TeV scale black holes thermodynamics with extra dimensions and quantum gravity effects

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ABSTRACT: TeV scale black hole thermodynamics in the presence of quantum gravity effects encoded in the existence of a minimal length and a maximal momentum is studied in a model universe with large extra dimensions.

KEYWORDS: Generalized Uncertainty Principle, Black Hole Thermodynamics, Extra Dimensions

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1. Introduction

In the last two decades, investigations revealed that the fundamental Planck scale in model universes with compactified Large Extra Dimensions (LEDs) might be as small as a TeV [1]. This is a promising feature with hope to observe TeV-scale black holes in experiments such as the LHC. Possible production and detection of TeV-scale black holes in colliders such as the LHC provide a suitable basis to test our comprehending of black hole physics at Planck scale and quantum gravity proposal. TeV scale black holes thermodynamics in the presence of quantum gravity effects encoded in the generalized uncertainty principle admitting a minimal measurable length and also in loop quantum gravity-based modified dispersion relations has been studied extensively in recent years (see for instance [2-10]). It has been shown that the leading order correction to the standard Bekenstein-Hawking entropy-area relation is generally a logarithmic correction term. For extra-dimensional gravity at TeV-scale, this leading order correction term leads to a significant change in the possibility of formation and detection of TeV black hole in the laboratories such as the LHC. Recently and in the context of doubly special relativity it has been argued that a particle’s momentum cannot be arbitrary imprecise leading to the nontrivial assumption of existence of maximal particle’s momentum of the order of the Planck momentum [11]. Existence of a maximal particle’s momentum has significant effects on the formulation of the TeV black hole thermodynamics and changes significantly the results of studies based on just the existence of a minimal measurable length. Our goal here is to see what is the mutual effects of the existence of a maximal momentum and a minimal length on the thermodynamics of a TeV-scale black hole and how these natural cutoffs affect the possible production of black holes in experiments such as the LHC. Especially we focus on the role played by the maximal momentum in this setup. With this motivation, we consider a generalized uncertainty principle (GUP) that admits the existence of both a minimal observable length and a maximal momentum in a model universe with large extra dimensions. Then we study
TeV scale black hole thermodynamics in this setup. Especially the final stage of evaporation of these black holes through Hawking radiation is treated with details and possible relation between spacetime dimensionality and final stage thermodynamical quantities is explored. Finally the role played by maximal momentum on the detectability of TeV black hole at the LHC is discussed.

2. Minimal Length and TeV-Scale Black Hole Thermodynamics

All approaches to quantum gravity proposal support the idea that there is a fundamental length scale of the order of the Planck length, which cannot be probed in a finite time. For example, in string theory, we cannot probe distances smaller than the string length. This minimal observable length can be realized by the generalized uncertainty principle (GUP) as a generalization of the standard Heisenberg uncertainty relation in the presence of gravitational effects. Hence, the standard uncertainty principle is replaced by the GUP that reflects quantum nature of gravity at very short distances phenomenologically and can be formulated by a generalized uncertainty relation as follows

$$\Delta x_i \geq \frac{1}{2\Delta p_i} + \alpha^2 L_p^2 \Delta p_i$$

(2.1)

where the fundamental Planck length is defined as $L_p = G_d^{-\frac{1}{d-2}}$, and $G_d$ is gravitational constant in $d$-dimensional spacetime which in the ADD scenario takes the form $G_d = G_4 R^{d-4}$, that $R$ is the size of extra dimension(s). In this paper natural units are used so that $\hbar = c = k_B = 1$. In the standard limit, $\Delta x \gg L_p$, one recovers the standard uncertainty relation, $\Delta x \Delta p \geq \frac{1}{2}$. It should be noted that the correction term in GUP formula (2.1) becomes noticeable when momentum and distances scales are nearby the Planck scale. In this section we use relation (2.1) to study thermodynamics of a TeV scale Schwarzschild black hole. Our main goal here is to see the relation between spacetime dimensionality and possible explanation of the logarithmic prefactor in black hole entropy relation. In this manner, we deduce numerical results for logarithmic prefactor in terms of spacetime dimensionality.

