Perturbation-Recovery Method for Recommendation

Jeongwhan Choi, Seoyoung Hong, Noseong Park, Sung-Bae Cho
{jeongwhan.choi,seoyoung.hong,noseong,pbcho}@yonsei.ac.kr
Yonsei University
Seoul, South Korea

ABSTRACT
Collaborative filtering is one of the most influential recommender system types. Various methods have been proposed for collaborative filtering, ranging from matrix factorization to graph convolutional methods. Being inspired by recent successes of GF-CF and diffusion models, we present a novel concept of blurring-sharpening process model (BSPM). Diffusion models and BSPMs share the same processing philosophy in that new information is discovered (e.g., a new image is generated in the case of diffusion models) while original information is first perturbed and then recovered to its original form. However, diffusion models and our BSPMs deal with different types of information, and their optimal perturbation and recovery processes have a fundamental discrepancy. Therefore, our BSPMs have different forms from diffusion models. In addition, our concept not only theoretically subsumes many existing collaborative filtering models but also outperforms them in terms of Recall and NDCG in the three benchmark datasets, Gowalla, Yelp2018, and Amazon-book. Our model marks the best accuracy in them. In addition, the processing time of our method is one of the shortest cases ever in collaborative filtering. Our proposed concept has much potential in the future to be enhanced by designing better blurring (i.e., perturbation) and sharpening (i.e., recovery) processes than what we use in this paper.

KEYWORDS
recommender systems, collaborative filtering, perturbation-recovery paradigm, blurring-sharpening process

1 INTRODUCTION
Recommender systems are one representative topic of information filtering. These days a non-trivial portion of the revenue of many global information technology (IT) companies is from advertising and recommendation. In this regard, recommender systems are of utmost interest in real-world environments. Among various technologies, collaborative filtering (CF) is one of the most popular approaches of recommender systems, and many CF-based methods have been proposed.

In particular, graph convolution-based CF methods currently show state-of-the-art accuracy [9, 15, 20, 23, 27]. They represent user-item interactions as a bipartite graph and apply the graph convolutional technology. Among various graph convolutional operations, they all use relatively simple linear or low-pass filters. Surprisingly, these approaches now beat other classical and deep learning-based methods.

In this paper, we propose a novel paradigm of Blurring-Sharpening Process Model (BSPM) for CF. Our blurring and sharpening processes are formulated as differential equations — we will also show that some of the existing graph convolution-based CF methods are special cases of our model.

![Figure 1: The comparison between diffusion models and our proposed BSPMs. Diffusion models, a recently proposed paradigm for deep generative task, outperform generative adversarial networks (GANs), variational autoencoders (VAEs), and many other generative models.](image)

![Table 1: Comparison of existing methods. Our BSPM not only combines the blurring and the sharpening processes but also interprets them in a continuous manner.](table)

| Model     | Blurring                        | Sharpening |
|-----------|--------------------------------|------------|
| LightGCN  | Discrete with heat equation     | X          |
| LT-OCF    | Continuous with heat equation   | X          |
| GF-CF     | Discrete with low pass & ideal filters | X          |
| BSPM      | Continuous with various filters | Continuous with a filter |

Our model is greatly inspired by GF-CF [27], which is simple and computationally efficient but shows state-of-the-art accuracy. GF-CF does not learn embedding vectors for users/items but directly processes the user-item interaction matrix to derive unknown user-item interactions. Although GF-CF has shown encouraging results, we found that our proposed perturbation-recovery paradigm popular in image generation can significantly enhance it. For instance, Diffusion models (or score-based generative models) [10, 18, 19, 28–30] show the state-of-the-art quality in the domain of image generation. In the diffusion models, specific types of stochastic differential equations (SDEs) are adopted to describe the forward and the backward
processes (cf. Fig. 1 (a)) — the backward process is considered as a generative model.

Our overall model design has a perturbation-recovery architecture, i.e., the blurring process corrupts (or perturbs) original information in the user-item interaction matrix, and the sharpening process tries to recover the original information in conjunction with promising additional information (cf. Fig. 1 (b)). We apply the blurring and the sharpening processes directly to the interaction matrix in a continuous manner whereas existing methods, such as GF-CF, apply certain (discrete) blurring filters to the matrix (cf. Table 1). To our knowledge, we are the first proposing the blurring-sharpening process paradigm for CF.

Therefore, the key in our model is how to define the blurring and the sharpening processes. Both of them are written as ordinary differential equations (ODEs) in our case (cf. Eqs. (8) and (13)). We customize various well-known blurring and sharpening functions proposed in various domains different from CF.

