Orbital Dynamics Of A Second Planet In HD 17156

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ABSTRACT

In this letter we report the possible existence of a second planet in the transiting extrasolar planet system HD 17156 and its interactive dynamics with the previously known planet. The analysis is achieved through the POFP optimization software which is based on a full integration of the system’s multiple-body Newtonian equations of motion. The two-planet solution yields a significantly improved fit to the previously published radial velocities. The two planets are strongly interacting and exchange angular momentum in a 5:1 mean motion resonance, yet remain stable as they mutually excite orbital eccentricities and periastron advances.

Subject headings: planetary systems – celestial mechanics – gravitation – instabilities – methods: N-body simulations – methods: numerical

1. Introduction

By virtue of its unusual characteristics, HD 17156b is one of the most valuable extrasolar planets for understanding planet formation and orbital dynamics. Discovered via the Doppler technique by the N2K consortium (Fischer et al. 2007), the planet was found to transit its host star by the TransitSearch.org collaboration (Barbieri et al. 2007). Additional transit observations and refined system parameters are given by Gillon et al. (2008), Narita et al. (2008), and Irwin et al. (2008). The planet radius and mass are approximately 1 R_{Jup}
and 3 $M_{\text{Jup}}$, giving a density of $\sim 3.5-4.0 \text{ g cm}^{-3}$, nearly three times that of Jupiter. More interesting however are the planet’s orbital characteristics: (i) The 21.2 d orbital period is roughly a factor 7 times longer than most other transiting planets; (ii) While most hot Jupiter planets have low eccentricities (the majority consistent with zero), HD 17156b has a very high eccentricity of $e=0.67$. Although the high eccentricity coupled with the small semimajor axis (0.16 AU) does not necessarily require the presence of a third body perturbing the planet’s orbit (Gillon et al. 2008), it suggests that such a body may be present. In this work we examine the published radial velocities of Fischer et al. (2007) and the transit times as given in Irwin et al. (2008) using a Newtonian (not Keplerian) 3-body integrator and conclude that a second planet is not only possible, but probable. Notably, while the two planets exhibit strong dynamical interaction leading to large eccentricities and periastron advances, they remain stable via an elegant exchange of orbital angular momentum.

2. Initial Analysis of Radial Velocities

Our initial investigation of the HD 17156 system began with a 1-planet fit to the Keck and Subaru radial velocity data from Fischer et al. (2007). We omitted the radial velocity data of Narita et al. (2008) since these data are influenced by the Rossiter-McLaughlin effect, but their inclusion has essentially no effect on the results. We reproduced the parameters of the planet HD 17156b as reported in Fischer et al. (2007), though we found that an additional 0.14 ms$^{-1}$ offset added to the Subaru data set gave a slightly better fit. We computed a power spectrum of the residuals of the radial velocities after removing the one-planet fit. A Monte Carlo technique was used to allow us to assess the significance of any peaks. The power spectrum indicated a possible peak with a period of approximately 115 days. This prompted us to undertake a much more thorough and realistic 3-body investigation described below. In addition to the power spectra, we computed Keplerian periodograms: for a given period the parameters of a Keplerian fit to the radial velocities were optimized ($K$, $e$, $\omega$, $T_0$, $\gamma$), i.e. the Lehmann–Filhés equation (see Hilditch 2001 for example). While a traditional Fourier power spectrum has no free parameters, it uses sines and cosines as basis functions. However, an eccentric orbit is far from being sinusoidal and as a consequence a Fourier power spectrum will require power at many periods to match the radial velocities. In contrast, by using the Lehmann–Filhés equation as the basis function we obtain an optimal periodogram. Furthermore, unlike a power spectrum, fitting a Keplerian orbit at each trial period allows one to use the error bars on each observation as weights. Thus no Monte Carlo or bootstrap sampling is necessary to assess the quality of the fit: the periodogram provides a $\chi^2$ versus

\footnote{see The Extrasolar Planets Encyclopedia by Jean Schneider at \url{http://exoplanet.eu}}
period directly. Such periodograms clearly showed the 21.2 d signal from planet b, and the residuals again indicated the presence of a possible second planet with a period of \(\sim 106-116\) d.

3. HD 17156c

To investigate characteristics of a possible second planet we used the Planetary Orbit Fitting Process (POFP), an optimization software written and operated in MATLAB — see Windmiller, Short & Orosz (2007) and Short, Windmiller & Orosz (2008). The POFP provides multi-body solutions based on a full integration of the Newtonian equations of motion.

