H-COUP Version 2:
a program for one-loop corrected Higgs boson decays
in non-minimal Higgs sectors

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We present the concept of H-COUP 2.0, a fortran program for numerical evaluation of decay rates of the Higgs boson with a mass of 125 GeV and the decay width with higher order corrections (next to leading order (NLO) for electroweak and scalar loop corrections, and next to NLO for QCD corrections) in the Higgs singlet model, four types of two Higgs doublet models with a softly-broken $Z_2$ symmetry and the inert doublet model. In the previous version (H-COUP 1.0), only a full set of the Higgs boson vertices are evaluated at one-loop level in a gauge invariant manner in these models. H-COUP 2.0 contains all the functions of H-COUP 1.0. After shortly introducing these extended Higgs models and discussing their theoretical and experimental constraints, we summarize formulae for the renormalized vertices and the decay rates. We then explain how to install and run H-COUP 2.0 with some numerical examples.

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I. INTRODUCTION

By the discovery of the Higgs boson at the LHC, exploring details of the scalar sector, which is responsible for the electroweak (EW) symmetry breaking, has become one of the most important subjects of high energy particle physics. The current situation clarified by collider experiments can be summarized by two important things, (i) properties of the discovered Higgs boson are consistent with those of the Standard Model (SM) Higgs boson within theoretical and experimental uncertainties, and (ii) other new particles have not yet been observed. These current experimental results can be explained within the minimal Higgs sector assumed in the SM, which is composed of one isospin scalar doublet.

On the other hand, the Higgs sector of the SM does not have any principle to determine its structure, differently from the gauge sector. Thus, non-minimal forms of the Higgs sector should also be considered as well, unless they are excluded by the current data. In addition, non-minimal Higgs sectors are often predicted in various new physics models which have been proposed to solve problems of the SM; i.e., the hierarchy problem as well as the existence of phenomena which cannot be explained in the SM such as neutrino oscillations, dark matter and baryon asymmetry of the Universe. Because the structure of the Higgs sector can strongly depend on each new physics scenario, exploring the shape of the Higgs sector is a key to determine the direction of new physics beyond the SM.

When additional Higgs bosons are discovered at future collider experiments, it provides direct evidence for a non-minimal Higgs sector. The structure of the non-minimal Higgs sector is then narrowed down by properties of additional Higgs bosons; e.g., electric charges, masses, couplings and so on. However, even if the second Higgs boson is not directly discovered, we can indirectly determine the structure of the Higgs sector by measuring deviations in observables for the discovered Higgs boson from the SM predictions such as couplings, the width, branching ratios and production cross sections. Currently, these observables are not sufficiently measured with enough accuracy for indirect searches for additional Higgs bosons. However, they are expected to be precisely measured at future experiments, such as the high-luminosity LHC (HL-LHC) [1, 2], the International Linear Collider (ILC) [3–6], the Future Circular Collider (FCC) [7], the Circular Electron Positron Collider (CEPC) [8] and the Compact Linear Collider (CLIC) [9]. For example, couplings of the discovered Higgs boson are expected to be measured with one percent level or better at the ILC with the center of mass energy of 250 GeV. Therefore, accurate calculations of the Higgs boson observables with radiative corrections are necessary to compare with their precisely measured values.
There have been many studies on radiative corrections to the vertex functions and decay rates of the Higgs boson $h(125)$ in various non-minimal Higgs sectors and new physics models in addition to the SM, where $h(125)$ represents the discovered Higgs boson with the mass of 125 GeV. In Table I, we summarize previous studies on one-loop corrections to the $hf\bar{f}$, $hVV$ ($V=W,Z$) and $hhh$ vertices as well as the decay rates for $h \to f\bar{f}$, $h \to VV$ in the SM, MSSM, two Higgs doublet models (THDMs) with a softly broken $Z_2$ symmetry, the Higgs singlet model (HSM) and the inert doublet model (IDM). One can numerically evaluate these vertex functions and decay rates with higher order corrections by using several public tools. For the SM and MSSM (next to MSSM), HDECAY [45, 46], FeynHiggs [47, 48] and HFOLD [49] (NMHDECAY [50, 51] and NMSSMCALC [52]) can compute decay width and branching ratios of Higgs bosons with EW corrections and QCD corrections. Regarding the extended Higgs models, 2HDMC [53] and SHDECAY [54] can give decay rates and total width, and branching ratios of Higgs bosons with QCD corrections in THDMs and the HSM. Also, 2HDECAY [55] can provide the decay rates and branching ratios with both EW corrections and QCD corrections in THDMs. Apart from these public tools, as a first tool to observables for $h(125)$ with one-loop EW corrections in various non-SUSY models with extended Higgs sectors, H-COUP 1.0 [56] had been published. Recently, next version of the program, H-COUP 2.0 has appeared, which evaluates the decay width and the decay branching rations of $h(125)$ with higher order corrections in addition to the full set of renormalized vertex functions of $h(125)$.

|       | $hf\bar{f}$ | $hVV$ | $hhh$ | $\Gamma(h \to f\bar{f})$ [QCD] | $\Gamma(h \to f\bar{f})$ [EW] | $\Gamma(h \to VV)$ |
|-------|-------------|-------|-------|-------------------------------|-------------------------------|-------------------|
| SM    | 10, 12      | 10, 13| 14    | 15, 16                        | 17, 20                        | 10, 12            |
| MSSM  | 21          | 22, 23| 24, 27| 17, 22, 28                    | 21                            | 20, 29            |
| THDMs | 30, 34, 36  | 37, 33, 34 | 30, 31, 38, 39 |                                     | 30, 39            |
| HSM   | 31, 40      | 41, 40, 41, 42 | 38, 39 |                                     | 38, 39            |
| IDM   | 31, 43, 44  | 44, 43, 44 |                                     | 39                            | 39                |

