THE 1.3MM FULL-STOKES POLARIZATION SYSTEM AT CARMA
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ABSTRACT

The CARMA 1.3mm polarization system consists of dual-polarization receivers that are sensitive to right- (R) and left-circular (L) polarization, and a spectral-line correlator that measures all four cross polarizations \((RR, LL, LR, RL)\) on each of the 105 baselines connecting the 15 telescopes. Each receiver comprises a single feed horn, a waveguide circular polarizer, an orthomode transducer (OMT), two heterodyne mixers, and two low-noise amplifiers (LNAs), all mounted in a cryogenically cooled dewar. Here we review the basics of polarization observations, describe the construction and performance of key receiver components (circular polarizer, OMT, and mixers—but not the correlator), and discuss in detail the calibration of the system, particularly the calibration of the \(R–L\) phase offsets and the polarization leakage corrections. The absolute accuracy of polarization position angle measurements was checked by mapping the radial polarization pattern across the disk of Mars. Transferring the Mars calibration to the well known polarization calibrator 3C286, we find a polarization position angle of \(\chi = 39.2 \pm 1.0\) for 3C286 at 225GHz, consistent with other observations at millimeter wavelengths. Finally, we consider what limitations in accuracy are expected due to the signal-to-noise ratio, dynamic range, and primary beam polarization.

\textit{Keywords:} instrumentation: interferometers; instrumentation: polarimeters; techniques: polarimetric; techniques: interferometric; polarization

1. INTRODUCTION

Millimeter-wave radiation from astronomical objects may be polarized by a number of processes, including many that involve magnetic fields (e.g., synchrotron radiation, Zeeman splitting, dust emission, etc.). This paper describes the design, calibration, and performance of the 1.3mm (210–270GHz) receiver system used for polarization measurements with the Combined Array for Research in Millimeter Astronomy (CARMA). CARMA is an 23-antenna aperture synthesis telescope; the 1.3mm receivers are installed on the six 10m-diameter and nine 6m-diameter antennas.

The CARMA system makes it possible to observe the polarized thermal emission from dust in protostellar cores with angular resolutions of \(\sim 1000\) AU in the nearest star-forming regions (Hull et al.\textsuperscript{2013b}, Krumholz \textit{et al.}\textsuperscript{2013}, Stephens \textit{et al.}\textsuperscript{2013}, Hull \textit{et al.}\textsuperscript{2014}, Davidson \textit{et al.}\textsuperscript{2014}, Wright \textit{et al.}\textsuperscript{2014}, Stephens \textit{et al.}\textsuperscript{2014}, Segura-Cox \textit{et al.}\textsuperscript{2015}) or toward evolved stars (Sabin \textit{et al.}\textsuperscript{2015}). The CARMA polarization system also has been used to measure Faraday rotation toward the active galactic nucleus in 3C84 (Plambeck \textit{et al.}\textsuperscript{2014}), constraining the mode of accretion onto the black hole in this source, and in Very Long Baseline Interferometer experiments that promise to probe the event horizons of nearby black holes (Johnson \textit{et al.}\textsuperscript{2014}).

The plan of this paper is as follows. We first review the measurement of polarization via the Stokes parameters, and we motivate our choice of crossed circular feeds for the CARMA system. We then describe the design and performance of the key receiver components—broadband waveguide circular polarizers, orthomode transducers, and mixers—and discuss in detail the calibration of the \(R–L\) phase offsets and polarization leakage terms. We describe measurements of the absolute polarization position angle (PA) using Mars, and transfer this calibration to source 3C286. Finally, we consider what limitations in accuracy are expected due to the signal-to-noise ratio (SNR), dynamic range, and primary beam polarization.

1.1. Stokes parameters

It is conventional to characterize the polarization properties of the incoming radiation field in terms of Stokes parameters. The Stokes parameters fully describe the characteristics of radiation that is fully polarized, partially polarized, or unpolarized. In an \(X–Y\) coordinate system where \(+x\) points North, \(+y\) points East,\textsuperscript{4} and radiation propagates toward us along the \(+z\) axis, the Stokes parameters are given in terms of the time-averaged products of the complex voltages \(E_x\) and \(E_y\) (Equations 2.47a–2.47d in Rybicki & Lightman\textsuperscript{1979}, Equations 1 in Hamaker & Bregman\textsuperscript{1996} and Equations 4.19 in Thompson \textit{et al.}\textsuperscript{2004}).

Note that, as in Figure 4.8 of Thompson \textit{et al.}\textsuperscript{2004} the \(X\) and \(Y\) axes are rotated 90\(^\circ\) counterclockwise relative to standard Cartesian axes.

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Thompson et al. [2004]:

\[
I = \langle E_X E_X^\ast \rangle + \langle E_Y E_Y^\ast \rangle \quad (1)
\]
\[
Q = \langle E_X E_X^\ast \rangle - \langle E_Y E_Y^\ast \rangle \quad (2)
\]
\[
U = \langle E_X E_Y^\ast \rangle + \langle E_Y E_X^\ast \rangle \quad (3)
\]
\[
V = -i \left( \langle E_X E_Y^\ast \rangle - \langle E_Y E_X^\ast \rangle \right) \quad (4)
\]

where \(E_X^\ast\) denotes the complex conjugate of \(E_X\).

Alternatively, as described in more detail in the Appendix, the Stokes parameters may be expressed in a circular polarization basis:

\[
I = \langle E_R E_R^\ast \rangle + \langle E_L E_L^\ast \rangle \quad (5)
\]
\[
Q = \langle E_R E_L^\ast \rangle + \langle E_L E_R^\ast \rangle \quad (6)
\]
\[
U = -i \left( \langle E_R E_L^\ast \rangle - \langle E_L E_R^\ast \rangle \right) \quad (7)
\]
\[
V = \langle E_R E_R^\ast \rangle - \langle E_L E_L^\ast \rangle \quad (8)
\]

Note that Equations 1–4 and 5–8 are appropriate for a single dish telescope. As described in Thompson et al. [2004], an interferometer like CARMA actually measures Stokes \textit{visibilities} from cross-correlations between antennas. For example, for telescopes \(m\) and \(n\), Stokes \(Q_{mn} = \langle E_{Xm} E_{Xn}^\ast \rangle - \langle E_{Ym} E_{Yn}^\ast \rangle\). These visibilities are Fourier transformed to produce images of Stokes \(I, Q, U, V\) toward the astronomical source. These are maps of radio brightness in units of Jansky/beam. Stokes \(I\) is always positive, but Stokes \(Q, U, V\) can be negative or positive (but not complex).

From the Stokes images, one can derive the fractional linear polarization \(\Pi_l\) and position angle \(\chi\):

\[
\Pi_l = \sqrt{Q^2 + U^2} \quad (9)
\]
\[
\chi = \frac{1}{2} \arctan \frac{U}{Q}, \quad 0 < \chi < \pi. \quad (10)
\]

1.2. Choice of crossed circular feeds.

For polarization measurements with an aperture synthesis array, the receivers may be configured either with crossed linear (\(E_X\) and \(E_Y\)) or crossed circular (\(E_R\) and \(E_L\)) feeds. For observations at 1.3 mm wavelength, the science goals are mostly to measure weak linear polarization; few sources have appreciable circular polarization. Linear polarization is derived from Stokes \(Q\) and \(U\). With crossed linear feeds, Stokes \(Q\) (Equation 2) is derived from the difference in power measured by the \(X\) and \(Y\) receivers. One must therefore take the difference of two large numbers (\(E_X E_X^\ast\) and \(E_Y E_Y^\ast\)) in order to measure a small number (\(Q\), which demands that the \(X\) and \(Y\) gains be extremely stable and well calibrated. The gain stability requirement is relaxed considerably if crossed circular feeds are used, since then both \(Q\) and \(U\) are derived from cross-products of \(E_R\) and \(E_L\). The magnitudes of these cross products are nearly zero if the source is weakly polarized, hence gain fluctuations are much less of a problem. For this reason, it is generally considered better to observe weak linear polarization with crossed circular feeds, and this is the approach that CARMA uses. For a more detailed discussion of the pros and cons of linear and circular feeds see Cotton [1998], as well as the summary (Section 7).

1.3. Observing modes

The 1.3 mm receiver system operates in three modes: single-polarization mode (\(LL\) or \(RR\)) dual-polarization \((LL\) and \(RR\) simultaneously), and full-Stokes \((LL, RR, LR, RL)\). Depending on the correlator setup, all three modes can use up to the full 8 GHz bandwidth of the CARMA correlator (4GHz per sideband), but with different combinations of correlator bands and polarizations. See Table 1 for the properties of the different observing modes in the wide-band (500 MHz/band) setup, which we used for observations of dust polarization.

There is no benefit to using dual-polarization mode for wide-band (continuum) observing. Typically the RCP receivers have higher noise temperatures than the LCP receivers, so it is better to use the available correlator bandwidth to sample a wider frequency range in LCP, rather than a narrower range in both LCP and RCP. The dual-polarization mode offers higher sensitivity only for spectral line observations, where it is beneficial to get independent information from the second polarization, even if the receiver temperature for this polarization is a little higher.

![1 mm dual-polarization receivers](image)

1 mm dual-polarization receiver module, installed inside the dewars on all the 10m and 6m antennas at CARMA. On the 10 m telescopes all these parts are cooled to 4 K. On the 6 m telescopes, to reduce the heat load on the 4 K cryocooler stages, the two WBA13 amplifiers are connected to the 15 K cryocooler stages, while all other parts are cooled to 4 K; the heat load from 15 K to 4 K through the stainless steel SMA connectors is only 1–2 mW.

