This paper describes a concept for the high-accuracy absolute calibration of the instrumental polarization introduced by the primary mirror of a large-aperture telescope. This procedure requires a small aperture with polarization calibration optics (e.g., mounted on the dome) followed by a lens that opens the beam to illuminate the entire surface of the mirror. The Jones matrix corresponding to this calibration setup (with a diverging incident beam) is related to that of the normal observing setup (with a collimated incident beam) by an approximate correction term. Numerical models of parabolic on-axis and off-axis mirrors with surface imperfections are used to explore its accuracy. © 2018 Optical Society of America

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1. **Introduction**

Astronomical polarimetry requires frequent calibration operations to remove the instrumental polarization introduced by the various optical elements encountered by the light beam along its path. This contamination is usually determined by placing calibration optics early in the light path, which is used to feed light in a known state of polarization into the instrument. By measuring the polarization of the light that comes out of the system, it is possible to characterize it in terms of its Jones matrix (or, alternatively, the Mueller matrix if one is using the Stokes formalism).

Obviously, we can only characterize and calibrate the optical elements that the beam encounters after the polarization calibration optics. Therefore, calibrating a telescope requires placing such optics at the telescope entrance, before the first reflection on the primary mirror (M1) occurs. This approach is being successfully employed for the Dunn Solar Telescope (at the Sacramento Peak Observatory, managed by the National Solar Observatory) and the German VTT on the island of Tenerife (at the Observatorio del Teide of the Instituto de Astrofísica de Canarias). In both cases, an array of linear polarizers and retarders are
slided in the light path, on top of the entrance window, for calibration operations. Separate calibrations are obtained for the telescope and the instrument, so that the former does not need to be done as frequently. The Jones matrix of the complete system is then obtained as the product of the telescope and instrument matrices.

Unfortunately, entrance window polarizers are not practical for apertures larger than $\sim 1$ m diameter. In the past this has not been a major concern because: a) solar telescopes have apertures that do not exceed 1 m; and b) large astronomical telescopes have not been used for polarization measurements. However, this scenario is starting to change. Polarimetry is proving to be a very powerful tool to explore a broad range of Astrophysical problems, resulting in a rapidly increasing interest to develop polarimeters for existing large telescopes. Second, the development of the Advanced Technology Solar Telescope (ATST)\textsuperscript{3,4} demands a reliable method to calibrate a large telescope for high accuracy spectro-polarimetry.

Solar telescopes have been calibrated in the past by observing magnetic structures and making assumptions on the underlying physics. This poses important challenges, however, especially when pushing the envelope towards new observational domains. Consider for example the ATST, which is intended to do polarimetry at the $\sim 10^{-4}$ accuracy level. One of the common assumptions that is usually made in solar polarimetry is to consider that the continuum radiation is unpolarized. However, scattering processes can polarize the continuum and generate signals of the order of $\sim 1\%$ in the blue side of the visible spectrum. It has been stated\textsuperscript{5} that “the direct observation of the polarisation of the continuous radiation still is a major outstanding observational challenge”. Considerations on the symmetry properties of Stokes profiles are not appropriate either. Gradients in the line-of-sight velocity or magnetic field introduce spurious asymmetries that can invalidate these assumptions. Furthermore, some physical processes operating in the atoms are known to induce Stokes asymmetries even in the absence of gradients. Spectral lines forming in the incomplete Paschen-Back regime become asymmetric.\textsuperscript{6} Moreover, the alignment-to-orientation conversion mechanism\textsuperscript{7} may also cause profile asymmetries.

In summary, it is very important to have an absolute calibration that does not rely on preconceived ideas on the objects under study. This is specially true when the instrumentation is being used to explore new scientific realms. The present work has been motivated by the challenge of calibrating the 4-meter primary mirror of the ATST to meet its very stringent polarization requirements. It might also be possible to use the calibration method proposed here in other existing large-aperture telescopes. However, the actual design of a practical implementation is beyond the scope of this paper. The main point of this work is to show that a calibration setup with inclined beam incidence can be used to measure the polarization properties of a large-aperture primary mirror. Geometrical effects can be calculated and removed from the measured Jones or Mueller matrices resulting in a good
approximation to such matrices in normal observing conditions.

