THE REDUCED HOMFLY-PT HOMOLOGY FOR THE CONWAY AND THE KINOSHITA-TERASAKA KNOTS

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ABSTRACT. In this paper we compute the reduced HOMFLY-PT homologies of the Conway and the Kinoshita-Terasaka knots and show that they are isomorphic.

1. INTRODUCTION

In this paper we use Rasmussen’s results in [4] to compute the reduced HOMFLY-PT homologies, defined by Khovanov and Rozansky [2], of the Conway and the Kinoshita-Terasaka knots. It turns out that these homologies are isomorphic. We also show that our calculations imply that the Khovanov-Rozansky $sl(N)$-homologies of these two knots are isomorphic for all $N \geq 2$. This result surprised us because the Floer knot homologies of these knots are non-isomorphic [3]. Since people have conjectured that for each knot there should exist a spectral sequence converging to the Floer knot homology with $E_2$-page isomorphic to the HOMFLY-PT homology, our result shows that the differentials of the conjectured spectral sequences for the Kinoshita-Terasaka and the Conway knot should be different.

We did our calculations in the summer of 2006 and simply put them in a drawer. Since then several people, who knew about the result, asked us to write it up, which is why we finally decided to write this small note.

We claim no original theoretical insights. We have simply used Rasmussen’s results. Since these calculations are hard and we had to use all sorts of tricks, this note might help other people to understand Rasmussen’s results and to do calculations themselves and it also gives the reduced HOMFLY-PT homologies of the aforementioned knots, which to our knowledge were not available before.

2. RASMUSSEN’S TOOLKIT

In this section we explain Rasmussen’s results [4] which enable us to do the computations. Note that we take the usual triple gradings $(a,q,t)$ from the HOMFLY-PT homology, using the conventions in [2], rather than Rasmussen’s gradings in [4]. We thank Rasmussen for explaining the conversion rules between the two gradings.

Given a knot $K$, denote its reduced HOMFLY-PT homology by $\overline{H}(K)$ and its reduced $sl(N)$ homology by $\overline{H}_N(K)$.

**Theorem 2.1.** (Thm. 2 in [4]) For each $N > 0$ there is a spectral sequence $(E_k(N), d_k(N))$ which starts at $\overline{H}(K)$ and converges to $\overline{H}_N(K)$.

The $(a,q,t)$-degree of the $d_k(N)$ is $(-2k, 2Nk, 1)$. Note that for $N$ big enough the differentials $d_k$ are zero.
Theorem 2.2. (Thm. 3 in [4]) There is a spectral sequence \((E_k(-1), d_k(-1))\) starting at \(\overline{H}(K)\) and converging to \(\mathbb{Q}\).

The \((a, q, t)\)-degree of \(d_k(-1)\) is \((2 - 2k, 2 - 2k, 2k - 1)\). Another result that will be useful for us is the following.

**Lemma 2.3.** (Lemma 6.2 in [4]) For any \(k\) the differentials \(d_k(-1)\) and \(d_1(N)\) anticommute.

For a two-component link \(L\) denote by \(\overline{H}_N(L, i)\) the \(sl(N)\) homology of \(L\) reduced w.r.t. to the link component \(i\). For \(j \neq i\), let

\[ \overline{H}_N(L, i) \xrightarrow{X_j} \overline{H}_N(L, i) \]

be the map induced by multiplication by \(X_j\) (i.e. on the \(j\)th component of \(L\)). Note that this map has \((q, t)\)-bidegree \((2, 0)\). Let \(\overline{H}_N(L)\) be the totally reduced \(sl(N)\) homology of \(L\).

**Lemma 2.4.** There is a long exact sequence

\[ \cdots \rightarrow \overline{H}_N(L, i) \xrightarrow{X_j} \overline{H}_N(L, i) \xrightarrow{(-1,1/2)} \overline{H}_N(L) \xrightarrow{(-1,1/2)} \overline{H}_N(L, i) \rightarrow \cdots \]

We have not given all the maps explicitly, since we do not need them, but we have given their bidegrees.

Let \(K_+\) be knot with a given positive crossing. Let \(K_-\) be the same knot except for that particular crossing which is now negative and let \(K_0\) be the two-component link obtained from \(K\) by the oriented resolution of the same crossing.

