RESEARCH ARTICLE

Operation optimization of variable frequency pumps in compound series-parallel heat transfer systems based on the power flow method

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Abstract

The optimal match of multiple operation parameters in heat transfer systems (HTSs) is the key to trade-off heat transfer and flow resistance for energy conservation. For a compound series-parallel HTS, this study applies the power flow model and the driving-resistance model to build the heat transfer and fluid flow constraints of the whole system directly, instead of individual components. Utilizing these constraints together with the Lagrange multiplier method offers the optimal operating frequencies of each variable frequency pumps (VFP) with the minimum pumping power consumptions under different heat loads. Operating the experiment platform with the optimized parameters shows that the heat load increment in parallel branches only needs to increase the operating frequency of VFP in the related hot-water loop, whereas the heat load increment in series branches needs to increases the operating frequencies of VSPs in both cold-water and the corresponding hot-water loops. When the heat load varies from 10 to 11 kW in the parallel branch, the downstream, and the upstream of the series branch, the total pumping power consumptions of all VFPs increase by 19.45%, 43.86%, and 39.99%, respectively. It means assigning the additional heat load in the parallel branch is more energy efficient.

KEYWORDS

compound series-parallel HTS, energy conservation, frequency optimization, power flow method, system constraints

1 | INTRODUCTION

Heat transfer systems (HTSs), especially the compound series-parallel systems, perform magnificent roles on industrial production and daily life, such as the central air conditioning system, power plant, chemical industry, and central heating system, where heat transfer enhancement and flow resistance reduction are two magnificent and restrictive objectives. Flow resistance may increase when enhancing heat transfer and vice versa. Therefore, for considering heat transfer and fluid flow characteristics collaboratively, the optimal match of multiple parameters, including structure parameters, requirement parameters, and operating parameters is the key in HTS optimizations.

There are many optimization methods in engineering practices. Using entropy concept is common in HTS optimizations. Wechsung et al. and Du et al. used energy analysis methods to optimize the heat exchanger network in a heating, ventilation, and air conditioning system. Naserbegi et al. proposed a novel exergy optimization
method for the Bushehr nuclear power plant, where the main objective is minimizing the exergy destruction. Florez-Orrego et al. proposed an exergy analysis model and optimized for an industrial ammonia synthesis unit. Besides, Li et al. used the exergy efficiency to analyze and optimize a combined cooling, heating, and power system. Caliskan et al. applied the exergy economic to analyze and evaluate the performance of a Maisotsenko cycle–based novel air cooler. Sangi et al. Ahmadi et al., and Sahraie et al. used the thermoeconomic as the criteria to analyze and optimize a building heating system, a Stirling heat pump, an irreversible three–heat-source absorption heat pump and a two-stage irreversible heat pump. Multiobjective optimization, which considers both the thermoeconomic and exergy, is also very common in energy system analysis and aims to improve exergetic efficiency. However, there are some limitations during the application of these criteria. For instance, the operating conditions of a HTS corresponding to the entropy generation minimization are not always the optimal ones. Besides, engineers will not always design a HTS based on entropy generation minimization in engineering applications, but need to satisfy its desired design requirements in confined space. Applying individual criterion cannot satisfy the various demands on performance and global constraints simultaneously.

On the other hand, Guo et al. proposed the concepts of entransy and entransy dissipation rate, which describe the heat transfer ability of a system and evaluates the irreversibility of a heat transfer process, respectively. Meanwhile, Chen et al. derived a general formula of entransy dissipation-based thermal resistance (EDTR) for different types of heat exchangers. On this basis, Xu et al. set up the optimization model for distinct heating networks, heat exchanger networks in spacecraft, and vapor-compression refrigeration system based on the entransy-balance equations. Besides, Chen et al. proposed an electrical circuit analogy method, that is, power flow method, to reflect the global heat transfer laws in HTSs for global optimization on HTSs. The method builds system constraints with equivalent thermal resistance network. Based on this method, Zhao et al. analyzed and optimized an absorption energy storage system and Wang et al. proposed a direct optimal control strategy for heat exchanger networks. These studies declare that the power flow method provides particular convenience on considering both constraints and performance requirements.

