Viscous modifications to quark and gluon distribution functions in quark-gluon plasma medium

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Viscous modifications to the thermal distributions of quark-antiquarks and gluons have been studied in a quasi-particle description of the quark-gluon-plasma medium created in relativistic heavy ion collision experiments. The model is described in terms of quasi-partons that encode the hot QCD medium effects in their respective effective fugacities. Both shear and bulk viscosities have been taken into account in the analysis and the modifications to thermal distributions have been obtained by modifying the energy momentum tensor in view of the non-trivial dispersion relations for the gluons and quarks. As an implication, dilepton production rate in the $q\bar{q}$ annihilation process has been investigated. Significant modifications have been observed in the dilepton production rate in the presence of interactions encoded in the QCD equation of state.

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I. INTRODUCTION

There are strong indications from relativistic heavy ion collider experiments (RHIC) at BNL concerning the creation of strongly coupled quark-gluon-plasma (QGP) that possess near perfectly fluidity. These interesting observations on the QGP are mainly corroborated by two of the most striking finding of the RHIC, viz., the large elliptic flow shown by QGP, and the large jet quenching at RHIC. The former led to the near perfect fluid picture and latter indicated towards the strongly coupled picture of the QGP. Preliminary results from heavy-ion collisions at the LHC [2, 3] reconfirm similar picture of the QGP. There are interesting possibilities for observing the other higher order flow parameters (dipolar and triangular etc.) at LHC, that are crucial for the quantitative understanding of collectivity and the viscous coefficients of the QGP [4, 5].

The strongly coupled picture of the QGP is seen to be consistent with the lattice simulations of the QCD equation of state (EoS) [6, 7]. The EoS in an important quantity that plays crucial role in deciding the bulk and transport properties of the QGP. Therefore, it need to be implemented in an appropriate way as a model for the equilibrium state of the QGP medium. Importantly, such modifications naturally encode the hydrodynamic behavior of the QGP (shear and bulk viscosities) are essential to understand and characterize its liquid state, and the hydrodynamic evolution in heavy-ion collisions. A tiny value of $\eta/s$ can be associated with the near perfect fluid picture and the strongly coupled nature of the QGP provided that the $\zeta/s$ is relatively smaller. Theoretical investigations suggest that this is true for the temperatures not very close to $T_c$ where bulk viscosity is large [13–15]. Several phenomenological and theoretical investigations do suggest that the QGP indeed possess a very tiny value of the $\eta/s$ [16].

Moreover, in certain situations, the temperature behavior of the $\zeta$ may lead to cavitation and it may cause the hydrodynamic evolution of the QGP to stop before the freeze out is actually reached [19–21]. Both the bulk and shear viscosities play vital role in deciding the observed properties of final state hadrons in the RHIC [22]. Furthermore, these transport coefficients have significant impact on the important phenomena such as heavy quark transport [23], photon and dilepton production in heavy ion collisions [24, 25]. All these investigations calls for an appropriate modeling of viscous modified thermal distribution functions of quarks and gluons in the QGP medium. Importantly, such modifications naturally encode hot QCD medium effects through the QGP EoS (described in terms of the quasi-particle approaches at high temperature).

The present analysis is devoted to obtain the viscous modified thermal distributions for quarks and gluons in the QGP medium, within the framework of transport theory, coupling it with a recently proposed effective fugacity quasi-particle model [28]. As an implication of these distribution functions, the dilepton production rate via $q\bar{q}$ annihilation process is analyzed, and significant mod-
ifications are obtained, as compared to those obtained by assuming that the QGP is an ultra-relativistic non-interacting gas of quarks and gluons (ideal QGP EoS). The shear and bulk viscosity qualitatively play the same role in both the case (employing realistic and the ideal QGP EoS). As we shall see that the quantitative differences are mainly induced by realistic QGP EoS.

The paper is organized as follows. Sec. II deals with a recently proposed quasi-particle description of hot QCD in terms of effective quasi-parton distribution functions along with how it modifies the kinetic theory definition of the energy momentum tensor. Furthermore, the modifications to the thermal distributions of the quasi-particles (quasi-gluons, and quasi-quarks) in the presence of dissipation that is induced by shear and bulk viscosity of the QGP, are obtained by coupling the kinetic theory with the hydrodynamic description of the QGP. In Sec. III, dilepton production rate is investigated employing these viscous modified thermal distribution functions, and interesting observation are discussed. Sec. IV articulates the conclusions and future directions.

