Local Causality and Completeness: Bell vs. Jarrett

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J.S. Bell believed that his famous theorem entailed a deep and troubling conflict between the empirically verified predictions of quantum theory and the notion of local causality that is motivated by relativity theory. Yet many physicists continue to accept, usually on the reports of textbook writers and other commentators, that Bell’s own view was wrong, and that, in fact, the theorem only brings out a conflict with determinism or the hidden-variables program or realism or some other such principle that (unlike local causality), allegedly, nobody should have believed anyway. (Moreover, typically such beliefs arise without the person in question even being aware that the view they are accepting differs so radically from Bell’s own.) Here we try to shed some light on the situation by focusing on the concept of local causality that is the heart of Bell’s theorem, and, in particular, by contrasting Bell’s own understanding with the analysis of Jon Jarrett which has been the most influential source, in recent decades, for the kinds of claims mentioned previously. We point out a crucial difference between Jarrett’s and Bell’s own understanding of Bell’s formulation of local causality, which turns out to be the basis for the erroneous claim, made by Jarrett and many others, that Bell misunderstood the implications of his own theorem.

I. INTRODUCTION

In 1964, J.S. Bell proved the result now known as Bell’s Theorem: any physical theory of a certain type must make predictions (for a certain class of experiment) which respect a so-called Bell Inequality. Quantum Mechanics (QM) predicts violations of the inequality, and subsequent experiments establish convincingly (though not without loopholes) that the Quantum Mechanical predictions are correct – i.e., the experiments establish that the type of theory Bell showed must respect the Inequality, cannot be empirically viable, i.e., cannot be true. But the question (which has given rise to an enormous literature) remains: what type of theory is it, exactly, that Bell’s Theorem (combined with the associated experiments) refutes?

Bell’s own view, expressed already in the opening lines of his 1964 paper and subsequently clarified and defended in virtually all of his later writings, was that “It is the requirement of locality ... that creates the essential difficulty.” (Bell, 1964, p. 14) By “locality” Bell here means the prohibition, usually taken to be an implication of special relativity (SR), of super-luminal (faster than light) causation. Bell thus took his own theorem to establish a troubling conflict between (the empirically verified predictions of) QM (i.e., between experiment) and SR:

“For me then this is the real problem with quantum theory: the apparently essential conflict between any sharp formulation and fundamental relativity. That is to say, we have an apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory...” (Bell, 1984, p. 172)

Most practicing physicists, however, (and most philosophers of physics) have disagreed with Bell and continued to believe in the unproblematic consistency of QM and SR. Where do they think Bell went wrong?

One can divide reasons for disagreement (with Bell’s own interpretation of the significance of his theorem) into two classes. First, there are those who assert that the derivation of a Bell Inequality relies not just on the premise of locality, but on some additional premises as well. The usual suspects here include Realism, Hidden Variables, Determinism, and Counter-Factual-Definiteness. (Note that the items on this list are highly overlapping, and often commentators use them interchangeably.) The idea is then that, since it is only the conjunction of locality with some other premise which is in conflict with experiment, and since locality is so strongly motivated by SR, we should reject the other premise. Hence the widespread reports that Bell’s theorem finally refutes the hidden variables program, the principle of determinism, the philosophical notion of realism, etc.

Here is how Bell responded to this first class of disagreement:

“My own first paper on this subject ... starts with a summary of the EPR argument from locality to deterministic hidden variables. But the commentators have almost universally reported that it begins with deterministic hidden variables.” (Bell, 1981, p. 157)

Here (a footnote, but also in the main text of the article in question) Bell goes out of his way to stress the overall logical structure of his two-part argument: first, an argument from locality and certain predictions of QM (namely, perfect anti-correlation for parallel spin measurements on a pair of spin 1/2 particles in the spin singlet state) to the existence of deterministic local hidden
variables; and then second, from such variables to the
inequality, i.e., to a disagreement with certain other pre-
dictions of QM. This whole first class of disagreement
with Bell, then, rests on a simple confusion about Bell’s
argument. 4

The more interesting and more subtle second class
of disagreement includes those who accept that the
empirically-violated Bell inequality can be derived from
Bell’s locality condition alone, but who argue that this
locality condition is too strong, i.e., that it smuggles in
some extra requirements beyond those minimally neces-
sary to respect SR’s prohibition on superluminal causa-
tion. At the head of this class is Jon Jarrett, whose 1983
PhD thesis and subsequent 1984 paper 5 argued that
Bell’s own local causality condition (which Jarrett calls
“strong locality”) is logically equivalent to the conjunc-
tion of two subsidiary conditions, which Jarrett described
respectively as “locality” and “completeness.”

Roughly speaking, Jarrett’s “locality” is the require-
ment that the outcome of a measurement on one particle
be independent of the type of measurement performed (at
spacelike separation) on a second particle (which, in the
interesting sorts of cases, is described by QM as being en-
tangled with the first particle). Jarrett’s “completeness” on
the other hand requires the outcome of the first mea-
surement to be independent of the outcome of the second,
spacelike separated measurement. He then argues that a
violation of “locality” would entail the ability to send
superluminal signals (Alice could learn something about
the setting of Bob’s measurement device by examining
the outcome of her own experiment) which would allow
Bob to transmit a signal across a spacelike interval, some-
thing clearly forbidden by SR. By contrast, a violation
of “completeness” indicates no conflict with SR. Thus,
Jarrett argues, in the face of the empirical data conflict-
ing with “strong locality” we may reject “completeness”
and thereby achieve — contra Bell — a kind of Peaceful
Coexistence between QM and SR. 6

Jarrett’s project has been widely hailed and widely dis-
cussed. It was the immediate stimulus for almost every-
thing in the “Bell literature” for about a decade after
its appearance, and continues to set a broadly influential
context for much ongoing work in this area. 7

The purpose of the present paper is to critically assess
Jarrett’s analysis and the conclusions it led him to, by
comparing his project side-by-side with Bell’s own discus-
sions of the relevant issues. In particular, I will claim that
Jarrett simply missed a crucial aspect (having something
to do with “completeness”) of Bell’s formulation of local
causality; this turns out to be the heart of the thinking
behind Jarrett’s (prima facie rather puzzling) terminol-
ogy for his two sub-conditions, as well as his central claim
that violations of his “completeness” criterion indicate no
conflict with special relativistic local causality. The main
conclusion is thus that, contrary to Jarrett and his fol-
lowers, Bell’s own local causality criterion is in no sense
“too strong.” And this of course undermines the attempt
to establish the Peaceful Coexistence of QM and SR, i.e.,
it supports Bell’s own interpretation of the meaning of
his theorem.

The following two sections present, respectively, Bell’s
own (final and most careful) formulation of the locality
premise, and then Jarrett’s analysis. Section IV includes
some comparative discussion, highlighting especially the
relation of Jarrett’s thinking to the EPR argument. A
brief final section then summarizes and concludes.

Before launching into this, however, it is appropriate
to briefly survey earlier criticisms of Jarrett’s analysis of
Bell’s locality concept. To my knowledge, the fullest crit-
ic discussion of Jarrett’s project is in Maudlin’s book.

