What Do We Know about Low–Energy $\bar{K}N$ Interactions?  
Need and Possibilities of New Experiments at DAΦNE.*

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Abstract

We present results from a dispersion–relation investigation on a coupled–channel analysis of $\bar{K}N$–initiated processes in the low– and intermediate–energy region. The analysis demonstrates both the effectiveness of these relations to constrain the parameters of the unmeasurable $\pi Y$ channels and the need for better data in the low–energy region, such as could be provided at a $\phi$–factory.

1. INTRODUCTION.

In this talk we shall discuss the quality of our knowledge of the parameters which describe $\bar{K}N$ low–energy interactions. The forthcoming beginning of operations at DAΦNE provides an interesting possibility for improving this knowledge,[1] but a motivation for further study has also emerged from our analysis of the $\pi Y$ channels’ parameters presented at the recent MENU ’97 Symposium in Vancouver[2].

2. GENERALITIES.

It is customary to analyse the low–energy $\bar{K}N$ interactions using a multichannel K–matrix (or, equivalently, M–matrix, with $M = K^{-1}$) formalism[3]

$$Q^\ell(T^\ell_I)Q^\ell = (K^\ell_I)^{-1} - iQ^{2\ell+1} ;$$ (1)

in this equation $K^\ell_I$ is a real Hermitian matrix, possibly depending on energy, and $Q$ is the diagonal matrix of the c.m. momenta $q_i$ for the different $S = -1$ channels. $I$ indicates the isospin and $\ell$ the orbital angular momentum of the partial wave considered. Neglecting the $\Lambda\pi\pi$ and $\Sigma\pi\pi$ channels, as done at these low energies for limited statistics experiments, each partial wave of the $\bar{K}N$ interactions is described by a matrix of dimensions $2 \times 2$ for $I = 0$ and $3 \times 3$ for $I = 1$. Note that in this isospin decomposition the $K^0 N$ channels are considered together with the $K^- N$ ones: this neglects mass difference effects which will be important at DAΦNE, since the charge–exchange threshold in $K^- p$ scattering is at a lab. momentum $k_L = 90$ MeV/c. This formalism has been used to analyse data from experiments carried out between threshold and a lab. momentum $k_L \approx 500$ MeV/c, and for this purpose S, P and D waves have been considered[4].

The matrix elements required for an analysis including up to $J = 3/2$ are 36, and if one takes into account their energy dependence, this increases further the number of parameters: for instance, in ref. [4a] they are 44.

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This problem has been circumvented either by analysing the data using only S waves (with only 9 matrix elements)\(^6\), and reducing consequently the highest laboratory momentum considered, or by introducing some theoretical constraints on the parameters\(^6\).

As we shall show, the parametrization of ref. [4a], which includes all waves up to \(J = 3/2\), badly violates obvious consistency conditions coming from dispersion relations. This strongly suggests the need for new analyses which could take advantage of the much better data DAΦNE is expected to make available, and which could incorporate adequate dispersion relation constraints.

We recall that the \(\pi N\) interaction at low energy, whose analyses make a heavy use of such constraints, is very well known, even if some uncertainty, of the order of a few percent, still exists on the \(\pi N\) coupling constant\(^7\). Our hope is that in a few years the joint work of experimentalists and theorists might lead, if not to a comparable quality in the knowledge of the \(KN\) interaction, at least to the elimination of the difficulties we are going to present here. The reasons for putting so much hope in DAΦNE are well known\(^8\): this machine will produce kaon pairs in quantities such as to lead to statistics orders of magnitude better than those of the experiments analysed in the past, and with an excellent momentum resolution.

Even if the main scientific goals of DAΦNE are direct CP violation in \(K_{L,S}\) decays, rare decays and bounds on CPT violation, the machine offers unique possibilities for measurements of angular distributions and polarization of final–state hyperons in \(\bar{K}N\) interactions, provided a strong scientific case for such experiments exists.

3. OUTLINE OF THE DISPERSIVE CALCULATIONS.

The analysis of ref. [4a], of about thirty years ago, is the only one which includes up to D waves and extends down to the \(\bar{K}N\) threshold, with a reasonable behaviour in the unphysical region below that threshold. We shall discuss here the inconsistencies put in evidence for that analysis by dispersion relations. We do not perform a similar analysis of parametrizations which only use S waves, because (i) it is obvious that their region of validity is significantly smaller, (ii) our method is particularly sensitive to P waves (and we anticipate that they will appear to be very poorly known), and (iii) one expects that the decuplet resonance \(\Sigma(1385)\) must play a significant role, as its non–strange partner \(\Delta(1232)\) does in the \(\pi N\) case.

