Sparse Reconstruction-Based Detection of Spatial Dimension Holes in Cognitive Radio Networks

Yahya H. Ezzeldin†, Radwa A. Sultan† and Karim G. Seddik‡

† Electrical Engineering Department, Alexandria University
‡ Electronics Engineering Department, American University in Cairo (AUC)

Email: yahya.ezzeldin@ieee.org, radwasultan@ieee.org, kseddik@aucegypt.edu

September 11, 2013
In Cognitive networks, transmission holes can present themselves in time, frequency as unused durations or empty bands.

In MIMO networks, spatial dimension holes can be present as well.

Spatial dimension holes refer to cases where the receiver antennas are more than transmitting antennas.

Energy Detectors are used to detect time and frequency holes without coordination with the primary users.

Spatial dimension holes cannot be detected by energy detectors as the band is energy-filled in time and frequency.
Motivation:
To detect the spatial dimension holes that are present in a MIMO primary system by extending the operation of the energy detector.

Contribution:
- Developing a spectrum-sensing algorithm using compressive sensing (CS) tools to detect the number of active primary users and hence detect the “spatial” spectrum holes.
- Using CS tools as well as the MMSE MIMO detector at the secondary network for decoding the primary users data for possible relaying of the primary user data.
System Model

Figure: System Model
Primary System Model

- Uplink of a multiuser MIMO (MU-MIMO) system is considered.
- The primary base station (PR-BS) is equipped with $N_{BS}$ antennas.
- The primary BS can demodulate a number of primary users uplink streams, that is less than or equal to the number of antennas at the BS.
- OFDM is used as a channel access technique with $L$ subcarriers.
- The number of active primary users is $N_P$ where $N_P \leq N_{BS}$.
- $x_i \in \mathbb{C}^L$ be a transmit OFDM symbol from the primary user $i$.
- IQ lattice constellations used such as: QPSK, 16QAM or 64QAM.
Secondary System Model

- **OFDM secondary user SU with** $N_S$ **antennas.**
- The SU can operate in the primary network to perform two goals:
  - Detect the activity of the primary users in terms of the maximum number of supported flows $N_{BS}$.
  - Detects the active primary users, decodes their transmitted symbols for possible relaying of the PU data.
- The received signal $y_j(k)$ at the $j$-th antenna of SU from the PU transmission on the $k$-th subcarrier is:

  $$y_j(k) = \sum_{i=1}^{N_P} h_{j,i}^s(k) \cdot x_i(k) + n_j(k), \quad k = 1, \ldots, L,$$

  where:
  - $h_{j,i}^s$: channel coefficients from the $i$-th PU to the $j$-th antenna of the SU.
  - $n_j$: i.i.d. complex Gaussian noise samples received at the $j$-th antenna
    $$\sim \mathcal{CN}(0, \sigma_n^2).$$
Compressive Sensing

- A technique for reconstructing sparse vector $\mathbf{v} \in \mathbb{C}^N$ from a small set of compressive measurements.
- Signal $\mathbf{x}$ is denoted as $K$-sparse if at most $K$ elements of $\mathbf{v}$ are non-zeros.
- Candes et al., demonstrated that reconstruction of a sparse vector $\mathbf{v}$ from underdetermined noisy measurements $\mathbf{r}$, is unique with negligible probability of error by $\ell_1$-minimization:

$$\mathbf{r} = \mathbf{A}\mathbf{v} + \mathbf{n}, \quad \mathbf{r} \in \mathbb{R}^M,$$

where: $\mathbf{A} \in \mathbb{C}^{M \times N}, M \leq N$

$$P1: \quad \min_{\tilde{\mathbf{v}}} ||\tilde{\mathbf{v}}||_{\ell_1}$$

subject to: $||\mathbf{r} - \mathbf{A}\tilde{\mathbf{v}}||_{\ell_2} \leq \epsilon$ (1)

- Techniques such as “subspace pursuit” and “orthogonal matching pursuit” for reconstructing sparse vectors exhibit computational complexity of $\mathcal{O}(NM)$ and $\mathcal{O}(N \log M)$, respectively.
Some signals have non-zero elements arranged in the form of blocks and hence denoted *Block Sparse Signals*.

Recognizing correlation structures in signals such as block sparsity allows for better reconstruction.

Block Sparse signals representation comes naturally in systems that use resource block allocation such as OFDM which is in question in this paper.
Block Sparse Reconstruction

\[ \mathbf{y} = \begin{bmatrix} H_1 & \cdots & H_i & \cdots & H_n \end{bmatrix} \mathbf{x} \]

- \( d \): size of the block.
- \( n \): the number of blocks.
- \( N \): total length of the block sparse vector (\( n \times d \)).

