ON THE CONSTANCY OF THE CHARACTERISTIC MASS OF YOUNG STARS

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ABSTRACT

The characteristic mass $M_c$ in the stellar initial mass function (IMF) is about constant for most star-forming regions. Numerical simulations consistently show a proportionality between $M_c$ and the thermal Jeans mass $M_J$ at the time of cloud fragmentation, but no models have explained how it can be the same in diverse conditions. Here we show that $M_c$ depends weakly on density, temperature, metallicity, and radiation field in three environments: the dense cores where stars form, larger star-forming regions ranging from GMCs to galactic disks, and the interiors of H II regions and super star clusters. In dense cores, the quantity $T^{3/2}n^{-1/2}$ that appears in $M_J$ scales with core density as $n^{0.25}$ or with radiation density as $U^{0.1}$ at the density where dust and gas come into thermal equilibrium. On larger scales, this quantity varies with ambient density as $n^{-0.05}$ and ambient radiation field as $U^{-0.033}$ when the Kennicutt-Schmidt law of star formation determines $U(n)$. In super star clusters with ionization and compression of prestellar globules, $M_c$ varies as the 0.13 power of the cluster column density. These weak dependencies on $n$, $U$, and column density imply that most environmental variations affect the thermal Jeans mass by at most a factor of $\sim 2$. Cosmological increases in $M_c$, which have been suggested by observations, may be explained if the star formation efficiency is systematically higher at high redshift for a given density and pressure, if dust grains are smaller at lower metallicity, and so hotter for a given radiation field, or if small prestellar cores are more severely ionized in extreme starburst conditions.

Subject headings: dust, extinction — stars: formation — stars: luminosity function, mass function

1. INTRODUCTION

The stellar initial mass function (IMF) has three properties that appear to be relatively robust in diverse environments: the power-law slope for masses between 1 and 10 $M_\odot$, originally measured by Salpeter (1955), the lower mass limit for the power law and the broad plateau below it before the brown dwarf regime (Miller & Scalo 1979; Scalo 1986; Rana 1987; Reid 1987), and the maximum mass of stars (Weidner & Kroupa 2004; Oey & Clarke 2005; Scalo 1979; Scalo 1986; Rana 1987; Reid 1987), and the maximum mass of stars (Weidner & Kroupa 2004; Oey & Clarke 2005; Scalo 1979; Scalo 1986; Rana 1987; Reid 1987). This paper considers the origin of the plateau, a range typically spanning a factor of $\sim 3$ on either side of a characteristic mass $M_c \sim 0.3 M_\odot$ where the IMF becomes relatively flat on a log $N$–log $M$ plot. Occasionally, an observed IMF has a small subpeak inside the plateau, but the mass of this peak could vary stochastically from region to region and with different mass binnings, calibration details, binary corrections, and completeness corrections. The existence of a broad IMF plateau defining a characteristic mass is robust, however. At lower mass the IMF drops into the brown dwarf regime, and at higher mass it drops into the stellar range for F, A, B, and O main-sequence types. Comprehensive reviews of cluster and field IMFs may be found in Scalo (1998), Kroupa (2002), and Chabrier (2003), a review of starburst IMFs is in Elmegreen (2005), and a review of galactic scale IMFs is in Elmegreen (2006). Recent reviews of the theory of the IMF are in Mac Low & Klessen (2004), Bonnell et al. (2007), and Larson (2007).

Observations and possible explanations for the IMF plateau are summarized in the next two sections. The observations suggest a remarkable uniformity in the characteristic stellar mass in spite of a wide range in environmental factors that should affect it. The explanations are still incomplete because the issue of constant $M_c$ is not usually addressed. In § 4, we show that the quantity $T^{3/2}n^{-1/2}$ that appears in the equation for the thermal Jeans mass for temperature $T$ and number density $n$ should be virtually independent of environment in three important cases: in dense cores where individual stars form, in the general ISM where star formation satisfies the Kennicutt (1998) relation, and in massive dense clusters where OB stellar radiation determines the pressure and temperature. In § 5, observations for a systematic increase in $M_c$ with redshift are reviewed, and explanations are offered for why this might be the case. Finally, we summarize our conclusions in § 6.

2. OBSERVATIONS OF THE IMF PLATEAU: UNIFORMITY IN THE LOCAL UNIVERSE

Observations of the mass range for IMF plateaus are collected in Table 1. Stochastic variations and systematic uncertainties in measured IMFs make the plateau imprecise, but we can define it well enough for our present purposes. As mentioned in the introduction, a typical IMF rises somewhat monotonically with approximately the Salpeter power-law slope from high-mass stars down to 0.5 or 1 $M_\odot$, and then it either levels off on a log-log plot or rises more slowly down to several tenths of a solar mass, at which point it decreases into the brown dwarf range. We consider the plateau to be the relatively flat part of such an IMF on a log-log plot, extending for about a full width at half maximum. The plateau for a lognormal approximation to the IMF is about the same as the plateau for a piecewise power-law approximation. For local clusters and the local field, the plateau extends between
mass range for the IMF plateau

| Region          | Lower Mass ($M_\odot$) | Upper Mass ($M_\odot$) | Reference          |
|-----------------|------------------------|------------------------|--------------------|
| Taurus          | 0.1                    | 0.8                    | Luhman (2004)      |
| $\rho$ Oph      | $\leq 0.1$             | 0.5                    | Luhman & Rieke (1999) |
| IC 348          | 0.1                    | 1                      | Luhman et al. (2003) |
| Cam 1           | 0.1                    | 1                      | Luhman (2007)      |
| Trapezium       | 0.1                    | 0.6                    | Muench et al. (2002) |
| Pleiades        | 0.2                    | 0.7                    | Bouvier et al. (1998) |
| Upper Sco OB    | $\leq 0.1$             | 0.6                    | Preibisch et al. (2002) |
| M35             | 0.2                    | 0.8                    | Barraod y Navascues et al. (2001) |
| Local Field     | 0.2                    | 0.5                    | Scalo (1986)       |
|                 | 0.1                    | 1                      | Rana (1987)       |
|                 | 0.1                    | 0.5                    | Kroupa et al. (1993) |
| 12 GCs*         | 0.14                   | 0.69                   | Paresce & de Marchi (2000) |
| M4 (GC)$^2$     | 0.14                   | 0.69                   | de Marchi et al. (2004a) |
| M15 (GC)$^3$    | 0.16                   | 0.57                   | Pasquali et al. (2004) |
| MW bulge        | ...                    | 0.5–0.7                | Holtzman et al. (1998) |
|                 | $\leq 0.15$            | 0.5                    | Zoccali et al. (2000) |

* For these Milky Way globular clusters, the initial mass function for $M < 0.75 M_\odot$ was found to be consistent with a lognormal having a peak mass of $M_p = 0.33 \pm 0.03$ (log $M_p = -0.5$) and a dispersion in the log-mass of $\sigma = 0.34 \pm 0.04$. The lower and upper limits of the plateau were taken to be the 1 $\sigma$ points on either side of the peak.

$^2$ De Marchi et al. (2004b) fit the mass function for M4 to a tapered power law, $M^{-\alpha}[1 - (M/M_r)^{\beta}]$ and find values of $\alpha$, $M_r$, and $\beta$ to be essentially the same as for the 12 globular clusters in Paresce & de Marchi (2000). Thus we take the same range for the plateau.

