On $M/(G_1, G_2)/1/V(MV)$ queue with two types of general heterogeneous service with Bernoulli feedback

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Abstract: This paper deals with the steady-state behavior of an $M/G/1$ queue with two types of general heterogeneous service to the arriving customers and Bernoulli feedback. We first derive the steady-state probability generating functions for the queue size distributions at a random epoch as well as at a departure epoch. Next, we derive the mean queue size at random epoch and the mean waiting time. Also, we obtain the mean busy period of this model and discuss some important particular cases.

1. Introduction

Queueing situations in which the idle server may take vacations encounter in computer, communication and manufacturing systems, etc. Many researchers have studied the vacation queueing systems in different frameworks and elaborated many applications of such vacation models.

Miller (1964) was the first to study the vacation model, where the server is unavailable for some random length of time for the $M/G/1$ queueing system. After Levy and Yechiali (1975) included several types of generalizations of the classical $M/G/1$ queueing system, this type of model has also been reported by a number of authors. When the service of a customer is unsuccessful, it may be retried again and again until a successful service completed. Takács (1963) was the first to study

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PUBLIC INTEREST STATEMENT

As the congestion problem in teletraffic, the range of applications has grown to include not only telecommunications and computer science, but also manufacturing, air traffic control, military logistics, call centers, supermarkets, inventories, hospitals, and many other areas that involve service systems whose demands are random. The mathematical analysis of the model established the indices that presumably relate the physical and stochastic parameters to certain performance measures, such as average response and waiting time, server utilization, system throughput, probability of buffer overflow, distribution function of response and waiting time, busy period of server. The system contains sufficient detail so that its performance measures reflect the behavior of the real system.
such a model, where the customer who completed their service are feedback instantaneously to the
tail of the queue with probability $p$ ($0 \leq p \leq 1$) or departs from the system forever with probability
$q = (1 - p)$. This mechanism is known as Bernoulli feedback. These queueing models arise in the
stochastic modeling of many real life situations. For example, in data transmission, a packet trans-
mitted from the source to the destination may be returned and it may go on until the packet is finally
transmitted. Boxma and Yechiali (1997) studied such a model where the service time of the unit
taken for its first service different from the subsequent service times of the unsuccessful units with
gated vacations. Choi, Kim, and Choi (2003) investigated this model in more depth. Recently,
Choudhury and Paul (2005) investigates an $\text{M}/\text{G}/1$ queueing system where the server provides two
phases of general heterogeneous service and Bernoulli feedback.

2. The mathematical model

Customer arrives at the system in a compound Poisson process. There is a single server who provides
two kinds of general heterogeneous one by one service to customers on a first come first served
basis. Before his service starts, each customer has the option to choose the first service with proba-
bility $q_1$ or the second service with probability $p_1$ where $q_1 + p_1 = 1$. As soon as the system becomes
empty, the server takes a vacation for a random length of time called vacation time to do other jobs,
which is uninterruptible. After returning from that vacation, there are two possibilities, viz. (1) he
keeps on taking vacations till he finds at least one unit in the queue (multiple vacations) or (2) he
may take only one vacation between two successive busy periods (single vacation). After completion
of first phase of service (FPS) or second phase of service (SPS) if the customer (unit) is not satisfied
with his service for certain reason or if he received unsuccessful service, then he may immediately
join the tail of the original queue as a feedback customer for receiving another regular service with
probability $\theta$ ($0 \leq \theta \leq 1$), otherwise he may depart forever from the system with probability $\theta'$ ($=1 - \theta$).
Let $B_1, B_2$ denotes the service time of SPS and SPS, respectively (Figure 1). Assume that each of $B_1, B_2$
has a general distribution with distribution function $B_i(x)$, $i = 1, 2$ respectively and probability density
function $b_i(x)$, Laplace Stieltjes transform (LST) $B_i^*(s)$ and finite moments $\mu_i^k, k \geq 1$ for $i = 1, 2$ (denot-
ing FPS and SPS respectively). The model under the investigation is depicted in the Figure 1.

