On The Low Frequency Quasi Periodic Oscillations of X-ray Sources

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Abstract. Based on the interpretation of the twin kilohertz Quasi Periodic Oscillations (kHz QPOs) of X-ray spectra of Low Mass X-Ray Binaries (LMXBs) to the Keplerian and the periastron precession frequencies at the magnetosphere-disk of X-ray neutron star (NS) respectively, we ascribe the low frequency Quasi Periodic Oscillations (LFQPO) and HBO (15-60 Hz QPO for Z sources or Atoll sources) to the periastron precession at some outer disk radius. The obtained conclusions include: all QPO frequencies increase with increasing the accretion rate. The obtained theoretical relations between HBO (LFQPO) frequency and the kHz QPO frequency are similar to the measured empirical formula. Further, the possible dynamical mechanism for QPO production is discussed.

Key words. X–rays: accretion disks — stars: neutron — X–rays: stars

1. Introduction

The QPO mechanisms of LMXB have been paid much attention with the discovery of kHz QPOs by the Rossi X-ray Timing Explorer (RXTE) since early 1996, briefly after its launch (van der Klis 1998, 2000, Wijnands 1999). The Z sources (Atoll sources), which are high (less) luminous neutron-star low-mass X-ray binaries (Hasinger & van der Klis 1989), typically show four distinct types of QPOs (van der Klis 1998, 2000). At present, these are the normal branch oscillation (NBO) $\nu_{\text{NBO}} \approx 5 - 20$ Hz, the horizontal branch oscillation (HBO) $\nu_{\text{HBO}} \approx 15 - 60$ Hz, and the kHz QPOs $\nu_{\text{kHz}} \approx 200 - 1200$ Hz that typically occur in pairs, upper frequency $\nu_{2}$ and lower frequency $\nu_{1}$. In several Atoll sources, nearly coherent burst oscillations $\nu_{\text{burst}} \approx 330 - 500$ Hz have also been detected during thermonuclear Type I X-ray bursts, which are considered as the spin frequency of NS or twice of them (see e.g., Strohmayer et al. 1996). Moreover, the low-frequency (0.1 - 10 Hz) QPOs have been detected in the black hole X-ray binaries, as well as three cases with higher frequencies, $\approx 67$ Hz in GRS 1915+105, $\approx 300$ Hz in GRO 1655-40 and $\approx 160 - 220$ Hz in XTE J1550-564, and the QPO properties of black-hole X-ray binaries show many similarities with those of neutron-star sources (see, e.g., Psaltis et al. 1998, 1999). All of these QPOs but the burst oscillation have centroid frequencies that increase with the inferred mass accretion rate $M$. Furthermore, the frequencies $\nu_{2}$ and $\nu_{1}$, as well as the frequencies $\nu_{2}$ and $\nu_{\text{HBO}}$ follows very similar relations in five Z sources, and the QPO frequencies of LMXB and BHC follow a tight and systematical correlation over three orders of magnitude in frequency (Psaltis et al. 1998, 1999).

However the frequency separation between the upper and the lower kHz QPOs $\Delta \nu \equiv \nu_{2} - \nu_{1}$ remains approximately constant, close to the inferred spin frequency of the neutron star, but however decreases systematically with instantaneous $M$ in some cases, e.g. Sco X-1 (van der Klis 1997), 4U1608–52 (Mendez et al. 1998a,b), 4U1735-44 (Ford et al. 1998) and 4U1728–34 (Mendez and van der Klis 1999), then in the latter the observed coherent burst frequency 364 Hz is higher than its maximum $\Delta \nu \sim 355$ Hz (Mendez and van der Klis 1999).

