Direction-dependent Corrections in Polarimetric Radio Imaging. I. Characterizing the Effects of the Primary Beam on Full-Stokes Imaging

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Abstract
Next generation radio telescope arrays are being designed and commissioned to accurately measure polarized intensity and rotation measures (RMs) across the entire sky through deep, wide-field radio interferometric surveys. Radio interferometer dish antenna arrays are affected by direction-dependent (DD) gains due to both instrumental and atmospheric effects. In this paper, we demonstrate the effect of DD errors of the parabolic dish antenna array on the measured polarized intensities of radio sources in interferometric images. We characterize the extent of polarimetric image degradation due to the DD gains through wide-band VLA simulations of representative point-source simulations of the radio sky at L band (1–2 GHz). We show that at the 0.5 gain level of the primary beam there is significant flux leakage from Stokes I to Q, U amounting to 10% of the total intensity. We further demonstrate that while the instrumental response averages down for observations over large parallactic angle intervals, full-polarization DD correction is required to remove the effects of DD leakage. We also explore the effect of the DD beam on the RM signals and show that while the instrumental effect is primarily centered around 0 rad-m⁻², the effect is significant over a broad range of RM requiring full polarization DD correction to accurately reconstruct the RM synthesis signal.

Key words: methods: analytical – methods: data analysis – methods: miscellaneous – polarization – techniques: interferometric – techniques: polarimetric

1. Introduction

The next generation of radio polarimetric surveys is part of a new era of wide-band, wide-area, full-Stokes continuum surveys on parabolic antenna interferometer arrays in the US and on the Square Kilometre Array precursor telescopes in Australia and South Africa.

Such surveys will collect data over very large instantaneous bandwidths in the GHz frequency regime and will require high-fidelity and high dynamic range imaging in all polarization states of the incoming radiation over the full field of view of the array antennas.

The response of radio antennas varies with both direction and frequency. For altitude-azimuth mounted antennas, the sky brightness distribution rotates with respect to the antenna primary beam (PB) as a function of the antenna parallactic angle. Consequently, for long integration observations during which the parallactic angle changes, the response of the array to a radio source includes an instrumental component that varies with time, frequency, and polarization. These variations corrupt the wide-field image by introducing image artifacts that are not removed by standard image deconvolution approaches, thereby limiting dynamic range and fidelity. The latter effect has been shown to be particularly important for the polarization response (Jagannathan et al. 2015).

The wide-band A-Projection (WB A-Projection or WB-AWP) algorithm offers a solution to account for direction-dependent (DD) effects across a large observing bandwidth and has been demonstrated to improve both dynamic range and image fidelity in wide-band total intensity images (Bhatnagar et al. 2013). In this paper, we examine the effects of wide-band DD errors on full-Stokes imaging performance based on ray-trace models of the JVLA L-band full-Stokes PB response. In a subsequent paper, we will assess the efficacy of the full-Stokes wide-band A-Projection algorithm in correcting for the DD effects. Using simulations of the sky brightness distribution, we demonstrate the limits of imaging fidelity that can be achieved with classical calibration and imaging, and the levels at which the full-Stokes A-Projection algorithm becomes necessary to enable high-fidelity and -dynamic full-Stokes imaging of wide-band observations.

1.1. Theory

The measurement equation (ME) for a single interferometer, calibrated for direction-independent (DI) terms, is given by

\[ V_{ij}^{\text{obs}}(\nu, t) = W_{ij}(\nu, t) \int M_{ij}(s, \nu, t) I(s, \nu) e^{ib_0 \cdot s} ds \]

(1)

where \( V_{ij}^{\text{obs}} \) is the visibility measured by a pair of antennas \( i \) and \( j \), with a projected separation of \( b_0 \). \( W_{ij} \) are the effective weights, and \( I(s, \nu) \) is the sky brightness distribution and is a function of direction \( s \) and frequency \( \nu \). \( I(s, \nu) \) is a full-Stokes vector of images of the sky and \( V_{ij}^{\text{obs}} \) is the vector of observed visibilities given by

\[ V_{ij}^{\text{obs}} = \begin{bmatrix} V_{ij}^{pp} \\ V_{ij}^{pq} \\ V_{ij}^{qp} \\ V_{ij}^{qq} \end{bmatrix} \quad \text{and} \quad I = \begin{bmatrix} I^{pp} \\ I^{pq} \\ I^{qp} \\ I^{qq} \end{bmatrix} \]

