into the eigenvalue analysis method [6]-[8], the frequency domain method [9][10], the time domain simulation method [11][12] and the security region method [13]-[15]. The first three methods can only analyze the security and stability of the power system under a specific operation mode (such as the output level of generators, load level, etc.), that is to say, for these three methods, once the operation mode is changed, operators have to recalculate and reconfirm whether or not the power system is stable under the new operating mode. Therefore, these three methods belong to the category of “point-by-point” method. Wind power is variable, intermittent and uncertain, which makes the operating modes of power systems become more complex and volatile. By means of “point-wise” method, we can only make the security and stability analysis in terms of several particular operating modes, but not all the possible operating modes of power systems. But from the perspective of security region, we can perform comprehensive analysis of the security and stability of power systems under various operation modes. In this sense, the method of security region has obvious advantages and significant meaning.

The main contributions of this paper are as follows. The Small Signal Stability Region (SSSR) of power systems with wind farm is established and the corresponding relation between the boundary of SSSR and the dominant oscillation mode of system is further studied. Studies show that the integration of wind power will deteriorate the small signal stability of the power system and the impact is also significantly influenced by the capacity of reactive power compensation of wind farms. In order to ensure the secure and stable operation of power system after the integration of wind power, it is suggested to reasonably equip the capacity of reactive power compensation in the wind farm based on the analysis of SSSR.

The rest of this paper is organized as follows. Section 2 introduces the model of Fixed-speed Induction Generator (FSIG) system. The definition and the calculation method of SSSR are given in Section 3. In Section 4, numerical examples are presented in detail. Conclusions are drawn in Section 5.

2. The Model of fixed-speed induction generator
The Fixed-speed Induction Generators is known to be inherently rugged, compact, fault tolerant and maintenance free generators[16]-[18]. However, FSIG need absorb reactive power from the power grid[19] and are less controllable, which has significant impacts on the stability of power systems[20]. As shown in figure 1, FSIG is generally composed of the wind turbine system, the gear box, the induction generator and the corresponding control system.

2.1. Wind turbine mathematical model
According to Betz law, the power characteristic of wind turbine is shown in the following equation (1). Where, \( P_w \) represents the wind power captured by wind turbine; \( A \) represents the area swept by wind turbine blades; \( R \) represents the radius of wind turbine blades; \( v \) represents the wind speed; \( \rho \) represents the air density; \( \omega_i \) represents the rotational speed of wind turbine; \( \lambda \) represents the tip speed ratio, \( \lambda = \omega_i R/v; \beta \) represents the blade pitch angle; \( C_p \) represents the wind energy conversion coefficient, which is a function of the tip speed ratio and blade pitch angle.
\[ P_w = \frac{1}{2} r AC \phi (\alpha, \beta) \nu^3 \]  

(1)

**2.2. Drive system model**

The drive system of FSIG is usually represented as a double mass model, as shown in equations (2) and (3). Where, \( \omega_g \) represents the rotational speed of generator; \( H_t \) and \( H_g \) are respectively the inertial time constant of wind turbine and generator; \( \theta_s \) represents the shafting torsional angle; \( K_s \) and \( D \) are respectively the shaft stiffness coefficient and damping coefficient; \( T_m \) represents the mechanical torque of wind turbine; \( T_e \) represents the electromagnetic torque of generator.

\[ 2H_t p w_i = T_m - K_s q_s - D(w_i - w_g) \]  

(2)

\[ 2H_g p w_g = K_s q_s + D(w_i - w_g) - T_e \]  

(3)

\[ p q = w_i - w_g \]  

(4)

\[ T_m = \frac{P_e}{w_i} \]  

(5)

**2.3. Induction generator model**

With the stator transients being neglected, the dynamic characteristics of induction generator can be expressed by equations (6) and (7). Where, \( T_0 \) represents the transient open circuit time constant of induction generator, \( X_s \) represents the transient reactance of induction generator. See also literature [4] for detailed model and corresponding derivation about the induction generator.

\[ p E_d = \frac{1}{T_0} [E_d + (X_s - X_s) i_d] s w_i E_q \]  

(6)

\[ p E_q = \frac{1}{T_0} [E_q - (X_s - X_s) i_q] - s w_i E_d \]  

(7)

The electromagnetic torque of induction generator can be calculated by equation (8).

\[ T_e = \varphi_d i_d - \varphi_q i_q \]  

(8)

