Fixed Order Interval Model for Multi Item Single Supplier Considering Lifetime and Minimum Order Quantity

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Abstract. This paper presents a material procurement model for multi-item single supplier material considering the life time and minimum order quantity. Procurement of several types of material (multi items) is carried out simultaneously from one supplier with consideration of the savings in ordering costs. The problem is when the order must be performed simultaneously, and how much material must be ordered to obtain a minimum total cost. The cost variables considered in this model are purchase costs, ordering costs, storage costs, and expired cost. This modeling was inspired by a real case found in a manufacturing company located in Bandung. Facts in the field show there were frequently waste due to material has expired. This happens because the material age is relatively short, while the order size must follow the minimum quantity stipulated by the vendor or supplier. Until now the authors have not found a model which fits to this case. However the authors found several articles which have similarities to, and the authors made it as comparison. The developed model departs from the basic model which be developed by several experts. The author modified little bit by adding the cost variable such as the expired cost. Then the authors engaged the model with completely algorithm. This model is a continuation of the single item model developed by Arifin D, and Charisma C, which has been presented and published in the IEOM Conference Proceedings 2018.

1. Introduction
Depart from the real problems faced by a manufacturing company located in Jalan Pajajaran Bandung, that one of the problems is the amount of material that has expired due to overstock while the material life time is relatively short. While the company must order according to the minimum order quantity set by the vendor. The authors were inspired to make material procurement models that could minimize total costs. Generally at the optimization procurement material models considered a few of costs such as purchase cost, ordering cost, storage cost, and stockout cost. But in the reality there are the another important cost namely expired cost. On that idea the authors developed a material procurement model by adding variable costs, namely expired costs.
2. The Developed Model

In deterministic situation and to avoid shortage the model can be described as follows,

![Figure 1. Model of EOI – multi item](image)

To formulate a mathematical model, we have some notations,

- \( P_i \) = purchase cost (price cost of item i)
- \( R_i \) = annual demand item i (in units)
- \( C \) = Ordering cost for all items
- \( c \) = order cost for each item i
- \( H_i \) = holding cost item i per unit per year
- \( Q_i \) = order quantity item i per each order (in unit)
- \( F \) = Fraction or proportion of the holding cost

\[
\begin{align*}
    m_i &= \frac{1}{T} \quad \text{number of orders per year} \\
    R &= \frac{R_i}{2m} \quad \text{average inventory in units} \\
    T &= \frac{1}{m} \quad \text{order interval (in years)} \\
    MOQ_i &= \text{Minimum Order Quantity for material i} \\
    d_i &= \frac{R_i}{N} \quad \text{usage rate per period during life time} \\
    N &= \text{number of working days} \\
    L_o &= \text{lead time Order} \\
    L_t &= \text{transport lead time} \\
    L_{f_i} &= \text{life time of item i (material age)} \\
    K_{d_i} &= \text{expired cost item i.}
\end{align*}
\]
The model was developed only considering the cost of material purchase, ordering costs, storage costs and expired costs.

**Total annual cost = purchase cost + order cost + holding cost + expired cost**

mathematically can be expressed follow

\[
TC(T) = \sum_{i=1}^{n} P_i R_i + \frac{C + \sum_{i=1}^{n} c_i}{T} + \frac{TF}{2} \sum_{i=1}^{n} P_i R_i + \frac{1}{T} \sum_{i=1}^{n} K d_i (MOQ_i - (L_f_i - L_t_i) d_i)
\]

Subject to \(Lt_i + T \leq Lf_i\) for all item \(i = 1, 2, 3,...n\).

To get the minimum cost, the equation above can be derivated to \(T\), found out

\[
T = \sqrt{\frac{2 \left[ C + \sum_{i=1}^{n} c_i + \sum_{i=1}^{n} K d_i (MOQ_i - (L_f_i - L_t_i) d_i) \right]}{F \sum_{i=1}^{n} P_i R_i}} = T^*
\]

This \(T\) value is the optimal ordering interval, we called as \(T^*\).

Test the second derivative,

\[
\frac{d^2 TC}{dT^2} = 2 \frac{C + \sum_{i=1}^{n} c_i}{T^3} + 2 \frac{\sum_{i=1}^{n} K d_i (MOQ_i - (L_f_i - L_t_i) d_i)}{T^3} > 0
\]

The second derivative indicates greater than zero indicating that optimal \(T\) is the minimum value.

Because the model must meet the requirements \((Lt_i + T) < Lf_i\) then to fulfill the feasibility of the solution researchers develop the following algorithm:

1. Make sure that the supplier sets the \(MOQ\) or not. If yes, continue to step 2) if not continue to step 3)
2. Calculate the initial \(T^*\) (optimal ordering interval) with formula eq. 2
3. Calculate the initial \(T^*\) with the formula (depending on step 1).

\[
T = \sqrt{\frac{2 \left[ C + \sum_{i=1}^{n} c_i \right]}{F \sum_{i=1}^{n} P_i R_i}}
\]

4. To avoid material deficiencies that have an impact on production losses, the \(T\) above must meet \(Lt_i + T \leq Lf_i\) (for all \(i = 1, 2, 3,... n\)) but if there is an item \(i\) with \(Lt_i + T > Lf_i\), then \(T\) needs revised with, \(T_{revised} = \min \{Lf_i - Lt_i\}\) and the next step of the revised \(T\) is used to calculate the number of cycles and the lot size ordered, \(Q_i = R_i \times T\)
5. Compare the order size \( Q_i \) to the Minimum Order Quantity \( MOQ_i \). If \( Q_i > MOQ_i \) for all \( i = 1, 2, 3, \ldots, n \)
   Set \( Q_i \) as an alternative booking size.
   But if there is item \( i = a \) with \( Q_i = a < MOQ_i = a \)
   Then set \( MOQ_i = a \) as the order size to be done for item \( i = a \).

