Possible Approach to Dynamical Supersymmetry Breaking via Nambu–Jona-Lasinio Model

Yifan Cheng$^{1,\ast}$

$^1$Institute of Physics, Academia Sinica, Taipei 115, Taiwan

Abstract. Supersymmetry is undoubtedly a popular candidate for physics beyond the Standard Model. However, the origin of soft supersymmetry breaking masses has been usually depicted intricately in the literature via extra hidden/mediating sectors. Thus, a simple theory for the generation of the soft masses would be more compelling. Here we discuss a possible approach to dynamical supersymmetry breaking via Nambu–Jona-Lasinio (NJL) Model, which has been missed since the first investigation of supersymmetric NJL model. We introduce a four-superfield interaction term that induces a real two-superfield composite with vacuum condensate. The latter has supersymmetry breaking parts, which we show to bear nontrivial solutions following a standard nonperturbative analysis for a NJL type model. The presence of the expected Goldstino state along with the supersymmetry breaking, is also verified. Moreover, the model gives rise to a composite spin one field, as its important characteristic.

1 Introduction

With the discovery of the Higgs boson, all the elementary particles which the Standard Model (SM) predicted, have been discovered. However, the Higgs sector, as well as the origin of the electroweak symmetry breaking in the SM, are not fully appreciated. As indicated in the literature, a dimension-six four-quark interaction can give interesting nonperturbative dynamics via strong interaction, and break symmetries [1]. That is the classic Nambu–Jona-Lasinio model [2], to which Higgs physics may correspond to the low energy effective theory with the Higgs doublet being identified as a two-fermion composite. Unfortunately, the beautiful idea of the top-mode SM [3–8] fails to accommodate the top quark mass phenomenologically [1].

On the other hand, though we have not seen any clear indication of supersymmetry or new physics, so long as phenomenology at the TeV scale is concerned, the supersymmetric standard model (SSM) is still a popular candidate theory beyond the SM being matched to the LHC results. The first supersymmetric version of Nambu–Jona-Lasinio model was introduced in the early eighties [9, 10], which generalizes the four-fermion interaction to a four-superfield interaction of the same dimension in the Kähler potential. However, it requires input soft supersymmetry breaking masses to have the dynamical (electroweak) symmetry breaking. Meanwhile, the origin of soft supersymmetry breaking masses in the phenomenological SSM is typically depicted through elaborated constructions of complicated and contrived models with extra supersymmetry breaking and mediating sectors [11].

$\ast$e-mail: yifancheng@gate.sinica.edu.tw
Following the line of thinking, it will be great to find a simple model which breaks supersymmetry and generates the soft mass dynamically under a NJL framework. This possibility has been missed in the literature, and has the potential importance for investigation. Therefore, we present here a new type of supersymmetric NJL model, which gives rise to a composite spin-one component field, as well as plausible signatures of dynamical supersymmetry breaking.

In Sec. II, we present the model and the supergraph derivation of the superfield gap equation, elaborating the extension of our framework of analysis [12, 13]. In Sec. III, we discuss the effective theory picture with the composite and the matching effective potential analysis performed at the component field level. Sec. IV is devoted to analysis of the nontrivial supersymmetry breaking solutions. In Sec. V, we focus on the existence of the Goldstino mode along with the supersymmetry breaking. Some remarks and conclusions will be presented in the last section.

2 The Model in Superfields

We start with the Lagrangian of single chiral superfield

$$\mathcal{L} = \int d^4\theta \left[ \Phi^\dagger \Phi + \frac{m_o}{2} \Phi \Phi^2 (\bar{\theta}) + \frac{m^*}{2} \Phi^+ \Phi^2 (\theta) - \frac{g^2_o}{2} \left( \Phi^\dagger \Phi \right)^2 \right],$$

in which we have suppressed multiplet (color) indices. Naively, if the bi-superfield condensate $\langle \Phi^\dagger \Phi | D \rangle$ develops, we would have a soft supersymmetry breaking mass $g^2_o \langle \Phi^\dagger \Phi | D \rangle$.