$^3$ Pasquali et al. (2004) fit a lognormal mass function to this globular cluster with peak mass $M_p = 0.3$ and dispersion $\sigma = 0.28$. The mass limits in the table are 1 $\sigma$ points around the peak.

Table 1: Mass Range for the IMF Plateau

$\sim 0.1$ and $\sim 1 M_\odot$. The observations in Table 1 constrain the midpoints of the plateau in each IMF to within a factor of $\sim 2$ in mass. Variations that are smaller than this are probably stochastic and in any case will not be identified with physical variations in this paper.

The characteristic mass $M_c$ is defined to be the midpoint of the plateau, again on a log-mass scale. With the above typical limits, it is log $M_c/M_\odot \sim -0.5 \pm 0.5$ or $M_c \sim 0.3 M_\odot$. This approximate value for the characteristic mass is observed in a surprisingly wide variety of regions and over a wide range of star formation epochs and rates. Among this diversity are included a high-latitude cloud with unusual abundances, Blanco 1 (Moraux et al. 2007), and some of the most remote clusters in the outer Milky Way, DIGEL 1 North and South (Yasui et al. 2008), which have IMFs like that in Orion. For Milky Way halo globular clusters, there is also a plateau with similar $M_c$. Paresce & de Marchi (2000) studied 12 globulars and found that all of them have the same IMF shape, even though they represent a wide range of metallicities, distances from the galactic center and plane, cluster radii, and concentration ratios; there was also no correlation between the IMF and the cluster disruption time. De Marchi et al. (2000) noted that globular cluster IMFs resemble that in the Pleiades. The only exceptions for globular clusters seem to be those that have age-dependent mass segregation, which can severely affect the present-day mass function (de Marchi et al. 2004a), and those that are highly dispersed by tidal forces, which can have an inverted present-day mass function (rising toward higher mass) because of a preferential loss of low-mass stars (de Marchi et al. 1999, 2006; Koch et al. 2004; de Marchi & Pulone 2007).

Distant globular clusters where the IMF is not observed directly should have about the same value of $M_c$ as local globular clusters in order to have remained bound after stellar evolution removed gas through massive stellar winds and supernovae (de Grijs & Parmentier 2007). Globular clusters with higher initial $M_c$ would presumably have been disrupted by such mass-loss processes, making $M_c \lesssim 0.5 M_\odot$ a selected value for survival. We do not know if $M_c$ was the same for all globular clusters, considering this selection effect (D. Pfenniger 2007, private communication).

In the Galactic bulge, Holtzman et al. (1998) found an IMF that rises with mass according to the Salpeter slope, $\sim -1.2$, down to $\sim 0.5 M_\odot$, and then turns over sharply. Zoccali et al. (2000) traced the bulge IMF to lower mass and found about the same slope as Holtzman et al. (1998) for $M > 0.5 M_\odot$ and a shallower slope, $\sim -0.33$, down to at least 0.16 $M_\odot$. This implies that $M_c \sim 0.3 M_\odot$ in the bulge, similar to the values seen in the globular clusters. The Galactic spheroid has an IMF with a plateau peaking at $\sim 0.6 M_\odot$ (Gould et al. 1997), slightly larger than the cluster plateau. The IMF in the local dwarf Spheroidal Ursa Minor has been traced down to 0.45 $M_\odot$ by Feltzing et al. (1999) and found to be indistinguishable from that of the globular cluster M92. The observed mass range in Ursa Minor is too small to see both sides of the plateau, but a significant increase in $M_c$ can be ruled out.

Chabrier (2003) compiled observations of IMFs for Milky Way disk and globular clusters and for the bulge, using a slightly different definition of $M_c$ and considering also binary star corrections. He suggested $M_c$ is slightly larger for the older systems, with a shift from $M_c \sim 0.08 M_\odot$ for the disk to $\sim 0.2 M_\odot$ for globulars and the bulge if the binary fraction in the bulge is significantly smaller than in the disk. These values are uncertain and the proposed shift is small, considering the large differences in star formation rates and luminosity densities when these systems formed. We return to these observations in § 5.

IMF variations under extreme environmental conditions such as circumnuclear starbursts are still uncertain. Elmegreen (2005) noted that most previous claims of top-heavy or high-$M_c$ IMFs in starbursts have been overturned by recent data. Bastian & Goodwin (2006) suggested that even the recent discussions about top-heavy IMFs in some super star clusters are premature, as these clusters appear unrelaxed and their masses uncertain.
Way, the Arches cluster near the Galactic center was reported to have a large \( M_c \) (Yang et al. 2002; Stolte et al. 2005), but it is poorly observed at low mass. The most recent observations by Kim et al. (2006) are incomplete below 1.3 \( M_\odot \). R136 in 30 Dor was also claimed to have a high \( M_c \approx 2 M_\odot \) (Sirianni et al. 2000), but extinction variations could affect this (Andersen & Zinnecker 2003; Andersen et al. 2005).

Most extragalactic regions are too far away to observe the IMF plateau directly, but limits can be placed on its range. Significantly higher \( M_c \) will produce too red a stellar population of giants without the corresponding main-sequence stars after the turnover age reaches the stellar lifetime at the lower limit of the power law (Charlot et al. 1993). Also, the oxygen abundance will be too high compared to solar after the turnover age if the plateau shifts upward by a factor of a few (Wang & Silk 1993). Significantly lower plateaus would produce too high a mass-to-light ratio in the disk.

The uniformity of \( M_c \) for local or normal star formation is difficult to understand considering the wide range of properties in molecular clouds. The radiation field, density, and temperatures for gas and dust are high in super star clusters and low in dispersed regions like Taurus, \( \rho \) Oph, and IC 348, yet all of these regions have about the same \( M_c \). Globular clusters, which presumably formed as super star clusters, also have about this \( M_c \). Indeed, the possible explanations for the plateau, discussed next, do not indicate why \( M_c \) should be so constant.

### 3. EXPLANATIONS FOR THE IMF PLATEAU

There is no convincing explanation for the constancy of \( M_c \) in these highly diverse regions. Although there are several proposals for the origin of \( M_c \), its invariance under varying environmental conditions is usually not addressed. These proposals include an accretion- or coagulation-driven scale-up of the opacity-limited mass (Field & Saslaw 1965; Rees 1976; Yoshii & Saio 1985; Murray & Lin 1996; Bate et al. 2003), the thermal Jeans mass at an inflection point in the effective equation of state (Larson 2005; Jappsen et al. 2005; see also Li et al. 2003), and the initial thermal Jeans mass in a marginally unstable cloud (Larson 1978; Tohline 1980; Klessen et al. 1998; Klessen & Burkert 2000, 2001). There are other models which relate the IMF to the properties of interstellar turbulence (Larson 1981; Fleck 1982; Elmegreen 1993; Padoan 1995; Padoan et al. 1997; Klessen 2001; Padoan & Nordlund 2002). Stellar winds may also play some role in self-limiting the stellar mass to a characteristic value (Larson 1982; Shu et al. 1987; Nakano et al. 1995; Adams & Fatuzzo 1996).

