Nonadiabatic quantum state engineering by time-dependent decoherence-free subspaces in open quantum systems

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Abstract
We extend the theory of quantum state engineering (QSE) to the open quantum systems which simultaneously suffer time-dependent decoherence and time-independent decoherence. A hybrid approach is proposed to effectively realize QSE by combination of time-dependent decoherence-free subspaces and shortcuts to adiabaticity theories. As an application, a concrete single-qubit QSE example with an open five-level system is presented. Numerical simulations confirm that the approach is reliable and robust against both of time-dependent decoherence and time-independent decoherence. Certain dissipation effects are no longer harmful but play a positive role in the QSE. In addition, the approach can be generalized to realize QSE in high-dimensional quantum systems. Therefore, this proposal is quite useful for quantum computation and quantum information processing in open systems.

1. Introduction
Realizing quantum state engineering (QSE) in a convenient high-effective way is a central ingredient for most quantum information science tasks [1–6]. However, in practice, the ideal robustness and the intended dynamics of QSE may be spoiled by decoherence effects due to the interaction between quantum system and environment. To cope with decoherence problems, especially for time-independent decoherence effects (such as spontaneous emission, thermal excitation, and dephasing), many proposals and strategies have been put forward [7–20]. For instance, the theory of quantum error-correcting codes (QECCs) offers a positive way to correct errors that occur during the information storage [8, 9]. In addition, the theory of decoherence-free subspaces (DFSs) [10–12] or noiseless subsystem [13] provides a crucial strategy for preventing time-independent collective decoherence by encoding quantum information into DFSs. Besides the QECCs and DFSs theories, another popular approach, so-called shortcuts to adiabaticity (STA) [14–18], is a significant technology for fighting decoherence effects. The critical idea of the STA is to accelerate the dynamics of quantum system by applying a designed coherent control, so that the system evolution on timescales is much shorter than decoherence times. Based on this novel idea, several methods have been proposed, such as counter-diabatic driving (equivalently, the transitionless quantum algorithm) [14–16], Lewis–Riesenfeld inverse engineering [17, 18], and so on. Up to now, STA is widely used for realizing QSE tasks in many physical systems [21–24] which relate to the weak time-independent decoherence.

On the other hand, by taking into account another extreme case, i.e. the time-dependent decoherence (such as, the effect of a squeezed vacuum reservoir, the damping of a harmonic cavity), the QECCs, DFSs and STA theories may be not valid, thus the engineering reservoir program [25–27] should be considered. The basic idea of the engineering reservoir program is to protect quantum information by driving the system to couple to an engineered reservoir. The engineering reservoir program has been developed for both trapped ions [26] and atomic two-level systems [27]. Particularly, Carollo et al proposed a way to observe the geometric phase by adiabatically manipulating the squeezed parameters of an engineered reservoir [28]. By following those works, several schemes were proposed to protect quantum information by means of engineering reservoir program [29–32]. Among them, the theory of time-dependent
decoherence-free subspaces (TDFSs) [32–35] provides a promising way to deal with time-dependent decoherence. The fundamental idea of the TDFSs theory is to make the system undergo a unitary evolution in TDFSs by utilizing the incoherent control and the coherent control. Here, we stress that, apart from protecting quantum information, the TDFSs theory can be used for realizing many interesting quantum information tasks, especially for QSE.

Both time-dependent decoherence and time-independent decoherence are extreme cases. With these two ideal circumstances, one may ask, what about the real situation? It seems a combination of these two cases may be more practical. Inspired by previous works, we here take into account both time-dependent decoherence and time-independent decoherence, and develop a hybrid approach by combining TDFSs and STA to engineer quantum states in open systems. Two coherent controls are designed to drive quantum states into the TDFS and realize the QSE, respectively. This approach has the following advantages: (1) different from the standard application of engineering quantum states, the quantum states in this work are encoded into the time-dependent basic vectors in TDFS which have a natural immunity to time-dependent decoherence. (2) A time evolution operator $U_{H_S}(t)$ is introduced to design the STA, thereby the system evolution on timescales is much shorter than time-independent decoherence times. Therefore, the approach is insensitive to time-independent decoherence. (3) Based on the reverse-engineering technique, the evolution of the system can be controlled accurately, and one can easily obtain the QSE in the optimal way. (4) Certain dissipative effects are no longer harmful. Instead, they become important and useful resources to the QSE. (5) The approach can be generalized to realize QSE in high-dimensional quantum systems. Therefore, this approach provides us with both an intuitive physical framework and a set of tools to efficiently engineer quantum states for open systems.

The rest of this paper is arranged as follows. In section 2, we briefly introduce the hybrid approach for nonadiabatic QSE, then discuss how to construct TDFS and how to drive the quantum state to evolve from the initial state to the target state in TDFS by utilizing coherent controls. In section 3, as an application of our approach, we show how to realize high fidelity single-qubit quantum operations in an open five-level system. In section 4, the influences of control parameters, decoherence effects, and imperfect initial states on the QSE are discussed. In section 5, we generalize the approach to the high-dimensional TDFS, and provided two alternative schemes to realize the population transfer in three- and four-dimensional TDFSs, respectively. Conclusions are given in section 6.

### 2. The hybrid approach for nonadiabatic QSE

In this section, we will introduce the hybrid approach for nonadiabatic QSE by taking into account both time-dependent decoherence and time-independent decoherence. For the sake of discussion, time-dependent decoherence and time-independent decoherence are expressed as $\mathcal{L}(\rho_t)$ and $\mathcal{D}(\rho)$, respectively. As shown in figure 1, the hybrid approach for nonadiabatic QSE can be divided into two parts: construction of the TDFS and inverse construction of the STA. The fundamental idea of part I is to deal with time-dependent decoherence $\mathcal{L}(\rho_t)$ by using the TDFSs theory. In this part, time-dependent decoherence $\mathcal{L}(\rho_t)$ is the important and useful resource [25–27], which helps us finding the potential TDFS. Then, a coherent control $H_0$ acting on the system is reversely designed to construct a TDFS ($H_{\text{DFS}}$) according to the TDFSs theory. As long as the TDFS is constructed, the system state in TDFS will undergo a unitary evolution, and the basic vectors of TDFS can be considered as computational vectors to realize quantum computing. On the other hand, the fundamental idea of part II is to suppress time-independent decoherence $\mathcal{D}(\rho)$ by using the STA theory. In this part, a coherent control $H_0$ acting on the system is introduced to design a STA, so that the system evolution on timescales is much shorter than time-independent decoherence times. Then, time-independent decoherence $\mathcal{D}(\rho)$ can be suppressed as much as possible. Finally, by combining two parts together, the nonadiabatic QSE can be realized rapidly. In the rest of this section, we will show how to construct the TDFS and inversely construct the STA by utilizing two coherent controls.

