Viscosity of Magnetic Fluid in Oscillation System in a Strong Magnetic Field

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Abstract. The article gives an assessment of viscosity and its increment (‘magnetoviscous’ effect) in a thin near-wall layer of the column of magnetic fluid that oscillates in the tube when applying a strong transverse magnetic field. Viscosity is calculated according to a formula derived from two different theoretical approaches. For calculation, previously published experimental results, which were discussed earlier on the assumption that there is no field dependence of viscosity, are used in the article. A comparative analysis of the “near-wall viscosity” estimates obtained using dynamic elasticity as well as the magnetization curve and the coefficient of static elasticity is carried out. The significance of the results obtained for ‘magnetoviscous’ effect for performing diagnosis of interparticle aggregation is indicated.

1. Introduction
Under certain conditions, in magnetic fluids (MFs), intense interaction of magnetic nanoparticles occurs with the formation and disintegration of aggregates, which leads to changes in their physical properties. Studying such processes in MFs is a difficult task, since conventional techniques for studying microscopic processes using electron or atomic force microscopes are applied under static conditions, while the aggregation processes are determined by both MF dynamics and magnetic fields structure. Therefore, studying aggregation processes in dynamic conditions is a topical task. Undoubtedly, when solving this task experimental and theoretical researches of the changes of MF viscosity in a magnetic field can be useful and productive.

[1, 2] present measurements and a theoretical analysis of the oscillations of MF-column confined by magnetic levitation in a tube in a strong magnetic field. Moreover, the calculations carried out using the ponderomotive elasticity model with the correction applied for the resistance of a moving viscous fluid are compared with the experimental magnetization curve. However, in the theoretical model, the MF viscosity invariance is assumed to be one of the approximations [2]. Consequently, the issue on the ‘magnetoviscous’ effect – the viscosity increment in the MF thin near-wall layer when applying a magnetic field – remains an open one.

In a shear flow, a moment of forces acts on a solid particle, which leads to its rotation. Magnetic field orients the magnetic moment of the particle and, if there is relation between the moment of the particle and the particle, hinders its free rotation. This leads to local gradients of the velocity of base-fluid near the particles and causes an increase of MF effective viscosity [3-7]. Saturation of the so-called ‘rotational’ viscosity occurs when a strong field rigidly orients the particles. The actual viscosity increment in strong magnetic fields for a magnetite sample with the volume concentration of 0.19–0.24 in Poiseuille flow through a capillary in a perpendicular magnetic field does not exceed 5–6% [8].

The lack of information on ‘magnetoviscous’ effect does not allow obtaining a comprehensive picture of the physical mechanisms of oscillatory motion of magnetic-fluid active elements in various
engineering devices and makes it difficult to draw on the acquired methodological experience to expand the means of controlling wear resistance and the magnetic colloid consumption.

In the present study, which is the continuation of work on low-frequency oscillations of MF in strong magnetic fields [1, 2], the ‘magnetoviscous’ effect in a thin near-wall layer when applying a strong transverse magnetic field is determined to expand physical representation of MF oscillatory flow in a magnetic field and to diagnose interparticle aggregation. The value of the ‘magnetoviscous’ effect was calculated by a formula derived from two different theoretical approaches. An array of published experimental results of comprehensive measurements of the oscillation frequency, oscillation damping coefficient, saturation magnetization and static displacement of MF-column for two samples of magnetic fluid discussed earlier on the assumption that there is no field dependence of viscosity is used in this work [2].

2. The samples parameters and the experiment brief description of the previously published studies

In relation to the task set, it is necessary to describe briefly the experimental setup and measurement procedure. The experimental setup designed to measure MF-column oscillations frequency was described in detail in [2] (Figure 1. Block diagram of the experimental setup No 1). In the work, we used a laboratory electromagnet FL-1; a tube with an internal diameter \( d = 12 \text{ mm} \) made of plexiglass was placed between the pole tips of the electromagnet. The tube axis passes horizontally (or vertically) through the centre of the pole gap and is parallel to the surface of the pole tips. The axis \( OZ \) coincides with the axis of the tube, and its beginning is in the centre of the MF-column in the equilibrium.

