Optical anisotropy measurement is essential for material characterization and biological imaging. In order to achieve single-shot mapping of the birefringence parameters of anisotropic samples, a novel polarized light imaging concept is proposed, namely quantitative polarization interference microscopy (QPIM). QPIM can be realized through designing a compact polarization-resolved interference microscopy system that captures interferograms bearing sample’s linear birefringence information. To extract the retardance and the orientation angle maps from a single-shot measurement, a mathematical model for QPIM is further developed. The QPIM system is validated by measuring a calibrated quarter-wave plate, whose fast-axis orientation angle and retardance are determined with great accuracies. The single-shot nature of QPIM further allows to measure the transient dynamics of birefringence changes in material containing anisotropic structures. This application is demonstrated by capturing transient retardance changes in a custom-designed parallel-aligned nematic liquid crystal-based device.

1. Introduction

Optical anisotropy is an intrinsic property of all materials.\textsuperscript{1,2} Liquid crystal (LC) materials commonly used in research and industry have high optical anisotropy.\textsuperscript{3} By measuring the birefringence parameters of LC materials, the performance of LC-based devices can be quantified.\textsuperscript{4–6} Recently, the existence of in-plane anisotropy in 2D materials,\textsuperscript{7} such as black phosphor\textsuperscript{8–10} and GaTe,\textsuperscript{11,12} has attracted great attention.\textsuperscript{13,14} It has been found that the stress exerted on 2D material\textsuperscript{15,16} and grain boundary\textsuperscript{17} can also cause optical anisotropy variations. Furthermore, optical anisotropy property can be used for defect detection on semiconductor wafers.\textsuperscript{18} For nanotubes, anisotropic optical scattering has been used for inferring their orientation variations, which can be applied to develop single particle tracking techniques.\textsuperscript{19} For biological structures, birefringence measurements can reveal the architecture of cytoskeletons,\textsuperscript{20} and can measure the orientation of single molecules in live cells.\textsuperscript{21}

Polarization, which describes the oscillation direction of a vector field, is an important property of electromagnetic fields.\textsuperscript{1,22,23} By measuring polarization state changes, we can probe the molecular-level birefringence in anisotropic specimens, even if the feature size is much smaller than the diffraction-limit.\textsuperscript{24} Polarized fluorescence microscopy has been widely used for single-molecule level observations of both material and biological structures. It has also been combined with 3D tracking\textsuperscript{25} and super-resolution imaging techniques.\textsuperscript{26,27} Polarization-coded structural illumination method has been developed to improve the temporal and spatial resolution of fluorescence microscopy.\textsuperscript{28} Frawley et al. have developed a polarization sensitive microscope system for the study of chiral molecules.\textsuperscript{29} Mazumder et al. have employed a polarization-resolved second harmonic generation imaging system for the imaging of biological structures such as collagens.\textsuperscript{30} Polarized Raman spectroscopy is another popular technique for anisotropic material characterizations.\textsuperscript{31}

Recently, several quantitative polarization imaging techniques have been developed. By replacing the polarization compensators with LC-based variable retarders in conventional Pol-scopes, Oldenbourg and co-workers\textsuperscript{32,33} quantified the 2D distributions of birefringence parameters. This advance has enabled researchers to observe single and bundled microtubules in cells\textsuperscript{34} and reveal the architecture of filamentous actins.\textsuperscript{14} However, the need to acquire multiple images at different retarder delays has limited
the applicability of current systems to measure fast dynamical events. For fast measurement of transient polarization dynamics, several single-shot polarization measurement techniques have been developed. Mu et al. proposed a method to measure the full Stokes matrix of the sample by applying a Wollaston prism array.[35] Ortega-Quijano et al. designed a novel method to simultaneously measure the linear dichroism angle of anisotropic samples.[36] However, Mu et al. have not verified their method’s applicability in microscopic imaging. Ortega-Quijano’s simultaneous measurement was only performed on a single point, which means mapping the sample anisotropy will require 2D scanning.