#### 2.1. Construction of the TDFS

The fundamental idea of the TDFSs theory is to make the system undergo a unitary evolution in TDFS by utilizing the incoherent control and the coherent control [32–35]. Regardless of the time-independent decoherence $\mathcal{D}(\rho)$, the dynamics of a $N$-dimensional open quantum system obeys the following Markovian master equation [28, 33, 35],
\[
\dot{\rho} = -i[H_0(t), \rho] + \mathcal{L}(\rho_t),
\]
where \(H_0(t)\) is the system Hamiltonian which can be designed by the coherent control, \(L_j(t)\) and \(\Gamma_j\) are the time-dependent Lindblad operator and the dissipation rate of the system which originates from the system-reservoir coupling respectively, and can be designed by the incoherent control \([25–32]\). The subspace
\[
\mathcal{H}_{\text{DFS}} = \text{Span}\{|\psi_1(t)\rangle, |\psi_2(t)\rangle, \ldots, |\psi_K(t)\rangle\},
\]
is the TDFS provided each basis vector of \(\mathcal{H}_{\text{DFS}}\) satisfies
\[
L_j|\psi_k(t)\rangle = \lambda_j|\psi_k(t)\rangle, \quad \forall \ j, k,
\]
where \(k = 1, 2, \ldots, K\). The effective Hamiltonian \(H_e(t)\) is invariant \([17, 18]\) under \(\mathcal{H}_{\text{DFS}}\), given by
\[
H_e(t) = G(t) + H_0(t) + \frac{i}{2} \sum_{j} [\lambda_j^* L_j - \lambda_j L_j^*],
\]
where
\[
G(t) = iU(t)^\dagger \dot{U}(t),
\]
and \(|\psi_{k'}(t)\rangle\) is a state orthogonal to the state \(|\psi_k(t)\rangle\). In other words, we should guarantee
\[
\langle \psi_{k'}(t)|H_e(t)|\psi_k(t)\rangle = 0, \quad \forall \ k', k.
\]
Substituting equations (4) and (5) into equation (6), we can find that if the form of Lindblad operator \(L_j(t)\) is fixed, one can construct the TDFS by designing the coherent control \(H_0(t)\) to satisfy the following relationship
\[
\langle \psi_{k'}(t)|H_0(t)|\psi_k(t)\rangle = -\frac{i}{2} \sum_j \lambda_j \langle \psi_{k'}(t)|L_j^*|\psi_k(t)\rangle
\]
\[-i\langle \psi_{k'}(t)|\psi_k(t)\rangle.
\]
Since the effective Hamiltonian \(H_e(t)\) is an invariant under TDFS, the basis vectors of TDFS are also the eigenstates of \(H_e(t)\). Then, \(H_e(t)\) can be rewritten as
\[
H_e(t) = \sum_{k=1}^{K} E_k |\psi_k(t)\rangle \langle \psi_k(t)|,
\]
\[
E_k = \langle \psi_k(t)|H_e(t)|\psi_k(t)\rangle.
\]
In this way, if the quantum state \( |\psi_i\rangle \) is initialized in TDFS, the final state \( |\psi_f\rangle \) will be in TDFS. This is remarkable since the TDFS provides a well closed space in the open quantum system, and the basic vectors of TDFS can be considered as computational vectors to realize quantum computing.

2.2. Inverse construction of the STA

As shown in figure 2, a central task in this work is to drive the quantum state to evolve from the initial state \( |\psi_i\rangle \) to the target state \( |\psi_f\rangle \) in the basis vectors of TDFS. In fact, according to the quantum adiabatic theorem \([36, 37]\), one can drive the eigenstates of an initial adiabatic Hamiltonian \( H(t) \) into those of a final Hamiltonian under the adiabatic condition. In order to gain an intuitional understanding of the dynamics of the adiabatic system, we introduce a time evolution operator \( U_{H(t)} \)

\[
U_{H(t)}(t) = \sum_{k=1}^{K} e^{i\vartheta_k [\tilde{\psi}_k(t)]} |\tilde{\psi}_k(t)\rangle \langle \tilde{\psi}_k(t)|,
\]

(9)

where \( |\tilde{\psi}_k(t)\rangle \) is the \( k \)-th instantaneous eigenstate of adiabatic Hamiltonian \( H(t) \) with a corresponding eigenvalue \( E_k \), \( \vartheta_k \) denotes the adiabatic phase. In addition, \( K \) is the dimensions of the constructed TDFS in subsection 2.1. More ambitiously, the STA theory (transitionless quantum driving \([14–16]\)) offers an effective way to realize the same evolution shown in equation (9) without the adiabatic condition by modifying the system’s Hamiltonian. And the modified effective Hamiltonian can be calculated by

\[
\tilde{H}_e(t) = iU_{H(t)}^\dagger(t)U_{H(t)}.
\]

(10)

In addition, the complete master equation of the engineered open system can be expressed as

\[
\dot{\rho} = -i[H_0(t) + \tilde{H}_e(t), \rho] + \mathcal{L}(\rho) + \mathcal{D}(\rho),
\]

(11)

where the engineered coherent control Hamiltonian \( \tilde{H}_e(t) \) reads

\[
\tilde{H}_e(t) = \tilde{H}_e(t) - H_e(t),
\]

(12)

and \( H_e(t) \) [see equation (8)] is the effective Hamiltonian for the constructed TDFS.

It is obvious that \( U_{H(t)} \) is the key element associated with the system dynamics. Provided the time evolution operator \( U_{H(t)} \) is fixed, one not only can accurately obtain the information about system evolution, but also can quickly get the modified effective Hamiltonian which is used to construct the STA. Moreover, the different choices of \( U_{H(t)} \) will induce different evolution paths which connect the initial state and the target state, and affect the feasibility in the practical realization. Therefore, we make some remarks on the design of the time evolution operator \( U_{H(t)} \) and the evolution paths:

(i) The time evolution operator \( U_{H(t)} \) can be expressed by using its instantaneous eigenstate \( |\tilde{\phi}_k(t)\rangle \) and eigenvalue \( \lambda_k(t) \),

\[
U_{H(t)}(t) = \sum_{k=1}^{K} e^{i\lambda_k(t) [\tilde{\phi}_k(t)]} |\tilde{\phi}_k(t)\rangle \langle \tilde{\phi}_k(t)| = \sum_{k=1}^{K} \lambda_k(t) |\tilde{\phi}_k(t)\rangle \langle \tilde{\phi}_k(t)|.
\]

(13)
In other words, by means of the reverse engineering technique [38], $U_{H_5}(t)$ can be constructed with a set of basic vectors $\{|\phi_1(t)\rangle, |\phi_2(t)\rangle, \ldots, |\phi_k(t)\rangle\}$, which satisfy the orthogonality and closure relations [39]. Moreover, the eigenvalue $\lambda_k(t)$ should satisfy $|\lambda_k(t)|^2 = 1$ to guarantee $U_{H_5}(t)U_{H_5}^\dagger(t) = 1$. In this way, reverse engineering technique offers us a positive way to accurately control the evolution of the system by designing different time evolution operators $U_{H_5}(t)$.

