\( \bar{B}_c \to \eta_c, \; \bar{B}_c \to J/\psi \) and \( \bar{B} \to D^{(*)} \) semileptonic decays including new physics.

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We apply the general formalism derived in N. Penalva \textit{et al.} [Phys. Rev. D 101, 113004 (2020)] to the semileptonic decay of pseudoscalar mesons containing a \( b \) quark. While present \( B \to D^{(*)} \) data give the strongest evidence in favor of lepton flavor universality violation, the observables that are normally considered are not able to distinguish between different new physics (NP) scenarios. In the above reference we discussed the relevant role that the various contributions to the double differential decay widths \( d^2\Gamma / (d\omega d\cos\theta_\ell) \) and \( d^2\Gamma / (d\omega dE_\ell) \) could play to this end. Here \( \omega \) is the product of the two hadron four-velocities, \( \theta_\ell \) is the angle made by the final lepton and final hadron three-momenta in the center of mass of the final two-lepton system, and \( E_\ell \) is the final charged lepton energy in the laboratory system. The formalism was applied there to the analysis of the \( \Lambda_b \to \Lambda_c \) semileptonic decay showing the new observables were able to tell apart different NP scenarios. Here we analyze the \( \bar{B}_c \to \eta_c\tau\bar{\nu}_\tau, \; \bar{B}_c \to J/\psi\tau\bar{\nu}_\tau, \; \bar{B} \to D\tau\bar{\nu}_\tau \) and \( \bar{B} \to D^*\tau\bar{\nu}_\tau \) semileptonic decays. We show that, as a general rule, the \( \bar{B}_c \to J/\psi \) observables, even including \( \tau \) polarization, are less optimal for distinguishing between NP scenarios than those obtained from \( \bar{B}_c \to \eta_c \) decays, or those presented in N. Penalva \textit{et al.} for the related \( \Lambda_b \to \Lambda_c \) semileptonic decay. Finally, we show that \( \bar{B} \to D \) and \( \bar{B}_c \to \eta_c \), and \( \bar{B} \to D^* \) and \( \bar{B}_c \to J/\psi \) decay observables exhibit similar behaviors.

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I. INTRODUCTION

The present values of the $R_{D^{(*)}}$ ratios ($\ell = e, \mu$)

$$R_D = \frac{\Gamma(\bar{B} \to D \tau \nu_\tau)}{\Gamma(\bar{B} \to D \ell \nu_\ell)} = 0.340 \pm 0.027 \pm 0.013 , \quad R_{D^*} = \frac{\Gamma(\bar{B} \to D^* \tau \nu_\tau)}{\Gamma(\bar{B} \to D^* \ell \nu_\ell)} = 0.295 \pm 0.011 \pm 0.008.$$  

are the strongest experimental evidence for the possibility of lepton flavor universality violation (LFUV). These values have been obtained by the Heavy Flavour Averaging Group (HFAG) [1] (see also Ref. [2] for earlier results), from a combined analysis of different experimental data by the BaBar [3, 4], Belle [5–8] and LHCb [9, 10] collaborations together with standard model (SM) predictions [11–13], and they show a tension with the SM at the level of 3.1 $\sigma$. However, taking only the latest Belle experiment from Ref. [8] the tension with SM predictions reduces to 0.8 $\sigma$ so that new experimental analyses seem to be necessary to confirm or rule out LFUV in $B$ meson decays. Another source of tension with the SM predictions is in the ratio

$$R_{J/\psi} = \frac{\Gamma(\bar{B}_c \to J/\psi \tau \nu_\tau)}{\Gamma(\bar{B}_c \to J/\psi \ell \nu_\ell)} = 0.71 \pm 0.17 \pm 0.18$$

recently measured by the LHCb Collaboration [14]. This shows a 1.8 $\sigma$ disagreement with SM results that are in the range $R_{J/\psi}^{SM} \sim 0.25 - 0.28$ [15–25].

If the anomalies seen in the data persist, they will be a clear indication of LFUV and new physics (NP) beyond the SM will be necessary to explain it. Since the data for the two first generations of quarks and leptons is in agreement with SM expectations, NP is assumed to affect just the last quark and lepton generation. Its effects can be studied with SM expectations, NP is assumed to affect just the last quark and lepton generation. Its effects can be studied in a phenomenological way by following an effective field theory model-independent analysis that includes different $b \to c \tau \nu_\tau$ effective operators: scalar, pseudo-scalar and tensor NP terms, as well as corrections to the SM vector and axial contributions. In the notation of Ref. [26] one writes

$$H_{\text{eff}} = \frac{4G_F|V_{cb}|^2}{\sqrt{2}} \left[(1 + C_{V_L})O_{V_L} + C_{V_R}O_{V_R} + C_{S_L}O_{S_L} + C_{S_R}O_{S_R} + C_TO_T + h.c.,\right]$$

with fermionic operators given by $(\psi_{L,R} = \frac{1}{2}[\gamma^\mu \psi])$;

$$O_{V,L,R} = (\bar{c}\gamma^\mu b_{L,R})(\bar{\ell}_L \gamma_\mu \nu_L), \quad O_{S,L,R} = (\bar{c} b_{L,R})(\bar{\ell}_R \nu_L), \quad O_T = (\bar{c} \sigma^{\mu\nu} b_L)(\bar{\ell}_R \sigma_{\mu\nu} \nu_L).$$

The corrections to the SM are assumed to be generated by NP that enter at a much higher energy scale, and which strengths at the SM scale are governed by unknown, complex in general, Wilson coefficients ($C_{V_L}, C_{V_R}, C_{S_L}, C_{S_R}$ and $C_T$ in Eq. (3) ) that should be fitted to data. Those analyses include the work of Ref. [27] or the more recent one in Ref. [26] from which we take the values for the Wilson coefficients that we use in the numerical part of this work.

The results of these analyses show that in fact NP can solve some of the present discrepancies. However, it is also found that different combinations of NP terms could give very similar results for the $R_{D^{(*)}}, R_{J/\psi}$ ratios. Thus, even though those ratios are our present best experimental evidence for the possible existence of NP beyond the SM, they are not good observables for distinguishing between different NP scenarios.

The relevant role that the various contributions to the two differential decay widths $d^2\Gamma/(d\omega d\cos \theta_\ell)$ and $d^2\Gamma/(d\omega dE_\ell)$ could play to this end was analyzed in detail in Refs. [28, 29]. Here, $\omega$ is the product of the two hadron four-velocities, $\theta_\ell$ is the angle made by the final lepton and final hadron three-momenta in the center of mass of the final two-lepton pair (CM), and $E_\ell$ is the final charged lepton energy in the laboratory frame (LAB).

Even in the presence of NP, it is shown that for any charged current semileptonic decay one can write [29]

$$\frac{d^2\Gamma}{d\omega d\cos \theta_\ell} = \frac{G_F^2|V_{cb}|^2 M^2 M^2}{16\pi^3} \left[\sqrt{\omega^2 - 1} - \frac{m_\ell^2}{q^2}\right]^2 A(\omega, \theta_\ell),$$

$$A(\omega, \theta_\ell) = \left.\frac{\sum|M|^2}{M^2(1 - \frac{m_\ell^2}{q^2})}\right|_{\text{unpolarized}} = a_0(\omega) + a_1(\omega) \cos \theta_\ell + a_2(\omega)(\cos \theta_\ell)^2,$$

$$\frac{d^2\Gamma}{d\omega dE_\ell} = \frac{G_F^2|V_{cb}|^2 M^2 M^2}{8\pi^3} C(\omega, E_\ell),$$

$$C(\omega, E_\ell) = \left.\frac{\sum|M|^2}{M^2}\right|_{\text{unpolarized}} = c_0(\omega) + c_1(\omega) \frac{E_\ell}{M} + c_2(\omega) \frac{E_\ell^2}{M^2}. $$
where $M, M'$ and $m_\ell$ are the masses of the initial and final hadrons and the final charged lepton respectively, $q^2$ is the four momentum transferred squared (related to $\omega$ via $q^2 = M^2 + M'^2 - 2M M' \omega$) and $M$ is the invariant amplitude for the decay. Note that at zero recoil $\theta$ is not longer defined and thus $a_1(\omega = 1)$ and $a_2(\omega = 1)$ vanish accordingly. The $a_{0,1,2}$ CM and $c_{0,1,2}$ LAB expansion coefficients are scalar functions that depend on $\omega$ and the masses of the particles involved in the decay. In the general tensor formalism developed in Refs. [28, 29], it is shown how they are determined in terms of the 16 Lorentz scalar $\tilde{W}$'s structure functions (SFs) that parameterize all the hadronic input. These $\tilde{W}$'s SFs depend on the Wilson coefficients ($C$'s) and the genuine hadronic responses ($W$'s), the latter being scalar functions of the actual form factors that parameterize the hadronic transition matrix elements for a given decay. The general expressions for he $a_{0,1,2}$ CM and $c_{0,1,2}$ LAB expansion coefficients in terms of the $\tilde{W}$'s SFs can be found in Ref. [29], where the hadron tensors associated with the different SM and NP contributions (including all possible interferences) are also explicitly given$^1$.

The fully developed formalism was applied in Ref. [29] to the analysis of the $\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau$ decay. The shape of the $d\Gamma(\Lambda_b \to \Lambda_c \mu \bar{\nu}_\mu)/d\omega$ differential decay width has already been measured by the LHCb Collaboration [30] and there are expectations that the $R_{\Lambda_b} = \Gamma(\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau)/\Gamma(\Lambda_b \to \Lambda_c \mu \bar{\nu}_\mu)$ ratio may reach the precision obtained for $R_D$ and $R_{D_s}$ [31]. With the use of Wilson coefficients from Ref. [26], fitted to experimental data in the $B$-meson sector, it is shown in Ref. [29] that, with the exception of $a_0$, all the other $a_{1,2}$ CM and $c_{0,1,2}$ LAB expansion coefficients are able to disentangle between different NP scenarios, i.e. different fits to the available data that otherwise give very similar values for the $R_{\Lambda_b}$, and $R_{D_s}, R_{J/\psi}$ ratios, or the corresponding $d\Gamma/d\omega$ distributions.

