$V_{td}$ FROM HADRONIC TWO-BODY $B$ DECAYS

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ABSTRACT

Certain hadronic two-body decays of $B$ mesons are dominated by penguin diagrams. The ratios of rates for several such decays, including $\Gamma(B^0 \to \overline{K}^0 K^0)/\Gamma(B^0 \to \phi K^0)$, $\Gamma(B^0 \to \overline{K}^0 K^{*0})/\Gamma(B^0 \to \phi K^{*0})$, $\Gamma(B^+ \to \overline{K}^0 K^+)/\Gamma(B^+ \to \phi K^+)$, and $\Gamma(B^+ \to \overline{K}^0 K^{*+})/\Gamma(B^+ \to \phi K^{*+})$, can provide information on the ratio of Cabibbo-Kobayashi-Maskawa (CKM) elements $|V_{td}/V_{ts}|$ in a manner complementary to other proposed determinations. SU(3) breaking effects cancel in some ratios. The cases of neutral $B$ decays are free of corrections from small annihilation terms.

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The charge-changing weak interactions of quarks are described in the electroweak theory \[1\] in terms of the unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix \( V \) \[2\]. The elements of this matrix are fundamental quantities in the theory. A major task of future experiments is to improve our present knowledge of these parameters. \( B \) physics experiments, from which very useful information on two of the couplings, \( V_{td} \) and \( V_{ub} \), was obtained \[3\], are expected to play in the future a crucial role towards this goal. Measurement of \( \Delta m_d \), the nonstrange neutral \( B \) meson mass difference, has provided through the box diagram mechanism the strongest constraint at present on \( V_{td} \) \[4, 5\]:

\[
0.12 < |V_{td}/V_{cb}| < 0.36 \quad (1)
\]

This range is consistent with unitarity of the CKM matrix and with the observed CP violation in the neutral \( K \) meson system. Another promising way to determine \( V_{td} \), or at least to further constrain it \[4\], is through measurement of radiative penguin decays, \( B^0 \to \rho^0 \gamma, B^0 \to \omega \gamma \), by comparing these rates to the measured rate of \( B^0 \to K^{*0} \gamma \) \[6\]. Alternatively, this may be achieved by improving the present lower limit on \( \Delta m_s \), the strange neutral \( B \) meson mass difference, or by a measurement of \( K^+ \to \pi^+ \nu \nu \) \[7\].

In this Letter we propose a method of measuring \( V_{td} \) which is based on comparing rates of strangeness-conserving and strangeness-changing two-body and quasi-two-body hadronic \( B \) decays to noncharmed mesons. For this purpose we consider decays which are dominated by QCD-penguin and by electroweak penguin operators. These operators, denoted respectively by \( Q_{3-6} \) and \( Q_{7-10} \) \[9\], appear in the effective Hamiltonian for charm-conserving hadronic \( B \) decays. The CKM coefficients of these operators, dominated by the \( t \) quark contributions, are given by \( V_{td}V_{tb}^* \) and \( V_{ts}V_{tb}^* \), corresponding to \( \Delta S = 0 \) and \( |\Delta S| = 1 \) decays, respectively. Thus, up to SU(3) breaking corrections, the ratios of corresponding decay rates is given approximately by \( |V_{td}/V_{ts}|^2 \) and could provide new measurements of \( V_{td} \).

Previous studies of penguin-dominated hadronic \( B \) decays \[10, 11, 12, 13, 14, 15\] calculated rates assuming factorization of penguin amplitudes, taking specific form factors and using representative values for the magnitude of CKM elements. These model-dependent calculations can serve as rough estimates. Our purpose is different. We will treat this class of processes in a model-independent manner, and will take ratios of \( \Delta S = 0 \) to \( |\Delta S| = 1 \) rates, in which a major part of the model-dependence is expected to cancel.