After defining our blurring and sharpening processes, we design two variants of BSPM: BSPM-LM and BSPM-EM. These variants differ from each other in how to connect the blurring and the sharpening processes. We then show that some of the popular existing methods are special cases of our method.

We conduct experiments with 3 benchmark datasets and 19 baselines. Surprisingly, our method beats all existing popular CF algorithms by large margins. There are no existing methods that are comparable to our method in all datasets.

Moreover, our proposed model can also be properly understood from the perspective of classical graph convolutional processing. Therefore, we emphasize that our proposed model has strong theoretical grounds, and it is not by chance that our model marks the best accuracy. Our contributions can be summarized as follows:

1. We design a perturbation-recovery concept, called the **Blurring-Sharpening Process Model** (BSPM).
2. Our BSPM directly perturbs (blurs) the user-item interaction matrix, and recovers (sharpens) the blurred matrix to derive unknown user-item interactions.
3. Our method outperforms all existing 19 popular CF methods in the three benchmark datasets.
4. To our knowledge, we are the first adopting the perturbation-recovery paradigm for CF. Therefore, one can consider that we propose a new paradigm for CF, and we think that it has much potential in the future by discovering better perturbation and recovery processes than ours.
5. One can download our codes and datasets from [here](#).

## 2 PRELIMINARIES & RELATED WORK

### 2.1 Collaborative Filtering

Let $R \in \{0, 1\}^{(|U| \times |V|)}$, where $U$ is a set of users and $V$ is a set of items, be an interaction matrix. $R_{u,v}$ is 1 iff an interaction $(u,v)$ is observed in data, or otherwise 0. We also define the normalized interaction matrix as $\bar{R} = U^{-\frac{1}{2}}RV^{-\frac{1}{2}}$, where $U = \text{Diag}(R_1)$, $V = \text{Diag}(1^T R)$, $1$ means a column vector of ones, and $^T$ means transpose. We also define the normalized item-item adjacency matrix as $\bar{R} = R^{T\bar{R}}R$.

**Matrix Factorization.** The most common CF paradigm is to learn latent features (also known as embedding vectors) to represent users and items. The dot product of user and item embedding vectors $\bar{e}_u^T \bar{e}_v$ approximates user $u$’s rating on item $i$, which is denoted as $r_{ui}$. Earlier CF models focused on low-rank matrix factorization [21], which aims to approximate the interaction matrix $R_{u,v}$. Singular value decomposition (SVD) was initially proposed to learn the feature matrices, followed by many other matrix factorization methods [5, 16, 24, 26, 33].

**Graph-based Methods.** From the perspective of the user-item interaction graph, the individual interaction history is equivalent to the first-order connectivity of the user. Thus, a natural extension is to mine the higher-order connectivity from the user-item graph structure. For example, the second-order connectivity of a user consists of similar users who have co-interacted with the same items. Fortunately, with the development and success of graph convolutional networks (GCNs) for modeling graph structure data in various machine learning areas, it recently became popular to adopt GCNs for CF [7, 9, 15, 27, 34, 38, 43].

Graph-based recommendation models have achieved remarkable results, but their efficiency remains unsatisfactory when confronted with large-scale recommendation scenarios. Therefore, improving the efficiency of graph-based methods while leaving high performance for recommendations has become a popular research question. Inspired by a simplified GCN (SGC) [41], LightGCN [15] outperformed NGCF [38] by removing the non-linear activation and feature transformation to improve both accuracy and efficiency. Its linear graph convolutional layer definition is as follows:

$$E(l + 1) = \bar{A}E(l),$$

where $E(0) \in \mathbb{R}^{|U| \times |V| \times D}$ is the learnable initial embedding matrix of users and items, $E(l)$ denotes the embedding matrix at $l$-th layer, and $\bar{A}$ is the normalized user-item adjacency matrix. LightGCN learns the initial embedding and uses the layer combination. The model prediction is defined as the dot product of the user’s and item’s final representation $\bar{e}_u^T \bar{e}_v$.

