UNBOUNDED ABSOLUTELY WEAKLY CAUCHY SEQUENCES AND APPLICATIONS

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ABSTRACT. Motivated by characterization of a reflexive Banach lattice in terms of unbounded absolutely weakly Cauchy (uaw-Cauchy) sequences, we consider operators between Banach lattices which maps uaw-Cauchy sequences to weakly (uaw- or norm) convergent sequences. This allows us to characterize KB-spaces and reflexive spaces in terms of these operators. Furthermore, we consider the unbounded Banach-Saks property as an unbounded version of the weak Banach-Saks property. There are many considerable relations between spaces possessing the unbounded Banach-Saks property with spaces fulfilled by different types of the known Banach-Saks property. In particular, we characterize order continuous Banach lattices and KB-spaces in terms of these relations, as well.

1. MOTIVATION AND INTRODUCTION

Let us first start with some motivation. It is proved in [7, Theorem 8] that, a Banach lattice E is reflexive if and only if every bounded uaw-Cauchy sequence in E is weakly convergent; this, in turn, implies that the identity operator on E possesses this property: it maps every bounded uaw-Cauchy sequence to a weakly convergent sequence. This motivates us to define operators which respect this property. The key point in this direction is the notion of uaw-Cauchy sequences. On the other hand, the authors in [3, 8] considered unbounded continuous operators between Banach lattices which respect, in a sense, unbounded null sequences. So, it would be interesting to consider pre-unbounded continuous operators by replacing uaw-null sequences with uaw-Cauchy sequences. This enables us to characterize KB-spaces and reflexive spaces in terms of these classes of operators. This is what our paper is about.

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In the second section, we consider pre-unbounded continuous operators and investigate their relations with unbounded continuous operators. Furthermore, we characterize \( KB \)-spaces and reflexive spaces in terms of them. In the third section, we consider the unbounded Banach-Saks property by replacing weak convergence in the definition of the weak Banach-saks property by \( uaw \)-convergence. We investigate its relations with other important types of the Banach-Saks property; namely, the Banach-saks property, the weak Banach-Saks property, the disjoint Banach-Saks property and the disjoint weak Banach-Saks property. There are, again, many interesting characterizations of order continuous Banach lattices and \( KB \)-spaces in terms of the relations between the unbounded Banach-Saks property and the known kinds of this property.

Before we proceed more, let us consider some preliminaries.

Suppose \( E \) is a Banach lattice. A net \((x_{\alpha})\) in \( E \) is said to be **unbounded absolute weak convergent** (\( uaw \)-convergent, for short) to \( x \in E \) if for each \( u \in E_+ \), \( |x_{\alpha} - x| \wedge u \to 0 \) weakly, in notation \( x_{\alpha} \xrightarrow{uaw} x \). \((x_{\alpha})\) is **unbounded norm convergent** (\( un \)-convergent, in brief) if \( \|x_{\alpha} - x\| \wedge u \to 0 \), in notation \( x_{\alpha} \xrightarrow{un} x \). Both convergences are topological. For ample information on these concepts, see [2, 5, 7]. Furthermore, we recall the notions of unbounded continuous operators which are defined in [3, 8], recently.

Suppose \( E, F \) are Banach lattices and \( X \) is a Banach space. A continuous operator \( T : E \to X \) is called sequentially unbounded continuous if for each bounded \( uaw \)-Cauchy sequence \((x_n) \subseteq E \), \( (T(x_n)) \) is weakly convergent. A continuous operator \( T : E \to F \) is said to be sequentially \( uaw \)-continuous if \( T \) maps every norm bounded \( uaw \)-null sequence into a \( uaw \)-null sequence. Moreover, recall that an operator \( T \) from \( E \) into \( X \) is \( uaw \)-Dunford-Pettis if for every norm bounded sequence \((x_n) \subseteq E \), \( x_n \xrightarrow{uaw} 0 \) implies that \( \|T(x_n)\| \to 0 \); see [3, 8] for a detailed exposition on these topics.

