Can extra dimensions accessible to the SM explain the recent measurement of anomalous magnetic moment of the muon? 1

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Abstract

We investigate whether models with flat extra dimensions in which SM fields propagate can give a significant contribution to the anomalous magnetic moment of the muon (MMM). In models with only SM gauge and Higgs fields in the bulk, the contribution to the MMM from Kaluza-Klein (KK) excitations of gauge bosons is very small. This is due to the constraint on the size of the extra dimensions from tree-level effects of KK excitations of gauge bosons on precision electroweak observables such as Fermi constant. If the quarks and leptons are also allowed to propagate in the (same) bulk (“universal” extra dimensions), then there are no contributions to precision electroweak observables at tree-level. However, in this case, the constraint from one-loop contribution of KK excitations of (mainly) the top quark to $T$ parameter again implies that the contribution to the MMM is small. We show that in models with leptons, electroweak gauge and Higgs fields propagating in the (same) bulk, but with quarks and gluon propagating in a sub-space of this bulk, both the above constraints can be relaxed. However, with only one Higgs doublet, the constraint from the process $b \rightarrow s\gamma$ requires the contribution to the MMM to be smaller than the SM electroweak correction. This constraint can be relaxed in models with more than one Higgs doublet.

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Theories with flat extra dimensions of size $\sim (\text{TeV})^{-1}$ in which SM gauge (and in some cases quark and lepton) fields propagate are motivated by SUSY breaking, gauge coupling unification, generation of fermion mass hierarchies and electroweak symmetry breaking by a composite Higgs doublet. In the effective 4$D$ theory, the extra dimensions “appear” in the form of Kaluza-Klein (KK) excitations of SM gauge bosons (and quarks and leptons). These KK states have masses quantized in units of $\sim 1/R$, where $R$ is the size of an extra dimension. In this paper, we study the contributions to anomalous magnetic moment of the muon (MMM) from these KK excitations. To be specific, we investigate whether this contribution can be significant enough to be relevant for an explanation of the $2.6\sigma$ discrepancy between the SM prediction and the new world-average value of the MMM: $a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} \approx (43\pm 16) \times 10^{-10}$. This discrepancy is (roughly) comparable to the electroweak correction to the MMM in the SM which is $15 \times 10^{-10}$. Of course, there are other new physics explanations of this discrepancy such as SUSY, non-minimal Higgs sector etc.

A priori, the one-loop contribution to the MMM from KK excitations of electroweak gauge bosons (and the muon) can be comparable to the SM electroweak correction to the MMM for two reasons. One is that the masses of these KK states (the compactification scale) need not be much larger than the electroweak scale. The other is the possible enhancement due to large number of KK states: in general, we get

$$a_{\mu}^{KK} \sim g_2^2 / (16\pi^2) \ m_{\mu}^2 \sum_n 1 / (n/R)^2 \sim a_{\mu}^{EW} (M_W R)^2 \sum_n 1/n^2$$

where $a_{\mu}^{KK}$ is the one-loop contribution to the MMM from KK states of $W$, $Z$ and $\gamma$ and $a_{\mu}^{EW} \sim g_2^2 / (16\pi^2) \ m_{\mu}^2 / M_W^2$ is the one-loop electroweak correction in the SM. The mass of the KK state with momentum $n_i/R$ in the $i^{th}$ extra dimension is given by $\approx n/R$, where $n^2 \equiv \sum_{i=1}^{i=\delta} n_i^2$ and $n_i$ is an integer (assume $\delta$ extra dimensions compactified on circles of radius $R$). Thus, $a_{\mu}^{KK} \sim a_{\mu}^{EW}$ if $R^{-1} \sim$ few 100 GeV and/or $\sum_n 1/n^2 \gg 1$.

There are, of course, constraints on $R^{-1}$ which depend on whether the SM quarks and leptons propagate in these extra dimensions or not.