Based on the relation (2.1), a simple calculation gives

$$\Delta p_i \simeq \frac{\Delta x_i}{\alpha^2 L_p^2} \left(1 - \sqrt{1 - \frac{\alpha^2 L_p^2 \Delta x_i^2}{\Delta x_i^2}}\right).$$

(2.2)

By a heuristic approach relying on Heisenberg uncertainty relation, one deduces the following equation for Hawking temperature of black holes [12]

$$T = \frac{d - 3}{2\pi} \Delta p_i,$$

(2.3)

which we have set the constant of proportionality equal to $\frac{(d-3)}{2\pi}$ as a calibration factor in a $d$-dimensional model (see [13] for instance). So, the modified black hole temperature based
on the GUP (2.1) becomes

\[ T = \left( \frac{d - 3}{2\pi} \right) \left( \frac{\Delta x_i}{\alpha^2 L_p^2} \right) \left[ 1 - \sqrt{1 - \frac{\alpha^2 L_p^2}{\Delta x_i^2}} \right]. \]  

(2.4)

In the vicinity of the black hole surface there is an inherent uncertainty in the position of any particle of about the Schwarzschild radius, \( r_s \). So, we can set

\[ \Delta x_i \approx r_s = \omega_d L_p m^{\frac{1}{d-3}}. \]  

(2.5)

In this relation, \( m \) and \( \omega_d \) (the dimensionless area coefficient) are given by

\[ \omega_d = \left( \frac{8\pi^{\frac{d-2}{2}} \Gamma \left( \frac{d-1}{2} \right)}{d-2} \right)^{\frac{1}{d-1}}, \quad m = \frac{M}{M_p}. \]  

(2.6)

where the fundamental Planck mass is defined as \( M_p = \frac{1}{L_p} \). Substituting equation (2.5) into equation (2.4), we find

\[ T = \left( \frac{d - 3}{2\pi} \right) \left( \frac{\omega_d m^{\frac{1}{d-3}}}{\alpha^2 L_p} \right) \left[ 1 - \sqrt{1 - \frac{\alpha^2}{\omega_d^2 m^{\frac{2}{d-3}}} \alpha^4 + O(\alpha^6)} \right]. \]  

(2.7)

To calculate an analytic form of black hole entropy, let us acquire a Taylor expansion of equation (2.7) around \( \alpha = 0 \),

\[ T \approx \left( \frac{d - 3}{16\pi L_p} \right) \left[ \frac{4}{\omega_d m^{\frac{1}{d-3}}} + \frac{\alpha^2}{\omega_d^2 m^{\frac{3}{d-3}}} \right], \]  

(2.8)

which can be rewritten as follows

\[ T \approx \left( \frac{d - 3}{16\pi L_p} \right) \left[ \frac{4}{\omega_d m^{\frac{1}{d-3}}} + \frac{\alpha^2}{\omega_d^2 m^{\frac{3}{d-3}}} \right], \]  

(2.9)

where we have saved only terms up to the second order of \( \alpha \). Now, the related entropy can be derived by the first law of black hole thermodynamics, that is, \( dS = \frac{dM}{T} \) as,

\[ S = \int_{M_{\text{min}}}^{M} dM \left( \frac{16\pi L_p}{d-3} \right) \left[ \frac{4}{\omega_d m^{\frac{1}{d-3}}} + \frac{\alpha^2}{\omega_d^2 m^{\frac{3}{d-3}}} \right]^{-1}, \]  

(2.10)

or

\[ S = \int_{M_{\text{min}}}^{M} dm \left( \frac{16\pi}{d-3} \right) \left[ \frac{4}{\omega_d m^{\frac{1}{d-3}}} + \frac{\alpha^2}{\omega_d^2 m^{\frac{3}{d-3}}} \right]^{-1}, \]  

(2.11)

where we have adopted the physical selection that the entropy vanishes at \( M_{\text{min}} \) where the black hole mass is minimized. With this selection, there is no black hole radiation for masses less than \( M_{\text{min}} \). Note that black hole temperature is indeterminate for \( M < M_{\text{min}} \), where

\[ M_{\text{min}} = \left( \frac{\alpha}{\omega_d} \right)^{d-3} M_p. \]  

(2.12)
Thus, Hawking evaporation process has to be stopped when the black hole mass reaches a Planck size remnant mass. In this situation, temperature of black hole has a maximum extremal value. We note that detailed thermodynamics of Planck size remnant is not well-known yet. In fact, the real thermodynamics of such an extreme system can be even out of the realm of standard thermodynamics. It seems that application of non-standard thermodynamics may be more reasonable at this scale (see for instance \[14\] and references therein).

