“Fog” Optimization via Virtual Cells in Cellular Network Resource Allocation

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Abstract—This work proposes a new resource allocation optimization framework for cellular networks using “fog” or neighborhood-based optimization rather than fully centralized or fully decentralized methods. In neighborhood-based optimization resources are allocated within virtual cells encompassing several base-stations and the users within their coverage area. As the number of base-stations within a virtual cell increases, the framework reverts to decentralized optimization, and as this number decreases it reverts to centralized optimization. We address two tasks that must be carried out in the fog optimization framework: forming the virtual cells and allocating the communication resources in each virtual cell effectively. We propose hierarchical clustering for the formation of the virtual cells given a particular number of such cells. Once the virtual cells are formed, we consider several optimization methods to solve the NP-hard joint channel access and power allocation problem within each virtual cell in order to maximize the sum rate of the entire system. We present numerical results for the system sum rate of each scheme under hierarchical clustering. Our results indicate that proper design of the fog optimization results in little degradation relative to centralized optimization even for a relatively large number of virtual cells. However, improper design leads to a significant decrease in sum rate relative to centralized optimization.

I. INTRODUCTION

The demand for increased capacity in cellular networks continues to grow, and is a major driver in the deployment of 5G systems. To increase cellular network capacity, the deployment of small cells has been proposed and is currently taking place [1]–[4]. The proximity of small cells to one another combined with their frequency reuse can cause severe interference to neighboring small cells and macrocells, which must be managed carefully to maximize the overall network capacity. Thus, powerful interference mitigation methods as well as optimal resource allocation schemes must be developed for 5G networks. In this work we investigate a flexible resource allocation structure for cellular systems where instead of each base-station serving all users within its own cell independently, several base-stations act cooperatively to create a virtual cell with joint resource allocation.

In order to design wireless networks that are composed of virtual cells we address in this work the following two design challenges: 1) Creating the virtual cells, i.e., cluster the base-stations into virtual cells. 2) Allocating the resources in each virtual cell. This resource allocation problem is a non-convex NP hard problem. In this work we address the uplink resource allocation problem for joint channel access and power allocation when there is only a single channel available in the system. The algorithms and fundamental ideas that we present can be extended to multi-channel systems as well as systems with interference cancellation, which is the subject of our current work.

Base-station and user clustering as part of a resource allocation strategy is discussed in the Cooperative Multi-Point (CoMP) literature, see for example [5]–[16]. The work [5] presents an extensive literature survey of cell clustering for CoMP in wireless networks. The clustering of base-stations and users can be divided into three groups: 1) Static clustering which considers a cellular network whose cells are clustered statically. Hence, the clustering does not adapt to network changes. Examples for static clustering algorithms are presented in [6]–[9]. 2) Semi-dynamic clustering, in which static clusters are formed but the cluster affiliation of users is adapted according to the networks changes. Examples for such algorithms are presented in [10]–[12]. 3) Dynamic clustering in which the clustering of both base-stations and users adapts to changes in the network. Examples for dynamic clustering algorithms are presented in [13]–[15].

Cell clustering as part of a resource allocation strategy in wireless networks is also investigated in the ultra-dense networks literature, see for example [17]–[22]. In particular, the work [17] considers K-means clustering of femto access points, where each access point serves one user. It then develops channel allocation schemes to maximize the sum rate of the downlink transmission, assuming constant power in every subchannel. The work [18] considers the downlink transmission in which users whose transmissions strongly interfere with one another are clustered. Then, assuming that there is small number of cells in each cluster, to maximize the throughput of the network, intra-cluster interference mitigation is performed based on inter-cell interference coordination. The work [19] also considers downlink transmission, where sets of base-stations and users are paired, and these pairs are partitioned into clusters. The interference in each cluster is then canceled using interference alignment in clusters; however,
this limits the maximal cluster size. The work [20] considers hierarchical clustering of cells with a non-cooperative game between clusters in which each cluster aims to maximize its throughput. It assumes perfect CSI of all the channels in the network and a fixed power allocation. The work [21] creates clusters by maximizing the inter-cluster interference, then allocates a different channel to each of the clusters. While these works address the clustering problem, they do not investigate the relation between the network clustering and resource allocation optimization scheme. Additionally, the effect of the number of clusters on the network sum rate was not analyzed. While the aforementioned works address the clustering problem for improving the performance of communication networks, they do not investigate the relation between the network clustering and resource allocation optimization scheme. Additionally, the effect of the number of clusters on the network sum rate is not analyzed.