2. The calibration setup

In this paper we shall consider two different configurations: the normal observing setup (OS) and the calibration setup (CS) proposed here. Fig 1 shows a schematic representation of the OS and CS for an on-axis M1 mirror. Our ultimate goal is to determine the Jones matrix of M1 in the OS ($M_{OS}$). A direct measurement of $M_{OS}$ would require polarization optics of the same diameter as the telescope aperture, which is not practical due to technical difficulties. The $M_{CS}$ matrix, on the other hand, can be determined by mounting appropriate calibration optics (and a mechanical control system) at a height $H_{cal}$ over M1 (as shown in Fig 1, right). A small aperture on the dome is probably an ideal location for it.

The CS requires some (small) amount of additional optics with respect to the OS. At least two lenses are required: one at the entrance to open the beam and another at the detector for imaging. These additional elements should be designed with stringent polarization requirements. Aberrations, chromatism and other image imperfections can be tolerated in these components, which will only be used for calibration. Even if they introduce some residual polarization, this should be easily measurable in the laboratory and removed from the M1 Jones matrix.

The size of the calibration optics affects the accuracy of the calibration. In the limit where it fills the entire telescope aperture, the CS and the OS are identical and no correction is needed. As the calibration aperture becomes smaller, the incidence angles increase resulting in larger corrections (and errors). The simulations presented in this paper consider the pessimistic limit in which the calibration optics has a diameter approaching zero (a pinhole aperture).

3. Basic relations

For the calculations in this paper it is convenient to use cylindrical coordinates with the vertical axis along the propagation direction of the incident light beam. The radial coordinate $\rho$ is the distance from the center of the aperture and $\phi$ is the azimuth angle measured from an arbitrary reference. Any given point on the surface of the M1 mirror at coordinates $(\rho, \phi)$ is characterized by its complex refraction index $N(\rho, \phi)$. We shall consider here the behavior of a monochromatic plane wave. This will allow us to describe the instrumental polarization of M1 in terms of its Jones matrix $M(\rho, \phi)$. Appendix 5 gives the Mueller equivalent of the most important Jones matrices derived in this work.

Locally, the behavior of M1 in the vicinity of $(\rho, \phi)$ may be approximated by a reflection on a flat mirror of homogeneous refraction index $N = n + i\kappa$. This process adopts a very simple form in the reference system of the plane of incidence (the plane formed by the incoming ray and the surface normal, see Fig 2). Let us denote the components in the plane of incidence
with the subindex $p$ and those perpendicular to it with $s$. In this frame, the Jones matrix of the reflection, $M_{ps}$, is simply:

$$M_{ps} = \begin{pmatrix} r_p & 0 \\ 0 & r_s \end{pmatrix}. \quad (1)$$

Assuming that the refraction index of air is 1, $r_p$ and $r_s$ are given by:

$$r_p = \frac{\sqrt{N^2 - \sin^2 \theta} - N \cos \theta}{\sqrt{N^2 - \sin^2 \theta} + N \cos \theta} \quad (2)$$

$$r_s = \frac{\cos \theta - \sqrt{N^2 - \sin^2 \theta}}{\cos \theta + \sqrt{N^2 - \sin^2 \theta}}, \quad (3)$$

where $\theta$ is the angle of incidence. This matrix can be transformed to the global reference frame $(\rho, \phi)$:

$$M(\rho, \phi) = R(-\phi)M_{ps}R(\phi), \quad (4)$$

where $R(\phi)$ is the usual rotation matrix. Writing down $M(\rho, \phi)$ explicitly:

$$M(\rho, \phi) = \begin{pmatrix} r_p \cos^2 \phi + r_s \sin^2 \phi & (r_p + r_s) \sin \phi \cos \phi \\ (r_p + r_s) \sin \phi \cos \phi & r_p \sin^2 \phi + r_s \cos^2 \phi \end{pmatrix}. \quad (5)$$

4. Mirror models

4.A. On-axis mirror

Consider an axisymmetric mirror illuminated by a collimated beam (OS). The angle of incidence $\theta$ is constant along concentric rings in the mirror ($\theta = \theta(\rho)$). For example, a parabolic mirror of focal length $F$ is characterized by the condition:

$$\tan \theta = \frac{\rho}{2F}. \quad (6)$$

Suppose that the complex refraction index is constant over the surface of the mirror (perfect mirror). In this case, $r_p$ and $r_s$ are only functions of $\rho$ (because $\theta = \theta(\rho)$). The dependences of $M(\rho, \phi)$ in Eq (5) are easily separable. The Jones matrix of a thin ring of radius $\rho$ is simply:

$$M(\rho) = \rho \int_0^{2\pi} M(\rho, \phi) d\phi = \rho \pi \begin{pmatrix} (r_s + r_p) & 0 \\ 0 & (r_s + r_p) \end{pmatrix}. \quad (7)$$

Eq (7) represents the Jones matrix of a non-polarizing system (identical reflectivity and retardance for both components of the electric field). This is a well-known property of axisymmetric systems. Notice, however, that the symmetry is broken if one observes away from
the center of the field of view. Imperfections in the mirror (irregularities in the refraction index caused by coating degradation, dust, etc) may also break the symmetry of the system and introduce instrumental polarization.

