Global dynamics of the Hořava–Lifshitz cosmological system

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Abstract

Using the qualitative theory of the differential equations we describe the global dynamics of the cosmological model based on Hořava–Lifshitz gravity in a Friedmann–Lemaître–Robertson–Walker space time with zero curvature and without the cosmological constant term.

Keywords Hořava–Lifshitz · Global dynamics · Cosmology · Poincaré compactification

1 Introduction

Ten years ago Hořava [1] put forward a theory of spacetime asymmetric gravity, which is similar to Lifshitz’s scalar field theory. If the spatial dimension in Lifshitz’s scalar field theory has a weight of one, then the time dimension has a weight of three. Therefore this theory is also known as Hořava–Lifshitz gravity. It ignited a great deal of research on the possible application of this theory in cosmology and black hole physics (see [2–5] or the review articles [6,7] and the references therein).

With or without detailed-balance conditions, Leon and Saridakis [8] carried out a detailed phase space analysis of Hořava–Lifshitz cosmology, and found that the universe governed by Hořava gravity had late-time solutions compatible with observations. They also presented several results on the stability of de Sitter solutions in Hořava–Lifshitz cosmology by using the central manifold theory [9]. Furthermore Saridakis [10] reviewed some general aspects of Hořava–Lifshitz cosmology and

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extracted cosmological equations from its basic version. He proved that bouncing and cycling can occur naturally by phase space analysis. However, he also showed that Hořava–Lifshitz gravity is affected by instability in its basic version with detailed perturbation analysis. For the stability of gravitational scalar mode Chen [11] briefly summarized the Hořava–Lifshitz theory of gravity and its modifications and its implications in cosmology.

In recent years Lepe and Saavedra [12] discussed some aspects of Hořava–Lifshitz cosmology with emphasis on some cosmological solutions that exist in general relativity (Friedmann cosmology), especially in the flat case that dust-driven evolution is the same in both cosmological theories. For Hořava–Lifshitz theory of gravitation Abreu et al. [13] explored a non-commutative version of the Friedmann–Robertson–Walker cosmological model, in which material content is described by ideal fluids, and the constant curvature of the spatial sections can be positive, negative or zero. Under the background spacetime of Friedmann–Lemaître–Robertson–Walker, Paliathanasis and Leon [14] divided the integrability of Hořava–Lifshitz scalar field into four cases according to the existence of cosmological constant term and the disappearance of space curvature. They followed the singularity analysis method to determine integrability. It is believed that their work will be very helpful to the integrability of the gravitational field equation in cosmology. In addition to the cosmological solutions in [14], two versions of Hořava–Lifshitz gravity were discussed for finding and analyzing the plane symmetric, static (non-static) solutions in Hořava–Lifshitz gravity [15]. They also studied the plane symmetry of the new modified version of Hořava gravity.

Here we shall describe the global dynamics of the Hořava–Lifshitz cosmological model in a Friedmann–Lemaître–Robertson–Walker spacetime with zero curvature and without the cosmological constant term. This dynamics is provided by the gravitational field equations in dimensionless variables given by

\[
\frac{dy_1}{dt} = \left(y_1^2 - 1\right)\left(3y_1 - \sqrt{6}y_3\right),
\]

\[
\frac{dy_2}{dt} = \left(3y_1^2 - 2\right)y_2,
\]

\[
\frac{dy_3}{dt} = -2\sqrt{6}y_1 f(y_3),
\]

with the power law potential \( f(y_3) = -y_3^2/(2n) \), where \( n \) is a natural number. See for instance equations (21)–(23) of [9] or equations (17)–(19) of [14] for more details. The description of dynamics of system (1) is given in Sect. 4, and the main conclusions in Sect. 5.

In the references [2,8,9,14,16] the authors either consider our three-dimensional differential system (1) or only the two-dimensional differential system formed by the first two equations of system (1), but for the three-dimensional system they only provide partial information on its dynamics, while for the two-dimensional system, taking into account all the results of these papers, they study completely its global dynamics together with the dynamics near the infinity using the Poincaré compactification. Here we characterize the global dynamics of the three-dimensional differential system (1) using the Poincaré compactification in dimension three.
The dynamics of system (1) also has been studied in subsection 3.3.2 of [9] using other coordinates, but there are three main differences with our study. First, in [9] many facts of the dynamics have been obtained numerically while our study is analytical, second in [9] the authors did not study the dynamics near the infinity, and the finite dynamics is only studied partially, our study describes completely the global dynamics of system (1).

