Efficient RCS Computation Over a Broad Frequency Band Using Subdomain MoM and Chebyshev Approximation Technique

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ABSTRACT The analysis of the target’s broadband electromagnetic scattering characteristics plays an important role in radar stealth and recognition areas. The traditional method of moments (MoM) needs to calculate the currents at each frequency point when analyzing the target. With the increase of the target electrical size and complexity, the radar cross section (RCS) of the target changes drastically with frequency, it is necessary to calculate the accurate frequency response with a small frequency interval, which is very time-consuming in calculation and not feasible. This paper proposes to combine the subdomain method of moments (SMoM) with the Chebyshev approximation technique (CAT) to efficiently analyze the electromagnetic scattering of arbitrary shaped objects over a broad frequency band. In the SMoM technique, the whole object can be divided into several regions, each of which can be independently solved by the method of moments (MoM). The CAT technique is employed to expand the unknown current coefficients into a Chebyshev series for achieving fast frequency sweeping. Instead of direct point-by-point simulations, it is only necessary to calculate the currents by SMoM at several Chebyshev-Gauss frequency sample points. Furthermore, the Chebyshev series are matched via the Maehly approximation to a rational function to improve the accuracy. Numerical examples show that the hybrid technique can greatly reduce the computation time without loss of accuracy.

INDEX TERMS Method of moments (MoM), Chebyshev approximation technique (CAT), hybrid analysis, fast frequency sweeping, Maehly approximation.

I. INTRODUCTION
Accurate radar cross section (RCS) prediction of complex objects over a broad frequency band plays an important role in radar imaging, target detection and recognition areas. The method of moments (MoM) has been a very useful tool in the past decades to solve electromagnetic scattering problems. However, the memory requirement is $O(N^2)$ and the computational complexity is $O(N^3)$ for a direct solver and $O(N^2)$ for an iterative one in the traditional MoM, where $N$ is the number of unknowns. For complex objects, the convergence of MoM based on an iterative solver may be very slow since the electrically large objects with small features will lead to the non-uniform meshes.

In order to efficiently analyze the multi-scale problems and the non-uniform meshes problems, many algorithms have been developed in recent years. One is a variety of precondition techniques, such as Block-diagonal [1], Incomplete LU (ILU) [2], Sparse approximate inverse (SAI) [3], Incomplete-Cholesky (IC) [4], and so on. The other is the Characteristic Basis Function methods (CBFM) [5], [6], the integral equation domain decomposition method (IE-DDM) [7] and subdomain MoM (SMoM) [8]. Among these methods, the SMoM introduces the idea of domain decomposition method (DDM), the electrically large size objects with small features is divided into several regions,
Numerical results are presented in Section III to demonstrate the accuracy and efficiency of the hybrid technique and the conclusions are provided in Section IV.

II. FORMULATIONS

A. ELECTRIC FIELD INTEGRAL EQUATION (EFIE)

For simplicity, we consider a combined object consisting of two cones which is illuminated by an incident plane wave $\mathbf{E}^i$, as shown in Fig. 1(a).

![Geometry of (a) the whole object and (b) two subdomains.](image)

In the proposed SMoM-CAT method, first of all, we divide the whole domain $\Omega$ into two separated subdomains $\Omega_1 + \Omega_{12}$ and $\Omega_2 + \Omega_{21}$, as shown in Fig. 1(b). Two artificial surfaces which are denoted by the blue part are introduced to close these two subdomains. $J_1$, $J_{12}$, $J_{21}$, and $J_2$ are the electric currents on the surfaces $\partial \Omega_1$, $\partial \Omega_1$, $\partial \Omega_2$, and $\partial \Omega_2$, respectively.

For subdomain 1, the electric field integral equation (EFIE) is given by

$$\left. \left( \mathbf{E}^i + \mathbf{E}^S_{12} \right) \right|_{\text{tan}} = \mathbf{E}^i_1 + \mathbf{E}^S_{12} + \mathbf{E}^S_{21} + \mathbf{E}^S_{\text{tan}} \right|_{\text{tan}} \quad \text{on } \Omega_1 + \tilde{\Omega}_{12}$$

(1)

where

$$E^S_\alpha = -L(J_\alpha) = -jk \eta_0 \int_{S_{\alpha}} \left[ J_\alpha(r') + \frac{1}{k} \nabla \cdot J_\alpha(r') \right] \mathbf{G}(r,r') \, dr'$$

(2)

and the subscript $\alpha$ stands for 1, 12, 21, and 2.