The thermal Jeans mass for an isothermal cloud is

\[
M_J = \left( \frac{kT}{G \rho} \right)^{1.5} \rho^{-0.5} = 0.9 \left( \frac{T}{10 \text{ K}} \right)^{1.5} \left( \frac{n_{H_2}}{10^4 \text{ cm}^{-3}} \right)^{-0.5} M_\odot.
\]

where \( k \) and \( G \) are the Boltzmann and gravitational constants, respectively, and where mass density \( \rho \) and number density \( n_{H_2} \) are related via the H\(_2\) mass, \( m_{H_2} \). The Bonner-Ebert critical mass for an external pressure \( P_{\text{ext}} \) is

\[
M_{\text{BE}} = 1.18 \left( k T / m_{H_2} \right)^{2} \left( G^2 P_{\text{ext}} \right)^{-0.5}.
\]

If we use a parameter \( \alpha \) to connect the internal and external pressures of the sphere, \( P_{\text{ext}} = \alpha k T / m_{H_2} \), then \( M_{\text{BE}} = 1.18 \alpha^{-1.2} M_J \). Magnetic fields are generally ignored for applications of these expressions to star formation, because the field eventually diffuses out. The minimum critical mass for stability is determined by \( M_J \) or \( M_{\text{BE}} \).

Without additional assumptions, these explanations for \( M_c \) are difficult to reconcile with the diversity of star formation conditions. The opacity-limited mass, in which opacity is limited by dust, should vary inversely with metallicity (Masunaga & Inutsuka 1999), so this explanation has difficulty with the similarity in \( M_c \) for modern disk clusters and halo globular clusters, which have \( \sim 10^{-10} \) times lower metallicity. A wind-limited mass is inconsistent with the apparent similarity between stellar mass functions and protostellar core mass functions (for recent observations and other references, see Alves et al. 2007; Ikeda et al. 2007; Nutter & Ward-Thompson 2007; Li et al. 2007; Walsh et al. 2007; Massi et al. 2007; see also Clark et al. 2007 for a critical note on that issue).

The appearance of a characteristic mass from an inflection point in the effective equation of state (EOS) has been demonstrated numerically for both the first and second generations of stars in the universe (Bromm et al. 2002; Clark et al. 2008) and present day conditions (Jappsen et al. 2005; Bonnell et al. 2006). For star formation in the solar neighborhood, Larson (1981, 2005) suggested that gas and grains couple thermally to \( T \approx 8 \text{ K} \) at a density of \( 10^{-19} \text{ g cm}^{-3} \) \( (n_{H_2} = 2.5 \times 10^4 \text{ cm}^{-3}) \), giving \( M_c \approx 2 M_\odot \). According to the model, the gas heats up with decreasing density below this coupling density, and it heats up with increasing density above the coupling density by equilibrating with a higher grain temperature. As a result, the effective adiabatic index is less than 1 at the beginning of the collapse, leading to fragmentation, and larger than 1 above the coupling density, leading to little fragmentation. The Jeans mass at the inflection point then ends up as the characteristic mass for star formation, \( M_c \). Whitworth et al. (1998) also suggested that \( M_c \approx M_J \) at the gas-dust coupling point. In Larson’s model, an increase in temperature with density also follows from the input of collapse energy. Masunaga & Inutsuka (2000) showed this collapse temperature increase, starting at a temperature of 5 K and a density of \( 10^{-17.5} \text{ g cm}^{-3} \) \( (n_{H_2} = 8 \times 10^3 \text{ cm}^{-3}) \), which gives \( M_c \approx 0.035 M_\odot \), considerably lower than \( M_c \). However, at the gas-dust coupling density in Masunaga & Inutsuka (2000), the temperature still decreased with increasing density because background starlight radiation was increasingly excluded. Thus, there is no inflection point there. Observations of prestellar dense cores confirm this temperature trend, showing decreasing temperatures with increasing densities up to at least \( 10^4 \text{ cm}^{-3} \) (Crapsi et al. 2007). Thus, the EOS inflection point may occur too late in the collapse to set \( M_c \approx M_J \) or it may not occur at all.

The other explanation for the characteristic mass, found in numerical simulations of various types (Klessen & Burkert 2000, 2001; Clark & Bonnell 2005; Bate & Bonnell 2005; Martel et al. 2006), is that it scales with the thermal Jeans mass \( M_J \) at the onset of collapse. In a statistical sense, the system retains knowledge of its initial average properties during gravitational contraction and the buildup of dense stellar clusters. We note, however, that these calculations are usually done with an isothermal EOS and do not take compressional heating or stellar feedback into account. They also describe evolutionary stages before the collapse energy significantly increases the core temperature, and thus correspond to a lower density and perhaps a higher temperature, giving a higher \( M_c \) than in the grain-gas coupling theory. According to Clark & Bonnell (2005), turbulent fragmentation makes unbound cores \( (M < M_J) \) that coalesce and become gravitationally bound when they reach \( M_c \). Then they fragment into lower mass stars in a regular way, preserving the initial sensitivity to \( M_c \). The coagulation process preserves the clump densities and \( M_c \) until the fragments become gravitationally bound, at which point their densities increase and they fragment gravitationally.
into stars of lower mass. The sensitivity to \( M_f \) in the pre-collapse cloud remains in the stars that form.

For both the EOS condition and the initial \( M_f \) condition, the environmental dependence of \( M_f \) should be examined. An important consideration is the dependence of dust temperature \( T_d \) on the local density of the star-forming clump, which determines \( M_f \) at grain-gas coupling. If \( T_d \propto n^{1/3} \), then \( M_f \) is about constant. Whether or not the EOS has an inflection point there does not matter, because as long as \( M_f \) is roughly constant near the beginning of the collapse, the IMF should have a constant \( M_f \) at the end of the collapse, according to Clark & Bonnell (2005) and others mentioned in the previous paragraph. A related dependency is that of gas and dust temperatures on the average density in the star-forming cloud or the average density in a region of the galaxy. These are more general considerations that should determine how \( M_f \) varies on larger scales. The value of \( M_f \) in a dense super star cluster should also be examined, because the pressure and temperature in a neutral prestellar clump depend mostly on the ionizing radiation field. These three environmental dependencies are discussed in the next section. Remarkably, they all give about the same \( M_f \) and show very little sensitivity to environment.

4. THREE REASONS FOR A CONSTANT \( M_f \) IN VARIOUS ENVIRONMENTS

4.1. High-Density Gas in Star-Forming Regions

The thermal Jeans mass depends on \( T^{3/2}n^{-1/2} \) in a molecular cloud. In general the temperature \( T \) and density \( n \) should be independent, making \( M_f \) vary with cloud conditions. At the high densities where stars form, however, grain-gas coupling is an important source of heating and cooling, making the gas and grain temperatures comparable. The primary source of gas heating at moderate to high density is the warm dust heated by starlight, provided the magnetic diffusion rate is not greatly elevated by temporary compression. The energy equation for the gas then has the heating rate from grain collisions equal the cooling rate from line emission.

