Temperature effects on the magnetoplasmon spectrum of a weakly modulated graphene monolayer

M Tahir¹, K Sabeeh² and A MacKinnon³

¹ Department of Physics, University of Sargodha, Sargodha, Pakistan
² Department of Physics, Quaid-i-Azam University, Islamabad 45320, Pakistan
³ Department of Physics, Imperial College London, South Kensington Campus, London SW7 2AZ, UK

E-mail: m.tahir@uos.edu.pk, ksabeeh@qau.edu.pk, kashifsabeeh@hotmail.com
and a.mackinnon@imperial.ac.uk

Received 30 May 2011, in final form 5 September 2011
Published 7 October 2011
Online at stacks.iop.org/JPhysCM/23/425304

Abstract

In this work, we determine the effects of temperature on the magnetoplasmon spectrum of an electrically modulated graphene monolayer as well as a two-dimensional electron gas (2DEG). The intra-Landau band magnetoplasmon spectrum within the self-consistent field approach is investigated for both the aforementioned systems. Results obtained not only exhibit Shubnikov–de Haas (SdH) oscillations but also commensurability oscillations (Weiss oscillations). These oscillations are periodic as a function of inverse magnetic field. We find that both the magnetic oscillations, SdH and Weiss, have a greater amplitude and are more robust against temperature in graphene compared to a conventional 2DEG. Furthermore, there is a $\pi$ phase shift between the magnetoplasmon oscillations in the two systems which can be attributed to Dirac electrons in graphene acquiring a Berry’s phase as they traverse a closed path in a magnetic field.

1. Introduction

Remarkable progress made in epitaxial crystal growth techniques has led to the fabrication of novel semiconductor heterostructures. These modern microstructuring techniques have made possible the realization of a two-dimensional electron gas (2DEG) system in semiconductor heterostructures. A 2DEG is a condensed matter system where a number of novel phenomena have been observed over the years. One such phenomenon was the observation of commensurability oscillations in physical properties when the 2DEG in the presence of a magnetic field is subjected to electric modulation. The electric modulation introduces an additional length scale in the system and the occurrence of these oscillations, commonly known as Weiss oscillations, is due to the commensurability of the electron cyclotron diameter at the Fermi energy and the period of the electric modulation. These oscillations were found to be periodic in the inverse magnetic field [1–3]. This type of electrical modulation of the 2D system can be carried out by depositing an array of parallel metallic strips on the surface or through two interfering laser beams [4, 5].

One of the important electronic properties of such a system is the collective excitations (plasmons). Weiss oscillations in the magnetoplasmon spectrum of a 2DEG have been the subject of continued interest [6–9]. Recently, the fabrication of crystalline graphene monolayers has generated a lot of interest. The study of this new material is not only of academic interest but there are serious efforts underway to investigate whether graphene can serve as the basic material for a carbon-based electronics that might replace silicon. Graphene has a honeycomb lattice of carbon atoms. The quasiparticles in graphene have a band structure in which electron and hole bands touch at two points in the Brillouin zone. At these Dirac points the quasiparticles obey the massless Dirac equation. In other words, they behave as massless Dirac fermions leading to a linear dispersion relation $\epsilon_k = v_k$ (with the characteristic velocity $v \simeq 10^6 \text{ m s}^{-1}$). This difference between the nature of the quasiparticles in graphene and those in a conventional...
2DEG has given rise to a host of new and unusual phenomena, such as anomalous quantum Hall effects and a π Berry’s phase [10–15]. These transport experiments have shown results in agreement with the presence of Dirac fermions. The 2D Dirac-like spectrum was confirmed recently by cyclotron resonance measurements and also by angle resolved photoelectron spectroscopy (ARPES) measurements in monolayer graphene [16, 17]. Recent theoretical work on graphene multilayers has also shown the existence of Dirac electrons with a linear energy spectrum in monolayer graphene [18]. Plasmons in graphene were studied as early as the 1980s [19] and more recently [20–23]. In this work, we investigate magnetoplasmons in a graphene monolayer when it is subjected to electric modulation at finite temperature. In a 2DEG system, as well as in graphene, plasmons emerge as a result of electron–electron interactions. In the presence of a magnetic field, these magnetoplasmons occur at frequencies that oscillate with the magnetic field. When this system is subjected to an electric modulation, broadening of the Landau levels occurs, resulting in both inter- and intra-Landau band magnetoplasmons. The former arise as a result of electronic transitions between different Landau bands, whereas the latter are due to transitions within a single Landau band. A study of magnetoplasmons at zero temperature was carried out by us in earlier work [24], where we showed that the intra-Landau band magnetoplasmons exhibit Weiss oscillations as a result of modulation. Currently, we are extending our earlier work by taking into account the effects of temperature on the magnetoplasmons in graphene. The motivation for the finite temperature calculation presented here is that it allows us to account for Shubnikov–de Haas (SdH) oscillations in addition to Weiss oscillations in a conventional 2DEG realized in semiconductor [24]. In this work, we also determine the finite temperature dispersion relation for magnetoplasmons in an electrically modulated [9].

