Spin-dependent electron transport through a parallel double-quantum-dot structure

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Electron transport properties in a parallel double-quantum-dot structure with three-terminals are theoretically studied. By introducing a local Rashba spin-orbit coupling, we find that an incident electron from one terminal can select a specific terminal to depart from the quantum dots according to its spin state. As a result, spin polarization and spin separation can be simultaneously realized in this structure. And spin polarizations in different terminals can be inverted by tuning the structure parameters. The underlying quantum interference that gives rise to such a result is analyzed in the language of Feynman paths for the electron transmission.

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I. INTRODUCTION

Spintronics is one of the most attractive investigation frontiers for both theoretical and experimental aspects due to the potential application of nanodevices, in which generating and controlling a spin-polarized current through a mesoscopic system is the major issue as demonstrated in a variety of relevant works$^{[1, 2]}$. One of the feasible techniques to achieve spin-polarized current is electrical spin injection, but the efficiency through ferromagnetic/ nonmagnetic semiconductor junctions is usually small due to the conductivity mismatch$^{[3, 4, 5]}$. An alternative method is the optical spin orientation technique$^{[6]}$, which is difficult to integrate with electronic devices.

Spin-orbit (SO) coupling is an important mechanism that influences the electron spin state, since it couples the spin degree of freedom of an electron to its orbital motion and vice versa. In particular, in low-dimensional structures Rashba SO interaction comes into play by exerting an electric potential to destroy the symmetry of space inversion in an arbitrary spatial direction$^{[7, 8, 9, 10]}$. Thus, based on the properties of Rashba interaction, electric control and manipulation of the spin state is feasible. Since Datta and Das proposed the concept of spin-field-effect transistor$^{[11]}$, effects of SO interaction on the electronic transport properties have been paid much attention$^{[12, 13, 14, 15, 16]}$. During the past two decades, a great number of studies have been devoted to improve the efficiency of spin polarization in the transport system based on the SO interaction but not under magnetic field or coupled FM leads$^{[1]}$. In the spin Hall devices, it has been observed that the pure spin current is induced in the transverse direction by the SO interaction by applying a longitudinal electric field in the two-dimensional electron (hole) system$^{[12, 13]}$. Very recently, Rashba interaction has been introduced to quantum dot (QD) systems, e.g., the Rashba interaction is locally applied to one QD in one arm of an Aharonov-Bohm (AB) interferometer$^{[10, 14, 16]}$. In these structures, with the interplay of the magnetic field and Rasha SO interaction, remarkable spin polarization comes into being during the electron transport process.

In the present work, we introduce Rashba SO interaction to act locally on one QD of a double QD AB ring, in which an additional lead is laterally coupled to another QD. In comparison with a single QD structure, such double QDs have more tunable parameters, and provide more Feynman paths for electron transmission. Furthermore, Rashba interaction can bring about the spin-dependent phase. Accordingly, it is expected that quantum interference in such a double QD structure can give rise to the spin-related electronic transport properties. The most interesting result we obtain is that an incident electron from one terminal can select a specific terminal to depart from the QDs according to its spin state. In other words, such a structure can realize the functions of spin polarization and spin separation simultaneously. And these functions can be tuned by the structure parameters. We find that the Rashba interaction and the terminal triplet play the crucial roles in creating such a feature.

The rest of the paper is organized as follows. In Sec. II, the electron Hamiltonian of second-quantization including the Rashba SO interaction in the double-QD structure is introduced first. Then a formula for linear conductance is derived by means of the nonequilibrium Green function technique. In Sec. III, the calculated results about the conductance spectra are shown. Then a discussion concerning the spin polarization and separation is given. Finally, the main results are summarized in Sec. IV.
II. MODEL

The parallel double QD structure that we consider is illustrated in Fig. 1. Apart from the left and right leads, another lead is applied to couple laterally to QD-2. A gate voltage is applied on QD-1 to induce a local Rashba SO interaction. The Hamiltonian to describe the electronic motion in the structure reads

\[ H = H_C + H_D + H_T, \]

(1)

The first term is the Hamiltonian for the noninteracting electrons in three leads:

\[ H_C = \sum_{\sigma, k, \alpha} \varepsilon_{k\alpha} c_{k\alpha}^\dagger c_{k\alpha}, \]

(2)

where \( c_{k\alpha}^\dagger \) (\( c_{k\alpha} \)) is an operator to create (annihilate) an electron of the continuous state \( |k\alpha\rangle \) in the lead-\( \alpha \) (\( \alpha = L, R, D \)), and \( \varepsilon_{k\alpha} \) is the corresponding single-particle energy. \( \sigma = \uparrow, \downarrow \) denotes the spin index. The second term describes electron tunneling in double QDs, which takes a form as

\[ H_D = \sum_{j, \sigma} \varepsilon_j d_{j\sigma}^\dagger d_{j\sigma} + \sum_j U_j n_{jL} n_{jR}, \]

