Privacy-Preserving Database Fingerprinting

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Abstract—When sharing sensitive relational databases with other parties, a database owner aims to (i) have privacy guarantees for the entries in the shared database, (ii) have liability guarantees (e.g., via fingerprinting) in case of unauthorized sharing of its database by the recipients, and (iii) provide a high quality (utility) database to the recipients. We observe that sharing a relational database with privacy and liability guarantees are orthogonal objectives. The former can be achieved by injecting noise into the database to prevent inference of the original data values, whereas, the latter can be achieved by hiding unique marks inside the database to trace malicious parties (data recipients) who redistribute the data without the authorization. In this paper, we achieve these two objectives simultaneously by proposing a novel entry-level differentially-private fingerprinting mechanism for relational databases.

At a high level, the proposed mechanism fulfills the privacy and liability requirements by leveraging the randomization nature that is intrinsic to fingerprinting and achieves desired entry-level privacy guarantees. To be more specific, we devise a bit-level random response scheme to achieve differential privacy guarantee for arbitrary data entries when sharing the entire database, and then, based on this, we develop an $\epsilon$-entry-level differentially-private fingerprinting mechanism. Next, we theoretically analyze the relationships between privacy guarantees, fingerprint robustness, and database utility by deriving closed form expressions. The outcome of this analysis allows us to bound the privacy leakage caused by attribute inference attack and characterize the privacy-utility coupling and privacy-fingerprint robustness coupling. Furthermore, we also propose a sparse vector technique (SVT)-based solution to control the cumulative privacy loss when fingerprinted copies of a database are shared with multiple recipients.

We experimentally show that our proposed mechanism achieves stronger fingerprint robustness than the state-of-the-art mechanisms (e.g., the fingerprint cannot be compromised even if a malicious recipient changes 80% of the entries in the fingerprinted database), and higher database utility compared to the simple composition of database perturbation under local differential privacy followed by fingerprinting (e.g., the statistical utility of the shared database by the proposed scheme is more than $10\times$ higher than perturbation followed by fingerprinting).

Index Terms—database, privacy, liability, fingerprinting

I. INTRODUCTION

Massive data collection and availability of relational databases (collection of data records with the same set of attributes) are very common in the current big data era. This results in an increasing demand to share such databases with (or among) different service providers (SPs), such as companies, research institutions, or hospitals, for the purpose of “do-it-yourself” calculations, like personal advertisements, social recommendations, and customized healthcare.

Most relational databases include personal data, and hence they usually contain sensitive and proprietary information, e.g., medical records collected as part of an agreement which restricts redistribution. This poses three major challenges in the course of database sharing with different SPs: (i) the database owner is obligated to protect the privacy of data entries in the shared database to comply with the privacy policy and ensure confidentiality, (ii) the database owner needs to prevent illegal redistribution of the shared databases, and eventually should be able to prosecute the malicious SPs who leak its data, and (iii) the shared database needs to maintain high utility to support accurate data mining and data analysis.

A. Current Status

Many works have attempted to address the challenges on privacy and liability in isolation. To address the privacy challenge, various data sanitization metrics are proposed, e.g., $k$-anonymity \cite{20}, $l$-diversity \cite{24}, $t$-closeness \cite{26}, and differential privacy \cite{16}. Among them, differential privacy has been developed as a de facto standard for responding to statistical queries from databases with provable privacy guarantees. It can also be used to share personal data streams or an entire database (i.e., identity query) in a privacy-preserving manner \cite{11,20}. Differentially-private mechanisms hide the presence or absence of a data record in the database by perturbing the query results with noise calibrated to the query sensitivity.

To protect copyright and deter illegal redistribution, different database watermarking and fingerprinting mechanisms are devised to prove database ownership (i.e., identification of the database owner from the shared database) \cite{1,39} and database possession (i.e., differentiating between the SPs who have received copies of the database) \cite{21,29,43}. In particular, when sharing a database with a specific SP, the database owner embeds a unique fingerprint (a binary string customized for the data recipient SP) in the database. The embedded fingerprint is typically hard to be located and removed even if a malicious SP attacks the fingerprinted database (to identify and distort the fingerprint).

In the literature, only a few works have attempted to combine database sanitization and fingerprinting during database sharing. In particular, \cite{8,17,24} propose inserting fingerprints into databases sanitized using $k$-anonymity and \cite{38} proposed embedding fingerprints into a database sanitized by the $(\alpha, \beta)$-privacy model \cite{37}. However, these works solve the aforementioned challenges in a two-stage (sequential) manner,
where data sanitization is conducted followed by fingerprinting. As a result, they end up changing a significant amount of entries in the database and they significantly compromise the utility of the shared database. We experimentally corroborate this in Section VII. These works also do not address the critical problem of controlling cumulative privacy loss if the same database is repeatedly shared with multiple SPs.

B. This Paper

In this work, we bring together data sanitization and fingerprinting in a unified data sharing algorithm, consider a stronger privacy model compared to previous works, and develop entry-level differentially-private fingerprinting for relational database sharing. In what follows, we briefly summarize the main contributions and insights of our work, and discuss its limitation caused by a unique requirement of DBMS (Database Management System) design.

Contributions. We observe that database fingerprinting is a randomized mechanism (which essentially performs bitwise randomization), and thus is naturally endowed with certain level of privacy protection. However, this hidden property (privacy protection) is ignored in the literature. In this paper, we harness this intrinsic randomness and transform it into a provable privacy guarantee. In particular,

- We propose a bit-level random response scheme, which fingerprints insignificant bits of data entries using pseudorandomly generated binary mark bits, to achieve $\epsilon$-entry-level differential privacy (Definition 2) for the entire database. Then, we devise an $\epsilon$-entry-level differentially-private fingerprinting mechanism (Algorithm 1) based on the bit-level random response scheme.
- We establish a comprehensive and solid theoretical foundation (Section V) to quantify the properties of the proposed $\epsilon$-entry-level differentially-private fingerprinting mechanism from 3 dimensions: (i) the privacy guarantee of it under attribute inference attack, (ii) the fingerprint robustness of the mechanism when it is subject to various attacks that can be launched against a fingerprinting mechanism, and (iii) the relationship among privacy and utility, and privacy and fingerprint robustness.
- We devise a sparse vector technique (SVT)-based solution (Algorithm 4) to control the cumulative privacy loss when fingerprinted databases are shared with multiple SPs.
- We evaluate and validate the proposed mechanism using a real-life database. Our experimental results show that the proposed mechanism (i) provides higher fingerprint robustness than a state-of-the-art database fingerprinting mechanism, (ii) provides higher database utility than the naïve solution, which first perturbs the database under local differential privacy guarantee, and then inserts the fingerprint, and (iii) achieves high database utility along with high privacy guarantee and fingerprint robustness even when the database is shared for multiple times.

Insights. This paper is the first to show the feasibility of considering privacy and liability in a unified mechanism to protect data privacy and prevent unauthorized data redistribution simultaneously. The proposed mechanisms can be used to guide a database owner to (i) generate privacy-preserving fingerprinted databases based on customized requirements on utility, privacy level, and fingerprint robustness and (ii) assess the privacy leakage under multiple database sharings and set the privacy budget accordingly in each sharing.

By harnessing the intrinsic randomness in fingerprinting to bridge privacy and liability, we believe that our work will draw attention to other challenges and urgent research problems along this direction including mitigating both attribute inference attacks and fingerprinting distortion that utilizes the correlations among data entries, addressing the collusion attack launched by allied malicious SPs, and improving the database utility via Bayesian denoising that leverages the data distributions. In Section VIII we provide potential solutions to each of these open problems.

Limitations. In this work, we consider the sharing of entire relational databases, where each data record (i.e., row) can be uniquely identified by an immutable pseudo-identifier (i.e., the primary key). This is a unique requirement of DBMS (Database Management System) design, and hence in this work, we do not consider membership inference attacks as they become irrelevant under these settings. We further discuss this in detail in Section III.

Paper organization. In Section II we review related works in the literature, which is followed by the privacy, system, and threat models in Section III. In Section IV we present the proposed entry-level differentially-private fingerprinting mechanism. Then, we theoretically investigate the relationships between database utility, fingerprint robustness, and privacy guarantees in Section V. We develop the SVT-based mechanism to share multiple fingerprinted databases under entry-level differential privacy in Section VI. We evaluate the proposed scheme via extensive experiments in Section VII. We provide further discussions and point out open problems with potential solutions in Section VIII. Finally, Section IX concludes the paper.

II. RELATED WORK

Quite a few works have studied the problem of protecting data privacy and ensuring liability in isolation when sharing databases. The works that are closest to ours include [5], [17], [24], [38]. Yet, these works embed watermark or fingerprint into an already sanitized database, instead of considering data sanitization and marking together (as a unified process). Thus, such sequential processing of database will result in significant degradation in utility. To be more specific, Bertino et al. [5] adopted the binning method [30] to generalize the database first, and then watermark the binned data to protect copyright. Kieseberg et al. [24] and Schrittweisner et al. [38] proposed fingerprinting a database generalized by $k$-anonymity. Gambús et al. [17] sanitized the database using the $(\alpha, \beta)$-anonymity model [37], which selects a true data record from the domain of the database with probability $\alpha$ and includes a fake data
Thus, the update of the primary key causes further changes in other tables.

be used to refer to another table keeping the applicant’s real name, email, etc.

applicant’s unique identification number. The identification number can then

applicant. Here, the primary key of the data record can be chosen as the

database in Section VII in which each data record represents a school

of potentially many other tables or rows in the system. The reason is that

the database in Section VII, we compare the proposed scheme

A. Privacy Model

Next, we provide our privacy model customized specifically

for relational databases with immutable primary keys (i.e., data

record pseudo-identifiers).

Definition 4 (ϵ-entry-level differential privacy). A randomized

mechanism \( M \) with domain \( \mathcal{D} \) satisfies ϵ-entry-level differential

privacy if for any two neighboring relational databases

\( R, R' \in \mathcal{D} \), and for all \( S \in \text{Range}(M) \), it holds that

\[
\Pr[M(R) = S] \leq e^\epsilon \Pr[M(R') = S], \quad \text{where } \epsilon > 0.
\]

Remark 1. Our proposed privacy model (Definition 4) is

adapted from the conventional notation of ϵ-differential privacy [10], which obfuscates the presence or absence of an entire row in \( R \). Since the database recipient can easily identify if an individual is present in \( R \) by directly checking its primary key, the conventional ϵ-differential privacy is not appropriate in the setting we consider. In contrast, our privacy model, which aims at obscuring the specific value of an arbitrary entry in \( R \), better suits the requirement of database management system (DBMS) design. As discussed in Section V-B, destroying pseudo-identifiers to prevent linkability or membership inference attacks becomes an ill-posed problem for our considered case of DBMS. Thus, we will focus on the attribute inference attacks instead of membership inference attacks in the paper.

B. Database Fingerprinting System

We present the system model in Figure 1. We consider a database owner with a relational database denoted as \( R \), who wants to share it with at most \( C \) SPs (e.g., to receive services, to help researchers, or for collaborative research). To prevent unauthorized redistribution of the database by a malicious SP (e.g., the \( c \)th SP in Figure 1), the database owner includes unique fingerprints in all shared copies of the database. The
fingerprint essentially changes different entries in $R$ at different positions (indicated by the yellow dots). The fingerprint bit-string customized for the $c$th SP ($SP_c$) is denoted as $f_{SP_c}$, and the database received by $SP_c$ is represented as $R_c$. Both $f_{SP_c}$ and $R_c$ are obtained using the proposed mechanism discussed in Section IV. We let $R$ represent a general instance of the privacy-preserving fingerprinted database.

Fig. 1: System model considered in this paper. All shared copies of the database meet entry-level differential privacy, fingerprint robustness, and database utility requirements.

