Time Dependence of Coherent $P^0\bar{P}^0$ Decays and $CP$ Violation at Asymmetric $B$ Factories

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Abstract

A generic formalism is presented for the time-dependent or time-integrated decays of any coherent $P^0\bar{P}^0$ system ($P^0 = K^0, D^0, B^0_d,$ or $B^0_s$). To meet various possible measurements at asymmetric $B$ factories, we reanalyze some typical signals of $CP$ violation in the coherent $B^0_d\bar{B}^0_d$ transitions. The advantage of proper time cuts is illustrated for measuring mixing parameters and $CP$ violation. We show that the direct and indirect $CP$ asymmetries are distinguishable in neutral $B$ decays to $CP$ eigenstates. The possibility to detect the $CP$-forbidden processes at the $\Upsilon(4S)$ resonance is explored in some detail.

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1. Introduction

It is known in particle physics that mixing between a neutral meson $P^0$ and its $CP$-conjugate state $\bar{P}^0$ provides a mechanism whereby interference in the decay amplitudes can occur, leading to the possibility of $CP$ violation [1]. To date, $K^0 - \bar{K}^0$ mixing and $B^0_d - \bar{B}^0_d$ mixing have been measured, and the $CP$ violating signal induced by $K^0 - \bar{K}^0$ mixing has been unambiguously established [2]. Compared with $B^0_d - \bar{B}^0_d$ mixing, $B^0_s - \bar{B}^0_s$ ($D^0 - \bar{D}^0$) mixing is expected to be quite large (very small) in the context of the standard electroweak model [3]. Today the $B^0_d - \bar{B}^0_d$ system is playing an important role in studying flavour mixing and $CP$ violation beyond the neutral kaon system. The $B^0_s - \bar{B}^0_s$ and $D^0 - \bar{D}^0$ systems are more interesting in practice for probing new physics that is out of reach of the standard model predictions.

A feasible way to study $CP$ violation arising from $P^0 - \bar{P}^0$ mixing is to measure the coherent decays of $P^0\bar{P}^0$ pairs produced at appropriate resonances. For instance,

$$\phi \to K^0\bar{K}^0, \quad \psi'' \to D^0\bar{D}^0, \quad \Upsilon(4S) \to B^0_d\bar{B}^0_d, \quad \Upsilon(5S) \to B^0_s\bar{B}^0_s.$$  

At present, efforts are underway to develop asymmetric $B$ factories at KEK and SLAC laboratories [4], while asymmetric $\phi$ factory options are also under consideration [5]. The main purpose of these machines is to probe $CP$ violation (and to test other discrete symmetries or conservation laws) by measuring the time-dependent transitions. By now some phenomenological analyses of coherent $K^0\bar{K}^0$ and $B^0_d\bar{B}^0_d$ decays have been made in the literature [6-8]. These works have outlined the main features of $CP$ violation in the $K^0 - \bar{K}^0$ or $B^0_d - \bar{B}^0_d$ systems, although many of their formulae and results rely on the characteristics of the system itself or some model-dependent approximations. A generic formalism, which can describe the common properties of coherent $P^0\bar{P}^0$ decays, is still lacking. In addition, little attention has been paid to the advantage of proper time cuts for measuring the mixing parameters and $CP$ asymmetries in coherent weak decays.

In this paper we shall present a generic and concise formalism for the time-dependent or time-integrated decays of any coherent $P^0 - \bar{P}^0$ system. This formalism should be very useful for phenomenological applications, because it is independent of the contexts of the standard model or its various non-standard extensions. To meet various possible measurements at the forthcoming $B$ factories, we shall carry out a reanalysis of the typical signals of $CP$ violation manifesting in $B^0_d - \bar{B}^0_d$ mixing, in $B^0_d$ vs $\bar{B}^0_d$ decays to $CP$ eigenstates, and in $CP$-forbidden transitions at the $\Upsilon(4S)$ resonance. Our analysis differs from the previous ones in the following three aspects: (1) we illustrate the advantage of proper time cuts for measurements of the mixing parameters and $CP$ violating asymmetries; (2) we highlight the distinguishable effect of direct $CP$ violation on $CP$ asymmetries in neutral $B$ decays; and (3) we explore in some detail the possibility to detect the $CP$-forbidden decays at $B$ factories. This work concentrates
on analytical studies. A more comprehensive discussion about the present topic, together with numerical (model-dependent) predictions, will be given elsewhere.

