Evaluation of natural frequency, Campbell diagram and forced torsional vibration of deepwell pumps for the marine, oil and gas industry

M. Bayat and H.L. Sørensen
Svanehøj A/S, Fabriksparken 6, 9230 Svenstrup J, Denmark

The corresponding author’s e-mail address: mba@svanehoj.com

Abstract. Deepwell marine and offshore pumps need to design in a way to present high performance and reliable lifetime. Deepwell pumps should be designed in a way to operate under offshore conditions. In order to investigate the resonance and find critical rotational speeds of the pump, it is needed to find the rotational natural frequency for undamped model. In order to avoid any resonance, the intersection points between natural frequency and excitation (1X and 2X) should not be in the interval of 90%-110% of operating speed, based on API standard (API 610). Deepwell pump structure is combination of motor, motor-shaft coupling and pumping configuration. Based on the size of tank and type of ship the length of whole deepwell pump can be more than 40 meters. Elastic rotating shafts connect with appropriate shaft-couplings with needed numbers of impellers and inducer. Small and linear angle of rotation theory are used to extract natural frequencies of deepwell pumps. Different damping scenarios are illustrated in detail. Holzer method is implemented to analyse torsional forced vibration. Free-free boundary conditions considered. Combination of discrete model, lumped mass model and finite element method are employed to present the Campbell diagram. Parametric strategy is used for forced torsional vibration analysis. It is found that how to model and consider the effective length of intermediate shaft in pumping configuration has effect on second natural frequency. It is seen that the first natural frequency of the whole system is highly depended on modelling of motor-shaft coupling. The results are compared when the whole pump is model by lumped mass model and finite element model. Results of this paper suggest that combination of three mentioned methods is acceptable and accurate. It is shown that Holzer method is suitable for torsional forced vibration analysis. The developed codes are verified with findings in the literature.

1. Introduction
Deepwell pump design includes the torsional and lateral vibrations investigation [1]. Torsional analysis is one of dynamic analyses’ procedure which is essential to be performed to avoid resonance and vibration issues to improve deepwell pump life and reliability [2]. Torsional analysis evaluates the twisting interaction between rotors and couplings and the torsional analysis should consider all potential excitation sources [3].

The torsional vibration of shaft has been recognized and documented since 1950 [4-8]. Several numerical methods have been developed and implemented to analyze torsional vibration of shaft such as: finite element method (FEM), discrete element method (DEM), Holzer method and lumped mass model (LMM) in [9-12] and references in there can be referred. A vertical pump contains inducer,
impellers, intermediate shafts which are connected by shaft coupling, bearing for each intermediate shaft, top shaft, shaft motor coupling and at the top motor is located as shown in Figure 1:

![Figure 1. Different part of vertical deepwell pump at Svanehøj](image)

A vertical pump consists of a conventional electrical motor mounted at the pump base (top), the input power convert to kinetic (centrifugal) energy in the liquid by accelerating the liquid with the impeller. Impeller is attached to a shaft in a needed length and for multi-stage configurations several impellers mounted on the same shaft to create higher pressure. At the bottom of the pump an inducer connected to shaft while the liquid enters through a suction branch. Vertical pumps work when liquid enters the pump at the bottom. From there it moves into the first stage impeller, which increase the liquid’s velocity. The liquid then enters immediately above the impeller, where this high velocity energy is converted into high pressure, and this process continues through all the stages of the pump. The liquid passes through a long vertical column pipe and leave the discharge branch. The rotating shaft inside the pipe is supported for each 650mm-1500mm intervals with guide bearings and pipe has supported approximately each 10m to the caisson pipe and finally caisson pipe supported by the tank if the caisson pipe is more than 15m.

It is vital to determine in advance the structural dynamics and follow the procedure as described in API 610 [1] regarding to rotational natural frequencies. Therefore, in this study the combination of discrete element method, lumped mass model, finite element and Holzer method are implemented. In-house codes programmed in MATLAB have been developed to calculate natural frequencies and present Campbell diagram. Holzer method is used for forced torsional analysis. It is revealed that combination of mentioned methods can be an appropriate procedure to predict natural frequency and perform vibration analysis. The developed codes are verified with findings in the literature.