II. VISCOUS MODIFICATION TO QUARK AND GLUON THERMAL DISTRIBUTION FUNCTIONS

The determination of transport properties of any fluid is subject to the matter of moving away from equilibrium followed by adopting either the transport theory approach or equivalently the field theory approach utilizing the well known Green-Kubo formulae [29]. Once these transport coefficients such as shear and bulk viscosities are known, it is pertinent to ask what kind of modifications are induced to the momentum distributions of the fluid degrees of freedom

Now, to obtain the modified distribution function of quarks and gluons which describe the viscous QGP, firstly we need an appropriate modeling of the equilibrium state of the QGP in terms of its degrees of freedom. To that end, we employ a recently proposed quasi-particle description of the QGP [28] as a model for its equilibrium state. This is followed by the linear perturbation induced in terms of shear and bulk viscous effects adopting the quadratic ansatz [12] (quadratic in terms of momentum dependence). To obtain, viscous corrections to the momentum distribution of quarks-antiquarks and gluon that constitute the QGP, kinetic theory expression for the energy momentum tensor, $T^\mu\nu$ needs to be equated with its hydrodynamic decomposition in the presence of viscosities. Let us first briefly review the quasi-particle model followed by the $T^\mu\nu$ obtained from this model.

A. The quasi-particle description of hot QCD

Let us now discuss the quasi-particle understanding of hot QCD medium effects employed in the present analysis, recently proposed by Chandra and Ravishankar [28]. This description has been developed in the context of the recent $(2+1)$-lattice QCD equation of state [31] at physical quark masses. There are more recent lattice results with the improved actions and more refined lattices [7], for which we need to re-visit the model with specific set of lattice data specially to define the effective gluonic degrees of freedom. This is beyond the scope of the present analysis. Henceforth, we will stick with the one set of lattice data utilized in the model [28].

The model initiates with an ansatz that the Lattice QCD EoS can be interpreted in terms of non-interacting quasi-partons having effective fugacities, $z_q$, $z_g$ which encode all the interaction effects, where $z_q$ denotes the effective gluon fugacity, and $z_g$, denotes the effective quark-fugacity respectively [28]. In this approach, the hot QCD medium is divided in to two sectors, viz., the effective gluonic sector, and the matter sector (light quark sector, and strange quark sector). The former refers to the contribution of gluonic action to the pressure which also involves contributions from the internal fermion lines. On the other hand, latter involve interactions among quark, anti-quarks, as well as their interactions with gluons. The ansatz can be translated to the form of the equilibrium distribution functions, $f_{eq} \equiv \{f_{eq}^g, f_{eq}^q, f_{eq}^\pi\}$ (this notation will be useful later while writing the transport equation in both the sector in compact notations) as follows,

$$f_{eq}^g = \frac{z_{g,q} \exp(-\beta E_p)}{1 + z_{g,q} \exp(-\beta E_p)},$$
$$f_{eq}^q = \frac{z_q \exp(-\beta \sqrt{p^2 + m^2})}{1 + z_q \exp(-\beta \sqrt{p^2 + m^2})}, \quad (1)$$

where $E_p = |p| \equiv p$ for gluons and light quarks, and $\sqrt{p^2 + m^2}$ for strange quarks ($m$ denotes the mass of the strange quark). The minus sign is for gluons and plus sign is for quark-antiquarks. The quarks and anti-quarks possess the same distribution functions since we are working at the zero baryon chemical potential. The determination of $f_{eq}$ achieved by fixing the temperature dependence of the effective fugacities $z_q$ and $z_g$ from the QGP EoS which in our case is the lattice QCD EoS (for details see [28]).

