Possibilities for measurement of nano-crystallites size with use of parametric X-ray radiation

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Abstract Spectral and angular properties of the parametric X-ray radiation (PXR) of relativistic particles emitted in the backward direction from a thin crystalline layer and polycrystal are considered. The shapes of the natural spectral peak of the PXR from a crystalline ball and spectral distribution of PXR from a polycrystal are estimated. It is shown that the natural spectral peak width determines the PXR spectral peak width emitted in backward direction from crystalline layer and grains with size of nanometer range. The method for measurements of crystalline layer thickness and crystalline grains size of nanometer range with use of spectral peak width of the PXR emitted in the backward direction is discussed.

1. Introduction
The first research of the width of the spectral peak of parametric X-ray radiation (PXR) emitted by relativistic charged particle moving in a singlecrystal has been performed at Kharkov [1]. It was found that the width is determined mainly by the experimental angular resolution and the natural width due to the crystal thickness is negligible in forward hemisphere. In research [2,3] it was demonstrated that the PXR spectral peak width from a singlecrystal dramatically reduces at observation in the backward direction relative the incident particle velocity. The multiple scattering effect on the spectral peak width was analyzed in [4]. It was shown in [5] that normalized spectral peak width of focused PXR emitted from a bent singlecrystal by channeling particles can reach order of value about $10^{-8}$. PXR from a polycrystal was first treated by Nasonov [6] and observed at Moscow [7]. Intense PXR from textured polycrystal was first studied at Hiroshima [8,9]. In these experiments, the PXR emitted by relativistic electrons in crystalline grains of a polycrystal has been identified. A possibility for application of the PXR natural spectral peak width for measurements of crystalline grains size was noted in [10]. Theoretical research of PXR from a polycrystal has been performed in [11]. In present paper we consider some features of the spectral peak of PXR emitted in the backward direction relative to the vector of incident particles velocity and possibilities for its applications for measurements of crystalline layer thickness and grains size in nanometer range. The consideration is performed in the frameworks of the Ter-Mikaelian theory of the PXR [12,13].

2. PXR yield in the backward direction
The general formula for absolute differential yield of PXR within the framework of the Ter-Mikaelian theory has been confirmed in a number of experiments with singlecrystals, see e.g. reviews [13-15]

$$\frac{dN}{d\Omega} = \frac{e^2 L}{2\pi \hbar c \omega} \left| k \right|^2 \left( k \times \vec{k}_\omega \right) \left( \left( \vec{k}_\perp - \vec{g}_\perp \right)^2 + k^2 \gamma_{\perp}^2 \right)^{-\frac{1}{2}},$$

(1)
where $dN$ is the number of quanta emitted in the solid angle $d\Omega$ by the particle with charge $e$ passed through the crystal of effective thickness $L$; $\chi_{\xi}(\omega)$ is the Fourier component of dielectric susceptibility; $\hbar$ is the Plank constant divided by $2\pi$; $\epsilon_0$ is the average permittivity; $\tilde{k} = k\Omega$ is the radiation wave vector; $k = \omega\sqrt{\epsilon_0}/c$; $\omega$ is the radiation frequency; $\Omega$ is the unit vector for the radiation direction; $\tilde{\xi} = \sqrt{\epsilon_0}V/c$ is the particle velocity $V$ in terms of the phase wave velocity in the crystal $c/\sqrt{\epsilon_0}$; $V[\tilde{\Omega}]$; $\tilde{\xi} = [\tilde{\xi}]$; $\tilde{\xi}^{-1} = 1 + \gamma_{e\Omega}^2/2$; $c$ is the light velocity; $\tilde{k}_0 = k\tilde{\xi} + \tilde{g}$; $\gamma_{e\Omega} = \left[\gamma^2 + |\chi_{\Omega}|^2\right]^{1/2}$ is the effective relativistic factor of the particle in the crystal with taking into account of the density effect [16]; $\gamma$ is the relativistic factor of the particle; $\chi_{\Omega} = \chi_{\Omega}(\omega)$ is the average dielectric susceptibility; $\tilde{g}$ is the reciprocal lattice vector of the crystallographic planes; $g = 2\pi/a$; $a$ is the distance between crystallographic planes, $\tilde{k}_\perp$ and $\tilde{g}_\perp$ are the components of vectors $\tilde{k}$ and $\tilde{g}$ perpendicular to $V$; $k = -\tilde{g} \cdot \tilde{V} / V\tilde{\xi}^{-1} - \tilde{V} \cdot \tilde{\Omega}$. The effective thickness $L$ of the target in the shape of crystalline slab with allowance of the radiation attenuation has form $L = T_v \tilde{t} \cdot \tilde{\Omega} \left[1 - \exp \left(-\frac{T}{T_v \tilde{t} \cdot \tilde{\Omega}}\right)\right]$, where $T_v = T_e(\omega)$ is the $e$-fold attenuation length of the radiation in the target; $T$ the slab thickness; $\tilde{t}$ is the unit vector perpendicular to the slab surface; $\tilde{v} = \tilde{V}/V$. In transparent crystal $L = \frac{T}{\tilde{t} \cdot \tilde{\Omega}}$.

