Development of electrical power transmission system linear hybrid state estimator based on circuit analysis techniques

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ABSTRACT

Electrical power systems are in rapid evolution, necessitating advanced monitoring, control, and protection aspects connected to an accurate knowledge of the state of the grid. State estimation is one-way such knowledge could be achieved by estimating voltage magnitudes and phase angles at all buses in electrical networks. Electric utilities are deploying advanced Phasor Measurement Units (PMUs) in grids with conventional Remote Terminal Units (RTUs). Integrating measurements from the two types of devices for state estimation is inevitable. In the past, hybrid state estimators have been based on nonlinear models associated with various challenges, such as using nonlinear state estimation algorithms with high computational time, slow convergence, and initialization problems. For this reason, there is a need to develop hybrid linear mathematical models that eliminate issues posed by nonlinearity models. This paper presents a novel mathematical formulation that gives a linear measurement model for hybrid state estimation based on the standard steady-state branch model. The developed model is then tested in MATLAB using three standard transmission test cases (IEEE 14-BUS, IEEE 30-BUS, and IEEE 57-BUS) with available measurements subjected to different errors. Notably, the performance of the developed model based on two different state estimation algorithms, Weighted Least Square (WLS) and Weighted Least Absolute Value (WLAV), are compared. The simulation results obtained for voltage magnitudes and phase angles for all buses in the three scenarios compare well with reference values. Using Normalized Cumulative Error (NCE) as a performance index, the developed linear measurement model gives less cumulative error than the conventional nonlinear model. With measurements subjected to bad data (such as negative values for measured variables), the developed model performs better using WLAV than the WLS estimation algorithm. The computational time for all three networks is notably less when using the WLAV algorithm than the WLS algorithm.

1. Introduction

Power systems are undergoing rapid evolution, and consequently, Energy Management Systems (EMS) require more reliable and accurate information regarding the operating status of the grid. The EMS is a real-time computer-based system comprising both hardware and software. One of the key hardware elements of EMS is the data acquisition subsystem in charge of collecting and transmitting data to control centers and transmitting control actions to field devices. The power system parameters acquired through the data acquisition subsystem include active and reactive power flow/injection, voltage, frequency, and circuit breaker positions. The EMS software includes Supervisory Control and Data Acquisition (SCADA), responsible for data acquisition, alarm processing, user interface updating, and execution of control actions. State estimator (SE) is also a key EMS application software which is a mathematical method that uses power system parameters available from data acquisition equipment to recreate values for all other unknown system state variables [1], [2]. Power system state variables consist of voltage magnitude and phase angle at all the buses within a given network. In the modern power system, data acquisition is made possible using either the SCADA system that employs Remote Terminal Unit (RTU) equipment or the synchrophasor system that uses Phasor Measurement Unit (PMU) equipment. Deployment of both RTU and PMU measurements in state estimation is inevitable due to the wide installation of

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the prior in many power systems across the world and also benefits associated with PMU measurements such as improved state estimation accuracy [3] and added advantages such as measurement of phase angle and time synchronization [4]. Therefore, using RTU and PMU measurements in state estimation results in Hybrid State Estimator (HSE). A key research area for HSE has been the formulation for combining RTU and PMU measurements [5].

There are three possible methods in the literature for combining RTU and PMU measurements in hybrid state estimation. In the first method, commonly referred to as a two-stage approach, a conventional nonlinear estimator processes RTU measurements. The state estimates from this stage are then integrated with PMU measurements in a linear estimator in the second stage, as shown in Fig. 1 [2]. Therefore, the results of the nonlinear state estimator and PMU measurements become a set of measurements that are linear functions of the estimated state vector. This method presents the advantage of being non-invasive with no significant modifications to the existing estimation software. However, it still relies on a nonlinear algorithm in the first stage, subject to various limitations such as initialization, slow convergence, and high computational burden.

The second method is integrated nonlinear HSE, where RTU and PMU measurements are combined into a common nonlinear HSE, as shown in Fig. 2. In this integrated method, voltage phasors from PMU are utilized in their polar coordinates (magnitude and phase angle). In contrast, the current phasors may be incorporated in polar, rectangular, pseudo voltage, pseudo flow measurements, and constrained formulations [5], [6]. This method modifies and eliminates the challenges of the two-stage HSE to accommodate PMU measurements.

The third method combines RTU and PMU measurements into an integrated linear state estimator, as shown in Fig. 3. Development of the linear estimator comprises various formulations for modeling RTU and PMU measurements. The algorithm is noniterative with a linear state estimator, eliminating challenges associated with nonlinear state estimators. Recent research in HSE has focused on developing a linear formulation combining RTU and PMU measurements in a single stage to eliminate nonlinear model challenges.

The resulting measurement model for HSE is either a nonlinear or linear model that necessitates an iterative or noniterative state estimation algorithm. Commercially available state estimation algorithms are Weighted Least Square (WLS) and Weighted Least Absolute Value (WLAV). The WLAV is more robust than WLS since it does not require a post-processing step to detect and discard bad data [7], [8]. This post-processing method increases the computational burden of the state estimator. Therefore, a SE that does not require a separate stage for bad data processing is preferred. The WLS method provides an optimal solution for the state estimation problem if the measurement consists of normally distributed Gaussian errors. However, if the measurements consist of bad measurements known as statistical outliers (very large errors), then the WLS method becomes highly unreliable [9]. The WLAV method presents a higher computational burden than WLS, especially with nonlinear measurement models [10]. Therefore, a linear measurement model for the state estimator would address the computational burden issue for WLAV since the estimation would not require an iterative state estimation algorithm.