The contributions of this study are trading off the heat transfer and flow resistance of a compound series-parallel HTS for energy conservation via experimental and theoretical researches. By building the global heat transfer and flow constraints with power flow method and flow resistance analysis, solving the equation sets established by Lagrange multiplier method offers the optimal operating frequencies of VFPs. Then, the optimal operation frequencies under different heat loads are studied experimentally.

## 2 | Physical and Mathematical Models of a Compound Series-Parallel HTS

Figure 1 shows a typical compound series-parallel HTS. There are three hot-water loops (right side) and two cold-water loops (left side). The cold water starts from the chiller and then flow through the thermostatic cold-water tank, which acts as the cold source. Then, it divides into two branches along with the flowing direction. One is the series branch, which heat exchanger 1 (HE1) and heat exchanger 2 (HE2) locate in. The other one is the parallel branch, which heat exchanger 3 (HE3) locates in. After flowing through HEs, the cold water from the two branches mixes and flows through the chiller to cycle again. All the loops contain electric control valves (ECV), variable frequency pump (VFP), differential pressure gauges (DPG), and flow meters (FM). Each hot-water loop includes a thermostat water tank, which acts as the heat source. In the inlets and the outlets of each HE, gluing a pair of thermocouples on the pipes ensures the measurement accuracy. A data acquisition system (DAS) collects the experiment data. Figure 2 gives the picture of the experimental compound series-parallel HTS platform.

### 2.1 | Heat transfer analysis by the power flow method

Figure 3 gives the $T$-$Q$ diagram of compound series-parallel HTS. $T_{hi,i}$ and $T_{ci,i}$ stand for the mean temperatures of hot fluid and cold fluid in each heat exchanger, respectively. $T_c$ is the lowest temperature in the system. $T_m$ stands for the characteristic temperature of the cold fluid. The heat transfer rate $Q_i$ in a heat exchanger is

$$Q_i = \frac{T_{hi,i} + T_{hi, out} - T_{ci,i} - T_{ci, out}}{R_{g,i}} = \frac{T_{hi,i} - T_{ci,i}}{R_{g,i}},$$  \hspace{1cm} (1)

where the subscript $in$ and $out$ are inlet and outlet for short, respectively, and $i$ is the number of heat exchanger. $R_{g,i}$ is the entransy dissipation-based thermal resistance (EDTR). For a counter-flow heat exchanger, the expression is as follows:

$$R_{g,i} = \frac{\xi_i}{2} \left[ \exp(KA\xi_i) + 1 \right] \frac{1}{mc_p},$$  \hspace{1cm} (2)

$$\xi_i = \frac{1}{mc_p} - \frac{1}{mc_p},$$  \hspace{1cm} (3)
where $K$ is the heat transfer coefficient, $A$ is the heat transfer area, $\xi$ is the flow arrangement factor, $\dot{m}$ is the mass flow rate, and $c_p$ is the constant pressure-specific heat. This study considers that the flow arrangement factor is unequal to zero. The subscript $h$ and $c$ are hot fluid and cold fluid for short, respectively. If the flow arrangement factor is equal to zero, then
the temperature differences $\Delta T_{am}$ at any position are the same at a heat transfer rate $Q_i$, which satisfies the following formula:

$$Q_i = (KA)_i \Delta T_{am,i},$$

where $(KA)_i$ is the thermal conductance of the heat exchanger. According to the definition of the entransy dissipation, the total entransy dissipation in the heat exchanger is as follows:

$$\Phi_{h,i} = (T_{h,i} - T_{c,i})Q_i = \Delta T_{am,i}Q_i.$$ (4)

Based on the definition of EDTR, the EDTR expression is as follows:

$$R_{g,i} = \frac{\Phi_{h,i}}{Q_i^2} = \frac{\Delta T_{am,i}}{Q_i} = \frac{1}{(KA)_i}.$$ (6)

Similar to the voltage variation of potential in an electric circuit, the variation of thermal potential, that is, temperature, in the HTS builds the relationships among temperatures, thermal resistances, and heat flow rates in the HTS.24