Let us now turn to the more general case of an imperfect mirror, defined as one with \( N = N(\rho, \phi) \). If this is the case then \( r_p \) and \( r_s \) vary across the mirror and Eq (7) is no longer valid. We seek to determine a suitable calibration by means of the (measurable) \( M^CS \) matrix. The differences between \( M^OS \) and \( M^CS \) are due to the different incidence angle of the beam (represented in Fig 2). We can expand \( r_p \) in a power series of \( \alpha \) as:

\[
 r_p(\theta_{CS}) = r_p(\theta_{OS} - \alpha) = r_p(\theta_{OS}) - \alpha \frac{dr_p}{d\theta}|_{\theta_{OS}} + \frac{\alpha^2}{2} \frac{d^2r_p}{d\theta^2}|_{\theta_{OS}} + \ldots ,
\]

and similarly for \( r_s \). Inserting this expansion into Eq (5) we obtain:

\[
 M^OS(\rho, \phi) \simeq M^CS(\rho, \phi) - \alpha \left( \begin{array}{cc} d_{1,p}\cos^2\phi + d_{1,s}\sin^2\phi & (d_{1,p} + d_{1,s})\sin\phi\cos\phi \\ (d_{1,p} + d_{1,s})\sin\phi\cos\phi & d_{1,p}\sin^2\phi + d_{1,s}\cos^2\phi \end{array} \right) \\
 + \frac{\alpha^2}{2} \left( \begin{array}{cc} d_{2,p}\cos^2\phi + d_{2,s}\sin^2\phi & (d_{2,p} + d_{2,s})\sin\phi\cos\phi \\ (d_{2,p} + d_{2,s})\sin\phi\cos\phi & d_{2,p}\sin^2\phi + d_{2,s}\cos^2\phi \end{array} \right) + \ldots ,
\]

where \( d_{i,p} \) and \( d_{i,s} \) have been introduced for notational simplicity:

\[
 d_{i} = \frac{d^i r_p}{d\theta}|_{\theta_{OS}} ,
\]

(10)

(and similarly for \( d_{i,s} \)). In the equations above, \( \alpha \), \( d_{i,p} \) and \( d_{i,s} \) are all functions of \((\rho, \phi)\). The angle \( \alpha \) can be easily determined from geometrical considerations. However, \( d_{i,p} \) and \( d_{i,s} \) are affected by imperfections in the mirror that change over time. Let us separate \( d_{i,p} \) into two components: a nominal \( \hat{d}_{i,p} \) derived from Eqs (2) and (10) with a theoretical refraction index \( N^nom \) (e.g., from manufacturer specifications), and an unknown \( \delta d_{i,p} \) due to coating degradation, dust accumulation, etc:

\[
 d_{i,p} = \hat{d}_{i,p} + \delta d_{i,p} ,
\]

(11)

(and similarly for \( d_{i,s} \)). Inserting this into Eq (9) and integrating over the entire mirror surface, we have:

\[
 M^OS = \int_0^{\rho_{max}} \int_0^{2\pi} \rho M^OS(\rho, \phi) d\rho d\phi = M^CS + \Delta M + \delta M
\]

(12)

where \( \rho_{max} \) is the radius of the M1 mirror. \( \Delta M \) can be calculated numerically as the integral of \( \Delta M(\rho, \theta) \):

\[
 \Delta M(\rho, \theta) = -\alpha \left( \begin{array}{cc} \hat{d}_{1,p}\cos^2\phi + \hat{d}_{1,s}\sin^2\phi & (\hat{d}_{1,p} + \hat{d}_{1,s})\sin\phi\cos\phi \\ (\hat{d}_{1,p} + \hat{d}_{1,s})\sin\phi\cos\phi & \hat{d}_{1,p}\sin^2\phi + \hat{d}_{1,s}\cos^2\phi \end{array} \right)
\]
Mirror imperfections are accounted for by the (measured) $M^{CS}$, whereas non-collimated incidence is accounted for by the (calculated) $\Delta M$. $\delta M$ is an unknown second-order term that couples mirror imperfections and non-collimated incidence. This term is small (as shown below) and may be neglected for our purposes here.

