A protocol for secure and deterministic quantum key expansion

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In all existing protocols of private communication with encryption and decryption, the pre-shared key can be used for only one time. We give a deterministic quantum key expansion protocol where the pre-shared key can be recycled. Our protocol costs less qubits and almost zero classical communication. Since the bit values of the expanded key is deterministic, this protocol can also be used for direct communication. Our protocol includes the authentication steps therefore we don’t worry about the case that Alice and Bob are completely isolated.

1. INTRODUCTION

Information processing with quantum systems enables us to do novel tasks which seem to be impossible with its classical counterpart [1–3]. Among all of the non-trivial quantum algorithms, quantum key distribution (QKD) [3–9] is one of the most important and interesting quantum information processing due to its relative low technical overhead: the only thing required there is quantum states preparing, transmission and measurement. It needs neither quantum memory nor collective quantum operation such as the controlled-NOT (CNOT) gate. Therefore, QKD will be the first practical quantum information processor [8]. QKD makes it possible for two remote parties, Alice and Bob to make unconditionally secure communications: they first build up a secure shared key and then use this key as the one-time-pad to send the private message. However, in the standard BB84 [3] protocol, at least half of the transmitted qubits are discarded due to the mismatch of preparation bases and measurement bases to the qubits. Also, the standard BB84 protocol does not include authentication. This makes it insecure in the case that Alice and Bob are completely isolated: Eavesdropper (Eve) may intercept all classical information and quantum information and the actual case there is that each of Alice and Bob are doing QKD with Eve separately.

In this Letter, we shall give an efficient protocol to expand the key deterministically or make direct communication, with authentication being included. Our protocol has the advantage of lower cost in both classical communication and quantum states transmission. Our protocol includes the authentication steps. The pre-shared key can be recycled in our protocol.

The requirement of pre-sharing a secret string is not a serious drawback of our protocol. In the case authentication is required for security, all protocols need a pre-shared secret string; in the case that authentication is thought to be unnecessary, our protocol need not pre-share anything initially: they may first use any standard QKD protocol to generate a secret random string and then use this string as the pre-shared string.

The initial version of QKD protocol [3] proposed by Bennett and Brassard is fully efficient by delaying the measurement. This delay requires the quantum memories which are very difficult technique. Another method is to assign significantly different probabilities to the different bases [10]. Although unconditional security of the scheme is given [10], it has a disadvantage that a larger number of key must be generated at one time. Roughly speaking, with the bases mismatch rate being set to \( \epsilon \), the number of qubits it needs to generate at one time is \( \epsilon^{-2} \) times of that of the standard BB84 [3]. In a recently proposed QKD protocol without public announcement of bases (PAB) [11,12], there is no measurement mismatch. However, the protocol in its present form has the disadvantage that one must make many batches of keys before any batch is used to encrypt and transmit classical message. Note that they must abort the pre-shared secret string after the key expansion. To really have an advantage in the efficiency, one should generate as many secret bits as possible at one time, by that protocol. Blindly generating too many secret bits at one time means a higher cost: First, the complexity of decoding the error correction code rises rapidly with the size of the code. Second, the quantum channel could be expensive. In practice, it could be the case that we don’t know how many secret bits are needed in the future communication. For example, a detective is sent to his enemy country Duba from the country VSA. He is scheduled to work in Duba for only one month and then come back to the headquarter in VSA. The so called secret bits will be useless after that month.

The existing protocols for quantum direct communication can save some cost of classical communications. Unfortunately, they are either insecure [18–20] or only quasisecure [21]. Moreover, all of them require quantum memory.

So far, it seems that our protocol is the unique one which has the advantage of lower cost of both quantum states transmission and classical communication while still holding the unconditional security.

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II. OUR PROTOCOLS AND SECURITY PROOF

We shall use the reduction technique. We first reduce the classical protocol to quantum protocol (the one uses perfect entangled pairs and quantum memories), and then reduce the quantum protocol back to classical protocol (the one without any entangled pair or quantum memory). We start with a trivial scheme, Protocol 1.

