Slope instability judgment criteria in FEM based on strength reduction method

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Abstract. The correct judgment of the instability state affects the reduction coefficient obtained by the strength reduction finite element method, and then affects the calculation result of the safety factor. Taking the Griffiths slope as an example, the Abaqus software was used to calculate the instability criteria using three types of criteria: non convergence of numerical iteration, sudden change in displacement of characteristic parts, generalized plastic strain or equivalent plastic strain penetration. The results show that there is a corresponding relationship between whether the numerical calculation is converged and whether the slope is unstable. When the calculation is terminated due to non-convergence, the strength reduction coefficient does not change much, and the setting of the finite element convergence condition has little effect on the result. Therefore, it is recommended to use finite element calculation for convergence as a basis for determining slope instability.

1. Introduction
The conventional finite element analysis method is difficult to directly evaluate the stability of the slope. Zienkiewicz[1] proposed the strength reduction method, which defines the ratio of the maximum shear strength that the soil can provide to the actual shear stress in the soil as the shear strength reduction factor. The smaller reduction factor $F_{\text{trial}}$ is first used, and then gradually increase the reduction factor $F_{\text{trial}}$ until it increases to a certain value, the slope is in the ultimate state, that is, the actual shear stress in the soil caused by the load is equal to the shear strength index after the soil shear strength is reduced according to the actual strength index. Duncan[2] pointed out that the slope safety factor can be defined as the degree to which the shear strength of the soil is reduced when the slope just reaches the critical failure state, that is, the shear strength reduction factor in the ultimate state is equivalent to the traditional slope stability safety factor.

At the beginning, the strength reduction method was restricted by calculation conditions and it was difficult to popularize. Griffiths[3] used Fortran language to code a finite element program in the finite element method to achieve a simpler slope strength reduction method analysis. With the development of computer hardware and numerical software, many commercial geotechnical software have been able to easily apply the strength reduction method. Compared with the limit equilibrium method, the strength reduction coefficient method can automatically obtain the critical slip surface of any shape and the corresponding minimum safety factor, and it can also truly reflect the slope instability and the development process of plastic zone.

2. Realization of strength reduction method in Abaqus
Although there is no strength reduction method in Abaqus, it can be easily achieved by setting the field variables[4,5].

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The essence of the basic principle of the strength reduction method is that the gradual decrease of
the strength parameters $c$ and $\phi$ causes the strength of the soil to be insufficient to bear the current
stress, which is reflected in the finite element as the gradual decrease of the strength parameters $c$ and
$\phi$ causing a plastic deformation as the stress of a unit exceeds the yield surface. The stress beyond the
yield surface will be transferred to the surrounding soil elements through the plastic deformation.
When the element with the stress exceeding the yield surface and forming a penetrating curved surface,
the soil will be instable.

In the Abaqus software, the field variable $F_{field}$ changes linearly with time increments. The
relationship between material parameters and field variables is as follows:

$$c' = F_{field}c$$

$$\tan \phi' = F_{field} \tan \phi$$

Among them, $c$ and $\phi$ are the shear strength indexes that the soil can provide, and $c'$, $\phi'$ are the
shear strength indexes of the soil after the strength reduction corresponding to the field variable at a
certain time increment.

Comparing equations (1) and (2), when the soil reaches the ultimate state, the shear strength
reduction coefficient $F_{trail}$ has the following relationship with the field variable $F_{field}$:

$$F_{trail} = \frac{1}{F_{field}}$$

In the calculation process of the Abaqus software, the field variable changes with the incremental
step time $t$. By setting the material parameters to change with the field variable, the material strength
reduction process is realized. Since the change of the incremental step time $t$ in the Abaqus software is
automatic, manual modification is no longer needed, and the strength can be automatically reduced.
Finally, based on a certain damage criterion, the corresponding incremental step time is found, and
then the safety factor can be obtained, and the corresponding sliding surface can be obtained.

