Examination of metamaterial solid immersion lenses for subwavelength optical manipulation

Andrey Novitsky$^{1,2}$, T. Repán$^2$, R. Malureanu$^2$, O. Takayama$^2$, E. Shkondin$^2$, A.V. Lavrinenko$^2$

$^1$ Department of Theoretical Physics and Astrophysics, Belarusian State University, Nezavisimosti Avenue 4, 220030 Minsk, Belarus
$^2$ DTU Fotonik, Technical University of Denmark, Ørsteds Plads 343, DK-2800 Kongens Lyngby, Denmark

E-mail: andreyvnovitsky@gmail.com; alav@fotonik.dtu.dk

Abstract. We theoretically inspect, whether a recently proposed metamaterial solid immersion lens (a cluster of closely packed nanospheres) possesses super-resolving properties that could be used for subwavelength optical manipulation. Harnessing the multiple scattering theory for assembly of electric dipoles, we investigate the evanescent-wave and point-source-radiation transmission through the slab of nanospheres to establish the mechanism of super-resolution. Our calculations do not confirm resolution of subwavelength details using the metamaterial solid immersion lens.

1. Introduction
Along with the known metamaterial devices beating the diffraction limit as hyperbolic metamaterials [1] and superlenses [2], the novel type of all-dielectric metamaterial solid immersion lens (mSIL) was proposed in [3, 4]. The authors of [3, 4] insist that nanoscopy of the mSIL improves its resolution beyond that known for the conventional solid immersion lenses. According to [3], two properties of mSILs are responsible for increase of the resolution. The first one is a conversion of evanescent waves to propagating ones allowing to detect an image in the far zone. Second property is associated with a guiding-like (non-diffractive) radiation propagation through a cluster of nanospheres. Our research verifies these features of mSILs, which can be important for subwavelength optical manipulation [5, 6].

2. Light propagation in densely packed assembly of nanospheres
Here we briefly discuss a scattering of a monochromatic electromagnetic wave with electric field $E_{\text{inc}}$ by a closely packed assembly of nanospheres, every second layer of which coincide [see a two-dimensional version of such a packing in Fig. 1 (a)]. In the Cartesian-like basis $\mathbf{b}_1 = R\mathbf{e}_x$, $\mathbf{b}_2 = \frac{R}{\sqrt{3}}\mathbf{e}_y$, $\mathbf{b}_3 = \frac{2\sqrt{2}R}{3}\mathbf{e}_z$, the lattice of nanospheres can be described as

$$r_{j_1,j_2,j_3} = j_1\mathbf{b}_1 + \text{Mod}(3j_1 - 2j_3, 6) - 6j_2]\mathbf{b}_2 + j_3\mathbf{b}_3, \quad (1)$$

where Mod$(3j_1 - 2j_3, 6)$ means the remainder of the division by 6. For a slab of nanospheres characterized by $-N_x/2 \leq j_1 \leq N_x/2$, $-N_y/2 \leq j_2 \leq N_y/2$ and $0 \leq j_3 \leq N_z$ one can calculate the total number of particles $(N_x + 1)(N_y + 1)(N_z + 1)$. 


3. Evanescent-to-propagating wave conversion under coherent excitation

We exploit the same radii $R = 7.5$ nm and refractive indices $n = 2.55$ of nanospheres as in [3] to make a proper comparison. Wavelength of the incident wave $\lambda = 470$ nm. Effective refractive index of the closely packed assembly is $n_{e,\text{eff}} = 2$. Since $R \ll \lambda$, the nanospheres can be treated as electric dipoles, which electric polarizability $\alpha = \alpha_0/(1 - 2ik_0^2\alpha_0/3) \approx \alpha_0$. Here $\alpha_0 = R^3(n^2 - 1)/(n^2 + 2)$ is a static electric polarizability. Applying definition of the dipole moment $p = \alpha E$ to a nanosphere in the lattice, one writes

$$p_{j_1,j_2,j_3} = \alpha \left[ E_{\text{inc}}(r_{j_1,j_2,j_3}) + \sum_{j_1',j_2',j_3'} \hat{G}_E(r_{j_1',j_2',j_3'} - r_{j_1,j_2,j_3})p_{j_1',j_2',j_3'} \right], \quad (2)$$

where $p_{j_1,j_2,j_3}$ is the dipole moment of the nanoparticle at the point $r_{j_1,j_2,j_3}$ and $\hat{G}_E$ is the electric Green function. Solving equation (2) for all nanoparticles, one can find the dipole moments and the field transmitted by the slab of nanospheres as $E_{\text{tr}}(r) = E_{\text{inc}}(r) + \sum_{j_1,j_2,j_3} \hat{G}_E(r - r_{j_1,j_2,j_3})p_{j_1,j_2,j_3}$. To accelerate the calculations, one can solve a two-dimensional problem, if some dipole moments are treated equal.

