Empirical Models for Estimation of Global Solar Radiation Using the Monthly Average Daily Sunshine Hours Data for Makurdi, Benue State.

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ABSTRACT
This paper proposes empirical correlation models for estimating global solar radiation using data of sunshine hours for the location of Makurdi in Benue State of Nigeria. The paper suggests extrapolation of the empirical models for other locations with similar climatic conditions. The proposed models are: Linear, Quadratic, Cubic, Exponential, Power and Logarithmic models. Each of the models is based on Angstrom-Prescott equations for estimating global solar radiation. Any of the models can ease the use of sophisticated equipments, which are expensive, delicate and sometimes develop faults during measurement. The results of the models show that the cubic model is the best with slightly higher coefficient of determination. The coefficient of determination of each of the models was found to be 0.952, 0.965, 0.967, 0.965, 0.948 & 0.924 respectively, while the absolute correlation was found to be unity. Errors evaluated include MBE, RMSE and MPE with minimal values. The percentage diffuse and direct solar radiations, clearness index and the diffuse fraction were also estimated using the models. The results of the estimations done using the proposed models indicate that there is an estimated average annual global solar radiation of 6056 MJ m⁻², monthly value of 505 MJ m⁻² and daily insolation of 16.82 MJ m⁻² sufficient enough for maximum solar radiation exploitation.

Keywords: Solar Radiation, Empirical Models, Diffuse Radiation, Direct Radiation

INTRODUCTION
Much of the current energy resources being harnessed is the fossil fuel energy whose rate of depletion is on the rise, particularly in Nigeria. It is therefore imperative to harness the solar energy resources to help meet the ever increasing energy demand in our country. The widely used renewable energy sources are solar and wind energies.

Solar radiation which arrives at the earth surface for every year is 160 times the world's proven fossil fuels reserves (Yoshida, et al. 2013). Nigeria is blessed with abundant amount of sunshine with an estimated 3000 hours of annual sunshine (Burari and Sambo, 2001).

To estimate the global solar radiation for Markurdi using the monthly average daily sunshine hours, several empirical correlation equations have been proposed for use in the course of this research work. They are based on the models developed by Angstrom (1924) and modified by Prescott (1940). And the diffuse solar radiation was estimated using the equation proposed by Page (1964). These equations have been frequently used to estimate the global and diffuse solar radiation in various places.

The results of the proposed models show a great deal of correlation with the measured data obtained from NIMET (Nigeria Meteorological Agency) Air Force Base, Markurdi. Statistical tests carried out on each of the models revealed minimal errors and high magnitude of coefficient of determination, R².

METHODOLOGY:
The monthly mean daily extraterrestrial radiation, $H_o$ was computed from equation (1) and recorded as $H_o$ in MJ m⁻² day⁻¹.

$$H_o = \frac{24}{\pi} \cdot 3600 \cdot \frac{G_{sc}}{365} \cdot (1 + 0.033 \cos \frac{\pi t}{180} + \frac{0.055}{\cos \frac{\pi t}{180}}).$$

(1)

where $G_{sc}$ is the solar constant ($G_{sc} = 1367$ W/m²)

Sunrise and sunset hour angle is given by Fayadh and Ghazi (1983) as:
where \( w = \cos^{-1}(-\tan \phi \tan \delta) \) \hspace{1cm} (2)

\( \phi \) = latitude = 7.68°N and \( \delta \) is declination angle

The value of the declination can be found from the equation of Cooper (1969) as:

\[
\text{Declination angle} \delta = 23.45 \sin \left( \frac{356}{365} \left( \frac{N + 214}{365} \right) \right)
\] \hspace{1cm} (3)

where \( N \) is the number of days of the year starting from first of January.

