Doubly-charged scalar in four-body decays of neutral flavored mesons

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Abstract: We study the four-body decays of neutral flavored mesons, including $K^0$, $D^0$, $B^0$, and $\bar{B}^0$. These processes, which could be induced by a hypothetical doubly-charged scalar particle, do not conserve the lepton number. Assuming, as an example, that the mass of the doubly-charged particle is 1000 GeV, and using the upper bounds of the couplings, we calculate the branching ratios of different channels. For $K^0 \rightarrow h_1^+ h_2^+ e^- e^-$, $D^0 \rightarrow h_1^- h_2^- e^+ e^-$, and $\bar{B}^0 \rightarrow h_1^+ h_2^+ e^- e^-$, it is of the order of $10^{-30}$, $10^{-32}$–$10^{-29}$, and $10^{-33}$–$10^{-28}$, respectively. Based on the experimental results for the $D^0 \rightarrow h_1^- h_2^- l^+_1 l^+_2$ channels, we also find the upper limit for the quantity $\frac{\Delta M}{M}$.

Keywords: doubly-charged scalar, four-body decay, neutral flavored meson

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1 Introduction

In a previous paper [1], we studied the lepton number violation decays of the $B^-_s$ meson induced by a doubly-charged Higgs boson. There are both experimental and theoretical motivations to study this kind of particle. Although the Higgs boson has been found, whether it is the one predicted by the Standard Model still needs more confirmation. It is possible that an extended Higgs sector exists, and that there are additional isospin multiplet scalar fields. For example, the $SU(2)_L$ triplet scalar, which contains a doubly-charged component, is introduced to generate small neutrino mass in the Type-II seesaw modes [2–5]. Generally, such a triplet representation is needed in the left-right symmetric models [6–8] to break the extended $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry in the Standard Model. The doubly-charged scalar also appears in other models, such as little Higgs models [9] and Georgi-Machacek model [10]. As it can decay into two leptons with the same charge, indicating lepton number violation, such processes for top quark, $\tau^-$ [11], and charged mesons, such as $K^-$, $D^-$, $D_s^-$, $B^-$ [12–15] have been investigated extensively. As the lower bound of the mass of the doubly-charged Higgs boson is around 800 GeV [16, 17], these low energy processes have extremely small branching ratios. Although it is not likely that these channels will be detected soon, as experiments collect more data, the upper limits of the branching ratios for such decay processes will become more stringent. One can also use them to derive further constraints for the effective short-range interactions [18].

In Ref. [1], we considered both the three-body and four-body decay channels of $B_s^-$ meson, in which the lepton number is not conserved. In this paper, we investigate the lepton number violation processes of the neutral flavored mesons induced by the doubly-charged Higgs boson. Contrary to the charged meson case, where the annihilation-type diagram and the two $W$ meson emitting diagram both contribute to the amplitude, in the case of the neutral meson, the light antiquark is just a spectator (see Fig. 1). Theoretically, this makes the calculation simpler, as there is no complicity brought by the cascade decay. As for the decay products, the two leptons have the same charge, and so do the two mesons. These decay modes have no equivalent in the Standard Model, which makes them also interesting experimentally.

These channels can also be induced by Majorana-type neutrinos. Their Feynman diagrams are similar to Fig. 1, but the $s$ channels should be replaced by $t$ channels. If the neutrino mass were around GeV, it could be produced on-shell, which has attracted much attention [19–22]. For the cases when the neutrino mass is very small or very large, the branching ratios will have the same order of magnitude as in the case of the doubly-charged Higgs boson [15, 23]. Therefore, the theoretical analy-
sis of the low energy processes induced by the doubly-charged Higgs boson also provides a useful complement to the Majorana neutrino scenario.

This paper is organized as follows. In Sec. 2, we give the Lagrangian which describes the couplings between the Higgs triplet and the Standard Model particles, and present the amplitudes and phase space integrals. In Sec. 3, we give the branching ratios of all decay channels and compare the results for $D^0$ with experimental data. We summarize our results in the last section. Some details of the meson wave functions are presented in the Appendix.

2 Theoretical formalism

The hypothetical Higgs triplet $\Delta$ in the $2 \times 2$ representation is defined as $\Delta$.

$$
\Delta = \begin{pmatrix}
\Delta^+ / \sqrt{2} & \Delta^{++} \\
\Delta^0 & -\Delta^+ / \sqrt{2}
\end{pmatrix}.
$$

(1)

It mixes with the usual $SU(2)_L$ Higgs doublet by a mixing angle $\theta_\Delta$, from which we define $s_\Delta = \sin \theta_\Delta$ and $c_\Delta = \cos \theta_\Delta$.