In order to perform a general analysis of two- and quasi-two-body penguin-dominated \( B \) decays, let us focus first on decays to two light pseudoscalars, generically denoted by \( B \to PP \). Assuming, at a first stage, flavour SU(3) for the strong interactions, it is very convenient \[16\] to replace the five SU(3) invariant amplitudes describing these processes \[17, 18, 19\] (see Table 1 below) by an overcomplete set of six quark diagrams \[12\], which we denote by \( T \) (tree), \( C \) (colour-suppressed), \( P \) (QCD-penguin), \( E \) (exchange), \( A \) (annihilation) and \( PA \) (penguin annihilation). The last three amplitudes, in which the spectator quark enters into the decay Hamiltonian, are expected to be suppressed by \( f_B/m_B \) (\( f_B \approx 180 \) MeV) and may be neglected to a good approximation. The presence of higher-order electroweak penguin contributions introduces no
new SU(3) amplitudes, and in terms of quark graphs merely leads to a substitution

\[ T \rightarrow t \equiv T + P^C_{EW}, \quad C \rightarrow c \equiv C + P_{EW}, \quad P \rightarrow p \equiv P - \frac{1}{3} P^C_{EW}, \]  

(2)

where \( P_{EW} \) and \( P^C_{EW} \) are colour-favored and colour-suppressed electroweak penguin amplitudes. To improve the precision of the analysis, one can then introduce first-order SU(3) breaking corrections in the amplitudes. In Ref. [21] we showed that this may be achieved in a most general manner through mass insertions in the above quark diagrams.

We will use the above analysis to write the amplitudes for the few processes of type \( B \rightarrow PP \) which obtain contributions from QCD- and electroweak penguin terms (in the combination (4) without any contributions from tree (T) and colour-suppressed (C) terms. For generality we include at this point smaller annihilation-like terms and SU(3) breaking terms. \( \Delta S = 0 \) amplitudes are denoted by unprimed quantities and \( |\Delta S| = 1 \) processes by primed quantities. We find (see Tables I and II in Refs. [16] and [21]):

\[ A(B^+ \rightarrow K^+\overline{K}^0) = p + p_3 + A, \]
\[ A(B^0 \rightarrow K^0\overline{K}^0) = p + p_3 + PA, \]
\[ A(B^+ \rightarrow \pi^+K^0) = p' + p'_1 + A', \]
\[ A(B_s \rightarrow K^0\overline{K}^0) = p' + p'_1 + p'_2 + PA'. \]  

(3)

Here \( p \) and \( p' \) are SU(3)-invariant amplitudes. The three SU(3)-breaking corrections \( p'_1, p'_2, p_3 \) are due, respectively, to a \( \bar{b} \rightarrow \bar{s} \) (rather than a \( \bar{b} \rightarrow \bar{d} \)) transition, an \( s \) (rather than a \( u \) or a \( d \)) spectator quark, and \( ss \) (rather than \( uu \) or \( dd \)) pair creation. These terms may be interpreted as form-factor and/or decay-constant corrections if one assumes factorization for penguin amplitudes. We neglect SU(3) breaking in the smaller \( A, PA \) (\( A', PA' \)) amplitudes.

The penguin amplitudes \( p, PA \) and \( p', PA' \) are dominated by the \( t \) quark contribution. We will neglect small \( u \) and \( c \) quark terms. They can affect the magnitude of \( p/p' \) [22] by as much as 30% for the smallest allowed values of \( |V_{td}/V_{ts}| \), but typically by at most 10% over most of the allowed range. The ratios \( p/p' \) and \( (p + PA)/(p' + PA') \) are then given simply in terms of the ratio of corresponding CKM matrix elements:

\[ \frac{p + PA}{p' + PA'} \approx \frac{p}{p'} \approx \frac{V_{td}V_{tb}^*}{V_{ts}V_{tb}^*} = \frac{V_{td}}{V_{ts}}. \]  

(4)

On the other hand, the ratio \( A/A' \) is given by a different CKM factor \( A/A' = V_{ud}/V_{us} \).