(2)

The super-scripts \( p \) and \( q \) represent the circular (\( R, L \)) or linear (\( X, Y \)) polarization states. \( M_{ij} \), referred to as the

4 All terms that can be/are assumed to be constant across the field of view.
Direction-dependent (DD) Mueller Matrix, encodes the mixing of the various elements of $I(s, \nu)$ in the measurement process. $M_{ij}$ can be written as an outer product of two antenna-based Jones matrices as $M_{ij} = E_i \otimes E_j^\dagger$, where $E_i$ describe the far-field electric field pattern for the antenna $i$. Figure 1 shows the amplitude of a complex-valued E-Jones matrix for a VLA antenna (in the circular polarization basis) at L band, computed using ray-tracing (for details, see Jagannathan et al. 2017). Similar to the DI G-Jones matrix, the diagonal elements of the E-Jones matrix represent the DD complex voltage gain patterns, and the off-diagonal elements represent the DD leakage patterns in the far field.

Equation (1) can be re-cast as

$$V_{ij}^{\text{Obs}}(\nu, t) = W_{ij}(\nu, t)F[(E_i(s, \nu, t) \otimes E_j^\dagger(s, \nu, t))I(s, \nu)],$$

(3)

where $F$ is the Fourier transform operator. $E_i$ and $E_j$ both vary as functions of frequency, time (for El-Az mount antennas), and direction in the sky. The effects of $E_i$ and the resulting mixing of polarization values encoded in $M_{ij}$ are characterized in the sections below. These effects due to $M_{ij}$ need to be calibrated and removed from $V_{ij}^{\text{Obs}}$ during imaging for a precise measurement of the true sky brightness distribution. That this is optimally done as part of the imaging process is discussed in Bhatnagar et al. (2008).

For the purpose of describing the calibration of the observed data for instrumental and atmospheric/iospheric effects, it is more convenient to write the right-hand side of Equation (1) as

$$V_{ij}^{\text{Obs}}(\nu, t) = W_{ij}(\nu, t)[A_{ij} * V_{ij}],$$

(4)

where $V_{ij} = F[I]$, and

$$A_{ij} = A_i \otimes A_j = F[E_i \otimes E_j^\dagger] = F[M_{ij}].$$

(5)

$A_i$ and $A_j$ are the time- and frequency-dependent aperture illumination functions of the two antennas. The diagonal terms of $A_i$ represent the complex antenna gains for the two polarizations, while the off-diagonal terms represent the polarization leakage across the antenna aperture. $\otimes$ is the outer convolution operator described in Bhatnagar et al. (2013). For identical antennas, the diagonal terms of $A_{ij}$ are the Fourier transforms of the true antenna far-field power pattern of the four correlation products (PBs).

It is worthwhile to note here that $A_i$ is the DD equivalent of the DI G-Jones matrix in the Hamaker et al. (1996) formulation and is the Fourier transform of the antenna far-field voltage pattern. Also note that $W_{ij}$ in the equation above is a scalar and has no impact on the Full-polarization Wide-band A-Projection imaging algorithm outlined below. Therefore, for brevity, we will drop it from the equations, but note that as with other imaging algorithms, the effects of various weighting schemes that modify $W_{ij}$ are admissible.

Full-Polarization Wide-band A-Projection belongs to a general class of radio interferometric DD imaging algorithms that correct for the terms inside the integral in Equation (1). Briefly, this is done by applying the inverse of the terms during convolutional gridding of the imaging process (transforming visibility data to the image domain). These algorithms, however, require a model for the antenna PB that includes all the dominant effects that need to be calibrated. Given a model for the baseline aperture illumination, $A_{ij}^M = FM_{ij}$, the A-Projection algorithm computes the image as $F[A_{ij}^M * V_{ij}^{\text{Obs}}]$ and the resulting images are normalized by an appropriate function of $F[W_{ij}(A_{ij}^M * A_{ij})]$ (see Bhatnagar et al. 2013 for details).

