1. INTRODUCTION

Micro-electro-mechanical systems (MEMS) based scanning mirrors are miniaturized mechatronic systems that consist of an optical micromirror and its actuation scheme to steer the laser beam. With abundant applications, MEMS mirrors receive much attention from pico projector (Yalcinkaya et al. (2006)) to the automotive lidar (Ito et al. (2013); Druml et al. (2018)) thanks to their compactness, robustness, scalability, easy means for integration, and cost-effectiveness (Patterson et al. (2004)). Especially for the automotive lidar application, high accuracy and precision scanning of MEMS mirrors is critical to ensure safety of drivers and passengers in various driving conditions such as highways and urban areas (Yoo et al. (2018)).

To ensure the performance of the MEMS mirror, accurate and precise characterization is crucial not only for inspection in production line but also for design of the MEMS mirror to identify the critical parameters to be improved (Brunner et al. (2019)). The characterization setup should provide the measurements in a standard unit, e.g., mirror angle in degree, for comparative analysis of a number of tests of multiple mirrors. For MEMS mirror characterization, the simplest method is an optical lever stage to convert the beam displacement on the PSD to a mirror angle measurement by a dedicated calibration procedure. Uncertainties in the angle measurement of the MEMS mirrors are analyzed considering the optical alignment, the characterization of the PSD, and the calibration data. By the proposed uncertainty analysis, the accuracy of the developed MEMS test bench shows up to 0.026° at the mirror angle of 15°.

This paper proposes a PSD based MEMS test bench for characterization of multiple 1D resonant MEMS scanning mirrors and analyzes potential uncertainties of the mirror angle measurement. The designed MEMS test bench provides a calibration procedure with a motorized stage, enabling accurate and precise mirror angle measurements. To evaluate the uncertainty of the angle measurements, measurement errors by the PSD, the optical alignment,
Fig. 1. Schematic diagram (top) and a picture (bottom) of the MEMS test bench. For the accurate alignment and calibration, the laser, the mirror, and the PSD are mounted on a 5 DoF optics mount, a 6 DoF mirror mount, and a 1 DoF motorized stage, respectively, and the calibration procedure are analyzed with their sensitivity functions. With measurement data from the calibration procedure, the angle measurements uncertainty is calculated to the characterization performance of the designed MEMS test bench.

2. DESIGN AND MODELING

2.1 Design of MEMS Test Bench

Fig. 1 illustrates a schematic diagram and a picture of the MEMS test bench. A single mode fiber laser source (SIFC635, 635 nm, 2.5 mW, Thorlabs, Newton, NJ, USA) is connected to a variable fiber attenuator (VOA630-FC, 630 nm, < 50 dB, Thorlabs) to secure class 1 operation. The attenuated laser shine on the MEMS mirror and focused by an adjustable fiber collimator (CFC-5X-A, Thorlabs) on the PSD (S5991-01, 2D PSD, 9×9 mm, Hamamatsu, Hamamatsu City, Japan), which provides high linearity in a wide range with relatively small nonlinear distortion. For precision alignment, the laser collimator and the mirror are mounted on a 5 degree of freedom (DoF) mount (K6X1, Thorlabs) and a 6 DoF mount (K6xXS, Thorlabs), enabling the correction in all possible axes of position and rotations. For correction of the error after replacement of the mirror, the mirror position and the beam position at the mirror is measured by a CMOS camera (DMK 33UX252, The Imaging Source, Bremen, Germany) with an objective (Telecentric Lens, x1, f = 40 mm, Edmund optics).

The photo current from PSD is processed by separated two modules, which are trans-impedance amplifiers (TIA) module and position process module. The TIA module changes the weak photo current to voltage and the position process module changes the voltage of the PSD electrodes to the position $x_{raw}, y_{raw}$ and intensity $SUM$ information. In the TIA module, the PSD is mounted with an angle of 45° to use a long diagonal axis for trajectory measurement and TIA are installed on the backside of PSD. To reduce electromagnetic interference (EMI) by the high voltage operation of the MEMS mirror, a pair of metal shield covers the PSD and TIA. The TIA module with PSD is attached on a CNC-machined steel mount and installed on a 1D motorized stage (VT-80 62309110, 25 mm range, 0.5 µm resolution, Physik Instrumente, Karlsruhe, Germany), which provides room for the mirror exchange and enable a calibration procedure to convert the PSD measurement to the mirror angle measurement.

