"Spaghetti" design for gravitational wave superconducting antenna

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Abstract. A new concept for detectors of gravitational wave radiation is discussed. Estimates suggest that strain sensitivity essentially better than that of the existing devices can be achieved in the wide frequency range. Such sensitivity could be obtained with devices about one meter long. Suggested device consists of multi-billion bimetallic superconducting wires ("spaghettis") and requires cryogenic operational temperatures (~0.3K in the case considered).

1. Introduction

To date, the most advanced approaches (LIGO, LISA) to gravitational wave (GW) detection measure change of the distances between macroscopic bodies. The longer the baseline the bigger the change. If the baseline distance is "2400 trillion miles, or one million times the distance to Neptune. … then a strong GW will briefly change that distance by less than the thickness of a human hair. We have perhaps less than a few tenths of a second to perform this measurement. And we don't know if this infinitesimal measurement will come next month, next year, or perhaps in thirty years." [1]. For the known sources of GW radiation, like the Crab Pulsar, the change will be far smaller. The difficulty of the task justifies searching for novel highly sensitive GW antennas [2]. In this note, we will describe a device based on the Tolman-Stewart (TS) effect: when a rotating coil is decelerating, an electric field \( E_{TS} = mg/e \) is generated in the conductor, which yields an electric current in the circuit containing the coil [3]. Here \( g \) is the deceleration (or, equivalently, acceleration) of the conductor, \( m \) is the rest mass of the electron, and \( e \) is its charge.

Gravitational waves accelerate charges by tidal gravitational forces, which are symmetric relative to the centre of mass of the charge carriers, generating a potential difference between the end of the conductor and its middle part. As early as 1976, Adler [4] noted that two different metallic conductors could be used to detect GWs by measuring the potential difference between the ends of the metals. Adler used the fact that acceleration in metallic wires acts not only on electrons but also on ions. By inhomogeneously deforming the lattice, it creates a gradient of chemical potential, which redistributes the electrons more strongly than the TS mechanism. This feature was first pointed out by Dessler et al. [5], confirmed experimentally by Beams [6], and recognized by many physicists, including Schiff [7]. The consensus [8] is that the electric field acting on electrons can be stronger than the TS field by a...
factor of $M/m$, where $M$ is the ionic mass. It also has an opposite sign, so that, say, in the gravity field of the earth the electrons will move opposite to the direction of $g$. The total electric field is therefore

$$E = -\alpha Mg/e + mg/e \approx -\alpha Mg/e.$$  \hspace{1cm} (1)$$

The dimensionless coefficient $\alpha$ depends on the parameters of the material, and is estimated to be $\alpha \approx 0.1 - 1$. Then, two wires of materials with different $\alpha M$ will develop different electric potentials in the GW field. Adler did not consider the noise factors such as the Johnson-Nyquist noise related to his very long antenna. We show below how to modify this approach by taking advantage of superconductivity. In particular, this will allow making the antenna much shorter, say, 1 m long, so that it will be feasible to cool it to sub-Kelvin cryogenic temperatures. We will discuss the inherent limitations of the sensitivity due to thermal fluctuations. It will be shown by using a reasonable experimental design, that GWs from the Crab Pulsar may become detectable.

2. Basic principle

We start with the antenna geometry shown in figure 1.

![Figure 1. The GW $h = h_0 \sin \omega t$ is incident perpendicularly to the plane of the two bimetallic conductors. Each conductor, A or B, is of length $L$. The ends of the conductors are connected (not shown) to close the loop.](image)

The force acting on a lattice ion at a distance $x$ from the centre of mass is [4]

$$F(x) = M \ddot{h} x / 2; \quad \ddot{h} = d^2 h / dt^2,$$  \hspace{1cm} (2)$$

where the dimensionless amplitude $h$ of the GW is defined via induced small coordinate deviation $\Delta x = xh / 2$. Then the analog of the Eq. (1) for the case of GW is

$$E(x) = \alpha M \ddot{h} x / 2e.$$  \hspace{1cm} (3)$$

The amplitude of the electromotive force $\mathcal{E}$ is the integral of the $E$-field over the closed loop [10, 11]:

$$\mathcal{E} = \oint_{A-B} E(x)dx = -(\hbar \omega / 2e)(\alpha_A M_A - \alpha_B M_B).$$  \hspace{1cm} (4)$$