We used a standard NJL gap equation analysis [9, 12, 13] applied to the soft supersymmetry breaking mass parameters. As a first step of the self-consistent Hartree approximation, we add to and subtract from the Lagrangian a superfield parameter

$$Y = y - \tilde{\eta} \theta^2 - \tilde{\eta}^* \bar{\theta}^2 - \tilde{m}^2 \theta^2 \bar{\theta}^2,$$

which contains not only the supersymmetry breaking parts but also the supersymmetric part. The Lagrangian can then be split into two parts as

$$\mathcal{L} = \mathcal{L}_o + \mathcal{L}_{\text{int}}$$

where

$$\mathcal{L}_o = \int d^4\theta \left[ \Phi^\dagger (1 + Y) \Phi + \frac{m_o}{2} \Phi^2 \delta^2 (\bar{\theta}) + \frac{m^*}{2} \Phi^+ \delta^2 (\theta) \right].$$

and

$$\mathcal{L}_{\text{int}} = \int d^4\theta \left[ -Y \Phi^\dagger \Phi - \frac{g^2_o}{2} \Phi^\dagger \Phi \Phi^\dagger \Phi \right].$$

The component $y$ contributes a (supersymmetry) wave function renormalization factor. Therefore, we introduce the renormalized superfield $\Phi_R \equiv \sqrt{Z} \Phi = \sqrt{1 + y} \Phi$ which gives the quantum effective action as

$$\Gamma = \Phi_R^\dagger \Phi_R (1 - \tilde{\eta} \theta^2 - \tilde{\eta}^* \bar{\theta}^2 - \tilde{m}^2 \theta^2 \bar{\theta}^2) + \frac{m_o}{2} \Phi^2 \delta^2 (\bar{\theta}) + \frac{m^*}{2} \Phi^+ \delta^2 (\theta)$$

$$-Y_R \Phi_R^\dagger \Phi_R - \frac{g^2_o}{2} \Phi_R^\dagger \Phi_R \Phi_R^\dagger \Phi_R + \Sigma_{\Phi_R \Phi_R} + \cdots,$$

in renormalized superfield, mass parameters and coupling constants. The superfield gap equation under the NJL framework is then given by

$$-Y_R + \sum_{\Phi_R \Phi_R}^{(\text{loop})}(p; \theta^2 \bar{\theta}^2) \bigg|_{\text{on-shell}} = 0;$$
in component form, we have

\[
\frac{y}{1+y} = \frac{\Sigma_{\text{loop}}(p)}{\text{on-shell}} = -g^2 \int_\epsilon \frac{(k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)}{(k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)^2 - 4|m|^2|\tilde{\eta}|^2},
\]

\[
\tilde{\eta} = \Sigma_{\tilde{\eta}}(p)_{\text{on-shell}} = g^2 \tilde{\eta} \int_\epsilon \frac{(k^2 - |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)}{(k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)^2 - 4|m|^2|\tilde{\eta}|^2},
\]

\[
\tilde{m}^2 = \Sigma_{\tilde{m}^2}(p)_{\text{on-shell}} = g^2 \int_\epsilon \frac{1}{(k^2 + |m|^2) (k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)^2 - 4|m|^2|\tilde{\eta}|^2}
\cdot \left\{[\tilde{m}^2(k^2 - |m|^2) + 2k^2|\tilde{\eta}|^2] (k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2) - 8k^2|m|^2|\tilde{\eta}|^2\right\}.
\]

where in accordance with the standard NJL analysis one uses the one-loop contribution to \(\Sigma_{\Phi\Phi}(p; \theta^2 \bar{\theta}^2)\) from the four-superfield interaction. The diagrammatic illustration of the renormalized superfield gap equation is given in Fig. 1.

Nontrivial solutions to the three coupled equations with nonvanishing \(\tilde{\eta}\) and/or \(\tilde{m}^2\) give supersymmetry breaking solutions. However, nontrivial \(y\) value only gives wave function renormalization to \(\Phi\) which does not change the qualitative answer to if supersymmetry breaking solution with the soft mass generation exists.

