Dynamic eccentricities in pushover analysis of asymmetric single-storey buildings

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Abstract  A new version about a method of documented application of pushover analysis on asymmetric single-storey buildings has been presented, recently. The main target of this new method was to rationally consider the coupling between torsional (about vertical axis) and translational vibrations of asymmetric single-storey building diaphragm under translational seismic excitation of its base. For this reason, the equivalent lateral static floor force according to the new method is applied using suitable dynamic eccentricities, eccentric to building Mass Centre, which are added with the accidental eccentricities, in such a way that the final design eccentricities move the application point of floor, external, lateral, static force away from the diaphragm Mass Centre. The origin point for measuring the dynamic eccentricities is called “Capable Near Collapse Centre of Stiffness \( CR_{sec} \)”. Also, the floor lateral static force must be oriented along the building principal axes which are called “Capable Near Collapse Principal Axes” \( I_{sec} \) and \( II_{sec} \), respectively. The appropriate dynamic eccentricities for the stiff-side and the flexible-one of the floor plan derive from extensive parametric analysis and are calculated by graphs and suitable equations. In the present work, a numerical example of an asymmetric single-storey building is presented to illustrate clearly and in detail the step by step application of the proposed pushover analysis method. It is a torsional sensitive reinforced concrete building designed in accordance with Eurocodes EN 1992-1 and 1998-1 for ductility class high (DCH). The proposed method is evaluated relative to the results of non-linear response history analysis (TH). The final results show that the proposed method of pushover analysis predicts with safety the displacement of the stiff side of the building as well as that of the flexible side.

1 Introduction

To seismically assess the seismic bearing capacity of a building, the basic method of analysis proposed by all contemporary seismic codes is the static non-linear method of analysis (pushover). For documented application of pushover analysis on asymmetric single-storey buildings, the lateral static floor load should be
applied with eccentricity relative to the Mass Centre of the diaphragm and should also be properly oriented along each horizontal principal axis of the building [1]. This is the only way to consider the phenomenon of the development of diaphragm torsional vibration about vertical axis coupled to translational ones, due to the developing inertial torsional (about Z-axis) moment of the building floor, a phenomenon occurring both in linear and in non-linear areas [1, 2, 3, 4]. Eurocode EN 1998-1 [5] suggests that the position of the Mass Centre (where the external lateral static floor force is applying) must be moved by the floor accidental eccentricity, but in this way, there is still the inability to obtain the inertial torsional moment of the diaphragm. Both in recent work [1] and the current one, where a numerical example is presented, the use of suitable dynamic eccentricities is proposed, in order to determine the point of application of the lateral static floor force in the plan. The calculation of, first, the starting point for the measurement of the dynamic eccentricities, second, the appropriate orientation direction of the lateral static floor force and third, the magnitude of the dynamic eccentricities are the threefold objective of the present paper, where the theoretical analysis has been given in [1].

2 Methodology

From the extensive parametric investigation [1] as well as by the recently international literature review [2, 3, 4], the main conclusions and the proposed methodology are summarized below:

1) The most appropriate point (almost independent of the loading) that must be the starting point for measuring the new dynamic eccentricities is the “Capable Near Collapse Centre of Stiffness” \( CR_{sec} \) resulting from the model whose all structural members have been provided with their secant stiffness \( EI_{sec} \) in the yield state of the members,

2) The most appropriate orientation of the horizontal static floor force of the pushover method (almost independent of the loading) is along the directions of the horizontal “Capable Near Collapse Principal Axes” \( I_{sec} \) and \( H_{sec} \) resulting for the above model of conclusion (1) using the secant stiffness \( EI_{sec} \) of the members,

3) The control of the building torsional sensitivity must be performed in the above model of conclusion (1) using the secant stiffness \( EI_{sec} \) of the members. The asymmetric single-story buildings that examined were divided into two categories: (a) buildings with torsional sensitivity when \( r_{1,sec} \) or \( r_{1,sec} \leq 1.10 \) \( r_m \) applies and (b) buildings without torsional sensitivity when \( r_{1,sec} \) and \( r_{1,sec} > 1.10 \) \( r_m \) applies, where \( r_{1,sec} \) and \( r_{1,sec} \) are the “Capable Near Collapse Torsional Radii” in respect to the axes \( I_{sec} \) and \( H_{sec} \) respectively, and \( r_m \) is the radius of gyration of the diaphragm,

4) The magnitude of the appropriate dynamic eccentricities \( e_{stiff} \) and \( e_{flex} \), for the building stiff and flexible side respectively, along each horizontal
“Capable Near Collapse Principal Axis” $I_{sec}$ and $H_{sec}$, have been determined from a statistical processing on the results of the extended parametric analysis and is given through graphs (Fig. 1-2) and equations (Eq. 1-4) (prediction lines with a suitable standard deviation).