The Lagrangian which describes the interaction between $\Delta$ and $W^-$ gauge boson or SM fermions has the following form

$$
\mathcal{L}_{\text{int}} = \frac{i}{2} h_{ij} \bar{\psi}_{iL} C \gamma_5 \sigma_{\mu \nu} \partial^\nu \Delta^\pm W^\pm_{\mu} - \sqrt{2} g_\Delta \Delta^\pm W^\pm_{\mu} + \frac{\sqrt{2}}{2} g_\Delta W^- \Delta - \frac{i}{2} g_\Delta \Delta^+ + \frac{ig s_\Delta}{\sqrt{2 m_w c_\Delta}} \Delta^+ (m_q \bar{q}_R \gamma_\mu q_L - m_q \bar{q}_L \gamma_\mu q_R) + \text{H.c.},
$$

(2)

where $C = i \gamma^2 \gamma^0$ is the charge conjugation matrix; $\psi_{iL}$ represents the leptonic doublet; $h_{ij}$ is the leptonic Yukawa coupling constant; $g$ is the weak coupling constant. The third and fourth terms represent the interactions between the singly-charged boson and the other particles. Compared with the second term, their contributions can be neglected.

If $q = q_3$, all four diagrams in Fig. 1 contribute to the decay:

$$
\mathcal{M}_A = \frac{g^3}{8 \sqrt{2 m_w^3}} V_{q_1 q_2} V_{q_2 q_3} s_\Delta h_{ij} (h_1 (p_1) h_2 (p_2)) (\bar{q}_3 q_3) \gamma_\mu (h (p)) \langle \text{lepton} \rangle
$$

$$
= \frac{g^3}{8 \sqrt{2 m_w^3}} V_{q_1 q_2} V_{q_2 q_3} s_\Delta h_{ij} f_{h_2} f_{h_2} \bar{q} \gamma_\mu (h_1 (p_1) | q_1 \gamma_\mu (1 - \gamma_5) Q | h (p) \rangle \langle \text{lepton} \rangle.
$$

(3)
\[
M_B = \frac{g^3}{8\sqrt{2m^2_W}} V_{q\bar{q}} V_{q\bar{q}_3} \frac{s_A h_{ij}}{m_A^2} f_{h_3}(p_3) \langle h_1(p_1) | h_2(p_2) | \bar{q}_3 \bar{q}_3 \rangle_{\nu^0} | h(p) \rangle \langle \text{lepton} 
\] 
\[
= \frac{g^3}{8\sqrt{2m^2_W}} \frac{1}{3} V_{q\bar{q}} V_{q\bar{q}_3} \frac{s_A h_{ij}}{m_A^2} f_{h_3}(p_3) \langle h_1(p_1) | \bar{q}_3 \bar{q}_3 \rangle_{\nu^0} | h(p) \rangle \langle \text{lepton}, \quad (4)
\]
\[
M_C = \frac{g^3}{8\sqrt{2m^2_W}} V_{q\bar{q}} V_{q\bar{q}_3} \frac{s_A h_{ij}}{m_A^2} f_{h_3}(p_3) \langle h_1(p_1) | h_2(p_2) | \bar{q}_3 \bar{q}_3 \rangle_{\nu^0} | h(p) \rangle \langle \text{lepton} 
\] 
\[
= \frac{g^3}{8\sqrt{2m^2_W}} V_{q\bar{q}} V_{q\bar{q}_3} \frac{s_A h_{ij}}{m_A^2} f_{h_3}(p_3) \langle h_2(p_2) | \bar{q}_3 \bar{q}_3 \rangle_{\nu^0} | h(p) \rangle \langle \text{lepton}, \quad (5)
\]
\[
M_D = \frac{g^3}{8\sqrt{2m^2_W}} V_{q\bar{q}} V_{q\bar{q}_3} \frac{s_A h_{ij}}{m_A^2} f_{h_3}(p_3) \langle h_1(p_1) | h_2(p_2) | \bar{q}_3 \bar{q}_3 \rangle_{\nu^0} | h(p) \rangle \langle \text{lepton} 
\] 
\[
= \frac{g^3}{8\sqrt{2m^2_W}} V_{q\bar{q}} V_{q\bar{q}_3} \frac{s_A h_{ij}}{m_A^2} f_{h_3}(p_3) \langle h_2(p_2) | \bar{q}_3 \bar{q}_3 \rangle_{\nu^0} | h(p) \rangle \langle \text{lepton}, \quad (6)
\]

where the factor \( \frac{1}{3} \) in \( M_B \) and \( M_D \) is introduced by the Fierz transformation; \( \langle \text{lepton} \rangle \) is the leptonic part of the transition matrix element; \( V_{q\bar{q}_3} \) is the Cabibbo-Kobayashi-Maskawa matrix element. The definition of the decay constant \( f_{h_3} \) of a pseudoscalar meson
\[
\langle h_1(p_1) | \bar{q}_3 \bar{q}_3 \rangle_{\nu^0} | h(p) \rangle = i f_{h_3} p_{h_3}^\mu \quad (7)
\]

is used. For vector mesons, it should be replaced by
\[
\langle h_1(p_1, \mu) | \bar{q}_3 \bar{q}_3 \rangle_{\nu^0} | h(p) \rangle = M_1 f_{h_3} \epsilon^{\mu\nu\rho\sigma} \langle \hat{h}_3 | p_{h_3}^\sigma | \hat{h}_3 \rangle_{\nu^0}, \quad (8)
\]

The values of the decay constants are given in Table 1. It should be pointed out that we have used the factorization assumption in Eqs. (3) - (6), which is not quite appropriate when both final mesons are light. However, as only the order of magnitude is important in such processes, we anticipate that the effects of nonfactorization and final meson interactions do not change the results significantly.

