Some elements of charge kinetics for particles and splinters in reference to the process of nuclear disintegration

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Abstract. This work is devoted to solution of some problems connected with kinetic equations of reproduction for charged particles and charged splinters of the fission inside of nuclear toroidal electromagnetic generator (nuclegen). There are discussed questions of solutions behaviour for charged equations. Special attention is paid to charge kinetics of the nuclegen under the influence of small random disturbances and stimulated by this influence to analysis of stochastic motion properties for the ionite gas.

1. Introduction
We consider a series of questions connected with composition and solution of kinetic equations for chain nuclear fission products into active zone of nuclear toroidal electric generator [1], [2]. Of course, by spelling of kinetic equations for the reproduction of charged particles and charged splinters of fission this consideration would be accompanied by great share of the convention. The main aim, which here is placed, consists in the description of general regularity of reproduction process and not at all the counting of exact charges quantity up to last electron and proton.

It is important to note, in our view, that on dynamics of growth of charged particles $Q$ number the governing action the growth of number prompt and delayed neutrons $N$ into active zone toroid will be exercising so far as the each new fission of nucleuses and to the formation of another charged particles and charged splinters.

2. Charge kinetics of a nuclear electric generator
Scaled demands at wide application of electric energy dictate the necessity to produce nuclear energy by means of fast-neutron electric generator. These nuclear devices are the most favourable due to their high values of the reproduction coefficient.

However, the use of fast-neutron electric generators challenges as there are several demands to maintain the nuclear kinetic process at a fitting level. Above all, it is necessary to keep high average energy of neutrons at the level of several hundred kilovolts [3], [4]. Here the energy spectrum of neutrons is defined by the properties of combustible fuel, raw materials and
construction materials in relation to the inelastic scattering of the neutrons ($^{238}$U has a big cross-section of inelastic scattering). In addition, to achieve criticality the operation of fast-neutron nuclear devices needs a high concentration of a combustible fuel in relatively small volumes. The latter is caused by low values of the fission cross-section in the area of high energies.

Therefore, our further reasoning is based on the admission that the nuclear device (nuclear electric generator or nuclegen) has a breeder basis. For simplicity and convenience all charged splinters and particles of both signs (ions, electrons, protons etc.) are called ionites, and their aggregate in quantity $Q$ is ionite gas (ionite gas cord). As in the case of neutrons, the balance of ionites is characterised by the ionite reproduction coefficient $f$. The reproduction coefficient $f$ is defined as the ratio of the number of ionites in any generation to the number of ionites in the previous generation.

The average lifetime of one generation of ionites $r$ is supposed to be proportional to the average lifetime of one generation of neutrons $l$:

$$r = \lambda + F(\varepsilon, \nu, \delta, \mu),$$

in which $\lambda > 0$ is a proportionality coefficient, $(\varepsilon, \nu, \delta, \mu)$ is a set of parameters characterizing the electromagnetic influence, air possible residual drag of the medium, dynamic and construction features respectively, all influencing the value $r$.

For all intents and purposes the dependence (1) may possess not a constant but a rather complex function form with all values included being dependent on time and spatial variables in some way. In work [2] to analyse the simplest charge kinetics value $\lambda = 1$ is fixed so that value $r$ coincides with value $l$ and is entirely defined by it. Apparently, this does not hold true what is evidenced by the following the correlation (1).

Unlike the neutron losses (capture without fission, leakage etc.), the losses of ionites, that control their lifetime, are determined by their neutralization degree (i.e. missed charges capture). The neutralization degree depends on the general field $(H, E)$, particles velocity etc. If $f = 1$, nuclear fission with the formation of ionites and neutrons occurs steadily at constant reaction rate ($k = 1$, $k$ is the coefficient of neutrons reproduction); its active zone is critical with a self-sustained chain reaction. This case implies $Q = \text{const}$, elements of inner electromagnetic field $(H_*, E_*)$ are also close to constant values.

Ignoring delayed ionites the density change of ionite gas $q$ in active zone within one generation equals to $fq - q = (f - 1)q$. If $r$ is an average lifetime of one ionite generation, it means that per time unit the density of charged lively particles $q$ changes in $(f - 1)/r$ times, i.e.

$$\frac{dq}{dt} = \frac{(f - 1)q}{r},$$

where $q(t) = q_0 \exp(t/T_*)$, with $q_0$ is initial ionite density, $T_* = r/(f - 1)$ is charge period of nuclear electric generator.