Also, let $V$ be the vacation time random variable, $v(x)$ be the probability density function and its
distribution function is $V(x)$ and LST is $V^*(s)$ so that the total service time required by a unit to com-
plete the service time period is given by,
\( B = \begin{cases} B_1 \; \text{with probability } q_1 \\ B_2 \; \text{with probability } p_1 \end{cases} \)

Therefore, the LST \( B^*(s) \) of \( B \) is given by,

\[
B^*(s) = q_1 B_1^*(s) + p_1 B_2^*(s)
\]

And the first two raw moments are,

\[
\beta_1 = q_1 \beta_1^{(1)} + p_1 \beta_2^{(1)} \]
\[
\beta_2 = q_1 \beta_1^{(2)} + p_1 \beta_2^{(2)}
\]

3. Steady-state equations

Assume that the system is in steady state condition and define that \( N_i(t) \) be the queue size at time \( t \), \( B_i^0(t) \) be the elapsed \( i \)th types of service at time \( t \) and \( V^0(t) \) be the elapsed vacation time at time \( t \), \( i = 1, 2 \). Let us introduce the following random variable,

\[
Y(t) = \begin{cases} 0 \; \text{if the system is idle at time “}t\text{”} \\ 1 \; \text{if the system is busy with FPS at time “}t\text{”} \\ 2 \; \text{if the system is busy with SPS at time “}t\text{”} 
\end{cases}
\]

Therefore, the supplementary variables \( B_i^0(t), B_i^0(t) \) and \( V^0(t) \) are introduced in order to obtain a bivariate Markov process \( \{ N_i(t), L(t) \} \), where

\[
L(t) = \begin{cases} 0 \; \text{if } Y(t) = 0, L(t) = V^0(t) \text{ if } Y(t) = 0, L(t) = B_i^0(t) \text{ if } Y(t) = 1, L(t) = B_i^0(t) \text{ if } Y(t) = 2 
\end{cases}
\]

And we define,

\[
U(t) = \text{Prob}[N_0(t) = 0, L(t) = 0]
\]
\[
V_i(x, t) = \text{Prob}[N_i(t) = n, L(t) = V^0(t); \; x < V^0(t) \leq x + dx]; \; x > 0, n \geq 0
\]
\[
P_{ni}(x, t) = \text{Prob}[N_i(t) = n, L(t) = B_i^0(t); \; x < B_i^0(t) \leq x + dx]; \; x > 0, n \geq 0, \; i = 1, 2
\]

Further, we assume that \( B_i(0) = 0, B_i(\infty) = 1, V(0) = 0, V(\infty) = 1, i = 1, 2 \) and \( B_i(x), V_i(x) \) are continuous at \( x = 0 \) so that \( \mu_i(x) = \frac{b_i(x)}{1 - B_i(x)}; \; i = 1, 2 \) and therefore we have,

\[
b_i(\Omega) = \mu_i(x) e^{-\int_0^x \mu_i(s) ds}, \; i = 1, 2
\]

Also \( \vartheta(x) = \frac{V_i(x)}{1 - V_i(x)} \) and therefore \( \vartheta(0) = \vartheta(x) - \int_0^x \vartheta(x) d(x) \).

Let us define the following PDF:

\[
P_i(x, z) = \sum_{n=0}^{\infty} P_{ni}(x) z^n, P_i(z) = \sum_{n=0}^{\infty} P_{ni} z^n, \; i = 1, 2, |z| \leq 1
\]
\[
V(x, z) = \sum_{n=0}^{\infty} V_n(x) z^n, V(z) = \sum_{n=0}^{\infty} V_n z^n, |z| \leq 1
\]
To analyze this model at stationary point of time, we can use the forward Kolmogorov equations, which under the steady-state conditions are:

\[
\frac{d}{dx} P_{n,1}(x) + (\lambda + \mu_n(x)) P_{n,1}(x) = \lambda P_{n-1,1}(x), \quad n \leq 0
\]  (1)

\[
\frac{d}{dx} P_{n,2}(x) + (\lambda + \mu_n(x)) P_{n,2}(x) = \lambda P_{n-1,2}(x), \quad n \leq 0
\]  (2)

\[
\frac{d}{dx} V_n(x) + (\lambda + \delta(x)) V_n(x) = \lambda V_{n-1}(x), \quad n \geq 0
\]  (3)

\[
\lambda U = \theta^2 \int_0^\infty (\delta(x) V_n(x)) dx + \theta^2 \int_0^\infty P_{1,0}(x) \mu_1(x) dx + \theta^2 \int_0^\infty P_{2,0}(x) \mu_2(x) dx
\]  (4)

