Higgs-Free Confinement Hierarchy in Five Colour QCD

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I consider the monopole condensate of five color QCD. The naive lowest energy state is unobtainable at one-loop for five or more colors due to simple geometric considerations. The consequent adjustment of the vacuum condensate generates a hierarchy of confinement scales in a natural Higgs-free manner. The accompanying symmetry hierarchy contains hints of standard model phenomenology.

§1. Introduction

It is already known,\(^1\)–\(^3\) that SU\((N)\) QCD can lower the energy of its vacuum with a monopole background field along the Abelian directions, where the Abelian components are equal in magnitude but orthogonal in real space.\(^2\),\(^3\) This orthogonality, while of no special consequence in SU\((3)\) QCD in three space dimensions, does have consequences when the number of Abelian directions is greater than three. As noted originally by Flyvberg,\(^2\) SU\((N \geq 5)\) QCD cannot realise its true minimum because four orthogonal vectors cannot fit in three dimensions. I shall call a system kept from reaching its true lowest energy state by a lack of spatial dimensions dimensionally frustrated.

The effective ground state energy of the monopole condensate is a non-analytic expression in the Cartan components \(H^{(1)}\). However, its form strongly suggests that the lowest physically available energy state is the one that leaves the effective condensate felt by each of the off-diagonal (valence) gluons equal in magnitude, or as close to it as can be realised. This is the motivation for my condensate ansatz. The lowest energy state allowed by it leaves the (matter-antimatter pair of) gluons associated with one root vector very strongly confined, some others very weakly confined, and the rest confined with identical intermediate strength. Examining the dynamics at intermediate energy scales where the strongly confined dynamics have dropped out finds an emergent three-colour QCD accompanied by gluons whose dynamics no longer conform to a special unitary symmetry group. They do however form two six-dimensional representations of SU\((3)\). One of these representations consists of the weakly confined valence gluons. Exactly one unconfined massless Abelian gauge field (photon) can be found by taking linear combinations of the accompanying Abelian gluons.
My treatment of the monopole condensate rests on the Cho-Faddeev-Niemi (CFN) decomposition.\(^4\)–\(^6\) I use the following notation:

The Lie group $SU(N)$ has $N^2 - 1$ generators $\lambda^{(j)}$, of which $N - 1$ are Abelian generators $A^{(i)}$. For simplicity, we specify the gauge transformed Abelian directions (Cartan generators) with

$$\hat{n}_i = U^\dagger A^{(i)} U. \tag{1}$$

In the same way, we replace the standard raising and lowering operators $E_{\pm \alpha}$ for the root vectors $\alpha$ with the gauge transformed ones

$$E_{\pm \alpha} \rightarrow U^\dagger E_{\pm \alpha} U, \tag{2}$$

where $E_{\pm \alpha}$ refers to the gauge transformed operator throughout the rest of this article.

Gluon fluctuations in the $\hat{n}_i$ directions are described by $c_{\mu}^{(i)}$. The gauge field of the covariant derivative which leaves the $\hat{n}_i$ invariant is

$$gV_{\mu} \times \hat{n}_i = -\partial_{\mu} \hat{n}_i. \tag{3}$$

In general this is

$$V_{\mu} = c_{\mu}^{(i)} \hat{n}_i + B_{\mu}, \quad B_{\mu} = g^{-1} \partial_{\mu} \hat{n}_i \times \hat{n}_i, \tag{4}$$

where summation is implied over $i$. $B_{\mu}$ can be assigned to non-Abelian monopoles, as indicated by the $\hat{n}_i$ describing the homotopy group $\pi_2[SU(N)/U(1)^{\otimes(N-1)}] = \pi_1[U(1)^{\otimes(N-1)}]$. The monopole field strength

$$H_{\mu\nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu} + gB_{\mu} \times B_{\nu}, \tag{5}$$

has only Abelian components, i.e.

