Modelling Dark Energy with Quintessence and a Cosmological 
Constant

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(January 10, 2022)

Abstract

In this talk we present a model of the universe in which dark energy is modelled explicitly with both a dynamical quintessence field and a cosmological constant. Our results confirm the possibility of a collapsing universe (for a given region of the parameter space), which is advantageous for an adequate formulation of both perturbative quantum field and string theories. We have also reproduced the measurements of modulus distance from supernovae with good accuracy.

I. INTRODUCTION

From 1998 to date several important discoveries in the astrophysical sciences have being made, which have given rise to the so called New Cosmology [1,2]. Amongst its more important facts we may cite:

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- Flat, critical density accelerating universe
- Early period of rapid expansion (inflation)
- Density inhomogeneities produced from quantum fluctuations during inflation
- Composition: 2/3 dark energy; 1/3 dark matter; 1/200 bright stars
- Matter content: (29 ± 4)% nonbarionic dark matter; (4 ± 1)% baryons, (0.1 — 5)% neutrinos
- $T_0 = 2.275 ± 0.001 K$
- $t_0 = 14 ± 1 Gyr$
- $H_0 = 72 ± 7 km.s^{-1} Mpc^{-1}$

It is a fact that the standard LCDM model, though rather simple from the theoretical point of view, can accommodate most of today’s astrophysical data. However, it still has several open questions (see, for instance, [3,4]), one of them being: could other (yet unknown) models fit the data equally well?

Alternative models should obviously consider the main components of the universe. There is much concern about unveiling the dark part of our universe, which implies that we don’t lack candidates. So, for dark matter we have neutrinos, axions, neutralinos; for dark energy: the cosmological constant, scalar fields (for example, quintessence), cosmic field defects, etc. So far, most models of dark energy have a rather phenomenological character, though a few proposals concerning the possible role of the dark energy field in the context of fundamental physics have appeared [5,6,7,8,9,10].

The recent dark energy scalar field research has several interesting features (see [4] for an extensive review). Many models have attractor or tracker behaviour, allowing, for a wide range of initial conditions, a subdominant field energy density at high redshifts (radiation and matter dominated eras).

In the simplest versions, scalar fields models of dark energy have a scalar field kinetic term, and the scalar field is coupled only to itself and gravity. So, the scalar field part of the model is fully characterized by the scalar field potential, with some broad constraints on the initial conditions for the field, if the attractor behaviour is realized.
Many different potentials have been used (see reviews [4,11]):
| Quintessence Potential | Reference |
|------------------------|-----------|
| $V_0 \exp(-\lambda \phi)$ | Ratra & Peebles (1988), Wetterich (1988), Ferreira & Joyce (1998) |
| $m^2 \phi^2, \lambda \phi^4$ | Frieman et al (1995) |
| $V_0/\phi^\alpha, \alpha > 0$ | Ratra & Peebles (1988) |
| $V_0 \exp(\lambda \phi^2)/\phi^\alpha, \alpha > 0$ | Brax & Martin (1999, 2000) |
| $V_0(\cosh \lambda \phi - 1)^p$, | Sahni & Wang (2000) |
| $V_0 \sinh^{-\alpha}(\lambda \phi)$, | Sahni & Starobinsky (2000), Ureña-López & Matos (2000) |
| $V_0(e^{\alpha \kappa \phi} + e^{\beta \kappa \phi})$ | Barreiro, Copeland & Nunes (2000) |
| $V_0(\exp M_p/\phi - 1)$, | Zlatev, Wang & Steinhardt (1999) |
| $V_0[(\phi - B)^\alpha + A]e^{-\lambda \phi}$, | Albrecht & Skordis (2000) |
In this talk I want to call the attention to exponential potentials, which have being often discarded on fine tuning arguments or (the simplest exponential) because they cannot produce the wanted transition from subdominant to dominant energy density (\cite{4}). However, as shown in \cite{12,13}, they have proved useful in describing several features in the history of the universe, from radiation decoupling to nowadays. Also, several authors have recently pointed out that the degree of fine tuning needed in these scenarios is no more than in others usually accepted \cite{14,15,12}.