Black hole entropy with some arbitrary numbers of spacetime dimensions with $\alpha = 1$ can be calculated as follows

\[
d = 4 \rightarrow S = 16\pi \left[ \frac{1}{4} M^2 - \frac{1}{64} \ln(16M^2 + 1) - \frac{1}{16} + \frac{1}{64} \ln(5) \right], \\
(2.13)
\]

\[
d = 5 \rightarrow S = 8\pi \left[ \frac{\sqrt{6}M^\frac{3}{2}}{9\sqrt{\pi}} - \frac{1}{6\pi M} + \frac{3\pi \arctan(\frac{\sqrt{6}M}{3\sqrt{\pi}})}{28} + \frac{\sqrt{144\pi}}{192} + \frac{3\pi}{64} - \frac{3\pi \arctan(2)}{128} \right], \\
(2.14)
\]

\[
d = 6 \rightarrow S = 16\pi \left[ \frac{3}{32} \left( \frac{12M^4}{\pi} \right)^\frac{1}{6} - \frac{1}{32} (18\pi M^2)^\frac{1}{6} + \frac{\pi}{64} \ln \left( 144M^2 \right)^\frac{1}{6} + (\pi^2)^\frac{1}{6} \right] \\
- \frac{\pi}{16} - \frac{5\pi^2}{192} \ln \left( 125\pi^2 \right)^\frac{1}{6} \right], \\
(2.15)
\]

\[
d = 7 \rightarrow S = 4\pi \left[ \frac{2M}{5} \left( \frac{M}{5\pi^2} \right)^\frac{1}{6} - \frac{1}{24} (5M^2\pi^2)^\frac{1}{6} + \frac{\pi}{128} (125M\pi^2)^\frac{1}{6} - \frac{5\pi^2}{512} \arctan(\frac{16M}{5\pi^2})^\frac{1}{6} \right] \\
- \frac{\pi^2}{16} - \frac{5\pi^2}{192} + \frac{5\pi^2}{256} \arctan(2), \\
(2.16)
\]

and so on. From a loop quantum gravity viewpoint, the leading order correction term to the Bekenstein-Hawking entropy of black hole has logarithmic nature [15]. Here we see that the logarithmic term is present just for even number of spacetime dimensionality (see also [16]).

The heat capacity of the black hole can also be achieved utilizing the following standard formula

\[
C = \frac{dM}{dT}. \\
(2.17)
\]

With $\alpha = 1$, we find

\[
C = (-16m\pi) \left[ \frac{4}{\omega_4 m^{d-3}} + \frac{3}{\omega_4^3 m^{d-3}} \right]^{-1}. \\
(2.18)
\]

Therefore, we were able to calculate some important thermodynamical quantities attributed to TeV black holes in a model universe with large extra dimensions where quantum gravity effects are taken into account through a generalized uncertainty principle that admits just a minimal measurable length. The physics of TeV scale black hole evaporation in the
presence of quantum gravity effects, especially existence of a minimal measurable length, powerfully support the idea that the final stage of these black holes evaporation is a stable, Planck-size remnant. These remnants may be a reliable candidate for dark matter.

Figures 1, 2 and 3 demonstrate some features of TeV black hole thermodynamics versus mass. In the minimal length GUP framework, evaporation of black hole stops once it reaches the stable Planck-size remnant and its entropy and the heat capacity in this stage are vanishing, but its temperature reaches a maximal value. So, evaporation of TeV scale black hole ends once black hole mass reaches a Planck-size remnant mass. The existence of this remnant has been considered as a possible solution to the information loss problem in black hole evaporation. It has been suggested also that these remnants may be a possible candidates for dark matter [7,16,17].