Finally, we get the transition amplitude
\[
M = M_A + M_B + M_C + M_D
\]
\[
= \frac{g^3 s_A h_{ij}}{8\sqrt{2m^2_W}} \left( V_{q\bar{q}} V_{q\bar{q}_3} + \frac{1}{3} V_{q\bar{q}} V_{q\bar{q}_3} \right) f_{h_3}(p_3) \times \langle h_1(p_1) | \bar{q}_3 \bar{q}_3 \rangle_{\nu^0} | h(p) \rangle
\]
\[
+ \langle h_2(p_2) | \bar{q}_3 \bar{q}_3 \rangle_{\nu^0} | h(p) \rangle \langle \text{lepton}, \quad (9)
\]

If \( q \neq q_3 \), only Fig. 1(a) and (b) contribute:
\[
M = M_A + M_B
\]

Table 1. Decay constants of mesons (in MeV). The values for \( \pi, K, D, \) and \( D_s \) are from Particle Data Group [24]; \( K^* \) and \( \rho \) are from Ref. [25]; \( D^* \) and \( D_s^* \) are from Ref. [26].

| \( f_\pi \) | \( f_K \) | \( f_{K^*} \) | \( f_\rho \) | \( f_D \) | \( f_{D_s} \) | \( f_{D_s^*} \) | \( f_{D^*} \) |
|---|---|---|---|---|---|---|---|
| 130.4 | 156.2 | 217 | 205 | 204.6 | 257.5 | 340 | 375 |
The partial decay width is obtained by evaluating the phase space integral
\[
\Gamma = \left(1 - \frac{1}{2} \delta_{h_1 h_2}\right) \left(1 - \frac{1}{2} \delta_{l_1 l_2}\right) \frac{\Delta s_{12}}{s_{12}} \frac{\Delta s_{34}}{s_{34}}
\times \int \frac{d\cos\theta_{12}}{s_{12}} \frac{d\cos\theta_{34}}{s_{34}} \int d\phi |M|^2,
\]
\[
\xi = \frac{\lambda^{1/2}(M^2, s_{12}, s_{34}) \lambda^{1/2}(s_{21}, M^2, m^2)}{\lambda^{1/2}(s_{34}, m_1^2, m_2^2)}
\times \lambda^{1/2}(s_{12}, M^2, M^2).
\]
We also use the definitions \( s_{12} = (p_1 + p_2)^2 \) and \( s_{34} = (p_3 + p_4)^2 \). The meanings of \( \delta_{h_1 h_2} \), \( \delta_{l_1 l_2} \), and \( \phi \) are shown in Fig. 2. \( \delta_{h_1 h_2} \) is 1 if \( h_1 \) and \( h_2 \) are identical particles, otherwise it is 0. The same is true for \( \delta_{l_1 l_2} \). The integral limits are
\[
s_{12} = \left[(M_1 + M_2)^2, (M_1 - M_2)^2\right],
\]
\[
s_{34} = \left[(m_1 + m_2)^2, (M - \sqrt{s_{12}})^2\right],
\]
\[
\phi \in [0, 2\pi], \quad \theta_{12} \in [0, \pi], \quad \theta_{34} \in [0, \pi],
\]
where \( M_2, m_1, \) and \( m_2 \) are the masses of \( h_2, l_1, \) and \( l_2 \), respectively.

Fig. 2. Kinematics of the four-body decay of \( h \) in its rest frame. \( P_1 \) and \( P_2 \) are respectively the momenta of \( h_1 \) and \( h_2 \) in their center-of-momentum frame; \( P_3 \) and \( P_4 \) are respectively the momenta of \( l_1 \) and \( l_2 \) in their center-of-momentum frame.

