Free Boson Representation of $DY_h(sl_2)_k$ and the Deformation of the Feigin-Fuchs

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Abstract

A realization of the Yangian double with center $DY_h(sl_2)_k$ of level $k(\neq 0, -2)$ in terms of free boson fields is constructed. The screening currents are also presented, which commute with $DY_h(sl_2)$ modulo total difference. In the $\hbar \rightarrow 0$ limit, the currents of Yangian double $DY_h(sl_2)_k$ becomes the Feigin-Fuchs realization of affine Lie $sl(2)_k$, while the screening currents of Yangian double $DY_h(sl_2)_k$ becomes the screening currents of the affine Lie algebra $sl(2)_k$.

1 Introduction

Yangians were proposed by Drinfeld as generalizations of classical loop algebra with nontrivial Hopf algebra structures [3, 4, 5]. From the physics point view, they are the symmetric algebra of the non-local currents of massive field theories [1]. Following the Faddeev-Reshetikhin-Takhtajan (FRT) formalism [4], as an associative algebras, Yangian can be defined through the Yang-Baxter relation (i.e. $RLL$-relations) with the structure constants determined by the rational solutions of the quantum Yang-Baxter equation (QYBE). However, this is just one part of the story, for physical applications, such as description the dynamical symmetry of the massive integrable quantum filed theories, the dual modules and the infinity dimensional representations are required [1, 21, 27], in another words, the Yangian double with center is emerged naturally as the dynamical symmetry of the integrable massive field theories. In fact, Yangian double with center were proved to have important applications in certain physical problems, especially in describing the dynamical symmetries of the perturbative integrable massive quantum field theories, calculating the correlation functions and form factors of some two-dimensional exactly solvable lattice statistical model and (1+1)-dimensional completely integrable quantum field theories [1, 21, 19, 27].

From the $RLL$ viewpoint, if one regards the Reshetikhin-Semenov-Tian-Shansky realization (RS) [25] as the affine extension of Faddeev-Reshetikhin-Takhtajan formalism. Yangian double with
center (the central extension for the quantum double of Yangian) \cite{14,15,17} are affine extensions of Yangian. Contrary to the quantum affine (corresponding to trigonometric solution of QYBE) case, the Yangian double with center is only recently obtained \cite{13,14,15}.

The free fields realization is a common used method in the quantum field theory. The free boson representations of $U_q(\mathfrak{sl}_2)$ with an arbitrary level have been obtained in Refs. \cite{26,22,23}. For the Yangian double with center, the free field representation of $DY_h(\mathfrak{sl}_2)$ with level $k(\neq 0 - 2)$ was constructed in \cite{20}, the level-1 and level-$k$ representation of $DY_h(\mathfrak{sl}_N)$ are obtained in \cite{12} and \cite{11}, respectively. The level-$k$ free field representation of $DY_h(\mathfrak{gl}_N)$ is also given in \cite{4}. However, all of these are the deformation of the Wakimoto modules, there is another kind of module-Feigin-Fucks module in the classical \cite{24} and the quantum affine case \cite{4,24}. In the case of Yangian double with center this kind of module is not given yet.

In this manuscript, we consider a representation of the level-$k$ ($\neq 0, -2$) Drinfeld currents in terms of the three free bosonic fields $\beta, \alpha$ and $\gamma$. In the classical limit $\hbar \to 0$, this module becomes the Feigin-Fuchs representation of the affine Lie algebra $\mathfrak{sl}(2)_k$. So this module can be regarded as a deformation of the so-called Feigin-Fuchs construction of the affine Lie algebras \cite{11,24}.

The manuscript is arranged as follows. Section 2, at first, we briefly review the Drinfeld new realization of the Yangian $Y(\mathfrak{sl}_2)$, then we give the defining relations of Drinfeld generators of the Yangian double with center $DY_h(\mathfrak{gl}_2)$. In section 3, the three free boson fields are introduced; and a free fields realization of $DY_h(\mathfrak{sl}_2)$ are given. The free fields realization of the screening currents is obtained in section 4.

