Motion analysis of kinetic impact projectiles for physics education in real context

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Received 15 April 2020, revised 21 July 2020
Accepted for publication 10 August 2020
Published 13 November 2020

Abstract
The article presents a proposal to contextualize the study of movement in introductory courses of university physics, as a contribution to decision making in social situations. For this, the use of kinetic impact projectiles and the actual data provided by official sources are considered. This information is used in an object motion model describing the kinematic characteristics of a spherical projectile (a rubber bullet). For these purposes, a Reynolds number Re ≫ 1 was used, which allows one to apply a nonlinear motion equation to find the velocity and impact energy per unit area of a projectile. The results and analysis of this model can generate an interesting discussion in the classroom about the need to build protocols for the use of kinetic impact projectiles, and the importance of using scientific knowledge in social conflicts.

Keywords: physics education, normalized energy, ocular damage, social situations, classical mechanics

(Some figures may appear in colour only in the online journal)

1. Introduction

Usually, introductory physics courses teach little contextualized content to everyday life and are disconnected from other disciplines. However, physics contributes significantly to understanding phenomena and situations related to, for example, the human body and its care [1].

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Contexts close to the student can be a motivation for learning as well as for developing the skills to solve their own quantitative discipline problems. Although conceptual understanding and problem solving in physics are fundamental, current scientific education promotes the development of skills that allow one to address interdisciplinary issues, which require the search for and evaluation of evidence to make sense of the information students receive from various sources [2]. Precisely, the information that has circulated internationally in recent months has shown Chile as the scene of a civil revolution. In this context, kinetic impact projectiles, commonly known as rubber bullets, have been used to dissolve protests, as in several countries [3].

As a result of the use of projectiles, some people are injured in different areas of the body with varying degrees of vulnerability. Some of these injuries have resulted from the use of kinetic impact projectiles that have impacted the ocular globe causing partial or total loss of vision. Moreover, the impact can produce extensive corneoscleral lacerations with either prolapse or loss of the intraocular contents, thus requiring early excision of the eye. In other circumstances, when the projectile has low energy, the impact can cause severe trauma injuries; however, there is a greater possibility that the eye can be saved [4].

According to various reports, the kinetic energy limit of the riot ammunition must be 122 J [5, 6]. Researchers have reported that impact energies below 20.3 J are low risk, as long as the projectile is large enough so that it does not perforate the eye. Between 40.7 J and 122 J is considered an energy range of dangerous impact, and for impacts above 122 J there would be a region of severe damage. However, other factors influence the potential damage from a shot to the human body, such as the separation range between the weapon and the subject, size, structure of the projectile, and shutter speed. Therefore, it is challenging to determine unique values for the regulation of use [7].

In Chile, a test performed in 2012 [8] was conducted on cartridges with 12 rubber bullets of 8 mm in diameter at distances between 5 and 30 m. It was concluded that there is a clear possibility of serious injury to the human body between 5 and 25 m away, including a burst eye. Over 30 m, slight injuries would be sustained but could still involve loss of the eye. Moreover, the experts warned that a larger dispersal distance of the pellets could affect more than one person. In the 30 m shot, only two of the 12 balls hit the target. Furthermore, according to data published by the institution responsible for the study, the report stated that the speed of the projectile was 380 m s$^{-1}$, which corresponds to supersonic speed in the conditions studied. Also, the weight of a pellet was 0.64 g. In turn, the supplier indicated, via a statement, that the shutter speed was 320 m s$^{-1}$. Thus, for the present study, both speed values are considered as a reference.

Based on the background, this article proposes an analysis from a physics perspective of the kinetic impact projectile in the human eye, contextualized with real data from the Chilean case. The aim is to contribute to citizen education in introductory courses in university physics, valuing the contribution that the discipline can make to social impact discussions. With the results of the analysis, the use of basic notions of classical physics can be provided to present evidence that favors decision making about healthcare and citizenship integrity.

