Energy loss of muons and taus through inelastic scattering on nuclei

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A hybrid model [1] was used to describe the energy loss of very high-energy taus and muons in matter due to inelastic scattering on nuclei. The model involves soft and semihard photonuclear interactions as well as the deep inelastic scattering including the weak neutral current processes. For the lepton scattering on nuclei all important nuclear effects, the shadowing, anti-shadowing, EMC and the nucleon binding, are taken into account. Approximating formulas for the muon and tau energy loss portion by the inelastic scattering on nuclei in water are given for wide energy range up to $10^9$ GeV.

1. Introduction

The muon inelastic scattering on nuclei contributes noticeably to the energy loss of cosmic rays muons. The influence of this interaction on the shape of ultra-high energy muon spectra at the great depth of a rock/water is still unknown in detail. The tau-lepton energy loss is of interest in view of ability of the atmospheric or extraterrestrial muon neutrinos to transform to the tau neutrinos which may in turn produce taus in $\nu N$ interactions.

In Ref. [1], the hybrid (two- and three-component) model was proposed to describe high-energy interactions of charged leptons with nuclei. Calculations of differential cross sections for lepton-nucleon inelastic scattering at the HERA energies were checked making a comparison with H1 and ZEUS measurements of electron and positron scattering on protons. With this model muon and tau energy loss spectra due to lepton-nuclear inelastic scattering were computed as well as the energy loss rate for leptons passing through standard rock. Now we apply the two-component version (2C) of the model [1] to study the energy loss in the scattering on nuclei of very-high energy muons and taus passing through water or rock. The difference in inelastic scattering of the oppositely charged leptons that might originate from the weak neutral current processes is considered.

2. Charged lepton inelastic scattering on the nuclei

The hybrid two-component (2C) model for inelastic interactions of high-energy muons and taus with nuclei involves photonuclear interactions at low and moderate momentum transfer squared as well as the deep inelastic scattering (DIS) processes at high $Q^2$. For virtuality $0 < Q^2 < 5$ GeV$^2$ the Regge based parametrization [2-3] for the electromagnetic structure function $F_2^\gamma$ was applied and the lepton-nucleon inelastic scattering cross section at $Q^2 \leq 5$ GeV$^2$ was computed with the formula

$$\frac{d^2\sigma}{dQ^2dy} = \frac{4\pi\alpha^2}{yQ^4} \left[ 1 - y - \frac{Q^2}{4E^2} + \frac{y^2}{2(1+R)} \left( 1 - \frac{2m_l^2}{Q^2} \right) \left( 1 + \frac{Q^2 E^2}{2y^2} \right) \right] F_2^\gamma(x, Q^2),$$

(1)

where the ratio $R = \sigma_L/\sigma_T$ is taken into account according to Ref. [4]; $x = Q^2/(2MEy)$. In the DIS range the cross section for the scattering of nonpolarized lepton on nonpolarized nucleon can be written in the form [5]

$$\frac{d^2\sigma}{dQ^2dy} = \frac{4\pi\alpha^2}{yQ^4} \left[ 1 - y - \frac{Q^2}{4E^2} + \frac{y^2}{2} \frac{y^2 m_l^2}{Q^2} \right] F_2^{NC} \pm \left( \frac{y^2}{2} - y \right) x F_3^{NC},$$

(2)
where we put \( R = Q^2/(Ey)^2 = 4M^2x^2/Q^2 \), that is equivalent to the Callan-Gross relation, \( F_2 = 2xF_1 \); signs “±” stand for \( \ell^\pm (\ell = \mu, \tau) \). In Eq. (2) used notations are:

\[
F_2^{NC} = F_2^\gamma - g_{V}^{\gamma Z}F_2^\gamma + \left( g_{V}^{\gamma Z} + g_{A}^{\gamma Z} \right) \eta_{\gamma Z}F_2^\gamma, \quad F_3^{NC} = - g_{A}^{\gamma Z}F_3^\gamma + 2g_{V}^{\gamma Z}g_{A}^{\gamma Z}F_3^\gamma; \quad (3)
\]