Very recently, researchers argued that linear GCN-based models resemble heat equations, which describe the law of thermal diffusive processes, i.e., Newton’s Law of Cooling [9, 40]. LT-OCF [9] redesigned LightGCN as a continuous diffusive process and outperforms LightGCN. LT-OCF also learns an optimal layer combination rather than relying on a pre-defined architecture. The heat equation is directly related to low pass filters and smoothness, which is one of the key operations in graph signal processing. GF-CF [27] was proposed from the perspective of the smoothness of graph signals. It is the special case of existing CF methods: the low-rank matrix factorization corresponds to the ideal low-pass filter, and LightGCN with infinitely embedding dimensionality corresponds to a first-order linear filter. Therefore, GF-CF proposed a simple model combining a linear filter and an ideal low-pass filter as follows:

$$\hat{R} = R(\hat{P} + \beta V^{-\frac{1}{2}}U^T \bar{V}^{-\frac{1}{2}}),$$

where $\hat{R}$ is an inferred interaction matrix, and $\hat{U}$ is the top-$k$ singular vectors of $\hat{R}$. GF-CF only needs matrix multiplication operations to calculate the recommendation scores thanks to its non-parametric architecture. Both LT-OCF and GF-CF use blurring processes with...
the heat equation and the low-pass filter, respectively, but no sharpening processes. In Table 1, we compare recent methods.

2.2 Diffusion Models

Fig. 1 (a) depicts the basic mechanism behind many diffusion models [10, 18, 19, 28–30]. The forward process is written as the following stochastic differential equation:

$$dx = f(x,t)dt + g(t)dw,$$

where $f(x,t) = f(t)x$, and its reverse SDE (i.e., backward process) is defined as follows:

$$dx = (f(x,t) - g^2(t)\nabla_x \log p_t(x))dt + g(t)dw,$$

where this reverse SDE process is a generative process. Depending on the types of $f$ and $g$, various sub-types of the diffusion model are defined.

In order to solve the reverse SDE, we need to know the gradient of the log-probability of the forward SDE (i.e., $\nabla_x \log p_t(x)$). We typically train a neural network, called score network, to approximate it (cf. Fig. 2). There exists a well-established theory for training the score network. After training the score network with the data, we formally compute it (cf. Fig. 2). There exists a well-established theory for training the score network. After training the score network with the data, we formally compute it (cf. Fig. 2). There exists a well-established theory for training the score network. After training the score network with the data, we formally compute it (cf. Fig. 2). There exists a well-established theory for training the score network. After training the score network with the data, we formally compute it (cf. Fig. 2).

2.3 Ordinary Differential Equations (ODEs)

The initial value problem (IVP) of ordinary differential equations can be written as follows:

$$x(T) = x(0) + \int_0^T f(x(t))dt,$$

where $x(0)$ is an initial value at time $t = 0$, and $f : \mathbb{R}^{\dim(x)} \rightarrow \mathbb{R}^{\dim(x)}$ is an ODE function describing the time-derivative of $x$, denoted $\frac{dx(t)}{dt}$. Therefore, integrating the time-derivative of $x$ until $t = T$ returns a solution $x(T)$ at time $t = T$.

$f$ is typically complicated in real-world applications and it is frequently impossible to find an analytical solution of $x(T)$. We then typically use ODE solvers, such as the Euler method, the Runge-Kutta method, the Dormand–Prince (DOPRI) method, and so on [11]. The Euler method is written as follows:

$$x(t + s) = x(t) + \tau \cdot f(x(t)),$$

where $\tau$ is a pre-configured step size.

Other ODE solvers use more complicated methods to update $x(t + \tau)$ from $x(t)$. For instance, the fourth-order Runge–Kutta (RK4) method uses the following method:

$$x(t + \tau) = x(t) + \frac{\tau}{6} \left(f_1 + 2f_2 + 2f_3 + f_4\right),$$

where $f_1 = f(x(t))$, $f_2 = f(x(t) + \frac{\tau}{2}f_1)$, $f_3 = f(x(t) + \frac{\tau}{2}f_2)$, and $f_4 = f(x(t) + \tau f_3)$.

In order to solve the above integral problem, therefore, we need to iterate one of the fixed-step ODE solvers $\lceil T/\tau \rceil$ times since each iteration updates $x(t)$ to $x(t + \tau)$. However, the DOPRI method is an adaptive solver, which dynamically adjusts the step-size $\tau$ depending on estimated potential errors. Therefore, the number of iterations is not deterministic for DOPRI. In general, DOPRI is considered one of the most advanced solvers. These solvers are already implemented on many deep learning platforms, such as PyTorch and TensorFlow. We test all those solvers for our experiments.

3 PROPOSED METHOD

We describe our BSPMs for CF, which consists of a blurring process and a sharpening process. Our method is greatly inspired by the recent successes of diffusion models for deep generative tasks. In fact, there already exists a research trend to use generative models for CF due to the similarity in them [3, 4, 6, 31, 35–37] — revealing hidden interactions between users and items means that we generate new interactions.