2 Phase portraits on the invariant planes

In order to study the phase portraits of system (1), we start studying the phase portraits of its invariant planes

\[ y_1 = \pm 1, \quad y_2 = 0, \quad y_3 = 0. \]

After we will study the local phase portraits of the finite and infinite equilibrium points, and finally the global phase portraits in the region \(-1 \leq y_1 \leq 1\), which is the interest region for cosmology.

2.1 The invariant plane \(y_1 = 1\)

On this plane system (1) becomes

\[
\frac{dy_2}{dt} = y_2, \quad \frac{dy_3}{dt} = \frac{\sqrt{6}}{n} y_3^2.
\]

(2)

The unique finite equilibrium point is \(q_{y_1,1} = (0, 0)\). It is a semi-hyperbolic equilibrium point and using Theorem 2.19 of [17] \(q_{y_1,1}\) is a saddle-node.

Based on the Poincaré transformation \(y_2 = 1/v, \quad y_3 = u/v\), on the local chart \(U_1\) (see for more details on the Poincaré compactification Chapter 5 of [17]) Eq. (2) become

\[
\dot{u} = \frac{\sqrt{6}}{n} u^2 - uv, \quad \dot{v} = -v.
\]

(3)

This system has the infinite semi-hyperbolic equilibrium point \(p_{y_1,1} = (0, 0)\). Applying to it Theorem 2.19 of [17] we obtain the saddle-node shown in Fig. 1a.

Similarly on the local chart \(U_2\) system (2) is

\[
\dot{u} = -\frac{\sqrt{6}}{n} u + uv, \quad \dot{v} = -\frac{\sqrt{6}}{n} v.
\]

(4)

Then the equilibrium point \(p_{y_1,2} = (0, 0)\) of system (4) is a hyperbolic stable node with eigenvalues \(-\sqrt{6}/n\) of multiplicity two.

Finally, joining the previous information on the studied three equilibrium points \(q_{y_1,1}, \quad p_{y_1,1}, \quad p_{y_1,2}\), together with the diametrically opposite equilibrium points \(p_{y_1,1}\) and \(p_{y_1,2}\) at infinity of \(p_{y_1,1}\) and \(p_{y_1,2}\), we obtain the global phase portrait of
Fig. 1  a The local phase portraits of the saddle-node $p_{y_1,1} = (0, 0)$ in the Poincaré disc. b The phase portraits of the invariant plane $y_1 = 1$

system (2) in the Poincaré disc of the invariant plane $y_1 = 1$ in Fig. 1b. Here we have also used that the straight lines $y_2 = 0$ and $y_3 = 0$ are invariant by the flow of system (2).

We note that the Poincaré compactification already has been previously used for studying the dynamics near the infinity of some versions of the Hořava–Lifshitz cosmologies. Thus, for instance, in the subsection 5.3.1.2 of [16] the authors studied the dynamics in a neighborhood of infinity of a model of flat universe reduced to dimension 2. In this paper we shall use the Poincaré compactification in dimensions 2 and 3.

### 2.2 The invariant plane $y_1 = -1$

On this plane system (1) reduces to

$$\frac{dy_2}{dt} = y_2, \quad \frac{dy_3}{dt} = -\sqrt[6]{n} y_3^2.$$

System (5) is similar to system (2), so the equilibrium point $q_{y_1,2} = (0, 0)$ is a saddle-node.

On the local chart $U_1$ system (5) is

$$\dot{u} = -\sqrt[6]{n} u^2 - uv, \quad \dot{v} = -v.$$

The semi-hyperbolic equilibrium point $p_{y_1,3} = (0, 0)$ is a saddle-node by Theorem 2.19 of [17], and its local phase portrait in the Poincaré disc is described in Fig. 2a.