The PXR differential yield in approximation of small angles $\delta, \alpha \ll 1$ in the vicinity of the PXR reflection center was derived from equation (1) with use of spherical coordinates in [15]

$$\frac{dN}{d\Omega} = \frac{z^2 n_p \chi_{\xi}(c\g) \left(\tilde{\xi}^{-1} - \cos(\theta_e - \delta)\right)^2}{137\epsilon_0^2 \xi^2 \sin^4 \phi} \left[\alpha^2 + \delta^2 \cos^2 \theta_e \right] \left(\delta^2 + \alpha^2 + \gamma_{e\Omega}^2\right)^2,$$

(2)

where $n_p = \frac{L}{a} \sin \phi$ is the number of crystallographic planes taking part in production of PXR, $z$ is the particle’s charge in units of the electron charge, $\phi$ is the Bragg angle between the crystallographic planes and vector $\tilde{V}$, $\chi_{\xi}(c\g) = \text{inv}(\omega)$, $\theta_e = 2\phi$ is the polar angle of the reflection center, $\delta, \alpha$ are coordinates of the observation direction in the plane of small angles that is perpendicular to the direction of the reflection center, $\delta$ is in the reaction plane containing vectors $\tilde{g}, \tilde{V}$ and $\alpha$ is perpendicular to the reaction plane. Formula (2) differs from the well known Feranchuk-Ivashin formula [17] e.g. by the factor in square brackets that provides proper description of experimentally observed asymmetry in the PXR reflection [1,16].

In order to consider properties of PXR emitted by electrons ($z = 1$) in the backward direction one can use spherical coordinates from [15] and derive specific formula for PXR differential yield in the backward direction in approximation of small angles $\tau, \delta, \rho \ll 1$ in the plane perpendicular to the incident particle velocity vector $\tilde{V}$

$$\left(\frac{dN}{d\Omega}\right)_{\text{bw}} = \frac{4n_p \chi_{\xi}(c\g)^2}{137\epsilon_0^2 \xi^2 \sqrt{\xi_0}} \left[\rho^2 + \gamma_{e\Omega}^2\right],$$

(3)

where

$$\rho^2 = \left|\delta - 2\rho\right|^2 = \delta^2 + 4\rho^2 - 4\delta \rho \cos \phi,$$

(4)
\( \rho \) is the angle between the observation direction \( \Omega \) and the PXR reflection center direction, \( \delta \) is the angle between the observation direction \( \Omega \) and vector \(-\vec{V}\), \( \tau = \pi - \theta \), \( \tau \) is the angle between vectors \( \vec{g} \) and \( \vec{V} \), \( \tau = \frac{\pi}{2} - \phi \), \( 2\tau = \pi - \theta \), \( \varphi \) is the angle between the reaction plane containing vectors \( \vec{g}, \vec{V} \) and the observation plane containing vectors \( \Omega, \vec{V} \). The number of crystallographic planes producing the PXR in the backward direction in a thin transparent crystal slab or layer of thickness \( T \) along the trajectory is \( n = \frac{T g}{2\pi} = \frac{T}{a} \).