1In the case of neural ordinary differential equations (NODEs), $f$ is approximated by a neural network, which means the time-derivative of $x$ is learned from data [node]. In this paper, however, $f$ is not a neural network but a blurring/sharpening function.
3.1 Overall Workflow

Our overall workflow is as simple as i) applying a continuous blurring process to the interaction matrix $R$ to derive its blurred matrix $B(T_k)$, and then ii) applying a continuous sharpening process to the blurred matrix to derive its sharpening matrix $S(T_k)$. After these processes, we can recommend items as we will shortly describe.

We also make it clear that in our method, there does not exist anything to train. During the processes, neural networks are not used at all and we do not learn user/item embedding vectors. The blurring and sharpening functions are all hand-crafted functions (without any trainable parameters) in our method. Therefore, our process is surprisingly simple and the overall computation can be done quickly. However, our method outperforms all existing popular methods by non-trivial margins.

**Meaning of Blurring.** The blurring process is a core of CF. Many graph-based CF methods use graph convolutional filters that correspond to blurring processes [1, 15, 27]. In general, popular items are recommended to users after this process.

**Meaning of Sharpening.** The sharpening process is an inverse of the blurring and therefore, it retrieves user-specific items — for general GCNs, similar sharpening processes are used to emphasize differences among node features [2, 8]. As we will show in our experiment section, it actually increases the overall recommendation accuracy while mostly decreasing the degree of recommended items. In other words, less popular items are also recommended to users in conjunction with popular items.

3.2 Blurring Process

Blurring processes can be written as follows and solved by the ODE solvers we reviewed in Sec. 2.3:

$$B(T_k) = B(0) + \int_0^{T_k} b(B(t))dt,$$

where $b : \mathbb{R}^{\text{dim}(B)} \to \mathbb{R}^{\text{dim}(B)}$ is a blurring function which approximates $\frac{dB(t)}{dt}$ and $B(0)$ is an interaction matrix $R$ in our setting. Therefore, $B(1)$ means a blurred interaction matrix.

The exact blurring process depends on how we define the function $b$. In various domains, similar blurring functions have been defined for various purposes. We introduce some key definitions among them that are widely used in various domains.

We first articulate that all the aforementioned notations can be naturally extended after considering the temporal nature of our proposed blurring-sharpening process. For instance, $B(t)$ means a blurred matrix of the original interaction matrix after $t$ following our proposed blurring process if $B(0) = R$.

**Heat equation.** The heat equation means the Newton’s law of cooling, which describes the rate of heat loss in a body. This concept is frequently used in image processing for blurring images. We use the following definition of $b$:

$$b_{\text{HE}}(B(t)) = k B(t)(\hat{P} - I),$$

where $k \in \mathbb{R}$ is a coefficient called heat capacity. This is a hyperparameter in our framework. This definition of $b$ has a resemblance to the low-pass filter in the field of graph convolutions.

3.3 Sharpening Process

Sharpening processes can also be written as follows and solved by the ODE solvers we reviewed in Sec. 2.3:

$$S(T_k) = S(0) + \int_0^{T_k} s(S(t))dt,$$

where $s : \mathbb{R}^{\text{dim}(S)} \to \mathbb{R}^{\text{dim}(S)}$ is a sharpening function which approximates $\frac{dS(t)}{dt}$, and $S(1)$ is a sharpened matrix from the input matrix $S(0)$. The sharpening function $s$ can be defined as follows:

$$s(S(t)) = -S(t)\hat{P},$$

where the negative sign is added to emphasize the difference from neighbors, i.e., sharpening.

3.4 Blurring-Sharpening Process Model (BSPM)

Using the blurring and sharpening processes, we can define a couple of variations of BSPM. In BSPM-LM in Fig. 3 (b), we use the two blurring processes but apply the sharpening process only to the heat equation-based blurring outcome. We then merge the sharpened interaction matrix with the matrix perturbed by the ideal low-pass filter. This variant can be written as follows and solved by the ODE
We already showed that GF-CF in Eq. (12) is a special case of BSPM. We note that our proposed BSPM does not include any training we do not learn any embedding vectors but directly process the interaction matrix. Adding $B_{HE}(T_b)$ to $\hat{R}$ is optional in our method and is called as residual connection.

