Test for the Myers-Chern-Simons Action

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Abstract

We present a generalization of the infinitesimal gauge transformation for nonabelian fields on the stack of branes up to the third order in Φ. We test the gauge invariance of the action up to the fifth order in Φ for D-instantons. This substantiates the Myers formula for the Chern-Simons term in action, which describes interaction with the RR fields of N coincident Dp branes.

1 Introduction

D-branes are by definition hypersurfaces, where open strings can end [1]. Since closed strings can interact with open strings, a separate D-brane can interact with closed strings. D-branes are dynamical objects. Low-energy dynamics of the brane is described by supersymmetric U(1) gauge theory, bosonic fields of this theory are adjoint scalars Φ and vector gauge field A_a. These fields can be obtained from the ten dimensional open string theory by the dimensional reduction. Scalars Φ describe transversal fluctuations of the brane.

The brane interaction with closed strings can be described by the low energy effective action. This action consists of two different parts. These are the Dirac-Born-Infeld (DBI) term, describing Yang-Mills theory, and the Wess-Zumino term (also known as the Chern-Simons action), governing the interaction of the Ramond-Ramond (RR) fields with branes. The difference between them is that the DBI action does not depend on the RR field and the Chern-Simons action is proportional to it. We will consider only the latter one. But both of them, apart from being invariant under the U(1) gauge transformation

\[ A \rightarrow A + P[\epsilon d\epsilon], \]  

with one scalar arbitrary gauge parameter \( \epsilon \), are also invariant under some other transformations, for example, a more general transformation

\[ A \rightarrow A + P[\lambda], \]
\[ B \rightarrow B - d\lambda, \]  

(2)

with a 1-form gauge parameter \( \lambda \). Obviously, transformation (1) is a particular case of (2) with exact \( \lambda \), i.e. \( \lambda = \epsilon d\epsilon \) for some \( \epsilon \).

If there are N coincident Dp-branes, U(1) groups extend to the U(N). The action in this case is a nontrivial generalization of the Abelian action. R.C.Myers in his work [2] derived this action for a particular case and proposed a general formula. The goal of this paper is to test the Myers action. As we know from string theory, the correct action is

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inevitably gauge invariant. Therefore we check the gauge invariance of the Myers action under gauge transformations like (2). The problem is that the generalization of (2) for the case of multiple D-branes is still unknown. We derive it up to the third order in the scalar field $\Phi$. The proper transformation looks as follows

$$
\Phi^k \rightarrow \Phi^k + \frac{1}{2} \partial_j \Lambda_j(0) \Phi^j + \frac{1}{2} \partial_j \Lambda^j(0) \Phi^i \Phi^j + \partial_j \Lambda_j(0) \left( \frac{1}{2} \Phi^i, \Phi^j \right) \Phi^j + \frac{1}{2} \left( \Phi^i, \Phi^j \right) \Phi^j + \beta \left( \left[ \Phi^i, \Phi^j \right] \right) + O(\Phi^4),
$$

and the coefficient $\beta$ can not be determined from our consideration. Possibly, it can be determined from the consideration of higher than fifth orders in $\Phi$ in the expansion of the action.

The paper is organized as follows: in section 2 we describe our notations and review properties of the Abelian Chern-Simons action. In section 3 we generalize all notations for the stack of branes and present the gauge transformation for the exact gauge parameter. In section 4 we derive the gauge transformation for general gauge parameter up to the third order in $\Phi$, analysing the action for the simplest case of D-instantons, which we expand up to the fifth order in $\Phi$. We test the transformation of the same form in the more difficult case of D2-branes in section 5. In Appendix we show that the simplest generalization of the Abelian action is not gauge invariant, and therefore, is not correct.

