Coherent Radiative Parton Energy Loss beyond the BDMPS-Z Limit

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Abstract It is widely accepted that a phenomenologically viable theory of jet quenching for heavy ion collisions requires the understanding of medium-induced parton energy loss beyond the limit of eikonal kinematics formulated by Baier-Dokshitzer-Mueller-Peigné-Schiff and Zakharov (BDMPS-Z). Here, we supplement a recently developed exact Monte Carlo implementation of the BDMPS-Z formalism with elementary physical requirements including exact energy-momentum conservation, a refined formulation of jet-medium interactions and a treatment of all parton branchings on the same footing. We document the changes induced by these physical requirements and we describe their kinematic origin.

1 Introduction

Over the last decade, experiments at RHIC [1–4] and at the LHC (see preliminary data in [5]) have demonstrated that the fragmentation pattern of highly energetic partons ($E_\perp \gtrsim 10$ GeV) is altered strongly when embedded in the dense QCD matter produced in ultra-relativistic nucleus-nucleus collisions. This is seen in particular in the inclusive high-$p_\perp$ hadronic spectra that are strongly suppressed by a factor up to 5 (7) at RHIC (LHC) compared to expectations from proton-proton spectra [6–9], and that stay suppressed over the entire high-$p_\perp$ range studied experimentally so far (up to $p_\perp \sim 100$ GeV at the LHC). Additional information about this jet quenching phenomenon comes from a broad range of jet-like particle correlation measurements [5], and most recently from the observation of strong modifications on the level of reconstructed jets in nucleus-nucleus collisions [10,11], where first steps towards a characterization of the entire medium-modified jet fragmentation pattern have been taken.

In general, both inelastic (giving rise to radiative energy loss) and elastic processes (giving rise to collisional energy loss) are expected to contribute to the strong modification of the internal structure of the parton shower, leading in particular to an energy degradation of the most energetic partons as well as effects on transverse momentum broadening and intra-jet multiplicity. However, the QCD-based analytical analysis of jet quenching remains restricted so far to kinematical limiting cases. In particular, studies of the dominant inelastic process of medium-induced gluon radiation have focussed so far on an eikonal high-energy approximation [12–17], according to which the energy of the projectile parton $E$ is taken to be much larger than the energy of the radiated gluon $\omega$, which in turn is treated in the collinear approximation with transverse momenta ($k_\perp$, $q_\perp$) much smaller than the gluon energy,

\begin{equation}
E \gg \omega \gg k_\perp, q_\perp \gg \Lambda_{\text{QCD}}.
\end{equation}

The seminal analyses of radiative parton energy loss by Baier, Dokshitzer, Mueller, Peigné, Schiff [12] and by Zakharov [13] (BDMPS-Z) are based on this approximation. This also applies to more recent analytical formulations (for an overview, see Ref. [18]) and to those Monte Carlo models [19–23] that aim at implementing the analytically known QCD-based calculations.

However, jet quenching phenomenology requires an understanding of the interaction between partonic projectile and QCD medium beyond the kinematic range (1). It is generally agreed that extrapolating calculations of parton energy loss from (1) to the full phenomenologically relevant kinematical range induces un-
certainties that are much larger than other known model-dependent differences [18]. In addition, the analytical calculations of parton energy loss based on (1) show qualitative features that are physically sensible only within this reduced kinematical range of validity. For instance, they conserve energy and momentum only up to corrections of order $O(\omega/E)$ and $O(k_\perp/\omega)$, an approximation that is justified for the range (1) but that can introduce large uncertainties in phenomenologically relevant kinematic regimes. An analogous conclusion applies to the treatment of recoil effects; collisional energy loss is negligible in the kinematics of (1) but may be sizeable in phenomenologically relevant kinematic regimes. Furthermore, the strong ordering of the energy fractions $E - \omega \gg \omega$ is an obstacle for treating all daughter partons subject to the same medium-modified dynamics. Also, within existing analytical treatments, the possibilities of studying the dependence of parton energy loss on properties of the medium is limited. For instance, it is difficult to vary the composition of the medium in terms of elastic and inelastic scattering centers, their hardness and their energy dependence.

