Gain Scheduling Control of Gas Turbine Engines: Absolute Stability by Finding a Common Lyapunov Matrix

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Abstract
This manuscript aims to develop and describe gain scheduling control concept for a gas turbine engine which drives a variable pitch propeller. An architecture for gain-scheduling control is developed that controls the turboshaft engine for large thrust commands in stable fashion with good performance. Fuel flow and propeller pitch angle are the two control inputs of the system. New stability proof has been developed for gain scheduling control of gas turbine engines using global linearization and LMI techniques. This approach guarantees absolute stability of the closed loop gas turbine engines with gain-scheduling controllers.

Nomenclature
N₁: Non-dimensional Fan Spool Speed
N₂: Non-dimensional Core Spool Speed
T: hrust (N)
TSFC: Thrust Specific Fuel Consumption
α: Scheduling Parameter
σ: Singular Value
λ: Eigenvalue

1 Introduction
The gain-scheduling approach is perhaps one of the most popular nonlinear control design approaches which has been widely and successfully applied in fields ranging from aerospace to process control [17, 11]. Gain-scheduling, specifically has been used for gas turbine engine control, some of these works are [2, 10, [4, 21, 25, 24]. In general, stability and control of gas turbine engines have been of interest to researchers and engineers from a variety of perspectives. Stability of axial flow fans operating in parallel has been investigated in [22]. An application of robust stability analysis tools for uncertain turbine engine systems is presented in [1]. Application of the Linear-Quadratic-Gaussian with Loop-Transfer-Recovery methodology

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to design of a control system for a simplified turbofan engine model is considered in [8]. A unified robust multivariable approach to propulsion control design has been developed in [7]. A simplified scheme for scheduling multivariable controllers for robust performance over a wide range of turbofan engine operating points is presented in [9].

In the previous work by authors [16, 14] controllers developed for single spool and twin spool turboshaft system. Those controllers were designed for small transients, and small throttle commands. In this work we develop a gain-scheduling control structure for JetCat SPT5 turboshaft engine using the method presented in [18, 17, 19, 20]. The controller is designed to be used for entire flight envelope of the twin spool turboshaft engine.

In this manuscript, first a linear representation of the turbofan system dynamics is developed. Then control theoretic concepts for gain-scheduling control of this model is presented. The developed controller can be used for the entire flight envelope of the engine with guaranteed stability. Finally the simulation results for gain scheduling control of a physics-based nonlinear model of the JetCat SPT5 turboshaft engine are presented.

2 Gain Scheduling Control Design

Consider the nonlinear dynamical system

\[
\begin{align*}
\dot{x}_p(t) &= f^p(x_p(t), u(t)), \\
y(t) &= g^p(x_p(t), u(t)),
\end{align*}
\]

where \(x^p \in \mathbb{R}^n\) is the state vector, \(u \in \mathbb{R}^m\) is the control input vector, \(y \in \mathbb{R}^k\) is the output vector, \(f^p(.)\) is an \(n\)-dimensional, and \(g^p(.)\) is an \(k\)-dimensional differentiable nonlinear vector functions. We want to design a feedback control such that \(y(t) \rightarrow r(t)\) as \(t \rightarrow \infty\), where \(r(t) \in D_r \subset \mathbb{R}^k\) is the output reference signals vector.

Assume that for each \(r(t) \in D_r\), there is a unique pair \((x^p_c, u_c)\) that depends continuously on \(r\) and satisfies the equations:

\[
\begin{align*}
0 &= f^p(x^p_c, u_c), \\
r &= g^p(x^p_c, u_c),
\end{align*}
\]

in case of a constant \(r\). \(x^p_c\) is the desired equilibrium point and \(u_c\) is the steady-state control that is needed to maintain equilibrium at \(x^p_c\).

**Definition 1.** The functions \(x^p_c(\alpha), u_c(\alpha)\), and \(r_c(\alpha)\) define an equilibrium family for the plant \((1)\) on the set \(\Omega\) if

\[
\begin{align*}
f^p(x^p_c(\alpha), u_c(\alpha), r_c(\alpha)) &= 0, \\
g^p(x^p_c(\alpha), u_c(\alpha)) &= r_c(\alpha), \quad \alpha \in \Omega.
\end{align*}
\]