The manuscript is organized as follows. Section 2 presents eye injury data upon which this paper is based to evaluate the risk of eye damage from the impact of kinetic impact projectiles. Section 3 discusses the projectile motion for describing the dynamics of a particle in a stochastic viscous fluid. Next, section 4 presents the numerical and analytical trajectories for projectiles fired directly at a target; the normalized energy is evaluated to estimate distances of eye injuries. Section 5 analyzes bounced projectile trajectories and their normalized energy by considering small angles and different coefficients of restitution. Finally, the scope of the results is discussed, and suggestions for their implementation in the university classroom.
2. Ocular damage from the impact of pellets

Several studies have investigated eye damage from impact with rubber bullets. In some cases, experiments have been conducted with animals to define an eye injury criterion; monkeys in Wiedenthal [9], pigs in Wyschpen [10] and Delori [11], and with human cadavers in Duma [12].

In Lavy et al [13], the authors state that when the eye is shot by a rubber bullet, it is very likely to lead to permanent injury due to loss of the eye. The authors analyzed the case of 42 patients injured in the eyes due to the use of rubber shells, of which 54% had cutaneous lacerations, 40% had hyphema, 38% had a fractured ocular globe, 33% had an orbital fracture (bones surrounding the eye), 26% had damage to the retina and, in 21%, the projectile remained inside the eye.

Further investigations have reported the possibility of an eye injury in terms of the distance of the shot [14]. From a distance of 20 m, there is a 35% chance of hurting someone in the body and a 2% chance of hitting the eyes; at 10 m away the probability is 50%, with a 4% probability of hitting the eyes; and 5 m away the probability is 80%, with a 9% probability of hitting the eyes.

Moreover, a study of pigs’ eyes allowed the evaluation of eye hazards by measuring intraocular pressure during the impact of high-velocity projectiles [15]. Although their tests were conducted at low speeds (range between 6.2 m s⁻¹, and 66.5 m s⁻¹), compared to those of a riot gun, they managed to establish risk probabilities, including globe rupture, depending on the diameter of the projectile. The authors investigated the correlation between intraocular pressure and normalized energy, defined as the kinetic energy divided by the cross-sectional area of the projectile. The results showed that the smaller the diameter of the projectile, the higher the probability of generating severe eye damage. It further states that a 50% risk of globe rupture occurs just over 36 000 J m⁻², 50% risk of retinal damage over 17 979 J m⁻², 50% of lens damage over 17 300 J m⁻², and a 50% risk of hyphema over 11 700 J m⁻². With these reference values, a comparison with the results obtained by analyzing the problem of impact on known real conditions can be promoted. Thus, students may offer recommendations for the use of such munitions, ensuring there is minimal risk for potential ocular damage.

3. Projectile motion model

When particles are transported in a fluid they experience forces and momentum in three directions. In many applications, the most important force acting on a particle is the one exerted in the opposite direction to its movement; this is called drag force \( \vec{F}_D \) and depends on the speed of the particle and its interaction with the environment. To determine the drag coefficient \( C_D \), it is important to consider the Reynolds number [16]. For spherical particles, the Reynolds number is defined as

\[
Re = \frac{DV}{\nu}
\]

where \( \nu = \rho / \mu \), \( \rho \) is the density of the medium, \( \mu \) accounts for the viscosity of the fluid, \( V \) is the terminal velocity of the body in the fluid, and \( D \) represents the characteristic length scale of the object in the cross-sectional plane [17].

Furthermore, one of the most precise relationships to predict the drag coefficient of spherical particles in Re-subcritical is the Clift and Gauvin model [18]:

\[
C_D (Re) = \frac{24}{Re} \left( 1 + 0.15 Re^{0.687} \right) + \frac{0.42}{1 + \frac{2350}{Re^{0.7}}}
\]

\[ (2) \]
Figure 1. The region $Re < 0.1$ is the Stokes regime, the region $0.1 < Re < 10^3$ is the intermediate regime, and for $10^3 < Re < 3 \times 10^5$ is the Newton regime.

