Multi-structure non-linear semi-parametric optimal weighting parameters estimation algorithm based on adaptive sparse representation deconstruction model and integrative COD comprehensive model

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Abstract. For the combined orbit determination (COD) satellite-network system which is made of GEO-IGSO-MEO satellite-constellation and LEO satellites, the satellites-trajectory are not precise enough, because the meticulous satellite dynamics model, credible observation data model and optimal parameters estimation method are not taking into consideration. The high-level adaptive sparse-deconstruction dynamics model, non-linear semi-parametric combined orbit determination comprehensive model is put forward. By using the above models, adaptive weighting iterative wavelets decomposition algorithm of sparse-representation deconstruction model and multi-structure non-linear semi-parametric optimization weighted estimation algorithm of integrative combined orbit determination comprehensive models are designed. Simulated calculation results show that results showed that the high-precision sparse representation method and the optimization model estimation method can improve COD precision, and when comprehensive model systematic & random error parameters and adaptive sparse-representation deconstruction model parameters are estimated by integrative nonlinear semi-parametric comprehensive model synchronously, COD ultimate trajectory calculation estimation precision can get significant improvement.

1. Introduction

Combined orbit determination is an effective data processing method to estimate space-based multi-satellite trajectory parameters synchronously by synthesizing GEO-satellites to LEO-satellite observations[1]. Meticulous satellite dynamic-model, credible measurement-data model & optimal parameters estimation method are three main factors to realize high-level COD results. The available estimation method seeks the accurate modeling of the trajectory-attitude dynamics model and to-be-estimated parameters optimization estimation. However, for inaccuracy trajectory models of satellites orbit perturbation, traditional experiential acceleration methods only eliminate a little dynamic modeling error [2-3]. For satellite-earth observation data models, the present research methods establish a whole parameter fusion model, but this kind of model doesn’t take a full account modeling to amalgamate error [4-5], and multi-parameter numerical computation method is only a near-linear estimation algorithm of least square method substantially.

The trajectory-perturbation attributions are comprehensively analyzed in the paper, a high-level adaptive sparse-deconstruction model of GEO-IGSO-MEO and LEOs dynamics is put forward. By deconstructing COD measurement-model systematic errors and other random errors, a compositive
nonlinear semi-parametric combined orbit determination comprehensive model is put forward, and Multi-structure non-linear semi-parametric optimization weighted estimation algorithm of integrative COD comprehensive models is designed.

2. Multi-satellite COD comprehensive model based on adaptive sparse deconstruction model and near-linear semi-parametric data model
Multi-satellite COD network system is made up of GEO-IGSO-MEO satellites constellation, COD ground command & control system, multi-LEO satellites, LEO STTC center. Assuming that space-based multi-satellite constellation includes 5 GEOs, 3 IGSOs, 4 MEOs, and certain amount of LEOs, and the specific composition is as follows in Figure. 1.

2.1 Adaptive sparse representation deconstruction dynamics model
For multi-satellite COD trajectory parameters solving problem, the classic dynamics and observation model is as follows:

\[
\begin{cases}
Y = G(X,t) + \epsilon \\
\dot{X} = F(X,t)
\end{cases}
\]

\[
X = [x_{G0}, \ldots, x_{G1}, \ldots, x_{I0}, \ldots, x_{I1}, \ldots, x_{M0}, \ldots, x_{M1}, \ldots, x_{U0}, \ldots, x_{U1}]^T, \\
\epsilon \text{ is observation error matrix; } G \text{ is computation matrix; } Y \text{ is observation matrix; } F \text{ is dynamics relationships.}
\]

Based on model parameter deconstruction theory, a multi-dimension deconstruction representation method is put forward as follows:

\[
\dot{X} = F(X,t) + \tilde{f}_{pa} + \tilde{f}_{sa} + \ldots
\]

Here \( F \) refers to perturbing force of traditional physics models; \( \tilde{f} \) refers to perturbing-force of estimated parameter models; \( e_p \) refers to trajectory perturbation noise. Supposing satellite perturbing force \( F \) can generate signal \( X_{pt} \) [7], then we can get the following equation:

\[
X_{pt} = \sum_{\nu} x_{pt} \phi_{\nu}
\]

If \( D \) is definite, the parameter matrix \( x_{pt} \) adaptive sparse-representation of is equal to solving the following question:

\[
(P_0 - \text{noise}) \min \left\| X_{pt} - D \alpha \right\|_2^2 + \lambda \| \alpha \|_1, \quad X_{pt} = D\alpha + z
\]