Molecular line emission cooling has been studied by Neufeld et al. (1995), who show in their Figure 2 the total cooling rate per \( H_2 \) molecule at various densities and temperatures. Their cooling rates per molecule are approximately constant at each temperature within the star-forming cloud or the average density in a region of the galaxy. These are more general considerations that should determine how \( M_f \) varies on larger scales. The value of \( M_f \) in a dense super star cluster should also be examined, because the pressure and temperature in a neutral prestellar clump depend mostly on the ionizing radiation field. These three environmental dependencies are discussed in the next section. Remarkably, they all give about the same \( M_f \) and show very little sensitivity to environment.

\[
\alpha \pi a^2 c_{\text{th}} n_{\text{H}_2} n_a 2k(T_d - T) = 10^{-2.65} n_{\text{H}_2}(T/10 \text{ K})^{2.5}
\]

for cgs units. We can now solve for the quantity appearing in \( M_f \),

\[
T^{3/2}n^{-1/2} = 10^{21.75} [\alpha \pi a^2 D2k\Delta T(8k/\pi n_{\text{H}_2})^{1/2}]^{3/4} n^{1/4}.
\]

In these equations, \( c_{\text{th}} = (8kT/\pi n_{\text{H}_2})^{1/2} \) is the three-dimensional thermal speed of collisions between gas and dust, \( a = 0.15 \) to 1 measures how well the gas atom thermalizes while on the grain (Tielens 2005, \( \Delta T = T_d - T \), and \( D = n_a/n_{\text{H}_2} \). The temperature difference \( \Delta T \) is assumed to be small and can be defined as independent of \( T \) at the coupling density.

Evidently, the Jeans quantity, \( T^{3/2}n^{-1/2} \), depends only weakly on density at the gas-grain coupling point, as \( n^{1/4} \). It depends weakly on grain properties too, because \( a^2D \) averaged over a grain size distribution proportional to \( a^{-3.5} \) (Mathis et al. 1977) depends mostly on the smallest grains and is therefore approximately independent of the dust-to-gas mass ratio and metallicity, which depends on the largest grains (see eq. [18] below). The metallicity dependence comes mostly from the gas cooling rate \( \Lambda \). If we assume \( \Lambda \propto Z^\alpha \) then \( M_f \propto (a^2D/Z)^{1/4} \) for metallicity Z. If the main coolants are optically thick, then the exponent \( \alpha \) should be small and the overall metallicity dependence should be weak, especially if \( a^2D \) decreases a little at lower \( Z \) along with \( \alpha \).

To emphasize the weakness of the \( n^{1/4} \) density dependence for \( M_f \), we note that \( n \propto T^2 \) at gas-grain coupling, and for typical grain properties, \( T \propto U^{0.2} \) with radiation field \( U \) (Tielens 2005, eq. [5.44]). Thus \( n \propto U^{0.4} \), and so the \( n^{1/4} \) dependence in the Jeans quantity is \( \propto U^{0.1} \). The Jeans mass at gas-grain coupling hardly depends on the most important environmental variable, the radiation field. For example, if the radiation field in the immediate vicinity of a prestellar clump increases by 3 orders of magnitude, the dust and gas temperatures at coupling both increase by a factor of \( \sim 4 \), the density at coupling increases by a factor of 16, but the Jeans mass increases by only a factor of 2.

An evaluation of \( M_f \) from equation (1) is somewhat uncertain, as it requires several assumptions about grain properties. The most important result for this paper is the extreme insensitivity of \( M_f \) to environment at the grain-coupling density. Still, the reader may be interested in the value of \( M_f \) derived in this way, so we make an attempt here. We assume that the size distribution of grains is \( d n_a/d a = n_a a^{-3.5} \) (Mathis et al. 1977), the maximum grain size is \( a_{\text{max}} \propto 2 \mu \text{m} \), the minimum grain size is \( a_{\text{min}} = 0.005 \mu \text{m} \), the grain specific density is \( \rho_g \approx 1 \text{ g cm}^{-3} \), and the total dust-to-gas mass ratio is 0.01. The grains are also assumed to be spherical. This is enough to derive the mean quantity

\[
\langle a^2D \rangle = 0.01 \left( m_{\text{H}_2}/\rho_g \right) \left( 3/4 \pi \right) (a_{\text{max}} a_{\text{min}})^{-1} \sim 9.6 \times 10^{-22} \text{ cm}^2
\]

that appears in equation (4). If we also take thermalization coefficient \( \alpha_a = 0.5 \), gas-grain temperature difference \( \Delta T = 2 \text{ K} \), and density \( n \sim 10^4 \text{ cm}^{-3} \) at coupling (which corresponds to a solution of eq. [4] for \( T = 9 \text{ K} \)), then \( M_f \approx 0.24 M_\odot \), which is a reasonable value for the characteristic mass.

The increasing dependence of \( M_f \) on \( n^{1/4} \) is too weak to imply that low-density regions of GMCs should produce more low-mass stars. The relative proportion of high- and low-mass stars depends more on the overall shape of the IMF, which is not discussed in this paper.

4.2. Ambient Gas in Star-Forming Regions

The ambient gas density in star forming regions varies from a low galactic average of \( \sim 1 \text{ cm}^{-3} \) or less in spiral galaxy disks to a high molecular cloud average of \( \sim 10^3 \text{ cm}^{-3} \) or more for giant molecular cloud complexes (GMCs) and inner-disk starburst regions. The characteristic mass in the IMF hardly varies throughout these regions, so there has to be some kind of regulation to keep \( T^{3/2}n^{-1/2} \) constant if \( M_f \propto M_\odot \). In the previous section, we discussed only the dense cores where small stellar groups and binary stars form, at densities close to the value where dust and gas become thermally coupled. Here we discuss the lower densities surrounding these cores, down to the ambient density in the galactic interstellar medium.
The most important regulator for star formation on scales that range from galactic disks to molecular clouds is the dynamical nature of the star formation process, which gives a rate per unit volume proportional to $n^{1.5}$ (e.g., Elmegreen 2002). This could be the origin of the Kennicutt (1998) law of star formation, sometimes called the Kennicutt-Schmidt (KS) law. The dynamical rate enters into the $n^{1.5}$ dependence, and the other factor of $n$ is from the mass per unit volume of available gas. There may be other dependencies on environmental parameters in this relation, such as the Mach number, mean magnetic field strength, or external radiation (Vázquez-Semadeni et al. 2003; Schaye 2004; Krumholz & McKee 2005), but these dependencies are difficult to observe directly. The KS law relating the star formation rate per unit area to the mass surface density of gas per unit area has been measured on a wide range of scales, from whole galaxies (Martin & Kennicutt 2001) to pieces of galaxies (Kennicutt et al. 2007). There are indications that either the coefficient or the power may vary with the local rate of shear (Vorobyov 2003; Luna et al. 2007), and there are uncertainties regarding the CO to H$_2$ conversion and other calibrations (Boissier et al. 2003), but the basic power-law form seems to be robust. Even models that assume a local star formation rate proportional to the first power of the molecular density, and then calculate the molecular fraction in various environments, recover the average KS relation with power 1.5 that we use here (Robertson & Kravtsov 2008).

The KS law is based on observations of column density, and here we consider volume density. This is a reasonable conversion if the gaseous scale height is about constant with radius in a galaxy, which is approximately true in our own Galaxy (e.g., Sanders et al. 1984). It is also reasonable if the photon mean free path in the near-UV is comparable to the disk thickness, which is also approximately true. The heating rate per grain depends on the mean intensity of radiation, which is the product of the volume emissivity and the photon mean free path averaged over all directions. The volume emissivity is about the star formation rate per unit area divided by the disk thickness, so if the mean free path is comparable to the disk thickness, then the mean intensity of radiation is proportional to the star formation rate per unit area, as assumed here.