Maudlin’s main points vis-a-vis Jarrett are as follows:
(a) Jarrett’s identification of his “locality” sub-condition
with the prohibition on superluminal signals is wrong; (b)
Jarrett’s identification of superluminal signaling with su-
perluminal causation is wrong; (c) Jarrett’s claim that a
violation of his “completeness” condition does not entail
any nonlocal causation is wrong. 8

The arguments for (a) and (b) are made clearly and
compellingly by Maudlin, who thus really demolished Jar-
rett’s erroneous identification of his “locality” with the
relevant requirements of SR. But the case for (c) is made
only indirectly, essentially by dismissing Jarrett and re-
asserting Bell’s claim to the contrary. So, to be a bit
more precise about the goal of the present paper, the
aim is to fill this gap by exploring in detail how Jarrett’s
“completeness” condition relates to Bell’s local causal-
ity criterion and how Jarrett’s misunderstanding of the
latter led him to the various erroneous conclusions.

I should note at the outset, however, that the view
to be presented here as Jarrett’s is almost certainly a
bit misleading as to his (or those I consider his follow-
ers’) fully-considered views. Jarrett does actually say all
the things I attribute to him, but my gloss will perhaps
minimize the extent to which Jarrett (I would argue, in-
consistently) also acknowledges points in conflict with
the views I will attribute to him. It is probably best,
therefore, to understand the “Jarrett” discussed here as
a rhetorically clarifying construct, which may or may not
correspond to the views of the actual Jon Jarrett.

II. Bell’s Concept of Local Causality

Bell’s fullest and evidently most-considered discussion
of local causality occurs in his last published paper, La
nouvelle cuisine (1990, 232-248). We will here essentially
follow that discussion, supplementing it occasionally with
things from his earlier papers.

Bell first introduces what he calls the “Principle of lo-
cal causality” as follows: “The direct causes (and effects)
of events are near by, and even the indirect causes (and
effects) are no further away than permitted by the veloc-
ity of light.” Then, referencing what has been reproduced
here as Figure 1, Bell elaborates: “Thus, for events in a
space-time region 1 ... we would look for causes in the
backward light cone, and for effects in the future light
cone. In a region like 2, space-like separated from 1, we would seek neither causes nor effects of events in 1. Of course this does not mean that events in 1 and 2 might not be correlated...” (1990, p. 239)

After remarking that this formulation “is not yet sufficiently sharp and clean for mathematics,” Bell then proposes the following version, referencing what has been reproduced here as Figure 2:

“A theory will be said to be locally causal if the probabilities attached to values of local beables in a space-time region 1 are unaltered by specification of values of local beables in a space-like separated region 2, when what happens in the backward light cone of 1 is already sufficiently specified, for example by a full specification of local beables in a space-time region 3...” (1990, 239-40)

Although Bell doesn’t immediately formulate this mathematically (which is curious, since he has just advertised it as a formulation which is “sufficiently sharp and clean for mathematics”), we may do so in a way that is clearly (as evidenced by what comes later in the paper) what he had in mind:

\[ P(b_1|B_3, b_2) = P(b_1|B_3). \]  (1)

Here \( b_i \) refers to some beable (or more precisely, its value) in region \( i \), and \( B_i \) refers to a “full specification” of beables in region \( i \). This simply asserts mathematically what Bell states in the caption of his accompanying figure: “full specification of [beables] in 3 makes events in 2 irrelevant for predictions about 1.” Note that Bell here uses the term (which he had earlier coined) “beable” (rhymes with “agreeable”) to denote whatever is posited, by the candidate theory in question, to be physically real:

“The beables of the theory are those elements which might correspond to elements of reality, to things which exist. Their existence does not depend on ‘observation’. Indeed observation and observers must be made out of beables.” (1984, p. 174)

For further discussion of “beables” see Bell’s The Theory of Local Beables (1975, pages 52-3), Beables for quantum field theory (1984, pages 174-6), and La nouvelle cuisine (1990, pages 234-5).

Bell then adds the following clarificatory remarks:

“It is important that region 3 completely shields off from 1 the overlap of the backward light cones of 1 and 2. And it is important that events in 3 be specified completely. Otherwise the traces in region 2 of causes of events in 1 could well supplement whatever else was being used for calculating probabilities about 1. The hypothesis is that any such information about 2 becomes redundant when 3 is specified completely.” (1990, p. 240)

It will be crucial to understand these remarks, so we shall briefly elaborate.

First, suppose that the region labeled 3 in Figure 2 sliced across the backwards light cone of 1 at an earlier time, such that it failed to “completely shield off from 1 the overlap of the backward light cones of 1 and 2.” See, for example, the region labeled 3∗ in Figure 3. Why then would a violation of Equation 1 fail to necessarily indicate the presence of nonlocal causation? Suppose we are dealing with a non-deterministic (i.e., irreducibly stochastic, genuinely chancy) theory. And suppose that some event (a beable we shall call “X”) comes into existence in the future of this region 3∗, such that it lies in the overlapping backwards light cones of 1 and 2. By assumption, the theory in question did not allow the prediction of this beable on the basis of a full specification of beables in 3∗. Yet once X comes into existence, it can (in a way that is perfectly consistent with local causality) influence events in both 1 and 2. And there is therefore the possibility that specification of events from 2 could allow one to infer something about X from which one could in turn infer more about goings-on in 1 than one could have inferred originally from just the full specification of beables in 3∗. In other words (Bell’s): “the traces in region 2 of causes [such as our X] of events in 1 could well supplement whatever else was being used for calculating probabilities about 1.” This mechanism for producing violations of Equation 1 without any superluminal causation, however, will clearly not be available so
events in region 3 of Figure 2 are of the backward light cones of 1 and 2. Thus, (following the language of Figure 2’s caption) even full specification of what happens in 3 does not necessarily make events in 2 irrelevant for predictions about 1 in a locally causal theory.

long as “region 3 completely shields off from 1 the overlap of the backward light cones of 1 and 2.”

Bell’s other clarification is also crucial. Suppose that events in region 3 of Figure 2 are not specified completely. We may denote such an incomplete description by $B_3$. Then does a violation of Equation 1 (but with $B_3 \rightarrow B_3$) necessarily imply the existence of any nonlocal causation? No, for it would then be possible that some event $X$ (again in the overlapping past light cones of 1 and 2) influences both 1 and 2 such that information about 2 could tell us something about $X$ which in turn could tell us something about 1 which we couldn’t infer from $B_3$.

We only need stipulate that the beables in region 3 which “carry” the causal influence from $X$ to 1 are (among) those omitted by $B_3$. But since there is, by definition, no such omission in the complete specification $B_3$, this eventuality cannot arise, and a violation of Equation 1 must indicate the existence of some nonlocal causation, i.e., causal influences not respecting Bell’s original “Principle of local causality” (as displayed in Figure 1).

It is worth noting that we cannot necessarily infer, from a violation of Equation 1 that $b_2$ exerts any direct or indirect causal influence on $b_1$. It might be, for example, that a violation of Equation 1 is produced by some $X$-like event lying in the future of region 3 which causally influences both 1 and 2. But in order to exert a local causal influence on 1, such an $X$ would have to lie outside the past light cone of 2 – and vice versa. What is ensured by a violation of Equation 1 is thus only that some violation of local causality (as sketched in Figure 1) is being posited somewhere by the theory in question. Whether something in 2 is exerting a causal influence on 1 (or vice versa, or neither) the mere violation of Equation 1 doesn’t permit us to say.

