Box-particle Cardinality Balanced Multitarget Multi-Bernoulli Filter for Multipath Multitarget Tracking in OTHR

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Abstract. This paper proposes a novel box-particle multipath cardinality balanced multitarget multi-Bernoulli (BP- MPCBMeMBer) filter for the problem of multitarget tracking in Over-the-horizon radar (OTHR). The proposed algorithm combines the multipath cardinality balanced multitarget multi-Bernoulli (MPCBMeMBer) filter with interval analysis to solve the problems of high computational complexity and nonlinear measurement models. First, establishment of the OTHR measurement model based on interval analysis, and then the box-particle filter is used to derive the BP-MPCBMeMBer based on the original MPCBMeMBer filter. Eventually, the simulation results are presented to verify the effectiveness of the BP-MPCBMeMBer filter.

Keywords: Box-particle filter, CBMeMBer filter, OTHR, Tracking.

1. Introduction

Over-the-horizon Radar (OTHR) system exploits the ionosphere reflections the high frequency electromagnetic waves to track targets. Since a target can produce multiple measurements from different ionosphere, one of the key problems in OTHR tracking system is the effect of multipath propagation. Conventional tracking algorithms designed for a single measurement per target. If the tracking algorithms can use all the measurements, the performance of OTHR tracking system will be greatly improved. Some traditional algorithms have been designed to solve the multipath problem such as multipath probabilistic data association (MPDA) [1], the multipath viterbi data association (MVDA) [2], Multiple Detection Multiple Hypothesis Tracker (MD-MHT) [3] and the Multiple Detection Joint Probabilistic Data Association (MD-JPDA) Filter [4]. Since the traditional algorithms require complex data association and high computational complexity, the tracking algorithms based on the random finite set (RFS) have attracted extensive attention recently. Many RFS tracking algorithms have been proposed, such as the Bernoulli filter [5], the probability hypothesis density (PHD) filter [6,7], the cardinality balanced multitarget multi-Bernoulli (CBMeMBer) filter [8,9], and the labeled multi-Bernoulli (LMB) filter [10-13].

Various RFS tracking approaches have been successfully used in OTHR to address the multipath target tracking problem. For the problem of multipath single target tracking, multipath Bernoulli filter based on the standard Bernoulli filter is proposed in [14]. In order to deal with the problem of multipath multitarget tracking problem, the first author has proposed multipath PHD (MPPHD) filter and multipath CBMeMBer (MPCBMeMBer) filter [15,16] based on the standard PHD filter and standard CBMeMBer filter. Due to the measurement models are nonlinear in OTHR tracking system, [16] has applied the particle filter to deal with the nonlinear problem. However, the particle filter implement has the problem of high computational complexity and hard to meet the real-time requirements. Recently, box-particle filter has been proposed in [17], which utilizes interval analysis theory to model measurements as intervals instead of the point observations. Then, [18,19] showed that the box-particle filter can reach the similar tracking performance as the traditional particle filter with less computational complexity. Therefore, this paper focuses on the problem of the multipath multitarget tracking and proposes a novel BP-MPCBMeMBer filter, which combines the advantages of MPCBMeMBer with interval analysis to solve the problems of high computational complexity and nonlinear measurement models in OTHR tracking system.
2. The Multipath CBMeMBer Filter for OTHR

2.1 Dynamic Model and Measurement Model in OTHR

Figure 1 shows the geometry of the radar sensors and target in OTHR tracking system. At time $k$, target state is defined by $x_k = [p(k), b(k), \dot{p}(k), \dot{b}(k)]^T$, where $p(k) = p_1(k), b(k)$, $\dot{p}(k)$ and $\dot{b}(k)$ denote the ground range, bearing, range rate and bearing rate respectively.