It is worth emphasizing that the effective fugacity is not merely a temperature dependent parameter which encodes the hot QCD medium effects. It is very interesting and physically significant, and can be understood in terms of effective number density of quasi-particles in hot QCD medium, and equivalently in terms of an effective Virial expansion [28]. Interestingly, its physical significance reflects in the modified dispersion relation both in
the gluonic and matter sector by looking at the thermodynamic relation of energy density \( \epsilon = -\partial_{\beta} \ln(Z) \). On thus find that the effective fugacities modify the single quasi-partron energy as follows,

\[
\begin{align*}
\omega_g &= p + T^2 \partial_T \ln(z_g) \\
\omega_q &= p + T^2 \partial_T \ln(z_q) \\
\omega_s &= \sqrt{p^2 + m^2} + T^2 \partial_T \ln(z_q).
\end{align*}
\]

(2)

This leads to the new energy dispersion for gluons (\( \omega_g \)), light-quark antiquarks (\( \omega_q \)) and strange quark-antiquarks, (\( \omega_s \)). These dispersion relations can be explained as follows. The second term in the right-hand side of Eq. (2), is like the gap in the energy-spectrum due to the presence of quasi-particle excitations. This makes the model more in the spirit of the Landau’s theory of Fermi-liquids. A detailed discussions regarding the interpretation and physical significance of \( z_q \) and \( z_q \) is discussed at a length in \[18, 28\]. Note, that the quasi-particle model is discussed at a length in \[28\].

B. Modification to the thermal distributions

Shear and bulk viscosities are essential to understand space-time evolution of the QGP during its hydrodynamic expansion. Physically, shear viscosity accounts for the entropy production during the anisotropic expansion of the system maintaining its volume constant, on the other hand bulk viscosity accounts for the entropy production while the volume of the system changes at constant rate (isotropic expansion). Since these transport coefficients are related to the non-equilibrium properties of the fluid, this requires to go beyond the equilibrium modeling of the fluid within linear response theory.

The general linear response (Chapman-Enskog) formalism assumes a small perturbation of the thermal equilibrium distribution (considering the small perturbation around the equilibrium distributions of the quark-antiquarks and gluons) as:

\[
f(\vec{p}, r) = f_0(\vec{p}) + \delta f.
\]

(3)

where

\[
f_0(\vec{p}) = \frac{z_g \exp(-\beta u^\mu p_\mu)}{1 + z_q \exp(-\beta u^\mu p_\mu)},
\]

de note the local thermal equilibrium distribution function in Eq. (5) in the absence of viscous effects and \( f_1 \) is the linear perturbation which encodes the viscous effects as described below. Here, \( g \) stand for quasi-gluons and \( q \) for quasi-quarks (we have also neglected the mass of the strange quark which is justified at high temperature), \( u^\mu \) is the 4-velocity of the fluid and \( \beta = 1/T \). The isotropic distribution, \( f_0(\vec{p}) \) reduced to \( f_0 \) in the local rest frame of the fluid (LRF). Again, the plus sign is for gluons and minus sign is for the quarks.

Now, using \( T \partial f_0 / \partial (u^\mu p_\mu) = -f_0(1 \pm f_0) \), the linear perturbation \( \delta f \) can be expressed as \[17\]:

\[
f(\vec{p}) = f_0(\vec{p}) + f_0(p)(1 \pm f_0(p)) f_1(\vec{p}).
\]

(5)

The perturbation \( f_1 \equiv \{ f_1g, f_1q \} \) (combined notation for quarks and gluons) can be thought of as a change in the argument of \( f_0 \) as \( (1 \pm f_0) \rightarrow (1 \pm f_0 - f_1(\vec{p}, r)) \) \[17\], and can be thought of as a local fugacity factor leading to following form of the near-equilibrium distributions:

\[
\begin{align*}
f_0(\vec{p}) &= \frac{z_g \exp(-\beta u^\mu p_\mu + f_1g)}{1 - z_g \exp(-\beta u^\mu p_\mu + f_1g)} \\
f_0(\vec{p}) &= \frac{z_q \exp(-\beta u^\mu p_\mu + f_1q)}{1 + z_q \exp(-\beta u^\mu p_\mu + f_1q)}
\end{align*}
\]

(6)

Note that Eq. (5) is obtained by expanding Eq. (6) and keeping only the linear term in the perturbation, \( f_1 \). Next, we discuss the Energy-Momentum tensor for the QGP fluid obtained from these distribution functions that is essential for determining the form of \( f_1 \) in terms of shear and the bulk viscosities.

