Is there any room for new physics in the muon g-2 problem?  

E. Bartoš²  
S. Dubnička³,  
A. Z. Dubničková⁴,  
E.A.Kuraev² and E.Zemlyanaya²

The muon is described by the Dirac equation and its magnetic moment is related to the spin by means of the expression

\[ \mu = g \left( \frac{e}{2m_\mu} \right) \vec{s} \]  

(1)

where the value of gyromagnetic ratio \( g \) is predicted (in the absence of the Pauli term) to be exactly 2.

However, interactions existing in nature modify \( g \) to be exceeding the value 2 because of the emission and absorption of:

- virtual photons (electromagnetic effects),
- intermediate vector and Higgs bosons (weak interaction effects)

---

¹Contribution on International Conference Hadron Structure '02, September 23.-27., 2002, Herlany, Slovakia
²Lab. of Theor. Physics, JINR, Dubna
³Inst. of Physics, Slovak Academy of Sciences, Bratislava, SR
⁴Dept. of Theor. Physics, Comenius University, Bratislava, SR
• vacuum polarization into virtual hadronic states (strong interaction effects).

In order to describe this modification of $g$ theoretically, the magnetic anomaly was introduced by the relation

$$ a_\mu \equiv \frac{g - 2}{2} = a^{(1)}_\mu \left( \frac{\alpha}{\pi} \right) + \left( a^{(2)\text{QED}}_\mu + a^{(2)\text{had}}_\mu \right) \left( \frac{\alpha}{\pi} \right)^2 + a^{(2)\text{weak}}_\mu + O \left( \frac{\alpha}{\pi} \right)^3 $$

(2)

where to every order Feynman diagrams (see Figs. 1-3) correspond and $\alpha = 1/137.03599976(50)$ is the fine structure constant.

Figure 1: The simplest Feynman diagram of an interaction of the muon with an external magnetic field.

The muon anomalous magnetic moment $a_\mu$ is very interesting object for theoretical investigations due to the following reasons:

i) it is the best measured quantity (BNL E–821 experiment) in physics

$$ a^{\text{exp}}_\mu = (116592040 \pm 86) \times 10^{-11} \text{ II} $$

(3)

ii) its accurate theoretical evaluation provides an extremely clean test of "Electroweak theory" and may give hints on possible deviations from Standard Model (SM)
iii) moreover, in near future the measurement in BNL is expected to be performed yet with an improved accuracy

$$\Delta a_{\mu}^{exp} = \pm 40 \times 10^{-11}$$  \hspace{1cm} (4)

i.e. it is aimed at obtaining a factor 2 in a precision above that of the last E–821 measurements.

At the aimed level of the precision (4) a sensibility will already exist to contributions

$$a_{\mu}^{(2,3)\text{weak}} = (152 \pm 4) \times 10^{-11},$$  \hspace{1cm} (5)

arising from single– and two–loop weak interaction diagrams. And so, if we compare theoretical evaluations of:

- QED contributions up to 8th order

$$a_{\mu}^{QED} = (116584705.7 \pm 2.9) \times 10^{-11}$$  \hspace{1cm} [2]

the single- and two-loop weak contributions

$$a_{\mu}^{(2,3)\text{weak}} = (151 \pm 4) \times 10^{-11}$$  \hspace{1cm} [3]
Figure 3: The third-order hadronic vacuum-polarization contributions to the anomalous magnetic moment of the muon.

\[ a^{(2,3)\text{weak}}_\mu = (153 \pm 3) \times 10^{-11} \] \[ a^{(2,3)\text{weak}}_\mu = (152 \pm 1) \times 10^{-11} \] 

strong int. contributions

\[ a^{\text{had}}_\mu = (7068 \pm 172) \times 10^{-11} \] \[ a^{\text{had}}_\mu = (7100 \pm 116) \times 10^{-11} \] \[ a^{\text{had}}_\mu = (7052 \pm 76) \times 10^{-11} \] \[ a^{\text{had}}_\mu = (7024 \pm 152) \times 10^{-11} \] \[ a^{\text{had}}_\mu = (7021 \pm 76) \times 10^{-11} \]

it is straightforward to see that the largest uncertainty is in \( a^{\text{had}}_\mu \).

Error is comparable, or in the best case 2x smaller than the weak interaction contributions.
So, in order to test the SM predictions for $a_\mu$ and to look for new physics in comparison with BNL E–821 experiment, one has still to improve an evaluation of $a^{\text{had}}_\mu$.

The most critical from all hadronic contributions are the light–by–light (LBL) meson pole terms (see Fig.4) and we have recalculated them in the paper [11].