(3)

where \( d_{j\sigma}^\dagger \) (\( d_{j\sigma} \)) is the creation (annihilation) operator of an electron in QD-\( j \) (\( j = 1, 2 \)), \( \varepsilon_j \) denotes the electron level in the corresponding QD. \( U_j \) represents the intradot Coulomb repulsion, and the interdot electron interaction is ignored since it is usually much smaller than the intradot one due to the screening effect. The last term in the total Hamiltonian describes the electron tunneling between the leads and QDs, which is given by

\[ H_T = \sum_{\sigma, j, k} (V_{jL}\sigma d_{j\sigma}^\dagger c_{kL}\sigma + V_{jR}\sigma d_{j\sigma} c_{kR}\sigma) + \sum_{\sigma, k} V_{2D}\sigma d_{2\sigma}^\dagger c_{k2\sigma} + H.c., \]

(4)

where \( V_{j\sigma}\alpha \) denotes the QD-lead coupling coefficients. They take the forms as: \( V_{1L}\sigma = V_\Phi e^{i\phi_0/4+i\sigma\varphi} \), \( V_{1R}\sigma = V_\Phi e^{i\phi_0/4-i\sigma\varphi} \), \( V_{2L}\sigma = V_\Phi e^{i\phi_0/4} \), and \( V_{2D}\sigma = V_\Phi \). This implies that the couplings between the L and R leads and the QDs have the uniform strength \( V \) but different phases. The phase \( \phi \) is associated with the magnetic flux \( \Phi \) threading the system by a relation \( \phi = 2\pi\Phi/\Phi_0 \), where \( \Phi_0 = h/e \) is the flux quantum. Besides, the spin-dependent phase \( \sigma\varphi \) arises from the electron spin precession induced by the Rashba SO coupling.

Next we turn to discuss the electron transport through this structure. With the Green function technique, the current flow in lead-\( \beta \) can be written as

\[ J_\beta = \sum_{\sigma} J_{\beta\sigma} = \frac{e}{h} \sum_{\alpha\sigma} \int d\omega T_{\beta\alpha,\sigma}(\omega) [f_\beta(\omega) - f_\sigma(\omega)], \]

(5)

where \( T_{\beta\alpha,\sigma}(\omega) = \text{Tr} \left[ \Gamma^\beta_{\sigma}(\omega) \Gamma^\alpha_{\sigma} G_0(\omega) \right] \) is the transmission function, describing electron tunneling ability between lead-\( \beta \) and lead-\( \alpha \) via double QDs, and \( f_\sigma(\omega) \) is the Fermi distribution function in lead-\( \alpha \). The QD-lead coupling matrix \( \Gamma^\alpha_{\sigma}\beta \) is defined as \( \Gamma^\alpha_{\sigma}\beta = 2\pi V_{\alpha\beta}\sigma \rho_\alpha(\omega) \), and the matrix elements are constants considering the two-dimensional nature of the density of states \( \rho_\alpha(\omega) \) in lead-\( \alpha \). By defining \( \Gamma_0 = 2\pi |V|^2 \rho_\alpha(\omega) \), we can write these matrices as

\[ \Gamma^L_{\sigma}\beta = \Gamma_0 \left[ \begin{array}{cc} 1 & e^{i\varphi + i\sigma\varphi} \\ e^{-i\varphi - i\sigma\varphi} & 1 \end{array} \right], \]

\[ \Gamma^R_{\sigma}\beta = \Gamma_0 \left[ \begin{array}{cc} 1 & e^{-i\varphi - i\sigma\varphi} \\ e^{i\varphi + i\sigma\varphi} & 1 \end{array} \right], \]

(6)

and \( \Gamma^\beta_{\sigma}\gamma = \frac{1}{2} (\Gamma^L_{\sigma}\beta + \Gamma^R_{\sigma}\beta) \) denotes the average QD-lead coupling. \( \Gamma^\alpha_{\sigma}\beta \), and \( \Gamma^\alpha_{\beta}\sigma \), the retarded and advanced Green functions, obey the relationship \( [\Gamma^\alpha_{\sigma}\beta] = [\Gamma^\beta_{\sigma}\alpha]^\dagger \). And the matrix elements of \( \Gamma^\alpha_{\sigma}\beta \) are defined as \( [\Gamma^\alpha_{\sigma}\beta]_{ij}(\omega) = \int_{-\infty}^{\infty} \langle \{ d_{i\sigma}(t), d_{j\beta}^\dagger(t) \} \rangle e^{i\omega t} dt \) with \( \langle \{ d_{i\sigma}(t), d_{j\beta}^\dagger(t) \} \rangle = \delta(t) \). The chemical potential of lead-L \( \mu_L \) is considered as the zero point of energy of the system, denoted as \( \mu_L = 0 \). \( \mu_R \) and \( \mu_D \), the chemical potentials in other two leads, are fixed at \( \mu_R = \mu_D = \mu_L - \delta \). Thereby, the electron injects from lead-L and departs from the QDs via the other two leads. Thus there are two channels for the electronic tunneling in this structure, i.e., the \( L \rightarrow R \) channel and \( L \rightarrow D \) channel. When the electron transport is in the linear regime and at low temperature, the current flow defined in Eq. 5 reduces to \( J_{\beta\sigma} = \frac{e^2}{h} T_{\beta L,\sigma} \langle \omega = 0 \rangle \).

