THE STRUCTURE & DYNAMICS OF MASSIVE EARLY-TYPE GALAXIES:  
ON HOMOLOGY, ISOTHERMALITY AND ISOTROPY INSIDE ONE EFFECTIVE RADIUS  

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ABSTRACT  

Based on 58 SLACS strong-lens early-type galaxies with direct total-mass and stellar-velocity dispersion measurements, we find that inside one effective radius massive elliptical galaxies with $M_{\text{eff}} \geq 3 \cdot 10^{10} M_\odot$ are well-approximated by a power-law ellipsoid with an average logarithmic density slope of $\gamma'_{\text{LD}} = -d \log (\rho_{\text{tot}})/d \log (r) = 2.085^{+0.025}_{-0.018}$ (random error on mean) for isotropic orbits with $\beta_c = 0, \pm 0.1$ (syst.) and $\sigma_c \leq 0.20^{+0.04}_{-0.02}$ intrinsic scatter (all errors indicate the 68% CL). We find no correlation of $\gamma'_{\text{LD}}$ with galaxy mass ($M_{\text{eff}}$), rescaled radius (i.e. $R_{\text{inst}}/R_{\text{eff}}$) or redshift, despite intrinsic differences in density-slope between galaxies. Based on scaling relations, the average logarithmic density slope can be derived in an alternative manner, fully independent from dynamics, yielding $\langle \gamma'_{\text{LD}} \rangle = 1.959 \pm 0.077$. Agreement between the two values is reached for $\langle \beta_c \rangle = 0.45 \pm 0.25$, consistent with mild radial anisotropy. This agreement supports the robustness of our results, despite the increase in mass-to-light ratio with total galaxy mass: $M_{\text{eff}} \propto L_{\text{eff}}^{3.63 \pm 0.05}$. We conclude that massive early-type galaxies are structurally close-to homologous with close-to isothermal total density profiles ($\leq 10\%$ intrinsic scatter) and have at most some mild radial anisotropy. Our results provide new observational limits on galaxy formation and evolution scenarios, covering four Gyr look-back time.  

Subject headings: gravitational lensing — galaxies: structure  

1. INTRODUCTION  

Understanding the internal structure of massive early-type galaxies (ETG) is essential, if we ever hope to fully understand hierarchical galaxy formation (e.g. Davis et al. 1985; Frenk et al. 1983; White & Frenk 1991) and the complex interplay between dark-matter and baryons. A number of tight relations, such as the Fundamental Plane (FP hereafter; Dressler et al. 1987; Djorgovski & Davis 1987), the color-magnitude relation (e.g. Visvanathan & Sandage 1977; Sandage & Visvanathan 1978; Bower et al. 1992) and the relation between black-hole and stellar spheroid masses (Kormendy & Richstone 1995; Magorrian et al. 1998; Ferrarese & Merritt 2000; Gebhardt et al. 2000), indicate that there must be physical processes that are not dominated by stochastic processes of hierarchical galaxy formation.

In the FP relation of ETGs both stars and dark-matter contribute to the mass and dynamics ($\sigma_c$) and total mass-to-light ratio ($M/L$). Hence, without knowledge of the dark-matter distribution and stellar populations, we remain clueless about whether changes in the FP are due to non-homology or due to changes in their stellar mass-to-light, or both (e.g. Hoírth & Madsen 1993; Prugniel & Simien 1997; Treu et al. 1999; Gerhard et al. 2001; Bertin et al. 2002; Cappellari et al. 2006; Nipoti et al. 2008; Graves et al. 2009). Given only first and second moments of the stellar velocity distribution, disentangling these effects has remained difficult, because of the mass-anisotropy degeneracy: steepening of the density profile and changes in the orbital anisotropy can offset each other to yield similar kinematic profiles (e.g. Gerhard 1993; Gerhard et al. 1998; Lokas & Mamon 2003). In studying the tilt of the FP (e.g. Renzini & Ciotti 1993), one can therefore not easily disentangle effects of mass structure (e.g. non-homology) from changes in stellar mass-to-light ratios, using only stellar kinematics.

