THE BATALIN-TYUTIN FORMALISM ON THE COLLECTIVE COORDINATES QUANTISATION OF THE SU(2) SKYRME MODEL

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Abstract

We apply The Batalin-Tyutin constraint formalism of converting a second class system into a first class system for the rotational quantisation of the SU(2) Skyrme model. We obtain the first class constraint and the Hamiltonian in the extended phase space. The vacuum functional is constructed and evaluated in the unitary gauge and a multiplier dependent gauge. Finally, we discuss the spectrum of the extended theory. The use of the BT formalism on the collective coordinates quantisation of the SU(2) Skyrme model leads an additional term in the usual quantum Hamiltonian that can improve the phenomenology predicted by the Skyrme model.

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1 Introduction

Most of the static properties of the Nucleon and the Delta are well reproduced by the Skyrme model\cite{1}, which describes baryons and their interactions through soliton solutions in a nonlinear type sigma model. If our goal is to improve the usual phenomenology predicted by the Skyrme model then, we believe that there are two simple ways to deal: first, in the classical sector, we can introduce higher order term in the derivatives of the pion field\cite{3}. This procedure leads the physical values to approach to the correct experimental values; second, in the quantum sector, we can analyze with more detail the process of the collective coordinates quantisation of the Skyrme model\cite{6}. Thus, the purpose of this paper is to develop the latter option, investigating with more rigorous the SU(2) Skyrme model in quantum mechanics. Some authors\cite{4} have pointed out that when the quantisation is performed with more care, an extra term appears in the usual Skyrme quantum Hamiltonian. The aim of the present work is to employ the BT\cite{9} constraint formalism\cite{10} with the objective to overcome the problem of operator ordering that occurs in the Dirac brackets of the collective coordinates operators\cite{5}.

The remaining part of this paper is organized as follows. In sec 2 we give a short outline of the BT formalism. In sec. 3 we apply the BT method in the SU(2) Skyrme model when we derive the first class constraints, an involutive extended Hamiltonian and the vacuum functional \( Z \) for two different gauge fixing. In sec 4 by imposing strongly the first class constraint, we obtain the usual mass spectrum with an additional term. Finally, in sec 5 we give the conclusions.

2 Brief review of the BT formalism

Let us consider a system described by a Hamiltonian \( H_0 \) in a phase space \((\phi^i(x), \pi_i(x))\) with \( i=1,\ldots,N \). We assume that the fields are bosonic( extension to include fermionic degrees of freedom and to the continuous case can be done in a straightforward way). We also suppose that this system contains a set of linearly independent bosonic second class constraints. Denoting \( T_\alpha = T_\alpha(\phi, \pi) \), with \( \alpha = 1,\ldots, M < 2N \), then the matrix

\[ \begin{bmatrix}
T_1 & T_2 & \cdots & T_M
\end{bmatrix} \]

\footnote{The Batalin Tyutin formalism is a powerful framework for the quantisation of gauge field theories that can be used when we depart with nontrivial Dirac brackets relations.}
\begin{equation}
\Delta_{\alpha\beta}(x, y) = \{T_{\alpha}(x), T_{\beta}(y)\},
\end{equation}

has a nonvanishing determinant. The inclusion of other constraints (i.e. fermion or first class) is a matter of technical detail and poses no problems in developing the formalism.

The general philosophy of the BT formalism is to convert second class constraint into first class ones. This is achieved by introducing new dynamical fields, one for each second class constraint. We note that the connection between the number of second class constraint and the new fields in one to one is to keep the same number of the physical degrees of freedom in the resulting extended theory. We denote these new fields by \(\Psi^\alpha(x)\) and assume that they have the following Poisson algebra

\begin{equation}
\{\Psi^\alpha(x), \Psi^\beta(y)\} = \omega^{\alpha\beta}(x, y),
\end{equation}

where \(\omega^{\alpha\beta}\) is an invertible field independent antisymmetric matrix. The new dynamical fields are introduced to extend the original phase space

