The Influence of Diameter-to-thickness Ratios on the Response and Failure for Locally-dented Circular Tubes Submitted to Cyclic Bending

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Abstract. This paper describes the cyclic bending response and failure of 6061-T6 aluminum alloy locally-dented tubes (abbreviation: 6061-T6 LDT) with different diameter-to-thickness ratios (abbreviation: Do/t ratio) of 16.5, 31.0 and 60.0. A small lateral local dent was created on the tube using a simple technique. A tube having a dent depth from very shallow to about 2.4 times the thickness of the tube wall is considered. From the first bending cycle, the moment-curvature relationship depicts a closed and stable loop. Due to small and localized dents, dent depth shows very small influence on its relationship. But, as the number of bending cycle increases, the ovalization-curvature relationship shows an increase and a ratchet-like distribution. However, the dent depth shows a dramatic effect to this relationship. Furthermore, for a certain Do/t ratio, five non-paralleled straight lines of five different dent depths were found for the controlled curvature-number of cycles required to ignite failure relationship on a log-log scale. Finally, an empirical formula was introduced to describe the aforementioned relationship. As a result, the experimental and analytical data were found to agree well.

1. Introduction

As we all know that the bending of a circular tube will lead to tube’s ovalization (i.e., the amount of change in outer diameter divided by the amount of original diameter, ΔDo/Do). In cyclic bending loads, ΔDo/Do slowly increases with the number of bending cycles. Finally, when a certain number of bending cycles is reached and ΔDo/Do reaches a certain critical value, the circular tube will undergo buckling or fracture failure. Thus, to completely understanding the behavior of circular tubes submitted to cyclic bending is a very important topic for the industry.

Research on the cyclic bending of smooth circular tubes has yielded significant results so far. In 1985, Professor Kyriakides and his research team designed a mechanical device to conduct the cyclic bending tests for various materials of circular tubes (6061-T6 aluminum alloy tube, 1018 steel tube, 304 stainless steel tube, and 1020 steel tube). Many experimental and theoretical studies have been carried out under monotonic or cyclic bending with or without internal or external pressure [1-6].

In 2010, Pan et al. began investigations on the response of the notched tube submitted to a cyclic bending load. Lee et al. [7] examined the changes in ΔDo/Do of SUS304 stainless steel sharp-notched tubes undertaken cyclic bending. In another study, Lee et al. [8] determined the collapse of SUS304 stainless steel sharp-notched tubes undertaken cyclic bending at different curvature rates; changes in both notch...
depth and curvature rate were also examined in this work. Chung et al. [9] investigated the stability of 6061-T6 aluminum alloy sharp-notched tubes submitted to cyclic bending. However, all of the above studies are circular circumferential notched tubes. Later, Lee et al. [10] studied the deterioration and failure of 6061-T6 aluminum alloy local sharp-notched tubes submitted to cyclic bending. In addition, the moment (M)-curvature (κ) and ΔD_o/D_o-κ relationships were analysed by the finite element analysis software “ANSYS”. Lee et al. [11] examined the behavior of SUS304 stainless steel local sharp-notched tubes with different notch depths at different notch directions undertaken cyclic bending. In their study, the M-κ and ΔD_o/D_o-κ relationships were also analysed by ANSYS. In addition, the controlled curvature-number of cycles required to ignite buckling relationship was carried out related descriptions. Experimental study of 6061-T6 LDTs with different D_o/t ratios of 16.5, 31.0 and 60.0 submitted to cyclic bending was demonstrated. This paper used the tube-bending device and curvature-ovalization measurement apparatus to perform the curvature-controlled cyclic bending tests. The quantities of M, κ and ΔD_o/D_o were measured by sensors on testing facilities. Additionally, the number of bending cycles required to ignite failure (N_f) was also recorded.

