Outage Bridging and Trajectory Recovery in Visible Light Positioning Using Insufficient RSS Information

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ABSTRACT Indoor positioning technology is vital for various location-aware applications while visible light positioning (VLP) is especially promising due to its ubiquitous and energy-efficient features. VLP has been widely investigated under the assumption of line of sight (LoS), yet, VLP signal blockage can happen frequently in a practical indoor environment and brings about outage problems to indoor localization/tracking services. However, this problem is usually overlooked or sidestepped in the existing works. Our work, for the first time, investigates the outage problem in a received signal strength (RSS)-based VLP system. Efficient algorithms for outage bridging and trajectory recovery are proposed by smartly fusing with insufficient RSS information. Specifically, a partial-RSS-assisted inertial navigation system (PRAINS) inspired by extended Kalman filter (EKF) is developed to bridge sporadic outage, while a bi-directional structured PRAINS (Bid-PRAINS) is developed to use both pre- and post- outage information to recover the lost trajectory information. To further deal with a more general situation when the system noise features are not pre-known and hard to be measured/estimated, a semi-parameterized RNN based learnable Kalman filter (SPR-LKF) is proposed in place of the EKF to learn the observation/transition noise features and optimize the estimation simultaneously through a recurrent neural network (RNN). Extensive tests show that the PRAINS/Bid-PRAINS has at least 62% accuracy improvement over the conventional inertial navigation system (INS)-only algorithm, while the proposed SPR-LKF/Bid-SPR-LKF can offer an even better accuracy gain of 70% even without pre-knowing the system noise feature.

INDEX TERMS Visible light positioning (VLP), recurrent neural network (RNN), received signal strength (RSS), outage bridging, extended Kalman filter (EKF).

I. INTRODUCTION

Indoor positioning (also known as indoor localization) with high accuracy and low cost is in urgent need. It would become one of the most exciting features of next-generation indoor wireless systems [1]. The potential applications include items and human tracking, robot control and navigation, and even industrial manufacturing [2]. Various technologies have been employed for indoor positioning purposes, such as ultra-wideband (UWB) [3], Wi-Fi [4], radio frequency identification (RFID) [5], ultrasound [6], Bluetooth [7], etc. Among them, visible light positioning (VLP) receives special attention due to the cheap, ubiquitous, long-lifetime and energy-efficient features of light-emitting diode (LED) infrastructure [8]. Photodiodes (PDs) and image sensors (IS) are usually used as receivers in a VLP system to detect either received signal strength (RSS), angle of arrival (AoA), or time of arrival (ToA), etc [9]–[11]. Among these techniques, RSS is a typically used VLP technique, where a PD is employed to detect RSS information from multiple reference LED sources. The position is then calculated based on trilateration methods (geometric methods), which finds the intersection of
multiple lines, circles, or balls determined by RSS according to the visible light propagation model [12]–[14].

An RSS-based positioning system operates relying on the condition of line of sight (LoS). Mathematically, at least three LoS signals should be detected for the RSS technique to uniquely determine the location of a user; otherwise, we call it VLP outage. Obviously, LoS condition could reduce the flexibility of the user and is prone to cause positioning/tracking service interruption. Existing VLP works usually sidestep the outage problem by assuming LoS condition is satisfied during an entire localization and navigation process. However, the blockage can happen randomly and very often in a practical indoor environment due to walls, furniture, and objects’ movement, resulting in sporadic VLP outages. When VLP outages, the insufficient RSS information is simply discarded while the localization/navigation services interrupt immediately until enough number of LoS links are restored. This severely influences the stability of the service and users’ experience.

To improve the situation, work [15] proposed a single LED based VLP system, which reduces outage probability. However, it requires a specially-designed circular LED with a marker and a high-resolution image sensor to support accurate geometric information extraction, which limits its applicability. Work [16] developed a tightly coupled visible-light/inertial positioning system which could support localization when an insufficient number of LEDs are captured. Yet, it requires several successive frames to reach an optimal location result which may cause unwanted time delay. A visual-inertial fusion approach based on extended Kalman filter (EKF) is reported in a recent work [17], which demonstrated to track objects accurately under LED shortage condition. Still, it is based on an image sensor. The investigation of the outage problem in RSS-based VLP is not found so far.

