Quantum and medium effects in (resonant) leptogenesis

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Abstract. Leptogenesis offers a very attractive explanation for the origin of the baryon asymmetry of the universe. Such scenarios based on leptonic CP-violation can be realized already within minimalistic seesaw extensions of the standard model. Apart from model building issues the answer to the question of whether a given particle theory can explain the observed baryon number density depends also on the detailed statistical evolution of the asymmetry. The CP-violation within a given model leads to an asymmetry only if it is accompanied by an out-of-equilibrium evolution in the early universe. Most existing analyses employ Boltzmann-like equations (BEs) to describe it. In this context fundamental issues arise which can be addressed in the framework of non-equilibrium quantum field theory (NEQFT). Here, the relevance of quantum and medium effects for thermal leptogenesis is investigated. Within the 2PI-formalism of NEQFT, questions such as that for the justification of the particle picture arise naturally in subsequent approximations when BEs are to be derived. This specific problem is particularly important in the case of resonant leptogenesis where the relevant particle states are almost degenerate in mass. It is found that Boltzmann like equations can (only) be obtained in certain cases. But it is then possible to account for corrections due to quantum and medium effects.

1. Introduction
Leptogenesis is today the most popular mechanism for the generation of the cosmological baryon asymmetry. Its prerequisites are realized by many conceivable extensions of the standard model of particle physics (SM). In particular they are met by moderate extensions where fermionic singlets, scalar or fermionic triplet fields are added to the theory. These seesaw extensions have the additional benefit of explaining the hierarchy between known light neutrino masses and those of charged SM fermions. A lot of attention was payed in the past to the implications of variations of the models and their parameters for leptogenesis. Fundamental issues related to the non-equilibrium statistical description, in turn, have been thoroughly investigated only recently. The progress here is manly based on the application of the 2PI-formalism of NEQFT which has helped to gain a deeper understanding of leptogenesis [1][2][3][4][5][6][7][8][9][10][11]. Also new results based on thermal quantum field theory and semi-classical analyses brought relevant new quantitative results [12][13][14][15][16][17]. Amongst the proposed leptogenesis scenarios the most favoured one is that of ‘thermal leptogenesis’ where two or more right-handed neutrino fields are added to the standard theory:

\[
\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2} \bar{N}_i \left( i \gamma^\mu \partial_\mu - M_i \right) N_i - h_{\alpha \bar{\ell}_\alpha} \bar{\ell}_\alpha \phi P_R N_i - h^\dagger_{\alpha \ell} \bar{N}_i \phi^\dagger P_L \ell_\alpha. \tag{1}
\]

Here \( N_i = N_i^c \) are the heavy Majorana fields, \( \ell_\alpha \) are the lepton doublets, \( \bar{\phi} \equiv i \sigma^2 \phi^* \) is the conjugate of the Higgs doublet, and \( h \) are the neutrino Yukawa couplings. The second ingredient is a standard cosmological evolution with high-temperature reheating. With heavy right handed neutrino masses \( M_i \) at a GUT-like scale, scenarios can comfortably be realized in which \( N_i \) stay relatively close to thermal...
equilibrium before they decay at $T \sim M_i$. The conventional analysis of this non-equilibrium process is based on cosmological 'rate equations' for the $(B - L)$-asymmetry $Y_L$ and the abundance of the heavy Majorana neutrinos $Y_{N_i}$ during this epoch. The coefficients for these equations ('reaction densities') are computed as Boltzmann-like collision integrals based on $S$-matrix amplitudes. The reaction densities for (inverse) decays, washout determine together with the $CP$-violating parameters $\epsilon_i$, the amount of asymmetry produced. In a perturbative approximation of order $h^4$ the graphs depicted in Fig. 1 contribute to the $CP$-violating decay. At the same order the two-body scattering processes in Fig. 2 need to be taken into account. We ignore further processes involving other SM fermions and gauge interactions as they are relevant but largely independent of the present analysis. Issues (specifically the double-counting problem) related to the insufficiency of this approach are usually fixed in an ad-hoc way to restore the compatibility with fundamental conservation laws. In equilibrium the requirement of unitarity of the $S$-matrix leads to well posed conditions between the amplitudes for decays and scattering which guarantee conserving equations but these are not applicable out-of-equilibrium. Also the question to which extent medium effects can be relevant arises naturally in the outlined picture of a dynamical asymmetry generation in the hot early universe.