2. Related works in hybrid state estimation

There has been growing interest in research in the mathematical formulation of HSE. The two-stage HSE is proposed in [11], which gives an alternative way of including PMU measurements in HSE, with the first stage considering state vectors in polar form. The second stage utilizes both measurement vector and state vector in rectangular coordinates. The authors in [12] propose a variant of the two-stage HSE, with the first stage comprising RTU and PMU measurements into a nonlinear estimator. The second stage consists of a nonlinear estimator based on PMU measurements and pseudo measurements obtained from previously estimated states. The authors in [13] also propose an enhanced state estimator based on a two-stage non-invasive HSE. The first stage gives output as estimated voltages in rectangular form. These voltages are used as pseudo measurements in the second stage, integrating them into a linear estimator with the state vector based on voltage in rectangular coordinates. The authors in [14] propose a two-step HSE, with the first step adopting a hybrid of SCADA/PMU measurements based on a
linear estimator. The second step is a linear estimator comprising PMU measurements and state vectors obtained from the previous step.

The authors propose the integrated nonlinear HSE in [15] to utilize the nonlinear HSE with a central coordinator HSE for processing estimates from local areas alongside PMU measurements. The algorithm implemented for state estimation is WLS, which necessitates using the largest Normalized Residual (LNR) test to identify bad data. In [16], integrated nonlinear HSE is developed based on pseudo flow measurements. The pseudo flow measurements are derived from PMU measurements to overcome convergence drawbacks associated with using PMU current phasors in the measurement Jacobian matrix. The authors in [17] formulated integrated nonlinear HSE to improve the bad data detection capability introduced by PMU measurements. The measurement vector comprises real power flow and power injection, with bus phase angle as the system vector. The authors in [18] propose a hybrid state estimator based on a nonlinear measurement model with the state vector used in their polar coordinates. Current phasors are transformed from polar form to rectangular format to establish a relationship with the state vector. The authors in [19] propose a hybrid constrained state estimator formulation that introduces a new set of state vectors comprising branch currents in polar form. The measurement model remains nonlinear, requiring the state vectors’ initialization in the state estimation algorithm. Although these proposed methods improve the state estimation performance compared to conventional state estimators based on RTU measurements, the nonlinear measurement model still presents the challenges associated with iterative state estimation algorithms.

The integrated linear HSE has been proposed in [20]. The authors developed a WLAV-based HSE capable of handling limited PMU measurements at refresh rates augmented with previously obtained RTU measurements. To achieve a linear measurement model that guarantees computational efficiency using WLAV, the authors propose a linearization approach for nonlinear measurements using first-order Taylor series expansion. The authors in [21, 22, 23, 24, 25, 26] propose an integrated linear HSE, an approach that is derived based on the equivalent split-circuit formulation for the power flow problem. The authors introduce new circuit models relating to the RTU and PMU measurements to formulate a linear measurement model. The equivalent circuit formulation method proposed is anchored on the modeling power system in terms of voltage and current variables. The equivalent circuit models are coupled with circuit analysis to transform SCADA measurements into linear functions of voltage and current variables. The authors in [27], [28] also propose a linear formulation for HSE where the state vector comprises complex voltage and a redundant state for the voltage phase angle. A Least Absolute Value (LAV) estimator is used in [27] for automatic rejection of bad data with simulations indicating the accuracy of LAV over WLS in the presence of bad data. Therefore, a key advantage in formulating an integrated linear state estimator is the capability of being implemented with the WLAV estimation criterion to improve computational efficiency and bad data identification and rejection. The Authors in [29] propose a linear state estimator that treats RTU and PMU data simultaneously, while the states are estimated in rectangular coordinates. Since the estimation criterion is based on WLS, the largest normalized residual test is used for bad data detection.

Therefore, this paper proposes the development of a novel mathematical formulation for an integrated linear HSE using a standard steady-state branch model typically used for power flow analysis. The proposed model is also robust against bad data through the WLAV estimation criterion. The developed HSE model is evaluated in three standard transmission test busses, IEEE 14 bus, IEEE 30 bus, and IEEE 57 bus, using WLS and WLAV state estimation methods. The main contributions of this paper are;

i. Derivation of an integrated linear measurement model for RTU and PMU measurements using a standard steady-state branch model consisting of transmission lines, transformers, and phase shifters.

Using a common branch model for the measurements coupled with circuit analysis techniques presents simplicity in the measurement model formulation compared to existing methods.

ii. A noniterative WLAV estimation criterion is developed based on the interior point method in MATLAB for the proposed linear measurement model. The WLAV eliminates the vulnerability of the HSE to bad data model, while the noniterative algorithm eliminates challenges of algorithm initialization, computational burden, and slow convergence.

Subsequent sections of this paper cover a conventional nonlinear measurement model basics in Section 3. Section 4 presents the proposed methodology used to formulate a linear hybrid measurement model illustrating the various steps to develop the model. In Section 5, the mathematical linear hybrid measurement model developed is evaluated using the three standard IEEE transmission systems through simulations performed in MATLAB. Section 6 presents the results and discussion comparing the performance of the proposed model to the conventional model, and finally, a conclusion is drawn in Section 7.

3. A conventional nonlinear measurement model

Power system state variables comprise bus voltage magnitude and phase angle at all buses. In contrast, the conventional measurements comprise power injection at the buses and power flow through a transmission line. Fig. 4 shows an equivalent circuit model for a transmission line connecting buses i and j [2].

