Modeling and control of the hydraulic actuator in a ladle furnace

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ABSTRACT

The dynamic behavior of the hydraulic actuator in a system for regulating the electrode’s position is crucial for the operation of a Ladle Furnace. This work aims to identify, model, and control the hydraulic actuator in the Ladle Furnace of ACINOX Las Tunas. For identifying the system, input signals of Pseudo-Random Binary type and black box models were used. As a result, three models were obtained, two reflecting the process’s asymmetric behavior according to the upward or downward movement. The third model approximates the process dynamic behavior around the operating point and includes the uncertainty caused by the weight variation during the electrode wear. The models obtained, with a fit greater than 85%, allow a better understanding of the study case behavior. In addition, these allowed the evaluation of the electrode’s weight variation and tuning of several controllers. The optimal one was a novel non-linear PI controller of guaranteed robustness. In future works, the use of a non-linear function could be evaluated to compensate for the asymmetric behavior of the process.

1. Introduction

The metallurgical searches for increasing energy efficiency of its production processes because the effective use of energy is becoming more important in the world owing to the energy crisis [1]. This industry is characterized by the wide use of Electric Arc Furnaces (EAF) in the liquid steel production process [2], representing approximately 30% of the production worldwide [3]. The Ladle Furnace (LF), also an electric arc furnace, is used for fine-tuning the temperature of the steel. All these furnaces are characterized by high electrical energy consumption. The automatic control systems of electrode position are essential for the efficiency of an EAF or LF [4]. In such control systems, the behavior of your hydraulic actuator is not always known exactly.

To control a specific industrial system, it is necessary to know the dynamics of the process [5]. In modeling hydraulic actuators, we can find two fundamental aspects; the first is given by the white box models or based on physical principles [6, 7, 8, 9], and the black box or empirical ones [10, 11, 12]. For modeling the hydraulic actuator of an EAF or LF, the most used variant is the black box in the frequency domain [1, 4, 11, 13, 14].

In [6, 11, 14], the asymmetric behavior of hydraulic actuators is reflected, which is associated with the difference between rising and down speed. The models presented in [1, 14] for hydraulic actuators in an EAF reflect the process parameters variations due to the electrode’s wear. Although the previously exposed nonlinearities vary the model parameters’ values, static nonlinearities also affect the process dynamics. These must be considered in the work of [15], in which the dead zone at the input and saturation at the system output is analyzed.

The reports consulted on hydraulic actuator models [1, 6, 11, 13, 14, 15] do not reflect all the dynamic and static nonlinearities that can occur in these systems. For example, in the bibliography consulted, the asymmetric behavior between the charge and discharge speed, the parametric uncertainty as a function of the variation in weight of the electrode, the dead zone, and the saturation is not analyzed. In the case of the hydraulic actuator of the Ladle Furnace (LF) of the ACINOX Las Tunas steelworks, Cuba, it is difficult to carry out actions to improve its efficiency because there is a model that describes the dynamic behavior of the system.

This work was focused on obtaining a process model of the study case for tuning controllers, comparing them, and selecting the best. Unlike previous results [1, 6, 11, 13, 14], this article delves into the study of the

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process for demonstrating its asymmetric behavior and obtaining models of variable parameters representative of both directions of the actuator displacement and the incidence of the weight of the electrodes.

The control strategies consulted for this type of system use intelligent controllers [16, 17], proportional-integral-derivative (PID) control [18, 19], non-linear PID control [2, 4, 20], and fractional order control [1, 4]. These control strategies do not consider all major requirements: load disturbance attenuation, robustness, control effort, and trade-offs. In this sense, the application of a novel method used for mold level control in continuous casting has been reported, which allows more comprehensive management of the requirements [21] and could be convenient for its application in an electrode positioning control of an LF.

In this work, the identification, modeling, and control of the hydraulic actuator of the ACINOX Las Tunas Ladle Furnace electrode position control system are carried out using MATLAB® as a support tool. The models obtained were determined experimentally under actual operating conditions. The nonlinearities and variation ranges of the parameters reflect with reasonable precision the asymmetric behavior between the speed of rising and fall, the parametric uncertainty as a function of the electrode's weight variation, the dead zone, and the saturation of the hydraulic actuator of a Ladle Furnace.

2. Materials and methods

Here is an overview of the LF, its control system of the position of the electrode, the hydraulic actuator, and the case study.

2.1. Control system of the position of the electrode

The basic operating principle of an LF consists of establishing and controlling the electric arc as a source of energy for transferring enough heat to make it possible to melt the materials involved in making the steel and raise the temperature of the liquid steel to desired values. Commonly, three graphite electrodes are used that are connected to the secondary of a three-phase transformer that supplies the electrical energy necessary for its operation. The electric arc appears when the electrode, through which the electrical energy circulates, is close to the metal.