3. PXR and Bragg frequencies in the backward direction

The general formula for PXR frequency \( \omega \) within the Ter-Mikaelian theory has been confirmed experimentally in [1,16]

\[
\omega = \frac{c g \vec{V}}{c - \sqrt{\epsilon \vec{V} \Omega}} = \frac{c g \cos \tau}{\sqrt{\epsilon \left( \frac{1}{\xi - \cos \theta} \right)}} ,
\]  
where \( \theta \) is the polar angle of the observation direction \( \Omega \). With use of the second-order Taylor’s formula one can find expression for deviation of the PXR frequency in the vicinity of the observation angle \( \theta \) and the angle of crystal alignment \( \tau \)

\[
\frac{d\omega}{\omega} = -\frac{\sin \theta - \tan \tau \cdot d\tau \cdot d\tau^2}{2} \left( 1 + \frac{\sin^2 \theta - \xi^{-1} \cos \theta}{\xi^{-1} - \cos \theta} \right) + \frac{\tan \tau \sin \theta}{\left( \frac{1}{\xi - \cos \theta} \right)^2} d\tau d\theta .
\]  
The first term in equation (6) is dominating for single crystal of fixed alignment \( \tau \) and at \( \theta \) not in the vicinity of \( \theta = 0, \pi \). In this case we obtain

\[
\Delta \omega = -\frac{\Delta \theta}{\tan(\theta/2)} .
\]  
Formula (7) for PXR spectral peak width \( \Delta \omega \) at experimental angular resolution \( \Delta \theta \) has been experimentally confirmed for PXR reflection emitted in forward hemisphere in [1]. In the vicinity of the exact backward direction at \( \theta = \pi, \tau = 0 \) three terms in equation (6) vanish and \( d\omega \) dramatically reduces. In such case the formula for normalized PXR frequency in backward direction is

\[
\frac{\omega - \omega_c}{\omega_c} = \frac{\delta^2 - \tau^2}{4 - \frac{\tau^2}{2}},
\]  
where

\[
\omega_c = \frac{c g}{\sqrt{\epsilon \left( \frac{1}{\xi - \cos \theta} + 1 \right)}}
\]  
is the frequency at \( \tau, \delta = 0 \). It is interesting to note that the frequency (8) increases at declination of the observation direction from exact backward direction \( \theta = \pi \), and decreases at declination of the crystal alignment from the one when the reciprocal lattice vector \( \vec{g} \) is parallel to \( \vec{V} \). The yield of PXR at \( \tau, \delta = 0 \) is zero, see equation (3).

Consider also the difference between the Bragg frequency in the observation direction \( \omega_b = -\frac{c g^2}{2\sqrt{\epsilon_g (\Omega g)}} \) and PXR frequency. The formula for the normalized difference in the vicinity of the PXR reflection center at \( \theta \) not in the vicinity of 0 or \( \pi \) has been derived with use of spherical coordinates in approximation of small angles in [18]. It is a function of square of the angle between the reflection center and observation direction \( \rho^2 = \alpha^2 + \delta^2 \),
Using spherical coordinates from [18], one can find specific formula for the difference in the backward direction. It is similar to one shown above

$$\frac{\omega_b - \omega}{\omega} = \frac{\gamma_{\text{eff}}^2 + \rho^2}{4 \sin^2 \phi}. \quad (10)$$

but the square of the angle between the reflection center and the observation direction is determined by equation (4). The Bragg frequency in the backward direction \( \omega_b = \frac{c_g}{\sqrt{E_0} \left(2 - \delta - r^2\right)} \) always exceeds the PXR frequency, the absolute difference is no less then \( \omega_b - \omega \geq \frac{c_g}{8 \sqrt{E_0}} \gamma_{\text{eff}}^2 \).

The transition radiation, bremsstrahlung, channeling radiation of frequency \( \omega_{\text{B}} \) can be diffracted in the backward direction. The frequency of any kind of diffracted radiation and the PXR frequency are separated by the frequency interval (11).