2. HARDWARE

Figure 1 is a photo of the 1.3 mm dual-polarization receiver module that is mounted in the cryogenically cooled dewar on each telescope. It includes a single feed horn, a waveguide circular polarizer (Section 2.1), an orthomode transducer (OMT; Section 2.2), two heterodyne mixers (Section 2.3), and two low-noise amplifiers (LNAs). The local oscillator (LO) and sky signals are combined using a mylar beamsplitter in front of the dewar window.
1.3 mm CARMA wide-band (500 MHz/band) observing modes. The total bandwidth of each mode is 8 GHz, and is equal to the number of correlator bands × the bandwidth per correlator band (BW/band) × the number of polarizations (Pol.) × the number of sidebands (SB). Observations with narrower bandwidths also are possible, and provide a greater number of channels.

### Table 1

| Mode       | Bands (#) | BW/band (MHz) | Pol. (#) | SB (#) | Total BW (GHz) | Cross-corr. | Channels (# per band) |
|------------|-----------|---------------|----------|--------|---------------|-------------|-----------------------|
| Single-pol | 8         | 500           | 1        | 2      | 8             | RR or LL    | 95                    |
| Dual-pol   | 4         | 500           | 2        | 2      | 8             | RR and LL   | 95                    |
| Full-Stokes| 4         | 500           | 2        | 2      | 8             | RR, LL, RL, LR | 47                    |

2.1. Waveguide Polarizer

The CARMA dual-polarization receivers use waveguide polarizers, operating at temperatures of 4 K, to convert incoming R and L polarized signals into X and Y linearly polarized signals. These polarizers have lower loss and are more compact than quarter waveplates mounted outside the dewars. Since the polarizers are permanently installed inside the dewars, they must operate over the full 210–270 GHz tuning range of the receivers. A polarizer consisting of a single quarter-wave retarder section would have much too narrow a bandwidth for this. However, it is possible to stack one or more half-wave retarder sections, rotated axially with respect to the quarter-wave retarder, to achieve broader bandwidth. Such multi-section polarizers were first described by Pancharatnam (1955) in the context of birefringent waveplates. The action of the multi-section design is most easily visualized on a Poincaré sphere (Figure 2).

The CARMA polarizer is similar to the 2-section waveguide polarizer developed by Kovac for the DASI experiment (Leitch et al. 2002; Kovac 2004). Whereas the (30 GHz) DASI polarizer used dielectric vanes as retarder sections, the much higher frequency (230 GHz) CARMA polarizer uses sections of faceted circular waveguide as retarders. The guide wavelengths differ parallel and perpendicular to the facets, causing a phase shift. The design of the polarizer is described in detail in CARMA Memo 54 (Plancke & Engargiola 2010). Figure 3 shows the waveguide layout, and Figure 4 gives the polarizer dimensions. The performance of the polarizer is characterized by the polarization leakages (see the discussion in Section 2). The leakages for ideal 1-, 2- and 3-section polarizers are shown in the left-hand panel of Figure 5; the leakages expected for a 2-section polarizer when machining tolerances are taken into account are shown in the right-hand panel. When one allows for machining tolerances, the performance of a 3-stage polarizer is no better than that of a 2-stage polarizer.

Note that a multi-section polarizer cannot be flipped end to end — this is because rotations do not commute on the surface of a sphere. On the CARMA polarizers, a thicker flange with a dot marks the OMT end.

As shown in Figure 6, the polarizers are manufactured by electroplating gold and the aluminum mandrel is dissolved with sodium hydroxide to leave the finished waveguide. All electroplating steps were done by A.J. Tuck Co. (Brookfield, CT).

We checked the performance of the polarizers by injecting a linearly polarized signal into the horn end of each polarizer. By definition, this linearly polarized wave is the superposition of equal amplitude R- and L-polarized waves. Thus, for an ideal polarizer one would measure equal powers out the X- and Y-ports of an OMT attached to the output port of the polarizer. Figure 7 shows the results of these tests. The input signal was tuned from 205–270 GHz. Two sets of power measurements were made, with the OMT and two power meters rotated by 90° for the second test in order to average out differences in the OMT loss and the power meter calibrations. The theoretically expected power split for our polarizer design is shown in the lower left panel of the figure.

The measured data for each polarizer were fitted to an analytic model (based on the one described in CARMA Memo 54) that allowed for errors in the angles and thicknesses of the faceted waveguide sections. Smooth curves in each panel show the R and L powers predicted by the model for that polarizer. The model is a simplification in that it assumes that the facet depths of each section are uniform and equal for both retarder sections. However, while the fits suggest that the thicknesses of the faceted retarder sections normally were within the ±0.0003″ allowed tolerance, the angles of these retarder sections often were outside the ±0.2° tolerance. Although it was difficult to inspect the inside of the waveguide on the completed polarizers, careful measurements with an optical comparator found angular offsets that were roughly consistent with the fitted results. The expected polarization leakages may be derived from the fitted dimensions; for most of the polarizers the predicted leakage amplitudes are less than 5% across the 210–270 GHz band.

2.2. Orthomode transducer

After the waveguide polarizer converts incoming R and L polarized signals into X and Y in the circular waveguide, these linearly polarized signals are coupled into separate rectangular waveguides by an orthomode transducer (OMT). As shown in Figure 3, the OMT employs a turnstile junction to split each of the two incoming polarizations into two opposite waveguide arms. These
arms recombine in $E$-plane Y-junctions.

The OMT is fabricated from four blocks that meet along a common edge. Initial tests of the design were made with a 20 GHz scale model (Navarrini et al. 2005 Navarrini & Plambeck 2006). To test the manufacturability at 1 mm, OMTs were then made by 4 independent machine shops (RAL, Custom Microwave, Protofab Inc., U of Arizona). These were tested by Alessandro Navarrini and Alberto Bolatto using a vector network analyzer at NRAO in Charlottesville, VA. Details of these tests are reported in CARMA Memo 32 (Navarrini et al. 2006), and are summarized in Navarrini & Plambeck (2006). The test results showed excellent input return loss (better than –15 dB) and polarization isolation (better than –40 dB). The transmission losses were initially 1–2 dB, a little higher than expected, but improved by several tenths of a dB when gaps in the tuning stub at the base of the turnstile were filled with indium. Unfortunately, the tuning stub is split between the 4 blocks, and getting the blocks to join tightly at the tuning stub is difficult.

Some of the prototype OMTs showed narrow resonances in their transmission curves. Simulations showed that these could be caused by small differences in the lengths of the opposite waveguide arms between the turnstile and the power combiner, perhaps due to misalignments of the 4 blocks (see Figure 19 in Navarrini et al. 2006). Energy coming from the turnstile junction reflects back from the power combiner if the two signals reaching it are not exactly 180° out of phase. Thus, standing waves can be set up between the turnstile and power combiner at frequencies where an integral number of half wavelengths fit within the waveguide arm.

Subsequently, a set of 30 OMTs were machined by Protofab Inc. (Petaluma, CA). These brass parts were gold plated. All but one of the OMTs were tested on a demonstration N4252A network analyzer at Agilent, Inc. (Santa Rosa, CA), in 2010 January and August.

Figure 2. Action of single- and 2-element circular polarizers visualized on a Poincaré sphere. The principal axes of the sphere correspond to Stokes $Q$, $U$, and $V$. The goal is to transform $R$ and $L$ circularly polarized radiation, at the north and south poles, to $X$ and $Y$ linear polarization, at opposite ends of the $Q$-axis on the equator. *Left:* Passing RCP radiation through a single quarter-wave retarder with its principal axes aligned at 45° to the $X$-direction corresponds to a rotation on the sphere that places the central frequency (230 GHz, green) exactly on the equator, but spreads neighboring frequencies (210 GHz, red; 260 GHz, blue) along an arc, leading to substantial ellipticity in their polarizations. *Right:* In a 2-stage polarizer the quarter wave section is followed by a half-wave retarder that rotates the center frequency back to the equator; the dispersion of this half-wave retarder nearly cancels chromatic errors produced by the quarter-wave section, thus broadening the bandwidth.

Figure 3. *Left:* Waveguide layout. The polarizer converts an $L$-polarized signal entering the horn end to a $Y$-polarized signal exiting the OMT end, while $R$ is converted to $X$. *Right:* Outer view of the polarizer. A thick flange with a dot indicates the OMT end. The dot is aligned with the polarizer’s $Y$-axis for serial numbers CP01–CP19, but (due to an unfortunate mixup) with the $X$-axis for CP20–CP36.
Figure 4. Polarizer dimensions (in inches).

Figure 5. Left: Theoretical polarization leakage for 1-, 2-, and 3-section waveguide polarizers (red, green, blue curves) using faceted circular retarder sections. The dashed green curve shows the expected performance of a 2-section polarizer when machining tolerances are taken into account. Right: Simulated leakages for 200 polarizers with machining tolerances of \( \pm 0.0003'' \) on the thickness of the faceted sections, and \( \pm 0.2^\circ \) on the angles of the flats. The green curve shows the mean result that is reproduced (dashed) in the left-hand panel.

We gratefully acknowledge Suren Singh for making these tests possible. Transmission measurements for the 15 polarizers ultimately used on the telescopes are shown in Figure 9. Average transmission losses are in the range 0.4 to 0.7 dB. Narrow (\(~150\text{MHz FWHM}[\text{full width at half maximum}]\)) resonances in the transmission, with losses of up to 2 dB, were often a problem, however. For the handful of OMTs measured both in January and August, the frequencies of the strongest resonances were identical. Often the resonances disappeared if we loosened the screws holding the 4 quadrants of the OMT together, and in some cases it was possible to retighten them in a sequence that maintained better performance. Loosening and tightening the screws changes the waveguide widths, hence the guide wavelengths, hence the electrical lengths between the turnstile junction and the Y-junction power combiners. The arms are about an inch long, so one expects resonances spaced by about 6 GHz, as observed.

