Compositional Verification of Smart Contracts Through Communication Abstraction (Extended)

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Abstract. Solidity smart contracts are programs that manage up to $2^{160}$ users on a blockchain. Verifying a smart contract relative to all users is intractable due to state explosion. Existing solutions either restrict the number of users to under-approximate behaviour, or rely on manual proofs. In this paper, we present local bundles that reduce contracts with arbitrarily many users to sequential programs with a few representative users. Each representative user abstracts concrete users that are locally symmetric to each other relative to the contract and the property. Our abstraction is semi-automated. The representatives depend on communication patterns, and are computed via static analysis. A summary for the behaviour of each representative is provided manually, but a default summary is often sufficient. Once obtained, a local bundle is amenable to sequential static analysis. We show that local bundles are relatively complete for parameterized safety verification, under moderate assumptions. We implement local bundle abstraction in SMARTACE, and show order-of-magnitude speedups compared to a state-of-the-art verifier.

1 Introduction

Solidity smart contracts are distributed programs that facilitate information flow between users. Users alternate and execute predefined transactions, that each terminate within a predetermined number of steps. Each user (and contract) is assigned a unique, 160-bit address, that is used by the smart contract to map the user to that user’s data. In theory, smart contracts are finite-state systems with $2^{160}$ users. However, in practice, the state space of a smart contract is huge—with at least $2^{2^{160}}$ states to accommodate all users and their data (conservatively counting one bit per user). In this paper, we consider the challenge of automatically verifying Solidity smart contracts that rely on user data.

A naive solution for smart contract verification is to verify the finite-state system directly. However, verifying systems with at least $2^{2^{160}}$ states is intractable. The naive solution fails because the state space is exponential in the number of
users. Instead, we infer correctness from a small number of representative users to ameliorate state explosion. To restrict a contract to fewer users, we first generalize to a family of finite-state systems parameterized by the number of users. In this way, smart contract verification is reduced to parameterized verification.

For example, consider Auction in Fig. 1 (for now, ignore the highlighted lines). In Auction, each user starts with a bid of 0. Users alternate, and submit increasingly larger bids, until a designated manager stops the auction. While the auction is not stopped, a non-leading user may withdraw their bid\(^5\). Auction satisfies Prop. 1: “Once stop() is called, all bids are immutable.” Prop. 1 is satisfied since stop() sets stopped to true, no function sets stopped to false, and while stopped is true neither bid() nor withdraw() is enabled. Formally, Prop. 1 is initially true, and remains true due to Prop. 1b: “Once stop() is called, stopped remains true.” Prop. 1 is said to be inductive relative to its inductive strengthening Prop. 1b. A Software Model Checker (SMC) can establish Prop. 1 by an exhaustive search for its inductive strengthening. However, this requires a bound on the number of addresses, since a search with all \(2^{160}\) addresses is intractable.

A bound of at least four addresses is necessary to represent the zero-account (i.e., a null user that cannot send transactions), the smart contract account, the manager, and an arbitrary sender. However, once the arbitrary sender submits a bid, the sender is now the leading bidder, and cannot withdraw its bid. To enable withdraw(), a fifth user is required. It follows by applying the results of [20], that a bound of five addresses is also sufficient, since users do not read each other’s bids, and adding a sixth user does not enable additional changes to leadingBid [20]. The bounded system, known as a harness, in Fig. 2 assigns the

\(^5\) For simplicity of presentation, we do not use Ether, Ethereum’s native currency.
zero-account to address 0, the smart contract account to address 1, the manager to address 2, the arbitrary senders to addresses 3 and 4, and then executes an unbounded sequence of arbitrary function calls. Establishing Prop. 1 on the harness requires finding its inductive strengthening. A strengthening such as Prop. 1b (or, in general, a counterexample violating Prop. 1) can be found by an SMC, directly on the harness code.

The above bound for Prop. 1 also works for checking all control-reachability properties of Auction. This, for example, follows by applying the results of [20]. That is, Auction has a Small Model Property (SMP) (e.g., [20,1]) for such properties. However, not all contracts enjoy an SMP. Consider Prop. 2: “The sum of all active bids is at least leadingBid.” Auction satisfies Prop. 2 since the leading bid is never withdrawn. To prove Auction satisfies Prop. 2, we instrument the code to track the current sum, through the highlighted lines in Fig. 1. With the addition of _sum, Auction no longer enjoys an SMP. Intuitively, each user enables new combinations of _sum and leadingBid. As a proof, assume that there are N users (other than the zero-account, the smart contract account, and the manager) and let $S_N = 1 + 2 + \cdots + N$. In every execution with N users, if leadingBid is N + 1, then _sum is less than $S_{N+1}$, since active bids are unique and $S_{N+1}$ is the sum of N + 1 bids from 1 to N + 1. However, in an execution with N + 1 users, if the i-th user has a bid of i, then leadingBid is N + 1 and _sum is $S_{N+1}$. Therefore, increasing N extends the reachable combinations of _sum and leadingBid. For example, if N = 2, then $S_3 = 1 + 2 + 3 = 6$. If the leading bid is 3, then the second highest bid is at most 2, and, therefore, _sum ≤ 5 $< S_3$. However, when N = 3, if the three active bids are \{1, 2, 3\}, then _sum is $S_3$. Therefore, instrumenting Auction with _sum violates the SMP of the original Auction.

Despite the absence of such an SMP, each function of Auction interacts with at most one user per transaction. Each user is classified as either the zero-account, the smart contract, the manager, or an arbitrary sender. In fact, all arbitrary senders are indistinguishable with respect to Prop. 2. For example, if there are exactly three active bids, \{2, 4, 8\}, it does not matter which user placed which bid. The leading bid is 8 and the sum of all bids is 14. On the other hand, if the leading bid is 8, then each participant of Auction must have a bid in the range of 0 to 8. To take advantage of these classes, rather than analyze Auction relative to all $2^{160}$ users, it is sufficient to analyze Auction relative to a representative user from each class. In our running example, there must be representatives for the zero-account, the smart contract account, the manager, and an (arbitrary) sender. The key idea is that each representative user can correspond to one or many concrete users.

Intuitively, each representative user summarizes the concrete users in its class. If a representative’s class contains a single concrete user, then there is no difference between the concrete user and the representative user. For example, the zero-account, the smart contract account, and the manager each correspond to single concrete users. The addresses of these users, and in turn, their bids, are known with absolute certainty. On the other hand, there are many arbitrary senders. Since senders are indistinguishable from each other, the precise address
of the representative sender is unimportant. What matters is that the representative sender does not share an address with the zero-account, the smart contract account, nor the manager. However, this means that at the start of each transaction the location of the representative sender is not absolute, and, therefore, the sender has a range of possible bids. To account for this, we introduce a predicate that is true of all initial bids, and holds inductively across all transactions. We provide this predicate manually, and use it to over-approximate all possible bids. An obvious predicate for \texttt{Auction} is that all bids are at most \texttt{leadingBid}, but this predicate is not strong enough to prove \textbf{Prop. 2}. For example, the representative sender could first place a bid of 10, and then (spuriously) withdraw a bid of 5, resulting in a sum of 5 but a leading bid of 10. A stronger predicate, that is adequate to prove \textbf{Prop. 2}, is given by \(\theta_U\): “Each bid is at most \texttt{leadingBid}. If a bid is not \texttt{leadingBid}, then its sum with \texttt{leadingBid} is at most \_sum.”

Given \(\theta_U\), \textbf{Prop. 2} can be verified by an SMC. This requires a new harness, with representative, rather than concrete, users. The new harness, Fig. 2 (now including the highlighted lines), is similar to the SMP harness in that the zero-account, the smart contract account, and the manager account are assigned to addresses 0, 1, and 2, respectively, followed by an unbounded sequence of arbitrary calls. However, there is now a single sender that is assigned to address 3 (line 15). That is, the harness uses a fixed configuration of representatives in which the fourth representative is the sender. Before each function call, the sender’s bid is set to a non-deterministic value that satisfies \(\theta_U\) (lines 6–10). If the new harness and \textbf{Prop. 2} are provided to an SMC, the SMC will find an inductive strengthening such as, “The leading bid is at most the sum of all bids.”

The harness in Fig. 2 differs from existing smart contract verification techniques in two ways. First, each address in Fig. 2 is an abstraction of one or more concrete users. Second, \texttt{msg.sender} is restricted to a finite address space by lines 13 to 15. If these lines are removed, then an inductive invariant must constrain all cells of \texttt{bids}, to accommodate \texttt{bids[msg.sender]}. This requires quantified invariants over arrays that is challenging to automate. By introducing lines 13 to 15, a quantifier-free predicate, such as our \(\theta_U\), can directly constrain cell \texttt{bids[msg.sender]} instead. Adding lines 13–15 makes the contract finite state. Thus, its verification problem is decidable and can be handled by existing SMCs. However, as illustrated by \textbf{Prop. 2}, the restriction on each user must not exclude feasible counterexamples. Finding such a restriction is the focus of this paper.

In this paper, we present a new approach to smart contract verification. We construct finite-state abstractions of parameterized smart contracts, known as \textit{local bundles}. A local bundle generalizes the harness in Fig. 2, and is constructed from a set of representatives and their predicates. When a local bundle and a property are provided to an SMC, there are three possible outcomes. First, if a predicate does not over-approximate its representative, a counterexample to the predicate is returned. Second, if the predicates do not entail the property, then a counterexample to verification is returned (this counterexample refutes the proof, rather than the property itself). Finally, if the predicates do entail the property, then an inductive invariant is returned. As opposed to deductive smart contract
solutions, our approach finds inductive strengthenings automatically [17,43]. As opposed to other model checking solutions for smart contracts, our approach is not limited to pre- and post-conditions [21], and can scale to $2^{160}$ users [24].

Key theoretical contributions of this paper are to show that verification with local bundle abstraction is an instance of Parameterized Compositional Model Checking (PCMC) [31] and the automation of the side-conditions for its applicability. Specifically, Theorem 3 shows that the local bundle abstraction is a sound proof rule, and a static analysis algorithm (PTGBuilder in Sec. 4) computes representatives so that the rule is applicable. Key practical contributions are the implementation and the evaluation of the method in a new smart contract verification tool SMARTACE, using SEAHORN [15] for SMC. SMARTACE takes as input a contract and a predicate. Representatives are inferred automatically from the contract, by analyzing the communication in each transaction. The predicate is then validated by SEAHORN, relative to the representatives. If the predicate is correct, then a local bundle, as in Fig. 2, is returned.

