Measuring the orbital angular momentum mode of vortex beam based on phase matching sequence

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Abstract—Measuring the orbital angular momentum (OAM) mode of vortex beams is of great significance in applications based on vortex beams. We propose a method of phase matching characteristics of vortex beams to measure the OAM mode. The method first uses a high-speed spatial light modulator (SLM) to sequentially load a set of helical phase sequence images, so that the vortex beam is modulated by this SLM. Then obtain the modulated optical field through pinhole filtering, and then the synchronized tilt phase modulation is performed by the 4f system and high-speed SLM. When the OAM mode of the vortex beam is opposite to the topological charge of the helical phase image, that is, the incident vortex beam degenerates to the fundamental Gaussian mode beam, and through our optical system, the output plane obtained a indicating spot with the position related to the OAM mode. The simulation and experimental results show that the method obtains a clear OAM mode indicating spot on the final output plane, which verifies the theoretical derivation.

1. Introduction

The vortex beam is a hollow ring beam with a continuous spiral phase wavefront. In 1992, Allen et al. first discovered that such beams carry OAM\cite{1}.

Because OAM has an infinite discrete value space, it can be used in large-capacity optical communications\cite{2}, and the spiral phase structure and a dark hollow ring light intensity distribution makes it applicable in the field of optical micro-manipulation\cite{3}. Especially it used in OAM multiplexing communication, when the channel terminal demultiplexes the information, and it needs to determine the OAM mode in the current optical field. Therefore, measuring the OAM mode is the key to the application based on vortex beam. Scholars have completed a lot of research on the detection of OAM mode, currently there are two main types of detection methods, one is the Interference method\cite{4-6}, which interferes with the vortex beam and another reference beam, and judges the OAM mode by the interference pattern characteristics obtained, however, any slight fluctuations related to the beam can easily cause the interference pattern to be unstable, this kind of method has high requirements on the experimental optical path; Another type of detection method is based on diffraction optical elements method, which is to pass the vortex beam through the aperture such as double slits\cite{7}, triangular holes\cite{8}, the ring gratings\cite{9,10}, then judge the OAM mode according to the characteristics of the diffracted spot. However, the traditional diffraction optical device is not flexible in application.

In recent years, scholars have proposed more OAM mode detection methods\cite{11-16}. One of the methods is us the optical device make the log-polar coordinate transformation to the vortex beam, so that the spiral wave-fronts of different OAM modes are converted into tilted plane wave-fronts with different phase gradients, then focus through the lens to obtain the focused spot, and it spatial position...
distribution is related to the tilt phase gradient, so as to judge the OAM mode. However, the experimental optical system of this method is not only complicated and difficult to align, which brings difficulties to OAM mode recognition.

In order to simplify the optical system, this paper experimentally verifies an OAM mode detection method based on the phase matching of vortex beam. By using a high-speed SLM to sequentially load a set of helical phase modulation sequence images, so that the vortex beam to be modulated by this SLM, When the OAM mode of vortex beam is opposite to the topological charge of helical phase image, then the phase matching is succeed, at this time the incident vortex beam degenerates to the fundamental Gaussian mode beam, that is, the hollow ring beam is transformed into a solid beam; When the phase is not matched, there is still a hollow vortex beam on the optical field. After the pinhole filtering, the bright spot is obtained under the condition of phase matched, otherwise, the pinhole has no bright spot passed through. After synchronized tilt phase modulation by the 4f system and high-speed SLM, there can obtain a tilted plane wave related to the OAM mode. Finally, after focusing by the lens, the indicating spot related to the OAM mode is obtained on the output plane. It provides a new idea for OAM mode detection.

2. Measurement principle

Generally, without considering the amplitude, the complex optical field of the beam carrying OAM propagating along the z-axis is expressed as:

\[ U_0 = \exp(i l \theta) \]  

Where \( \theta \) is the azimuth angle on the beam section, \( l \) is the spiral topology charge, its value represents the number of \( 2\pi \) cycles experienced by one rotation of the azimuth angle, and the symbol represents the direction of rotation, \( \exp(i l \theta) \) represents the spiral phase factor of the beam. For vortex beams, the measurement of the OAM mode is the measurement of the spiral phase factor of the vortex beam.