5) The proposed method for the documented application of pushover analysis to asymmetric single-storey buildings uses the design eccentricities. Noting that the design eccentricities $e_1$, $e_2$ (Eq. 5-6) and $e_3$, $e_4$ (Eq. 7-8) are used for loading along the “Capable Near Collapse Principal Axis” $H_{sec}$ and $I_{sec}$ respectively, all measured from the “Capable Near Collapse Centre of Stiffness” $CR_{sec}$ and with positive direction towards the Mass Centre CM (see Fig. 4). The calculation of the design eccentricities is achieved by the simultaneous reception of the new dynamic eccentricities resulting from Eq. (1-4) and of the floor accidental eccentricities $e_a$ obtained in such a way that the final position of the horizontal static floor force to be more eccentric relative to the Mass Centre CM.

The dynamic eccentricities along each horizontal direction, $I_{sec}$ and $H_{sec}$ are given by:

For buildings with Torsional Sensitivity, $r_{I,sec}$ or $r_{H,sec} \leq 1.10 \ r_m$:

$$e_{stiff} = 0.046 \cdot e_R - 0.11 \cdot r_m$$

$$e_{flex} = 0.84 \cdot e_R + 0.12 \cdot r_m$$

(1)

(2)

For buildings without Torsional Sensitivity $r_{I,sec}$ and $r_{H,sec} > 1.10 \ r_m$:

$$e_{stiff} = 0.043 \cdot e_R - 0.05 \cdot r_m$$

$$e_{flex} = 0.83 \cdot e_R + 0.17 \cdot r_m$$

(3)

(4)

In equations (1-4), $e_R$ is the static (or stiffness) eccentricity, regarding the $CR_{sec}$ position in the plan, relative to the examined horizontal direction $I_{sec}$ or $H_{sec}$. The design eccentricities along each horizontal direction, $I_{sec}$ and $H_{sec}$, are given by:

$$e_1 = e_{flex,I} + e_{a,I}$$

$$e_2 = e_{stiff,I} - e_{a,I}$$

$$e_3 = e_{flex,II} + e_{a,II}$$

$$e_4 = e_{stiff,II} - e_{a,II}$$

(5)

(6)

(7)

(8)

In Eq. (5-8), $e_{stiff}$ and $e_{flex}$ are the dynamic eccentricities from eq. (1-4) while the accidental eccentricities are determined from the equations $e_{a,I} = \pm (0.05 \sim 0.10) \cdot L_I$ and $e_{a,II} = \pm (0.05 \sim 0.10) \cdot L_{II}$ according to EN1998-1, where $L_I$ and $L_{II}$ are the maximum floor plan dimension normal to the loading direction.

Considering the two signs ($\pm$) of application of the lateral static floor forces, eight separate pushover analyses of the building are performed. The final results from the spatial action of the earthquake are computed as proposed by Eurocode EN 1998-1, i.e. by the SRSS combinations on the results of the eight separate pushover analyses (sixteen loading combinations). These combinations are carried out in that step of the separate pushover analyses where the "seismic target-displacement" of
the Mass Centre CM of the diaphragm occurs. It is noted that the capacity curve of the building, along the horizontal axis under consideration ($I_{sec}$ or $H_{sec}$), is given by the corresponding building shear base and the displacement of the diaphragm point, where the lateral static floor force is applied.

Fig. 1: Normalized dynamic eccentricities $e_{stiff}/r_m$ for the stiff side of plan

Fig. 2: Normalized dynamic eccentricities $e_{flex}/r_m$ for the flexible side of plan

It is clarified that, in order to use the new design eccentricities, the following must already have been calculated: (a) the position of the “Capable Near Collapse Centre of Stiffness” $C_{sec}$ which is the origin for measurement them, from Eq. (9a,b) (see [6, 7]), (b) the orientation of the horizontal “Capable Near Collapse Principal Axes” $I_{sec}$ and $H_{sec}$, which determines the orientation of the lateral
loading, from Eq. (10) (see [6, 7]) and (c) the two building “Capable Near Collapse Torsional Radii” \( r_{1,\text{sec}} \) and \( r_{2,\text{sec}} \), relative to the horizontal axes \( I_{\text{sec}} \) and \( II_{\text{sec}} \), through which the torsional sensitivity of the building is determined, from Eq. (11a,b), [8]. All the above, are calculated in the building model, whose structural members have been provided with their secant stiffness \( EI_{\text{sec}} \) at their yield state. Also, the radius of gyration of the diaphragm must be calculated from the relation 
\[
r_{m} = \sqrt{\frac{J_{m}}{m}}.
\]