1. Energy-momentum tensor

Importantly, the kinetic theory definition of the energy-momentum tensor \( T^{\mu\nu} \) obtained by employing the above expression for the \( f(\vec{p}) \), as a requirement for the continuity of \( T^{\mu\nu} \), should reproduce the correct hydro-dynamical decomposition of the \( T^{\mu\nu} \). To achieve this requirement, we have to closely look at the the kinetic theory definition of \( T^{\mu\nu} \) and appropriately revise it. Notably, the expression of \( T^{\mu\nu} \) thus obtained in terms of \( f(\vec{p}) \) must capture the hot QCD medium effects in terms of non-trivial dispersion relations and the effective fugacities.

The energy density and the pressure can be obtained in terms of quasi-gluons and quasi-quarks in our quasi-
particle model \([23]\) as,
\[
\epsilon = \int \frac{d^3\overrightarrow{p}}{8\pi^3} \left( v_g \omega_g f_g^{eq} + v_q \omega_q f_q^{eq} \right)
\]
\[
\mathcal{P} = -\frac{1}{\beta} v_g \int \frac{d^3\overrightarrow{p}}{8\pi^3} \ln(1 - z_g \exp(-\beta p))
\]
\[
\quad + \frac{1}{\beta} v_q \int \frac{d^3\overrightarrow{p}}{8\pi^3} \ln(1 + z_q \exp(-\beta p)).
\]
(7)

We use the notation \(\nu_q = 2(N_c^2 - 1)\) for gluonic degrees of freedom, \(\nu_q = 4 \times 2 \times N_c \times 2\) for quarks in the present case.

In kinetic theory \(T^{\mu \nu}\) is obtained from the single particle momentum distributions as,
\[
T^{\mu \nu} = \sum_{g,q} \int \frac{d^3\overrightarrow{p}}{(2\pi)^3} \omega f(\overrightarrow{p}) \mu^{\mu} \nu^{\nu},
\]
(8)

It is emphasized in \([13]\), the above expression of \(T^{\mu \nu}\) can not simply be utilized in the present case, since it does capture correctly the non-trivial dispersions of quasi-particles. In other words, the thermodynamic consistency condition is not satisfied with this expression of \(T^{\mu \nu}\) yielding incorrect expressions for energy density and the pressure.

This issue has recently been addressed in \([13]\) by arguing for a modified form of the \(T^{\mu \nu}\), in the similar spirit as it is done in the effective mass quasi-particle models \([12]\):
\[
T^{\mu \nu} = \sum_{g,q} \left\{ \int \frac{d^3\overrightarrow{p}}{2(2\pi)^3} \omega p^{\mu} p^{\nu} f(\overrightarrow{p}) + \int \frac{d^3\overrightarrow{p}}{2(2\pi)^3} \omega p^{\mu} f_0(p) \right\} + \int \frac{d^3\overrightarrow{p}}{(2\pi)^3} \omega p^{\mu} u^{\nu} f_0(p),
\]
(9)

One can clearly realize the presence of the factors, \(T^2 \delta^{\mu \nu}\) and \(T^2 \pi^{\mu \nu}\) in the expression for \(T^{\mu \nu}\) in Eq. \(9\). The second term in the right-hand side of Eq. \(9\) ensures the correct expression for the pressure, and the third term ensures the correct expression for the energy density, and hence the definition of \(T^{\mu \nu}\) incorporates the thermodynamic consistency condition correctly. In view of the reliability of the quasi-particle descriptions of hot QCD for temperature beyond the QCD transition temperature, we may ignore the strange quark mass effects. In this case the QGP can be described by massless quasi-gluons, and massless quasi-quarks having non-trivial dispersion relations. Therefore, in Eq. \(9\), \(\omega \equiv (\omega_g, \omega_q)\), and summation is over the gluons and quarks.

On the other hand, the fluid dynamic definition of \(T^{\mu \nu}\) in the presence of shear and bulk viscous effects is given as,
\[
T^{\mu \nu} = \epsilon u^{\mu} u^{\nu} - (p + \Pi) \delta^{\mu \nu} + \pi^{\mu \nu},
\]
(10)

where \(\Pi\) and \(\pi^{\mu \nu}\) are the shear and bulk part of the viscous stress tensor.

