Spin-orbit coupling and the up-down differential transverse flow in intermediate-energy heavy-ion collisions

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To explore the strength, isospin dependence, and density dependence of the in-medium spin-orbit coupling that is poorly known but relevant for understanding the structure of rare isotopes, the nucleon spin up-down differential transverse flow is studied systematically by varying the beam energy and impact parameter of Au+Au collisions within a spin-isospin dependent transport model recently developed for investigating spin-related phenomena in intermediate-energy heavy-ion collisions. The optimal reaction conditions for delineating and disentangling effects of different terms in the spin-orbit coupling are discussed.

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I. INTRODUCTION

The importance of nuclear spin-orbit interaction was first recognized after Goeppert-Mayer [1] and Haxel et al [2] used it to explain successfully the magic numbers 65 years ago. The in-medium spin-orbit interaction is also known to be critical for determining properties of drip-line nuclei [3], the astrophysical r-process [4], and the location of stability island for superheavy elements [5, 6]. However, despite of the extensive studies of the shell structure and other properties of finite nuclei, there exists major uncertainties and interesting puzzles regarding the strength, density dependence, and isospin dependence of the spin-orbit coupling. For instance, the strength of the spin-orbit coupling has only been constrained between 80 and 150 MeVfm [7], by studying nuclear structures [8–9]. While the structure of ”bubble nuclei” [10–12] provides an interesting opportunity for studying the density dependence of the spin-orbit coupling, the density range and gradient explored are rather small compared to that reached in nuclear reactions at intermediate energies. The isospin dependence of the spin-orbit coupling refers to the relative strength of the isospin-like nucleons with respect to the isospin-unlike ones. It was found that the kink of the charge radii of lead isotopes favors a similar coupling strength for the isospin-like and isospin-unlike nucleons [13, 14], while strong experimental evidences of a decreasing spin-orbit coupling strength with increasing neutron excess were reported [15, 16].

The so-call ”Spin Hall Effect” [17] due to the spin-orbit coupling is expected to be a general phenomenon during particle transport processes. In nuclear reactions, effects of the spin-orbit coupling have already been studied at both low [18, 21] and ultra-relativistic energies [21]. To investigate the spin-dependent phenomena and explore the strength, isospin dependence, and density dependence of the in-medium spin-orbit interaction, we have recently developed a spin-isospin dependent transport model SIBUU12 [22] by incorporating the spin degree of freedom and the spin-dependent nucleon potentials into the isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) transport model [23, 24] for intermediate-energy heavy-ion collisions. The nucleon spin up-down differential transverse flow was found to be sensitive to the variation of parameters for the spin-orbit coupling in Au+Au reaction at a beam energy of 50 MeV/nucleon and an impact parameter of b = 8 fm [22]. However, the strength, density dependence, and isospin dependence will all affect the spin up-down differential flow, and it is thus difficult to extract clear information about different components of the spin-orbit coupling with a single reaction. In this work, we perform a systematic study of the nucleon spin up-down differential transverse flow by varying the beam energy and impact parameter of Au+Au collisions. The optimal reaction conditions and the possibility of delineating and disentangling effects of different terms in the spin-orbit coupling by using high transverse momentum nucleons are discussed.

The paper is organized as follows. In Sec. II we outline the spin-orbit interaction and the spin-dependent mean-field potentials within Hartree-Fock calculations. In Sec. III we describe how the spin degree of freedom and the spin-dependent potentials are incorporated into the SIBUU12 transport model. In Sec. IV we present results of our systematic studies on the spin up-down differential transverse flow and discuss the possibility of delineating and disentangling effects of the strength, isospin dependence, and density dependence of the spin-orbit coupling. Finally, a summary is given at the end.