Similarly, for subdomain 2, the EFIE is given by

$$\left. \left( \mathbf{E}^i_2 + \mathbf{E}^S_{21} \right) \right|_{\text{tan}} = \mathbf{E}^i_2 + \mathbf{E}^S_{21} + \mathbf{E}^S_{12} + \mathbf{E}^S_1 \text{ on } \Omega_2 + \tilde{\Omega}_{21}$$

(3)

B. CHEBYSHEV APPROXIMATION TECHNIQUE BASED ON SMoM (SMoM-CAT)

For a numerical solution of the EFIEs, these two subdomains can be discretized into a number of small triangular patches, and the equivalent currents on each subdomain can be expanded by the RWG basis functions $f_k(r)$:

$$\begin{align*}
J_1(r) &= \sum_{k=1}^{N_1} I_{1,k} f_{1,k}(r), \quad r \in \Omega_1 \\
J_{12}(r) &= \sum_{k=1}^{N_{12}} I_{12,k} f_{12,k}(r), \quad r \in \tilde{\Omega}_{12}
\end{align*}$$

(4)

Each of which can be solved independently by MoM. Since only one subdomain needs to be calculated at a time, the amount of storage can be drastically reduced. However, if the wide-band RCS computation is of interest, the SMoM requires to repeatedly solving the electric field integral equations (EFIEs) at each frequency point, which leads to a lot of computation time. The asymptotic waveform evaluation (AWE) [9] and model-based parameter estimation (MBPE) [10] are proposed to achieve fast frequency sweeping. In the AWE and MBPE technique, the current coefficients in wide-band frequency response are interpolated by a low-order rational function, namely Padé approximation. Some other model order reduction (MOR) techniques, such as the impedance matrix interpolation technique [11] and the Cauchy method [12], are applied for fast frequency or angular sweeping as well. Recently, the Chebyshev approximation technique (CAT) is present to interpolate the frequency response by the Chebyshev polynomials [13]–[17]. Among the above techniques, the AWE is quite popular and applicable to the CBF and MoM its fast solvers [18]–[21]. However, if the wide-band RCS computation is of interest, the SMoM is much easier to integrate into the SMoM code and it does not require computing the high-order derivative matrices.

In this paper, a novel hybrid SMoM-CAT is proposed to achieve fast wide-band RCS prediction of arbitrary 3D objects. In the proposed SMoM-CAT technique, the whole object is divided into several subdomains and the initial surface currents on these subdomains are obtained at several Chebyshev-Gauss frequency points. The modified excitation vectors of each subdomain can be updated by coupling the other subdomains’ contributions and the new currents on these subdomains are recomputed at these Chebyshev-Gauss points. An iterative procedure is implemented until the desired accuracy is satisfied. Finally, the currents at any frequency can be obtained by the Maehly approximation. Distinguished from the prior publications [7], [8], [13]–[17], the implementation of SMoM-CAT method is presented as follows:

1) The SMoM technique introduces the artificial surfaces. Therefore, the currents on the real surfaces and the artificial surfaces over a broad frequency band are explored.

2) Both the currents on the real surfaces and artificial surfaces are employed to compute the coefficients of Chebyshev series and Maehly approximation.

3) The wide-band RCS of SMoM-CAT with different orders $L$ are explored and the memory and computation time of SMoM-CAT with different orders $L$ are discussed.

The remainder of this paper is organized as follows. In Section II, the method of establishing the electric field integral equation (EFIE) for connected objects and the principle of SMoM algorithm are introduced; the formula and solution procedure of the proposed SMoM-CAT algorithm are given. Numerical results are presented in Section III to demonstrate...
where $N_1, N_{12}, N_2, N_{21}$ are the number of basis functions, $I_{1,k}$, $I_{2,k}$ denote the current coefficients on the real surface and artificial surface of subdomain 1, respectively, $I_{21,k}, I_{2,k}$ stand for the current coefficients on the real surface and artificial surface of subdomain 2.

Substituting (4), (5) into (1), (3) and applying the Galerkin’s procedure results in the following linear systems for each subdomain:

\[
\begin{align*}
\begin{bmatrix} \tilde{Z}_1 \end{bmatrix} (k) & = \begin{bmatrix} \tilde{I}_1 \end{bmatrix} (k) = \tilde{V}_1 (k) - \Delta \tilde{V}_1 (k) \quad \text{on } \Omega_1 + \tilde{\Omega}_{12} \\
\begin{bmatrix} \tilde{Z}_2 \end{bmatrix} (k) & = \begin{bmatrix} \tilde{I}_2 \end{bmatrix} (k) = \tilde{V}_2 (k) - \Delta \tilde{V}_2 (k) \quad \text{on } \tilde{\Omega}_2 + \tilde{\Omega}_{21}
\end{align*}
\]  

(6)

where $\tilde{I}_1, \tilde{I}_2$ are the column vectors with the size of $(N_1 + N_{12}) \times 1$, $(N_2 + N_{21}) \times 1$ for the unknown amplitudes of the basis functions on subdomain 1 and subdomain 2, respectively.