TABLE I. Summary for studies on radiative corrections to the Higgs boson couplings at one-loop level as well as Higgs boson decay rate including at next-to-leading order (NLO). For the $h \to f\bar{f}$, we separately show the works for the NLO QCD corrections and EW corrections.
(H-COUP.1.0), a full set of vertices for \( h(125) \) can be evaluated at one-loop level in the improved on-shell scheme for NLO EW in these models. By extending the H-COUP.1.0 functionalities, we completed the calculations of all the decay rates of \( h(125) \) as H-COUP.2.0. Therefore, H-COUP.2.0 contains all the functions of H-COUP.1.0.

Physics results obtained by preliminary version of H-COUP.2.0 have been presented in Refs. [38, 39] where NLO EW and NLO QCD corrections were implemented. We note that, with a process to make a public version of the H-COUP.2.0 program, we added NNLO-QCD corrections to the \( h \to q\bar{q}, gg, \gamma\gamma \) modes. We also added \( h \to \mu\mu \) for the completeness of the list of the decay modes.

This article is organized as follows. In Sec. II, we briefly review the extended Higgs models, and define input parameters for each model. In Sec. III, we discuss renormalized vertices and decay rates of \( h(125) \) based on Refs.[16, 32–34, 38–41, 43] which are implemented in H-COUP.2.0. In Sec. IV, the structure of H-COUP.2.0 is explained. In Sec. V, the installation and how to run H-COUP.2.0 are described with some numerical examples. Summary of this manual is given in Sec. VI.

II. MODELS AND CONSTRAINTS

In this section, we define the Higgs sectors of the HSM, the THDMs and the IDM. In particular, we uniformly and compactly introduce mass eigenstates of the scalar fields and free input parameters in each model. Since all the models are exactly the same as those in H-COUP.1.0, see the manual of Ver.1 [56] for details of definitions and descriptions about the models such as Lagrangian and some formulae.

In the all models covered in this manual, mass eigenstates of scalar fields are commonly represented as follows;

\[ h : \text{the discovered CP-even Higgs boson with the mass 125 GeV}, \]
\[ H : \text{another CP-even Higgs boson}, \]
\[ A : \text{a CP-odd Higgs boson}, \]
\[ H^\pm : \text{a pair of singly charged Higgs bosons}. \]  

H-COUP.2.0 incorporates some theoretical constraints, i.e., the tree-level unitarity bound, the triviality bound, the vacuum stability bound (tree level and improved by renormalization group equations (RGEs)) and the true vacuum condition, as well as an experimental constraint by the
EW S and T parameters, which are exactly the same as those in H-COUP_1.0. Detailed descriptions for the constraints are given in the manual of H-COUP_1.0 [56].

A. HSM

The Higgs sector of the HSM is composed of the SM Higgs field $\Phi$, i.e., the isospin doublet Higgs field with hypercharge $Y = 1/2$, and an isospin singlet scalar field $S$ with $Y = 0$. Detailed definitions of descriptions about the HSM are given in Refs. [40] [41], whose notation is the same as the notation in this article. After the EW symmetry breaking, there appear two physical scalar states $h$ and $H$ by the mixing of neutral components of $\Phi$ and $S$. The Higgs potential has 8 free parameters. Two of them, the mass of $h(125)$ $m_h$, and the vacuum expectation value (VEV) $v$ of the doublet field $\Phi$ are fixed, i.e., $m_h = 125$ GeV and $v \simeq 246$ GeV. Moreover, the VEV of the singlet field, can be absorbed by the field redefinition [57]. Here, the following 5 parameters are chosen as input free parameters;

$$m_H, \alpha, \lambda_S, \lambda_{\Phi S}, \mu_S,$$

(2)

where $m_H$ and $\alpha$ are the mass of $H$ and the mixing angle between $h$ and $H$, respectively. We define the range of $\alpha$ as $-\pi/2 \leq \alpha \leq \pi/2$. The remaining three parameters are the original parameters given in the potential.

B. THDMs

THDMs contain two isospin doublet Higgs fields $\Phi_1$ and $\Phi_2$ with $Y = 1/2$. In these models with a softly broken $Z_2$ symmetry, the two scalar fields are assigned different $Z_2$ charges with each other. H-COUP_2.0 covers four types of THDMs with different Yukawa interactions [58] [60], which are called Type-I, Type-II, Type-X and Type-Y. Please see Refs. [16] [32] [33] for details of the models. In the THDMs, three neutral scalar fields ($h, H$ and $A$) and a pair of singly charged scalar fields ($H^{\pm}$) appear as mass eigenstates. We choose the following 6 parameters as input free parameters,

$$m_H, m_A, m_{H^{\pm}}, \sin(\beta - \alpha), \tan \beta, M^2,$$

(3)

where $\sin(\beta - \alpha) \geq 0$ and $\tan \beta > 0$ are taken, $m_H, m_A, m_{H^{\pm}}$ represent masses of the additional Higgs bosons, and $\alpha$ ($\beta$) is a mixing angle of CP-even (CP-odd) scalar components, and $M^2$ is a
parameters describing the soft breaking scale of the $Z_2$ symmetry. When we take $\sin(\beta - \alpha)$ and $\tan \beta$ as input parameters, we also have to specify the sign of $\cos(\beta - \alpha)$.

