Pre-big bang in M-theory*

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Abstract

We discuss a simple cosmological model derived from M-theory. Three assumptions lead naturally to a pre-big bang scenario: (a) 11-dimensional supergravity describes the low-energy world; (b) non-gravitational fields live on a three-dimensional brane; and (c) asymptotically past triviality.

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1 Introduction and preliminaries

Over the last two decades superstring theory [1] and, most recently, M-theory [2] have emerged as the most promising candidates for the theory of quantum gravity. From a cosmological point of view, the key theoretical question to be addressed is whether a successful inflationary model can be constructed from superstring/M-theory. In superstring theories duality relations between different regions of the moduli space provide a much richer setting to investigate inflationary models than Einstein theory. Moreover, the field content of superstring/M-theory

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in a given region of the moduli space is determined unambiguously on theoretical grounds. This provides a firm guidance in the search for inflationary models which is absent in Einstein theory. On the astrophysical/cosmological side a strong test for superstring/supergravity theories is their compatibility with the standard model of early universe. Observational cosmology becomes more and more efficient in constraining cosmological parameters and the spectrum of primordial perturbations. Gravitational wave experiments may provide new and complementary constraints. In the near future, cosmology will be the main laboratory to test whether superstring/M-theory does really describe our universe or is just a nice mathematical construction [3]. In view of these developments superstring/M-theory cosmology is a subject worth investigating.

In the cosmological setting, essential features of superstring/M-theory are the presence of the dilaton, axion, Ramond-Ramond (R-R) forms and various moduli fields, in addition to higher-curvature terms that appear in the low-energy effective actions. The presence of these fields and string duality relations have a profound impact on cosmological scenarios. For instance, simple low-energy string models with graviton and dilaton lead to the so-called pre-big bang (PRBB) scenario [4] which is structurally different from standard Einstein cosmology. In the PRBB scenario different branches of the solution are related by time reflection and internal transformations – the scale factor duality [5]. The universe evolves from a weakly coupled string vacuum state first to a radiation-dominated and then to a matter-dominated Friedmann-Robertson-Walker (FRW) geometry through a region of strong coupling and large curvature.

If M-theory is the ultimate theory of quantum gravity the low-energy world is described by the 11-dimensional supergravity action [2, 6]

\[ S^{(11)} = \frac{1}{l_{pl,11}^9} \int d^{11}x \sqrt{-g^{(11)}} \left[ R^{(11)}(g^{(11)}) - \frac{1}{48} F_{a_1...a_4} F^{a_1...a_4} + \right. \\
\left. - \frac{1}{12^4 \sqrt{-g^{(11)}}} e^{a_1...a_3 b_1...b_4 c_1...c_4} A_{a_1...a_3} F_{b_1...b_4} F_{c_1...c_4} \right], \tag{1} \]

where \( a_i, b_i, c_i = 0 \ldots 10 \), \( F_{a_1...a_4} = 4 \partial_{[a_1} A_{a_2...a_4]} \) is the 4-form field strength of the antisymmetric 3-form potential \( A_{a_1...a_3} \), and \( g^{(11)} \) denotes the determinant of the 11-dimensional metric \( g_{ab} \). Equation (1) describes the low-energy limit of M-theory. (Here and throughout the paper we use natural units and set \( l_{pl,11} \) such that the four-dimensional Planck length is \( l_{pl,4} = 1 \).

Starting from equation (1) several different M-theory cosmological models have been proposed in the literature. In particular, the idea of brane world has emerged in the works of Lukas et al [7] and Randall and Sundrum [8]. According to this model our four-dimensional universe emerges as the world volume of a 3-brane in a higher-dimensional spacetime. A more standard approach [9] deals with different classes of cosmological solutions that reduce to solutions of string dilaton gravity. The analysis of four-dimensional isotropic and homogeneous cosmologies derived from M-theory and type IIA superstring theory (see last paper in [9, 10])...
has found that form-fields associated with the Neveu/Schwarz-Neveu/Schwarz (NS-NS) and
R-R sectors play a different and crucial role in determining the dynamical behavior of the
solutions: the NS-NS fields, such as the axion, tend to forbid inflation whereas the R-R fields
have the opposite effect [10].

The purpose of this paper is to investigate what cosmological scenario follows from three
simple assumptions:

(a) M-theory is the correct description of nature;
(b) non-gravitational fields live on a three-dimensional brane propagating in the 11-dimen-
sional spacetime;
(c) the universe originated in the vacuum of the theory (asymptotically past triviality, APT).