Besides the very limited VLP works related to outage problem, we also refer to global positioning system (GPS) works to get a more comprehensive understanding of this issue. Similar GPS outage problems can happen in tunnels, beside buildings and overpasses, or when strong electronic interference is present. In such cases, a common feasible solution is to integrate an inertial navigation system (INS) with GPS [18]–[21]. When GPS outages, a user’s path can be predicted by adding up successive position displacements from the known location, based on the INS data and the object motion model. Kalman filter (KF) is usually adopted to make the prediction. However, in these works, GPS and INS only serve as the mutual complement. In other words, the system simply switches in different states where each approach works independently at a time, therefore, no performance gain can be obtained through INS/VLP information fusion. Besides, INS has a primary problem of rapid performance degradation as the GPS outage time increases, due to sensors’ bias error drift, scale factor instability, and misalignment [22].

To solve this problem, work [23] proposed a vision-aided Inertial measurement unit (IMU) where the successive frames from a camera are used to calculate the translation between two frames, by which the drift of IMU can be controlled in an outage period. Yet, the detecting and matching of point features can be computationally complex and time-consuming. Alternatively, researchers search the help from the artificial neural network (ANN), such as radial basis function (RBF) neural network [24] and Dempster Shafer Neural Network (DSNN) algorithm [25]. RBF and DSNN are trained to predict GPS data to compensate for the INS error during GPS outage. However, they only consider the situation of either free outage or full outage, regardless of the cases of partial outage. Meanwhile, their model cannot deal with the situation where noise parameters of the model are not pre-known or hard to be measured.

After reviewing these related works, it is concluded that the outage problem in VLP systems is rarely investigated, and no proper solution exists or can be transferred/learned from GPS. To fill this gap, our idea is to fully exploit the partial RSS information. Specifically, though one or two RSS values are insufficient for uniquely determining a position, it still provides constraints which can refine the location estimation. Moreover, partial outage with one or two RSS detected happens much more often than full outage (no RSS detected) in practice. Then, instead of simply discarding insufficient RSS information, algorithms can be developed to fuse partial RSS data with the immediate INS data. In this way, real-time RSS measurements could help to relieve cumulative errors of the INS-only algorithm, while INS data could compensate for the insufficient RSS information, thus to support continuous and accurate localization services.

The main contribution and advantages of our work are summarized as follows:

1) The outage problem is investigated for the first time in an RSS-based VLP system, where two applications under two scenarios are considered, leading to four algorithms accordingly as listed in Table 1.
2) PRAINS is proposed for outage bridging by smartly fusing the insufficient RSS data and INS data when system noise features are pre-measured or can be estimated.
3) SPR-LKF with an RNN structure that could learn noise features through fast training is further proposed for outage bridging in more general cases without knowing system noise features.
4) Bi-directional Bid-PRAINS and Bid-SPR-LKF are proposed based on PRAINS and SPR-LKF, respectively, where the specially designed bi-directional

| Applications          | System Noise Feature Known | System Noise Feature Unknown |
|-----------------------|-----------------------------|-----------------------------|
| Outage Bridging       | PRAINS                      | SPR-LKF                     |
| Trajectory Recovery   | Bid-PRAINS                  | Bid-SPR-LKF                  |
structure could make use of both pre- and post-outage information for trajectory recovery.

5) The proposed four algorithms are proved to support long-term VLP outage without suffering large cumulative errors. The impact of outage duration on the algorithms performance is also investigated.

II. SYSTEM OVERVIEW

A. RSS-BASED VLP SYSTEM

Considering a typical indoor RSS-based VLP system as shown in Fig.1 (a), where common LEDs are mounted on the ceiling. Each LED is assigned with a unique frequency or an ID as a label that is then modulated on its lightwave and broadcast in free space. At the receiver side, the mixed lightwave from multiple LED sources is detected by PDs, which can be differentiated through signal processing. Then, the RSS corresponding each LED source is measured. The distance between a light source and a receiver can be estimated based on the visible light propagation model, and the user’s location can then be obtained using the trilateration method as illustrated in Fig.1 (b) by finding the intersection of multiple balls. Due to the blockage of obstacles and limited coverage of LED, a part of users may not be able to receive a sufficient number of RSS at a moment, which causes VLP outage. In our system, positioning is in no longer simple ON (free outage) or OFF (outage) states at a certain moment, but is further divided into three states as the example depicted in Fig.1 (a), including free outage with more than three RSS detected (UGV 1), full-outage with no RSS detected (UGV 4), and partial outage with one or two RSS detected (UGV 2 and 3). This is more practical because partial outage is more often than full outage due to random movement and dynamic environment. Thus, solving the partial outage problems could greatly improve the users’ experience of the system.