To fast-image the birefringence distribution, we propose a method to combine polarization measurement with quantitative phase microscopy (QPM). With the ability to precisely measure complex field measurements, QPM provides an efficient way to accurately retrieve structural and functional information of live cells and fabricated material structures.[37–40] Recent efforts have also focused on utilizing QPM for imaging anisotropic samples. Wang et al. and Kim et al.[41,42] first mapped the sample birefringence distributions in terms of the Jones matrix representations that require multiple image acquisitions. However, the retardance and orientation angle distributions were not determined quantitatively. Later on, a quadriwave lateral shearing interferometer-based technique[43] and an integrated phase and birefringence imaging technique[44] were developed to quantitatively measure the birefringence parameters, that is, retardance and orientation angle, through multiple image acquisitions. However, as of today, no single-shot quantitative birefringence imaging technique has been demonstrated.

In this article, we propose a novel polarization microscopy technique, called quantitative polarization interference microscopy, namely QPIM, to quantify 2D retardance and orientation angle distributions of linear birefringent materials. QPIM is achieved through a common-path interferometric imaging system with circularly polarized laser illumination and a comprehensive imaging algorithm. It can retrieve the retardance and the orientation angle maps of anisotropic samples from a single-frame image acquisition. We have validated our QPIM technique by characterizing a calibrated quarter-wave plate sample. QPIM has many important imaging applications such as characterizing LC materials. LC-based devices are widely used in optical imaging and testing. Ellipsometers are usually used to characterize LC devices, but they have a very low throughput and a small field of view.[45] Faster LC device characterization techniques can benefit manufacturers and researchers to optimize the fabrication and performance of these devices. Using QPIM, we are able to recover the retardance and the orientation angle distributions of a customized parallel-aligned LC sample with a 1.22 × 1.54 mm field of view within a few milliseconds. The single-shot nature of our system has enabled us to observe fast transient changes in the LC devices due to applied voltage changes. This demonstration has implications for many other imaging and metrology applications that involve fast dynamics of optical anisotropy.

2. System Design

The experiment setup for QPIM is shown in Figure 1, where a fiber-coupled single-mode 633 nm He-Ne laser is used as the illumination source. After the fiber, the laser beam is collimated with a diameter of approximately 5 mm. A circular polarizer is used to convert the collimated laser beam into right-handed circular polarization before illuminating the sample. After transmitting through the sample, the beam is then collected by an objective lens (Olympus, 4X, numerical aperture (NA) = 0.16). A Wollaston prism (WP10P, Thorlabs), placed at the intermediate image plane, decomposes the sample beam into horizontally and vertically polarized beams (oscillating along x- and y-axes, respectively). These two beams (i.e., the ordinary beam and extraordinary beams) are symmetrically separated along the optical axis with a separation angle of 19.69° at the wavelength of 633 nm (the separation angle originates from Thorlabs’ specification literature of the Wollaston prisms). A cross section of the Wollaston prism for illustrating its working principle is shown as an inset figure, where we have defined the x-y-z coordinate system. The transmitted light from the crossed polarizers passes through another Wollaston prism placed at 45° to the LP, to obtain four separated beams. After passing through a 4f system and imaging on a CMOS camera (Pointgrey, FL3-U3-13Y3M-C; full frame 1024 × 1280 pixels; pixel size 4.8 × 4.8 μm) at the final image plane, closely after the LP. For intensity matching, the transmission axis of the linear polarizer is set at 45° with respect to the x-y plane. The measurement involves recording fringe patterns (i.e., an interferogram), which is then used to retrieve the complex fields with a Fourier transform method.[45] In order to extract the birefringence parameters, we have developed a new polarization recovery algorithm as described in Section 3.