In Figure 1, an electrode control system of an LF is illustrated. In these, the supply current or the value of the arc impedance can be used as a controlled variable; depending on the installation characteristics or the process requirements, the arc impedance control is the most used. Any deviation from the optimal arc length contributes to increased energy consumption and decreases the system's efficiency [1].

There are frequent process disturbances that cause length variations of the electric arc, for example, the addition of materials, and the blowing of gases, among others [1]. The controller moves the electrode up or down to attenuate these disturbances and maintains the desired arc length. The mass of the electrodes is constantly changing since they are consumed during the fusion process, causing variations in the dynamic parameters of the hydraulic actuator.

The models proposed for EAF and LF are varied [2, 22, 23, 24, 25, 26], and the objectives pursued with them are dissimilar. Most of these models are notable for their complexity and are unsuitable for controller design and tuning. In addition, it is possible to find simple dynamic models through identification [1, 27].

Several studies [2, 11, 14, 27] propose to identify the $G_{sp}$ process, represented in Figure 2, separating the identification of the $G_{sh}$ hydraulic actuator from the $G_{se}$ electrical subsystem, where $z^*(s)$ represents the reference signal, $e(s)$ the error at the controller input, $u(s)$ the controller output, $D(s)$ the disturbance at the input of the system, $L(s)$ the position of the electrode and $Z(s)$ the impedance at the output of the system.

2.2. Hydraulic actuator

Hydraulic transmission is the most used for moving the electrodes of electric arc furnaces. Compared to the electromechanical one, this transmission has several advantages. It is not a complex mechanical transmission; it has high speeds and can move large masses with high accelerations [10].

A hydraulic system is a set of individual components interconnected to provide the desired form of hydraulic transfer. Its basic structure comprises the hydraulic power supply, control elements, actuator, and other components such as pipe measuring devices. Hydraulic actuators transform the hydraulic energy provided by the pump into mechanical

![Figure 1. General diagram of a control system for the position of the electrodes of an LF.](image-url)
energy [28]. They can be grouped into linear (cylinders) or rotary (motors) depending on the movement and work.

Cylinders are actuators that transform hydraulic energy into a linear force, being used where large thrust forces are required for displacement. The fluid leaves and enters through a single chamber in single-acting cylinders, as in the study case. Its movement is carried out by forces external to the hydraulic system itself, such as the force of gravity. According to the system’s dynamic characteristics, single-acting cylinders can be used to achieve important requirements, such as high precision in positioning, short travel times, normal positioning behavior, and good dynamic response quality [29].

A process model that reflects its dynamics is very useful for selecting a good control strategy and tuning the controller, attending to load disturbance attenuation, robustness, control effort, and the trade-offs [5]. Also, the process dynamics of the study case are characterized by nonlinearities such as friction forces, valve dynamics, oil compressibility, and the influence of loads [8].

In the case of the models in the frequency domain, the authors are divided between the third-order models with an integrator that are governed by Eq. (1) [7, 10, 30], and the second order with an integrator and transport delay whose behavior is given by (2), [11, 14]. Of these two variants, the one shown in (2) is the most used for actuators in the control system of the electrode position of an LF.

\[
L(s) = \frac{K}{s(T_1s + 1)(T_2s + 1)}
\]  
\[
L(s) = \frac{K}{s(T_2s + 1)e^{-\alpha T_2}}
\]

Where \(T, T_1, \) and \(T_2\) are time constants, \(r\) the delay, and \(K\) the gain.

In identifying this system type, two kinds of signals can be distinguished [30]; the first is the input signal sent to the system that the actuator must reproduce (the signal sent to the servo-valve). The second is the output signal that indicates how said actuator behaves (the signal that registers the position sensor).

For implementing the model with simulation tools, it is necessary to consider that the static nonlinearities must be included in the path between the input signal and the output signal. The dead zone, visible experimentally, is given because the electrodes do not begin to move until the signal from the controller does not exceed a certain threshold. In addition, saturation appears when the physical characteristics of the installation limit the actuator travel.

2.3. The case study

The object of study is the ladle furnace of the ACINOX Las Tunas steelworks, Cuba, which has the general scheme represented in Figure 1. It works with a three-phase alternating current, has a production capacity of 60 t, and has an approximate consumption of 82 kWh/t. Among its subsystems are the electrodes hydraulic system, which is made up of the tank, the hydraulic pumps, the cylinders, the conduits, the control valves, the electrode-holder arms, and the electrodes of 300 mm diameter.