We will see that these simple postulates lead naturally to a PRBB cosmological scenario,
where two different branches of the solution are related by internal symmetries of the model
and the universe evolves from a weakly coupled string vacuum state to a decelerated FRW
geometry through a state with large curvature. Although our model is probably too simple to
give an accurate or even acceptable description of our universe, we believe it is nevertheless
interesting and may represent a good starting point to discuss the PRBB scenario in M-theory.

2 The model

Assumption (a) implies that the low-energy world is described by equation (1). Following
Witten [2] we assume that the 11th dimension is compactified on a circle $S_1$ of radius $R_{S_1}$. Carrying out a Kaluza-Klein reduction we find

\[ S^{(10)} = \frac{1}{l_{pl,10}^8} \int d^{10}x \sqrt{-g^{(s)}} \left[ e^{-\Phi_{10}} \left( R^{(10)}(g^{(s)}_{mn}) + (\nabla \Phi_{10})^2 - \frac{1}{12} H_{mnp} H^{mnp} \right) - \frac{1}{48} F_{mnpq} F^{mnpq} - \frac{1}{384 \sqrt{-g^{(11)}}} \epsilon^{m_1 m_2 n_1 \ldots n_4 p_1 \ldots p_4} B_{m_1 m_2} F_{n_1 \ldots n_4} F_{p_1 \ldots p_4} \right] , \]  

(2)

where we have rescaled the ten-dimensional metric as $g_{ab} = R_{S_1}^{-1} g_{ab}^{(s)} (a, b \neq 10)$ and we have defined the dilaton by $R_{S_1} = e^{\Phi_{10}/3}$. $H_{mnp}$ and $F_{mnpq}$ are the field strengths of the potentials $B_{np}$ and $A_{npq}$, respectively. Note that we have ignored the 1-form potential that arises from the
Kaluza-Klein reduction. Equation (2) is the effective action for massless type IIA superstring.
The first line of equation (2) corresponds to the NS-NS sector and the second line corresponds
to the R-R sector.
Assumption (b) implies, for consistency, that the ten-dimensional geometry must be of the form $M_4 \times C_6$ and that the only non-trivial components of the field strengths are those associated with $M_4$. $C_6$ is a six-dimensional compact space which we assume to be a generic (Ricci flat) Calabi-Yau space, or, for sake of simplicity, a six-dimensional torus. Upon compactification on the six-dimensional internal space $C_6$ we find $(l_{pl,4} = 1)$

$$S = \int d^4x \sqrt{-g} \left[ e^{-\Phi_4} \left( R^{(4)}(g_{\mu\nu}) + (\nabla \Phi_4)^2 - 6 (\nabla \beta)^2 - \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} - \frac{1}{48} e^{6\beta} F_{\mu\nu\lambda\kappa} F^{\mu\nu\lambda\kappa} \right) \right],$$

where the radius of the internal space is $R_{C_6} = e^\beta$ and the four-dimensional dilaton field is $\Phi_4 = \Phi_{10} - 6\beta$. The field equation for the four-form $F^{\mu\nu\lambda\kappa}$ can be solved and the 3-form $H^{\mu\nu\lambda}$ can be dualized. The final result is

$$S = \int d^4x \sqrt{-g} \left[ e^{-\Phi_4} \left( R^{(4)}(g_{\mu\nu}) + (\nabla \Phi_4)^2 - 6 (\nabla \beta)^2 - \frac{1}{2} e^{2\Phi_4} (\nabla \sigma)^2 - \frac{1}{2} Q^2 e^{-6\beta} \right) \right],$$

where $\sigma$ is the pseudo-scalar axion field dual to the 3-form, $H^{\mu\nu\lambda} = e^{\Phi_4} e^{\mu\nu\lambda\kappa} \nabla_\kappa \sigma$, and $F^{\mu\nu\lambda\kappa} = Q e^{-6\beta} e^{\mu\nu\lambda\kappa}$. Equation (4) describes the world as seen by the four-dimensional observer and is our starting point to investigate M-theory cosmology.

