Quark sea asymmetries of the octet baryons

Neetika Sharma and Harleen Dahiya

Department of Physics,
Dr. B.R. Ambedkar National Institute of Technology,
Jalandhar, 144011, India

The effects of “quark sea” in determining the flavor structure of the octet baryons have been investigated in the chiral constituent quark model (χCQM). The χCQM is able to qualitatively generate the requisite amount of quark sea and is also known to provide a satisfactory explanation of the proton spin and related issues in the nonperturbative regime. The Bjorken scaling variable $x$ has been included phenomenologically in the sea quark distribution functions to understand its implications on the quark sea asymmetries like $\bar{d}(x) - \bar{u}(x)$, $\bar{d}(x)/\bar{u}(x)$ and Gottfried integral for the octet baryons. The results strengthen the importance of quark sea at lower values of $x$. 

arXiv:1005.5655v1 [hep-ph] 31 May 2010
After the first direct evidence for the point-like constituents in the nucleon \[^{[1]}\], identified as the valence quarks with spin-1/2 in the naive constituent quark model (NQM) \[^{[2,4]}\], a lot of experiments have been conducted to probe the structure of the proton in the deep inelastic scattering (DIS) experiments. Surprisingly, the DIS results in the early 80’s \[^{[5]}\] indicated that the valence quarks of the proton carry only about 30% of its spin and is referred to as the “proton spin crisis” in the NQM. These results provided the first evidence for the proton being composed of three valence quarks surrounded by an indistinct sea of quark-antiquark pairs (henceforth referred to as the “quark sea”). In the present day, the study of the composition of hadrons can be said to be primarily the study of the quark sea and gluons and is considered as one of the active areas in hadronic physics.

The conventional expectation that the quark sea perhaps can be obtained through the perturbative production of the quark-antiquark pairs by gluons produces nearly equal numbers of \(\bar{u}\) and \(d\) and is considered as one of the active areas in hadronic physics. In the present day, the study of the composition of hadrons can be said to be primarily the study of the quark sea valence quarks surrounded by an indistinct sea of quark-antiquark pairs (henceforth referred to as the “quark sea”).

80’s \[^{[5]}\] indicated that the valence quarks of the proton carry only about 30% of its spin and is referred to as the structure of the proton in the deep inelastic scattering (DIS) experiments. Surprisingly, the DIS results in the early spin-1/2 in the naive constituent quark model (NQM) \[^{[2–4]}\], a lot of experiments have been conducted to probe the \(\pi\) sea is believed to originate from process such as virtual pion production. It is suggested that in the deep inelastic scattering (DIS) experiments coming from the different sea quarks for the octet baryons can also be compared. To understand the relation of the Bjorken scaling variable and quark sea, it would be significant to study its implications in the region \(x < 0.3\) which is a relatively clean region to test the quark sea structure as well as to estimate their structure functions and related quantities \[^{[26]}\].

The key to understand the “proton spin crisis”, in the \(\chi\)CQM formalism \[^{[11]}\], is the fluctuation process

\[
q^+ \rightarrow GB + q^+ \rightarrow (q\bar{q}') + q^+ ,
\]

(1)

\(q\bar{q}' + q'\) constitute the “quark sea” \[^{[11][13][15]}\]. The effective Lagrangian describing interaction between quarks and a nonet of GBs, can be expressed as

\[
\mathcal{L} = g_8 \bar{q} \left( \Phi + \frac{\zeta q'}{\sqrt{3}} I \right) q = g_8 \bar{q} (\Phi') q ,
\]

(2)

where \(\zeta = g_1/g_8\), \(g_1\) and \(g_8\) are the coupling constants for the singlet and octet GBs, respectively, \(I\) is the \(3 \times 3\) identity matrix. In terms of the SU(3) and axial U(1) symmetry breaking parameters, introduced by considering \(M_u > M_{u,d}\),
$M_{K,\eta} > M_\pi$ and $M_{\eta'} > M_{K,\eta}$ [11, 12, 13], the GB field can be expressed as

