Reduced Visibility of Rabi Oscillations in Superconducting Qubits

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Coherent Rabi oscillations between quantum states of superconducting micro-circuits have been observed in a number of experiments, albeit with a visibility which is typically much smaller than unity. Here, we show that the coherent coupling to background charge fluctuators [R. W. Simmonds et al., Phys. Rev. Lett. 93, 077003 (2004)] leads to a significantly reduced visibility if the Rabi frequency is comparable to the coupling energy of micro-circuit and fluctuator. For larger Rabi frequencies, transitions to the second excited state of the superconducting micro-circuit become dominant in suppressing the Rabi oscillation visibility. We also calculate the probability for Bogoliubov quasi-particle excitations in typical Rabi oscillation experiments.

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Possible applications in quantum information processing have renewed the interest in quantum coherent phenomena of micrometer-scale Josephson junction (JJ) circuits [1]. During the past years, several experiments have demonstrated Rabi oscillations of a macroscopic variable including charge \[2,\] phase \[3,\] a combinations of both \[4,5,\] and flux \[6,\] which persist up to several microseconds \[7,\] However, in many of the reported experiments, the Rabi oscillation visibility is significantly smaller than unity even at times short compared to the decoherence time. Characteristic values are as small as 10\% (Ref. [8]) and 50\% (Refs. [9,10,11]), respectively, which indicates either substantial leakage out of the computational basis or a pronounced randomization of the state occupation over short time-scales. In the following, we discuss three possible mechanisms that result in a reduced visibility, namely (i) coherent coupling to a background fluctuator which induces transitions between the eigenstates of the superconducting micro-circuit \[8,\] (ii) population of higher excited states caused by a strong driving field, i.e., the AC current or voltage applied to drive Rabi oscillations; and (iii) the excitation of Bogoliubov quasi-particles.

Background charge fluctuators are presently considered one of the dominant source of decoherence in superconducting qubits (squbits) \[8,9,10,11,\] However, the theoretical analysis of fluctuators has so far been focused on fluctuators with an incoherent dynamics which are coupled to the squbit by an Ising-like interaction. Motivated by the results of Refs. [7] and [12], we show that the coherent dynamics of a squbit-fluctuator system with a transverse exchange coupling leads to a reduced visibility and beating in the Rabi oscillation signal of the squbit. For Rabi oscillation frequencies large compared to the squbit-fluctuator coupling strength \(J,\) the fluctuator does not provide an efficient mechanism for visibility reduction on short time-scales. We show that, in this regime, coupling to the second excited squbit state suppresses the visibility to 0.7 for Rabi frequencies larger than 150 MHz in the experiment in Ref. [7]. This mechanism persists for adiabatic switching, in stark contrast to the leakage scenarios discussed previously \[13,14,\] Finally, we calculate the excitation rate of Bogoliubov quasi-particles and show that the quasi-particle excitation probability is small as long as the JJ is in a zero-voltage state.

**Coherent squbit-fluctuator coupling.** While fluctuators, trapped charges in bi-stable potentials, have so far mainly been considered as source of decoherence for squbits, recent experiments provide strong evidence that some fluctuators are coupled coherently to a squbit. An important conclusion drawn from the anti-crossings in the microwave spectra of a phase-squbit \[7,\] and the subsequent observation of real-time oscillations \[12,\] is that the squbit-fluctuator coupling has a transverse component rather than the Ising-form considered in previous theoretical analysis \[3,11,\] Because of the transverse coupling component, fluctuators induce transitions between different squbit basis states, randomizing the occupation probabilities. In order to analyze the dynamics of a coupled squbit-fluctuator system, we introduce a pseudo-spin notation and define \(\hat{s}_z = (|1\rangle\langle1| - |0\rangle\langle0|)/2\) and \(\hat{s}_x = (|1\rangle\langle0| + |0\rangle\langle1|)/2\) in terms of the ground (first excited) state \(|0\rangle, (|1\rangle)\) of the squbit, such that \(|s_z = 1\rangle\) represents the ground state. Similarly, we define the pseudo-spin states \(|I_z = 1\rangle\) and \(|I_z = 1\rangle\) as the ground and first excited states of the fluctuator, respectively. Rabi oscillations between the squbit basis states are driven by an AC current in resonance with the level splitting \(\hbar \omega_{10}\) of the squbit. In our pseudo-spin notation, the AC current acts as transverse magnetic field \(b_x\) and, in the co-rotating frame,