Each of the three electrodes is positioned by its Simelt electrode control system. The latter manipulates the control valve for adjusting the flow from the hydraulic tank to the cylinder, thus producing a vertical movement. This subsystem is an essential part of the LF. The electrode's position determines the arc's length directly and, consequently, the impedance of the arc.

3. Results and discussion

The correct identification of a process implies knowing its characteristics for designing the experiments that will provide the data for obtaining a model that describes the system.

3.1. Identification and process modeling

System identification by experimental data processing is defined as the deduction of the mathematical models of the system from the analysis of experimental data, measurements, and observations [31]. Its theoretical basis is supported, in part, by the theory of dynamic systems and signals of a stochastic nature and by mathematical methods and algorithms for estimating the parameters involved. In general, this process involves the following five steps: Obtaining input-output data; Previous treatment of the registered data; Choice of the model structure; Obtaining the model parameters and Model validation.

The identification methods can be classified in various ways depending on the model represented. Nonparametric methods can define a model utilizing tables or graphs using a non-finite number of samples and a vector of parameters. On the contrary, parametric methods seek to describe the interaction of the variables of a process through functions or mathematical equations. In addition, they require a choice of model structure, where the adjustment and estimation of the model parameters are left to the criterion. The analysis can be performed both in the time and frequency domains. In both cases, the aim is to reduce the error between the model and the real process.

The model obtained through identification always approximates the real process. The similarity will depend on several factors, such as getting adequate data, region of operation, set of selected structures, and others. The identification methods can make it possible to obtain a simple mathematical model representing the system's dynamics and meeting the requirements for its use in the control design.

3.1.1. Getting input-output data

For modeling the case study, it is recognized that the system presents variable parameters over time. However, it is estimated that the parameters vary very slowly over time, and therefore the system can be considered linear at each operating point. Carrying out several offline experiments using special signals could be enough to offer simple models that reproduce the process behavior with a better approximation than previous models.

In identifying processes through special signals, the most frequently used are steps and pseudo-random binary sequences (PRBS) [32, 33]. Random signals have, concerning others, such as the step, the advantage that their amplitude can be minimal, reducing the degree of disturbance introduced into the plant object of identification.

An excellent combination of signals for identification can include applying a step in the control variable beforehand and the PRBS. The step responses allow estimating three important parameters with more significant precision: the stationary gain, the delay time, and the dominant time constant. The first two can check and correct the identification result obtained through the PRBS. The third makes it possible to select the PRBS commutation period more accurately, following the practical rule that the commutation period should be 5 to 7 times less than the lowest time constant of the process [32, 33, 34].

In designing the PRBS signal to be used in the experiments, the system was excited with a step-type signal whose amplitude corresponded to the maximum value that can be had at the process input. The response to this excitation was processed, and the dominant time constant was obtained, which provides the necessary information for the correct design of the pseudo-random signals.
The diagram shown in Figure 3 was used to carry out the experiments. To generate the excitatory signals, a PC with the MATLAB® software tool was used. The signal was sent to the TDA NI USB 6008 data acquisition card, which generates a 0–5 V analog output signal converted to the ±10 V range with a transducer connected to the plant. Then, another converter converts the ±10 V signal to ±150 mA to power the single-acting servo-valve, TR7 / W4 DB 200 / 0.5 004 SERIES # 4370 governs the hydraulic electrode position system.

A linear position transducer measures the movement made by the hydraulic cylinder. It is calibrated to generate a minimum output value of 4 mA when the cylinder is initially positioned. In comparison, the maximum value of 20 mA is reached when it reaches its mechanical stop at 177 cm. The transducer’s output signal is suitable for measurement with the data acquisition card, using a current-to-voltage converter that converts the signal from 4-20 mA to 0–10 V. This signal is sent to an analog input card, reaches the PC for storage, and further processing.

In the review of the works of [35, 36, 37, 38] and through the observation and analysis of the case study itself, the static nonlinearities in the process were determined: dead zone and saturation. The dead zone of the process is determined by checking the cylinder begins to move when voltage values greater than 0.87 V in the upward movement and less than -0.76 V in the downward trend are present at its input. In the case of saturation, the physical characteristics of the hydraulic cylinder limit its displacement to the range between 0 and 177 cm.

The four experiments described below were designed and planned for obtaining the desired models:

- **Experiment 1.** With a maximum weight of 606 kg, the electrode has its initial position at the bottom of the stroke. The PRBS signal generated in the PC with two repetitions produces variations from 0 to 10 V at the input of the Simelt Card. The system’s response is measured by the position transducer, stored together with the PRBS signal in the PC. Both input and output signals are graphed in Figure 4a.