The real and reactive power injection at bus i are expressed by Eq. (1) and Eq. (2), respectively. The real and reactive power flow from bus i to bus j is expressed by Eq. (3) and Eq. (4), respectively.

\[
P_i = V_i \sum_{j=1}^{n} V_j |(G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j))| \tag{1}
\]

\[
Q_i = V_i \sum_{j=1}^{n} V_j |(G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j))| \tag{2}
\]

\[
P_{ij} = V_i^2 (s_{ji} + g_{ij}) - V_i V_j |(g_{ij} \cos(\theta_i - \theta_j) + b_{ij} \sin(\theta_i - \theta_j))| \tag{3}
\]

\[
Q_{ij} = V_i^2 (b_{ij} + h_{ij}) - V_i V_j |(g_{ij} \sin(\theta_i - \theta_j) - b_{ij} \cos(\theta_i - \theta_j))| \tag{4}
\]

Where,

- \(V_i, \theta_i\) are the voltage magnitude and phase angle at bus i
- \(G_{ij} + B_{ij}\) is the complex \(j^{th}\) element of the bus admittance matrix
- \(g_{ij} + b_{ij}\) is the series admittance between buses i and j
- \(s_{ji} + h_{ij}\) is the admittance of the shunt branch connected to bus i
- \(s_{ij} + h_{ij}\) is the admittance of the shunt branch connected to bus j

The presence of trigonometric functions sine and cosine in the power equations introduce nonlinearity. Therefore, using RTU measured variables in state estimation gives a nonlinear state estimation model in Eq. (5) as the objective function.

\[
z = h(x) + e \tag{5}
\]
4. Proposed linear HSE measurement model formulation

The proposed mathematical formulation of a linear hybrid measurement model involves using a branch model equivalent circuit expanded in subsection 4.1. RTU and PMU measurements are used to develop a linear hybrid measurement model through transformation and circuit analysis techniques. Subsections 4.2 and 4.3 give RTU and PMU measurements mathematical formulations, respectively, with the integration of both mathematical formulations illustrated in subsection 4.4. Fig. 5 shows the block diagram for the proposed hybrid state estimator, which captures the various stages of developing a linear hybrid measurement model and state estimator. The RTU measured variables are voltage magnitude, real/reactive power injections, and power flows. The considered PMU measured variables are voltage phasor and current phasor. In this paper, the power system state variables are taken as real and imaginary voltages in rectangular form.

PMU measurements have higher reporting rates than RTU measurements, with a PMU capable of measuring voltage and current phasor at 20 to 60 times in a second. At the same time, an RTU updates its measurements every 2 to 4 seconds. This paper assumes that PMU and RTU measurements are taken at the same snapshot in the proposed estimator. However, various methods proposed in the literature could compensate for the time skew, such as RTU measurement forecasting to augment the PMU measurements at the instant when only PMU measurements are received [20], [30]. The authors in [31] also propose using an optimal buffer to address the reporting rates disparity.

4.1. Standard steady state branch model

A standard steady-state branch model is shown in Fig. 6, consisting of a transmission line, transformer, and phase shifter. The transmission line comprises series impedance and charging susceptance, in series with a phase-shifting transformer [32], [33]. The branch model links two buses, $k$ and $m$, where:

- $V_k, V_m$ represent complex terminal voltages at buses $k$ and $m$, respectively
- $r$ is the magnitude of the transformer tap ratio
- $\delta_{k/m}$ is transformer phase shift angle
- $y_s$ is series admittance/series component
- $b_s$ is charging susceptance/shunt component
- $i_{km}$ is complex current flow from bus $k$ to bus $m$
- $i_{mk}$ is complex current flow from bus $m$ to bus $k$

RTU and PMU meters may be located at either the buses for measurement of voltage magnitude, real/reactive power injections, and voltage phasors, or on the line/branch linking buses $k$ and $m$ to measure real/reactive power flow current phasors.

4.2. RTU linear measurement model formulation

The linear mathematical formulation for RTU measurements involves; transforming measured RTU variables into current variables, expressing current flow within a transmission line in terms of series and
shunt components, and deriving a linear measurement model based on electrical circuit analysis.

**i. Transformation of measured RTU variables into real and imaginary current variables**

Real/reactive power injections at a bus and real/reactive power flow (Measured RTU variables) through a branch are transformed into complex current measurements based on the complex power equation in Eq. (6).

\[ V^* = P + jQ \]  

\[ V \] is a complex voltage, \( I^* \) is a complex current conjugate, \( P \) is real power, and \( Q \) is reactive power. The complex current in Eq. (6) is expressed as given in Eq. (7)

\[ I^* = \frac{P + jQ}{V} \]  

\[ I \]

As shown in Fig. 6, an RTU meter installed at bus \( k \) measures real and reactive power injection as well as the voltage magnitude; therefore, in terms of these measured variables, the real and imaginary current injected at bus \( k \) is given in Eq. (8) and Eq. (9) respectively. The terms, \( i^R_k \) and \( i^I_k \) represent real current and imaginary current injected at bus \( k \), \( V^R_k \) and \( V^I_k \) represent real and imaginary voltages at bus \( k \). Finally, the terms \( P_k \), \( Q_k \) and \( V_k \) denote real power injection, reactive power injection and voltage magnitude measured at bus \( k \) using RTU meters.

\[ i^R_k = \frac{P_k}{|V_k|^2} V^R_k + \frac{Q_k}{|V_k|^2} V^I_k \]  

\[ i^I_k = \frac{P_k}{|V_k|^2} V^I_k - \frac{Q_k}{|V_k|^2} V^R_k \]  

An RTU meter installed along a branch connecting bus \( k \) with bus \( m \) measures real and reactive power flow and voltage magnitude at bus \( k \). Therefore, the real and imaginary current flow from bus \( k \) to bus \( m \) is given in Eq. (10) and Eq. (11), where, \( i^R_{km} \) and \( i^I_{km} \) represent real current and imaginary current flow from bus \( k \) to bus \( m \), \( V^R_k \) and \( V^I_k \) represent real and imaginary voltages at bus \( k \). The terms \( P_{km} \) and \( Q_{km} \) denote real power flow and reactive power flow measured from bus \( k \) to bus \( m \), while \( V_k \) is the voltage magnitude measured at bus \( k \).

\[ i^R_{km} = \frac{P_{km}}{|V_k|^2} V^R_k + \frac{Q_{km}}{|V_k|^2} V^I_k \]  

\[ i^I_{km} = \frac{P_{km}}{|V_k|^2} V^I_k - \frac{Q_{km}}{|V_k|^2} V^R_k \]  