3 Numerical results

The Bethe-Salpeter method has certain advantages when calculating the form factors, especially in the case when both initial and final mesons are heavy. In the first step, the wave functions of the mesons, which include relativistic corrections, are obtained by solving numerically the corresponding instantaneous Bethe-Salpeter equation. Their pole structure is important for describing the properties of heavy mesons. Subsequently, the form factors for the physically allowed region are calculated using Eq. (13) without any analytic extension. Although the instantaneous approximation is reasonable for the double heavy mesons and acceptable for the heavy-light mesons, it results in large errors for the light mesons, such as \( \pi \) and \( K \). For example, when we change the parameters by \( \pm 5\% \), the form factors at \( Q^2 = 0 \) for the channels with heavy mesons change by less than \( 10\% \), while for those with \( \pi \) or \( K \), the errors can be larger than \( 50\% \). For processes with light mesons, such as \( B \to \pi \rho \), other methods are more appropriate, for example the light-cone sum rules. Nevertheless, we use this approximation also for the light mesons as the decay channels we consider are related to new physics, for which the branching ratios are expected to be very small, and only the order of magnitude is important.

The parameters of the doubly-charged Higgs boson have no definite values at present, only the lower or upper limits from experiments are available. For example, the latest results of the ATLAS and CMS Collaborations \([16, 17]\) show that the mass of \( \Delta^{++} \) is larger than 800 GeV. From Ref. \([12]\), the upper limit for \( s_{\Delta} \) is 0.0056. The constraints for the coupling \( h_{\mu\mu} \) can be extracted from the \( e^+ e^- \) annihilation process \([29]\):
\[
\frac{\alpha}{\pi} \leq 9.7 \times 10^{-6} \text{GeV}^{-2}.
\]
For \( h_{\mu\mu} \), the Muon g-2 experiment provides the limit \([30]\):
\[
\frac{\alpha h_{\mu\mu}}{m_\mu^2} \leq 3.4 \times 10^{-6} \text{GeV}^{-2}.
\]
The \( h_{\mu\mu} \) is related to \( \mu^- \to e^- e^- e^- \) and \( \mu^- \to e^- \gamma \) processes \([12]\), which give \( \frac{\alpha h_{\mu\mu}}{m_\mu^2} \leq 3.2 \times 10^{-11} \text{GeV}^{-2} \) and \( \frac{\alpha h_{\mu\mu}}{m_\mu^2} \leq 2.0 \times 10^{-10} \) \text{GeV}^{-2}, respectively. Taking \( m_{\Delta} = 1000 \) GeV as an example, we can estimate the upper limits of the quantity \( \left( \frac{\alpha h_{\mu\mu}}{m_\mu^2} \right)^2 \) for the \( ee \) and \( \mu\mu \) cases as \( 3.0 \times 10^{-16} \) and \( 1.1 \times 10^{-16} \), respectively. For the \( e\mu \) case, following the method applied in Ref. \([12]\), we let \( h_{ee} \) and \( h_{\mu\mu} \) equal to their upper bound, and get \( h_{ee} \leq 1.1 \times 10^{-16} \), which leads to \( \left( \frac{\alpha h_{ee}}{m_\Delta^2} \right)^2 \leq 3.3 \times 10^{-27} \).

For \( K^0 \), there are only three channels allowed by the phase space, namely \( \pi^+ \pi^- l_i^+ l_i^- \) (\( l_i = e, \mu \)). The corresponding diagrams are Fig. 1(a)–(d). The \( \pi^+ \pi^- e^- e^- \) channel has the largest branching ratio, which is of the order of \( 10^{-30} \) (see Table 2). Experimentally, \( Br(K^+ \to \pi^- l_i^+ l_i^-) \leq 10^{-10} \) \([31]\), which is the most precise result for lepton number violation. However, lepton number violation in four-body decay channels of this particle has not been experimentally found. In Refs. \([32, 33]\), the channels \( K_{l_1 l_2} \to \pi^+ \pi^- e^- e^- \) are investigated. We hope that the \( K_{l_1 l_2} \to \pi^+ \pi^- l_i^+ l_i^- \) channels will be experimentally studied in the future.

For \( D^0 \), the final mesons can be pseudoscalars or vectors. The results for the case when \( h_1 \) and \( h_2 \) are both pseudoscalars, that is \( \pi \pi, \pi K \), or \( KK \), are given in Table 3. The largest value is of the order of magnitude of \( 10^{-29} \). We note that the Fermilab E791 Collaboration presented the upper limits of the branching ratios for these channels \([34]\), which are of the order of \( 10^{-5} \). By comparing the theoretical predictions and experimental data, we find the upper limit of the constant \( \left( \frac{\alpha h_{ee}}{m_\Delta^2} \right)^2 \) of the
order of $10^4$ GeV$^{-2}$. One can also extract this upper limit from the three-body decay processes, such as $D^- \rightarrow \pi^- e^- e^-$, which gives about $10^2$ GeV$^{-2}$ by using the results in Ref. [12]. The branching ratios of $D^0$ decay channels, where $h_1$ and $h_2$ are 0$^{-1}$ or 1$^{-1}$, are given in Table 4; the largest value has the order of magnitude of $10^{-29}$. The results for $B^0$ and $\bar{B}^0$ are given in Tables 5–10. The largest value is of the order of $10^{-28}$. In Ref. [35], the four-body decay channel $B^- \rightarrow D^0 \pi^+ \mu^- \mu^-$ was measured to have a branching ratio of less than $1.5 \times 10^{-6}$. There are no experimental values available at present for the neutral B meson decay channels. However, as LHCb is continuing to run, more data will be available. We expect that the LHCb Collaboration will detect such decay modes and will set more stringent constraints on the parameters of doubly-charged Higgs boson. Besides, the future B-factories, such as Belle-II, will also have the possibility of providing more information about these channels.