For undefined terminology and concepts, we refer the reader to [1]. All operators in this note, are assumed to be continuous, unless otherwise stated, explicitly.

### 2. Pre-unbounded operators

**Definition 1.** Suppose \( E \) and \( F \) are Banach lattices and \( T : E \to F \) is a continuous operator. \( T \) is said to be

(i) Pre-unbounded continuous if for each bounded \( uaw \)-Cauchy sequence \((x_n) \subseteq E \), \( (T(x_n)) \) is weakly convergent.

(ii) Pre-\( uaw \)-continuous if for each bounded \( uaw \)-Cauchy sequence \((x_n) \subseteq E \), \( (T(x_n)) \) is \( uaw \)-convergent.
(iii) Pre-uaw-Dunford-Pettis if for each bounded uaw-Cauchy sequence \((x_n) \subseteq E\), 
\((T(x_n))\) is norm convergent.

Observe that in parts (i) and (iii) of the definition, we can replace Banach lattice 
\(F\) with a Banach space \(X\), too. These operators are abundant in the category of all continuous operators. We shall show that when \(E\) is a \(KB\)-space, every sequentially unbounded continuous operator on \(E\) is pre-unbounded continuous; or every uaw-Dunford-Pettis operator (\(M\)-weakly compact operator) is pre-uaw-Dunford-Pettis. Moreover, when \(E\) is reflexive, every continuous operator on \(E\) is pre-unbounded continuous.

First of all, we consider the following; it states that in \(KB\)-spaces, we have an obvious relation between different kinds of unbounded continuous operators and the corresponding types of pre-unbounded continuous operators. It is an easy combination of [7, Theorem 4] and [5, Theorem 4.6].

**Proposition 2.** Suppose \(E\) is a \(KB\)-space, \(F\) is a Banach lattice, and \(X\) is a Banach space. Then we have the following observations.

(i) Every sequentially unbounded continuous operator \(T : E \to X\) is pre-unbounded continuous.

(ii) Every sequentially uaw-continuous operator \(T : E \to F\) is pre-uaw-continuous.

(iii) Every uaw-Dunford-Pettis operator \(T : E \to X\) is pre-uaw-Dunford-Pettis.

**Remark 3.** Note that being \(KB\)-space is essential in Proposition 2 and cannot be removed. Consider the identity operator \(I\) on \(c_0\); it is unbounded continuous and also uaw-continuous. Nevertheless, by considering the sequence \(u_n = (1, \ldots, 1, 0, \ldots)\), we see that \(I\) is neither pre-unbounded continuous nor pre-uaw-continuous. Moreover, consider the operator \(T : c_0 \to \ell_1\) defined via \(T((x_n)) = (\frac{x_n}{n})\). \(T\) is uaw-Dunford-Pettis; for if \((x_n) \subseteq c_0\) is norm bounded and uaw-null, then it is weakly null by [7, Theorem 7] so that \(T((x_n))\) is weakly null in \(\ell_1\) and therefore norm null by the Schur property. But, it fails to be pre-uaw-Dunford-Pettis; again, by considering the sequence \((u_n)\) as before.

To obtain some connections for the other direction, we need the following useful fact. It is an extension of [5, Lemma 9.10].

**Lemma 4.** Suppose \(E\) is a Banach lattice and \((x_\alpha) \subseteq E\) is a net such that \(x_\alpha \xrightarrow{uaw} x\) and \(x_\alpha \xrightarrow{w} y\). Then \(x = y\).
Proof. We may assume that \( y = 0 \). By \([1\] Theorem 4.37\), for each \( \varepsilon > 0 \) and for each \( f \in E'_+ \), there exists some \( u \in E_+ \) such that \( f(|x_\alpha - x| - |x_\alpha - x| \wedge u) < \varepsilon \), for each \( \alpha \). But for sufficiently large \( \alpha \), \( |x_\alpha - x| \wedge u \to 0 \) so that \( x_\alpha \wedge u \to x \); this means that \( x = 0 \). \( \square \)

Now, we have the following.