(ii) According to equation (13), the evolution state of the system at the time $t$ can be expressed as

$$|\Psi_t\rangle = U_{H_5}(t)|\Psi_0\rangle = \sum_{k=1}^{K} \lambda_k(t)\langle\tilde{\phi}_k(0)|\tilde{\phi}_k(t)\rangle.$$  \hspace{1cm} (14)

As shown in figure 2, we can drive the quantum state to evolve from the initial state $|\Psi_0\rangle$ to the target state $|\Psi_f\rangle$, when $t = t_f - t_i$ [here, $t_i$ ($t_f$) means the initial (final) time]. In this scenario, the evolution paths depend on $\sum_{k=1}^{K} \lambda_k(t)\langle\tilde{\phi}_k(0)|\tilde{\phi}_k(t)\rangle$. Then, the inherent flexibility of the reverse-engineering technique also allows one to easily obtain the engineered coherent control Hamiltonian $\tilde{H}_5(t)$ and evolution paths, thus one can choose the optimal evolution path to realize the QSE on demand.

(iii) For the two-dimensional TDFS, the associated time evolution operator $U_{H_5}$ can be expressed directly as

$$U_{H_5} = \exp(i\gamma n \cdot \sigma) = \begin{pmatrix} \cos \gamma + i \sin \gamma \cos \theta & \sin \gamma \sin \theta e^{-i\theta} \\ i \sin \gamma \sin \theta e^{i\theta} & \cos \gamma - i \sin \gamma \cos \theta \end{pmatrix},$$  \hspace{1cm} (15)

where $n = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ represents the unit vector, $\gamma(\theta, \phi)$ is a time-dependent phase parameter, and $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ is the vector of Pauli operator in the TDFS. However, for the higher-dimensional TDFS, associated time evolution operator $U_{H_5}$ could not be easily expressed by using Pauli operators. In this case, it is advisable to construct the time evolution operator $U_{H_5}$ by means of the reverse engineering method.

For example, realizing a complete population transition between $|0\rangle$ and $|2\rangle$ without disturbing the quasi-stable state $|1\rangle$ in three-dimensions TDFS, is a central ingredient for QSE. Here, we show how to construct associated time evolution operator $U_{H_5}$ to realize such population transfer. First, we introduce a set of new basic vectors

$$\begin{align*}
|\tilde{\phi}_1(t)\rangle &= \cos \beta |0\rangle - \sin \beta |2\rangle, \\
|\tilde{\phi}_2(t)\rangle &= \sin \alpha \sin \beta |0\rangle + \cos \alpha |1\rangle + \sin \alpha \cos \beta |2\rangle, \\
|\tilde{\phi}_3(t)\rangle &= \cos \alpha \sin \beta |0\rangle - \sin \alpha |1\rangle + \cos \alpha \cos \beta |2\rangle,
\end{align*}$$  \hspace{1cm} (16)

where $\alpha$ and $\beta$ are real time-dependent parameters and we have omitted the explicit time dependence of these parameters for the sake of simplicity. One can find that the above basic vectors satisfy the orthogonality relation and closure relation, and the introduced basic vector $|\tilde{\phi}_1(t)\rangle$ connects the initial state $|0\rangle$ with the target state $|2\rangle$ by choosing the following boundary condition

$$\cos \alpha_{t_f} = \cos \beta_{t_f} = 1 \rightarrow \cos \alpha_{t_i} = \cos \beta_{t_i} = 0.$$  \hspace{1cm} (17)

Then, under the basis vectors $\{|0\rangle, |1\rangle, |2\rangle\}$, the time evolution operator $U_{H_5}$ reads

$$U_{H_5} = \sum_{k=1}^{3} \lambda_k(t)|\tilde{\phi}_k(t)\rangle\langle\tilde{\phi}_k(t)|$$

$$= \begin{pmatrix} \lambda_1 \cos \beta & \lambda_2 \sin \alpha \sin \beta & \lambda_3 \cos \alpha \sin \beta \\ 0 & \lambda_2 \cos \alpha & -\lambda_3 \sin \alpha \\ -\lambda_1 \sin \beta & \lambda_2 \sin \alpha \cos \beta & \lambda_3 \cos \alpha \cos \beta \end{pmatrix},$$  \hspace{1cm} (18)

where $\lambda_k$ is the eigenvalue of time evolution operator $U_{H_5}$ which satisfies $|\lambda_k|^2 = 1$. Notice that equations (15) and (18) are the primary results to be used in following work.

3. High fidelity single-qubit QSE in open system

As an application of our approach, we will show how to realize high fidelity single-qubit quantum operations in an open five-level system. As shown in figure 3(a), besides two coherent fields ($\Omega_1, \Omega_2$), the
The five-level system is driven by two squeezed vacuum reservoirs ($\hat{a}_1$, $\hat{a}_2$). Under the Markov approximation, the master equation describing the dynamics of the open system reads [25–29]

$$\dot{\rho} = -i[H_0(t) + \hat{H}_0(t), \rho] + \mathcal{D}(\rho)$$

$$+ \sum_{j=1}^{2} \Gamma_j \left[ L_j(t)\rho L_j^\dagger(t) - \frac{1}{2} L_j^\dagger(t)L_j(t), \rho \right].$$

(19)

Here, $\Gamma_j$ is the dissipation rate caused by the coupling to the $j$th squeezed vacuum reservoir, the corresponding Lindblad operator $L_j(t)$ is [25–29]

$$L_j(t) = \cosh[r_j(t)]\hat{S}_j + \sinh[r_j(t)] \exp[i\Theta_j(t)]\hat{S}_j^\dagger,$$

(20)

where $r_j(t) > 0$ and $\Theta_j(t) \in [0, 2\pi]$ are the amplitude and phase of the squeezed vacuum reservoir, respectively; and $S_1 = |1\rangle \langle 0| + |0\rangle \langle -1|$, $S_2 = |2\rangle \langle 0| + |0\rangle \langle -2|$. The Hamiltonian of the five-level system interacting with two coherent control fields ($\Omega_1$, $\Omega_2$) can be written as

$$H_0(t) = \Omega_1(t)|1\rangle \langle -1| + \Omega_2(t)|2\rangle \langle -2| + \text{H.c.}$$

(21)

It is obvious that the Lindblad operators $L_1$ and $L_2$ give two orthogonal decoherence-free eigenvectors

$$|\psi_1(t)\rangle = c_{\gamma_1}(t)|-1\rangle - s_{\gamma_1}(t) \exp[i\Theta_1(t)]|1\rangle,$$

$$|\psi_2(t)\rangle = c_{\gamma_2}(t)|-2\rangle - s_{\gamma_2}(t) \exp[i\Theta_2(t)]|2\rangle,$$

