DOA Estimation Based on LSTM Neural Network with Uniform Linear Antenna Array

Thanh Han Trong*, Ngo Duc Nam, Tran Van Hung

Abstract—Radio Direction Finding system is a system that determines the direction or coordinates of radio signal sources. The main function of this system is to determine the direction of arrival (DOA) of an incident radio wave. DOA information plays an important role in array signal processing and has many applications in communications, radar, seismic survey... In this paper, with the simulated signal data set obtained at the linear antenna array (ULA), the Long Short-Term Memory (LSTM) network model is used to estimate the DOA. The performance of the method is evaluated based on the RMSE parameter and compared with MUSIC, DNN algorithms in different cases such as: deviation of incident angle of radiation sources, signal-to-noise ratio (SNR).

Index Terms—Radio Direction Finding; Direction of Arrival; Long-Short Term Memory; Uniform Linear Antenna Array.

1. Introduction

In radio communication systems, besides basic parameters such as frequency, amplitude, phase... information about the direction of the incoming wave also needs to be determined precisely. The antenna arrays commonly used in radio direction finding systems are the Uniform Linear Antenna Array (ULA), Uniform Circular Antenna Array (UCA), Uniform Rectangular Antenna Array (URA) [1]. Information about the direction of the incident wave will help increase the ability to recover the transmission channel, coordinate synchronization, automatically adjust the radiation graph in the necessary direction to increase the quality of the received signal.

Several techniques have been developed in the DOA estimation problem MUSIC [2]–[4], ESPRIT [5], TFBMP [6], [7]. In which, the MUSIC algorithm is an algorithm that has been developed for a long time and has many practical applications. In recent years, artificial intelligence algorithms, especially deep learning algorithms, have been studied and applied to determine DOA with high accuracy and speed [8]. The deep learning methods do not need to compute the signal features in the prediction process, so the real-time estimation process will be shorter and more applicable. Deep Neural Network (DNN) [9]–[11] and Convolutional Neural Network (CNN) [12]–[15] using Adam’s optimization function were used to estimate the angle of arrival with fairly accurate results. The Radial Basis Function Neural Network (RBFNN) [16] can also estimate the DOA with good accuracy under favorable environmental conditions ....

This paper focuses on research and development of a simulation database of the signals received from the ULA antenna array. From the obtained data set, the Long-Short Term Memory (LSTM) algorithm is proposed to be applied to calculate the DOA of incoming signals. From the obtained results, the performance of the proposed method will be evaluated and compared with other typical methods.

2. Materials and methods

2.1. Uniform Linear Antenna Array and Signal Model

This study uses a uniform linear antenna array (ULA) with M elements. The structure of the ULA antenna array is shown in Fig. 1. The element in the antenna system acts as an omnidirectional source. These antennas operate in phase with each other to create a
unique radiation direction so that the signal sent to the processor is kept in phase and amplitude in comparison with the signal arriving at the antenna system. Furthermore, the ULA antenna array has several advantages over other types of antenna arrays as shown in [1], [6].

Assuming that the incoming signal is in the same azimuth plane as the antenna array, the signal transmitted to the antenna array is illustrated as Fig. 1.

The antenna array (ULA) used in this study has \( M \) elements, equally spaced with a distance of \( d \). Assume that the system has \( K \) incoming signal sources with wavelength \( \lambda \). The received signal at each antenna element is the sum of all incoming signals at the same time. The mathematical representation of the signal received at the \( m_{th} \) antenna element is described as in Eq.1., where \( s_k(t) \) and \( \theta_k \) are the complex amplitude and the DOA of the \( k_{th} \) source \((k = 1, 2, \ldots, K)\).

\[
x_m(t) = \sum_{k=1}^{K} s_k(t)e^{-j\frac{2\pi}{\lambda}(m-1)d\sin\theta_k} + n_m(t) \tag{1}
\]

Where \( m = 1, 2, \ldots, M \) and \( n_m(t) \) is the noise received at the \( m_{th} \) antenna element of the array.