For subdomain 1, the impedance matrix $\tilde{Z}_1$, the excitation vector $\tilde{V}_1$, and the modified excitation vector $\Delta \tilde{V}_1$ can be expressed as:

\[
\begin{align*}
\tilde{Z}_1 & = \begin{bmatrix} Z_{1,1} & Z_{1,12} \\
Z_{12,1} & Z_{12,12} \end{bmatrix} \\
\tilde{V}_1 & = \begin{bmatrix} \tilde{V}_1 \\
\tilde{V}_{12} \end{bmatrix} \\
\Delta \tilde{V}_1 & = \begin{bmatrix} \Delta V_1 \\
\Delta V_{12} \end{bmatrix}
\end{align*}
\]  

(7)

(8)

(9)

The elements of $Z_{1,1}, Z_{1,12}, Z_{12,1}, Z_{12,12}, V_1, V_{12}, \Delta V_1, \Delta V_{12}$ are given by:

\[
\begin{align*}
Z_{\alpha, \beta, mk} & = \langle f_{\alpha, m} (r), L (f_{\beta, k} (r)) \rangle \\
V_{\alpha, m} & = \langle f_{\alpha, m} (r), E_{\alpha}^0 (r) \rangle \\
\Delta V_{\alpha, m} & = \langle f_{\alpha, m} (r), \left[ \sum_{k=1}^{N_1} I_{2,k} L (f_{2,k} (r)) \\
& \quad + \sum_{k=1}^{N_{12}} I_{21,k} L (f_{21,k} (r)) \right] \rangle
\end{align*}
\]  

(10)

(11)

(12)

where $\alpha$ denotes the subscript 1, 12, and $\beta$ also stands for the subscript 1, 12.

Similarly, for subdomain 2, the impedance matrix $\tilde{Z}_2$, the excitation vector $\tilde{V}_2$, and the modified excitation vector $\Delta \tilde{V}_2$ can be expressed as:

\[
\begin{align*}
\tilde{Z}_2 & = \begin{bmatrix} Z_{2,2} & Z_{2,21} \\
Z_{21,2} & Z_{21,21} \end{bmatrix} \\
\tilde{V}_2 & = \begin{bmatrix} \tilde{V}_2 \\
\tilde{V}_{21} \end{bmatrix} \\
\Delta \tilde{V}_2 & = \begin{bmatrix} \Delta V_2 \\
\Delta V_{21} \end{bmatrix}
\end{align*}
\]  

(13)

(14)

(15)

The elements of $Z_{2,2}, Z_{2,21}, Z_{21,2}, Z_{21,21}, V_2, V_{21}, \Delta V_2, \Delta V_{21}$ are given by:

\[
\begin{align*}
Z_{\alpha, \beta, mk} & = \langle f_{\alpha, m} (r), L (f_{\beta, k} (r)) \rangle \\
V_{\alpha, m} & = \langle f_{\alpha, m} (r), E_{\alpha}^0 (r) \rangle \\
\Delta V_{\alpha, m} & = \langle f_{\alpha, m} (r), \left[ \sum_{k=1}^{N_1} I_{2,k} L (f_{2,k} (r)) \\
& \quad + \sum_{k=1}^{N_{12}} I_{21,k} L (f_{21,k} (r)) \right] \rangle
\end{align*}
\]  

(16)

(17)

(18)

where $\alpha$ denotes the subscript 2, 21, and $\beta$ also stands for the subscript 2, 21.

Next, the $Q + 1$ Chebyshev–Gauss frequency points $\tilde{k}_q$ which are the roots of the Chebyshev polynomial $T_{Q+1}(x)$ are computed and then the Chebyshev–Gauss sampling points $k_q$ are obtained by transforming the $\tilde{k}_q$ from the interval $[-1, 1]$ to the desired band $[k_q, k_b]$ as:

\[
\tilde{k}_q = \cos \left( \frac{\pi (2q + 1)}{2 (n + 1)} \right)
\]  

(19)

\[
k_q = \frac{1}{2} \left( \tilde{k}_q (k_b - k_a) + (k_b + k_a) \right)
\]  

(20)

where $q = 0, 1, \ldots, Q$. And $Q$ denotes the truncated order of the Chebyshev series.

Secondly, the initial currents $	ilde{I}_1^0 (k_q) = [I_1^0 (k_q), I_{12}^0 (k_q)]^T$ on $\Omega_1 + \tilde{\Omega}_{12}$ and $	ilde{I}_2^0 (k_q) = [I_2^0 (k_q), I_{21}^0 (k_q)]^T$ on $\tilde{\Omega}_2 + \tilde{\Omega}_{21}$ at the certain $Q + 1$ Chebyshev–Gauss frequency points can be calculated by MoM:

\[
\begin{align*}
\tilde{Z}_1^0 (k_q) \tilde{I}_1^0 (k_q) & = \tilde{V}_1^0 (k_q)
\end{align*}
\]  

(21)

where the subscript $\alpha$ stands for 1, and 2.

