Radiative Corrections in the LHC and ILC Era

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Abstract. In this talk I will present a brief review of the role of radiative corrections in present and future collider experiments with emphasis on the CERN Large Hadron Collider (LHC). After a general discussion of the importance, typical features and status of higher-order calculations I will discuss their impact on the example of Higgs boson production in association with heavy quarks and precision studies of W and Z bosons at the LHC.

1. Why radiative corrections?
With the CERN Large Hadron Collider (LHC) currently successfully colliding protons at a center-of-mass (CM) energy of 7 TeV the discovery of the Higgs boson and of physics Beyond the Standard Model (BSM) may be imminent. While some direct signals of BSM physics may be relatively easy to detect, more likely scenarios are signals overwhelmed by large Standard Model (SM) backgrounds and/or indirect signals in form of minute deviations from SM predictions. In the latter two cases, full advantage has to be taken of the ability of the LHC to not only probe the energy but also the precision frontier. Moreover, the identification of newly discovered particles will rely on high-precision measurements of their properties, such as masses and couplings. Many aspects of the interpretation of LHC data will rely on the precise knowledge of total production rates and kinematic distributions, including radiative corrections, i.e. higher-order terms beyond leading order (LO) in perturbation theory.

In the past two decades, the two pillars of precision physics, availability of high-precision collider experiments, such as the CERN LEP/SLAC SLC $e^+e^-$ colliders, the DESY HERA ep and the Fermilab Tevatron $p\bar{p}$ collider, and the predictive power of the SM beyond LO in perturbation theory, have made it possible

- to test the SM of electroweak (EW) and strong interactions as a fully-fledged Quantum Field Theory (QFT) at the per mille and per cent level, respectively,
- to constrain the mass of the SM Higgs boson,
- to exclude or severely constrain many new physics extensions of the SM, and
- to measure SM input parameters with high precision.

Figures 1 and 2 illustrate the impressive agreement of precision measurements at the Z resonance at LEP/SLC and above the W-pair production threshold at LEP-II, respectively, with SM predictions.

Electroweak and QCD radiative corrections will also play an important role in all aspects of the physics program at the LHC or a future International Linear $e^+e^-$ Collider (ILC), i.e.
where \( f \) functions.

The partonic colliders can be written as:

\[
\sum_{ij} \frac{1}{1 + \delta_{ij}} dx_1 dx_2 \{ f_{i/h_1}(x_1; \mu_f) f_{j/h_2}(x_2; \mu_f) d\hat{s}_{ij}(\hat{s}, \mu_r, \mu_f) + x_1 \leftrightarrow x_2 \} \mathcal{O}
\]  

(1)

where \( f_{i/h_1} \) denotes the PDF of parton \( i \) in hadron \( h_1 \) and \( \mathcal{O} \) a set of kinematic cuts or jet functions. \( \mu_r \) and \( \mu_f \) are the renormalization and factorization scales, respectively, and \( x_{1,2} \) denote the momentum fractions of the hadron momentum carried by the partons. The partonic CM energy (\( \hat{s} \)) is related to the hadronic CM energy by \( \hat{s} = x_1 x_2 S \). Naturally, at hadron colliders

**Discovery**: in modeling signal and background processes for Higgs and new physics searches to distinguish the new from the known, and in searching for indirect signals of BSM physics in quantum loops,

**Identity**: in precisely extracting model parameters, such as masses and couplings of the underlying model from data,

**Precision**: in reducing systematic errors by improving studies of effects of selection/analysis of data, by using single \( W \) and \( Z \) production cross sections as luminosity monitor, and by constraining Parton Distribution Functions (PDFs) using, e.g., the \( W \) charge asymmetry, and \( \gamma \) and jet production processes, and last but not least

**Fundamentals**: in probing and exploring the QFT structure of the SM, which is interesting in its own right.

Before I present examples of the role of radiative corrections at the LHC, I will briefly review the general structure and typical features of radiative corrections. This discussion follows closely Ref. [3].