![Figure 4: Meson (M) pole diagrams in the third order hadronic light–by–light scattering contributions to $a^{\text{had}}_\mu$.](image)

More concretely we have evaluated contributions of the scalar $\sigma$, $a_0$ and pseudoscalar $\pi^0$, $\eta$, $\eta'$ mesons ($M$) in the framework of the linearized extended Nambu–Jona–Lasinio model

$$\mathcal{L}_{q\bar{q}M} = g_M \bar{q}(x) [\sigma(x) + i\pi(x)\gamma_5] q(x).$$

The reason for the latter are predictions of series of recent papers

$$a^{\text{LBL}}_\mu = (+52 \pm 18) \times 10^{-11} \quad [12]$$
$$a^{\text{LBL}}_\mu = (+92 \pm 32) \times 10^{-11} \quad [13]$$
\[
\alpha_{\mu}^{LBL} = (+79.2 \pm 15.4) \times 10^{-11} \quad [14]
\]
\[
\alpha_{\mu}^{LBL} = (+83 \pm 12) \times 10^{-11} \quad [15]
\]
\[
\alpha_{\mu}^{LBL}(\pi_0) = (+58 \pm 10) \times 10^{-11} \quad [16]
\]

which differ in the magnitude.

Moreover, in these papers only the pseudoscalar pole contributions were considered.

We include the scalar meson \((\sigma, a_0)\) pole contributions as well.

Current methods in a description of the \(\gamma^* \rightarrow M \gamma^*\) transition form factors are ChPT and the vector–meson–dominance (VMD) model.

Here the corresponding transition form factors by the constituent quark triangle loops with colourless and flavourless quarks with charge equal to the electron one are represented.

An application of a similar modified constituent quark triangle loop model for a prediction of the pion electromagnetic form factor behaviour was carried out in [17] where also a comparison with the naive VMD model prediction was demonstrated.

The mass of the quark in the triangle loop is taken to be:

\[
m_u = m_d = m_q = (280 \pm 20) \text{ MeV}
\]
determined [18] in the framework of the chiral quark model of the Nambu–Jona–Lasinio type by exploiting the experimental values of the pion decay constant, the \(\rho\)-meson decay into two-pions constant, the masses of pion and kaon and the mass difference of \(\eta\) and \(\eta'\) mesons.

The unknown strong coupling constants of \(\pi^0, \eta, \eta'\) and \(a_0\) mesons with
quarks are evaluated in a comparison of the corresponding theoretical two-photon widths with experimental ones.

The $\sigma$-meson coupling constant is taken to be equal to $\pi^0$-meson coupling constant as it follows from the corresponding Lagrangian.

The $\sigma$-meson mass is taken to be $m_{\sigma}=(496 \pm 47)$ MeV as an average of the values recently obtained experimentally from the decay $D^+ \rightarrow \pi^- \pi^+ \pi^+$[19] and excited $\Upsilon$ decay [20] processes.

As a result we present explicit formulas for $a^{LBL}_\mu(M) (M = \pi^0, \eta, \eta', \sigma, a_0)$ in terms of Feynman parametric integrals of 10-dimensional order, which subsequently are calculated by MIKOR method.

As a result one finds

\begin{align*}
    a^{LBL}_\mu(\pi^0) &= (81.83 \pm 16.50) \times 10^{-11} \\
    a^{LBL}_\mu(\eta) &= (5.62 \pm 1.25) \times 10^{-11} \\
    a^{LBL}_\mu(\eta') &= (8.00 \pm 1.74) \times 10^{-11} \\
    a^{LBL}_\mu(\sigma) &= (11.67 \pm 2.38) \times 10^{-11} \\
    a^{LBL}_\mu(a_0) &= (0.62 \pm 0.24) \times 10^{-11}.
\end{align*}

(6)

So, the total contribution of meson poles in LBL is

\begin{equation}
    a^{LBL}_\mu(M) = (107.74 \pm 16.81) \times 10^{-11},
\end{equation}

where the resultant error is the addition in quadrature of all partial errors of (6).

Together with the contributions of the pseudoscalar meson ($\pi^\pm, K^\pm$) square loops and constituent quark square loops (Fig.5) taken from Hayakawa and Bijnens it gives
Figure 5: Third order hadronic light–by–light scattering contribution to $a^\mu_{\text{had}}$ (A) and class of pseudoscalar meson square loop diagrams (B) and quark square loop diagrams (C) contributing to (A).

$$a^\mu_{\text{LBL}}(\text{total}) = (111.20 \pm 16.81) \times 10^{-11}.$$  \hspace{1cm} (8)

The others 3-loop hadronic contributions derived from the hadronic vacuum polarizations ($V_P$) were most recently evaluated by Krause\cite{21}

$$a^\mu_{\text{VP}} = (-101 \pm 6) \times 10^{-11}.$$  \hspace{1cm} (9)

Then the total 3-loop hadronic correction is

$$a^\mu_{\text{had}}^{(3)} = a^\mu_{\text{LBL}}(\text{total}) + a^\mu_{\text{VP}}^{(3)} = (10.20 \pm 17.28) \times 10^{-11}$$  \hspace{1cm} (10)

where the errors have been again added in quadratures.

