ABSTRACT

The aim of this study is to apply a Chebyshev polynomial approximation of the compressor map for dynamic modelling and control of centrifugal compressors. The results are compared to those from an approximation based on the third order polynomials and a compressor map derived from first principles. In the analysis of centrifugal compressors, a combination of dynamic conservation laws and static compressor map provides an insight into the surge phenomenon, whose avoidance remains one of the objectives of compressor control. The compressor maps based on the physical laws provide accurate results, but require a detailed knowledge about the properties of the system, such as the geometry of the compressor and gas quality. Third order polynomials are usually used as an approximation for the compressor map, providing simplified models at the expense of accuracy. Chebyshev polynomial approximation provides a trade-off between the accuracy of physical modelling with the ease of use provided by third order polynomial approximation.

INTRODUCTION

A compressor map shows the relationship between the mass flow through the compressor, its speed and the pressure ratio. Usually delivered by the manufacturer of the equipment, it is obtained via a series of experiments for known operating conditions, such as inlet temperature, pressure, and gas composition. However, the operating conditions are subject to change due to both external and internal factors. Taking into account that the performance of a compressor changes there is a need for effective methods of calculation of the compressor characteristics which can be then used for dynamic modelling or optimization. Previous work has demonstrated that Chebyshev polynomials have shown better performance than traditional third order polynomials in providing approximations for the compressor map.

One of the phenomena associated with centrifugal compressors is surge, a dynamic instability consisting in the oscillations of the flow inside the compressor. It results in strong vibrations which can damage the equipment. Therefore, the avoidance of surge remains one of the issues discussed both by researchers [1] and industrial practitioners [2].

The dynamic model is a starting point for various applications, including compressor control and surge protection. In [1] and [3] the relationships between compressor control and surge protection were analyzed, comparing PID and MPC controllers. Transient behaviour analysis in case of disturbances from electrical grid was analyzed in [2] where the responses of the compressor during driver trips were analyzed and in [4] where a control algorithm protecting against such trips was proposed. A model-based approach was also used in [5] for analysis of the behaviour of a degrading subsea compressor. Further model-based approaches to compressor control are described in [6].

The dynamic process models used in those works use a compressor map in form of a polynomial approximation of second or third order. In some cases, however, physical models are used, e.g., in [7,8] which were used as the source of data for this work.
or in [9] where an adaptive procedure for the calculation of a compressor map was developed.

This work combines the dynamic process model from [8] with Chebyshev polynomial approximation of a compressor map to provide a more accurate framework compared to those based on other polynomial approximations that also do not require knowledge about the geometry of the compressor. Chebyshev polynomials offer a promising approach when examining the behaviour of the compressor close to the surge condition, where the compressor can be difficult to control.

The first part of the paper presents the model of the compressor which is then used for validation of the approximations of the compressor map. Three ways of calculation of the compressor map are described: a physical model used as benchmark for the models based on third order polynomials and Chebyshev polynomials. Their influence on the dynamic process model is presented in the subsequent section using two control frameworks: speed control and speed control combined with surge protection. The paper ends with a brief summary and conclusions.

**COMPRESSOR SYSTEM**

The compressor system considered in this work is depicted in Figure 1 and consists of a centrifugal compressor with variable speed drive, a discharge tank, close coupled valve, passages inside the compressor, and an outlet valve. The parameters of the system are shown in Tab. 1 [8].

The system from Figure 1 can be described by three first-order ordinary differential equations [8, 10, 11] derived from mass balance in the tank, momentum balance in the pipe and momentum balance on the compressor shaft, respectively (Eq. (1))

\[
\dot{p} = \frac{d_{01}}{V_p} (m - m_i)
\]

\[
m = A_1 (p_2 - p)
\]

\[
\dot{\omega} = \frac{1}{J} (\tau_i - \tau_c)
\]

where \(p\) - pressure in the tank in Pa, \(m\) - mass flow through the compressor in kg s\(^{-1}\), \(p_2\) - pressure downstream of the compressor in Pa, \(\omega\) - angular velocity of the compressor in rad s\(^{-1}\), \(\tau_i\) - torque of the motor, \(\tau_c\) - torque of the centrifugal compressor.

\[\rho = \frac{3\pi N}{60}\] where \(N\) is the compressor speed in rpm. The variable \(m_i\) denotes the mass flow through the outlet valve in kg s\(^{-1}\)}
and is modelled as:

\[ m_t = k_t \sqrt{p - p_{01}} \]  

(2)

where \( p_{01} \) denotes the external pressure. The compressor torque \( \tau \) is given by equation \( \tau = |m| r^2 s \omega \) where \( r_2 = \frac{D_1}{2} \) is the radius of impeller exit, whereas \( \tau \) denotes the external torque applied to the shaft.

