Research of the Mathematical Model of Heated Greenhouses

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Abstract. Large-scale greenhouse production is currently developing along the path of introducing technologies for intensive cultivation of vegetable crops and using automated control systems for technological processes based on micro and minicomputers. One of important technological processes is watering and feeding plants with mineral fertilizers. The need to automate this process is associated with laboriousness of process of preparing solutions, accurately maintaining given concentration of substances in them, timely supply and uniform dosing over the entire area of greenhouse under various disturbances of external environment. The article shows a technological scheme of a heated greenhouse of the simplest design. A method for constructing balance models is presented; model can be transformed for more complex schemes. Three main models in space state (continuous, discrete, operator) are considered. Information about processes is recorded discretely, therefore, we first identify discrete form of the model, then we obtain parameters of the discrete form. Further, using equation of constraints, a matrix of discrete and continuous forms, we obtain transfer functions for the internal air temperature. Due to simplicity of setting parameters, it can be easily reproduced for many different designs and systems of greenhouse complexes, as well as for various scenarios of changing environmental conditions. Proposed balanced dynamic models allow the most complete use of computers for studying heat consumption modes of heated greenhouses, as well as for synthesis of temperature controllers. In this case, vector-matrix algebra is used as a mathematical apparatus, standard programs for which are widely used on computers for various purposes.

1. Introduction
In conventional greenhouses, due to the large area of translucent surfaces, significant heat losses occur, compensation of which requires a certain fuel consumption in the heating system. Greenhouses can be heated with hot water, steam, heated air, infrared radiation or combustion products. When creating a solar greenhouse, first of all, we need to take care of a significant reduction in heat loss through the use of thermal insulation. In addition, it is necessary to ensure the capture of the maximum possible amount of solar energy and the accumulation of excess heat [1,2,3]. A significant difference between greenhouses and other types of protected ground structures is the ability to create favourable conditions not only for grown plants, but also for service personnel and technological equipment. As a result, labour productivity and production culture in greenhouses increase, and the seasonal nature of agricultural work...
disappears [4,5]. In a greenhouse, in contrast to small-sized shelters and greenhouses, all agrotechnical measures can be carried out without violating the integrity of the fencing, as well as various mechanisms for caring for plants can be widely used. In heated greenhouses, they often strive to increase their energy efficiency, both by means of automation and by design changes. In this case, the greenhouse is considered as an object with lumped parameters of the thermal state, averaged over the volume and area of the fencing.

2. Materials and methods
Approximately half of the southern sky should be visible from the greenhouse, or sunlight entering the greenhouse at an angle of 45° should reach the lower edge of its rear wall. The end walls should be fully or partially transparent if the greenhouse is oriented to the south or When constructing models that reflect the essential energy features of the greenhouse, the most effective functional-identification approach [6,7]. Table 1 shows the indicators that affect the microclimate of the greenhouse.

| No. | Criterion description |
|-----|-----------------------|
| 1   | the indoor temperature will usually be higher than the outdoor temperature (even too high in case of sun exposure) |
| 2   | the soil temperature rises sufficiently, often even to too high values, at which the germination of seeds of some plants stops. The soil in the greenhouse does not freeze |
| 3   | the amount of lighting entering the greenhouse is almost half that of the outdoors one |
| 4   | the effect of wind is almost completely eliminated, which significantly increases comfort for humans and only partially for plants |
| 5   | different smells may appear |
| 6   | air exchange decreases, plants may lack carbon dioxide (CO2) |
| 7   | in the greenhouse, the humidity is higher than is necessary for the plants, since with prolonged humidity, mold formation and fungal growth are observed |
| 8   | the penetration of pests into the greenhouse is also difficult, but if they do get into the room, they begin to multiply in favourable conditions of the greenhouse, which makes a very depressing impression on some people |
| 9   | it is difficult for beneficial insects to access plants, since they can enter the greenhouse only through doors, ventilation hatches or vents |
| 10  | the greenhouse protects from natural rains, and the receipt of moisture by plants depends entirely on the person who cares for them |
| 11  | in an unheated greenhouse, excess humidity, as a rule, does not create additional difficulties for people |
| 12  | with insolation, as a rule, favorable conditions are created in the greenhouse for people to stay |
| 13  | due to the presence of plants, the air in the greenhouse contains more oxygen than in the apartment |