## 2 Abelian Chern-Simons Action

In the type IIA and IIB theories massless modes of the open string sector are the gravitational field $G$, Calb-Ramond 2-form field $B$, Ramond-Ramond poli-form $C$ and dilaton scalar $\varphi$. Respectively, in the type IIA case the RR field is the sum of odd forms and in the type IIB – the sum of even forms. The massless mode of open strings is a one-form gauge field. If we consider a brane, which breaks translational invariance in the bulk, some components of this one–form become scalar fields $\Phi$ on the brane, and other induce the gauge field on the brane with field strength $F$.

The low energy action for the brane is described by the sum of the Dirac-Born-Infeld and Chern-Simons actions

$$
S = S_{DBI} + S_{CS} =
$$

$$
= - \int_{W_{p+1}} d^{p+1}\sigma \left( e^{-\varphi} \sqrt{-\det \left( P[G + B]_{ab} + F_{ab} \right)} \right) + \int_{W_{p+1}} P[Ce^{B+F}],
$$

where $W_{p+1}$ is the $p+1$ dimensional brane world-volume, and $P[\ldots]$ is a pull-back operation, which we determine later. We will split the bulk co-ordinates $(X^\mu, \mu = 0, \ldots, n)$ into two parts: co-ordinates along the brane, which coincide with the co-ordinate system on the brane $(x^a = X^a, a = 0, \ldots, p)$ and those orthogonal to it $(X^i, i = p + 1, \ldots, n)$. This means we use the so called "static gauge". The position of the brane in the bulk is described by the set of the scalar fields $\Phi(x^a)$. From the brane point of view, closed string fields are the fields on the brane, depending on scalars $\Phi$. We consider them as series in $\Phi$, for example

$$
B_{\mu|x} = B_{\mu}(x^a, 0) + \partial_i B_{\mu}(x^a, 0) \Phi^i(x^a) + \ldots
$$

1Our notation slightly differs from that of R.C.Myers, namely, we set $T_p \rightarrow 1$, $\mu_p \rightarrow 1$, $iA \rightarrow A$ and $i\lambda \rightarrow 1$. We also have the different sign in front of $F$. 

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From now on we omit the explicit dependence of all the fields on co-ordinates on the brane $x^a$.

Since $F$ is a form on the brane, and the closed string fields $B$ and $C$ are forms in the bulk, there are two different types of forms: forms in the bulk and forms on the world-volume. Respectively, we have distinctive differentials $d^{brane}$ and $d$, and different wedge products $\wedge^{brane}$ and $\wedge$. We are bound to convert bulk tensors into world-volume tensors. To this end we apply the pull-back operation $P[\ldots]$. The pull-back acts, for example, on the 3-form as follows

$$P[C^{(3)}]_{abc} = C^{(3)}_{\mu
u\lambda}(X)\partial_\mu X^a \partial_\nu X^b \partial_\lambda X^c \big|_{X=X_{brane}} =$$

$$= C^{(3)}_{ab} (\Phi) + C^{(3)}_{abi}(\Phi)\partial_i \Phi^j + C^{(3)}_{aib}(\Phi)\partial_i \Phi^j + C^{(3)}_{abc}(\Phi)\partial_a \Phi^j \partial_b \Phi^j +$$

$$+ C^{(3)}_{ibj}(\Phi)\partial_a \Phi^i \partial_c \Phi^k + C^{(3)}_{ijc}(\Phi)\partial_a \Phi^i \partial_b \Phi^j \partial_c \Phi^k.$$  

The pull-back is the homomorphism, that is

$$P[U \wedge W] = P[U] \wedge^{brane} P[W].$$  

This means we can rewrite the Chern-Simons term in another manner

$$S = \int_{W_{p+1}} P[C] e^{P[B]} + F.$$  

The Chern-Simons action is invariant with respect to the gauge transformations of two different types [3]:

1) Ramond-Ramond field transformations

$$C \rightarrow C + e^{-B} d\epsilon,$$  

2) transformation, leaving combination $P[B] + F$ invariant

$$A \rightarrow A + P[A],$$

$$B \rightarrow B - d\Lambda.$$  

This latter transformation does not change the combination $P[B] + F$, since $F = d^{brane} A$ and

$$d^{brane} P[\ldots] = P[d\ldots].$$

We investigate its generalization for the nonabelian case. If $\Lambda = d\epsilon$, transformation (10) is just $U(1)$ transformation (9).

