Station Layout Optimization Genetic Algorithm for Four Stations TDOA Location

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Abstract: Station layout has a direct impact on localization precision for four-station TDOA (time difference of arrival) location system. In order to optimize the overall location precision of targets in certain area, genetic algorithm for station layout optimization is presented. GDOP is regarded as the measurement rule of location precision. Then the multi-object programming mathematical model of station layout optimization is built. The optimal solution is searched by genetic algorithm. Consequently, the optimal layout of station is obtained. The simulation provides the GDOP distribution of two station layouts. One is the initial station layout, and the other is the optimal station layout. The simulation results show that for the optimal station layout, the effect of improving the location precision is obvious, and also indicate the validity of genetic algorithm.

1. Introduction

Time difference location is a passive location method, which uses multi-channel measurement of time difference data to achieve three-dimensional positioning of the target. Because of its passive work, high positioning accuracy and other advantages, it has been widely used in radar, wireless communication, underwater acoustic and other fields.

For TDOA Localization system, the Localization error of the target is closely related to the geometric relationship between the target and the measuring station. With the increase of the number of measuring stations, it is an effective way to improve the localization accuracy to optimize the station layout when the time difference measurement error and the self-localization accuracy of time difference and the stations layout, and empirically describe optimal stations layout. Aiming at the problem of multi-stations and multi-position parameters optimization, this paper uses genetic algorithm and multi-objective programming to establish a model and construct a judgment function so as to improve the overall localization accuracy of a certain area.
2. Basic principle

Suppose that the coordinates of the three measuring stations are \([x_i, y_i, z_i]\), \(i=0,1,2\) in the rectangular coordinates of the earth's center. Receiving the target information from a certain coordinate on the earth's surface, we can get two time-difference data corresponding to two time-difference equations. Simultaneously, the WGS-84 earth's surface equation can be established to set up the target localization equations.

\[
\begin{align*}
    r_1 - r_0 &= c \cdot \Delta t_1 \\
    r_i - r_0 &= c \cdot \Delta t_i \\
    r_0 &= \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} \\
    r_i &= \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}
\end{align*}
\]

In the equations, \(r_0, r_i\) are the distance from the target to three measuring stations, \(c\) is the electromagnetic wave velocity, \(\Delta t_1, \Delta t_i\) are the time difference between the signal from the target to the measuring station 1 and other measuring stations, respectively.

3. Stations layout optimization introduction

Layout optimization is a multi-objective optimization model, aiming at multiple objective functions in a region. The mathematical model is

\[
\min F(X) = \min [f_1(X), f_2(X) \cdots f_n(X)]
\]

s. t. \(G(X) \leq 0\)

In the formula, \(X = [x_1, x_2, \cdots, x_m]\) is decision variables, \(G(X) = [g_1(X), g_2(X), \cdots, g_p(X)]\) is constraint condition, \(g_i(X) (i = 1, 2, \cdots, p)\) can be linear functions or Nonlinear function, unconstrained multi-objective programming when \(p=0\) is used.

4. Geometrical Dilution of Precision(GDOP)

Localization accuracy is one of the most important indicators of localization system. In this article, Geometrical Dilution of Precision(GDOP) is used as a measure of positioning accuracy.

The differential between the two sides of the first two forms of formula (1) is obtained:

\[
\begin{align*}
    \text{d}X = \text{Cd}\Delta t_i + U_i dX_i - U_0 dX_0 (i = 1,2) \\
    a_{ix} &= \frac{x - x_i}{r_i}, a_{iy} = \frac{y - y_i}{r_i}, a_{iz} = \frac{z - z_i}{r_i}, i = 0,1,2,3; \\
    dX &= [dx \ dy \ dz]^T; \\
    dX_i &= [dx_i \ dy_i \ dz_i]^T; \ i = 0,1,2,3; \\
    U_i &= [a_{ix} \ a_{iy} \ a_{iz}]^T; \ i = 0,1,2,3;
\end{align*}
\]
Differentiating the last formula in formula (1)
\[
\frac{x}{N} dx + \frac{y}{N} dy + k \frac{z}{N} dz = 0
\] (5)

In the formula:
\[
k = \frac{1}{(1 - e^2)^2}
\] (6)

Combination formula (3) and (4)
\[
AdX = Cd\Delta T + dU
\] (7)

\[
A = \begin{bmatrix}
a_{1x} - a_{0x} & a_{1y} - a_{0y} & a_{1z} - a_{0z} \\
a_{2x} - a_{0x} & a_{2y} - a_{0y} & a_{2z} - a_{0z} \\
a_{3x} - a_{0x} & a_{3y} - a_{0y} & a_{3z} - a_{0z} \\
\end{bmatrix}
\] (8)

\[
d\Delta T = \begin{bmatrix}
d\Delta t_1 \\
d\Delta t_2 \\
d\Delta t_3 \\
0
\end{bmatrix}
\]
\[
dU = \begin{bmatrix}
U_1^T dX_1 - U_0^T dX_0 \\
U_2^T dX_2 - U_0^T dX_0 \\
U_3^T dX_3 - U_0^T dX_0 \\
0
\end{bmatrix}
\]

\[
dX = A^{-1}(Cd\Delta T + U)
\] (9)

The covariance matrix of the location error of the target emitter is assumed to be 0 and uncorrelated.
\[
P_{dX} = E(dXdX^T) = A^{-1}[e^2E(d\Delta Td\Delta T^T) + E(dUdU^T)][A^{-1}]^T
\] (10)

GDOP is:
\[
GDOP = \sqrt{P_{dX}(1,1) + P_{dX}(2,2) + P_{dX}(3,3)}
\] (11)

5. Genetic Algorithm

Genetic algorithm has the characteristics of robustness, high parallelism, randomness and adaptability. The essence of genetic algorithm is to use the evolution of population to achieve the optimal search process.