On the local chart $U_2$ system (5) becomes

$$\dot{u} = \frac{\sqrt[6]{n}}{n} u + uv, \quad \dot{v} = \frac{\sqrt[6]{n}}{n} v.$$

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2.3 The invariant plane $y_2 = 0$

On this plane system (1) can be rewritten as

$$\frac{dy_1}{dt} = (y_1^2 - 1) \left(3y_1 - \sqrt{6}y_3\right), \quad \frac{dy_3}{dt} = \frac{\sqrt{6}}{n} y_1 y_3^2.$$  \hspace{1cm} (9)
This system has three equilibrium points $O_{y_{2.0}} = (0, 0)$, $q_{y_{2.1}} = (1, 0)$, $q_{y_{2.2}} = (-1, 0)$, which are semi-hyperbolic. In order to apply Theorem 2.19 of [17], the linear part at each equilibrium point of system (9) must be written into its real Jordan normal form. So we do the following transformation to the linear part of $O_{y_{2.0}}$

$$y_1 = \frac{\sqrt{6}}{3}(y - x), \quad y_3 = -x.$$  

Then Eq. (9) becomes

$$\dot{x} = -\frac{2}{n}x^3 + \frac{2}{n}x^2y,$$

$$\dot{y} = 3y - \frac{2}{n}x^3 + \frac{1}{2}\left(\frac{1}{n - 1}\right)x^2y + 4xy^2 - 2y^3.$$  

(10)

Let $y = g(x) = 2x^3/3n + 4(n - 1)x^5/(9n^2) + O(x^5)$ be the solution of the second equation of system (10). Substituting $y = g(x)$ into the first equation of system (10), we obtain $\dot{x} = -2x^3/n + O(x^5)$. So by Theorem 2.19 of [17], $O_{y_{2.0}} = (0, 0)$ is a saddle point. Applying again Theorem 2.19 of [17] to the equilibrium points $q_{y_{2.1}} = (1, 0)$ and $q_{y_{2.2}} = (-1, 0)$, we obtain that they are saddle-nodes.

On the local chart $U_1$ system (9) becomes

$$\dot{u} = u \left[3\left(v^2 - 1\right) + \frac{\sqrt{6}}{n}u\left(1 + n - nv^2\right)\right],$$

$$\dot{v} = v\left(3 - \sqrt{6}u\right)\left(v^2 - 1\right).$$  

(11)
Its equilibrium points are \( p_{y_2,1} = (0, 0) \), \( p_{y_2,2} = \left( \sqrt{6n}/(2(1 + n)), 0 \right) \). The equilibrium point \( p_{y_2,1} \) is a hyperbolic stable node with eigenvalues \(-3\) of multiplicity two, and \( p_{y_2,2} \) is a hyperbolic unstable saddle with eigenvalues \(3\) and \(-3 + 3n/(1 + n)\). On the local chart \( U_2 \) system (9) writes

\[
\dot{u} = v^2 \left( \sqrt{6} - 3u \right) - \frac{\sqrt{6}(1 + n)}{n} u^2 + 3u^3, \quad \dot{v} = -\frac{\sqrt{6}}{n} uv. \tag{12}
\]

The \( p_{y_2,3} = (0, 0) \) is a linearly zero equilibrium point, i.e. its linear part is identically zero. Its topological index is zero by the Poincaré-Hopf theory (see Theorem 6.30 of [17] for more details). In order to study the local phase portraits of the equilibrium point \((0, 0)\) of system (12) we apply the blow-up techniques (see [18] for more details). We do a vertical blow-up by introducing the transformation \( w = v/u \), then we have

\[
\dot{u} = u^2 \left[ -\frac{\sqrt{6}}{n} + \left( \sqrt{6} - 3u \right) \left( w^2 - 1 \right) \right], \quad \dot{w} = -uw \left( \sqrt{6} - 3u \right) \left( w^2 - 1 \right). \tag{13}
\]