C. IDM

The Higgs sector of the IDM consists of two isospin doublet Higgs fields $\Phi$ and $\eta$ with $Y = 1/2$. This model has unbroken $Z_2$ symmetry, so that $\Phi$ and $\eta$ with different $Z_2$ charges do not mix their components. As a result, there are five types of scalar particles, i.e., $h$, $H$, $A$ and $H^\pm$, where $h$ ($H$, $A$ and $H^\pm$) is the original component of $\Phi$ ($\eta$). In H-COUP, the following five parameters,

$$m_H, m_A, m_{H^\pm}, \mu_2, \lambda_2,$$

(4)

are taken as for input parameters, where $\mu_2$ and $\lambda_2$ are coefficient parameters of the quadratic and quartic terms of $\eta$ in the potential, respectively. Details of definitions, formulae and descriptions for the IDM are given in Refs. [43].

III. RENORMALIZED VERTICES AND DECAY RATES

In this section, renormalized vertex functions for the discovered Higgs boson $hf \bar{f}$, $hVV$ ($V = W$ or $Z$) and $hhh$ at one-loop are defined. Subsequently, analytical expressions of the decay rates with higher order corrections are described, i.e., $h \rightarrow f \bar{f}$, $h \rightarrow V f \bar{f}$, $h \rightarrow gg$ and $h \rightarrow V\gamma$ ($V = \gamma$ or $Z$). These quantities are output parameters of the H-COUP 2.0.

Here, we outline the renormalization scheme for calculations of radiative corrections in H-COUP. All Feynman diagrams are computed in the 't Hooft-Feynman gauge, and the UV divergences are renormalized by applying the improved on-shell scheme [34] for EW corrections. In this scheme, the gauge dependence arising from a mixing of scalar fields is got rid of by utilizing the pinch technique [34, 61, 62]. On the other hand, for the NLO and NNLO QCD corrections to the Higgs decay processes, the $\overline{\text{MS}}$ scheme is applied.

Apart from the UV divergences, for the decay of $h \rightarrow f \bar{f}$ and $h \rightarrow VV^*$ and also $hf \bar{f}$ and $hVV$ vertex functions, IR divergences appear in Feynman diagrams with a virtual photon, which are cancelled with contributions from real photon emission. For the decay of $h \rightarrow f \bar{f}$ and $h \rightarrow ZZ^*$, virtual photon loop corrections can be separated from weak corrections and analytical formulae for the total corrections (virtual photon loop corrections plus real photon emissions) have already known in the SM. Since these QED corrections at the NLO for extended Higgs models are common with those of the SM, the analytical formulae of the total QED corrections are simply implemented.
in H-COUP_2.0. Related to these treatment of the QED corrections to $h \to f \bar{f}$ and $h \to VV^*$, virtual photon loop corrections are switched off in evaluations of the $h f \bar{f}$ and $h ZZ$ vertex functions. On the other hand, photon loop corrections and weak corrections to the $h \to W f \bar{f}$ are not separable. Therefore, virtual photon corrections and contributions of real photon emissions are individually evaluated. The latter is evaluated by using the phase space slicing method \cite{63}, thus photon phase space is divided into the soft region and the hard region. While the analytical expressions are implemented in H-COUP for contributions with soft photon, the numerical values for contributions with hard photon are evaluated by Madgraph5_aMC@NLO \cite{64} with default values for SM parameters in H-COUP_2.0.

### A. Renormalized vertex functions

The renormalized $h f \bar{f}$ and $hVV$ vertices are expressed in terms of form factors as

$$
\hat{\Gamma}_{hff}(p_1^2, p_2^2, q^2) = \hat{\Gamma}_{hff}^S + \gamma_5 \hat{\Gamma}_{hff}^P + \hat{\Gamma}_{hff}^V + \hat{\Gamma}_{hff}^T + \hat{\Gamma}_{hff}^{PT},
$$

where $p_\mu^1$ and $p_\nu^2$ for the $h f \bar{f}$ ($hVV$) vertex are defined as incoming momenta of fermion and antifermion (two weak gauge bosons), and $q^\mu$ denotes the outgoing momentum of the Higgs boson. In contrast to these vertices, the $hhh$ vertex $\hat{\Gamma}_{hhh}(p_1^2, p_2^2, q^2)$ is a scalar function. The renormalized scalar functions $\hat{\Gamma}_{hXX}$ are commonly divided into two parts

$$
\hat{\Gamma}_{hXX}^i(p_1^2, p_2^2, q^2) = \Gamma_{hXX}^i,\text{tree} + \Gamma_{hXX}^i,\text{loop}(p_1^2, p_2^2, q^2),
$$

where the loop part $\Gamma_{hXX}^i,\text{loop}$ is further decomposed into 1PI diagram contributions and counterterm contributions, i.e. $\Gamma_{hXX}^i,\text{loop} = \Gamma_{hXX}^i,\text{1PI} + \delta \Gamma_{hXX}^i$. The tree-level contributions for each vertex function are written by

$$
\Gamma_{hff}^1 = \frac{-m_f}{v} \kappa_f, \quad \Gamma_{hVV}^1 = \frac{2m_V^2}{v} \kappa_V, \quad \Gamma_{hhh}^1 = \frac{-3m_h^2}{v} \kappa_h,
$$