The abovementioned (a), (b) and (c) calculation steps are implemented as follows:

a) From a temporary static linear analysis of the model with a static unit floor moment about vertical axis, e.g. \( M_{z} = 1.00 \text{kNm} \), the displacements \( u_{x,\text{CM}M_{z}} \) and \( u_{y,\text{CM}M_{z}} \) of the Mass Centre CM and the rotation \( \theta_{z,M_{z}} \) of the diaphragm are calculated. The coordinates \( x_{CR,\text{sec}}, y_{CR,\text{sec}} \) of the “Capable Near Collapse Centre of Stiffness” \( CR_{\text{sec}} \), relative to the Mass Centre, are calculated from Eq. 9\((a,b)\):
\[
\begin{align*}
x_{CR,\text{sec}} &= -u_{y,\text{CM}M_{z}}/\theta_{z,M_{z}}, & y_{CR,\text{sec}} &= +u_{x,\text{CM}M_{z}}/\theta_{z,M_{z}}
\end{align*}
\]

b) From a temporary static linear analysis of the model with a lateral static unit force \( F_{x} = 1.00 \text{kN} \) that is located on \( CR_{\text{sec}} \) along \( x \)-axis, the displacement \( u_{x,F_{x}} \) of \( CR_{\text{sec}} \) along the \( x \)-axis is calculated. Similar, from a temporary static linear analysis with a lateral static unit force \( F_{y} = 1.00 \text{kN} \) that is located on \( CR_{\text{sec}} \) along \( y \)-axis, the displacements \( u_{y,F_{y}} \) and \( u_{x,F_{y}} \) of \( CR_{\text{sec}} \) along the \( y \)-axis and \( x \)-axis respectively are calculated. The orientation (angle \( \alpha \)) of the building horizontal “Capable Near Collapse Principal Axes” \( I_{\text{sec}} \) and \( II_{\text{sec}} \), relative to the initial \( x \) and \( y \) axes respectively, is calculated from Eq. (10):
\[
\tan 2\alpha = \frac{2u_{x,F_{y}}}{u_{x,F_{x}} - u_{y,F_{y}}}
\]

c) From a temporary static linear analysis of the model with a lateral static unit force \( F_{y} = 1.00 \text{kN} \) that is located on \( CR_{\text{sec}} \) along axis \( II_{\text{sec}} \), the displacement \( u_{y,F_{y}} \) of \( CR_{\text{sec}} \) along the \( II_{\text{sec}} \) axis is calculated. Also, from a second temporary static linear analysis, where the lateral static unit force \( F_{x} = 1.00 \text{kN} \) is located on \( CR_{\text{sec}} \) along axis \( I_{\text{sec}} \), the displacements \( u_{x,F_{x}} \) of \( CR_{\text{sec}} \) along the \( I_{\text{sec}} \) axis is calculated. With known rotation \( \theta_{z,M_{z}} \) due to the static unit floor moment \( M_{z} = 1.00 \text{kNm} \) about vertical axis, the building “Capable Near Collapse Torsional Radii” \( r_{1,\text{sec}} \) and \( r_{2,\text{sec}} \) along the two axes \( I_{\text{sec}} \) and \( II_{\text{sec}} \) respectively are calculated from Eq. (11a,b):
\[
\begin{align*}
r_{1,\text{sec}} &= \frac{u_{y,F_{y}}}{\theta_{z,M_{z}}}, & r_{2,\text{sec}} &= \frac{u_{x,F_{x}}}{\theta_{z,M_{z}}}
\end{align*}
\]
3 Numerical example

In this section, an example of an asymmetric single-storey r/c building is presented to illustrate clearly and in detail the application of the proposed pushover analysis method using design eccentricities and also for evaluation purposes.

