Abstract

We analyse the condensation of closed string tachyons on the $C/Z_N$ orbifold. We construct the potential for the tachyons up to the quartic interaction term in the large $N$ limit. In this limit there are near marginal tachyons. The quartic coupling for these tachyons is calculated by subtracting from the string theory amplitude for the tachyons, the contributions from the massless exchanges, computed from the effective field theory. We argue that higher point interaction terms are also of the same order in $1/N$ as the quartic term and are necessary for existence of the minimum of the tachyon potential that is consistent with earlier analysis.
1 Introduction

The condensation of closed string tachyons has been studied in some specific models. For closed string on nonsupersymmetric $C/Z_N$ orbifold, there are tachyons in the twisted sectors. Being in the twisted sectors, these tachyons are localised at the fixed point of the orbifold. This gives a more manageable set up for the study of condensation of these tachyons. In [1] D-brane probes were used to follow this process and it was shown that the end result of the condensation with generic tachyonic perturbations is flat space. There are however specific perturbations which drive $C/Z_N$ to $C/Z_{N-2}$ or other lower nonsupersymmetric orbifolds [2, 6]. Condensation of tachyons for a more general background namely the twisted circles confirms these observations [8]. For other studies on condensation of localized closed string tachyons see [3, 5, 7, 9].

In this paper we consider the problem of tachyon condensation for Type II theory on the $C/Z_N$ orbifold in the large $N$ limit. In this limit there are tachyons which become almost marginal and it makes sense to write an effective action involving the tachyons and the other massless particles (graviton, dilaton) while integrating out the massive string modes. The aim is to compute the effective tachyon potential for large $N$. With this as the guideline, we construct the tachyon potential upto the quartic interaction term. The procedure followed is along the lines of [10]. The four point amplitude for the twisted sector tachyons (of $m^2 = -1/N$) is first calculated [11]. In the large $N$ limit when the tachyons are nearly massless we take the zero momentum limit of this amplitude. The nonderivative quartic coupling for the tachyons is then obtained by subtracting the contribution of the massless exchanges from the string four point amplitude. The four point amplitude with massless exchanges, which in this case are the graviton and the dilaton is obtained from the low energy effective field theory of tachyons coupled to these massless fields.

The quartic coupling for the tachyon potential is found to be of the order $1/N$. We get the the height of the potential to the lowest order in $1/N$. We expect this minimum to correspond to the $C/Z_{N-k}$ orbifold.

However there are various points which show that the higher point interaction couplings are also comparable to the quartic coupling. One being that, with a quartic potential having global $O(2)$ symmetry, we expect a particle of positive $(mass)^2 = -2m^2$, where $m^2$ is the mass of the tachyon, and the
usual Goldstone boson. However these modes are not present in the spectrum of closed string on $C/Z_N$. Furthermore if we stick to the predicted height of the tachyon potential [4], we find that there is a mismatch by a factor of $1/N$ in the height of the potential, when the minimum is expected to correspond to the $C/Z_N$ orbifold. However since the above modes are absent in the tree-level spectrum of closed string on $C/Z_N$ orbifold we conclude that the higher point amplitudes are also of the order $1/N$. This includes the term $\phi^N$ allowed by the twist symmetry. These higher order terms modify the potential and the spectrum already to the lowest order in $1/N$.

Furthermore, the four point coupling is subject to field redefinitions. One can make the offshell contact term as large or small as one wishes and can also change the sign. This makes the coupling nonuniversal and the existence of the minimum is not clear in this approach if one truncates the potential upto the quartic term. We elaborate on these points in section 5.

This paper is organised as follows. In section 2, we compute the spectrum for closed strings on $C/Z_N$ orbifold and show that there are tachyons. In section 3, using the conformal field theory of $C/Z_N$ orbifold we review the calculation of the four point amplitude of the tachyons in the large $N$ limit where the tachyon is nearly marginal. In section 4, we find the OPE of two tachyon vertices and show that the only intermediate massless exchanges are the graviton and dilaton. In section 5, from the effective field theory of the tachyon coupled to the graviton and dilaton we compute the exact contribution of these massless exchanges to the four point tachyon amplitude. The quartic coupling for the tachyon is then obtained after subtraction of the massless exchanges from the string theory amplitude and write down the potential upto the quartic interaction term. We give our conclusions in section 6.