where scaling factors $\kappa_X$ for each extended Higgs model are summarized in Table \ref{table:HSM} and other form factors become zero at tree level, namely $\Gamma_{hVV}^{2,\text{tree}} = \Gamma_{hVV}^{3,\text{tree}} = \Gamma_{hff}^{a,\text{tree}} = 0 (a \neq S)$. Explicit formula for the loop contributions of each vertex are give in Refs. \cite{34, 40}, Refs. \cite{33, 34, 41}, and Ref. \cite{43} for the HSM, THDMs and the IDM, respectively.
κ_f \kappa_V \kappa_h

| HSM  | $c_\alpha$ | $c_\alpha$ | $c_\alpha^3 + 2s_\alpha^2 c_\alpha^2 (c_\alpha \lambda_{4FS} - s_\alpha \mu_S^2)$ |
|-------|-----------|-----------|--------------------------------------------------|
| THDMs | $s_{\beta-\alpha} + \xi_f c_{\beta-\alpha}$ | $s_{\beta-\alpha}$ | $s_{\beta-\alpha} + \left(1 - \frac{M_{\phi}^2}{m_{\chi}^2}\right) c_{\beta-\alpha} \left(2s_{\beta-\alpha} + c_{\beta-\alpha} \left(\frac{1}{t_\beta} + t_\beta\right)\right)$ |
| IDM   | 1         | 1         | 1                                                 |

TABLE II. Scaling factors for Higgs couplings in the extended Higgs models at tree level. The factor $\xi_f$ in the THDMs varies in accordance with structure of Yukawa interactions, which is given in Table III.

| THDMs                          | $\xi_u$ | $\xi_d$ | $\xi_e$ |
|-------------------------------|---------|---------|---------|
| Type-I                        | $\cot \beta$ | $\cot \beta$ | $\cot \beta$ |
| Type-II                       | $\cot \beta - \tan \beta - \tan \beta$ |
| Type-X (lepton specific)      | $\cot \beta$ | $\cot \beta$ | $- \tan \beta$ |
| Type-Y (flipped)              | $\cot \beta - \tan \beta$ | $\cot \beta$ |

TABLE III. The $\xi_f (f = u, d, e)$ factors appearing in Table II

B. Higgs decay rates

The decay rates for $h \to f \bar{f}$ with higher order corrections can be schematically described as

$$
\Gamma(h \to f \bar{f}) = \Gamma_0(h \to f \bar{f}) \left[ 1 + \Delta_{EW}^f + \Delta_{QCD}^f \right],
$$

where $\Gamma_0$ denotes the formula at the LO, i.e.,

$$
\Gamma_0(h \to f \bar{f}) = \frac{N_c^f}{8\pi} m_h (\Gamma_{hff}^{S, \text{tree}})^2 \left(1 - \frac{4m_{\chi}^2}{m_h^2}\right)^{3/2},
$$

with $N_c^f = 3(1)$ for quark (lepton), and $\Delta_{EW}^f$ and $\Delta_{QCD}^f$ denote the EW corrections and the QCD corrections to $h \to ff$, respectively. Hereafter, for all decay modes of the Higgs bosons, we commonly denote the contributions of EW (QCD) corrections as $\Delta_{EW(QCD)}^X$. The EW corrections $\Delta_{EW}^f$ at the NLO can be further divided into the QED corrections (radiative corrections of a photon) and weak corrections (all the other EW loop corrections) as $\Delta_{EW}^f = \Delta_{QED}^f + \Delta_{\text{Weak}}^f$. For the decay into leptons $f = \ell$, the NLO QED correction $\Delta Q_{\text{QED}}^\ell$ is given in the on-shell scheme by [11, 12, 65]

$$
\Delta_{\text{QED}}^\ell = \frac{\alpha_{\text{em}}}{\pi} Q_\ell^2 \left(\frac{9}{4} + \frac{3}{2} \log \frac{m_\ell^2}{m_h^2}\right),
$$

(11)
and for the decay into quarks, the NLO QED corrections is given in $\overline{\text{MS}}$ scheme by \[66\]

$$\Delta_{\text{QED}}^q = \frac{\alpha_{\text{em}}}{\pi} Q_q^2 \left( \frac{17}{4} + \frac{3}{2} \log \frac{\mu^2}{m_h^2} \right), \quad (12)$$

where $\mu$ is taken to be $m_h$. Whereas, the weak corrections can be commonly expressed in terms of the renormalized Higgs vertex functions as

$$\Delta_{\text{weak}}^f = \frac{2}{\Gamma_{S,\text{tree}}^{h_{ff}}} \text{Re} \left[ \left( \Gamma_{S,\text{loop}}^{h_{ff}} + 2m_f \Gamma_{V,\text{loop}}^{h_{ff}} + m_h^2 \right) \frac{1 - m_f^2}{m_h^2} \Gamma_{T,\text{loop}}^{h_{ff}} \right] (m_f^2, m_f^2, m_h^2) - \Delta r, \quad (13)$$

where $\Delta r$ denotes the radiative correction to the muon decay \[67\].