- **Experiment 2.** With a maximum weight of 606 kg, the electrode has its initial position at the top of the path. The PRBS signal generated in the PC with two repetitions produces variations from -10 to 0 V at the input of the Simelt Card. Figure 4b illustrates the input and output signals obtained during this experiment.

- **Experiments 3 and 4.** These were carried out to identify the influence of the weight of the electrodes on the dynamic behavior. In experiment 3, a 606 kg electrode (maximum weight from statistical analysis) was used, and in experiment 4, a 208 kg electrode for maximum wear (minimum weight from statistical analysis) was used. In both cases, a typical operating point of the electrode position control system was taken as the initial position of the electrodes, approximately 47 cm above the lower position.

The PRBS signal generated in the PC with ten repetitions produces variations of ±3 V at the input of the Simelt Card, obtaining up and down commands. The selected range responds to the maximum average variation of the plant around its specific operating point. The system’s responses are graphed in Figure 5, observing an upward position trend. The applied signal generates an asymmetric behavior in both directions of the movement since the upward speed is higher than the low speed.

All experiments’ input and output data were stored at 20 samples per second. Then, following the steps involved in the identification process, the data is processed using the facilities provided by the MATLAB® identification toolbox. In addition, the data to be used for identifying and validating were selected. In experiments one and two, the first sequence of PRBS signals was used for identifying and the second for validating. Similarly, the estimation and validation data were taken for experiments three and four.

Once the data treatment process was completed, the model structure was selected. In the case of hydraulic actuators, several consulted reports propose models that follow the behavior of Eqs. (1) and (2) [1, 7, 10, 12]. In this research, the structure of Eq. (2), of second order with integrator and delay, was selected as it was the most used in the main reports of hydraulic actuators for EAF [1, 11, 14, 39]. Furthermore, the low-order model Eq. (2) structure is suitable for various tuning methods of PID controllers without affecting the adequate representation of the dynamic behavior observed in this type of system.

3.1.2. Obtaining the model parameters

Experiments one and two are processed in the MATLAB® identification toolbox. The models that describe the system’s behavior in Figure 4 are obtained from them. Eq. (3) describes the model representing the upward displacement, while Eq. (4) describes the downward displacement model.

\[
G_{up}(s) = \frac{1.74}{s(0.018s + 1)} e^{-0.015s} \tag{3}
\]

\[
G_{down}(s) = \frac{0.57}{s(0.018s + 1)} e^{-0.014s} \tag{4}
\]

When analyzing the values of the parameters of both models, it is

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**Figure 3.** Scheme of the configuration used to carry out the experiments. (1) PC, (2) TDA NI USB 6008 card, (3) Voltage to Voltage converter, (4) SIMELT (Spoon Oven Electric Control System) card, (5) Hydraulic transmission of electrodeposition, (6) Linear position transducer, and (7) Current to Voltage Converter.
Figure 4. a) Response of the hydraulic system in the first experiment with a PRBS signal of amplitude 0–10V (electrodes rising); b) Response of the hydraulic system in the second experiment with a PRBS signal of amplitude -10 to 0V (electrodes going down).

Figure 5. System response to a PRBS signal around the operating point (±3V) considering the weight of the electrodes. a) Experiment 3 with a maximum weight of the electrodes (606 kg) and b) Experiment 4 with a minimum weight of the electrodes (228 kg).
observed that the displacement direction does not significantly influence the $T$ and $r$ values. However, the $K$ value is approximately three times higher in Eq. (3) than in (4). This difference in the gain value for the rise and fall of the electrode is a direct representation of the difference in the speed at which they respond to the same input signal. This can be seen by simply observing Figure 4a) and b), in which PRBS signals of equal amplitude and opposite directions are used. In the first, the hydraulic actuator performs the maximum displacement is less than 30 s, while in the second, the displacement is approximately half for the same period.

The results of expressions (3) and (4) allow us to consider that the EAF hydraulic actuator model has one variable and two constant parameters. However, it varies much slower than the dynamics of the process. This statement induces the consideration of the system as Linear of Variable Parameters (LVP) [39, 40]. The direction of the upward or downward movement causes a variation of the dynamic gain of the model (2) in the range corresponding to $K \in [K_{\min}, K_{\max}]$ such that:

$$K \in [0.57, 1.74]$$

(5)

The data from experiments three and four graphed in Figure 5 are processed in the identification toolbox. From them, the models that describe the system’s behavior when it moves in both directions are obtained:

$$G_{\text{max}}(s) = \frac{0.363}{s(0.1s + 1)} e^{-0.011}$$

(6)

$$G_{\text{min}}(s) = \frac{0.40}{s(0.04s + 1)} e^{-0.006}$$

(7)