4. Natural spectral peak of the PXR emitted in the backward direction from a crystalline slab

From classical point of view, the PXR is the wavetrain of electromagnetic oscillations. The number of oscillations \( n \) in the wavetrain is determined by the effective number of crystallographic planes crossed by the particle and taking part in production of the PXR. In present paper we deal with PXR emitted in the backward direction from a crystal or crystallite that are thin enough to neglect both multiple scattering of electrons and attenuation of radiation. The PXR in backward direction is produced at crystallographic planes aligned almost perpendicularly the particle trajectory. The number of oscillations \( n \) of permanent amplitude in the wavetrain from a thin crystal of thickness along the particle trajectory \( T \) is \( n = \frac{T_g}{2\pi} \). One can obtain spectral distribution of the radiation intensity \( I(s) \) of the wavetrain with use of Fourier transformation. It is described by the well known function

$$I(s) \sim \pi^2 n^2 \frac{\sin^2 s}{s^2}, \quad (12)$$

where \( s = \frac{\Delta \omega n \pi}{\omega_x} \). The full width at half of maximum (FWHM) of the central peak in (12) is

$$\frac{\Delta \omega}{\omega_x}_{\text{nat}} = 0.89 n^{-1}. \quad (13)$$

Therefore one can determine the crystal slab thickness as \( T = 0.89 \cdot a \left( \frac{\Delta \omega}{\omega_x}_{\text{nat}} \right)^{-1} \). Normalized spectral distribution (12) is shown in figure 1.

5. Spectral peak of the PXR emitted in the backward direction from a crystalline ball

Crystallites in a polycrystal can have different forms. Calculations of X-rays scattering on a crystalline ball are described in [19]. Let us estimate spectral properties of the PXR emitted in the backward direction by relativistic particles crossing a ball-shaped crystal of radius \( R \). We will suppose that the radius of the cylinder \( \lambda \gamma_{\text{eff}} \) (where \( \lambda = 2a \), see (9)) around of the particle trajectory where the PXR is produced is less then the crystallite ball radius \( R \), \( \lambda \gamma_{\text{eff}} < R \). Also, we will believe that noticeable yield of the PXR in backward direction can provide the crystallographic planes that are almost perpendicular to the particle’s trajectory only.
The number of crystallographic planes $n$ crossed by the particle is a function of the impact parameter $r$, 
$$n(r) = n_D \sqrt{1 - \left(\frac{r}{R}\right)^2},$$ 
where $n_D = \frac{2R}{a}$, $0 \leq r \leq R$. Integrating (12) over the ball cross section 
$$I_b(s) \sim 2\pi^2 n_D^2 \int_0^s \sin^2 \left(\frac{\Delta \omega}{\omega} \pi n(r)\right) \frac{2\pi r dr}{\pi R^2}$$ 
one can find spectral distribution of the PXR emitted in the backward direction from a crystalline ball

$$I_b(s) \sim 2\pi^2 n_D^2 \int_0^s \sin^2 \left(\frac{\Delta \omega}{\omega} \pi n_D\right) \frac{2\pi r dr}{\pi R^2}.$$

where $s = \frac{\Delta \omega n_D}{\omega}$. Factor 2 is included for normalization at $s = 0$. The full width at half of maximum (FWHM) of the central peak in the distribution (14) is

$$\left(\frac{\Delta \omega}{\omega}\right)_{B=nat} = 1.11 \cdot n_D^{-1},$$

where $n_D$ is the diameter of the ball in units of the interplanar spacing. Thus, the natural spectral peak width from the crystalline ball exceeds in 1.25 times the one from the slab (13) of thickness equal to the ball diameter. One can determine the crystalline ball diameter $D = 1.11 \cdot a \cdot \left(\frac{\Delta \omega}{\omega}\right)_{B=nat}$ after measurement of the natural spectral peak width of the PXR emitted in the backward direction. Note that natural spectral peak shapes (12, 14) are independent of incident particle energy. Normalized spectral distribution (14) is shown in figure 1.