We aim to achieve three main goals in this system, i.e.,

(i) Privacy guarantee for each data entry, i.e., a data analyst cannot distinguish between $r_i[t]$ and $r_i[t]'$ by inferring its received copy $\tilde{R}$. (ii) High fingerprint robustness in order to successfully extract a malicious SP’s fingerprint (even if the malicious SP tries to distort the fingerprint to mitigate detection) if the database is redistributed without authorization. (iii) High data utility for the fingerprinted database in order to support accurate database queries and data mining tasks.

C. Potential Threats

Since we consider developing a mechanism to simultaneously achieve data privacy and liability guarantees, we also need to address the corresponding threats from these two aspects. In particular, the malicious SP can

- Adopt sophisticated learning methods to infer the original values of each data entry (in the shared database) by using its prior knowledge or other revealed data entries. In particular, we consider an adversary who knows all data entries except for one, and it will use advanced learning methods to infer the original value of the unknown data entry. In Section VIII, we discuss how to augment our proposed mechanism to defend against attribute inference attacks that leverage the correlations among data entries.
- Conduct various attacks to distort the embedded fingerprint bit-strings, e.g., random bit flipping attack (which changes bits of randomly selected data entries), subset attack (which randomly removes fractions of data records from the database), and correlation attack (which changes the data entries by taking advantage of data correlations available from other resources). In Section V, we discuss these attacks in detail and derive closed-form robustness expression achieved by our mechanism.

IV. PRIVACY-PRESERVING FINGERPRINTING

In this section, we present the proposed privacy-preserving fingerprinting mechanism. First, we discuss the design principles of the mechanism. Next, we develop a general condition for bit-level random response to achieve entry-level differentially-private database release (or sharing). Then, we propose a concrete mechanism built upon the bit-level random response scheme to achieve provable privacy guarantees during fingerprint insertion.

A. Principles of Mechanism Design

First of all, we notice that all fingerprinting schemes achieve liability guarantees on each shared copy by flipping different collections of randomly selected bits of the data entries using a certain probability. The collections of selected bits vary for different SPs and their fingerprinted values are determined by the unique fingerprint bit-strings of the SPs. Thus, we observe that database fingerprinting schemes are randomized mechanisms, which essentially perform bitwise-randomization, i.e., change the data values by introducing noise at the bit-level of the data entries instead of directly perturbing the data entries (i.e., introducing noise at the enter-level). As a consequence, we also establish our entry-level differentially-private fingerprinting scheme by conducting bitwise-randomization, and to achieve provable privacy guarantee, we calibrate the flipping probability based on the sensitivities of the data entries.

On the contrary, the entrywise-randomization adopted by the conventional differentially-private output perturbation mechanisms, e.g., [11], [20], are infeasible as a building block of a fingerprinting mechanism, because they substantially change all data entries by adding noises drawn from some probability distributions. Although local differential privacy via randomized response only changes each data entry with a particular probability determined by the privacy budget and the number of possible instance of the entries [4], connecting this probability with the randomly generated fingerprint bit-string is not straightforward. This is because randomly changing each bit of each data entry (by fingerprinting) may not lead to the identical random effect required by local differential privacy. Hence, it is also not suitable for designing a database fingerprinting mechanism.

Based on the principle of bitwise-randomization, we consider achieving entry-level differential privacy for the release of the entire database by using bit-level random response. In particular, when sharing a database with a specific SP, the values of selected insignificant bits of selected data entries are determined by XORing them with random binary variables, which is different for different data sharing instances (with various SPs). Such modification of bit positions in the database using different binary values can also be considered as inserting different fingerprints, which can be used to accuse
a malicious SP if there is data leakage. Moreover, to achieve high utility for the shared database, we simultaneously achieve entry-level differential privacy and fingerprinting, instead of fingerprinting a differentially-private database (or sanitizing an already fingerprinted database). This will be further elaborated in Section V and validated in Section VII.

B. Privacy-preserving Database Release via Bit-level Randomization

Traditional differential privacy guarantees that the released statistics computed from a database (such as mean or histogram) are independent of the absence or presence of an individual. However, in this work, we consider the release (sharing) of the entire fingerprinted database and the existence of a particular individual can be easily determined by checking its corresponding primary key in the released copy (as discussed in Section III-B). Therefore, in this work, we consider the privacy of database entries (i.e., attributes of individuals). Many works have also studied the problem of achieving differential privacy for entries in a database, e.g., [12], [20], [22], [35]. Our work differs from them as we can achieve additional liability for entries in a database, e.g., [12], [20], [22], [35].

We use MD5 to generate a 128-bits fingerprint string, because if the database owner shares L copies of its database, then as long as L ≥ ln C, the fingerprinting mechanism can thwart exhaustive search and various types of attacks. Usually, a 64-bits fingerprint can provide high robustness [29].

Now, we define the bit-level random response scheme.

Definition 5 (Bit-level random response scheme). A bit-level random response scheme (pseudorandomly) selects some bits of some data entries in the database and changes the bit values of such entries by conducting an XOR operation on them with independently generated random binary mark bits, denoted as B, where B ∼ Bernoulli(p).

Existing database fingerprinting schemes only mark the insignificant bits of the data entries to introduce differential privacy in the database. In this paper, we assume that the kth to the last bit of an entry is its kth insignificant bit. If the kth insignificant bit of attribute t of data record r_i (represented as r_i[t, k]) is selected, then the bit-level random response scheme changes its value as r_i[t, k] ⊕ B, where ⊕ stands for the XOR operator, and B is a Bernoulli random variable with parameter p.

As discussed in Section III-B, the modified database should have small amount of error, because both database owner and data recipient SPs expect high utility for the shared (and received) relational databases. Thus, to obtain high database utility, we let the bit-level random response scheme only change the last K bits of data entries, i.e., k ∈ [1, K]. As a result, we can develop the following condition for such a scheme to achieve ε-entry-level differential privacy on the entire database. The proof is deferred to Appendix B.

Theorem 1. Given a relational database R with sensitivity $\Delta$ (Definition 3), a bit-level random response scheme, which only changes the last K bits of data entries, satisfies ε-entry-level differential privacy if $K = \lceil \log_2 \Delta \rceil + 1$ and $p \geq \frac{1}{e^{\epsilon K} + 1}$.

C. An ε-Entry-level Differentially-Private Fingerprinting Mechanism

Due to the randomness involved in the bit-level random response scheme, for any given p and R, the output databases will vary for each different run. However, in order to detect the guilty SP who is responsible for the leakage of a database, it is required that the fingerprinted database shared with a specific SP must be unique and it can be reproduced by the database owner even if the mark bits, i.e., B’s, are generated randomly. In this section, we discuss how to develop an instantiation of an ε-entry-level differentially-private fingerprinting mechanism based on the bit-level random response scheme, i.e., a mechanism that satisfies Theorem 1 and at the same time, is reproducible when sharing a fingerprinted copy with any specific SP using a given Bernoulli distribution parameter p.

First, we collect all fingerprintable bits in R, i.e., all insignificant bits (the last K bits) of all entries, in a set $\mathcal{P}$: $\mathcal{P} = \{ r_i[t, k] | i \in [1, N], t \in [1, T], k \in [1, \min\{K, K_t]\} \}$, where N is the number of data record in R, and $K_t$ represents the number of bits to encode the tth attribute in R. When the database owner wants to share a fingerprinted copy of R with an SP with a publicly known external ID denoted as ID_{external}, it first generates an internal ID for this SP denoted as ID_{internal}. We will elaborate the generation of ID_{internal} in Section VI-A. Then, the database owner generates the unique fingerprint for this SP using a cryptographic hash function, i.e., $f = Hash(y|ID_{internal})$, where y is the secret key of the database owner and | represents the concatenation operator. We use L to denote the length of the generated fingerprint.

The database owner also has a cryptographic pseudorandom sequence generator $\mathcal{U}$, which selects the data entries and their insignificant bits, and determines the mask bit $x$ and fingerprint bit $f$ (which is an element of the fingerprint bit-string f) to obtain the Bernoulli random variable (i.e., $B = x \oplus f$). To be more specific, for each $r_i[t, k]$ in $\mathcal{P}$, the database owner sets the initial seed as $s = \{ r_i[t, k], PreyKey[t] \}$. If $\mathcal{U}_i(s) \mod \frac{1}{2^p} = 0 \ (p = \frac{1 - e^{\epsilon K}}{e^{\epsilon K} + 1})$, then $r_i[t, k]$ is fingerprinted. Next, the database owner decides the value of mask bit $x$ by checking if $\mathcal{U}_2(s)$ is even or odd, and sets the fingerprint index $l = \mathcal{U}_3(s) \mod L$. It obtains the mask bit $B = x \oplus f(l)$, and finally changes the bit value of $r_i[t, k]$ with $r_i[t, k] \oplus B$. We summarize the steps to generate a fingerprinted database $\mathcal{M}(R)$ for SP $ID_{external}$ in Algorithm 1.

Theorem 2. Algorithm 1 is ε-entry-level differentially-private.

Remark 2. Please refer to Appendix C for the proof. The proposed database fingerprinting scheme is different from the existing ones discussed in Section II as all existing schemes fingerprint each selected bit by replacing it with a new value obtained from the XOR of pseudorandomly generated mask bit x and fingerprint bit f. As a result, the new value is indepen-
Algorithm 1: Generate $\mathcal{M}(\mathbf{R})$ for SP $ID_{\text{external}}$.

**Input:** The original database $\mathbf{R}$, the privacy budget $\epsilon$, number of changeable bits $K$, the Bernoulli distribution parameter $p = \frac{\epsilon}{\lceil K \rceil}$, database owner’s secret key $Y$, and pseudorandom number sequence generator $U$.

**Output:** $\epsilon$-differentially-private fingerprinted database $\mathcal{M}(\mathbf{R})$.

1. Construct the fingerprintable set $\mathcal{P}$.
2. Generate the internal ID, i.e., $ID_{\text{internal}}$ for this SP (will be elaborated in Section V-A).
3. Generate the fingerprint string, i.e., $f = Hash(Y||ID_{\text{internal}})$.
4. For all $r_i[t,k] \in \mathcal{P}$ do
   5. Set pseudorandom seed $s = \{Y|\mathbf{R},P\text{myKey}|t[k]\}$.
   6. If $U_d(s) \mod \lceil \frac{1}{p} \rceil = 0$ then
      7. Set mask bit $x = 0$, if $U_d(s)$ is even; otherwise $x = 1$.
   8. Set fingerprint index $l = U_d(s) \mod L$.
   9. Let fingerprint bit $f = f(l)$.
   10. Obtain mark bit $B = x \oplus f$.
   11. Set $r_i[t,k] = r_i[t,k] \oplus B$. {insert fingerprint}
5. Return the fingerprinted database $\mathcal{M}(\mathbf{R})$.

V. ASSOCIATING PRIVACY, FINGERPRINT ROBUSTNESS, AND DATABASE UTILITY

In the previous section, we have presented a mechanism that achieves provable privacy guarantee when fingerprinting a database. Here, we investigate its impact on the database utility and fingerprint robustness, and also establish the connection between $p$ (the Bernoulli distribution parameter, which represents the probability of changing one insignificant bit of a data entry) and entry-level differential privacy guarantee ($\epsilon$), fingerprint robustness, and utility of shared databases. We show a graphical relationships between these in Figure 2 where the arrow means “leads to”. Clearly, we can obtain the high-level conclusion that differential privacy and fingerprint robustness are not conflicting objectives that can be achieved at the same time, whereas, at the cost of database utility.

A. Privacy against General Attribute Inference Attacks

After receiving the fingerprinted database $\mathcal{M}(\mathbf{R})$, a malicious SP can leverage sophisticated learning methods to infer the original value of each data entry. In this section we show that under our proposed privacy model (i.e., entry-level differential privacy), the malicious SP’s inference capability can never exceed a certain threshold.