Next, the boundary conditions are enforced on the artificial surfaces $\tilde{\Omega}_{12}$ and $\tilde{\Omega}_{21}$ by:

\[
\begin{align*}
\sum_{k=1}^{N_{12}} & I_{21,k} (k_q) \langle f_{21,k} (r), L (f_{21,k} (r)) \rangle \\
& = - \sum_{k=1}^{N_{12}} I_{21,k} (k_q) \langle f_{12,k} (r), L (f_{12,k} (r)) \rangle \\
& \quad - \sum_{k=1}^{N_{12}} I_{21,k} (k_q) \langle f_{21,k} (r), L (f_{21,k} (r)) \rangle
\end{align*}
\]  

(22)

(23)

Then the new $Q + 1$ excitation vectors $\tilde{V}_1^1 (k_q)$ and $\tilde{V}_2^1 (k_q)$ of subdomain 1 and subdomain 2 can be updated by:

\[
\begin{align*}
\tilde{V}_1^1 (k_q) & = \tilde{V}_1^0 (k_q) - \Delta \tilde{V}_1^1 (k_q) \\
& = [\tilde{V}_1^0 (k_q) - \Delta \tilde{V}_1^1 (k_q), \tilde{V}_{12}^0 (k_q) - \Delta \tilde{V}_{12}^1 (k_q)]^T \\
\tilde{V}_2^1 (k_q) & = \tilde{V}_2^0 (k_q) - \Delta \tilde{V}_2^1 (k_q) \\
& = [\tilde{V}_2^0 (k_q) - \Delta \tilde{V}_2^1 (k_q), \tilde{V}_{21}^0 (k_q) - \Delta \tilde{V}_{21}^1 (k_q)]^T
\end{align*}
\]  

(24)

(25)
Thirdly, the new currents $\tilde{I}_1(k_q) = [I_1^1(k_q), I_1^2(k_q)]^T$ on $\Omega_1 + \tilde{\Omega}_{12}$ and $\tilde{I}_2(k_q) = [I_2^1(k_q), I_2^2(k_q)]^T$ on $\Omega_2 + \tilde{\Omega}_{21}$ at the certain $Q+1$ frequency points can be obtained by:

$$
\tilde{Z}_a(k_q) \tilde{I}_a^1(k_q) = \tilde{V}_a^1(k_q) \quad (26)
$$

Next, the iterative procedure is carried out from (21) to (26) by using “i” instead of “1”. The convergence criterion is that the maximum errors of the currents at $Q+1$ frequency points on two subdomains are checked whether they are satisfied the desired accuracy:

$$
I_{\text{error}}^i(k_q) = \max \left( \frac{\|\tilde{I}_1^i(k_q) - \tilde{I}_1^{i-1}(k_q)\|}{\|\tilde{I}_1^{i-1}(k_q)\|}, \frac{\|\tilde{I}_2^i(k_q) - \tilde{I}_2^{i-1}(k_q)\|}{\|\tilde{I}_2^{i-1}(k_q)\|} \right) \quad (27)
$$

Fourthly, the currents $\tilde{I}_1(k)$ and $\tilde{I}_2(k)$ on $\Omega_1 + \tilde{\Omega}_{12}$ and $\Omega_2 + \tilde{\Omega}_{21}$ at any frequency point can be approximated as:

$$
\tilde{I}_a(k) \approx \sum_{q=0}^{Q} m_{a,q} T_q(\tilde{k}_q) - \frac{1}{2} m_{a,0} \quad (28)
$$

where $T_q(x)$ denotes the Chebyshev polynomial which satisfies the recursion relation in [8], $m_{a,q}$ are the expanding coefficients. The elements of $m_{1,q}$ on $\Omega_1 + \tilde{\Omega}_{12}$ can be expressed as:

$$
m_{1,\beta,q} = \frac{2}{Q+1} \sum_{q=0}^{Q} I_{1,\beta,q}(k_q) T_q(\tilde{k}_q), \quad \beta = 1, 2, \ldots, N_1 \quad (29)
$$

$$
m_{12,\beta,q} = \frac{2}{Q+1} \sum_{q=0}^{Q} I_{12,\beta,q}(k_q) T_q(\tilde{k}_q), \quad \beta = 1, 2, \ldots, N_2 \quad (30)
$$

where $N_1$ and $N_2$ are the number of unknowns on $\Omega_1$ and $\tilde{\Omega}_{12}$, respectively.