Figure 1. Comparison of spectral distribution of PXR emitted in the backward direction from a crystalline slab and crystalline ball. The distribution from the slab is calculated by equation (12) and shown by dashed line. The distribution from the ball is calculated by equation (14) and shown by solid line. Thickness of the slab is equal to the diameter of the ball, $n = n_D$. Normalized spectral distributions are shown as a functions of $s = \frac{\Delta \omega n_D}{\omega}$. 

6. Influence of finite experimental angular resolution

We will suppose that an X-ray spectrometric detector with circular aperture is installed at observation angle 180° and experimental angular resolution is restricted by value $\delta < \delta_{max}$. The value $\delta_{max}$ is determined mainly by finite sizes of the beam spot at the target and the aperture of the detector, the divergence of the incident particle beam, and also by multiple scattering of electrons in the target. Considering (8) one can see that such restriction leads to the restriction of the spectral range observed.
by the detector due to finite angular resolution. Radiation is absent at frequencies out of the range

\[-\frac{\tau^2}{2} < \frac{\omega - \omega_x}{\omega_x} < -\frac{\tau^2}{2} + \frac{\delta^2}{4}\]

(16)
of full width \(\delta^2 / 4\). At \(\delta_{\text{max}} = \sigma \gamma \omega^4\) the full width of the range due to finite angular resolution is

\[\frac{\Delta \omega}{\omega_x}_{\text{ang}} = \frac{\sigma^2 \gamma^2}{4}\]

(17)

For example, at \(\sigma = 0.2\) the full width of the spectral range is \(\frac{\Delta \omega}{\omega_x}_{\text{ang}} = 10^2 \gamma^2\).

The spectral distribution within the range (the spectral peak shape) can have different forms depending on the alignment of \(\tilde{\tau}\). The frequency \(\omega_{\text{max}} = \omega_x(1 + \sigma^2 \gamma^2 / 4)\) is the maximum PXR frequency that can be observed from a polycrystal by the detector installed in the backward direction.

7. PXR from polycrystal in the backward direction

Let us consider spectral properties of the PXR emitted in the backward direction at \(\theta = \pi\) from a thin polycrystal with the reciprocal lattice vector \(\tilde{g}\) of grains randomly distributed in solid angle \(4\pi\). The PXR yield is proportional to normalized number of the reciprocal lattice vectors \(\frac{2\pi d\tau}{4\pi} = \frac{d\omega}{2\omega_x}\), see (8,9). With this expression and \(\rho^2 = 2\tau^2\), one can derive from equation (3) with use of equations (4,8)

\[
\frac{dN}{d\Omega d\omega} = \frac{4n|\chi_{\tilde{g}}(cg)|^2}{137x^2} \left[\frac{1}{2\omega_x} \right]^{-8x^2} (1-8x^2)^2 \gamma^2,
\]

(18)

where \(x = \frac{\omega - \omega_x}{\omega_x} \gamma^2\), \(x < 0\). Formula (18) describes spectral distribution of the intensity of the PXR emitted at \(\theta = \pi\) by relativistic particles from a thin polycrystal. The distribution has the shape of the asymmetrical spectral peak at \(\omega < \omega_x\), \(x < 0\) shown in figure 2. The maximum of the peak is at \(x = -\frac{1}{8}\), \(\frac{\omega - \omega_x}{\omega_x} = -\frac{\gamma^2}{8}\). The similar maximum was obtained in equation (18) in work [11]. The full width at half of maximum (FWHM) of the peak (18) is

\[\Delta x = \frac{1}{\sqrt{2}}, \quad \left(\frac{\Delta \omega}{\omega_x}\right)_{\text{polycr}} = \frac{\gamma^2}{\sqrt{2}}.
\]

(19)

Figure 2. The shape of the spectral peak of the PXR emitted in the backward direction from a polycrystal at \(\theta = \pi\). The spectral distribution is calculated by equation (18). Normalized spectral distribution is shown as a function of \(x = \frac{\omega - \omega_x}{\omega_x} \gamma^2\).