3.1 Building characteristics

Consider the building that seems on Fig. 3 which is a reinforced concrete (r/c) asymmetric single-storey building with construction materials C25/30 for the concrete and B500c for the steel of average strengths $f_{cm} = 33$ MPa and $f_{ym} = 550$ MPa respectively. In Table 1, the elastic and inertial characteristics of the building as well as the torsional sensitivity check of its non-linear model (in which all members are supplied with their secant stiffness $E_k$ at their yield state) are presented. The building is characterized as torsional sensitive ($\frac{r_1}{r_m}$ or $\frac{r_{II}}{r_m} < 1.10$). The position of the Mass Centre CM coincides with the geometric center of the floor plan. The building consists of three wing parts with different orientations between them. A rigid diaphragm of thickness 17 cm joins the wings and extends outwards forming a 2 m perimetric cantilever. The X-Y wing consists of coupled walls (3 walls about X and 6 walls about Y direction coupled with T-beams), the $135^\circ$ wing consists of five frames (2 frames along $135^\circ$ and 3 frames normal to $135^\circ$ direction) and the -$135^\circ$ wing consists of seven frames, two of them coupled with r/c flexural walls (4 frames along -$135^\circ$ and 3 frames normal to -$135^\circ$ direction). The two wing parts at orientations $135^\circ$ and -$135^\circ$ are continuous, along their normal direction, with a common frame while the third X-Y wing part relates to the previous wing parts via Tee beams. The structural system of the building consists of rectangular columns of dimensions 0.50/0.50 m, beams with Tee section 0.30/0.60/1.60/0.17 m and flexural walls with orthogonal section 0.30/1.50 m. All the perimetric walls have one boundary or middle barbell (column 0.50/0.50 m) to satisfy the anchorage length of normal beam longitudinal bars. The height of the buildings is 3 m.

| Table 1: Elastic and inertial characteristics and torsional sensitivity control in the non-linear model of asymmetric single-storey building |
|---------------------------------------------------------------|
| Static eccentricity $e_{Rx}, e_{Ry}$ (m) : 6.29, -0.68        |
| Static eccentricity $e_{RII}, e_{RII}$ (m) : 6.02, 1.95       |
| Mass $m$ (tn) : 1103                                          |
| Mass moment of inertia $J_m$ (tn·m$^2$) : 222958              |
| Radius of gyration $r_m$ (m) : 14.22                          |
| Torsional radius $r_{Isec}$ & $r_{IIsec}$ (m) : 13.32 & 16.27 |
| Ratio $r_{Isec}/r_m$ & $r_{IIsec}/r_m$ : 0.94 & 1.14          |
3.2 Building design

The design is performed according to the provisions of Eurocodes EN1992-1 and EN1998-1. The building system is characterized as wall-equivalent dual according to EN1998-1 (§5.1.2). The design model of the building is also torsional sensitive ($n_{des}/n_m = 0.96$). The building has an importance factor $\gamma_i=1$ and is designed for effective peak ground acceleration $a_g = 0.24g$, soil $D$, ductility class high (DCH) and total behavior factor $q=3$. The details of the longitudinal and confinement steel reinforcements can be found in authors.

![Fig. 3: Floor plan of asymmetric single-story buildings](image)
3.3 Non-linear model

For the application of non-linear analyses, the structural members of the building model are provided with the secant moments of inertia \( I_{sec} \) (at their yield). The secant stiffness \( EI_{sec} \) is taken as a constant value over the entire length of each structural member, is equal to the arithmetic average of the \( EI_{sec} \) values of its two end cross-sections for positive and negative bending and is given by EN 1998-3 [9]:

\[
EI_{sec} = \frac{M_y}{\theta_y} \cdot \frac{L_v}{3}
\]

For the calculation of chord rotation at yield \( \theta_y \), the equations (A.10b) και (A.11b) of EN1998-3 are used for columns-beams and walls respectively:

\[
\theta_y = \varphi_y \frac{L_v + a_v \cdot z}{3} + 0.0013 \left( 1 + \frac{1.5 \cdot h}{L_v} \right) + 0.13 \cdot \varphi_y \frac{d_h \cdot f_{ym}}{\sqrt{E_{cm}}} \tag{13}
\]

\[
\theta_y = \varphi_y \frac{L_v + a_v \cdot z}{3} + 0.002 \left( 1 - \frac{0.125 \cdot L_v}{h} \right) + 0.13 \cdot \varphi_y \frac{d_h \cdot f_{ym}}{\sqrt{E_{cm}}} \tag{14}
\]