The rest of the paper is structured as follows. Sec. 2 reviews parameterized verification. Sec. 3 presents MicroSol, a subset of Solidity with network semantics. Sec. 4 relates user interactions to representatives. We formalize user interactions as Participation Topologies (PTs), and define PT Graphs (PTGs) to over-approximate PTs for arbitrarily many users. Intuitively, each PTG over-approximates the set of representatives. We show that a PTG is computable for every MicroSol program. Sec. 5 defines local bundles and proves that our approach is sound. Sec. 6 evaluates SMARTACE and shows that it can outperform VERX, a state-of-the-art verification tool, on all but one VERX benchmark.

2 Background

In this section, we briefly recall Parameterized Compositional Model Checking (PCMC) [31]. We write $u = (u_0, \ldots, u_{n-1})$ for a vector of $n$ elements, and $u_i$ for the $i$-th element of $u$. For a natural number $n \in \mathbb{N}$, we write $[n]$ for $\{0, \ldots, n-1\}$. Labeled Transition Systems. A labeled transition system (LTS), $M$, is a tuple $(S, P, T, s_0)$, where $S$ is a set of states, $P$ is a set of actions, $T : S \times P \rightarrow 2^S$ is a transition relation, and $s_0 \in S$ is an initial state. $M$ is deterministic if $T$ is a function, $T : S \times P \rightarrow S$. A (finite) trace of $M$ is an alternating sequence of states and actions, $(s_0, p_1, s_1, \ldots, p_k, s_k)$, such that $\forall i \in [k] \cdot s_{i+1} \in T(s_i, p_{i+1})$. A state $s$ is reachable in $M$ if $s$ is in some trace $(s_0, p_1, \ldots, s_k)$ of $M$; that is, $\exists i \in [k+1] \cdot s_i = s$. A safety property for $M$ is a subset of states (or a predicate$^6$) $\varphi \subseteq S$. $M$ satisfies $\varphi$, written $M \models \varphi$, if every reachable state of $M$ is in $\varphi$.

Many transition systems are parameterized. For instance, a client-server application is parameterized by the number of clients, and an array-manipulating program is parameterized by the number of cells. In both cases, there is a single control process that interacts with many user processes. Such systems are called

$^6$ Abusing notation, we refer to a subset of states $\varphi$ as a predicate and do not distinguish between the syntactic form of $\varphi$ and the set of states that satisfy it.
synchronized control-user networks (SCUNs) [31]. We let \( N \) be the number of processes, and \([N]\) be the process identifiers. We consider SCUNs in which users only synchronize with the control process and do not execute code on their own.

An SCUN \( N \) is a tuple \((SC, SU, P_1, P_S, T_I, T_S, c_0, u_0)\), where \( SC \) is a set of control states, \( SU \) a set of user states, \( P_1 \) a set of internal actions, \( P_S \) a set of synchronized actions, \( T_I : SC \times P_1 \rightarrow SC \) an internal transition function, \( T_S : SC \times SU \times P_S \rightarrow SC \times SU \) a synchronized transition function, \( c_0 \in SC \) is the initial control state, and \( u_0 \in SU \) is the initial user state. The semantics of \( N \) are given by a parameterized LTS, \( M(N) := (S, P, T, s_0) \), where \( S := SC \times (SU)^N \), \( P := P_1 \cup (P_S \times [N]) \), \( s_0 := (c_0, u_0, \ldots, u_0) \), and \( T : S \times P \rightarrow S \) such that:

1. If \( p \in P_1 \), then \( T((c, u), p) = (T_I(c, p), u) \), and
2. If \( (p, i) \in P_S \times [N] \), then \( T((c, u), (p, i)) = (c', u') \) where \((c', u'_i) = T_S(c, u_i, p)\), and \( \forall j \in [N] \{i\} \cdot u'_j = u_j \).

*Parameterized Compositional Model Checking (PCMC).* Parameterized systems have parameterized properties [16,31]. A \( k \)-universal safety property [16] is a predicate \( \varphi \subseteq SC \times (SU)^k \). A state \((c, u)\) satisfies predicate \( \varphi \) if \( \forall \{i_1, \ldots, i_k\} \subseteq [N] \cdot \varphi(c, u_{i_1}, \ldots, u_{i_k}) \). A parameterized system \( M(N) \) satisfies predicate \( \varphi \) if \( \forall N \in \mathbb{N} \cdot M(N) \models \varphi \). For example, *Prop. 1* (Sec. 1) of SimpleAuction (Fig. 1) is 1-universal: “For every user \( u \), if \text{stop()} \) has been called, then \( u \) is immutable.”

Proofs of \( k \)-universal safety employ compositional reasoning, e.g., [2,16,31,33]. Here, we use PCMC [31]. The keys to PCMC are uniformity—the property that finitely many neighbourhoods are distinguishable—and a compositional invariant—a summary of the reachable states for each equivalence class, that is closed under the actions of every other equivalence class. For an SCUN, the compositional invariant is given by two predicates \( \theta_C \subseteq SC \) and \( \theta_U \subseteq SC \times SU \) satisfying:

**Initialization** If \((c_0, u_0) \in \theta_U \); \n**Consecution 1** If \( c \in \theta_C \), \((c, u) \in \theta_U \), \( p \in P_S \), and \((c', u') \in T_S(c, u, p) \), then \((c', u') \in \theta_U \); \n**Consecution 2** If \( c \in \theta_C \), \((c, u) \in \theta_U \), \( p \in P_C \), and \( c' = T_I(c, p) \), then \((c', u) \in \theta_U \); \n**Non-Interference** If \( c \in \theta_C \), \((c, u) \in \theta_U \), \((c, v) \in \theta_U \), \( u \neq v \), \( p \in P_S \), and \((c', u') = T_S(c, u, p) \), then \((c', v) \in \theta_C \).

By PCMC [31], if \( \forall c \in \theta_C \cdot \forall \{(c, u_1), \ldots, (c, u_k)\} \subseteq \theta_U \cdot \varphi(c, u_1, \ldots, u_k) \), then \( M \models \varphi \). This is as an extension of Owicki-Gries [33], where \( \theta_C \) summarizes the acting process and \( \theta_U \) summarizes the interfering process. For this reason, we call \( \theta_C \) the inductive invariant and \( \theta_U \) the interference invariant.

## 3 MicroSol: Syntax and Semantics

This section provides network semantics for MicroSol, a subset of Solidity\(^7\). Like Solidity, MicroSol is an imperative object-oriented language with built-in communication operations. The syntax of MicroSol is in Fig. 3. MicroSol restricts Solidity to a core subset of communication features. For example, MicroSol does not

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\(^7\) [https://docs.soliditylang.org/](https://docs.soliditylang.org/)
respectively. Similarly, we use client
languages. Each typed variable is further classified as either
state
variables correspond to dictionaries and object pointers in other object-oriented
variables cannot cast to address variables. Mapping and contract-reference vari-
contract reference constructs.

For simplicity of presentation, each transaction is given as a global transition.
internal action, and finally, the users are updated through synchronized actions.

Throughout this section, we illustrate MicroSol using
Auction (Appendix B), that extends our semantics to a wider set of smart contracts.
In our evaluation (Sec. 6), we use a superset of MicroSol, called MiniSol (see
Appendix B), that extends our semantics to a wider set of smart contracts. Throughout this section, we illustrate MicroSol using Auction in Fig. 1.

A MicroSol smart contract is similar to a class in object-oriented programming, and consists of variables, and transactions (i.e., functions) for users to call. A transaction is a deterministic sequence of operations. Each smart contract user has a globally unique identifier, known as an address. We view a smart contract as operating in an SCUN: the control process executes each transaction sequentially, and the user processes are contract users that communicate with the control process. Users in the SCUN enter into a transaction through a synchronized action, then the control process executes the transaction as an internal action, and finally, the users are updated through synchronized actions. For simplicity of presentation, each transaction is given as a global transition.

A constructor is a special transaction that is executed once after contract creation. Calls to new (i.e., creating new smart contracts) are restricted to constructors. Auction in Fig. 1 is a smart contract that defines a constructor (line 6), three other functions (lines 8, 17, and 25), and four state variables (lines 2–3).

MicroSol has four types: address, numeric (including bool), mapping, and contract reference. Address variables prevent arithmetic operations, and numeric variables cannot cast to address variables. Mapping and contract-reference variables correspond to dictionaries and object pointers in other object-oriented languages. Each typed variable is further classified as either state, input, or local. We use role and data to refer to state variables of address and numeric types, respectively. Similarly, we use client and argument to refer to inputs of address
and numeric types, respectively. In Auction of Fig. 1, there is 1 role (manager), 2 contract data (leadingBid and stopped), 1 mapping (bids), 1 client common to all transactions (msg.sender), and at most 1 argument in any transaction (amount).

Note that in MicroSol, user denotes any user process within a SCUN. A client is defined relative to a transaction, and denotes a user passed as an input.

**Semantics of MicroSol.** Let $C$ be a MicroSol program with a single transaction $tr$ (see Appendix A for multiple transactions). An $N$-user bundle is an $N$-user network of several (possibly identical) MicroSol programs. The semantics of a bundle is an LTS, $\text{lts}(C, N) := (S, P, f, s_0)$, where $S_C := \text{control}(C, [N])$ is the set of control states, $S_U := \text{user}(C, [N])$, is the set of user states, $s_\bot$ is the error state, $S \subseteq (S_C \cup \{s_\bot\}) \times (S_U)^N$ is the set of LTS states, $P := \text{action}(C, [N])$ is the set of actions, $f : S \times P \rightarrow S$ is the transition function, and $s_0$ is the initial state. We assume, without loss of generality, that there is a single control process$^8$.

Let $\mathbb{D}$ be the set of 256-bit unsigned integers. The state space of a smart contract is determined by the address space, $\mathcal{A}$, and the state variables of $C$. In the case of $\text{lts}(C, N)$, the address space is fixed to $\mathcal{A} = [N]$. Assume that $n$, $m$, and $k$ are the number of roles, data, and mappings in $C$, respectively. State variables are stored by their numeric indices (i.e., variable 0, 1, etc.). Then, $\text{control}(C, \mathcal{A}) \subseteq \mathcal{A}^n \times \mathbb{D}^m$ and $\text{user}(C, \mathcal{A}) \subseteq \mathcal{A} \times \mathbb{D}^k$. For $c = (x, y) \in \text{control}(C, \mathcal{A})$, $\text{role}(c, i) = x_i$ is the $i$-th role and $\text{data}(c, i) = y_i$ is the $i$-th datum. For $u = (z, y) \in \text{user}(C, \mathcal{A})$, $z$ is the address of $u$, and $\text{map}(u) = y$ are the mapping values of $u$.