Let \(\Omega \subset \mathbb{R}^{m+n}\) be the region of interest for all possible system state and control vector \((x^p, u)\) during the system operation, and denote \(x^p_i\) and \(u^*_i\), \(i \in I = 1, 2, ..., l\), as a set of (constant) operating points located at some representative (and properly separated) points inside \(\Omega\). Introduce a set of \(l\) regions \(\Omega_i\) centered at the chosen operating points \((x^p_i, u^*_i)\), and denote their interiors as \(\Omega^*_i\), such that \(\Omega^*_j \cap \Omega^*_k = \emptyset\) for all \(j \neq k\), and \(\bigcup_{i=1}^l \Omega_i = \Omega\). The linear model around each equilibrium point is

\[
\begin{align*}
\dot{x} &= A^p_i(x - x^p_i) + B^p_i(u - u^*_i), \\
y &= C^p_i(x - x^p_i) + D^p_i(u - u^*_i) + y^*_i,
\end{align*}
\]
where the matrices are obtained as follows

\[
\begin{align*}
A_p^i &= \frac{\partial f_p}{\partial x_p}(x_p^i,u_p^i), \quad \forall (x^p,u) \in \Omega_i, \\
B_p^i &= \frac{\partial f_p}{\partial u}(x_p^i,u_p^i), \quad \forall (x^p,u) \in \Omega_i, \\
C_p^i &= \frac{\partial g_p}{\partial x_p}(x_p^i,u_p^i), \quad \forall (x^p,u) \in \Omega_i, \\
D_p^i &= \frac{\partial g_p}{\partial u}(x_p^i,u_p^i), \quad \forall (x^p,u) \in \Omega_i.
\end{align*}
\]

(5)

Here we assume that the common boundary of two regions \(\Omega_j \) and \(\Omega_2 \) belongs to only one of \(\Omega_j \) and \(\Omega_2 \). Note that at each moment, \((x^p,u)\) belongs to only one \(\Omega_i \).

Performing linearizations at a series of trim points gives a linearization family described by

\[
\begin{align*}
\delta \dot{x}^p &= A^p(\alpha)\delta x^p + B^p(\alpha)\delta u, \\
\delta y &= C^p(\alpha)\delta x^p + D^p(\alpha)\delta u.
\end{align*}
\]

(6)

where

\[
\begin{align*}
\delta x^p &= x^p - x_e^p(\alpha) \\
\delta y &= y - y_e(\alpha), \\
\delta u &= u - u_e(\alpha), \quad \forall \alpha \in \Omega.
\end{align*}
\]

(7)

Gain scheduled controller for plant \([6]\), is designed as follows. First, a set of parameter values \(\alpha_i \) are selected, which represent the range of the plant’s dynamics, and a linear time-invariant controller for each is designed. Then, in between operating points, the controller gains are linearly interpolated such that for all frozen values of the parameters, the closed loop system has excellent properties, such as nominal stability and robust performance. To guarantee that the closed loop system will retain the feedback properties of the frozen-time designs, the scheduling variables should vary slowly with respect to the system dynamics \([18]\).

The parameter \(\alpha \) called the scheduling variable in gain scheduling control. \(A^p(\alpha), B^p(\alpha), C^p(\alpha), \) and \(D^p(\alpha) \) are the parameterized linearization system matrices and \(x_e^p(\alpha), u_e(\alpha), \) and \(y_e(\alpha) \) are the parameterized steady-state system variables, which form the equilibrium manifold of system \([1]\). The subscript ‘e’ stands for steady-state throughout this paper.

Let the controller have the following structure

\[
\begin{align*}
\dot{x}^c &= A^c(\alpha)\delta x^c + B^c(\alpha)[\delta y - \delta r], \\
\delta u &= C^c(\alpha)\delta x^c + D^c(\alpha)[\delta y - \delta r], \quad \forall \alpha \in \Omega.
\end{align*}
\]

(8)

where

\[
\begin{align*}
\delta x^c &= x^c - x_e^c(\alpha) \\
\delta r &= r - r_e(\alpha), \quad \forall \alpha \in \Omega.
\end{align*}
\]

(9)

A standard realization of the parameterized controller can be written in the following form with the reference signal explicitly displayed

\[
\begin{bmatrix}
\dot{x}^c \\
\delta u
\end{bmatrix} =
\begin{bmatrix}
A^c(\alpha) & B^c(\alpha) \\
C^c(\alpha) & D^c(\alpha)
\end{bmatrix}
\begin{bmatrix}
\delta x^c \\
\delta y \\
\delta r
\end{bmatrix}, \quad \forall \alpha \in \Omega.
\]

(10)

Controller has the general form

\[
\begin{align*}
\dot{x}^c(t) &= f^c(x^c(t), y(t), r(t)), \\
u(t) &= g^c(x^c(t), y(t), r(t)),
\end{align*}
\]

(11)

with the input and output signals corresponding to the nonlinear plant \([1]\).
The objective in linearization scheduling is that the equilibrium family of the controller \( (11) \) match the plant equilibrium family, so that the closed loop system maintains suitable trim values, and second the linearization family of the controller is the designed family of linear controllers \( [17] \).