The common factor \( u \) of system (13) can be eliminated by rescaling the time \( udt = d\tau \). So we get

\[
u' = u \left[ -\frac{\sqrt{6}}{n} + \left( \sqrt{6} - 3u \right) \left( w^2 - 1 \right) \right], \quad w' = -w \left( \sqrt{6} - 3u \right) \left( w^2 - 1 \right). \tag{14}
\]

where the prime represents the derivative with respect to the time \( \tau \). Note that system (14) has three equilibrium points \( p_{y_2,4} = (0, -1) \), \( p_{y_2,5} = (0, 0) \) and \( p_{y_2,6} = (0, 1) \) on \( u = 0 \). Therefore both the equilibrium points \( p_{y_2,4} \) and \( p_{y_2,6} \) are hyperbolic stable nodes with eigenvalues \(-\sqrt{6}/n\) and \(-2\sqrt{6}\), the equilibrium point \( p_{y_2,5} \) is a hyperbolic unstable saddle with eigenvalues \(-\sqrt{6}(1 + n)/n\) and \(\sqrt{6}\). The local phase portraits around these three equilibrium points are shown in Fig. 4a. Considering that there is a time rescaling \( d\tau = udt \) between systems (13) and (14), so the local phase portraits of system (13) is shown in Fig. 4b. Moreover, all points on the \( w \)-axis, i.e. \( u = 0 \), are singularities of system (13). Thus the local phase portraits of system (12) is shown in Fig. 4c, and then the local phase portrait at the origins of \( U_2 \) and \( V_2 \) for \( y_2 = 0 \) can be found in Fig. 4d.

In summary joining the previous information we obtain the global phase portraits in Fig. 5 in the Poincaré disc of the plane \( y_2 = 0 \) restricted to the strip \(-1 \leq y_1 \leq 1\).
The local phase portraits of the equilibrium points in system (14), (13) and (12), respectively. 

The local phase portraits at the origin of $U_2$ and $V_2$ for $y_2 = 0$

2.4 The invariant plane $y_3 = 0$

On this plane system (1) writes

$$\frac{dy_1}{dt} = 3y_1 \left(y_1^2 - 1\right), \quad \frac{dy_2}{dt} = y_2 \left(3y_1^2 - 2\right).$$  \hspace{1cm} (15)
This system has three equilibrium points \( O_{y_3,0} = (0, 0) \), \( q_{y_3,1} = (1, 0) \) and \( q_{y_3,2} = (-1, 0) \). Then \( O_{y_3,0} \) is a hyperbolic stable node with eigenvalues -3 and -2, both \( q_{y_3,1} \) and \( q_{y_3,2} \) are unstable hyperbolic nodes with eigenvalues 1 and 6.

On the local chart \( U_1 \) system (15) becomes

\[
\dot{u} = uv^2, \quad \dot{v} = 3v (v^2 - 1).
\]  

(16)

So all the infinity of this system is filled of equilibrium points. We note that removing \( v \) of system (16) doing the change of time \( vdt = d\tau \), we get the system

\[
u' = uv, \quad v' = 3 (v^2 - 1),
\]  

(17)

which has no infinite equilibrium points.

On the local chart \( U_2 \) system (15) writes

\[
\dot{u} = -uv^2, \quad \dot{v} = v (-3u^2 + 2v^2).
\]  

(18)

Again doing the change of variable \( d\tau = vdt \) system (18) becomes

\[
u' = -uv, \quad v' = -3u^2 + 2v^2.
\]  

(19)
The $p_{3,1} = (0, 0, 0)$ is also a linearly zero equilibrium point of system (19). Applying again the blow-up techniques of [18] to $p_{3,1}$, then we obtain

$$u' = -u^2 w, \quad w' = 3u(w^2 - 1),$$  \hspace{1cm} (20)

after doing the change of variable $w = v/u$. Rescaling the time of system (20) we get

$$\dot{u} = -uw, \quad \dot{w} = 3(w^2 - 1).$$  \hspace{1cm} (21)

Hence system (21) has two equilibrium points $p_{3,2} = (0, -1)$ and $p_{3,3} = (0, 1)$ on $u = 0$. Therefore both the equilibrium points $p_{3,2}$ and $p_{3,3}$ are hyperbolic unstable saddles with eigenvalues 1, $-6$ and $-1, 6$, respectively. The local phase portraits around these two equilibrium points is shown in Fig. 6a. Considering that there is time $d\tau = udt$ to rescale between systems (20) and (21), the local phase portraits of system (20) is shown in Fig. 6b. In addition, all points on the $w$-axis, i.e. $u = 0$ are the singularities of the system (20). Therefore the local phase portraits of system (19) is shown in Fig. 6c, and then the local phase portraits at the origins of $U_2$ and $V_2$ of $y_3 = 0$ can be found in Fig. 6d.