(22)

where $c_{\gamma_j}(t) = \cosh[r_j(t)]/\sqrt{\cosh^2[r_j(t)]}$, $s_{\gamma_j}(t) = \sinh[r_j(t)]/\sqrt{\cosh^2[r_j(t)]}$, (j = 1, 2). Then, $|\psi_1(t)\rangle$ and $|\psi_2(t)\rangle$ can be chosen as the basis to construct the TDFS. The complete Hilbert space $\mathcal{H}$ of the open system can be obtained by adding a complementary subspace $\mathcal{H}_{\text{DFS}}^\perp$, which is spanned by the following orthogonal normalized bases,

$$|\psi_1^\perp(t)\rangle = s_{\gamma_1}(t)|-1\rangle + c_{\gamma_1}(t) \exp[i\Theta_1(t)]|1\rangle,$$

$$|\psi_2^\perp(t)\rangle = s_{\gamma_2}(t)|-2\rangle + c_{\gamma_2}(t) \exp[i\Theta_2(t)]|2\rangle,$$

$$|\psi_0^\perp(t)\rangle = |0\rangle.$$

(23)

Substituting equations (21)–(23) into equation (7), the coherent control fields can be designed as follows

$$\Omega_1(t) = \left(\frac{\dot{\Theta}_1 \sinh[2r_1]}{2} - \frac{i\dot{r}_1}{\cosh[2r_1]}\right) \exp[i\Theta_1],$$

$$\Omega_2(t) = \left(\frac{\dot{\Theta}_2 \sinh[2r_2]}{2} - \frac{i\dot{r}_2}{\cosh[2r_2]}\right) \exp[i\Theta_2],$$

(24)

where $\dot{r}_j(t)$ and $\dot{\Theta}_j(t)$ are the time partial derivative for the squeezed parameters $r_j(t)$ and $\Theta_j(t)$, respectively. Up to now, we have successfully constructed a two-dimensional TDFS, and the effective Hamiltonian $H_e(t)$
is written as

\[ H_\varepsilon(t) = \sum_{k=1}^{2} E_k |\psi_k(t)\rangle \langle \psi_k(t)|, \]

\[ E_k = -\tilde{\Omega}_k(t) \sin^2[r_k(t)], \Delta E = E_2 - E_1. \]  \hspace{1cm} (25)

As shown in figure 3(b), the five-level open system in the TDFS can be considered as an effective two-dimensions closed system, and the energy difference between the TDFS states \(|\psi_2(t)\rangle\) and \(|\psi_1(t)\rangle\) is \(\Delta E\).

Next, we start to design the coherent control Hamiltonian \(H_0(t)\) for realizing high fidelity single-qubit quantum operations. For a convenient discussion, \(|\psi_2(t)\rangle\) and \(|\psi_1(t)\rangle\) are encoded as \(|1\rangle\) and \(|0\rangle\), respectively. According to equation (15), under the two-dimensions basis vectors \({|1\rangle, |0\rangle}\), the time evolution operator \(U_{H_0}\) is expressed as

\[ U_{H_0} = \exp(i\gamma \mathbf{n} \cdot \sigma) = \begin{pmatrix} \cos \gamma + i \sin \gamma \cos \theta & i \sin \gamma \sin \theta e^{-i\phi} \\ i \sin \gamma \sin \theta e^{i\phi} & \cos \gamma - i \sin \gamma \cos \theta \end{pmatrix}. \] \hspace{1cm} (26)

In fact, \(U_{H_0}\) can also be constructed directly by means of the reverse engineering method with its instantaneous eigenstates

\[ |\tilde{\phi}_1(t)\rangle = \sin \left( \frac{\theta}{2} \right) e^{i\phi} |0\rangle + \cos \left( \frac{\theta}{2} \right) |1\rangle, \]

\[ |\tilde{\phi}_2(t)\rangle = \cos \left( \frac{\theta}{2} \right) e^{i\phi} |0\rangle - \sin \left( \frac{\theta}{2} \right) |1\rangle, \] \hspace{1cm} (27)

and corresponding eigenvalues \(\lambda_1 = \exp(i\gamma)\), \(\lambda_2 = \exp(-i\gamma)\). Then, the effective Hamiltonian \(\tilde{H}_\varepsilon(t)\) is written as

\[ \tilde{H}_\varepsilon(t) = iU_{H_0}^\dagger(t) U_{H_0}(t) = \tilde{\Omega}_x \sigma_x + \tilde{\Omega}_y \sigma_y + \tilde{\Omega}_z \sigma_z, \] \hspace{1cm} (28)

with

\[ \tilde{\Omega}_x = \left[ -\dot{\gamma} \sin \theta \cos \phi - \dot{\theta} \sin \gamma (\sin \gamma \sin \phi + \cos \gamma \cos \theta \cos \phi) \right. \]

\[ + \dot{\phi} \sin \gamma \sin \theta (\cos \gamma \sin \phi - \sin \gamma \cos \theta \cos \phi)] \],

\[ \tilde{\Omega}_y = \left[ -\dot{\gamma} \sin \theta \sin \phi + \dot{\theta} \sin \gamma (\sin \gamma \cos \phi - \cos \gamma \cos \theta \sin \phi) \right. \]

\[ - \dot{\phi} \sin \gamma \sin \theta (\sin \gamma \cos \theta \sin \phi + \cos \gamma \cos \phi) \],

\[ \tilde{\Omega}_z = [ -\dot{\gamma} \cos \theta + \dot{\theta} \sin \gamma \cos \gamma \sin \theta + \dot{\phi} \sin^2 \gamma \sin^2 \theta ], \] \hspace{1cm} (29)

where \(\dot{\gamma}(\dot{\theta}, \dot{\phi})\) is the time partial derivative for the time-dependent phase parameters \(\gamma(\theta, \phi)\). The engineered coherent control Hamiltonian \(H_0(t)\) is expressed as

\[ \tilde{H}_0(t) = \tilde{\Omega}_x \sigma_x + \tilde{\Omega}_y \sigma_y + \tilde{\Omega}_z \sigma_z, \tilde{\Omega}_z = \tilde{\Omega}_2 - \frac{\Delta E}{2}. \] \hspace{1cm} (30)

When the initial state \(|\Psi_i\rangle\) is fixed, for example, the system is initially in \(|0\rangle\), according to equations (14) and (26), the final state will be

\[ |\Psi_f\rangle = (\cos \gamma - i \sin \gamma \cos \theta) |0\rangle + i \sin \gamma \sin \theta e^{-i\phi} |1\rangle, \] \hspace{1cm} (31)

with the initial conditions \(\gamma_i = \cos \theta_i = 1\) or \(\gamma_i = 1\). We stress that by picking out appropriate phase parameters, one can perform any trajectory on the Bloch sphere to realize the QSE.