If we take into account the most recent evaluation \cite{22} of the lowest–order hadronic vacuum–polarization contribution to the anomalous magnetic moment of the muon

$$a^\mu_{\text{had}}^{(2)} = (7021 \pm 76) \times 10^{-11}$$  \hspace{1cm} (11)

the pure QED contribution up to 8th order...
and the single- and two-loop weak interaction contribution, finally one gets the SM theoretical prediction of the muon anomalous magnetic moment value to be

\[
a_{\mu}^{th} = (116591888.9 \pm 78.1) \times 10^{-11}.
\]  

(13)

Comparing this theoretical result with experimental one finds

\[
a_{\mu}^{exp} - a_{\mu}^{th} = (151 \pm 116) \times 10^{-11}
\]

(14)

which implies a reasonable consistency of the SM prediction for the anomalous magnetic moment of the muon with experiment.

However, one expects in near future a 2x lowering of the error in BNL E-821 experiment and then there can appear still a room for a new physics beyond the SM.

On the other hand, we see some possible improvements for theoretical value, which as a result could diminish the difference between theoretically estimated value and the measured value in E-827 experiment.

The first improvements we see still in the lowest-order hadronic vacuum-polarization diagram contributions (Fig.2), which can be expressed by the integral

\[
a_{\mu}^{(2)had} = \frac{1}{4\pi^3} \int_{4m_{\pi}^2}^{\infty} \sigma^{tot}(s) K_{\mu}(s) ds;
\]

(15)
where $\sigma_h(s)$ stands for the total cross section $\sigma(e^+e^- \rightarrow had)$ and

$$K_\mu(s) = \int_0^1 \frac{x^2(1-x)}{x^2 + (1-x)s/m_\mu^2} dx.$$  

(16)

We have indications, that the $e^+e^- \rightarrow K^+K^-$ and $e^+e^- \rightarrow K^0\bar{K}^0$ data at the $\phi$-resonance region, measured at the Novosibirsk, are inconsistent with analyticity.

They have to be systematically shifted to the larger values and so they will give larger positive contributions approaching the theoretically estimated value to the experimental one.

The same can be said about the contribution of the processes $e^+e^- \rightarrow \pi\gamma$, $e^+e^- \rightarrow \eta\gamma$ and $e^+e^- \rightarrow \eta'\gamma$, which were not estimated up to now as there is unknown a behaviour of corresponding transition form factors in the time-like region.

We have elaborated the unitary and analytic model which solves the latter problem.

The last contribution which according to our knowledge was not considered up to now is $K^0$ meson square loop diagram in the third order hadronic light-by-light scattering contribution to $a_\mu^{had}$.

We hope to realize all these ideas before obtaining the final result at the BNL E–821 experiment with the precision

$$\Delta a_\mu^{exp} = \pm 40 \times 10^{-11}.$$  

References

[1] G.W. Bennett et al., Phys. Rev. Lett. 89 (2002) 101804-1.
[2] V. W. Hughes and T. Kinoshita, Rev. Mod. Phys. 71 (2) (1999) S133.

[3] A. Czarnecki, W. Marciano, Nucl. Phys. B(Proc. Suppl.) 76 (1999) 245.

[4] G. Degrassi and G. F. Giudice, Phys. Rev. D58 (1998) 53007.

[5] M. Knecht, S. Peris, M. Perrottet, E. de Rafael, hep-ph/0205102.

[6] T. Kinoshita et al., Phys. Rev. D31 (1985) 2108.

[7] J.A. Casas et al., Phys. Rev. D32 (1985) 736.

[8] L. Martinovič and S. Dubnička, Phys. Rev. D 42 (1990) 884.

[9] S. Eidelman, F. Jegerlehner, Z. Phys. C67 (1995) 585.

[10] S. Narison, Phys. Lett. 513 B (2001) 53.

[11] E. Bartoš, A.Z. Dubničková, S. Dubnička, E.A. Kuraev, E. Zemlyanaya, Nucl. Phys. B632 (2002) 330.

[12] M. Hyakawa, T. Kinoshita, I. Sanda, Phys. Rev. D54 (1996) 3137.

[13] J. Bijnens et al., hep-ph/0112255

[14] M. Hyakawa, T. Kinoshita, Phys. Rev. D57 (1998) 465, hep-ph/0112102

[15] M. Knecht and A. Nyffeler, hep-ph/0111058

[16] I. Blokland, A. Czarnecki and K. Melnikov, hep-ph/0112117

[17] S. Dubnička, G. Georgios, V.A. Meshcheryakov, Can. J. Phys. 63 (1985) 1357.
[18] M. Nagy, M. K. Volkov and V. L. Yudichev, Proc. of Int. Conf. "Hadron Structure 2000", Stara Lesna, Slovak Republic, 2.–7.10. 2000, Eds: A.-Z. Dubničková, S. Dubnička and P. Stríženec, Comenius Univ., Bratislava (2001) p. 188.

[19] E.M. Aitala et al., Phys. Rev. Lett. 86 (2001) 70.

[20] T. Komada, M. Ishida and S. Ishida, Phys. Lett. 508 B (2001) 31.

[21] B. Krause, Phys. Lett. 390 B (1997) 392.

[22] S. Narison, Phys. Lett. 513 B (2001) 53.