The differential hadronic cross section for a \( h_1 h_2 \rightarrow N \) particles scattering process at hadron colliders can be written as:

\[
d\sigma(S) = \sum_{ij} \frac{1}{1 + \delta_{ij}} dx_1 dx_2 \{ f_{i/h_1}(x_1; \mu_f) f_{j/h_2}(x_2; \mu_f) d\hat{s}_{ij}(\hat{s}, \mu_r, \mu_f) + x_1 \leftrightarrow x_2 \} \mathcal{O}
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where \( f_{i/h_1} \) denotes the PDF of parton \( i \) in hadron \( h_1 \) and \( \mathcal{O} \) a set of kinematic cuts or jet functions. \( \mu_r \) and \( \mu_f \) are the renormalization and factorization scales, respectively, and \( x_{1,2} \) denote the momentum fractions of the hadron momentum carried by the partons. The partonic CM energy (\( \hat{s} \)) is related to the hadronic CM energy by \( \hat{s} = x_1 x_2 S \). Naturally, at hadron colliders

![Figure 1](image1.png)

**Figure 1.** EW precision data are compared with SM predictions. Taken from Ref. [1].

![Figure 2](image2.png)

**Figure 2.** Precise measurements of \( W \)-pair production cross sections (\( \sigma_{WW} \)) at LEP-II and SM predictions including EW \( \mathcal{O}(\alpha) \) corrections. A precise \( W \) mass measurement of \( M_W = 80.376 \pm 0.033 \text{ GeV} \) [1] has been extracted from \( \sigma_{WW} \). Taken from Ref. [2].
one is first concerned with the impact of QCD corrections. A QCD prediction of the partonic cross section $d\sigma_{ij}$ for the process $ij \rightarrow N$ at fixed order in the strong coupling constant $\alpha_s$ in perturbation theory exhibits the following general structure:

$$d\sigma_{ij}(\mu_r,\mu_f) = \alpha_s^k(\mu_r)[d\tilde{\sigma}^{LO} + \alpha_s(\mu_r)d\tilde{\sigma}^{NLO}(\mu_r,\mu_f) + \alpha_s^2(\mu_r)d\tilde{\sigma}^{NNLO}(\mu_r,\mu_f) + \ldots]$$

(2)

where $\alpha_s^k d\tilde{\sigma}^{LO}$ constitutes the LO prediction, $\alpha_s^k[d\tilde{\sigma}^{LO} + \alpha_s d\tilde{\sigma}^{NLO}]$ the next-to-leading order (NLO) prediction of $d\tilde{\sigma}_{ij}$, and so on. Radiative corrections in general consist of virtual, i.e. quantum loop, corrections and the real radiation of extra partons. For instance, the NLO contribution in Eq. (2) reads

$$d\tilde{\sigma}^{NLO} = d\sigma_{\text{virtual}}(ij \rightarrow N) + d\sigma_{\text{real}}(ij \rightarrow N + 1).$$

(3)

After employing a suitable renormalization procedure to remove UltraViolet (UV) divergences from $d\sigma_{\text{virtual}}$ and a regularization procedure for InfraRed (IR) singularities to facilitate their cancellation between $d\sigma_{\text{virtual}}$ and $d\sigma_{\text{real}}$, $d\tilde{\sigma}^{NLO}$ is finite, but may exhibit numerically large logarithms. For example, soft gluon radiation in single $W$ and $Z$ boson production at hadron colliders gives rise to the occurrence of large logarithms in the transverse momentum ($p_T$) of the $W$ and $Z$ bosons of the form $L = \alpha_s/\pi \ln(q_T^2/\Lambda^2)$, $V = W, Z$, which can be resummed to all orders in perturbation theory. For instance, an expansion of $d\tilde{\sigma}_{ij}$ in $\alpha_s^k L^n$

$$d\tilde{\sigma}_{ij} \propto 1 + \alpha(L^2 + L + 1) + \alpha^2(L^4 + L^3 + L^2 + L + 1) + \ldots$$

(4)

reads after resummation

$$d\tilde{\sigma}_{\text{res}} \propto C(\alpha) \exp[Lg_1(\alpha L) + g_2(\alpha L) + \alpha g_3(\alpha L) + \ldots] R(\alpha),$$

(5)

where $g_{1,2,...}$ are computable functions.