In conclusion from the previous information and taking into account that the straight lines $y_1 = 0$ and $y_2 = 0$ are invariant under the flow of system (15), we obtain the global phase portraits restricted to the band $-1 \leq y_1 \leq 1$ in Fig. 7 in the Poincaré disc of the plane $y_3 = 0$.

3 Phase portraits inside the Poincaré ball restricted to $-1 \leq y_1 \leq 1$

We divide the Poincaré ball restricted to $-1 \leq y_1 \leq 1$ into four regions:

$$R_1 : \ y_2 \leq 0, \ y_3 \geq 0. \quad R_2 : \ y_2 \leq 0, \ y_3 \leq 0. \quad R_3 : \ y_2 \geq 0, \ y_3 \geq 0. \quad R_4 : \ y_2 \geq 0, \ y_3 \leq 0.$$  

Since system (1) is invariant under the symmetries with respect to the $y_2$-axis, i.e. ($y_1, y_2, y_3$) $\rightarrow$ ($-y_1, y_2, -y_3$), and with respect to the origin ($y_1, y_2, y_3$) $\rightarrow$ ($-y_1, -y_2, -y_3$), we only need to study the phase portraits in the region $R_1$. Putting together the phase portraits of the invariant planes $y_1 = \pm 1$, $y_2 = 0$ and $y_3 = 0$ we obtain the phase portraits in the boundary of the region $R_1$ in Fig. 8. The three-dimensional cartesian coordinate system in this paper is defined as follows: we consider the $y_1y_2$-plane as the horizontal plane in $\mathbb{R}^3$, in which the direction of the $y_2$-axis is horizontally to the right. If the $y_2$-axis is rotated $90^\circ$ counterclockwise we get the $y_1$-axis. The $y_3$-axis is vertically upward, then $y_1y_2y_3$ constitutes a three dimensional left-handed cartesian coordinate system.

Note that the original system (1) admits three finite equilibrium points $(-1, 0, 0)$, $(0, 0, 0)$ and $(1, 0, 0)$. The dynamical behavior of the system inside the region $R_1$ depends on the behavior of the flow in the following five planes

$$y_1 = ay_3, \ y_1 = \pm a, \ y_1 = 0, \ y_3 = 0,$$
Fig. 6 a–c The local phase portraits of the equilibrium points in system (21), (20) and (19), respectively. d The local phase portraits at the origins of $U_2$ and $V_2$ for $y_3 = 0$

where $a = \sqrt{6}/3$.

These five planes divide the region $R_1$ into six different subregions $G_i$, $i = (1, 2, \ldots, 6)$ (see Fig. 9 for more details).
Fig. 7 The phase portrait of the invariant plane $y_3 = 0$ restricted to $-1 \leq y_1 \leq 1$

Fig. 8 Phase portrait in the boundary of the region $R_1$. $N$ denotes the North Pole of the Poincaré ball

4 Dynamics in the interior of the region $R_1$

In Table 1 we describe the behavior of $\dot{y}_1$, $\dot{y}_2$ and $\dot{y}_3$ in the six subregions $G_i$ for $i = 1, 2, \ldots, 6$. From this table we get that the orbits from the subregion $G_1$ must go
Fig. 9 The six subregions of $R_1$ in the Poincaré ball