The remainder of this paper is organized as follows. Section II presents the problem formulation that we analyze in this work. Sections III-V present several algorithms for joint channel access and power allocation within virtual cells. In particular, Section III proposes a joint channel access and power allocation scheme. Section IV proposes a channel access and power allocation scheme based on an alternating user-centric optimization. Section V proposes a channel access and power allocation scheme based on an alternating BS-centric optimization. Section VI describes the method for forming the virtual cells in an optimal manner. Section VII presents numerical results of the average system sum rate for all of our proposed methods. Finally, VIII summarize our results and discuss future work.

II. PROBLEM FORMULATION

We consider a communication network that comprises a set of base-stations (BSs) \( B \) and a set of users \( U \). The users communicate with the BSs and interfere with the transmission of one another. Each user \( u \in U \) has a power constraint of \( P_u \) dBm. The BSs and users are clustered into virtual cells which must fulfill the following characteristics.

A. Virtual Cells

**Definition 1 (Virtual BS):** Let \( b_1, \ldots, b_n \) be \( n \) BSs in a communication network, we call the set \( \{b_1, \ldots, b_n\} \) a virtual BS.

**Definition 2 (Proper clustering):** Let \( B \) be a set of BSs, \( U \) be a set of users. Denote \( V = \{1, \ldots, V\} \). For every \( v \), define the sets \( B_v \subseteq B \) and \( U_v \subseteq U \). We say that the set \( V \) is a proper clustering of the sets \( B \) and \( U \) if \( B_v \) is a partition of the sets \( B \) and \( U \). That is, \( \bigcup_{v \in V} B_v = B \) and \( \bigcup_{v \in V} U_v = U \). Additionally, \( B_{v_1} \cap B_{v_2} = \emptyset \) and \( U_{v_1} \cap U_{v_2} = \emptyset \) for all \( v_1, v_2 \in V \) such that \( v_1 \neq v_2 \).

**Definition 3 (Virtual cell):** Let \( B \) be a set of BSs, \( U \) be a set of users, and \( V \) be a proper clustering of \( B \) and \( U \). For every \( v \in V \) the virtual cell \( C_v \) is composed of the virtual BS \( B_v \) and the set of users \( U_v \).

This condition ensures that every BS and every user belongs to exactly one virtual cell.

Let \( V \) be a proper clustering of the set of BSs \( B \) and the set of users \( U \), and let \( \{C_v\}_{v \in V} \) be the set of virtual cells that \( V \) creates. In each virtual \( C_v \), we assume that the BSs that compose the virtual BS \( B_v \) allocate their resources jointly.

B. The Uplink Resource Allocation Problem

In each virtual cell we consider the uplink resource allocation problem in which all the BSs in the virtual cell jointly optimize the channel access and power of the users within the virtual cell. Further, we consider single user detection in which every BS \( b \) decodes each of its codewords separately. That is, suppose that users \( u_1 \) and \( u_2 \) are both served by BS \( b \), then \( b \) decodes the codeword of \( u_1 \) treating the codeword of \( u_2 \) as noise, and decodes the codeword of \( u_2 \) treating the codeword of \( u_1 \) as noise.

While each user can communicate with all the BSs in its virtual cell, it follows by [23] that given a power allocation scheme, the maximal communication rate for each user is achieved when the message is decoded by the BS with the highest SINR for this user.

Denote by \( h_{u,b} \) the channel coefficient of the channel from user \( u \in U \) to BS \( b \), and let \( P_u \) be the transmit power of user \( u \). Further, let \( \sigma_b^2 \) denote the noise power at BS \( b \) and let \( W \) denote the bandwidth of the channel. The uplink resource allocation problem in each virtual cell \( C_v \), ignoring interference from other virtual cells, is given by:

\[
\begin{align*}
\max & \sum_{b \in B_v} \sum_{u \in U_v} \gamma_{u,b} W \log_2 \left( 1 + \frac{|h_{u,b}|^2 P_u}{\sigma_b^2 + J_{u,b}} \right) \\
\text{s.t.} & \quad 0 \leq P_u \leq P_u^* \quad \forall u \in U_v, \\
& \quad \sum_{\tilde{u} \in U_v, \tilde{u} \neq u} |h_{\tilde{u},b}|^2 P_{\tilde{u}} = J_{u,b} \quad \forall u \in U_v, b \in B_v, \\
& \quad \gamma_{u,b} \leq 1 \quad \forall u \in U_v, \\
& \quad \gamma_{u,b} \in \{0, 1\} \quad \forall u \in U_v, b \in B_v. 
\end{align*}
\]

This is a mixed-integer programming problem that is NP-hard. The next three sections present, respectively, three different methods to approximate the solution of this problem for a given virtual cell. The first method translates this problem from a mixed-integer programming problem to an equivalent problem with continuous variables. The second method approximates the optimal solution by solving a user-centric channel access problem, and a power allocation problem, alternately. The third method approximates the optimal solution by solving a BS-centric channel access problem, and a power allocation problem, alternately. The algorithm to define the virtual cells from all base-stations and users in the system that satisfies the conditions defined in Section II-A is described in Section VI.