Note that everything in the above discussion refers to some particular candidate physical theory. For example, there is a tendency for misplaced skepticism to arise from Bell’s use of the concept of “beables” in the formulation of local causality. This term strikes the ears of those influenced by orthodox quantum philosophy as having a metaphysical character and/or possibly committing one (already, in the very definition of what it means for a theory to respect relativistic local causality) to something unorthodox like “realism” or “hidden variables.” Such concerns, however, are based on the failure to appreciate that the concept “beable” is theory-relative. “Beable” refers not to what is physically real, but to what some candidate theory posits as being physically real. Bell writes: “I use the term ‘beable’ rather than some more committed term like ‘being’ or ‘beer’ to recall the essentially tentative nature of any physical theory. Such a theory is at best a candidate for the description of nature. Terms like ‘being’, ‘beer’, ‘existent’, etc., would seem to me lacking in humility. In fact ‘beable’ is short for ‘maybe-able’.” (1984, p. 174)

Similar considerations apply to the notion of “completeness” that is, as stressed above, essential to Bell’s formulation. A complete specification of beables in some spacetime region simply means a specification of everything (relevant) that is posited by the candidate theory in question. There is no presumption that such a full specification actually correspond to what really exists in the relevant spacetime region, i.e., no presumption that the candidate theory in question is true. And the same goes for the probabilities in Equation 1 that Bell’s locality criterion is formulated in terms of. These should be read not as empirical frequencies or subjective measures of expectation, but as the fundamental dynamical probabilities described by the candidate theory in question (which we assume, without loss of generality, to be irreducibly stochastic).

Since all the crucial aspects of Bell’s formulation of locality are thus meaningful only relative to some candidate theory, it is perhaps puzzling how Bell thought we could say anything about the locally causal character of Nature. Wouldn’t the locality condition only allow us to assess the local character of candidate theories? It is important to understand that the answer is essentially (at least initially): Yes! Indeed, note that Bell begins the formulation with “A theory will be said to be locally causal if...” (emphasis added). Let us state it openly and explicitly: Bell’s locality criterion is a way of distinguishing local theories from nonlocal ones:

“I would insist here on the distinction between analyzing various physical theories, on the one hand, and philosophising about the unique real world on the other hand. In this matter of causality it is a great inconvenience that the real world is given to us once only. We cannot know [by looking] what would have happened if something had been different. We cannot repeat an experiment changing just one variable; the hands of the clock will have moved, and the moons of Jupiter. Physical theories are more amenable in this respect. We can calculate the consequences of changing free elements in a theory, be they only initial conditions, and so can explore the causal structure of the theory. I insist that [my concept of local causality] is primarily an analysis of certain kinds of physical theory.”
How then did Bell think we could end up saying something interesting about Nature? That is precisely the beauty of Bell’s theorem, which shows that no theory respecting the locality condition (no matter what other properties it may or may not have – e.g., hidden variables or only the non-hidden sort, deterministic or stochastic, particles or fields or both or neither, etc.) can agree with the empirically-verified QM predictions for certain types of experiment. That is (and leaving aside the various experimental loopholes), no locally causal theory in Bell’s sense can agree with experiment, can be empirically viable, can be true. Which means the true theory (whatever it might be) necessarily violates Bell’s locality condition. Nature is not locally causal. [13]

For ease of future reference and to fix some terminology, it will be helpful to lay out here a bit more explicitly the type of setup involved in the Bell experiments, and to indicate precisely how one gets from locality as formulated by Bell to the somewhat different-looking mathematical condition (sometimes called “factorizability”) from which standard derivations of Bell’s inequality proceed.

The setup relevant to Bell’s theorem involves a particle source which emits pairs of spin-correlated particles, and two spatially separated devices each of which allows measurement of one of several spin components on the respective incident particle. (In actual experiments, the particles are typically photons with polarization playing the role of “spin.”) Two experimenters, traditionally Alice and Bob, man the two devices. We use the symbols \( \hat{a} \) and \( \hat{b} \) to refer, respectively, to the “settings” of Alice’s and Bob’s apparatus (one usually thinks here of an axis in space along which the polarizer or Stern-Gerlach magnetic field is oriented), and \( A \) and \( B \) refer to the “outcomes” of their respective spin-component measurements. Finally, we will use the symbol \( \lambda \) to refer to the “state of the particle pair.” The scare-quotes around the various terms here are an advertisement for the following discussion.

First, note that all of the symbols just introduced refer to beables. There is a tendency in the literature for all of these things (the apparatus settings, the outcomes, and the physical state of the particle pair) to remain needlessly abstract. But all of these things are perfectly concrete, at least relative to some particular candidate theory. The setting of Alice’s apparatus, for example, refers to something like the spatial orientation of a Stern-Gerlach device, or some sort of knob or lever on some more black-box-ish device. Thus, this “setting” ultimately comes down to the spatial configuration of some physically real matter, i.e., it must be reflected somehow in the beables posited by any serious candidate theory.

Likewise, the outcome of Alice’s experiment is not some ethereal event taking place in Alice’s consciousness or some other place about which serious candidate physical theories fail to speak directly. Rather, the outcome should be thought of as being displayed in the post-measurement position of a pointer (or the arrangement of some ink on a piece of paper, etc.) – in short, the outcome too is just a convenient way of referring to some physically real and directly observable configuration of matter, and so will necessarily be reflected in the beables posited by any serious candidate theory. The case with \( \lambda \) is a little different, because this is not something to which we have any sort of direct observational access. But this only means there is significantly more freedom about what sort of beables \( \lambda \) might refer to in various different candidate theories.

The basic space-time structure of the setup in question is sketched in Figure 4, and the overall logic is explained in the caption. The idea is simply to apply Bell’s locality condition to the two measurement outcomes \( A \) and \( B \) in order to assert that the probability assigned to a given outcome (by a locally causal candidate theory) should be independent of both the setting and outcome of the distant experiment. That is, in a locally causal theory, we must have

\[
P(A|\hat{a}, \hat{b}, B, \lambda) = P(A|\hat{a}, \lambda) \tag{2}
\]

and

\[
P(B|\hat{a}, \hat{b}, A, \lambda) = P(B|\hat{b}, \lambda). \tag{3}
\]

It is then trivial to apply the definition of conditional probability to arrive at the conclusion that the joint probability (for outcomes \( A \) and \( B \)) factorizes:

\[
P(A, B|\hat{a}, \hat{b}, \lambda) = P(A|\hat{a}, \lambda) \times P(B|\hat{b}, \lambda). \tag{4}
\]
Bell writes: “Very often such factorizability is taken as the starting point of the analysis. Here we have preferred to see it not as the formulation of ‘local causality’, but as a consequence thereof.” (1990, p. 243)

It is also important that the apparatus settings \( \hat{a} \) and \( \hat{b} \) are in some sense “free.” This is often discussed in terms of Alice and Bob making literal last-minute free-will choices about how to orient their devices. What is actually required for the proof is merely the assumption that \( \hat{a} \) and \( \hat{b} \) are (stochastically) independent of the particle pair state \( \lambda \). This comes up in the course of the derivation of Bell’s inequality when we write an expression for predicted empirical frequency of a certain joint outcome as a weighted average over the candidate theory’s predictions for a given \( \lambda \) – that is,

\[
E(A, B|\hat{a}, \hat{b}) = \int d\lambda P(A, B|\hat{a}, \hat{b}, \lambda) P(\lambda)
\]

(5)

where \( P(\lambda) \) is the probability that a given particle pair state \( \lambda \) is produced by the preparation procedure used at the source.

If the probability distribution \( P(\lambda) \) actually depended on \( \hat{a} \) or \( \hat{b} \) – as one would expect if the particle pair exerted some causal influence on the settings (or vice versa!) or if the particle pair and the settings were mutually causally influenced in some non-trivial way by events farther back in the past – then the above expression for the empirical frequency would be invalid and the derivation of the Bell inequality wouldn’t go through.