Similarly, actions are determined by the address space, $\mathcal{A}$, and the input variables of $tr$. Assume that $q$ and $r$ are the number of clients and arguments of $tr$, respectively. Then $\text{action}(C, \mathcal{A}) \subseteq \mathcal{A}^q \times \mathbb{D}^r$. For $p = (x, y) \in \text{action}(C, \mathcal{A})$, $\text{client}(p, i) = x_i$ is the $i$-th client in $p$ and $\text{arg}(p, i) = y_i$ is the $i$-th argument in $p$. For a fixed $p$, we write $f_p(s, u)$ to denote $f((s, u), p)$.

The initial state of $\text{lts}(C, N)$ is $s_0 := (c, u) \in \text{control}(C, [n]) \times \text{user}(C, [n])^N$, where $c = (0, 0)$, $\forall i \in [N] \cdot \text{map}(u)_i = 0$, and $\forall i \in [N] \cdot \text{id}(u)_i = i$. That is, all variables are zero-initialized and each user has a unique address.

An $N$-user transition function is determined by the (usual) semantics of $tr$, and a bijection from addresses to user indices, $\mathcal{M} : \mathcal{A} \rightarrow [N]$. If $\mathcal{M}(a) = i$, then address $a$ belongs to user $u_i$. In the case of $\text{lts}(C, N)$, the $i$-th user has address $i$, so $\mathcal{M}(i) = i$. We write $f := \mathcal{M} \cdot f_p$, and given an action $p$, $f_p$ updates the state variables according to the source code of $tr$ with respect to $\mathcal{M}$. If an assert fails or an address is outside of $\mathcal{A}$, then the error state $s_\bot$ is returned. If a require fails, then the state is unchanged. Note that $f$ preserves the address of each user.

For example, $\text{lts}(\text{Auction}, 4) = (S, P, f, s_0)$ is the 4-user bundle of Auction. Assume that $(c, u)$ is the state reached after evaluating the constructor. Then $\text{role}(c, 0) = 2$, $\text{data}(c, 0) = 0$, $\text{data}(c, 1) = 0$, and $\forall i \in [4] \cdot \text{map}(u)_i = 0$. That is, the manager is at address 2, the leading bid is 0, the auction is not stopped, and there are no active bids. This is because variables are zero-indexed, and stopped

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$^8$ Restrictions place on new ensure that the number of MicroSol smart contracts in a bundle is a static fact. Therefore, all control states are synchronized, and can be combined into a product machine.
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is the second numeric variable (i.e., at index 1). If the user at address 3 placed a bid of 10, this corresponds to \( p \in P \) such that \( \text{client}(p, 0) = 3 \) and \( \text{arg}(p, 0) = 10 \). A complete LTS for this example is in Appendix A.

**Limitations of MicroSol.** MicroSol places two restrictions on Solidity. First, addresses are not numeric. We argue that this restriction is reasonable, as address manipulation is a form of pointer manipulation. Second, \texttt{new} must only appear in constructors. In our evaluation (Sec. 6), all calls to \texttt{new} could be moved into a constructor with minimal effort. We emphasize that the second restriction does not preclude the use of abstract interfaces for arbitrary contracts.

### 4 Participation Topology

The core functionality of any smart contract is communication between users. Usually, users communicate by reading from and writing to designated mapping entries. That is, the communication paradigm is shared memory. However, it is convenient in interaction analysis to re-imagine smart contracts as having rendezvous synchronization in which users explicitly participate in message passing. In this section, we formally re-frame smart contracts with explicit communication by defining a (semantic) participation topology and its abstractions.

A user \( u \) participates in communication during a transaction \( f \) whenever the state of \( u \) affects execution of \( f \) or \( f \) affects a state of \( u \). We call this influence. For example, in Fig. 1, the sender influences \texttt{withdraw} on line 19. Similarly, \texttt{withdraw} influences the sender on line 22. In all cases, the influence is witnessed by the state of the contract and the configuration of users that exhibit the influence.

Let \( C \) be a contract, \( N \in \mathbb{N} \) be the network size, \( (S, P, f, s_0) = \text{lts}(C, N) \), and \( p \in P \). A user with address \( a \in \mathbb{N} \) influences transaction \( f_p \) if there exists an \( s, r, r' \in \text{control}(C, [N]) \), \( u, u', v, v' \in \text{user}(C, [N])^N \), and \( i \in [N] \) such that:

1. \( \text{id}(u_i) = a \);
2. \( \forall j \in [N] \cdot (u_i = v_j) \iff (i \neq j) \);
3. \( (r, u') = f_p(s, u) \) and \( (r', v') = f_p(s, v) \);
4. \( (r = r') \Rightarrow (\exists j \in [N] \setminus \{i\} \cdot u'_j \neq v'_j) \).

That is, there exists two network configurations that differ only in the state of the user \( u_i \), and result in different network configurations after applying \( f_p \). In
practice, \(f_p\) must compare the address of \(u_i\) to some other address, or must use the state of \(u_i\) to determine the outcome of the transaction. The tuple \((s, u, v)\) is a witness to the influence of \(a\) over transaction \(f_p\). A user with address \(a \in \mathbb{N}\) is influenced by transaction \(f_p\) if there exists an \(s, s' \in \text{control}(C, [N])\), \(u, u' \in \text{user}(C, [N])^N\), and \(i \in [N]\) such that:

1. \(\text{id}(u_i) = a\);
2. \((s', u') = f_p(s, u)\);
3. \(u'_i \neq u_i\).

That is, \(f_p\) must write into the state of \(u_i\), and the changes must persist after the transaction terminates. The tuple \((s, u)\) is a witness to the influence of transaction \(f_p\) over user \(a\).

**Definition 1 (Participation).** A user with address \(a \in \mathbb{N}\) participates in a transaction \(f_p\) if either \(a\) influences \(f_p\), witnessed by some \((s, u, v)\), or \(f_p\) influences \(a\), witnessed by some \((s, u)\). In either case, \(s\) is a witness state.

Smart contracts facilitate communication between many users across many transactions. We need to know every possible participant, and the cause of their participation—we call this the participation topology (PT). A PT associates each communication (sending or receiving) with one or more participation classes, called explicit, transient, and implicit. The participation is explicit if the participant is a client of the transaction; transient if the participant has a role during the transaction; implicit if there is a state such that the participant is neither a client nor holds any roles. In the case of MiniSol, all implicit participation is due to literal address values, as users designated by literal addresses must participate regardless of clients and roles. An example of implicit participation is when a client is compared to the address of the zero-account (i.e., \(\text{address}(0)\)) in Fig. 1.

**Definition 2 (Participation Topology).** A Participation Topology of a transaction \(f_p\) is a tuple \(\text{pt}(C, N, p) := (\text{Explicit}, \text{Transient}, \text{Implicit})\), where:

1. \(\text{Explicit} \subseteq \mathbb{N} \times [N]\) where \((i, a) \in \text{Explicit}\) iff \(a\) participates during \(f_p\), with \(\text{client}(p, i) = a\);
2. \(\text{Transient} \subseteq \mathbb{N} \times [N]\) where \((i, a) \in \text{Transient}\) iff \(a\) participates during \(f_p\), as witnessed by a state \(s \in \text{control}(C, [N])\), where \(\text{role}(s, i) = a\);
3. \(\text{Implicit} \subseteq [N]\) where \(a \in \text{Implicit}\) iff \(a\) participates during \(f_p\), as witnessed by a state \(s \in \text{control}(C, [N])\), where \(\forall i \in \mathbb{N}, \text{role}(s, i) \neq a\) and \(\text{client}(p, i) \neq a\).

For example, Fig. 4a shows a PT for any function of Fig. 1 with 4 users. From Sec. 1, it is clear that each function can have an affect. The zero-account and smart contract account are both implicit participants, since changing either account’s address to 3 would block the affect of the transaction. The manager is a transient participant and the sender is an explicit participant, since the (dis)equity of their addresses is asserted at lines 9, 18, and 26.

Def. 2 is semantic and dependent on actions. A syntactic summary of all PTs for all actions is required to reason about communication. This summary
is analogous to over-approximating control-flow with a “control-flow graph” [3].

This motivates the Participation Topology Graph (PTG) that is a syntactic over-approximation of all possible PTs, independent of network size. A PTG has a vertex for each user and each action, such that edges between vertices represent participation classes. In general, a single vertex can map to many users or actions.

PTG edges are labeled by participation classes. For any contract \( C \), there are at most \( m \) explicit classes and \( n \) transient classes, where \( n \) is the number of roles, and \( m \) is the maximum number of clients taken by any function of \( C \). On the other hand, the number of implicit classes is determined by the PTG itself. In general, there is no bound on the number of implicit participants, and it is up to a PTG to provide an appropriate abstraction (i.e., \( L \) in Def. 3). The label set common to all PTGs is \( AP(C) := \{ explicit@i \mid i \in [n] \} \cup \{ transient@i \mid i \in [m] \} \).

**Definition 3 (Participation Topology Graph).** Let \( L \) be a finite set of implicit classes, \( V \subseteq N \) be finite, \( E \subseteq V \times V \), and \( \delta \subseteq E \times (AP(C) \cup L) \). A PTG for a contract \( C \) is a tuple \((V, E, \delta), \rho, \tau\), where \((V, E, \delta)\) is a graph labeled by \( \delta, \rho \subseteq action(C, N) \times V \), and \( \tau \subseteq action(C, N) \times N \times V \), such that for all \( N \in \mathbb{N} \) and for all \( p \in action(C, [N]) \), with \( pt(C, N, p) = (Explicit, Transient, Implicit) \):

1. If \((i, a) \in Explicit\), then there exists a \((p, u) \in \rho \) and \((p, a, v) \in \tau\) such that \((u, v) \in E \) and \( \delta((u, v), explicit@i)\);
2. If \((i, a) \in Transient\), then there exists a \((p, u) \in \rho \) and \((p, a, v) \in \tau\) such that \((u, v) \in E \) and \( \delta((u, v), transient@i)\);
3. If \(a \in Implicit\), then there exists a \((p, u) \in \rho\), \((p, a, v) \in \tau\), and \( l \in L\) such that \((u, v) \in E \) and \( \delta((u, v), l)\).

In Def. 3, \( \tau \) and \( \rho \) map actions and users to vertices, respectively. An edge between an action and a user indicates the potential for participation. The labels describe the potential participation classes. As an example, Fig. 4b is a PTG for Fig. 1, where all actions map to \( sc \), the zero-account maps to vertex 0, the smart contract account maps to vertex 1, and all other users map to \( * \). The two implicit classes have the label \( implicit@0 \) and \( implicit@1 \), respectively.