Protocol 1, Classical protocol
Alice and Bob share a secret key, i.e., $g$-bit random string, $G$. Alice wants to send an $N$-bit classical binary string $s$ to Bob, $g > N$. She chooses first $N$ bits from $G$ and denotes this substring as $b$. She prepares an $N$-qubit string $q$ which is in the quantum state $|b ⊕ s⟩$, and sends these $N$ qubits to Bob. Here $⊕$ is the summation modulo 2. Suppose the values of the $i$th element in string $b$ and $s$ are $b_i$ and $s_i$, respectively, given any value of $b_i ⊕ s_i$, she just prepares the $i$th quantum state $|b_i ⊕ s_i⟩$ accordingly. All qubit states in $q$ are prepared in $Z$ basis. Bob measures each of qubits in $Z$ basis and obtain an $N$-bit classical string, taking $⊕$ operation of this string and string $b$ he obtains the message string. Alice and Bob discards string $b$.

This is just classical private communication with one-time-pad. Obviously, the message string $s$ is perfectly secure no matter how noisy the quantum channel is. Though there are bit-flip errors in to the transmitted message, there is no information leakage. In this protocol, the one-time-pad cannot be recycled. Since all qubits are prepared in $Z$ basis, Eve in principle can have full information of $b⊕s$ without disturbing the quantum string $q$ at all. For the purpose of recycling the one-time-pad, we reduce it to our Protocol 2, a quantum protocol. Later on, we shall classicize Protocol 2.

Protocol 2: Secure communication with recyclable quantum one-time-pad.
Alice and Bob share $g$ pairs of (exponentially) perfect entangled pairs of $|φ^+⟩ = \frac{1}{\sqrt{2}}(|00⟩ + |11⟩)$ for convenience we shall call this pair state as EPR pair. Alice wants to send $N$-bit classical binary string $s$ to Bob. According to each individual bit information, she prepares an $N$-qubit quantum state $|s⟩$, all of them being prepared in $Z$ basis. She chooses her halves of first $N$ pairs from $g$ pairs and number them from 1 to $N$. We denote these $N$ pairs by $E$, Alice’s halves of $E$ by $E_A$, Bob’s halves of $E$ as $E_B$. To each of the $i$th qubit in $|s⟩$ and $i$th qubit in $E_A$, she takes CNOT operation with the $i$th qubit in $E_A$ being the controlled qubit and the $i$th qubit in $|s⟩$ as the target qubit. $i$ runs from 1 to $N$. She sends those $N$ target qubits to Bob. Bob takes a CNOT operation to each of the $i$th received qubit and the $i$th qubit of $E_B$, with the received qubit being the target qubit and the qubit in $E_B$ being controlled qubit. Bob takes a measurement in $Z$ basis to each of the target qubit and obtain a classical string. He uses this string as the message from Alice. The message $s$ in this protocol is as secure as that in Protocol 1.

Proof. Imagine the case that Alice measures each qubits in $E_A$ in $Z$ basis in the beginning, then protocol 2 is identical to Protocol 1. However, no one except Alice knows whether she has taken the measurement. Therefore she can choose not to measure her halves of entangled pairs. This is just Protocol 2. In Protocol 2, $N$ EPR pairs have been used as a quantum shared key, however, we don’t have to discard them after the message $s$ has been decrypted. Instead, Alice and Bob may do purification to those $N$ pairs, given the information of bit-flip rate and phase-flip rate. After the purification, the outcome pairs can be re-used as (almost) perfect entangled pairs. So the next question is on how to do the purification efficiently. The bit-flip rate is defined as the percentage of pairs which have been changed into state $|ψ^+⟩ = \frac{1}{\sqrt{2}}(|01⟩ + |10⟩)$ or state $|ψ−⟩ = \frac{1}{\sqrt{2}}(|01⟩ − |10⟩)$; phase-flip rate is defined as the percentage of pairs which have been changed into state $|φ−⟩ = \frac{1}{\sqrt{2}}(|00⟩ − |11⟩)$ or state $|ψ−⟩$. Or mathematically, if we consider the Pauli channel consisting of the following operations:

$$σ_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(1)