2. Formalism of coherent $P^0\bar{P}^0$ decays

The time-dependent wave function for a $P^0_{\text{phys}}\bar{P}^0_{\text{phys}}$ pair at rest can be written as

$$\frac{1}{\sqrt{2}} \left[ |P^0_{\text{phys}}(K, t)\rangle \otimes |\bar{P}^0_{\text{phys}}(-K, t)\rangle + C|P^0_{\text{phys}}(-K, t)\rangle \otimes |\bar{P}^0_{\text{phys}}(K, t)\rangle \right] ,$$

where $K$ is the three-momentum vector of the $P$ meson, and $C = -$ or $+$ is the charge-conjugation parity of the $P^0_{\text{phys}}\bar{P}^0_{\text{phys}}$ pair. The proper time evolution of an initially ($t = 0$) pure $P^0$ or $\bar{P}^0$ is given by

$$|P^0_{\text{phys}}(t)\rangle = g_+(t)|P^0\rangle + (q/p)g_-(t)|\bar{P}^0\rangle ,$$
$$|\bar{P}^0_{\text{phys}}(t)\rangle = (p/q)g_-(t)|P^0\rangle + g_+(t)|\bar{P}^0\rangle ,$$

where the mixing parameters $p$ and $q$ connect the flavour eigenstates $|^{(-)}P^0\rangle$ to the mass eigenstates $|P_{1,2}\rangle$ through $|P_1\rangle = p|P^0\rangle + q|\bar{P}^0\rangle$ and $|P_2\rangle = p|P^0\rangle - q|\bar{P}^0\rangle$; and

$$g_\pm(t) = \frac{1}{2} e^{-i(m \pm \frac{1}{2})t} \left[ e^{+i(\Delta m - \Delta \Gamma)t} \pm e^{-i(\Delta m - \Delta \Gamma)t} \right] .$$

Here we have defined $m = (m_1 + m_2)/2$, $\Delta m = (m_2 - m_1)$, $\Gamma = (\Gamma_1 + \Gamma_2)/2$, and $\Delta \Gamma = (\Gamma_1 - \Gamma_2)$, where $\Gamma_{1,2}$ and $m_{1,2}$ are the widths and masses of $P_{1,2}$.

Now we consider the case that one of the two $P$ mesons (with the momentum $K$) decays to a final state $f_1$ at proper time $t_1$ and the other (with $-K$) to $f_2$ at $t_2$. $f_1$ and $f_2$ may be either hadronic or semileptonic states. After a lengthy calculation, the joint decay rate for having such an event is given as

$$R(f_1, t_1; f_2, t_2)_C \propto |A_{f_1}|^2 |A_{f_2}|^2 e^{-\Gamma t_1} \left[ \frac{1}{2} |\xi_C + \zeta_C| e^{-\Delta \Gamma t_C} + \frac{1}{2} |\xi_C - \zeta_C| e^{\Delta \Gamma t_C} - \left( |\xi_C|^2 - |\zeta_C|^2 \right) \cos(\Delta m t_C) + 2\text{Im}(\xi^*_C \zeta_C) \sin(\Delta m t_C) \right] ,$$

where

$$A_{f_i} = \langle f_i | H | P^0 \rangle , \quad \bar{A}_{f_i} = \langle f_i | H | \bar{P}^0 \rangle , \quad \rho_{f_i} = \frac{\bar{A}_{f_i}}{A_{f_i}} , \quad (i = 1, 2) ;$$

and

$$t_C = t_2 + C t_1 , \quad \xi_C = (p/q) + C(q/p) \rho_{f_1} \rho_{f_2} \quad \zeta_C = \rho_{f_2} + C \rho_{f_1} .$$

The time-independent decay rate is obtainable from Eq. (4) by integrating $R(f_1, t_1; f_2, t_2)_C$ over $t_1$ and $t_2$:

$$R(f_1, f_2)_C \propto |A_{f_1}|^2 |A_{f_2}|^2 \left[ \frac{|\xi_C + \zeta_C|^2}{2(1 + y)(1 + Cy)} + \frac{|\xi_C - \zeta_C|^2}{2(1 - y)(1 - Cy)} - \frac{1 - Cx^2}{(1 + x^2)^2} \left( |\xi_C|^2 - |\zeta_C|^2 \right) + \frac{2(1 + C)x}{(1 + x^2)^2} \text{Im}(\xi^*_C \zeta_C) \right] ,$$

$$C = \frac{m_1 - m_2}{m_1 + m_2} , \quad y = \frac{\Gamma_1}{\Gamma} , \quad x = \frac{1}{1 + Cy} , \quad C_x = \frac{1}{1 + Cx} .$$
where \( x = \Delta m / \Gamma \) and \( y = \Delta \Gamma / (2 \Gamma) \) are two measurables of the \( P^0 - \bar{P}^0 \) system. Note that Eqs. (4) and (7) are useful at both symmetric and asymmetric flavour factories.

An asymmetric \( e^+e^- \) collider running at the threshold of production of \((P^0_{phys}\bar{P}^0_{phys})_C\) pairs will offer the possibility to measure the decay-time difference \( t_- = (t_2 - t_1) \) between \( P^0_{phys} \rightarrow f_1 \) and \( \bar{P}^0_{phys} \rightarrow f_2 \). It is usually difficult to measure the \( t_+ = (t_2 + t_1) \) distribution in either linacs or storage rings, unless the bunch lengths are much shorter than the decay lengths [4,5,9]. Hence it is more practical to study the \( t_- \) distribution of the joint decay rates. Here and hereafter we use \( t \) to denote \( t_- \) for simplicity. Integrating \( R(f_1, t_1; f_2, t_2)_C \) over \( t_+ \), we obtain the decay rates (for \( C = \pm \)) as follows:

\[
R(f_1, f_2; t)_- \propto |A_{f_1}|^2 |A_{f_2}|^2 e^{-\Gamma t} \left[ \frac{1}{2} |\xi_+ + \zeta_-|^2 e^{-y \Gamma t} + \frac{1}{2} |\xi_- - \zeta_+|^2 e^{y \Gamma t} 
- \left( |\xi_-|^2 - |\zeta_+|^2 \right) \cos(x \Gamma t) + 2 \text{Im} \left( \xi_+ \zeta_- \right) \sin(x \Gamma t) \right] \tag{8}
\]

and

\[
R(f_1, f_2; t)_+ \propto |A_{f_1}|^2 |A_{f_2}|^2 e^{-\Gamma t} \left[ \frac{|\xi_+ + \zeta_-|^2}{2(1 + y)} e^{-y \Gamma t} + \frac{|\xi_- - \zeta_+|^2}{2(1 - y)} e^{y \Gamma t} 
- \frac{|\xi_+|^2 - |\zeta_+|^2}{\sqrt{1 + x^2}} \cos(x \Gamma t + \phi_x) + \frac{2 \text{Im} \left( \xi_+ \zeta_- \right)}{\sqrt{1 + x^2}} \sin(x \Gamma t + \phi_x) \right] \tag{9}
\]

where \( \phi_x = \arctan x \) signifies a phase shift. One can check that integrating \( R(f_1, f_2; t)_C \) over \( t \), where \( t \in (-\infty, +\infty) \), will lead to the time-independent decay rates \( R(f_1, f_2)_C \) in Eq. (7). Eqs. (8) and (9) are two basic formulae for investigating coherent \( B^0 \bar{B}^0 \) (or \( K^0 \bar{K}^0 \)) decays at asymmetric \( B \) (or \( \phi \)) factories.

Another possibility is to measure the time-integrated decay rates of \((P^0_{phys}\bar{P}^0_{phys})_C\) with a proper time cut, which can sometimes increase the sizes of \( CP \) asymmetries [10]. In practice, appropriate time cuts can also suppress background and improve statistic accuracy of signals. If the decay events in the time region \( t \in [+t_0, +\infty) \) or \( t \in (-\infty, -t_0] \) are used, where \( t_0 \geq 0 \), the respective decay rates can be defined by

\[
\hat{R}(f_1, f_2; +t_0)_C \equiv \int_{+t_0}^{+\infty} R(f_1, f_2; t)_C dt , \\
\hat{R}(f_1, f_2; -t_0)_C \equiv \int_{-\infty}^{-t_0} R(f_1, f_2; t)_C dt . \tag{10}
\]

From Eqs. (8) and (9) we obtain

\[
\hat{R}(f_1, f_2; \pm t_0)_- \propto |A_{f_1}|^2 |A_{f_2}|^2 e^{-\Gamma t_0} \left[ \frac{|\xi_- \pm \zeta_-|^2}{4(1 + y)} e^{-y \Gamma t_0} + \frac{|\xi_- \mp \zeta_-|^2}{4(1 - y)} e^{y \Gamma t_0} 
- \frac{|\xi_-|^2 - |\zeta_-|^2}{2\sqrt{1 + x^2}} \cos(x \Gamma t_0 + \phi_x) \pm \frac{2 \text{Im} \left( \xi_- \zeta_- \right)}{2\sqrt{1 + x^2}} \sin(x \Gamma t_0 + \phi_x) \right] \tag{11}
\]
and
\[
\hat{R}(f_1, f_2; \pm t_0) \propto |A_{f_1}|^2 |A_{f_2}|^2 e^{-\Gamma t_0} \left[ \frac{\xi_+ + \zeta_+}{4(1 + y^2)} e^{-y\Gamma t_0} + \frac{\xi_+ - \zeta_+}{4(1 - y^2)} e^{y\Gamma t_0} - \frac{|\xi_+|^2 - |\zeta_+|^2}{2(1 + x^2)} \cos (x\Gamma t_0 + 2\phi_x) + \frac{\text{Im}(\xi_+ \zeta_+)}{1 + x^2} \sin (x\Gamma t_0 + 2\phi_x) \right] .
\]

(12)

It is easy to check that
\[
\hat{R}(f_1, f_2; +0) + \hat{R}(f_1, f_2; -0) = R(f_1, f_2)_C .
\]

(13)

One can observe that in \( \hat{R}(f_1, f_2; \pm t_0)_C \) different terms are sensitive to the time cut \( t_0 \) in different ways. Thus it is possible to enhance a \( CP \) violating term (and suppress the others) via a suitable cut \( t_0 \).