In addition to the radial velocity data, we used POFP to fit the 4 transit times as listed in Irwin et al. (2008). The inclusion of the transit times was crucial because it not only greatly improved the period of the known planet b, but the precise timings strongly constrain the orbital configuration and interaction between the planets. Since the system transits, we assumed the second planet’s orbit is edge-on and co-planar with the first. A careful reading of Fischer et al. (2007) led us to use caution with the Subaru radial velocities, thus we considered two data set combinations: The first consisted of the 24 high-quality radial velocities from Keck plus the 4 observed transit times, and the second consisted of the 33 combined Keck and Subaru radial velocities plus the 4 transit times. Following Fischer et al. (2007), we set the stellar mass to \(1.2\, M_\odot\) and the stellar jitter to \(3\, \text{ms}^{-1}\). Using POFP we fit the two data sets with a single planet model giving two solutions denoted here by POFP1 K+TT (Keck plus transit times) and POFP1 K+S+TT (Keck, Subaru, plus transit times). This provided a baseline set of models to compare against when considering a two-planet solution. The orbital parameters are given in Table 1, along with the Fischer et al. (2007) and Irwin et al. (2008) solutions.

We then fit using the Keck and Subaru radial velocities plus transit times with a two-planet model, giving the solution POFP2. Because the POFP2 solution is fully Newtonian and the planets interact, the tabulated parameters are valid only at a specific time, which is taken to be the time of the first Keck radial velocity measurement. The fitness for the 1-planet and 2-planet solutions is given in Table 2. Note that in this table only the following are optimized: the 1-planet model using Keck radial velocities, the 1-planet model using Keck+Subaru velocities, and the 2-planet model using Keck+Subaru velocities; in all cases the transit times were used. These optimized values are shown in bold-face; the other entries result when these models are evaluated and matched against the listed data set and are given for comparison. Some reduced \(\chi^2\) values are less than 1.0, suggesting that the \(3\, \text{ms}^{-1}\) jitter estimate of Fischer et al. (2007) is slightly too large. The key point of Table 2 is the following: including two planets significantly improves the fit in all cases (Keck only,
Keck+TT, Keck+Subaru, Keck+Subaru+TT).

In our 2-planet solution we found that the inner planet maintains an almost constant semi-major axis of 0.1596 AU, a slowly oscillating eccentricity having a period of 141.1 yr with \( e \) between 0.665 and 0.670, and a full rotation of its line of apsides every 33,050 yr. Thus, there are two long term cycles of approximately 141.1 yr and 33,050 yr. Meanwhile, the outer planet HD 17156c exhibits an oscillation in eccentricity with a pronounced amplitude, varying between \( e = 0.10 \) and \( e = 0.50 \), in a near-mirror image of the eccentricity oscillation of planet b (see Fig. 1). These complementary oscillations of eccentricities provide a very interesting example of planetary angular momentum interchange. The semi-major axis of planet c varies between \( \sim 0.46 \) and 0.49 AU and the position of its periastron completes a 141.1 yr cycle in step with the change in the eccentricities. Furthermore, for each 141.1 yr cycle, there will be an orbit of maximum eccentricity for planet c. Over 33,050 yr the periastron of this maximum eccentricity orbit will advance slowly and synchronously with the precession of planet b’s periastron. As a result of this resonance coupling between the orbits, the stability of this system is maintained by keeping an approximately 0.14 AU buffer between the two planets’ orbital paths. This is illustrated in Fig. 2, showing the change in the orbital configuration caused by planetary interaction. Over a span of half the 141.1 yr cycle we see planet c’s eccentricity change dramatically. The lower left panel shows a much larger advance in time and is a snapshot of a maximum eccentric orbit of planet c; notice the simultaneous precession of both planets’ periastra at the longer 33,050 yr cycle.

In addition to any goodness of fit analysis, the two-planet model must show long-term stability. We have used the symplectic integration package\(^2\) HNBody of Rauch and Hamilton (2002) to do a long term integration of this system. Over a time interval of 100 million days (\( \sim 274,000 \) yr), we have verified stability and that the planets exhibit a 5:1 mean motion resonance. Following the lead of Barnes (2006), we plotted \( e_b e_c \sin(\Delta \omega) \) vs. \( e_b e_c \cos(\Delta \omega) \) in Fig. 3, where \( \Delta \omega \) is the difference in the longitude of the planets’ periastra. In this figure, if the angle between the periastra was constant, all the points would be co-linear, extending radially outward from the origin. The closed loop shape arises because the angle between periastra grows and spans a full \( 2\pi \) every 141.1 yrs; there is no libration. From inspection of the figure, the maximum of \( e_b e_c \) occurs when \( \Delta \omega = 0 \), and since \( e_b \approx 0.67 \), the orbits of planet c reach their maximum eccentricity when the periastra are aligned. Hence the specific cases illustrated in Fig. 2 are in fact typical.

