The effect of new physics on \( R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{hadrons})} \)

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**Abstract**

Motivated by the \( \sim 2\sigma \) discrepancy between the experimental value for \( R_b \) and the theoretical prediction of the Standard Model, we examine the effect of new physics on the \( Z-b\bar{b} \) vertex. Using general results for both new scalars and gauge bosons, we show that two conditions must be satisfied in order for new physics to give observable deviations in the vertex. In particular, the fermion in the loop must transform chirally under \( SU(2) \times U(1) \) and it must be massive compared to the exchanged boson. We examine the implications of these results on the 2 Higgs Doublet model and the Left-Right Symmetric model.

PACS numbers: 12.12.Lk, 12.60.Cn, 12.60.Fr, 13.38.Dg
I. INTRODUCTION

In the last few years, precision electroweak experiments have improved to the point where they are now providing highly sensitive tests of the Standard Model (SM). While the SM is generally in excellent agreement with experiment, recent results on the left-right asymmetry $A_{LR}$ at SLC \[1\] and $R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$ measured at LEP \[2\] indicate a possible disagreement at the 2 to 2.5 $\sigma$ level. Although the discrepancy between $A_{LR}$ and the LEP asymmetry results is difficult to accommodate, $R_b$ presents the interesting possibility that new physics beyond the SM may show up non-obliquely.

A common approach to studying new physics is to assume that the dominant effect comes from oblique corrections. In this manner, deviations from the SM may be parametrized, for example, in terms of the parameters $S$, $T$ and $U$ \[3\] or $\epsilon_{N1}$, $\epsilon_{N2}$ and $\epsilon_{N3}$ \[4\]. Contributions from box and vertex diagrams with heavy new particles are typically small. However, in the context of the SM, the $Z$-$b$-$\bar{b}$ vertex receives an important contribution from heavy top loops \[5–7\], leading us to speculate whether new physics may also play a role in direct corrections.

While all fermion vertices may receive corrections from new particles, we choose to focus on the $b$ vertex. After all, in the SM, this is the only vertex that receives a large correction. We take the heavy top as a hint that the third family is somehow singled out, and that this characteristic may persist even beyond the SM. For instance, in the 2 Higgs Doublet (2HD) model, while the additional physical Higgs particles will lead to new vertex contributions, the only ones of importance lie in the third family, as it is the only family with significant Yukawa couplings. Nevertheless, our analysis is easily generalized to include vertex corrections to the first two families as well.

The ratio $R_b$ provides an excellent test of the $b$ vertex both because it is fairly insensitive to QCD corrections and because the oblique corrections are mostly cancelled in the ratio. The latter allows us to focus solely on the $b$ vertex without worrying about the effect of new physics on the oblique parameters. Experimentally, the LEP collaborations have measured $R_b$ to be $R_b = 0.2208 \pm 0.0024$ \[2\], which disagrees with the theoretical value of $\approx 0.215$ for $m_t \approx 174$ GeV. In this letter, we explore whether vertex corrections from new physics can bring the theoretical predictions closer to the experimental result. In particular, we examine the effect of both new scalars and new gauge bosons on the $b$ vertex and hence their impact on $R_b$. We find that, although large contributions are possible from new physics, they depend on the presence of massive chiral fermions in the vertex — either the top or possible new fermions.
II. THE RATIO $R_b$ AND THE $b$ VERTEX

We work in the * scheme [8], where the partial width of the $Z$ into fermion pairs is given by

$$
\Gamma(Z \to f \bar{f}) = \frac{\alpha_s}{6s^2 c^2} N_c \beta M_Z Z_{Z^*} \left[ (1 - x)((a_L^f)^2 + (a_R^f)^2) + 6x a_L^f a_R^f \right],
$$

where $x = (m_b/M_Z)^2$ and $\beta = \sqrt{1 - 4x}$. $N_c$ is the color factor and $a_L^f$ and $a_R^f$ are the left- and right-handed fermion couplings to the $Z$ gauge boson. Since we take the ratio of widths to get $R_b$, most oblique and QCD corrections cancel. Thus we need not worry about effects such as the propagator renormalization hidden in $Z_{Z^*}$. The weak mixing angle in the * scheme, $s^*$, will pick up oblique corrections. However such effects turn out to be fairly small.