$$H_{\mu\nu}^{(i)} \hat{n}_i = H_{\mu\nu}, \tag{6}$$

where $H_{\mu\nu}^{(i)}$ has the eigenvalue $H^{(i)}$. Since I am only concerned with magnetic backgrounds, $H^{(i)}$ is considered the magnitude of a background magnetic field $H^{(i)}$. The field strength of the Abelian components $c_{\mu}^{(i)}$ also lies in the Abelian directions as expected and is shown by

$$\mathbf{F}_{\mu\nu} = \mathbf{F}_{\mu\nu}^{(i)} \hat{n}_i. \tag{7}$$

Defining

$$F_{\mu\nu}^{(i)} = \partial_{\mu} c_{\nu}^{(i)} - \partial_{\nu} c_{\mu}^{(i)}, \tag{8}$$

the Lagrangian of the Abelian and monopole components is

$$-\frac{1}{4} (F_{\mu\nu}^{(i)} \hat{n}_i + H_{\mu\nu})^2. \tag{9}$$
The dynamical degrees of freedom (DOF) perpendicular to \( \hat{n}_i \) are denoted by \( X_\mu \), so if \( A_\mu \) is the gluon field then

\[
A_\mu = V_\mu + X_\mu = e^{(i)}_\mu \hat{n}_i + B_\mu + X_\mu, \tag{10}
\]

where

\[
X_\mu \perp \hat{n}_i, \quad X_\mu = g^{-1} \hat{n}_i \times D_\mu \hat{n}_i, \quad D_\mu = \partial_\mu + g A_\mu \times . \tag{11}
\]

Because \( X_\mu \) is orthogonal to all Abelian directions it can be expressed as a linear combination of the raising and lowering operators \( E_{\pm \alpha} \), which leads to the definition

\[
X_\mu^{(\pm \alpha)} \equiv E_{\pm \alpha} \text{Tr}[X_\mu E_{\pm \alpha}], \tag{12}
\]

so

\[
X_\mu^{(- \alpha)} = X_\mu^{(+ \alpha)^\dagger}. \tag{13}
\]

\( H_\mu^{(\alpha)} \), defined by

\[
H_\mu^{(\alpha)} = \alpha_j H_\mu^{(j)}, \tag{14}
\]

is the monopole field strength tensor felt by \( X_\mu^{(\alpha)} \). I also define the background magnetic field

\[
H^{(\alpha)} = \alpha_j H^{(j)}, \tag{15}
\]

whose magnitude \( H^{(\alpha)} \) is \( H^{(\alpha)}_{\mu \nu} \)'s non-zero eigenvalue.

**The Vacuum State of five-color QCD**

The one-loop effective energy of five-color QCD is given by\(^2\),\(^3\)

\[
\mathcal{H} = \sum_{\alpha > 0} \| H^{(\alpha)} \|^2 \left[ \frac{1}{5 g^2} + \frac{11}{48 \pi^2} \ln \frac{H^{(\alpha)}}{\mu^2} \right] \tag{16}
\]

which is minimal when

\[
H^{(\alpha)} = \mu^2 \exp \left( -\frac{1}{2} - \frac{48 \pi^2}{55 g^2} \right). \tag{17}
\]

This neglects an alleged imaginary component\(^7\) which has been called into serious question recently\(^3\),\(^8\)–\(^13\) with growing evidence to suggest that it is only an artifact of the quadratic approximation. Taking this to be the case, I employ the Savvidy vacuum. This can be criticised for lacking Lorentz covariance but I argue that it is likely to match the true vacuum at least locally.

Since

\[
\| H^{(1,0,0,0)} \| = \| H^{(1)} \|,
\]

\[
\| H^{(1/2, 1/\sqrt{2}, 0)} \|^2 = \frac{1}{4} \| H^{(1)} \|^2 + \frac{3}{4} \| H^{(2)} \|^2 \pm \sqrt{3} \frac{H^{(1)} \cdot H^{(2)}}{2},
\]

\[
\| H^{(1/2, 1/\sqrt{12}, 1/\sqrt{6}, 0)} \|^2 = \frac{1}{4} \| H^{(1)} \|^2 + \frac{1}{12} \| H^{(2)} \|^2 + \frac{2}{3} \| H^{(3)} \|^2 \pm \sqrt{2} \frac{H^{(1)} \cdot H^{(2)}}{3},
\]