In this work we apply the general formalism of Ref. [29] to the study of the semileptonic $P_b \to P_c$ and $P_b \to P_c^*$ decays, with $P_0$ and $P_1$ pseudoscalar mesons ($\bar{B}_c$ or $\bar{B}$ and $\eta_c$ or $D$, respectively) and $P_2^*$ a vector meson ($J/\psi$ or $D^*$).

For the case of $\bar{B} \to D^{(*)}$ decays, the hadronic matrix elements are relatively well known. In fact, there exist some experimental $q^2$-shape information [4, 5], which can be used to constrain the transition form factors. They are then computed using a heavy quark effective theory parameterization that includes corrections of order $d$ distribution, we present in Sec. III for the first time details of the LAB

In fact, full general expressions for both LAB and CM decay distributions, decomposed in helicity contributions of the outgoing charged lepton, can also be found in [29].
II. $B_c \to \eta_c$ AND $B_c \to J/\psi$ SEMILEPTONIC DECAY RESULTS

In this section we present the results for the $B_c \to \eta_c \tau \bar{\nu}_\tau$ and $B_c \to J/\psi \tau \bar{\nu}_\tau$ semileptonic decays. For the NP terms we use the Wilson coefficients corresponding to Fits 6 and 7 in Ref. [26]. As for the form factors we shall use two different sets obtained in different theoretical approaches. The first approach is the NRQM evaluation of Ref. [17]. There, five different interquark potentials are used: AL1, AL2, AP1 and AP2 taken from Refs. [42, 43], and the BHAD potential from Ref. [44]. All the form factors are obtained without the need to rely on quark field level equations of motion and their expressions can be found in Appendix C. As in Ref. [17], we will take as central values the results corresponding to the AL1 potential. The deviations from this result obtained with the other four potentials will provide an estimate of the theoretical error associated to the form factors determination in this type of models. These errors will be shown in the corresponding figures below as the narrower lighter uncertainty bands. To evaluate the theoretical error associated to the Wilson coefficients for each of Fits 6 and 7, we use different sets of models. These errors will be shown in the corresponding figures below as the narrower lighter uncertainty bands. To evaluate the theoretical error associated to the Wilson coefficients for each of Fits 6 and 7, we use different sets obtained in different theoretical approaches. The first approach is the NRQM evaluation of Ref. [26]. As for the form factors we shall use two different sets obtained in different theoretical approaches. The first approach is the NRQM evaluation of Ref. [26].

The second set of form factors we shall use are the ones evaluated in Ref. [21] within a perturbative QCD (pQCD) factorization approach. In this latter case only vector and axial-vector form factors have been obtained. They have been evaluated in the low $q^2$ region and extrapolated to higher $q^2$ values using a model dependent parameterization. These form factors have been used in the two recent calculations of Refs. [26, 41] where the rest of form factors needed (scalar, pseudoscalar or tensor ones) are determined using the quark level equations of motion of Ref. [45]. In Ref. [21], the authors give the theoretical uncertainties for the vector and axial form factors at $q^2 = 0$. However neither correlations, nor errors for the parameters used in the low $q^2$–extrapolation are provided. Thus, in this case we will only show the error band stemming from the Wilson coefficients, even though larger uncertainties are to be expected.

A. Results with an unpolarized final $\tau$ lepton

We begin with the results corresponding to an unpolarized final $\tau$ lepton. In Fig. 1, we show the $d\Gamma/d\omega$ differential distribution for $B_c \to \eta_c \tau \bar{\nu}_\tau$ and $B_c \to J/\psi \tau \bar{\nu}_\tau$ reactions. As can be seen in the plots, the $\omega$ values accessible in the transitions are around $\omega \sim 1.2$ at most, while for the similar $B \to D^{(*)} \tau \bar{\nu}_\tau$ reactions the available phase-space is larger, and $\omega$ varies from 1 to 1.35–1.40.

In both $B_c$ decays, the NRQM form factors from Ref. [17] lead to larger total widths. We note that already $d\Gamma/d\omega$ for the decay into $\eta_c$ computed with this latter set of form factors discriminate between NP Fits 6 and 7. Looking at the rest of cases shown in the figure, though NP effects are clearly visible, we see that this observable would not be

|                | SM [NRQM] | [pQCD] | NP Fit 6 [NRQM] | [pQCD] | NP Fit 7 [NRQM] | [pQCD] |
|----------------|-----------|--------|----------------|--------|----------------|--------|
| $\mathcal{R}_{\eta_c}$ | $\frac{B(B_c \to \eta_c \tau \bar{\nu}_\tau)}{B(B_c \to \eta_c \mu \bar{\nu}_\mu)}$ | 0.349$^{+0.006}_{-0.007}$ | 0.309 | 0.452$^{+0.034}_{-0.030}$ | 0.40$^{+0.03}_{-0.03}$ | 0.384$^{+0.024}_{-0.018}$ | 0.40$^{+0.04}_{-0.03}$ |
| $\mathcal{R}_{J/\psi}$ | $\frac{B(B_c \to J/\psi \tau \bar{\nu}_\tau)}{B(B_c \to J/\psi \mu \bar{\nu}_\mu)}$ | 0.266$^{+0.006}_{-0.004}$ | 0.289 | 0.306$^{+0.007}_{-0.007}$ | 0.342$^{+0.013}_{-0.015}$ | 0.301$^{+0.005}_{-0.007}$ | 0.326$^{+0.008}_{-0.009}$ |

TABLE I. $\mathcal{R}_{\eta_c}$ and $\mathcal{R}_{J/\psi}$ ratios obtained in the SM and with NP effects from Ref. [26]) and the sets of form factors from Refs. [17] [NRQM] and [21] [pQCD].

2 Note that in Ref. [21] they work with different form factor decompositions than those used here. The relations between our form factors and theirs can be obtained straightforwardly.
able to distinguish between the two NP scenarios examined in this work. Evaluating the SM predictions for a final massless charged lepton ($\mu$ or $e$), we obtain the $R_{\eta_c}$ and $R_{J/\psi}$ ratios collected in Table I. The systematic uncertainties due to the inter-quark potential in the NRQM scheme are largely canceled out in the ratios, as can be inferred from the SM predictions. Predictions with the NRQM and pQCD form factors differ by approximately 10%, except for NP due to the inter-quark potential in the NRQM scheme are largely canceled out in the ratios, as can be inferred from the SM predictions. For the latter ratio, we mentioned above that $\Gamma^{NRQM}(B_c \to J/\psi \nu \bar{\nu}) \geq \Gamma^{pQCD}(B_c \to J/\psi \nu \bar{\nu})$ and thus, the massless lepton modes of the $B_c \to J/\psi$ semileptonic decay calculated with NRQM form factors must also be larger than when pQCD form factors are used. Moreover, the difference has to be greater than for the $\tau$ mode to explain $R_{J/\psi}$ ratios collected in Table I. The systematic uncertainties are reduced compared to those observed in some regions of the differential distributions in Fig. 1. We also note that the NRQM $R_{\eta_c}$ and $R_{J/\psi}$ ratios are systematically bigger and smaller, respectively, than those obtained with pQCD form factors. For the $R_{\eta_c}$ case evaluated with the NP Fit 7 where an increase of only 10% is found. In fact, at the $\eta_c$ level of ratios, only the NRQM $R_{\eta_c}$ discriminates between NP Fits 6 and 7. One can compare the values for $R_{J/\psi}$ with the only available experimental measurement quoted above. In this case we see all predictions fall short of the present central experimental value by almost 2$\sigma$, adding in quadratures the errors given in Eq. (2). The agreement is slightly better when using the pQCD form-factor set. The improvement is not significant, however, within the present accuracy in the data and, as we explain below, there are some inconsistencies in the corresponding pQCD form factors for this decay.

[3] The kinematical treatment is fully relativistic, but close to $q^2 = 0$, the transition matrix elements are sensitive to large momentum components of the non-relativistic meson wave-functions.
Now we discuss results for the $d^2\Gamma/(d\omega d\theta)$ and $d^3\Gamma/(d\omega dE_e)$ double differential distributions. The $a_{0,1,2}$ CM angular and $c_{0,1,2}$ LAB energy expansion coefficients are shown in Figs. 2 and 3, and Figs. 4 and 5 for the $\eta_c$ and $J/\psi$ decay modes, respectively.

For the $B_c \to \eta_c \tau \bar{\nu}_\tau$ decay, both sets of form factors lead to qualitatively very similar results. As in Ref. [29] for the $\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau$ semileptonic decay, we find that, with the exception of $a_0$, all the other expansion coefficients serve the purpose of giving a clear distinction between NP Fits 6 and 7. Thus, different fits that otherwise give similar $d\Gamma/d\omega$ decay widths, can be told apart by looking at these CM angular or LAB energy observables. In addition, we also observe $\omega$--shapes for all $B_c \to \eta_c$ SM and NP coefficients similar to those obtained in Ref. [29] for the $\Lambda_b \to \Lambda_c$ transition, except $a_0$ that here grows with $\omega$ while for the baryon decay it is a decreasing function of $\omega$.
FIG. 4. $\bar{B}_c \to J/\psi \tau \bar{\nu}_\tau$ decay: CM $a_{0,1,2}$ angular expansion coefficients as a function of $\omega$. We show results obtained with both, NRQM (upper panels) and pQCD (lower panels) form factors from Refs. [17] and [21], respectively. The beyond of the SM scenarios Fits 6 and 7 are taken from Ref. [26]. Uncertainty bands as in Fig. 1.

