Energy-conserving schemes for wind farm aerodynamics

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Abstract. Computational demands compel researchers to use coarse grids for the study of wind farm aerodynamics, which necessitates the use of accurate numerical schemes. Energy-conserving (EC) schemes are designed to enforce the conservation of Kinetic Energy (KE), an invariant property of incompressible flows. These schemes are numerically stable and free from artificial dissipation, even on coarse grids and could be used as an alternative to high-order pseudo-spectral schemes. This article details tests on EC schemes in the context of Large Eddy Simulations (LES). Results suggest that the accuracy of EC schemes is influenced by the Subgrid Scale model used for the LES. EC methods use central schemes that lead to dispersion, which is more apparent with a less-dissipative Scale Similarity model. Whereas, the purely dissipative Smagorinsky’s model reduces the dispersion and generates a smoother solution but at the cost of accuracy in terms of predicting the KE. Although the impact of EC schemes and SGS models on the LES of wind farms is yet to be assessed, a simple LES of a model wind farm uncovers that EC schemes are quite suitable for wind farm aerodynamics.

1. Introduction

Wind farm aerodynamics is the study of wind turbine wakes and their interaction with each other and with the atmospheric boundary layer (ABL). These interactions govern the wakes’ velocities and ultimately the power that turbines would produce when exposed to these wakes. They also determine the turbulence intensity that affects a turbine’s loading and hence, its longevity [1, 2].

The knowledge of wind farm aerodynamics is central to the design of wind farms, which involves determining the placement of wind turbines to generate the maximum possible power for a specified set of atmospheric conditions, over a defined piece of land. The industry currently relies on simple engineering models (EM) to gauge power production. EMs are quite efficient at delivering mean statistics on power with minimal computational effort, but rely on pre-tuning with experimental data from wind farms, which can be inadequate. Further, the simplicity of EMs makes them unusable for discerning the physics of wind farm aerodynamics [3].

On the other hand, Direct Numerical Simulation (DNS) is very comprehensive and accurate, but is computationally unworkable for wind farm aerodynamics. Thus, as a trade-off between computational demands and accuracy, researchers must resort to Large Eddy Simulation (LES). Apart from precise modelling of the ABL, subgrid scales (SGS) and the turbine, the accuracy of LES is, to a great extent, determined by the numerical schemes used for spatio-temporal discretisation.

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Pseudo-spectral schemes are often favoured because they are accurate and have low or no numerical dissipation, which could dominate the error on the generic coarse grids used for the LES of wind farms. But as a drawback, these schemes are computationally expensive. On the other hand, Energy-Conserving (EC) schemes are free from numerical dissipation (see Section 2), while being numerically stable even on coarse grids. Their ease of implementation with a Cartesian-Finite Volume approach makes them a promising alternative to pseudo-spectral schemes [4].

2. Conservation of energy in incompressible flows
In the presence of periodic or non-penetrative boundaries, the convective and diffusive terms of the incompressible Navier-Stokes (NS) equations possess a property called symmetry. The equations read,

\[ \int_{\Gamma} \mathbf{u} \cdot \mathbf{n} \, d\Gamma = 0 \quad \text{and} \]

\[ \int_{\Omega} \frac{\partial \mathbf{u}}{\partial t} \, d\Omega + \int_{\Gamma} \mathbf{uu} \cdot \mathbf{n} \, d\Gamma = -\int_{\Gamma} p \, \mathbf{n} \, d\Gamma + \int_{\Gamma} \nu \nabla \mathbf{u} \cdot \mathbf{n} \, d\Gamma, \]

where Ω and Γ represent volume and surface integrals, respectively and \( \mathbf{u} \) is the velocity vector.

In a discrete setting, the symmetries translate into a skew-symmetric convective operator, \( C \) and a symmetric, positive-definite diffusive operator, \( D \), when rendered as matrices [5].