2 Closed string spectrum

We compute the spectrum of the closed string on the $C/Z_N$ orbifold. We will be concerned with the NS sector as it is in this sector that tachyons appear. In the RNS formulation of superstring, we consider $x^\mu$ and $\Psi^\mu$ as world sheet fields corresponding to the nonorbifolded directions. For the orbifolded directions we have the complex $X, \bar{X}, \psi, \bar{\psi}$ fields as defined below,
\[ X = X^8 + iX^9, \quad \bar{X} = X^8 - iX^9; \quad \psi = \psi^8 + i\psi^9, \quad \bar{\psi} = \psi^8 - i\psi^9 \] (1)

The \( Z_N \) group action on \( C \) defines the following boundary conditions on \( X, \bar{X} \) and \( \psi, \bar{\psi} \) for the closed string in the NS sector,

\[
X(\sigma + 2\pi, \tau) = e^{2\pi i \frac{\Delta}{N}} X(\sigma, \tau) \\
\bar{X}(\sigma + 2\pi, \tau) = e^{-2\pi i \frac{\Delta}{N}} \bar{X}(\sigma, \tau) \\
\psi(\sigma + 2\pi, \tau) = e^{2\pi i (\frac{\Delta}{N} + \frac{1}{2})} \psi(\sigma, \tau) \\
\bar{\psi}(\sigma + 2\pi, \tau) = e^{-2\pi i (\frac{\Delta}{N} - \frac{1}{2})} \bar{\psi}(\sigma, \tau) \quad (2)
\]

These boundary conditions give the following mode expansions for the world sheet scalars,

\[
\partial_z X(z) = -i \sum_{m=-\infty}^{\infty} \frac{\alpha_{m-k} + \Delta}{z^{m+1}+\frac{\Delta}{N}} \\
\partial_z \bar{X}(z) = -i \sum_{m=-\infty}^{\infty} \frac{\bar{\alpha}_{m+k} + \Delta}{z^{m+1}+\frac{\Delta}{N}} \quad (3)
\]

\[
\partial_{\bar{z}} X(\bar{z}) = -i \sum_{m=-\infty}^{\infty} \frac{\tilde{\alpha}_{m+k} + \Delta}{z^{m+1}+\frac{\Delta}{N}} \\
\partial_{\bar{z}} \bar{X}(\bar{z}) = -i \sum_{m=-\infty}^{\infty} \frac{\tilde{\bar{\alpha}}_{m-k} + \Delta}{z^{m+1}+\frac{\Delta}{N}} \quad (4)
\]

and for the fermions,

\[
\psi(z) = \sum_{r \in \mathbb{Z} + \frac{1}{2}} \frac{\psi_{r-k}}{z^{r+\frac{1}{2}}+\frac{\Delta}{N}} \\
\bar{\psi}(z) = \sum_{r \in \mathbb{Z} + \frac{1}{2}} \frac{\bar{\psi}_{r+k}}{z^{r+\frac{1}{2}}+\frac{\Delta}{N}} \quad (5)
\]

\[
\bar{\psi}(\bar{z}) = \sum_{r \in \mathbb{Z} + \frac{1}{2}} \frac{\bar{\psi}_{r+k}}{z^{r+\frac{1}{2}}+\frac{\Delta}{N}} \\
\psi(\bar{z}) = \sum_{r \in \mathbb{Z} + \frac{1}{2}} \frac{\psi_{r-k}}{z^{r+\frac{1}{2}}+\frac{\Delta}{N}} \quad (6)
\]

From the usual OPEs of the world sheet fields and the mode expansions we get the following canonical commutation relations,

\[ ^1 \alpha' = 2 \text{ in all calculations} \]
\[
\begin{align*}
[a_{m-\frac{k}{N}}, \bar{a}_{n+\frac{k}{N}}] &= (m - \frac{k}{N})\delta_{m+n,0} \quad [a_{m}, a_{n}] = mn^{\mu\nu}\delta_{m+n,0} \\
[\tilde{a}_{m+\frac{k}{N}}, \tilde{a}_{n-\frac{k}{N}}] &= (m + \frac{k}{N})\delta_{m+n,0} \quad [\tilde{a}_{m}, \tilde{a}_{n}] = mn^{\mu\nu}\delta_{m+n,0}
\end{align*}
\]

\[
\begin{align*}
\{\psi_{r-\frac{k}{N}}, \bar{\psi}_{s+\frac{k}{N}}\} &= \delta_{r+s,0} \quad \{\psi_{r}^{\mu}, \psi_{s}^{\nu}\} = \eta^{\mu\nu}\delta_{r+s,0} \\
\{\tilde{\psi}_{r+\frac{k}{N}}, \tilde{\psi}_{s-\frac{k}{N}}\} &= \delta_{r+s,0} \quad \{\tilde{\psi}_{r}^{\mu}, \tilde{\psi}_{s}^{\nu}\} = \eta^{\mu\nu}\delta_{r+s,0}
\end{align*}
\]

The holomorphic part of the energy momentum tensor for the world sheet fields corresponding to the orbifolded directions is given by,

\[
T_{zz}(z) = -\partial X \partial \bar{X} - \frac{1}{2} \bar{\psi} \partial \psi - \frac{1}{2} \psi \partial \bar{\psi}
\]  

(11)