B. VISIBLE LIGHT PROPAGATION MODEL

The radiation of a light source follows the common Lambertian radiation pattern given by [26],

$$G_{RSS} = \frac{A}{2\pi d^2} \cos^m(\phi) T_s(\phi) g(\phi) \cos(\phi),$$

where $0 \leq \phi \leq \phi_c$, $m$ is the Lambertian order, $A$ is the physical area of the PD, $d$ is the distance between a transmitter and a receiver, $\phi$ is the angle of incidence and $\phi$ is the angle of irradiance, $\phi_c$ denotes the field of view of the PD receiver; $T_s(\phi)$ and $g(\phi)$ are the gains of an optical filter and an optical concentrator, respectively.

Further, the received light current is calculated as $I = \gamma P_t G_{RSS}$, where $\gamma$ is the responsivity of the PDs; $P_t$ denotes the transmit power of the LEDs.

III. PROBLEM FORMULATION AND PRAINS ALGORITHM

As has been seen, at least three RSS measurements are required to uniquely determine a 3D location in the conventional RSS-based trilateration. Yet, in the challenging indoor environment, sporadic blockages are inevitable due to furniture, walls, and human activities. As introduced in Section I, the reported integrated localization/navigation systems are either a simple “switching approach” without fusion, or has high computational complexity. In this section, an efficient Partial-RSS-Assisted INS (PRAINS) algorithm is proposed to fuse partial RSS and INS information. The framework
of extended Kalman filter (EKF) is adopted in the PRAINS algorithm as its sequential structure is suitable for temporal prediction types of applications (e.g., tracking). Besides, by fusing with real-time RSS measurements, the major problems of INS mentioned in Section I, including sensor drifting and cumulative error, can be relieved or eliminated.

A. INERTIAL NAVIGATION SYSTEM

IMU is widely found in mobile devices nowadays and usually used to provide pedestrian navigation. The acceleration along x-axis and y-axis at time $t$, i.e., $a' = [a'_x, a'_y]^T$ can be measured by an embedded IMU periodically with a sampling interval of $\Delta t$. Let $X^t = [p^t_x, p^t_y, v^t_x, v^t_y, 1]^T$ denote a user’s state at $t$, where $p^t_x$, $p^t_y$, $v^t_x$, and $v^t_y$ represent the user’s x-axis and y-axis positions and x-axis and y-axis velocities, respectively. Noting that, $I$ in $X^t$ is an argumented constant for the convenience of matrix manipulation later. The transition from state $X^{t-1}$ to state $X^t$ can be described by a kinetic model:

$$X^t = F \cdot X^{t-1} + B \cdot a' + n_X,$$  

(3)

where $F$ is the transition matrix given in Eq. (4), $B$ is the control matrix given by Eq. (5), and $n_X$ represents the transition noise with zero mean and covariance matrix $Q$ given in Eq. (6).

$$F = \begin{bmatrix}
1 & 0 & \Delta t & 0 & 0 \\
0 & 1 & 0 & \Delta t & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix},$$  

(4)

$$B = \begin{bmatrix}
\frac{1}{2} \Delta t^2 \\
0 \\
\Delta t \\
0 \\
0
\end{bmatrix},$$  

(5)

$$Q = \begin{bmatrix}
Q_p & 0 & 0 & 0 \\
0 & Q_v & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix},$$  

(6)

The observation matrix regarding velocity ($H_v$) is given by

$$H_v = \begin{bmatrix}
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}. $$  

(9)

RSS is modeled by the Lambertian model, as shown in Eq.(2). Let Lambertian order $m = 1$ as in [26], Eq.(2) can be further simplified as

$$RSS = \frac{C}{d^4} = C(||P_X - P_l||_2)^{-2}. $$  

(10)

where the constant $C = \frac{(m+1)ACdh^m}{2\pi}$ when $0 \leq \varphi \leq \varphi_c$; $|| \cdot ||_2$ is l2-norm; $P_X = (p_x, p_y)$ and $P_l = (p_{lx}, p_{ly})$ denote the location of user and the corresponding LED lamp. For numerical stability, we take the decibel of the RSS value during the calculation, that is,

$$f(P_X) = 10 \cdot \log_{10} (RSS) = -20 \cdot \log_{10} (C(||P_X - P_l||_2).$$  

(11)