The 1-\( \sigma \) confidence limits for the parameters of the 2–planet solution were calculated following Press et al. (1986) and are given in Table 3. Note that we quote the parameter

\(^2\)http://janus.astro.umd.edu/HNBody/
values to more significant digits than is warranted by the uncertainties so that others can exactly reproduce our 2-planet POPP2 numerical solution. Also note that while the short-term orbital characteristics of planet c can be determined, the timescales of the much longer precession cycles are poorly constrained by the data (e.g. an earlier POPP2 solution yielded long term cycles of 212.7 yr and 37,700 yr).

The question remains, does HD 17156c exist? Noting the small number of observations, the short time span of these observations, and the small mass of the inferred second planet, caution must be taken. Using the F statistic on the fitness values from Table 2 to test if the extra parameters of the two-planet model are warranted, we find that the probability for including those additional parameters is greater than 99.9%. While the mass of HD 17156c is poorly determined, it is not consistent with zero, further supporting the F statistic result. The two-planet model also is physically realizable as a stable system. These facts led to our characterization of HD 17156c as not only possible, but probable. Obviously, to resolve the remaining uncertainty, additional observations are required. In §4, we point out observations that would allow distinguishing between the one and two-planet models.

4. Discussion

Additional radial velocity and transit time data would greatly help confirm and constrain specific parameters of the second planet. However, the general dynamics of the 3-body interactions are already well determined. Using the current observations and our two-planet POPP model we can make several observable predictions. The divergence between the velocities derived from the single-planet reproduction and the two-planet solutions is shown in Fig. 4. In about 2 years, the differences in radial velocity predictions between the two models become larger than 10 ms$^{-1}$ at the peaks of the graphs, occurring at the time of periastron of planet b. In particular, for the epoch of planet b’s periastron at HJD 2455266.04 (UT 2010 Mar 10), a difference of 18 ms$^{-1}$ is predicted.

Another observable is planet b’s time of transit. While this is slowly advancing on the 33,050 yr cycle, there is a much larger and rapid modulation in planet b’s instantaneous period, i.e. the time between sequential transits. The near-future predicted instantaneous period is shown in Fig. 5. Planet-planet interactions, especially when near apastron of planet b, result in sharp changes in the instantaneous period. The average time interval between successive conjunctions is $\sim$26.2 d which is $\sim$24% longer than planet b’s period. Thus conjunctions close to the apastron of planet b occur roughly every $\sim$5 orbits. The exchange of angular momentum at these times causes the jumps in period of planet b shown in Fig. 5. A longer $\sim$1000 d cycle results from the longitude of planet c shifting by -6.75 degrees during
each successive apastral conjunctions. A predicted O-C diagram using the mean period at the current epoch is shown in the lower panel of Fig. 5. Unlike the instantaneous period changes, an O-C diagram includes the sum of the effects of the period changes since the fiducial epoch, and thus is akin to the integral of the instantaneous period. The deviations from a linear ephemeris are quite large and thus monitoring planet b’s transit times should reveal the presence of planet c and be very helpful in further refinement of the system’s parameters. Finally, planet c may in principle also be detected via the transit method. Unfortunately, however, planet c is not expected to transit the host star if its orbit is coplanar with that of planet b (e.g. $i = 86.5^\circ$, Irwin et al. 2008).

Whereas the parameters of the probable planet c are poorly constrained, the qualitative dynamical aspects of such a system are of great interest. This case provides an example of a strong dynamical interaction between a high eccentricity “hot Jupiter” planet and a second planet found further away from the star. The predicted near-term observables should provide guidance to future observations of this system and may provide a possible explanation to any variations seen in observed transit times.

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Table 1. HD 17156 Planet Parameters

| Solution      | Epoch$^a$ (HJD-2450000) | P (days) | $T_0^b$ (HJD-2450000) | e     | $\omega$ (deg) | K (ms$^{-1}$) | a (AU) | Mass$^d$ (M$_{\text{Jup}}$) |
|---------------|--------------------------|----------|----------------------|-------|----------------|--------------|--------|-----------------------------|
| Fischer et al. 2007 | ...                       | 21.2     | 3738.529             | 0.67  | 121.3          | 273.8        | 0.15   | 3.12                        |
| Irwin et al. 2007 (K+S+TT) | ...                       | 21.2169  | 3738.605             | 0.67  | 121.3          | 273.8        | 0.160  | 3.13                        |
| POPF2 K+TT   | ...                       | 21.2167  | 3738.614             | 0.67  | 121.4          | 273.8        | 0.160  | 3.12                        |
| POPF2 Planet b | 3746.7582                 | 21.2144  | 3738.593             | 0.67  | 120.9          | 273.0        | 0.160  | 3.13                        |
| and Planet c | 3746.7582                 | 111.394  | 3736.880             | 0.136 | 273.6          | 2.6          | 0.481  | 0.07                        |