The results (6) and (7) allow us to affirm that the parameters of the EAF hydraulic actuator model are variable, although they vary much slower than the dynamics of the process. This statement corroborates the consideration given above as an LPV system [41, 42, 43, 44, 45], according to (3) and (4). The direction of the upward or downward movement causes parameters variations of the model (2) in the ranges corresponding to $K \in [K_{\min}, K_{\max}]$, $T \in [T_{\min}, T_{\max}]$, and $r \in [r_{\min}, r_{\max}]$ such that (8):

$$K \in [0.363, 0.4]; T \in [0.04, 0.1]; r \in [0.0046, 0.018]$$

(8)

The variations ranges given in (8) for parameters of the LVP model (2) indicate that the three parameters undergo variations due to the weight change of the electrodes. However, the largest variations occur in the values of $T$ and $r$.

### 3.2. Model validation

In this section, the model obtained for the case study is validated. To this end, it was established as a selection criterion that the percentage of adjustment between the output values of the model and the validation data must be greater than 90 % in the first and second experiments, corresponding to asymmetric behavior. For the third and fourth experiments, the selection criteria were equal to or greater than 80 % since these consider a single model with the same structure of (2) for displacement in both directions around a specific operating point, using electrodes with maximum and minimum weight.

Other elements usually used, and in the case of the study, they will be taken into account in a complementary way, are the Final Prediction Error (FPE), the Mean Squared Error (MSE), and the residual model; all of them provided by the System Identification Toolbox.

In the estimation and validation process of the model, the set of data obtained through the previously described experimental route was used. Before proceeding to the estimation and subsequent validation, the data was processed to remove means and trends.

Figure 6 shows the experimental data, its subsequent treatment, and the selection of the sections to be used for estimation and validation, in the case of upward displacement and downward displacement, Figure 6 a) and b), respectively.

Likewise, Figure 7 a) and b) show the experimental data and the selection of the sections to be used in the estimation and validation of the model, taking into account the maximum and minimum weights of the electrodes.

In the case of experiments one and two, the models obtained are governed by Eqs. (3) and (4) and presented an adjustment of 92.43 % and 95.02 %, respectively, Figure 8 a) and b).

For models of Eqs. (6) and (7), the adjustments are 85.05 % and 85.91 %, Figure 9 a) and b), respectively. These values are considered adequate around a specific operating point, considering (3) and (4) different system dynamics depending on the movement direction.

The final Prediction Error criterion provides a measure of model quality by simulating the situation where the model is tested on a different data set. The mean squared error tells you how close a regression line is to a set of points. Table 1 shows the FPE and MSE values for the models obtained from the upward and downward displacement of the electrodes, as well as their maximum and minimum weight.

The residual associated with the data and a given model are ideally independent of the input for the model to correctly describe the system. The rule is that if the correlation functions go significantly outside the confidence intervals, do not accept the corresponding model as a good description of the systems. Figure 10 a) and b) show the residual models for scrolling up and scrolling down.

In the case of Figure 11 a) and b), the residual models correspond to the minimum and maximum weight of the electrodes.

According to the results derived from the validation criteria applied, the models obtained were taken as adequate since the percentages of adjustments are higher than those previously selected, and the residual models are not dispersed in any cases.

### 3.3. Control strategy

For evaluating possible improvements in the control strategy using the model, the influence of parameter variations of the $G_{\text{SP}}$ subsystem on the control loop behavior of which it is part is analyzed. To do this, the behavior of said loop is simulated with a proportional controller of gain equal to 1.1947. The selected increase corresponds to the controller used in the case study.

The control system is not only made up of the hydraulic subsystem; it is also necessary to take into account the electrical subsystem, with the transfer function (9) for the case study:

$$G_{\text{Sp}}(s) = \frac{0.4043}{(1.1782s + 1)(0.17614s + 1)}$$

(9)

Furthermore, a linear interpolation of the parameter variations between the maximum and minimum values given in (6) and (7) is assumed. In that case, the medium $G_{\text{SP}}$ values can be inferred, given in the following transfer function (10):

$$G_{\text{min}}(s) = \frac{0.3815}{s(0.075 + 1)} e^{-0.011}$$

(10)

From the above, it follows those Eqs. (11), (12), and (13) are transfer functions of the $G_{\text{SP}}$ process, corresponding to the minimum, medium and maximum values of the electrodes weight, respectively:

$$G_{\text{Sp}}(s) = \frac{0.1617}{s(0.04s + 1)(1.1782s + 1)(0.17614s + 1)} e^{-0.0046}$$

(11)

$$G_{\text{Sp}}(s) = \frac{0.1542}{s(0.075 + 1)(1.1782s + 1)(0.17614s + 1)} e^{-0.011}$$

(12)

$$G_{\text{Sp}}(s) = \frac{0.1468}{s(0.1s + 1)(1.1782s + 1)(0.17614s + 1)} e^{-0.011}$$

(13)
Figure 12 shows the responses obtained for the models given in (11), (12), and (13), with a proportional controller proposed for the control scheme represented in Figure 2. Furthermore, in Figure 12, it is possible to observe that value changes of the model parameters cause moderate variations between the system transient responses after exciting it to the same change in the reference.