\[
\eta_{\gamma Z} = \frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha} \frac{Q^2}{M_Z^2 + Q^2}, \quad g_{V}^{\gamma Z} = \frac{1}{2} + 2\sin^2 \theta_W, \quad g_{A}^{\gamma Z} = -\frac{1}{2}, \quad (4)
\]

where \( G_F \) is the Fermi coupling constant, \( M_Z \) is the \( Z^0 \) mass, \( \theta_W \) is the weak-mixing angle. The structure functions \( F_2^\gamma, F_3^\gamma \) represent the weak neutral current (NC) contribution and \( F_2^{\gamma Z}, F_3^{\gamma Z} \) take account of the electromagnetic and weak current interference. The nucleon structure functions, \( F_2^\gamma, F_3^{\gamma Z}, F_2^\gamma, F_3^{\gamma Z}, F_3^{\gamma Z} \), are defined in the quark-parton picture:

\[
\left[ F_2^\gamma, F_2^{\gamma Z}, F_2^\gamma \right] = \sum_{q} \left[ e_q^2, 2e_qg_q^V, g_q^V + g_q^A \right] (q + \overline{q}), \quad \left[ F_3^{\gamma Z}, F_3^{\gamma Z} \right] = \sum_{q} \left[ 2e_qg_q^A, 2g_q^Vg_A^q \right] (q - \overline{q}), \quad (5)
\]

where \( g_q^V = \pm \frac{1}{2} - 2e_q\sin^2 \theta_W, g_A^q = \pm \frac{1}{2} \). Sign (+) corresponds to \( u, c, t \) (\( d, s, b \))-quarks. For the range of \( Q^2 > 5 \text{ GeV}^2 \) the electroweak nucleon SFs are computed with the CTEQ6 [6] and the MRST2002 [7] sets of the parton distributions. Linear fits for the nucleon SFs are used in \( 5 < Q^2 < 6 \text{ GeV}^2 \) range. For the scattering on nuclei the nucleon shadowing, anti-shadowing as well as EMC effect are taken into account according to Ref. [8,9] (see also [10] and [11] for details).

3. Some results

The energy loss spectra for lepton passing a substance with nuclear weight \( A \) can be derived from the differential cross-section:

\[
N_{0}\frac{d\sigma^{TA}}{dy} = N_{0} y \int_{Q_{\text{min}}^2}^{Q_{\text{max}}^2} dQ^2 \frac{d^2\sigma^{TA}}{dQ^2 dy}, \quad y = \frac{E - E'}{E}, \quad (6)
\]

where \( N_0 = N_A/A \). The energy loss rate due to the lepton-nucleus interactions is defined as

\[\text{Figure 1.} \quad \text{NC contribution to the cross section of the } \tau^\pm \text{-proton inelastic scattering (left) and that of the } \tau^\pm \text{-A scattering in standard rock (A=22) (right) at } E = 1 \text{ PeV, } Q^2 > 10^4 \text{ GeV}^2.\]
Figure 2. Left panel: Energy loss due to the inelastic scattering in water of muons (upper lines) and taus (bottom). Right panel: NC contribution to the lepton energy loss due to $\ell^\pm$-nucleus inelastic scattering in water.

$$b_n^{(\ell)}(E) \equiv -\frac{1}{E} \frac{dE}{dh} = N_0 \int_{y_{\min}}^{y_{\max}} y \frac{d\sigma^{\ell A}}{dy} dy.$$  \hspace{1cm} (7)