**Theorem 1.** Let \( C \) be a contract with a PTG \((G, \rho, \tau)\), \( G = (V, E, \delta) \), and \( \delta \subseteq E \times (AP(C) \cup L) \). Then, for all \( N \in \mathbb{N} \) and all \( p \in action(C, [N]) \), \( pt(C, N, p) = (Explicit, Transient, Implicit) \) is over-approximated by \((G, \rho, \tau)\) as follows:

1. If \( Explicit(i, a) \), then \( \exists (u, v) \in E \cdot \rho(p, u) \land \tau(p, a, v) \land \delta((u, v), explicit@i)\);
2. If \( Transient(i, a) \), then \( \exists (u, v) \in E \cdot \rho(p, u) \land \tau(p, a, v) \land \delta((u, v), transient@i)\);
3. If \( Implicit(a) \), then \( \exists (u, v) \in E \cdot \exists l \in L \cdot \rho(p, u) \land \tau(p, a, v) \land \delta((u, v), l)\).

For any PT, there are many over-approximating PTGs. The weakest PTG joins every user to every action using all possible labels and a single implicit class. Fig. 4b shows a simple, yet stronger, PTG for Fig. 1. First, note that there are two implicit participants, identified by addresses 0 and 1, with labels \( implicit@0 \) and \( implicit@1 \), respectively. Next, observe that any arbitrary user can become the manager. Finally, the distinctions between actions are ignored. Thus, there are three user vertices, two which are mapped to the zero-account.
and smart contract account, and another mapped to all other users. Such a PTG is constructed automatically using an algorithm named $\text{PTGBuilder}$.

$\text{PTGBuilder}$ takes a contract $C$ and returns a PTG. The implicit classes are $L := \{ \text{implicit}@a \mid a \in \mathbb{N} \}$, where $\text{implicit}@a$ signifies implicit communication with address $a$. PTG construction is reduced to taint analysis [23]. Input address variables, state address variables, and literal addresses are tainted sources. Sinks are memory writes, comparison expressions, and mapping accesses. $\text{PTGBuilder}$ computes $(\text{Args}, \text{Roles}, \text{Lits})$, where (1) $\text{Args}$ is the set of indices of input variables that propagate to a sink; (2) $\text{Roles}$ is the set of indices of state variables that propagate to a sink; (3) $\text{Lits}$ is the set of literal addresses that propagate to a sink. Finally, a PTG is constructed as $(G, \rho, \tau)$, where $G = (V, E, \delta)$, $\rho \subseteq \text{action}(C, N) \times V$, $\tau \subseteq \text{action}(C, N) \times N \times V$, $sc$, and $*$ are unique vertices:

1. $V := \text{Lits} \cup \{ \text{sc}, * \}$ and $E := \{ (\text{sc}, v) \mid v \in V \setminus \{ \text{sc} \} \}$;
2. $\delta := \{ (e, \text{explicit}@i) \mid e \in E, i \in \text{Args} \} \cup \{ (e, \text{transient}@i) \mid e \in E, i \in \text{Roles} \} \cup \{ ((\text{sc}, a), \text{transient}@a) \mid a \in \text{Lits} \}$;
3. $\rho := \{ (p, \text{sc}) \mid p \in \text{action}(C, N) \}$;
4. $\tau := \{ (p, a, *) \mid p \in \text{action}(C, N), a \in N \setminus \text{Lits} \} \cup \{ (p, a, a) \mid p \in \text{action}(C, N), a \in \text{Lits} \}$.

$\text{PTGBuilder}$ formalizes the intuition of Fig. 4b. Rule 1 ensures that every literal address has a vertex, and that all user vertices connect to $sc$. Rule 2 over-approximates explicit, transient, and implicit labels. The first set states that if an input address is never used, then the client is not an explicit participant. This statement is self-evident, and over-approximates explicit participation. The second and third set make similar claims for roles and literal addresses. Rules 3 and 4 define $\rho$ and $\tau$ as expected. Note that in MicroSol, implicit participation stems from literal addresses, since addresses do not support arithmetic operations, and since numeric expressions cannot be cast to addresses.

By re-framing smart contracts with rendezvous synchronization, each transaction is re-imagined as a communication between several users. Their communication patterns are captured by the corresponding PT. A PTG over-approximates PTs of all transactions, and is automatically constructed using $\text{PTGBuilder}$. This is crucial for PCMC as it provides an upper bound on the number of equivalence classes, and the users in each equivalence class (see Appendix C).

## 5 Local Reasoning in Smart Contracts

In this section, we present a proof rule for the parameterized safety of MicroSol programs. Our proof rule extends the existing theory of PCMC. The section is structured as follows. Sec. 5.1 introduces syntactic restrictions, for properties and interference invariants, that expose address dependencies. Sec. 5.2, defines local bundle reductions, that reduce parameterized smart contract models to finite-state models. We show that for the correct choice of local bundle reduction, the safety of the finite-state model implies the safety of the parameterized model.

### 5.1 Guarded Properties and Split Invariants

Universal properties and interference invariants might depend on user addresses. However, PCMC requires explicit address dependencies. This is because address
dependencies allow predicates to distinguish subsets of users. To resolve this, we introduce two syntactic forms that make address dependencies explicit: guarded universal safety properties and split interference invariants. We build both forms from so-called address-oblivious predicates that do not depend on user addresses.

For any smart contract \( C \) and any address space \( A \), a pair of user configurations, \( u, v \in \text{user}(C,A)^k \), are \( k \)-address similar if \( \forall i \in [k] \cdot \text{map}(u_i) = \text{map}(v_i) \). A predicate \( \xi \subseteq \text{control}(C,A) \times \text{user}(C,A)^k \) is address-oblivious if, for every choice of \( s \in \text{control}(C,A) \), and every pair of \( k \)-address similar configurations, \( u \) and \( v \), \( \xi(s,u) \Leftrightarrow \xi(s,v) \). Prop. 1 and Prop. 2 in Sec. 1 are address-oblivious.

A guarded \( k \)-universal safety property is built from a single \( k \)-user address-oblivious predicate. The predicate is guarded by constraints over its \( k \) user addresses. Each constraint compares a single user’s address to either a literal address or a role. This notion is formalized by Def. 4, and illustrated in Ex. 1.

**Definition 4 (Guarded Universal Safety).** For \( k \in \mathbb{N} \), a guarded \( k \)-universal safety property is a \( k \)-universal safety property \( \varphi \), given by a tuple \( (L,R,\xi) \), where \( L \subseteq \mathbb{N} \times [k] \) is finite, \( R \subseteq \mathbb{N} \times [k] \) is finite, and \( \xi \) is an address-oblivious \( k \)-user predicate, such that:

\[
\varphi(s,u) := \left( \bigwedge_{(a,i) \in L} a = \text{id}(u_i) \right) \land \left( \bigwedge_{(i,j) \in R} \text{role}(s,i) = \text{id}(u_j) \right) \Rightarrow \xi(s,u)
\]

Note that \( A_L := \{a \mid (a,i) \in L \} \) and \( A_R := \{i \mid (i,j) \in R \} \) and define the literal and role guards for \( \varphi \).

**Example 1.** Consider the claim that in Auction of Fig. 1, the zero-account cannot have an active bid. This claim is stated as **Prop. 3**: For each user process \( u \), if \( \text{id}(u_0) = 0 \), then \( \text{map}(u_0)_0 = 0 \). That is, **Prop. 3** is a guarded 1-universal safety property \( \varphi_1(s,u) := (0 = \text{id}(u_0)) \Rightarrow (\text{map}(u_0)_0 = 0) \). Following Def. 4, \( \varphi_1 \) is determined by \( (L_1, \emptyset, \xi_1) \), where \( L_1 = \{(0,0)\} \) and \( \xi_1(s,u) := \text{map}(u_0)_0 = 0 \).

The second set is \( \emptyset \) as there are no role constraints in **Prop. 3**. If a state \((s,u)\) satisfies \( \varphi_1 \), then \( \forall \{i\} \subseteq [N] \cdot \varphi_1(s,(u_i)) \).

The syntax of a split interference invariant is similar to a guarded safety property. The invariant is constructed from a list of address-oblivious predicates, each guarded by a single constraint. The final predicate is guarded by the negation of all other constraints. Intuitively, each address-oblivious predicate summarizes the class of users that satisfy its guard. The split interference invariant is the conjunction of all (guarded predicate) clauses. We proceed with the formal definition in Def. 5 and a practical illustration in Ex. 2.

**Definition 5 (Split Interference Invariant).** A split interference invariant is an interference invariant \( \theta \), given by a tuple \( (A_L,A_R,\zeta,\mu,\xi) \), where \( A_L = \{l_0, \ldots, l_{m-1}\} \subseteq \mathbb{N} \) is finite, \( A_R = \{r_0, \ldots, r_{n-1}\} \subseteq \mathbb{N} \) is finite, \( \zeta \) is a list of
\(m\) address-oblivious 1-user predicates, \(\mu\) is a list of \(n\) address-oblivious 1-user predicates, and \(\xi\) is an address-oblivious 1-user predicate, such that:

\[
\psi_{\text{Lits}}(s, u) := \left( \bigwedge_{i=0}^{m-1} \text{id}(u_0) = l_i \right) \Rightarrow \zeta_i(s, u)
\]

\[
\psi_{\text{Roles}}(s, u) := \left( \bigwedge_{i=0}^{n-1} \text{id}(u_0) = \text{role}(s, r_i) \right) \Rightarrow \mu_i(s, u)
\]

\[
\psi_{\text{Else}}(s, u) := \left( \bigwedge_{i=0}^{m-1} \text{id}(u_0) \neq l_i \right) \land \left( \bigwedge_{i=0}^{n-1} \text{id}(u_0) \neq \text{role}(s, r_i) \right) \Rightarrow \xi(s, u)
\]

\[
\theta(s, u) := \psi_{\text{Roles}}(s, u) \land \psi_{\text{Lits}}(s, u) \land \psi_{\text{Else}}(s, u)
\]

Note that \(A_L\) and \(A_R\) define literal and role guards of \(\theta\), and that \(|u| = 1\).