For the equilibrium conditions of plant \( (1) \) and controller \( (11) \) to match, there must exist a function \( x_c^e(\alpha) \) such that
\[
\begin{align*}
0 &= f^e(x_c^e(\alpha), y_c(\alpha), r_c(\alpha)), \\
u_c(\alpha) &= g^e(x_c^e(\alpha), y_c(\alpha), r_c(\alpha)), \quad \forall \alpha \in \Omega, \\
\end{align*}
\]
where
\[
\begin{align*}
A^e(\alpha) &= \frac{\partial f^e}{\partial x^e}(x_c^e(\alpha), y_c(\alpha), r_c(\alpha)), \\
B^e(\alpha) &= \frac{\partial f^e}{\partial y}(x_c^e(\alpha), y_c(\alpha), r_c(\alpha)), \\
C^e(\alpha) &= \frac{\partial g^e}{\partial x^e}(x_c^e(\alpha), y_c(\alpha), r_c(\alpha)), \\
D^e(\alpha) &= \frac{\partial g^e}{\partial y}(x_c^e(\alpha), y_c(\alpha), r_c(\alpha)), \quad \forall \alpha \in \Omega.
\end{align*}
\]
So the controller family has the form
\[
\begin{align*}
\dot{x}^e &= A^e(\alpha)[x^e - x_c^e(\alpha)] + B^e(\alpha)[y - r], \\
u &= C^e(\alpha)[x^e - x_c^e(\alpha)] + D^e(\alpha)[y - r] + u_c(\alpha), \quad \forall \alpha \in \Omega.
\end{align*}
\]
Note that \( r_c(\alpha) = y_c(\alpha) \), as a result \( \delta y - \delta r = y - r \). The scheduling parameter \( \alpha \) is treated as a parameter throughout the design process, and then it becomes a time-varying input signal to the gain-scheduled controller implementation through the dependence \( \alpha(t) = p(y(t)) \). Thus the gain-scheduled controller becomes
\[
\begin{align*}
\dot{x}^e &= A^e(p(y))[x^e - x_c^e(p(y))] + B^e(p(y))[y - r], \\
u &= C^e(p(y))[x^e - x_c^e(p(y))] + D^e(p(y))[y - r] + u_c(p(y)).
\end{align*}
\]
Linearization of \( (15) \) about an equilibrium specified by \( \alpha \) gives
\[
\begin{align*}
\delta \dot{x}^e &= A^e(\alpha)\delta x^e + B^e(\alpha)[y - r] - [A^e(\alpha)\frac{\partial x_c^e(\alpha)}{\partial \alpha}] \times \frac{\partial p}{\partial y}(y_c(\alpha))(y - r), \\
\delta u &= C^e(\alpha)\delta x^e + D^e(\alpha)[y - r] + [\frac{\partial u_c(\alpha)}{\partial \alpha} - C^e(\alpha)\frac{\partial x_c^e(\alpha)}{\partial \alpha}] \times \frac{\partial p}{\partial y}(y_c(\alpha))(y - r).
\end{align*}
\]
Comparing this to \( (10) \), we see there are additional terms, which we refer to them as hidden coupling terms \( [17] \).

In order to get rid of these terms we have to design the controller such that
\[
\begin{align*}
A^e(\alpha)\frac{\partial x_c^e(\alpha)}{\partial \alpha} &= 0, \\
\frac{\partial u_c(\alpha)}{\partial \alpha} - C^e(\alpha)\frac{\partial x_c^e(\alpha)}{\partial \alpha} &= 0.
\end{align*}
\]
It is not always easy to come up with solutions to satisfy this condition. In order to make the design process easier, we can augment integrators at the plant input, so there is no need for equilibrium control value. By augmenting integrators at the plant \( (1) \) input we obtain
\[
\begin{bmatrix}
\dot{x}^p(t) \\
\dot{u}(t)
\end{bmatrix} = \begin{bmatrix}
f^p(x^p(t), u(t)) \\
-\eta_c u(t)
\end{bmatrix} + \begin{bmatrix}
0 \\
\eta_c \times I
\end{bmatrix} v(t).
\]
The new controller has the general form
\[
\begin{align*}
\dot{x}^c(t) &= f^c(x^c(t), y(t), r(t)), \\
v(t) &= g^c(x^c(t), y(t), r(t)),
\end{align*}
\]
with the input and output signals corresponding to the nonlinear plant \([1]\).

Combining (18) and (19) leads to

\[
\begin{bmatrix}
\dot{x}^p(t) \\
\dot{u}(t) \\
\dot{x}(t)
\end{bmatrix} = \begin{bmatrix}
f_p(x^p(t), u(t)) \\
-f_c(x^c(t), g_p(x^p(t), u(t)), r(t))
\end{bmatrix} + \begin{bmatrix}
0 \\
\eta_c 	imes I \\
0
\end{bmatrix} v(t).
\]

Then the closed loop nonlinear system is

\[
\dot{x}(t) = f(x(t), r(t)) + Bg(x(t), r(t)) = F(x(t), r(t))
\]

