Numerical values of $f^F$, $f^D$, $f^S$ coupling constants in $SU(3)$ invariant interaction Lagrangian of vector-meson nonet with $1/2^+$ octet baryons

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Abstract

It is demonstrated how an utilization of all existing experimental information on electric and magnetic nucleon form factors, to be described by the Unitary and Analytic (U&A) nucleon electromagnetic structure model in space-like and time-like regions simultaneously, can provide numerical values of $f^F$, $f^D$, $f^S$ coupling constants in $SU(3)$ invariant interaction Lagrangian of the vector-meson nonet with $1/2^+$ octet baryons. The latter, together with universal vector-meson coupling constants $f_V$, play an essential role in a prediction of $1/2^+$ octet hyperon electromagnetic form factors behaviors.

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I. INTRODUCTION

Recently the BESIII Collaboration has published [1] new results on the proton electromagnetic (EM) form factors (FFs) in the time-like region by measuring the process $e^+e^- \rightarrow p\bar{p}$ with very high precision at 12 different center-of-mass energies from $4.9836 \text{ GeV}^2$ to $13.4762 \text{ GeV}^2$. In such way the obtained results enriched a set of already existing number of experimental points on proton EM FFs in the space-like and time-like regions substantially. Moreover, the BESIII Collab. and before also the BaBar Collab. [2], measuring the polar angular distribution $F(\cos \theta_p)$ of created protons at few different energies, have been able to determine a separate information on the proton electric and on the proton magnetic FFs in time-like region for the first time.

On the other hand a very successful microscopic model for the proton EM FFs in the near-threshold time-like region has been elaborated [3] to be based on the assumption that the behavior of the EM FFs as a function of the energy is given by the initial or final state interaction between proton and antiproton in the processes $e^+e^- \leftrightarrow p\bar{p}$. In this approach the antinucleon-nucleon potential is constructed in the framework of chiral effective field theory [4] and fitted to results of partial-wave analysis of existing $\bar{p}p$ scattering data. As a result predictions of the proton EM FFs $G_E^p(t)$ and $G_M^p(t)$, and also their ratios are in good agreement with existing data up to $t = 3.986 \text{ GeV}^2$, i.e. at the region of a validity of the model under consideration.

Next intentions of the BESIII Collab. are to extend such data measurements for $e^+e^- \rightarrow Y\bar{Y}$ processes, where $Y$ are hyperons to be members of the $1/2^+$ octet baryons $p, n, \Lambda, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-$. In a preparation of such a program it can be useful to dispose with some model predictions for hyperon EM FF behaviors and subsequently also of the corresponding differential and total cross-sections.

But the model presented in [3] is not applicable for such model predictions of hyperon EM FFs as it is based on the existence of baryon-antibaryon scattering data, what is unattainable in the case of hyperons.

In this paper we demonstrate a scheme in the framework of which one can do such predictions. As one can see further, an important role in the prediction of hyperon EM FFs play the advanced Unitary and Analytic ($U&A$) nucleon EM structure model, respecting
the $SU(3)$ symmetry and OZI rule violation, then a knowledge of the universal vector-meson coupling constants $f_V$ to be extracted from the experimental values of vector-meson lepton widths

$$\Gamma(V \rightarrow e^+ e^-) = \frac{\alpha^2 m_V}{3} \left( \frac{f_V^2}{4\pi} \right)^{-1}$$

with $\alpha = \frac{1}{137}$ to be the QED fine structure coupling constant and $m_V$ the vector-meson mass, and a knowledge of the numerical values of $f^F, f^D, f^S$ coupling constants in $SU(3)$ invariant Lagrangian

$$L_{VBB} = \frac{i}{\sqrt{2}} f^F [\bar{B}^a_{\gamma\mu} B^a_{\gamma\mu} - \bar{B}^a_{\gamma\mu} B^a_{\gamma\mu}] (V^a_{\mu})_{\alpha} +$$
$$+ \frac{i}{\sqrt{2}} f^D [\bar{B}_{\gamma\mu} B^a_{\gamma\mu} + \bar{B}_{\gamma\mu} B^a_{\gamma\mu}] (V^a_{\mu})_{\alpha} +$$
$$+ \frac{i}{\sqrt{2}} f^S \bar{B}_{\gamma\mu} B^a_{\gamma\mu} \omega^0_{\mu}$$

describing strong interactions of the nonet of vector-mesons with $1/2^+$ octet baryons, where $B, \bar{B}$ and $V$ are baryon, anti-baryon and vector-meson octuplet matrices and $\omega^0_{\mu}$ is omega-meson singlet.

The advanced $U&A$ nucleon EM structure model represents a harmonious unification of the vector-meson pole contributions and cut structure of EM FFs, which represent the so-called continua contributions in nucleon EM FFs.

The nucleon EM FFs are analytic functions in the whole complex plane of their variable $t$, besides cuts on the positive real axis from the lowest branch point $t_0$ to $+\infty$.

The shape of nucleon EM FFs is directly related with an existence of complex conjugate pairs of poles on unphysical sheets of the Riemann surface in $t$-variable, corresponding to unstable true neutral vector-mesons $\rho(770), \omega(782), \phi(1020); \rho'(1450), \omega'(1420), \phi'(1680); \rho''(1700), \omega''(1650), \phi''(2170)$ with the quantum numbers of the photon to be revealed explicitly in the process of electron-positron annihilation into hadrons.

As a result the complex nature of nucleon EM FFs for $t > t_0$ in time-like region is secured by imaginary parts of the vector-meson poles on unphysical sheets of the Riemann surface in $t$-variable and the cuts on the positive real axis of the first, so-called physical sheet, of the Riemann surface, whereby imaginary part of the EM FFs are given by a difference of
the FF values on the upper boundary of the cuts and the FF values on the lower boundary of the cuts.

Every nucleon electric FF is canonically normalized to the nucleon electric charge and every nucleon magnetic FF is normalized to the nucleon magnetic moment.

All these nucleon EM FFs govern the asymptotic behaviors as predicted by the quark model of hadrons to be proven also in the framework of the QCD [9].

The advanced $U$&$A$ nucleon EM structure model depends on the coupling constants ratios $(f_{VNN}/f_V)$ to be defined for every of nine above-mentioned unstable true neutral vector-mesons in VMD model. However, not all nine of them appear explicitly in the advanced $U$&$A$ nucleon EM structure model as free parameters. In the process of a construction of the VMD model to be automatically normalized and governing the correct asymptotic behavior, some of these coupling constants ratios can be expressed through the table mass values of the true neutral vector-mesons under consideration and other coupling constant ratios. And just here some freedom exists in the choice, which of the coupling constants ratios $(f_{VNN}/f_V)$ will be left as free parameters of the model.

The problem is that not for all nine true neutral vector-mesons under consideration an experimental information on the lepton width $\Gamma(V \to e^+e^-)$ exists. The latter is known [8] only for ground state vector-mesons $\rho(770)$, $\omega(782)$ and $\phi(1020)$. For all the first and second excited states the Rev. of Part. Physics [8] declares, that their lepton decays are seen, but they are not precisely measured experimentally up to now.

Just for this reason in the present advanced nucleon EM structure model we keep the coupling constant ratios $(f_{\rho NN}/f_{\rho})$, $(f_{\omega NN}/f_{\omega})$, $(f_{\phi NN}/f_{\phi})$ and also $(f_{\omega NN}/f_{\omega'})$, $(f_{\phi NN}/f_{\phi'})$, for which some model estimations of $f_{\omega'}$ and $f_{\phi'}$ exist [10], as free parameters to be determined in comparison of the present advanced nucleon EM structure model with all existing data on nucleon EM FFs in space-like and time-like regions simultaneously.

The estimation of $f_{\omega'}$, $f_{\phi'}$ (and also $f_{\rho'}$ for a determination of $f_{\rho' NN}$ from $(f_{\rho' NN}/f_{\rho'})$ to be completely given by the table mass values of vector-mesons under consideration) in [10] will be only a model admixture in subsequent determination of the $1/2^+$ octet hyperon EM FF behaviors.
II. EXISTING EXPERIMENTAL INFORMATION ON NUCLEON EM FFS

The EM structure of the nucleons (iso-doublet compound of the proton and neutron), as revealed experimentally for the first time in elastic unpolarized electron-proton scattering in the middle of the last century, is completely described by four independent scalar functions, the electric $G_E^p(t), G_E^n(t)$ and the magnetic $G_M^p(t), G_M^n(t)$ FFs, dependent of one variable to be chosen as the squared momentum transferred $t = -Q^2$ of the virtual photon $\gamma^*$. The experimental information on these functions consists of 11 following different sets of data on:

- the ratio $\mu_p G_E^p(t)/G_M^p(t)$ in the space-like ($t < 0$) region from polarization experiments \[1\]-\[15\]
- $G_E^p(t)$ in the space-like ($t < 0$) region \[16\]
- $|G_E^p(t)|$ in the time-like $t > 0$ region; only from exp. in which $|G_E^p(t)| = |G_M^p(t)|$ is assumed
- $G_M^p(t)$ in the space-like ($t < 0$) region \[16\]-\[23\]
- $|G_M^p(t)|$ in the time-like $t > 0$ region \[1\],\[2\],\[24\]-\[32\]
- $|G_E^p(t)/G_M^p(t)|$ in the time-like $t > 0$ region \[1\],\[2\]
- $G_E^n(t)$ in the space-like ($t < 0$) region \[33\]-\[39\]
- $|G_E^n(t)|$ in the time-like $t > 0$ region; only from exp. in which $|G_E^n(t)| = |G_M^n(t)|$ is assumed
- $G_M^n(t)$ in the space-like ($t < 0$) region \[33\]-\[40\]-\[46\]
- $|G_M^n(t)|$ in the time-like $t > 0$ region \[28\]
- the ratio $\mu_n G_E^n(t)/G_M^n(t)$ in the space-like ($t < 0$) region from polarization experiments on the light nuclei \[47\],\[48\]

from which not all are equally trustworthy.
The most reliable, from all above-mentioned data, are considered to be the experimental points on the ratio \( \mu_p G_E^p(t)/G_M^p(t) \) \([11]-[15]\) in the space-like region, which have been extracted from simultaneous measurement of the transverse component

\[
P_t = \frac{\hbar}{I_0}(-2)\sqrt{\tau(1+\tau)}G_{Ep}G_{Mp} \tan \theta/2
\]
and the longitudinal component

\[
P_l = \frac{\hbar(E_e + E_e')}{I_0 m_p} \sqrt{\tau(1+\tau)}G_{M}^2 \tan^2 \theta/2
\]

of the recoil proton’s polarization in the polarized electron scattering plane of the polarization transfer process \( e^- p \rightarrow e^- p \), where \( h \) is the electron beam helicity, \( I_0 \) is the unpolarized cross-section excluding \( \sigma_{Mott} \) and \( \tau = -\frac{t}{4m_p^2} \), by means of the relation

\[
\mu_p G_E^p = \frac{P_t (E_e + E_e')}{P_l 2m_p} \tan \theta/2.
\]

The data \([11]-[15]\) clearly demonstrate that a general belief in the dipole behavior of the proton electric FF \( G_E^p(t) \) in the space-like region to be obtained by the Rosenbluth method from the process of the elastic scattering of unpolarized electrons on unpolarized protons \( e^- p \rightarrow e^- p \) described by the differential cross-section in the laboratory system

\[
\frac{d\sigma^{lab}(e^- p \rightarrow e^- p)}{d\Omega} = \frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)} \frac{1}{1 + \left(\frac{2E}{m_B}\right) \sin^2(\theta/2)}
\]

\[
\cdot \left[ \frac{G_E^2(t) - \frac{t}{4m_p^2} G_M^2(t)}{1 - \frac{t}{4m_p^2}} - 2 \frac{t}{4m_B^2} G_M^2(t) \tan^2(\theta/2) \right]
\]

with the QED the fine structure constant \( \alpha = 1/137 \), the incident electron energy \( E \) and the scattering angle \( \theta \), before the polarization experiments \([11]-[15]\) have been carried out, is no more valid.

This is a reason, why further we ignore all older data on \( G_E^p(t) \) in the space-like region and we take into account only the newest data from MAMI \([16]\) at the region \(-0.5524GeV^2 < t < -0.0152GeV^2\), where the application of the Rosenbluth method is still justified.

On the second place to be concerned of the reliability are the data on the proton magnetic FF \( G_M^p(t) \) \([16]-[23]\) in the space-like region, which have been obtained by the Rosenbluth method from experimental information on the differential cross-section describing elastic scattering of unpolarized electrons on unpolarized protons. As one can see from the expression of the differential cross-section \( (6) \), with increased negative values of \( t \), the proton
magnetic FF is dominant in comparison with the proton electric FF and so, the data on $G_M^p(t)$ extracted by the Rosenbluth method are reliable.

Less reliable are data \[1,2,24-32\] on absolute value of the magnetic FF $|G_M^p(t)|$ in the time-like $t > 0$ region (the same is concerned also of the neutron magnetic FF $|G_M^n(t)|$ in the time-like $t > 0$ region) as the most of them are extracted from the total cross-section of the electron-positron annihilation process into proton-antiproton pair

$$\sigma_{tot}(e^+e^- \rightarrow pp) = \frac{4\pi\alpha^2\beta_p}{3t} [\left| G_M^p(t) \right|^2 + \frac{2m_p^2}{t} \left| G_E^p(t) \right|^2]$$ (7)

with $\beta_p = \sqrt{1 - \frac{4m_p^2}{t}}$, or from the total cross-section of the antiproton-proton annihilation into electron-positron pair

$$\sigma_{tot}(\bar{p}p \rightarrow e^+e^-) = \frac{2\pi\alpha^2}{3p_{c.m.}\sqrt{t}} [\left| G_M^p(t) \right|^2 + \frac{2m_p^2}{t} \left| G_E^p(t) \right|^2]$$ (8)

with $p_{c.m.}$ to be the antiproton momentum in the c.m. system, by and assumption $|G_E^p(t)|=|G_M^p(t)|$, which is exactly valid only at the proton-antiproton threshold following directly from the definition of both FFs, or by an assumption that for high values of $t$ $|G_E^p(t)|=0$ and then the total cross-sections under consideration give information only on $|G_M^p(t)|$, what is not well-founded.

Very promising method of a determination of $|G_E^p(t)|$ and $|G_M^p(t)|$ is demonstrated in \[1,2\] by a measurement of the distribution of $\theta_p$, the angle between the proton momentum in the $pp$ rest frame, and the momentum of the $pp$ system in the $e^+e^-$ c.m. frame and by a subsequent fitting of the obtained experimental points by the expression

$$F(\cos \theta_p) = N_{norm} \left[ 1 + \cos^2 \theta_p + \frac{4m_p^2}{t} R^2 (1 - \cos^2 \theta_p) \right]$$ (9)

to be obtained from

$$\frac{d\sigma(t)}{d\Omega} = \frac{\alpha^2\beta_p C}{4t} \left[ |G_M^p(t)|^2 (1 + \cos^2 \theta_p) + \frac{4m_p^2}{t} |G_E^p(t)|^2 \sin^2 \theta_p \right]$$ (10)

with the Coulomb correction factor $C$, where

$$R = \frac{|G_E^p(t)|}{|G_M^p(t)|}$$ (11)

and
FIG. 1: Data on the ratio $\mu_p G_E^p(t)/G_M^p(t)$ in the space-like ($t < 0$) region from polarization experiments [11]-[15].

$$N_{norm} = \frac{2\pi \alpha^2 \beta L}{4t} \left[ 1.94 + 5.04 \frac{m_p^2}{t} R^2 |G_M^p(t)|^2 \right]$$  (12)

is the overall normalization factor and $L$ is the integrated luminosity.

However, the latter method is in the phase of a development and the ratios $R$ with $|G_M^p(t)|$ at only several values of the energy $t$ have been determined [1],[2] up to now.

The existing data on $G_E^n(t), G_M^n(t)$ in the space-like region and the data on $\mu_n G_E^n(t)/G_M^n(t)$ are even more model dependent as they have been extracted from the scattering processes of electrons on light nuclei by various theoretical model ingredients.

All these data are graphically presented in Figs. (1-11).

Despite of all their shortcomings they will be further analyzed by one advanced U&A nucleon EM structure model simultaneously.

III. ADVANCED UNITARY AND ANALYTIC NUCLEON EM STRUCTURE MODEL

First we shall demonstrate, how it is possible in the advanced nucleon EM structure model to keep the coupling constants ratios $(f_\rho_{NN}/f_\rho), (f_\omega_{NN}/f_\omega), (f_{\phi NN}/f_\phi)$ and also
FIG. 2: Proton electric FF data in the space-like ($t < 0$) region from MAMI [16].

FIG. 3: Proton electric FF data in the time-like ($t > 0$) region from exps. in which $|G_p^E(t)| = |G_p^M(t)|$ has been assumed $(f_{\omega'NN}/f_\omega), (f_{\phi'NN}/f_\phi)$ as free parameters of the model to be then subsequently determined numerically in comparison of the advanced U&A model with all existing data simultaneously.

