An Algorithmic Introduction to Savings Circles

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Abstract

Rotating savings and credit associations (roscas) are informal financial organizations common in settings where communities have reduced access to formal financial institutions. In a rosca, a fixed group of participants regularly contribute sums of money to a pot. This pot is then allocated periodically using lottery, aftermarket, or auction mechanisms. Roscas are empirically well-studied in economics. They are, however, challenging to study theoretically due to their dynamic nature. Typical economic analyses of rosca stop at coarse ordinal welfare comparisons to other credit allocation mechanisms, leaving much of rosca's ubiquity unexplained. In this work, we take an algorithmic perspective on the study of rosca. Building on techniques from the price of anarchy literature, we present worst-case welfare approximation guarantees. We further experimentally compare the welfare of outcomes as key features of the environment vary. These cardinal welfare analyses further rationalize the prevalence of rosca. We conclude by discussing several other promising avenues.

1 Introduction

Rotating saving and credit associations (roscas) are financial institutions common in low- and middle-income nations, as well as immigrant and refugee populations around the world. In a rosca, a group of individuals meet regularly for a defined period of time. At each meeting, members contribute a sum of money into a pot, which is then allocated via some mechanism, such as a lottery or an auction. Recipients often use this money to purchase durable goods (e.g., farming equipment, appliances, and vehicles), to buffer shocks (e.g. an unexpected medical expense), or to pay off loans. Roscas often exist outside of legal frameworks and do not typically have a central authority to resolve disputes or enforce compliance. Instead, they provide a decentralized mechanism for peer-to-peer lending, where members who receive the pot earlier borrow from those who receive it later. They also create a structure for mutual support and community empowerment.

Roscas are used in over 85 countries and are especially prevalent in contexts where communities have reduced access to formal financial institutions (Aredo 2004; Bouman 1995; Klonek 2002; La Ferrara 2002). Roscas account for about one-half of Cameroon’s national savings. Likewise, over one in six households in Ethiopia’s highlands participate in ekub, the region’s variant of rosca (Bouman 1995). Due to their ability to provide quick, targeted support within communities, rosca and other mutual aid organizations often play an instrumental role when communities experience shocks and disasters (Chevedden 2021; Mesch et al. 2020; Travlou 2020).

Roscas are well-studied in the economics literature, with economic theory on the subject pioneered by Besley, Coate, and Loury (1993); Kovsted and Lyk-Jensen (1999); Kuo (1993) (see Appendix A for further related works). This line of work seeks to explain how roscas act as insurance, savings, and lending among members. While such studies have deepened our understanding of rosca, they are typically constrained in two main ways. First, the standard economic approach solves exactly for equilibria, which can be especially difficult due to the dynamic nature of rosca. Second, much of the existing theory focuses on coarse-grained comparisons between the welfare of rosca and other mechanisms for allocating credit. In part due to these coarse comparisons, this work often concludes that rosca allocate credit suboptimally, leaving open the question of why rosca are prevalent in practice.

In this work, we initiate an algorithmic study of roscas. Viewing roscas through the lens of approximation and using techniques from the price of anarchy literature, we study the welfare properties of rosca outcomes without directly solving for them. We specifically quantify the allocative efficacy of roscas: how well do roscas coordinate saving and lending among participants with heterogeneous investment opportunities? We show roscas enable a group’s lending and borrowing in a way that approximately maximizes the groups’ total utility. We do so under a wide range of assumptions on both participants’ values for investment and the mechanisms used for allocating the pots. This robustness may provide one explanation for their prevalence.

Our work builds on the saving and lending formulation of Besley, Coate, and Loury (1994). We assume each participant seeks to purchase an investment, such as a durable good, but can only do so upon winning the roscas’s pot. We analyze the welfare properties of typical pot allocation mechanisms, such as “swap roscas,” where the participants are given an initial (e.g. random) allocation and then swap
positions in an after-market through bilateral trade agreements. We also study the price of anarchy in auction-based rosicas where, during each meeting, participants bid to decide a winner among those who have not yet received a pot. During each round, participants must weigh the value of investing earlier against the utility loss from spending to win that round.