At tree-level, the couplings are given simply by $a_L^f = T_3(f) - Q(f)s^2_\ast$ and $a_R^f = -Q(f)s^2_\ast$. However, even in the context of the minimal SM, the experimental data is accurate enough that it is important to take SM loops into account. Since we are interested in additional vertex corrections beyond the SM, we separate out the new physics and parametrize the left- and right-handed couplings by

$$
a_{L,R}^f = a_{L,R}^{f,\text{SM}} + \frac{\alpha_s}{4\pi s^2_\ast} \delta a_{L,R}^f,
$$

where $a_{L,R}^{f,\text{SM}}$ include the SM vertex corrections[4]. For $f \neq b$, the corrections are small but non-negligible. However, for the $b$ vertex, $a_L^b$ receives an important non-universal contribution due to heavy top loops. Although there is now evidence for a top mass $m_t = 174 \pm 17$ Gev [9], we wish to leave the top mass as a free parameter so we may study the effects of $m_t$ on the vertex. Hence we take $a_{L,SM}^b = a_{L,SM}^b(m_t = 0)$ and instead incorporate $m_t$ into $\delta a_L^b$ according to

$$
\delta a_L^b = [a_{L,SM}^b(m_t) - a_{L,SM}^b(m_t = 0)] + \cdots
$$

where $\cdots$ signifies the contribution from new physics.

To be precise, we note that in general the SM vertex correction may have imaginary parts due to both $Z$ and $W$ loops. However, since we know $m_t > M_Z/2$, this imaginary

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To be general, the dipole form factors should also be included. However, in the absence of left-right mixing, they are suppressed by a factor of $m_b/M_W$ and may be ignored. Even when mixing is present, the dipole contributions are often suppressed, as we will point out later.
part arises only from the $Z$ loop and may be incorporated entirely in $a_L^b$ itself. We always assume this is done so that $\delta a_L^b$ is real. Furthermore, since we expect any new particles to be heavy, both $\delta a_L^b$ and $\delta a_R^b$ will remain real in the presence of new physics.

For the SM prediction of $R_b$, we take $R_b^{\text{SM}}(m_t = 0) = 0.2179 \pm 0.0004$ where the uncertainty mainly arises from the determination of $m_b$. Then to linearized order, and ignoring the $b$-quark mass, $\delta a_{L,R}^b$ shift this prediction according to

$$R_b = R_b^{\text{SM}} + R_b^{\text{SM}} (1 - R_b^{\text{SM}}) \frac{\alpha_s}{2\pi s_*^b} \frac{a_L^b \delta a_L^b + a_R^b \delta a_R^b}{(a_L^b)^2 + (a_R^b)^2} \approx 0.2179 - 0.0021 \delta a_L^b + 0.00038 \delta a_R^b.$$  \hfill (4)

In the SM, $\delta a_R^b = 0$, whereas $\delta a_L^b \sim m_t^2/4M_W^4$ for large $m_t$. In Fig. 1, we show where the $1\sigma$ contour of $R_b$ lies in the $\delta a_R^b$-$\delta a_L^b$ plane. Due to the subtraction of (3), the origin corresponds to the SM with an unphysical $m_t = 0$. Other values of $m_t$ are indicated on the figure, showing the $\sim 2\sigma$ disagreement for $m_t \approx 174$ GeV.

The forward-backward asymmetry, $A_{FB}^b$, is also sensitive to the left- and right-handed couplings of the $b$ quark. From $A_{FB}^{0,b} = \frac{3}{4}A_e\alpha_b$, we find

$$A_{FB}^{0,b} = (A_{FB}^{0,b})^{\text{SM}} + (A_{FB}^{0,b})^{\text{SM}} \frac{\alpha_s}{\pi s_*^b} \frac{a_L^b a_R^b}{(a_L^b)^2 - (a_R^b)^2} \left[ a_R^b \delta a_L^b - a_L^b \delta a_R^b \right].$$ \hfill (5)

Since $a_L^b$ is about five times larger than $a_R^b$, $R_b$ and $A_{FB}^b$ give complimentary information on the couplings — $R_b$ is sensitive to $\delta a_L^b$ whereas $A_{FB}^b$ is sensitive to $\delta a_R^b$. In addition, $R_b$, unlike $A_{FB}^b$, is almost insensitive to oblique corrections. Thus at present, we focus only on $R_b$, although we expect improved measurements of the forward-backward asymmetry as well as $A_{FB}^{\text{pol}}(b)$ from SLC in the future.