On scales smaller than the galactic scale height, the $\rho^{1.5}$ dependence for the star formation rate is still a reasonable assumption, considering the dynamical nature of the processes involved. The use of a local radiation field proportional to this rate is also reasonable because star formation generally dominates the background heat sources. We discuss below how a time dependence might change the results.

We now ask how the dust temperature varies with density on the scales where the KS law operates. The dust temperature for general dust composition varies with grain size $a$ and ambient radiation field $G_0$ as (Tielens 2005, eq. [5.44])

$$T_d = 33.5(a/1 \mu m)^{-0.2}(G_0/10^4)^{0.2} K$$

where $G_0$ is measured in units of the Habing field, $1.6 \times 10^{-3}$ erg cm$^{-2}$ s$^{-1}$. This relation assumes that the Planck mean efficiency of grain absorption scales inversely with wavelength (a wavelength-squared dependence would change the exponents to 1/6).

In a star-forming region the radiation field $G_0$ will be proportional to the star formation rate, which is proportional to $n^{1.5}$ by the KS law. Thus, $T_d \propto n^{0.3}$. At the locally high densities of star formation, where the dust and gas temperatures are comparable, the environmentally dependent Jeans mass quantity, $T^{3/2}n^{-1/2}$, is therefore dependent on ambient density only weakly, as $n^{-0.05}$. Using the KS law again, this corresponds to an $M_f \propto G_0^{-0.03}$ dependence, which is also very weak. For an increase in star formation rate and radiation field by 3 orders of magnitude, $M_f$ decreases by only a factor of 1.3 when the gas and dust temperatures are comparable, even at densities that are not the thermal coupling density. At thermal coupling, $M_f$ increases by a factor of 2, from the previous subsection. The necessary precision to observe these variations in $M_f$ is not available yet, explaining why the observed characteristic mass for star formation is so constant, spanning the range from ambient field regions like Taurus to intense starburst regions like interacting galaxies.

The cores where individual stars form are much denser than the ambient ISM discussed above, so the constancy of $M_f$ on large scales may seem irrelevant to the IMF. However, if star formation operates at the dynamical rate for a wide range of scales, as seems to be the case (Elmegreen & Efremov 1996; Elmegreen 2007), then the star formation rate should be proportional to $n^{1.5}$ for a wide range of densities, possibly even including the inner cores of dense clusters, where the IMF is determined. Similarly, the radiation field should scale with the local star formation rate for a range of densities, considering time and space averages of this field. In this case, the constancy of $M_f$ from the KS relation would also apply to cloud cores. An interesting exception should occur at the onset of star formation, because the density can be high before the radiation field or temperature are high, lowering $M_f$ below the characteristic value. This would seem to be a problem for inactive regions like the Pipe Nebula (Alves et al. 2007) that have cold and dense prestellar cores with a characteristic mass a factor of $\sim 3$ above $M_f$ for stars. However, these cores began forming from lower density material in the Pipe Nebula, and at that time, the discussions in Clark & Bonnell (2005) apply: $M_f$ in the turbulent medium has an important influence on $M_c$ even before self-gravity becomes important on small scales. Indeed, the Pipe Nebula cores near the mass function turnover are not gravitationally bound (Lada et al. 2008), so the $M_f$ argument of § 4.1 does not apply to them yet. The difficult question is not whether $M_f$ is constant, which appears to be the case in a variety of situations, but when $M_c \sim M_f$ is established in the life cycle of a molecular cloud.

The value of $M_f$ for average GMC and lower density conditions depends on the radiation field that is expected for a star formation rate given by the KS law. The star formation rate $\dot{M}$ is related to the total IR luminosity as $\dot{M} = 2 \times 10^{-10}(L_{IR}/L_\odot) M_\odot$ yr$^{-1}$ (Kennicutt 1998). This equation assumes that all of the radiation from young stars comes out in the IR, so this luminosity is the total from the star-forming region. The Kennicutt (1998) relation in which the star formation rate per unit area depends on the $\sim 1.5$ power of the column density of gas was converted by Elmegreen (2002) into a relation between the star formation rate per unit volume and the 1.5 power of the volume density. It assumes that the average ISM density is comparable to the threshold tidal density in the galaxy, which is approximately true everywhere, and that the disk has an exponential light profile with a flat rotation curve. This gives the result $\dot{M}/\text{volume} \sim 0.012\rho(G_0)^{1/2}$ in cgs units. Thus, the luminosity density from star formation is

$$L/\text{volume} = 3.8 \times 10^{15} \rho(G_0)^{1/2}$$

in cgs units. The volume emissivity is this luminosity density divided by $4\pi$, and the radiation field $G_0$ is the volume emissivity times the path length. For a path length of $L = 1$ kpc, which is about one optical depth, $G_0 = 1.9 \times 10^{-3} n_H^{1.5}$ erg cm$^{-2}$ s$^{-1}$, where $n_H$ is the ambient density measured in molecules per
cubic centimeter for convenience in scaling to higher densities. The Habing radiation field is $1.6 \times 10^{-3}$ erg cm$^{-2}$ s$^{-1}$, so this value from the star formation rate is 1.2 times the Habing field for a KS relation at unit molecular density. The dust temperature from equation (5) is therefore 5.5 K for $a/1 \mu$m = 1. In general, the dust temperature from star formation heating alone is

$$T_d = 5.5(a/1 \mu m)^{-0.2}(L/1 \text{ kpc})^{0.2}n_\text{H}_2^{0.3}$$

based on the KS law with the above assumptions. For ambient radiation there would be other sources in addition to star formation, so this temperature is low compared to the observed ambient $T_d$. This equation is useful for scaling to higher star formation densities and their correspondingly higher radiation fields.

At this point in the evaluation of $M_f$, we need to assume some density where new star formation occurs; the density used in the previous paragraph is the average in the region producing the radiation, which is assumed to follow the KS law. We designate the density in the radiating star formation region $n_\text{rad}$, and the density in the new star-forming region, where $M_f$ is to be determined, $n_\text{sf}$. If new star formation occurs in a typical molecular cloud core, $n_\text{sf} \sim 10^4$ cm$^{-3}$. Putting $T_d$ and $n_\text{sf}$ into equation (1), we get

$$M_f = 0.37\left(\frac{n_\text{rad}}{1 \text{ cm}^{-3}}\right)^{0.45}\left(\frac{n_\text{sf}}{10^4 \text{ cm}^{-3}}\right)^{-0.5}\left(\frac{L}{1 \text{ kpc}}\right)^{0.3}M_\odot.$$ (8)

If the radiation path length, $L$, is about constant and $n_\text{sf} \propto n_\text{rad}$, we obtain the very weak scaling of $M_f$ with density discussed earlier in this subsection. The radiation path length should scale inversely with $n_\text{rad}$, however, as smaller regions of star formation typically have higher densities. For Larson-law scaling or for constant absorption on the line of sight, this would be an exact inverse relation. The density of new star formation should also depend on $n_\text{rad}$, as pressure from the H II region around the radiating stars comes to equilibrium with the pressure in the new star formation region (see § 4.3 for more on this situation). For this latter relation, we take an ionizing luminosity per unit volume proportional to the star formation rate per unit volume, which scales with $n_\text{rad}^{-2}$ according to the KS law, and then the density of the H II region should scale with the square root of this, as $n_\text{rad}^{-0.25}$. If we consider all of this, i.e., $L \propto n_\text{rad}^{-1}$ and $n_\text{sf} \propto n_\text{rad}^{0.75}$, we get a Jeans mass scaling as $M_f \propto n_\text{rad}^{-2.25}$. This is a weak dependence on environmental density, but not as weak at the $M_f \propto n^{-0.05}$ relation derived at the beginning of this subsection. For $n_\text{rad}$ varying by a factor of $10^5$ from the ambient medium to the average density in a GMC, $M_f$ decreases by a factor of $\sim 5$ in this expression.