The NNLO QCD corrections to $h \to q\bar{q}$ are expressed in the $\overline{\text{MS}}$ scheme in a limit neglecting contributions with quark masses as \[68\]-\[70\]

$$\Delta_{\text{QCD}}^q = 5.67 \frac{\alpha_s(m_h)}{\pi} + (35.94 - 1.36 N_f) \left( \frac{\alpha_s(m_h)}{\pi} \right)^2$$

$$+ \frac{\kappa_t}{\kappa_q} \left( \frac{\alpha_s(m_h)}{\pi} \right)^2 \left\{ 1.57 - \frac{2}{3} \log \frac{m_h^2}{m_t^2} + \frac{1}{9} \log \left( \frac{\tilde{m}_Q(m_h)^2}{m_h^2} \right) \right\}, \quad (14)$$

where $N_f$ is the active flavor number and the renormalization scale is taken to be at the mass of the Higgs boson, $\mu = m_h$, in this expression. While the first and the second terms are common in the extended Higgs models and the SM, the third term, which comes from the top loop contributions at the NNLO, contains the ratio of scaling factors of Yukawa couplings $\kappa_t/\kappa_q$. When we apply the QCD corrections, we regard the quark mass as the running $\overline{\text{MS}}$ mass in the Yukawa couplings appeared in Eq. (10), which is also evaluated at $\mu = m_h$. The running quark masses are calculated from the $\overline{\text{MS}}$ mass at $\mu = m_q$ by using the relation \[71\], i.e., $m_q(m_h) = \tilde{m}_q(m_q)c[\alpha_s(m_h)/\pi]/c[\alpha_s(m_q)/\pi]$, where the function $c$ can be found up to the three loop level in Refs. \[72\], \[75\]. In H-COUP 2.0, two options for the computations of $\Gamma_0(h \to q\bar{q})$ can be selected: the one is computations with current masses for quarks, and the other is computations used $\overline{\text{MS}}$ mass for the Yukawa couplings.

The decay rates for $h \to V f \bar{f}$ can be expressed as the same manner with $h \to f \bar{f}$, i.e.,

$$\Gamma(h \to V f \bar{f}) = \Gamma_0(h \to V f \bar{f}) \left[ 1 + \Delta_{\text{EW}}^V + \Delta_{\text{QCD}}^V \right]. \quad (15)$$

Here the decay rate at the LO, $\Gamma_0(h \to V f \bar{f})$, is presented in terms of a fraction $\epsilon_V = m_V/m_h$ by \[76\]

$$\Gamma(h \to V f \bar{f}) = \sqrt{2} G_f m_h c_V \left( \frac{\Gamma_{1,\text{tree}}^{hVV}}{\Gamma_{hVV}} \right)^2 F(\epsilon_V), \quad (16)$$
where the factor $C_V$ is $C_V = 4(v_f^2 + a_f^2)$ for the $Z$ boson and $C_V = 1$ for the $W$ boson, and the function $F(\epsilon_V)$ is written as

$$F(\epsilon_V) = \frac{3(1 - 8\epsilon_V^2 + 20\epsilon_V^4)}{\sqrt{4\epsilon_V^2 - 1}} \arccos \left( \frac{3\epsilon_V^2 - 1}{2\epsilon_V^4} \right)$$

$$- (1 - \epsilon_V^2) \left( \frac{47}{2} \epsilon_V^2 - \frac{13}{2} + \frac{1}{\epsilon_V^2} \right) - 3(1 - 6\epsilon_V^2 + \epsilon_V^4) \log \epsilon_V. \quad (17)$$

For $h \rightarrow Z f \bar{f}$, further separation of the EW correction $\Delta_{\text{EW}}^Z$ into the QED part and the weak part can be performed as $\Delta_{\text{EW}}^Z = \Delta_{\text{QED}}^Z + \Delta_{\text{Weak}}^Z$, similar to $h \rightarrow f \bar{f}$. The NLO QED correction $\Delta_{\text{QED}}^Z$ is given by the same expression to the SM [77] in the $m_f \rightarrow 0$ limit; i.e.,

$$\Delta_{\text{QED}}^Z = Q_f^2 \frac{3\alpha_{\text{em}}}{4\pi}, \quad (18)$$

since the QED corrections only appear in the vertex of the off-shell $Z$ boson with a pair of fermions, which does not have new physics effects in the massless limit of decaying fermions. In contrast to $h \rightarrow Z f \bar{f}$, for $h \rightarrow W f \bar{f}$, such separation cannot be done because the Feynman diagrams with a virtual photon are accompanied by virtual $W$ bosons. The weak corrections to $h \rightarrow Z f \bar{f}$ and the EW corrections to $h \rightarrow W f \bar{f}$, namely $\Delta_{\text{Weak}}^Z$ and $\Delta_{\text{EW}}^W$ are expressed in terms of renormalized vertices of $hVV$ and $hf\bar{f}$ as well as other contributions; e.g. oblique corrections to the off-shell weak boson and box diagrams for $h \rightarrow Vf\bar{f}$. The explicit formulae can be found in Ref. [39]. On the other hand, the NLO QCD correction to $h \rightarrow Vq\bar{q}$ in the $\overline{\text{MS}}$ scheme is commonly presented by [77]

$$\Delta_{\text{QCD}}^V = C_F \frac{3\alpha_s(m_h)}{4\pi}, \quad (19)$$

with $C_F = 3/4$. In H-COUP_2.0 the three-body-decays of the Higgs boson $\Gamma(h \rightarrow V f \bar{f})$ are implemented. However, four body decays $\Gamma(h \rightarrow 4f)$ are not included. They are calculated with NLO EW and NLO QCD corrections in HSM [78] and THDMs [79].

The loop induced decays of the Higgs boson are also evaluated in H-COUP_2.0, i.e., $\Gamma(h \rightarrow gg)$ and $\Gamma(h \rightarrow V\gamma)$ ($V = Z, \gamma$), including higher order QCD corrections. Analytical formulae for these processes can be found in Refs. [39, 40], Refs. [33, 39, 80], and Refs. [39, 43, 80] for the HSM, THDMs and the IDM, respectively.