These equations are to be solved under the following boundary conditions,

\[
P_{n,1}(0) = q_1 \int_0^\infty P_{n,1}(x) \mu_1(x) dx + q_1 \theta \int_0^\infty P_{n+1,1}(x) \mu_1(x) dx + q_1 \theta \int_0^\infty (\delta(x) V_n(x)) dx + q_1 \theta \int_0^\infty (\delta(x) V_{n+1}(x)) dx
\]  (5)

\[
P_{0,1}(0) = q_1 \int_0^\infty P_{1,0}(x) \mu_1(x) dx + q_1 \theta \int_0^\infty P_{1,1}(x) \mu_1(x) dx + q_1 \theta \int_0^\infty (\delta(x) V_0(x)) dx + q_1 \theta \int_0^\infty (\delta(x) V_1(x)) dx + q_1 \theta \int_0^\infty (\delta(x) V_{1,1}(x)) dx + \lambda U q_1
\]  (6)

\[
V_n(0) = \int_0^\infty P_{n,1}(x) \mu_1(x) dx + \int_0^\infty P_{n,2}(x) \mu_2(x) dx, \quad n \geq 0
\]  (7)

\[
P_{n,2}(0) = p_1 \int_0^\infty P_{n,1}(x) \mu_1(x) dx + p_1 \theta \int_0^\infty P_{n+1,1}(x) \mu_1(x) dx + p_1 \theta \int_0^\infty (\delta(x) V_n(x)) dx + p_1 \theta \int_0^\infty (\delta(x) V_{n+1}(x)) dx + p_1 \theta \int_0^\infty (\delta(x) V_{n,1}(x)) dx + p_1 \theta \int_0^\infty (\delta(x) V_{n+1,1}(x)) dx + \lambda U p_1
\]  (8)

\[
P_{0,2}(0) = p_1 \int_0^\infty P_{1,0}(x) \mu_1(x) dx + p_1 \theta \int_0^\infty P_{1,1}(x) \mu_1(x) dx + p_1 \theta \int_0^\infty (\delta(x) V_0(x)) dx + p_1 \theta \int_0^\infty (\delta(x) V_1(x)) dx + p_1 \theta \int_0^\infty (\delta(x) V_{1,2}(x)) dx + \lambda U p_1
\]  (9)

4. The steady queue size distribution

Proceeding in the usual manner with Equations (1)–(4) we obtain,

\[
P_1(x, z) = P_1(0, z) e^{-\left(\lambda - \lambda Z\right) x - \int_0^x \mu_1(t) dt}
\]  (10)

\[
V(x, z) = V(0, z) e^{-\left(\lambda - \lambda Z\right) x - \int_0^x \delta(t) dt}
\]  (11)
\[ P_2(x, z) = P_2(0, z)e^{-(1/\lambda)xt} \]  

Solving for \( P_1(0, z) \), \( P_2(0, z) \) and \( V(0, z) \) we have

\[ P_1(0, z) = \frac{\lambda U(z - 1)q_1}{(1 + V'(\lambda - \lambda z))(z\theta + \theta')(p_1B'_2[\lambda - \lambda z] + q_1B'_1[\lambda - \lambda z]) - z} \]  

\[ P_2(0, z) = \frac{\lambda U(z - 1)p_1}{(1 + V'(\lambda - \lambda z))(z\theta + \theta')(p_1B'_2[\lambda - \lambda z] + q_1B'_1[\lambda - \lambda z]) - z} \]  

\[ V(0, z) = \frac{\lambda U(z - 1)(p_1B'_2[\lambda - \lambda z] + q_1B'_1[\lambda - \lambda z])}{(1 + V'(\lambda - \lambda z))(z\theta + \theta')(p_1B'_2[\lambda - \lambda z] + q_1B'_1[\lambda - \lambda z]) - z} \]  

