Confinement in $SU(N_c)$ Gauge Theory with a Massive Dilaton

Mohamed Chabab$^{a,b}$ and Latifa Sanhaji$^a$

$^a$) LPHEA, Physics Department, Faculty of Science - Semlalia P.O. Box 2390 Cadi-Ayyad University, Marrakech, Morocco.
and
$^b$) Centro de Fisica Torica, Departamento de Fisica, Universidade de Coimbra, 3004-516 Coimbra, Portugal.

Abstract

Following a recently proposed confinement generating scenario [6], we provide a new string inspired model with a massive dilaton and a general dilaton-gluon coupling. By solving analytically the equations of motion, we derive a new class of confining interquark potentials, which includes most of the QCD motivated potential forms given in the literature.
1 Introduction

To describe the confinement of quarks and gluons, several low energy effective models have been proposed. The most popular ones are: Color dielectric models [1, 2, 3], the constituent models with non relativistic quark and a confining potential [4], and the dual Landau-Ginzberg Model [5]. Recently, the extension of gauge field theories by inclusion of dilatonic degrees of freedom has evoked considerable interest. Particularly, dilatonic Maxwell and Yang-Mills theories which, under some assumptions, possess stable and finite energy solutions [18]. Indeed, in theories with dilaton fields, the topological structure of the vacuum is drastically changed compared to the non dilatonic ones. It is therefore of great interest to investigate the vacuum solutions induced by r-dependent dilaton field, through a string inspired effective theory which may reproduce the main features of QCD, in particular, the quark confinement. Recall that the dilaton is an hypothetical scalar particle appearing in the spectrum of string theory and Kaluza-Klein type theories [19]. Along with its pseudo scalar companion, the axion, they are the basis of the discovery F-theory compactification [20] and of the derivation of type IIB self duality [21]. The main features of a dilaton field is its coupling to the gauge fields through the Maxwell and Yang-Mills kinetic term. In particular, in string theory, the dilaton field determines the strength of the gauge coupling at tree level of the effective action. In this context, Dick [6] observed that a superstring inspired coupling of a massive dilaton to the 4d $SU(N_c)$ gauge fields provides a phenomenologically interesting interquark potential $V(r)$ with both the Coulomb and confining phases. The derivation performed in [6] is phenomenologically attractive since it provides a new confinement generating mechanism. In this context, a general formula of a quark-antiquark potential, which is directly related to the dilaton-gluon coupling function, has been obtained in [7]. The importance of this formula is manifest since it generalizes the Coulomb and Dick potentials, and it may be confronted to known descriptions of the confinement, particularly, those describing the complex structure of the vacuum in terms of quarks and gluons condensates. Moreover a generalized version of Dick model with both a massive and massless dilaton has been proposed in [10].

In this paper, we shall propose a new effective coupling of a massive dilaton to chromoelectric and chromomagnetic fields subject to the requirement that the Coulomb problem still admits an analytic solution. Our main interest concerns the derivation of a new family of confining interquark potentials. As a by product, we shall set up a theoretical basis to various QCD motivated

\footnote{there is a missing factor $\frac{1}{1+4\delta}$ in second term of Eq.(12) and Eq.(13). Also, the second term of Eq.(14) should be multiplied by $q^4$.}
quark potentials used in the literature. The later would gain in credibility if they can emerge from low energy effective theories.

The plan of this work is as follows: In section 2, we will develop our model and derive the main equations of motion. Particularly, emphasis will be put on the equations of a massive dilaton in the asymptotic regime. The latter should show the long range behaviour of the solutions, and consequently is connected with the confining phase. Section 3 will be devoted to the existence of analytical solutions from which we shall extract a new class of interquark potentials whose magnitude grows with the separation between the quark and antiquark. The main features of these potentials will be presented along with their connection to some popular phenomenological ones. Finally our conclusion will be drawn in section 4.