Figure 1 illustrates the charge-dependent NC contribution $\left( \frac{d\sigma^{+Z}}{dy} \right) / \frac{d\sigma^{-Z}}{dy} - 1)$ to the lepton inelastic scattering on protons (left panel) or on the nuclei ($A=22$) (right panel) at $E = 1$ PeV, $Q^2 > 10^4$ GeV$^2$. The effect of the $Z^0$ exchange is fairly seen just for very large $Q^2$. In Fig. 2 the left panel, presented are the muon and tau energy loss $b_n(E)$ due to the inelastic interactions with nuclei in water (the solid lines). For comparison the calculations according to the vector-meson dominance model [12] are shown (dashed). The right panel of Fig. 2 shows the relative difference of the energy loss, $\left( b_n^{+Z} - b_n^{-Z} \right) / b_n^{-Z}$, calculated for the $\ell^+$ and $\ell^-$ particles. For whole energy range the NC contribution to the $b_n^{(\ell)}(E)$ is too little ($10^{-4}$) to be of practical interest.

Table 1. The muon and tau energy loss in standard rock ($A = 22$)

| $E$, GeV | This work | Ref. [13] | Ref. [10] | Ref. [14] |
|----------|-----------|-----------|-----------|-----------|
| Muon     |           |           |           |           |
| 10$^5$   | 0.62      | 0.60      | 0.68      | 0.70      |
| 10$^6$   | 0.82      | 0.80      | 0.90      | 1.08      |
| 10$^8$   | 1.53      | 1.50      | –         | 2.25      |
| 10$^9$   | 2.16      | 2.15      | –         | 3.10      |
| Tau      |           |           |           |           |
| 10$^5$   | 0.13      | 0.12      | –         | 0.14      |
| 10$^6$   | 0.19      | 0.18      | –         | 0.21      |
| 10$^8$   | 0.41      | 0.40      | –         | 0.50      |
| 10$^9$   | 0.65      | 0.60      | –         | 0.72      |

In the second column of the Table 1 the 2C model calculation of the muon and tau energy loss in standard rock and that are presented along with recent predictions [10, 13, 14]. One can see that present calculations of
tau-lepton energy loss in standard rock are similar. As concerns to muons, this work result for $b_n^{(\mu)}$ as well as that by Ref. \cite{13} differs apparently at $E > 10^6$ GeV from the predictions of Ref. \cite{14}.

The energy dependence of the inelastic scattering energy loss rate of muons and taus traveling through water may be fitted for the range ($10^2 - 10^9$) GeV with formula ($\ell = \mu, \tau$):

$$b_n^{(\ell)}(E) = (c_0 + c_1 \eta + c_2 \eta^2 + c_3 \eta^3 + c_4 \eta^4) \cdot 10^{-6} \text{ cm}^2/\text{g}, \quad \eta = \log(E/1 \text{ GeV}),$$

where coefficients are:

$\mu$:

$c_0 = 1.06416$, $c_1 = -0.64629$, $c_2 = 0.20394$, $c_3 = -0.02465$, $c_4 = 0.0013$;

$\tau$:

$c_0 = 0.35697$, $c_1 = -0.24437$, $c_2 = 0.07403$, $c_3 = -0.00940$, $c_4 = 0.00051$.

4. Conclusions

Recent calculations \cite{11, 10, 13, 14} of the energy loss in the tau-nuclear interactions are compatible at least for lepton energy up to $10^9$ GeV. However there is the discrepancy between predictions for high-energy behavior of the muon energy loss, $b_n^{(\mu)}(E)$, in Refs. \cite{13} and \cite{11}) on the one side, and that of Ref. \cite{14} on the other side, likely due to diverse ways in considering of the nuclear effects and high $Q^2$ processes.

The neutral current ($Z^0$ exchange) contribution to energy loss of muons and taus is found to be negligible both in water and standard rock on whole energy range (up to $10^{12}$ GeV). Though the ratio of the cross section for inelastic scattering of $\tau^-$ to that of $\tau^+$ is a sizeable for $Q^2 > 10^4$ GeV$^2$ at not too large energies, the effect for the energy loss ($\Delta b_n \sim 10^{-4} \cdot b_n$) seems too small in the cosmic ray physics context.

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