HE1 is in series connection with HE2. The temperature $T_{h,1}$ decreases to $T_{c,1}$ because of the heat $Q_1$ flowing through the EDTR $R_{g,1}$ and the temperature difference is as follows:

$$T_{h,1} - T_{c,1} = Q_1 R_{g,1}.$$ (7)

Then, the temperature $T_{c,1}$ drop to $T_c$ owing to the heat flowing through another thermal resistance $R_{s,1}$, that is, the series connection thermal resistance and the temperature difference is as follows:

$$T_{c,1} - T_c = Q_1 R_{s,1} = \frac{Q_1}{2m_{c1}c_{p1}}.$$ (8)

The same analysis is applicable for HE2, but the series connection thermal resistance is different. The temperature differences are as follows:

$$T_{h,2} - T_{c,2} = Q_2 R_{g,2}$$ (9)

$$T_{c,2} - T_c = Q_2 R_{s,2} = Q_2 \left( \frac{1}{2m_{c1}c_{p2}} + \frac{Q_1}{Q_2 m_{c1}c_{p2}} \right)$$ (10)

HE3 is in parallel connection with HE1 and HE2. The temperature $T_{h,3}$ decreases to $T_{c,3}$ because of the heat $Q_3$ flowing through the EDTR $R_{g,3}$. Then, the temperature $T_{c,3}$ drops to $T_c$ owing to the heat flowing through another thermal resistance $R_{p,3}$, that is, the parallel connection thermal resistance. The temperature differences are as follows:

$$T_{h,3} - T_{c,3} = Q_3 R_{g,3}$$ (11)

$$T_{c,3} - T_c = Q_3 R_{p,3} = \frac{Q_3}{2m_{c3}c_{p3}}$$ (12)

Then, through the processes above, the heat flows meet together at the lowest temperature $T_c$ of the HTS. At last, it flows through a thermal motive $\epsilon_p$, which makes the temperature increase from $T_c$ to $T_m$ as following:

$$\epsilon_p = T_m - T_c = \frac{\sum_{i=1}^{3} Q_i}{2(m_{c1} + m_{c3})c_p}$$ (13)

Integrating Equations (7-13) gives the inherent mathematical relationship of the heat transfer processes in the whole HTS:

$$T_{h,i} - Q_i R_{g,i} - Q_i R_{s,i} + \epsilon_p = T_m, i = 1, 2$$ (14)

$$T_{h,3} - Q_3 R_{g,3} - Q_3 R_{p,3} + \epsilon_p = T_m$$ (15)

Figure 4 provides the equivalent thermal resistance network diagram of compound series-parallel HTS. Equations (14) and (15) give the global heat transfer constraints of the whole system. There exists no intermediate parameters in Equations (14) and (15). What’s more, building the optimization functions with the constraint above can also satisfy the various performance requirements, such as power consumption minimization, heat transfer maximization, total mass minimization, and so on.

### 2.2 Fluid flow analysis by the driving-resistance method

#### 2.2.1 Pipeline model

Figure 5 shows the characteristic parameter distributions in the pipeline. The equation of continuity declares that the
mass flow rate in each hot-water loop is the same, which makes the characteristic parameters constant (right side in Figure 5). Under slight flow rate variation, the pressure head $H$ caused by the pipeline is as follows:

$$H = h_s + d_h \dot{m}^2,$$  \hspace{1cm} (16)

where $h_s$ stands for the static pressure head, $d_h$ is the characteristic parameter of the pipeline, $\dot{m}$ is the mass flow rate, and $i$ is the number of hot-water loop.

The mass flow rates are different in main and branches of the cold-water loop, which makes the characteristic parameters variable (left side in Figure 5). To simplify the problem, embedding the static pressure head into the coefficient $d$ provides the relationship between pressure head $H$ and mass flow rate $\dot{m}$:

$$H_c = d \dot{m}_{ci}^2,$$  \hspace{1cm} (17)

Then, the pressure head and mass flow rate of the cold-water loop satisfy the following formulas:

$$H_{c1} = d_{12} \dot{m}_{c1}^2 + d_{01} (\dot{m}_{c1} + \dot{m}_{c3})^2$$  \hspace{1cm} (18)

$$H_{c3} = d_3 \dot{m}_{c3}^2 + d_{01} (\dot{m}_{c1} + \dot{m}_{c3})^2,$$  \hspace{1cm} (19)

where $H_{c1}$ and $H_{c3}$ stand for the pressure heads of series branch and parallel branch, respectively. $d_{01}$, $d_{12}$, and $d_3$ are the characteristic parameters in the main and branch.