Neglecting annihilation-like amplitudes and SU(3) breaking terms leads to the approximate relations

\[ A(B^+ \rightarrow K^+\overline{K}^0) \approx A(B^0 \rightarrow K^0\overline{K}^0), \]  

(5)

\[ A(B^+ \rightarrow \pi^+K^0) \approx A(B_s \rightarrow K^0\overline{K}^0), \]  

(6)

\[ \frac{A(B^0 \rightarrow K^0\overline{K}^0)}{A(B_s \rightarrow K^0\overline{K}^0)} \approx \frac{V_{td}}{V_{ts}}, \]

(7)
The amplitude equality (8) for $\Delta S = 0$ transitions, which follows from the $\Delta I = 1/2$ property of the $\bar{b} \to \bar{d}$ penguin operator, can be used to test the magnitude of the small annihilation terms which were neglected. Eq. (8), for $|\Delta S| = 1$ amplitudes, is expected to be more sensitive to the SU(3) breaking term $p_2'$. In Eq. (5) and strange neutral $B$ decay amplitudes, we neglect only SU(3) breaking terms, while in Eq. (8) for charged $B$ mesons also $A$, $A'$ must be neglected.

To evaluate the precision of the relations (5–8), one must estimate the relative contributions of the neglected terms. This can be done in a model-dependent manner, for instance by assuming factorization and specific models for form factors [11]. A rough estimate of the $A$, $A'$ terms based on their $f_B/m_B$ suppression was obtained in Ref. [21]: $A/p = O(1/5)$, $A'/p' = O((1/5)^3)$. The first ratio may be an overestimate if the annihilation amplitude $A$ is further suppressed, for instance by a helicity argument.

SU(3) breaking terms in penguin amplitudes are generally expected to lead to no more than 30% corrections. In the ratios of amplitudes (7) and (8), the numerators and denominators contain different types of SU(3) breaking terms and it is difficult to argue for cancellation effects in a model-independent manner. As will be shown below, such a cancellation occurs in $B$ decays involving vector mesons in the final state.

Let us consider quasi-two-body decays of the type $B \to PV$ and $B \to VV$, where $V$ stands for a charmless vector meson. For completeness, and in order to treat these processes in the above SU(3) breaking framework using quark diagrams, we digress at this point to discuss the equivalence between a description of these processes in terms of SU(3) reduced amplitudes and quark graphs.

In the weak Hamiltonian governing $B$ meson decays to pairs of charmless final states [16, 17, 18, 19], a $\bar{b}$ quark undergoes a transition to one light quark [a 3 under flavour SU(3)] and two light antiquarks ($3^*$), leading to operators transforming as $3^*$, 6 and $15^*$. Penguin operators with the SU(3) structure $\bar{b} \to \bar{d}$ and $\bar{b} \to \bar{s}$ transform only as $3^*$. The independent amplitudes for symmetric and antisymmetric final states [17] composed of two flavour octets are summarized in Table 1.

The decays of (spinless) $B$ mesons to pairs of flavour octet pseudoscalar mesons $P$ are characterized by the reduced amplitudes in the left-hand (symmetric) column of Table 1, since the mesons are produced in an S-wave and the final state is symmetric under the interchange of the two final mesons. Decays to one pseudoscalar octet and one vector ($V$) octet involve both columns. Decays to two vector mesons involve symmetric amplitudes for $S$- and $D$-wave final states and antisymmetric amplitudes for $P$-wave final states.

The relation between graphs for $B \to PP$ decays and the reduced matrix elements in the left-hand column of Table 1 was noted in the Appendix of Ref. [16]. A corresponding expansion is possible for $B \to PV$ and $B \to VV$ amplitudes. The need for both columns of Table 1 in describing $B \to PV$ decays arises from the distinction between processes in which the spectator quark enters either $P$ or $V$.