| decay channel | upper limit of $Br$ | Exp. bound on $Br$ [34] | $\frac{2\Delta m_{ee}}{m_{ee}}$/GeV$^{-2}$ |
|---------------|---------------------|-------------------------|---------------------------------------------|
| $D^0 \rightarrow \pi^- \pi^- e^- e^+$ | $1.8 \times 10^{-29}$ | $< 1.12 \times 10^{-5}$ | $< 42754$ |
| $D^0 \rightarrow \pi^- \pi^- \mu^+ \mu^+$ | $7.2 \times 10^{-30}$ | $< 2.9 \times 10^{-5}$ | $< 21080$ |
| $D^0 \rightarrow \pi^- \pi^- e^+ e^+$ | $4.1 \times 10^{-30}$ | $< 7.9 \times 10^{-5}$ | $< 25371$ |
| $D^0 \rightarrow \pi^- K^- e^+ e^+$ | $7.1 \times 10^{-29}$ | $< 20.6 \times 10^{-5}$ | $< 29548$ |
| $D^0 \rightarrow \pi^- K^- \mu^+ \mu^+$ | $2.7 \times 10^{-29}$ | $< 39.0 \times 10^{-5}$ | $< 39855$ |
| $D^0 \rightarrow \pi^- K^- e^+ e^+$ | $1.5 \times 10^{-30}$ | $< 21.8 \times 10^{-5}$ | $< 21683$ |
| $D^0 \rightarrow K^- K^- e^+ e^+$ | $6.1 \times 10^{-30}$ | $< 15.2 \times 10^{-5}$ | $< 86661$ |
| $D^0 \rightarrow K^- K^- \mu^+ \mu^+$ | $2.3 \times 10^{-30}$ | $< 9.4 \times 10^{-5}$ | $< 67045$ |
| $D^0 \rightarrow K^- K^- e^+ e^+$ | $1.3 \times 10^{-30}$ | $< 5.7 \times 10^{-5}$ | $< 38177$ |

| decay channel | upper limit of $Br$ |
|---------------|---------------------|
| $D^0 \rightarrow \pi^- \rho^- e^- e^+$ | $2.8 \times 10^{-30}$ |
| $D^0 \rightarrow \pi^- \rho^- \mu^+ \mu^+$ | $9.9 \times 10^{-31}$ |
| $D^0 \rightarrow \pi^- \rho^- e^+ e^+$ | $5.8 \times 10^{-41}$ |
| $D^0 \rightarrow \pi^- K^{*-} e^- e^+$ | $4.8 \times 10^{-30}$ |
| $D^0 \rightarrow \pi^- K^{*-} \mu^+ \mu^+$ | $1.6 \times 10^{-30}$ |
| $D^0 \rightarrow \pi^- K^{*-} e^+ e^+$ | $9.5 \times 10^{-41}$ |
| $D^0 \rightarrow \rho^- K^- e^+ e^+$ | $1.4 \times 10^{-29}$ |
| $D^0 \rightarrow \rho^- K^- \mu^+ \mu^+$ | $4.2 \times 10^{-30}$ |
| $D^0 \rightarrow \rho^- K^- e^+ e^+$ | $2.6 \times 10^{-40}$ |

| decay channel | upper limit of $Br$ |
|---------------|---------------------|
| $B^0 \rightarrow \pi^+ \pi^+ e^- e^-$ | $7.1 \times 10^{-30}$ |
| $B^0 \rightarrow \pi^+ \pi^+ \mu^- \mu^-$ | $2.8 \times 10^{-30}$ |
| $B^0 \rightarrow \pi^+ \pi^+ e^- e^-$ | $1.6 \times 10^{-40}$ |
| $B^0 \rightarrow \pi^+ K^+ e^- e^-$ | $2.7 \times 10^{-31}$ |
| $B^0 \rightarrow \pi^+ K^+ \mu^- \mu^-$ | $1.1 \times 10^{-31}$ |
| $B^0 \rightarrow \pi^+ K^+ e^- e^-$ | $6.2 \times 10^{-42}$ |
| $B^0 \rightarrow \pi^+ K^+ \mu^- \mu^-$ | $7.8 \times 10^{-29}$ |
| $B^0 \rightarrow \pi^+ D^+ e^- e^-$ | $3.0 \times 10^{-29}$ |
| $B^0 \rightarrow \pi^+ D^+ \mu^- \mu^-$ | $1.7 \times 10^{-39}$ |
| $B^0 \rightarrow \pi^+ D^+ e^- e^-$ | $6.2 \times 10^{-30}$ |
| $B^0 \rightarrow \pi^+ D^+ \mu^- \mu^-$ | $2.4 \times 10^{-30}$ |