### 2.2.2 Pump model

For a variable frequency pump, the mass flow rate $\dot{m}$, the frequency $\omega$, and the pressure head $H$ satisfy the following equation:

$$H_{pu,i} = a_0 \omega_i^2 + a_1 \omega_i \dot{m}_i \rho + a_2 \dot{m}_i^2, \hspace{1cm} i = 1,2,3,c,$$  \hspace{1cm} (20)

where $\rho$ is the fluid density and $a_0$, $a_1$, and $a_2$ are the characteristic parameters of the pumps.

### 3 Identification of the characteristic parameters

For the characteristic parameters of each hot-water loop, the following experimental steps can identify their values. First, run the pump with the frequency of 45 Hz and the valve opening of 100% at steady state for 10 min. Then, run the pump with the frequencies of 40, 35, 30, and 25, respectively, at steady state. At last, fit the data with Equation (16). Table 1 lists the characteristic parameters of hot-water pipeline. For the characteristic parameter in the cold-water loop, running the pump with two different frequencies, 20 and 25 Hz, offers four equations. Solving three equations will give the characteristic parameters and the forth equation validates the results. Table 2 shows the characteristic parameters of the cold-water loop.

For the characteristic parameters of each pump, the following experimental steps can identify their values. First, run the pump under the rated frequency, 50 Hz, with a certain valve opening, 100%, at steady state for 10 minutes. Secondly, repeat the first step with a series of valve opening, 80%, 60%, 50%, 40%, 30%, and 20%, respectively. Thirdly, fitting the mass flow rate and the pressure head with Equation (20) gives the characteristic parameters. Running the pump with different frequencies, 45, 40, and 35 Hz, respectively, will validate the values. Table 3 shows the characteristic parameters of each pump.

### 4 Optimization of the compound series-parallel HTS

For a given HTS with fixed pipeline structures, HEs and VFPs, seeking for the optimal operating frequencies of VFPs to minimize the total pumping power consumption at a given heat load is effective for energy conservation. Based on the
power consumption of a VFP and ignoring the mechanical loss, the total power consumption of all VFPs in the HTS is

\[
Pt = \sum_{i=1}^{3} P_{ti} = \sum_{i=1}^{3} \left( \dot{m}_{i} g H_{pu,i} + (\dot{m}_{c1} + \dot{m}_{c3}) g H_{pu,c} \right). \tag{21}
\]

The driving and the resistance of fluid in the system should be equal at the steady state, which means the pressure heads of pumps should be equal to those of pipelines as following

\[
H_{pu,i} = H_{i}, \quad i = 1, 2, 3, c \tag{22}
\]

Therefore, the pumping power consumption minimization with the global heat transfer and flow constraints is actually a Lagrange conditional extremum problem. Then, the Lagrange function is constructed as following:

\[
\Pi = Pt + a_{i}(T_{hi} - Q_{Rg,i} - Q_{Rsi} + \varepsilon_{p} - T_{m}) + \beta_{i}(H_{pu,i} - H_{i}) + \gamma_{1}(H_{pu,c} - H_{c1}) + \gamma_{2}(H_{pu,c} - H_{c3}), \quad i = 1, 2, 3 \tag{23}
\]

Making the partial derivatives of the Lagrange function with respective to \( \dot{m}_{i}, \omega_{i}, \alpha_{c}, \tilde{m}_{i1}, \tilde{m}_{i3}, a_{i}, \beta_{i}, \gamma_{1}, \gamma_{2} \) \((i = 1, 2, 3)\) equal to zero (as shown in Appendix A) provides the optimization equations. There are 17 equations with 17 unknown parameters, that is, \( \dot{m}_{i}, \omega_{i}, \alpha_{c}, \tilde{m}_{i1}, \tilde{m}_{i3}, a_{i}, \beta_{i}, \gamma_{1}, \gamma_{2} \) \((i = 1, 2, 3)\) totally. Solving these equations simultaneously will obtain the optimal operating parameters to minimize the pumping power consumption with the collaborative consideration of heat transfer and flow characteristic.

