A TEST OF THE EQUIVALENCE PRINCIPLE BY LONG-BASELINE NEUTRINO-OSCILLATION EXPERIMENTS

Kazuhito Iida, Hisakazu Minakata and Osamu Yasuda

Department of Physics, Tokyo Metropolitan University
1-1 Minami-Osawa Hachioji, Tokyo 192-03, Japan

Abstract

We show that a breakdown of the universality of the gravitational couplings to different neutrino flavors can be tested in long-baseline neutrino-oscillation experiments. In particular we have analyzed in detail a proposed experiment at SOUDAN 2 with $\nu_\mu$ beams from the Fermilab Main Injector. It turns out that we can study both masses of neutrinos and such a breakdown with sensitivity to the order of $10^{-14}$ by investigating the energy spectrum of the resulting muons.
It has been pointed out\textsuperscript{1,2} that a breakdown of the universality of the gravitational couplings to different neutrino flavors could lead to neutrino oscillations. In particular the authors of Ref. 2 studied the possibility in which the solar neutrinos (see, e.g., Ref. 3) can be used to test this kind of breakdown of the universality. Since the flux of the solar neutrinos is relatively small and the energy spectrum is beyond our control, the utility of the solar neutrino for this purpose is limited. In this paper we propose a possible long-baseline experiments of neutrino oscillations to test the breakdown of the universality of the gravitational couplings to neutrinos. In case of the long-baseline experiments with an accelerator, we have a larger flux than that of solar neutrinos and in principle we can change the energy spectrum of the neutrino beams, and therefore more information, if any, can be obtained on neutrino oscillations. As we will see, the breakdown of the universality of the gravitational couplings to neutrinos of different flavors leads to a violation of Einstein’s equivalence principle (see, e.g., Ref. 4) which states that all the laws of physics must take on their familiar special-relativistic forms in any and every local Lorentz frame, anywhere and any time in the universe. In our case, we show that we can probe the magnitude of the breakdown of Einstein’s equivalence principle to the order of $10^{-14}$, assuming that there are neutrino mixings. Among various experiments to test the equivalence principle (see e.g., Ref. 5 for a review), there have been few tests of Einstein’s equivalence principle for neutrinos\textsuperscript{6}. The universality of the gravitational couplings that we study in this paper is of different type from these experiments in the past, so our discussions here are complementary to them.

In this paper we assume that there are two neutrino flavors which have different couplings to gravity and that the eigenstates of these different gravitational couplings do not coincide with those of the electroweak flavors. Throughout the present discussions we consider neutrino oscillations between two flavors for simplicity. Let us start with the following Lagrangian of two
kinds of neutrinos

\[ \mathcal{L} = e(G_1) \nabla_1 [i e^{a \mu} (G_1) \gamma_a D_\mu (G_1) - m_1] \nu_1 \\
+ e(G_2) \nabla_2 [i e^{a \mu} (G_2) \gamma_a D_\mu (G_2) - m_2] \nu_2 \\
+ \text{(interaction terms with electroweak gauge bosons)}, \tag{1} \]

where we have included mass terms to keep generality, \( e^{a \mu} (G_i) \) \((i = 1, 2)\) are the vierbein fields of some background metric with different Newton constants \( G_i \) \((i = 1, 2)\), and \( e(G_i) \equiv \det e^{a \mu} (G_i) \). For simplicity we assume that the eigenstates of the gravitational couplings coincide with those of the masses. Notice that even if these neutrinos are massless, we cannot rotate these two fields so that these are the eigenstates of the electroweak theory, since the gravitational coupling terms are not invariant under the rotation in the flavor space. Note also that each term in (1) is consistent with local Lorentz invariance, general covariance, and the CPT invariance, as the gravitational couplings to particles and anti-particles are the same. Since the gravitational couplings for these two kinds of neutrinos are different, even if we choose a coordinate system in which the Dirac equation for \( \nu_1 \) in (1) becomes the one in a flat space-time, the Dirac equation for \( \nu_2 \) in the same coordinate system does not necessarily do so. Thus Einstein’s equivalence principle is violated in (1). The situation here is similar to that of the gauge theory where the gauge invariance is explicitly broken, and physics does depend on which gauge we choose. So we are forced to choose one particular coordinate system from which we start. The most natural choice in our case seems to be the coordinate system which is at rest on the Earth. This is because neutrinos which we observe are created from the accelerator and annihilated near the detector, and both equipments are fixed on the Earth. So we take the coordinate system which are moving together with the Earth, and we choose as our background the so-called interior Schwarzschild metric (see, e.g., chapter 11.7 in Ref. 7) whose curvature is entirely caused by the
gravitational field due to the Earth. We will discuss the issue of the choice of the coordinate systems again later.