**ii. Expression of real and imaginary current flow in terms of transmission line components**

The branch admittance matrix (\( Y_{br} \)) relates complex current at both ends of a transmission line with the complex terminal voltages as shown in Eq. (12), where \( i_{km} \) and \( i_{mk} \) are complex current flow from bus \( k \) and bus \( m \), respectively. \( V_k \) and \( V_m \) are the terminal complex voltages at bus \( k \) and bus \( m \), respectively. The branch admittance matrix (\( Y_{br} \)) is expressed in matrix form as given in Eq. (13) based on the parameters of the branch model given in Fig. 6. For simplicity, the four elements of the branch admittance matrix are labeled as shown in Eq. (14), where \( Y_{ff} \) and \( Y_{ft} \) are the complex branch admittance matrix elements relating complex current flow from bus \( k \) to complex terminal voltages at bus \( k \) and bus \( m \), respectively. The terms \( Y_{ff} \) and \( Y_{ft} \) are complex admittance elements relating complex current flow from bus \( m \) to complex voltages at bus \( k \) and bus \( m \), respectively.

\[ \begin{bmatrix} i_{km} \\ i_{mk} \end{bmatrix} = Y_{br} \begin{bmatrix} V_k \\ V_m \end{bmatrix} \]  

\[ Y_{br} = \begin{bmatrix} y_{ff} & y_{ft} \\ y_{tf} & y_{tt} \end{bmatrix} = \begin{bmatrix} y_{ikm} & -\frac{1}{\tau_e} \end{bmatrix} \begin{bmatrix} y_{ikm} & -\frac{1}{\tau_e} \end{bmatrix} \]  

\[ Y_{ff} = \begin{bmatrix} Y_{ff} \\ Y_{ft} \end{bmatrix}, \quad Y_{ft} = \begin{bmatrix} Y_{tf} \\ Y_{tt} \end{bmatrix} \]  

Converting the complex currents in Eq. (12) into real and imaginary current parts, the current flow from both ends is given in terms of the transmission line components, as shown in Eq. (15).

\[ \begin{bmatrix} i^R_{km} \\ i^I_{km} \\ i^R_{mk} \\ i^I_{mk} \end{bmatrix} = \begin{bmatrix} y^R_{ff} & y^R_{ft} & -y^I_{ff} & -y^I_{ft} \\ y^I_{ff} & y^I_{ft} & y^R_{ff} & y^R_{ft} \\ y^I_{ff} & y^I_{ft} & y^I_{ff} & y^I_{tt} \\ y^R_{ff} & y^R_{ft} & y^I_{ff} & y^I_{tt} \end{bmatrix} \begin{bmatrix} V^R_k \\ V^I_k \\ V^R_m \\ V^I_m \end{bmatrix} \]  

Where \( i^R_{km} \) and \( i^I_{km} \) are real and imaginary current flow from bus \( k \) to bus \( m \), respectively, \( i^R_{mk} \) and \( i^I_{mk} \) are real and imaginary current flow from bus \( m \) to bus \( k \), respectively, \( y^R_{ff} \), \( y^I_{ff} \), \( y^R_{ft} \), \( y^I_{ft} \) are real parts of the complex branch admittance matrix, \( y^I_{ff} \), \( y^I_{ft} \), \( y^I_{ff} \), \( y^I_{tt} \) are imaginary parts of the complex branch admittance matrix, \( V^R_k, V^I_k, V^R_m, V^I_m \) are real and imaginary parts of the complex terminal voltages at bus \( k \) and bus \( m \).
ii. Derivation of linear measurement model based on electrical circuit analysis

Fig. 7 shows complex current injection into a bus $k$ based on RTU measured variables and complex current flow to other buses through respective branches. Based on Kirchhoff’s Current Law (KCL), the algebraic sum of current injection at a bus equals zero, considering currents flowing away from the bus to be positive and currents flowing into the bus to be negative. Thus, the real and imaginary algebraic current injected at bus $k$ is given in Eq. (16) and Eq. (17), respectively.

\[
0 = I_k^R = \left\{ \sum_{j=1}^{n} (\gamma_{fji}) + \sum_{i=1}^{n} (\gamma_{kii}) - \frac{P_k}{|V_k|^2} \right\} v_k^R + \left\{ \sum_{i=1}^{n} (\gamma_{fji}) + \sum_{i=1}^{n} (\gamma_{kii}) \right\} v_m^R \\
+ \left\{ \sum_{i=1}^{n} (-\gamma_{fji}) - \frac{P_k}{|V_k|^2} \right\} v_k^I + \left\{ \sum_{i=1}^{n} (-\gamma_{fji}) + \sum_{i=1}^{n} (\gamma_{kii}) \right\} v_m^I \tag{16}
\]

\[
0 = I_k^I = \left\{ \sum_{i=1}^{n} (\gamma_{fji}) + \sum_{i=1}^{n} (\gamma_{kii}) + \frac{Q_k}{|V_k|^2} \right\} v_k^R + \left\{ \sum_{i=1}^{n} (\gamma_{fji}) + \sum_{i=1}^{n} (\gamma_{kii}) \right\} v_m^R \\
+ \left\{ \sum_{i=1}^{n} (-\gamma_{fji}) - \frac{Q_k}{|V_k|^2} \right\} v_k^I + \left\{ \sum_{i=1}^{n} (-\gamma_{fji}) + \sum_{i=1}^{n} (\gamma_{kii}) \right\} v_m^I \tag{17}
\]

An illustration of algebraic current flows through a transmission line connecting bus $k$ and bus $m$, considering the transmission line series and shunt components, is shown in Fig. 8. The measured variables within the branch are real and reactive power flow.