Using the mode expansions and the canonical commutation relations one gets the zero mode part of the energy momentum tensor,

\[
L_0 = \sum_{n=0}^{\infty} [a_{-n-\frac{k}{N}}, \bar{a}_{n+\frac{k}{N}} + \frac{1}{2}(n + \frac{k}{N})] + \sum_{n=1}^{\infty} [\tilde{a}_{-n+\frac{k}{N}}, \tilde{a}_{n-\frac{k}{N}} + \frac{1}{2}(n - \frac{k}{N})] + \sum_{r\geq 1/2} \left[ (r + \frac{k}{N})\psi_{r-\frac{k}{N}} \bar{\psi}_{r+\frac{k}{N}} + (r - \frac{k}{N})\bar{\psi}_{r-\frac{k}{N}} \psi_{r+\frac{k}{N}} - r \right]
\]

(12)

Where we have included the contributions from the normal ordering constant. The contribution to the zero point energy from the fields on the orbifolded complex plane is now given by,

\[
E_{\text{orb}} = \frac{1}{2N} \frac{k}{8} + \sum_{n=0}^{\infty} n - \sum_{r\geq 1/2} r
\]

\[
= \frac{1}{2N} - \frac{1}{8}
\]

(13)
In NS-sector in the light cone gauge, we have six real periodic bosons and six antiperiodic fermions which contributes an amount of $-3/8$ to the zero point energy. Adding the zero point energies for all the directions, for the left moving part, we get,

$$E_L = -\frac{1}{2}(1 - \frac{k}{N})$$  \hspace{1cm} (14)

It may seem that for $1/2 < k/N < 1$, the $r = 1/2$ term from the second fermionic part in (12) contributes an additional $(1/2 - k/N)$ after normal ordering. Therefore, for this case we should have,

$$E_L = -\frac{k}{2N}$$  \hspace{1cm} (15)

However, since normal ordering is only a prescription for removing infinities from the zero point energy, we must choose one so that the zero point energy is consistent with that which is obtained from the world sheet conformal field theory. It will be seen in the next section, that a choice of the prescription, where we keep the $r = 1/2$ as it is, is consistent with the dimensions of the twist operators. The mass spectrum is thus given by,

$$M^2 = n + \tilde{n} - (1 - \frac{k}{N}) \quad \text{for} \quad 0 < k/N < 1$$  \hspace{1cm} (16)

Where $n$ and $\tilde{n}$ are level numbers which are no longer integers for the twisted sectors. The ground state of the NS sector is thus tachyonic. For the $(N - k)th$ sector we have a tachyon of $(mass)^2 = -k/N$. Some of the excited states are also tachyonic their masses are given by,

$$\bar{\psi}_{\frac{1}{2}, \frac{k}{N}} |0\rangle \equiv -\frac{k}{N} \quad \text{marginal for,} \quad N \to \infty$$  \hspace{1cm} (17)

$$\alpha_{-\frac{1}{2}, \frac{k}{N}} |0\rangle \equiv -(1 - \frac{3k}{N}) \quad \text{tachyonic for,} \quad 3k < N$$  \hspace{1cm} (18)

From the moding of the oscillators it can be seen that there are no massless states in the twisted sector, as the oscillators are moded by $k/N$ and the zero
point energy is $-k/2N$. The massless states arise from the untwisted sector and these are the usual graviton, dilaton and the antisymmetric second rank tensor. In the following sections we will construct the tachyon potential in the large $N$ limit where the tachyons become marginal.

3 Four point amplitude from CFT

In this section we review the computation of the four point amplitude for the tachyons that we found in the spectrum [11]. This gives the four point tachyon amplitude with all the massless and the massive exchanges. In the next section we will compute the exact contribution from the massless exchanges. Subtracting this from the amplitude computed here gives the effective four point coupling for the tachyon field.

The vacuum for the twisted sector that is labeled by $k/N$ is created from the untwisted vacuum by the action of the bosonic twist fields, $\sigma_{\pm k/N}$ and the fermionic twist fields $s_{\pm k/N}$.

The OPEs of these twist fields with the world sheet fields, $X, \bar{X}, \psi, \bar{\psi}$ are given by [11],

\[
\begin{align*}
\partial_z X(z) \sigma_+(w, \bar{w}) &\sim (z - w)^{(1 - \frac{k}{N})} \tau_+(w, \bar{w}) \\
\partial_z \bar{X}(z) \sigma_+(w, \bar{w}) &\sim (z - w)^{-\frac{k}{N}} \tau'_+(w, \bar{w}) \\
\partial_{\bar{z}} X(\bar{z}) \sigma_+(w, \bar{w}) &\sim (\bar{z} - \bar{w})^{-\frac{k}{N}} \bar{\tau}_+(w, \bar{w}) \\
\partial_{\bar{z}} \bar{X}(\bar{z}) \sigma_+(w, \bar{w}) &\sim (\bar{z} - \bar{w})^{-(1 - \frac{k}{N})} \bar{\tau}'_+(w, \bar{w})
\end{align*}
\]  

