Anomaly Cancellations and String Symmetries in the Effective Field Theory

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Abstract
This contribution briefly describes some developments of the use of string symmetries and anomaly cancellation mechanisms to include string loop corrections in the construction of the low-energy effective supergravity of superstrings.

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1 Effective field theory

The purpose of the effective low-energy field theory is to describe the dynamics of massless string modes in the low-energy domain where string massive states have only virtual effects. This energy range can be characterized by an ultraviolet physical cutoff $M_{uv}$, which is smaller than the lightest massive mode of the string theory. The effective field theory is specified by a local lagrangian density, a Wilson effective lagrangian $L_{\text{eff}}$. In string perturbation theory which is not expected to lead to spontaneous breaking of supersymmetry \(^1\), the effective low-energy field theory of a superstring will be a $N = 1$ supergravity.

Consider a certain amplitude $A(p_1, p_2, \ldots)$ for a physical process involving only massless external string modes, computed in string perturbation theory:

$$A(p_1, p_2, \ldots) = \sum_{L \geq 0} A^{(L)}(p_1, p_2, \ldots),$$

where $L$ is the string loop order. Its arguments are external momenta, helicity or internal quantum numbers attached to the external states and also parameters which must be introduced to define string perturbation theory. For instance, the presence of massless modes requires the introduction of an infrared cutoff $\Lambda$ to regulate loops. This arbitrary scale parameter may be identified with the ultraviolet cutoff $M_{uv}$ of the effective field theory, but this is not necessary. In the limit where all energies and momenta in (1) are small compared with $M_{uv}$, it is expected that (1) is reproduced by the same amplitude computed perturbatively in the quantum field theory defined by the effective lagrangian $L_{\text{eff}}$. In correspondance with expansion (1), this effective lagrangian will have a formal expansion in string-loop order:

$$L_{\text{eff}} = \sum_{k \geq 0} L_{\text{eff}}^{(k)}.$$

At string tree-level in (1) ($L = 0$) and in the low-energy limit, the amplitude $A^{(0)}(p_1, p_2, \ldots)$ can be obtained using the tree-level effective lagrangian $L_{\text{eff}}^{(0)}$ in which the effect of massive string modes is hidden in non-renormalisable interactions. At this order, $A^{(0)}$ and $L_{\text{eff}}^{(0)}$ do not depend on $\Lambda$ and the amplitude $A^{(0)}(p_1, p_2, \ldots)$ is the sum of all tree diagrams obtained with $L_{\text{eff}}^{(0)}$. The knowledge of string tree amplitudes allows then in principle to construct $L_{\text{eff}}^{(0)}$.

In the low-energy limit, the string one-loop contribution to (1), $A^{(1)}(p_1, p_2, \ldots)$, corresponds in the effective field theory to two classes of contributions. Firstly, the sum of the relevant one-loop diagrams obtained using $L_{\text{eff}}^{(0)}$ only. Secondly, the new interactions described by $L_{\text{eff}}^{(1)}$, which are formally already ”string one-loop”, lead to a number of tree diagrams generated by $L_{\text{eff}}^{(0)} + L_{\text{eff}}^{(1)}$ and containing one vertex present in $L_{\text{eff}}^{(1)}$. Since $A^{(1)}$ will depend on the infrared cutoff $\Lambda$, the effective lagrangian will also depend on $\Lambda$ starting with the one-loop term $L_{\text{eff}}^{(1)}$.

\(^1\) For a discussion of the status of supersymmetry breaking in superstrings, see the contribution by D. Lüst [4].
In general, a Feynman diagram of the effective lagrangian (2) with $\ell$ loops will have a "string-loop-order" given by adding to $\ell$ the orders of all vertices, as defined by the expansion (2). Summing all diagrams up to "string-loop-order" $L_{\text{max}}$ will provide the low-energy limit of the amplitude (1) computed up to $L_{\text{max}}$ string loops.