1 Quarks and leptons confined to 4$D$

Suppose the quarks and leptons are localized on a “3-brane”, i.e., do not propagate in the extra dimensions (the “bulk”) in which the gauge bosons and Higgs fields propagate. Then translation

5It is possible that there are extra dimensions of size much larger than $\sim (\text{TeV})^{-1}$ in which gravity and SM singlet fields (for example, right-handed neutrino) only propagate. In this case, the contribution to the MMM from KK excitations of graviton (or right-handed neutrino) is also important. Contributions to the MMM from “warped” extra dimensions with and without SM gauge fields propagating in them have also been studied.
invariance in the extra dimensions is broken by couplings of fields on the brane to fields in the bulk – in this case, the couplings of quarks and leptons to gauge bosons. In momentum space, conservation of extra dimensional momentum is violated. Thus, in the effective 4D theory vertices with only one KK state, for example, tree-level couplings of KK gauge bosons to leptons and quarks, are allowed. These couplings result in a contribution to muon decay, atomic parity violation (APV) etc. at tree-level from Feynman diagrams with exchange of KK W’s and Z’s instead of (zero-mode) W and Z as in the SM [3, 14]. Thus, these contributions are $\approx \sum_n M_W^2 / (n/R)^2$ compared to the SM.

Then, precision electroweak measurements ($G_F$ etc.) imply an upper limit on $\sum_n M_W^2 / (n/R)^2$ of about a few percent. and, in turn, $a_{\mu}^{KK} \ll a_{\mu}^{EW}$ (see Eq. (1)) [3].

2 Universal extra dimensions

Suppose quarks and leptons are also in the (same) bulk – the extra dimensions are “universal”. Then, extra dimensional momentum is conserved, i.e., in the effective 4D theory, “KK number” is conserved. Thus, there are no vertices with only one KK state; in particular, couplings of (zero-modes of) quarks and leptons to KK gauge bosons are not allowed. This implies that there are no tree-level effects on muon decay, APV, $e^+e^- \rightarrow \mu^+\mu^-$ etc. [15].

However, there are constraints on $R^{-1}$ from one loop contribution to T from KK excitations of top quark, W and Higgs [15]:

$$T \approx 0.83 \sum_n m_t^2 / (n/R)^2 \left[1 - 0.75 m_t^2 / (n/R)^2 + ..\right] - (0.25 + 0.034\delta) \sum_n M_W^2 / (n/R)^2$$

$$-0.043 \sum_n M_H^2 / (n/R)^2 \left[1 + O\left(M_H^2 / (n/R)^2\right)\right]$$

(2)

Here, $\delta$ is the number of extra dimensions and $M_H$ is the Higgs mass. We have assumed one Higgs doublet as in the minimal SM. The contribution of KK states of top quark is positive and dominates over that of KK states of W and Higgs which is negative. Thus, we get a constraint from the 2$\sigma$ upper limit $T \lesssim 0.4$ (for $M_H \lesssim 250$ GeV). In the above equation, the contribution of KK states of W and Higgs at each level (i.e., for each value of n) depends on $\delta$ and $M_H$. In addition, the sum over KK states $\sim \sum_n 1/n^2$ depends on $\delta$ – in fact, $\sum_n 1/n^2$ diverges for $\delta \geq 2$ and so has to be regulated by a cut-off. Thus, the constraint on $R^{-1}$ depends on $\delta$ and $M_H$. In any case, using $m_t/M_W \approx 2.1$ and assuming $\delta \leq 6$ and $M_H \leq 250$ GeV, we get the constraint $(M_W R)^2 \sum_n 1/n^2 \approx 0.1$ (if we neglect $O\left(R/n\right)^4$ terms). The constraint is a little bit weaker if we allow larger $\delta$ and $M_H$. Nevertheless, this implies that $a_{\mu}^{KK} \ll a_{\mu}^{EW}$.

6The SM contribution to the vacuum polarization diagrams for W and Z and hence the 2$\sigma$ limit on T depends weakly (logarithmically) on $M_H$. 

