The Bose-Einstein effect
and the joint WW decay

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Abstract

The influence of the Bose - Einstein interference effect on the joint WW hadronic decay is discussed. It is shown that the weight method incorporating this effect into Monte Carlo generators produces in a natural way an excess of average multiplicity as compared to the independent decay of two W bosons. The quantitative results for the average multiplicity and momentum distribution of charged pions, obtained with a simple parametrization of weights compatible with the observed shape of the "Bose - Einstein ratio", agree well with the existing data.

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1 Introduction

With the advent of LEP data on the $e^+e^- \rightarrow WW$ process one started to discuss in detail possible effects which could influence the final state obtained from double hadronic decay and break the simple factorization picture. The original motivation was the concern on the possibility to use these data for the precision measurements of $W$ mass, crucial for the tests of the standard model. Although the original suggestions of possible large $W$ mass shifts [1] seem to be rather exaggerated (see refs.[2,3], and references quoted therein), there are other possible observables which discriminate between the existing models of space - time development of the hadronization process.

There are two proposed effects which can make the final state from double hadronic $WW$ decay significantly different from a simple superposition of two systems coming from the decays of two $W$ bosons. First, there is a Bose - Einstein interference (called often ”HBT effect” [4], and here denoted by ”BE effect”) for pairs of identical pions coming from two different $W$-s. Second, there may be so-called ”colour reconnection” processes, in which a colour neutral hadron is formed by coloured partons from two $W$-s.

This note is devoted to the BE effect. We refer only occasionally to some results on the colour reconnection effect [1,2,5]. In Section II we discuss qualitatively the average multiplicity of $WW$ decay. In Section III we present quantitative results obtained when implementing the BE effect into Monte Carlo (MC) generators by our weight method [6]. We conclude with Section IV.

2 Multiplicity as a signal for the BE effect: a qualitative discussion

The BE effect can make the final state of the joint $WW$ hadronic decay different from a simple superposition of two systems coming from independent decays of two $W$-s. Its average multiplicity $\bar{n}_{WW}$ may be not just twice the average multiplicity from single $W$ decay $\bar{n}_W$. Moreover, one may predict which one is bigger.

For the $WW$ production at the energies near to the threshold the decay products of both $W$-s are formed in the same space - time region. Therefore the transition amplitudes to the hadronic final states are not just products of two decay amplitudes. The symmetrization enhances the probabilities to obtain final states where the momenta of identical pions are close in momentum space. This is more likely for higher multiplicities, where many pions are slow in the CM frame. Thus one may expect that $\Delta \bar{n} \equiv \bar{n}_{WW} - 2\bar{n}_W > 0$. Moreover, the excess should be located at low CM momenta, since the decay products of different $W$-s with high CM momenta are rarely close in momentum space.

This argument is qualitative and in general one cannot prove that the excess is significant. Nevertheless, it seems strange to claim that ”there is no reason to expect the excess of multiplicity from the BE effect”[5]. There exists a specific algorithm imitating the BE effect in MC generators which does not change the multiplicities [7,8,9], but it has no theoretical justification. The excess of multiplicity in joint $WW$ decay seems to be a natural result of the BE effects.

This should be contrasted with the consequences of the colour reconnection effects,
which lead naturally to the deficit of multiplicity [5] due to the colour screening. Thus
the net result of these two effects may well be negligible (the so called ”BE conspiracy” [9]).
However, the data may also show that one of the effects dominates. For this reason it is useful to formulate the quantitative predictions. It will be done for the BE effect in
the next Section within the framework of the weight method, for which one can calculate easily the multiplicity excess.

Let us remind here shortly the basics of the weight method [10]. The probability to
produce any final state corrected for the BE effect may be approximated as a product
of the original probability (without the BE effects) and the weight $W(n)$ calculated as a
sum of $n!$ terms, each being a product of two-particle weight factors:

$$W(n) = \sum_{\{P_n\}} \prod_{i=1}^{n} w_{i,k_i}.$$  \hspace{1cm} (1)

More precisely, the global BE weight is a product of weights calculated for each kind
of identical particles. The shape of a two-particle weight factor should be fitted to the
data. A simple one-parameter Gaussian form is often used:

$$w_{i,k}(p_i, p_k) = \exp[-(p_i - p_k)^2/2\sigma^2].$$  \hspace{1cm} (2)

The weight method meets obvious practical difficulties for a large number of particles
$n$, as the factorial increase of number of terms makes the computing impossibly long.
There are some methods to overcome this difficulty [11,12,13,14,15]. We employ here the
recent version of our method [6,3], which uses the clustering algorithm with the cluster
size defined by a parameter $\epsilon$, and yields the weight as a product of cluster weight factors.
For clusters with more than five particles these factors are calculated approximately. The
employed method is time saving and we regard it as a more reliable approximation of
formula (1) than the previous ones.