The neglect of graphs in which the spectator quark participates in the weak inter-
Table 1: Reduced amplitudes for hadronic decays of $B$ mesons to pairs of charmless final states.

| Symmetric | Antisymmetric |
|-----------|---------------|
| $\langle 27|15^*|3 \rangle$ | $\langle 10^*|15^*|3 \rangle$ |
| $\langle 8_s|15^*|3 \rangle$ | $\langle 8_A|15^*|3 \rangle$ |
| $\langle 8_s|6|3 \rangle$ | $\langle 10|6|3 \rangle$ |
| $\langle 8_s|3^*|3 \rangle$ | $\langle 8_A|6|3 \rangle$ |
| $\langle 1|3^*|3 \rangle$ | $\langle 8_A|3^*|3 \rangle$ |

action was shown in Ref. [10] to be equivalent to two relations between reduced matrix elements:

$$\langle 27|15^*|3 \rangle \leftrightarrow \langle 8_s|15^*|3 \rangle , \quad \langle 8_s|3^*|3 \rangle \leftrightarrow \langle 1|3^*|3 \rangle . \quad (9)$$

The reduced amplitudes which are related to one another have the same flavour structure of the effective weak Hamiltonian. The neglect of spectator interactions is equivalent to forbidding contractions between the SU(3) index of the initial spectator quark and the indices associated with the weak Hamiltonian. One must necessarily get relations among final-state amplitudes involving the same Hamiltonian structure.

A corresponding set of relations can be seen for the antisymmetric amplitudes:

$$\langle 10^*|15^*|3 \rangle \leftrightarrow \langle 8_A|15^*|3 \rangle , \quad \langle 10|6|3 \rangle \leftrightarrow \langle 8_A|6|3 \rangle . \quad (10)$$

As for the symmetric amplitudes, three independent reduced matrix elements remain when interactions with the spectator quark are neglected. In the graphical approach, these correspond to tree, colour-suppressed, and penguin amplitudes.

When considering vector mesons we shall need to discuss the $\phi$, an octet-singlet mixture. Aside from the special case of $B_s \rightarrow \phi\phi$, which we discuss separately, all additional amplitudes of interest arising from the singlet component of the $\phi$ will involve a final-state octet. These consist of terms $\langle 8'|15^*|3 \rangle$, $\langle 8'|6|3 \rangle$, and $\langle 8'|3^*|3 \rangle$, where the prime denotes an amplitude involving one final-state singlet and one final-state octet meson.

We expect processes such as $B^+ \rightarrow \pi^+\phi$ to be highly suppressed since the $\phi$ must be connected to the rest of the diagram by at least three gluons, a photon, a $Z$, or a $W^+W^-$ pair. The last three (colour-favored electroweak penguin) processes [13], to which we shall return, lead to about 10% corrections to the QCD-dominated penguin amplitudes to be considered here. If the three-gluon and colour-favored electroweak penguin processes are assumed to be zero (a good approximation), one obtains relations between each of the three $8'$ amplitudes noted above and those in Table 1 involving the same Hamiltonian structure.

In the case of $B_s \rightarrow \phi\phi$, a new amplitude of the form $\langle 1'|3^*|3 \rangle$ may be related to the others by the condition that both $\phi$'s should be connected by quark lines either to
one another (as in the “penguin annihilation” diagrams of Ref. [16], considered here to be small), or to the rest of the diagram.

To form all the penguin-dominated processes of the type $B \to PV$ and $B \to VV$, we make the following observations:

1) In $b \to d$ transitions, one must consider only those decays in which an $s\bar{s}$ pair is produced from the vacuum. The production of a $u\bar{u}$ pair leads to an effective transition $\bar{b} \to \bar{d}u\bar{u}$ which can also arise from tree-type processes. The production of a $d\bar{d}$ pair can lead to mesons containing $d\bar{d}$ which are impossible to distinguish from those containing $u\bar{u}$ (and hence which can be produced by colour-suppressed tree-type processes.)

2) In $b \to s$ transitions, one can consider production of an $s\bar{s}$ pair or a $d\bar{d}$ pair from the vacuum. A $u\bar{u}$ pair leads again to an effective transition which can also arise from tree-type processes.