$^a$Time at which tabulated parameters are valid for the POPF2 solutions

$^b$Time of periastron passage

$^c$Assuming a stellar mass of 1.2 M$_{\odot}$ from Fischer et al. 2007

Table 2. Fitness$^a$ of Solutions$^b$

| Solution       | Number of parameters | rms (ms$^{-1}$) | Keck RV only N=24 | Keck+TT N=28 | Keck+Subaru RV only N=33 | Keck+Subaru+TT N=37 |
|----------------|----------------------|------------------|-------------------|--------------|---------------------------|---------------------|
| Fischer et. al. 2007 | ...                   | 3.97             | ...               | ...          | (1.37; 1.08)$^c$           | ...                 |
| POPF2 K+TT     | 5                    | 3.65             | 26.47 (1.39)      | 27.45 (1.19) | 30.41 (1.13)              | 33.12 (1.07)        |
| POPF2 K+S+TT   | 6                    | 3.75             | 26.49 (1.39)      | 29.99 (1.30) | 30.48 (1.13)              | 31.51 (1.02)        |
| POPF2 K+S+TT   | 11                   | 3.14             | 16.56 (1.18)      | 17.90 (0.99) | 19.96 (0.91)              | 21.38 (0.82)        |

$^a$Reduced $\chi^2$ values are in parentheses.

$^b$Optimized solutions are shown in bold; other values show the solution evaluated against the specific data set.

$^c$Note Fischer et al. (2007) give two values for the reduced $\chi^2$.

Table 3. HD 17156 POPF2 Error Estimates$^a$

| Planet       | P (days) | $T_0$ (HJD) | e     | $\omega$ (deg) | K (ms$^{-1}$) | a (AU) | Mass (M$_{\text{Jup}}$) |
|--------------|----------|-------------|-------|----------------|--------------|--------|-------------------------|
| Planet b     | +1$\sigma$ | +0.0031     | +0.05 | +0.003         | +0.14        | +1.77  | +0.0001                 | +0.02                  |
|              | -1$\sigma$ | 21.2144     | 3738.593 | 0.669614     | 120.89       | 272.987 | 0.159505                | 3.125                  |
| Planet c     | +1$\sigma$ | +0.43       | +0.58 | +0.0029       | +0.17        | +0.79  | +0.001                  | +0.021                 |
|              | -1$\sigma$ | 111.3937    | 3736.880 | 0.136492     | 273.57       | 2.580  | 0.481478                | 0.068                  |

$^a$Full precision is given so the initial conditions of the solution are exactly specified.
Fig. 1.— The cyclical nature of the planets’ orbital properties is seen in this POFP2 integration spanning $10^6$ d. The panels show, from top to bottom, the eccentricity of planet b ($e_b$), the eccentricity of planet c ($e_c$), and the longitude of periastron of planet c ($\omega_c$). The 141.1 yr (51,550 d) periodicity is manifest in the small modulation of $e_b$ (upper panel) and the much larger changes in $e_c$ and $\omega_c$. The anti-phasing of the eccentricities is a result of the exchange of angular momentum between the planets.
Fig. 2.— The orbital configuration of the POPP2 two-planet solution is shown at four separate epochs. Clockwise from the upper left, the panels show the advance of planet c’s cycle (141.1 yr) at approximately 1/4 cycle intervals. The lower left panel illustrates the precession of planet b’s orbit, on its much longer cycle of 33,050 yr. The cross marks the periastron of planet c’s orbit, and the arrow in the lower right corner marks the direction to the observer.
Fig. 3.— The resonant coupling of the eccentricities of the two planets is shown, following the suggestion of Barnes (2006) of plotting the product of the instantaneous eccentricities times the sine and cosine of the angle between the two planets’ periastra ($\Delta \omega$). The orbits were sampled every 50,000 days over the $10^8$ d interval of the HNBody integration. Since the eccentricity of planet b is almost constant, the figure shows that the orbit of maximum eccentricity of planet c occurs when the periastra of the two planets are approximately aligned, i.e., along the abscissa where $\Delta \omega \sim 0$. 
Fig. 4.— The difference between the radial velocity predictions of the POFP one-planet and two-planet solutions is shown. The bottom panel is an enlargement of a 50-day interval from the upper panel. The largest differences (peaks in the lower panel) occur around the time of planet b’s periastron, and become readily measurable around HJD 2455200 (∼ 2010 Jan).
Fig. 5.— **Upper panel:** The *instantaneous* period of planet b in the POFP2 two-planet solution plotted against time. Angular momentum exchange between the planets results in jumps in the orbital period. Changes of up to \sim 3 \text{ min} in the transit-to-transit period are predicted. **Lower panel:** A predicted O-C diagram for the times of transit of planet b, with the transits times from Irwin et al. (2007) superposed, using the ephemeris $T(E) = 2453738.32783 + 21.216159 E$ (HJD).