A new approach to the gender pay gap decomposition by economic activity

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Abstract

The aim of this paper is to present an original approach to estimate the gender pay gap. We propose a model-based decomposition, similar to the most popular approaches, where the first component measures differences in group characteristics and the second component measures the unexplained effect; the latter being the real gap. The novel approach incorporates model selection and bias correction. The pay gap problem in a small area context is considered in this paper, although the approach is flexible to be applied to other contexts.

Specifically, the methodology is validated for analysing wage differentials by economic activities in the region of Galicia (Spain) and by analysing simulated data from an experimental design that imitates the generation of real data. The good performance of the proposed estimators is shown in both cases, specifically when compared with those obtained from the widely used Oaxaca-Blinder approach.

1 Introduction

Gender wage inequality is a documented fact that happens in almost all of the industrialized countries (Juhn et al., 1993, Oaxaca and Ransom, 1994, 1998, Aláez and Ullibarri, 2001, Moral-Arce et al., 2011, Fortin et al., 2011) and interest in the quantification of this gap has increased in recent years.

In the case of the Spanish labor market, a number of actions were taken to provide women with real access to employment with full social and economic rights, with special emphasis on the reconciliation of family and working life. This and other reasons led a massive incorporation of women into the labor market, but it has not resulted in equality, that is, women’s unemployment rates are higher in most sectors, the jobs that women take do not involve the same degree of responsibility or decision-making power and women’s participation is limited to a few sectors of the economy. But more important is that, for a similar job, it has been shown that men have better wages than women (Fernández et al., 2000; Aláez and Ullibarri, 2001; Aláez et
The gender pay gap ($GPG$), calculated as the difference between men’s average hourly earnings and women’s average hourly earnings, usually expressed as a percentage of the men’s average hourly earnings, is widely used as the key indicator to study the progress in the European Union (EU), see Leythienne and Ronkowski (2018), among others. Moreover, most researchers in this field (Jann, 2008, Moral-Arce et al., 2011, Anastasiade and Tillé, 2017a, 2017b, Hlavac, 2018) work with the transformed variable to logarithms, $Y = \log(W)$, where $W$ is the hourly wages. The rationale behind the use of the logarithmic transformation is that the salaries have a skew distribution. However, the skewness is also exhibited on the log scale. Finding the optimal transformation is not straightforward. As an alternative, Graf et al. (2019) propose considering a much more flexible distribution called generalized beta of the second kind (GB2). In this work, we follow the ideas of most researchers. We are aware that it would be interesting to incorporate this proposal, but it goes beyond the objectives of this paper.

Typically the sample estimators are used directly as the GPG measure because the sample size are often very high. Instead, a more formal definition of the GPG is used here, as we deal with scenarios with small sample sizes. It is given, using expectations, as follows:

$$GPG = \frac{E(W_m) - E(W_w)}{E(W_m)},$$

where the subscripts $m$ are the men’s wage and $w$ the women’s wage.

The most widely used approach to analyze the gender gap is the decomposition due to Oaxaca (1973) and Blinder (1973); Specifically, Eurostat applies this approach to decompose the estimator of $GPG$ (Leythienne and Ronkowski, 2018). The Oaxaca-Blinder (OB) decomposition breaks down the difference $\Delta = E(Y_m) - E(Y_w)$ into two components: $\Delta = Q + U$, where the first component explains the difference between the observed productive characteristics, such as education or work experience, and the second accounts for differences in the structure of the model (Anastasiade and Tillé, 2017a, 2017b). The U component is known as the unexplained component and is usually considered to be the wage discrimination by gender in the labor market. By instances, Popli (2013) states that the unexplained wage gap includes the effect of labour market discrimination, unobservable variables and omitted variables. The methodology presented in this paper accounts for the bias from unobservable or omitted variables.

So, the $GPG$ decomposition is:

$$GPG_Q = \frac{GPG}{\Delta} \quad \text{and} \quad GPG_U = \frac{GPG}{\Delta},$$

which represent the explained and unexplained part of the $GPG$, respectively. The OB method, including the definitions of $Q$ and $U$, is briefly revised in Section 2.

Despite its widespread use, the standard Oaxaca-Blinder decomposition has important limitations; firstly, too simple parametric functions are often used for the regression model; secondly, the results may depend heavily on the set of explanatory variables selected; and thirdly, the unexplained variability is not always taken into account and generates bias.

This paper aims to adapt the methodology to estimate $GPG_Q$ and $GPG_U$ in small areas, which are disaggregated levels of the population, and specifically analyze the wage differentials by economic activities in the region of Galicia (Spain). Often in small area contexts, the sample
sizes are too small, which implies the standard estimators are unreliable. In particular, in our application for 25% of the economic activities, we have information on less than 26 male workers, being the sample sizes for women even smaller.

In fact, no official estimators of wages are provided at the economic activity level because direct estimates have very low accuracy. The latter is a typical SAE (Small Area Estimation) problem. The most extended SAE approach is to give a model-based estimator, where auxiliary information in terms of explanatory variables is incorporated. Besides, the model is often defined as a mixed effect model where a random effect represents the areas (see Rao and Molina (2015)).

The proposal here is to combine SAE methodology and OB methodology by deriving a decomposition inspired by [1], adapted to a small area scenario in such a way that for a given area \(d\), \(\text{GPG}_d\) is also decomposing into the \(\text{GPG}_{Qd}\) and \(\text{GPG}_{Ud}\). These components are efficiently estimated using an approach in three steps that use SAE techniques and overcome OB’s drawback, commented above. First, by doing model selection using an AIC criterion, and second, by including a bias correction term that accounts for omitted variable bias as in many applications, the implicit assumption that all confounding variables are included is not easy to attain. Confidence intervals for \(\text{GPG}_{Qd}\) and \(\text{GPG}_{Ud}\) are obtained for each area.

Furthermore, to validate the methodology proposal, a simulation study is conducted. It has a particular design that mimics the generation of data of the real application at hand. The estimators obtained with the novel approach are compared with those using the OB approach, showing that the formers outperform the latter.

We organize the remainder of the paper as follows. Section 2 revises the OB approach and SAE models, while Section 3 details the novel proposal. The estimation of the gender gap by economic activities in the region of Galicia (Spain) is presented in Section 4 and the simulation study in Section 5. Finally, the main conclusions and a brief discussion of future research is given in Section 6.

2 Background

We use the notation which is usually considered for the OB decomposition and SAE models. The beta parameter appears in both formulations, but with a different role in each one.

2.1 Pay gap decomposition

Let \(Y_g = \log(W_g)\) be the natural log of hourly earnings for \(g = m, w\); men and women, respectively. And let \(X_g ; (X_{g1}, \ldots, X_{gp})\) represent the set of available explanatory variables. Typically, these explanatory variables are the education level, the age of the worker or the work experience.