In particular, we consider a malicious SP who has access to $\mathbf{R}/r_i[t]$ (i.e., the original values of all data entries except
the $t$th attribute of the $i$th data record, $r_i[t]$, and its inference capability (denoted as $\text{InfCap}$) is defined as
\[
\text{InfCap} = \Pr(r_i[t] = \zeta_1 | \mathcal{M}(R), R_{/r_i[t]}),
\]
which is the posterior probability of the unknown entry $r_i[t]$ takes a specific value $\zeta_1$. $\text{InfCap}$ covers a wide-range of inference attacks using learning-based techniques, because most of the learning frameworks give outputs in terms of posterior probabilities, e.g., Bayesian inference and deep learning. In particular, we can reach to the following proposition about the inference capability of a malicious SP (the proof is shown in Appendix F).

**Proposition 1.** No matter what learning-based inference attack the malicious SP conducts, its inference capability can never be higher than $\frac{\psi_1^e}{\psi_1^e + \Psi}$, i.e., $\text{InfCap} \leq \frac{\psi_1^e}{\psi_1^e + \Psi}$, where
\[
\psi = \frac{\Pr(r_i[t] = \zeta_1 | R_{/r_i[t]})}{\Pr(r_i[t] = \zeta_2 | R_{/r_i[t]})}
\]
is the malicious SP’s prior knowledge on the ratio between the probabilities of the unknown entry $r_i[t]$ taking different values (i.e., $\zeta_1$ and $\zeta_2$) given all other entries are known.

For a given $\psi$, $\frac{\psi_1^e}{\psi_1^e + \Psi}$ decreases as $\epsilon$ decreases, it means that the higher the entry-level differential privacy guarantee (smaller $\epsilon$) the lower the inference capability of malicious SPs. In Appendix M-C we consider a similar adversary proposed in [31], empirically investigate its inference capability, and compare it with our derived upper bound.

The above considered adversary who knows the entire database except one data entry is a standard threat model in the differential privacy literature. In some of the real-world attacks, an adversary can also utilize some publicly known auxiliary information (e.g., correlation among data entries [11], [31], [42], [44] or social connections [20], [25]) to improve its inference capability.

Some works have attempted to augment the conventional differentially-private mechanisms to also make them robust against inference attacks utilizing the auxiliary information. For example, Liu et al. [31] propose to augment the Laplace mechanism with a dependence coefficient, which computes the query sensitivity of correlated data. Chanyaswad et al. [11] reinforce the Gaussian mechanism by considering the row- and column-wise covariance matrix in database. Ji et al. [20] enhance random response of binary data by modeling the correlation between binary data using log-associations.

Our adopted privacy model can also be easily augmented to achieve robustness against attribute inference attacks that use additional information about data correlations. In particular, if a malicious SP knows the pairwise data entry correlations (measured in terms of pairwise joint distributions between columns), it can improve \(\text{InfCap}\) by factoring $\psi$, i.e.,
\[
\psi = \frac{\Pr(r_i[t] = \zeta_1 | R_{/r_i[t]})}{\Pr(r_i[t] = \zeta_2 | R_{/r_i[t]})} = \frac{\prod_{k \neq i} \Pr(r_i[t] = \zeta_1 | R_{/r_i[t]})}{\prod_{k \neq i} \Pr(r_i[t] = \zeta_2 | R_{/r_i[t]})}.
\]
Then, to achieve a provable privacy guarantee against attribute inference attacks using data correlations, the database owner can further augment the proposed entry-level differentially private mechanism by involving the auxiliary information of the joint probability distribution of any pair of data entries. In the mechanism design, i.e., adjusting $p$ accordingly based on the value of $\psi$ and $\epsilon$. In Section VIII we provide a mathematically viable approach to achieve such augmentation.

The goal of attribute inference attack using data correlations is to compromise data privacy. In Section VII C3 we discuss a counterpart of it to compromise the fingerprint robustness; the malicious SP can also leverage the discrepancy between data correlations before and after fingerprint insertion to distort the embedded fingerprint bits. Also, in Section VIII we provide a thorough discussion on how the database owner can also utilize data correlations to mitigate both attribute inference attack and correlation-based attacks target on fingerprints.

**B. Database Utility**

Fingerprinting naturally changes the content of the database, and thus degrades the utility. Here, we evaluate the utility of a fingerprinted database from both data accuracy and data correlation perspectives: we quantify the impact of Algorithm I on the accuracy of each fingerprinted data entry and the joint probability distribution of any pair of data entries. The theoretical analyses are summarized in the following propositions. These considered utility metrics are application independent, and in general, the higher the accuracy of data entries and pairwise joint distributions, the better the task-specific application utilities, e.g., classification accuracy and mean square error. We empirically validate this statement by evaluating the proposed scheme by focusing on task-specific application utilities in Section VII.

**Proposition 2.** Let $r_i[t]$ and $\tilde{r}_i[t]$ be the original and the fingerprinted values of the $t$th attribute of the $i$th row. Then, the expected error caused by fingerprinting, i.e.,
\[
E_{B \sim \text{Bernoulli}(p)} \left[ r_i[t] - \tilde{r}_i[t] \right],
\]
falls in the range of $[0, \Delta p]$, where $\Delta$ is the sensitivity of a pair of neighboring relations (see Definition 3), and $p$ is the probability of a mark bit $B$ taking value 1, i.e., the probability of changing one insignificant bit of a data entry.

The proof is in Appendix E. Clearly, the higher the value of $p$, the larger the expected absolute difference between a fingerprinted data entry and the original value. This suggests that the database owner can set the value of $p$ based on
its requirement of data entry accuracy when generating a fingerprinted database, which achieves a certain level of entry-level differential privacy, and vice versa. This leads us to the following corollary.

**Corollary 1.** Define fingerprint density as \( \|M(R) - R\|_{1,1} \), where \( \| \cdot \|_{1,1} \) is the matrix \((1, 1)\)-norm which sums over the absolute value of each entry in the matrix. Then, we have 
\[
\mathbb{E}_{\tilde{B} \sim \text{Bernoulli}(p)} \left[ \|M(R) - R\|_{1,1} \right] \in [0, \Delta pNT].
\]

In Section VI-B we will exploit the concept of fingerprint density to develop a SVT-based solution to share fingerprinted databases with multiple SPs.

**Proposition 3.** Let \( \Pr(R[t] = \pi, R[z] = \omega) \) and \( \Pr(R[t] = \pi, R[z] = \omega) \) be the joint probability of the \( t \)th attribute taking value \( \pi \) and the \( z \)th attribute taking value \( \omega \) before and after fingerprinting, respectively. Then, \( \Pr(R[t] = \pi, R[z] = \omega) \) falls in the range of 
\[
\frac{1}{2} \left( 1 - (1 - p)^2K + P_{\text{max}}(R[t], R[z]) \right) \left( 1 - (1 - p)^K \right)^2,
\]
where \( P_{\text{min}}(R[t], R[z]) \) (or \( P_{\text{max}}(R[t], R[z]) \)) denotes the minimum (or maximum) joint probability of the \( t \)th and \( z \)th attributes in the original database.

The proof is in Appendix C. By marginalizing over \( R[t] \) and \( R[z] \), we can have the following corollary about the impact of fingerprinting on the marginal distributions.

**Corollary 2.** Let \( \Pr(R[t] = \pi) \) and \( \Pr(R[t] = \pi) \) be the marginal probability of the \( t \)th attribute taking value \( \pi \) before and after fingerprinting, respectively. Then, \( \Pr(R[t] = \pi) \) belongs to 
\[
\left\{ \frac{1}{2} \left( 1 - (1 - p)^2K + P_{\text{max}}(R[t]) \right) \left( 1 - (1 - p)^K \right)^2, \right\}
\]
where \( P_{\text{max}}(R[t]) \) (or \( P_{\text{max}}(R[t]) \)) denotes the minimum (or maximum) marginal probability of the \( t \)th attribute in the original database.

Clearly, when \( p \) is small, both joint distributions and marginal distributions will be close to that of the original databases. This means that the fingerprinted database will have higher statistical utility for smaller values of \( p \).

### C. Fingerprint Robustness

Although, Li et al. [29] have attempted to analyze fingerprint robustness by studying the false negative rate (i.e., the probability that the database owner fails to extract the exact fingerprint from a pirated database), they do not establish the direct connection between the robustness and the tuning parameter (the fingerprinting ratio, which can be interpreted as a counterpart of \( p \) in our work) in their mechanism.

In this paper, we investigate the robustness of the proposed fingerprinting mechanism against three attacks, i.e., the random bit flipping attack [1], [14], [29], subset attack [10], [14], [29], [43], and correlation attack [27], [43]. These attacks are all well-established attacks for database fingerprinting, and have been widely studied to investigate the robustness of a database fingerprinting mechanism in the literature. In the following, we quantitatively analyze the relationship between \( p \) (the probability of changing one insignificant bit of a data entry) and fingerprint robustness against these three attacks.

1) **Robustness Against Random Flipping Attack:** In random flipping attack, a malicious SP flips each of the \( K \) last bits of data entries in \( R \) with probability \( \gamma_{\text{rnd}} \) with the goal of distorting the data in the fingerprinted positions. In [21], the authors have empirically shown that the malicious SP ends up being uniquely as accurate as long as the extracted fingerprint from the leaked database has more than 50% matches with the malicious SP’s fingerprint.

However, as the database owner shares more fingerprinted copies of its database with different SPs, to uniquely hold the correct malicious SP responsible, it requires more bit matches between the extracted fingerprint and the malicious SP’s fingerprint. Thus, the number of bit matches (denoted as \( D, D \leq L \)) should be set based on the number of fingerprinted sharings of the same database. Please refer to Appendix H for the discussion of how to determine \( D \) given the number of times a database is shared (with different SPs) and the fingerprint length \( L \).

Given \( D \), we evaluate the robustness of the proposed fingerprinting mechanism against random bit flipping attack in terms of the probability (denoted as \( P_{\text{robst-rnd}} \)) that the database owner successfully extracts any \( D \) fingerprint bits of the malicious SP. Let the \( t \)th bit of the fingerprint string be embedded \( w_l \) times in \( R \) (with the probability \( \left( \frac{1}{2} \right)^{w_l} \)).

To extract this fingerprint bit correctly from a copy of \( R \) that is compromised by the random bit flipping attack, the database owner needs to make sure that at most \( \left( \frac{w_l}{2} \right) \) bits in \( R \) that are marked by the \( t \)th bit of the fingerprint string are flipped by the malicious SP, which happens with probability
\[
p_l = \sum_{w_l=0}^{\left( \frac{w_l}{2} \right)} \left( \frac{1}{2} \right)^{w_l} \gamma_{\text{rnd}} \left( 1 - \gamma_{\text{rnd}} \right)^{w_l - r}.
\]

Let \( m \) be the number of fingerprinted bit positions in the database \( (m \leq NK) \) received by the malicious SP, and define set \( W \) as \( W = \{w_1, w_2, \ldots, w_L > 0| \sum_{l=1}^{L} w_l = m \} \). Let also \( L_D \) be the collection of any \( D \) bits of the malicious SP’s fingerprint \( (|L_D| = D) \). Then, by marginalizing all possible instances of malicious SP’s fingerprint, all collections of \( D \) fingerprint bits of it, and the number of fingerprinted bits \( m \), we have the closed form expression of \( P_{\text{robst-rnd}} \) in terms of \( p \) as
\[
P_{\text{robst-rnd}} = \sum_{m=1}^{NK} \left( \sum_{w_l \in W, |l| \in [1, L]} \sum_{l \in L_D} p_l \left( \frac{1}{2} \right)^{w_l} \right) (NK - m) (1 - 2p)^{NK - m}. \]

To show that higher \( p \) leads to more robustness against the random bit flipping attack, it is equivalent to show that \( P_{\text{robst-rnd}} \) is monotonically increasing with \( p \) \((0 < p < 0.5)\) Detailed analysis is shown in Appendix H.