Nevertheless, before that we would like to note that the nucleon EM FFs $G_E^p(t), G_M^p(t), G_E^n(t), G_M^n(t)$ are very suitable for extraction of an experimental information...
on the nucleon EM structure from the earlier mentioned physical quantities (3)-(9). But for a construction of various nucleon EM structure models the flavour-independent iso-scalar and iso-vector parts $F_{1s}^N(t), F_{1v}^N(t), F_{2s}^N(t), F_{2v}^N(t)$ of the Dirac and Pauli FFs to be defined by a parametrization of the matrix element of the nucleon EM current...
FIG. 6: Data on the ratio $|G_E^p(t)/G_M^p(t)|$ in the time-like ($t > 0$) region [1],[2]

FIG. 7: Neutron electric FF data in the space-like ($t < 0$) region [33]-[39]

\[ < N|J_{\mu}^{EM}|N> = e\bar{u}(p')\{\gamma_{\mu}F_1^N(t) + \frac{i}{2m_N}\sigma_{\mu\nu}(p' - p)\nu F_2^N(t)\}u(p) \]  

are more suitable.
FIG. 8: Neutron electric FF data in the time-like ($t > 0$) region from an assumption $|G_E^n(t)| = |G_M^n(t)|$

FIG. 9: Neutron magnetic negative FF data in the space-like ($t < 0$) region [33], [40]–[46]
FIG. 10: Neutron magnetic FF data in the time-like ($t > 0$) region \[28\]

FIG. 11: Data on the ratio $\mu_n G^n_F(t)/G^n_M(t)$ in the space-like ($t < 0$) region from polarization experiments on the light nuclei \[47, 48\]
Both sets of these FFs are related as follows

\[ G_E^p(t) = \left[ F_{1s}^N(t) + F_{1v}^N(t) \right] + \frac{t}{4m_p^2} \left[ F_{2s}^N(t) + F_{2v}^N(t) \right] \]

\[ G_M^p(t) = \left[ F_{1s}^N(t) + F_{1v}^N(t) \right] + \left[ F_{2s}^N(t) + F_{2v}^N(t) \right] \]

\[ G_{E}^m(t) = \left[ F_{1s}^N(t) - F_{1v}^N(t) \right] + \frac{t}{4m_n^2} \left[ F_{2s}^N(t) - F_{2v}^N(t) \right] \]

\[ G_{M}^m(t) = \left[ F_{1s}^N(t) - F_{1v}^N(t) \right] + \left[ F_{2s}^N(t) - F_{2v}^N(t) \right] \]

whereby experimental fact of a creation of true neutral vector-meson resonances with quantum numbers of the photon in \( e^+e^- \rightarrow \text{hadrons} \) is in the first approximation taken into account by a saturation of \( F_{1s}^N(t), F_{2s}^N(t) \) with neutral iso-scalar vector mesons \( \omega(782), \phi(1020), \omega'(1420), \phi'(1680), \omega''(1650), \phi''(1710) \) and \( F_{1v}^N(t), F_{2v}^N(t) \) with neutral iso-vector vector-meson resonances \( \rho(770), \rho'(1450), \rho''(1700) \) in the corresponding VMD FF parametrization in the zero-width approximation.

For the sake of a generality let us consider FF \( F(t) \) with a normalization \( F(0) = F_0 \), the asymptotic behavior \( F(t)_{t \rightarrow \infty} \sim 1/t^n \) and to be saturated with \( n \)-true neutral vector mesons \( V_n \). Then in the framework of the standard VMD model in the zero-width approximation one can write

\[
F(t) = \frac{m_1^2}{m_1^2 - t} a_1 + \frac{m_2^2}{m_2^2 - t} a_2 + \cdots + \frac{m_n^2}{m_n^2 - t} a_n,
\]

where \( a_n = (f_{V_nNN}/f_{V_n}) \), \( f_{V_nNN} \) is the coupling constant of an interaction of the \( n \)-th vector-meson with nucleons and \( f_{V_n} \) is the universal vector-meson coupling constant to be determined numerically from the lepton width of the \( n \)-th vector-meson \( [11] \). Now requiring the normalization of \( (15) \) at \( t = 0 \) one gets the following equation for coupling constant ratios \( (f_{iNN}/f_i) \)

\[
\sum_{i=1}^{n} (f_{iNN}/f_i) = F_0.
\]

Then transforming \( (15) \) into a common denominator one gets in the numerator polynomial of \( t^{n-1} \) degree. In order to achieve the required asymptotic behavior of \( F(t) \) one has to put some coefficients in the polynomial of the numerator, starting from the highest degree \( t^{n-1} \), to be zero, and one comes to another \( m - 1 \) equations

\[
\sum_{i=1}^{n} m_i^{2r} (f_{iNN}/f_i) = 0; \quad r = 1, 2, \ldots, m - 1
\]
for $n$-coupling constant ratios. As a result a solution of the system of equations (16), (17) will lead $m$ coupling constant ratios to be given through the table masses of vector mesons and all additional coupling constant ratios $(f_{m+1}NN/f_{m+1}), \ldots, (f_nNN/f_n)$, which will be considered to be free parameters of the model.

The general solution of the system of eqs. (16), (17) for $n > m$ leads to the FF to be saturated by $n$-vector-meson resonances in the form suitable for the unitarization

$$F(t) = F_0 \frac{\prod_{j=1}^{m} m_j^2}{\prod_{j=1}^{m} (m_j^2 - t)} + \sum_{k=m+1}^{n} \left\{ \sum_{j=1}^{m} \frac{m_k^2}{(m_k^2 - t)} \prod_{j \neq i, j=1}^{m} m_j^2 \prod_{j \neq i, j=1}^{m} (m_j^2 - m_k^2) \right\} (f_{kNN}/f_k)$$

and for $n = m$

$$F(t) = F_0 \frac{\prod_{j=1}^{m} m_j^2}{\prod_{j=1}^{m} (m_j^2 - t)}$$

for which the required asymptotic behavior $F(t)_{t \to \infty} \sim 1/t^m$ and for $t = 0$ the normalization $F(0) = F_0$ are fulfilled automatically.

Now these general results will be applied to the flavor-independent iso-scalar and iso-vector parts of the Dirac and Pauli nucleon FFs $F_{1s}^N(t), F_{1v}^N(t), F_{2s}^N(t), F_{2v}^N(t)$ by means of which all required properties of the nucleon $U&\Lambda$ electromagnetic structure model like

- experimental fact of a creation of unstable vector-meson resonances in $e^+e^-$ annihilation processes into hadrons
- the analytic properties
- the reality conditions
- the unitarity conditions
- normalizations
- asymptotic behaviors
are achieved.

However, before the latter we have to specify normalizations and asymptotic behaviors of flavor-independent iso-scalar and iso-vector parts of the Dirac and Pauli nucleon FFs.

The nucleon EM FFs are normalized as follows

\[ G_E^p(0) = 1; G_M^p(0) = \mu_p; G_E^n(0) = 0; G_M^n(0) = \mu_n \]  

(20)

with \( \mu_p \) and \( \mu_n \) to be proton and neutron magnetic moments, respectively.

The asymptotic behaviors of nucleon EM FFs are

\[ G_E^p(t)_{|t\to\infty} = G_M^p(t)_{|t\to\infty} = G_E^n(t)_{|t\to\infty} = G_M^n(t)_{|t\to\infty} \sim \frac{1}{t^2}. \]  

(21)

Then from (14) one obtains the normalizations of iso-scalar and iso-vector parts of the Dirac and Pauli nucleon FFs

\[ F_{1s}^N(0) = F_{1v}^N(0) = \frac{1}{2}; F_{2s}^N(0) = \frac{1}{2}(\mu_p + \mu_n - 1); F_{2v}^N(0) = \frac{1}{2}(\mu_p - \mu_n - 1) \]  

(22)

and their asymptotic behaviors are

\[ F_{1s}^N(t)_{|t\to\infty} = F_{1v}^N(t)_{|t\to\infty} \sim \frac{1}{t^2}; F_{2s}^N(t)_{|t\to\infty} = F_{2v}^N(t)_{|t\to\infty} \sim \frac{1}{t^3}. \]  

(23)

If we apply (18) to \( F_{1s}^N(t) \) with \( m = 2 \) and \( n = \omega'', \phi'', \omega', \phi', \omega, \phi \) one obtains
\[ F_{1s}^N(t) = \frac{1}{2} \frac{m_{\omega'}^2 m_{\phi'}^2}{(m_{\omega'}^2 - t)(m_{\phi'}^2 - t)} + \]

\[ + \left\{ \frac{m_{\phi'}^2 m_{\omega}^2}{(m_{\phi'}^2 - t)(m_{\omega}^2 - t)} \frac{(m_{\phi}^2 - m_{\omega}^2)}{(m_{\phi'}^2 - m_{\omega}^2)} + \frac{m_{\phi'}^2 m_{\omega}^2}{(m_{\phi'}^2 - t)(m_{\omega}^2 - t)} \frac{(m_{\phi}^2 - m_{\omega}^2)}{(m_{\phi'}^2 - m_{\omega}^2)} - \frac{m_{\omega'}^2 m_{\phi'}^2}{(m_{\omega'}^2 - t)(m_{\phi'}^2 - t)} \right\} \left( f_{\phi NN}^{(1)} / f_{\phi'} \right) + \]

\[ + \left\{ \frac{m_{\phi'}^2 m_{\omega}^2}{(m_{\phi'}^2 - t)(m_{\omega}^2 - t)} \frac{(m_{\phi}^2 - m_{\omega}^2)}{(m_{\phi'}^2 - m_{\omega}^2)} + \frac{m_{\phi'}^2 m_{\omega}^2}{(m_{\phi'}^2 - t)(m_{\omega}^2 - t)} \frac{(m_{\phi}^2 - m_{\omega}^2)}{(m_{\phi'}^2 - m_{\omega}^2)} - \frac{m_{\omega'}^2 m_{\phi'}^2}{(m_{\omega'}^2 - t)(m_{\phi'}^2 - t)} \right\} \left( f_{\phi NN}^{(1)} / f_{\phi} \right). \]

