Physics input for modelling superfluid neutron stars with hyperon cores

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ABSTRACT
Observations of massive ($M \approx 2.0 M_\odot$) neutron stars (NSs), PSRs J1614−2230 and J0348+0432, rule out most of the models of nucleon–hyperon matter employed in NS simulations. Here, we construct three possible models of nucleon–hyperon matter consistent with the existence of 2 $M_\odot$ pulsars as well as with semi-empirical nuclear matter parameters at saturation, and semi-empirical hypernuclear data. Our aim is to calculate for these models all the parameters necessary for modelling dynamics of hyperon stars (such as equation of state, adiabatic indices, thermodynamic derivatives, relativistic entrainment matrix, etc.), making them available for a potential user. To this aim a general non-linear hadronic Lagrangian involving $\sigma \omega \rho \phi \sigma^*$ meson fields, as well as quartic terms in vector-meson fields, is considered. A universal scheme for calculation of the $\ell = 0, 1$ Landau Fermi-liquid parameters and relativistic entrainment matrix is formulated in the mean-field approximation. Use of this scheme allows us to obtain numerical tables with the equation of state, Landau quasi-particle effective masses, adiabatic indices, the $\ell = 0, 1$ Landau Fermi-liquid parameters, and the relativistic entrainment matrix for the selected models of nucleon–hyperon matter. These data are available online and suitable for numerical implementation in computer codes modelling various dynamical processes in NSs, in particular, oscillations of superfluid NSs and their cooling.

Key words: stars: interiors – stars: neutron – stars: oscillations.

1 INTRODUCTION
Neutron stars (NSs), being massive, compact, rapidly rotating objects, with central density up to 10 times normal nuclear density ($\rho_0 \approx 2.8 \times 10^4$ g cm$^{-3}$, corresponding to baryon number density $n_0 \approx 0.16$ fm$^{-3}$), are promising sources of gravitational waves, associated with axial-symmetry breaking stellar pulsations, triggered by various types of instabilities (Andersson et al. 2011, 2013). Modelling NS dynamics requires hydrodynamical description of its liquid core, of density ranging from $\sim 0.5 \rho_0$ at the outer edge of the core to $\sim 10 \rho_0$ at the centre of the most massive stars. It is expected that the core layer up to $2-3 \rho_0$, called the outer core, consists of nucleons (mostly neutrons) and leptons (electrons and muons), while at higher density (inner core), the matter is expected to contain also hyperons. We are then dealing with a baryon matter, consisting of more than two baryon species (to be contrasted with nuclear matter in the outer core), with an admixture of leptons required by weak-interaction equilibrium and charge neutrality. At least some of the baryon species are thought to be superfluid.

To study dynamics of a multisuperfluid nucleon–hyperon (NH) matter, one needs not only the equation of state (EOS), involving various thermodynamic derivatives, but also a (symmetric) relativistic entrainment matrix $Y_\mu$ (hereafter subscripts $i, j$ run over all baryon species), describing non-dissipative interaction between superfluids due to strong interaction of baryons. A method of the calculation of $Y_\mu$ for a mixture of NH superfluids was presented in the limiting case of zero temperature ($T = 0$) in Gusakov, Kantor & Haensel (2009a) and then generalized to non-zero $T$ in Gusakov, Kantor & Haensel (2009b). Strong interactions between baryons were included using relativistic extension (Baym & Chin 1976) of the Landau theory of Fermi liquids.

Numerical results of Gusakov et al. (2009a,b) were obtained employing a basic version of the relativistic mean-field model (RMF; see Glendenning 2000, 1985 and references therein). This RMF model involved the baryon octet, interacting via coupling to scalar ($\sigma$), vector ($\omega^\mu$), and vector-isovector ($\rho^\mu$) meson fields; here $\mu$ and $i$ indices denote the space–time and isospin components of the field, respectively. Unfortunately, the model used in Gusakov et al. (2009a,b) is not consistent with up-to-date hypernuclear data.

In this paper, we replace the $\sigma \omega \rho \phi$ Lagrangian by a more general non-linear model involving two additional hidden-strangeness mesons (Bednarek & Manka 2009, and references therein). This RMF model includes the hyperon octet, interacting via coupling to scalar ($\sigma$), vector ($\omega^\mu$), and vector-isovector ($\rho^\mu$) meson fields; here $\mu$ and $i$ indices denote the space–time and isospin components of the field, respectively. Unfortunately, the model used in Gusakov et al. (2009a,b) is not consistent with up-to-date hypernuclear data.

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(Demorest et al. 2010; Antoniadis et al. 2013). They are consistent with semi-empirical saturation parameters of nuclear matters, binding energies of \(\Lambda\) and \(\Sigma^+\) hyperons in nuclear matter deduced from hypernuclei and reproduce potential well for \(\Sigma^-\) in nuclear matter deduced from the \(\Sigma^-\) atoms. As shown in several recent papers (Bednarek et al. 2012; Weissenborn, Chatterjee & Schiffner-Bielich 2012a,b), all these constraints can be simultaneously satisfied by introducing an additional vector-meson field \(\phi\) coupled only to hyperons, resulting in a strong hyperon repulsion at high densities and/or allowing for breaking of SU(6) symmetry in the vector-mesons coupling to hyperons. Therefore, instead of a (too) simple \(\sigma\omega\pi\phi\sigma^*\) model, used in Gusakov et al. (2009a,b), we consider at least the \(\sigma\omega\phi\sigma^*\) one. In order to get a better fit to a larger number of semi-empirical hypernuclear parameters (e.g. to describe a weak \(\Lambda-\Lambda\) interaction following from ‘Nagara’ event; see Takahashi et al. 2001), an additional scalar meson \(\sigma^*\) can be included, leading to a \(\sigma\omega\phi\sigma^*\) model. For the general \(\sigma\omega\phi\sigma^*\) Lagrangian that includes quartic terms in vector-meson fields, we develop a calculational scheme for the \(f_{ij}^{\phi}\) Landau Fermi-liquid parameters, and associated with them matrix \(Y_i\), as well as for the \(f_{ij}^{\phi}\) Landau parameters needed for calculation of various thermodynamic derivatives. Numerical calculations of \(f_{ij}^{\phi}\), \(f_{ij}^{\phi}\) and \(Y_i\) are done for three selected models of dense NH matter consistent with existence of 2\(M_p\) pulsars as well as with semi-empirical nuclear matter parameters at saturation and semi-empirical hypernuclear data. For these models, we also present EOS, Landau effective masses of baryons and adiabatic indices. This data provide all microphysics input allowing one to model dynamics of superfluid NSs. All numerical results are available online.

The plan of this paper is as follows. Basic definitions and relations for superfluid NH mixture are recapitulated in Section 2. The \(\sigma\omega\phi\sigma^*\) Lagrangian for the baryon octet is presented in Section 3.1. The Dirac equations for baryons and their solutions in the RMF approximations are given in Section 3.2. The equations for the meson fields in the presence of baryon currents are given in Section 3.3. Section 3.4 presents expressions for thermodynamic functions. Landau parameters \(f_{ij}^{\phi}\) and \(f_{ij}^{\phi}\) are derived in Sections 4 and 5, respectively. Numerical results are collected in Section 6. Three up-to-date RMF models of NH matter are presented in Section 6.1. The EOSs for these models as well as the parameters of NS configurations with maximum mass \(M_{\text{max}}\) are compared in the same Section 6.1. Particle fractions for NH matter in beta equilibrium, adiabatic indices and the speed of sound, all as functions of baryon number density, are compared in Section 6.2. Landau effective masses are calculated in Section 6.3. Numerical results for the Landau Fermi-liquid parameters and entrainment matrix are presented in Section 6.4. Stability of the ground state of NH matter is briefly discussed in Section 6.5. Section 7 contains summary of our results. Detailed information about the coupling constants for the three RMF models employed in this paper is given in Appendix A. The way of calculating EOSs for these three models is reviewed in Appendix B. Adiabatic indices are discussed in Appendix C. Finally, a description of publicly available online numerical material containing the results of our calculations is given in Appendix D.

2 Basic Definitions and Relations

Here, we briefly review the Landau Fermi-liquid theory (see, e.g., Baym & Pethick 1991; Pines & Nozieres 1999) generalized to the case of relativistic one-component liquid by Baym & Chin (1976) and extended to relativistic mixtures by Gusakov et al. (2009a). For the sake of compactness of notation, we use the convention \(\hbar = c = 1\), where \(\hbar\) is the Planck constant and \(c\) is the speed of light; we also assume that the metric tensor is \(\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)\). Unless otherwise stated, all quantities and relations are given in the reference system associated with normal fluid of leptons. In this reference system, the four-velocity of the normal fluid is \(u^\mu = (1, 0, 0, 0)\).

We are dealing with an uniform mixture of baryon species. Let us first assume that all baryon species are normal (no superfluid gaps). This means that if we start with a (reference) system of bare non-interacting baryons and then the interaction is slowly switched on, the system of bare non-interacting baryons transforms adiabatically into a system of Landau quasi-particles. This system of quasi-particles retains essential properties of a mixture of ideal Fermi gases. Namely the distribution function of quasi-particles in the momentum space is the same as that of an ideal ‘reference’ system. The number of quasi-particles is equal to the number of particles. The Landau Fermi-liquid theory establishes therefore a one-to-one correspondence between the states of a system of quasi-particles and those of the real system. The quasi-particle states will be labelled by momentum \(p\) and spin \(s\), \(ps\). As we will deal with spin unpolarized systems, the quantities under consideration are spin independent and can be replaced by spin-averaged ones.

In the ground state, distribution function of quasi-particle species \(i\) is then a filled Fermi sphere,

\[ n_{i0}(p) = \theta(p_{F_i} - p), \]

where \(p_{F_i}\) is the Fermi momentum for quasi-particles, coinciding with that for bare non-interacting particles, so that the number density \(n_i = p_{F_i}^3/3\pi^2\). \(\theta(x)\) is the step function: \(\theta(x) = 1\) if \(x > 0\) and \(0\) otherwise. Subscript 0 will denote the quantities in the ground state of the system.