The pressure downstream of the compressor \( p_2 \) is defined using the compressor map \( \psi(m, \omega) \) and the pressure drop across the close-coupled valve (CCV) \( \psi_v(z, m) \)

\[ p_2 = (\psi(m, \omega) - \psi_v(z, m)) p_{01}. \]  

(3)

The formula for \( \psi(m, \omega) \) was derived in [8] and is briefly described in the next section. The pressure drop \( \psi_v(z, m) p_{01} \) is a function of valve opening \( z \) and mass flow rate \( m \), and was chosen as manipulated variable for surge control. The CCV is assumed to have a linear relationship between flow and valve opening \( z \), and adjustment of the position \( z \) of the CCV modifies the overall relationship between speed \( \omega \), mass flow rate \( m \) and overall pressure ratio \( \frac{p_2}{p_{01}} \). The derivation of (1) and the analysis of transient behaviour including the dynamic pressure rise can be found in [11, 12].

**COMPRESSOR MAP**

This section presents three ways of calculating the compressor map. In the first approach, proposed in [8], the model is derived from first principles using the assumption that a compressor system can be modelled as an isentropic process in series with an isobaric process including enthalpy increase [7]. The next approach, widely used in the literature, uses third order polynomials to approximate the compressor map [4, 7, 13]. Finally, the Chebyshev approximation is described which so far has not been used in dynamic modelling of a compressor.

**Physical modelling**

The calculation of a physical model of the compressor map is based on energy transfer in a compressor system taking account of various losses inside the compressor ([7, 9]), but most authors agree that the most important losses are incidence and friction losses. In [8], there are also efficiency losses which will be taken into account in this simulation.

Table 2 presents the incidence and friction losses for a centrifugal compressor. The variables in Tab. (2) are given as

\[ D_1 = \sqrt{D_{h1}^2 + D_{t1}^2}, \]  

(4)

\[ r_1 = \frac{D_1}{2}, \]  

(5)

\[ A_1 = \pi \left( \frac{D_1}{2} \right)^2 - \left( \frac{D_{h1}}{2} \right)^2, \]  

(6)

\[ \alpha_{2b} = \tan \left( \frac{D_1 \tan \beta_{1b}}{\sigma D_2} \right), \]  

(7)

\[ C_f = 4 \cdot 0.3164 (Re)^{-0.25}, \]  

(8)

where \( D_1, r_1, A_1 \) - diameter, radius and area of the impeller inlet, \( \alpha_{2b} \) - blade angle at the diffuser and \( C_f \) - friction loss coefficient [8]. The formula (8) was derived for turbulent flow in smooth pipes with \( Re < 100000 \), nonetheless it has been shown it is also valid for compressor modelling with \( Re = \text{const} = 100000 \) [8]. The values of passages lengths, \( l_i \) and \( l_o \), were found using optimization toolbox in Matlab in order to fit the characteristics to the map available in [8].

The isentropic efficiency of a compressor is then defined as

\[ \eta_i(w, \omega) = \frac{\Delta h_{0c,\text{ideal}}}{\Delta h_{0c,\text{ideal}} + \Delta h_{\text{loss}}} \]  

(9)

where \( \Delta h_{0c,\text{ideal}} = \sigma r^2 c^2 \omega^2 \) is the ideal specific enthalpy (J kg\(^{-1}\)) delivered to the fluid and \( \Delta h_{\text{loss}} = \Delta h_i + \Delta h_{f1} + \Delta h_d + \Delta h_{f2} \) describes the losses in the compressor in form of specific enthalpies.