**Lemma 2.5.** (Lemma 7.6 in [4]) There is a long exact sequence

\[ \cdots \rightarrow \overline{H}_N(K_-) \xrightarrow{(N,-1/2)} \overline{H}_N(K_0) \xrightarrow{(N,-1/2)} \overline{H}_N(K_+) \xrightarrow{(-2N,2)} \overline{H}_N(K_-) \rightarrow \cdots \]

Let us do a simple example to illustrate the exact sequences above. By abuse of notation we always identify the homology with the Poincaré polynomial. Thus, multiplying the homology by a polynomial means multiplying the Poincaré polynomial by that polynomial.

Recall (see [4]) that the positive Hopf link, \(\mathcal{H}^+\), has reduced HOMFLY-PT homology equal to

\[ aq^{-1} + q(qNt-1)^2(q^{-N+2} + q^{-N+4} + \cdots + q^{N-2}) .\]

Note that it does not matter which component we choose for reduction in this case. Multiplication by \(X\) maps the generator of degree \(q^i\) to the generator of degree \(q^{i+2}\) in \(q^{-N+2} + q^{-N+4} + \cdots + q^{N-2}\). Therefore the kernel of

\[ \overline{H}_N(\mathcal{H}^+, i) \xrightarrow{X_j} \overline{H}_N(\mathcal{H}^+, i) \]

is given by the generators \(aq^{-1} + q^{N-1}(q^Nt-1)^2\) and the cokernel by the generators \(aq^{-1} + q^{-N+3}(q^Nt-1)^2\). Using the exact sequence (1) we see that

\[ \overline{H}(\mathcal{H}^+) = a^2 t^{-5/2} + at^{-1/2} + aq^2 t^{-3/2} .\]

Let \(K_+\) be the positive trefoil \(\mathcal{T}^+\), then (see [4])

\[ \overline{H}(K_+) = a^2 q^{-2} + a^2 q^2 t^{-2} + a^4 t^{-3} .\]

Note that \(K_0 = \mathcal{H}^+\) and \(K_-\) is the unknot. One now easily checks that the long exact sequence (2) holds.
There is also a useful variant of the exact sequence \(1\). Let \(L_-\) and \(L_+\) be the two-component link diagrams which differ by the sign of one crossing between different components and \(K\) the knot diagram obtained by resolving that crossing respecting the orientations.

**Lemma 2.6.** There is a long exact sequence

\[
\cdots \to \overline{H}_N(L_-) \xrightarrow{(N,-1/2)} \overline{H}_N(K) \xrightarrow{(N,-1/2)} \overline{H}_N(L_+) \xrightarrow{(-2N,2)} \overline{H}_N(L_-) \to \cdots
\]

Notice that \(X\) acts as zero on \(\overline{H}_N(K)\) and

\[
\overline{H}_N(K) = \text{Cone}(\overline{H}_N(K) \xrightarrow{0} \overline{H}_N(K)) = \overline{H}_N(K)(q^{-1}t^{1/2} + qt^{-1/2}).
\]

3. Computations

In this section we compute the HOMFLY-PT homologies of the Conway and the Kinoshita-Terasaka knots. Resolving and changing a particular crossing of a diagram results in simpler diagrams where homology can be computed more easily. We then use the long exact sequence \(2\) recursively to obtain the homology of the initial diagram. The spectral sequences of Theorems 2.1 and 2.2 and the knowledge of the \(sl(N)\) homology of the initial diagram help us to unambiguously identify isomorphisms and zero maps in the long exact sequences \(2\) and \(3\).

The HOMFLY-PT polynomial of the Conway and Kinoshita-Terasaka knots is

\[
\mathcal{P}(K_{\text{Conway}}) = \mathcal{P}(K_{\text{KT}}) = a^{-4}(q^{-4} - q^{-2} + 2 - q^2 + q^4) + a^{-2}(-q^{-6} - 2q^{-2} - 2q^2 - q^6) + (q^{-6} + 2q^{-2} + 1 + 2q^2 + q^6) + a^2(-q^{-4} + q^{-2} - 2 + q^2 - q^4),
\]

and their reduced Khovanov homologies are isomorphic and given by

\[
\overline{Kh}(K_{\text{Conway}}) = \overline{Kh}(K_{\text{KT}}) = q^8 t^{-5} + 2q^6 t^{-4} + 2q^4 t^{-3} + 3q^2 t^{-2} + (3 + q^2) t^{-1} + 3 + 2q^{-2} + (2q^{-2} + 2q^{-4}) t + (3q^{-4} + q^{-6}) t^2 + 3q^{-6} t^3 + 2q^{-8} t^4 + 2q^{-10} t^5 + q^{-12} t^6.
\]

From Theorem 2.1 it follows that \(\dim \overline{H}(K_{\text{Conway}}) \geq 33\) and \(\dim \overline{H}(K_{\text{KT}}) \geq 33\).