The inclusion of higher orders in the perturbative series in $\alpha_s(\mu_r^2)$ decreases the dependence on the unknown scales $\mu_r$ and $\mu_f$, and thus increases the reliability of the QCD prediction. Typical features of LO, NLO, NNLO and resummed QCD calculations are:

- **LO**: provides an order of magnitude estimate, but has large theoretical uncertainties due to the renormalization/factorization scale dependence.
- **NLO**: provides a reliable cross section prediction (reduced scale dependence), can considerably reduce or enhance the LO cross section and may distort the shape of kinematic distributions. As a result of the latter, reweighting of LO distributions to estimate the effect of NLO corrections with so-called K-factors, i.e. $K = \sigma_{\text{NLO}}/\sigma_{\text{LO}}$, may not be a good approximation.
- **NNLO**: provides reliable cross section predictions (reduced scale dependence) and a reliable estimate of the theoretical uncertainties due to dependence on $\mu_r$ and $\mu_f$.
- **Resummation**: large logarithmic corrections are tamed, reduced scale dependence, provides an estimate of unknown fixed-order corrections.

It is quite remarkable that already at the Tevatron experimental studies of some SM processes, e.g., $p\bar{p} \rightarrow W + X$ and jet+$X$, require precise theoretical predictions even beyond NLO QCD.

Electroweak corrections also play an important role, especially in EW gauge boson production, such as $Z$ boson production at LEP-I/SLC, $W$-pair production at LEP-II, and the production of one, two or three EW gauge bosons at hadron colliders. Electroweak corrections are of special interest since they exhibit a rich structure of different energy scales due to the presence of massive particles, such as the Higgs, weak gauge bosons and top and bottom quarks. Even at hadron colliders and in QCD dominated processes NLO EW corrections can be numerically at least as important as NNLO QCD corrections. At high-energies they may even be the dominant corrections due to soft/collinear virtual and real radiation of $W$ and $Z$ bosons resulting in so-called EW Sudakov logarithms.
2. State-of-the-art of higher-order QCD and EW corrections

In recent years impressive progress has been made in all aspects of higher-order QCD and EW calculations, i.e. in fixed-order, complete matrix element calculations of multi-leg processes, in the resummation of large logarithms in multi-scale processes and in the interface or merging of these two approaches. A recent review including an extensive list of references of the status of QCD calculations can be found, e.g., in Refs. [3, 4] and of EW calculations in Ref. [3]. The following very condensed overview of the status of higher-order calculations also includes examples of state-of-the-art calculations:

- **multi-loops and multi-legs**: Fixed-order calculations for $2 \rightarrow 2, 3, 4, 5$ processes at NLO QCD, $2 \rightarrow 2, 3$ processes at NNLO QCD (fully differential cross sections), and $2 \rightarrow 4$ processes at NLO EW. Examples: $pp, pp \rightarrow W + 3, 4$ jets [5, 6, 7, 8], $pp, pp \rightarrow t\bar{t}b\bar{b}$ [9, 10], $pp \rightarrow 4b$ [11], $e^+e^- \rightarrow 5$ jets [12] at NLO QCD; $pp, pp \rightarrow H$ (with decays) [13, 14], Drell-Yan (DY) production of single $W$ or $Z$ bosons (with decays) [15, 16], $e^+e^- \rightarrow 3$ jets [17, 18] at NNLO QCD; $e^+e^- \rightarrow 4f$ [19, 20] at NLO EW.