## 2 Yangian and Yangian double with center

Yangian is a Hopf algebra, there are three realizations of Yangian $Y(g)$: Drinfeld realization \cite{3,7}, Drinfeld new realization \cite{3}, and $R$-matrix realization or FRT realization \cite{18,7}. The isomorphism between the Drinfeld new realization and the $R$-matrix (or FRT) realization can be established by using the Ding and Frenkel correspondence \cite{3}. Here, we only give the definition of the Drinfeld new realization of Yangian $Y(\mathfrak{sl}_2)$, for the Yangian double $DY(\mathfrak{sl}_2)$ is defined in terms of quantum double of the Drinfeld new realization, while the Yangian double with center is the central extension of the Yangian double. There is the following theorem:

**Theorem 1** \cite{3} Yangian $Y(\mathfrak{sl}_2)$ is a Hopf algebra generated by the symbols $e_m, f_m, h_m \ m \geq 0$ satisfying the relations

\[
[h_m, h_n] = 0, \quad [e_m, f_n] = h_{m+n}, \\
[h_0, e_m] = 2e_m, \quad [h_0, f_m] = -2f_m, \\
[h_{m+1}, e_n] - [h_m, e_{n+1}] = \hbar\{h_m, e_n\}, \\
[h_{m+1}, f_n] - [h_m, f_{n+1}] = -\hbar\{h_m, f_n\}, \\
[e_{m+1}, e_n] - [e_m, e_{n+1}] = \hbar\{e_m, e_n\}, \\
[f_{m+1}, e_n] - [f_m, e_{n+1}] = -\hbar\{f_m, e_n\}. \tag{2.1}
\]

Here $\hbar$ is a formal variable and $\{a, b\} = ab + ba$.

The central extension of the Yangian double \cite{13,14} is a Hopf algebra over $C[[\hbar]]$ generated by $\{e_m, f_m, h_m, c, d | m \in \mathbb{Z}\}$. The generating functions are
\[ e^\pm(u) = \pm \sum_{m \geq 0} e_m u^{-m-1}, \quad f^\pm(u) = \pm \sum_{m \geq 0} f_m u^{-m-1}, \]
\[ h^\pm(u) = 1 \pm \hbar \sum_{m \geq 0} h_m u^{-m-1}, \quad (2.2) \]

and
\[ e(u) = e^+(u) - e^-(u), \quad f(u) = f^+(u) - f^-(u), \]
\[ \delta(u - v) = \sum_{n+m=-1} u^n v^m. \]

satisfying the following relations.

\[ [d, \chi(u)] = \frac{d}{du} \chi(u), \quad \chi(u) = e(u), \quad f(u), \quad h^\pm(u) \]
\[ e(u)e(v) = \frac{u-v+\hbar}{u-v-\hbar} e(v)e(u), \]
\[ f(u)f(v) = \frac{u-v-\hbar}{u-v+\hbar} f(v)f(u), \]
\[ h^\pm(u)e(v) = \frac{u-v+\hbar}{u-v-\hbar} e(v)h^\pm(u), \]
\[ h^+(u)f(v) = \frac{u-v-(1+c)\hbar}{u-v+(1-c)\hbar} f(v)h^+(u), \]
\[ h^-(u)f(v) = \frac{u-v-\hbar}{u-v+\hbar} f(v)h^-(u), \]
\[ [h^\pm(u), h^\pm(v)] = 0, \]
\[ h^+(u)h^-(v) = \frac{u-v+\hbar}{u-v-\hbar} u - v - (1+c)\hbar \]
\[ \frac{u-v+(1-c)\hbar}{u-v-(1-c)\hbar} h^-(v)h^+(u), \]
\[ [e(u), f(v)] = \frac{1}{\hbar} (\delta(u - (v + hc))h^+(u) - \delta(u - v)h^-(v)), \]

and the coproduct can be found [13, 15].