**Proposition 5.** Suppose \( E \) and \( F \) are Banach lattices. Then we have the following observations.

(i) Every sequentially uaw-continuous operator \( T : E \to F \) which is also pre-unbounded continuous, is sequentially unbounded continuous.

(ii) Every sequentially unbounded continuous operator \( T : E \to F \) which is also pre-uaw-continuous, is sequentially uaw-continuous.

(iii) Suppose \( T : E \to F \) is pre-uaw-Dunford-Pettis and also so is either sequentially unbounded continuous or sequentially uaw-continuous. Then \( T \) is uaw-Dunford-Pettis.

Proof. (i). Suppose \( (x_n) \subseteq E \) is uaw-null. By the assumption, \( T(x_n) \xrightarrow{\text{uaw}} 0 \). Moreover, \( T(x_n) \xrightarrow{\text{w}} x \) for some \( x \in F \). By Lemma \([\text{I}]\) \( x = 0 \).

(ii). Suppose \( (x_n) \subseteq E \) is uaw-null. By the assumption, \( T(x_n) \xrightarrow{\text{w}} 0 \). Moreover, \( T(x_n) \xrightarrow{\text{uaw}} x \) for some \( x \in F \). By Lemma \([\text{I}]\) \( x = 0 \).

(iii). Suppose \( (x_n) \subseteq E \) is uaw-null. By the assumption, \( T(x_n) \to x \) for some \( x \in F \) so that \( T(x_n) \xrightarrow{\text{w}} x \) and \( T(x_n) \xrightarrow{\text{uaw}} x \). Now, if \( T \) is either sequentially unbounded continuous or sequentially uaw-continuous, we conclude that \( x = 0 \). \( \square \)

Now, we have the following simple ideal properties.

**Proposition 6.** Let \( E, F, G \) be Banach lattices and \( X, Y \) be Banach spaces. Then, we have the following observations.

(i) If \( T : E \to X \) is pre-unbounded continuous and \( S : X \to Y \) is continuous, then \( ST \) is also pre-unbounded continuous.

(ii) If \( T : E \to F \) is pre-uaw-continuous and \( S : F \to G \) is sequentially uaw-continuous, then \( ST \) is also pre-uaw-continuous.

(iii) If \( T : E \to F \) is pre-uaw-continuous and \( S : F \to X \) is sequentially unbounded continuous, then \( ST \) is also pre-unbounded continuous.

**Theorem 7.** For a Banach lattice \( E \), the following are equivalent.

(i) \( E \) is reflexive.
(ii) For every Banach space $X$, every continuous operator $T : E \to X$ is pre-unbounded continuous.

Proof. $(i) \Rightarrow (ii)$. Suppose $E$ is reflexive and $T : E \to X$ is a continuous operator. Assume that $(x_n)$ is a bounded uaw-Cauchy sequence in $E$. So, by [7, Theorem 8], it is weakly convergent to $x \in E$. This means that the identity operator $I$ on $E$ is pre-unbounded continuous. Note that $T = TI$ and use Proposition 6.

$(ii) \Rightarrow (i)$. Put $X = E$ and $T = I$; the identity operator on $E$. This means that $I$ is pre-unbounded continuous. Again, [7, Theorem 8], yields the result. □

Theorem 8. For a Banach lattice $E$, the following are equivalent.

$(i)$ $E$ is a $K$-B-space.

$(ii)$ For every Banach lattice $F$, every sequentially uaw-continuous operator $T : E \to F$ is pre-uaw-continuous.

Proof. $(i) \Rightarrow (ii)$. Suppose $E$ is a $K$-B-space and $T : E \to F$ is a sequentially uaw-continuous operator. Assume that $(x_n)$ is a bounded uaw-Cauchy sequence in $E$. So, by [7, Theorem 4] and [5, Theorem 4.6], it is uaw-convergent to $x \in E$. This means that the identity operator $I$ on $E$ is pre-uaw-continuous. Note that $T = TI$ and use Proposition 6.

