Resurrecting power law inflation in the light of Planck results

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It is well known that a canonical scalar field with an exponential potential can drive power law inflation (PLI). However, the tensor-to-scalar ratio in such models turns out to be larger than the stringent limit set by recent Planck results. Power law inflation can also be realized in a k-inflation model such as the one proposed by Armendariz-Picon \textit{et al.} \cite{1}. Although, the scalar spectral index and the tensor-to-scalar ratio for this model are within Planck limits, it is difficult to reconcile the large value of the non-gaussianity parameter $f_{\text{NL}}^\text{equil}$ in this model with the bounds set by the Planck data. We propose a new model of power law inflation for which the scalar spectra index, the tensor-to-scalar ratio and the non-gaussianity parameter $f_{\text{NL}}^\text{equil}$ are in excellent agreement with Planck results. Inflation, in this model, is driven by a non-canonical scalar field with an \textit{inverse power law} potential. The Lagrangian for our model is structurally similar to that of a canonical scalar field and has a power law form for the kinetic term. A simple extension of our model resolves the graceful exit problem which usually afflicts models of power law inflation.

I. INTRODUCTION

It is widely believed that the early universe underwent a brief ‘inflationary phase’ during which its expansion rapidly accelerated. The inflationary paradigm was originally suggested to resolve the horizon, flatness and singularity problems of Big Bang cosmology \cite{2}. An attractive byproduct of the paradigm was its ability to provide an elegant causal mechanism to seed the formation of large scale structure in the universe \cite{3}. An important early prediction of inflation – that of a nearly scale invariant spectrum of perturbations – has been spectacularly confirmed by experiments of the Cosmic Microwave Background (CMB) including the current Planck mission \cite{4, 5}. Indeed, increasingly accurate measurements of the CMB have significantly reduced the number of theoretical models in the inflationary zoo \cite{6–12}, and one hopes that current and future CMB experiments will further help in pointing the way forward for inflationary model building.

An inflationary model is usually characterized by observables including: (1) the scalar spectral index $n_s$, (2) tensor to scalar ratio $r$, (3) the non-gaussianity parameter $f_{\text{NL}}$. Other important inflationary parameters include the running of scalar spectral index $n_{\text{run}} \equiv d n_s / d \ln k$ and the spectral index for tensor perturbations $n_t$. Recent CMB data from Planck combined with the large angle polarization data from the Wilkinson Microwave Anisotropy Probe (WMAP) (henceforth Planck+WP) place strong bounds on these parameters: $n_s = 0.9603 \pm 0.0073$ and $r < 0.11$ at 95\% CL \cite{4}. Since Planck data does not indicate any statistically significant running of the spectral index \cite{4}, it is interesting to investigate the viability of power law inflation for which $n_{\text{run}}$ \textit{identically vanishes}. Although, there do exist other models with constant $n_s$ \cite{13}, our analysis will be confined to the simplest case, \textit{viz.} power law inflation.

Power law inflation (PLI) with $a \propto t^q$, $q > 1$ arises in the context of a canonical scalar field with an exponential potential \cite{14–16}. The tensor-to-scalar ratio in canonical PLI, originally determined in \cite{17}, turns out to be larger than the limits set by Planck data, which rules out this class of models. PLI can also arise in a K-inflation scenario such as the one proposed by Armendariz-Picon \textit{et al.} \cite{1}. This model has acceptable $n_s$ and $r$ but still comes into tension with Planck data \cite{18} on account of the large value of the non-gaussianity parameter $f_{\text{NL}}^\text{equil}$, determined in the equilateral limit \cite{19}.

In this paper we propose a new PLI model in which inflation is driven by a non-canonical scalar field \cite{20}. In this model observables such as $n_s$, $r$ and $f_{\text{NL}}^\text{equil}$ lie well within the limits set by the Planck data.

This paper is organized as follows. Sec. II briefly reviews power law inflation, with Sec. II A focussing on canonical PLI while Sec. II B discusses PLI based on K-inflation. Our new model, based on a non-canonical Lagrangian with an \textit{inverse power law} potential, is described in Sec. III. It is well known that PLI models suffer from a \textit{graceful exit problem} since inflation continues forever in such models precluding the possibility of reheating. Our new model does not share this drawback since it can accommodate a graceful exit, as demonstrated in Sec. IV. Our main results are summarized in Sec. V.

II. POWER LAW INFLATION

In a spatially flat Friedmann-Robertson-Walker (FRW) universe

$$ds^2 = dt^2 - a^2(t) \left[ dx^2 + dy^2 + dz^2 \right],$$

power law expansion corresponds to

$$a(t) \propto t^q,$$
where \( q > 1 \) for power law inflation (PLI). The slow roll parameters \( \varepsilon, \delta \) are defined as

\[
\varepsilon \equiv -\frac{\dot{H}}{H^2}, \quad \delta \equiv \varepsilon - \frac{\dot{\varepsilon}}{2H\varepsilon},
\]

(3)

where \( H \equiv \dot{a}/a \). It is easy to see that for power law expansion \( H = q/t \) so that

\[
\delta = \varepsilon = q^{-1}.
\]

(4)

Slow roll PLI corresponds to \( \varepsilon << 1 \) which occurs when \( q >> 1 \).