### TABLE 1 Characteristic parameters of hot-water pipeline

| Hot-water loop | \( h_{si} \) | \( d_{hi} \) |
|---------------|--------|--------|
| Loop 1       | 1.22   | 300.08 |
| Loop 2       | 0.84   | 240.00 |
| Loop 3       | 0.88   | 400.00 |

### TABLE 2 Characteristic parameters of the cold-water pipeline

| Parameters | \( d_{01} \) | \( d_{12} \) | \( d_{3} \) |
|------------|-------------|-------------|-------------|
| Values     | 116.27      | 225.53      | 108.65      |

### TABLE 3 Characteristic parameters of the pumps in the heat transfer system

| Pump       | \( a_{0i} \) | \( a_{1i} \) | \( a_{2i} \) |
|------------|-------------|-------------|-------------|
| VFP1       | 0.0074      | -0.053      | -2.66       |
| VFP2       | 0.0071      | -0.092      | -2.32       |
| VFP3       | 0.0072      | -0.054      | -5.91       |
| VFPc       | 0.018       | 1.10        | -61.15      |

### 5 EXPERIMENTAL RESULTS AND DISCUSSIONS

As the thermal conductance is variable with fluid flow rate (in fact, it is the heat transfer coefficient that varies with flow rate), the iteration method can determine the experimental thermal conductance on the premise of not measuring the heat transfer coefficient. Figure 6 provides the flow chart of iteration process.

First, assuming the initial thermal conductances of each HE and calculate the corresponding optimal frequencies of VFPs. Second, operating the frequencies on the experimental platform at steady state for 40 minutes provides the updated thermal conductances of each HE. Third, if the thermal conductance deviations between the updated and the initial values are within 5%, which is the measurement uncertainty, the experimental operating parameters are the optimal case. Otherwise, recalculating the optimal frequencies with the newly updated thermal conductance and conduct the experiment again until the deviation is within 5%.

### 5.1 Experiment with the optimal operating frequencies

The given heat loads of HE1, HE2, and HE3 are 4000W, 3000W, and 3000W while the related hot-water temperatures in tanks are fixed at 40°C, 40°C, and 35°C, respectively. The initial thermal conductances of each HE are assumed 1000, 1600, and 1000 W/K, respectively. The temperature in the cold-water tank is fixed at 25°C. Calculate the optimal frequencies of each VFP and operate the experimental platform at steady state for

\[ \text{Updated thermal conductance} \]

\[ \text{Optimal operating parameters} \]

\[ \frac{|(kA)_{\text{exp}} - (kA)_{\text{hand}}|}{(kA)_{\text{exp}}} \leq 5\% \]
40 min. Table 4 shows the deviation variations in thermal conductances in each iteration. The final deviations of three loops are all within 5%, which determines the actual thermal conductances. Table 5 lists the optimal operating frequencies, pressure heads, and minimum power consumptions of VFPs. Even the heat load of HE2 is lower than that of HE1 but the power consumption of VFP2 is much larger than that of VFP1. The reason is that VFP2 locates in the downstream position of the series branch and the inlet temperature of cold-water becomes higher.

5.2 | Experiment with variable heat loads in the parallel branch

The heat load of HE3 varies from 3000 W to 4000 W with the interval of 250 W. The temperatures in the hot-water and the cold-water tanks and the heat loads of HE1 and HE2 are same with the experiment in Section 5.1. Figure 7 shows the optimal operating frequencies of VFPs. When the heat load of HE3 increases, the operating frequencies of VFP1, VFP2, and VFPc keep nearly constant but that of VFP3 increases obviously. Figure 8 provides the power consumption variations in each pump. When the heat load of HE3 increases, the pumping power consumptions of the cold loop, hot loop 1, and hot loop 2 keep constant basically. The pumping power consumption of the hot loop 3 increases apparently, which leads to the total pumping power consumption increment.