A significant circumstance of the obtained dependences of the transverse value of the magnetic field \( H_x(z) \) is the presence of a linear section on the curves at the level of \( z = 57.5 \text{ mm} \), which makes it possible to consider the magnetic field intensity gradient in this area \( \Delta H_x / \Delta z = \text{const.} \). In a strong and inhomogeneous magnetic field, MF-column takes a form close to a cylindrical one. In the conducted experiment, the distance between the bases of the cylinder is \( b = 115 \text{ mm} \). In the view of the assumptions concerning the tube ‘thinness’, magnetic field at the points of the MF free surface is tangential to it, i.e. it has only the tangential component \( H_t \) (perpendicular to the surface of the pole tips) and the magnetic field intensity gradient is directed perpendicularly to the surface along the \( OZ \) axis to its beginning. The maximum magnetic field in the centre between the poles of the electromagnet is 900 kA/m.

Method of static displacement of the MF-column is also considered in [2]; the essence of this method is that static pressure is applied to the MF-column. Figure 2. shows a block diagram of the experimental setup No. 2. In this case, to create a hydrostatic pressure, a communicating vessel in the form of a U-shaped tube filled with distilled water is used and it is connected with a MF-column by a flexible pipe.

The samples of magnetic fluids MF-1 and MF-2 are synthesized on the basis of finely dispersed magnetite – \( \text{Fe}_3\text{O}_4 \) which is stabilized by a surfactant – oleic acid \( \text{C}_8\text{H}_{17}\text{CH=CH(CH}_2\text{)}_7\text{COOH} \). In the sample MF-1, aviation kerosene TS-1 was used as a dispersive medium – a carrier liquid; in the sample MF-2, undecane \( \text{C}_{11}\text{H}_{24} \) (a hydrocarbon from the alkane class) was carrier liquid. The objects of the research were synthesized in the Fundamental Scientific Research Laboratory of Applied Ferrohydrodynamics of Ivanovo State Power Engineering University.

The densities of the samples MF-1 and MF-2 and their saturation magnetization are \( \rho = 1,245 \text{ kg/m}^3 \) and \( \rho = 1,227 \text{ kg/m}^3 \), \( M_s = 39.5 \text{ kA/m} \) and \( M_s = 40.4 \text{ kA/m} \), respectively. The shear viscosity \( \eta \) is measured using Brookfield DV2T viscosimeter; its values for MF-1 and MF-2 is 34.8 mP∙s and 30.4 mPa∙s respectively at a shear rate of 79.2 1/s.

3. Theoretical justification

Within the framework of continuum mechanics, i.e. neglecting the dispersion of the system and the interaction of the dispersed particles with each other, the equation of damped harmonic oscillations can be written in the form:

\[
M_a \frac{d^2 \xi}{dt^2} + r \frac{d \xi}{dt} + k_r \xi = 0 ,
\]
where $\xi$ is the displacement from the equilibrium position of the centre of gravity of the MF-column in the tube, $r^*$ is the fluid active resistance on the inner surface of the tube, and $k_p$ is the ponderomotive elasticity coefficient.

An expression for the ponderomotive elasticity coefficient $k_p$ is obtained in [1]:

$$k_p = \mu_0 \frac{\pi d^2}{2} \left( M_s \frac{\partial H}{\partial z} \right)_{z=b/2},$$

in which $\mu_0$ is the magnetic constant, $M_s$ and $\partial H / \partial z$ are the magnetization and the magnetic field intensity gradient at the MF-column base, respectively, $d$ is the tube diameter, and $b$ is the length of the MF-column.

An expression for fluid active resistance $r^*$ is given in [9]:

$$r^* = \pi d b \sqrt{2 \eta \rho \omega / 2},$$

Expression (3) was first obtained by Helmholtz. The limitation is the ratio of the tube circumference $\pi d$ to the length of the viscous wave $\lambda' = 2\pi \sqrt{2 \eta / \rho \omega}$ at which it exceeds 10. In the case under consideration, at the maximum frequency of 15 Hz applied in the experiment, this ratio is $\sim 7.5$.

In equation (1), $M_\omega$ can be represented as follows [9]:

$$M_\omega = m \left( 1 + \frac{2}{d} \sqrt{2 \eta / \rho \omega} \right),$$

where $m$ is the mass of fluid in the tube.

The second term in the brackets of formula (4) is relatively small ($\approx 0.1$); if it is multiplied by the mass of the MF, it represents the so-called ‘added mass’ determined by fluid viscosity.