$$
\Phi' = \begin{pmatrix}
\frac{\alpha K^+}{\sqrt{2}} & \frac{\alpha K^0}{\sqrt{2}} & -\beta \frac{2a}{\sqrt{2}} + \zeta \frac{\sqrt{3}}{\sqrt{2}} \\
-\frac{\alpha K^-}{\sqrt{2}} & \frac{\alpha K^0}{\sqrt{2}} & -\beta \frac{2a}{\sqrt{2}} + \zeta \frac{\sqrt{3}}{\sqrt{2}} \\
\frac{\pi^0}{\sqrt{2}} + \frac{\beta}{\sqrt{6}} + \zeta \frac{\sqrt{3}}{\sqrt{2}} & \frac{\pi^0}{\sqrt{2}} + \frac{\beta}{\sqrt{6}} + \zeta \frac{\sqrt{3}}{\sqrt{2}} & \frac{\pi^0}{\sqrt{2}} + \frac{\beta}{\sqrt{6}} + \zeta \frac{\sqrt{3}}{\sqrt{2}}
\end{pmatrix}
$$

and $q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$.

The parameter $a = |g_8|^2$ denotes the probability of chiral fluctuation $u(d) \rightarrow d(u) + \pi^+(-)$, whereas $\alpha^2 a, \beta^2 a$ and $\zeta^2 a$ respectively denote the probabilities of fluctuations $u(d) \rightarrow s + K^{-}(-), u(d, s) \rightarrow u(d, s) + \eta$, and $u(d, s) \rightarrow u(d, s) + \eta'$. For the sake of simplification, the GB field can also be expressed in terms of the quark contents of the GBs and is expressed as

$$
\Phi' = \begin{pmatrix}
\phi_{uud}u\bar{u} + \phi_{udd}d\bar{u} + \phi_{usd}s\bar{u} \\
\phi_{uds}d\bar{u} + \phi_{ddd}d\bar{d} + \phi_{ssd}s\bar{s} \\
\phi_{usd}u\bar{d} + \phi_{dsd}d\bar{s} + \phi_{ssd}s\bar{s}
\end{pmatrix},
$$

where

$$
\phi_{uud} = \frac{1}{2} + \frac{\beta}{6} + \frac{\zeta}{3}, \quad \phi_{ssd} = \frac{2\beta}{3} + \frac{\zeta}{3}, \quad \phi_{usd} = \phi_{dsd} = \phi_{sds} = -\beta \frac{3}{3} + \frac{\zeta}{3},
$$

$$
\phi_{du} = \phi_{ud} = \frac{1}{2} + \frac{\beta}{6} + \frac{\zeta}{3}, \quad \phi_{ud} = \phi_{du} = 1, \quad \phi_{us} = \phi_{ds} = \phi_{su} = \phi_{sd} = \alpha.
$$

The sea quark content of the baryon can be calculated in $\chi$CQM by substituting for every constituent quark $q \rightarrow \sum P_q q + |\psi(q)|^2$, where $\sum P_q$ is the transition probability of the emission of a GB from any of the $q$ quark and $|\psi(q)|^2$ is the transition probability of the $q$ quark. The flavor structure for the baryon of the type $B(xyz)$ is expressed as $2P_x x + P_y y + 2|\psi(x)|^2 + |\psi(y)|^2$ and for the type $B(xy)$ it is expressed as $P_x x + 2P_y y + 2|\psi(x)|^2 + |\psi(y)|^2 + |\psi(z)|^2$, where $x, y, z = u, d, s$. The sea quark distribution function for the octet baryons $p, \Sigma^+,$ $\Sigma^0,$ and $\Xi^0$ have been presented in Table I.