Obviously, RSS value (in dB) is non-linearly correlated with $P_X$. In order to apply the Kalman filter framework, we specify the time $t$ for $P_X$ as $P_{X}^t$ and linearize $f(P_X)$ by conducting the first-order Taylor expansion at point $(P_{X}^{t-1})$,

$$f(P_{X}^t) \approx f(P_{X}^{t-1}) + (P_{X}^t - P_{X}^{t-1}) \cdot \frac{\partial f}{\partial P_{X}}|_{P_{X}^{t-1}} \cdot (P_{X}^t - P_{X}^{t-1})$$

$$+ \frac{1}{2} (P_{X}^t - P_{X}^{t-1}) \cdot \frac{\partial^2 f}{\partial P_{X}^2} \cdot (P_{X}^t - P_{X}^{t-1}).$$  

(12)

Submit Eq. (10) into (11) and expand it as in (12), RSS observation at time $t$ from the $i$-th LED is linearized and represented as

$$Z_{RSS,i}^t = H_{RSS,i}^t \cdot X^t, $$  

(13)

where $H_{RSS,i}^t$ is the observation matrix regarding $i$-th RSS given by

$$H_{RSS,i}^t = \begin{bmatrix}
A_i' & B_i' & 0 & 0 & C_i'
\end{bmatrix},$$  

(14)

with $A_i'$, $B_i'$ and $C_i'$ are given by

$$A_i' = \frac{40(p_{lx,i} - p_{lx,i}^{t-1})}{\ln 10 \cdot ||P_X - P_l||^2_{12}},$$  

(15)

$$B_i' = \frac{40(p_{ly,i} - p_{ly,i}^{t-1})}{\ln 10 \cdot ||P_X - P_l||^2_{12}},$$  

(16)

$$C_i' = 10 \cdot \log_{10} \left( \left(p_{lx,i}^{t-1} - p_{lx,i}^{t-1}\right) \cdot A_i - p_{ly,i}^{t-1} \cdot B_i \right).$$  

(17)

Then, Eq. (7) at time $t$ can be rewritten as

$$Z^t = H^t \cdot X^t + n_Z^t $$

$$= \begin{bmatrix}
H_v \\
H_{RSS,1} \\
\vdots \\
H_{RSS,n}
\end{bmatrix} \cdot X^t + n_Z^t $$

$$= \begin{bmatrix}
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix} \cdot \begin{bmatrix}
p_{x}^{t-1} \\
p_{y}^{t-1} \\
\vdots \\
\vdots \\
\vdots \\
v_{x}^{t-1} \\
v_{y}^{t-1}
\end{bmatrix} + n_Z^t. $$  

(18)
where $i = 1 \cdots n$ and $n$ equals to 1 or 2 during partial outage period. The transition noise $n_X$ and the observation noise $n_Z$ are drawn from Gaussian distributions $N(0, Q)$ and $N(0, R)$, respectively.

Referring to Kalman filter [30], the primary state estimation $\hat{X}^t$ with covariance matrix of $\Sigma^t_X$, measurement residual $Z^t_{\text{res}}$, and Kalman gain $K$ are calculated as the first step,

$$\hat{X}^t = FX^{t-1} + Bd^{-1}$$
$$\Sigma^t_X = F\Sigma^t_{\text{X}}F^T + Q^t,$$
$$Z^t_{\text{res}} = Z^t - H\hat{X}^t,$$
$$K = \Sigma^t_XH^T(H\Sigma^t_{\text{X}}H^T + R)^{-1}.$$  

Then, the optimal prediction of state vector $X^t$ can be updated by

$$X^t = \hat{X}^t + KZ^t_{\text{res}},$$

with a covariance of

$$\Sigma^t_X = \hat{\Sigma}^t_X - KH\hat{\Sigma}^t_X.$$  

The workflow is illustrated in Fig. 2. In this way, partial RSS information and realtime INS information are effectively fused by the proposed PRAINS algorithm. The specific workflow is summarized in Algorithm 1.

![FIGURE 2. Schematic diagram of Kalman filter workflow.](image)

**Algorithm 1** PRAINS Algorithm

**Initialization:** $F, B$

while $t < T$ do

1. Calculate $H_{\text{RSS},i}^t$ according to Eqs.(14-17).
2. Calculate $Z^t_{\text{res}}$ according to Eq.(18).
3. Update $\hat{X}^t$ and $\Sigma^t_X$ according to Eqs.(19-24).