The fact that \( \varepsilon \) is identically constant could be viewed as a drawback of PLI since it makes exit from inflationary expansion difficult. PLI models are therefore somewhat incomplete since an exit mechanism needs to be added in order for a decelerating phase to succeed inflation. A resolution of this quandary, in the form of a reasonable exit mechanism, will be discussed later in Sec. IV. For the moment we shall assume that such an exit mechanism will not significantly alter the power law nature of the solution (2) during inflation.

This paper shall focus on the action

\[
S[\phi] = \int d^4x \sqrt{-g} L(X, \phi),
\]

(5)

where the Lagrangian density \( L(\phi, X) \) can, in general, be an arbitrary function of the field \( \phi \) and the kinetic term

\[
X = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi.
\]

(6)

For a generic \( L(\phi, X) \), besides the two slow roll parameters (3) it is convenient to introduce a third slow roll parameter \( \sigma \) defined as [21]

\[
\sigma = \frac{\dot{c}_s}{H c_s},
\]

(7)

where \( c_s \) is the speed of sound of the scalar field [22]

\[
c_s^2 \equiv \left[ \frac{\partial L/\partial X}{\partial L/\partial \dot{X}} + (2X)(\partial^2 L/\partial X^2) \right].
\]

(8)

Slow roll inflation requires not only \( \varepsilon << 1 \) and \( |\delta| << 1 \) but also \( |\sigma| << 1 \). For the canonical scalar field the value of \( \sigma \) is identically zero and this is also the case for kinetically driven PLI [1] as well as the non-canonical model [20] studied in this paper. Some of the results of this paper will remain valid for any PLI model based on the action (5) but for which \( \sigma \) identically vanishes. The PLI scenario for the case when \( \sigma \) is non-zero but \( |\sigma| << 1 \) will also be briefly mentioned in this paper.

Broadly speaking, the functional form of the Lagrangian density \( L(\phi, X) \) can be divided into two types: (1) \( L(\phi, X) = F(X) - V(\phi) \) and (2) \( L(\phi, X) = F(X)V(\phi) \), although other possibility also exist, for instance DBI models [23]. Canonical scalar field models are the simplest possible formulation of the first type with \( L(\phi, X) = X - V(\phi) \), while the model proposed by Armendariz-Picon et al. [1] is of the second type. Our non-canonical model [20] is of the first type with \( F(X) \propto X^\alpha \). Power law inflation, within the framework of these three scalar field models, is discussed below.

### A. Power law inflation from a canonical scalar field

It is well known that a canonical scalar field with the Lagrangian density [15]

\[
L(X, \phi) = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V_0 \exp \left( -\sqrt{\frac{g}{q}} \left( \frac{\phi}{M_p} \right) \right),
\]

(9)

can drive power law expansion with \( a \propto t^q \), in a spatially flat FRW space-time (\( M_p \equiv 1/\sqrt{8\pi G} \)).

The power spectrum of scalar curvature perturbation \( R_k \) is defined as

\[
P_S(k) = \left( \frac{k^3}{2\pi^2} \right) |R_k|^2,
\]

(10)

while the tensor power spectrum is

\[
P_T(k) = 2 \left( \frac{k^3}{2\pi^2} \right) |h_k|^2,
\]

(11)

where \( h \) is the amplitude of the tensor perturbation. The scalar spectral index \( n_s \) and the corresponding spectral index for the tensor perturbations are defined as

\[
n_s - 1 = \frac{\ln P_S}{\ln k},
\]

(12)

\[
n_T = \frac{\ln P_T}{\ln k}.
\]

(13)

Furthermore, the tensor-to-scalar ratio is defined as

\[
r = \frac{P_T}{P_S}.
\]

(14)

For the canonical PLI model (9), in the slow roll limit, one finds

\[
n_T = n_s - 1 \simeq -\frac{2}{q} \quad (15)
\]

\[
r \simeq \frac{16}{q} \quad (16)
\]

Note that when slow roll is not assumed one gets \( n_s = 1 - 2/(q - 1) \). Planck data in combination with the large angle polarization data from WMAP requires the value of the scalar spectral index \( n_s \) to lie in the range \([0.945 - 0.98]\) at 95% CL [4]. This restricts \( q \) in the power law solution \( a(t) \propto t^q \) to the range

\[
38 \lesssim q \lesssim 101. \quad (17)
\]

With \( q \) within the above range, the tensor-to-scalar ratio \( r = 16/q \) lies in the range \( 0.16 < r < 0.43 \). This is well above the limit set by Planck data which indicate \( r < 0.12 \) at 95% CL when BAO data is also included [4]. Therefore one concludes that PLI based on a canonical scalar field with an exponential potential is in considerable tension with Planck data.
B. Power law k-inflation (K-PLI)