Since we are interested in homogeneous and isotropic cosmologies we impose the metric ansatz

$$ds^2_{(4)} \equiv g_{\mu\nu}(x)dx^\mu dx^\nu = -N^2(t)dt^2 + a(t)^2 d\Omega_{3k}, \quad N(t) > 0$$

where $d\Omega_{3k}$ is a maximally symmetric three-dimensional unit metric with curvature $k = 0, \pm 1$, respectively. Moreover, in the spirit of the APT postulate, we assume that the four-dimensional metric (5) is spatially flat, i.e., we set $k = 0^1$. By substituting equation (5) in equation (4) and requiring for consistency that the modulus field $\beta$, the dilaton $\Phi_4$, and the axion $\sigma$ depend only on $t$, the density action per comoving volume in the physical spacetime becomes

$$S = \int dt \left[ \frac{1}{\mu} \left( 3\alpha^2 - \dot{\phi}^2 + 6\dot{\beta}^2 + \frac{1}{2} \sigma^2 e^{2(3\alpha + \phi)} \right) - \frac{1}{2\mu} Q^2 e^{3\alpha - \phi - 6\beta} \right],$$

where $\alpha(t) = \ln[a(t)]$, and we have defined the ‘shifted dilaton’ field $\phi = \Phi_4 - 3\alpha$ and the Lagrange multiplier $\mu(t) = N e^\phi > 0$. Finally, equation (6) can be cast in the canonical form

$$S = \int dt \left[ \dot{\alpha} p_\alpha + \dot{\phi} p_\phi + \dot{\beta} p_\beta + \dot{\sigma} p_\sigma - \mathcal{H} \right],$$

where the Hamiltonian is

$$\mathcal{H} = \mu H , \quad H = \frac{1}{24} \left[ 2p_\alpha^2 - 6p_\phi^2 + p_\beta^2 + 12 Q^2 e^{3\alpha - \phi - 6\beta} \left( 1 + \frac{p_\sigma^2}{Q^2} e^{-9\alpha + \phi + 6\beta} \right) \right].$$

The total Hamiltonian $\mathcal{H}$ is proportional to the non-dynamical variable $\mu$ which enforces the constraint $H = 0$.

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1Note that the APT, in its general form [11], admits more general initial states than spatially flat spacetime, namely, generic perturbative solutions of the low-energy string action which lead to gravitational instability.
We now make a further assumption. We assume that, at least at the beginning of the evolution, the NS-NS axion field is negligible w.r.t. the R-R 4-form field, namely, we consider a volume in the four-dimensional spacetime which is nearly devoid of axions. Quantitatively, we require

$$\frac{|Q|}{p_\sigma} \gg 1.$$  

(Note that $p_\sigma$ is constant.) The dynamical behaviour at the beginning of the evolution is described by the solution [10]

$$\begin{align*}
\alpha &= \alpha_0 - \frac{1}{4} \ln \cosh (\kappa T) - \alpha_1 T, \\
\phi &= \phi_0 - \frac{1}{4} \ln \cosh (\kappa T) - \phi_1 T, \\
\beta &= \beta_0 + \frac{1}{4} \ln \cosh (\kappa T) - \beta_1 T,
\end{align*}$$

where

$$T(t) = \int_{t_0}^t \mu(t') dt', \quad t > t_0.$$  

The constants of motion are related by (we choose $\kappa > 0$ for simplicity)

$$\begin{align*}
\kappa^2 + 6\alpha_1^2 - 2\phi_1^2 + 12\beta_1^2 - 2H &= 0, \\
3\alpha_0 - \phi_0 - 6\beta_0 - 2 \ln \left( \frac{\kappa}{|Q|} \right) &= 0, \\
3\alpha_1 - 6\beta_1 - \phi_1 &= 0.
\end{align*}$$

The dynamics of the model is determined essentially by $\alpha_1$. The constant $\kappa$ determines the scale of the time evolution and can be reabsorbed in the solution by defining the parameter

$$\tau = \kappa T$$

and the constants $\xi = \alpha_1/\kappa$, $\chi = \phi_1/\kappa$, $\rho = \beta_1/\kappa$ and $Q = Q/\kappa$. The constants $\alpha_0$, $\beta_0$ and $\phi_0$ are initial conditions for the phase space coordinates $\alpha$, $\beta$ and $\phi$, respectively.

The evolution proceeds monotonically in the gauge parameter $\tau$. The latter is related to the proper time of the four-dimensional world by the relation

$$t_\varepsilon(\tau) = \int_{t_0}^\tau d\tau' e^{-\phi(\tau')}.$$  

Since the integrand in Eq. (15) is positive defined, the time evolution in the cosmic time $t_\varepsilon$ follows the evolution in the gauge parameter $\tau$.