Figure 1 shows a flow diagram of a heated greenhouse of the simplest design. The model obtained for such a scheme based on the proposed method can be transformed for more complex schemes. [8,9].
Three main forms of the state-space model:

continuous

\[ X = AX + BU(t) + CF(t), \quad X(t_0) = X_0; \]  

discrete

\[ X[k + 1] = \Phi X[k] + BU[k] + CF[k], \quad X[0] = X_0; \]  

operator

\[ X(p) = W_U(p)U(p) + W_F(p)F(p), \]  

where \( A = [n \times n] \) – dimensional dynamic matrix; \( n \) – dynamic order of the model; \( X = [n \times 1] \) – dimensional state vector; \( B = [n \times m] \) – dimensional matrix of controls; \( m \) – number of independent controls; \( U = [m \times 1] \) – control vector; \( t \) – time; \( C = [n \times r] \) – dimensional matrix of perturbations; \( r \) – number of independent disturbances; \( F = [r \times 1] \) – dimensional vector of disturbances; \( k \) – step number; \( \Phi = [n \times n] \) – dimensional matrix of transition to one step; \( W_U(p) = \Phi(p)B \), \( W_F(p) = \Phi(p)C \) – transfer matrices for control and disturbance, defined through the operator image of the transition matrix.

3. Results and discussion

For the given greenhouse scheme, the return heat carrier temperature is \( x_2 = \theta_2 \); average temperature of heating devices is \( x_2 = \theta_{on} \); average air temperature in the greenhouse is \( x_3 = \theta_u \); average temperature of the fencing is \( x_4 = \theta_{ok} \); coolant temperature at the heat exchanger outlet is \( u = \theta_1 \); temperature of outdoor \( f = \theta_n \). Obviously, for the considered model \( n = 4 \), \( m = 1 \), \( r = 1 \).

As a rule, information about the processes is recorded discretely, therefore, at first it is convenient to identify the discrete form of model (2), which we will reduce to the “input-output” form:

\[ X[k + 1] = \{\Phi, B, C\} \begin{cases} X[k] \\ U[k] \\ f[k] \end{cases}, \]  

or

\[ \omega[k] = PV[k], \]  

where \( \omega[k] = X[k + 1] \) – output vector; \( P = \{\Phi, B, C\} \) – matrix of unknown parameters; \( V^T[k] = \{X^T[k], U[k], f[k]\} \) – vector of input signals; index “\( T \)” - transposition.

As an algorithm, we use the stochastic approximation [1]

\[ \hat{P}_j^T[k] = \hat{P}_j^T[k - 1] + g_j[k](\omega_j[k] - \hat{P}_j^T[k - 1] \times V_j[k]V_j[k]); \]  

\[ g_j[k] = \gamma/[k(V_j^T[k - 1]V_j[k - 1])], \quad j = 1, 4, \]  

where \( \gamma \) – algorithm gain coefficient; \( j \) – numbers of rows of matrix \( P \).

As a result of identification, the parameters of the discrete form of the model were obtained:
Using the equation of connection of matrices of discrete and continuous forms of state space models (1), (2)

\[
\Phi = e^{A\tau}, \quad B = A^{-1}(e^{A\tau} - I)B, \quad C = A^{-1}(e^{A\tau} - I)C,
\]

where \(\tau\) – sampling interval; \(I = [n \times n]\) – dimensional unit matrix, we will have parameters of model (1) for a time scale \(K_M = 1200\):

\[
A = \begin{pmatrix}
-3.007 & 0.315 & 0 & 0 \\
1.197 & -2.481 & 0.057 & 0.903 \\
0 & 0.0344 & -3.255 & -0.573 \\
0 & 0.019 & 0.709 & -3.119 \\
\end{pmatrix};
\]

\[
B = \begin{pmatrix}
0.4579 \\
0 \\
0 \\
\end{pmatrix}; \quad C = \begin{pmatrix}
0 \\
0.13 \\
0.0147
\end{pmatrix}.
\]

As can be seen from the transfer matrices, the construction of the operator form is reduced mainly to the construction of the matrix \(\Phi(p)\). It can be obtained through a dynamic matrix \(A\):

\[
\Phi(p) = (pI - A)^{-1},
\]

and the inverse matrix as follows:

\[
(pI - A)^{-1} = \frac{1}{\varphi(p)} G(p),
\]

where \(\varphi(p)\) – characteristic polynomial of the matrix\(A\); \(G(p)\) – operator adjoint matrix.