FIG. 5. $\bar{B}_c \to J/\psi \tau \bar{\nu}_\tau$ decay: Same as in Fig. 4 but for the LAB $c_{0,1,2}$ energy expansion coefficients.

The corresponding results for a decay into $J/\psi$ are shown in Figs. 4 and 5. There are two distinct features in this case. First, the utility of these observables to distinguish between Fits 6 and 7 and, in some cases, between those NP predictions and the SM results, is not as good as in the $\eta_c$ case. This happens to be true independent of the form-factor set used. Second, the results obtained with the two form-factor sets turn out to be very different in most cases, with only $a_0$ and $a_1$ showing a similar qualitative behavior. In order to better understand this discrepancy, we show in Fig. 6 all the form factors defined in Eqs. (A1) and (A2), and that correspond to decays into both $\eta_c$ and $J/\psi$. In the left panel we give the results obtained with the AL1 NRQM of Ref. [17]. We see the results are close to expectations from Eq. (A7) based on HQSS. In the middle panel we give the results obtained using the form factors from Ref. [21] and the quark level equations of motion from Ref. [45]. Large violations of HQSS are already seen for
$h_{A_2}$ and $h_{A_3}$ (where no quark level equations of motion are involved), also for $h_{T_3}$ and, to a lesser extent, for $h_P$. These HQSS violations are related, at least in part, to the fact that the $A_0(q^2)$, $A_1(q^2)$ and $A_2(q^2)$ axial form factors evaluated in Ref. [21], and in terms of which the $h_{A_{1,2,3}}$ ones are determined, do not respect the $q^2 = 0$ constraint

$$A_0(0) = \frac{M + M'}{2M'} A_1(0) - \frac{M - M'}{2M'} A_2(0)$$

(7)

Even though $q^2 \geq m^2$ for a final $\tau$, taking the wrong values of the form factors at $q^2 = 0$ affects the determination of the values at larger $q^2$. In the right panel of Fig. 6 we see the effect of imposing the above constraint on $A_0(0)$. The $h_P$ form factor is now in agreement with HQSS expectations and things improve for $h_{A_2}$, $h_{A_3}$ and $h_{T_3}$. Note that this restriction also corrects the divergences at $q^2 = 0$ that otherwise appear for $h_{A_2}$ and $h_{A_3}$ and which signatures are clearly visible in the middle panel at large recoils.

In Fig. 7 we now show the forward-backward asymmetry in the CM frame evaluated with the form factors from the NRQM of Ref. [17]. This asymmetry is given by the ratio

$$A_{FB} = \frac{a_1(\omega)}{2a_0(\omega) + 2a_2(\omega)/3}$$

(8)

For the decay into $J/\psi$, and as it is the case for other observables, the SM result falls into the error band of Fit 6. However, for the two decays, this observable is also able to distinguish between Fits 6 and 7, in particular for the $\eta_c$ mode.

4 Note that when we use the form factors from Ref. [21], we determine $h_{A_{1,2,3}}$ from $A_{0,1,2}$ and the relation in Eq.(7) has to be satisfied in order for $h_{A_{2,3}}$ not to diverge at $q^2 = 0$. 

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FIG. 6. Different $h_a$ form factors defined in Eqs. (A1) and (A2) for $\bar{B}_c \to \eta_c$ and $\bar{B}_c \to J/\psi$ semileptonic decays. Left panel: Results obtained with the NRQM of Ref. [17] using the AL1 potential. Middle panel: Results obtained from the form factors of Ref. [21] and the use of quark-level equations of motion from Ref. [45]. Right panel: Same as the middle panel but forcing Eq. (7) on $A_0(0)$.
FIG. 7. Forward-backward asymmetry in the CM reference frame for the $B_c \to \eta_c \tau \bar{\nu}_\tau$ (left) and $B_c \to J/\psi \tau \bar{\nu}_\tau$ (right) decays. The results have been obtained with the form factors evaluated with the NRQM of Ref. [17]. Uncertainty bands as in Fig. 1.

B. Results with a polarized final $\tau$ lepton

In this section we collect the results corresponding to the $B_c \to \eta_c \tau \bar{\nu}_\tau$ and $B_c \to J/\psi \tau \bar{\nu}_\tau$ decays with a polarized $\tau$ (well defined helicity $h = \pm 1$ in the CM or LAB frames). In this case, and for simplicity, we will only present results obtained with the use of the NRQM form factors from Ref. [17].

In Fig. 8 we show the $d\Gamma/d\omega$ differential decay width for the $B_c \to \eta_c \tau \bar{\nu}_\tau$ decay with a final $\tau$ with well defined helicity in the CM reference frame (left panel) and in the LAB frame (right panel). The corresponding results for the $B_c \to J/\psi \tau \bar{\nu}_\tau$ decay are presented in Fig. 9. The negative helicity contribution is dominant in all cases except for the $\eta_c$ CM distributions obtained both in the SM and in Fit 6. This, unexpected feature, also occurs for the polarized $B \to D \tau \bar{\nu}_\tau$ decay (see Appendix D). We see that both CM and LAB $\tau$ negative-helicity distributions obtained in $\eta_c$ decays clearly discriminate between SM and different NP scenarios. On the other hand, for the $B_c \to J/\psi \tau \bar{\nu}_\tau$ decay $d\Gamma/d\omega$ is not an efficient tool for that purpose, even taking into account information on the outgoing $\tau$ polarization.

As shown in Ref. [29], for a polarized final $\tau$ lepton with well defined helicity $h = \pm 1$, the CM angular and LAB energy distributions are respectively determined by

$$\frac{2 \sum |M|^2}{M^2(1 - \frac{m_{\tau}^2}{E_{\tau}^2})} = a_0(\omega, h) + a_1(\omega, h) \cos \theta_\ell + a_2(\omega, h)(\cos \theta_\ell)^2$$

$$\frac{2 \sum |M|^2}{M^2} = \frac{1}{2} \left( c_0 + c_1 \frac{E_\ell}{M} + c_2 \frac{E_\ell^2}{M^2} \right) - \frac{h M}{2 p_\ell} \left( \tilde{c}_0 + [c_0 + \tilde{c}_1] \frac{E_\ell}{M} + [c_1 + \tilde{c}_2] \frac{E_\ell^2}{M^2} + [c_2 + \tilde{c}_3] \frac{E_\ell^3}{M^3} \right)$$

FIG. 8. CM (left) and LAB (right) helicity decomposition of the $d\Gamma(B_c \to \eta_c \tau \bar{\nu}_\tau)/d\omega$ differential decay width, calculated with the NRQM form factors from Ref. [17]. Uncertainty bands as in Fig. 1.
In the latter equation, \( p_\ell \) is the final charged lepton three-momentum modulus measured in the LAB frame. The general expressions of the \( a_{0,1,2}(\omega, h) \) and \( \tilde{c}_{0,1,2,3}(\omega) \) coefficients in terms of the W SFs can be found in Ref. [29]. In Figs. 10 and 11 we present the results for the \( \bar{B}_c \rightarrow J/\psi \tau \bar{\nu}_\tau \) decay with a polarized \( \tau \) with positive (upper panels) and negative (lower panels) helicity. They have been evaluated with the NRQM form factors from Ref. [17]. Uncertainty bands as in Fig. 1.

We see that even taking uncertainties into account, Fits 6 and 7 provide distinct predictions for all non-zero angular coefficients that also differ from the SM results, with the exception of \( a_0(\omega, h = +1) \), for which SM and NP Fit 6 results overlap below \( \omega \leq 1.1 \). We also observe that for this decay, the relations

\[
a_0(\omega, h = -1) = -a_2(\omega, h = -1), \quad a_1(\omega, h = -1) = 0
\]  

are satisfied because of angular momentum conservation. Since both the initial and final hadrons have zero spin, the virtual particle exchanged (a W boson in the SM) should have helicity zero. In the CM system this corresponds to a zero spin projection along the quantization axis defined by its three-momentum in the LAB frame, the same axis that is defined by the final hadron LAB (or CM) three-momentum. Thus, in the CM system, the angular momentum of the final lepton pair measured along that axis must be zero. As a consequence, the CM helicity of a final \( \tau \) lepton...
emitted along that direction, which corresponds to either \( \theta_\ell = 0 \) or \( \theta_\ell = \pi \), must equal that of the \( \bar{\nu}_\tau \), the latter being always positive. This means that a negative helicity \( \tau \) cannot be emitted in the CM system when \( \theta_\ell = 0 \) or \( \pi \). Looking at Eq. (9), this implies that \( a_0(\omega, h = -1) = -a_2(\omega, h = -1) \) and \( a_1(\omega, h = -1) = 0 \).

Besides, at zero recoil CM and LAB frames coincide and angular momentum conservation requires the helicity of the \( \tau \) lepton to equal that of the anti-neutrino. This implies \( a_0(\omega = 1, h = -1) = 0 \), since \( a_2(\omega = 1) \) vanishes, and also the cancellation of Eq. (10) at zero recoil for \( h = -1 \). In fact, the LAB \( d^4\Gamma(\bar{B}_c \rightarrow \eta_c \tau\bar{\nu}_\tau)/(d\omega dE_\ell) \) distribution should cancel for \( h = -1 \) and any value of \( \omega \) when \( E_\ell \) equals its maximum value\(^5\) for that particular \( \omega \). The reason is that this maximum \( E_\ell \) value corresponds necessarily to \( \theta_\ell = \pi \) and in that case the helicity of the \( \tau \) is the same in both CM and LAB frames. Since \( h = -1 \) is forbidden in the CM for that specific kinematics it is also forbidden in the LAB. Note that any violation of these results will require negative helicity anti-neutrinos which means NP contributions with right-handed neutrinos. The possible role of such beyond the SM terms in the explanation of the LFU ratio anomalies have been considered in Refs. [46–56] and their existence has not been discarded by the available \( B \rightarrow D^{(*)} \) data \[57\].