Occasionally an OMT that worked well at room temperature developed resonances when cooled to 4 K, probably because of differential thermal contraction between the screws and the 4 OMT quadrants. We were able to detect these resonances by measuring the receiver noise temperature across the 1–10 GHz IF passband over a range of LO frequencies. A lossy resonance in the OMT shows up as a narrow spike in the noise temperature, and this spike moves through the IF as the LO is stepped to different frequencies, forming what we termed “boat wakes” in the noise temperature plots. A good example is shown in
The mixers use ALMA (Atacama Large Millimeter-submillimeter Array) Band 6 SIS (superconductor-insulator-superconductor) tunnel junctions fabricated at the University of Virginia by Arthur Lichtenberger. These are arrays of 4 SIS devices in series, offering greater dynamic range and higher saturation power than a single SIS device. Most of our mixers were constructed using “C12,L56” or “C14,L56” devices from wafer 1489.

Although at ALMA these devices are used in sideband-separating mixers (Kerr et al. 2013), the CARMA mixers are a simple double sideband (DSB) design. An advantage of this design is that the local oscillator can be coupled into the mixer via a beamsplitter mounted outside the dewar. Although a DSB mixer folds together signals arriving in the sidebands above and below the local oscillator frequency so that they appear at the same IF frequency, a phase-switching pattern applied to the local oscillators allows the correlator to separate the upper and lower sideband signals in cross correlation spectra. Unfortunately, this is possible only for signals that are common to a pair of telescopes, not for atmospheric noise that is independent for each telescope. Thus, DSB mixers allow each correlator segment to process twice its bandwidth in sky frequencies, but noise from both sidebands appears in the spectrum of each sideband.

A magnet mounted behind the SIS junction is used to tune out Josephson tunneling through the insulating barrier, which otherwise adds noise and makes it difficult to tune the mixers stably. On ALMA an electromagnet is used for this purpose, so the field can be adjusted to cancel the Josephson tunneling very precisely; for simplicity, the CARMA mixers use just a tiny permanent magnet for this purpose. The magnet clings to a steel 4-40 set screw in the aluminum block attached to the mixer. The set screw may be adjusted (at room temperature) to position the magnet to get about the right field.

One of the steps in optimizing the tuning of an SIS mixer is to adjust the LO power level. In the CARMA system the LO power cannot be adjusted independently for the RCP and LCP mixers. The LO is linearly polarized. It is coupled in through a beamsplitter. As shown in Figure 7, a linearly polarized signal injected into the module will be coupled about equally—but not perfectly equally—into the LCP and RCP mixer blocks. However, mixers sometimes prefer to operate at different power levels. It would be possible to install a rotatable quarter-wave plate between the LO waveguide horn and the beamsplitter to produce any elliptical polarization, which would offer the flexibility of adjusting the power levels independently on the two mixers, but to keep the system as simple as possible this is not done. Instead, the procedure is to optimize the LO power for the LCP mixer only; the RCP mixer must live with whatever it gets. Thus the RCP noise temperatures at CARMA tend to be slightly higher than the LCP noise temperatures.

Figure 11 displays the DSB receiver temperatures as a function of LO frequency for the mixers installed on the telescopes. These noise temperatures were measured in the lab using 295 K ambient and 77 K cold loads. They are based on total power measurements of the entire 1–9 GHz IF band using a broadband power meter, with no bandpass limiting filters. No corrections were made for optics losses in the dewar or for noise contributions from warm amplifiers. The best devices have DSB receiver temperatures of approximately 40 K over much of the 210–270 GHz band. The median receiver temperature, shown by the thick black curve in the plot, is slightly greater than 50 K from 210–240 GHz.

The average power and DSB noise temperature across the IF passband, at an LO frequency of 220 GHz, are shown in Figure 12. The usable part of the IF band is between about 2 and 8.5 GHz.

3. Calibrating Polarization Observations

One approach to calibrating polarization data is through the use of Mueller matrices. A Mueller matrix is a transfer function between the observed and the actual Stokes parameters. This is the approach discussed, for example, by Sault et al. (1995) and by Helles et al. (2001b). We do not use this approach because CARMA data typically are analyzed with the MIRIAD software package, which breaks the calibration into separate passband, gain, $R-L$ phase, and leakage steps.

The passband and gain calibrations are handled by MIRIAD tasks MF CAL and SELFCAL. The passbands generally are stable for the duration of each 6–8 hour observation, and the gain phases of the $R$ and $L$ channels track each other very closely as well. The $R$ vs. $L$ gain amplitudes vary substantially only if one receiver is mistuned; generally amplitude variations are due to pointing or focus errors, which apply equally to the $R$ and $L$ channels.

5 This font signifies the data reduction package MIRIAD itself; a task, procedure, or keyword within MIRIAD; or a Python task. MIRIAD (Sault et al. 1995) is one of the standard data-reduction software packages for (sub)millimeter-wave interferometry.

6 While we are actually measuring the $R-L$ phases at CARMA, the quantity is known in MIRIAD as XYphase.
Figure 7. Test results for all the circular polarizers used on the telescopes. A linearly polarized signal (which can be represented as a superposition of LCP and RCP with equal amplitude) was injected into the “horn” end of the polarizer, and the normalized power emerging from the R (red) and L (blue) outputs of the OMT are plotted as a function of frequency. The power levels would be perfectly equal for an ideal polarizer; the expected power split for a 2-section polarizer of our design is shown in the lower left panel. The data for each polarizer were fitted to an analytical model that allowed for errors in the angles and thicknesses of the faceted sections. $f$ is the depth of the facets (the target dimension is 0.0060′′), $\Theta_3$ is the angle of the half-wave section relative to the Y axis of the OMT (the target value is 15.0°), and $\Theta_2$ is the angle between the quarter-wave and half-wave sections (the target value is 59.5°).

The $R-L$ phase calibration corrects for the phase difference between the $R$ and $L$ channels on each telescope (i.e., the $R-L$ phase) caused by delay differences in the receiver, underground cables, and correlator cabling. The $R-L$ phase is not a single number, but is a function of frequency, due to fiber- and cable-length differences. That is, the same piece of the IF in different correlator sections can have completely different $R-L$ phases. See Section 4 for a detailed discussion of $R-L$ phase and position-angle calibration.

The leakage corrections compensate for cross-coupling between the $R$ and $L$ channels, caused by imperfections in the polarizers and OMTs, reflections inside the dewars, and cross-coupling of IF signals in the analog electronics both preceding and within the correlator. See Section 5 for a detailed discussion of leakage correction.

4. $R-L$ PHASE CALIBRATION

For circularly polarized feeds, the phase difference between the $R$ and $L$ channels is what measures the position angle of an incoming linearly polarized signal. An $R-L$ phase difference of $2\chi$ corresponds to a linearly polarized position angle $\chi$. The ordinary passband correction done with MFCAL analyzes only on the parallel-hand signals $LL$ and $RR$; it does not solve for the $R-L$ phase difference. In order to fit the $R-L$ phase it is necessary to observe a linearly polarized source with a known polarization position angle. $R-L$ phase calibration on astronomical sources is difficult at millimeter wavelengths, since most calibrators are weakly polarized and their polarization position angle varies on time scales of weeks or months. At centimeter wavelengths 3C286 is the usual polarization calibrator. It is approximately 15% polarized, and its position angle has been stable for decades (Perley & Butler 2013). Unfortunately, its flux density at 230GHz is only about 0.4 Jy.

4.1. Grid calibration

For example, a 180° rotation of the $R-L$ phase leads to a 90° rotation of $\chi$. This is because of the circular-to-linear conversion: an $R-L$ phase change of 90° will transform the radiation from linear ($R$ and $L$ in phase) to circular (90° out of phase); another 90° change will transform the radiation from circular to linear (180° out of phase, with a position angle perpendicular to the original linear orientation).
Not only is it impractical to calibrate the $R-L$ phase using astronomical sources, but matters are further complicated because in the CARMA system there is not a single $R-L$ phase difference for each telescope. Rather, there is a different $R-L$ delay—hence, a phase slope as a function of frequency—for each correlator band.

These band-dependent delays arise in a number of places in the signal path: (1) on the front end, the path lengths from the input port of the OMT to the $R$ and $L$ outputs differ by 0.1 inch.\(^8\) (2) There are chromatic effects in the polarizer and OMT that cause slight differences in the phase shifts for signals in the upper and lower sidebands—see Section 4.2 for further discussion. (3) There are differences in the $R$ and $L$ fiber lengths from the receivers to the correlator. (4) In the block downconverters, there are cable-length differences between the upper (>5 GHz) path (the path with the mixer) and the lower (<5 GHz) path (without the mixer).

To overcome these various difficulties, we use observations of artificial linearly polarized noise sources to calibrate the $R-L$ phase. The noise sources are created by inserting wire grid polarizers (Figure 13) into the beams of the 10 m telescopes. With the grid in place, one linear polarization reaching the receiver originates from the sky, while the other originates from a room temperature load. Since the room temperature load is much hotter than the sky, the receiver sees thermal noise that is strongly polarized (Figure 14).

By default, the master observing script performs grid observations approximately every 45 minutes during full polarization observing tracks. The calibration requires between 1 and 2 minutes: 45 seconds for the grids to move in and out of the beams, and 30 seconds of integration.