We can summarize the present results as follows. For star formation in dense cores that are exposed to radiation and pressure from a surrounding region that may range in density from the ambient value in a galaxy to the average in a GMC, the temperature and local density scale together in such a way that the thermal Jeans mass is approximately constant and comparable to the characteristic mass in the IMF. All that has been assumed is that the radiation density scales with the surrounding star formation rate density, and that the star formation rate scales with the gas density multiplied by the dynamical rate, i.e., according to the Kennicutt-Schmidt law written in three-dimensional form. If the path length for the accumulated flux of this radiation is constant, and the local density for star formation scales with the surrounding density, $n$, then $M_f$ scales extremely weakly with density, as $n^{-0.05}$. If the path length scales inversely with surrounding density, whether by Larson’s laws for star formation in GMCs or by a constancy of the absorption, and if the local density for star formation scales with the surrounding ionization density, then $M_f$ scales as $n^{-0.225}$. In both cases, the KS law is a unifying factor that helps to preserve a nearly constant $M_f$ in a wide range of environments.

### 4.3. Compressed Globules in H II Regions

H II regions provide a very different environment for star formation compared to the dark neutral clouds considered in the previous two subsections. It is important to consider how $M_f$ varies for the most extreme conditions when neutral clouds and cloud pieces inside and adjacent to H II regions are heated, compressed, and ionized by strong radiation fields. Such conditions should apply to any cluster massive enough to form O-type stars, including super star clusters and young globular clusters. Inside and surrounding the cloud cores where these clusters form, there should be neutral globules, proplyds, elephant trunks, and other neutral material that is still forming stars. Each neutral piece will be heated and compressed by the first massive stars that form in the cluster, giving an $M_f$ specific to that region. Before these first massive stars appear, the results of the previous two subsections should apply.

We use the analysis of Bertoldi & McKee (1990) to determine the critically stable mass for gravitational collapse in a pressure-confined globule with an ionized boundary. This mass, given in their equation (5.5) for nonmagnetic clouds, is analogous to $M_f$ or $M_\text{crit}$ in the previous discussion and so presumably is related to the characteristic mass in the IMF for this environment. We also use the bolometric luminosities $L$ and Lyman continuum ionization rates $S$ for massive stars from Vacca et al. (1996), which were integrated over an IMF in Elmegreen (2007, Fig. 1) to give these quantities as functions of the cluster mass, $M$. For clusters more massive than $\sim 10^3 M_\odot$, where O-type stars are present, the integrals give nearly linear dependences between both $L$ and $S$ and the cluster mass,

$$L \sim 10^6 (M/10^3 M_\odot),$$

$$S_{49} \sim 10 (M/10^3 M_\odot),$$ (9)

where $S_{49}$ is the ionization rate in units of $10^{49}$ s$^{-1}$.

The critically stable mass from equation (5.5) in Bertoldi & McKee (1990) is

$$M_\text{crit} = 47.2 c^{14/3} \left[\frac{S_{49}}{(R/1 \text{ pc})^2}\right]^{-1/3} \phi M_\odot,$$ (10)

where $\phi$ includes a combination of factors of order unity, and $R$ is the distance to the star with ionization rate $S_{49}$, in units of parsecs. This equation considers the pressurized boundary of the near-spherical globule, where the pressure is determined by the ionization front. The thermal speed in the neutral cloud is $c$, in km s$^{-1}$ (based on $c^2 = kT/m_\text{H}_2$). This thermal speed depends on the dust temperature in the neutral gas, which depends on the radiation field $G_0$ according to equation (5). Tielens (2005) writes $G_0 = 2.1 \times 10^2 (L/10^4 L_\odot) (R/1 \text{ pc})^{-2}$. Thus,

$$T_d \sim 15.5 \left(\frac{a}{1 \mu \text{m}}\right)^{-0.2} \left[\frac{L/10^4 L_\odot}{(R/1 \text{ pc})^2}\right]^{0.2} K,$$

$$= 38.9 \left(\frac{a}{1 \mu \text{m}}\right)^{-0.2} \left[\frac{M/10^2 M_\odot}{(R/1 \text{ pc})^2}\right]^{0.2} K$$ (11)
where we have used equation (9). Inserting this dust temperature into the sound speed we get

\[
c = 0.36 \left( \frac{a}{1 \mu m} \right)^{-0.1} \left( \frac{M/10^3 M_\odot}{(R/1 \text{ pc})^2} \right)^{0.1} \text{ km s}^{-1}. \quad (12)
\]

Thus the critical mass is

\[
M_{\text{crit}} = 0.2 \left( \frac{a}{1 \mu m} \right)^{-0.47} \left( \frac{M/10^3 M_\odot}{(R/1 \text{ pc})^2} \right)^{0.13} M_\odot. \quad (13)
\]

This critical mass is similar to the observed characteristic mass of star formation and is only weakly dependent on cluster mass \(M\) and size \(R\). Note that for heating and compression inside a cluster core, the mean distance to the ionizing source, \(R_c\), is the cluster size as written here.

A magnetic field in the globule will stabilize it, so the critical mass for collapse will be larger than in equation (13) when a field is present (e.g., Bertoldi & McKee 1990). This is true for all of the critical masses derived in this paper. The field will eventually weaken because of ambipolar diffusion, so the nonmagnetic result is most relevant for the final state.

Another consideration for \(H\alpha\) regions is the complete ionization of small globules before they get compressed. Such ionization would deplete the low-mass part of the IMF and shift the characteristic mass to a higher value, regardless of \(M_{\text{crit}}\). This is possible for the smallest globules, according to equation (4.5) in Bertoldi & McKee (1990), where the mass limit for complete ionization may be written

\[
M_{\text{ionize}} < 0.0189 \left( \frac{n}{10^3 \text{ cm}^{-3}} \right)^{-5} \left( \frac{S_{49}}{(R/1 \text{ pc})^2} \right)^{3/2} M_\odot = 19 \left( \frac{n}{10^3 \text{ cm}^{-3}} \right)^{-5} \left( \frac{M/10^3 M_\odot}{(R/1 \text{ pc})^2} \right)^{3/2} M_\odot. \quad (14)
\]

This expression is not very useful because of the strong dependences on the initial globule density, \(n\), and the cluster mass, \(M\), and size, \(R\). Still, the mass limit is comparable to \(M_{\text{crit}}\) when \(n \sim 2.3 \times 10^3 \text{ cm}^{-3}\), a reasonable value for a piece of the GMC before ionization compression.