The intra-Landau band plasmon spectrum is determined within the Ehrenreich–Cohen self-consistent field (SCF) approach [29]. Following [24] the plasmon dispersion relation is given by solving

\[
\epsilon(n, x_0) = \hbar \omega_0 \sqrt{n} + F_n \cos(Kx_0),
\]

where \(\omega_0 = v\sqrt{2eB/\hbar}\) is the cyclotron frequency of Dirac electrons in graphene, \(n\) is an integer, \(F_n = \frac{1}{2} V_0 e^{-n/2}[L_n(\chi) + L_{n-1}(\chi)]\), \(\chi = \frac{1}{2} K^2 p^2\), \(x_0 = \frac{\pi}{2} k_y\), \(L_n(\chi)\) are Laguerre polynomials and \(l = \sqrt{\hbar/eB}\) is the magnetic length. The Landau level spectrum for Dirac electrons in graphene \(\epsilon_n = \sqrt{n}\omega_0\) is significantly different from the spectrum for electrons in a conventional 2DEG, which is given as \(\epsilon(n) = \hbar \omega_0 (n + 1/2)\), where \(\omega_0\) is the cyclotron frequency.

The intra-Landau band plasmon spectrum is determined within the Ehrenreich–Cohen self-consistent field (SCF) approach [29]. Following [24] the plasmon dispersion relation is given by solving

\[
1 = \frac{2\pi e^2}{kq} \frac{1}{\pi a^2} \sum_{n,n'} C_{nn'} \left( \frac{q^2}{2eB} \right) [I_1(\omega) + I_1(-\omega)],
\]

for \(\omega\), where

\[
I_1(\omega) = \int_0^a \frac{dx_0}{\epsilon(n, x_0)} \frac{f[\epsilon(n, x_0)]}{\epsilon(n, x_0) - \epsilon(n, x_0 + x'_0) + \omega},
\]

\(x'_0 = \frac{\pi}{2} q_y, q\) is the 2D wavenumber, \(k\) is the background dielectric constant and \(f[\epsilon(n, x_0)]\) is the Fermi distribution function. This result was obtained previously in [24] where it was used to obtain the zero temperature dispersion relation. Here we are interested in temperature effects to account for SdH oscillations, in addition to Weiss oscillations, in the magnetoplasmon spectrum. We proceed as follows to determine the finite temperature dispersion relation. In

2. Temperature effects on the magnetoplasmon spectrum of weakly modulated graphene

We consider 2D Dirac electrons in graphene moving in the \((x-y)\) plane. A magnetic field \((B)\) is applied along the \(z\) direction perpendicular to the graphene plane. This system is subjected to weak electric modulation along the \(x\) direction. We employ the Landau gauge and write the vector potential as \(A = (0, Bx, 0)\). Employing the tight-binding approach it has been shown that the Dirac electrons, in the absence of a periodic potential and magnetic field, obey the 2D Dirac-like Hamiltonian [26, 27]. The effect of the magnetic field can be included through the minimal coupling with the result that the 2D, single-electron, Dirac-like Hamiltonian in the Landau gauge is [12, 25–28]

\[
H_0 = v \sigma \cdot (-i \hbar \nabla + eA).
\]