The configuration of the long-baseline experiment we will discuss is depicted in Fig. 1, and the neutrino beams go underneath the ground along the geodesics. First let us consider the Dirac equation of left-handed neutrinos without any flavor in the interior Schwarzschild background:

\[(i e^a \gamma_a D_\mu - m)\psi = 0, \quad (2)\]

where \(e^a_\mu\) is the vierbein of the interior Schwarzschild metric

\[ds^2 = (e^0_t)^2 dt^2 - (e^1_r)^2 dr^2 - (e^2_\theta)^2 d\theta^2 - (e^3_\varphi)^2 d\varphi^2 \quad (3)\]

and is given by

\[e^0_t = \frac{3}{2} \sqrt{1 - \frac{\alpha}{R}} - \frac{1}{2} \sqrt{1 - \frac{\alpha r^2}{R^2}}, \quad e^1_r = \frac{1}{\sqrt{1 - \frac{\alpha r^2}{R^2}, \quad e^2_\theta = r, \quad e^3_\varphi = r \sin \theta.} \quad (4)\]

\[D_\mu \psi = (\partial_\mu - \frac{1}{2} \omega_{\mu ab} \sigma^{ab}) \psi\]

is the covariant derivative acting on a spinor \(\psi\), \(\omega_{\mu ab}\) is the spin connection given by \(e^b_\nu \omega^a_{\mu b} = \partial_\mu e^a_\nu\), and \(\alpha\) in (4) is the Schwarzschild radius.

One typical dimensionless parameter in our case is \(ER\), where \(E\) is the energy of the neutrino, and \(R\) is the radius of the Earth. For \(E=10\) GeV and \(R=6,400\) Km, \(ER \sim 3 \times 10^{23}\), and derivative terms in the spin connections are all of the order of \(1/ER\), so we will neglect them throughout this paper. In this approximation the Dirac equation becomes

\[\left[i(e^0_t)^{-1} \gamma^0 \frac{\partial}{\partial t} + i(e^1_r)^{-1} \gamma^1 \frac{\partial}{\partial r} + i(e^2_\theta)^{-1} \gamma^2 \frac{\partial}{\partial \theta} + i(e^3_\varphi)^{-1} \gamma^3 \frac{\partial}{\partial \varphi} - m\right] \psi = 0 \quad (5)\]

Since we consider neutrinos in the ultra relativistic limit \(E \gg m\), we have only to discuss (3) along the geodesics for massless fields. The geodesics of the interior Schwarzschild metric is given by

\[\frac{R^2 \cos^2 \delta}{r^2} = \sin^2 \varphi + \frac{3\alpha}{2R} (\cos^2 \delta - \sin^2 \varphi) \quad (6)\]
to the order of $\alpha/R$, where $\delta$ is half of the angle $\angle AOB$ in Fig. 1. We solve (5) on the plane $\theta = \pi/2$, and we remove the time dependence by $\psi(\vec{x}, t) = e^{-iEt} \chi(\vec{x})$. On the geodesics (6), (5) becomes

$$\left[ (e^0_1)^{-1}E\gamma^0 - m + \frac{1}{1+w^2} \left( (e^1_1)^{-1}w\gamma^1 + \gamma^3 \right) \frac{i}{r} \frac{d}{d\varphi} \right] \chi = 0,$$

where $w \equiv d\ln r(\varphi)/d\varphi$, $r = r(\varphi)$ is defined in (5), and we have taken only the tangential component into consideration. Following the convention of the Dirac matrices by Bjorken and Drell, and denoting $\chi^T \equiv (\chi_1, \chi_2, \chi_3, \chi_4)$, it is easy to show that (7) can be rewritten as