### 3 The Effective Theory Picture

Following the general effective theory picture of the NJL-type models, we modify the model Lagrangian by adding to it

\[
\mathcal{L}_s = \int d^4 \theta \frac{1}{2} (\mu U + g_o \Phi \Phi)^2,
\]

where \(U\) is an ‘auxiliary’ real superfield and mass parameter \(\mu\) taken as real and positive (for \(g_o^2 > 0\)). The equation of motion for \(U\), from the full Lagrangian \(\mathcal{L} + \mathcal{L}_s\) gives

\[
U = \frac{g_o}{\mu} \Phi \Phi,
\]

showing it as a superfield composite of \(\Phi \Phi\) and \(\Phi\). Moreover, the condition indicates that the model with \(\mathcal{L} + \mathcal{L}_s\) is equivalent to that of \(\mathcal{L}\) alone. Expanding the term in \(\mathcal{L}_s\), we have a cancellation of the dimension six interaction in the full Lagrangian, giving it as

\[
\mathcal{L}_{\text{eff}} \equiv \mathcal{L} + \mathcal{L}_s = \int d^4 \theta \left[ \Phi \Phi + \frac{\mu^2}{2} U^2 + \mu g_o U \Phi \Phi + \frac{m_o}{2} \Phi^2 \delta(\bar{\theta}) + \frac{m_s}{2} \Phi^2 \delta(\theta) \right].
\]

Obviously, if \(U|_D\) develops a vacuum expectation value (VEV), the \(\mu g_o U \Phi \Phi\) term gives a soft supersymmetry breaking mass of \(m_o^2 = -\mu g_o \langle\ U|_D\rangle\), and supersymmetry is spontaneously broken. All of the above looks very much like the standard features of NJL-type model.

The real superfield \(U\) can be expanded into the following components,

\[
U(x, \theta, \bar{\theta}) = \frac{C(x)}{\mu} + \sqrt{2} \theta \frac{X(x)}{\mu} + \sqrt{2} \bar{\theta} \bar{\chi}(x) + \theta \theta \frac{N(x)}{\mu} + \bar{\theta} \bar{\theta} \bar{N}(x) + \sqrt{2} \theta \theta \bar{\theta} \bar{\bar{\theta}} D(x).
\]
The $\mu$ factor is put to set the mass dimensions right. Though $U$ does contain a vector component $v_\mu$, please note that it differs from the usually studied ‘vector superfield’ which is a gauge field supermultiplet. The effective Lagrangian in component form is then given by

$$L_{\text{eff}} = (1 + g_o C) \left[ A^* \Box A + i (\partial_\mu \bar{\psi}) \bar{\sigma}^\mu \psi + F^* F \right] + \frac{m_0}{2} (2 A F - \psi \psi) + \frac{m_\psi^2}{2} (2 A^* F^* - \bar{\psi} \psi)$$

+ $\mu C D - \mu \chi \lambda - \mu \bar{\chi} \bar{\lambda} + NN^* - \frac{\mu^2}{2} v^\nu v_\nu - \mu g_o \psi \lambda A^* - \mu g_o \bar{\psi} \bar{\lambda} A + \mu g_o D A^* A$

- $i g_o \bar{\psi} \bar{\sigma}^\mu \chi \partial_\mu A + i \frac{g_o}{2} (\partial_\mu \bar{\psi}) \bar{\sigma}^\mu \chi A - g_o \bar{\chi} \bar{\psi} F^* + g_o N A F^*$

+ $i g_o \bar{\psi} \bar{\sigma}^\mu \partial_\mu A^* - i \frac{g_o}{2} \bar{\sigma}^\mu \chi \partial_\mu \psi - g_o \bar{\chi} \bar{\psi} F + g_o N^* A^* F$

- $\frac{\mu g_o}{\sqrt{2}} \eta^{\mu \nu} v_\mu A^* \partial_\nu A + \frac{\mu g_o}{\sqrt{2}} \eta^{\mu \nu} v_\mu i(\partial_\nu A^*) A - \frac{\mu g_o}{\sqrt{2}} \eta^{\mu \nu} v_\mu \partial_\nu \bar{\psi} \bar{\chi} \psi$ . \hspace{1cm} (11)

In accordance with the ‘quark-loop’ approximation in the (standard) NJL gap equation analysis and our particular supergraph calculation scheme above in particular, we consider plausible nontrivial vacuum solution with nonzero vacuum expectation values (VEVs) for the composite scalars $C$, $D$ and $N$.

