DBI ACTION FROM CLOSED STRINGS AND D-BRANE SECOND QUANTIZATION

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Abstract

Brane-like vertex operators play an important role in a world-sheet formulation of D-branes and M theory. In this paper we derive the DBI D-brane action from closed NSR string with brane-like states. We also show that these operators carry RR charges and define D-brane wave functions in a second quantized formalism.
1 Introduction

One of the most challenging problems in string theory is to find a way to quantize background and, as a part of this big problem, how to describe second quantization of different branes. In a canonical world-sheet formulation branes are not on the same footing as perturbative string states, which are produced by vertex operators (quantum objects) in open and closed sectors and are second-quantized. To find quantum operator(s) which can be interpreted as brane(s) creation (annihilation) operator(s) will be very important. Some time ago two- and five-forms brane-like vertex operators in NSR superstring theory were introduced in [1] and in a recent paper [2] detailed proof has been given that these operators are physical, i.e. BRST-invariant and BRST-nontrivial. While these vertex operators are proven to be physical, clearly there are no massless two-forms or five-forms in perturbative spectrum of an open string, so the question arises what is the actual role of these physical states in superstring theory. It has been suggested in the papers [3], [4] that these vertex operators and the closed string brane-like vertices which can be constructed from the open ones carry crucial information about nonperturbative physics of strings, D-branes and M-theory, rather than being related to perturbative string dynamics. The first hint comes from the superalgebraic arguments as the zero momentum parts of these operators appear as two and fiveform central terms in a picture changed space-time superalgebra [1]. Since the p-form central terms in the SUSY algebra are always related to topological charges of p-branes [5] this gave the first indication that these new states may be related to brane dynamics. The essential property of these operators is the ghost-matter mixing i.e. they appear only as higher BRST-cohomologies breaking the discrete symmetry of the picture-changing [6]. Using the formalism of brane-like states one can study the dynamics of branes and M-theory [4] as well as of superstring theory in curved backgrounds such as $AdS_5 \times S^5$ [3]. The fact that one has a matter-ghost mixing in the presence of these operators leads to the the world-sheet logarithmic CFT [4],[7] which agrees with earlier suggestions that string theory with second-quantized D-branes (and other backgrounds in general) must be described by world-sheet logarithmic CFT [8] where logarithmic operators describe background collective coordinates. It seems quite natural to assume that by this reason brane-like vertex operators must describe D-brane collective coordinates.

In this paper we shall further analyse these brane-like vertex operators and find several properties which strongly support the idea to use them as creation-annihilation operators for extended solitonic objects (D-branes). To get it we must see that our vertex operators
have exactly the same number of physically independent polarizations as the number of transverse directions, otherwise we shall not be able to describe the correct number of D-brane collective coordinates. At the same time we have to show that there is a correct coupling to the RR field. In this paper we shall only discuss the case of $D3$ brane and respective vertices. We shall show that close string brane-like vertex corresponding to the $D3$ brane has precisely 6 physical degrees of freedom which is what we need for $D3$ brane. But it is open string brane-like vertex operator which has the desired coupling to the RR field. From the low energy effective action point of view, we will show that the close string brane-like vertices generate the bulk (square root of the determinant) terms in the DBI action, while open string brane-like vertices correspond to the RR terms. The fact that one must have two type of vertex operators to describe second-quantized solitonic object in string theory is rather unusual and deserves further investigation. We also want to stress here that the brane operators are not usual creation-annihilation operators in a sense that adding more brane operators does not lead to creation of additional branes but rather generates interaction for brane collective coordinates.

The paper is organized as follows. In the next section we shall review basic facts about open and closed string brane-like states. In a section 3 we will derive the DBI effective action straight from the sigma-model with closed string brane-like states by computing appropriate correlation functions on the sphere. It is remarkable that the effective action will be shown to have the $1/g_{s}^{2}$ dilaton dependence even though technically the computations are performed in closed string theory on the sphere. As we shall argue the “open-string” dilaton dependence follows from the logarithmic nature of the world-sheet CFT when the pair of closed string brane-like operators produce a logarithmic cut effectively producing disk from the sphere. We shall be also able to reproduce the correct D-brane tension from correlators of the closed string brane-like states. In a section 4, we compute the disc correlators of open string vertices with the Ramond Ramond insertion on the disc with Neumann boundary condition showing that the open string brane operators are carriers of the RR-charges. In conclusion we discuss obtained results and unsolved problems which have to be addressed in a future.

2 Brane-like states

Brane-like states are described by physical vertex operators, existing at selected nonzero pictures only, i.e. the vertices with superconformal ghost matter mixing which cannot be removed by a picture changing transformation. Unlike usual perturbative vertex operators,
such as a graviton or a photon, the brane-like states do not account for any point-like string excitation but describe the dynamics of nonperturbative extended objects in string theory, such as branes and solitons. These brane-like vertex operators appear in both open and closed superstring theories. In the open string sector these vertices, the two-form and the five-form are given by:

\[
V^\pm_5(k) = H_{m_1 \ldots m_5}(k) \oint \frac{dz}{2i\pi} e^{-3\phi \psi_{m_1} \ldots \psi_{m_5}} e^{ikX(z)}
\]

\[
V^0_2(k) = H_{m_1 m_2}(k) \oint \frac{dz}{2i\pi} (\psi_{m_1} \psi_{m_2} + C_{m_1 m_2}) e^{ikX(z)}
\]