This might seem like cause for concern, especially considering that (as displayed in Figure 4) the past light cones of \( \hat{a} \) and \( \hat{b} \) overlap with the region containing \( \lambda \) – and \( \lambda \) by definition is supposed to contain a complete specification of beables in this region. Given all that, one wonders how \( \hat{a} \) and \( \hat{b} \) could possibly not be causally influenced by \( \lambda \) (in a locally causal theory).

Here we see the reason for another aspect of Bell’s carefully-phrased formulation of locality, which requires that beables in (the relevant) “region 3” (which will of course be two different regions for the two measurements) be “sufficiently specified, for example by a full specification...” Here there is the implication that a specification could be sufficient without being complete. For example, some candidate theory (and this is actually true of every serious extant candidate theory) might provide a specification of the state of the particle pair which is sufficient in the relevant sense, even though it leaves out some fact (say, the millionth digit of the energy of some relic microwave background photon that happens to fly into the detection region just prior to the measurement) which actually exists in the relevant spacetime region. Such a fact could then be allowed to determine the setting \( \hat{a} \) without introducing even the slightest evidence for the problematic sort of correlation between \( \hat{a} \) and \( \lambda \). Indeed, this is just an exaggerated version of what happens in the actual experiments, where carefully-isolated and independent pseudo-random-number generators are used to produce the settings at the two stations.

Finally, as Shimony, Horne, and Clauser have pointed out,

“In any scientific experiment in which two or more variables are supposed to be randomly selected, one can always conjecture that some factor in the overlap of the backward light cones has controlled the presumably random choices. But, we maintain, skepticism of this sort will essentially dismiss all results of scientific experimentation. Unless we proceed under the assumption that hidden conspiracies of this sort do not occur, we have abandoned in advance the whole enterprise of discovering the laws of nature by experimentation. [Hence, the extra] supposition needed [to derive Bell’s inequality from Bell’s locality condition] is no stronger than one needs for experimental reasoning generically, and nevertheless just strong enough to yield the desired inequality.” [17]

So in the end there is really nothing worth worrying about here, i.e., nothing which, in the face of the experimental data conflicting with Bell’s inequalities, one might reasonably reject as an alternative to rejecting Bell’s local causality. What is important is that Equation [16] (along with the “freedom” or “no conspiracies” assumption just discussed) entails the Bell inequality. The derivation is standard and will not be repeated here. [18] We will instead simply note that (starting in 1975) Bell almost always referred to the empirically testable inequality as the “locality inequality.” One can hopefully now appreciate why.

One final caveat. There is a sense in which our verbal description of the meanings of the relevant beables \( (\hat{a}, \hat{b}, A, B, \lambda) \) is potentially confusing. For example, there is no sense in which Bell makes some dubious “extra assumption” that, e.g., particles fly from the “particle source” to the measurement devices. A theory which posits no particle beables, but only (say) waves or a wave function or whatever is perfectly fine and the formal locality criterion will apply to it just the same way. Only the words would need to change. Relatedly, many papers in the Bell literature raise issues about hidden variables associated not with the particle pair, but with the measurement devices. Have we excluded such variables, since our \( \hat{a} \) refers only to some knob setting on Alice’s device and \( \lambda \) refers only to the state of the particle pair? No. The only important distinction here is that \( \hat{a} \) and \( \lambda \) refer to beables which are “free” (or “random”) in the above sense, while the variables \( \lambda \) are somehow set by past events.

This is no doubt a fuzzy distinction, but nothing important hinges on it. The point is that any microscopic features of Alice’s device (hidden or not) – or anything else relevant to the candidate theory’s predictions for the probabilities in question – can be included under the “setting” variables \( (\hat{a}, \hat{b}) \) or the “particle state” variables \( (\lambda), \)
whichver seems more natural. In short, Bell’s formulation of locality is significantly more general than might otherwise be suggested by some of the words used to describe it.

III. JARRETT’S ANALYSIS

Jon Jarrett’s influential analysis of Bell’s locality criterion begins with Equations 2 and 3 which he dubs “strong locality.” (Of course, we are innocuously changing – and occasionally simplifying – Jarrett’s notation to make it consistent with that introduced above.) For simplicity, let us focus the discussion on Equation 2. Jarrett defines two sub-conditions which, he subsequently proves, are jointly equivalent to this “strong locality.”

The first sub-condition Jarrett dubs “locality.” (It is also sometimes referred to as “simple locality” or “parameter independence” or “remote context independence” in the literature.)

\[ P(A|\hat{a}, \hat{b}, \lambda) = P(A|\hat{a}, \lambda). \] (6)

As Jarrett explains, “Locality requires that the probability for the outcome \([A]\) ... be determined ‘locally’; i.e., that it depend only on the state \(\lambda\) ... of the two-particle system and on the state \([\hat{a}]\) of the measuring device. In particular, that probability must be independent of which (if any) component of spin the distant measuring device is set to measure.”

Jarrett’s second sub-condition, which he dubs “completeness” (and is also known as “predictive completeness,” “outcome independence,” “remote outcome independence” and “conditional outcome independence”) is the following:

\[ P(A|\hat{a}, \hat{b}, B, \lambda) = P(A|\hat{a}, \hat{b}, \lambda) \] (7)

which “asserts the stochastic independence of the two outcomes in each pair of spin measurements.”

It is easy to see that, indeed, “locality” and “completeness” are jointly equivalent to Bell’s locality condition as expressed in Equation 2. First, \(P(A|\hat{a}, \hat{b}, B, \lambda)\) is, under the assumption of “completeness,” equal to \(P(A|\hat{a}, \hat{b}, \lambda)\). And this, in turn, is equal to \(P(A|\hat{a}, \lambda)\) under the assumption of “locality.” So “locality” and “completeness” jointly entail “strong locality.” And likewise “strong locality” clearly entails both “locality” and “completeness.” So the two sub-conditions are, indeed, equivalent to Equation 2.

The alleged significance of this decomposition, however, emerges only from Jarrett’s discussion of the physical interpretation of his two sub-conditions.

First, Jarrett argues that “locality” is equivalent to the prohibition of superluminal signaling, and hence expresses just what relativity requires of other theories. As was mentioned in the introduction, both steps of this argument have been found wanting. First, it is only in combination with some assumptions about the controllability of various beables (notably \(\lambda\)) that Jarrett’s “locality” is equivalent to the prohibition on superluminal signaling. There is at least one extent, empirically viable theory (Bohmian Mechanics) which violates Jarrett’s “locality” condition and yet doesn’t permit the possibility of superluminal signaling, precisely because the relevant states cannot (as a matter of principle, as predicted by the theory) be sufficiently controlled. And second, it is dubious to claim that the prohibition of superluminal signals adequately captures relativity’s fundamental speed limit. This would, for example, render Bohmian Mechanics consistent with SR despite its need to postulate a dynamically privileged reference frame – a “gross violation of relativistic causality” according to Bell. (1984, p. 171) But since these problems have been discussed elsewhere in the literature, we leave them aside here and focus instead on Jarrett’s physical interpretation of his second sub-condition, “completeness.”

Jarrett elaborates the meaning of his “completeness” condition with the following example:

“A simple (and incorrect) model for the Bell-type correlated spin phenomena may serve as a useful illustration. Suppose, purely for the sake of illustration, that spin is correctly represented as an ordinary classical angular momentum. Suppose further that when a pair of particles is prepared in the singlet state, the spin vectors for the two particles are aligned exactly anti-parallel to each other. Moreover, given an ensemble of such two-particle systems, suppose that each direction in space is equally likely to be the direction of alignment for an arbitrarily selected member of the ensemble. Finally, if the unit vector \([\hat{a}]\) gives the direction along which the axis of the Stern-Gerlach apparatus is aligned ... and if \([\hat{s}]\) is the spin vector of the particle ... then the outcome of that measurement is +1 if \([\hat{a} \cdot \hat{s} > 0]\) and −1 if \([\hat{a} \cdot \hat{s} < 0]\).”