We now address the issue of how large can $\delta a_{L,R}^b$ become. Since we have separated out an explicit loop factor of $\alpha_s/4\pi s_*^b$, both $\delta a_L^b$ and $\delta a_R^b$ are generically of order 1. From Eq. (4), we note that $\delta a_L^b$ will affect $R_b$ at about the $1\%$ level — the same as the present experimental precision. Thus the $\delta a_L^b$ contribution to $R_b$ is of roughly the same importance as that of the oblique parameters $S$, $T$ and $U$ to other electroweak observables. $\delta a_R^b$, on the other hand, has a much smaller effect on $R_b$. At the same time, in most models, additional right-handed currents are either suppressed or not present. Thus $\delta a_R^b$ is often of much less importance.

While the parameter $\epsilon_b$ has been introduced to describe the $Z \to b\bar{b}$ vertex, it is defined in relation to the partial width, $\Gamma(Z \to b\bar{b})$ instead of the ratio $R_b$. Thus the extraction of

\footnote{In terms of $S$ and $T$, the oblique correction to $R_b$ is given by $\delta R_b \approx 0.00014S - 0.00008T$.}
\( \epsilon_b \) from experimental data is more sensitive to the determination of \( \alpha_s(M_Z) \) as well as the treatment of oblique corrections. Nevertheless, assuming \( \delta a_R^b \approx 0, \epsilon_b \) is related to the vertex correction by \( \epsilon_b \approx -\frac{\alpha_s}{2\pi s_w^2} \delta a_L^b \).

III. NEW SCALARS AND \( \delta A_{L,R} \) — THE 2HD MODEL

The 2HD model is one of the simplest extensions of the SM. In this model, an additional Higgs doublet is introduced, leading to additional vertex corrections with physical charged and neutral Higgs loops. In the 2HD model, separate Higgs doublets give masses to the up- and down-type quarks. Due to large couplings to the \( t \)-quark in the vertex, \( R_b \) has been used to rule out most of the small \( \tan \beta \) region of parameter space \([10,11]\) where \( \tan \beta = v_2/v_1 \) with \( v_2 \) and \( v_1 \) giving masses to up- and down-type quarks respectively.

Although we include the 2HD model in our discussion, we also allow for more general scalar interactions. In the simplest case, we add a single new scalar, \( \phi \) with arbitrary isospin and electric charge, to the SM. In order to give a direct contribution to the \( Z-b-\bar{b} \) vertex, \( \phi \) must couple to the \( b \)-quark. We first investigate \( \phi \) with a left-handed coupling

\[
\mathcal{L}_Y = g \lambda_{L} \bar{b}_L \phi F_R + \ldots + \text{H.c.,} \tag{6}
\]

where \( F_R \) may be either an ordinary or a new right-handed quark. While isospin and electric charge must be conserved in the above interaction, we only need the explicit term written above (in general, \( \phi \) may be a member of a larger multiplet, filling out some \( SU(2) \times U(1) \) representation). Let \( T_3(\phi) \) and \( Q(\phi) \) denote the third component of isospin and charge of the new scalar and similarly \( \{ T_3(F_L), T_3(F_R) \} \) and \( Q(F) \) for the quark \( F \). In general we allow \( F_L \) and \( F_R \) to transform under different representations of \( SU(2) \) so we may accommodate both vector and chiral particles\(^3\). By isospin conservation, the above quantities must satisfy the relation \( T_3(b) = T_3(\phi) + T_3(F_R) \), as indicated in Fig. 2a. Note that when \( SU(2)_L \times U(1)_Y \) is broken, scalars carrying different isospin may mix. Although we avoid such cases, they may be treated similarly without much further complication.