KCL is applied at nodes $a$ and $b$ of the transmission line. Eq. (18) and Eq. (19) give the real and imaginary algebraic current at node $a$ considering current flow to the node to be negative and the current flow away from the bus taken to be positive. Eq. (20) and Eq. (21) give real and imaginary current injections at node $b$.

\[
0 = I_a^R = \left\{ y_{fji}^R - \frac{P_{km}}{|V_k|^2} \right\} v_k^R + \left\{ -\gamma_{fji}^R - \frac{Q_{km}}{|V_k|^2} \right\} v_m^R \\
+ \left\{ \gamma_{fji}^R - \frac{Q_{km}}{|V_k|^2} \right\} v_k^I - \gamma_{fji}^I v_m^I \tag{18}
\]

\[
0 = I_a^I = \left\{ y_{fji}^I + \frac{Q_{km}}{|V_k|^2} \right\} v_k^R + \left\{ y_{fji}^R - \frac{P_{km}}{|V_k|^2} \right\} v_m^R + \left\{ -\gamma_{fji}^R - \frac{Q_{km}}{|V_k|^2} \right\} v_k^I + \gamma_{fji}^I v_m^I \tag{19}
\]

\[
0 = I_b^R = \left\{ y_{fji}^R - \frac{P_{km}}{|V_k|^2} \right\} v_k^R + \left\{ \gamma_{fji}^R - \frac{Q_{km}}{|V_k|^2} \right\} v_m^R + \left\{ y_{fji}^R - \frac{P_{km}}{|V_k|^2} \right\} v_k^I - \gamma_{fji}^I v_m^I \tag{20}
\]

\[
0 = I_b^I = \left\{ y_{fji}^I + \frac{Q_{km}}{|V_k|^2} \right\} v_k^R + \left\{ y_{fji}^R - \frac{P_{km}}{|V_k|^2} \right\} v_m^R + \left\{ \gamma_{fji}^R - \frac{Q_{km}}{|V_k|^2} \right\} v_k^I + \gamma_{fji}^I v_m^I \tag{21}
\]

Considering state variables as real and imaginary voltages at the respective buses and based on KCL, the algebraic sum of currents entering a bus is zero. Therefore, the zero values for both complex current injection and complex current flow are related to the state variables by constant terms giving rise to an RTU-based linear measurement model in Eq. (22).

\[
\begin{bmatrix}
 0 \\
 0 \\
 0 \\
 0 \\
 0
\end{bmatrix} = 
\begin{bmatrix}
 H_{11} & H_{12} & H_{13} & H_{14} & v_k^R \\
 H_{21} & H_{22} & H_{23} & H_{24} & e_k^R \\
 H_{31} & H_{32} & H_{33} & H_{34} & e_k^I \\
 H_{41} & H_{42} & H_{43} & H_{44} & e_m^R \\
 H_{51} & H_{52} & H_{53} & H_{54} & e_m^I
\end{bmatrix} \\
\begin{bmatrix}
 v_k^R \\
 e_k^R \\
 v_k^I \\
 e_k^I \\
 e_m^R \\
 e_m^I
\end{bmatrix} \tag{22}
\]

where $v_k^R, v_k^I, e_k^R$ and $e_k^I$ represent real and imaginary forms of the state variables respectively at the respective buses. $H$ matrix represents various coefficients of the state variables, which are constant terms. $e_k^R, e_k^I, e_m^R$ and $e_m^I$ are real and imaginary errors within the currents resulting from measured values.

4.3. PMU linear measurement model formulation

PMU measurement for state estimation comprises both voltage phasors and current phasors. The measurement model adopted considers state variables in rectangular coordinates. Therefore, the measured phasor variables are converted into a rectangular form to obtain a linear relationship for the phasors with respect to state variables. Each of the phasor measurements is converted separately to derive a linear relationship.

i. Voltage Phasor measurements

The conversion of measured voltage phasor measurements to rectangular form results in the measurement model given in Eq. (23). The voltage measurements are linked to state variables via an identity matrix.
where: \( V_{\text{bus}}^R \) and \( V_{\text{bus}}^I \) are real and imaginary voltage values obtained from measured phasor voltages
\( i_k^R \) and \( i_k^I \) are state variables in rectangular format
\( e_k^R \) and \( e_k^I \) are errors resulting from real and imaginary voltages at bus \( k \)
\( e_{\text{bus}}^R \) and \( e_{\text{bus}}^I \) are errors resulting from real and imaginary voltages at bus \( m \).

### 4.4. Linear HSE measurement model integration

A linear hybrid measurement model integrates models formulated based on RTU and PMU measurements. In hybrid state estimation, both models are integrated to leverage the advantages associated with both types of measurements. Integrating the developed linear RTU and PMU models results in a hybrid linear measurement model in Eq. (23). The model parameters are obtained from RTU and PMU linear model formulation under sections 4.2 and 4.3. Eq. (26) may also represent the linear measurement model with \( H \) as a constant matrix relating the measurements to the estimated state variables, \( x \). The formulation gives a linear measurement model; therefore, a linear algorithm can be applied to determine power system state variables, eliminating challenges associated with nonlinear algorithms.

\[
\begin{bmatrix}
    i_k^R \\
    i_k^I \\
    i_{\text{bus}}^R \\
    i_{\text{bus}}^I \\
\end{bmatrix}
\begin{bmatrix}
    Y_{ff}^R & Y_{fi}^R & -Y_{if}^R & -Y_{if}^R \\
    Y_{fi}^R & Y_{ff}^R & Y_{if}^R & -Y_{if}^R \\
    Y_{if}^R & -Y_{if}^R & Y_{ff}^R & Y_{fi}^R \\
    Y_{if}^R & -Y_{if}^R & -Y_{if}^R & Y_{ff}^R \\
\end{bmatrix}
\begin{bmatrix}
    i_k^R \\
    i_k^I \\
    i_{\text{bus}}^R \\
    i_{\text{bus}}^I \\
\end{bmatrix}
\begin{bmatrix}
    e_k^R \\
    e_k^I \\
    e_{\text{bus}}^R \\
    e_{\text{bus}}^I \\
\end{bmatrix}
\]

(Eq. 24)

### 5. Evaluation of the proposed linear HSE model in MATLAB

The evaluation of the proposed linear HSE model describes the state estimation criterion adopted and the simulation description.