Figure 9 shows the outlet temperature variations in HE1, HE2, and HE3, respectively. When the heat load of HE3 increases, the hot-water and the cold-water outlet temperatures of HE1 and HE2 keep constant basically, but the hot-water and the cold-water outlet temperatures of HE3 increase by 1.69°C and 1.42°C with the proportions of 6.25% and 4.78% comparing with that of 3000W. As the parallel branch is independent and the heat load increment has no influence on other branch, thus, the pumping power consumption increment happens only in the hot-water loop of the parallel branch. That is, there is no need to increase the frequencies of VFPs in the series branches if the heat load in the parallel branch increases in practical application.

5.3 | Experiment with variable heat loads in the series branch

Locating in the downstream position of the series branch, the heat load of HE2 varies from 3000 W to 4000 W with the

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**Table 4** Thermal conductance and deviations variations between two iterations during the optimization process

| Parameters | HE1 (W/K) | HE2 (W/K) | HE3 (W/K) |
|------------|-----------|-----------|-----------|
| Initial values | 1000      | 1600      | 1000      |
| Iteration 1 (W/K) | 912.95    | 1479.77   | 906.65    |
| Deviations (%)   | 9.10%     | 7.81%     | 9.79%     |
| Iteration 2 (W/K) | 910.23    | 1472.22   | 914.12    |
| Deviations (%)   | 0.22%     | 0.51%     | 0.82%     |

**Table 5** Optimal operating frequencies and other results of all VFPs

| Pumps | VFP1 | VFP2 | VFP3 | VFPc |
|-------|------|------|------|------|
| Frequency (Hz) | 20.25 | 30.30 | 23.12 | 25.44 |
| Pressure head (m) | 2.95  | 6.07  | 3.73  | 7.95  |
| Power consumption (W) | 10.25 | 40.59 | 10.80 | 77.77 |
interval of 250 W. The heat loads of HE1 and HE3, the temperatures in the hot-water tanks and in the cold-water tank are same with the experiment in Section 5.1. Figure 10 provides the optimal operating frequencies of each VFP. As the heat load of HE2 goes up, both the frequencies of VFP2 and VFPc increase obviously. The operating frequency of VFP3 keeps nearly constant. The operating frequency of VFP1 decreases slightly. Figure 11 shows the variations in the power consumptions of each pump. When the heat load of HE2 increases, both the power consumptions of VFPc and VFP2 increase apparently. The power consumptions of VFP1 and VFP3 keep essentially constant.

Figure 12 indicates the outlet temperature variations in HE1, HE2, and HE3 with the heat load of HE2. When the heat load of HE2 increases, the frequency of VFP2 increases apparently but the outlet temperatures change slightly. Therefore, the frequency of VFPc increases, which provides larger cold-water mass flow rate. Owing to the series connection with HE2, the cold-water mass flow rate in the HE1 will increase. Then frequency of VFP1 and the outlet temperature of hot-water decrease slightly to ensure the constant heat load. HE3 is in the parallel branch, where the mass flow rate of cold-water increases owing to the increment of the frequency of VFPc. Thus, the outlet temperature of cold-water decreases slightly to ensure the constant heat load. The frequency of VFP3 and the outlet temperature of hot-water keep constant. Above all, the heat load increment in the downstream of series branch affects both the VFP frequencies of the cooling loop and the related hot loop. However, there are few influences on the VFP frequencies of the parallel branch and the upstream of the series branch. That is, both the VFP frequencies of the cold loop and the related hot loop need to increase while the heat load in the downstream of series branch increases.

Locating in the upstream position of the series branch, the heat load of HE1 varies from 4000 W to 5000 W with the interval of 250 W. Both the heat loads of HE2 and HE3, the temperatures in hot-water tanks and in the cold-water tank are same as the experiment in Section 5.1. Figure 13 provides the optimal operating frequencies of each VFP. As the heat load of HE1 increases, all the frequencies of VFP1, VFP2, and VFPc increase. The frequency of VFP3 keeps nearly constant. Figure 14 gives the pumping power consumption variations. When the heat load of HE1 increases, the power consumption of VFP3 changes little and those of VFP1, VFP2, and VFPc increase obviously, which leads to the increments of the total pumping power consumption.
There are few influences on the frequencies of the VFPs in parallel branch. That is, all the frequencies of the VFPs in the cold loop and hot loops need to increase if the heat load in the upstream position of the series branch increases.