Figure 2 shows the $M_{ij}$ matrix in circular polarization basis for an EVLA antennas in L band computed using the ray-tracing code (for details, see Jagannathan et al. 2017). The diagonal elements of this matrix are the antenna gain pattern for the sky signal in all polarization products $(RR, RL, LR, and LL)$, while the off-diagonal elements encode direction-dependent
where, $L$ is the geographical latitude of the antenna. The WB-AWP algorithm corrects for this parallactic angle-, direction-, and frequency dependence of $A$ for Stokes-$I$ and -$V$ imaging.

In the sections below, we characterize the time and frequency-dependent errors introduced into Stokes $Q$ and $U$ for circularly polarized feeds from an analysis including the effects of the off-diagonal terms of the Mueller matrix. We perform a suite of simulations that characterize the effects of a polarized and unpolarized off-axis source. We show that corrections for the time- and frequency-dependent PB in all polarization including the effects of polarization leakages will be necessary (e.g., using a full-Stokes version of the WB-AWP algorithm) for accurate reconstruction of the true full-Stokes flux density vector in wide-field, wide-band imaging. In a subsequent paper, we will layout the framework and workings of the full-Stokes WB-AWP algorithm that corrects for the entire Mueller matrix during image reconstruction.

Linear polarized intensity can be expressed as

$$ P = \Pi_L I e^{i\psi}, $$

where $\Pi_L$ is the fractional linear polarization and $\psi$ the electric vector polarization angle (EVPA). $\Pi_L$ and $\psi$ are defined in terms of the observed Stokes parameters as

$$ \Pi_L = \sqrt{Q^2 + U^2} / I, $$
$$ \psi = \frac{1}{2} \tan^{-1} \left( \frac{U}{Q} \right). $$

The ME in the image plane can be cast in Stokes basis as

$$ I^M = \sum_k M^k \cdot I $$

where $k$ is an index over time, frequency, and baseline. The vector $I$ is the true incident full-Stokes sky brightness distribution. $I^M$ is the measured apparent sky brightness distribution. $M^k$ encodes the response of the interferometer to the incident polarization vector. Expanding $M^k$ in Equation (11) shows explicitly the dependence on the incident polarization.

$$ I^M = \sum_k \begin{pmatrix} M_{11}^k I + M_{12}^k Q + M_{13}^k U + M_{14}^k V \\ M_{21}^k I + M_{22}^k Q + M_{23}^k U + M_{24}^k V \\ M_{31}^k I + M_{32}^k Q + M_{33}^k U + M_{34}^k V \\ M_{41}^k I + M_{42}^k Q + M_{43}^k U + M_{44}^k V \end{pmatrix}. $$

The diagonal of the Mueller matrix encodes the response of the interferometer pair to each individual Stokes parameter. The off-diagonal elements encode the leakage between Stokes parameters. $M_{1Q}$ and $M_{1U}$ encode the amount of flux leaking from $I$ into $Q$ and $U$ respectively. The magnitude of leakage is $\sim 5 \times 10^{-7}$ of $I$ for the VLA at L band at the half-power point (Figure 3), and grows with increasing distance from the beam center. Typical linear polarized intensity of astrophysical sources is at the level of a few percent of Stokes-$I$ flux. Hence, the flux leakage results in a fractional error of up to 100% in the wide-band full-Stokes measurements of typical astrophysical signals. It is noteworthy that the Mueller matrix element $M_{3Q}^k$ representing the mutual coupling between Stokes-$Q$ and $I$, has exactly the same magnitude as $M_{1Q}^k$, within