The augmented linear family of systems for (18) becomes

\[
\begin{bmatrix}
\dot{x}^p(t) \\
\dot{u}(t)
\end{bmatrix} = \begin{bmatrix}
A^p(\alpha) & B^p(\alpha) \\
0 & -\eta_c \times I
\end{bmatrix} \begin{bmatrix}
\delta x^p \\
\delta u
\end{bmatrix} + \begin{bmatrix}
0 \\
\eta_c \times I
\end{bmatrix} v(t),
\]

\[
\delta y = \begin{bmatrix}
C^p(\alpha) & D^p(\alpha)
\end{bmatrix} \begin{bmatrix}
\delta x^p \\
\delta u
\end{bmatrix}.
\]

Now, the control realization for this system is

\[
\dot{x}^c = A^c(\alpha)x^c + B^c(\alpha)[y - r],
\]

\[
v = C^c(\alpha)x^c + D^c(\alpha)[y - r], \quad \forall \alpha \in \Omega.
\]

One of the options for control design is to set controller matrices as follows

\[
A^c(\alpha) = A^c = -\epsilon_c I, \quad B^c(\alpha) = B^c = I,
\]

\[
C^c(\alpha) = -K_i(\alpha), \quad D^c(\alpha) = -K_p(\alpha),
\]

which is a kind of PI control, where \(K_i(\alpha)\) is the integral gain matrix, and \(K_p(\alpha)\) is the proportional gain matrix.

Hence the control for the augmented system has the final form

\[
\begin{bmatrix}
\dot{x}^c \\
v
\end{bmatrix} = \begin{bmatrix}
-\epsilon_c I & I \\
K_i(\alpha) & -K_p(\alpha)
\end{bmatrix} \begin{bmatrix}
x^c \\
y_r
\end{bmatrix}, \quad \forall \alpha \in \Omega.
\]

With these choices for control matrices, the control input is

\[
x^c = \int (-\epsilon_c x^c + (y - r)) \, d\tau,
\]

\[
v = -K_i(\alpha)x^c - K_p(\alpha)(y - r), \quad \forall \alpha \in \Omega.
\]

Figure [1] shows schematically how the gain scheduling controller works.

The linearized closed loop system (22) with controller (23) becomes

\[
\begin{bmatrix}
\delta x^p \\
\delta u \\
\dot{x}^c
\end{bmatrix} = \begin{bmatrix}
A(\alpha) & B(\alpha) & 0 \\
0 & -\eta_c I + D(\alpha)D^p(\alpha) & \eta_c C(\alpha) \\
B^c(\alpha)C^p(\alpha) & B^c(\alpha)D^p(\alpha) & A^c(\alpha)
\end{bmatrix} \begin{bmatrix}
\delta x^p \\
\delta u \\
x^c
\end{bmatrix} + \begin{bmatrix}
0 \\
-\eta_c D^c(\alpha) \\
-B^c(\alpha)
\end{bmatrix} \delta r, \quad \forall \alpha \in \Omega.
\]

\[
(27)
\]
For the case where we have simplified output $\delta y = \delta x^p$, (i.e. $C^p(\alpha) = I, D^p(\alpha) = 0$) the linearized closed loop system (22) with controller (25) becomes

$$\begin{bmatrix}
\dot{\delta x}^p \\
\dot{\delta u}^c \\
\dot{x}^c
\end{bmatrix} =
\begin{bmatrix}
A^p(\alpha) & B^p(\alpha) & 0 \\
-\eta_c K_p(\alpha) & -\eta_c K_i(\alpha) & -\epsilon_c I \\
0 & -\epsilon_c I & -I
\end{bmatrix}
\begin{bmatrix}
\delta x^p \\
\delta u^c \\
x^c
\end{bmatrix} +
\begin{bmatrix}
0 \\
\eta_c K_p(\alpha) \\
-\eta_c I
\end{bmatrix}
\begin{bmatrix}
\delta r
\end{bmatrix}, \quad \forall \alpha \in \Omega.
$$

(28)

2.1 Stability Analysis

In closed loop system (28), let $\delta r = 0$, and consider the unforced linear time varying system

$$\dot{x} = A_{cl}(\alpha)x, \quad x(0) = x_0, \quad \forall \alpha \in \Omega,$n

$$\delta y = \delta x^p.$$

Assumption 1. The matrix $A_{cl}$ is bounded and Lipschitz continuous as follows

$$||A_{cl}(t)|| \leq k_A, \quad \forall t > 0,$n

$$||A_{cl}(t) - A_{cl}(\tau)|| \leq L_A ||t - \tau||, \quad \forall t, \tau > 0.$$

(30)

Assumption 2. The constant eigenvalues of matrix $A_{cl}(\alpha)$ are uniformly bounded away from the closed complex right-half plane for all constant $\alpha$.

Theorem 1. Consider system (29), under assumptions 1 and 2, then there exists constants $m, \lambda, \epsilon > 0$ such that if

$$||\dot{y}(t)|| \leq \epsilon_y, \quad \forall t \in [0, T],$$

then

$$||x(t)|| \leq me^{-\lambda t}||x_0||, \quad \forall t \in [0, T].$$

(32)

To analyse the stability of the nonlinear closed-loop system, we use a technique known as ”global linearization” developed in [3].