3. The Polarization Recovery Algorithm

After a plane wave transmits through a transparent anisotropic sample, the spatial variation of the birefringence properties will mostly induce a change in the wavefront of the illumination wave. Therefore, compared with light intensity, the complex field...
reveals more information of the sample, such as sample thickness and birefringence distributions. Conventional polarization microscopes give qualitative sample information as they only measure the modified intensity maps. In order to recover the polarization parameters (i.e., retardance and orientation angle) in a quantitative fashion, multiple intensity measurements and a complex system are currently deployed.\cite{24,32,33} Interferometric microscopy such as QPM allows us to retrieve the complex scattered field from the sample in a single shot. Integrating polarization-sensitive optical elements in an interferometric microscope will enable us to retrieve the birefringence information from the complex electric field due to the interference of the ordinary and the extraordinary sample beams. The retardance and the orientation angle distributions are coupled in the real and the imaginary parts (or amplitude and phase) of the transmitted field. In Section 6, we introduce an algorithm to recover both the retardance and the orientation angle distributions of the sample from the measured interferogram.

After a Fourier transform of the interferogram, we obtained the 0th order (i.e., direct current (DC) term), +1st order and −1st order terms. The +1st (or the −1st) order term gives the complex electric field, \( U(x, y) \):

\[
U(x, y) = B(x, y) + iC(x, y)
\]

where \( B(x, y) \) and \( C(x, y) \) are the real and the imaginary parts of the retrieved electric field. After the formulation (see details in Methods, Derivation of the Interferogram), the retardance distribution of the sample, \( \Delta(x, y) \), can be recovered as

\[
\Delta(x, y) = \cos^{-1}\left[ -\frac{2C(x, y)}{A(x, y)} \right]
\]

where \( A(x, y) \) is the intensity of the 0th order term. Furthermore, the orientation angle distribution, \( \psi(x, y) \), can be calculated as

\[
\psi(x, y) = \frac{1}{2} \cos^{-1}\left[ \frac{2B(x, y)}{\sqrt{A^2(x, y) - 4C^2(x, y)}} \right]
\]

Note that if the retardance of the anisotropic sample is larger than 2\( \pi \), it will be necessary to unwrap the recovered retardance to obtain the correct retardance distribution (the detailed algorithm is described in the Experimental Section, Derivation of the Interferogram, centered around Equation (14)). By extracting the real and the imaginary parts of the complex field, we obtain two equations to decouple the retardance and the orientation angle. The intensity distribution in the 0th order image is used to eliminate the effect of the intensity transmittance through the sample. Equations (2) and (3) have allowed us to recover the retardance and the orientation angle maps with a single image acquisition. The accuracy and efficiency of our algorithm will be explored in Section 4.

### 4. Experimental Results

To verify the imaging concept of QPIM, we first measured a calibrated 633 nm zero-order quarter-wave plate (WPQ10ME-633, Thorlabs). We rotated the wave plate’s fast axis from 0 to 180° with respect to the x-axis and acquired one interferogram for each 10° increment. Applying our imaging algorithm on this interferogram, we retrieved the retardance and the wave plate orientation angle (i.e., the angle between the x-axis and the fast axis of quarter-wave plate). In Figure 2a, the recovered average retardance as a function of the sample rotation angle is plotted in red circles, and the actual retardance is plotted in a blue dashed line. From this measurement, the retardance of the quarter-wave plate is determined to be \((0.51 \pm 0.02) \pi \) rad which is in a good agreement with the actual retardance of 0.5 \( \pi \). The recovered orientation angle (in red circles) and the actual orientation angle (in a blue dashed line) as a function of the rotation angle is presented in Figure 2b. Again, we have observed a close match between the measured and the actual values, and the standard deviation of the difference between the recovered and the actual orientation angles is 3.11° (≈54 mrad). These results have shown that our system can measure retardance and orientation angle simultaneously with an error around 50–60 mrad. Note that the precision of the rotation stage (CRM1, Thorlabs) is 2°. To further improve the measurement accuracy, the sample tilt and defocus effect and the precision of the rotation stage may need to be taken into consideration.\cite{46,47}