The current flow in lead-\( \beta \) is proportional to the linear conductances

\[ G_{\beta L, \sigma} = \frac{e^2}{h} T_{\beta L, \sigma}(\omega)|_{\omega = 0}, \]

(7)

where \( \beta = R \) or \( D \). It is obvious that the linear conductances are associated with the Green functions, which can be solved by means of the equation-of-motion method. By a straightforward derivation, we obtain the matrix form of retarded Green function

\[ [G^\alpha_{\sigma}\beta](\omega) = \left[ (z - \varepsilon_1) S_{1\sigma} + i \Gamma_0 \cos(\frac{\varphi}{2} + \sigma\varphi) + i \Gamma_0 \cos(\frac{\varphi}{2} - \sigma\varphi) \right]^{-1} \]

(8)

with \( z = \omega + i0^+ \) and \( \Gamma_D = [\Gamma^D_{\beta\sigma}]_{22} \). And \( S_{\sigma\sigma'} = \frac{-\varepsilon_2 - U_\sigma}{\varepsilon_2 - U_\sigma + \Gamma_D/2} \) accounts for the contribution of the Coulomb interaction up to the second-order approximation. The average electron occupation number in QD is determined by the relations \( \langle n_{j\sigma} \rangle = -\frac{1}{\pi} \int d\omega \text{Im}[G^\sigma_{\sigma\beta}]_{jj} \). So the Green function can be numerically resolved by iteration technique.
III. NUMERICAL RESULTS AND DISCUSSIONS

With the formulation developed in the previous section, we can perform the numerical calculation to investigate the linear conductance spectra of the two-channel AB interferometer. Prior to the calculation, we need to introduce a parameter $t_0$ as the units of energy.

First of all, we are concerned with the adjustment of a magnetic field on the linear conductances. We choose the coupling between QD-2 and lead-D $\Gamma_D = \Gamma_0 = 2t_0$, and consider the QD levels $\varepsilon_1 = \varepsilon_2 = 0$. In such a simple case, the Hamiltonian is independent of the electron spin, hence $G_{3L,\sigma} = G_{3L,\sigma} = G_{3L}$. Besides, the Hubbard $U$ does not affect the electron transport since $\varepsilon_j \sigma \psi_j = 0$ according to the treatment of the electron interaction given above. An interesting result shown in Fig.2(a) is that the two conductances, $G_{RL}$ and $G_{DL}$, do not vary with $\phi$ in phase. In particular, when $\phi = n\pi$, the magnetic phase being a multiple of $\pi$, the peak of one conductance ($G_{RL}$ or $G_{DL}$) just encounters the valley of another one.

When Rashba interaction comes into play, the linear conductance becomes spin dependent. In Fig.2(b) and (c) the linear conductances versus the magnetic phase $\phi$ are plotted in the absence of Rashba interaction. We choose the coupling between QD-2 and lead-$D$ $\Gamma_D = \Gamma_0 = 2t_0$. In Fig.2(b), the linear conductance becomes spin dependent. In order to perform the numerical calculation, we can rewrite the Green function

$$G^{r}_{11,\sigma} = \frac{g_{j\sigma}^{-1}}{g_{1\sigma}^{-1} + \Gamma_{12,\sigma}^{-1} \Gamma_{21,\sigma}^{-1}} = \sum_{j=0}^{\infty} g_1(-g_{1\sigma}g_{2\sigma} \Gamma_{12,\sigma} \Gamma_{21,\sigma})^j,$$

we can then express the transmission probability amplitude $t_{RL,\sigma}(1,1)$ as a summation of Feynman paths of all orders, i.e.,

$$t_{RL,\sigma}(1,1) = \sum_{j=0}^{\infty} V_{R1\sigma} g_{1\sigma}(-g_{1\sigma}g_{2\sigma} \Gamma_{12,\sigma} \Gamma_{21,\sigma})^j V_{L1\sigma} = \sum_{j=0}^{\infty} t_{RL,\sigma}^{(j)}(1,1).$$

Here $g_{1\sigma} = (z-\varepsilon_1+i\Gamma_0)^{-1}$ and $g_{2\sigma} = (z-\varepsilon_2+i\Gamma_0 + \frac{\Delta \Gamma}{2})^{-1}$. They are the Green functions of individual QD when another QD is removed from the structure. It is obvious that higher-order Feynman paths have more complicated forms. By the same token, we can expand $t_{RL,\sigma}(1,2)$ as a summation of Feynman paths, which is given by

$$t_{RL,\sigma}(1,2) = \sum_{j=1}^{\infty} iV_{R1\sigma}(-g_{1\sigma}g_{2\sigma} \Gamma_{12,\sigma})^j V_{L2\sigma} = \sum_{j=1}^{\infty} t_{RL,\sigma}^{(j)}(1,2).$$