Figure 15 provides the total pumping power consumption variations with the total heat load increments happening in the parallel branch and the downstream and the upstream of the series branch, respectively. For the heat load increments in different positions of the series branch, the total pumping power consumptions are basically the same. However, the total pumping power consumptions of that heat load increment happens in the series branch are always higher than that happens in the parallel branch. The total pumping power consumption increases by 19.45%, 43.86%, and 39.99% while the total heat loads increase from 10 to 11 kW for the three cases, respectively. That is, for the whole HTS in engineering application, assigning the additional heat load in the parallel branch is better for energy conservation.

6 | CONCLUSIONS

Based on the newly proposed power flow method together with the physical models of VFPs and pipelines, this study
built the global heat transfer and fluid flow constraints of a compound series-parallel HTS. With these constraints, applying Lagrange multiplier method provides the optimal operating frequencies of each VFP in the HTS to minimize the total pumping power consumption, where both the heat transfer and the flow resistance characteristics are taken into account for energy conservation.

After the identification of the characteristic parameters of HEs, VFPs, and pipelines, operating the experiment platform with the optimized frequencies of VFPs shows that only the parallel branch need to increases the VFP frequency and the series branches need no VFP frequencies increment while the heat load in the parallel branch increases. Both the VFP frequencies of the cold loop and related hot loop in the series branch need to increase when heat loads in the series branch (including downstream position and upstream position) increase but that of hot loop in the parallel branch needs no increment. For the three cases mentioned above, that is, heat load increments happen in the parallel branch, in the downstream position and upstream position of the series branch, the total pumping power consumption increases by 19.45%, 43.86%, and 39.99%, respectively, when the total heat load varies from 10 to 11 kW. It declares that undertaking the additional heat load increment with the parallel branch is more energy saving.

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CONFLICT OF INTEREST

None declared.

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NOMENCLATURES

- $K$: heat transfer coefficient, W/(m$^2$·K)
- $m$: mass flow rate, kg/s
- $P_t$: pumping power consumption, W
- $Q$: heat transfer rate, W
- $R_s$: entransy dissipation-based thermal resistance (EDTR), K/W
- $R_p$: thermal resistance in series, K/W
- $R_p$: thermal resistance in parallel, K/W
- $T$: temperature, °C
- $W$: uncertainty
- $x$: independent parameter
- $\alpha$, $\beta$, $\gamma$: Lagrange multiplier
- $\epsilon$: thermal motive, °C
- $\xi$: flow arrangement factor, K/W
- $\Pi$: Lagrange function
- $\rho$: density, kg/m$^3$
- $\omega$: frequency of VFPs, Hz
- $A$: heat transfer area, m$^2$
- $a$: characteristic parameters of VFPs
- $c_p$: specific heat capacity at constant pressure, J/(kg·K)
- $d$: characteristic parameters of pipeline
- $g$: acceleration of gravity, 9.8 m/s$^2$
- $h_s$: static pressure head, m
- $H$: pressure head, m
- $c$: cold fluid
- $h$: hot fluid
- $i$: heat exchanger number
- $in$: inlet
- $m$: characteristic parameter of cold fluid
- $out$: outlet
- $p$: parallel connection
- $pu$: pump
- $s$: series connection

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APPENDIX

A. PARTIAL DERIVATIVES OF THE LAGRANGE FUNCTION

\[
\frac{\delta \Pi}{\delta m_i} = \alpha_i \left( \frac{Q_i}{2 m_i} + \frac{Q_i (\exp(\Delta t_i) + 1)}{2 m_i} - \frac{Q_i (\exp(kA_i) \exp(kA_i) (\frac{1}{m_i} - \frac{1}{m_i})^2}{c_p m_i (\exp(kA_i) - 1)^2} \right) \\
+ \beta_i (2a_i \dot{m}_i - 2d_i \dot{m}_i + a_{1i} \omega_i) + g \left( a_{2i} \dot{m}_i + a_{1i} \dot{m}_i + a_{3i} \omega_i + a_{4i} \omega_i + a_{5i} \omega_i \right) \\
+ g m_i (2a_i \dot{m}_i + a_{1i} \dot{m}_i + a_{3i} \omega_i) = 0
\]  
(A1)

