Evaluation of losses in transmission of machinery for development of mineral deposits in conditions of variable load

I E Zvonarev, S L Ivanov
Saint-Petersburg Mining University, 2, 21st line, Saint-Petersburg, 199106, Russia
E-mail: ZVano@mail.ru

Abstract. The influence of individual elements of machines transmissions on the operation of the whole system is shown. The approach of determining the resource of operation of systems elements based on the energy theory is presented. The formulas for determining the total energy resource of the reducer are given. The influence of individual elements of the system on each other is indicated. The principle of researching the system by the method of equivalent circuits is substantiated. The weakest places of transmission (gears, bearing supports and shafts) are determined. A mathematical model of a mechanical transmission was developed. To test the adequacy of the mathematical model, the stand for obtaining experimental data was designed. The description of the stand and the principle of its operation are given. Experimental data are presented. A comparative analysis of modeling and experimental data is carried out and the adequacy of the developed mathematical model is proved. The principle of determining the resource of the system as a whole for the element with the minimal resource of work is suggested.

1. Introduction
In general, the mechanical transmission of the machinery is an elastic multimass oscillatory system consisting of elements with elastic, inertial and dissipative properties.

The energy source of the mechanical transmission (gearbox) will consist of the energy sources of its constituent units. The failure occurs due to the depletion of the energy source of one unit of the daisy chain of transmission elements.

2. Theory research
The service life of the transmission of the machine and the resources of its elements is defined by the ratio of the value of the energy source to the value of the power loss in the transmission as a whole or in its elements, respectively [1]. The energy source of the gear to failure is given by [2]:

\[ E = \sum_{j=1}^{l} \left[ \sum_{i=1}^{n} \left( \frac{A_{i,j} \cdot N_{\text{lim}}^{1-m_{i,j}}}{1-m_{i,j}} \right) \cdot u_{j} \right] + \sum_{k=1}^{p} P'_{k} \cdot N_{k} \]  

(1)

where \( N_{\text{lim}} \) – limit of the number of cycles of the input element (reduction link); \( u_{j} \) – gear ratio of the transmission of the \( j \)th stage; \( A_{i,j} \) – constant coefficient of the curve of the energy source of the \( i \)th level of the \( j \)th stage; \( m_{i,j} \) – index of the curve of the energy source, respectively, \( P'_{k} \) – specific work of
the losses, attributed to one cycle of load, in the power shafts and base elements of the transmission and similar losses; \( N_k \) – corresponding number of cycles.

When the transmission is exposed to external dynamic actions, its elements dissipate additional energy. These additional losses are related to the transmission of the variable component of the load, which in turn changes its value depending on the accuracy of matching of the frequency of load application and the system frequency. It should be noted that the transmission behaves as a whole unit, having a common resonance frequency (frequencies). It is explained by the binding of transmission elements into a unified kinematic chain, in which the individual elements have a mutual influence on each other [3]. Additionally, the presence of dissipative losses both in separate parts and in the kinematic pairs is of paramount importance. The instantaneous external load can be given by

\[
R_i(t) = R_p + \sum_{i=1}^{n} A_i \sin(\omega_i t + \alpha_i),
\]

where \( R_p \) – constant component of the load; \( A_i \) – amplitudes of the frequency components of the load; \( \omega_i \) – frequency of the variable components of the load; \( \alpha_i \) – phase displacement of the periodic components. The number of expressions \( n \) that determine the nature of the external load is determined by the need to bring the calculated load into proximity with the actual load.

There is a well-known qualitative match of the characteristics of the internal friction in the material and in the joints. Therefore, the effective methods for solving vibration problems for systems with internal friction in the material are equally effective for solving vibration problems with internal friction in the joints [4].

The behavior of the transmission with the dynamic external action can pretty exactly be determined by the equivalent circuits. Application of simplified equivalent circuits is possible in determining the resonant frequency and evaluating the gain ratio of an external variable load. The use of individual universal schemes, corresponding to the main nodes of the transmission and generation of the equivalent circuit of the transmission from them as a whole, allow evaluating the losses element by element and thus take into account the expenditure of energy source of the various elements of the system. The most common nodes are gears with elements of their fastening to shafts, bearing supports and shafts. In this case, the refinement of the parameters of such uniform circuits leads to an improvement of the information about transmission received through the serial or parallel connection of the local schemes based on the kinematics of the transmission.

The rheological description of internal friction can be represented by the viscoelastic Voigt model, guided mainly by considerations of simplicity of the differential equations of motion. For such system the following expression would be fair:

\[
J \ddot{\varphi} + \beta \dot{\varphi} + c \varphi = M,
\]

where \( c \) – elastic member; \( \beta \) – viscous drag coefficient; \( J \) – moment of inertia; \( M \) – external moment.

When calculating the ratio \( \beta \), the following expression is used:

\[
\beta = \psi c / \pi \omega \, ,
\]

where \( \psi \) – absorption coefficient, \( \omega \) – resonant frequency of vibration of the system. Damping studies have shown that \( \psi \) does not depend on the oscillation frequency [5].