### 3 Nonabelian Chern-Simons Action

If we consider a stack of branes, the situation is more difficult. Now the gauge theory on the stack of branes is nonabelian, and the action can acquire new features. For example, the theory celebrates the so called "dielectric effect", i.e. the interaction of branes with the RR field of higher than $p + 1$ dimension emerges [5]. Now we want to generalize the action [5]. In order to determine the action for the nonabelian case, we must introduce trace with some ordering and determine $P[\ldots]$. We also can add some terms vanishing in the Abelian case.

The generalization of the pull-back is straightforward – we must substitute the covariant derivatives for the usual ones.
\[ \partial_a \Phi^i \rightarrow D_a \Phi^i = \partial_a \Phi^i + [A_a, \Phi^i] \]

The most natural ordering is \( \text{STr} \{ \ldots \} \) (see [4]). This is a symmetric trace, i.e. it is symmetrical in \( \Phi^i, D_a \Phi^i \) and \( F_{ab} \). With this prescription pull-back operation remains homomorphism

\[ \text{STr} \left\{ P[A] \wedge \text{brane} P[B] \right\} = \text{STr} \left\{ P[A \wedge B] \right\}. \]  

Thus, the most natural generalization of the action (8) is

\[ S = \int_{W_{p+1}} \text{STr} \left\{ P[C] e^{P[B]+F} \right\}. \]  

However, it is impossible to recover the gauge invariance for this action (see the Appendix). Hence, this action is not correct. Using T-duality, R.C.Myers proposed another action for the stack of branes [5]

\[ S = \int_{W_{p+1}} \text{STr} \left\{ P[e^{i \Phi^i \Phi} C e^{-B-F}] \right\}. \]  

Our aim here is to find the generalization of transformation (11). The generalization of the \( U(1) \) transformations (1) is the infinitesimal \( U(N) \) transformations

\[ A \rightarrow A + P[d\epsilon] + [A, \epsilon], \]

\[ \Phi \rightarrow \Phi + [\Phi, \epsilon], \]  

since the finite \( U(N) \) transformations are

\[ A \rightarrow G \partial G^{-1} + GAG^{-1}, \]

\[ \Phi \rightarrow G \Phi G^{-1}. \]  

One obtains (13) for \( G = e^{-\epsilon} \). Thus, for the exact \( \Lambda = d\epsilon \), one knows the gauge transformation

\[ A_a \rightarrow A_a + \Lambda_a(\Phi) + \Lambda_i(\Phi) \partial_i \Phi^i + \]

\[ + [A_a, \Lambda_i(0) \Phi^i + \frac{1}{2} \partial_i \Lambda_i(0) \Phi^i \Phi^j + O(\Phi^3)], \]

\[ \Phi^k \rightarrow \Phi^k + [\Phi^k, \Lambda_i(0) \Phi^i + \frac{1}{2} \partial_i \Lambda_i(0) \Phi^i \Phi^j] + O(\Phi^4). \]  

Now we determine gauge transformations for the generic \( \Lambda \). We perform our calculations in the case of \( D \)-instantons.

### 4 \( D \)-instantons

The action (14) has the simplest form in the case of \( D(-1) \) branes. This action is

\[ S = \text{STr} \left\{ e^{i \Phi^i \Phi} C e^{-B} \right\}. \]  