8. Measurement of the crystalline layer thickness

There are two main reasons for broadening of the spectral peak of the PXR emitted in the backward
direction from a crystalline slab or layer: i. because of finite thickness of the slab or layer (13) and ii.
because of finite experimental angular resolution (17). The total spectral peak width \( \frac{\Delta \omega}{\omega_{\pi}} \) can be estimated as

\[
\left( \frac{\Delta \omega}{\omega_{\pi}} \right)_{\text{tot}} = \sqrt{\left( \frac{\Delta \omega}{\omega_{\pi}} \right)_{\text{ang}}^2 + \left( \frac{\Delta \omega}{\omega_{\pi}} \right)_{\text{nat}}^2}.
\]

Measurements of the slab or layer thickness is possible at condition

\[
\frac{\Delta \omega}{\omega_{\pi}}_{\text{nat}} > \frac{\Delta \omega}{\omega_{\pi}}_{\text{ang}}.
\]

From equations (13,17,21) one can find that broadening because of the slab thickness dominates at

\[
T < \frac{3.6a}{\sigma^2} \gamma_{\text{eff}}^2.
\]

For example, at \( \sigma = 0.2 \), typical inter-planar distance in a crystal \( a = 3 \cdot 10^{-8} \text{ cm} \), and \( \gamma_{\text{eff}} = 20 \) that corresponds to electron beam energy about 10 MeV the broadening because of the slab thickness can dominate up to crystal thickness about \( T \sim 10 \mu m \) at normalized spectral peak width up to \( 3 \cdot 10^{-5} \). But real possibilities for measurements of so narrow spectral peak width from a thick crystal are restricted by energy resolution of X-ray spectrometers and increasing of \( \delta \) in so thick target due to multiple electron scattering. Besides, there exist more traditional methods for measurements of thickness of thick crystals. However, generation of PXR in thick singlecrystal is interesting for creation of X-ray source with high spectral density [2-4].

The method seems much more convenient for measurements of crystalline layers of nanometer range. At well executed inequality (21), the crystalline layer or slab thickness may be estimated as

\[
T = 0.89 \cdot a \left( \frac{\Delta \omega}{\omega_{\pi}} \right)_{\text{exp}}^{-1}.
\]

For example, at the layer thickness in the range 3-300 nm and \( a = 3 \cdot 10^{-8} \text{ cm} \) the normalized PXR spectral peak width measured experimentally \( \left( \frac{\Delta \omega}{\omega_{\pi}} \right)_{\text{exp}} \) should be in the range of about

\[
0.89 \cdot 10^{-3} - 0.89 \cdot 10^{-3}
\]

respectively. At the layer thickness 60 nm the widths are

\[
\left( \frac{\Delta \omega}{\omega_{\pi}} \right)_{\text{nat}} = 2.5 \cdot 10^{-3}, \quad \left( \frac{\Delta \omega}{\omega_{\pi}} \right)_{\text{ang}} = 2.5 \cdot 10^{-5}.
\]

9. Measurement of the crystalline grains size in a polycrystal

Discuss some details concerned to determination of the crystalline grains size in a thin polycrystal by the method of measurement of the PXR spectral peak width. There are three main sources for broadening of spectral peak of the PXR emitted in the backward direction from grains of a thin polycrystal: i. because of grains (balls) size (15) ii. because of finite experimental angular resolution (17) and iii. because of arbitrary alignments of grains (19). The effect of the multiple electron scattering can be neglected in a thin target. The total spectral peak width \( \left( \frac{\Delta \omega}{\omega_{\pi}} \right)_{\text{tot}} \) can be estimated as

\[
\left( \frac{\Delta \omega}{\omega_{\pi}} \right)_{\text{tot}} = \sqrt{\left( \frac{\Delta \omega}{\omega_{\pi}} \right)_{\text{ang}}^2 + \left( \frac{\Delta \omega}{\omega_{\pi}} \right)_{\text{B-nat}}^2 + \left( \frac{\Delta \omega}{\omega_{\pi}} \right)_{\text{polyer}}^2}
\]
Measurements of the grains size is possible at condition

\[
\frac{(\Delta \omega)}{\omega_x}_{\text{nat}} > \left( \frac{(\Delta \omega)}{\omega_x}_{\text{B-ang}} + \left( \frac{(\Delta \omega)}{\omega_x}_{\text{polycr}} \right)^2 \right)^{1/2}.
\]  