**Figure:** Block Sparse Model
A signal is $k$-block-sparse if at most $k$ blocks are non-zero.

For block sparse signals, we can use a reconstruction problem that more reflects its properties:

$$P2 : \min_{\tilde{x}} \sum_{i=1}^{n} \|\tilde{x}_i\|_{\ell_2}$$

subject to : $\|y - H\tilde{x}\|_{\ell_2} \leq \epsilon$

where $\tilde{x}_i$ represent the elements of vector $\tilde{x}$ from indices $(i - 1)d + 1$ to $id$
Outline

1. Introduction
2. System Model
3. Compressive Sensing
4. Proposed Sensing Strategy
5. Results and Evaluation
6. Conclusion
Proposed Sensing Strategy

Activity Pattern Detection

To detect spatial activity, we customize the convex problem (1), with \( \tilde{x} \in \mathbb{C}^N \) being the target vector to be recovered.

**(Step 1)** Solve the convex problem for \( \tilde{x} \in \mathbb{C}^{N_P} \):

\[
\min_{\tilde{x}} \sum_{i=1}^{N_P} \|\tilde{x}_i\|_{\ell^2}
\]

subject to: \( \|y - H\tilde{x}\|_{\ell^2} \leq \frac{1}{2} \sigma_n^2 \cdot N_S \cdot L \)
Proposed Sensing Strategy

Activity Pattern Detection

(Step 2) Take binary decisions on elements in $\tilde{x}$ as follows:

$$\hat{x}(i) = \begin{cases} 
\tilde{x}(i) & \text{if } |\tilde{x}(i)| \geq \rho \\
0 & \text{otherwise,}
\end{cases}$$

where:

$\hat{x}$: output from the decision operation.

$\rho$: norm of points equidistant form origin and the nearest symbol in the alphabet $\mathbb{A}'$.

Figure: QPSK Constellation with threshold contour showing points to be zeroed out.
Proposed Sensing Strategy

Activity Pattern Detection

(Step 3) Construct the activity vector $\mathbf{a}$ where:

$$\mathbf{a} = \begin{bmatrix} a_1, & a_2, & \ldots, & a_N \end{bmatrix}^T$$

$$a_i = \begin{cases} 1 & \text{if } \|\hat{x}_i\|_0 \geq L/2 \\ 0 & \text{otherwise,} \end{cases}$$

where:

$L$ : Number of subcarriers shared by the users.

(Step 4) Set Sparsity Pattern $(S)$ as the indices of the non-zero elements of $\mathbf{a}$. 
Proposed Sensing Strategy

Demodulating Active Users Symbols

Based on the activity pattern has been detected, we use an MMSE detector to detect the sparse subset of the transmitted signal, $\tilde{x}_s$

$$
\tilde{x}_s = (H_s^H H_s + \sigma_n^2 I)^{-1} H_s^H y
$$

where:

- $H_s$ subset of $H$ of the columns, indexed by the sparsity pattern $(S)$.
- $y$: received signal vector at the $N_S$ antennas of the SU.
- $\sigma_n^2$: noise variance at SU.
Simulation Setup

- Max number of active primary users, $N_P = 8$ (As in LTE Advanced).
- Number of subcarriers $N_{SC} = 72$ subcarriers
- Symbols are QPSK modulated.
- Number of Active PU changes with each simulation.
- Number of antennas at SU changes with each simulation.
Simulation Results

Figure: Activity Detection using $\ell_2/\ell_1$ CS detector and MMSE detector.
Simulation Results

The probability of Error is due to false alarm except for very low SNR, which provides more protection for the primary users.

![Graph showing Probability of Error, misdetection and false alarm in detecting Activity using CS Detector](image)

Figure: Probability of Error, misdetection and false alarm in detecting Activity using CS Detector where $N_P = 2$, $N_{BS} = 8$, $N_S = 4$. 

**Introduction**

**System Model**

**Compressive Sensing**

**Proposed Sensing Strategy**

**Results and Evaluation**

**Conclusion**
Simulation Results

Using genie-aided MMSE decoder with CS, outperforms CS Decoder and MMSE decoder for different activity profiles.

Figure: Symbol Error Rate (SER) using $\ell_2/\ell_1$ CS decoder, MMSE decoder and CS-MMSE detector for (i) 2 out of 8 active users and (ii) 4 out of 8 active users.
Simulation Results

Figure: Symbol Error Rate (SER) using $\ell_2/\ell_1$ CS decoder, MMSE decoder and CS-MMSE detector for (i) 6 out of 8 active users and (ii) 8 out of 8 active users.
Outline

1. Introduction
2. System Model
3. Compressive Sensing
4. Proposed Sensing Strategy
5. Results and Evaluation
6. Conclusion
We have considered the spatial dimension holes in an uplink MU-MIMO primary system.

We proposed a spatial dimension activity detector based on compressive sensing tools and reconstruction of block-sparse signals.

We have shown that the proposed detector outperforms activity detection based on the MMSE estimator.

We shown that using the proposed detector to aid MMSE decoder in a genie-aided model provides reliable decoding results.
Thank You

Yahya H. Ezzeldin

Email: yahya.ezzeldin@ieee.org

http://www.yezzeldin.com