Figure 1 shows the drag coefficient as a function of the Reynolds number for spherical particles. When $Re \ll 1$ it is called the Stokes regime (or progressive flow) [19]. In this regime, the terms of inertia of the flow are negligible with regard to viscous flow terms and remain attached to the sphere. Equation (2) approaches $C_D = 24/Re$. In the range $0.1 < Re < 10^3$ there is intermediate speed, and the drag coefficient of the area continues to decrease as the number increases the Reynolds number, although the rate of decrease is lower than that of the Stokes regime. Finally, the drag coefficient becomes almost constant in the Newton regime, $10^3 < Re < 3 \times 10^5$, with a minimum of 0.38 in $5 \times 10^3$ and a maximum of 0.50 in $7 \times 10^4$. The value $Re > 3 \times 10^5$ corresponds to a critical transition and a supercritical regime, where the boundary layer and the wake behind the sphere are turbulent. The value $Re = 3 \times 10^5$ is called the critical Reynolds number, in which the drag crisis occurs and noticeably reduces the drag coefficient [16, 20].

The projectile motion model describes the kinematic characteristics of a spherical bullet submerged in air. As a first approach, we work within the Newton regime since the projectile propagates in the air at high speed and the ratio of the density and viscosity of the air, $\nu = \rho/\mu$, is of the order $10^4$. Thus, the drag force exerted by the air on the projectile is proportional to the square of the velocity in the form $\vec{F}_d \propto -|\vec{v}| \vec{v}$. Therefore, the dynamic model is given by

$$\frac{d}{dt} \vec{v} = -\frac{C_D}{2} \rho A |\vec{v}| \vec{v} + m \vec{g}$$

(3)

where $\vec{v} = v_x(t) \hat{i} + v_y(t) \hat{j}$ denotes the velocity of the spherical projectile, $m$ and $A = (\pi/4)D^2$ are the mass and cross-sectional area of the sphere, respectively, $C_D$ accounts for the drag coefficient, and $\vec{g} = -g \hat{j}$ is the gravity acceleration where $|\vec{g}| = g = 9.81$ m s$^{-2}$. Thus, the equations of motion for the velocity components of a rubber bullet are given by

$$\frac{d}{dt} v_x = -\alpha \sqrt{v_x^2 + v_y^2} v_x,$$

(4a)
\[
\frac{dv_x}{dt} = -\alpha \sqrt{v_x^2 + v_y^2} v_x - g
\]

(4b)

here \( \alpha = (C_D/2m)\rho A \), the drag coefficient is \( C_D = 0.5 \) [21] (for bodies of a spherical shape, see figure 1), and the initial conditions for the velocity components are \( v_{x0} = v_0 \cos \theta_0 \) and \( v_{y0} = v_0 \sin \theta_0 \). The terminal velocity is calculated by eliminating the temporal derivatives in equation (4), i.e. \( dv_x/dt = 0 \) and \( dv_y/dt = 0 \), which leads to

\[
V = \sqrt{\frac{2mg}{C_D\mu A}}
\]

(5)

By replacing the density value \( \rho = 1.22 \text{ kg m}^{-3} \) and viscosity \( \mu = 1.50 \times 10^{-5} \text{ m}^2 \text{ s}^{-1} \) of air, and the mass \( m = 0.64 \text{ g} \) and diameter \( D = 8 \times 10^{-3} \text{ m} \) of a projectile, which is considered as a sphere, it is found that the terminal velocity is \( V = 20.24 \text{ m s}^{-1} \). The density and the viscosity of the air were considered under conditions of 20 °C and 1 atm of pressure. Then, the corresponding Reynolds number (1) is equal to \( Re = 1.3 \times 10^4 \), which confirms the assumption of this model (3).

It should be noted that there is a set of forces that were not considered within the framework of this quadratic model [22], because their influence is not significant and we sought to simplify the analysis for initial physics courses. On the one hand, the presence of wind and the buoyancy effects are at least an order of magnitude smaller than the weight of the sphere. Furthermore, the possible rotation of the rubber bullet and the Magnus effect on it were not considered. Finally, D’Alembert forces [23, 24] are not included in this analysis, since it was considered an inertial frame of reference. Analysis and considerations to address in the classroom are reviewed below.