2. Polarization Effect of the Antenna PB Off-axis

The continuous visibility full-polarization vector-field $V_{ij} = F I$ is sampled by the interferometer via $A_{ij}$ as in Equation (4) to give $V_{ij}^{\text{obs}}$. $A_{ij}$ terms introduce the effects of the antenna far-field pattern in the observed data, which if ignored, limits the imaging performance of the instrument (Rau et al. 2016). The $A_{ij}$ varies with frequency, time (due to relative rotation of the sky for Alt-Az mount antennas, or as time dependent antenna pointing errors, structural deformations, etc.), and polarization. The dominant variation of $A_{ij}$ with time for an Alt-Az mounted antenna is given by the change in parallactic angle $\chi$, defined as the angle between the source’s hour circle and the great circle passing through the source and the zenith as viewed by the antenna. For a source at decl. $\delta$, the parallactic angle changes with hour angle $h$, as

$$ \chi = \tan^{-1} \left( \frac{\cos L \cdot \sin h}{\sin L \cdot \cos \delta - \cos L \cdot \sin \delta \cdot \cos h} \right). $$
calibration errors. However, the amount of flux leakage is modulated by the intensity of the Stokes parameters. The amount of flux leaking from Stokes $Q$ into $I$ is $M_{Q}^{I} \cdot Q$. For a typical linear polarization of a few percent, the magnitude of flux leaking from Stokes $Q$ into $I$ is $10^{-10}$, which will be of concern only for very high dynamic range imaging. However, in cases of highly polarized emission, such as those that may be seen in extended radio jets of some radio stars, leakage from $Q$ to $I$ of the order of $10^{-3}$ may occur in uncorrected images.

We can isolate the effects of leakage by recasting Equation (11) into a sum of the diagonal and off-diagonal elements:

$$I = \sum_{i,j} M_{i}^{j} \cdot I_{j} + \sum_{j \neq i} M_{i}^{j} \cdot I_{j}.$$  (13)

The first term is the direction-dependent PB responses in each Stokes. The second term isolates the direction-dependent leakage between Stokes flux leakage, which for linear polarization is dominated by the direction-dependent Stokes $I$ leakage. Note that, for the time-variable component, all terms can be ignored and the second term can be represented as a constant only for very short “snapshot” observations with Alt-Az antennas. Observations with equatorial mount antennas or antennas with a third axis of motion to maintain a fixed parallactic angle (McConnell et al. 2016) allow for a simple correction in the form of a direction-dependent flux subtraction post imaging. This technique was used to good effect for the Canadian Galactic Plane Survey (Taylor et al. 2003), using the equatorial mount antennas of the Dominion Radio Astronomy Observatory synthesis radio telescope. For observations covering a large parallactic range with Alt-Az antennas, these effects must be corrected as part of the deconvolution and imaging stage.

3. Simulations

To examine the characteristics of the errors in polarized signals and their effect on astrophysical measurements, we carried out a suite of simulations. In each case, we consider a simple point source located at an off-axis point in the PB. This simple model allows for an intuitive understanding and interpretation of the resulting PB effects. Three cases were considered. In the first case, we consider the effect on an unpolarized point source. We then consider two polarized sources, one with zero Faraday Rotation Measure (RM) and the second with an RM of 100 rad m$^{-2}$. These cases allow us to explore the effects of the frequency dependence of the Mueller terms on FRM synthesis. The simulations in all these cases utilized the ray-traced PBs using the implementation in the CASA R&D code base.

We simulated observations over an hour angle range of $-4$ hr to $+4$ hr and spanning 1 GHz bandwidth in two frequency resolutions, the first with eight 128 MHz wide spectral windows across the VLA L band (1–2 GHz) to understand continuum polarization imaging fidelity. The second with 64 spectral windows with a bandwidth of 16 MHz each to understand the effects of antenna PB on FRM synthesis. The sources were at decl. $+70^\circ$. An image domain sky model was created following Equation (11). The sky model was then used to create visibilities given by Equation (1) for the VLA in the C-Configuration. The multiplication of the Mueller Matrix with the sky models was carried out in a standalone python routine and the prediction from the image plane to model VLA visibilities was carried out within the CASA framework. A block diagram of the simulation framework is provided in Figure 4.

3.1. An Off-axis Point Source As Viewed by a Single Interferometric Baseline

We examine the effect of the PB off-axis through two simulations. The first is of an unpolarized point source of 1 Jy flux density located at the half power of the PB at the reference
frequency of 1.5 GHz. The second simulation is of a polarized point source at the same location with 1 Jy flux density in total intensity and a frequency-independent fractional linear polarization ($\Pi_L$) of 5% with a polarization position angle (EVPA) of 22°.