Birefringence properties are widely explored for making precision optical devices such as LC devices and spatial light modulators (SLMs), but there is always a need for better metrology methods to characterize such polarization-sensitive devices. Ellipsometry-based techniques are normally used for inspecting those devices, but they have a low throughput and a small field of view.\cite{44-46} Polarization-sensitive optical coherence tomography (OCT) systems\cite{48} can be potentially applied to mapping the birefringence distributions, but its point scanning imaging mode will still limit the throughput. On the other hand, our QPIM
can achieve full-field and high-speed mapping of the birefringence distributions with a single-shot image capture, which will be specifically validated with our experiments for the characterization of LC samples in the Section 6.

First, we illustrate the retardance and the orientation angle parameters in an LC sample with the help of Figure 3a. When the light polarization (extraordinary beam) is along the long axis of an LC molecule, the refraction index is $n_x$, while the refraction index is $n_o$ when the light polarization (ordinary beam) is orthogonal to the long axis (i.e., along the short axis). The LC molecule is not necessarily oriented in the observation $x$-$y$ plane, thus we need to first project the LC molecule orientation into the $x$-$y$ plane. Then, the angle between the $x$ axis and the long axis is defined as the orientation angle $\phi$. The phase difference between ordinary and extraordinary beams in the $x$-$y$ plane is the retardance $\Delta$. In our experiment, we measure the $x$-$y$ plane distributions of $\phi$ and $\Delta$. A custom-made single layer nematic-LC sample (prepared by Hamamatsu Photonics; refer to Methods, Sample Preparation, for LC sample preparation), as shown in Figure 3b, is characterized with our system. In this sample, the LC molecules are uniformly distributed between two quartz glass plates and divided into two regions. Each region has a separate pair of electrodes; the device can be considered as an SLM with just two macro pixels. The LC sample thickness is 20 $\mu$m, whereas the birefringence (difference of $n_x$ and $n_o$) is 0.2. Therefore, the maximum possible retardance is 39.7 rad.

After mounting the sample on our QPIM system and connecting the electrodes, we obtained the interferogram image of the LC boundary region (note that this is the boundary of the two macro pixels) as shown in Figure 3c, where a zoom-in image clearly shows the fringes. The right-hand side of the interferogram corresponds to the region, where a zero-centered rectangular voltage wave is applied through a function generator (DS345, Stanford Research Systems). The voltage wave has an amplitude of 3 V and a frequency of 1 kHz. A 2D Fourier transform of the interferogram is performed and the logarithm of its magnitude is shown in Figure 3d. By selecting and shifting the 1st order into the center and performing an inverse Fourier transform, we retrieve the complex sample field. The sample retardance is recovered using Equation (2), and the result is shown in Figure 3e. As the retardance is much larger than $2\pi$, we have developed a method (discussed in the Methods, Derivation of the Interferogram) combined with the Goldstein algorithm to unwrap the retardance for obtaining the correct measurement result as shown in Figure 3f. The average retardance difference between the left and the right regions is 26.4 rad. With no voltage applied to the left region, the retardance of this region remains at the original value of 39.7 rad. The LC molecules in the right region are elevated due to the external electric field, thus resulting in a decrease of the retardance down to 13.3 rad. The orientation angle distribution is retrieved using Equation (3), which is shown in Figure 3g. From this image, we found the average orientation angle distribution is about 116.3° with a standard deviation of 1.5°. In Figure 3h, we...
show a quiver plot of the retardance and the orientation angle for a 24 × 18 µm region across the boundary, indicated in the white box in Figure 3f. Figure 3h shows that the orientation angle is uniformly distributed, and it only deviates around the boundary.

