Managing Flow of Liquid Light

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Strongly coupled light-matter systems can carry information over long distances and realize low threshold polariton lasing, condensation and superfluidity. These systems are highly non-equilibrium in nature, so constant nonzero fluxes manifest themselves even at the steady state and set by a complicated interplay between nonlinearity, dispersion, pumping, dissipation and interactions between the various constituents of the system. Predicting the flow velocities even for a simple drive configuration has been challenging and no analytical spatially nonuniform solutions to the system were previously known. Based on the mean-field governing equations of lasers or polariton condensates, we develop a theoretical approach for engineering and controlling the velocity profiles by manipulating the spatial pumping and dissipation in the system. We present analytically exact pumping and dissipation profiles that lead to a large variety of spatially periodic density and velocity profiles. Our approach opens the way to the controllable implementation of laser or polariton flows for ultra fast information processing and integrated circuits.

The full exploitation of polariton’s potential for technological applications requires engineering microstructures with certain high quality trapping profiles with precise parameters and realising quasiparticles flow and density control. The goal is to engineer polaritonic lattices excited by a pumping potential using specially designed sample fabrication or manipulation techniques to control laser spot\textsuperscript{[10]}, an incidence pumping angle\textsuperscript{[29]} or laser regime (coherent or incoherent)\textsuperscript{[29]} with emphasis on velocity control. There are several proposed ways of organising such structures including optically imprinting using a spatial light modulator\textsuperscript{[12,14]} etching the sample surface depositing metallic patterns periodically\textsuperscript{[25,26]}, use confined strain-induced traps\textsuperscript{[25]} surface acoustic waves\textsuperscript{[21]}, direct fabrication with the gold deposition technique\textsuperscript{[26]} by using hybrid air gap microcavities\textsuperscript{[20]} by coupled mesas etched during the growth of the microcavities\textsuperscript{[20]} by micropillars\textsuperscript{[31]} and many others\textsuperscript{[12–35]}. Polariton condensation has been recently achieved in a one-dimensional strong lead halide perovskite lattice\textsuperscript{[30–32]} paving the way for all-optical integrated logic circuits operating at room temperature. Polariton lasing has been achieved using organic semiconductor materials in a planar dielectric microcavity for ultra low thresholds\textsuperscript{[33]}. Oligofluorenes can be synthesized and processed into films of a desired thickness, and therefore desired spatially controlled dissipation. Furthermore, spatial anisotropy of dissipation was shown to lead to an effective control of sign and intensity of the coupling strength between any two neighbouring condensates\textsuperscript{[34]}. However, unlike optical lattices of equilibrium ultra-cold BECs, the periodic trapping of nonequilibrium condensates provides flows between the lattice sites that are difficult to predict and challenging to control. In this Letter, we show how to fully control the velocity and density of polariton condensates by using spatially varying dissipation and pumping profiles. In particular, we find wide classes of analytical expressions for pumping and dissipation that lead to the stable periodic variation in polariton density and flow velocity. Figure 1 shows the schematic of the experimental setup with uniform incoherent injection and nonuniform spatially varying dissipation that lead to the desired flow pattern. Analytical solutions of the Ginzburg-Landau equations with a periodically varying dissipation. Our starting point is a generic laser or nonequilibrium condensate model in the form of the complex Ginzburg-Landau equation (cGLE) with saturable nonlinearity that results from the Maxwell-Bloch equations:

\[
\frac{\partial \Psi}{\partial t} = -\frac{1}{2} \Delta \Psi + |\Psi|^2 \Psi + g n_R \Psi + i(n_R - \gamma(x))\Psi, \tag{1}
\]

where $\Psi(x,t)$ is the complex-valued order parameter, $\gamma(x)$ is the spatially varying linear losses due to imperfect confinement, $n_R$ is the density distribution of noncondensed reservoir particles (such as hot excitons for polariton condensates) that gives gain saturation $n_R = P(x,t)(1 + b |\Psi|^2)^{-1}$, $P(x,t)$ is the gain, $b$ characterizes the relative strengths of

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FIG. 1. Schematic of flow management approach presented in our paper. The top yellow plate represents the source of the incoherent uniform pumping across the system. Grey layers denote the semiconductor microcavity, where the formation of the exciton-polariton BEC takes place. On the top layer of this structure we show the target flow profile, represented by the color gradient and black arrows that indicate the direction of the flow. Below the structure we show the dissipation induced and controlled by methods described in the main text that leads to such a flow profile.