In BSPM-EM in Fig. 3 (c), we merge the heat equation-based and the ideal low-pass filter-based blurring outcomes as early as before the sharpening process begins. We then apply the sharpening process. This can be written as follows and solved by the ODE solvers we reviewed in Sec. 2.3:

\[
\begin{align*}
B_{HE}(T_b) &= B(0) + \int_0^{T_b} b_{HE}(B(t)) dt, \\
B_{IDL}(T_b) &= B(0) + \int_0^{T_b} b_{IDL}(B(t)) dt, \\
S(T_b) &= S(0) + \int_0^{T_b} s(S(t)) dt, \\
\hat{R} &= \begin{cases} 
S(T_b) + B_{IDL}(T_b) + B_{HE}(T_b), & \text{if residual,} \\
S(T_b) + B_{IDL}(T_b), & \text{otherwise,}
\end{cases}
\end{align*}
\]

where $B(0) = R$, $S(0) = B_{HE}(T_b)$, and $\hat{R}$ is an inferred interaction matrix. Adding $B_{HE}(T_b)$ to $\hat{R}$ is optional in our method and is called as residual connection.

### 3.5 Direct Inference without Training

We note that our proposed BSPM does not include any training phase, which drastically reduces the total computation time. Since we do not learn any embedding vectors but directly process the interaction matrix $R$, there is no training process. Solving Eq. (15) or (16) is enough to infer unknown user-item interactions, and there exist many ODE solvers which can solve Eqs. (15) and (16) efficiently. As a matter of fact, our method is one of the fastest CF methods. In addition, our method shows the best accuracy in almost all cases for our experiments.

### 3.6 Comparison with Other Methods

We already showed that GF-CF in Eq. (12) is a special case of BSPM. We will show that other popular CF algorithms are also special cases of our model: i) LightGCN is one of the most influential algorithms for linear graph-based CF. Shen et al. [27] already proved that LightGCNs with infinite-dimensional embeddings are theoretically the same as a one-step heat equation process, which is equivalent to our blurring process with $T_k = 1$, the Euler method with a step size of 1, and $k = 1$. LightGCN also does not have any sharpening processes. ii) LT-OCF is a continuous generalization of LightGCN. Therefore, our ODE-based blurring process with the heat equation conceptually corresponds to the key idea of LT-OCF although it has several other contributions as well. No sharpening processes are used in LT-OCF. iii) It is obvious that GF-CF is equivalent to the blurring process with $b_{GF-CF}$ and the Euler method of $T_b = s = 1$ to solve it.

### 4 EXPERIMENTS

In this section, we describe our experimental environments and results. The following software and hardware environments were used for all experiments: Ubuntu 18.04 LTS, Python 3.6.6, PyTorch 1.9.0, NumPy 1.18, SciPy 1.5, sparsesvd 0.2.2, torchdiffeq 0.2.2, CUDA 11.4, and NVIDIA Driver 470.42, and i9 CPU, and NVIDIA RTX A6000.

#### 4.1 Experimental Environments

##### 4.1.1 Datasets and Baselines.

In our experiments, we use the three benchmark datasets that are the most frequently used in the literature: Gowalla, Yelp2018, and Amazon-book [7, 15, 38]. We summarize the dataset statistics in Table 3. Fig. 4 shows the long tail characteristic of the datasets. We compare our proposed BSPM with the following baseline models:

1. MF-BPR [26], Neu-MF [16], CMN [12], HOP-Rec [42], GC-MC [34], Multi-VAE [22], GRMF [25], ENMF [5], NGCF [38], DGCF [39] are all popular methods for CF.
2. NIA-GCF [32] explicitly models the relational information between neighbor nodes and exploits the heterogeneous nature of the user-item bipartite graph.
3. LR-GCCF [7] and LightGCN [15] remove feature transformation and non-linear activations of NGCF.
widely used ranking metrics: Recall@20 and NDCG@20. All items process and RK4 for the sharpening process.

Figure 4: The long-tail characteristic in all datasets.

(4) UltraGCN [23] is a simplified CF that skips infinite layers of message passing for an efficient recommendation.

(5) LT-OCF [9] extends linear GCNs based on ordinary differential equations.

(6) GF-CF [27] achieves competitive or better performance against deep learning-based methods.

(7) LinkProp [13] proposes a new linkage score for link prediction on a bipartite graph.

4.1.2 Evaluation Metrics and Hyperparameters. We adopt the two widely used ranking metrics: Recall@20 and NDCG@20. All items that do not have any interactions with a user are recommendation candidates for the user. To keep the comparison fair with previous studies, we use the same datasets and the same train/test splits.

For other baselines, we use the recommended hyperparameters and for our method, we test the following hyperparameters:

- For solving the integral problems of the blurring/sharpening processes, we consider the following ODE solvers: the Euler method, RK4, and DOPRI. However, we found that RK4 and DOPRI produce almost the same results in our preliminary experiments so we test only the Euler method and RK4.
- For the blurring process, the number of steps \( \frac{T_k}{T} \) for solvers is in \([1,2,3,4]\), and the terminal time \( T_b \) is set to 1 to 5.
- For the sharpening process, the number of steps \( \frac{T_k}{T} \) is in \([1,2,3,4]\), and the terminal time \( T_s \) is set to 1 to 5.
- The size of \( \beta \) is in \([0.0,0.1,\ldots,1.0]\).
- The heat capacity \( k \) is in \([0.1,\ldots,1.0]\).