Our technical contributions are as follows: For swap rosicas, we prove that all outcomes guarantee at most a factor 2 loss. For auction-based rosicas, we give full-information price of anarchy results: we study second-price sequential rosicas, we prove that all outcomes guarantee at most a factor 2. For Auction-based rosicas, we give a price of anarchy of $2$.

## Algorithm 1: Rosca Multi-Round Allocation

**Constants:** $n$: the number of participants and rounds in the rosca. $p_0$: amount contributed by each participant to the pot in each round of the rosca.

**Inputs:** Valuations $v$, where $v_i^t$ indicates the value to participant $i$ of winning the pot in round $t$. Alloc an allocation mechanism.

For each round $t \in \{1, 2, \ldots, n\}$

1. Each participant contributes $p_0$ into the pot
2. Alloc selects the winning participant (who has not yet won a round)
3. The winning participant receives the pot worth $np_0$
4. Optional: Some participants make payments based on Alloc, which is redistributed to the others as rebates.

### 2.1 Roscas with Payments

A variety of different pot allocation mechanisms are common in practice (see Ardenner, 1964; Bouman, 1995). This work considers rosicas where participants make payments to influence their allocations, and assumes as a first-order approximation that participants are rational. Payments in rosicas take the form $p = (p_1, \ldots, p_n)$, where $p_i = (p_1^i, \ldots, p_n^i)$. As participants’ abilities to save money over time are typically limited, we assume participants’ utilities are additively separable across rounds, but possibly nonlinear in money. That is, participant $i$ with value vector $v_i$ has utility for allocation $x_i$ and payments $p_i$ given by

$$u_i^v(x_i, p_i) = x_i \cdot v_i - \sum_t C(p_i^t),$$

for some disutility function $C$ that is both increasing and satisfies $C(0) = 0$. In a given round, $p_i^t$ could be positive, if the allocation mechanism requires $i$ to make payments, or negative, if a different participant’s payments are redistributed to $i$. We refer to the latter as rebates, and assume all payments are redistributed each round, i.e., $\sum_t p_i^t = 0$ for all $t$.

A participant who makes positive payments in round $t$ has less money to spend in round $t$, and one who receives rebates in the form of negative payments has more to spend. The function $C(\cdot)$ describes participants’ preferences for these changes in wealth. A more precise interpretation of $C(p_i^t)$ is as follows: assume that each participant $i$ has a per-round...
income of \( w \). Without participating in the rosca, they would receive a utility \( U(w) \) from consumption of that income, for some increasing consumption utility function \( U \). Upon contributing \( p_0 \) into the rosca pot each round, the participant’s baseline consumption utility is \( U(w - p_0) \). If the rosca’s allocation procedure requires additional payments (or distributes rebates) of \( p'_i \), a participant’s utility from consumption becomes \( U(w - p_0 - p'_i) \). The disutility function \( C \) then represents the participant’s difference in utility for consumption,

\[
C(p) = U(w - p_0) - U(w - p_0 - p),
\]

which is increasing.

A large body of anthropological and empirical work on rosca shows that participants in the same rosca tend to have similar economic circumstances (see [Ardener 1964; Aredo 2004; Mequenat 1996]). So, following the theory literature, we assume \( U \) and \( w \) (and hence \( C \)) are identical across participants, even if the value for receiving the pot differ between participants ([Besley, Coate, and Loury 1994; Kovsted and Lyk-Jensen 1999; Klonner 2001]). It is typical to assume consumption utility \( U \) is weakly concave, and hence \( C \) is weakly convex ([Anderson and Baland 2002; Klonner 2003]). The special case of quasilinear utilities, where \( C(p) = p \), is especially well-studied in the algorithmic game theory literature.