Let us rewrite the equation of damped oscillations (1) in the standard form:

$$\frac{d^2 \xi}{dt^2} + 2 \beta \frac{d \xi}{dt} + \omega_0^2 \xi = 0$$

where $2 \beta = r^*/M_\omega = r^* / m \left( 1 + \frac{2}{d} \sqrt{2 \eta / \rho \omega} \right) = 2 \beta_\text{Helm} / \left( 1 + \frac{2}{d} \sqrt{2 \eta / \rho \omega} \right), \quad \beta_\text{Helm} = \frac{1}{d} \sqrt{2 \eta \omega / \rho}$.

In case of transition of $\omega \to \nu$:

$$\beta_\text{Helm} = \frac{2}{d} \sqrt{\frac{\pi \eta \nu}{\rho}}$$

Expression (6) obtained on the basis of the Helmholtz formula (3) allows calculating oscillations damping coefficient.

In equation (5)

$$\omega_\text{damping} = \frac{k_p}{M_\omega}.$$  

The general solution of equation (5) has a known form $\xi = C e^{-\beta t} \cos(\omega t + \varphi)$, where $C, \varphi$ are arbitrary constants, $\beta$ is the oscillations damping coefficient and the oscillation frequency $\omega$ is expressed as follows:

$$\omega = \sqrt{\omega_0^2 - \beta^2},$$

After algebraic transformations, relation (8) takes the form:

$$\omega^2 = \frac{k_p}{m \left( 1 + \frac{2}{d} \sqrt{2 \eta / (\rho \omega)} \right)} - \frac{\beta_\text{Helm}^2}{\left( 1 + \frac{2}{d} \sqrt{2 \eta / (\rho \omega)} \right)^2},$$

or

$$4\pi^2 \nu^2 = \frac{4 k_p}{\pi \rho b d \left( 1 + \frac{2}{d} \sqrt{\eta / \pi \nu} \right)} - \frac{\beta_\text{Helm}^2}{\left( 1 + \frac{2}{d} \sqrt{\eta / \pi \nu} \right)^2}$$

Neglecting the second term in the right-hand side of (10) and taking into account (2), we obtain:
There is another approach to solving this problem; it is based on the application of the law of energy conservation in the oscillatory system [2]. As oscillations occur, the oscillatory energy of MF-column with ponderomotive elasticity decreases. Let us take the value of kinetic energy as maximum $E_{k0}$ at the initial time. Taking into account the formula for energy dissipation due to the reciprocating flow of a viscous fluid column in a tube [10], the potential energy of the oscillation motion after a quarter period can be represented as follows:

$$
\frac{\pi^2}{2} \nu^2 b^2 \rho + 2 \nu b d \sqrt{\pi^3 \eta_0 v^3} = \mu_0 \frac{\pi d^2}{2} \left( M_s \frac{\partial H_z}{\partial z} \right)_{z=b/2}
$$

(11)

where $\Delta z_0$ and $\Delta z_{01}$ are displacement amplitudes at the initial moment and after a quarter period.

If we make a transition in the left side of (12): $\Delta z_{01} \rightarrow \Delta z_0$ and $k_p \rightarrow (k_p + \delta_p)$, and require that the equality was not formally upset, we get:

$$
\left( k_p + \delta_p \right) \Delta z^2_0 = \frac{k_p \Delta z^2_0}{2} - \frac{\pi^2}{2} b d \sqrt{\frac{\omega \eta}{2}} \frac{\Delta z^2}{2}.
$$

(12)

In this case, $\delta_p$ is the correction for ponderomotive elasticity coefficient, resulting from the viscous fluid flow:

$$
\delta_p = -\frac{\pi^2}{2^{1/2}} \nu b d \sqrt{\pi^3 \eta_0 v^3}
$$

(14)

However, the formula obtained for $\delta_p$ gives an ‘upper’ estimate (the maximum value after a quarter period). The viscous elasticity coefficient $k_\eta$ is the ‘half-period average’ value of the harmonic function of the maximum value, i.e. $k_\eta = \frac{2}{\pi} \delta_p$, therefore:

$$
k_\eta = -\frac{2}{\pi} \nu b d \sqrt{\pi^3 \nu v^3 \eta_\eta}
$$

(15)