The main question we address in this Letter is how to engineer a given velocity profile using the controls available in our system such as the spatially varying pumping or dissipation. This control will be achieved by observing that nonlinear Eq. (3) yields the exact solutions in terms of $\rho$ for spatially constant pumping and a given periodic velocity profiles from some known classes of periodic functions. Such density when substituted into Eq. (4) allows one to specify the spatially varying dissipation that leads to the given supercurrent.

In what follows we consider a one-dimensional velocity profile $\mathbf{u} = u(x)$. Equation (4) is the second order nonlinear ordinary differential equation which for a given periodic expression for $u(x)$ relates to the steady state of the Gross-Pitaevskii equation (GPE) that describes quasi-one-dimensional dilute gas of the Bose - Einstein condensate (BEC) trapped in a periodic optical lattice (a standing light wave). The GPE for such gas reads

$$i\psi_t = -\frac{1}{2}\psi_{xx} + \beta|\psi|^2 \psi + V(x)\psi,$$

where $\psi(x,t)$ is the macroscopic wavefunction of the Bose-Einstein condensate and $V(x)$ is the periodic potential describing the modulation of the standing wave.

The steady state of Eq. (5) with $\psi = \sqrt{\rho} \exp[is - i\Omega t]$ and a constant phase $\hat{S}$ satisfies $\hat{\mu} = V(x) - (\sqrt{\rho})_{xx}/2\sqrt{\rho} + \beta \rho$, which reduces to Eq. (3) if we let $2V(x) = u(x)^2$, $\hat{\mu} = \mu$ and $\hat{\rho} = \rho$. So the trap used in equilibrium condensates can be thought of as the postulated expressions for the velocity profiles. The exact expressions for the density, therefore, can be obtained similarly to the exact solution of the GPE in the presence of the periodic external potential. Once we know the density profile, Eq. (4) gives the expression for the spatial dissipation that leads to that density and velocity

$$\gamma(x) = \frac{\rho}{P_x - \sigma P - u_x - \rho_s u_x / \rho_x}.$$

A large class of periodic variations of velocity profiles can be described as $u(x)^2 = -2V_0\sin^2(x,k)$, where $\sin(x,k)$ denotes the Jacobi elliptic sine function with an elliptic parameter $0 < k < 1$ and $V_0 < 0$ characterizes the depth of the periodic variation. Denoting $r(x) = \sqrt{\rho(x)}$, Eq. (3) becomes

$$\mu r^2(x) = -\frac{r^2(x) r''(x)}{2} + \beta r^2(x) - V_0\sin^2(x,k) r^4(x).$$

The solutions to the nonlinear ordinary differential Eq. (7) can be obtained by expanding $r(x)$ in terms of the various Jacobi elliptic functions ($sn$, $dn$, $cn$), substituting $r(x)$ into Eq. (7) and equating equal powers of these functions to zero to get the expressions for the parameters.

**Defocusing regime:** $\beta = 1$. Firstly, we consider $r^2(x) = A\sin^2(x,k) + B$, which leads to the following conditions $\mu = \frac{1}{2}(1 + k^2 + 3B - \frac{B_0}{V_0^2})$, $0 = B(1 + \frac{B}{B_0}) (k^2 + V_0 + Bk^2)$, $A = k^2 + V_0$. These conditions are satisfied in the following three cases. 1) $B = 0$ and $A = k^2 + V_0$ with $\mu = (1 + k^2)/2$ result in

$$\rho(x) = (V_0 + k^2) \sin^2(x,k), \quad -k^2 \leq V_0 < 0; \quad (8)$$
2) $B = -A = -(k^2 + V_0)$ with $\mu = 1/2 - k^2 - V_0$ results in
\[ \rho(x) = -(V_0 + k^2)\text{cn}^2(x, k), \quad V_0 \leq -k^2; \] 
(9)

3) $B = -(V_0 + k^2)/k^2 = -A/k^2$ with $\mu = -1 - V_0/k^2 + k^2/2$ results in
\[ \rho(x) = -(V_0 + k^2)/k^2 \frac{\text{dn}^2(x, k)}{k^2}, \quad V_0 \leq -k^2. \] 
(10)