Scalar $C$ couples to kinetic terms of components of $\Phi$; $c \equiv \langle C \rangle$ hence contributes to a supersymmetric wave function renormalization of the latter. It is the supersymmetric part of $\Sigma_{\phi \phi^\dagger}^{(lopp)} (p; \theta^2, \bar{\theta}^2)$ in our gap equation calculation in the previous section, with $\Sigma_{\phi \phi^\dagger} = g c$. Furthermore, with $n \equiv \langle N \rangle$ and $d \equiv \langle D \rangle$, we can identify $-g n$ and $-\mu g d$ corresponding to the supersymmetry breaking masses $\bar{\eta}$ and $\bar{m}^2$ of $\Phi$, as appeared in Eq. 6.

One can easily obtain the minimum condition for the effective potential following the Weinberg tadpole method [14, 15]. It is interesting to see that the effective potential analysis for (the components of) the composite superfield $U$ gives results equivalent to the superfield gap equations. Please also note that our analysis here is in direct matching with the corresponding discussion for the NJL case presented in Ref. [10], though for a superfield theory instead.

4 Supersymmetry Breaking Solutions

Since $y$ parameter does not correspond to any physical quantity, it may be considered of little interest. Our focus is if solutions of nontrivial supersymmetry breaking masses $\bar{\eta}$ and $\bar{m}$ exist.

With all variables and parameters expressed in terms of dimensionless counterparts normalized to cut-off scale $\Lambda$ given by $G = \frac{\eta^2 \Lambda^2}{16 \pi^2}$, $s = \frac{\bar{m}^2}{\Lambda^2}$ and $t = \frac{m_\psi^2}{\Lambda^2}$, the last two gap equations corresponding to the generation of soft masses in Eq. 6 are equivalent to

$$\frac{1}{G(s, t, z)} = \frac{s + 2 t (1 - z)}{s + 2 t z (1 - z)^2} + \frac{2 t (1 - z)}{s + 2 t z (1 - z)^2} t \ln \left[ 1 + \frac{1}{t} \right]$$

$$- \frac{s + 2 t (1 - z^2)}{s + 2 t z (1 - z)^2} \left[ s + t (1 + z^2) \right] \ln \left[ 1 + \frac{1}{s + t (1 + z)^2} \right] , \hspace{1cm} (12)$$

for $z = \frac{|\bar{\eta}|}{\bar{m}}$ respectively.

The two equations for positive and negative (but equal) values of $z$ of course collapse to one at $z = 0$, indicating a vanishing $|\bar{\eta}|$. We illustrate the solutions for $\bar{m}^2$ in the case of vanishing $|\bar{\eta}|$ in Fig. 2. For more details, we see that solutions for nontrivial $\bar{m}^2$ for the case is given by the reduced form of Eq.(12) as $\frac{1}{G} = 1 - s \ln \left[ 1 + \frac{1}{s} \right]$, obviously giving solution for $0 < s < 1$ for the strong enough
coupling $G > 1$. It can be seen from the numerical plot that the value of the $\tilde{m}^2$ solution rises fast with increasing $G$. However, nonzero $t$ has a strong limiting effect. It increases the critical coupling needed for a nontrivial solution to $s$ substantially.

As to looking for solution with nontrivial $\tilde{\eta}$, it is more tricky and requires a careful analysis scanning the numerical results. Again, we check plots of the effective coupling $G$ as given in Eq.(12) versus $s$ simultaneously for positive and negative values of $y$ of fixed magnitude, at a fixed input $t$ value, in Fig. 3. Numerically, where the two curves (dubbed $G_+$ and $G_-$, respectively) intersect within the window of interest gives a solution. We find that solution exists in general, though some of the features of the solution locations are not somewhat peculiar and not easy to understand.

5 The Massless Goldstino Corresponding to Supersymmetry Breaking

In accordance with the supersymmetry breaking, there should be a massless Goldstino state, the existence of which can be verified via diagonalization of the fermionic mass matrix.