The day-length \( (s) \) is given by: \( Iqbal, 1983 \)

\[
s = \frac{2}{15} \cos^{-2} \omega \sin \phi \]

\hspace{1cm} (4)

**Analysis on each of the models:**

The Angstrom-Prescott models have constants evaluated with the aid of Microsoft Excel Programme. The models are given by the following relations:

\[
\frac{H_m}{H_0} = a + b \left( \frac{N}{365} \right) \hspace{1cm} \text{(Linear Models)} \hspace{1cm} (5)
\]

\[
\frac{H_m}{H_0} = a + b \left( \frac{N}{365} \right) + c \left( \frac{N}{365} \right)^2 \hspace{1cm} \text{(Quadratic Model)} \hspace{1cm} (6)
\]

\[
\frac{H_m}{H_0} = a + b \left( \frac{N}{365} \right) + c \left( \frac{N}{365} \right)^2 + d \left( \frac{N}{365} \right)^3 \hspace{1cm} \text{(Cubic Model)} \hspace{1cm} (7)
\]

\[
\frac{H_m}{H_0} = e^{b \left( \frac{N}{365} \right)} \hspace{1cm} \text{(Exponential Model)} \hspace{1cm} (8)
\]

\[
\frac{H_m}{H_0} = a \left( \frac{N}{365} \right)^b \hspace{1cm} \text{(Power Model)} \hspace{1cm} (9)
\]

\[
\frac{H_m}{H_0} = a + b \ln \left( \frac{N}{365} \right) \hspace{1cm} \text{(Logarithmic Model)} \hspace{1cm} (10)
\]

\( Tijjani \text{and} \)Musa, 2011.

\[ y = 0.5483x + 0.14 \]

\[ R^2 = 0.9527 \]

\[ y = 0.2244e^{0.13x} \]

\[ R^2 = 0.9634 \]

\[ y = 0.6 \]

\[ y = 0.4 \]

\[ y = 0.2 \]

\[ y = 0.0 \]

Figure 1: Correlation of the Linear Model

Figure 2: Correlation of the Exponential Model

\[ y = 0 \]

\[ y = 0.5 \]

\[ y = 1 \]
RESULTS OF ANALYSIS

The evaluation of the constants results into the formulation of the proposed models that can be used to estimate the global solar radiation of the research location Makurdi and other areas with similar climatic conditions. They are given by the following relations:

\[
\frac{H_{\text{global}}}{H_0} = 0.140 + 0.648 \left( \frac{H_0}{H_{\text{global}}} \right) \tag{11}
\]

\[
\frac{H_{\text{global}}}{H_0} = 0.327 - 0.168 \left( \frac{H_0}{H_{\text{global}}} \right) + 0.852 \left( \frac{H_0}{H_{\text{global}}} \right)^2 \tag{12}
\]

\[
\frac{H_{\text{global}}}{H_0} = -0.064 + 2.426 \left( \frac{H_0}{H_{\text{global}}} \right) - 4.701 \left( \frac{H_0}{H_{\text{global}}} \right)^2 + 3.862 \left( \frac{H_0}{H_{\text{global}}} \right)^3 \tag{13}
\]

\[
\frac{H_{\text{global}}}{H_0} = 0.224 + 0.424 \log \left( \frac{H_0}{H_{\text{global}}} \right) \tag{14}
\]

\[
\frac{H_{\text{global}}}{H_0} = 0.737 \left( \frac{H_0}{H_{\text{global}}} \right)^{0.259} \tag{15}
\]

\[
\frac{H_{\text{global}}}{H_0} = 0.0576 + 0.2961 \log \left( \frac{H_0}{H_{\text{global}}} \right) \tag{16}
\]

The diffuse solar radiation was estimated using the equation proposed by Page (1964) given by:

\[
\frac{H_d}{H_m} = 1.0 - 1.43 \left( \frac{H_m}{H_0} \right) \tag{17}
\]

RESULTS AND DISCUSSION

The results are presented in Table 1 as well as Figures 1-16. This is followed by the discussion of the results.