As shown by Armendariz-Picon et. al. [1], power law inflation, $a \propto t^q$, can also arise in a K-Inflation model governed by the Lagrangian density:

$$\mathcal{L}(X, \phi) = f(\phi) (-X + X^2),$$

(18)

where $f(\phi) \propto \phi^{-2}$. During PLI the kinetic term $X$ is a constant

$$X = \frac{2 - \gamma}{4 - 3\gamma},$$

(19)

where

$$\gamma = 1 + w \equiv \frac{2}{3q},$$

(20)

$w$ being the equation of state parameter for the scalar field. Substituting (18) and (19) into the expression for the speed of sound in (8), one gets

$$c_s^2 = \frac{\gamma}{8 - 3\gamma} \equiv \frac{1}{3(4q - 1)},$$

(21)

while the slow roll parameter $\varepsilon$ defined in Eq. (3), is given by $\varepsilon = 3\gamma/2$. Therefore, the slow roll limit corresponds to $\gamma << 1$ and this gives $X \simeq 0.5$ and $c_s^2 \simeq \gamma/8 = 1/(12q) \ll 1$. For this model, it turns out that the scalar spectral index is exactly same as the one for the canonical scalar field $\nu$. Eq. (15), whereas the tensor-to-scalar ration is given by [22]

$$r \simeq \frac{16c_s^2}{q} = \left(\frac{8}{\sqrt{3}}\right) \left(\frac{1}{q^{5/2}}\right),$$

(22)

For the range of $q$ described by Eq. (17), $r$ lies in the range $0.005 < r < 0.02$, which is well within the Planck bound $r < 0.12$ at 95% CL [4]. Therefore, the $\{n_s, r\}$ results for K-PLI are consistent with Planck data. Let us now check whether the non-Gaussianity parameter $f_{\text{NL}}^{\text{equil}}$ in K-PLI is consistent with Planck data [18].

For a non-canonical model with the Lagrangian $\mathcal{L}(X, \phi)$, the non-gaussianity parameter $f_{\text{NL}}$ in the equilateral limit at leading order is given by [19]

$$f_{\text{NL}}^{\text{equil}} = \frac{5}{81} \left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma}\right) - \frac{35}{108} \left(\frac{1}{c_s^2} - 1\right),$$

(23)

where

$$\begin{align*}
\lambda &= X^2 \mathcal{L}_{,XX} + \frac{2}{3} X^3 \mathcal{L}_{,XXX} \\
\Sigma &= X \mathcal{L}_{,X} + 2X^2 \mathcal{L}_{,XX}
\end{align*}$$

(24)

Note that we use the WMAP sign convention in defining $f_{\text{NL}}^{\text{equil}}$ which is opposite to that defined in [19]. From Eq. (24), it follows that for the model (18)

$$\frac{2\lambda}{\Sigma} = \frac{4}{6\lambda - 1}.$$

(25)

In the slow roll limit ($\gamma << 1$) one finds $X \sim 1/2$, $c_s^2 \sim \gamma/8$, so that $2\lambda \simeq \Sigma$ and (23) reduces to [19]

$$f_{\text{NL}}^{\text{equil}} = \frac{-170}{81\gamma} = -\left(\frac{85}{27}\right) q.$$  

(26)

Since the relation between $n_s$ and $q$ remains exactly the same as in the canonical case, namely (15), the Planck-based restriction on the allowed range of $q$, described by (17), holds for K-PLI as well. Eqs. (17) and (26) imply

$$-318 \lesssim f_{\text{NL}}^{\text{equil}} \lesssim -118.$$  

(27)

The minimal value of $q$ for which $n_s$, $r$ are in good agreement with Planck data is $q \simeq 50$, for which $n_s \simeq 0.96$, $r \simeq 0.013$. However in this case one finds $f_{\text{NL}}^{\text{equil}} \simeq -157$ which appears to conflict with the Planck-derived value [18] $f_{\text{NL}}^{\text{equil}} = -42 \pm 75$. One therefore concludes that, although $\{n_s, r\}$ for K-PLI do satisfy CMB constraints, the large value of $f_{\text{NL}}^{\text{equil}}$ predicted by this model is in some tension with Planck results.

III. A NEW MODEL OF POWER LAW INFLATION

Consider a non-canonical scalar field with Lagrangian density [20, 24]:

$$\mathcal{L}(X, \phi) = X \left(\frac{X}{M^4}\right)^{\alpha - 1} - V(\phi),$$

(28)

where $\alpha$ is dimensionless while $M$ has dimensions of mass. For $\alpha = 1$, Eq. (28) reduces to the usual canonical scalar field Lagrangian $\mathcal{L}(X, \phi) = X - V(\phi)$. Therefore (28) can be viewed as a natural generalization of the standard canonical scalar field Lagrangian.

It is interesting to note that, in contrast to the canonical case, chaotic inflation with $\alpha > 1$ and $V(\phi) = \lambda \phi^4$ in (28), agrees quite well with CMB data even for $\lambda$ as large as unity [20]; also see Ref. [24].