2 String gauge symmetries and anomalies

An important help in the construction of the effective field theory $\mathcal{L}_{\text{eff}}$ is provided by string symmetries which leave a physical amplitude like (1) invariant at each order of string perturbation theory. These symmetries strongly constrain the form of the effective lagrangian, even if they are not in general symmetries of $\mathcal{L}_{\text{eff}}$, which is not a physical object. At string tree-level, the invariance of $\mathcal{A}^{(0)}$ implies the invariance of $\mathcal{L}^{(0)}_{\text{eff}}$. In general however, this symmetry of $\mathcal{L}^{(0)}_{\text{eff}}$ can be anomalous: some one-loop diagrams which contribute to the effective description of $\mathcal{A}^{(1)}$ do not respect the symmetry. Then, the invariance of $\mathcal{A}^{(1)}$ imposes that the effective contributions generated by $\mathcal{L}^{(1)}_{\text{eff}}$ cancel the one-loop anomaly and restore the string symmetry. Since the knowledge of $\mathcal{L}^{(0)}_{\text{eff}}$ is sufficient to compute the one-loop anomalous diagrams, the requirement of anomaly cancellation gives a strong constraint on the form of $\mathcal{L}^{(1)}_{\text{eff}}$. In some cases, the anomaly-cancellation condition is strong enough to determine completely the one-loop terms $\mathcal{L}^{(1)}_{\text{eff}}$. This procedure can be in principle pursued order by order, except if the existence of non-renormalisation theorems (similar to the Adler-Bardeen theorem) terminates the argument at the one-loop order.

Green and Schwarz [2] found the first example of string symmetries realized in this "anomaly-cancellation mode" in ten-dimensional (heterotic or type I) superstrings, which possess space-time [the Lorentz group $SO(1,9)$] and gauge [$E_8 \times E_8$ or $SO(32)$] symmetries. Both symmetries are anomalous in the tree-level effective lagrangian and their restoration requires specific contributions in $\mathcal{L}^{(1)}_{\text{eff}}$. The argument can be summarized as follows, considering for simplicity gauge symmetries only.

1) The theory contains massless fermions, described by Majorana-Weyl spinors, which couple chirally to gauge fields. The effective tree-level lagrangian generates then chiral gauge anomalies through one-loop anomalous diagrams with six external gauge fields. The formal expression of the anomaly factorises for gauge groups $E_8 \times E_8$ and $SO(32)$, a necessary requirement to be able to cancel it.

2) The theory also contains an antisymmetric tensor field $b_{\mu\nu} = -b_{\nu\mu}$. In the tree-level effective lagrangian $\mathcal{L}^{(0)}_{\text{eff}}$, this field appears through its gauge invariant curl

$$H_{\mu\nu\rho} = \partial_{[\mu} b_{\nu\rho]} - \frac{\kappa}{\sqrt{2}} \omega_{\mu\nu\rho},$$

involving the gauge Chern-Simons form $\omega_{\mu\nu\rho}$ suitably normalised. The tree-level lagrangian contains a term proportional to $H_{\mu\nu\rho} H^{\mu\nu\rho}$, and then an interaction of the form

$$\partial^\rho b^\nu \omega_{\mu\nu\rho},$$

which couples $b_{\mu\nu}$ to two gauge fields.

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3) The one-loop contribution \( \mathcal{L}_{\text{eff}}^{(1)} \) will contain precisely the terms necessary to cancel the gauge anomaly: a coupling of \( b_{\mu\nu} \) with four gauge fields, and a contact interaction involving six gauge fields. These terms in \( \mathcal{L}_{\text{eff}}^{(1)} \) have to be gauge variant, and their variation is specified by the chiral anomaly computed using the tree-level lagrangian. The sum \( \mathcal{L}_{\text{eff}}^{(0)} + \mathcal{L}_{\text{eff}}^{(1)} \) is then gauge non invariant.

In four dimensions, only \( U(1) \) gauge symmetries can be realised in the anomaly-cancellation mode: only abelian (or mixed abelian–nonabelian) chiral anomalies factorise. Gauge anomaly cancellation can then only appear in string vacua with gauge groups containing at least a \( U(1) \) factor. The cancellation mechanism, described by Dine, Seiberg and Witten [3], is closely analogous to ten-dimensional case mentioned above. The same coupling (4) is present at tree-level. To cancel the gauge anomaly, \( \mathcal{L}_{\text{eff}}^{(1)} \) contains in particular a gauge-variant term of the form

\[
b_{\mu\nu} \partial^\mu A^\nu
\]

\((A_\mu \text{ is the abelian gauge field})\), and the chiral anomaly generated by the triangle diagram is cancelled using the exchange of the antisymmetric tensor. The one-loop contribution \( \mathcal{L}_{\text{eff}}^{(1)} \) plays the rôle of a Fayet-Iliopoulos term which gives a mass to the abelian vector multiplet removing the anomalous \( U(1) \) symmetry from the low-energy symmetry content of the model.