\begin{equation}
(\phi, \pi) \oplus (\Psi).
\end{equation}

The new first class constraints of the system in the extended phase space (3) are denoted by \(\tilde{T}_\alpha\). Of course, these depend on the new fields \(\Psi^\alpha\), written as

\begin{equation}
\tilde{T}_\alpha = \tilde{T}_\alpha(\phi, \pi, \Psi),
\end{equation}

and satisfy the boundary condition

\begin{equation}
\tilde{T}_\alpha(\phi, \pi, 0) = T_\alpha(\phi, \pi),
\end{equation}

where the right hand side of (5) is just the original set of second class constraint. The characteristic of the new constraints is that they are assumed to be strongly involutive, i.e.

\begin{equation}
\{\tilde{T}_\alpha, \tilde{T}_\beta\} = 0.
\end{equation}

The solution of (6) can be achieved by considering \(\tilde{T}_\alpha\) expanded as a power series expansion is
\[ \tilde{T}_\alpha = \sum_{n=0}^{\infty} T_{\alpha}^{(n)}, \quad (7) \]

where \( T_{\alpha}^{(n)} \) is a term of order \( n \) in \( \Psi \). Compatibility with the boundary condition \((3)\) requires that \( T_{\alpha}^{(0)} = T_\alpha \). The first order correction term in the infinite series \((9)\) is

\[ T_{\alpha}^{(1)}(x) = \int dy X_{\alpha\beta}(x, y) \Psi_\beta(y), \quad (8) \]

and the involutive bracket \((6)\) leads the follow relation

\[ \int dzdz' X_{\alpha\mu}(x, z) \omega^{\mu\nu}(z, z') X_{\nu\beta}(z', y) = -\Delta_{\alpha\beta}(x, y). \quad (9) \]

This determines \( T_{\alpha}^{(1)} \). Equation \((9)\) does not give \( X_{\alpha\mu} \) univocally, because it also contains the still unknown \( \omega_{ab} \). What we usually do is to choose \( \omega_{ab} \) in such a way that the new fields are unconstrained. It is opportune to mention that this procedure is not always possible to be done \((2)\). It is possible to show \((4)\) that when \( X_{\alpha\beta} \) does not depend on \( \phi \) and \( \pi \), only \( T_{\alpha}^{(1)} \) contributes in the series \((7)\), defining the first class constraint.

The next step in the BT formalism is that any dynamical function \( H(\phi, \pi) \) (in instance, the Hamiltonian) has also to be properly modified in order to be strongly involutive with the first class constraints \( \tilde{T}_\alpha \). Denoting the strongly involutive function by \( \tilde{H} \), then we have

\[ \{ \tilde{H}, \tilde{T}_\alpha \} = 0, \quad (10) \]

subject to the boundary condition \( \tilde{H}(\phi, \pi, 0) = H_c(\phi, \pi) \), where \( H_c \) is the canonical Hamiltonian. The general solution \((1)\) for the involutive Hamiltonian \( \tilde{H} \), which can be expanded in an infinite series, is

\[ \tilde{H} = H_c + \sum_{n=1}^{\infty} H^{(n)}, \quad (11) \]

where \( H^{(n)} \) is given by

\[ H^{(n)} = -\frac{1}{n} \int dxdydz \Phi^i(x) \omega_{ij}(x, y) X^{jk}(y, z) G_k^{(n-1)}(z), \quad (n \geq 1), \quad (12) \]

and the generating functions \( G_k^{(n)} \), in the case that only \( T_{\alpha}^{(1)} \) contributes in the series \((7)\), read
Here, $\omega_{ij}$ and $X_{ij}$ are the inverse matrices of $\omega_{ij}$ and $X_{ij}$ respectively. This concludes this brief review on the BT construction of the first class system which is strongly involutive.