We are not going to describe here this method in details. Let us remind only two
important approximations used. First, as in the earlier papers [7,12,13,15], only so called
”direct pions” are taken into account since the decay products of the long-living resonances
are born far away from the collision area and the parameter $\sigma$ of formula (2) would
be very small for pairs including such particles. Second, a simple rescaling of weights by a
factor $cV^n$ [12] is used instead of refitting all the free parameters of the MC code (fitted to
the data without the BE effect). This allows to restore the average multiplicity increased
by the weights, which are naturally larger for large multiplicities.

It is this second approximation which allows to estimate easily the multiplicity excess
due to the BE effect. For the MC without the BE effect the final state of the hadronic
$WW$ decay is a simple superposition of two $W$ decay product systems. This means that
the generating function of the multiplicity distribution

$$G(z) = \sum P(n)z^n.$$  \hspace{1cm} (3)

should be just the square of the generating function for a single decay

$$G_{WW}(z) = |G_W(z)|^2.$$  \hspace{1cm} (4)
Rescaling the distribution $P(n)$ by $c V^n$ factors rescales the argument of $G$ by $V$ (the normalization factor $c$ is irrelevant since any $G(z)$ has to fulfill the equation $G(1) = 1$). This does not spoil the relation (4). Rescaling may be interpreted as refitting the parameter which controls the density of particles from a single string. Thus the same value of the rescaling parameter $V$ should be used for single- and double $W$ decay.

In the single decay this value is fitted to restore exactly the average multiplicity obtained without weights (and compatible with the data). However, for the double decay such a reduction is insufficient: the BE weight for a final state from the double decay is always bigger than the product of weights for two independent decays. Since weights enhance the states with high multiplicity, an excess of multiplicity appears. It will be presented quantitatively in the next section.

Let us conclude this discussion with a remark on the energy dependence. For the energies far above the threshold both $W$-s move apart with nearly the speed of light and the formation points of their hadronic decay products are separated by sizeable distances. Thus both the BE effect and the colour reconnection effect should be greatly reduced and the factorization of the final state should be a very good approximation. Later on we comment shortly on the possible parametrization of the energy dependence of the BE effect.

## 3 Results and comparison with data

We have generated many samples of 100 000 events for different channels of the $e^+e^- \rightarrow W^+W^-$ process at 172 GeV by the default PYTHIA/JETSET generator. First, we have chosen the channel where one of the $W$-s decays leptonically, and the other one decays into two hadronic jets. We have generated a sample of events without weights and samples with weights given by our approximation to the formula (1) (with the two-particle weight factors given by (2) and various values of $\sigma$ and $\epsilon$). Then we have chosen the values of the parameters $\sigma = 0.2$ GeV and the ratio $\epsilon/\sigma^2 = 10$ (at the lower limit of the range of values, for which the results do not depend on this ratio [6]). With these values we have found the average multiplicity with BE weights by 20% larger than without weights. To restore the original multiplicity of charged pions we rescaled the weights by a $c V^n$ factor with $V = 0.920$ and $c = 1.798$. This last value guaranteed that the average value of the rescaled weight is just 1. Rescaling reduces the discrepancy between the average multiplicities to less than 1%. Also the inclusive distributions are now not significantly affected by the BE weights.

The choice of the value of $\sigma$ was determined by the experimental shape of BE enhancement at the $Z^0$ peak (where the data are much better than for the $W$ decay, and the shape is expected to be similar). For simplicity we use the same quantity as recently employed for the BE analysis of the LEP2 data [16]: the ”double ratio”, defined as

\[ R_{++}^{++}(Q) = (\rho_{2}^{++} / \rho_{2}^{+-})^{exp} / (\rho_{2}^{++} / \rho_{2}^{+-})^{MC}, \]  

where $\rho_{2}^{ik}$ are the two-particle densities in $Q = \sqrt{-(p_1 - p_2)^2}$ for the pairs of pions with different charge combinations. The superscripts ”exp” and ”MC” denote the data and a MC, in which the BE effect is not included. In Fig.1 we show the data from ref. [16] and
the analogous "double ratio" of the MC results with weights (rescaled) to those without weights. It is obvious that our weight method with the chosen value of $\sigma$ describes the data well.