From Equations (11) and (15), we obtain,

\[ P_1(z) = \int_0^\infty P_1(x, z)dx = \frac{q_1(B'_1[\lambda - \lambda z] - 1)U}{(1 + V'(\lambda - \lambda z))(z\theta + \theta')(p_1B'_2[\lambda - \lambda z] + q_1B'_1[\lambda - \lambda z]) - z} \]  

From Equations (11) and (15), we obtain,

\[ V(z) = \int_0^\infty V(x, z)dx = \frac{(p_1B'_2[\lambda - \lambda z] + q_1B'_1[\lambda - \lambda z])(V'(\lambda - \lambda z) - 1)U}{(1 + V'(\lambda - \lambda z))(z\theta + \theta')(p_1B'_2[\lambda - \lambda z] + q_1B'_1[\lambda - \lambda z]) - z} \]  

Also from Equations (12) and (14), we obtain,

\[ P_2(z) = \int_0^\infty P_2(x, z)dx = \frac{p_1(B'_1[\lambda - \lambda z] - 1)U}{(1 + V'(\lambda - \lambda z))(z\theta + \theta')(p_1B'_2[\lambda - \lambda z] + q_1B'_1[\lambda - \lambda z]) - z} \]  

The unknown probability \( U \) can be determined by utilizing the normalizing condition which is equivalent to \( U + P_1(1) + P_2(1) + V(1) = 1 \) and the utilization factor of this system \( \rho \) is given by,

\[ \rho = \frac{P_1(1) + P_2(1) + V(1)}{P_1(1) + P_2(1) + V(1)} = \frac{U}{P_1(1) + P_2(1) + V(1)} < 1 \]

which is the steady-state probability that the system is idle and also we have \( \frac{\rho}{\lambda} < 1 \) which is the stability condition under which the steady-state solution exist. Now, we let \( P_1(z) = U + P_1(z) + P_2(z) + V(z) \) denote the steady state PGF of the system size distribution at a random epoch, then adding the Equations (16)–(18) we obtain on simplifying,

\[ P(z) = P_1(z) + P_2(z) + V(z) \]  

Let \( P(z) = P_1(z) + P_2(z) + V(z) \) be the PGF of the queue size distribution at a random epoch then,

\[ P(z) = \frac{(p_1B'_2[\lambda - \lambda z] + q_1B'_1[\lambda - \lambda z])V'[\lambda - \lambda z] - 1}{(1 + V'[\lambda - \lambda z])(z\theta + \theta')(p_1B'_2[\lambda - \lambda z] + q_1B'_1[\lambda - \lambda z]) - z} \]  

Now, let us denote \( P_0(z) \) as the PGF of the queue size distribution at departure epoch of this model. Then we get,

\[ P_0(z) = U + zP(z) = \frac{2(1 - 2\theta - 4\lambda\beta_1)z + (3\lambda\beta_1 + 2\theta - 1)(1 + V'[\lambda - \lambda z])(z\theta + \theta') + (3\lambda\beta_1 + 2\theta - 1)V'[\lambda - \lambda z]}{(5\lambda\beta_1 + 2\theta - 1)(1 + V'[\lambda - \lambda z])(z\theta + \theta')(p_1B'_2[\lambda - \lambda z] + q_1B'_1[\lambda - \lambda z]) - z} \]
5. The mean queue size at a random epoch and the mean waiting time

Let \( L_q \) denote the mean queue size at a random epoch for an \( M/G/1 \) queueing system with two types of general heterogeneous service with multiple vacations and Bernoulli feedback then we obtain from (19) the mean queue size; where \( L_q = \frac{d}{dz} P_q (z) \bigg|_{z=1} = \lambda \left[ Y^{(1)} + \beta \right] (v) \bigg|_{0} = -Y^{(1)}. \) We derive the LST of the waiting time distribution of a test customer for this model, to obtain this we follow the same approach used in Kleinrock (1975) for solving the LST of the time in the system from the PGF of the distribution function of the number of the system in a regular \( M/G/1 \) queue. Let \( W^*_q(s) \) be the LST of the distribution function of the waiting time of a test customer of this model, then utilizing the standard argument of Kleinrock (1975) (for instance see p. 197) we may write,

\[
W^*_q(\lambda - \lambda z) B^*(\lambda - \lambda z) = P_q(z)
\]