In recent papers (Bolton et al. 2006; Treu et al. 2006; Koopmans et al. 2006; Bolton et al. 2007; Gavazzi et al. 2007; Bolton et al. 2008a), we analyzed a subsample of well-selected gravitational lens systems from the SLACS Survey, showing that massive elliptical galaxies have on average close-to isothermal density profiles, with some minor, but noticeable, intrinsic scatter between their logarithmic density slopes (e.g. Koopmans et al. 2006; Treu & Koopmans 2004). They follow the classical FP (Treu et al. 2006, as well as a tight Mass Fundamental Plane (MFP; Bolton et al. 2007, 2008b), where galaxy surface brightness is replaced by surface density. In all observable respects, they follow the trends of normal elliptical galaxies (Bolton et al. 2006; Treu et al. 2006; Bolton et al. 2008a; Treu et al. 2009) and these lens-based results can thus be extended to non-lens galaxies in the same parameter space (e.g. Hyde & Bernardi 2008; Mandelbaum et al. 2008).

In this letter we study the total mass-density profile of massive early-type galaxies inside one effective radius, using the full SLACS sample of 58 gravitational lens systems with high-fidelity HST-ACS observations. We examine the intrinsic scatter in density slopes that is allowed by the sample, whether the slope correlates with other global parameters and we set limits on the level of orbital anisotropy in these system. In Section 2, we present the logarithmic density slopes of the SLACS early-type lens galaxies, based on...
two different methods, one based on lensing and dynamics and one based on scaling relations, that explicitly include their average density profile as a free parameter. Comparing these two values allows us to set limits on their average orbital anisotropy. In section 3, we summarize our results and conclusions. Throughout this paper, we make use of the sample of 58 SLACS single-lens systems from Bolton et al. (2008a) and take all quantities from that paper. If not mentioned otherwise, all masses are in units of $10^{10}M_\odot$. We assume $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $H_0 = 100$ km/s/Mpc.

2. THE DENSITY PROFILE OF MASSIVE ETGS

To determine the logarithmic slope of the total density profile, we use two alternative methods: (i) through combining SDSS-based stellar velocity dispersions and lensing-based total masses, and (ii) through scaling relations between luminosity, mass and rescaled radius, which does not require a measured stellar velocity dispersion. Whereas a single-power law mass model is an approximation, Koopmans & Treu (2003; Treu & Koopmans 2004; Barnabè et al. 2009) show that two component models can robustly be approximated by a single power-law component given current data quality.

2.1. Derivation from Lensing and Stellar Dynamics

To derive the logarithmic density slopes, we follow Treu & Koopmans (2002; Koopmans & Treu 2003; Treu & Koopmans 2004; Koopmans et al. 2006). First the lensing mass inside the critical curves is determined from the lens models in Bolton et al. (2008a), which is nearly invariant under changes in the density profile (Kochanek 1991), hence the assumed density profile during this step is not relevant in further steps (see Koopmans et al. (2006)). Subsequently we solve the spherical Jeans equations – assuming this mass as external constraint and an Einstein radius equal to the deflection angle of an equivalent spherical mass distribution – for a luminosity-density profile that follows either a Hernquist (1990) or Jaffe (1983) profile, embedded as trace-component inside the total density profile $\rho_{\text{tot}} \propto r^{-\alpha}$ (i.e. a power-law profile). The half-light radius of the projected luminosity-density profile is set equal to the observed effective radius (Bolton et al. 2008a). We take seeing (FWHM=1.5 arcsec) into account. We vary the slope $\gamma_{\text{LD}}$ over a range of 1.1 to 2.9 and compare the predicted velocity dispersion inside the 3-arcsec diameter SDSS fiber with the observed value. The error in the measured velocity dispersion is by far the most dominant source of uncertainty. Hence, the likelihood $dP/d\gamma_{\text{LD}} \propto e^{-\chi^2/2}$ is determined from the $\chi^2$ mismatch between the model and observed velocity-dispersion values.