Many of the above-mentioned limitations arise from the use of analytical techniques and could be avoided if one formulated the dynamics of jet quenching in terms of a Monte Carlo algorithm. For instance, exact energy-momentum conservation or the democratic treatment of all partonic splittings are implemented easily in Monte Carlo approaches while their inclusion in analytical formulations is complicated. Also, Monte Carlo techniques enhance naturally the versatility in testing different scattering properties of the medium or in interfacing with different hadronisation models. In practice, if one wants to improve the applicability of a calculational framework with Monte Carlo techniques, it is an obvious prerequisite to establish with which accuracy and in which kinematic range a Monte Carlo algorithm reproduces by construction a defined calculational framework. With this motivation, we have established recently a MC algorithm that faithfully accounts for the dominant medium-induced quantum interference effects present in the BDMPS-Z formalism [24,25]. In the present work, we document how results of this MC version of the BDMPS-Z formalism are altered, when the MC simulation outside the kinematic range (1) is supplemented by various physical requirements.

2 Extending the MC algorithm beyond the BDMPS-Z limit

In the following, the setting in which the MC algorithm reproduces quantitatively all features of the BDMPS-Z formalism will be referred to as BDMPS-Z limit. This algorithm is described in detail in [25]. In the BDMPS-Z limit, the medium is simulated as a source of elastic and inelastic scattering centres, characterized by the mean free paths $\lambda_{\text{inel}}$ and $\lambda_{\text{elas}}$ and the corresponding differential cross sections

$$\frac{d\sigma_{\text{elas}}}{dq} \propto C_R \frac{1}{(q^+ + \mu^2)^2} \theta(2\mu - |q_\perp|),$$

(2)

and

$$\frac{d\sigma_{\text{inel}}}{d\omega dq d\kappa} \propto C_R \frac{d\sigma_{\text{elas}}}{dq} \frac{1}{\omega} \delta(k_\perp - q_\perp).$$

(3)

With this choice, the BDMPS-Z transport coefficient that is typically used to characterize the medium reads $\hat{Q}_{MC} \equiv \mu^2/\lambda_{\text{elas}}$. Here, $\lambda_{\text{elas}} \equiv 1/n\sigma_{\text{elas}}$ and $n$ denotes the density of scattering centers. In the following, we study extensions of the BDMPS-Z baseline algorithm that account for the following physical requirements:

1. energy-momentum conservation

In the BDMPS-Z limit (1), the projectile parton energy remains unchanged by gluon emission. Also, the transverse momentum accumulated by the gluon is independent of the gluon energy. Here, we extend this MC algorithm to include exact local energy and momentum conservation of all partonic interactions and splittings.

2. refined formulation of jet-medium interactions

In the BDMPS-Z limit, transverse momentum transfers are bounded by $q_\perp \leq 2\mu$ to ensure a multiple soft scattering scenario. Here, we consider extensions that vary the scale and energy dependence of elastic and inelastic interactions with the medium. In particular

(a) beyond the soft multiple scattering approximation

The extended MC algorithm will allow for momentum transfers in the extended range $q_\perp \leq \omega$.

(b) energy-dependent cross section

In general, the scattering cross sections can acquire an energy dependence, e.g. due to phase space restrictions. We consider a scenario with energy dependent mean free paths

$$\lambda_{\text{elas}}(\omega) \propto \frac{\omega^2 + \mu^2}{\omega^2}$$

(4)

and

$$\lambda_{\text{elas}}(E_{\text{proj}}) \propto \frac{\lambda_{\text{elas}}(E_{\text{proj}})}{\ln(E_{\text{proj}}/\omega_{\text{min}})}.$$  

(5)

As explained in Ref. [25], the cut-off dependence of the MC algorithm on the kinematic range $[\omega_{\text{min}}, E_{\text{proj}}]$ of the gluon energy $\omega$ does not add to the uncertainties of the BDMPS-Z limit, since it can be absorbed in a logarithmic dependence of the inelastic mean free path.
3. **no overlap of formation times**

The somewhat surprising finding of [25] was that in order to reproduce the BDMPS-Z limit exactly, one has to allow for overlapping gluon formation times. Firstly, this complicates numerical implementations considerably. Secondly, it still remains to be shown whether the effect of overlapping formation times persists in more complete calculations of multiple medium-induced gluon radiation (for a first step towards such calculations, see Refs. [26–28]). It is therefore interesting to quantify the difference between an exact implementation of the BDMPS-Z results, and a simplified ‘no overlap’ implementation in which only one gluon can be formed at a time.