Assume that samples of sizes \(n_m, n_w\) are available for men and women, respectively, and the following models:

\[
Y_{gi} = X_{gi} \beta_g + \epsilon_{gi}, \quad i = 1, \ldots, n_g, \quad g \in (m, w);
\]

where \(\beta_g = (\beta_{g1}, \ldots, \beta_{gp})'\) is an unknown vector of parameters and \(\epsilon_{gi}\) is independent, with \(E(\epsilon_g) = 0\).

Then,

\[
\Delta = E(Y_m) - E(Y_w) = X_m \beta_m - X_w \beta_w,
\]
where, $\bar{X}_m$ and $\bar{X}_w$ are the corresponding sample means.

Now, let $\beta_r$ be the auxiliary coefficient; then, adding and subtracting $X_m \beta_r$ and $X_w \beta_r$, the above difference can be written as:

$$\Delta = (\bar{X}_m - \bar{X}_w)\beta_r + X_m(\beta_m - \beta_r) + X_w(\beta_r - \beta_w).$$

Frequently, the discrimination is directed toward one of the groups (women, in our case), which determines the value of the auxiliary coefficient, so that $\beta_r = \beta_m$ and then,

$$\Delta = (\bar{X}_m - \bar{X}_w)\beta_m + X_w(\beta_m - \beta_w).$$

where

$$Q = (\bar{X}_m - \bar{X}_w)\beta_m,$$

is the component that is explained by group differences in the predictors (the “quantify effect” or “the explained part”) and

$$U = X_w(\beta_m - \beta_w).$$

is the unexplained component, usually attributed to discrimination. Notice that $U$ also captures all the potential effects of differences in unobserved variables.

The estimation of the decomposition is straightforward. Let $\hat{\beta}_m$ and $\hat{\beta}_w$ be the least-squares estimates for $\beta_m$ and $\beta_w$ obtained separately from the two samples. Then,

$$\hat{\Delta} = \bar{Y}_m - \bar{Y}_w = \hat{Q} + \hat{U},$$

$$\hat{Q} = (\bar{X}_m - \bar{X}_w)\hat{\beta}_m,$$

$$\hat{U} = X_w'(\hat{\beta}_m - \beta_w).$$

More details and other decompositions can be seen in Jann (2008) and Hlavac (2018).

### 2.2 SAE models

The wage estimation by small areas is a typical SAE problem as it is assumed that the sample sizes are too low in many areas. The most extended approach in SAE problems is to give a model-based estimator that usually includes random effects due to the areas. To clarify the presentation we have only considered linear models, as these are widely used in SAE applications and are well suited to analyzing the problem at hand. Nevertheless, the proposal in the paper can be adapted to consider other additive non-linear models as those which include monotone and spline models such as those in Opsomer et al. (2008) and Rueda and Lombardía (2012).

Specifically, the so-called nested error regression model with sampling weights, which, for the $i$-th worker in area $d$, is defined as follows:

$$Y_{di} = X_{di}\beta_d + Z_{di}u_d + \epsilon_{di}, d = 1, \ldots, D, i = 1, \ldots, n_d;$$

where $Y_{di}$ is the (log) wage, $X_{di} = (X_{1di}, \ldots, X_{pdi})$ is a $p$ vector of explanatory variables, $\beta_d$ is the vector of unknown parameters; $Z_{di}$ are the random effects, with $Z_{di}$ being a $q$ dimensional vector also defined from the explanatory variables, and $u_d \in \mathcal{N}(0, \Sigma_u)$, where $\Sigma_u$ is a matrix of dimension $q$. $u_d$ is independent of the model error $\epsilon_{di}$, which are assumed to be independent
\(N(0, \sigma_e^2 w_i^{-1})\), where \(w_i\) is the sampling weight for individual \(i\) in area \(d\). In these models the covariance of the response variable in area \(d\) is \(V_d = Z_d \Sigma_u Z_d + \Sigma_e\) where \(\Sigma_e = \sigma_e^2 W_d^{-1}\) and \(W_d = \text{diag}(w_i)_{n_d \times n_d}\). Two particular interesting cases are considered by Battese et al. (1988) and Dempster et al. (1981).

In SAE applications, researchers are mainly interested in deriving estimators for the response variable in small areas; they usually work with the conditional means as the parameters of interest, which are defined as follows:

\[
\mu_d = E(Y_d | u_d) = \bar{X}_d \beta_d + \bar{Z}_d u_d, \quad d = 1, \ldots, D
\]

The estimation process starts with the variance components \((\Sigma_u, \Sigma_e)\), which are estimated by the residual maximum likelihood (REML) method; the empirical best linear unbiased estimators of \(\beta_d\) and the empirical best linear unbiased predictors of \(u_d\) are then obtained. Furthermore, estimators for \(\mu_d\) are derived as follows:

\[
\hat{\mu}_d = \bar{X}_d \hat{\beta}_d + \bar{Z}_d \hat{u}_d, \quad d = 1, \ldots, D
\]

For details on the estimation process and further learning about SAE methods, the reader is referred to the monographs of Jiang (2007) and Rao and Molina (2015).

3 The novel proposal

Assume that we have wage data for men and women and explanatory variables from samples from \(D\) small areas, and that we are interested in deriving estimates for the two components of the GPG.

A three-step approach is proposed to derive GPG estimators. The first step is a model selection step; in the second step, preliminary area-specific estimators for \(GPG_Q\) and \(GPG_U\) are obtained from a GPG decomposition adapted to small areas; and finally, in the third step, a Monte-carlo algorithm is consider to derive corrected bias \(GPG_Q\) and \(GPG_U\) estimators and confidence intervals.

First Step: Model Selection

There are different approaches to model selection, and the task is complicated when random effects are present. The discussions by Lombardia et al. (2017), where the GAIC is first presented, and by Fan and Li (2012), where iterative regularization methods are proposed, are very instructive. Besides proposing a method for making the model selection, a list of candidate models must be defined. From the methodological point of view, it can be as large as the practitioner decides. From the computational and interpretative points of view, a reduced list of candidate models are recommended.