Within the normal Landau Fermi-liquid theory, the energy of the system is a functional of the quasi-particle distribution functions, \(n_i(p)\). The validity of the quasi-particle description of an excited state is restricted to vicinity of the Fermi surfaces. This means that \(\delta n_i(p) = n_i(p) - n_{i0}(p)\) is non-zero for \(|p - p_{F_i}| \ll p_{F_i}\). The energy \(E\) of an excited state of the system can be expressed in terms of \(\delta n_i(p)\) by expanding the functional \(E[n_i(p)]\) around \(E_0\) (see, e.g., Baym & Pethick 1991; Pines & Nozieres 1999),

\[ E - E_0 = \sum_{p\neq 0} \epsilon_{i0}(p) \delta n_i(p) + \frac{1}{2} \sum_{p\neq p', i\neq j} f_{ij}^{\phi}(p, p') \delta n_i(p) \delta n_j(p'), \]

(2)

where third-order terms in \(\delta n_i(p)\) have been neglected. Here, \(\epsilon_{i0}(p)\) and \(f_{ij}^{\phi}(p, p')\) are, respectively, energy of an \(i\)-quasi-particle in the ground state and the (spin-averaged) quasi-particle interaction – a central object in the Landau Fermi-liquid theory. Because of the isotropy of the ground state, \(\epsilon_{i0}(p)\) depends only on \(|p| = p\). This needs not to be so for a quasi-particle in an excited state of the system; in the latter case, the quasi-particle energy is given by (Baym & Pethick 1991; Pines & Nozieres 1999)

\[ \epsilon_i(p) = \epsilon_{i0}(p) + \sum_{p'\neq p} f_{ij}^{\phi}(p, p') \delta n_j(p'). \]

(3)

Near the Fermi surface, the function \(\epsilon_{i0}(p)\) can be expanded into a series in powers of the quantity \(p - p_{F_i}\) and approximated by a linear form,

\[ \epsilon_{i0}(p) \approx \mu_i + \nu_i (p - p_{F_i}), \]

(4)

where \(\mu_i = \epsilon_{i0}(p_{F_i})\) is the relativistic (i.e. including the rest energy) chemical potential or, equivalently, the Fermi energy of...
quasi-particle species $i$ and $v_{fi} = (\partial \varepsilon_{fi}(p)/\partial p)_{p=p_i}$ is the velocity of quasi-particles on the Fermi surface. The Landau quasi-particle effective mass $m_i^*$ is introduced through the relation

$$v_{fi} = p_{fi}/m_i^*.$$  

(5)

Within the region of validity of the Landau Fermi-liquid theory, the magnitude of momentum arguments of the quasi-particle interaction $f_{ij}(p,p')$ can be approximated as $|p| \approx p_{fi}$ and $|p'| \approx p_{fj}$, respectively. Therefore, the momentum dependence of $f_{ij}$ can be expanded into Legendre polynomials $P_{ij}(\cos \theta)$,

$$f_{ij}(p,p') = \sum \ell f_{ij}^{\ell}(P_{ij}(\cos \theta),$$  

(6)

where $\theta$ is the angle between $p$ and $p'$, and $f_{ij}^{\ell}$ are the Landau Fermi-liquid parameters, $f_{ij}^{\ell} = f_{ij}^{\ell,0}$. The dimensionless Landau parameters $F_{ij}^{\ell}$ are defined through

$$F_{ij}^{\ell} = \sqrt{N_{fi}N_{fj}}f_{ij}^{\ell}, \quad N_{fi} = m_i^* p_{fi}/\pi^2,$$  

(7)

where $N_{fi}$ is the density of $i$-quasi-particle states at the Fermi surface.

The effective mass $m_i^*$ in the relativistic theory can be expressed in terms of the Landau parameters $F_{ij}^{\ell}$ (see equation 24 of Gusakov et al. 2009a),

$$\frac{\mu_i}{m_i^*} = 1 - \frac{1}{3} \sum_{j} \frac{\mu_j}{\sqrt{m_j^* m_i^*}} \left( \frac{p_{fi}}{|p_{fi}|} \right)^{3/2} F_{ij}^{\ell}.$$  

(8)

Let us pass now to the superfluid baryons. A mixture of baryon superfluids is described in terms of Bogoliubov quasi-particles. At $T = 0$, all baryons are paired into Cooper pairs and the energy gaps for (Bogoliubov) i-quasi-particles at the Fermi surface are $\Delta_i$ (for the sake of simplicity, we restrict ourselves to isotropic gaps). As long as $\Delta_i \ll \mu_i - m_i$, the energy gaps will not affect the main formulas, e.g. the particle current densities $j_i$ will be related to the distribution functions by the same expression as in the case of a normal Fermi liquid, see Leggett (1965, 1975).

We consider excited states of the system associated with uniform superfluid flows, each of them with macroscopic velocity $\mathbf{V}_s$. The macroscopic flow velocity of species $i$ is related to the total momentum of a Cooper pair $2 \mathbf{Q}_i$ by

$$\mathbf{V}_s = \frac{\mathbf{Q}_i}{m_i},$$  

(9)

where $m_i$ is a free (in vacuum) mass of baryon $i$. In the linear approximation in $\mathbf{Q}_i$ ($|\mathbf{Q}_i| \ll |p_{fi}|$), the current densities $j_i$ are connected with $\{\mathbf{Q}_i\}$ by (Gusakov et al. 2009a)

$$j_i = \sum_j Y_{ij} \mathbf{Q}_j.$$  

(10)

The relativistic entrainment matrix, $Y_{ij}$, is symmetric, $Y_{ij} = Y_{ji}$, and fulfils a sum rule

$$\sum_j Y_{ij} n_j = n_i.$$  

(11)

In the case of vanishing temperature ($T = 0$), the entrainment matrix can be expressed in terms of the $F_{ij}$ Landau parameters (Gusakov et al. 2009a),

$$Y_{ij} = \frac{n_i}{m_i^*} \delta_{ij} + \left( \frac{n_i n_j}{m_i^* m_j^*} \right)^{1/2} F_{ij}^{\ell},$$  

(12)

where $\delta_{ij}$ is the Kronecker delta. The non-diagonal elements of $Y_{ij}$ describe the superfluid entrainment, a non-dissipative interaction between the superfluid baryon flows.

The more complex expression for $Y_{ij}$, valid at arbitrary temperature, was formulated by Gusakov et al. (2009b). It also involves the Landau parameters $F_{ij}$ (see equations 41– 43 and 45 of that reference).

3 THE $\sigma$-MODEL

We use a non-linear model of Bednarek & Manka (2009).

3.1 Lagrangian

The Lagrangian density $\mathcal{L}$ of the strongly interacting baryon system can be split into two basic components, $\mathcal{L}_{BM}$ involving baryon terms affected by the meson fields and $\mathcal{L}_{3M}$ involving exclusively meson fields.

We consider Dirac spinor fields for baryons $\Psi_i$, depending on space–time point $\tau$. The contravariant coordinates of $x$ are $x^\mu = (x^0, x^1, x^2, x^3)$ and the contravariant derivative $\partial^\mu = \partial/\partial x^\mu$. The $\mathcal{L}_{BM}$ component of the Lagrangian density is then

$$\mathcal{L}_{BM} = \sum_i \overline{\Psi}_i \left( i \gamma_\mu \partial^\mu - m_i - g_{\omega\sigma} \gamma_\sigma \omega_i - g_{\phi\nu} \gamma_\nu \phi_i \right) + \frac{1}{2} g_{\omega\sigma} \tau_\alpha \gamma_\sigma \omega_i + g_{\omega\sigma} \gamma_\sigma + \frac{1}{8} \gamma_\sigma \gamma_\sigma,$$  

(13)

where $\gamma_\mu$ are Dirac matrices and $\psi_i = \Psi_i \gamma_0$ is an adjoint Dirac spinor. The components of $\tau_\alpha$ ($\alpha = 1, 2, 3$) are Pauli matrices acting in the isospin space. The parameters $g_{\omega\nu} (m = \sigma, \sigma^*, \omega, \rho, \phi)$ are coupling constants of the meson fields to the baryon fields.

Baryon densities and currents are sources of the meson fields. We assume spatially uniform and time-independent sources and therefore resulting meson fields are $x$ independent. Meson fields and their interactions generate the meson Lagrangian $\mathcal{L}_M = \mathcal{L}_M^{(S)} + \mathcal{L}_M^{(V)}$, where $S$ and $V$ refer to the scalar and vector mesons, respectively. The scalar meson contribution is

$$\mathcal{L}_M^{(S)} = - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{2} m_\omega^2 \sigma^* \sigma - \frac{1}{3} \bar{\sigma} \gamma_\sigma \sigma + \frac{1}{6} \bar{\sigma} \gamma_\sigma \gamma_\sigma \sigma,$$  

(14)

where the coupling constants $g_3$ and $g_4$ determine the strength of $\sigma$ meson self-interaction.

The vector-meson contribution includes terms quadratic and quartic in vector-meson fields,

$$\mathcal{L}_M^{(V)} = \frac{1}{2} m_\omega^2 (\rho_\omega \rho_\omega^* + \phi_\omega \phi_\omega^*) + \frac{1}{2} m_\rho^2 (\rho_\rho \rho_\rho^* + \phi_\rho \phi_\rho^*)$$

$$+ \frac{1}{2} m_\phi^2 (\phi_\phi \phi_\phi^*) - \frac{1}{4} c_5 (\rho_\omega \rho_\omega^*)^2$$

$$+ \frac{1}{4} c_8 (\phi_\rho \phi_\rho^*)^2 + \frac{1}{4} c_3 (\phi_\phi \phi_\phi^*)^2 + \frac{1}{4} c_7 (\omega_\omega \omega_\omega^*)^2$$

$$+ \frac{3}{4} c_3 (\phi_\phi \phi_\phi^*) (\omega_\omega \omega_\omega^*) + \frac{1}{4} (g_{\rho\omega} g_{\omega\omega})^2 \mathcal{A}_V (\phi_\rho \phi_\rho^*)^2$$

$$- \frac{1}{2} (g_{\rho\omega} g_{\omega\omega})^2 \mathcal{A}_V (\phi_\rho \phi_\rho^*) (\phi_\phi \phi_\phi^*)$$

$$+ (g_{\rho\omega} g_{\omega\omega})^2 \mathcal{A}_V (\phi_\rho \phi_\rho^*) (\omega_\omega \omega_\omega^*),$$  

(15)
In equations (14) and (15), \( m_0, m_+, m_- \) and \( m_\sigma \) are the corresponding meson masses. Three additional parameters in equation (15), \( c_3, \tilde{c}_3 \) and \( \Lambda_V \) determine the strength of the quartic vector-meson terms. (Note that in Bednarek & Manka 2009, it was assumed that \( c_3 = c_3 \).) For less general models considered by us in Section 6, only the underlined terms in equation (15) are taken into account, that is, we put \( \tilde{c}_3 = \Lambda_V = 0 \).

### 3.2 The microscopic state of baryons in the RMF approximation

The equations of motion for the baryon fields \( \Psi_i \) are obtained as Euler–Lagrange equations from \( \mathcal{L} \),

\[
\frac{\partial \mathcal{L}}{\partial \Psi_i} = \left( \gamma_{ij} \partial^\mu - m_i - g_{\omega \rho} \omega^\mu - g_{\phi} \gamma_{0} \phi^\mu - \frac{1}{2} g_{\rho} \tau_3 \gamma_{0} \rho^\mu + g_{\omega} \gamma_{0} \omega^\mu + g_{\lambda} \gamma_{3} \sigma^\mu + g_{\mu} \gamma_{0} \mu^\mu \right) \Psi_i = 0. \tag{16}
\]

These are Dirac equations for baryons coupled to meson fields. We look for the macroscopic states of NH matter which are uniform in space and stationary. In the RMF approximation, the meson fields in \( \mathcal{L} \) are replaced by their \( x \)-independent mean values. Therefore, solutions \( \Psi_i \) of equation (16) are the eigenstates of the four-momentum \( p_i \),

\[
\Psi_i = \Psi_i(p_i^\mu) e^{-ip_i \cdot \nu}. \tag{17}
\]

After putting Ansatz (17) into the equation of motion (16), we solve it using standard methods for the Dirac equation. In this way, we find the Dirac equation eigenvalues of the energy, \( e_i \), at fixed values of the uniform meson fields,

\[
e_i(p^\mu) = \frac{g_{\omega \rho} \omega^\mu + g_{\phi} \gamma_{0} \phi^\mu}{\sqrt{p_{\rho}^2 + m_i^2}} \frac{\left[ \left( p - g_{\omega \rho} \omega + g_{\phi} \gamma_{0} \phi \right) - m_i \gamma_{0} \gamma_{\lambda} \rho^\mu + g_{\omega} \gamma_{0} \omega^\mu + g_{\lambda} \gamma_{3} \sigma^\mu + g_{\mu} \gamma_{0} \mu^\mu \right]}{\sqrt{\left( m_i - g_{\omega} \gamma_{0} \sigma + g_{\lambda} \gamma_{3} \sigma^\mu + g_{\mu} \gamma_{0} \mu^\mu \right)^2}}, \tag{18}
\]

where \( I_3 \) is the third component of the isospin of baryon \( i \), with \( I_3 = -1/2 \) (the subscript \( n \) stands for neutrons).

