An AdS/CFT calculation of screening in a hot wind

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One of the challenges in relating experimental measurements of the suppression of the number of J/ψ mesons produced in heavy ion collisions to lattice QCD calculations is that whereas the lattice calculations treat J/ψ mesons at rest, in a heavy ion collision a c ¯c pair can have a significant velocity with respect to the hot fluid produced in the collision. The putative J/ψ finds itself in a hot wind. We present the first rigorous non-perturbative calculation of the consequences of a wind velocity v on the screening length Ls for a heavy quark-antiquark pair in hot N = 4 supersymmetric QCD. We find Ls(v, T) = f(v)[1 − v2]1/3/πT with f(v) only mildly dependent on v and the wind direction. This Ls(v, T) ∼ Ls(0, T)/√T velocity scaling, if realized in QCD, provides a significant additional source of J/ψ suppression at transverse momenta which are high but within experimental reach.

Twenty years ago, Matsui and Satz suggested that because the attraction between a quark and an antiquark is screened in a deconfined quark-gluon plasma, the production of J/ψ mesons should be suppressed in sufficiently energetic nucleus-nucleus collisions relative to that in proton-proton or proton-nucleus collisions, since the screened interaction between a c and ¯c immersed in a quark-gluon plasma would not bind them [1]. In the intervening years, marked progress on many fronts has not changed this basic qualitative picture. On the experimental side, we now have data from the NA50 and NA60 experiments at the CERN SPS and from the PHENIX experiment at RHIC that demonstrate the existence of a suppression [2]. On the theoretical side, we now have ab initio calculations of the temperature-dependent potential between a color singlet heavy quark and antiquark separated by a distance L [3, 4]. This potential is as at T = 0 for small L, but begins to weaken for L larger than some Ls and flattens at larger L. The potentials obtained in these lattice calculations can be crudely characterized as indicating Ls ∼ 0.5/T in hot QCD with two flavors of light quarks [3] and Ls ∼ 0.7/T in hot QCD with no dynamical quarks [3]. Furthermore, lattice QCD calculations of the Minkowski space J/ψ spectral function itself have now been done in quenched QCD [3], and early results in QCD with dynamical quarks have also been reported [6]. These studies indicate that the J/ψ meson ceases to exist as a bound state above a temperature somewhere between 1.5 Tc and 2.5 Tc, in agreement with conclusions drawn based upon the screening potential between static quarks [3].

The multifaceted challenge, now, is to make quantitative contact between the lattice calculations and data from heavy ion collisions. One significant difficulty is that the lattice calculations treat a quark-antiquark pair in the quark-gluon plasma rest frame, whereas in a heavy ion collision a c ¯c pair is not produced at rest. This challenge becomes more acute in higher energy collisions: at LHC energies, c ¯c pairs which if produced in vacuum would yield J/ψ mesons with transverse momenta many times their rest mass will be copious. Even in collisions at SPS and RHIC energies, the collective flow developed by the hot medium in which the c ¯c pair finds itself is considerable. A rigorous determination of the v-dependence of the screening length Ls(T) for a heavy quark-antiquark pair in a quark-gluon plasma moving with velocity v would therefore be a significant advance. We provide one, albeit for hot N = 4 super Yang-Mills theory.

N = 4 super Yang-Mills (SYM) theory is a conformally invariant theory with two parameters: the rank of the gauge group Nc and the ’t Hooft coupling λ = g2SYM/Nc. We shall define the screening length Ls below, based upon an analysis of a fundamental Wilson loop describing the dynamics of a color-singlet quark-antiquark “dipole” moving with velocity v along, say, the x3-direction through the hot strongly interacting N = 4 SYM plasma. In the rest frame of the dipole, which sees a hot wind blowing in the x3-direction, the contour C of the Wilson loop is given by a rectangle with large extension T in the t-direction, and short sides of length L along some spatial direction. Evaluating this Wilson loop (whether ultimately in QCD or in N = 4 SYM as we do here) will teach us about the L-dependent color singlet quark-antiquark potential and hence allow us to define a screening length in the presence of a hot wind.