To \( F_{1s}^N(t) \) with \( m = 2 \) and \( n = \rho'', \rho', \rho \) one obtains

\[ F_{1s}^N(t) = \frac{1}{2} \frac{m_{\rho'}^2 m_{\rho}^2}{(m_{\rho'}^2 - t)(m_{\rho}^2 - t)} + \]

\[ + \left\{ \frac{m_{\rho'}^2 m_{\rho}^2}{(m_{\rho'}^2 - t)(m_{\rho}^2 - t)} \frac{(m_{\rho}^2 - m_{\rho}^2)}{(m_{\rho'}^2 - m_{\rho}^2)} + \frac{m_{\rho'}^2 m_{\rho}^2}{(m_{\rho'}^2 - t)(m_{\rho}^2 - t)} \frac{(m_{\rho}^2 - m_{\rho}^2)}{(m_{\rho'}^2 - m_{\rho}^2)} - \frac{m_{\rho''}^2 m_{\rho'}^2}{(m_{\rho''}^2 - t)(m_{\rho'}^2 - t)} \right\} \left( f_{\rho NN}^{(1)} / f_{\rho} \right). \]

To \( F_{2s}^N(t) \) with \( m = 3 \) and \( n = \omega'', \phi'', \omega', \phi', \omega, \phi \) one obtains
$$F_{2s}^N(t) = \frac{1}{2}(\mu_p + \mu_n - 1) \frac{m_{\omega'}^2 m_{\rho}^2 m_{\phi}^2}{(m_{\omega'}^2 - t)(m_{\rho}^2 - t)(m_{\phi}^2 - t)} + (26)$$

$$+ \left\{ \frac{m_{\omega'}^2 m_{\rho}^2 m_{\phi}^2}{(m_{\omega'}^2 - t)(m_{\rho}^2 - t)(m_{\phi}^2 - t)} \right\} \left( \frac{m_{\omega'}^2 - m_{\omega}^2}{(m_{\omega'}^2 - m_{\omega}^2)(m_{\omega'}^2 - m_{\omega}^2)} + \frac{m_{\omega'}^2 m_{\phi}^2 m_{\phi}^2}{(m_{\omega'}^2 - t)(m_{\phi}^2 - t)(m_{\phi}^2 - t)} \right) +$$

$$+ \left\{ \frac{m_{\omega'}^2 m_{\rho}^2 m_{\phi}^2}{(m_{\omega'}^2 - t)(m_{\rho}^2 - t)(m_{\phi}^2 - t)} \right\} \left( \frac{m_{\omega'}^2 - m_{\omega}^2}{(m_{\omega'}^2 - m_{\omega}^2)(m_{\omega'}^2 - m_{\omega}^2)} + \right\} +$$

$$+ \left\{ \frac{m_{\omega'}^2 m_{\rho}^2 m_{\phi}^2}{(m_{\omega'}^2 - t)(m_{\rho}^2 - t)(m_{\phi}^2 - t)} \right\} \left( \frac{m_{\omega'}^2 - m_{\omega}^2}{(m_{\omega'}^2 - m_{\omega}^2)(m_{\omega'}^2 - m_{\omega}^2)} + \right\} +$$

$$+ \left\{ \frac{m_{\omega'}^2 m_{\rho}^2 m_{\phi}^2}{(m_{\omega'}^2 - t)(m_{\rho}^2 - t)(m_{\phi}^2 - t)} \right\} \left( \frac{m_{\omega'}^2 - m_{\omega}^2}{(m_{\omega'}^2 - m_{\omega}^2)(m_{\omega'}^2 - m_{\omega}^2)} + \right\} +$$

$$+ \left\{ \frac{m_{\omega'}^2 m_{\rho}^2 m_{\phi}^2}{(m_{\omega'}^2 - t)(m_{\rho}^2 - t)(m_{\phi}^2 - t)} \right\} \left( \frac{m_{\omega'}^2 - m_{\omega}^2}{(m_{\omega'}^2 - m_{\omega}^2)(m_{\omega'}^2 - m_{\omega}^2)} + \right\} +$$

$$+ \left\{ \frac{m_{\omega'}^2 m_{\rho}^2 m_{\phi}^2}{(m_{\omega'}^2 - t)(m_{\rho}^2 - t)(m_{\phi}^2 - t)} \right\} \left( \frac{m_{\omega'}^2 - m_{\omega}^2}{(m_{\omega'}^2 - m_{\omega}^2)(m_{\omega'}^2 - m_{\omega}^2)} + \right\} +$$

$$+ \left\{ \frac{m_{\omega'}^2 m_{\rho}^2 m_{\phi}^2}{(m_{\omega'}^2 - t)(m_{\rho}^2 - t)(m_{\phi}^2 - t)} \right\} \left( \frac{m_{\omega'}^2 - m_{\omega}^2}{(m_{\omega'}^2 - m_{\omega}^2)(m_{\omega'}^2 - m_{\omega}^2)} + \right\}$$

Application of (19) to $F_{2s}^N(t)$ with $m = 3$ and $n = \rho''$, $\rho', \rho$ leads to

$$F_{2s}^N(t) = \frac{1}{2}(\mu_p - \mu_n - 1) \frac{m_{\omega'}^2 m_{\rho}^2 m_{\phi}^2}{(m_{\omega'}^2 - t)(m_{\rho}^2 - t)(m_{\phi}^2 - t)} + (27)$$

The expressions (24)-(27) are automatically normalized and they govern the asymptotic behaviors as predicted by the quark model of hadrons.

An unitarization of the model, i.e. an incorporation of the correct analytic properties of
the nucleon EM FFs, is achieved by the non-linear transformations

\begin{equation}
\begin{aligned}
t &= t_0^s + \frac{4(t_1^s - t_0^s)}{1/V(t) - V(t)}; \quad t = t_0^v + \frac{4(t_1^v - t_0^v)}{1/W(t) - W(t)}; \\
t &= t_0^s + \frac{4(t_2^s - t_0^s)}{1/U(t) - U(t)}; \quad t = t_0^v + \frac{4(t_2^v - t_0^v)}{1/X(t) - X(t)};
\end{aligned}
\end{equation}

respectively and a subsequent inclusion of the nonzero values of vector-meson widths.

In non-linear transformations \(t_0^s = 9m_\pi^2\), \(t_0^v = 4m_\pi^2\), \(t_1^s\), \(t_1^v\), \(t_2^s\), \(t_2^v\) are the square-root branch points, as it is transparent from the inverse transformations

\begin{equation}
V(t) = i\sqrt{\left(\frac{t_1^s - t_0^s}{t_0^s}\right)^{1/2} + \left(\frac{t - t_0^s}{t_0^s}\right)^{1/2} - \sqrt{\left(\frac{t_1^s - t_0^s}{t_0^s}\right)^{1/2} - \left(\frac{t - t_0^s}{t_0^s}\right)^{1/2}}} \end{equation}

and similarly for \(W(t), U(t), \) and \(X(t), \) which map the corresponding four-sheeted Riemann surfaces always into one \(V-, W-, U-, \) and \(X-\) plane.

Practically let us demonstrate the unitarization on the case of the Dirac iso-scalar FF \(24\). The non-linear transformation \(t = t_0^s + \frac{4(t_1^s - t_0^s)}{1/V(t) - V(t)}\) implies also the relations \(m_r^2 = t_0^s + \frac{4(t_1^s - t_0^s)}{1/V(t) - V(t)}\) and \(0 = t_0^s + \frac{4(t_1^s - t_0^s)}{1/V(t) - V(t)}\).

Then every term

\begin{equation}
\frac{m_r^2}{m_r^2 - t} = \frac{m_r^2 - 0}{m_r^2 - t} = \frac{1 - V_N^{-2}}{1 - V_N^{-2}} \cdot \frac{(V_N - V_{r_0})(V_N + V_{r_0})(V_N - 1/V_{r_0})(V_N + 1/V_{r_0})}{(V - V_{r_0})(V + V_{r_0})(V - 1/V_{r_0})(V + 1/V_{r_0})}
\end{equation}

in \(24\) is factorized into asymptotic term and on the so-called finite-energy term (for \(|t| \to \infty\) it turns out to be a real constant) giving a resonant behavior around \(t = m_r^2\).