Figure 5 shows the RR and LL amplitudes for the unpolarized source at the half-power point at 1.5 GHz as a function of parallactic angle for a single baseline. The traces show data at six frequencies in steps of 0.2 GHz from 1 GHz in blue to 2 GHz in red. The differences in amplitude with frequency are due to the changing width of the PB across the large fractional bandwidth. The source at 0.5 PB gain at 1.5 GHz is at 0.8 PB gain at 1.0 GHz and 0.2 PB gain at 2.0 GHz. This introduces a steep false spectral index that is corrected for in WBA Projection. The similar but oppositely curved shapes of the RR and LL signals at each frequency arises from beam squint caused by the RR and LL beams having different pointing centers in the sky from the off-axis feed geometry of the VLA antennas. The effect of the squint is maximum at the half power of the PB and reduces in magnitude as the source moves away from the half-power point in either direction.

Figure 6 shows the amplitude of the cross-hand (RL, LR) visibilities for a baseline for the unpolarized source. The polarization signal arises from the leakage of flux from the total intensity of the parallel hands RR and LL into the cross-hands, RL and LR. The leakage signal is a strong function of both parallactic angle and frequency. Figure 7 shows the same quantities for the 5% polarized source placed at the same beam position. These traces show the combined effect of Stokes I leakage and the other Mueller matrix terms that are dependent on $Q$ and $U$. The combined effects create a strong modulation of the polarized signal from less than 1% to over 7%.

3.2. Polarization Fidelity and Observing Interval

The reconstructed flux density full-Stokes vector $I^M$ during imaging results from a summation over time as given in Equation (13). Over the course of an observation with total interval $\Delta t$, the parallactic angle will swing through a corresponding interval in $\Delta \chi$. To show the effects of the duration of an observation on the error incurred in polarimetric flux density, we extended the simulations over 24 hr and examine net polarization (Equation 13) as a function of the parallactic angle range $\Delta \chi$. In Figures 8–10, we show the observed fractional $Q$ and $U$ intensities ($Q^M/I^M$, $U^M/I^M$) and the total fractional polarization $\Pi_L^M$ as a function of the parallactic angle interval $\Delta \chi$, for both the unpolarized and polarized sources at the half-power point of the PB at 1.5 GHz. In the plots, the families of solid lines represent the response to the unpolarized source, while the dotted lines represent the polarized source with true $\Pi_L = 0.05$. The colors represent frequencies as in the previous figures. For very small $\Delta \chi$, the error in fractional polarization can be very large. At the high frequency end of the band the fraction leakage is $>0.25$ for $\Delta \chi < 50^\circ$, and remains high until $\Delta \chi$ approaches $180^\circ$. At $\Delta \chi = 180^\circ$, the net instrumental polarization sums to zero at all frequencies, and the true fractional polarization in all Stokes
is retrieved. However, for typical observing intervals from a few minutes to a few hours, where the parallactic angle range is much less than $180^\circ$, instrumental polarization will be significantly larger than the true polarized signals over most of the PB.

### 3.3. Effect on RM Synthesis

At frequencies of a few GHz and below, the incident linear EVPA of a polarized source may be a complicated function of frequency due to Faraday effects either intrinsic to the source or due to propagation effects through the intervening Galactic or extragalactic media. In the simplest scenario of an intervening Faraday screen, the rotation of the EVPA is proportional to the square of the wavelength. Burn (1966) introduced the Faraday Dispersion Function (FDF), a measure of the polarization, as a function of Faraday depth $\phi$ to measure the distribution of Faraday components effecting the polarization vector. This technique and associated analysis is becoming widely used as a powerful means to study the astrophysics of the Faraday effects in radio sources and intervening space. Brentjens & de Bruyn (2005) introduced a window function and demonstrated that for a band-limited interferometric measurement, the FDF can be approximated by

$$\tilde{F}(\phi) \approx K \sum_{i} W_i P_i(\lambda^2) e^{-2i(\lambda_0^2 - \lambda^2)}$$