Using the expressions for the transfer matrices \(W_U(p)\) and the transition matrix \(W_f(p)\), we obtain the transfer functions for the temperature of the internal air, the components \(X_i\) of the state vector (for the convenience of subsequent calculations, the components \(\tilde{\vartheta}_w, \tilde{\vartheta}_{ok}\) were swapped).

\[
W_U^{x_i}(p) = \frac{1.218p + 1}{3.79p^4 + 5.79p^3 + 7.753p^2 + 4.574p + 1};
\]

\[
W_f^{x_i}(p) = \frac{2.535p^3 + 6.693p^2 + 5.782p + 1}{3.79p^4 + 5.79p^3 + 7.753p^2 + 4.574p + 1}.
\]

Thus, we examined the methodology for constructing balanced models of heated greenhouses. But before moving on to examples of their use, we will show the sequence in calculating the accuracy for these models [10].

The modeling results are comparable with the real state of the greenhouse. Based on the results of this comparison, one can calculate the accuracy matrix

\[
\Omega = [M([X[k] - X_u[k]](X[k] - X_u[k])^T)].
\]
Assuming that the vector of simulation errors obeys the Gaussian law, we write an expression for the joint density of the error probability distribution based on the parameters $\Phi$, $B$, $C$ of the model and the accuracy matrix $\Omega$:

$$
\pi(\varepsilon|\Phi, B, C, \Omega) = (2\pi)^{-\frac{N}{2}}|\Omega|^{\frac{1}{2}} \times
\exp\left\{-\frac{1}{2}(X[k + 1] - \Phi X[k] - BU[k] - CF[k])^T \times
\Omega (X[k + 1] - \Phi X[k] - BU[k] - CF[k])\right\}.
$$

(14)

Expressions (15) and (14) allow, given the admissible vector of modeling errors $\varepsilon_D^T = [|e_{D1}|, |e_{D2}|, |e_{D3}|, |e_{D4}|]$, to calculate the confidence coefficient for the obtained identification result

$$
\Pi(\varepsilon) = 2\Phi(\varepsilon^T \Omega \varepsilon),
$$

(15)

where $\Phi(\cdot)$ – Laplace function for which special tables are compiled [1].

The value of the confidence coefficient in our case is: $\Pi(\varepsilon) = 0.96$.

We will show several examples of the use of models of heated greenhouses for the synthesis of temperature controllers on a computer.

Let us synthesize a stabilizing temperature controller for model (1). The stabilization law in this case:

$$
u(t) = -L^T X(t),
$$

(16)

where $L$ – vector of controller gains, which must be determined based on the requirements for the transient process in the automatic temperature control system.

The required stabilization quality can be achieved by choosing the coefficients $d_1, ..., d_4$ of the characteristic polynomial $\varphi(p)$ of the matrix $A$:

$$
\varphi(p) = p^4 + d_1p^3 + d_2p^2 + d_3p + d_4.
$$

(17)

When choosing the parameters of the controller according to law (7), we must provide

$$
\det(A - BL) = p^4 + d_1p^3 + d_2p^2 + d_3p + d_4 = 0.
$$

(18)

The desired vector $L$ – solution to the modal control problem [4]:

$$
L = ((B, BA, BA^2BA^3)^{-1})^T \times \left(\begin{array}{cccc}
1 & 0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right)^{-1} (\bar{a} - \bar{d}),
$$

(19)

where $\bar{a}, \bar{d}$ – vectors composed of actual and specified values of the characteristic polynomial $\varphi(p)$.

Let us take $d_1 = 9.45$, $d_2 = 8.11$, $d_3 = 120.0$, $d_4 = 131.0$. Then, after substituting all the values of the parameters and the coefficients of the stabilizing controller into formula (20)

$$
L^T = [0.3; -0.985; -9.513; 3.12].
$$

The stabilization law described by formula (17) can be implemented on analog computers [11,12].

It is also possible to synthesize a digital stabilizing regulator if necessary. For this we use the model (2).