| Baryon   | $\bar{u}$     | $\bar{d}$     | $\bar{s}$     |
|----------|---------------|---------------|---------------|
| $p$(uud) | $a(2\phi_{uud}^2 + \phi_{uud}^2 + \phi_{uud}^2)$ | $a(2\phi_{udd}^2 + 2\phi_{udd}^2 + \phi_{udd}^2)$ | $a(2\phi_{usd}^2 + 2\phi_{usd}^2 + \phi_{usd}^2)$ |
| $\Sigma^+$(uud) | $a(2\phi_{uud}^2 + \phi_{uud}^2 + \phi_{uud}^2)$ | $a(2\phi_{udd}^2 + 2\phi_{udd}^2 + \phi_{udd}^2)$ | $a(2\phi_{usd}^2 + 2\phi_{usd}^2 + \phi_{usd}^2)$ |
| $\Sigma^0$(uds) | $a(\phi_{uud}^2 + \phi_{uud}^2 + \phi_{uud}^2)$ | $a(\phi_{udd}^2 + 2\phi_{udd}^2 + \phi_{udd}^2)$ | $a(\phi_{usd}^2 + 2\phi_{usd}^2 + \phi_{usd}^2)$ |
| $\Xi^0$(uus) | $a(\phi_{uud}^2 + 2\phi_{uud}^2 + 2\phi_{uud}^2)$ | $a(\phi_{udd}^2 + \phi_{udd}^2 + 2\phi_{udd}^2)$ | $a(\phi_{usd}^2 + \phi_{usd}^2 + 2\phi_{usd}^2)$ |

TABLE I. The sea quark distribution functions for the octet baryons. The expressions for other octet baryons can be obtained through isospin symmetry.

There are no simple or straightforward rules which could allow incorporation of $x$–dependence in $\chi$CQM. To this end, instead of using an $ab initio$ approach, we have phenomenologically incorporated the $x$–dependence getting clues from Eichten et al. [10, 11, 12, 13], Isgur [3] et al. and Le Yaouanc et al. [14]. The $x$–dependent sea quark distribution functions can be now expressed as $\bar{u}^B(x) = \bar{u}^B(1 - x)^{10}, \bar{d}^B(x) = \bar{d}^B(1 - x)^{10}, \bar{s}^B(x) = \bar{s}^B(1 - x)^{10}$ which together with the valence quark distribution functions give the flavor structure of the baryon as

$$
q^B(x) = q^B_{val}(x) + q^B(x),
$$

where $q = u, d, s$. Using the sea quark distribution functions from Table I, the quark sea asymmetries $\bar{u}(x) - \bar{d}(x)$ and $\bar{d}(x)/\bar{u}(x)$ can also be calculated at different $x$ values. We have already discussed the inclusion of $x$-dependence in detail and compared our results with the experimental data for the case of nucleon in Ref. [27]. In the present communication however, we have extended our calculations to the case of other octet baryons for which experimental data is not yet available.

The $x$–dependence of the structure functions $F_1$ and $F_2$ can be calculated from

$$
F_2^B(x) = x \sum_{u,d,s} c_0^2 [q^B(x) + \bar{q}^B(x)],
$$

$$
F_1^B(x) = \frac{1}{2x} F_2^B(x),
$$
where $e_q$ is the charge of the quark $q$ ($e_u = \frac{2}{3}$ and $e_d = e_s = -\frac{1}{3}$). In terms of the quark distribution functions, the structure function $F_2$ for any baryon can be expressed as

$$F_2^B(x) = \frac{4}{9} x (u^B(x) + \bar{u}^B(x)) + \frac{1}{9} x (d^B(x) + \bar{d}^B(x) + s^B(x) + \bar{s}^B(x)) \cdot \tag{9}$$

Several important quantities can be obtained from the structure functions of different isospin multiplets. For example, for the case of proton and neutron we have

$$\frac{F_P^p(x) - F_P^n(x)}{x} = \frac{4}{9} (u^p_{val}(x) - u^n_{val}(x)) + 2\bar{u}^p(x) - 2\bar{u}^n(x) + \frac{1}{9} (d^p_{val}(x) + s^p_{val}(x) - d^n_{val}(x) - s^n_{val}(x)) + 2\bar{d}^p(x) + 2\bar{s}^p(x) - 2\bar{d}^n(x) - 2\bar{s}^n(x)] \cdot \tag{10}$$

The Gottfried integral $I_G^p$ can be expressed in terms of the sea quarks as follows

$$I_G^p = \int_0^1 \frac{F_P^p(x) - F_P^n(x)}{x} dx = \frac{1}{3} + \frac{2}{3} \int_0^1 \left[ \bar{u}^p(x) - \bar{d}^p(x) \right] dx, \tag{11}$$

where we have used the following normalization conditions

$$\int_0^1 u^p_{val}(x) dx = 2, \quad \int_0^1 d^p_{val}(x) dx = 1, \quad \int_0^1 s^p_{val}(x) dx = 0, \quad \int_0^1 \bar{u}^p(x) dx = 1, \quad \int_0^1 \bar{d}^p(x) dx = 2, \quad \int_0^1 \bar{s}^p(x) dx = 0, \quad \int_0^1 \bar{u}^n(x) dx = 1, \quad \int_0^1 \bar{d}^n(x) dx = 2, \quad \int_0^1 \bar{s}^n(x) dx = 0. \tag{12}$$