2) **Robustness Against Subset Attack:** In subset attack, the malicious SP generates a pirated database by selecting each data record in \( R \) for inclusion (in the pirated database)
with probability $\gamma_{\text{sub}}$. This attack is shown to be much weaker than the random bit flipping attack \cite{21, 29, 43}. According to \cite{29} (page 40), the subset attack cannot succeed (i.e., distorting even one fingerprint bit) unless the malicious SP excludes all the rows fingerprinted by at least one fingerprint bit. Thus, we measure the robustness of the proposed fingerprinting mechanism against subset attack using the probability (denoted as $P_{\text{rbst}_{\text{sub}}}$) that the malicious SP fails to exclude all fingerprinted rows involving a particular fingerprint bit (note that our analysis can be easily generalized for excluding a fraction of fingerprinted rows).

Since the probability that a specific row is fingerprinted by a specific fingerprint bit is $1 - (1 - p/L)^{KT}$, we have the closed form expression of $P_{\text{rbst}_{\text{sub}}}$ in terms of $p$ (the probability of changing an insignifican bit of an entry) as

$$P_{\text{rbst}_{\text{sub}}} = 1 - \sum_{n=1}^{N} \left( \binom{N}{\gamma_{\text{sub}}} \right)^n \left( 1 - (1 - p/L)^{KT} \right)^n \left( 1 - (1 - p/L)^{KT(N-n)} \right) = 1 + \left( 1 - (1 - p/L)^{KT} \right) - \left( 1 - (1 - p/L)^{KT} + \gamma_{\text{sub}}(1 - p/L)^{KT} \right)^N.$$

Clearly, the larger the probability of $p$, the less the difference between $1 - (1 - p/L)^{KT}$ and $1 - (1 - p/L)^{KT} + \gamma_{\text{sub}}(1 - p/L)^{KT}$, which suggests that $P_{\text{rbst}_{\text{sub}}}$ monotonically increases with $p$.

3) Robustness Against Correlation Attack: In \cite{21}, the authors identify a correlation attack against database fingerprinting mechanisms, which takes advantage of the intrinsic correlation between data entries in the database to infer and compromise the potentially fingerprinted bit positions. In particular, the malicious SP changes the insignificant bits of entries in $\mathbf{R}$ if the data entries satisfy

$$\Pr(\mathbf{R}[^{\pi}] = \pi, \mathbf{R}[^{\omega}] = \omega) - \Pr(\mathbf{R}[^{\pi}] = \pi, \mathbf{R}[^{\omega}] = \omega) \geq \tau, \forall \omega, \forall \mathbf{R},$$

where $\tau$ is a predetermined parameter for this attack.

Similar to \cite{21}, we adopt the confidence gain of the malicious SP (denoted as $G$) to analyze the robustness of the proposed fingerprinting mechanism against the correlation attack. The confidence gain measures the knowledge of a potentially fingerprinted data entry under correlation attack over random guess. To be more specific, $G$ is defined as the ratio between the probability that a specific entry (whose original $t$th attribute takes value $\pi$) will be selected to be compromised in the correlation attack and the probability that such entry will be selected to be compromised in the random bit flipping attack. Mathematically, this can be shown as

$$G = \frac{1 - \Pi_{\mathbf{R} \in \{1, \ldots, N\}} \Pi_{\mathbf{r} \in \mathbf{R}} \Pr \left( \left| \Pr(\mathbf{R}[^{\pi}] = \pi, \mathbf{R}[^{\omega}] = \omega) - \Pr(\mathbf{R}[^{\pi}] = \pi, \mathbf{R}[^{\omega}] = \omega) \right| \leq \tau \right)}{(1 - (1 - p/L)^{KT})^{\gamma_{\text{sub}}}}.$$

In \cite{21}, the authors have shown that $G$ decreases as the percentage of fingerprinted entries increases (when considering the fingerprinting mechanism developed in \cite{29}). In Appendix \cite{1} we present the similar analysis and show that $G$ also decreases as $p$ increases when our proposed entry-level differentially-private fingerprinting mechanism is used. As a result, this implies that the robustness of our proposed fingerprinting mechanism also increases with $p$. In Section \text{VI-A}, we will discuss how to augment our proposed mechanism to mitigate the correlation attacks.

VI. SHARING MULTIPLE FINGERPRINTED DATABASES

A major challenge in practical use of differential privacy is that data privacy degrades if the same statistics are repeatedly calculated and released using the same differentially-private mechanism. The same is true for sharing a database with multiple SPs. If different fingerprinted copies of the same database are shared multiple times, the average of them may converge to the original database, which implies that privacy guarantee of Algorithm \cite{1} degrades linearly with number of sharrings, as it is used to share the same database repeatedly.

However, in practice, the database owner releases its database only to a limited number of SPs, and for each released copy, it will have certain data privacy and fingerprint robustness requirements. According to Figure \text{2} these requirements can both be fulfilled if the utility of the shared database is compromised to a certain extent (but not significantly as will be corroborated in Section \text{VII}). Based on Section \text{V}, we know that the database utility is readily be characterized by the fingerprint density, as defined in Corollary \cite{1} because the higher the fingerprint density, the lower the database utility, and the database owner can control the utility of repeatedly shared databases using fingerprint density. As a result, we let the database owner only share a fingerprinted database, $M(\mathbf{R})$, if its fingerprint density, $\|M(\mathbf{R}) - \mathbf{R}\|_1$, is beyond a predetermined publicly known threshold, $\Gamma$, in order to meet the database owner’s requirement of high data privacy guarantee and fingerprint robustness.

As discussed in Section \text{V-C}, the fingerprinted database $M(\mathbf{R})$ customized for a particular SP depends on an internal ID assigned by the database owner to the corresponding SP. Since the internal ID of the SP is an input for inserting the fingerprint (Algorithm \cite{1}), whether $\|M(\mathbf{R}) - \mathbf{R}\|_1$ is higher than $\Gamma$ also depends on the assigned internal ID. As a consequence, when an SP queries the database, the database owner needs to keep generating a new internal ID for it until the resulting fingerprint density is above $\Gamma$. Moreover, this process (i.e., internal ID generation and fingerprint density comparison with the threshold) also needs to be performed in a privacy-preserving manner. The reason is that according to Section \text{V} (specifically, Proposition \text{2} and Corollary \text{1}), fingerprint density provides additional knowledge about the fingerprint robustness and differential privacy guarantee. If a malicious SP accurately knows that its received database has fingerprint density higher than a threshold $\Gamma$, it can estimate the percentage of changed entries due to fingerprinting, and it can further distort the fingerprint via a correlation attack \cite{21}.

The above discussion inspires us to resort to the sparse vector technique (SVT) \cite{16, 33}, that only releases a noisy query result when it is beyond a noisy threshold, to design a mechanism for sharing multiple entry-level differentially-private fingerprinted databases and at the same time controlling the cumulative privacy loss. The unique benefit of SVT is that it can answer multiple queries while paying the cost of privacy only for the ones satisfying a certain condition, e.g., when the result is beyond a given threshold. In Section \text{VI-A}, we present an intermediate step which considers only one SP, determines its internal ID, and conducts the comparison between the resulting fingerprint density and threshold under differential privacy guarantee. In Section \text{VI-B}, we show how
to compose this intermediate step for $C$ times to determine the internal IDs for $C$ SPs and share different fingerprinted databases with them.

A. Intermediate Step: Determining Internal ID for One SP

As elaborated earlier, the database owner needs to assign an internal ID to a SP in order to achieve $||M(R) - R||_{1,1} > \Gamma$ for the purpose of simultaneously meeting data privacy and fingerprint robustness requirements. To achieve differential privacy for this intermediate step, we perturb both $||M(R) - R||_{1,1}$ and $\Gamma$, and consider the noisy comparison $||M(R) - R||_{1,1} + \mu > \Gamma + \rho$, where $\mu$ and $\rho$ are Laplace noises. Establishing the noisy comparison is a standard approach in SVT (see [16] page 57, and [33] page 639).

Next, we formally present the intermediate step. When the database owner receives a query from a new SP (suppose that this SP is the $c$th SP and $c \in [1, C]$), it generates an instance of internal ID for the $c$th SP via $ID^{c}_{\text{internal}} = Hash(K[c][i])$, where $i \in \{1, 2, 3, \cdots \}$ denotes the sequence number of this trial to generate $ID^{c}_{\text{internal}}$. Then, the database owner generates a copy of fingerprinted database by calling Algorithm 1 with the internal ID set as $ID^{c}_{\text{internal}}$ in Line 2. Similarly, we denote the fingerprinted database generated for the $c$th SP at the $i$th trial as $M^{c}_{i}(R)$. Next, the database owner conducts the noisy comparison $||M^{c}_{i}(R) - R||_{1,1} + \mu_{i} > \Gamma + \rho_{i}$, where $\mu_{i} \sim Lap(\frac{2}{\epsilon})$ and $\rho_{i} \sim Lap(\frac{\Delta}{\epsilon})$. Here, $\epsilon_{2}$ and $\epsilon_{3}$ are the privacy budgets used to control the accuracy of the noisy comparison. If $||M^{c}_{i}(R) - R||_{1,1} + \mu_{i} > \Gamma + \rho_{i}$ holds, then the database owner returns a symbol $\top$ and immediately terminates the intermediate step. This means that $ID^{c}_{\text{internal}}$ generated at the $i$th trial for the $c$th SP can lead to a fingerprinted database satisfying the data privacy and fingerprint robustness requirements. Otherwise, the database owner returns a symbol $\bot$, increase $i$ by 1, and continues the process. We summarize this intermediate step in Algorithm 2. This entire process is differentially-private, and is proved in the following theorem. It is noteworthy that the algorithm 2. This entire process is differentially-private, and is proved in the following theorem. It is noteworthy that the algorithm 2. This entire process is differentially-private, and is proved in the following theorem. It is noteworthy that the algorithm 2.

Algorithm 2: Determine the Internal ID for One SP

```
1 forall i \in \{1, 2, 3, \cdots \} do
2 \quad Generate an instance of internal ID for the $c$th SP via $ID^{c}_{\text{internal}} = Hash(K[c][i])$.
3 \quad Generate $M^{c}_{i}(R)$ by calling Algorithm 1 with $ID^{c}_{\text{internal}}$ and privacy budget $\epsilon$.
4 \quad Sample $\mu_{i} \sim Lap(\frac{2}{\epsilon})$ and $\rho_{i} \sim Lap(\frac{\Delta}{\epsilon})$.
5 \quad if $||M^{c}_{i}(R) - R||_{1,1} + \mu_{i} \geq \Gamma + \rho_{i}$ then
6 \quad \quad Output $a_{i} = \top$. (the $i$th trial meets the requirements)
7 \quad else
8 \quad \quad Output $a_{i} = \bot$. (the $i$th trial does not meet the requirements)
```

Theorem 3. Algorithm 2 is $(\epsilon_{2} + \epsilon_{3})$-entry-level differentially private.

We show the proof of Theorem 3 in Appendix B. Note that although we allocate the privacy budget $\epsilon$ in the course of generating the fingerprinted database for the SP at Line 5 in Algorithm 2, it does not contribute to the total privacy loss. This is because here, we only determine $ID^{c}_{\text{internal}}$ that is used for fingerprint insertion, but the numerical fingerprinted database has not been shared yet.

B. Composition of Intermediate Steps: Releasing Multiple Numerical Fingerprinted Databases

In the previous section, we have presented an intermediate step, in which, to guarantee that an SP receives a copy of fingerprinted database, the database owner keeps generating an instance of internal ID for it until the noisy comparison result is “TRUE”. Here, we compose the intermediate steps for $C$ times to determine the internal IDs for $C$ SPs, and at the same time, share the corresponding fingerprinted databases (generated using their final internal IDs) with them.

We first restate the advanced composition theorem, which provides a tight cumulative privacy loss for adaptive composition of differentially-private mechanisms (see [16] page 49), in context of our adopted privacy model, i.e., entry-level differentially privacy.