| decay channel | upper limit of $Br$ |
|---------------|---------------------|
| $B^0 \rightarrow \pi^+ \pi^+ e^- e^-$ | $1.4 \times 10^{-40}$ |
| $B^0 \rightarrow \pi^+ \pi^+ \mu^- \mu^-$ | $5.2 \times 10^{-30}$ |
| $B^0 \rightarrow \pi^+ \pi^+ e^- e^-$ | $2.0 \times 10^{-30}$ |
| $B^0 \rightarrow \pi^+ K^+ e^- e^-$ | $1.2 \times 10^{-40}$ |
| $B^0 \rightarrow \pi^+ K^+ \mu^- \mu^-$ | $5.3 \times 10^{-30}$ |
| $B^0 \rightarrow \pi^+ D^+ e^- e^-$ | $2.1 \times 10^{-30}$ |
| $B^0 \rightarrow \pi^+ D^+ \mu^- \mu^-$ | $1.2 \times 10^{-40}$ |
| $B^0 \rightarrow \pi^+ D^+ e^- e^-$ | $7.2 \times 10^{-29}$ |
| $B^0 \rightarrow \pi^+ D^+ \mu^- \mu^-$ | $2.9 \times 10^{-29}$ |
| $B^0 \rightarrow \pi^+ D^+ e^- e^-$ | $1.6 \times 10^{-39}$ |
Table 6. The upper limit of $Br$ for $0^-1^-$ decay channels of $\bar{B}^0$.

| decay channel | upper limit of $Br$ | decay channel | upper limit of $Br$ |
|---------------|---------------------|---------------|---------------------|
| $B^0 \rightarrow \pi^+ \rho^+ e^- e^-$ | $6.4 \times 10^{-30}$ | $B^0 \rightarrow \rho^+ D^+_s e^- e^-$ | $3.8 \times 10^{-31}$ |
| $B^0 \rightarrow \pi^+\rho^- e^- e^-$ | $2.5 \times 10^{-30}$ | $B^0 \rightarrow \rho^+ D^+_s \mu^- \mu^-$ | $1.5 \times 10^{-31}$ |
| $B^0 \rightarrow \pi^+\rho^- e^- e^-$ | $1.5 \times 10^{-30}$ | $B^0 \rightarrow \rho^+ D^+_s \mu^- \mu^-$ | $8.7 \times 10^{-42}$ |
| $B^0 \rightarrow \pi^+ K^+ + e^- e^-$ | $5.1 \times 10^{-31}$ | $B^0 \rightarrow K^+ D^+ e^- e^-$ | $4.0 \times 10^{-30}$ |
| $B^0 \rightarrow \pi^+ K^+ e^- e^-$ | $2.0 \times 10^{-31}$ | $B^0 \rightarrow K^+ D^+ e^- e^-$ | $1.6 \times 10^{-30}$ |
| $B^0 \rightarrow \pi^+ K^+ e^- e^-$ | $1.2 \times 10^{-41}$ | $B^0 \rightarrow K^+ D^+ e^- e^-$ | $9.0 \times 10^{-41}$ |
| $B^0 \rightarrow \rho^+ K^+ e^- e^-$ | $1.8 \times 10^{-32}$ | $B^0 \rightarrow K^+ D^+ e^- e^-$ | $6.9 \times 10^{-30}$ |
| $B^0 \rightarrow \rho^+ K^+ e^- e^-$ | $7.2 \times 10^{-33}$ | $B^0 \rightarrow K^+ D^+ e^- e^-$ | $2.7 \times 10^{-30}$ |
| $B^0 \rightarrow \rho^+ K^+ e^- e^-$ | $4.1 \times 10^{-43}$ | $B^0 \rightarrow K^+ D^+ e^- e^-$ | $1.5 \times 10^{-40}$ |
| $B^0 \rightarrow \rho^+ D^+ e^- e^-$ | $4.7 \times 10^{-39}$ | $B^0 \rightarrow D^+ D^+ e^- e^-$ | $4.2 \times 10^{-31}$ |
| $B^0 \rightarrow \rho^+ D^+ e^- e^-$ | $1.8 \times 10^{-39}$ | $B^0 \rightarrow D^+ D^+ e^- e^-$ | $1.6 \times 10^{-31}$ |
| $B^0 \rightarrow \rho^+ D^+ e^- e^-$ | $1.1 \times 10^{-39}$ | $B^0 \rightarrow D^+ D^+ e^- e^-$ | $9.2 \times 10^{-42}$ |
| $B^0 \rightarrow \rho^+ D^+ e^- e^-$ | $1.3 \times 10^{-38}$ | $B^0 \rightarrow D^+ D^+ e^- e^-$ | $4.2 \times 10^{-29}$ |
| $B^0 \rightarrow \rho^+ D^+ e^- e^-$ | $4.9 \times 10^{-39}$ | $B^0 \rightarrow D^+ D^+ e^- e^-$ | $1.6 \times 10^{-39}$ |
| $B^0 \rightarrow \rho^+ D^+ e^- e^-$ | $2.8 \times 10^{-39}$ | $B^0 \rightarrow D^+ D^+ e^- e^-$ | $9.1 \times 10^{-40}$ |
| $B^0 \rightarrow \rho^+ D^+ e^- e^-$ | $1.5 \times 10^{-39}$ | $B^0 \rightarrow D^+ D^+ e^- e^-$ | $1.9 \times 10^{-29}$ |
| $B^0 \rightarrow \rho^+ D^+ e^- e^-$ | $6.0 \times 10^{-30}$ | $B^0 \rightarrow D^+ D^+ e^- e^-$ | $7.3 \times 10^{-30}$ |
| $B^0 \rightarrow \rho^+ D^+ e^- e^-$ | $3.5 \times 10^{-40}$ | $B^0 \rightarrow D^+ D^+ e^- e^-$ | $4.2 \times 10^{-40}$ |