For the fermionic two-spinors $\chi$ and $\lambda$, the chirality conserving part of their self-energy diagrams gives rise to kinetic terms. However, to find the mass eigenvalues is complicated. One needs first to take a unitary transformation on the hermitian matrix and kinetic terms to diagonalize it. Denote the eigenvalues by $N_{f_l}$ and $N_{f_r}$, and the diagonalizing matrix by $T$. The canonically normalized fermionic modes are given by

$$
\begin{pmatrix}
  f_r \\
  f_l
\end{pmatrix} = \begin{pmatrix}
  \frac{1}{\sqrt{N_{f_l}}} & 0 \\
  0 & \frac{1}{\sqrt{N_{f_r}}}
\end{pmatrix} T \begin{pmatrix}
  \chi \\
  \lambda
\end{pmatrix}.
$$

(13)

Only the mass matrix for the canonically modes can be diagonalized to give the mass eigenvalues. The mass matrix $M_f$ for $f_r$ and $f_l$ is hence given by

$$
M_f = \begin{pmatrix}
  \sqrt{N_{f_l}} & 0 \\
  0 & \sqrt{N_{f_r}}
\end{pmatrix} T \begin{pmatrix}
  M_{k,l} \\
  T^\tau \begin{pmatrix}
  \sqrt{N_{f_l}} & 0 \\
  0 & \sqrt{N_{f_r}}
\end{pmatrix}
\end{pmatrix},
$$

(14)

where $M_{k,l} = \begin{pmatrix}
  0 & \mu \\
  \mu & 0
\end{pmatrix} + \Omega$, the first part being the tree-level mass while the last is the matrix for chirality-flipping pieces of self-energy diagrams. We have

$$
det M_f = N_{f_l} N_{f_r} det M_{k,l}.
$$

(15)

In the case that the matrix of kinetic terms has the full rank, a zero determinant of $det M_f$ or equivalently $det M_{k,l}$ shows the existence of a massless Goldstino state, which is to be expected from the supersymmetry breaking. For the chirality-flipping diagrams (see Fig. 4), dropping the $p$-dependent parts, we have the mass terms

$$
\Omega_{s,s} = -\frac{g^2 \tilde{m}^4}{\tilde{\eta}} |m| I_{sr}(|m|^2, m_{d,s}^2, m_{d,s}^2) + \frac{1}{2\tilde{\eta}} (g^2 I_c + \tilde{m}^2 g^2 I_{sr}) ,
$$

$$
\Omega_{s,t} = 2\mu g^2 \tilde{m}^2 |m|^2 I_{sr}(|m|^2, m_{d,s}^2, m_{d,s}^2) - \mu g^2 I_{sr} ,
$$

$$
\Omega_{s,u} = -\mu g^2 \tilde{m}^2 |m|^2 I_{sr}(|m|^2, m_{d,s}^2, m_{d,s}^2) ,
$$

(16)

where $I_{sr}(|m|^2, m_{d,s}^2, m_{d,s}^2)$ is the integral of the product of three Feynman propagators with the mass-squares as specified. $I_{sr}$ and $I_c$ are integrals of the gap equations [cf. the second and third integrals in Eqs.(6)]. It is straightforward to calculate the determinant of the mass matrix $M_{k,l}$. By applications of the gap equations, one can easily show that the determinant is exactly zero. Hence, we have verified
the existence of a Goldstino mode for the supersymmetry breaking solution with $\tilde{\eta} \neq 0$. For the $\tilde{\eta} = 0$ case, only the off-diagonal term is nonzero, which is a result one can see even simply from the $U(1)_R$ symmetry considerations in the Lagrangian. Supersymmetry is really a local/spacetime symmetry. The Goldstino would eaten up by the gravitino which would then be massive.

6 Conclusion

We have shown how the NJL mechanism can be used to break supersymmetry dynamically. Our model is very simple with only one chiral superfield. As the bi-superfield condensate $\Phi^\dagger \Phi$ develops VEV beyond a critical coupling, soft supersymmetry breaking mass can be generated.

The spin one vector boson $v_\mu$ is another important characteristic of the model. We have constructed a supersymmetric version of the NJL model which gives rise to a composite spin one component field. To our best knowledge, it is the first one of this kind.