The computer controls the motorized stage and records the PSD data by a DAQ module (U2531A, 2 MSps, Keysight, Santa Rosa, CA, USA). The mirror control signal is applied to a custom-made high voltage driver circuit to drive the MEMS mirror with a high voltage up to 150 V (Brunner et al. (2019)).

2.2 Ray Tracing Model of MEMS Test bench

Fig. 1a also contains the global coordinate system for the beam propagation description from the laser to the PSD. The rotation axis of the MEMS mirror is set at the origin of the global coordinate system. The mirror surface at the zero mirror angle is defined by a normal vector $n_{norm}$ and a position vector $p_{norm}$. The surface plane of the scanning mirror with a mechanical mirror angle $\theta_m$ can be describe as

$$0 = n_{m}^T(\theta_m)(r - p_{m0}(\theta_m)),$$

$$n_{m}(\theta_m) = R(\theta_m, r_{m})n_{m0},$$

$$p_{m}(\theta_m) = R(\theta_m, r_{m})p_{m0},$$

where $a^T$ denotes transpose of the vector $a$, $r$ denotes position variables, i.e. $r = [x \ y \ z]^T$, $n_{m}(\theta_m)$ and $p_{m}(\theta_m)$ denote the normal vector and the position vector of the mirror surface with a mechanical mirror angle $\theta_m$, respectively. $R(\theta_m, r_{m})$ is a 3D rotation matrix with a mechanical mirror angle $\theta_m$ along the rotation axis of the mirror $r_m = [r_{m,x} \ r_{m,y} \ r_{m,z}]^T$. From the laser position vector $p_l$ and the laser direction vector $u_l$, the position of the
mirror reflection \( p_m \) and the direction of reflection \( u_{lm} \) is obtained by
\[
\begin{align*}
\hat{p}_m(\theta_m) &= p_i + \frac{n_i(\theta_m)(p_m(\theta_m) - p_i)}{n_i(\theta_m)u_i}, \\
\hat{u}_{lm}(\theta_m) &= u_i - 2\left(\hat{u}_i n_m(\theta_m)\right) n_m(\theta_m).
\end{align*}
\]
(4)
(5)
Besides, the surface of the PSD is defined as
\[
0 = n_i^T\left(r - p_g\right),
\]
(6)
where \( p_g \) and \( n_i \) denotes the position and the normal vector of the PSD surface, respectively. The reflected laser beam by the scanning mirror coincides the surface of PSD at
\[
\hat{p}_{lm}(\theta_m) = p_{lm}(\theta_m) + \frac{n_i^T\left(p_g - p_{lm}(\theta_m)\right)}{n_i^T\hat{u}_{lm}(\theta_m)}u_{lm}(\theta_m).
\]
(7)
\( p_{lm} \) already indicates the position of the beam spot on the PSD surface in the global coordinate system. The beam position output of the PSD can be transformed from the 3D points to a 2D coordinate system by
\[
\hat{p}_{op}(\theta_m) = k E \hat{R}_{rev}(p_{op}(\theta_m) - p_i),
\]
(8)
where \( \hat{R}_{rev} \) is a rotation matrix that change the surface of the PSD to one of 2D coordinate system, and \( E \) is a \( 2 \times 3 \) matrix to eliminate the unnecessary dimension. \( k \) denotes the scaling factor of the PSD, which is discussed further in Sec. 3.1.

2.3 Angle Measurement and Calibration Procedure
Assume that the reflected beam with the zero mirror angle is orthogonal to the projected trajectory on the PSD. Then the mirror angle is obtained by the beam displacement \( p_o \) from the zero angle as
\[
\theta_m = \frac{1}{2} \tan^{-1}\left(\frac{\hat{p}_m(\theta_m)}{D_{op}}\right),
\]
(9)
\[
p_o(\theta_m) = a^T_{py}(p_{op}(\theta_m) - p_{op}(0)),
\]
(10)
where \( D_{op} \) denotes an operation distance from the mirror surface to the PSD and \( p_{op} \) is the unit vector of \( y \) direction of the PSD which is parallel to the ideal beam trajectory on the PSD. The beam displacement is defined by a projection on the scanning axis on the PSD, i.e. the inner product of the beam displacement and the beam scan direction. An accurate measurement of the operation distance is difficult because the space between PSD to the mirror is only a few mm excluding the packages of the mirror and the shield of the PSD.