- **multi-scales**: Logarithmic enhanced corrections of the form $\alpha^k \ln^n(Q_1^2/Q_2^2)$, $Q_1^2 > Q_2^2$ are resummed up to NNLL accuracy. Examples: $q_T$ distributions in $pp, pp \rightarrow H$ [21] and in DY [22]; threshold corrections in top-pair production (for a review see, e.g., [23]); EW Sudakov logarithms in 4-fermion processes (for a review see, e.g., Ref. [24]). The thrust distribution in $e^+e^-$ collisions has even been determined at $N^3LL$ accuracy using an effective field theory (SCET) [25].

- **hard-soft transition**: Fixed-order NLO matrix element calculations are interfaced/matched or merged with parton-shower Monte Carlo (MC) programs (e.g., MC@NLO [26] and POWHEG [27]) or multi-purpose MC programs are improved with matrix element corrections in kinematic regions where the parton shower is not sufficient.

Many of these state-of-the-art calculations required new and innovative ideas to be able to handle a large number of potentially numerically unstable one-loop diagrams, new multi-loop integrals, and a complex structure of IR singularities. These new developments include (see, e.g., Ref. [3] for more details and references)

- methods for automation and numerical evaluation of one-loop integrals include sector decomposition (IR) and contour deformation (thresholds), and new tensor reductions (to avoid spurious singularities due to vanishing Gram determinants),
- methods for dealing with IR singularities include phase space slicing and dipole or antenna subtraction, and
- methods that obtain one-loop amplitudes for multi-leg processes directly from poles and cuts reducing them to lower-leg/lower-point amplitudes (for a review see, e.g., Ref. [28]).

The so-called Les Houches wish list of higher-order calculations, recently updated in Ref. [29] and shown in Table 1, not only nicely illustrates the enormous progress in recent years in providing reliable predictions for processes that have been identified to be essential for the LHC physics program, but also that there is still much work to do.

3. Radiative corrections and the search for the Higgs boson

The Higgs boson, predicted as a direct consequence of $W$ and $Z$ boson mass generation in the SM via spontaneous symmetry breaking of the SU(2)$_L$ $\otimes$ U(1)$_Y$ gauge group, so far eluded direct observation. The mass of the SM Higgs boson is severely constrained ($95 \% \text{ C.L.}$): $M_H < 185 \text{ GeV}$ [1] by global fits to EW precision data, $M_H > 114.4 \text{ GeV}$ [1] and $M_H \notin [158, 175] \text{ GeV}$ [30] by direct searches at LEP and the Tevatron, respectively. Moreover, the Tevatron may be able to exclude the entire mass range preferred by EW precision data. It is expected that if the SM Higgs boson exists, it will not escape detection at the LHC. Direct
Table 1. The Les Houches 2009 wish list [29] (slightly modified) for processes which require NLO or NNLO QCD precision at the LHC (\(V = \gamma, Z\) or \(W\)). The NLO QCD calculations of processes 1-12 are completed or close to completion. The need for EW corrections is not reflected in this list. For references see [29].