II. SPIN-ORBIT INTERACTION AND SPIN-DEPENDENT MEAN-FIELD POTENTIALS

We begin with the Skyrme type effective nuclear spin-orbit interaction between two nucleons at position \( \vec{r}_1 \) and...
\[ V_{so} = iW_0(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{k} \delta(\vec{r}_1 - \vec{r}_2)\vec{k}', \]  

(1)

where \( W_0 \) is the strength of the spin-orbit coupling, \( \vec{\sigma}_{1(2)} \) is the Pauli matrix, \( \vec{k} = (\vec{p}_1 - \vec{p}_2)/2 \) is the relative momentum operator acting on the right with \( \vec{p}' = -i\nabla \), and \( \vec{k}' \) is the complex conjugate of \( \vec{k} \). Within the framework of Hartree-Fock calculation, the above nuclear interaction leads to the time-even and time-odd spin-dependent mean-field potentials as

\[
U_q^{s\text{-even}} = -\frac{W_0}{2}(\nabla \cdot (\vec{J} + \vec{J}_q)) + \frac{W_0}{2}(\nabla \rho + \nabla \rho_q) \cdot (\vec{p} \times \vec{\sigma}),
\]

(2)

\[
U_q^{s\text{-odd}} = -\frac{W_0}{2}\vec{p} \cdot [\nabla \times (\vec{s} + \vec{s}_q)] - \frac{W_0}{2}\vec{p} \cdot [\nabla \times (\vec{j} + \vec{j}_q)],
\]

(3)

where

\[
\rho = \sum_i \phi_i^* \phi_i,
\]

(4)

\[
\vec{s} = \sum_i \sum_{\sigma,\sigma'} \phi_i^* \langle \sigma | \vec{r} | \sigma' \rangle \phi_i,
\]

(5)

\[
\vec{j} = \frac{1}{2i} \sum_i (\phi_i^* \nabla \phi_i - \phi_i \nabla \phi_i^*),
\]

(6)

\[
\vec{J} = \frac{1}{2i} \sum_i \sum_{\sigma,\sigma'} (\phi_i^* \nabla \phi_i - \phi_i \nabla \phi_i^*) \times \langle \sigma | \vec{r} | \sigma' \rangle,
\]

(7)

are respectively the number, spin, momentum, and spin-current densities, with \( \phi_i \) being the wave function of the \( i \)th nucleon. It is noteworthy that although only time-even potentials are considered in the study of spherical nuclei, time-odd potentials should be included in the study of deformed nuclei. In heavy-ion collisions with all kinds of geometric asymmetries, both time-even and time-odd potentials are necessary, otherwise the Galilean invariance will be broken and the frame-dependent spurious spin polarization will appear.

The second term in Eq. (2) is usually called the spin-orbit potential written in the form of \( \vec{W}_q \cdot (\vec{p} \times \vec{\sigma}) \) with the form factor expressed as

\[
\vec{W}_q = \frac{W_0}{2}(\nabla \rho + \nabla \rho_q).
\]

(8)

In Ref. (26) a density-dependent nuclear spin-orbit interaction is generalized, and the form factor of the spin-orbit potential has the form of

\[
\vec{W}_q = \frac{W_0}{2}\nabla (\rho + \rho_q) + \frac{W_1}{2}\rho \nabla (\rho - \rho_q) + (2 + \gamma)(2\rho_q)\gamma \nabla \rho_q + \frac{W_1}{4} (\gamma \rho^{\gamma - 1} (\rho - \rho_q) \nabla \rho, \]  

(9)

where \( W_1 \) and \( \gamma \) are the two additional parameters. To reproduce the kink of the charge radii with the increasing mass of lead isotopes, the isospin dependence of the nuclear spin-orbit interaction was investigated, and in Ref. (12) the form factor of the spin-orbit potential is written as

\[
\vec{W}_q = \frac{W_0}{2}(1 + \chi_w)\nabla \rho + \frac{W_0}{2}\nabla \rho_q, (q \neq q') \]

(10)

where \( \chi_w \) is the parameter to mimic the isospin dependence. For the relativistic mean-field model, the non-relativistic expansion of the Dirac equation usually gives different density and isospin dependence of the spin-orbit coupling compared to the Skyrme-Hartree-Fock functional. In order to take the main physics of the density and isospin dependence of the spin-orbit coupling into consideration while making the form as simple as possible, the form factor can be generally written as

\[
\vec{W}_q = \frac{W_0^*(\rho)}{2}(a \nabla \rho + b \nabla \rho_q)(q \neq q') \]