In addition, it was thought previously that MF viscosity does not depend on the magnetic field intensity. Assuming such a dependence, let us make a replacement of $\eta$ by $\eta_\eta$:

$$
k_\eta = -2 \nu b d \sqrt{\pi^3 \nu v^3 \eta_\eta \rho}
$$

(16)

Then equation (17) in [2] can be written as follows:

$$
\mu_0 \frac{\pi d^2}{2} \left( M_s \frac{\partial H_z}{\partial z} \right)_{z=b/2} = \pi^2 \nu^2 \nu b d ^2 + 2 \nu b d \sqrt{\pi^3 \nu v^3 \eta_\eta \rho}
$$

(17)

Equation (17) coincides with expression (11) obtained above. After elementary algebraic operations with equation (17), it is easy to obtain a formula for calculating viscosity:

$$
\eta_\eta = \frac{1}{\nu^4} \left[ \frac{\mu_0 d M_s}{4b\pi \sqrt{\eta \rho}} \left( \frac{\partial H_z}{\partial z} \right)_{z=b/2} - \frac{\sqrt{\eta \rho} \cdot d \cdot v^2}{2} \right]
$$

(18)

Under the action of hydrostatic pressure, due to the difference in water levels in the elbows of the U-shaped tube, the MF column shears to equilibrium:

$$
\rho_\eta g \Delta h = 4k_e \Delta z / \pi d^2
$$

(19)

where $k_e$ is the static elasticity coefficient, $\rho_\eta$ is water density, $\Delta h$ is the difference in water levels in the elbows of the U-shaped tube, $\Delta z$ is the MF-column displacement under hydrostatic pressure, $g$ is the gravity factor.

The coefficient of static elasticity $k_e$ is a numerical equivalent of the coefficient of ponderomotive elasticity $k_p$ and it can be used to determine the force impact of inhomogeneous magnetic field on a magnetized fluid.

Let us obtain from (19):

$$
k_e = \pi d^2 \rho_\eta g (\Delta h / 4 \Delta z)
$$

(20)
Equation (17) can be rewritten as follows:
\[
\pi d^2 \rho_s \varphi (\Delta h / 4 \Delta z) = \pi^3 v' \rho bd^2 + 2bd \sqrt{\pi^3 v' \eta_\mu \rho}
\] (21)

Then the formula for calculating viscosity in the fluid near-wall layer is as follows:
\[
\eta_\mu = \left[ \frac{d \rho_s g (\Delta h / \Delta z)}{8b \sqrt{\pi^3 v' \rho}} - d\sqrt{\pi^3 v' \rho} \right] \] (22)

4. The results of calculations

Figures 1 and 2 show the graphs of dependences of magnetization of fluids MF-1 and MF-2 on \(H^t\) in the vicinity of magnetic saturation, taken from the article [2]. Experimental data of \(M\) are obtained for a magnetic field of \(\leq 750\) kA/m. The circles in the figures indicate magnetization values corresponding to the reciprocal magnitude of the magnetic field \(1/H_\mu\): 1/800 m/kA and 1/900 m/kA. The crosses show the magnetization values corresponding to the reciprocal of the magnetic field intensity at the edges of MF-column \(1/H_t\): 1/619 m/kA and 1/686 m/kA. The formulas in the figures, which analytically reflect the linear approximation of the dependence under consideration, allow us to calculate the numerical values of \(M_t\) and obtain these values \(M'_t\) and \(M''_t\) for the values of \(1/H_\mu\) and \(1/H_t\). Taking into account the obtained values of \(M_t\) using formula (18) the viscosity values \(\eta_\mu\) and \(\eta''_\mu\) in the near-wall layer in a strong transverse magnetic field are calculated.

**Figure 1.** The dependence of \(M\) on \(1/H\) for sample MF-1. Block diagram of the experimental setup.

**Figure 2.** The dependence of \(M\) on \(1/H\) for sample MF-2.

The obtained results for \(\eta''_\mu\) give an ‘upper’ estimate, since in this case the values of \(M_t\) chosen for the maximum magnetic field in the interpolar gap \(H_\mu\) are used. At the same time, the results \(\eta''_\mu\) give a ‘lower’ estimate, since they are calculated for the minimum value of the magnetic field intensity related to the boundaries of the MF-column.