For the $\Gamma(h \rightarrow gg)$, the QCD corrections up to NNLO in the $\overline{\text{MS}}$ scheme are implemented in H-COUP_2.0. The analytic expression for $m_h^2/m_t^2 \rightarrow 0$ is taken [68, 81],

$$\Delta_{\text{QCD}}^g = \frac{215}{12} \frac{\alpha_s(m_h)}{\pi} + \left( \frac{\alpha_s(m_h)}{\pi} \right)^2 \left( 156.8 - 5.7 \log \frac{m_t^2}{m_h^2} \right), \quad (20)$$
where the active flavor number $N_f$ and the renormalization scale $\mu$ have been taken to be $N_f = 5$ and $\mu = m_h$, respectively. Typically, the corrections of the NLO contribution (the first term) and the NNLO contribution (the second terms) are about 70% and 20% to the LO contributions, respectively. For the $\Gamma(h \rightarrow \gamma\gamma)$, the QCD corrections up to NNLO are implemented in the limit $m_t \rightarrow \infty$ in the program. In this process, the QCD corrections are only implemented to the top loop diagrams; because that to the another quark loop contributions are numerically negligible. Thus, top loop contributions denoted as $(\Gamma_{h\gamma\gamma}^{\text{loop}})_t$ are modified at the amplitude level as \cite{68, 82}

\begin{equation}
(\Gamma_{h\gamma\gamma}^{\text{loop}})_t \rightarrow (\Gamma_{h\gamma\gamma}^{\text{loop}})_t \left[ 1 - \frac{\alpha_s(\mu)}{\pi} - \left( \frac{\alpha_s(\mu)}{\pi} \right)^2 \left( \frac{31}{24} + \frac{7}{4} \log \frac{\mu^2}{m_t^2} \right) \right],
\end{equation}

where we take $\mu = m_h/2$, following Ref. \cite{68}. Apart from $h \rightarrow \gamma\gamma$, for $h \rightarrow Z\gamma$, only NLO corrections, which are given by the second terms in Eq. (21), are applied in H-COUP.2.0. Typical size of the NLO QCD corrections to the LO contributions is $O(0.1)\%$, so that the NNLO corrections can be negligible.

IV. STRUCTURE OF H-COUP.2.0

The structure of H-COUP.2.0 is schematically shown in Fig. 1. Differently from H-COUP.1.0, the model and the order of calculations are specified from the command line interface (see Sec. V). H-COUP.2.0 then reads the model independent (global) and model dependent input parameters, where the former is the SM inputs and the squared momenta of the renormalized form factors, which are commonly used in all the model files. The SM parameters and their default values are summarized in Table IV. In this table, $\Delta \alpha_{\text{em}}$ denotes the shift of the fine structure constant given at the zero energy $\alpha_{\text{em}}$ to that given at the Z boson mass $\alpha_{\text{em}}(m_Z)$, i.e.,

\begin{equation}
\alpha_{\text{em}}(m_Z) = \frac{\alpha_{\text{em}}}{1 - \Delta \alpha_{\text{em}}}.
\end{equation}

The strong coupling constant $\alpha_s(m_Z)$ is given at the Z boson mass. For the calculation of the Higgs boson decay rates, we have to use the strong coupling constant at different energy scale $\mu$ such as the Higgs boson mass as discussed in Sec. III, which is calculated by using the RGE running. For the bottom and charm quark masses, we show both the on-shell and $\overline{\text{MS}}$ masses, where the former masses can be derived from the latter by perturbative calculations. In H-COUP.2.0, we simply quote the value of these on-shell masses from \cite{83}.

\footnote{In Ref. \cite{68}, validity for taking the renormalization scale at $\mu = m_h/2$ is also discussed.}
For the squared momenta, their input values are only used to output values of the renormalized form factors of the Higgs boson, so that users who are interested in the width and the branching ratios of the Higgs boson do not need to take care of these parameters. For details of the treatment of the squared momenta, see Ref. [56]. The model dependent parameters and their default values are summarized in Tables V, VI and VII for the HDM, the THDMs and the IDM, respectively. Here, we note that in H-COUP 1.0 the type of Yukawa interactions (Type-I, -II, -X and -Y) can be specified from the input file of the THDM, but in H-COUP 2.0 it can now be specified from the command line interface. Therefore, the “Type” parameter in the THDMs in H-COUP 1.0 disappears in H-COUP 2.0.

In the computation block, tree-level Higgs boson couplings, 1PI diagrams and counterterms are calculated under the fixed model and input parameters. These calculations are then used to compute the decay rates of the Higgs boson.
### TABLE IV. Input global SM parameters. All these parameters are defined by double precision, and their input values are taken from particle data group [83].

| Parameter | Definition | Description | Default value |
|-----------|------------|-------------|---------------|
| $m_Z$     | mz         | $Z$ mass    | 91.1876 GeV   |
| $\alpha_{em}$ | alpha_em   | Fine structure constant | $137.035999139^{-1}$ |
| $G_F$     | G_F        | Fermi constant | $1.1663787 \times 10^{-5}$ GeV$^{-2}$ |
| $\Delta \alpha_{em}$ | del_alpha | Shift of $\alpha_{em}$ | 0.06627 |
| $\alpha_s(m_Z)$ | alpha_s     | Strong coupling | 0.1185 |
| $m_h$     | mh         | Higgs boson mass | 125.1 GeV |
| $m_t$     | mt         | On-shell $t$ mass | 173.1 GeV |
| $m_b$     | mb         | On-shell $b$ mass | 4.78 GeV |
| $\bar{m}_b(m_b)$ | mb_ms       | MS $b$ mass | 4.18 GeV |
| $m_c$     | mc         | On-shell $c$ mass | 1.67 GeV |
| $\bar{m}_c(m_c)$ | mc_ms       | MS $c$ mass | 1.27 GeV |
| $m_\tau$  | mtau       | $\tau$ mass | 1.77686 GeV |
| $m_{\mu}$ | mmu        | $\mu$ mass | 0.1056583745 GeV |

