Dynamics of a vortex system in a layered high-temperature superconductor under a pulsed current impact

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Abstract. By using the Monte Carlo method, electric field relaxation has been numerically studied in a 2D layered HTS with defects. Time dependences of the electric field strength inside the sample for an instantaneous rise of electric current have been calculated for different temperatures. The influence of temperature on the shape of such dependences has been demonstrated. The impact of a rectangular current pulse on the vortex system has been studied. The value of the response field strength and its dependence on the amplitude and duration of the pulse have been calculated as well as the impact of pulse duration on the shape of the response.

1. Introduction
One of the application areas of both low-temperature and high-temperature superconductors are the development of high-speed current-limiting devices as well as current switches. In current limiters it is assumed to limit the sharp impulse of current during a short circuit. High-speed current switches can be used, for example, for impulse output of power from the energy storage systems. It is obvious that during a sharp pulse of current flowing through a superconductor, a highly non-equilibrium magnetic flux distributions in the superconductors (that is non-equilibrium Abrikosov vortices states) are created. Nowadays, great attention is paid to the study of processes of creation and relaxation of such non-equilibrium vortex systems. For example, in paper [1], transport characteristics of a $2H$-NbSe$_2$ superconductor have been measured, and a history dependence associated with the so-called “peak effect” has been demonstrated. The existence of metastable vortex lattice states has also been shown. Paper [2] is dedicated to studying the vortex lattice in the Bragg glass state. An exponential time dependence of voltage in the sample under the impact of pulsed current has been demonstrated with the exponent characterizing the deviation from equilibrium. In paper [3], energy dissipation induced in the sample by the vortex motion has been studied by using short and long current pulses.

A significant number of studies have been dedicated to the influence of current rise-rate $dI/dt$ on the current-voltage ($I$-$V$) curve. Some research has been done on MgB$_2$ samples (e.g. in [4]), as well as on high-temperature BSCCO [5] and YBCO [6] superconductors (HTS). The $I$-$V$ curve irreversibility and staircase behaviour have been demonstrated. As has been demonstrated in theoretical and experimental papers [4–7], the $I$-$V$ curve irreversibility becomes generally more prominent with the growth of $dI/dt$ and may be due to the granular structure of samples, the vortex dynamics under strong pinning conditions [3], and different pinning regimes in a polycrystalline sample. In paper [5], voltage relaxation has been observed after an instantaneous stop in changing the current during $I$-$V$ curve
measurements. It has been shown that the sign of such relaxation depends on the sign of \( \frac{dl}{dr} \). Magnetic field has been demonstrated to suppress the \( I-V \) curve hysteresis.

Since the effects considered in the mentioned papers can generally be associated with the dynamics of the vortex system of the sample in the mixed state and the vortex interactions with pinning cites, a detailed study of the vortex structure dynamics is an important task. The use of the Monte Carlo method is rather convenient in this case. It has already been successfully used to calculate magnetic and transport characteristics of HTS materials under applied transport current or external magnetic field \( (H) \) in papers [8–11]. For example, an S-like \( I-V \) curve peculiarity has been observed for a superconductor with ferromagnetic defects in magnetic field [10,11]. The mentioned peculiarity could not be reproduced when reversing the current rise, which is an example of hysteresis. The method developed by the authors of papers [8–11] can also be used to study the relaxation effects in a vortex system under the impact of instantaneous rise and fall of electric current through the sample. The purpose of this paper is to numerically study the vortex dynamics under the impact of a pulsed current depending on temperature and the parameters of the pulse. We assume for simplicity that \( H=0 \), i.e. the vortices are generated by the self-field of transport current.

2. Method for calculations. The critical current of HTS with defects at different temperatures

In this paper, the electric field strength – current density \( (E-j) \) curves and energy losses in the superconductor have been calculated by using the method developed in [8,9]. Transport losses are due to the movement of vortex lines under the impact of transport current and can be obtained by calculating the number of opposite-sign vortex pairs that have annihilated in the sample centre. By using the value of energy released in those annihilations (which equals the work of Lorenz force during vortex-antivortex movement to the centre of the superconductor) and knowing the value of transport current, one can calculate the electric field strength \( E \) in the superconductor. Since there is no real time in the simulations, a normalization of the calculated \( E-j \) curve to an experimental one is needed.