Example 2. To establish \(\varphi_1\) from Ex. 1, we require an adequate interference invariant such as Prop. 4: The zero-account never has an active bid, while all other users can have active bids. That is, Prop. 4 is a split interference invariant:

\[
\theta_1(s, u) := (\text{id}(u_0) = 0 \Rightarrow (\text{map}(u_0))_0 = 0) \land (\text{id}(u_0) \neq 0 \Rightarrow (\text{map}(u_0))_0 \geq 0)
\]

Following Def. 5, \(\theta_1\) is determined by \(\text{Inv} = (A_L, \emptyset, (\xi_1), \emptyset, \xi_2)\), where \(A_L = \{0\}\), \(\xi_1\) is defined in Ex. 1, and \(\xi_2(s, u) := \text{map}(u_0)_0 \geq 0\). The two instances of \(\emptyset\) in \(\text{Inv}\) correspond to the lack of role constraints in \(\theta_1\). If \(\text{Inv}\) is related back to Def. 5, then \(\psi_{\text{Roles}}(s, u) := T\), \(\psi_{\text{Lits}}(s, u) := (\text{id}(u_0) = 0) \Rightarrow (\text{map}(u_0))_0 = 0\), and \(\psi_{\text{Else}}(s, u) := (\text{id}(u_0) \neq 0) \Rightarrow (\text{map}(u_0))_0 \geq 0\).

\[\square\]

### 5.2 Localizing a Smart Contract Bundle

A local bundle is a finite-state abstraction of a smart contract bundle. This abstraction reduces smart contract PCMC to software model checking. At a high level, each local bundle is a non-deterministic LTS and is constructed from three components: a smart contract, a candidate interference invariant, and a neighbourhood. The term candidate interference invariant describes any predicate with the syntax of an interference invariant, regardless of its semantic interpretation. Sets of addresses are used to identify representatives in a neighbourhood.

Let \(A\) be an \(N\)-user neighbourhood and \(\theta_U\) be a candidate interference invariant. The local bundle corresponding to \(A\) and \(\theta_U\) is defined using a special relation called an \(N\)-user interference relation. The \(N\)-user interference relation (for \(\theta_U\)) sends an \(N\)-user smart contract state to the set of all \(N\)-user smart contract states that are reachable under the interference of \(\theta_U\). A state is reachable under the interference of \(\theta_U\) if the control state is unchanged, each address is unchanged, and all user data satisfies \(\theta_U\). For example, lines 6–10 in Fig. 2 apply a 4-user interference relation to the states of \texttt{Auction}. Note that if the interference relation for \(\theta_U\) fails to relate \((s, u)\) to itself, then \((s, u)\) violates \(\theta_U\).
Definition 6 (Interference Relation). Let $N \in \mathbb{N}$, $C$ be a contract, $S = \text{control}(C, N) \times \text{user}(C, N)^N$, and $\theta_U$ be a split candidate interference invariant. The $N$-user interference relation for $\theta_U$ is the relation $g : S \rightarrow 2^S$ such that $g(c, u) := \{(c, v) \in S \mid \forall i \in [N] \cdot id(u_i) = id(v_i) \land \theta_U(s, v_i)\}$.

Each state of the local bundle for $A$ and $\theta_U$ is a tuple $(s, u)$, where $s$ is a control state and $u$ is an $N$-user configuration. The $N$ users in the local bundle correspond to the $N$ representatives in $A$, and therefore, the address space of the local bundle can be non-consecutive. The transition relation of the local bundle is defined in terms of the (global) transaction function $f$. First, the transition relation applies $f$. If the application of $f$ is closed under $\theta_U$, then the interference relation is applied. Intuitively, $\theta_U$ defines a safe envelop under which the interference relation is compositional.

Definition 7 (Local Bundle). Let $C$ be a contract, $A = \{a_0, \ldots, a_{N-1}\} \subseteq \mathbb{N}$ be an $N$-user neighbourhood, $\theta_U$ be a candidate split interference invariant, and $g$ be the $N$-user interference relation for $\theta_U$. A local bundle is an LTS $\text{local}(C, A, \theta_U) := (S, P, f, s_0)$, such that $S := \text{control}(C, A) \times \text{user}(C, A)^N$, $P := \text{action}(C, A)$, $s_0 := (c_0, u)$, $c_0 := (0, 0)$, $\forall i \in [N] \cdot id(u_i) = a_i \land \text{map}(u_i) = 0$, and $\hat{f}$ is defined with respect to $M : A \rightarrow [N]$, $M(a_i) = i$, such that:

$$\hat{f}((s, u), p) := \begin{cases} g(s', u') & \text{if } (s', u') = [C]_M((s, u), p) \\ [C]_M((s, u), p) \text{ otherwise} & \end{cases}$$

Example 3. We briefly illustrate the transition relation of Def. 7 using Auction of Fig. 1. Let $A_1 = \{0, 1, 2, 3\}$ be a neighbourhood, $\theta_1$ be as in Ex. 2, $g$ be the 4-user interference relation for $\theta_1$, and $(S, P, f, s_0) = \text{local}(C, A_1, \theta_1)$. Consider applying $\hat{f}$ to $(s, u) \in S$ with action $p \in P$, such that $s = \{\text{manager} \rightarrow 2; \text{leadingBid} \rightarrow 0\}$, $\forall i \in [4] \cdot \text{map}(u_i) = 0$, and $p$ is a bid of 10 from a sender at address 3.

By definition, if $(s', v) = \hat{f}(s, u, p)$, then the leading bid is now 10, and the bid of the sender is also 10, since the sender of $p$ was not the manager and the leading bid was less than 10. Clearly $(s', v) \in g(s', v)$, and therefore, $g(s', v) = \hat{f}(s, u, p)$. A successor state is then selected, as depicted in Fig. 5a. This is done by first assigning an arbitrary bid to each representative, and then requiring that each bid satisfies $\theta_1$ relative to $s'$. In Fig. 5a, a network is selected in which $\forall i \in [4] \cdot id(v_i) = i$. As depicted in Fig. 5a, $\theta_1$ stipulates that the zero-account must satisfy $\xi_1$ and that all other users must satisfy $\xi_2$.

In Fig. 5b, a satisfying bid is assigned to each user. The choice for $d_0$ was fixed since $\xi_1(s, v_0)$ entails $d_0 = 0$. For $d_1$ to $d_3$, any non-negative value could have been selected. After the transaction is executed, $\text{map}(u'_0) = 0$, $\text{map}(u'_1) = 1$, $\text{map}(u'_2) = 2$, $\text{map}(u'_3) = 3$, and $s' = \{\text{manager} \rightarrow 2; \text{leadingBid} \rightarrow 10\}$. Then $(s', u') \in \hat{f}(s, u)$, as desired. Note that $(s', u')$ is not reachable in $\text{lts}(C, 4)$. □

Ex. 3 motivates an important result for local bundles. Observe that $(s', u') \models \theta_1$. This is not by chance. First, by the compositionality of $\theta_1$, all user configurations reached by $\text{local}(C, A_1, \theta_1)$ must satisfy $\theta_U$. Second, and far less obviously,
by choice of $A_1$, if all reachable user configurations satisfy $\theta_1$, then $\theta_1$ must be compositional. The proof of this result relies on a saturating property of $A_1$.

A neighbourhood $A$ is saturating if it contains representatives from each participation class of a PTG, and for all role guards $(A_R \subseteq N)$ and literal guards $(A_L \subseteq N)$ of interest. Intuitively, each participation class over-approximates an equivalence class of $C$. The number of representatives is determined by the equivalence class. In the case of PTGBuilder, a saturating neighbourhood contains one address for each participation class. For an implicit class, such as $\text{implicit@x}$, $x$ is literal and must appear in the neighbourhood. All other addresses are selected arbitrarily. The saturating property of $A_1$ is depicted in Fig. 5b by the correspondence between users and participation classes ($A_R = \emptyset$, $A_L = \{0\}$).

Definition 8 (Saturating Neighbourhood). Let $A_R, A_L \subseteq N$, $C$ be a contract, $(G, \rho, \tau)$ be the PTGBuilder PTG of $C$, and $G = (V, E, \delta)$ such that $A_R$ and $A_L$ are finite. A saturating neighbourhood for $(A_R, A_L, (G, \rho, \tau))$ is a set $A_{\text{Exp}} \cup A_{\text{Trans}} \cup A_{\text{Impl}}$ s.t. $A_{\text{Exp}}, A_{\text{Trans}}, A_{\text{Impl}} \subseteq N$ are pairwise disjoint and:

1. $|A_{\text{Exp}}| = \{|i \in N \mid \exists e \in E \cdot \delta(e, \text{explicit@i})\}$,
2. $|A_{\text{Trans}}| = \{|i \in N \mid \exists e \in E \cdot \delta(e, \text{transient@i})\} \cup |A_R|$,
3. $A_{\text{Impl}} = \{x \in N \mid \exists e \in E \cdot \delta(e, \text{implicit@x})\} \cup A_L$.

A saturating neighbourhood can be used to reduce compositionality and $k$-safety proofs to the safety of local bundles. We start with compositionality. Consider a local bundle with a neighbourhood $A^+$ where $A^+$ contains a saturating neighbourhood, the guards of $\theta_U$, and some other address $a$. The neighbourhood $A^+$ contains a representative for: each participation class; each role and literal user distinguished by $\theta_U$; an arbitrary user under interference (i.e., $a$). We first claim that if $\theta_U$ is compositional, then a local bundle constructed from $\theta_U$ must be safe with respect to $\theta_U$ (as in Ex. 3). The first claim follows by induction. By Initialization (Sec. 2), the initial users satisfy $\theta_U$. For the inductive step, assume that all users satisfy $\theta_U$ and apply $\hat{f}_P$. The users that participate in $\hat{f}_P$ maintain $\theta_U$ by Consequence (Sec. 2). The users that do not participate also maintain $\theta_U$ by Non-Interference (Sec. 2). By induction, the first claim is true. We also claim that for a sufficiently large neighbourhood—say $A^+$—the converse is also true. Intuitively, $A^+$ is large enough to represent each equivalence class imposed by both the smart contract and $\theta_U$, along with an arbitrary
user under interference. Our key insight is that the reachable control states of the local bundle form an inductive invariant $\theta_C$. If the local bundle is safe, then the interference relation is applied after each transition, and, therefore, the local bundle considers every pair of control and user states $(c, u)$ such that $c \in \theta_C$ and $(c, u) \in \theta_U$. Therefore, the safety of the local bundle implies Initialization, Consecution, and Non-Interference. This discussion justifies Theorem 2.

**Theorem 2.** Let $C$ be a contract, $G$ be a PTG for $C$, $\theta_U$ be a candidate split interference invariant with role guards $A_R$ and literal guards $A_L$, $A$ be a saturating neighbourhood for $(A_R, A_L, G)$, $a \in \mathbb{N} \setminus A$, and $A^+ = \{a\} \cup A$. Then, $\text{local}(C, A^+, \theta_U) \models \theta_U$ if and only if $\theta_U$ is an interference invariant for $C$.