Up to fifth power in \( \Phi \), the action is of the form

\[ S = \text{STr} \left\{ (C^{(0)} + i \Phi^i \Phi (C^{(0)} B + C^{(2)}) + \frac{1}{2} (i \Phi^i \Phi)^2 (\frac{1}{2} C^{(0)} B^2 + C^{(2)} B + C^{(4)}) \right\}. \]
We consider infinitesimal gauge transformation as a series in $\Phi$ and, in general, it derivatives
\[
\Phi \rightarrow \Phi + \delta(2)\Phi + \delta(3)\Phi + \ldots
\] (20)
The gauge invariance imposes equations for $\delta(3)\Phi$, simplest of them are the follows
\[
\begin{align*}
\text{STr} \left\{ \delta_k C^{(0)}(\Phi) \delta(\Phi) - C^{(0)}(\Phi) \left[ \Phi^i, \Phi^j \right] \delta_j \Lambda_i(0) \right\} &= 0, \\
\text{STr} \left\{ \frac{1}{2} \left[ \Phi^i, \Phi^j \right] \delta_k \left( C^{(0)}(\Phi) B_{ji}(\Phi) \right) \right\} &+ \left[ \Phi^i, \delta(\Phi) \right] C^{(0)}(\Phi) B_{ji}(\Phi) \delta_j(\Phi) \\
&- \frac{1}{3} \left[ \Phi^i, \Phi^j \right] \left[ \Phi^m, \Phi^n \right] C^{(0)}(\Phi) \left( B(\Phi) \wedge d\Lambda(0) \right)_{nmji} = 0, \\
\text{STr} \left\{ \frac{1}{2} \left[ \Phi^i, \Phi^j \right] \delta_k C^{(2)}_{ji}(\Phi) \delta(\Phi) \right\} &+ \left[ \Phi^i, \delta(\Phi) \right] C^{(2)}_{ji}(\Phi) \delta_j(\Phi) \\
&- \frac{1}{8} \left[ \Phi^i, \Phi^j \right] \left[ \Phi^m, \Phi^n \right] \left( C^{(2)}(\Phi) \wedge d\Lambda(0) \right)_{nmji} = 0.
\end{align*}
\] (21)
This gives a system determining $\delta(\Phi)$
\[
\begin{align*}
\partial_k C^{(0)}(0) & \text{STr} \left\{ \delta(\Phi) \Phi^k \right\} - \partial_k C^{(0)}(0) \partial_j \Lambda_i(0) \text{STr} \left\{ \left[ \Phi^i, \Phi^j \right] \Phi^k \right\} = 0, \\
\partial_k \partial_m C^{(0)}(0) & \text{STr} \left\{ \delta(\Phi) \Phi^{km} \right\} - \frac{\partial_k \partial_m C^{(0)}(0)}{2!} \partial_j \Lambda_i(0) \text{STr} \left\{ \left[ \Phi^i, \Phi^j \right] \Phi^{km} \right\} = 0, \\
\partial_k \partial_m \partial_n C^{(0)}(0) & \text{STr} \left\{ \delta(\Phi) \Phi^{kmn} \right\} - \frac{\partial_k \partial_m \partial_n C^{(0)}(0)}{3!} \partial_j \Lambda_i(0) \text{STr} \left\{ \left[ \Phi^i, \Phi^j \right] \Phi^{kmn} \right\} = 0, \\
\text{STr} \left\{ \frac{1}{2} \left[ \Phi^i, \Phi^j \right] \delta_k \left( C^{(0)}(0) B_{ji}(0) \right) \delta(\Phi) \right\} &+ \left[ \Phi^i, \delta(\Phi) \right] \partial_k \left( C^{(0)}(0) B_{ji}(0) \right) \delta_j(\Phi) \\
&- \frac{1}{8} \left[ \Phi^i, \Phi^j \right] \left[ \Phi^m, \Phi^n \right] \left( \partial_k \left( C^{(0)}(0) B(0) \right) \wedge d\Lambda(0) \right)_{nmji} = 0, \\