\[
\| H^{(1/2, 1/\sqrt{2}, 1/\sqrt{2}, 0)} \|^2 = \frac{1}{4} \| H^{(1)} \|^2 + \frac{1}{4} \| H^{(2)} \|^2 + \frac{1}{4} \| H^{(3)} \|^2 \pm \sqrt{2} \frac{H^{(1)} \cdot H^{(2)}}{3},
\]

\[
\| H^{(1/2, 1/\sqrt{2}, 1/\sqrt{6}, 0)} \|^2 = \frac{1}{4} \| H^{(1)} \|^2 + \frac{1}{12} \| H^{(2)} \|^2 + \frac{2}{3} \| H^{(3)} \|^2 \pm \sqrt{2} \frac{H^{(1)} \cdot H^{(2)}}{3},
\]

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\| H^{(1/2, 1/\sqrt{12}, 1/\sqrt{6}, 0)} \|^2 = \frac{1}{4} \| H^{(1)} \|^2 + \frac{1}{12} \| H^{(2)} \|^2 + \frac{2}{3} \| H^{(3)} \|^2 \pm \sqrt{2} \frac{H^{(1)} \cdot H^{(2)}}{3},
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\| H^{(1/2, 1/\sqrt{2}, 1/\sqrt{2}, 0)} \|^2 = \frac{1}{4} \| H^{(1)} \|^2 + \frac{1}{4} \| H^{(2)} \|^2 + \frac{1}{4} \| H^{(3)} \|^2 \pm \sqrt{2} \frac{H^{(1)} \cdot H^{(2)}}{3},
\]

\[
\| H^{(1/2, 1/\sqrt{12}, 1/\sqrt{6}, 0)} \|^2 = \frac{1}{4} \| H^{(1)} \|^2 + \frac{1}{12} \| H^{(2)} \|^2 + \frac{2}{3} \| H^{(3)} \|^2 \pm \sqrt{2} \frac{H^{(1)} \cdot H^{(2)}}{3},
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\| H^{(1/2, 1/\sqrt{2}, 1/\sqrt{2}, 0)} \|^2 = \frac{1}{4} \| H^{(1)} \|^2 + \frac{1}{4} \| H^{(2)} \|^2 + \frac{1}{4} \| H^{(3)} \|^2 \pm \sqrt{2} \frac{H^{(1)} \cdot H^{(2)}}{3},
\]

\[
\| H^{(1/2, 1/\sqrt{12}, 1/\sqrt{6}, 0)} \|^2 = \frac{1}{4} \| H^{(1)} \|^2 + \frac{1}{12} \| H^{(2)} \|^2 + \frac{2}{3} \| H^{(3)} \|^2 \pm \sqrt{2} \frac{H^{(1)} \cdot H^{(2)}}{3},
\]

\[
\| H^{(1/2, 1/\sqrt{2}, 1/\sqrt{2}, 0)} \|^2 = \frac{1}{4} \| H^{(1)} \|^2 + \frac{1}{4} \| H^{(2)} \|^2 + \frac{1}{4} \| H^{(3)} \|^2 \pm \sqrt{2} \frac{H^{(1)} \cdot H^{(2)}}{3},
\]

\[
\| H^{(1/2, 1/\sqrt{12}, 1/\sqrt{6}, 0)} \|^2 = \frac{1}{4} \| H^{(1)} \|^2 + \frac{1}{12} \| H^{(2)} \|^2 + \frac{2}{3} \| H^{(3)} \|^2 \pm \sqrt{2} \frac{H^{(1)} \cdot H^{(2)}}{3},
\]
\[ \pm \frac{1}{2\sqrt{3}} \mathbf{H}^{(1)} \cdot \mathbf{H}^{(3)} + \frac{\sqrt{2}}{3} \mathbf{H}^{(2)} \cdot \mathbf{H}^{(3)}, \]

\[ \left\| \mathbf{H}^{(0, -\frac{1}{\sqrt{3}}, -\frac{2}{\sqrt{6}}, 0)} \right\|^2 = \frac{1}{3} \left\| \mathbf{H}^{(2)} \right\|^2 + \frac{2}{3} \left\| \mathbf{H}^{(3)} \right\|^2 - \frac{2\sqrt{2}}{3} \mathbf{H}^{(2)} \cdot \mathbf{H}^{(3)}, \]