The elements of $m_{2,q}$ on $\Omega_2 + \tilde{\Omega}_{21}$ can be obtained by:

$$
m_{2,\beta,q} = \frac{2}{Q+1} \sum_{q=0}^{Q} I_{2,\beta,q}(k_q) T_q(\tilde{k}_q), \quad \beta = 1, 2, \ldots, N_2 \quad (31)
$$

$$
m_{21,\beta,q} = \frac{2}{Q+1} \sum_{q=0}^{Q} I_{21,\beta,q}(k_q) T_q(\tilde{k}_q), \quad \beta = 1, 2, \ldots, N_2 \quad (32)
$$

Finally, the Maehly approximation is employed to achieve a better accuracy by a rational function:

$$
\tilde{I}_a(k) \approx \frac{\sum_{i=0}^{L} a_{\alpha,\beta,i} T_i(\tilde{k})}{1 + \sum_{j=1}^{M} b_{\alpha,\beta,j} T_j(\tilde{k})} \quad (33)
$$

where $L$ and $M$ are the order of unknown coefficients $a_{\alpha,\beta,i}$ and $b_{\alpha,\beta,j}$, respectively.

The coefficients in the numerator and the denominator of the Maehly approximation on $\Omega_1 + \tilde{\Omega}_{12}$ are obtained by:

$$
\sum_{j=1}^{M} b_{1,\beta,j} (m_{1,\beta,L+i+j} + m_{1,\beta,L+i-j}) = -2m_{1,\beta,L+i} \quad (34)
$$

$$
\sum_{j=1}^{M} b_{12,\beta,j} (m_{12,\beta,L+i+j} + m_{12,\beta,L+i-j}) = -2m_{12,\beta,L+i} \quad (35)
$$

$$
a_{1,\beta,i} = m_{1,\beta,i} + \frac{1}{2} \sum_{j=1}^{M} b_{1,\beta,j} (m_{1,\beta,j+i} + m_{1,\beta,j-i}) \quad (36)
$$

$$
a_{12,\beta,i} = m_{12,\beta,i} + \frac{1}{2} \sum_{j=1}^{M} b_{12,\beta,j} (m_{12,\beta,j+i} + m_{12,\beta,j-i}) \quad (37)
$$

The coefficients of the Maehly approximation on $\Omega_2 + \tilde{\Omega}_{21}$ can be obtained by:

$$
\sum_{j=1}^{M} b_{2,\beta,j} (m_{2,\beta,L+i+j} + m_{2,\beta,L+i-j}) = -2m_{2,\beta,L+i} \quad (38)
$$

$$
\sum_{j=1}^{M} b_{21,\beta,j} (m_{21,\beta,L+i+j} + m_{21,\beta,L+i-j}) = -2m_{21,\beta,L+i} \quad (39)
$$

$$
a_{2,\beta,i} = m_{2,\beta,i} + \frac{1}{2} \sum_{j=1}^{M} b_{2,\beta,j} (m_{2,\beta,j+i} + m_{2,\beta,j-i}) \quad (40)
$$

$$
a_{21,\beta,i} = m_{21,\beta,i} + \frac{1}{2} \sum_{j=1}^{M} b_{21,\beta,j} (m_{21,\beta,j+i} + m_{21,\beta,j-i}) \quad (41)
$$

A flow chart is shown to help readers better understand the procedure of the hybrid SMoM-CAT method, as illustrated in Fig. 2.

III. NUMERICAL RESULTS

In this section, three examples are presented to show the accuracy and efficiency of the proposed SMoM-CAT method for fast wide-band scattering analysis of arbitrarily shaped 3D object. All the computations were carried out serially on a PC with 4.2 GHz Intel CPU and 8GB RAM.

A. A FOUR PATCH ARRAY

To show the accuracy of the SMoM-CAT algorithm for sparse objects, the first example we consider is a four PEC patch array, and each patch element has a size of 2.5 cm $\times$ 2.5 cm, as shown in Fig. 3. The gap between the two adjacent elements is 0.5 cm. The incident direction of the plane wave is $\theta_{inc} = 60^\circ$ and $\varphi_{inc} = 0^\circ$. In the SMoM and SMoM-CAT analysis, the whole array is divided into four subdomains. Fig. 4 shows the $\theta\theta$ polarized monostatic RCS from 2 GHz to 12 GHz by using three different orders of SMoM-CAT ($L = 1, 3, 5$). The numerical results of SMoM-CAT are compared to that of point-by-point SMoM simulations, we can see that the 1st order approximation cannot get the correct results.
results, and the 3rd order approximation becomes better but is still not accurate around 8.5 GHz. The 5th order approximation can produce satisfactory results over the entire band. Fig. 5 illustrated that the SMoM and SMoM-CAT results and they agree well with the MoM solutions. The step frequency of MoM, SMoM and SMoM-CAT are all 100 MHz.