Similarly, for the case of other octet baryons the following normalization conditions

$$\int_0^1 u^\Sigma^+(x) dx = 2, \quad \int_0^1 d^\Sigma^+(x) dx = 0, \quad \int_0^1 s^\Sigma^+(x) dx = 1, \quad \int_0^1 u^\Xi^0(x) dx = 1, \quad \int_0^1 d^\Xi^0(x) dx = 1, \quad \int_0^1 s^\Xi^0(x) dx = 1, \quad \int_0^1 u^\Xi^-(x) dx = 1, \quad \int_0^1 d^\Xi^-(x) dx = 0, \quad \int_0^1 s^\Xi^-(x) dx = 2. \tag{13}$$

lead to the other Gottfried integrals in terms of the sea quarks

$$I_G^{\Sigma^+} = \int_0^1 \frac{F_2^{\Sigma^+}(x) - F_2^{\Xi^0}(x)}{x} dx = \frac{1}{3} + \frac{2}{9} \int_0^1 \left[ 4\bar{u}^{\Sigma^+}(x) + \bar{d}^{\Sigma^+}(x) - 4\bar{u}^{\Xi^0}(x) - \bar{d}^{\Xi^0}(x) \right] dx, \tag{14}$$

$$I_G^{\Xi^0} = \int_0^1 \frac{F_2^{\Xi^0}(x) - F_2^{\Xi^-}(x)}{x} dx = \frac{1}{3} + \frac{2}{9} \int_0^1 \left[ 4\bar{u}^{\Xi^0}(x) + \bar{d}^{\Xi^0}(x) - 4\bar{d}^{\Xi^+}(x) - \bar{u}^{\Sigma^+}(x) \right] dx,$$

$$I_G^{\Xi^-} = \int_0^1 \frac{F_2^{\Xi^-}(x) - F_2^{\Xi^0}(x)}{x} dx = \frac{1}{3} + \frac{2}{9} \int_0^1 \left[ \bar{u}^{\Xi^-}(x) - \bar{d}^{\Xi^-}(x) \right] dx.$$

It is clear from Eqs. (11) and (14), the flavor symmetric sea leads to the Gottfried sum rule $I_G = \frac{1}{3}$ with $\bar{u}^B = \bar{d}^B$.

After having detailed the contribution of the quark sea and the various asymmetries in the octet baryons of different quark structure, we now discuss the variation of these quantities with the Bjorken variable $x$. For the numerical calculation of the sea quark distribution functions of the octet baryons, we have used the same set of input parameters as detailed in our earlier calculations [13, 16, 27, 29]. In Fig. 1, we have shown the variation of the sea quark distributions $x\bar{u}(x)$, $x\bar{d}(x)$ and $x\bar{s}(x)$ with the Bjorken scaling variable $x$ for $p(uud)$, $\Sigma^+(uus)$, $\Sigma^0(uds)$ and $\Xi^0(uus)$. From a cursory look at the plots, one can easily find out that

$$\bar{d}^P(x) > \bar{u}^P(x) > \bar{s}^P(x),$$
\[ d_{\Sigma^+}(x) > \bar{u}_{\Sigma^+}(x) \approx \bar{s}_{\Sigma^+}(x), \]
\[ d_{\Sigma^0}(x) > \bar{u}_{\Sigma^0}(x) > \bar{s}_{\Sigma^0}(x), \]
\[ d_{\Xi^0}(x) > \bar{u}_{\Xi^0}(x) > \bar{s}_{\Xi^0}(x), \]

showing a clear quark sea asymmetry as observed in the DIS experiments [6, 17, 18]. These distributions clearly indicate that our results pertaining to the quark sea asymmetry seem to be well in line with the expected results.