\text{STr} \left\{ \frac{1}{2} \left[ \Phi^i, \Phi^j \right] \delta_k C^{(2)}_{ji}(0) \delta(\Phi) \right\} &+ \left[ \Phi^i, \delta(\Phi) \right] \partial_k C^{(2)}_{ji}(0) \delta_j(\Phi) \\
&- \frac{1}{8} \left[ \Phi^i, \Phi^j \right] \left[ \Phi^m, \Phi^n \right] \left( \partial_k C^{(2)}(0) \wedge d\Lambda(0) \right)_{nmji} = 0.
\end{align*}
\] (22)
The most general form of the $\delta(\Phi)$ is
\[
\delta(\Phi) = (\alpha_1 \left[ \Phi^i, \Phi^k \right], \left[ \Phi^i, \Phi^k \right], \Phi^k + \alpha_2 \left[ \Phi^i, \Phi^k \right], \Phi^k + \alpha_3 \left[ \Phi^k, \Phi^i \right], \Phi^k + \\
+ \beta_1 \left[ \Phi^i, \Phi^k \right], \Phi^k \right] + \beta_2 \left[ \left[ \Phi^i, \Phi^k \right], \Phi^k \right] + \beta_3 \left[ \left[ \Phi^k, \Phi^i \right], \Phi^k \right] + \\
+ \gamma_1 \left[ \left[ \Phi^i, \Phi^k \right], \Phi^k \right] + \gamma_2 \left[ \left[ \Phi^i, \Phi^k \right], \Phi^k \right] + \gamma_3 \left[ \left[ \Phi^k, \Phi^i \right], \Phi^k \right] \right) \delta_j \Lambda_i(0),
\] (23)
where $\left[ \Phi^i, \Phi^k \right]$ is anticommutator
\[
\left[ \Phi^i, \Phi^k \right] \equiv \Phi^i \Phi^k - \Phi^k \Phi^i.
\] (24)
We can put some $\beta$'s to zero, using the Jacobi identity
\[
\left[ \left[ \Phi^i, \Phi^k \right], \Phi^j \right] + \left[ \left[ \Phi^i, \Phi^k \right], \Phi^j \right] + \left[ \left[ \Phi^i, \Phi^k \right], \Phi^j \right] \equiv 0,
\] (25)
and put some $\gamma$'s to zero, using the identity
\[
\left[ \left[ \Phi^i, \Phi^k \right], \Phi^j \right] + \left[ \left[ \Phi^i, \Phi^k \right], \Phi^j \right] + \left[ \left[ \Phi^k, \Phi^i \right], \Phi^j \right] \equiv 0.
\] (26)
Substituting (23) into (22), one obtains the restrictions on coefficients $\alpha$, $\beta$ and $\gamma$
\[
\begin{align*}
\begin{cases}
\alpha_1 + \alpha_3 = 1 \\
\beta_1 + \beta_3 = \alpha_2 = \gamma_1 = \gamma_3 = 0
\end{cases}
\end{align*}
\] (27)
This system with the requirement that the transformation is the generalization of the transformation (17) for $d\Lambda \neq 0$ gives
\[
\begin{align*}
\begin{cases}
\gamma_1 = \gamma_3 = \alpha_2 = \beta_1 = \beta_3 = 0 \\
\alpha_1 = \alpha_3 = \frac{1}{2} \\
\gamma_2 = -\frac{1}{4}
\end{cases}
\end{align*}
\] (28)
So we determined all but one coefficients. Possibly, $\beta_2$ can be determined from the higher power expansion of the action. Now we test this transformation with the $D2$-branes action.