3.1. The Kinoshita-Terasaka knot. A diagram of the KT knot is given in Figure 1. Changing the encircled crossing we get the unknot, while resolving it results in the two-component link diagram \(K_0\) depicted in Figure 2. Notice that \(K_0\) is the pretzel link \(P(3,-2,2,-3)\) which is am-

![Figure 1: The Kinoshita-Terasaka knot](image-url)
phicheiral \((K_0 = K_0^\phi)\). This diagram corresponds to the (non-alternating) link \(L_{10n36}\) in Thistlethwaite’s link table \([1]\).

Taking the oriented resolution and changing the encircled crossing of \(K_0\) we obtain the diagrams \(M_0\) and \(M_+\) respectively, depicted in Figure 3. The diagram \(M_0\) corresponds to the pretzel knot \(P(3,−1,2,−3)\) and is isotopic to the Pretzel knot \(P(3,−2,−3)\), which in turn is the mirror of the knot 8\(_{20}\) in Rolfsen table \([1]\). We have

\[
\overline{H}(M_0) = \overline{H}(8_{20}^1) = a^4 q^3 t^{-11/2} + (a^2 q^5 + a^4 q)t^{-9/2} + (a^4 q^{-1} + a^2 q^3)t^{-7/2}
\]

\[
+ (2a^2 q + a^4 q^{-3})t^{-5/2} + (q^3 + 2a^2 q^{-1})t^{-3/2} + (2q + a^2 q^{-3})t^{-1/2}
\]

\[
+ (2q^{-1} + a^2 q^{-5})t^{1/2} + q^{-3}t^{3/2}.
\]

The diagram \(M_+\) is the connected sum of the (positive) Hopf link with the connected sum of the positive trefoil \(T^+\) and the negative trefoil \(T^-\). From Lemma 7.8 of \([4]\) for connected sums it follows that

\[
\text{H}(M_+) = \text{H}(T^+) + \text{H}(T^-)
\]
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\begin{align*}
\mathcal{H}(M_+) = \mathcal{H}(M^+) \otimes \mathcal{H}(T^+) \otimes \mathcal{H}(T^-) \\
= a^2q^{-3} + aq^3t^{-2} + a^2q^{-3}t^{-1} + 3aq^{-1} + a^{-1}qt + aq^{-5}t^2 + a^{-1}q^{-3}t^3 \\
+ |N - 1| \left( \alpha^3q^3 t^{-5} + a^2q^5 t^{-4} + a^2q^{-1}q^{-1} + 3a^2qt^{-2} + qt^{-1} + a^2q^{-3} + q^{-1}t \right).
\end{align*}

We can now determine \( \mathcal{H}(K_0) \) using the long exact sequence (3) where the diagrams \( K_0, M_0 \) and \( M_+ \) correspond to \( L_-, K \) and \( L_+ \) respectively. It reads

\[ \cdots \longrightarrow \mathcal{H}(K_0) \xrightarrow{\alpha^{-1/2}} \mathcal{H}(M_0) \xrightarrow{\alpha^{-1/2}} \mathcal{H}(M_+) \xrightarrow{\alpha^{-2/2}} \mathcal{H}(K_0) \longrightarrow \cdots \]

From Equations (3) and (5) and comparing degrees we have that \( t^{-1/2}a^4q^3, t^{-1/2}a^4q^{-1} \) and \( t^{-1/2}q \) are in the kernel of the map \( \mathcal{H}(M_0) \xrightarrow{\alpha^{-1/2}} \mathcal{H}(M_+) \) and that \( t^{-5}a^4q^2(q^{N-3} + \cdots + q^{5-N}), t^{-4}a^2q^4(q^{N-3} + \cdots + q^{3-N}), t^{-3}aq^6(q^{N-3} + \cdots + q^{5-N}), t^{-2}a^2q^8(q^{N-3} + \cdots + q^{3-N}), t^{-1}aq^{10}(q^{N-3} + \cdots + q^{3-N}), t^{-2}aq^{12}(q^{N-3} + \cdots + q^{3-N}), t^{-1}aq^{14}(q^{N-3} + \cdots + q^{3-N}) \) and \( t^{-3}a^{-1}q^{-3} \) are in the cokernel. Using this we can form a first list of guaranteed and possible generators of \( \mathcal{H}(K_0) \). Since \( K_3 = K_0 \) the polynomial of \( \mathcal{H}(K_0) \) has to be invariant under the transformation \( \psi(a,q,t) = (a^{-1},q^{-1},t^{-1}) \). To have this symmetry we need to promote some possible generators to generators of \( \mathcal{H}(K_0) \) and discard possible generators not paired by \( \psi \). Then we apply the exact sequence (3) again to the newly promoted generators to obtain a new list which is in Table 1.