4. **democratic treatment of all partons**

In the BDMPS-Z limit, one neglects elastic interactions of partons with energy $O(E)$ (since elastic momentum transfers can be neglected in the kinematical range (1)), and one neglects inelastic branchings of radiated gluons (since the calculation focuses on the medium modifications of the most energetic parton) [25]. Here, we include both. It is assumed that the new mean free paths can be approximated by adjusting the colour factors of the BDMPS-Z ones,

$$
\lambda_{\text{elas}}^\beta = \lambda_{\text{elas}}, \quad \lambda_{\text{elas}}^\eta = \frac{C_A}{C_F} \lambda_{\text{elas}},
\lambda_{\text{inel}}^\beta = \left( \frac{C_F}{C_A} \right)^2 \lambda_{\text{inel}}, \quad \lambda_{\text{inel}}^\eta = \lambda_{\text{inel}}. \quad (6)
$$

Scenarios without democratic treatment will be referred to as standard in the following. Subjecting all partons to the same medium-dependent dynamics may be regarded as a first step towards describing jet-like observables. It obviously influences the inclusive gluon spectrum and angular distribution, but it also provides additional information for a study of the suppression pattern of single-inclusive hadron spectra. In particular, instead of calculating the energy loss of the “projectile parton” as in formalisms based on the BDMPS-Z limit, one can now characterise the energy fraction carried by the most energetic partonic fragment in the ensemble, irrespective of whether this fragment is the projectile parton or some of the other (mostly gluonic) fragments.

In the following, we discuss the impact of these model improvements on different observables.

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1We restrict the study of this democratic treatment of all parton branchings to simulations in which exact energy-momentum conservation is implemented. The (partonic) multiplicity and the total energy of the ensemble would grow unphysically large in a scenario in which energy-momentum conservation is treated only approximately according to (1) while all partons are subject to the same dynamics of medium-induced rescattering and splitting.
We understand this by observing that the requirement $k_\perp \leq \omega$ leads to a minimal formation time

$$\tau = \frac{3 \frac{2\omega}{k^2_\perp} \geq \frac{3}{\omega} \frac{2}{\omega} = \tau_0. \quad (8)$$

Since the formation time has to be smaller than the medium length $L$, gluons with energy smaller than $\omega_0 = 6/L$ cannot be radiated².

If one allows radiated gluons to branch again (‘democratic scenario’), the large-$\omega$ tail of the spectrum remains unchanged since energetic gluons have long formation times, which inhibits further emissions. At very low energies, however, the part of the spectrum that was inaccessible in the standard scenario gets populated by gluons that moved into that region by radiating away parts of their energy, see Fig. 1.

Compared to the extensions of the MC algorithm discussed so far, lifting the soft scattering approximation leads to a quantitatively much more important increase in the number of energetic gluons. This is so, since a gluon can decohere much faster than in the soft scattering case, when a rather substantial transverse increase in the number of energetic gluons. This is so, since the formation time has to be smaller than the medium length $L$, gluons with energy smaller than $\omega_0 = 6/L$ cannot be radiated².

In the standard scenario, the spectrum at low gluon energy is still suppressed, since the condition $q_\perp \leq \omega$ is a more stringent bound than $q_\perp \leq 2\mu$, which is used in the BDMPS-Z simulation. This leads to the suppression of low-energy gluons in the standard scenario. The shape of the gluon spectrum changes to an approximate $\omega^{-3/2}$ dependence at large energies and is nearly flat for small energies. In the democratic scenario the suppression of soft gluons is overcome by the enhanced radiation leading to an increase in the entire $\omega$-range.

Rescaling the mean free paths to take into account their energy dependence leads to a slight increase in radiation around energies below about 1 GeV (Fig. 1), while requiring that only one gluon can be formed at a time leads to the expected suppression of gluon radiation over the entire energy range. The size of the effect depends on the relation between the elastic and inelastic mean free paths and is only mild in this example.

In summary, the most dramatic changes in the gluon spectrum come from imposing energy and momentum conservation and from relaxing the soft scattering approximation. We note in this context that there is no a priori knowledge as to whether jet-medium interactions are dominated entirely by soft momentum exchanges with $q_\perp < 2\mu$, or whether the $\propto 1/q_\perp^4$-tail of hard perturbative processes makes a numerically significant contribution. Both cases may be regarded as corresponding to different, a priori conceivable properties of the dense QCD matter produced in heavy ion collisions. The strong sensitivity of the medium-induced gluon radiation pattern to such differences in the nature of medium-induced momentum transfers may be regarded as a phenomenologically wanted feature, as it points to the possibility of differentiating between different dynamical mechanisms of jet-medium interactions.