Note also that, as a result of \( a_1(\omega, h = -1) \) being zero for the \( P_b \rightarrow P_c \) decays, the forward-backward asymmetry in the CM system \( (A_{FB} \text{ shown in the left panel of Fig. 7}) \) can only originate from positive helicity \( \tau \)’s. For the same reason, for massless charged leptons \( (\ell = e, \mu) \), \( A_{FB} \) vanishes in the SM for transitions between pseudoscalar mesons.

At maximum recoil, we also observe the approximate result

\[
 a_0(\omega_{\text{max}}, h = +1) - a_1(\omega_{\text{max}}, h = +1) + a_2(\omega_{\text{max}}, h = +1) \approx 0,
\]

that can be readily inferred from the corresponding figures and which corresponds to a small probability of CM positive helicity \( \tau \)’s emitted at \( \theta_\ell = \pi \). This result has a dynamical origin and it is due to the fact that our main contribution selects negative chirality for the final charged lepton\(^6\). A \( \tau \) lepton emitted with positive helicity in the CM frame and with \( \theta_\ell = \pi \) will also have positive helicity in the LAB frame. However, close to maximum recoil its momentum in the LAB is very large and helicity almost equals chirality, hence the cancellation. Note that this result is independent of the spin of the hadrons involved as long as negative chirality lepton current operators are dominant. This behavior can already be seen in the polarized results for the \( \Lambda_b \rightarrow \Lambda_c \) decay shown in Ref. [29].

---

\(^5\) The maximum \( E_\ell^+ \) and minimum \( E_\ell^- \) energy values allowed to the final charged lepton for a given \( \omega \) are

\[
 E_\ell^\pm = \frac{(M - M'\omega)(q^2 + m_\ell^2) \pm M'\sqrt{\omega^2 - T(q^2 - m_\ell^2)}}{2q^2}.
\]

\(^6\) Note that only the \( O_{S_L, S_R} \) and \( O_{T} \) NP terms in Eqs. (3) and (4) select positive chirality for the final charged lepton.
Besides, and for the same reason, the LAB $d^2\Gamma/(d\omega dE_\ell)$ distribution, proportional to the quantity given in Eq. (10), should approximately cancel for $h = +1$ when $E_\ell$ is large enough, i.e. when maximum recoil is approached.

In Fig. 11 we present the results for the $\tilde{\omega}_{0,1,2,3}(\omega)$ coefficients associated to this decay. We observe that $\tilde{\omega}_0(\omega)$ and $\tilde{\omega}_1(\omega)$ are able to distinguish between the two NP fits from Ref. [26] considered in the present work. The other two observables $\tilde{\omega}_2(\omega)$ and $\tilde{\omega}_3(\omega)$, available from the polarized $d^2\Gamma(\bar{B}_c \to \eta_c \tau \bar{\nu}_\tau)/(d\omega dE_\ell)$ distribution, turn out to be very small and negligible when compared with $\tilde{\omega}_1$ and $\tilde{\omega}_2$, respectively (see the plots in Fig. 11). Therefore, these two additional coefficients have little relevance in the discussion of the NP Fits 6 and 7, for which the NP tensor Wilson coefficient $|C_T|$ $\sim 10^{-2}$ is quite small. As discussed in Ref. [29], $\tilde{\omega}_2$ and $\tilde{\omega}_3$ are, however, optimal observables to restrict the validity of NP schemes with larger $|C_T|$ values.

To conclude this section, in Figs. 12 and 13 we collect the corresponding results for the $\bar{B}_c \to J/\psi \tau \bar{\nu}_\tau$ decay. In this case, no angular momentum related restriction is in place for $\omega_{0,1,2}(\omega, h = -1)$ since the final hadron has spin one and there are three possible helicity states. However, one can see that the approximate relation in Eq. (13) is indeed satisfied. Also the discussion above about $d^2\Gamma/(d\omega dE_\ell)$ approximately canceling for $h = +1$ near maximum recoil also applies for this decay.

Although the $\bar{B}_c \to J/\psi$ decay is perhaps easier to measure experimentally, as a general rule we find the $\bar{B}_c \to J/\psi$ observables, also in the case of a polarized $\tau$, are less optimal for distinguishing between NP Fits 6 and 7 than those discussed above for $\bar{B}_c \to \eta_c$ decays, or those presented in Ref. [29] for the related $\Lambda_b \to \Lambda_c$ semileptonic decay.

III. $\bar{B} \to D$ AND $\bar{B} \to D^*$ SEMILEPTONIC DECAY RESULTS WITH AN UNPOLARIZED FINAL $\tau$ Lepton

We present now results for the $\bar{B} \to D\tau \bar{\nu}_\tau$ and $\bar{B} \to D^* \tau \bar{\nu}_\tau$ semileptonic decays. As in the previous section, we shall use the Wilson coefficients and form factors corresponding to Fits 6 and 7 in Ref. [26]. The form factors are taken from Ref. [32], but in Ref. [26] not only the Wilson coefficients but also the $1/m_{b,c}$ and $1/m^2_{h}$ corrections to the form factors are simultaneously fitted to experimental data. To estimate the theoretical uncertainties, for each fit, we shall use different sets of Wilson coefficients and form factors, selected such that the $\chi^2$ merit function computed in [26] changes at most by one unit from its value at the fit minimum. With those sets, for each of the observables that we calculate we determine the maximum deviations above and below their central values. These deviations will give us the 1$\sigma$ theoretical uncertainty and it will be shown as an error band in the figures below.

We start by showing in Fig. 14 the $d\Gamma/d\omega$ differential decay width. Both NP fits give similar results that differ from the SM distribution. The corresponding predictions for the $R_D$ and $R_{D^*}$ ratios are given in Table II. The ratios obtained with NP are in agreement with present experimental results, though they are located at the high-value corner.
FIG. 13. LAB charged lepton energy expansion coefficients $\hat{c}_{0,1,2,3}(\omega)$ for the polarized $\bar{B}_c \to J/\psi \tau \bar{\nu}_\tau$ decay. We also show the $(c_0 + \hat{c}_1)$, $(c_2 + \hat{c}_3)$ and $(c_1 + \hat{c}_2)$ sums in the third top, second and fourth bottom panels, respectively. All the functions have been evaluated with the NRQM form factors from Ref. [17]. Uncertainty bands as in Fig. 1.

FIG. 14. $d\Gamma(\bar{B} \to D\tau \bar{\nu}_\tau)/d\omega$ (left) and $d\Gamma(\bar{B} \to D^*\tau \bar{\nu}_\tau)/d\omega$ (right) differential decay widths, as a function of $\omega$ and in units of $10|V_{cb}|^2$ ps$^{-1}$. We show SM predictions and full NP results obtained using the Wilson coefficients and form factors corresponding to Fits 6 and 7 of Ref. [26]. Uncertainty bands obtained as explained in the main text.

| $\mathcal{R}_D$ | $\mathcal{R}_{D^*}$ |
|----------------|---------------------|
| $|V_{cb}|^2$ | $|V_{cb}|^2$ |
| $0.300 \pm 0.005$ | $0.251 \pm 0.004$ |
| $0.405 \pm 0.048$ | $0.302 \pm 0.014$ |
| $0.389 \pm 0.045$ | $0.306 \pm 0.013$ |

TABLE II. $\mathcal{R}_D$ and $\mathcal{R}_{D^*}$ ratios obtained in the SM and with NP effects from Ref. [26].
of the allowed regions, since they were fitted in Ref. [26] to the previous HFLAV world average values quoted in [2]\textsuperscript{7}. Again, we notice that for these quantities both fits are equivalent within errors and other observables are needed in order to decide between different NP explanations of the experimental data.

FIG. 15. $\bar{B} \to D \tau \bar{\nu}_\tau$ decay: $a_{0,1,2}$ CM angular and $c_{0,1,2}$ LAB energy expansion coefficients as a function of $\omega$. Uncertainty bands as in Fig. 14.

FIG. 16. $\bar{B} \to D^* \tau \bar{\nu}_\tau$ decay: $a_{0,1,2}$ CM angular and $c_{0,1,2}$ LAB energy expansion coefficients as a function of $\omega$. Uncertainty bands as in Fig. 14.

These observables can be the $a_{0,1,2}$ and $c_{0,1,2}$ coefficients in the CM angular $d^2\Gamma/(d\omega d\cos\theta_{\ell})$ and LAB energy $d^2\Gamma/(d\omega dE_{\ell})$ distributions. They are shown in Figs. 15 and 16 for the $\bar{B} \to D \tau \bar{\nu}_\tau$ and $\bar{B} \to D^* \tau \bar{\nu}_\tau$ decays, respectively.

\textsuperscript{7} In the latest HFLAV average [1], a measurement by the BaBar collaboration [58] is omitted, because it does not allow for a separation of the different isospin modes.
With the only exception of $a_0$, all of them can be used to distinguish between the two fits. However, for the $\bar{B} \to D^*$, and similar to what happened for the $\bar{B}_c \to J/\psi$ decay, SM results for some of these coefficients fall within the error band of those obtained from NP Fit 6. In fact the $\omega$—shape patterns exhibited in Figs. 15 and 16 for the $\bar{B} \to D^*$ reactions are qualitatively similar to those found in Sec. II for the $\bar{B}_c$ decays.

We stress that the LAB $d^2\Gamma(\bar{B} \to D^{(*)}\tau\bar{\nu}_\tau)/(d\omega dE_\ell)$ differential decay widths are reported for the very first time in this work. Though, as shown in [29], CM and LAB unpolarized distributions provide access to equivalent dynamical information (invariant functions $A(\omega)$, $B(\omega)$ and $C(\omega)$ defined in Eq. (14) of that reference), it should be explored if the LAB observables could be measured with better precision.