3) In decays producing an $s\bar{s}$ meson, we demand that it be the $\phi$, since the $\phi$ appears to be composed mostly of $s\bar{s}$ and its couplings appear to approximately respect the Okubo-Zweig-Iizuka (OZI) rule forbidding disconnected quark diagrams. This rule is less likely to hold for processes involving $\eta$ and $\eta'$ (which in any case are not pure $s\bar{s}$) [23, 24].

Using these simple rules it is straightforward to write expressions similar to (3) for all $PV$ and $VV$ penguin-dominated decay modes. Here again charged $B$ decay amplitudes involve corrections from $A$ and $A'$ terms, while the likely smaller $PA$ and $PA'$ amplitudes contribute to nonstrange and strange neutral $B$ decays. When neglecting these terms one obtains a set of equalities between $B^0$ and $B^+$ amplitudes. These relations, including Eq. (7), are given in Table 2. The amplitude equalities, which are free of SU(3) breaking corrections, follow from $\Delta I = 1/2$ and $\Delta I = 0$ selection rules of $\bar{b} \to \bar{d}$ and $\bar{b} \to \bar{s}$ transitions, respectively. Note that in all cases an $s\bar{s}$ pair is created in the vacuum. The equalities in Table 2 can be used to test the small magnitude of annihilation terms which were neglected. In Table 2 and subsequently, we shall distinguish between $B \to PV$ and $B \to VP$ decays by adopting the convention that the second meson is the one containing the spectator quark. In Refs. [11, 12] the $B \to PV$ penguin amplitudes are found to be very small as a result of model-dependent dynamical cancellations. In these cases the colour-favored electroweak penguin contributions involving $\phi$ production by the neutral weak current are no longer negligible.

One may obtain a set of relations involving $B^0$ and $B_s$ $\bar{b} \to \bar{d}$ and $\bar{b} \to \bar{s}$ transitions, which determine $V_{td}/V_{ts}$. These relations, given in Table 3 and containing Eq. (7), do not require neglect of annihilation terms. They may be affected, however, by SU(3) breaking effects which are implicit in the table. The $q\bar{q}$ pair created out of the vacuum in each process is indicated in the table and represents a possible SU(3) breaking correction term of type $p_3$ ($p'_3$). (An example of appreciable form-factor effects in comparing $d\bar{d}$ and $s\bar{s}$ production is given in Ref. [13].) Other SU(3) breaking corrections are a type-$p'_1$ term which occurs in $\bar{b} \to \bar{s}$ transitions and a type-$p_2$ ($p'_2$) which contributes to $B_s$ decays. We note that the type-$p_3$ ($p'_3$) terms cancel in ratios of $\Delta S = 0$ and $|\Delta S| = 1$ amplitudes, such as $A(B^0 \to \bar{K}^{*0}K^0)/A(B^0 \to \phi K^0)$ and $A(B^0 \to \bar{K}^{*0}K^0)/A(B^0 \to \phi K^{*0})$, which are therefore expected to provide a better
Table 2: Relations involving $B^0$ and $B^+$ decays dictated by selection rules associated with dominance of penguin amplitudes.

| $B^0$ decay | $B^+$ decay |
|-------------|-------------|
| $\bar{K}^0K^0$ | $\bar{K}^0K^+$ |
| $\bar{K}^0K^{*0}$ | $\bar{K}^0K^{*+}$ |
| $\bar{K}^{*0}K^0$ | $\bar{K}^{*0}K^+$ |
| $\bar{K}^{*0}K^{*0}$ | $\bar{K}^{*0}K^{*+}$ |
| $\phi K^0$ | $\phi K^+$ |
| $\phi K^{*0}$ | $\phi K^{*+}$ |

*aSmall amplitude in some models.*

measure of $V_{td}/V_{ts}$ than other ratios.

The last line of Table 3 involves the decay $B_s \to \phi\phi$, which can occur only in even partial waves (S and D). One must then separate even from odd partial waves in the other decays if one wants to compare them with the $B_s \to \phi\phi$ rate. One can then write, for example, taking account of identical particle effects,

$$\tilde{\Gamma}(B_s \to \phi\phi) = 2\tilde{\Gamma}(B^0 \to \phi K^{*0})|_{S+D},$$

where $\tilde{\Gamma} \equiv \Gamma/(\text{phase space})$. All other relations among $VV$ decays hold separately for each partial wave.