2
3 Quarks and gluon in a subspace of the bulk

To relax the constraint from the $T$ parameter, we need to reduce the (positive) contribution of KK states of top quark to $T$ compared to the (negative) contribution from KK states of $W$ and Higgs. This can be done by allowing the top quark to propagate in fewer extra dimensions than the electroweak particles. One possibility is to have only third generation quarks stuck on a 3-brane. Then, in the weak basis, only third generation quarks couple to KK gluons. Thus, when we perform a unitary rotation on the quarks to go to mass basis, there is flavor violation at this KK gluon vertex \[16\]. This results in contribution to, for example, $B - \bar{B}$ mixing from tree-level exchange of KK gluons. If we assume CKM-like mixing among down-type quarks, then the coefficient of the $\Delta B = 2$ operator is given by

\[ \sim (V^{\star}_{tb}V_{td})^2 g_{QCD}^2 \sum 1/(n/R)^2. \]

Then, the constraint that this coefficient should be smaller than that in the SM which is

\[ \sim (V^{\star}_{tb}V_{td})^2 g_{QCD}^4 / (16\pi^2) 1/m_t^2 \]

implies that

\[ (M_W R)^2 \sum 1/n \ll 1 / g_{QCD}^2 g_{QCD}^4 / (16\pi^2) \ll 1. \]

In turn, this shows that $a_{\mu}^{KK} \ll a_{\mu}^{EW}$.

Also, if there are some extra dimensions in which gluons propagate but the first generation quarks do not, then there is a tree-level contribution to $p\bar{p} \rightarrow \text{jets}$ from exchange of KK excitations of gluons. This results in the constraint $R - 1 \sim 1$ TeV from CDF \[18\]: such a large $R^{-1}$ will result in $a_{\mu}^{KK} \ll a_{\mu}^{EW}$.

If Higgs fields are on a 3-brane (or in general, propagate in a sub-space of the bulk in which electroweak gauge bosons propagate), then there is a mass-mixing term between zero-mode $W$, $Z$ and their KK excitations. This mixing is due to the coupling $H^{\dagger}W_{\mu}^{(0)}W_{\mu}^{(n)}$, where $W_{\mu}^{(n)}$ denotes a KK state of $W$ with mass $n/R$ and $H$ is the SM Higgs field \[17, 16\]. Due to this mixing, the physical $W$ and $Z$ masses are shifted from the values given by the (tree-level) expression in the SM, i.e., from $M_W^{SM} = gv/2$ and $M_Z^{SM} = \sqrt{g^2 + g'^2}v/2$, where $v \approx 246$ GeV and $g$, $g'$ are the $SU(2)$ and $U(1)_Y$ gauge couplings. The relative shift in the masses is given by \[ \sim \sum_n M_W^{SM}^2 (or M_Z^{SM}^2) / (n/R)^2, \]
whereas the tree-level expression for muon decay (in terms of $v$) is the same as in the SM since only zero-mode of $W$ contributes in this model \[16\]. As a result precision data on $W$, $Z$ masses give an upper limit on $\sum_n M_W^2 / (n/R)^2$ of about a few percent. From Eq. \[11\], we see that this constraint results in $a_{\mu}^{KK} \ll a_{\mu}^{EW}$. So, to get $a_{\mu}^{KK} \sim a_{\mu}^{EW}$, we have to assume that Higgs fields propagate in the same extra dimensions as the electroweak gauge bosons.