Our consistency test is based on the evaluation of the \(PBB\) couplings. We recall that flavour SU(3) symmetry provides definite predictions for these quantities\(^9\) in terms of one of them, \(G_{\pi^-pn}\), and of a parameter \(\alpha = F/(F + D)\) which expresses the weight of the antisymmetric octet in the direct product \(8 \times 8\).

The \(PBB\) coupling constants are usually evaluated by forward dispersion relations, and in the case of the \(S = -1\) sector a slightly different situation occurs for \(\pi YY'\) couplings with respect to the \(KYN\) ones. In the case of \(KYN\) couplings, since \(K^\pm N\) total cross sections are known up to very high energies, one can write once–subtracted dispersion relations for the spin–averaged forward amplitude \(C(\omega)\), since its imaginary part is directly proportional to \(\sigma_{tot}\) via the optical theorem. This amplitude turns out to be S–wave dominated at very low energies, and this has made it possible to determine these couplings from low–energy analyses which only included the S waves\(^10\). Instead, in the case of the \(\pi \Sigma\Sigma\) and \(\pi \Lambda\Sigma\) couplings, the amplitudes can only be estimated in the region of validity of the multichannel formalism, and the previous application of the optical theorem is clearly of no use. Therefore one must use rapidly convergent dispersion relations, and this leads to choose the amplitude \(B(\omega)/\omega\), even under crossing between \(s\) and \(u\) channels, for elastic \(\pi \Lambda\) and \(\pi \Sigma\) scattering. It has to be noted that these amplitudes, beside being rapidly decreasing as \(\omega \to \infty\), exhibit a strong dependence on the P waves close to threshold.

We also calculated the product of these couplings from the \(\pi \Lambda \to \pi \Sigma\) reaction, using this time the amplitude \(C(\omega)/\omega\); this channel has been considered here for the first time, while the two elastic processes were already studied by other authors\(^11\). However, the novelty of
our calculation is not only in this part, and in the update of the inputs for the higher–energy ranges of the dispersive integrals, but also in the fact that for the first time we analysed the stability of the results with respect to the energy at which the dispersion relations were evaluated.

This provides an important consistency check of the parametrization of the $S = -1$ meson–barion interactions with fixed–$t$ analyticity: we found indeed that $\pi Y$ amplitudes showed marked inconsistencies and, in order to investigate their sources, we analysed separately the contributions from each partial wave in the parametrization of the low–energy region, and in particular their dependences on the energy at which the dispersions relations were evaluated. Such a separation is useful in building a priority scale for future experiments to be performed, for instance, at DAΦNE.

4. DESCRIPTION OF THE RESULTS.

For what concerns our calculations of the $\pi Y Y'$ couplings, here we shall rapidly summarise the results of ref. [2], to which we also refer for the details on the evaluation of the dispersive integrals.

In the case of $\pi \Lambda$ elastic scattering we found that the value for $G^2_{\pi \Lambda \Sigma}/4\pi$ evaluated at the $\bar{K}N$ threshold was in agreement with the similar calculation by Chang and Meiere [11]. However, as we varied the energy at which the dispersion relation was evaluated, we found significant variations in the coupling as well, almost entirely due to the $P_{11}$ wave, the remaining contributions having a smooth energy dependence, and accounting at the $\bar{K}N$ threshold for about 28.5 % of the value of $G^2_{\pi \Lambda \Sigma}/4\pi$.