(11)

with \( W_0^*(\rho) = W_0(\rho/\rho_0)^{\gamma} \). In the above, \( W_0, \gamma, a, \) and \( b \) are the corresponding parameters for the strength, density dependence, and isospin dependence of the spin-orbit coupling, and \( \rho_0 = 0.16 \text{ fm}^{-3} \) is the saturation density. As mentioned in the introduction, the strength of the spin-orbit coupling has been constrained between 80 and 150 MeVfm\(^5\), while its density and isospin dependence are still under hot debate. We will use \( W_0 = 150 \text{ MeVfm}^5 \), \( a = 2 \), \( b = 1 \), and \( \gamma = 0 \) in the following calculations unless stated otherwise. A similar generalization to all the spin-dependent mean-field potentials leads to

\[
U_q^{s\text{-even}} = -\frac{W_0^*(\rho)}{2}[\nabla \cdot (a\vec{J}_q + b\vec{J}_q')]
\]

(12)

\[
U_q^{s\text{-odd}} = \frac{W_0^*(\rho)}{2}a\vec{p} \cdot [\nabla \times (a\vec{s}_q + b\vec{s}_q')]
\]

(13)

III. INCORPORATING THE SPIN DEGREE OF FREEDOM INTO THE IBUU TRANSPORT MODEL

The isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) transport model has been very successful in studying intermediate-energy heavy-ion collisions, especially the isospin effects. However, in the previous studies, spin effects were neglected as only spin-averaged quantities were studied. To introduce spin effects into the IBUU model, each nucleon now has an additional degree of freedom, i.e., a unit vector representing the expectation value of its spin \( \vec{\sigma} \). The spin, momentum, and spin-current densities can be calculated by using the test
particle method\cite{27,28} similar to the number density $\rho$, i.e.,

$$
\rho(\vec{r}) = \frac{1}{N_{\text{test}}} \sum_i \delta(\vec{r} - \vec{r}_i), \quad (14)
$$

where $N_{\text{test}}$ is the test particle number. In addition, the equations of motion in the presence of the spin-dependent mean-field potentials can now be written as

$$
\frac{d\vec{r}}{dt} = \frac{\vec{p}}{m} + \frac{W_{0}^s(\rho)}{2} \times (a\nabla \rho_a + b\nabla \rho_q)
- \frac{W_{0}^s(\rho)}{2} \nabla \times (a\delta_q + b\delta_q^s), \quad (18)
$$

$$
\frac{d\vec{p}}{dt} = -\nabla U_q - \nabla U_{q_{s=\text{even}}} - \nabla U_{q_{s=\text{odd}}}, \quad (19)
$$

$$
\frac{d\vec{\sigma}}{dt} = W_{0}^s(\rho)[(a\nabla \rho_a + b\nabla \rho_q) \times \vec{p}] \times \vec{\sigma}
- W_{0}^s(\rho)[\nabla \times (a\vec{q} + b\vec{q}^s)] \times \vec{\sigma}. \quad (20)
$$

The spin-independent mean-field potential $U_q$ is fitted to give the binding energy $E_0(\rho_0) = -16$ MeV for normal nuclear matter and symmetry energy $E_{\text{sym}}(\rho_0) = 30$ MeV at saturation density. The incompressibility $K_0$ and the slope parameter $L$ of the symmetry energy are set to be 230 MeV and 60 MeV, respectively.

In simulating heavy-ion collisions we set $z$ as the beam axis. The distance between the centers of the two colliding nuclei in the $x$ direction is the impact parameter $b$. We refer in the following a nucleon with its spin in the $+y$ ($-y$) direction as a spin-up (spin-down) nucleon. Since initially the spins of nucleons are randomly distributed, the second terms in Eqs. (12) and (13) are most important. During the collision process, as shown in Fig. 1 of Ref.\cite{22}, the second term in Eq. (12) (13) is repulsive (attractive) for spin-up nucleons and attractive (repulsive) for spin-down ones. Within the framework of the SIBUU transport model \cite{22} and the time-dependent Hartree-Fock model \cite{19}, the time-odd contribution (Eq. (13)) is larger than the time-even contribution (Eq. (12)). The net spin-dependent potential is thus more attractive (repulsive) for spin-up (down) nucleons. This is similar to the symmetry potential which is repulsive (attractive) for neutrons (protons).

Besides the spin-dependent mean-field potential, the spin during nucleon-nucleon scatterings should also be treated with care. It was found that the spin of a nucleon may flip after nucleon-nucleon scatterings\cite{29} from spin-dependent nucleon-nucleon interactions. Although it was shown that the spin-flip probability is appreciable and dependent on the energy and momentum transfer\cite{30}, it is still not well determined due to the lack of accurate knowledge regarding in-medium spin-related nucleon-nucleon interactions. In the present work, as a conservative approximation we randomize the final spins of the two colliding nucleons after each scattering. In addition, the nucleon phase space distribution is further sorted according to nucleon spin to properly treat the spin- and isospin-dependent Pauli blocking.