A macroscopic spatially uniform stationary state for baryons, under given constraints on baryon currents, \( j_i \), and baryon densities, \( n_i \), is obtained by filling lowest Dirac energy eigenstates. The distribution function of the occupied Dirac states coincides then with distribution function of the Landau i-quasi-particles. Therefore, in the RMF approximation, the quasi-particle energy of a baryon species \( i \) is equal to the Dirac equation eigenvalue, \( e_i(p^\mu) \). In particular, the particle current density can be expressed through \( e_i(p^\mu) \) as

\[
j_i = \sum_{p^\mu} \frac{\partial e_i(p^\mu)}{\partial p^\mu} n_i(p^\mu). \tag{19}
\]

### 3.3 Field equations for meson fields in the presence of baryon currents

Meson fields are calculated assuming a uniform stationary state of baryons. The field equations for mesons are the Euler–Lagrange equations obtained from \( \mathcal{L} \). The baryon fields enter the source terms in the meson field equation. In the RMF approximation, the source term is replaced by a mean value calculated in the uniform stationary state of the baryon system described in Section 3.2. Both source terms and meson fields are \( x \)-independent. Equations for meson fields can be written as

\[
m_0^2 \sigma = -g_{\phi} \sigma^2 - g_{\phi} \sigma^3 + \sum_i g_{\mu} R_i (m_i - g_{\mu} \gamma_{0} \sigma + g_{\lambda} \gamma_{3} \sigma^\mu),
\]

\[
g_{\omega \rho} \omega + g_{\phi} \gamma_{0} \phi + g_{\mu} \gamma_{0} \mu^\mu \tag{20}
\]

\[
m_0^2 \sigma^* = \sum_i g_{\mu} R_i (m_i - g_{\mu} \gamma_{0} \sigma + g_{\lambda} \gamma_{3} \sigma^\mu),
\]

\[
g_{\omega \rho} \omega + g_{\phi} \gamma_{0} \phi + g_{\mu} \gamma_{0} \mu^\mu \tag{21}
\]

\[
\left[ m_0^2 + A_\omega (\omega \times \omega) + A_\phi (\phi \times \phi) \right] \omega^\mu = \sum_i g_{\omega \rho} I_{\omega \rho} j_i^\mu \tag{22}
\]

\[
\left[ m_0^2 - B_\omega (\omega \times \omega) + B_\phi (\phi \times \phi) \right] \phi^\mu = \sum_i g_{\phi} I_{\phi} j_i^\mu \tag{23}
\]

\[
\left[ m_0^2 + C_\omega (\omega \times \omega) + C_\phi (\phi \times \phi) \right] \mu^\mu = \sum_i g_{\mu} I_{\mu} j_i^\mu \tag{24}
\]

Here,

\[
R_i(x, y) = \sum_{p^\mu} \frac{1}{\sqrt{(p - y)^2 + x^2}} n_i(p). \tag{25}
\]

In case of the \( \omega \rho \phi \sigma \) Lagrangian of Bednarek & Manka (2009), described in Section 3.1, the constants \( \lambda_\omega, \ldots, \lambda_\phi \) in equations (22)–(24) are given by expressions

\[
A_\omega = \frac{3}{2} \tilde{c}_3 - \Lambda_V (g_{\omega \rho} g_{\omega \rho}), \quad A_\phi = \frac{3}{2} \tilde{c}_3 - \Lambda_V (g_{\phi} g_{\phi}), \tag{26}
\]

\[
B_\omega = 2 \Lambda_V (g_{\omega \rho} g_{\omega \rho}), \quad B_\phi = \tilde{c}_3, \quad B_\mu = A_\phi, \tag{27}
\]

\[
C_\omega = \frac{3}{2} \tilde{c}_3 - \Lambda_V (g_{\omega \rho} g_{\omega \rho}), \quad C_\phi = C_\omega, \tag{28}
\]

As we already emphasized above, for less general RMF models considered in Section 6, the only non-zero constant is \( \Lambda_\sigma = \tilde{c}_3 \) (\( \tilde{c}_3 = \Lambda_V = 0 \)). Neglecting \( \sigma^* \) and \( \phi \) mesons, equations (20)–(28) correctly reproduce the \( \omega \rho \phi \sigma \) model of Glendenning (Glendenning 2000; Gusakov et al. 2009a).

### 3.4 Chemical potential, energy density and pressure

In this section, we assume that there is no baryon currents in the system, that is, \( \mathbf{u} = \rho_3 = \phi = 0 \). Knowledge of the particle energy (18) allows one to immediately find the relativistic chemical potential \( \mu_i \),

\[
\mu_i = e_i(p_{\nu}^\mu) = g_{\omega \rho} \omega^\mu + g_{\phi} \gamma_{0} \phi^\mu + \sqrt{p_{\nu}^2 + \left[ \left( m_i - g_{\omega} \gamma_{0} \sigma + g_{\lambda} \gamma_{3} \sigma^\mu \right) \right]^2}. \tag{29}
\]

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and the Landau effective mass $m_i^*$ (cf. equation 47 of Gusakov et al. 2009a),

$$m_i^* = \left[ \frac{\partial^2 \mathcal{E}(\mathbf{p})}{\partial \mathbf{p}^2} \right]_{p=p_i} = \sqrt{\frac{p_i^2}{\rho} + (m_i - g_{\sigma_1} \sigma - g_{\sigma^*} \sigma^*)^2}. \tag{30}$$

The energy density $\rho$ (also termed density in what follows) can be obtained from the Lagrangian of Section 3.1 in the same way as it was done, e.g. in Glendenning (2000). The result is

$$\rho = -\langle \mathcal{L} \rangle + \sum_i \left[ g_{\text{col}} \omega_i n_i + g_{\omega_1} \omega_i I_{3i} \rho_i^0 n_i + g_{\phi} \rho_i^0 n_i \right]$$

$$+ R_\xi (m_i - g_{\sigma_1} \sigma - g_{\sigma^*} \sigma^*, \ p_{Vi})$$

$$+ \sum_{i=\text{e, } \mu} \sum_{j=\text{e, } \mu} R_\xi (m_i, \ p_{Vi}), \tag{31}$$

where the summation is performed over all baryon species $i$ and lepton species $l = e, \mu$. Here

$$\langle \mathcal{L} \rangle = \frac{1}{2} m_\omega^2 \sigma^2 - \frac{1}{3} g_\omega \sigma^2 - \frac{1}{4} g_{\omega^*} \sigma^2 + \frac{1}{2} m_{\omega^*}^2 (\omega^*)^2$$

$$+ \frac{1}{2} m_\rho^2 (\rho,)^2 + \frac{1}{2} m_{\omega^*}^2 (\omega^*)^2 - \frac{1}{2} m_{\rho^*}^2 (\rho^*)^2$$

$$+ \frac{1}{4} c_3 (\omega^*)^4 + \Lambda V (g_{\omega \sigma_1} g_{\omega^*})^2 (\omega^*)^2 (\rho^*)^2$$

$$- \frac{1}{2} \Lambda V (g_{\omega \sigma_1} g_{\omega^*})^2 \left( \frac{3}{4} c_3 \right) (\omega^*)^2 (\rho^*)^2$$

$$- \frac{1}{2} \Lambda V (g_{\omega \sigma_1} g_{\omega^*})^2 \left( \frac{3}{4} c_3 \right) (\psi)^2 (\phi)^2$$

$$+ \frac{1}{4} c_3 (\rho^*)^4 + \frac{1}{4} \Lambda V (g_{\omega \sigma_1} g_{\omega^*})^2 + \frac{1}{4} c_3 \frac{1}{4} (\phi)^4, \tag{32}$$

and

$$R_\xi (x, y) = \frac{1}{V} \int_0^1 p^2 \sqrt{p^2 + x^2} \, dp. \tag{33}$$

Now the pressure $P$ can be expressed through $\rho$ and $\mu_k$ by the following standard formula,

$$P = -\rho + \sum_k \mu_k n_k, \tag{34}$$

where the subscript $k$ runs over all particle species (baryons and leptons).

## 4 Derivation of Expression for $f_{1j}^i$

To calculate the Landau parameter $f_{1j}^i$, we have to create a uniform baryon current in the system. For that we shift the distribution function $n_{\text{col}}(\mathbf{p})$ (the step function) of a baryon species $i$ by a small vector $\mathbf{Q}$. Note that, in the linear approximation in $\mathbf{Q}$, the scalars $\sigma, \sigma^*, \omega, \omega^*, \rho_3, \rho^*_3$ and $\phi, \phi^*$ remain the same as in the absence of baryon currents.

Following the derivation of equation 43 of Gusakov et al. (2009a), one obtains

$$j_i = \frac{n_i}{m_i} (\mathbf{Q} - g_{\text{col}} \omega - g_{\omega_1} I_{3i} \rho^0_3 - g_{\phi} \phi^0_3), \tag{35}$$

where $m_i^*$ is given by equation (30). Equation (35) should be supplemented by the expressions for $\omega, \rho_3$ and $\phi$. These expressions can be found from equations (22)–(24),

$$m_i^2 \omega = \sum_i g_{\text{col}} j_i, \tag{36}$$

$$m_i^2 \rho_3 = \sum_i g_{\omega_1} I_{3i} j_i, \tag{37}$$

$$m_i^2 \phi = \sum_i g_{\phi} j_i, \tag{38}$$

where we defined the effective meson masses

$$m^2_{\omega} = m^2_{\omega} + A_{\omega} (\omega^0)^2 + A_{\rho} (\rho^0)^2 + A_{\phi} (\phi^0)^2, \tag{39}$$

$$m^2_{\rho} = m^2_{\rho} + B_{\omega} (\omega^0)^2 + B_{\rho} (\rho^0)^2 + B_{\phi} (\phi^0)^2, \tag{40}$$

$$m^2_{\phi} = m^2_{\phi} + C_{\omega} (\omega^0)^2 + C_{\rho} (\rho^0)^2 + C_{\phi} (\phi^0)^2, \tag{41}$$

and made use of the fact that, for example, $\omega_i \omega^0 = (\omega^0)^2$ with the accuracy to linear terms in $\mathbf{Q}$ ($\omega^0$ is also independent of $\mathbf{Q}$ in the linear approximation).