\[ \left\| \mathbf{H}^{(0, 0, -\frac{\sqrt{2}}{\sqrt{3}}, -\frac{\sqrt{2}}{\sqrt{6}})} \right\|^2 = \frac{3}{8} \left\| \mathbf{H}^{(3)} \right\|^2 + \frac{5}{8} \left\| \mathbf{H}^{(4)} \right\|^2 - \frac{\sqrt{15}}{4} \mathbf{H}^{(3)} \cdot \mathbf{H}^{(4)}, \]

\[ \left\| \mathbf{H}^{(0, 0, -\frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{6}})} \right\|^2 = \frac{3}{8} \left\| \mathbf{H}^{(2)} \right\|^2 + \frac{1}{24} \left\| \mathbf{H}^{(3)} \right\|^2 + \frac{5}{8} \left\| \mathbf{H}^{(4)} \right\|^2 - \sqrt{\frac{1}{16} \mathbf{H}^{(2)} \cdot \mathbf{H}^{(3)}} \]

\[ \left\| \mathbf{H}^{(0, \pm \frac{1}{\sqrt{3}}, -\frac{2}{\sqrt{6}}, \frac{\sqrt{2}}{\sqrt{12}})} \right\|^2 = \frac{1}{4} \left\| \mathbf{H}^{(1)} \right\|^2 + \frac{1}{12} \left\| \mathbf{H}^{(2)} \right\|^2 + \frac{1}{24} \left\| \mathbf{H}^{(3)} \right\|^2 + \frac{5}{8} \left\| \mathbf{H}^{(4)} \right\|^2 \]

\[ \pm \frac{2}{3} \mathbf{H}^{(1)} \cdot \mathbf{H}^{(2)} + \frac{1}{\sqrt{24}} \mathbf{H}^{(1)} \cdot \mathbf{H}^{(3)} + \frac{1}{\sqrt{16}} \mathbf{H}^{(2)} \cdot \mathbf{H}^{(3)} \]

\[ \pm \frac{5}{\sqrt{8}} \mathbf{H}^{(1)} \cdot \mathbf{H}^{(4)} + \frac{\sqrt{5}}{\sqrt{24}} \mathbf{H}^{(2)} \cdot \mathbf{H}^{(4)} + \frac{\sqrt{5}}{\sqrt{48}} \mathbf{H}^{(3)} \cdot \mathbf{H}^{(4)}, \]

it follows that

\[ \left\| \mathbf{H}^{(i)} \right\| = \left\| \mathbf{H}^{(j)} \right\|, \quad \mathbf{H}^{(i)} \perp \mathbf{H}^{(j)}, \quad i \neq j, \]

which means that the chromomagnetic field components must be equal in magnitude but mutually orthogonal in the lowest energy state. However three dimensional space can only accommodate three mutually orthogonal vectors. Since the number of Cartan components is always \( N - 1 \) in \( SU(N) \) it follows that QCD with more than four colors cannot achieve such an arrangement.

One could substitute the Cartan basis \( \mathbf{H}^{(i)} \) but this leads to intractable equations that cannot be solved analytically. It is reasonable to expect that the lowest attainable energy state is only slightly different from (16) and that this difference is due to the failure of mutual orthogonality. I therefore propose the ansatz that all Cartan components are equal in magnitude to what they would be in the absence of dimensional frustration, and that their relative orientations in real space are chosen so as to minimise the energy. In practice this means that three of the four are mutually orthogonal and the remaining one is a linear combination of those three. This remainder will increase the effective energy through its scalar products with the mutually orthogonal vectors but not all scalar products contribute equally. This follows from the form of the root vectors in Eq. (18). This means that the orientation of the remaining real space vector in relation to the mutually orthogonal ones impacts the effective energy.

A little thought reveals that the lowest energy state should have only one scalar product contribute to it. The problem of finding the lowest available energy state therefore reduces to finding the scalar product that contributes to it the least. The
those that feel the background will be confined with different strengths and therefore at different length scales. Different valence gluons and even different quarks (in the fundamental representation) will be confined with intermediate strength. The remainder will be confined with intermediate strength. The confining potential felt by quarks is fundamentally different from that of the valence gluons, and will not be treated in this paper.