3 The BT formalism on the SU(2) Skyrme model

The Lagrangian of the SU(2) Skyrme model performed in a semi-classical collective coordinates expansion reads

$$L = -M + \lambda Tr[\partial_0 A \partial_0 A^{-1}] = -M + 2\lambda \dot{a}^i \dot{a}^i,$$

where $M$ is the soliton mass, $\lambda$ is the inertia moment and $A$ is a SU(2) matrix which can be expanded as $A = a^0 + a^i \tau$. The primary constraint is

$$T_1 = a^i a^i - 1 \approx 0.\quad (15)$$

Introducing the conjugate momentum

$$\pi^i = \frac{\partial L}{\partial \dot{a}^i} = 4\lambda \dot{a}^i,$$

we can now rewrite the canonical Hamiltonian in the form

$$H_c = \pi^i \dot{a}^i - L = 4\lambda \dot{a}^i \dot{a}^i - L = M + 2\lambda \dot{a}^i \dot{a}^i = M + \frac{1}{8\lambda} \sum_i \pi^i.\quad (17)$$

Then, the standard quantization is made where we replace $\pi^i$ by $-i\partial/\partial a_i$ in (17), leading to

$$H = M + \frac{1}{8\lambda} \sum_{i=0}^{3} (\frac{-i}{\partial a_i^2}).\quad (18)$$
A typical polynomial wavefunction \((a^0 + ia^1)^l\) is an eigenvector of the Hamiltonian \([18]\), with the eigenvalues given by \([3]\)

\[
E = M + \frac{1}{8\lambda} l(l + 2), \quad l = 1, 2, \ldots.
\]

Following the usual Dirac standard procedure\([3]\) we find a secondary constraint,

\[
T_2 = a^i \pi_i \approx 0 \quad (20)
\]

obtained by conserving \(T_1\) with the total Hamiltonian

\[
H_T = H_c + \lambda_c T_1, \quad (21)
\]

where \(\lambda_c\) is a Lagrange multiplier. No further constraints are generated via this iterative procedure. The constraints \(T_1\) and \(T_2\) are second-class, satisfying the Poisson algebra

\[
\Delta_{\alpha,\beta} = \{T_\alpha, T_\beta\} = -2\epsilon_{\alpha\beta} a^i a^i, \quad \alpha, \beta = 1, 2 \quad (22)
\]

where \(\epsilon_{\alpha\beta}\) is the antisymmetric tensor normalized as \(\epsilon_{12} = -\epsilon^{12} = -1\).

In order to convert this system into first-class one, the first step is to transform \(T_\alpha\) into the first-class by extending the phase space. Following the BT formalism\([3]\), we introduce new auxiliary coordinates\([3]\) \(b^j\) to convert the second-class constraint \(T_\alpha\) into the first-class one in the extended phase space, and consider that the Poisson algebra of these new coordinates is given by

\[
\{b^\alpha, b^\beta\} = \omega^{\alpha\beta}, \quad (23)
\]

where \(\omega^{ij}\) is an antisymmetric matrix. Then, the modified constraint in the extended phase space is given by

\[
\tilde{T}_\alpha (a^i \pi_i, b^j) = T_\alpha + \sum_{n=1}^\infty T_\alpha^{(n)}; \quad T_\alpha^{(n)} \sim (b^j)^n, \quad (24)
\]

satisfying the boundary condition

\(^2\)This wave function is also eigenvector of the spin and isospin operators, written as \([3]\)

\[
J^k = \frac{1}{2}(a_0 \pi_k - a_k \pi_0 - \epsilon_{klm} a_l \pi_m) \quad \text{and} \quad I^k = \frac{1}{2}(a_k \pi_0 - a_0 \pi_k - \epsilon_{klm} a_l \pi_m).
\]

\(^3\)We now rewrite the auxiliary fields defined in \([3]\) as \(b^\alpha\).
\[ \tilde{T}_\alpha(a^i \pi_i, 0) = T_\alpha. \] (25)