Equation (35) should be solved together with equations (36)–(38). To proceed further, let us multiply (35) by $g_{\text{col}}$ and sum it over $i$. Then, using equation (36), one obtains

$$m_i^2 \omega = \sum_i \frac{n_i}{m_i} \left( \mathbf{Q} - g_{\text{col}} \omega - g_{\omega_1} I_{3i} \rho_3 - g_{\phi} \phi \right). \tag{42}$$

Similarly,

$$m_i^2 \rho_3 = \sum_i \frac{n_i}{m_i} g_{\omega_1} I_{3i} \left( \mathbf{Q} - g_{\text{col}} \omega - g_{\omega_1} I_{3i} \rho_3 - g_{\phi} \phi \right). \tag{43}$$

$$m_i^2 \phi = \sum_i \frac{n_i}{m_i} g_{\phi} \left( \mathbf{Q} - g_{\text{col}} \omega - g_{\omega_1} I_{3i} \rho_3 - g_{\phi} \phi \right). \tag{44}$$

Equations (42)–(44) can be rewritten in the matrix form,

$$\begin{bmatrix}
 m^2_{\omega} + \frac{n_i}{m_i} g_{\text{col}} & \sum_i \frac{n_i}{m_i} g_{\omega_1} I_{3i} & \sum_i \frac{n_i}{m_i} g_{\phi} \\
 \sum_i \frac{n_i}{m_i} g_{\omega_1} g_{\omega_1} g_{\omega_1} & \sum_i \frac{n_i}{m_i} g_{\omega_1} I_{3i} & \sum_i \frac{n_i}{m_i} g_{\phi} I_{3i} \\
 \sum_i \frac{n_i}{m_i} g_{\omega_1} g_{\omega_1} g_{\omega_1} & \sum_i \frac{n_i}{m_i} g_{\omega_1} I_{3i} & \sum_i \frac{n_i}{m_i} g_{\phi} I_{3i}
\end{bmatrix} \begin{bmatrix}
 \omega \\
 \rho_3 \\
 \phi
\end{bmatrix} = \begin{bmatrix}
 \sum_i \frac{n_i}{m_i} g_{\text{col}} I_{3i} \\
 \sum_i \frac{n_i}{m_i} g_{\omega_1} I_{3i} \\
 \sum_i \frac{n_i}{m_i} g_{\phi} I_{3i}
\end{bmatrix} \mathbf{Q}. \tag{45}$$

The solution to equation (45) can be presented as

$$\omega = \sum_j \alpha_{\omega j} \mathbf{Q} j, \tag{46}$$

$$\rho_3 = \sum_j \alpha_{\rho j} \mathbf{Q} j, \tag{47}$$

$$\phi = \sum_j \alpha_{\phi j} \mathbf{Q} j, \tag{48}$$

where the coefficients $\alpha_{\omega j}, \alpha_{\rho j}$ and $\alpha_{\phi j}$ can be determined from equation (45) using the methods of linear algebra. Using equations (35) and (46)–(48), one can calculate the entrainment matrix $Y_{ij}$.

$$Y_{ij} = \frac{n_i}{m_i} \left( g_{\omega j} \alpha_{\omega j} - g_{\omega j} I_{3i} \alpha_{\rho j} - g_{\phi j} \alpha_{\phi j} \right). \tag{49}$$

and, consequently, the Landau parameters $f_{1j}^i$ (see equation 46 of Gusakov et al. 2009a or equation 12).
5 DERIVATION OF EXPRESSION FOR $f^i_0$

Here, we will closely follow the derivation of section IIIIC in Gusakov et al. (2009a). Let us consider a system without baryon currents (i.e. $\omega = \phi = \rho = 0$). To calculate the Landau parameters $f^i_0$, we slightly vary the Fermi momentum $p_i$ by a small quantity $\Delta p_i$, so that the distribution function of a quasi-particle species $i$ will become $n_i(p_i) = \theta(p_i + \Delta p_i - p)$. This will shift the energy of baryons $e_i(p)$ by a small quantity $\delta e_i(p)$. At $p = p_i$, the expression for $\delta e_i(p)$ takes the form (see equation 18)

$$
\delta e_i(p) = g_{ii} \delta \omega_i + g_{ii} \delta \phi_i + g_{ii} \delta \rho_i - \left( m_i/m^*_i - g_{ii} \sigma/m^*_i + g_{ii} \sigma^*/m^*_i \right) \times (g_{ii} \delta \sigma + g_{ii} \delta \sigma^*),
$$

(50)

where we used equation (30) for the Landau effective mass $m^*_i$. On the other hand, it follows from the Landau theory of Fermi liquids that

$$
\delta e_i(p) = \sum f^{(i)}_0 \delta n_i,
$$

(51)

where $\delta n_i \equiv p_i^i \Delta p_i/\pi^2$. Comparing equations (50) and (51), one can calculate the parameters $f^{(i)}_0$. For that we need to express the variations $\delta \omega_i$, $\delta \phi_i$, $\delta \rho_i$, and $\delta \rho_i^0$ through $\delta n_i$. We start with the quantities $\delta \omega_i$ and $\delta \sigma^*$. They can be found from the linearized equations (20) and (21),

$$
\left( m^2_o + 2 g_o \sigma + 3 g_o \sigma^2 \right) \delta \sigma = -\sum g_{ss} \frac{\partial R_s(x,0)}{\partial x} \left[ m_s/m^*_s - g_{ss} \sigma/m^*_s - g_{ss} \sigma^*/m^*_s \right] \delta n_s,
$$

(52)

$$
\left( m^2_o + 2 g_o \sigma + 3 g_o \sigma^2 \right) \delta \sigma^* = -\sum g_{ss} \frac{\partial R_s(x,0)}{\partial x} \left[ m_s/m^*_s - g_{ss} \sigma/m^*_s - g_{ss} \sigma^*/m^*_s \right] \delta n_s,
$$

(53)

In the matrix form, equations (52) and (53) can be rewritten as

$$
\begin{pmatrix}
\left( m^2_o + 2 g_o \sigma + 3 g_o \sigma^2 \right) I_{\delta \sigma}
\end{pmatrix}
\begin{pmatrix}
\delta \sigma
\end{pmatrix}
= \left( \sum g_{ss} \left[ m_s/m^*_s - g_{ss} \sigma/m^*_s - g_{ss} \sigma^*/m^*_s \right] \delta n_s
\right),
$$

(54)

where we defined

$$
I_{\delta \sigma} = \sum g_{ss} \frac{\partial R_s(x,0)}{\delta x} |_{x = \mu_i - \delta \mu_i \sigma^* - \delta \mu_i \sigma^*},
$$

(55)

$$
I_{\delta \sigma^*} = \sum g_{ss} \frac{\partial R_s(x,0)}{\delta x} |_{x = \mu_i - \delta \mu_i \sigma^* - \delta \mu_i \sigma^*},
$$

(56)

$$
I_{\delta \sigma^*} = \sum g_{ss} \frac{\partial R_s(x,0)}{\delta x} |_{x = \mu_i - \delta \mu_i \sigma^* - \delta \mu_i \sigma^*},
$$

(57)

The solution to the system (54) can be easily found. To calculate the quantities $\delta \omega^0, \delta \rho^0$ and $\delta \phi^0$, we have to linearize the corresponding equations (22)–(24). The result is

$$
m^2_o \delta \omega^0 + 2 \left( A_o \omega^0 \delta \omega^0 + A_o \rho^0 \delta \rho^0 + A_o \phi^0 \delta \phi^0 \right) \omega^0
$$

$$
= \sum g_{ii} \delta n_i,
$$

(58)

$$
m^2_o \delta \rho^0 + 2 \left( B_o \omega^0 \delta \omega^0 + B_o \rho^0 \delta \rho^0 + B_o \phi^0 \delta \phi^0 \right) \rho^0
$$

$$
= \sum g_{ii} I_{\delta \rho} \delta n_i,
$$

(59)

$$
m^2_o \delta \phi^0 + 2 \left( C_o \omega^0 \delta \omega^0 + C_o \rho^0 \delta \rho^0 + C_o \phi^0 \delta \phi^0 \right) \phi^0
$$

$$
= \sum g_{ii} I_{\delta \phi} \delta n_i,
$$

(60)

where the meson effective masses $m^*_o, m^*_p$ and $m^*_\phi$ are given by equations (39)–(41). In the matrix form, the system of equations (58)–(60) is presented as

$$
\begin{pmatrix}
\left( m^2_o + 2 A_o \omega^0 \right)^2
\end{pmatrix}
\begin{pmatrix}
\delta \omega^0
\end{pmatrix}
= \left( \sum g_{ii} \left[ m_i/m^*_i - g_{ii} \sigma/m^*_i - g_{ii} \sigma^*/m^*_i \right] \delta n_i
\right),
$$

(61)

The solution to this matrix equation can also be easily obtained. Schematically, expressions for $\delta \omega_i$, $\delta \sigma^*$, $\delta \rho_i$, $\delta \rho_i^0$ and $\delta \phi^0$ can be written as

$$
\delta \omega_i = \sum \beta_{ij} \delta n_j,
$$

(62)

$$
\delta \sigma^* = \sum \beta_{ij} \delta n_j,
$$

(63)

$$
\delta \rho_i = \sum \beta_{ij} \delta n_j,
$$

(64)

$$
\delta \rho_i^0 = \sum \beta_{ij} \delta n_j,
$$

(65)

$$
\delta \phi^0 = \sum \beta_{ij} \delta n_j,
$$

(66)

where we assume that the quantities $\beta_{ij}, \ldots, \beta_{ij}$ have been already calculated from equations (54) and (61). Finally, taking into account equations (62)–(66) and comparing equations (50) and (51), one finds the following expression for the Landau parameters $f^{(i)}_0$,

$$
f^{(i)}_0 = g_{ii} \beta_{ii} + g_{ii} I_{\delta \phi} \beta_{ij} + g_{ii} \delta \phi^0 - \left( m_i/m^*_i - g_{ii} \sigma/m^*_i - g_{ii} \sigma^*/m^*_i \right) \left( g_{ii} \beta_{ij} + g_{ii} \delta \phi^0 \right).
$$

(67)

It can be shown (see, e.g., Gusakov et al. 2009a) that these parameters are directly related to the derivatives $\partial \mu_i/\partial n_j$, which should be taken at fixed particle number densities $n_k (k \neq j)$. Namely one has the following relation

$$
\frac{\partial \mu_i(n_{i+}, \ldots, n_{j+})}{\partial n_j} = \frac{\partial \mu_j(n_{i+}, \ldots, n_{j+})}{\partial n_i} = f^{(i)}_0 + \frac{1}{N_{ii}} \delta n_j,
$$

(68)
model, \( n_s = 0.145 \text{ fm}^{-3} \). The nuclear matter incompressibility, \( K_N = 281 \text{ MeV} \) is on the high side of semi-empirical evaluations. The Dirac effective nucleon mass in symmetric nuclear matter at saturation point is rather small, \( m_{\text{Dirac}} = 0.634m_N \), where \( m_N \) is chosen to be \( m_N = 938 \text{ MeV} \) for this model. The nuclear symmetry energy, \( E_{\text{sym}} = 36.9 \text{ MeV} \), is higher than typical semi-empirical evaluations. Extension of TM1 to NH matter includes the vector \( \phi \) meson and the scalar \( \sigma^* \) meson. The breaking of the SU(6) symmetry is even stronger than for the GM1 B model and corresponds to \( z = 0.2 \). In addition to fitting the upper limits on \( P_{\Lambda} \) and \( U_{\Sigma^+}^{\text{max}} \) potential well depths, this model also fits a weak \( \Lambda - \Lambda \) attraction, \( U_{\Lambda\Lambda}^{\text{max}} = -5.0 \text{ MeV} \) (Takahashi et al. 2001), and assumes \( U_{\Sigma^+}^{\text{max}} \approx U_{\Lambda}^{\text{max}} \approx 2U_{\Sigma^-}^{\text{max}} \approx -10.0 \text{ MeV} \) (Schaffner et al. 1994). Maximum allowable mass is 2.056 \( M_\odot \).