Let us introduce the matrix:

$$
S = \left(\begin{array}{c}
H \\
H\Phi \\
H\Phi^2 \\
H\Phi^3
\end{array}\right),
$$

(20)
where \( H = [0 \ 0 \ 0 \ 1] \) – output matrix for model (2). Using the matrix \( S \), we perform the canonical transformation of model (2):

\[
\tilde{X}[k+1] = \tilde{\Phi}X[k] + \tilde{B}U[k] + \tilde{C}F[k];
\]

\[
\tilde{X}(k) = SX[k]; \quad \tilde{\Phi} = S^{-1}\Phi S;
\]

\[
\tilde{B} = S^{-1}B; \quad \tilde{C} = S^{-1}C.
\] (22)

As a result of transformations of expressions (21), (22), we obtain the following matrix structure:

\[
\tilde{\Phi} = \begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-\tilde{\phi}_4 - \tilde{\phi}_3 - \tilde{\phi}_2 - \tilde{\phi}_1
\end{pmatrix};
\quad \tilde{B} = \begin{pmatrix}
\tilde{b}_1 \\
\tilde{b}_2 \\
\tilde{b}_3 \\
\tilde{b}_4
\end{pmatrix};
\quad C = \begin{pmatrix}
\tilde{c}_1 \\
\tilde{c}_2 \\
\tilde{c}_3 \\
\tilde{c}_4
\end{pmatrix}.
\]

It is easy to determine the coefficients of the stabilizing digital controller in the basis of the transformed system (22);

\[
k_{p,i} = d_{i}' - \tilde{\phi}_i,
\] (23)

where \( d_{i}' \) – the required values of the characteristic polynomial of the matrix \( \Phi \).

Taking \( d_1' = 0.08, \ d_2' = 0.12, \ d_3' = 0.18, \ d_4' = 0.24 \) will have for system (22), (23) \( k_{p,1} = 0.0796, \ k_{p,2} = 0.107, \ k_{p,3} = 0.238, \ k_{p,4} = -0.206 \).

The canonical form of model (21), (22) allows synthesizing a digital combined controller. For this, using the structure of the matrix \( \tilde{\Phi} \), we reduce the vector equation (21) to the scalar form:

\[
y[k] + \sum_{i=1}^{4} \phi_i y[k-i] = \sum_{i=1}^{4} (\beta_i u[k-i] + \gamma_i f[k-1]);
\]

\[
\beta = \Phi \times \tilde{B};
\]

\[
\gamma = \Phi \times \tilde{C};
\]

\[
\Phi * = \begin{pmatrix}
1 & 0 & 0 & 0 \\
\tilde{\phi}_1 & 1 & 1 & 1 \\
\tilde{\phi}_2 & \tilde{\phi}_3 & \tilde{\phi}_2 & \tilde{\phi}_1 & 1
\end{pmatrix},
\] (26)

where \( y[k] = \tilde{x}_4[k] - \tilde{x}_{ts} \) – regulation error, which is the system output; \( \tilde{x}_{ts} \) – task setting.

The equation of the combined digital regulator is found from the condition that the current control error is zero:

\[
u[k] = \beta_{-1} \left( \sum_{i=2}^{4} u[k-i] \beta_i - \sum_{i=1}^{5} (\phi_i y[k-i] - \gamma_i f[k-i]) \right).
\] (27)

As a result of calculations by formulas (21), (22), (24, 27), the following parameters of the digital combined controller were obtained:

\[
\beta^r = [-2.058; \ 1.724; \ 1.311; \ -0.093];
\]

\[
\gamma^r = [10.623; \ -12.113; \ -31.77; \ 5.674].
\]

Note that the parameters of digital controllers are synthesized in a transformed basis, i.e. for the state \( \tilde{X} \). Therefore, to pass to the true state vector, it is necessary to apply the inverse transformation \( X = S^{-1}\tilde{X} \).
4. Conclusion
Greenhouses differ from each other both in terms of the time period of operation (while they are divided into spring-summer and year-round), and in appearance. The design of the greenhouse depends not only on the possibilities and imagination, as one might think, but also on what kind of plants are going to be grown. Some greenhouses are optimally suited for specific plant varieties, others have a wider range of uses. The presented model of a greenhouse for controlling microclimate parameters, developed for use in computerized yield control systems for finding optimal operating modes. Due to the simplicity of setting the parameters, it can be easily reproduced for many different designs and systems of greenhouse complexes, as well as for various scenarios of changing environmental conditions. In this case, vector-matrix algebra is used as a mathematical apparatus, standard programs for which are widely used on computers for various purposes.

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