(25)

From equations (17,19) one can find that \( (\Delta \omega/\omega_x)_{\text{polycr}} \gg (\Delta \omega/\omega_x)_{\text{ang}} \) in popular experimental case \( \sigma < 1 \).

In such case it is possible to neglect term \( (\Delta \omega/\omega_x)_{\text{ang}} \) in (24,25). Then from inequality (25) we obtain the condition for the grain size \( G = 2R \) that can be measured

\[
G < 1.11 \sqrt{2a \gamma_{\text{eff}}^2}.
\]

At this condition the natural spectral peak gives dominating contribution to the width of emitted spectral peak. At well executed inequality (25) the size of grains may be estimated as

\[
G = 1.11 \cdot a \cdot \left( \frac{(\Delta \omega/\omega_x)_{\text{exp}}}{a} \right)^{-1},
\]

(27)

where \( (\Delta \omega/\omega_x)_{\text{exp}} \) is the experimentally measured FWHM of the PXR peak. For example, at \( a = 3 \cdot 10^{-8} \text{cm} \) and \( \gamma_{\text{eff}} = 20 \) that corresponds to electron beam energy about 10 MeV the broadening because of the crystallites size dominates at \( G < 190 \text{nm} \). At grains of size in the range 30-150 nm the experimentally observed PXR spectral peak width \( (\Delta \omega/\omega_x)_{\text{exp}} \) should be in the range of about

\[
1.11 \cdot 10^{-2} - 2.22 \cdot 10^{-3}
\]

respectively. At the grains size \( G = 60 \text{ nm} \) and \( \sigma = 0.2 \) the widths are

\[
(\Delta \omega/\omega_x)_{\text{nat}} = 5.5 \cdot 10^{-3}, \quad (\Delta \omega/\omega_x)_{\text{polycr}} = 1.8 \cdot 10^{-3}, \quad (\Delta \omega/\omega_x)_{\text{ang}} = 2.5 \cdot 10^{-5}.
\]

10. Results and discussion

The main result of this paper is: There is the real possibility for measurements of the crystal layer thickness and crystalline grains size in polycrystal in nanometer range with use of the natural spectral peak width of the PXR emitted in the backward direction. Besides, note possibilities to study by PXR other objects of nanometer size range e.g. fullerene-like structures.

The influence of the density effect is taken into account in present consideration due to use in calculations the effective relativistic factor \( \gamma_{\text{eff}} \). The influence of diffracted transition radiation should be insignificant at electron beam energy \( E_e < mc^2 \omega_x/\omega_p \). The yield of the PXR from a polycrystal (18) has to be multiplied by the factor equal to doubled number of the same type crystallographic planes in crystallites. Concrete grains shape and distribution of grains by sizes in polycrystal has to be taken into account. Also, the plane effect [20] can increase the PXR yield and width in backward direction especially at low electron beam energy. Besides, diffraction of channeling radiation is possible in backward direction at channeling of electrons in crystallographic axes and planes as well as diffraction of bremsstrahlung at frequencies \( \omega \gtrsim \omega_p (1 + \gamma_{\text{eff}}^2/4) \) exceeding the maximum PXR frequency \( \omega_p \).

Experimental research at an electron accelerator with energy of about ten(s) MeV is desirable to check possibilities of the method. The electron beam should excite PXR in a thin crystalline or polycrystalline layer. Textured polycrystal can be used as a target to increase the PXR yield. Target
should be installed in a goniometer to provide optimal alignment of the target relative to the electron beam. A crystal-diffraction X-ray spectrometer can be used for measurements of the PXR spectra at observation angle $\theta = 180^\circ$.

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