Once a neutral globule reaches pressure equilibrium with its ionized boundary, it either collapses quickly or is stable, according to whether its mass is greater or less than \(M_{\text{crit}}\). It is unstable, it cannot be ionized significantly because the collapse time is much faster than the ionization time. Using equation (4.10a) in Bertoldi & McKee (1990) for the ionization time and converting \(S_{49}\) and \(c\) into cluster mass \(M\) as before, we get the ionization time

\[
t_{\text{ionize}} \sim 10^6 \left( \frac{a}{1 \mu m} \right)^{0.12} \left( \frac{M/10^3 M_\odot}{(R/1 \text{ pc})^2} \right)^{-0.32} \left( \frac{M_{\text{globule}}}{1 M_\odot} \right)^{0.4} \text{ yr}. \quad (15)
\]

for \(M_{\text{globule}}\) in \(M_\odot\). The collapse time of a compressed globule is much smaller than this. Equation (3.33c) in Bertoldi & McKee (1990) gives the globule pressure in terms of \(c\) and \(S_{49}/R^2\). Converting these variables into cluster mass \(M\) and dividing by \(kT\) gives

\[
n_{\text{compressed}} = 1 \times 10^6 \left( \frac{a}{1 \mu m} \right)^{0.45} \left( \frac{M/10^3 M_\odot}{(R/1 \text{ pc})^2} \right)^{0.15} \times \left( \frac{M_{\text{globule}}}{1 M_\odot} \right)^{-0.2} \text{ cm}^{-3}. \quad (16)
\]

The corresponding dynamical time (within a factor of 2 of the collapse time) is

\[
t_{\text{dyn}} = (G\rho)^{-0.5} = 6 \times 10^6 \left( \frac{a}{1 \mu m} \right)^{-0.225} \left( \frac{M/10^3 M_\odot}{(R/1 \text{ pc})^2} \right)^{-0.75} \times \left( \frac{M_{\text{globule}}}{1 M_\odot} \right)^{0.1} \text{ yr}. \quad (17)
\]

The dynamical time is always much less than the evaporation time in pressure equilibrium, so if the globule does not get ionized immediately, before the implosion, it will collapse to a dense core with a much higher pressure and greater self-shielding ability. Then a star will likely form.

5. Extreme Star Formation: Low Metals and High Redshifts

The above considerations apply to star formation in the local universe. If we go to high redshift or very low metallicity, the situation changes. Detailed numerical simulations predict zero-metallicity (Population III) stars to be massive, \(M > 20 M_\odot\) (Abel et al. 2002; Bromm et al. 2002; Yoshida et al. 2006; O’Shea & Norman 2007). The lack of zero-metal stars in the Milky Way is consistent with this (see review in Beers & Christlieb 2005). Thus, \(M_\star\) must have been higher at extremely low metal abundance (Tumlinson 2006).

The critical abundance for the transition in \(M_\star\) is debated. Extremely metal-poor subgiant stars in the Galactic halo have masses below \(1 M_\odot\) (Christlieb et al. 2002; Beers & Christlieb 2005). The most extreme of these stars have \([\text{Fe/H}] < -7\), although carbon and oxygen are still relatively high, \(\sim 10^{-3}\) times solar. This unusual abundance pattern could be produced by pair-instability supernovae in Population III (Heger & Woosley 2002) or mass transfer from close binary companions (Ryan et al. 2005; Komiya et al. 2007). There are hints for an increasing binary fraction with decreasing metallicity for these stars (Lucatello et al. 2005). Some models suggest that low-mass star formation becomes possible once atomic fine-structure line cooling from carbon and oxygen becomes effective (Bromm et al. 2001; Frebel et al. 2007), setting a value for the transition metallicity \(z_{\text{crit}}\) at around \(10^{-3.5} Z_\odot\). However, for cold initial conditions and \(n < 10^3 \text{ cm}^{-3}\), \(H_2\) is the dominant coolant, suggesting that the transition is determined by other physical processes, such as dust formation (Jappsen et al. 2007). An alternative view is that low-mass star formation is the result of dust-induced fragmentation at high densities and late stages in protostellar collapse (Schneider et al. 2002, 2006; Omukai et al. 2005). The transition metallicity is then in the range \(10^{-6} \leq Z_{\text{crit}} \leq 10^{-3} Z_\odot\), with the resulting IMF plateau clearly falling below 1 \(M_\odot\) at higher abundances (Clark et al. 2008).

Observations that \(M_\star\) increases with redshift come from global population studies of distant galaxies. These are not the hypothesized Population III stars, but normal stars that are observed in present-day mass functions. For example, van Dokkum (2008) studied mass-to-light ratios and \(U-V\) colors for early-type galaxies in the range 0.02 < \(z < 0.83\). These are highly evolved galaxies, and the mass-to-light ratio depends mostly on the present-day mass function near 1 \(M_\odot\). The best-fit IMF for this sample is flat at 1 \(M_\odot\), constrained by the observed high rate of change of the mass-to-light ratio within this redshift range. This means that the IMF plateau has to include 1 \(M_\odot\) within its range, unlike the local IMF, which begins to steepen into the Salpeter power law at this
mass. Van Dokkum (2008) also noted that a flat IMF at $M_\odot$ is consistent with the observed shallow rate of change of the Balmer absorption strength over this redshift range. Constraints from cosmic background starlight and the local luminosity density of galaxies also suggest an upward shift in $M_\star$. Fardal et al. (2007) found that a shallow power-law slope is not enough to simultaneously fit these two measurements, but a “paunchy” IMF with a peak at $\sim 5 M_\odot$ is required. In a third study, Davé (2008) compared the star formation rate per unit mass out to $z = 2$ using three different surveys with that expected from cosmological simulations that assume a fixed $M_\star$. There was a clear discrepancy that was explained by an increase in $M_\star$ with redshift as $0.5(1+z)^2 M_\odot$, making $M_\star$ larger by a factor of $\sim 9$ at $z = 2$. For extreme metal-poor stars in the Milky Way, Komiyama et al. (2007) suggested that $M_\star \sim 5 M_\odot$ from the fractions of these stars that are C-rich with and without $s$-process elements. C-rich extreme metal-poor stars are probably surviving low-mass binary members, so models of stellar evolution and binary mass transfer were involved in their analysis.

Other studies of cosmological structure formation also suggest that the IMF shifts toward more massive stars, but most of these studies consider a shallower slope in the power-law part of the IMF above a few solar masses, not a possible change in $M_\star$, which could also be happening (or happening instead of a shallower slope). These shallower slopes include the starburst phases of massive elliptical galaxies ( Pipino & Matteucci 2004; Nagashima et al. 2005b) and clusters of galaxies (Renzini et al. 1993; Loewenstern & Mushotsky 1996; Chiosi 2000; Moretti et al. 2001; Tornatore et al. 2004; Romeo et al. 2006; Portinari et al. 2004; Nagashima et al. 2005a). They also include the Milky Way and M31 bulges, which appear to have had shallow IMFs at intermediate to high mass because of their large [Fe/H] abundances (Ballero et al. 2007). In the central parsec of the Galaxy, the IMF for young stars may be shallower as well (Nayakshin & Sunyaev 2005; Paumard et al. 2006; Maness et al. 2007). Shallow in these studies means an IMF slope of $\Gamma \sim -1$ to $-1.1$, where the Salpeter slope is $-1.35$. Recent theoretical models of flat IMFs are given in Klessen et al. (2007).

Evidently, the observations suggest that $M_\star$ was higher at a redshift of $\sim 2$, perhaps by a factor of $\sim 10$, even for stars that are not zero-metallicity. What could have caused this increase if $M_\star$ is as independent of environment as the previous sections suggest? To attempt to answer this question, we consider how $M_\star$ might have increased in the early universe for each of the three models in § 4.