The EOSs for these models, \( P = P(\rho) \), are plotted in Fig. 1. The way they are obtained is briefly discussed in Appendix B. One notices that for \( \rho \gtrsim 2 \times 10^{15} \text{ g cm}^{-3} \), the EOS TM1C is the softest one. And still, it yields the highest value of \( M_{\text{max}} \). This apparent paradox can be explained as follows. \( M_{\text{max}} \) is a functional of the EOS, \( M_{\text{max}}[\rho(\rho < \rho_{\text{max}})] \), but the EOS for \( \rho \) greater than the maximum central density in stable NSs does not affect the value of \( M_{\text{max}} \). TM1C is actually the stiffest for \( \rho \lesssim 1.4 \times 10^{15} \text{ g cm}^{-3} \), which is quite close to the maximum central density \( \rho_{\text{max}} \approx 1.85 \times 10^{12} \text{ g cm}^{-3} \) for stable NSs based on this EOS. Therefore, while TM1C is the softest EOS for \( \rho \gtrsim 2 \times 10^{15} \text{ g cm}^{-3} \), it is irrelevant for the value of \( M_{\text{max}} \).

### 6.1 RMF models, EOSs and \( M_{\text{max}} \)

We consider three RMF models of NH matter (we will call them GM1A, GM1 B and TM1C), which are specific realizations of \( \sigma\omega\rho\phi\sigma^* \) model of Bednarek & Manka (2009). The parameters of the models are given in the Appendix A. Below, we give their brief characteristics. For all the models the binding energy per nucleon at saturation is \( B_1 = -16.3 \text{ MeV} \). Moreover, they all reproduce the semi-empirical depths of potential wells for hyperons at rest in symmetric nuclear matter at saturation density, \( U_{\Lambda}^{(N)} = -28 \text{ MeV} \), \( U_{\Sigma^+}^{(N)} = -18 \text{ MeV} \), \( U_{\Sigma^-}^{(N)} = 30 \text{ MeV} \) (e.g. Millener, Dover & Gal 1988). The parameters of NS configuration with \( M_{\text{max}} \) for non-rotating NS models are given in Table 1 for GM1A, GM1 B and TM1C models.

**Table 1.** Parameters of non-rotating NS models with maximum allowable mass. The columns are (from left to right): RMF model of NH matter, maximum stellar mass in units of the solar mass, corresponding radius of the star in km, central density in g cm\(^{-3} \), central baryon number density in fm\(^{-3} \), the ratio of (minus) strangeness number density \( S \) to baryon number density \( n_B \). \( n_S = -S/n_B = -(n_\Lambda + 2n_\Sigma - 2n_\Xi)/n_B \) in the centre of the star, the same ratio but averaged over the whole star.

| RMF model | \( M_{\text{max}} \) (\( M_\odot \)) | \( R(M_{\text{max}}) \) (km) | \( \rho_{\text{max}}/10^{15} \) (g cm\(^{-3} \)) | \( n_{\text{max}} \) (fm\(^{-3} \)) | \(-0.5S\hbar c^{\text{\prime}}\) | \(-<S>_B\) |
|-----------|-----------------------------|----------------------|--------------------------|-------------------------|----------------------------|-------------------|
| GM1A      | 1.994                       | 12.05                | 2.00                     | 0.923                   | 0.607                      | 0.143             |
| GM1 B     | 2.015                       | 11.45                | 2.28                     | 1.018                   | 0.671                      | 0.181             |
| TM1C      | 2.056                       | 12.51                | 1.85                     | 0.856                   | 0.493                      | 0.093             |

where \( N_\odot \) is defined by equation (7).

### 6 NUMERICAL RESULTS

#### 6.2 Particle fractions, adiabatic indices and the speed of sound

In Fig. 2, we show the particle fractions of constituents of NH matter, \( y_i = n_i/n_\odot \), as functions of baryon number density \( n_B \). Three panels correspond to three RMF models (GM1 A, GM1 B and TM1C). Dot–dashed vertical lines correspond to the maximum baryon number density reachable in stable non-rotating NSs, see Table 1. The order of appearance of hyperons with increasing density is identical for all EOSs. The corresponding thresholds are presented in Table 2. The first hyperon to appear is \( \Lambda \), the second hyperon is \( \Sigma^- \). The third hyperon, \( \Sigma^0 \), appears only in model GM1 B and exists only in configurations close to the \( M_{\text{max}} \) one, thus playing a marginal role in stable stars. A large repulsive potential energy of \( \Sigma^- \) in nuclear matter makes its threshold density very high, from \( 9n_\odot \) for TM1C to more than \( 10n_\odot \) for GM1 A. Therefore, \( \Sigma^- \) are absent in stable NSs.

An important quantity characterizing dynamic properties of stellar matter is the adiabatic index

\[
\gamma = \frac{P + \rho}{P} \frac{\delta P}{\delta \rho},
\]

where \( \delta P \) is a small deviation of the pressure \( P \) from its equilibrium value caused by a small variation \( \delta \rho \) of the energy density \( \rho \). This index is related to the speed of sound \( s \) by the equality \( s = [\gamma P/(P + \rho)]^{1/2} \). The ratio \( \delta P/\delta \rho \) in equation (69) should be calculated under a number of additional conditions (such as quasi-neutrality, chemical equilibrium, etc.), which differ depending on a time-scale \( \tau \) of a physical process under consideration. The resulting adiabatic indices \( \gamma \) will also be different.

Here, we consider three adiabatic indices: equilibrium adiabatic index \( \gamma_{\text{eq}} \) (Haensel, Potekhin & Yakovlev 2007), frozen adiabatic index \( \gamma_{\text{fz}} \) (Haensel et al. 2007) and ‘partly frozen’ adiabatic index

\[
\gamma_{\text{fz}} = \frac{1}{\omega \sin \pi \tau}
\]

1 The most natural example of such process is the NS oscillations. Then, \( \tau \sim 1/\omega \), where \( \omega \) is the oscillation frequency.
Physics input for NSs with hyperon cores

Figure 1. Pressure $P_{35} = P/10^{35}$ versus density $\rho_{15} = \rho/10^{15}$ for three models of NH matter considered in this paper. Right-hand panel: overall plots of EOSs. For $\rho_{15} > 2$, the TM1C EOS is the softest and GM1’B the stiffest. Left-hand panel: lower density, $\rho_{15} < 2$, segments of the EOSs. The ordering of the EOSs according to their stiffness depends on the density interval. For further discussion of this effect and its impact on the value of $M_{\text{max}}$ see the text.

Figure 2. Particle fractions $y_i = n_i/n_b$ versus baryon number density $n_b$ for three EOSs, GM1A, GM1’B and TM1C. The vertical dot–dashed lines correspond to the maximum baryon number density reached in stable non-rotating NSs for a given EOS, see Table 1. For further details see Section 6.2.

Table 2. Thresholds $n_b^{(k)}$ of appearance of particles $k = \mu, \Lambda, \Xi^-$ and $\Xi^0$ for which $n_b^{(k)} < n_{b,\text{max}}$ (see the last column). Only model GM1’B admits the existence of $\Xi^0$ hyperons in stable NSs.

| Model  | $n_b^{(\mu)}$ (fm$^{-3}$) | $n_b^{(\Lambda)}$ (fm$^{-3}$) | $n_b^{(\Xi^-)}$ (fm$^{-3}$) | $n_b^{(\Xi^0)}$ (fm$^{-3}$) | $n_{b,\text{max}}$ (fm$^{-3}$) |
|--------|-----------------|-----------------|-----------------|-----------------|-----------------|
| GM1A   | 0.1271          | 0.3472          | 0.4076          | --              | 0.923           |
| GM1’B  | 0.1272          | 0.3669          | 0.4438          | 0.9750          | 1.018           |
| TM1C   | 0.1090          | 0.3466          | 0.4622          | --              | 0.856           |

$\gamma_{\text{part}}$. In Fig. 3, they are shown by, respectively, dot–dashed, solid and dashed lines as functions of $n_b$ for the three models of NH matter adopted in this paper.

The index $\gamma_{\text{eq}}$ naturally appears in the situation when the dynamical process of interest is very slow. This means that $\tau \gg \tau_{\text{strong}}$ and $\tau \gg \tau_{\text{weak}}$, where $\tau_{\text{strong}}$ and $\tau_{\text{weak}}$ are the characteristic time-scales of ‘fast’ (due to strong interaction) and ‘slow’ (due to weak interaction) reactions of particle mutual transformations, which move the system towards full thermodynamic equilibrium (see, e.g., Yakovlev et al. 2001; Kantor & Gusakov 2009).

The index $\gamma_{\text{int}}$ can be introduced (e.g. Haensel et al. 2007) in the opposite limit, when $\tau \ll \tau_{\text{strong}}$ and $\tau \ll \tau_{\text{weak}}$. In that case, the process is so fast that all the reactions are effectively ‘frozen’ on a dynamical time-scale $\tau$. Mathematically, this means that the particle fractions $y_i$ remain constant during this process for any particle species $i = e, \mu, n, p, \Lambda, \ldots$:

$y_i = n_i/n_b = \text{constant}$. Finally, the index $\gamma_{\text{part}}$ is introduced in the intermediate case, when $\tau_{\text{weak}} \gg \tau \gg \tau_{\text{strong}}$. In that case, the matter is in equilibrium with respect to the fast reactions, while slow reactions (such as, e.g., Urca reactions; see Haensel et al. 2007) are frozen. In stable NSs, the fast reactions are $p + \Xi^- \leftrightarrow \Lambda + \Lambda$ and $n + \Xi^0 \leftrightarrow \Lambda + \Lambda$. The adiabatic indices are considered in more detail in Appendix C.
\gamma and \nu, \pi, \Delta for GM-type models are shown in Fig. 1. 
\gamma_{\text{eq}} (dot-dashed lines) assumes equilibrium with respect to the ‘fast’ reactions, while ‘slow’ reactions are frozen (see Section 6.2 for details). Vertical dotted lines indicate thresholds of appearance of (from left to right) \Lambda, \Xi^-, \Xi^0 and \Sigma^- hyperons. Vertical dot-dashed lines show the maximum baryon number density in a non-rotating NS of a maximum allowable mass (see also Table 1).

As follows from Fig. 3, each time, when a hyperon species appears as \nu_b increases, we see a sharp drop of the equilibrium adiabatic index \gamma_{\text{eq}}. Such drops reflect the fact that appearance of hyperons makes the EOS softer. The magnitude of the hyperon threshold drops decreases with increasing density, with the largest drop at the threshold for \Lambda’s. This is not surprising and is related to the increasing number of baryon species with growing density (the more the baryon species, the less sensitive is \gamma_{\text{eq}} to the appearance of additional hyperon species).

In contrast to \gamma_{\text{eq}}, \gamma_{\nu} and (\gamma_{\text{part,fr}}) which is practically indistinguishable from \gamma_{\nu}, does not drop sharply near the hyperon thresholds: the influence of hyperon thresholds is less pronounced if we consider rapid processes with \tau \ll \tau_{\text{strong}} and \tau \ll \tau_{\text{weak}}.

