Complex morphological filtering algorithm based on multi-structure elements

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Abstract: Morphological filter is a kind of nonlinear filter. Based on the erosion and dilation operations a complex morphological filter was proposed with multi-structure elements. In order to protect the image detail effectively, a noise detect method was introduced firstly. The noise pixels are removed using the proposed filter, while signal pixels hold their pixel value and are left unprocessed. The experimental results demonstrate that the proposed algorithm has good filtering performance, which can preserve edge details effectively during denoising. For both gray image and binary image with salt & pepper noise pollution, the proposed algorithm is superior to the conventional morphological methods in the aspect of objective performance evaluation and subjective visual effect. Meanwhile, the proposed method can also be extended to the color image processing.

Keywords: nonlinear filter; noise detection; multi-structural elements; morphological filtering

1. Introduction
In engineering applications, images are often affected by various noises during acquisition and transmission, degrading the final image quality. In order to facilitate the subsequent processing of images (such as edge detection, image segmentation, image recognition, etc.) [1], image filtering has become a key part of image preprocessing. The challenge is to remove noise and effectively protect the edge details of the image. Salt and pepper noise is the most common kind of noise, which usually appears as the gray-scale extremum of the image neighborhood, and the gray value of the noise pixel is independent of the gray value of the adjacent uncontaminated pixels. The median filtering method is often used for the removal of salt and pepper noise [2], but the traditional median filtering is affected by the filtering window, and it cannot be considered at the same time in terms of detail protection and noise removal. The morphological filter created by G. Matheron and J. Serra [3] in the early 1980s is a new type of nonlinear filter developed from mathematical morphology. Due to its fast parallel implementation, It has received extensive attention in the field of signal processing and image analysis [4-5]. The morphological filter is based on the set structure characteristics of the image, and uses a predefined structural element (corresponding to a filter window) to match or partially correct the signal to achieve the purpose of extracting the signal and suppressing noise[6-7]. Based on the traditional morphological transformation, a complex morphological filter based on multi-structural elements is constructed. At the same time, in order to protect the image details more effectively, the
noise is first detected before filtering, and only the noise points are processed during filtering, and the pixel values are kept unchanged for non-noise points. Simulation results show that the proposed method has better filtering performance. Compared with the traditional filtering method, this method can achieve better visual effects and objective evaluation indicators for both grayscale images and binary images. At the same time, the method can be applied to color image processing.

2. The proposed method

2.1 Noise model

First, we give a theoretical model of salt and pepper noise, whose expression is as follows [8]:

\[ y_{i,j} = \begin{cases} S & \text{with probability } p \\ x_{i,j} & \text{with probability } 1-p \end{cases} \quad S \in [0,255] \]  

(1)

In the above formula, \( x_{i,j} \) is a real image, and \( y_{i,j} \) is a noisy image. The noise density \( p \in [0,1] \). Figure 1 below shows the experimental results of adding a noise to a constant image. It is obvious from its histogram that the salt and pepper noise exhibits gray-scale extremum.

![Figure 1](image)

(a) Constant image  
(b) Noise image  
(c) Histogram of (a)  
(d) Histogram of (b)

**Figure 1.** Adding salt and pepper noise (50%) to the constant image

2.2 Noise detection

Let matrix \( u_{M_1 \times M_2} \) denote the noise gray image to be detected of size \( M_1 \times M_2 \). Use \( A \) to represent the collection of all pixels inside, \( A = \{(i, j) | 1 \leq i \leq M_1, 1 \leq j \leq M_2; M_1, M_2 \in Z^+ \} \). \( \Psi_{M_1 \times M_2} \) is a noise identification matrix whose elements are represented by “1” and “0”.

2.2.1 The first noise detection:

Let \( S_{i,j}^w \) denote the size \( w \times w \) ( \( w \) is odd). The center point is the window of \( u(i, j) \). The initial detection is performed for each pixel point \( u(i, j) \) in the image according to the adaptive median method [9], and the result is identified by “1” or “0”, namely:
\[ \Psi_{i,j} = \begin{cases} 1 & u(i, j) \text{ is the candidate noise point} \\ 0 & \text{otherwise} \end{cases} \]  

(2)

Estimate the noise density \( \tau \) of the image as follows:

\[ \tau = \frac{1}{M_1 M_2} \sum_{i=1}^{M_1} \sum_{j=1}^{M_2} \Psi_{i,j} \]  

(3)

If \( \tau \) exceeds a given threshold \( \tau_0 \), the candidate noise point is treated as a true noise point; otherwise, it is secondarily detected.