Table I. Candidate parallel components for vacuum condensate. The column on the left is for parallel vectors, the column on the right is for antiparallel vectors. $\Delta H$ should be multiplied by $H^2_{5/96\sqrt{2}}$.

| $H^{(\alpha)}$ | $\Delta H$ | $H^{(\alpha)}$ | $\Delta H$ |
|---------------|-------------|---------------|-------------|
| $H^{(1)} = +H^{(2)}$ | 1.06381 | $H^{(1)} = -H^{(2)}$ | 1.06381 |
| $H^{(1)} = +H^{(3)}$ | 0.857072 | $H^{(1)} = -H^{(3)}$ | 0.857072 |
| $H^{(1)} = +H^{(4)}$ | 0.715651 | $H^{(1)} = -H^{(4)}$ | 0.715651 |
| $H^{(2)} = +H^{(3)}$ | 1.01655 | $H^{(2)} = -H^{(3)}$ | 0.656584 |
| $H^{(2)} = +H^{(4)}$ | 0.882589 | $H^{(2)} = -H^{(4)}$ | 0.577976 |
| $H^{(3)} = +H^{(4)}$ | 1.00042 | $H^{(3)} = -H^{(4)}$ | 0.540983 |

six candidates are

$$H^{(1)} \cdot H^{(2)}, H^{(1)} \cdot H^{(3)}, H^{(1)} \cdot H^{(4)}, H^{(2)} \cdot H^{(3)}, H^{(2)} \cdot H^{(4)}, H^{(3)} \cdot H^{(4)}.$$ \hspace{1cm} (20)

As can be seen from Table I, $H^{(3)} = -H^{(4)}$ (antiparallel) yields the lowest effective energy when all other scalar products are zero.

Substituting this result into (18) finds that all $H^{(\alpha)}$ have the same magnitude except for those that couple to $H^{(4)}$, namely $H^{(\pm,\pm,\mp,\sqrt{\sqrt{8}})}$, where $\mp$ indicates that there are several possible values. The other background field strengths are

$$\|H^{(\alpha)}\|^2 = H^2.$$ \hspace{1cm} (21)

Of those $H^{(\alpha)}$ that do couple to $H^{(4)}$, the strongest is

$$\|H^{(0,0,-\sqrt{\sqrt{2}},\sqrt{\sqrt{2}})}\|^2 = H^2 \left(1 + \frac{\sqrt{15}}{4}\right),$$ \hspace{1cm} (22)

and the weakest are

$$\|H^{(\pm,\pm,\pm,\sqrt{\sqrt{8}})}\|^2 = H^2 \left(1 - \sqrt{\frac{5}{48}}\right).$$ \hspace{1cm} (23)

Remember the negative signs are affected by $H^{(3)}, H^{(4)}$ being antiparallel.

Assuming the dual superconductor model of confinement, \cite{4,14-17} it follows that different valence gluons and even different quarks (in the fundamental representation) will be confined with different strengths and therefore at different length scales. Those that feel the background $H^{(0,0,-\sqrt{\sqrt{2}},\sqrt{\sqrt{2}})}$ will be confined the most strongly, those that feel the backgrounds of the form $H^{(\pm,\mp,\pm,\sqrt{\sqrt{8}})}$ will be confined least strongly. The remainder will be confined with intermediate strength. The confining potential felt by quarks is fundamentally different from that of the valence gluons, \cite{3,18,19} and will not be treated in this paper.

At highest energy then, we have the full dynamics of $SU(5)$ QCD. Moving down to some intermediate energy however, finds that the dynamics associated with the
root vector \( (0, 0, -\frac{\sqrt{3}}{\sqrt{8}}, \frac{\sqrt{5}}{\sqrt{8}}) \) are confined out of the dynamics. The remaining gluons interact among themselves. Moving to lower energy scales we find that those dynamics are all removed in their turn except for those corresponding to the root vectors \( (?, ?, \frac{1}{\sqrt{24}}, \sqrt{\frac{5}{8}}) \), almost leaving an \( SU(2) \) gauge field interaction. I say ‘almost’ because I shall later demonstrate that the form of the monopole condensate is sufficiently different from the \( SU(2) \) condensate to alter the dynamics, producing three confined \( U(1) \) gauge fields, one unconfined \( U(1) \) gauge field that may be identified with the photon, and three copies of the valence gluons of \( SU(2) \). At lowest energies only the unconfined gauge field remains. In this way a hierarchy of confinement scales and effective dynamics emerges naturally, without the introduction of any ad hoc mechanisms like the Higgs field.