In Fig. 17 we show the CM forward-backward asymmetry (Eq. (8)). The shape in each case is very similar to what we obtained respectively for $\bar{B}_c \to \eta_c$ and $\bar{B}_c \to J/\psi$ decays, see Fig. 7, with very close values at maximum recoil and significantly smaller errors.

![Fig. 17. Forward-backward asymmetry in the CM reference frame for the $\bar{B} \to D$ (left) and $\bar{B} \to D^*$ (right) decays. Uncertainty bands as in Fig. 14.](image1)

In Fig. 18, we show the theoretical predictions for $\mathcal{R}(A_{FB})$ ratios defined in Eq. (14) for the $\bar{B}_c \to J/\psi$, $\bar{B} \to D^*$ and $\Lambda_b \to \Lambda_c$ semileptonic decays, as a function of $\omega$. Errors bands have been calculated as in Figs. 1 and 14 for the $\bar{B}_c$ and $\bar{B}$ meson decays. The $\Lambda_b$ plot has been taken from Fig. 4 of Ref. [29].

To minimize experimental and theoretical uncertainties, it was proposed in Ref. [29] to pay attention to the ratio $\mathcal{R}(A_{FB})$, defined as

$$\mathcal{R}(A_{FB}) = \frac{(A_{FB})_{\tau}^{NP}}{(A_{FB})_{\ell=e,\mu}^{SM}} = \left[ \frac{a_1}{2a_0 + 2a_2/3} \right]_{\tau}^{NP} \left[ \frac{a_1}{2a_0 + 2a_2/3} \right]_{\ell=e,\mu}^{SM}$$

(14)

In Fig. 18, we show the theoretical predictions for $\mathcal{R}(A_{FB})$ for the $\bar{B}_c \to J/\psi$, $\bar{B} \to D^*$ and $\Lambda_b \to \Lambda_c$ semileptonic decays, with the latter taken from Ref. [29] where details of the LQCD form factors used in the calculation can be found. Note that for $\bar{B}_c \to \eta_c$ and $\bar{B} \to D$ decays, the denominator in Eq. (14) vanishes in the massless lepton limit ($m_\ell \to 0$), since $a_1(\omega) = a_1(\omega; h = -1) + a_1(\omega; h = +1)$, and the negative helicity contribution is zero (Eq. (11)), while the positive helicity one is proportional to $m_\ell$. 

![Fig. 18. $\mathcal{R}(A_{FB})$ ratios defined in Eq. (14) for the $\bar{B}_c \to J/\psi$, $\bar{B} \to D^*$ and $\Lambda_b \to \Lambda_c$ semileptonic decays, as a function of $\omega$. Errors bands have been calculated as in Figs. 1 and 14 for the $\bar{B}_c$ and $\bar{B}$ meson decays. The $\Lambda_b$ plot has been taken from Fig. 4 of Ref. [29].](image2)
The ratio $R(A_{FB})$ can be measured by subtracting the number of events seen for $\theta_{L} \in [0, \pi/2]$ and for $\theta_{L} \in [\pi/2, \pi]$ and dividing by the total sum of observed events, in each of the $H_{\tau \rightarrow H_{c} c \bar{\nu}_{\tau}}$ and $H_{b \rightarrow H_{c} c (\mu) \bar{\nu}_{c (\mu)}}$ reactions. We expect that this observable should be free of a good part of experimental normalization errors. On the theoretical side, we see in Fig. 18 that predictions for this ratio have indeed small uncertainties, and that this quantity has the potential to establish the validity of the NP scenarios associated to Fit 7, even more if all three reactions shown in Fig. 18 are simultaneously confronted with experiment.

For completeness, $B \rightarrow D^{(*)}$ results with a polarized final $\tau$ are given in Appendix D. Roughly, the same qualitative features that we have discussed for the polarized $B_{c} \rightarrow \eta_{c}$ and $B_{c} \rightarrow J/\psi$ semileptonic decays are also found in this case.

IV. CONCLUSIONS

We have shown the relevant role that the $a_{0,1,2}(\omega)$ CM and $c_{0,1,2}(\omega)$ LAB scalar functions, in terms of which the CM $d^{2}\Gamma/(d\omega d \cos \theta_{L})$ and LAB $d^{2}\Gamma/(d\omega dE_{L})$ differential decay widths are expanded, could play in order to separate between different NP scenarios that otherwise give rise to the same $R_{D^{(*)}}, R_{\eta_{c},J/\psi}$ ratios. The scheme we have used is the one originally developed in Ref. [29], and applied there to the analysis of the $\Lambda_{b} \rightarrow \Lambda_{c} \tau \bar{\nu}_{\tau}$ decay, that we have extended in this work to the study of the $B \rightarrow D, B \rightarrow D^{*}, B_{c} \rightarrow \eta_{c}$ and $B_{c} \rightarrow J/\psi$ meson reactions.

The analysis of the $B_{c} \rightarrow \eta_{c}, J/\psi$ transitions is novel. We have obtained results from a NRQM scheme, consistent with the expected breaking pattern of HQSS from $B \rightarrow D^{(*)}$ decays [59], estimating the systematic uncertainties caused by the use of different inter-quark potentials. As a general rule, the $B_{c} \rightarrow J/\psi$ observables, even involving $\tau$ polarization, are less optimal for distinguishing between NP scenarios than those obtained from $B_{c} \rightarrow \eta_{c}$ decays, or those discussed in Ref. [29] for the related $\Lambda_{b} \rightarrow \Lambda_{c}$ semileptonic decay. We have also found qualitative similar behaviors for $B \rightarrow D$ and $B_{c} \rightarrow \eta_{c}$, and $B \rightarrow D^{*}$ and $B_{c} \rightarrow J/\psi$ decay observables.

We have also drawn the attention to the ratio $R(A_{FB})$, defined in Eq. (14) and shown in Fig. 18 for $B_{c} \rightarrow J/\psi$, $B \rightarrow D^{*}$ and $\Lambda_{b} \rightarrow \Lambda_{c}$ decays, as a promising quantity, both from the experimental and theoretical points of view, to shed light into details of different NP scenarios in $b \rightarrow c \tau \bar{\nu}_{\tau}$ transitions.

One should notice however that the effective Hamiltonian of Eq. (3), despite excluding right-handed neutrino terms, contains five, complex in general, NP Wilson coefficients. While one of them can always be taken to be real, that still leaves nine free parameters to be determined from data. Even assuming that the form factors were known, and therefore the genuinely hadronic part (W) of the $\bar{W}$ SFs, it would be difficult to determine all NP parameters just from the study of a unique reaction. As shown in Ref. [29], for decays with an unpolarized final $\tau$ lepton, the CM $d^{2}\Gamma/(d\omega d \cos \theta_{L})$ and LAB $d^{2}\Gamma/(d\omega dE_{L})$ differential decay widths are completely determined by only three independent functions which are linear combinations of the $\bar{W}$ SFs, the latter depending on the NP Wilson coefficients. This means that $a_{0,1,2}(\omega)$ and $c_{0,1,2}(\omega)$ contain the same information. For the case of polarized final $\tau$'s, the CM $d^{2}\Gamma/(d\omega d \cos \theta_{L})$ and LAB and $d^{2}\Gamma/(d\omega dE_{L})$ distributions provide complementary information giving access to another five independent linear combinations of the $\bar{W}$'s [29]. But in this case it is the experimental measurement of the required polarized decay that could become a very difficult task. We think it is therefore more convenient to analyze data from various types of semileptonic decays simultaneously (e.g. $B \rightarrow D, B \rightarrow D^{*}, \Lambda_{b} \rightarrow \Lambda_{c}, B_{c} \rightarrow \eta_{c}, B_{c} \rightarrow J/\psi...$), considering both the $e/\mu$ and $\tau$ modes. The scheme presented in [29] is a powerful tool to achieve this objective.

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Appendix A: Form Factors for $P_{b}(0^{-}) \rightarrow P_{c}(0^{-})$ and $P_{b}(0^{-}) \rightarrow P_{c}^{*}(1^{-})$ transitions

For these two transitions we use the standard definitions of the form factors taken from Ref. [32]8.

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8 Note however that within the conventions of Ref. [29], that we follow here, our hadronic matrix elements are dimensionless and they should be compared to those given in [32] divided by a $\sqrt{2M\sqrt{2M}^2}$ factor.
\( P_b \to P_c \)

\[
\langle P_c; \vec{p}'|\bar{c}(0)b(0)|P_b; \vec{p}\rangle = \frac{1 + \omega}{2} h_S(\omega)
\]

\[
\langle P_c; \vec{p}'|\bar{c}(0)\gamma_5 b(0)|P_b; \vec{p}\rangle = \langle P_c; \vec{p}'|\bar{c}(0)\gamma^\alpha \gamma_5 b(0)|P_b; \vec{p}\rangle = 0
\]

\[
\langle P_c; \vec{p}'|\bar{c}(0)\gamma^\alpha b(0)|P_b; \vec{p}\rangle = \frac{1}{2} (v^\alpha + v'^\alpha) h_+(\omega) + \frac{1}{2} (v^\alpha - v'^\alpha) h_-(\omega)
\]

\[
\langle P_c; \vec{p}'|\bar{c}(0)\sigma^{\alpha\beta} b(0)|P_b; \vec{p}\rangle = \frac{i}{2} (v^\alpha v^\beta - v'^\alpha v'^\beta) h_T(\omega)
\]

\[
\langle P_c; \vec{p}'|\bar{c}(0)\sigma^{\alpha\beta} \gamma_5 b(0)|P_b; \vec{p}\rangle = \frac{1}{2} \epsilon^{\alpha\beta\delta\eta} v'_\delta v_\eta h_T(\omega)
\]