The results in Figure 1 show that most values lie around a slope of two, which is that of an isothermal mass profile ($\rho \propto r^{-2}$). A joint analysis of the sample yields:

$$\gamma'_{\text{LD}} \equiv -d\log(\rho_{\text{tot}})/d\log(r) = 2.085^{+0.025}_{-0.018} \ (68\% \text{ CL}), \ (1)$$

in the range of radii $0.2 - 1.3 R_{\text{eff}}$ and for $\beta_r = 0$. The dependence on orbital anisotropy is small with the slopes varying mildly over $\beta_r = \pm 0.50$ (see Fig 1). Based on changing the luminosity density profile, seeing, etc., we estimate an $\sim 0.1$ systematic error. We note two points: (i) a more detailed two-dimensional kinematic analyses of six SLACS galaxies (see Czoske et al. 2008; Barnabè et al. 2009) agree with these results, and (ii) comparing the density slopes of the 14 systems that overlap with Koopmans et al. (2006), we find an average increase of $\gamma'_{\text{LD}}$ by 6%. This difference can be attributed to minor model improvements, the use of better HST images, leading to an average decrease of $R_{\text{eff}}$ by 13%, and an improved derivation of the stellar velocity dispersion, leading to an increase by +3%. In particular the latter leads to an average increase in $\gamma'_{\text{LD}}$, explaining most of the difference. Hence currently we are limited by systematics.

An intrinsic spread of $\sigma_{\gamma_{\text{LD}}} = 0.20^{+0.04}_{-0.02}$ (i.e. $\sigma_{\gamma_{\text{LD}}}/\gamma_{\text{LD}} = 0.10^{+0.02}_{-0.01}$) of 68% CL is derived, assuming Gaussian intrinsic and error distributions (see Koopmans et al. 2006; Barnabè et al. 2009), consistent with the scatter found in Koopmans et al. (2006), Jiang & Kochanek (2007; Barnabè et al. 2009). Despite the uniformity of the sample, differences between galaxies are present, which could partly be physical (see e.g. Gerhard et al. 2001), partly due to systematics, or due to small uncorrelated contributions from the environment and large scale structure (Auger 2008; Treu et al. 2009; Guimarães & Sodré 2009). Conservatively it should therefore be regarded as an upper limit on physical variations.

2.2. Derivation from Scaling Relations

A second method to derive the ensemble-average density profile is to assume a scaling relation between the observables, luminosity, effective radius and Einstein radius and Einstein mass:

$$\alpha \log(L_{\text{eff}}) = \log \left[ M_{\text{Einst}} \left( \frac{R_{\text{eff}}}{R_{\text{Einst}}} \right)^{(3-\gamma_{\text{LD}})} \right] + \delta \ (2)$$

This relation assumes that the density profile scales as a power-law with $\rho \propto r^{-\gamma_{\text{LD}}}$ and that mass and light scale with
$M_{\text{eff}} \propto L_{\text{eff}}^\alpha$ with $M_{\text{eff}} \equiv M_{\text{inst}}(R_{\text{eff}}/R_{\text{inst}})^{(3-\gamma_{\text{SR}})}$. The idea is that for a fixed luminosity, $M_{\text{inst}}$ scales with $R_{\text{inst}}/R_{\text{eff}}$ and thus provides an ensemble-averaged density slope.