4. Numerical analyses

Let us now test the validity of our approach by numerical simulation. First, we should judge the validity of the constructed two-dimensional TDFS by using the designed coherent control \(H_0(t)\). For this purpose, we numerically calculate the populations and the purity of the system. The population for a quantum state \(|\tilde{\phi}\rangle\) is given through the relation \(P_{\tilde{\phi}} = \langle |\tilde{\phi}\rangle |\rho(t)| \tilde{\phi}\rangle\), while purity is given through the relation purity = \(\text{tr}\rho^2(t)\)
where $\rho(t)$ is the density operator of the system at the time $t$. For simplicity, we consider the initial state $|\Psi_0\rangle = |0\rangle$ and scale all the parameters with respect to the dissipation rate $\Gamma_1 = \Gamma$. Without loss of generality, both the squeezed parameters $r_j(t)$ and $\Theta_j(t)$ are set to depend on time linearly [28]

$$
\begin{align*}
  r_j(t) &= u_j t + \epsilon, \\
  \Theta_j(t) &= \upsilon_j t.
\end{align*}
$$

(32)

In the first example, we choose a set of parameters

$$
\begin{align*}
  u_2 &= 2u_1 = \Gamma, \\
  v_2 &= 2v_1 = \Gamma, \\
  \epsilon &= 0.1\Gamma, \\
  \Gamma_2 &= \Gamma_1 = 2\Gamma.
\end{align*}
$$

(33)

The above parameters mean that our approach works in the non-adiabatic mechanism since the change rates of the squeezed parameters and the dissipation rate $\Gamma$ are basically on the same magnitude. Furthermore, different from previous studies associated with the collective decoherence [11, 12, 28, 33, 34], we consider a more realistic case where the squeezed parameters for the two squeezed vacuum reservoirs are different.

In figure 4(a), the populations and the purity of the time-dependent states $|0\rangle$ and $|1\rangle$ are plotted by numerically solving the master equation (19) [here, we have ignored the effect of time-independent decoherence $D(\rho)$] with the designed coherence control field $H_0$ and without the designed coherence control field. One can see that the quantum state of the system stabilizes in the $|0\rangle$ under the coherent control $H_0$, while the quantum state of the system rapidly changes over time without the coherent control. The behavior of the parameters mentioned above can be illustrated by studying the purity of the system. As shown in figure 4(b), we find that when the coherent control is applied, the purity does not change over time and the evolution of the quantum state is unitary. However, the purity rapidly decays over time without the coherent control. In other words, the quantum state will become a complex mixed state. The results obtained here coincide with our previous predictions. Therefore, the constructed two-dimensional TDFS work well under the designed coherence control, and the time-dependent states $|0\rangle$ and $|1\rangle$ have a natural immunity to the time-dependent decoherence $L(\rho_i)$.

Second, we should judge the validity of the population engineering by using the designed coherent control $H_0(t)$. As shown in equation (31), if the system is initially in $|0\rangle$, the target state is $|\Psi_{t}\rangle = (\cos \gamma - i \sin \gamma \cos \theta |0\rangle + i \sin \gamma \sin \theta e^{-it}|1\rangle$ after an engineering time $T = t_f - t_i$. One can pick out appropriate phase parameters to realize the QSE. For example, we can easily realize the population inversion from $|0\rangle\langle 0|$ to $|1\rangle\langle 1|$ along two different paths by choosing the following boundary conditions

$$
\begin{align*}
  \text{path A: } & \sin \gamma_{t_i} = \cos \theta_{t_i} = 1 \rightarrow \cos \gamma_{t_f} = \cos \theta_{t_f} = 0, \\
  \text{path B: } & \cos \gamma_{t_i} = 1 \rightarrow \sin \gamma_{t_f} = \sin \theta_{t_f} = 1.
\end{align*}
$$

(34)

For an intuitive grasp of the performance of the population engineering, the populations of the time-dependent states $|0\rangle$ and $|1\rangle$ are plotted in figure 5 by numerically solving the master equation (19), where we have ignored the effect of time-independent decoherence $D(\rho)$, and assumed $\gamma = \gamma_t + \gamma_0, \theta = \theta t$ and $\phi = 0$ for the sake of simplicity. It is clear from figure 5 that whether the evolution is along path A or path B, we can accurately control the system evolution, and a complete population inversion takes place when $T = 1/\Gamma$, which is consistent with our design. It is worth to note that the time $T$ for a complete population inversion can be significantly shortened in two ways: increasing the dissipation rate $\Gamma$ or increasing the change rates of the control parameters. For the first case, the dissipation effect $\Gamma$ is no longer harmful. Actually, the dissipation effect is the important and useful resource to the QSE. For the second case, the approach can work well even in the non-adiabatic mechanism. Therefore, the population engineering performs well under the designed coherence control $H_0(t)$. On the other hand, from an experimental view point, the dynamics of the $\hat{H}_0(t)$ with different parameters should be investigated. In figures 5(c) and (d), we plot the time evolution of the coherent control $\hat{H}_0(t)$ with different evolution paths. It is clear that the shapes of the Rabi frequency of $\hat{H}_0(t)$ in figures 5(c) and (d) are simple, which are available within present-day experimental techniques [40, 41]. Thus, the coherent control fields in our approach can be realized in practice. Furthermore, comparing figure 5(c) with figure 5(d), we find that path A may be better than path B, since one can obtain the same goal by using a simpler coherent control. We stress that in our approach, the inherent flexibility of the reverse-engineering technique allows us to easily obtain the evolution paths [as shown in equation (14)], thus one can choose the optimal evolution path to realize the QSE on demand.

In the above discussion, the effects of the time-independent decoherence $D(\rho)$ have not been considered carefully. Thus, we should investigate the influences of time-independent decoherence, e.g. the damping and

8
Figure 4. (a) The populations of the system versus time, with the coherent control $H_0(t)$ and without the coherent control. (b) The purity of the system versus time with the coherent control $H_0(t)$ and without the coherent control. The parameters are based on equation (33).

dephasing of the excited state of the system, on our approach. The complete master equation for the open system can be expressed as

$$\dot{\rho} = -i[H_0(t) + \tilde{H}_0(t), \rho]$$

$$+ \sum_{j=1}^{2} \Gamma_j \left[ L_j(t)\rho L_j^\dagger(t) - \frac{1}{2} \{ L_j(t)L_j^\dagger(t), \rho \} \right]$$

$$+ \sum_{j,k=1}^{2} \gamma^k_j \left[ L^k_j \rho L^k_j - \frac{1}{2} \{ L^k_j L^k_j, \rho \} \right],$$

where $L^k_j$ is the Lindblad operator for the time-independent decoherence $D(\rho)$. In the present system, the damping of the excited state can be described by the Lindblad operators $L^1_1$ and $L^2_1$, while the dephasing effect can be described by the Lindblad operators $L^1_2$ and $L^2_2$,