The grid data are identified by setting the purpose keyword to P in the data header. Grid observations are done at the end of phase calibration observations. Although the sky signal includes flux from the calibration source, this is negligible compared with the huge signal from the ambient load, and thus does not affect the measured $R-L$ phase.\(^9\)

Since the polarized noise is local to each telescope, it does not show up in cross-correlations with other telescopes. However, it leads to a strong $LR$ autocorrelation (or "cross-auto") signal, which is the cross-correlation of the $L$ and $R$ channels from a single antenna. The MIRIAD task $XYAUTO$ averages together all the $LR$ autocorrelation data for the grid observations in each dataset to create a channel-by-channel passband correction for the 10 m telescopes. Rewriting the data with UVCAT or UVCAL applies these corrections. In the new dataset, the phases of all the $R$ and $L$ channels are equal on the 10 m telescopes. This means that a linearly polarized signal reaching the receiver with the same PA as the noise source will produce $LR$ and $RL$ correlations with phases of zero. One of these 10 m telescopes must then be used as the reference antenna for the regular passband correction performed with MFCAL. The passband correction synchronizes the $R$ phases on all telescopes with the $R$ phase of the reference antenna, and the $L$ phases on all telescopes with the $L$ phase for the reference antenna. Since the $R$ and $L$ phases of the reference antenna are equal, then the $R$ phases of all antennas equal the $L$ phases on all antennas.

Absolute position angle of the grids. MIRIAD computes the polarization position angle of a source as

\(^8\) A distance of 0.1 in is a couple of wavelengths at 230 GHz. However, both the incoming radiation and the LO travel through this same length, and it is only the difference in phase shift at these two frequencies that matters. For an IF frequency of 5 GHz, this differential phase shift is $\sim 20^\circ$.

\(^9\) For example, 3C279 has a brightness of $\sim 10 \text{ Jy}$, and is about 10% polarized. The antenna gains of the 10 m telescopes are about 65 Jy/K, so in terms of brightness temperature, the flux from the 3C279 is roughly 15 mK. This is two to three orders of magnitude smaller than the (polarized) temperature difference between sky and ambient load, which is of order 100 K even at low elevation in bad weather.
Figure 9. Transmission loss (S21) measurements of the OMTs used on the telescopes, made with an Agilent N4252A network analyzer. Losses in waveguide transitions to the OMT have been calibrated out. Red curves are the transmission from the circular input to the “short” arm of the OMT (rectangular waveguide port closer to the input); black curves are transmission to the “long” arm. The step size is 18.75 MHz. The lossy resonances are roughly 150 MHz wide at the half-power points.

\[ \chi = q + \text{evector}, \] where \( q \) is the source’s parallactic angle and the \( uv \)-variable \( \text{evector} \) is defined, for linear feeds, as the position angle of the X-feed relative to the local vertical. For circular feeds, \( \text{evector} \) is interpreted as the position angle \( \chi \) for which RCP and LCP radiation are in phase with one another, which is determined by the angle of the wire grid noise source.

With the grids in place and the telescopes pointed at zenith, noise reaching the receivers is vertically polarized. However, because of the reflection off the tertiary mirror, a vertically polarized signal in the receiver cabin corresponds to a horizontally polarized (\( \chi = 90^\circ \)) source on the sky (Figure 13). Consequently, in datasets from CARMA the MIRIAD \( uv \)-variable \( \text{evector} \) is set to 90°.

4.2. Systematic limits to R–L phase accuracy

The grid calibration of \( R–L \) phase is susceptible to a number of limitations that an astronomical calibration would not be.

USB and LSB are averaged together. The autocorrelation averages together the \( R–L \) phase differences in the upper and lower sidebands. In normal cross-correlation spectra between antennas \( m \) and \( n \), the LSB and USB are separated by demodulating the phase-switching pattern between the \( m \) and \( n \) local oscillators. This is not possible for the \( LR \) autocorrelation spectrum for a single antenna, since both the RCP and LCP channels use a common LO.

The delays that contribute to the \( R–L \) phase difference are mostly at the IF frequency, and thus produce precisely the same phase shifts for signals in the LSB and USB. The only exception is the difference in delay from the beamsplitter to the RCP and LCP mixers. There is a 0.1 in length difference through the OMT for the two polarizations, and the waveguide is slightly dispersive,
Figure 10. Receiver noise temperature vs. IF frequency, measured at 46 different LO frequencies, from 210 GHz (bottom) to 255 GHz (top) in 1 GHz steps. In this plot resonances in the OMT show up as sharp peaks that march across the IF band, forming “boat wakes.” In this example mixer 54-22 was mounted on the short arm of OMT10. The handful of OMTs that showed this behavior were disqualified for use on the array.

so the delay is slightly different for the USB and LSB. A quick calculation shows that this effect is small: the phase shift of the LSB relative to the LO is at most 2°

Figure 11. DSB receiver temperatures as a function of LO frequency for the mixers used on the array (except 4 that are not shown because their data were lost in a disk crash). These are based on total power measurements of the entire 1–9 GHz IF passband with a broadband power meter, with no band limiting filters. The median receiver temperature is shown by the thick black curve.

so the delay is slightly different for the USB and LSB. A quick calculation shows that this effect is small: the phase shift of the LSB relative to the LO is at most 2°

Figure 12. IF output power and DSB noise temperature as a function of IF frequency, from an average of 22 mixers. Noise temperatures were based on measurements of the IF power made with an Agilent E4407 spectrum analyzer, using a 3 MHz resolution bandwidth.

greater than the phase shift of the USB relative to the LO. The waveguide polarizer also is slightly chromatic, so the phase shifts through it will differ slightly for the upper and lower sidebands.

**Common noise.** Noise that is common to both the R and L channels will generate an LR autocorrelation signal even when the wire grid is out of the beam. For example, noise radiated out the input of one mixer could be transmitted out through the OMT, polarizer, and feed horn, reflect back into the module from the dewar window (as the opposite circular polarization), and be coupled into the opposite mixer. If the R and L mixers had independent local oscillators with different phase switch patterns, these signals could be rejected in the LR autocorrelations; however the local oscillator is shared between the two polarizations in the CARMA receivers, so this is not possible.

In order to derive the R–L phase with high accuracy, the signal from the polarized noise source must be much larger than the background level. To test this, we compared the LR amplitudes in typical 1 mm weather with and without the grids in place. The amplitudes are ~20 times higher with the grids in. This corresponds to an rms error of ~1/20 radian, about 3°, which leads to an uncertainty of 1.5° in position angle. Of course, in poor weather or at low elevation the uncertainty can be greater because the contrast between sky and ambient is less, so the polarized noise level is lower.

**Leakage.** There is always some crosstalk between the R and L channels due to polarization leakage. As described in Section 5, the leakages exhibit considerable frequency structure. Ideally one would solve for R–L phase and leakage in an iterative way, but this is not easily accomplished with MIRIAD, since there is no simple way of applying

11 The guide wavelength is $\lambda_g = \lambda_0 / \sqrt{1 - (\lambda_0 / 2a)^2}$, where $\lambda_0$ is the wavelength of the radiation in free space and $a$ is the broad dimension of the waveguide. The dispersion is greatest at the low end of the band. Taking an LO frequency of 210 GHz and an IF of 8 GHz, the phase shift is 36.2° for the LSB and 34.8° for the USB.
leakage corrections on a channel-by-channel basis. This means that the leakages can introduce ripples into the $R-L$ phase calibration.

**Phase-lock loop (PLL) jitter.** Each correlator band uses separate digitizers for $R$ and $L$. The digitizers are not run off of a single 1 GHz clock, but instead are run off of independent clocks that are phase-locked to a common reference signal. One systematic effect that limits our ability to measure absolute position angle is rapid variation in $R-L$ phase solutions caused by jitter (phase noise) in the 1 GHz PLL outputs. See Figure 15 for plots of the $R-L$ phase residuals (after correction), which show 1–2° variations on very short (10 s) timescales. This effect should average out for most astronomical measurements.

5. **LEAKAGE CALIBRATION**

Leakage corrections compensate for cross-coupling between the $R$ and $L$ channels, caused by imperfections in the receivers or crosstalk in the analog electronics that precede the correlator. Leakages are measured in terms of voltages (1% [or 0.01] leakage in voltage corresponds to $10^{-4}$ in power), and are defined in the following way (Thompson et al. 2004, Equation 4.42):

$$v'_R = v_R + D_R v_L$$
$$v'_L = v_L + D_L v_R$$

where $v'_R$ and $v'_L$ are the observed signals, $v_R$ and $v_L$ are the true signals, $D_R$ is the leakage from $L$ into $R$, and $D_L$ is the leakage from $R$ into $L$.\(^\dagger\) Note that the leakages are complex numbers.

Leakages are calibrated by observing a strong source (usually the gain calibrator) over a range of parallactic angle. The calibrator may be polarized or unpolarized. Since the telescopes have altitude-azimuth (alt-az) mounts, the

\(^\dagger\) In MIRIAD, $D_x \to D_R$, and $D_y \to D_L$. 

---

**Figure 13.** Left: Photo of a wire grid polarizer in the receiver cabin of a 10m telescope, with the telescope pointed at zenith. The wires in the grid are vertical. Right: Photo of the tertiary mirror with the 10m telescope pointed at zenith. Horizontally polarized radiation from the sky reflects off the tertiary such that it is vertically polarized at the receiver. The receiver with its wire grid rotates in elevation along with the telescope, so this correspondence is preserved at all elevations.

**Figure 14.** A schematic showing how the wire grid polarizers produce a highly polarized noise source by reflecting ambient radiation into the receiver.