5.1. Parameter determination

Population size, variable range, maximum iteration number, selection replication probability, crossover probability and mutation probability are all parameters in genetic algorithm.

It is necessary to encode the parameters of the actual problem to use genetic algorithm, that is, the parameters of the actual problem are converted into genetic space and the chromosomes are composed of genes according to a certain structure. In this algorithm, the typical parameter coding methods are binary coding and real number coding. Among them, the binary coding principle is simple and universal, the target measurability is not strong, the coding length is limited; the real number coding directly uses a real number to represent each parameter, and these real numbers in series form a chromosome, intuitive and convenient, will not appear inadequate precision problems, but the need for Design specialized genetic operations.

In base station layout using genetic algorithm, real coding is adopted.

5.2. Initialization

In the stations layout, there are four stations, each station position is three-dimensional coordinates, to achieve the optimal base station layout needs to determine twelve variables. If the population number is 100, the population size of the genetic algorithm is $100 \times 12$

5.3. Fitness function

Population fitness is the criterion of evaluating the individual quality of the whole population, and is the basis of deciding whether the individual of the population will continue to reproduce or be
eliminated. In the process of searching, genetic algorithm is based on the fitness function, using the fitness function of the individual of the population as the judgment basis, using the fitness function of the individual of the population to evolve. It is very important to determine fitness function of population. The objective of the optimal layout of the station is to make the positioning accuracy higher, that is, to minimize GDOP. Therefore, the population fitness function can be expressed as:

\[ \text{Fitness} = \frac{\sum_{i=1}^{N} \text{GDOP}_{i}}{N} \]  

(12)

Formula: N indicates the number of flight path test points.

5.4. Selection process

The base station layout optimization scheme based on genetic algorithm, the selection is to select two questions from the population to ensure the quality of the individual population. The basic idea is to generate a random number in \([0, \text{Sum}/M]\) and define it as pointer P. The individuals in M are selected by random traversal sampling according to their position in \([p, p+1, p+2, \ldots, p+M-1]\). When executing the selection operation, the probability of individual selection is expressed as:

\[ F(x_i) = \frac{f(x_i)}{\sum_{i=1}^{M} f(x_i)} \]  

(13)

Formula: \( F(x_i) \) is the probability of \( x_i \) selection of population individuals, and \( f(x_i) \) is the fitness function value of \( x_i \) of population individuals.

5.5. Cross operation

The purpose of population crossover operation is to increase the diversity of populations, and obtain better individuals. The two parents of the population crossing are \( X_1 \) and \( X_2 \).

\[
\begin{align*}
X'_1 &= rX_1 + (1-r)X_2 \\
X'_2 &= rX_2 + (1-r)X_1
\end{align*}
\]  

(14)

Formula: the probability of crossover is \( r \in [0,1] \). When the crossover probability changes adaptively, the diversity of the population can be improved and the global search ability of the population can be improved.

In the proposed genetic algorithm, \( P_c \) will increase or decrease due to the population fitness. According to this idea, the cross probability can be expressed as:

\[
P_m = \begin{cases} 
P_{\text{max}} - \frac{(P_{\text{max}} - P_{\text{min}})(f-f_{\text{av}})}{f_{\text{max}}-f_{\text{av}}} & f \geq f_{\text{av}} \\ 
P_{\text{max}} & f < f_{\text{av}} \end{cases}
\]  

(15)

In the formula: \( P_{\text{max}} \) and \( P_{\text{min}} \) are the maximum and minimum crossover probability, \( f_{\text{av}} \) is the average population fitness; \( f \) is the parameter fitness; \( f_{\text{max}} \) is the maximum population fitness.

6. Simulation and analysis

The simulation experiment assumes that the target area ranges from 121.5 to 123.5 degrees east longitude, 21.5 to 23.5 degrees north latitude and 70 km elevation.

By using the above-mentioned optimization method based on genetic algorithm, the optimization result X can be obtained.

\[
X = \left( \begin{array}{cccc}
22.8371 & 122.9713 & 22.8138 & 122.9735 \\
22.8071 & 123.0074 & 22.8758 & 123.0110
\end{array} \right)
\]

The time difference measurement error of the setting position system is 10ns, and the measurement station assumes that the errors in all directions of x, y, Z are 5m in Geodetic rectangular coordinates. Figure 2 is the GDOP contour map of the initial value. Figure 3 is the optimized GDOP contour map. The dots in the picture are the locations of the four measuring stations.
Figure 2. Initial location GDOP isoline

Figure 3. Location of GDOP contour after optimization
As can be seen from Fig. 2 and 3, compared with the initial value, i.e. the initial layout form, the overall positioning accuracy of the layout optimization results obtained by using the genetic algorithm is obviously improved, and the coverage of the whole target area is guaranteed at the same time.

7. Conclusion

In this paper, GDOP is used as a measure of positioning accuracy. Considering the improvement of positioning accuracy in practical engineering, a four-station TDOA positioning optimization method based on multi-objective programming is proposed. The simulation results show that the genetic algorithm can not only ensure the coverage of the system, but also optimize the overall positioning accuracy of the target area. In practice, according to the experience information, each station can set up different small-scale optimization areas to improve the optimization efficiency.

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