The form of the perturbations \(f_1\) to the thermal distributions of gluons and quarks can be obtained in terms of the \(\Pi\) and \(\pi^{\mu \nu}\) by relating the two definitions (kinetic theory and fluid dynamic) of the \(T^{\mu \nu}\). The two definitions can be matched through the following quadratic ansatz for \(f_1(\overrightarrow{p})\) \([12]\),
\[
f_1(\overrightarrow{p}) = \frac{1}{(\epsilon + P)T^2} \left( \frac{p^{\mu} p^{\nu}}{2} C_1 \pi^{\mu \nu} + \frac{C_2}{5} p^{\mu} p^{\nu} \delta_{\mu \nu} \Pi \right),
\]
(11)

where the coefficients \(C_1\) and \(C_2\) are obtained by the matching of the two definitions of \(T^{\mu \nu}\) in the local rest frame of the fluid (LRF). This follows from the fact that shear and bulk viscosities are Lorentz invariant quantities and can conveniently be obtained in the LRF of the fluid.

Next, utilizing the notations in Eq. \(4\), and matching right-hand sides of Eq. \(9\) and Eq. \(11\) in the LRF, we obtain,
\[
\Pi \delta^{ij} + \pi^{ij} = \frac{\nu_q}{ST^3} \int \frac{d^3\overrightarrow{p}}{(2\pi)^3} \omega p^{i} p^{j} f_q(1 + f_q) \times \left( C_1 \Pi_{lm} + \frac{C_2}{5} \Pi \right)
\]
\[
- \frac{\nu_g}{ST^3} \int \frac{d^3\overrightarrow{p}}{(2\pi)^3} \omega p^{i} p^{j} f_g(1 - f_g) \times \left( C_1 \Pi_{lm} + \frac{C_2}{5} \Pi \right).
\]
(12)

The integral over the momentum in the above equations can be expressed as in \([12]\): \(I_{g,q} (\delta^{ij} \delta^{lm} + \delta^{ij} \delta^{jm} + \delta^{im} \delta^{jl})\) (the subscripts \(g\) and \(q\) are used to distinguish the gluonic and the matter sector), where
\[
I_g = \int \frac{d^3\overrightarrow{p}}{8\pi^3} \omega_p p^{i} f_g(1 + f_g)
\]
\[
I_q = \int \frac{d^3\overrightarrow{p}}{8\pi^3} \omega_p p^{i} f_g(1 - f_g)
\]
(13)

Now, from Eq. \(12\) in the gluonic sector,
\[
C_1 = C_2 = \frac{1}{I_g},
\]
(14)

and in the matter sector,
\[
C_1 = C_2 = \frac{1}{I_q}.
\]
(15)

The viscous modified thermal distributions of gluons and quarks in the QGP in terms of \(I \equiv (I_{g,q})\),
\[
f(\overrightarrow{p}) = f_{eq} + \frac{f_{eq}}{ST^3} \left( \frac{p^{\mu} p^{\nu}}{2l} \pi^{\mu \nu} + \frac{p^{\mu} p^{\nu} \Delta_{\mu \nu} \Pi}{5l} \right).
\]
(16)
As mentioned earlier, $f_{eq} \equiv (f_g, f_q)$.

Let us discuss the validity of the above expression of the viscous modified thermal distributions. The validity criterion is simply $(f - f_{eq}) \ll f_{eq}$ (near equilibrium condition). In other words, for the validity of our formalism, the viscous corrections ($\pi^{\mu \nu}$ and $\Pi$) must induce small corrections to the equilibrium distribution of the gluons and quarks. This translates to the condition,

$$\frac{p^\mu p^\nu \pi_{\mu \nu}}{2} + \frac{p^\mu p^\nu \delta_{\mu \nu} \Pi}{5} \ll ST^3 (1 \pm f_{eq}) I.$$  \hfill (17)

Next, we consider a case, where the integral displayed in Eq. \[13\] can be solved analytically. In the limit $T^2 \partial f(z_{q,g}) / \partial p \ll 1$ (high temperature limit), we can obtain analytic expressions for $I_g$ and $I_q$ as,

$$I_g = \frac{4\nu_0 T^3}{\pi^2 S} \text{PolyLog}[5, z_g]$$

$$I_q = -\frac{4\nu_0 T^3}{\pi^2 S} \text{PolyLog}[5, -z_q].$$ \hfill (18)

