STRONG-FIELD TESTS OF f(R)-GRAVITY IN BINARY PULSARS

P.I. DYADINA, S.O. ALEXEYEV, K.A. RANNU
Sternberg Astronomical Institute, Lomonosov Moscow State University, Universitetsky Prospekt, 13, Moscow 119991, Russia

In this work we develop the PPK approach for a class of analytic f(R)-models of gravity. We use data from the double binary pulsar system PSR J0737-3039. We obtain restrictions on parameters of this class of f(R)-models and show that f(R)-gravity is not ruled out by the observations in strong field regime.

1 Introduction

General relativity (GR) is a very beautiful theory which allows to go beyond the Newtonian picture of the world and explains many unaccounted phenomena. However our understanding of fundamental laws still has several shortcomings. The accelerated expansion of the Universe (i.e., dark energy) has been found from cosmological observations recently. Moreover already in 1930s the problem of galactic rotation curves arose. One way to unriddle these puzzles is to add yet unknown particles and look for them on LHC and in cosmic rays. Another way is to expand GR by including additional corrections in terms of the Ricci scalar in the Lagrangian. This method underlies f(R)-gravity.

2 f(R)-gravity

f(R)-gravity is actually a family of theories, each of them is defined by a different function of the Ricci scalar. In the simplest case the function equals to the scalar; that is GR. We can explain dark matter, dark energy and inflation by different models of f(R)-gravity. The action of f(R)-gravity has the following form:

$$S = \int d^4x \sqrt{-g} [f(R) + \kappa L_m]$$

where $\kappa = 16\pi G/c^4$ is the coupling coefficient, $g$ is the determinant of the metric tensor, $L_m$ is the standard matter Lagrangian, $f(R)$ is an analytical function of the general form. This function can be expanded in a series in terms of the Ricci scalar:

$$f(R) = \sum_n \frac{f^{(n)}(R_0)}{n!} (R - R_0)^n \simeq f_0 + f_0' R + \frac{1}{2} f_0'' R^2 + \ldots,$$

where

$$f_0 = \text{const}, \quad f_0' = \frac{df(R)}{dR} \bigg|_{R=0}, \quad f_0'' = \frac{d^2f(R)}{dR^2} \bigg|_{R=0}.$$
Table 1: Post-Newtonian parameters

| PPN parameter | Physical meaning | Experimental value |
|---------------|------------------|--------------------|
| \( \gamma \)  | space-curvature produced by unit restmass | \( 1 \pm 2.3 \times 10^{-5} \) |
| \( \beta \)    | nonlinearity in the superposition law for gravity | \( 1 \pm 8 \times 10^{-5} \) |

The flat Minkowskian background is recovered for \( R = R_0 = 0 \). GR is recovered in the limit \( f_0 = 0, f_0' = 4/3, f_0'' = 0^7 \). Hereafter we assume \( f_0 = 0, f_0' = 4/3 \) whereas \( f_0'' \) is a free parameter. Our purpose is to restrict the possible value of this free parameter \( f_0'' \).

However any theory of gravity should be verifiable. Naturally, there are many other ways for testing theories of gravity but in this work we applied only PPN and PPK formalisms to \( f(R) \)-gravity.

3 Parametrized post-Newtonian formalism

Parametrized post-Newtonian formalism was originally developed to compare various metric theories with each other and GR \(^8\). The post-Newtonian limit (PPN) is established in the framework of the asymptotically flat space-time background and small velocities. Motion of matter should obey the hydrodynamics equations for the perfect fluid. Distinctions between GR and other theories of gravity are reflected via the set of 10 post-Newtonian parameters. Each parameter is responsible for its effect. However, the considered \( f(R) \) gravity model is the conservative theory and, in this case, only two parameters \((\gamma, \beta)\) are not equal to zero (see table 1) \(^{10}\).

Drewing an analogy between the scalar-tensor gravity and the higher order theories of gravity, Capozziello and Troisi \(^6\) developed the PPN formalism for \( f(R) \)-gravity. The similarity between the non-minimally coupled scalar models (Lagrangian of Brans-Dicke type\(^11\).\(^12\)) and the models of gravity with higher order curvature corrections have been discussed since 1983 \(^13\). Basing on this similarity Capozziello and Troisi \(^6\) obtained the Eddington's parameters for \( f(R) \)-gravity in analytical form:

\[
\gamma_{PPN} = -1 = - \frac{f''(R)^2}{f'(R) + 2f''(R)^2}, \quad \beta_{PPN} = -1 = \frac{1}{4} \left[ \frac{f'(R)f''(R)}{2f'(R)^2 + 3f''(R)^2} \right] dR_{PPN} \frac{d\phi}{dR},
\]

where \( f(R) \) is an arbitrary function of \( R \). Using the expansion (2), we carried out the Eddington's parameters for the considered model of \( f(R) \)-gravity:

\[
\gamma_R = -1 = - \frac{(f_0'')^2}{f_0 + 2(f_0'')^2}, \quad \beta_R = -1 = \frac{1}{4} \left[ \frac{f_0''(f_0')^3}{2(f_0'')^3 + 20f_0''(f_0')^4 + 11(f_0'')^2(f_0')^2 + 6(f_0'')^4} \right].
\]

Using the fact that \( f(R) \)-gravity recovers GR at \( f_0'' = 4/3 \) and the observational values of parameters \( \gamma_{PPN} \) and \( \beta_{PPN} \) \(^{10}\) (see. table 1), we can impose restrictions on the value of \( f_0'' \) by solving the system of equations (5):

\[
\gamma_{PPN} : \quad -0.0055 \leq f_0'' \leq 0, \quad \beta_{PPN} : \quad -7 \leq f_0'' \leq 0
\]