Due to the existence of the phase singularity in the spiral phase center, the beam carrying OAM forms a hollow ring-shaped structure during the propagation process, which is vortex beam. The phase matching method uses this ring-shaped light intensity distribution characteristic for OAM detection, and its principle is: When the OAM mode of vortex beam is opposite to the topological charge of the phase matching sequence, that means the incident vortex beam degenerates to the fundamental Gaussian mode beam, that is, the phase matching is succeed. If the OAM mode of vortex beam is opposite to the topological charge of the phase matching sequence, the spiral phase factor of the vortex beam is not canceled, and the output optical field is still a hollow ring-shaped vortex beam, that is, the phase is not matched. The mathematical expression of this process is:

\[ U_i = U_0 \exp(i l \theta) = \exp(i l \theta) \exp(i l \theta) = \begin{cases} \exp(i(l+L)\theta) & l+L=0 \\ \exp(i(l+L+1)\theta) & l+L\neq0 \end{cases} \]  

Where, the phase matching sequence is represented by \( \exp(i l \theta) \), and the OAM mode of the beam to be measured is set within the range of \( L \), that is, \( L = -n \sim +n(\pm l < |l|) \).

Placing a pinhole \( P(x_p,y_p) \) at the center of the output optical field \( U_1 \) after matching, The mathematical expression of this process is:

\[ U_2 = U_1 P(x_p,y_p) = P(x_p,y_p) \begin{pmatrix} 1 & (x_p^2 + y_p^2) < r_p^2 \\ 0 & \text{other} \end{pmatrix} \]  

Where \( x_p \) and \( y_p \) represent the coordinates on the plane where the pinhole is located, and \( r_p \) represents the radius of the pinhole.

In the case of phase matching succeed, there is a bright spot passing though from the pinhole, in the case of phase unmatched, only the central dark spot can pass through the pinhole, then \( U_2 \) is:

\[ U_2 = \begin{cases} \delta(x,y) & l+L=0 \\ 0 & l+L\neq0 \end{cases} \]  

So far, it can be judged whether the phase matching is succeed according to whether there is a bright spot passing through the pinhole, so as to identify the OAM mode. However, due to the fixed
position of the pinhole, the spatial position of the bright spot passing through the pinhole is also fixed, and the OAM mode cannot be obtained directly through the spatial position here. If the vortex beams of different OAM modes correspond to the output spots of different spatial positions, and the OAM mode can be directly judged according to the spatial positions.

When the phase matching is succeed, the bright light spot passing through the pinhole can be regarded as a point light source, using a Fourier lens to perform a Fourier transform on it, we will obtain a plane wave parallel to the z-axis:

$$\mathcal{F}[U_3](x, y) = \delta(x)$$

Where $$\mathcal{F}$$ represents the Fourier transform operator.

Using the tilt modulation grating to modulate the plane wave, and then it will transform a tilted plane wave after modulation, which is represented by $$U_3$$ as:

$$U_3 = \exp[-i2\pi(a_f(x, y))], a = \frac{D_x}{N_L} \quad (7)$$

Where the tilt modulation grating is represented by $$\exp[-i2\pi(f \cdot a)]$$, $$a$$ is the parameter of tilt modulation, $$D_x$$ is the length of the horizontal interval of the light field space, and $$N_L$$ is the number of spiral phase sequence images, divide the optical field space into $$N_L$$ equal parts in the horizontal direction, and the fixed interval of each part is $$D_x/N_L$$, treat the fixed interval as a constant $$C$$, then the tilt modulation parameter is $$a = l \cdot C$$, that is, the parameter $$a$$ takes the topological charge $$l$$ as a variable and $$a$$ is proportional to $$l$$.

After the tilted plane wave passes through the Fourier lens, that is, performing a Fourier transform on $$U_3$$, and the output optical field $$U_4$$ is:

$$U_4 = \mathcal{F}^{-1} U_3 = \mathcal{F}^{-1} \{ \exp[-i2\pi(f \cdot a)] \} = \delta(x-a_3y) \quad (6)$$

$$U_4$$ is the focus spot with the lateral position offset related to $$l$$, so that the OAM mode of the vortex beam can be directly judged according to the lateral position of the focus spot.

### 3. Experiment

In order to verify the theoretical derivation, the method proposed in this paper was verified experimentally.