Jarrett then makes what amounts to the following point: for this model, we clearly have that Alice’s particle (from a randomly selected member of the ensemble) is equally likely to be found in the state \(A = +1\) and \(A = -1\) states along any arbitrary direction \(\hat{a}\). Thus, for example,

\[ P(A = +1|\hat{a}, \hat{b}, \lambda) = \frac{1}{2} \] (8)

where \(\lambda\) is the singlet state – evidently meaning, in the context of this example, the state description according to which the particle has some definite but completely unknown spin direction \(\hat{s}\).

On the other hand, it is built into the model that, for \(\hat{a} = \hat{b}\), the outcomes of Alice’s and Bob’s measurements will be perfectly anti-correlated. Hence, if we additionally specify the outcome of Bob’s experiment, the
outcome of Alice’s is fixed. Suppose, for example, that $B = +1$. Then $A = +1$ is forbidden, i.e.,

$$P(A = +1|\hat{a}, \hat{b}, B, \lambda) = 0. \quad (9)$$

Comparing the previous two equations, we see that “completeness is clearly violated.” [22]

Jarrett elaborates:

“The probabilities specified for this model are grounded in a blatantly incomplete description of the two-particle state. In the context of this model, if the theory assigns probabilities only on the basis of the occupancy by the two-particle system of the singlet state, then conditioning on the outcome $[B]$ of a $[\hat{b}]$-component spin measurement on [Bob’s particle] may well yield a different probability for the outcome of a spin measurement on [Alice’s particle] than would have been given by the corresponding unconditioned probability (i.e., 1/2). This is so because, if the outcome of the measurement on [Bob’s particle] is $+1$ [and if $\hat{a} = \hat{b}$], then it may be inferred that $[\hat{a} \cdot \hat{s}_A < 0]$ (with probability 1), where $[\hat{s}_A]$ is the spin of Alice’s particle.... The outcome of the measurement on [Bob’s particle thus] provides information about [Alice’s particle] which was not included in the incomplete state description $[\lambda].$” [23]

The important conclusion is this: the fact that the probability assigned to a certain outcome for Alice’s experiment depends, in violation of Equation [7] on the outcome of Bob’s experiment, does not mean that there is any relativistically-forbidden superluminal causal influence (e.g., from Bob’s outcome to Alice’s). That is the point of the illustrative example, in which (by assumption) the outcome of Alice’s experiment is determined exclusively by factors (namely $\hat{a}$ and $\hat{s}_A$) which are present at her location. No nonlocal causal influence exists. Instead, the violation of Equation [7] indicates only that we were dealing with incomplete state descriptions, such that Bob’s outcome provides some information (usefully supplementing what was already contained in $\lambda$) which warrants an updating of probabilities.

On the basis of this example, Jarrett thus urges the following physical interpretation of his two sub-conditions: a violation of “locality” would allow the possibility of sending superluminal signals, and hence indicates clearly the existence of some relativity-violating superluminal causal influences; on the other hand, a violation of “completeness” does not indicate the existence of any relativity-violating influences, but instead suggests only that the state descriptions of the theory in question are not complete. It is clear that these physical interpretations of the two conditions were the basis for Jarrett’s decision to name them “locality” and “completeness” respectively.

Here is Jarrett’s summary of the cash value of this decomposition vis-a-vis Bell’s theorem and the associated experiments: these together provide very strong evidence “that strong locality cannot be satisfied by any empirically adequate theory. Since locality is contravened only on pain of a serious conflict with relativity theory (which is extraordinarily well-confirmed independently), it is appropriate to assign the blame to the completeness condition. ...[O]ne must conclude that certain phenomena simply cannot be adequately represented by any theory which ascribes properties to the entities it posits in such a way that no measurement on the system may yield information which is both non-redundant (not deducible from the state descriptions) and predictively relevant for distant measurements. That ‘information’ is not (neither explicitly nor implicitly) contained in the ‘incomplete’ state description.” [24]

IV. COMPARISON

The fundamental origin of the disagreement between Bell and Jarrett should now be clear: the two authors do not understand (e.g.) Equation [2] in the same way. For Bell, the variables $\lambda$ in this formula (together with $\hat{a}$ as per the previous discussion) constitute a complete (or perhaps merely sufficient) specification of beables in some space-time region that has the same relation to Alice’s experiment that region 3 (of Figure 1) had to region 1. Jarrett, by contrast, is agnostic about the completeness of the description afforded by $\lambda$.

Strictly speaking, therefore, Jarrett’s decomposition of Equation [2] is not a decomposition of Bell’s locality condition, but, rather, a decomposition of some sort of no-correlation condition

$$P(b_1|\bar{B}_3, b_2) = P(b_1|\bar{B}_3) \quad (10)$$

which is analogous to Equation [11] except that, following the notation of Section II, the variables $\bar{B}_3$ are not assumed to provide a complete specification of beables in the relevant spacetime region. But as we have already discussed in Section II, and as Bell was perfectly aware, a violation of Equation [10] does not necessarily indicate the presence of any nonlocal causation in the candidate theory in question. Indeed, it is precisely to close off the avenue eventually taken by Jarrett (that is, blaming a violation of this “no-correlation” condition on the incompleteness of the specified beables) that Bell specifically stresses the importance that “events in [the relevant region] be specified completely.” (1990, p. 240)

This confusion – this departure from Bell’s actual locality criterion – is the ultimate basis for Jarrett’s choice of terminology, and also for any initial plausibility of his project of establishing “Peaceful Coexistence” by showing that a violation of the locality criterion needed for Bell’s inequality (in particular, a violation of his “completeness” sub-condition) need not indicate any conflict with relativistic local causality.
Of course, one could simply apply Jarrett’s decomposition strategy to Bell’s actual locality condition. That is, it is true that, as Jarrett claimed to have shown, Bell’s locality condition can be decomposed into two Jarrett-like sub-conditions—namely, our Equations 8 and 9 but now with the requirement (inherited from Bell’s actual locality condition) that \( \lambda \) provide a complete (or sufficient) specification of relevant beables (as posited by some candidate theory whose locality is being assessed by the locality condition). But this decomposition fails to have any of the physical implications urged by Jarrett. In particular, a violation of the (strengthened) “completeness” condition

\[
P(A|\hat{a}, \hat{b}, B, \lambda) = P(A|\hat{a}, \hat{b}, \lambda)
\]  

(formally equivalent to Jarrett’s “completeness” sub-condition, but now, with Bell but contra Jarrett, with the insistence that \( \lambda \) and \( \hat{a} \) jointly provide a complete description of beables in some spacetime region through the past of \( A \) which divides \( A \) off completely from the overlap of the past light cones of \( A \) and \( B \)) has absolutely nothing to do with the completeness of state descriptions, but instead indicates the presence of some nonlocal causation (in violation of the causal structure outlined in Figure 1) in the candidate theory in question.

The most that could be said to distinguish the two sub-conditions is that, since \( \hat{b} \) is (by definition) controllable and \( B \) (most likely) isn’t, a violation of \( \| \) is (all other things being equal) more likely to yield the possibility of superluminal signaling than a violation of \( \| \). But that only matters if we drop what Bell calls “fundamental relativity” and instead read SR instrumentally, as prohibiting superluminal signalling but allowing in principle superluminal causation (so long as it can’t be harnessed by humans to transmit messages). That is, at best, a dubious and controversial reading of SR, as already mentioned.