According to the AdS/CFT correspondence [8], in the large-Nc and large-λ limits the thermal expectation value ⟨WF(C)⟩ for the Wilson loop in the absence of a wind velocity can be calculated using the metric for a 5-dimensional curved space-time describing a black hole in anti-deSitter (AdS) space [3]. Calling the fifth dimension r, the black hole horizon is at some r = r0 and we add a probe D3-brane extended along the x1, x2, x3 directions at some r = Λ ≫ r0. The external quarks described by the Wilson loop are open strings ending on the probe brane. The prescription for evaluating ⟨WF(C)⟩ is that we must find the extremal action surface in the five-dimensional AdS spacetime whose boundary at r = Λ is the contour C in Minkowski space R5,1. ⟨WF(C)⟩ is then given by exp[isos(C)] = exp[iE(C)T], with S the ac-
tion of the extremal surface $\Sigma$. For the time-like Wilson loop we analyze, $S(\Sigma)$ is proportional to the time $T$, meaning that $E(\Sigma)$ can be interpreted as the energy of the dipole. In the limit of infinitely heavy quarks, i.e. $\Lambda \to \infty$, $E(\Sigma) \propto \Lambda$ but this divergence comes from the $L$-independent self-energy of the quark or antiquark taken separately. We are only interested in the $L$-dependent part of $E(\Sigma)$, which is finite in the $\Lambda \to \infty$ limit, and we take this limit henceforth $\dagger$. 

To describe a hot wind in the $x_3$-direction, we boost the five-dimensional AdS black hole metric, obtaining
\begin{equation}
    ds^2 = -Adt^2 + 2Bdtdx_3 + Cdx_3^2 + r^2(dx_1^2 + dx_2^2) + f^{-1}dr^2
\end{equation}
where $f = \frac{c^2}{R^2}(1 - \frac{\ell^2}{r^2})$ with $R$ the curvature radius of the AdS space and where we have defined
\begin{equation}
    A = \frac{r_1^2}{R^2} - \frac{r_3^2}{r^2R^2}, \quad B = \frac{r_1^2}{R^2} + \frac{r_3^2}{r^2R^2}, \quad C = \frac{r_1^2}{R^2} + \frac{r_3^2}{r^2R^2}
\end{equation}
and $r_1 = r_0^2 \sin^2 \eta = \frac{r_0^2}{R^2}$ and $r_3 = r_0^2 \cos^2 \eta = r_0^2 \gamma^2 v^2$, with $v$ the velocity of the wind, $\eta$ its rapidity, and $\gamma = 1/\sqrt{1 - v^2}$. Here, the temperature $T$ of the Yang-Mills theory at $v = 0$ is given by the Hawking temperature of the black hole, $T = \frac{r_0}{\pi R^2}$, and $R$ and the string tension $1/2\pi\alpha'$ are related to the ‘t’Hooft coupling by $\frac{\alpha'}{\pi} = \sqrt{\lambda}$. 

The short side of $C$ can be chosen to lie in the $(x_1, x_3)$ plane, at an angle $\theta$ between the quark and the antiquark, and the boundary conditions are imposed at $y = \pm \infty$.

Thus, the boundary condition for $x_3(\Sigma)$ can be taken as $x_3(\pm \frac{\ell}{2} \sin \theta) = \pm \frac{\ell}{2} \cos \theta$. For the bulk coordinate $r(\sigma)$, we implement the requirement that the world sheet has $C$ as its boundary by imposing $r(\pm \frac{\ell}{2} \sin \theta) = \Lambda \to \infty$. It proves convenient to define $y( r/\rho_0) \equiv z( x_3/\rho_0) R^2 = x_3\pi T$ and hence $\ell \equiv Lr_0/R^2 = L\pi T$ and to define a rescaled $\tilde{\sigma} = \sigma r_0/R^2$ and promptly drop the tilde. The boundary conditions become
\begin{equation}
    y \left( \pm \frac{\ell}{2} \sin \theta \right) = \infty \quad z \left( \pm \frac{\ell}{2} \sin \theta \right) = \pm \frac{\ell}{2} \cos \theta .
\end{equation}