\[
\hat{H} = \delta \hat{I}_z + J \left( \hat{s}_z \hat{I}_x + \hat{s}_y \hat{I}_y \right) + b_x \hat{s}_x,
\]

where \(\delta = \hbar (\omega_{eg} - \omega_{10})\) is the detuning of the fluctuater level splitting \(\hbar \omega_{eg}\) relative to the squbit. The ansatz for the transverse squbit-fluctuator exchange cou-
pling with coupling strength $J$ is microscopically motivated from the observation that the critical current of a JJ may depend on the position of a fluctuator in its bi-stable potential [17]. As shown below, a fluctuator influences the squbit dynamics strongly if $|\delta| < J$ or $|\delta \pm b_z| < J$. For the JJ in Ref. [4], $J/\hbar \simeq 25$ MHz while the typical level spacing between different fluctuator resonances is of order 60 MHz. This allows us to restrict our analysis to one fluctuator with minimum $|\delta|$ or $|\delta \pm b_z|$ first. For the experimental temperature $T \simeq 20$ mK $\ll \hbar \omega_{q}/KB, \hbar \omega_{ef}/KB$, at the beginning of the Rabi pulse, $t = 0$, both squbit and fluctuator are in their ground-state and $|\psi(0)⟩ = |↓↓⟩$ in the product basis $|s_z, I_z⟩$ of the squbit-fluctuator system. The experimentally accessible quantity is the probability $p_{1}(t)$ for the squbit to occupy its first excited state as a function of Rabi pulse duration, $p_{1}(t) = \sum_{I_{z}=\pm 1} |⟨\uparrow; I_{z} | \psi(t)⟩|^{2}$. While the energy level splitting $\hbar \omega_{ef}$ of a fluctuator is fixed, the squbit level splitting $\hbar \omega_{q}$ can be tuned via the DC bias current through the JJ, which allows one to measure squbit Rabi oscillations for varying $\delta$ and $b_z$.

We now calculate the dynamics of $|\psi(0)⟩ = |↓↓⟩$ and $p_{1}(t)$ as a function of $\delta$ and $b_z$. The squbit-fluctuator exchange coupling gives rise to a linear dependence of the eigenenergies on $J$ for $|\delta| \lesssim J$ and $|\delta \pm b_z| \lesssim J$, where cross-relaxation processes between squbit and fluctuator reduce the visibility of the squbit Rabi oscillations [16]. While $p_{1}(t)$ is readily obtained from integration of the Schrödinger equation for arbitrary $b_z, \delta,$ and $J$, here we focus on the two cases $\delta = 0$ and $\delta \pm b_z = 0$, where the cross-relaxation between squbit and fluctuator is most efficient. For $\delta = 0$, we find

$$p_{1}(t)|_{\delta = 0} = \frac{1}{2} \left[ 1 - \cos \left( \frac{Jt}{2\hbar} \right) \cos \left( \frac{\sqrt{J^{2} + 4b_{z}^{2}t^{2}}}{2\hbar} \right) \right]$$

$$= \frac{1}{2} - \sin \left( \frac{Jt}{2} \right) \sin \left( \frac{\sqrt{J^{2} + 4b_{z}^{2}t^{2}}}{2\hbar} \right) \sqrt{1 + \frac{b_{z}^{2}}{J^{2}}}.$$

For $|b_z|/J \gg 1$, the exact expression for $p_{1}(t)$ is too long to be presented here, but for $|b_z|/J \gtrsim 2$ it is well approximated by

$$p_{1}(t)|_{\delta \pm b_z = 0} \approx \frac{1}{2} \left[ 1 - \cos \left( \frac{Jt}{4\hbar} \right) \cos \left( \frac{b_{z}t}{\hbar} \right) \right].$$

Equations (2) and (3) describe the Rabi oscillations of a squbit in the presence of a fluctuator. The transverse coupling to a fluctuator introduces an additional Fourier component which leads to beating of the Rabi-oscillation signal, in agreement with experimental results, where typically $|b_z|/J \gtrsim 2$. For short time-scales, the $J$-dependent factor in Eqs. (2) and (3) leads to a decrease in Rabi oscillation visibility. In particular, the first maximum in $p_{1}(t)$ is reduced relative to unity. For $|b_z|/J \gtrsim 2$, we obtain $p_{1}(t = \pi\hbar/b_z) \approx 1 - (\pi J/4b_z)^{2}$ and $p_{1}(t = \pi\hbar/b_z) \approx 1 - (\pi J/8b_z)^{2}$ for $\delta = 0$ and $\delta \pm b_z = 0$, respectively [10]. While a resonant fluctuator reduces $p_{1}(t = \pi\hbar/b_z)$ to 0.85 for $|b_z|/J \gtrsim 2$, Rabi oscillations with amplitude 1 are predicted to emerge for $|b_z|/J \gg 1$, in stark contrast to experimental results where the first local maximum of $p_{1}(t)$ is of order 0.5 even for large $b_z$ [7].