the channel operation $σ_x$ or $σ_y$ causes a bit-flip, the channel operation $σ_y$ or $σ_z$ will cause a phase-flip. One direct way to know the bit-flip rate and phase-flip rate is to let Alice and Bob randomly take some samples of those pairs and then measure the samples in $Z$ ( {$|0⟩, |1⟩$}) or in $X$ ( {$|±⟩ = \frac{1}{\sqrt{2}}(|0⟩ ± |1⟩)$}) basis in each side, and obtain the statistical values of those flip rates for the remained pairs. However, in testing the phase-flip rates with samples of those used EPR pairs, the corresponding message bits must be discarded because once the bit values of EPR pairs are announced, Eve has a way to attack encrypted message bits. Moreover, we want to reduce the protocol back to classical protocol therefore we don’t directly sample the entangled pairs. We can have a better way for the error test. Consider the initial state of an entangled pair and the quantum state of message bit $|χ_A⟩$,

$$|h_0⟩ = |φ^+⟩ ⊗ |χ_A⟩$$

(2)

In the most general case $|χ_A⟩ = α|0⟩ + β|1⟩$ and $|α|^2 + |β|^2 = 1$. In our protocol 2, there will be Alice’s CNOT operation, transmission and Bob’s CNOT operation to the message qubit. In transmission, the encrypted quantum state of message bit could bear a flipping error of $σ_x, σ_z$ or $σ_y$. It is easy to see that, after Bob’s CNOT operation, $σ_x$ error of transmission channel will cause a $σ_z$ error to the the message qubit only, $σ_z$ error of transmission channel will cause a $σ_x$ error to the EPR pair and $σ_y$ error to the message qubit, while $σ_y$ error of transmission channel will cause a $σ_y$ error to the EPR pair and $σ_y$ error to the message qubit. That is to say, the final state will be

$$|h_f⟩ = |φ^+⟩ ⊗ (σ_x|χ_A⟩)$$

(3)
given a $\sigma_z$ flip to the encrypted message qubit in transmission:

$$|h_f\rangle = (\sigma_z|\phi^+\rangle) \otimes (\sigma_z|\chi_A\rangle). \quad (4)$$

given a $\sigma_z$ flip to the encrypted qubit in transmission; and

$$|h_f\rangle = (\sigma_z|\phi^+\rangle) \otimes (\sigma_y|\chi_A\rangle) \quad (5)$$
given a $\sigma_y$ flip to the encrypted qubit in transmission. We now show eq.(4). The other two equations can be shown in a similar way. Consider the initial state defined by eq.(2). After the CNOT operation done by Alice, the state is changed to

$$|h_0\rangle = \frac{1}{\sqrt{2}}(|00\rangle \otimes |\chi_A\rangle) + \frac{1}{\sqrt{2}}(|11\rangle \otimes (\alpha|1\rangle + \beta|0\rangle)). \quad (6)$$

Suppose there is a phase-flip to the encrypted qubit during the transmission, the total state is then changed to

$$|h_0\rangle = \frac{1}{\sqrt{2}}(|00\rangle \otimes (\alpha|0\rangle - \beta|1\rangle)) + \frac{1}{\sqrt{2}}(|11\rangle \otimes (\alpha|1\rangle + \beta|0\rangle)).$$

After Bob take the CNOT operation, the final state is changed to

$$|h_f\rangle = |\phi^+\rangle \otimes |\chi_A\rangle = (\sigma_z|\phi^+\rangle) \otimes (\sigma_z|\chi_A\rangle). \quad (8)$$