The action (3) now takes the form
\begin{equation}
    \frac{S}{T} = E = K \int_0^T d\sigma \mathcal{L}
\end{equation}
with the constant prefactor given by $K = \sqrt{\lambda}T$ and with the Lagrangian
\begin{equation}
    \mathcal{L} = \sqrt{\left( y'^4 - \cosh^2 \eta \right) \left( 1 + \frac{y'^2}{y^2 - 1} \right) + z'^2 (y^4 - 1)}
\end{equation}

\begin{figure}[h]

\caption{String world sheet for wind with velocity $v = 0.7$ blowing at an angle $\theta = 45^\circ$ relative to the dipole. The solution has integration constants $p = 1.325$ and $q = 1.109$, which correspond to $\theta = 45^\circ$ and $\ell = 0.689$. (This $\ell$ is the maximum possible for this $v$ and $\theta$.) $\sigma = x_1$ extends from $-(\ell/2)\sin \theta$ to $(\ell/2)\sin \theta$. (a) $y(\sigma)$. (b) $z(\sigma) - \sigma$ is the deviation of the string world sheet away from $z = \sigma$, the straight line at $\theta = 45^\circ$ between the quark and the antiquark.

Eqs. (7) and (8) with the boundary conditions (4) can be integrated numerically and in Fig. 1 we present an example. Somewhat counterintuitively, the projection of the string world sheet onto the $(x_1, x_3)$-plane is not a straight line connecting the quark and the antiquark, but rather has a sinusoidal form. This behavior arises for all values of $\theta$ except $\theta = 0$ or $\pi/2$ (wind parallel or perpendicular to the dipole) for which the projection is indeed a straight line. Furthermore, we see that even though there is a wind blowing in the $z$-direction, $y(\sigma)$ is even and $z(\sigma)$ is odd for any angle $\theta$, meaning that the string world sheet is not dragged at all by this wind. This conclusion is antithetical to that for the world sheet of an isolated string ending on a single quark or antiquark, analyzed in Ref. [12], and in qualitative agreement with the conclusion that “meas feel no drag” reached in a different context in Ref. [13]. The $\theta = \pi/2$ case is particularly
we need only analyze the shape of the world sheet in the several values of \( \cosh \).

The energy can be written as

\[
E = K \int_{y_c}^\infty dy \frac{y^4 - \cosh^2 \eta}{(y^4 - 1)(y^4 - y_c^4)},
\]

and can be made finite by subtracting the self-energy of an isolated quark and antiquark.

In Fig. 2a we plot \( \ell \) as a function of \( q \) from Eq. (9) for several values of \( \cosh^2 \eta \). Analysis of Eq. (9) shows that, for any \( \cosh^2 \eta \), \( \ell \) decreases at large \( q \) like \( \ell \approx 1.198/\sqrt{q} \). This corresponds to an energy \( E \propto -K/\sqrt{q} \approx -K/\ell \propto -\sqrt{\lambda}/L \), and thus describes a quark and antiquark separated by a small distance \( L \) interacting via an attractive Coulomb potential. Small values of \( L \) can also be achieved by choosing \( q \to 0 \), as \( \ell \propto q \) at small \( q \). These world sheets have \( E \propto \text{const} + L^2 \), meaning that these small \( L \) solutions have much higher energy than those describing Coulomb attraction, and so are not of interest. Since \( \ell \) is everywhere positive and goes to zero for both small and large \( q \), at some \( q_m \) it has a maximum value \( \ell_{\text{max}} \) as illustrated in Fig. 2a. For \( \ell > \ell_{\text{max}} \), no extremal world sheet bounded by the Wilson loop \( C \) exists, with the exception of disconnected world sheets “hanging from” the quark or the antiquark Wilson line separately, describing their self energies. Hence, for \( \ell > \ell_{\text{max}} \) there is no \( L \)-dependent potential between the quark and antiquark \([11]\). We can therefore define a screening length \( L_s \) by \( L_s \equiv \ell_{\text{max}}/\pi T \)[20]. Analysis of (9) shows that for \( v = 0 \), \( q_m = 0.96 \) and \( \ell_{\text{max}} = 0.869 \) are numbers of order 1 while for large \( \cosh^2 \eta \), \( q_m \approx 1.24 \cosh \eta \) and \( \ell_{\text{max}} \approx 0.743/\sqrt{\cos \eta} = 0.743(1-v^2)^{1/4} \). This motivates writing the screening length as