A similar system was analyzed in Ref. [18] using different numerical and approximate analytical approaches. Here, we provide an analytical solution of Eq. (4) for $|b_z|/J \gg 1$ where, to leading order in $J, H \simeq \delta I_{z} + \delta J b_{z} I_{z} + b_{z} s_{z}$. Then, the sixteen coupled differential equations in Eq. (4) decouple into sets of four differential equations. We calculate $p_{1}(t) = \sum_{I_{z}=\pm 1} |⟨\uparrow; I_{z} | \tilde{\rho}(t) | \uparrow; I_{z}⟩|$ from the explicit solution of Eq. (4). For $\delta = 0$ and $\tilde{\rho}(0) = |↓↓⟩⟨↓↓|$, we find

$$p_{1}(t) \approx \frac{1}{2} - \frac{1}{4} \left[ \left( 1 - \frac{\gamma}{\sqrt{\gamma^{2} - J^{2}}} \right) - (\gamma + \sqrt{\gamma^{2} - J^{2}}) e^{-\gamma t + \sqrt{\gamma^{2} - J^{2}} t/2\hbar} + (1 + \frac{\gamma}{\sqrt{\gamma^{2} - J^{2}}} ) e^{-\gamma - \sqrt{\gamma^{2} - J^{2}} t/2\hbar} e^{-b_{z}t/\hbar} \right].$$

Note the intriguing dependence of $p_{1}(t)$ on the fluctuator decoherence rate $\gamma/\hbar$ and the coupling strength $J$ (Fig. 1). For $\gamma/\hbar \ll 1$,

$$p_{1}(t) \approx \frac{1}{2} \left[ 1 - e^{-\gamma t/2\hbar} \cos \left( \frac{Jt}{2\hbar} \right) \cos \left( \frac{b_{z}t}{\hbar} \right) \right]$$

shows the beating already derived in Eq. (2), but with a finite damping rate $\gamma/2\hbar$. In contrast, for $\gamma/\hbar \gg 1$,

$$p_{1}(t) \approx \frac{1}{2} \left[ 1 - e^{-4J^{2}t/\hbar^{2}} \cos \left( \frac{b_{z}t}{\hbar} \right) \right]$$

exhibits single-frequency oscillations with a damping rate $4J^{2}/\hbar^{2}$ (see also Ref. [15]). This can be understood from the observation that, for $\gamma \gg J$, the phase of the fluctuator is randomized on a time-scale short compared to $\hbar/J$ and the mean field $J \langle \tilde{I}_{z} \rangle$ acting on the squbit is averaged out to leading order, which restores the single-frequency oscillations in Eq. (7).

We discuss next to which extent far off-resonant fluctuators with $|\delta| \gg |b_z| \gg J$ reduce the Rabi oscillation visibility. Because of the perturbative parameter $J/|\delta| \ll 1$, the visibility reduction in $p_{1}(t) \simeq (1 -
J^2/4\delta^2) \sin^2(b_xt/2h) is small for a single off-resonant fluctuator. More generally, a large number of off-resonant fluctuators interacts with the qubit, and the problem is closely related to the dynamics of an electron spin coupled to a bath of nuclear spins by the hyperfine contact interaction [10]. To leading order in the coupling constants, a set \{i\} of off-resonant fluctuators with coupling constants \(J_i\) and detunings \(\delta_i\) reduces the Rabi oscillation visibility by \(\sum_i J_i^2/4\delta_i^2\). Introducing the distribution function \(P(E)\) of the fluctuator level splittings, for constant \(J_i = J\),

\[
\sum_i J_i^2/4\delta_i^2 = \frac{J^2}{4} \int_{|E - \hbar\omega_{10}| > |b_x|, E > E_c} \frac{dE P(E)}{(\hbar\omega_{10} - E)^2}, \tag{8}
\]