The global supersymmetrization of this anomaly cancellation mechanism is very simply described using a linear multiplet [4, 5], which contains the antisymmetric tensor \( b_{\mu\nu} \), a real scalar and a Majorana spinor. The real linear superfield \( L \) is defined by the supersymmetric constraints \( \mathcal{D} \mathcal{D} L = \overline{\mathcal{D}} \overline{\mathcal{D}} L = 0 \). A supersymmetric lagrangian generalizing the expressions (3) and (4) is

\[
\int d^4 \theta F(L - \Omega),
\]

omitting the explicit dependence on chiral superfields and superpotential terms. \( \Omega \) is the supersymmetric generalisation of the Chern-Simons form and the superfield \( L - \Omega \) contains [3]. Gauge invariance requires

\[
\delta L = \delta \Omega.
\]

In general, \( \Omega \) is a fixed linear combination of the Chern-Simons forms of all factors of the gauge group, which is supposed to contain an anomalous \( U(1) \) factor [with superfield \( \tilde{V} \)]. The chiral anomaly can be represented by the non-local expression

\[
c \int d^2 \theta WW \mathcal{P}_L \tilde{V} + h.c.,
\]

where \( \mathcal{P}_L \) is the chiral (non-local) projector and \( W \) is the chiral gauge curvature superfield. Since \( \delta \tilde{V} = \Lambda + \overline{\Lambda} \), its gauge variation is

\[
c \int d^2 \theta WW \Lambda + h.c. = - \frac{c}{4} \int d^4 \theta (\Lambda + \overline{\Lambda}) \Omega,
\]
using simple identities. The one-loop contribution to the effective lagrangian which cancels this anomaly is then clearly of the form

$$-\frac{c}{4} \int d^4 \theta \left( L - \Omega \right) \bar{V},$$

(8)

the introduction of $L$ being necessary to avoid unwanted non-abelian anomalies. This superfield expression contains the required coupling proportional to $b_{\mu \nu} \partial^\mu \bar{V}^\nu$. The one-loop corrected effective lagrangian is then

$$\mathcal{L}_{eff} = \int d^4 \theta \left[ F(L - \Omega) - \frac{c}{4}(L - \Omega) \bar{V} \right],$$

(9)

omitting superpotential terms. The supergravity generalisation of this globally supersymmetric lagrangian is, in the superconformal formalism,

$$\mathcal{L}_{eff} = \left[ S_0 \bar{S}_0 F \left( \frac{L - \Omega}{S_0 \bar{S}_0} \right) - \frac{c}{4}(L - \Omega) \bar{V} \right]_D,$$

(10)

where $S_0$ is the chiral compensating multiplet (this is ”old minimal supergravity”) and $[..]_D$ denotes the real vector density formula of superconformal tensor calculus.

It is well known that the antisymmetric tensor can be transformed into a pseudoscalar with a duality transformation. Its supersymmetric version, which will be discussed in the last section, transforms the linear multiplet into a chiral one.

3 Kähler symmetry

The coupling of a chiral matter–super-Yang-Mills system to supergravity is naturally invariant under Kähler transformations. In the superconformal approach, the lagrangian density is [6]

$$\mathcal{L} = \left[ S_0 \bar{S}_0 e^{-\mathcal{K}/3} \right]_D + [S^3 \omega + fW]_F,$$

(11)

where the Kähler potential $\mathcal{K}(\Sigma, \Sigma e^V)$ is a real function of the chiral multiplets $\Sigma$, and the introduction of the gauge vector multiplet $V$ ensures gauge invariance of the theory. $W$ is the gauge curvature chiral multiplet and the function $f$ which appears in the chiral density $[..]_F$ is a holomorphic funcion of $\Sigma$. Gauge transformations, with chiral parameter $\Lambda$ act according to

$$\Sigma \rightarrow e^{i\Lambda} \Sigma, \quad \Sigma \rightarrow \Sigma e^{-i\Lambda}, \quad e^V \rightarrow e^{i\pi} e^V e^{-i\Lambda},$$

(12)

so that $\Sigma e^V \Sigma$ is gauge invariant.