A careful study of the plots brings out several interesting points. As already mentioned in the introduction, the sea quarks do not contribute at higher values of \(x\), therefore in Fig. 1 we have taken the region \(x = 0 – 0.5\). Beyond this \(x\) region the contribution of the sea quarks is negligible and the contributions should be completely dominated by the valence quarks. The difference between the various sea distributions is observed to be maximum at \(x \approx 0.1\). As the value of \(x\) increases, the difference between the sea contributions decreases in all the cases which is in line with the observations of other models [20, 22, 24].

The general aspects of the variation of the magnitudes of the sea quark distribution functions \(\bar{u}(x), \bar{d}(x)\) and \(\bar{s}(x)\) for the octet baryons are able to explain some of the well known experimentally measurable quantities, for example, \(d_B(x) - \bar{u}_B(x), \bar{d}_B(x)/\bar{u}_B(x)\) and the Gottfried integral. These quantities not only provide important constraint on a model that attempts to describe the origin of the quark sea but also provide a direct determination of the presence of significant amount of quark sea in the low \(x\) region. In Fig. 2 the \(\chi\)CQM results for the \(\bar{d}_B(x) - \bar{u}_B(x)\) and the Gottfried integrals have been plotted at different \(x\) values. It is clear from the plots that when \(x\) is small \(\bar{d}_B(x) - \bar{u}_B(x)\) asymmetries are large implying the dominance of sea quarks in the low \(x\) region. In fact, the sea quarks dominate only in the region where \(x\) is smaller than 0.3. At the values \(x > 0.3\), \(\bar{d} - \bar{u}\) tends to 0 implying that there are no sea quarks in this region. The contribution of the quark sea in the case of \(\Sigma^0\) is particularly interesting because of its flavor structure which has equal numbers of \(u, d\) and \(s\) quarks in its valence structure. Unlike the other octet baryons, where the \(\bar{d}(x) - \bar{u}(x)\) asymmetry decreases continuously with the \(x\) values, the asymmetry in this case first increases and then for values of \(x > 0.1\) it decreases. However, it is interesting to observe the the asymmetry peak in this case which matches with our other predictions where the contribution of the quark sea is maximum at \(x \approx 0.1\).

A measurement of the Gottfried integral [6, 17] for the case of nucleon has shown a clear violation of Gottfried sum rule from \(1/3\) which can find its explanation in a global quark sea asymmetry \(\int_0^1 (\bar{d}(x) - \bar{u}(x)) dx\). Similarly, for the case of \(\Sigma^+, \Sigma^0\), and \(\Xi^0\), the Gottfried sum rules should read \(I_{G}^{\Sigma^+\Sigma^0} = 1/3\), \(I_{G}^{\Xi^0\Xi^+} = 1/3\) and \(I_{G}^{\Xi^0\Xi^0} = 1/3\) if the quark sea was symmetric. However, due to the \(\bar{d}(x) - \bar{u}(x)\) asymmetry in the case of octet baryons, a lower value of the Gottfried integrals is obtained. We have plotted the results in Fig. 2. In the case of nucleon the results are in good agreement with the experimental data [17] as already presented in [27]. The quality of numerical agreement in the other cases can be assessed only after the data gets refined. Further, this phenomenological analysis strongly suggests an important role for the quark sea at low value of \(x\). New experiments aimed at measuring the flavor content of the other octet baryons are needed for profound understanding of the nonperturbative properties of QCD.

To summarize, in order to investigate the effects of “quark sea”, we have calculated the sea quark distribution functions for the octet baryons in the chiral constituent quark model (\(\chi\)CQM). The Bjorken scaling variable \(x\) has been incorporated phenomenologically to enlarge the scope of model and to understand the range of \(x\) where quark sea effects are important. Implications of the quark sea have also been studied to estimate the quark sea asymmetries like \(\bar{d}(x) - \bar{u}(x), \bar{d}(x)/\bar{u}(x)\) and Gottfried integral. The results justify our conclusion regarding the importance of quark sea at small values of \(x\).

In conclusion, the results obtained for the quark distribution functions reinforce our conclusion that \(\chi\)CQM is able to generate qualitatively as well as quantitatively the requisite amount of quark sea. This can perhaps be substantiated by a measurement of the quark distribution functions of the other octet baryons.