To measure allocative performance of a rosca, we study the participants’ total utility:

\[
\text{WEL}^x(x, p) = \sum_i u_i^x(x_i, p_i).
\]

Following the interpretation of \( C \) in terms of consumption utility \( U \), \( \text{WEL}^x(x, p) \) represents the gain in utility to all participants for a given allocation \( x \) and payments \( p \), above the baseline total utility of \( n^2U(w - p_0) \), obtained by each of the \( n \) participants obtaining utility \( U(w - p_0) \) for \( n \) rounds. Among all possible matchings \( x \) and payment profiles \( p \in \mathbb{R}^n \times n \), the optimal welfare-outcome is given by the maximum-weight matching \( x^* = \text{argmax}_x \sum x_i \cdot v_i \) and payments \( p^* = (0, \ldots, 0) \), whose welfare is denoted \( \text{OPT}^x = \text{WEL}^x(x^*, p^*) \).

To quantify the inefficiency of a rosca outcome \((x, p)\), we study the approximation ratio \( \text{OPT}^x/\text{WEL}^x(x, p) \). When rosca outcomes are equilibria of auctions, as in Section 3 this ratio is also known as the price of anarchy (PoA).

**Roadmap.** The remainder of this paper proceeds as follows: In Section 3 we prove a constant-approximation for auction rosca. We do the same for swap rosca in Section 4. Both sets of results focus on quasilinear utilities, where \( C(p) = p \), and hence all welfare loss comes from allocative inefficiency. We extend these results to nonlinear utilities in the supplement. In Section 4 we further conduct experiments to study the impact of nonlinear utility on swap rosca welfare. We give directions for future work in Section 5.

### 3 Auction Roscas

Auctions are a common mechanism for allocating pots in rosca ([Ardener 1964; Bouman 1995; Klonner 2003]). Two major sources of variety in auction rosca are (1) when bids are solicited from participants and (2) the type of auction run for the bidding process. The bidding may occur either at the beginning, in which case a single (up-front) auction determines the full schedule of pot allocations, or sequentially, in which case a separate auction is held each period to determine the allocation for the corresponding pot. We consider sequential first- and second-price (equivalently, ascending- and descending-price) auctions, as well as up-front all-pay style auctions. Payments are typically redistributed as rebates among all of the non-winning participants.

The fact that outcomes depend on participants’ bidding behavior complicates our analysis. We assume participants play a Nash equilibrium (NE) of the rosca’s auction game. That is, their bidding strategy maximizes their utility given the bidding strategies of other participants. Our analysis will use the smoothness framework of [Syrgkanis and Tardos 2013], along with new arguments to handle rosca-specific obstacles. We assume participants have quasilinear utilities.

#### 3.1 Proof Template: Up-Front Roscas

We begin our analysis by considering rosca with up-front bidding. In an up-front rosca, each participant \( i \) submits a bid \( b_i \) at the beginning of the rosca. Participants pay their bids, and are then assigned pots in decreasing order of their bids, with the highest participant receiving the pot in round 1, and so on. Each participant \( i \)’s payments are redistributed even among the other participants in the form of reduced per-period payments into the rosca. Under quasilinear utilities, it is not relevant to the participants’ utilities what round payments are made; the only relevant outcome is total payments, which we write as \( p_i = \sum t p_i' \) when context allows.

We can further assume per-period payments remain fixed and that the participants receive the redistributed payments up-front in the form of a rebate. We decompose the participants’ total payments into their gross payments \( \hat{p}_i \) and rebates \( \hat{r}_i \), with \( p_i = \hat{p}_i - \hat{r}_i \). Formally:

**Definition 1.** In an up-front rosca with quasilinear participants, each participant \( i \) submits a bid \( b_i \), with \( b = (b_1, \ldots, b_n) \). Let \( r_i \) denote the rank of participant \( i \)’s bid. Allocations are \( x_i^t(b) = 1 \) if \( t = r_i \) and 0 otherwise. Participant \( i \)’s gross payment is \( \hat{p}_i(b) = b_i \), and their rebate is \( \hat{r}_i(b) = \sum_{i' \neq i} b_{i'}/(n-1) \).