Gravdahl in [8] includes also efficiency loss

\[ \Delta \eta_{\text{loss}} = \Delta \eta_s + \Delta \eta_{f1} + \Delta \eta_v + \Delta \eta_d \]  

(10)

where \( \Delta \eta_s = 0.3 \frac{l_v}{D_1} \) is the clearance loss, \( \Delta \eta_{f1} = 0.03 \) is the backflow loss, \( \Delta \eta_v \in [0.02, 0.06] \) is the volute loss, and \( \Delta \eta_d \) is the diffusor loss [8].

Finally the efficiency of the compressor can be calculated as

\[ \eta_i(m, \omega) = \frac{\Delta h_{0c,\text{ideal}}}{\Delta h_{0c,\text{ideal}} + \Delta h_{\text{loss}}} - \Delta \eta_{\text{loss}} \]  

(11)

and the pressure rise for positive mass flow in a compressor is

\[ \psi_+(m, \omega) = \frac{p_2}{p_{01}} = \left( 1 + \frac{\eta_i(m, \omega) \Delta h_{0c,\text{ideal}}}{T_{01} c_p \omega} \right)^\frac{\tau}{\tau_t}. \]  

(12)

In order to model and control the surge, the characteristics are extended to negative mass flow:

\[ \psi(m, \omega) = \begin{cases} c_m m^2 + \psi_+(0, \omega), & m \leq 0 \\ \psi_+(m, \omega), & m > 0 \end{cases} \]  

(13)

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where \( c_n \) defines the slope on the negative side of the characteristics. Here \( c_0 = 5 \) and was fitted to the data available in [8]. The resulting compressor map can be found in Figure 4a) (solid colour lines). The dashed red line shows the surge line calculated as a linear interpolation between the inflection points on the positive side of the characteristics.

Detailed description and derivation of (12) and (13) and can be found in [8]. The author of [8] states also that the theoretical results are similar to experimental values from [14].

**Compressor map approximation**

The characteristics derived from physical principles provide a good result. However, the calculation requires detailed knowledge of the geometry of the compressor, e.g., the dimensions of the impeller or the blade angle. Therefore it might be difficult to use it in industrial applications. In this work, due to its accuracy, it was used as a baseline for third order polynomial approximation and Chebyshev polynomial approximation.

Since the data for approximation come from a continuous physical model, it is assumed that the measurements can be done for five compressor speeds, \( N \), ranging from 25000 rpm to 55000 rpm, and for 20 mass flow data points from the interval \([-0.2, 0.8] \). Three selected compressor speeds \( N \) with respective mass flow \( m \) values are depicted with circles in Fig. 4a).

**Third order polynomial approximation** This section examines the construction of compressor characteristics by means of third order polynomials using the method recommended in [7]. The algorithm for the experimental setup is as follows:

1. For each rotational velocity \( N \) find at least four values of pressure.
2. Find an approximation in form of third order polynomial as function of mass flow.
3. Take the coefficients of the approximating polynomials for mass raised to a given power and find an approximation as function of rotational speed.

The last step ensures that the characteristics are continuous with respect not only to mass flow, but also to the rotational speed \( N \). The resulting characteristics have the form

\[
\psi(m, N) = c_0(N) + c_1(N)m + c_2(N)m^2 + c_3(N)m^3
\]

(14)

where

\[
c_j(N) = c_{j0} + c_{j1}N + c_{j2}N^2 + c_{j3}N^3.
\]

(15)

The Equation (14) with coefficients given by Eq. (15) can be rewritten in vector-matrix form as [4]:

\[
\psi(m, N) = \begin{bmatrix} \frac{N-\mu_1}{\mu_2} \\ \frac{N-\mu_1}{\mu_2}^2 \\ \frac{N-\mu_1}{\mu_2} \\ 1 \end{bmatrix} \begin{bmatrix} m^3 \\ m^2 \\ m \\ 1 \end{bmatrix} W
\]

(16)

where \( W \) is a matrix \( 4 \times 4 \) and \( \mu_1, \mu_2 \) are the scaling parameters, the mean and standard deviation of \( N \), respectively, improving the numerical performance of the approximation [15]. The parameters \( c_{ij} \) are then calculated from Eq. (16). The resulting compressor map is depicted in Figure 4c) (solid colour lines) with \( \mu_1 = 40000, \mu_2 = 12990 \).