This completes the proof. Although there could be phase-flips to the transmitted qubits, as we have shown already, in principle, there is no information leakage of the original message. Therefore we disregard those phase-flips to the message qubits. Note that the model of Pauli channel and classical statistics work perfectly here [13–15], given arbitrary channel noise, including any type of collective noise. Therefore if we know the bit-flip rate and phase-flip rate of the channel, we can deduce exactly the flipping rate of those used EPR pairs. Therefore we can simply mix some of qubits (test qubits) in transmitting the message qubits. We don’t do any CNOT operations (quantum encryption or decryption) to those test qubits. Half of the test qubits should be prepared in $X$ basis and half of the test qubits should be prepared in $Z$ basis. All of the test qubits should be mixed randomly with the message qubits. Bob needs to know the measurement bases of each qubits so as not to destroy any message qubits. Bob also needs to know which qubits are for testing and the original state of each test qubits so as to see the flip-rates of transmission. Therefore, besides $N$ EPR pairs, they must also share a classical string $b'$ for the information of bases, positions and bit values of each test qubits. Suppose after reading the test qubits, Bob finds the error rate to those test qubits in $X$ bases is $t_0$. Then they may safely assume $(t_0 + \delta)N$ phase-flips to the used EPR pair. $\delta$ is a very small number. The probability that the phase-flip rate of those used EPR pairs is larger than $t_0 + \delta$ is exponentially small. As we have shown earlier, there is not bit-flip error to the used entangled pairs. Therefore they may purify the used pairs by the standard purification protocol [13,16] which costs only $N\cdot H(t_0 + \delta)$ pairs, $H(x) = -x \log_2 x - (1-x) \log_2 x$.

Since their purpose is to re-use those pairs securely for private communication in the future instead of really reproducing the perfect EPR pairs, they need not really complete the full procedure of the purification. Instead, as it has been shown in Ref [13], except Alice herself, no one knows it if she measures all EPR pairs in $Z$ bases in the beginning of the protocol. Therefore the CSS code can be classicalized [13] if the purpose is for security of private communication instead of the real entanglement purification. Consequently, the initially shared EPR pairs before running the protocol can be replaced by a classical random string and after they run the protocol they recycle the random string by a classical Hamming code with the phase-error rate input being $t_0 + \delta$. Protocol 3 can help them to do quantum key expansion efficiently, without any quantum memory or entanglement resource: 1. Alice and Bob pre-share a secret classical random string $G$. They are sure that the bit-flip rate and phase-flip rate of the physical channel are less than $t_x - \delta$ and $t_z - \delta$, respectively. (In quantum cryptography, the knowledge of flipping rates of physical channel does not guarantee the security in any sense.) They choose two Hamming code $C_x$ and $C_z$ which can correct $(t_x + \delta)M$ bits and $(t_z + \delta)M$ bits of error, respectively. We suppose $t_x + \delta < 11\%$ and $t_z + \delta < 11\%$. 2. Alice plans to send $N$ deterministic bits, string $s$ to Bob. Alice and Bob take an $M$–bit string $b$, an $M'$–bit substring $b'$, a 200-bit substring $c$ and a 200-bit substring $d$ from $G$, from left to right. Here $M = \frac{N}{1-H(t_0+\delta)}$. 3. Alice expands the message string $s$ to $S$ by Hamming code $C_x$. Obviously, there are $M$ bits in the expanded string $S$. She encrypts the expanded string $S$ with string $b$, i.e., she prepares an $M$–qubit quantum state $|S_b\rangle = |S \oplus b\rangle$ in $Z$ basis. All these encrypted message qubits are placed in order. She also produces a $2N = 2k$ test qubits and mix them with those qubits in $|S_b\rangle$. The position, bit value and preparation basis of each test qubits are determined by substring $b'$. This requires substring $b'$ including $M' = \left(\frac{M + 2k}{2k}\right) + 4k$ bits. The bit values (0 or 1), position and bases ($X$ or $Z$) of those test qubits must be totally random, since $b'$ is random. After the mixing, she has a quantum sequence $q$ which contains $M + 2k$ qubits. 4. Alice transmits sequence $q$ to Bob. 5. Bob reads $b'$. After receives sequence $q$ from Alice, he measures each of them in the correct bases. He then separates the test bits and message bits, with their original positions in each string being recovered. Bob reads the test bits and check the error rate (authentication). If he finds the bit-flip rate $t_{x0} > t$ or phase-flip rate $t_{z0} > t$ on the test bits, he sends substring $c \oplus d$ to Alice by classical communication and abort the protocol with string $c$ being deleted from $G$. If he finds the bit-flip rate $t_{x0} \leq t$ and
phase-flip rate $t_{e0} \leq t$ on the test bits, he sends substring $c$ to Alice by classical communication and continues the protocol. 6. Bob deletes $c$ from $G$. He decrypts the encrypted expanded message string by $b$ and then decodes it by Hamming code $C_s$ and obtains the message string. The probability that Bob’s decoded string is not identical to the original message string $s$ is exponentially close to 0. The key expansion part (or communication part) has been completed now. 7. Alice reads the 200-bit classical message from Bob. If it is not $c$, she aborts the protocol with string $c$ being deleted from $G$. (This is also authentication.) If it is $c$, she deletes substring $c$ from $G$ and carries out the next step. 8. Alice and Bob replace $b$ by the coset of $b + C_z$ as the recycled string.

