Modeling of Sedimentation Process in Water

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Abstract. Sedimentation process occurs in container of water reservoir. In the sedimentation process will be observed how the fluid flow occurs and how sedimentation occurs in water. The sedimentation is influenced by the density of the water type and the speed of water in the container of water reservoir. This study aims to find out how to model the mixture of solid and liquid particles (suspension) on the container of water by way of modeling it into a mathematical form that is by using differential equations. In this case, the factors affecting the distribution of the sludge concentration will be used as a reference for modeling the mathematical equations will then be solved using differential equations. The results of this study will show that the equation model of mixed solid and liquid particles (suspension), which then is called the momentum transport equation.

1. Introduction

The water supply company is a regional-owned company engaged in the processing and clean water industry for public society. Some facilities in clean water processing include: intake, raw water tower, clarifier, fast mixing building (Rapid Mixing), slow mixing building, filtration building, and reservoir. All of these equipments can be operated by computer system. The water supply company also use chemicals such as chlorine and alum in water processes [1]. There are several stages of clean water process: filtering and precipitation, coagulation, flocculation, sedimentation, filtration and disinfection. One of the water treatment process is sedimentation. From the problem of the sedimentation process in water, some researchers performed the analysis of the sedimentation process in water by modeling the sedimentation process into the model of mathematics [2, 3].

Model plays an important role in the field of science. Usually in terms of economics to save time and cost or other valuable commodities. Modeling can avoid the risk of real system damage. Thus a model is required when an experiment with a real system becomes blocked because it is expensive, dangerous or something that is impossible to do. Mathematical models are ideal representations of real systems that are expressed or expressed in terms of symbols and mathematical statements. In other words, the mathematical model represents a system in the form of quantitative and logical relations, in the form of a mathematical equation. In artificial mathematical models of events or natural events are described through a set of mathematical equations. In this case, modeling will change the sedimentation process in water into Mathematical model to make it easier to find solutions to the problem.
2. Materials and Methods

Fluid Mechanics

Fluid mechanics is the discipline within the broad field of applied mechanics that is concerned with the behavior of liquids and gases at rest or in motion. It covers a vast array of phenomena that occur in nature (with or without human intervention), in biology, and in numerous engineered, invented, or manufactured situations. There are few aspects of our lives that do not involve fluids, either directly or indirectly [6]. A fluid is defined as a substance that deforms continuously when acted on by a shearing stress of any magnitude [4]. A shearing stress (force per unit area) is created whenever a tangential force acts on a surface as shown by the figure in the margin. Classification of fluid flows are uniform, one, two and three dimensional flows, viscous and inviscous flows, laminar and turbulent flows and incompressible and compressible flows [5].

Sedimentation is a solid-liquid separation using gravity precipitation to set aside suspended-solid. The suspension is a fluid mixture containing solid particles. Or in other words, a heterogeneous mixture of liquids and solids dissolved in the liquid. Examples of suspensions are mud, flour, mist, paint and others. One example of such suspension is mud. Mud is a liquid, mixture between water and soil. Mud occurs when the soil is wet. Geologically, mud is a mixture of water and particles of sediment. Factors that influence the Sediment are concentration, pressure, density, surface tension, viscosity, and velocity.

Mathematical models can be defined as equations, equations that model some real situations. Research can help model the situation into the form of a differential equation. Technology can be used to help solve the equation (computer program gives answer). Answers based technology would be interpreted or communicated in accordance with real situations.

The Differential Equation

To solve a partial differential equation for a dependent variable, certain conditions are required which means that the independent variables must be determined at certain values of independent variables. If the independent variables are spatial coordinates such as speed, the conditions are called boundary conditions. If the independent variable is time, the conditions are called initial conditions. The boundary condition is the velocity component to the perpendicular in the non-condensed flow. In the non-viscous flow in which viscosity is neglected, the velocity vector has a tangential direction to the boundary. These differential equations will be derived using cartesian coordinates.