(19)

where, $\tau_+, \tau'_+, \bar{\tau}_+, \bar{\tau}'_+$ are excited twist fields. Using (19) and the OPE of the twist fields with the energy momentum tensor, the world sheet dimension of the bosonic twist fields are found to be,

\[
h_{\sigma} = \bar{h}_{\sigma} = \frac{1}{2N} \frac{k}{(1 - \frac{k}{N})}
\]

(20)

Bosonising the world sheet fermions,

\[
\begin{align*}
\psi(z) &= -i \sqrt{2} e^{iH(z)} \\
\bar{\psi}(z) &= -i \sqrt{2} e^{-iH(z)}
\end{align*}
\]  

(21) (22)
giving the fermionic twist fields as, $s_\pm = e^{\pm i\frac{k}{N}H(z)}$. From this we get the following OPEs of the fermionic fields with the twist fields.

\[
\bar{\psi}(z)s_+(w) = -i\sqrt{2}e^{-iH(z)}e^{i\frac{k}{N}H(w)} \\
\sim -i\sqrt{2}(z-w)\frac{k}{N}e^{-i(1-\frac{k}{N})H(z)}[1 + (z-w)\partial H(z)] \quad (23)
\]

Similarly,

\[
\bar{\psi}(z)s_+(w) = -i\sqrt{2}e^{iH(z)}e^{i\frac{k}{N}H(w)} \\
\sim -i\sqrt{2}(z-w)\frac{k}{N}e^{i(1+\frac{k}{N})H(z)}[1 + (z-w)\partial H(z)] \quad (24)
\]

For the fermionic string, vertices for the twist fields in the $(-1,-1)$ and $(0,0)$ pictures are given by,

\[
V_{(-1,-1)}^+(z,\bar{z}) = e^{-\phi}e^{-\tilde{\phi}}s_+s_+\sigma_+e^{ik\cdot x}(z,\bar{z}) \quad (25)
\]

\[
V_{(0,0)}^+(z,\bar{z}) = e^{\phi}T_Fe^{\tilde{\phi}}\tilde{T_F}V_{(-1,-1)}^+(z,\bar{z}) \quad (26)
\]

Where,

\[
T_F = -\frac{1}{4}(\partial X\bar{\psi} + \partial \bar{X}\psi) - \frac{1}{2}\partial x.\Psi \quad (27)
\]

Note that the dimension of the vertex gives the mass of the tachyon,

\[
M^2 = -(1 - \frac{k}{N}) \quad (28)
\]

This corresponds to the ground state tachyons in the twisted sector. For the near marginal tachyons, in the large $N$ limit, which are in the $(N-k)th$ sector, the vertex operator in the $(-1,-1)$ picture is,

\[
V_{(-1,-1)}^+(z,\bar{z}) = e^{-\phi}e^{-\tilde{\phi}}e^{i(1-\frac{k}{N})H(z)}e^{-i(1-\frac{k}{N})\tilde{H}(\bar{z})}\sigma_+e^{ik\cdot x}(z,\bar{z}) \quad (29)
\]
The four point amplitude for these lowest lying tachyons can now be computed by taking two vertices in the \((0, 0)\) picture and two in the \((-1, -1)\) picture.

\[
C \int_C d^2z \left< V_{(-1,-1)}(z, \bar{z}) \right| e^{\phi T_F e^{\phi \tilde{T}_F} V_{(-1,-1)}(1)} \left| V_{(-1,-1)}(0) \right> (30)
\]

The constant \(C = g_c^4 C_s^2\). Where \(C_s^2\) is related to \(g_c\) by,

\[
C_s^2 = \frac{4\pi}{g_c^2} \quad (31)
\]

This amplitude can now be computed and is given by [11],

\[
I = C(k_1, k_3)^2 \int_C d^2z \frac{|z|^{-2-s}|1-z|^{-2-t}}{|F(z)|^2} (32)
\]

Where, \(F(z)\) is the hypergeometric function,

\[
F(z) \equiv F\left(\frac{k}{N}, 1 - \frac{k}{N}; 1; z\right) = \frac{1}{\pi} \int_{0}^{1} dy y^{-\frac{k}{2N}} (1-y)^{-\left(1-\frac{1}{N}\right)} (1-yz)^{-\frac{k}{2N}} \quad (33)
\]

and, \(s = -(k_1 + k_2)^2\), \(t = -(k_2 + k_3)^2\), \(s = -(k_3 + k_1)^2\).

In the large \(N\) approximation,

\[
F(z) \sim 1 + \frac{k}{N} (z + \frac{1}{2} z^2 + \frac{1}{3} z^3 + \ldots) + O((k/N)^2) \quad (34)
\]

Note that the terms proportional to \(k/N\) in (34) shift the \(s\)-channel pole.