Here \(X^m, m = 0, \ldots, 9\) are 10d space-time coordinates, \(\psi_m\) are their superpartners on the worldsheet, \(\phi\) and \(\chi\) are bosonized superconformal ghost fields. BRST-invariance and BRST nontriviality of the states (1) has been proven in [2]. The open string five-form state can be expressed at pictures -3 and +1 (but with no version at picture zero). The picture +1 version must also include the b-c ghost counterterms \(B_{m_1 \ldots m_5}\) and \(C_{m_1 \ldots m_5}\) (carrying the reparametrization ghost numbers -1 and +1) in order to insure its BRST-invariance; the picture -3-version requires no b-c ghost terms. The precise form of these ghost counterterms has been given in [2]. In this paper we will only need the expression for \(B_{m_1 \ldots m_5}\):

\[
B_{m_1 \ldots m_5} = \frac{1}{10} (cT_\chi) \left[ (\partial \phi - \partial \chi) \partial b e^{2\phi - \chi} \psi_{m_1} \ldots \psi_{m_5} \psi \partial^2 X \right]
\]

The hat operators \((cT_\chi)_n\) are defined by acting on any local operator \(A(w)\) as

\[
(cT_\chi)_n A(w) = \lim_{u \to w} \oint \frac{dz}{2i\pi} (z - u)^n : cT : (z) A(u)
\]

\[
T_\chi = \frac{1}{2} ((\partial \chi)^2 - \partial^2 \chi)
\]

Analogously, the BRST invariant expression for the two-form vertex at picture zero must contain the c-ghost two-form \(C_{m_1 m_2}\). BRST nontriviality of the vertices (1) can be proven
straightforwardly by showing that they have non-vanishing correlation functions with other physical vertex operators, however the BRST non-triviality imposes important constraints on the five-form $H_{m_1...m_5}$. Indeed, it is possible to construct the operator

$$W_5 = H_{m_1...m_5} \oint dz_2 i\pi e^{-3\phi} \psi_{m_1} \cdots \psi_{m_5} (\psi_m \partial X^m) e^{ikX}$$

such that

$$\{Q_{BRST}, W_5 \} = -\frac{3}{2} H_{m_1...m_5} \oint dz_2 i\pi e^{-3\phi} \psi_{m_1} \cdots \psi_{m_5} e^{ikX}$$

$$+ iH_{m_1...m_5} \oint dz_2 i\pi \partial^2 e^{-3\phi} \partial \chi (k\psi) \psi_{m_1} \cdots \psi_{m_5} e^{ikX}$$

$$\equiv V_5^{(-3)}(k) + iH_{m_1...m_5} \oint dz_2 i\pi \partial^2 e^{-3\phi} \partial \chi (k\psi) \psi_{m_1} \cdots \psi_{m_5} e^{ikX}$$

Therefore the $V_5^{(-3)}$ vertex operator is BRST-nontrivial only if the second term in the commutator is non-vanishing. This is obviously equivalent to the condition

$$k[m_6 H_{m_1...m_5}] \neq 0$$

or

$$dH^{(5)} \neq 0$$

which means that the $V_5$-operator is physical if the five-form $H(k)$ is not closed. In fact, this condition has a simple physical meaning: below we will see that $dH$ plays the role of the D-brane wavefunction in the second-quantized formalism which of course must not vanish. Analogous condition can be shown to appear for the five-form at the $+1$-picture, though in that case the form of the $W_5$-operator is much more complicated. We will not present it here for the sake of shortness.

Apart from the open string vertices (1) brane-like states are also present in the closed string sector. They can be constructed straightforwardly by either taking the brane-like forms (1) as holomorphic and antiholomorphic parts of the vertices, or taking the brane-like left parts and photonic right parts. Depending on the contraction of the space-time indices, the closed string brane-like states may describe various branes and brane configurations. In this paper we shall explore the closed string brane-like states given by

$$V_5^{cl.(-3)} = H_{m_1...m_6}(k) \int d^2 z e^{-3\phi-\bar{\phi}} \psi_{m_1} \cdots \psi_{m_5} \bar{\psi}_{m_6} e^{ikX}$$
where $H_{m_1...m_6}$ is antisymmetric 6-form field. Complex conjugated part must be added here to insure the overall unitarity of the amplitudes. In this paper we restrict ourselves to considering only a totally antisymmetric rank 6 representation of the Lorentz group, as it is this representation that will yield the desired D3-brane dynamics. The meaning of other possible irreducible rank 6 representations in (8) is of course an interesting question and we hope to consider it in the future. It is possible that these representations correspond to more complicated brane configurations (such as brane bound states) but in any case we do not discuss any of these questions in our present paper. Below we will show that the physical meaning of the 6-form is that it determines the space-time location of the D3-brane, after imposing appropriate BRST constraints on $H$.

### 3 DBI action from closed string brane like states

In this section we derive the DBI action for the D3-brane from the closed string sigma-model with the 6-form brane-like states (8). To obtain the low energy effective action one first of all has to specify the number of physical degrees of freedom associated with the 6-forms $H_{m_1...m_6}(k)$ left after fixing the gauge symmetry associated with the BRST conditions on $H$.