$$L^1_1 = |1\rangle\langle 0| + |0\rangle\langle -1| + | - 1\rangle\langle 1|,$$

$$L^2_1 = |2\rangle\langle 0| + |0\rangle\langle -2| + | - 2\rangle\langle 2|,$$

$$L^1_2 = |1\rangle\langle 1|, L^2_2 = | - 2\rangle\langle -2|.$$  

In equation (35), $\gamma^1_1$ represents the rate of the damping from level $|1\rangle$ to $|0\rangle$ or $|-1\rangle$, $\gamma^2_1$ represents the rate of the damping from level $|2\rangle$ to $|0\rangle$ or $|-2\rangle$, $\gamma^1_2$ and $\gamma^2_2$ denote the dephasing rate of the states $|1\rangle$ and $|2\rangle$, respectively. The fidelity vs the decay rates $\gamma_1$ and $\gamma_2$ is presented in figure 6, where we have assumed $\gamma^1_1 = 2\gamma^1_2 = \gamma_1$ and $\gamma^2_1 = 2\gamma^2_2 = \gamma_2$ for the sake of convenience. It is clear from figure 6 that the fidelity is also insensitive to the time-independent decoherence. For a wide range of parameters, we can get a high fidelity (above 0.99). Meanwhile, the fidelity will be slightly affected by the large dephasing effects of the excited state. The influence of the damping will be more evident than that of the dephasing. This is due to the fact that the damping will directly break the superposition of the states in TDFS according to the TDFSs.
Figure 5. Evolution of the populations of the system and the shapes of the designed coherence control $\hat{H}_0(t)$ with different evolution paths. For (a) and (c), the system evolves along path A with the parameters $\dot{\gamma} = 0, \dot{\theta} = \gamma_0 = \pi \Gamma / 2$. For (b) and (d), the system evolves along path B with the parameters $\dot{\theta} = \dot{\gamma} = \pi \Gamma / 2, \gamma_0 = 0$. The populations are based on equation (19), while the shapes of the designed coherence control $\hat{H}_0(t)$ are based on equation (30).

Figure 6. The fidelity vs the decay rates $\gamma_1$ and $\gamma_2$. The fidelity is plotted by numerically solving the master equation (35).

theory. Nevertheless, the dephasing effects is almost restrained owing to advantages of the STA. As discussed above, our approach also shows a good robust feature for the time-independent decoherence.

Besides the decoherence effects, the errors produced in the preparation of the initial state is another severe problem in quantum information process [42–44]. Thus, we further test the sensitivity of our approach to the imperfect initial state. For a convenient discussion, we consider two realistic cases: (i) the initial state $|\Phi^\prime\rangle_A$ is in the TDFS, while there exists a small population in the state $|\bar{1}\rangle$, (ii) the initial state $|\Phi^\prime\rangle_B$ is not perfectly in the TDFS, there exists a small population in the state $|0^-\rangle$ which is in the complementary subspace $H_{\perp DFS}$,

$$
|\Phi^\prime\rangle_A = \sqrt{1 - \eta^2} |0\rangle + \eta |1\rangle,
$$

$$
|\Phi^\prime\rangle_B = \sqrt{1 - \eta^2} |0\rangle + \eta |0^-\rangle,
$$

where $\eta$ is a small deviation constant.
Figure 7. The fidelity as a function of $t$ and $\eta$ for different imperfect initial states. (a) The initial state $|\Psi_{t,i}\rangle_A$ is in the TDFS, but there exists a small population in the state $|\bar{1}\rangle$. (b) The initial state $|\Psi_{t,i}\rangle_B$ is not perfectly in the TDFS, there exists a small population in the state $|\bar{0}\rangle$.

In figure 7(a), we can clearly see that when the initial state is $|\Psi_{t,i}\rangle_A$, a high fidelity (above 0.99) can be achieved for a wide range of parameters. In addition, although the deviation $\eta$ still plays a negative role in the current approach, the fidelity is independent of the evolution time $t$. This means that we can accurately control the evolution of the system for the whole process and obtain the quantum state at any time when $\eta$ is small. Figure 7(b) shows that when the initial state is $|\Psi_{t,i}\rangle_B$, a high fidelity (above 0.98) can also be achieved for a wide range of parameters. However, comparing figure 7(b) with figure 7(a), we find that the area of a high fidelity in figure 7(b) is less than that in figure 7(a) and the fidelity simultaneously depends on the deviation $\eta$ and the evolution time $t$. More specifically, the fidelity increases with the evolution time $t$ for a fixed $\eta$. In principle, even if the initial state is imperfect, a high fidelity can be obtained by prolonging the evolution time in this case. Consequently, the imperfect initial state would not affect the fidelity significantly. As discussed above, our approach shows a good robust feature for imperfect initial states and various dissipation cases.

5. High fidelity single-qudit QSE in open system

Recent studies showed that $d$-dimensional systems (for instance, qudits $d \geq 3$) are better suited for certain quantum communication and quantum information processing tasks [45–47]. For example, Bruß et al [45] studied optimal eavesdropping in quantum cryptography with three-dimensional states and showed that their scheme is more secure against symmetric attacks than protocols using two-dimensional states. Furthermore, Cerf et al [46] investigated quantum cryptographic schemes relying on encoding the key into qudits and found that the quantum cryptographic scheme using $d$-dimensional bases is more secure against individual attacks than the BB84 scheme [48] using two-dimensional bases. Therefore, from a practical point of view it is important and interesting to demonstrate how to implement our approach in the high-dimensional quantum systems.

Before elaborating on the detailed examples of high fidelity single-qudit QSE in high-dimensional system, we make some remarks on construction of the TDFS and inverse construction of the STA, which are two key parts of our hybrid approach as mentioned in section 2. (1) Based on the TDFSs theory, the
high-dimensional TDFS can be constructed by using a coherent control $H_0$. From an experimental view point, the Hamiltonian terms of the introduced coherent control $H_0$ should not signally grow with the (high-dimensional quantum) system size and be available within presentday experimental techniques. (2) The goal of our approach is driving the quantum state to evolve from the initial state $|\Psi_i\rangle$ to the target state $|\Psi_f\rangle$ in the basis vectors of TDFS. Thus, the designed system’s evolution operator $U_{H_0}(t)$ must connect the initial state $|\Psi_i\rangle$ with the target state $|\Psi_f\rangle$, and in principle the involved transition $|\Psi_i\rangle \rightarrow |\Psi_f\rangle$ should be related to an interesting quantum computation or QSE. (3) By means of the reverse engineering technique, $U_{H_0}(t)$ can be constructed with a set of basic vectors $\{|\phi_1(t)\rangle, |\phi_2(t)\rangle, \ldots, |\phi_N(t)\rangle\}$, which satisfy the orthogonality and closure relations [39]. Therefore, in the design process of inverse construction of the STA, the first step is introducing appropriate basic vectors $|\phi_i(t)\rangle$ to construct the time evolution operator $U_{H_0}(t)$ according to the goal of QSE. Once the time evolution operator $U_{H_0}(t)$ is fixed, the modified effective Hamiltonian which is used to construct the STA can be obtained easily. Finally, the nonadiabatic QSE can be rapidly realized in high-dimensional system. As application examples, we now demonstrate how to realize high fidelity single-qubit quantum operations in an open $(2N+1)$-level system, and provide two alternative QSE schemes in three- and four-dimensional TDFSs, respectively.