To obtain \( \tilde{T}_\alpha \) we follow the procedure discussed in sec. 3. The first order correction term in the infinite series\(^9\) is given by
\[ T^{(1)}_\alpha = X_{\alpha \beta} b^\beta, \] (26)
and the first-class constraint algebra of \( \tilde{T}_\alpha \) requires the condition as follows
\[ X_{\alpha \mu} \omega^{\mu \nu} X_{\beta \nu} = -\Delta_{\alpha \beta}. \] (27)

A possible choice for \( \omega^{\mu \nu} \) and \( X_{\alpha \beta} \) satisfying (23) and (27) is\(^9\)
\[ \omega^{\mu \nu} = 2 \epsilon^{\mu \nu}, \] (28)
\[ X_{\alpha \beta} = \begin{pmatrix} 1 & 0 \\ 0 & a^i a^i \end{pmatrix}. \] (29)

As was emphasized in Ref.\(^9\), there is a natural arbitrariness in this choice, which corresponds to the canonical transformation in the extended phase space. Using (24), (25), (26), (28) and (29), the new set of constraints is found to be
\[ \{ \tilde{T}_\alpha, \tilde{T}_\beta \} = 0. \] (31)

Thus, we have all first-class constraints in the extended phase space by applying the BT formalism systematically. We observe further that the terms in the series (24) for \( n > 1 \) are redundant. This completes the conversion of the second-class constraints \( T_\alpha \) to first-class ones \( \tilde{T}_\alpha \).

Next, we derive the corresponding involutive Hamiltonian in the extended phase space. Considering what we have seen in sec. 3 and that we also have \( T^{(n)} = 0 \) for \( n > 1 \), the corrections that give \( \tilde{H} \) can be written as
\[ \tilde{H}^{(n)} = -\frac{1}{n} b^\mu \omega_{\mu\nu} X^{\nu\rho} G_\rho^{(n-1)}, \quad (n \geq 1), \] (32)

where \( G_\rho^{(n)} \) is given by (13) and

\[ \omega_{\mu\nu} = -\frac{1}{2} \epsilon_{\mu\nu}, \] (33)

\[ X^{\nu\rho} = \begin{pmatrix} 1 & 0 \\ 0 & -\frac{1}{a^i a^i} \end{pmatrix}. \] (34)

Using the expression for the second-class constraints \( \tilde{T}_\alpha \) (eqs. (13) and (20)) and the Hamiltonian (17) as well as (32), (33) and (34), it is possible to compute the terms appearing in the power series of the involutive Hamiltonian

\[
\tilde{H} = M + \frac{1}{8\lambda} \left[ 1 - \frac{b_1^i a^i}{a_i a^i} + \left( \frac{b_1^i a^i}{a_i a^i} \right)^2 - \left( \frac{b_1^j a^j}{a_i a^i} \right)^3 + \ldots \right] \pi_j \pi_j \\
- \frac{1}{4\lambda} b_1^2 \left[ 1 - \frac{b_1^i a^i}{a_i a^i} + \left( \frac{b_1^j a^j}{a_i a^i} \right)^2 - \left( \frac{b_1^j a^j}{a_i a^i} \right)^3 + \ldots \right] \pi_i \\
+ \frac{1}{8\lambda} b_2^2 \left[ 1 - \frac{b_1^i a^i}{a_i a^i} + \left( \frac{b_1^j a^j}{a_i a^i} \right)^2 - \left( \frac{b_1^j a^j}{a_i a^i} \right)^3 + \ldots \right] a^i a^i. \] (35)

If the convergence ratio, \( R = \left| \frac{b_1^i a^i}{a_i a^i} \right| \), is less than one, the extended canonical Hamiltonian can be summed in a geometric series

\[
\tilde{H} = M + \frac{1}{8\lambda} \frac{(a^i a^i)}{a_i a^i + b^i \pi_j \pi_j} - \frac{1}{4\lambda} \frac{(a^i a^i) b^2}{a_i a^i + b^i \pi_j} + \frac{1}{8\lambda} \frac{(a^i a^i)^2 (b^2)^2}{a_i a^i + b^1}, \] (36)

which is involutive with the first-class constraints,

\[ \{ \tilde{T}_\alpha, \tilde{H} \} = 0 \quad (\alpha = 1, 2). \] (37)