3 Free bosons

For the following usage, we first introduce a Heisenberg algebra \( \mathcal{H} \) generated by \( \alpha_n, \beta_n, \gamma_n, n \in Z \) and \( P_\alpha, P_\beta, P_\gamma, Q_\alpha, Q_\beta, Q_\gamma \) satisfying the commutation relations

\[ [\alpha_m, \alpha_n] = k + 2 m \delta_{m+n,0}, \quad [P_\alpha, Q_\alpha] = \frac{k+2}{2}, \]
\[ [\beta_m, \beta_n] = -\frac{k}{2} m \delta_{m+n,0}, \quad [P_\beta, Q_\beta] = -\frac{k}{2}, \]
\[ [\gamma_m, \gamma_n] = \frac{k}{2} m \delta_{m+n,0}, \quad [P_\gamma, Q_\gamma] = \frac{k}{2}, \quad (3.1) \]
with \( k \in C(\neq 0, -2) \). The other commutators vanish identical. Denote the vacuum states with \( \alpha, \beta, \gamma \)-charges \( l, s, t \) as \( |l; s, t) \) :

\[
|l; s, t\rangle = e^{\sum Q_\alpha + \frac{i}{2}Q_\beta + \frac{i}{2}Q_\gamma} |0\rangle,
X_n |0\rangle = 0, \quad n > 0, \quad P_X |0\rangle = 0,
\]

for \( X = \alpha, \beta, \gamma \), and as \( \mathcal{F}_{l,s,t} \) the Fock space constructed on \( |l; s, t) \):

\[
\mathcal{F}_{l,s,t} = \{ \prod \alpha_m \prod \beta_n \prod \gamma_r |l; s, t) \} \tag{3.2}
\]

with \( m, n, r \in \mathbb{Z}_{<0} \). It is convenient to introduce generating functions \( X(u; A, B) \) \( X = \alpha, \beta, \gamma \) as

\[
X(u; A, B) = \sum_{n > 0} \frac{X_n}{n} (u + Ah)^n - \sum_{n > 0} \frac{X_n}{n} (u + Bh)^{-n} + \log(u + Bh)P_X + Q_X. \tag{3.3}
\]

and use the notation \( X(u; A, A) = X(u; A) \) for concise. The normal ordered product \( : \) is defined as moving the positive frequencies to the right of the negative ones. While the difference operator \( \alpha \partial_u f \) is defined as:

\[
\alpha \partial_u f(u) = \frac{f(u + \alpha \hbar) - f(u)}{\hbar}.
\]

Then we have the following results:

**Proposition 1**: The currents \( e(u), f(u) \) and \( h^\pm(u) \) of the Heisenberg algebra \( \mathcal{H} \) acting on \( \mathcal{F}_{l,s,t} \) can be defined by

\[
e(u) = -\frac{1}{\hbar} : \exp \left\{ \sum_{n > 0} \frac{1}{n} (\alpha_n + \beta_n)((u - (k + 1)\hbar)^{-n} - (u - (k + 2)\hbar)^{-n}) \right\}
\]
\[
\times \left( \frac{u - (k + 2)\hbar}{u - (k + 1)\hbar} \right)^{P_\alpha + P_\beta}
\]
\[
- \exp \left\{ \sum_{n > 0} \frac{1}{n} (\alpha_n - \beta_n)((u - (k + 1)\hbar)^{-n} - (u - (k + 2)\hbar)^{-n}) \right\}
\]
\[
\times \exp \left\{ \frac{2}{k} (\beta + \gamma)(u; -(k + 1), -(k + 2)) \right\} : , \tag{3.4}
\]

\[
f(u) = \frac{1}{\hbar} : \exp \left\{ \sum_{n > 0} \frac{1}{n} (\alpha_n + \beta_n)((u - 2\hbar)^{-n} - (u - \hbar)^{-n}) \right\} \left( \frac{u - \hbar}{u - 2\hbar} \right)^{P_\alpha + P_\beta}
\]
\[
\times \exp \left\{ \sum_{n > 0} \frac{1}{n} \gamma_n((u - 2\hbar)^{-n} - u^{-n}) \right\} \left( \frac{u}{u - 2\hbar} \right)^{P_\beta}
\]
\[
\times \exp \left\{ \frac{2}{k} (\beta + \gamma)(u; -1, -2) \right\}
\]
\[
- \exp \left\{ \sum_{n > 0} \frac{1}{n} (\alpha_n - \beta_n)((u - (k + 2)\hbar)^{-n} - (u - (k + 3)\hbar)^{-n}) \right\}
\]
So we can denote the Heisenberg algebra \( H \) by

\[
\text{Theorem 2} \quad \text{By analytic continuation, the Heisenberg algebra } H \text{ of the currents } e(u), f(u), h^\pm(u) \text{ and the operator } d \text{ are homomorphism (2.3) with } c = k \text{ on } F_{l,s,t}.
\]