Various theoretical models have been proposed to account for the QPO phenomena in X-ray NS systems. For the high frequency of kHz QPOs and its proportional relation to the accretion rate, the upper kHz QPO $\nu_{2}$ is simply considered to originate from the Keplerian orbital frequencies at the preferred radius close to the compact object, which exhibit the inner accretion flows, however the lower kHz QPO $\nu_{1}$ is attributed to the beat of such frequency with the stellar spin $\nu_{s}$ (Miller et al. 1998). Recently, the general relativistic effects are invoked to account for kHz QPOs by Stella and Vietri (1999) (SV99 for short), which can explain the varied kHz QPOs separation $\Delta \nu$. However, it is also a common phenomenon for the separation $\Delta \nu$ to be approximately constant. The
range of \( \Delta \nu \) is also quite narrow across different sources (\( \Delta \nu \approx 250 - 360 \) Hz) with a nearly coherent frequency of \( \nu_{\text{burst}} \approx 330 - 590 \) Hz. In some cases (such as 4U 1702-43 and KS 1731-260) the burst frequency is consistent, to within the errors, with the frequency separation \( \Delta \nu \), or twice its value \( 2\Delta \nu \). So the approximately constant separation in some sources seems also to make the purely GR precession model in difficulty. Although many other viable new ideas are also proposed (see Psaltis & Norman 2000, Osherovich & Titarchuk 1999, Titarchuk et al. 1998, Klein et al. 1996), there has not yet been any model satisfactorily to explain all observed QPO phenomena of LMXBs until now. The mechanisms of kHz QPOs of LMXBs are still debated and open problems.

Moreover, HBO frequency \( (\nu_{\text{HBO}} \approx 15 - 60 \) Hz), first discovered in GX 5-1 in 1985, is interpreted to be the beat frequency between the Keplerian frequency of the magnetosphere-disk and the stellar spin frequency by the standard beat frequency model (BFM) (Alpar and Shaham 1985, Lamb et al. 1985). Later, it was considered to be the nodal precession of Lense-Thirring effect in the disk (Vetri and Stella 1998).

In the model of SV99 (Stella & Vietri 1999), the twin kHz QPOs are ascribed to the Keplerian frequency and the periastron frequency of material orbiting the neutron star at some disk radius, i.e., \( \nu_2 = \nu_K = (M/4\pi^2r^3)^{1/2} \) and \( \nu_1 = \nu_K[1 - (1 - 6M/r^2)^{1/2}] \), where \( r \) is the Schwarzschild coordinate distance and \( M \) is the gravitational mass of neutron star (We set the unity of the speed of light and the gravitational constant \( c = G = 1 \) throughout this paper). It is commonly believed that the kHz QPOs are produced close to the innermost stable circular orbit (ISCO) or the surface of NS, which provides a probe to detect the accretion flow around the non-Newtonian strong gravity region (van der Klis 2000, Stella & Vietri 1999).

In this article, we concentrate on the explanation of LFQPO and HBO (for Atoll sources, 15 - 60 Hz QPO is supposed to be the same mechanism as HBO of Z sources (see also Psaltis et al. 1999)), and their relations to the twin kHz QPOs, then neglect the detail of the physical mechanism for QPO production at the present stage.

2. The Model

The neutron star magnetosphere and the accretion flow are complicated, especially close to the innermost stable circular orbit (ISCO) \( R_I = 3R_s \), where \( R_s = 2M \sim 3m \) (km) (one solar mass corresponds to about 3 km of the Schwarzschild radius) is the Schwarzschild radius calculated with the gravitational mass \( M \) (\( m = M/M_\odot \) is the mass in unit of solar mass). The motion of the disk matter will be influenced not only by gravitational field but also by the magnetic field. For simplicity, we assume that the disk matter to exhibit QPO are mainly dominated by the Schwarzschild gravitational field with a slightly eccentric orbit \( (e \simeq 0) \). Therefore, these QPO frequencies are conveniently arranged as follows through defining the parameter \( y = \frac{\nu_2}{\nu_0} \), which is the ratio of the Schwarzschild radius to the instantaneous disk radius.

Here we assume that HBO \( (\nu_{\text{HBO}} \approx \nu_{QPO}) \) is a periastron precession frequency of the accreted orbiting materials at some outer disk boundary, and the twin kHz QPOs are ascribed to the Keplerian frequency and the periastron frequency of the orbiting material at some inner disk radius. There exists a scaling factor to connect two radii, which will be determined by the well consistent data of kHz QPO and HBO (LFQPO).