The strategy adopted in this paper is to define a reduced list of candidate models and then to use the GAIC statistic to select a model from this list. In particular, for applications with a reduced number and well-known, explanatory variables, we recommend the use of prior knowledge to decide an initial selection of fixed and random effects. Typically, in small area applications, the variable determining the areas is modelled using random effects. Other candidate models may be defined by considering the interactions of that variable, with other explanatory variables.
The group (men/women) with the larger sample size is used to select the model. In GPG applications, it is usually that of men \((m)\). The models from which we make the choice differ in the explanatory variables selected and the type of effects (fixed or random) that describe the model and are formulated as the general model \((2)\), as follows:

\[
M : Y_{mdi} = X_{mdi} \beta_{md} + Z_{mdi} \mu_{md} + \epsilon_{mdi}, \quad d = 1, \ldots, D, i = 1, \ldots, n_d.
\]

The selection of the best model is done using an AIC statistic (Akaike, 1973); specifically, we use an adapted version for unit level models of the \(xGAIC\) proposed by Lombardía et al. (2017) with the following expression for the model \(M\)

\[
\text{\(xGAIC\)} = -2 \log(l_x(M)) + xGDF
\]

where \(l_x(M)\) is the quasi-loglikelihood of the model \(M\), which considers the focus on the random effect and the total variability and \(xGDF\) is a measure of the complexity of the model, as follows:

\[
\log(l_x(M)) = -\frac{1}{2} D \log(2\pi) - \frac{1}{2} \log |V_{Ym}| - \frac{1}{2} (Y_m - \mu_m)' V^{-1}_{Ym} (Y_m - \mu_m)
\]

and

\[
xGDF = \sum_{d=1}^{D} \sum_{i=1}^{n_d} \sum_{j=1}^{n_d} V^{-1}_{mdij} Cov(\hat{\mu}_{mdi}, Y_{mdj})
\]

where \(Y_m\) is the \(\sum_{d=1}^{D} n_d \times 1\) vector with elements \(Y_{mdi}\), \(\mu_m\) is the vector of the conditional means \(\mu_m = E(Y_m | u)\) with elements \(\mu_{mdi} = X_{mdi} \beta_{md} + Z_{mdi} \mu_{md}\), \(V_{Ym}\) is the conditional covariance matrix, which is diagonal with elements \(V_{md}\) according to Section 2.2 and \(V^{-1}_{mdij}\) is the \(ij\)-element of \(V_{Ym}\).

Parametric bootstrap is used to estimate \(xGDF\), as the analytic values are difficult to obtain in a similar way to Lombardía et al. (2017). The model with the lowest value of \(xGAIC\) is selected.

**Second Step: GPG decomposition. Preliminary estimators.**

Let \(X_m, Z_m\) and \(X_w, Z_w\) be the design matrices representing the explanatory variables and the random effects on the selected model for men and women, respectively. For a given area, \(d\), consider \(\Delta_d = \mu_{md} - \mu_{wd}\), which is decomposed as follows:

\[
\Delta_d = \mu_{md} - \mu_{wd} = \bar{X}_{md} \beta_{md} + \bar{Z}_{md} \mu_{md} - \bar{X}_{wd} \beta_{wd} - \bar{Z}_{wd} \mu_{wd}.
\]

\[
\Delta_d = Q_d + U_d
\]

\[
Q_d = (X_{md} - X_{wd}) \beta_{md} + (Z_{md} - Z_{wd}) \mu_{md},
\]

\[
U_d = X_{wd} (\beta_{md} - \beta_{wd}) + Z_{wd} (\mu_{md} - \mu_{wd}).
\]

and,

\[
GPGQ_d = GPG_d \frac{Q_d}{\Delta_d} \quad \text{and} \quad GPGU_d = GPG_d \frac{U_d}{\Delta_d}.
\]

where

\[
GPG_d = \frac{E(W_{md}) - E(W_{wd})}{E(W_{md})},
\]
Small area estimators for \( Y_{gd} \), \( \Delta_d \), \( U_d \) and \( Q_d \) (as in Rao and Molina, 2015) and estimators for \( E(W_{gd}), g \in (m, w) \) \( d = 1, ..., D \), are defined as follows:

\[
\hat{\Delta}_d = \bar{X}_{md}\hat{\beta}_{md} + \bar{Z}_{md}\hat{u}_{md} - \bar{X}_{wd}\hat{\beta}_{wd} - \bar{Z}_{wd}\hat{u}_{wd}, \\
\hat{Q}_d = (\bar{X}_{md} - \bar{X}_{wd})\hat{\beta}_{md} + (\bar{Z}_{md} - \bar{Z}_{wd})\hat{u}_{md}, \\
\hat{U}_d = \hat{\Delta}_d - \hat{Q}_d. 
\]  

(3)

\[
\hat{E}(W_{gd}) = \exp(\hat{Y}_{gd}), g \in (m, w), d = 1, ..., D; 
\]  

(4)

From (4), we have that:

\[
\hat{GPG}_d = \frac{\hat{E}(W_{md}) - \hat{E}(W_{wd})}{\hat{E}(W_{md})}, 
\]  

(5)

**Third Step: bias corrected estimators and confidence intervals.**

Omitted variable bias is taken into account as in many applications, the implicit assumption that all confounding variables are included is not easy to attain. The bias in \( Q_d \) is derived by splitting the men sample into two parts \( I \) times. The value of \( B_{id} \), for iteration \( i = 1, ..., I \), is expected to be zero when comparing the populations representing these two parts. A non-zero estimated value is assumed to be due to the omitted variables and is denoted by \( \hat{B}_{id} \), for iteration \( i \). From these values, the bias in \( Q_d \) is estimated, and bias-corrected estimators for \( GPG_{Qd} \) and \( GPG_{Ud} \) are derived, as explained below. Iterations with no men or women in any of the activities are discarded.

For each iteration \( i = 1, ..., I \) repeat:

1. Randomly split the data \( S \) in two parts of size \( \frac{I}{2} \): \( S_1^i \) and \( S_2^i \). The individuals of \( S_1^i \) are selected randomly, in such a way that those that have not been selected form part of \( S_2^i \). Therefore, the union of these two sets is the complete sample. In the following, subindex 1 or 2 indicates the data coming from \( S_1^i \) or \( S_2^i \), respectively.

2. The model selected is fitted to men and women’s data in \( S_1^i \), and \( \hat{Q}_{1id}, \hat{\Delta}_{1id}, \) and \( \hat{GPG}_{1id}^i \) are calculated according to (3) and (5).

3. For each area \( d \) and iteration \( i \), the bias is calculated considering men’s data from \( S_1^i \) and \( S_2^i \), as follows:

\[
\hat{B}_{id}^i = (\bar{X}_{1md} - \bar{X}_{2md})\hat{\beta}_{1md} + (\bar{Z}_{1md} - \bar{Z}_{2md})\hat{u}_{1md}. 
\]

Then, the final bias term is estimated as:

\[
\hat{B}_d = \frac{1}{I} \sum_{i=1}^{I} \hat{B}_{id}^i, 
\]

and the corrected bias estimates of \( Q_d \) and \( U_d \) are:

\[
\hat{Q}_d^B = \hat{Q}_d + \hat{B}_d, \\
\hat{U}_d^B = \hat{\Delta}_d - \hat{Q}_d^B, 
\]

where \( \hat{Q}_d \) and \( \hat{\Delta}_d \) are from the complete sample, as in (3).
Now, the corrected bias estimates of the explained and unexplained parts of $GPG$ are derived from the latter and from (5) as:

$$\hat{GPG}_{Qd} = \hat{GPG}_d \frac{\hat{Q}_d}{\hat{\Delta}_d} \quad \text{and} \quad \hat{GPG}_{Ud} = \hat{GPG}_d \frac{\hat{U}_d}{\hat{\Delta}_d}$$  \hspace{1cm} (6)

Moreover, confidence intervals are also derived as follows:

From $S_i$, $i = 1, \ldots, I$, and (4), we have that:

$$\hat{Q}_d^i = \hat{Q}_{1d}^i + \hat{B}_d, \quad \text{and} \quad \hat{U}_d^i = \hat{\Delta}_{1d}^i - \hat{Q}_d^i.$$

$$\hat{GPG}_{1d}^i = \frac{E(W_{1md}) - E(W_{1wd})}{E(W_{1md})},$$

$$\hat{GPG}_{Qd}^i = \hat{GPG}_{1d}^i \frac{\hat{Q}_d^i}{\Delta_{1d}^i},$$

$$\hat{GPG}_{Ud}^i = \hat{GPG}_{1d}^i \frac{\hat{U}_d^i}{\Delta_{1d}^i},$$

Now, let us define:

$$V(\hat{GPG}_{Qd}) = \frac{1}{I} \sum_{i=1}^I \left( \hat{GPG}_{Qd}^i - \hat{GPG}_{Qd}^B \right)^2,$$

$$V(\hat{GPG}_{Ud}) = \frac{1}{I} \sum_{i=1}^I \left( \hat{GPG}_{Ud}^i - \hat{GPG}_{Ud}^B \right)^2,$$

and,

$$\hat{GPG}_{Qd}^B = \frac{1}{I} \sum_{i=1}^I \hat{GPG}_{Qd}^i,$$

$$\hat{GPG}_{Ud}^B = \frac{1}{I} \sum_{i=1}^I \hat{GPG}_{Ud}^i,$$

while the confidence intervals are derived as follows:

$$\hat{GPG}_{Qd} \pm Z_{\alpha/2} \sqrt{V(\hat{GPG}_{Qd})} \quad \hat{GPG}_{Ud} \pm Z_{\alpha/2} \sqrt{V(\hat{GPG}_{Ud})}$$  \hspace{1cm} (7)

The aim of this document is to provide an easy-to-apply methodology for constructing confidence intervals to allow for inferences to be made. The post-selection inference is an interesting issue that could be considered in the proposed methodology following the ideas of Berk et al. (2013), Lee et al. (2016) and Tibshirani et al. (2016), among others; but it is outside the scope of this document.
4 Estimation of the GPG by economic activities in Galicia.

The data come from the Structure of Earnings Survey (SES) conducted by the Spanish National Institute of Statistics (INE) in 2014. The subset considering in this paper, corresponds to the Galicia region at the Northwest of Spain. The survey is conducted in a similar way to wage structure surveys in other European countries; it uses a two-level sampling of national companies, a stratified sampling in the first stage for local units and systematic sampling to select workers at those units. The economic activities are defined by the Statistical Classification of Economic Activities in the European Community (NACE09 Rev.2) at the division level.

Galicia has an economy strongly linked to natural resources, and its labor market has experienced growth in recent years. The GPG, it has been reduced from 15.5% in 2010 to 13.5% in 2016. This latter value is below that of the EU (16.3%), and below that of Spain (15.1%) in that year.

The survey collects information on non-self-employed workers who work in establishments with at least 10 employees and covers a wide range of private sectors (industry, construction, commerce, catering business, transport, financial intermediation, . . .) excluding the primary sector. The total sample size in Galicia being 10276. The list of explanatory variables considered in this study are included in Table 1, the selection being similar to that in Aláez et al., (2000, 2001, 2003), De la Rica et al. (2008) or Moral-Arce et al. (2011).

The economic activities, which are defined by the Statistical Classification of Economic Activities in the European Community (NACE09 Rev.2) at the division level, are the areas of interest. Tables 10 and 11 in the Appendix include the list of codes and the description of the NACE09 at the division level, excluding the primary sector. In the present study, we have information for \( D = 78 \) of the 84 NACE09’s. SAE methods are appealing because the sample sizes in some activities are very small. Specifically, the quartiles of the sample sizes distribution in the areas are \( Q_1 = 26 \), \( Q_2 = 50 \) and \( Q_3 = 104 \), for men, and \( Q_1 = 15 \), \( Q_2 = 26 \) and \( Q_3 = 50 \), for women.

Table 2 gives the main statistics for men and women regarding labor characteristics. The numbers in the table show a greater percentage of women in higher education levels, 41% against 33% of men, and that almost half the men (48%) are employed in the manufacturing industry or in public administration, while public administration (41%) and the retail trade (16%) are the activities with higher percentages for women. Regarding their type of work, women are more frequently on part-time work (27% vs 10% for men).

The distribution of the response variable \( Y \), the wage per hour in logarithm, is shown in Figure 1 for men and women; the shape of both curves being quite similar, though that of men is shifted to the right, which indicates an increase in salaries. Specifically, the quartiles are \( Q_1 = 7.6 \), \( Q_2 = 9.6 \) and \( Q_3 = 13.6 \) for men and \( Q_1 = 6.4 \), \( Q_2 = 8.1 \) and \( Q_3 = 12.1 \) for women. Finally, Figure 2 shows important differences in mean wages per hour across activities and between sexes. These differences range from 9 euros in the median of the activity retail trade and the maximum of 18 euros for those employed in finance and insurance activities.

The candidate list of models have been built as follows; First, the explanatory variables more often used in similar studies have been selected and have been described in Table 1. Next, the candidate models are defined combining fixed effect for Experience, Age, Education, Occupation, Contract, Type of work, Size of the enterprise, and Market with the effects by the Economic Activities. The latter can be incorporated in the model using fixed effects for aggregated levels (\( X_{19} \) to \( X_{27} \)), or fixed effects for each activity (\( X_{28} \) to \( X_{104} \), or a random
Table 1: Variables in the survey, in brackets the category used as a reference. For the variable Economic activity for two digits, the reference category is the activity 7, because in Galicia there are not people working in the activity 6 (see Appendix).

| Name                        | Variable | Description                                                                 |
|-----------------------------|----------|-----------------------------------------------------------------------------|
| Wage per hour               | Y        | Gross hourly earnings from employment                                       |
| Gender                      | X        | Men, Women                                                                  |
| Experience                  | X₁       | Number of years in the actual enterprise                                   |
| Age                         | X₂       | Years of the employee                                                       |
| Education                   | X₃       | (Primary)                                                                   |
|                             | X₄       | Secondary                                                                   |
|                             | X₅       | Higher                                                                      |
| Occupation                  | X₆       | (Professionals and managers)                                                |
|                             | X₇       | Tecnic                                                                      |
|                             | X₈       | Operators                                                                   |
|                             | X₉       | Services and sales workers                                                  |
|                             | X₁₀      | Non skilled workers                                                         |
| Contract                    | X₁₁      | (Long term), Short term                                                     |
| Type of work                | X₁₂      | (Full time), Part time                                                      |
| Size of the enterprise      | X₁₃      | (Between 10 to 19 employees)                                                |
|                             | X₁₄      | Between 20 to 49 employees                                                  |
|                             | X₁₅      | Between 50 to 99 employees                                                  |
|                             | X₁₆      | Between 100 to 199 employees                                                |
|                             | X₁₇      | Between 200 to 499 employees                                                |
|                             | X₁₈      | More than 500 workers                                                       |
| Market                      | X₁₉      | (Local or regional)                                                         |
|                             | X₂₀      | National                                                                    |
|                             | X₂₁      | International                                                               |
| Aggregated economic activity| X₂₂      | (Energy)                                                                    |
|                             | X₂₃      | Manufacturing industry                                                      |
|                             | X₂₄      | Construction                                                                |
|                             | X₂₅      | Retail trade                                                                |
|                             | X₂₆      | Transportation, storage and Accommodation                                   |
|                             | X₂₇      | Information and Communication                                               |
|                             | X₂₈      | Finance and insurance                                                       |
|                             | X₂₉      | Professionals                                                               |
|                             | X₃₀      | Public Administration                                                       |
|                             | X₃₁      | Other service                                                               |
| Economic activity for two  | X₃₂      | NACE9 Rev. 2, division level, two digits                                     |
| digits                      | X₃₃      | (7)                                                                         |

