Regulation of the stop points throughput capacity in urban public transport

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Abstract. The article considers the problem of determining the throughput capacity of bus lines, which is usually determined by the limited throughput capacity of stopping points. Currently, the throughput capacity of a stop point is usually defined as the inverse mathematical expectation of the average service time for one transportation vehicle. To account such factors as random disturbances in the transport process, appropriate correction coefficients are applied. However, it has not been proved that the considered coefficients provide acceptable results for practice in all cases. In this article, in order to study the throughput capacity, the work of a stopping point is described as a multi-channel single-phase queuing system (QS). An algorithm is proposed for simulating a public transport stopping point model based on the principle of consecutive submitting requests. The recommendations formulated for the throughput capacity regulation of stop points are illustrated by using an example of a specific stop point.

1. Introduction

One of the most important parameters of the transport system is the throughput capacity of lines, which is usually determined by the stop points throughput capacity in urban public transport system [1, 2, 3, 4, 5]. The throughput capacity of a bus lane depends on the critical stop points throughput capacity. As a rule, it is the stop point with the largest passenger traffic [2].

Currently, the throughput capacity of a stop point is usually defined as the inverse mathematical expectation of the average service time for one transportation vehicle. To account such factors as random disturbances in the transport process, appropriate correction coefficients are applied. For example, in the work [6] the coefficient of the incoming request flow unevenness is used, which is defined as the ratio of the maximum recorded intensity of the transportation vehicles flow to the average intensity for the calculation period. In the work [2, see 27-13] the coefficient of service request refusal probability and the coefficient of passengers service time variation at the stop point are used. However, it has not been proved that the considered coefficients provide acceptable results for practice in all cases.

In the works [5,6,8,9,10] it was established that the stopping places at the stop points have unequal productivity: the first place has the highest productivity, after that the productivity of each subsequent place decreases. Therefore, the concept of “effective number of stopping places” is applied, which corresponds to the conditional stopping places number with the throughput capacity of the first stopping place.
2. Problem description
In order to study the throughput capacity of a stop point, we will describe it as a multi-channel single-phase queuing system (QS) (figure 1). To the system, a flow of service requests with intensity $\lambda$ arrives (passengers boarding and alighting the vehicles). The service channel is the stopping place of the fleet at the stop point. $\mu_i$ - performance (service intensity) of the i channel (stop place). We will not impose restrictions on the queue length and waiting time in this QS.

![Figure 1. Diagram of a stop point as a QS](Image)

The performance (intensity of service) of the service channel has a random character, which is described by one of the statistical laws, for example, gamma distribution, log-normal distribution [5], exponential distribution [7, 8], Erlang distribution [3], etc.

Given the unequal performance of stopping places, the flow of service requests has an intensity:

$$\mu = \sum_{i=1}^{m} \mu_i = k_m \mu',$$  

(1)

where:
- $m$ - number of occupied service channels;
- $\mu_i$ - performance (service intensity) of the i channel;
- $k_m$ - reduction factor to the first channel intensity of service ($\mu'$) with n occupied service channels ($k_1 = 1; k_2 \leq 2; ...$).

Previous studies have shown that the random process of service requests in the QS under consideration is not the simplest, since it is described not by an exponential distribution, but by a gamma distribution. In this regard, to calculate the parameters of the QS we use simulation modeling [10,11,12].

3. Problem solution
We formulate the methodology of the simulating model of public transport stop point, based on the principle of consecutive requests submission [10].

There are two random processes in the system: requests received and requests served.

The time to submit $i$ request is calculated as follows:

$$t_i^\lambda = t_{i-1}^\lambda + \tau_i^\lambda,$$

(2)

$\tau_i^\lambda$ - random time interval between submitted requests in the system, determined in accordance with the given distribution law.

The time to serve the request completion:

$$t_j^\mu = t_i^b + \tau_j^\mu,$$

(3)
where: \( t_j^b \) - service start time (submit a service request);

\( \tau_j^\mu \) - random required time to serve request, determined in accordance with the given distribution law.

Figure 2 shows a diagram of the change in the state of the QS. Events that change the state of the system are the submission and completion of service requests. A queue in the system occurs if the number of unserved requests exceeds the number of service channels. The requests set A located in the system is described by the following relation:

\[
A(I,T^i,T^s,T^o),
\]

Where: \( I \) - Id service request;

\( T^i,T^s,T^o \) - time of the request submission, inception of the request service and request service completion.

In the system \( n \) service channels, \( t_j^\mu \) - The time required for \( j \) service channel to become available (\( j = \overline{1,n} \)). For each channel, there are specific statistical laws of submitting and serving requests.