Next, we present our main result: a sound proof rule for $k$-universal safety. As in Theorem 2, Theorem 3 uses a saturating neighbourhood $A^+$. This proof rule proves inductiveness, rather than compositionality, so $A^+$ does not require an arbitrary user under interference. However, a $k$-universal property can distinguish between $k$ users at once. Thus, $A^+$ must have at least $k$ arbitrary representatives.

**Theorem 3.** Let $\varphi$ be a $k$-universal safety property with role guards $A_R$ and literal guards $A_L$, $C$ be a contract, $\theta_U$ be an interference invariant for $C$, $G$ be a PTG for $C$, $A = A_{\text{Exp}} \cup A_{\text{Trans}} \cup A_{\text{Impl}}$ be a saturating neighbourhood for $(A_R, A_L, G)$. Define $A^+ \subseteq \mathbb{N}$ such that $A \subseteq A^+$ and $|A^+| = |A| + \max(0, k - |A_{\text{Exp}}|)$. If $\text{local}(C, A^+, \theta_U) \models \varphi$, then $\forall N \in \mathbb{N} \cdot \text{lts}(C, N) \models \varphi$.

Theorem 3 completes Ex. 2. Recall $(\varphi_1, \theta_1, A_1)$ from Ex. 3. Since $\varphi_1$ is 1-universal and $A_1$ has one explicit representative, it follows that $A^+ = A_1 \cup \emptyset$. Using an SMC, $\text{local}(C, A^+, \theta_1) \models \varphi_1$ is certified by an inductive strengthening $\theta_1^*$. Then by Theorem 3, $C$ is also safe for $2^{160}$ users. Both the local and global bundle have states exponential in the number of users. However, the local bundle has 4 users (a constant fixed by $C$), whereas the global bundle is defined for any number of users. This achieves an exponential state reduction with respect to the network size. Even more remarkably, $\theta_1^*$ must be the inductive invariant from Sec. 2, as it summarizes the safe control states that are closed under the interference of $\theta_1$. Therefore, we have achieved an exponential speedup in verification and have automated the discovery of an inductive invariant.

### 6 Implementation and Evaluation

We implement smart contract PCMC as an open-source tool called **SMARTACE**, that is built upon the Solidity compiler. It works in the following automated steps: (1) consume a Solidity smart contract and its interference invariants; (2) validate the contract’s conformance to MiniSol; (3) perform source-code analysis and transformation (i.e., inheritance inlining, devirtualization, PTGBuilder); (4) generate a local bundle in LLVM IR; (5) verify the bundle using SeaHorn [15].

In this section, we report on the effectiveness of SMARTACE in verifying real-world smart contracts. A full description of the SMARTACE architecture and of
Table 1: Experimental results for SMARTACE. All reported times are in seconds.

| Name       | Prop. LOC | SmartACE | VerX |
|------------|-----------|----------|------|
|            | Time      | Inv. Size | Users | Time |
| Alchemist  | 3 401     | 7         | 0     | 7    | 29  |
| ERC20      | 9 599     | 12        | 1     | 5    | 158 |
| Melon      | 16 462    | 30        | 0     | 7    | 408 |
| MRV        | 5 868     | 2         | 0     | 7    | 887 |
| Overview   | 4 66      | 4         | 0     | 8    | 211 |
| PolicyPal  | 4 815     | 26        | 0     | 8    | 20,773 |
| Zebi       | 5 1,209   | 8         | 0     | 7    | 77  |
| Zilliqa    | 5 377     | 8         | 0     | 7    | 94  |
| Brickblock | 6 549     | 13        | 0     | 10   | 191 |
| Crowdsale  | 9 1,198   | 223       | 0     | 8    | 261 |
| ICO        | 8 650     | 371       | 0     | 16   | 6,817 |
| VUToken    | 5 1,120   | 19        | 0     | 10   | 715 |
| Mana       | 4 885     | —         | —     | —    | 41,409 |
| Fund       | 2 38      | 1         | 0     | 6    | —   |
| Auction    | 1 42      | 1         | 1     | 5    | —   |
| QSPStaking | 4 1,550   | 3         | 7     | 8    | —   |

Benchmarks and Setup. To answer the above research questions, we used a benchmark of 89 properties across 15 smart contracts (see Tab. 1). Contracts Alchemist to Mana are from VerX [34]. Contracts Fund and Auction were added to offset the lack of parameterized properties in existing benchmarks. The QSPStaking contract comprises the Quantstamp Assurance Protocol\[10\] for which we checked real-world properties provided by Quantstamp. Some properties require additional instrumentation techniques (i.e., temporal [34] and aggregate [17] properties). Aggregate properties allow SMARTACE to reason about the sum of all records within a mapping. In Tab. 1, Inv. Size is the clause size of an interference invariant manually provided to SMARTACE and Users is the maximum number of users requested by PTGBuilder. All experiments were run on an Intel® Core i7® CPU @ 2.8GHz 4-core machine with 16GB of RAM on Ubuntu 18.04.

RQ1: Compliance. To assess if the restrictions of MiniSol are reasonable, we find the number of compliant VerX benchmarks. We found that 8 out of 13 bench-
marks are compliant after removing dead code. With manual abstraction, 4 more benchmarks complied. Brickblock uses inline assembly to revert transactions with smart contract senders. We remove the assembly as an over-approximation. To support Crowdsale, we manually resolve dynamic calls not supported by SMARTACE. In ICO, calls are made to arbitrary contracts (by address). However, these calls adhere to effectively external callback freedom [12,34] and can be omitted. Also, ICO uses dynamic allocation, but the allocation is performed once. We inline the first allocation, and assert that all other allocations are unreachable. To support VRFToken, we replace a dynamic array of bounded size with variables corresponding to each element of the array. The function _calcTokenAmount iterates over the array, so we specialize each call (i.e., _calcTokenAmount_{1,2,3,4}) to eliminate recursion. Two other functions displayed unbounded behaviour (i.e., massTransfer and addManyToWhitelist), but are used to sequence calls to other functions, and do not impact reachability. We conclude that the restrictions of MiniSol are reasonable.

**RQ2: Effectiveness.** To assess the effectiveness of SMARTACE, we determined the number of properties verified from compliant VERX contracts. We found that all properties could be verified, but also discovered that most properties were not parameterized. To validate SMARTACE with parameterized properties, we conducted a second study using auction, as described on our development blog\(^{11}\). To validate SMARTACE in the context of large-scale contract development, we performed a third study using QSPStaking. In this study, 4 properties were selected at random, from a specification provided by Quantstamp, and validated. It required 2 person days to model the environment, and 1 person day to discover an interference invariant. The major overhead in modeling the environment came from manual abstraction of unbounded arrays. The discovery of an interference invariant and array abstractions were semi-automatic, and aided by counterexamples from SEAHORN. For example, one invariant used in our abstraction says that all elements in the array powersOf100 must be non-zero. This invariant was derived from a counterexample in which 0 was read spuriously from powersOf100, resulting in a division-by-zero error. We conclude that SMARTACE is suitable for high-assurance contracts, and with proper automation, can be integrated into contract development.

**RQ3: Performance.** To evaluate the performance of SMARTACE, we compared its verification time to the reported time of VERX, a state-of-the-art, semi-automated verification tool. Note that in VERX, predicate abstractions must be provided manually, whereas SMARTACE automates this step. VERX was evaluated on a faster processor (3.4GHz) with more RAM (64GB)\(^{12}\). In each case, SMARTACE significantly outperformed VERX, achieving a speedup of at least 10x for all but 2 contracts\(^{13}\). One advantage of SMARTACE is that it benefits

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\(^{11}\) [http://seahorn.github.io/blog/](http://seahorn.github.io/blog/)

\(^{12}\) We have requested access to VERX and are awaiting a response.

\(^{13}\) We compare the average time for VERX to the total evaluation time for SMARTACE.
from state-of-the-art software model checkers, whereas the design of \textsc{VerX} requires implementing a new verification tool. In addition, we suspect that local bundle abstractions obtained through smart contract PCMC are easier to reason about than the global arrays that \textsc{VerX} must quantify over. However, a complete explanation for the performance improvements of \textsc{SmartACE} is challenging without access to the source code of \textsc{VerX}. We observe that one bottleneck for \textsc{SmartACE} is the number of users (which extends the state space). A more precise PTGBuilder would reduce the number of users. Upon manual inspection of \textsc{Melon} and \textsc{Alchemist} (in a single bundle), we found that user state could be reduced by 28%. We conclude that \textsc{SmartACE} can scale.

7 Related Work

In recent years, the program analysis community has developed many tools for smart contract analysis. These tools range from dynamic analysis \cite{19,42} to static analysis \cite{27,30,26,39,13,32,25,40,5} and verification \cite{21,17,41,29,34,38}. The latter are most related to \textsc{SmartACE} since their focus is on functional correctness, as opposed to generic rules (e.g., the absence of reentrancy \cite{14} and integer overflows). Existing techniques for functional correctness are either deductive, and require that most invariants be provided manually (i.e., \cite{17,41}), or are automated but neglect the parameterized nature of smart contracts (i.e., \cite{28,29,34,38}). The tools that do acknowledge parameterization employ static analysis \cite{25,5}. In contrast, \textsc{SmartACE} uses a novel local reasoning technique that verifies parameterized safety properties with less human guidance than deductive techniques.

More generally, parameterized systems form a rich field of research, as outlined in \cite{4}. The use of SCUNs was first proposed in \cite{11}, and many other models exist for both synchronous and asynchronous systems (e.g., \cite{9,36,37}). The approach of PCMC is not the only compositional solution for parameterized verification. For instance, environmental abstraction \cite{6} considers a process and its environment, similar to the inductive and interference invariants of \textsc{SmartACE}. Other approaches \cite{35,10} generalize from small instances through the use of ranking functions. The combination of abstract domains and SMPs has also proven useful in finding parameterized invariants \cite{2}. The addresses used in our analysis are similar to the scalar sets of \cite{18}. Most compositional techniques require cutoff analysis—considering network instances up to a given size \cite{7,20,22}. Local bundles avoid explicit cutoff analysis by simulating all smaller instances, and is similar to existing work on bounded parameterized model checking \cite{8}. \textsc{SmartACE} is the first application of PCMC in the context of smart contracts.

8 Conclusions

In this paper, we present a new verification approach for Solidity smart contracts. Unlike many of the existing approaches, we automatically reason about smart contracts relative to all of their clients and across multiple transactions. Our
approach is based on treating smart contracts as a parameterized system and using Parameterized Compositional Model Checking (PCMC).

