We discuss the criteria for teleporting coherent states from simple considerations about information exchange during the teleportation process.

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I. INTRODUCTION

Quantum teleportation has emerged in recent years as a major paradigm of theoretical and experimental quantum information. The initial approaches using discrete variables have been extended to continuous quantum variables. Though there is a general agreement about the main ideas of quantum teleportation, discussions have appeared about the significance and the evaluation criteria of real, and thus imperfect, teleportation experiments.

Here our basic requirement for successful teleportation will be that the information content of the teleported state is higher than the information content of any (classical or quantum) copy of the input state, that he tries to compare with the initial state that was teleported by Alice, knowing that this state is a pure coherent state $|\alpha\rangle$. The quality of this comparison may be characterized by a generalized fidelity, which is the conditional probability $P(\alpha|I)$ that $|\alpha\rangle$ was sent, given the available information $I$. This quantity may eventually be averaged over the set of initial states $|\alpha\rangle$. The difference with the “usual” fidelity is simply that $I$ does not need to be a quantum state, it can as well be classical information.

II. CLASSICAL VS QUANTUM FIDELITY

Considering first the usual case where the output is a quantum state, the density matrix of the teleported state can be expanded on a coherent state basis $|\beta\rangle$, where the probability to reconstruct the state $|\beta\rangle$ is denoted as $P(\beta)$. The (standard) fidelity is then simply:

$$F_{\text{quant}} = \int d^2 \beta P(\beta) |\langle \beta | \alpha \rangle|^2 = \int dx dy P(x, y) \times \exp (-\frac{(x-x_0)^2}{2} - \frac{(y-y_0)^2}{2})$$  \hspace{1cm} (1)

where $\alpha = (x_0 + iy_0)/2$, $\beta = (x + iy)/2$, and the vacuum noise variance has been normalized to 1. In a gaussian noise hypothesis, $P(x, y)$ is a normalized gaussian function of $x$ and $y$, centered on the values $x_0$ and $y_0$. The $x$ and $y$ variances are the equivalent input noise $N_X^\text{out}$ and $N_Y^\text{out}$ in the teleportation process, which are discussed in ref. [8]. One has thus:

$$P(x, y) = \frac{1}{2\pi N_X^\text{out} N_Y^\text{out}} e^{-\frac{(x-x_0)^2}{2N_X^\text{out}}} - \frac{(y-y_0)^2}{2N_Y^\text{out}}$$  \hspace{1cm} (2)

We obtain by carrying out the integration:

$$F_{\text{quant}} = \frac{2}{\sqrt{(2 + N_X^\text{out})(2 + N_Y^\text{out})}} e^{-\frac{(x_0-x_b)^2}{2(2 + N_X^\text{out})} - \frac{(y_0-y_b)^2}{2(2 + N_Y^\text{out})}}$$  \hspace{1cm} (3)

The fidelity is thus strongly peaked on the condition $(x_0 = x_b, y_0 = y_b)$, which is obtained for unity gain $(g_T = 1)$ in the teleportation scheme. Assuming that this condition is satisfied (this is easy to do in practice), one obtains:

$$F_{\text{quant}} = \frac{2}{\sqrt{(2 + N_X^\text{out})(2 + N_Y^\text{out})}}$$  \hspace{1cm} (4)

This quantity is clearly relevant for characterizing quantum teleportation, and can reach the value $F_{\text{quant}} = 1$ when $N_X^\text{out} = N_Y^\text{out} = 0$, i.e. when the teleportation noise is zero. For getting a qualitative idea of the classical limit of the teleportation process, let us consider the case where the input beam is split in two equal parts, and two homodyne measurements shifted by $\pi/2$ are done on each part. The splitting introduces a vacuum fluctuation mode $v_1$. Then the measured quantities are used to reconstruct the input state, which introduces another vacuum fluctuations mode $v_2$. It can be shown (see e.g. [8]) that an optimized measurement procedure will give:

$$X_{\text{out}} = X_{\text{in}} + X_{v_1} + X_{v_2}$$
$$Y_{\text{out}} = Y_{\text{in}} - Y_{v_1} + Y_{v_2}$$  \hspace{1cm} (5)

From this equation one gets the classical limits:

$$N_X^\text{out} = (\Delta X_{v_1})^2 + (\Delta X_{v_2})^2 = 2$$
$$N_Y^\text{out} = (\Delta Y_{v_1})^2 + (\Delta Y_{v_2})^2 = 2$$
$$F_{\text{quant}} = 1/2$$  \hspace{1cm} (6)

This corresponds to having twice the shot noise, or using the terminology of ref. [8], two “qduies”, one being associated with the measurement stage, and the other one with the reconstruction stage. On the other hand, by using EPR beams [3], the fluctuations of the two stages are perfectly correlated for one quadrature, and anticorrelated for the other one, yielding ideally $N_X^\text{out} = N_Y^\text{out} = 0$. 