(A1)

\[
\text{with } v^\alpha = p^\alpha/M \text{ and } v'^\alpha = p'^\alpha/M' = (p^\alpha - q^\alpha)/M', \text{ the quadrivelocity of the initial and final hadrons, which have masses } M \text{ and } M', \text{ respectively, } \omega = (v \cdot v') \text{ and } \epsilon_{0123} = +1.
\]

\( P_b \to P_c^* \)

\[
\langle P_c^*; \vec{p}', r|\bar{c}(0)b(0)|P_b; \vec{p}\rangle = 0
\]

\[
\langle P_c^*; \vec{p}', r|\bar{c}(0)\gamma_5 b(0)|P_b; \vec{p}\rangle = -\frac{1}{2} (\epsilon_\sigma \cdot v) h_P(\omega)
\]

\[
\langle P_c^*; \vec{p}', r|\bar{c}(0)\gamma^\alpha b(0)|P_b; \vec{p}\rangle = \frac{i}{2} \epsilon^{\alpha\beta\delta\eta} \epsilon_r v_\delta v_\eta h_T(\omega)
\]

\[
\langle P_c^*; \vec{p}', r|\bar{c}(0)\sigma^{\alpha\beta} b(0)|P_b; \vec{p}\rangle = \frac{\omega + 1}{2} \epsilon^\alpha h_{A_1}(\omega) - \frac{(\epsilon_\sigma \cdot v)}{2} \left[ v^\alpha h_{A_2}(\omega) + v'^\alpha h_{A_3}(\omega) \right]
\]

\[
\langle P_c^*; \vec{p}', r|\bar{c}(0)\sigma^{\alpha\beta} \gamma_5 b(0)|P_b; \vec{p}\rangle = -\frac{i}{2} \epsilon^{\alpha\beta} \delta_\sigma \left\{ \epsilon_r \left[ (v^\alpha + v'^\alpha) h_{T_1}(\omega) + (v^\alpha - v'^\alpha) h_{T_2}(\omega) \right] + v^\alpha v'^\alpha (\epsilon_\sigma \cdot v) h_{T_3}(\omega) \right\}
\]

(A2)

where \( r \) is the helicity of the final vector meson, with \( \epsilon_\sigma \) its corresponding polarization vector. In short,

\[
\langle P_c^*; \vec{p}', r|\bar{c}(0)\Gamma^{(\alpha\beta)} b(0)|P_b; \vec{p}\rangle = T^{(\alpha\beta)}_{\mu}(\omega) \epsilon^\mu_{\mu}
\]

(A3)

with \( \Gamma^{(\alpha\beta)} = 1, \gamma_5, \gamma^\alpha \gamma_5, \sigma^{\alpha\beta} \) and \( \sigma^{\alpha\beta} \gamma_5 \) and \( T^{(\alpha\beta)}_{\mu} \) read from Eq. (A2).

The form factors are real functions of \( \omega \) greatly constrained by HQSS near zero recoil \( (\omega = 1) [32, 59] \). Indeed, all factors in Eqs. (A1) and (A2) have been chosen such that in the heavy quark limit each form factor either vanishes or equals the leading-order Isgur-Wise function\(^9\)

\[
\hat{h}_- = h_{A_2} = h_{T_2} = h_{T_3} = 0, \quad h_+ = h_V = h_{A_1} = h_{A_3} = h_S = h_P = h_T = h_{T_1} = \xi
\]

(A7)

The hadron tensors and \( \bar{W} \) SFs introduced in Ref. [29] are straightforwardly obtained from Eq. (A1) in the case of \( P_b \to P_c^* \) transitions, while for decays into vector mesons, we use

\[
\sum_r \langle P_c^*; \vec{p}', r|\bar{c}(0)\Gamma^{(\alpha\beta)} b(0)|P_b; \vec{p}\rangle \langle P_c^*; \vec{p}', r|\bar{c}(0)\Gamma^{(\rho\lambda)} b(0)|P_b; \vec{p}\rangle^* = T^{(\alpha\beta)}_{\mu} T^{(\rho\lambda)}_{\nu} (-g^\mu\nu + \eta^\mu v'^\nu)
\]

(A8)

The explicit expressions for the \( \bar{W} \) SFs in terms of the above form factors and the Wilson coefficients are given in the following appendix.

---

\(^9\) These relations trivially follow from

\[
\langle P_c^{(*)}; \vec{p}', (r)|\bar{c}(0)\Gamma b(0)|P_b; \vec{p}\rangle = -\frac{1}{2} \epsilon(\omega) \text{Tr} \left[ R^{(c)}_{\nu\rho} \Gamma H^{(b)}_{\nu\rho} \right]
\]

(A4)

where the pseudoscalar and vector mesons are represented by a superfield, which has the right transformation properties under heavy quark and Lorentz symmetry [32, 59]

\[
H^{(Q)}_{\nu} = \frac{1 + \gamma^5}{2} \left( V^{(Q)}_{\nu} - F^{(Q)}_{\nu} \gamma_5 \right)
\]

(A5)

and \( R^{(Q)}_{(1)} = \gamma^0 H^{(Q)}_{\nu} \gamma_5 \gamma^0 \). For \( B_c \to \eta_c, J/\psi \) transitions, the appropriate 4 \times 4 field accounts also for the heavy anticharm quark both in the initial and final mesons [60]

\[
H^{(Q)}_{\nu} = \frac{1 + \gamma^5}{2} \left( V^{(Q)}_{\nu} - F^{(Q)}_{\nu} \gamma_5 \right) \frac{1 - \gamma^5}{2}
\]

(A6)
Appendix B: Hadron tensor $\bar{W}$ SFs for the $P_b \to P_c \ell^- \bar{\nu}_\ell$ and $P_b \to P_c^* \ell^- \bar{\nu}_\ell$ decays

We compile here the $\bar{W}$ SFs introduced in Ref. [29] for the particular meson decays studied in this work. As shown in that reference, these $\bar{W}$ SFs determine the LAB $d\Gamma/(d\omega dE_\ell)$ and CM $d\Gamma/(d\omega d\cos \theta_\ell)$ differential decay widths, for the full set of NP operators in Eq. (3), for generally complex Wilson coefficients, and for the case where the final charged lepton has a well defined helicity in either reference frame.

1. $P_b \to P_c \ell^- \bar{\nu}_\ell$

In this case, the SFs related to the SM currents are

$$\bar{W}_1 = \bar{W}_3 = 0, \quad \bar{W}_2 = \frac{|C_V|^2}{r} F_2^\ell, \quad \bar{W}_4 = \frac{|C_V|^2}{4r} (F_+ - F_-)^2, \quad \bar{W}_5 = \frac{|C_V|^2}{r} F_+ (F_- - F_+)$$

(B1)

where

$$F_+ = \frac{1}{R} \left( h_+ - \frac{1 - r}{1 + r} h_- \right), \quad F_- = \frac{1}{R} \left( h_- - \frac{1 - r}{1 + r} h_+ \right) = \frac{1 - r^2}{1 + r^2 - 2r\omega} (F_0 - F_+)$$

$$F_0 = \frac{2r(1 + \omega)}{R(1 + r)^2} \left[ h_+ - \frac{1 + r \omega - 1}{1 - r \omega + 1} h_- \right]$$

(B2)

with $r = M'/M$ and $R = 2\sqrt{T}/(1 + r)$, and we have also introduced the $F_0$ form-factor in the definition of $F_-$, as commonly done in this type of calculations. In addition,

$$\bar{W}_{1SP} = |C_S|^2 \left( \frac{1 + \omega}{2} \right)^2 h_5^S, \quad \bar{W}_{11} = C_V C_S \frac{1 + \omega}{2\sqrt{r}} h_S F_+, \quad \bar{W}_{12} = C_V C_S \frac{1 + \omega}{2\sqrt{r}} h_S (F_- - F_+)$$

$$\bar{W}_{13} = -C_T^* C_S \frac{1 + \omega}{2r} h_S h_T, \quad \bar{W}_{14} = -C_T^* C_V \frac{h_T F_+}{r^{3/2}}, \quad \bar{W}_{15} = C_T^* C_V \frac{h_T (F_+ - F_-)}{2r^{3/2}}, \quad \bar{W}_{16} = \bar{W}_{17} = 0$$

$$\bar{W}_1^T = \frac{|C_T|^2}{4r} (\omega^2 - 1) h_T^2, \quad \bar{W}_2^T = \frac{|C_T|^2}{4r^2} (1 + r^2 - 2r\omega) h_T^2, \quad \bar{W}_3^T = \frac{|C_T|^2}{4r^2} h_T^2,$$\n
$$\bar{W}_4^T = -\frac{|C_T|^2}{4r^2} (1 - r\omega) h_T^2, \quad \bar{W}_5^T = 0.$$ (B3)

As derived in Ref. [29], the tensor $\bar{W}$ SFs accomplish:

$$2\bar{W}_1^T + \bar{W}_2^T + (1 - 2r\omega + r^2)\bar{W}_3^T + 2(1 - r\omega)\bar{W}_4^T = 0$$

(B4)