\[
\nu_1(y) = (1 - \sqrt{1 - 3y}) \times \nu_2(y),
\]
\[
\nu_2(y) = \nu_0 y^{3/2},
\]
\[
\nu_{\text{HBO}}(y) = \nu_1(\phi y) = \nu_0 (1 - \sqrt{1 - 3\phi y}) \times (\phi y)^{3/2},
\]
\[
\nu_0 \equiv \frac{11300}{m} \text{ (Hz)},
\]

where \( r = 1.9 \times 10^6 \text{ (cm)} B_8^{1/7} M_{17}^{-2/7} m^{1/7} R_6^{12/7} \) is the radius of NS magnetosphere-disk (see e.g., Shapiro and Teukolsky 1983, Frank et al. 1992), which is inversely related to the accretion rate and proportionally related to the magnetic field strength of the star, where \( M_{17} \), \( B_8 \) and \( R_6 \) are the mass accretion rate, the star magnetic field and the radius in unit of \( 10^{17} \text{ g/s} \), \( 10^8 \text{ G} \) and \( 10^6 \text{ cm} \) respectively. \( \phi \) is a scaling parameter to connect the radii of the NS magnetosphere-disk and the outer disk, but here we suppose it to be \( 0.3 \sim 0.4 \) for the reason of the best consistent.

From Eq.(3) and Eq.(4), the parameter \( y \) can be expressed as follows,

\[
y = \frac{\nu_1}{\nu_2} \left( \frac{2 - \nu_1}{\nu_2} \right), \quad y = 0.2 \left( \frac{\nu_2}{1000} \right)^{3/2},
\]

or from Eq.(5), the parameter \( y \) can also be approximately acquired by means of the Taylor expansion to the first order of \( \phi y \) because the maximum value of \( \phi y \) is \( 0.4/3 \sim 0.13 \),

\[
y \simeq \phi^{-1} \left( \frac{2\nu_{\text{HBO}}}{3\nu_0} \right)^{2/5}.
\]

At the innermost stable circular orbit (ISCO), the parameter \( y = \frac{1}{3} \) and the twin kHz QPO frequencies are given as \( \nu_1(1/3) = \nu_2(1/3) = 2175(\text{Hz})/m \), corresponding to 1200 (Hz) for 1.9 solar masses. The parameter \( y \) can also be written as \( y = \frac{R}{R_{\text{ISCO}}} = \left( \frac{R}{R_{\text{ISCO}}} \right) M_{17}^{2/7} \), where \( M_x = M/(M) \) is the ratio between the instantaneous accretion rate \( \dot{M} \) and the long-term averaged accretion rate \( \langle \dot{M} \rangle \), and the latter determines the corotation magnetosphere radius \( R_{\text{ISCO}} = 1.9 \times 10^6 (\text{cm}) B_8^{1/7} M_{17}^{-2/7} m^{1/7} R_6^{12/7} \) with the nearly unity fastness (see e.g., Shapiro and Teukolsky 1983, Frank et al. 1992), where \( \langle \dot{M} \rangle_{17} \) is the long-term averaged mass accretion rate in the unit of \( 10^{37} \text{ g/s} \).

The relations \( \nu_{\text{HBO}} \) vs. \( \nu_2 \) is plotted in FIG.1, together with the well measured five Z-source samples, and it is
shown that the agreement between the model and the observed QPO data is quite well for the selected values of the NS mass about 2.0 $M_\odot$ and $\phi = 0.4$, which are the free parameters in the Eqs.(1-3).

In FIG.2, we take the low-frequency QPO $\nu_{QPO}$ of Atoll-sources, as well as several other neutron star system and a number of the black hole binaries as the similar mechanism to HBO of the Z-sources (Psaltis et al. 1999), and plot the theoretical curve with respect to lower kHz QPO. We find that the remarkably consistent correlations between the theoretical expectation and the detected data is obtained, which is extending over nearly three orders of magnitude in the QPO frequency. However one interesting fact is that the theoretical curves are weakly related to the star masses because $\nu_1$ and $\nu_{QPO}$ are all inversely related to the NS mass from Eqs.(1) and (3). Their ratio $\nu_1/\nu_{QPO} = \phi^{-3/2}(1 - \sqrt{1 - 3\phi})/(1 - \sqrt{1 - 3\phi})$ has no direct correlation with star mass, and further we have, for $\phi = 0.4, 1 \leq \text{LOG}(\nu_1) - \text{LOG}(\nu_{QPO}) \leq 1.25$, which means that there is no much variation in $\nu_{QPO}$ vs. $\nu_1$ plot even if mass changes one magnitude order. We plot the theoretical curves with the parameter conditions $m = 5.0 M_\odot$ and $\phi = 0.4$ (short dash line), as well as $m = 10.0 M_\odot$ and $\phi = 0.4$ (long dash line), and find that the theoretical curves are insensitive to the mass parameter. However the mass parameter will influence on the maximum frequency of the lower kHz QPO (at ISCO) because it is inversely related to mass. For example, if 300 Hz QPO in GR 1655-40 were the maximum frequency, then the mass of this source would be 7.0 solar masses. It is also the fact that $\nu_{\text{HBO}}$ increases slower with $\nu_1$ when $\nu_1$ is higher.

