Quark mass dependence of H-dibaryon

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The H-dibaryon is the exotic multiquark state with baryon number 2 and strangeness $-2$. The existence of the deeply bound H-dibaryon is excluded by the observation of the double hypernuclei. However the recent Lattice QCD simulations have found the bound state below the $\Lambda\Lambda$ threshold with large quark masses by HALQCD and NPLQCD collaborations. In this talk, the quark mass dependence of the H-dibaryon mass is discussed using the pionless effective field theory (EFT) where a bare H-dibaryon field is coupled with two-baryon states. We determine the parameters in this theory by fitting the recent Lattice QCD results in the SU(3) limit. As a result, we obtain the attractive scattering length at the physical point where the H-dibaryon is unbound.

KEYWORDS: Exotic hadron, H-dibaryon, Pionless effective field theory, Baryon-baryon interactions

1. Introduction

The H-dibaryon investigated by R. L. Jaffe [1] is the exotic multiquark state which has baryon number 2 and strangeness $-2$. It was predicted as the flavor-singlet dihyperon state with $J^P = 0^+$ and the mass $M_H = 2150$ MeV that is 80 MeV below the $\Lambda\Lambda$ threshold. Recently the H-dibaryon has been studied with the Lattice QCD simulations by the HALQCD and NPLQCD collaborations. HALQCD found the bound states with the pion mass $m_\pi = 470-1170$ MeV in the SU(3) limit [2, 3] and resonances with the the SU(3) breaking [4]. NPLQCD found the bound states with $m_\pi = 390$ MeV in the SU(3) breaking case [5, 6]. These Lattice QCD simulations show that the H-dibaryon bounds at large pion mass regions.

The H-dibaryon has been also studied by the experimental approaches. Observation of the double hypernuclei $^6\Lambda\Lambda$He in the NAGARA event constrains the binding energy of the H-dibaryon as $B_H < 6.93$ MeV because the decay process $^6\Lambda\Lambda$He $\rightarrow ^4$He $+ H$ was not found [7, 8]. This constraint excludes a deeply bound state. Belle collaboration investigated the H-dibaryon in the $\Upsilon(1S)$ and $\Upsilon(2S)$ decays and found no peak structure near the $\Lambda\Lambda$ threshold [9]. STAR collaboration studying the $\Lambda\Lambda$ correlation in the heavy ion collision obtained the attractive scattering length of $\Lambda\Lambda$ which implies no bound state of $\Lambda\Lambda$ [10, 11]. Hence the current experimental analysis shows absence of the H-dibaryon so far.

The difference between the Lattice QCD results in the large quark mass regions and the experiments in the physical point can be connected by studying the quark mass dependence of the H-dibaryon’s properties. In the literature, there have been two interesting studies of the quark mass dependence. One approach is the chiral extrapolation technique. In Refs. [12, 13], the mass of the bare H-state was evaluated up to the $m_\pi^{3/2}$ order with the one-loop of the Nambu-Goldstone bosons, and the H-dibaryon was shown to be unbound at the physical point. However the couplings to baryon-baryon channels were not considered. The other study is in the framework of the chiral effective field theory [14–16], where the baryon-baryon scattering was investigated. In this study, the H-dibaryon

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was unbound at the physical point. However the bare H-dibaryon field considered in the chiral extrapolation approach was not included here.

In this talk, the quark mass dependence of the H-dibaryon mass is discussed using the low energy effective field theory (EFT) technique. The scattering length of the baryon-baryon scattering with the quark mass dependence is calculated by the pionless EFT [17, 18], where two-baryon states are coupled with a bare H-dibaryon field. The EFT is applicable to the scattering amplitude of the two-baryon system near the threshold. In this study, we focus on the flavor singlet channel. We determine the parameters in this theory by fitting the recent Lattice QCD results in the SU(3) limit by the HALQCD.

2. Formalism

The pionless EFT is applicable near the two-baryon threshold with a large length scale of the bound state. When the length scale $\ell_B = (2\mu B)^{-1/2}$ is much larger than the interaction range, e.g. the pion wavelength $\lambda_\pi = 1/m_\pi$, it is possible to describe the baryon-baryon interaction as the contact term. Here $\mu$ is the reduced mass of the two baryons, $B$ is the binding energy and $m_\pi$ is the pion mass. The length scale $\ell_B$ and the pion wavelength $\lambda_\pi$ in the Lattice QCD simulations are summarized in Table I. In all the case the ratio of $\ell_B$ and $\lambda_\pi$ is $\lambda_\pi/\ell_B < 1$ and therefore we apply the pionless EFT to describe the baryon-baryon scattering at these quark mass regions.