The theory (11) is invariant under the Kähler transformation

$$\begin{align*}
\omega & \rightarrow e^{-\varphi(\Sigma)} \omega, \\
S_0 & \rightarrow e^{\varphi(\Sigma)/3} S_0, \\
\mathcal{K} & \rightarrow \mathcal{K} + \varphi(\Sigma) + \bar{\varphi}(\Sigma)
\end{align*}$$

(13)
\( V, W \) and \( f \) being unaffected. This Kähler transformation is a formal symmetry which indicates that the lagrangian (11) only depends on the function

\[ \mathcal{G} = \mathcal{K} + \log \omega \]

[choose \( \varphi = \log \omega \) in (13)]. Notice that the quantity \( S_0 e^{-K/3} \), which appears in the lagrangian (13), is analogous to the argument \( \Sigma e^V \) of \( \mathcal{K} \) itself. Also the Kähler invariant combination \( S_0 \Sigma_0 e^{-K/3} \) is similar to the gauge invariant \( \Sigma e^V \Sigma \). The function \( \mathcal{K} \) is then a Kähler connection in the same way as \( V \) is the gauge connection. \( \mathcal{K} \) is a composite multiplet which enters algebraically in lagrangian (13). Since the theory (13) is both Kähler and gauge invariant, the composite Kähler connection will appear in fermion covariant derivatives, together with gauge potentials (in \( V \)) and also sigma-model covariantization of kinetic terms.

It is then natural to consider potential anomalies of Kähler symmetry, or more generally of sigma-model local symmetries [7, 8]. The supersymmetric formalism sketched in the previous section translates directly to these cases. For instance, a Kähler anomaly would correspond to the chiral \( F \)-density

\[ c_K [WW \mathcal{P}_L \mathcal{K}]_F \]  

\( (c_K \) is a numerical coefficient), replacing \( \tilde{V} \) in (7) by the Kähler connection \( \mathcal{K} \). And the effective lagrangian for the theory with the linear multiplet including the anomaly-cancelling Green-Schwarz one-loop term is [8, 9]:

\[ \mathcal{L}_{eff} = \mathcal{L}_{eff}^{(0)} + \mathcal{L}_{eff}^{(1)}, \]

\[ \mathcal{L}_{eff}^{(0)} = \frac{1}{4} c_K [(L - \Omega) \mathcal{K}]_D, \]

\[ \mathcal{L}_{eff}^{(1)} = -\frac{i}{4 c_K} [(L - \Omega) \mathcal{K}]_D, \]

omitting again the superpotential. The tree-level lagrangian \( \mathcal{L}_{eff}^{(0)} \) is written in a Kähler invariant form: the variable \( X \) is invariant and the dots denote a possible dependence on other invariant functions of the chiral multiplets.

Kähler symmetry is a property of supergravity couplings. In the superconformal approach, it is directly related to the chiral internal \( U(1) \) part of the conformal superalgebra. Its relation with superstrings has to do with the fact that certain string symmetries act on the massless fields of the effective theory with Kähler transformations. An example is target-space duality in (2, 2) orbifolds or Calabi-Yau strings. Considering a idealized model with a unique (1, 1) modulus \( T \), the Kähler connection for this modulus would be

\[ \mathcal{K}_T = -3 \log(T + \mathcal{T}). \]

Target-space duality acts on \( T \) according to

\[ T \rightarrow \frac{aT - ib}{icT + d}, \quad ad - bc = 1. \]
On the connection,
\[ \mathcal{K}_T \rightarrow \mathcal{K}_T + 3 \log |icT + d|^2, \]
which is a particular Kähler transformation. A Kähler anomaly would then also be a target-space duality anomaly, and the fact that target-space duality is a quantum string symmetry implies that the effective lagrangian should include anomaly-cancelling terms of the form introduced in (10).

The component expansion of the loop-corrected effective lagrangian (16) contains, besides the anomaly-cancelling terms, corrections to the gauge kinetic terms which depend on the Kähler connection. A string one-loop computation of gauge kinetic terms [i.e. a string amplitude (1) with two external gauge fields and an arbitrary number of moduli] is then able to directly establish the existence of the quantum correction \( \mathcal{L}_\text{eff}^{(1)} \). This calculation of threshold corrections, performed in the context of (2, 2) symmetric orbifolds (10) (see also (11, 12)), has in fact been at the origin of the use of Kähler anomaly cancellation for loop-corrected effective supergravities, as developed in ref. [8]. It should however be mentioned that the inclusion of the complete set of (untwisted) moduli present in generic (2, 2) symmetric orbifold, as in (10) and [8], leads to a situation more complicated than the simple example considered here.