| process                                      | relevant for                                      |
|----------------------------------------------|--------------------------------------------------|
| 1. \(pp \rightarrow VV + \text{jet}\)       | \(ttH\), new physics                             |
| 2. \(pp \rightarrow H + 2\) jets            | \(H\) production by VBF                          |
| 3. \(pp \rightarrow \bar{t}t\bar{b}b\)      | \(ttH\)                                          |
| 4. \(pp \rightarrow \bar{t}t + 2\) jets     | \(ttH\)                                          |
| 5. \(pp \rightarrow VV \bar{b}b\)           | VBF\(\rightarrow H \rightarrow VV, ttH\), new physics |
| 6. \(pp \rightarrow VV + 2\) jets           | VBF\(\rightarrow H \rightarrow VV\)              |
| 7. \(pp \rightarrow V + 3\) jets            | various new physics signatures                    |
| 8. \(pp \rightarrow VVV\)                   | SUSY trilepton searches                           |
| 9. \(pp \rightarrow \bar{t}t \rightarrow 6\) fermions | background to Higgs                               |
| 10. \(pp \rightarrow \bar{t}t\j\)          | background, \(tt\@NNLO\)                         |
| 11. \(pp \rightarrow bbb\)                  | background to MSSM Higgs                         |
| 12. \(pp \rightarrow V + 4\) jets           | \(tt\), various new physics signatures           |
| 13. \(pp \rightarrow WW\bar{j}\)           | top, various new physics signatures               |
| 14. \(pp \rightarrow t\bar{t}t\)            | various new physics signatures                    |
| 15. NNLO \(pp \rightarrow \bar{t}t\)        | SM benchmark                                      |
| 16. NNLO to VBF and \(Z/\gamma + \text{jet}\)| background to Higgs                               |
| 17. NNLO QCD+NLO EW for \(W/Z\)              | SM benchmark                                      |

searches for the Higgs boson require that the underlying production as well as background processes are well under theoretical control. All SM Higgs production processes at the Tevatron and the LHC and most background process are at least known at NLO QCD and for some processes, such as \(gg \rightarrow H\), NNLO QCD corrections had to be included to obtain reliable predictions. In addition, for many processes and observables also the impact of EW corrections has been studied and resummed calculations of soft gluon radiation have been made available. A caveat of many of these higher-order calculations is that they did not include decays of the final-state particles, as observed in the experiments, so that much of the recent and ongoing theory effort goes now into remedying this situation.

Once the Higgs boson is observed, it will be crucial to measure its properties, such as the Yukawa couplings to fermions. The production of Higgs bosons in association with top and bottom quarks, \(pp \rightarrow \bar{t}tH, b\bar{b}H\), plays a special role in this endeavour, since it directly probes the top and bottom-quark Yukawa coupling. \(b\bar{b}H\) production can even be the dominant Higgs production channel in the Minimal Supersymmetric extension of the SM (MSSM) because of the possibility of enhanced Yukawa couplings, \(g^{MSSM}_{b\bar{b}h(h^0, H^0)} = \frac{(-\sin \alpha, \cos \alpha)_{\cos \beta}^{SM}}{\cos \beta} g^{SM}_{b\bar{b}h}\) (in the MSSM there are three neutral \((h^0, H^0, A^0)\) and a charged Higgs boson \((H^\pm)\)). For the extraction of top and bottom-quark Yukawa couplings at the LHC the predictions for the underlying Higgs production processes need to be under good theoretical control. For instance, the study of Ref. [31] finds that the top-quark Yukawa coupling can be determined with a 15% precision when LHC and ILC measurements are combined. The inclusion of NLO QCD corrections to \(t\bar{t}h\) [32, 33, 34, 35] and \(b\bar{b}h\) [36, 37, 38, 39, 40, 41] production at the LHC resulted in a reduced scale dependence. For instance, the remaining theoretical uncertainty from scale variation of the total \(t\bar{t}h\) production rate at NLO QCD is estimated to be about \(10 - 15\%\) at the Tevatron and \(15 - 20\%\) at the LHC.
For $\mu_r = \mu_f = \mu$ in the range $m_t < \mu < 2m_t$ the $O(\alpha_s)$ corrections reduce the LO cross section at the Tevatron, $K = \sigma_{NLO}/\sigma_{LO} = 0.7 - 0.95$, but at the LHC for $m_t + M_t/2 < \mu < 4m_t + 2M_h$ they lead to an enhancement, $K = 1.2 - 1.4$ [34, 35]. Although $t\bar{t}h$ production is considered to be experimentally very difficult due to its small production rate and large QCD background, recent studies that consider boosted Higgs bosons and top quarks [42] suggest that the situation is not as hopeless as originally thought, which triggered renewed theoretical and experimental interest in this process (see also Ref. [29]). Recently the decay $h \rightarrow b\bar{b}$ has been included in the NLO QCD prediction of $t\bar{t}H$ [29] which is the dominant decay for a light Higgs boson ($M_h < 135$ GeV). Also important background processes to $pp \rightarrow t\bar{t}h$ and $pp \rightarrow b\bar{b}h$ with $h \rightarrow b\bar{b}$, such as $pp \rightarrow t\bar{t}bb$ and $pp \rightarrow b\bar{b}bb$ have been made available at NLO QCD as shown in Figures 3 and 4, respectively.