#### i. State estimation method

A well-established and robust state estimation algorithm known as Weighted Least Absolute Value (WLAV) is considered in this paper to determine the solution of the developed linear measurement model [3, 34]. WLAV minimizes the weighted sum of absolute values of the measurement residuals. Given a linear measurement model developed in Eq. (26), the solution for vector \( x \) (state variables) is given by the solution of the state estimation optimization problem in Eq. (27) with the objective function minimizing the sum of the weighted absolute value of the measurement residuals. The optimization problem is solved using linear programming, one of the most successful forms of optimization in power system problems [35].

\[
\min \sum w_i |r_i|
\]

(Eq. 27)

subject to:

#### s.t.
\[ z = Hx + r \]

where \( w \) is a vector of various measured variables weights and \( r \) as the measurement residue. The optimization problem is transformed to a standard linear programming format to use existing MATLAB linear programming solvers, with the interior point method adopted in determining the solution. A standard linear programming format [35] is given in Eq. (28).

\[
\begin{align*}
\min & \quad c^T y \\
\text{s.t.} & \quad z = Hx + r \\
& \quad y \geq 0
\end{align*}
\]

(Eq. 28)

Transforming the state estimation problem given in Eq. (27) into standard linear programming format involves decomposing each of the vectors \( x \) and \( r \) into two positive vectors (\( x^+ \) and \( x^- \)) and (\( r^+ \) and \( r^- \)) as given in Eq. (29). The vectors are decomposed to ensure the variables are strictly nonnegative.

\[
\begin{align*}
\min & \quad c^T |x^+ - x^-| \\
\text{s.t.} & \quad z = H(x^+ + x^-) + r^+ - r^- \\
& \quad x^+, x^-, r^+, r^- \geq 0
\end{align*}
\]

(Eq. 29)

Defining vector \( y \) as having positive vector variables;

\[
\begin{bmatrix}
    x^+ \\
    x^- \\
    r^+ \\
    r^-
\end{bmatrix}
\]

and \( c^T = w^T = [0 \quad 0 \quad 1 \quad -1] \). Eq. (29) may be written as given in Eq. (30), which represents a standard linear programming problem format as depicted in Eq. (28).

\[
\begin{bmatrix}
    z = H^+ x^+ - H^- x^- + r^+ - r^- = \begin{bmatrix} H & -H & I & -I \end{bmatrix} \begin{bmatrix} x^+ \\
    x^- \\
    r^+ \\
    r^-
\end{bmatrix} = My = b
\end{bmatrix}
\]

(Eq. 30)

where

\[
M = \begin{bmatrix} H & -H & I & -I \end{bmatrix}, \quad z = b.
\]

#### ii. Simulation description

The developed linear hybrid measurement model is simulated in MATLAB using MATPOWER. The simulations are performed on an HP
Table 1. Standard deviation for various measurement types.

| Measurement Type       | Standard Deviation (σ) |
|------------------------|------------------------|
| Voltage magnitude      | 0.01                   |
| Real power injection   | 0.02                   |
| Reactive power injection | 0.04                |
| Real power flow        | 0.02                   |
| Reactive power flow    | 0.04                   |
| Voltage phasor         | 0.001                  |
| Current phasor         | 0.001                  |

Table 2. RTU measurement description.

| Measurement type          | IEEE 14 BUS | IEEE 30 BUS | IEEE 57 BUS |
|---------------------------|-------------|-------------|-------------|
| Voltage Magnitude         | 5           | 14          | 28          |
| Real power injection      | 9           | 16          | 34          |
| Reactive power injection  | 9           | 16          | 34          |
| Real power flow from a bus| 11          | 25          | 50          |
| Reactive power flow from a bus | 11          | 25          | 50          |
| Real power flow to a bus  | 6           | 16          | 32          |
| Reactive power flow to a bus | 6           | 16          | 32          |
| Total RTU measurements (m)| 57          | 128         | 260         |
| Number of states to be estimated (n)| 27        | 59         | 113         |
| Measurement Redundancy (2^n) | 2.11       | 2.17        | 2.30        |

Computer with an Intel processor core i7 CPU @ 1.80 GHz, 1.99 GHz, and 8 GB RAM. Load flow is performed to obtain various RTU and PMU measurements. Random error is added to each load flow measurement, as shown in Eq. (31). Standard deviation reflects the expected accuracy of the corresponding meter. The voltage magnitude meter is assigned maximum weight for RTU meters, with the reactive power injection/reactive power flow meter assigned least weight. Real power injection and reactive power flow are assigned a weight between the voltage magnitude and the reactive power values [9]. The weights are based on various standard deviations. A smaller standard deviation indicates a higher weight, corresponding to a more reliable measurement, while a large standard deviation indicates a low weight, corresponding to a less reliable measurement [36]. PMU devices provide relatively higher accurate measurements than RTU. Therefore, a much lower standard deviation than RTU measurements is assigned. The standard deviation (σ_i) values used are obtained in literature [37], [38] and are given in Table 1.

\[ Z_{i} = (σ_i \times \text{rand} (1) + \text{Actual value}_i) \]  

where:

- \( Z_{i} \) is the \( i^{th} \) measurement with added random error
- \( σ_i \) is the standard deviation of \( i^{th} \) measurement
- \( \text{rand} \) denotes a random number between 0 and 1
- \( \text{Actual value}_i \) is the actual value of \( i^{th} \) measurement based on load flow results.

Three standard transmission system test cases are considered, IEEE 14-BUS, IEEE 30-BUS, and IEEE 57-BUS. The number and type of measurement considered for each test case are given for RTU measurements to ensure enough redundancy. For instance, in the IEEE-14 bus system, the network comprises 14 buses and 20 branches. The network is assumed to have five voltage magnitude meters, nine power injection meters, eleven power flow meters from a bus, and six power flow meters to a bus. Power meters are assumed to measure both real and reactive values. The number of RTU meters for each measurement type are randomly determined and are given in Table 2.