Consider the operator $V^{cl,(+1)}_5$. As for the BRST- nontriviality of its left part, it still requires the condition

$$k_{[m_7}H_{m_1...m_5]m_6} \neq 0$$

where the brackets imply total antisymmetrization. The condition (9) is easily derived in full analogy with (5). Indeed, in the closed string case the role of the $W_5$-operator is played by

$$W_5 = H_{m_1...m_6} \int d^2 z e^{i 4 \phi - \phi \bar{\psi} \psi} (e^{i \psi \partial X} \bar{\psi} \psi) e^{ikX}$$

and

$$\{Q_{BRST}, W_5 \} = \frac{i}{2} H_{m_1...m_6} \int \frac{dz}{2 i \pi} \partial^2 c e^{i 4 \phi - \phi \bar{\psi} \psi} (k_n \bar{\psi} \psi) e^{ikX}$$

Again, the BRST nontriviality means that the second term does not vanish.
At the same time, the condition for the BRST-invariance of the antiholomorphic part leads to the constraint

\[ k_{m_1} H_{m_1 \ldots m_6}(k) = div H^{(6)} = 0 \]  

(10)

The constraints (9), (10) have simple geometrical meaning. They imply that for any given polarization \( m_1, \ldots m_6 \) of the \( V_5 \) vertex, its momentum must be transverse to the \( m_1, \ldots m_6 \) directions so the vertex effectively propagates in 4 dimensions for any given polarizations. The BRST constraints (9), (10) on \( H \) largely reduce the number of independent physical degrees of freedom. To identify the degrees of freedom, note that the number of independent components of a general 6-form in 10 dimensions is equal to \( \frac{10!}{4!6!} = 210 \). The BRST condition (10) means that the 6-form \( H^{(6)} \) can be locally expressed as divergence of the 7-form which reduces the number of independent components to

\[ N = \frac{10!}{3!7!} - \frac{10!}{2!8!} + \frac{10!}{1!9!} - 1 = 84 \]  

(11)

This number is still reduced by constraints induced by the nontriviality conditions (9). To calculate it note that the conditions (9) induce the set of the gauge transformations

\[ H^{(6)} \rightarrow H^{(6)} + d\Lambda^{(5)} \]  

(12)

The number of gauge transformations is given by

\[ P = \frac{10!}{5!5!} - \frac{10!}{4!6!} + \frac{10!}{3!7!} - \frac{10!}{2!8!} + \frac{10!}{1!9!} - 1 = 126 \]  

(13)

However, what we need is not the full set of these gauge transformations but only those consistent with the constraints (9). This implies that we need to include only the gauge transformations that can be written in terms of the Laplacian of the 6-forms. The number of transformations to exclude is then given by the number of seven-forms that are closed and not divergences of the eight-forms; it is easy to check that this number is given by

\[ Q = 2 \times \frac{10!}{4!6!} - \frac{10!}{5!5!} - \frac{10!}{3!7!} = 48 \]  

(14)

Therefore the total number of independent gauge transformations is equal to \( 126 - 48 = 78 \) number of physical degrees of freedom left after imposing the BRST constraints is given by

\[ N - P + Q = 84 - 78 = 6 \]  

(15)
Below we shall see that they correspond to 6 transverse fluctuations of a D3-brane. Having identified the number of the degrees of freedom, we can choose the gauge

$$V_5^{(-3)} = \varepsilon \psi_{[t_1...t_5} \chi^{(k^\perp)} \psi_{t_5]} \overline{\psi}_{t_6} e^{i k^\perp \cdot X} + c.c. \quad (16)$$

where the space-time indices are split in the 4 + 6 way: \(m = (a, t), a = 0, ...3; t = 4, ...9\) and \(k^\perp \cdot X = k_a X_a\). Note that the 4+6 splitting is achieved as a result of a particular choice of the gauge (16) fixing the 6 physical degrees of freedom. Indeed this gauge has a SO(1,3) \(\times\) SO(6) isometry. Choosing the gauge (16) is equivalent here to fixing one particular polarization of the vertex (8), for which the momentum is orthogonal to 6 space-time indices. Therefore the vertex (8) effectively propagates in four-dimensional space-time. This situation is quite different, for example, from the case of photons, for which the transversality condition does not reduce the effective dimensionality in which they propagate. Indeed, for the gauge choice (16) only one polarization is admissible, while while in the photonic case there is nothing fixing polarization. Technically, the difference between the photonic and the brane-like case occurs because a single polarization condition for photons \((ke(k)) = 0\) is replaced by two BRST conditions (9), (10) which altogether are much stronger than transversality constraints for standard operators of U(1) gauge fields.

Now we are ready to begin the computation of the low energy effective action. It is easy to see that the three-point function of the \(V_5\)-operators is zero, as the three-point correlator of the NSR fermions vanishes:

$$\langle: \psi_{t_1}...\psi_{t_5} : (z_1) : \psi_{s_1}...\psi_{s_5} : (z_2) \psi_{u_1}...\psi_{u_5} : (z_3) \rangle = 0$$

Therefore the first nonvanishing contribution to the beta-function and low-energy equations of motion comes from the 4-point correlator of the \(V_5\)-vertices. We will consider the limit of a slowly changing \(\lambda^\perp\)-field in which only the massless poles of the Veneziano amplitude are important. We will also need a picture-changed version of the \(V_5^{(-3)}\)-operator to insure correct ghost number balance in the holomorphic part of the 4-point correlator. Acting on \(V_5^{(-3)}\) with picture changing transformation we get the picture \(-2\)-representation of the 5-form:

$$V_5^{(-2)} = H_{m_1...m_5}(k) \int \frac{dz}{2i\pi} e^{i\phi} e^{i(\chi \cdot 3\phi)} \partial \chi \psi_{m_1}...\psi_{m_5} e^{i k \cdot X} (z) + c.c. \quad (17)$$