3. Instability determination of strength reduction method in Abaqus

Correctly judging the instability state affects the reduction coefficient obtained by the strength
reduction finite element method, which in turn affects the calculation result of the safety factor. There
are mainly three types of instability criteria for the strength reduction finite element method, namely:
numerical iteration non-convergence criterion, characteristic part displacement mutation criterion,
ge generalized plastic strain or equivalent plastic strain penetration criterion.

3.1. Analysis model

In order to facilitate the discussion and comparison, the case in Griffiths [3] was used for strength
reduction finite element analysis with Abaqus software, and the above instability criteria were
compared.

Example 1 is a homogeneous slope, with slope height $H=10$ m, slope angle $\beta=30^\circ$, soil weight $\gamma=20$
kN/m$^3$, cohesion $c=5$ kPa, and internal friction angle $\phi=30^\circ$.

The finite element model of the slope was established, and the geometric dimensions were the same
as those in the literature [3]. The slope top and the toe to the model boundary were all taken as 12 m,
and the soil thickness below the slope toe was taken as 10 m. The elastic modulus $E=100$ MPa was
assumed to eliminate the calculation result error caused by the mesh division. We adopted the same
mesh division as the literature [3], and adopted the reduced integral plane strain unit CPE4R. We
constrained the horizontal displacement of the left and right ends of the model, and the horizontal and
vertical displacement of the bottom end. The finite element model is shown in Figure 1(a).
For the engineering problem of actually calculating the safety factor of the slope, the strength reduction factor $F_{\text{trial}} \geq 1$, that is, the strength reduction is performed according to the measured strength index of the soil, and the actual safety factor of the slope is calculated, but for the finite element calculation, the practical significance is not considered. The strength reduction coefficient can be less than 1, that is, the reduction coefficient plays a role in expanding the strength index, and the strength index after reduction tends to be safer.

The calculation is carried out in two steps: the first step is to apply vertical downward gravity to the slope. In order to successfully complete the gravity loading step, most areas are in an elastic state after gravity loading, and the strength reduction calculation is smoothly excessive, using the parameters when $F_{\text{trial}}=0.5$, as cohesive force $c=10$ kPa, internal friction angle $\phi=49^\circ$; the second step is to reduce the strength parameters, we set the final state to $F_{\text{trial}}=2.0$, and the corresponding soil strength parameter is cohesion force $c=2.5$ kPa, internal friction angle $\phi=16.7^\circ$.

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The final total displacement contour of the finite element includes the displacement caused by the gravity load in the first part, and the position of the sliding surface cannot be judged. Since the trend of soil slope instability can be reflected in the incremental displacement, it can be judged by calculating the incremental displacement of the last incremental step of termination. The contour map of the last step of the finite element calculation is shown in Figure 3. From this figure, the position of the sliding surface can be judged, as shown in Figure 1(b)\textsuperscript{3}, the sliding face ratio is also very close.
As can be seen from Figure 1(b), the value of $\nu$ has a greater impact on the distribution of the plastic zone. When $\nu$ is set to a smaller value, the plastic zone is larger than when the value is larger. When $\nu$ is larger, the plastic zone is still small until the calculation is terminated due to non-convergence. However, when the $\nu$ is smaller, the plastic zone will even be penetrated. Therefore, the generalized plastic strain or equivalent plastic strain penetration criterion cannot accurately judge the instability of the soil slope.

3.2. Influence of sudden change of displacement of characteristic points

The slope vertices and toe points were respectively selected as of characteristic points, and the relationship between the calculated strength reduction coefficient $F_{\text{trial}}$ and the vertical displacement $U$ of the characteristic points was drawn as a curve, as shown in Figure 4.