Equations (4a) and (4b) allow one to find, by numerical integration regarding time, the velocities \( v_x \) and \( v_y \), and further, the position of the projectile on the \( x \)-axis and the \( y \)-axis. Students can use this model to analyze different cases at straight and low angle shots, putting forward approaches to evaluate relevant physical parameters of a rubber bullet impact generating body injuries. In this case, the analyses are focused on ocular trauma in conformity with the arguments of section 2.

Finally, figure 2 shows the comparison between an ideal trajectory and the quadratic model (3) for \( v_0 = 5 \text{ m s}^{-1} \). Figure 2(a) shows that both trajectories are comparable; however, if the initial velocity is \( v_0 = 320 \text{ m s}^{-1} \), figure 2(b) shows that the influence of the drag force on the sphere must be considered.

4. Analysis of ‘direct’ projectile trajectories

In a first analysis, the calculations used an initial angle of injection of \( \theta_0 = 0^\circ \), and initial velocities of \( v_0 = 320 \text{ m s}^{-1} \) and \( v_0 = 380 \text{ m s}^{-1} \) corresponding to the supplier and the police reported data, respectively. Moreover, it is assumed that the projectile is fired horizontally at an initial height of 1.7 m measured from the floor, disregarding the dispersion due to the transmitted momentum of the other pellets. Other studies [25–27] detail essential considerations to understand various dispersion models of cartridge bullets fired to multiple distances. It should also be noted, regarding the time of the phenomena description, that all the analyses concern the time taken from the exit of the shotgun riot to the target, leaving aside the physical aspects when the bullet is inside the barrel. For this we implemented the fourth order Runge–Kutta algorithm with a temporary step of \( \Delta t = 0.01 \text{ s} \). All numerical and analytical simulations were performed using MATLAB 2020a software.
Figure 2. A comparison between the ideal trajectory and the Newton model for an initial velocity of (a) \( v_0 = 5 \text{ m s}^{-1} \) and \( \theta = 0^\circ \), (b) \( v_0 = 320 \text{ m s}^{-1} \) and \( \theta = 0^\circ \).

Figure 3. The projectile trajectory graph, \( y = f(x) \). Initial velocities of \( v_0 = 320 \text{ m s}^{-1} \) and \( v_0 = 380 \text{ m s}^{-1} \) were used, with a firing angle of \( \theta = 0^\circ \).

Figure 3 shows the trajectory of the rubber bullet; the curve of the height versus the \( x \)-position shows how the bullet begins to descend, losing altitude. As can be seen, for the initial speeds of 320 m s\(^{-1}\) and 380 m s\(^{-1}\), the projectile has descended only 16 cm and 11 cm, respectively, when it travels 40 m. This implies that in a perfect shot, a person standing may suffer serious eye injury if they shoot straight at the face at close distances up to 40 m, which is much more than indicated by the official documents mentioned above. At this point, the model allows one to establish correlations between the data reported in the literature for eye damage in terms of speed and energy per unit area of impact at different shooting distances. This information would be valuable to establish protocols for the use of riot shotguns in public prevention, among others.

4.1. An analytical solution: small angles

At this point, it is necessary that the student develops skills needed to operate any programming language that supports mathematical calculations to solve equations such as (4). The teacher must encourage their students to do this on a cross-cutting basis. On the other hand, the teacher must instruct students on how to work out the movement problem analytically; thus, the student can evaluate the scope of the proposed model and their considerations at the time of its description. Accordingly, by addressing the launch of the projectile considering small \( \theta \) angles, it is correct to assume that the horizontal velocity is, on average, much higher than the vertical
velocity, that is, $v_x(t) \gg v_y(t)$. Therefore in this approximation, the equations of motion (4) are given by