4 Radiated energy

In the BDMPS-Z limit, the radiative energy loss $\Delta E$ is uniquely defined as the energy radiated off the projectile parton and is given by the first moment of the gluon spectrum. It has a quadratic path length dependence characteristic of the LPM-effect. As shown in Fig. 2 this behaviour is reproduced to high accuracy by the MC algorithm. In the standard scenario and for sufficiently small medium path length $L$, conservation of energy and momentum has only a mild effect on the energy loss, as mainly low energy emissions are affected. But for larger in-medium path length, deviations from the BDMPS-Z formalism due to four-momentum conservation are more pronounced. In particular, for medium path lengths $L$ that exceed the typical coherence length $L_c$ of the most energetic gluons $\omega \sim E_{proj}$, the average parton energy loss is known to grow linearly in the BDMPS-Z formalism [24,25], while it is bound to $\Delta E < E_{proj}$ by exact energy momentum conservation. Here, we did not explore further this trivial and known effect that sets in only at very large in medium path lengths where it plays an important if not dominant role [24].

Relaxing the soft scattering assumption, on the other hand, not only significantly increases the energy loss by enhancing energetic radiation, but also changes the path length dependence from quadratic to linear (except for very small $L$). The reason is that harder momentum transfers can destroy coherence leading to quasi-incoherent emission of energetic gluons.

Energy-dependent mean free paths and no overlapping formation times lead to a slight enhancement and a somewhat bigger reduction of the energy loss, respectively, without changing the $L$-dependence much. When all these extensions of the MC algorithm are included, one finds that the removal of the soft scattering approximation induces the most significant changes both in the $L$-dependence and the absolute magnitude of $\Delta E$.

If one allows radiated partons to split again (democratic treatment), one can define the radiative energy

²The factor 3 in Eq. 8 arises from the observation that in QCD-based calculations of destructive interference terms, the phase has to become larger than $\sim 3$ in order for the gluon to decohere from the projectile [25].
loss alternatively as the difference between the energy of the incident projectile parton and the energy of the most energetic parton at the end of the cascade. This definition is arguably more physical, as a unique correspondence between the initial projectile parton and the most energetic final parton exists only in the BDMPS-Z limit. (In general, the most energetic parton may very well be one of the gluons radiated off the projectile quark.) Obviously, the so defined energy loss cannot be larger than the energy radiated away by the projectile quark. This is clearly seen on the lower panel of Fig. 2. The energy loss is smaller, but the qualitative picture of how extensions of the MC algorithm affect the result is very similar to the standard scenario.

A more differential way of looking at the energy loss is by calculating the distribution of the energy loss at a fixed path length $L$ as shown in Fig. 3. The distributions all fall steeply with a rather significant probability for no energy loss at all. In terms of differences between the different modifications the results reflect closely the findings discussed for the total energy loss. In the democratic scenario, virtually the only difference between the two definitions of the energy loss is that the distributions with the energy loss derived from the most energetic fragment at some point break off. This is most prominent in the case without overlapping formation times: Here the distribution stops at $\epsilon = 0.5$ because it is very unlikely to radiate more than one gluon per event. In both scenarios, removing the soft scattering assumption induces the largest modifications.

**Fig. 2** MC results for the radiative energy loss in the standard scenario (top), where the energy loss is defined as the energy radiated away by the projectile quark, and democratic scenario (bottom), where the energy loss is defined as the difference between the energy of the most energetic fragment and the incoming quark energy. Note that in the lower panel the 'BDMPS + energy conservation' calculation has the standard definition of the energy loss (parameters as in Fig. 1).

**Fig. 3** MC results for the distribution of the fractional energy loss $\epsilon = \Delta E/E_{proj}$ in the standard (top) and democratic (bottom) scenarios for the parameters of Fig. 1. The different definitions of $\Delta E$ in both scenarios are explained in the text. The probability of no parton energy loss has a discrete weight $p_0 \delta(\epsilon)$ and is included in the first bin. Note that in the lower panel the 'BDMPS + energy conservation' calculation has the standard definition of the energy loss.
democratic scenario - 0.5 GeV < ω < 1 GeV

standard scenario - 0.5 GeV < ω < 1 GeV

Fig. 4 MC results for the transverse momentum distribution ($1/N\, dN/dk^2$) in one low and one high gluon energy bin in the standard and democratic scenario (parameters as in Fig. 1).