2.2.2 The second noise detection

For all candidate noise points that satisfy \( \Psi_{i,j} = 1 \), define the mean of all signal points in window \( w \times w \):

\[ \bar{u}(i, j) = \frac{1}{W} \sum_{r=-\lfloor w/2 \rfloor}^{\lfloor w/2 \rfloor} u(i + r, j + r) \]  

(4)

Where \( W \) represents the number of all signal points in window \( w \times w \), and \( u(i + r, j + r) \) represents the signal point in the window.

If the absolute difference between the gray values \( u(i, j) \) and \( \bar{u}(i, j) \) of the current candidate noise point exceeds a certain threshold, it is regarded as a true noise point, namely:

\[ \Psi_{i,j} = \begin{cases} 1 & |u(i, j) - \bar{u}(i, j)| > T \quad \text{and} \quad \Psi_{i,j} = 1 \\ 0 & \text{otherwise} \end{cases} \]  

(5)

In this paper, \( T \) is defined by the mean square error of all signal points in window \( w \times w \):

\[ T = \frac{1}{W} \sum_{r=-\lfloor w/2 \rfloor}^{\lfloor w/2 \rfloor} [u(i + r, j + r) - \bar{u}(i, j)]^2 \]  

(6)

2.3 Multi-structural element composite morphological filter

The traditional morphological filters mainly use morphological open operations, closed-loop, and combinations of them [10-11].

2.3.1 Gray-scale corrosion

Let \( f(x, y) \) denote the input image, \( B(i, j) \) represents a structural element (function), gray-scale corrosion can be defined as:

\[ (f \Theta B)(x, y) = \min \left\{ f(x+i, y+j) - B(i, j) \mid (x+i, y+j) \in D_f ; (i, j) \in D_B \right\} \]  

(7)

In above formula, \( D_f \) and \( D_B \) are the domains of functions \( f \) and \( B \), respectively. The displacement parameter \( (x+i, y+j) \) must be included in the domain of the function \( f \). Corrosion of gray-scale images can yield two results: if all structural elements are positive, the output image will tend to be darker than the input image; the effect will be diminished after brightly etched in areas smaller than the structural elements. The degree of attenuation depends on the gray value surrounding the luminance region and the shape and amplitude of the structural element itself[12].

Corrosion operation is performed point by point. The operation result of a point is the difference between the gray value of the point in a local range and the corresponding point in the structural element, and the minimum value is selected[13]. It can be seen that after the etching operation, the
gray value of the point where the gray value of the edge portion is relatively large is lowered, so that the edge shrinks to the area where the gray value is higher than the adjacent area.

2.3.2 Gray-scale expansion

Gray-scale expansion is defined as:

\[
(f \oplus B)(x, y) = \max \left\{ f(x-i, y-j) + B(i, j) \mid (x-i, y-j) \in D_f, (i, j) \in D_B \right\}
\] (8)

In above formula, \(D_f\) and \(D_B\) are the domains of functions \(f\) and \(B\), respectively. The displacement parameter \((x-i, y-j)\) must be included in the domain of the function \(f\). Since the expansion operation is the maximum value of \(f + B\) selected in the neighborhood defined by the shape of the structural element. Thus, generally, the expansion processing method for gray-scale images can obtain two results: if all structural elements are positive, the output image will tend to be brighter than the input image; black detail reduction or removal depends on the value and shape of structural element correlation in the expansion operation.

The expansion operation is performed point by point. The calculation involves the gray value of the point around it and the value of the structural element. It is actually the sum of the gray value of the point in the local range and the corresponding point in the structural element value[14]. Therefore, after the expansion operation, the edges are extended.

2.3.3 Gray-scale open operation

The opening operation of the image is to first perform the etching operation on the gray-scale image with the structural element, and then perform the expansion operation on the obtained result, namely:

\[
f \ominus B = (f \ominus B) \oplus B
\] (9)

2.3.4 Gray-scale closed operation

The closing operation of the image is to first expand the gray-scale image with the structural element, and then perform the etching operation on the obtained result, namely:

\[
f \odot B = (f \odot B) \ominus B
\] (10)

Since the corrosion removes less bright detail and there is no recovery in subsequent expansion, in practical applications, the gray-scale opening operation is often used to remove small bright spots relative to structural elements while retaining all grays. Degrees and large bright area characteristics are unchanged. Similarly, the expansion operation removes less dark detail, and these details are not recovered in the erosion operation[15]. Therefore, the closed operation of the gray-scale image is often used to remove smaller dark details while maintaining the overall gray level and Large, bright areas are unaffected.