Our auction rosca analyses all follow from a two-step argument. First, we use or modify the smoothness framework of [Syrgkanis and Tardos 2013] to obtain a tradeoff between participants’ utilities and their gross payments. Without rebates, typical auction analyses conclude by noting that high payments imply high welfare. However, because gross payments in rosca are redistributed, it could happen that both gross payments and rebates are high, but welfare is low. Our second step is to rule out this problem. For up-front rosca, we can demonstrate both steps simply.

The first step follows from Lemma A.20 of [Syrgkanis and Tardos 2013]:

**Lemma 1.** With quasilinear participants, any Nash equilibrium \( b \) of any up-front rosca with values \( v \) satisfies

\[
\sum_i u_i^v(b) \geq \frac{1}{2} \text{OPT}(v) - \sum_i \hat{p}_i(b).
\]
The left hand side of (1) is the equilibrium welfare. It therefore suffices for the second step to upper bound the gross payments on the right hand side.

**Lemma 2.** Let \( b \) be a Nash equilibrium of an up-front rosca with quasilinear participants and values \( v \). Then, for any participant \( i \), \( \hat{p}_i(b) \leq v_i \cdot x_i(b) \).

**Proof.** Assume for some \( i \) that \( \hat{p}_i(b) > v_i \cdot x_i(b) \). Then participant \( i \) must be overbidding. They could improve their utility by bidding \( 0 \), which, in an up-front rosca, does not change their rebates: \( \hat{r}_i(0, b_{-i}) = \hat{r}_i(b) > v_i \cdot x_i(b) - \hat{p}_i(b) + \hat{r}_i(b) \).

Since \( \sum_i v_i \cdot x_i(b) \) is equal to the equilibrium welfare, Lemmas 1 and 2 together imply the following.

**Theorem 1.** With quasilinear participants, every Nash equilibrium of an up-front rosca has PoA at most 4.

### 3.2 Sequential Roscas

We now consider rosca with separate sequentially-held first- and second-price auctions for each pot as opposed to the single-auction format from the previous section.

**Definition 2.** A first-price rosca runs a first-price auction in each round.

That is, if the highest-bidding participant in round \( t \) among those who have not yet won is participant \( i^* \), with bid \( b_{i^*} \), then \( x_{i^*} = 1 \), \( x_i = 0 \) for all other participants \( i \).

The gross payments are \( \hat{p}_{i^*} = b_{i^*} \) and \( \hat{p}_i = 0 \) otherwise. The rebates are \( \hat{r}_i = b_{i^*} / (n - 1) \) for all \( i \neq i^* \).

**Definition 3.** A second-price rosca runs a second-price auction in each round. That is, if the highest-bidding participant in round \( t \) among those who have not yet won is participant \( i^* \), with second-highest bid \( b_{(2)} \), then \( x_{i^*} = 1 \), \( x_i = 0 \) for all other participants \( i \).

The gross payments are \( \hat{p}_{i^*} = b_{(2)} \) and \( \hat{p}_i = 0 \) otherwise. The rebates are \( \hat{r}_i = b_{(2)} / (n - 1) \) for all \( i \neq i^* \).

Sequential auctions require a monitoring scheme in which the auctioneer discloses information about participants’ bids after each round. Our results will hold for any deterministic monitoring scheme. A key subtlety is that participants’ actions are now behavioral strategies: that is, at each stage, participants observe the disclosed history of play so far and can condition their future bids on this history. We denote the vector of behavioral strategies by \( a = (a_1, \ldots, a_n) \), and denote by \( b_i \) participant \( i \)’s bid in a round \( t \).