In a variation of this simple scenario at least two of the heavy Majorana neutrinos are almost degenerate in mass $M_1 \simeq M_2$. This specific choice (which can for instance be motivated as a consequence of a radiatively broken symmetry) entails the benefit that $\epsilon_{1,2}$ can be sufficiently enhanced to lower the mass scale for the hypothetical heavy neutrino to $\sim 1$ TeV. Thermal leptogenesis can therefore be reconciled with lower reheating temperatures and the masses $M_{1,2}$ become accessible to collider experiments. The resonant contribution to the $CP$-violating parameter is maximized if the mass splitting $|M_1 - M_2|$ is comparable to the width $\Gamma_{1,2}$. In this regime peculiar issues related to the definition of quasi-particle states arise. These become evident when a first principle computation within NEQFT is performed as the spectrum of the quasi-particle excitations defining $N_{1,2}$ has to be specified in due course [3]. In the same regime oscillations between the Majorana neutrinos can become relevant and the simple description in terms of rate equations and a few reaction densities can prove to be untenable.

2. Nonequilibrium QFT

The 2PI-formalism in NEQFT represents a suitable framework for the analysis of quantum statistical processes out-of-equilibrium. It is known to yield consistent transport equations at a given truncation of a
perturbative expansion and to evade the double-counting problem. Furthermore it yields equations which take medium effects into account. Upon a number of controllable approximations simple rate equations may be obtained which can be compared with conventional results. The most straight-forward access to leptogenesis is via the divergence

$$D_{\mu} j^\mu_L(x)$$

of the expectation value of the lepton current operator

$$j^\mu_L(x) = \left\langle \sum_{\alpha,a} \bar{\ell}^a_\alpha(x) \gamma^\mu \ell^a_\alpha(x) \right\rangle = - \sum_{\alpha,a} \text{tr} \left[ \gamma_\mu S^{\alpha\alpha}_{aa}(x,x) \right].$$

The second equality uses the definition of the closed-time-path (CTP) propagator for leptons (the indices are for flavour (greek) and $SU(2)$ (latin)),

$$S^{\alpha\beta}_{ab}(x,y) = \left\langle T C \bar{\ell}^a_\alpha(x) \ell^b_\beta(y) \right\rangle.$$  

(2a)

Similar relations hold for other propagators such as those of leptons and Higgs. For the full lepton propagator $S$ (here in matrix notation) the 2PI-formalism provides a self-consistent Schwinger-Dyson equation

$$S^{-1}(x,y) = S_0^{-1}(x,y) - \Sigma(x,y),$$  

(3)

where the lepton self-energy $\Sigma$ is obtained by functional differentiation of the 2PI effective action $\Gamma_{2\text{PI}}$ with respect to $S$. $\Gamma_{2\text{PI}}$ can be approximated by the three-loop truncation in Fig. 3(a),(b).

![Figure 3](https://example.com/figure3.png)

**Figure 3.** Two- (a) and three-loop (b) contributions to the 2PI effective action and the corresponding contributions to the lepton self-energy (c), (d). At this order all processes contributing to the $S$-matrix elements squared find their equivalent in the 2PI-derivation.

The divergence of the lepton current leads, upon gradient expansion and Wigner-transformation, to (the subscripts $>$, $<$ refer to greater (lesser) components of the ‘Wightman-propagators’):

$$D_{\mu} j^\mu_L(t) = 2 \int_0^\infty \frac{dp_0}{2\pi} \int \frac{d^3p}{(2\pi)^3} \text{tr} \left\{ \left[ \Sigma_{\alpha\beta}^\alpha (t,p) S_{\alpha\beta}^\beta (t,p) - \Sigma_{\alpha\beta}^{\alpha\beta} (t,p) S_{\alpha\beta}^{\alpha\beta} (t,p) \right] - \left[ \Sigma_{\alpha\beta}^{\alpha\beta} (t,p) S_{\alpha\beta}^\beta (t,p) - \Sigma_{\alpha\beta}^{\alpha\beta} (t,p) S_{\alpha\beta}^{\alpha\beta} (t,p) \right] \right\}. $$