There is an additional factor of \((k_1, k_2)^2\), due to which the contact term from any of the terms of (34) apart from 1, would atleast be of \(O((k/N)^2)\). With this observation, the integral can now be performed for \(F(z) \rightarrow 1\).

\[
I = C^2 \pi (k_1, k_3)^2 \frac{\Gamma\left(-\frac{2}{N}\right) \Gamma\left(-\frac{1}{2}\right) \Gamma\left(1 + \frac{s}{2} + \frac{t}{2}\right)}{\Gamma\left(-\frac{2}{N} - \frac{1}{2}\right) \Gamma\left(1 + \frac{s}{2}\right) \Gamma\left(1 + \frac{t}{2}\right)}
\]

\[
= -4\pi g_c^2 \times \frac{1}{4} (u - 2m^2)^2 \left( \frac{1}{s} + \frac{1}{t} \right) \frac{\Gamma\left(1 - \frac{s}{2}\right) \Gamma\left(1 - \frac{t}{2}\right) \Gamma\left(1 + \frac{s}{2} + \frac{t}{2}\right)}{\Gamma\left(1 - \frac{s}{2} - \frac{t}{2}\right) \Gamma\left(1 + \frac{s}{2}\right) \Gamma\left(1 + \frac{t}{2}\right)} \quad (35)
\]
Now using $s + t + u = 4m^2$,

$$I = -4\pi^2 g_c^2 \left( \frac{(t-2m^2)^2}{s} + \frac{(s-2m^2)^2}{t} + 3(s + t) - 8m^2 \right)$$

(36)

We have to expand the gamma functions. Now since we are interested in the order $O(\frac{1}{N^2})$ of the amplitude, any correction to the expansion of the gamma functions that when the limit $s, t \to 0$ is taken, will be at least of order $O(\frac{1}{N})$. But the factor multiplying the gamma functions is already of order $O(\frac{1}{N})$ except for the pole terms. So we can take the contribution of the gamma functions to be 1. Thus we can write the string amplitude in the zero momentum limit as,

$$I \sim -4\pi^2 g_c^2 \left( \frac{(t-2m^2)^2}{s} + \frac{(s-2m^2)^2}{t} + 3(s + t) - 8m^2 \right)$$

(37)

### 4 OPE of two tachyon vertices

In this section we compute the OPE of two tachyon vertices with one in the $(0, 0)$ picture and another in the $(-1, -1)$ picture and find the couplings of tachyon to the massless particles.

The OPE we wish to find is,

$$V_{(-0,0)}^-(z, \bar{z}) V_{(-1,-1)}^+(w, \bar{w}) = \left[ \frac{1}{4} (\partial \tilde{X} \bar{\psi} + \partial \bar{X} \psi) + \frac{1}{2} \partial x. \Psi \right] \left[ \frac{1}{4} (\partial \tilde{X} \bar{\psi} + \partial \bar{X} \tilde{\psi}) + \frac{1}{2} \partial x. \tilde{\Psi} \right]$$

$$\times s_- s_- \sigma_- e^{ik.x}(z, \bar{z}) \times e^{-\phi} e^{-\bar{\phi}} s_+ s_+ \sigma_+ e^{ip.x}(w, \bar{w})$$

$$= \left[ \frac{1}{4} (\partial \tilde{X} \bar{\psi} + \partial \bar{X} \tilde{\psi}) + k. \Psi \right] \left[ \frac{1}{4} (\partial \tilde{X} \bar{\psi} + \partial \bar{X} \tilde{\psi}) + k. \tilde{\Psi} \right]$$

$$\times s_- s_- \sigma_- e^{ik.x}(z, \bar{z}) \times e^{-\phi} e^{-\bar{\phi}} s_+ s_+ \sigma_+ e^{ip.x}(w, \bar{w})$$

(38)

Now, the following OPEs are necessary to compute (38).
\begin{align*}
e^{ik.x}(z, \bar{z})e^{ip.x}(w, \bar{w}) & \sim |z - w|^{2k_p} e^{i(p+k).x}(w, \bar{w}) [1 + (z - w)(k - p)_\mu \partial x^\mu + (\bar{z} - \bar{w})(k - p)_\mu \bar{\partial} x^\mu + |z - w|^2 (k - p)_\mu (k - p)_\nu \partial x^\mu \bar{\partial} x^\mu] \\
\end{align*}

For the fermionic twist fields,

\begin{align*}
s_-(z)s_+(w) &= e^{-i \frac{k}{N} H(z)} e^{i \frac{k}{N} H(w)} \\
&\sim (z - w)^{-\left(\frac{k}{N}\right)^2} [1 - (z - w)\frac{2k}{N} \partial H(z)] \tag{40} \\
\tilde{s}_-(z)\tilde{s}_+(w) &= e^{-i \frac{k}{N} H(\bar{z})} e^{i \frac{k}{N} H(\bar{w})} \\
&\sim (\bar{z} - \bar{w})^{-\left(\frac{k}{N}\right)^2} [1 - (\bar{z} - \bar{w})\frac{2k}{N} \partial H(\bar{z})] \tag{41} \\
\end{align*}