2. $P_b \to P_c^* \ell^- \bar{\nu}_\ell$

In this case, the $\bar{W}$ SFs related to the SM currents are

$$\bar{W}_1 = \frac{|C_V|^2}{4r} (\omega^2 - 1) h_V^2 + \frac{|C_A|^2}{4r} (\omega + 1)^2 h_A^2,$$

$$\bar{W}_2 = -\frac{|C_V|^2}{4r^2} (1 + r^2 - 2r\omega) h_V^2 + \frac{|C_A|^2}{4r^2} (\omega + 1)^2 \left( h_A^2 - 2 \frac{\omega - r}{\omega + 1} h_A (r h_{A_2} + h_{A_3}) + \frac{\omega - 1}{\omega + 1} (r h_{A_2} + h_{A_3})^2 \right)$$

$$\bar{W}_3 = \frac{\text{Re}[C_V C_A^*]}{r} (\omega + 1) h_V h_A,$$\n
$$\bar{W}_4 = -\frac{|C_V|^2}{4r^2} h_V^2 + \frac{|C_A|^2}{4r^2} (\omega + 1)^2 (h_A^2 - h_{A_3}) \left( h_A + \frac{1 - \omega}{1 + \omega} h_{A_3} \right)$$

$$\bar{W}_5 = \frac{|C_V|^2}{2r^2} (1 - r\omega) h_V^2 - \frac{|C_A|^2}{2r^2} \left\{ (1 + \omega) (h_{A_1} - r h_{A_2} - h_{A_3}) [(1 + \omega) h_{A_1} - (\omega - r) h_{A_3}] + (1 + \omega) (r h_{A_2} + h_{A_3}) (h_{A_1} - (1 - r) h_{A_3}) \right\}. \quad \text{(B5)}$$
The rest of NP $\tilde{W}$ SFs are

$$\tilde{W}_{SP} = \frac{|C_P|^2}{4} (\omega^2 - 1) h_P^2$$

$$\tilde{W}_{I1} = \frac{C_A C_P^*}{2r} (\omega^2 - 1) \left[ r h_{A_2} + h_{A_3} - \frac{\omega - r}{\omega - 1} h_{A_1} \right] h_P$$

$$\tilde{W}_{I2} = \frac{C_A C_P^*}{2r} (\omega + 1) \left[ \omega h_{A_1} + (1 - \omega) h_{A_3} \right] h_P$$

$$\tilde{W}_{I3} = -\frac{C_P C_T^*}{2r} (\omega^2 - 1) h_P T_1$$

$$\tilde{W}_{I4} = \frac{C_P C_T^*}{2r} \left[ (1 - r\omega) T_2 + (\omega - r) T_3 \right] h_V - \frac{C_A C_P^*}{2r^2} \left\{ (\omega + 1) h_{A_1} \left[ (r - \omega) T_1 + r T_2 + T_3 \right] + (\omega^2 - 1) (r h_{A_2} + h_{A_3}) T_1 \right\}$$

$$\tilde{W}_{I5} = -\frac{C_P C_T^*}{2r} \left[ (T_2 + \omega T_3) h_V + \frac{C_A C_T^*}{2r^2} \left\{ (\omega^2 - 1) h_{A_1} T_1 - (\omega + 1) h_{A_3} \left[ \omega T_1 - T_3 \right] \right\} \right\}$$

$$\tilde{W}_{I6} = \frac{C_P C_T^*}{2r} (\omega^2 - 1) h_V (r T_2 + T_3) - \frac{C_A C_T^*}{2r} (\omega + 1) h_{A_1} \left[ (1 - r\omega) T_2 + (\omega - r) T_3 \right]$$

$$\tilde{W}_{I7} = -\frac{C_P C_T^*}{2r} (\omega^2 - 1) h_V T_3 + \frac{C_A C_T^*}{2r} (\omega + 1) h_{A_1} \left[ T_2 + \omega T_3 \right]$$

$$\tilde{W}_1^T = \frac{|C_T|^2}{4} (\omega^2 - 1)^2 T_1^2$$

$$\tilde{W}_2^T = \frac{|C_T|^2}{4r^2} \left[ (r^2 - 2r\omega + 1)(\omega^2 - 1) T_1^2 + (1 - r^2)(T_2^2 - T_3^2) + 2r(1 - \omega)(\omega T_2 + T_3) T_2 + 2(r - \omega)(T_2 + \omega T_3) T_3 \right]$$

$$\tilde{W}_3^T = \frac{|C_T|^2}{4r^2} \left[ (\omega^2 - 1)(T_1^2 - T_3^2) - (T_2 + \omega T_3)^2 \right]$$

$$\tilde{W}_4^T = \frac{|C_T|^2}{4r^2} \left[ (1 - r\omega)(T_2^2 + T_3^2) + 2(\omega - r) T_2 T_3 + (\omega^2 - 1)(2T_3^2 - (1 - r\omega) T_3^2) \right]$$

$$\tilde{W}_5^T = 0$$

with the tensor $\tilde{W}_{1,2,3,4}^T$ SFs satisfying Eq. (B4), and

$$T_1 = \frac{(\omega + 1) h_{T_1} + (\omega - 1) h_{T_2}}{\omega^2 - 1}, \quad T_2 = -\frac{(\omega + 1) h_{T_1} + (\omega - 1) h_{T_2}}{\omega^2 - 1}, \quad T_3 = \frac{(\omega + 1) h_{T_1} - (\omega - 1) h_{T_2}}{\omega^2 - 1}$$

(B7)

Although $T_1, T_2$ and $T_3$ behave as $\pm 1/(\omega - 1)$ in the heavy quark limit, the corresponding $\tilde{W}_{1,2,3,4}^T$ SFs are finite at zero recoil, as they should be, with their values being given by $|C_T|^2 \left\{ 1, -\frac{r^2 + 6r + 1}{4r^2}, -\frac{1}{4r^2}, -\frac{3r + 1}{4r^2} \right\}$, respectively.

**Appendix C: Evaluation of the $\bar{B}_c \to \eta_c$ and $\bar{B}_c \to J/\psi$ semileptonic decay form factors within the NRQM of Ref. [17]**

Within the NRQM calculation of Ref. [17], and with the global phases used in the present work, we obtain the following expressions for the different form factors.

1. $\bar{B}_c \to \eta_c$

For the pseudoscalar-pseudoscalar $\bar{B}_c \to \eta_c$ transition we have

$$F_+ = \frac{1}{2M} \left( V_0^0 + \frac{V_0^3 E' - M}{|\vec{q}|} \right), \quad F_- = \frac{1}{2M} \left( V_0^0 + \frac{V_0^3 E' + M}{|\vec{q}|} \right),$$

$$h_S = \frac{S}{(\omega + 1)\sqrt{M M'}}; \quad h_T = -\sqrt{\frac{M'}{M}} \frac{T^{03}}{M |\vec{q}|},$$

with $F_\pm$ defined in Eq. (B2), and where $V_\mu$, $S$ and $T^{\mu\nu}$ stand for the NRQM matrix elements of the vector, scalar and tensor $b \to c$ transition currents, respectively. Besides, $E' = \sqrt{M'^2 + \vec{q}'^2}$ is the energy of the final meson that has
three-momentum $-\vec{q}$ in the LAB frame. Note that $\vec{q}$ is the LAB three-momentum transferred and, for the purpose of calculation, we take it along the positive $Z$ axis. For the matrix elements one has the results

\[
\begin{align*}
V^0 &= \sqrt{2M'2E'} \int d^3p \frac{1}{4\pi} \left[ \phi^{(n_\text{c})}(\vec{p}\vec{\bar{p}}) \right] \phi^{(B_c)} \left( |\vec{p} - \frac{1}{2} \vec{q}| \right) \sqrt{\frac{E_c E_b}{4E_c E_b}} \left( 1 + \left( \frac{1}{2} |\vec{q}| - p_z \right) \cdot \left( \frac{1}{2} \vec{q} - \vec{p} \right) \right), \\
V^3 &= \sqrt{2M'2E'} \int d^3p \frac{1}{4\pi} \left[ \phi^{(n_\text{c})}(\vec{p}\vec{\bar{p}}) \right] \phi^{(B_c)} \left( |\vec{p} - \frac{1}{2} \vec{q}| \right) \sqrt{\frac{E_c E_b}{4E_c E_b}} \left( \frac{1}{2} |\vec{q}| - p_z \right) + \frac{1}{2} |\vec{q}| - p_z \right) \right), \\
S &= \sqrt{2M'2E'} \int d^3p \frac{1}{4\pi} \left[ \phi^{(n_\text{c})}(\vec{p}\vec{\bar{p}}) \right] \phi^{(B_c)} \left( |\vec{p} - \frac{1}{2} \vec{q}| \right) \sqrt{\frac{E_c E_b}{4E_c E_b}} \left( 1 - \left( \frac{1}{2} |\vec{q}| - p_z \right) \cdot \left( \frac{1}{2} \vec{q} - \vec{p} \right) \right), \\
T^{03} &= i\sqrt{2M'2E'} \int d^3p \frac{1}{4\pi} \left[ \phi^{(n_\text{c})}(\vec{p}\vec{\bar{p}}) \right] \phi^{(B_c)} \left( |\vec{p} - \frac{1}{2} \vec{q}| \right) \sqrt{\frac{E_c E_b}{4E_c E_b}} \left( \frac{1}{2} |\vec{q}| - p_z \right) + \frac{1}{2} |\vec{q}| - p_z \right) \right),
\end{align*}
\]

(C2)

Here, $\hat{\phi}$ stands for the orbital part of the meson wave functions in momentum space and $\hat{E}_f = E_f + m_f$, with $m_f, E_f$ the mass and relativistic energy of the quark with flavor $f$. The corresponding three-momenta are $(|\vec{q}|/2 - \vec{p})$ and $(-|\vec{q}|/2 - \vec{p})$ for the quarks $b$ and $c$, respectively.