For illustration, Fig. 4 shows the equilibrium speed of sound \nu_{\text{eq}} = [\nu_{\text{eq}} P/(P + \rho)]^{1/2} for the three EOSs considered in this paper.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure3}
\caption{Three adiabatic indices versus \nu_b for the three selected models of NH matter. \gamma_{\text{eq}} (dot-dashed lines) is calculated assuming full thermodynamic equilibrium; \gamma_{\nu} (solid lines) is obtained under assumption that all reactions of particle mutual transformations are frozen (completely frozen matter composition); \gamma_{\text{part,fr}} (dashed lines) assumes equilibrium with respect to the ‘fast’ reactions, while ‘slow’ reactions are frozen (see Section 6.2 for details). Vertical dotted lines indicate thresholds of appearance of (from left to right) \Lambda, \Xi^-, \Xi^0 and \Sigma^- hyperons. Vertical dot-dashed lines show the maximum baryon number density in a non-rotating NS of a maximum allowable mass (see also Table 1).}
\end{figure}

6.3 Effective masses

Our results for normalized Landau effective masses \nu_{ij} = \nu_i/m_i (see equation 30) are shown in Fig. 5.

For all EOSs at all densities \nu_n > \nu_p, and, moreover, at densities relevant to stable NSs \nu_n > \nu_p > \nu_\Lambda > \nu_{\Xi^-}. At the same time, one notices a systematic differences in \nu_{ij}(\nu_b) curves (hereafter \nu_{ij} = \nu_{ij}(\nu_b) curves)

For GM1A, all hyperon \nu_{ij} curves are very flat, ranging from 0.7 to 0.85. For GM1B model, the values of \nu_{ij} are systematically smaller, about 0.6–0.75. The strongest Fermi-liquid effect and density dependence are obtained for the TM1C model, \nu_{ij} fall into the range 0.5–0.65.

We conclude that there is a significant model dependence of \nu_{ij}(\nu_b) for hyperons. This fact reflects limitations of our knowledge of the N–H and H–H interactions in dense NH matter.

6.4 Landau Fermi-liquid parameters \nu_{ij}^{(L)}, \nu_{ij}^{(P)} and entrainment matrix \nu_{ij}^{(Y)}

Our results for dimensionless Landau Fermi-liquid parameters \nu_{ij}^{(L)} = \nu_{ij}^{(L)} and \nu_{ij}^{(P)} = \nu_{ij}^{(P)} are collected in Figs 6 and 7, respectively. The results for normalized dimensionless entrainment matrix \nu_{ij}^{(Y)} are shown in Fig. 8.

\begin{table}
\centering
\begin{tabular}{|c|c|c|}
\hline
\nu_{ij} & \nu_{ij}^{(L)} & \nu_{ij}^{(P)} \\
\hline
\hline
\end{tabular}
\caption{Landau Fermi-liquid parameters for GM-type models}
\end{table}

Landau parameters \nu_{ij}^{(L)} (hereafter \nu_{ij} = n, p) for GM-type models have similar values and density dependence, those for TM1C model are smaller and are more similar to Gusakov et al. (2009a) results. In contrast to Gusakov et al. (2009a), Landau parameters \nu_{ij}^{(P)} are all positive except for \nu_{ij}^{(P)} in the model TM1C and \nu_{ij}^{(P)} in the model GM1A near the threshold for the \Sigma^- hyperon.

Nucleon \ell = 1 Landau parameters \nu_{ij}^{(L)} are not so much model sensitive, they are quite similar for the three models developed here and also for the model of Gusakov et al. (2009a). We find that, as a rule, the \ell = 1 Landau parameters are negative. A few ones which are positive, remain very small. \nu_{ij}^{(P)} dominates in magnitude over remaining Landau parameters, but \nu_{ij}^{(P)} becomes comparable to it at

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure4}
\caption{Equilibrium sound speed \nu_{\text{eq}} = [\nu_{\text{eq}} P/(P + \rho)]^{1/2} (in units of speed of light c) versus \nu_b for three models of NH matter.}
\end{figure}
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Figure 5. The normalized Landau effective masses $m^{*}_i/m_i$ versus $n_b$ for three EOSs. Each curve is marked by a corresponding baryon species index $i = n, p, \Lambda, \ldots$. Other notations are the same as in Fig. 3.

Figure 6. The dimensionless Landau Fermi-liquid parameters $F_0^{ij}$ versus baryon number density for GM1A (upper panels), GM1'B (middle panels) and TM1C (bottom panels) RMF models. Each curve is marked by the corresponding symbol $ij$. Other notations are the same as in Fig. 3.
the largest densities. Model dependence of Landau parameters with $i$ and $j$ indices is more significant. 

The entrainment matrix elements $Y_{ij}$ are also not very model dependent. The bundle of $Y_{ij}$ is bound by $Y_{i\Lambda\Lambda}$ from above and by (negative) $Y_{n\Lambda\Lambda}$ from below. $Y_{i-2}$ is significantly smaller than $Y_{i\Lambda\Lambda}$. Non-diagonal matrix elements $Y_{ij}$ with $i \neq j$ are significantly smaller than $Y_{ii}$ or $Y_{jj}$, and are usually negative; if positive, they are close to zero. On the opposite, diagonal elements $Y_{ii}$ are positive.

### 6.5 Stability with respect to $\ell = 0$ and 1 deformations of the Fermi surfaces

A small $\ell = 0$ deformation of the $i$-Fermi surface induces a small perturbation $\delta n_i$ of particle number density $n_i$, and vice versa. We consider only long-wave (uniform) perturbations that preserve electric charge neutrality of the system (in order to exclude the stabilizing effect of the Coulomb energy; see Gusakov et al. 2009a for details). Then the stability requirement is equivalent to the positive definiteness of the quadratic form $\sum_{i=0}^{\ell} n_i \delta n_i$, where the indices $k$ and $m$ run over all particle species, except for electrons. The matrix $A_{km}$ is expressible in terms of the Fermi-liquid parameters $F_{ij}$ (see, e.g., Gusakov et al. 2009a, and references therein). Stability with respect to perturbation $\delta n_i$ imposes a number of conditions on $F_{ij}$. We checked that these stability conditions are satisfied within the liquid NS core, i.e. for $n_b > 0.1$ fm$^{-3}$, for all considered models.

A small $\ell = 1$ deformation of the $i$-superfluid Fermi surface keeps the value of $n_i$ unchanged but induces a uniform superfluid current associated with $Q_i$. The change of the energy density associated with superfluid currents is given by a quadratic form $\frac{1}{2} \sum_{i=0}^{\ell} Y_{ij} Q_i Q_j$, see Gusakov et al. (2009a). Stability of the ground state is equivalent to the positive definiteness of the matrix $Y_{ij}$, implying a number of conditions on the parameters $F_{ij}$. We checked that these conditions are satisfied for all the three models and at all densities relevant to NSs.
Recently, Gulminelli, Raduta & Oertel (2012) and Gulminelli et al. (2013) pointed out the first-order phase transition associated with the appearance of strangeness in dense baryonic matter. This first-order phase transition is signalled by a spinodal instability of an uniform baryon matter (nΛ-matter in Gulminelli et al. 2012, npΛe-matter in Gulminelli et al. 2013). In our calculations, the matrix $A_{ij}$ of Section 6.5 is positive definite at $n_b > 0.1 \, \text{fm}^{-3}$ and appearance of $\Lambda$ is continuous (second-order phase transition): we do not find spinodal instability associated with appearance of strangeness that would indicate a phase-separation instability. However, in contrast to Gulminelli et al. (2012, 2013), we consider exclusively baryon matter with no trapped neutrinos and close to beta equilibrium. Therefore, our particle fractions $y_i$ in equilibrium result from the weak-interaction equilibrium conditions (see equation B1), and are functions of $n_b$, $y_j = y_j^{(eq)}(n_b)$. Our trajectory $y_j = y_j^{(eq)}(n_b)$ does not cross a spinodal instability region.

7 SUMMARY OF RESULTS

We develop a general scheme for calculation of the $\ell = 0, 1$ Landau Fermi-liquid parameters, valid for a broad class of non-linear RMF models of dense baryon matter. A non-linear Lagrangian that we consider involves the octet of baryons coupled to the $\sigma\omega\rho\phi\sigma^*$ mesons. It includes quartic terms in meson fields.

Knowledge of the Landau Fermi-liquid parameters is crucial for modelling NSs because it allows one to directly calculate the following important quantities: (i) the thermodynamic derivatives $\partial \mu_i / \partial n_j$ (see equations 68 and D1), where $\mu_i$ and $n_i$ are the relativistic chemical potential and the number density of particle species $i$ and $j$, respectively; (ii) the relativistic entrainment matrix $Y_{ij}$, both at zero temperature (see equation 12) and at finite temperatures (see Gusakov et al. 2009b); this is a basic parameter for superfluid NSs.

The developed general scheme has been applied to study in detail three up-to-date specific RMF models of NH matter, which are consistent with the existence of $2 M_\odot$ pulsars (PSR J1614−2230 and J0348+0432; see Demorest et al. 2010; Antoniadis et al. 2013) and with semi-empirical nuclear and hypernuclear data. These models allow for the presence of (maximum) three hyperon species in stable NSs. Two of the models (GM1A and GM1B) predict the appearance of (with increasing density) $\Lambda$ and $\Xi^-$ hyperons, while the model TM1C predicts also appearance of $\Xi^0$ hyperons close to a maximum density reachable in stable NSs for this model.
It is interesting that, in contrast to, e.g. the paper by Gusakov et al. (2009a), $\Sigma$ hyperons do not appear in stable NSs for the selected RMF models because of their large repulsive potential energy in nuclear matter.

For all models, we calculated and analysed the Landau Fermi-liquid parameters $F_{\rho}^{(i)}$ and $F_{\omega}^{(i)}$ as functions of the baryon number density $n_b$, entrainment matrix $Y_{\rho}(n_b)$ at $T = 0$, EOS [pressure versus density relation $P(\rho)$], particle number densities $n_i(n_b)$, adiabatic indices and Landau effective masses.

All obtained numerical results for the three RMF models constructed by us are available online as a public domain at: http://www.ioffe.ru/astro/NSG/HEOS/hyp.html. This data source contains all necessary information to model dynamics of superfluid NSs, e.g. their oscillations and cooling. The description of the online material is presented in the Appendix D.

