NEUTRINO OSCILLATIONS AND NEUTRINOLESS DOUBLE BETA DECAY

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The relation between neutrino oscillation parameters and neutrinoless double beta decay is studied, assuming normal and inverse hierarchies for Majorana neutrino masses. For normal hierarchy the crucial dependence on $U_{e3}$ is explored. The link with tritium beta decay is also briefly discussed.
There is now convincing evidence for neutrino masses and lepton mixings from oscillation experiments. Neutrino masses can be either of the Dirac type or of the Majorana type. In the case of Majorana masses the neutrinoless double beta decay ($0\nu\beta\beta$ decay) is allowed \[1\]. Such a decay has not yet been observed and only an upper limit on the related mass parameter $M_{ee}$ (to be defined below) is available, $M_{ee} < 0.2 \text{ eV}$ \[2\] (in this paper $M_{ee}$ is always expressed in eV). Several experiments have been proposed to lower this limit by one or even two orders and eventually discover the $0\nu\beta\beta$ decay, thus revealing the Majorana nature of neutrinos. Therefore, the subject of the relation between oscillation parameters and $M_{ee}$ has been studied by many authors \[3–5\]. Here we turn to the question in order to clarify the link between the lepton mixings and the predictions for $M_{ee}$.

In fact, the random extraction of the relevant neutrino parameters is very useful in this case. In particular, we will see that for $U_{e3} \lesssim 0.05$ the four solutions to the solar neutrino problem may give quite different predictions for the mass parameter $M_{ee}$. Since also the bound $U_{e3} < 0.2$ \[1\] is expected to be lowered in the future, phenomenological relations between $M_{ee}$ and $U_{e3}$ are welcome. We consider normal and inverse hierarchies for the Majorana masses of three active neutrinos. Recent evidence for cosmological dark energy eliminates most of the motivations for considering the degenerate spectrum, which was before relevant for hot dark matter (see for example \[7\]).

Let us now define the mass parameter $M_{ee}$ as

$$M_{ee} = |U_{e1}^2 e^{2i\alpha} m_1 + U_{e2}^2 e^{2i\beta} m_2 + U_{e3}^2 m_3|,$$

(1)

where $U_{ei}$ ($i = 1, 2, 3$) are the moduli of the elements in the first row of the lepton mixing matrix $U$. This matrix can be parametrized as the standard form of the CKM matrix (with one phase $-\delta$ in entry 1-3) times $\text{diag}(e^{i\varphi_1}, e^{i\varphi_2}, 1)$. Two relative phases $\alpha = \varphi_1 + \delta$ and $\beta = \varphi_2 + \delta$ appear in $M_{ee}$. Moreover, $m_i$ are positive Majorana masses. From ref. \[8\] we get

$$U_{e2}^2 = (\sin^2 \theta_s)(1 - U_{e3}^2),$$

(2)

where $\theta_s$ is the solar neutrino mixing angle, and due to the unitarity of $U$ we have

$$U_{e1}^2 = 1 - U_{e2}^2 - U_{e3}^2 = (\cos^2 \theta_s)(1 - U_{e3}^2),$$

(3)
Therefore, neglecting $U_{e3}^2$ with respect to 1, eqn.(1) can be written as

$$M_{ee} = |(\cos^2 \theta_s)e^{2i\alpha}m_1 + (\sin^2 \theta_s)e^{2i\beta}m_2 + U_{e3}^2 m_3|. \quad (4)$$

In this way $M_{ee}$ depends on seven neutrino parameters. As said above the mixing $U_{e3}$ is bounded (by the CHOOZ experiment),

$$U_{e3} < 0.2. \quad (5)$$

In order to determine the masses $m_i$ we have to distinguish between the normal mass hierarchy, $m_1 \ll m_2 \ll m_3$, and the inverse mass hierarchy, $m_1 \simeq m_2 \gg m_3$. In the normal hierarchy case

$$m_3 = \sqrt{\Delta m^2_a + m_1^2}, \quad (6)$$

$$m_2 = \sqrt{\Delta m^2_s + m_1^2}, \quad (7)$$

and for $m_1$ we take $10^{-5} \sqrt{\Delta m^2_s} < m_1 < 10^{-1} \sqrt{\Delta m^2_s}$, with $\Delta m^2_a = (1 - 6) \times 10^{-3}$eV$^2$ for atmospheric neutrinos, and $\Delta m^2_s$ reported in Table I (in eV$^2$) together with $\sin \theta_s$ for solar neutrinos. These values come from ref. [9]. LMA, SMA, LOW are the large mixing angle, small mixing angle, low mass matter (MSW) solutions, and VO is the vacuum solution. The best global fit of solar neutrino data is given by the LMA solution, although the other solutions are not ruled out [7]. The value $\sin \theta_s = 0.71 (\theta_s = \pi/4)$ means maximal mixing. For inverse hierarchy one has $m_1 \simeq m_2 \simeq \sqrt{\Delta m^2_a}$.