Theorem 4 (Advanced Composition). For all $\epsilon, \delta, \delta' \geq 0$, the $C$-fold composition of $(\epsilon, \delta)$-entry-level differentially private mechanisms satisfy

\[ (\sqrt{2C \ln(1/\delta')} \epsilon + C_1(\epsilon^c - 1), C \delta + \delta') \]-entry-level differential privacy.

Theorem 4 can be proved by following the exact procedures of proving the advanced composition theorem for conventional differential privacy with a simple switch of the privacy model. In Theorem 4, $\delta'$ is determined by the database owner to ensure the desired $\delta'$-approximate max divergence between two random variables (16 page 43). Using this, we propose Algorithm 4 (shown in Appendix K), which outputs the numerical fingerprinted databases to $C$ SPs, and all shared database copies meet the database owner’s requirements for data privacy and fingerprint robustness. The workflow of Algorithm 4 is similar to that of Algorithm 2, the only difference is that Algorithm 4 outputs the numerical fingerprinted database generated by the final internal ID of each SP instead of the symbol $\top$. We include a box around Line 8 in Algorithm 4 to highlight this. Whereas, Algorithm 2 just identifies one proper internal ID for each SP. Note that if the database owner wants to share its database with more than $C$ different SPs, it can reduce the value of the fingerprint density threshold $\Gamma$, which, however, compromises the privacy and fingerprint robustness of the shared databases, because reducing $\Gamma$ increases utility of the shared database.
Theorem 5. Algorithm 4 is $(\epsilon_0, \delta_0)$-entry-level differentially private (defined in Definition 4) with \( \epsilon_0 = \sqrt{2C\ln(\frac{1}{\delta})}(\epsilon + \epsilon_2 + \epsilon_3) + C\left(\epsilon(e^\epsilon - 1) + (e^\epsilon + \epsilon_3)(e^{2e\epsilon + \epsilon_3} - 1)\right)\), and \( \delta_0 = 2\delta^\prime \).

We show the proof of Theorem 5 in Appendix C.

Privacy budget allocation. In practice, given the cumulative privacy budget \( \epsilon_0 \) and \( \delta^\prime \) specified by the database owner, we need to decide how to assign the values of \( \epsilon, \epsilon_2, \) and \( \epsilon_3 \). Since \( \epsilon \) is used to obtain the fingerprinted database, its value should be determined based on the specific database of interest. Specifically, as we have shown in Figure 3, a lower \( \epsilon \) leads to lower database utility and a higher \( p \) value (the probability of changing one insignificant bit of a data entry), which, in turn, increases fingerprint robustness. As a result, the database owner can decide \( \epsilon \) based on its requirements about database utility and fingerprint robustness.

Furthermore, we note that \( (\epsilon_2 + \epsilon_3) \) is used to obtain the internal IDs of SPs. Once \( \epsilon \) is decided, the database owner can solve for \( (\epsilon_2 + \epsilon_3) \) numerically, i.e.,

\[
(\sqrt{2C\ln(\frac{1}{\delta})} - C)(\epsilon_2 + \epsilon_3) + C(\epsilon_2 + \epsilon_3)e^{\epsilon_2 + \epsilon_3} = \epsilon_0 - (\sqrt{2C\ln(\frac{1}{\delta})} - C)e^{-C\epsilon^\prime}
\]

Suppose the numerical solution is \( (\epsilon_2 + \epsilon_3) = \epsilon^\prime \). Then, we need to allocate \( \epsilon^\prime \) to \( \epsilon_2 \) and \( \epsilon_3 \). We observe that \( \epsilon_2 \) and \( \epsilon_3 \) control the accuracy of noisy comparison, i.e., the comparison between the perturbed fingerprint density and the perturbed threshold, \( ||M_0(R) - R||_{\mu_1 + \mu_2} \geq \Gamma + \rho_i \) (or equivalently, \( ||M_0(R) - R||_{\mu_1} \geq \Gamma - \rho_i - \mu_i \)) in both Algorithms 2 and 4.

To boost the accuracy of the noisy comparison, we minimize the variance of the difference between \( \rho_i \) and \( \mu_i \). Since they are both Laplace random variables, the variance of their difference is \( 2(\frac{\Delta}{\epsilon^\prime})^2 + 2(\frac{\Delta}{\epsilon^\prime})^2 \). Clearly, given \( \epsilon^\prime \), the variance is minimized when \( \epsilon_2 = \epsilon_3 = \epsilon^\prime / 2 \). Note that in classical SVT, the database owner does not respond to all the queries, i.e., it merely reports “1” if the considered noisy comparison is “FALSE” (see [16] page 55), however, this is not user-friendly in database sharing, especially, when the database owner still has more privacy budget remained. In our proposed SVT-based solution, we make sure all SPs get their fingerprinted databases as long as they are among the top \( C \) SPs sending the query request. This is achieved by letting the database owner keep generating new internal IDs for the SPs, until the noisy comparison turns out to be “TRUE”, and this approach does not violate the design principle of SVT.

VII. EXPERIMENTS

Here, we evaluate the developed differentially-private relational database fingerprinting mechanism under both single and multiple database sharing scenarios. Note that some experiment results are deferred to Appendix M-A.

A. Experiment Setup

The database we used for the evaluations is the nursery school application database [2], which contains data records of 12960 applicants. Each applicant has 8 categorical attributes, e.g., “form of the family” (complete, completed, incomplete, or foster), and “number of children in the family” (1, 2, 3, or more). Each data record belongs to one of the five classes, i.e., “not_recom”, “recommend”, “very_recom”, “priority”, and “spec_prior”, indicating the decisions for the applicants.

Database encoding. To fingerprint the categorical data, we first represent all possible values of each attribute as an integer starting from 0. In the considered database, the maximum integer representation of a data entry is 4 (we do not fingerprint the labels, which will be used in a classification task to evaluate the utility of the fingerprinted database). We also perform zero padding to make sure that the binary representations of different data entries have the same length. Note that our proposed mechanism is not limited to discrete or categorical attributes only. If a relation has continuous attributes, we can first quantize the entry values into non-overlapping ranges, and then represent each range interval using an integer [21].

Sensitivity Control. Since the integer representations of data entries vary from 0 to 4, the sensitivity is \( \Delta = 4 \). Thus, the proposed mechanism needs to fingerprint \( K = \log_2 4 + 1 = 3 \) (see Theorem 1) least significant bits of each data entry. However, in the experiments, we consider sensitivity \( \Delta = 1 \) with the assumption that the different entries in a pair of neighboring nursery databases can change by at most by 1, otherwise, it introduces a rare event (i.e., data record that occurs with very low probability) in the database. Our approach to control the sensitivity is similar to the restricted sensitivity [6] (which calculates sensitivity on a restricted subset of the database, instead of quantifying over all possible data records) and smooth sensitivity [23] (which smooths the data records after partitioning them into non-overlapping groups). Note that it has been widely recognized that rare events or outliers consume extra privacy budget, and this is a common problem in differentially-private database queries [12], [15], [27], [32].

Controlling local and global sensitivity in differential privacy is a separate topic, and it is beyond the scope of this paper.

Post-processing. After fingerprinting a chosen database (R), some entries may have integer representations that are outside the domain of the considered database. Thus, we also conduct a post-processing on the resulting database (M(R)) to eliminate the data entries that are not in the original domain. Otherwise, the SP which receives the database can understand that these entries are changed due to fingerprinting.

B. Evaluations for One-time Sharing

We first consider the scenario, where the database owner only releases the fingerprinted database to one SP. Thus, only Algorithm 1 is invoked.
1) Fingerprint Robustness: Among common attacks against database fingerprinting mechanisms (i.e., random flipping attack, subset attack, and super set attack [29]), random flipping attack is shown to be the most powerful one [29, 43], thus, we investigate the robustness of the proposed mechanism against this attack. In Section VIII, we will discuss how to make the proposed mechanism robust against more sophisticated attacks, e.g., the collusion attack [43] and correlation attack [21]. In particular, in favor of the malicious SP (and in order to show the robustness of our mechanism), we let the malicious SP randomly flip 80% of the bit positions in its received copy of fingerprinted database \(M(R)\), i.e., \(\overline{R}\) (the compromised database) is obtained by flipping 80% bits in \(M(R)\). Then, we measure the fingerprint robustness using the number of bit matches between the malicious SP's fingerprint and the one extracted from \(\overline{R}\).

We compare the fingerprint robustness with a state-of-the-art database fingerprinting mechanism proposed in [29]. The reason we choose this mechanism is because it also has high robustness, e.g., the probability of detecting no valid fingerprint due to random bit flipping attack is upper bounded by \((|SP| - 1)/2^{|P|}\) (here \(|SP|\) is the number of SPs who have received the fingerprinted copies). This mechanism controls the fingerprint density using fingerprint fraction (i.e., the fraction of fingerprinted data entries in \(R\)) denoted as \(\lambda\).

We present the comparison results in Table I. In particular, we vary the privacy budget of our entry-level differentially-private fingerprinting mechanism, i.e., \(\epsilon\) in Algorithm 1, from 1 to 7, the corresponding probability of changing an insignificant bit as \(p = 1/(e^{1/\epsilon} + 1)\) (according to Theorem 3.3), and set the fingerprint fraction (\(\lambda\)) used in [29] to be the percentage of changed data entries caused by our proposed mechanism under varying \(\epsilon\) followed by post-processing. Clearly, our proposed mechanism (the highlighted column in Table I) achieves higher fingerprint robustness. Note that the database utility of the two mechanisms are identical in each row in Table I. As \(p\) (or \(\lambda\)) decreases (i.e., less bit positions are marked by either fingerprint mechanisms), the number of matched fingerprint bits between the extracted fingerprint from \(R\) and the malicious SP's fingerprint also decreases. However, the robustness of the proposed scheme remains significantly high compared to [29]. Especially, when \(p \leq 0.0025\), even if the malicious SP changes 80% of the bits in \(M(R)\), it is still not able to compromise more than half of the fingerprint bits (i.e., 64 out of 128) generated by the proposed mechanism. It has been empirically shown that as long as the malicious SP does not distort more than half of the fingerprint bits, it will end up being uniquely identifiable (by the fingerprint detection algorithm of the database owner) [29]. In contrast, when \(\lambda \leq 0.24\%), the malicious SP can compromise more than half of the fingerprint bits generated by [29], which means that it will not be accused although it has illegally redistributed the data. In addition to higher fingerprint robustness, our proposed mechanism also achieves privacy guarantee at the same time.

In Section VIII, we derive the robustness (denoted as \(P_{\text{rbst}_{\text{rnd}}}\)) of our mechanism against random bit flipping attack in terms of flipping probability \(\gamma_{\text{rnd}}\). In Appendix H, we show that \(P_{\text{rbst}_{\text{rnd}}}\) is monotonically increasing with \(p\) (\(p \in (0, 0.5)\)), i.e., we showed how the robustness of the proposed mechanism against random bit flipping attack increases with the increasing number data entries are changed due to fingerprinting. By letting \(D = 64\) (threshold of number of matched fingerprint bits, see Section VIII), \(\gamma_{\text{rnd}} = 0.8\), and setting \(p\) as the values reported in Table I, we plot the numerical value of \(P_{\text{rbst}_{\text{rnd}}}\) in Figure B in Appendix M-B. In the same figure, the green line is the percentage of bit matches obtained by our mechanism as a result of our empirical evaluation (i.e., values in the highlighted columns in Table I divided by 128). We observe that both lines in the figure almost overlap, which validates our theoretical findings. Besides, we have also evaluated the averaged error of entries in the fingerprinted database, and we observe that empirical value is close to \(\Delta p\) as proved in Proposition 2.

In addition to fingerprint robustness, in Appendix M-C, we also evaluate the privacy guarantee of the proposed mechanism by empirically investigating the inference capability of an adversary that is formulated in [31]. Our experimental results show that attacker's inference capability is always bounded by our theoretical findings in Proposition 1 under varying \(\epsilon\), which suggests the robustness of our mechanism against attribute inference attack.