The formulae given above are applicable to all coherent decays of the \( K^0 - \bar{K}^0 \), \( D^0 - \bar{D}^0 \), \( B_d^0 - \bar{B}_d^0 \), and \( B_s^0 - \bar{B}_s^0 \) systems. Denoting the decay amplitudes of \( P_n \to f_i \) by \( A_{f_i}^{(n)} \) \( (n, i = 1, 2) \) and the ratio of \( A_{f_i}^{(2)} \) to \( A_{f_i}^{(1)} \) by \( \eta_{f_i} \), one can also express the joint decay rates in terms of \( A_{f_i}^{(n)} \) and \( \eta_{f_i} \) through the following transformations:
\[
A_{f_i} = \frac{1}{2p} \left[ A_{f_i}^{(1)} + A_{f_i}^{(2)} \right] , \quad \bar{A}_{f_i} = \frac{1}{2q} \left[ A_{f_i}^{(1)} - A_{f_i}^{(2)} \right] , \quad \rho_{f_i} = \frac{p}{q} \frac{1 - \eta_{f_i}}{1 + \eta_{f_i}} .
\]

(14)

Such notations are usually favoured in the \( K^0 - \bar{K}^0 \) system [6]. In the following we shall apply the above formalism to the coherent \( (B_d^0_{\text{phys}} \bar{B}_d^0_{\text{phys}})_C \) decays and \( CP \) violation at the \( \Upsilon(4S) \) resonance, a basis of the forthcoming \( B \) factories [4,9].

### 3. Signals of \( CP \) violation at \( B \) factories

The unique experimental advantages of studying \( b \)-quark physics at the \( \Upsilon(4S) \) resonance are well known. For symmetric \( e^+e^- \) collisions the produced \( B \) mesons are almost at rest and their mean decay length is only about \( 20 \mu \)m, a distance which is insufficient for identifying the decay vertices or measuring the decay time difference [9,10]. If the colliding \( e^+ \) and \( e^- \) beams have different energies, the product of collisions will move with a significant relativistic boost factor in the laboratory \( \Upsilon(4S) \) (along the direction of the more energetic beam). This can cause the two \( B \) mesons far apart in space, such that the distance between their decay vertices becomes measurable. It is then possible to study the time distribution of the joint decay rates and \( CP \) asymmetries.

#### A. \( CP \) violation in \( B_d^0 - \bar{B}_d^0 \) mixing

\(^1\text{For a moving \( \Upsilon(4S) \) system, the momentum of the \( B \) mesons in the \( \Upsilon(4S) \) rest frame can be ignored. This safe approximation has been discussed in Refs. [9,10].}\)
We first consider the joint decays \((B_{d,\text{phys}}^0 \bar{B}_{d,\text{phys}}^0)_C \rightarrow (l^+ X^0_{d})(l^+ X^0_{d})\), which lead to dilepton events in the final states. Keeping the \(\Delta Q = \Delta B\) rule and \(CPT\) symmetry, we have

\[
\langle l^+ X_i^- | H | B_d^0 \rangle = \langle l^- X_i^+ | H | B_d^0 \rangle \equiv A_{ti}^i; \\
\langle l^- X_i^+ | H | B_d^0 \rangle = \langle l^+ X_i^- | H | B_d^0 \rangle = 0 ,
\]

where \(i = a\) or \(b\). Subsequently we use \(N_C^{\pm \pm}(t)\) and \(N_C^{\mp -}(t)\) to denote the time-dependent like-sign and opposite-sign dilepton numbers, respectively. Similarly, let \(\hat{N}_C^{\pm \pm}(\pm t_0)\) and \(\hat{N}_C^{\mp -}(\pm t_0)\) denote the time-integrated dilepton events with the time cut \(t_0\). With the help of Eqs. (8) and (9), we obtain

\[
N_{C}^{++}(t) \propto |p/q|^2 |A_{ta}|^2 |A_{tb}|^2 e^{-\Gamma t} \left[ \cos(y \Gamma t) - \cos(x \Gamma t) \right], \\
N_{C}^{--}(t) \propto |q/p|^2 |A_{ta}|^2 |A_{tb}|^2 e^{-\Gamma t} \left[ \cos(y \Gamma t) - \cos(x \Gamma t) \right], \\
N_{C}^{+-}(t) \propto 2 |A_{ta}|^2 |A_{tb}|^2 e^{-\Gamma t} \left[ \cos(y \Gamma t) + \cos(x \Gamma t) \right],
\]

and

\[
N_{C}^{++}(t) \propto |p/q|^2 |A_{ta}|^2 |A_{tb}|^2 e^{-\Gamma t} \left[ \frac{\cos(y \Gamma t) + y \sinh(y \Gamma t)}{1 - y^2} \right] \\
N_{C}^{--}(t) \propto |q/p|^2 |A_{ta}|^2 |A_{tb}|^2 e^{-\Gamma t} \left[ \frac{\cos(y \Gamma t) + y \sinh(y \Gamma t)}{1 - y^2} \right] \\
N_{C}^{+-}(t) \propto 2 |A_{ta}|^2 |A_{tb}|^2 e^{-\Gamma t} \left[ \frac{\cos(y \Gamma t) + y \sinh(y \Gamma t)}{1 - y^2} \right].
\]

If \(y\) is not very small in comparison with \(x\), its size and sign should be (in principle) determinable from the above two equations.\(^2\)

Since both \(N_{C}^{\pm \pm}(t)\) and \(N_{C}^{\mp -}(t)\) are even functions of proper time \(t\), one finds

\[
\hat{N}_{C}^{++}(+t_0) = \hat{N}_{C}^{++}(-t_0) \equiv \frac{1}{2} \hat{N}_{C}^{++}(t_0), \quad \hat{N}_{C}^{--}(+t_0) = \hat{N}_{C}^{--}(-t_0) \equiv \frac{1}{2} \hat{N}_{C}^{--}(t_0).
\]