III. JOINT CHANNEL ACCESS AND POWER ALLOCATION

This section introduces the first resource allocation scheme that we present in this paper. This scheme solves the power allocation and channel access problem jointly within a given
virtual cell and is found by converting (1) to an equivalent continuous variable problem.

A. An Equivalent Continuous Variable Resource Allocation Problem

We can represent the problem (1) by an equivalent problem with continuous variables. Suppose that instead of sending a message to the best BS a user sends messages to all BSs. The signal of user $u \in \mathcal{U}_u$ is then given by $X_u = \sum_{b \in \mathcal{B}_u} X_{u,b}$ where $X_{u,b}$ is the part of the signal of user $u$ intended to be decoded by BS $b$. Let $P_{u,b}$ be the power allocation of the part of the signal of user $u$ intended to be decoded by BS $b$; i.e. $P_{u,b} = EX_{u,b}^2$. We will argue that (1) can then be written in the following equivalent form:

$$\max \sum_{b \in \mathcal{B}_u} \sum_{u \in \mathcal{U}_u} W \log_2 \left(1 + \frac{|h_{u,b}|^2 P_{u,b}}{\sigma_b^2 + J_{u,b}}\right)$$

s.t.:

$$\sum_{b \in \mathcal{B}_u} P_{u,b} \leq P_u, \quad \forall u \in \mathcal{U}_u,$$

$$\sum_{(\tilde{u},\tilde{b}) \in \mathcal{U}_u \times \mathcal{B}_u, (\tilde{u},\tilde{b}) \neq (u,b)} |h_{\tilde{u},\tilde{b}}|^2 P_{\tilde{u},\tilde{b}} = J_{\tilde{u},\tilde{b}}, \quad \forall \tilde{u} \in \mathcal{U}_u, \tilde{b} \in \mathcal{B}_u,$$

$$0 \leq P_{u,b}, \quad \forall u \in \mathcal{U}_u, b \in \mathcal{B}_u.$$  (2)

The equivalence of (1) and (2) is argued as follows. First, the solution of (1) can be achieved by the solution of (2) by setting $X_{u,b} = 0$ whenever $\gamma_{u,b} = 0$, and $EX_{u,b}^2 = P_u$ whenever $\gamma_{u,b} = 1$; thus the maximal sum rate (2) upper bounds the maximal sum rate (1). On the other hand, it follows by (2) that the maximal sum rate of (1) cannot be larger than that of (1). Thus, the two problems (1) and (2) are equivalent.

B. Solving an Approximation of the Continuous Variable Resource Allocation Problem

Denote:

$$\text{SINR}_{u,b}(P) = \frac{|h_{u,b}|^2 P_{u,b}}{\sigma_b^2 + \sum_{(\tilde{u},\tilde{b}) \in \mathcal{U}_u \times \mathcal{B}_u, (\tilde{u},\tilde{b}) \neq (u,b)} |h_{\tilde{u},\tilde{b}}|^2 P_{\tilde{u},\tilde{b}}},$$  (3)

where $P = (P_{u,b})_{(u,b) \in \mathcal{U}_u \times \mathcal{B}_u}$ is the matrix of the transmission power.

Using the high SINR approximation 24

$$\log(1 + z) \geq \alpha(z_0) \log z + \beta(z_0),$$  (4)

where

$$\alpha(z_0) = \frac{z_0}{1 + z_0}, \quad \beta(z_0) = \log(1 + z_0) - \frac{z_0}{1 + z_0} \log z_0,$$

yields the following approximated iterative problem:

$$\mathbf{P}^{(m)} = \arg \max \sum_{b \in \mathcal{B}_u} \sum_{u \in \mathcal{U}_u} W \log_2 \left(1 + \frac{|h_{u,b}|^2 P_{u,b}}{\sigma_b^2 + \sum_{(\tilde{u},\tilde{b}) \in \mathcal{U}_u \times \mathcal{B}_u, (\tilde{u},\tilde{b}) \neq (u,b)} |h_{\tilde{u},\tilde{b}}|^2 P_{\tilde{u},\tilde{b}}\right) + \beta(m)$$

s.t.:

$$\sum_{b \in \mathcal{B}_u} P_{u,b} \leq P_u, \quad 0 \leq P_{u,b}, \quad \forall u \in \mathcal{U}_u, b \in \mathcal{B}_u,$$  (6)

where $\alpha^{(m)}_{u,b} = \alpha(\text{SINR}_{u,b}(\mathbf{P}^{(m-1)})), \quad \beta^{(m)}_{u,b} = \beta(\text{SINR}_{u,b}(\mathbf{P}^{(m-1)}))$ and $\alpha^{(0)}_{u,b} = 1, \beta^{(0)}_{u,b} = 0$ for all $u \in \mathcal{U}_u$ and $b \in \mathcal{B}_u$. It is left to solve the problem (6). Transforming the variables of the problem using $P_{u,b} = \exp(g_{u,b})$ yields the equivalent convex problem:

$$\ln(\mathbf{P}^{(m)}) = \arg \max \sum_{b \in \mathcal{B}_u} \sum_{u \in \mathcal{U}_u} W \mathbf{a}_{u,b}^{(m)} \left[\frac{g_{u,b}}{\ln 2} + \log_2(|h_{u,b}|^2)\right]$$

$$- \log_2 \left(\frac{\sigma_b^2 + \sum_{(\tilde{u},\tilde{b}) \in \mathcal{U}_u \times \mathcal{B}_u, (\tilde{u},\tilde{b}) \neq (u,b)} |h_{\tilde{u},\tilde{b}}|^2 \exp(g_{\tilde{u},\tilde{b}})}{\ln 2 + W \mathbf{a}_{u,b}^{(m)}}\right)$$

s.t.:

$$\sum_{b \in \mathcal{B}_u} \exp(g_{u,b}) \leq P_u, \quad \forall u \in \mathcal{U}_u.$$  (7)

Since the problem (7) is convex with nonempty interior, its duality gap is zero. Define

$$f_{u,b}(P, \lambda, \alpha) = \frac{W \alpha_{u,b}}{\ln 2 + W \sum_{(\tilde{u},\tilde{b}) \in \mathcal{U}_u \times \mathcal{B}_u, \alpha_{\tilde{u},\tilde{b}} \mathbf{P}_{\tilde{u},\tilde{b}} |h_{\tilde{u},\tilde{b}}|^2} \text{SINR}_{u,b}(P)$$

Let $\lambda_u$ be the Lagrangian multiplier associated with the power constraint of user $u$ in (7). By (24) we can find the power allocation that achieves the Lagrangian dual function of (7) by the following fixed point iteration:

$$P^{(m, s+1)}_{u,b} = f_{u,b}(P^{(m, s)}, \lambda_u, \alpha^{(m)}).$$

We can then solve the dual problem of (7) by optimizing the Lagrangian dual function over $\lambda_u$ using gradient methods. Alternatively, by the convexity of (7), the following KKT conditions are sufficient for optimality:

$$P^{(m)}_{u,b} = f_{u,b}(P^{(m)}, \lambda_u, \alpha^{(m)}),$$

$$0 = \lambda_u \left(\sum_{b \in \mathcal{B}_u} P^{(m)}_{u,b} - P_u\right), \quad \sum_{b \in \mathcal{B}_u} P^{(m)}_{u,b} \leq P_u, \quad \lambda_u \geq 0,$$

for all $u \in \mathcal{U}_u$. Define the following fixed point iteration:

$$P^{(m, s+1)}_{u,b} = f_{u,b}(P^{(m, s)}, \lambda_u^{(s)}, \alpha^{(m)}),$$  (8)
where $\lambda_u^{(s)} = 0$ if
\[ \sum_{b \in B_v} f_u,b(P^{(m,s)}, 0, \alpha^{(m)}) \leq T_u, \] (9)
otherwise $\lambda_u^{(s)}$ is chosen such that $\sum_{b \in B_v} P^{(m,s+1)}_{u,b} = T_u$. While the conditions for the convergence of (8) presented in [25] are not fulfilled, in practice the convergence of (8) is observed. Note that whenever (8) converges, it converges to an optimal point of (7).

IV. USER CENTRIC RESOURCE ALLOCATION

This section presents the second resource allocation scheme that we consider. This scheme is composed of an alternating algorithm for the resource allocation problem (4). This algorithm is user-centric in that it starts from a maximal power allocation in which every user transmits with its maximal power. Then, every user chooses the receiving BS to be the one with the maximal SINR for this user. Then, we alternate between the power allocation and channel access problems as described in Algorithm 1 below.