In the case of $\pi \Sigma$ elastic scattering, for which one has to use only the (crossing–even) combination of isospin amplitudes $2B_1 - B_0$, in order to eliminate the unknown amplitude $B_2$ in both the $s$ and $u$ channels, both $\Lambda$ and $\Sigma$ poles are present in the Born term: thus a reasonable approach to the determination of $G^2_{\pi \Sigma \Sigma}/4\pi$ is to combine the dispersion relation for this channel with the one for the $\pi \Lambda$ channel (calculated at the same c.m. energy), so that the $\Lambda$–pole term in the first relation is exactly canceled by the $\Sigma$–pole one in the second (this was done as well by Chan and Meiere [11], although only at the $\bar{K}N$ threshold). Our results for the $\pi \Sigma \Sigma$ coupling agree with Chan–Meiere’s at the $\bar{K}N$ threshold, but away from it a complicated energy dependence appears, which in part can be ascribed both to the poor matching of Kim’s analysis to the input used for the intermediate–energy range of the integrals (at the upper end of the energy interval covered by our calculations), and (at the lower end of that interval) to the $\Lambda$ pole falling on the cut between the $\pi \Lambda$ and $\pi \Sigma$ thresholds, so that the M–matrices can not describe correctly the real part of the amplitude in this energy region. Between these two extremes, the coupling shows a hump close to the $\bar{K}N$ threshold, which is again, as in the previous case, mostly due to the $P_{11}$ wave.

Finally, when the $\pi \Lambda \rightarrow \pi \Sigma$ reaction is analysed, one finds several inconsistencies: first, the product $G_{\pi \Lambda \Sigma}G_{\pi \Sigma \Sigma}/4\pi$ has a value close to zero at the $\bar{K}N$ threshold, in clear contrast with the values found by us and by ref. [11] for the separate couplings at the same energy. Second, the product shows a marked dependence on the energy, going rapidly from positive to negative values: again, this seems to be mostly due to the $P_{11}$ wave, whose contribution is negative over most of the range and rapidly decreasing.

However, if the $J = 1/2$ $P$ waves are dropped from all three calculations, not only the couplings obtained show much smaller variations in the central part of the energy range covered by our calculations, but also the average values obtained from the three channels are rather consistent with each other (and, incidentally, also with the expectations from flavour SU(3) symmetry and the known value of $G^2_{\pi NN}/4\pi$). This seems to suggest that these waves were very poorly determined in Kim’s analysis, at least as far as their matrix elements in the $\pi Y$ channels are concerned.

We have therefore analysed also the $\bar{K}N$ elastic scattering $B$ amplitudes by the same
approach. It is convenient to recall again here that the dispersive evaluations of the $KY\Lambda N$ couplings have usually involved the amplitudes $C(\omega)$, mainly sensitive to the $S$ waves close to threshold. Nevertheless, an old paper [12] had indeed found an energy dependence in $G^2_{K\Lambda N}$, which perhaps could have had something to do with the problems we have just pointed out with Kim's $J = 1/2$ $P$ waves.

The calculation of $G^2_{K\Lambda N}/4\pi$ and $G^2_{K\Sigma N}/4\pi$ was performed using the pure $I = 0$ and 1 isospin combinations of $KN$ $B$ amplitudes, using for the crossed $KN$ channels the VPI phase shift analysis [13]. We calculated a dispersion relation for the difference $\text{Re}B(\omega_1) - \text{Re}B(\omega_2)$, where $B, B'$ are the $KN, KN$ $B$ amplitudes, and $\omega_2$ was kept fixed while $\omega_1$ varied in the energy range covered by Kim's parametrization (unphysical range included). Calculations were carried out for four values of $\omega_2$, corresponding to $KN$ c.m. energies of 1,468, 1,500, 1,530 and 1,560 MeV: the results were very similar, so that we only present here those referring to the first value, i.e. to $\omega_2 = 548$ MeV.

![Graphs showing results on the couplings to the $\Sigma$ pole in, respectively, (first row, from left to right) $\pi\Lambda \rightarrow \pi\Lambda$, $\pi\Sigma \rightarrow \pi\Sigma$ and $\pi\Lambda \rightarrow \pi\Sigma$, to be compared (second row) with the product of the first two, and results for the couplings to the $\Lambda$ and $\Sigma$ poles in the $I = 0$ and 1 channels of $K\Lambda N$ scattering.](image)

Fig. 1. Results on the couplings to the $\Sigma$ pole in, respectively, (first row, from left to right) $\pi\Lambda \rightarrow \pi\Lambda$, $\pi\Sigma \rightarrow \pi\Sigma$ and $\pi\Lambda \rightarrow \pi\Sigma$, to be compared (second row) with the product of the first two, and results for the couplings to the $\Lambda$ and $\Sigma$ poles in the $I = 0$ and 1 channels of $K\Lambda N$ scattering.