Theorem 2. Consider nonlinear system (21), and assume there are a family of equilibrium points $(x_e, r_e)$ such that $F(x_e, r_e) = 0$. Then $A_{nl}^{e_i} = \frac{\partial F}{\partial x} \in S, \forall x$, where $S$ is a polytope, and it is described by a list of its vertices, i.e. in the form

$$S := \text{Co}\{A_{nl}^{e_1}, ..., A_{nl}^{e_L}\},$$

(33)

where $A_{nl}^{e_i}$'s are obtained by linearizing nonlinear system (21) near equilibrium points (steady state condition), and also non-equilibrium points (transient condition). Now, assume their exist a common symmetric positive definite matrix $P = P^T > 0$ such that:

$$PA_{nl}^{e_i} + A_{nl}^{e_iT} P < 0, \quad \forall i \in \{1, 2, ..., L\}.$$

(34)

then system (21) is absolutely stable. Since by design $A_{cl}(\alpha) \in S, \forall \alpha$, then system (29) is also stable.
If there are no hidden coupling terms involving $δy$, then the design of a stabilizing linear controller family can be assumed to guarantee stability of the linearized closed-loop system in a neighborhood of every $α ∈ Ω$. The closed-loop system is not restricted to remain in a neighborhood of any single equilibrium, but is assumed to be slowly-varying and to have initial state sufficiently close to some equilibrium in $S$. Then the conclusion is that the closed-loop system remains in a neighborhood of the equilibrium manifold [17]. Using results developed in [18], we can figure out if a system is slowly-varying or not. Here we rewrite theorem Theorem 12 from [17]:

**Theorem 3.** For plant (1), suppose the gain-scheduled controller (15) is such that there are no hidden coupling terms and the eigenvalues of the linearized closed-loop system satisfy $\text{Re}[λ] ≤ −\epsilon < 0$ for every $α ∈ Ω$. Then given $ρ > 0$ there exist positive constants $µ$ and $γ$ such that the response of the nonlinear closed-loop system satisfies the following property. If the exogenous signal $||\dot{r}(t)|| < µ$, for $t ≥ 0$, and if for some $α ∈ Ω$,

$$
\left| \begin{array}{c}
x_p(0) \\
u(0) \\
x_c(0)
\end{array} \right| - \left| \begin{array}{c}
x_p(α) \\
u_c(α) \\
0
\end{array} \right| < γ,
$$

then

$$
\left| \begin{array}{c}
x_p(t) \\
u(t) \\
x_c(t)
\end{array} \right| - \left| \begin{array}{c}
x_p(p(y(t))) \\
u_c(p(y(t))) \\
0
\end{array} \right| < ρ, \forall t ≥ 0.
$$

### 2.2 Integration Anti-Windup

It is desirable to have integral action in the controller since the presence of an integral term eliminates steady state error in the controlled variable. Since the allowable values for control inputs are limited, if any controller reaches its limit, and error is produced form the difference of the control signal and the actual limited signal applied to the plant. This phenomenon is known as integral wind-up. Because of this, Integral Wind-Up Protection (IWUP) is used to reduce the effect of the integral term of the controller.

An approach to IWUP from [13, 5] was adopted for our control problem. The main idea with this approach is to decrease the error seen by the integrators. This allows the integrator to increase to an appropriate value and decrease the size of the instantaneous change in magnitude when the controller becomes saturated. First, the generated control signal to the fuel-metering valve is subtracted from the saturation value. The resulting difference is then amplified by an integral feedback gain (IFB) and subtracted from the input to the integrator. The IFB is empirically tuned to provide adequate performance. The IFB is not gain scheduled, a constant value is sufficient for good performance.

### 3 Turboshaft Engine Example

We apply the developed gain-scheduling controller to a physics-based model of a turboshaft engine driving a variable pitch propeller developed in [9, 15]. For a standard day at sea level condition we found five equilibrium points for linearizing the dynamics near them. The linearization matrices for these five equilibrium points and steady state values of the engine variables and control parameters are:

- **Equilibrium Point 1 (Full Thrust):**
  $u^*_1 = 1.0$, $u^*_2 = 16$ (deg), $x^*_1 = 1.0$, $x^*_2 = 0.9524$, $T^* = 255.8685$ (N), $α^* = 1.3810$, and the matrices are

  $$
  A_1 = \begin{bmatrix}
  -5 & 0 \\
  3.5 & -2.3
  \end{bmatrix}, \quad B_1 = \begin{bmatrix}
  1.4 & 0 \\
  0.63 & -0.085
  \end{bmatrix}, \quad C_1 = I,
  $$

  $$
  K_i_1 = \begin{bmatrix}
  0.7 & 0.7 \\
  0.7 & 0.6
  \end{bmatrix}, \quad K_p_1 = \begin{bmatrix}
  1.2 & 1.2 \\
  1.2 & 1.2
  \end{bmatrix}.
  $$