To further verify the accuracy of the proposed SMoM-CAT algorithm for sparse objects, we extract the currents at the non-Gaussian sampling point calculated by the SMoM-CAT method to calculate the bistatic RCS and compare it with the calculation result of MoM. Fig. 6 shows the bistatic results obtained at the non-Gaussian sampling point calculated at 7 GHz according to the SMoM-CAT method at L = 5,
the incident direction of the plane wave is $\theta_{\text{inc}} = 60^\circ$ and $\varphi_{\text{inc}} = 0^\circ$, the bistatic acceptance angle is $\theta_{\text{sca}} = 0 \sim 360^\circ$ and $\varphi_{\text{sca}} = 0^\circ$, the polarization is $\theta \theta$ polarization. Compare its bistatic results with the results of MoM calculations we can see that the two are well fitted, and the accuracy of the proposed algorithm for sparse objects is further verified.

**B. A MISSILE MODEL**

To show the accuracy of the SMoM-CAT algorithm for connected objects, the second example we consider a missile model. The length, width and height of the missile model are 3.63 cm, 1.49 cm, and 0.95 cm, respectively. In the SMoM and SMoM-CAT technique, the whole missile model is divided into six subdomains and these subdomains are displayed in different colors, as shown in Fig. 7. The incident direction of the planewave is $\theta_{\text{inc}} = 60^\circ$ and $\varphi_{\text{inc}} = 0^\circ$. Fig. 8 shows the $\theta \theta$ polarized monostatic RCS from 2 GHz to 18 GHz obtained from the SMoM-CAT with three different order ($L = 4, 7, 10$). Compared to the SMoM solutions, we can observe that the 3rd order approximation gives incorrect results across the entire frequency band. The 5th order approximation can give better results over the entire band but the results are not accurate at 4 to 10GHz. When the 7th order approximation is applied, we can obtain the results with good accuracy over the entire band. Fig. 9 illustrated that the SMoM and SMoM-CAT results and they agree well with the MoM solutions. The frequency sweeping increment is 100 MHz for the SMoM-CAT, the SMoM and the MoM methods.

To further verify the accuracy of the proposed SMoM-CAT algorithm for connected objects, we extract the currents at the non-Gaussian sampling point calculated by the SMoM-CAT method at $L = 7$, the bistatic RCS was obtained by using the current at a frequency of 10 GHz and compared with the calculation result of MoM. As shown in Fig. 10, the incident direction of the plane wave is $\theta_{\text{inc}} = 60^\circ$ and $\varphi_{\text{inc}} = 0^\circ$, the bistatic acceptance angle is $\theta_{\text{sca}} = 0 \sim 360^\circ$ and $\varphi_{\text{sca}} = 0^\circ$, the polarization is $\theta \theta$ polarization, we can see from the figure that the results are in good agreement,
and the accuracy of the proposed algorithm connected objects is further verified.

C. AN AIRCRAFT MODEL

To show the accuracy of the SMoM-CAT algorithm for connected objects, the third example we consider an aircraft model. The length, width and height of the aircraft model are 0.95 m, 0.69 m and 0.195 m, respectively. In the SMoM and SMoM-CAT technique, the whole aircraft model is divided into four subdomains and these subdomains are displayed in different colors, as shown in Fig. 11. The incident direction of the plane wave is $\theta_{\text{inc}} = 90^\circ$ and $\phi_{\text{inc}} = 0^\circ$. Fig. 12 shows the $\theta\theta$ polarized monostatic RCS from 0.2 GHz to 1.2 GHz obtained from the SMoM-CAT with three different order ($L = 4, 7, 10$). Compared to the SMoM solutions, we can observe that the 4th order approximation gives accurate results from 0.2 GHz to 1.2 GHz. It provides incorrect results out of these bands. The 7th order approximation can give better results over the entire band except the peak around 0.4 GHz is not correct. When the 10th order approximation is applied, we can obtain the results with good accuracy over the entire band. Fig. 13 illustrated that the SMoM...
and SMoM-CAT results and they agree well with the MoM solutions. The frequency sweeping increment is 10 MHz for the SMoM-CAT, the SMoM and the MoM methods.

For example C, we also extract the currents at the non-Gaussian sampling point calculated by the SMoM-CAT method to calculate the bistatic RCS and compare it with the calculation result of MoM. Fig. 14 shows the bistatic results obtained at the non-Gaussian sampling point calculated at 0.7 GHz according to the SMoM-CAT method at L = 11, the incident direction of the plane wave is \( \theta_{\text{inc}} = 90^\circ \) and \( \varphi_{\text{inc}} = 0^\circ \), the bistatic acceptance angle is \( \theta_{\text{sca}} = 0^\circ \sim 360^\circ \) and \( \varphi_{\text{sca}} = 0^\circ \), the polarization is \( \theta\theta \) polarization. Compare its bistatic results with the results of MoM calculations we can see that the two curves are well fitted.