The time-integrated observables of \(CP\) violation and \(B_d^0 - \bar{B}_d^0\) mixing can be defined as:

\[
A_{C}^{+-}(t_0) \equiv \frac{\hat{N}_{C}^{++}(t_0) - \hat{N}_{C}^{--}(t_0)}{\hat{N}_{C}^{++}(t_0) + \hat{N}_{C}^{--}(t_0)}, \quad S_{C}^{+-}(t_0) \equiv \frac{\hat{N}_{C}^{++}(t_0) + \hat{N}_{C}^{--}(t_0)}{\hat{N}_{C}^{++}(t_0) - \hat{N}_{C}^{--}(t_0)}.
\]

Using Eqs. (11) and (12) we obtain

\[
A_{C}^{+-}(t_0) = A_{C}^{+-}(t_0) = \frac{|p/q|^2 - |q/p|^2}{|p/q|^2 + |q/p|^2},
\]

\(^2\)A more detailed discussion has been given by Dass and Sarma in Ref. [7], where only the case of \(C = -\) was taken into account.
which is independent of the time cut $t_0$. The nonvanishing $A^C_\pm(t_0)$ implies CP violation in $B_d^0 - B_d^0$ mixing. In addition, we find

$$
S^+_\pm(t_0) = \frac{|p/q|^2 + |q/p|^2}{2} \frac{\cosh(y \Gamma t_0) + y \sinh(y \Gamma t_0) - z \cos(x \Gamma t_0 + \phi_x)}{\cosh(y \Gamma t_0) + y \sinh(y \Gamma t_0) + z \cos(x \Gamma t_0 + \phi_x)} ,
$$

$$
S^-_\pm(t_0) = \frac{|p/q|^2 + |q/p|^2}{2} \frac{(1 + y^2) \cosh(y \Gamma t_0) + 2 y \sinh(y \Gamma t_0) - z^2 \cos(x \Gamma t_0 + 2 \phi_x)}{(1 + y^2) \cosh(y \Gamma t_0) + 2 y \sinh(y \Gamma t_0) + z^2 \cos(x \Gamma t_0 + 2 \phi_x)} ,
$$

(21)

where $z = (1 - y^2)/\sqrt{1 + x^2}$.

In the context of the standard model, $|q/p| \approx 1$ and $y \approx 0$ are two good approximations [3]. Thus Eq. (21) is simplified as

$$
S^+_\pm(t_0) \approx \frac{\sqrt{1 + x^2} - \cos(x \Gamma t_0 + \phi_x)}{\sqrt{1 + x^2} + \cos(x \Gamma t_0 + \phi_x)} \quad t_0 = 0 \quad \frac{x^2}{2 + x^2} ,
$$

$$
S^-_\pm(t_0) \approx \frac{1 + x^2 - \cos(x \Gamma t_0 + 2 \phi_x)}{1 + x^2 + \cos(x \Gamma t_0 + 2 \phi_x)} \quad t_0 = 0 \quad \frac{3 x^2 + x^4}{2 + x^2 + x^4} .
$$

(22)

We show the evolution of $S^C_\pm(t_0)$ with $t_0$ in Fig. 1, where the experimental input is $x \approx 0.7$ [2]. One observes that an appropriate time cut can significantly increase the ratio of the same-sign dilepton events to the opposite-sign ones. Practically time cuts should be a useful way to enhance the signals of $B_d^0 - B_d^0$ mixing, only if the cost of the total number of events is not too large.

**B. CP asymmetries in $B_d^0$ vs $\bar{B}_d^0$ decays to CP eigenstates**

Neutral $B$ decays to CP eigenstates are favoured in both theory and experiments to study quark mixing and CP violation. At the $\Upsilon(4S)$ resonance, the produced $B_{d,\text{phys}}^0$ and $\bar{B}_{d,\text{phys}}^0$ mesons exist in a coherent state until one of them decays. Thus one can use the semileptonic decay of one $B_d$ meson to tag the flavour of the other meson decaying to a flavour-nonspecific hadron state. Let us consider the joint transitions $(B_{d,\text{phys}}^0 \bar{B}_{d,\text{phys}}^0)_C \to (l^\pm X^\pm)\bar{f}$, where $f$ denotes a hadronic CP eigenstate such as $J/\psi K_S, D^+ D^-$, or $\pi^+ \pi^-$. To a good degree of accuracy in the standard model, we have $|q/p| \approx 1$ and $y \approx 0$. With the help of Eqs. (8) and (9), the time-dependent decay rates are given as

$$
R(l^\mp, f; t)_- \propto |A_l|^2 |A_f|^2 e^{-\Gamma t} \left[ \frac{1 + |\lambda_f|^2}{2} \pm \frac{1 - |\lambda_f|^2}{2} \cos(x \Gamma t) \mp \text{Im} \lambda_f \sin(x \Gamma t) \right]
$$