Algorithm 1
1: Input: $\delta > 0$
2: Set $n = 0$, $\delta_0 = 2\delta$, $R_0 = 0$;
3: Set $P_u^{(1)} = T_u$ for all $u \in U_v$;
4: while $\delta_n > \delta$ do
5: $n = n + 1$;
6: For every $u \in U_v$ and $b \in B_v$ calculate
\[ J_{u,b}^{(n)} = \sum_{\hat{u} \in U_v, \hat{u} \neq u} [h_{\hat{u},b}]^2 P_u^{(n)}, \quad \forall u \in U_v, b \in B_v; \]
7: For every $u \in U_v$, calculate
\[ b_u^{(n)} = \arg \max_{b \in B_v} \frac{|h_{u,b}|^2 P_u^{(n)}}{\sigma_b^2 + J_{u,b}^{(n)}}; \]
8: For every $u \in U_v$ and $b \in B_v$ set $\gamma_{u,b}^{(n)} = \|\{b \neq b_u^{(n)}\}$;
9: Calculate the sum rate
\[ R_n = \sum_{b \in B_v} \sum_{u \in U_v} \gamma_{u,b}^{(n)} W \log_2 \left( 1 + \frac{|h_{u,b}|^2 P_u^{(n)}}{\sigma_b^2 + J_{u,b}^{(n)}} \right); \]
10: $\delta_n = R_n - R_{n-1}$;
11: Calculate the optimal power allocation $(P^{(n+1)}_{u,b})_{u \in U_v}$, given the channel allocation $(\gamma_{u,b}^{(n)})_{(u,b) \in U_v \times B_v}$ by solving:
\[ (P^{(n+1)}_{u,b})_{u \in U_v} = \arg \max_{P_u} \sum_{b \in B_v} \sum_{u \neq \hat{u} \in U_v} \gamma_{u,b}^{(n)} W \log_2 \left( 1 + \frac{|h_{u,b}|^2 P_u}{\sigma_b^2 + J_{u,b}} \right) \]
s.t.: $0 \leq P_u \leq T_u, \quad \forall u \in U_v,$
\[ \sum_{\hat{u} \in U_v, \hat{u} \neq u} |h_{\hat{u},b}|^2 P_{\hat{u}} = J_{u,b}, \quad \forall u \in U_v, b \in B_v; \] (10)
12: end while

It is left to solve the problem (10). First, we rewrite this problem by defining the function $b_u^{(m)} : U \rightarrow B$ to be the BS that decodes the message of user $u$, it follows that $\gamma_{u,b}^{(m)} = 1$.

The optimization problem (10) can then be written as,
\[ \max \sum_{u \in U_v} W \log_2 \left( 1 + \frac{|h_{u,b}^{(u)}|^2 P_u}{\sigma_b^{(u)} + \sum_{\hat{u} \in U_v, \hat{u} \neq u} |h_{\hat{u},b}^{(u)}|^2 P_{\hat{u}}} \right) \]
s.t.: $0 \leq P_u \leq T_u, \quad \forall u \in U_v,$
\[ \delta_n = R_n - R_{n-1}; \]
(11)

Denote:
\[ \text{SINR}_{u,b}^{(u)}(P) = \frac{|h_{u,b}^{(u)}|^2 P_u}{\sigma_b^{(u)} + \sum_{\hat{u} \in U_v, \hat{u} \neq u} |h_{\hat{u},b}^{(u)}|^2 P_{\hat{u}}}, \] (12)
where $P = (P_u)_{u \in U}$ is the vector of the power transmission.

We solve the problem (11) approximately by applying the high SINR approximation (4). This yields the problem
\[ P^{(m)} = \arg \max_{P \in U_v} \sum_{u \in U_v} W \text{log}_2 \left( 1 + \frac{|h_{u,b}^{(u)}|^2 P_u}{\sigma_b^{(u)} + \sum_{\hat{u} \in U_v, \hat{u} \neq u} |h_{\hat{u},b}^{(u)}|^2 P_{\hat{u}}} \right) \]
s.t.: $0 \leq P_u \leq T_u, \quad \forall u \in U_v,$
\[ \delta_n = R_n - R_{n-1}; \]
(13)

where $\alpha_u^{(m)} = \alpha \text{SINR}_{u,b}^{(u)}(P^{(m-1)})$ and $\beta_u^{(m)} = \beta \text{SINR}_{u,b}^{(u)}(P^{(m-1)})$, and $\alpha^{(0)} = 1$, $\beta^{(0)} = 0$ for all $u \in U_v$.

Now, substituting $P_u = \exp(g_u)$ yields the equivalent problem
\[ \text{ln}(P^{(m)}) = \arg \max_{\sum_{u \in U_v} W \text{log}_2 \left( 1 + \frac{|h_{u,b}^{(u)}|^2 \exp(g_u)}{\sigma_b^{(u)} + \sum_{\hat{u} \in U_v, \hat{u} \neq u} |h_{\hat{u},b}^{(u)}|^2 \exp(g_{\hat{u}})} \right) \}
\[ \text{exp}(g_u) \}
\[ \text{exp}(g_u) \]
\[ \text{exp}(g_u) \]
\[ \text{exp}(g_u) \]
(14)