For many of the $B^0$ and $B_s$ decays, the flavour of the decaying meson cannot be ascertained from the final state because it is observed in a CP eigenstate or can be produced from both the neutral $B$ and its charge-conjugate (via mixing). In all cases aside from $B^0 \to \phi K^0$ and $B_s \to \bar{K}^0\phi$ useful information on $|V_{td}/V_{ts}|$ still may be obtained from time-integrated decay rates, summed over a process and its charge-conjugate. One can isolate the process $B^0 \to \bar{K}^0K^0$, which is to be compared with $B^0 \to \phi K^0$, by identifying the flavour of the initial $B$ by tagging and that of the decaying one by the charge of the kaon in $\bar{K}^{*0} \to K^-\pi^+$, and taking account of the known amount of $B^0\bar{B}^0$ mixing. The rate for $B_s \to \bar{K}^0\phi$, which requires observing time-dependent $B_s - \bar{B}_s$ oscillations, is one of those found to be small and susceptible to colour-favored electroweak penguin terms in some models.

Finally, another set of relations can be obtained for $B^+$ decays, in which the ratio of $\bar{b} \to \bar{d}$ and $\bar{b} \to \bar{s}$ amplitudes is given by $V_{td}/V_{ts}$. These relations, including Eq. (8), are summarized in Table 4 which also labels the quark pair produced from the vacuum. Again, SU(3) breaking is more likely to affect those relations in which the quark pair produced is not the same in the $\bar{b} \to \bar{d}$ and $\bar{b} \to \bar{s}$ transitions. In this respect, the amplitude ratios $A(B^+ \to \bar{K}^{*0}K^+)/A(B^+ \to \phi K^+)$ and $A(B^+ \to \bar{K}^{*0}K^{*+})/A(B^+ \to \phi K^{*+})$ are expected to give more precise information on $|V_{td}/V_{ts}|$ than other ratios. These relations may be slightly affected by contributions from annihilation amplitudes.
Table 3: Summary of amplitude relations between $\bar{b} \to \bar{d}$ and $\bar{b} \to \bar{s}$ penguin-dominated $B^0$ and $B_s$ decays. Ratios of $\bar{b} \to \bar{d}$ and $\bar{b} \to \bar{s}$ processes are given by $V_{td}/V_{ts}$. The pair produced out of the vacuum in each class of decay is indicated. Entries indicate final states. Rows indicate amplitudes related to one another.

| $B^0$ decays | $B_s$ decays |
|---------------|---------------|
| $\bar{b} \to \bar{d}$ | $\bar{b} \to \bar{d}$ | $\bar{b} \to \bar{s}$ |
| $s\bar{s}$ pair | $s\bar{s}$ pair | $s\bar{s}$ pair |
| $K^0\bar{K}^0$ | $K^0\phi\ a$ | $K^0\bar{K}^0\ a$ |
| $\bar{K}^0K^{*0}$ | $\phi\ K^0\ a$ | $K^0\bar{K}^{*0}$ |
| $K^{*0}\bar{K}^{*0}$ | $\phi\ K^{*0}\ a$ | $K^{*0}\bar{K}^{*0}\ a$ |

$^a$Small amplitude in some models.