§3. Intermediate energy dynamics

In constructing the hierarchical picture above, we began with \( SU(5) \) and finished with \( U(1) \) but had no apparent gauge group governing the dynamics at the intermediate energy scale. The dynamics of this energy scale will prove to be quite interesting.

To facilitate the discussion I introduce a notation inspired by the Dynkin diagram of \( SU(5) \). The root vectors implicitly specified in Eq. (18) are all linear combinations of a few basis vectors, which according to Lie algebra representation theory can be chosen for convenience. I take the basis vectors

\[
(1, 0, 0, 0), \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0, 0\right), \left(0, -\frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}}, 0\right), \left(0, 0, -\sqrt{3}, \sqrt{\frac{5}{8}}\right),
\]

which I shall each represent by

\[
OXXX, XOXX, XXOX, XXXO,
\]

respectively. The remaining root vectors are sums of these basis vectors. In this notation their representation contains an ‘O’ if the corresponding basis vector is included and ‘X’ if it is not. For example the root vector

\[
\left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0, 0\right) = (1, 0, 0, 0) + \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0, 0\right),
\]

is represented by

\[
OOXX = OXXX + XOXX.
\]

When convenient, a ‘?’ is used to indicate that either ‘O’ or ‘X’ might be substituted.

In addition to its brevity, this notation has the nice feature of making obvious which root vectors can be combined to form other root vectors because there are no root vectors with an ‘X’ with ‘O’s on either side. There is no OXXO for example.

The confinement of \( X_\mu^{0, 0, -\frac{\sqrt{3}}{\sqrt{8}}, \frac{\sqrt{5}}{\sqrt{8}}} \), the valence gluon corresponding to XXXO, out of the dynamics directly affects only those remaining valence gluons that couple
to it, those of root vectors of the form ??O?. The remaining gluons, corresponding to the root vectors OOXX, XOXO and OXXX (collectively given by ??XX), may still undergo the full set of interactions available to them at higher energies. It is easy to see that these are the root vectors that comprise the group $SU(3)$, to which the other valence gluons couple forming two six dimensional representations. Subsequent discussion shall extend the X,O,? notation to include the valence gluons corresponding to a root vector. Whether it is the gluon or the root vector that is meant will be clear from context.

Consider the beta function, or to be less imprecise, the scaling of the various gluon couplings. I shall now demonstrate that the loss to confinement of the root vector XXXO causes unequal corrections to the running of the couplings for different gluons. Since this is only an introductory paper the following analysis is only performed to one-loop.

The gluons ??XX, corresponding to the above-mentioned $SU(3)$, retain their original set of interactions. Performing the standard perturbative calculation\footnote{20} therefore yields the standard result for $SU(5)$ QCD. The remaining gluons do not. The absence of the maximally confined XXXO restricts their three-point vertices to those of $SU(4)$, since all root vectors are now of the form ???X. The same is not true of the four-point interactions, but the exceptions do not contribute to the scaling of the coupling constant at one-loop.\footnote{20} We have then that the $SU(3)$ subgroup’s coupling scales differently from the rest of the unconfined gluons when the maximally confined valence gluons XXXO drop out.

The beta function is proportional to the number of colors in pure QCD at one-loop, so as the length scale increases, the coupling among gluons within the $SU(3)$ subgroup initially grows faster than the couplings involving the other gluons. As noted above, the $SU(3)$ couplings will initially scale as in the five-color theory, while the remainder scale as though there were only four colors. This specific behaviour must soon change due to both non-perturbative contributions and because the non-$SU(3)$ gluons have a weaker coupling. A detailed understanding requires a non-perturbative analysis well beyond the scope of this letter. Indeed, the application of one-loop perturbation theory at anything other than the far ultraviolet is questionable in itself. The point remains that the $SU(3)$ subgroup ??XX separates from the remaining gluons by its stronger coupling strength.