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REFERENCES

Andersson N., Ferrari V., Jones D. I., Kokkotas K. D., Krishnan B., Read J. S., Rezzolla L., Zink B., 2011, Gen. Rel. Grav., 43, 409
Andersson N. et al., 2013, Class. Quantum Grav., 30, 193002
Antoniadis J. et al., 2013, Science, 340, 448
Baym G., Chinn S. A., 1976, Nucl. Phys. A, 262, 257
Baym G., Pethick C., 1991, Landau Fermi-Liquid Theory: Concepts and Applications. Wiley-VCH, Weinheim
Bednarek I., Manka R., 2009, JI. Phys. G: Nucl. Phys., 36, 095201
Bednarek I., Haensel P., Zdunik J. L., Bejger M., Manka R., 2012, A&A, 543, A157
Demorest P. B., Pennucci T., Ransom S. M., Roberts M. S. E., Hessels J. W. T., 2010, Nature, 467, 1081
Glendenning N. K., 1985, ApJ, 293, 470
Glendenning N., 2000, Compact Stars. Springer-Verlag, New York
Glendenning N. K., Moszkowski S. A., 1991, Phys. Rev. Lett., 67, 2414
Gulminelli F., Raduta A. R., Oertel M., 2012, Phys. Rev. C, 86, 025805
Gulminelli F., Raduta A. R., Oertel M., Margueron J., 2013, Phys. Rev. C, 87, 055809
Gusakov M. E., 2007, Phys. Rev. D, 76, 083001
Gusakov M. E., Kantor E. M., 2008, Phys. Rev. D, 78, 083006
Gusakov M. E., Kantor E. M., Haensel P., 2009a, Phys. Rev. C, 79, 055806
Gusakov M. E., Kantor E. M., Haensel P., 2009b, Phys. Rev. C, 80, 015803
Haensel P., Potekhin A. Y., Yakovlev D. G., 2007, Neutron Stars 1: Equation of State and Structure. Springer, New York
Kantor E. M., Gusakov M. E., 2009, Phys. Rev. D, 79, 043004
Leggett A. J., 1965, Phys. Rev., 140, 1869
Leggett A. J., 1975, Rev. Mod. Phys., 47, 331
Millener D. J., Dover C. B., Gal A., 2008, Phys. Rev. C, 38, 2700
Pines D., Nozières P., 1999, Theory of quantum liquids. Westview Press, Boulder
Schaffner J., Dover C. B., Gal A., Greiner C., Millener D. J., Stocker H., 1994, Ann. Phys., 235, 35
Schaffner-Bielich J., Gal A., 2000, Phys. Rev. C, 62, 034311
Shen H., Yang F., Toki H., 2006, Prog. Theor. Phys., 115, 325
Sugahara Y., Toki H., 1994, Nucl. Phys. A, 579, 557
Takahashi H. et al., 2001, Phys. Rev. Lett., 87, 212502
Weissenborn S., Chatterjee D., Schaffner-Bielich J., 2012a, Phys. Rev. C, 85, 065802

APPENDIX A: COUPLING CONSTANTS

Here, we discuss the coupling constants for models GM1A, GM1’ and TM1C. The main parameters characterizing these models are summarized in Table A1. The actual values of hyperon and meson masses which were used in all calculations are presented in Table A2 (note that the masses of baryons in each isomultiplet are assumed to be the same). For all the models $\Lambda_{\nu} = \bar{c} = 0$. The data which are not included in Table A1 are the depths of potential wells for hyperons in symmetric nuclear matter at saturation density (e.g. Millener et al. 1988; Schaffner-Bielich & Gal 2000; Shen, Yang & Toki 2006; Weissenborn et al. 2012b),

$$U^{i(N)}_\Lambda = -28.0 \text{ MeV}, \quad U^{i(N)}_\omega = -18.0 \text{ MeV},$$

$$U^{i(N)}_\sigma = 30.0 \text{ MeV},$$

(A1) which are the same for all three models. In addition, the model TM1C, which allows for the presence of $\sigma^*$ meson, fits also the weak $\Lambda - \Lambda$ attraction (Takahashi et al. 2001),

$$U^{i(\Lambda)}_\omega = -5.0 \text{ MeV},$$

(A2) and assumes (Schaffner et al. 1994)

$$U^{i(\Sigma)}_\omega \approx U^{i(\Sigma)}_\omega \approx 2U^{i(\Lambda)}_\omega \approx 2U^{i(\Lambda)}_\omega = -10.0 \text{ MeV}.$$ (A3)

Using these data, one can calculate various coupling constants for the models GM1A, GM1’ and TM1C. Most of them are listed in Table A3. The remaining constants are related to those from Table A3 by the following conditions (see, e.g., equation 11 of Weissenborn et al. 2012a)

$$g_{\omega N} = \frac{\sqrt{3}}{\sqrt{2} + \sqrt{3}} g_{\omega N}, \quad g_{\omega \Sigma} = g_{\omega \Lambda}, \quad g_{\omega \Xi} = \frac{2\sqrt{3} z}{\sqrt{2} + \sqrt{3} z} g_{\omega N},$$

(A4)

$$g_{\rho N} = 0, \quad g_{\rho \Sigma} = g_{\rho N}, \quad g_{\rho \Xi} = g_{\rho N},$$

(A5)

$$g_{\sigma N} = \frac{\sqrt{6} z - 1}{\sqrt{2} + \sqrt{3} z} g_{\omega N}, \quad g_{\phi \Lambda} = -g_{\phi \Xi} = -\frac{1}{\sqrt{2} + \sqrt{3} z} g_{\omega N},$$

$$g_{\rho \Xi} = -\frac{1 + \sqrt{6} z}{\sqrt{2} + \sqrt{3} z} g_{\omega N}. $$

(A6)

Let us briefly describe how we calculated the coupling constants presented in Table A3. The constants $g_\omega, g_\sigma, g_\rho N, g_\omega N$ and $g_\omega N$ can be expressed through the parameters $n_b, B, E_{sym}, K_s$ and $m_{\psi}$, respectively. The corresponding consideration is similar to that presented in section 4.8 of Glendenning (2000) with the only exception that in our case $\phi$ can be non-vanishing even for pure nucleon matter (because $g_{\rho N}$ is non-zero and is related to $g_{\omega N}$ by equation A6).

The constants $g_{\omega N}$, where $i = \Lambda, \Sigma, \Xi$, can be obtained from the requirement that the energy of an $i$-hyperon (with the momentum $p = 0$) in the symmetric nuclear matter at saturation is equal to

$$m_i + U^{i(N)}_i = g_{\omega N} \omega^0 + g_{\rho \phi} \phi^0 + (m_i - g_{\omega N} \sigma).$$ (A7)
Table A1. Various physical parameters for three models: GM1A, GM1’B and TM1C. In the table, \( m_N \) is the nucleon mass; \( n_i \) the saturation density; \( B_i \) binding energy per nucleon; \( E_{\text{sym}} \) the symmetry energy; \( K_a \) nuclear matter incompressibility and \( m_{\phi/N}^0/m_N^0 \) the Dirac effective mass in units of \( m_N \). All these quantities are given at saturation point. Further, \( c_3 \) is the coupling constant characterizing non-linear interaction of \( \omega \)-mesons; \( z \) is the parameter introduced in Weissenborn et al. (2012a) to describe deviation of a given model from the SU(6)-symmetric case [the SU(6) value is \( z = 1/\sqrt{6} \)]. Finally, the last column indicates that \( \sigma^\pm \)-mesons are only allowed for TM1C model.

| Model  | \( m_N \) (MeV) | \( n_s \) (fm\(^{-3}\)) | \( B_i \) (MeV) | \( E_{\text{sym}} \) (MeV) | \( K_a \) (MeV) | \( m_{\phi/N}^0/m_N^0 \) | \( c_3 \) | \( z \) | \( \sigma^\pm \) |
|--------|-----------------|-----------------|----------------|----------------|----------------|-----------------|--------|--------|----------------|
| GM1A   | 938.919         | 0.153           | -16.3          | 32.5           | 300.0          | 0.0             | 1/\sqrt{6} | No     | No     |
| GM1’B  | 938.919         | 0.153           | -16.3          | 32.5           | 240.0          | 0.7             | 0.3      | No     | No     |
| TM1C   | 938.0           | 0.153           | -16.3          | 36.9           | 281.0          | 0.634           | 0.2      | Yes    |        |

Table A2. Masses (in MeV) of hyperons and mesons adopted in all calculations.

| Mass    | \( m_\Lambda \) | \( m_\Sigma \) | \( m_\Sigma^+ \) | \( m_\Sigma^- \) | \( m_\Xi^0 \) | \( m_\Xi^- \) | \( m_\Omega^- \) | \( m_\Omega^0 \) | \( m_\Omega^+ \) |
|---------|-----------------|----------------|-----------------|-----------------|----------------|----------------|-----------------|-----------------|-----------------|
| MeV     | 1115.63         | 1193.12        | 1318.1          | 511.198         | 783.0          | 770.0          | 1020.0          | 975.0           |

Table A3. Coupling constants for the models GM1A, GM1’B and TM1C.

| Model    | \( g_3 \) (fm\(^{-1}\)) | \( g_4 \) | \( g_\sigma^\Lambda \) | \( g_\sigma^\Sigma \) | \( g_\sigma^\Xi \) | \( g_\sigma^\Omega \) | \( g_\sigma^\Delta \) | \( g_\sigma^\Sigma \) | \( g_\sigma^\Omega \) |
|----------|-----------------|-------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| GM1A     | 9.840           | -6.693| 8.897                | 10.617               | 8.198                | 5.435                | 3.603                | 2.844                | -                  |
| GM1’B    | 16.324          | -31.985| 9.096                | 10.558               | 8.198                | 6.241                | 4.368                | 4.275                | -                  |
| TM1C     | 7.684           | -2.224| 10.038               | 12.300               | 9.275                | 7.733                | 6.037                | 6.320                | 0.0                |

To use this formula one should first calculate the fields \( \sigma \), \( \omega^0 \) and \( \phi^0 \) using, respectively, equations (20), (22) and (24). Note that nucleons do not generate the \( \sigma^- \)-field and \( \rho_i^0 = 0 \) is symmetric nuclear matter.

Finally, following Yang & Shen (2008), the constants \( g_{\sigma^i} \) (\( i = \Lambda, \Sigma \)) for the model TM1C are calculated assuming \( U_{\sigma^i}^{(\Xi)} = \gamma = 10 \) MeV (see equation A3). To calculate them we consider symmetric matter at \( n = n_s \), composed of equal number of \( \Sigma^- \) and \( \Xi^0 \) hyperons and add one more \( i \)-hyperon with momentum \( p = 0 \) to this system. Then the energy of this \( i \)-hyperon will be

\[
m_i + U_{\sigma^i}^{(\Xi)} = g_{\sigma^i} \omega^0 + g_{\sigma^i} \phi^0 + m_i - g_{\sigma^i} \sigma - g_{\sigma^i} \sigma^e,
\]

so that \( g_{\sigma^i} \) can be easily found from this formula provided that the fields \( \sigma, \sigma^e, \omega^0 \) and \( \phi^0 \) are already calculated from equations (20)–(22) and (24). Note that \( \rho_i^0 = 0 \) in symmetric matter is composed of \( \Sigma^- \) and \( \Xi^0 \) hyperons. The constant \( g_{\sigma^\Sigma} \) is set equal to \( g_{\sigma^\Lambda} \).

**APPENDIX B: EQUATION OF STATE**

Here, we consider beta-equilibrated NH matter. The condition of beta equilibrium implies the following relations between the relativistic chemical potentials (e.g. Haensel et al. 2007),

\[
\mu_i = \mu_e - q_i \mu_e, \quad \mu_\Sigma = \mu_\mu,
\]

where \( q_i \) is the charge of baryon species \( i \) in units of the proton charge. These equations should be supplemented by the quasi-neutrality condition,

\[
\sum_i q_i n_i = n_e - n_\mu = 0.
\]

Together with equations (29)–(34) and the field equations (20)–(24), these relations allow us to find all thermodynamic quantities as functions of baryon number density \( n_b \), as well as to determine the function \( P(n_b) \).

**APPENDIX C: ADIABATIC INDICES**

Here, we describe in more detail the calculation of the adiabatic indices \( \gamma_{\text{eq}} \), \( \gamma_{\text{eq}} \) and \( \gamma_{\text{part}} \). All the indices are given by equation (69), which can be represented as

\[
\gamma = \frac{n_b}{P} \frac{\delta P}{\delta n_b},
\]

where we make use of the fact that in thermodynamic equilibrium \( P + \rho = \mu \rho_0 \) and that for small deviations from thermodynamic equilibrium \( \delta P = \mu \delta \rho_0 \) (see, e.g., Gusakov 2007; Gusakov & Kantor 2008).