Initial and Boundary Conditions

For the problem sedimentation in this research is assumed as follows:

(i) Assume sedimentation is mud
(ii) Viscous effects are negligible
(iii) It is assumed that water flows and does not rotate, incompressible flow, and laminar flow
(iv) Cohesion force and adhesion force are not considered

3. Results and Discussion

Two-Phase Flow Modeling of a Dense Suspension

Liquid-solid mixtures (suspensions) are important in a variety of industrial fields, such as oil and gas refinement, paper manufacturing, food processing, slurry transport, and wastewater treatment. A suspension is a mixture of solid particles and a liquid. The dynamics of a suspension can be modeled by a momentum transport equation for the mixture. It uses the following equation of momentum transport
The Differential Continuity Equation

To derive the differential continuity equation, the infinitesimal element is utilized. It is a small control volume into and from which the fluid flows. It is shown in the $xy$-plane with depth $dz$. Let us assume that the flow is only in the $xy$-plane so that no fluid flows in the $z$-direction. Since mass could be changing inside the element, the mass that flows into the element minus that which flows out must equal the change in mass inside the element. This is expressed as

$$
\rho u dy dz - (\rho u + \frac{\partial (\rho u)}{\partial x} dx) dy dz + \rho v dx dz - (\rho v + \frac{\partial (\rho v)}{\partial y} dy) dx dz = \frac{\partial}{\partial t} (\rho dx dy dz) \tag{1}
$$

where the density $\rho$ is allowed to change across the element. Simplifying the above, recognizing that the elemental control volume is fixed, results in:

$$
\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = -\frac{\partial \rho}{\partial t} \tag{2}
$$

Differentiate the products and include the variation in the $z$-direction. The equation is described and the result is the differential continuity equation. The differential continuity equation for an incompressible flow is

$$
\frac{D \rho}{Dt} + \rho \nabla \cdot V = 0 \tag{3}
$$

The Differential Momentum Equation

The differential continuity equation derived in the differential continuity equation contains the three velocity components as the dependent variables for an incompressible flow. If there is a flow of interest in which the velocity field and pressure field are not known, such as the flow around a turbine blade or over a weir, the differential momentum equation provides three additional equations since it is a vector equation containing three component equations. The four unknowns are then $u$, $v$, $w$, and $p$ when using a rectangular coordinate system. The four
equations provide us with the necessary equations and then the initial and boundary conditions allow a tractable problem.

Apply Newton’s second law to the element of Fig. 2, assuming that no shear stresses act in the \( z \)-direction (we will simply add those in later) and that gravity acts in the \( z \)-direction only:

\[
\begin{align*}
&\left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} dx\right)dydz - \sigma_{xx} dydz + \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y} dy\right)dxdz - \tau_{xy} dxdz = \rho dxdydz \frac{Du}{Dt} \\
&\left(\sigma_{yy} + \frac{\partial \sigma_{yy}}{\partial y} dy\right)dxdz - \sigma_{yy} dxdz + \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x} dx\right)dydz - \tau_{xy} dydz = \rho dxdydz \frac{Dv}{Dt}
\end{align*}
\]

The equation above is described and the result is the Navier–Stokes equations. The Navier–Stokes equations can be written in vector form as

\[
\frac{\rho}{D} \frac{DV}{Dt} = -\nabla p + \mu \nabla^2 V + \rho g
\]  

**The Mixture Model Equation**

A mixed model (or an algebraic slip model) is a simplified formulation of a multiphase flow equation (two phases). The mixed model consists of continuity and momentum equations for mixed and continuity equations for dispersion or separate phases. If in the fluid flow there is a solid particle which is assumed to be a mud to form sedimentation, then there is a series of new equations found in the Navier-Stokes equation. The new equation is the effect of sedimentation on the flow of fluid flowing in a container. In the same way as deriving the differential continuity equation, we find an equation describing the solid particles present in a fluid stream.

This equation is called the momentum transport equation for two-phase flow of suspension that can solve and calculate how much sedimentation in a container of water reservoir.

The momentum transport equations can be written as

\[
\rho \frac{DV}{Dt} = -\nabla p - \nabla(\rho m_p(1 - m_p)V_{slip} V_{slip}') + \mu \nabla^2 V + \rho g
\]
where $V$ is the mass averaged mixture velocity ($m/s$), $p$ denotes the pressure (Pa), $g$ refers to the acceleration of gravity ($m/s^2$), $m_p$ is the dimensionless particle mass fraction, $V_{slip}$ gives the relative velocity between the solid and the liquid phases ($m/s$) and $\rho$ is mixture density.

This equation is called the momentum transport equation for two-phase flow of suspension. It can solve and calculate how many sedimentation in a container of water reservoir.

4. Conclusion
From the results and discussion, it shows that the problems in the real world can be modeled by using Mathematical Model, in this case is the problem of sedimentation on the container of water reservoir in the sedimentation container at the time of clean water process. The problem can be modeled into Mathematical form, which is to be an equation called the momentum transport equation. The momentum transport equation is

$$\rho \frac{DV}{Dt} = -\nabla p - \nabla \left(\rho m_p (1 - m_p) V_{slip} V'_{slip} + \mu \nabla^2 V + \rho g\right).$$

It also shows that mathematics can be applied in everyday life.

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