Figure 4. LO and NLO predictions for the invariant mass distribution of the two leading b-jets in $pp \rightarrow b\bar{b}b\bar{b}$ production at the LHC. The bands are obtained by varying $\mu_r$ ($\mu_f$ is fixed) and again the scale dependence is much less pronounced at NLO QCD. Taken from Ref. [11].

4. Electroweak precision physics with $W$ and $Z$ bosons
The $W$ and $Z$ production process are one of the theoretically best understood, most precise probes of the SM. As illustrated in Figs. 1 and 2 precision measurements and calculations of $Z$ boson observables at LEP-I/SLC and of $W$-pair production cross sections at LEP-II probed the SM at the per mille level. For instance, for the precision measurement of the $W$ mass at LEP-II the theoretical uncertainty of predictions for the total $W$-pair production cross section ($\delta\sigma_{WW}^{theory}$) had to better than 1% above the $W$-pair threshold, which was achieved by including EW $O(\alpha)$ to $e^+e^- \rightarrow WW \rightarrow 4f$, resulting in $\delta\sigma_{WW}^{theory} = 0.4\%$ at $\sqrt{s}=200$ GeV [43]. At a future ILC, precision measurements at the $W$-pair threshold require an even more dedicated theory effort, resulting in the calculation of the complete EW one-loop corrections to $e^+e^- \rightarrow Wf$ [19, 20].

Precision measurements of the $W$ mass at the Tevatron recently surpassed the LEP-II precision and will continue to improve (for a review see, e. g., [44]). At the LHC, a measurement of $M_W$ with $\delta M_W = 15$ MeV (or even 7 MeV) is anticipated. Besides precision measurements of SM parameters, $M_W$ and the effective weak mixing angle $\sin^2 \theta_W$, $W$ and $Z$ boson observables...
constrain the SM Higgs boson mass and are sensitive to signals of BSM physics. For example, as illustrated in Fig. 5, the virtual presence of new particles in loop corrections may lead to observable deviations from SM predictions of EW observables, so that ever more precise measurements and SM and MSSM predictions of $M_W$ (together with a precise knowledge of $m_t$) could indicate that the MSSM is preferred over the SM. Moreover, BSM physics could manifest itself as resonances of new, heavy gauge bosons in invariant mass distributions and forward-backward asymmetries at high energies, and as anomalous triple and quartic EW gauge boson self-couplings in the production of two and three EW gauge bosons.

In view of the importance and rich physics potential of EW gauge boson production processes, it is not surprising that these processes have been well studied not only experimentally but also theoretically. For hadron collider physics, for instance, NLO QCD and EW calculations are available for the production of one, two and three gauge bosons, as well as fully differential NNLO QCD predictions for the DY process. As mentioned earlier, an interesting signature of EW corrections at higher energies $\sqrt{s} \gg M_{W,Z}$ is their enhancement by EW Sudakov logarithms of the form $\alpha^l \ln^N \left( \frac{s}{M_{W,Z}^2} \right)$ (with $1 \leq N \leq 2l$ and $l = 1, 2, \ldots$ denotes the number of loops). They are remnants of UV singularities after renormalization and of soft/collinear emission of virtual and real $W/Z$ bosons. EW logarithmic corrections to 4-fermion processes are known up to 2-loop $N^3LL$ order and are available in form of compact analytical formula (for a review see, e.g., Ref. [24]). The $W/Z$ boson masses act as a physical cut-off parameter and real $W/Z$ radiation is usually not included, since it leads to a different initial/final state. However, as shown on the example of a number of observables in Ref. [48] it may be the case that the experimental setup requires their inclusion, which can result in a reduction of the impact of the logarithms of virtual origin. A recent study [49] discusses in more detail this partial cancellation of real and virtual corrections.