In writing this we neglected the contribution of the connectors between two bimetallic wires in Fig.1. Time-periodic electric field $E = E_0 \exp(i \omega t)$ generates the following total current density in a superconductor [11, 12]:

$$j = j_n + j_s = (\sigma_1 - i \sigma_2)E,$$  \hspace{1cm} (5)$$

where

$$\sigma_1 = (n_e e^2 \tau)[m(1 + \omega^2 \tau^2)], \quad \sigma_2 = n_e e^2 / (m \omega) + n_e e^2 (\omega \tau)^2 /[m \omega(1 + \omega^2 \tau^2)].$$  \hspace{1cm} (6)$$

In Eq. (6), $n_e$ and $n_s$ are superconducting and normal electron densities, $\omega$ is the frequency, and $\tau$ is the relaxation time of the normal electrons (typically, $\tau \approx 10^{-11}$ s). The smallness of $\omega \tau$ allows keeping only the first term in $\sigma_2$ of Eqs. (6) with the result

$$j \approx j_s = -i \frac{n_e e^2}{m \omega} E.$$  \hspace{1cm} (7)$$

Notice that in the case of normal metals ($n_s = 0$ in Eq. (6)), the dominant contribution to the current (5) would come from the $\sigma_1$-term, and the current would be smaller than (7) by a factor of $\omega \tau$, i.e., ten orders of magnitude smaller than the superconductor response at 100 Hz frequency range. We
assume for simplicity that the ratio $n_s / m$ in (7), as well as the wire cross sections $S$ are the same for both materials $A$ and $B$, so that

$$E = \frac{\mathcal{F}}{(4L)} = (\hbar L / 8e)\left(\alpha_A M_A - \alpha_B M_B\right).$$  

(8)

The requirement of having the same current flow through all parts of our circuit implies via Eq. (7) that the electric field $E$ in (8) is also spatially constant. For the harmonic GW mentioned in the caption to figure 1 with amplitude $h_0$ and frequency $\omega$, the current amplitude is:

$$I = |jS| = [n_e \omega h_0 LS / (8m)]\left[\alpha_A M_A - \alpha_B M_B\right]$$  

(9)

- this expression is immediately applicable to periodic sources, such as the Crab Pulsar.

3. Inductance and magnetic energy

In the above calculations we neglected the magnetic inductance effects. To justify such a procedure the magnetic energy: $E_{\text{mag}} = LI^2 / 2$ (where $L$ is the circuit inductance) should not exceed the kinetic energy associated with the current flow: $E_{\text{kin}} = LSnmv^2$. Otherwise, the energy would be transferred to the magnetic field rather than to the charge motion. For a single loop in figure 1 their ratio is

$$E_{\text{mag}} / E_{\text{kin}} \sim \mu_0 L e^2 n_s^2 v^2 / (2 m S)$$  

(10)

(where $\mu_0 = 4\pi \cdot 10^{-7}$ Henry/meter). Substituting $n_s \sim 10^{22}\text{cm}^{-3}$ into Eq. (10), we find that $E_{\text{mag}} / E_{\text{kin}} \sim 1$ when $S \sim 10^{-4} m^2$, or the wire diameter is about 0.1$\mu$m. This choice has the extra advantage of using superconducting materials most effectively, since the diameter of the wire is close to the London penetration depth $\lambda$ of typical superconductors. Substituting $S$ and $[(\alpha M / m)_A - (\alpha M / m)_B] \sim 10^4$ (which one can expect, say, for vanadium-lead bimetallic pair) into Eq. (10), we find $I \approx 2 \cdot 10^{-3} (\omega / \text{Hz}) h_0$. Thus, for $h_0 \sim 10^{-26}$ and $\omega \sim 2\pi(60)\text{Hz}$ we have $I \approx 0.7 \cdot 10^{-25}$ [Amp] in a single loop. In principle, one can imagine a design made of a big number $N$ of bimetallic wires with currents in opposite direction in neighboring conductors. The wires can be merged into a single read-out circuit, as shown in figure 2.

