Abstract

The axial strange form factor $F_A^s$ of the nucleon is assumed to be dominated at low momentum transfer by the isoscalar axial vector mesons $f_1(1285)$ and $f_1(1420)$. The importance of the $a_0\pi N$-triangular vertex correction is demonstrated.

1 Introduction

The EMC measurements of the longitudinally polarized deep inelastic structure function $g_1^p(x)$ of the proton[1] raised the question of the spin distribution among quarks and gluons. In the framework of the “naive” parton model the proton spin is expressed in terms of the spin polarization distribution of quarks and antiquarks

$$\Delta q(x) = q^\uparrow(x) - q^\downarrow(x) + \bar{q}^\uparrow(x) - \bar{q}^\downarrow(x)$$

as

$$\Delta \Sigma = \Delta u + \Delta d + \Delta s, \quad \text{with} \quad \Delta q = \int_0^1 dx \Delta q(x).$$

In fact, the EMC has measured the integral

$$\Gamma_1^p(Q_0^2 = 10.7 \text{GeV}^2) = \int_{0.01}^{0.7} dx g_1^p(x, Q_0^2) = \frac{1}{18}(4\Delta u + \Delta d + \Delta s)(1 - \frac{\alpha_s(Q_0^2)}{\pi}).$$

Note that $\Delta q(Q^2) = \text{const.}$ in the perturbative regime of QCD because of chiral symmetry arguments. In combining (2) with the experimental value of the isovector axial coupling constant $g_A$ known from neutron $\beta$ decay,

$$g_A^p = -g_A^\pi = \Delta u - \Delta d,$$

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on the one side, and with the \( F/D \) ratio extracted from hyperon \( \beta \) decay
\[
\frac{1}{\sqrt{3}}(3F - D) = \frac{1}{\sqrt{3}}(\Delta u + \Delta d - 2\Delta s),
\]
(4)
on the other side, the result is obtained, that the fraction of the proton spin carried by the quarks is surprisingly small (see [2] for details). The integral in (2) can alternatively be expressed as a combination of the axial isovector \( (g_A) \), axial hypercharge \( (G_A^{(8)}) \), and axial flavor singlet \( (G_A^{(0)}) \) form factors as follows:
\[
\int_0^1 dx \, g_1^p(x) = \frac{1}{12\sqrt{3}}(\sqrt{3}g_A + 2G_A^{(8)} + 4\sqrt{2}G_A^{(0)})(1 - \frac{\alpha_s(Q^2)}{\pi})
= \frac{1}{6}\left(\frac{g_A}{2} + \frac{5}{\sqrt{3}}G_A^{(8)} + 2G_A^{(0)}\right)(1 - \frac{\alpha_s(Q^2)}{\pi}),
\]
(5)
where (5) is based on the (presumably rather strong) assumption of SU(3) symmetry leading to the relations
\[
G_A^s = \sqrt{\frac{2}{3}}G_A^{(0)} - 2\sqrt{\frac{1}{3}}G_A^{(8)}, \quad \frac{2}{\sqrt{3}}G_A^{(8)} = \frac{1}{3}(3F - D).
\]
(6)
The nucleon matrix elements of the axial hypercharge \( (A_\mu^{(8)}) \) and flavor singlet \( (A_\mu^{(0)}) \) quark currents are normalized according to:
\[
\langle N \mid A_\mu^{(0,8)} \mid N \rangle = \langle N \mid \bar{\Psi}_\gamma \gamma_5 \lambda^{(0,8)}_A \Psi \mid N \rangle = G_A^{(0,8)}\bar{U}_\gamma \gamma_5 U,
\]
(7)
where \( \lambda^{(0,8)} \) denote the corresponding Gell-Mann matrices, \( U \) the nucleon Dirac spinor [3], and \( \Psi \) stands for the flavor quark SU(3) spinor. The strange axial nucleon form factor \( G_A^s \) is introduced as
\[
\langle N \mid \bar{s}_\gamma \gamma_5 s \mid N \rangle = G_A^s\bar{U}_\gamma \gamma_5 U.
\]
(8)
In the case of a vanishing strange axial form factor, eq. (5) reduces to the so called Ellis-Jaffe sum rule [4]. Now the surprise is the significant deviation of the measured value for \( \Gamma_1^p \) from the Ellis-Jaffe sum rule, attributed to a small flavor singlet axial form factor or, alternatively, to a significant nonvanishing value of \( G_A^s \). Different scenarios have been discussed in the literature in this respect. Here, we would like to point out the one presented in [5] where due to a non-trivial structure of the hadronic vacuum the flavor singlet axial charge is “eaten” by instanton effects. The axial form factors introduced above enter the neutral axial vector current according to (for details see, e.g., ref. [6])
\[
\langle N \mid A_\mu \mid N \rangle = \langle N \mid \bar{\Psi}_\gamma \gamma_5 \lambda_3 \psi + \frac{1}{4}\bar{s}_\gamma \gamma_5 s + \frac{1}{4}\bar{c}_\gamma \gamma_5 c \mid N \rangle
= -\frac{g_A}{2}\bar{U}_\gamma \gamma_5 \tau_3 U + \frac{G_A^s}{4}\bar{U}_\gamma \gamma_5 U.
\]
(9)
Here, the notation \( \bar{\psi} = (u, d) \) has been used and the \( \bar{c}c \) content of the nucleon has been neglected in the last line of (9). Thus in the light of the proton spin problem, the theoretical investigation of the strange axial nucleon form factor becomes important.