The electric field strength \( E \) is related to the pair annihilation energy and the magnetic field \( H_l \) created by the transport current on the edge of the sample by the following equation: 
\[
E = \alpha \frac{Q}{H_l},
\]
where \( \alpha \) is the proportionality coefficient. It is assumed that a certain number of annihilated pairs corresponds to the start of the flux flow regime and to the critical electric field strength of 1 \( \mu \)V/cm (used in most experiments). Based on this, the coefficient \( \alpha \) can be determined. During calculations of the \( E(t) \) dependence, the released energy is memorized regularly over a certain number of Monte Carlo steps and it is assumed that the number of annihilated pairs is proportional to the calculation time. Accordingly, the proportionality coefficient \( \alpha \) is recalculated as \( \alpha \to \alpha \frac{\Delta t}{\Delta T} \), where \( \Delta t \) is the time interval between two neighbouring points on the \( E(t) \) dependence, and \( \Delta T \) is the calculation time of one point on the \( E-j \) curve. The time equivalent of one Monte Carlo step has been estimated as \( \sim 10^{-10} - 10^{-8} \) s from calculations of relaxation of residual magnetization in HTS. For further analysis, we need to know the values of current density \( j \) that corresponds to the values of electric field strength \( E \) of 1 \( \mu \)V/cm, 2 \( \mu \)V/cm, and 3 \( \mu \)V/cm in the sample. We also need to know how those values depend on temperature and the pinning parameters (namely the defect potential well depth) in the HTS. To reduce the influence of calculation error, all simulations have been carried out for the same defect configuration with density \( n_d = 6.7 \times 10^9 \) \( \text{cm}^{-2} \) (the total number of defects per sample is 2020). The sample width \( d = 6 \) \( \mu \)m.

Figure 1(a) shows a series of \( E-j \) curves for a sample with defects of potential well depth \( c = 0.01 \) eV at different temperatures \( T \). It can be seen that with the increasing temperature, the slope of the \( E-j \) curves decreases, which is in a qualitative agreement with experiment. The thermal dependences of critical current density \( j_c \) (determined by the 1 \( \mu \)V/cm criterion) are presented in figure 1(b) for different values of defect potential well depth. It can be seen that the critical current density decreases when the potential well depth becomes comparable to \( T \). From this point on, the potential well depth was kept constant at \( c = 0.01 \) eV for convenience.
3. The time dependences of electric field strength at instantaneous rise of current

Let us consider the case when a current of critical density $j_c$ (which we have determined from the $E$-$j$ curves) is momentarily switched on (which is equivalent to a rectangular current pulse of infinite duration). Figure 2(a) shows the superconductor’s response curve in the form of time dependences of the electric field inside the sample at different temperatures. These dependences have peculiarities: a sharp increase in the field strength up to values significantly exceeding 1 $\mu$V/cm in the initial section of the plot. Then the electric field slowly saturates to an average value that also exceeds 1 $\mu$V/cm, at least for the considered calculation time. The sharp increase in the initial section was also observed on the time dependences of the number and energy of vortices in the sample so it can be explained by an avalanche-like process of vortices penetrating the sample and further annihilating their counterparts in the sample centre. These peculiarities disappear at temperatures higher than 10 K (in figure 2(a), they are already not visible as the electric field saturates almost immediately) since the vortices can no longer be effectively pinned by defects at this temperature, and there is no avalanche-like process.

Figure 2(b) shows the main response parameters: $E_{\text{pulse}}$ is the average electric field value for large time values, $t_f$ is the front duration, i.e. the time passed from the moment of switching on the current until the electric field saturated. For each time dependence, $t$ is shown as the number of Monte Carlo steps.

Figures 3(a) and (b) show the thermal dependences of the mentioned response parameters. Additionally, three different pulse amplitudes have been considered: the values of current density that correspond to $E = 1$ $\mu$V/cm, 2 $\mu$V/cm, and 3 $\mu$V/cm on the pre-calculated $E$-$j$ curves (figure 1(a)). It can be seen that in all three cases, the thermal dependence of $t_f$ (figure 3(a)) is descending in average, and the sharpest decrease can be seen at $T = 10$ K, i.e. the temperature value at which the $E(t)$ peculiarities disappear (figure 2(a)). In figure 3(b), $E_{\text{pulse}}$ shows a weak temperature dependence, and the value of electric field jump $dE$ decreases rather sharply with the increasing temperature.