where $i$ is the channel of observation within a finite bandwidth, $W_i$ is the weight per channel, $K$ is the inverse of the summed weights, and $\lambda_0$ is the mean observing wavelength. Equation (14) is a Fourier transform of the weighted complex polarized intensity. The observed FDF is a convolution of the true FDF with the Rotation Measure Spread Function (RMSF) or the Faraday depth point-spread function given by

$$R(\phi) \approx K \sum_{i} W_i e^{-2i(\lambda_0^2 - \lambda_i^2)}.$$  

The FWHM of the main lobe of the RMSF is a metric of the “resolution” in Faraday space and is approximately given by

$$\Delta \phi \approx \frac{2\sqrt{3}}{\Delta \lambda} \text{ rad m}^{-2},$$  

with the wavelength $\lambda$ in meters. For a band spanning 1–2 GHz, $\Delta \phi \approx 50 \text{ rad m}^{-2}$. The shape and range of the FDF of the instrumental leakage will be determined by $\Delta \phi$ and by the complexity of the wavelength dependence of the time-averaged $M_{Q}$ and $M_{U}$ relative to $M_{R}$. A smooth variation with frequency will produce an instrumental signal largely confined to low values of RM.

FDFs of the 1–2 GHz off-axis response for the point source in our three simulations are plotted in Figures 11–13. For these plots, the colors now indicate increasing observing intervals of the parallactic angle $\Delta \chi$, ranging from $\Delta \chi = 36^\circ$ (blue) to $\Delta \chi = 180^\circ$ (red).

The FDF of the net Stokes $I$ leakage into $Q$ and $U$ from the unpolarized source at the half-power point of the beam is shown in Figure 11. The spectrum shows a central peak around $RM = 0 \text{ rad m}^{-2}$. The “side-lobes” are largely from the RMTF determined by the overall bandwidth and the channelization. The amplitude of the central peak decreases with increasing $\Delta \chi$ as the net instrument response reduces. The signal vanishes entirely at $\Delta \chi = 180^\circ$, as expected. Figure 12 shows the FDF of $P^M$ (the measured polarized intensity) of the source with $\Pi_L = 0.05$ and $RM = 0 \text{ rad m}^{-2}$. At small $\Delta \chi$, the true
polarized signal is almost entirely removed. The trace at \( \Delta \chi = 36^\circ \) shows a very weak maximum shift slightly to negative RM. As \( \Delta \chi \) get larger, \( P^M \) increases and approaches a better approximation of the true polarization, reaching \( P^M = P \) at \( \Delta \chi = 180^\circ \). This result demonstrates that, for sources with low RM, the instrumental polarization for observations with parallactic angle intervals less than \(-100^\circ\) will significantly corrupts the true FDF signal.

Figure 13 shows FDF for the case of a polarized source with RM of 100 rad m\(^{-2}\). The signal from the source is present at close to the correct amplitude and at the correct RM for all parallactic angle intervals. For shorter intervals \( (\Delta \chi < 100^\circ) \), there are strong spurious signals at RM = 0 of similar amplitude to the source signal, and both the peak polarization and the RM of the peak of the signals from the polarized source differ from the true value by about 5\%. As in the other cases, as \( \Delta \chi \) approaches 180\(^\circ\), the instrumental effect vanishes and the signal from the source approaches its true value.

Faraday rotation synthesis thus offers the possibility to distinguish instrumental effects from true polarization even in data uncorrected for off-axis effects for sources with large RM that are well separated in Faraday depth from the instrumental signal. For the VLA at L band, RM significantly larger than about 50 rad m\(^{-2}\) is required. While RM values of this magnitude are not uncommon in the plane of the Milky Way Galaxy due to the dense Galactic magneto-ionic medium (Brown et al. 2003), at higher galactic latitudes, such large RM values are the exception rather than the rule (Taylor et al. 2009). Moreover, broadband polarimetry reveals complex FDFs with signals over a range of RM in a significant fraction of sources, arising from internal Faraday effects (O’Sullivan et al. 2012). Upcoming wide-area surveys will require high precision polarimetry and Faraday Rotation synthesis over the full range of RM. Wide-band, off-axis polarization corrections therefore will be essential.