Effect. Moreover, when a random effect is used to model the activities, the interactions with Experience, Education, and Occupation may also be considered.

Regarding the notation, $u_d$ is the random effect for the Economic Activities, $u_{dj}, j = 1, 2$ for the interaction with Education, $u_{dk}, k = 1, 2, 3, 4$ for the interaction with Occupation; and $v_d$ for the interaction with Experience. Only a selection of the most interesting models is presented in Table 3 to simplify the exposition. Specifically, eight models labeled as $M1$ to $M8$ are presented,
Table 2: Summary of labor force characteristics of the people in the survey by gender.

|                  | Men  | Women |
|------------------|------|-------|
| Experience (mean years) | 11.16 | 10.43 |
| Education (%)     |      |       |
| Primary           | 0.19 | 0.13  |
| Secondary         | 0.48 | 0.46  |
| Higher            | 0.33 | 0.41  |
| Contract (%)      |      |       |
| Long-term         | 0.77 | 0.77  |
| Short-term        | 0.23 | 0.23  |
| Type of work (%)  |      |       |
| Full time         | 0.90 | 0.73  |
| Part time         | 0.10 | 0.27  |
| Economic activity (%) |      |       |
| Energy            | 0.03 | 0.01  |
| Manufacturing industry | 0.27 | 0.13 |
| Construction      | 0.09 | 0.01  |
| Retail trade      | 0.13 | 0.16  |
| Transportation and storage and Accommodation | 0.08 | 0.07 |
| Information and Communication | 0.03 | 0.02 |
| Finance and insurance | 0.03 | 0.03 |
| Professionals     | 0.10 | 0.14  |
| Public Administration | 0.21 | 0.41 |
| Other services    | 0.02 | 0.02  |

Figure 1: Density of ln(wage/hour) by sex in Galicia.

models $M6$ and $M7$ are defined with fixed effects, and the rest with a combination of fixed and random effects. Models defined by other combinations of effects have been considered but have
Figure 2: Wage per hour by sex and economic activity in Galicia. The codes for the activities are: 2 Energy, 3 Manufacturing industry, 4 Construction, 5 Retail trade, 6 Transportation and storage and Accommodation, 7 Information and Communication, 8 Finance and insurance, 9 Professionals, 10 Public Administration and 11 Other services.

been discarded because the results are far from being better than those obtained with other models.

| Label | Fixed effects | Random Effects | Fixed parameters (n0) | Random parameters (n0) |
|-------|---------------|----------------|-----------------------|------------------------|
| M1    | X1-X18        | ud             | 18                    | 1                      |
| M2    | X1-X18        | vd             | 18                    | 1                      |
| M3    | X1-X18        | ud, vd         | 18                    | 2                      |
| M4    | X1-X18        | udj            | 18                    | 2                      |
| M5    | X1-X18        | vdk            | 18                    | 4                      |
| M6    | X1-X27        | -              | 27                    | 0                      |
| M7    | X1-X18, X28-X104 | -   | 95                    | 0                      |
| M8    | X1-X18, X28-X104 | vd  | 95                    | 1                      |

Table 3: Models used to fit the data.

In order to explore differences between models, the estimated bias for each activity has been calculated for each candidate model and is shown in Figure 3. The activities are sorted in increasing order of bias according to the M1 model. Figure 3 shows important bias in some activities: 88 = Social work activities without accommodation, 90 = Creative, arts and entertainment activities or 39 = Remediation activities and other waste management services, among others. Moreover, the bias corrected estimator of the explained and unexplained parts of the GPG for each economic activity and model, as shown in the expression (6), are shown in Figures 4 and 5, respectively, sorting the activities by the values of $\hat{GPG}_{Qd}$ and $\hat{GPG}_{Ud}$ for
These Figures show that while estimates for $\hat{GPG}_Q$ are similar between models, there are important differences in $\hat{GPG}_U$ across models.

The model selected, using the proposal described in Section 3, is M1. The parameter estimates for M1 are shown in Table 4, which explains that the wage of a worker increase with higher education, working in an enterprise with a national, UE or international market, and belonging to a large enterprise. The coefficients related to occupations show negative signs as Professionals and managers is the reference category. The results are consistent with those from other studies (Aláez et al. (2003), De la Rica (2008) and Moral-Arce et al. (2011)).

Pay gap estimators have been obtained using the model M1 and the proposal in Section 3. In Galicia, the pay gap between men and women ($\hat{GPG}$) is estimated to be of 11.3%, where 2.8% is due to the observed quantify effect ($\hat{GPG}_Q$) and 8.5% to discrimination ($\hat{GPG}_U$), as Table 5 shows.

Moreover, Figures 6 and 7 show $\hat{GPG}_Q$ and $\hat{GPG}_U$, respectively, for each activity, $d$, with their confidence intervals. Activities are ordered by the increasing value of the estimator.

It is worth commenting on the results from activities 84 and 85, which correspond to the Public Administration and Education. They are useful for validating the procedure because no discrimination is expected in these cases as the Administration control the wages (Aláez et al., 2000, Moral-Arce et al., 2011). Figure 6 shows no significant differences between the characteristics of men and women, for activities 84 and 85; while Figure 7 shows, for 84 and 85, no significative differences from those of Galicia. The results are consistent with what would be expected.

On the other hand, the results show very high discrimination in eight activities: 30, 28,
Figure 4: \( \hat{GPG}_Qd \) for all the models considered and for each activity \( d \) defined by the NACE09.

Figure 5: \( \hat{GPG}_Ud \) for all the models considered and for each activity \( d \) defined by the NACE09.