We now reconstruct the potential $V(\phi)$ which can drive power law inflation for the Lagrangian density (28). The energy density, $\rho_s$, and pressure, $p_s$, are given by

$$\rho_s = \left(\frac{\partial \mathcal{L}}{\partial X}\right)(2X) - \mathcal{L},$$

(29)

$$p_s = \mathcal{L}, \ X \equiv \frac{1}{2} \phi^2.$$  

(30)

Substituting for $\mathcal{L}$ from (28) results in

$$\rho_s = (2\alpha - 1) X \left(\frac{X}{M^4}\right)^{\alpha - 1} + V(\phi),$$

(31)
For the power law solution \( a(t) \propto t^q \), the Friedmann equations

\[
\left( \frac{\dot{a}}{a} \right)^2 = \left( \frac{8\pi G}{3} \right) \rho_\phi, \tag{32}
\]

\[
\frac{\ddot{a}}{a} = -\left( \frac{4\pi G}{3} \right) (\rho_\phi + 3 p_\phi), \tag{33}
\]

imply

\[
\rho_\phi = \frac{3 M_p^2 q^2}{t^2}, \tag{34}
\]

\[
p_\phi = w_\phi \rho_\phi, \tag{35}
\]

where the equation of state parameter \( w_\phi \) is related to \( q \) as in (20), namely

\[
q = \frac{2}{3(1 + w_\phi)}. \tag{36}
\]

From Eqs. (31) to (36) we get

\[
\frac{d}{dt} \left( \frac{\phi}{M_p} \right) = M_p \left( \frac{q^2 \alpha^4 (\alpha - 1)}{\alpha} \right)^\frac{1}{2} [M_p t]^{-\frac{5}{2}}, \tag{37}
\]

\[
V(\phi) = 3 M_p^4 q^2 \left( 1 - \frac{(2\alpha - 1)w_\phi}{2\alpha} \right) [M_p t]^{-2}, \tag{38}
\]

where

\[
\mu = M_p M_\alpha. \tag{39}
\]

For a canonical scalar field (\( \alpha = 1 \)) integrating (37) gives \( t(\phi) \propto \exp \left[ \phi/(M_p \sqrt{2q}) \right] \). Substitution in (38), results in the usual exponential potential for canonical PLI viz. \( V(\phi) = V_o \exp \left[ -\sqrt{2/q(\phi/M_p)} \right] \). Interestingly, when \( \alpha \neq 1 \), Eqs. (37) and (38) imply the inverse power law potential\(^1\) [25]

\[
V(\phi) = \frac{V_o}{(\phi/M_p)^s}; \quad \text{where} \quad s = \frac{2\alpha}{\alpha - 1}. \tag{40}
\]

In the above equation the constant \( V_o \) is given by

\[
V_o = M^4 \left( 1 - \frac{(2\alpha - 1)w_\phi}{2\alpha} \right) \left( \frac{s^2 \alpha (3q) 2^{2\alpha - 1}}{\alpha 6^\alpha} \right)^\frac{1}{\alpha - 1}. \tag{41}
\]

Note that since \( w_\phi \) and \( q \) are related via (36), while \( \alpha \) and \( s \) are related via (40), \( V_o \) is a function of three independent quantities, namely \( M, \alpha \) and \( q \). We therefore conclude that an inverse power law potential of the form (40) leads to power law expansion \( a(t) \propto t^q \). It is important to note that since the parameter \( s \) in (40) does not depend of the value of \( q \), therefore it is the amplitude of the potential, \( V_o \), and not its shape, which determines the value of \( q \) in \( a(t) \propto t^q \).

In the slow roll limit, \( \varepsilon \ll 1, |\delta| \ll 1 \), the slow roll parameters (3) can be related to the potential \( V \) as follows [20]:

\[
\varepsilon \simeq \varepsilon_v = \frac{1}{\alpha} \left( \frac{3 M_p^4}{V} \right)^{\alpha - 1} \left( \frac{M_p V'}{V} \right)^{2\alpha} \left( \frac{M_p}{V^2} \right)^{\frac{2\alpha}{\alpha - 1}}, \tag{42}
\]

where the subscript in \( \varepsilon_v \) indicates that this parameter depends only on the potential. For the inverse power law potential (40), one finds \( \varepsilon_v \simeq 1/q \). Note that in the canonical case \( \alpha = 1 \), and \( \varepsilon_v \) reduces to its well known form

\[
\varepsilon_v^{(c)} = \frac{M^2}{2} \left( \frac{V'}{V} \right)^2. \tag{43}
\]

During slow roll, the power spectrum of scalar curvature perturbations in our model (28) acquires the form [20]

\[
\mathcal{P}_s(k) = \left( \frac{1}{12\pi^2 c_s^2} \right) \left( \frac{\alpha 6^\alpha}{M_p^4 (\alpha - 1)} \right)^\frac{1}{\alpha - 1} \times \left\{ \frac{1}{M_p^{14\alpha - 8}} \left( \frac{V(\phi)^{5\alpha - 2}}{V'(\phi)^{2\alpha}} \right) \right\}^\frac{1}{\alpha - 1} \tag{44}
\]

where \( c_s \) is the speed of sound defined in (8), which, for our model (28) turns out to be

\[
c_s = \frac{1}{\sqrt{2\alpha - 1}}. \tag{45}
\]