(37)
• Equilibrium Point 2:
  $u_1^* = 0.7$, $u_2^* = 16$ (deg), $x_1^* = 0.8826$, $x_2^* = 0.6263$, $T^* = 181.9711$ (N), $\alpha^* = 1.0822$, and the matrices are

$$A_2 = \begin{bmatrix} -2.83 & -0.0008 \\ 1.20 & -2.10 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1.14 & 0 \\ 0.78 & -0.054 \end{bmatrix}, \quad C_2 = I,$$

$$K_{i2} = \begin{bmatrix} 0.6 \\ 0.6 \\ 0.5 \end{bmatrix}, \quad K_{p2} = \begin{bmatrix} 1.1 & 1.1 \\ 1.1 & 1.1 \end{bmatrix}.$$  

(38)

• Equilibrium Point 3 (Cruise):
  $u_1^* = 0.4685$, $u_2^* = 16$ (deg), $x_1^* = 0.7264$, $x_2^* = 0.5$, $T^* = 70.5125$ (N), $\alpha^* = 0.8818$, and the matrices are

$$A_3 = \begin{bmatrix} -1.9 & 0.061 \\ 0.45 & -1.1 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 1.57 \\ 0.3 & -0.023 \end{bmatrix}, \quad C_3 = I,$$

$$K_{i3} = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.4 \end{bmatrix}, \quad K_{p3} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$  

(39)

• Equilibrium Point 4:
  $u_1^* = 0.3$, $u_2^* = 16$ (deg), $x_1^* = 0.5327$, $x_2^* = 0.3678$, $T^* = 38.155$ (N), $\alpha^* = 0.6473$, and the matrices are

$$A_4 = \begin{bmatrix} -0.85 & 0.032 \\ 0.32 & -0.64 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 1.1 \\ 0.17 & -0.011 \end{bmatrix}, \quad C_4 = I,$$

$$K_{i4} = \begin{bmatrix} 0.4 \\ 0.4 \\ 0.3 \end{bmatrix}, \quad K_{p4} = \begin{bmatrix} 0.8 \\ 0.8 \end{bmatrix}.$$  

(40)

• Equilibrium Point 5 (Idle):
  $u_1^* = 0.145$, $u_2^* = 16$ (deg), $x_1^* = 0.295$, $x_2^* = 0.161$, $T^* = 7.317$ (N), $\alpha^* = 0.3361$, and the matrices are

$$A_5 = \begin{bmatrix} -0.38 & -0.0008 \\ 0.26 & -0.34 \end{bmatrix}, \quad B_5 = \begin{bmatrix} 0.7 \\ 0.1 & -0.0024 \end{bmatrix}, \quad C_5 = I,$$

$$K_{i5} = \begin{bmatrix} 0.3 \\ 0.3 \\ 0.2 \end{bmatrix}, \quad K_{p5} = \begin{bmatrix} 0.6 \\ 0.6 \end{bmatrix}.$$  

(41)

Other controller parameters are set to

$$\epsilon_c = 1, \quad \eta_c = 3, \quad Q = 3 \times I.$$  

(42)

To show the stability of the closed loop system, 20 different (10 equilibrium, and 10 non-equilibrium) linearizations have been used, to solve inequality \[34\], in Matlab with the aid of YALMIP [12] and SeDuMi [23] packages. The numerical value for the common matrix $P$ is:

$$P = \begin{bmatrix} 0.639 & 0.035 & 0.121 & -0.015 & -0.073 & -0.036 \\ 0.034 & 0.391 & 0.036 & -0.002 & -0.103 & -0.029 \\ 0.121 & 0.036 & 0.184 & -0.048 & -0.029 & -0.017 \\ -0.015 & -0.002 & -0.048 & 0.130 & 0.028 & 0.022 \\ -0.073 & -0.103 & -0.029 & 0.028 & 0.322 & 0.028 \\ -0.036 & -0.029 & -0.017 & 0.022 & 0.028 & 0.298 \end{bmatrix}.$$  

(43)
To show that the designed gain scheduled controller works properly on JetCat engine we used it to control the engine from idle to cruise condition and then again back to idle condition in a stable manner and with good performance. Simulation results are shown in figures 2 to 21.

Figure 2 shows the history of the norm of closed-loop system matrix $||A_{cl}(t)||$, and its rate of change $||\dot{A}_{cl}(t)||$. As it can be seen the figure shows the boundedness of these two variables in accordance with Assumption 1 where $k_{A} = 7.4539$, and $L_{A} = 1.0106$. Figure 3 shows the history of the closed-loop system matrix eigenvalues $\lambda(A_{cl}(\alpha))$. As it is apparent, all the six eigenvalues remain negative with the time change of the scheduling parameter $\alpha$, and hence satisfies assumption 2 of the stability theorem.