Figure 1: Temperature of black hole versus its mass in different spacetime dimensions. Mass is in the units of the Planck mass.
Figure 2: Entropy of black hole versus its mass in different spacetime dimensions.

Figure 3: Heat Capacity of black hole versus its mass in different spacetime dimensions.

In which follows we firstly present a discussion on the issue of maximal momentum and then we reconsider the issue of TeV black hole thermodynamics in a model universe with large extra dimensions and in the presence of the quantum gravity effect through a GUP that admits both a minimal observable length and a maximal particle's momentum. We focus mainly on the role played by the maximal momentum and we comment on the
production of TeV scale black hole at the LHC in the presence of these quantum gravity effects. We also compare our results with those results that are obtained by excluding the possibility of having a natural cutoff on particle’s momentum.

3. A GUP with minimal length and Maximal momentum

In the context of the Doubly Special Relativity (DSR), one can show that a test particles’ momentum cannot be arbitrarily imprecise. In fact there is an upper bound for momentum fluctuations [11]. As a nontrivial assumption, this may lead to a maximal measurable momentum for a test particle [18]. In this framework, the GUP that predicts both a minimal observable length and a maximal momentum can be written as follows [18]

\[ \Delta x \Delta p \geq \frac{1}{2} \left[ 1 + \left( \frac{\alpha}{\sqrt{\langle p^2 \rangle}} + 4\alpha^2 \right) \Delta p^2 + 4\alpha^2 \langle p \rangle^2 - 2\alpha \sqrt{\langle p^2 \rangle} \right]. \]  

(3.1)

Since \((\Delta p)^2 = \langle p^2 \rangle - \langle p \rangle^2\), by setting \(\langle p \rangle = 0\) for simplicity, we find

\[ \Delta x \Delta p \geq \frac{1}{2} \left[ 1 - \alpha(\Delta p) + 4\alpha^2(\Delta p)^2 \right]. \]  

(3.2)

This GUP that contains both minimal length and maximal momentum is the primary input of our forthcoming arguments. Before treating our main problem, first we show how maximal momentum arises in this setup. The absolute minimal measurable length in our setup is given by \(\Delta x_{\min}(\langle p \rangle = 0) \equiv \Delta x_0 = \frac{3\alpha}{2}\). Due to duality of position and momentum operators, it is reasonable to assume \(\Delta x_{\min} \propto \Delta p_{\max}\). Now, saturating the inequality in relation (3.2), we find

\[ 2(\Delta x \Delta p) = \left( 1 - \alpha(\Delta p) + 4\alpha^2(\Delta p)^2 \right). \]  

(3.3)

This results in

\[ (\Delta p)^2 - \frac{(2\Delta x + \alpha)}{4\alpha^2} \Delta p + \frac{1}{4\alpha^2} = 0. \]  

(3.4)

So we find

\[ (\Delta p_{\max})^2 - \frac{(2\Delta x_{\min} + \alpha)}{4\alpha^2} \Delta p_{\max} + \frac{1}{4\alpha^2} = 0. \]  

(3.5)

Now using the value of \(\Delta x_{\min}\), we find

\[ (\Delta p_{\max})^2 - \frac{1}{\alpha} \Delta p_{\max} + \frac{1}{4\alpha^2} = 0. \]  

(3.6)

The solution of this equation is

\[ \Delta p_{\max} = \frac{1}{2\alpha}. \]  

(3.7)

so, there is an upper bound on particle’s momentum uncertainty. As a nontrivial assumption, we assume that this maximal uncertainty in particle’s momentum is indeed the maximal measurable momentum. This is of the order of Planck momentum.
4. TeV-Scale Black Hole Thermodynamics with Natural Cutoffs

Now we focus on the TeV-scale black hole thermodynamics in the presence of both a minimal measurable length and a maximal momentum as phenomenological, natural cutoffs. By assuming a model universe with $d$ dimensions and based on the relation (3.2), by a simple calculation we find

$$\Delta p_i \simeq \left( \frac{\Delta x_i + \gamma}{4 \alpha^2} \right) \left[ 1 - \sqrt{1 - \frac{4 \alpha^2}{(\Delta x_i + \gamma)^2}} \right],$$