(23)

and

$$
R(l^\mp, f; t)_+ \propto |A_l|^2 |A_f|^2 e^{-\Gamma t} \left[ \frac{1 + |\lambda_f|^2}{2} \pm \frac{1}{\sqrt{1 + x^2}} \frac{1 - |\lambda_f|^2}{2} \cos(x \Gamma |t| + \phi_x) \mp \frac{1}{\sqrt{1 + x^2}} \text{Im} \lambda_f \sin(x \Gamma |t| + \phi_x) \right],
$$

(24)
where $\lambda_f = (q/p)\rho_f$. The time-dependent $CP$ asymmetries, defined by

$$A_C(t) = \frac{R(l^-, f; t) - R(l^+, f; t)}{R(l^-, f; t) + R(l^+, f; t)} ,$$

(25)

can be explicitly expressed as

$$A_-(t) = U_f \cos(x \Gamma t) + V_f \sin(x \Gamma t) ,$$

$$A_+(t) = \frac{1}{\sqrt{1 + x^2}} [U_f \cos(x \Gamma t + \phi_x) + V_f \sin(x \Gamma t + \phi_x)] ,$$

(26)

where

$$U_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} , \quad V_f = \frac{-2 \text{Im}\lambda_f}{1 + |\lambda_f|^2} .$$

(27)

We find that $A_C(t)$ contains both the direct $CP$ asymmetry in the decay amplitude ($|\lambda_f| \neq 1$ or $U_f \neq 0$) and the indirect one from interference of mixing and decay ($\text{Im}\lambda_f \neq 0$ or $V_f \neq 0$). Measuring the time distribution of $A_\pm(t)$ can distinguish between these two sources of $CP$ violation [8].

There are two ways to combine the time-integrated decay events (with the time cuts $\pm t_0$), leading to two types of $CP$ asymmetries:

$$A^{(1)}_-(t_0) = \frac{[\hat{R}(l^-, f; +t_0) + \hat{R}(l^-, f; -t_0)] - [\hat{R}(l^+, f; +t_0) + \hat{R}(l^+, f; -t_0)]}{[\hat{R}(l^-, f; +t_0) + \hat{R}(l^-, f; -t_0)] + [\hat{R}(l^+, f; +t_0) + \hat{R}(l^+, f; -t_0)]} ,$$

(28)

$$A^{(2)}_-(t_0) = \frac{[\hat{R}(l^-, f; +t_0) + \hat{R}(l^+, f; -t_0)] - [\hat{R}(l^+, f; +t_0) + \hat{R}(l^-, f; -t_0)]}{[\hat{R}(l^-, f; +t_0) + \hat{R}(l^+, f; -t_0)] + [\hat{R}(l^+, f; +t_0) + \hat{R}(l^-, f; -t_0)]} .$$

With the help of Eqs. (11) and (12), we obtain

$$A^{(1)}_-(t_0) = \frac{\cos(x \Gamma t_0 + \phi_x)}{\sqrt{1 + x^2}} U_f , \quad A^{(2)}_-(t_0) = \frac{\sin(x \Gamma t_0 + \phi_x)}{\sqrt{1 + x^2}} V_f ;$$

(29)

and

$$A^{(1)}_+(t_0) = \frac{\cos(x \Gamma t_0 + 2\phi_x)}{1 + x^2} U_f + \frac{\sin(x \Gamma t_0 + 2\phi_x)}{1 + x^2} V_f , \quad A^{(2)}_+(t_0) = 0 .$$

(30)

Clearly the asymmetries $A^{(1)}_-(t_0)$ and $A^{(2)}_-(t_0)$ signify direct and indirect $CP$ violation, respectively. They can be separated from each other on the $\Upsilon(4S)$ resonance. In Fig. 2 we show the ratios $A^{(n)}_-(t_0)/A^{(n)}_-(0)$ ($n = 1$ and 2) as functions of $t_0$. A proper time cut can certainly increase the $CP$ asymmetries (at some cost of decay events). In practice, it can also suppress background and improve statistic accuracy of signals. Note that a suitable cut of decay time is (in principle) able to isolate the direct or indirect $CP$ asymmetry in $A^{(1)}_+(t_0)$. For example,

$$A^{(1)}_+ \left( \frac{\pi}{x \Gamma} - \frac{2\phi_x}{x \Gamma} \right) = -\frac{1}{1 + x^2} U_f , \quad A^{(1)}_+ \left( \frac{\pi}{2x \Gamma} - \frac{2\phi_x}{x \Gamma} \right) = \frac{1}{1 + x^2} V_f .$$

(31)

At symmetric $B$ factories, it is possible to measure a large $CP$ asymmetry $A^{(1)}_+(0)$. 8
C. CP-forbidden transitions

We finally consider the $CP$-forbidden decay modes

$$(B_{d,\text{phys}}^0 \bar{B}_{d,\text{phys}}^0)_{\pm} \rightarrow (f_a f_b)_{\pm},$$

where $f_{a,b}$ denote the $CP$ eigenstates with the same or opposite $CP$ parities. It should be emphasized that for such decays $CP$ violating signals can be established by measuring the joint decay rates other than the decay rate asymmetries [3]. In practice, this implies that neither flavour tagging nor time-dependent measurements are necessary.