Analogous picture-changing transformation can be done for the left (brane-like) part of the closed-string 6-form. To compute the four-point function of the \(V_5^{\text{closed}}\), one has to take
Here the indices $s, t, u, v$ the left (5-form) part of two operators at picture -2 while the left part of the remaining two at picture +1. Then the contribution to the four-point function will be given by

$$A_4(p, k, q, l) = \langle \partial \chi e^{X - 3\phi} \partial \chi \psi_{t_1} ... \psi_{t_5} (\partial X_{m_6} i(p^\dagger \bar{\psi}) \psi_{t_6} e^{ip^\dagger X}(z_1, \bar{z}_1))$$

$$c \partial \chi e^{X - 3\phi} \partial \chi \psi_{s_1} ... \psi_{s_5} (\partial X_{s_6} i(k^\dagger \psi) \psi_{s_6} e^{ik^\dagger X}(z_2, \bar{z}_2))$$

$$e^{-\bar{\phi}} B_{u_1 ... u_3} \bar{\psi}_{u_4} e^{iq^\dagger X}(z_3, \bar{z}_3) e^{-\bar{\phi}} B_{v_1 ... v_3} \bar{\psi}_{v_4} e^{il^\dagger X}(z_4, \bar{z}_4) >$$

$$\times \epsilon_{[t_1 ... t_5} \lambda_{a_6]}(p^\dagger) \epsilon_{[s_1 ... s_5} \lambda_{b_6]}(k^\dagger) \epsilon_{[u_1 ... u_5} \lambda_{c_6]}(q^\dagger) \epsilon_{[v_1 ... v_5} \lambda_{d_6]}(l^\dagger) > + c.c. \quad (18)$$

Here the indices $s, t, u, v = 4, ..., 9$ and the B-fiveform, carrying the fermionic ghost number -1, has been defined above in (2), (3).

For simplicity let us first consider the case when only one out of six components of $\lambda_t$ is nonzero; for instance one can take

$$\lambda_4 \equiv \lambda; \lambda_{5, 6, 7, 8, 9} = 0.$$

It will then be straightforward to generalize it to the case when all the components are nonzero. First, let us calculate the antiholomorphic photonic part of this correlator (which of course is holomorphic in the complex conjugated part). Simple calculation gives

$$A_{R}(p, k, q, l) = \frac{1}{(z_1 - \bar{z}_2)^2(z_3 - \bar{z}_4)^2} - \frac{(k^\dagger p^\dagger + k^\dagger + q^\dagger + l^\dagger)}{(z_1 - \bar{z}_2)(z_3 - \bar{z}_4)} \frac{1}{(z_1 - \bar{z}_2)(z_3 - \bar{z}_4)}$$

$$- \frac{1}{(z_1 - \bar{z}_3)(z_2 - \bar{z}_4)} + \frac{1}{(z_1 - \bar{z}_4)(z_2 - \bar{z}_3)} \} \quad (19)$$

Here and elsewhere we only consider the kinematic part of the amplitude, dropping the factor of $\prod_{i, j} |z_i - \bar{z}_j|^k i^j$ since we are only interested in cotributions from the massless poles.

Next, calculation of the holomorphic matter part gives

$$A_{R}^{\text{matter}}(p, k, q, l) = \frac{1}{(z_1 - \bar{z}_2)(z_3 - \bar{z}_4)} - \frac{1}{(z_1 - \bar{z}_3)(z_2 - \bar{z}_4)} + \frac{1}{(z_1 - \bar{z}_4)(z_3 - \bar{z}_2)}$$

$$\times \left( \frac{p_a}{(z_1 - z_3)^2} + \frac{k_a}{(z_2 - z_3)^2} + \frac{l_a}{(z_2 - z_4)^2}\right) \left( \frac{p_a}{(z_1 - z_4)^2} + \frac{k_a}{(z_2 - z_4)^2} + \frac{q_a}{(z_3 - z_4)^2}\right)$$

$$\times \delta(p^\dagger + k^\dagger + q^\dagger + l^\dagger) \quad (20)$$

Next, we have to calculate the ghost contribution. It is given by the correlator

$$A_{L}^{\text{host}} = \oint \frac{du}{2i\pi} \oint \frac{dw}{2i\pi} (u - z_3)^7 (w - z_4)^7 < ce^{X - 3\phi} \partial \chi(z_1) ce^{X - 3\phi} \partial \chi(z_2)$$

$$b \partial \phi e^{2\phi - \chi}(\partial \phi - \partial \chi)(z_3) b \partial \phi e^{2\phi - \chi}(\partial \phi - \partial \chi)(z_4) c(\partial \chi \partial \chi - \partial^2 \chi)(u) c(\partial \chi \partial \chi - \partial^2 \chi)(w) > \quad (21)$$
Technically this part is the most complicated since it involves evaluating two tedious contour integrals entering the definition (3) of the hat operators. Upon calculating these integrals, the final answer is greatly simplified after fixing the Koba-Nielsen’s measure:

\[
\begin{align*}
    z_1 & \to \infty \\
    z_2 &= z \\
    z_3 &= 1 \\
    z_4 &= 0
\end{align*}
\]  

(22)
multiplying by the \( SL(2, C) \) FP determinant

\[
\text{det}(SL(2, C)) = |z_1 - z_3|^2 |z_1 - z_4|^2 |z_3 - z_4|^2 \sim |z_1|^4
\]