2) Utility of the Shared Database: Here, to experimentally show the utility guarantees of the proposed entry-level differentially-private fingerprint mechanism, we compare the utility of it with that achieved by a two-stage approach, i.e., first perturbing the entire database under local differential privacy with the identical \(\epsilon\) and then fingerprinting the perturbed database using [29] as before. We conduct the comparison from two perspectives: (i) application task-independent comparison, which considers the change of variance of each attribute caused by our mechanism and the two-stage approach and the accuracy of specific SQL queries after fingerprinting, and (ii) application task-specific comparison, where we use fingerprinted databases (ours and the two-stage approach) to do classification and principal component analysis (PCA).

**Application task-independent comparison.** Please refer to Appendix M-D for the setup and detailed results of these experiments. Here, we only highlight the empirical findings. Our proposed mechanism always achieve higher utility, i.e., the changes in invariance of each attribute caused by our

**TABLE I:** Comparison of fingerprint robustness achieved by us and [29] when random flipping attack changes 80% bit positions in \(M(R)\). Each row in the table corresponds to the same database utility for both mechanisms.

| \(\epsilon\) | \(p\) set by us | \(\lambda\) set in [29] | bit matches obtd. by us | bit matches obtd. by [29] |
|------------|------------------|-------------------------|-------------------------|-------------------------|
| 0.0025     | 0.0025           | 0.0025                  | 128                     | 128                     |
| 0.0047     | 0.0047           | 0.0047                  | 120                     | 96                      |
| 0.0180     | 0.0180           | 0.0180                  | 106                     | 82                      |
| 0.0067     | 0.0067           | 0.0067                  | 84                      | 72                      |
| 0.0025     | 0.0025           | 0.0025                  | 71                      | 50                      |
| 0.0009     | 0.0009           | 0.0009                  | 67                      | 30                      |
mechanism are generally 10 times smaller than that caused by the two-stage approach, and our approach also achieves higher SQL query accuracy given different choices of \( \epsilon \). These imply that our proposed mechanism can also achieve higher utility for various task-specific applications, because in data mining or machine learning, the results usually depend on the intrinsic data distributions, and more accurate variance (or covariance matrices) lead to better performance. We will validate this experimentally in the following.

**Task-specific comparison.** As discussed in Section VII-A, data records belong to 5 classes. To perform classification, we adopt a multi-class support vector machine (SVM) classifier and use 65\% of data records for training and the rest for testing. We evaluate the utility of various fingerprinted databases by comparing the **fingerprinted testing accuracy** (i.e., SVM classifier trained on fingerprinted training data and then tested on the original testing data) with the **original testing accuracy** (i.e., SVM classifier trained on the original training data and then tested on the original testing data). Thus, the less the difference between fingerprinted testing accuracy and original testing accuracy, the higher the database utility.

The database utility for PCA is defined as the total deviation, 
\[
\text{TTL}_{\text{DEV}} = \sum_{i=1}^{8} |\lambda_i - \tilde{v}_i^T C \tilde{v}_i|, \]
where \( C \) is the empirical covariance matrix obtained from the original (non-fingerprinted) database, \( \lambda_i \) values are the eigenvalues of \( C \), and \( \tilde{v}_i \) vectors are the eigenvectors of the empirical covariance matrix obtained from fingerprinted database. Specifically, \( \text{TTL}_{\text{DEV}} \) quantifies the deviation of the variance (of the fingerprinted database) from \( \lambda_i \) in the direction of the \( i \)th component of \( C \). The smaller the deviation (\( \text{TTL}_{\text{DEV}} \)) is, the higher the utility.

In Figure 6(a) and (b) (see Appendix M-E) by varying \( \epsilon \) from 0.25 to 2, we compare the task-specific database utilities for classification and PCA achieved by our mechanism and the two-stage approach. Clearly, our proposed mechanism achieves higher database utilities in all considered applications. As future work, we will also investigate database utility in more sophisticated tasks, such deep learning and federated learning, where stochastic gradient descent will be iteratively calculated using the fingerprinted databases.

**C. Evaluations for Multiple Sharing**

Next, we consider the scenario where at most 100 SPs query the entire database over time in a sequential order (i.e., \( C = 100 \) in order to control the cumulative privacy loss). As discussed in Section VII, the database owner performs the noisy comparison \( ||\mathcal{M}_{\delta}^\epsilon(R) - R||_{1,1} + \mu_i \geq \Gamma + \rho_i \), where \( \Gamma \) is the fingerprint density, \( \mu_i \) and \( \rho_i \) are Laplace noises) to determine the proper internal IDs for the SPs to generate the fingerprinted databases. In the experiment, we set \( \Gamma = (\frac{1}{2} - \frac{1}{\sqrt{12}})\Delta p N K \). The rationality is that according to Corollary 1, the expected value of \( ||\mathcal{M}_{\delta}^\epsilon(R) - R||_{1,1} \) falls in the range of \( [0, \Delta p N T] \). Since we do not have any assumption on the database, and the pseudorandom number generator \( \mathcal{U} \) generates each random number with equal probability, we approximately model \( ||\mathcal{M}_{\delta}^\epsilon(R) - R||_{1,1} \) as a uniformly distributed random variable in the range of \( [0, \Delta p N T] \). Then, its mean and standard deviation are \( \frac{1}{2}\Delta p N T \) and \( \frac{\sqrt{12}}{2}\Delta p N K \).

Moreover, we consider the cumulative privacy loss as \( \epsilon_0 = 40 \) and \( \delta_0 = 2 \times 10^{-3} \). If \( \epsilon_0 < 40 \) and the database owner still wants to generate fingerprinted databases with the identical privacy and fingerprint robustness guarantees as when \( \epsilon_0 = 40 \), it will end up sharing its database with fewer number of SPs. To achieve a decent database utility, we set the privacy budget to generate the entry-level differentially-private fingerprinted database as \( \epsilon = 0.5 \). Then, by solving \( \epsilon \) numerically, we have \( \epsilon_2 + \epsilon_3 = 0.002 \) approximately.

| \( \epsilon_2 : \epsilon_3 \) | 9 : 1 | 7 : 1 | 5 : 1 | 3 : 1 | 1 : 1 |
|---|---|---|---|---|---|
| Total No. of trials | 181 | 177 | 173 | 165 | 156 |

**TABLE II: Impact of the ratio between \( \epsilon_2 \) and \( \epsilon_3 \) on the total number of internal ID generation trials for 100 SPs.**

In Figure 7, we plot the fingerprint densities of the shared databases when \( \epsilon_2 = \epsilon_3 \), i.e., \( ||\mathcal{M}_{\delta}^\epsilon(R) - R||_{1,1} \) as a result of sharing with 100 SPs. In particular, the red line represents the result achieved by us and the blue line represents the result achieved by the two-stage approach (discussed in Section VII-B3), which is executed for 100 times in order to share the database with 100 SPs. For each execution of the two-stage approach, we let it have the same privacy and fingerprint robustness guarantees as ours. Clearly, our mechanism not only achieves lower fingerprint density, but it also makes the fingerprint density have smaller variance (the variance of
results of ours and the two-stage approach are evaluated as 317 and 871, respectively). It means that the fingerprinted databases generated by us have more stable utilities, and are promising for real-life applications.

VIII. DISCUSSION

Our work is a first step in uniting differential privacy and database fingerprinting. We believe that it will draw attention to other challenges and urgent research problems, which we plan to investigate in the future. We list some of the potential extensions of this work in the following.

A. Augmenting the entry-level differentially-private mechanism against attribute inference attack using data correlation

As discussed in Section VA the database owner can further augment the proposed mechanism to achieve robustness against correlation based attribute inference attack by involving the auxiliary information of the fingerprinting mechanisms in the design mechanism. Such augmentation can be obtained by taking a closer observation at the proof of Theorem 1. In particular, one intermediate step in the proof of Theorem 1 considers deriving the upper bound of \( \sum_{i,t,k} p_{\text{r}_{i,t,k}} \leq 1 \) by using the correlations among attributes and plugging in the Bernoulli parameter \( p \) (the probability of changing an insignificant bit in an entry in the database), this specific term can be upper bounded by

\[
\prod_{k=1}^{K} \Pr(r_{i,t,k} | r_{i,t,k} = r_{i,t,k}) \leq \prod_{k=1}^{K} (1 - p) \times \sum_{r_{i,t,k}} \max_{r_{i,t,k}} \Pr(r_{i,t,k} | r_{i,t,k})
\]

where the inequality \( \leq \) is achieved by following the similar steps presented in the proof of Theorem 1. Letting the new bound less than \( \epsilon \), and then set the value of \( p \) accordingly, the proposed mechanism can achieve provable privacy guarantees against attribute inference attacks that use data correlation. We will also work along this direction in future work.

B. Mitigation of correlation attacks

Ji et al. \([21]\) have developed a mitigation technique to alleviate the correlation attacks against database fingerprinting. Their technique modifies a fingerprinted database to make sure that it has similar column- and row-wise joint distributions with the original database. Since their technique only changes the non-fingerprinted data entries and it can be applied as a post-processing step after any fingerprinting mechanism, it can also be utilized following our mechanism to defend against the correlation attacks. In case of such an integration, our differential privacy guarantee will still hold because of the immunity property of differential privacy for post-processing \([16]\).

C. Incorporation with collusion-resistant fingerprinting

In this work, we do not consider the collusion attacks (where multiple malicious SPs ally together to generate a pirated database from their unique fingerprinted copies) on the fingerprinted databases. Several works have proposed collusion-resistant fingerprinting mechanisms in the literature \([7], [8]\), \([36], [41]\). To develop a differentially-private and collusion-resistant fingerprinting mechanism, one potential solution is to replace the fingerprint generation step (i.e., Line 3 of Algorithm 1) with the Boneh-Shaw (BS) codes \([7]\) and decide \( p \) (the probability of changing one insignificant bit of an entry) based on \( \epsilon_1 \) and the number of 1’s in the BS codeword. We will explore this extension in future work.

D. Improving database utility by utilizing data distribution

In many real world applications, especially in machine learning and data mining, data records are usually assumed to follow a prior distribution, i.e., they are drawn i.i.d. from an underlying (multivariate) distribution denoted as \( d_{\text{prior}} \). In this case, we can further improve the utility (e.g., accuracy) of the fingerprinted database by formulating a statistical estimation problem \([3]\) as

\[
\text{R}^* = \arg \min_{\text{R}} \mathbb{E}[d_{\text{prior}}(\mathbb{E}[M(\text{R}) - \text{R}])]
\]

where \( \text{R}^* \) is interpreted as the Bayesian denoised fingerprinted database, which is closer to the original database. Note that \( \text{R}^* \) has the same privacy guarantee as \( M(\text{R}) \) because of the post-processing immunity property of differential privacy.

IX. CONCLUSIONS

In this paper, we have proposed a novel mechanism that unites provable privacy and database fingerprinting for sharing relational databases. To this end, we have first devised a bit-level random response scheme to achieve \( c \)-entry-level differential privacy guarantee for the entire database, and then developed a concrete entry-level differentially-private database fingerprinting mechanism on top of it. We have also provided the closed form expressions to characterize the connections between database utility, privacy protection, and fingerprint robustness. Finally, we have developed a SVT-based solution to share entry-level differentially-private fingerprinted databases with multiple recipients, and at the same time, control the cumulative privacy loss due to repeated sharing. Experimental results on a real relational database show that under the same database utility requirement, our mechanism provides stronger fingerprint robustness (along with privacy guarantee) than other mechanisms. Under the same privacy and fingerprint robustness guarantee, our mechanism achieves higher utility than database perturbation followed by fingerprinting.

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[6] [36], [41]. To develop a differentially-private and collusion-resistant fingerprinting mechanism, one potential solution is to replace the fingerprint generation step (i.e., Line 3 of Algorithm 1) with the Boneh-Shaw (BS) codes \([7]\) and decide \( p \) (the probability of changing one insignificant bit of an entry) based on \( \epsilon_1 \) and the number of 1’s in the BS codeword. We will explore this extension in future work.