In this study we present a \( f_1 \)-vector meson dominance (VMD) model for this form factor and demonstrate the importance of a lowest 1-loop vertex correction.
2 The Structure of the $f_1NN$ Vertex

In the spirit of the VMD ansatz\cite{7} and in analogy to the $A_1$ dominance of the isotriplet axial current exploited in\cite{8} we suggest the following representation for the strange axial form factor of the nucleon at low energies:

$$F_s^A(t) = \frac{G_s^A}{\bar{g}} \left\{ - \frac{g_D m_D^2}{m_D^2 - t} F_D(t) \sin \epsilon + \frac{g_E m_E^2}{m_E^2 - t} F_E(t) \cos \epsilon \right\}. \tag{10}$$

where $\bar{g} = g_E \cos \epsilon - g_D \sin \epsilon$. Here, "$D$" and "$E$" label respectively the constants related to the $f_1(1285)$ and $f_1(1420)$ mesons. In (10) the deviation of the mixing angle between the isoscalar of the octet and the flavor singlet state in the axial vector meson nonet from the ideal mixing angle of $\theta_0 = 35.3^\circ$ has been made explicit according to:

$$f_1(1285) = \frac{\bar{u}u + \bar{d}d}{\sqrt{2}} \cos \epsilon - \bar{s}s \sin \epsilon, \quad \tag{11}$$
$$f_1(1420) = \frac{\bar{u}u + \bar{d}d}{\sqrt{2}} \sin \epsilon + \bar{s}s \cos \epsilon. \quad \tag{12}$$

The angle $\epsilon$ has been determined experimentally in\cite{9} to be $\epsilon = 15^\circ \pm 10^\circ$. The latter equation shows that a significant violation of the OZI rule is observed in the axial vector meson nonet\cite{10}. By chiral symmetry arguments it is possible to parametrize the coupling constants of the $f_1$ mesons as $g_D = g_\omega \cos \epsilon - g_\phi \sin \epsilon$ and $g_E = g_\omega \sin \epsilon + g_\phi \cos \epsilon$. The values for $g_\omega$ and $g_\phi$ are taken from\cite{11}.