Here \(\sigma = [\sigma_x, \sigma_y]\) are the Pauli matrices and \(v\) characterizes the electron velocity. The complete Hamiltonian of our system is represented as

\[
H = H_0 + V(x),
\]

where \(H_0\) is the unmodulated Hamiltonian and \(V(x)\) is the 1D periodic modulation potential along the \(x\) direction modelled as

\[
V(x) = V_0 \cos(Kx),
\]

where \(K = 2\pi/a\), \(a\) is the period of modulation and \(V_0\) is the constant modulation amplitude. Applying standard perturbation theory to determine the first order correction to the unmodulated energy eigenvalues in the presence of modulation, we obtain

\[
\epsilon(n, x_0) = \hbar \omega_0 \sqrt{n} + F_n \cos(Kx_0),
\]

where \(\omega_0 = v\sqrt{2eB/\hbar}\) is the cyclotron frequency of Dirac electrons in graphene, \(n\) is an integer, \(F_n = \frac{1}{2} V_0 e^{-n/2}[L_n(\chi) + L_{n-1}(\chi)]\), \(\chi = \frac{1}{2} K^2 p^2\), \(x_0 = \frac{\pi}{2} k_y\), \(L_n(\chi)\) are Laguerre polynomials and \(l = \sqrt{\hbar/eB}\) is the magnetic length. The Landau level spectrum for Dirac electrons in graphene \(\epsilon_n = \sqrt{n}\omega_0\) is significantly different from the spectrum for electrons in a conventional 2DEG, which is given as \(\epsilon(n) = \hbar \omega_0 (n + 1/2)\), where \(\omega_0\) is the cyclotron frequency.

The intra-Landau band plasmon spectrum is determined within the Ehrenreich–Cohen self-consistent field (SCF) approach [29]. Following [24] the plasmon dispersion relation is given by solving

\[
1 = \frac{2\pi e^2}{kq} \frac{1}{\pi a^2} \sum_{n,n'} C_{nn'} \left( \frac{q^2}{2eB} \right) [I_1(\omega) + I_1(-\omega)],
\]

for \(\omega\), where

\[
I_1(\omega) = \int_0^a \frac{dx_0}{\epsilon(n, x_0)} \frac{f[\epsilon(n, x_0)]}{\epsilon(n, x_0) - \epsilon(n, x_0 + x'_0) + \omega},
\]

\(x'_0 = \frac{\pi}{2} q_y, q\) is the 2D wavenumber, \(k\) is the background dielectric constant and \(f[\epsilon(n, x_0)]\) is the Fermi distribution function. This result was obtained previously in [24] where it was used to obtain the zero temperature dispersion relation. Here we are interested in temperature effects to account for SdH oscillations, in addition to Weiss oscillations, in the magnetoplasmon spectrum. We proceed as follows to determine the finite temperature dispersion relation. In
the regime of weak modulation, the distribution function \( f[ε(n, x_0)] \) can be expressed as

\[
f[ε(n, x_0)] \approx f(ε_n) + F_n f'(ε_n) \cos(Kx_0),
\]

where \( f'(x) = \frac{d}{dx}f(x) \). Substituting the above expression for \( f[ε(n, x_0)] \), the integral over \( x_0 \) may be written as

\[
I_1(ω) = K^{-1} \int_0^{2π} \left[ \frac{f(ε_n) + f'(ε_n)F_n \sin(\frac{1}{2}Kx_0') \sin \theta}{2F_n \sin(\frac{1}{2}Kx_0') \sin \theta + \omega} \right] dθ
\]

where \( f(ε_n) \) may be written as

\[
\sum_{n} \delta(ε_n - ε) f(ε_n) \int_0^{2π} \frac{\sin \theta dθ}{\Phi_n \sin \theta + \omega}
\]

and is also a term with cosines in the numerator, but the resulting integral is easily shown to be zero by symmetry around \( \theta = \frac{π}{2} \). The first integral in (9) can be solved by rewriting it in terms of a contour integral around a unit circle to obtain

\[
\int_0^{2π} \frac{dθ}{\Phi_n \sin \theta + \omega} = \begin{cases} \frac{2π}{\sqrt{\omega^2 - \Phi_n^2}} & \text{if } \Phi_n > 1 \\ 0 & \text{if } -1 < \omega < 1 \\ -\frac{2π}{\sqrt{\omega^2 - \Phi_n^2}} & \text{if } \Phi_n < -1 \end{cases}
\]

(10)

We note here that as \( \omega \) must be real and (6) requires us to take the principal value the contributions from the poles cancel when \(|\omega| < |\Phi_n| \). However, this is not the integral we need as it will cancel in the expression \( I(ω) + I(−ω) \). The second integral in (9) may be deduced from (10) using

\[
\Phi_n \int_0^{2π} \frac{\sin \theta dθ}{\Phi_n \sin \theta + \omega} = 2π - \omega \int_0^{2π} \frac{dθ}{\Phi_n \sin \theta + \omega}
\]