Now, \( s = \lambda (1 - z) \) in the above equation we get \( W^*_q(s) \) and having the LST of the waiting time distribution we can easily derive the LST of the distribution function for the response time. The response time \( W^*_p \) is the time interval from the arrival time of a tagged customer to the time when it leaves the system after service completion that means, waiting time plus service time. If \( W^*_q(s) \) be the LST of the distribution function of \( W_q \) then,

\[
W^*_q(s) = \frac{2(1 - 2 \theta - 4 \lambda \beta_1)(1 - \frac{1}{\lambda}) + (\lambda \beta_1 + \beta_2)(3 \lambda \beta_1 + 2 \theta - 1)(1 + V'(s))(1 - \frac{1}{\lambda}) + (5 \lambda \beta_1 + 2 \theta - 1)(1 - \frac{1}{\lambda}) V'(s))}{(5 \lambda \beta_1 + 2 \theta - 1)(1 + V'(s))(1 - \frac{1}{\lambda}) \lambda \beta_1 + q \beta_2 - (1 - \frac{1}{\lambda})}
\]

If \( E(W^*_p) \) be the mean response time of a test customer in this model then,

\[
E(W^*_p) = -\frac{dW^*_q(s)}{ds} \bigg|_{s=0} = Y^{(1)} + \beta_1
\]

6. Mean busy period

We obtain the mean busy period for our model. Here, we define the busy period as the length of the time interval during which the server remains busy and this continues till the instant when the server becomes free again. This busy period is equivalent to the ordinary busy period generated by the units which arrive during the vacation period plus an idle period, which we may call as generalized idle period and define,

\[
T_b = \text{length of the generalized idle period}
\]
\[
T_q = \text{length of the busy period}
\]

Since \( T_b \) and \( T_q \) generate an alternating renewal process, therefore, we may write,

\[
\frac{E(T_b)}{E(T_q)} = \frac{\text{Prob} [T_b]}{1 - \text{Prob} [T_b]}
\]

and, \( \text{Prob} [T_b] = \text{Prob} \{ \text{the server is busy with FPS} \} + \text{Prob} \{ \text{the server is busy with SPS} \} \)

\[
= \frac{\lambda \left( q_1 \beta_1^{(1)} + p_1 \beta_2^{(1)} \right)}{5 \lambda \beta_1 + 2 \theta - 1}
\]

\[
E(T_b) = E(V) + \frac{1}{\lambda}
\]

Hence, we have \( E(T_b) = \frac{\beta_1^{(1)} (1 + 2z V)}{4 \lambda \beta_1 + 2 \theta - 1} \)
7. Numerical insights
Let us consider that the FPS and SPS time distributions follow exponential distribution with probability distribution function \( B_i(x) = 1 - e^{-\mu_i x}, x > 0, i = 1, 2 \) with \( \mu_1^{(1)} = \frac{1}{\theta_1} \) and finite moment \( \mu_2^{(1)} = \frac{1}{\theta_2} \) for \( i = 1, 2 \) and the vacation time random variable follows the Erlang distribution with probability distribution function \( v(x) = \frac{x^{k-1} e^{-x/\nu}}{(k-1)!}, x > 0 \) with mean \( \frac{\nu}{k} \).

In this section, we analyze and compare the performance of the proposed model by varying the arrival rate, feedback probability and expected vacation time. We first analyze the mean busy period for the proposed queueing system with respect to the arrival rate of the customer and feedback probability. The result is graphically illustrated in Figure 2. We observed that the mean busy period decreases with the increase in arrival rate (or, feedback probability) for different values of feedback probability (or, arrival rate). Figure 3 depicts the mean busy period as the arrival rate of the customer decreases with the increase in the arrival rate (or, feedback probability) for different values of feedback probability (or, arrival rate). A sharp increase in the busy period is observed as the expected vacation time of the server increases for fixed arrival rate of the customer. On the other hand, the expected busy period decreases with the increase in the arrival rate for fixed feedback probability.