3. Linear theory of charge kinetics

As mentioned above delayed neutrons play a crucial role and have a practical application in the controlled process of nuclear chain reaction. The fact that delayed neutrons along with other $\beta^-$ descendants contribute into ionite gas formation is obvious.

Let $\gamma$ is delayed ionite part (charged $\beta^-$ descendants), $(1 - \gamma)$ is instant ionite part. The reproduction coefficient $f$ can be noted as a coefficient consisting of the two parts: $f = f_0 + f_*$, where $f_0 = (1 - \gamma)f$, $f_* = \gamma f$. After $\beta^-$-disintegration decay of splinters the delayed ionites become instant. This means that their effective lifetime $r_\ast$ equals the sum: $r_\ast = r_\beta + r_0$, where $r_\beta$ is delay time ($\beta^-$-disintegration time), $r_0$ is instant ionite life time. Hence, average effective lifetime of $r$ ionite generation equals

$$r = \gamma r_\ast + (1 - \gamma) r_0 \approx r_0 + \gamma r_\ast.$$
Since $\gamma r, \gg r_0$, the charge period of nuclear electric generator $T_* = r/(f - 1)$ is determined by the average time of ionite delay.

The simplest linear theory of charge kinetics needs more thorough consideration. The density of ionite gas $q$ is admitted to be the blend of densities of charged particles and fission splinters of the two types mentioned above, i.e. $q = q_m + q_z$. Here $q_m$ is instant ionite density, $q_z$ is delayed ionite density. Then instead of the correlation (2) ignoring the presence of delayed ionites the system of two kinetic equations is possible to be written. These equations describe how the dynamics of ionite gas density (concentration) changes in an nuclear electric generator:

$$\frac{dq_m}{dt} = \frac{[(1 - \gamma) f - 1]}{r} q_m + \frac{q_z}{r_*}, \quad (\ast) \quad \frac{dq_z}{dt} = \gamma f q_m - \frac{q_z}{r_*} \quad (\ast\ast)$$

with initial conditions

$$q_m \big|_{t=0} = q_{m0}, \quad \frac{dq_m}{dt} \big|_{t=0} = q_m^{'0}, \quad q_z \big|_{t=0} = q_{z0}, \quad \frac{dq_z}{dt} \big|_{t=0} = q_z^{'0}. \quad (4)$$

The right parts of equations (3) should be commented. In equation (3$\ast$) coefficient $q_m$ characterizes the density change of instant ionite number in lifetime of one generation. The second summand characterizes the growth of value $q_m$ at the cost of $q_z$ by the transition of the latter into the category of instant ionites. Equation (3$\ast\ast$) describes the density change of delayed ionites. In this equation the first summand shows the increase of the concentration of delayed ionites at the cost of formation instant ionites; the second one, on the contrary, shows the decrease of the concentration of the amount of delayed ionites within their lifetime (i.e. within the time of $\beta^-$-decay). As well seen in the system (3) the density (the concentration of the amount) of delayed ionites $q_z$ creates the feedback in relation to the density of instant ionites $q_m$.

Following some simple transformations, the system of equations (3) is reduced to a homogenous differential equation of the second order with constant coefficients. The equation describes how to regulate kinetics peculiarities of a nuclear electric generator: $\ddot{x} + \delta_1 \dot{x} - \delta_2 x = 0$, in which $x = q_m$ (or $x = q_z$),

$$\delta_1 = \frac{r + r_* - (1 - \gamma) f r_*}{rr_*}, \quad \delta_2 = \frac{f - 1}{rr_*}. \quad (5)$$

The characteristic equation has two real roots of a clearly determined sign ($f \geq 1$):

$$\lambda_1 = -\frac{\delta_1 + (\delta_1^2 + 4\delta_2)^{1/2}}{2} \geq 0, \quad \lambda_2 = -\frac{\delta_1 - (\delta_1^2 + 4\delta_2)^{1/2}}{2} < 0.$$