(11)

\[
= \begin{cases} 2π \left[ 1 - \frac{\omega}{\sqrt{\omega^2 - \Phi_n^2}} \text{sgn}\left( \frac{\omega}{\Phi_n} \right) \right] & \text{if } |\omega| > |\Phi_n| \\ 2π & \text{if } |\omega| < |\Phi_n| \end{cases}
\]

(12)

We note that the first line of (12) may be expanded in the form

\[
\Phi_n \int_0^{2π} \frac{\sin \theta dθ}{\Phi_n \sin \theta + \omega} = -\frac{π}{\omega^2} + O\left( \frac{1}{\omega^4} \right)
\]

(13)

Substituting this back into (5) and concentrating on intra-Landau level plasmons by setting \( c_{m'} = b_{m'} \) we obtain

\[
1 = \frac{2πe^2}{kq} \sum_{n} f'(ε_n) \int_0^{2π} \frac{dθ}{\sqrt{ω^2 - \Phi_n^2}} \left[ 1 - \frac{\omega}{\sqrt{ω^2 - \Phi_n^2}} \text{sgn}\left( \frac{ω}{\Phi_n} \right) \right]
\]

\[
\approx \frac{2πe^2}{kq} \sum_{n} \left[ -f'(ε_n) \right] \frac{\Phi_n^2}{ω^2}
\]

(14)

As long as \(|\omega| > |\Phi_n|\) and the case \(|\omega| < |\Phi_n|\) need not concern us. We note at this point that \( V_0, F_n \) and \( \omega \) are of similar orders of magnitude (meV), but that, in the long wavelength (small \( q_f \)) limit \( x_0' \to 0 \) (recall that \( q_f = \frac{π}{2} \)). We are therefore justified in taking the leading term in the expansion in (15), which can be solved to give the intra-Landau band plasmon excitation frequency (\( \tilde{ω} \)) as

\[
\tilde{ω}^2 = \frac{4π^2}{kq^2} \sum_{n} \sin^2\left( \frac{πx_0'}{a} \right) \times G_ε.
\]

(16)

with \( G_ε = \sum \frac{F_n^2}{ω^2} \times [−f'(ε_n)] \). This result will be valid except for low frequencies, where \( G_ε \) may acquire an \( ω \) dependence and cease to exist before \( ω = 0 \).

The resulting physics can be made more transparent by considering the asymptotic expression for the intra-Landau band magnetoplasmon spectrum, where analytic results can be obtained in terms of elementary functions.

To obtain an expression for the intra-Landau band plasmon spectrum we employ the following asymptotic expression for the Laguerre polynomials:

\[
\exp^{-\chi/2} L_n^{(\chi)} → \frac{1}{\sqrt{πn^2}} \cos\left( 2\sqrt{n\chi} - \frac{π}{4} \right).
\]

(17)

Note that the asymptotic results are valid when many Landau levels are filled. We now take the continuum limit:

\[
n → \frac{ε^2}{h^2ω_ε^2}, \quad \sum_{n=0}^{∞} \rightarrow \int_0^{∞} \frac{2π dε}{h^2ω_ε^2}.
\]

(18)

\( G_ε \), which appears in the expression for \( ω^2 \) in equation (16), can be expressed as

\[
G_ε = \frac{V_0^2}{π} \int_0^{∞} \frac{2π dε}{h^2ω_ε^2} \frac{βg(ε)}{[g(ε) + 1]^2} \left[ \frac{1}{2} \frac{χ}{n} \right] \times \cos^2\left( 2\sqrt{n\chi} - \frac{π}{4} \right)
\]

(19)

where \( g(ε) = \exp[β(ε - ε_F)] \), \( ε_F = ℏvK_f, K_f = \sqrt{2πn_D}/b \) is the number density, \( χ = \frac{1}{b} K_f^2 β^2 = 2n^2/β/h \) and \( β = 1/K_f \). Assuming that the temperature is sufficiently low that \( β^{-1} ≪ ε_F \) and substituting \( ε = ε_F + sβ^{-1} \), we rewrite the above integral as

\[
G_ε = \frac{V_0^2}{π} \frac{2π^2}{h^2ω_ε^2} \frac{βg(ε)}{[g(ε) + 1]^2} \int_0^{∞} dε \frac{e^ε}{[e^ε + 1]^2}
\]

\[
\times \cos^2\left( 2\sqrt{εF}/hω_ε - \frac{π}{4} + \frac{2√{εF}β^{-1}}{hω_ε} \right)
\]