(4.1)

where $\gamma \equiv \alpha^2$. Since $T = \frac{d - 3}{2\pi} \Delta p_i$, we find

$$T = \left( \frac{d - 3}{2\pi} \right) \left( \frac{\Delta x_i + \gamma}{4 \alpha^2} \right) \left[ 1 - \sqrt{1 - \frac{4 \alpha^2}{(\Delta x_i + \gamma)^2}} \right]$$

(4.2)

Once again, we assume that in the vicinity of the black hole horizon surface, the inherent uncertainty in the position of any particle is of the order of Schwarzschild black hole radius, $r_s$. So we have $\Delta x_i \approx r_s = \omega d L_p m^{\frac{1}{d - 3}}$. Using this result we find

$$T = \left( \frac{d - 3}{2\pi} \right) \left( \frac{\omega d m^{\frac{1}{d - 3}} + B}{4 \alpha^2 L_p} \right) \left[ 1 - \left( \frac{4 \alpha^2}{(\omega d m^{\frac{1}{d - 3}} + B)^2} - \frac{(4 \alpha^2)^2}{8(\omega d m^{\frac{1}{d - 3}} + B)^4} \right) \right],$$

(4.3)

where $\alpha = \alpha_0 L_p$ and $B \equiv \frac{\alpha_0^2}{2}$. To obtain an analytic form for black hole entropy, we expand (4.3) in a Taylor series about $\alpha = 0$. This gives the following result up to $O(\alpha^2)$

$$T = \left( \frac{d - 3}{2\pi} \right) \left( \frac{\omega d m^{\frac{1}{d - 3}} + B}{4 \alpha^2 L_p} \right) \left[ 1 - \left( 1 - \frac{4 \alpha^2}{2(\omega d m^{\frac{1}{d - 3}} + B)^2} - \frac{(4 \alpha^2)^2}{8(\omega d m^{\frac{1}{d - 3}} + B)^4} \right) \right],$$

(4.4)

which can be rewritten as

$$T = \left( \frac{d - 3}{16\pi L_p} \right) \left[ \frac{4}{(\omega d m^{\frac{1}{d - 3}} + B)} + \frac{A}{(\omega d m^{\frac{1}{d - 3}} + B)^3} \right],$$

(4.5)

where we have set $A \equiv 4 \alpha^2_0$. This result gives the modified black hole temperature based on GUP (3.2). To compute entropy of TeV black hole in this setup, we use the first law of classical black hole thermodynamics $ds = \frac{dM}{T}$ but now with modified temperature. We find

$$S = \int_{M_{min}}^{M} dm \left( \frac{16\pi}{d - 3} \right) \left[ \frac{4}{(\omega d m^{\frac{1}{d - 3}} + B)} + \frac{A}{(\omega d m^{\frac{1}{d - 3}} + B)^3} \right]^{-1},$$

(4.6)

In the same way as the previous section, we adopt the physical selection that the evaporation process terminates when the mass of the radiating black hole reaches a Planck-size remnant of mass $M_{min}$. In this phase the entropy of black hole vanishes while its temperature reaches a maximum, finite value. In which follows we present the entropy of TeV black hole for some spacetime dimensions for $\alpha = 1$

$$d = 4 \rightarrow S = 16\pi \left[ \frac{1}{4} M^2 + \frac{1}{8} M - \frac{1}{16} \ln(16M^2 + 8M + 5) - \frac{1}{8} + \frac{1}{16} \ln(13) \right]$$