The simulation algorithm for the stop point functioning:

1. System initialization:
   - number of service channels;
   - the distribution law of the requests’ incoming flow;
   - the distribution law of required time to serve request for each channel;
   - time for channel to become available is set to 0 (all channels are free).

2. To determine the random point time (appropriately distributed) when the request is submitted, the calculation of requests cortege attributes is carried out \( a_k \) as follows:

2.1. time of request submission

\[
t_k^i = t_i.
\]

2.2. Determining the time when serve the request is begun. To do this, select the service channel with the least time to become available \( t_j^\mu = Min(t_i^\mu), i = \overline{1,n} \).

Thus, the service start time:

\[
\tau_k^a = \begin{cases} 
\tau_k^i, & \text{if } \tau_k^i \geq \tau_j^\mu \\
\tau_j^\mu, & \text{if } \tau_k^i < \tau_j^\mu
\end{cases}
\]

(5)

2.3. To determine the time of service request completion.

\[
t_k^o = t_k^s + \tau_j^\mu,
\]

(6)

where: \( \tau_j^\mu \) - the required time to serve request at \( j \) channel, determined in accordance with a specific random law.

time for \( j \) channel to become available \( t_j^\mu = t_k^o \).

3. Verification of the simulation completion condition \( t_i > T \). If the condition is met, the simulation is completed; start calculate simulation results.

Otherwise, go to step 2.

Calculation of the functioning parameters of QS according to the results of simulation modeling is carried out as follows:

1. The probability of requests absence in the system:
\[ P_0 = \frac{T_0}{T}, \]  

where:  
\( T_0 \) - the total time where the system was in the state  \( S_0 \)  during the simulation;  
\( T \) - total duration of the simulation.  

2. Queue probability in QS:  
\[ P_{nt} = \sum_{i>n} T_i / T, \]  

where:  
\( T_i \) - the total time where the system was in the state  \( S_i \).  

3. Average queue length:  
\[ l_{nt} = \frac{\sum_{i>n} iT_i}{\sum_{j>n} T_i}, \]  

4. The total time when the system was in the state  \( S_i \)  
\[ T_i = \sum_{s_{j-1}, j=i} (t_j - t_{j-1}), \]  

4. Results analysis  
Using the example of a specific stop point, we will formulate recommendations for its throughput capacity regulation. Based on the request service time sample, the parameters of the vehicles random service process at the stop point are given in table (1). It was found that, as in other studies, service channels have unequal performance: The average service time for a request increases with increasing channel number. The random process submits to gamma distribution.  

![Diagram of changes in the state of the QS (with three channels)](figure2.png)  
where:  
\( S_j, j = 1, \infty \) - QS status determined by the number of requests in the system;
For certainty, we consider the case of Poisson flow of requests arriving at the QS under consideration, although in general terms the proposed simulation model is applicable for any distribution law of random vehicles flow arriving to a stop point.

We will use the following machine algorithm to simulate random processes of submitting requests and their serving [8]:

\[
x = \sum_{i=1}^{b} \left( -\frac{1}{\lambda} \ln R_i \right) = -\frac{1}{\lambda} \ln \left( \prod_{i=1}^{b} R_i \right),
\]

(11)

where: \( R_i \) - random numbers with uniform distribution.

We will use this machine algorithm (11) for both exponential (the simplest random process) and gamma distribution. In the case of the exponential distribution, \( b = 1 \).

**Table 1.** The distribution parameters of serving vehicles time at the stop point

| Distribution parameters | Rate       | Calculated value |
|-------------------------|------------|-----------------|
| Alpha (a)               | 9.098      | 8.9, 9.2, 9.6   |
| Beta (b)                | 4.893      | 5.0, 5.0, 5.0   |
| Expected value, \( \kappa \) | 44.51      | 46.22, 48.10   |
| Standard deviation      | 14.6       |                |
| Dispersion              | 218.0      |                |

Considering the simulation results of a stop point with varying the number of stopping places from 1 to 3. Figure 3 shows the dependences of the probability of the requests absence in the system \( P_o \) and the queue occurrence probability \( P_s \) for the considered options of stop point when the requests intensity varies from 5 requests per hour to the throughput capacity limit. The following limit values are obtained for the throughput capacity of the considered configuration options for a stop point: 80 units per hour for one stopping place, 155 units per hour for two places and 230 units per hour for three places.

**Figure 3** - Parameters of the stop point functioning for a different number of stopping places (from 1 to
Therefore, in practice, the application of the throughput maximum capacity of a stop point is unreasonable. At the stop point, a line of vehicles will constantly be present, which will block the road network. Wherefore, the stopping point must have a reserve of throughput capacity for the smooth functioning of the routes network.