The parameters \( \tilde{c}_{ij}, i, j = 0, 1, 2, 3 \), were found using least squares algorithm implemented as Matlab ‘polyfit’ function. The implementation is based on solving a system of algebraic equations \( Vc = b \) where \( c \) is the vector of unknown coefficients, here \( \tilde{c}_{ij} \), \( b \) is the vector of values, here the pressure ratio, and \( V \) is the Vandermonde matrix [15] calculated from the vector of independent variable \( x \), here mass flow in the first step and compressor speed in the second step.

Figure 4c) shows that the polynomial approach is not effective. For speeds below 45000 rpm, the curves differ from the physical map. This set of curves does not predict surge and therefore Fig. 4c) does not show a surge line. The maxima are shifted to the point where \( m = 0 \) which changes the shape of the surge line. This is the result of the flatness of the original curves in this region. A more in-depth analysis of this issue can be found in [16].

**Chebyshev approximation** Chebyshev approximation was proposed as the third order polynomials proved ineffective for this particular system. In [15] it was proved that this approach has better numerical stability than classic approximation method implemented among the others in Matlab.

This work uses ‘chebfun’ package, an open-source Matlab toolbox developed by researchers from Oxford University [17]. It was primarily designed for univariate functions analysis and then extended to two variables. The two-dimensional version is described by formula (17) from [17, 18]

\[
f(x,y) \approx p_M(x,y) = \sum_{i=0}^{r-1} \sum_{j=0}^{n-1} a_{ij}(M)T_i(y)T_j(x). \tag{17}
\]

The parameters \( n \) and \( r \) denote the numbers of measurements for \( x \) and \( y \), while \( a_{ij}(M) \) are the coefficients of the approximation as functions of the parameter \( M \). The functions \( T_i(x) \) are Chebyshev polynomials given by the trigonometric formula

\[
T_i(x) = \cos(k \cos^{-1}(x)). \tag{18}
\]

The algorithm proposed in [18] adjusts the values of \( a_{ij}(M) \) so that predicted values of \( p_M(x_i,y_j) \), \( i=0,\ldots,r-1, j=1,\ldots,n-1 \) are close in a least squares sense to the measured values \( f(x_i,y_j) \). The parameter \( M \) is called the rank of approximation and describes the number of terms used to calculate the coefficients \( a_{ij}(M) \) on the right hand side of (17) [18]. \( M \) can take values between 1 and \( \min\{r,n\} \).
Applied to modelling of a compressor map, the variables \(x_i\) and \(y_j\) will be mass flow \(m_i\) and rotational velocity \(N_j\) which can be measured. The values \(f(x_i, y_j)\) denote in this case the pressure ratio. The parameters \(r\) and \(n\) are the numbers of data points available. There are two assumptions about \((x_i, y_j)\) which must be taken into account. The formula (17) assumes that \((x_i, y_j) \in [-1, 1]^2\) and \(x_i = \cos \left( \frac{2\pi}{r} \right), y_j = \cos \left( \frac{2\pi}{N} \right)\). Therefore, the measurement must be taken according to these two formulas. Moreover, it is necessary to provide a linear scaling for mass flow and rotational velocity:

\[
x_i = 2m_i - 0.6, \\
y_j = \frac{1}{15000}N_j - \frac{8}{3}.
\]

The parameters \(r\) and \(n\) used for approximation depend on the number of measurements, here \(r = 5, n = 20\). The resulting characteristics are depicted in Figure 4b) (solid colour lines).

Figure 4b) shows that the Chebyshev polynomials provide a more accurate approximation than the third order polynomials. The shape of the speed curves is preserved in the whole range of \(N\) which results in an accurate surge line approximation. Reference [16] gives a comparison with third order polynomials.

**DYNAMIC MODELLING AND CONTROL**

The three compressor maps, derived from first principles, approximated using third order polynomials, and approximated using Chebyshev polynomials, were then applied in dynamic modelling of the whole system (compressor, pipework and valves) described with Eq. (1). The analysis is divided in three parts:

1. The equilibrium points of the system (1) are calculated using the three compressor maps to verify their usefulness in choosing the right operating point.
2. The behaviour of the system (1) is analyzed when there is no pressure drop across the CCV and the external torque is the manipulated variable for speed control.
3. The behaviour of the system (1) is analyzed when the speed control is combined with surge protection with pressure drop across the CCV as manipulated variable.