5 Transverse momentum distribution

Fig. 4 shows the transverse momentum distribution of radiated gluons in two energy bins for the cases that subsequent branchings of radiated gluons are ('democratic scenario') or are not ('standard scenario') allowed. The transverse momentum is defined relative to the incident projectile’s direction.

In the standard scenario, energy and momentum conservation cuts out the parts of the distribution with $q_\perp > \omega$, as can be seen in the low energy bin (Fig. 4 (a)). At this gluon energy removing the soft scattering constraint restricts the $q_\perp$ range, which shifts the transverse momentum distribution towards smaller $k_\perp$ values. Forbidding formation times to overlap, on the other hand, leads to an overall suppression without affecting the shape of the gluon distribution much. Finally, the sum of all effects is dominated by energy-momentum conservation. This changes drastically at higher gluon energies (Fig. 4 (b)). Here, conservation of energy and momentum and making the cross sections energy dependent has no effect at all on the transverse momentum distribution. The assumption that formation times cannot overlap again leads to a suppression but does not change the shape. In the case without soft scattering approximation, on the other hand, the more energetic gluons can profit from the tail of the elastic scattering cross section and can acquire significantly higher transverse momenta. This feature is also visible when all modifications are considered together.

In the democratic scenario the most striking feature is that the distribution extends to much larger transverse momenta. This is natural as in this scenario gluons radiated off gluons start out with a sizeable transverse momentum relative to the projectile or jet axis. In return another effect becomes visible, namely the smearing out of the peak which has to do with the fact that gluons can radiate other gluons and thus move to lower energies. Again, the most significant modifications compared to the BDMPS-Z limit result from imposing energy and momentum conservation and from...
removing the soft scattering approximation for interactions between jet and medium.

6 Summary and Conclusions

The present study documents for exemplary cases how elementary physical requirements (such as energy-momentum conservation or the democratic treatment of all scattering processes) and different physical assumptions about the jet-medium interaction can affect the simulation of jet quenching phenomena. We emphasize that the significant differences between the model cases explored here should not be viewed as systematic uncertainties of jet quenching simulations. Rather, energy-momentum conservation and a dynamical treatment of all scattering and branching processes on an equal footing are clearly sensible physical requirements. To the extent to which imposing these requirements changes the result of a simulation, a simulation without them should not be regarded as reliable. This prompts us to conclude that these requirements are prerequisites if one wants to employ Monte Carlo simulations of parton energy loss for separating between different pictures of jet-medium interactions, such as the two different pictures described in section 2. Our study also further supports the argument [18] that the BDMPS-Z formalism (that does not impose exact energy-momentum conservation and that is restricted to describing the medium-induced radiation of a nominally leading parton) cannot be regarded as a phenomenologically viable approximation for the full dynamics relevant for jet-medium interactions, although it is an interesting, analytically accessible limiting case of the full dynamics of parton energy loss.

The present study also explains how one can understand qualitatively (and to some extent even quantitatively) the deviations from the BDMPS-Z parton energy loss in the high-energy limit (1) induced by imposing the physical requirements listed in section 2. In particular, the role of medium-induced destructive interference depends sensitively on the nature of the jet-medium interaction. Harder momentum transfers from the medium can destroy coherence on shorter length-scales and result in quasi-incoherent inelastic interactions with enhanced energy loss and a linear dependence of the average parton energy loss on the medium length. This strong sensitivity is of significant interest as it may help to constrain the nature of the jet-medium interactions. Our discussion in sections 3-5 has also identified simple physical origins for the effects resulting from supplementing the results of the BDMPS-Z formalism with exact energy-momentum conservation, democratic treatment of all partons and no overlap between gluon formation times.

While our study illustrates the insufficiency of a parton energy loss calculation based on the eikonal limit (1) alone, we note that the extension of eikonal kinematics to the full kinematic range is not unique. Thus our study cannot provide a unique solution to the question of how the full kinematical range relevant for jet-medium interactions should be modeled. To achieve a formulation valid in the entire phase space, one may want to start therefore from a formulation that is consistent with the known results in the eikonal limit, but that does involve neither eikonal approximations nor concepts that are only meaningful in eikonal kinematics. Such an approach is taken for instance in a new version of the medium-modified final state parton shower JEWEL (for first results, see Ref. [29]) that aside of other phenomenologically wanted features also accounts for the physical requirements listed in section 2. More generally, we hope that the present study contributes to clarifying those physical features that a phenomenologically viable MC simulation of parton energy loss must satisfy.

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