In order to evade all these constraints on $R^{-1}$, we assume that all quarks and gluons propagate in the same extra dimensional space, namely, $\delta_e$ extra dimensions. Whereas, all leptons, electroweak gauge bosons and Higgs doublet propagate in $\delta - \delta_e$ additional extra dimensions. As mentioned before, to evade the constraint from tree-level effects of KK states on electroweak observables involving leptons, the leptons and electroweak gauge bosons have to propagate in the same bulk. For simplicity, assume that all extra dimensions are compactified on circles of the same radius $R$. 
A possible motivation for such a set-up is to explain $g_{\text{EW}}^2 \ll g_{\text{QCD}}^2$ at the cut-off (and hence at the scale $\sim M_Z$) as we discuss below. The gauge coupling of the effective 4D theory and the dimensionless gauge coupling of the fundamental $(4 + \delta)D$ theory, $g_{(4+\delta)D}$, are related at the cut-off $M$ by $g_{4D}^2 \sim g_{(4+\delta)D}^2/(MR)^\delta$ and thus if $\delta_c < \delta$, then $g_{4D,\text{EW}}^2$ is reduced (more) by extra dimensional volume than $g_{4D,\text{QCD}}^2$. Therefore, $g_{4D,\text{EW}}^2 \ll g_{4D,\text{QCD}}^2$ at the cut-off even though the fundamental (dimensionless) gauge coupling, $g_{(4+\delta)D}$, might be same for QCD and electroweak gauge groups. Between the cut-off and the energy scale $R^{-1}$, $g_{4D,\text{QCD}}^2$ and $g_{4D,\text{EW}}^2$ evolve with a power-law due to the contribution of the KK states \[2\]: if $\delta_c < \delta$, then this power-law evolution is more important for $g_{4D,\text{EW}}^2$ than for $g_{4D,\text{QCD}}^2$ since there are fewer KK states with color. This evolution leads to an additional difference between $g_{4D,\text{QCD}}^2$ and $g_{4D,\text{EW}}^2$ at the scale $\sim M_Z$. This is to be contrasted to the “standard” picture with the gluon and electroweak gauge bosons propagating in the same bulk. In this picture, due to power-law evolution (which is equally important for both gauge couplings), it is possible that the two effective 4D couplings, i.e., $g_{4D,\text{QCD}}^2$ and $g_{4D,\text{EW}}^2$, unify at the cut-off $M$ (even though $M$ might not be much larger than $R^{-1}$) \[2\]. Whereas, with $\delta_c < \delta$, as discussed above, the fundamental dimensionless QCD and electroweak gauge couplings can “unify” at the cut-off, even though the effective 4D gauge couplings do not.

There are two cases within this type of model which we now discuss.

### 3.1 Case $\delta_c = 0$: quarks and gluon on a brane

In this case, there are no KK excitations of the top quark and hence Eq. \[2\] gives $T \approx -(0.35 - 0.8) (M_W R)^2 \sum_n 1/n^2 \sim -(0.2 - 0.3)$ which is the 2σ lower limit on $T$: this also depends weakly on $M_H$. The range $0.35 - 0.8$ corresponds to varying $\delta$ and $M_H$ (assuming $\delta \leq 6$ and $M_H \approx 250$ GeV). This implies that $(M_W R)^2 \sum_n 1/n^2 \sim 1$ so that $a_{\mu}^{KK} \sim a_{\mu}^{\text{EW}}$ is barely allowed by the constraint from $T$ (see Eq. \[I\]).

However, there is a stronger constraint from inclusive hadronic decays of $B$. The reason is that KK states of $W$ contribute at tree-level to hadronic weak decays, but not to semileptonic weak decays since (zero-mode) leptons do not couple to KK states of gauge bosons. Again, this additional contribution to the amplitude is $\approx (M_W R)^2 \sum_n 1/n^2$ compared to the SM. The SM prediction for the ratio of the rates of hadronic and semileptonic inclusive $B$ decays, to be specific $\Gamma(b \rightarrow c\bar{u}d)/\Gamma(b \rightarrow c\ell\nu)$, agrees with experiment to within $\sim 20\%$ at 1σ (combining theory and experiment errors) \[19\]. Thus, we get $(M_W R)^2 \sum_n 1/n^2$ (the ratio of KK to SM $W$ amplitude) is at most about 10% so that $a_{\mu}^{KK} \ll a_{\mu}^{\text{EW}}$.
In this case, the constraints from both $T$ and hadronic $B$ decays can be relaxed as we discuss below.