6. Calculate total costs using the revised \( T \) above. Finished.

Model assumptions
1. The level of demand for goods is known with certainty and is constant over time.
2. The cost structure is assumed to remain unchanged during the planning period and there is no discount factor.
3. The order interval is always the same for each period.
4. The number of orders varies according to order.
5. The number of items ordered comes together
6. Assumed sufficient storage space capacity for each material arrival.

3. Example of Model Application

Suppose there are 5 types of material provided by the same vendor, the order will be ordered simultaneously with the goal of obtaining savings in ordering costs when compared to the respective orders. Data regarding needs, prices and other parameters as in the table below. It is assumed that the number of days in a year is 360 days, the order dost for all items \( C = 10 \) per each order, and the storage cost \( F = 15\% \).

| item | \( R \)  | \( P \)  | \( Kd \) | \( Lf \) | \( Lt \) | handling | \( MOQ \) |
|------|--------|--------|--------|--------|--------|----------|--------|
| 1    | 1000   | 15     | 15     | 90     | 14     | 1        | 250    |
| 2    | 1500   | 10     | 10     | 100    | 14     | 1        | 325    |
| 3    | 1200   | 15     | 15     | 90     | 14     | 0.5      | 250    |
| 4    | 1300   | 10     | 10     | 100    | 14     | 1        | 300    |
| 5    | 1200   | 15     | 15     | 120    | 14     | 1        | 400    |

| item | \( R \) | \( P \) | \( Kd \) | \( Lf \) | \( Lt \) | handling | \( MOQ \) |
|------|--------|--------|--------|--------|--------|----------|--------|
| 1    | 1000   | 15     | 15     | 90     | 14     | 1        | 250    |
| 2    | 1500   | 10     | 10     | 100    | 14     | 1        | 325    |
| 3    | 1200   | 15     | 15     | 90     | 14     | 0.5      | 250    |
| 4    | 1300   | 10     | 10     | 100    | 14     | 1        | 300    |
| 5    | 1200   | 15     | 15     | 120    | 14     | 1        | 400    |

Solution,
\[
T = \sqrt{\frac{2}{F} \left( C + \sum_{i=1}^{n} c_i + \sum_{i=1}^{n} Kd_i \cdot (MOQ_i - (Lf_i - Lt_i) \cdot d_i) \right) + \sum_{i=1}^{n} P_i R_i}
\]
\[
= \sqrt{\frac{2}{0.15} \left[ 10 + 4.5 + 1283 \right] \left( \frac{3}{79000} \right)} = 0.468 \text{ year}
\]

The \( T \) value that minimizes the total cost is \( T = 168 \) days.
To ensure that \( T = 168 \) is the least cost,
Table 2. Number of material ordered

| Item | R     | T (dys) | Q initials col. 2 x col.3 | MOQ | Number of material ordered |
|------|-------|---------|---------------------------|-----|---------------------------|
| 1    | 1000  | 104     | 288,8                     | 250 | 288,8                     |
| 2    | 1500  | 104     | 433,3                     | 325 | 433,3                     |
| 3    | 1200  | 104     | 346,6                     | 250 | 346,6                     |
| 4    | 1300  | 104     | 375,5                     | 300 | 375,5                     |
| 5    | 1200  | 104     | 346,6                     | 400 | 400                       |

Calculate total annual cost by revised $T = 104$ days.

Table 3. Total cost comparison between $T^*$ with revised $T$

| $T$  | Purchase cost | Order cost | Holding cost | Expired cost | Total cost |
|------|---------------|------------|--------------|--------------|------------|
| 104  | 79000         | 50,4       | 1706,4       | 4455,9       | 85212,65   |
| 168  | 79000         | 30,9       | 2784,75      | 2730,4       | 84546,0    |

Differences | 0 | 19,5 | -1078,35 | 1725,47 | 666,62 |

Optimal $T$ is obtained at $T = 168$ days with a total annual cost of 84546.0. The calculated value of $T$ with equation 1 results in the same value by enumerating in figure 2, so it can be said that the optimum $T$ formulation in the equation 2 is valid.

Return to the total cost model in the equation 1, requires $L_{ti} + T < L_{fi}$, for all items $i = 1, 2, 3, \ldots n$. In this case the conditions are not fulfilled, so $T$ is revised to $T = 104$ days. This is intended to prevent shortages due to expiration. Therefore the order interval is made every 104 days with the number of orders for each material as in table 2 column 8. The consequence of this change causes a cost difference between the optimal 168 days and the cost at $T = 104$ days. At the cost of ordering an increase of 19.5 while the cost of storage decreased by 1078.35 and the expiration fee increased by 1725.47, while the total cost increased to 666.62. This problem arises due to the minimum order quantity from the supplier.

4. Conclusions and Recommendations

1. Model for determining the optimal order interval for multi item single suppliers taking into account the life time and minimum order quantity obtained as shown in the presentation (equation 2).

2. Because the developed model does not accommodate any shortages, the optimization is achieved 100% if the calculated $T$ meets the requirements of $L_{ti} + T < L_{fi}$, for all items $i = 1, 2, 3, \ldots n$. Meanwhile, when in the practice the conditions are not met, the problem solving is followed by the algorithm developed above.

3. The optimization model developed by authors are categorized as semi heuristic optimization. First the optimal solution is obtained from the equation 2 then then continued by the algorithm heuristically.

4. The model developed includes a simple model and still has drawbacks among them, the model is deterministic, does not accommodate shortages.

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