$$\frac{dv_x}{dt} = -\alpha v_x^2, \quad (6a)$$

$$\frac{dv_y}{dt} = -\alpha v_x v_y - g. \quad (6b)$$

Thus, considering the initial conditions $x(0) = x_0, y(0) = y_0$, and by directly integrating (6), the closed-form solutions for the velocity can be written as

$$v_x(t) = \frac{1}{(\alpha t + 1/v_x0)}, \quad (7a)$$

$$v_y(t) = \frac{\tan(\theta_0) - ((\alpha/2)t + 1/v_x0)gt}{\alpha t + 1/v_x0}, \quad (7b)$$

and for the position as

$$x(t) = x_0 + \frac{\ln(\alpha v_x0 t + 1)}{\alpha}, \quad (8a)$$

$$y(t) = y_0 - \left(\frac{2}{\alpha v_x0} + t\right)\frac{gt}{4} + \left(\frac{g}{2\alpha v_x0} + v_y0\right)\frac{\ln(\alpha v_x0 t + 1)}{\alpha v_x0}. \quad (8b)$$

These analytical solutions for low angles will allow students to analyze multiple projectile launches to evaluate potential body damage when an impact occurs. Figure 4 shows the similarities between analytical and numerical solutions under the approximation of small angles. Thus, both solutions allow students to describe the movement of the projectile correctly while analyzing the physical parameters associated with eye injuries. Consistently, the analysis then developed is valid for both cases.
4.2. Normalized energy for a direct shot

Similar behavior will have kinetic energy that depends on the projectile mass and the square of
the velocity, \( E_c = \frac{mv^2}{2} \). A rubber bullet of mass 0.64 g fired at an initial speed of 320 m s\(^{-1}\)
will have a kinetic energy of 32.8 J; in the same projectile with an initial speed of 380 m s\(^{-1}\),
the kinetic energy will be 46.2 J. In both cases, the single pellet energy is below the energy
limit of 122 J. However, since the cartridge contains 12 bullets with similar energy, assuming
the same structural and physical conditions, the riot gunshot weapons exceed the limits and are
lethal at short distances.

Reviewing the firing length suggested, which is 30 m, the bullets will have decreased their
speeds to approximately 154 m s\(^{-1}\) and 180 m s\(^{-1}\) when they have been fired at initial speeds of
320 m s\(^{-1}\) and 380 m s\(^{-1}\), respectively. The projectiles have reduced their speed by more than
half for both cases; they will have a kinetic energy of 7.5 J, and 10.4 J, respectively. Considering
the 12 bullets of a cartridge, the kinetic energy will be 90 J, and 124.8 J, the magnitude of which,
that although their current assessment, is very close to the internationally recommended value
of 122 J. This fact would suggest that 30 m is a safe distance that significantly reduces the
probability of an impact causing personal injuries. However, before taking this statement for
granted, some considerations have to be made. First, the value of 122 J is a reference, not an
exact limit that discriminates between severe or mild damage. Besides, essential characteristics
such as the caliber of the ammunition, mass, bullet dimensions, or materials of the pellets used
to define this value are different from those studied in this case. For these reasons, defining a
degree of injury only by the kinetic energy of the shot would be unsuitable.