The symmetry reduction that takes place in this model is suggestive of boson mass generation but there appears to be no obvious specific mechanism. Kondo et al. have argued for the spontaneous generation of mass through various non-trivial mechanisms.\cite{3},\cite{12},\cite{21} This is consistent with the well-studied correlation between confinement and chiral symmetry breaking (see Refs. 22–25 and references therein).

\section*{4. The emergence of QED}

Neglecting off-diagonal gluons, the equality $H^{(3)} = -H^{(4)}$ allows the change in variables

\[
\begin{align*}
    c^{(3)}_\mu \hat{n}_3 &\rightarrow \frac{1}{2} (c^{(3)}_\mu \hat{n}_3 + c^{(4)}_\mu \hat{n}_4) + \frac{1}{2} (c^{(3)}_\mu \hat{n}_3 - c^{(4)}_\mu \hat{n}_4) = \frac{1}{\sqrt{2}} (A_\mu + Z^0_\mu),
\end{align*}
\]
Substituting Eqs. (28) into the Abelian dynamics (9) finds that the antisymmetric combination \( Z^0_\mu \) couples to the background

\[
H(\hat{n}_3 - \hat{n}_4),
\]

but the symmetric combination \( A_\mu \) does not. This is a key point because it is the monopole background that provides the confinement, not only in Yang-Mills theories as described earlier, but even in compact \( U(1) \) gauge theories.\(^{26}\) \( A_\mu \) is unconfined not just because it is a \( U(1) \) gauge field, which is obviously insufficient since \( Z^0_\mu \) is confined, but because it disengages from the monopole condensate. Since neutral weak currents are short range and the electromagnetic field is long range the natural interpretation of these combinations are the electroweak \( Z^0_\mu \) for the symmetric combination and the photon for the antisymmetric combination.

The rotation from \( c^{(3)}_\mu \hat{n}_3, c^{(4)}_\mu \hat{n}_4 \) to \( Z^0_\mu, A_\mu \) in interactions with valence gluons is only meaningful if the gluon in question couples to both \( c^{(3)}_\mu \hat{n}_3 \) and \( c^{(4)}_\mu \hat{n}_4 \). Otherwise the combination of \( Z^0_\mu \) and \( A_\mu \) is ill-defined because it is not unique, i.e. if the valence gluon couples to \( c^{(3)}_\mu \hat{n}_3 \) but not to \( c^{(4)}_\mu \hat{n}_4 \) then arbitrary multiples of \( c^{(4)}_\mu \hat{n}_4 \) may be added to the interaction term, yielding arbitrary mixtures of \( Z^0_\mu \) and \( A_\mu \). The gluons for which this occurs are of the form \( ??OX \). Consequently the concept of coupling to \( A_\mu \) with a conserved charge is only meaningful in the low energy effective theory in which all \( ??OX \) have been confined out of the dynamics.

This is consistent with the requirement that the physical QED \( U(1) \) generators not be part of a larger \( SU(N) \) gauge group. At high energies this condition obviously fails. It is only at relatively low energies when the rest of the gluon field is either removed or restricted by confinement that the condition may be considered true. The \( SU(3) \) subgroup corresponding to \( ??XX \) is not a threat to this argument because it does not couple to these generators. Note that if \( A_\mu \) had been extracted from \( SU(5) \) using the Higgs mechanism then the resultant \( U(1) \) monopole solutions would spoil this interpretation. It is pleasing that the more natural mechanism should have such an advantage.

§5. Summary

I have studied the long known but generally ignored result that QCD with five or more colors has an altered vacuum state due to the limited dimensionality of space, a condition dubbed ‘dimensional frustration’. Attempting to identify the physical vacuum encounters an intractable set of non-analytic equations but a well-motivated ansatz enabled further analysis. Assuming the dual superconductor model, a range of confinement scales emerged with one root vector being confined more strongly than all the rest, and others less tightly. The remaining gluons exhibit unconventional dynamics at intermediate energy scales because only some of them couple to the XXXO, which is most strongly confined. At intermediate energies, a subset
of these intermediate gluons represent $SU(3)$ and have stronger interactions among themselves.