Table 7. The upper limit of $Br$ for $1^-1^-$ decay channels of $\bar{B}^0$.

| decay channel | upper limit of $Br$ | decay channel | upper limit of $Br$ |
|---------------|---------------------|---------------|---------------------|
| $B^0 \rightarrow \rho^+ \rho^- e^- e^-$ | $1.3 \times 10^{-30}$ | $B^0 \rightarrow \rho^+ D^+_s e^- e^-$ | $2.2 \times 10^{-42}$ |
| $B^0 \rightarrow \rho^+ \rho^- e^- e^-$ | $5.0 \times 10^{-31}$ | $B^0 \rightarrow K^+ K^+ e^- e^-$ | $1.2 \times 10^{-42}$ |
| $B^0 \rightarrow \rho^+ \rho^- e^- e^-$ | $3.0 \times 10^{-41}$ | $B^0 \rightarrow K^+ K^+ e^- e^-$ | $4.4 \times 10^{-30}$ |
| $B^0 \rightarrow \rho^+ \rho^- e^- e^-$ | $4.6 \times 10^{-42}$ | $B^0 \rightarrow K^+ K^+ e^- e^-$ | $2.6 \times 10^{-40}$ |
| $B^0 \rightarrow \rho^+ \rho^- e^- e^-$ | $1.7 \times 10^{-32}$ | $B^0 \rightarrow D^+ D^+ e^- e^-$ | $2.0 \times 10^{-29}$ |
| $B^0 \rightarrow \rho^+ \rho^- e^- e^-$ | $9.9 \times 10^{-43}$ | $B^0 \rightarrow D^+ D^+ e^- e^-$ | $7.7 \times 10^{-30}$ |
| $B^0 \rightarrow \rho^+ \rho^- e^- e^-$ | $2.0 \times 10^{-28}$ | $B^0 \rightarrow D^+ D^+ e^- e^-$ | $4.6 \times 10^{-40}$ |
| $B^0 \rightarrow \rho^+ \rho^- e^- e^-$ | $7.6 \times 10^{-29}$ | $B^0 \rightarrow D^+ D^+ e^- e^-$ | $2.0 \times 10^{-28}$ |
| $B^0 \rightarrow \rho^+ \rho^- e^- e^-$ | $4.6 \times 10^{-39}$ | $B^0 \rightarrow D^+ D^+ e^- e^-$ | $7.8 \times 10^{-29}$ |
| $B^0 \rightarrow \rho^+ \rho^- e^- e^-$ | $9.7 \times 10^{-32}$ | $B^0 \rightarrow D^+ D^+ e^- e^-$ | $4.6 \times 10^{-30}$ |
| $B^0 \rightarrow \rho^+ \rho^- e^- e^-$ | $3.7 \times 10^{-32}$ | $B^0 \rightarrow D^+ D^+ e^- e^-$ | $4.6 \times 10^{-30}$ |

Table 8. The upper limit of $Br$ for $0^-0^-$ decay channels of $\bar{B}^0$.