One can prove

1. if \(m_r^2 - \Gamma_r^2/4 < t_m \Rightarrow V_{r_0} = -V_{r_0}^*
2. if \(m_r^2 - \Gamma_r^2/4 > t_m \Rightarrow V_{r_0} = 1/V_{r_0}^*

which lead in the case 1. to the expression

\begin{equation}
\frac{m_r^2}{m_r^2 - t} = \left(\frac{1 - V_N^{-2}}{1 - V_N^{-2}}\right) \cdot \frac{(V_N - V_{r_0})(V_N - V_{r_0}^*)(V_N - 1/V_{r_0})(V_N - 1/V_{r_0}^*)}{(V - V_{r_0})(V - V_{r_0}^*)(V - 1/V_{r_0})(V - 1/V_{r_0}^*)}
\end{equation}

and in the case 2. to the following expression

\begin{equation}
\frac{m_r^2}{m_r^2 - t} = \left(\frac{1 - V_N^{-2}}{1 - V_N^{-2}}\right) \cdot \frac{(V_N - V_{r_0})(V_N - V_{r_0}^*)(V_N + V_{r_0})(V_N + V_{r_0}^*)}{(V - V_{r_0})(V - V_{r_0}^*)(V + V_{r_0})(V + V_{r_0}^*)}.
\end{equation}
Lastly, introducing the non-zero width of the resonance by a substitution

\[ m_r^2 \rightarrow (m_r - i\Gamma_r/2)^2 \]  

(33)
i.e. simply one has to rid of ”0” in sub-indices of the previous two expressions, one gets:
in the 1. case

\[ \frac{m_r^2}{m_r^2 - t} \rightarrow \left(1 - V^2\right)^2 \frac{(V_N - V_r)(V_N - V_r^*)(V_N - 1/V_r)(V_N - 1/V_r^*)}{(V - V_r)(V - V_r^*)(V - 1/V_r)(V - 1/V_r^*)} = \]  

(34)

= \left(1 - \frac{V^2}{V_N^2}\right)^2 L_r(V)

and in the 2. case

\[ \frac{m_r^2}{m_r^2 - t} \rightarrow \left(1 - V^2\right)^2 \frac{(V_N - V_r)(V_N - V_r^*)(V_N + V_r)(V_N + V_r^*)}{(V - V_r)(V - V_r^*)(V + V_r)(V + V_r^*)} = \]  

(35)

= \left(1 - \frac{V^2}{V_N^2}\right)^2 H_r(V).

Then for every iso-scalar and iso-vector Dirac and Pauli FF one obtains just one analytic and smooth from \(-\infty\) to \(+\infty\) function in the forms
dependent on 2 free physically interpretable parameters ($t_{1s}^{1v}$) and

$$
F_{1s}^{N}[V(t)] = \left( \frac{1 - V^2}{1 - W_N^2} \right)^4 \left\{ \frac{1}{2} H_{\phi''}(V) H_{\phi''}(V) + \right. $$

$$
+ \left[ H_{\phi''}(V) H_{\omega''}(V) \frac{(C_{1s}^{1s} - C_{1s}^{1s})}{(C_{1s}^{1s} - C_{1s}^{1s})} + H_{\omega''}(V) H_{\omega''}(V) \frac{(C_{1s}^{1s} - C_{1s}^{1s})}{(C_{1s}^{1s} - C_{1s}^{1s})} - 

$$

$$
-H_{\omega''}(V) H_{\phi''}(V) \right\} (f_{\phi''NN}/f_{\omega'}) + 

$$

$$
+ \left[ H_{\phi''}(V) L_{\phi''}(V) \frac{(C_{1s}^{1s} - C_{1s}^{1s})}{(C_{1s}^{1s} - C_{1s}^{1s})} + H_{\omega''}(V) L_{\omega''}(V) \frac{(C_{1s}^{1s} - C_{1s}^{1s})}{(C_{1s}^{1s} - C_{1s}^{1s})} - 

$$

$$
-H_{\omega''}(V) H_{\phi''}(V) \right\} (f_{\phi''NN}/f_{\omega'}) + 

$$

$$
+ \left[ H_{\phi''}(V) L_{\phi''}(V) \frac{(C_{1s}^{1s} - C_{1s}^{1s})}{(C_{1s}^{1s} - C_{1s}^{1s})} + H_{\omega''}(V) L_{\phi''}(V) \frac{(C_{1s}^{1s} - C_{1s}^{1s})}{(C_{1s}^{1s} - C_{1s}^{1s})} - 

$$

$$
-H_{\omega''}(V) H_{\phi''}(V) \right\} (f_{\phi''NN}/f_{\omega'}) \} 
$$

dependent on 5 free physically interpretable parameters,

$$(f_{\omega'NN}/f_{\omega'}), (f_{\phi'NN}/f_{\phi'}), (f_{\omega''NN}/f_{\omega''}), (f_{\phi''NN}/f_{\phi''}), t_{1s}^{1v}$$

$$
F_{1v}^{N}[W(t)] = \left( \frac{1 - W^2}{1 - W_N^2} \right)^4 \left\{ \frac{1}{2} L_{\rho}(W) L_{\rho'}(W) + \right. $$

$$
+ \left[ L_{\rho'}(W) L_{\rho''}(W) \frac{(C_{1s}^{1v} - C_{1s}^{1v})}{(C_{1s}^{1v} - C_{1s}^{1v})} + L_{\rho}(W) L_{\rho''}(W) \frac{(C_{1s}^{1v} - C_{1s}^{1v})}{(C_{1s}^{1v} - C_{1s}^{1v})} - 

$$

$$
-L_{\rho}(W) L_{\rho'}(W) \right\} (f_{\rho''NN}/f_{\rho}) \} 
$$

dependent on 2 free physically interpretable parameters ($f_{\rho''NN}/f_{\rho}$) and $t_{1s}^{1v}$
\[
F_{2n}^N[U(t)] = \left( \frac{1 - U^2}{1 - U_{2n}^2} \right)^6 \left\{ \frac{1}{2} (\mu_p + \mu_n - 1) H_{\omega''}(U) H_{\phi''}(U) H_{\omega'}(U) + \right.
\]
\[+ \left[ H_{\phi''}(U) H_{\omega'}(U) H_{\phi'}(U) \frac{(C_{2s}^{\omega''} - C_{2s}^{\omega})(C_{2s}^{\phi''} - C_{2s}^{\phi})}{(C_{2s}^{\omega''} - C_{2s}^{\phi})(C_{2s}^{\phi''} - C_{2s}^{\phi})} + \right.
\]
\[+ H_{\omega''}(U) H_{\phi''}(U) H_{\phi'}(U) \frac{(C_{2s}^{\phi''} - C_{2s}^{\phi})(C_{2s}^{\omega''} - C_{2s}^{\omega})}{(C_{2s}^{\phi''} - C_{2s}^{\omega})(C_{2s}^{\omega''} - C_{2s}^{\omega})} + \right.
\]
\[+ H_{\omega''}(U) H_{\phi''}(U) H_{\phi'}(U) \frac{(C_{2s}^{\phi''} - C_{2s}^{\phi})(C_{2s}^{\phi''} - C_{2s}^{\phi})}{(C_{2s}^{\phi''} - C_{2s}^{\phi})(C_{2s}^{\phi''} - C_{2s}^{\phi})} - \]
\[+ H_{\omega''}(U) H_{\phi''}(U) H_{\phi'}(U) \left( f_{\phi,NN}^{(2)} / f_{\phi'} \right) + \]
\[+ \left[ H_{\phi''}(U) H_{\omega'}(U) L_{\omega}(U) \frac{(C_{2s}^{\omega''} - C_{2s}^{\omega})(C_{2s}^{\phi''} - C_{2s}^{\phi})}{(C_{2s}^{\omega''} - C_{2s}^{\phi})(C_{2s}^{\phi''} - C_{2s}^{\phi})} + \right.
\]
\[+ H_{\omega''}(U) H_{\phi'}(U) L_{\omega}(U) \frac{(C_{2s}^{\phi''} - C_{2s}^{\phi})(C_{2s}^{\omega''} - C_{2s}^{\omega})}{(C_{2s}^{\phi''} - C_{2s}^{\omega})(C_{2s}^{\omega''} - C_{2s}^{\omega})} + \right.
\]
\[+ H_{\omega''}(U) H_{\phi'}(U) L_{\omega}(U) \frac{(C_{2s}^{\phi''} - C_{2s}^{\phi})(C_{2s}^{\phi''} - C_{2s}^{\phi})}{(C_{2s}^{\phi''} - C_{2s}^{\phi})(C_{2s}^{\phi''} - C_{2s}^{\phi})} - \]
\[+ H_{\omega''}(U) H_{\phi'}(U) L_{\phi}(U) \left( f_{\phi,NN}^{(2)} / f_{\phi} \right) \}
\]

dependent on 4 free physically interpretable parameters
\((f_{\phi,NN}^{(2)} / f_{\phi'}), (f_{\omega,NN}^{(2)} / f_{\omega}), (f_{\phi,NN}^{(2)} / f_{\phi}), t_{in}^{2s}\)

\[
F_{2n}^N[X(t)] = \left( \frac{1 - X^2}{1 - X_{2n}^2} \right)^6 \left\{ \frac{1}{2} (\mu_p - \mu_n - 1) L_{\mu}(U) L_{\phi''}(U) H_{\phi'}(U) + \right. \]
\[\left. \right. \]
\[\}
\]

\[
(39)
\]