(4.7)
\[d = 5 \rightarrow S = 8\pi \left[ \frac{\sqrt{6}M^{\frac{7}{2}}}{9\sqrt{6\pi}} + \frac{M}{8} - \frac{1}{8}\sqrt{6\pi M} + \frac{3}{64}\pi ln(32M + 8\sqrt{6\pi M} + 15\pi) + \frac{3}{16}\pi \arctan \left( \frac{64\sqrt{M} + 8\sqrt{6\pi})}{96\sqrt{6\pi}} \right) - \frac{\pi \sqrt{144}}{192} \right. \]
\[+ \frac{9\pi}{64} - \frac{3\pi}{64} ln(39\pi) - \frac{3\pi}{16} \arctan \left( \frac{3}{2} \right) \right] \tag{4.8}\]

\[d = 6 \rightarrow S = \frac{16\pi}{3} \left[ \frac{3}{32} \left( \frac{12M^4}{\pi} \right)^{\frac{1}{2}} + \frac{M}{8} - \frac{1}{8}(18\pi M^2)^{\frac{1}{2}} + \frac{1}{8}(12\pi^2 M)^{\frac{1}{2}} \right. \]
\[+ \frac{3\pi}{16} ln \left( \frac{144M^2}{\pi^2} \right)^{\frac{1}{2}} + \left( \frac{96M}{\pi} \right)^{\frac{1}{2}} + 5 \right) - \frac{\pi}{2} \arctan \left( \frac{8M^{\frac{1}{2}} + (\frac{2\pi}{3})^\frac{1}{2}}{2\sqrt{2}} \right) - \frac{3}{8} + \frac{\pi}{6} - \frac{3\pi}{6} ln(13) + \frac{\pi}{2} \arctan \left( \frac{108\frac{1}{3}}{2\sqrt{2}} \right) \right] \tag{4.9}\]

\[d = 7 \rightarrow S = 4\pi \left[ \frac{2}{25} \left( \frac{125M^6}{\pi^2} \right)^{\frac{1}{2}} + \frac{M}{8} - \frac{1}{6}(5\pi^2 M^3)^{\frac{1}{2}} + \frac{1}{16}\sqrt{5\pi^2 M} \right. \]
\[+ \frac{3}{32} \left( 125\pi^6 M \right)^{\frac{1}{2}} - \frac{55\pi^2}{256} ln \left( \frac{16\sqrt{5M}}{\pi} + 8 \left( \frac{125M}{\pi} \right)^{\frac{1}{2}} + 25 \right) - \frac{71\pi^2}{384} \right. \]
\[ - \frac{5\pi^2}{64} \arctan \left( \frac{4(125M)^{\frac{1}{2}} + 5}{10} \right) + \frac{55\pi^2}{256} ln(65) + \frac{5\pi^2}{64} \arctan \left( \frac{1}{2} \right) \right] \tag{4.10}\]

and so on.

The heat capacity of black hole can be computed by using the relation \( C = \frac{dM}{dT_H} \)

\[\alpha = 1 \rightarrow C = -\frac{16\pi}{\omega_d} m^{\frac{d-4}{2}} \left[ \frac{4}{(\omega_d m^{\frac{d-1}{2}} + \frac{1}{2})^2} + \frac{12}{(\omega_d m^{\frac{d-1}{2}} + \frac{1}{2})^4} \right]^{-1} \tag{4.11} \]

Hence we were able to compute some important thermodynamical properties of black holes by utilizing a new version of GUP admitting both a minimal length and a maximal momentum. The main ingredient of this analysis is the existence of a stable, Planck-size remnant. Figures 4, 5 and 6 show the temperature, entropy and heat capacity of the black hole versus its mass in the presence of both minimal length and maximal momentum. Similar to the previous section, the evaporation process of black hole ends up in a stable, Planck-size remnant with vanishing entropy and heat capacity but its temperature reaches a maximal value. We focus on the role played by the natural cutoff on momentum in the next section.
Figure 4: Temperature of black hole versus its mass in the presence of both minimal length and maximal momentum in different spacetime dimensions. Mass is in the unit of the Planck mass.

Figure 5: Entropy of black hole versus its mass in the presence of both minimal length and maximal momentum in different spacetime dimensions.