When the applied voltage increases, the LC molecules tilt toward the z-axis, thus resulting in a decrease of sample retardance. If the elevation angle of the LC molecules is 90°, the LC sample will essentially become isotropic with zero retardance. However, in practice, the elevation angle will not become 90° even when maximum allowable voltage is applied. Knowing the elevation angle distributions is important for many applications, such as characterizing linearly photopolymerized liquid crystal polymer films designed to increase the field of view of liquid crystal displays.\[4\] Next, we measured the LC sample’s birefringence response to voltage changes. As we increased the applied voltage of the LC sample right region from 2.3 to 5 V, we acquired images for every 0.1 V increment. With our imaging algorithm, both the retardance and the orientation angle distributions were retrieved. In Figure 4a, we show the recovered retardance map for the initial applied 2.3 V voltage. In the right region, an area is selected, as indicated in the white box, for illustrating the birefringence responses to voltage. The spatially averaged retardance and orientation angle values of this selected area are plotted as a function of the applied voltage as shown in Figure 4b,c. In those plots, the blue curves represent the 2nd order polynomial fits. In Figure 4b, we observe an expected decrease in retardance, which tends to saturate as a function of time. The standard deviation of the difference of experimental values and the 2nd order polynomial fitting curve is less than 90 mrad. Figure 4c shows the observation plane orientation angle at 115.8 ± 3.1 degrees, which does not change as expected. Typically, the retardance of a LC sample can be characterized by simply measuring the intensity of transmitted light as a function of voltage when the LC is placed between two linear polarizers with a 45° transmission axes mismatch. We performed such an experiment using QPIM and have presented the results in the blue dashed line in Figure 4d, where the right vertical axis represents the intensity. While the intensity measurements cannot be used to directly obtain the quantitative retardance curve, our system readily provides quantitative retardance which can be used to obtain the intensity curve in the red dashed line that matches well with the blue curve. This experiment again confirms the quantitative nature of QPIM.

When the applied voltage on the LC sample is suddenly changed, there is a fast transient change in its birefringence distribution, which can be captured with QPIM due to its high-speed imaging capability. To demonstrate this capability, we recorded interferograms at 150 frames per second (fps) while changing the applied voltage to the right region from 2 to 3 V with the function generator. The integration time of the camera is approximately 7 ms. The recovered retardance distributions in natural logarithm scale for time points at 1.85, 1.92, and 1.95 s are shown in Figure 5a–c, where the imaging area shows the boundary between voltage-controlled and uncontrolled region. Note that we took the natural logarithm of the unwrapped retardance for better observation of the small retardance variances on the right side. The time sequence of the recovered average.

Figure 4. Measured voltage response of the retardance and the orientation of custom-built LC sample. a) The recovered retardance map when the initial 2.3 V voltage is applied to the right region. b) The voltage response of the recovered average retardance for the region shown in (a). The red dots are the experimentally measured values, and the blue line is the fitted 2nd order polynomial curve. c) The voltage response of the recovered average orientation angle for the region shown in (a). The red dots are the experimentally measured values, and the blue line is a constant orientation at 115.8°. d) The blue dashed curve is the measured LC sample intensity transmission versus the applied voltage using two polarizers whose orientation mismatch is 45°. The red dashed curve is the intensity transmission, calculated from measured retardance values, versus the applied voltage.
Figure 5. The transient dynamics of the retardance distribution due to a sudden voltage change created through the function generator. a)–c) The unwrapped retardance distributions (in natural logarithm scale) of a small boundary region for time points at 1.85, 1.92, and 1.95 s, respectively. d) The time sequence of the recovered average retardance of the right region. The red dots represent the calculated retardance values, while the blue curve is after the average filter. The scale bar denotes 50 μm.

retardance is shown in Figure 5d, from which we noticed that the retardance transition time is between 0.05 and 0.1 s, which is caused either by the electrical circuit or by the mechanical rotation of LC molecules. The time-lapse video of the retardance distribution dynamics is available in the Supporting Information.