Using the algorithms and principles laid down in [1], a mathematical model of the mechanical transmission was developed. The value of the absorption coefficient was determined as the ratio of power losses in the kinematic pair with an average load per cycle to the amplitude of the variable component of the transmitted power. Relevant factors of vibrations diffusion in the shafts, keyways and splines were taken in accordance with the recommendations of [6-7]. The values of the required power were determined using the developed universal models of elements that consist of the
transmission node. The coefficient of viscous drag, the frequency of forced and natural oscillations of the system determine the dynamic factor and changes of the amplitude of load cycles.

3. Experimental research
To test the adequacy of the developed model, the simulation results were compared with the data obtained experimentally on the SPMI stand [4, 8-10]. The stand represented a united, kinematically interconnected chain of units consisting of two DC machines, an engine and a generator and two identical gearboxes of type RM-650-3-7ts installed between them, which were the object under study. In the scheme, one of them worked as the reduction unit, the other — as the speed-increase unit. The frequency of the external dynamic load supplied to the system was 0; 6.7; 9.8; 14.3 Hz with a variation load factor $K_H$ 0; 0.8; 2.1. The term “variation load factor” means the ratio of the amplitude of the power supplied to the system to the average power supplied to the generator. The losses in the generator and in the engine were considered in the model as additional and equal to relevant losses, experimentally determined during the calibration of the stand. Figure 1 shows the simulation results and experimental data.

![Figure 1. The values of power losses in the G-T-D system at 9.8 Hz of load frequency depending on the variation factor $K_H$.](image)

As seen from the comparison of curves, the modeling results agree well with the experimentally derived data. The mismatch does not exceed 6 %. The calculated value of losses under dynamic loading is somewhat higher than the experimental one. The best match of the experimental and calculated values for a particular system can be achieved by adjusting the values of the additional losses in the model that characterize manufacturing and assembling inaccuracies of a real transmission. Thus, the developed mathematical model adequately reflects the behavior of the system under the influence of dynamic load of varying intensity defined by $K_H$ values.
Figure 2. The values of losses in the G-T-D system under the working load and variation factors KH = 0 and KH = 0.8

Figure 2 shows a histogram of losses in the transmission elements with an average load level of 30 kW and various dynamic factors.

Figure 3. The values of power losses in the G-T-D system under the working load of 30 kW and variation factor KH = 0.8 for various loading frequencies

Figure 3 shows the dynamics of the power loss changes in the transmission against frequency changes in the external load.
The calculation shows that the resonance frequency of the system is equal to 4.04 Hz.

4. Conclusion
After analyzing the experimental data provided and the calculation results, it can be concluded that the developed model allows evaluating with sufficient accuracy the power loss in dynamic loading
conditions both in the transmission as a whole and in its components. The implementation of such approach makes it possible to evaluate the transmission resource at the design stage for the actual conditions of the machine, to specify the frequency and nature of scheduled preventive maintenance, to identify the number and composition of the spare parts at a mining enterprise.

If the external load is presented with a sequence diagram of load, the effect of different load cycles on the durability of the part is usually estimated by the coefficient \( a_R \), which is called the sum of relative damages and is calculated from a linear hypothesis of summation. Based on the linear hypothesis and considering the functional relationship of stresses, arising in the elements of the transmission system under the action of external forces and relative power corresponding to the magnitude of the external influence, one may write:

\[
a_p = \zeta \cdot a_f \geq 0.2 ,
\]

where \( a_f = 0.9-1.15 \) – coefficient reflecting the influence of the loading rate in the range from 1 to 1000 Hz, that is approximated by two straight portions, the inflection point 50 Hz corresponds to a coefficient which equals one; \( \zeta \) – coefficient characterizing the shape of the loading unit that is defined by expression

\[
\zeta = \sum_{i=1}^{l} \frac{P_{ai}}{P_{a_{max}}} t_i ,
\]

where \( t_i \) – ratio of the number of pulses of \( ith \) amplitude to the number of cycles in the sequence diagram block.

Using the thus calculated coefficient \( a_R \), the original life of the element may be defined as

\[
T = \frac{a_p E}{\Delta P} .
\]

In its turn, the original life of the entire transmission will be determined by the minimum of its element's original life.

References
[1] Ivanov S L, Kolomiitsev M D 1995 Memoirs Mining Institute 142 113-120
[2] Ivanov S L 1997 Mining journal 11 29-30
[3] Dokukin A V, Krasnikov Yu D, Khurgin Z Ya 1978 Statistical Machine building 239
[4] Rivin E I 1966 Machine building 204
[5] Nguyen Ph Th 1981 Memoirs Mining Institute 87 20-23
[6] Kim H, Oh K, Ko K, Kim P, Yi K 2016 Journal of Mechanical Science and Technology 30(2) 603-610
[7] Zvonarev I E, Ivanov S L, Shishlyannikov D I 2016 Procedia Engineering 150 618-625
[8] Chu J, Wang G 2015 Chemical Engineering Transactions 46 1147-1152
[9] Vinogradov A B, Gnezdov N E, Zhuravlev S V, Sibirtsev A N 2015 Russian Electrical Engineering 86(3) 139-146
[10] Vesnin A V, Sistuk V O, Bogachevskiy A O 2015 Metallurgical and Mining Industry 7(3) 272-275