(20)

with the result

\[
G_ε = \frac{V_0^2}{π} \frac{2π^2}{h^2ω_ε^2} \left[ \frac{1}{2} \frac{χ}{n} - 4A \left( \frac{T}{T_W} \right) \cos^2\left( 2\sqrt{εF}/hω_ε - \frac{π}{4} \right) \right],
\]

(21)

where

\[
A \left( \frac{T}{T_W} \right) = \frac{T}{T_W} / \sinh\left( \frac{T}{T_W} \right)
\]
Both Weiss and SdH oscillations is given by magnetoplasmon dispersion relation that takes into account the absence of scattering and with gap $D$ is the characteristic damping temperature of the Weiss oscillations. Substituting the expression for $G_c$ in equation (16), the asymptotic expression for the intra-Landau spectrum is obtained as

$$\omega^2 = \frac{2e^2}{kq\pi T} \frac{V_0^2 \cos^2 \frac{\omega_0}{2\pi} \sqrt{\chi}}{\pi \hbar \omega_g \sqrt{\chi}} \sin^2 \left( \frac{\pi}{a} (x'_0) \right)$$

$$\times \left[ 2 - 2A \left( \frac{T}{T_W} \right) + 4A \left( \frac{T}{T_W} \right) \right]$$

$$\times \cos^2 \left( \frac{2\pi \sqrt{\chi}}{\hbar \omega_g} - \frac{\pi}{4} \right).$$

As the above expression is only valid at high temperature ($K_B T \gg \omega_g/2\pi$) it is not able to account for the SdH oscillations occurring in the magnetoplasmon spectrum. To take this into account we use the following expression for the density of states at low magnetic fields in the absence of impurity scattering:

$$D(\varepsilon) = \frac{\varepsilon}{\pi^2 \hbar^2 \omega_g^2} \left[ 1 + 2 \cos \left( \frac{2\pi \varepsilon^2}{\hbar^2 \omega_g^2} \right) \right].$$

This expression for $D(\varepsilon)$ was obtained in [30, 31] in the absence of scattering and with gap $\Delta = 0$. The sum can now be expressed in the continuum approximation as $\sum_{n=0}^{T} 2\pi T \int D(\varepsilon) d\varepsilon$. Therefore, the intra-Landau band magnetoplasmon dispersion relation that takes into account both Weiss and SdH oscillations is given by

$$\omega^2 = \frac{2e^2}{kq\pi T} \frac{V_0^2 \cos^2 \frac{\omega_0}{2\pi} \sqrt{\chi}}{\pi \hbar \omega_g \sqrt{\chi}} \sin^2 \left( \frac{\pi}{a} (x'_0) \right)$$

$$\times \left[ 2 - 2A \left( \frac{T}{T_W} \right) + 4A \left( \frac{T}{T_W} \right) \right]$$

$$\times \cos^2 \left( \frac{2\pi \sqrt{\chi}}{\hbar \omega_g} - \frac{\pi}{4} \right)$$

$$+ 4 \frac{2 T}{T_{SdH}} \frac{2 \pi \sqrt{\chi}}{\hbar \omega_g} \cos \left( \frac{2\pi \varepsilon_0^2}{\hbar^2 \omega_g^2} \right) \cos^2 \left( \frac{2\pi \sqrt{\chi}}{\hbar \omega_g} - \frac{\pi}{4} \right),$$

(23)

where

$$T = \frac{4\pi^2 \varepsilon_0^2 K_B T}{\hbar \omega_g^2}$$

and

$$T_{SdH} = \frac{\hbar^2 \omega_g^2}{4\pi^2 \varepsilon_0^2 K_B}$$

is the characteristic damping temperature of the SdH oscillations.