And the following results are straightforward:

\[
\begin{align*}
h^+(u) &= \exp\left\{ \sum_{n>0} \frac{\gamma_n}{n} [(u-(k+2)\hbar)^n - (u-k\hbar)^n] \right\} \left( \frac{u-k\hbar}{u-(k+2)\hbar} \right)^{\frac{P}{r}}, \quad (3.5) \\
h^-(u) &= \exp\left\{ \sum_{n>0} \frac{2}{nk}(\beta_n + \gamma_n) [(u-(k+1)\hbar)^n - (u-k\hbar)^n] \right\} \\
&\quad \times \exp\left\{ -\sum_{n>0} \frac{1}{n} (\alpha_n + \beta_n) [(u-(k+3)\hbar)^n - (u-(k+1)\hbar)^n] \right\} \\
&\quad \times \exp\left\{ \sum_{n>0} \frac{2}{(k+2)n} (\alpha_n + \beta_n + \gamma_n) [(u-(k+3)\hbar)^n - (u-(1-h)^n)] \right\}, \quad (3.7)
\end{align*}
\]

and the operator by

\[
d = \frac{2}{k+2} d_\alpha - \frac{2}{k} d_\beta + \frac{2}{k} d_\gamma, \tag{3.8}
\]

in which

\[
\begin{align*}
d_\alpha &= \alpha_1 P_\alpha + \sum_{n \in \mathbb{Z}_{>0}} \alpha_{n-1} \alpha_n, \\
d_\beta &= \beta_1 P_\beta + \sum_{n \in \mathbb{Z}_{>0}} \beta_{n-1} \beta_n, \\
d_\gamma &= \gamma_1 P_\gamma + \sum_{n \in \mathbb{Z}_{>0}} \gamma_{n-1} \gamma_n.
\end{align*}
\]

And the following results are straightforward:

To state the results

\[
\begin{align*}
e(u) &\to \left( \sqrt{\frac{k+2}{2}} \partial \phi_1(u) + \sqrt{\frac{k}{2}} i \partial \phi_2(u) \right) \exp\left\{ \sqrt{\frac{2}{k}} (i \phi_2(u) - \phi_0(u)) \right\}, \\
f(u) &\to \left( \sqrt{\frac{k}{2}} \partial \phi_1(u) - \sqrt{\frac{k+2}{2}} i \partial \phi_2(u) \right) \exp\left\{ -\sqrt{\frac{2}{k}} (i \phi_2(u) - \phi_0(u)) \right\}, \\
\frac{1}{\hbar} (h^+(u) - h^-(u)) &\to -\sqrt{\frac{k}{2}} \partial \phi_0(u),
\end{align*}
\]

where \( \phi_i(u) \phi_j(v) = \delta_{i,j} \ln(u-v) \), and the operator \( d \) is \( L_{-1} \) of Virasoro algebra.

It is easy to show
\[ [\chi(u), P_\beta + P_\gamma] = 0, \quad \chi(u) = e(u), \quad f(u), \quad h^\pm(u). \]

so in the following, we restrict the Fock space \( \mathcal{F}_{l,s,t} \) to the \( s = -t \) sector without loss of generality, on this sector the currents \( e(u), f(u) \) and \( h^\pm(u) \) are single valued so that the expansion such as \( 2.2 \) makes sense.