where the integral is evaluated for all energies with \(|E - \hbar\omega_{10}| > |b_x|\) (off-resonance condition) larger than a lower energy cut-off \(E_c\) given by the fluctuator decoherence rate. Evaluating the integral for the distribution function \(P(E) \propto 1/E\) characteristic for fluctuator level spacings (1/f-noise), we find \(\sum_i J_i^2/4\delta_i^2 \approx 2J^2P(\hbar\omega_{10})/4|b_x| \approx 0.2J/|b_x|\) for the experimental parameters of Ref. [7]. If cross correlations between fluctuators can be neglected, the reduction of the first maximum in \(p_1(t)\) is obtained by summing the contributions from resonant fluctuators with \(\delta \approx 0\) and \(\delta \pm b_x \approx 0\) [Eqs. (2) and (3)] and off-resonant fluctuators [Eq. (8)]. An upper bound is given by \(1 - p_1(t = \pi\hbar/b_x) \leq 2J^2P(\hbar\omega_{10})/4|b_x| + (3/32)(\pi J/b_x)^2\), which decreases with increasing \(|b_x|\). This shows that, even when both resonant and off-resonant fluctuators are taken into account, the small visibility \(p_1(t = \pi\hbar/b_x) \leq 0.5\) observed in the limit of large Rabi frequencies, \(|b_x|/\hbar \gg J/\hbar\) in Ref. [7] cannot be effected by fluctuators.

\[\text{Energy shifts induced by AC driving field. – Exploring the visibility reduction at a time-scale of 10 ns requires Rabi frequencies } |b_x|/\hbar \gtrsim 100 \text{ MHz. We show next that, in this regime, transitions to the second excited qubit state lead to an oscillatory behavior in } p_1(t) \text{ with a visibility smaller than } 0.7. \]

The analysis of the AC current, transitions to \(|2\rangle\) can be neglected as long as \(|b_x| \ll \hbar\Delta = \hbar(\omega_{10} - \omega_{21}) \approx 0.03\hbar\omega_{10}\). However, for \(b_x\) comparable to \(\hbar\Delta\), the applied AC current strongly couples \(|1\rangle\) and \(|2\rangle\) because \(\langle 2|\phi(1)|1\rangle \neq 0\), where \(\phi\) is the phase operator. For typical parameters, \(|b_x|/\hbar\Delta\) varies from 0.05 to 1, depending on the irradiated power \[7, 10\]. Taking into account the second excited state of the phase-qubit, the qubit Hamiltonian in the rotating frame is

\[
\hat{H} = -\hbar\Delta\omega|2\rangle\langle 2| + b_x \left( |1\rangle\langle 1| + \sqrt{2}|1\rangle\langle 2| + \text{H.c.} \right)/2. \tag{9}
\]

In the following, we neglect fluctuators, which is valid for the short-time dynamics if \(|b_x| \gg J\).

The time-evolution of \(|\psi(0)\rangle = |0\rangle\) is readily calculated by integration of the Schrödinger equation. Expanding \(|\psi(t)\rangle\) in \(|b_x|/2\hbar\Delta\omega\), we find that

\[
p_1(t) = \left[1 - \frac{3}{2} \frac{b_x}{2\hbar\Delta\omega}\right]^2 \sin^2 \frac{b_x [1 - (b_x/2\hbar\Delta\omega)] t}{\hbar} \tag{10}
\]

exhibits single-frequency oscillations with reduced visibility [Fig. 2(b)]. Part of the visibility reduction can be traced back to leakage into state \(|2\rangle\). More subtly, off-resonant transitions from \(|1\rangle\) to \(|2\rangle\) induced by the driving field lead to an energy shift of \(|1\rangle\), such that the transition from \(|0\rangle\) to \(|1\rangle\) is no longer resonant with the driving field, which also reduces the visibility. For \(|b_x|/2\hbar\Delta\omega = 1/3\), corresponding to \(|b_x|/\hbar = 150 \text{ MHz} \) in Ref. [7], Eq. (10) predicts a visibility of 0.7. Note that the off-resonant transition to \(|2\rangle\) discussed here is induced by a large amplitude of the driving field and not by additional frequency components that result, e.g., from non-adiabatic switching of the driving field [12] or current noise in the microcircuit [20]. Rather, the visibility reduction in Eq. (10) corresponds to a steady-state solution of the Schrödinger equation and persists even for adiabatic switching [21]. Whether such off-resonant transitions are responsible for the reduced Rabi oscillation visibility can be tested experimentally from the dependence of the Rabi frequency and amplitude on \(b_x\). Equation (10) predicts a non-linear behavior of the visibility.
dependence of the Rabi frequency on $b_x$, while the visibility is predicted to decrease quadratically with $b_x$. Decreasing visibility with increasing driving field has indeed been observed in some Rabi oscillation experiments [22], but further quantitative analysis is required to determine the explicit functional dependence.