Besides, the other transmission coefficients $t_{RL,\sigma}(2,2)$ and $t_{RL,\sigma}(2,1)$ have the similar expansions as $t_{RL,\sigma}(1,1)$ and $t_{RL,\sigma}(1,2)$. The lowest-order Feynman paths in above equations are $t_{RL,\sigma}^{(0)}(1,1) = V_{R1\sigma} g_{1\sigma} V_{L1\sigma}$ and $t_{RL,\sigma}^{(0)}(2,2) = V_{R2\sigma} g_{2\sigma} V_{L2\sigma}$. All Feynman paths contribute to the linear conductances coherently, the lowest-order ones are the leading terms, though.

As shown in Fig.2(a), the contribution of the zero-order paths has the similar oscillation to the exact linear conductance with the magnetic adjustment. Therefore, we can only take into account the zero-order paths, i.e., $t_{RL,\sigma}^{(0)}(1,1)$ and $t_{RL,\sigma}^{(0)}(2,2)$, to analyze the quantum interference that cause the spin-dependent electron transport property shown in Fig.2(b) and (c). The linear conductance almost oscillate in phase. At $\phi = (n + \frac{1}{2})\pi$, both reach their respective maxima. This means that around the points $\phi = (n + \frac{1}{2})\pi$ the incident electron from lead-L can select a specific lead to leave the QDs according to its spin state. Thus, we implement the spin polarization and spin separation simultaneously in such a structure at a specific magnetic field. In addition, by adjusting the magnetic field, the orientations of spin polarization in lead-R and lead-D can be just interchanged.

The underlying physics that gives rise to the spin dependent electron transport property shown in Fig.2 is quantum interference. In order to obtain an intuitive picture about the quantum interference, we analyze the electron transmission in the language of Feynman path. To do so, we need firstly to rewrite the electron transmission function in a form as $T_{RL,\sigma}(\omega) = \sum_{j,l=1}^{2} t_{RL,\sigma}(j,l)^2$, where the electron transmission probability amplitudes are defined as $t_{RL,\sigma}(j,l) = V_{Rj\sigma} G_{j\sigma} V_{Ll\sigma}$ with $V_{j\sigma\sigma} = V_{j\sigma\sigma} V_{j\sigma\sigma} \sqrt{2\pi \rho_j(\omega)}$. Following the expansion of the Green function

$$G^{r}_{11,\sigma} = \frac{g_{j\sigma}^{-1}}{g_{1\sigma}^{-1} + \Gamma_{12,\sigma}^{-1} \Gamma_{21,\sigma}^{-1}} = \sum_{j=0}^{\infty} g_1(-g_{1\sigma}g_{2\sigma} \Gamma_{12,\sigma} \Gamma_{21,\sigma})^j,$$

we can then express the transmission probability amplitude $t_{RL,\sigma}(1,1)$ as a summation of Feynman paths of all orders, i.e.,

$$t_{RL,\sigma}(1,1) = \sum_{j=0}^{\infty} V_{R1\sigma} g_{1\sigma}(-g_{1\sigma}g_{2\sigma} \Gamma_{12,\sigma} \Gamma_{21,\sigma})^j V_{L1\sigma} = \sum_{j=0}^{\infty} t_{RL,\sigma}^{(j)}(1,1).$$

Here $g_{1\sigma} = (z-\varepsilon_1+i\Gamma_0)^{-1}$ and $g_{2\sigma} = (z-\varepsilon_2+i\Gamma_0 + \frac{\Delta \Gamma}{2})^{-1}$. They are the Green functions of individual QD when another QD is removed from the structure. It is obvious that higher-order Feynman paths have more complicated forms. By the same token, we can expand $t_{RL,\sigma}(1,2)$ as a summation of Feynman paths, which is given by

$$t_{RL,\sigma}(1,2) = \sum_{j=1}^{\infty} iV_{R1\sigma}(-g_{1\sigma}g_{2\sigma} \Gamma_{12,\sigma})^j V_{L2\sigma} = \sum_{j=1}^{\infty} t_{RL,\sigma}^{(j)}(1,2).$$

Besides, the other transmission coefficients $t_{RL,\sigma}(2,2)$ and $t_{RL,\sigma}(2,1)$ have the similar expansions as $t_{RL,\sigma}(1,1)$ and $t_{RL,\sigma}(1,2)$. The lowest-order Feynman paths in above equations are $t_{RL,\sigma}^{(0)}(1,1) = V_{R1\sigma} g_{1\sigma} V_{L1\sigma}$ and $t_{RL,\sigma}^{(0)}(2,2) = V_{R2\sigma} g_{2\sigma} V_{L2\sigma}$. All Feynman paths contribute to the linear conductances coherently, the lowest-order ones are the leading terms, though.