Obviously, the ideal (most effective) mode of functioning is the absence of a queue at the stop point. The lines of vehicles at the stop point lead to negative processes that significantly reduce the throughput capacity of the road network and the level of road safety. It is proposed to use the criterion of the queue absence to determine the stop point throughput capacity by using standard statistical probability levels (values) 10%, 5% and 1% of queue occurrence. A similar approach was applied in HCM 2010 [2], which regulate the probability of vehicle service denial (all stopping places are occupied) under the condition of normal distribution of passenger service time.

Table 3 shows the parameters of the throughput capacity of a stop point, obtained by the results of simulation, for the configuration of a stop point with one, two or three stopping places and probability levels 0.01, 0.05 and 0.1 of queue occurrence. The probability of the requests absence in the system \( P_o \) and the probabilities of the queue of two \( P_{s2} \) and three \( P_{s3} \) and four \( P_{s4} \) requests are given. From the table we can see that the throughput capacity of a stop point at a probability level of queue occurrence \( P_{s1} = 0.01 \) ensures the functioning of the stop point virtually without queues. Only 1% of the time at the stop point will be a queue, the length of which does not exceed one vehicle. At the probability level of queue occurrence \( P_{s1} = 0.05 \) in the 1% of the time system, a queue of two requests will be observed. At the \( P_{s1} = 0.1 \) assumes the system will operate with a queue in 10% of the time, at 2-3% of the time the queue will consist of two requests, and at 1% of the time - from three requests.

Thus, the throughput capacity of the system, based on their probability level of queue occurrence at 1%, should be established for cases where the queue of vehicles at the stop point has a significant effect on road traffic. For example, if the stop point is located immediately after the intersection, there is no possibility of forming a queue of vehicles on the road network. The probability level of queue occurrence of 10% is recommended to be used in cases where the queue of vehicles at a stop point does not lead to the occurrence of congestion on the road network.

To formulate the procedure for regulation the throughput capacity of stop points during the formation (optimization) of the transit program (Public transit supply):

- determine a list of critical stop points (stop points with the highest passenger traffic) for each of the directions of the route network;
- run a survey of these stop points to calculate the parameters of random processes of submitting and serving requests;
- use the analytical or simulation model, to calculate the throughput capacity parameters of the considered stop points;

Evaluate the calculation results for routes with insufficient stop points throughput capacity. The decision to be made is to adjust the transit supply, increase stopping places or reduce traffic intensity by increasing the capacity of fleet.
Table 2. Parameters of the stop point throughput capacity

| Intensity, unit/h | Probability of requests absence $P_o$ | Probability of queue occurrence |
|------------------|---------------------------------------|---------------------------------|
|                  | One or more requests $P_{s1}$ | Two or more requests $P_{s2}$ | Three or more requests $P_{s3}$ | Four or more requests $P_{s4}$ |
| One place        |                                |                                |                                |                                |
| 11               | 0.872                           | 0.01                           | 0.001                          | 0.000                          |
| 22               | 0.726                           | 0.05                           | 0.008                          | 0.001                          |
| 31               | 0.617                           | 0.10                           | 0.024                          | 0.006                          |
| Two places       |                                |                                |                                |                                |
| 34               | 0.641                           | 0.01                           | 0.001                          | 0.000                          |
| 57               | 0.469                           | 0.05                           | 0.012                          | 0.003                          |
| 74               | 0.358                           | 0.10                           | 0.031                          | 0.010                          |
| Three places     |                                |                                |                                |                                |
| 62               | 0.447                           | 0.01                           | 0.002                          | 0.000                          |
| 100              | 0.256                           | 0.05                           | 0.015                          | 0.004                          |
| 120              | 0.189                           | 0.10                           | 0.037                          | 0.014                          |

5. Conclusion
1. The developed mathematical model, which consider the stop point of urban public transport as a multi-channel single-phase queuing system, allows us to determine the dependence of the stop point throughput capacity on the number of stopping places and the parameters of the random process of fleet serving, furthermore, regulate the intensity of fleet traffic over network sections.
2. The methodology for regulating the throughput capacity of urban public transport stop points, based on modeling the processes of boarding and alighting passengers by using the apparatus of queuing theory by means of the proposed method of simulation, allows us to solve this problem with various variants of the considered random processes.
3. According to the results of the calculations, it is recommended that the stop point throughput capacity is set according to the criterion of the queue absence, using standard statistical levels (values) of 10%, 5% and 1%.

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