This approach is slightly different from [Bolton et al. (2008a)], who assumed a SIE or constant M/L mass profile in deriving the MFP, but similar to [Rusin & Kochanek (2005)] who focused on the FP and density slope. It allows us to derive the density slope independently from assumptions about stellar dynamics. The resulting values are $\alpha = 1.363 \pm 0.056$, $\delta = -0.959 \pm 0.050$ and for the best-fit ensemble-average logarithmic density slope

$$\langle \gamma'_{\text{SR}} \rangle = 1.959 \pm 0.077.$$  

This result is close to that derived based on dynamics models, although it assumes nothing about the dynamical structure of these galaxies (e.g. isotropy). The value of $\alpha \approx 1/\eta'$, with $\eta'$ as defined and given in [Bolton et al. (2008a)]. The difference between $\langle \gamma'_{\text{SR}} \rangle$ and $\langle \gamma'_{\text{LD}} \rangle$ of $\sim 0.1$ implies that on average anisotropy can not be very large (see [2, 4]).

2.3. Correlations of Slope and Galaxy Properties

To assess whether $\gamma'_{\text{LD}}$ correlates with global galaxy quantities or cosmic time, we plot them against effective mass, rescaled ($R_{\text{inst}}/R_{\text{eff}}$) radius and redshift. The results are shown in Figure 2. We find the following linear gradients:

$$
\begin{align*}
\frac{d\gamma'_{\text{LD}}}{ds_{\text{SIE}}} &= (-1.6 \pm 8.3) \cdot 10^{-4} \\
\frac{d\gamma'_{\text{LD}}}{dM_{\text{eff}}} &= (-0.6 \pm 14.5) \cdot 10^{-4} \\
\frac{d\gamma'_{\text{LD}}}{d(R_{\text{inst}}/R_{\text{eff}})} &= (0.10 \pm 0.16) \\
\frac{d\gamma'_{\text{LD}}}{dz} &= (-0.05 \pm 0.43),
\end{align*}
$$

with $s_{\text{SIE}}$ in units of km/s and $M_{\text{eff}}$ in units of $10^{10} M_\odot$. No weights on the points are used to avoid the brighter low-redshift galaxies from dominating the fits. We find no correlations at any significant level. Similar results were found by [Koopmans et al. (2006)] and recently by [Auger et al. (2008); Treu et al. (2009); Guzmán & Sodre (2009)], based on studies of their environment and mass along the line-of-sight. Overall, the inner regions ($R \leq R_{\text{eff}}$) of massive early-type ($> L$) galaxies are remarkably homologous and simple. This result is unlike some of the more detailed results found by the SAURON collaboration for $\leq L$, galaxies (e.g. [Cappellari et al. (2004); Emsellem et al. (2007)]), which show a range of different kinematic signatures (e.g. counter-rotating cores, triaxiality, fast versus slow rotators, etc.). We find that these complexities, at least at the high-mass end, does not seem to affect the derivation of their mass distributions and scaling relations (see e.g. [Barnabè et al. 2009], for a discussion), although the overlap between the samples in mass is small. The analyses of six SLACS lens system ([Czoske et al. 2008; Barnabè et al. 2009]), based on VLT VIMOS-IFU data, shows that the majority of the SLACS galaxies are slow rotators predominantly supported by random stellar motions.

2.4. Limits on Orbital Anisotropy

Of the two methods to derive the average density slope, one depends on dynamics and the other does not. In the former, we assumed $\beta_r = 0$. If the galaxies, however, on average have a different value of $\beta_r$, the average value of $\gamma'_{\text{LD}}$ can be either too large or to small, since we would attribute a higher/lower dispersion to a steeper/shallower density slope and not to radial/tangential orbital anisotropy.

![Figure 2](image-url)  

The median values of $\gamma'_{\text{LD}}$ versus redshift. The blue symbols are S0 galaxies and the magenta colored system is an E/S0 galaxy. The dashed curves is given at $\gamma'_{\text{LD}} \equiv 2.085$ for reference with the grey box being $\pm 10\%$ intrinsic scatter. The thin red line is the best linear fit (i.e. shown curved in the log-plots).