As shown in Fig.2(a), the contribution of the zero-order paths has the similar oscillation to the exact linear conductance with the magnetic adjustment. Therefore, we can only take into account the zero-order paths, i.e., $t_{RL,\sigma}^{(0)}(1,1)$ and $t_{RL,\sigma}^{(0)}(2,2)$, to analyze the quantum interference that cause the spin-dependent electron transport between lead-R and lead-L, as shown in Fig.2. The phase difference between the two zero order paths is $\Delta \theta_\sigma = [\phi + 2\sigma \phi + \theta_1 - \theta_2]$, where the phase $\theta_\sigma$ arises from the Green function $g_{\sigma\sigma}$, i.e., $g_{\sigma\sigma} = |g_{\sigma\sigma}| e^{i\theta_\sigma}$. According to the above relation, we can realize that $\phi$, $\sigma\phi$, and $\theta_1 - \theta_2$ are the phases associated with the magnetic field, Rashba interaction, and the QD parameters, respectively. They interplay to influence the quantum interference. And it should be noticed that the Rashba interaction gives rise to the spin dependence of the phase, hence to lead to the spin-dependent conductance. For the cases shown in Fig.2 the Rashba strength $\sigma = \frac{\pi}{2}$ and both $\theta_1$ and $\theta_2$ are equal to $\frac{\pi}{2}$ due to QD levels being fixed at the Fermi level. Then the change of $\phi$ influences the quantum
interference and brings out spin polarization. In the case of \( \phi = (n + \frac{1}{2})\pi \), the opposite-spin electrons will undergo distinct quantum interference. For example, when \( \phi = \frac{1}{2}\pi \) the interference for spin-up electron between the two zero-order Feynman paths is destructive with \( \Delta \theta_1 = \pi \), but for the spin-down electron it is constructive since \( \Delta \theta_1 = 0 \); However, in the case of \( \phi = \frac{3}{2}\pi \) the situation just becomes opposite, namely, the interference for spin-up electron is constructive whereas it is destructive for the spin-down electron. In contrast, in the case of \( \phi = \pi \) it can be readily seen that the quantum interference and the conductance does not depend on the spin freedom. Taking the case of \( \phi = \pi \) as an example, one has \( \Delta \theta_1 = \frac{3}{2}\pi \) and \( \Delta \theta_1 = \frac{1}{2}\pi \). As a result, when the opposite-spin electrons tunnel through the two paths, they undergo the same quantum interference and no spin polarization occurs.

With the similar method, we can analyze \( T_{DL,\sigma}(\omega) \) by writing out \( T_{DL,\sigma}(\omega) = \left| \sum_{l=1}^{2} t_{DL,\sigma}(2,l) \right|^2 \). Here lead-R plays the same role as lead-D in the above case to act on the quantum coherence. It is useful for us to give the explicit forms of the lowest-order Feynman paths herein. They are \( t_{DL,\sigma}(2,2) \) and \( t_{DL,\sigma}(2,1) \), with Fig.3(b) shows the contribution of the lowest-order paths to the conductance, which presents the in-phase oscillation to the exact conductance spectrum. So we can analyze the quantum interference by taking only the lowest-order Feynman paths into account. The phase difference between \( t_{DL,\sigma}(2,2) \) and \( t_{DL,\sigma}(2,1) \) is \( \nu_{0a,\sigma} = [\phi + 2\sigma \varphi + \theta_1 + \frac{\pi}{2}] \), the phase difference between \( t_{DL,\sigma}(2,2) \) and \( t_{DL,\sigma}(2,1) \) is \( \nu_{0b,\sigma} = [\theta_1 + \frac{\pi}{2}] \), and the phase difference between \( t_{DL,\sigma}(2,1) \) and \( t_{DL,\sigma}(2,1) \) is \( \nu_{ab,\sigma} = [\phi + 2\sigma \varphi] \). With this phase relations, we can find that, when \( \phi = \frac{3}{2}\pi \), \( \nu_{0a,\sigma} = 2\pi \), \( \nu_{0b,\sigma} = \pi \), and \( \nu_{ab,\sigma} = \pi \) which give rise to the constructive interference for spin-up electron, but for the spin-down electron it is destructive since \( \nu_{0a,\sigma} = \pi \) and \( \nu_{0b,\sigma} = 0 \). Besides, with the help of these analysis, it can also be found that the peak of \( G_{RL,\sigma} \) encounters the valley of \( G_{DL,\sigma} \) and vice versa.