For the bosonic twist fields,

\begin{align*}
\sigma_-(z)\sigma_+(w) &\sim |z - w|^{-2\frac{1}{N}(1 - \frac{k}{N})} [1 + \ldots] \tag{42} \\
\end{align*}

Using these, the OPE of the compact part of $T_F$ with the twist fields is,

\begin{align*}
(\partial X \bar{\psi} + \partial \bar{X} \psi)s_+\sigma_+ \sim (z - w)^{-1} e^{-i(1 - \frac{k}{N})H(z)} + \tau_+ e^{i(1 + \frac{k}{N})H(z)} \tag{43} \\
\end{align*}

This OPE includes higher twist operators and hence does not contain massless states which we are looking for. The massless state is obtained from the $(k, \Psi k, \bar{\Psi})$ term in the expansion (38) with the other twist fields contracted. The coupling for the term is,

\begin{align*}
V_{\mu\nu}(k) = k_\mu k_\nu \tag{44} \\
\end{align*}

This term is completely symmetric in the indices. It thus corresponds to the massless vertex for the graviton and the dilaton in the $(-1, -1)$ picture, which is the symmetric part of

\begin{align*}
e^{-\phi} e^{-\bar{\phi}} \bar{\Psi}_\mu \Psi_{\nu} e^{i(k+p).x} \tag{45} \\
\end{align*}
The four point tachyon scattering amplitude with a massless graviton and dilaton exchange is given by,

\[
A_4 = V_{\mu\nu}(p_1) \frac{1}{q^2} [\delta_{\mu\alpha} \delta_{\nu\beta}] V_{\alpha\beta}(p_3)
\]

\[
= p_1^\mu p_1^\nu \frac{1}{q^2} [\delta_{\mu\alpha} \delta_{\nu\beta}] p_3^\alpha p_3^\beta
\]

\[
= -\frac{(p_1 \cdot p_3)^2}{s} = -\frac{1}{4} \frac{(u - 2m^2)^2}{s} \tag{46}
\]

where, \( q^2 = -(p_1 + p_2)^2 = s \) and \( p_1^2 = -m^2 \). We have chosen the configuration of the momenta for the tachyon fields such that it matches with the original configuration used in (30) for convenience. Namely \( p_1 \) and \( p_3 \) corresponds to the momenta of the external \( \phi^* \) field corresponding to the \( V^- \) vertices. We still have to add the \( t \)-channel contribution to (46). Adding this we have,

\[
A_4 = -\frac{1}{4} \left[ \frac{(u - 2m^2)^2}{s} + \frac{(u - 2m^2)^2}{t} \right] \tag{47}
\]

This reproduces the poles which we have found in (37) as expected apart from a factor of \( 2\pi \) which comes in (37) from the integral over the vertex position.

5 Amplitude from effective field theory

In the previous section we have seen that the massless exchanges in the four point amplitude of the twisted sector tachyons are the graviton and the dilaton. In this section we calculate the tachyon four point amplitude with these massless exchanges, namely the graviton and dilaton from the effective field theory.

The action for the complex tachyon coupled to graviton and dilaton is given by,

\[
S = \frac{1}{\kappa^2} \int d^Dx \sqrt{(-g)} [-R + \frac{4}{D-2} \Phi \partial^2 \Phi + \frac{1}{2} \phi^*(-\partial^2)\phi + \frac{1}{2} m^2 e^{\frac{1}{4\phi^*}} \phi^* \phi] \tag{48}
\]

\]
Expanding $g_{\mu\nu}$ about the flat metric,

$$g_{\mu\nu} = \delta_{\mu\nu} + \kappa h_{\mu\nu}$$  \hspace{1cm} (49)

and rescaling the dilaton field by,

$$\Phi \rightarrow \sqrt{\frac{8}{D-2}} \Phi$$  \hspace{1cm} (50)

We have,

$$S = \frac{\kappa}{2} \int d^Dx \left[ \frac{1}{2} h_{\mu\nu}(\partial^2 + \cdots)h_{\mu\nu} - \frac{1}{2} \Phi \partial^2 \Phi + \frac{1}{2} \phi^*(-\partial^2 + m^2)\phi - h_{\mu\nu} T_{\mu\nu} + T \Phi \right]$$  \hspace{1cm} (51)

The couplings of the graviton and dilaton to the complex scalar field are now,

$$-\kappa h_{\mu\nu} T_{\mu\nu} \quad \text{and} \quad \kappa T \Phi$$  \hspace{1cm} (52)