Our main theoretical contribution is to show that PCMC offers an exponential reduction for k-universal safety verification of smart contracts. That is, verification of safety properties with k arbitrary clients.

The theoretical results of this paper are implemented in an automated Solidity verification tool SMARTACE. SMARTACE is built upon a novel model for smart contracts, in which users are processes and communication is explicit. In this model, communication is over-approximated by static analysis, and the results are sufficient to find all local neighbourhoods, as required by PCMC. The underlying parameterized verification task is reduced to sequential Software Model Checking. In SMARTACE, we use the SEAHORN verification framework for the underlying analysis. However, other Software Model Checkers can potentially be used as well.

Our approach is almost completely automated – SMARTACE automatically infers the necessary predicates, inductive invariants, and transaction summaries. The only requirement from the user is to provide an occasional interference invariant (that is validated by SMARTACE). However, we believe that this step can be automated as well through reduction to satisfiability of Constrained Horn Clauses. We leave exploring this to future work.

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A The Auction LTS

In this section we formally define the bundle of $C := \text{Auction}$, from Fig. 1. We no longer assume that $C$ consists of a single transaction. This means that each input must also specify the name of the function to execute. In the case of $C$, the function name are defined by $\mathcal{T} := \{\text{constructor}, \text{bid}, \text{withdraw}, \text{stop}\}$.

The control states are $\text{control}(C, A) \subseteq (A \times \mathbb{D}) \times (A \times \mathbb{D})$, where $A$ is the address space. For each state $(a_1, d_1, d_2, \text{aux}_1) \in \text{control}(C, A)$, $a_1$, $d_1$, and $d_2$ correspond to the manager, leadingBid, and stopped, respectively. An auxiliary variable, $\text{aux}_1$, is added to indicate whether the constructor has been called. When $\text{aux}_1$ is 0, then the constructor has not been called and $\text{msg.sender}$ must be called in the next transaction. When $\text{aux}_1$ is 1, then the constructor has been called, and may not be called again. For simplicity, $\text{data}(s, 2) = \text{aux}_1$.

The user states are $\text{user}(C, A) \subseteq A \times \mathbb{D})$. For each user $(x, y_1) \in \text{control}(C, A)$, $x$ is the user’s address and $y_1$ is the user’s bid.

The inputs are $\text{action}(C, A) \subseteq \mathcal{T} \times (A \times \mathbb{D})$, where $\mathcal{T}$ is the transactional semantics for $A$. For any $p = (t, x_1, x_2, y_1) \in \text{action}(C, A)$, the interpretation of $p$ depends on $t$. In all cases, $x_1 \in \text{control}(C, A)$. If $t = \text{constructor}$, then $x_2$ represents $\text{mgr}$. If $t = \text{bid}$, then $y_1$ represents $\text{amount}$. In all other cases, $x_2$ and $y_1$ are unused.

For the rest of this section, we restrict our discussion to $\text{lt}(C, N)$. This means that $A = [N]$, and that $\mathcal{M}$ is the identity function for $[N]$. We now present transactional semantics for $f = [\mathcal{C}]_{\mathcal{M}}$. We assume that $N \geq 2$, so that the zero-account and smart contract account are defined.

\[
\begin{align*}
f((s, u), (t, x_1, x_2, y_1)) &= \begin{cases} (s, u) & \text{if } x_1 = \text{id}(u_0) \\
(s, u) & \text{if } x_1 = \text{id}(u_1) \\
g_1((s, u), (x_1, x_2)) & \text{if } t = \text{constructor} \\
g_2((s, u), (x_1, y_1)) & \text{if } t = \text{bid} \\
g_3((s, u), (x_1)) & \text{if } t = \text{withdraw} \\
g_4((s, u), (x_1)) & \text{if } t = \text{stop} \end{cases} \\
g_1((s, u), (x_1, x_2)) &= \begin{cases} (s, u) & \text{if } \text{data}(s, 2) \neq 0 \\
(s', u) & \text{else, s.t. } \text{role}(s', 0) = x_2, \text{data}(s', 0) = \text{data}(s, 0), \\
 & \text{data}(s', 1) = \text{data}(s, 1), \text{and } \text{data}(s', 2) = 1 \end{cases} \\
g_2((s, u), (x_1, y_1)) &= \begin{cases} (s, u) & \text{if } \text{data}(s, 2) \neq 1 \\
(s, u) & \text{if } \text{role}(s, 0) = x_1 \\
(s, u) & \text{if } \text{data}(s, 0) \geq y_1 \\
(s, u) & \text{if } \text{data}(s, 1) = 1 \\
(s, u') & \text{else, s.t. } \forall i \in N : \text{id}(u'_i) = \text{id}(u_i) \land i \neq x_1 \Rightarrow \\
 & \text{map}(u'_i) = \text{data}(u_i) \land i = x_1 \Rightarrow \text{map}(u'_i) = (y_1) \end{cases} \end{align*}
\]
Recall from Sec. 3 that a reverted transaction (such as a failed require statement) is treated as a no-op. In \( f \), these no-ops correspond to the cases that send \((s, u)\) to \((s, u)\). Most of these cases, such as the second case of \( g_2 \), are derived directly from the source code (line 10 for this case). However, the first two cases for \( f \), along with the first case for each \( g_i \), are special.

The first case for \( f \) guards against transactions sent from the zero-account. An important observation is that the sender, \( x_1 \), is compared to \( \text{id}(u_0) \), rather than 0. This is because \( M \) is used to map all literal addresses to users. For the first case of \( f \), \( M \) first maps \( 0 \) to \( u_0 \), and then \( \text{id}(u_0) \) maps \( u_0 \) back to \( 0 \)^14. Note that the second case of \( f \) is similar to the first case, in that the second case guards against transactions from the smart contract account.

The first case of \( g_1 \) guards against calls to constructor after the constructor has already the called. Similarly, the first case of \( g_2, g_3, \) and \( g_4 \), each guard against calls to non-constructor functions before the constructor has been called. These cases ensure that the contract is constructed once and only once.

## B MiniSol: Syntax and Semantics

MiniSol is an extension of MicroSol that provides more communication primitives. Specifically, MiniSol introduces: transactions with multiple calls; a currency called Ether; a built-in time primitive. The grammar for MiniSol is presented in Fig. 6. As in MicroSol, the address type in MiniSol is non-arithmetic, and mappings must relate addresses to integers. Note, however, that MiniSol does allow for multi-dimensional mappings. Throughout the section, we illustrate MiniSol using the MiniSol program in Fig. 7.

In MiniSol, a transaction consists of one or more user calls. As in MicroSol, the first call is performed by a user that is neither the zero-account nor a smart contract account. Each subsequent call is made by the account of the smart contract currently executing a functions (e.g., line 48 in Fig. 7). As in most object-oriented languages, each function has a visibility modifier (see \( \langle \text{Vis} \rangle \) in

^14 This indirection allows for users to be readdressed in Def. 1
Fig. 6: The formal grammar of the MiniSol language.

Fig. 7: A MiniSol program that allows a FundManager to raise funds inside a Fund. The funds are forwarded to a predetermined destination after a duration.

Fig. 6). Functions with a public visibility can be called by users (e.g., line 12 in Fig. 7), whereas functions with an internal visibility can only be executed as part
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| Expression | Interpretation |
|------------|----------------|
| role(s, i) | The value of the i-th address-typed state variable. |
| data(s, 0) | The current value of block.number. |
| data(s, 1) | The current value of block.timestamp. |
| data(s, 2 + i) | The value of the i-th numeric-typed state variable. |
| id(u_i) | The address of the i-th user. |
| map(u_i)_0 | The balance of the i-th user. |
| map(u_i)_1+j | The j-th mapping value of the i-th user. |
| client(p, 0) | The value of tx.origin. |
| client(p, 1) | The value of msg.sender. |
| client(p, 2 + i) | The value of the i-th address-typed input variable. |
| arg(p, 0) | The value of msg.value. |
| arg(p, 1 + i) | The value of the i-th numeric-typed input variable. |

Table 2: The interpretation of a control state s, a user configuration u, and an action p in MiniSol.

of an ongoing call (e.g., the function at line 8 called at line 5 in Fig. 7). In either case, the inputs tx.origin and msg.sender expose the addresses of the first and most recent user to call a function during the current transaction, respectively.

A currency called Ether is integrated into the MiniSol language. Each user has a balance of Ether, that is exposed through the built-in balance function (e.g., line 14 in Fig. 7). A user that is neither the zero-account nor a smart contract account can spontaneously acquire Ether through minting. A smart contract account can receive Ether through a function marked payable (e.g., line 4 in Fig. 7). The amount of Ether sent to a payable function is accessible through the msg.value input (e.g., line 9 in Fig. 7). There are two ways that one smart contract can send Ether to another smart contract. If a function is called by name, then the .value modifier is used (e.g., line 48 in Fig. 7). Otherwise, a built-in transfer function (e.g., line 43 in Fig. 7) can be used to call a special unnamed function, called a fallback function (e.g., line 4 in Fig. 7).

MiniSol also exposes a built-in time primitive to each function. In MiniSol, time is measured with respect to the number of transactions that have been executed. Transactions are executed in batches, known as blocks. The block.number input exposes the number of blocks executed so far. The block.timestamp input exposes the real-world time at which the last block was executed (e.g., line 42 in Fig. 7). MiniSol enforces that block.timestamp is non-decreasing.

The semantics of a MiniSol program are similar to those of a MicroSol program. For a MiniSol program C with transaction tr, the transition function [C]_M is still determined by the (usual) semantics of the single transaction tr. However, the state space and actions of each MiniSol program also include information about the first caller of each transaction, Ether, and time. The changes are outlined in Tab. 2.
C Uniformity and Participation

In this section we show that MicroSol (and MiniSol) satisfy the uniformity assumptions of PCMC given in Sec. 2 (i.e., there are finitely many finite neighbourhoods). First, we show that if the set of implicit users is finite, then each transaction of the network is performed against a neighbourhood of bounded size. Second, we show that the users within the given neighbourhood are interchangeable (up to address). Finally, we show that if all transactions can be reduced to a finite set of addresses. From these results, it follows that all MicroSol transactions are uniform.

C.1 Reduction to a Bounded Neighbourhood

Let $C$ be a contract. This section proves that if a PT over-approximation for $C$ is finite, then every transaction of $C$ can be computed by projecting the network onto a neighbourhood of bounded size. To do this, a precise definition must first be given of an over-approximation and a projection. The notion of over-approximation generalizes a PTG from Sec. 4.