Remark 1. Our cost of qubit-transmission is less than half of that in BB84 protocol. Our cost of classical communication is almost zero. Remark 2. After the protocol, string $b'$ and $d$ can be re-used safely. In our protocol, even Alice announces $b',d$, Eve’s information about message $s$ is 0. Therefore the mutual information between $s$ and $\{b',d\}$ is $I(s: \{b',d\}) = 0$. Therefore, if message $s$ is announced while $\{b',d\}$ is not announced, Eve’s information about $\{b',d\}$ must be also 0. Consequently, Eve’s information to $\{b',d\}$ must be zero after the protocol. Remark 3. If we want to reduce the number of pre-shared qubits, we can use fewer test bits, i.e., reduce the value of $r$. In our protocol, the total qubits needed is $r^{-1}$ times of that of BB84 protocol. To avoid a too large key expansion at one time, we can choose to raise the value of $\delta$, given a small $r$.

III. EXISTING PROTOCOLS OF DIRECT COMMUNICATION WITH QUBITS ARE INSECURE.

Our protocol cannot be replaced by any existing direct communication protocol [17–21] with quantum states. The insecurity of existing direct communication protocols have been pointed out already for the case of noisy channel [18,19]. Here we show that these protocols are not exponentially secure even with noiseless quantum channel. We suppose that there are $m$ test qubits and $N$ message qubits. Consider the best case that they find no error to the test bits. Even in such a case, the message is still polynomially insecure: Eve has non-negligible probability to obtain a few bits information to the message. For example, Eve just intercepts one qubit in transmission and measures it in $Z$ basis ($\{|0\rangle, |1\rangle\}$) and then resends it to Bob. Suppose the physical channel itself is noiseless. Obviously, There is a probability of $N/(N + m)$ that Bob finds no error to the test bits while Eve has one bit information about the message. In particular, in certain cases, 1 bit leakage of message is disastrous [14]. Such type of direct private communication is insecure even with noiseless quantum channel, since the zero error of test bits only guarantees less than $\delta$ errors of the message bits, it does not guarantee zero phase-flip of the message bits. In principle, there is no way to verify zero phase-flip error of the untested bits by looking at the test bits only. The insecurity of existing protocols is due to the lack of privacy amplification step, which is the main issue of the security of private communication. One cannot directly append a privacy amplification step here since this may change the message bits therefore destroy the message. One of the non-trivial point of our protocol is that the transmitted message bits in our protocol is unconditionally secure without any privacy amplification, no matter how noisy the channel is. There we only need to correct the bit-flip errors in the message. This does not change the message itself.

IV. DISCUSSIONS

Our protocol can obviously be used for both key expansion and direct communication. In the security proof, we have used a pre-condition that Eve has zero information to the preshared string $G$. However, strictly speaking, this condition does not hold in our real protocol. First, as we have argued that the pre-shared string can be generated by standard BB84 QKD protocol where Eve’s information to the shared key is exponentially close to 0 instead of strict 0. Second, Eve’s information to the recycled string is also exponentially close to 0 rather than 0. Eve’s exponentially small prior information is not a problem to the security of classical private communication. However, here we have used quantum states to carry the classical message, Eve may store her quantum information about the pre-shared (or recycled) secret string and directly attacks the decoded message or the updated key finally. With the universality of quantum compossibility [22], we know that Eve’s a exponentially small amount of prior information about the pre-shared string or the recycled string will only cause an exponentially small amount of information about the private message or the updated shared string. Therefore our protocol is unconditionally secure in the real case that Eve has exponentially small amount of information to the pre-shared key.

ACKNOWLEDGMENTS

I am very grateful to Prof. H. Imai for his long term supports. I thank D. Leung and H.-K. Lo for pointing out ref [22].

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