**Table 1: Statistical analysis of data**

|         | Linear | Quadratic | Cubic | Exponential | Power | Logarithmic |
|---------|--------|-----------|-------|-------------|-------|-------------|
| $R^2$   | 0.0952 | 0.965     | 0.967 | 0.965       | 0.948 | 0.924       |
| MBE     | -0.050 | -0.0167   | 0.014 | -0.036      | -0.0011 | 0.0150     |
| RMSE    | 0.431  | 0.058     | 0.405 | 0.434       | 0.0037 | 0.0521     |
| MPE     | 0.28%  | 0.07%     | -0.12% | 0.19%       | -0.002% | -0.083%    |

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Figure 7. Comparison of monthly variation of the measured and calculated Global Solar Radiation (Cubic Model)

Figure 8. Comparison of monthly variation of the measured and calculated Global Solar Radiation (Linear Model)

Figure 9. Comparison of monthly variation of the measured and calculated Global Solar Radiation (Quadratic Model)

Figure 10. Comparison of monthly variation of the measured and calculated Global Solar Radiation (Exponential Model)
Figure 11. Comparison of monthly variation of the measured and calculated Global Solar Radiation (Power Model)

Figure 12. Comparison of monthly variation of the measured Global Solar Radiation with the linear and quadratic models

Figure 13. Comparison of monthly variation of the measured Global Solar Radiation with the power and Logarithmic models

Figure 14. Comparison of monthly variation of the measured Global Solar Radiation with the Cubic and Exponential models
Figure 15. Comparison of monthly variation of the measured Global Solar Radiation with the calculated linear, cubic and quadratic models.

Figure 16. Comparison of monthly variation of the measured Global Solar Radiation with the calculated exponential, logarithmic and power models.

Figure 17. Comparison of monthly variation of the measured clearness index with the calculated linear, cubic and quadratic models.

Figure 18. Comparison of monthly variation of the measured clearness index with the calculated exponential, power and logarithmic models.
Figure 19. Comparison of monthly variation of the measured clearness index with the calculated index from the quadratic model

Figure 20. Comparison of monthly variation of the measured clearness index with the calculated index from the exponential model

Figure 21. Comparison of monthly variation of the measured clearness index with the calculated index from the linear models

Figure 22. Comparison of monthly variation of the percentage diffuse radiation to the percentage of direct radiation
DISCUSSION

Figures 7-11 show the comparison of Measured and Calculated Global Solar Radiation for each of the Models. Figures 12-14 compare the measured and calculated global solar radiation for the location for each of two models. Figures 15-18 compare the calculated clearness index of the models with the measured values, while Figures 19-21 compare the measured clearness index with that calculated by each of the individual models. Figure 22 compares the percentage of diffuse solar radiation with the direct solar radiation.

The results reveal the correlation between the calculated and measured data. Low solar insolation is indicated by trough seen in the graphs due to the overcast experienced between May and October in the raining season. The gaps between the lines of graph depict the disparity between estimated and measured data. Obviously due to error in model/instrument used in measurement. The gap between May and October depicts heavy presence of cloud cover with high diffusion fraction in rainy season. Where the lines cross in the graphs of solar radiation between October and November signify change into dry season and is observed by a clearness index $K_T$ that is high.

Further observation shows that when the percentage of the diffuse radiation is high, the percentage of the direct radiation is low. This is similar to the variation of the diffuse fraction and clearness index. Variations are also due to the cloudy nature of the atmosphere and the presence of particles in the atmosphere, which strongly attenuate solar radiation during the months of November, December and January.

CONCLUSION

The results obtained using the proposed models are in much correlation with the measured data by NIMET. Hence, the models are recommended for use to estimate the global solar radiation in Makurdi, Benue State and other regions with similar climatic conditions. The results of the estimations done using the proposed models show that there is an estimated average annual global solar radiation of 6056MJm$^{-2}$, monthly value of 505MJm$^{-2}$ and daily insolation of 16.82MJm$^{-2}$ sufficient enough for maximum solar radiation exploitation.

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