### 5.1. Construction of the TDFS in high-dimensional open system

The potential level configuration of the $(2N+1)$-level system is depicted in figure 8(a). Besides coherent fields ($\Omega_1, \Omega_2, \Omega_3, \ldots, \Omega_N$), the $(2N+1)$-level system is driven by squeezed vacuum reservoirs $(\hat{a}_1, \hat{a}_2, \hat{a}_3, \ldots, \hat{a}_N)$. The master equation describing the dynamics of the open system reads [25–29]

$$\dot{\rho} = -i[H_0(t) + \hat{H}_0(t), \rho] + D(\rho) + \sum_{j=1}^{N} \Gamma_j \left[ L_j(t) \rho L_j(t)^\dagger - \frac{1}{2} \{ L_j(t)^\dagger L_j(t), \rho \} \right],$$

(38)

where the Lindblad operator $L_j(t)$ is

$$L_j(t) = \cosh[\tau_j(t)] S_j + \sinh[\tau_j(t)] \exp[i\Theta_j(t)] S_j^\dagger,$$

(39)

$\tau_j(t) > 0$ and $\Theta_j(t) \in [0, 2\pi]$ are the amplitude and phase of the squeezed vacuum reservoir, respectively; and $S_j = |j\rangle \langle 0| + |0\rangle \langle -j|$. It is obvious that the decoherence-free eigenvector of Lindblad operator $L_j(t)$

$$|\psi_j(t)\rangle = c_j(t)|-j\rangle - s_j(t) \exp[i\Theta_j(t)]|j\rangle,$$

(40)

can be chosen as the basis to construct the TDFS, where $c_j(t) = \cosh[\tau_j(t)]/\sqrt{\cosh[2\tau_j(t)]}$, $s_j(t) = \sinh[\tau_j(t)]/\sqrt{\cosh[2\tau_j(t)]}$, ($j = 1, 2, 3, \ldots, N$). The Hamiltonian of the $(2N+1)$-level system interacting with $N$ coherent control fields ($\Omega_1, \Omega_2, \Omega_3, \ldots, \Omega_N$) can be written as

$$H_0(t) = \sum_{j=1}^{N} \Omega_j(t) |j\rangle \langle -j| + \text{H.c.}$$

(41)

Similar to the procedure of the single-qubit quantum operation in section 3, one can construct a $N$-dimensional TDFS ($|\psi_1(t)\rangle, |\psi_2(t)\rangle, |\psi_3(t)\rangle, \ldots, |\psi_N(t)\rangle$) by designing the coherent control field $\Omega_j(t)$.
as follows

\[ \Omega_{\xi}(t) = \left( \frac{\dot{\Theta}_j \sinh[2\tau_j]}{2} - \frac{ir_j}{\cosh[2\tau_j]} \right) \exp[i\Theta_j], \quad (42) \]

where \( r_j(t) \) and \( \dot{\Theta}_j(t) \) are the time partial derivative for the squeezed parameters \( r_j(t) \) and \( \Theta_j(t) \), respectively. We emphasize that the designed coherent control Hamiltonian (42) is simple and is available within present day experimental techniques. Meanwhile, the effective Hamiltonian \( H_{e}(t) \) is written as

\[ H_{e}(t) = \sum_{k=1}^{N} E_k |\psi_k(t)\rangle \langle \psi_k(t)|, \]

\[ E_k = -\dot{\Theta}_k(t) \sinh^2[r_k(t)]. \quad (43) \]

As shown in figure 8(b), the \((2N + 1)\)-level open system in the TDFS can be considered as an effective \( N \)-level closed system. Up to now, the \( N \)-dimensional TDFS has been constructed successfully.

### 5.2. Inverse construction of the STA in high-dimensional TDFS

Based on the STA theory which was mentioned in subsection 2.2, one can design an effective Hamiltonian \( \tilde{H}_{e}(t) \) to realize high fidelity single-qudit quantum operations. In addition, the inherent flexibility of the reverse engineering technique allows one to realize the QSE on demand. Without loss of generality, we first generalize our approach to the three-dimensional TDFS. It is well known that the stimulated Raman adiabatic passage (STIRAP) [47] is a central ingredient for QSE in three-dimensional system, which aims to realize a complete population transition between \( |0\rangle \) and \( |2\rangle \) without disturbing the quasi-stable state \( |1\rangle \). However, due to the limit of adiabatic conditions, the traditional STIRAP process requires large pulse areas and long interaction time. In this case, the ideal robustness and the intended dynamics may be spoiled by the vast amount of accumulation of perturbations and decoherence due to noise and undesired interactions. Here, we inversely construct a STA to overcome the time-dependent decoherence and provide an alternative fast population transfer scheme. First, we introduce a set of new basic vectors as shown in equation (16). Then, according to equation (18), under the basis vectors \( \{|0\rangle, |1\rangle, |2\rangle\} \), an alternative time evolution operator \( U_{\tilde{H}_e} \) reads

\[ U_{\tilde{H}_e} = \sum_{k=1}^{N} \lambda_k(t) |\phi_k(t)\rangle \langle \phi_k(t)| \]

\[ = \begin{pmatrix}
\cos \beta & \sin \alpha & \sin \beta & \cos \alpha & \sin \beta \\
0 & \cos \alpha & -\sin \alpha & \cos \alpha & \cos \beta \\
-\sin \beta & \sin \alpha & \cos \beta & \cos \alpha & \cos \beta
\end{pmatrix}, \quad (44) \]

where we have assumed \( \lambda_1 = \lambda_2 = \lambda_3 = 1 \) for the sake of simplicity. Substituting equation (44) into equation (10), we can obtain the effective Hamiltonian \( \tilde{H}_{e}(t) \)

\[ \tilde{H}_{e}(t) = iU_{\tilde{H}_e}^\dagger(t)\dot{U}_{\tilde{H}_e}(t) \]

\[ = \begin{pmatrix}
0 & i\alpha \sin \beta & i\beta \\
-i\alpha \sin \beta & 0 & -i\alpha \cos \beta \\
-i\beta & i\alpha \cos \beta & 0
\end{pmatrix} \]

\[ = \Omega_1 |0\rangle \langle 1| + \Omega_2 |0\rangle \langle 2| + \Omega_3 |1\rangle \langle 2| + \text{H.c.} \quad (45) \]

where \( \Omega_1 = \dot{\alpha} \sin \beta, \Omega_2 = i\dot{\beta}, \Omega_3 = -i\dot{\alpha} \cos \beta \), and \( \dot{\alpha}(\dot{\beta}) \) is the time partial derivative for the time-dependent phase parameters \( \alpha(\beta) \). Physically, the effective Hamiltonian (45) can be considered as a three-level system driven by three coherent fields \( \Omega_1, \Omega_2 \) and \( \Omega_3 \). In this case, once \( \alpha \) and \( \beta \) satisfy the boundary condition

\[ \cos \alpha_{\xi} = \cos \beta_{\xi} = 1, \quad \cos \alpha_{\xi} = \cos \beta_{\xi} = 0, \quad (46) \]

we can achieve a nearly perfected population transfer from \( |\psi_{\xi}(t_1)\rangle = |0\rangle \) to \( |\psi_{\xi}(t_1)\rangle = |2\rangle \) without disturbing the quasi-stable state \( |1\rangle \). One can pick out appropriate \( \alpha \) and \( \beta \) to realize the fast population transfer and control accurately the evolution of the system.