Especially interesting results are obtained if we model dark energy using both a scalar field and a cosmological constant.

The cosmological constant can be incorporated into the quintessence potential as a constant which shifts the potential value, especially, the value of the minimum of the potential, where the quintessence field rolls towards. Conversely, the height of the minimum of the potential can also be regarded as a part of the cosmological constant. Usually, for separating them, the possible nonzero height of the minimum of the potential is incorporated into the cosmological constant and then set to be zero. The cosmological constant can be provided by various kinds of matter, such as the vacuum energy of quantum fields and the potential energy of classical fields and may also be originated in the intrinsic geometry. So far there is no sufficient reason to set the cosmological constant (or the height of the minimum of the quintessence potential) to be zero, especially when the ultimate fate of our universe is more sensitive to the presence of the cosmological constant (or the nonzero height of the minimum of the quintessence potential) than any other matter content, even though the cosmological constant may be extremely tiny and undetectable at all in present time (\cite{16}. In particular, some mechanisms to generate a negative cosmological constant have been pointed out, in the context of spontaneous symmetry breaking \cite{17,18}.
II. THE MODEL

We consider a model consisting of a three-component cosmological fluid: matter, scalar field (quintessence with an exponential potential) and a negative cosmological constant. We point out that we model dark energy with both the quintessence field and the negative cosmological constant, resulting positive our effective cosmological constant, in agreement with experimental data [19]. ”Matter” means barionic + cold dark matter, with no pressure, and the scalar field is minimally coupled and noninteracting with matter, so the action is:

\[
S = \int d^4x \sqrt{-g} \left\{ \frac{c^2}{16\pi G} (R - 2\Lambda) + L_\phi + L_m \right\},
\]

(2.1)

where \(\Lambda\) is the cosmological constant, \(L_m\) is the Lagrangian for the matter degrees of freedom and the Lagrangian for the quintessence field is given by

\[
L_\phi = -\frac{1}{2}\phi,\phi - V(\phi).
\]

(2.2)

This model cannot be used from the very beginning of the universe, but only since decoupling of radiation and dust. Thus, we don’t take into account inflation, creation of matter, nucleosynthesis, etc. We apply the same technique of adimensional variables we used in [20] (this allows to determine the integration constants without additional assumptions). We use the dimensionless time variable \(\tau = H_0 t\), where \(t\) is the cosmological time and \(H_0\) is the present value of the Hubble parameter. In this case \(a(\tau) = \frac{a(t)}{a(0)}\) is the scale factor. Then we have that, at present \((\tau = 0)\)

\[
a(0) = 1,
\]

\[
\dot{a}(0) = 1,
\]

\[
H(0) = 1,
\]

(2.3)

Considering a homogeneous and isotropic universe, and using the experimental fact of a spatially flat universe [21], the field equations derivable from (2.1) are

\[
\left(\frac{\ddot{a}}{a}\right)^2 = \frac{2}{9}\sigma^2 \left\{ \frac{\ddot{D}}{a^3} + \frac{1}{2}\dot{\phi}^2 + \tilde{V}(\phi) + \frac{3}{2} \frac{\dot{\Lambda}}{\sigma^2} \right\},
\]

(2.4)
\[
2 \ddot{a} + \left(\frac{\dot{a}}{a}\right)^2 = -\frac{2}{3} \sigma^2 \left\{ \frac{1}{2} \dot{\phi}^2 - \bar{V}(\phi) - \frac{3}{2} \frac{\dot{\Lambda}}{\sigma^2} \right\},
\]

(2.5)

and

\[
\ddot{\phi} + 3 \frac{\dot{a}}{a} \dot{\phi} + \bar{V}'(\phi) = 0,
\]