In contrast, if the kinetic energy is expressed in terms of the cross-section of the projec-
tile, the students will be capable of performing correlations between possible human body
injuries and normalized energy impacts. This comparison is significant, since a projectile with
kinetic energy acting over a smaller impact section will have higher bone and tissue drilling
capacity. In the present case, for rubber bullets, the projected area will be a circle of radius
8 mm. Thus, the energy is normalized by their cross-sectional area and then compared with
reported values in the literature obtained from experimental investigations of ocular trauma
produced by similar kinetic projectiles. Figure 5 shows the behavior of kinetic energy per unit
area through distance, and the box shows a zoom of the earliest values. Realize that up to 30 m
a person impacted in the face will absorb kinetic energy of approximately 165 000 J m\(^{-2}\) and
210 000 J m\(^{-2}\) when the projectile has been fired at an initial speed of 320 m s\(^{-1}\) and
380 m s\(^{-1}\), respectively. On the one hand, the information provided in 2012 states that to 30 m
the probability of globe rupture exists. According to the literature [15], a 50% risk of
globe rupture occurs just over 36 000 J m\(^{-2}\). Curves show a projectile reaching this value
from distances near 60.4 m and 67.6 m away when projectiles are fired at 320 m s\(^{-1}\) and
380 m s\(^{-1}\), respectively. As previously discussed, the energy per unit area of other eye injuries is
17 979 J m\(^{-2}\) for 50% risk of retinal damage, over 17 300 J m\(^{-2}\) for 50% risk of lens damage,
and over 11 700 J m\(^{-2}\) for 50% risk of hyphema. However, these energy values are reached at
over 75 m and 85 m, respectively; almost 2.5 and 3 times over the recommended fired distance
to avoid injuries. This evidence is necessary to discuss recommendations for use.

5. Analysis of bounced projectile trajectories

On the other hand, it is also possible to supplement the analysis by simulating what happens
with the impact energy if the projectile bounces off the ground before reaching its target. Based
on the above model, the trajectory of a projectile bouncing in an inelastic collision after being
fired with an initial angle of \( \theta = 2^\circ \) below the horizontal to 320 m s\(^{-1}\) has been estimated. The
coefficient of restitution of rubber is around 0.8. It is recognized that this parameter depends on properties such as the stiffness, toughness, strength, and hardness of the two bodies involved in the collision. Therefore, it is worth mentioning that in the introductory physics courses, the microscopic composition of the ground and projectile to describe the kinetic energy after a bounce is neglected. However, further courses, such as the science of materials, will allow the student to address a rigorous description of it. Thus for the estimate, the model uses the values of 0.4 to 0.9 for this parameter. These cases can describe a deterrence situation where rubber bullets go straight to the ground.

To analyze the bounce trajectory, initially, the model describes the motion until the projectile impacts with the ground. Then, the velocities and energies of the bullet before and after the collision are calculated using the restitution coefficient. Finally, the bounce angle, obtained from the inverse tangent of the velocity component’s relationship, and the velocity after ground impact will be the new initial conditions for application of the model one more time. Figure 6 shows that the maximum extent in the horizontal and vertical directions decreases with smaller coefficients of restitution. Likewise, the normalized energy has a similar curve tendency but different values in each bounce case, as is shown in figure 7.

5.1. Normalized energy for an indirect shot

It can be realized from the energy curve that even ricochets have enough energy to cause eyeball injuries, even ocular rupture. However, these values must be correlated with the trajectory to determine whether these projectiles hit the face or other body parts. In the latter case, the normalized energies must be those that cause specific damage to bone, skin, and limbs, among others.

Table 1 shows the normalized energy and vertical height reached by a projectile to distances between 45 m and 80 m for the restitution coefficients 0.4, 0.6, 0.8, and 0.9. After the bounce, the projectile reaches a maximum height of 76.5 cm for a $C_R = 0.9$ with energy of 183 98.7 J m$^{-2}$. If people are 70 m away from the shooter, under these conditions, the projectile would not strike the victim’s face but could hurt a child.
Figure 6. The trajectory of a projectile fired below the horizontal line producing one bounce. The initial velocity of the rubber bullet is $v = 320 \text{ m s}^{-1}$, and the firing angle is $\theta = -2^\circ$.

Figure 7. The normalized energy of a projectile fired below the horizontal line producing one bounce. The initial velocity of the rubber bullet is $v = 320 \text{ m s}^{-1}$, and the firing angle is $\theta = -2^\circ$. The insert shows a detailed view for normalized energy in the order $10^4 \text{ J m}^{-2}$.