5. Conclusions

In this article, we have proposed and demonstrated a novel optical anisotropy imaging technique, QPIM, for fast mapping the birefringence distributions of anisotropic samples. A mathematical model has been developed to retrieve the retardance and the orientation angle distributions from the single-shot interferogram measurements. The single-shot nature of QPIM allows for fast mapping of the retardance and the orientation angle distributions—a feature that has unique potential for many material metrology applications. We have experimentally demonstrated one such metrology application of QPIM, that is, characterizing LC samples. We envision that QPIM will find promising applications in high-speed imaging of fast dynamics in anisotropic materials and biological samples.

6. Methods

6.1. Derivation of the Interferogram

Jones calculus\[^{[50]}\] is applied to trace the complex electric field in the QPIM system. A mathematical algorithm based on digital holography is proposed to extract the birefringence parameters from the measured complex electric field. First, the right-handed circularly polarized illumination light, $E_{in}$, can be expressed as a Jones vector:

$$E_{in} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

(4)

The axes of Wollaston prism are aligned parallel to the $x$ and $y$ axes as shown in Figure 1, and the orientation of the LP is set at 45° to the $x$ axis in the $x$-$y$ plane. Then, the Jones matrices of the Wollaston prism and the linear polarizer are determined to be

$$J_{\text{Wollaston}} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & 1 \end{pmatrix}, \quad J_{\text{LP}} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

(5)

where $k = k_0 \sin \alpha$, $k_0 = 2\pi/\lambda$, $\lambda$ is the wavelength of the laser in free space, and $\alpha$ is the divergence angle of the Wollaston prism. $\alpha$ and the 4f system following the Wollaston prism determine the period of the interferogram captured on the camera. Next, we derive the Jones matrix of the birefringence sample.

The birefringence of the sample contains two parameters: retardance and orientation angle. Taking a single LC molecule oriented in the $x$-$y$ plane as an example (Figure 3a), at each point the retardance, $\Delta(x, y)$, is defined as the difference between the extraordinary light phase delay, $\phi_e(x, y)$, and the ordinary light phase delay, $\phi_o(x, y)$; that is, $\Delta(x, y) = \phi_e(x, y) - \phi_o(x, y)$. At each point, the orientation angle, $\varphi(x, y)$, is defined as the angle between the long axis of the LC molecule and the $x$-axis. The retardance and orientation angle at each sample point will be retrieved in QPIM, thus giving a 2D distribution for each parameter. Note that as the size of a single LC molecule is beyond the diffraction limit of the microscope, the measured birefringence parameters are the spatially average values within one diffraction spot. At the
end, the Jones matrix of the sample is determined to be

$$J_{\text{sample}} = \begin{pmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{pmatrix} \begin{pmatrix} \exp(i\varphi) & 0 \\ 0 & \exp(i\varphi) \end{pmatrix} \begin{pmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{pmatrix}$$

(6)

where we neglect \((x, y)\) for simplicity. Finally, the electric field Jones vector at the detector plane is calculated as

$$E_{\text{out}} = J_{\text{Wollaston}} J_{\text{sample}} E_{\text{in}}$$

(7)

where the intermediate Jones matrix \(J_m\) of the Wollaston prism and the sample can be written as

$$J_m = J_{\text{Wollaston}} J_{\text{sample}} = \begin{pmatrix} e^{i(\varphi_o + k\lambda)} \cos^2 \varphi + e^{i(\varphi_o + k\lambda)} \sin^2 \varphi & -e^{i(\varphi_o + k\lambda)} \sin \varphi \cos \varphi + e^{i(\varphi_o + k\lambda)} \sin \varphi \cos \varphi \\ -e^{i(\varphi_o + k\lambda)} \sin \varphi \cos \varphi + e^{i(\varphi_o + k\lambda)} \sin \varphi \cos \varphi & e^{i(\varphi_o + k\lambda)} \sin^2 \varphi + e^{i(\varphi_o + k\lambda)} \cos^2 \varphi \end{pmatrix}$$