2 The model

We propose an effective field theory defined by the general Lagrangian:

$$L(\phi, A) = -\frac{1}{4F(\phi)} G^a_{\mu\nu} G^{a\mu\nu} + \frac{1}{2} \partial_\mu \Phi \partial^\mu - V(\phi) + J^a_\mu A^a_\mu$$  \hspace{1cm} (1)$$

where the coupling function $F(\phi)$ depends on the dilaton field and $V(\phi)$ denotes the non-perturbative scalar potential of $\phi$. $G^{\mu\nu}$ is the field strength in the language of 4d gauge theory.

Several forms of the function $F(\Phi)$ appeared in different theoretical frameworks: $F(\Phi) = e^{-k\Phi}$ as in string theory and Kaluza-Klein theories [19]; $F(\Phi) = \frac{\Phi}{f}$ in the Cornwall-Soni model parameterizing the glueball-gluon coupling [8, 9]. As to Dick model, $F(\Phi)$ is given by $F(\Phi) = k + \frac{f^2}{4\Phi}$. The constant $f$ is a characteristic scale of the strength of the dilaton/glueball-gluon. By using the formal analogy between the Dick problem and the Eguchi-Hansen one [22], we noted in [7] that $f$ is similar to the $4dN = 2$ Fayet-Ilioupoulos coupling in the Eguchi-Hansen model. It may be interpreted as the breaking scale of the $U(1)$ symmetry rotating the dilaton field.

Now, to analyze the problem of the Coulomb gauge theory augmented with dilatonic degrees of freedom in (1), we proceed as follows: first, we consider a point like static Coulomb source which is defined in the rest frame by the current:

$$J^a_\mu = g\delta(r)C_a \nu^a_0 = \rho_a \nu^a_0$$  \hspace{1cm} (2)$$

$C_a$ represents the expectation value of the $SU(N)$ generator $\chi_a$ for a normalized
spinor in Coulomb space. They satisfy the algebraic identity:

\[ \sum_{a=1}^{N_c^2-1} C_a^2 = \frac{N_c - 1}{2N_c} \]  

The equations of motions, inherited from the model (1) and emerging from the static configuration (2) are given by:

\[ [D_{\mu}, F^{-1}(\Phi)G^{\mu\nu}] = J^\nu \]  

and

\[ \partial_\mu \partial^\mu \Phi = -\frac{\partial V(\Phi)}{\partial \Phi} - \frac{1}{4} \frac{\partial F^{-1}(\Phi)}{\partial \Phi} G^\mu_\nu G^\nu_\mu \]  

By setting \( G_0^i_a = E^i_\chi_a = -\nabla^i \Phi_a \), we obtain, after some straightforward algebra, the simplified expressions:

\[ \frac{d\Phi_a}{dr} = r^{-2} F(\Phi(r)) \left( -\frac{g}{4\pi} C_a \right) \]  

\[ \Delta \Phi = \frac{\partial V(\Phi)}{\partial \Phi} - \tilde{\alpha} \frac{\partial F}{\partial \Phi} \]  

with \( \tilde{\alpha} = \frac{g^2}{32\pi^2} \left( \frac{N_c - 1}{2N_c} \right) \). We then derive the important formula of [7, 11],

\[ \Phi_a(r) = \frac{-gC_a}{4\pi} \int dr \frac{F(\Phi(r))}{r^2} \]  

which shows that the quark confinement appears if the following condition is satisfied:

\[ \lim_{r \to \infty} rF^{-1}(\Phi(r)) = \text{finite} \]  

Then, the interquark potential reads as,

\[ U(r) = \Phi_a(r) \left( \frac{-g}{4\pi} C_a \right) = 2\tilde{\alpha} \int \frac{F(\Phi(r))}{r^2} dr \]  

At this stage, note that the effective charge is defined by,

\[ Q_{eff}^a(r) = \left( g \frac{C_a}{4\pi} \right) F(\Phi(r)) \]
thus the chromo-electronic field takes the usual standard form:

$$E_a = \frac{Q_{eff}(r)}{r^2}$$

Therefore, it is the running of the effective charge that makes the potential stronger than the Coulomb potential. Indeed if the effective charge did not run, we recover the Coulomb spectrum.