In the output block, H-COUP 2.0 tells us if a given configuration determined by the input parameters is allowed or excluded. If a given parameter choice is excluded, a message “Excluded by XXX” appears, where “XXX” can be perturbative unitarity, vacuum stability, triviality, true vacuum conditions and/or ST parameters. In the both cases, the output file is generated in the output directory. H-COUP 2.0 provides the decay branching ratios and the total width as well as outputs given in H-COUP 1.0 (the renormalized form factors).

### V. INSTALLATION AND HOW TO RUN

In order to run the H-COUP program, users need to install a Fortran compiler (GFortran is recommended) and LoopTools [84] in advance. One can download the LoopTools package from [84], and see the manual for its installation.

One can download the H-COUP program on the following webpage

http://www-het.phys.sci.osaka-u.ac.jp/~hcoup

In the following, we describe how to run H-COUP 2.0 in order.

1. Unzip the HCOUP-2.0.zip file:
### HSM

| Parameters | $m_H$ | $\alpha$ | $\mu_S$ | $\lambda_S$ | $\lambda_{\Phi S}$ | $\Lambda$ |
|------------|-------|----------|---------|-------------|-------------------|---------|
| H-COUP def. | mbh   | alpha    | mu      | lam         | lam_phris         | cutoff  |
| Default value | 500 GeV | 0.1      | 0       | 0.1         | 0                 | 3 TeV   |

TABLE V. Input parameters in the HSM. All these parameters are defined by double precision.

### THDM

| Parameters | $m_{H^\pm}$ | $m_A$ | $m_H$ | $M^2$ | $s_{\beta-\alpha}$ | $\text{Sign}(c_{\beta-\alpha})$ | $\tan \beta$ | $\Lambda$ |
|------------|-------------|-------|-------|-------|-------------------|-------------------------------|-------------|---------|
| H-COUP def. | mch         | ma    | mbh   | bmsq  | sin_ba            | sign(+1 or -1) tanb          | cutoff      |         |
| Default value | 500 GeV    | 500 GeV | 500 GeV | (450 GeV)$^2$ | 1                     | 1                           | 1.5            | 3 TeV   |

TABLE VI. Input parameters in the THDMs. All these parameters are defined by double precision except for $\text{sign}$ which is defined by integer, and can be either 1 or $-1$.

### IDM

| Parameters | $m_{H^\pm}$ | $m_A$ | $m_H$ | $\mu_2^2$ | $\lambda_2$ | $\Lambda$ |
|------------|-------------|-------|-------|-----------|-----------|---------|
| H-COUP def. | mch         | ma    | mbh   | mu2sq     | lam2      | cutoff  |
| Default value | 500 GeV    | 500 GeV | 500 GeV | (500 GeV)$^2$ | 0.1       | 3 TeV   |

TABLE VII. Input parameters in the IDM. All these parameters are defined by double precision.

```
$ unzip HCOUP-2.0.zip
```

Then, the HCOUP-2.0 directory (HCOUP-2.0/) is created. In this directory, one can find 3 files (Makefile, README, main.F90) and 4 directories as follows:

```
$ ls
Makefile README main.F90 inputs/ models/ modules/ outputs/
```

Each directory contains the following files:

- *inputs/* (input files for the model dependent/global parameters)
  - in_hsm.txt (input file for the HSM)
  - in_thdm.txt (input file for the THDMs)
  - in_idm.txt (input file for the IDM)
  - in_sm.txt (global input file for the SM parameters)
  - in_momentum.txt (global input file for momenta)
• outputs/ (output files generated by H-COUP)
  out_hsm.txt, outBR_hsm.txt (output files for the HSM)
  out_thdm.txt, outBR_thdm.txt (output files for the THDMs)
  out_idm.txt, outBR_idm.txt (output files for the IDM)
  out_sm.txt, outBR_sm.txt (output files for the SM)
• models/ (main Fortran90 files of H-COUP)
  HCOUP_HSM.F90 (main file for the HSM)
  HCOUP_THDM.F90 (main file for the THDMs)
  HCOUP_IDM.F90 (main file for the IDM)
• modules/ (module files of H-COUP)

We note that users do not need to touch the files in models/ and modules/, but only need to modify the input files in inputs/.

2. Open Makefile by an editor and replace “PATH_TO_LOOPTOOLS” appearing in the line “LPATH” by the correct path to the library file of LoopTools (libooptools.a).

3. To compile the code, execute

```
$ make
```

in the HCOUP-2.0 directory. Then, an executable file “hcoup” is generated.

4. To run the H-COUP program, execute

```
$ ./hcoup
```

Then, you are asked,

Which model? (1=HSM, 2=THDM-I, 3=THDM-II, 4=THDM-X, 5=THDM-Y, 6=IDM)

in the command line. You can specify the model by typing the number. You are further asked,

Which order for EW? (0=LO, 1=NLO)

and

---

2 Initially this directory is empty.
Which order for QCD? (-1=LO(quark mass: OS), 0=LO(quark mass: MSbar), 1=NLO, 2=NNLO)

in order. You can specify the order of calculations by typing the numbers, see also Sec. [III B] for details of quark masses.