A symmetry-preserving discretisation ensures that the kinetic energy (KE) of an incompressible flows remains unchanged in the absence of molecular viscosity, which is an invariant property of inviscid incompressible flows. Owing to this, a symmetry-preserving discretisation is also known as an energy-conserving discretisation. Mathematically,

\[ \frac{d}{dt} \int_{\Omega} k \, d\Omega = -\nu \int_{\Omega} \| \nabla \mathbf{u} \|^2 \, d\Omega, \]

where \( k \) is the KE and \( \nu \) is the kinematic viscosity that is assumed to be constant. The integrand on the right-hand side is the Frobenius product of tensor \( \nabla \mathbf{u} \) with itself.

Equation 3 indicates that the KE of an incompressible flow decreases with time due to viscous dissipation alone or remains unchanged for inviscid flows. Additionally, the KE is unaffected by the convective operator (due to skew-symmetry) and the pressure gradient, \( P \). The latter necessitates the use of a staggered grid for discretisation [6].

Compliance with equation 3 in a discrete sense, ensures the absence of artificial dissipation for spatial operators. All dissipation is physical (laminar or turbulent) and arises from the term \( -\nu \int_{\Omega} \| \nabla \mathbf{u} \|^2 \, d\Omega \), which represents the dissipation through molecular viscosity. Lack of artificial dissipation is achieved by using central schemes instead of upwind schemes. Such a discretisation is important to prevent the numerical diffusion from overwhelming the effect of molecular diffusion in a DNS or the SGS model in an LES, which can dampen the energy spectrum [4].

Combining the above with energy-conserving Runge-Kutta methods, preserves the energy-conserving features of the spatial discretisation with temporal integration. In addition, a completely EC code (spatial and temporal) is unconditionally stable for any mesh size and time step, which is desirable for wind farm simulations that are conducted on coarse grids for computational efficiency [4].

3. Energy-conservation in LES
In LES, only the large energy-containing scales of a turbulent flow are resolved using the filtered NS equations, while the small dissipative scales are modelled mathematically. In implicit LES, the dissipative scales are modelled with numerical dissipation introduced with suitable numerical
schemes. Whereas, in explicit LES that this article concerns, the dissipative scales are modelled with a subgrid (SGS) model. The equations for explicit LES read,

\[ \int_{\Gamma} \tilde{u} \cdot n \, d\Gamma = 0 \quad \text{and} \quad \int_{\Omega} \frac{\partial \tilde{u}}{\partial t} \, d\Omega + \int_{\Gamma} (\tilde{u} \tilde{u}) \cdot n \, d\Gamma = -\int_{\Gamma} \tilde{p} n \, d\Gamma + \int_{\Gamma} \nu \nabla \tilde{u} \cdot n \, d\Gamma - \int_{\Gamma} \tau \cdot n d\Gamma, \]

where \( \tau \) is the subgrid tensor, which represents the interaction between the modelled dissipative scales and the resolved large scales, the KE of which is \( k_{\tilde{u}} \). \( \tilde{u} \) stands for the filtered or resolved velocity field.

Equation 3 is not suitable for describing the dissipation of KE in an LES because of two reasons, only the KE of the resolved scales obtained from equations 4 & 5 is known in an LES and apart from molecular viscosity, the subgrid tensor also partakes in dissipation.

Using a procedure similar to that described in ref. [4], one arrives at the following equation that dictates the evolution of only the KE carried by the resolved scales, in the presence of only periodic or non-penetrative boundaries; in an LES,

\[ \frac{d}{dt} \int_{\Omega} k_{\tilde{u}} \, d\Omega = -\nu \int_{\Omega} \| \nabla \tilde{u} \|^2 \, d\Omega - \int_{\Omega} (\nabla \cdot \tau) \cdot \tilde{u} \, d\Omega, \]

where \( \psi_v + \psi_s = \psi \), are obtained through post-processing and characterise the contribution of molecular viscosity and the subgrid tensor, and their total contribution to the change in KE, respectively (eqn. 6 is not solved). It is obvious from equation 6 that the KE carried by the resolved scales is dynamically altered by dissipative scales (represented in an LES through the subgrid tensor).