Let us consider first the normal hierarchy. The results of the calculation (2500 points extracted) are in Figs. 1-2, where we plot $\log_{10} M_{ee}$ versus $U_{e3}$. For $U_{e3} > 0.1$ the SMA, LOW and VO solutions give similar values for $M_{ee}$, while the LMA solution provides also higher values. However, the LMA solution gives $M_{ee}$ almost constant, because the $m_3$-term is negligible even for $U_{e3} \simeq 0.2$, while for the other solutions $M_{ee}$ decreases for smaller $U_{e3}$, till the $m_3$-term becomes negligible. In particular, for $U_{e3} \lesssim 0.05$ the LMA solution is clearly distinguished from the SMA solution (and also from the others). A similar behaviour happens for the LOW solution with respect to the VO solution, for $U_{e3} \lesssim 0.02$. In order to clarify this aspect
we have checked the results on a linear plot. In Fig. 3 we report the lower LMA bound and the upper SMA bound as well as the lower LOW bound and the upper VO bound for $M_{ee}$. The lower LMA bound can be obtained from the expression

$$M_{ee} \simeq m_2 \sin^2 \theta_s - m_3 U_{e3}^2$$

and the upper SMA bound from $M_{ee} \simeq m_1 + m_3 U_{e3}^2$. We can thus predict, for $U_{e3} \lesssim 0.05$, that $4 \times 10^{-4} < M_{ee} < 8 \times 10^{-3}$ for the LMA solution, while $M_{ee} < 4 \times 10^{-4}$ for the SMA solution (and also the LOW solution). For $U_{e3} \lesssim 0.01$ we get $3 \times 10^{-5} < M_{ee} < 4 \times 10^{-4}$ for the LOW solution, while $M_{ee} < 3 \times 10^{-5}$ for the VO solution. We now comment about the cancellations appearing in Figs. 1-2. They are obtained when the $m_2$-term and/or the $m_1$-term are comparable with the $m_3$-term. Of course, this happens for different $U_{e3}$ values (and the relevant phase tuned around $\pi/2$), according to the different solar solutions. Note also that in the SMA case the $m_1$-term can easily exceed the $m_2$-term. In the other cases, for $U_{e3} \simeq 0$, we have $M_{ee} \simeq (\sin^2 \theta_s)m_2 \simeq (\sin^2 \theta_s)\sqrt{\Delta m_a^2}$.

For the inverse hierarchy two main results can be drawn out. One is with respect to the normal hierarchy, namely in the region $10^{-2} < M_{ee} < 10^{-1}$ only the inverse hierarchy is possible, while $M_{ee} < 10^{-2}$ for the normal hierarchy. The other result is that the SMA solution gives clean bounds, $3 \times 10^{-2} < M_{ee} < 8 \times 10^{-2}$. All solutions have the same upper bound $M_{ee} < 8 \times 10^{-2}$, not depending on $U_{e3}$, which is easily understood since the $m_3$-term is negligible for inverse hierarchy. The basic features of the inverse hierarchy case can be obtained by using the approximation

$$M_{ee} \simeq (\cos^2 \theta_s \pm \sin^2 \theta_s)m_{1,2}$$

in the CP-conserving case ($\alpha = 0$, $\beta = 0, \pi/2$). In fact, the plus sign ($\beta = 0$) gives $M_{ee} \simeq m_{1,2} \simeq \sqrt{\Delta m_a^2}$, while the minus sign ($\beta = \pi/2$) gives $M_{ee} \simeq \sqrt{\Delta m_a^2}\cos2\theta_s$. For small mixing $\theta_s \simeq 0$ one has $M_{ee} \simeq \sqrt{\Delta m_a^2}$, while for large mixing $\theta_s \simeq \pi/4$ the value $M_{ee} \simeq 0$ is allowed by cancellations, so that the full range $0 \leq M_{ee} \leq \sqrt{\Delta m_a^2}$ is covered. Of course, if maximal mixing is excluded, a lower bound appears also for the LMA, LOW and VO solutions.