Then, output files are generated in the output directory. If a given set of the input parameters is excluded by some of the constraints, a message appears in the command line. An example of the generated output file in outputs/ is shown in Fig. 2.

5. One can change the model-dependent input parameters by modifying the in_hsm.txt, in_thdm.txt and in_idm.txt files in the input directory. One can also change the SM parameters and the squared momenta of the renormalized Higgs vertices by modifying the in_sm.txt and in_momentum.txt files in the input directory. These two files are commonly used to all the model files for each extended Higgs model. In Fig. 3 we show an example of the input file for the HSM (in_hsm.txt).

As a physics example computed by H-COUP 2.0, we also present branching ratios of the SM-like Higgs boson in four types of THDMs in Fig. 4 where the NLO-EW and NNLO-QCD corrections are taken into account.

VI. SUMMARY

In this paper, the concept and the manual of H-COUP 2.0 have been presented, which is a set of fortran programs for numerical evaluation of decay rates of the Higgs boson with a mass of 125 GeV and the decay width with higher order corrections (NLO for EW and scalar loop corrections, and NNLO for QCD corrections) for various models of extended Higgs sectors. In H-COUP 2.0, in addition to the SM, the Higgs singlet model, four types of two Higgs doublet models with a softly-broken $Z_2$ symmetry and the inert doublet model are implemented. H-COUP 2.0 contains all the functions of H-COUP 1.0 where a full set of the Higgs boson vertices are evaluated at one-loop level in a gauge invariant manner in these models. We have briefly introduced these models with their theoretical and experimental constraints, and we have summarized formulae for the renormalized vertices and the decay rates. After the explanation of the structure of the program, we have demonstrated how to install and run H-COUP 2.0 with some numerical examples.
| BLOCK MODEL # |
|---------------|
| 1 1 # HSM |

| BLOCK BSINPUTS # |
|------------------|
| 1 1.00000000E+00 # alpha |
| 2 0.00000000E+00 # lambda_{phi S} |
| 3 1.00000000E+00 # lambda_S |
| 4 0.00000000E+00 # mu_S (GeV) |

| BLOCK SMINPUTS # |
|------------------|
| 1 7.29735257E-03 # alpha_em |
| 2 1.16637870E-05 # Fermi constant |
| 3 1.18100000E-01 # alpha_s |
| 4 1.27000000E+00 # mc(mc) MSbar |
| 5 4.18000000E+00 # mb(mb) MSbar |
| 6 1.67000000E+00 # mc On-shell |
| 7 4.78000000E+00 # mb On-shell |

| BLOCK MASS # |
|---------------|
| 4 5.66262421E-01 # mc(mh) MSbar |
| 5 2.79078561E+00 # mb(mb) MSbar |
| 6 1.73100000E+02 # mt |
| 13 1.05658374E-01 # mmu |
| 15 1.77866000E+00 # mtau |
| 23 9.11876000E+01 # mz |
| 24 8.09388629E+00 # mw (calculated) |
| 25 1.25100000E+02 # mh |
| 35 5.00000000E+02 # mH |

| BLOCK CONSTRAINTS # |
|----------------------|
| 0 3.00000000E+03 # The cutoff scale (GeV) |
| 1 0 # Vacuum stability at tree level [0=OK, 1=No] |
| 2 0 # Tree-level unitarity [0=OK, 1=No] |
| 3 0 # S and T parameters [0=OK, 1=No] |
| 4 0 # True vacuum [0=OK, 1=No] |
| 5 0 # Vacuum stability (RGE improved with the cutoff scale) [0=OK, 1=No] |
| 6 0 # Triviality (with the cutoff scale) [0=OK, 1=No] |

# Decay width of the SM-like Higgs boson by H-COUP #

| DECAY Width |
|-------------|
| BR 25 0.38892264E-02 # EW:NLO QCD:NNLO |

| # |
| BR  | NDA | ID1 | ID2 | |
|-----|-----|-----|-----|-----|
| 2.5665990E-02 | 2 | 4 | -4 | # BR(h -> c c^-) |
| 5.97839285E-01 | 2 | 5 | -5 | # BR(h -> b b^-) |
| 2.49351532E-04 | 2 | 13 | -13 | # BR(h -> mu- mu+) |
| 6.91400587E-02 | 2 | 15 | -15 | # BR(h -> tau- tau+) |
| 8.17011648E-02 | 2 | 21 | 21 | # BR(h -> g g) |
| 2.38951877E-03 | 2 | 22 | 22 | # BR(h -> gam gam) |
| 1.64468878E-03 | 2 | 22 | 23 | # BR(h -> gam Z) |
| 2.2850951E-02 | 2 | 23 | 23 | # BR(h -> Z Z*) |
| 1.98520240E-01 | 2 | 24 | -24 | # BR(h -> W+ W-) |

FIG. 2. Example of the output file (outBR_hsm.txt)
Input parameters for the HSM

500.d0 ! m_H in GeV
0.1d0 ! alpha
0.4d0 ! lambda_{phi \ S}
0.1d0 ! lambda_S
0.0d0 ! mu_S in GeV
3.d3 ! cutoff in GeV

FIG. 3. Example of the input file (in_hsm.txt)

FIG. 4. Branching ratios of the 125-GeV Higgs boson as a function of tan\beta in the Type-(I, II, X, Y) THDMs for sin(\beta - \alpha) = 0.99 with cos(\beta - \alpha) > 0, where we take M = m_H = 300 GeV and m_A = m_{H^\pm} = 600 GeV.
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