Figure 2. Left - Cross sectional view of wire arrangement. Right - Schematics of multi-wire interconnections for non-invasive current measurement via SQUID. Since the flow of current in materials $A$ and $B$ takes place in opposite directions, two semi-circles constitute a full current loop for inductive pick up.

One can, in principle, choose $N$ as big as required to make the current detectable.

4. Sensitivity

The sensitivity of the antenna is determined by the signal-to-noise ratio: at unknown frequency $I_{\text{signal}} / I_{\text{noise}}$ should be at least $>1$ for detection. Comprehensive consideration of all noise factors is beyond the scope of this paper. However, thermodynamic Johnson-Nyquist noise is unavoidable and we consider it as a first approximation. It is generated by unpaired electrons, and has magnitude

$$\langle I_{\text{noise}} \rangle = [4 (k_B T / R_n) \delta \nu]^{1/2} [11,12],$$  

where $k_B$ is the Boltzmann constant, $R_n$ is the resistance to normal (unpaired) electrons in superconductors, and $\delta \nu$ is the measurement bandwidth:

$$I_{\text{signal}} / I_{\text{noise}} \sim \alpha_n S L \omega (\alpha M / m) h_0 / [8(4(k_B T / R_n)\delta \nu)^{1/2}] > 1.$$  

(11)
This estimate is for a single loop. For the \( N \) loops signal will scale linearly with \( N \), and noise will scale as \( N^{1/2} \), thus signal to noise will scale as \( N^{1/2} \). That means the amplitude \( h_0 \) can be no smaller than \( \frac{\Delta}{\sqrt{\Delta T}} \). Since \( \Delta = \Delta(T) \) is the BCS gap in the spectrum of single-electron excitations [11,12], at \( T << T_c \), \( \Delta(T) \approx 3.52k_B T_c \):

\[
h_0 \approx \left[ 16(k_B T \delta v)^{1/2} / e_n (\alpha M / m) N^{1/2} \right] \exp(-0.88 T_c / T). \tag{12}
\]

At the parameters \( L \sim 1m, S \sim 10^{-14}m^2, n_s \sim 10^{28}m^{-3}, \omega \sim 2\pi(60)Hz, \alpha M / m \sim 10^4, \rho \sim 0.1m\Omega cm, T \sim 0.3K \), and \( N \sim 10^{12} \): \( h_0 \approx 0.5 \cdot 10^{-22} (\delta v / Hz)^{1/2} \exp(-0.88 T_c / T) \). Since \( T_c \geq 5K \) (for the V-Pb bimetallic pair), at \( T \sim 0.3K \) the exponential factor is \( \sim 7 \cdot 10^4 \), and thus \( h_0 \approx 2 \cdot 10^{-29} (\delta v / Hz)^{1/2} \), making the Crab Pulsar GW radiation detectable. At the chosen number of wires \( N \) the value of the current signal to be detected is \( I_{signal} \sim 10^{-13}A \), which, in principle, can be readily done.

5. Discussion and conclusions

To summarize, a concept for a superconducting GW is proposed. The antenna transforms part of the GW energy into the motion of superfluid electrons which is then detected electronically. The technical realization of the antenna may require some technological efforts, but there is no showstopper. For example, each “spaghetti” should have \( \sim 0.1 \mu m \) diameter. With the chosen number \( N \sim 10^{12} \), the total cross-section of the current loop is \( \sim 10^{-2}m^2 \). With \( \sim 1Hz \) bandpass filter that will allow direct detection of Crab Pulsar, since signal-to noise will be \( >1 \). Moreover, because in this and similar cases the source frequency is known with high accuracy, one can use the lock-in amplification, and detect signals much smaller than the noise level. That brings in an opportunity to reduce the number of spaghettis and making the device easier to fabricate. The fabrication of the device will require thin film deposition and nanolithography methods. Another possibility is the eventual use of superconducting nanotubes as spaghettis. The estimates we provided above demonstrate that very high sensitivity could be obtained with these devices. They are non-resonant and may be used for a wide variety of sources. Hopefully, in parallel with other recognized efforts [2], the suggested superconducting device will become efficient for a most challenging physics measurement – the detection of gravitational waves.

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