The structure of the moduli space is conveniently described in the plane $(\xi, \rho)$. The physical points are determined in this plane by the two branches of the hyperbola (12) (see figure 1)

$$(+) \text{ branch: } \rho = \frac{3}{5} \xi + \frac{1}{5} \sqrt{4\xi^2 + \frac{5}{12}}; \quad (-) \text{ branch: } \rho = \frac{3}{5} \xi - \frac{1}{5} \sqrt{4\xi^2 + \frac{5}{12}}.$$  

(The notation will be clarified soon.) The $(+)$ and $(-)$ branches are characterized by a few distinctive kinematical and dynamical properties:
Figure 1: Parameter space for the R-R four-form dominated solution. The physical points are represented by the two branches of the hyperbola (12). The PRBB and POBB phases correspond to the portions of the upper and lower hyperbola in the colored regions, respectively.

Table 1.

|                          | (+) branch                  | (-) branch                  |
|--------------------------|-----------------------------|-----------------------------|
| 4D effective coupling $g$| Increasing                  | Decreasing                  |
| Initial 4D effective coupling| Weak, perturbative          | Strong, non-perturbative    |
| 11D curvature $K$        | Increasing                  | Decreasing                  |
| Initial curvature scale  | Arbitrarily small           | Arbitrarily large           |

where $g \equiv e^{\phi}$ and $K \equiv R^{(11)}_{\mu\nu\rho\sigma} R^{(11)}{}^{\mu\nu\rho\sigma}$. The (+) and (-) branches remind one of the PRBB and post-big bang (POBB) branches in the PRBB scenario [4]. In the standard dilaton gravity models of PRBB [4] the two branches coincide with accelerated and decelerated scale factors, respectively. Here, the presence of extra fields (the R-R 4-form) makes the picture more complicated. For each of the two branches we find three different dynamical behaviours of the external geometry according to the value of $\xi$: expanding for $\xi < -1/4$ (accelerated for the (+) branch and decelerated for the (-) branch), contracting for $\xi > 1/4$ (accelerated for the (+) branch and decelerated for the (-) branch) and bouncing for $-1/4 < \xi < 1/4$ (accelerated at early and late times for the (+) branch and decelerated for the (-) branch). The qualitative behaviour of the external scale factor is represented in figure 2. The Hubble parameter and the deceleration parameter of the four-dimensional world are

$$H(t_c) = -\frac{e^{\phi_0 - \chi_\tau}}{\cosh^{1/4}(\tau)} \left( \xi + \frac{1}{4} \tanh(\tau) \right), \quad (16)$$

\footnote{We do not consider here the ‘fine-tuned’ cases $\xi = \pm1/4$.}
Figure 2: Qualitative behaviour of the external scale factor for the (+) branch (top figures) and 
(−) branch (bottom figures) for: (a) $\xi < -1/4$, (b) $-1/4 < \xi < 1/4$, and (c) $\xi > 1/4$. Note the 
symmetry $(+ \rightarrow (-)$ and $\xi \rightarrow -\xi$.

and

$$q(t_c) = -\frac{1}{\xi + \tanh(\tau)/4} \left( \xi + \chi + \frac{1}{2} \tanh(\tau) - \frac{1}{4 \cosh^2(\tau)(\xi + \tanh(\tau)/4)} \right), \quad (17)$$

respectively. For $\tau \to \infty$ we have

$$H(t_c) \approx -2^{1/4} e^{\phi_0} (\xi + 1/4) e^{-\tau(\chi + 1/4)}, \quad (18)$$

and

$$q(t_c) \approx -\frac{1}{\xi + 1/4} (\xi + \chi + 1/2). \quad (19)$$

Note that the deceleration parameter is always finite.