It is sometimes suggested that the relation between Bell and Jarrett on this point is one of basic agreement, since both held that the validity of Equation 7 has something to do with the completeness of the physical theory in question. This is doubly incorrect. First, the mathematical condition in question is, for Jarrett, a formulation of completeness; it is, for Bell, (part of) a formulation of local causality which functions appropriately as such only if the relevant symbols in the formula stand for a complete specification of the relevant beables. According to Jarrett, Equation 7 is supposed to tell us whether completeness holds. For Bell, on the other hand, the equation (correctly understood) already presupposes that completeness holds. (If it doesn’t hold, the condition is useless for his purposes, and states only that the two outcomes are uncorrelated.) And second, where Jarrett (and most subsequent commentators) regard the “completeness” in question as a property of theories, Bell regards his “completeness” as a property of a certain specification of beables relative to some candidate theory. In this context, the separate question of whether the theory itself is complete (i.e., whether its posited beables capture everything that really exists) simply doesn’t come up.

Note also that, even on his own terms, i.e., even leaving aside his departure from Bell’s actual locality criterion, Jarrett’s formulation of completeness actually fails as a criterion for assessing the completeness of the relevant physical state descriptions.

Consider again Equation 7 where for the moment we follow Jarrett and interpret \( \lambda \) here as providing some kind of state description, but not necessarily a complete one. Jarrett has shown (with the example discussed in Section III) that a violation of this condition can sometimes be blamed on the use of incomplete state descriptions, with no implication of any superluminal causation. But it is equally easy to display an example in which violation of Jarrett’s condition cannot be blamed on incomplete state descriptions, but instead indicates the presence of superluminal causation.

Consider a toy model discussed by Maudlin, in which each particle in the pair is indeterminate (in regard to its spin along any particular direction) until one of the particles encounters a spin-measurement apparatus; at this point, this “first” particle flips a coin to decide whether to emerge from the +1 or the −1 port of the apparatus, and sends an instantaneous tachyon signal to the other particle in the pair, instructing it as to how it should behave in order to give rise to the correct quantum mechanical joint outcomes. Assuming, as before, that \( \hat{a} = \hat{b} \), and that Bob’s particle arrives at its detector first with the result \( B = +1 \), this model makes the same predictions, Equations 8 and 9 as the local deterministic model discussed by Jarrett. And so this model, like Jarrett’s, violates Jarrett’s “completeness” condition. And yet clearly with this model there is no blaming that violation on the use of incomplete state descriptions—instead here it is obvious (by construction) that the violation is due to the presence of superluminal causal influences (in the particular form of “tachyon signals”).

This should not be surprising. We have already argued that, if one follows Bell in requiring \( \lambda \) to constitute a complete state description, then a violation of Jarrett’s “completeness” can only be understood as indicating the presence of nonlocal causation. The point here is that, even if we follow Jarrett in remaining agnostic about the completeness of the description afforded by \( \lambda \), we cannot necessarily say that a violation of “completeness” is compatible with relativity’s prohibition on superluminal causation. It might be (as shown by Jarrett’s model). But it might not be (as shown by Maudlin’s).

The correct conclusion is therefore as follows: a violation of Jarrett’s “completeness” condition (where we are openly agnostic about the completeness of the state description \( \lambda \)) means either that we have relativity-violating nonlocal causation, or that we were dealing with incomplete state descriptions. And this dilemma is precisely that posed already in 1935 by Einstein, Podolsky, and Rosen: either we concede Bohr’s claim that the QM description of states is complete and accept the re-
ality of the nonlocal causation present in that theory; or we reject the completeness claim and adopt a different (“hidden variables”) theory which (the authors thought) could restore locality. 29

Jarrett also sees a similarity between his conclusion and that of EPR (namely that QM is not complete). But where EPR considered this a defect to be corrected in some alternative “hidden variable” theory (which they hoped would also restore local causality), Jarrett argues that incompleteness is not a defect of orthodox QM, but, rather, a fact of nature:

“By separating out the relativistic component of the strong locality condition ... there emerges a clarification of that class of theories excluded by the Bell arguments: the class of theories which satisfy completeness. Although the term ‘incompleteness’ may conote a defect (as if, as was the case for the model discussed [above], all incomplete theories may be ‘completed’), incomplete theories (e.g., quantum mechanics) are by no means ipso facto defective. On the contrary, when the result of Bell-type experiments are taken into account, the truly remarkable implication of Bell’s Theorem is that incompletion, in some sense, is a genuine feature of the world itself.” 30

This sort of claim is echoed also in Jarrett’s later writings. 31

Ballentine and Jarrett also give an argument that the “completeness” of interest to EPR is a stronger form of Jarrett’s “completeness” in the sense that the former entails the latter. (This occurs in the context of their agreeing with EPR that QM is incomplete, which they establish by arguing that, since the correct conclusion from Bell/experiment is that Jarrett’s “completeness” condition fails, then the stronger EPR completeness condition must also fail, just as the EPR authors claimed.) But this association is mistaken, since the argument Ballentine and Jarrett display sneaks in the additional premise of (EPR’s version of) local causality. 32 And this is the premise that does (literally) all the work in the EPR argument.

There is thus no apparent sense at all in which EPR’s completeness has anything to do with Jarrett’s, except that it was precisely in the face of QM’s violation of Jarrett’s “completeness” that EPR argued (correctly) that QM was a non-local theory which, perhaps, could be replaced by a locally causal alternative theory by adding hidden variables (or jetisoning the description in terms of wave functions entirely). Indeed, at the end of the day, it is pretty clear that Jarrett’s condition has nothing to do with the completeness of physical state descriptions. In concluding (from Bell’s Theorem and the associated experiments) that reality itself is “in some sense” incomplete, Jarrett makes clear that he is no longer using the term with anything like its ordinary meaning – namely, a description which leaves nothing (relevant) out. By what standard, exactly, could “the world itself” be supposed to have left something out?

It is worth stepping back, therefore, and clarifying what, if anything, one can say about “completeness.” There are two related senses on the table. First of all, a theory may be said to be “complete” or “incomplete” in relation to external physical reality. In this sense, a theory is complete if and only if it captures or describes everything (relevant) that in fact really exists. This is of course just the sense of completeness of interest to EPR. Their argument, in essence, was that relativistic local causality (which they simply took for granted) combined with certain empirical predictions of QM entailed the existence of some “elements of reality” which had no counterpart in the QM description of reality. That is, the QM description left something out; it was hence an incomplete theory.

Since this usage of “completeness” involves a comparison between theories and external reality (to which our best access is precisely through theories!), there is a tendency for it to be regarded as “metaphysical”. Perhaps this apparently metaphysical flavor is responsible for Jarrett’s suppression of Bell’s requirement that λ contain a complete specification of the relevant beables. Since there’d be no way to verify whether a given specification was or wasn’t complete in this sense, one might think, Bell’s requirement is meaningless and might as well just be dropped.

But this attitude fails to appreciate one of Bell’s important advances – namely, that his formulation of local causality is a criterion for assessing the locality of candidate theories. As already discussed in Section II, Bell’s “complete specification of beables” simply does not mean a specification that captures everything which in fact really exists; rather, it means a specification which captures everything which is posited to exist by some candidate theory. There is thus nothing the least bit metaphysical or obscure about Bell’s requirement. For any unambiguously formulated candidate theory, there should be no question about what is being posited to exist. And so there will be no ambiguity about what a complete description of relevant beables should consist of, and hence no ambiguity about the status – vis-a-vis local causality – of a given well-formulated theory.