There are still a lot of work to do to truly realize the theoretical features and properties of the supersymmetry breaking solutions. We do not have a definite answer to the relative stability issue of the different vacuum solutions, partly due to the theoretical constraint of nonperturbative analysis. However, NJL analog indicates preference of the (super)symmetry breaking solution at beyond couplings, not to say even metastable supersymmetry breaking solutions should be of great interest, especially as it comes from such a simple model and gives directly only soft supersymmetry breaking masses to the basic chiral superfield. Possibility of the dynamical supersymmetry breaking approach should not be overlooked, and needs more investigation, so that we can fully justify the value of it.

$$\Phi_R^\dagger - Y_R \Phi_R + \Phi_R^\dagger \Phi_R = 0$$

Figure 1. The renormalized superfield gap equation, with $Y_R = \frac{y}{1+y} - \tilde{\eta} \bar{\theta}^2 - \bar{\tilde{\eta}} \theta^2 - \tilde{m}^2 \theta^2 \bar{\theta}^2$.

Figure 2. Numerical plot of nontrivial solutions to the soft mass gap equation with $|\tilde{\eta}| = 0$. Coupling parameter $G = \frac{N g^2 \Lambda^2}{16 \pi^2}$ is plotted against the normalized soft mass parameter $s = \frac{\tilde{m}^2}{\Lambda^2}$ for $t = \frac{\tilde{m}^2}{\Lambda^2}$ values of 0 (red), 0.1 (blue), 0.2 (pink), 0.4 (orange), 0.5 (green), from the lowest to the highest curves, respectively. Here $N$ is the ‘color’ factor for the case of the basic chiral superfield $\Phi$ being an $SO(N)$ or $SU(N)$ multiplet not shown explicitly in the calculation, and $\Lambda$ is the model cutoff scale. Notice that the critical coupling increases from $G = 1$ for nonzero values of the input supersymmetric mass $m$. 
To our best knowledge, it is the first one of this kind. A supersymmetric version of the NJL model which gives rise to a composite spin one component field.

Figure 1. \( \tilde{\eta} \)

We have shown how the NJL mechanism can be used to break supersymmetry dynamically. Our conclusion is very simple with only one chiral superfield. As the bi-superfield condensate \( \Phi \) we have

\[ \text{Figure 3. An illustrative of intersecting point solutions, with } G \text{ versus } s. \]

\[ \text{Figure 4. Diagrams for fermion masses.} \]

References

[1] For a review, see G. Cvetic, and references therein, Rev. Mod. Phys. 71, 513 (1999).
[2] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); \textit{ibid.} 124, 246 (1961).
[3] Y. Nambu, Enrico Fermi Institute Report No. 89-08 (1989), \textit{Bootstrap Symmetry Breaking in Electroweak Unification}.  
[4] V.A. Miransky, M. Tanabashi and K. Yamawaki, Phys. Lett. B 221, 177 (1989).
[5] V.A. Miransky, M. Tanabashi and K. Yamawaki, Mod. Phys. Lett. A 4, 1043 (1989).
[6] W.J. Marciano, Phys. Rev. Lett. 62, 2793 (1989).
[7] W. J. Marciano, Phys. Rev. D 41, 219 (1990).
[8] W.A. Bardeen, C.T. Hill and M. Lindner, Phys. Rev. D 41, 1647 (1990).
[9] W. Buchmuller and S. T. Love, Nucl. Phys. B 204, 213 (1982).
[10] W. Buchmuller and U. Ellwanger, Nucl. Phys. B 245, 237 (1984).
[11] See for example D.J.H. Chung, L.L. Everett, G.L. Kane, S.F. King, J.D. Lykken and L.T. Wang, Phys. Rept. 407, 1 (2005).
[12] G. Faisel, D.W. Jung and O.C.W. Kong, JHEP 1201, 164 (2012).
[13] Y.-M. Dai, G. Faisel, D.W. Jung and O.C.W. Kong, Phys. Rev. D 87, 085033 (2013).
[14] S. Weinberg, Phys. Rev. D 7, 2887 (1973).
[15] R.D.C. Miller, Nucl. Phys. B 228 (1983) 316.