5. Discussion and Results

The final stage of a TeV scale black hole evaporation in the presence of quantum gravity effect encoded in the existence of a minimal measurable length is a stable remnant. In
in the presence of both minimal length and maximal momentum in different spacetime dimensions. This phase the entropy and heat capacity are zero, but the temperature reaches a maximal value that depends explicitly on the spacetime dimensionality. This temperature increases when the dimensionality of spacetime increases. When we incorporate also the Doubly Special Relativity motivated maximal momentum as a natural cutoff to a test particle’s momentum, the overall behavior of thermodynamical quantities are the same as the case with just a minimal length, but now the maximum value of temperature in final stage of evaporation is less than the case with just a minimal length cutoff. This is physically reasonable since existence of high momentum cutoff suppresses the contribution of highly excited states that were not forbidden in ordinary situation. The black hole remnant in a model universe with large extra dimensions is hotter than its 4-dimensional counterpart. On the other hand, by increasing the number of extra dimensions, the minimum mass of black hole remnant increases. This means that possibility of creation and detection of black holes in the LHC or any high energy physics laboratory decreases by increasing the number of spacetime dimensions. This is because the minimum energy for creation of black hole in extra dimensions increases.

In figures 7, 8 and 9 we compare thermodynamical quantities of the black hole computed once with just a minimal length and in the other case with both minimal length and maximal momentum to highlight the role played by natural cutoff on particle’s momentum. Figure 7 shows that the temperature of black hole decreases by considering the maximal momentum. Figure 8 shows that by increasing the number of spacetime dimensionality, the minimum mass increases. Also this figure shows that as a result of natural cutoff on momentum, entropy of the black hole increases relative to the case that this cutoff is ignored. As figure 9 shows, the heat capacity of black hole increases in magnitude when we consider the effect of momentum natural cutoff. In summary, when we consider both minimal length and maximal momentum, $M_{\text{min}}$ is smaller than the case that we consider just
Figure 7: Temperature of black hole versus its mass in different spacetime dimensions. Mass is in the unit of the Planck mass (solid lines for minimal length GUP and dashed-lines for minimal length and maximal momentum GUP).

Figure 8: Entropy of black hole versus its mass in different spacetime dimensions (solid lines for minimal length GUP and dashed-lines for minimal length and maximal momentum GUP).

the minimal length. This means that possibility of creation and detection of black holes at the LHC increases in this situation. We note that all previous studies in this direction were neglected the possible existence of a natural cutoff on a test particle’s momentum. Here we considered this phenomenological aspect of quantum gravity and we have shown that it leads to more probable creation and detection of TeV scale black holes at the LHC. Tables
**Figure 9:** Heat Capacity of black hole versus its mass in different spacetime dimensions (solid lines for minimal length GUP and dashed-lines for minimal length and maximal momentum GUP).

**Table 1:** GUP-corrected maximum temperature and minimum mass of black hole for $\alpha = 1$ in scenarios with large extra dimensions. Mass is in units of the Planck mass and temperature is in units of the Planck energy (supposing $M_p = 1$ TeV).

| $d$ | $M_{\alpha=1}^\text{min}(\text{TeV})$ | $T_{\alpha=1}^\text{max}(\text{TeV})(\text{min})$ | $T_{\alpha=1}^\text{max}(\text{TeV})(\text{minmax})$ |
|-----|-----------------------------------|------------------------------------------|------------------------------------------|
| 4   | 0.5                               | 0.1                                      | 0.08                                     |
| 5   | 1.18                              | 0.2                                      | 0.15                                     |
| 6   | 2.09                              | 0.3                                      | 0.23                                     |
| 7   | 3.08                              | 0.4                                      | 0.31                                     |
| 8   | 3.94                              | 0.5                                      | 0.38                                     |
| 9   | 4.51                              | 0.6                                      | 0.46                                     |
| 10  | 4.72                              | 0.7                                      | 0.54                                     |
| 11  | 4.56                              | 0.8                                      | 0.61                                     |

min: GUP with just a minimal length

minmax: GUP with both minimal length and maximal momentum

1 and 2 summarize our results for $\alpha = 1$ and $\alpha = \frac{2}{\pi}$. 
Table 2: GUP-corrected maximum temperature and minimum mass of black hole for $\alpha = \frac{2}{\pi}$ in scenarios with large extra dimensions. Mass is in units of the Planck mass and temperature is in units of the Planck energy (supposing $M_p = 1$ TeV).