![Transmitter-receiver model in OTHR](image)

**Fig 1.** Transmitter-receiver model in OTHR

Since the distance between the radar sensors and the target is large, we can presume a linear and discrete-time state equation with the form

$$x_k = Fx_{k-1} + u_{k-1}$$

where $u_{k-1}$ is a white Gaussian noise and the matrix $F$ is given by

$$F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $T$ is the sampling interval.

As shown in Fig.1, the OTHR system exploits the ionosphere reflections of the high frequency electromagnetic waves to track targets. For simplicity we generally assume that there are two ionospheres (E and F). Therefore, there are four possible propagation modes. It means that one target can produce multiple measurements from different propagation paths at the same time.

The OTHR measurements include slant range $R_g = r_1 + r_2$, Doppler $R_r$ and azimuth $Az = \pi/2 - \theta$ of the form $z_k = [R_g(k), R_r(k), Az(k)]^T, z_k \in \mathbb{R}$, where $\mathbb{R}$ is space of slant coordinates. Hence the measurement models of OTHR system can be expressed as [16]

$$z_k = \begin{cases} h_1(x_k) + \omega_{i,j} & \text{if mode EE} \\ h_2(x_k) + \omega_{i,j} & \text{if mode EF} \\ h_3(x_k) + \omega_{i,j} & \text{if mode FF} \\ \text{clutter} & \text{otherwise} \end{cases}$$

where $\omega_{i,j}$ is white Gaussian noise, $h_i(\cdot)$ is the measurement function for $i$th propagation mode

$$h_i(x) = \begin{cases} \sqrt{p/2} + h_i^2 + \sqrt{(\rho/2)^2 - d \sin(b)/2 + h_i^2} \\ \sqrt{p/2} + h_i^2 - \sqrt{(\rho/2)^2 - d \sin(b)/2 + h_i^2} \\ \sin^{-1}(\rho \sin(b)/2 \sqrt{(\rho/2)^2 + h_i^2}) \end{cases}$$

$$\text{if mode EE}$$

$$\text{if mode EF}$$

$$\text{if mode FF}$$

$$\text{otherwise}$$

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2.2 The RFS Measurement Model in OTHR

In the traditional tracking algorithms, it is presumed that a target produces at most one measurement at the same time. In OTHR system, multiple measurements can be generated by one target due to multipath effect. Therefore, the RFS measurement model of MPCBMeMBer filter is different from the standard CBMeMBer filter. It is necessary to give the RFS measurement model. In OTHR system, a finite set measurement can be denoted as \( Z_k = \{ z_{k,1}, z_{k,2}, \ldots, z_{k,N_k} \} \), where \( z_{k,i} \) are the received measurements at time \( k \). As shown in Fig.1 the OTHR system uses multiple ionospheric reflection signals to track targets. Thus the OTHR RFS measurements can be given by

\[
Z_k = \Theta_{k,1}(x_k) \cup \Theta_{k,2}(x_k) \cup \Theta_{k,3}(x_k) \cup \Theta_{k,4}(x_k) \cup \Gamma_k
\]

where \( \Theta_{k,l}(x_k) \), \( l=1, \ldots, 4 \) is the measurements from the \( l \)th propagation path, \( \Gamma_k \) is the RFS of clutter. The following algorithms are based on this OTHR RFS measurement model.

2.3 The MPCBMeMBer Filter

This subsection will introduce the multi-Bernoulli RFS, which is relevant for this paper.

A multi-Bernoulli RFS \( X \) can be completely described by the independent Bernoulli RFSs parameter set \( \{ (r^{(i)}, p^{(i)}) \}_{i=1}^M \) , where \( r^{(i)} \in (0,1) \) and \( p^{(i)} \) denote the existence probability and space probability density. The mean cardinality of a multi-Bernoulli RFS is \( \sum_{i=1}^M r^{(i)} \), and the multitarget probability density \( \pi \) is

\[
\pi(\{x_1, \ldots, x_N\}) = \prod_{i=1}^M (1-r^{(i)}) \sum_{i_1=1}^{M} \prod_{i=2}^M r^{(i_1)} p^{(i_1)}(x_i) / 1-r^{(i_1)}
\]