(4)

The lepton self-energies are functionals of the Majorana-neutrino- and Higgs-propagators $\mathcal{S}$ and $\Delta$ as depicted in Fig. 3(c),(d). These are governed by their respective Schwinger-Dyson equations similar to Eq. (3). Further simplification of the evolution equation for the lepton current can be achieved if their solution yields additional analytic information about the propagators. In particular, if the spectral dependence of the propagators is that of a sharply peaked on-shell state, as is the case for leptons and Higgs. It is then valid to introduce a ‘quasi-particle approximation’ and the degrees of freedom in the propagators $S$, $\Delta$ are reduced to those of one-particle distribution functions $f_\ell$ and $f_\phi$ which depend only on space, time and the three-momenta. The approximation of the Majorana neutrino propagator $\mathcal{S}$ is more intricate because it can deviate from equilibrium and the finite spectral width can be relevant.
The vertex contribution to the CP-violating parameter arises as cut of the 2PI graph in Fig. 3(a). This diagram yields also the \( s \times u \) contributions to \( \ell \phi \leftrightarrow \bar{\ell} \phi \) as well as the \( u \times t \) contribution to \( \ell \ell \leftrightarrow \phi \phi \). To obtain the self-energy contribution, the off-diagonal elements of the Majorana neutrino propagator have to be taken into account. Furthermore an ’extended quasi-particle approximation’ needs to be employed in order to obtain the \( s \times s \) and \( u \times u \) contributions to \( \ell \phi \leftrightarrow \bar{\ell} \phi \) as well as \( u \times u \) and \( t \times t \) contributions to \( \ell \ell \leftrightarrow \phi \phi \) scattering from the setting-sun graph in Fig. 3. To a large extent, the answer to the question if conventional Boltzmann-like equations are applicable for leptogenesis depends on whether such quasi-particle approximations for the Majorana neutrino propagator are justified.

3. Non-resonant Leptogenesis

If the heavy Majorana neutrino masses are hierarchical and deviations from thermal equilibrium not too large, the quasi-particle approximation (and a ’Kadanoff-Baym’ ansatz which relates the statistical dependence of the propagators to the distribution functions) can be applied. In a homogeneous and isotropic FRW-universe, it is then possible to reduce the quantum kinetic equations to a system of rate equations for the abundances which may be compared to conventional results:

\[
\frac{sH}{z} \frac{dY_L}{dz} = \sum_i \langle \epsilon_i \gamma_D^{N_i} \rangle \left( \frac{Y_{N_i}}{\nu_{eq}^{N_i}} - 1 \right) - \frac{Y_L}{2Y_{eq}} \sum_i \langle \gamma^{W_i} \rangle + 4 \langle \gamma^{\ell \phi} \rangle + 4 \langle \gamma_{\ell \bar{\ell}} \rangle ,
\]

\[
\frac{sH}{z} \frac{dY_{N_i}}{dz} = - \langle \gamma^{D_i} \rangle \left( \frac{Y_{N_i}}{\nu_{eq}^{N_i}} - 1 \right) .
\]

Here we introduced decay reaction densities \( \langle \epsilon_i \gamma_D^{N_i} \rangle, \langle \gamma^{W_i} \rangle \) by defining the averages:

\[
\langle X \gamma_D^{N_i} \rangle \equiv \int d\Pi^{\ell_i\ell_i}_{\nu_i\nu_i} (2\pi)^4 \delta(k - p) X |M|_{N_i}^2 f_{\nu_i}^{eq} f_{\ell_i}^{eq} , \quad \langle X \gamma^{W_i} \rangle \equiv \langle X (1 - f_{\nu_i}^{eq}) \gamma_D^{N_i} \rangle ,
\]

where \( f_{\ell i} \equiv (1 - f_{\ell i}^{eq} + f_{\ell i}^{eq}) \), and \( d\Pi^{(ij)}_{\nu_i\nu_i} \) represents the complete phase space element. Similar for scattering:

\[
\langle X \gamma^{\ell \phi} \rangle \equiv \int d\Pi^{\ell \phi}_{\nu_i\nu_i} (2\pi)^4 \delta(k + p - r) X |M|_{ab+ij}^2 (1 \pm f_{\nu_i}^{eq}) (1 \pm f_{\ell_i}^{eq}) f_{\ell_i}^{eq} f_{\ell_i}^{eq} .
\]