Table 1 gives the applied combination of values of magnetic field parameters: \(H_\mu\) is the magnetic field intensity in the center between the electromagnet poles, \(H_t\) is the magnetic field intensity and \(\Delta H_t / \Delta z\) is the intensity gradient at the base of the MF-column. It also shows the oscillation frequency of the MF-column \(v\), \(M'_t, M''_t\), the calculated values \(\eta'_\mu, \eta''_\mu\), the increments of viscosity in the magnetic field (‘magnetoviscous effect’) \(\Delta \eta'_\mu, \Delta \eta''_\mu\), and the relative values of these parameters \(\Delta \eta'_\mu / \eta\), and \(\Delta \eta''_\mu / \eta\).
Based on the ‘upper’ and ‘lower’ estimates of the ‘near-wall viscosity’, their contribution to the oscillation damping coefficient is calculated. Table 2 shows the experimental values of the damping coefficient \( \beta \), taken from [2]. It also presents the values of \( \beta_{HE}^{\text{m}} \) and \( \beta_{HE}^{*} \), calculated by formula (6) using \( \eta_{H} \) and \( \eta_{H}^{*} \), respectively.

### Table 2. System attenuation rating

| Sample | \( H_0 \), kA/m | \( H_\infty \), kA/m | \( \Delta h/\Delta z \), MA/m² | \( v \), Hz | \( \beta \), c⁻¹ | \( \beta_{HE}^{*} \), c⁻¹ | \( \beta_{HE}^{\text{m}} \), c⁻¹ |
|--------|-----------------|-------------------|--------------------------|-------|----------------|----------------|----------------|
| MF-1   | 800             | 619               | 14.2                     | 14.5  | 0.008         | 0.012         | 0.013         |
| MF-2   | 900             | 686               | 15.9                     | 15.5  | 0.010         | 0.014         | 0.015         |

Table 3 shows \( H_s \), which is the magnetic field intensity; \( v \), which is the oscillation frequency of the MF-column; \( \Delta h/\Delta z \), which is the ratio of the difference in water levels in the elbows of the U-shaped tube \( \Delta h \) to the displacement of the MF-column under the action of static pressure \( \Delta z \); the values \( \eta_{H} \) calculated by formula (22); the increments of viscosity in the magnetic field (‘magnetoviscous’ effect) \( \Delta \eta_{H} \); the relative value of this parameter \( \Delta \eta_{H} / \eta \).

### Table 3. Viscosity determination using statistical experiment data

| Sample | \( H_s \), kA/m | \( v \), Hz | \( \Delta h/\Delta z \) | \( \eta_{H} \), Pa·s | \( \Delta \eta_{H} \), Pa·s | \( \Delta \eta_{H} / \eta \) | \( \beta \), s⁻¹ | \( \beta_{HE}^{*} \), s⁻¹ |
|--------|-----------------|-------|-----------------|-----------------|-----------------|-----------------|-------|----------------|
| MF-1   | 900             | 15.5  | 160             | 0.063           | 0.033           | 0.001           | 9.4   | 8.8            |
| MF-2   | 900             | 15.3  | 158             | 0.057           | 0.022           | 0.001           | 9.5   | 7.8            |

Comparing the values of the parameters: \( \eta_{H} \), \( \eta_{H}^{*} \) and \( \eta_{H}^{*} \); \( \Delta \eta_{H} \), \( \Delta \eta_{H}^{*} \) and \( \Delta \eta_{H}^{*} \); \( \Delta \eta_{H} / \eta \), \( \Delta \eta_{H}^{*} / \eta \) and \( \Delta \eta_{H}^{*} / \eta \) given in Tables 1, 2, and 3, it is possible to see their quantitative close proximity, which confirms the validity of the theoretical approaches used when obtaining formulas (18) and (22) as well as the soundness of the assumption of quantitative close proximity of the coefficients of ponderomotive and static elasticity introduced in [1, 2].