In [17], [18], the signals received at the antenna array will be passed through a preprocessor before being processed to calculate DOA information. Therefore, the correlation matrix of size \( M \times M \) of the received signals at the antenna array can be represented as follows:

\[
R_{xx} = E \left[ x(t)x^H(t) \right] = ASA^H + R_n \tag{2}
\]

Where \( A \) is a matrix of size \( M \times K \) including the elements represented as follows:

\[
a_{m,k} = e^{-j\frac{2\pi}{\lambda}(m-1)d\sin\theta_k} \tag{3}
\]

\([\cdot] \) and \([\cdot]^H \) are the expectation and the Hermitian transpose, respectively; \( S \) and \( R_N \) are correlation matrices of size \( K \times K \) of signal and noise, respectively, and are represented as follows:

\[
S = E \left[ s(t)s^H(t) \right] \tag{4}
\]

\[
R_n = E \left[ n(t)n^H(t) \right] \tag{5}
\]

From there, Eq.2 can be rewritten as

\[
R_{xx} = ASA^H + \sigma_{\text{noise}}^2I \tag{6}
\]

Where \( I \) is an identity matrix of size \( M \times M \), \( \sigma_{\text{noise}}^2 \) is the noise power. The correlation matrix is also called as the Hermitian matrix. This matrix is used as input for the DOA estimation models.

2.2. DOA Estimation

2.2.1. Long Short-Term Memory networks

Long Short-Term Memory (LSTM) is an artificial recurrent neural network that takes the form of a sequence of repeating modules and contains feedback connections. This network is often used in problems where the input is a data string such as speech or video... An LSTM unit consists of an input and an output port. They have a 4-layer structure that interacts with each other in a very specific way as depicted in Fig. 2.

In the \( t^{th} \) state of the LSTM model, the general parameters of the network model have been presented in [19] with the input and output as:

Output: \( c_t, h_t, c \) is called cell state, \( h \) is hidden state.

Input: \( c_{t-1}, h_{t-1}, x_t \), where \( x_t \) is the input in the \( t^{th} \) state of the model, \( c_{t-1}, h_{t-1} \) is the output of the previous state.

This study proposes an LSTM model to estimate DOA as shown in Fig. 3.

This network is designed with 1 input layer, 2 hidden layers and 1 output layer. The signal received at the antenna is processed at the pre-processing unit to obtain the correlation vector. That correlation vector is the input to the LSTM network. The output layer with \( n \) elements is used to estimate the DOA. Detailed description of the classes is presented in the following section.

2.2.2. Data Pre-Processing

The LSTM network is trained with a large amount of data. In order to reduce the input bias and variation of the signal, the signal preprocessing is carried out with the input signal received at the antenna array and the output as a correlation matrix \( R_{xx} \) of size \( M \times M \):

\[
R_{xx} = \begin{bmatrix}
    r_{1,1} & r_{1,2} & \cdots & r_{1,M} \\
    r_{2,1} & r_{2,2} & \cdots & r_{2,M} \\
    \vdots & \vdots & \ddots & \vdots \\
    r_{M,1} & r_{M,2} & \cdots & r_{M,M}
\end{bmatrix}
\tag{7}
\]

Since \( R_{xx} \) is a Hermitian matrix, the upper triangular matrix and the lower triangular matrix carry the same information. According to the methods published in [10], [20], the upper triangular matrix has enough information to estimate the DOA. Therefore, to reduce the amount of information for the input of the network, this study only uses the upper triangular matrix of \( R_{xx} \) and then transforms it into a vector \( r \) of length \( M \times (M - 1) \).

\[
r = \begin{bmatrix} R(r_{1,2}), R(r_{1,3}), \ldots, R(r_{M-1,M}), \\
I(r_{1,2}), I(r_{1,3}), \ldots, I(r_{M-1,M}) \end{bmatrix}^T
\tag{8}
\]
Where $r_{i,j}$ is the $(i, j)^{th}$ element in the matrix $R_{xx}$. Since $r_{i,j}$ is a complex number, it cannot be put directly into the network for calculation. Therefore, before putting into the training network, each element $r_{i,j}$ will be represented into 2 components, the real part $R$ and the imaginary part $I$.

2.2.3. Data Labeling

The input is defined as vector $r(\theta, \Delta_j)$ as Eq.8 with 2 incoming signals at the two angles $\theta$ and $\theta + \Delta_j$, respectively, in which $\Delta_j$ is the angular difference between the two incident sources with $\theta_{min} \leq \theta \leq \theta_{max} - \Delta_j$.