The PolyLog[n, x] function appearing in Eq. \[13\] is having the series representation, $\text{PolyLog}[n, x] = \sum_{k=1}^{\infty} \frac{x^k}{k^n}$ (convergence of the series is subject to the condition that $|x| \leq 1$). The Stefan-Boltzmann (SB) limit (employment of ideal QGP EoS) is obtained only asymptotically (by putting $z_g \equiv 1$) in right-hand side of Eq. \[13\]. It can easily be seen that $I_g$ and $I_q$ are of the order of unity in the case of ideal EoS. This is also realized in \[12\]. To see the difference in these two cases, we plot the quantities, $I_{gg} = \frac{15 z_g^2 S}{4 \pi^2 \nu_0}$ and $I_{qq} = \frac{15 I_q z_q^2 S}{4 \pi^2 \nu_0}$ for the ideal QGP EoS, and (2+1)-flavour lattice QCD EoS (temperature dependence of $z_g$ and $z_q$ are taken from Ref. \[28\] in Fig. \[11\]). Here, we use the identities $\text{PolyLog}[5, 1] = \zeta(5)$, and $-\text{PolyLog}[5, -1] = \frac{11}{12} \zeta(5)$ to obtain $I_g$ and $I_q$ in the case of ideal EoS.

Clearly $I_{gg}$ and $I_{qq}$ will approach to their SB limit that is $\zeta(5)$ asymptotically. The interaction effects are significant even at $3.5 T_c$. Therefore, one can not simply ignore these effects while obtaining the viscous modified forms of the thermal distributions of gluons and quarks in the QGP medium. This crucial observation has been realized in the case of effective mass quasi-particle model in \[12\].

Next, we shall investigate the significance of such viscous modified thermal distributions of gluons and quarks in the context of dilepton production.

III. DILEPTON PRODUCTION VIA $q\bar{q}$ ANNihilation

The dilepton production in the QGP medium has dominant contributions from the $q\bar{q}$ annihilation process via the mechanism, $q\bar{q} \rightarrow \gamma^* \rightarrow l^+ l^-$. The kinetic theory expression for the dilepton production rate for a given dilepton mass and momentum is given by \[20\],

$$\frac{dN}{d^2xd^4p} = \int \int d^4p_1 \frac{d^4p_2 \sigma(M^2)}{(2\pi)^4} f(E_1, T) f(E_2, T) \times \frac{M^2 g^2 \sigma(M^2)}{2E_1E_2} \delta^4(P - p_1 - p_2),$$ \hfill (19)

where the 4-momenta $p_{1,2} = (E_{1,2}, \vec{p}_{1,2})$ are of quark and anti-quark respectively with $E_{1,2} = \sqrt{p_{1,2}^2 + m^2} \approx |\vec{p}_{1,2}|$, if one neglects the quark masses. The quantity $M^2 = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2$ is the invariant mass of the intermediate virtual photon. The quantity $f(E, T)$ is the quark (anti-quark) distribution function in thermal equilibrium, $f(E, T) = \frac{1}{1 + e^{|E| / T} - 1}$ (this form is in view of the effective quasi-particle model based on realistic QGP EoS). In the case of ideal QGP EoS the factor $z_q$ will be replaced by unity, as done in most of works on dilepton production in the QGP medium in the literature. As we shall argue that the EoS effects are quite significant even if we take high temperature limit of quark (anti-quark) distribution functions. Recall from the previous section that the realistic EoS strongly influence the viscous modified portion of the thermal distributions of gluons, and quarks (anti-quarks).

Here, $g$ is the degeneracy factor, and $\sigma(M^2)$ is the thermal dilepton production cross section. Here, $P = p_0 = E_1 + E_2$, $\vec{p} = \vec{p}_1 + \vec{p}_2$ is the 4-momentum of the dileptons. In the present analysis we are interested in the invariant masses that are larger compared to the temperature, $T$. In this limit, we can take the high temperature limit of quark (anti-quark) equilibrium thermal distribution functions (replacing Fermi-Dirac distribution with...
classical Maxwell-Boltzmann distribution in the case of Ideal QGP EoS as
\[
f(E, T) \rightarrow z_q \exp(-\frac{E}{T}),
\]
where \(E = |\vec{p}| \equiv p\). The form will remain the same for the quarks and antiquarks since the baryon chemical potential is zero here. It is straightforward to observe from Eq. (19) that the effects coming from the EoS are of the order \(z_q^2\) (this quantity is quite significant even at 2\(T_c\)). In other words, the dilepton production rate is modulated by a factor \(z_q^2\). Let us now proceed to explore the impact of EoS and the viscous modifications to the dilepton production rate.