Apart from providing ever more precise predictions, theorists are also increasingly getting involved in the usage of their calculations in the experimental analysis and in determining the residual theoretical uncertainty. This is especially important when observables need to be under control at the per cent or even per mille level in the relevant kinematic regions. The latter is the case for the transverse mass distribution of the lepton pair ($M_T$) in single $W$ production from which $M_W$ is extracted at the Tevatron. Figure 6 illustrates the combined impact of EW and QCD NLO corrections on $M_T$. As shown in the lower panel the ambiguity in combining these two NLO calculations results into to small differences which can only be resolved by a calculation of the two-loop $O(\alpha^2\alpha_s)$ corrections to single $W$ production. Such a calculation will be necessary, if such small differences affect the $W$ mass at the few per cent level. The question whether $W$ and $Z$ boson observables at the Tevatron and the LHC are under theoretical control at the required level of precision is presently also under investigation in the context of the
Figure 6. Impact of combined EW and QCD NLO corrections on the $W$ transverse mass distribution using MC@NLO/HORACE. The percent difference shown in the lower panel (solid black and dashed green curves) is due to an ambiguity in performing this combination. Taken from Ref. [46].

Milano/Fermilab $W$ mass workshop [50]. The efforts in providing a definite answer include an assessment of uncertainties of the available QCD predictions due to:

- QCD factorization/renormalization scale dependence,
- different treatments of non-perturbative QCD contributions and different implementation of subleading logarithms for $q_T \sim 20$ GeV in soft-gluon resummation,

and of EW calculations due to:

- EW input scheme dependence and different implementations of higher-order corrections,
- QED scale dependence of the PDFs, and
- unknown higher-order EW corrections.

Results of earlier studies can be found in Refs. [51, 52] and a report of the findings of the Milano/Fermilab $W$ mass workshop is in preparation [50].

It is also important to quantify the uncertainty when using a theory prediction in the experimental analysis that does not include all known higher order corrections, for instance, as done in Ref. [53] where predictions of PHOTOS, which includes multiple photon radiation, in the experimental analysis and not HORACE, which in addition also provides the full EW one-loop corrections. The study of predictions of the total $W$ production cross section at the LHC in Ref. [53] finds $\delta \sigma_W/\sigma_W = -0.904 \pm 0.109\%(-0.322 \pm 0.014\%)$ at 7 (14 TeV) due to missing $O(\alpha)$ EW corrections and $\delta \sigma_W/\sigma_W = 2.86 \pm 0.10\%(2.24 \pm 0.11\%)$ due to missing NNLO QCD corrections.

5. Conclusions

Radiative corrections are needed to fully exploit the potential of present and future collider experiments to discover the Higgs boson and/or signals of new physics and, once detected, to determine their properties to reconstruct the underlying theory. In view of high precision measurements at the LHC and a future ILC, SM predictions have to be under control at an unprecedented level of precision, requiring the calculation of QCD and EW radiative corrections to multi-leg processes, in some case up to the NNLO level (and NNLL accuracy). New techniques and automated tools have been and are being development to meet this challenge. Apart from checking off the Les Houches wish list, the interpretation of LHC data will require close collaboration between theorists and experimentalists to make the best use of these higher-order calculations. LHC physics results are already being compared to NLO and NNLO predictions and first steps have been made towards combining multi-purpose MCs that are needed for modeling of hadronization and underlying event (such as HERWIG and PYTHIA) and higher-order (parton-level) calculations.
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Appendix
A number of tools for higher-order calculations are publicly available and a partial list is provided in the following (please see the program’s webpages for references). There are many more Monte Carlo programs publicly available that calculate higher-order corrections for specific processes. The list below is intended to be a starting point for loop practitioners to be rather than an exhaustive collection of available codes.