In Eq. (44), the quantity on the right hand side must be evaluated at the sound horizon exit \( a H = c_s k \). Substituting the expression for \( V(\phi) \) from Eq. (40) into Eq. (44) and using Eq. (41), we get

\[
\mathcal{P}_s(k) = \left( \frac{q}{24\pi^2 c_s} \right) \left( \frac{1 - (2\alpha - 1)w_\phi}{2\alpha} \right)^\frac{1}{\alpha - 1} \times \left( \frac{V(\phi)}{M_p^4} \right)_{a H = c_s k} \tag{46}
\]

Since \( a(t) \propto t^q \), we can write \( a(t) = a_i (t/t_i)^q \). Therefore, at sound horizon exit \( a = c_s k \) one gets

\[
M_p t = (M_p t_i) \left( \frac{t_i k_s}{a_s q} \right)^\frac{1}{\alpha - 1} \left[ c_s \right]^\frac{1}{\alpha - 1} \left( \frac{k}{k_s} \right)^\frac{1}{\alpha - 1}, \tag{47}
\]

where \( k_s \) is an arbitrary wavenumber which can be set as the pivot scale. Substituting the above expression in Eq. (38) we get

\[
\left( \frac{V(\phi)}{M_p^4} \right)_{a H = c_s k} = g \left[ c_s \right]^{-\frac{2}{\alpha - 1}} \left( \frac{k}{k_s} \right)^{-\frac{2}{\alpha - 1}}, \tag{48}
\]

\(^1\) In the canonical context (\( \alpha = 1 \)), the potential \( V \propto \phi^{-\frac{2}{4}} \) gives rise to ‘intermediate’ inflation [26, 27] with \( \alpha \propto \exp (A t^p) \) where \( p = 4/(4 + s) \). Such models are in tension with the Planck+WP+BAO results [4].
where \( g \) is defined as
\[
g = 3 q^2 \left( \frac{1 - (2\alpha - 1)w_s}{2\alpha} \right) \left[ (M_p t_i)^{q-1} \left( \frac{t_i k_\ast}{a_i q} \right) \right]^{-\frac{2}{q-1}}. \tag{49}
\]

From Eqs. (46) and (48), the expression for the scalar power spectrum turns out to be
\[
P_s(k) = A_s \left( \frac{k}{k_\ast} \right)^{-\frac{2}{q-1}}, \tag{50}
\]
where
\[
A_s = \left( \frac{q}{24\pi^2 c_s} \right)^{\alpha-1} \left( \frac{1 - (2\alpha - 1)w_s}{2\alpha} \right) g \left[ c_s \right]^{-\frac{2}{q-1}}. \tag{51}
\]

In the slow roll limit \((q \gg 1)\), the above expression simplifies to
\[
A_s \simeq \left( \frac{q g}{24\pi^2 c_s} \right). \tag{52}
\]

The expression for the tensor perturbation in terms of the potential \( V(\phi) \) in our model (28) is exactly the same as the one for the canonical scalar field in the slow roll limit and is given by [22]
\[
P_T(k) = \left( \frac{2}{3\pi^2} \right) \left( \frac{V(\phi)}{M_p^2} \right)_{aH = k}. \tag{53}
\]

It is important to note that unlike the expression (44) which one evaluates at the sound horizon exit \( a H = c_s k \), the tensor power spectrum expression (53) must be evaluated at the horizon exit \( a H = k \) [22]. Since the expression (48) is valid for any value of \( c_s \), we can evaluate the quantity \( V(\phi)/M_p^2 \) at horizon exit by setting \( c_s = 1 \) in Eq. (48). Therefore, at the horizon exit \((a H = k)\), we find that
\[
\left( \frac{V(\phi)}{M_p^2} \right)_{aH = k} = g \left( \frac{k}{k_\ast} \right)^{-\frac{2}{q-1}}, \tag{54}
\]
where \( g \) is defined in Eq. (49). Substituting Eq. (54) in Eq. (53) gives
\[
P_T(k) = A_T \left( \frac{k}{k_\ast} \right)^{-\frac{2}{q-1}}, \tag{55}
\]
where
\[
A_T = \left( \frac{2}{3\pi^2} \right) g. \tag{56}
\]

From Eqs. (50) and (55) we get
\[
n_T = n_s - 1 = -\frac{2}{q-1}, \tag{57}
\]
where \( n_s \) and \( n_T \) are defined in Eqs. (12) and (13), respectively. Note that the above expression for \( n_s \) and \( n_T \)
is independent of the parameter \( \alpha \) in the Lagrangian (28). In other words, one gets the same result for \( n_s \) and \( n_T \) for canonical \((\alpha = 1)\) and non-canonical \((\alpha \neq 1)\) PLI. It is also interesting to note that although we started with the expression (44) and (53) which presumes slow roll, the result (57) is exact and one gets the same result even without imposing the slow roll condition!