19, 17, 42, 33, 29 and 10, almost all of them are from the industrial sector (see Figure 7); while differences between the characteristics due to gender are not significantly higher for these
|          | Estimate | Std. Error | p-value |
|----------|----------|------------|---------|
| Intercept| 2.299    | 0.033      | 0.000   |
| Experience \(X_1\) | 0.007    | 0.001      | 0.000   |
| Age \(X_2\) | 0.005    | 0.000      | 0.000   |
| Secondary \(X_3\) | 0.015    | 0.011      | 0.188   |
| Higher \(X_4\) | 0.122    | 0.014      | 0.000   |
| Technic \(X_5\) | -0.248   | 0.015      | 0.000   |
| Operators \(X_6\) | -0.439   | 0.016      | 0.000   |
| Services and sales workers \(X_7\) | -0.504   | 0.019      | 0.000   |
| Non-skilled workers \(X_8\) | -0.527   | 0.016      | 0.000   |
| Short term contract \(X_9\) | -0.098   | 0.010      | 0.000   |
| Part time work \(X_{10}\) | -0.017   | 0.013      | 0.210   |
| Between 20 to 49 workers \(X_{11}\) | -0.029   | 0.012      | 0.028   |
| Between 50 to 99 workers \(X_{12}\) | 0.047    | 0.014      | 0.003   |
| Between 100 to 199 workers \(X_{13}\) | 0.042    | 0.014      | 0.011   |
| Between 200 to 499 workers \(X_{14}\) | 0.081    | 0.014      | 0.000   |
| More than 500 workers \(X_{15}\) | 0.113    | 0.016      | 0.000   |
| National Market \(X_{17}\) | 0.052    | 0.010      | 0.000   |
| UE Market \(X_{18}\) | 0.076    | 0.018      | 0.001   |
| International Market \(X_{19}\) | 0.072    | 0.016      | 0.000   |

\[ \sigma_u^2 = 0.021 \]

Table 4: Coefficient estimates for men and for M3.

|          | Estimation | CI 95%       |
|----------|------------|--------------|
| \( \widehat{GPG}_Q \) | 0.028      | (-0.018, 0.07) |
| \( \widehat{GPG}_U \) | 0.085      | (0.057, 0.14) |
| \( \widehat{GPG} \) | 0.113      | -            |

Table 5: Decomposition of gender pay gap in Galicia.

activities than for Galicia (Figure 6).

Taking as reference the \( \widehat{GPG} \) of the region of Galicia, Table 9 shows the activities with higher \( \widehat{GPG}_d \) than Galicia, sorted from highest to lowest. The activity at the top is *Electricity, gas, steam and air conditioning supply* and it is also one of the activities where there are more discrimination. The second activity at the top is *Manufacture of wearing apparel*, and this is an example that high wage differences are not necessarily accompanied by discrimination. This activity is very important in the Galician business sector and employs approximately 8,600 workers, 1% of the total number of employed people in Galicia. On the other hand, note that only ten activities out of 74 have positive discrimination significantly higher than the region’s mean. These activities employ 8.2% of the workers in the region. It is also interesting to note that there are two activities in which discrimination is significantly negative (in favor of women). These are *Warehousing and support activities for transportation* and *Security and investigation activities* that belong to the service sector where women have a more significant presence than men.
Figure 6: $\hat{GPG}_{Qd}$ with their confidence interval (95%) and for each activity $d$ defined by the NACE09. The horizontal continuous line is the $GPG_Q$ for Galicia.

Figure 7: Discrimination ($\hat{GPG}_{Ud}$) with their confidence interval (95%) and for each activity $d$ defined by the NACE09. The horizontal continuous line is the $GPG_U$ for Galicia.
5 Simulations

The aim of this section is to validate the novel proposal and compare it with the classical approach of OB. Moreover, the estimation error that results when important explanatory variables are omitted, is also illustrated.

The simulated experiment imitates SES real data. The 30 activities which have the largest sample sizes and the explanatory variables in Table 1 are considered. The sample sizes take values from 21 to 111 for men and 7 to 82 for women. The model used to generate the data is defined as follows:

\[ Y_{gdi} = X_{1gdi} \beta_{1gdi} + X_{2gdi} \beta_{2gdi} + X_{3gdi} \beta_{3gdi} + X_{4gdi} \beta_{4gdi} + X_{5gdi} \beta_{5gdi} + X_{6gdi} \beta_{6gdi} + X_{7gdi} \beta_{7gdi} + \epsilon_{gdi} \]

where \( \epsilon_{gdi} \sim N(0, \sigma_{gdi}^2) \), with \( \sigma_{gdi}^2 = 0.1 \) and \( \beta_{jgdi}, j = 1, \ldots, 7 \) are the estimated values using model M6.

Note that model (8) is much more complex than any of the models in the candidate list. The generating model includes a fixed parameter for each activity and explicative variable, giving a model with \( 8 \times 30 = 240 \) parameters; being the candidate models much more simple.

For each \( d \), \( GPG_d, GPG_Qd \) and \( GPG_Ud \) are obtained using (1), (2) and (5) and are assumed to be true values.

The models in Table 6 combine fixed and random effects, as in the real case. Estimators for GPG components and each model are obtained using proposal in Section 3.

| Label | Fixed effects | Random Effects | Fixed parameters (n⁰) | Random parameters (n⁰) |
|-------|---------------|----------------|-----------------------|-----------------------|
| MS1   | \( X_1, X_3-X_8 \) | \( u_d \)      | 7                     | 1                     |
| MS2   | \( X_1, X_3-X_8 \) | \( v_d \)      | 7                     | 1                     |
| MS3   | \( X_1, X_3-X_8 \) | \( u_d, v_d \) | 7                     | 2                     |
| MS4   | \( X_1, X_3-X_8 \) | \( u_{dj} \)   | 7                     | 2                     |
| MS5   | \( X_1, X_3-X_8 \) | \( u_{dk} \)   | 7                     | 4                     |
| MS6   | \( X_1, X_3-X_8, X_{19}-X_{27} \) | -              | 16                    | 0                     |
| MS7   | \( X_1, X_3-X_8, X_{28}-X_{56} \) | -              | 36                    | 0                     |
| MS8   | \( X_1, X_3-X_8, X_{28}-X_{56} \) | \( v_d \)      | 36                    | 1                     |

Table 6: Models used to fit the data.

Now, the empirical mean square error (EMSE) of \( \hat{GPG}_{Qd} \) and \( \hat{GPG}_{Ud} \) for all the models and for each activity \( d \) are as follows:

\[
EMSE(\hat{GPG}_{Qd}) = \frac{1}{100} \sum_{i=1}^{100} \left( \hat{GPG}^{(i)}_{Qd} - GPG_{Qd} \right)^2
\]

\[
EMSE(\hat{GPG}_{Ud}) = \frac{1}{100} \sum_{i=1}^{100} \left( \hat{GPG}^{(i)}_{Ud} - GPG_{Ud} \right)^2
\]

where \( \hat{GPG}^{(i)}_{Qd} \) and \( \hat{GPG}^{(i)}_{Ud} \) are the estimations for each iteration \( i = 1, \ldots, 100 \).
Finally, we calculate the confidence intervals for $GPG_{Qd}$ and $GPG_{Ud}$ from (7) and give the percentage of times that the true values $GPG_{Qd}$ and $GPG_{Ud}$ are in the corresponding confidence interval.