Following the same approach as given above for a graphene monolayer, we can obtain the intra-Landau band magnetoplasmon spectrum for a conventional 2DEG at finite temperature with the result

$$\omega^2 = \frac{4e^2 m^* \omega_c}{\hbar q \pi} \sin^2 \left( \frac{\pi}{a} (x'_0) \right) \times G_c,$$

(24)

where $m^*$ is the standard electron mass, $G_c = \sum_n F_n^2(C) \times [-f^c(\varepsilon(n))]$, and $F_n(C) = V_0 e^{-x^2/2L_n(\chi)}$ is the modulation width of the conventional 2DEG. The corresponding asymptotic result is

$$\omega^2 = \frac{4e^2 m^* \omega_c}{kq2\pi \hbar \sqrt{\chi} \omega_g \sqrt{\chi}} \sin^2 \left( \frac{\pi}{a} (x'_0) \right) \left[ 1 - A \left( \frac{T}{T_W} \right) \right]$$

$$+ 2A \left( \frac{T}{T_W} \right) \cos^2 \left( 2 \sqrt{\frac{\chi}{\hbar \omega_g}} - \frac{\pi}{4} \right)$$

$$+ 4 \frac{2 T}{T_{SdH}} \frac{2 \pi \varepsilon_0^2}{\hbar \omega_g} \cos \left( \frac{2\pi \varepsilon_0^2}{\hbar^2 \omega_g^2} \right) \cos^2 \left( \frac{2\pi \sqrt{\chi}}{\hbar \omega_g} - \frac{\pi}{4} \right),$$

(25)

where

$$\frac{T}{T_W} = \frac{4\pi^2 K_B T}{\hbar \omega_g \sqrt{\chi}},$$

$$\frac{T}{T_{SdH}} = \frac{2\pi^2 K_B T}{\hbar \omega_g \sqrt{\chi}},$$

$$\chi = \frac{K^2 F}{2} = \frac{2\pi^2}{b},$$

$$b = \frac{e \alpha d^2}{h}.$$

Now we can compare the exact results for the temperature dependent magnetoplasmon in modulated graphene, as given in equation (23) in terms of the elementary functions [32, 33], with that of a conventional 2DEG derived in equation (25). The differences are the following.

- The standard electron energy eigenvalues scale linearly with the magnetic field whereas those for Dirac electrons in graphene scale as the square root.
- The temperature dependence of the Weiss oscillations, $A(T/T_W)$, is clearly different from that of the standard 2DEG, $A(T/T_W)$.
- The temperature dependence of SdH oscillations in graphene, $\frac{T}{T_{SdH}} / \sinh(\frac{T}{T_{SdH}})$, is different from that of the standard 2DEG, $\frac{T}{T_{SdH}} / \sinh(\frac{T}{T_{SdH}})$.
- The density of states term that contains the cosine function responsible for the SdH oscillations has a different dependence in each of the systems: $\cos(2\pi \varepsilon_0^2/\hbar^2 \omega_g^2)$ and $\cos(2\pi \sqrt{\chi}/\hbar \omega_g - \pi)$, respectively.

These differences will give contrasting results for the temperature dependent magnetoplasmon spectrum, as we discuss in section 3.

3. Discussion of results

The finite temperature intra-Landau band magnetoplasmon dispersion relation for electrically modulated graphene and 2DEG given by equations (23) and (25) are the central results of this work. These results allow us to see clearly the effects...
of temperature on the Weiss and SdH oscillations in the magnetoplasmon spectrum of the two systems. We find that these two types of oscillations have different characteristic damping temperatures. This is easily understood if we realize that these oscillations have quite different origins. The SdH oscillations arise due to the discreteness of the Landau levels and their observation requires that the thermal energy $k_B T$ of the electrons at temperature $T$ has to be smaller than the separation between the levels. Weiss oscillations are related to the commensurability of two lengths: the size of the cyclotron orbit and the period of the modulation. These oscillations will be observed if the spread in the cyclotron diameter is smaller than the period of the modulation. To clarify the effects of temperature we present the magnetoplasmon spectrum in graphene and 2DEG at two different temperatures in figures 1 and 2.