Moving on to the tensor-to-scalar ratio, we find from Eqs. (14), (50), (52), (55) and (56) that
\[
r = \frac{16}{q \sqrt{2\alpha - 1}}. \tag{58}
\]

Note that in obtaining the above result we have used Eq. (52) instead of Eq. (51) as the difference between the two is insignificant in the slow roll limit \( q \gg 1 \).

The tensor-to-scalar ratio does depend on the parameter \( \alpha \) in the model (28). In fact \( r \) decreases as

\[2\] It is clear that in the slow roll limit (57) reduces to (15). The fact that \( n_T = n_s - 1 \equiv -2/q \) in the slow roll regime, is a generic result for any power law inflation model based on the action (5) but for which the parameter \( \sigma \) defined in Eq. (7) vanishes, which corresponds to models with a constant speed of sound. This simply follows from the fact that for any inflationary model based on the action (5), in the slow roll limit, one gets [22]: \( n_s - 1 \equiv -4\varepsilon + 2\delta - \sigma \) and \( n_T = -2\varepsilon \), where the slow roll parameters \( \varepsilon \), \( \delta \) and \( \sigma \) are defined in Eqs. (3) and (7), respectively. For the PLI model, since \( \varepsilon \equiv 1/q \) and \( \delta = \varepsilon \) one gets \( n_T = n_s - 1 \equiv -2/q \) whenever \( \sigma \) vanishes, which is the case for PLI models discussed in this paper. For PLI models with \( \sigma \neq 0 \) but \( |\sigma| < 1 \) one gets \( n_T \neq n_s - 1 \), an example of this scenario is discussed in [28].
$\alpha$ is increased. It is for this reason that non-canonical inflation described by (28) fares better than canonical PLI (9) which gives a larger value of $r$ than implied by Planck. For example, when $q = 50$, Eq. (15) and (16) give $n_s = 0.96$ and $r = 0.32$, respectively, for the canonical PLI model (9), whereas $r = 0.096$ and $n_s = 0.96$ for $q = 50$ and $\alpha = 6$ in (28). Joint constraints on $n_s$ and $r$ obtained from Planck+WP+BAO data are shown in the Fig. 1. The non-canonical version of PLI is clearly in agreement with observations.

From (45), (57) and (58), one finds that in the slow roll regime ($q >> 1$)

$$r = -8c_s n_T = 8c_s (1 - n_s) .$$

This is the consistency relation for non-canonical PLI, and it distinguishes the class of models studied in this paper from canonical PLI for which $r = -8n_T$.

A. Non-gaussianity parameter $f_{NL}^\text{equil}$

Next we carry out a simple estimate of non-gaussianity in our model. One finds, from (28) and (24) that

$$\frac{\lambda}{s} - \frac{\alpha - 1}{3} .$$

Substituting this in (23) and using Eq. (45), we get

$$f_{NL}^\text{equil} \simeq -0.57(\alpha - 1) \simeq -0.28 (c_s^{-2} - 1) .$$

For $\alpha = 6$, the above expression gives $f_{NL}^\text{equil} \simeq -2.8$, which is in excellent agreement with the Planck result $f_{NL}^\text{equil} = -42 \pm 75$ [18]. It is easy to see that the Planck bounds on $f_{NL}^\text{equil}$ effectively translate into $1 < \alpha \leq 208$ for our model. However, as noted earlier, canonical PLI with $\alpha = 1$ is excluded on the basis of its values of $\{n_s, r\}$. Indeed, the relationship between $n_s, r$ and $c_s$ in (59), together with the Planck constraints: $f_{NL}^\text{equil} = -42 \pm 75$, $n_s \in [0.945 - 0.98]$ and $r < 0.12$ at 95% CL, allow one to infer that the sound speed for PLI should lie in the range

$$0.05 \lesssim c_s \lesssim 0.75$$

which is valid when $c_s$ is constant. Canonical PLI (9) clearly does not satisfy the above criteria since $c_s = 1$ in this model.

For the K-PLI model discussed in section II B, $c_s^2 \simeq \gamma/s = 1/(12q)$ and since $n_s - 1 \simeq -2/q$, one gets $n_s = 1 - 24c_s^2$. The Planck bound on $n_s$ then translates into $c_s \lesssim 0.05$ which is just outside of the allowed range (62) casting doubts on the viability of this model.

For the new PLI model (28) discussed in this paper $c_s = (2\alpha - 1)^{-1/2}$, and the range (62) is easily satisfied when $\alpha \gtrsim 2$. We therefore conclude that CMB observations favor our new PLI model over both canonical PLI and K-PLI.

IV. EXIT FROM POWER LAW INFLATION

A central drawback of power law inflation is that, since the universe accelerates forever, it cannot accommodate deceleration in the form of the radiative and matter dominated epochs which succeed inflation. A possible way out of this dilemma is to assume that the potential driving PLI approximates a more general potential which allows exit from inflation. Here we will demonstrate this possibility for the non-canonical PLI model described in the preceding section.