It is important to mention that with an SGS model that accurately predicts the interaction between the resolved and the subgrid scales, \( \psi \) will always equal \( \frac{d}{dt} K_{\tilde{u}} \), which in the limit of a DNS should respectively, equal the left and right hand sides of equation 3. However, there is nothing such as an accurate SGS model. So, in an actual simulation with an Energy-Conserving code, we would only expect the \( \frac{dK_{\tilde{u}}}{dt} \) to be predicted accurately because the code is expected to conserve kinetic energy. However, \( \psi \), which is obtained through post-processing, will not necessarily equal \( \frac{dK_{\tilde{u}}}{dt} \) due to various reasons, which include SGS model and the order of accuracy of the mathematical operators used to approximate the right hand side of equation 6 (2nd order for this article).

Various methods of parametrising \( \tau \) have been proposed based on empirically deduced properties of the dissipative scales or theoretical approximations [7, 8]. It is fair to speculate that the evolution of KE dictated by 6, would be dependent on \( \tau \) that appears in the filtered momentum equation and influences the evolution of the velocity field, in turn governing the field’s KE. The aim of this article is to shed light on the interplay between the use of an EC discretisation, the SGS model and accuracy, in the context of LES.

4. Methodology

All results presented in this article were obtained with the Energy-Conserving Navier-Stokes (ECNS) code developed by the Energy Research Centre of the Netherlands [9]. The ECNS uses second-order spatial and fourth-order Runge-Kutta temporal, energy-conserving discretisation to solve the incompressible NS equations on a uniform, staggered Cartesian grid. A sufficiently small time-step (mention in the relevant section) has been used for all simulations to ensure that the resultant error is purely spatial.

SGS models have been compared over the course of two numerical experiments, the roll-up of shear layer in 2 dimensions (2-D) and the decay of a Taylor-Green vortex in 3-D. The SGS models used are, Smagorinsky’s eddy-viscosity model (SM) [10] and Bardina’s scale-similarity model (BM) [11]. SM can be pre-tuned by specifying the value of \( C_S \), the Smagorinsky constant,
to which the eddy-viscosity is directly proportional. Two values of $C_S$ are examined here, 0.17 corresponding to homogeneous isotropic turbulence as suggested by Lilly [12] and 0.1 that is apt for anisotropic flows and leads to lower dissipation [13].

BM relies on relating the stresses at the grid-cell’s scale and the same filtered spatially at another test-scale; the ratio of the two scales or $\beta$ governs the model’s dynamics [7]. The test scale should not be large in comparison to the grid-cell for the hypothesis of scale-similarity to hold. Two test-scales are considered here, one equal in size to the grid-cell ($\beta = 1$) and the other twice its size ($\beta = 2$).

The results from Section 5 & 6 will delineate the interplay between LES and energy-conservation. Finally, the ECNS will be employed to simulate a model wind farm using actuator disks to represent the wind turbines. This is not a rigorous validation exercise, which will follow later, but merely a qualitative analysis of the code’s ability to resolve wake evolution and wake-turbine interaction.

5. Shear layer roll-up in 2-D

LES is employed to repeat the detailed simulations done by Brown and Minion on a perturbed doubly-periodic shear layer and the ensuing vorticity, at a Reynolds number of 10000 [14]. Although turbulence is intrinsically 3-D, a 2-D verification of the ECNS will provide insight into numerical inconsistencies that could be adverse in 3-D.

![Figure 1. Contours of vorticity with different SGS models. The spurious wiggles disappear with higher SGS dissipation (Top-right: Solution from ref. [14]).](image-url)

All simulations were done on a $[0,1]^2$ domain using grids with $32^2$, $64^2$, $128^2$ and $256^2$ cells, but only the results for the $64^2$ case are presented here. Due to the unavailability of Direct Numerical Simulation (DNS), a reference solution was obtained with the ECNS on a $320^2$ grid with a time-step of $10^{-4}$ s. In terms of spatial resolution, $256^2$ and $320^2$ grids produced identical results, with respect to earlier simulations by Brown and Minion [14]. Following a thorough investigation, the time-step was set to $\delta t = 0.005s$ to avert any temporal error. The vorticity
ψ = dK/dt (both joule s⁻¹) is clearly enforced on finer grids. N is the number of grid cells in either direction and N² is the total number of cells.