Next, we generalize our approach to four-dimensional TDFS and also provide an alternative population transfer scheme. Similar to the design steps of the two-dimensional (three-dimensional) TDFS, we assume
the eigenstates of the four-dimensional TDFS read
\begin{align}
|\tilde{\phi}_1(t)⟩ &= \cos \alpha \cos \beta |0⟩ + \cos \alpha \sin \beta |1⟩ \\
&\quad + \sin \alpha \cos \beta |2⟩ + \sin \alpha \sin \beta |3⟩, \\
|\tilde{\phi}_2(t)⟩ &= \cos \alpha \sin \beta |0⟩ - \cos \alpha \cos \beta |1⟩ \\
&\quad + \sin \alpha \sin \beta |2⟩ - \sin \alpha \cos \beta |3⟩, \\
|\tilde{\phi}_3(t)⟩ &= \sin \alpha \cos \beta |0⟩ + \sin \alpha \sin \beta |1⟩ \\
&\quad - \cos \alpha \cos \beta |2⟩ - \cos \alpha \sin \beta |3⟩, \\
|\tilde{\phi}_4(t)⟩ &= \sin \alpha \sin \beta |0⟩ - \sin \alpha \cos \beta |1⟩ \\
&\quad - \cos \alpha \sin \beta |2⟩ + \cos \alpha \cos \beta |3⟩,
\end{align}

where \( \alpha, \beta \in [0, \pi/2] \) are real parameters. It can be found that the eigenstates in equation (47) satisfy the orthogonality relation and closure relation, and the initial state will be \( |\psi_1(t_i)⟩ = |\tilde{\phi}_1(t_i)⟩ = |0⟩ \) if \( \alpha(t_i) = \beta(t_i) = 0 \).

Then, under the basis vectors \( \{|0⟩, |1⟩, |2⟩, |3⟩\} \), an alternative time evolution operator \( U_{\mathcal{H}_S} \) reads
\begin{align}
U_{\mathcal{H}_S} = \sum_{k=1}^{4} \lambda_k |\tilde{\phi}_k(t)⟩⟨\tilde{\phi}_k(t)|
\end{align}

where we also have assumed the \( \lambda_k = 1 \). The effective Hamiltonian \( \tilde{H}_s(t) \) can be obtained by taking into account equations (10) and (48),
\begin{align}
\tilde{H}_s(t) &= \begin{pmatrix}
0 & \Omega_S & \Omega_P & 0 \\
\Omega_S^* & 0 & 0 & \Omega_P \\
\Omega_P^* & 0 & 0 & \Omega_S \\
0 & \Omega_P^* & \Omega_S^* & 0
\end{pmatrix},
\end{align}

where \( \Omega_S = -i\beta, \Omega_P = -i\alpha \). So far, an alternative population transfer scheme has been constructed. Once \( \alpha \) and \( \beta \) satisfy the boundary condition
\begin{align}
\beta(t_i) = \alpha(t_i) = 0, \beta(t_f) = \alpha(t_f) = \frac{\pi}{2},
\end{align}

we can achieve a nearly perfected population transfer from \( |\psi_1(t_i)⟩ = |0⟩ \) to \( |\psi_1(t_f)⟩ = |3⟩ \).

Similar to the design steps of inverse construction of the STA in two-dimensional, three-dimensional (four-dimensional) TDFS, one can easily construct the STA for the \( N \)-dimensional TDFS by introducing appropriate basic vectors \( \{|\tilde{\phi}_1(t)⟩, |\tilde{\phi}_2(t)⟩, \ldots, |\tilde{\phi}_N(t)⟩\} \) and associated time evolution operator \( U_{\mathcal{H}_S}(t) \).

For any quantum system whose effective Hamiltonian is possible to be simplified into the forms in equations (28), (45) and (49) (the basic vectors for the simplified Hamiltonian can be arbitrary dressed states as long as the dressed states satisfy the orthogonality and closure relations in the TDFS), the approach can be applied straightforwardly. Therefore, our approach is reliable in high-dimensional quantum systems, which might lead to a useful step toward realizing fast, noise-resistant and safe quantum communication (quantum information processing) for high-dimensional quantum systems in current technology.

6. Conclusion

We have generalized the QSE to open quantum systems which simultaneously suffer time-dependent decoherence and time-independent decoherence. A hybrid approach has been presented to effectively realize the QSE by combination of the TDFSs and STA theories. Moreover, a concrete single-qubit QSE example with an open five-level system is presented to show the validity of our approach. Numerical simulation showed that our approach can work well even under various decoherence cases, including the time-dependent decoherence induced by the coupling to the squeezed vacuum reservoirs, the
time-independent decoherence caused by the damping and dephasing of the excited states, and the imperfect initial states. We can fast realize high fidelity single-qubit quantum operations in the proposed open system. In addition, we have demonstrated how to implement our approach in the high-dimensional quantum systems, and provided alternative schemes in three- and four-dimensional TDFSs, respectively. Our approach works well in high-dimensional quantum systems, which might lead to a useful step toward realizing fast, noise-resistant and safe quantum communication (quantum information processing) in current technology.

As shown above, this approach has some distinguishing advantages and features. Firstly, the quantum states in the QSE have a natural immunity to the time-dependent decoherence, since they are encoded by using the time-dependent basic vectors in TDFS. Secondly, during the evolution of the system, the QSE is governed by the STA; in this case, the system evolution on timescales is much shorter than time-independent decoherence times, thus the time-independent decoherence can be restrained effectively. Thirdly, the inherent flexibility of the reverse-engineering technique allows one to easily obtain the engineered coherent control Hamiltonian $\tilde{H}_0(t)$ and the evolution paths [see equations (12) and (14)], thus one can choose the optimal evolution path to realize the QSE on demand. In addition, dissipative effects (especially, time-dependent decoherence) are no longer harmful, they are the important and useful resources to the QSE. Finally, the approach can be generalized to realize QSE in high-dimensional quantum systems. Therefore, this approach provides us with both an intuitive physical framework and a set of tools to efficiently engineer quantum states for open systems.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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