The weak correlation to the mass of the source (as well as to the magnetic field strength) in $\nu_{QPO}$ vs. $\nu_1$ plot seems to hint that the QPO phenomena of Atoll sources, Z-sources, as well as other neutron star system and black hole binaries are intimately related to the specific radii of the disk and the ratio between them, which may reflect the common property of the accretion flow around the gravitational sources. However the mechanism to account for this is unclear.

From Eqs.(1), (3) and (3), we can derive the theoretical relations between QPO frequencies in the following,

$$\nu_{\text{HBO}} \simeq 500 \phi^{5/2} (Hz) \left( \frac{\nu_1}{500} \right) \left[1 - 0.19(1 - \phi) \left( \frac{m \nu_1}{500} \right)^{2/5} \right],$$

$$\nu_{\text{HBO}} \simeq 297.3 \phi^{5/2} (Hz) \left( \frac{\nu_1}{1000} \right)^{5/3} \left[1 + 0.15 \phi \left( \frac{m \nu_2}{1000} \right)^{2/3} \right],$$

$$\nu_1 \simeq 297.3 \left( \frac{\nu_2}{1000} \right)^{5/3} \left[1 + 0.15 \left( \frac{m \nu_2}{1000} \right)^{2/3} \right].$$

If the orbit scaling factor $\phi = 0.4$ is set, we have the following relations,

$$\nu_{\text{HBO}} \simeq 50.6 (Hz) \left( \frac{\nu_1}{500} \right) \left[1 - 0.11 \left( \frac{m \nu_1}{500} \right)^{2/5} \right],$$

$$\nu_{\text{HBO}} \simeq 30.1 \left( \frac{\nu_1}{1000} \right)^{5/3} \left[1 + 0.06 \left( \frac{m \nu_2}{1000} \right)^{2/3} \right].$$

As a comparison, we list the empirical relations between HBO and twin kHz QPO frequencies (Psaltis et al. 1998, Psaltis et al. 1999),

$$\nu_{\text{HBO}} \simeq (42 \pm 3 \ Hz)(\nu_1/500 \ Hz)^{0.95 \pm 0.16},$$

$$\nu_{\text{HBO}} \simeq 13.2 a_2 \left( \frac{\nu_1}{500 \ Hz} \right) \left( \frac{\nu_2}{1 \ kHz} \right)^{b_2},$$

with $a_2 \approx 4.6$, and $b_2 \approx 1.8$. Therefore the consistency between Eq.(4) and Eq.(12), as well as Eq.(1) and Eq.(3), is apparently found. Further, we can also obtain the NS mass formula represented by the kHz QPO frequencies from Eqs.(1-2),

$$m = 2.17 \times \left( \frac{\nu_1}{500} \right)^{3/2} \left( \frac{\nu_2}{1000} \right)^{-5/2} \left[1 - \frac{\nu_1}{2 \nu_2} \right]^{3/2} \left(10 \right).$$

According to Eq.(14), we apply the detected twin kHz QPO data (van der Klis 2000) to calculate the mass of the star and obtain the mass average value $1.86 M_\odot$ for the six Z-sources and eleven Atoll sources, which is close to the measured NS mass of the X-ray binary Cygnus X-2 $1.78 \pm 0.23 M_\odot$ (Orosz & Kuulkers 1999).

3. On the dynamical mechanism of HBO and lower kHz QPO

If we ascribe HBO and lower kHz QPO to the periastron precession of the accreted matter in the different orbits now there arise the following problems. Firstly, how is the dynamical mechanism to produce the QPO in the X-ray spectrum? Secondly, if HBO is ascribed to the periastron frequency at the NS magnetosphere boundary, why do we not detect the Keplerian orbital frequency? And further, what are the reasons for the simultaneous occurrence of the twin kHz QPOs? In order to answer these questions, we need to describe our point of view on the dynamical mechanisms of QPOs.