2. $B_c \rightarrow J/\psi$

For the pseudoscalar-vector $\tilde{B}_c \rightarrow J/\psi$ transition we now have

\[
\begin{align*}
h_V &= \frac{M' \sqrt{2}}{|\vec{q}| \sqrt{M' M''}}, \quad h_P = \frac{M' P_{\lambda=0}}{|\vec{q}| \sqrt{M' M''}}, \\
h_A_1 &= \frac{\sqrt{2}}{\omega + 1} \frac{A_{\lambda=-1}^1}{\sqrt{M' M''}}, \quad h_{A_2} = -\frac{M' A_{\lambda=-1}^0}{|\vec{q}| \sqrt{M' M''}} - \frac{E' A_{\lambda=-1}^0}{|\vec{q}| \sqrt{M' M''}} + \sqrt{2} \frac{M' A_{\lambda=-1}^1}{|\vec{q}| \sqrt{M' M''}}, \\
h_{A_3} &= -\frac{M' T_{\lambda=0}^2}{|\vec{q}|^2 \sqrt{M' M''}} - \sqrt{2} \frac{E' A_{\lambda=-1}^1}{M' \sqrt{M' M''}}, \\
T_1 &= \frac{M' T_{\lambda=0}^1}{|\vec{q}| \sqrt{M' M''}}, \quad T_2 = \sqrt{2} \frac{M' T_{\lambda=0}^1}{|\vec{q}| \sqrt{M' M''}}, \quad T_3 = \sqrt{2} \frac{M' T_{\lambda=0}^3}{|\vec{q}| \sqrt{M' M''}},
\end{align*}
\]

(C3)

with $T_{1,2,3}$ defined in Eq. (B7) and $V_1^\mu, A_1^\mu, P_\lambda$ and $T_{\lambda\mu}^\nu$ the NRQM matrix elements of the vector, axial, pseudoscalar and tensor $b \rightarrow c$ transition currents, respectively. Here, $\lambda$ is the polarization of the final $J/\psi$ meson. We use states that have well defined spin in the $Z$ direction in the $J/\psi$ rest frame. Since the $J/\psi$ three-momentum equals $-\vec{q}$ (which is directed along the negative $Z$ axis), $\lambda$ coincides with minus the helicity, the latter being the same in the CM and LAB frames. We obtain the following expressions for the matrix elements

\[
\begin{align*}
V_{\lambda=-1}^1 &= -\frac{1}{\sqrt{2}} \sqrt{2M'2E'} \int d^3p \frac{1}{4\pi} \left[ \phi^{(J/\psi)}(\vec{p}\vec{\bar{p}}) \right] \phi^{(B_c)} \left( |\vec{p} - \frac{1}{2} \vec{q}| \right) \sqrt{\frac{E_c E_b}{4E_c E_b}} \left( -\frac{1}{2} |\vec{q}| - p_z \right) \right), \\
A_{\lambda=0}^1 &= \sqrt{2M'2E'} \int d^3p \frac{1}{4\pi} \left[ \phi^{(J/\psi)}(\vec{p}\vec{\bar{p}}) \right] \phi^{(B_c)} \left( |\vec{p} - \frac{1}{2} \vec{q}| \right) \sqrt{\frac{E_c E_b}{4E_c E_b}} \left( -\frac{1}{2} |\vec{q}| - p_z \right) \right), \\
A_{\lambda=-1}^1 &= \frac{1}{\sqrt{2}} \sqrt{2M'2E'} \int d^3p \frac{1}{4\pi} \left[ \phi^{(J/\psi)}(\vec{p}\vec{\bar{p}}) \right] \phi^{(B_c)} \left( |\vec{p} - \frac{1}{2} \vec{q}| \right) \sqrt{\frac{E_c E_b}{4E_c E_b}} \left( 1 + \frac{2p_z^2}{E_c E_b} \right) \\
&\times \left( 1 + \frac{2(-\frac{1}{2} |\vec{q}| - p_z) \cdot (\frac{1}{2} |\vec{q}| - p_z)}{E_c E_b} \right),
\end{align*}
\]
\[ P_{\lambda=0} = \sqrt{2M^2E^2} \int d^3p \frac{1}{4\pi} \begin{bmatrix} \phi^{(J/\psi)}(\bar{p}|p) \end{bmatrix}^* \phi^{(B_c)}(\bar{q}-\frac{1}{2}\bar{q}') \left( \frac{1}{2} |\bar{q}'| - p_z - \frac{1}{2} |\bar{q}'| - p_z \right) \frac{\sqrt{\bar{E}_c\bar{E}_b}}{E_c E_b} \]

\[ T_{\lambda=0}^{12} = \sqrt{2M^2E^2} \int d^3p \frac{1}{4\pi} \begin{bmatrix} \phi^{(J/\psi)}(\bar{p}|p) \end{bmatrix}^* \phi^{(B_c)}(\bar{q}-\frac{1}{2}\bar{q}') \left( \frac{1}{2} |\bar{q}'| - p_z - \frac{1}{2} |\bar{q}'| - p_z \right) \frac{\sqrt{\bar{E}_c\bar{E}_b}}{E_c E_b} \times \left( 1 - 2\frac{1}{2} |\bar{q}'| - p_z \right) + \frac{1}{2} |\bar{q}'| - p_z \right) \frac{\sqrt{\bar{E}_c\bar{E}_b}}{E_c E_b} \]

\[ T_{\lambda=-1}^{01} = \int d^3p \frac{1}{4\pi} \begin{bmatrix} \phi^{(J/\psi)}(\bar{p}|p) \end{bmatrix}^* \phi^{(B_c)}(\bar{q}-\frac{1}{2}\bar{q}') \left( \frac{1}{2} |\bar{q}'| - p_z - \frac{1}{2} |\bar{q}'| - p_z \right) \frac{\sqrt{\bar{E}_c\bar{E}_b}}{E_c E_b} \times \left( 1 - 2\frac{1}{2} |\bar{q}'| - p_z \right) + \frac{1}{2} |\bar{q}'| - p_z \right) \frac{\sqrt{\bar{E}_c\bar{E}_b}}{E_c E_b} \]

\[ T_{\lambda=-1}^{23} = \int d^3p \frac{1}{4\pi} \begin{bmatrix} \phi^{(J/\psi)}(\bar{p}|p) \end{bmatrix}^* \phi^{(B_c)}(\bar{q}-\frac{1}{2}\bar{q}') \left( \frac{1}{2} |\bar{q}'| - p_z - \frac{1}{2} |\bar{q}'| - p_z \right) \frac{\sqrt{\bar{E}_c\bar{E}_b}}{E_c E_b} \times \left( 1 - 2\frac{1}{2} |\bar{q}'| - p_z \right) + \frac{1}{2} |\bar{q}'| - p_z \right) \frac{\sqrt{\bar{E}_c\bar{E}_b}}{E_c E_b} \]

\[ (C4) \]

Appendix D: Results for the $B \to D\tau\bar{\nu}_\tau$ and $B \to D^*\tau\bar{\nu}_\tau$ decays for the case of a polarized final $\tau$

![Graphs showing helicity decomposition of differential decay width](image)

**FIG. 19.** CM (left) and LAB (right) helicity decomposition of the $d\Gamma/d\omega$ differential decay width with a polarized $\tau$. We show distributions for $B \to D\tau\bar{\nu}_\tau$ (top) and $B \to D^*\tau\bar{\nu}_\tau$ reactions (bottom), which have been evaluated with Wilson coefficients and form factors from Ref. [26]. Uncertainty bands as in Fig. 14.

In this appendix we collect in Figs. 19–23, results for $B \to D\tau\bar{\nu}_\tau$ and $B \to D^*\tau\bar{\nu}_\tau$ decays where the final $\tau$ has well defined helicity in the CM or LAB frames. All observables have been evaluated with the NP Wilson coefficients of Fits 6 and 7 and form factors from Ref. [26].

We obtain predictions that are qualitatively similar to those discussed in Sec. II for $B_c \to \eta_c$ and $B_c \to J/\psi$...
semileptonic decays. We would like to stress that unlike the unpolarized case, where all the accessible observables could be determined either from the CM or LAB distributions, in the polarized case, the LAB and CM charged lepton helicity distributions provide complementary information. Actually both differential distributions $d^2\Gamma/(d\omega d\cos\theta_L)$ and $d^2\Gamma/(d\omega dE_L)$ should be simultaneously used to determine the five new independent functions $A_H, B_H, C_H, D_H$ and $\mathcal{E}_H$, which appear for the case of a polarized final $\tau$ (see Eq. (23) of Ref. [29]).

![Graphs showing CM angular expansion coefficients for $\bar{B} \to D\tau\bar{\nu}_\tau$ decay with a polarized $\tau$ with positive (upper panels) and negative (lower panels) helicity. They have been evaluated with the Wilson coefficients and form factors from Ref. [26]. Uncertainty bands as in Fig. 14.](image1)

![Graphs showing LAB charged lepton energy expansion coefficients $\tilde{c}_{0,1,2,3}(\omega)$ for the polarized $\bar{B} \to D\tau\bar{\nu}_\tau$ decay. We also show the $(c_0 + \tilde{c}_1)$, $(c_1 + \tilde{c}_2)$ and $(c_2 + \tilde{c}_3)$ sums in the third top, second and fourth bottom panels, respectively. All quantities have been evaluated with the Wilson coefficients and form factors from Ref. [26]. Uncertainty bands as in Fig. 14.](image2)
FIG. 22. CM angular expansion coefficients for the $\bar{B} \to D^*\tau\bar{\nu}_\tau$ decay with a $\tau$ with positive (upper panels) and negative (lower panels) helicity. They have been evaluated with the Wilson coefficients and form factors from Ref. [26]. Uncertainty bands as in Fig. 14.

FIG. 23. LAB charged lepton energy expansion coefficients $\hat{c}_{0,1,2,3}(\omega)$ for the polarized $\bar{B} \to D^*\tau\bar{\nu}_\tau$ decay. We also show the $(c_0 + \hat{c}_1)$, $(c_1 + \hat{c}_2)$ and $(c_2 + \hat{c}_3)$ sums in the third top, second and fourth bottom panels, respectively. All quantities have been evaluated with the Wilson coefficients and form factors from Ref. [26]. Uncertainty bands as in Fig. 14.

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