(8)

We assume the intensity transmission coefficient at each point on the sample to be \(I(x, y)\). With the illumination intensity distribution of \(I_{\text{in}}(x, y)\), which will be assumed uniform, the intensity recorded on the CCD is supposed to be

$$I(x, y) = \tau(x, y) I_{\text{in}}(x, y) E_{\text{out}}^* E_{\text{in}}^*$$

$$= \tau(x, y) I_{\text{in}}(x, y) \left[ 2 - \cos k x \cos 2\varphi(x, y) \sin \Delta(x, y) \\
-\sin k x \cos \Delta(x, y) \right]$$

(9)

Note that there is also a background intensity distribution that can be eliminated by subtracting a calibration image. Through recording \(I(x, y)\), namely the interferogram image, we can retrieve the complex field \(U(x, y)\) from the 1st order signal (detailed in Section 1, Supporting Information), thus giving

$$U(x, y) = \tau(x, y) I_{\text{in}}(x, y) (\cos 2\varphi(x, y) \sin \Delta(x, y))$$

$$-i \cos \Delta(x, y)$$

(10)

We can also recover the original 0th order signal, that is, the DC term, as

$$A(x, y) = 2\tau(x, y) I_{\text{in}}(x, y)$$

(11)

The real part of the complex field is

$$B(x, y) = \tau(x, y) I_{\text{in}}(x, y) \cos 2\varphi(x, y) \sin \Delta(x, y)$$

(12)

The imaginary part of the complex field is

$$C(x, y) = -\tau(x, y) I_{\text{in}}(x, y) \cos \Delta(x, y)$$

(13)

With Equations (11)–(13), Equations (2)–(3) can be derived and the retardance and orientation angle distributions can be obtained.

For samples like the LC device, the retardance is often quite large (over 2\(\pi\)), but the retardance recovered from Equation (2) ranges from 0 to \(\pi\) due to the property of the function \(\cos^{-1}\).

The unwrap algorithm we often used is designed to process the wrapped phase, which is retrieved from \(\tan^{-1}\), ranging from \(-\pi/2\) to \(\pi/2\). To unwrap the retardance, we should do the following operation to make the retrieved retardance compatible with unwrap algorithms

$$\Delta_2(x, y) = \tan^{-1}\left( -\frac{B(x, y)}{C(x, y) \cos 2\varphi(x, y)} \right)$$

(14)

where \(\cos 2\varphi(x, y)\) is calculated with Equation (3). It is easier to unwrap \(\Delta_2(x, y)\), since it ranges from \(-\pi/2\) to \(\pi/2\) so that we can directly use Goldstein’s algorithm.

### 6.2. Sample Preparation

The LC sample used to verify our method is specially designed at Hamamatsu Photonics. An LC cell was created by bonding two quartz glass plates with 20 \(\mu\)m cell gap in set. Both sides of the glass plates were coated with a wide-range anti-reflective coating, a high-resistance indium tin oxide (ITO) conductive layer, and a polyimide (PI) alignment layer on internal surface of the cell. The steps for filling the cell with LC are as follows:

1) PI layer was rubbed and assembled for parallel alignment of the LC.
2) Arbitrary electrodes were fabricated on the ITO layer with femtosecond laser processing.
3) Inserted a cell with single filling hole into a vacuum chamber, placed a LC material in a container under the cell.
4) Evacuated the vacuum chamber and heated the container to a temperature above the LC clearing point.
5) After the chamber reached the required vacuum level and temperature, contacted the LC to the filling hole of the cell.
6) Released the pressure in the chamber to fill the cell with LC.
7) Removed the cell from the chamber and sealed the filling hole with UV glue.
8) All electrodes were wired to electrical cable to supply voltage, respectively.

### Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest

The authors declare no conflict of interest.

Keywords

instrumentation, interference microscopy, measurement and metrology, microscopy, polarimetric imaging

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