![FIGURE 3. Schematic diagram of Bid-PRAINS algorithm for trajectory completion.](image)

**Algorithm 2** Bid-PRAINS Algorithm

1. Calculate $X^t_{\text{fw}}, \Sigma^t_{X_{\text{fw}}}$ and $X^t_{\text{bw}}, \Sigma^t_{X_{\text{bw}}}$ respectively according to Algorithm 1.
2. Fuse them to get optimal estimation $\hat{X}^t$ and $\Sigma^t_X$ according to Eq. (25) and (26).
3. Repeat Step.1 and 2 until all missing trajectory points are recovered.

C. BI-DIRECTIONAL PRAINS FOR TRAJECTORY RECONSTRUCTION

Apart from the interruption of real-time localization service, sporadic outages also bring about trajectory missing problem. This problem is commonly found for household and industrial robots or unmanned ground vehicles, where the trajectory requires to be recorded under some circumstances. In an indoor VLP system, the missing trajectory is usually short-term which makes it possible to be recovered with high accuracy. To address this issue, a Bid-PRAINS algorithm is proposed based on PRAINS algorithm, to recover the lost trajectory using corresponding historical INS and RSS data.

As the name suggests, Bid-PRAINS consists of a structure of two-way PRAINS (forward and backward paths) as illustrated in Fig. 3. The specially designed bi-directional structure of the Bid-PRAINS makes it possible to leverage both the pre-outage and post-outage information. Importantly, the final estimated state is not obtained by simply taking the average of the estimators from two directions, but by calculating the possibility density function (PDF) of the fused estimators based on the Bayesian theory. Specifically, the PDF of the optimal estimation can be obtained by multiplying two PDFs of two Gaussian distributed estimators of forward and backward paths, i.e., $X^t_{\text{fw}}$ with the covariance of $\Sigma^t_{X_{\text{fw}}}$ and $X^t_{\text{bw}}$ with $\Sigma^t_{X_{\text{bw}}}$. The optimal estimation with mean and covariance being $X^t$ and $\Sigma^t_X$ is given by

$$X^t = (X^t_{\text{fw}}\Sigma^t_{X_{\text{fw}}} + X^t_{\text{bw}}\Sigma^t_{X_{\text{bw}}})(\Sigma^t_{X_{\text{fw}}} + \Sigma^t_{X_{\text{bw}}})^{-1},$$
$$\Sigma^t_X = (\Sigma^t_{X_{\text{fw}}}\Sigma^t_{X_{\text{bw}}})(\Sigma^t_{X_{\text{fw}}} + \Sigma^t_{X_{\text{bw}}})^{-1}.$$  

![FIGURE 3. Schematic diagram of Bid-PRAINS algorithm for trajectory completion.](image)

IV. SEMI-PARAMETERIZED RNN BASED LEARNABLE KALMAN FILTER

As can been seen, PRAINS and Bid-PRAINS algorithms require the knowledge of system noise, thus they may not work well when transition noise covariance $Q$ and observation noise covariance $R$ are unknown or hard to be measured accurately. To address this issue, we further propose a semi-parameterized learnable Kalman filter (SPR-LKF) which exploits the inherent recurrent structure of a Kalman filter. Unlike the common VLP works which applied fully-parameterized ANN where the channel
gain, various noise, system errors are all modeled by ANN indiscriminately, we use a semi-parameterized RNN model. For one thing, fully-parameterized ANNs are widely used in the fields of computer vision and natural language processing (NLP) because the studied issues in these fields are usually not well-modeled, while our VLP system involves many well-investigated models which can be utilized. For another, a semi-parameterized RNN structure has two major advantages in terms of performance: (1) A fully-parameterized model is prone to be overfitted to noise when the training data is not sufficient, while the semi-parameterized ANN can alleviate the noise influence to the model, and has better generalization; (2) For a fully-parameterized ANN, input parameters may have different units and their dynamic ranges vary greatly, leading to the difficulty for normalization and training process. In contrast, normalization is not required in our semi-parameterized ANN.

**A. STRUCTURE OF SPR-LKF**

The proposed SPR-LKF has two phases, i.e., training and estimation. In the training phase, SPR-LKF is formulated by semi-parameterizing the introduced PRAINS model which involves the kinetic model and the RSS model. Specifically, the covariance matrices of transition and observation noises, i.e., \( Q \) and \( R \), are treated as learnable parameters and can be updated by error backpropagation \[31\]. The whole computing graph is depicted as Fig. 4. Compared with the workflow of PRAINS given in Fig. 2, a recurrent structure is formed by adding a backpropagation path of \( Q \) and \( R \) as highlighted, through which \( Q \) and \( R \) can be learned and optimized at each step.

