The QCD equation of state from improved staggered fermions

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We calculate the equation of state in 2+1 flavor QCD at finite temperature with physical strange quark mass and almost physical light quark masses using lattices with temporal extent $N_\tau = 8$. Calculations have been performed with two different improved staggered fermion actions, the asqtad and p4 actions. Overall, we find good agreement between results obtained with these two $O(a^2)$ improved staggered fermion discretization schemes. A comparison with earlier calculations on coarser lattices is performed to quantify systematic errors in current studies of the equation of state. We also present results for observables that are sensitive to deconfining and chiral aspects of the QCD transition on $N_\tau = 6$ and 8 lattices. We find that deconfinement and chiral symmetry restoration happen in the same narrow temperature interval.

1 Introduction

A detailed and comprehensive understanding of the thermodynamics of quarks and gluons, e.g. of the equation of state is most desirable and of particular importance for the phenomenology of relativistic heavy ion collisions. In particular, the interpretation of recent results from RHIC on jet quenching, hydrodynamic flow, and charmonium production rely on an accurate determination of the energy density and pressure as well as an understanding of both the deconfinement and chiral transitions. For vanishing chemical potential, which is appropriate for experiments at RHIC and LHC, lattice calculations of the EoS can be performed with an almost realistic quark mass spectrum. In addition, calculations at different values of the lattice cutoff allow for a systematic analysis of discretization errors and will soon lead to a controlled continuum extrapolation of the EoS with physical quark masses.

2 Improved actions and the calculational setup

Studies of the QCD equation of state are most advanced in lattice regularization schemes that use staggered fermions. In this case, improved actions have been developed that reduce $O(a^2)$ discretization effects efficiently. Such a reduction is mandatory because a quantitative lattice determination of the EoS requires rather coarse lattices as one has to take into account that pressure and energy density are dimension 4 operators, which are difficult to calculate on fine lattices. At tree level which is relevant for the high temperature phase both the asqtad and the p4 actions were found to give rise to only small deviations from the asymptotic ideal gas limit already on lattices with temporal extent $N_\tau = 6$. At $N_\tau = 8$ the differences from the continuum Stefan-Boltzmann value are at the 1% level. At moderate values of the temperature non-perturbative effects contribute to the cutoff dependence. In particular in the hadronic phase at
low temperature, the breaking of flavor (taste) symmetry, inherent to staggered fermions away from the continuum limit, leads to an $O(a^2)$ distortion of the pseudoscalar hadron spectrum and may influence the thermodynamics in the confined phase. In order to judge the importance of different effects that contribute to the cutoff dependence of thermodynamic observables, we have performed calculations with both p4 and asqtad actions which deal with these systematic effects in different ways. We extend previous lattice calculations on $N_\tau = 6$ lattices (asqtad\cite{3} and p4\cite{4}) to $N_\tau = 8$ lattices\cite{5}. In these studies the bare strange quark mass ($m_s$) was tuned such that the zero temperature kaon mass acquired a constant value of 500 MeV (p4) and a slightly larger value for asqtad (570 MeV). The light quark mass was held at a fixed ratio to the strange one of $m_l = 0.1m_s$ leading to a Goldstone pion mass of about 220 MeV at $T = 0$. The temperature scale was set by using the Sommer parameter $r_0 = 0.469(7) \text{ fm}$,\cite{8} which is determined from the shape of the heavy quark potential.

3 The trace anomaly, pressure and energy density

The basic thermodynamic quantity most convenient to calculate on the lattice is the trace anomaly in units of the fourth power of the temperature $\Theta^{\mu\nu}/T^4$. This is given by the derivative of $p/T^4$ with respect to the temperature,

$$\frac{\Theta^{\mu\nu}(T)}{T^4} \equiv \frac{\epsilon - 3p}{T^4} = T \frac{\partial}{\partial T}(p/T^4).$$  \hspace{1cm} (1)

As the pressure is given by the logarithm of the partition function, $p/T = V^{-1} \ln Z$, the calculation of the trace anomaly requires only the evaluation of rather simple expectation values. In Fig.\cite{1} we show results for $\Theta^{\mu\nu}/T^4$ obtained with both the asqtad and p4 actions. The new $N_\tau = 8$ results\cite{5} have been obtained on lattices of size $32^4 \times 8$ and the additional zero temperature calculations, needed to carry out the necessary vacuum subtractions, have been performed on $32^4$ lattices. The $N_\tau = 6$ results are taken from Ref. 3 (asqtad) and Ref. 4 (p4), respectively.