3 The scenario

A physical description of our universe requires a large universe at late times. This is achieved
in the region of the moduli space $\xi < -1/4$. Since we are assuming that at small times the
4-form potential is dominant with respect to the axion potential term, the dynamics at the early stages of the evolution is described by the solution above. The APT assumption (c) 
then implies that the early universe must be described by the (+) branch solution of the hyperbola. Indeed, for the (−) branch of the hyperbola the 11-dimensional curvature blows up at early times and approaches zero at large times. The converse is true for the (+) branch.
The APT postulate requires that the evolution starts in a low-energy state, i.e. in a state with small curvature. Clearly, the lower branch does not satisfy this requirement. If we restrict
attention to the region $\xi < -1/4$ the similarity of the $(+)$ and $(-)$ branches to the PRBB and POBB branches of the PRBB scenario is complete. Table 2 summarizes the kinematical and dynamical properties of the plus and minus branch for $\xi < -1/4$:

| Time evolution | $(+)$ branch | $(-)$ branch |
|----------------|--------------|--------------|
| $\dot{a} > 0$, $\dot{a} > 0$ | $\dot{a} > 0$, $\dot{a} < 0$ |
| Hubble parameter | $H > 0$, $H > 0$ | $H > 0$, $H < 0$ |
| Event/Particle horizon | Decreasing/- | $-/increasing$ |
| 11D Curvature | Increasing | Decreasing |
| Initial curvature scale | Arbitrarily small | Arbitrarily large |
| 4D effective coupling $g$ | $\dot{g} > 0$, $g_i = 0$, $g_e = \infty$ | $\dot{g} < 0$, $g_i = \infty$, $g_e = 0$ |
| 4D coupling $g^{(4)}$ | $\dot{g}^{(4)} > 0$, $g_i^{(4)} = 0$ | $\dot{g}^{(4)}$ undefined, $g_e^{(4)} = 0$ |

where $g^{(4)} = e^{\Phi_4}$, $H = \dot{a}/a$ and a dot denotes differentiation with respect to cosmic time $t_c$.

The $(+)$ branch enjoys all the properties of the PRBB branch of the standard PRBB model [4]. So our model leads to a picture of the evolution of the Universe which does indeed describe a PRBB scenario. According to the latter, the (four-dimensional) universe starts in an expanding weak-coupled, low-curvature regime with growing curvature and growing four-dimensional string coupling $g^{(4)}$. The expansion is accelerated and continues until the strongly coupled regime with large curvature is approached. Here non-perturbative effects enter into play and possibly induce a transition to the POBB $(-)$ branch (graceful exit). The expansion is now decelerated and both the Hubble parameter and the four-dimensional coupling constant $g^{(4)}$ vanish at large times.

The consistency of this picture requires that the contribution of the axion to the Hamiltonian (8) remains subdominant during the PRBB phase. The axion potential term can be shown to be monotonically decreasing in the region of the moduli space $\rho > \xi + 1/3$. Therefore, a consistent description of the dynamics constrains the physical solutions to the range $\xi < -17/48 \approx -0.354$ of the $(+)$ branch. This region of the moduli space coincides essentially with the region $\xi < -1/4$ so an asymptotically past trivial patch of spacetime with dominant 4-form, which is initially expanding, is likely to continue its (accelerated) expansion until it reaches the strong-coupling regime. Therefore, we conclude that a patch of spacetime with dominant 4-form, which is initially expanding and asymptotically past trivial, potentially evolves to a (spatially flat) homogeneous, accelerated expanding universe with finite negative deceleration parameter and infinite curvature (PRBB phase).

Up to now we have not discussed the dynamical behavior of the internal space and of the 11th-dimension. The $\xi < -1/4$ region of the $(+)$ branch is characterized by an expanding internal space for $\xi \leq -3/4 - 1/\sqrt{3}$, and a bouncing internal space for $-3/4 - 1/\sqrt{3} < \xi <$

8
−1/4. In both cases the internal space becomes exponentially large when the strongly coupled region is approached. The ratio between the two scale factors is

$$\frac{a}{R_{C_6}} = e^{\alpha_0 - \beta_0} \frac{e^{\tau (\rho - \xi)}}{\sqrt{\cosh(\tau)}}. \quad (20)$$

In the region $\rho > \xi + 1/2$ of the moduli space $(\xi, \rho)$ the scale factor of the internal space grows at a slower pace than the external scale factor. Therefore for $\xi < -7/12$ the (+) branch solution leads to $a/R_6 \gg 1$ at large times. Since $\xi < -7/12 < -17/48$, the demand that the size of the six-dimensional internal space is compactified with respect to external space is consistent with the axion potential remaining subdominant during the PRBB phase. Now let us turn to the 11th dimension. We have

$$\frac{a}{R_{S_1}} = e^{-(\alpha_0 + \phi_0 + 6\beta_0)/2} \frac{e^{2\xi \tau}}{\sqrt{\cosh(\tau)}}. \quad (21)$$

In the region $\xi < 1/4$ the ratio $a/R_{S_1}$ tend to zero at large times. The size of the 11th dimension becomes much larger than the size of the four-dimensional world both for the (expanding) (+) and (−) branches. This implies that a weakly coupled 11-dimensional universe reaches the strong-coupling regime in a state with four large dimensions (where matter exists), one extra-large dimension, and six compactified internal dimensions. From the point of view of a four-dimensional observer, the universe is five-dimensional at the end of the PRBB evolutionary phase. This picture shows some resemblance to the brane world picture of Lukas et al [7], and of Randall and Sundrum [8].