$(ii) \Rightarrow (i)$. Put $F = E$ and $T = I$; the identity operator on $E$. This means that $I$ is pre-uaw-continuous. Again, [7, Theorem 4] and [5, Theorem 4.6] would complete the proof. □

The following results are similar to [8, Proposition 3] and [8, Corollary 4].

Proposition 9. Suppose $E$ and $F$ are Banach lattices such that $F'$ is order continuous. Then every pre-uaw-continuous operator $T : E \to F$ is pre-unbounded continuous.

Corollary 10. Suppose $E$ is a Banach lattice and $F$ is an AM-space. Then an operator $T : E \to F$ is pre-unbounded continuous if and only if it is pre-uaw-continuous.

Now, we state a version of Theorem 7 in which the part $(ii) \Rightarrow (i)$ is improved.

Theorem 11. Suppose $E$ is a Banach lattice. Then the following are equivalent.

$(i)$ $E$ is reflexive.

$(ii)$ Every continuous operator $T : E \to c_0$ is pre-unbounded continuous.

Proof. $(i) \Rightarrow (ii)$. It is done by Theorem 7.
(ii) ⇒ (i). Suppose not; so, by [6, Theorem 2.4.15], $E$ contains a lattice copy of either $\ell_1$ or $c_0$. Moreover, by [6, Proposition 2.3.11], there exists a positive projection $P : E \to \ell_1$. Therefore, the restriction of $P$ to $\ell_1$ is the identity operator on $\ell_1$ which is not pre-unbounded continuous. Now, suppose $(e_n)$ is the standard basis of $\ell_1$ which is certainly disjoint in $E$ so that uaw-null by [7, Lemma 2] but it can be easily seen that it is not weakly convergent in $\ell_1$. Furthermore, by [6, Theorem 2.4.12], there exists a positive projection $P : E \to c_0$. The restriction of $P$ to $c_0$ is the identity operator which is not pre-unbounded continuous; use the uaw-Cauchy sequence $(u_n)$ defined via $u_n = (1, \ldots, 1, 0, \ldots)$. Note that $(u_n)$ is in fact absolutely weakly Cauchy in $c_0$ so that absolutely weakly Cauchy in $E$. Thus, $(u_n)$ is uaw-Cauchy in $E$. But it is obvious that it is not certainly weakly convergent. \[\square\]

Combining Theorem 11 and Corollary 10, we have the following.

**Corollary 12.** Suppose $E$ is a Banach lattice. Then the following are equivalent.

(i) $E$ is reflexive.

(ii) Every continuous operator $T : E \to c_0$ is pre-uaw-continuous.

Observe that every pre-uaw-Dunford-Pettis operator is pre-unbounded continuous. Therefore, we have the following.

**Corollary 13.** Suppose $E$ is a Banach lattice. If every continuous operator $T : E \to c_0$ is pre-uaw-Dunford-Pettis, then $E$ is reflexive.

Note that the converse of Corollary 13 is not true, in general. The inclusion map $i : \ell_2 \to c_0$ is not pre-uaw-Dunford-Pettis operator.

### 3. Unbounded Banach-Saks Property

Suppose $E$ is a Banach lattice. $E$ is said to have the **unbounded Banach-Saks property** (UBSP, for short) if for every norm bounded uaw-null sequence $(x_n) \subseteq E$, there is a subsequence $(x_{n_k})$ whose Cesàro means is convergent. Moreover, recall that $E$ possesses the disjoint Banach-Saks property (DBSP, for short) if every bounded disjoint sequence in $E$ has a Cesàro convergent subsequence; $E$ has the disjoint weak Banach-Saks property (DWBSP, in brief) if every disjoint weakly null sequence in $E$ has a Cesàro convergent subsequence. Furthermore, $E$ possesses the weak Banach-Saks property (WBSP, in brief) if for every weakly null sequence $(x_n)$, it has a subsequence which is Cesàro convergent. For a brief but comprehensive context in this subject, see [4]. In the following lemma, we collect some preliminary facts.
Lemma 14.  

(i) Suppose $E'$ is order continuous. If $E$ possesses $WBSP$, then it satisfies $UBSP$.