Now we discuss the mass parameter $m_\beta$, related to tritium beta decay, which is defined as

$$m_\beta^2 = U_{e1}^2 m_1^2 + U_{e2}^2 m_2^2 + U_{e3}^2 m_3^2.$$
Using eqns. (2), (3) and neglecting again $U_{e3}^2$ with respect to 1, we obtain

$$m_\beta^2 = (\cos^2 \theta_s)m_1^2 + (\sin^2 \theta_s)m_2^2 + U_{e3}^2m_3^2.$$  \hspace{1cm} (10)

There are no cancellations for $m_\beta$, so that it is sufficient to evaluate $M_{ee}$ in $U_{e3} \simeq 0$ and $U_{e3} \simeq 0.2$. In the normal hierarchy case, for $U_{e3} \simeq 0.2$ we get $m_\beta \simeq U_{e3}m_3 \simeq U_{e3}\sqrt{\Delta m^2_s}$. For $U_{e3} \simeq 0$ the SMA solution gives $m_\beta \simeq m_1 \ll \sqrt{\Delta m^2_s}$, while the other solutions give $m_\beta \simeq (\sin \theta_s)m_2 \simeq (\sin \theta_s)\sqrt{\Delta m^2_s}$. For inverse hierarchy all solutions give $m_\beta \simeq m_{1,2} \simeq \sqrt{\Delta m^2_s}$, not depending on $U_{e3}$. The experimental limit on $m_\beta$ is now $m_\beta < 2.2$ eV (see [1]) and it is hard to lower this limit by one order. However, the maximum value allowed by the previous discussion is $m_\beta \simeq 8 \times 10^{-2}$ eV, so that the impact of neutrino oscillations on the prediction for $m_\beta$ cannot be checked.

In conclusion, we have studied the prediction for $M_{ee}$ obtained by varying neutrino parameters, within the experimental ranges, for the normal and inverse mass hierarchy cases. For normal hierarchy the main result is that for $U_{e3} \lesssim 0.05$ the LMA solution is clearly distinguished from the other solutions. Moreover, for $U_{e3} \lesssim 0.01$ the LOW solution is distinguished from the VO solution. This means that if the LMA solution is confirmed, and even if $U_{e3}$ is very small (similar to $V_{cb}$ or $V_{ub}$), the GENIUS II (10 t) experiment should find the $0\nu\beta\beta$ decay unless neutrinos are Dirac particles. Instead, if another solution is confirmed and $U_{e3} \lesssim 0.1$, then the GENIUS project will not be able to decide about the neutrino nature. For inverse hierarchy $M_{ee}$ could be higher by one order, with respect to normal hierarchy, and the $0\nu\beta\beta$ decay be possibly found also by the GENIUS I (1 t) and MOON experiments.

In this paper we have taken $0 < \theta_s \leq \pi/4$. However, for LOW and VO solutions, part of the range $\pi/4 < \theta_s < \pi/2$ (the so-called dark side of neutrino parameter space [11]) is allowed (see for example [12]), so that for normal hierarchy the related regions in $M_{ee}$ can overlap also for $U_{e3} \simeq 0$. Of course, progress in the determination of neutrino oscillation parameters will sharpen the predictions on $M_{ee}$ for both hierarchies.
|          | LMA                  | SMA                  | LOW                 | VO                  |
|----------|----------------------|----------------------|---------------------|---------------------|
| $\Delta m^2_s$ | $(0.15 - 1.5) \times 10^{-4}$ | $(0.4 - 1) \times 10^{-5}$ | $(0.3 - 2.5) \times 10^{-7}$ | $(0.3 - 10) \times 10^{-10}$ |
| $\sin \theta_s$ | $0.40 - 0.71$       | $0.02 - 0.05$       | $0.53 - 0.71$       | $0.43 - 0.71$       |

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FIG. 1. $\log_{10} M_{ee}$ vs $U_{e3}$ for the LMA and SMA solutions with the normal hierarchy
FIG. 2. $\log_{10} M_{ee}$ vs $U_{e3}$ for the LOW and VO solutions with the normal hierarchy
FIG. 3. Upper SMA bound (a) and lower LMA bound (b) as well as upper VO bound (c) and lower LOW bound (d) for $M_{ee}$ with the normal hierarchy.