As noted earlier, the potential $V \propto \phi^{-s}$ drives PLI in the non-canonical setting (28). The following slight change to this potential allows for PLI together with the possibility of a ‘graceful exit’ from inflation:

$$V(\phi) = V_0 \left[ \left( \frac{\phi}{M_p} \right)^{-s/2} - \left( \frac{\phi}{M_p} \right)^{s/2} \right]^2 ; \quad s = \frac{2\alpha}{\alpha - 1} .$$

In Fig. 2 this potential is plotted for $\alpha = 6$. The potential has two branches: the left corresponds to $\phi < M_p$ and gives rise to PLI with $V \propto \phi^{-s}$, while the right branch ($\phi > M_p$) yields the potential commonly associated with chaotic inflation, namely $V \propto \phi^4$. The potential in (63) has a minimum at $\phi = M_p$ where $V(\phi) = 0$. Oscillations about this minimum allow for the universe to reheat [9]. The viability of inflation from both branches of $V(\phi)$ is discussed below.
A. Power law inflation from the left branch of $V(\phi)$

The left branch is characterized by $\phi \leq M_p$. At $\phi = M_p$, the potential in (63) vanishes and from (31) one finds that the equation of state of the scalar field is

$$w_\phi = \frac{\rho_\phi}{\rho_0} = \frac{1}{2\alpha - 1} > 0,$$  \hspace{1cm} (64)$$
thus ensuring that the inflationary epoch has ended.

To determine precisely when inflation ends one needs to take a look at the slow roll parameter $\varepsilon$. Substituting $V(\phi)$ from (63) into (42) we get

$$\varepsilon_v = \left[ \frac{1}{\alpha} \left( \frac{3 M^4}{V_0} \right)^{\alpha - 1} \left( \frac{s(\phi_N^2 - \phi_s^2)}{\phi_N \sqrt{2}} \right)^{2\alpha} \right] \frac{\phi}{\phi_N^2},$$  \hspace{1cm} (65)$$
where $\phi_N = \phi/M_p$ and for PLI it is necessary that $V_0$ and $M$ be related via Eq. (41). Eq. (65) implies $\varepsilon_v \simeq 1/q$ when $\phi_N << 1$, and the slow roll regime ($q >> 1$) is satisfied. This simply reflects the fact that $V \propto \phi^s$ when $\phi_N << 1$. As the field rolls further down its potential, the value of $\varepsilon_v$ increases until $\varepsilon_v = 1$ which is reached when

$$\phi = \phi_\ast = 0.91 M_p,$$  \hspace{1cm} (66)$$
for $\alpha = 6$ and $q = 50$. Therefore at this point inflation ends for this model.

Next, it is important to ascertain that the new potential (63) does not alter the power law nature of the solution at roughly 60 e-folds from the end of inflation. Otherwise, $n_s$ and $r$ will differ from (57) and (58), respectively.

The value of the field 60 e-folds from the end of inflation is determined by noting that

$$\frac{d\phi}{dN} = -\frac{\dot{\phi}}{H},$$  \hspace{1cm} (67)$$
where $N$ is the number of e-folds counted from the end of inflation. In the slow roll regime

$$H^2 \simeq \frac{V(\phi)}{3 M_p^2},$$  \hspace{1cm} (68)$$
Using Eq. (2.28) of Ref. [20], one finds from (67) and (68)

$$\frac{d\phi_N}{dN} = \left[ \left( \frac{6 M^4}{V_0} \right)^{\alpha - 1} \left( \frac{s(\phi_N^2 - \phi_s^2)}{\alpha \phi_N} \right) \right]^{\frac{1}{2\alpha - 1}} \times \left( \frac{1}{\phi_N^{s/2} - \phi_s^{s/2}} \right)^{\frac{2\alpha}{2\alpha - 1}}.$$  \hspace{1cm} (69)$$
Integrating the above equation with the condition that at $N = 0$, $\phi = \phi_\ast$, we find for $\alpha = 6$ and $q = 50$

$$\phi = 0.24 M_p \text{ at } N = 60, \hspace{1cm} (70)$$
Substituting the above value of $\phi$ in Eq. (65) gives

$$\varepsilon_v = \frac{1.1}{q} \text{ at } N = 60, \hspace{1cm} (71)$$
which should be compared with $\varepsilon_v = q^{-1}$ for the power law model (40). For $N = 70$, equations (65) & (69) give $\varepsilon_v = 1.07/q$. We therefore conclude that the potential (63) does not significantly alter the power law nature of the solution $a(t) \propto t^3$ between 60 & 70 e-folds from the end of inflation. Consequently, $n_s$ and $r$ described by (57) and (58), remain valid for the potential (63) when the field rolls down it along the left branch.