Secondly, we consider $r^2(x) = a_1\text{cn}(x, k) + b_1$ which yields the following conditions on the coefficients: $V_0 = -3k^2, \mu = \frac{1}{8}(1 + k^2) + \frac{6a_1^2}{k^2}, b_1 = \frac{4a_1^2}{k^2}$. This leads to two possibilities for amplitude $a_1$: $a_1 = k^2/2$, and $a_1 = -k^2/4$, resulting in
\[ \rho(x) = k^2/4 (1 \pm \text{cn}(x, k)). \] 
(11)

Finally, we consider $r^2(x) = a_2\text{dn}(x, k) + b_2$ that gives the conditions on the parameters $V_0 = -3k^2, \mu = \frac{1}{8}(1 + k^2) + \frac{6a_1^2}{k^2}, b_1 = \frac{4a_1^2}{k^2}$. This leads to $a_2 = \pm 1/4, \mu = 1/2 + k^2/8$ with the expression on the amplitude
\[ \rho(x) = \frac{1}{4} (1 \pm \text{dn}(x, k)), \] 
(12)
or $2) a_2 = \sqrt{1 - k^2}/4, \mu = 1 - k^2/2$ with $k' = \sqrt{1 - k^2}$ leading to
\[ \rho(x) = k'/4 (k' + \text{dn}(x, k)). \] 
(13)

The dissipation profile that leads to these density and flow profiles is determined by Eq. (6). The representative examples of these solutions are depicted in Fig. 2.

**Focusing regime:** $\beta = -1$. The focusing case yields some different sets of solutions. Taking $r^2(x) = \text{Asn}^2(x, k) + B$ leads to $\mu = \frac{1}{2}(1 + k^2 - 3B + \frac{2b_0}{V_0 + k^2}), 0 = B(\frac{2b_0}{V_0 + k^2} - 1)(V_0 + k^2 - Bk^2), A = -(V_0 + k^2)$. The possible choices are 1) $B = 0$ with $\mu = (1 + k^2)/2$ leading to
\[ \rho(x) = -(V_0 + k^2)\text{sn}^2(x, k), \quad -k^2 \geq V_0, \] 
(14)

2) $B = -(k^2 + V_0)$, with $\mu = 1/2 - k^2 - V_0$ leading to
\[ \rho(x) = (V_0 + k^2)\text{cn}^2(x, k), \quad -k^2 \leq V_0 < 0, \] 
(15)
or 3) $B = (V_0 + k^2)/k^2$ with $\mu = -1 - V_0/k^2 + k^2/2$ leading to
\[ \rho(x) = \frac{(V_0 + k^2)}{k^2} \frac{\text{dn}^2(x, k)}{k^2}, \quad -k^2 \leq V_0 < 0. \] 
(16)

Taking $r^2(x) = a_1\text{sn}(x, k) + b_1$ gives us a set of conditions: $V_0 = -3k^2, \mu = \frac{1}{8}(1 + k^2) - \frac{6a_1^2}{k^2}, 0 = -\frac{a_1^2}{k^2}(16a_1^2 - k^2)(16a_1^2 - k^2), b_1 = \frac{4a_1^2}{k^2}$.

If 1) $a_1 = \pm k^2/4, \mu = 1/8 - k^2/4$, we get
\[ \rho(x) = \frac{k^2}{4} (1 \pm \text{sn}(x, k)), \] 
(17)
or 2) $a_1 = \pm k/4, \mu = -1/4 + k^2/8$, so that
\[ \rho(x) = \frac{1}{4}(1 \pm \text{sn}(x, k)). \] 
(18)

Finally, we consider $r^2(x) = a_2\text{dn}(x, k) + b_2$ which yields $V_0 = -3k^2, \mu = \frac{1}{8}(1 + k^2) + 6a_1^2, 0 = \frac{a_1^2}{k^2}((16a_1^2 - k^2)(16a_1^2 - k^2) - 1), b_2 = -\frac{4a_1^2}{k^2}$. The only nontrivial solution exists for $a_2 = \sqrt{1 - k^2}/4$. Let $k' = \sqrt{1 - k^2}$, and with $\mu = 1/4 + k^2/4$ the solution becomes
\[ \rho(x) = \frac{k'}{4} (\text{dn}(x, k) - k'). \] 
(19)

For any given uniform pumping $P$, the corresponding dissipative profile is given by Eq. (6) with some representative examples of these solutions shown in Fig. 2.