\[
\frac{\delta \Pi}{\delta a_i} = \beta_i (a_i \dot{m}_i + 2a_0 \dot{m}_i) + g m_i (a_i \dot{m}_i + 2a_0 \dot{m}_i) = 0
\]  
(A2)

\[
\frac{\delta \Pi}{\delta m_{c1}} = \alpha_i \left( \frac{Q_i}{2 m_{c1}} - \frac{Q_i (\exp(\Delta t_i) + 1)}{2 m_{c1}} \right) \\
+ g (a_{2i} \dot{m}_{c1} + \dot{m}_{c3})^2 + a_{0i} \omega_i + a_{1i} \omega_i + a_{2i} \dot{m}_{c1} + \dot{m}_{c3}) + g (m_{c1} + \dot{m}_{c3}) (a_{1i} \omega_i + a_{2i} \dot{m}_{c1} + \dot{m}_{c3}) \\
- \gamma_i (2d_{1i} \dot{m}_{c1} - a_{1i} \omega_i - 2a_{2i} \dot{m}_{c1} + \dot{m}_{c3} + 2d_{10} \dot{m}_{c1} + \dot{m}_{c3}) \\
+ \gamma_i (a_{1i} \omega_i + 2a_{2i} \dot{m}_{c1} + \dot{m}_{c3}) - 2d_{10} \dot{m}_{c1} + \dot{m}_{c3} = 0, (i = 1, 2)
\]  
(A3)

\[
\frac{\delta \Pi}{\delta m_{c3}} = \alpha_i \left( \frac{Q_i}{2 m_{c3}} - \frac{Q_i (\exp(\Delta t_i) + 1)}{2 m_{c3}} \right) \\
+ g (a_{2i} \dot{m}_{c1} + \dot{m}_{c3})^2 + a_{0i} \omega_i + a_{1i} \omega_i + a_{2i} \dot{m}_{c1} + \dot{m}_{c3}) + g (m_{c1} + \dot{m}_{c3}) (a_{1i} \omega_i + a_{2i} \dot{m}_{c1} + \dot{m}_{c3}) \\
+ \gamma_i (a_{1i} \omega_i + 2a_{2i} \dot{m}_{c1} + \dot{m}_{c3}) - 2d_{10} \dot{m}_{c1} + \dot{m}_{c3} = 0
\]  
(A4)

B. ERROR ANALYSIS

To evaluate the validations and accuracies of experiments, the uncertainties analysis is essential. Assuming that \(x_1, x_2, x_3, \ldots, x_n\) is independent variables and they satisfy the following relationship,

\[
R = f(x_1, x_2, x_3, \ldots, x_n),
\]  
(A6)

then the uncertainty of \(R\) is as following:

\[
W_R = \sqrt{\left( \frac{\partial R}{\partial x_1} W_{x_1} \right)^2 + \left( \frac{\partial R}{\partial x_2} W_{x_2} \right)^2 + \cdots + \left( \frac{\partial R}{\partial x_n} W_{x_n} \right)^2}
\]
(A7)

where \(W_{x_i}\) is the uncertainty of \(x_i\).

The instruments accuracies are as following: the 0.5\% of the volume flow meter with 20 L/min full range and the 0.2\% of the pressure transducer with 350 kPa full range. Then, the uncertainties of basic measured parameters are as follows:

\[
W_V = 20 \times 0.5\% = 0.1 \text{ L/min}
\]  
(A8)

\[
W_P = 350 \times 0.2\% = 0.7 \text{ kPa}
\]  
(A9)

Among all the experimental cases, heat transfer rates of 4000W, 3000W, and 3000W have the minimum volume flow rate and pressure differential but the maximum uncertainties. Table A1 shows the uncertainties of pump energy consumptions.

**TABLE A1**  Uncertainties of the experimental parameters

| Parameters | \(P_1\) | \(P_2\) | \(P_3\) | \(P_4\) |
|------------|--------|--------|--------|--------|
| Uncertainty (%) | 4.87   | 9.99   | 6.03   | 13.03  |