**FIGURE 4. Schematic diagram of SPR-LKF workflow.**

The loss function in the training phase is defined as the summation of the mean square error (MSE) between the predicted state and the ground-true state at each time step \( t \),

\[
E = \frac{1}{N} \sum_{t=0}^{N-1} ||X^t - \hat{X}^t||_2^2,
\]

where \( N \) is the total number of time steps of a path and \( X^t \) is the ground-true state at \( t \)-th step.

When system noise features \( Q \) and \( R \) are learned, SPR-LKF turns into the estimation phase. The forward path of SPR-LKF illustrated in Fig. 4 can make estimation in real-time. Similar to the traditional KF, LKF firstly takes the state vector \( X^{t-1} \) and the corresponding covariance matrix \( \Sigma_X^{t-1} \) as inputs and calculates \( \hat{X}^t \) and \( \Sigma_X^t \) at current time step based on the motion model with \( F, B, a_t \) and the transition noise featured by \( Q^t \). Then, the estimation (\( \hat{X}^t \) and \( \Sigma_X^t \)) is further refined by combining with observations (featured by \( R^t \) and \( Z^t \)), after which the final estimation is obtained.

Besides, referring to the Bid-PRAINS algorithm, a bi-directional SPR-LKF, namely, Bid-SPR-LKF, is developed specially for the purpose of trajectory recovery without knowing the system noise features.

**B. POSITIVE SEMI-DEFINITE CONSTRAINTS OF PARAMETERS**

Yet, during the training process of the LKF, failures are reported in the backpropagation process, because the neural net cannot ensure the updated covariance matrices to be positive semi-definite. A special technique called “particle approximation” is proposed to guarantee the positive semi-definite property of the obtained the covariance matrices during the learning process.

Specifically, two sets of particles are generated to approximate the statistics of transition and observation noises, respectively. Each set has \( S_Q \) and \( S_R \) particles, whose values are recorded in two matrices denoted as \( \Phi_Q \) with dimension of \( S_Q \times 2 \) and \( \Phi_R \) of with dimension of \( S_R \times (2 + n) \). \( n \) is the number of detected RSS with each row representing a particle. These particles can be seen as samples drawn from the estimated noise distribution. Noting that, here we use 1000 particles for distribution approximation, i.e., \( S_Q = S_R = 1000 \).

According to the definition, \( Q \) and \( R \) can be represented as

\[
Q = \frac{1}{N_Q} (\Phi_Q - m_{\Phi_Q})^T \cdot (\Phi_Q - m_{\Phi_Q}), \quad (28)
\]

\[
R = \frac{1}{N_R} (\Phi_R - m_{\Phi_R})^T \cdot (\Phi_R - m_{\Phi_R}), \quad (29)
\]

where

\[
m_{\Phi_Q} = \frac{1}{S_Q} \sum_{i=0}^{S_Q-1} \Phi_Q(i), \quad (30)
\]

\[
m_{\Phi_R} = \frac{1}{S_R} \sum_{i=0}^{S_R-1} \Phi_R(i), \quad (31)
\]

are the mean of particle matrices \( \Phi_Q \) and \( \Phi_R \), respectively. As can be seen, instead of updating the exact \( Q \) and \( R \) directly at each step, we could update the particles instead of two particle matrices. Then, the covariance matrices obtained based on these updated particles could naturally satisfy the constraints of positive semi-definite property. The SPR-LKF algorithm and Bid-SPR-LKF algorithm with “particle approximation” are summarized in Algorithm 3 and Algorithm 4, respectively.
Algorithm 3 SPR-LKF Algorithm

Initialization: \( F, B, \Phi_D, \Phi_R, LOSS, Phase \)

\[
\text{while } \text{Phase} = \text{training}, \text{loss} < \text{LOSS} \text{ do }
\]

1. Calculate \( H_{\text{RSS},i} \) according to Eqs.(14-17).
2. Calculate \( Z_t^i \) according to Eq.(18).
3. Update \( \hat{X}_t \) and \( \Sigma_t^X \) according to Eqs.(19-24).
4. Calculate loss according to Eq.(27).
5. Error backpropagation and update trainable parameters according to [31], [32].
6. Update \( Q \) and \( R \) according to Eqs.(28-31).

\[
\text{while } \text{Phase} = \text{estimation} \text{ do }
\]

1. Calculate \( H_{\text{RSS},i} \) according to Eqs.(14-17).
2. Calculate \( Z_t^i \) according to Eq.(18).
3. Update \( \hat{X}_t \) and \( \Sigma_t^X \) according to Eqs.(19-24).