In figure 1 we show the magnetoplasmon energy as a function of the inverse magnetic field in a graphene monolayer at two different temperatures: $T = 1$ and 12 K. The following parameters were employed for graphene [20–25]: $k = 3$, $n_D = 3.16 \times 10^{16} \text{ m}^{-2}$, $v = 10^6 \text{ m s}^{-1}$, $a = 380 \text{ nm}$ and $V_0 = 1.0 \text{ meV}$. We also take $q_s = 0$ and $q_c = 0.01 k_F$, with $k_F = (2\pi n_D)^{1/2}$. Weiss oscillations superimposed on the SdH oscillations can be clearly seen in both the curves. In equation (23) the terms containing the characteristic temperature for Weiss oscillations $T_W$ are mainly responsible for Weiss oscillations whereas terms containing $T_{\text{SdH}}$ are responsible for the SdH oscillations. The intra-Landau band plasmons have frequencies comparable to the bandwidth and they occur as a result of the broadening of the Landau levels due to the modulation in the system. This type of intra-Landau band plasmons accompanied by regular oscillatory behaviour in $1/B$ of the SdH type was first predicted in [34, 35] for a tunnelling planar superlattice where the overlap of the electron wavefunction in adjacent quantum wells provides the mechanism for the broadening of the Landau levels. The SdH oscillations occur as a result of the emptying of electrons from successive Landau levels when they pass through the Fermi level as the magnetic field is increased. The amplitude of these oscillations is a monotonic function of the magnetic field when the Landau bandwidth is independent of the band index $n$. In the density modulated case, the Landau bandwidths oscillate as a function of the band index $n$, with the result that, in the plasmon spectrum of the intra-Landau band type, there is a new kind of oscillation named the Weiss oscillation which is also periodic in $1/B$ but with a different period and amplitude from the SdH type oscillation. At $T = 12 \text{ K}$ we find that the SdH oscillations are washed out while the Weiss oscillations persist. From equation (23) we see that $T_W$ and $T_{\text{SdH}}$ set the temperatures at which these oscillations will be damped. For the 2DEG the magnetoplasmon energy as a function of inverse magnetic field is presented in figure 2 at two different temperatures: $T = 0.3$ and 3 K. For a conventional 2DEG (a 2DEG at a GaAs–AlGaAs heterojunction) we use the following parameters [1–9]: $m^* = 0.07m_e$ ($m_e$ is the electron mass), $k = 12$ and $n_D = 3.16 \times 10^{15} \text{ m}^{-2}$ with the modulation strength and the period as in the graphene system. In this case we find that the SdH oscillations in the magnetoplasmon spectrum die out at $T = 3 \text{ K}$, while they are present at a higher temperature (12 K) in graphene.

To compare the results for the two systems we show in figure 3 the magnetoplasmon spectrum as a function of inverse magnetic field ($\frac{1}{B} = \frac{e}{B \sigma}$, where $B = \frac{n_D}{\rho \sigma}$ = 0.0046 T) for both graphene (solid line at temperature 5 K) and the 2DEG (dotted and dashed line at temperature 1 K and 5 K respectively). The oscillations in the conventional 2DEG have been damped out strongly at 5 K but these are well resolved.
significant and have large amplitude in graphene at the same temperature. This confirms that graphene oscillations are robust and enhanced against temperature compared to those in a conventional 2DEG. The magnetic oscillations, SdH and Weiss, have a greater amplitude in graphene compared to a 2DEG. This can be attributed to the larger characteristic velocity, \( v \sim 10^6 \text{ m s}^{-1} \) of electrons in graphene compared to the Fermi velocity of standard electrons as well as the smaller background dielectric constant \( k \) in graphene compared to a conventional 2DEG. The temperature at which we expect the Weiss oscillations to dampen is determined by comparing the characteristic temperatures for Weiss oscillations in the two systems: 

\[
T^*_W / T_W = v_F / v.
\]

The ratio of the characteristic temperatures:

\[
SdH / T = \frac{\hbar}{K m^*} \sim 10^{-6} \text{ m}^2 / \text{V s}
\]

for \( n_p = 3.16 \times 10^{15} \text{ m}^{-2} \) whereas it is \( \sim 0.74 \) for \( n_p = 3.16 \times 10^{16} \text{ m}^{-2} \). Hence SdH oscillations are damped at a higher temperature in graphene compared to a 2DEG. Furthermore, our results show that these oscillations in the magnetoplasmon spectrum differ in phase by \( \pi \) in the two systems, which is due to quasiparticles in graphene acquiring a Berry’s phase of \( \pi \) as they move in the magnetic field [12–15].

### 4. Conclusions

We have determined the finite temperature intra-Landau band magnetoplasmon frequency for both electrically modulated graphene and for a 2DEG in the presence of a magnetic field, by employing the SCF approach. We find that the magnetic oscillations (SdH and Weiss) in the magnetoplasmon spectrum in a graphene monolayer have a higher amplitude compared to the conventional 2DEG realized in semiconductor heterostructures. Moreover, these oscillations persist at a higher temperature in graphene compared to the 2DEG. Hence they are more robust against temperature in graphene. Furthermore, the \( \pi \) Berry’s phase acquired by the Dirac electrons leads to a \( \pi \) phase shift in the magnetoplasmon spectrum in a graphene monolayer compared to a 2DEG.