The calculation of the curvature at yield \( \varphi_y \) and yield moment \( M_y \) of element end-sections was carried out with the module Section Designer of the analysis program SAP2000 [10]. The unconfined and confined model for the concrete follows the constitutive relationship of the uniaxial model proposed by Mander et al. [11], while the steel reinforcement is represented by the simple (parabolic at strain-hardening region) model of SAP2000. The axial force of the vertical resisting elements that is used for the calculation of \( \varphi_y \) is determined from the (seismic) combination \( G+0.3Q \), where \( G \) is the permanent and \( Q \) is the live vertical load, respectively. The shear span \( L_v \) was assumed equal to the half clear length \( L_{cl} \) of the structural elements along the frame bending planes except the strong direction of walls and the direction of columns with cantilever bending (six columns of the second wing 135°), where it was considered equal to \( L_{cl} \). The secant stiffness \( EI_{sec} \) is calculated as percentage of the geometric stiffness \( EI_g \). It is equal to an average of 11% for the columns along frame bending planes, 17% for columns along cantilever bending planes, 11% for the strong wall direction, 12% for the weak wall direction and 10% for the beams, where the average modulus \( E_{cm} \) of concrete C25/30 was considered equal to 31 GPa. For the calculation of the plastic capacity \( \theta_{pl} \) in terms of chord rotations, the analysis software uses the relation \( \theta_{pl} = (\varphi_a - \varphi_y) \cdot L_{pl} \), where \( L_{pl} \) is calculated from Eq. (A.9) of EN 1998-3. In the non-linear analysis model, point plastic hinges were inserted at each end-section of all structural members. \( P-M_z-M_I \) hinges and \( M_I \) hinges are used for vertical elements and beams respectively.

The asymmetric single-storey building has static eccentricities \( e_{RI} = 6.02 \) and \( e_{RI} = 1.95 \) m [6, 7] along the horizontal axes \( I_{sec} \) and \( J_{sec} \) (position of the CR_{sec} relative to CM) which are rotated by -24° relative to the Cartesian x, y axes [6,7] (Fig. 3) and the building is characterized as torsional sensitive since \( \tau_{sec}/r_m = 0.94 < 1.10 \) applies [8], where the torsional radius \( \tau_{sec} \) refers to CR_{sec} (Table 1).
The periods of the three coupled modes are $T_1 = 0.363$ sec, $T_2 = 0.326$ sec and $T_3 = 0.226$ sec.

In the non-linear analysis model, the accidental eccentricity along the axes $L_{sec}$ and $H_{sec}$ is taken with a value equal to 5% of the maximum plan dimension normal to the loading direction (see Fig. 5):

$$e_{aI} = \pm 0.05 \cdot L_1 = \pm 0.05 \cdot 40.33 = \pm 2.02 \text{ m}$$
$$e_{aII} = \pm 0.05 \cdot L_2 = \pm 0.05 \cdot 31.72 = \pm 1.59 \text{ m}$$

where $L_{1,sec} = 40.33 \text{ m}$ and $L_{II,sec} = 31.72 \text{ m}$ are the maximum plan dimensions.

### 4 Calculation of Dynamic and Design Eccentricities

The calculation of the dynamic eccentricities $e_{stiff}$ and $e_{flex}$ (Eq. 1-2 for buildings with torsional sensitivity) along each horizontal “Capable Near Collapse Principal Directions” $L_{sec}$ and $H_{sec}$ as well as of the design eccentricities $e_1, e_2$ (Eq. 5-6) along the direction $L_{sec}$ and $e_3, e_4$ (Eq. 7-8) along the direction $H_{sec}$, which are used for the application of the proposed pushover analysis method on the asymmetric single-storey building (see Fig. 4), is performed step by step as follows:

- **Stiffness eccentricity (CR sec):** $e_{R_{1,sec}} = 6.02$ and $e_{R_{II,sec}} = 1.95 \text{ m}$
- **Storey Mass:** $m = 1103 \text{ tn}$
- **Mass moment of inertia:** $I_m = 222958 \text{ tn} \cdot \text{m}^2$
- **Radius of gyration:** $r_m = \sqrt{I_m/m} = \sqrt{222958/1103} = 14.22 \text{ m}$
- **Min torsional radius:** $r_{I,sec} = 13.32 \text{ m}$
- **Torsional Sensitivity:** $r_{I,sec}/r_m = 0.94 < 1.10 \rightarrow$ Torsional sensitive
- **Accidental Eccentricity (Eq. 15-16):** $e_{a_{I,sec}} = 2.02 \text{ m}$ and $e_{a_{II,sec}} = 1.59 \text{ m}$
- **Dynamic Eccentricities (Eq. 1-2):**
  - $e_{stiff,sec} = 0.046 \cdot e_{R_{I,sec}} - 0.11 \cdot r_m = 0.046 \cdot 6.02 - 0.11 \cdot 14.22 = -1.29 \text{ m}$
  - $e_{stiff,II,sec} = 0.046 \cdot e_{R_{II,sec}} - 0.11 \cdot r_m = 0.046 \cdot 1.95 - 0.11 \cdot 14.22 = -1.47 \text{ m}$
  - $e_{flex,sec} = 0.84 \cdot e_{R_{I,sec}} + 0.12 \cdot r_m = 0.84 \cdot 6.02 + 0.12 \cdot 14.22 = 6.76 \text{ m}$
  - $e_{flex,II,sec} = 0.84 \cdot e_{R_{II,sec}} + 0.12 \cdot r_m = 0.84 \cdot 1.95 + 0.12 \cdot 14.22 = 3.35 \text{ m}$
- **Design Eccentricities (Eq. 5-8):**
  - $e_1 = e_{flex,sec} + e_{a_{I,sec}} = 6.76 + 2.02 = 8.78 \text{ m}$ from $CR_{sec}$ to the flexible side of plan along $L_{sec}$
  - $e_2 = e_{stiff,sec} - e_{a_{III,sec}} = -1.29 - 2.02 = -3.31 \text{ m}$ from $CR_{sec}$ to the stiff side of plan along $L_{sec}$
  - $e_3 = e_{flex,II,sec} + e_{a_{II,sec}} = 3.35 + 1.59 = 4.93 \text{ m}$ from $CR_{sec}$ to the flexible side of plan along $H_{sec}$
\[ e_4 = e_{\text{stiff,IIsec}} - e_{a,IIsec} = -1.47 - 1.59 = -3.06 \text{ m} \]

from CR_{sec} to the stiff side of plan along II_{sec}

5 Seismic assessment

In this section, the proposed method of documented application of pushover analysis on the asymmetric single-story building is described in detail. All results by pushover analysis compare with the seismic demand ones (target displacement) by nonlinear response history analysis which is also referred.

5.1 Proposed method of pushover analysis

According to the proposed method of pushover analysis, the procedure to be performed is shown in Fig. 4. In this figure the proposed methodology is appropriately formulated and performed as described below:

1) The appropriate dynamic eccentricities along each horizontal “Capable Near Collapse Principal Directions” \( I_{sec} \) or \( II_{sec} \) are calculated by Eq. (1-2) which are then introduced into Eq. (5-8) to determine the design eccentricities \( e_1, e_2, e_3, e_4 \) used for the positioning of the lateral static floor force along the axes \( II_{sec} \) and \( I_{sec} \) respectively (as described in detail in section 4).

2) In total, for both horizontal directions \( I_{sec} \) and \( II_{sec} \), eight pushover analyses are obtained considering the two signs (+, -) of application of the lateral static floor loads,

3) The displacement results along the horizontal “Capable Near Collapse Principal Directions” \( I_{sec} \) or \( II_{sec} \) of the eight separate pushover analyses are combined with the SRSS rule, in that step of the analyses where the seismic target-displacement at the lateral load application point is achieved, and from the sixteen combinations the envelope is taken.

For comparison purposes, Fig. 5 also shows the process of applying the pushover analysis according to EN 1998-1, i.e. by applying the lateral static floor force on the position of the Mass Centre moved by the floor accidental eccentricity. It is worthy note that the locations in the plan of the proposed pushover method lateral static forces are in fully disagreement with Eurocode EN 1998-1.

Also, in Fig. 6, by the recently international literature using a fully different methodology, the pushover analysis according to the “corrective eccentricity method” by Bosco et al. (2017) [4] is illustrated, where the corrective eccentricities \( e_{IIsec} \) and \( e_{Isec} \) for the building stiff sides (for loading along the horizontal axes \( II_{sec} \) and \( I_{sec} \) respectively) have been calculated, plus the accidental eccentricities. Thus, our investigative results are very compatible with Bosco’s ones; however, the investigative results disappoint Eurocode EN 1998-1.
In Figures 4, 5 and 6 the sixteen SRSS combinations of the eight separate pushover analyses, from which the envelope of the displacement results along the horizontal axes $I_{sec}$ and $II_{sec}$ is calculated, are as follows:

1. $(1) \oplus (5), (1) \oplus (6), (1) \oplus (7), (1) \oplus (8)$
2. $(2) \oplus (5), (2) \oplus (6), (2) \oplus (7), (2) \oplus (8)$
3. $(3) \oplus (5), (3) \oplus (6), (3) \oplus (7), (3) \oplus (8)$
4. $(4) \oplus (5), (4) \oplus (6), (4) \oplus (7), (4) \oplus (8)$

Fig. 4: Proposed method of pushover analysis for the single-storey building.
Fig. 5: Pushover analysis according to EN 1998-1 for the single-storey building

Fig. 6: Pushover analysis according to the “corrective eccentricity method”, Bosco et al (2017)
The capacity curves resulted from the pushover analyses, with lateral static floor force along the horizontal axes $I_{sec}$ and $II_{sec}$ and on the positions defined by the accidental eccentricities (according to EN 1998-1, Fig. 5) and by the design eccentricities (according to the proposed pushover method, Fig. 4), are presented in Fig. 7 and 8, respectively.

Fig. 7: Capacity curves of pushover analysis according to EN 1998-1

Fig. 8: Capacity curves of pushover analysis according to the proposed method
5.2 Time History analysis

In the context of the current work, the "seismic target-displacement" is calculated by performing non-linear response history analysis (TH). According to EN 1998-1, the TH analysis is performed in a (non-linear) model resulting from the simultaneous movement of the Mass Centre CM by each accidental eccentricity along the horizontal “Capable Near Collapse Principal Directions” $I_{\text{sec}}$, $II_{\text{sec}}$ (from eq. 15-16) which constitute the appropriate principal directions. Of the four sign combinations of the two accidental eccentricities $e_{a,1}$ and $e_{a,2}$, four displaced CM positions are defined. Three pairs of horizontal accelerograms [1] consisting of five artificial accelerograms (created by Seismosoft [12]) are used that have similar characteristics with the Hellenic tectonic faults [13]. Each pair is rotated about the vertical axis successively per 22.5°, to find the worst seismic load state [14] of the 16 TH analyses obtained for each pair. Finally, the envelope of the displacements along the axes $I_{\text{sec}}$, $II_{\text{sec}}$ from all these TH analyses was the "seismic target-displacement" for each control position in plan.

6 Results of non-linear analysis methods

The seismic inelastic displacements of the building along the “Capable Near Collapse Principal Axes” $I_{\text{sec}}$ and $II_{\text{sec}}$ resulting from the non-linear analysis methods of sections 5.1 and 5.2, i.e. the non-linear response history analysis (TH), the proposed pushover analysis, the pushover analysis according to EN 1998-1 as well as the “corrective eccentricity method” of pushover analysis (Bosco et al [4]), are presented here for comparison purposes. The results are illustrated in Fig. 9 in terms of plan inelastic displacement profile.

We observe that, relative to the displacement results from TH analysis (seismic target-displacement), the displacement of the stiff side $u_{I_{\text{sec}}}$ along the horizontal axis $I_{\text{sec}}$ resulted from the pushover analysis according to EN 1998-1 is lower by 12%. Similarly, the displacements $u_{II_{\text{sec}}}$ and $u_{II_{\text{sec}}}$ of the flexible side along the horizontal axes $I_{\text{sec}}$ and $II_{\text{sec}}$ are a little lower by 4% and 2% respectively. On the contrary, the displacement of the stiff side $u_{I_{\text{sec}}}$ along the horizontal axis $II_{\text{sec}}$ resulted from all pushover method of analysis is predicted with safety. We also notice that the proposed method of pushover analysis provides the displacement of the stiff side $u_{I_{\text{sec}}}$ along the horizontal axis $I_{\text{sec}}$ with a safety margin of 11% and the displacements $u_{I_{\text{sec}}}$ and $u_{II_{\text{sec}}}$ of the flexible side along the horizontal axes $I_{\text{sec}}$ and $II_{\text{sec}}$ also with a safety margin 2% and 1% respectively.

It is worthy noted that, relative to the displacement results of the TH analysis, the “corrective eccentricity method” of pushover analysis gives non-conservative results by 10% for the displacement $u_{I_{\text{sec}}}$ of the stiff side along the $I_{\text{sec}}$ axis. Similar results, such as the EN1998-1 pushover method, apply to the displacements $u_{I_{\text{sec}}}$ and $u_{II_{\text{sec}}}$ of the flexible side along the axes $I_{\text{sec}}$ and $II_{\text{sec}}$, i.e. lower by 4% and
2%, respectively. This is since only the accidental eccentricity is used for loading in order to predict the flexible side displacements.