**3. Small signal stability region of power systems**

**3.1. Definition**

The Small Signal Stability Region of power system is a collection of steady operation points which can maintain small signal stability of the power system. Usually the SSSR can be defined in parameter space or power injection space. In this paper, SSSR of configuration \( i \) is defined in power injection space as follows:

\[ \Omega_{SSSR} (i) = \{ p | \text{all eigenvalues of } J(p) \text{ possess negative real parts}, \ p \in \sigma^{2n} \subset R^{2n} \} \]  

(9)

Where, \( p \) represents the column vector of node active power injection of power system; \( \sigma^{2n} \) represents the node active power injection space considering the output limit of generators; \( R^{2n} \) represents the 2n-
dimensional real space; $n$ is the number of nodes injecting active power in the power network topology $i$; \( \Omega_{\text{SSSR}}(i) \) is only determined by the power network topology $i$.

So far, researchers have carried out wide exploration and extensive research on the construction techniques of SSSR, the boundary properties and relative application of SSSR and so forth. Studies have shown that the boundary of SSSR are mainly composed of three kinds of local bifurcation, namely saddle-node bifurcation, Hopf bifurcation and singularity induced bifurcation [21].

3.2. Calculation method
The method to calculate SSSR can generally be divided into two categories, namely the analytical method and the fitting method. The analytical method has the advantage of fast calculation speed, but it depends on the system model and has lower calculation accuracy. By contrast, the fitting method can guarantee more accurate domain boundary, but its calculation speed is much slower because of large quantities of calculation in the process of searching boundary points. In order to accurately understand and grasp the boundary properties of the SSSR of power system with induction generator based wind farm connected, the paper adopts the fitting method. See also literature [12] for more detailed steps of the fitting method.

4. Case studies
4.1. Introduction
Based on the PowerFactory/Digsilent software, the model of the power system with induction generator based wind farm is established and the calculation of SSSR is realized by DPL language provided by the software. The 4-generator 11-bus system is used as an example, the diagram of which is shown in figure 2. In this paper, the following two scenarios are considered: A. without wind power; B. replace synchronous generator G1 with a FSIG of the same capacity as G1. The model parameters of synchronous generator, load, grid structure and so on can be found in literature [4].

![Figure 2. Diagram of the 4-generator 11-bus system.](image)

4.2. Parameter space of security region
For Scenario A and Scenario B, perform the eigenvalue analysis respectively to get the main electromechanical oscillation modes, as shown in Table 1. As shown in the table, for Scenario A, without connecting the wind farm, there are three main electromechanical oscillation modes of which two are local oscillation modes and one is inter-area oscillation mode; for Scenario B, after the induction generator takes place of synchronous generator G1, there are four main electromechanical oscillation modes of which two are local oscillation modes and the another two are inter-area oscillation modes.

Through the eigenvalue analysis method, the most critical factors on the damping characteristics of the dominant mode after the wind farm is connected into the power grid is analyzed to determine the parameter space of SSSR. figure 3 and figure 4 respectively shows how the dominant eigenvalue of system changes with the active power output of wind farm and the capacity of reactive power compensation and gives corresponding relation curves. Graphical results show that both the active power of wind farm and the capacity of reactive power compensation have a significant influence on the stability.
behavior of low frequency oscillations in power system. Therefore, these two factors should be taken into account when calculating the SSSR of system.

Table 1. The main electromechanical oscillation modes.

| Scenario | Mode  | Frequency (Hz) | Damping Ratio | Involved Units |
|----------|-------|----------------|---------------|----------------|
| A        | 1     | 1.0003         | 0.08978       | G3/G4          |
|          | 2     | 0.9675         | 0.09162       | G1/G2          |
|          | 3     | 0.4811         | 0.03199       | G1/G2/G3/G4    |
| B        | 2     | 0.6301         | 0.12376       | FSIG/G2       |
|          | 3     | 0.5265         | 0.15405       | FSIG/G2/G3/G4 |
|          | 4     | 0.1808         | 0.04952       | FSIG/G2/G3/G4 |

Figure 3. Real part of the dominant eigenvalue for different output of the wind farm.

Figure 4. Real part of the dominant eigenvalue for different reactive power compensation.

4.3. SSSR of Power Systems with FSIG based Wind Farm

For Scenario B, with the integration of wind farm to the power system, we select the active power of induction generator, the active power of synchronous generator G2 and the capacity of reactive power compensation of induction generator to establish the parameter space and calculate the SSSR of system in 3-dimensional space, as shown in figure 5. The results show that considering the physical limitations of node power injection, the SSSR of system is formed by several pieces of smooth edges.