$$\frac{1}{ir} \frac{d}{d\varphi} \left( \chi_1 + i\chi_2 \right) = \sqrt{(e^0_1)^{-2}E^2 - m^2} \frac{1+w^2}{1+(e^1_1)^{-2}w^2} U \sigma_3 U^{-1} \left( \chi_1 + i\chi_2 \right),$$

where $U$ is a certain $2 \times 2$ matrix. It turns out that a derivative term $r^{-1}dU^{-1}/d\varphi$ is of order $1/ER$ which is extremely small, and hence we get

$$\frac{1}{ir} \frac{d}{d\varphi} \left( \nu \right) = \sqrt{(e^0_1)^{-2}E^2 - m^2} \frac{1+w^2}{1+(e^1_1)^{-2}w^2} \sigma_3 \left( \nu \right),$$

where $(\nu, \bar{\nu}) \equiv (\chi_1 + i\chi_2, \chi_3 - i\chi_4)(U^{-1})^T$. $\nu$ and $\bar{\nu}$ correspond to the forward-going and the backward-going energy solutions, and we will consider only the forward-going solution in the following. $\alpha$ in eq. (4) is the Schwarzschild radius of the Earth which is about 9 mm, so we expand (9) to the first order in $\alpha/r$. Thus, in the ultra relativistic limit $E \gg m$, we obtain

$$\frac{1}{i} \frac{d\nu}{dx} = B \left[ 1 - \frac{m^2}{2E^2} + \frac{\alpha}{R} \left( \frac{3}{2} - \cos^2 \delta - 2x^2 \cos^2 \delta \right) \right] \nu,$$

where we have defined a variable $x \equiv \tan(\varphi - \pi/2)$, and $B \equiv ER \cos \delta$ is a very large number. Now let us go back to the Lagrangian (1) with two kinds of neutrinos. This time we assume that the vierbein fields in the Lagrangian (1) are those of the interior Schwarzschild background (4). From the previous
discussion, it is straightforward to see that the Dirac equation for (1) is given by
\[
\frac{1}{i} \frac{d}{dx} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = B \left[ 1 - \frac{m_2^2 + m_1^2}{4E^2} + (f_1 + f_2) \Phi \left( \frac{3}{2} - \cos^2 \delta - 2x^2 \cos^2 \delta \right) + \Delta(x) \sigma_3 \right] \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix},
\]
(11)
where
\[
\Delta(x) \equiv \Delta m^2 \left( 1 - \frac{m_2^2 - m_1^2}{4E^2} + \frac{\Delta f \Phi}{3} \cos^2 \delta - 2x^2 \cos^2 \delta \right).
\]
(12)
Here \( \Delta m^2 \equiv m_2^2 - m_1^2 \) is the difference of the masses, we have defined the Newton potential \( \Phi \equiv -GM/R \) on the surface of the Earth, and we have also defined the difference \( \Delta f = f_2 - f_1 \) of the dimensionless gravitational couplings of the two neutrino species
\[
\begin{pmatrix} f_1 \Phi \\ f_2 \Phi \end{pmatrix} = -\begin{pmatrix} \alpha_1/2R \\ \alpha_2/2R \end{pmatrix} = -\begin{pmatrix} G_1M/R \\ G_2M/R \end{pmatrix}.
\]
(13)
The equation (11) can be easily integrated from \( x = -\tan \delta \) to \( x = \tan \delta \).

Now let us introduce the flavor eigenstates \( \nu_a, \nu_b \) of the weak interaction by
\[
\begin{pmatrix} \nu_a \\ \nu_b \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}.
\]
(14)
Then the probability of detecting a different flavor \( \nu_b \) at a distance \( L \) after producing one neutrino flavor \( \nu_a \) is given by
\[
P(\nu_a \rightarrow \nu_b) = \sin^2 \theta \sin^2 \left[ \left( \frac{\Delta m^2}{4E^2} + \frac{\Delta f \Phi}{\alpha_2/2R} \right) \left( 1 + \frac{L^2}{6R^2} \right) \right] E L,
\]
(15)
where \( L \equiv 2R \sin \delta \) is the distance AB in Fig. 1. This formula applies to the transition between \( \nu_\mu \) and \( \nu_\tau \), where no MSW effect \(^9\) is expected to occur.