Using $m_t \approx 2.1M_W$ in Eq. (2), we get

$$T \approx (M_W R)^2 \left[ 3.9 \sum_n \frac{\delta_c}{n^2} - (0.35 - 0.8) \sum_n \frac{\delta}{n^2} \right], \quad (3)$$

where (and from now on) $\sum_n^{\delta'} 1/n^2$ denotes sum over KK states of a particle with momentum in only $\delta'$ of the $\delta$ extra dimensions. As discussed before, the coefficient $\sim (0.35 - 0.8)$ of the $W$ and Higgs contribution to $T$ at each KK level depends on $\delta$ and $M_H$ – we have allowed $\delta \leq 6$ and $M_H \leq 250$ GeV. It is clear that there can be a cancelation between the contributions of top quark and $W+$Higgs, i.e., $T \sim 0$ if the following condition is satisfied:

$$\frac{\sum_{\delta} 1/n^2}{\sum_{\delta_c} 1/n^2} \sim (5 - 10), \quad (4)$$

where the RHS depends on $\delta$ and $M_H$. In other words, this case is intermediate between the case in section 2, where the contribution from top quark is dominant and the case in section 3.1, where there is only the $W$ and Higgs contribution.

Also, in this case, the additional contribution to hadronic $B$ decays (relative to SM) is given by

$$r_B \equiv \frac{W_{KK \text{ amplitude}}}{W \text{ amplitude in SM}} \approx (M_W R)^2 \sum_n^{\delta - \delta_c} \frac{1}{n^2} \quad (5)$$

since only KK excitations of $W$ with momentum in the $(\delta - \delta_c)$ extra dimensions in which quarks do not propagate contribute. From Eq. (3), to get $a_{\mu}^{KK} \sim a_{\mu}^{EW}$, we require

$$(M_W R)^2 \sum_n^{\delta} \frac{1}{n^2} \sim 1. \quad (6)$$

Thus, even if Eq. (3) is satisfied, the contribution to hadronic $B$ decays can be smaller than $\sim 10\%$ in the amplitude (as required by the agreement between theory and experiment) provided the following condition is satisfied

$$\sum_n^{(\delta - \delta_c)} \frac{1}{n^2} \lesssim 1/10 \sum_n^{\delta} \frac{1}{n^2}. \quad (7)$$

Therefore, for given $\delta$, it might be possible to evade the constraints from $T$ and hadronic decays (while getting $a_{\mu}^{KK} \sim a_{\mu}^{EW}$) by an appropriate choice of $\delta_c$ and $M_H$ such that Eqs. (4) and (7) are satisfied.

\footnote{Strictly speaking $-0.3 \lesssim T \lesssim 0.4$ is allowed at 2$\sigma$ (again depending weakly on $M_H$), but for simplicity, we assume $T \sim 0$.}
Next, we discuss the values of the parameters $\delta, \delta_c, R^{-1}$ and the cut-off $M$ for which this is possible.

We begin with direct bounds on $R^{-1}$ from collider searches. In this case, there are KK excitations of quarks which appear as heavy stable quarks at hadron colliders and searches at CDF imply $R^{-1} \gtrsim 300$ GeV (for $\delta_c = 1$) \cite{13}. The direct lower bound on $R^{-1}$ will be (slightly) larger for $\delta_c > 1$ since there will be more KK modes at the lowest level, i.e., with $n = 1$.