Next, we employ the result obtained in Eq. (16) for the viscous modified quark (antiquark) distribution function \(f(\vec{p})\), and take its high temperature limit, and analyze shear and bulk viscous contributions one by one. In this limit, the viscous modified quark (anti-quark) distribution functions become,
\[
f(\vec{p}) = z_q \exp(-\frac{\mu}{T}) \left[ 1 + \frac{(1 - z_q \exp(-\frac{\mu}{T}))}{5} \right].
\]

Note that the first term in the above equation accounts for the equilibrium part of the quark (anti-quark) thermal distribution, the second encodes the shear viscous effects, and the third one encodes the bulk viscous effects. Our aim here is not to make any quantitative statements regarding the impact of \(\eta\) and \(\zeta\), rather highlight the role of realistic EoS (non-trivial dispersion relations) to the dilepton rate. The former needs a complete study coupling the analysis with the hydrodynamic evolution of the QGP and it will be taken up in the near future.

Now, the effects of viscosities on the production rate of dileptons, we employ Eq. (21) to Eq. (19), and rewrite the dilepton production rate in the Component form as,
\[
\frac{dN}{d^4x dp} = \frac{dN^{(0)}}{d^4x dp} + \frac{dN^{(\eta)}}{d^4x dp} + \frac{dN^{(\zeta)}}{d^4x dp}.
\]

The notations \(\eta\) and \(\zeta\) are introduced since \(\pi^{\mu \nu}\), \(\Pi\) involve them as the first order transport coefficient in their definitions. These three terms in Eq. (22) have already been computed for the Ideal QGP EoS in [43, 54], and straight-forward to compute in our case (difference are there in the definition of the distribution functions). The first term is given by the following integral,
\[
\frac{dN^{(0)}}{d^4x dp} = \int \int \frac{d^3\vec{p}_1}{(2\pi)^3} \frac{d^3\vec{p}_2}{(2\pi)^3} z_q^2 \exp(-\frac{E_1 + E_2}{T}) \times M^2 g^2 \sigma(M^2) \frac{2}{E_1 E_2} \delta(p - p_1 - p_2).
\]

This integral is well known in the literature in the case of \(z_q = 1\). Since \(z_q\) is independent of the of the momentum of the particles, so the integral can be evaluated in the same way as [53].

\[
\frac{dN^{(0)}}{d^4x dp} = \frac{z_q^2}{2} \frac{M^2 g^2 \sigma(M^2)}{2\pi^3} \exp(-\frac{p_0}{T}).
\]

The modification to rate due to the shear viscosity (at first order \(\pi^{\alpha \beta} \equiv 2\eta \sigma^{\alpha \beta} = 2\eta\), where \(\sigma^{\alpha \beta}\) is the Navier-Stokes tensor) can be obtained from the following equation,
\[
\frac{dN^{(\eta)}}{d^4x dp} = \int \int \frac{d^3\vec{p}_1}{(2\pi)^3} \frac{d^3\vec{p}_2}{(2\pi)^3} z_q^2 \exp(-\frac{E_1 + E_2}{T}) \times M^2 g^2 \sigma(M^2) \left[ \frac{\eta}{214} ST^3 \right] \times \delta(p - p_1 - p_2).
\]

Following the analysis of [27], we obtain the following expression for the shear viscous correction of the rate,
\[
\frac{dN^{(\eta)}}{d^4x dp} = \frac{z_q^2}{4\nu_4 T^3 PolyLog[5, -z_q]/\pi^2 S} \times M^2 g^2 \sigma(M^2) \exp(-\frac{\eta}{T}) \times \frac{2}{3} \left[ \frac{\eta}{2ST^3} \pi^0 p^0 \eta \sigma_{\alpha \beta} \right].
\]