(ii) $UBSP$ implies $DBSP$.

(iii) Suppose $E$ is either an $AM$-space or an atomic order continuous Banach lattice. Then $UBSP$ implies $WBSP$.

Proof. (i). Consider [7, Theorem 7] and the proof would be complete.

(ii). Consider [7, Lemma 2] which asserts that every disjoint sequence is $uaw$-null.

(iii). In both cases, it can be verified easily that the lattice operations are weakly sequentially continuous so that weak convergence implies $uaw$-convergence. □

Remark 15. Observe that order continuity assumption in Lemma 14(i) is essential and can not be removed. For example $\ell_1$ possesses $WBSP$ but it fails $UBSP$; consider the standard basis sequence $(e_n)$ which is $uaw$-null by [7, Lemma 2].

Now, we have the following two useful statements.

Lemma 16. Every $AM$-space possesses $DBSP$.

Proof. Suppose $E$ is an $AM$-space and $(x_n) \subseteq E$ is a norm bounded disjoint sequence in $E$. Then for each subsequence $(x_{n_k})$ from $(x_n)$, we have

$$\lim_{n \to \infty} \left\| \frac{1}{n} \sum_{k=1}^{n} x_{n_k} \right\| = \lim_{n \to \infty} \left\| \frac{1}{n} \sum_{k=1}^{n} x_{n_k} \right\| = \lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} \left\| x_{n_k} \right\| = \frac{1}{n} \sum_{k=1}^{n} \left\| x_{n_k} \right\| \leq \frac{1}{n} \to 0.$$ □

For a Banach lattice $E$, by Lemma 14 $UBSP$ implies $DBSP$. In the following, we show that, in order continuous Banach lattices, these notions agree.

Theorem 17. A $\sigma$-order complete Banach lattice $E$ with $DBSP$ possesses $UBSP$ if and only if $E$ is order continuous.

Proof. Suppose $E$ is order continuous and $(x_n)$ is a bounded $uaw$-null sequence in $E$. By [7, Theorem 4] and [2, Theorem 3.2], there are a subsequence $(x_{n_k})$ of $(x_n)$ and a disjoint sequence $(d_k)$ such that $\|x_{n_k} - d_k\| \to 0$. By passing to a subsequence, we may assume that $\lim_{m \to \infty} \frac{1}{m} \sum_{i=1}^{m} d_i = 0$. Now, the result follows from the following identity.

$$\left\| \frac{1}{m} \sum_{i=1}^{m} x_{n_i} - \frac{1}{m} \sum_{i=1}^{m} d_i \right\| = \frac{1}{m} \sum_{i=1}^{m} \left\| x_{n_i} - d_i \right\| \to 0.$$

For the converse, Suppose on contrary, $E$ is not order continuous. So by [1, Theorem 4.51], it contains a lattice copy of $\ell_\infty$. Note that $\ell_\infty$ possesses $DBSP$ by Lemma 16 but it fails $UBSP$; it is known that $\ell_\infty$ does not have $WBSP$ so that
it fails to have UBSP by Lemma 14. Therefore, there exists a $uaw$-null sequence in $\ell_\infty$ such that no subsequence of it has a convergent Cesàro means. Since the lattice operations in $\ell_\infty$ are weakly sequentially continuous and by using [7, Theorem 7], we conclude that this sequence is absolutely weakly null. So, it is absolutely weakly null in $E$. This, in turn, implies that the sequence is $uaw$-null in $E$. Thus, $E$ fails to have UBSP, as well.

\begin{proof}
\end{proof}

**Theorem 18.** A Banach lattice $E$ with WBSP possesses UBSP if and only if $E'$ is order continuous.

**Proof.** The backward implication is proved in Lemma 14. For the converse, observe that $\ell_1$ possesses WBSP but it fails to have UBSP; note that $\ell_1$ does not possess DBSP. Now, assume that $E'$ is not order continuous so that it contains a lattice copy of $\ell_1$. Suppose $(e_n)$ is the standard basis in $\ell_1$ which is certainly disjoint so that disjoint in $E$. Therefore, $(e_n)$ is $uaw$-null in $E$ by [7, Lemma 2]. Nevertheless, it does not possess any Cesàro convergent subsequence in $E$ which is a contradiction. \hfill \Box

Combining Theorem 17 with [4, Proposition 6.15], we have the following.