## 4 Screening currents

In this section, we construct two screening currents of the algebra \( \mathcal{D}Y(sl_2) \). In the classical limit \( \hbar \to 0 \), they become the screening currents of the affine Lie algebra \( sl(2)_k \). Let us next consider the following operators.

\[
\xi(u) = S^+(u)^{-1}, \quad S^+(u) = \exp\{(\alpha + \beta)(u; -(k + 2))\}.
\]

We have

\[
\xi(u)S^+(v) = -S^+(v)\xi(u) \sim \frac{1}{u-v}. \tag{4.1}
\]

Here \( \sim \) means that the relation is equivalent up to modulo regular terms. The fields \( \xi(u) \) and \( S^+(u) \) are single valued on \( \mathcal{F}_{l,s,s} \) \( s \in \mathbb{Z} \). From (4.1), the zero-modes \( \xi_0 = \oint du 2\pi i u \xi(u) \) and \( Q^+_0 = \oint du S^+(u) \) anticommute \( \{\xi_0, Q^+_0\} = 0 \). Note also \( \xi_0^2 = 0 = Q^+_0^2 \). In addition, the following equations hold in the sense of analytic continuation.

**Proposition 2**

\[
\begin{align*}
&h^\pm(u)S^+(v) = S^+(v)h^\pm(u) \sim 0, \\
f(u)S^+(v) = -S^+(v)f(u) \sim 0, \\
e(u)S^+(v) = -S^+(v)e(u) \\
&\quad \sim_1 \partial_v \left( \frac{1}{u-v+\hbar} \exp\left\{ \frac{2}{k}(\beta + \gamma)(u; -(k + 2), -(k + 3)) \right\} \\
&\quad \times \exp\{((\alpha + \beta)(u; -(k + 2), -(k + 3))\} \right).
\end{align*}
\]

Therefore the zero-mode \( \eta_0 \) commutes with the action of \( \mathcal{D}Y(sl_2) \). So the current \( S^+(u) \) is the screening operator of \( \mathcal{D}Y(sl_2) \). we then restrict the Fock space \( \mathcal{F}_{l,s,s} \) to the kernel of \( \mathcal{D} \), and hence arrive at the deformation of the Feigin-Fuchs modules

\[
\mathcal{F}_l = \bigoplus_{s \in \mathbb{Z}} \text{Ker}(Q^+: \mathcal{F}_{l,s,s} \to \mathcal{F}_{l,s,s+1}). \tag{4.2}
\]

For the level \( -k \) \( U_q(\tilde{sl}_2) \), the \( q \)-deformation of the Feigin-Fuchs modules were obtained in \([2,3]\).

Next we consider another screening operator defined by

\[
S(u|L,M) = \frac{1}{\hbar} : \exp\{\sum_{n>0} \frac{1}{n}(\alpha_n + \beta_n)(u-h)^{-n} - u^{-n}\} \left( \frac{u}{u-h} \right)^{P_{\alpha} + P_{\beta}}.
\]
\[ - \exp \left\{ \sum_{n>0} \frac{1}{n} (\alpha_n + \beta_n) [(u-h)^n - u^n] \right\} \]
\[ \times \exp \left\{ - \sum_{n>0} \frac{2}{(k+2)n} \alpha_{-n} (u-h)^n \right\} e^{-\frac{2}{k+2} Q_{\alpha}} \]
\[ \times (u-2h) \frac{1}{2} (P_{\beta} + P_{\gamma}) \prod_{l=1}^{L} \left( \frac{u - (2 + (k+2)l)h}{u - (k+2)lh} \right) \frac{1}{2} (P_{\beta} + P_{\gamma}) \]
\[ \times \exp \left\{ \sum_{l=1}^{L} \sum_{n>0} \frac{2}{kn} (\beta_n + \gamma_n) [(u - (k+2)lh) - n] \right\} \]
\[ \times (u - (2 + (k+2)lh)) \frac{1}{2} (P_{\alpha} + P_{\beta} + P_{\gamma}) \]
\[ \times \exp \left\{ \sum_{m=0}^{M} \sum_{n>0} \frac{1}{n} (\alpha_n + \beta_n + \gamma_n) [(u - (k+2)mhh) - n] \right\} \]
\[ \times (u - (2 + (k+2)mhh)) \frac{1}{2} (P_{\beta} + P_{\gamma}) \]

with \( L, M \in \mathbb{Z}_{>0} \).
The following results are based on the direct calculation:

**Proposition 3**

\[ h^+(u)S(v)_{[L, M]} = S(v)_{[L, M]} h^+(u) \sim 0, \]
\[ e(u)S(v)_{[L, M]} = S(v)_{[L, M]} e(u) \sim 0, \]
\[ S(u)_{[L, M]} S^+(v) = S^+(v) S(u)_{[L, M]} \]
\[ \sim_1 \partial_v \left( \frac{1}{u-v+(k+2)h} \right) \exp \left\{ (\alpha + \beta)(v; -(k+3), -(k+2)) \right\} \]
\[ \times \exp \left\{ - \sum_{n>0} \frac{2}{(k+2)n} \alpha_{-n} (v)^n \right\} e^{-\frac{2}{k+2} Q_{\alpha}} \]
\[ \times (v-h) \frac{1}{2} (P_{\beta} + P_{\gamma}) \prod_{l=1}^{L} \left( \frac{v - (1 + (k+2)lh)}{v - (1 + (k+2)lh)} \right) \frac{1}{2} (P_{\beta} + P_{\gamma}) \]
\[ \times \exp \left\{ \sum_{l=1}^{L} \sum_{n>0} \frac{2}{kn} (\beta_n + \gamma_n) [(v - (1 + (k+2)lh) - n) \right\} \]
\[ \times \exp \left\{ \sum_{m=0}^{M} \sum_{n>0} \frac{1}{n} (\alpha_n + \beta_n + \gamma_n) [(v - (1 + (k+2)mhh) - n) \right\} \]
\[ [S(u)_{[L, M]}, P_{\beta} + P_{\gamma}] = 0. \]

In addition, in the limit \( L, M \to \infty \),
\[ h^-(u)S(v) = S(v)h^-(u) \sim 0, \]
\[ f(u)S(v) = S(v)f(u) \sim 0. \]

\[ \sim_k \partial_v \left( \frac{1}{u - v} \exp \left\{ - \sum_{n>0} \frac{2}{(k + 2)n} \alpha_n (v - h)^n \right\} e^{-\frac{2}{k+2}Q_\alpha} \times (v - 2h) \hat{z}^{(P_\beta + P_\gamma)} \prod_{l=1}^L \left( \frac{v - (2 + (k + 2)l)h}{v - (k + 2)lh} \right) \hat{z}^{(P_\beta + P_\gamma)} \times \exp \left\{ \sum_{l=1}^L \sum_{n>0} \frac{2}{kn} (\beta_n + \gamma_n) [(v - (k + 2)lh)^{-n}] \right. \right. \]
\[ \times \exp \left\{ \sum_{m=0}^M \sum_{n>0} \frac{1}{n} (\alpha_n + \beta_n + \gamma_n) [(v - (k + 2)mh)^{-n}] - (v - (2 + (k + 2)mh)^{-n}] \right\} \]

where \( S(u) = \lim_{L, M \to \infty} S(u)[L, M] \).

Therefore the screening charge \( Q = \oint \frac{du}{2\pi i} S(u)[L, M] \) commutes with all the currents in \( DY(sl_2) \) and \( Q_+ \) in the limit \( L, M \to \infty \). The charge \( S \) yields a linear map \( S : \mathcal{F}_l \to \mathcal{F}_{l-2} \).

We have constructed a free field representation of the level-\( k \) Drinfeld currents for the Yangian double \( DY(sl_2) \) and screening operators. As a result, we have obtained a deformation of the level-\( k \) Feigin-Fuchs modules.

A possible application of the results is a calculation of correlation functions in massive integrable quantum field theory such as higher spin \( SU(2) \) invariant Thirring model and in higher spin XXX spin chains. For this purpose, one has to make a precise identification of the space of states with the Feigin-Fuchs modules. We hope to discuss these problems in future publication.

**Acknowledgments:** One of the authors (Ding) would like to thanks Prof. B. Y. Hou, prof K. Wu and Prof. Z. Y. Zhu for fruitful discussion, and he is supported in part by the “China postdoctoral Science Foundation”.

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