**Non-interacting case:** $\beta = 0$. The case of $g_P = b_1 = 1$. Here, few fewer physically relevant solutions. For $r^2(x) = \text{Asn}^2(x, k) + B$, we get $2\mu A^2 = -V_0 b_1^2 + AB + AB^2, 0 = (\mu B + A/2)B, 2\mu A^2 = -3Ak^2B + A^2(1 + k^2) - 4ABV_0, V_0 = -k^2$, which leads to the following possibilities. 1) $B = 0$ and $\mu = (1 + k^2)/2$ gives (with $A > 0$):
\[ \rho(x) = A \text{sn}^2(x, k), \] 
(20)
or 2) $2\mu = -A/B = 1$ which yields ($A < 0$):
\[ \rho(x) = -A \text{cn}^2(x, k), \] 
(21)
and 3) $2\mu = -A/B = k^2$ results in ($A < 0$):
\[ \rho(x) = -A \text{dn}^2(x, k)/k^2. \] 
(22)

For $r^2(x) = a_1\text{cn}(x, k) + b_1$, or $r^2(x) = a_1\text{dn}(x, k) + b_1$ setting the terms before the basic functions to zero imposes equations:

- $\mu a_1^2 = -V_0 b_1^2 + b_0 b^2 + (1 - 2k^2) a_1^2/8, \mu a_1^2 = -V_0 b_1^2 + (1 - 2k^2) a_1^2/8, \forall \text{cn-type and } \mu a_1^2 = -V_0 b_1^2 + k^2 - 2a_1^2/8, \mu b_1^2 = -b_0 b_1^2 + k^2 - 2a_1^2/8, \forall \text{dn-type with two common equations } V_0 a_1^2 + (3a_1^2 k^2)/8 = 0, \text{ and } 4V_0 a_1 b_1 + a_1 b_1 k^2 = 0, \text{ which are compatible only with } b_1 = 0, V_0 = -3k^2/8$, giving the resulting values for $k = 1, \mu = 1/4$. The final solutions in both cases take the form of a solitary wave
\[ \rho(x) = a_1 \text{sech}(x), \quad a_1 > 0. \] 
(23)

Next, we consider the stability of the presented solutions by evolving them dynamically according to Eq. (1) with initial noise. The instability takes place where the dissipative profile becomes strongly negative in some spatial regions, so start representing the generation of quasiparticles that can not be compensated by the remaining positive dissipative regions. This leads to the uncontrollable growth of particles in the repulsive ($\beta = 1$) and non-active ($\beta = 0$) regimes; in attractive regime ($\beta = -1$) the system either exhibits oscillations due to the interplay between attractive forces and fast saturation response or undergoes the transition to another stationary solution.
We present two stability diagrams for $dn(x,k)$ solutions using the direct numerical integration of Eq. (1) in Fig. 3(a) and of Eq. (2) in Fig. 3(b). The stability diagrams for other types of solutions are similar. Note, that our analytical solutions were derived for Eq. (2). When the same dissipative profile was used in Eq. (1) one can expect that the behaviour changes when densities become large and the use of the Taylor expansion in Eq. (2) is no longer justified.

Figures 3 (a-c) demonstrate the behavior of the solutions governed by Eq. (10): stable evolution (a), evolution to a different time-independent solution (b) and unlimited growth of the density because of the lack of the gain saturation. Figures 3 (d-f) demonstrate the behavior of the solutions governed by Eq. (16): stable evolution (d), evolution of the initial analytical solution towards a quasiperiodic one and time-dependent oscillations (f) that appear due to the high negative values of the dissipation which in combination with the attractive self-interactions lead to the steepening of the density profile which increases the saturation nonlinearity and reduces the density. The larger is the negative dissipation the higher is the density and, therefore, the higher is the frequency of the time oscillations.

Notoriously few exact analytical solutions are known for the laser and nonequilibrium condensates evolution equations. The cGLSE-type systems are pattern-forming systems with a complex interplay of gain, dissipation, nonlinearities due to self interactions and gain saturation. This makes the engineering of the flow and density profiles in such systems challenging. However, such control is necessary for implementation of various proposals from topological insulators to optical/photonic/polaritonic transistors and other information processing devices. In this Letter we proposed a method for realising periodic modulations of the density and velocity profiles in a wide variety of laser/condensate systems governed by the cGLSE with gain saturation and obtained the analytical representation for the required pumping and dissipation controls.

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The data that supports the findings of this study are available within the article.

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