One of the characteristics of QD is its tunable level with respect to the Fermi level, which can alter the phases of Feynman paths taking part in the quantum interference. Therefore, we now turn to discuss the variation of the spin-polarized current flow with the shift of QD level. In the absence of a magnetic field but in the presence of Rashba interaction with \( \varphi = \frac{1}{2}\pi \), we calculate the linear conductances as functions of \( \epsilon_1 = \epsilon_2 = \epsilon_0 \), which are shown in Fig.3(a) and (b). From these spectra one can find the notable and opposite spin polarizations in the \( L \rightarrow R \) and \( L \rightarrow D \) channels except at the vicinity of \( \epsilon_0 = 0 \). Besides, in either channel the spin polarization flips over when \( \epsilon_0 \) passes through the Fermi level. These results can be readily understood by analyzing the quantum interference following the above argument. Namely, we will discuss the quantum interference by taking only the lowest-order Feynman paths into account. This is supported by the calculated results shown in Fig.4(c) and (d), which are the contributions of the lowest-order Feynman paths to the conductances. They show that the spin polarizations in analogy with the exact ones shown in Fig.3(a) and (b), respectively. As for the Feynman paths from lead-R the phase difference \( \Delta \theta_1 = 2\sigma \varphi + \theta_1 - \theta_2 \), in which \( \theta_1 - \theta_2 = \tan^{-1} \frac{\nu_0}{\nu_\omega} - \tan^{-1} \frac{\nu_{0a,\sigma}}{\nu_{0b,\sigma}} \) is nonzero when \( \epsilon_0 \) departs from the Fermi level. As a result, when electron tunnels through \( t_{DL,\sigma}^{(0)}(1,1) \) and \( t_{DL,\sigma}^{(0)}(2,2) \), the phase difference is associated with the electron spin. For example, at the point of \( \epsilon_0 = 2t_0 \) where the spin polarization is very striking, we can obtain \( \Delta \theta_1 = \frac{2\pi}{\nu_{0a,\sigma}} \) and \( \Delta \theta_1 = \frac{2\pi}{\nu_{0b,\sigma}} \); Conversely, at the point of \( \epsilon_0 = -2t_0 \), \( \Delta \theta_1 = \frac{2\pi}{\nu_{0a,\sigma}} \) and \( \Delta \theta_1 = \frac{2\pi}{\nu_{0b,\sigma}} \). These distinct phase differences just lead to opposite quantum interference for the opposite-spin electrons in lead-R, hence the notable spin polarization in the electron transport through this lead. Furthermore, \( \epsilon_0 \) passing through the Fermi level inverses the spin polarization. As for the quantum interference in the other channel, the situation is more complicated since there are three low-order Feynman paths to be taken into account. We can find \( \nu_{ab,\sigma} = (\pm \frac{1}{2}\varphi) \) independent of the adjustment of \( \epsilon_0 \). Besides, \( \nu_{0b,\sigma} = \frac{\pi}{2}\sigma \), \( \nu_{0a,\sigma} = \frac{\pi}{4}\sigma \), and \( \nu_{0a,\sigma} = \frac{\pi}{4}\sigma \), when \( \epsilon_0 = 2t_0 \); \( \epsilon_0 = -2t_0 \) corresponds to \( \nu_{0b,\sigma} = \frac{\pi}{2}\sigma \), \( \nu_{0a,\sigma} = \frac{\pi}{4}\sigma \), and \( \nu_{0a,\sigma} = \frac{\pi}{4}\sigma \). The contribution of the three low-order paths to the conductance is displayed in Fig.4(d), which agrees with the calculations of these phase differences. Based on these analysis, it is also clear that for the same nonzero \( \epsilon_0 \) the spin polarization in different channels are opposite to each other. Therefore, with the tuning of gate voltage, the spin-up polarized current in lead-R and spin-down polarized current in lead-D can come into being simultaneously. It is worth mentioning that the spin-dependent conductance spectra shown Fig.4 are obtained in the absence of a magnetic field, which indicates that an external field is not indispensable to cause the spin related quantum interference. Without a magnetic field one can still fulfill the required result of the spin-dependent electron transport by tuning the QD parameters.

It should be pointed out that, the third lead(lead-D) is essential to realize the spin polarization in the absence of a magnetic field. For instance, when \( \Gamma_D = 0 \), \( g_{1\sigma} \) and \( g_{2\sigma} \) correspond to the same phase(\( \theta_1 - \theta_2 \equiv 0 \)), which makes the total phase difference to be nothing to do with the electron spin. Hence only in a multi-terminal structure the electron transport with the spin-polarization effect can be fulfilled. And for the two-terminal case and in the absence of magnetic field, spin polarization is impossible.

As is known, so far it is still a formidable challenge
to fabricate experimentally an QD structure consisting of identical QDs. Thereby it is necessary for us to investigate the influence of the fluctuations of structure parameters on the spin polarization and separation during the electron transport process. We then calculate the conductance spectra with the fluctuated structure parameters, i.e., the QD levels and QD-lead couplings. The corresponding results are shown in Fig.5. From Fig.5(a)-(b), we find that the increase of the difference of QD levels brings out a little weakening of the spin polarization when $\epsilon_0$ is below the Fermi level, whereas it strengthens the spin polarization for the case of $\epsilon_0$ above the Fermi level. However, the fluctuation of QD levels can not result in the remarkable destruction of the spin polarization and separation. On the other hand, according to the results in Fig.5(c)-(d) we can see that the fluctuations of QD-lead couplings are not able to induce the change of the electron transport in principle until the fluctuations exceed 30%. Therefore, our conclusion is that the spin polarization and separation in this structure does not require an absolute uniformity of the structure parameters.