In case of the transition between \( \nu_e \) and \( \nu_\mu \), we have to take the MSW effect \(^9\) into consideration, and the Dirac equation is modified as
\[
\frac{1}{i} \frac{d}{dx} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = B \left( \begin{pmatrix} \Delta(x) \cos 2\theta - \frac{G_F N_e}{\sqrt{2}E} \\ \Delta(x) \sin 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \right),
\]
(16)
where $G_F$ is the Fermi coupling constant, and $N_e$ is the density of electrons in the Earth. (16) can be solved in the same way as before by introducing the variables

$$\Delta_N(x) \cos 2\theta_N = \Delta(x) \cos 2\theta - \frac{G_F N_e}{\sqrt{2} E}$$

$$\Delta_N(x) \sin 2\theta_N = \Delta(x) \sin 2\theta. \quad (17)$$

Note that $\theta_N$ does depend on the variable $x$ in this case. It is easy to integrate (17), and we have the transition probability of detecting $\nu_e$ at a distance $L$ from the source of $\nu_\mu$ beams

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta_N(x = \tan \delta) \sin^2 \left( \int_{-\tan \delta}^{\tan \delta} dx \Delta_N(x) \right), \quad (18)$$

where we have used the fact $\theta_N(x = \tan \delta) = \theta_N(x = -\tan \delta)$ and $\Delta_N, \theta_N$ are defined through (17). We have performed the integration in the exponent in (18) numerically.

Here we would like to comment again on the dependence of our results on the choice of the coordinate systems. As we mentioned earlier, equations (9) and (11) depend on how we choose a coordinate system, since the Newton potential terms in (9) and (11) are changed if we switch to a coordinate system which moves with acceleration relative to the Earth. In Ref. 10 it was argued that one could derive stronger bound on a breakdown of the equivalence principle by using the contribution to the Newton potential from the supergalactic cluster. However, this argument holds only when one assumes the Lagrangian (11) in the coordinate system which is at rest in the supergalactic cluster. Such a coordinate system is different from our coordinate system, i.e., a different choice of gauge is taken in Ref. 10. Throughout this paper we take the background (4), and we make our analyses below using this choice of the background. Since any other contribution to the Newton potential is negative definite, the absolute value of $\Phi$ in (14) with our ansatz is the
smallest among all possibilities. Hence our choice gives the most conservative bound on $\Delta f$.

Let us now consider a proposed long-baseline neutrino-oscillation experiment which will be performed with $\nu_\mu$ beams from the Fermilab Main Injector\textsuperscript{11}. Our discussions here are analogous to those by Bernstein and Park\textsuperscript{12}. Although there may be several factors which cause the systematic errors as has been emphasized in Ref. 12, we will not discuss this issue in this paper. We assume that the Fermilab Main Injector neutrino beams have an energy spectrum which is given by Fig. 6.27 in Ref. 11, but we extrapolate the graph in Ref. 11 naively for neutrinos of the energy larger than 70 GeV. The average of the energy of the neutrino beams is typically from 10 GeV to 20 GeV. If Einstein’s equivalence principle is violated, the higher the energy of the neutrino beams becomes, the more probability of neutrino oscillations we have (see eq. (15)), unlike in case of neutrino-oscillations due to masses.

We choose a value of the distance $L=800$ Km which is motivated by the SOUDAN 2 experiment. We mainly consider the so-called disappearance experiments\textsuperscript{12,11}, in which the initial $\nu_\mu$ flux at some short distance and the $\nu_\mu$ flux at the detector are measured by detecting muons which are created from charged-current interactions. The probability of detecting muons at a distance $L$ is given by $1 - \epsilon(\nu_\mu \rightarrow \nu_a) \ (a = e$ or $\tau)$ where

$$
\epsilon(\nu_\mu \rightarrow \nu_a) \equiv \frac{\int_{\epsilon(q)}^{\epsilon(q)_{\text{max}}} \epsilon(q) \, dq \int_{E_\text{min}(q)}^{E_\text{max}(q)} \frac{d\sigma(E,a)}{dq} F(E) P(\nu_\mu \rightarrow \nu_a) n_T(q) \, dE}{\int_{\epsilon(q)}^{\epsilon(q)_{\text{max}}} \epsilon(q) \, dq \int_{E_\text{min}(q)}^{E_\text{max}(q)} \frac{d\sigma(E,a)}{dq} F(E) n_T(q) \, dE} \quad (a = e \text{ or } \tau).
$$