Now, the third term which is the correction to the rate due to the bulk viscosity (at first order, \(\Pi \equiv -\zeta \Theta\), where \(\Theta = \partial_\alpha u^\alpha\) is the expansion rate of the fluid) can be evaluated from the following expression,
\[
\frac{dN^{(\zeta)}}{d^4x dp} = \int \int \frac{d^3\vec{p}_1}{(2\pi)^3} \frac{d^3\vec{p}_2}{(2\pi)^3} z_q^2 \exp(-\frac{E_1 + E_2}{T}) \times M^2 g^2 \sigma(M^2) \left[ \frac{2\zeta}{10ST^3} \right] \times \delta(p - p_1 - p_2).
\]

This integral can be evaluated using the analysis of [35] as,
\[
\frac{dN^{(\zeta)}}{d^4x dp} = \frac{z_q^2}{4\nu_4 T^3 PolyLog[5, -z_q]/\pi^2 S} \times M^2 g^2 \sigma(M^2) \exp(-\frac{\eta}{T}) \times \frac{2}{3} \left[ \frac{2\zeta}{10ST^3} \pi^0 p^0 \eta \sigma_{\alpha \beta} \Theta \right] - \frac{2\zeta}{5} \frac{\eta}{4ST^3} M^2 \Theta.
\]

The full expression for the rate displayed in Eq. (23) can be obtained by combining Eq. (22). These expressions reduces to the those obtained in [20] (the expressions obtained by employing the ideal QCD EoS) by substituting \(\zeta_q = 1\) (in this case \(\nu_4 \approx 1\) as already described in [12]).

If we ignore the viscous corrections, it is obvious that the EoS induced modifications appear as a factor, \(z_q^2\).
FIG. 2. Behavior of $z_q^2$ as function of $T/T_c$ is shown along with its SB limit ($z_q \to 1$). The temperature dependence of the effective quark fugacity, $z_q$ is taken from Ref. [28].

On the other hand, the shear and bulk viscous corrections to dilepton production rate gets a factor of

$$R_q = \frac{-z_q^2 \pi^2}{\eta_q \text{Polylog}[5, -z_q]}$$

(whose SB limit is $16/15\zeta(5)$), as a modification from the EoS. We have plotted both of these factors, employing the quasi-particle model for (2+1)-flavor QCD [28] in Fig. 2 and Fig. 3. On looking at the temperature behavior of both these factors, we can safely say that all the three terms in the dilepton rate in Eqs. (24), (26), and (28) get significant modifications from the QGP EoS. From Figs. 2 and 3 both $z_q^2$ and $R_q$ approach their respective SB limit only asymptotically.

Let us discuss the interesting observations that can be made out, based on the results of the dilepton production rate obtained in the viscous environment and the realistic EoS. Since the quantities, $\eta$ and $\zeta$ are the phenomenological parameters for the QGP, and assumed to be same for realistic and ideal QGP EoSs. Employing the realistic EoS for the QGP. As an implication, the impact of them is demonstrated on the dilepton production via $q\bar{q}$ annihilation in RHIC. The realistic QGP EoS also induces significant modifications to the viscous modified thermal distributions of the gluons and quark-antiquarks that constitute the QGP. The effects are equally significant in deciding the dilepton production rate in the viscous QGP medium. In particular, even in the high temperature regime, where the hot QCD medium effects are weaker, the realistic EoS and viscosities play crucial role.

Finally, coupling the present analysis to the relativistic hydrodynamic evolution of the QGP and impact of the temperature dependence of the shear and bulk viscosities on the dilepton production rate will be matters of future investigation. In this concern, it would be of interest to exploit a recently reported ECHO-QGP code [36] which is a (3+1)-d relativistic viscous hydro code for studying the physics of the QGP in the near future.

IV. CONCLUSIONS

In conclusion, the form of viscous modified thermal distribution functions for quasi-quarks and quasi-ghons are obtained in the QGP medium by systematically employing the realistic EoS for the QGP. As an implication, the impact of them is demonstrated on the dilepton production via $q\bar{q}$ annihilation in RHIC. The realistic QGP EoS also induces significant modifications to the viscous modified thermal distributions of the gluons and quark-antiquarks that constitute the QGP. The effects are equally significant in deciding the dilepton production rate in the viscous QGP medium. In particular, even in the high temperature regime, where the hot QCD medium effects are weaker, the realistic EoS and viscosities play crucial role.

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