Table 4: Summary of amplitude relations between $\bar{b} \to \bar{d}$ and $\bar{b} \to \bar{s}$ penguin-dominated $B^+$ decays. Ratios of $\bar{b} \to \bar{d}$ and $\bar{b} \to \bar{s}$ processes are given by $V_{td}/V_{ts}$. The pair produced out of the vacuum in each class of decay is indicated. Entries indicate final states. Rows indicate amplitudes related to one another.

| $\bar{b} \to \bar{d}$ | $\bar{b} \to \bar{s}$ |
|---------------------|---------------------|
| $s\bar{s}$ pair | $d\bar{d}$ pair | $s\bar{s}$ pair |
| $\bar{K}^0\ K^{*+}$ | $K^0\pi^+$ | |
| $\bar{K}^0K^{*+}\ a$ | $K^0\rho^+\ a$ | |
| $\bar{K}^{*0}K^+$ | $K^{*0}\pi^+$ | $\phi\ K^+$ |
| $\bar{K}^{*0}\ K^{*+}$ | $K^{*0}\rho^+$ | $\phi\ K^{*+}$ |

$^a$Small amplitude in some models.
In order to get a feeling for the level at which (typical SU(3) corrections may affect these relations, let us refer to a specific model [11] for a calculation of the above two ratios of $B^+$ amplitudes. Assuming factorization of penguin amplitudes, the authors find the following dependence on the $\phi$ coupling and mass:

$$A(B^+ \to \phi K^+) \simeq \text{Const. } V_{ts}(g_{\phi}/m_{\phi}) \, , \quad A(B^+ \to \phi K^{*+}) \simeq \text{Const.' } V_{ts}g_{\phi} \, ,$$  \hspace{1cm} (12)

where the coupling $g_{\phi}$ is obtained from the $\phi \to e^+e^-$ rate. Correspondingly, we then obtain in this model (cancelling form factors which are equal to a couple of percent)

$$A(B^+ \to K^{*0}K^+) \simeq \frac{V_{td}g_{K^*/m_{K^*}}}{V_{ts}g_{\phi}/m_{\phi}} \, , \quad A(B^+ \to \bar{K}^{*0}K^+) \simeq \frac{V_{td}g_{K^*}}{V_{ts}g_{\phi}} \, ,$$  \hspace{1cm} (13)

where the $K^*$ weak decay constant $g_{K^*}$ is obtained from the rate of $\tau \to K^*\nu$. Using the measured $\phi \to e^+e^-$ and $\tau \to K^*\nu$ rates, we find

$$\frac{A(B^+ \to K^{*0}K^+)}{A(B^+ \to \phi K^+)} = 0.97 \frac{V_{td}}{V_{ts}} \, , \quad \frac{A(B^+ \to \bar{K}^{*0}K^+)}{A(B^+ \to \phi K^{*+})} = 0.85 \frac{V_{td}}{V_{ts}} \, .$$  \hspace{1cm} (14)

The inclusion of electroweak penguin effects [13] in which the $\phi$ couples directly to the neutral current raises the two coefficients in (14) by about 10%. That is, in this particular model-calculation, these two ratios of $\bar{b} \to \bar{s}$ and $\bar{b} \to \bar{d}$ amplitudes measure $|V_{td}/V_{ts}|$ to within better than 10%.

The branching ratios for the $\bar{b} \to \bar{s}$ modes are expected to be typically of order $10^{-5}$; $B(B^+ \to \phi K^{*+})$ may be a few times $10^{-5}$ [14] [12]. [An exception occurs for the $B \to PV$ processes (the second line in Tables 2–4), whose rates are unpredictably small in some models.] Corresponding $\bar{b} \to \bar{d}$ branching ratios are expected to be about a factor of 8 to 70 smaller, depending on where in the range $|V_{td}/V_{ts}|$ lies. A few times $10^7 B$'s are thus expected to provide useful information on this ratio of CKM elements.

In summary, we have presented a general analysis of penguin-dominated two- and quasi-two-body hadronic $B$ decays to noncharmed pseudoscalar and vector mesons. The ratios of rates for corresponding strangeness-conserving and strangeness-changing processes measure the CKM ratio of elements $|V_{td}/V_{ts}|$. We have shown that corrections from annihilation graphs are absent in neutral $B$ decays, and certain SU(3)-breaking effects can be avoided in some cases. This method of determining $|V_{td}|$ is complementary to other methods proposed in the past, and contains numerous possibilities for estimating corrections in view of the large number of relations which may be studied.

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