We now look for the vacuum functional \( Z \). A consistent way of doing this is by means of the path integral formalism in Faddev-Senjanovic [10]. Let us identify the new variables \( b^\mu \) as a canonically conjugate pair \((\phi, \pi_\phi)\) in the Hamiltonian formalism,
\begin{align*}
  b^1 & \to 2\phi, \\
  b^2 & \to \pi_\phi,
\end{align*}
(satisfying (23), (28) and (29)). Then, the general expression for the vacuum functional reads
\begin{equation}
Z = N \int [d\mu] \exp \left\{ i \int dt (\dot{a}^i \pi_i + \dot{\phi} \pi_\phi - \tilde{H}) \right\},
\end{equation}
with the measure \([d\mu]\) given by
\begin{equation}
[d\mu] = [d\phi][d\pi_\phi] \prod_{\alpha,\beta=1}^2 \delta(\bar{T}_\alpha) \delta(\chi_\beta) |\text{det}\{\bar{T}_\alpha, \chi_\beta\}|,
\end{equation}
where the Hamiltonian is now expressed in terms of \((\phi, \pi_\phi)\) instead of \(b^\mu\).

The gauge fixing conditions \(\chi_\beta\) are chosen so that the determinant occurring in the functional measure is nonvanishing. Let us now compute \(Z\) in different gauges. First, we consider the unitary gauge
\begin{align*}
  \chi_1 &= a^i a^i - 1, \\
  \chi_2 &= a^i \pi_i.
\end{align*}

The vacuum functional \(Z\) takes the form
\begin{equation}
Z = N \int [da^i][d\pi_i][d\phi][d\pi_\phi] \delta(a^i a^i - 1 + 2\phi) \\
\delta(a^i \pi_i - a^i a^i \pi_\phi) \delta(a^i a^i - 1) \delta(a^i \pi_i) \exp \left\{ i \int dt (\dot{a}^i \pi_i + \dot{\phi} \pi_\phi - M - \frac{1}{8\lambda} \frac{a^i a^i}{a^i a^i + 2\phi} \pi_j \pi_j + \frac{1}{4\lambda} \frac{a^i a^i}{a^i a^i + 2\phi} \pi_\phi a^j \pi_j - \frac{1}{8\lambda} \frac{(a^i a^i)^2}{a^i a^i + 2\phi} \pi_\phi^2) \right\}.
\end{equation}

We note that due \(\delta(a^i a^i - 1)\) in (42) the Faddev-Senjanovic determinant can be absorbed in the normalization. The \(\phi\) and \(\pi_\phi\) integrations are trivially performed. After exponentiating the delta function \(\delta(a^i \pi_i)\) with Fourier variable \(\xi\), we obtain
Performing the integration over $\pi_i$, we finally obtain

$$Z = N \int [da^i][d\pi_i][d\xi] \delta(a^i a^i - 1) \exp\{i \int dt (\dot{a}^i \pi_i - M - \frac{1}{8\lambda} \pi_i \pi_i - \xi a^i \pi_i)\}. \quad (43)$$

where we have absorbed the integration over $\xi$ into the normalization. Expression (14) is, therefore, seen to reproduce the original Lagrangian (14) subject to the constraint (15). This result shows, without doubt, the consistency of the theory. Now, let us consider a multiplier dependent gauge

$$\chi_1 = \pi_\phi + p_1, \quad \chi_2 = \phi. \quad (45)$$

The vacuum functional $Z$ takes the form

$$Z = N \int [da^i][d\pi_i][d\phi][d\pi_\phi] \delta(a^i a^i - 1 + 2\phi) \delta(a^i \pi_i - a^i a^i \pi_\phi) \delta(\pi_\phi + p_1) \delta(\phi)|2a^i a^i| \exp\{i \int dt (\dot{a}^i \pi_i + \dot{\phi} \pi_\phi + \dot{\lambda}^1 p_1 - M