The \( b \) vertex correction due to the scalar \( \phi \) may easily be evaluated. We find

\[
\delta a_L^b(\phi) = \lambda_L^2 \left[ (T_3(b) - Q(b)s_w^2)\Theta - (T_3(\phi) - Q(\phi)s_w^2)(\Theta + \Psi) \right. \\
+ (T_3(F_R) - T_3(F_L)) \Delta] (M_Z^2; M_{\phi}^2, m_F^2) , \tag{7}
\]

\(^3\)By chiral, we mean chiral under the SM gauge group.
where the functions $\Theta$, $\Psi$ and $\Sigma$ are given in terms of finite combinations of Passarino-Veltman functions \[12\] by

$$
\Theta(q^2; M^2, m^2) = B_1(0; m^2, M^2) + [2C_{24} - \frac{1}{2} - m^2C_0 + q^2(C_{22} - C_{23})](0, q^2, 0; M^2, m^2, m^2)
$$

$$
\Psi(q^2; M^2, m^2) = -B_1(0; m^2, M^2) - 2C_{24}(0, q^2, 0; m^2, M^2, M^2)
$$

$$
\Delta(q^2; M^2, m^2) = m^2C_0(0, q^2, 0; M^2, m^2, m^2)
$$

(8)

For degenerate masses present in the $C$ functions, use of Passarino-Veltman identities allows us to rewrite $\Theta$ and $\Psi$ to give

$$
\Theta(q^2; M^2, m^2) = q^2[C_{12} + 2C_{22} - C_{23}](0, q^2, 0; M^2, m^2, m^2)
$$

$$
\Psi(q^2; M^2, m^2) = q^2[2C_{22} - C_{23}](0, q^2, 0; m^2, M^2, M^2)
$$

(9)

showing that they vanish in the limit $q^2 \to 0$. This should come at no surprise since it is a consequence of the vector Ward identity. On the other hand, $\Delta$ is non-vanishing in this limit, and for small $q^2$ has the expansion $\Delta(q^2; M^2, m^2) = \Delta_0(x) + O(q^2/M^2)$ where

$$
\Delta_0(x) = \frac{x}{1-x} + \frac{x}{(1-x)^2}\log x
$$

(10)

In the same spirit as the $STU$ parameters, we may consider an expansion in inverse powers of $M_\phi^2$, the new physics scale. In this case, since $\Theta$ and $\Psi$ are suppressed relative to $\Delta$ by a factor of $q^2/M_\phi^2 = M_F^2/M_\phi^2$, the lowest order expression for $\delta a_L^b(\phi)$ becomes simply

$$
\delta a_L^b(\phi) = \lambda_L^b(T_3(F_R) - T_3(F_L))\Delta_0(m_F^2/M_\phi^2) + \cdots
$$

(11)

This expression allows us to make a few observations. First of all, since it vanishes when $F_L$ and $F_R$ carry the same isospin, chiral fermions in the loop are necessary in order to get large shifts in $\delta a_L^b$ (and hence $R_b$). For vector fermions, the Ward identity ensures the vertex correction is suppressed by the factor $M_F^2/M_\phi^2$, giving another example of the decoupling theorem. Secondly, $\Delta_0$ vanishes in the limit $m_F \to 0$, so contributions from light fermions are suppressed as well. Both conditions are necessary in order for the new physics to generate sizable effects in the vertex. Finally, since $-1 < \Delta_0(x) \leq 0$ for all $x$, the sign of $\delta a_L^b(\phi)$ is completely determined (to lowest order), and hence the direction of the shift in $R_b$ is fixed simply by the isospins of the quark $F$.

For a scalar $\chi$ that interacts with right-handed $b$-quarks, similar expressions may be derived for $\delta a_R^b$. In this case, as shown in Fig. 2b, isospin conservation demands $T_3(\chi) = -T_3(F_L)$. The result is similar to (7) and (11), but with $L \leftrightarrow R$:
\[ \delta a^b_R(\chi) = \lambda^2_R(-Q(b)s^2_s)(\Theta - (T_3(\chi) - Q(\chi)s^2_s)(\Theta + \Psi) + (T_3(F_L) - T_3(F_R))M^2_ZM^2_R + m^2_F) \]
\[ = \lambda^2_R(T_3(F_L) - T_3(F_R))\Delta_0(m^2_t/M^2_Z) + \cdots \quad (12) \]

Note that \( \chi \) may be the same scalar as \( \phi \), provided \( T_3(b) = T_3(F_R) - T_3(F_L) \) for consistency among the left and right Yukawa interactions. This is indeed the case for the charged Higgs loop in the 2HD model where \( F = t \).