**Figure 15.** A sequence of cross-auto ($LR$) grid calibration data on antenna C3, taken at 10 second intervals, after the $R-L$ phase correction has been applied. The 1–2° residuals are caused by phase jitter in the digitizer clocks.
source’s polarization will appear to vary, in the frame of the receivers, as the telescopes track it across the sky, modulating the LR and RL cross correlation amplitudes in a predictable way. The component of the cross correlation amplitude that does not vary with parallactic angle must then be due to instrumental leakage.

MIRIAD task GPCAL fits observations of the calibrator in order to solve simultaneously for the antenna gains vs. time, the polarization leakages, and the source polarization. Given good weather and normal antenna performance, a 4–6 hr observation for which the source parallactic angle varies by more than about 60° yields a reliable leakage solution.

Unfortunately, MIRIAD allows for only a single pair of leakage corrections $D_x$ and $D_y$ for each telescope. To solve for channel-by-channel leakages requires one to run GPCAL many times, specifying each channel range in turn and saving the results. We have written a library of Python routines to handle this chore, and have accumulated a library of leakage solutions covering many frequency ranges.

**Expected leakage amplitudes.** Ideally, the polarization leakages would be due exclusively to imperfections in the waveguide polarizers. Figure 5 shows that we expect leakage amplitudes of ~2–3% for polarizers that are within the expected dimensional tolerances. From the polarizer test data in Figure 13, we inferred that many of the polarizers were not within these tolerances; however, the leakage vs. frequency expected from each polarizer can be computed if its dimensions are known, using the software described in Plambeck & Engargiola (2010).

Figure 16 shows the band-averaged leakages for all telescopes based on observations obtained from 2011–2015. In some cases dewars were swapped, so fewer datasets were used. The dashed curve on each plot shows the theoretical leakage expected for the particular circular polarizer on that telescope, based on the fits to the polarizer dimensions shown in Figure 13. In most cases the leakage amplitudes are substantially larger than expected from the polarizer alone, although for a few telescopes (C1, C2, C8, C15) the polarizer curve appears to form a lower bound to the measured leakages.

With higher frequency resolution, one finds that in most cases the leakages have substantial frequency structure, with periods of a few × 100 MHz to a few GHz. Figure 17 shows examples of this structure for C2, one of the best antennas, and for C13, one of the worst. The ripples in the leakages suggest that cross coupling of the unwanted polarization takes place via multiple paths with different delays. We discuss possible sources of cross-coupling in the sections below.

**Cross-coupling in the block downconverter.** One source of leakage ripples is cross-coupling of IF signals in the correlator room, probably in the block downconverters. Each of the 8 analog downconverters assigned to a telescope obtains its input from a 4-way switch on that telescope’s block downconverter. The 4 inputs to the switch are the low-band RCP, low-band LCP, high-band RCP, and high-band LCP, where “low-band” is the 1–5 GHz piece of the IF band, and “high-band” is the 5–9 GHz piece (which has been downconverted to 1–5 GHz). Although nominally the switch provides 45 dB isolation between ports, this is not adequate if the RCP and LCP power levels differ substantially. For example, if the RCP power is 15 dB greater than the LCP power, then the relative level of RCP coupled into the LCP IF is ~30 dB, which means that $|V_R| = 0.03|V_L|$. This RCP signal beats with the RCP voltage at the mixer, producing a 6% peak-peak ripple in $D_L$. Meanwhile, the relative level of LCP in the RCP IF is ~60 dB, which produces just 0.2% ripple in $D_R$.

Thus, cross-coupling in the block downconverter manifests itself as leakage ripples in one polarization, but not the other. An example of this behavior is shown in Figure 18. In this observation 2 correlator sections were centered at 2.5 GHz in the IF, while the other 2 were centered at 7.5 GHz. Prominent leakage ripples are seen only in $D_L$, and only in the 1–5 GHz section of the IF band. Evidently the RCP and LCP power levels are more closely balanced above 5 GHz.

Figure 19 shows that the ripples can be reduced by equalizing the RCP and LCP power levels. For these observations the RCP power into the block downconverter on telescope C9 was initially 9 dB greater than the LCP power. Installing a 10 dB attenuator on the RCP input to the block downconverter substantially reduced the ripple in $D_L$ for the the low-band sections, and caused no deterioration in performance above 5 GHz or in $D_R$.

**Reflections in the receiver.** The ripples that we attribute to cross-coupling in the block downconverters have periods of about 250 MHz. Many antennas, particularly the 6 m antennas (C7–C15), also have leakage ripples with a period of about 1 GHz (see the results for C13 plotted in Figure 17, which corresponds to cross-coupling with a 1 nsec delay). We hypothesize that this is caused by reflections inside the dewar. In the 6 m dewars the front of the feed horn is approximately 5 cm behind the dewar window, and the path lengths through the components are roughly 5 cm for the horn, 2 cm for the waveguide polarizer, and 3 cm for the OMT. Thus, an RCP signal that reflected off the RCP mixer would travel back through a 15 cm path to the dewar window (as RCP), Reflection off the window would convert it to LCP. It then would travel through another 15 cm path to the LCP mixer, accounting for a total delay of 30 cm/c, or 1 nsec.

To test this hypothesis, we attempted to reduce the reflection from the RCP mixer in antenna C9 by changing the mixer bias (see Figure 20). Reducing the reflection off of one mixer should reduce the leakage ripples for the opposite polarization. For the data shown in red in Figure 20 the RCP mixer was biased to 13 mV. This is above the superconducting energy gap, so no RCP astronomical signals would have been downconverted to the IF: and indeed, the RCP leakages (red curves, lower panel) are just noise. However, at this bias there should be a better impedance match between the SIS mixer and waveguide, reducing the magnitude of the signal reflecting back from the mixer. And, indeed, the LCP leakage ripples (red curves, upper panel) appear to be reduced, consistent with the reflection hypothesis.

We then tried to reduce the reflection from the dewar...
Figure 16. Leakage amplitudes vs. frequency for all telescopes, derived from 12 datasets obtained over a 3.5 year period. Each point shows the leakage amplitude for a single 0.5 GHz wide correlator window. Red points indicate $D_R$; blue points, $D_L$. The dashed curve in each plot shows the leakage expected from the circular polarizer on that antenna, based on the polarizer measurements shown in Figure 7. Fewer data are plotted for some antennas because of receiver swaps.

window (a double convex Teflon lens) by tilting it with a special clamp ring. It was physically possible to tilt the lens by only about 5°, and this had no apparent effect on the leakages. In another experiment, we attempted to worsen the reflection from the dewar window on C7—an antenna with relatively small leakage ripples—by installing a flat 0.010 in thick piece of Mylar just in front of the window. Again, however, there was no discernable effect on the leakages. Finally, to confirm that the ripples originate in the dewar and not elsewhere in the telescope or correlator room, we physically swapped the dewars between antennas C9 and C10. This swapped the leakage pattern between these antennas, confirming that the problem does originate in the dewar.

Reproducibility of the leakage solutions. Since the CARMA receivers have no moving parts, the leakages are expected to be stable and reproducible. Figure 21 presents a histogram of the scatter in leakage solutions from several different datasets obtained over a two-month interval. In this case the standard deviation of the Re and Im parts of the leakage terms was $\sim 0.009$; however, in data taken a few days apart the standard deviation can be as small as $\sim 0.002$.

Factors limiting the reproducibility are reflections from the mixers—which will depend on their physical temperature and on their voltage and current bias—and the relative RCP and LCP power levels in the block downconverters, which will depend on the tuning and correlator
Figure 17. Leakage amplitudes vs. frequency for antennas C2 and C13 over the frequency range 214–224 GHz. The frequency resolution is typically 50 MHz. In most cases there is excellent reproducibility in the leakages obtained months or years apart. Dashed curves show the leakage amplitude expected from the circular polarizers on these telescopes.

Figure 18. Leakage amplitudes for antennas C2 and C3 plotted as a function of sky frequency, derived from a dataset where two correlator sections were positioned above 5 GHz in the IF (outer two columns, “LSB-high” and “USB-high”), and two below 5 GHz (middle two columns, “LSB-low” and “USB-low”); note that the lowest sky frequency corresponds to the highest IF frequency in the LSB. Black curves are $D_R$ and blue curves are $D_L$. Large ripples are evident only in $D_L$, and only below IF frequencies of 5 GHz (the “low” bands), indicative of cross-coupling in the block downconverters.

There is an ambiguity in the leakage terms because they always appear in pairs in expressions for the LR and RL cross-correlations (Sault et al. 1995). For example, the observed $RL^*$ cross correlation between antennas $m$ and $n$ is given by:

$$
\langle v_{Rm}^*v_{Ln}^* \rangle = \langle (v_{Rm} + D_{Rm}v_{Lm})(v_{Ln} + D_{Ln}v_{Rn})^* \rangle
$$

(13)

$$
= \langle v_{Rm}v_{Ln}^* \rangle + D_{Rm}\langle v_{Lm}v_{Ln}^* \rangle + D_{Ln}\langle v_{Rm}v_{Rn}^* \rangle
$$

(14)

$$
= \langle v_{Rm}v_{Ln}^* \rangle + \frac{1}{2} f_{mn}(D_{Rm} + D_{Ln})
$$

(15)

This expression is unchanged if one adds an arbitrary complex number $c = a + jb$ to all the $D_R$ terms, and subtracts its conjugate $c^* = a - jb$ from all the $D_L$ terms:

$$(D_{Rm} + a + jb) + (D_{Ln} - a + jb)^* = D_{Rm} + D_{Ln}^*.$$  

(16)
Figure 19. $D_L$ leakage amplitude vs. frequency for antenna C9 before (black) and after (red) attenuating the RCP input to the block downconverter by 10 dB. The frequency setup is similar to that in Figure 18. Lowering the RCP power level in the downconverter reduced coupling of RCP into LCP, hence reduced the magnitudes of the leakage ripples in $D_L$ for the two “low-band” correlator sections. The $D_R$ leakages were unchanged, as were the leakages on all other telescopes.