Table 1 Dynamical behavior in six different subregions

| Subregions | Corresponding region | Increase or decrease |
|------------|----------------------|----------------------|
| $G_1$      | $y_1 < y_3, \sqrt{6}/3 < y_1 < 1, y_3 > 0$ | $\dot{y}_1 > 0, \dot{y}_2 < 0, \dot{y}_3 > 0$ |
| $G_2$      | $y_1 > y_3, \sqrt{6}/3 < y_1 < 1, y_3 > 0$ | $\dot{y}_1 < 0, \dot{y}_2 < 0, \dot{y}_3 > 0$ |
| $G_3$      | $y_1 < y_3, 0 < y_1 < \sqrt{6}/3, y_3 > 0$ | $\dot{y}_1 > 0, \dot{y}_2 > 0, \dot{y}_3 > 0$ |
| $G_4$      | $y_1 > y_3, 0 < y_1 < \sqrt{6}/3, y_3 > 0$ | $\dot{y}_1 < 0, \dot{y}_2 > 0, \dot{y}_3 > 0$ |
| $G_5$      | $y_1 < y_3, -\sqrt{6}/3 < y_1 < 0, y_3 > 0$ | $\dot{y}_1 > 0, \dot{y}_2 > 0, \dot{y}_3 < 0$ |
| $G_6$      | $y_1 < y_3, -1 < y_1 < -\sqrt{6}/3, y_3 > 0$ | $\dot{y}_1 > 0, \dot{y}_2 > 0, \dot{y}_3 < 0$ |

to the equilibrium point $N$, this is represented as follows

$$G_1 \longrightarrow N.$$ 

In a similar way and taking into account that $y_2 = 0$ and $y_3 = 0$ are invariant planes, we obtain that

$$G_1 \longrightarrow N \quad \text{and} \quad G_6 \longrightarrow G_5 \longrightarrow G_3 \longrightarrow N,$$

If we define the subregions

$$R_{1+} = \{(y_1, y_2, y_3) \in R_1 : 0 < y_1 < 1\},$$

$$R_{1-} = \{(y_1, y_2, y_3) \in R_1 : -1 < y_1 < 0\},$$
the orbits obtained from Table 1 says that the orbits of system (1) contained in the region $R_{1+}$ have $\alpha$-limit at the equilibrium point $(1, 0, 0)$ and $\omega$-limit at the infinite equilibrium point $N$; the orbits of system (1) contained in the region $R_{1-}$ have $\alpha$-limit and $\omega$-limit at the infinite equilibrium point $N$.

This completes the description of all the qualitative dynamics of system (1).

5 Conclusions

From the references [2,8,9,14,16] we obtain the physical meaning of our variables $y_1, y_2$ and $y_3$. Thus we have that

$$
\begin{align*}
    y_1 &= \frac{\dot{\phi}}{2\sqrt{6}H}, \\
    y_2 &= \frac{\mu}{4(3\lambda - 1)a^2 H}, \\
    y_3 &= -\frac{V'(\phi)}{V(\phi)},
\end{align*}
$$

where $\lambda$ is a dimensionless constant, $V(\phi)$ is the potential, $\mu$ is a constant, and $H = \dot{a}/a$ is the Hubble parameter, $a$ is the dimensionless scale factor for the expanding universe.

We note that the phase portrait described in Fig. 7, except near the infinity, was already obtained in [8], this phase portrait is the one of system (1) restricted to $y_3 = 0$ and to $-1 \leq y_1 \leq 1$.

Here we have described the phase portrait of system (1) in $-1 \leq y_1 \leq 1$ for all values of $y_3$. This phase portrait is given in Fig. 8 and using the fact that system (1) is invariant under the two symmetries $(y_1, y_2, y_3) \rightarrow (-y_1, y_2, -y_3)$ and $(y_1, y_2, y_3) \rightarrow (-y_1, -y_2, -y_3)$, we provide the complete global phase portrait of system (1) in $-1 \leq y_1 \leq 1$. This phase portrait shows that generically (in the sense that except for initial conditions having $y_3 = 0$) the final evolution in forward time of the orbits of system (1) with $-1 \leq y_1 \leq 1$ tend to infinity, at the equilibrium points located at the north and south poles of the Poincaré ball. Taking into account (22) this implies that the Hubble parameter $H$ tends to zero when the time tends to infinity for the cosmological model based on Hořava–Lifshitz gravity in a Friedmann–Lemaître–Robertson–Walker space-time with zero curvature and without the cosmological constant term.

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