At a macroscopic level, the ECNS does render the large scale structures correctly with 64² cells, as shown by the contours of vorticity in figure 1. An error analysis revealed that the spatial accuracy is indeed second order (juxtaposed with the 320² solution, not shown here). One easily observes the dispersion stemming from the central EC discretisation schemes that is most apparent with BM (β = 1), which by construction is not very dissipative [11]. Using SM that is purely dissipative by construction, helps dampen the wiggles by smoothing the solution for a better representation on a coarse grid. But in doing so, the large scale structures are rendered incorrectly in terms of an elliptical rather than circular structure of the large vortex. A comparison of the ||e||₂ and ||e||₂ norms of the velocity (and vorticity) field reveals that BM is more accurate than SM despite the wiggles. Although the ||e||₂ error in the velocity field is reduced by a mere 2% with the BM relative to SM with 32² cells, the reduction is nearly 50% with 256² cells.

The ECNS is constructed to enforce KE conservation in a discrete setting for an inviscid flow. An analysis of KE dissipation as dK/dt, reveals that equation 6 is satisfied by all SGS models in the limit of fine grid resolution, assessed using the reference solution obtained on a 320² grid. However, on coarse grids, the decay is predicted more correctly with BM or with SM using a smaller Cₛ, which makes the latter less dissipative. In fact, BM is quite accurate even on a 64² grid. Further, SM shows a large deviation near t = 0 s, which reduces only when the simulation becomes less dynamic near t = 2 s, as shown in figure 2.

Mathematically, the subgrid tensor alters the velocity field through the momentum equation, which in turn determines the KE, albeit with some error. Whereas, ψ is obtained through post-processing but also depends on the subgrid tensor. SM regularises the velocity field to be easily rendered on a coarse grid, in turn ensuring accurate post-processing, which is noticeable by the closeness of ψ and dK/dt for Cₛ = 0.17. But this leads to a large inaccuracy in predicting either of them correctly, as compared to the reference solution obtained with grid of 320² cells.

Compliance with equation 6 is purely an aspect of post-processing. An EC-LES merely prevents numerical dissipation from altering the velocity field and hence its KE, which must ideally be dissipated through the subgrid tensor.

6. Decaying Taylor-Green vortex in 3-D
2-D tests revealed that an EC spatial discretisation will lead to a dispersive solution that will however, be more accurate than a solution obtained with a dissipative model, which will be free
from spurious wiggles. But the true nature of turbulence can only be interpreted with a 3-D test case.

This experiment records the decay of an initially isotropic Taylor-Green vortex in a $[-\pi, \pi]^3$ domain, at a Reynolds number of 1600. This case is particularly interesting as the flow is initially laminar and transitions into turbulence with time [15, 16]. Data on the rate of decay of turbulence KE, $\epsilon$ or $-\frac{dK}{dt}$, from earlier DNS studies serves as a reference. LES was done on $16^3$, $32^3$, $48^3$ and $64^3$ uniform grids, with a time step of $\delta t = 0.001s$.

Visualising the vorticity is made easy with the Q-criterion, a positive value of which indicates a dominance of vorticity over shear [17]. Figure 3 depicts how the flow transitions from being laminar with vortex tubes into small-scale turbulence. Colourbars are not provided for terseness as the amount of KE reduces with time and also because the authors wish to emphasise the nature of the flow rather than its KE.

A comparison between BM, $\beta = 1$ and SM, $C_S = 0.17$, reveals the qualitative effects of SGS dissipation on the velocity (indirectly vorticity) field in figure 4. Although the SM renders a smoother field due to increased dissipation, the field’s KE as suggested by the colour bars is different from that predicted by BM, which is in fact more accurate.