Where,

$$T_{\mu\nu} = -\frac{1}{2} [\partial_{\mu} \phi^* \partial_{\nu} \phi + \partial_{\nu} \phi^* \partial_{\mu} \phi] + \frac{1}{2} \delta_{\mu\nu} [\partial_\alpha \phi^* \partial_\alpha \phi + m^2 |\phi|^2]$$

$$T = \sqrt{\frac{2}{D-2}} m^2 \phi^* \phi$$  \hspace{1cm} (53)

The tachyon-graviton vertex and the graviton propagator in the harmonic gauge are given by,

$$V_{\mu\nu}(p, k) = i\kappa \left[ -\frac{1}{2} (p_\mu k_\nu + p_\nu k_\mu) + \frac{1}{2} \delta_{\mu\nu}(k.p - m^2) \right]$$  \hspace{1cm} (54)

$$\Delta_{\mu\nu\alpha\beta}(q^2) = \frac{1}{q^2} \left[ \delta_{\mu\alpha} \delta_{\nu\beta} + \delta_{\mu\beta} \delta_{\nu\alpha} - \frac{2}{D-2} \delta_{\mu\nu} \delta_{\alpha\beta} \right]$$  \hspace{1cm} (55)
The four point amplitude for four massless scalar scattering with a graviton exchange is,

\[ A^g_4 = V_{\mu\nu}(p_1,p_2)\Delta_{\mu\nu\alpha\beta}(q^2)V_{\alpha\beta}(p_3,p_4) \]

\[ = \frac{\kappa^2}{q^2}[(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3) - (p_1.p_2)(p_3.p_4) + m^2(p_1.p_2 + p_3.p_4) - \frac{D}{D - 2}m^4] \]

Similarly, for the dilaton exchange we have,

\[ A^d_4 = \frac{\kappa^2}{q^2} \frac{2}{D - 2}m^4 \]

Therefore, the four point tachyon amplitude with graviton and dilaton exchange is,

\[ A_4 = A^g_4 + A^d_4 \]

\[ = \frac{\kappa^2}{q^2}[(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3) - (p_1.p_2)(p_3.p_4) + m^2(p_1.p_2 + p_3.p_4) - \frac{D}{D - 2}m^4] \]

\[ = -\frac{\kappa^2}{48}[(u - 2m^2)^2 + (t - 2m^2)^2 - s^2] \]

\[ = -\frac{\kappa^2}{2}\left[\frac{(t - 2m^2)^2}{s} + t - 2m^2\right] \]

In writing (58) from the previous step, we have used \( p_i^2 = -m^2 \) and put in \( q^2 = (p_1 + p_2)^2 = -s \). From (58) to (59) we have used \( s + t + u = 4m^2 \) which uses the mass shell conditions. Now including the t-channel process we get,

\[ A_4 = -\frac{\kappa^2}{2}\left[\frac{(t - 2m^2)^2}{s} + \frac{(s - 2m^2)^2}{t} + (t + s) - 4m^2\right] \]

Comparing with the pole term of the string amplitude (37), we see that the pole is due to graviton exchange. This also relates \( \kappa \) to \( g_c \) which is found to be,
After subtraction, the nonderivative quartic term and the derivative terms left behind are,

\[-4\pi^2 g_c^2 [-4m^2 + 2(s + t)]\]  \hspace{1cm} (62)

5.1 The Potential

We can now write down the potential for the tachyon upto the quartic term. It may be noted that the sign of the quartic term is to be fixed relative to the sign of the pole terms which has to be positive. So we get the quartic coupling as,

\[\lambda = (4\pi^2 g_c^2) \times (-4m^2)\]  \hspace{1cm} (63)

which is positive since \(m^2 = -k/N\). The tachyon potential is now,

\[V(\phi^* \phi) = \frac{1}{2} m^2 (\phi^* \phi) + \frac{\lambda}{4} (\phi^* \phi)^2\]

\[= -\frac{k}{2N} (\phi^* \phi) + 4\pi^2 g_c^2 \frac{k}{N} (\phi^* \phi)^2\]  \hspace{1cm} (64)

The potential has a minimum at \(|\phi|^2 = 1/(16\pi^2 g_c^2)\) which is of \(O(1)\). We expect the nearest minimum to correspond to the \(C/Z_{N-k}\) orbifold. In this case the height of the potential from \(C/Z_N\) to \(C/Z_{N-k}\) is given by,

\[\Delta = \frac{1}{64\pi^2 g_c^2 N} \frac{k}{N}\]  \hspace{1cm} (65)

One may compare this, upto normalizations, with the conjectured height [4] which is,
\[ \Delta = 4\pi \left( \frac{1}{N-k} - \frac{1}{N} \right) \sim 4\pi \frac{k}{N^2} \] (66)

This shows that the perturbative result (65) is off by a factor of \(1/N\). At this point we are not in a position to trust this perturbative result. Higher point amplitudes will most likely modify this.