Since the problem (14) is convex with non empty interior, its duality gap is zero and the KKT conditions are sufficient for the points to be primal and dual optimal. The KKT conditions for (14), after substituting $P_u = \exp(g_u)$, are
\[ P^{(m)} = \frac{W \alpha_u^{(m)}}{\lambda_u \ln 2 + W \sum_{\hat{u} \in U_v, \hat{u} \neq u} \frac{\alpha_u^{(m)} |h_{u,b}^{(u)}|^2}{\sigma_b^{(u)} + \sum_{\hat{u} \in U_v, \hat{u} \neq u} |h_{\hat{u},b}^{(u)}|^2 P_{\hat{u}}^{(m)}}}; \]
\[ 0 = \lambda_u \left( P_u^{(m)} - T_u \right), \quad P_u^{(m)} \leq T_u, \quad \lambda_u \geq 0, \] (15)
for all $u \in U_v$. Thus, the power allocation for user $u$ must fulfill the equation:
\[ P_u^{(m)} = \min \left\{ T_u, \frac{\alpha_u^{(m)} \text{SINR}_{u,b}^{(u)}(P^{(m)})}{\sum_{\hat{u} \in U_v, \hat{u} \neq u} \alpha_u^{(m)} \text{SINR}_{u,b}^{(u)}(P^{(m)}) |h_{\hat{u},b}^{(u)}|^2} \right\}. \] (16)
By [25] we have that we can solve (16) iteratively. Let \( P_{u}^{(m,s)} \) be the transmitting power of user \( u \) at the \( s \)th iteration. Then the update rule
\[
P_{u}^{(m,s+1)} = \min \left\{ \bar{P}_{u}, \sum_{\bar{a} \in \mathcal{U}_{u}, \bar{a} \neq u} \alpha_{u}^{(m)} \frac{\text{SINR}_{u,b}(\alpha_{u}^{(m)} P_{u}^{(m,s)}, |h_{u,b}(\bar{a})|^2)}{\sum_{b \in B_u} |h_{u,b}(\bar{a})|^2} \right\}
\]
converges to the solution of (16), both synchronously and asynchronously, provided that it exists. The solution is guaranteed because of the strong convexity of (14).

We note that we can also solve (13) by solving the problem (6) using the initial values: \( \alpha_{u,b}^{(0)} = \mathbb{1}_{\{b = b(u)\}} \) and \( \beta_{u,b}^{(0)} = 0 \) for all \( (u, b) \in \mathcal{U}_{e} \times B_{c} \).

V. BASE-STATION CENTRIC RESOURCE ALLOCATION

This section presents the third and final resource allocation scheme that is included in this paper. This scheme is composed of an alternating algorithm for the resource allocation problem (1). This algorithm optimizes the resource allocation in a BS centric manner. In particular it starts from a maximal power allocation in which every user transmits with its maximal power. Then, every BS chooses the transmitting user to be the one with the maximal SINR for this BS. Then, we alternate between the power allocation and channel access problems as described in Algorithm 2.

It is left to solve the problem (18). We rewrite this problem by defining the function \( u(b) : B_{c} \rightarrow \mathcal{U}_{e} \) to be the user that transmit to BS \( b \), it follows that \( \gamma_{u(b),b} = 1 \). The optimization problem (18) is then be written as,
\[
\max \sum_{b \in B_{e}} W \log_{2} \left( 1 + \frac{|h_{u(b),b}|^2 P_{u(b),b}}{\sigma_{b}^2 + \sum_{b \in B_{e}, b \neq b} |h_{u(b),b}|^2 P_{u(b),b}} \right)
\]
s.t.: \( 0 \leq \sum_{b \in B_{e}, u(b) = u} P_{u,b} \leq \bar{P}_{u}, \forall u \in \mathcal{U}_{e} \).

Denote:
\[
\text{SINR}_{u(b),b}(P) = \frac{|h_{u(b),b}|^2 P_{u(b),b}}{\sigma_{b}^2 + \sum_{b \in B_{e}, b \neq b} |h_{u(b),b}|^2 P_{u(b),b}},
\]
where \( P = (P_{u(b),b})_{b \in B_{e}} \) is the vector of the power allocations. We solve the problem (17) approximately by applying the high SINR approximation (4). This yields the problem
\[
P^{(m)} = \arg \max_{P} \sum_{b \in B_{e}} W, \text{ s.t. } 0 \leq \sum_{b \in B_{e}, u(b) = u} P_{u,b} \leq \bar{P}_{u}, \forall u \in \mathcal{U}_{e},
\]
the initial values: \( \alpha_{u,b}^{(0)} = \mathbb{1}_{\{u = u(b)\}} \) and \( \beta_{u,b}^{(0)} = 0 \) for all \( (u, b) \in \mathcal{U}_{e} \times B_{c} \).

VI. FORMING THE VIRTUAL CELLS

This section presents the clustering approaches that create the virtual cells within which the resource allocation algorithms defined in the previous three sections operate. We consider two methods to cluster the BSs. The first is a hierarchical clustering of the BS according to a minimax linkage criteria. To evaluate the performance of this clustering method we compare it to an exhaustive search over all the possible clusters.