The amplitudes squared obtained here agree with those obtained in S-matrix theory but medium effects are implied by the quantum statistical factors and the term \( f_{\ell i} \). Further such contributions show up in the (momentum dependent) CP-violating parameters. The contributions to \( \epsilon_i \) read

\[
\epsilon_i^S(q, p) = - \sum_{j \neq i} \frac{M_i \Gamma_j}{M_j^2 - M_i^2} \int \frac{d\Omega}{4\pi} (1 - \cos \theta) f_{\ell j}^{eq} ,
\]

\[
\epsilon_i^V(q, p) = - \sum_{j \neq i} \frac{\Gamma_j}{M_j} \int \frac{d\Omega}{4\pi} \frac{1 - \cos \theta}{1 + \cos \theta + 2M_j^2/M_i^2} f_{\ell j}^{eq} ,
\]

where the same dependence on the distributions, \( f_{\ell j}^{eq} \) enters. The reaction density \( \langle \gamma^{\ell \phi} \rangle \) has to be compared with the RIS-subtracted two-body scattering rate. In the narrow width limit:

\[
\langle \gamma^{\ell \phi} \rangle = \left\langle \left[ 1 - \sum_i \frac{\pi |M|_{N_i}^2}{4 |M|_{\ell \phi + \ell \bar{\ell}}} \delta(s - M_i^2) \right] \gamma^{\ell \phi} \right\rangle.
\]

A peculiar medium effect becomes relevant if, at high temperatures, the thermal Higgs mass exceeds that of the heavy Majorana neutrinos \( m_\ell + m_\phi > M_i \). In this case the Majorana neutrino decay
becomes impossible but decays of the Higgs into a Majorana neutrino and a lepton can become allowed instead. This process violates CP due to the purely thermal one-loop contributions in Fig. 4. To obtain quantitative results for thermal corrections to the CP-violating parameters, which become relevant at high temperatures, it needs to be taken into account. The main difference compared to Majorana neutrino decay consist in the replacement of \( f_{\ell \phi} = (1 - f_{\ell \phi}^e + f_{\ell \phi}^\nu) \rightarrow (f_{\ell \phi}^e + f_{\ell \phi}^\nu) \). The simple picture of temperature dependent thermal masses is altered by the modified dispersion relations obtained for leptons in the framework of a hard thermal loop resummation \([18, 13]\) or if the collinear enhancement effects described in \([15, 16, 17]\) are taken into account. At the present order (in \( \mu_\ell / T, \Delta f_{N_i} \) and \( \epsilon_i \)) Eqs. (5) can also be obtained within thermal quantum field theory if the RIS subtraction is generalized to include quantum statistical factors and to compensate the double-counting related to Higgs-decays. The CP-violating parameters (i.e. the amplitudes that enter the Boltzmann equations) can be computed using retarded/advanced products in the real time formalism \([12]\) or in the imaginary time formalism \([13, 14]\). Differences appear at higher order in the expansion in these quantities \([5]\). Further differences arise when cuts through the internal Majorana neutrino line \([10]\) or flavour effects are to be taken into account \([9]\).