On the $\Upsilon(4S)$ resonance, the typical $CP$-forbidden channels include $(B_{d,\text{phys}}^0 \bar{B}_{d,\text{phys}}^0)_{-} \rightarrow (f_f)_{+}$ with $f = J/\psi K_S, J/\psi K_L, D^{+}D^{-}$, and $\pi^{+}\pi^{-}$. Taking $f = X_{cc}K_S$ for example, where $X_{cc}$ denote all the possible charmonium states that can form the odd $CP$ eigenstates with $K_S$ (see Fig. 3), we have the safe approximation $\rho_{X_{cc}K_S} \approx -1$. In addition, we take $y \approx 0$ and $q/p \approx e^{-2i\beta}$, where $\beta$ corresponds to an inner angle of the Kobayashi-Maskawa unitarity triangle [2]. With the help of Eqs. (7) and (8), one obtains the branching fractions

$$B(X_{cc}K_S, X_{cc}K_S; t)_{-} \propto B_{X_{cc}K_S}^2 \sin^2(2\beta) e^{-\Gamma t} [1 - \cos(x t)] ,$$

$$B(X_{cc}K_S, X_{cc}K_S; t)_{+} \propto B_{X_{cc}K_S}^2 \sin^2(2\beta) \frac{x^2}{1 + x^2} ,$$

where $B_{X_{cc}K_S}$ denotes the branching ratio of $B_{d}^0 \rightarrow X_{cc}K_S$. Clearly the above joint decay rates are forbidden by $CP$ symmetry ($\beta = 0$ or $\pm \pi$). In practice, summing over the available final states $X_{cc}K_S$ can statistically increase the number of decay events:

$$B_{-} \equiv \sum_{X_{cc}} B(X_{cc}K_S, X_{cc}K_S; t)_{-} \propto \frac{x^2}{1 + x^2} \sin^2(2\beta) \sum_{X_{cc}} \left( B_{X_{cc}K_S}^2 \right) .$$

Just above the $\Upsilon(4S)$ resonance, an interesting type of $CP$-forbidden channels should be $(B_{d,\text{phys}}^0 \bar{B}_{d,\text{phys}}^0)_{+} \rightarrow [(X_{cc}K_S)(X_{cc}K_L)]_{-}$. Neglecting $CP$ violation in the kaon system, we have $\rho_{X_{cc}K_L} \approx -\rho_{X_{cc}K_S} \approx 1$ and $B_{X_{cc}K_S} \approx B_{X_{cc}K_L}$ to a good degree of accuracy. From Eqs. (7) and (9), it is straightforward to obtain

$$B(X_{cc}K_S, X_{cc}K_L; t)_{+} \propto B_{X_{cc}K_S}^2 \sin^2(2\beta) e^{-\Gamma t} \left[ 1 - \frac{\cos(x t) + \phi_x}{\sqrt{1 + x^2}} \right] ,$$

$$B(X_{cc}K_S, X_{cc}K_L; t)_{-} \propto B_{X_{cc}K_S}^2 \sin^2(2\beta) \frac{3x^2 + x^4}{1 + 2x^2 + x^4} .$$

Summing over the possible states $(X_{cc}K_S)(X_{cc}K_L)$, we find

$$B_{+} \equiv \sum_{X_{cc}} B(X_{cc}K_S, X_{cc}K_L; t)_{+} \propto \frac{3x^2 + x^4}{1 + 2x^2 + x^4} \sin^2(2\beta) \sum_{X_{cc}} \left( B_{X_{cc}K_S}^2 \right) .$$

In Fig. 4 we show the relative sizes of the effective branching fractions $B_{-}$ and $B_{+}$ in the region $0.17 \leq \sin(2\beta) \leq 0.99$, limited by the current data [11]. For our purpose, the suitable $X_{cc}$ states
include $J/\psi$, $\psi'$, $\psi''$, $\eta_c$, $\eta_c'$, etc\footnote{Note that $\psi' \to J/\psi\pi\pi$, $\psi'' \to D\bar{D}$, and $\eta_c' \to \eta_c\pi\pi$.}. Since such channels occur through the same tree-level quark diagram (see Fig. 3), their branching ratios $B_{X_cK_S}$ (or $B_{X_cK_L}$) are expected to be the same order. Thus a combination of the possible decays $(B_{d,\text{phys}}^0\bar{B}_{d,\text{phys}}^0)_C \to X_cK_S$ or $X_cK_L$ should increase the decay events of a single mode by several times. In a similar way one can study the joint transitions $(B_{d,\text{phys}}^0\bar{B}_{d,\text{phys}}^0)_C \to X_cK_S\pi^0$ or $X_cK_L\pi^0$. For a more detailed discussion about $CP$ violation in the semi-inclusive decays $B^0_d$ vs $\bar{B}^0_d \to (\bar{c}c)K_S$ or $(\bar{c}c)K_L$, we refer the reader to Ref. [12].