The $K\Lambda N$ coupling constant $G^2_{K\Lambda N}/4\pi$ exhibits a bump-like structure for a $Kn$ c.m.
energy of about 1,300 through 1,450 MeV, and a general trend to increase with c.m. energy over the whole energy range analysed by us, oscillating around a value of 19.5 in the central part of this range. The bump is coming from the contribution of the $D_{03}$ wave, while the average slope comes mostly from the $u$–channel $KN$ contributions. No significant effect can be attributed to the $P_{01}$ wave contribution.

In the case of the $K\Sigma N$ coupling, $g_{K\Sigma N}^2/4\pi$ exhibits a dependence on the $K\Sigma$ c.m. energy peaking around threshold, which is almost completely eliminated when the contributions from the $P_{11}$ wave is dropped out.

From these calculations it transpires that the only parametrisation of the $S = -1$, low-energy $PB$ scattering, which includes waves up to $J = 3/2$, and appears to give a reliable description of the structure of the $\pi\Lambda$ and $\pi\Sigma$ channels in the $K\Sigma$ unphysical region, is inconsistent with dispersion relations, and that also the VPI partial–wave analysis of $KN$ scattering\cite{13} seems to meet some difficulties, at least for the combination $B_1 - B_0 = 2B_{K^+p} - B_{K^+n}$. However, if one considers the many measurements possible in low–energy $KN$–initiated reactions, and the few ones actually performed in all previous experiments, it is possible to hold the optimistic view that with DAΦNE the situation might significantly improve. Before discussing this in some detail, let us recall that the use of more sophisticated tools of analysis, like those proposed in ref. [1c], can also shed more light on this subject.

Going to the experimental perspectives, it has to be stressed that the parameters of the $K$–matrices are determined at the lowest energies from very few sources, measured usually like those proposed in ref. [1c], and (but without resolving the channels) $K^-p \rightarrow \Sigma^+\Lambda$ and $K^-p \rightarrow \pi^0\Sigma^+$, while the $K^+\Sigma^0$ contribution is unphysical.

5. FUTURE PERSPECTIVES.

DAΦNE will make possible to analyse all these reactions and, with the calorimetric techniques employed by KLOE, to resolve all charged and neutral channels (and detecting the $\gamma$ from $\Sigma^0$ decay to separate $\pi^0(\pi^0)\Sigma^0$ from $\pi^0(\pi^0)\Lambda$), not only measuring the rates but also the differential distributions, at least to the statistical level to extract their Lagrange coefficients $L_1$ and $L_2$. The extremely good efficiency and spatial resolution of a KLOE–like apparatus, of much smaller dimensions than its “big brother” due to the much shorter decay length of the $K^\pm$ with respect to that of the $K^0_L$, will also allow measurements of the polarizations of the final–state $\Lambda$ and $\Sigma^+$ hyperons from the angular distributions of their decay products. The same measurements will also be possible for all $K^0_Lp$–initiated processes, and, replacing hydrogen with deuterium, for $K^0_Ln$– and $K^-n$–initiated ones as well. Already this amount of experimental information at laboratory momenta from about 110 down to about 90 MeV/c (the detector itself will act as a “moderator” for $K^\pm$) should lead to a much better knowledge of the $K$–matrices and to more reliable calculations of the quantities related to them, such as the $PBB$ coupling constants for the $S = -1$ sector and the zero–energy values of the crossing–even amplitudes $C(\omega, t)$, which can lead to an estimate of the $KN\sigma$–terms\cite{14}. Other interesting experiments will already be possible with existing detectors: KLOE\cite{15} will surely be able to register all interactions of both $K^\pm$ and $K^0_L$ with the $^4$He filling its wire chamber, interactions never observed before at such low lab. momenta, DEAR\cite{16} will measure the $K$ lines of kaonic hydrogen (and deuterium) giving independent information on the $KN$ S–wave scattering lengths (with CCDs covering much lower $\gamma$–ray energies they could also think about investigating the $P$ waves through the study of the L lines as well), and FINUDA\cite{17}, though starting with a much narrower scope than KLOE, will anyway be able to make some high quality measurements, in particular of the $K^0_Lp$ charge–exchange processes taking place in the hydrogen of its plastic scintillators\cite{18}.

We hope with the present discussion to have offered at least part of the background we were referring to for the motivation of new, perhaps less fashionable than others, but nonetheless still very interesting experiments, several of which, despite the folklore about low–energy
physics having been already adequately explored in the past, were never done before, and quite certainly not accomplishable anywhere else but at a \( \phi \)-factory.

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