The multitarget tracking can be seen as a Bayesian filter process, which propagates the multitarget posterior density. The MPCBMeMBer filter is one of the Bayesian filter algorithms, which approximates the multipath multitarget posterior density by parameters \( \pi=\{ (r^{(i)}, p^{(i)}) \}_{i=1}^M \). As with standard Bayesian filtering, the MPCBMeMBer filter also needs two steps: the prediction and update. Since the prediction step is identical to the classic CBMeMBer filter, we only introduce the update step.

If the multitarget predicted density is a multi-Bernoulli RFS with the form \( \pi_{k-1}=\{ (r^{(i)}_{k-1}, p^{(i)}_{k-1}) \}_{i=1}^{M_{k-1}} \) at time \( k \), then the multipath multitarget posterior density can be approximated by

\[
\pi_k = \left\{ (r^{(i)}_{k,k}, p^{(i)}_{k,k}) \right\}_{i=1}^{M_{k-1}} \cup \left\{ \left( \left\{ r_{i,k}^{(i)}(W), p_{i,k}^{(i)}(z;W) \right\}_{k_{i}} \right) \right\}_{k_{i}=1}^{K_{i}}
\]

where

\[
r_{i,k}^{(i)}(z) = \frac{r^{(i)}_{i,k-1} p^{(i)}_{i,k-1} |q_{i,k}^4|}{1 - r^{(i)}_{i,k-1} + r^{(i)}_{i,k-1} p^{(i)}_{i,k-1} |q_{i,k}^4|}
\]

\[
p^{(i)}_{i,k} = p^{(i)}_{i,k-1}(x) \frac{|q_{i,k}^4|}{p^{(i)}_{i,k-1} |q_{i,k}^4|}
\]

\[
r_{i,k}^{(i)}(z) = \frac{\omega_{i,k}}{d_{i,k}^W} \sum_{i=1}^{M_{k-1}} \left[ r^{(i)}_{i,k-1} (1 - r^{(i)}_{i,k-1}) p^{(i)}_{i,k-1} |q_{i,k}^4| \right]
\]

\[
p^{(i)}_{i,k} = \frac{\sum_{i=1}^{M_{k-1}} r^{(i)}_{i,k-1} p^{(i)}_{i,k-1} |q_{i,k}^4|}{\sum_{i=1}^{M_{k-1}} (1 - r^{(i)}_{i,k-1}) p^{(i)}_{i,k-1} |q_{i,k}^4|}
\]
and where
\[ q_{D,k} = 1 - p_{D,k} \]  \hspace{1cm} (12)
\[ \omega_w = \frac{\prod_{w=1}^{w=c_k} d_w}{\sum_{w'=1}^{w=c_k} \prod_{w'=1}^{w=c_k} d_w} \]  \hspace{1cm} (13)
\[ d_w = \delta_{|x|} + \sum_{j=1}^{M_{x,k-1}} \frac{r_{k-1,j}^{(0)} p_{k-1}^{(i)}[\psi_w]}{1 - r_{k-1,j}^{(0)} + r_{k-1,j}^{(0)} q_{D,k}^2} \]  \hspace{1cm} (14)

3. The Box-particle MPCBMeMBer Filter

Since the problem of non-linear measurements problem in OTHR, particle MPCBMeMBer (P-MPCBMeMBer) filter has proposed in [16]. However, the P-MPCBMeMBer filter has the disadvantage of high computational complexity. This subsection will propose a novel BP-MPCBMeMBer filter, which combines the advantages of BP-MPCBMeMBer filter and interval analysis to solve the problem of high computational complexity and nonlinear measurement models.

The interval measurements vector denoted by \( [Z_k] = \{[z_{k,1}], [z_{k,2}], \ldots [z_{k,N_k}] \} \), the BP-MPCBMeMBer filter is described as follows. Note that since the prediction step is the same as the classic filter, we only give the update step.