4. Resonant Case

According to established reasoning it is sufficient to modify the CP-violating parameters such that the divergence, also present in Eq. (8a), for \( M_j \rightarrow M_i \) is regulated by the finite width of the Majorana neutrinos:

\[
\epsilon_i^{S, vac} = - \sum_{j \neq i} \text{Im} \left\{ \frac{(h^\dagger h)^2_{ij}}{(h^\dagger h)_{ii} (h^\dagger h)_{jj}} \right\} \frac{R}{R^2 + A^2}, \quad \text{with } R = M_j^2 - M_i^2 \frac{M_i}{M_j} \Gamma_j, \tag{9}
\]

with (e.g.) \( A = M_i/M_j \). However, as outlined in Sec. 2, the applicability of simple Boltzmann-like equations depends on the validity of the quasi-particle approximation. Staying within the quasi-particle picture, an improved expression for the CP-violating parameters may be derived if there is only a mild degeneracy of the Majorana neutrino masses \([19]\):

\[
\epsilon_i^S(q, p) = - \sum_{j \neq i} \text{Im} \left\{ \frac{(h^\dagger h)^2_{ij}}{(h^\dagger h)_{ii} (h^\dagger h)_{jj}} \right\} \frac{R}{R^2 + A^2} \frac{p L_p}{p q}, \tag{10}
\]

with \( A = q L_p/(M_i M_j) \). It takes into account a ‘collisional broadening’ of the Majorana neutrino propagator. If the two states get so close that their spectral functions have significant overlap, the approximation by two \( \delta \)-like peaks becomes impossible and the dynamics of the flavour off-diagonals of the Majorana neutrino propagator \( \mathcal{S} \) cannot be reduced to that of its diagonal elements. It is possible to find analytic solutions for \( \mathcal{S} \) if one assumes that the expansion and washout effects are negligible \([20]\). This leads to an immediate result for the time dependence of the lepton asymmetry \( n_L = j_0 \) in the
degenerate limit \([21]\):

\[
n_L(t) = -\frac{\text{Im}\{(h^\dagger h)^{12}\}}{8\pi} \frac{M_1 M_2 \Delta M_{12}^2}{(\Delta M_{12}^2)^2 + (M_1 \Gamma_1 - M_2 \Gamma_2)^2} \\
\times 2 \int d\Omega \frac{N_{12}^2 \delta(p - k)}{(E_{N_1} - p - k)^{\Gamma_{\ell \phi}/2}} \frac{\Gamma_{\ell \phi}}{E_{N_1} - E_{N_2}} \\
\times \left[ \sum_{i=1,2} \frac{1 - e^{-M_i \Gamma_i/E_{N_i} t}}{M_i \Gamma_i/E_{N_i}} - 4 \Re \left( \frac{1 - e^{-i(N_1 - N_2)h - \Gamma t/4}}{\Gamma/2 + 2i(N_1 - N_2)} \right) \right],
\]

(11)

where \(\Delta M_{12}^2 = (M_1^2 - M_2^2)\). \(\Gamma\) is the average of the Majorana neutrino decay rates, see \([21]\) for details. The energy conserving Dirac-delta is here replaced by a Breit-Wigner curve dependent on the sum of the thermal widths of leptons and Higgs \(\Gamma_{\ell \phi}\). The additional square bracket in Eq. (11) is related to coherent flavour transitions between the Majorana neutrino states \(N_1\) and \(N_2\). As advertised above, this term can become relevant if the size of the mass splitting \(\Delta M = |M_i - M_j|\) is of the same order as the width \(\Gamma\). Under similar assumptions density matrix like equations have been obtained in \([22]\), as well based on analytic solutions for \(\gamma\). They describe the oscillations between the degenerate states and how they source the CP-asymmetry. Note that although the relative size of the thermal corrections to the Majorana neutrino masses (not described here) is generically small it can lead to peculiar effects in the degenerate limit if they are of the same order as the mass splitting, so that their effect should be taken into account in such cases. Oscillatory effects are particularly important if the asymmetry production is mainly due to the mechanism of leptogenesis via neutrino oscillations \([23]\).

The application of NEQFT methods to leptogenesis as it evolved in recent years put leptogenesis on a solidly footing. The employed 2PI-formalism represents an adequate basis for the description of this strikingly elegant link between the fundamental micro-physics of CP-violation and the macroscopic dominance of baryons in the universe. The treatment confirmed conventional results in the limit of zero temperature and led to a corrected account of medium effects otherwise. Beyond the simplest scenarios where the ‘Boltzmann-limit’ is applicable it can lead to more elaborate kinetic equations which govern the generation of an asymmetry. These increasingly challenge numerical methods as they are in the class of integro-differential equations and further reduction is not always possible.

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