![Figure 3](image)

**Figure 3.** Iso-surfaces representing regions with a value of Q-criterion equal to 0.5 $s^{-2}$, which are coloured by kinetic energy (joule) at various instances using Bardina’s model with $\beta = 1$. $t = 0$: initial, $t = 3$: laminar, $t = 8$: in transition and $t = 18$: small-scale turbulence.

But which model is more accurate can only be deduced by the evolution of KE with time (see figure 5). Although SM renders a smoother field, BM with $\beta = 1$ is more accurate than SM at estimating $\epsilon$, and more so on a $64^2$ grid. Also, a more accurate peak dissipation is obtained with BM. A deviation from the reference solution at $t = 0$ with SM as noticed in 2D testing (marked with a box), is also also apparent in 3D, although diminishing with grid resolution. Further, BM with $\beta = 2$ is as accurate as BM with $\beta = 1$ on a finer grid. This happens because using $\beta = 2$ on a coarse grid, sets the test-scale to a relatively larger value at which the hypothesis of
Figure 4. Iso-surfaces representing regions with a value of $Q$-criterion equal to $0.5 \text{ s}^{-2}$, which are coloured by kinetic energy (joule) at $t = 14\text{s}$, using Bardina’s model with $\beta = 1$ and Smagorinsky’s model with $C_S = 0.17$.

scale-similarity may not hold true, as opposed to a finer grid on which scale-similarity would be plausible with $\beta = 2$.

Figure 5. The evolution of $\epsilon$ with time on $32^2 \& 64^2$ grids obtained with two variations of Bardina’s and Smagorinsky’s models.

To summarise, it is evident that the ability of an EC-LES to correctly predict the velocity field and hence its KE is, to a great extent, determined by the SGS model. A more dissipative model, such as SM, may render a smooth solution of a coarse grid, which may not be accurate. Having said that, it is important to mention that the basic assumptions of SM lack experimental support, as opposed to those of BM [18]. However, the ability of SM to generate agreeable mean flow statistics and its ease of implementation, have made the SM and its higher-order extensions dominant in wind farm LES codes.

7. Relevance to wind farm aerodynamics
A simple and common approach employed by wind farms codes is to predict a turbine’s output by using the local velocity field (calculated numerically) and turbine’s thrust coefficient. Thus, it is advantageous to obtain an accurate velocity field, which obviously will have the correct KE.
However, the results presented in the previous sections hint at a trade-off between using an EC spatial discretisation and obtaining a smooth velocity field that could be represented on a coarse grid without spurious wiggles. An SGS model that not only helps conserve the KE in the context of an LES, but also produces an accurate velocity field on a coarse grid, would be most lucrative for the LES of wind farms.

![Isosurfaces of Q-criterion coloured by streamwise velocity before (above) and after (below) wake-interaction](image)

Although the authors are currently investigating the above in detail, this article provides a quick insight into the capabilities of an EC-LES through the simulation of a model wind farm. Six turbines of diameter, \( D = 100 \) m at a hub height of 100 m are simulated over a 12D-by-5.5D area in an ABL-like inflow of height 4D, defined with a simple boundary layer profile [19],

\[
u(y) = y^{0.14},
\]

with the no-slip boundary condition at \( y = 0 \). A precursor simulation was performed to let the ABL inflow develop before assigning it to the inflow boundary. The ABL’s maximum velocity was set to \( u_{in} = 15 \) ms\(^{-1}\), and the flow conditions were adjusted to deliver a Reynolds number greater than \( 10^7 \) based on \( D \). All the other boundaries are assigned the outflow boundary condition, which enables placing the turbines close to the boundary without any effect on the solution. This is advantage over pseudo-spectral methods that are although more accurate but require periodic boundaries and hence, a larger computational domain to prevent the downstream wakes from influencing the inflow conditions due to periodicity.