The number of terms to retain in the Taylor expansion of Eq (13) depends on the particular telescope configuration and the accuracy required. Typical examples are presented below in which $\Delta M$ can be neglected entirely (on-axis mirror) or needs to be calculated up to second order (off-axis mirror, see §4.B).

In the reminder of this section I present the results of numerical simulations that provide some insight into the various terms that are involved in the calibration procedure. The parameters of the simulation are listed in Table 1. They represent a 4-m on-axis telescope with a silver coating on the M1 mirror. The coating has been degraded so that the complex refraction index fluctuates over the mirror surface as $N(\rho, \phi) = N^{av}[1 + N^{f} \cos(\phi/2)]$ ($N^{av}$ is the average refraction index and $N^{f}$ is the amplitude of the fluctuation). This choice has been made to represent a pessimistic scenario that induces a considerable amount of instrumental polarization. The discretization of the simulation considers 100 points in $\rho$ and 200 in $\phi$.

The Jones matrices $M^{OS}$ and $M^{CS}$, obtained by applying Eq (5) to each area element of the mirror, are:

$$M^{OS} = -0.94 \exp(0.64i) \left[ 1 + \begin{pmatrix} 0.00 & -0.06 \exp(1.72i) \\ -0.06 \exp(1.72i) & 4.20 \times 10^{-5} \exp(0.40i) \end{pmatrix} \right], \quad (14)$$

$$M^{CS} = M^{OS} + 1.56 \times 10^{-5} \exp(0.78i) \begin{pmatrix} 1.00 & -0.07 \exp(0.28i) \\ -0.07 \exp(0.28i) & -0.76 \exp(0.67i) \end{pmatrix}. \quad (15)$$

The “irregularities” introduced in the refraction index of the mirror break the symmetry and give rise to polarizing effects in $M^{OS}$, with off-diagonal terms of approximately 6%. Fortunately, the calibration setup matrix $M^{CS}$ is an excellent approximation to $M^{OS}$, with a maximum difference of $\sim 10^{-5}$. It is then possible to calibrate the telescope almost down to the $10^{-5}$ level without even having to correct for the inclined incidence (i.e., neglecting $\Delta M$ in Eq [12])

4.B. **Off-axis mirror**

An off-axis mirror can be represented by a larger “equivalent” on-axis mirror with a variable refraction index, as depicted in Fig 3. The equations derived in §4.A above are still valid in
the \((ρ, φ)\) reference frame. One simply needs to set the reflectivity to zero outside the shaded area of the figure \((r > r_{\text{max}})\). This can be accomplished, e.g. by setting the refraction index to 1.

This type of mirrors is slightly more complicated to calibrate. Even a perfectly coated mirror will produce instrumental polarization. A simulation with the parameters listed in Table 2 yields the following matrices:

\[
M^{\text{OS}} = -0.97 \exp(0.59i) \left[ 1 + \begin{pmatrix}
0.00 & 0.00 \\
0.00 & -3.51 \times 10^{-2} \exp(1.61i)
\end{pmatrix} \right], \quad (16)
\]

\[
M^{\text{CS}} = M^{\text{OS}} + 1.24 \times 10^{-2} \exp(2.22i) \begin{pmatrix}
1.00 & 0.00 \\
0.00 & 0.95 \exp(3.11i)
\end{pmatrix}. \quad (17)
\]

\(M^{\text{CS}}\) is now significantly different from \(M^{\text{OS}}\) and we need to calculate the correction term \(\Delta M\) (Eqs [12] and [13]). Let us consider for the moment that the refraction index is perfectly known \((δd_{i,p} = δd_{i,s} = 0\) in Eq [11]). Calculating \(\Delta M\) with Eq (13) up to second order we have that:

\[
M^{\text{CS}} + \Delta = M^{\text{OS}} + 2.22 \times 10^{-4} \exp(2.65i) \begin{pmatrix}
1.00 & 0.00 \\
0.00 & 0.43 \exp(0.44i)
\end{pmatrix}. \quad (18)
\]