In this study, a labeling method called one hot encoding with multiple labels is used to label the data. $[y(\theta, \Delta_j)]_{\text{label}}$ is the corresponding label of the incoming signals. With 121 outputs corresponding to incoming angles in the range $[-60^\circ \div 60^\circ]$ with a jump of $1^\circ$, $[y(\theta, \Delta_j)]_{\text{label}}$ is defined as:

$$
\text{label} = \begin{cases} 
1, & \text{at } \theta \\
1, & \text{at } \theta + \Delta_j \\
0, & \text{otherwise}
\end{cases}
$$

(9)

For example, with two incoming signals, the label form will be $[1 \ 0 \ 1 \ldots \ 0 \ 0]$. Therefore, the output of the LSTM network corresponding to the input $r(\theta, \Delta_j)$ is $y(\theta, \Delta_j)$

2.2.4. RMSE

This study uses the root mean square error function (RMSE) to evaluate the performance of the model and algorithm.

$$
RMSE = \sqrt{\frac{1}{NK} \sum_{k=1}^{K} \sum_{n=1}^{N} (\theta_k - \hat{\theta}_{k,n})^2}
$$

(10)

Where $K$ is the number of incident sources, $N$ is the number of trials, $\theta_k$ and $\hat{\theta}_{k,n}$ are the incident angle of the source $k_{th}$ and the estimated angle of $k_{th}$ source at the $n_{th}$ trial.

3. Experiments

3.1. Simulation establishment

This study uses a 10-element ULA antenna array with $d = \lambda/2$. The incoming signal has a frequency of 2GHz. Assuming the incoming signals are uncorrelated with each other, simulation data is generated according to Eq.1 with $SNR = 10dB$, snapshot $= 400$.

For the LSTM network, the size of the input layer is 90 nodes, the number of hidden layers is two where each layer has 256 states. Assume that the incoming signals are in the range $[-60^\circ, 60^\circ]$, and the angular resolution is equal to 1$^\circ$. Therefore, the number of nodes of the output layer will be 121. The spatial spectrum has also an angular resolution of 1$^\circ$, so there is a total of 121 grids with $\theta_1 = -60^\circ, \theta_2 = -59^\circ, \ldots, \theta_{121} = 60^\circ$.

The training samples are generated by considering two signals separated by $\Delta_\theta$, where $\Delta_\theta$ is taken from the set $\Delta = \{2^\circ, 4^\circ, 6^\circ, \ldots, 40^\circ\}$. This $\Delta$ set includes cases where two signals are very close to each other and far apart. When the DOA of the first signal is created by sampling in the $[-60^\circ, 60^\circ - \Delta]$ with the sampling distance is 1$^\circ$, then the DOA of the second signal will be $\theta + \Delta$. Then the unilateral vectors are calculated as the input of Eq.8 and the corresponding Labels as in Eq.9.

SNRs for both signals are 10dB, each signal sample is collected 100 times with random noise. Therefore, there are 200000 covariance vectors created and they are input data for LSTM networks. For the training process, the learning rate is 0.001, Batch Size is 32, and the number of Epochs is 300. In addition, the network uses the ADAM Optimization Algorithm to optimize the time and predictability of the algorithm.

3.2. Simulation results

In this section, the results obtained from the LSTM method are presented in different cases and compared...
with some other DOA methods such as MUSIC and DNN [9].

In the first experiment, the performance of the proposed method will be assessed in 2 cases: the incoming signals are in training set (at angle $10^\circ$ and $12^\circ$) and the incoming signals are not in training set (at angle $10^\circ$ and $11^\circ$) with $SNR_s = 10dB$.

![Fig. 4: The signal spectrum of the incoming signals at $10^\circ$ and $12^\circ$ based on LSTM and MUSIC method](image1)

The simulation results are shown in Fig. 4 and Fig. 5, showing that the algorithm successfully estimates DOA of two signal sources with a separated signal spectrum. For the MUSIC algorithm, with very close incident angles, the signal spectrum fluctuates slightly. Obviously, with the angles of incidence so close together, LSTM performs better than the MUSIC method.

In the next experiment, the proposed method is executed with two signals with angle difference $\Delta\theta = 10^\circ$ and the obtained results will be compared to DNN method. When the DOA of the first signal is in the range $[-30^\circ, 20^\circ]$ with a jump $= 1^\circ$, the DOA of the second signal will be $\theta + \Delta\theta$. The results described in Fig.6 indicate that both LSTM and DNN (with the same training data set) give the estimated results with good accuracy and are close to the actual results, however, the LSTM is somewhat finer than the DNN method.