The KK states of electroweak gauge bosons with momentum in the $(\delta - \delta_c)$ extra dimensions (in which quarks do not propagate) couple to zero-mode quarks (with the same strength as in SM), but not to zero-mode leptons. Thus, at hadron colliders, the signatures of these KK states will be similar to that of $W'$ and $Z'$ decaying to dijets. The CDF search excludes hadronic decays of $W'$ in the mass range $300 < M_{W'} < 420$ GeV at 95\% confidence level \cite{18}. For the KK states of $W$, the branching ratio to quarks is slightly larger than that for $W'$ since the leptonic decays are absent. Also, since there are 2 $(\delta - \delta_c)$ relevant states at the lowest KK level (i.e., with $n = 1$), the production cross-section can be larger than that for $W'$ of same mass. We will assume the limit $R^{-1} \gtrsim 400$ GeV from a combination of the heavy stable quark and $W'$ searches.

In turn, this implies that $\sum_n^\delta 1/n^2 \gtrsim 25$ to give $a^K_W \sim a^{EW}_\mu$ (see Eq. (3)). Thus, $\delta = 1, 2$ will not suffice since $\sum_n^\delta 1/n^2 = \pi^2/3$ for $\delta = 1$ and is log-divergent (but, $O(1)$) for $\delta = 2$. The sum over KK states of a SM particle $\sum_n^\delta 1/n^2$ is power-divergent if $\delta \geq 3$. It can be approximated by an integral (provided $M \gg R^{-1}$): $\sum_n^\delta 1/n^2 \approx S_\delta/(\delta - 2) (MR)^{\delta-2}$, where the KK sum is cut-off at the mass scale $M$ (i.e., $n_{\text{max}}/R \approx M$) and $S_\delta = 2\pi^{\delta/2}/\Gamma[\delta/2]$ (the surface area of a unit-radius sphere in $\delta$ dimensions). Thus, for $\delta \geq 3$, we get

$$a^K_W a^{EW}_\mu \approx x (M_W R)^2 S_\delta/(\delta - 2) (MR)^{(\delta-2)},$$

where $x$ is a factor of $O(1)$ which includes two effects as follows. One effect is from the loop integral. The reason is that in the case of SM electroweak correction there is only one heavy particle in the loop ($W$) whereas in the case of the KK contribution both the particles in the loop ($W_{KK}$ and $\nu_{KK}$) are heavy. Hence the ratio of the contribution from a KK state of mass $n/R$ to the SM electroweak contribution is not simply the ratio of heaviest masses in the loop, i.e., it is $(M_W/ (n/R))^2$ only up to a factor of $O(1)$. In principle, $x$ can be computed in the effective 4D theory. However, we see from Eq. (8) that this factor can be absorbed into the definition of the cut-off $M$. The other effect included in the factor $x$ is that $a^{EW}_\mu$ includes the $W$ and $Z$ loop contributions in the SM, whereas $a^K_W$ includes loop contributions from KK states of $W$, $Z$ and the photon.

There is an upper limit on $(MR)^\delta$ from the condition that the effective 4D theory at the cut-off $M$ remain perturbative. The loop expansion parameter at the cut-off $M$ is given by $\sim N_{KK} N_i g_2^2/(16\pi^2)$. Here $N_i$ is the number of “colors” and $N_{KK} \approx S_\delta/\delta (MR)^\delta$ is the number of KK states lighter than $M$, i.e, the total number of KK states in the effective 4D theory. We
cannot trust perturbative calculations if this factor is larger than 1. Therefore, we impose the constraint
\[ N_{KK} \approx S_\delta / \delta (MR)^\delta \sim 16\pi^2 \] (9)
(QCD effective coupling is smaller since number of KK states of gluons and quarks is smaller).
Thus, the upper limit on \( M \) is not much larger than \( R^{-1} \). Nonetheless, we have checked if \( N_{KK} \gg 1 \) (i.e., \( M \) is close to the upper limit of Eq. (9)), then the sum over KK states can still be approximated by an integral.