The formulas for speed and surge controllers are taken from [8].

**Equilibrium calculation**

The analysis of equilibrium points provides an insight into the behaviour of the system and is used for controller design [19]. The equilibrium points are defined as constant solutions of a system of nonlinear differential equations [19]. Therefore, the following system of algebraic equations derived from Eq. (1) should be solved:

\[
m - m_0 = 0, \\
p_2 - p = 0, \\
\tau_2 - \tau_0 = 0.
\]

As the default pressure drop across the CCV is equal to zero, inserting \(p_2 = \psi \cdot p_{01}\) into the second equation gives

\[
p = \psi (m, \omega) \cdot p_{01}.
\]

At the same time, using the mass flow flow through the outlet valve (2) in the first equation gives

\[
p = \frac{m^2}{\sum c} + p_{01}.
\]

Gravdahl in [8] takes the speed \(N = 50000\) and this was also assumed in this work. Then it is possible to solve the system consisting of Eq. (20) and Eq. (21) graphically looking for an intersection point over the whole range of possible mass flows \(m\). The plots used for calculation are in Figure 2 and the numerical results are gathered in Tab. 3 where the external torque was calculated from the last equation in (19).

Figure 2 shows that all the results are close to the default value obtained from physical compressor map and depicted with solid line. This is true also in case of third order polynomial approximation (dashed line), because the calculations are based on the values of the characteristics and not on the shape. The plot shows, nevertheless, that the Chebyshev approximation (dotted line) is closer to the original map than the approximation with third order polynomials.

However, the lack of constrains about the surge line results in an operating point to the left of the surge line [8]. Figure 2 shows that the compressor discharge pressure increases with increasing mass flow. Even though such an operating point is not feasible in under normal circumstances, i.e., without active surge control, it is useful for validation of the behaviour of the system using various approximations and calculated equilibria were taken as operating points in the controller simulations.

**Control**

One of the objectives of the control system in a centrifugal compressor is to protect the equipment against surge phenomenon using the surge controller. There are multiple ways of

| \(m_0\) [kg s\(^{-1}\)] | \(p_{20}\) [Pa] | \(\tau_0\) [Nm] |
|----------------|----------|---------|
| Physical model | 0.1037   | 219517.64 | 2.0019  |
| Chebyshev model| 0.1035   | 219173.16 | 1.999   |
| Polynomial model| 0.1034  | 218825.72 | 1.996   |

Table 3. EQUILIBRIUM POINT CALCULATED USING THREE APPROXIMATIONS
designing the surge controller [1]. In this work, the approach introduced and described in [8] was adopted. There are two separate control loops:

1. Speed control with compressor speed as process variable and the applied torque as manipulated variable (Figure 3)
2. Surge control with mass flow as process variable and the pressure drop across the CCV as manipulated variable (Figure 7)

Such a framework, where the speed set-point is fixed regardless of the mass flow and pressure, allows validation of the behaviour of the system both during surge and with surge control in a large area of the compressor map. However, this approach differs from the industrial configurations where the speed controller gets its set-point from a master process control loop that calculates it from the default values obtained from the physical model. This section validates their behaviour in dynamic modelling of a compressor train from Figure 1.

**Speed control** The control loop from Figure 3 presents the process control loop in a compressor system 1. Gravdahl in [8] proposes to use a PI controller of form:

$$\Delta \tau_e = -k_p \Delta \omega - k_i \int_0^t \Delta \omega(y) dy$$

(22)

where $k_p$, $k_i > 0$ and $\Delta \omega = \omega - \omega_0$. Then $\tau_e = \Delta \tau + \tau_0$ where $\tau_0$ is given in Tab. 3. The parameters of the controller were chosen as $k_p = 0.1$ and $k_i = 0.07$ [8].

As the equilibrium point is on the left side of the surge line, the system without surge protection is expected to go into surge, i.e., oscillations in mass flow and pressure, and therefore speed, are expected. As long as the desired speed is not achieved, the controller will adjust the torque in order to change $\omega$. Assuming that the speed $\omega_0$ is reached, the system moves close to the desired speed curve. If the mass flow $m > m_0$, then $\psi(m, \omega_0) > \psi(m_0, \omega_0)$ and, in consequence, as the derivative $m$ is positive according to Eq. (1), the mass flow increases until $\psi(m, \omega_0)_{p_{11}} = \psi(m_0, \omega_0)_{p_{11}}$. Further increase will result in $\psi(m, \omega_0)_{p_{11}} < p_p$ and the flow is reversed. On the other hand, if $m < m_0$ then $\psi(m, \omega_0) < \psi(m_0, \omega_0)$ so the mass flow decreases until $\psi(m, \omega_0) = \psi(m_0, \omega_0)$ which is on the negative side of the speed curve, and the flow is reversed again.