There will of course still be difficult questions about how to decide whether a given candidate theory is true, and hence whether the particular sort of non-local causation contained in it accurately describes some aspect of Nature. But the miracle of Bell’s argument is that we need not know which theory is true, in order to know that the true theory (whatever it turns out to be) will have to exhibit non-local, super-luminal causation. There is thus no escaping Bell’s conclusion that some sort of non-local causation (in violation of the structure displayed in Figure 1) exists in Nature – in apparent conflict with what most physicists take to be the requirements of SR.
V. DISCUSSION

In the previous section, we interpreted Jarrett as claiming that a violation of his “completeness” condition in no way implied the presence of non-local causation, but instead only implied that the state descriptions used in the test had been incomplete. This is both fair and unfair – fair because Jarrett does hang his entire case for the plausibility of his terminology and his physical interpretation of the two sub-conditions on precisely this view, but also unfair because Jarrett also later seems to acknowledge that, ultimately, his “completeness” condition has to be understood very differently. For example, he remarks in a footnote that “completeness, too, has the character of a ‘locality’ condition.” And the trend in the Bell literature since Jarrett’s paper has certainly been to concede that a violation of Jarrett’s “completeness” cannot be quite so trivially written off (as involving a mere updating of information in the face of having previously used incomplete state descriptions), but rather must be understood as indicating some sort of non-locality or “holism” or “non-separability” or non-causal “passion at a distance.”

We do not, therefore, wish to claim that Jarrett (and those who follow him in thinking his decomposition is in some way or other helpful in understanding Bell’s locality condition and/or in establishing the peaceful coexistence of SR and QM (24)) fully commits precisely the mistake presented (through the stark contrast to Bell’s views) in the previous section. Rather, we intend only the weaker claim that Jarrett et al. have been led, by Jarrett’s initial analysis, down a path which is obviously untenable once one clearly understands Bell’s own formulation of local causality (including especially the parallel status – namely, both are beables – of “settings” and “outcomes,” and the crucial distinction between superluminal causation and superluminal signaling).

It turns out to be a rather subtle question whether or not SR genuinely requires local causality in the sense of Figure 1. But if one grants this (and virtually all physicists and commentators do), then it really is possible to establish an

“essential conflict between any sharp formulation of QM and fundamental relativity. That is to say, we have an apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory...” (Bell, 1984, p. 172)

The widespread claims to the contrary – i.e., the claims that instead Bell’s theorem refutes only some already-dubious, dogmatic, philosophically-motivated program to restore “determinism” or “classicality” or “realism” (and I mean here both classes of such claims mentioned in the introduction) – turn out inevitably to have their roots in a failure to appreciate some aspect of Bell’s own arguments.

There is, in particular, a tendency for a relatively superficial focus on the relatively formal aspects of Bell’s arguments, to lead commentators astray. For example, how many commentators have too-quickly breezed through the prosaic first section of Bell’s 1964 paper (p. 14-21) – where his reliance on the EPR argument “from locality to deterministic hidden variables” is made clear – and simply jumped ahead to section 2’s Equation 1 (p. 15), hence erroneously inferring (and subsequently reporting to other physicists and ultimately teaching to students) that the derivation “begins with deterministic hidden variables”? (1981, p. 157) Likewise, we have here explored in detail a similar case of too-quickly accepting some formal version of a premise used in Bell’s derivation (such as “factorizability”) while failing to appreciate the rich conceptual context that gives it the precise meaning Bell intended.

Our final conclusion, therefore, is a plea – directed at physicists in general, but commentators on Bell’s theorem, textbook writers, and students in particular – to simply read (and not just read, but read) Bell’s writings. They are truly a model of clarity and physical insight, and almost always convey the essential ideas much more lucidly and tersely than anything in the secondary Bell literature. (I have no doubt this applies even to the current essay!) Bell himself, in the preface to the first edition of his compiled papers (Speakable and Unspeakable in Quantum Mechanics) suggests that “even quantum experts might begin with [chapter] 16, ‘Bertlmann’s socks and the nature of reality’, not skipping the slightly more technical material at the end.” It is hard to disagree with that advice, although a strong case could be made also for Bell’s 1990 essay (written after the first edition of the book, and hence included only in the more recent second edition) ‘La nouvelle cuisine,’ in which the central importance and meaning of “local causality” is emphasized in lucid detail.

If more physicists would only study Bell’s papers instead of relying on dubious secondary reports, they would, I think, come to appreciate that there really is here a serious inconsistency to worry about. A much higher-level inconsistency between quantum theory and (general) relativity has been the impetus, in recent decades, for enormous efforts spent pursuing (what Bell once referred to as) “presently fashionable ‘string theories’ of ‘everything’.” (1990, p. 235) How might a resolution of the more basic inconsistency identified by Bell shed light on (or radically alter the motivation and context for) attempts to quantize gravity? We can’t possibly know until (perhaps long after) we face up squarely to Bell’s important insights.

[1] John S. Bell, Speakable and Unspeakable in Quantum Mechanics, 2nd ed., Cambridge University Press, 2004. Subsequent references to Bell’s writings in the text will be...
given in-line with the year of the referenced paper and page numbers from the book.

[2] For a review of recent experiments and associated loopholes, see, e.g., Abner Shimony, “Bell’s Theorem”, The Stanford Encyclopedia of Philosophy (Fall 2006 Edition), Edward N. Zalta (ed.), URL = http://plato.stanford.edu/archives/fall2006/entries/bell-theorem.

[3] See, for example: N. David Mermin, “What is quantum mechanics trying to tell us?” AmJPhys, 66(9), September 1998, pg 753-767; Marek Zukowski, “On the paradoxical book of Bell,” Stud. Hist. Phil. Mod. Phys., 36 (2005) 566-575; A. Zeilinger, “The message of the quantum,” Nature 438, 743 (8 December, 2005); Daniel Styer, The Strange World of Quantum Mechanics (page 42), Cambridge, 2000; George Greenstein and Arthur Zajonc, The Quantum Challenge (Second Edition), Jones and Bartlett Publishers, Sudbury, Massachusetts, 2006; John Townsend, A Modern Approach to Quantum Mechanics, McGraw-Hill, 1992; Herbert Kroemer Quantum Mechanics, Prentice Hall, New Jersey, 1994; Richard Liboff, Introductory Quantum Mechanics (2nd edition), Addison-Wesley, Reading, Massachusetts, 1992.

[4] For further discussion, see any of Bell’s papers and, e.g.: Tim Maudlin, Quantum Non-Locality and Relativity (Second Edition), Blackwell, Malden, Massachusetts, 2002; Travis Norsen, “Bell Locality and the Nonlocal Character of Nature,” Found. Phys. Lett., 19(T), 633-655 (Dec. 2006).

[5] Jon Jarrett, “On the Physical Significance of the Locality Conditions in the Bell Arguments,” Nous 18 (1984) 569-589.

“Peaceful Coexistence” is Abner Shimony’s term: “Metaphysical problems in the foundations of quantum mechanics,” International Philosophical Quarterly 18, 3-17.

[7] See, for example, M.L.G. Redhead, Incompleteness, Non-locality, and Realism: A Prolegomenon to the Philosophy of Quantum Mechanics Oxford, 1987; J. Cushing and E. McMullin, eds., Philosophical Consequences of Quantum Theory, Notre Dame, 1989 (see especially the contributions of Paul Teller and Don Howard); Brandon Fogel, “Formalizing the separability condition in Bell’s theorem,” Stud. Hist. Phil. Mod. Phys., 38 (2007), 920-937.