(i) **Equilibrium adiabatic index \( \gamma_{\text{eq}} \)**

As it is discussed in Section 6.2, in that case the ratio \( \delta P/\delta n_b \) should be calculated in full thermodynamic equilibrium, that is, under conditions (B1) and (B2). In this situation, \( P \) can be presented as only a function of \( n_b \), while other particle number densities can be expressed through \( n_b \) by means of equations (B1) and (B2). In other words, one can calculate \( \gamma_{\text{eq}} \) from the following formula,

\[
\gamma_{\text{eq}} = \frac{n_b}{P} \frac{\delta P}{\delta n_b}.
\]

(ii) **Frozen adiabatic index \( \gamma_{\text{f}} \)**

In this case all reactions of particle mutual transformations are frozen, that is, \( Y_i = n_i/n_b = \text{constant} \) for any particle species \( i \). The quasi-neutrality condition (B2) is then automatically satisfied and \( \gamma_{\text{f}} \) can be calculated from the formula

\[
\gamma_{\text{f}} = \frac{n_b}{P} \frac{\delta P}{\delta n_b}.
\]

(iii) **Partly frozen adiabatic index \( \gamma_{\text{part}} \)**

In this case, all the slow reactions due to weak interaction (in particular, those with leptons \( e \) and \( \mu \)) are frozen, which means that \( Y_e = \text{constant} \).
\[ y_\mu = \text{constant}. \] (C5)

In contrast, the reactions due to strong interaction are so fast that the matter is always in equilibrium with respect to them. Here are these fast reactions

\[ p + \Sigma^- \leftrightarrow \Lambda + \Lambda, \] (C6)
\[ n + \Sigma^0 \leftrightarrow \Lambda + \Lambda, \] (C7)
\[ p + \Sigma^- \leftrightarrow n + \Lambda, \] (C8)
\[ n + \Sigma^0 \leftrightarrow n + \Lambda, \] (C9)
\[ n + \Sigma^+ \leftrightarrow p + \Lambda, \] (C10)
\[ \Sigma^+ + \Sigma^- \leftrightarrow \Lambda + \Lambda. \] (C11)

and the corresponding conditions of equilibrium

\[ \mu_p + \mu_{\Sigma^-} = 2\mu_\Lambda, \] (C12)
\[ \mu_n + \mu_{\Sigma^0} = 2\mu_\Lambda, \] (C13)
\[ \mu_p + \mu_{\Sigma^-} = \mu_n + \mu_\Lambda, \] (C14)
\[ \mu_{\Sigma^0} = \mu_\Lambda, \] (C15)
\[ \mu_n + \mu_{\Sigma^+} = \mu_p + \mu_\Lambda, \] (C16)
\[ \mu_{\Sigma^+} + \mu_{\Sigma^-} = 2\mu_\Lambda. \] (C17)

In stable NSs, only \( \Lambda, \Sigma^- \), and (for the model GM1'B) \( \Sigma^- \), hyperons can be present, so only the first two conditions, (C12) and (C13), are relevant.

The final two conditions that should be taken into account are the conservation of electric charge (B2) and the strangeness fraction

\[ y_S = S/n_b, \] (C18)

where \( S = \sum_i s_i n_i \) is the strangeness number density and \( s_i \) is the strangeness of particle species \( i \). The condition (C18) follows from the observation that strangeness is conserved in reactions (C12)–(C17) (while other reactions are frozen).

The conditions (B2), (C4), (C5) and (C12)–(C18) allow one to express the pressure as a function of only four variables \( n_b, y_e, y_\mu \) and \( y_S \), and to present adiabatic index \( \gamma_{\text{part}} \) in the form

\[ \gamma_{\text{part}} = \frac{n_b}{P} \frac{\partial P(n_b, y_e, y_\mu, y_S)}{\partial n_b}. \] (C19)

**APPENDIX D: DESCRIPTION OF ONLINE MATERIAL**

The results of our numerical calculations are summarized in a number of files that can be found on the web: [http://www.ioffe.ru/astro/NSG/heos/hyp.html](http://www.ioffe.ru/astro/NSG/heos/hyp.html). We briefly describe them here.

1. Files GM1_A.dat, GM1_B.dat and TM1C.dat contain data concerning the pressure \( P \), energy density \( \rho \) (both in MeV fm\(^{-3}\)) and particle number densities \( n_i \) (in fm\(^{-3}\)) for three models GM1A, GM1'B and TM1C, studied in this paper. Each file consists of 13 columns for 13 parameters listed in Table D1.

2. Files GM1A_Fields.dat, GM1B_Fields.dat and TM1C_Fields.dat contain data concerning the values of meson fields (in MeV) at different baryon number densities \( n_b \) (see Table D2).

3. Files GM1A_gamma.dat, GM1B_gamma.dat and TM1C_gamma.dat contain data concerning the values of adiabatic indices \( \gamma_{\text{eq}}, \gamma_{\text{part}} \) and \( \gamma_{\text{fr}} \) at different baryon number densities \( n_b \) (see Table D3).

4. Files GM1A_Mass.dat, GM1B_Mass.dat and TM1C_Mass.dat contain data concerning the values of Landau masses

| Column number | Parameter | Dimension |
|---------------|-----------|-----------|
| 1             | \( n_b \)  | fm\(^{-3}\) |
| 2             | \( p \)    | MeV fm\(^{-3}\) |
| 3             | \( \rho \)  | MeV fm\(^{-3}\) |
| 4             | \( n_e \)   | fm\(^{-3}\) |
| 5             | \( n_\mu \) | fm\(^{-3}\) |
| 6             | \( n_\Lambda \) | fm\(^{-3}\) |
| 7             | \( n_{\Sigma^-} \) | fm\(^{-3}\) |
| 8             | \( n_{\Sigma^0} \) | fm\(^{-3}\) |
| 9             | \( n_{\Sigma^+} \) | fm\(^{-3}\) |

| Column number | Parameter | Dimension |
|---------------|-----------|-----------|
| 1             | \( n_b \)  | fm\(^{-3}\) |
| 2             | \( \gamma_{eq} \) | Dimensionless |
| 3             | \( \gamma_{part} \) | Dimensionless |
| 4             | \( \gamma_{fr} \) | Dimensionless |

| Column number | Parameter | Dimension |
|---------------|-----------|-----------|
| 1             | \( n_b \)  | fm\(^{-3}\) |
| 2             | \( m_{s_1} \) | g |
| 3             | \( m_{s_2} \) | g |
| 4             | \( m_{s_3} \) | g |
| 5             | \( m_{s_4} \) | g |
| 6             | \( m_{s_5} \) | g |
| 7             | \( m_{s_6} \) | g |
| 8             | \( m_{s_7} \) | g |
| 9             | \( m_{s_8} \) | g |

| Column number | Parameter | Dimension |
|---------------|-----------|-----------|
| 1             | \( n_b \)  | fm\(^{-3}\) |
| 2             | \( m_{s_1} \) | g |
| 3             | \( m_{s_2} \) | g |
| 4             | \( m_{s_3} \) | g |
| 5             | \( m_{s_4} \) | g |
| 6             | \( m_{s_5} \) | g |
| 7             | \( m_{s_6} \) | g |
| 8             | \( m_{s_7} \) | g |
| 9             | \( m_{s_8} \) | g |
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Table D5. A schematic structure of the files GM1A_F0.dat, GM1’_B_F0.dat, TM1C_F0.dat, GM1A_F1.dat, GM1’_B_F1.dat, TM1C_F1.dat, GM1A_Entr.dat, GM1’_B_Entr.dat and TM1C_Entr.dat.

| Column number | Parameter          |
|---------------|--------------------|
| 1             | \( n_b \)          |
| 2             | \( n_m \)          |
| 3             | \( n_p \)          |
| 4             | \( n_{\Lambda} \)  |
| 5             | \( n_{\Xi}^{-} \)  |
| 6             | \( n_{\Xi}^{0} \)  |
| 7             | \( n_{\Xi}^{+} \)  |
| 8             | \( n_{\Lambda} \)  |
| 9             | \( n_{\Sigma}^{-} \) |
| 10            | \( n_{\Sigma}^{+} \) |
| 11            | \( p_{\Lambda} \)  |
| 12            | \( p_{\Xi}^{-} \)  |
| 13            | \( p_{\Xi}^{0} \)  |
| 14            | \( p_{\Xi}^{+} \)  |
| 15            | \( p_{\Lambda} \)  |
| 16            | \( p_{\Sigma}^{-} \) |
| 17            | \( \Lambda_{\Lambda} \) |
| 18            | \( \Lambda_{\Xi}^{-} \) |
| 19            | \( \Lambda_{\Xi}^{0} \) |
| 20            | \( \Lambda_{\Xi}^{+} \) |
| 21            | \( \Lambda_{\Sigma}^{-} \) |
| 22            | \( \Lambda_{\Sigma}^{0} \) |
| 23            | \( \Lambda_{\Sigma}^{+} \) |
| 24            | \( \Xi_{\Xi}^{-} \)  |
| 25            | \( \Xi_{\Xi}^{0} \)  |
| 26            | \( \Xi_{\Xi}^{+} \)  |
| 27            | \( \Xi_{\Sigma}^{-} \) |
| 28            | \( \Xi_{\Sigma}^{0} \) |
| 29            | \( \Xi_{\Sigma}^{+} \) |
| 30            | \( \Sigma_{\Xi}^{-} \) |
| 31            | \( \Sigma_{\Xi}^{0} \) |
| 32            | \( \Sigma_{\Xi}^{+} \) |
| 33            | \( \Sigma_{\Sigma}^{-} \) |
| 34            | \( \Sigma_{\Sigma}^{0} \) |
| 35            | \( \Sigma_{\Sigma}^{+} \) |

Effective masses \( m_\ast \) [equation (30)] at different baryon number densities \( n_b \) (see Table D4).

(5) Files GM1A_F0.dat, GM1’_B_F0.dat and TM1C_F0.dat contain dimensionless Landau parameters \( F_{ij}^0 \) (see equation 7). Note that \( F_{ij}^0 = F_{ji}^0 \), so only \( 8 \times 9/2 = 36 \) matrix elements are independent and presented in these files (37 columns in each file; the first column is \( n_b \) in fm\(^{-3}\)). An actual column number containing the Landau parameters with indices \( i \) and \( j \) can be found from Table D5. For instance, Landau parameters \( F_{ij}^0 \) and effective masses \( m_\ast \) allows one to calculate the important thermodynamic derivatives, \( \partial \mu_i(n_n, \ldots, n_{\Sigma^+})/\partial n_j \). As follows from equations (7) and (68),

\[
\frac{\partial \mu_i(n_n, \ldots, n_{\Sigma^+})}{\partial n_j} = \frac{\partial \mu_j(n_n, \ldots, n_{\Sigma^+})}{\partial n_i} = \frac{\pi^2 \hbar^3}{\sqrt{m_i m_j p_{\mu_i} p_{\mu_j}}} \left( F_{ij}^0 + \delta_{ij} \right).
\]  

(D1)

(6) Files GM1A_F1.dat, GM1’_B_F1.dat and TM1C_F1.dat contain dimensionless Landau parameters \( F_{ij}^1 \) and have exactly the same structure as the files with \( F_{ij}^0 \) (see Table D5).

(7) Files GM1A_Entr.dat, GM1’_B_Entr.dat and TM1C_Entr.dat contain the symmetric entrainment matrix \( Y_{ij} \) (see equation 12) ordered in the same way as in the case of Landau parameters \( F_{ij}^0 \) and \( F_{ij}^1 \) (see Table D5).

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