The universe emerges from the graceful exit in a five-dimensional state and starts evolving according to the (−) branch. During the POBB evolution the 11th dimension continues to expand at a faster pace than the four-dimensional world. So we expect one extra-large dimension at POBB late times as well. During the POBB phase the six-dimensional internal space is also expanding faster than the four-dimensional scale factor. However, this should not be disturbing because the NS-NS axion potential term will eventually dominate the R-R 4-form potential. If this occurs, at late POBB times the dynamics is described by the solution [10]

$$\alpha = \tilde{\alpha}_0 + \frac{1}{2} \ln [\cosh (\tau)] - \tilde{\xi} \tau, \quad p_\alpha = 3 \tanh (\tau) - 6 \tilde{\xi},$$

$$\phi = \tilde{\phi}_0 - \frac{1}{2} \ln [\cosh (\tau)] + 3 \tilde{\xi} \tau, \quad p_\phi = \tanh (\tau) - 6 \tilde{\xi}, \quad (22)$$

$$\beta = \tilde{\beta}_0 + \frac{\tilde{p}_\beta}{12} \tau, \quad \sigma = \tilde{\sigma}_0 + \frac{1}{\tilde{p}_\sigma} \tanh (\tau),$$

where

$$1 - 12 \tilde{\xi}^2 + \frac{\tilde{p}_\beta^2}{12} - 2H = 0, \quad (23)$$

$$3 \tilde{\alpha}_0 + \tilde{\phi}_0 - \ln |\tilde{p}_\sigma| = 0. \quad (24)$$
The structure of moduli space of the NS-NS solution is similar to the structure of the moduli space of the R-R solution. The physical points are determined in the plane $(\tilde{\xi}, \tilde{p}_\beta)$ by the two branches of the hyperbola (23) (see figure 3)

$$\tilde{p}_\beta = \pm 2\sqrt{3}\sqrt{12\tilde{\xi}^2 - 1}. \quad (25)$$

The left $(-)$ branch and the right $(+)$ branch are characterized by the same kinematical and dynamical properties of table 1. The dynamical behaviour of the scale factor of the external space is similar to that of the 4-form dominated solution. The $(-)$ branch describes a solution with expanding four-dimensional scale factor for $\tilde{\xi} \leq -1/2$ and bouncing (first contracting then expanding) four-dimensional scale factor for $-1/2 < \tilde{\xi} < -1/2\sqrt{3}$. The $(+)$ branch describes bouncing (first contracting then expanding) and contracting four-dimensional scale factor for $1/2\sqrt{3} < \tilde{\xi} < 1/2$ and $\tilde{\xi} \geq 1/2$, respectively. The sign of $\tilde{p}_\beta$ determines the behaviour of the internal space. Positive values of $\tilde{p}_\beta$ describe solutions with expanding internal dimensions and $\tilde{p}_\beta < 0$ solutions with shrinking internal space. The ratio between the two scale factors is

$$\frac{a}{R_{C_6}} = e^{\tilde{\alpha}_0 - \tilde{\beta}_0} e^{-\tau(\tilde{p}_\beta/12 + \tilde{\xi})} \sqrt{\cosh(\tau)}. \quad (26)$$

When the axion starts dominating the dynamics of the R-R POBB solution the 11-dimensional universe may find itself either in the $(-)$ or in the $(+)$ branch. Accordingly, the four-dimensional world may either: (a) continue its decelerated expansion; (b) contract for a while and then resume its (decelerate) expansion (bouncing solutions); or (c) start an accelerated contracting phase. If we live in the axion-dominated phase, a physical expanding universe at large times requires that the axion-dominated four-dimensional world continues its expansion in the (a) region of the $(−)$ branch. This is achieved by $\tilde{\xi} < -1/2$. From Eq. (26) we find that
the ratio \( a/R_{C_6} \) becomes exponentially large for the \((-)\) branch solution. As a consequence, the expansion of the internal dimensions of the R-R POBB phase is eventually halted. Since the size of the six-dimensional internal space is already compactified to small scales by the R-R PRBB phase the internal space remains compactified at late times.