A. Base-Station Clustering

1. Hierarchical clustering - Minimax linkage [26]: Let \( d : \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R} \) be the Euclidean distance function.

Definition 4 (Radius of a set around point): Let \( S \) be a set of points in \( \mathbb{R}^2 \), the radius of \( S \) around \( s_i \in S \) is defined as
\[
r(s_i, S) = \max_{s_j \in S} d(s_i, s_j).
\]

Algorithm 2

1: Input: \( \delta > 0 \)
2: Set \( n = 0, \delta_{n} = 2\delta, R_{0} = 0; \)
3: Set \( P_{u}^{(1)} = \bar{P}_{u} \), for all \( u \in \mathcal{U}_{e}; \)
4: while \( \delta_{n} > \delta \) do
5: \( n = n + 1; \)
6: For every \( b \in B_{c} \) and \( u \in \mathcal{U}_{e} \) calculate
\[
J_{u,b}^{(n)} = \sum_{\bar{u} \in \mathcal{U}_{e}, \bar{u} \neq u} \frac{|h_{u,b}|^2 P_{\bar{u},b}^{(n)}}{\sigma_{b}^2 + J_{\bar{u},b}^{(n)}};
\]
7: For every \( u \in \mathcal{U}_{e} \), calculate
\[
u_{u}^{(n)} = \arg \max_{u \in \mathcal{U}_{e}} \frac{|h_{u,b}|^2 P_{u,b}^{(n)}}{\sigma_{b}^2 + J_{u,b}^{(n)}};
\]
8: For every \( b \in B_{c} \) and \( u \in \mathcal{U}_{e} \) set \( \gamma_{u,b}^{(n)} = \mathbb{1}_{\{u = \nu_{u}^{(n)}\}} \);\)
9: Calculate the sum rate
\[
R_{n} = \sum_{b \in B_{c}, u \in \mathcal{U}_{e}} \gamma_{u,b}^{(n)} W \log_{2} \left( 1 + \frac{|h_{u,b}|^2 P_{u,b}^{(n)}}{\sigma_{b}^2 + J_{u,b}^{(n)}} \right);
\]
10: \( \delta_{n} = R_{n} - R_{n-1}; \)
11: Calculate the optimal power allocation \( P_{u}^{(n+1)}(\gamma_{u,b}^{(n)})_{u \in \mathcal{U}_{e}} \) given the channel allocation \( (\gamma_{u,b}^{(n)})_{(u,b) \in \mathcal{U}_{e} \times B_{c}} \) by solving:
\[
(P_{u,b}^{(n+1)})_{u \in \mathcal{U}_{e}, b \in B_{c}} = \arg \max_{P} \sum_{b \in B_{c}, u \in \mathcal{U}_{e}} \gamma_{u,b}^{(n)} W \log_{2} \left( \frac{|h_{u,b}|^2 P_{u,b}}{\sigma_{b}^2 + J_{u,b}} \right),\]
s.t.: \( 0 \leq \sum_{b \in B_{c}} P_{u,b} \leq \bar{P}_{u}, \forall u \in \mathcal{U}_{e}, \)
\[
\sum_{\bar{u} \in \mathcal{U}_{e}, \bar{u} \neq u} |h_{u,b}|^2 P_{\bar{u},b} = J_{u,b}, \forall u \in \mathcal{U}_{e}, b \in B_{c} ; \]
12: end while
Definition 5 (Minimax radius): Let $S$ be a set of points in $\mathbb{R}^l$, the minimax radius of $S$ is defined as $r(S) = \min_{s_i \in S} r(s_i, S)$.

Definition 6 (Minimax linkage): The minimax linkage between two set of points $S_1$ and $S_2$ in $\mathbb{R}^l$ is defined as $d(S_1, S_2) = r(S_1 \cup S_2)$.

Note that $d(\{s_1\}, \{s_2\}) = r(\{s_1\} \cup \{s_2\}) = d(s_1, s_2)$.

The agglomerative hierarchical clustering algorithm using the minimax linkage criterion is as follows:

Input: $C_n = \{\{s_1\}, \ldots, \{s_n\}\}$ and $d(\{s_i\}, \{s_j\}) = d(s_i, s_j)$, $\forall s_i, s_j \in S = \{s_1, \ldots, s_n\}$.

For all $k = n - 1, \ldots, 1$
1) Find $(S_1, S_2) = \arg \min_{G,H \in C_k} d(G, H)$.
2) Update $C_k = C_{k+1} \bigcup \{S_1 \cup S_2\} \setminus \{S_1, S_2\}$.
3) Calculate $d(S_1 \cup S_2, G)$ for all $G \in C_k$.

We perform the hierarchical clustering over the set of BS locations to create the virtual BSs.