Using these results, we now examine the 2HD model in greater detail. Focusing only on the charged Higgs, \( H^+ \), the vertex correction \( \delta a^b_{L,R}(H^+) \) may be calculated from (7) and (12) using \( \lambda_L = m_t\cot\beta/\sqrt{2}M_W \) and \( \lambda_R = m_b\tan\beta/\sqrt{2}M_W \). For small values of \( M_{H^+} \), the exact expressions need to be used since the expansion factor \( M^2_Z/M^2_{H^+} \) is not sufficiently small. Nevertheless, the vertex is typically still dominated by the isospin splitting term

\[ \delta a^b_{L,R}(H^+) \approx \mp \lambda^2_{L,R}\Delta_0(m^2_t/M^2_{H^+}) \quad , \quad (13) \]

where the top sign corresponds to \( \delta a^b_L \). Because of the Yukawa couplings, the left- (right)-handed interactions dominate for small (large) \( \tan\beta \). This is evident in Fig. 1 where we have shown how \( \delta a^b_L \) and \( \delta a^b_R \) are shifted in the 2HD model relative to the SM with \( m_t = 150 \text{ GeV} \). Note that, regardless of \( \tan\beta \), the prediction for \( R_b \) from the charged Higgs is always decreased compared to the SM.

This figure also shows that since \( R_b \) is less sensitive to changes in \( \delta a^b_R \) it more easily rules out the small \( \tan\beta \) region of the 2HD model. Furthermore, for large \( \tan\beta \), the neutral Higgs couplings will become important, and the neutral Higgs sector will also play a role \[ \text{(13)}. \] In principle, the \( H^+ \) loop also generates dipole form factors proportional to \( \lambda_L\lambda_R = m_\phi m_t/2M^2_W \). However, as this is independent of \( \tan\beta \), the dipole terms have no enhancement and may safely be neglected. In order to demonstrate how strong the constraints are, we show the 99\%C.L. excluded region for several values of \( m_t \) in Fig. 2.

While the 2HD model essentially describes the Higgs sector of the minimal supersymmetric standard model (MSSM), the \( Z-b\bar{b} \) vertex in the latter model picks up additional contributions from both neutralino and chargino loops \[ \text{(14,15)}. \] Since these contributions may have either sign \[ \text{(14)}, \] the MSSM, unlike the 2HD model, actually allows for predictions of \( R_b \) in closer agreement with experiment \[ \text{(16)}. \]

#### IV. NEW GAUGE BOSONS

We now turn to the effect of new gauge bosons on the \( b \) vertex. In a similar vein to the previous section, we consider the addition of a new gauge boson, \( V_L \) or \( V_R \), with either a left-
or a right-handed coupling to the $b, V-b\bar{F}$. When $V_L = W$ and $F = t$, this reproduces the SM vertex correction. However, we again allow for the possibility that $F$ is a new quark. For example, in the $SU(3) \times U(1)$ model of [17], the ordinary quark doublet, $(t, b)$, is extended by the addition of a charge $5/3$ quark, $T$, to fill out a $SU(3)$ anti-triplet $(b, t, T)$.

Working in 't-Hooft-Feynman gauge, the result for a $V_L$ is given by

$$\delta a^L_3(V_L) = [(T_3(b) - Q(b)s^2)\frac{1}{2}\Phi + (T_3(V) - Q(V)s^2)[B_0(0; M^2, M_V^2) - \frac{1}{2}(\Phi + \Lambda)]$$

$$+ (T_3(F_R) - T_3(F_L))\Xi](M^2; M_V^2, m^2_F) \, .$$

Since isospin is conserved at the vertex, we must ensure $T_3(b) = T_3(F_L) + T_3(V)$. Once again, we have ignored isospin mixing that, however, would generally arise once $SU(2) \times U(1)$ is broken. The functions are given by

$$\frac{1}{2}\Phi(q^2; M^2, m^2) = (1 + \frac{x}{2})\Theta(q^2; M^2, m^2) - q^2[C_0 + C_{11}](0, q^2, 0; M^2, m^2, m^2)$$

$$\frac{1}{2}\Lambda(q^2; M^2, m^2) = (1 + \frac{x}{2})\Psi(q^2; M^2, m^2) + q^2C_{11}(0, q^2, 0; m^2, M^2, M^2)$$

