Multiplication of Qubits in a Doubly Resonant Bichromatic Field

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Multiplication of spin qubits arises at double resonance in a bichromatic field when the frequency of the radio-frequency (rf) field is close to that of the Rabi oscillation in the microwave field, provided its frequency equals the Larmor frequency of the initial qubit. We show that the operational multiphoton transitions of dressed qubits can be selected by the choice of both the rotating frame and the rf phase. In order to enhance the precision of dressed qubit operations in the strong-field regime, the counter-rotating component of the rf field is taken into account.

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Theoretical models of quantum computations assume the existence of an ideal two-level quantum system (qubit) and the possibility of an exact description of the qubit’s interaction with external electromagnetic fields [1]. It is known that the resonant interaction between electromagnetic radiation and qubit induces Rabi oscillations, which are the basis for quantum operations. The Rabi frequency \( \omega_R \) is defined by the amplitude of the electromagnetic field and usually is much smaller than the energy difference \( \omega_0 \) (in frequency units) between the qubit’s states. The “dressing” of qubit by the electromagnetic field splits each level into two giving rise to two new qubits with energy difference \( \omega_R \). The spectrum of the multilevel “qubit + field” system consists of three lines at the frequencies \( \omega_0 \) and \( \omega_0 \pm \omega_R \) (the Mollow triplet [2]). The second low-frequency electromagnetic field with the frequency close to the Rabi frequency could induce an additional Rabi oscillation on dressed states of new qubits. These qubits are attracting interest because their coherence time is longer than that of the initial qubit [3–5]. The results of studies of qubits dressed by bichromatic radiation formed by fields with strongly different frequencies are important for a wide range of physical objects, including, among others, nuclear and electron spins, double-well quantum dots, flux and charge qubits in superconducting systems. In NMR [6, 7], EPR [5, 8, 9] and optical resonance [10] such investigations are used in the development of line-nar- rowing methods.

In this letter, we describe the multiplication of spin qubits at double resonance in a bichromatic field with strongly different frequencies. We then show that the operational multiphoton transitions of dressed qubits can be selected by the choice of both the rotating frame and the phase of the low-frequency field. Two important examples of such transitions in the rotating and doubly rotating frames are presented.

Let an electron spin qubit be in three fields: a microwave (mw) one directed along the \( x \) axis of the laboratory frame, a radio-frequency (rf) one directed along the \( z \) axis, and a static magnetic one also directed along the \( z \) axis. The Hamiltonian of the qubit in these fields can be written as follows:

\[
H = H_0 + H_\perp(t) + H_\parallel(t). \tag{1}
\]

Here \( H_0 = \omega_0 \sigma_z \) is the Hamiltonian of the Zeeman energy of a spin in the static magnetic field \( B_0 \), where \( \omega_0 = \gamma B_0 \), and \( \gamma \) is the electron gyromagnetic ratio. Moreover, \( H_\perp(t) = 2\omega_1 \cos(\omega t + \phi) \sigma_x \) and \( H_\parallel(t) = 2\omega_2 \cos(\omega_0 t + \psi) \sigma_z \) are the Hamiltonians of the spin interaction with linearly polarized mw and rf fields, respectively. \( B_1 \) and \( B_2 \), \( \omega_1 \) and \( \omega_2 \), and \( \phi \) and \( \psi \) denote the respective amplitudes, frequencies, and phases of the mw and rf fields. Finally, \( \omega_1 = \gamma B_1 \) and \( \omega_2 = \gamma B_2 \) stand for the Rabi frequencies, whereas \( \sigma^x, \sigma^y, \sigma^z \) are the components of the spin operator.

The evolution of the system with the Hamiltonian (1) is described by the Liouville equation for the density matrix \( \rho \):

\[
i \frac{\partial \rho}{\partial t} = [H, \rho] \tag{2}
\]
At where we have:

\[ H_1 = U_1^\dagger H U_1 = \Delta s^z + (\omega_1/2)(s^+ + s^-) + 2\omega_2\cos(\omega_{rf}t + \psi)s^z \]

and \( \Delta = \omega_0 - \omega \). The mw phase \( \varphi = 0 \) and the counter-rotating component of the mw field is neglected. We also assume that the exact resonance condition is fulfilled \( \Delta = 0 \), and that \( \omega_1, \omega_{rf} \gg \omega_2 \). Upon rotation of the frame around the \( y \) axis by the angle of \( \pi/2 \) (\( e^{\frac{i\pi}{2}t} \)), we obtain:

\[ i\dot{\rho}_2 = [H_2, \rho_2], \]

where \( H_2 = U_2^\dagger H U_2 = \omega_1 s^z - \omega_2 \cos(\omega_{rf}t + \psi)(s^+ + s^-) \).

Now, we pass to the interaction representation by choosing the frame rotating with frequency \( \omega_1 \) around the \( z \) axis (\( e^{i\omega_1 t} \)). In this frame we have:

\[ i\dot{\rho}_3 = [H_3, \rho_3], \]

where \( H_3 = U_3^\dagger H U_3 = -(\omega_2/2)s^+(e^{i\delta t}e^{-i\psi} + e^{-i\omega_{2rf}t}e^{i\psi}) \). The diagonalization of the Hamiltonian in the effective frame can be eliminated by the Krylov—Bogoliubov—Mitropolsky method [5, 11, 12]. Averaging over the period \( 2\pi/\omega_{rf} \), we obtain the following effective Hamiltonian up to the second order in \( \omega_2/\omega_{rf} \):

\[ H_3 \rightarrow H_{\text{eff}} = H_{\text{eff}}^{(1)} + H_{\text{eff}}^{(2)}. \]

In the above equation we have put:

\[ H_{\text{eff}}^{(1)} = \langle H_3(t) \rangle = -(\omega_2/2)s^+(e^{i\delta t}e^{-i\psi} + \text{h.c.}), \]

\[ H_{\text{eff}}^{(2)} = \frac{i}{2}\int dt \langle [H_3(t) - \langle H_3(t) \rangle, H_3(t) \rangle \rangle = \Delta_{BS}s^z. \]

The symbol \( \langle \ldots \rangle \) denotes time averaging: \( \langle A(t) \rangle = \frac{1}{T} \int^T_0 A(t) \, dt \), where \( T = 2\pi/\omega_{rf} \) and \( \Delta_{BS} \approx \omega_2^2/4\omega_{rf} \) is the Bloch—Siegelert-like frequency shift.