In Table 2, it can be observed, quantitatively, that the affectation causes increases of seconds in the establishment time $t_s$. In addition, there are non-negligible variations in global performance indexes, such as the Integral Square Error (ISE) and the Integral Absolute Error (IAE). IAE and ISE variations show how controller efficiency changes as process parameters vary due to changes in the electrode weight. In addition, it confirms the relevance of using the model for tuning the controller and analyzing the system's behavior.

The analysis of the simulation results using the obtained model with parametric variations due to the changes in the electrode's weight is carried out by evaluating the performance of other controllers. The comparison considers the requirements for disturbances attenuation, robustness, control effort, obtaining a good trade-off between the control effort and the disturbances attenuation, and nonlinearities of the system.

The control strategies consulted do not satisfy all the requirements raised previously; for example, those that achieve a good disturbances attenuation do not guarantee sufficient robustness and vice versa. For this reason, it is considered a novel method applied to mold level control in continuous steel casting because it allows more comprehensive management of the requirements [21].

The control application reported in [21] is based on the concept of a non-linear PID controller in a robustness region (NPID-RR) presented in [44] and which expresses that a controller $C(s)$ is NPID-RR when the control law is based on the following general expression of an NPID (14):

$$ u(\cdot, t) = K(\cdot) e(t) + k(\cdot) \int e(t) dt + k_d(\cdot) \dot{e}(t) \tag{14} $$

Where $u(\cdot, t)$ is the controller output, $K(\cdot)$, $k(\cdot)$, and $k_d(\cdot)$ are the variable gains, and $e(t)$ is the control error, but if and only if, the sets of
controller gain values describing a continuous path contained in a region given by robustness constraints. The robustness can be measured utilizing the sensitivity $M_s$ and the complementary sensitivity $M_t$; it is assumed that $M_s = M_t$, and that it is delimited in a given region between a minimum gain controller ($K_{\text{min}}$, $K_{\text{min}}^i$, $K_{\text{min}}^d$) corresponding to a value of $M_{\text{min}}$ and another with profit values ($K_{\text{max}}$, $K_{\text{max}}^i$, $K_{\text{max}}^d$) corresponding to the robustness required for a value of $M_{\text{max}}$, then it must be satisfied that $K(\cdot) \in [K_{\text{min}}^d, K_{\text{max}}^d]$, $k_i(\cdot) \in [K_{\text{min}}^i, K_{\text{max}}^i]$, $k_o(\cdot) \in [K_{\text{min}}^o, K_{\text{max}}^o]$, and $M_{\cdot} \in [M_{\text{min}}, M_{\text{max}}]$.

The control law of an NPID-RR with $k_d(\cdot) = 0$, used in [21], is given by the expression (15):

$$C(s) = \left(1 + \frac{K_{\text{min}}}{s}\right) \cdot \Delta K(\epsilon) + K_{\text{min}} + K_{i_{\text{rate}}} + \Delta k_{i_{\text{rate}}}$$

(15)

The minimum values $K_{\text{min}}$, $K_{i_{\text{min}}}$ of the proportional and integral gains correspond to a PI controller with the desired variation in steady-state ($\Delta H_{\text{e}}$) and a known maximum sensitivity value ($M_{\text{max}}$). Furthermore, $\Delta k_{i_{\text{min}}}$ is the residual value of a function evaluated in the nominal design corresponding to $K_{\text{min}}$, $K_{i_{\text{min}}}$. This function represents the functional relationship $\varnothing$ between the values of the integral gain $k_i = k_{i_{\text{min}}} + \Delta k_i$ and the proportional gain $K = K_{\text{min}} + \Delta K$, that is, $\Delta k_i = \varnothing(\Delta K)$.

It is assumed that $\varnothing$ is a linear variation ratio $k_{i_{\text{rate}}}$ between $K$ and $K_{i_{\text{rate}}}$ in the decision space $K_{\text{min}} \leq K \leq K_{\text{max}}$, $K_{i_{\text{min}}} \leq K_{i_{\text{max}}} \leq K_{\text{max}}$. The maximum values guarantee the highest disturbance attenuation for the desired robustness in the presence of the process uncertainty $M_{\text{max}}$. For an efficient trade-off between robustness and load disturbance attenuation, the ratio $k_{i_{\text{rate}}}$ guarantees PI controllers that minimize the IAE in the selected robustness region, $M_{\text{min}} \leq M_{\cdot} \leq M_{\text{max}}$.