$$- [B_0(q^2; M^2, M^2) - B_0(0; M^2, M^2)]$$

$$\Xi(q^2; M^2, m^2) = 2m^2C_0(0, q^2, 0; m^2, M^2, M^2) - m^2C_0(0, q^2, 0; M^2, m^2, m^2)$$

$$+ \frac{x}{2}[\Theta + \Psi + \Delta](q^2; M^2, m^2) \, .$$

As before, $x = m^2/M^2$. The terms proportional to $x$ come from the would-be Goldstone boson in 't Hooft-Feynman gauge as can be seen from comparison with (11) and (8).

We note that the vertex is finite except for the universal piece arising from $B_0(0; M_V^2, M_V^2)$ in the non-abelian term. This is a well known situation and is removed by a counterterm of a similar form in the on-shell scheme [18–21]. The treatment is similar for the * scheme although the subtraction is instead the momentum dependent term $B_0(q^2; M_V^2, M_V^2) \Xi$.

Taking $F = t$ in (14) reproduces the SM top contribution to the $Z \rightarrow b\bar{b}$ vertex [3]. In particular, these three vertex functions have been given before in [18], although in a form reduced to the elementary scalar integrals $B_0$ and $C_0$. Prior to numerical evaluation of these functions, we prefer the above expressions both because of their conciseness and because they explicitly demonstrate the vanishing of $\Phi$ and $\Lambda$ as $q^2 \rightarrow 0$. In the limit $m_F \rightarrow 0$, we verify that both $\Phi$ and $\Lambda$ reduce to the well known expressions for the vertex in the massless limit [3][18][22].

Similar to the scalar case, this shows that in order for a new gauge interaction to generate large vertex corrections, the internal fermion, $F$, must be both massive and chiral. The vanishing of $\Xi$ as $m_F \rightarrow 0$ can be understood through helicity conservation in the massless limit since its coefficient in (14) indicates that this term arises from the difference in isospin between $F_L$ and $F_R$. Expanding to lowest order in $q^2/M^2_V$ gives
\[ \delta a_L^R(V_L) = (T_3(F_R) - T_3(F_L))\Xi_0(m_t^2/M_W^2) + \cdots, \]  

where

\[ \Xi_0(x) = \frac{x(-6 + x)}{2(1 - x)} - \frac{x(2 + 3x)}{2(1 - x)^2} \log x. \]  

We note that \( \Xi_0(x) < 0 \) for \( x \gtrsim 0.1 \) with \( \Xi_0(x) \sim -x/2 \) for large \( x \). Therefore the direction of the shift in \( R_b \) is again determined by the isospins of \( F \), at least for the interesting case when \( F \) is heavy. For the SM, we take \( F = t \) and find \( \delta a_L^b(W) \sim x/4 \), leading to the asymptotic behavior \( \epsilon_b \sim -G_F m_t^2/4\sqrt{2}\pi^2 [4] \).

For a \( V_R \), we interchange left- and right-handed couplings and find

\[ \delta a_R^b(V_R) = [(-Q(b)s_w^2)^2\frac{1}{2} \Phi + (T_3(V) - Q(V)s_w^2)[B_0(0; M_L^2, M_R^2) - \frac{1}{2}(\Phi + \Lambda)] \\
+ (T_3(F_L) - T_3(F_R))\Xi](M_L^2; M_L^2, M_R^2), \]  

where now the condition \( T_3(V_R) = -T_3(Q_R) \) must be satisfied. As an example of a right-handed interaction, consider the left-right symmetric (LRS) model [23–25]. There are two charged gauge bosons in this model, \( W_L \) and \( W_R \), which may mix with mixing angle \( \zeta \) to form the mass eigenstates \( W_{1,2} \). In general both \( W_1 \) and \( W_2 \) have left- and right-handed couplings. However, as the mixing is constrained to be small [26,27], \( W_1 \) is mostly \( W_L \) and similarly for \( W_2 \). In this case, the contributions from \( W_{1,2} \) separate with \( \delta a_L^b(W_1) \) identical to the SM case. For \( W_2 \), on the other hand, although \( F = t \) is chiral, it turns out that \( \delta a_R^b(W_2) \) is small since in this case \( m_t < M_{W_2} \) [27] and hence \( \Xi_0 \) is suppressed.