Calibration of the MEMS test bench is a procedure to estimate the operation distance \( D_{op} \), by multiple measurements of a scan trajectory. Fig. 2 describes the basic idea of calibration procedure by moving the PSD unit while the MEMS mirror is running at a fixed amplitude. Assume that the mechanical scan trajectory is a periodic function with an amplitude \( \Theta_m \) and a zero offset. An estimated operation distance can be obtained by
\[
D_{op} = a(\Theta_m) p_{pp}(d_{pp}, \Theta_m) - d_{pp},
\]
(11)
\[
p_{pp} = u_i^T(p_{pp}(\Theta_m) - p_{pp}(\Theta_m)),
\]
(12)
where \( p_{pp} \) denotes the peak to peak displacement of the PSD by the motorized stage. \( a(\Theta_m) \) is the slope \( a(\Theta_m) \), defined as
\[
a(\Theta_m) = \frac{1}{\Theta_m} \sum_{i=1}^{n} \Theta_m.
\]
The estimation of operation distance \( D_{op} \) is obtained by solving a linear regression of
\[
\begin{align*}
\hat{p}_{op}(\Theta_m) &= k E \hat{R}_{rev}(p_{op}(\Theta_m) - p_i),
\end{align*}
\]
(13)
with multiple measurements of \( p_{op} \) with various \( d_{pp} \). The estimation of the slope \( a \) is automatically obtained by the linear regression as well. Then the measured mechanical mirror angle with the calibration can be written as
\[
\theta_m = \frac{1}{2} \tan^{-1}\left(\frac{p_o(\Theta_m)}{D_{op}}\right).
\]

3. UNCERTAINTY ANALYSIS
In this section, potential error sources and their uncertainties are analyzed for the evaluation of the mirror angle measurement. The errors in the PSD measurement, misalignment of the optical setup, and the calibration error can be considered as dominant error sources in the MEMS test bench. By assuming that all error sources are independent, the standard deviation of the beam displacement error can be written as
\[
\sigma_{p_o} = \sigma_{p_{pp}}^2 + \sigma_{opt}^2,
\]
(14)
where \( \sigma_{p_{pp}} \) and \( \sigma_{opt} \) denote the standard deviation caused by PSD and optical alignment, respectively. The errors of the calibration and the mirror angle measurement are analyzed based on the errors of the beam displacement. The details of each error source are discussed in the following subsections.

3.1 Uncertainty in PSD measurement
Since the PSD is the main sensor of the MEMS test bench, errors in the PSD measurement are important to be analyzed. Three error sources can be considered in the PSD measurements, which are a scale error, an accuracy error, and a precision error. First, the scale error of the PSD is the errors in scaling of the PSD position calculation, described by \( k_{s} \) from \( k = k_{s}(1 + k_{d}) \), where \( k_{d} \) denotes the nominal scaling factor. This scale error can be caused by the inaccurate the calibration of the PSD, temperature variation, and gain mismatch in analog unit (Yoo et al. (2019)). Secondly, the accuracy error is the error due to the distortion of the PSD measurements, which can be defined as a residual position error or nonlinear distortion after the correction of the scaling error. Last, the precision error is an uncertainty of the PSD measurement by the noise of the signal. The precision error changes by the bandwidth and can be improved by averaging in offline analysis such as the calibration procedure.

Fig. 3 illustrates the measured accuracy error and precision errors of a PSD along the two diagonal axes by the PSD.

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First, the height of laser beam, mirror, and the diagonal axis of the PSD are at the same height and beam does not change the height, which leads to $d_{x_l} = d_{x_m} = d_{x_p}$ and $\delta_{x} = \delta_{y_m} = 0$. Second, the laser beam from the MEMS mirror is perpendicular to the y axis of the PSD, i.e. $\delta_{y_p} = 0$ and $\delta_{y_m} = 0$, and is parallel to the moving axis of the motorized stage. Third, the laser beam shines at the center of the mirror where the rotational axis is located, i.e. $d_{y_d} = 0$. Lastly, the operation distance is less than 16.4 mm to cover the mirror angle of 15° with a reasonable accuracy of the PSD.

The alignment procedure is to fix each source of errors in the local coordinate systems of the laser beam, MEMS mirror, and the PSD. For the height alignment, the laser and the PSD are aligned with a long distance over 20 cm, and the height and angle error of the laser are adjusted. Tip tilt errors of the mirror, $\delta_{x_m}$ and $\delta_{y_m}$, are roughly aligned with the laser, and are finely aligned by minimizing the variation of the beam displacement along the movement of the motorized stage. The alignment of the PSD tilt angle $\delta_{y_p}$ is defined by the CNC-machined solid mount, and the beam center on the mirror surface can be aligned by the images of the MEMS mirror and the beam spot by the camera. The operation distance is discussed further in the following subsection about the calibration procedure.