2. Model description
The deepwell pump with the motor at the top can be modeled as a configuration which consist of three general sections: Motor, Motor coupling and Pump sections as shown in shown in Figure 2:

![Figure 2. General sections of vertical pump at Svanehøj](image)
Each section in Figure 2, is going to model with different procedure. It can be modeled as series of disks and rotational springs. The disks will represent the effect of mass and rotational springs are going to represent of stiffness of the model. In more details, pumps can be represented as:

Motor – MotorShaftCoupling – Shaft-disk–…-Shaft-disk

2.1. Motor model
The discrete element method (DEM) is employed. And it is modelled as concentrated inertia load. The second moment of inertia of the motor is considered as discrete inertia.

2.2. MotorShaftCoupling model
The lumped mass model (LMM) is employed to model the MotorShaftCoupling structure. Based on the type of the motor shaft coupling a combination of rotational springs and disks in series form as it is shown in

![Figure 3. Spacer coupling [13], and modelling as series of springs and disks](image)

2.3. Pump configuration model
The FEM and DEM are employed to model the all shafts, shaft couplings, impellers and inducer. In order to model shafts, the FEM is used. DEM is employed to model the mass effect of impellers and inducer. This section of pump can be modeled as combination of rotating disk and rotational spring as shown in Figure 4:

![Figure 4. Disk-spring model from the end of MotorShaftCoupling to the end of the pump](image)

Based on the Lagrange’s equations for a shaft with the length of $L$ subjected to rotation angle of $\theta_1$ and $\theta_2$ for each end, the shaft stiffness and inertia can be represented as:

$$K_{\text{element,FEM}} = K_{sh} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad I_{\text{element,FEM}} = \frac{2}{3} I_{sh} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad K_{sh} = \frac{GJ}{L}, \quad I_{sh} = \rho J$$

(1)

where $G$ is shear modulus, $J$ is second moment of area and $\rho$ is density.

3. Details of methodology

3.1. Free torsional vibration analysis
Three main sections: Motor, MotorCoupling and pump configuration have been modelled as disk-spring system. The stiffness and mass matrix for disk-spring system can be presented in matrix format. The free-free condition is considered for whole system as shown in Figure 5:

![Figure 5. Free-free condition for whole system](image)
The Lagrange’s equations, where one makes use of scalar quantities such as kinetic energy and potential energy are used to obtain the equations of motion. The Lagrange’s equations for a system with \( N \) degrees of freedom can be described by a set of \( N_{\text{lag}} \), and using generalized coordinates: \( q_{\text{lag},j} \) while \( \text{lag.}_j = 1,2, \ldots N_{\text{lag}} \). The general form of equations can be written as:

\[
\frac{d}{dt} \frac{\partial T_{\text{lag},j}}{\partial \dot{q}_{\text{lag},j}} - \frac{\partial T_{\text{lag},j}}{\partial q_{\text{lag},j}} + \frac{\partial U_{\text{lag},j}}{\partial \dot{q}_{\text{lag},j}} = Q_{\text{lag},j} \quad \text{lag.}_j = 1,2, \ldots N_{\text{lag}}. \tag{2}
\]

where \( q_{\text{lag},j} \) are the generalized velocities, \( T_{\text{lag},j} \) is the kinetic energy of the system, \( V_{\text{lag},j} \) is the potential energy of the system, \( D_{\text{lag},j} \) is the Rayleigh dissipation function, and \( Q_{\text{lag},j} \) is the generalized force that appears in the \( \text{lag.}_j \) th equation. More details are referred to [5].

By implementing mentioned equations in Lagrange equation (3), and writing potential and kinetic energy equations of motion can be written as:

\[
\begin{align*}
I_1 \ddot{\theta}_1 + k_{MCL}(\theta_1 - \theta_2) &= T_{\text{motor}} \\
I_{MCL,1} \ddot{\theta}_2 + k_{MCL}(\theta_2 - \theta_3) + k_{MCM}(\theta_2 - \theta_3) &= 0 \\
I_{MCL,2} \ddot{\theta}_3 + k_{MCM}(\theta_3 - \theta_2) + k_{MCM}(\theta_3 - \theta_4) &= 0 \\
\frac{1}{6} I_{sh-1}(2\dot{\theta}_4 + \dot{\theta}_5) + k_{MCM}(\theta_4 - \theta_3) + k_{sh-1}(\theta_4 - \theta_5) &= 0 \\
\frac{1}{6} I_{sh-2}(2\dot{\theta}_5 + \dot{\theta}_6) + k_{sh-2}(\theta_5 - \theta_4) + k_{sh-2}(\theta_5 - \theta_6) &= 0 \\
\frac{1}{6} I_{sh-3}(2\dot{\theta}_6 + \dot{\theta}_7) + k_{sh-3}(\theta_6 - \theta_5) + k_{sh-3}(\theta_6 - \theta_7) &= 0 \\
& \vdots \\
\frac{1}{6} I_{sh-n-1}(2\dot{\theta}_{n-1} + \dot{\theta}_n) + \frac{1}{6} I_{sh-n-2}(2\dot{\theta}_{n-2} + \dot{\theta}_{n-1}) + \frac{1}{6} I_{sh-n-3}(2\dot{\theta}_{n-3} + \dot{\theta}_{n-2}) + k_{n-5}(\theta_{n-1} - \theta_{n-2}) + k_{n-4}(\theta_{n-1} - \theta_{n-3}) + k_{n-4}(\theta_{n-1} - \theta_n) &= 0
\end{align*}
\]

The general Mass matrix \( \bar{M} \) and Stiffness matrix \( \bar{K} \) can be calculated and then the natural frequencies can be obtained by solving the equation (for natural frequency vector \( \omega \)) as:

\[
\det[\bar{K} - \omega^2 \bar{M}] = 0
\]

It can be mentioned that FEM is used to model the shafts and the kinetic energy from fluid momentum in the impeller geometry is neglected in Equation (2), however, to achieve more accurate results, the fluid kinetic energy should be considered. Based on API [14], the undamped train torsional natural frequencies are recommended and then if needed we need to do other simulation such as shaft stress and transient analysis (refer to Figure2-2 in [14]). In order to plot Campbell diagram, the undamped natural frequencies are used. The damping effect are considered for force response analysis.

3.2. Forced torsional vibration analysis

In order to investigate the actual influence of the excitation where the intersect point between torsional natural frequencies’ lines and 1X (and 2X) excitation placed within the ± 10% of operating speed range, the forced response analysis should be performed. Based on ANSI/API 610 [10], the stress analysis and forced response are accomplished to demonstrate that the resonances have no adverse effect on the complete train. In following, the implemented damping strategy and procedure for Holzer-type problem are demonstrated in detail.

3.2.1. Damping strategy

In general, it is difficult to quantify the source of a system’s damping and it is even more difficult to determine the value of the damping accurately. Therefore, the damping forces are usually derived through test by measuring damping ratio \( \xi \)[15]. In API 610 [1], the value of damping ratio is presented. Classical damping is implemented which is well-suited for linear system. Classical damping represents the ideal condition of damping where: \( \bar{C} \bar{M}^{-1} \bar{K} = \bar{K} \bar{M}^{-1} \bar{C} \).

By neglecting the stiffness proportional damping in Rayleigh damping system which is a linear combination of mass and stiffness we can find the damping matrix as [15]:

\[
\bar{C} = \begin{bmatrix}
\frac{1}{2} \bar{M} & 0 \\
0 & \frac{1}{2} \bar{M}
\end{bmatrix} \bar{K}
\]
\[
\ddot{C} = \alpha \ddot{M} + \beta \dddot{K} \rightarrow \ddot{C} = \alpha \ddot{M}, \alpha = 2\xi\omega_1 = \frac{4\pi\xi}{T_1}
\]  

(5)

The mass proportional damping widely used as a damping model. It can be noted that the stiffness proportional damping usually should be neglected for its significant influence on limit time increment, reasonable mass proportional damping becomes more important in actual engineering [15]. It can be added that, in explicit method, the stability limit of time increment will be reduced significantly because of the introduction of stiffness proportional damping, so it is proposed to be neglected by some software, such as ABAQUS [16].

3.2.2. Postprocessing

Holzer method is employed to determine the rotational angle, torque and stresses of pump in each element. For a linear mass-dashpot-spring system as shown in Figure 5, by using equilibrium equations for disks and shaft for left and right hand sides (which are designated by superscripts L and R) and implementing continuity conditions (Figure 7), we can have:

\[
\theta_n = \theta_{n-1} - \frac{T_n^{\text{com.}}}{K_n^{\text{com.}}}, T_n^{\text{com.}} = \sum_{j=1}^{n} \left( \omega_j^2 \theta_j - \omega_i \theta_j + MO_j \right), K_n^{\text{com.}} = k_n + i\omega_c n
\]

\[\text{Figure 6. Chain-type mass-dashpot-spring system} \quad \text{Figure 7. An element between two disks} \]

\(T_n^{\text{com.}}\) and \(K_n^{\text{com.}}\) are called as the complex mass and stiffness. \(MO\) is the applied external moment on disk, where \(MO_j\) represents the applied external torque on disk \(j\). It can be highlighted that if the rotation at first disk is unknown, we need to gain the parametric strategy and analysis in MATLAB to find the rotation in first disk and then implement the mentioned procedure.