In this research, the test systems are assumed to be completely observable via RTU measurements as per the measurement redundancy provided for each test case in Table 2. Including PMU measurement is necessary to evaluate the proposed linear HSE model’s performance in estimation accuracy. The test case systems are assumed to be partially observable through PMUs. From the existing studies, for complete system observability based on PMUs only, an optimal number of 4, 10, and 17 PMUs are required for IEEE 14 bus, IEEE 30 bus, and IEEE 57 bus, respectively [39, 40, 41, 42]. Therefore, each test case considers a random number of PMUs for partial observability (less than the optimal number). Consequently, 2 PMUs are chosen for IEEE 14-bus and randomly placed on buses 2 and 6, while for IEEE 30-bus, 4 PMUs are chosen and randomly placed on buses 6, 10, 12, and 25. For IEEE 57-bus, 10 PMUs are chosen and randomly placed on buses 2, 27, 30, 31, 35, 43, 46, 48, 50, and 52. The selected PMUs are assumed to measure the voltage phasor at the installed bus and all current phasors for the branches incident to that bus. Based on the selected PMUs, the number of available PMU measurements is given in Table 3, with the specific meter location in Table 4.

The flow chart for the proposed state estimation process is shown in Fig. 9, where the same procedure is followed for all three test cases. Three scenarios are employed in the simulation,

a. Simulation of a conventional nonlinear measurement model incorporating only RTU measurements using the WLS algorithm.
b. Simulation of developed hybrid linear measurement model incorporating RTU and PMU measurements using the WLS algorithm.
c. Simulation of developed hybrid linear measurement model incorporating RTU and PMU measurements using the WLAV algorithm.

The robustness of WLAV over conventional WLS is tested through the manual introduction of single bad data into a few randomly selected RTU and PMU measurements for each test case. Bad data is characterized by a large standard deviation more than five times the normal standard deviation or negative values for the measured variables such as voltage magnitudes [2]. The bad data parameters introduced into the measurements are given in Table 5.

Performance evaluation for the three different test cases is based on Normalized Cumulative Error (NCE), which is the mean of the sum of the absolute difference between actual values and estimated values given by Eq. (32) [38]. The slack bus phase angle is chosen as the arbitrary reference angle with a value of 0°. For a system containing n buses, the estimated state vector comprises (2n − 1) elements, with n bus volt-
age magnitudes and \((n-1)\) phase angles. A lower value of NCE indicates higher accuracy of the state estimation process. Graphs showing reference and estimated values for the state variables are also generated. A comparison between computation time when using both estimation algorithms is also presented.

\[
NCE = \frac{1}{2n} \sum_{j=1}^{2n-1} |Actual \ value_j - Estimated \ value_j|
\]  

(32)

Where

\(n\) is the number of buses in each test case.

Actual value, is the reference value based on load flow results for \(i^{th}\) measurement.

Estimated value, is the estimated state value for \(i^{th}\) measurement.

6. Results and discussion

The results obtained are classified into two categories, with the first category giving estimation results considering all measurements used with random errors, also known as standard Gaussian errors. The second category considers a few highly erroneous measurements, also known as bad data.

6.1. Results based on measurements with normal Gaussian error

The estimated states in this paper are finally presented as bus voltage magnitude and phase angle for all three test cases.

i. Estimated bus voltage magnitude values

The reference/estimated voltage magnitudes for IEEE 14-BUS, IEEE 30-BUS, and IEEE 57-BUS are shown in Fig. 10, Fig. 11, and Fig. 12, respectively. The figures compare the three test cases’ estimated voltage magnitudes at each bus. The reference value/expected value is the bus voltage magnitude per load flow result. State estimation with higher accuracy is expected to track the reference value closely. The figures also present estimated values considering the conventional nonlinear state estimator based on the WLS method, the proposed linear state estimator based on the WLS method, and finally proposed linear estimator based on a more robust WLAV method. It is observed that the estimated voltage magnitude based on the proposed model closely trails the reference value for all the test cases. As the size of the system increases, the

| Table 5. Bad data description. |
|-----------------------------|
| **Bus Type** | **RTU measurements** | **PMU measurements** |
| IEEE 14 BUS | Voltage magnitude at bus 13 made negative | The real current value for branches 6-13 changed by five times the normal standard deviation |
| IEEE 30 BUS | Voltage magnitude at bus 1 changed by five times the normal standard deviation | The real voltage value at bus 6 made negative |
| IEEE 57 BUS | Voltage magnitude at bus 1 changed by five times the normal standard deviation | The real voltage value at bus 2 made negative |

![Fig. 10. Reference/Estimated bus voltage magnitude for IEEE 14-BUS.](image)

![Fig. 11. Reference/Estimated bus voltage magnitude for IEEE 30-BUS.](image)
conventional model gives estimated values that deviate much from the reference values.

### ii. Estimated bus phase angle values

The reference/estimated phase angle absolute values for the three test cases are shown in Fig. 13, Fig. 14, and Fig. 15, respectively. Estimated phase angle values based on the proposed model also closely track the reference values.

### iii. Normalized cumulative error and computational time

NCE values are presented in Table 6 for the three different test cases for the overall performance evaluation of the proposed linear hybrid model. NCE values show that the developed linear measurement model gives the least cumulative error when using WLS or WLAV than the conventional nonlinear model based on WLS. Table 7 gives computational time with only measurements with Gaussian errors considered. It is observed that the conventional nonlinear model requires more time to compute the system states than the proposed model using either WLS or WLAV.