Among the test configurations, the best configuration set in each data is as follows: In Gowalla, \( \frac{T_b}{T} = 1, T_b = 1, \frac{T_s}{T} = 1, T_s = 2.4 \), the heat capacity \( k = 1.0 \), and \( \beta = 0.21 \). In Yelp2018, \( \frac{T_b}{T} = 1, T_b = 1, \frac{T_s}{T} = 1, T_s = 3.2, \) the heat capacity \( k = 1.0 \), and \( \beta = 0.3 \). In Amazon-book, \( \frac{T_b}{T} = 1, T_b = 1, \frac{T_s}{T} = 2, T_s = 2.2 \), the heat capacity \( k = 1.0 \), and \( \beta = 0 \). It is the best to use the Euler method for the blurring process and RK4 for the sharpening process.

4.2 Experimental Results

In Table 4, we summarize the overall accuracy in terms of Recall and NDCG. Our specific choices of baselines cover almost all representative CF methods, and the three datasets are widely used in the literature. As reported, our method clearly marks the best accuracy in all cases. In particular, BSPM-LM is the best method for Amazon-book, and BSPM-EM is the best method for Yelp2018 and Gowalla. In many cases, BSPM-LM and BSPM-EM mark the best and the second-best methods respectively, and their differences are not significant. LT-OCF also works well for Gowalla. GF-CF marks second place in Yelp2018.

Among the tested baselines, UltraGCN, LinkProp, LT-OCF, and GF-CF work well in some cases. However, only LinkProp is comparable to our method for Gowalla and Amazon-book — however, the accuracy gap between our method and LinkProp is still non-trivial. For Yelp2018, GC-CF shows good scores. However, there are no existing methods that are comparable to our proposed method in all datasets. Therefore, we consider that our proposed concept of BSPM opens a new era of CF.

LightGCN is worse than recent methods such as LT-OCF and GF-CF, but its value is that it is the first method showing that simple linear convolutions work better than non-linear ones. Being inspired by it, many methods have been proposed, including ours. Our blurring and sharpening processes are all linear operations. One can adopt non-linear sharpening processes, but in general, linear operations show reliable recommendations.
4.3 Ablation and Sensitivity Studies

We report some selected key sensitivity and ablation study results. It is the case that our model is not significantly sensitive to a hyperparameter if not reported in this subsection. In general, our model is sensitive to the hyperparameters of the sharpening process and we focus on them.

4.3.1 Sensitivity on $T_s$. By varying the terminal integral time $T_s$ of the sharpening process, we investigate how the model accuracy changes in Fig. 5. For Gowalla and Amazon-book, $T_s$ around 2.4 produces the best outcomes. After a certain point, however, the model accuracy drastically decreases in all datasets. It is obvious that applying the sharpening too much (i.e., $T_s$ is too large) is not helpful in the perspective of CF since the sharpening process emphasizes user-specific information (rather than collaborative information).

4.3.2 Sensitivity on $\tau$. By varying the number of steps $\tau$ for ODE solvers, we test our model's performance in varying $\tau$ in Table 4, depending on the used sharpening function type. As shown, they do not show reliable performance in comparison with our main model.

4.3.3 Ablation study. As ablation study models, we test the models with only a blurring process, denoted 'Only Blurring (HE)', 'Only Blurring (IDL)', and 'Only Blurring (GF-CF)' in Table 4, depending on the number of steps $\tau$. For other datasets, it is best not to use the residual connection.

4.4 Efficacy of the Sharpening Process

As mentioned earlier, we are the first proposing to use the sharpening process for CF. Therefore, we conduct in-depth analyses on how the sharpening process contributes. Other metrics for CF to measure the quality of recommendation, called beyond-accuracy metrics. For these analyses, we use the following beyond-accuracy metrics: novelty [44] and item coverage [17]. The item coverage refers to the extent of items a recommender system can predict. A novel item for a user is one the user has little or no knowledge about it [14].

The novelty measures the unexpectedness of recommended items relative to their global popularity. Using these metrics provides a broader picture of the contribution by the sharpening process. The results are presented in Figs. 8 (b), (d), (f), the blurring-sharpening process.