\[
L_s = \frac{f(v)}{\pi T}(1-v^2)^{1/4}, \tag{11}
\]

in so doing defining the function \( f(v) \). We plot \( f(v) \) in Fig. 2b, and find that its velocity dependence is mild meaning that the dominant \( v \)-dependence of \( L_s \) is the factor \( (1-v^2)^{1/4} \). For general \( \theta \), the results are very similar: the interaction is screened for \( L/L_s \) is given by (11) with a slightly different \( f(v) \), shown in Fig. 2b for a wind blowing parallel to the dipole. As the angle \( \theta \) between wind and dipole changes from \( 0^\circ \) to \( 90^\circ \), \( f(v) \) interpolates between the two curves in Fig. 2b.

Our central result is that, in \( N = 4 \) SYM theory, the dominant dependence of the screening length of a dipole in a hot wind on the wind velocity is \( L_s(v,T) \sim L_s(0,T)/\sqrt{v} \), with the remaining weak dependence described by the function \( f(v) \) in Fig. 2b. The dominant velocity dependence suggests that \( L_s \) should be thought of as \( \propto (\text{energy density})^{-1/4} \), since the energy density increases like \( \gamma^2 \) as the wind velocity is boosted. It turns out that \( L_s(0,T) \) is within a factor of two of that for QCD. Although it would be interesting to see whether \( f(0) \) is closer to that in QCD in more QCD-like theories with gravity duals, given the availability of reliable lattice calculations of the screening potential in QCD itself this is not a pressing issue. It would certainly be interesting to see whether \( L_s(v,T) \sim L_s(0,T)/\sqrt{v} \) persists in other theories with gravity duals, as this would support its applicability to QCD [21].

If the velocity-scaling of \( L_s \) that we have discovered holds for QCD, it will have qualitative consequences for quarkonium suppression in heavy ion collisions. For illustrative purposes, consider the explanation of the \( J/\Psi \)}
suppression seen at SPS and RHIC energies proposed in Refs. \([7, 14]\) and lattice calculations of the \(qar{q}\)-potential indicate that the \(J/\Psi(1S)\) state dissociates at a temperature \(\sim 2.1T_c\) whereas the excited \(\chi_c(2P)\) and \(\Psi'(2S)\) states cannot survive above \(\sim 1.2T_c\); so, if collisions at both the SPS and RHIC reach temperatures above \(1.2T_c\) but not above \(2.1T_c\), the experimental facts (comparable anomalous suppression of \(J/\Psi\) production at the SPS and RHIC) can be understood as the complete loss of the “secondary” \(J/\Psi\)’s that would have arisen from the decays of the excited states, with no suppression at all of \(J/\Psi\)’s that originate as \(J/\Psi\)’s. Taking eq. (11) at face value, the temperature \(T_{\text{diss}}\) needed to dissociate the \(J/\Psi\) decreases \(\propto (1 - v^2)^{1/4}\). As can be seen from Fig. 3 this indicates that \(J/\Psi\) suppression at RHIC will increase markedly (as the \(J/\Psi(1S)\) mesons themselves dissociate) for \(J/\Psi\)’s with transverse momentum \(p_T\) above some threshold that is at most \(\sim 9\) GeV and would be \(\sim 5\) GeV if the temperatures reached at RHIC are \(\sim 1.5T_c\). These illustrative considerations point to a novel quarkonium suppression pattern at transverse momenta above 5 GeV, a regime that is within experimental reach of future high-luminosity runs at RHIC and that will be studied thoroughly at the LHC. If the temperatures reached at the LHC are, say, \(\sim 3T_c\), the LHC could discover \(\Upsilon\) suppression, but only at high enough \(p_T\).