Figure 4 shows the history of the scheduling parameter $\alpha(t) = ||x(t)||$ and its rate of change $\dot{\alpha}(t) = \frac{x^{T}\dot{x}}{||x(t)||}$, and hence satisfies assumption 2 of the stability theorem.

Figure 5 shows the history the norms of the output vector and its rate of change $||y(t)||$.
rate. This satisfies the condition of Theorem 1 with $\epsilon_y = 0.0025$. Using formulas from [18], we can compute $m = 496.7476$, and $\lambda = 0.5271$.

Figure 6: History of the unforced closed-loop system Lyapunov function $V(t)$, and its rate of change $\dot{V}(t)$

Figure 7: Rate of change of reference signals ($\dot{r}$)

Figure 8 shows the history of the quadratic time varying Lyapunov function of the unforced closed loop system (29). As it is apparent, $V(t) = \delta X^T P(t) \delta X$, is decrescent and bounded from above and below. The history of $V$, shows that it is non-positive for all $t > 0$, so the exponential stability of the slowly varying system (29) with a gains-scheduling controller is guaranteed. Figure 7 shows the rate of change of the reference signals for the outputs of the system. The outputs in this simulation are core and spool speed. $||\dot{r}|| < 0.15$, which corresponds to the assumption of the $||\dot{r}||$ boundedness in the theorem 3.

Figure 8: Plant states: core and fan spool speeds

Figure 9: Controller states

Figure 8 shows the history of the plant states which are core and fan spool speeds. Figure 9 shows the time histories of the controller states. Figure 10 shows the phase plot for core and fan spool dynamics. Figure 11 shows the time history of the fan and core spool accelerations, i.e. $\dot{N}_1$ and $\dot{N}_2$.

Figures 12 and 13 show the core and fan spool speeds tracking their reference signals. Figure 14 shows the history of thrust and it is following its reference command from idle to cruise condition and then back to the idle for standard day, sea level condition. Figure 15 shows the control inputs to the augmented system,
Figure 10: Core and fan spool speeds vs. core and fan spool accelerations

Figure 11: Core and fan spools accelerations

Figure 12: Output: core spool speed and its reference signal

Figure 13: Output: fan spool speed and its reference signal

$v(t) = [v_1(t), v_2(t)]^T$, each element corresponding to one of the control inputs to the original system. To keep the engine dynamics within the limits of the operation for the available engine model, some limits have been defined on the augmented system control input, $-0.18 \leq v(t) \leq 0.18$. This limits will help to keep the fuel control input non-negative and also limits the rate of the control, $\dot{u}(t)$.

Figure 16 shows time rate of fuel and prop pitch angle inputs. Figure 17 shows fuel flow and propeller pitch angle histories as control inputs. For better performance and also to keep the engine in the safe range of operation limits has been defined for the augmented control inputs. Figures 18 and 19 show the baseline fuel controller integral ($K_i(\alpha)$) and proportional ($K_p(\alpha)$) gain matrices histories. These gains have been obtained by interpolation using the previously deigned fixed-gain controllers, each one corresponding to a equilibrium point of the engine. The numerical values of these gains are mentioned in equations (38) to (40).

Figure 20 shows JetCat SPT5 turboshaft engine compressor map. In this map the approximate stall line and also the operating line for this simulation has been shown. The engine operates in a safe region with a big stall margin during its acceleration from idle to cruise and again decelerating back to idle condition. Figure 21 shows the histories of turbine temperature, thrust specific fuel consumption (TSFC), compressor
Using gain-scheduling control technique, this simulation shows the possibility of controlling the linear parameter varying model of the turboshaft engine for a large transient throttle. This case study, simulates the engine accelerating from idle thrust to the cruise condition and then decelerates back to the idle condition in the standard day sea level condition. To tune the controller we need to trade off between the settling time and overshoot percentage. For large throttle transients, the existing controller regulates the controlled variable, additional limiters, will be developed to protect critical engine variables from exceeding physical bounds and to ensure safer operation.

4 Conclusions

We developed a gain-scheduling controller with stability guarantees for nonlinear gas turbine engine systems. Using global linearization and LMI techniques, we showed guaranteed absolute stability for closed loop gas pressure ratio and corrected air flow rate.
turbine engine systems with gain-scheduling controllers. Simulation results presented to show the applicability of the proposed controller to the nonlinear physics-based JetCat SPT5 turboshaft engine model for large transients from idle to cruise condition and vice versa.

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References

[1] A. E. Ariffin and N. Munro. Robust control analysis of a gas-turbine aeroengine. *IEEE Transactions on Control Systems Technology*, 5(2):178, 1997.
[2] G. Balas. Linear parameter-varying control and its application to a turbofan engine. *International Journal of Robust and Nonlinear Control*, 12(9):763, 2002.

[3] S. Boyd, L. El Gauoui, E. Feron, and V. Balakrishnan. *Linear Matrix Inequalities in System and Control Theory*. SIAM, Philadelphia, 1994.