4 Axion–antisymmetric tensor duality

We have up to now discussed anomaly cancellation using the linear multiplet. This is a natural approach since the Chern-Simons form is the crucial object in this mechanism and gauge invariance of the tree-level effective lagrangian associates the Chern-Simons form with the antisymmetric tensor, as in (3). As already mentioned, duality can always be used to transform the antisymmetric tensor into a pseudoscalar or the linear superfield into a chiral one. Suppose for instance that we want to apply duality to lagrangian (3). This theory is equivalent with

\[ \int d^4 \theta \left[ F(U) - (S + \overline{S})(U + \Omega) \right], \]

where \( U \) is a real vector superfield and \( S \) is chiral. The equation of motion for \( S \) indicates that \( U + \Omega \) is linear, and hence (17) and (3) are equivalent. Or solving the equation of motion for \( U \),

\[ \frac{\partial}{\partial U} F(U) = S + \overline{S}, \]

allows to express \( U \) as a function of \( S + \overline{S} \) and leads to the equivalent theory

\[ \int d^4 \theta \left[ F(U) - (S + \overline{S})U \right]_{U=U(S+\overline{S})} \]

\[ + \frac{i}{4} \left( \int d^2 \theta SWW + \text{h.c.} \right) \]

\[ \equiv \int d^4 \theta G(S + \overline{S}) + \frac{1}{4} \left( \int d^2 \theta SWW + \text{h.c.} \right). \]

\[ \text{See also (13), (14) and (16).} \]
The comparison of (5) and (9) shows immediately that the addition of the abelian
gauge anomaly cancelling term in the one-loop effective lagrangian is equivalent to the
substitution
\[ G(S + \mathcal{S}) \rightarrow G(S + \mathcal{S} + \frac{c}{4}\tilde{V}) \] (20)
in eq. (19), as observed in ref. [3]. The case of Kähler anomaly is similar. The chiral
theory dual to \( \mathcal{L}_{eff}^{(0)} \) in eq. (14) is
\[ [UF(X_U, \ldots) - (S + \mathcal{S})U]_{D} + \frac{1}{4}[SWW]_{D}, \] (21)
where \( U \) is the function of \( S + \mathcal{S} \) such that
\[ \frac{\partial}{\partial U} UF(X_U, \ldots) = S + \mathcal{S}. \]
The loop correction \( \mathcal{L}_{eff}^{(1)} \) in (16) corresponds then to the substitution
\[ S + \mathcal{S} \rightarrow S + \mathcal{S} + \frac{1}{4}c_K \mathcal{K}. \] (22)

Notice however that the duality transformation does not respect the string loop
expansion of the effective Wilson lagrangian. The expansion (2), which has been ap-
plied in the linear multiplet formalism, is resummed by the duality transformation of
the linear multiplet \( L \) into the chiral superfield \( S \). This observation suggests that the
formal equivalence of \( L \) and \( S \) does not necessarily mean that the choice of formulating
the effective theory of superstrings either with \( S \) or with \( L \) is indifferent. Expansion
(2) could apply to one version of the theory only, which would then allow for an easier
and more natural field theory interpretation of physical quantities computed in string
perturbation theory.

The calculations performed in (2, 2) string models suggest that the fields contained
in the linear multiplet are in closer relationship to string physical parameters. More
precisely, a study of the \( E_8 \) sector of some (2, 2) orbifolds [13, 3] shows that the scalar
component \( C \) of the linear multiplet is directly related to the renormalised, physical
\( E_8 \) gauge coupling constant \( g_R \):
\[ \frac{1}{\kappa^2(C)} - \frac{C(E_8)}{16\pi^2} = \frac{1}{g_R^2} \] (23)
\[ [C(E_8) = 30 \text{ is the } E_8 \text{ quadratic Casimir}]. \]
On the other hand, the scalar component \( s \) of the chiral multiplet \( S \) is related to the bare, unphysical gauge coupling constant appearing in the Wilson effective lagrangian \( \mathcal{L}_{eff} \), in the term
\[ -\frac{1}{4} \frac{1}{g_W^2} F_{\mu\nu}^{A} F^{A}_{\mu\nu}. \]
Then,
\[ \frac{1}{2} \langle s + \mathcal{S} \rangle = \frac{1}{g_W^2} \] (24)
Duality can then be viewed as a transformation from bare to physical quantities, or as a renormalisation-group transformation from some unified string coupling to a low-energy running coupling.

More realistic theories need however to be considered in order to decide of the choice of the most appropriate set of low-energy fields, which would provide the most natural interpretation of string perturbative calculations. This is also needed to decide whether the linear multiplet is of special interest in the discussion of superstring effective supergravities.

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