We find that the results with asqtad and p4 formulations are in good agreement. In particular, both actions yield consistent results in the low temperature range (see Fig.\cite{1} (right)), in which $\Theta^{\mu\nu}/T^4$ rises rapidly, and at high temperature, $T \gtrsim 300$ MeV. This is also the case for the cutoff dependence in these two regimes. At intermediate temperatures, $200 \text{ MeV} \lesssim T \lesssim 300$ MeV, the two actions show differences and cutoff effects are more pronounced. In the transition region the cutoff effects can be well accounted for by a shift of the $N_\tau = 6$ data towards lower temperatures. This reflects the cutoff dependence of the transition temperature and may also....

Figure 1: The trace anomaly, $(\epsilon - 3p)/T^4$ on $N_\tau = 6$ and 8 lattices. Results are obtained with the p4 and asqtad actions. The right panel shows the low temperature part of the trace anomaly in detail. Here we show fits to the data, as well as a comparison with the resonance gas model (dashed and dash-dotted lines).
subsume residual cutoff dependencies of the zero temperature observables used to determine the temperature scale in the transition region.

In Fig. 2 we show results for the energy density and three times the pressure. Using Eq. 1, the pressure is obtained by integrating $\Theta^{\mu\nu}/T^5$ over the temperature. Crosses with error bars indicate the systematic error on the pressure that arises from different integration schemes. The energy density ($\epsilon$) is then obtained by combining results for $p/T^4$ and $(\epsilon - 3p)/T^4$. Finally, we show in in Fig. 2(right) $p/\epsilon$ as function of $\epsilon$. From a spline fit to that data we also calculate the square of the velocity of sound, $c_s^2$.

4 The QCD transition

The bulk thermodynamic observables $p/T^4$, $\epsilon/T^4$ discussed in the previous sections are sensitive to the change from hadronic to quark-gluon degrees of freedom that occur during the QCD transition; they thus reflect the deconfining features of this transition. In a similar vein, the temperature dependence of quark number susceptibilities gives information on thermal fluctuations of the degrees of freedom that carry a net number of light or strange quarks, i.e., $\chi_q \sim \langle N^2_q \rangle$, with $N_q$ denoting the net number of quarks carrying the charge $q$. Quark number susceptibilities change rapidly in the transition region as the carriers of charge, strangeness or baryon number are heavy hadrons at low temperatures but much lighter quarks at high temperatures.

Another important aspect of the QCD transition is the spontaneous breaking (restauration) of the chiral symmetry. In Fig. 3(right) we show the quantity $\Delta_{l,s}$,

$$\Delta_{l,s}(T) = \frac{\langle \bar{\psi}\psi \rangle_{l,T} - \frac{m_l}{m_s} \langle \bar{\psi}\psi \rangle_{s,T}}{\langle \bar{\psi}\psi \rangle_{l,0} - \frac{m_l}{m_s} \langle \bar{\psi}\psi \rangle_{s,0}},$$

which is defined in terms of light and strange quark chiral condensates evaluated at zero and non-zero temperature, respectively. This is an appropriate observable that takes care of the additive renormalizations in the chiral condensate by subtracting a fraction, proportional to $m_l/m_s$, of the strange quark condensate from the light quark condensate. To remove the multiplicative renormalization factor we divide this difference at finite temperature by the corresponding zero temperature difference, calculated at the same value of the lattice cutoff.

It is evident from Fig. 3 that $\Delta_{l,s}(T)$ varies rapidly in the same narrow temperature range as the bulk thermodynamic observables and in particular the light quark number fluctuations as shown in Fig. 3(left). Based on this agreement we conclude that the onset of liberation of light quark and gluon degrees of freedom (deconfinement) and chiral symmetry restoration occur in
Figure 3: The light quark number susceptibility, calculated on lattices with temporal extent $N_\tau = 6$ and 8 (left) and the subtracted chiral condensate normalized to the corresponding zero temperature value (right). The band corresponds to a temperature interval $185 \text{ MeV} \leq T \leq 195 \text{ MeV}.

the same temperature range in QCD with almost physical values of the quark masses, i.e., in a region of the QCD phase diagram where the transition is not a true phase transition but rather a rapid crossover.

Furthermore, we note that the observed cutoff effects in the chiral condensate can to a large extent be absorbed in a common shift of the temperature scale. A global shift of the temperature scale used for the $N_\tau = 6$ data sets by 5 MeV for the p4 and by 7 MeV for the asqtad action makes the $N_\tau = 6$ and 8 data sets coincide almost perfectly. This is similar in magnitude to the cutoff dependence observed in $(\epsilon - 3p)/T^4$ and again seems to reflect the cutoff dependence of the transition temperature as well as residual cutoff dependencies of the zero temperature observables used to determine the temperature scale.

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