While we have focused on the contributions to \( a_{L,R} \), the \( W_L-W_R \) mixing will induce dipole form factors at the vertex. This increases \( R_b \) by

\[ \frac{\delta R_b}{R_b} = (1 - R_b)\frac{\alpha_s}{4\pi s_w^2}|\zeta|^2 \frac{m_t^2/M_W^2}{(a_L^b)^2 + (a_R^b)^2} \sim 0.05|\zeta|^2, \]  

which is negligibly small [4]. Thus, unlike the 2HD model, \( R_b \) does not provide any additional constraints for the charged gauge boson mixing in the LRS model.

V. SUMMARY

It is usually assumed that the dominant effects of new physics show up only obliquely at the current energy scales. We have examined the validity of this assumption in more detail

\[ ^4 \text{This mixing effect, however, is important in the decay } b \to s\gamma [28]. \]
for the case of the $b$ vertex. In particular, we have considered the effects of both new scalars and new gauge bosons on the vertex corrections, $\delta a_{L,R}^b$. By using a model-independent approach, we find that in general two conditions must be satisfied by the new particles in order for the vertex correction to compete with the effects of the oblique corrections. First of all, the fermion in the vertex must be chiral, and secondly it must be massive compared to the boson in the loop. These conditions are required to avoid the vector Ward identity which would otherwise constrain the corrections to be small.

Both $R_b$ and $A_{FB}^b$ may be used to constrain the $b$ vertex corrections. Both observables are complimentary since the former is mostly sensitive to $\delta a_L^b$ while the latter is mostly sensitive to $\delta a_R^b$. However $R_b$ is especially useful as most of the oblique effects are cancelled in the ratio, thus allowing us to study the vertex independently of the oblique corrections that undoubtably arise when new physics is present.

We have applied the general results to both the 2HD and the LRS model. For the 2HD model, we find strong restrictions on the small $\tan \beta$ regime, but for the LRS model no useful constraints are obtained. While the top is present in the loop in both cases, the contrast is mainly due to the behavior of the Yukawa coupling in the 2HD model, $\lambda_L \sim \cot \beta$, which enhances the contribution for small $\tan \beta$.

A more careful treatment, including the effects of isospin mixing may be undertaken in specific models. We have done this for the $SU(3) \times U(1)$ model and found that the effects are quite small since the new quark $T$ is a $SU(2) \times U(1)$ vector. While isospin mixing plays a role, the additional terms are proportional to both $M_{W}^{2}/M_{Y}^{2}$ and the mass splitting between the new gauge bosons, $Y$, and the new scalars. Thus, we feel these general results without the incorporation of isospin splitting are sufficient to present a good understanding of new physics and the $b$ vertex.

When both conditions for large vertex corrections are satisfied, the sign of the contribution to $\delta a_{L,R}^b$ is fixed by the isospin of the fermion in the loop. This may provide a clue to what possible new physics may arise in order to bring $R_b$ into closer agreement with experiment. For instance, while the 2HD model always predicts a smaller $R_b$ than the SM, the additional contributions in the MSSM may lead corrections of either sign. This is also the case for $b \rightarrow s\gamma$ in these two models. Of course it remains to be seen whether the experimental discrepancy in $R_b$ will hold up in the future or not.

This work was supported in part by the National Science Foundation under Grant No. PHY-916593, and by the Natural Science and Engineering Research Council of Canada.
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FIGURES

FIG. 1. The 1σ contour for $R_b$ in the $\delta a^b_L - \delta a^b_R$ plane. The SM predictions with a heavy top are given by the solid line with $\delta a^b_R = 0$. Also included in the figure are the small and large tan $\beta$ behavior of the vertex corrections in the 2HD model in the case where $m_t = 150$ GeV.

FIG. 2. Yukawa interactions for new scalars $\phi$ and $\chi$.

FIG. 3. The 99% C.L. excluded region in the $M_{H^+} - \tan \beta$ plane for the 2HD model. Note that the SM with $m_t > 200$ GeV is already excluded at 99% C.L. by $R_b$, thus excluding the 2HD model with such a heavy top as well.
Figure 1
Figure 2
Figure 3
This figure "fig1-1.png" is available in "png" format from:

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