dependent on 1 free physically interpretable parameter \(t_{in}^{2s}\), where
\[ L_r(V) = \frac{(V_N - V_r)(V_N - V_r^*)(V_N - 1/V_r)(V_N - 1/V_r^*)}{(V - V_r)(V - V_r^*)(V - 1/V_r)(V - 1/V_r^*)}, \]  
\[ C_{r_1}^{1s} = \frac{(V_N - V_r)(V_N - V_r^*)(V_N - 1/V_r)(V_N - 1/V_r^*)}{-(V_r - 1/V_r)(V_r - 1/V_r^*)}, \ r = \omega, \phi \]  
\[ H_l(V) = \frac{(V_N - V_l)(V_N - V_l^*)(V_N + V_l)(V_N + V_l^*)}{(V - V_l)(V - V_l^*)(V + V_l)(V + V_l^*)}, \]  
\[ C_{l_1}^{1s} = \frac{(V_N - V_l)(V_N - V_l^*)(V_N + V_l)(V_N + V_l^*)}{-(V_l - 1/V_l)(V_l - 1/V_l^*)}, \ l = \omega'', \phi'', \omega', \phi' \]  
\[ L_k(W) = \frac{(W_N - W_k)(W_N - W_k^*)(W_N - 1/W_k)(W_N - 1/W_k^*)}{(W - W_k)(W - W_k^*)(W - 1/W_k)(W - 1/W_k^*)}, \]  
\[ C_{k_1}^{1v} = \frac{(W_N - W_k)(W_N - W_k^*)(W_N - 1/W_k)(W_N - 1/W_k^*)}{-(W_k - 1/W_k)(W_k - 1/W_k^*)}, \ k = \rho'', \rho', \rho \]  
\[ L_r(U) = \frac{(U_N - U_r)(U_N - U_r^*)(U_N - 1/U_r)(U_N - 1/U_r^*)}{(U - U_r)(U - U_r^*)(U - 1/U_r)(U - 1/U_r^*)}, \]  
\[ C_{r_2}^{2s} = \frac{(U_N - U_r)(U_N - U_r^*)(U_N - 1/U_r)(U_N - 1/U_r^*)}{-(U_r - 1/U_r)(U_r - 1/U_r^*)}, \ r = \omega, \phi \]  
\[ H_l(U) = \frac{(U_N - U_l)(U_N - U_l^*)(U_N + U_l)(U_N + U_l^*)}{(U - U_l)(U - U_l^*)(U + U_l)(U + U_l^*)}, \]  
\[ C_{l_2}^{2s} = \frac{(U_N - U_l)(U_N - U_l^*)(U_N + U_l)(U_N + U_l^*)}{-(U_l - 1/U_l)(U_l - 1/U_l^*)}, \ l = \omega'', \phi'', \omega', \phi' \]  
\[ L_k(X) = \frac{(X_N - X_k)(X_N - X_k^*)(X_N - 1/X_k)(X_N - 1/X_k^*)}{(X - X_k)(X - X_k^*)(X - 1/X_k)(X - 1/X_k^*)}, \]  
\[ C_{k_2}^{2v} = \frac{(X_N - X_k)(X_N - X_k^*)(X_N - 1/X_k)(X_N - 1/X_k^*)}{-(X_k - 1/X_k)(X_k - 1/X_k^*)}, \ k = \rho', \rho \]  
\[ H_{\rho''}(X) = \frac{(X_N - X_{\rho''})(X_N - X_{\rho''}^*)(X_N + X_{\rho''})(X_N + X_{\rho''}^*)}{(X - X_{\rho''})(X - X_{\rho''}^*)(X + X_{\rho''})(X + X_{\rho''}^*)}, \]  
\[ C_{\rho''}^{2v} = \frac{(X_N - X_{\rho''})(X_N - X_{\rho''}^*)(X_N + X_{\rho''})(X_N + X_{\rho''}^*)}{-(X_{\rho''} - 1/X_{\rho''})(X_{\rho''} - 1/X_{\rho''}^*)}. \]
since in a fitting procedure of all existing data by means of this version of the $U$&$A$ nucleon EM structure model simultaneously one finds

\[
\begin{align*}
(m_\omega^2 - \Gamma_\omega^2/4) &< t_{in}^{1s}; 
(m_\phi^2 - \Gamma_\phi^2/4) < t_{in}^{1s}; 
(47) \\
(m_\omega'^2 - \Gamma_\omega'^2/4) &> t_{in}^{1s}; 
(m_\phi'^2 - \Gamma_\phi'^2/4) > t_{in}^{1s}; 
(48) \\
(m_\omega''^2 - \Gamma_\omega''^2/4) &> t_{in}^{1s}; 
(m_\phi''^2 - \Gamma_\phi''^2/4) > t_{in}^{1s}; 
(49) \\
(m_\rho^2 - \Gamma_\rho^2/4) &< t_{in}^{1v}; 
(m_\rho'^2 - \Gamma_\rho'^2/4) < t_{in}^{1v}; 
(m_\rho''^2 - \Gamma_\rho''^2/4) < t_{in}^{1v}; 
(50)
\end{align*}
\]

IV. RESULTS OF THE ANALYSIS OF EXISTING NUCLEON EM FF DATA

We have collected 534 reliable experimental points on the nucleon EM structure from more than 40 independent experiments as they are represented graphically in Figs. 1-11. They have been analysed simultaneously by means of the 9 resonance $U$&$A$ model of the nucleon EM structure as formulated in the previous Section. The minimum of the $\chi^2=2214$ has been achieved with the values of 12 free parameters of the model with a clear physical meaning as they are presented in Table 1.
Table 1. The numerical values of the free parameters of the nucleon $U&A$ EM structure model, respecting SU(3) symmetry, formulated in the previous section

\begin{align*}
  t_{in}^{1s} &= (1.0442 \pm 0.0200) GeV^2 \\
  t_{in}^{2s} &= (1.0460 \pm 0.1399) GeV^2 \\
  t_{in}^{1v} &= (2.9506 \pm 0.5326) GeV^2 \\
  t_{in}^{2v} &= (2.3449 \pm 0.7656) GeV^2 \\
  (f_{\omega NN}^{(1)}/f_{\omega}) &= (1.5717 \pm 0.0022) \\
  (f_{\phi NN}^{(1)}/f_{\phi}) &= (-1.1247 \pm 0.0011) \\
  (f_{\omega NN}^{(1)}/f_{\omega'}) &= (0.0418 \pm 0.0065) \\
  (f_{\phi NN}^{(1)}/f_{\phi'}) &= (0.1879 \pm 0.0010) \\
  (f_{\omega NN}^{(2)}/f_{\omega}) &= (-0.2096 \pm 0.0067) \\
  (f_{\phi NN}^{(2)}/f_{\phi}) &= (0.2657 \pm 0.0067) \\
  (f_{\phi NN}^{(2)}/f_{\phi'}) &= (0.1781 \pm 0.0029) \\
  (f_{\rho NN}^{(1)}/f_{\rho}) &= (0.3747 \pm 0.0022)
\end{align*}

whereby the results are not very sensitive on the position of the effective inelastic thresholds $t_{in}^{1s}, t_{in}^{2s}, t_{in}^{1v}, t_{in}^{2v}$.

The corresponding description of the data by means of this $U&A$ model with numerical values of parameters given in Table 1. is graphically shown in Figs. 12-19.

Of course one could not expect that a description of such gigantic set of data to be obtained from so much independent experiments, every of them to be charged with corresponding statistical and systematical errors, will be consistent with rules of a standard statistics. We have been able to reduce the total $\chi^2$ on 522 degrees of freedom only to the value 4.24.

The results of the analysis can be summarized as follows:

- a perfect description (see Fig. 12) of the most reliable nucleon EM structure data, i.e. the data on the ratio $\mu_p G_E^p(t)/G_M^p(t)$ in the space-like region to be obtained in polarization experiments, is achieved.
• description of all other existing data (see Figs. 13-19) is quite reasonable too, besides an inconsistency of the data on neutron EM FFs in the time-like region with all other data on nucleon EM FFs, indicating that the total-cross section of $e^+e^- \rightarrow n\bar{n}$ is considerably larger than it has been found in FENICE experiment [28] in Frascati. So, we are coming to the same conclusions as it was pointed out by one of the authors in papers [50], [51] published more than 25 years ago.

• again the existence of the zero of the proton electric FF $G_E(t)$ approximately at $t_z = -13$ GeV$^2$ is confirmed, which has been predicted in the paper [52] for the first time.

• electric and magnetic mean square charge radii of the nucleons are determined to be $<r_{Ep}^2> = (0.7182 \pm 0.0369)$ fm$^2$, $<r_{Mp}^2> = (0.7573 \pm 0.0133)$ fm$^2$, $<r_{En}^2> = (-0.1162 \pm 0.0369)$ fm$^2$, $<r_{Mn}^2> = (0.8312 \pm 0.0195)$ fm$^2$, whereby the obtained electric root mean square charge radius of the proton is consistent with the value $<r_{Ep}^2> = 0.84184 \pm 0.00067$ fm determined in the muon hydrogen atom spectroscopy [53] and in this way the electric proton charge radius puzzle is definitely solved. Recently to the same conclusions came also Ulf.-G.Meissner with collaborators in [54].

• electric mean square charge radius of the neutron is found to be almost identical with the value given by Rev. of Part. Physics [8].