Next we proceed to determine the values of the parameters $V_0$ and $M$ in our model using CMB normalization viz. $P_s(k_s) = 2.2 \times 10^{-9}$ at the pivot scale $k_s = 0.05\ Mpc^{-1}$ [4]. We assume that the pivot scale exits the horizon at $\sim 60$ e-folds from the end of inflation. Since $r = P_r/P_s$, one finds from (53), (58) and (63) that

$$\frac{V_0}{M_p^4} = \left( \frac{24 \pi^2 P_s(k_s)}{q \sqrt{2\alpha - 1}} \right) \left( \frac{1}{(\phi_s/M_p)^{-s/2} - (\phi_s/M_p)^{s/2}} \right)^2,$$  \hspace{1cm} (72)$$
where $\phi_s$ is the field value 60 e-folds from the end of inflation. For $\alpha = 6$ and $q = 50$, Eqs. (70) and (72) give

$$V_0 = 1.08 \times 10^{-10} M_p^4,$$  \hspace{1cm} (73)$$
Substituting this value in (41), we get

$$M = 2.3 \times 10^{-4} M_p.$$  \hspace{1cm} (74)$$
From (63), (70) and (73), one gets $V(\phi) \simeq 3.14 \times 10^{-9} M_p^4$ and $H_s \simeq 3.2 \times 10^{-5} M_p$ for $\alpha = 6$ and $q = 50$. Here $V$ and $H$ are evaluated at roughly 60 e-folds from the end of inflation.

B. Chaotic inflation from the right branch of $V(\phi)$

The right branch of the potential (63), corresponds to $\phi > M_p$ and $V \propto \phi^s$ (see Fig. 2). The non-canonical version of chaotic inflation based on (28) was described in detail in Ref. [20]. Since $s = 2\alpha/(\alpha - 1)$, Eqs. (3.13) and (3.23) of [20] imply

$$n_s = 1 - \frac{2(4\alpha - 3)}{4N(\alpha - 1) + 2\alpha - 1},$$
$$r = \frac{16\sqrt{2\alpha - 1}}{4N(\alpha - 1) + 2\alpha - 1}.$$  \hspace{1cm} (75)$$
For $\alpha = 6$, the above equation gives $n_s = 0.965$ and $r = 0.04$, respectively, at 60 e-folds from the end of inflation.
i.e. at $N = 60$. These values are consistent with the Planck results [4].

The value of the model parameters $V_o$ and $M$ was earlier fixed in the context of PLI which arises from the left branch of (63). This procedure yielded $V_o$ and $M$ given by Eqs. (73) and (74), respectively. It is interesting to verify whether these value of $V_o$ and $M$ are consistent with inflation being realized from the right branch of (63) i.e., when $\phi > M_p$ and $V \propto \phi^s$. For inflation with this potential, the value of $V_o$ can be fixed using the CMB normalized value $P_g(k_s) = 2.2 \times 10^{-9}$, for a given value of $M$, and vice versa. However, we assume that $V_o$ and $M$ are related via Eq. (41) and therefore either $V_o$ or $M$ can be fixed using the CMB normalization.

Since $P_r = r P_g$, it turns out from Eq. (53) and (63) that

$$\frac{V_o}{M^4_p} = \left( \frac{3 \pi^2}{2} \frac{r P_g(k_s)}{2} \right)^2 \left( \frac{1}{(\phi_i/M_p)^s/2 - (\phi_i/M_p)\phi_i/2} \right), \quad (76)$$

where $\phi_i$ is the value of the field at 60 e-folds from the end of inflation when inflation is realized from the right branch of potential (63) and $r$ is given by Eq. (75). For $\alpha = 6$ and $q = 50$, it turns out that $\phi_i \approx 2 M_p$. On substituting this value of $\phi_i$ in Eq. (76), we get $V_o \approx 3.3 \times 10^{-10} M_p^4$ and consequently Eq. (41) gives $M \approx 3 \times 10^{-4} M_p$. These values are nearly close to those required by PLI in Eqs. (73) and (74), respectively. In fact when $\alpha = 4$ and $q = 60$, the CMB normalized value of $M$ as determined from both the branches of the potential (63) is approximately the same and is given by $M \approx 2 \times 10^{-4} M_p$.

We therefore conclude that both the branches of the potential (63) give rise to viable inflationary models which are compatible with CMB results.

\section{V. CONCLUSIONS}

Power law inflation in a spatially flat universe can be realized in a number of distinct ways:

(i) By a minimally coupled canonical scalar field with an exponential potential [15] examined in Sec. II A.

(ii) By k-inflation [1] examined in Sec. II B.

(iii) By a scalar field with a non-canonical kinetic term and with an inverse power law potential (40), examined in Sec. III.

Both (i) and (ii) appear to be in tension with recent CMB data. Values of $n_s$ and $r$ in scenario (i) conflict with Planck data, while in scenario (ii), the absolute value of the non-Gaussianity parameter $f_{NL}^{\text{equil}}$ appears to be too large to be accommodated by recent CMB observations.

By contrast, our new model (iii) gives values for $n_s$, $r$ and $f_{NL}^{\text{equil}}$ which are in excellent agreement with CMB data.

Conventionally, power law inflation ($a \propto t^q$, $q > 1$) possesses a graceful exit problem, since cosmic acceleration never ends in this model. Sec. IV provides a remedy to this problem by adding another branch to the inverse power law potential (40). The new potential, given by (63), accommodates PLI along its left branch ($\phi < M_p$) and chaotic inflation along its right branch ($\phi > M_p$). Oscillations of the scalar field at the minimum value of $V(\phi)$ allow the universe to reheat. Thus the PLI model presented by us in this paper appears to be a viable model of inflation in all respects.

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