To solve the equations of motion (6) and (7), we need to fix two of the four unknown quantities $\Phi(r)$, $F(\Phi)$, $V(\Phi)$ and $\Phi_a(r)$ in our model. We set $V(\Phi)$ to $V(\Phi) = \frac{1}{2}m^2\Phi$ and we introduce a new coupling function:

$$F(\Phi) = \left(1 - \frac{\beta \Phi^2}{f^2}\right)^{-n}$$

Then the equation (7) becomes:

$$\Delta \Phi = m^2\Phi - 2n\frac{\tilde{\alpha}_s}{r^4} \left(1 - \frac{\beta \Phi^2}{f^2}\right)^{-(n+1)} \frac{\beta \Phi}{f^2}$$

This equation is very difficult to solve analytically. However since we are usually interested by the large distance behaviour of the dilaton field and its impact on the Coulomb problem, an analytical solution of (11) in the asymptotic regime is very satisfactory. Indeed, it is easily shown that the following function:

$$\Phi = \frac{f^2}{\beta} - \left(\frac{\beta}{f^2}\right)^{\frac{n}{n+1}} \left(\frac{2n\alpha_s}{m^2}\right)^{\frac{1}{n+1}} \left(\frac{1}{r}\right)^{\frac{n}{n+1}}$$

solves (11) at large $r$. Therefore, thanks to the master formula (11), we derive the potential,

$$\Phi_a(r) = \frac{gC_a}{4\pi} \left(\frac{2n\alpha_s}{m^2f^2}\right)^{\frac{n}{n+1}} \frac{n + 1}{3n - 1} r^{\left(\frac{3n - 1}{n+1}\right)}$$

By imposing the condition (9), we obtain a family of confining interquark potentials if $n \geq \frac{1}{3}$. If moreover, we use the criterion of Seiler [21] which states that the magnitude of confining potentials, can not grow more rapidly than linear, then the values of $n$ are constrained to the range $n \leq 1$. Therefore the confinement in our model (1) appears for the coupling function $\frac{1}{F(\Phi)}$ with $n \in \left[\frac{1}{3}, 1\right]$. Such class of confining potentials is very attractive. Indeed, by selecting specific values of $n$, we may reproduce several popular QCD motivated
interquark potentials: Indeed if \( n = 1 \), we recover the confining linear term of Cornell potential [13]. Martin’s potential \((V(r) \sim r^{0.1})[14]\) corresponds to \( n = \frac{11}{29} \), while Song-Lin interquark potential [15] and Motyka-Zalewski potential [16], with a long range behaviour scaling as \( \sqrt{r} \), are obtained by setting \( n \) to \( \frac{3}{5} \). Turin potential [17] is recovered for \( n = \frac{5}{9} \). We see then, that these phenomenological potentials, which gained credibility only through their confrontation to the hadron spectrum, can now have a theoretical basis since they can be derived from the low energy effective theory.

3 Conclusion

In this paper we have found a family of electric solutions corresponding to a string inspired effective gauge theory with a massive dilaton varying with \( r \) and a new coupling function \( F(\Phi) = \left(1 - \frac{\Phi^2}{r^2}\right)^{-\frac{1}{n}} \). By constraining the values of \( n \) by both the Seiler criterion and by condition of Eq.(9) we have shown the existence of a class of confining interquark potentials. The latter are phenomenologically interesting since they reproduce, through selecting specific values of \( n \), several QCD motivated potentials which successfully describe meson and baryon spectra. Clearly these popular potentials would gain in credibility since they emerge from an low energy effective theory, and at the same time fit well the hadron spectrum.

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