- \frac{1}{8\lambda} a^i a^i \pi_j \pi_j + \frac{1}{4\lambda} a^i a^i + \frac{1}{2\phi} \pi_\phi a^i \pi_j

- \frac{1}{8\lambda} (a^i a^i)^2 \pi_\phi^2)\}. \quad (46)$$

The $\phi$ and $p_1$ integrations are trivially done. Then, the Faddeev-Senjanovic determinant can be absorbed in the normalization. Exponentiating the delta function $\delta(a^i \pi_i - a^i a^i \pi_\phi)$ with Fourier variable $\xi$, and doing successively the $\pi_i$ and $\pi_\phi$ integrations, we obtain

$$Z = N \int [da^i][d\lambda^1] \delta(a^i a^i - 1) \exp\{i \int dt [-M + 2\lambda \dot{a}^i \dot{a}^i - \frac{1}{32\lambda} - (4\lambda \lambda^1 + 1)\frac{\dot{\lambda}^1}{2}]\}, \quad (47)$$
where we have absorbed the $\xi$ integration into the normalization. We observe that this expression differs from (44), but both are expected to yield identical S-matrix elements by the Fradkin-Vilkovisky theorem [11]. The expression (47) illustrates the generality of the BT-Formalism, since it can not be obtained by conventional quantisation methods.[10]

4 The spectrum of the extended theory

In order to obtain the spectrum of the extended theory, the constraints (30) are treated as strong equations. Consequently, we can replace the canonical conjugate pair $(b^1, b^2)$ by the collective coordinates $(a_i, \pi_i)$ in the Hamiltonian (36), which reads

$$\tilde{H} = M + \frac{1}{8\lambda} a_i^i \pi_j \pi_j - \frac{1}{4\lambda} a_i^i \pi_i \pi_i.$$  (48)

Now $\pi_j$ describes a free momentum particle and its representation on the collective coordinates space $a^i$ is given by

$$\pi_j = -i \frac{\partial}{\partial x_j}.  \quad \text{(49)}$$

If we substitute the expression (49) in (48), we obtain

$$\tilde{H} = M - \frac{1}{8\lambda} a_i^i \partial_j \partial_j + \frac{1}{4\lambda} a_i^i \partial_j + \frac{1}{4\lambda} a_i^i \partial_i \partial_j,$$  (50)

with the eigenvalues given by

$$E = M + \frac{1}{8\lambda} [l(l + 2) + l(l - 2)].$$  (51)

Comparing expression (51) with (19) we see that an extra term appears in the last equation. We can observe that for the Nucleon state, $l=1$, the extra term is negative and according to A. Toda [13], this result improves the physical values given by the Skyrme model.

\footnote{Remember that the eigenvectors are written in a polynomial form $(a^0 + ia^1)^l$.}
5 Conclusions

In this work we have applied the Batalin-Tyutin formalism to the rotational quantisation of the SU(2) Skyrme model. We note that the conventional Dirac method\cite{8} presents operator ordering difficult and BT formalism, in principle, overcomes these problems. We have obtained the involutive Hamiltonian, and when we impose strongly the new involutive constraints, we get the usual mass spectrum of the theory plus an additional term. This result has been found by many authors\cite{4} using different procedures. Thus, as we have remarked in the introduction, the combined results of introducing higher derivative terms \cite{3,14}, and an adequate quantisation which contains the constrains information, can lead the Skyrme model to predict, with more success, the physical parameters of the mesons and baryons. In a future paper we intend to apply the Batalin-Tyutin formalism on the quantisation of Skyrme model with the inclusion of higher derivative terms.

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