In order to have a visual comparison of how SSSR changes before and after the wind farm is connected to grid as well as to reveal how the wind farm influences the stability behavior of system low frequency oscillation, we select the active power injection of generators in the sending area as key parameters and get two-dimensional cross sections of SSSR as shown in figure 6. It can be seen that when the synchronous generator is replaced by the induction generator of the same capacity as the former, the SSSR of system is reduced significantly, that is to say, the stability behavior of system low frequency oscillation gets worse after the wind farm is connected. Besides, it is noteworthy that within the top and bottom limitations of node injection and without the integration of wind farm, the boundary of SSSR corresponds to the high output power in the sending area. Then with the integration of wind farm, the boundary of SSSR becomes two part. One corresponds to the high output power in the sending area and the other corresponds to the low output power in the sending area, which means when the wind power is low, it is also possible to cause low frequency oscillation in the system.
For the system with wind power, perform eigenvalue analysis respectively on the different boundary points of SSSR to obtain the frequency of dominant oscillation mode, as shown in the figures below. Seen from figure 6 and figure 7, it is known that considering the physical limitations of node injection, after the induction generator takes place of the synchronous generator, the SSSR of system is composed of two pieces of boundaries that respectively correspond to two different oscillation modes. When the output of wind power is low, the original inter-area oscillation mode(mode 3 in table 1) in the system acts as the key factor that mainly influences the stability behavior of system low frequency oscillation; When the output of wind power is high, power exchange among areas is mainly limited by the inter-area oscillation mode(mode 4 in table 1) caused by the interconnection of wind power. Therefore, by means of SSSR, on the one hand, dispatching operators can easily judge whether or not the system satisfies the constraints of small signal stability by checking if the operating point is in the SSSR, avoiding the complex power flow calculation and the tedious eigenvalue analysis. On the other hand, the relative position of current operating point in SSSR is useful to determine the system mode in which the low frequency oscillation is more likely to occur, thus providing suggestion on relevant control measures. Moreover, it is important to note that due to the variability of wind power, the advantages of SSSR described above are highlighted in the condition of large-scale wind power interconnection.

For Scenario B, get one point respectively on both sides of the boundaries, namely point a (540,400) and point b (720,720). Apply a small disturbance on load 7, then do the time domain simulation and the results are presented in figure 8. As for the oscillating curve of the rotational speed of generator, make the Prony analysis and obtain that the frequency of dominant oscillation mode in point a is 0.3773Hz while that of point b it is 0.1871Hz. The results are consistent with the above analysis.
In order to study the influence of the capacity of reactive power compensation in wind farm on the system low frequency oscillation, we select the active power output of wind farm and the capacity of reactive power compensation as the key parameters and then obtain the two-dimensional cross section of SSSR as shown in figure 9. The graphical results show that the capacity of reactive power compensation in wind farm has a significant influence on the SSSR of system. When the output of wind power is low, the variable range of the capacity of system reactive power compensation is larger, while the output of wind power is high, this range reduces significantly. By means of SSSR established on the parameter space of the active power output of wind farm and the capacity of reactive power compensation, it is easy to determine the capacity of reactive power compensation that maximizes the active power output of wind farm with all the stability constraints of system low frequency oscillation satisfied.

Perform the eigenvalue analysis for the critical points on the boundary of SSSR, then get the frequency of dominant oscillation mode as shown in figure 10. It turns out that with low output of wind power and high capacity of reactive power compensation, the frequency of oscillation mode corresponding to the SSSR boundary is high, while the output of wind power increases, this frequency will step down.

![Figure 9. SSSR of Scenario B.](image1)

![Figure 10. The dominant oscillation mode.](image2)

5. Conclusions
Based on the methodology of security region, the SSSR of power system with induction generator based wind farm is established in the paper, then the corresponding relation between the boundary of SSSR and the dominant oscillation mode is further studied. Results show that the wind farm takes place of the traditional synchronous power plant, the SSSR of system will narrow down, in other words, the characteristics of system low frequency oscillation deteriorate. When the output of wind power is high, the security and stability of system mainly depend on the inter-area oscillation mode caused by the connection of wind farm. Both the active power output of wind farm and the capacity of reactive power compensation have a significant influence on the SSSR of system. SSSR is an useful analytical tool and give a guideline to choose the capacity of reactive power compensation in the power grid with induction generator based wind farm connected.

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