After the alignment procedure, residual uncertainties in the optical alignment influence the uncertainty of the beam displacement. Four main sources of the optical alignment error can be considered, which are the mirror tilt error $\delta_{y_m}$, the mirror tip error $\delta_{x_m}$, the PSD tilt error $\delta_{y_p}$, and the laser lateral misalignment $d_{y_d}$. The laser lateral misalignment $d_{y_d}$ defines the misalignment of the laser beam to the mirror rotation center with the incident angle $\alpha_m$. By applying the global coordinate system, the position and direction vectors of the laser, the MEMS mirror, and the PSD are defined as follows.

$$p_l = \begin{pmatrix} d_{x_m} \cos(\pi - 2\delta_{x_m}) + d_{x_l} \cos(\pi/2 - 2\delta_{y_m}) \\ d_{x_m} \sin(\pi - 2\delta_{x_m}) + d_{x_l} \sin(\pi/2 - 2\delta_{y_m}) \end{pmatrix},$$

$$u_l = \begin{pmatrix} d_{x_m} \cos(\pi - 2\delta_{x_m}) \\ d_{x_m} \sin(\pi - 2\delta_{x_m}) \end{pmatrix},$$

$$p_p = \begin{pmatrix} -D_{x_p} \sin(\pi/2 - \delta_{x_p}) \cos(\pi/2 - \delta_{y_p}) \\ -D_{x_p} \sin(\pi/2 - \delta_{x_p}) \sin(\pi/2 - \delta_{y_p}) \end{pmatrix},$$

$$p_{opt} = \begin{pmatrix} \sin(\pi/2 - \delta_{x_p}) \cos(\pi - (\alpha_m + 2\delta_{y_m})) \\ \sin(\pi/2 - \delta_{x_p}) \sin(\pi - (\alpha_m + 2\delta_{y_m})) \end{pmatrix},$$

$$r_{psd} = \begin{pmatrix} \sin(\delta_{y_m} \sin(\pi - \alpha_m) \cos(\pi - \alpha_m + 4\delta_{x_m})) \\ \sin(\delta_{y_m} \sin(\pi - \alpha_m) \sin(\pi/2 - 2\delta_{y_m}) \cos(\pi/2 - \delta_{y_p}) \cos(\pi/2 - \delta_{y_m})) \end{pmatrix}. \quad (17)$$

where $d_{x_m}$ denotes the distance from the laser to the mirror, which is set to 24 mm. $\delta_{x_p}$ denotes the tip angle of the PSD, i.e. 7°. For the simulation, the incident angle $\alpha_m$ is set to 20°. The operation distance $D_{x_p}$ is set to 9.618 mm to match the value with the calibration result (c.f. Sec. 4). The uncertainties of the optics misalignment can be written as...
For the uncertainty analysis of the MEMS test bench, this estimated accuracy of PSD is used instead of the measured accuracy of PSD in Sec. 3.1. That is because it is directly measured by the MEMS test bench, and it considers filtering of the scanning trajectory measurements. Therefore the estimated accuracy of PSD is regarded more realistic to be used for the uncertainty analysis.

### 3.4 Uncertainty in Angle Measurement

From (13), the uncertainty of the measured mechanical angle of the MEMS mirror can be rewritten as

\[
\sigma_m^2 = \sigma_{\text{psd,acc}}^2 + \sigma_{\text{cal}}^2 + \sigma_{\text{op}}^2
\]

where \(\sigma_{\text{psd,acc}}\), \(\sigma_{\text{cal}}\), and \(\sigma_{\text{op}}\) denote the uncertainty of the angle measurements contributed by the PSD, the optical alignment, and the calibration. The other error sources except for parameters from calibration are already available based on the prior measurements and assumptions. This uncertainty of the mirror angle measurement can be calculated by the calibration process of the measurements, discussed in the following section.

### 4. CALIBRATION AND UNCERTAINTY EVALUATION

This section describes the measurement results of the proposed calibration procedure and derives the uncertainty of the MEMS test bench based on the estimated parameters from the calibration. First, the calibration measurements are analyzed to estimate the operation distance, the variance of operation distance, and the accuracy error of PSD. For the experiment, a MEMS mirror in (Brunner et al. (2019)) is used for the evaluation of the MEMS test bench and the calibration procedure. While the mirror is running with an unknown amplitude, the beam trajectory measurements by the PSD are recorded for 23 measurement points from 0 to 22 mm. The peak to peak displacement for each point is obtained by averaging 42 periods of the mirror scanning trajectories.