Figure 20. Leakage amplitudes as a function of frequency for antenna C9 derived from 9 different datasets. For the data shown in red the voltage bias on the RCP mixer was set above the SIS junction’s superconducting energy gap so that the junction behaved as a resistive load. This reduced reflections of RCP signals from this mixer, which lowered the magnitude of the LCP leakage ripples. The RCP leakages for this dataset were then just noise, since there was no signal coming from the RCP receiver.

By convention, MIRIAD chooses this complex offset $c$ such that $\sum \text{Re}(D_{Rm}) = \sum \text{Re}(D_{Lm})$ and $\sum \text{Im}(D_{Rm}) = -\sum \text{Im}(D_{Lm})$. As a consequence, the absolute leakages may change if some antennas are missing from the data set. When plotted in the complex plane, $D_R$ and $D_L$ have a reflection symmetry across the imaginary axis for most antennas (see Figure 22). That is, $\text{Re}(D_R) = -\text{Re}(D_L)$ and $\text{Im}(D_R) = \text{Im}(D_L)$. Thus, the MIRIAD convention causes the average of the imaginary parts of the leakages to be close to 0. If an antenna like C3, for which the leakages have a large imaginary offset, is omitted from the solution, then the imaginary parts of all the other antennas will increase by about 0.01. One must therefore be cautious when comparing leakages derived from different data sets.

6. SYSTEMATIC LIMITATIONS TO POLARIZATION MEASUREMENTS

There are several effects that limit the accuracy of polarization measurements made with CARMA. (1) The accuracy is limited by the signal-to-noise ratio; noise bias leads to an overestimate of the polarization fraction in the low-SNR case. (2) There is a dynamic range limit: for a bright source that is weakly polarized, errors in the leakage calibrations can give false detections of polarization. (3) The absolute position angle accuracy depends on the grid calibrations. (4) For extended (non-point-like) sources, the polarization varies across the beam. These effects are discussed in the sections below.

6.1. Signal-to-noise limitations: debiasing polarimetric images

Polarization measurements have a positive bias because the polarization $P = \sqrt{Q^2 + U^2}$ is always positive, even though the Stokes parameters $Q$ and $U$ from which $P$ is derived can be either positive or negative. This bias has
The probability density function (PDF) for the observed polarization $P_c$ of a signal with true polarization $P$, is given by the Rice distribution (Killeen et al. 1986; Equation B1, and Vaillancourt 2006; Equation 6):

$$
\text{PDF}(P_c|P, \sigma_P) = \frac{P}{\sigma_P} I_0 \left( \frac{PP_c}{\sigma_P^2} \right) \exp \left[ -(P^2 + P_c^2)/2\sigma_P^2 \right].
$$

(18)

Thus, given the observed polarization $P_c$, one can calculate the true polarization $P$ by finding the maximum (i.e., the most probable value) of the PDF in Equation 18.

For very significant polarization detections ($P \geq 5\sigma_P$), the simple high-SNR limit is valid (see Vaillancourt 2006; Equation 12):

$$
P_c \approx \sqrt{Q^2 + U^2 - \sigma_P^2}.
$$

(19)

However, for low-SNR detections ($P \lesssim 5\sigma_P$), things are not so simple, and one must use Equation 18 to calculate the debiased polarization intensity.

The position angle $\chi$ and uncertainty $\delta\chi$ (calculated using standard error propagation) of the incoming radiation are

$$
\chi = \frac{1}{2} \arctan \left( \frac{U}{Q} \right),
$$

(20)

$$
\delta\chi = \frac{1}{2} \frac{\sigma_P}{P_c}.
$$

(21)
I.e., for a detection $P_e = 2 \sigma_p$, the uncertainty in the position angle is $1/4$ of a radian, or $\pm 14^\circ$.

6.2. Limitations due to leakage uncertainties.

The accuracy of polarization measurements also is limited by uncertainties in the polarization leakage corrections. These uncertainties affect bright sources as well as weak ones; thus they provide a dynamic range limit for polarization measurements with CARMA.

To test the effects of leakage variations on the solutions of polarization position angle and fraction, we performed simulations using UVGEN. As discussed above in Section 5, the Re and Im parts of the leakages vary by up to a few $\times$ 0.01 from track to track. Thus, we set up the simulations to probe the effects of Gaussian random errors in the leakages with standard deviations between 0.01 and 0.03. The simulated $uv$ data included thermal noise consistent with typical 1 mm weather at CARMA: system temperatures of 300 K and opacity $\tau \approx 0.3$. The data comprised 8 GHz of continuum bandwidth, and included a point source with a flux of 1 Jy and at an elevation of 30°. We used actual leakage terms from CARMA polarization data as the initial set of values.

We performed two tests, examining how variations in leakage terms affect the PA and the polarization fraction ($\%$) of (1) a snapshot observation (for example, when measuring the polarization of the passband calibrator, which is only observed for 10 min), and (2) a longer, $\sim 6$ hr observation of the calibrator or the science target.

The simulations proceeded as follows:

- Used UVGEN to simulate leakage-free data: either a 10 min snapshot, or a long track covering $\pm 3$ hr around transit
- Copied “real” leakages (from a typical 1 mm CARMA polarization track) into the data file
- Rewrote the data to apply the leakages, thus corrupting the data
- Copied a negated version of the above leakages into the dataset after varying the leakages’ Re and Im parts using Gaussian random errors with standard deviations between 0.01 and 0.03
- Used UVFLUX to find the $\%$ and PA of the source
- Ran a Monte-Carlo test for point sources with 0%, 1%, and 10% polarization, and calculated the standard deviation of the polarization fraction and PA for each value of the leakage error

Copying negated leakages into the dataset corrects the corrupted data exactly. However, copying a set of leakages whose Re and Im parts were modified slightly result in slight differences in the calculated PA and $\%$. Performing these simulations gave us a handle on how robust our calculated PA and $\%$ values were to typical leakage variations.

See Figure 23 for the results. Unsurprisingly, the scatter in PA and $\%$ is larger for snapshot observations than for full tracks. Additionally, the scatter in PA is much larger for weakly polarized sources. Looking at the plots of the scatter in $\%$, we conclude that the instrumental polarization caused by the typical leakage errors of $\sim 0.01$ is $\sim 0.3\%$ for a snapshot observation and $\sim 0.1\%$ for a longer track. The corresponding variations in position angle are $\sim 8^\circ$ and $\sim 3^\circ$ for a 1% polarized source.

Note that very small variations in the leakages can cause drastic changes to the measured polarization position angles if a source is very weakly polarized ($P_e/I \lesssim 0.5\%$). For example, when analyzing CARMA data toward the protoplanetary nebula CRL 618 (see Sabin et al. 2014), the position angles varied by up to $90^\circ$ from night to night depending on how exactly we reduced the data. The scatter in the Re and Im parts of the leakage solutions on the two nights was $\sim 0.01$, which is standard; however, in this case these slight differences led to significantly different Q and U maps. This was not a problem for the TADPOL sources (see Hull et al. 2013, 2014), which were on average at least a few percent polarized, and which tended to be much fainter sources where the ability to detect polarization was limited by the SNR instead of by dynamic range.

We therefore urge caution when interpreting the position angles of sources with polarization fractions of $< 0.5\%$.

6.3. Absolute position-angle accuracy

Even for a strongly polarized source, there is an uncertainty in PA due to the absolute accuracy of the $R$–$L$ phase calibration. As discussed in Section 4, polarized noise sources in the cabins of the 10 m telescopes are used to measure the $R$–$L$ phase. One of the 10 m telescopes (usually C1, since it has relatively small leakages) is used as the reference antenna for the passband calibration. This transfers its $R$–$L$ phase calibration to all the other telescopes, including the 6 m telescopes, which are not equipped with polarized noise sources.

Observations of an astronomical source with a stable, well defined PA are required in order to check the accuracy of the PA measurements. At centimeter wavelengths, 3C286 is the usual polarization calibrator: its position angle has been stable for decades (Perley & Butler 2013). However, the PA of 3C286 increases slowly with frequency, from $\chi = 33^\circ$ at $\lambda \gtrsim 3.7$ cm to $\chi = 36^\circ$ at $\lambda 0.7$ cm. At millimeter wavelengths, Agudo et al. (2012) measured $\chi = 37.3 \pm 0.8^\circ$ at $\lambda 3$ mm and $\chi = 35.1 \pm 0.7^\circ$ at $\lambda 1.3$ mm. ALMA commissioning results at $\lambda 1.3$ mm gave $\chi = 39^\circ$ (Stuart Corder, priv. comm., 2013).

Fortunately, observations of the polarized thermal emission from a rocky planet or satellite provide an absolute standard by which the PA of 3C286 can be measured. The planet must be well resolved by the synthesized beam. The polarization is expected to be radial with respect to
the center of the planet’s disk.\textsuperscript{14}

The radial polarization pattern has been seen before in, for example, observations of the Moon (Heiles & Drake 1963; Davies & Gardner 1966; White & Cogdell 1973; Heiles & Drake 1983) found that the maximum polarization of the Moon was \( \sim 20\% \) when observed at \( \lambda \) 21 cm. The Moon is much too large to fit into the primary beams of millimeter wave telescopes like CARMA or ALMA, however. Mars is a better choice for these telescopes.