The experimental setup is shown in Fig.1: Schematic diagram of experimental optical system is shown in Fig.1(a), the beam emitted by the He-Ne laser becomes a quasi-plane wave after passing through the beam expand system. After passing through the SLM1 loaded with the helical phase image, the vortex beam is generated and irradiated on the SLM2. Here, SLM2 and SLM3 are connected to the same computer PC2, the purpose is to achieve synchronization between them two; PC2 load on a set the vortex beam is generated and irradiated on the SLM2. Here, SLM2 and SLM3 are connected to the same computer PC2, the purpose is to achieve synchronization between them two; PC2 load on a set

First, verify the phase matching result of the vortex beam and the spiral phase sequence image. As shown in Fig.2. When the vortex beam with $$l=5$$ passes through the SLM loaded with the spiral phase image of $$l=5$$, obtained the fundamental Gaussian mode beam with $$l=0$$, that is, obtained the solid beam; when the vortex beam with $$l=5$$ passes through the SLM loaded with the spiral phase image of $$l=-3$$, obtained the vortex beam with $$l=2$$, which is still a ring-shaped beam with a dark area at the center. This is consistent with the theoretical derivation.
Experiment to verify the OAM mode detection method proposed in this paper. In this experiment, the range of topological charge of helical phase image loaded on SLM2 is \( L = -5 \sim 5 \), this is also the scope of the detectable OAM mode. The simulation and optical experiment are the detection of the OAM mode of the vortex beam with \( l = -5 \sim 5 \).

The simulation result is shown in Fig.3(a), the position of the bright spot in the figure indicates the OAM mode of the vortex beam currently. When the bright spot is at the center of the figure, it means OAM mode \( l = 0 \), when it is in the left half of the center position, the OAM mode number is negative, when it is in the right half of the center position indicates that the OAM mode number is positive, and the farther away from the center position, the greater the absolute value of the OAM mode number. If the pinhole is in an ideal state, use one pixel to characterize the pinhole in the simulation, and we will obtain a light spot with one pixel on the output plane. However, in order to consider the actual effect, the pinhole is enlarged, and we use a pinhole of \( 2 \times 2 \) pixels in the simulation. Therefore, it can be seen from the simulation results that, in addition to the light spot that appears at the correct indication position, there is still weak light intensity at other positions, This is because the pinhole aperture is larger than the unmatched vortex light ring light intensity, causing the pinhole to not only pass through the dark area of the ring light intensity center, but also part of the light energy.
leaks to the output plane to form a weak light intensity.

The optical experimental results are shown in Fig.3.(b). It can be seen that the indicator light spot appears at the correct OAM position, and in the non-indicating position, there is still weak light leakage. This is also caused by the larger size of the pinhole hole, which is mutually verified with the simulation results. The light leakage caused by the pinholes brings about a decrease in the contrast of the output plane, which affects the final interpretation result, but within a certain threshold range, correct interpretation can be performed. Except for the indicator spot indicating the OAM mode, there is always zero-order diffraction in the detection result, this is because optical experiments have high requirements for equipment, and the SLM3 we used in the experiment is not well on the performance of phase modulation, so the amplitude and phase modulation exists at the same time.

4. Conclusion

Based on the above discussion, the following conclusions are drawn:

(1) We studies the detection method of OAM mode, and proposes an OAM mode detection method based on phase matching sequence. This method modulates the vortex beam by loading the SLM of the spiral phase sequence images. Then, obtained the light spot in case the phase matching succeed from the pinhole filtering, after passing through the optical system, we obtained an indicator light spot whose position is related to the OAM mode on the output plane. The results show that this method can judge the OAM mode of the vortex beam according to the position of the indicated light spot.

(2) The scope of the detection of OAM mode is limited by the range of topological charge of the helical phase sequence images. During the experiment detection process, the larger the range of helical phase sequence, the larger the detection range of the OAM mode, that means the more sequence images need to be prepared in advance, and the amount of work to be prepared in the early stage is complicated. The method to detect the OAM mode is by a time sequential manner, the helical phase images with different topological charges loaded on the SLM dynamically, that is equivalent to different signals that do not overlap each other on the time axis. Even if there are vortex beams with different OAM modes in the optical field, they can be phase-matched with different signals on the time axis one by one, then to obtain the OAM modes. Therefore, this method is also suitable for multi-mode OAM detection.

(3) The disadvantage of this paper is that there exists zero-order diffraction in the results of optical experiments, which is related to the phase modulation performance of the SLM equipment. If SLM with better phase modulation performance can be used in the experiment, the zero-order diffraction can be reduced or eliminated.
Acknowledgments
This work was financially supported by the National Natural Science Foundation of China(61705178).

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