(23)
and integrating over \( z \). After fixing the \( SL(2, C) \) gauge, expression for the antiholomorphic part of the correlator becomes

\[
A_R(p, k, q, l) = (p^\perp k^\perp)\delta(p^\perp + k^\perp + q^\perp + l^\perp)\frac{1}{z_1^2} (1 + \frac{1}{\bar{z}(\bar{z} - 1)})
\]

(24)
Multiplying it by the holomorphic matter part and the FP determinant we obtain the correlator without the holomorphic ghost factor given by:

\[
A_R \times A_{\text{matter}} \times \text{det}(SL(2, C))(p, k, q, l) = z_1^{-3} (1 + \frac{1}{z(z - 1)})^5 (1 + \frac{1}{\bar{z}(\bar{z} - 1)})
\]

(25)
\times (kp)\delta(p^\perp + k^\perp + q^\perp + l^\perp)\left(\frac{k}{(z - 1)^2} + l_a\left(\frac{k}{z^2} + q_a\right) + O(z^{-4})\right)

Evaluating the contour integrals and the ghost holomorphic part, we obtain

\[
A_{\text{ghost}} = z_1^3 F(z) + O(z_1^2)
\]

(26)
Since \( A_R \times A_{\text{matter}} \times \text{det}(SL(2, C))(p, k, q, l) \) behaves as \( z_1^{-3} \) as we fix the gauge \( z_1 \to \infty \), only the \( z_1^3 \) term in the \( A_{\text{ghost}} \) contributes to the correlator. Evaluation of the \( F(z) \) function gives

\[
F(z) = 4z^3(z - 1)^3 \left\{ \left( \frac{5}{z(z - 1)} - 1 \right) + \frac{1}{(z - 1)^2} - \frac{1}{z^2} \right. \\
+ \left. \frac{z^5}{(z - 1)^2} + \frac{(z - 1)^5}{z^2} \right\}(\frac{2}{z - 1} + 1)(-\frac{2}{z} + 1)z^3(z - 1)^3
\]
$$\begin{align*}
+4z^3(z - 1)^3\{(1 - z)(-\frac{1}{(z - 1)^2} + (\frac{1}{z - 1} - 1)(-\frac{1}{z - 1} + 5) + \frac{1}{2}(-\frac{1}{z - 1} + 5)^2
\end{align*}$$
$$\begin{align*}
-\frac{1}{2}(\frac{1}{(z - 1)^2} + 5)) + z(-\frac{1}{z^2} + 1 - (\frac{1}{z} + 1)(5 + \frac{1}{z}) + \frac{1}{2}(5 + \frac{1}{z})^2 - \frac{1}{2}(5 + \frac{1}{z}^2)^2)\}
\end{align*}$$
$$\begin{align*}
\times\{\frac{1}{(z - 1)^2}(\frac{2}{z} + 1) + \frac{1}{z^2}(\frac{2}{z - 1} - 1) - (\frac{2}{z} - 1)(\frac{2}{z - 1} + 1)(\frac{1}{z + 1} + \frac{1}{z})\}
\end{align*}$$
$$\begin{align*}
+4z^3(z - 1)^3\{(\frac{1 - z}{z^2} - \frac{2}{z} + \frac{1 - z}{z} - 1) + (9z - 3)(-\frac{1}{z^2} + (\frac{1}{z} + \frac{1}{z - 1})(\frac{2}{z} + 1))
\end{align*}$$
$$\begin{align*}
+(9z + 6)(-\frac{1}{(z - 1)^2} + (\frac{1}{z} + \frac{1}{z - 1})(\frac{2}{z} + 1))
\end{align*}$$
$$\begin{align*}
+4z^3(1 - z)^3(\frac{1 - z}{z^2} - \frac{2}{z} + \frac{1 - z}{z} - 1)
\end{align*}$$
(27)

Finally, collecting all the pieces together and integrating over $z$ gives:

$$\begin{align*}
A(p^\perp, k^\perp, q^\perp, l^\perp) &= \int d^2z\{A_R \times A_L^{\text{matter}} \times A_L^{\text{ghost}} \times \text{det}(SL(2, C)) + \text{c.c.}\}
\end{align*}$$
$$\begin{align*}
= \{((k^\perp p^\perp)[(k^\perp)^2 - (p^\perp)^2 + (q^\perp k^\perp) + (q^\perp p^\perp)] + \frac{1}{2}(k^\perp p^\perp)^2
\end{align*}$$
$$\begin{align*}
\times \lambda(p^\perp)\lambda(k^\perp)\lambda(q^\perp)\lambda(l^\perp)\delta(p^\perp + k^\perp + q^\perp + l^\perp)\}\Gamma(0)
\end{align*}$$
(28)

where the $\Gamma(0)$ factor accounts for the massless pole in the $z$ integration.