**IV. RESULTS AND DISCUSSIONS**

In our previous work with the SIBUU12 transport model\cite{22}, we already found that the spin up-down differential transverse flow is a useful probe of the in-medium spin-orbit coupling. Here we study more systematically the spin-dependent transverse flow. One of our purposes is to find the beam energy and the collision centrality at which the spin up-down differential transverse flow is the largest and most easily observable. We are also interested in disengaging effects of the strength, isospin dependence, and density dependence of the spin-orbit coupling. In our calculations, we stop the simulations when the projectile and the target fall apart. 200,000 events are generated for each beam energy and impact parameter.

**A. Up-down differential transverse flow and its dependence on the spin-orbit coupling, beam energy, and impact parameter**

![FIG. 1](Color online) Transverse flow of Au+Au collisions at an impact parameter of 8 fm and beam energies of 50, 100, 200, and 300 MeV/nucleon (left panel), and at a beam energy of 100 MeV/nucleon and impact parameters of 4, 8, and 12 fm (right panel).

The nucleon transverse flow is normally revealed by examining the nucleon average transverse momentum in the reaction plane versus rapidity. For ease of following discussions, we first illustrate in Fig. 1 the spin-averaged nucleon transverse flow and its dependence on the beam
energy and impact parameter. We note that in our code the projectile (target) is put on the \(-x\) \((+x)\) side in the reaction plane. Thus, the sign of the transverse flow is negative as shown in Ref. 22, different from the convention that some others may have used. The main features seen here are consistent with those found in previous studies in the literature 28 31 32. In particular, as a result of the competition between the generally attractive mean-field potential in the energy range considered and the repulsive nucleon-nucleon scatterings, the transverse flow first increases fast then slowly with increasing beam energy. At a given beam energy of 100 MeV/nucleon, the transverse flow is largest at \(b = 4\) fm and changes sign at \(b = 12\) fm.

\[ F_{ud}(y_r) = \frac{1}{N(y_r)} \sum_{i=1}^{N(y_r)} \sigma_i(p_x)_i , \quad (21) \]

where \(N(y_r)\) is the number of nucleons with rapidity \(y_r\), and \(\sigma_i\) is \(1(-1)\) for spin-up (spin-down) nucleons. The spin up-down differential transverse flow maximizes the effects of the opposite spin-dependent potentials for spin-up and spin-down nucleons while canceling out largely spin-independent contributions. The left panel of Fig. 2 displays the spin up-down differential transverse flow for Au+Au collisions at an impact parameter of \(8\) fm and beam energies of 50, 100, 200, and 300 MeV/nucleon. Different from the spin-averaged transverse flow, the slope of the differential one first increases then decreases with increasing collision energy, and the largest spin up-down differential transverse flow is observed at a beam energy of about 100 MeV/nucleon. We note that the angular momentum increases with increasing beam energy, leading to stronger spin-dependent potentials. On the other hand, the violent nucleon-nucleon scatterings randomize the spin direction and turn to wash out the effects from the spin-dependent potentials, especially at higher energies. The competition of the above effects leads to the non-monotonical behavior of the slope of \(F_{ud}\) with respect to the beam energy. In the right panel of Fig. 2 we show the results at a beam energy of 100 MeV/nucleon and an impact parameter of 4, 8, and 12 fm. It is seen that the spin up-down differential flow is larger for peripheral collisions where the statistical errors are also larger. This is understandable as the second terms in Eqs. 12 and 13 generally increase with the increasing nucleon density gradient, and the latter becomes larger near the edge of a nucleus.