It can be seen from the figure that as the strength reduction coefficient increases, the soil strength index decreases until the calculation is terminated due to non-convergence. The vertical displacements of the slope toe and the slope top feature points show the law of larger displacement when the
Poisson's ratio is small, which is in accordance with the elastic theory. The vertical displacement of the characteristic point of the slope toe has no obvious sudden change. When the Poisson's ratio is larger, the vertical displacement even has a slight decrease trend. The displacement of the characteristic point of the slope top has a more obvious sudden change, but the strength reduction coefficient of the displacement abrupt transition point varies greatly. The end point of the horizontal section of the curve is selected as the mutation point. The intensity reduction coefficient $F_{trial}$ is shown in Table 1.

| Poisson’s ratio $\nu$ | 0.25 | 0.26 | 0.30 | 0.31 |
|----------------------|------|------|------|------|
| field variable $F_{trial}$ | 0.9090 | 0.9564 | 1.0672 | 1.1015 |

It can be seen from Table 1 that the distribution of the plastic zone will affect the sudden change of the characteristic point displacement on the top of the slope to determine the strength reduction coefficient $F_{trial}$ results. As the Poisson's ratio increases, the plastic zone range decreases, and the strength reduction coefficient $F_{trial}$ judged from these increases. There is a general trend, and the method needs to determine the displacement mutation point based on experience. There is no clear index as the basis for judgment, and there is a certain empirical error.

### 3.3. Influence of non-convergence of numerical iteration

Table 2 summarizes the strength reduction coefficient $F_{trial}$ when the numerical calculation does not converge and the calculation is terminated.

| Poisson’s ratio $\nu$ | 0.25 | 0.26 | 0.30 | 0.31 |
|----------------------|------|------|------|------|
| field variable $F_{trial}$ | 1.4665 | 1.4639 | 1.4726 | 1.4844 |

It can be seen from Table 2 that with the increase of Poisson's ratio, although the distribution of the plastic zone changes, the difference in the strength reduction coefficient $F_{trial}$ is small when the calculation is terminated due to non-convergence, and this method does not need to determine the displacement mutation point based on experience.

There are many factors that cause calculation non-convergence in finite element. Some scholars question whether there are other man-made result deviations based on non-convergence of numerical calculation as the basis for slope failure. This can be explained by comparing the same model with different settings.

Before discussing this issue, firstly, we assumed that the finite element model used in the calculation is correct. If the model is incorrectly built or the finite element program used is defective, the non-convergence of the finite element numerical calculation is meaningless in itself. The calculation step used in the above model calculation is 1, the initial increment step was 0.1, and the minimum increment step was $1 \times 10^{-5}$. The minimum increment step was the key to judging convergence. The minimum increment steps were further reduced, and were set to $1 \times 10^{-6}$ and $1 \times 10^{-7}$ respectively, and the comparing calculation results as shown in Table 3.

| Poisson’s ratio $\nu$ | 1 $\times 10^{-5}$ | 1 $\times 10^{-6}$ | 1 $\times 10^{-7}$ |
|----------------------|------------------|------------------|------------------|
| 0.25                 | 1.4665           | 1.4665           | 1.4665           |
| 0.26                 | 1.4639           | 1.50435          | 1.50435          |
| 0.30                 | 1.47264          | 1.47264          | 1.47264          |
| 0.31                 | 1.48440          | 1.48441          | 1.48441          |

The default minimum increment step in Abaqus is $1 \times 10^{-5}$. After the minimum increment step is further reduced to $1 \times 10^{-6}$ and $1 \times 10^{-7}$, the strength reduction coefficient changes little when the calculation is terminated due to non-convergence, indicating that the default incremental step can
guarantee sufficient calculation accuracy. The calculation results also show that the setting of the finite element convergence conditions has little effect on the results. Therefore, the convergence of the finite element calculation can be used as the criterion for slope instability.

4. Conclusion
After the calculation and analysis, the following conclusions are obtained:

(1) The value of \( \nu \) has a greater impact on the distribution of the plastic zone. But when \( \nu \) is set to a smaller value, the plastic zone will even be penetrated. Therefore, the generalized plastic strain or equivalent plastic strain penetration criterion cannot accurately judge the slope instability;

(2) Using the distribution of the plastic zone needs to determine the displacement mutation point based on experience, so there’s a certain empirical error;

(3) The strength reduction coefficient when the calculation is terminated due to non-convergence does not change much, indicating that the default incremental step can ensure sufficient calculation accuracy, and the calculation result also shows that the setting of the finite element convergence conditions has little effect on the results, and it is recommended to use the finite element calculation convergence as the basis for the instability determination.

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