We conclude that, the proposed pushover methodology of the present paper is more accurate than the pushover method that uses the “corrective eccentricities”; however the later method, that is based on the corrective eccentricities, certainly drives in compatible results with our parametric analysis. On the contrary, the pushover analysis according to EN 1998-1 is exceptionable.

![Plan displacement profile](image)

**Fig. 9:** Plan displacement profile along the “Capable Near Collapse Principal Axes” $H_{sec}$ (left) and $I_{sec}$ (right) resulted from the non-linear methods of analysis.

### 7 Conclusions

In the current work, a proposed method of documented application of pushover analysis on asymmetric single-story buildings has been presented in detail. To clarify and evaluate the method, a single-storey r/c building has been assessed. The building is double-asymmetric and also is torsional sensitive. For the application of the method, the non-linear model of the building has been formed, in which the structural members have been provided with their secant stiffness $E I_{sec}$ (in their yield state). Then, the following have been calculated: (a) the plan floor position of the “Capable Near Collapse Centre of Stiffness” $C R_{sec}$, (b) the orientation of the horizontal “Capable Near Collapse Principal Axes” $I_{sec}$ and $H_{sec}$, (c) the “Capable Near Collapse Torsional Radii” $r_{1,sec}$ and $r_{II,sec}$ relative to the horizontal axes $I_{sec}$ and $H_{sec}$ as well as the radius of gyration $r_m$ of the diaphragm, and (d) the torsional sensitivity of the model according to the relationship $r_{1,sec}$ or $r_{II,sec} \leq 1.10 r_m$.

Finally, using the previous data, the dynamic eccentricities plus the accidental ones, namely the design eccentricities, have been calculated from eq. (1-4) and (5-8).
respectively. The process of applying the proposed method of pushover analysis, using the design eccentricities $e_1$, $e_2$ and $e_3$, along each horizontal direction $I_{sec}$ and $II_{sec}$ respectively, has been illustrated in detail in Fig. 4. In the context of this work, the seismic target-displacement of each control point of the diaphragm has been calculated by non-linear response history analysis (TH). The floor plan inelastic displacement profile along each horizontal direction $I_{sec}$ and $II_{sec}$ resulting from TH analysis has been compared with the corresponding ones from pushover analysis according to EN 1998-1, from the proposed method of pushover analysis and from the pushover that use the “corrective eccentricities” [4]. The main conclusions are the following:

1) The application of the pushover analysis method according to EN 1998-1 (Fig. 5) results in non-conservative displacement $u_{I_{sec}}$ (by 12%) of the building stiff side along the horizontal axis $I_{sec}$. Also, the displacements of the building flexible sides along both the horizontal directions $I_{sec}$ and $II_{sec}$ are a little lower. This due to wrong location in the plan of lateral static floor forces during the pushover procedure.

2) The application of the proposed method of pushover analysis (Fig. 4) predicts with safety the displacement $u_{I_{sec}}$ of the building stiff side along the horizontal axis $I_{sec}$ (by 11%) as well as the displacements $u_{I_{sec}}$ and $u_{II_{sec}}$ of the building flexible side along the horizontal axes $I_{sec}$ and $II_{sec}$ (by 1% and 2% respectively).

3) The “corrective eccentricity method” that have been proposed by Bosco et al (2012) (Fig. 6), in which the floor lateral loading is applying with less eccentricity than the design eccentricity of the proposed pushover method, gives non-conservative results (by 10%) for the displacement $u_{I_{sec}}$ of the stiff side along the $I_{sec}$ axis. Also, the displacements of the flexible side along both the horizontal axes $I_{sec}$ and $II_{sec}$ remain a little lower, as for EN1998-1, since “corrective eccentricity” is not used for the flexible side, except the accidental eccentricity. These approaches or differences are small in the present case study, but on the other buildings maybe non-small.

Therefore, the proposed method for the documented application of the pushover analysis using suitable design eccentricities is a rational way to predict with safety the (real) coupling between the torsional vibrations with the translational ones of an asymmetric single-storey building under pure translational seismic excitation of its base, especially as regards the displacements of the stiff sides of the building. The proposed pushover method of the present paper is more accurate than the pushover method that uses the “corrective eccentricities”; however, the later method that is based on the corrective eccentricities certainly drives in compatible results with our parametric analysis, general, but the present paper gives more accuracy. On the contrary, the locations in the plan of lateral static floor forces into the frame of pushover analysis according to EN 1998-1 are fully inadequate.
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