It is assumed that the plasma blob in the state of the Keplerian motion at the NS magnetosphere-disk boundary and the outer disk boundary (both radii are presumed to be connected with each other by a best consistent factor 0.4), and the periastron motion of the plasma blob will collide the other plasma blobs in the orbits or surrounding plasma background. The collision of the plasma blobs will heat up the temperature of colliding blobs and make them more luminous than the surrounding background, which accounts for the detected QPO in the X-ray power spectrum. If the cooling time scale ($\tau$) of the luminous blob is too short, we either detect the sharp (high Q) QPO profile or nothing. If $\tau$ is sufficient longer than the orbital period and blob is sufficient luminous, then we would have chance to detect the Keplerian frequency because the longer luminous time than the orbital period makes the X-ray modulation in the orbital period possible. Otherwise, the convenient short luminous time just makes us detect the periastron period signal. Therefore, it is meaningful to estimate the cooling time scale of the luminous plasma blob after colliding.
Here the thermal bremsstrahlung cooling mechanism is assumed, and the thermal energy density of the plasma blob of the completely ionized hydrogen is
\[
\epsilon = 2nkT, \tag{15}
\]
where \( n \) is the ion density and \( k \) is Boltzman constant. If we consider the thermal bremsstrahlung radiation to account for the decay of the plasma blob, then the total power per unit volume emitted by the bremsstrahlung is given by (Rybicki and Lightman 1979)
\[
\frac{de}{dt} = 1.4 \times 10^{-27} n_i n_e Z^2 T^{1/2} g_B, \tag{16}
\]
where \( n_i \) and \( n_e \) are ion and electron number density respectively, and \( Z \) is the proton number. Here \( g_B \) is about 1.2, an averaged Gaunt factor (Rybicki and Lightman 1979). For simplicity, here we just consider the completely ionized Hydrogen plasma, which means \( n_i = n_e = n \) and \( Z = 1 \). The particle number density can be calculated by the mass accretion rate
\[
n = \frac{\dot{M}}{2\pi r_p^2 v_{ff} m_H}, \tag{17}
\]
where \( \pi r_p^2 = \frac{4}{3} \pi r^3 \) is the polar cap area at radius \( r \) (Shapiro and Teukolsky 1983), and \( v_{ff} \) is the free fall velocity and \( m_H \) is the hydrogen mass,
\[
v_{ff} = \sqrt{\frac{2GM}{r}} = 1.64 \times 10^{10} (cm/s)(\frac{M}{M_\odot})^{1/2} r_6^{-1/2}, \tag{18}
\]
where \( r_6 \) is the radius in unit of 10^6 cm. The orbital period is expressed as
\[
P_{orb} = 2\pi r \sqrt{\frac{r}{GM}} = 5.4 \times 10^{-4} (s)(\frac{M}{M_\odot})^{-1/2} r_6^{3/2}. \tag{19}
\]
The cooling time scale \( \tau \) of the heated hot plasma is,
\[
\tau = \frac{\epsilon}{\dot{e}} = 1.2 \times 10^{29} kT^{1/2} \text{n}^{-1}
\]
\[
= 2.83 \times 10^{-3} (s)(\frac{M}{M_\odot})^{1/2} R_6^{-3/2} T_8^{-1/2} M_{17}^{-1}, \tag{20}
\]
where \( T_8 \) is the temperature in unit of 10^8 K. So the ratio parameter \( (\delta) \) between the cooling time scale and the orbital period
\[
\delta \equiv \frac{\tau}{P_{orb}} = 5.23 (\frac{M}{M_\odot}) T_8^{1/2} M_{17}^{-1} \frac{R_6}{r}. \tag{21}
\]
If the parameter \( \delta \) is more than unity, the cooling time of the plasma blob after collision exceeds over the orbital period. In this case, the observer should have a chance to detect the orbital period modulation of X-ray signal. Otherwise, if the parameter \( \delta \) is less than unity for the usual thermal temperature and mass accretion rate, it is difficult to detect the orbital modulation of X-ray or the orbital frequency QPO. Therefore, the theoretically expected minimum detected orbital frequency would be determined by the condition \( \delta = 1 \), which gives the orbital frequency \( \nu_2 = 811 (Hz) T_8^{-1/4} M_{17}^{1/2} R_6^{-3/2} = 441 \) (Hz) if the parameters are chosen as \( R_6 = 1.5, T_8 = 1 \) and \( M_{17} = 1 \). However we stress that the temperature of the X-ray source is proportionally related to the mass accretion rate, so we cannot obtain the direct dependence of the minimum detected orbital frequency on the mass accretion rate or the temperature.