Tables 7 and 8 present the results of the simulations, EMSE values and coverage rates of confidence intervals, respectively. Table 7 shows the mean of the EMSE of $\hat{GPG}_{Qd}$ and $\hat{GPG}_{Ud}$ obtained from the models $MS_1$ to $MS_8$; the OB model and the model selected by xGAIC (XG) are also included. The lefthand part of the table corresponds to the model with all the explanatory variables and the righthand side shows the results when the information relative to education ($X_3$ and $X_4$) is removed.

The best models, in terms of EMSE, are $MS_5$ and XG. When relevant information is missed out the error increases in all the models, but the increase is higher for OB and the MS6 model. The conclusions are similar regarding the coverage rates of 95% confidence intervals in Table 8. From these results, it can be said that the proposed methodology is robust to the potential effects of differences in unobserved variables; to be precise, much more than the OB approach.

In addition, in Tables 7 and 8 you can see that the models selected by the xGAIC have a good behavior, because they are among those that have a lower EMSE and a greater coverage. When the model is fitted with all the variables, the xGAIC selects the MS5 model 82% of times, 15% of times $MS_4$ and 3% $MS_8$. When $X_3$ and $X_4$ are simultaneously removed from the working model, the xGAIC selects the model $MS_5$ 98% of times and 2% the model $MS_8$. Finally, this study highlights the gain of our methodology with respect to the OB methodology, especially for estimating $GPG_{Ud}$.

|                | Without $X_3, X_4$ |                | Without $X_3, X_4$ |
|----------------|--------------------|----------------|--------------------|
| $GPG_Q$        | $GPG_U$            | $GPG_Q$        | $GPG_U$            |
| OB             | 0.0049             | 0.0067         | 0.0060             | 0.0089             |
| MS1            | 0.0046             | 0.0045         | 0.0054             | 0.0057             |
| MS2            | 0.0057             | 0.0066         | 0.0067             | 0.0076             |
| MS3            | 0.0060             | 0.0048         | 0.0066             | 0.0058             |
| MS4            | 0.0045             | 0.0044         | 0.0054             | 0.0057             |
| MS5            | 0.0044             | 0.0044         | 0.0052             | 0.0054             |
| MS6            | 0.0049             | 0.0067         | 0.0061             | 0.0079             |
| MS7            | 0.0046             | 0.0048         | 0.0053             | 0.0060             |
| MS8            | 0.0060             | 0.0049         | 0.0065             | 0.0060             |
| XG             | 0.0045             | 0.0043         | 0.0052             | 0.0054             |

Table 7: EMSE of $GPG_Q$ and $GPG_U$ for the model with and without $X_3$ and $X_4$. OB is the Oaxaca-Blinder model and XG is the model selected by the xGAIC.
Table 8: Coverage of the $GPG_Q$ and $GPG_U$ confidence intervals for the model with and without $X_3$ and $X_4$. OX is the Oaxaca-Blinder model and XG is the model selected by the $xGAIC$

6 Conclusions

The first contribution of this paper, and the most important from the theoretical point of view, is the derivation of a novel approach to estimate the components of the gender pay gap in small areas. This methodology is consistent with that used by Eurostat (Leythienne and Ronkowski, 2018), it also generates confidence intervals for the explained and unexplained part of the GPG, includes a bias correction and considers linear mixed models and a selection of models option. Moreover, this novel approach is robust against potential differences in unobserved variables.

The second, and most important contribution from the practical point of view, is that the application of the methodology to estimate wage discrimination in economic activities in Galicia reveals important differences in the region and among activities.

In Galicia, 25% of the pay gap is explained by the differences between characteristics and 75% is due to discrimination. In other words, if there were no discrimination and the characteristics of men and women had been paid at the same prices, the wage differential would be reduced to 2.8%.

Regarding the activities, a high pay gap due to differences in the characteristics is shown in such activities as 66 and 14, while a high pay gap due to discrimination is shown in other activities, such as 28, 19, 17, among others.

The existence of salary differences by sex continues to be a real problem, according to the results obtained. The $GPG$ indicator, together with the explained and unexplained gap by economic activities, allow for a better identification and interpretation of the causes of the gender pay gap. As a consequence, policy actions can be better targeted.

Finally, to determine whether two workers have the same qualification and value, useful information, such as the total work experience, knowledge of languages or the awards or publications obtained, among many others, is still missing. The inclusion of this information as an explanatory variable would likely derived in more accurate pay gap estimators.