D. Improving database utility by utilizing data distribution

In many real world applications, especially in machine learning and data mining, data records are usually assumed to follow a prior distribution, i.e., they are drawn i.i.d. from an underlying (multivariate) distribution denoted as \( d_{\text{prior}} \). In this case, we can further improve the utility (e.g., accuracy) of the fingerprinted database by formulating a statistical estimation problem \([3]\) as

\[
\text{R}^* = \arg \min_{\text{R}} \mathbb{E}[d_{\text{prior}}(\mathbb{E}[M(\text{R}) - \text{R}])]
\]

where \( \text{R}^* \) is interpreted as the Bayesian denoised fingerprinted database, which is closer to the original database. Note that \( \text{R}^* \) has the same privacy guarantee as \( M(\text{R}) \) because of the post-processing immunity property of differential privacy.
APPENDIX A
FREQUENTLY USED NOTATIONS
In Table III, we list frequently used notations in the paper.

| Notations | Descriptions |
|-----------|--------------|
| R         | original database |
| R         | a neighboring database of R |
| A(R)      | fingerprinted database |
| R         | leaked (pirated) database |
| r_i       | the ith row of R |
| r_i[t,k]  | the kth insignificant bit of the ith attribute of r_i |
| p         | the probability of changing an insignificant bit in an entry in the database |
| B         | mark bit to fingerprint a bit position |
| B         | B ~ Bernoulli(p) |
| ε,ε_1,ε_2 | privacy budgets |
| Δ         | database sensitivity |

TABLE III: Frequently used notations in the paper.

APPENDIX B
PROOF OF THEOREM 1

Proof. Since we consider neighboring databases that have only a pair of different data entries which differ by at most Δ, it requires K = \left\lceil \log_2 \Delta \right\rceil + 1 bits to encode the difference. Then, by applying Definition 4, we have

\[
\Pr \left( \mathcal{M}(R) = \overline{R} \right) \approx \prod_{k=1}^{K} \frac{1}{p} 
\]

where (a) can be obtained by assuming (without loss of generality) that R and \( \overline{R} \) differ at the tth attribute of the i th row, and thus the probability ratio at other entries cancel out. \( r_i[t,k] \) (or \( \overline{r}_i[t,k] \)) represents the kth least significant bit of the tth attribute of \( r_i \) (or \( \overline{r}_i \)), \( B_{i,t,k} \) (or \( \overline{B}_{i,t,k} \)) is the random mark bit fingerprinted on \( r_i[t,k] \) (or \( \overline{r}_i[t,k] \)), and \( \overline{F}_i[t,k] \) is the identical result of the bit-level random response at this bit position. (b) is because each of the last K bits of entry \( r_i[t] \) (or \( \overline{r}_i[t] \)) are changed independently with probability p, and (c) can be obtained by applying \( u \oplus v = (1-u)v + u(1-v) \) for any binary variable u and v. Then, by making \( \prod_{k=1}^{K} \frac{1}{p} \leq \varepsilon \), we complete the proof.

APPENDIX C
PROOF OF THEOREM 2

Proof. Since the value of \( \mathcal{U}_s \) (the jth random value generated by \( \mathcal{U} \)) is uniformly distributed for a seed s \( \in [0,1] \), we have \( \Pr(\mathcal{U}_s \mod \left\lceil \frac{1}{2p} \right\rceil = 0) = \frac{1}{\left\lceil \frac{1}{2p} \right\rceil} > 2p \). Similarly, \( \Pr(x = 0) = \frac{1}{2} \), thus, for any given fingerprint bit f, we also have \( \Pr(B = 1, \mathcal{U}_s \mod \left\lceil \frac{1}{2p} \right\rceil = 0) \geq \frac{1}{2}2p = p \), which suggests that each \( r_i[t,k] \) will be changed (i.e., XORed by 1) with probability higher than p, and this satisfies the condition in Theorem 1.

APPENDIX D
THE FINGERPRINT EXTRACTION ALGORITHM

Algorithm 3 summarizes the main steps of extracting the fingerprint bit strings from a leaked fingerprinted database (discussed in Section IV.D).

Algorithm 3: Fingerprint extraction procedure

Input : The original database R, the leaked database \( \overline{R} \), the Bernoulli distribution parameter p, database owner’s secret key, \( \mathcal{K} \), pseudorandom number sequence generator \( \mathcal{U} \), and a fingerprint template.

Output: The extracted fingerprint from the leaked database.

1. Initialize \( c_0(l) = c_1(l) = 0 \), \( \forall l \in [1, L] \).
2. Construct the fingerprintable set \( \mathcal{F} \).
3. forall \( \overline{F}_i \in \mathcal{F} \) do
   4. Set pseudorandom seed \( s = \left\{ r_i \right\} PseudKey[l,k] \).
   5. if \( \mathcal{U}_s \mod \left\lceil \frac{1}{2p} \right\rceil = 0 \) then
      6. Set mark bit \( x = 0 \), if \( \mathcal{U}_s(x) \) is even; otherwise \( x = 1 \).
      7. Set fingerprint index \( l = \mathcal{U}_s(x) \mod L \).
      8. Recover mark bit \( B = \overline{F}_i[t,k] \oplus r_i[t,k] \).
      9. Recover fingerprint bit \( f_i = x \oplus B \).
   10. \( c_1(l) ++ \), if \( f_i = 1 \); otherwise \( c_0(l) ++ \).
   11. forall \( l \in [1, L] \) do
      12. \( f(l) = 1 \), if \( c_1(l) > c_0(l) \); otherwise \( f(l) = 0 \).
   13. Return the extracted fingerprint bit string f.

APPENDIX E
PROOF OF PROPOSITION 2

Proof. The fingerprinting mechanism only changes the last K bits of selected data entries, thus, we have

\[
\mathbb{E}\left( r_i[t,k] - r_i[t,k] \right) = \mathbb{E}\left( \sum_{k=1}^{K} r_i[t,k]2^{r_i[t,k]} - r_i[t,k]2^{r_i[t,k]} \right) = \mathbb{E}\left( \sum_{k=1}^{K} r_i[t,k]2^{r_i[t,k]} \oplus B \right)2^{r_i[t,k]} - B - r_i[t,k]2^{r_i[t,k]} \right) = \mathbb{E}\left( \sum_{k=1}^{K} \left( 1 - r_i[t,k] \right)2^{r_i[t,k]} - r_i[t,k]2^{r_i[t,k]} \right) \right| p = \mathbb{E}\left( \sum_{k=1}^{K} \left( \overline{r}_i[t,k]2^{r_i[t,k]} - r_i[t,k]2^{r_i[t,k]} \right) \right| p.
\]
Since $\sum_{k=1}^{K} r_k[t, k]2^{[t, k]-1}$ is the decimal representation of the complement of the last $K$ bits of $r_t$, and according to Definition 3, $r_k[t, k]2^{[t, k]-1}$ falls in the range of $[-\Delta, \Delta]$, so its absolute value falls in $[0, \Delta]$, which completes the proof.

APPENDIX H

PROOF OF PROPOSITION

Proof of Proposition

Mathematically, we have

$$\text{InfCap} = \Pr[r_t[t] = 1] - 1 - \Pr(r_t[t] = 0)$$

$$= \Pr(\mathcal{M}(R) | r_t[t] = 1) \Pr(r_t[t] = 1) - \Pr(\mathcal{M}(R) | r_t[t] = 0) \Pr(r_t[t] = 0)$$

$$= \frac{\Pr(\mathcal{M}(R) | r_t[t] = 1)}{\Pr(\mathcal{M}(R))} \Pr(r_t[t] = 1) - \frac{\Pr(\mathcal{M}(R) | r_t[t] = 0)}{\Pr(\mathcal{M}(R))} \Pr(r_t[t] = 0)$$

As a result, it is sufficient to show that $f(m)$ is monotonically increasing with $m$. First, we observe that $p_t(w_t) = \sum_{q=0}^{\min(2w_t, 0) \gamma_{\text{rand}}(1 - \gamma_{\text{rand}}) \gamma_{\text{rand}}^q} = \text{the cumulative distribution function of a binomial distribution function, which is monotonically increasing with } w_t, \text{ thus the multiplication of all } p_t(w_t) \text{ is increasing with } m = \sum w_t. \text{ Second, it is easy to check that the cardinality of } \mathcal{W} \text{ is } m!S(w, L), \text{ where } S(w, L) \text{ represents Stirling number of the second kind (i.e., the number of ways to partition a set of } w \text{ objects into } L \text{ non-empty subsets) [19]. Since } m \text{ grows faster then } L^{w_1} \text{ as } m \text{ increases, we can conclude that } f(m) \text{ is monotonically increasing with } m \text{ (the number of fingerprinted bit positions).}$

When $0 < p < 0.5$, $P_{\text{bstd.rand}}$ can be characterized as the summation of monotonically increasing functions with respect to $m$ and $p$, which suggests that the higher the value of $p$, the more robust the proposed fingerprinting mechanism is against the random bit flipping attack.

APPENDIX I

ANALYSIS OF $G$ IN THE CORRELATION ATTACK

As per proposition 3, $Pr[\mathcal{R}[t] = \pi, \mathcal{R}(z) = \omega] = Pr[\mathcal{R}[t] = \pi, \mathcal{R}(z) = \omega] = \text{falls in the range of } [0, \max(A, B)]$, where $A$ and $B$, respectively, are

First, we show how to determine $D$ (number of bit matches with the malicious SP’s fingerprint) given $C$ (number of times a database can be shared), and $L$ (length of fingerprinting string). If the database owner shares its database with $C$ different SPs. To make the extracted fingerprint have the most bit matches with the malicious SP, it requires that the probability of having more than $D$ bit matches is higher than $1/C$, i.e., \((L/2)+(L/2)^{L-D} \geq 1/C\), which can be solved analytically.

In order to show $P_{\text{bstd.rand}}$ is monotonically increasing with $p$, we define function $f(m) = \sum_{q=0}^{w_t \gamma_{\text{rand}}(1 - \gamma_{\text{rand}}) \gamma_{\text{rand}}^q}$ (m is embedded as a parameter of set $\mathcal{W}$, i.e., $\mathcal{W} = \{w_1, w_2, \ldots, w_L > 0 \text{ if } \sum_{w_1} w_t = m\}$. Then, $P_{\text{bstd.rand}}$ represents the expected value of $f(m)$, $m \sim \text{Binomial}[NK, T, 2p]$. As a result, it is sufficient to show that $f(m)$ is monotonically increasing with $m$. First, we observe that $p_t(w_t) = \sum_{q=0}^{\min(2w_t, 0) \gamma_{\text{rand}}(1 - \gamma_{\text{rand}}) \gamma_{\text{rand}}^q} = \text{the cumulative distribution function of a binomial distribution function, which is monotonically increasing with } w_t, \text{ thus the multiplication of all } p_t(w_t) \text{ is increasing with } m = \sum w_t. \text{ Second, it is easy to check that the cardinality of } \mathcal{W} \text{ is } m!S(w, L), \text{ where } S(w, L) \text{ represents Stirling number of the second kind (i.e., the number of ways to partition a set of } w \text{ objects into } L \text{ non-empty subsets) [19]. Since } m \text{ grows faster then } L^{w_1} \text{ as } m \text{ increases, we can conclude that } f(m) \text{ is monotonically increasing with } m \text{ (the number of fingerprinted bit positions).}$

When $0 < p < 0.5$, $P_{\text{bstd.rand}}$ can be characterized as the summation of monotonically increasing functions with respect to $m$ and $p$, which suggests that the higher the value of $p$, the more robust the proposed fingerprinting mechanism is against the random bit flipping attack.

\[ P_{\text{bstd.rand}} = \sum_{q=0}^{w_t \gamma_{\text{rand}}(1 - \gamma_{\text{rand}}) \gamma_{\text{rand}}^q} \]
Appendix J
Proof of Theorem 3
Proof. Suppose that Algorithm 2 terminates with l outputs (it takes l tries to determine $ID_{\text{internal}}^c$, leading to a “TRUE” condition for the noisy comparison), i.e., $a = [a_1, a_2, \cdots, a_l] = \{1\}^{l-1} \cup \{2\}$. By defining
\[
\begin{align*}
  f_i(R, z) &= \Pr(||M_i(R) - R||_{1,1} + \mu_i - \Gamma + z_i), \\
  g_i(R, z) &= \Pr(||M_i(R) - R||_{1,1} + \mu_i \geq \Gamma + z_i),
\end{align*}
\]
where $z_i$ is an instance of $\rho_i$ generated at Line 5 in Algorithm 2. Then, we can have
\[
\Pr\left(\text{DetermineTheInternalIDforOneSP}(R) = a\right) = \frac{\int_{-\infty}^{\infty} \Pr(\rho_i = z_i) \prod_{i=1}^{l} f_i(R, z_i) g_i(R, z_i) dz_i}{\int_{-\infty}^{\infty} \Pr(\rho_i = z_i) \prod_{i=1}^{l} f_i(R', z_i) g_i(R', z_i) dz_i} = \Delta,
\]
where $\Delta$ is obtained by changing all the integration variables, i.e., $z_i$'s, to $(z_i - \Delta)$'s, $\forall i \in \{1, 2, 3, \cdots\}$. Next, we investigate the three parts of the integrand in the numerator of $\Delta$ separately.