| \( t^i \) | guaranteed | possible (from \( \mathcal{H}(M_0) \)) | possible (from \( \mathcal{H}(M_+) \)) |
|---|---|---|---|
| \( t^{-3} \) | \( a^3q^3 \) | \( aq \) | \( aq^3 \) |
| \( t^{-4} \) | \( aq^3 \) | \( a^{-1}q^{-3} \) | \( aq^{-3} \) |
| \( t^{-5} \) | \( a^3q^{-1} + aq^3 + a^2q^2(q^{N-4} + \cdots + q^{5-N}) \) | \( aq^3 \) | \( aq^{-3} \) |
| \( t^{-2} \) | \( aq^3(q^{N-3} + \cdots + q^{N+3}) \) | \( a^{-1}q^3 + a^{-1}q^{-3} \) | \( a^{-1}q^{-3} \) |
| \( t^{-1} \) | \( aq^3(q^{N-4} + \cdots + q^{N+4}) \) | \( a^{-1}q^3 + a^{-1}q^{-3} \) | \( aq^{-3} \) |
| \( t^0 \) | \( a^{-1}q^3 + a^{-1}q^{-3} \) | \( a^{-1}q^3 + a^{-1}q^{-3} \) | \( aq^{-3} \) |
| \( t^1 \) | \( a^{-2}q^2(q^{N-3} + \cdots + q^{N+3}) \) | \( a^{-1}q^{-1} \) | \( a^{-1}q^{-1} \) |
| \( t^2 \) | \( a^{-2}q^2(q^{N-4} + \cdots + q^{N+4}) \) | \( a^{-1}q^{-1} \) | \( a^{-1}q^{-1} \) |
| \( t^3 \) | \( a^{-2}q^2(q^{N-5} + \cdots + q^{N+5}) \) | \( a^{-1}q^{-1} \) | \( a^{-1}q^{-1} \) |
| \( t^4 \) | \( a^{-1}q^{-5} \) | \( a^{-1}q^{-5} \) | \( a^{-1}q^{-5} \) |
| \( t^5 \) | \( a^{-1}q^{-3} \) | \( a^{-1}q^{-3} \) | \( a^{-1}q^{-3} \) |

The dimension of \( E_\infty(1) \) has to be 1, living in homological degree 0. A straightforward computation shows that we already have this convergence in the column of guaranteed generators of \( \mathcal{H}(K_0) \). By inspection we see that if we promote the generator \( t^{-3}a^3q^3 \) in the column of possible generators from \( \mathcal{H}(M_0) \) than it would survive in \( E_\infty(1) \). Therefore the exact sequence (3) and the symmetry under \( \psi \) imply that \( t^{-3}a^3q^3, t^{-2}a^3q^3, t^{-1}a^{-1}q^{-3} \) and \( t^{-3}a^{-1}q^{-3} \) must be discarded from the list of possible generators of \( \mathcal{H}(K_0) \). We present the updated list in Table 2.
Using the exact sequence (1) we obtain

$$K_h(K_0) = a^3 q^3 t^{-5} + a q^5 t^{-4} + (a^3 q^{-1} + a^2 q^2 [N-2]) t^{-3} + (2aq + q^4 [N-2]) t^{-2}$$

$$+ (aq^{-3} + a^{-1} q^{-1} + a^{-1} q^{-2} [N-2]) t + (2a^{-1} q^{-1} + a^{-2} q^{-2} [N-2]) t^2$$

$$+ (a^{-3} q + a^{-2} q^{-2} [N-2]) t^3 + a^{-1} q^{-5} t^4 + a^{-3} q^{-3} t^5. \tag{8}$$