[4] F. Bruzelius, C. Breitholtz, and S. Pettersson. Lpv-based gain scheduling technique applied to a turbofan engine model. In *Proceedings of the 2002 International Conference on Control Applications*, page 713–718, 2002.

[5] J. Csnak, R. D. May, J. S. Litt, and T.-H. Guo. Control design for a generic commercial aircraft engine. In *Proceedings of the 46th AIAA/ASME/SAE/ASEE Joint Propulsion Conference*, Hartford, CT, 2010. AIAA-2010-6629.

[6] N. Fitzgerald, M. Pakmehr, E. Feron, J Paduano, and A. Behbahani. Physics-Based Dynamic Modeling of a Turboshaft Engine Driving a Variable Pitch Propeller. *to be submitted to Journal of Aircraft*, 2012.

[7] D. K. Frederick, S. Garg, and S. Adibhatla. Turbofan engine control design using robust multivariable control technologies. *IEEE Transactions on Control Systems Technology*, 8(6):961, 2000.

[8] S. Garg. Turbofan engine control system design using the LQG/LTR methodology. In *Proceedings of the American Control Conference*, Pittsburgh, PA, 1989.

[9] S. Garg. A simplified scheme for scheduling multivariable controllers. *IEEE Control Systems Magazine*, 17(4):24, 1997.

[10] W. Gilbert, D. Heurion, J. Bernussou, and D. Boyer. Polynomial LPV Synthesis Applied to Turbofan Engines. *Control Engineering Practice*, 18(9):1077, 2010.

[11] D. J. Leith and W. E. Leithead. Survey of Gain-Scheduling Analysis and Design. *International Journal of Control*, 73(11):1001, 2000.

[12] J. Löfberg. Yalmip: A toolbox for modeling and optimization in MATLAB. In *Proceedings of the CACSD Conference*, Taipei, Taiwan, 2004. URL [http://users.isy.liu.se/johanl/yalmip](http://users.isy.liu.se/johanl/yalmip).

[13] S. Martin, I. Wallace, and D. G. Bates. Development and validation of a civil aircraft engine simulation model for advanced controller design. *Journal of Engineering for Gas Turbines and Power*, 130(5), 2008.

[14] M. Pakmehr, N. Fitzgerald, G. Kiwada, J. D. Paduano, E. Feron, and A. Behbahani. Decentralized adaptive control of a turbofan system with partial communication. In *Proceedings of the 46th AIAA/ASME/SAE/ASEE Joint Propulsion Conference*, Nashville, TN, 2010. AIAA-2010-6835.

[15] M. Pakmehr, N. Fitzgerald, J. Paduano, E. Feron, and A. Behbahani. Dynamic modeling of a turboshaft engine driving a variable pitch propeller: a decentralized approach. In *Proceedings of the 47th AIAA/ASME/SAE/ASEE Joint Propulsion Conference*, San Diego, California, 2011.

[16] M. Pakmehr, M. Mounier, N. Fitzgerald, G. Kiwada, J. D. Paduano, E. Feron, and A. Behbahani. Distributed control of turbofan engines. In *Proceedings of the 45th AIAA/ASME/SAE/ASEE Joint Propulsion Conference*, Denver, Colorado, 2009. AIAA-2009-5532.

[17] W. J. Rugh and J. S. Shamma. Research on gain scheduling. *Automatica*, 36(10):1401, 2000.

[18] J. S. Shamma. *Analysis and Design of Gain Scheduled Control Systems*. PhD thesis, MIT, 1988.

[19] J. S. Shamma. Gain scheduling. In *DISC Summer School on Identification of Linear Parameter-Varying Systems*, pages 6988–6993, Koningshof, Veldhoven, The Netherlands, 2006.
[20] J. S. Shamma. Overview of LPV systems. In J. Mohammadpour and C. W. Scherer, editors, *Control of Linear Parameter Varying Systems with Applications*, Chapter 1, pages 3–26. Springer, New York, NY, 2012.

[21] L. Shu-qing and Z. Sheng-xiu. A modified LPV modeling technique for turbofan engine control system. In *Proceedings of 2010 International Conference on Computer Applications and System Modeling (ICCASM)*, page V5–99–V5–102, 2010.

[22] J. S. Simon. Modeling and stability analysis of axial flow fans operating in parallel. Master’s thesis, MIT, 1985.

[23] J. F. Sturm, O. Romanko, and I. Pólik. Sedumi (self-dual-minimization): A matlab toolbox for optimization over symmetric cones. 2001. URL http://sedumi.ie.lehigh.edu.

[24] D. Yu, Xu Z. Zhao, H., Y. Sui, and J. Liu. An approximate non-linear model for aeroengine control. *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, 255(12):1366, 2011.

[25] H. Zhao, J. Liu, and D. Yu. Approximate nonlinear modeling and feedback linearization control of aeroengines. *Journal of Engineering for Gas Turbines and Power*, 133(11):111601, 2011.