Algorithm 4 Bid-SPR-LKF Algorithm

1. Calculate \( X_{\text{fw}}^t, \Sigma_{X_{\text{fw}}}^t \) and \( X_{\text{bw}}^t, \Sigma_{X_{\text{bw}}}^t \) respectively according to Algorithm 3.
2. Fuse them to get optimal estimation \( X_t^i \) and \( \Sigma_t^X \) according to Eq. (25-26)
3. Repeat Step.1 and 2 until all missing trajectory points are recovered.

V. SIMULATION RESULTS AND ANALYSIS

Considering an indoor square area of 100 × 100 \( m^2 \) where the ceiling height is 4m and LEDs are deployed every 2m on the ceiling. The PD receivers on the UGV robots face straight up to the ceiling as illustrated in Fig.1. The normal axes of a transmitter and a receiver are parallel, leading to \( \phi = \psi \) and \( \cos(\phi) = \cos(\psi) = h/d \). When the optical parameters take typical values as in [26], the constant \( C \) in Eq.(10) equals to 0.7443. Noting that, the rotation angle of the PD receiver doesn’t affect the effectiveness of our algorithm. It only leads to a different value of \( C \).

The ground-true paths are generated based on the traditional kinetic model with transition noise, as given in Eq.(3). Specifically, the robot is assumed to start from the origin of the testing area with an initial speed of \( (v^0_x, v^0_y) = (0 \, m/s, 0.2 \, m/s) \). A random acceleration \( a_x, a_y \in (-0.3 \, m/s^2, 0.3 \, m/s^2) \) is applied at each sampling moment to simulate the random resistance and internal/external forces acted on the object. Other parameters are given in Table 2.

During the movement of the user, three states, free outage, partial outage, and full outage are simulated.

TABLE 2. Simulation settings.

| Parameters                      | Settings   |
|---------------------------------|------------|
| Indoor area                     | 100 × 100m²|
| LED lamp installing interval    | 2m         |
| Field of View (FoV)             | 2π/3       |
| Radius of sensing               | 2m         |
| Trajectory length per path      | 50 steps   |
| Sampling interval               | 1s         |

A. PRAINS ALGORITHM EVALUATION

Firstly, the performance of the proposed PRAINS and Bid-PRAINS algorithms is evaluated by 10000 randomly generated paths, for both VLP outage bridging and trajectory reconstruction. Obviously, the full outage and the INS-only algorithm should have the same performance in our system, because they either do not receive additional RSS information or discard the insufficient RSS information.

Fig. 5 shows a sample result under one out of 10000 randomly generated paths with a 30-step outage, which compares the positioning results before and after applying the proposed PRAINS algorithm. As shown, the true trajectory is depicted in blue while the positioning results in the free-outage period and outage period (including full or partial outage) are marked in red and yellow, respectively. Compared with the conventional method based on INS-only algorithms (Fig. 5(b)), the proposed PRAINS algorithm (Fig. 5(c) and (d)) more accurately localizes the user.
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During partial outage period. Especially when 2 RSS is detected, PRAINS could even achieve comparable results as in the free-outage scenario.

Then, Bid-PRAINS can be applied to recover the missing trajectory information using corresponding historical INS and RSS data. As the baseline, the positioning result of the conventional uni-directional INS (Uni-INS) algorithm is shown as Fig. 6 (a), which has an obvious offset from the true trajectory. Firstly, a bi-directional structure is introduced where the bi-directional INS (Bid-INS) largely improves the recovery accuracy as given in 6 (b), which proves the effectiveness of the bi-directional structure. Bid-PRAINS further enhance the accuracy of trajectory reconstruction, compared with Bid-INS, as shown in Fig. 6 (c) and (d). As expected, 2 RSS-assisted Bid-PRAINS is more accurate than Bid-PRAINS with 1 RSS.

Meanwhile, in a practical VLP system, outages can happen sporadically with varying lengths of the period. As one can expect, the outage period also affects the algorithm performance. Thus, the impact of the outage duration on the performance improvement over the conventional INS-only system is investigated. Since the randomly generated testing paths vary greatly, resulting in a large difference in absolute improvement measured in meters, here, we calculated the improvement percentage per path as a more reasonable evaluation metric. Specifically, the improvement percentage for PRAINS is defined as \(\frac{E_{\text{INS}} - E_{\text{PRAINS}}}{E_{\text{PRAINS}}}\), where \(E_{\text{INS}}\) and \(E_{\text{PRAINS}}\) represent the positioning error using INS-only algorithm and PRAINS algorithm, respectively. The improvement percentage for other proposed algorithms is defined similarly. Fig.7 depicts the mean accuracy under 10000 random paths, which also proves the universal validity regardless of outage period. As shown, when outage duration ranges from 5 to 40 steps, the performance improvement (in percentage) of PRAINS increases sharply from 62% to 92.5% with 1 RSS, and from 85% to 98% when 2 RSS is received. Meanwhile, for all the tests, the maximum error of PRAINS is smaller than that of the conventional INS-only algorithm.