**Corollary 19.** A $\sigma$-order complete Banach lattice $E$ with DWBSP possesses UBSP if and only if $E$ and $E'$ are order continuous.

**Proof.** The "if" part is done by Theorem 17 and [4, Proposition 6.15]. For the other direction, suppose $E$ is a Banach lattice with DWBSP. Note that neither $\ell_\infty$ nor $\ell_1$ have UBSP; although, both of them possess DWBSP. Now, suppose either $E$ or $E'$ does not have an order continuous norm. Therefore, $E$ contains a lattice copy of either $\ell_\infty$ or $\ell_1$. In the former case, a similar argument as we had in the second part of the proof of Theorem 17, we have the desired result. For the latter case, consider an argument similar to the proof of Theorem 18. \hfill \Box

**Lemma 20.** Suppose $E$ is a Banach lattice with order continuous norm whose lattice operations are weakly sequentially continuous. Then every norm bounded sequence in $E$ possesses a $uaw$-Cauchy subsequence.

**Proof.** Suppose $(x_n) \subseteq E$ is norm bounded. By [1, Theorem 4.72], there exists a subsequence $(y_n)$ of $(x_n)$ which is either weakly Cauchy or is equivalent to the standard basis of $\ell_1$. For the former case, since the lattice operations are weakly sequentially continuous, $(x_n)$ is also absolutely weakly Cauchy so that $uaw$-Cauchy. For the latter
case, $\ell_1$ is embeddable in $E$. But in this case, it should be lattice embeddable; otherwise, by [1, Theorem 4.69], $E'$ is order continuous so that by [1, Theorem 4.25], $(y_n)$ possesses a weakly Cauchy subsequence which is in contradiction with this fact that $(y_n)$ is equivalent to the standard basis of $\ell_1$. Note that the standard basis of $\ell_1$ is uaw-null by [7, Lemma 2].

\[\square\]

**Remark 21.** Note that the assumptions in Lemma [20] are essential. Consider $\ell_{\infty}$; the lattice operations are weakly sequentially continuous but its norm is not order continuous. It is easy to see that the sequence $(u_n)$ given by $u_n = (0, \ldots, 0, 1, \ldots)$, with the 0 occupying the first $n$ positions, does not possess any uaw-convergent subsequence. Moreover, consider $L^1[0,1]$; it is an order continuous Banach lattice whose lattice operations are not weakly sequentially continuous. The Rademacher functions $(r_n)$ does not have any uaw-convergent subsequence.

Finally, note that by a similar argument as we had in Lemma [20] to any subsequence of $(x_n)$, we can find a further uaw-Cauchy subsequence; since uaw-convergence is topological, we conclude that $(x_n)$ is in fact uaw-Cauchy.

It is easy to see that BSP implies UBSP. For the converse, we have the following.

**Theorem 22.** Suppose $E$ is an order continuous Banach lattice whose lattice operations are weakly sequentially continuous. Furthermore, assume that $E$ possesses UBSP. Then $E$ has BSP if and only if $E$ is a KB-space.

**Proof.** Suppose $E$ is a KB-space and $(x_n)$ is a bounded sequence in $E$. By Lemma [20] there exists a subsequence $(y_n)$ of $(x_n)$ which is uaw-Cauchy. By [5, Theorem 4.6] and [7, Theorem 4], $(y_n)$ is uaw-convergent. So, by passing to a further subsequence, we may assume that its Cesàro means is convergent. The other implication is trivial since every Banach space with BSP is reflexive. \[\square\]

**Corollary 23.** Suppose $E$ is a KB-space whose lattice operations are weakly sequentially continuous with UBSP. Then $E$ is reflexive.

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