The independence of the second method from dynamics allows us therefore to place constraints on the average orbital anisotropy of the ensemble of galaxies ($\beta_r$). As indicated in Fig. 3, good agreement is found for a positive value of $\beta_r \approx 0.45 \pm 0.25$, in agreement with results from [Gerhard et al. (2001)]. The galaxies are at most mildly radial anisotropic, as also found in [Czoske et al. (2008); Barnabè et al. (2009)] from a more detailed two-integral and two-dimensional analysis of SLACS systems.

3. RESULTS & CONCLUSIONS

We have presented a complete analysis of the inner mass density profile of 58 early-type galaxies from the SLACS survey, focusing on their logarithmic density slope. We find the follow results for galaxies with $M_{\text{eff}} \geq 3 \cdot 10^{10} M_\odot$:

- Based on lensing and stellar dynamics constraints, the average inner logarithmic density slope in their inner $0.2-1.3 R_{\text{eff}}$ is $\langle \gamma'_{\text{LD}} \rangle = 2.085^{+0.015}_{-0.005}$ (random error on mean; 68% CL; for isotropic orbits, $\beta_r = 0$, ±0.01 (syst.) and a small (±0.1) dependence on anisotropy for a range of $\beta_r = \pm 0.50$. An intrinsic scatter in $\gamma'_{\text{LD}}$ between individual galaxies, of $\sigma_{\gamma'} \lesssim 0.20^{+0.04}_{-0.02}$ (68% CL) is still allowed by our data. This should be regarded as an upper limit on physical variations, since it must include residual systematic effects. However, we believe part of it to be due to real differences between ETG density profile slopes (see also [Barnabè et al. 2009]).

- A second independently-derived value of the average inner logarithmic density slope is found from scaling relations, yielding $\langle \gamma'_{\text{SR}} \rangle = 1.959 \pm 0.077$. This value is completely independent from dynamics and therefore a robust sanity check of the lensing plus dynamics results.

- Since $\langle \gamma'_{\text{SR}} \rangle$ is independent of orbital anisotropy, we can set a weak limit $\langle \beta_r \rangle = 0.45 \pm 0.25$ on the average
anisotropy of these systems. This shows that massive galaxies are at most mildly radial anisotropic.

- No correlation of $\gamma'_{LD}$ is found with either galaxy mass or redshift, nor with radius over which this slope is measured ($0.2$–$1.3$ $R_{e}$), implying that these results are genuine and widely applicable.

- Taking the density slope into account, an increase in luminosity is found with increasing galaxy mass: $M_{\text{eff}} \propto I_{\text{eff}}^{1.36 \pm 0.06}$ in agreement with Bolton et al. (2008b).

Based on these numerical results, we conclude that massive early-type galaxies with total masses $M_{\text{eff}} \gtrsim 3 \cdot 10^{10} M_{\odot}$ are structurally and dynamically very similar in their inner regions (roughly one effective radius), over a look-back time of about 4 Gyr, with stellar and dark-matter adding up to a combined close-to-isothermal density profile but having some, although relatively little, intrinsic scatter between their logarithmic density slopes. This bulge-halo conspiracy occurs despite (i) a clear increase in their total $M/L$ inside one effective radius with galaxy mass (as found in this paper and in Bolton et al. (2008b), leading to a tilt in the FP, and (ii) a very complex hierarchical formation history. Moreover, from the agreement between two different determinations of their average density slope – one dependent on dynamics and the other not – we also find that these galaxies are also at most mildly radially anisotropy. Our results are based on single-component mass models, which provide a good description of the available data for systems. In forthcoming publications, we will address two-component models where stellar and dark matter are modeled separately.

Our results on the near homology, isothermality and isotropy of massive early-type galaxies provide new constraints on theoretical models and numerical simulations of the formation of early-type galaxies, their subsequent evolution and the understanding of the FP. Whereas these models/simulations should match these scaling relations, they should also match their intrinsic scatter.

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