In this study, a four baryons contact term and a bare H-dibaryon field in flavor singlet channel are introduced to describe the baryon-baryon scattering amplitudes in spin singlet and strangeness $S = -2$ sector. The tree-level Feynman diagrams are shown in Fig. 1. Here two coupling constants appears in the vertices, $\lambda_0$ and $g_0$ for the contact term and the bare H-dibaryon field, respectively, where they are fitted by the Lattice data. $d_i (i = 1, 2, 3 = \Lambda\Lambda, N\Xi, \Sigma\Sigma)$ is the coefficient for the flavor degeneracy of the two baryon states in the SU(3) breaking region,

$$d_1 = \frac{1}{\sqrt{8}}, d_2 = \frac{2}{\sqrt{8}}, d_3 = \sqrt{\frac{3}{8}}, \quad \sum_{i=1}^{3} d_i^2 = 1 \,. \quad (1)$$

The primary contribution to the quark mass dependence comes from the change of the two-body

![Fig. 1. Tree-level diagrams of the four baryons contact term and the bare H-dibaryon field.](image)
phase space, which is built in the scattering equation. Additional quark mass dependence can be introduced in the interaction parameters. Here we parameterize the mass difference \( v_0 \) as

\[
v_0 = M_H^{(0)} - 2M_\Lambda, \quad M_H^{(0)} = M_H - \sigma_H(m_\pi^2/2 + m_K^2),
\]

where \( M_\Lambda \) and \( m_K \) are the mass of \( \Lambda \) and kaon, respectively \([12, 13]\). Parameters \( M_H \) and \( \sigma_H \) are also fitted by the Lattice data.

The scattering amplitude is obtained by solving the Lippmann-Schwinger equations with the contact and bare H-dibaryon field terms. The diagonal component of the amplitude for the SU(3) breaking is given by

\[
f_{ii}(E) = \frac{\mu_i}{4\pi} \sum_{\ell=1}^3 d_\ell^2 \left( \frac{\Lambda_0 + \frac{g_0^2}{E - \nu_0 + i0^+}}{\Lambda - \kappa_\ell \tan^{-1} \frac{\Lambda}{\kappa_\ell}} \right)^{-1}.
\]

\( \mu_i \) is the reduced mass of two-baryon states for \( i = 1, 2, 3 \), \( \kappa_\ell \) is given by \( \kappa_\ell = \sqrt{-2\mu_\ell(E - \Delta_\ell)} \), where \( \Delta_1 = 0, \Delta_2 = M_N + M_\Sigma - 2M_\Lambda \) and \( \Delta_3 = 2M_\Xi - 2M_\Lambda \). \( M_N \) and \( M_\Sigma \) are the mass of nucleon and \( \Xi \), respectively. In this study, the SU(3) symmetric interaction is used. However the SU(3) breaking effect is given by the mass differences of the baryons. We introduce the momentum cutoff \( \Lambda = 300 \) MeV in the loop integral. The flavor singlet scattering amplitude for the SU(3) limit is obtained by \( \Sigma_{i=1}^3 f_{ii}(E) \). The binding energy is obtained as poles of the scattering amplitudes.

3. Numerical results

In this study, let us focus on the results which is given by using only the SU(3) limit data by HALQCD [3]. From the Lattice data of binding energies in Table I, the parameters are determined as \( \Lambda_0 = -1.3 \times 10^{-5} \) MeV\(^2\), \( g_0 = 2.4 \) MeV\(^{-1}\), \( M_H = 19783 \) MeV and \( \sigma_H = -1.5 \times 10^{-3} \) MeV\(^{-1}\), respectively. We obtain the negative \( \Lambda_0 \) which indicates that the contact term in Fig. 1 works attractively. In addition the bare mass \( M_H \) is large. The large \( M_H \) suppresses the bare dibaryon field term of the scattering amplitude in Eq. (3) and therefore this term plays a minor role in this system.

From the scattering amplitudes with these fitting parameters, we calculate the quark mass dependence of the scattering length. In Table II, the obtained scattering lengths at the point corresponding to the recent Lattice data and the physical point are summarized. We note that the scattering length is the value of the baryon-baryon singlet channel for the SU(3) limit and of the \( \Lambda \Lambda \) channel for the SU(3) breaking. For the SU(3) limit, we obtain the values being close to the Lattice result \([19]\). This indicates that the quark mass dependence of the H-dibaryon and the low-energy baryon-baryon interaction can be well modeled by the present EFT. We also attempt to extrapolate the EFT to the physical point. We obtain the attractive scattering length at the physical point. Hence the H-dibaryon is not bound. A large magnitude of the scattering length supports the validity of the extrapolation using the contact interaction model.

4. Summary

The quark mass dependence of the H-dibaryon has been studied. The baryon-baryon scattering was described by the pionless EFT which was applicable near the thresholds. The scattering amplitude was obtained by solving the Lippmann-Schwinger equation with the four baryon contact term and the couplings to the bare H-dibaryon field. The coupling constant of the EFT was fitted by the recent Lattice QCD simulations. By using the Lattice data for the SU(3) limit, the parameters were fixed and the scattering length were obtained. At the physical point we found no bound state. Our model will be improved by introducing the 8s and 27 components, and the quark mass dependence of the coupling constants.
Table II. Predicted scattering lengths $a$ at the point corresponding to the Lattice data by HALQCD [3] and the physical point. $M_{B(A)}$ is the baryon (A) mass at the point. Positive (negative) $a$ shows the repulsive (attractive) scattering length.

| Point                  | $a$ [fm] |
|------------------------|----------|
| SU(3) limit            |          |
| HAL-1 ($M_B = 1749$)  | 1.40     |
| HAL-2 ($M_B = 1484$)  | 1.49     |
| HAL-3 ($M_B = 1161$)  | 1.71     |
| SU(3) breaking         |          |
| Physical ($M_A=1116$) | $-3.77$  |

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