Using the exact sequence (1) we obtain

$$K_h(K_0) = (qt^{-1/2} + q^{-1/2}) \left[ a^3 q^3 t^{-5} + a q^5 t^{-4} + a^3 q^{-1} t^{-3} + 2aq^{-2} + (a^{-1} q^3 + aq)t^{-1} \right.$$

$$+ (a^{-1} q^3 + aq^{-3}) + (aq^{-3} + a^{-1} q^{-1}) + 2a^{-1} q^{-1} t^2 + a^{-3} q^{-3} t^3 + a^{-1} q^{-5} t^4 + a^{-3} q^{-3} t^5$$

$$+ \left. q^{-1} t^{1/2} (2a^3 q^{-1} t^{-3} + aq t^{-2} + 2a^3 q^{-5} + 2aq^{-3} + a^{-1} q^{-1} + a^{-1} q^{-1} + a^{-1} q^{-5}) \right]$$

$$+ q^{-1} t^{1/2} (a q^5 + a^{-1} q^7 + aq + 2a^{-1} q^3 + a^{-1} q + a^{-3} q^5 + a^{-1} q^{-1} + a^{-3} q). \tag{9}$$

To determine whether the remaining possible generators are generators of $K_h(K_0)$ we use the spectral sequence $E_k(2)$. The reduced Khovanov homology of $K_0$ (computed from KhoHo [5] with $q \rightarrow q^{-1}$ to agree our conventions) is

$$K_h(K_0) = a^9 t^{-5} + q^7 t^{-4} + q^5 t^{-3} + 2q^4 t^{-2} + (q^3 + q)t^{-1} + 2(q + q^{-1})$$

$$+ (q^{-1} + q^{-3}) t^2 + 2q^{-3} t^3 + q^{-5} t^3 + q^{-7} t^4 + q^{-9} t^5. \tag{7}$$

To have $E_k(2) \Rightarrow K_h(K_0)$ we have to promote the generators $t^{-2} aq, t^{-1} aq, t^{-1} a^{-1} q^3, a^{-1} q^3, aq^{-3}, t a q^{-3}, t a^{-1} q^{-1}$ and $t^2 a^{-1} q^{-1}$ (recall that in our conventions $d_k(N) = a^{-2k} q^{2k+l}$). A simple computation shows that $E_k(2)$ collapses after the second page i.e. $E_2(2) = E_{\infty}(2) = K_h(K_0)$ and that by promoting the four remaining generators the spectral sequence $E_k(2)$ would not converge to $K_h(K_0)$. We leave the details to the reader. After rearranging some terms we have that

$$K_h(K_0) = a^3 q^3 t^{-5} + a q^5 t^{-4} + (a^3 q^{-1} + a^2 q^2 [N-2]) t^{-3} + (2aq + q^4 [N-2]) t^{-2}$$

$$+ (aq^{-3} + a^{-1} q^{-1} + a^{-1} q^{-2} [N-2]) t + (2a^{-1} q^{-1} + a^{-2} q^{-2} [N-2]) t^2$$

$$+ (a^{-3} q + a^{-2} q^{-2} [N-2]) t^3 + a^{-1} q^{-5} t^4 + a^{-3} q^{-3} t^5. \tag{8}$$

Using the exact sequence (1) we obtain

$$K_h(K_0) = (qt^{-1/2} + q^{-1/2}) \left[ a^3 q^3 t^{-5} + a q^5 t^{-4} + a^3 q^{-1} t^{-3} + 2aq^{-2} + (a^{-1} q^3 + aq)t^{-1} \right.$$

$$+ (a^{-1} q^3 + aq^{-3}) + (aq^{-3} + a^{-1} q^{-1}) + 2a^{-1} q^{-1} t^2 + a^{-3} q^{-3} t^3 + a^{-1} q^{-5} t^4 + a^{-3} q^{-3} t^5$$

$$+ \left. q^{-1} t^{1/2} (2a^3 q^{-1} t^{-3} + aq t^{-2} + 2a^3 q^{-5} + 2aq^{-3} + a^{-1} q^{-1} + a^{-1} q^{-1} + a^{-1} q^{-5}) \right]$$

$$+ q^{-1} t^{1/2} (a q^5 + a^{-1} q^7 + aq + 2a^{-1} q^3 + a^{-1} q + a^{-3} q^5 + a^{-1} q^{-1} + a^{-3} q). \tag{9}$$
Finally, comparing $\frac{1}{2} \overline{\Pi}(K_0)$ to $\overline{\Pi}(\text{unknot})$ gives

$$
\overline{\Pi}(K_{KT}) = a^2 q^4 t^5 + (q^6 + a^2 q^2) t^{-4} + (2a^2 + q^4) t^{-3} + (3q^2 + a^2 q^{-2} + q^4) t^{-2}
+ (a^{-2} q^6 + a^2 q^{-4} + a^{-2} q^{-4} + 2 + q^2) t^{-1} + (3 + 3q^{-2} + a^{-2} q^2 + a^{-2} q^4)
+ (3a^{-2} q^2 + q^{-2} + 2a^{-2} + q^{-4}) t + (a^{-4} q^4 + q^{-6} + 2a^{-2} + q^{-4} + a^{-2} q^{-2}) t^2
+ (3a^{-2} q^{-2} + a^{-4} q^2 + a^{-2} q^{-4}) t^3 + (2a^{-4} + a^{-2} q^{-4}) t^4
+ (a^{-4} q^2 + a^{-2} q^{-6}) t^5 + a^{-4} q^{-4} t^6.
$$