4. Summary

In keeping with the experimental efforts to study flavour mixing and $CP$ violation, we have presented a generic formalism for the time-dependent and time-integrated decays of all possible $P^0$ – $\bar{P}^0$ systems. This formalism is useful for various phenomenological applications at the forthcoming flavour factories, where a large amount of coherent $P^0\bar{P}^0$ events will be produced and accumulated. In our calculations, the $\Delta Q = \Delta P$ rule and $CPT$ symmetry have been assumed. Relaxing these two constraints one can obtain the more general formulae, which should be useful for searching for $CPT$ violation or $\Delta Q = \Delta P$ breaking in the $K^0 - \bar{K}^0$ [6] and $B^0 - \bar{B}^0$ systems [13,14].

To meet various possible measurements of $CP$ violation at asymmetric $B$ factories, we have carried out a reanalysis of three types of $CP$ violating signals manifesting in the coherent $B^0_d\bar{B}^0_d$ transitions at the $\Upsilon(4S)$ resonance. Although some comprehensive works have been done on this topic, our present one differs from them in several aspects. We illustrate the advantage of proper time cuts for measuring the $B^0_d - \bar{B}^0_d$ mixing parameters and $CP$ asymmetries. It is shown that direct and indirect $CP$ violating effects are distinguishable in neutral $B$ decays to $CP$ eigenstates. In addition, we explore the possibility to measure some $CP$-forbidden processes as a direct test of $CP$ symmetry breaking at $B$ factories.

A more detailed study of the topic under discussion, together with some numerical predictions or estimates, is in preparation.

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References

[1] T. D. Lee and C. S. Wu, Annu. Rev. Nucl. Sci. 16 (1966) 511.

[2] Particle Data Group, M. Aguilar-Benitez et al., Phys. Rev. D50 (1994) 1173.

[3] I. I. Bigi, V. A. Khoze, N. G. Uraltsev, and A. I. Sanda, in CP Violation, edited by C. Jarlskog (World Scientific, Singapore, 1988), p. 175.

[4] KEK Report 92-3 (1992); SLAC Report SLAC-400 (1992).

[5] D. Cline, in the Proceedings of the XXVI International Conference on High Energy Physics, Dallas, August 1992.

[6] M. Hayakawa and A. I. Sanda, Phys. Rev. D48 (1993) 1150; C. D. Buchanan et al., Phys. Rev. D45 (1992) 4088; J. Bernabéu, F. J. Botella, and J. Roldán, Phys. Lett. B211 (1988) 226; I. Dunietz, J. Hauser, and J. L. Rosner, Phys. Rev. D35 (1987) 2166.

[7] I. I. Bigi and A. I. Sanda, Phys. Lett. B194 (1987) 307; I. Dunietz and T. Nakada, Z. Phys. C36 (1987) 503; H. J. Lipkin, Phys. Lett. B219 (1989) 474; M. Gronau, Phys. Rev. Lett. 63 (1989) 1451; G. V. Dass and K. V. L. Sarma, Int. J. Mod. Phys. A7 (1992) 6081; A8 (1993) 1183 (E); A. Acuto and D. Cocolicchio, Phys. Rev. D47 (1993) 3945.

[8] H. Fritzsch, D. D. Wu, and Z. Z. Xing, Phys. Lett. B328 (1994) 477; D. Du and Z. Z. Xing, Phys. Rev. D47 (1993) 2825; Z. Z. Xing and D. Du, Phys. Lett. B276 (1992) 511.

[9] G. J. Feldman et al., in the Proceedings of High Energy Physics in the 1990’s, edited by S. Jensen (World Scientific, Singapore, 1988), p. 561; K. Berkelman et al., preprint CLNS 91-1050 (1991).

[10] R. Aleksan et al., Phys. Rev. D39 (1989) 1283.

[11] A. Ali and D. London, CERN-TH.7398/94 (to appear in Z. Phys. C).

[12] J. Bernabéu and C. Jarlskog, Phys. Lett. B301 (1993) 275; H. J. Lipkin, preprint SLAC-373 (1991).

[13] M. Kobayashi and A. I. Sanda, Phys. Rev. Lett. 69 (1992) 3139; G. V. Dass and K. V. L. Sarma, Phys. Rev. Lett. 72 (1994) 191.

[14] Z. Z. Xing, Phys. Rev. D50 (1994) R2957; D. Colladay and V. A. Kostelecky, preprint IUHET 285 (1994).
Figure 1: Ratios of the same-sign dilepton events to the opposite-sign ones as functions of the time cut $t_0$ at the $\Upsilon(4S)$ resonance.

Figure 2: Ratios of the $CP$ asymmetries $A^{(n)}_{-}(t_0)$ to $A^{(n)}_{-}(0)$ as functions of the time cut $t_0$ on the $\Upsilon(4S)$ resonance.
Figure 3: Quark diagrams for $B_d^0$ versus $\bar{B}_d^0$ decays to $X_{cc}K_{S,L}$. Here $X_{cc}$ denote the possible charmonium states that can form the odd (even) $CP$ eigenstates with $K_S$ ($K_L$).

Figure 4: Relative sizes of the effective branching fractions $B_C$ (in arbitrary units) as functions of the $CP$ violating angle $\beta$ at the $\Upsilon(4S)$ resonance.