The following default parameters are considered in order to find the optimal values of expected busy period as in Figures 2 and 3,

- Figure 2: \( q_1 = 0.7, \mu_2 = 2, \mu_1 = 3, E(v) = 0.4 \)
- Figure 3: \( q_1 = 0.6, \mu_2 = 2, \mu_1 = 10, \theta = 0.4 \).

8. Remarks
By specifying vacation time random variable as well as service time random variables, we obtained some results by considering some particular cases of our model.

Case I: \( M(G_1, G_2)^{1/V} (MV) \) queue with Erlangian vacation time.

Suppose that vacation time is an \( k \)-Erlang, i.e. \( E_v \), with probability density function:

\[
v(x) = \frac{dV(x)}{dx} = \frac{(kv)^{k-1} e^{-kvx}}{(k-1)!}; \quad k > 0, v \geq 1
\]

Also \( V^*(\lambda, z) = \left( \frac{v k}{v k + \lambda (1-z)} \right)^k \) and \( E(V) = \frac{1}{\varphi} U = 1 - \left( \frac{2 \mu_1^{(1)}}{1 + 5 \mu_1^{(1)} - 2 \theta} \right) \) then we have,

\[
P_1(z) = \frac{(B_1^*[\lambda - \lambda z] - 1) q_1 U}{(z \theta + \theta') \left( \frac{v k}{v k + \lambda (1-z)} \right)^k + 1 \left( p_1 B_2^*[\lambda - \lambda z] + q_1 B_1^*[\lambda - \lambda z] \right) - z}
\]

\[
V(z) = \frac{\left( \frac{v k}{v k + \lambda (1-z)} \right)^k - 1 U(p_1 B_2^*[\lambda - \lambda z] + q_1 B_1^*[\lambda - \lambda z])}{(z \theta + \theta') \left( \frac{v k}{v k + \lambda (1-z)} \right)^k + 1 \left( p_1 B_2^*[\lambda - \lambda z] + q_1 B_1^*[\lambda - \lambda z] \right) - z}
\]

\[
P_2(z) = \frac{(B_2^*[\lambda - \lambda z] - 1) p_1 U}{(z \theta + \theta') \left( \frac{v k}{v k + \lambda (1-z)} \right)^k + 1 \left( p_1 B_2^*[\lambda - \lambda z] + q_1 B_1^*[\lambda - \lambda z] \right) - z}
\]

Further from Equation (20), we get the PGF of the queue size distribution at a random epoch,
Case II: \( M/\mu/1 \) \( G_1/G_2 \) queue with Exponential vacation time.

As special case of this model, we consider the case of Markovian vacation time random variable with mean \( E(V) = \frac{1}{\mu} \), then the PGF of the various queue size distribution of this model can be obtained by putting \( k \neq 1 \) in Equations (22)–(24):

\[
P(z) = \frac{\left( \frac{v_k}{\mu k + \lambda - \lambda z} \right)^k (p_1 B_2' [\lambda - \lambda z] + q_1 B_1' [\lambda - \lambda z]) - 1}{(z \theta + \theta')(\frac{v_k}{\mu k + \lambda - \lambda z} + 1) (p_1 B_2' [\lambda - \lambda z] + q_1 B_1' [\lambda - \lambda z]) - z}
\]

\[
P_1(z) = \frac{(v + \lambda(1-z))q_1 U(B_2' [\lambda - \lambda z]) - 1}{(z \theta + \theta)(\lambda(1-z) + 2V)(p_1 B_2' [\lambda - \lambda z] + q_1 B_1' [\lambda - \lambda z]) - Z(1 - Z)}
\]

\[
V(z) = \frac{\lambda(1-z)U(p_1 B_2' [\lambda - \lambda z] + q_1 B_1' [\lambda - \lambda z])}{Z(1-Z)} - (z \theta + \theta')(\lambda(1-z) + 2V)(p_1 B_2' [\lambda - \lambda z] + q_1 B_1' [\lambda - \lambda z])
\]

\[
P_2(z) = \frac{(v + \lambda(1-z))p_1 U(B_2' [\lambda - \lambda z]) - 1}{(z \theta + \theta)(\lambda(1-z) + 2V)(p_1 B_2' [\lambda - \lambda z] + q_1 B_1' [\lambda - \lambda z]) - Z(1 - Z)}
\]
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