### Table 6. Performance evaluation based on NCE for measurements with normal Gaussian errors.

| Bus Type | Normalized Cumulative Error Values |
|----------|-----------------------------------|
|          | Conventional nonlinear SE based on WLS | Conventional nonlinear SE based on WLAV | Conventional nonlinear SE based on WLS | Conventional nonlinear SE based on WLAV |
| IEEE 14 BUS | 0.0068 | 0.0027 | 0.0039 |
| IEEE 30 BUS | 0.0536 | 0.0075 | 0.0122 |
| IEEE 57 BUS | 0.2202 | 0.0043 | 0.0020 |

### Table 7. Computational time for measurements with normal Gaussian errors.

| Bus Type | Nonlinear model using WLS | Proposed linear model using WLS | Proposed linear model using WLAV |
|----------|----------------------------|---------------------------------|---------------------------------|
| IEEE 14 BUS | 0.0003348 secs | 0.001769 secs | 0.001694 secs |
| IEEE 30 BUS | 0.013758 secs | 0.002734 secs | 0.001856 secs |
| IEEE 57 BUS | 0.669491 secs | 0.5878 secs | 0.3271 secs |
6.2. Results based on measurements with bad data

i. Estimated bus voltage magnitude values considering bad data

Fig. 16, Fig. 17, and Fig. 18 show the reference/estimated voltage magnitude values for IEEE 14-BUS, IEEE 30-BUS, and IEEE 57-BUS, respectively, when some of the measurements consist of bad data. It is observed that the estimated values are highly affected when using the conventional nonlinear model compared to the proposed model. As the system size increases, the proposed model performs better when the WLAV method is employed than WLS.

ii. Estimated bus phase angle values considering bad data

Fig. 19, Fig. 20, and Fig. 21 show the reference/estimated bus absolute phase angles for IEEE 14-BUS, IEEE 30-BUS, and IEEE 57-BUS, respectively, with some measurements having bad data. The performance of the proposed linear model based on the WLAV method gives better results than the WLS method, even as the system size increases.
Conventional system values.

Table 8. Performance evaluation considering measurements with bad data.

| Bus Type        | Normalized Cumulative Error Values | Conventional nonlinear SE based on WLS | Proposed linear HSE based on WLS | Proposed linear HSE based on WLAV |
|-----------------|-----------------------------------|----------------------------------------|---------------------------------|----------------------------------|
| IEEE 14 BUS     | 1.5691                            | 0.0053                                 | 0.0049                          |                                  |
| IEEE 30 BUS     | 0.0665                            | 0.0561                                 | 0.0122                          |                                  |
| IEEE 57 BUS     | 5.9784                            | 0.4728                                 | 0.0022                          |                                  |

Table 9. Computational time for measurements with bad data.

| Bus Type      | Conventional nonlinear model using WLS | Proposed linear model using WLS | Proposed linear model using WLAV |
|---------------|----------------------------------------|---------------------------------|---------------------------------|
| IEEE 14 BUS   | 0.008625 secs                          | 0.001955 secs                  | 0.001391 secs                  |
| IEEE 30 BUS   | 0.011536 secs                          | 0.003121 secs                  | 0.002306 secs                  |
| IEEE 57 BUS   | 0.375381 secs                          | 0.006266 secs                  | 0.003219 secs                  |

It is also observed that the conventional nonlinear model is highly affected by bad data in the measurements.

iii. Normalized cumulative error and computational time

The overall performance of the conventional nonlinear and proposed linear model is shown in Table 8. It is observed that the performance of the proposed linear model based on the WLAV method gives the best performance of all three test cases. The NCE values for the conventional nonlinear model give a high value indicating less accurate estimated values. The WLAV method takes less computational time to give the system estimates, as shown in Table 9.

6.3. Discussion

For state estimation considering measurements with Gaussian errors, the performance of the conventional nonlinear model is comparable to the proposed linear model for a small test system like the IEEE 14-BUS system. With an increase in system size, the performance of the nonlinear model is negatively affected, as evident in Figs. 8 and 11. Including PMU measurements in the state estimation and the proposed linear model greatly improves the state estimator performance. The performance of the proposed model is also evident in the NCE values, as the proposed linear model gives a lower NCE value signifying a higher estimation accuracy.

Both nonlinear and proposed linear models are negatively affected for measurements with bad data included when the WLS method is employed. Consequently, this shows the necessity for a more robust state estimation method to identify and eliminate bad data in the state estimation process. The proposed linear model based on WLAV remains stable even in the presence of bad data showing the robustness of the WLAV estimation method. The main challenge associated with the WLAV method is computational time. Still, with the development of the proposed linear model, this has been eliminated, and the computational time while using WLAV is the least, as shown in Table 7 and Table 9.

Comparing the proposed linear HSE with the recently developed linear HSE in [29], the PMU functions are reduced from three to two in the proposed methodology by utilizing PMU voltage phasors and PMU current line flow in rectangular coordinates. The proposed methodology integrates transformers and phase shifters in the model formulation other than considering the transmission line separately. The proposed model also uses a robust WLAV criterion that does not require a post-processing stage for bad data detection and elimination. The simplicity of the proposed formulation in attaining a linear HSE model is therefore evident in the presented methodology.

7. Conclusion

This paper develops a novel linear hybrid measurement model based on RTU and PMU measurements using a standard steady-state branch model. The developed model is then validated in hybrid state estimation using three standard transmission IEEE test networks based on two commercially available state estimation algorithms (WLS and WLAV). The states are estimated in rectangular coordinates, then used to obtain voltage magnitude and phase angles at all network buses. The results demonstrate that the developed linear measurement model gives higher accuracy than the existing nonlinear model. It further examines the performance of the developed model, with some measurements being highly erroneous based on WLS and WLAV, with the latter indicating better estimation and computational time performance. Therefore, the developed linear measurement model can be used in hybrid state estimation with different estimation algorithms eliminating the challenges associated with nonlinearity in state estimation. In this paper, the PMUs are randomly placed in the system, and future work will focus on the optimal placement of the PMU to evaluate the state estimation performance.

Declarations

Author contribution statement

Mercy Ndinda Kiio: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.
Cyrus W. Wekesa, Stanley I. Kamau: Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data.

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Data availability statement

The data associated with this study is available online at https://matpower.org.

Declaration of interests statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.