Wherein: \( X_{pt} \) refers to noised signal (\( \| \cdot \|_2^2 \leq \epsilon^2 \)), and \( \lambda \) seeks the best combination of allowable error and parameters sparseness. When the above model sparseness is met, the model (3) can be simply indicated as:
In the light of Lagrange least square principle, equation (4) is equal to finding the solution of the below non-linear minimization problem:

\[
\arg \min_{\alpha} \left[ \frac{1}{2} \| X_{\alpha} - D \alpha \|^2 + \lambda \| \varepsilon \|^2 \right]
\]  

(5)

2.2 Non-linear semi-parametric COD comprehensive model

According to model \( Y = G(X,t) + \varepsilon \), if we consider estimated measurement systematic errors \( g(a) \), the probably model errors \( s \) and other random errors \( \varepsilon \) of non-linear factors, the credible non-linear semi-parametric measurement data model of GEO-IGSO-MEO constellation and LEOs can be expanded:

\[
Y = G(X,t) + g(a) + s + \varepsilon
\]  

(6)

When adaptive sparse-representation deconstruction representation and non-linear semi-parametric measurement data model are summarized, and 5GEO-4IGSO-3MEO constellation and M LEOs are considered as background, then the traditional COD model (1) can extend and refine to an integrated and comprehensive COD model as follows:

\[
\begin{align*}
Y_i(t) &= G_i(X_i,t) + g_{i,a}(t) + s_i + e_i(t) \\
& \quad \text{...} \\
Y_{i2}(t) &= G_{i2}(X_{i2},t) + g_{i2,a}(t) + s_{i2} + e_{i2}(t) \\
E_i(t) &= G_i(X_i,t) + g_{i2,a}(t) + s_i + e_i(t) \\
& \quad \text{...} \\
Y_{i2}(t) &= G_{i2}(X_{i2},t) + g_{i2,a}(t) + s_{i2} + e_{i2}(t) \\
X_1 &= F_{i1}(X_i,t) + F_{i2}(X_i,t) \\
& \quad \text{...} \\
X_2 &= F_{i1}(X_i,t) + F_{i2}(X_i,t) \\
X_{ii}(t) &= X_{ii}^0 \\
X_{ii}(t) &= F_{i1}(X_{ii}(t)) + F_{i2}(X_{ii}(t)) + e_{i2}(X_{ii}(t)) \\
& \quad \text{...} \\
X_{ii}(t) &= F_{i1}(X_{ii}(t)) + F_{i2}(X_{ii}(t)) + e_{i2}(X_{ii}(t)) \\
X_{ii}(t) &= X_{ii}^e
\end{align*}
\]  

(7)

3. Multi-structure non-linear semi-parametric optimal weighting parameters estimation algorithm

3.1 Adaptive weighting iterative wavelets decomposition algorithm of dynamics sparse representation deconstruction model

Taking advantage of the high-precision wavelet deconstruction fusion processing method, the trajectory perturbation signal \( X_{\alpha} \) can be resolved as wavelet subspace \( V_0, W_0, W_1, \ldots, W_{j-1} \). Assuming we can get function \( \phi(t) \) and \( \psi(t) \) [8], based on two-scale wavelet transform algorithm \( X_{\alpha} \) is decomposed as the following equation:

\[
X_{\alpha} = \sum_j d_j \psi_j(t) + \sum_j d_{j-1} \psi_{j-1}(t) + \sum_j d_{j-2} \psi_{j-2}(t) + \sum_j c_{j-2} \phi_{j-2}(t)
\]

Supposing \( \rho(a) = \| e \| = \sum_j \rho(a_j) \| e_j \| \cdot \rho(a_j) \), the non-linear minimization problem (5) can be transformed to next minimization solution:

\[
\arg \min_{\alpha} \left[ \frac{1}{2} \| X_{\alpha} - \sum_j \phi_j \alpha_j \|_2^2 + \sum_j \lambda_j \rho(\alpha_j) \right]
\]  

(8)

So adaptive weighting iterative wavelets decomposition algorithm form to solve dynamics sparse representation deconstruction model of non-linear minimization problem shows as follows:
\[
\begin{align*}
\begin{cases}
\text{Given } \alpha^{(0)}, D \\
\alpha^{(k+1)} = Q(\alpha^{(k)}) D^T \left( \lambda I_{n \times n} + DQ(\alpha^{(k)}) D^T \right)^{-1} X_n
\end{cases}
\end{align*}
\]

(9)

3.2 Multi-structure non-linear semi-parametric optimization weighted estimation algorithm of integrative COD comprehensive models