In Figs. 20-25 it is clearly demonstrated that the nucleon EM FFs represented by the $U&A$ model indeed fulfil the reality condition $G^*(t) = G(t^*)$, i.e. they all are real functions from $-\infty$ up to the lowest branch point $t_0 = 4m_n^2$ on the positive real axis.

The imaginary parts of all nucleon EM FFs are different from zero just only from the lowest branch point at $t = 0.0784GeV^2$ to $+\infty$ and their behaviors are given by the unitarity conditions of the corresponding FFs.

V. NUMERICAL VALUES OF THE $f^F$, $f^D$, $f^S$ AND $f^{F'}$, $f^{D'}$, $f^{S'}$ COUPLING CONSTANTS

The $SU(3)$ invariant Lagrangian (2) of vector-meson nonet $\rho^-, \rho^0, \rho^+, K^{*-}, K^{*0}, \bar{K}^{*0}, K^{*+}, \omega, \phi$ with $1/2^+$ octet baryons $p$, $n$, $\Lambda$, $\Sigma^+$, $\Sigma^0$, $\Sigma^-$, $\Xi^0$, $\Xi^-$ provides the following
FIG. 12: Prediction of proton electric to magnetic FFs ratio behavior in space-like region by $U\&A$ model respecting $SU(3)$ symmetry and its comparison with existing data.

FIG. 13: Prediction of proton electric FF behavior by $U\&A$ model respecting $SU(3)$ symmetry and its comparison with existing data.
FIG. 14: Prediction of proton magnetic FF behavior by $U\&A$ model respecting $SU(3)$ symmetry and its comparison with existing data.

FIG. 15: Prediction of the absolute value of proton electric to magnetic FFs ratio behavior in time-like region by $U\&A$ model respecting $SU(3)$ symmetry and its comparison with existing data.
FIG. 16: Prediction of the absolute value of neutron electric to magnetic FFs ratio behavior in time-like region by $U\&A$ model respecting $SU(3)$ symmetry.

FIG. 17: Prediction of neutron electric FF behavior by $U\&A$ model respecting $SU(3)$ symmetry and its comparison with existing the data.
FIG. 18: Prediction of neutron magnetic FF behavior by U&A model respecting $SU(3)$ symmetry and its comparison with existing data.

FIG. 19: Prediction of neutron electric to magnetic FFs ratio behavior in space-like region by U&A model respecting $SU(3)$ symmetry and its comparison with existing data.
FIG. 20: Prediction of real (solid line) and imaginary (dashed line) parts of the proton electric FF by $U\&A$ model respecting $SU(3)$ symmetry.

expressions

$$f^{(1)}_{\rho NN} = \frac{1}{2}(f^D_1 + f^F_1)$$  \hspace{0.5cm} (51)
$$f^{(1)}_{\omega NN} = \frac{1}{\sqrt{2}} f^S_1 \cos \theta - \frac{1}{2\sqrt{3}} (3f^F_1 - f^D_1) \sin \theta$$
$$f^{(1)}_{\phi NN} = \frac{1}{\sqrt{2}} f^S_1 \sin \theta + \frac{1}{2\sqrt{3}} (3f^F_1 - f^D_1) \cos \theta$$

and

$$f^{(2)}_{\rho NN} = \frac{1}{2}(f^D_2 + f^F_2)$$  \hspace{0.5cm} (52)
$$f^{(2)}_{\omega NN} = \frac{1}{\sqrt{2}} f^S_2 \cos \theta - \frac{1}{2\sqrt{3}} (3f^F_2 - f^D_2) \sin \theta$$
$$f^{(2)}_{\phi NN} = \frac{1}{\sqrt{2}} f^S_2 \sin \theta + \frac{1}{2\sqrt{3}} (3f^F_2 - f^D_2) \cos \theta$$

where angle $\theta = 43.8^0$ and it is given by the Gell-Mann-Okubo quadratic mass formula

$$m_{\phi(1020)}^2 \cos^2 \theta + m_{\omega(782)}^2 \sin^2 \theta = \frac{4m_{K^*}^2(892) - m_{\rho(770)}^2}{3}.$$  \hspace{0.5cm} (53)
Fig. 21: Prediction of real (solid line) and imaginary (dashed line) parts of the proton magnetic FF by U&A model respecting SU(3) symmetry.

From the SU(3) invariant Lagrangian of the first excited vector-meson nonet $\rho^{-}, \rho^{0}, \rho^{+}, K^{*-}, K^{*0}, K^{*+}, \omega, \phi$ with $1/2^{+}$ octet baryons $p, n, \Lambda, \Sigma^{+}, \Sigma^{0}, \Sigma^{-}, \Xi^{0}, \Xi^{-}$

$$L_{V'BB} = \frac{i}{\sqrt{2}} f^F [\bar{B}_\beta \gamma_\mu B^\beta_\gamma - \bar{B}^\beta_\gamma \gamma_\mu B^\alpha_\beta] (V'_\gamma)_{\alpha} +$$

$$+ \frac{i}{\sqrt{2}} f^D [\bar{B}^\beta_\gamma \gamma_\mu B^\alpha_\beta + \bar{B}^\alpha_\gamma \gamma_\mu B^\beta_\gamma] (V'_\gamma)_{\alpha} +$$

$$+ \frac{i}{\sqrt{2}} f^S \bar{B}^\alpha_\beta \gamma_\mu B^\beta_\alpha \omega_{\mu}'$$

another two systems of expressions

$$f^{(1)}_{\rho'NN} = \frac{1}{2} (f^{D'}_{1} + f^{F'}_{1})$$

$$f^{(1)}_{\omega'NN} = \frac{1}{\sqrt{2}} f^{S'}_{1} \cos \theta' - \frac{1}{2\sqrt{3}} (3f^{F'}_{1} - f^{D'}_{1}) \sin \theta'$$

$$f^{(1)}_{\phi'NN} = \frac{1}{\sqrt{2}} f^{S'}_{1} \sin \theta' + \frac{1}{2\sqrt{3}} (3f^{F'}_{1} - f^{D'}_{1}) \cos \theta'$$

and
FIG. 22: Prediction of a phase difference of the proton electric and magnetic FFs by U&A model respecting $SU(3)$ symmetry.

\begin{align*}
\rho_{NN}^{(2)} f &\equiv \frac{1}{2} (f_D + f_F') \\
\omega_{NN}^{(2)} f &\equiv \frac{1}{\sqrt{2}} f_S' \cos \theta' - \frac{1}{2\sqrt{3}} (3f_F' - f_D') \sin \theta' \\
\phi_{NN}^{(2)} f &\equiv \frac{1}{\sqrt{2}} f_S' \sin \theta' + \frac{1}{2\sqrt{3}} (3f_F' - f_D') \cos \theta'
\end{align*}

are obtained, where angle $\theta' = 50.3^0$ and it is again given by the Gell-Mann-Okubo quadratic mass formula

$$m_{\rho'(1680)}^2 \cos^2 \theta' + m_{\omega'(1420)}^2 \sin^2 \theta' = \frac{4m_{K\omega'(1680)}^2 - m_{\rho'(1450)}^2}{3},$$

however, with the first excited states of the corresponding particles.

So, if one knows numerical values of the coupling constants on the left-hand side of the expressions (51), (52) and (55), (56) one can find numerical values of all $f_F, f_D, f_S$ and $f_F', f_D', f_S'$ coupling constants in both $SU(3)$ invariant Lagrangians (3) and (54), needed for a prediction of EM FFs behaviors of all 1/2 octet hyperons $\Lambda, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-$. 

For (51) and (52) it is straightforward to determine them from the numerical values in Table 1., besides $(f_{\rho NN}^{(2)}/f_\rho)$ which is calculated by means of the relation.
FIG. 23: Prediction of real (solid line) and imaginary (dashed line) parts of the neutron electric FF by U&A model respecting $SU(3)$ symmetry.

\[
(f^{(2)}_{\rho NN}/f_{\rho}) = \frac{1}{2}(\mu_p - \mu_n - 1) \frac{C_{\rho'}^{2\nu} C_{\rho}^{2\nu}}{(C_{\rho'}^{2\nu} - C_{\rho}^{2\nu})(C_{\rho'}^{2\nu} - C_{\rho'}^{2\nu})} = 2.8956, \quad (58)
\]

and the values $f_\rho = 4.9582$, $f_\omega = 17.0620$, $f_\phi = -13.4428$ to be calculated from existing data on lepton width $\Gamma(V \to e^+e^-)$ by means of the relation $\Gamma(V \to e^+e^-) = \frac{\alpha^2 m_V}{3} \left(\frac{f_L^2}{4\pi}\right)^{-1}$.