Using the on-shell momentum conservation it is convenient to add to this expression the piece given by

$$\begin{align*}
B = -\left\{\frac{1}{4}(k^\perp s)(p^\perp)^2 - \frac{3}{4}(q^\perp s)(p^\perp k^\perp) + \frac{1}{4}(p^\perp s)(k^\perp s) - \frac{1}{8}(p^\perp k^\perp s)^2\right\}
\end{align*}$$
$$\begin{align*}
\times \lambda(p^\perp)\lambda(k^\perp)\lambda(q^\perp)\lambda(l^\perp)\delta(s)\Gamma(0)
\end{align*}$$
(29)

where

$$\begin{align*}
s = p^\perp + k^\perp + q^\perp + l^\perp
\end{align*}$$
(30)

Adding this piece corresponds to adding full derivative terms to the space-time effective action. Then the expression for the amplitude becomes

$$\begin{align*}
A(p^\perp, k^\perp, q^\perp, l^\perp) = \{(p^\perp)^2(k^\perp q^\perp) - (k^\perp p^\perp)(p^\perp q^\perp)\lambda(p^\perp)\lambda(k^\perp)\lambda(q^\perp)\lambda(l^\perp)\}\delta(s)\Gamma(0)
\end{align*}$$
(31)
This concludes the evaluation of the the 4-point function (limited to the massless pole contribution). In the position space, the worldsheet RG equations and the low energy equations of motion following from this amplitude are given by

\[
\frac{d\lambda}{d(\log \Lambda)} = -\partial_a \partial^a \lambda + \partial_b \partial^b \lambda \partial_a \lambda \partial^a \lambda - \partial_a \partial_b \lambda \partial^a \lambda \partial^b \lambda = 0 \quad (32)
\]

where \( a = 0, \ldots, 3 \). It is easy to check that this EOM follows from the effective action given by

\[
S(\lambda) = \int d^4x \sqrt{\det(\eta_{ab} + \partial_a \lambda \partial^a \lambda)} \quad (33)
\]

Generalization of this result for the case when all the six components of \( \lambda \) are nonzero is completely straightforward, and all the computations are quite analogous, though the answer quite predictably follows from considerations of the Lorentz invariance. In the general case, we compute the amplitude to be

\[
A(p^\perp k^\perp, q^\perp, l^\perp) = \left\{(p^\perp)^2 (k^\perp q^\perp) + 2(k^\perp q^\perp)(k^\perp p^\perp) + (p^\perp q^\perp)(p^\perp k^\perp)\right\} \\
\times (\lambda_t(k^\perp) \lambda_t(k^\perp)) \lambda_s(k^\perp) \lambda_s(k^\perp) \delta(p^\perp + k^\perp + q^\perp + l^\perp) \Gamma(0) \quad (34)
\]

The corresponding low energy equations of motion are given by

\[
-\partial_b \partial^b \lambda_t + 2\partial_a \partial_b \lambda_s \partial^a \lambda_t \partial^b \lambda^s \\
+ \partial_a \partial_b \lambda_t \partial_a \lambda_s \partial^b \lambda^s + \partial_b \partial^b \lambda_s \partial_a \lambda_s \partial^a \lambda_t = 0 \quad (35)
\]

These equations of motion can be easily shown to follow from the quartic term in the expansion of the DBI action for the D3-brane:

\[
S = \int d^4 x \sqrt{\det(\eta_{ab} + \partial_a \lambda \partial_b \lambda')} \quad (36)
\]

This concludes the derivation of the DBI action from sigma-model with the brane-like states; however, this derivation has been done in the absence of the dilaton background. In the next section we will calculate the impact of the dilaton field on the effective action (36).
4 Dilaton coupling and brane tension

In the presence of the dilaton background the low energy effective Lagrangian (36) is modified as

\[ e^{\alpha \varphi} \sqrt{\det(\eta_{ab} + \partial_a \lambda_t \partial_b \lambda^t)} \]

. The problem now is to determine the coefficient \( \alpha \). In closed string theory one usually has \( \alpha = -2 \) for the dilaton coupling with the NS-NS fields and \( \alpha = 0 \) for the coupling with Ramond-Ramond fields. In our case, however, the situation is different. To compute the dilaton coupling one has to consider the three-point function \( < V_5^{\text{closed}} V_5^{\text{closed}} V_\varphi > \) This correlator gives the first order term in the dilaton expansion and which is enough to read off \( \alpha \). The simple computation gives us

\[ < V_5^{\text{closed}(+1,-1)}(p^\perp, z_1, \bar{z}_1) V_5^{\text{closed}(-3,+1)}(k^\perp, z_2, \bar{z}_2) V^{0,0}_\varphi(q^\perp, z_3, \bar{z}_3) > = \frac{(k^\perp q^\perp) \lambda_t (p^\perp) \lambda_t (k^\perp) \varphi(q^\perp)}{|z_1 - z_2|^2 |z_1 - z_3|^2 |z_2 - z_3|^2} \delta(k^\perp + p^\perp + q^\perp) \] (37)

where the dilaton operator is taken at the \((0,0)\)-picture:

\[ V_\varphi(k) = \varphi(k)(\partial X^m + i(k \bar{\psi}) \bar{\psi}^m)(\bar{\partial} X^\bar{m} + i(k \psi) \bar{\psi}\bar{\psi}^\bar{m})(\eta_{mn} - k_m \bar{k}_n - k_n \bar{k}_m) \]

\[ k^2 = \bar{k}^2 = 0 \]
\[ (k \bar{k}) = 1 \] (38)

The crucial point is that it is only the X-part of the dilaton operator that contributes to these correlators; the \( \psi \)-part vanishes since at least one of the NS fermions in the dilaton operator, \((q^\perp \psi)\) always has a polarization orthogonal to \( \psi \)'s in the brane-like operators and therefore has no partners to be contracted with. As a result, the relative normalization of this correlator is one half of those of the dilaton with the usual perturbative superstring vertices. As a result, the overall dilaton coupling must be proportional to \( e^{-\varphi} \) and the effective action in the presence of the dilaton is given by:

\[ S_{\text{eff}}(\lambda) = \int d^4x e^{-\varphi} \sqrt{\det(\eta_{ab} + \partial_a \lambda_t \partial_b \lambda^t)} \] (39)