| decay channel | upper limit of $Br$ | decay channel | upper limit of $Br$ |
|---------------|---------------------|---------------|---------------------|
| $B^0 \rightarrow \pi^+ K^- e^- e^-$ | $2.5 \times 10^{-31}$ | $B^0 \rightarrow K^+ D^+_s e^- e^-$ | $1.8 \times 10^{-40}$ |
| $B^0 \rightarrow \pi^+ K^- e^- e^-$ | $9.8 \times 10^{-32}$ | $B^0 \rightarrow K^+ D^+_s e^- e^-$ | $2.3 \times 10^{-32}$ |
| $B^0 \rightarrow \pi^+ K^- e^- e^-$ | $3.2 \times 10^{-32}$ | $B^0 \rightarrow K^+ D^+_s e^- e^-$ | $9.2 \times 10^{-33}$ |
| $B^0 \rightarrow \pi^+ K^- e^- e^-$ | $1.3 \times 10^{-32}$ | $B^0 \rightarrow K^+ D^+_s e^- e^-$ | $5.3 \times 10^{-44}$ |
| $B^0 \rightarrow \pi^+ K^- e^- e^-$ | $7.3 \times 10^{-43}$ | $B^0 \rightarrow K^+ D^+_s e^- e^-$ | $2.4 \times 10^{-30}$ |
| $B^0 \rightarrow \pi^+ K^- e^- e^-$ | $5.6 \times 10^{-29}$ | $B^0 \rightarrow K^+ D^+_s e^- e^-$ | $9.5 \times 10^{-31}$ |
| $B^0 \rightarrow \pi^+ K^- e^- e^-$ | $5.8 \times 10^{-32}$ | $B^0 \rightarrow K^+ D^+_s e^- e^-$ | $5.4 \times 10^{-41}$ |
| $B^0 \rightarrow \pi^+ K^- e^- e^-$ | $2.2 \times 10^{-29}$ | $B^0 \rightarrow K^+ D^+_s e^- e^-$ | $1.3 \times 10^{-28}$ |
| $B^0 \rightarrow \pi^+ K^- e^- e^-$ | $1.3 \times 10^{-29}$ | $B^0 \rightarrow K^+ D^+_s e^- e^-$ | $5.2 \times 10^{-29}$ |
| $B^0 \rightarrow \pi^+ K^- e^- e^-$ | $8.1 \times 10^{-30}$ | $B^0 \rightarrow K^+ D^+_s e^- e^-$ | $2.9 \times 10^{-30}$ |
| $B^0 \rightarrow \pi^+ K^- e^- e^-$ | $3.2 \times 10^{-30}$ | $B^0 \rightarrow K^+ D^+_s e^- e^-$ | $2.9 \times 10^{-30}$ |
Wave functions of mesons

With the instantaneous approximation, the Bethe-Salpeter wave function of the meson fulfills the full Salpeter equations [36]

\[
(M - \omega_L) \varphi_{\mu}^{+}(q_L) = \lambda^{+}_{\mu} \eta_{\mu}(q_L) \Lambda^{2}_{\mu},
\]

\[
(M + \omega_L) \varphi_{\mu}^{-}(q_L) = -\Lambda^{-}_{\mu} \eta_{\mu}(q_L) \Lambda^{2}_{\mu},
\]

(A1)

\[
\varphi_{\mu}^{+}(q_{\perp}) = \varphi_{\mu}^{-}(q_{\perp}) = 0,
\]
where $q_±^2 = q^2 - P^2 / M^2$, $\omega_1 = \sqrt{m_1^2 - q_1^2}$, and $\omega_2 = \sqrt{m_2^2 - q_2^2}$; $m_1$ and $m_2$ are the masses of quarks and antiquarks, respectively; $\Lambda^2 = \frac{1}{8} \left[ \frac{2}{3} \omega_1 \omega_2 + \omega_1^2 \right] (q_1 + m_i) / 2$ is the projection operator. In the above equation, we have defined

$$
\eta_{\mu}(q_\perp) = \int \frac{d^3k_\perp}{(2\pi)^3} V(P; q_\perp, k_\perp) \varphi_\mu(k_\perp),
$$

and

$$
\varphi_{\perp}^{\pm}(q_\perp) = \Lambda^2 \frac{P}{M} \varphi_{\perp}(q_\perp) \frac{P}{M} \Lambda^2,
$$

where $\varphi_{\perp}(q_\perp)$ is the wave function, which is constructed using $q_\perp$, $P$, and the polarization vector. Here we only show the expression for the positive energy part of the wave function. For the 1$^-$ state, it has the form

$$
\varphi_{1-}^{++}(q_\perp) = (q_\perp \cdot e) \left[ A_1(q_\perp) + \frac{P}{M} A_2(q_\perp) + \frac{q}{M} A_3(q_\perp) \right]
$$

$$
+ \frac{Pq}{M^2} A_4(q_\perp) + Mf \left[ A_5(q_\perp) + \frac{q}{M} A_6(q_\perp) \right].
$$

For the 0$^-$ state, it has the form

$$
\varphi_{0-}^{++}(q_\perp) = \left[ B_1(q_\perp) + \frac{P}{M} B_2(q_\perp) + \frac{q}{M} B_3(q_\perp) + \frac{Pq}{M^2} B_4(q_\perp) \right] \gamma_\nu.
$$

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