Next simulation shows the results of 3 methods LSTM, MUSIC, and DNN with SNR between -10dB to 10dB, a jump of 2dB and the angles of incidence $31^\circ$, $41^\circ$ (in the training set) and $30^\circ$, $39^\circ$ (Not in the training set). The results are shown in Fig. 7 and Fig. 8, evaluated by the RMSE formula with 1000 trials.

![Fig. 6: Comparison between actual and estimated DOA values of two signals separated by $\Delta\theta = 10^\circ$ at $SNR = 10dB$ based on two methods of LSTM and DNN.](image2)

Obviously, with high SNR values, all algorithms can accurately determine DOA with $RMSE < 0.5$. However, it can be seen that LSTM can do better than the others even in case of low SNR. The result presented in Fig. 8 indicated that although the 2 incoming signals are not in the training set, the LSTM still work very well, the RMSE error is lower than that of the two methods DNN and MUSIC. To examine the effect of noise on the LSTM method, a test with 51 samples with $\Delta\theta = 10^\circ$ at low SNR values was performed. The results presented in Fig. 9 show that even though the SNR is low, the performance of the LSTM method is still quite good.

To evaluate the resolution in the DOA estimation of the LSTM algorithm, we assume that the two incoming signals differ by an amount $\Delta\theta$ with $SNR = -10dB$. The estimated results are shown in Table 1. With two algorithms MUSIC and DNN, the results are given in Tables 2 and 3. From those tables, it can be seen that
if we consider the RMSE of desired results less than 2 degrees, LSTM method has the highest result with the resolution approximately 2 degrees.

Fig. 7: The RMSE (degrees) of the proposed LSTM-based DOA estimation algorithm, the standard MUSIC algorithm, and the DNN method with different SNR values for incident angles in the training set.

Fig. 8: The RMSE (degrees) of the proposed LSTM-based DOA estimation algorithm, the standard MUSIC algorithm, and the DNN method with different SNR values for the incident angles not in training set.

Fig. 9: Comparison between the actual and estimated DOA values of the two signals of the LSTM method.

### TABLE 1: The angles resolution of LSTM (SNR = -10dB)

| $\Delta \theta$ | Input (degree) | Output (degree) | RMSE  |
|-----------------|----------------|-----------------|-------|
| 2°              | 30             | 32              | 29.99 | 34.24 | 1.58  |
| 4°              | 30             | 34              | 31.38 | 36.21 | 0.98  |
| 6°              | 30             | 36              | 29.96 | 38.66 | 0.47  |
| 8°              | 30             | 38              | 30.82 | 34.46 | 1.58  |

### TABLE 2: The angles resolution of MUSIC (SNR = -10dB)

| $\Delta \theta$ | Input (degree) | Output (degree) | RMSE  |
|-----------------|----------------|-----------------|-------|
| 2°              | 30             | 32              | 31.83 | 59.91 | 19.78 |
| 4°              | 30             | 34              | 31.84 | 50.36 | 11.64 |
| 6°              | 30             | 36              | 29.65 | 32.84 | 2.25  |
| 8°              | 30             | 38              | 31.12 | 40.38 | 1.86  |

### TABLE 3: The angles resolution of DNN (SNR = -10dB)

| $\Delta \theta$ | Input (degree) | Output (degree) | RMSE  |
|-----------------|----------------|-----------------|-------|
| 2°              | 30             | 32              | 30.82 | 54.46 | 15.89 |
| 4°              | 30             | 34              | 29.96 | 46.82 | 9.07  |
| 6°              | 30             | 36              | 29.97 | 37.6  | 1.13  |
| 8°              | 30             | 38              | 31.08 | 38.9  | 0.994 |

### 4. Conclusion

This paper uses the LSTM network to estimate the DOA with the ULA antenna system. The obtained simulation results show that the model works more accurately than typical algorithms such as MUSIC and DNN algorithms in both low SNR cases as well as when radiation sources are quite close. However, the LSTM algorithm is still limited as in the case where the deviation between the angles is not in the training set, the error is still quite high. In the future, it can be developed to work with other antenna systems such as UCA or increase the accuracy.
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