The wind turbines are modelled as actuator disks, which are introduced into the computational domain with an Immersed Interface Approach (see ref. [4]). A turbine’s effect on the flow is modelled as a volume force (divided by the density) in the NS equations using,

\[
f_x = \frac{1}{2} C_T u_{in}^2,
\]
where $C_T$ is the turbine’s axial thrust coefficient, set to 0.8 in this case. For simplicity, a turbine’s tangential thrust and the subsequent rotation of its the wake are ignored.

A uniform Cartesian grid with $128 \times 64 \times 64$ cells, with an overall grid-resolution of 0.08D\(^1\), is used for the computations and the turbines have a spanwise and streamwise separation of 2.5D and 4D, respectively, which had to be kept low to resolve the wakes with a more computational cells. Smagorinsky’s model with $C_S = 0.17$ is used to model the subgrid tensor. To make the simulation more realistic, the turbines are staggered slightly (0.5D, spanwise) to obtain partial wake-interactions, where the wake of an upstream turbine interacts partially with a downstream turbine.

Figure 6 depicts isosurfaces of Q-criterion that are coloured by streamwise velocity, along with slices depicting contours of streamwise velocity, both before and after wake-interaction. Despite a simple SGS model and 2\(^{nd}\) order discretisation, the ECNS is able to render basic features of a wake. There is reduction in axisymmetry with downstream distance due to the interaction between the ground and the expanding boundary layer. Further, the wake expands significantly even after partial interaction with a downstream turbine. Finally, the wake from the first turbine is quite smooth till 1D-1.5D, following which it breaks down into turbulent structures.

The presence of "rings" in figure 6 is a start-up effect because the simulation was initialised with an ABL inflow to avert numerical instability.

Figure 7 displays plots of streamwise velocity against height from the ground in a plane perpendicular to the first turbine and passing through its vertical diameter. After the ABL encounters the first turbine, there is an abrupt reduction in velocity with simultaneous wake expansion. The deficit increases gradually till the wake interacts with the second turbine, following which there is a further increase in deficit. However, after this the wake begins to recover and its velocity nearly increases to that of the ABL by 10D.

These observations are congruous with experimental and numerical studies on wakes, which conclude that a wake’s is swifter beyond 5D due to increased turbulent mixing [20, 21]. Had there been any artificial dissipation, the wake should have recovered before 10D, as reported by previous studies [22].

Future publications will incorporate detailed studies with more accurate turbine and ABL modelling.

\(^1\) Overall grid-resolution or $\Delta = \sqrt[3]{\Delta_x \Delta_y \Delta_z}$
8. Conclusion and future work

Through a series of test cases, this article elucidates the benefits and drawbacks of energy-conserving methods, which the authors seek to develop for the LES of wind farms, as an alternative to pseudo-spectral schemes. The following conclusions were drawn.

An EC spatial scheme is based on central discretisation to prevent numerical dissipation, but incurs dispersion leading to a velocity field with wiggles, and more so with a non-dissipative SGS model. A dissipative SGS model, like Smagorinsky’s model, can dampen these wiggles to produce a smooth solution, albeit with reduced accuracy. The dispersion arises from the convective term and can be reduced by increasing the order of scheme used to formulate the discrete convective operator. Further, the authors are giving a thought to higher-order SGS models, like the scale-dependent versions of Smagorinsky’s model or a blend of Smagorinsky’s and Bardina’s model.

Even with a simple model of the turbine, the ECNS can account for basic wake properties like reduction in axisymmetry due to the atmospheric boundary layer, recovery of velocity deficit with distance from the rotor and the response of a wake following a wake-turbine interaction. However, a proper validation of the ECNS would involve a more accurate model of the turbine, for example a non-uniformly loaded actuator disk with tangential induction. In addition, the ABL must be modelled with a more reliable approach, like Monin-Obukov’s Similarity model, which is often used in the LES of wind farms. These modifications to the ECNS and subsequent wind farm simulations will be the theme for future publications by the authors.

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