Figure 4a), where the physical model was used, shows also that the surge phenomenon occurs before the desired speed set-point has been reached (for $m \approx 0.18$ kg s$^{-1}$ and $N \approx 47000$ rpm the slope of the response changes and oscillations begin). Such behaviour is defined by the parameters of the system, e.g. the inertia of the shaft and the parameters of the controller. Smaller moment of inertia allows faster speed changes and, in consequence, avoidance of surge [20]. The speed of the compressor $\omega$ depends also on mass flow rate and, therefore, on the compressor map. Hence the approximation of the characteristics should be accurate in the whole range of compressor speed. Figure 4a) shows the default response of the system with physics-based compressor map. The initial point was chosen on the left side of the surge line, but, nonetheless, the system goes into surge and the oscillations are visible, as the mass flow oscillates between -0.15 and 0.49 (solid line in Figure 5b)). This results in oscillations in the compressor discharge pressure (solid line in Figure 5a)) and compressor speed (solid line in Figure 5c)).

The third order polynomial approximation does not predict the compressor surge, even though the value of equilibrium point was close to the default value. The comparison of the responses in Figures 4c) and 4b) suggests better performance of the compression system based on third order approximation of the compressor map, as there is no surge phenomenon visible. Dashed lines in Figures 5a), 5b), and 5c), show the compressor discharge pressure, mass flow, and compressor speed, respectively, for this case. However, the comparison with the default system responses with physical map indicates that it is the opposite and the third
order polynomial approximation does not provide sufficient accuracy. Due to the fact that the maxima of the characteristics are shifted to \( m = 0 \) for \( N \leq 50000 \) rpm, the speed curves regarded as functions of mass flow are flat (Figure 4c), so the flow is not reversed.

The dynamic process model based on Chebyshev polynomial approximation, however, shows similar behaviour to the system with physics-based compressor map (Figure 4b). The mass flow oscillates between -0.15 and 0.5 (dotted line in Figure 5b)), which is close to the default result. The oscillations in the compressor discharge pressure and speed are also visible (dotted lines in Figures 5a) and 5c), respectively). Figures 5a), 5c), and 5b) show also that the dynamic process model based on Chebyshev compressor map enters surge at the same time as physical model which confirms its accuracy.

**Speed control and surge protection** The last part of the analysis verifies how the Chebyshev polynomial approximation behaves when the surge protection is included. The surge controller was based on [8] as a proportional controller

\[
\psi_k(z,m) = k_v(z)\Delta m \tag{23}
\]

where \( \Delta m = m - m_0 \) and \( k_v(z) > 0 \), \( z \) denotes the opening of the CCV. The form of Eq. (23) implies that there is no pressure drop across the CCV for \( \Delta m = 0 \). For the theoretical analysis of the properties of the system with both controllers (22) and (23) the reader is referred to [8].

The control law given by (23) ensures that the pressure drop is adjusted if the mass flow \( m \) is different than the operating point \( m_0 \), decreasing or increasing \( \psi(m,\omega) \) if \( m > m_0 \) or \( m < m_0 \), respectively. The physical interpretation is that the proposed control law flattens the speed curve around the operating point. Therefore, the choice of \( k_v \) depends on the characteristics around \( m_0 \). In order to obtain flat characteristics, the formula must be fulfilled:

\[
\psi(m,\omega_0) - \psi_r(m) = \psi(m_0,\omega_0) \tag{24}
\]

which, taking (23) into account, can be rewritten as

\[
k_v(m - m_0) = \psi(m,\omega_0) - \psi(m_0,\omega_0) \tag{25}
\]