- Some automated tools for the calculation of tree-level amplitudes and cross sections:
  - Madgraph/MadEvent: madgraph.hep.uiuc.edu
  - SUSY-Madgraph: www.thphys.uni-heidelberg.de/~plehn/smadgraph
  - O’Mega/Whizard: projects.hepforge.org/whizard
  - Sherpa/Amegic++: www.montecarlonet.org/index.php?p=Projects/sherpa
  - Helac/Phegas: helac-phegas.web.cern.ch/helac-phegas/helac-phegas.html
  - Comphep: comphep.sinp.msu.ru
  - Alpgen: tin.home.cern.ch/mlm/alpgen
  - carlomat: kk.us.edu.pl/carlomat.html

- Some tools for manipulating strings of Dirac spinors and γ matrices:
  - FORM: www.nikhef.nl/~form
  - Tracer.m: library.wolfram.com/infocenter/MathSource/2987

- Some tools for NLO calculations:
  - FeynArts: www.feynarts.de, FormCalc: www.feynarts.de/formcalc, and LoopTools: www.feynarts.de/looptools
  - Cut tools: www.ugr.es/~pittau/CutTools
  - Golem95: lappweb.in2p3.fr/lapth/Golem/golem95.html
  - Samurai: websupport1.citytech.cuny.edu/faculty/gossola/index2.html

- Collections of NLO calculations:
  - MCFM: mcfm.fnal.gov
  - GRACE: minami-home.kek.jp

- NLO QCD calculations interfaced with parton showers:
  - MC@NLO: https://www.hep.phy.cam.ac.uk/theory/webber/MCatNLO
  - POWHEG: https://twiki.cern.ch/twiki/bin/view/Main/PowhegBOX

- A collection of scalar one-loop integrals can be found, e. g., in Refs. [54, 55].

References
[1] J. Alcaraz et al. [ ALEPH and CDF and D0 and DELPHI and L3 and OPAL and SLD Collaboration ], arXiv:0911.2604 [hep-ex], and updates from http://lepewwg.web.cern.ch/LEPEWG.
[2] J. Alcaraz et al. [ ALEPH and DELPHI and L3 and OPAL and LEP Electroweak Working Group Collaborations ], hep-ex/0612034.
[3] E. Laenen, D. Wackeroth, Ann. Rev. Nucl. Part. Sci. 59, 367-396 (2009).
[4] T. Gehrmann, PoS D 152010, 004 (2010) [arXiv:1007.2107 [hep-ph]].
[5] C. F. Berger et al., ArXiv:1009.2338 [hep-ph].
[6] C. F. Berger et al., Phys. Rev. D 80, 074036 (2009) [arXiv:0907.1984 [hep-ph]].
[7] K. Melnikov and G. Zanderighi, Phys. Rev. D 81, 074025 (2010) [arXiv:0910.3671 [hep-ph]].
[8] R. K. Ellis, K. Melnikov and G. Zanderighi, Phys. Rev. D 80, 094002 (2009) [arXiv:0906.1445 [hep-ph]].
[9] A. Bredenstein, A. Denner, S. Dittmaier and S. Pozzorini, Phys. Rev. Lett. 103, 012002 (2009) [arXiv:0905.0110 [hep-ph]].
[10] G. Bevilacqua, M. Czakon, C. G. Papadopoulos, R. Pittau, M. Worek, JHEP 0909, 109 (2009). [arXiv:0907.4723 [hep-ph]].