The results are

\[
\begin{align*}
  f_{\rho NN}^{(1)} & = 1.8578 \quad (59) \\
  f_{\omega NN}^{(1)} & = 26.8163 \\
  f_{\phi NN}^{(1)} & = 15.1191
\end{align*}
\]

and

\[
\begin{align*}
  f_{\rho NN}^{(2)} & = 14.3570 \quad (60) \\
  f_{\omega NN}^{(2)} & = -3.5762 \\
  f_{\phi NN}^{(2)} & = -3.5718
\end{align*}
\]

However, for excited vector meson coupling constants in (55) and (56) a big problem appeared. There are no data (see [8]) on $\Gamma(V \to e^+e^-)$ for $\omega'(1420), \rho'(1450), \phi'(1680)$.
in order to determine $f_{\rho'}$, $f_{\omega'}$ and $f_{\phi'}$. The first two are calculated from lepton widths estimated by Donnachie and Clegg [10] $f_{\rho'} = 13.6491$, $f_{\omega'} = 47.6022$ and the third one for $\phi'$ is determined to be $f_{\phi'} = -33.6598$ from the relations $f_{\rho'}^2 : f_{\omega'}^2 : f_{\phi'}^2 = 1 : 1 : 2$ following (see [55]) from the quark structure of the corresponding vector mesons and the electric charges of the three constituent quarks from which these vector mesons are compound.

Having numerical values of $f_{\rho'}$, $f_{\omega'}$, $f_{\phi'}$ and calculating missing in Table 1. coupling constant ratios for excited vector mesons by the relations

$$
(f_{\rho'}^{(1)} / f_{\rho'}) = \frac{1}{2} \left( \frac{C_{\rho'}^{1v}}{C_{\rho'}^{1v} - C_{\rho'}^{1v}} - \frac{C_{\rho'}^{1v} - C_{\rho'}^{1v}}{C_{\rho'}^{1v} - C_{\rho'}^{1v}} \right) \left( \frac{f_{\rho NN}^{(1)}}{f_{\rho NN}^{(1)}} \right) = 0.7635
$$

(61)

$$
(f_{\rho'}^{(2)} / f_{\rho'}) = -\frac{1}{2} (\mu_p - \mu_n - 1) \frac{C_{\rho'}^{2v} C_{\rho'}^{2v}}{C_{\rho'}^{2v} - C_{\rho'}^{2v}) \frac{C_{\rho'}^{2v} - C_{\rho'}^{2v}}{C_{\rho'}^{2v} - C_{\rho'}^{2v}} = -1.3086,
$$

(62)
FIG. 25: Prediction of a phase difference of the neutron electric and magnetic FFs by U&A model respecting $SU(3)$ symmetry.

\[
(f_{\omega'''NN}/f_{\omega'}) = \frac{1}{2} (\mu_p + \mu_n - 1) \frac{C_{\omega'}^2 C_{\phi'}^2}{(C_{\phi'}^2 - C_{\omega'}^2)(C_{\omega'''}^2 - C_{\omega''}^2)} - \frac{(C_{\phi'}^2 - C_{\omega'}^2)(C_{\omega''}^2 - C_{\omega'''}^2)}{(C_{\phi'}^2 - C_{\omega'}^2)(C_{\omega'''}^2 - C_{\omega''}^2)} (f_{\omega NN}/f_{\omega}) - \frac{(C_{\phi'}^2 - C_{\omega'}^2)(C_{\omega''}^2 - C_{\omega'''}^2)}{(C_{\phi'''}^2 - C_{\omega'}^2)(C_{\omega''}^2 - C_{\omega'''}^2)} (f_{\phi NN}/f_{\phi}) - \frac{(C_{\phi'''}^2 - C_{\omega''}^2)(C_{\omega''}^2 - C_{\phi''})}{(C_{\phi'''}^2 - C_{\omega'}^2)(C_{\omega''}^2 - C_{\omega'''}^2)} (f_{\phi NN}/f_{\phi'}) = -0.5771
\]

one obtains the results

\[
\begin{align*}
  f_{\rho'''}_{NN} & = 10.4211 \\
  f_{\omega'''}_{NN} & = 1.9900 \\
  f_{\phi'''}_{NN} & = -6.3247
\end{align*}
\]
\[ f_{\rho^2}^{(2)} = -17.8612 \]  
\[ f_{\omega^2}^{(2)} = -27.4712 \]  
\[ f_{\phi^2}^{(2)} = -5.9948 \]

Now, by a solution of the systems of algebraic equations (51), (52), (55) and (56) according to three unknown constants \( f^F, f^D, f^S \) one comes to the following expressions

\[ f_1^F = \frac{1}{2} \left[ \sqrt{3} \left( f_{\phi NN}^{(1)} \cos \theta - f_{\omega NN}^{(1)} \sin \theta \right) + f_{\rho NN}^{(1)} \right] \]  
\[ f_1^D = \frac{1}{2} \left[ 3 f_{\rho NN}^{(1)} - \sqrt{3} \left( f_{\phi NN}^{(1)} \cos \theta - f_{\omega NN}^{(1)} \sin \theta \right) \right] \]  
\[ f_1^S = \sqrt{2} \left( f_{\omega NN}^{(1)} \cos \theta + f_{\phi NN}^{(1)} \sin \theta \right) \]  

\[ f_2^F = \frac{1}{2} \left[ \sqrt{3} \left( f_{\phi NN}^{(2)} \cos \theta - f_{\omega NN}^{(2)} \sin \theta \right) + f_{\rho NN}^{(2)} \right] \]  
\[ f_2^D = \frac{1}{2} \left[ 3 f_{\rho NN}^{(2)} - \sqrt{3} \left( f_{\phi NN}^{(2)} \cos \theta - f_{\omega NN}^{(2)} \sin \theta \right) \right] \]  
\[ f_2^S = \sqrt{2} \left( f_{\omega NN}^{(2)} \cos \theta + f_{\phi NN}^{(2)} \sin \theta \right) \]  

\[ f_1'^F = \frac{1}{2} \left[ \sqrt{3} \left( f_{\phi NN}^{(1)} \cos \theta' - f_{\omega NN}^{(1)} \sin \theta' \right) + f_{\rho NN}^{(1)} \right] \]  
\[ f_1'^D = \frac{1}{2} \left[ 3 f_{\rho NN}^{(1)} - \sqrt{3} \left( f_{\phi NN}^{(1)} \cos \theta' - f_{\omega NN}^{(1)} \sin \theta' \right) \right] \]  
\[ f_1'^S = \sqrt{2} \left( f_{\omega NN}^{(1)} \cos \theta + f_{\phi NN}^{(1)} \sin \theta \right) \]  

\[ f_2'^F = \frac{1}{2} \left[ \sqrt{3} \left( f_{\phi NN}^{(2)} \cos \theta' - f_{\omega NN}^{(2)} \sin \theta' \right) + f_{\rho NN}^{(2)} \right] \]  
\[ f_2'^D = \frac{1}{2} \left[ 3 f_{\rho NN}^{(2)} - \sqrt{3} \left( f_{\phi NN}^{(2)} \cos \theta' - f_{\omega NN}^{(2)} \sin \theta' \right) \right] \]  
\[ f_2'^S = \sqrt{2} \left( f_{\omega NN}^{(2)} \cos \theta + f_{\phi NN}^{(2)} \sin \theta \right) \]
and by a substitution of numerical values (59), (60), (64), (65), respectively, one finds the numerical values

\[ f_1^F = -5.69470 \] \hspace{1cm} (70)
\[ f_1^D = 9.4103 \]
\[ f_1^S = 42.1706 \]

\[ f_2^F = 7.08952 \] \hspace{1cm} (71)
\[ f_2^D = 21.6245 \]
\[ f_2^S = -7.1465 \]

\[ f_1^{F'} = 0.38580 \] \hspace{1cm} (72)
\[ f_1^{D'} = 20.45639 \]
\[ f_1^{S'} = -5.0842 \]

\[ f_2^{F'} = 6.0577 \] \hspace{1cm} (73)
\[ f_2^{D'} = -41.7801 \]
\[ f_2^{S'} = -31.3390 \]

of all scrutinized coupling constants in SU(3) invariant interaction Lagrangians of vector-meson nonets with 1/2+ octet baryons, which allow us to predict behaviors of EM FFs of all 1/2+ octet hyperons in space-like and time-like regions, including their real and imaginary parts, phase differences of electric and magnetic FFs and corresponding differential and total cross-sections, in which EM structure of hyperons is planned to be measured. Predictions on all above mentioned quantities will be given in the next paper which will be published elsewhere.

VI. CONCLUSIONS

All existing 11 sets of data on nucleon EM FFs in space-like and time-like region from more than 40 different experiments has been analysed, from which 534 reliable experimental
points are here reasonably described by 9 resonance $U\&A$ model of nucleon EM structure, which respects the SU(3) symmetry and OZI rule violation. The analysis revealed the following results:

- an existence of the zero of $G_{E_p}(t)$ around $t_z = 13$ GeV$^2$ is again confirmed
- the value of the proton charge $rms$ radius coincides with the value obtained in the muon hydrogen atom spectroscopy experiment and in this way the existing puzzle is definitively removed
- the value of the neutron charge mean squared radius is identical with the value given by Rev. of Part. Physics
- the coupling constants in the SU(3) invariant Lagrangian of the vector meson nonet interaction with $1/2^+$ octet baryons $f^F, f^D, f^S$ are determined numerically, which allow to predict all quantities describing the EM structure of $1/2^+$ octet hyperons.

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