4. Verification and pump analysis

4.1. Verification of the developed codes

In order to verify the developed codes, several examples from literature and textbooks are selected and analysed. The numerical results obtained from Eqs. (4) and (6) can be verified by comparing the natural frequencies and torque-frequency analysis for free and forced vibration analysis, respectively.

4.1.1. Natural frequencies for free vibration analysis

The torsional natural frequencies a flywheel through a rigid mechanical coupling as shown in Figure 8 has been calculated by using FEM and Holzer method by [17].

\[\text{Figure 8. Real mechanical system and given data to the system to analyse [17]} \]

The natural frequencies are calculated by [17] based on Holzer method in Error! Reference source not found. and based on FEM in ANSYS [17] in Figure 9:
Table 1: Torsional natural frequencies by Holzer method [17]

| F (Hz) | 𝐹 (Nm) | 𝜃₁ (rad) | 𝜃₂ (rad) | 𝜃₃ (rad) |
|--------|---------|-----------|-----------|-----------|
| 26.64  | 0       | 1         | 0.2996    | -0.7463   |
| 72.4   | 0       | 1         | -0.4173   | 0.4438    |

Figure 9. First (26.473 Hz) and second (71.75 Hz) torsional natural frequencies by FEM in ANSYS [17]

The results from the developed code are: 

The calculated results from developed code and results from [17], have good agreement as shown in Error! Reference source not found.

Table 2. Comparison results from Quiroga et al. (2019) and developed in-house code

| Natural Frequency (Hz) | \( f_1 \) | \( f_2 \) |
|------------------------|-----------|-----------|
| Results by using Holzer method by [17] | 26.64 | 72.4 |
| Results by using FEM by [17] | 26.47 | 71.75 |
| Results from developed code | 26.42 | 72.41 |

4.1.2. Example for torque-frequency analysis

The following torsional system has been analysed by [18] as shown in Figure 10:

The torque-frequency curve for damped system presented in Figure 11, and the results from the developed code is in Figure 12:

4.2. Pump analysis

The torsional analysis for offshore process and cargo (OPC) 200 pump with the length of approximately 25 m has been done, different parts of OPC 200 are shown in Figure 13. The considered material for OPC 200 is stainless steel AISI 316 with the specification as shown in Table 3.

Table 3: The specifications of OPC 200

| \( I_{motor} = 4.8 \, \text{kgm}^2 \); | \( \text{Operating speed} = 2700 \text{rpm} \); | \( \text{Number}_{\text{shaft coupling}} = 9 \); | \( I_{shaft \, coupling} = 1.725E - 9 \, \text{kgm}^2 \); | \( D_{out \, coup} = 67 \text{mm} \) |
Poisson’s ratio = 0.3; \hspace{3em} Young’s modulus = 2.13E11 Pa; \hspace{3em} Density = 7800 kg/m³;

\begin{tabular}{l l l}
Number\_impeller & 1; & \hspace{3em} I\_impeller & 0.5845 kgm²; & \hspace{3em} L\_impeller & 90mm \\
L\_Top\_shaft & 275.5mm; & \hspace{3em} L\_adjusting\_1 & 801mm; & \hspace{3em} L\_adjusting\_2 & 2952mm; \\
L\_intermediate\_shaft & 3m; & \hspace{3em} Number\_intermediate\_shaft & 6; & \hspace{3em} Number\_impeller & 1; \\
L\_shaft\_up\_impeller & 2.637m; & \hspace{3em} I\_impeller\_shaft & 479mm; & \hspace{3em} L\_coupling & 171mm; \\
\end{tabular}

The whole pump is modelled by 101 rotating disks and 100 springs. The first disk represents the motor, and next two disks are related to the MotorShaftCoupling and the other 98 disks simulating others such as: top shaft, intermediate shafts, shaft coupling, impeller shaft and impeller. Shaft has been divided into small elements with nodes in relevant positions such as geometry changes and locations with added inertia from impellers and shaftCouplings. Each intermediate shaft is modelled with 10 disks and 9 springs.