For obtaining $K(\epsilon)$ the non-linear function presented in Figure 9 is proposed. It is a linear function in sections, combined with dead zone and saturation as $\Delta K(\epsilon) = K(\epsilon) - K_{\text{min}}$ for $C(s)$. The dead zone allows a linear law to close to a steady state for avoiding unnecessary switching such as can be produced by measurement noise. Saturation is a

Table 1. FPE and MSE values for the obtained models represent the displacement up and down and the maximum and minimum weight of the electrodes.

| Model | FPE | MSE |
|-------|-----|-----|
| $G_{\text{up}}(s)$ | 7.979 | 6.81 |
| $G_{\text{down}}(s)$ | 0.3683 | 0.3018 |
| $G_{\text{max}}(s)$ | 0.382 | 0.3628 |
| $G_{\text{min}}(s)$ | 0.384 | 0.3745 |

Figure 8. Response of the models of equations a) (4) and b) (5), corresponding to the electrode’s upward and downward displacement, respectively.

Figure 9. Response of the models of equations a) (6) and b) (7), corresponding to the maximum and minimum weight of the electrodes, respectively.
requirement of the NPI-RR, because of the robustness constraint \( M_{i - \text{max}} \) induces the maximum values \( K_{\text{max}}^*, K_{i \text{max}}^* \). Then \( K(e) \) is:

\[
K(e) = \begin{cases} 
K_{\text{min}}^* \mid e \mid \leq e_1 \\
K_{\text{min}}^* + (K_{\text{max}} - K_{\text{min}}) \mid e - e_1 \mid > e > -e_2 \\
K_{\text{min}}^* + (K_{\text{max}} - K_{\text{min}}) \mid e \mid < e < e_2 \\
K_{\text{min}}^* + (K_{\text{max}} - K_{\text{min}}) \mid e \mid \geq e_2
\end{cases}
\]

(16)

Where \( K_{\text{rate}} \) is the rate of variation of \( K \), \( e \) is the control error and \( e_1, e_2 \) are its values for the vertices of Figure 13.

The method proposes \( e_1 = \max \{ \Delta H_{\text{max}} \} \) and \( K_i = K_{i \text{max}}^* \). For determining \( e_2, K_0 \) and \( K_{\text{rate}} \) the decision space (\( e_2 < e_1 < \epsilon_{\text{max}} \)) and (\( K_i < K_0 < K_{\text{max}}^* \)) is defined. The parameter \( \epsilon_{\text{max}} \) is obtained from the response of the PI controller with PI and \( K_{\text{min}}^* \) under the critical disturbance of the process (\( d_i \)).

For tuning (16), achieving an adequate trade-off between disturbances attenuation and control effort, a multi-objective function is used with three global performance indexes such as ISE, IAE, and the Total Variation (TV).

a) The ISE, IAE, and TV values are determined to obtain the control system’s response subjected to \( d_i \) at a time \( t_1 \) using the PI controller corresponding to \( K_{\text{min}}^*, K_{i \text{min}}^* \) in a total time of simulation \( t_{\text{sim}} = t_1 + 2T_s \), and with a sampling time \( t_{\text{samp}} \):

\[
\text{ISE} = \int_0^{t_{\text{sim}}} \epsilon^2(t)dt
\]

(17)

\[
\text{IAE} = \int_0^{t_{\text{sim}}} |\epsilon(t)|dt
\]

(18)

\[
\text{TV} = \sum_{i=1}^{m} |u_{i+1} - u_i|
\]

(19)

b) \( K_{\text{rate}} \) is calculated by expression (20), selecting \( e_0(i) = e_1 + (\epsilon_{\text{max}} - e_1) \cdot i \) and \( K_0(q) = K_i + (K_{\text{max}} - K_i) \cdot q \) given the sample steps \( i, q (i = e, 1(e^{-1}), q = 0, 1) \) within the decision space (\( e_0(i) < \epsilon_{\text{max}} < K_{\text{max}}^* \)) and (\( K_i < K_0(q) < K_{\text{max}}^* \)).

\[
K_{\text{rate}}(i, q) = \frac{K_0(q) - K_i}{e_0(i) - e_1}
\]

(20)

c) Eqs. (17), (18), and (19) are determined for obtaining the control system response under conditions like point a) but using the control law given in (15) and tuned by (16) with \( e_0(i) \) and \( K_{\text{rate}}(i, q) \).