Similar results are observed in Bid-PRAINS for trajectory recovery. Accuracy improvement percentage of Bid-PRAINS increases from 78% to 93% and from 90% to 98% for 1 RSS and 2 RSS, respectively. The reason is that the conventional INS-only system makes predictions relying on the previous state only, whose chain effect causes error cumulation along with the increasing outage time if no real-time measurements are involved for correction. In such a case, involving partial real-time measurements, no matter 1 or 2 RSS, can greatly relieve cumulative error, especially in a long outage period. 2RSS-assisted PRAINS can even achieve almost the same performance as that of a free-outage scenario.

B. SPR-LKF ALGORITHM EVALUATION

In this section, SPR-LKF is validated and evaluated under the condition that system noise features are not known. 20000 paths under the same setting as above are generated for training and testing. Our model is pre-trained on 10000 randomly generated paths with outage steps. It is optimized by...
Adam optimizer [32] with a momentum of 0.9. The batch size is set to be 32. In the pre-training process, the learning rate is set to be $10^{-3}$. It is worth mentioning that the training loss converges quickly to near-optimum within around 2000 iterations by using Adam optimizer, which is much faster than the common stochastic gradient descent (SGD) optimizer [33]. After noise features are captured through training process, the proposed SPR-LKF could make estimation as PRAINS in real time. Both pre-training and testing are implemented with Pytorch r1.0 on Dell Precision T7500 workstation with Intel Xeon 5600 processor, 40GB memory and NVIDIA GTX 1080 GPU.

As in Section V A, we examine the performance of the well-trained SPR-LKF under free-outage, full-outage (INS only), 1 RSS and 2 RSS-assisted scenarios. The results are recorded and analyzed. Firstly, Fig. 8 compares their positioning performance with a 30-step outage period.

The corresponding cumulative error during an outage for 4 testing settings, i.e., free-outage, full-outage (INS only), SPR-LKF (1 RSS) and SPR-LKF (2 RSS), are 1.06, 109.059, 4.626 and 4.214 respectively. Obviously, SPR-LKF greatly enhances the positioning accuracy during VLP outage period.

Meanwhile, as expected, the well-trained SPR-LKF is found to have better performance than the PRAINS, which shows the advantage of data-driven probabilistic approaches over the conventional predefined probabilistic model. SPR-LKF algorithm could learn and optimize the parameters through both outage and free outage data to make the optimal estimation.

C. BID-SPR-LKF ALGORITHM EVALUATION

Similarly, SPR-LKF can be implemented as a bi-directional filter with the same structure as Bid-PRAINS for trajectory reconstruction. Fig. 9 gives an example of a reconstructed
under the condition when no system noise features are pre-known or measured. The end-to-end RNN structure enables LKF to learn the system features efficiently in a short period. The proposed algorithms demonstrate effectiveness through extensive tests with randomly generated paths. Significant improvements can be observed compared with the conventional INS-only algorithm under different settings.

VI. CONCLUSION

In the existing RSS-based VLP systems, a sufficient number of LoS links are commonly assumed during the entire period of localization, which is unlikely to be satisfied all the time in a practical dynamic indoor environment, leading to interruption of localization. In this work, we for the first time investigate the approach of providing continuous localization services during VLP outage by smartly fusing the partial RSS with INS information. Both the issues of outage bridging and trajectory recovery are considered. Specifically, PRAINS are developed for position estimation during outage bridging and trajectory recovery are considered. Specif-

The impact of outage duration on the accuracy improvement is also investigated. Compared with PRAINS in Fig. 7, SPR-LKF in Fig. 10 offers an averagely higher enhancement under the same condition. For example, SPR-LKF with 1 RSS provides 70% to 97% accuracy boost, while the improvement in SPR-LKF is more accurate than SPR-LKF (labeled in green dash line).

The proposed algorithms demonstrate effectiveness through extensive tests with randomly generated paths. Significant improvements can be observed compared with the conventional INS-only algorithm under different settings.

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