We see that the six \( \lambda^t \)-fields, emerging from the 6-form \( H(k) \) upon imposing the BRST constraints (9) determine the location of the D3-brane in the space-time, as has been already
It is remarkable that the effective action (39) derived from the closed string sector that we have just computed, has an open string D-brane-like dilaton dependence. Below we will give some heuristic arguments showing that such an open-closed string transmutation is closely related to the logarithmic properties of the brane-like states. Namely, it has been observed previously that closed string brane-like states (8), integrated over their four-dimensional momenta, constitute a pair of logarithmic operators. This means that the worldsheet logarithmic singularities will appear in the OPE of two such states, after the appropriate momentum integration. Therefore the insertion of two integrated closed string brane-like vertices produces one logarithmic branch point on the worldsheet. Appearance of the branch point means that two brane-like insertions cut a hole on the worldsheet which cannot be removed by any metric redefinition. Such a logarithmic hole effectively changes the Euler character of the worldsheet by 1. But the dilaton dependence of the effective action is determined from the tree-point correlator of the dilaton with two brane-like insertions. As a result one obtains an open string dilaton dependence from closed string scattering amplitudes. Geometrically, the mechanism of the "wrong" dilaton coupling that we observed, is quite analogous to the anomalous dilaton dependence of the RR-fields, which again is determined from the three-point function of the dilaton with two RR-vertices. The difference, however, is that in the case of the RR-dilaton interaction each of the RR-insertions pins a separate worldsheet hole, as each of the RR-vertices changes boundary conditions for fermions, creating two separate cuts. As a result, in the RR case the effective Euler character is changed by two units, contrary to the brane-like case where it takes at least a couple of vertices to create a cut.

5 Open String Brane-like Vertices and D-brane wavefunctions

So far we were considering the brane-like states in the closed string sector, showing that in the low energy limit their correlations leads to the "gravitational" part of the DBI action, involving the induced gravity in the worldvolume. Now it is important to understand the role of the brane-like states in the open-string sector. Below we'll try to show that this role is that the open string brane-like states account for the terms with the Ramond-Ramond charges carried by D-branes. Namely, we shall try to show that the open string vertices mentioned above. The 4-dimensional momentum $k$ corresponds to the worldvolume degrees of freedom in the DBI action.
(1) can be understood as creation operators for the RR charge sources. The full action for D-branes also contains the terms involving the coupling of the worldvolume form to the Ramond-Ramond potential. It is well-known that D-branes can be understood as closed string solitons, carrying the RR-charges. We are going to show that the vertices (1) also can be regarded as quantum operators charged with the RR gauge fields. To clarify the relation of the operators (1) to the standard interpretation of D-branes as the RR charge sources, one can think of the following simple analogy with the QED. One can describe an electron as a classical object with a certain charge density. In QED, however, the electron described in terms of a quantum wavefunction $\psi$, giving rise to creation operators in the second quantized formalism. Then the fact of the electron carrying the $U(1)$ charge follows from its interaction term with the gauge field in the QED action, given by $\sim \psi A \psi$ where $A$ is the gauge potential. Therefore in order to show the relation of the vertices (1) to the Ramond-Ramond charges one has to consider their interaction with the RR vertices on a disc, showing that relevant terms in the effective action have the structure $\sim dH(5)dH(5)A_{\text{RR}}$, where $H(5)$ is the space-time five-form of the $V_5$ brane-like vertex. Note that the role of the wavefunction must be played by the gauge-invariant $dH$ field since, due to the BRST conditions (6) the $H$ five-form is defined up to a gauge transformation by the derivative of a four-form. In other words, one needs to show that in the effective action the $H(5)$-field couples to the RR gauge potential, rather than the RR field strength. To show this we have to calculate the disc correlator of a RR vertex operator with two five-forms or the two-forms, inserted on the disc boundary. To insure the correct ghost number balance (the sum of left and right ghost numbers must be equal to $-2$ on the disc) one has to consider correlators

$$A_{5-5-\text{RR}} = \langle V_5^{(-3)}(k; \tau_1)V_5^{(+1)}(p; \tau_2)V_{\text{RR}}^{(+1/2,-1/2)}(q; z, \bar{z}) \rangle$$

or

$$A_{2-2-\text{RR}} = \langle V_2^{(-2)}(k; \tau_1)V_2^{(0)}(p; \tau_2)V_{\text{RR}}^{(1/2,-1/2)}(q; z, \bar{z}) \rangle$$

i.e. the Ramond-Ramond vertex operator must be taken at the mixed $(+\frac{1}{2}, -\frac{1}{2})$-picture. Here and elsewhere we shall consider the case of Neumann boundary conditions on the disc. The expression for the RR vertex in this picture is given by

$$V_{\text{RR}}^{+1/2,-1/2}(q) = e^{\frac{i}{4}\phi - \frac{i}{4}b\sum_\alpha \sum_\beta (\partial X_m + \frac{i}{4}(q\psi)^m \gamma_{m_1...m_p})_{\alpha\beta} F_{m_1...m_p}(q)}$$

where $F^{\text{RR}}$ is the Ramond-Ramond p-form field strength. Now, since expressions for the 5-form brane-like states at any of the two pictures do not contain any dependence on the
derivatives of $X$ or the momentum (except for the exponent $e^{ikX}$), and the RR-vertex $V^{+1/2,-1/2}_{RR}$ is linear in $\partial X$ and $q$, it is clear that the the overall amplitude is linear in momentum as well, i.e. it has the structure

$$S_{HHF} \sim qHHF_{RR} \sim dHHF_{RR} \sim dHdHA_{RR}$$

i.e. it has precisely the form we are looking for; here $A_{RR}$ stands for the RR gauge potential.