In general, the blurring process can be considered as a collaborative step, where a user’s interactions with items are mixed with its neighbors.
We compare BSPM-EM and BSPM-LM with and without the sharpening process in terms of Hits@20 to see how the sharpening process changes recommended items. For all datasets, more items are accurately recommended after the sharpening process. Hits@20 increases from 1.13 to 1.16 for Gowalla, from 0.63 to 0.65 for Yelp2018, and from 0.58 to 0.60 for Amazon-book. Fig. 9 shows case studies for several users on Amazon-book. After the sharpening process, a few more items are accurately recommended for users A and B.

Interestingly, those additional hits, highlighted in orange in Fig. 9, after the sharpening process has relatively low node degrees. After the sharpening procedure, six correct recommendations (i.e., hits) are added for user B in Fig. 9 (b). When only the blurring process was performed, the recommended items’ degrees are as high as 1389 and 466, while the degrees of the extra hits after the sharpening process are as low as less than 85, i.e., those additional hits are not popular items but specific to the user. On Amazon-book, the average node degree of hits is 48.20 when only the blurring process is used, but it drops to 33.10 when the sharpening process is added. The overall histograms of degrees are in Fig. 10, and we can observe similar patterns. Fig. 10 shows that the ratio of hits with low item degree increases after sharpening process. These results suggest that the model with the sharpening process accurately recommends less popular but user-specific items.

The long-tail problem, i.e., how to recommend more user-specific items, is a major challenge in the recommender system. Benchmark data also has long-tail characteristics, as shown in Fig. 4, making it difficult to recommend user-specific items. Our case study results show that the sharpening process allows for user-specific item recommendations while also improving accuracy.
5 CONCLUSIONS & FUTURE WORK

We presented a novel paradigm of blurring-sharpening process model (BSPM) for CF. Our work is greatly inspired by the recent two research breakthroughs: i) CF-CF and ii) diffusion models for generating fake images. As in diffusion models, we adopt the perturbation-recovery paradigm to discover new information. As in GF-CF, we do not learn embedding vectors but directly process the interaction matrix. After defining our BSPM paradigm, we also design a couple of variants to enhance the recommendation accuracy further. In addition, our BSPM is one of the fastest methods ever designed for CF since it does not include any training phase but directly infers unknown user-item interactions. In our experiments with 19 baselines and 3 benchmark datasets, our method marks the best accuracy by large margins only except in one case. Since our method is not only the most accurate but also one of the fastest methods, it has a significant impact on real-world CF applications.

In the future, we hope that it can be further improved by adopting better blurring and sharpening processes since we have focused on designing the overall architecture by customizing popular filters.

REFERENCES

[1] Muhammet Balcilar, Guillaume Renton, Pierre Héroux, Benoit Gaüzère, Sébastien Adam, and Paul Honeine. 2021. Analyzing the Expressive Power of Graph Neural Networks in a Spectral Perspective. In ICLR.

[2] Deyu Bo, Xiaowei Wang, Chuan Shi, and Huawei Shen. 2021. Beyond Low-frequency Information in Graph Convolutional Networks. In AAAI.

[3] Dong-Kyu Chae, Jin-Soo Kang, Sang-Wook Kim, and Jaeho Choi. 2019. Rating Augmentation with Generative Adversarial Networks towards Accurate Collaborative Filtering. In TheWebConf (former WWW).

[4] Dong-Kyu Chae, Jin-Soo Kang, Sang-Wook Kim, and Jung-Tae Lee. 2018. CFGAN: A Generic Collaborative Filtering Framework Based on Generative Adversarial Networks. In CIKM.

[5] Chong Chen, Min Zhang, Yongfeng Zhang, Yiqun Liu, and Shaoping Ma. 2020. Efficient Neural Matrix Factorization without Sampling for Recommendation. ACM Trans. Inf. Syst. 38, 2, Article 14 (2020), 28 pages.

[6] Honglong Chen, Shuai Wang, Nan Jiang, Zhe Li, Na Yan, and Leyi Shi. 2021. Trust-aware generative adversarial network with recurrent neural network for recommender systems. International Journal of Intelligent Systems 36, 2 (2021), 778–795.

[7] Lei Chen, Le Wu, Richang Hong, Kun Zhang, and Meng Wang. 2020. Revisiting Graph-based Collaborative Filtering. A Linear Residual Graph Convolutional Network Approach. In AAAI.

[8] Eli Chien, Jianhao Peng, Pan Li, and Olgica Milenkovic. 2021. Adaptive Universal Generalized PageRank Graph Neural Network. In ICLR.

[9] Jiequn Chen, Jinming Yeon, and Neseong Park. 2021. LT-OCF: Learnable-Time ODE-based Collaborative Filtering. In CIKM.