5 $D2$-branes

We test the gauge invariance of the action for the $D2$-branes up to third power in $\Phi$. Consider the field configuration with $B = 0$, $A = 0$, $C^{(1)} = 0$ and $C^{(5)} = 0$. The action is

$$S = \int W_3 \text{STr} \left\{ P \left[ C^{(3)} + i\Phi i\Phi (C^{(3)} \wedge (B + F)) \right] \right\}. \quad (29)$$

The variation of the action due to the "dielectric" effect is as follows

$$\delta S = \delta \int W_3 \text{STr} \left\{ P \left[ i\Phi i\Phi (C^{(3)} \wedge (B + F)) \right] \right\} =$$

$$= \int W_3 \text{STr} \left\{ i\Phi i\Phi \left( C \wedge \delta (B + F) \right) \right\} + \frac{1}{2} \left[ \Phi_i, \Phi^j \right] \partial_{ij} \Lambda^k \partial_k \Phi^k + \frac{1}{2} \left[ \Phi_i, \Phi^k \right] C_{jbc} \partial_{i} \Lambda_{ab} \partial_{a} \Phi^k +$$

$$+ \left[ \Phi^i, \Phi^j \right] C_{jbc} \partial_{ia} \partial_{a} \Phi^k + \left[ \Phi^i, \Phi^k \right] C_{jbc} \partial_{ia} \Lambda^k \partial_{a} \Phi^i \right\} e^{abc} d^3 V. \quad (30)$$

The gauge invariance of the actions puts the condition (see the Appendix)

$$\delta S = \delta A S + \delta_\Phi S + \delta S S = 0, \quad (31)$$

which gives the system

$$\left\{ \begin{array}{l}
\text{STr} \left\{ \partial_k C_{abc}(0) \delta (3) \Phi^k + \frac{i}{2} \left[ \Phi_i, \Phi^j \right] \partial_k C_{abc}(0) \left( \partial_i \Lambda_j(0) - \partial_j \Lambda_i(0) \right) \Phi^k \right\} e^{abc} = 0, \\
\text{STr} \left\{ C_{jbc}(0) \partial_\delta (3) \Phi^j + \left[ \Phi_i, \Phi^j \right] \left( C_{jbc}(0) \Phi^k \partial_k \partial_a \Lambda_i(0) \right) \right\} + \\
\left[ \Phi^i, \Phi^j \right] C_{jbc}(0) \partial_i \Phi^j \partial_k \Lambda^k(0) + \\
\left[ \Phi^i, \Phi^j \right] C_{jbc}(0) \left( \partial_i \Lambda_k(0) - \partial_k \Lambda_i(0) \right) \right\} e^{abc} = 0, \\
\text{STr} \left\{ \partial_\mu C_{abc}(0) \Phi^k \delta_{(2)} \Phi^m \right\} e^{abc} = 0, \\
\text{STr} \left\{ \partial_\delta C_{jbc}(0) \left( \delta_{(2)} \Phi^k \partial_a \Phi^j + \Phi^k \partial_a \delta_{(2)} \Phi^j \right) + \\
\partial_\delta C_{jbc}(0) \left[ \Lambda_i(0) \partial_a \Phi^j + \partial_i \Lambda_a(0) \Phi^j, \Phi^j \right] \Phi^k + \\
\left[ \Phi^i, \Phi^j \right] \Phi^k \partial_\delta C_{jbc}(0) \left( \partial_a \Lambda_i(0) - \partial_i \Lambda_a(0) \right) \right\} e^{abc} = 0, \\
\text{STr} \left\{ C_{jbc}(0) \partial_\delta \partial_{(2)} \Phi^k \partial_a \Phi^j + C_{jbc}(0) \partial_\delta \Phi^k \left[ \Lambda_i(0) \partial_a \Phi^j + \partial_i \Lambda_a(0) \Phi^j, \Phi^j \right] \Phi^k + \\
\left[ \Phi^i, \Phi^j \right] C_{jbc} \left( \partial_\delta \Lambda_k(0) - \partial_k \Lambda_i(0) \right) \partial_a \Phi^k + \\
\frac{1}{2} \left[ \Phi^i, \Phi^j \right] C_{jbc}(0) \left( \partial_\delta \Lambda_k(0) - \partial_k \Lambda_i(0) \right) \partial_a \Phi^k \right\} e^{abc} = 0.
\end{array} \right. \quad (32)$$

It is easy to check that the gauge transformations found in section 4 solve these equations.
6 Conclusions

We obtained the manifest expression for the gauge transformation for arbitrary gauge parameter up to the third power in $\Phi$. The fact that for the Myers-Chern-Simons action it is possible to restore gauge invariance, supports its form. However, our consideration does not allow one to restore it unambiguously. It would be interesting to test the same gauge invariance with the nonabelian DBI action.