Finally, we have to point out that the appropriate Rashba coupling strength is crucial for the effect of spin polarization and separation in such a three-terminal double-QD structure. To illustrate this, we see the cases of zero magnetic field and $\varphi = \frac{\pi}{2}$, thus $\Gamma_{2\sigma} = \Gamma_{21\sigma} = 0$. Accordingly, the contributions of the higher-order paths become zero due to the destructive quantum interference. In such a case $T_{RL,\sigma} = |t_{RL,\sigma}^{(0)}(1,1) + t_{RL,\sigma}^{(0)}(2,2)|^2$, and the phase difference resulting from the opposite-spin electrons passing through these two paths is $\Delta \theta_\sigma = |\pi + \theta_1 - \theta_2|$, which is independent of the electron spin. Alternatively, $T_{DL,\sigma} = |t_{DL,\sigma}^{(0)}(2,2)|^2$, which is also irrelevant to the spin index but shows a Breit-Wigner lineshape in the linear conductance spectrum. Therefore, in this case the quantum interference does not involve the electron spin, and no spin polarization occurs.

So far we have not discuss the effect of electron interaction on the spin-dependent conductance spectra, though it is included in our theoretical treatment. Now we incorporate the electron interaction into the calculation of the conductance spectra. We wonder whether it can destroy the property of spin polarization and separation. We deal with the many-body terms by employing the second-order approximation, since we are not interested in the electron correlation here. And a uniform Coulomb repulsion $U = 3z_0$ is assumed for both QDs. Figure 7 shows the calculated conductance spectrum. It is found that in the linear conductance spectra are separated into two groups due to the Coulomb repulsion, and they are symmetric about the center of insulating band $\epsilon_0 = -U/2$, which arises from the electron-hole symmetry. Clearly, in such an approximation the Rashba-related spin polarization and separation effect remain.

With regard to the many-body effect, we should emphasize the following point. Our calculation indicates that the electron interaction does not destroy the spin polarization and separation effect in this structure. However, we have taken the many-body terms into account within the approximations only to second order and did not consider the electron correlation, e.g., the Kondo effect. Thus one can pay attention to the influence of Kondo resonance on the spin polarization and separation by setting the QDs in the Kondo regime. We would like to point out that in this system, the Kondo effect can destroy the phenomena of spin polarization and separation. It is known that in equilibrium, the influence of Kondo effect on electron transport is to renormalize the QD levels to coincide with the Fermi level of the system, which gives rise to the Kondo resonance. However, according to the Feynman path theory above such a property will bring out the invariability of the phase of $g_{\sigma}$ (i.e., $\theta_\sigma \approx -\pi/2$), which will modify the quantum interference in this system and restrain the spin polarization and separation. Up to now, we can make a conclusion that only in the case of relatively small Coulomb repulsion, the properties of spin polarization and separation can be observed.

We now turn to investigate the electron transport properties of this structure in the case of finite bias voltage, to clarify the effect of finite bias voltage on the spin polarization and separation. The corresponding numerical results in Fig.7 describe the change of the current in the drain (lead-R and lead-D) with the increase of bias voltage. In Fig.7(a), the non-interacting case, we can see that with the adjustment of the bias voltage the the properties of spin polarization and separation remain in this electron transport process. Besides, when the many-body effect is taken into account to the second order of the equation of motion, the similar results to the non-interacting case are found, as shown in Fig.7(b).

**IV. SUMMARY**

In conclusion, we have theoretically investigated the electron transport properties in a parallel double-QD structure with three-terminals. By introducing the local Rashba spin-orbit coupling, we find that an incident electron from one terminal can select a specific terminal to depart from the QDs according to its spin state. As a result, the functions of spin polarization and separation can be simultaneously realized in this structure. And spin polarizations in different channels can be inverted by tuning the structure parameters. The underlying quantum interference that gives rise to such results is analyzed in the language of Feynman paths for the electron transmission. We find that the total phase differences between any two low-order Feynman paths associated with the Rashba interaction, the applied magnetic field, and the QD scattering. In partic-
ular, the Rashba interaction provides a spin-related phase, which is the origin of the spin-dependent electron transport properties as we have reported. Besides, it should be noted that a magnetic field and the QD parameters can adjust the phase difference on the equal footing. Therefore, an applied magnetic field is not indispensable to realize the spin polarization in this structure. Instead of it, we can also obtain these results via the adjustment of the QD parameters such as the QD levels. In addition, by the detailed analysis on the quantum interference, we find that for the appropriate Rashba interaction strength (i.e., $\varphi \sim \pi/4$), it is possible to result in the spin polarization and separation. And either of three leads is absolutely necessary in the absence of magnetic field. However, for a two-terminal structure it is impossible to obtain the spin polarization without the magnetic field. When the approximation of many-body effect is considered to second order, it is found that the phenomena of spin polarization and separation remain. On the basis of this feature, we propose that such a structure can be considered as a device prototype to manipulate the spin freedom.