\[ F' = \frac{dF_{ud}}{dy_r} , \quad (22) \]

where \(y_r/y_r^{beam}\) is the rapidity. Similar to the calculation of the flow parameter for the direct flow in Ref. 33, we use a 3rd-order polynomial to fit the \(s\)-shaped \(F_{ud}\) curve to get its slope \(F'\). Moreover, to measure quantitatively effects from varying \(W_0\) we define a sensitivity parameter as

\[ \delta = \frac{F'_{150} - F'_{80}}{F'_{150} + F'_{80}} , \quad (23) \]

where \(F'_{150}\) (\(F'_{80}\)) is the flow parameter \(F'\) from \(W_0 = 150\) (80) MeVfm\(^2\). The values of \(F'_{150}\) and \(F'_{80}\) as well as \(\delta\) at
different beam energies are listed in Table I. It is seen that the value of the sensitivity parameter \( \delta \) is appreciable and approximately a constant within the statistical errors from 50 to 200 MeV/nucleon.

| \( E_{\text{beam}} \) (AMeV) | \( 50 \) | \( 100 \) | \( 200 \) |
|----------------------|-------|-------|-------|
| \( F_{150} \)        | 3.60 ± 0.19 | 4.49 ± 0.35 | 2.25 ± 0.22 |
| \( F'_{50} \)        | 2.23 ± 0.13 | 2.60 ± 0.02 | 1.33 ± 0.23 |
| \( \delta \)         | 0.23 ± 0.04 | 0.27 ± 0.04 | 0.26 ± 0.07 |

TABLE I: Slope parameters for spin up-down differential transverse flow at \( W_0 = 150 \) and 80 MeVfm\(^5\) as well as their relative difference \( \delta \) (Eq. (23)) in Au+Au collisions at an impact parameter of \( b = 8 \) fm with different beam energies.

![Graph showing spin up-down differential transverse flow for neutrons and protons with different ratios of isospin-like and isospin-unlike couplings](image)

FIG. 4: (Color online) Spin up-down differential transverse flows for neutrons and protons with different ratios of isospin-like and isospin-unlike couplings, i.e., \( a = 2 \), \( b = 1 \) (a, c, e) and \( a = 1 \), \( b = 2 \) (b, d, f) for Au+Au collisions at a beam energy of 50 (a, b), 100 (c, d), and 200 (e, f) MeV/nucleon with impact parameter \( b = 8 \) fm.

The isospin dependence of the spin-orbit coupling can be studied by comparing the spin up-down differential transverse flow for neutrons and protons using different strength of isospin-like and isospin-unlike coupling, i.e., different values of \( a \) and \( b \) in Eqs. (12) and (13). Here we use typical values of \( (a = 2 \) and \( b = 1 \)) and \( (a = 1 \) and \( b = 2 \)), and the results for different beam energies are shown in Fig. 4. We note that the system is globally neutron rich, and the \( \nabla \rho_n \) and \( \nabla \times j_n \) are generally larger than the \( \nabla \rho_p \) and \( \nabla \times j_p \), respectively. It is seen that with the parameter set corresponding to a stronger isospin-like coupling, i.e., \( (a = 2 \) and \( b = 1 \)), the \( F_{\text{ud}} \) of neutrons is larger than that of protons. While the difference of \( F_{\text{ud}} \) between neutrons and protons becomes smaller with the stronger isospin-unlike coupling, i.e., \( (a = 1 \) and \( b = 2 \)).

To quantify the sensitivity, we define the relative difference \( \delta' \) as

\[
\delta' = \frac{F_n' - F_p'}{F_n' + F_p'}
\]

where \( F_n' (F_p') \) is the value of the flow parameter \( F' \) of the spin up-down differential transverse flow for neutrons (protons). The values of \( F_n' \) and \( F_p' \) as well as \( \delta' \) for different beam energies in Fig. 4 are summarized in Table II.

We can see that the sensitivity is still appreciable at beam energies of 50 and 100 MeV/nucleon, as the difference of \( \delta' \) from the two parameter sets are significantly larger than the statistical error. Considering the magnitude of \( F_{\text{ud}} \), the different neutron and proton spin up-down differential transverse flows for mid-central Au+Au collisions at the beam energy of about 100 MeV/nucleon might be the best probe of the isospin dependence of the spin-orbit coupling.