If at time \( k \), the multitarget predicted density \( \pi_k = \{ (L_k, \pi_{k,1}^{(i)}) \}_{i=1}^{M_{x,k-1}} \) is given, where the probability density \( p_{k-1}^{(i)} \) is composed of a set of box particles \( \{ \omega_k^{(i)} [x_{k-1}^{(i)}] \}_{i=1}^{M_{x,k-1}} \), i.e.,

\[ p_{k-1}^{(i)} = \sum_{j=1}^{M_{x,k-1}} \omega_k^{(i)} U_{[x_{k-1}^{(i)}]}(x) \]  \hspace{1cm} (15)

where \( [x_{k-1}^{(i)}] \) is a box particle, \( U_{[x_{k-1}^{(i)}]}(x) \) is the uniform PDF.

Then, the multipath multitarget posterior density \( \pi_k = \{ (L_k, \pi_{k,2}^{(i)}) \}_{i=1}^{M_{x,k-1}} \cup \{ \left( \bigcup_{w=c_k} \{ (w, \pi_{k,1}, \pi_{k,2}^{(i)}) \} \right) \}_{w=c_k} \) can be computed as follow:

\[ r_{L,k}^{(i)} = \frac{r_{L,k}^{(i)} q_{D,k}^2}{1 - r_{k-1,j}^{(0)} + r_{k-1,j}^{(0)} q_{D,k}^2} \]  \hspace{1cm} (16)
\[ p_{L,k}^{(i)} = p_{k-1}^{(i)} ([x_{k-1}^{(i)}]) \]  \hspace{1cm} (17)

\[ r_{U,k}^{(i)}(y) = \frac{\omega_k^{(i)} \sum_{j=1}^{M_{x,k-1}} r_{k-1,j}^{(0)} (1 - r_{k-1,j}^{(0)}) \sum_{j=1}^{M_{x,k-1}} \omega_k^{(i)} U_{[x_{k-1}^{(i)}]}([x_{k-1}^{(i)}])}{(1 - r_{k-1,j}^{(0)} + r_{k-1,j}^{(0)} q_{D,k}^2)^2} \]  \hspace{1cm} (18)
\[ p_{U,k}(x,y) = \frac{\sum_{i=1}^{M_{x,k-1}} \sum_{j=1}^{M_{x,k-1}} r_{k-1,j}^{(0)} \omega_k^{(i)} U_{[x_{k-1}^{(i)}]}([x_{k-1}^{(i)}]) / (1 - r_{k-1,j}^{(0)})}{\sum_{i=1}^{M_{x,k-1}} \sum_{j=1}^{M_{x,k-1}} \omega_k^{(i)} U_{[x_{k-1}^{(i)}]}([x_{k-1}^{(i)}]) / (1 - r_{k-1,j}^{(0)})} \]  \hspace{1cm} (19)
\[
d_w = \delta_{ij} + \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_j^{(i)} \psi_w (x_{k+1}^{(i)}, x_k^{(j)}) / (1 - r_{i,j}^{(i,j)} g_{D,k})
\]

where \( \psi_w \) can be seen formula (21).

\[
\ell_{x,y}(x) = \frac{g_{x,y}([z] | x_{k+1}^{(i)})}{\lambda e([z])}
\]

where \( g_{x,y}([z] | x_{k+1}^{(i)}) \) is the likelihood of \( i \)th propagation path.