In § 4.1, the thermal Jeans mass, $M_J$, was shown to have a dependence on $(a^2 D)/Z^{3/4} n^{1/4}$ for grain radius $a$, dust-to-gas particle number ratio $D$, density $n$, and metallicity dependence $Z^\alpha$ that appears in the gas cooling expression. We noted how this would not depend much on metallicity as collisional processes bias the average value $(a^2 D)$ toward small grains while metallicity, containing the average of $a^2 D$, has a greater weight for large grains. However, when the relative dust mass gets low, $(a^2 D)$ starts to drop. If we consider a Mathis et al. (1977) grain size distribution $dn_d/da = n_d a^{-3.5}$, and a relative dust mass abundance proportional to the metallicity, $Z$, then $Z/Z_\odot = (a_{\max}/a_{\min})^{1/2} / A_{\odot}$, for $Z = n_d/a_{\min}$ and $A = 1 - (a_{\min}/a_{\max})^{1/2}$. The minimum and maximum grain sizes are denoted by subscripts. Solving for $A$ and integrating over the grain size distribution again, we get

$$\frac{(a^2 D)_\odot}{(a^2 D)_Z} = \frac{Z/Z_\odot}{(Z/Z_\odot)(A_{\odot}/\xi) + (a_{\min}/a_{\max})^{1/2}}$$

for constant $a_{\min}$. For normal metallicities, when $Z/Z_\odot > (\xi/A_{\odot})(a_{\min}/a_{\max})^{1/2}$, $(a^2 D) \propto n_d$ and is nearly independent of $Z/Z_\odot$, as mentioned above. For very low metallicities, $Z/Z_\odot < (\xi/A_{\odot})(a_{\min}/a_{\max})^{1/2}$, $(a^2 D)$ decreases with $Z/Z_\odot$. Unless $\xi$ in the cooling rate is larger than 1, i.e., the molecular gas cooling rate depends sensitively on metallicity, $(a^2 D)/Z^\alpha$ decreases at very low metallicity. This lowers $M_J$, which is opposite to the effect observed in the early universe. Thus, lower gas cooling at small $Z$ does not necessarily produce larger $M_J$ at the grain-gas coupling density.

In § 4.2, ambient ISM conditions were considered rather than dense cores. $M_J$ was shown to be insensitive to density and radiation field as long as the Kennicutt star formation law is satisfied. If the star formation rate is much higher for a given density than it is locally, then the dust temperature can be higher for a given density, and $M_J$ is higher. This requires a higher efficiency of star formation. Such deviations from the Kennicutt relation have been suggested for ultraluminous infrared galaxies (Graciá-Carpio et al. 2008). Thus, higher efficiencies could have caused the observed $M_J$ increase. The results in § 4.2 also depend on equation (3), which is the local relation between grain temperature and radiation field. If the metallicity is much lower and the grains are systematically smaller and hotter, $M_J$ will be higher for a given relation between radiation field and density. Observations of 66 starburst galaxies by Engelbracht et al. (2008) showed $T_d \propto Z^{-0.2}$ down to $Z/Z_\odot \sim 0.1$ (their Fig. 5), although $T_d$ appears to drop with decreasing $Z/Z_\odot$ below that. Setting $M_J \propto T_d^{3/2}$ for this Engelbracht et al. (2008) relation, we get $M_J \propto Z^{-0.3}$. If the Kennicutt relation still holds, thus, decreases in metallicity could correspond to increases in $M_J$ because of increased ambient grain temperatures at the same density and radiation field. It is not clear if this trend should continue for $Z/Z_\odot < 0.1$, where the Engelbracht et al. (2008) power-law relation stops. Equations (5) and (18) suggest that it should. In that case, the extremely metal-poor Milky Way halo stars investigated by Komiyama et al. (2007), which have $[Fe/H] < -2.5$, would have higher $M_J$ by more than a factor of $10^{0.25 \times 0.3} = 5.6$, which is about the $M_J$ shift that they observe.

Section 4.3 considers $M_J$ in highly ionized regions. The same dependence of $M_J$ on $Z$ should result if the grains become systematically hotter for lower metallicity with a given radiation field. A more prominent effect in strong radiation fields may be the complete ionization of prestellar globules, even if $M_J$ itself does not change much. Equation (14) indicates that sufficiently small globules are destroyed by ionization before they collapse to a star. We were inconclusive about the importance of this effect because it depends sensitively on the global density and on the column density of the surrounding cloud. If the cosmological increase in $M_J$ depends more on star formation rate and star formation density than it does on metallicity, then the ionization of small prestellar globules could be the primary explanation for the lack of low-mass stars. For example, if $M_J$ is high in starbursting mergers during the formation of an elliptical galaxy, and the metallicity is somewhat solar, then the ionization of prestellar globules could be an important factor. Metallicity could still play a role because dust affects the scattering of ionizing photons.

For all of these scenarios, it is difficult to understand why Milky Way globular clusters, with metallicities $[Fe/H] \sim -1.5$ (Binney & Tremaine 1987), have characteristic masses that appear similar to those of modern metal-rich super star clusters. According to the $M_J \propto Z^{-0.3}$ relation above, $M_J$ should be larger by a factor of $\sim 3$ in globular clusters if evaporation of prestellar cores is not effective, and larger still if it is affected by prestellar
evaporation. A factor of 3 may be too small to notice after a Hubble time of mass segregation and evaporation in these clusters. However, such a shift is consistent with that suggested by Chabrier (2003) for the Milky Way bulge and globular clusters, as noted in § 1. More observations of the characteristic stellar mass as a function of metallicity or average grain size are needed.

6. CONCLUSIONS

The characteristic mass for star formation, which appears as a plateau in the IMF, is nearly constant over a wide range of conditions, ranging from the outer part of the Milky Way, to dense molecular cores, to super star clusters. This constancy is explained as the result of very weak dependencies for the thermal Jean mass $M_J$ on density, metallicity, and radiation field in three fundamental environments: dense cloud cores, the ambient ISM, and the vicinity of highly ionizing super star clusters. Dense cores give a constant $M_J$ because of the way the molecular cooling rate scales with density and temperature. The ambient ISM has a constant $M_J$ because of star formation feedback with the Kennicutt-Schmidt law. Dense ionizing clusters have constant $M_J$ because of interdependencies between the pressure and radiation field, which is coupled to the dust temperature. In all cases, $M_J$ varies by only a factor of ~4 or less in the most extreme situations.

These three cases are representative of a wide range of conditions for star formation. The values of $M_J$ are about the same for each, and in all cases, this value may be identified with the characteristic mass of star formation, $M_*$. The fact that $M_J$ is so invariant with environment, even for different ways of looking at environment, seems to explain the constancy of the characteristic mass in the IMF.

Observations of real increases in $M_J$ under cosmological conditions have been summarized. The environment for star formation is not well understood in this case, but there are some fundamental ways in which the physical parameters are expected to change in extreme conditions. One possibility is that the efficiency of star formation increases, so the radiation field is stronger for a given density and pressure. Another possibility is that the minimum grain temperature increases for a given radiation field as a result of a decreasing maximum grain size at low metallicities. A third possibility is that high radiation densities in super starburst conditions evacuate low-mass prestellar cores. Observations of any correlations between the lower part of the IMF and environmental conditions at high redshift would be useful in understanding the characteristic mass.

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