The existence of the minimum detected orbital frequency about 441 (Hz) can help us to understand why HBO (15 - 60 Hz QPO) has no the correspondent orbital frequency (about 300 Hz or lower) like the upper twin kHz QPO. It indicates that the cooling time scale is inversely related to the radius \( r \) from Eq. (21), so there exists a critical radius where the cooling time scale is comparable to the orbital period. If the orbital radius is larger than this critical radius, the cooling time scale will be shorter than the orbital period, which will result in the low efficiency in detecting the periodic X-ray signal modulation.

4. Discussion and Conclusions

As a conclusion, it is remarked that the model described here is simply and roughly one, and many physical details are neglected, such as NS spin induced gravitomagnetic effect, NS quadrupole induced nodal precession (Vietri and Stella 1998), the self-gravity of the disk, magnetosphere structure and magnetic axis inclination, the spiral-in effect of the accreted matter, the origination and influence of the non-zero eccentricity, etc.. Especially, the non-zero eccentricity should have somewhat effects on the QPOs, but the origination mechanism is still unknown. At least, we can speculate that the motion of the accreted matter in the disks might not exactly described by a free test particle in a circular orbit of a purely gravitational field. Consideration of these factors will construct our future exploration and understanding of QPOs in LMXBs.

The parallel line phenomenon in the diagram of the X-ray luminosity vs. kHz QPO frequency is seen across the sources and within the individual sources (see e.g., van der Klis 2000). For the parallel line phenomenon across the sources, we try to ascribe it to the magnetic field strength of the different sources (Z-sources and Atoll sources). For the high luminosity Z-sources and low luminosity Atoll sources, the luminosities vary more than two magnitude orders from \( \sim 1% \dot{M}_{Edd} \) to \( \dot{M}_{Edd} \) (Eddington accretion rate), however the QPO frequencies are very homogenous varied from 500 (Hz) to 1200 (Hz) for the upper kHz QPO. If the QPO frequency is only related to the instantaneous luminosity of the star, the homogenous distribution of the kHz QPO frequencies cannot be satisfactorily explained (van der Klis 2000). The other factor should be taken into account, and here we prefer the magnetic influence of the sources. The magnetic fields of the Z-sources are stronger than that of the Atoll sources, so the strong magnetic fields equilibrate the influence of the high luminosities of the Z-sources. As a result, the disk radii of the Z-sources and Atoll sources are similar, which will account for the homogenous distribution of the kHz QPO frequencies. Our
proposed QPO frequencies are directly related to the radii of the accreted disks, which are determined by the ratio of the instantaneous accretion rate to the long-term averaged accretion rate, not only by the instantaneous accretion rate itself. However the long-term averaged accretion rate matches the magnetic field strength of the star from the magnetic evolution scenario (Zhang et al. 1997, Cheng & Zhang 1998, 2001), this point of view is consistent with the recent expectation by van der Klis that the disk accretion rate normalized by its own long-term averaged, $\eta = \dot{M}/\langle \dot{M} \rangle$, determines most of the phenomenology of QPO (van der Klis 2001).

Finally we stress that the proposed model has also implications that the QPO phenomenon in LMXB and BHC is likely to reflect the fundamental relativistic motion of matter in the vicinity of the NS strong gravitational field. As discussed, if the model were successfully to explain the observed QPOs in LMXBs, we can determine or constrain the NS parameters, such as the magnetosphere, magnetic field, radius and mass, by means of the QPO detections.

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Fig. 1. HBO frequency versus the upper kHz QPO frequency for five Z sources of LMXBs (Psaltis et al. 1998, 1999 and references therein). Error bars are not plotted for the sake of clarity. The model presents a well consistency for the nearly circular orbit of NS mass about 2.0 solar mass with the scaling parameter $\phi = 0.4$.

Fig. 2. The lower kHz QPO frequency versus the low QPO frequency for the Z-sources, Atoll sources and other sources (Psaltis et al. 1999). Error bars are not plotted for the sake of clarity. That theoretical curves are weakly related to the NS mass parameter is found. The solid(dot) line represents the parameter condition with $m=2.0$ and $\phi = 0.4$ ($\phi = 0.33$); the short(long) dash line represents the parameter condition with $m=5.0(10.0) \, M_\odot$ and $\phi = 0.4$. 