Hence, considering the starting conditions (4) the kinetics equations have the following solutions as functions of time:

$$q_m(t) = \left(\frac{q_m^{'0} - \lambda_2 q_m^0}{\lambda_1 - \lambda_2}\right) e^{\lambda_1 t} + \left(\frac{\lambda_1 q_m^0 - q_m^{'0}}{\lambda_1 - \lambda_2}\right) e^{\lambda_2 t},$$

$$q_z(t) = \left(\frac{q_z^{'0} - \lambda_2 q_z^0}{\lambda_1 - \lambda_2}\right) e^{\lambda_1 t} + \left(\frac{\lambda_1 q_z^0 - q_z^{'0}}{\lambda_1 - \lambda_2}\right) e^{\lambda_2 t}. \quad (6)$$

In solutions (5) for $q_m$ and $q_z$, the second summand in the right part quickly approaches to zero. Disregarding it, growth dynamics of density (of the number) of ionites $q_m$ and $q_z$ is supposedly defined by the first summand in the solutions (5). As mentioned above, delayed ionites slow the transient process significantly, and the charge period $T_* = r/(f - 1)$ is almost entirely defined
by the average time of delay \( r_s \). This feature is also notable in the correlations (5). Indeed, the following approximations:

\[ r_s \sim r, \quad \delta_1 \sim \frac{2 - f}{r}, \quad \delta_2 \sim \frac{f - 1}{r^2}, \]

prove the evaluations: \( q_m(t), q_z(t) \sim e^{\lambda_1 t} \sim e^{t/T_s} \).

4. Asymptotic stability and stochastic instability

Unperturbed deterministic kinetic system (3) is considered here:

\[ \dot{x} = Ax, \quad x = \begin{pmatrix} q_m \\ q_z \end{pmatrix}, \quad x(0) = x_0 \quad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad (7) \]

where

\[ a_{11} = (1 - \gamma) \frac{f - 1}{r}, \quad a_{21} = \gamma f \frac{r}{r}, \quad a_{12} = a_{22} = \frac{1}{r_s}, \]

which at the origin of coordinates (at zero point identifiable with a two-dimensional zero) has the stability position: \( 0 = A0 \).

The stability position (zero point) is stable (Lyapunov stable) if for any neighborhood \( W \) : \( 0 \in W \) there is a neighborhood \( W_0 \) for which equation solutions are (7): \( x(t) \in W, t \geq 0 \). If, moreover, \( x(t) \to 0 \) at \( t \to \infty \), then the stability position is called asymptotically stable.

Then it is assumed that the system (7) is perturbed by small white noise

\[ \dot{X} = AX + \varepsilon \dot{w}, \quad X(0) = x_0, \quad (8) \]

where \( w(t) \) is a two-dimensional Wiener process, \( \varepsilon > 0 \) is a small numerical parameter. The problem of trajectories exit of process \( X(t) \) from limited region \( D \) with stability position is of great interest. The appearance of a small parameter in system (8) can by explained with an assumption that the noise intensity is low compared to deterministic factors affecting the system dynamics.

If matrix \( A \) is stable (Hurwitz matrix), i.e. its own numbers \( \lambda_1 \) and \( \lambda_2 \) have negative real parts. For this condition it is necessary and satisfactory that coefficients \( \delta_1, \delta_2 \) of characteristic equation \( \lambda^2 + \delta_1 \lambda - \delta_2 = 0 \) fulfilled the following inequalities (see (5)): \( \delta_1 > 0, \delta_2 < 0 \). It is obvious that the inequalities are held if \( f < 1 \). As matrix \( A \) is considered as stable, the stability position of the unperturbed system (origin of coordinates) is asymptotically stable:

\[ |x(t)| = |e^{At} \cdot x_0| \leq |e^{At}| \cdot |x_0| \to 0 \quad \text{when} \quad t \to \infty. \]

Introduce random value \( \tau = \min \{ t : X(t) \notin D \} \) is the time before system (8) destruction, where \( X(t) \) is its solution with initial condition \( x_0 \) or in other way \( \tau \) is the first moment of process exit from \( D \).

5. Conclusions

The description of solutions behaviour of charged equations in the form of ionite gas motion is realized and stochastic charge kinetics equations are investigated in detail.

References

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