We can rewrite Eq. (8) in terms of \( N_{KK} \) (using \( N_{KK} \approx S_\delta / \delta (MR)^\delta \)) as follows:
\[ \frac{a^K_{\mu}}{a^{EW}_{\mu}} \approx x (MR)^2 S_\delta / (\delta - 2) \left( \delta / S_\delta N_{KK} \right)^{(\delta-2)/\delta} \] (10)
In Fig. 1, we plot \( a^K_{\mu}/a^{EW}_{\mu} \) as a function of \( N_{KK} \) for different values of \( \delta \) and for \( R^{-1} = 400 \) GeV. From Eq. (9), the upper limit on \( N_{KK} \) is about 16\(\pi^2 \approx 160 \). We have chosen \( x = 1 \) for this plot; as mentioned before, if \( x \neq 1 \), then it can be absorbed into the definition of the cut-off, or in other words, in the value of \( N_{KK} \) (see Eq. (10)). Also, in this figure, we have not imposed the constraint from \( \text{BR}(B \to X_s \gamma) \) (see the discussion later). We see from this figure that it is possible to have \( a^K_{\mu} \sim a^{EW}_{\mu} \) for \( \delta \geq 3 \) as long as \( N_{KK} \gtorder 40 \).

We now discuss the values of \( \delta_c \) for which the constraints from \( T \) and hadronic \( B \) decays can be satisfied.

Since \( \sum_\delta 1/n^2 = \pi^2/3 \) for \( \delta = 1 \), it is clear from Eq. (3) that if \( (\delta - \delta_c) = 1 \), then \( r_B \leq 13\% \) provided \( R^{-1} \geq 400 \) GeV. Thus, it is possible to (barely) satisfy the constraint from hadronic \( B \) decays by choosing \( \delta_c = \delta - 1 \).

For given values of \( \delta \) and \( \delta_c \), in general, it should be possible to choose the value of \( M_H \) such that \( T \approx 0 \) (see Eq. (3) or Eq. (4)). Note that a small non-zero value of \( T \) is allowed (at 2\(\sigma \) level, \( -0.3 \lesssim T \lesssim 0.4 \) as mentioned before) and similarly the constraint from hadronic \( B \) decays is also not very precise. Thus, for given values of other parameters, there can be a range of values of \( \delta_c \) and \( M_H \) which is consistent with \( T \) and with hadronic \( B \) decays.

Next, we discuss the constraints from measurements of \( \Gamma(b \to s \gamma)/\Gamma(b \to c\nu) \). The semileptonic decay is not affected by the KK states of \( W \) since these states do not couple to (zero-mode) leptons. The amplitude for the process \( b \to s \gamma \) gets a contribution from KK states of \( W \sim g_2^0 / (16\pi^2) m_b V_{tb}^* V_{ts} m_t^2 \sum_\delta (n/R)^4 \) (the factor of \( m_t^2 \) reflects GIM cancelation). This is to be compared to the SM contribution \( \sim g_2^0 / (16\pi^2) m_b V_{tb}^* V_{ts} 1/M_W^2 \): the ratio is \( \sim (M_W m_t R^2)^2 \sum_\delta 1/n^4 \).

\[ \text{A caveat is that as mentioned earlier, if } x \neq 1, \text{ then the results in Fig. 1 really correspond to a “redefined” value of } N_{KK} (\text{ or in other words, the cut-off } M). \text{ Then, the cut-off appearing in the sum over KK states in Eq. (3) is, strictly speaking, different than that in Fig. 1. Thus, for a given value of } N_{KK} \text{ in Fig. 1 the value of } M_H \text{ which gives } T \approx 0 \text{ is modified. However, the two values of cut-off and hence the two values of } M_H \text{ differ by a factor of at most } O(1). \]
Figure 1: The contribution to the MMM from KK states of the electroweak gauge bosons relative to the SM electroweak correction as a function of the total number of KK states (see Eq. (10)) and for $\delta = 3, 4, 5,$ and $6,$ where $\delta$ is the number of extra dimensions in which the leptons and electroweak gauge bosons propagate. We assume $x = 1,$ $R^{-1} = 400$ GeV and do not impose the constraint from the process $b \rightarrow s\gamma$ since this constraint depends sensitively on the Higgs content (see text).
Thus, even if Eq. (6) is satisfied, i.e., the KK contribution to $a_\mu$ is comparable to the SM electroweak correction, the $W_{KK}$ contribution to $b \to s\gamma$ is smaller than in the SM. This is due to (a) an additional suppression by a factor of $1/(m_t R)^2 \sim 5$ (since $R^{-1} \sim 400 \text{ GeV}$) in the case of $b \to s\gamma$ (as compared to $a_\mu$) and (b) $\sum_n 1/n^4 < \sum n^2 1/n^2$.