2. Experiments

2.1. Experimental device

Fig. 1(a) schematically shows the experiment executed by a specially built tube-bending machine. This facility was set up to conduct monotonic, and cyclic bending tests. A detailed description of the experimental setup is included in some of the papers (e.g., Lee et al. [8], Chung et al. [9]). Pan et al. [12] designed a new apparatus to measure the κ and ΔD_o/D_o of the tube shown in Fig. 1(b). Two side-inclinometers in the apparatus were used to detect the changes in the angle of the tube during cyclic bending. The magnitude of κ can be obtained by an easy calculation according to angle changes. The version of the calculation can be discovered in the paper by Pan et al. [12]. The value of ΔD_o/D_o can also be measured in the central portion of the apparatus that included a magnetic detector and a magnetic block.

![Figure 1](image1.png)

Figure 1. Schematic drawings of (a) the tube-bending machine and (b) curvature-ovalization measurement apparatus.

2.2. Material and Specimens

A round tube made of 6061-T6 aluminum alloy is used in this study and its chemical composition is Mg 0.916, Si 0.733, Cu 0.293, Ti 0.268, Fe 0.256, Mn 0.132, Zn 0.0983, Cr 0.0682, Ni 0.0056, Pb 0.005, Sn < 0.001, and Al remainder. The mechanical properties of this material are: ultimate tensile strength of 310 MPa, 0.2% offset yield strength of 286 MPa, percent elongation of 23%.

The original smooth 6061-T6 aluminum alloy tube has a length L_o = 800 mm, an outer diameter D_o = 35.0 mm, and a wall thickness t = 3.0 mm. The original circular tubes were processed on the outside surface to give the desired D_o/t ratios of 16.5, 31.0 and 60.0, as shown in Fig. 2. However, the inner
radiiuses of all tested tubes were intact (29.0 mm). Next, the tube with a certain \(D_o/t\) ratio was again treated on the outer surface to achieve the desired dent shape and depth. Fig. 3(a) and Fig. 3(b) respectively depict a picture and a schematic drawing of dent production. The upper mould was in contact with the surface of the tube and the pressure was applied to create a dent. In present study, five dent depths \((a)\) were took into account: 0.0, 0.3, 0.6, 0.9 and 1.2 mm. Note that \(a = 0.0\) mm indicates a tube with a smooth surface.

![Figure 2](image1.png)

**Figure 2.** A schematic drawing of tube’s dimensions for \(D_o/t\) ratios of 16.5, 31.0 and 60.0.

![Figure 3](image2.png)

**Figure 3.** Process a dent on the 6061-T6 aluminum alloy tube.

### 2.3. Test Procedures

The bending experiment was carried out under curvature control conditions with a loading curvature of 0.03 m \(^{-1}\)/s. Two load cells (Fig. 1(a)) settled in the tube-bending machine were used to measure \(M\). The amounts of \(\kappa\) and \(\Delta D_o/D_o\) were measured by the curvature-ovalization measurement apparatus, as shown in Fig. 1(b). In addition, the sum of \(N_I\) was also recorded.

### 3. Experimental Result and Discussion

#### 3.1. Mechanical Response

Figs. 4(a)-4(o) respectively show a typical experimentally determined \(M-\kappa\) curves for 6061-T6 LDTs, with \(D_o/t\) = 16.5, 31.0, 60.0 and \(a = 0.0, 0.3, 0.6, 0.9\) and 1.2 mm, when submitted to cyclic bending. The tubes were cycled between \(\kappa = \pm 0.4\) m \(^{-1}\). As can be seen from Figs. 4(a)-4(e) that the \(M-\kappa\) relationships were summarized as an almost closed and stable hysteresis loops from the first bending cycle. However, it was observed Figs. 4(f)-4(j) that from the first bending cycle, the \(M-\kappa\) relationships were considered to be a closed and stable hysteresis loop for \(a = 0.0, 0.3\) and 0.6 mm. The \(M-\kappa\) relationships were closed hysteresis loops, but became stable after several cycles of bending for other \(a\). Additionally, from Figs. 4(k)-4(o), the \(M-\kappa\) relationships were closed and stable hysteresis loops from the first bending cycle for \(a = 0.0\) and 0.3 mm. However, the \(M-\kappa\) relationships were closed hysteresis loops, but became stable after a few bending cycles for other \(a\).
Figure 4. Experimental $M$-$\kappa$ curves for 6061-T6 LDTs with $D_0/t = 60.0$ and $a = (a) 0.0$, (b) 0.3, (c) 0.6, (d) 0.9 and (e) 1.2 mm, $D_0/t = 31.0$ and $a = (f) 0.0$, (g) 0.3, (h) 0.6, (i) 0.9 and (j) 1.2 mm, $D_0/t = 16.5$ and $a = (k) 0.0$, (l) 0.3, (m) 0.6, (n) 0.9 and (o) 1.2 mm submitted to cyclic bending.