Fig. 5 illustrates measured peak to peak displacements, the estimated affine function, and the residual PSD errors from the estimation. The zero crossing (red circle) of the estimated affine function indicates the estimated operation distance of the mirror, which \(D_m = 9.618\) mm. The standard deviation of the estimated operation distance is obtained by the covariance of the estimation, i.e. \(\sigma_{D_m} = 6.55\) \(\mu\)m. The estimated accuracy of the PSD \(\delta_{\text{psd,acc}}\) is 2.01 \(\mu\)m since the estimated slope \(\hat{s}\) is 3.26.

Fig. 6 shows the angle measurement uncertainty along the mechanical mirror angle and the contribution of each uncertain components in PSD and optical alignment. The left figure illustrates the uncertainty of the mirror angle measurement uncertainty along two holes with a distance of 34 mm, \(\hat{\Delta} = 9.618\) mm. The angle of the MEMS mirror can be rewritten as

\[
\alpha = \tan^{-1}\left(\frac{y_m - y_m^0}{x_m^0 - x_m^0}\right)
\]

along the mechanical mirror angle. For \(\hat{\Delta}\), \(\delta_{\text{psd,acc}}\), and \(\delta_{\text{op}}\), the sensitivity functions of the uncertainties in the optical misalignment along the mechanical mirror angle. For \(\delta_{\text{psd,acc}}\), \(\delta_{\text{op}}\), and \(\delta_{\text{op}}\), the sensitivity functions are mainly even functions while \(\delta_{\text{psd,acc}}\) is mixed with an odd function, showing asymmetric sensitivity.

The uncertainties of each component can be obtained by the defined alignment procedure. For example, \(\sigma_{\text{psd,acc}}\) and \(\sigma_{\text{cal}}\) can be obtained by accuracy error of PSD between the stage movement of 20 mm, i.e. \(\tan^{-1}(\sigma_{\text{psd,acc}}/20) \approx 0.027^\circ\). \(\delta_{\text{psd,acc}}\) can be given by the tolerance of the solid block, e.g. 25 \(\mu\)m along two holes with a distance of 34 mm, i.e. \(\tan^{-1}(0.025/34) \approx 0.017^\circ\). \(\delta_{\text{op}}\) can be obtained by the quarter of the 1/10 beam diameter at the mirror, i.e. 0.032 \(\cos\hat{\alpha}\) \(\approx 0.303\) mm.

### 3.3 Uncertainties of Calibration and Estimation of PSD Accuracy Error

The standard deviation of the estimated operation distance \(\sigma_{\text{op}}\) is obtained by the variance calculation of the residual errors in the linear regression. During the regression, uncertainties in optical alignment and the scaling errors of the PSD do not influence the estimated operation distance since they only change the estimated slope \(\hat{a}\). The precision of PSD can be neglected by a large number of averaging as well. Then the uncertainty of the estimated operation distance can be written as

\[
\sigma_{\text{op}}^2 = (\hat{a}(\Theta_m))^2 \sigma_{\text{psd,acc}}^2 + \sigma_{\text{op}}^2
\]

Since \(\sigma_{\text{op}}^2 > 0\), the accuracy of PSD can be estimated by the calibration procedure as

\[
\delta_{\text{psd,acc}} = \frac{\sigma_{\text{op}}}{|\hat{a}(\Theta_m)|} > \delta_{\text{psd,acc}}.
\]
This paper proposes a MEMS test bench based on a PSD and analyzes its uncertainty to ensure highly accurate and precise mirror angle measurement for a 1D resonant MEMS mirror. The MEMS test bench is equipped for a PSD module mounted on a motorized stage, enabling a precision conversion from the beam displacement to the mechanical mirror angle of the MEMS mirror. In the MEMS test bench, the laser and the MEMS mirror are mounted by a 5 DoF and a 6 DoF manual mount, respectively, to compensate for all potential misalignment in the optical path. For the uncertainty analysis, the optical path of the MEMS test bench is modeled as vector equations to derive sensitivity functions of the angle measurements for the uncertainties of the PSD, the alignment of the optical setup, and the calibration procedure. The analysis based on the measured calibration procedure reveals that the developed MEMS test bench shows up to 0.026° of angle measurement uncertainty at 15° mechanical mirror angle, showing capability of the accurate and precise characterization of multiple resonant MEMS mirrors.

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