d) Steps b) and c) are repeated for the \( e_0(i) \) and \( K_0(q) \) required to evaluate (21).
\[ \text{where } f_k \text{ represents (17), (18), and (19) respectively, } w_k \text{ is the vector of weights such that } w_k > 0 \text{ and } \sum_{k=1}^{3} w_k = 1. \text{ The } w_k \text{ values depend on the design context.} \]

Considering the steps described above, for the adjustment of the NPI-RR controller under conditions of the medium weight of the electrodes and \(1.4 \leq M_e = M_i \leq 2\), the control law given in expression (22) was obtained:

\[ C(s) = \left(1 + \frac{0.2216}{s}\right) \Delta K(e) + 1.0554 + \frac{0.0224}{s} \]  \hspace{1cm} (22)

in addition, the tuned non-linear function appears (23):

\[ k(e) = \begin{cases} 
1.0554 + [-6.7935(e + 0.0444)] & \text{if } 0.0444 > e > -0.3231 \\
1.0554 + [-6.7935(e - 0.0444)] & \text{if } 0.0444 < e < 0.3231 \\
3.2812 & \text{if } |e| \geq 0.444
\end{cases} \]  \hspace{1cm} (23)

Figure 13 illustrates the system responses with various controllers subjected to reference changes, load disturbances, and the progressive decrease of the electrode weight. Among the controllers are the
A proportional controller used in the plant (P Empirical), the NPI-RR controller, the adjusted one using the MATLAB PID Tuner design tool (24), the AMIGO with $M_i = 1.4$ (25), and with $M_i = 2.0$ (26) and the adjusted with the Lambda method (27):

$$C(s) = 4.2134 \left( 1 + \frac{0.0095}{s} \right)$$ (24)

$$C(s) = 1.7526 \left( 1 + \frac{0.0974}{s} \right)$$ (25)

$$C(s) = 3.8503 \left( 1 + \frac{0.1719}{s} \right)$$ (26)

$$C(s) = 2.5794 \left( 1 + \frac{0.0714}{s} \right)$$ (27)

Table 3 shows the robustness of the controller, the performance indexes variations IAE, ISE, TV, and the multi-objective function $f_{mult}$; in the latter, IAE and ISE have twice the weight of the TV. These variations reflect differences in the performance of the controllers. Notably, the NPI-RR controller provides the best performance because it is the one that achieves the minimum value of the multi-objective function. Therefore, it guarantees the desired robustness and the best trade-off between the load disturbances attenuation and the control effort.

Table 3. Comparison of various controller’s performance.

| Method      | Robustness | IAE  | ISE  | TV  | $f_{mult}$ |
|-------------|------------|------|------|-----|------------|
| P (Empirical) | Unknown | 359.15 | 181.65 | 15.92 | 219.5       |
| PI (PID Tuner) | Unknown | 143.28 | 61.63  | 23.77 | 86.71       |
| PI (AMIGO) | $M_i = 1.4$ | 177.18 | 55.23  | 22.12 | 97.39       |
| PI (AMIGO) | $M_i = 2.0$ | 208.81 | 53.64  | 37.65 | 112.51      |
| PI (Lambda) | Unknown | 166.07 | 54.60  | 22.09 | 92.69       |
| NPI-RR | $M_i = 1.4 \rightarrow 2.0$ | 115.64 | 38.71  | 73.62 | 76.46       |

4. Conclusions

This article provides information on modeling techniques for EAF actuators, specifically LF actuators. Based on this knowledge and classical methods of process identification, experiments were designed and carried out to obtain the models of a hydraulic actuator for a ladle furnace. Also, control strategies were evaluated, and a novel one was selected.

By applying the combination of a signal for experimental identification, given by the previous application of a step in the control variable and later pseudo-random binary signals, several models were obtained for describing the dynamic behavior with a fit greater than 85% of the hydraulic actuator. The study shows that it is an LPV system, corroborating the asymmetric behavior of the actuator, whose upward displacement speed can be three times higher than in the downstream direction due to changes in the model’s gain. Also, it was shown that the electrode weight variation causes variations in all model parameters, affecting the dynamic behavior of the control system.

Several tuned controllers with different methods were evaluated to compensate for the process dynamics. The novel use of a non-linear controller of guaranteed robustness or NPI-RR provides the best trade-off between load disturbances attenuation and control effort. In the future, it is recommended to evaluate the possibility that the NPI-RR controller uses an asymmetric optimal non-linear function for improving the compensation of the process’s asymmetric behavior. In addition, implementing the results presented may contribute to the energy efficiency of this process.

Declarations

Author contribution statement

José Ricardo Núñez Alvarez: Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Yeizabet Nápoles-Báez, Guillermo González-Yero: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.