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16. At this point the present calculation becomes qualitatively distinct (even in the \(v \rightarrow 1\) limit) from the calculation of the light-like Wilson loop used to determine the “jet quenching parameter” \(\tilde{q}\) in Refs. \([10, 11]\). Although we can apply our calculation of screening in a hot wind in the \(v \rightarrow 1\) limit, it does not reduce to the calculation of \(\tilde{q}\) in this limit. In this paper, we calculate the screening potential between infinitely massive (mass \(\propto \Lambda\), where we have taken \(\Lambda \rightarrow \infty\)) test quarks. To reproduce the calculation of \(\tilde{q}\), we must instead first take the \(v \rightarrow 1\) limit at finite \(\Lambda\), and only then are free to take \(\Lambda \rightarrow \infty\). The two limits do not commute. The string world sheet which in our screening calculation is time-like becomes space-like for \(r_0^2 > \Lambda^2\); meaning for high enough wind velocity. Whereas a time-like world sheet (and \(\langle W \rangle \sim \exp[\langle S\rangle\]) has a sensible physical interpretation as the calculation of an interaction energy and hence yields information about screening, a space-like world sheet (and \(\langle W \rangle \sim \exp[-S]\)
with \( S \) real) has a sensible physical interpretation in the context of high energy scattering in an eikonal approximation [10]. Finally, it is possible to show that upon starting with \( \eta \) and \( \Lambda \) finite and taking the \( \Lambda \rightarrow \infty \) limit while keeping \( r_0^4 \gamma^2 \gg \Lambda^4 \), one does obtain the jet quenching parameter \( \hat{q} \) of Ref. [10]. In calculating \( \hat{q} \) using this limiting procedure, there is only a single extremal world sheet: the “trivial world sheet”, found and discarded in Ref. [10], does not even arise.

[17] With the exception of a wind parallel to the dipole (\( \theta = 0 \)), which can be formulated easily as a special case.

[18] If \( p \) and hence \( z' \) were nonzero, the boundary condition \( z(+\ell/2) = z(-\ell/2) \) would then require that there be point(s) at which \( z' = 0 \). From (7) and (8) we see that at such a point \( y'^2 \) would be negative. Hence, \( p = 0 \).

[19] In this theory as in QCD one expects a residual attraction that falls exponentially with \( L \) for \( L > L_s \). Seeing such effects (which are nonperturbative in \( \alpha' \)) requires analysis beyond extremizing the Nambu-Goto action.

[20] Alternatively, we could define \( L_s \) as the length \( L_c \) below which the nontrivial extremal world sheet has less energy than two disconnected world sheets hanging from the quark and antiquark Wilson lines separately. \( L_c \) is of order 10% smaller than our \( L_s \) at \( v = 0 \); they become equivalent at high enough \( v \).

[21] In a hot wind, the large-spin mesons in the confining, nonsupersymmetric theory studied by Peeters, Sonnenchein and Zamaklar [13] dissociate beyond a maximum wind velocity. The relation between the size \( L \) of these mesons and their dissociation velocity \( v \) is consistent with \( L \propto (1 - v^2)^{1/4} \), in qualitative agreement with the result we have obtained analytically in a simpler setting.

[22] Any \( J/\Psi \) mesons formed by recombination will have transverse momenta much lower than those at which our calculation is relevant. Also, it is only at much higher \( p_T \) that one has to take into account the possibility that \( J/\Psi \) mesons could form outside the hot medium [15].