| $d$  | $M_{\text{min}}^{\alpha=\frac{2}{\pi}} (\text{TeV})$ | $T_{\text{max}}^{\alpha=\frac{2}{\pi}} (\text{TeV})(\text{min})$ | $T_{\text{max}}^{\alpha=\frac{2}{\pi}} (\text{TeV})(\text{minmax})$ |
|------|----------------------------------------------------|-------------------------------------------------|-------------------------------------------------|
| 4    | 0.32                                               | 0.16                                            | 0.12                                            |
| 5    | 0.48                                               | 0.31                                            | 0.25                                            |
| 6    | 0.54                                               | 0.47                                            | 0.37                                            |
| 7    | 0.51                                               | 0.62                                            | 0.5                                             |
| 8    | 0.41                                               | 0.78                                            | 0.62                                            |
| 9    | 0.3                                                | 0.94                                            | 0.74                                            |
| 10   | 0.2                                                | 1.09                                            | 0.87                                            |
| 11   | 0.12                                               | 1.25                                            | 0.99                                            |

References

[1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 429, 263 (1998)  
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 436, 257 (1998)  
L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999).

[2] K. Nozari and A. S. Sefiedgar, Gen. Relat. Gravit. 39, 501 (2007).

[3] A. J. M. Medved, Class. Quant. Grav. 22, 133 (2005)  
G. Gour and A. J. M. Medved, Class. Quant. Grav. 20, 3307 (2003)  
A. J. M. Medved, Class. Quant. Grav. 20, 2147 (2003).

[4] R. J. Adler $et$ $al$, Gen. Rel. Grav. 33, 2101 (2001).

[5] K. Nozari and S. H. Mehdipour, Mod. Phys. Lett. A 20, 2937 (2005).

[6] G. Amelino-Camelia, M. Arzano, Y. Ling and G. Mandanici, Class. Quant. Grav. 23, 2585 (2006).

[7] K. Nozari and S. H. Mehdipour, Europhys. Lett. 84, 20008 (2008)  
K. Nozari and S. H. Mehdipour, Class. Quant. Grav. 25, 175015 (2008).

[8] W. Kim, E. J. Son and M. Yoon, JHEP 0801, 035 (2008).

[9] L. Xiang and X. Q. Wen, JHEP 0910, 046 (2009).

[10] R. Banerjee and S. Ghosh, Phys. Lett. B, 688, 224 (2010).

[11] J. Magueijo and L. Smolin, Phys. Rev. Lett. 88, 190403 (2002)  
J. Magueijo and L. Smolin, Phys. Rev. Lett. D 67, 044017 (2003)  
J. Magueijo and L. Smolin, Phys. Rev. D 71, 026010 (2005)  
J. L. Cortes and J. Gamboa, Phys. Rev. D 71, 065015 (2005).

[12] H. Ohanian and R. Ruffini, *Gravitation and Spacetime*, 2nd edition (W. W. Norton) 1994, p. 481.

[13] M. Cavaglià and S. Das, Class. Quant. Grav. 21, 4511 (2004)  
M. Cavaglià, $et$ $al$, Class. Quant. Grav. 20, L205 (2003).
[14] K. Nozari and S. H. Mehdipour, Chaos Solitons and Fractals, 39, 956 (2009).

[15] A. J. M. Medved, Elias C. Vagenas, Phys. Rev. D 70, 124021 (2004).

[16] K. Nozari and A. S. Sefiedgar, Gen. Relat. Gravit. 39, 501 (2007)
K. Nozari, Astrophys. Space Science, DOI 10.1007/s10509-012-1078-6

[17] K. Nozari, Astropart. Phys. 27, 169 (2007).

[18] A. F. Ali, S. Das and E. C. Vagenas, Phys. Rev. D 84, 044013 (2011)
P. Pedram, K. Nozari and S. H. Taheri, JHEP 1103, 093 (2011)
K. Nozari and A. Etemadi, Phys. Rev. D 85, 104029 (2012) [arXiv:1205.0158].