**ACKNOWLEDGMENTS**

H.D. would like to thank Department of Science and Technology, Government of India, for financial support.

---

[1] E.D. Bloom *et al.*, Phys. Rev. Lett. 23, 930 (1969); M. Breidenbach *et al.*, Phys. Rev. Lett. 23, 935 (1969).
[2] A. De Rujula, H. Georgi and S.L. Glashow, Phys. Rev. D 12, 147 (1975).
[3] N. Isgur, G. Karl and R. Koniiik, Phys. Rev. Lett. 41, 1269 (1978); N. Isgur and G. Karl, Phys. Rev. D 21, 3175 (1980); P. Geiger and N. Isgur, Phys. Rev. D 55, 299 (1997); N. Isgur, Phys. Rev. D 59, 034013 (1999).
[4] A. Le Yaouarn, L. Oliver, O. Pene and J.C. Raynal, Phys. Rev. D 12, 2137 (1975); 15, 844 (1977).
FIG. 1. Sea quark distribution functions as a function of Bjorken scaling variable $x$ for the $p$, $\Sigma^+$, $\Sigma^0$, and $\Xi^0$, respectively.
FIG. 2. Sea quark asymmetry and Gottfried integrals as a function of Bjorken scaling variable $x$ for the octet baryons.

[17] E866/NuSea Collaboration, E.A. Hawker et al., Phys. Rev. Lett. 80, 3715 (1998); J.C. Peng et al., Phys. Rev. D 58, 092004 (1998); R. S. Towell et al., Phys. Rev. D 64, 052002 (2001).
[18] HERMES Collaboration, K. Ackerstaff et al., Phys. Rev. Lett. 81, 5519 (1998).
[19] M. Alberg, E.M. Henley, and G.A. Miller, Phys.Lett. B 471, 396 (2000); S. Kumano and M. Miyama, Phys. Rev. D 65, 034012 (2002); Fu-Guang Cao and A. I. Signal, Phys. Rev. D 68, 074002 (2003); F. Huang, R.-G. Xu, B.-Q. Ma, Phys. Lett. B 602, 67 (2004); B. Pasquini, S. Boffi, Nucl. Phys. A 782, 86 (2007).
[20] M. Wakamatsu, Phys. Rev. D 44, R2631 (1991); M. Wakamatsu, Phys. Rev. D 46, 3762 (1992); H. Weigel, Phys. Rev. D 55, 092004 (1997); M. Wakamatsu and T. Kubota, Phys. Rev. D 57, 5755 (1998); M. Wakamatsu, Phys. Rev. D 67, 034005 (2003).
[21] Yong Ding, Rong-Guang Xu, and Bo-Qiang Ma, Phys. Rev. D 71, 094014 (2005); Lijing Shao, Yong-Jun Zhang, and Bo-Qiang Ma, Phys. Lett. B 686, 136 (2010).
[22] L.A. Trevisan, C. Mirez, T. Frederico, and L. Tomio, Eur. Phys. J. C 56, 221 (2008); Yunhua Zhang, Lijing Shao, and Bo-Qiang Ma, Phys. Lett. B 671, 30 (2009); Yunhua Zhang, Lijing Shao, and Bo-Qiang Ma, Nucl. Phys. A 828, 390 (2009).
[23] A.I. Signal and A.W. Thomas, Phys. Rev. D 40, 2832 (1989).
[24] J. Alwall and G. Ingelman, Phys. Rev. D 71, 094015 (2005).
[25] M. Glück, E. Reya, and A. Vogt, Z. Phys. C 67, 433 (1995); M. Glück, E. Reya, M. Stratmann, and W. Vogelsang, Phys. Rev. D 53, 4775 (1996); D. de Florian, C.A. Garcia Canal, and R. Sassot, Nucl. Phys. B 470, 195 (1996).
[26] CDHS Collaboration, H. Abramowicz et al., Z. Phys. C 17, 283(1983); Costa et al., Nucl. Phys. B 297, 244 (1988).
[27] H. Dahiya and M. Gupta, Eur. Phys. J. C 52, 571 (2007).
[28] K. Gottfried, Phys. Rev. Lett. 18, 1174 (1967).
[29] C. Amsler et al., Phys. Lett. B 667, 1 (2008).