Let us now turn to the more general case of an off-axis mirror with surface irregularities for which we have only an imperfect knowledge of the average refraction index. Again, we use a refraction index with an angular dependence \(N(ρ, φ) = N^{\text{av}}[1 + N^{\text{f}} \cos(φ/2)]\). Furthermore, we do not know exactly the average refraction index of the mirror \(N^{\text{av}}\), but only an approximation \(N^{\text{nom}}\). This approximate value will be used in the calculation of \(\hat{d}_{i,p}, \hat{d}_{i,s}\) and \(\Delta M\) (Eqs [11] and [13]). I carried out several experiments with different values of \(N^{\text{nom}}\) and \(N^{\text{f}}\) to determine the sensitivity of the calibration to these parameters. Some of the calculations include only the first-order dependence of \(\Delta M\) on \(α\) (first term in the right-hand side of Eq [13]). These are denoted by \(\Delta M^{*}\) (as opposed to \(\Delta M\), which considers the second-order term). Table 3 lists the results of these experiments. The first two rows give the highest polarizing term in \(M^{\text{OS}}\) for each simulation. The third to fifth rows show how the calibration error decreases with successive levels of approximation. By comparing the second and fifth rows, we can see that the calibration is able to reduce the instrumental polarization by an amount between one and two orders of magnitude.

5. Conclusions

This paper introduces a new concept to calibrate telescopes for astronomical polarimetry. The proposed method is particularly useful for modern large-aperture telescopes, for which
it is probably the only practical procedure (at least for purely instrumental calibration). An accurate absolute calibration will be crucial for the new weak-signal science that the ATST will open. Existing night-time telescopes may also take advantage of this calibration procedure.

The Jones (and Mueller) matrix of an on-axis mirror is almost unaffected by the non-collimated incidence of the beam in the CS. For the particular configuration considered in §4.A, the Jones matrix obtained from the calibration is good to almost $10^{-5}$ with no need for any additional correction (see Eq [15]). This is in spite of the relatively large mirror imperfections in the simulation, which generate polarizing terms of the order of 6%.

An off-axis mirror suffers more instrumental polarization due to the asymmetric configuration. A mirror that produces instrumental polarization of a few percent can be calibrated to reach the $10^{-4}$ level (see Table 3). In addition to measuring the Jones matrix of the calibration setup ($M^{CS}$), it is also necessary to calculate the correction term $\Delta M$. This calculation is straightforward, though, and $\Delta M$ does not need to be recalculated unless the mirror is recoated or it degrades to a point where its average refraction index changes significantly. Note that the $10^{-4}$ calibration accuracy includes some uncertainty on the average refraction index of the mirror.

Appendix A: Mueller formalism

I have used in this paper the Jones matrix formalism, which deals directly with the components of the electric field of the light wave. Sometimes, however, the Mueller formalism is more adequate, especially when dealing with partially polarized or non-monochromatic light. Many researchers are more familiar with the Mueller matrices and the Stokes parameters. For these reasons it is probably useful to provide the Mueller equivalent\textsuperscript{9} of the Jones matrices derived in this work.

Eq (14) becomes:

$$M^{OS} = \begin{pmatrix}
0.89 & -3.42 \times 10^{-5} & 1.65 \times 10^{-2} & -2.19 \times 10^{-6} \\
-3.42 \times 10^{-5} & 0.88 & 5.49 \times 10^{-7} & 0.11 \\
1.65 \times 10^{-2} & 5.49 \times 10^{-7} & 0.89 & -1.45 \times 10^{-5} \\
2.19 \times 10^{-6} & -0.11 & 1.45 \times 10^{-5} & 0.88
\end{pmatrix}. \quad (A1)$$

Eq (15) becomes:
$$M^{CS} = M^{OS} + 2.23 \times 10^{-5} \times \begin{pmatrix} 0.30 & 1.00 & -6.77 \times 10^{-2} & 6.42 \times 10^{-2} \\ 1.00 & 0.30 & -1.80 \times 10^{-2} & 5.45 \times 10^{-2} \\ -6.77 \times 10^{-2} & -1.80 \times 10^{-2} & 0.30 & 0.45 \\ -6.42 \times 10^{-2} & -5.45 \times 10^{-2} & -0.45 & 0.30 \end{pmatrix}.$$ 

Eq (16) becomes:

$$M^{OS} = \begin{pmatrix} 0.94 & -1.93 \times 10^{-3} & 2.29 \times 10^{-7} & 0.00 \\ -1.93 \times 10^{-3} & 0.94 & 0.00 & 0.00 \\ 2.29 \times 10^{-7} & 0.00 & 0.94 & 3.29 \times 10^{-2} \\ 0.00 & 0.00 & -3.29 \times 10^{-2} & 0.94 \end{pmatrix}.$$ 