Evaluating quantum teleportation of coherent states

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We consider now the case where the output is directly obtained from the measurement outcome, without really reconstructing a quantum state. However, a quite similar reasoning can be applied: one can guess that the input state is $|\beta\rangle$ with a probability $P_{cl}(\beta)$, and the conditional probability to obtain the correct answer $|\alpha\rangle$ given the available information $P_{cl}(\beta)$ is:

$$F_{\text{class}} = \int d^2 \beta P_{cl}(\beta) |\langle \beta | \alpha \rangle|^2 = \int dx \, dy P_{cl}(x, y) \times \exp\left(-\frac{(x-x_a)^2}{4} - \frac{(y-y_a)^2}{4}\right)$$

(7)

with the same definitions as before. The difference with the previous case is that the $x$ and $y$ variances are now the noises $N_X^m$ and $N_Y^m$ associated with the measurement, rather than with the full reconstruction procedure. By the same calculation as above, one obtains thus:

$$F_{\text{class}}^{g\gamma=1} = \frac{2}{\sqrt{(2 + N_X^m)(2 + N_Y^m)}}$$

(8)

Considering again the case where the input beam is split in two equal parts, and two homodyne measurements shifted by $\pi/2$ are done on each part, the classical measurement outcome $X_m$ and $Y_m$ are given by:

$$X_m = X_{in} + X_{v1}$$

$$Y_m = Y_{in} - Y_{v1}$$

(9)

However, contrary to the previous case, the measurement noise must take into account not only the equivalent input noise $N_X^v = N_Y^v = 1$ which is due to the beamsplitting process, but also the noise in the input mode, which is $N_X^m = N_Y^m = 1$ for a coherent state input. One gets thus again $N_X^m = N_Y^m = 2$, and therefore $F_{\text{class}}^{g\gamma=1} = 1/2$.

### III. DISCUSSION

These two cases correspond to two different views on the teleportation process, which in some sense are associated to either an “Heisenberg” (i.e. operatorial) or a “Schroedinger” (i.e. quantum state) picture.

In the first (Heisenberg-type) view, the teleportation process is described by the operatorial equations already given above:

$$X_{out} = X_{in} + X_{meas} + X_{rec}$$

$$Y_{out} = Y_{in} - Y_{meas} + Y_{rec}$$

(10)

where $\text{meas}$ and $\text{rec}$ correspond respectively to the measurement and reconstruction procedures. Perfect teleportation is by definition obtained for $X_{out} = X_{in}$, $Y_{out} = Y_{in}$. Correspondingly, the noise in the input beam $(X_{in}, Y_{in})$ does not contribute to the equivalent input noise in eq. (4). The classical limit is thus twice the shot noise (two “quduties”), and perfect teleportation is obtained when the measurement and reconstruction noises perfectly compensate each other. This point of view is the one used in ref. [8], and fits naturally with previous work on QND criteria [9].

In the second (Schroedinger-type) view, the state is first measured. As said above, the input noise is now relevant, and it is equal to shot-noise for a coherent state. This input noise plus the beam-splitting noise gives again a classical limit equal to twice the shot-noise. On the other hand, in this picture there is no extra noise associated to the reconstruction: given a measured $\beta$, one can exactly reconstruct the coherent state $|\beta\rangle$, by using a deterministic translation of the vacuum.

Some confusions, in particular in the previous version of this note, may have been due to mixing up these two point of views.

### IV. CONCLUSION

Finally, we note that purification procedures [10,11] or recently demonstrated entanglement criteria [12] are compatible with the $F = 1/2$ limit. However, following ref. [8], we give below two arguments that question the meaning of quantum teleportation of coherent states for small transmission efficiency of the EPR beams.

First, the intensity of one EPR beam can be measured in order to use that information to reduce the noise of the second beam [13]. Then the noise of the corrected beam can be reduced below shot-noise only when the losses on each beam are less than 50%. This example is closely related to the non-separability argument of ref. [8], which also requires that “conditional squeezing” can be obtained on one EPR beam, given a measurement that is done on the other one. The requirement that the losses on each EPR beam are less than 50% is not compatible with the $F = 1/2$ boundary, which can tolerate arbitrarily large losses, but would be a consequence of the requirement $F > 2/3$.

Second, a possible use for quantum teleportation is to send a quantum state from Alice to Bob for quantum cryptography purposes. In that case, one must worry about the amount of information which can be eavesdropped during the teleportation process. For simplicity, let us consider again a teleportation scheme using EPR beams, with a finite degree of squeezing, and transmission losses. It is assumed that Eve is able to perfectly eavesdrop the classical channel, and that she has full access to the losses along at least one “transmission arm” of the EPR beam (this is a strong hypothesis, but it is usually done for evaluating the security of standard quantum cryptography). The simplest solution for Eve is to build her own teleported state, and she will be successful if this state has an equivalent noise smaller than the one achieved by Bob. It can be shown simply, and it is physically obvious, that as long as the EPR channel efficiency $\eta$ is smaller than 1/2, Eve can obtain a teleported copy of the input state which is better than the one obtained.
by Bob. Such low values of $\eta$ can be obtained for values of $F$ larger than $1/2$, but smaller than $2/3$.

It has been shown in [11] that purifications procedure can be initiated as soon as $F > 1/2$, and may lead to high fidelity values. However, as long as such procedures are not used, the above arguments lead to the conclusion that quantum teleportation with $F < 2/3$ may have severe limitations as a quantum communication tool.

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