3.2. The Conway knot. We follow the same method as in the calculation for the Kinoshita-Terasaka knot. A diagram of the Conway knot is given in Figure 4.

Figure 4: The Conway knot

Changing the (negative) encircled crossing we obtain the unknot, while resolving it results in the two-component link diagram $L_0$ of Figure 5 and corresponds to the Pretzel link $P(3, -2, -3, 2)$ which corresponds to the link $L_{10n59}$ in Thistlethwaite’s table [1] and is amphicheiral. Using KhoHo [5] we find that

$$
\overline{Kh}(L_{10n59}) = \overline{Kh}(L_{10n36}),
$$

with $\overline{Kh}(L_{10n36})$ given in Equation 7.

Changing and taking the oriented resolution of the encircled crossing of $L_0$ we obtain the diagrams $N_0$ and $N_+ \text{ of Figure 6}$. The diagram $N_+$ is isotopic to the connected sum of the (positive) Hopf link and the connected sum of positive trefoil and negative trefoil. The diagram $N_0$ is isotopic to the Pretzel knot $P(3, -3, 2)$ which in turn corresponds to the mirror image of the knot $8_{20}$. The diagrams $N_0$ and $N_+$ are therefore isotopic to the diagrams $M_0$ and $M_+$ of Subsection 3.1 respectively. This means that $\overline{\Pi}(L_0) = \overline{\Pi}(K_0)$ that is

$$
\overline{\Pi}(P(3, -2, -3, 2)) = \overline{\Pi}(P(3, -2, 2, -3)).
$$

Since the other diagram obtained in the first step from the diagram for the Conway knot is the unknot (as in Subsection 3.1) we see that the HOMFLY-PT homologies of the Conway and
Figure 5: Diagram $L_0$ obtained by taking the oriented resolution of the encircled crossing of the diagram of the Conway knot of Figure 1. The (negative) encircled crossing is used for the long exact sequence $\text{(3)}$.

Figure 6: Diagrams $N_+$ and $N_0$ obtained by taking the oriented resolution of the encircled crossing of the diagram $L_0$ of Figure 5.

Kinoshita-Terasaka knots are isomorphic and given by

$$\Pi(K_{\text{Conway}}) = \Pi(K_{\text{KT}}) = a^2 q^4 t^{-5} + (q^6 + a^2 q^2) t^{-4} + (2 a^2 + q^4) t^{-3} + (3 q^2 + a^2 q^{-2} + q^4) t^{-2}$$
$$+ (a^{-2} q^6 + a^2 q^{-4} + a^{-2} q^4 + 2 + q^2) t^{-1} + (3 + 3 q^2 + a^{-2} q^2 + a^{-2} q^4)$$
$$+ (3 a^{-2} q^2 + q^{-2} + 2 a^{-2} + q^{-4}) t + (a^{-4} q^4 + q^{-6} + 2 a^{-2} + q^{-4} + a^{-2} q^{-2}) t^2$$
$$+ (3 a^{-2} q^{-2} + a^{-4} q^2 + a^{-2} q^{-4}) t^3 + (2 a^{-4} + a^{-2} q^{-4}) t^4$$
$$+ (a^{-4} q^{-2} + a^{-2} q^{-6}) t^5 + a^{-4} q^{-4} t^6.$$ 

Note that $d_k(N) = 0$ for $N \geq 3$ for $\Pi(K_{\text{Conway}})$ and $\Pi(K_{\text{KT}})$ for degree reasons. This implies that $\Pi_N(K_{\text{Conway}}) = \Pi_N(K_{\text{KT}})$ for all $N \geq 3$. We already knew that the same holds for $N = 2$ by direct computation. Since the knot Floer homology of these two knots differ $\text{(3)}$, this fact
might be interesting for someone trying to find a relation (e.g. a spectral sequence) between Khovanov-Rozansky homology and knot Floer homology.

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