After the canonical transformation \( \rho_3 \rightarrow \rho_4 = U_4^\dagger \rho_3 U_4, U_4 = e^{-i(\delta t - \psi)s^z} \), the equation

\[ i\dot{\rho}_3 = [H_{\text{eff}}, \rho_3] \]

is transformed into

\[ i\dot{\rho}_4 = [H_{\text{eff}}, \rho_4], \]

where \( H_4 = U_4^\dagger H_{\text{eff}} U_4 = (\delta + \Delta_{BS})s^z - (\omega_2/2)(s^+ + s^-) \).

The diagonalization of the Hamiltonian \( H_4 \) by means of the rotation operator \( U_5 = e^{-i\xi s^z} \) yields:

\[ i\dot{\rho}_5 = [H_5, \rho_5]. \]

Here \( H_5 = \varepsilon s^z, \varepsilon = \sqrt{(\omega_1 - \omega_{rf} + \Delta_{BS})^2 + \omega_2^2} \) is the frequency of the Rabi oscillations between the spin states dressed simultaneously by the mw and rf field while \( \xi = -\omega_2/\varepsilon, \cos\xi = (\omega_1 - \omega_{rf} + \Delta_{BS})/\varepsilon \).

By using Eqs. (2)—(9), the density matrix in the laboratory frame (LF) can be written as:

\[ \rho(t) = U_1 U_2 U_3 U_4 U_5 e^{-iH_{\text{rf}}t} \rho_3(0) \]

\[ \times e^{iH_{\text{rf}}t} U_5^\dagger U_4^\dagger U_3^\dagger U_2^\dagger U_1^\dagger, \]

where

\[ \rho_3(0) = U_5^\dagger U_4^\dagger U_3^\dagger U_2^\dagger U_1^\dagger(0) \]

\[ \times \rho(0) U_1 U_2 U_3 U_4 U_5(0), \]

and \( U_1(0) = 1, U_3(0) = 1, U_4(0) = e^{-i\nu s^z} \).

Initially, the qubit is in the ground state and \( \rho(0) = 1/2 - s^z \). The absorption signal \( \nu(t) = \text{Tr}(\rho(t)(s^+ - s^-))/2i \) in the laboratory frame can be derived from Eqs. (10) and (11):

\[ \nu(t) = \langle \langle 1|\rho(t)|2 \rangle - 2\langle |\rho(t)|1 \rangle \rangle /2i \]

\[ = (1/2)\sin\xi \cos\xi \cos\psi \sin\omega t \]

\[ + (1/16)\{4\sin\xi \sin\psi[\cos(\omega + \psi)t - \cos(\omega - \psi)t] \]

\[ + 4\sin\xi \cos\xi \cos\psi[\sin(\omega - \psi)t + \sin(\omega + \psi)t] \]

\[ + 2\sin^2\xi \sin^2\psi[\cos(\omega + \omega_{rf})t + \cos(\omega - \omega_{rf})t] \]

\[ + ((\cos\xi - 1)^2 + (\cos^2\xi - 1)\cos 2\psi) \]

\[ \times [\sin(\omega + \omega_{rf} - \psi)t - \sin(\omega - \omega_{rf} + \psi)t] \]

\[ + (\cos^2\xi - 1)\sin 2\psi \]

\[ \times [\cos(\omega - \omega_{rf} + \psi)t + \cos(\omega + \omega_{rf} - \psi)t] \]

\[ + ((\cos\xi + 1)^2 + (\cos^2\xi - 1)\cos 2\psi) \]

\[ \times [\cos(\omega + \omega_{rf} + \psi)t - \cos(\omega - \omega_{rf} - \psi)t]. \]

\[ \nu(t) = \langle \langle 1|\rho(t)|2 \rangle - 2\langle |\rho(t)|1 \rangle \rangle /2i \]

\[ = (1/2)\sin\xi \cos\xi \cos\psi \sin\omega t \]

\[ + (1/16)\{4\sin\xi \sin\psi[\cos(\omega + \psi)t - \cos(\omega - \psi)t] \]

\[ + 4\sin\xi \cos\xi \cos\psi[\sin(\omega - \psi)t + \sin(\omega + \psi)t] \]

\[ + 2\sin^2\xi \sin^2\psi[\cos(\omega + \omega_{rf})t + \cos(\omega - \omega_{rf})t] \]

\[ + ((\cos\xi - 1)^2 + (\cos^2\xi - 1)\cos 2\psi) \]

\[ \times [\sin(\omega + \omega_{rf} - \psi)t - \sin(\omega - \omega_{rf} + \psi)t] \]

\[ + (\cos^2\xi - 1)\sin 2\psi \]

\[ \times [\cos(\omega - \omega_{rf} + \psi)t + \cos(\omega + \omega_{rf} - \psi)t] \]

\[ + ((\cos\xi + 1)^2 + (\cos^2\xi - 1)\cos 2\psi) \]

\[ \times [\cos(\omega + \omega_{rf} + \psi)t - \cos(\omega - \omega_{rf} - \psi)t]. \]