It is worth mentioning that there are energy parameters that should impact the eye producing lens and retina injuries, but in most cases without globe rupture. This fact means that the projectile behaves as a blunt object causing a closed globe injury [4]. After the bounce, there is only the likelihood of producing global rupture for a $C_R = 0.8$ and $C_R = 0.9$ at a horizontal distance of 50 m, and 55 m, respectively, when the values of the normalized energy take values higher than 36 000 J m$^{-2}$. For other combinations of position and $C_R$, this will result in blunt blows without globe rupture. The bounce height ranges from 8 cm to 80 cm, depending on the coefficient of restitution and properties of the ground and projectile.

Similarly, table 2 shows the normalized energy and maximum height reached by a projectile fired with $\theta = -2^\circ$ and an initial velocity of 380 m s$^{-1}$. In this case, when the restitution coefficient is 0.9, the projectile reaches a maximum height of 81.8 cm at 75 m from the shot. As expected, when the speed increases, the number of cases in which an open globe eye injury
Table 1. Normalized energy and maximum height after a rebound; \( v = 320 \text{ m s}^{-1} \) and \( \theta = -2^\circ \).

| \( C_R \) | \( 0.4 \) | \( 0.6 \) | \( 0.8 \) | \( 0.9 \) |
|----------|----------|----------|----------|----------|
| \( x \) (m) | \( E_n \) (J m\(^2\)) | \( y \) (cm) | \( E_n \) (J m\(^2\)) | \( y \) (cm) | \( E_n \) (J m\(^2\)) | \( y \) (cm) | \( E_n \) (J m\(^2\)) | \( y \) (cm) |
| 45 | 122.000 | 7.9 | 271.741 | 9.3 | 488.271 | 8.5 | 611.549 | 9.6 |
| 50 | 9612.0 | 20.8 | 214.436 | 28.1 | 384.974 | 29.9 | 489.586 | 30.0 |
| 55 | 7527.7 | 17.3 | 169.273 | 39.7 | 301.096 | 47.5 | 385.921 | 48.6 |
| 60 | — | — | 134.073 | 42.4 | 234.928 | 59.9 | 301.778 | 64.0 |
| 65 | — | — | 104.746 | 33.1 | 188.436 | 65.0 | 235.426 | 73.8 |
| 70 | — | — | 828.266 | 9.1 | 147.519 | 62.4 | 183.987 | 76.5 |
| 75 | — | — | — | — | 116.320 | 49.1 | 147.810 | 71.4 |
| 80 | — | — | — | — | 925.31 | 23.4 | 116.550 | 55.6 |

Table 2. Normalized energy and maximum height after a rebound; \( v = 380 \text{ m s}^{-1} \) and \( \theta = -2^\circ \).

| \( C_R \) | \( 0.4 \) | \( 0.6 \) | \( 0.8 \) | \( 0.9 \) |
|----------|----------|----------|----------|----------|
| \( x \) (m) | \( E_n \) (J m\(^2\)) | \( y \) (cm) | \( E_n \) (J m\(^2\)) | \( y \) (cm) | \( E_n \) (J m\(^2\)) | \( y \) (cm) | \( E_n \) (J m\(^2\)) | \( y \) (cm) |
| 45 | 171.960 | 2.3 | 382.168 | 3.5 | 671.099 | 4.7 | 844.168 | 5.2 |
| 50 | 135.823 | 18.9 | 299.127 | 23.6 | 531.953 | 25.0 | 684.530 | 23.9 |
| 55 | 105.797 | 24.3 | 240.490 | 37.0 | 415.387 | 43.5 | 541.437 | 42.7 |
| 60 | 8486.2 | 15.7 | 187.418 | 45.7 | 333.324 | 56.7 | 421.943 | 59.6 |
| 65 | — | — | 146.788 | 35.9 | 264.993 | 66.2 | 326.531 | 72.9 |
| 70 | — | — | 118.166 | 35.9 | 209.843 | 70.3 | 260.192 | 80.0 |
| 75 | — | — | 9212.9 | 11.1 | 162.212 | 66.6 | 206.479 | 81.8 |
| 80 | — | — | — | — | 129.239 | 54.0 | 163.848 | 76.4 |

occurs is higher. Even more, for larger angles below the horizontal, the bounce distance will be closer to the firing officer; therefore, both bounce heights and impact energy will be more significant, thus increasing the likelihood of an open globe injury. These are excellent exercises for the students to work out these trends for themselves.