Here $E$ and $q$ are the energy of the incident $\nu_\mu$ and the outgoing muon, respectively, $F(E)$ is the flux of neutrino beams, $n_T(q)$ is the effective number of target nucleons, we have modeled the detection efficiency function $\epsilon(q)$ for muons with a step function, as in Ref. 12, and we have used the $y$ distribution for deep-inelastic scattering of neutrinos to determine the cross section $d\sigma(E,q)/dq$ of charged-current interaction\textsuperscript{13}. $P(\nu_\mu \rightarrow \nu_e \text{ or } \nu_\tau)$ in (19) is either $P(\nu_\mu \rightarrow \nu_e)$ or $P(\nu_\mu \rightarrow \nu_\tau)$, depending on whether we consider
oscillations $\nu_\mu \leftrightarrow \nu_e$ or $\nu_\mu \leftrightarrow \nu_\tau$.

We have studied the quantity $\epsilon$ for various cases. In case of $\nu_\mu \leftrightarrow \nu_\tau$ oscillations, there is no MSW effect, and we have obtained the results for $\epsilon(\nu_\mu \rightarrow \nu_\tau)$ from (13) and (19). We have plotted, assuming no signal for beam attenuation up to $\epsilon$, the excluded region of the $(\sin^2 2\theta, \Delta f/10^{-14})$ plane for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations with the detection threshold energy $E_{\text{th}}=10$ GeV of muons in Fig. 2. We have also calculated the excluded region for $E_{\text{th}}=5$ and 20 GeV, and the results are almost similar to the case of $E_{\text{th}}=10$ GeV. In Fig. 3 we have plotted the excluded region of the $(\sin^2 2\theta, -\Delta f/10^{-14})$ plane for $\nu_\mu \leftrightarrow \nu_e$ with $E_{\text{th}}=10$ GeV. To get this region we have used (18) and (19), where the MSW effect is taken into account. We have used a value for the density of electrons: $N_e = 8.05 \times 10^{23}$ electrons/cm$^3$ which is constant on the entire trajectory of neutrino beams in the present case (see, e.g., Ref. 14). Comparing Fig. 2 with Fig. 3, we observe that there is a slight difference in $\nu_\mu \leftrightarrow \nu_e$ oscillations because of the MSW effect. We have shown the result with negative $\Delta f$ in Fig. 3, because (17) shows that negative $\Delta f$ gives larger $|\Delta N(x)|$ and therefore $\epsilon(\nu_\mu \rightarrow \nu_e)$ with negative $\Delta f$ is larger than that with the same $\Delta m^2$ and positive $\Delta f$. Notice that $\epsilon(\nu_\mu \rightarrow \nu_\tau)$ with $\Delta m^2 = 0$ and $-\Delta f$ is the same as $\epsilon(\nu_\mu \rightarrow \nu_\tau)$ with $\Delta m^2 = 0$ and $\Delta f$, as is obvious from (15).