\[
p_{x,y}([z] | [x]) \approx \int_{[z]} p_{x}(z - h_{1}([x])) dz = \int_{[z]} L_{h_{1}([z])}([z] \cap h_{1}([x]) + [\varepsilon]) / |[\varepsilon]|
\]

\[
\psi_w = \begin{cases} 
q_{D,k}^3 P_{D,k}^{2} \sum_{j=1}^{M_x} \ell_{z,i} & \text{if } W = \{z_i\} \\
q_{D,k}^2 P_{D,k} \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \ell_{z,i} \ell_{z,j} & \text{if } W = \{z_i, z_j\} \\
p_{D,k} \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \sum_{k=1}^{M_z} \ell_{z,i} \ell_{z,j} \ell_{z,k} & \text{if } W = \{z_i, z_j, z_k\} \\
p_{D,k} \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \sum_{k=1}^{M_z} \sum_{l=1}^{M_y} \ell_{z,i} \ell_{z,j} \ell_{z,k} \ell_{z,l} & \text{if } W = \{z_i, z_j, z_k, z_l\}
\end{cases}
\]

4. Numerical Simulations

In this subsection, the proposed BP-CBMeMBer filter’s performance is compared with the P-CBMeMBer filter through one Monte Carlo simulation scenario in OTHR multitarget tracking system, and each implementation is demonstrated using the same simulation scheme that described in the following.

In the tracking scheme, it is assumed that three targets appear and disappear at different times in OTHR tracking system. The total simulation time is 800 seconds with sampling period \( T = 20s \). Target 1 and 2 appear throughout the experiment with initial state \( x_i = (1100km, 0.10472rad, 0.15km/s, 8.72665e-05rad/s) \) and \( x_2 = (1170km, 0.11472rad, -0.14km/s, 7.72665e-05rad/s) \) respectively, and target 3 appears at scan \( t = 160s \) and disappears at \( k = 480s \) with the initial states \( x_3 = (1170km, 0.05km/s, 0.15701rad, -8.72665e-05rad/s) \). Clutter is modeled as a Poisson RFS with the intensity function \( \kappa_z(z) = \lambda V_u(z) \), we suppose that there are 5 clutter measurements each scan. And four possible propagation modes have the same target survive probability ( \( p_{s,k} = 0.98 \) ) and detection probability ( \( p_{d,k} = 0.6 \) ). It models the interval measurements with an interval length \( \Delta = [\Delta \rho, \Delta b, \Delta \rho, \Delta b]' \), where \( \Delta \rho = 60m \), \( \Delta b = 0.03^o \), \( \Delta \rho = 0.01m/s \) and \( \Delta b = 0.001^o / s \) are the lengths of intervals in the ground range, bearing, range rate and bearing rate respectively. We use the optimal subpattern assignment (OSPA) metric [20] to evaluate the tracking performance. Other simulation parameters are same as [16]. The true target trajectory is shown in Fig. 2.
Fig 2. The true target trajectory.

Fig 3. Average of OSPA distance for the P-MPCBMeMBer filter and BP-MPCBMeMBer filter

Fig 4. Average of estimated target number for the P-MPCBMeMBer filter and BP-MPCBMeMBer filter

Table 1. Comparison of the number of particles and runtime

| Algorithm        | Number of particles | Runtime (sec) |
|------------------|---------------------|---------------|
| P-MPCBMeMBer     | 3000                | 378.33        |
| BP-MPCBMeMBer    | 50                  | 142.62        |

Fig.3 and Fig.4 show the average of the OSPA metric and estimated target number comparison between the proposed MP-CBMeMBer filter and P-CBMeMBer filter respectively. It can be seen that the performance of the MP-CBMeMBer filter performs similarly to the P-CBMeMBer filter, it means that both filters can track the multipath multitarget effectively in OTHR system.

Table 1 shows computational time and particles number for the proposed BP-CBMeMBer filter and P-CBMeMBer filter. It demonstrates the P-CBMeMBer filter requires 3000 traditional particles, corresponds to the average computational time over 378s. However, the proposed BP-CBMeMBer filter just requires 50 box particles, corresponds to the average computational time 142s. It means that the proposed BP-CBMeMBer filter can reach the similar performance as the P-CBMeMBer filter with less computational time, and can deal with nonlinear problems of multipath multitarget effectively in OTHR system.
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