By implementing the mentioned information in in-house code, the natural frequencies are calculated as shown in Figure 14:

As it can be seen from Campbell diagram (Figure 15) there is no interaction between horizontal lines (first and second natural frequencies) and inclined lines (excitation, 1X and 2X), in a yellow area. Based on API 610 [1], the yellow area is ± 10% of operating speed range. This simulation shows there is no interface and critical resonance.

5. Conclusions
In this work the combinations of discrete element, lumped mass model, finite element model and Holzer’s method are implemented to perform rotational vibration analysis for deepwell pump. The
shafts and couplings are modelled by FEM and for another parts DEM and LMD are used. The in-house codes for free and forced vibration analysis have been verified with well-known results in the literature. It is concluded that the series of rotating disks and springs can be a good model for torsional analysis of deepwell pumps. It is considerable if the length of sections for each spring model should be small and comparable with the length of shaft coupling. Beside the disk-spring model the value of inertia and stiffness have important roles to present torsional analysis.

References
[1] ANSI/API Standard 610 10th edition 2004 Centrifugal pumps for petroleum, Petrochemical and natural gas industries
[2] Quiroga J, Bohórquez O, Ardila J, Bonilla V, Cortés N and Martinez E 2019 Torsional natural frequencies by Holzer method IOP Conf. Series: Journal of Physics: Conf. Series 1160 (2019) 012008
[3] Simons S, Hinchliff M, White B, Talabisco G, Kurz R and Ji M 2019 Compressor System Design and Analysis Compression Machinery for Oil and Gas chapter 11 pp 427–447
[4] Kelly P and Menday M 2010 Tribology and Dynamics of Engine and Powertrain Fundamentals, Applications and Future Trends chapter 27 pp 839–856
[5] Meirovitch L 1997 Principles and Techniques of Vibrations Prentice-Hall Int., Inc., New Jersey.
[6] Rao S S 2010 Tribology and Dynamics of Engine and Powertrain Fundamentals, Applications and Future Trends chapter 27 pp 839–856
[7] Meirovitch L 1997 Principles and Techniques of Vibrations Prentice-Hall Int., Inc., New Jersey.
[8] Nestorides E A 1958 Handbook on Torsional Vibration London: Cambridge University Press
[9] Pandya H R, Pandya J G and Acharya G D 2018 Vibration Analysis of Pump Shaft Using Finite Element Analysis Software: A Review Trends in Mech. Eng. & Tech. 8 (1) pp 46–54
[10] El-Gazzar D M 2017 Finite element analysis for structural modification and control resonance of a vertical pump Alexandria Eng. J. 56 pp 695–707
[11] Corbo M A and Malanoski S B 1996 Practical design against torsional vibration Proceedings of the 25th Turbomachinery Symposium Texas pp 189–222
[12] Wachel J C and Szenasi F R 1993 Analysis Of Torsional Vibrations In Rotating Machinery in Proc. 22nd Turbomachinery Symposium pp 127–151
[13] John crane company (2019), https://www.johncrane.com/. Accessed 2019
[14] API publication 684 edition 1-1996 Tutorial on the API standard paragraphs covering rotor dynamics and balancing: an introduction to lateral critical and train torsional analysis and rotor balancing
[15] Xiaoming C, Jin D and Yungui L 2015 Mass proportional damping in nonlinear time-history analysis In: Proc. of 3rd Int. Con. Mat., Mech. and Manuf. Eng., IC3ME 2015, June 27-28, Guangzhou, China pp 567–571
[16] ABAQUS User Manual 2016, ABAQUS Inc
[17] Quiroga J, Bohórquez O, Ardila J, Bonilla V, Cortés N and Martinez E 2019 Torsional natural frequencies by Holzer method IOP Conf. Series: Journal of Physics: Conf. Series 1160-012008, doi:10.1088/1742-6596/1160/1/012008
[18] Thomson W T 1972 Vibration Theory and Applications Prentice-Hall, Allen & U, New edition3

Acknowledgments
Authors wishing to acknowledge assistance or encouragement from colleagues, special work by technical staff or financial support from Svanehøj A/S.