### Acknowledgments

One of us (KS) would like to acknowledge the support of the Higher Education Commission (HEC) of Pakistan through project No. 20-1484/R&D/09. MT would like to acknowledge the support of the Pakistan Higher Education Commission (HEC).

### References

[1] Winkler R W, Kotthaus J P and Ploog K 1989 Phys. Rev. Lett. 62 1177
[2] Vasilopoulos P and Peeters F M 1989 Phys. Rev. Lett. 63 2120
[3] Peeters F M and Vasilopoulos P 1992 Phys. Rev. B 46 4667
[4] Gerhardts R R, Weiss D and Klitzing K v 1989 Phys. Rev. Lett. 62 1173
[5] Weiss D, Klitzing K v, Ploog K and Weimann G 1989 Europhys. Lett. 8 179
[6] Balev O G, Studart N and Vasilopoulos P 2000 Phys. Rev. B 62 15834
[7] Kushwaha M S 2001 Surf. Sci. Rep. 41 1 and references therein
[8] Kushwaha M S and Sakaki H 2004 Phys. Rev. B 69 155331
[9] Cui H L, Fessatidis V and Horing N J M 1989 Phys. Rev. Lett. 63 2598
[10] Novoselov K S, Geim A K, Morozov S V, Jiang D, Katsnelson M I, Grigorieva I V, Dubonos S V and Firsov A A 2005 Nature 438 197
[11] Zhang Y, Tan Y W, Stormer H L and Kim P 2005 Nature 438 201
[12] Zheng Y and Ando T 2002 Phys. Rev. B 65 245420
[13] Gusynin V P and Sharapov S G 2005 Phys. Rev. Lett. 95 146801
[14] Perez N M R, Guinea F and Castro Neto A H 2006 Phys. Rev. B 73 125411
[15] Katsnelson M I, Novoselov K S and Geim A K 2006 Nature Phys. 2 620
[16] Deacon R S, Chuang K-C, Nicholas R J, Novoselov K S and Geim A K 2007 Phys. Rev. B 76 081406(R)
[17] Zhou S Y, Gweon G-H, Graf J, Fedorov A V, Spataru C D, Diehl R D, Kopelevich Y, Lee D H, Louie S G and Lanzara A 2006 Nature Phys. 2 595
[18] Partoens B and Peeters F M 2007 Phys. Rev. B 75 193402
[19] Shung K W-K 1986 Phys. Rev. B 34 979
[20] Lin M-F and Shyu F-L 2000 J. Phys. Soc. Japan 69 607
[21] Apalkov V, Wang X-F and Chakraborty T 2007 Int. J. Mod. Phys. B 21 1167
[22] Hwang E H and Das Sarma S 2007 Phys. Rev. B 75 205418
[23] Wang X-F and Chakraborty T 2007 Phys. Rev. B 75 033408
[24] Tahir M and Sabeeh K 2007 Phys. Rev. B 76 195416
[25] Matulis A and Peeters F M 2007 Phys. Rev. B 75 125429
[26] Castro Neto A H, Guinea F, Peres N M R, Novoselov K S and Geim A K 2009 Rev. Mod. Phys. 81 109 and references therein
[27] Abergela D S L, Apalkov B, Berashevicha J, Ziegler K and Chakraborty T 2010 Adv. Phys. 59 261 and references therein
[28] Slonczewski J C and Weiss P R 1958 Phys. Rev. 109 272
[29] Ehrenreich H and Cohen M H 1959 Phys. Rev. 115 786
[30] Gusynin V P and Sharapov S G 2005 Phys. Rev. B 71 125124
[31] Sharapov S G, Gusynin V P and Beck H 2004 Phys. Rev. B 69 075104
[32] Dingle R B 1952 Proc. R. Soc. A 211 517
[33] Ishihara A and Smrcka L 1986 J. Phys. C: Solid State Phys. 19 6777
[34] Que W M and Kirchzenow G 1989 Phys. Rev. Lett. 62 1687
[35] Que W M and Kirchzenow G 1987 Phys. Rev. B 36 6596