Now let us turn to the 11th dimension. For the axion-dominated POBB phase the ratio between the size of the 11th dimension and the scale factor of the four-dimensional world is

\[
\frac{a}{R_{S_1}} = e^{-\left(\tilde{\alpha}_0 + \tilde{\phi}_0 + 6\tilde{\beta}_0\right)/2} e^{-\tau(\tilde{\phi}_3/4 + \tilde{\xi})}.
\]

(27)

The four-dimensional scale factor increases at a faster pace than the eleven-dimensional radius for the \((-)\) branch. Eventually, for large times we have \( a/R_{S_1} \gg 1 \). However, when the axion starts dominating the dynamics, the ratio \( a/R_{S_1} \) is very small because of the R-R PRBB phase. Therefore, the universe can be tuned to remain five-dimensional on very long time scales.

### 4 Justifying the PRBB-POBB transition

The M-theory PRBB model which is described in the previous section requires a transition at high-curvature scales from the \((+)\) branch to the \((-)\) branch. This transition, the so-called graceful exit [12], is typical of the PRBB scenario. One of the main unsolved problems of string cosmology is actually understanding the mechanism responsible for the transition from the inflationary PRBB phase with increasing curvature to the deflationary POBB phase with decreasing curvature. Since in the PRBB phase the curvature is increasing monotonically, the graceful exit necessarily involves a high-curvature, strongly coupled, regime where higher derivatives and string loop terms must be taken into account [13]. In the usual PRBB scenario it has been shown that for any choice of the (local) dilaton potential no cosmological solutions that connect smoothly the PRBB and POBB phases exist at classical level [14]. In contrast, quantum effects may induce a transition from the PRBB phase to the POBB phase. A number of quantum string cosmology models have been investigated in the literature [15]. The outcome of these investigations is that the quantum PRBB-POBB transition probability is generally finite and non-zero. In the quantum cosmology context, the transition from the PRBB phase to the POBB phase is described by a scattering of the PRBB wavefunction by an effective potential barrier that mimics the strongly coupled regime of the theory [15]. In the simplest models the PRBB and the POBB phases are identified, in the weakly coupled region of the phase space, by stationary eigenfunctions of the Wheeler-de Witt (WDW) equation with opposite momentum, say, \( \psi^+ \) and \( \psi^- \). The PRBB-POBB transition amplitude is then given by the product \( A = (\psi^+, \psi^-) \) in the Hilbert space.

In our model the transition from the \((+)\) branch to the \((-)\) branch may be explained by a similar mechanism which involves reflection of wavefunctions. Let us define the gauge-
invariant canonical pairs

\[ X = \frac{1}{5} \left[ -6(5\alpha + 6\beta + \phi) + \frac{5p_\alpha + 3p_\beta - 3p_\phi}{\kappa} \arccosh \left( \frac{\kappa}{|Q|} e^{-\frac{(3\alpha - 6\beta - \phi)}{2}} \right) \right], \]

\[ P_X = -\frac{1}{48} (5p_\alpha + 3p_\beta - 3p_\phi), \]

\[ Y = \frac{1}{5} \left[ -12 (\beta + \phi) + \frac{p_\beta - 6p_\phi}{\kappa} \arccosh \left( \frac{\kappa}{|Q|} e^{-\frac{(3\alpha - 6\beta - \phi)}{2}} \right) \right], \]

\[ P_Y = \frac{1}{12} (p_\beta - 6p_\phi), \]

where

\[ \frac{1}{16} (-p_\alpha + p_\beta - p_\phi)^2 + Q^2 e^{3\alpha - 6\beta - \phi} = \kappa^2. \]

The canonical variables above can be completed by the pair \((T, H)\), where

\[ T = \frac{1}{\kappa} \arccosh \left( \frac{\kappa}{|Q|} e^{-\frac{(3\alpha - 6\beta - \phi)}{2}} \right), \]

(30)

to give a complete set of canonical variables. Using this canonical set the gauge-fixed density action reads

\[ S_{\text{eff}} = \int dt \left[ \dot{X} P_X + \dot{Y} P_Y - H_{gf} \right], \]