(2.6)

where the dot means derivative in respect to \( \tau \) and,

\[
\bar{V}(\phi) = \bar{B}^2 e^{-\sigma \phi},
\]

(2.7)

\[
\bar{X} = \frac{X}{H_0^2},
\]

(2.8)

\[
(\text{except for } D = \frac{D}{a_0^2 H_0^2} = \frac{\rho_{ma}}{H_0^2}),
\]

with \( \rho_{ma} \) - the present density of matter, \( \sigma^2 = \frac{12\pi G}{c^2} \) and \( B^2 \) - a generic constant.

Applying the Noether Symmetry Approach [22,23,24,25], it can be shown that the new variables we should introduce to simplify the field equations are the same used in [12]:

\[
a^3 = uv,
\]

(2.9)

and

\[
\phi = -\frac{1}{\sigma} \ln \left( \frac{u}{v} \right),
\]

(2.10)

In these variables the field Eqs. (2.4-2.6) may be written as the following pair of equations

\[
\frac{\ddot{u}}{u} + \frac{\ddot{v}}{v} = \bar{B}^2 \sigma^2 \frac{u}{v} - \sigma^2 \bar{V}_0,
\]

(2.11)

and

\[
\frac{\ddot{u}}{u} - \frac{\ddot{v}}{v} = -\sigma^2 \bar{B}^2 \frac{u}{v},
\]

(2.12)

where:

\[
\bar{V}_0 = -\frac{3}{2} \frac{\dot{\Lambda}}{\sigma^2}.
\]

(2.13)
Combining of Eqs. (2.11) and (2.12) yields

\[ \ddot{u} = -\frac{\sigma^2 \bar{V}_0}{2} u, \]  

(2.14)

and

\[ \ddot{v} = -\frac{\sigma^2 \bar{V}_0}{2} v + \sigma^2 \bar{B}^2 u. \]  

(2.15)

The solutions of the equations (2.14) and (2.15) are found to be

\[ u(\tau) = u_1 \sin(\sigma \sqrt{\frac{\bar{V}_0}{2}} \tau) + u_2 \cos(\sigma \sqrt{\frac{V_0}{2}} \tau), \]  

(2.16)

and

\[ v(\tau) = \left\{ v_2 + \frac{\bar{B}^2}{2 \bar{V}_0} u_2 \right\} \cos(\sigma \sqrt{\frac{V_0}{2}} \tau) + \left\{ v_1 + \frac{\bar{B}^2}{2 \bar{V}_0} u_1 \right\} \sin(\sigma \sqrt{\frac{V_0}{2}} \tau), \]  

(2.17)

where \( u_1, u_2, v_1 \) and \( v_2 \) are the integration constants.

In finding the integration constants we use the equations (2.3) and field equations evaluated at \( \tau = 0 \). Finally, using \( \Omega_{m0} + \Omega_{Q0} + \Omega_\Lambda = 1 \) and the ansatz

\[ \bar{B}^2 = n \bar{V}_0, \]  

(2.18)

where \( n \) is a positive real number, then the above integration constants can be written in the following way:

\[ u_2^{(\pm)} = \pm \sqrt{\frac{2 - q_0 - 1.5 \Omega_{m0} - 3 \Omega_\Lambda}{-3n \Omega_\Lambda}}, \]  

(2.19)

\[ v_2^{(\pm)} = \frac{1 - \frac{n}{2} u_2^{(\pm)}}{u_2^{(\pm)}}, \]  

(2.20)

\[ u_1^{(\pm)} = \frac{\sqrt{3} - [\pm] \sqrt{1 + q_0 - 1.5 \Omega_{m0}}}{\sqrt{-3 \Omega_\Lambda}} u_2^{(\pm)}, \]  