At this point, the teacher can ask students some further variants of the problem, such as:

- Investigate biomechanical properties of the skin or other tissue to determine possible body injuries in terms of normalized energy values. Depending on the course or education, studies and analyses can be performed experimentally or by using experimental data from the literature.
- To use the analysis model for other projectiles or materials, implying that after the ricochet, higher lengths are reached, and the probability of eye damage persists.
- Develop experimental activities to simulate the case of projectiles, with large balls thrown at low speeds to avoid damage. Thus, one can experimentally test the relationship between firing angles, and the horizontal and vertical ranges achieved by the projectile.
- To implement active learning with modeling instruction, favorable attitude changes are obtained in the introductory courses at university level [28, 29].
• Improve the model, introducing rotation of the rubber bullet, Magnus effect, or considering D'Alambert forces in a non-inertial system.

Additionally, the proposed numerical model can be implemented by the teacher or student using free mathematical calculation programs such as Geogebra or Scilab that allow 2D and 3D visualizations. In practice, the goal may be for the student to be introduced to more specialized scientific programs used for more complex calculations, such as Mathematica or MATLAB. For the latter, there are free trial and online versions with benefits for students. Another option is to migrate the code from MATLAB to GNU Octave to obtain very similar commands for numerical analysis.

6. Conclusions

An analysis of a real and contextualized case to promote the use of evidence in physics classes was presented. The obtained results using basic concepts of introductory physics represent reliable support for the analysis of eye damage from the correlation between the energy per unit area of impact in the eyeball surroundings of the orbital cavity.

For this purpose, the study first implemented a scientific exploration of the cinematic of a spherical projectile using coupled ordinary differential equations (ODEs) built based on the value of the Reynolds number, \( \text{Re} \gg 1 \). The ODEs were integrated numerically to find the speed and position of the projectile, allowing one to compare the energy per unit of area with reported values for ocular injuries. In addition, since there is a wide range of possible shots, it intends to make an approach based on the description of the movement for small angles, which gives students an essential tool to discern other projectile movements. Also, we can see the ranges of a direct or bounce shot on a surface such as the ground.

Projectile kinetic impact cases with official reported data about the type of weapons used to fire rubber bullets of 8 mm in diameter and 0.64 g were analyzed. The study takes into account air friction, air density, and loss of height due to gravity. The impact energy per unit area is explained in the cases of straight and angle-shots, 0° and –2°, respectively; the latter contemplates a rebound with different restitution indexes. These analyses conclude that the shotguns should not be used to directly shoot at distances of less than 70 m at risk of causing open or closed ocularglobe injuries when a direct or indirect impact by rebound occurs.

The study also shows that it is necessary to use normalized energy data and correlate it with the biophysical characteristics of the physiology of each part of the human body by considering the location of the possible impact. This fact allows one to determine whether a weapon may or may not be appropriate for use in civil protests. Studies should review not only evidence of direct firing, but also angled trajectories with a positive and negative inclination in terms of horizontal and vertical distances, as well as the coefficient of restitution on a projectile rebound.

Postgraduate courses can address the same problem with more comprehensive complexities and approaches. For instance, since a single cartridge contains 12 shells interacting with each other at the time of the shot, it is suggested to approach the study with statistical mechanics and turbulence to investigate collisions between projectiles. Also, the analysis could expand further in biophysics courses and health sciences to other areas of the human body and damage.

Since this contribution is for students to discuss real cases in the classroom by applying fundamental physics, there is the potential for research in science education. Qualitative research regarding opinion and reflection of the problem, or on how the use of scientific knowledge is used for decision making, can be valuable proposals to accomplish this purpose. The authors intend to continue this work, reporting the results of its implementation in the classroom.
Acknowledgments

E Mosso was supported by Convenio Marco Formación Inicial Docente, UTA 1856.

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