First, we have $\Pr(\rho_i = z_i - \Delta) \leq e^{\epsilon_3} \Pr(\rho_i = z_i)$, as $\rho_i$ is attributed to a Laplace distribution whose parameter is calibrated using $\Delta$.

Second, suppose $R$ and $R'$ differs at $r_{ij}$ and $r_{ij}'$. Then, $||M_i(R') - R'||_{1,1} - ||M_i(R) - R||_{1,1} = |r_{ij}' - r_{ij}| - |r_{ij} - r_{ij}'| \leq \Delta$, where $r_{ij}'$ (or $r_{ij}'$) is the fingerprinted version of $r_{ij}$ (or $r_{ij}'$). The equality follows from that for any specific SP (which uniquely determines a pseudorandom seed), Algorithm 1 will select exactly the same bit positions in both $R$ and $R'$ to insert the fingerprint, and all selected bits except for the different entry between $R$ and $R'$ will also be replaced with the exact same bit values. The inequality is because both $|r_{ij} - r_{ij}'|$ and $|r_{ij}' - r_{ij}'|$ are upper bounded by $\Delta$. As a result, for the second part in $\Delta$, we obtain $\prod_{i=1}^{l} f_i(R', z_i) g_i(R', z_i) dz_i = \Pr(||M_i(R') - R||_{1,1} + \mu_i - \Delta < \Gamma + z_i) = f_i(R', z_i)$, and the inequality holds since we replace $||M_i(R) - R||_{1,1}$ by a smaller value, i.e., $||M_i(R') - R'||_{1,1}$, which decreases the probability.

Third, since $\mu_i$ is a Laplace noise, which is also calibrated using $\Delta$, we have $g(R, z - \Delta) = \Pr(||M_i(R) - R||_{1,1} + \mu_i - \Delta < \Gamma + z_i - \Delta) \leq e^{\epsilon_3} g(R_i(R') - R||_{1,1} + \mu_i - \Delta < \Gamma + z_i - \Delta) = e^{\epsilon_3} g(R, z_i) z_i) dz_i$. Hence, $\Delta \leq \frac{\int_{-\infty}^{\infty} \prod_{i=1}^{l} f_i(R, z_i) g_i(R, z_i) dz_i}{\int_{-\infty}^{\infty} \prod_{i=1}^{l} f_i(R', z_i) g_i(R', z_i) dz_i} = e^{\epsilon_3 + \epsilon_3}$.

Appendix K
Database Sharing with Multiple SPs
In Algorithm 4, we summarizes the steps of sharing fingerprinted databases with $C$ SPs (discussed in Section VI-B).

Algorithm 4: Share Fingerprinted Databases with $C$ SPs
Input: Original database $R$, fingerprinting scheme $M$, sequence of number of SPs, i.e., $\{1, 2, \cdots, C\}$. threshold $\Gamma$, and privacy budget $\epsilon$, $\epsilon_2$, $\epsilon_3$ and $\delta'$.
Output: $\{a_1, a_2, a_3, \cdots, a_C\}$
1 Set count = 0.
2 forall $c \in \{1, 2, \cdots, C\}$ do
3 forall $t \in \{1, 2, 3, \cdots\}$ do
4 Generate an instance of internal ID for the $t$th SP via $ID_{\text{internal}} = Hash\left(\epsilon\right)$.
5 If $\Pr\left(||M'_i(R) - R||_{1,1} + \mu_i < \Gamma \right) > \delta'$
6 Sample $\mu_i \sim \text{Lap}\left(\frac{\mu_i}{\epsilon_2}\right)$ and $\rho_i \sim \text{Lap}\left(\frac{\lambda}{\epsilon_3}\right)$.
7 if $||M'_i(R) - R||_{1,1} + \mu_i \geq \Gamma$ then
8 Output $a = M'_i(R)$
9 else
10 Output $a_i = \perp$.

Appendix L
Proof of Theorem 5
Proof. Algorithm 4 is the composition of $C$ rounds of Algorithm 2 together with $C$ rounds of Algorithm 1. According to Theorem 4, $C$ rounds of Algorithm 2 and $C$ rounds of Algorithm 1 are $\left(\sqrt{2C \ln\left(\frac{1}{\delta'}\right)}(\epsilon_2 + \epsilon_3) + C(\epsilon_2 + \epsilon_3)(e^{\epsilon_3 + \epsilon_3} - 1), \delta'\right)$-differentially private and $\left(\sqrt{2C \ln\left(\frac{1}{\delta'}\right)}\epsilon + C(e^{\epsilon_3} - 1), \delta'\right)$-differentially private, respectively. Then, by simple composition of those two, we complete the proof.

Appendix M
Detailed Experiment Results: Tables and Plots
A. Pairwise Difference in Each Class
In Table IV, we summarize the results of the fraction of pairwise absolute differences taking a specific value in each class. Note that this observation helps us to control the sensitivity as discussed in Section VII-A.

| Class | not_rec | rec | rec
|-------|--------|-----|-----|
| 1     | 93.75% | 2.5% | 0   |
| 2     | 0      | 0   | 0   |
| 3     | 0      | 0   | 0   |
| 4     | 0      | 0   | 0   |

TABLE IV: Fraction of pairwise absolute differences between instances of attributes.

B. Validating $P_{\text{robust}}$ Empirically
In Figure 3, we plot the numerical value of $P_{\text{robust}}$ varies with different values of $p$ (reported in Table I).

C. Privacy Evaluation against Attribute Inference
Here, we investigate the robustness of our proposed entry-level differentially-private fingerprinting scheme when it is subject to the attribute inference attack discussed in Section V-A. Specifically, we aim to show that the attacker’s inference capability for our considered database (in Section VII-A) is upper bounded by our theoretical findings in Proposition 1.
Fig. 3: Theoretical and empirical values of $P_{\text{rbst}}$ under different $p$.

In particular, we consider the classical adversary in conventional differential privacy, which assumes data entries within a dataset (database) are independent, i.e., data entries are mutually independent and identically distributed (similar to the “independent tuple assumption” adopted in [31] - page 5 left column). Also, according to [31], under this adversary model, the auxiliary information $R_{r_i[t]}$ (all the original entries in the database except for $r_i[t]$) can be used to serve as sampling values of $r_i[t]$, which can be used to estimate the posterior probability $Pr(r_i[t] = \xi_1)$, where $\xi_1$ is any possible value that $r_i[t]$ can take. Then, according to statistical inference, the occurrence frequency of each value of $\xi_1$ is an unbiased estimator of $Pr(r_i[t] = \xi_1)$. Thus, the attacker’s inference capability (InfCap) defined in (1) can be empirically computed from $R_{r_i[t]}$ (note that [31] applies the similar attack approach). In Figure 4, we compare the empirical results of InfCap for the inference of $r_i[t]$ when $t$ (the attribute) is chosen as “finance” or “housing” with the theoretical upper bound. Clearly, the considered adversary cannot have higher inference capability than our derived upper bound. Note that we discuss how to deal with attribute inference attack that uses data correlation in Section VIII-A (we will further study it in future work).

Fig. 4: Empirical inference capability of the attacker v.s. upper bound.

D. Experiment Results on Task-independent Database Utility

In this experiment, we consider a lower privacy budget (which provides a higher privacy guarantee and higher fingerprint robustness, as demonstrated in Figure 2), and let $\epsilon$ take values 0.25, 0.5, 0.75, and 1. We do not consider moderate to high values of $\epsilon$ as in the previous section, because they return much higher utility values.

We measure the general database utility as the change in the variance of each attribute caused by both mechanisms. Table VII summarizes the fraction of changed data entries and the change of variance under varying $\epsilon$. The column for $\epsilon = \infty$ (highlighted in blue) records the original variance of each attribute in $R$, columns highlighted in red are the results achieved by our mechanism, and the non-highlighted columns are the results obtained by applying local differentially-private perturbation followed by fingerprinting (i.e., the two-stage approach we use for comparison). It is clear that given the same privacy guarantee and fingerprint robustness, our mechanism is able to maintain higher database utility by making less changes in $R$, i.e., the changes in variance caused by our mechanism are generally 10 times smaller than that caused by the approach we use for comparison. This suggests that by merging differential privacy and fingerprinting as a unified mechanism, we can approximately boost the statistical utility of the released nursery database by 10 times compared to the two-stage approach.

We also investigate the accuracy of the released database queries by considering the following customized queries: (i) $Q_1$: the data records which have more than 3 children (“more”) and the social conditions are slightly problematic (“slightly_prob”), and (ii) $Q_2$: the data record whose parents’ occupation are “usual” and the finance condition are inconvenient (“incov”), i.e.,

\begin{align*}
Q_1 & : \text{SELECT PmyKey FROM Nursery WHERE} \\
& \text{children = more AND social = slightly_prob} \\
Q_2 & : \text{SELECT PmyKey FROM Nursery WHERE} \\
& \text{parent = usual AND finance = incov}
\end{align*}

By varying $\epsilon$ from 0.25 to 2, we show the query accuracy (i.e., the fraction of matched data records with the query results obtained from the original database) achieved by our mechanism and that by the two-stage approach in Figure 5. Again, we observe that our proposed mechanism can achieve more accurate database queries than the two-stage approach.
TABLE V: Comparison of database utility (measured in terms of the change of variance of each attribute in the database). Column colored in blue is the original variance of each attribute in $R$. Columns colored in red are the change of variance caused by our proposed mechanism under various privacy budgets. Non-highlighted columns are the change of variance caused by applying local differentially-private perturbation followed by fingerprinting proposed in [29].

E. Experiment Results on Task-specific Database Utility

In Figure 6, we show the fingerprinted database utility in the considered task-specific applications.

![Fingerprinted Database Utility](image)

(a) SVM Classification.  (b) PCA.

Fig. 6: Database utility in task-specific applications. (a) SVM: difference between fingerprinted testing accuracy and original testing accuracy versus $\epsilon$. (b) $\text{TTL} \text{DEV}$ versus $\epsilon$.

F. Experiment Results on Fingerprint Densities Comparison under Multiple Sharings

In Figure 7, we plot the fingerprint densities of the shared databases when $\epsilon_2 = \epsilon_3$, i.e., $||M(R) - R||_1$ as a result of sharing with 100 SPs.

![Fingerprint Densities Comparison](image)

Fig. 7: Comparison of fingerprint densities of 100 fingerprinted databases achieved by the proposed mechanism and the two-stage approach.