4 Parametrized post-Keplerian formalism

Parametrized post-Keplerian formalism (PPK) was created to link the arrival time of the pulses and their time of radiation in the frame of a pulsar \(^{14}\). \(^{15}\). PPK is a strong-field analogue of...
Table 2: Parameters of PSR 0737-3039

| Parameter | Physical meaning                      | Experimental value |
|-----------|---------------------------------------|--------------------|
| $P_b$ (day) | orbital period                        | 0.10225156248(5)   |
| $e$        | eccentricity                          | 0.0877775(9)       |
| $\omega$ (deg/yr) | secular advance of the periastron | 1.415032(1)        |
| $\gamma$ (ms) | time dilation parameter              | 0.3856(26)         |
| $s$        | Shapiro delay parameter               | 0.99974(-39, +16)  |
| $r$ (µs)   | Shapiro delay parameter               | 6.21(33)           |
| $R = \frac{m_1}{m_2} = \frac{2x}{x_1}$ | mass ratio                        | 1.0714(11)         |

The PPN formalism. It includes such effects as the Einstein time delay, Römer time delay, Shapiro time delay and the effects of aberration. The general form of these corrections is model-independent, therefore all possible manifestations of the extended gravity model can be expressed through the 8 post-Keplerian parameters $\dot{\omega}$, $\gamma$, $\dot{P}_b$, $r$, $s$, $\delta_8$, $\delta_1$, $\dot{x}$. However, in this work we considered only those parameters that have the most accurate measurements, so we didn't take into account the last three of them.

It should be noted that different theories of gravity can give different predictions for PPK parameters. We should compare predictions of the theory and the values of these parameters obtained from observations. Thus we have a powerful instrument for testing extended gravity models in the strong field limit.

The analytical form of the first derivative of the orbital period for the considered model of f(R) gravity was obtained by De Laurentis and Capozziello. And other PPK parameters were obtained for the first time in our work for this model of f(R) gravity.

The parameters depend only on the orbit eccentricity, projection of the semi-major axis of the pulsar orbit, orbital period, masses of the pulsar and its companion and also the parameter $f''_0$ of the f(R) gravity model. All of them, except the parameter and masses of the model,
can be obtained from observations. In our work we used the data for binary pulsar J0737-3039 which was presented in the article by Kramer and his colleagues\textsuperscript{17}. It is the only known double binary pulsar. It is the smallest period that the known systems of this type may have. The extraordinary closeness of system components, small orbital period and also the fact that we see almost edge-on system allow to investigate the manifestation of relativistic effects with the highest precision. Also it is possible to measure semi-major axis of the orbit for each of components of the system J0737-3039 and hence their ratio is equals:

\[
\frac{a_2}{a_1} = \frac{m_2}{m_1} = R,
\]

i.e. the ratio of the masses can be measured directly!

5 Test of f(R)-gravity

And now we proceed directly to the method of testing models of gravity\textsuperscript{15}. We constructed curves on the plane, where the ordinate displays the possible values of the companion masses \(m_2\), and the abscissa displays possible values of the masses of the pulsar \(m_1\). Each parameter specifies the curve. The point of intersection of all curves on this plane within the measurement accuracy will display the values of the pulsar and companion masses. However, if curves diverge within some model of gravity, it does not speak in favor of the model.

All the results are presented in the corresponding figures. For GR all the curves intersect within the measurement accuracy (see fig. 1)\textsuperscript{17}. Let us to recall that GR is recovered in the limit \(f_0 = 0, f_0' = 4/3, f_0'' = 0\). Then we begin to change \(f_0''\) and we can see that at some point \(f_0'' = 0.05772\) the curves start to diverge (see fig. 1, fig. 2, fig. 3). That is the limitation that we receive for this parameter from the binary pulsar data:

\[
-0.05772 \leq f_0'' \leq 0.
\]
6 Conclusions

In this work we impose restrictions on the considered model of $f(R)$-gravity from the observations in the strong and weak field limits. For our aims we used the data of double bynary pulsar system and accurate measurements of the PPN parameters in the Solar System, respectively. We show that the observational data of double pulsar system give the following limit on a value of parameter $f_0^m$:

$$-0.05772 \leq f_0^m \leq 0.$$  \hspace{2cm} (10)

This parameter characterizes the contribution of the quadratic curvature correction in the action of $f(R)$-gravity. It is important to note that the obtained restriction on the possible values of $f_0^m$ is small but at the same time it can not be considered negligible even within the measurement accuracy. This result allows the realization of GR as well as its extensions, including quadratic curvature corrections.

At the same time it is possible to receive the limitations on the value $f_0^m$ from Eddington parameters measurements in the Solar system. The parameter $\gamma_{PPN}$ gives a better limit than the parameter $\beta_{PPN}$:

$$\gamma_{PPN} : \quad -0.0055 \leq f_0^m \leq 0, \quad \beta_{PPN} : \quad -7 \leq f_0^m \leq 0,$$  \hspace{2cm} (11)

Thus, more strict limitation on the model parameters follows from the experiments in the solar system than from the data of bynary pulsar systems. On the one hand, it can be connected with the fact that measurement accuracy in the Solar system is much better than in the systems with the pulsar. On the other hand, in a system with a compact object gravity is much stronger $\left(2GM/(c^2R)_{PSR} \approx 0.2\right)$, than in the solar system $\left(2GM/(c^2R)_{SUN} \approx 10^{-6}\right)$, therefore, the contribution of corrections type $R^2$ should be more prominent.

Since $f(R)$-gravity is one of the ways to describe dark energy and dark matter, then obtaining the experimental constraints on the parameters of such models is an important step in solving these fundamental problems.
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