3.4. **Effects of PB Squash**

The effects of the antenna PB on polarimetric imaging described in the sections above is based on ray-traced antenna models that account only for the leading order terms in phase (e.g., *squint*). Measured PB from holography (Perley 2016) of the VLA antennas shows that the PB response is antenna dependent with different antennas displaying beam *squash* in addition to well known beam *squint* of the VLA. While the PB *squint* is a linear term in phase, which represents a displacement of the beam centers of \( R \) and \( L \) beams, PB *squash* is a quadratic term in phase (Heiles et al. 2001). *Squash* is symmetric and is the superposition of *defocus* and *coma*, along with the addition of polarized emission from the sub-reflector feed legs. All of these effects alter the measured linear polarization response of the antenna.

The most noticeable effect is the redistribution of power from the main lobe to the side-lobes and changes to the peaks of the two opposing polarization lobes in Stokes \( Q \) and \( U \). If the quadrupolar symmetry is altered, the flux leakage from Stokes \( I \) leaking into the linear polarizations would not reduce to zero even when \( \Delta \chi = 180^\circ \). Figure 14, shows the leakage as a fraction of Stokes \( I \) for different intervals of parallactic angle, for an ideal beam and for a *squashed* beam. The ideal antenna
Stokes $Q$, $U$ and fractional polarization are plotted in solid lines in blue, green, and red, while the squashed beam Stokes $Q$, $U$ are plotted in dotted blue, green, and red respectively. The squashed beam shows higher levels of leakage for smaller parallactic angle intervals and does not average to zero $\Delta \chi = 180^\circ$. The residual flux at $\Delta \chi = 180^\circ$ is $\sim 0.5\%$ of Stokes $I$.

The squashed beam thus exacerbates the problem of leakage correction and removes the symmetry that averages out of the leakage over long integrations. The $A$-Projection algorithm, which can naturally account for the squashed term with an appropriate aperture model offers the ideal solution for achieving noise-limited high-fidelity imaging in polarization over the full PB.

4. Conclusion

The next generation of radio polarimetric surveys utilizing parabolic dishes, spanning wide bandwidths and wide-area in all Stokes parameters cannot achieve high fidelity in polarimetric imaging using conventional imaging algorithms. The effects of direction-dependent gain and polarization leakage in parabolic dishes with off-axis feed location is shown to create spurious polarized signals and significantly corrupt the polarization response at levels similar to or exceeding the true sky polarization. The off-axis effects from Stokes $I$ leakage can be ameliorated to some degree by observations over long averaging time intervals that provide a range in parallactic angle $\Delta \chi > 100^\circ$. The net effect of the off-diagonal terms of the Mueller matrix averages to zero for $\Delta \chi = 180^\circ$. However, note that due to the time variability of the PSF, time averaging will lead to deconvolution errors during image reconstruction. Observations that do not span such long time intervals or cover exactly symmetric hour angles about the meridian, the polarization leakage does not vanish. We also show that, for realistic beam models with measurable second-order errors (beam squashed), averaging of polarization leakage is not a viable solution.

The off-axis instrumental polarization response of the antenna in Faraday space is shown to be close to RM of $0$ rad m$^{-2}$. For a typical RM signature of astrophysical sources at GHz frequencies, the off-axis leakage response alters the Faraday dispersion spectrum, introducing spurious signals similar in scale to the sky polarization and corrupting the true sky RM signature. Our simulations show that achieving polarimetric imaging simultaneously over wide-fields and wide-bands for next generation deep and wide surveys will not be possible through conventional imaging methods that do not include DD corrections.

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Software: CASSBEAM, CASA, seaborn, matplotlib.

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Figure 14. Plotted is the leakage flux at the half power of PB at 1.5 GHz, as a fraction of Stokes $I$ for different parallactic angle intervals $\Delta \chi$. The dotted (squashed) and solid (ideal) lines in red represent leakage into polarized intensity $I_L$, the blue lines represent the Stokes $U$ and the green lines represent the Stokes $Q$.