(31)

where we have fixed the gauge \(T = t - t_0\). The effective Hamiltonian \(H_{gf}\) vanishes on-shell. The Schrödinger equation is

\[ \dot{H}_{gf} \Psi(X,Y; t) = i \frac{\partial}{\partial t} \Psi(X,Y; t). \]

(32)

Since the effective Hamiltonian of the system is identically zero the wave functions do not depend on \(t\). An orthonormal basis in the Hilbert space is given by the set of eigenfunctions of the gauge invariant observables,

\[ \dot{P}_X = -i \frac{\partial}{\partial X}, \quad \dot{P}_Y = -i \frac{\partial}{\partial Y}, \]

(33)

with eigenvalues \(x\) and \(y\), respectively,

\[ \Psi(X,Y) = \frac{1}{2\pi} e^{i(x X + y Y)}. \]

(34)

As was expected, in the low-energy limit the wavefunctions are free plane waves in the two-dimensional \((X,Y)\) space with respect to the Hilbert product

\[ (\psi_1, \psi_2) = \int dX dY \, \psi_1^* \cdot \psi_2. \]

(35)
Quantum high-curvature effects generate a potential $V(X,Y)$. So we expect that when quantum effects are properly taken into account scattering and reflection of wavefunctions occurs in the $(X,Y)$ space. Classically, the relation between the moduli parameters and the gauge-invariant variables is

$$
\alpha_0 = -\frac{5}{48}X - \frac{1}{4} \ln |Q|, \quad \beta_0 = -\frac{1}{16}X + \frac{1}{12}Y + \frac{1}{4} \ln |Q|, \quad \phi_0 = \frac{1}{16}X - \frac{1}{2}Y - \frac{1}{4} \ln |Q|,
$$

$$
\xi = \frac{1}{\kappa}P_X, \quad \rho = \frac{1}{5\kappa}(3P_X + P_Y), \quad \chi = -\frac{3}{5\kappa}(P_X + 2P_Y),
$$

where

$$
\frac{12}{5}(P_Y^2 - 4P_X^2) = \kappa^2 - 2H.
$$

A reflection in the $(X,Y)$ space with respect to the $X$ plane ($Y \to -Y$) is equivalent, in the moduli space $(\xi,\rho)$, to the transformation $\xi \to \xi$, $\rho \to 6\xi/5 - \rho$, i.e., to a change of branch in the moduli space. Therefore, high-curvature quantum effects may induce a transition from the PRBB branch to the POBB branch, in complete analogy with the standard PRBB scenario.

5 Concluding remarks

In this paper we have discussed a cosmological scenario which emerges from (low-energy) M-theory. We have found that the APT postulate and the confinement of the non-gravitational fields on a 3-brane propagating in the 11-dimensional spacetime lead naturally to a PRBB model. In the latter the four-dimensional universe undergoes first a phase of accelerated expansion (PRBB) and then a phase of decelerated expansion (POBB) connected by a state with large curvature.

Our model is clearly too simple to have any pretense of describing the real world. A (partial) list of potential problems and drawbacks contains the following issues: (a) there is no stabilization of the extra-dimensions; (b) three-dimensional spatial curvature has not been considered; (c) when the full phase space of the model (4-form + axion) is considered the dynamics is (probably) non-integrable so at a given evolutionary stage chaotic behaviour may appear; (d) chaotic behaviour certainly appears when all supergravity form fields are excited [16], so the overall dynamics may be qualitatively different; (e) fine-tuning issues may appear when trying to fit the observed cosmological parameters to the model; (f) as in the standard PRBB scenario, we do not know of any convincing mechanism that could induce the graceful exit, etc.

Though the list above could include many other potential problems, we still believe that some interesting information can be extracted from our model. Firstly, we have learned that the PRBB scenario fits naturally in a model which is not (at least explicitly) invariant under scale factor duality. This provides some evidence about the generality of the cosmological
PRBB picture in models derived from M-theory and/or string theory. Scale factor duality may be an accidental symmetry of the simplest dilaton gravity systems that does not exist in more complex models\(^3\), still PRBB seems to be quite a generic feature. Secondly, R-R forms are recognized to be essential ingredients in the construction of a viable model of the observed universe. Finally, and most importantly, we have seen that PRBB and brane-world cosmologies may be compatible in principle. In this context, it would be worth trying to implement and reinterpret the PRBB scenario in the LOW [7] and/or RS [8] brane world models.

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