(2.21)

and
respectively. Effective quintessence potential $\bar{W}(\phi)$ from field equations (2.4) or (2.5) and equations (2.7) and (2.13) can be written

$$\bar{W}(\phi) = \bar{B}^2 e^{-\sigma \phi} - \bar{V}_0,$$  \hspace{1cm} (2.23)

so the ansatz (2.18) establishes an interesting relationship between the value $\bar{V}(0) = \bar{B}^2$ of the exponential potential

$$\bar{V}(\phi) = \bar{B}^2 e^{-\sigma \phi},$$  \hspace{1cm} (2.24)

and the value $\bar{W}(\infty) = -\bar{V}_0$ towards which $\bar{W}(\phi)$ asymptotes. Other ansatze could have been taken, however, this one notably simplifies equations (for instance, eq.(2.17)). Also, as we will see later, many of the cosmological parameters result independent of $n$, avoiding fine tuning respect to this parameter.

III. ANALYSIS OF RESULTS

Since $\sqrt{1 + q_0 - 1.5\Omega_{m_0}}$ should be real (see equation (2.21)) then, the following constrain on the present value of the deceleration parameter follows

$$q_0 \geq -1 + 1.5\Omega_{m_0}.$$

(3.1)

It can be noticed that the constants (and, consequently, the solutions) depend on 4 physical parameters: $\Omega_{m_0}$, $\Omega_{\Lambda}$, $q_0$ and on the positive real number $n$.

After making a detailed study, it was determined that the only relevant cosmological magnitude that has a sensible dependence on parameter $n$ is the state parameter $\omega$. We used $\Omega_{m_0} = 0.3$ and $q_0 = -0.44$.

Figure 1 shows the evolution of the scale factor for $\Omega_{\Lambda} = -0.15$. It can be shown both algebraically and graphically that the evolution of the universe is independent of $n$, but not
written for the sake of simplicity. This results favour the formulation of both quantum field and string theories. A breakdown of perturbative quantum field theory in spacetimes with accelerated expansion is known to occur [24]. On the other hand, an eternally accelerating universe seems to be at odds with string theory, because of the impossibility of formulating the S-matrix. In a deSitter space the presence of an event horizon, signifying causally disconnected regions of space, implies the absence of asymptotic particle states which are needed to define transition amplitudes. It is also interesting that Sen and Sethi, using an ansatz for the scale factor that produces future deceleration, obtain from field equations that the quintessence potential should be a double exponential plus a constant [27].

Figure 2 shows the behaviour of the deceleration parameter as function of the redshift \( z \) for the same values of the parameters. This figure shows an early stage of deceleration and a current epoch of acceleration. A transition from an accelerated phase to a decelerated one is seen approximately for \( z = 0.5 \). We appreciate an increase of the deceleration parameter upon increasing the value of \( z \). This points at a past epoch in the evolution when gravity of the dark energy was attractive. As follows from figure 1, acceleration is not eternal: in the future \( q > 0 \) again, which gives rise to the collapse.

Figure 3 shows the evolution of the state parameter of the effective quintessence field \( \omega_\phi \). It’s noticeable that the effective quintessence field has state parameter \( \omega_\phi \) near \(-1\) today, which means that its behaviour is similar to the “pure” cosmological constant, as a vacuum fluid. If we are to explain the very desirable for today’s cosmology recent and future deceleration obtained in our model, it’s important to look at the dynamical quintessence field. We see that in the recent past \( \omega_\phi > 0 \), which implies that quintessence field behaved (or simply was) like ordinary attractive matter, giving rise to the logical deceleration. In the future this will happen again \( (\omega_\phi > 0) \), with the consequent deceleration.

Now we proceed to analyze how our solution reproduces some experimental results. With this purpose, in Fig. 4 we plot the distance modulus \( \delta(z) \) vs redshift \( z \), calculated by us and the one obtained with the usual model with a constant \( \Lambda \) term. The relative deviations are of about 0.5%.
So far, we have investigated one of the several possible branches of the solution, leaving for the future the investigation of the others.
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