where the right side is the compressor map translated along the vertical axis and the left side is a line with the slope \( k_v \). For mass flow between zero and \( m_s \), denoting when the surge occurs, the characteristics are strictly increasing which means that the extrema are reached at the ends of the interval \([0,m_s]\) (Figure 6). Therefore, the objective is to find minimal \( k_v \) such that

\[
k_v(m - m_0) = \psi(m,\omega_0) - \psi(m_0,\omega_0) \tag{26}
\]
Figure 5. DYNAMIC RESPONSES OF THE SYSTEM WITH SPEED CONTROLLER, OBTAINED IN THREE CASES: PHYSICAL MODEL (solid line), CHEBYSHEV MODEL (dotted line), POLYNOMIAL MODEL (dashed line): 5a) COMPRESSOR DISCHARGE PRESSURE, 5b) MASS FLOW, 5c) COMPRESSOR SPEED

Figure 6. GRAPHICAL CALCULATION OF \( k_v \) USING THE SPEED CURVE FOR THE OPERATING POINT

where

\[
\tilde{m} = \begin{cases} 
0 & \text{if } |\psi(0, \omega_0) - \psi(m_0, \omega_0)| \geq \psi(m_s, \omega_0) - \psi(m_0, \omega_0) \\
 m_s & \text{if } |\psi(0, \omega_0) - \psi(m_0, \omega_0)| < \psi(m_s, \omega_0) - \psi(m_0, \omega_0). 
\end{cases}
\]  

This approach gave \( k_v = 0.74 \) both for physical compressor map and the characteristics based on Chebyshev polynomials. Because of its shape, the third order polynomial approximation provides characteristics that are flat enough to remove the surge without the controller and was included without modification in the simulations. Therefore no surge is expected in all cases. The results are depicted in Figs. 8a) and 8b) along with modified compressor maps for physical model and Chebyshev approximation.

Figures 8a) and 8b) show that the responses of the system with Chebyshev polynomial approximation is similar to the original. The compressor maps are modified by the CCV according to Eq. (3). It provides a new, flatter compressor map. The maxima are shifted to the left which allows surge avoidance. This is also confirmed in Figs. 10b),10a) and 10c) where the mass flow, compressor discharge pressure and the compressor speed are presented. Figure 9 shows the drop \( \Psi_v(m) \) (the transient period at the beginning is caused by the choice of the initial conditions). As long as the mass flow is larger than desired, \( \Psi_v(m) > 0 \) and the behaviour of the compressor is defined by the modified compressor maps (dotted lines in Figures 8a) and 8b)). When the desired mass flow is reached, \( \Psi_v(m) = 0 \) and the compressor enters steady-state.

CONCLUSIONS

Dynamic models are used for various purposes as they provide an insight into the behaviour of the system, so they can be used for controller design and optimization before implementation in industrial frameworks. However, their performance de-
Figure 7. FEEDBACK LOOP WITH MASS FLOW RATE AS PROCESS VARIABLE (PV) AND PRESSURE DROP ACROSS THE CCV AS MANIPULATED VARIABLE (MV)

Figure 8. RESPONSE (in blue) OF THE SYSTEM WITH: 8a) PHYSICS-BASED AND 8b) CHEBYSHEV POLYNOMIAL APPROXIMATION COMPRESSOR MAP WITH SURGE AND SPEED CONTROLLER PLOTTED AGAINST THE ORIGINAL CHARACTERISTICS (solid colour line) AND THE FLATTER CHARACTERISTICS (dotted black lines)

Figure 9. DROP ACROSS THE CCV FOR: PHYSICAL MODEL (SOLID LINE), CHEBYSHEV MODEL (DOTTED LINE), AND POLYNOMIAL MODEL (DASHED LINE)

not require the knowledge about the geometry of the system. The accuracy of Chebyshev polynomial approximation makes it then useful in dynamic modelling. The outcome of the simulation is close to the results obtained from physical modelling. As the Chebyshev polynomials are able to capture the behaviour of the compressor for all mass flow, including the surge area, they can be used for tuning the anti-surge controller.

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Figure 10. DYNAMIC RESPONSES OF THE SYSTEM WITH SPEED AND SURGE CONTROLLERS: PHYSICAL MODEL (solid line) AND CHEBYSHEV MODEL (dotted line): 10a) COMPRESSOR DISCHARGE PRESSURE, 10b) MASS FLOW, 10c) COMPRESSOR SPEED.

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