Figs. 5(a)-5(o) respectively show a typical experimentally determined $\Delta D_0/D_0$-$\kappa$ curves for 6061-T6 LDTs, with $D_0/t = 16.5, 31.0, 60.0$ and $a = 0.0, 0.3, 0.6, 0.9$ and 1.2 mm, when submitted to cyclic bending. The tubes also cycled between $\kappa = \pm 0.4$ m$^{-1}$. It was demonstrated that the $\Delta D_0/D_0$-$\kappa$ relationships displayed a ratchet and an increasing trend when the number of bending cycles increased. For a certain $D_0/t$ ratio, larger $a$ caused an asymmetrical appearance of the $\Delta D_0/D_0$-$\kappa$ relationship. Moreover, a larger $D_0/t$ ratio or $a$ in the dented tubes caused larger $\Delta D_0/D_0$. 
In this study, the liner relationships between ln amount of C and ln α were found in Figs. 8(a)-8(c) and Figs. 9(a)-9(c). Therefore, we write

\[ \ln C = C_0 - \beta (\alpha t) \]  

in which C and α are the material parameters related to material properties. The quantity of C is the amount of \( \kappa / \kappa_0 \) when \( N_t = 1 \), and \( \alpha \) is the slope of the line in the log-log plot. In this study, the liner relationships between ln C and \( \alpha t \) and ln α and \( \alpha t \) were respectively found in Figs. 8(a)-8(c) and Figs. 9(a)-9(c). Therefore, we write

\[ \log \kappa / \kappa_0 = \log C - \alpha \log N_t, \]  

in which \( C \) and \( \alpha \) are the material parameters related to material properties. The quantity of C is the amount of \( \kappa / \kappa_0 \) when \( N_t = 1 \), and \( \alpha \) is the slope of the line in the log-log plot. In this study, the liner relationships between ln C and \( \alpha t \) and ln α and \( \alpha t \) were respectively found in Figs. 8(a)-8(c) and Figs. 9(a)-9(c). Therefore, we write

\[ \ln C = C_0 - \beta (\alpha t) \]  

 Kyriakides and Shaw [1] introduced an empirical formula for describing the \( \kappa / \kappa_0 - N_t \) relationship of the materials they tested as
and

\[ \ln \alpha = \alpha_0 - \gamma (a/t), \]  

(3)
in which \( C_0, \beta, \alpha_0 \) and \( \gamma \) are material parameters. The quantities of \( C_0 \) and \( \beta \) were the intercepts and slopes in Figs. 8(a)-8(c) for \( D_o/t = 16.5, 31.0 \) and \( 60.0 \), respectively, and quantities of \( \alpha_0 \) and \( \gamma \) were the intercepts and slopes in Figs. 9(a)-9(c) for \( D_o/t = 16.5, 31.0 \) and \( 60.0 \), respectively.