2.3.5 Morphological filter

Combining open and closed operations eliminates noise. If you use a small structural element to open an operation and then close an image, it is possible to remove a similar noise structure in the image that is smaller than the structural element[16]. In order to simultaneously remove peak (positive pulse) and bottom valley (negative pulse) noise in the image, open-closed (OC) and closed-close (CO) filters can be constructed in open and closed cascades, which are defined as follows:

\[
OC(f(i, j)) = (f \ominus B \odot B)(x, y)
\]

\[
CO(f(i, j)) = (f \odot B \ominus B)(x, y)
\]
2.3.6 Composite morphological filter
In morphological filtering, the selection of structural elements plays a key role in the filtering results and performance. Using a fixed structural element to construct a filter operator, the noise immunity is limited, and it is difficult for a single structural element to detect the edges of various geometric shapes [17-18]. Based on this, this paper constructs a new composite morphological filter \( \psi(f(x, y)) \) whose expression is:

\[
\psi(f(x, y)) = \frac{1}{2}((f \circ B_1 \bullet B_2)(x, y) + (f \bullet B_1 \circ B_2)(x, y))
\]  

(13)

among them \( B_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \), \( B_2 = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \)

In order to protect the image details more effectively, the noise is first detected by the method of Section 1.2 before filtering. Only the noise points are processed during the filtering, and the pixel values are kept unchanged for the non-noise points. At the same time, the method can be applied to color images. Based on the RGB space, the three components of R, G and B in the color image to be processed are filtered by the equation (13), and then the filtering results of the three components can be superimposed.

3. Experimental results and analysis
In order to verify the filtering performance of the proposed method, this section deals with the image of salt and pepper noise pollution. In Experiment 1, a grayscale image "rice" and a binary image "imagetech" were used as test objects. The morphological filters such as open, closed, open-close, and closed-open are used for processing, and the peak signal-to-noise ratio (PSNR) is used as an objective criterion for evaluating the filtering performance. The experimental results are shown in Figure 2 and Figure 3. Table 1 gives the corresponding results of the objective performance evaluation indicators.
As can be seen from the above experimental results in Figure 2 - 3, the open operation removes the small bright spots (salt noise) relative to the structural elements while retaining all gray levels and large bright area features. The closed operation removes less dark detail (pepper noise) while the relative gray level and larger bright areas remain unaffected. The open-close and close-open filters simultaneously filter out salt and pepper noise. As can be seen from Table 1, the closed-close filter can achieve better results than the other three operations.

Experiment 2, using the grayscale image "lena" as the test object. The open-closed, closed-open filters and the methods herein are used for processing. The experimental results are shown in Figure 4 below.

Table 1. Comparison of objective performance evaluation indicators (PSNR) of different morphological filters

| testing object         | open operation | Closed operation | Open-close filter | Close-open filter |
|------------------------|----------------|------------------|-------------------|-------------------|
| Gray-scale image “rice”| 18.6607        | 19.1454          | 21.3279           | 21.6610           |
| Binary image “imagetech”| 15.9032        | 22.7749          | 27.7963           | 27.8318           |

As can be seen from the above experimental results in Figure 2 - 3, the open operation removes the small bright spots (salt noise) relative to the structural elements while retaining all gray levels and large bright area features. The closed operation removes less dark detail (pepper noise) while the relative gray level and larger bright areas remain unaffected. The open-close and close-open filters simultaneously filter out salt and pepper noise. As can be seen from Table 1, the closed-close filter can achieve better results than the other three operations.
It can be seen from the filtering results in Figure 4 that all three morphological filters can effectively remove the salt and pepper noise, and the method of this method has some improvement over the visual effect of the open-close or close-open filter alone. The original image is moderately bright and dark.

In Experiment 3, the color image "peppers" was selected as the test object. Under the condition of different noise density pollution, the method is filtered by the method of this paper. The experimental results are shown in Figure 5.
It can be seen from Figure 5 that for the color images with different density noise pollution, the method can achieve good filtering effect.

4. Conclusion
A complex morphological filter is constructed by using multiple structural elements based on the traditional morphological transformation. At the same time, before the filtering, the noise detection process is added, which effectively avoids the diffusion of noise points in the filtering process. The experimental results demonstrate the effectiveness of the proposed method. Compared with the traditional morphological filtering method, this method can achieve better visual effects and can be further applied to color image filtering.

Acknowledgments
This work was supported in part by Scientific Research Fund of Sichuan Provincial Education Department, China under Grant No.18ZB0509, by Science and Technology Program of Dazhou, China under Grant No.KJJ2015001, and by a Grant from the Key Programs of Sichuan University of Arts and Science, China under Grant No.2017KZ003Z, which are greatfully acknowledged.

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