There are various indications that higher point interaction terms in the amplitude are in fact important and are not of order less than \(1/N\). We may note that with a potential up to the quartic coupling having global \(O(2)\) symmetry, in the spontaneously broken theory there is a massless scalar corresponding to the Goldstone boson and a massive particle of mass, \(-2m^2 = 2k/N\). This means that in the spectrum of \(C/Z_{N-k}\) to which the theory is supposed to flow, there must be a massless and a massive scalar of mass \(2k/(N - k)(\sim 2k/N + \cdots;\text{for large } N)\). However the spectrum does not contain these scalars. The absence of the massless Goldstone particle indicates that the \(O(2)\) symmetry of the tachyon potential has to be broken. This can only happen if the correlation functions of \(N\) twist operators are also of order \(1/N\).

This fact may also be seen as follows. The three point graviton vertex and the two tachyon and one graviton vertex, both have two powers of momentum (Figure 1). The four point tachyon amplitude with a graviton exchange has two positive powers of momentum.

![Diagram](image)

Figure 1: (A) Two tachyon one graviton vertex, (B) Three graviton vertex, (C) Six-point tachyon amplitude with graviton exchange.

With the addition of two more external tachyons, using (A), the positive power of momentum for the six point amplitude (C), remains two. Of
course with a three graviton vertex insertion, we can introduce more negative powers, but there are tree diagrams where we can have two positive powers of momentum. The \( N \) point contact term is obtained by subtracting these graviton exchanges from the full \( N \) point tachyon amplitude from string theory. The graviton exchange diagrams as mentioned contains two positive powers of momentum, which in the onshell limit give terms proportional to \( m^2 \sim O(1/N) \). It is thus very likely that the subtracted term would give a \( 1/N \) dependence as the leading part.

We further see that the dilaton field redefinitions such as,

\[ \Phi \rightarrow \Phi + c\phi^* \phi \tag{67} \]

where \( c \) is a constant, can change the value of the contact term and can even change the sign. The minimum thus depends crucially on the expectation value of the dilaton field which when becomes large, makes this perturbative analysis anyway unreliable. A similar observation was made in [9], in the context of closed string tachyon condensation with Rohm’s Compactification. In general the existence of the minimum is independent of field redefinitions. The fact that we are able to change the nature of the potential by field redefinitions implies the potential upto the quartic term does not shed much light on the minimum of the theory. It is argued in various approaches that the Type II theory on the \( C/Z_N \) orbifold ultimately upon closed string tachyon condensation goes to the Type II theory on flat space. Our analysis does not give a proof of this observation. If we assume this to be true, that a stable minimum exists, then following the above arguments, we may conclude that the higher point terms are indeed necessary and are of the order \( 1/N \).

6 Conclusion

We have studied the condensation of closed string tachyons for Type II strings on the \( C/Z_N \) orbifold. We constructed the potential for the tachyons upto the quartic term in the large \( N \) limit by subtracting the massless exchanges from the four point tachyon amplitude computed from string theory. We expect the minimum of the potential for the near marginal tachyons for the
$k$-th twisted sector, in the large $N$ limit to correspond to the $C/Z_{N-k}$ orbifold. When compared to the conjectured value for the height of the potential for the $C/Z_N$ orbifold, we find a mismatch by a factor of $1/N$. However, we have argued that the higher point amplitudes are indeed important and are of the same order in $1/N$ as the quartic term. A potential up to the quartic term after spontaneous symmetry breaking gives masses which are not there in the spectrum for closed string on $C/Z_{N-k}$ to which the $C/Z_N$ theory is expected to flow. This leads us to conclude that the higher point amplitudes including the global $O(2)$ breaking term, $\phi^N$, must all be of order $1/N$ so that the potential gives a mass spectrum, consistent with that of the $C/Z_{N-k}$ orbifold. We have also argued that field redefinitions can alter the contact term and can even change the sign. If the theory can be deformed so that the minimum can be reliably reached in perturbation theory, then, a direct approach such as the one discussed in this paper can ascertain whether this minimum has the required properties.

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**Note added**: The computation of the quartic coupling for the twisted sector tachyon is done recently by Atish Dabholkar, Ashik Iqubal and Joris Raeymaekers in *Off-Shell Interactions for Closed-String Tachyons* [hep-th/0403238]. It is pointed out correctly by these authors that additional contributions to the four point contact term will also come from the massive untwisted states with momentum along the orbifold plane, $C/Z_N$. This modifies the $1/N$ dependence of the quartic term which we computed in this paper to a more suppressed $1/N^3$ dependence. However with this modification, the expectation that the large $N$ approximation may be used to study the RG flow due to tachyon condensation from the $C/Z_N$ orbifold to lower nonsupersymmetric orbifolds is even further weakened. The conclusions of this paper thus remain unaltered.
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