The memory requirement and total CPU time of the SMoM-CAT technique with three different orders for the patch array, the missile model, and the aircraft model are illustrated in Table 1, Table 2, and Table 3, respectively. We can observe that the memory requirement and CPU time are gradually rising as we increase the order of the SMoM-CAT. The reason is that the higher order is chosen, the more currents at Chebyshev-Gauss sampling frequency points are needed to be computed and stored.

### TABLE 1. CPU time and memory requirements of SMoM-CAT for wideband RCS simulation of patch array at different orders.

| problems   | Methods           | No. of frequency points | No. of Moments | Memory (MB) | CPU time (s) |
|------------|-------------------|-------------------------|----------------|-------------|--------------|
| Patch array| SMoM-CAT (1st order) | 101                     | 4              | 79.60       | 230          |
|            | SMoM-CAT (3rd order) |                         | 10             | 238.80      | 691          |
|            | SMoM-CAT (5th order) |                         | 16             | 398.00      | 1,152        |

### TABLE 2. CPU time and memory requirements of SMoM-CAT for wideband RCS simulation of Missile model at different orders.

| problems   | Methods           | No. of frequency points | No. of Moments | Memory (MB) | CPU time (s) |
|------------|-------------------|-------------------------|----------------|-------------|--------------|
| Missile model| SMoM-CAT (3rd order) | 181                     | 10             | 205.92      | 515          |
|            | SMoM-CAT (5th order) |                         | 16             | 343.20      | 824          |
|            | SMoM-CAT (7th order) |                         | 22             | 480.48      | 1122         |

### TABLE 3. CPU time and memory requirements of SMoM-CAT for wideband RCS simulation of Aircraft model at different orders.

| problems   | Methods           | No. of frequency points | No. of Moments | Memory (MB) | CPU time (s) |
|------------|-------------------|-------------------------|----------------|-------------|--------------|
| Aircraft model| SMoM-CAT (4th order) | 101                     | 13             | 3038.76     | 9,064        |
|            | SMoM-CAT (7th order) |                         | 22             | 5317.83     | 15,863       |
|            | SMoM-CAT (10th order) |                         | 31             | 7596.91     | 22,661       |

FIGURE 13. Variations of the RCS of an aircraft model with frequency.

FIGURE 14. Bistatic RCS of an aircraft model by MoM and SMoM-CAT.
Table 4 summarizes the number of unknowns, the average number of iterations, the memory requirement, and the total CPU time of the SMoM-CAT, the SMoM, and the MoM, respectively. From Table 4, we can see that, compared with the SMoM, the SMoM-CAT can achieve 84%, 87.8%, and 69% reduction on CPU time for the patch array, the missile, and the aircraft, respectively.

### IV. CONCLUSIONS AND DISCUSSIONS

The hybrid SMoM-CAT technique is presented for fast analyzing the scattering problem of 3D PEC objects over a broad frequency band. The SMoM is employed to flexibly model the object with small subdomains and each subdomain can be handled independently. For efficiently predicting the wide-band RCS, the Chebyshev approximation technique is combined with the SMoM to significantly reduce the computation time and maintain good accuracy. Since the surface currents of all subdomains over the desired frequency band have been obtained, the proposed SMoM-CAT can be also used to calculate the bistatic RCS at any frequency point.

### REFERENCES

[1] Ö. Ergül and L. Gürel, “Efficient solution of the electric and magnetic current combined-field integral equation with the multilevel fast multipole algorithm and block-diagonal preconditioning,” *Radio Sci.*, vol. 44, no. 6, pp. 1–15, Dec. 2009.

[2] H. Wang, L. Xu, J. Li, and B. Li, “An inverse-based multifrontal block incomplete LU preconditioner for the 3-D finite-element eigenvalue analysis of lossy slow-wave structures,” *IEEE Trans. Microw. Theory Techn.*, vol. 63, no. 7, pp. 2094–2106, Jul. 2015.

[3] J. B. Liu, Z. R. Li, H. Zhang, and J. X. Su, “The application of sparse approximate inverse preconditioner in the fast solution of volume-surface integral equation,” in *Proc. Int. Appl. Comput. Electromagn. Soc. Symp. (ACES)*, Suzhou, China, 2017, pp. 1–2.

[4] T. Tsibouraya, Y. Okamoto, and Z. Meng, “Improvement of block IC preconditioner using fill-in technique for linear systems derived from finite-element method including thin elements,” *IEEE Trans. Magn.*, vol. 54, no. 3, pp. 1–4, Mar. 2018, Art no. 7202504.