![Graph](image.png)

**Fig.1.** The double ratio (4) from the data of Ref. [15] (black points) and from our MC (crosses). The errors for MC, not shown for transparency, are of the order of 0.01.

Next, we generate the samples of events in which both $W$ s decay into two hadronic jets. As argued in the previous section, the rescaling of weights should be done with the same value of the parameter $V$ as for a single $W$ hadronic two-jet decay. For simplicity we use the same value $\sigma = 0.2$ GeV for all pairs of identical pions, as the energy is not far above the threshold (as we argue later, the effective value of $\sigma$ for pairs of pions from different $W$-s should decrease with energy). The value of $c$ is again adjusted to give the average weight equal one. To the seven events with anomalous high weight values we ascribed a limiting value of 500 to avoid excessive fluctuations in some histograms. The histograms shown in this paper are not affected by this cut.

In Fig.2 we present the distributions of CM momenta for charged pions with- and without weights. One sees that the sample with weights shows a clear excess of pions for momenta below 1.5 GeV. The global excess $\Delta \pi$ integrated over momenta is about 2.1 charged pions per event. Since without weights we find the average multiplicity just twice the multiplicity of a single $W$ decay, this excess should be attributed to the BE effect. Let us note that the results are quite sensitive to the value of $\sigma$: reducing it by 30% we get a four times smaller excess! We estimate the uncertainty of the value of $\sigma$ to be about 10%, resulting in the uncertainty of $\Delta \pi$ of about 40%. The obtained value of $\Delta \pi = 2.1 \pm 0.9$ is in a very good agreement with the averaged preliminary data from all the LEP2 experiments [17], which give $\Delta \pi = 2.4 \pm 1.8$. Obviously, much better data are needed to draw any definite conclusions, but the results look encouraging.
The distribution of CM momenta \( p[GeV/c] \) for the \( e^+e^- \rightarrow W^+W^- \rightarrow 4j \) (in thousands of pions) without weights (diamonds) and with rescaled weights (crosses).

The fact that the excess appears at small momenta agrees also with some preliminary data [17]. Let us note that our estimate of \( \Delta\pi \) is much larger than the estimates of the (negative) shift of the multiplicity due to the colour reconnection (CR) effect [5]. This justifies \textit{a posteriori} our procedure of calculating the BE effect neglecting the possible CR effects. If the future results confirm the positive shift of multiplicity of similar size as seen now, it will be a strong argument for our method of implementing the BE effect, and against an important rôle of the CR effects.

4 Conclusions and outlook

As a new application of our method of implementing the BE effect in MC generators [6], we have calculated the excess of the charged pion multiplicity in the joint hadronic \( WW \) decay over the value expected from independent decays of two \( W \)-s. The value of this excess is significantly larger than the deficit expected from the colour reconnection effects [5], and agrees very well with the preliminary data [17]. The excess is located at small CM momenta.

If the preliminary data are confirmed, it will be a decisive argument for using the weight method and not the momentum shifting methods [7,8,9] for implementing the BE effects in MC generators. In fact, any observed excess leads to the same conclusion: the momentum shifting method does not affect the multiplicity and the colour reconnection effects produce a deficit instead of an excess.

Both effects mentioned above are expected to decrease at higher energy. For the BE effect a simple approximate method to estimate this decrease would be to define an effective source size \( R_{eff} = hc/\sigma_{eff} \) for pairs of pions from different \( W \)-s as given by

\[ R_{eff}^2 = R_W^2 + 4\beta^2\gamma^2 c^2 \tau^2 \]  

(6)
where $\beta$ and $\gamma$ are the velocity and the Lorentz factor of each $W$ in the CM frame at given energy. $R_W$, given by $hc/\sigma$, defines the effective size of $W$, as measured by the BE effect in the $W$ (or $Z$) decay, and should be used for pairs coming from the same $W$. We do not present here any definite predictions for the energy dependence, as it is not quite clear if $\tau$ should represent $W$’s lifetime or rather the hadronization time for $W$ decay products; this problem should be solved experimentally.

A more direct way to investigate the BE effect for the joint $WW$ decay is to subtract the distributions for the joint $WW$ decay and for the single $W$ decay to get the separated BE effect for pairs from two different $W$-s. However, this has not given yet conclusive results [16]. Some preliminary data suggest that at 172 Gev the value of $R_{\text{eff}}$ for such pairs should be similar to that of $R$, whereas other data show a suppression instead of BE enhancement at low $Q$ (!). Certainly better data are needed to clarify the situation. In any case the conclusions drawn from the multiplicity excess/deficit and from the low $Q$ enhancement/suppression should be compatible.

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