[8] Op cit., pp. 93-98

[9] Similar points are made in Jeremy Butterfield, “Bell’s Theorem: What it Takes” Brit. J. Phil. Sci., 43 (1992) 41-83. Butterfield, however, (unlike Maudlin) ends up accepting both the validity/meaningfulness of Jarrett’s analysis and Jarrett’s ultimate conclusion about Peaceful Coexistence, even though he disagrees with some of Jarrett’s arguments. Since I disagree with Butterfield’s conclusions, I find him a less convincing overall critic of Jarrett’s project than Maudlin.

[10] Note that Bell stresses the need for a complete specification of beables in the relevant space-time region already in his 1975 paper The theory of local beables: “Now my intuitive notion of local causality is that events in 2 should not be ‘causes’ of events in 1, and vice versa. But this does not mean that the two sets of events should be uncorrelated, for they could have common causes in the overlap of their backward light cones. It is perfectly intelligible then that if \[B_3\] in [our Equation 1] does not contain a complete record of events ... it can be usefully supplemented by information from region 2. So in general it is expected that \[P(b_1|B_3, b_2) \neq P(b_1|B_3)\]. However, in the particular case that \[B_3\] contains already a complete specification of beables ... supplementary information from region 2 could reasonably be expected to be redundant.” (1975, p. 54) Emphasis in original. This is especially relevant since we will eventually criticize Jarrett for failing to appreciate (in his 1984 paper) this particular aspect. So it shouldn’t be thought that we are criticizing him for something Bell only understood and clarified later. It is worth noting, however, that there are some interesting differences between Bell’s 1975 and 1990 formulations of local causality; these will be explored elsewhere, though, since they do not bear on Jarrett’s analysis.

[11] This is why, despite being an improvement over Jarrett’s terminology for the two sub-conditions to be discussed in Section III, Abner Shimony’s terminology (“parameter independence” for what Jarrett calls “locality” and “outcome independence” for what Jarrett calls “completeness”) is also dubious. For the terminology implies that a violation of one of the conditions entails that the event in question causally depends on the distant “parameter” or “outcome” respectively. But this need not be the case.

[12] Determinism is simply a special case in which all probabilities are either zero or unity.

[13] This sometimes comes as a shock to adherents of orthodox quantum theory, who are used to thinking of their own theory – especially in its allegedly relativistic variants – as perfectly consistent with SR. But the nonlocality of orthodox QM is quite obvious, if one knows where to look. The key here is that the theory is not defined exclusively by the Schrödinger (or equivalent) dynamical equation, but also by some version of a collapse postulate. And this latter is explicitly nonlocal. Indeed, orthodox (collapse) QM is even more nonlocal than certain alternative theories, like Bohmian Mechanics, which are often maligned precisely for displaying an obvious kind of nonlocality. The simplest type of example which suffices to make this point is the “Einstein’s Boxes” scenario. (See Travis Norsen, AmJPhys 73(2), Feb. 2005, pages 164-176.) Bella explains beautifully how this scenario manifests the nonlocal causation inherent in orthodox QM: “Suppose … we have a radioactive nucleus which can emit a single \(\alpha\)-particle, surrounded at a considerable distance by \(\alpha\)-particle counters. So long as it is a chance for a particular counter that …”
observations, that some non-local causation is occurring in Nature.) The point here is that orthodox QM’s account of the “Einstein’s Boxes” scenario involves non-local causation; whether any nonlocality in fact occurs in Nature when one performs the indicated experiment involving an α-particle, however, is a very different question. If, for example, Bohmian Mechanics (rather than orthodox QM) is true, the answer would be no.

[14] A candidate theory which posited no beables corresponding to such things as knobs and levers should not, and probably could not, be taken seriously. Bell stresses in his very first discussion of beables that: “The beables must include the settings of switches and knobs on experimental equipment ... and the readings of instruments.” (1975, p. 52) For elaboration of the sense of the term “serious” being used here, see Bell’s (1986, pp. 194-5).

[15] See Bell, 1990, pp. 243-4.
[16] See Bell’s discussion in his 1977, pages 100-104.
[17] A. Shimony, M.A. Horne, and J.F. Clauser, “An Exchange on Local Beables,” Dialectica, 39 (1985) 86-110
[18] See, e.g., Bell 1975, pages 55-57.
[19] Bell, 1981, p. 150
[20] Jon Jarrett, op cit., p. 573
[21] Ibid., p. 578
[22] Ibid., p. 580
[23] Ibid., p. 580
[24] Ibid., p. 585
[25] See Bell’s discussions of “controllable” beables and causation vs. signaling in his 1975, pp. 60-61; 1984, p. 171; and 1990, pp. 244-246.
[26] See, e.g., p. 153 of Harvey Brown’s half of “Nonlocality in Quantum Mechanics,” Michael Redhead and Harvey Brown, Proceedings of the Aristotelian Society, Supplementary Volumes, Vol. 65 (1991), pp. 119-159.
[27] See Maudlin, op cit., pg 82. After presenting this model as a simple example of how the observed spin correlations might arise, Maudlin uses it to counterexample the claim, made by Don Howard and others, that Jarrett’s “completeness” and “locality” can be mapped, respectively, onto the “separability” and “locality” conditions which emerge from some of Einstein’s comments about local causality. As Maudlin points out, the toy model is perfectly separable in the sense of Einstein, and yet violates what Howard et al. would have us take as a mathematical formulation of Einstein’s “separability” (namely, Jarrett’s “completeness.”) Curiously, however, Maudlin does not mention this model in his (earlier) discussion of Jarrett.

[28] Of course, since Alice’s and Bob’s measurements are, by hypothesis, space-like separated, there is no relativistically unambiguous meaning to “first.” But that is really the whole point. This model explicitly involves anti-relativistic superluminal causation, so part of the model is that relativity is wrong and there exists some dynamically privileged reference frame which gives an unambiguous meaning to this “first” (and also to the “instantaneous” in the description of the tachyon signal).

[29] A. Einstein, B. Podolsky, and N. Rosen, “Can quantum-mechanical description of physical reality be considered complete?” Phys. Rev. 47 (1935), 777-780. For some more recent discussion, see T. Norsen, “Einstein’s Boxes,” op cit. and references therein.

[30] Op cit., p. 585
[31] L.E. Ballentine and Jon P. Jarrett, “Bell’s theorem: Does quantum mechanics contradict relativity?” Am.J.Phys. 55(8) (August 1987), 696-701; Jon Jarrett, “Bell’s Theorem: A Guide to the Implications” in J. Cushing and E. McMullin, eds., op cit.

[32] “But this prediction was made without in any way disturbing particle L, since the R device is at spacelike separation from it...” This mistake was also noted by Andrew Elby, Harvey R. Brown, and Sara Foster in “What Makes a Theory Physically Complete?” Found. Phys. 23(7), 971-985 (1993). Note also that this error leads Ballentine and Jarrett to remark, in passing, that EPR’s concept of completeness entails determinism all by itself, which is surely a misrepresentation of the worries of Einstein and his followers. See Section 1 of Bell’s 1981, pp. 139-145, and the important footnote 10 – already partially quoted in the introduction – on p. 157.

[33] Jon Jarrett, op cit., p. 589
[34] For a systematic review of the recent literature, see Berkovitz, Joseph, “Action at a Distance in Quantum Mechanics”, The Stanford Encyclopedia of Philosophy (Spring 2007 Edition), Edward N. Zalta (ed.), URL = http://plato.stanford.edu/archives/spr2007/entries/qm-action-distance/
[35] Much of Maudlin’s excellent book, op cit., is dedicated to exploring just this question.