The intermediate $SU(3)$ symmetry, the low energy $SU(2)s$, and the single unconfined photon are tantalising hints of standard model phenomenology, but this work is a long way from having reproduced it. If future work along these lines does reproduce it, the $W^\pm_\mu$ would be identified with the off-diagonal generators of $??OO$. These interact with the $SU(3)$ subgroup $??XX$, corresponding to direct interaction between the $W^\pm_\mu$ and the QCD gluons, which does not occur in the standard model. However to my knowledge, it has never been experimentally tested either. It predicts anomalous scattering of $W^\pm_\mu$ when fired at deep inelastic scattering energies into proton targets.

It could be reasonably objected that the $W^\pm_\mu$ and $Z^0_\mu$ are not confined, but this is simply the current understanding of the standard model. Experimentally we know that they have only short lifetimes and are never observed to propagate freely over significant distances. As such it can be argued that they are confined, but very loosely.

There is considerable work to follow from the humble beginning presented here. The dynamics of the fundamental representation have yet to be studied. Recent work on the non-Abelian Stokes’ theorem$^{18,19}$ demonstrates that quark confinement is not synonymous with gluon confinement. It would be particularly interesting to see whether a non-confined pair of ‘leptons’ emerged.

Dimensional frustration is a natural, almost inevitable, means of generating a hierarchy in QCD with five or more colors without resorting to contrived symmetry breaking methods such as the Higgs field. Even a simplistic analysis such as this finds a rich phenomenology, with further complexity expected at higher loop.

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References

1) G. K. Savvidy, Phys. Lett. B 71 (1977), 133.
2) H. Flyvbjerg, Nucl. Phys. B 176 (1980), 379.
3) M. L. Walker, J. High Energy Phys. 01 (2007), 056.
4) Y. M. Cho, Phys. Rev. D 21 (1980), 1080.
5) L. D. Faddeev and A. J. Niemi, Phys. Lett. B 449 (1999), 214.
6) S. Li, Y. Zhang and Z.-Y. Zhu, Phys. Lett. B 487 (2000), 201.
7) N. K. Nielsen and P. Olesen, Nucl. Phys. B 144 (1978), 376.
8) J. Honerkamp, Nucl. Phys. B 48 (1972), 269.
9) Y. M. Cho, M. L. Walker and D. G. Pak, J. High Energy Phys. 05 (2004), 073.
10) Y. M. Cho and M. L. Walker, Mod. Phys. Lett. A 19 (2004), 2707.
11) Y. M. Cho and D. G. Pak, Phys. Rev. D 65 (2002), 074027.
12) K.-I. Kondo, Phys. Lett. B 600 (2004), 287.
13) D. Kay, A. Kumar and R. Parthasarathy, Mod. Phys. Lett. A 20 (2005), 1655.
14) Y. Nambu, Phys. Rev. D 10 (1974), 4262.
15) S. Mandelstam, Phys. Rep. 23 (1976), 245.
16) A. M. Polyakov, Nucl. Phys. B 120 (1977), 429.
17) G. 't Hooft, Nucl. Phys. B 190 (1981), 455.
18) K.-I. Kondo and Y. Taira, Mod. Phys. Lett. A 15 (2000), 367.
19) K.-I. Kondo and Y. Taira, Prog. Theor. Phys. 104 (2000), 1189.
20) P. H. Frampton, Quantum Field Theories (Benjamin-Cummings, California, 1987).
21) S. Kato et al., Phys. Lett. B 632 (2006), 326.
22) Y. Hatta and K. Fukushima, Phys. Rev. D 69 (2004), 097502.
23) F. Karsch, E. Laermann and A. Peikert, Nucl. Phys. B 605 (2001), 579.
24) L. G. Yaffe and B. Svetitsky, Phys. Rev. D 26 (1982), 963.
25) R. D. Pisarski and F. Wilczek, Phys. Rev. D 29 (1984), 338.
26) K. G. Wilson, Phys. Rev. D 10 (1974), 2445.