[5] V. V. S. Prakash and R. Mittra, “Characteristic basis function method: A new technique for efficient solution of method of moments matrix equations,” *Microw. Opt. Technol. Lett.*, vol. 36, no. 2, pp. 95–100, Jan. 2003.

[6] R. Mittra and K. Du, “Characteristic basis function method for iteration-free solution of large method of moments problems,” *Prog. Electromagn. Res. B*, vol. 6, pp. 307–336, 2008.

[7] Z. Peng, X.-C. Wang, and J.-F. Lee, “Integral equation based domain decomposition method for solving electromagnetic wave scattering from non-penetrable objects,” *IEEE Trans. Antennas Propag.*, vol. 59, no. 9, pp. 3328–3338, Sep. 2011.

[8] X. Wang, S. Zhang, Z.-L. Liu, and C.-F. Wang, “A SAIM-FAFFA method for efficient computation of electromagnetic scattering problems,” *IEEE Trans. Antennas Propag.*, vol. 64, no. 12, pp. 5507–5512, Dec. 2016.

[9] C. J. Reddy, M. D. Deshpande, C. R. Cockrell, and F. B. Beck, “Fast RCS computation over a frequency band using method of moments in conjunction with asymptotic waveform evaluation technique,” *IEEE Trans. Antennas Propag.*, vol. 46, no. 8, pp. 1229–1233, Aug. 1998.

[10] X. Wang and D. H. Werner, “Improved model-based parameter estimation approach for accelerated periodic method of moments solutions with application to the analysis of convoluted frequency selected surfaces and metamaterials,” *IEEE Trans. Antennas Propag.*, vol. 58, no. 1, pp. 122–131, Jan. 2010.

[11] X. C. Wei and E. P. Li, “Wide-band EMC analysis of on-platform antennas using impedance-matrix interpolation with the moment-method-physical optics method,” *IEEE Trans. Electromagn. Compat.*, vol. 45, no. 3, pp. 552–556, Aug. 2003.

[12] J. Yang and T. K. Sarkar, “Interpolation/extrapolation of radar cross-section (RCS) data in the frequency domain using the cauchy method,” *IEEE Trans. Antennas Propag.*, vol. 55, no. 10, pp. 2844–2851, Oct. 2007.

[13] M. S. Chen, Z. X. Huang, W. Sha, and X. L. Wu, “Fast and accurate radar cross-section computation over a broad frequency band using the best uniform rational approximation,” *IET Microw., Antennas Propag.*, vol. 2, no. 2, pp. 200–204, Mar. 2008.

[14] J. Ling, S.-X. Gong, X. Wang, B. Lu, and W.-T. Wang, “A novel two-dimensional extrapolation technique for fast and accurate radar cross section computation,” *IEEE Antennas Wireless Propag. Lett.*, vol. 9, pp. 244–247, 2010.

[15] H.-L. Dong, P.-F. Zhang, J. Ma, B. Zhao, and S.-X. Gong, “Fast and accurate analysis of broadband RCS using method of moments with loop-tree basis functions,” *IET Microw. Antennas Propag.*, vol. 9, no. 8, pp. 775–780, Jan. 2015.

[16] Y.-R. Jeong, I.-P. Hong, Y. B. Park, H.-J. Chun, Y. J. Kim, and J.-G. Yook, “Fast scattering analysis over a wide frequency band using Clenshaw–Lord-type Pade–Chebyshev approximation,” *IET Microw., Antennas Propag.*, vol. 10, no. 3, pp. 245–250, Feb. 2016.

[17] X. Wang, S. Zhang, H. Xue, S.-X. Gong, and Z.-L. Liu, “A Chebyshev approximation technique based on AIM-PO for wideband analysis,” *IEEE Antennas Wireless Propag. Lett.*, vol. 15, pp. 93–97, 2016.

[18] Y. Sun, Y. Du, and Y. Shao, “Fast computation of wideband RCS using characteristic basis function method and asymptotic waveform evaluation technique,” *J. Electron.*, vol. 27, no. 4, pp. 453–457, Jul. 2010.

[19] A. A. Kucharski, “Wideband characteristic basis functions in radiation problems,” *Radio Eng.*, vol. 21, no. 2, pp. 590–596, Jun. 2012.

[20] X. Wang, S.-X. Gong, J.-L. Guo, Y. Liu, and P.-F. Zhang, “Fast and accurate wide-band analysis of antennas mounted on conducting platform using AIM and asymptotic waveform evaluation technique,” *IEEE Trans. Antennas Propag.*, vol. 59, no. 12, pp. 4624–4633, Dec. 2011.

[21] B.-Y. Wu and X.-Q. Sheng, “Application of asymptotic waveform evaluation to hybrid FE-BI-MLFMA for fast RCS computation over a frequency band,” *IEEE Trans. Antennas Propag.*, vol. 61, no. 5, pp. 2597–2604, May 2013.
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