7 Appendix

Tables 10 and 11 show the codes and the description of the Statistical Classification of Economic Activities in the European Community (NACE09 Rev.2) at the division level, excluding the primary sector.
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| NACE09 | $\hat{GPG}_d$ | $\hat{GPG}_{Qd}$ | $\hat{GPG}_{Qd}$ | $\hat{GPG}_{Qd}$ | $\hat{GPG}_{C}$ | $\hat{GPG}_{Ud}$ | $\hat{GPG}_{Ud}$ | $\hat{GPG}_{Ud}$ |
|--------|----------------|-----------------|-----------------|-----------------|----------------|----------------|----------------|----------------|
| 35     | 0.40           | 0.08            | -0.02           | 0.18            | 0.32           | 0.17           | 0.17           | 0.47           |
| 14     | 0.35           | 0.21            | 0.12            | 0.30            | 0.17           | -0.03          | 0.37           |                |
| 96     | 0.32           | 0.14            | 0.01            | 0.26            | 0.19           | 0.05           | 0.33           |                |
| 19     | 0.28           | -0.00           | -0.32           | 0.31            | 0.28           | 0.12           | 0.45           |                |
| 17     | 0.27           | 0.01            | -0.11           | 0.13            | 0.27           | 0.16           | 0.39           |                |
| 61     | 0.27           | 0.13            | 0.05            | 0.21            | 0.15           | 0.05           | 0.24           |                |
| 28     | 0.25           | -0.05           | -0.23           | 0.14            | 0.29           | 0.13           | 0.44           |                |
| 65     | 0.24           | 0.12            | -0.02           | 0.26            | 0.11           | -0.05          | 0.28           |                |
| 94     | 0.22           | 0.03            | -0.12           | 0.17            | 0.21           | -0.07          | 0.48           |                |
| 50     | 0.21           | 0.08            | -0.20           | 0.36            | 0.14           | -0.05          | 0.32           |                |
| 29     | 0.21           | 0.00            | -0.08           | 0.08            | 0.21           | 0.14           | 0.28           |                |
| 45     | 0.20           | 0.12            | -0.01           | 0.24            | 0.08           | -0.07          | 0.23           |                |
| 10     | 0.20           | -0.01           | -0.07           | 0.05            | 0.21           | 0.11           | 0.31           |                |
| 13     | 0.19           | 0.05            | -0.04           | 0.14            | 0.14           | 0.00           | 0.28           |                |
| 18     | 0.19           | 0.05            | -0.03           | 0.12            | 0.15           | 0.05           | 0.24           |                |
| 69     | 0.19           | 0.13            | -0.00           | 0.26            | 0.07           | -0.23          | 0.36           |                |
| 86     | 0.19           | 0.04            | -0.02           | 0.10            | 0.15           | 0.04           | 0.26           |                |
| 27     | 0.19           | -0.02           | -0.20           | 0.16            | 0.21           | 0.02           | 0.40           |                |
| 95     | 0.19           | 0.07            | -0.07           | 0.22            | 0.11           | -0.02          | 0.25           |                |
| 16     | 0.18           | -0.01           | -0.12           | 0.10            | 0.19           | 0.09           | 0.29           |                |
| 81     | 0.18           | 0.05            | -0.00           | 0.11            | 0.12           | 0.04           | 0.21           |                |
| 21     | 0.17           | 0.06            | -0.02           | 0.14            | 0.09           | -0.02          | 0.20           |                |
| 74     | 0.16           | 0.09            | -0.03           | 0.20            | 0.07           | -0.16          | 0.29           |                |
| 30     | 0.16           | -0.15           | -0.34           | 0.04            | 0.32           | 0.13           | 0.51           |                |
| 88     | 0.15           | 0.10            | -0.26           | 0.47            | 0.07           | -0.62          | 0.75           |                |
| 15     | 0.14           | 0.03            | -0.09           | 0.16            | 0.11           | -0.07          | 0.29           |                |
| 73     | 0.14           | 0.07            | -0.02           | 0.17            | 0.06           | -0.11          | 0.22           |                |
| 37     | 0.14           | 0.01            | -0.17           | 0.20            | 0.11           | -0.08          | 0.30           |                |
| 58     | 0.14           | 0.08            | -0.05           | 0.21            | 0.06           | -0.10          | 0.22           |                |
| 68     | 0.13           | 0.01            | -0.18           | 0.19            | 0.14           | -0.15          | 0.43           |                |
| 38     | 0.13           | 0.04            | -0.03           | 0.11            | 0.09           | -0.03          | 0.22           |                |
| 75     | 0.12           | 0.04            | -0.19           | 0.27            | 0.07           | -0.30          | 0.44           |                |
| 79     | 0.12           | -0.02           | -0.24           | 0.21            | 0.16           | -0.20          | 0.52           |                |

Table 9: Observed gap, $\hat{GPG}_{Qd}$ and $\hat{GPG}_{Ud}$ with the confidence interval for the 33 activities with the highest observed gaps.
| NACE09 | Description |
|--------|-------------|
| 6      | Extraction of crude petroleum and natural gas |
| 7      | Mining of metal ores |
| 8      | Other mining and quarrying |
| 9      | Mining support service activities |
| 10     | Manufacture of food products |
| 11     | Manufacture of beverages |
| 12     | Manufacture of tobacco products |
| 13     | Manufacture of textiles |
| 14     | Manufacture of wearing apparel |
| 15     | Manufacture of leather and related products |
| 16     | Manufacture of wood and of products of wood and cork, except furniture |
| 17     | Manufacture of paper and paper products |
| 18     | Printing and reproduction of recorded media |
| 19     | Manufacture of coke and refined petroleum products |
| 20     | Manufacture of chemicals and chemical products |
| 21     | Manufacture of basic pharmaceutical products and pharmaceutical preparations |
| 22     | Manufacture of rubber and plastic products |
| 23     | Manufacture of other non-metallic mineral products |
| 24     | Manufacture of basic metals |
| 25     | Manufacture of fabricated metal products, except machinery and equipment |
| 26     | Manufacture of computer, electronic and optical products |
| 27     | Manufacture of electrical equipment |
| 28     | Manufacture of machinery and equipment n.e.c. |
| 29     | Manufacture of motor vehicles, trailers and semi-trailers |
| 30     | Manufacture of other transport equipment |
| 31     | Manufacture of furniture |
| 32     | Other manufacturing |
| 33     | Repair and installation of machinery and equipment |
| 35     | Electricity, gas, steam and air conditioning supply |
| 36     | Water collection, treatment and supply |
| 37     | Sewerage |
| 38     | Waste collection, treatment and disposal activities; materials recovery |
| 39     | Remediation activities and other waste management services |
| 41     | Construction of buildings |
| 42     | Civil engineering |
| 43     | Specialised construction activities |
| 45     | Wholesale and retail trade and repair of motor vehicles and motorcycles |
| 46     | Wholesale trade, except of motor vehicles and motorcycles |
| 47     | Retail trade, except of motor vehicles and motorcycles |
| 49     | Land transport and transport via pipelines |
| 50     | Water transport |
| 51     | Air transport |
| 52     | Warehousing and support activities for transportation |
| 53     | Postal and courier activities |
| 55     | Accommodation |
| 56     | Food and beverage service activities |

Table 10: List of codes for the NACE09 at division level, excluding the primary sector.
| NACE09 | Description |
|--------|-------------|
| 58     | Publishing activities |
| 59     | Motion picture, video and television programme production, sound recording and music |
| 60     | Programming and broadcasting activities |
| 61     | Telecommunications |
| 62     | Computer programming, consultancy and related activities |
| 63     | Information service activities |
| 64     | Financial service activities, except insurance and pension funding |
| 65     | Insurance, reinsurance and pension funding, except compulsory social security |
| 66     | Activities auxiliary to financial services and insurance activities |
| 68     | Real estate activities |
| 69     | Legal and accounting activities |
| 70     | Activities of head offices; management consultancy activities |
| 71     | Architectural and engineering activities; technical testing and analysis |
| 72     | Scientific research and development |
| 73     | Advertising and market research |
| 74     | Other professional, scientific and technical activities |
| 75     | Veterinary activities |
| 77     | Rental and leasing activities |
| 78     | Employment activities |
| 79     | Travel agency, tour operator and other reservation service and related activities |
| 80     | Security and investigation activities |
| 81     | Services to buildings and landscape activities |
| 82     | Office administrative, office support and other business support activities |
| 84     | Public administration and defence; compulsory social security |
| 85     | Education |
| 86     | Human health activities |
| 87     | Residential care activities |
| 88     | Social work activities without accommodation |
| 90     | Creative, arts and entertainment activities |
| 91     | Libraries, archives, museums and other cultural activities |
| 92     | Gambling and betting activities |
| 93     | Sports activities and amusement and recreation activities |
| 94     | Activities of membership organisations |
| 95     | Repair of computers and personal and household goods |
| 96     | Other personal service activities |
| 97     | Activities of households as employers of domestic personnel |
| 98     | Undifferentiated goods- and services-producing activities of private households for own use |
| 99     | Activities of extraterritorial organisations and bodies |

Table 11: List of codes for the NACE09 at division level, excluding the primary sector (continuation).