Table 3 presents the values of the oscillation damping coefficients \( \beta \) and \( \beta_{HE} \) measured and calculated by formula (6) using \( \eta_{H} \). The numerical values of these parameters are in the same correlation as the correlation of the values of \( \beta \), \( \beta_{HE}^{*} \), and \( \beta_{HE}^{\text{m}} \) given in Table 2. The calculation results indicate that, firstly, the obtained values of the ‘near-wall viscosity’ do not lead to an excess of the experimental result \( \beta \), which, in its turn, can be regarded as physical justification of the proposed model theory; secondly, the presence of an excess of \( \beta \) over \( \beta_{HE}^{*} \), \( \beta_{HE}^{*} \), and \( \beta_{HE}^{\text{m}} \) by \( \sim 15 \% \) actually reflects the presence of other energy dissipation mechanisms in the oscillatory system (in particular, the emission of elastic energy into the structural elements of the setup and the environment).

The limitation of the applied theories indicated above allows preferring the values of the calculated parameters belonging to the highest magnetic field intensity of 900 kA/m, at which the maximum value of the oscillation frequency and the minimum length of the viscous wave were obtained. At the same time, in order to interpret the differences in the values of the ‘near-wall viscosity’ of the samples...
MF-1 and MF-2, data on the characteristic features of their structure, which are absent in the published work [2], are necessary.

There is one fundamental difference between the considered techniques for determining the ‘near-wall viscosity’ of a MF in a strong magnetic field, which is considered further. When applying a static (hydrostatic) technique for determining the elasticity coefficient of an oscillating system and combining it with a technique for determining the dynamic elasticity coefficient obtained using measurements of the oscillation frequency, there is no need to make complex measurements of magnetic field intensity and intensity gradient as well as to obtain a magnetization curve. Moreover, the measurements mentioned are characterized by a large total error. The only condition that must be fulfilled is that all magnetic, pressure and temperature parameters must be identical in the experiments for static and dynamic measurements.

Obtaining information on the ‘near-wall viscosity’ for MFs synthesized applying new technologies is essential in order to diversify carrier medium, concentration and sizes of magnetic nanoparticles as well as in connection with the problem of aggregation in magnetic colloids. Additional possibilities are emerging for the diagnosis of interparticle aggregation to the occurrence of which ‘magnetoviscous’ effect in the thin near-wall layer turns out to be very sensitive when applying a strong magnetic field to MF.

5. Conclusion
The results of the research are as follows:

– Estimation of the viscosity increment (‘magnetoviscous’ effect) in a thin near-wall layer of a MF-column oscillating in the tube when a strong transverse magnetic field is applied;
– Calculation of the value of ‘near-wall viscosity’ is obtained by a formula derived from two different theoretical approaches;
– An array of previously published experimental results was used; it includes complex measurements of oscillation frequency, oscillation damping coefficient, saturation magnetization, and static displacement of the MF-column for two fluid samples;
– Based on the ‘upper’ and ‘lower’ estimates of the ‘near-wall viscosity’ obtained using the magnetization curve as well as the results of static displacement of the MF-column, their contribution to the oscillation damping coefficient is calculated;
– Additional possibilities are emerging for the diagnosis of interparticle aggregation to the occurrence of which ‘magnetoviscous’ effect in the thin near-wall layer turns out to be very sensitive when applying a strong magnetic field to MF;
– The calculations do not take into account the transition thin layer of the viscous wave passage, since it is assumed that the entire MF-column is involved in the oscillatory motion;
– The applicability of the relations obtained for the evaluation of can be extended, for example, by increasing the magnetic field intensity gradient, which allows obtaining a higher oscillation frequency of a MF-column.

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References
[1] Polunin V M, Ryapolov P A, Platonov V V and Kuzko A E 2016 Acoustic Journal 62 302-307
[2] Polunin V M, Ryapolov P A, Platonov V B, Sheldeshova E V, Karpova G V and Arcifyev I M 2017 ACOUST PHYS+ 63 371–379
[3] Mc Taque J P 1969 J. Chem. Phys. 51 133-136
[4] Rosensweig R E, Kaiser R and Miskolezy G. 1969 J. Colloid Interface Sci. 29 680-686
[5] Shliomis M. I. 1971 JETP. 6 (12) 2411-2418
[6] Shliomis M I 1974 Adv. Phys. 112 427-459
[7] Cebers A O 1975 MHD 4 37-44.
[8] Maiorov M M 1980 MHD 4 11-18
[9] Rzhevkin S N 1960 Lectures on the Theory of Sound (Moscow: Publishing house of Moscow state University) 336
[10] Landau L D and Lifshits E M 1988 Theoretical Physics. Hydrodynamics (Moscow: Science) 6 736