Next, the material parameters \( C_0, \beta, \alpha_0 \) and \( \gamma \) were assumed to be related to the \( D_o/t \) ratios. In this study, we tried to find the linear relationships between the above material parameters and \( D_o/t \) ratios. After a lot of attempts, the linear relationships were obtained in Figs. 10(a)-10(d). Thus, the linear relationships can be written to be

\[ \ln C_0 = a_1 \ln D_o/t + a_2, \]  

(4)
\[ \ln \beta = b_1 \ln D_o/t + b_2 \]  

(5)
\[ 1/\ln \alpha_0 = c_1 \ln D_o/t + c_2 \]  

(6)
and

\[ 1/\gamma = d_1 (D_o/t) + d_2 \]  

(7)
where \( a_1, a_2, b_1, b_2, c_1, c_2, d_1 \) and \( d_2 \) are material constants. The values of \( a_1, b_1, c_1 \) and \( d_1 \) were the slopes in Figs 10(a)-10(d), respectively. The amounts of \( a_2, b_2, c_2 \) and \( d_2 \) were the intercepts in Figs. 10(a)-10(d), respectively. These material constants, \( a_1, a_2, b_1, b_2, c_1, c_2, d_1 \) and \( d_2 \) were determined to be -1.356, 4.551, -3.098, 9.684, 0.287, -1.574, 2.161 and -29.657, respectively.

Figure 8. \( \ln C-\ln t \) relationship for \( D_o/t = (a) 16.5, (b) 31.0 \) and (c) 60.0.

Figure 9. \( \ln \alpha-\ln t \) relationship for \( D_o/t = (a) 16.5, (b) 31.0 \) and (c) 60.0.

Figure 10. (a) \( \ln C_0-\ln D_o/t \) relationship, (b) \( \ln \beta-\ln D_o/t \) relationship, (c) \( 1/\ln \alpha_0-\ln D_o/t \) relationship, and (d) \( 1/\gamma-\ln D_o/t \) relationship.
Finally, Eqs. (1)-(7) were used to simulate the experimental finding in Figs. 7(a)-7(c). The simulated results of \( \kappa/\kappa_0/N_t \) relationships for 6061-T6 LDTs with \( D_o/t = 16.5, 31.0 \) and 60.0 under cyclic bending on a log-log scale are respectively depicted in Figs. 7(a)-7(c) in solid lines. Good agreement has been achieved between the experimental and simulation results.

4. Conclusions
The response and failure of 6061-T6 LDTs with different \( D_o/t \) ratios and \( a \) subjected to cyclic bending were examined. According to the experimental and simulation results, this study draws the following conclusions:

1. The experimental \( M-\kappa \) relationship for 6061-T6 LDTs with any \( D_o/t \) ratio or \( a \) displayed a closed and stable hysteresis loop from the first bending cycle or after a few bending cycles. When \( a \) is less than \( t \), the magnitudes of \( M \) are almost equal at the maximum and minimum curvatures. When \( a \) is close to or larger than \( t \), due to the two sides of the dent coming into contact during reverse bending, the magnitude of \( M \) at the minimum curvature may smaller than that at the maximum curvature.

2. The experimental \( \Delta D_o/D_o-\kappa \) relationship for 6061-T6 LDTs with any \( D_o/t \) ratio or \( a \) revealed an increase and a ratchet-like distribution in the number of bending cycles. The \( \Delta D_o/D_o-\kappa \) relationships were symmetrical for \( a = 0.0 \) mm, but asymmetrical for \( a \neq 0.0 \) mm. Moreover, the tubes with a larger \( D_o/t \) or \( a \) led to more asymmetrical trend and a larger \( \Delta D_o/D_o \).

3. The empirical formula of Eq. (1) introduced by Kyriakides and Shaw [1] was modified to describe the \( \kappa/\kappa_0-N_t \) relationship for 6061-T6 LDTs with different \( D_o/t \) ratios undertaken cyclic bending. According to the experimental data, the forms of the material parameters, \( C \) and \( \alpha \), were respectively introduced in Eqs. (2) and (3). In addition, The forms of the material parameters, \( C_o, \beta, \alpha \), and \( \gamma \), were respectively introduced in Eqs. (4)-(7). The analytical data described by Eqs. (1)-(7) were very consistent with the experimental findings (Figs. 7(a)-7(c)).

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