Eq (17) becomes:

$$M^{CS} = M^{OS} + 2.34 \times 10^{-2} \times \begin{pmatrix} 6.92 \times 10^{-3} & 5.85 \times 10^{-2} & 0.00 & 0.00 \\ 5.85 \times 10^{-2} & 6.92 \times 10^{-3} & 0.00 & 0.00 \\ 0.00 & 0.00 & 2.96 \times 10^{-2} & -1.00 \\ 0.00 & 0.00 & 1.00 & 2.96 \times 10^{-2} \end{pmatrix}.$$ 

Eq (18) becomes:

$$M^{CS} + \Delta M = M^{OS} + 1.82 \times 10^{-4} \times \begin{pmatrix} -0.97 & -0.13 & -1.78 \times 10^{-5} & 0.00 \\ -0.13 & -0.97 & 0.00 & 1.13 \times 10^{-5} \\ -1.78 \times 10^{-5} & 0.00 & -1.00 & 0.72 \\ 0.00 & -1.13 \times 10^{-5} & -0.72 & -1.00 \end{pmatrix}.$$ 

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Table 1. Simulation of a 4-m on-axis mirror

| Parameter              | Value                        |
|------------------------|------------------------------|
| $\rho_{\text{max}}$   | 2 m                          |
| $F$                    | 8 m                          |
| $H_{\text{cal}}$      | 10 m                         |
| $N_{\text{av}}$       | 0.2 + 3.4i                   |
| $N_f$                  | 0.50                         |
| $\max(|M_{\text{OS}} - M_{\text{CS}}|)$ | $1.56 \times 10^{-5}$       |
Table 2. Simulation of a 4-m off-axis mirror

| Parameter | Value       |
|-----------|-------------|
| $r_{\text{max}}$ | 2 m        |
| $p_{\text{max}}$ | 6 m        |
| $F$       | 8 m        |
| $H_{\text{cal}}$ | 10 m      |
| $N^{\text{av}}$ | 0.2 + 3.4i |
| $N^{f}$  | 0.0        |
| $\max(|M^{OS} - M^{CS}|)$ | $1.24 \times 10^{-2}$ |
Table 3. Simulation of a 4-m off-axis mirror with surface irregularities

|                      | $N^f = 0.10$ | $N^f = 0.10$ | $N^f = 0.25$ | $N^f = 0.25$ |
|----------------------|--------------|--------------|--------------|--------------|
|                      | $N^{av}$     | $N^{av}$     | $N^{av}$     | $N^{av}$     |
| $|M^{OS}_{2,2} - |M^{OS}_{1,1}|$  | 2.02 x 10^{-3} | 2.02 x 10^{-3} | 2.25 x 10^{-3} | 2.25 x 10^{-3} |
| $|M^{OS}_{1,2}|$  | 1.50 x 10^{-2} | 1.50 x 10^{-2} | 3.86 x 10^{-2} | 3.86 x 10^{-2} |
| $\max(|M^{OS} - M^{CS}|)$  | 1.25 x 10^{-2} | 1.25 x 10^{-2} | 1.26 x 10^{-2} | 1.26 x 10^{-2} |
| $\max[|M^{OS} - (M^{CS} + \Delta M^*)|]$  | 4.04 x 10^{-3} | 3.89 x 10^{-3} | 3.71 x 10^{-3} | 2.59 x 10^{-3} |
| $\max[|M^{OS} - (M^{CS} + \Delta M)|]$  | 2.84 x 10^{-4} | 4.18 x 10^{-4} | 7.11 x 10^{-4} | 1.80 x 10^{-3} |
Fig. 1. Left: Normal observing configuration (OS), with a collimated incident beam. Right: Calibration configuration (OS), with an inclined (diverging) incident beam. navarrof1.eps.

Fig. 2. Incidence angles $\theta$ and $\alpha$ for the OS and CS, respectively. Solid (dashed) represent rays in the OS (CS). The dotted line represents the normal to the mirror surface. navarrof2.eps.

Fig. 3. Schematic representation of the equivalent on-axis mirror. Left: Lateral view. The thick line represents the actual off-axis mirror of radius $r_{max}$. The thin line represents the equivalent on-axis mirror. Right: Top-down view. The shaded area is the actual off-axis mirror. navarrof3.eps.