However, there is a larger contribution to the amplitude for $b \to s\gamma$ from loops with KK states of charged would-be-Goldstone boson (WGB) as follows. With only one Higgs doublet, the zero-mode charged WGB is unphysical since it becomes the longitudinal component of $W$. However, the excited (KK) states of charged WGB are physical and have masses $\sim n/R$ (assuming $1/R \gg M_H$). The coupling of the KK states of charged WGB to right-handed top quark is the same as for the zero-mode charged WGB (i.e., longitudinal $W$) and is $\lambda_t \approx \sqrt{2} m_t / v$. Thus, the loop diagram with KK states of charged WGB and top quark gives an amplitude for $b \to s\gamma$ given by $\sim \lambda_t^2 / (16\pi^2) m_b V_{tb}^* V_{ts} \sum_n (n/R)^2$. The ratio of this contribution to the SM amplitude is $\sim (m_t R)^2 \sum_n 1/n^2$ and is clearly larger than the $W_{KK}$ contribution. Moreover, if Eq. (6) is satisfied, i.e., $a_\mu^{KK} \sim a_\mu^{EW}$, then this additional amplitude for $b \to s\gamma$ is also comparable to the SM amplitude. Such a large contribution to the process $b \to s\gamma$ is not allowed since the SM prediction for the rate agrees with experiment to within $\sim 20\%$ (combining theory and experiment $1\sigma$ errors). Thus, with one Higgs doublet (and assuming no other new physics contributions to $b \to s\gamma$), we see that the constraint from $B \to X_s \gamma$ rules out the possibility that the contribution to the MMM from KK states of electroweak gauge bosons is comparable to the SM electroweak correction.

However, if the Higgs sector is non-minimal, then the contribution of KK states of charged Higgs to the process $b \to s\gamma$ is modified. For example, in the 2-Higgs-doublet model (Model-II) [20], a cancelation occurs between the contribution of KK states of physical charged Higgs and the contribution of the KK states of the charged WGB mentioned above. This opens up the possibility that $a_\mu^{KK} \sim a_\mu^{EW}$. Note that the loop contribution to $a_\mu$ from KK states of charged Higgs is negligible in this model since it is suppressed by additional powers of $m_{\mu}$ compared to the contribution due to KK states of $W$.

4 Conclusions

In summary, we have revisited the contributions to the anomalous magnetic moment of the muon (MMM) in models with flat extra dimensions of size $\sim \text{TeV}^{-1}$ accessible to the SM particles. In particular, we analyzed a model with colored particles (quarks and gluon) propagating in a subspace of the bulk in which non-colored particles (leptons, Higgs and electroweak gauge bosons) propagate. In such a model, the constraints on the size of the extra dimensions from tree-level processes and the $T$ parameter (which have been analyzed previously) can be evaded. Thus, in this model the contribution to the MMM from the KK states of electroweak gauge bosons can
be (potentially) comparable to the SM electroweak correction (unlike in other types of models). However, with only one Higgs doublet, once the constraint from contribution of KK states to the process $b \rightarrow s\gamma$ (which has not been considered before) is imposed, we have shown that such a large contribution to the MMM is not possible. The $b \rightarrow s\gamma$ constraint can be relaxed in models with more than one Higgs doublet and then a large contribution to the MMM becomes possible.

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