The hierarchical clustering is of great relevance to our setup since it enjoys an important property that both the K-means clustering and the spectral clustering lack, namely, the number of clusters can be changed without disassembling all the clusters in the networks. Thus, the number of virtual BSs can be easily adapted according to current state of the network.

b) Exhaustive Search: As previously written, to evaluate the performance of hierarchical clustering we performed exhaustive search over all the possible clustering. In this way, for each number of clusters (virtual cells) we produced all the possible clusters and in the end chose the clustering that yielded the maximal sum rate of the network, considering interference from other virtual cells, given that particular number of clusters.

B. Users’ Affiliation with Clusters

To create the virtual cells, each user is affiliated with its closest BS, and belongs to the cluster its affiliated BS belongs to; this way every virtual BS and it associated users compose a virtual cell.

It is easy to verify that this formation of the virtual cells fulfills the requirement presented in Section II-A.

VII. NUMERICAL RESULTS

This section presents Monte Carlo simulation results that compares the three resource allocation schemes for both the hierarchical clustering and the exhaustive search over all possible clustering. We set the following parameters for the simulation: the network is comprised of 6 BSs and 30 users which were uniformly located in a square of side 2500 meter. The channel bandwidth was 1 MHz and the carrier frequency was 1800 MHz. The noise power received by each BS was $-174$ dBm/Hz, and the maximal power constraint for each user was 23 dBm. Finally, we simulated the channel gains according to a free space model $h_{u,b} = \frac{\lambda}{4\pi d_{(u,b)}}$, where $\lambda$ is the signal wavelength, and $d(u,b)$ is the distance between user $u$ and BS $b$. We averaged the results over 500 measurements, in each we generated randomly the locations of the BSs and users.

In the simulation we compared the sum rate achieved by each of the clustering and resource allocation methods presented in this paper. We note that while the resource allocation ignored the interference caused by other virtual cells, the sum rate of the network was calculated considering this interference.

The simulation results are shown in Fig. 1 for the following cases:

- Exhaustive - alternating UCB and Hierarchical - alternating UCB (user choose base-station) lines refer to the resource allocation scheme presented in Section IV when performing exhaustive search over all the possible clusters, and hierarchical clustering, respectively.
- Exhaustive - alternating BCU and Hierarchical - alternating BCU (base-station choose user) lines refer to the resource allocation scheme presented in Section V when performing exhaustive search over all the possible clusters, and hierarchical clustering, respectively.
- Exhaustive - joint and Hierarchical - joint lines refer to the resource allocation scheme presented in Section III when performing exhaustive search over all the possible clusters, and hierarchical clustering, respectively.

Figure 1 leads to several interesting insights and conclusions, first, it confirms the expectation that as the number of virtual cells decreases the average sum rate increases. The only line that does not show this is the Hierarchical - alternating BCS line in which interference from outer virtual cells was more severe. For a larger number of virtual cells the BS centric resource allocation outperformed the other two methods, however, for a smaller number of virtual cells the user centric resource allocation outperformed the other two methods. As for the joint channel and power allocation, even though its performance did not exceed the other two, the algorithm can be improve in the future by choosing better initial values for $\alpha_{u,b}$ and $\beta_{u,b}$. The initial values of the two other resource allocation methods are two examples for

![Fig. 1. Sum rate as a function of the number of clusters.](image-url)
such initial values. Finally, comparing the performance of the exhaustive search over all the possible virtual cells of a certain size and the hierarchical clustering leads to the conclusion that the hierarchical clustering can improve the sum rate of the network compared to the independent resource allocation by each BS (i.e. the current fully distributed optimization method), however, this depends on the resource allocation scheme. Thus, future evaluation and the development of resource allocation and clustering schemes must go hand in hand since they both significantly affect the performance of the network.

VIII. CONCLUSION AND FUTURE WORK

This work addresses the role of virtual cells in resource allocation for future wireless networks. It proposes methods for two design aspects of this optimization; namely, forming the virtual cells and allocating the communication resources in each virtual cell effectively. We present three types of resource allocation schemes: the first converts the mixed integer resource allocation problem into a continuous resource allocation problem and then finds an approximate solution, the second alternates between the power allocation and channel access problems when the channel allocation is carried out in a user-centric manner, finally, the third resource allocation scheme we present alternates between the power allocation and channel access problems when the channel allocation is carried out from a base-station centric perspective. We also propose the use of hierarchical clustering in the clustering of the base-stations to form the virtual cells, since changing the number of virtual cells only causes local changes and does not force a recalculation of all the virtual base-stations in the network. Finally, we present numerical results for all these methods and discuss the merits of the resource allocation and clustering schemes. We note that the results presented in this paper can be extended to the multi channel setup, we also note that other hierarchical clustering algorithms can be considered in order to improve the overall network performance.

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