For the COD solution based on 5GEO-4IGSO-3MEO constellation and M LEOs, the fundamental hypothesis is as follows:

\[
Y_i = \begin{bmatrix}
Y_{j_1}(t_1) \\
\vdots \\
Y_{j_1}(t_n) \\
Y_{j_2}(t_1) \\
\vdots \\
Y_{j_2}(t_n) \\
\vdots \\
Y_{j_2}(t_1) \\
\vdots \\
X_{j_1}(t_1) \\
\vdots \\
X_{j_1}(t_n) \\
\vdots \\
X_{j_2}(t_1) \\
\vdots \\
X_{j_2}(t_n)
\end{bmatrix}, \quad G_i = \begin{bmatrix}
G_{j_1}(t_1) \\
\vdots \\
G_{j_1}(t_n) \\
G_{j_2}(t_1) \\
\vdots \\
G_{j_2}(t_n) \\
\vdots \\
G_{j_2}(t_1) \\
\vdots \\
F_{j_1}(t_1) \\
\vdots \\
F_{j_1}(t_n) \\
\vdots \\
F_{j_2}(t_1) \\
\vdots \\
F_{j_2}(t_n)
\end{bmatrix}, \quad \epsilon_i = \begin{bmatrix}
\epsilon_{j_1}(t_1) \\
\vdots \\
\epsilon_{j_1}(t_n) \\
\epsilon_{j_2}(t_1) \\
\vdots \\
\epsilon_{j_2}(t_n) \\
\vdots \\
\epsilon_{j_2}(t_1) \\
\vdots \\
\gamma_{j_1}(t_1) \\
\vdots \\
\gamma_{j_1}(t_n) \\
\vdots \\
\gamma_{j_2}(t_1) \\
\vdots \\
\gamma_{j_2}(t_n)
\end{bmatrix}
\]

For the integrated COD comprehensive models (7), solving trajectory parameters \(X\) is equivalent to finding the minimum value based on \(\rho_j (j = 1, \ldots, M) \quad 0 < \rho_j < 1, \sum M \rho_j = 1\):

\[
\arg \min_{X} \sum_{j=1}^{M} \rho_j \sigma_j^{-2} \left\| Y_j - G_j(X) \right\|_2^2
\]

(10)

Namely, \(\text{MSEM}(\hat{X}) = \frac{1}{2} \left[ \sum_{j=1}^{M} V_j^T V_j \right]^{-1} \left[ \sum_{j=1}^{M} (V_j^T t(t_j^T V_j)^{-1} W_j)^T \left( V_j^T t(t_j^T V_j)^{-1} W_j \right)^T \right] + \frac{1}{2} \sum_{j=1}^{M} L_j \left( V_j^T + W_j^T \right) L_j^T \).

Wherein: \(V_j = \sqrt{\rho_j \sigma_j} \left( \frac{\partial G_j(t_j)}{\partial X_j} \right)_{t \in \mathbb{R}} \) and \(W = \sqrt{\rho \sigma} \left( \frac{\partial G_j(t_j)}{\partial X_j} \right)_{t \in \mathbb{R}} \).

For the integrated COD comprehensive model (7), assuming \(e\) is Gaussian white noise[9], solving trajectory parameters \(X\), semi-parametric COD comprehensive model systematic errors \(a\), random error \(s\) and adaptive sparse representation deconstruction model parameters \(\lambda\) are equal to solving the two minimum-value problem:

\[
\begin{align*}
\arg \min_{X, a} \sum_{j=1}^{M} \rho_j \sigma_j^{-2} \left\| Y_j - G_j(X) - g_j(a) - s_j \right\|_2^2 \\
\arg \min_{X, a, s} \sum_{j=1}^{M} \rho_j \sigma_j^{-2} \left\| \hat{X} - F_{j_1}(X_j) - F_{j_2}(X_j) \right\|_2^2
\end{align*}
\]

(11)  (12)

If \(\sigma_j^2\) and \(\sigma^2\) are given, based on the above models and algorithms design, the detailed multi-structure non-linear semi-parametric optimization weighted estimation algorithm as follows:

**Step 1** giving trajectory parameters original iterative matrix \(X^{(0)}\), adaptive sparse-representation coefficients initial-value vector \(X^{(0)}\), \(\delta > 0\), \(\lambda\) and initial weight-vector \(\rho^{(0)}\), and taking advantage of satellite orbit adaptive sparse representation deconstruction dynamics models, computing all instantaneous state vectors \(X^{(0)}(t)\);