Dispersion relations with one subtraction have now been assumed for the $f_1(1285)NN$ and $f_1(1420)NN$ strange form factors, respectively,

$$F_{D/E}(t) = 1 + \frac{t}{\pi} \int_{\text{threshold}}^\infty dt' \frac{3(F_{D/E}(t'))}{t'(t' - t - i\epsilon)}. \tag{13}$$

Now, the imaginary part of $F_{D/E}(t)$ can be calculated from the Feynman graphs of Fig. 1 by means of the Cutkosky rules\cite{12}. The most important $f_1NN$ vertex correction is due to the $(f_1(1285) \to a_0(980) + \pi)$ decay channel with a $(37 \pm 7)\%$ branching ratio. It is especially important for the modification of the one-body isoscalar axial vector current of in-medium nucleons, a fact demonstrated in\cite{13}. There it was shown for the case of the Fermi gas model that the $a_0\pi$ exchange current reduces the one-body matrix element of the isoscalar axial vector current by about 10%. The effect is determined by the ratio $\sin \epsilon / (g_D G_A^s)$ and acquires importance mainly because of the above mentioned significant violation of the OZI rule in the axial vector meson nonet. We use the value $G_A^s = \frac{1}{2} G_A^s = -0.13 \pm 0.04$ in accordance with\cite{2}.

### 3 Results and Discussion

To describe the $f_1(1285)a_0\pi$ vertex we use the following effective Lagrangian density

$$\mathcal{L}_{Da_0\pi} = g_{Da_0\pi} f_1^A \partial_A \vec{\phi}_\pi \cdot \vec{a}_0. \quad \tag{14}$$
The value of $g_{D\alpha_0\pi}$ extracted from the corresponding partial width is up to a sign $|g_{D\alpha_0\pi}| = 5.3$. The coupling constant of the $\alpha_0(980)$ meson to the nucleon is identified to that of the scalar-isotriplet channel of the Bonn potential\cite{14} as $g_{\alpha_0NN} = 3.73$, and a pseudoscalar form of the $\pi N$ vertex is assumed. From the graphs in Fig. 1 we obtain for the absorptive part of the $f_1NN$ form factor the following expressions for its low frequency part ($(m_{\alpha_0} + m_{\pi})^2 \leq t \leq 4m_N^2$):

$$\Im(F_D(t)) = c \frac{m_Nt - (m_{\alpha_0}^2 - m_{\pi}^2)(\sqrt{t} - m_N)}{t\sqrt{|t - 4m_N^2|}} \arctan A(t),$$

where

$$A(t) = \sqrt{|t - 4m_N^2|} \sqrt{t - 2(m_{\alpha_0}^2 + m_{\pi}^2) + (m_{\alpha_0}^2 - m_{\pi}^2)^2/t} \Big/ (t - m_{\alpha_0}^2 - m_{\pi}^2),$$

$$c = \frac{g_{D\alpha_0\pi}g_{\alpha_0NN}g_{\pi NN}}{8\pi g_DG_A^s}.$$  

The high frequency part ($4m_N^2 \leq t \leq \infty$) is simply obtained from (15) by replacing \arctan\(A(t)\) by $\frac{1}{2} \ln \frac{1 - A(t)}{1 + A(t)}$. It is small for unitarity arguments.

In Fig. 2 the ratio of $F_A^s(t)$ including the vertex contribution to the one ($F_{A,0}^s(t)$) without is presented. At higher momentum transfers, the vertex contribution is on the level of several percent and thus non negligible. We expect that the $\alpha_0\pi N$ triangular vertex correction to the strange axial form factor will increase in case the $\alpha_0\pi$ interaction is included. The relevant T-matrix amplitude can be evaluated within the framework presented in\cite{17}. The main conclusion of our study is that the strong violation of the OZI rule in the axial vector meson nonet implies

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Fig. 1: Triangle vertex correction to the strange axial form factor of the nucleon.

Fig. 2: Ratio of the strange axial form factor with and without vertex correction.