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8 Appendix

It is easy to see that in contrast to the Myers action (14), it is impossible to construct the gauge transformations, generalizing (2), for the action (13). We shall examine the case of 2-branes in the case $B = 0$, $C^{(1)} = 0$ and $A = 0$ up to the third power in $\Phi$. In this case

$$S = \int W_3 \text{STr} \left\{ P [C^{(3)}(\Phi)] \right\},$$

(33)

and

$$\delta S = \delta A S + \delta \Phi S$$

(34)
does not vanish. Implicitly we have

$$\delta_A S = 3 \int W_3 \text{STr} \left\{ [\Lambda_i \partial_i \Phi^i + \partial_i \Lambda_i \Phi^i, \Phi^i] \left( 2 C^{(3)}_{jbc}(0) \partial_i \Phi^k + \partial_i C^{(3)}_{jbc}(0) \Phi^k \right) \right\} \epsilon^{abc} d^3 V,$$

(35)

and

$$\delta_\Phi S = \int W_3 \text{STr} \left\{ \partial_k C_{abc}(0) \delta(3) \Phi^k + \partial_k \partial_m C_{abc}(0) \Phi^k \delta(2) \Phi^m + 3 C_{jbc}(0) \partial_a \delta(3) \Phi^j + + 3 \partial_k C_{jbc}(0) \left( \delta(2) \Phi^k \partial_a \Phi^j + \Phi^k \delta(2) \Phi^j \partial_a \Phi^k \right) + 6 C_{jbc}(0) \partial_a \delta(2) \Phi^i \partial_b \Phi^j \right\} \epsilon^{abc} d^3 V.$$

(36)

If the action (33) was correct, we could chose $\delta(2) \Phi$ and $\delta(3) \Phi$ to cancel the full $\delta S$

$$\delta S = \delta A S + \delta \Phi S = 0$$

(37)

However, we want to show, there is no proper $\delta(2) \Phi$, and the action (33) is not exact. As $C(0)$ and $\partial C(0)$ are independent coefficients, we have the sets of equations for determination of $\delta(3) \Phi$ and $\delta(2) \Phi$

$$\begin{align*}
\text{STr} \left\{ \partial_k C_{abc}(0) \delta(3) \Phi^k \right\} &\epsilon^{abc} = 0, \\
\text{STr} \left\{ C_{jbc}(0) \partial_k \delta(3) \Phi^j \right\} &\epsilon^{abc} = 0.
\end{align*}$$

(38)
In a similar manner since \( \Phi \)'s are arbitrary, the trace vanishes only if
\[
\partial_i L \Phi - \Lambda_i(0) \partial_i \Phi^i - \partial_i \Lambda_n(0) \Phi^i = 0.
\]
Equation (***) of the system (39) gives the same condition. As $\Phi$'s and their derivatives are independent matrices, one has conditions

$$\begin{align*}
\left\{ \begin{array}{l}
\partial_a L_i - \partial_i \Lambda_a(0) = 0, \\
(\partial_a L_i^c) \partial_c \Phi^i + L_i \partial_a \Phi^i - \Lambda_i(0) \partial_a \Phi^i = 0.
\end{array} \right. 
\end{align*}$$

(50)

One observes, that

$$\partial_a L_i^c = \delta_a^c N_i,$$

(51)

for some $N_i$. The only possibility is $N_i = \text{const}$, but since we explore the part, proportional to $\Lambda$, we have $N_i \equiv 0$. Now the system (50) gives

$$\partial_a \Lambda^i(0) - \partial_i \Lambda_a(0) = 0$$

(52)

Thus, if $\Lambda$ does not satisfy this restriction, one can not restore the gauge invariance without changing the action (12).

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