One important feature is the dependence of $\epsilon$ on the detection threshold energy $E_{\text{th}}$ of muons. As we mentioned earlier, it is more advantageous to look at neutrinos (and therefore outgoing muons) of higher energy to investigate a breakdown $\Delta f$ of the universality of the gravitational couplings. We have shown $\epsilon(\nu_\mu \rightarrow \nu_e)$ as a function of $E_{\text{th}}$ in Figs. 4 and 5 with $\sin^2 2\theta = 0.5$ and with $\Delta m^2 = 1 \times 10^{-2}$ eV$^2$, $0.5 \times 10^{-2}$ eV$^2$, and 0. The parameters are $\Delta f = 0$ for Fig. 4 and $\Delta f = -0.5 \times 10^{-14}$ for Fig. 5, respectively. $\epsilon(\nu_\mu \rightarrow \nu_e)$ with $\Delta f = 0$ in Fig. 4 decreases as $E_{\text{th}}$ increases, since the argument of the phase (=the mass squared) in case of ordinary MSW effect (see (18)) is suppressed by the neutrino energy $E_\nu$ ($\propto 1/E_\nu$). Fig. 5 shows,
on the other hand, that $\epsilon(\nu_\mu \rightarrow \nu_e)$ in the presence of $\Delta f (= -0.5 \times 10^{-14})$ has a conspicuous difference from the case with $\Delta f = 0$. Thus, if we look at the $E_{\text{th}}$-dependence of $\epsilon(\nu_\mu \rightarrow \nu_e)$, we can determine $\Delta f$ with higher accuracy. Because of the interaction term $G_F N_e$ with matter, $\epsilon(\nu_\mu \rightarrow \nu_e)$ is in general larger than $\epsilon(\nu_\mu \rightarrow \nu_\tau)$, but the qualitative features for the results of $\nu_\mu \leftrightarrow \nu_\tau$ oscillations are the same as those of $\nu_\mu \leftrightarrow \nu_e$. The dependence of $\epsilon$ on $E_{\text{th}}$ is also very useful, if we can change the energy of the neutrino beams. Namely, by comparing the results with different two values of energy of the neutrino beams, we could establish the existence of non-zero value of $\Delta f$. If the statistics is good enough, then we could even measure masses of neutrinos, by subtracting the effects of $\Delta f \neq 0$ from the data. This is one advantage that long-baseline experiments have over solar neutrino experiments.

Another important point is the expected numbers of observed muons. In case of the planned experiments at SOUDAN 2, we estimate the numbers to be approximately 16,000 events/year for $E_{\text{th}} = 10$ GeV and 6,000 events/year for $E_{\text{th}} = 20$ GeV, which are significantly larger than those of solar neutrino experiments where typical numbers are several hundreds events/year. So also in this aspect long-baseline experiments are promising.

In this paper we have proposed long-baseline experiments to test the universality of the gravitational couplings of neutrinos, and we found that we could probe the dimensionless parameter $\Delta f$ as small as $10^{-14}$ which is smaller by a few orders of magnitudes than the upper limit on a breakdown of the equivalence principle from different types of experiments. Although we have not evaluated systematic errors in detail, we hope our analysis will stimulate and motivate long-baseline experiments in the near future.

**Noted Added**

Toward the completion of our paper, we became aware of the work by Pantaleone, Halprin and Leung\textsuperscript{15}, where the similar topics has been discussed.
from a slightly different viewpoint.

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**Figures**

1. The cross section of the Earth. The accelerator is located at point A, and the SOUDAN 2 detector is located at point B. The distance $L$ between the points A and B is about 800 Km, and the radius $R$ of the Earth is about 6,400 Km. The curve from A to B in the Earth is geodesics in the background of the interior Schwarzschild metric.

2. The excluded region in the $(\sin^2 2\theta, \Delta f/10^{-14})$ plane for $\nu_\mu \leftrightarrow \nu_\tau$ with $\epsilon=1\%$ (solid), 3\% (dashed) and 10\% (dotted line). The detection threshold energy of muons is 10 GeV. The upper and right side of the curves is excluded.

3. The excluded region in the $(\sin^2 2\theta, -\Delta f/10^{-14})$ plane for $\nu_\mu \leftrightarrow \nu_e$ with $\epsilon=1\%$ (solid), 3\% (dashed) and 10\% (dotted line). The detection threshold energy of muons is 10 GeV. The upper and right side of the curves is excluded.
4. The transition probability $\epsilon(\nu_\mu \rightarrow \nu_e)$ as a function of the detection threshold energy $E_{th}$ of muons for $\sin^2 2\theta = 0.5$ and $\Delta f = 0$, which is solely due to the standard MSW effect. The solid and dashed curves have parameters $\Delta m^2 = 1 \times 10^{-2} \text{eV}^2$ and $0.5 \times 10^{-2} \text{eV}^2$, respectively.

5. The transition probability $\epsilon(\nu_\mu \rightarrow \nu_e)$ as a function of the detection threshold energy $E_{th}$ of muons for $\sin^2 2\theta = 0.5$ and $\Delta f = -0.5 \times 10^{-14}$. The solid, dashed and dotted curves have parameters $\Delta m^2 = 1 \times 10^{-2} \text{eV}^2$, $0.5 \times 10^{-2} \text{eV}^2$, and 0, respectively.