![Graphs](image_url)

**Figure 1.** Transport characteristics of HTS samples with defects of density $n_d = 6.7 \times 10^9$ cm$^{-2}$: (a) $E$-$j$ curves of a sample with a fixed defect potential well depth $c = 0.01$ eV at different temperatures from 0.2 K to 70 K; (b) The $j_c$ versus $T$ dependence for different defect potential well depths from 0.005 eV to 0.1 eV.
Figure 2. Time dependences of electric field strength in the sample for an instantaneous rise of current (a rectangular pulse of infinite duration): (a) for different temperatures from 0.1 K to 10 K; (b) for $T = 0.1$ K with the main response parameters shown: $t_f$ is the front duration (the time necessary for the electric field to achieve a constant value), $E_{\text{pulse}}$ is the average electric field value, $dE$ is the value of field jump during rise of current (relative to $E_{\text{pulse}}$).

Figure 3. Thermal dependences of the response parameters for the instantaneous rise of current of densities corresponding to the electric field values of 1 μV/cm, 2 μV/cm, and 3 μV/cm obtained from $E$-$j$ curves: (a) the front duration $t_f$; (b) the average field strength $E_{\text{pulse}}$ and field jump $dE$. 
4. Finite rectangular current pulses

Let us now consider the case when a rectangular current pulse of finite duration is acting on the sample. For that, we have calculated the response curves for pulses of 3 different amplitudes (same as before) and 2 different duration times: a short pulse (duration time equivalent of 7000 Monte Carlo steps) and a long pulse (time duration of 200000 steps). The temperature was kept constant this time at \( T = 1 \) K. The response curve shapes are shown in figures 4 (a), (b) and (c) for each pulse amplitude in ascending order. The thick dashed lines in all figures are for short pulses and the thin solid lines are for long pulses. The insets in each figure show the scaled parts of the responses to short pulses. If we compare the shapes of the response curves for long pulses to the ones presented in figure 2, it can be seen that the initial parts of the dependences match.

It can also be seen that in the case of short current pulse there is no saturation on the response curve, which means that the pulse duration is less than the front duration. It should be noted that increasing the amplitude of the pulse only changes the peak value of electric field strength and has no effect on the time it takes for the field to drop to zero when the current is switched off (fall time). This can be clearly seen in the insets in each figure. In the case of a long pulse, the electric field saturation can be seen throughout the pulse duration, and after the current is switched off, the field strength drops to zero. Just as for the case of short pulse, the fall time does not depend on the amplitude of the pulse. Moreover, it appears to be of the same value as for the short pulse. It should be noted that, for now, the nonzero fall time is merely a peculiarity occurring in our simulations which requires further research.
Figure 4. $E(t)$ curves (response curves) for finite rectangular current pulses of 3 different amplitudes and 2 duration times. The thick dashed lines denote the response to the short pulse (7000 Monte Carlo steps long), and the thin solid lines denote the response to the long pulse (200000 steps long). The scaled insets show the response to short pulses in more detail. Solid horizontal lines schematically show the current pulses of each duration time.

5. Conclusion

By using the Monte Carlo method, time dependences of the electric field strength in the superconductor have been calculated for an instantaneous rise of current of a given value. These dependences have demonstrated a sharp increase right after the current is switched on with a further saturation to an equilibrium value that exceeds the one that corresponds to this current of $E-j$ curves. The sharp field jumps have been shown to disappear with the increase of temperature. Thermal dependences of the response parameters of HTS to the instantaneous rise of current have been obtained for different current values. The response curves of the superconductor to rectangular current pulses of different amplitudes and durations have been calculated. It has been shown that for pulse durations exceeding the front duration, the response curve represents the initial section of the electric field versus time dependence for an instantaneous rise of current. It has also been shown that the relaxation time does not depend on the duration of the initial pulse for the considered amplitudes and durations of pulses. The results provided by our simulations can be useful for understanding of physical processes occurring in non-equilibrium vortex systems as well as for picking out the most optimal parameters (such as temperature regimes and maximum current loads) in the design of superconducting high-speed current limiters and switchers.

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