TABLE II: Slope parameters for neutron and proton spin up-down differential transverse flow as well as their relative difference \( \delta' \) (Eq. (24)) in Au+Au collisions at impact parameter \( b = 8 \) fm with different beam energies.

| \( E_{\text{beam}} \) (AMeV) | \( 50 \) | \( 100 \) | \( 200 \) |
|----------------------|-------|-------|-------|
| \( F_n' \)          | 4.17 ± 0.69 | 3.41 ± 0.53 | 5.62 ± 0.35 |
| \( F_p' \)          | 2.59 ± 0.36 | 3.58 ± 0.34 | 2.55 ± 0.33 |
| \( \delta' \)       | 0.23 ± 0.06 | 0.02 ± 0.09 | 0.38 ± 0.06 |

B. Disentangle effects of the strength and density dependence of the spin-orbit coupling with high transverse momentum nucleons

Within our framework, we use a parameterized form of the spin-orbit coupling, i.e., \( W_0(\rho/\rho_0)\gamma \), with \( W_0 \) representing the strength and \( \gamma \) representing the density dependence of the spin-orbit coupling. In our previous work [22] we have shown that the spin up-down differential transverse flow is sensitive to both the strength and the density dependence of the spin-orbit coupling. Thus, it is difficult to probe both \( W_0 \) and \( \gamma \) simultaneously using a single reaction. In this subsection, we investigate if additional information can be obtained by restricting our analyses to high transverse momentum \( (p_T) \) nucleons which are known to be mostly emitted from high density region in the earlier stage of heavy-ion collisions. In the SIBUU12 model, a nucleon is considered free when its local density is below \( \rho_0/8 \). Since the number of high \( p_T \) nucleons is very small, to have enough statistics and ensure that our qualitative conclusions are insensitive to the specific values of the \( p_T \) cut, we analyzed a million events for each case and used the...
the beam energy with the flow parameter about the high-density phase of the reaction, including the time-integrated effects of the spin-dependent mean-field potentials before their emission.

For a comparison, we first show in Panel (a) of Fig. 7 the flow parameter $F'$ of the spin up-down differential transverse flow as a function of the beam energy with $\gamma = 0$ and $\gamma = 1$, respectively.

It is seen that the $F'$ is slightly higher with $\gamma = 0$ especially at lower beam energies, as nucleons are mostly emitted from subsaturation density regions especially at lower beam energies. Since the spin-orbit coupling is stronger with $\gamma = 0$ than $\gamma = 1$ at subsaturation densities, the $F'$ is thus larger for $\gamma = 0$ than for $\gamma = 1$ as already shown in Ref. [22]. However, it is clear that the observed effect is too small to be practically useful. With the high $p_T$ cut, it is interesting to see in the right panel of Fig. 7 that the signal becomes much stronger especially at lower beam energies. There is a cross-over in $F'$ around $E_{beam} = 150$ MeV/nucleon associated with the cross-over of the function $W_0(\rho/\rho_0)^\gamma$ shown in the inset of Fig. 7. This is consistent with the observation from Fig. 6 that the average density where high-$p_T$ nucleons are emitted is smaller (higher) than $\rho_0$ in reactions at beam energies below (above) 150 MeV/nucleon.

In this way, the strength and the density dependence of the spin-orbit coupling $W_0(\rho/\rho_0)^\gamma$ can thus be determined separately by considering the spin up-down differential transverse flow of high-$p_T$ nucleons at their corresponding beam energies.

V. SUMMARY

In summary, we have studied more systematically the spin up-down differential transverse flow in intermediate-energy heavy-ion collisions. Its potential for probing the in-medium spin-orbit interaction is examined in some detail. For the Au+Au collisions studied here, we found that the largest and most reliable spin up-down differential transverse flow occurs in mid-central reactions at the beam energy of about 100 MeV/nucleon. In this optimal reaction, the strength and the isospin dependence of the
spin-orbit coupling can be extracted by measuring the slope of the differential flow and compare the slopes of neutrons and protons. Moreover, the spin up-down differential transverse flow of high-$p_T$ nucleons at different beam energies can be used to delineate the density dependence of the spin-orbit coupling. Although we consider all the nucleons in the present study, the conclusion remains the same if only protons are considered when only charged particles are measured. Similarly, the differential flows for neutrons and protons may also be replaced by those for $t$ and $^3$He that are experimentally more easy to measure. While currently no spin-related observable has been measured in heavy-ion reactions at intermediate energies, our exploratory study demonstrates clearly that the density and isospin dependence of the in-medium spin-orbit coupling poses some interesting questions worth investigating both theoretically and experimentally. We believe that the spin-isospin physics in heavy-ion collisions at intermediate energies will provide complementary information about the poorly known in-medium spin-orbit force affecting many novel features of rare isotopes.

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