[10] Prafulla Dhariwal and Alexander Nichol. 2021. Diffusion Models Beat GANs on Image Synthesis. In NeurIPS.

[11] J.R. Dormand and P.J. Prince. 1980. A family of embedded Runge-Kutta formulae. J. Comput. Appl. Math. 6, 1 (1980), 19 – 26.

[12] Travis Ebesu, Bin Shen, and Yi Fang. 2018. Variational Autoencoders for Collaborative Filtering. In TheWebConf (former WWW).

[13] Jonathan Ho, Ajay Jain, and Pieter Abbeel. 2020. Denoising Diffusion Probabilistic Models. In NeurIPS.

[14] Mouzhi Ge, Carla Delgado-Battenfeld, and Dietmar Jannach. 2010. Beyond Matrix Factorization for Recommender Systems. Computer 42, 8 (2009), 36–37.

[15] Dawen Liang, Rahul G. Krishnan, Matthew D. Hoffman, and Tony Jebara. 2018. Variational Autoencoders for Collaborative Filtering. In TheWebConf (former WWW).

[16] Tae Yong Kong, Taeri Kim, Jinsung Jeon, Yeon-Chang Lee, Neseong Park, and Sang-Wook Kim. 2022. Linear, or Non-Linear, That is the Question! In WSDM. 517–525.

[17] Y. Koren, R. Bell, and C. Volinsky. 2009. Matrix Factorization Techniques for recommender systems. In KDD, Vol. 20.

[18] Y. Koren, R. Bell, and C. Volinsky. 2009. BPR: Bayesian Personalized Ranking from Implicit Feedback. In IAT.

[19] Jiaying Mao, Jingming Zhu, Xiaoyu Xiao, Miaofeng Wang, and Xiuqiang He. 2021. UltraGCN: Ultra Simplification of Graph Convolutional Networks. In CIKM.

[20] Xiaofeng Mao, Han Liu, Meng Liu, Zhaochun Ren, Tian Gan, and Liqiang Nie. 2020. LARA: Attribute-to-Feature Adversarial Learning for New-Item Recommendation. In recsys.

[21] Jiayao Sun, Yingxue Zhang, Wei Guo, HuiFeng Guo, Ruiming Tang, Xiuqiang He, Chen Ma, and Mark Coates. 2020. Neighbor Interaction Aware Graph Convolutional Networks for Recommendation. In Sigr.

[22] Y. Tai, Lui Anh Tuan, and Su Jeong-Hui. 2018. Latent Relational Metric Learning via Memory-based Attention for Collaborative Ranking. In TheWebConf (former WWW).

[23] Rianne van den Berg, Thomas N. Kipf, and Max Welling. 2017. Graph Convolutional Matrix Completion. In KDD.

[24] Yiqi Wang, Ying Xiong, and Ming-Feng Tsai. 2018. GraphGAN: Graph Representation Learning with Generative Adversarial Nets. In AAAI.

[25] Yang Song, Conor Durkan, Ian Murray, and Stefano Ermon. 2020. Maximum Likelihood Training of Score-Based Diffusion Models. In NeurIPS.

[26] Yang Song and Stefano Ermon. 2020. Improved Techniques for Training Score-Based Generative Models. In NeurIPS.

[27] Yang Song, Jiascha Sohl-Dickstein, Diederik P Kingma, Abhishek Kumar, Stefano Ermon, and Ben Poole. 2021. Score-Based Generative Modeling through Stochastic Differential Equations. In ICLR.

[28] Yang Song and Stefano Ermon. 2021. Improved Techniques for Training Score-Based Generative Models. In NeurIPS.

[29] Yang Song and Stefano Ermon. 2020. Improved Techniques for Training Score-Based Generative Models. In NeurIPS.

[30] Yang Song, Jascha Sohl-Dickstein, Diederik P Kingma, Abhishek Kumar, Stefano Ermon, and Ben Poole. 2021. Score-Based Generative Modeling through Stochastic Differential Equations. In ICLR.

[31] Xiaofeng Sun, Jingfeng Zhang, Yuejun Hung, Yiqin Li, and Shaoping Ma. 2020. Efficient Neural Matrix Factorization without Sampling for Recommendation. ACM Trans. Inf. Syst. 38, 2, Article 14 (2020), 28 pages.

[32] Jianing Sun, Yingxue Zhang, Wei Guo, HuiFeng Guo, Ruiming Tang, Xiuqiang He, Chen Ma, and Mark Coates. 2020. Neighbor Interaction Aware Graph Convolutional Networks for Recommendation. In Sigr.