**Step 2** assuming COD comprehensive model systematic errors initial-values vector \(a^{(0)}\), random model error initial-values vector \(s^{(0)}\) and weight-vector \(\rho^{(0)}\), and combining \(X^{(0)}(t)\) of step 2 results, then we can compute vector \(X = \left[ \begin{array}{c} X^{(0)}(t) \\ a^{(0)} \end{array} \right] \) using Gauss-Newton iterative algorithm;
Step 3 computing $V'(\hat{x}^0), W_j(\hat{x}), G_j(\hat{x}), V'_j, V'_j$, MSEM($\hat{x}$) of $\hat{x}$, and solving $\hat{\rho}$ which meets $\text{tr}(\text{MSEM}(\hat{x}, \hat{\rho})) = \min \text{tr}(\text{MSEM}(\hat{x}, \hat{\rho}))$;

Step 4 solving dynamic trajectory models the minimum value of parameters $x^{(l)}$, and then again $X_2 = [x^{(l)} x^{(l)}]^{\top}$ using weighting iterative wavelets decomposition algorithm;

Step 5 solving weight value $\hat{\rho}$ resembling step 2-4 which meets $\min \text{tr}(\text{MSEM}(\hat{x}, \hat{\rho}))$;

Step 6 accomplishing above all iterative computation steps until parameters $x_n$, $x_n$, $x_n$ approaches to convergence, so iterative-result of $\hat{x}_n$ is the final expectant value of GEO-IGSO-MEO constellation and LEOs trajectory parameters.

4. Simulation computation and results analysis

4.1 Simulation data and important assumptions

COD system: LEOs($U_1, U_2, U_3$), GEO-IGSO-MEO constellation($G_1, G_2, I_1, I_2, I_3, M_1, M_2$);

Simulation data: gaining range-sum data based on satellite trajectory ephemeris, and then overlaying systematic errors $A \frac{\cos 2 \pi / T}{T}$ and random Gauss white noises;

Adaptive sparse representation model: gaining based on standard wavelet function symmlet-8.

4.2 COD simulation conditions and results

Discussing 5 GEO-4 IGSO-3 MEO constellation and LEOs COD, one is solving trajectory parameters with not adopting non-linear semi-parametric COD comprehensive model; the other is using multi-structure non-linear semi-parametric optimization weighted estimation algorithm of integrative COD comprehensive models. Concrete simulation conditions and calculation results are shown in Table 1.

According to COD precision results of Case 1 and Case 2, trajectory and model errors can be restrained commendably adopting non-linear semi-parametric optimization weighted estimation algorithm synchronously, and final trajectory estimation-precision is further getting better markedly.

Table 1. COD simulation conditions and calculation results

| CASE | Observation data(available data) | Simulation systematic error | COD models computation mode | LEOs type | LEO COD precision(RMS) |
|------|---------------------------------|-----------------------------|-----------------------------|-----------|-------------------------|
| 1    | 172800 (5880)                   | A=20                        | COD with not considering sparse parameters and nonlinear semi-parametric models | $U_1$     | 3.82  4.65  3.75  7.0907 |
|      |                                 |                             | COD by non-linear semi-parametric optimal weighting estimation algorithm of integrative comprehensive models | $U_2$     | 12.25 13.55 10.15 20.8971 |
|      |                                 |                             |                             | $U_3$     | 8.36  12.8  4.35  15.8938 |
| 2    | 172800 (5880)                   | A=20                        |                             | $U_1$     | 1.82  2.61  1.38  3.4683 |
|      |                                 |                             |                             | $U_2$     | 8.16  5.65  3.36  10.4784 |
|      |                                 |                             |                             | $U_3$     | 5.16  7.38  2.36  9.3091  |

5. Conclusions

This paper dissected the existing deficiency of COD satellite dynamics trajectory model, measurement data model and optimization weighted estimation algorithm, then a high-level adaptive sparse-deconstruction model is put forward. And then, Multi-structure non-linear integrative COD comprehensive models are constructed by sparse-representation deconstruction model and non-linear semi-parametric combined measurement model. Based on the adaptive weighting-iterative wavelets
decomposition algorithm of deconstruction model, multi-structure non-linear semi-parametric optimization weighted estimation algorithm of integrative COD comprehensive models is designed.

The COD simulation conditions and calculation results show that if satellites trajectory fundamental model is only used, it will produce relatively big error, and is hard to satisfy high-precision requirements. If we consider trajectory parameters, system and model errors by using multi-structure non-linear semi-parametric optimization weighted estimation algorithm of integrative COD comprehensive models, the trajectory precision of LEOs can be improved evidently.

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