**Definition 9 (PT Over-Approximation).** For $N \in \mathbb{N}$ and $p \in \text{action}(C, [N])$, the tuple $(E, T, I) \subseteq \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ over-approximates the PT, $pt(C, N, p) = (\text{Explicit}, \text{Transient}, \text{Implicit})$, when $\{i \mid \text{Explicit}(i, x)\} \subseteq E$, $\{i \mid \text{Transient}(i, x)\} \subseteq T$, and $\text{Implicit} \subseteq I$. Furthermore, $(E, T, I)$ is a PT over-approximation of $C$ when $(E, T, I)$ over-approximates all PT's of $C$.

Note that when a PT Over-Approximation is finite, the over-approximation corresponds to a \textsc{PTGBuilder} PTG. This is because each label in a \textsc{PTGBuilder} PTG corresponds to precisely one user, with a specific address. Therefore, the users that appear in a PT over-approximate also over-approximate the local neighbourhood of a transaction.

**Definition 10 (View).** Let $(E, T, I)$ be a PT over-approximation of $C$, $s \in \text{control}(C, [N])$, and $p \in \text{action}(C, [N])$. The $(E, T, I)$-view for $s$ and $p$ is:

$$\text{view}(E, T, I, s, p) := \{\text{client}(p, i) \mid i \in E\} \cup \{\text{role}(s, i) \mid i \in T\} \cup I$$

In Sec. 5.2 we use sets of addresses to represent local neighbourhoods. Intuitively, each address corresponds to the user (or a representative of the user) with the given address. In other words, the users in a bundle are projected onto the neighbourhood, as defined below in Def. 11.

**Definition 11 (Projection).** Let $A_2 \subseteq A_1 \subseteq \mathbb{N}$, $N = |A_1|$, $M = |A_2|$, and $u \in \text{user}(C, A_1)^N$. Then $v \in \text{user}(C, A_2)^M$ is a projection of $u$ onto $A_2$, written $v = \pi_{A_2}(u)$, if there exists a total, injective, order-preserving mapping, $\sigma[M] \rightarrow [N]$ such that $A_2 = \{\text{id}(v_i) \mid i \in [M]\}$ and $\forall i \in [M] \cdot v_i = u_{\sigma(i)}$. 
Notice that as $\sigma$ in Def. 11 is order-preserving, each projection is unique (with respect to $A$). This means that projections can be compared directly. The following theorem shows that each transaction can be reduced to a neighbourhood projection of finitely many clients.

**Theorem 4.** Let $(E,T,I)$ be a PT over-approximation of $C$ with $N_0 = |E| + |T| + |I|$ finite. Then for all $N \geq N_0$ with $(S,P,f,s_0) = \text{its}(C,N)$, and for all $(s,u) \in S$, $p \in P$, if $(s',u') = f(s,u,p)$ and $A = \text{view}(E,T,I,s,p)$, then $\forall i \in [N] \cdot \text{id}(u_i) \notin A \Rightarrow u_i = u_i'$ and $(s',\pi_A(u')) = \llbracket tr \rrbracket_{N_0} (s',\pi_A(u),p)$.

**Proof.** Theorem 4 draws two conclusions. (1) The first conclusion claims that $\forall i \in [N] \cdot \text{id}(u_i) \notin A \Rightarrow u_i = u_i'$. When $\text{id}(u_i) \in A$, this holds trivially. Assume that $\text{id}(u_i) \notin A$. Then $u_i$ is not a participant. Then $f_p$ has no influence on $u_i$. By definition, $u_i = u_i'$, as desired. (2) The second conclusion claims that $(s',\pi_A(u')) = \llbracket tr \rrbracket_{N_0} (s',\pi_A(u),p)$. By construction, all participants of $f_p$ are in $A$. Then $\pi_A(u)$ contains all clients which influence $f_p$, while $\pi_A(u')$ contains all clients which are influenced by $f_p$. In addition, $\llbracket tr \rrbracket$ is the semantic interpretation of $tr$. The source text of $tr$ is defined independent from the size of the network, therefore $\llbracket tr \rrbracket_N$ and $\llbracket tr \rrbracket_{N_0}$ must perform the same sequence of synchronized and internal actions against the same set of clients.

### C.2 The Interchangeability of Users

Let $C$ be a contract. This section shows that arbitrary users in $C$ are interchangeable. We rely on the MicroSol restriction that address values are not numeric.

To discuss the interchangeability of users, we require a notion of swapping users. Let $A$ be an address space, $u,u' \in \text{user}(C,A)^N$, and fix $x,y \in A$. Then $u'$ is an $(x,y)$ address swap of $u$, written $\text{swap}(u,x,y)$, if $\forall i \in [N]$: (1) $\text{id}(u_i) = y \Rightarrow \text{id}(u'_i) = x$; (2) $\text{id}(u_i) = x \Rightarrow \text{id}(u'_i) = y$; (3) $\text{id}(u_i) \notin \{x,y\} \Rightarrow u'_i = u_i$; (4) $\text{map}(u'_i) = \text{map}(u_i)$. A similar notion is defined for the roles of a control process. Specifically, let $s,s' \in \text{control}(C,A)$. Then, $s'$ is an $(x,y)$ address swap of $s$, if $\forall i \in [N]$: (1) $\text{role}(s,i) = y \Rightarrow \text{role}(s',i) = x$; (2) $\text{role}(s,i) = x \Rightarrow \text{role}(s',i) = y$; (3) $\text{role}(s,i) \notin \{x,y\} \Rightarrow \text{role}(s',i) = \text{role}(s,i)$. The extensions to action$(C,A)$ and control$(C,A) \times \text{user}(C,A)^N$ are trivial.

**Lemma 1.** Let $C$ be a contract, $N \in \mathbb{N}$, $(S,P,f,s_0) = \text{its}(C,N)$, $p \in P$, and $pt(C,N,p) = (\text{Explicit}, \text{Transient}, \text{Implicit})$. For every $s,t \in S$ and every $x,y \in \mathbb{N} \setminus \text{Implicit}$, $f(\text{swap}(t,x,y),\text{swap}(p,x,y)) = \text{swap}(f(s,p),x,y)$.

Lemma 1 says that address swaps commute with MicroSol transactions. The lemma follows directly from the semantics of MicroSol, since MicroSol restricts address comparisons to (dis)equality.

Lemma 1 extends to address bijections. If $A_1,A_2 \subseteq \mathbb{N} \setminus \text{Implicit}$ are finite, transactions are also preserved under any address bijection $\sigma : A_1 \rightarrow A_2$. Intuitively, $\sigma$ is constructed using a sequence of swaps from $A_1$ to $A_2$. The definition of $\sigma$ is inductive, with respect to the size of $A_1$. 
Theorem 5. Let $\mathcal{C}$ be a contract, $N \in \mathbb{N}$, $(S, P, f, s_0) = \text{its}(\mathcal{C}, N)$, $p \in P$, $pt(\mathcal{C}, N, p) = (\text{Explicit}, \text{Transient}, \text{Implicit})$, and $A_1, A_2 \subseteq \mathbb{N} \setminus \text{Implicit}$ be finite. If $\sigma : A_1 \to A_2$ is bijective, then $f_{\sigma(p)}(\sigma(s)) = \sigma^{-1}(f_p(s))$.

Proof. This follows by induction over the size of $A_1$. As $\sigma$ is a correspondence, then $|A_1| = |A_2|$. In the base case, $|A_1| = 1$, $A_1 = \{a_1\}$ and $A_2 = \{a_2\}$ with $a_1 \not\in \text{Implicit}$ and $a_2 \not\in \text{Implicit}$. Then $\sigma(s) = \sigma^{-1}(s) = \text{swap}(s, a_1, a_2)$ and $\sigma(p) = \sigma^{-1}(p) = \text{swap}(p, a_1, a_2)$. Now assume that the inductive argument holds up to $k \geq 1$, and that $|A_1| = k + 1$. Then fix some $a_1 \in A_1$, and let $A_1' = A_1 \setminus \{a\}$ and $A_2' = A_1 \setminus \{\sigma(a)\}$. Clearly $\tau : A_1' \to A_2'$, as defined by $\tau(x) = \sigma(x)$ is also a correspondence. Then $\sigma(s) = \text{swap}(\tau(s), a, \sigma(a))$, $\sigma^{-1}(s) = \text{swap}(\tau^{-1}(s), a, \sigma(a))$, $\sigma(p) = \text{swap}(\tau(p), a, \sigma(a))$, and $\sigma^{-1}(p) = \text{swap}(\tau^{-1}(p), a, \sigma(a))$. By the inductive hypothesis, $\tau$ and $\tau^{-1}$ are also reducible to sequences of swaps. \hfill \Box

C.3 Reduction to a Finite Address Set

Let $\mathcal{C}$ be an contract. This section proves that if the assumptions of Theorem 4 holds for $\mathcal{C}$, then by the results of Theorem 5, the state space of each neighbourhood can be reduced to a finite state space. Recall that all non-address state is already over a finite domain.

Theorem 6. Let $(E, T, I)$ over-approximate PT’s of $\mathcal{C}$ with $N_0 = |E| + |T| + |I|$ finite and let $I \subseteq \mathcal{A}_0 \subseteq \mathcal{A}^* \subseteq \mathbb{N}$ with $|\mathcal{A}_0| = N_0$. Then for all $s \in \text{control}(\mathcal{C}, \mathcal{A}^*)$, and for all $p \in \text{action}(\mathcal{C}, \mathcal{A}^*)$, with $A = \text{view}(E, T, I, s, p)$, there exists a correspondence $\sigma : (\mathcal{A} \setminus I) \to (\mathcal{A}_0 \setminus I)$ such that $f_p(s, u) = \sigma^{-1}(f_{\sigma(p)}(\sigma(s), \sigma(u)))$.

Proof. This proof follows in two parts. First, it is shown that $\sigma$ exists. This follows quite trivial. By construction, $|A| = N_0$, $|\mathcal{A}_0| = N_0$, $I \subseteq \mathcal{A}$, and $I \subseteq \mathcal{A}_0$, therefore $|A \setminus I| = |\mathcal{A}_0 \setminus I|$. Then a correspondence $\sigma : A \to A_0$ can be constructed arbitrarily. As $\sigma$ does not map to or from $I$, Theorem 5 applies to $\sigma$. As $I$ is finite, Theorem 4 also applies. The conclusion of Theorem 6 follows as a direct consequence. \hfill \Box

From Theorem 6, every MicroSol transaction can be simulated using a finite address space. By Theorem 5, this simulation also preserves $k$-universal safety properties. This means that there are finitely many distinguishable neighbourhoods, and that these neighbourhoods are subsumed by $\mathcal{A}$. Therefore, all MicroSol transactions are uniform, as desired.