Figure 23 shows 225 GHz polarization observations of Mars made with CARMA on 04 May 2014. The planet was \( 14.2'' \) in diameter and the synthesized beam was \( 2.9'' \times 1.7'' \). Observations of Mars, 3C279, and 3C286 were interleaved over a 3 hour long track. Leakages were derived from the 3C279 data. A careful analysis of the polarization vectors (Figure 25) shows that they are skewed by about 1.8° from the radial direction.

**PA depends on the choice of reference antenna.** Unfortunately, a different choice of reference antenna yields a slightly different absolute position angle. As shown in column 5 of Table \( \text{2} \) for the 04 May 2014 data the offset ranges from \( -0.5^\circ \) for C5 up to \( 8.7^\circ \) for C6. These discrepancies are unlikely to originate from leakages; C3 has the worst leakages (magnitudes of order 10\%), but its PA deviation is no larger than average. The deviations may originate in the optics between the wire grids and the primary mirrors of the 10 m telescopes, but further tests should be made.

For each choice of reference antenna we also computed the PA of 3C286. Column 10 of Table \( \text{2} \) shows the 3C286 position angles after subtracting the corresponding Mars radial deviation. The answers are surprisingly consistent, with an average PA value of 39.9°. Using another leakage solution derived from 3C279 observations two days earlier, on 02 May 2014, gave a similar result: 40.0° for the PA of 3C286.

Table 3 summarizes Mars polarization observations on two different days: 04 May 2014 (see Table \( \text{2} \)) and 17 Jan 2014. The difference in the final corrected PA for 3C286 between the two days gives some indication of the uncertainty in the calibration. For the January observation Mars was observed over a narrow range in

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**Figure 23.** Errors in PA and fractional polarization as a function of leakage errors.
parallactic angle, and the planet’s disk was less well-resolved by the synthesized beam, so it is tempting to assume that those data are less reliable. On the other hand, primary beam polarization (Section 6.4) might have skewed the results of the May observations, when the planet’s disk was larger. Taking the average of the two measurements, we conclude that the polarization position angle of 3C286 at 225 GHz is approximately 39.2° ± 1°.

USB – LSB position angles. For Faraday rotation measurements, one measures the difference in a source’s polarization position angle in the LSB and USB. Slight differences in the $R-L$ phase difference are expected at these two frequencies because of chromatic effects in the waveguide polarizers. It is not possible to derive the USB and LSB $R-L$ phases from the wire grid noise source, since the sidebands cannot be separated in noise that is local to each antenna.

Column 4 in Table 3 shows the difference in the Mars radial deviation for the USB and LSB. Depending on the choice of antenna used for the $R-L$ phase calibration, this value ranges from 0.12 to 2.81°. Column 8 shows the difference in PA measured on 3C286 for the USB and LSB. For reasons that are still unclear, there is a systematic difference of about 1.2° between the Mars and 3C286 offsets. We have higher confidence in the 3C286 values, since 3C286 is a point source.

6.4. Primary-beam Polarization

If a source is not perfectly on-axis, then additional distortions are introduced across the primary beams of the telescopes. To check for these variations in the instrumental polarization, we observed BL Lac (a bright, highly polarized quasar) at 16 offset positions, eight of which were 12′′ and eight of which were 24′′ from the field center.15 Our aim was to characterize the “beam squint” and “beam squash.”

Beam squint arises when the feeds of the telescope are tilted with respect to the optical axis of the primary reflector, causing the LCP and RCP responses to be slightly offset from the symmetry axis. Beam squint is discussed in Chu & Turrin (1973); Adatia & Rudge (1975); Rudge & Adatia (1978). Squint manifests itself as a four-lobed “cloverleaf” pattern in the linearly polarized Stokes $Q$ and $U$ maps. This phenomenon is discussed in Napier (1994, 1999); the term “beam squint” was coined in Heiles et al. (2001a).

Maps of both squint and squash have been made by Robishaw (2008) for the Green Bank Telescope (GBT) and Heiles et al. (2001a) for Arecibo.

Additionally, there is a large body of work on wide-field polarimetric calibration and imaging with the Karl G. Jansky Very Large Array (VLA). Uson & Cotton (2008) primarily discuss Stokes $V$ (squint) calibration. Cotton & Perley (2010) discuss both off-axis circular (squint) and linear instrumental polarization, and present a model for correcting for both. They measure the off-axis instrumental polarization for various frequencies using the method we employ at CARMA, where all antennas move to various offset positions simultaneously.

A more robust method of removing polarization artifacts is beam holography, where one or more “reference antennas” remain pointed at a strong source while the remaining antennas move in a raster pattern, thus measuring the primary beam of each dish in all four Stokes parameters $IQUV$ (see, e.g., Corder & Wright 2006; Lamb & Corder 2008), who map the total power [Stokes $I$] beam at CARMA. After mapping all four beams for all antennas, one can then use those beam models to correct the full-Stokes data.

Beam squash. While the CARMA system is almost always used for measuring linear polarization, we are still able to make maps of Stokes $V$, and are thus able to measure squash. Beam squint is normally characterized using the squint angle $\Psi_s$ (Rudge & Adatia 1978; Equation 25; see also Napier 1994):

15 The analysis presented below only uses the data taken 12′′ from the field center.
Figure 25. Analysis of deviation from radial of the Mars polarization data shown in Figure 24. The red circle indicates the diameter of Mars. Radial deviations are computed only at pixels where the polarized intensity is > 0.3 Jy bm$^{-1}$. The color scale on the deviation map runs from –10 to 10°. Values are weighted by polarized intensity when computing the mean deviation.

\[ \Psi_s = \arcsin \left( \frac{\lambda \sin \theta_0}{4\pi F} \right) \]  
\[ \approx \frac{\lambda \theta_0}{4\pi F}, \]  

(22)

(23)

where \( \lambda \) is the observing wavelength, \( \theta_0 \) is the angular offset between the feed and the telescope’s optical axis, and \( F \) is the focal length of the primary reflector. The approximation in Equation 23 is good when the angular offset \( \theta_0 \) is small, which is usually the case.

The CARMA 6 m antennas have a Cassegrain design, with feed horn offset by \( \Delta \text{az} = 7.126' \) and \( \Delta \text{el} = 1.91' \) (Plambeck 2000). Using Equation 23 with an effective focal length of \( F = 1186 \) inches, the theoretical squint produced by these offsets should be incredibly small: \( \approx 0.002'' \). The measured squint is significantly larger (\( \approx 0.45'' \); see Figure 26 right panel). The feeds in the 10 m antennas are on-axis (and should thus be squint-free); however, there are multiple reflections in the optical path, including three curved mirrors with \( \sim 90' \) bends that are all in different planes. The odd number of off-axis reflections may be the cause of the 10 m squint, which is even worse than the 6 m squint (\( \approx 2.16'' \); see Figure 26 left panel).

In order to fit for the squint we solved for Stokes \( V \) at each \((12'')\) offset position and for each 30 s integration on BL Lac. Note that we applied the gain and leakage corrections solved at the center position only. We calculated the offsets in the frame of the dish (in azimuth and elevation) by de-rotating the offset position (in RA and DEC) by the parallactic angle \( \chi \) associated with the given integration. In order to derive the squint, we fit the data to the difference of two circular Gaussians \( G_2 - G_1 \):

\[ G_1 = \exp\left(-\frac{[(\text{az} - \Delta \text{az}/2)^2 + (\text{el} - \Delta \text{el}/2)^2]}{(2\sigma^2)}\right) \]  
\[ G_2 = \exp\left(-\frac{[(\text{az} + \Delta \text{az}/2)^2 + (\text{el} + \Delta \text{el}/2)^2]}{(2\sigma^2)}\right), \]  

(24)

(25)

where \( \sigma = \text{FWHM}/(2\sqrt{2\log 2}) \), and FWHM is the full-width at half-maximum of the primary beam at \( \lambda 1.3 \) mm (\( 30'' \) for the 10 m antennas and \( 56'' \) for the 6 m antennas); \( \Delta \text{az} \) and \( \Delta \text{el} \) are the vertical and horizontal components of the offset between the RCP and LCP beams, and the angle of the beam offset in the frame of the dish \( \theta = \arctan (\Delta \text{el}/\Delta \text{az}) - 90' \), measured east from north (or counterclockwise from vertical, in the reference frame of the antenna). We assume that the amplitudes of the
Gaussians are identical, and that the two beams are offset by equal and opposite amounts ($\pm \Delta z/2$ and $\pm \Delta \ell/2$) from the pointing center.

**Beam squash.** Telescopes with native circular feeds like CARMA should see squash in both Stokes $Q$ and $U$, which are both combinations of the cross-polarizations $RL$ and $LR$ (see Equations 6 and 7). For telescopes like the GBT and Arecibo with native linear feeds, it should be easier to see squash in Stokes $U$, which is calculated using the cross-polarizations $XY$ and $YX$ (see Equation 8) and is thus unaffected by $XX$ and $YY$ gain variations that can plague Stokes $Q$ (see Equation 2). Maps of beam squash have been made in Stokes $U$ for the GBT [Robishaw 2008] and in both Stokes $Q$ and $U$ for Arecibo [Heiles et al. 2001a] note that for Arecibo the gain variations had a minimal effect, allowing excellent maps of both Stokes parameters.

We see no evidence in the $Q$ or $U$ CARMA maps for a quadrupolar squash pattern, which should have twice the frequency of the squint as one moves around the dish. However, variations in $Q$ and $U$ do lead to squint-like variations (i.e., a single crest and trough) in the polarization fraction and position angle.