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FIG. 1: A schematic of the parallel double QD structure with three terminals (labeled as L, R, and D). An electric field is applied to QD-1 to induce the local Rashba interaction; Φ indicates a magnetic flux penetrating the ring.

FIG. 2: The linear conductances versus the magnetic phase φ. The structure parameters take the values as ε_j = 0 and Γ_0 = 2t_0. (a) In the absence of Rashba interaction. (b) and (c) In the presence of Rashba interaction with the strength ϕ = 1/4π.

FIG. 3: A comparison between the exact conductance and the contributions of the lowest-order Feynman paths to the corresponding conductances.

FIG. 4: (a) and (b) The linear conductances versus the QD level ε_0 in the presence of Rashba interaction with ϕ = 1/4π. (c) and (d) The contribution of the lowest-order Feynman paths.

FIG. 5: The conductance spectra in the presence of the fluctuated QD parameters. (a) and (b) The conductances with the QD levels different from each other. The results of ε_1 = ε_0 + δ and ε_2 = ε_0 - δ with δ = 0, 0.1t_0, and 0.5t_0 are shown, respectively. (c) and (d) The conductances with the fluctuation of the QD-lead couplings. The fluctuation is measured by the quantity ΔΓ = [(\sum_j |\Gamma_j|^2/N - (\sum_j |\Gamma_j|^2/N)^2)]^{1/2} with x_j = |\Gamma_j|^2/N, and the cases of ΔΓ = 5% and = 10% are investigated.

FIG. 6: The conductances versus the QD levels in the presence of electron interaction with U = 3t_0 in both QDs.

FIG. 7: The current versus the finite bias voltage. (a) corresponds to the noninteracting case, and (b) is the result in the presence of many-body effect.
(a) $\epsilon_j = 0, \Gamma_0 = 2t_0, \phi = 0$

(b) $\epsilon_j = 0, \Gamma_0 = 2t_0, \phi = \pi / 4$

(c) $\epsilon_j = 0, \Gamma_0 = 2t_0, \phi = \pi / 4$

Conductance ($e^2/h$)

$\phi$ (in units of $\pi$)
Contribution to conductance \( \frac{e^2}{h} \):

(a) \( \varepsilon_j = 0, \Gamma_0 = 2t_0, \phi = \frac{\pi}{4} \)

\[ |t^{(0)}_{\uparrow \uparrow} (1,1) + t^{(0)}_{\uparrow \uparrow} (2,2)|^2 \]

(b) \( \varepsilon_j = 0, \Gamma_0 = 2t_0, \phi = \frac{\pi}{4} \)

\[ |t^{(1)}_{\uparrow \uparrow} (2,1) + t^{(0)}_{\uparrow \uparrow} (2,2)|^2 \]
Conductance ($e^2/h$)

(a) $\varepsilon_j = \varepsilon_0, \Gamma_0 = 2t_0$
- $\phi = 0, \varphi = \pi / 4$
- $G_{RL,\uparrow}$
- $G_{RL,\downarrow}$

(b) $\varepsilon_j = \varepsilon_0, \Gamma_0 = 2t_0$
- $\phi = 0, \varphi = \pi / 4$
- $G_{DL,\uparrow}$
- $G_{DL,\downarrow}$

(c) $|t_{RL,\uparrow}^{(0)} (1,1) + t_{RL,\downarrow}^{(0)} (2,2)|^2$
- $|t_{RL,\downarrow}^{(0)} (1,1) + t_{RL,\downarrow}^{(0)} (2,2)|^2$

(d) $|t_{DL,\uparrow}^{(1)} (2,1) + t_{DL,\downarrow}^{(0)} (2,2)|^2$
- $|t_{DL,\downarrow}^{(1)} (2,1) + t_{DL,\downarrow}^{(0)} (2,2)|^2$

$\varepsilon_0$ (in units of $t_0$)
(a) $G_{RL,\uparrow}, \delta = 0.0t_0$

(b) $G_{DL,\uparrow}, \delta = 0.0t_0$

(c) $G_{RL,\uparrow}, \Delta_\Gamma = 0$

(d) $G_{DL,\uparrow}, \Delta_\Gamma = 0$

Conductance (e$^2$/h) vs. $\varepsilon_0$ (in units of $t_0$)
(a) $\varepsilon_j = \varepsilon_0, \Gamma_0 = 2t_0, \phi = 0, \varphi = \pi/4, U = 3t_0$

Conductance ($e^2/h$)

\begin{align*}
\varepsilon_0 \text{ (in units of } t_0) \quad &
\end{align*}
(a) \( \varepsilon_j = 0, \Gamma_0 = 2t_0 \)
\[ \phi = 0, \varphi = \frac{\pi}{4}, U = 0 \]

(b) \( \varepsilon_j = 0, \Gamma_0 = 2t_0 \)
\[ \phi = 0, \varphi = \frac{\pi}{4}, U = 3t_0 \]