3. Evanescent-to-propagating wave conversion under coherent excitation

An evanescent wave is described by $E_{\text{inc}} = E_0 \exp \left( ik_x x - \sqrt{k_x^2 - k_0^2} z \right) e_y$, where $E_0$ is the field amplitude and $k_x > k_0$ is the transverse wavenumber. We apply the two-dimensional calculation technique, because the field $E_{\text{inc}}(x, z)$ depends only on the two coordinates and the dipoles of nanospheres in the $y$-direction can be chosen approximately equal for the fixed $x$ and $z$.

Expected dependence of the transmitted field $E_{\text{tr}}(k_x) \sim \exp(-\sqrt{k_x^2 - k_0^2} z)$ for a continuous slab shown in Fig. 1(b) violates, if all nanospheres are coherently excited. Coherent excitation corresponds to the in-phase oscillation of the nanoparticles and can be obtained at $k_x R = 2\pi m$, where $m$ is an integer number. Radiation of coherently emitting dipoles constructively interfere to enhance transmission. In Fig. 1(b) we indeed observe the increase of the transmission at the wavenumbers multiple to the reciprocal lattice number $2\pi/R$. More precisely, the dispersion equation $(k_x - 2\pi m/R)^2 + k_z^2 = n_{e,\text{eff}}^2 \omega^2/c^2$ of the wave in a periodic system provides a number of allowed bands $2\pi m/R - n_{e,\text{eff}}\omega/c < k_x < 2\pi m/R + n_{e,\text{eff}}\omega/c$ shown by boxes in Fig. 1(b).
For a continuous medium, one has a single band $m = 0$. Thus, evanescent-to-propagating-wave conversion exists, but only in conditions of coherent excitation (in contrast to any evanescent wave in [3]). Transverse distribution of the transmitted electric field and oscillating character of the dipole moment shown in Fig. 1(c) are in agreement with [3].

4. Propagation of subwavelength-source radiation

We model a subwavelength source as a couple of point dipoles spaced at the distance substantially less than $\lambda$. According to [3], even at the distance of $6R$ we should clearly see two distinct light spots at the output of the slab. Here we exploit a two-dimensional point source, which electric field equals $E_{PS}(u_1, u_2) = G_{2D}(u_1, u_2)p_s$, where $u = (x, 0, z)$, $p_s = p_0e_y$ is the dipole moment and $G_{2D}$ is the electric Green function.

When a single source is close to the slab of nanospheres, it induces electric dipole moments. In the vicinity to the source the nanospheres are strongly coupled to its radiation and one observes a high peak in Fig. 2(a) near $z = 0$. This peak rapidly vanishes and the source cannot be noticed at the output. Dipole moments are excited at the sides of the composite slab (see the inset). Lateral size of the slab is likely not important, because the maximum source radiation should be anyway observed near $j_1 = 0$, if guiding as in [3] takes place.

Electric fields of two point sources can be written as $E_{inc} = E_{PS}(u - u_+) + E_{PS}(u - u_-)$, where $u_\pm = (\pm d/2, 0, z_0)$ determine positions of the sources. Taking $d = 6R$ as in [3], we observe the oscillating distribution of electric dipole moment $p$ across the slab of nanospheres, what is the sign of the field propagation (see Fig. 2(b)). However, we do not see images of the sources at the output: transverse distribution of the transmitted electric field $E$ in Fig. 2(b) does not show peaks corresponding to the sources neither in the near nor in the far field. Thus, in contrast to the results of [3] the assembly of nanospheres is not able to resolve subwavelength features.

5. Conclusion

We have demonstrated that in spite of the evanescent-to-propagating wave conversion the subwavelength imaging cannot be achieved for the planar slab of nanospheres. We explain this by small efficiency of the conversion. Thus, the novel approach of optical manipulation at subwavelength scale is not justified.

References

[1] Jacob Z, Alekseyev L V and Narimanov E 2006 Opt. Express 14 8247
[2] Pendry J B 2000 Phys. Rev. Lett. 85 3966
[3] Fan W, Yan B, Wang Z and Wu L 2016 Sci. Adv. 2 e1600901
[4] Zhu H, Fan W, Zhou S, Chen M and Wu L 2016 ACS Nano 10 9755
[5] Yang A H J, Moore S D, Schmidt B S, Klug M, Lipson M and Erickson D 2009 Nature 457 71
[6] Gao D, Ding W, Nieto-Vesperinas M, Ding X, Rahman M, Zhang T, Lim C T and Qiu C-W 2017 Light: Science & Applications 6 e17039