Excitation of quasi-particles.— Transient high-frequency components in the switching pulses induce leakage to states outside the computational basis. Bogoliubov quasi-particle excitations represent a large class of excited states which is often ignored for the discussion of leakage because the excitation gap is large. For both charge-based and phase-based squbits, the excitation of quasi-particles during the measurement process has been quantified and is known to limit the decoherence time $\Gamma$ [22,23,24]. In phase-squbits, quasi-particles trigger leakage to finite-voltage states or to the excited squbit states [2] or [3] by incoherent tunneling across the JJ at a rate determined by the single-particle tunneling rate and the total number of quasi-particles [24].

While the number of quasi-particles created during read-out can be decreased by increasing experimental waiting times [24]. Bogoliubov quasi-particles are also excited during an experimental cycle by high-frequency Fourier components of an external current or voltage pulse with $\omega > 2\Delta/h$, where $\Delta$ is the energy gap of the superconductor. Microscopic mechanisms for quasi-particle excitation include, e.g., dissipative tunneling through the JJ. We calculate the excitation rate $\Gamma$ of quasi-particle pairs for an oscillating current component $\delta I_\omega \cos(\omega t)$ with $\omega > 2\Delta/h$. Using a semiclassical approximation, the high-frequency current induces oscillations $\delta \phi(t)$ of the superconducting phase around its equilibrium value $\phi_0 = \arcsin(I/I_c) \approx \pi/2$, where $I$ and $I_c$ are the DC bias current and critical current of the JJ, respectively. Amplitude and frequency of $\delta \phi(t)$ are determined by the externally imposed current, $\delta \phi(t) = -\left(\delta I_\omega/(\hbar \omega^2 C)\right) \cos \omega t$, where $C$ is the JJ capacitance. Dissipative tunneling through the JJ – processes in which a quasi-particle pair is excited under absorption of energy $\hbar \omega$ from the bias current – is described by both the normal current and the Josephson cosine-term [22]. To lowest non-vanishing order in $(\delta I_\omega)/(\hbar \omega^2 C)$, the time-averaged power dissipation $\bar{P}$ at the JJ is [25]

$$
\bar{P} = \frac{1}{2} \left( \frac{\delta I_\omega}{C \omega} \right)^2 e \frac{1}{\hbar \omega} \left[ I_n(\hbar \omega) + I_2(\hbar \omega) \cos \phi_0 \right]
$$

$$
\simeq \frac{1}{2} \left( \frac{\delta I_\omega}{C \omega} \right)^2 e \frac{1}{\hbar \omega} I_n(\hbar \omega),
$$

(11)

with the standard expressions for the normal current $I_n$ and the Josephson cosine-term $I_2$. For $\hbar \omega \to 2\Delta^+$, $I_n$ is determined by the normal-state resistance $R_n$ of the JJ [26]. $I_n(2\Delta^+) = (\pi/4R_n)(2\Delta/e)$. The second line of Eq. (11) is valid for squbits biased close to the critical current, where $\phi_0 \approx \pi/2$. The quasi-particle excitation rate is obtained from $\bar{P}$ via [27]

$$
\Gamma = \frac{\bar{P}}{\hbar \omega} = \frac{\pi \delta I_\omega^2}{8 \hbar C^2 R_n \omega^3}.
$$

(12)

$\Gamma$ is negligibly small unless $C \leq 1 \text{fF}$ and $\delta I_\omega \geq 1 \text{pA}$. With typical parameters $R_n = 29 \Omega$, $C = 1 \text{fF}$, and $\omega = 4 \text{KkB}/\hbar$, we obtain $\Gamma = 1 \text{msec}^{-1} \times (\delta I_\omega/\text{pA})^2$.

For instantaneous switching, a current pulse with amplitude $I_{ac} = 10 \text{nA}$ and duration $T = 10 \text{msec}$, $\delta I_{ac} \approx I_{ac}/\omega T = 1 \text{pA}$ for $\omega = 4 \text{KkB}/\hbar$. These values show that quasi-particle excitation by dissipative tunneling cannot explain the substantial visibility reduction evidenced in current Rabi oscillation experiments for phase squbits. However, the excitation of Bogoliubov quasi-particles is relevant in view of the ultimate goal of reducing quantum gate errors to less than $10^{-4}$.

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