**Variation in polarization fraction and position angle across the primary beam.** While we saw no evidence of squash in the data, we did see squint-like behavior in $Q$, $U$, and the PA and polarization fraction derived from them. This behavior is not expected theoretically, but is seen at Arecibo and discussed in various publications and technical memos including Heiles (1999); Heiles et al. 2001a, 2003; Heiles & Crutcher (2005).

We fit a sinusoid to find the peak-to-peak variation in the calculated PA and polarization fraction of both the 10 m and 6 m dishes. The 6 m dishes showed very little variation in PA and polarization fraction, whereas the 10 m dishes showed more (possibly because observations were further out into the primary beam). The 10 m antennas had peak-to-peak variations of $\sim 8^\circ$ in PA and $\sim 0.02$ in polarization fraction (0.02 represents a $\sim 25\%$ variation in the mean of the BL Lac polarization fraction). See Figure 2.

**7. SUMMARY**

In this overview of the 1.3 mm dual polarization receiver system built for CARMA, we described the design and performance of the key hardware components—circular polarizers, orthomode transducers, and mixers—and discussed the calibration of the system for polarization observations, focusing particularly on the calibration of the $R$–$L$ phase offsets and the polarization leakage terms. The leakages were found to exhibit considerable frequency structure, which we attribute to reflections inside the dewars and to cross-coupling in the analog block downconverter, which precedes the correlator.

We discussed limitations on the accuracy of polarization measurements made with this system, including the effects of signal-to-noise, leakage uncertainties, and primary beam polarization. The absolute accuracy of polarization position angle measurements was checked by mapping the radial polarization pattern across the disk of Mars. Transferring the Mars calibration to 3C826, we measured a polarization position angle for 3C826 at 225 GHz of $39.2^\circ \pm 1^\circ$.
Figure 26. 6 m and 10 m beam squint, fit using data positions offset by 12\arcsec from the pointing center. The solid curve is the best-fit squint model (see Equations 24 and 25).

Figure 27. Variations of PA and polarization fraction across the 6 m and 10 m primary beams, fit using data from positions offset by 12\arcsec from the pointing center. The solid curves are sinusoids fit to the data to find the peak-to-peak variations.

Lessons learned. In retrospect, we are not sure that the choice of circularly polarized feeds was a good one. In a dual polarization system with circularly polarized feeds, signals that reflect off a mixer can bounce back from the dewar window with the opposite polarization, introducing frequency-dependent cross-coupling between the two polarization channels. Accurately characterizing these ripples in the polarization leakages is extremely time-consuming, since it requires observations of a strong calibrator over a wide parallactic angle range. Although the leakage ripples in the CARMA system tended to average out over the full passband, and were not a severe...
limitation for continuum polarization observations of dust or extragalactic synchrotron sources, these ripples did hamper our ability to search for polarization in spectral lines.

In contrast, in a dual polarization system with linearly polarized feeds,\textsuperscript{16} reflections would not cause cross-coupling between the two polarizations ($XY$ and $YX$), but instead would create ripples in the receiver bandpass ($XX$ and $YY$), which are easily calibrated with a brief observation of a bright continuum source. Of course, as discussed in Section\textsuperscript{11} crossed-linear feeds have their own problems. Specifically, in order to detect weak linear polarization with crossed linear feeds, the receiver gains must be extremely stable. The newest interferometer with crossed-linear receivers is ALMA. The promising continuum polarization results obtained thus far with ALMA suggest that gain stability is not a significant limitation; further tests of the ALMA system will reveal whether there are other issues unique to crossed-linear systems that were not encountered at CARMA.

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APPENDIX: CONVERTING FROM LINEAR TO CIRCULAR STOKES PARAMETERS

Here we will work through the details of converting the Stokes parameters from their commonly seen linear forms to their circular forms. As we described in Section\textsuperscript{11} in an $X-Y$ coordinate system where $+x$ points North, $+y$ points East, and radiation propagates toward us along the $+z$ axis, the Stokes parameters for crossed-linear feeds are:

\begin{align}
I &= \langle EX E_X^* \rangle + \langle EY E_Y^* \rangle \\
Q &= \langle EX E_X^* \rangle - \langle EY E_Y^* \rangle \\
U &= \langle EX E_Y^* \rangle + \langle E_X E_Y \rangle \\
V &= -i(\langle EX E_Y^* \rangle - \langle E_X^* E_Y \rangle)
\end{align}

where $E_X^*$ denotes the complex conjugate of $E_X$. These equations are consistent with any number of textbooks and publications, including Equations 2.47a–2.47d in Rybicki & Lightman\textsuperscript{1979} Equations 1 in Hamaker & Bregman\textsuperscript{1996} Equations 4.19 in Thompson et al.\textsuperscript{2004} and others.

In radio astronomy $E_R$ is defined using the IEEE (Institute of Electrical and Electronics Engineers) convention: the phase $\phi$ of the sinusoid $E_Y$ lags the sinusoid $E_X$ by $90^\circ$. Thus, a right-circularly polarized wave (one with pure $E_R$) can be represented in the following way:

\begin{align}
E_Y &= E_X e^{-i\frac{\phi}{2}} \\
&= -iE_X \tag{30}. \end{align}

As defined and discussed in Contopoulos & Jappel\textsuperscript{1974}; Hamaker & Bregman\textsuperscript{1996}; IEEE\textsuperscript{1997}, this lag results in right-circularly polarized (RCP) radiation whose phasor rotates counter-clockwise as viewed by the receiver.

Additionally, in Contopoulos & Jappel\textsuperscript{1974} the IAU deemed Stokes $V$ to be positive if the signal has net RCP. To check this, we plug $E_X = e^{i\phi}$ and a lagging $E_Y = e^{i(\phi + \frac{\pi}{2})}$ into the crossed-linear equation for Stokes $V$ (Equation\textsuperscript{29}), and we find that indeed, Stokes $V$ is positive for RCP radiation.

To convert from linear to circular we use the linear-to-circular coordinate transform reported in Equation 17 of Hamaker et al.\textsuperscript{1996}, and further elucidated in Section 3 of Hamaker & Bregman\textsuperscript{1996} (note that we have chosen the upper sign convention in Hamaker & Bregman\textsuperscript{1996} which is the convention used in Hamaker et al.\textsuperscript{1996} as well as in the references they quote):

\begin{equation}
C_A = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & i \\
1 & -i
\end{bmatrix}. \tag{32}
\end{equation}

## Hamaker & Bregman\textsuperscript{1996} explicitly refer to the above matrix as circular-r, clearly defining the order of their matrix (top row $\rightarrow R$, bottom row $\rightarrow L$). We can then convert from $[E_X, E_Y]$ to $[E_R, E_L]$:

\begin{equation}
\begin{bmatrix}
E_R \\
E_L
\end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & i \\
1 & -i
\end{bmatrix} \begin{bmatrix}
E_X \\
E_Y
\end{bmatrix}. \tag{33}
\end{equation}

$E_R$ and $E_L$ are thus:

\begin{align}
E_R &= \frac{1}{\sqrt{2}} (E_X + iE_Y) \\
E_L &= \frac{1}{\sqrt{2}} (E_X - iE_Y)
\end{align}

\textsuperscript{16} We note that, if desired, a beamsplitter can still be used to couple the local oscillator into the mixers in such a system, simply by rotating the plane of polarization of the LO with a waveguide twist or half wave plate so that it is oriented at 45° to the OMT axes.
\[ E_R = \frac{1}{\sqrt{2}} (E_X + iE_Y) \]  \hfill (34)
\[ E_L = \frac{1}{\sqrt{2}} (E_X - iE_Y) \]  \hfill (35)

As a check, we substitute \( E_Y = -iE_X \) (Equations 30 and 31), which describe how \( E_Y \) lags \( E_X \) in RCP radiation, into the above expression for \( E_L \). We find that \( E_L = 0 \), as expected.

One can also invert matrix \( C_A \) to obtain the matrix required to convert from circular back to linear:
\[
\begin{bmatrix}
  E_X \\
  E_Y \\
\end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -i & i \end{bmatrix} \begin{bmatrix} E_R \\
  E_L \end{bmatrix} ;
\]  \hfill (36)

this yields expressions for \( E_X \) and \( E_Y \):
\[
E_X = \frac{1}{\sqrt{2}} (E_R + E_L) \]  \hfill (37)
\[
E_Y = \frac{1}{\sqrt{2}} (-iE_R + iE_L) \]  \hfill (38)

We can substitute these expressions for \( E_X \) and \( E_Y \) (Equations 37 and 38) into the crossed-linear Stokes parameters (Equations 26 and 29). We find the crossed-circular Stokes parameters to be:
\[
I = \langle E_R E_R^* \rangle + \langle E_L E_L^* \rangle \]  \hfill (39)
\[
Q = \langle E_R E_L^* \rangle + \langle E_L E_R^* \rangle \]  \hfill (40)
\[
U = -i \left( \langle E_R E_L^* \rangle - \langle E_L E_R^* \rangle \right) \]  \hfill (41)
\[
V = \langle E_R E_R^* \rangle - \langle E_L E_L^* \rangle . \]  \hfill (42)

Note that occasionally the Stokes parameters are defined as \( \frac{1}{2} \times \) the expressions listed in Equations 39–42 (e.g., Equations 1–4 in Roberts et al. 1994). When this is the case, one can express the four cross-correlations of \( R \) and \( L \) in terms of the Stokes parameters:
\[
E_R E_R^* = I + V \]  \hfill (43)
\[
E_L E_L^* = I - V \]  \hfill (44)
\[
E_R E_L^* = Q + iU \]  \hfill (45)
\[
E_R^* E_L = Q - iU . \]  \hfill (46)

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