In general case, the expression for this cubic term involving the higher spin gauge fields, may be complicated and involve various nontrivial ways of contraction of indices to insure the overall gradient invariance. The latter is guaranteed by the BRST conditions on $H^{(5)}$. One only has to insure that the overall contribution is nonzero and is not reduced to just a topological Chern-Simons term. At this point we do not yet have a complete classification of all the RR-charges coupling to the vertices (1). For instance, it is not yet clear if each of the vertices (1) couples to a particular RR-form, or to several forms at the same time. To clarify this point, one has to compute all the set of three-point disc correlators of the vertices (1) with all the RR-forms which has not yet been done, as in general these correlators involve rather lengthy combinations of the cubic terms. This computation is currently in progress; we hope to present it soon in our future paper. In this paper we consider only one precise example of such an interaction of the vertices (1) to the RR-fields, demonstrating the coupling of the brane-like forms to the Ramond-Ramond charges. Namely, we consider the correlator of the RR five-form with two brane-like two-form insertions on the disc. Note that the c-ghost term in the expression (1) for the two – form at picture zero does not contribute to the correlator in this case and can be neglected. Using the expression (1) for the two-forms at pictures $-2$ and $0$ and for the RR 5-form at picture $+1/2, -1/2$ we obtain after simple calculation:

$$A_{2-2-RR} = <V^{(-2)}_2(p)V^{(0)}_2(k)V^{+1/2,-1/2}_{RR-5}>(q) >$$

$$\sim Tr(\gamma^{m_1m_2...m_5}\gamma^{n_1n_2n_3n_4})q_m F_{m_1...m_5}^{RR}(q)H_{n_1n_2}(p)H_{n_3n_4}(k)$$

Evaluating the gamma-matrix trace and making Fourier transform we easily find corresponding terms in the effective action:

$$S_{2-2-RR} \sim \int d^{10} X H_{m_1m_2}(dH)_{m_3m_4m_5}F_{RR}^{m_1...m_5} + H \wedge dH \wedge RR$$

The second term is just the topological CS-term while the first one reflects the fact that the $H$ 3-form field is a quantum wavefunction that carries the RR charge of a $D3$-brane.
The gradient invariance of this term can be easily proven: there is a manifest invariance under the gauge transformations of the Ramond-Ramond field while the invariance under the gauge transformations of the two-form: $H^{(2)} \to H^{(2)} + d\Lambda^{(1)}$ can be easily shown by partial integration and using the Maxwell’s conditions on $F_{RR}$: $\text{div}(F) = \partial_m F_{RR}^{m_1...m_5} = 0$ which are the BRST constraints for the Ramond-Ramond vertex operator. After partial integration we can write the first term of (44) as

$$S_{2-2-RR} \sim \int d^{10}X \partial_m H_{mn}(dH)_{pqr}A_{RR}^{npqr}$$

(45)

From the structure of this amplitude we conclude that indeed the two-form state is the carrier of the 5-form of the RR-charge; this means that the three-form brane-like field $dH_{mnp}$ constitute a wavefunction for the D3-brane. One needs, however, to consider the correlations of the two-forms with other Ramond-Ramond fields in order to point out the full set of the RR-charges generated by the open string two-forms. The main conclusion from this computation is that the brane-like states can be regarded as creation operators for the D-branes.

6 Conclusion

We have demonstrated that the open and close string brane-like vertex operators describe the dynamics of D-branes in the second quantized formalism. Closed string vertices induce the gravity in the D-brane worldvolume while the open string operators account for the coupling with the RR gauge fields. It is remarkable that the DBI action with open string dilaton coupling appears as a result of the closed string computation. We argue that this anomalous dilaton coupling is related to the logarithmic properties of the closed string brane-like states, such as the creation of the logarithmic branch point on the worldsheet by the pair of brane-like vertex operators. The logarithmic behavior of vertex operators in string theory occurs when their wavefunction (space-time field) satisfies the so-called "logarithmicity criterium", namely, in the on-shell limit it should asymptotically behave as $H(k) \sim k^{-N}$ with $N > 6$. The asymptotic behavior is determined from the worldsheet beta-function equation involving the corresponding vertices. Unlike the usual vertex operators, the brane-like states satisfy the above criterium. In our next paper, currently in progress, we shall provide the detailed analysis of the LCFT properties of the brane-like states, to peculiarities of the worldsheet RG flow involving vertex operators with the ghost-matter mixing. There are many unanswered questions about the relation of the ghost-matter mix-
ing and of the brane-like states to non-perturbative dynamics of D-branes and M-theory; there is also a multitude of implications for future work in this direction. One particular problem is giving the full classification of the RR charges generated by the open string brane-like operators. Our hope is that the ghost-matter mixing principle, apparently allowing us to consider the D-branes in the operator formalism and to develop an alternative approach to non-perturbative brane dynamics, is not confined to string theory, but also to gauge theories. It is possible that the ghost-matter mixing may be a universal phenomenon in various physical theories, containing crucial information about their non-perturbative dynamics.

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