As with up-front roscas, we first derive a tradeoff between utility and gross payments, and second consider the impact of rebates. The sequential format complicates both steps. Our first step will follow from a novel composition argument, where we show that both first- and second-price rosca inherit a tradeoff from their single-item analogs. For second-price rosca, a standard no-overbidding assumption then bounds the auction’s rebates and implies a welfare bound. For first-price rosca, we give a more involved analysis that bounds overbidding and yields unconditional guarantee. Overbidding can both occur in equilibrium and harm welfare, so such an analysis is necessary.

Observe that, first- and second-price rosca can be thought of as the sequential composition \( n \) single-item auctions, with a rule excluding past winners. Formally:

**Definition 4 (Round-Robin Composition).** Given a single-item auction \( M \), the \( n \)-item round-robin composition of \( M \) is a multi-round allocation mechanism for \( n \) items using the following procedure: During each round \( t \), each participant \( i \) who has not yet been allocated an item submits a bid \( b_i^t \). The mechanism then runs \( M \) among the remaining participants to determine the allocation and payments for that round.

The following definition of “smoothness,” adapted from Syrgkanis and Tardos (2013), lets us characterize both first- and second-price rosca with the same framework. For our purposes, it applies to any auction where in round \( t \), each bidder who has not yet won submits a real-valued bid \( b_i^t \), which we term sequential single-bid auctions. Note that this includes single-item auctions. We will show that smoothness of single-item auctions implies smoothness of their round-robin composition.

**Definition 5.** Let \( M \) be a sequential single-bid auction. We say \( M \) is \((\lambda, \mu_1, \mu_2)\)-smooth if for every value profile \( v \) and action profile \( a \), there exists a randomized action \( a_i^*(a_i, v) \) for each \( i \) such that:

\[
\sum_i (v_i \cdot x_i(a_i^*(a_i, v), a_{-i}) - p_i(a_i^*(a_i, v), a_{-i}) \geq \lambda \OPT(v) - \mu_1 \sum_i p_i(a_i) - \mu_2 \sum_i B_i(a),
\]

where \( B_i(a) \) is \( i \)'s bid in the round where they win, or 0 if no such round exists.

Syrgkanis and Tardos (2013) show that single-item first-price and second-price auctions are \((1 - 1/e, 1, 0, 1)\)-smooth, respectively. However, the smoothness result they prove for a form of sequential composition fails to hold for round-robin composition, due to the cardinality constraint on allocations as in our setting. Here, we instead give a new composition argument tailored specifically to the rosca setting, that relies on values decreasing in time. Our composition result follows the following useful definition:

**Definition 6.** A single-item mechanism \( M \) with allocation rule \( x \) and payments \( p \) is strongly individually rational (IR) if (1) for every profile of actions \( a \), \( x(a) = 0 \) only if \( \hat{p}_i(a) = 0 \), and (2) there exists an action \( \perp \) such that for all \( i \) and \( a_{-i} \), \( p_i(\perp, a_{-i}) = 0 \).

**Lemma 3.** Let \( M \) be a strongly individually-rational single-item mechanism. If \( M \) is \((\lambda, \mu_1, \mu_2)\)-smooth for \( \lambda \leq 1 \) and \( \mu_1, \mu_2 \geq 0 \), then its round-robin composition is \((\lambda, \mu_1 + 1, \mu_2)\)-smooth as long as \( \lambda v_i^t \geq v_i^{t+1} \) for all \( i \) and \( t \).

Our proof of this lemma, presented in the supplement, augments the main idea from the Syrgkanis and Tardos (2013) composition result with ideas from Kesselheim, Kleinberg, and Tardos (2015), who consider smoothness of nonsequential mechanisms for cardinality-constrained allocation environments. As a corollary of Lemma 3, we obtain that first- and second-price rosca are respectively \((1 - 1/e, 2, 0)\) and \((1, 1, 1)\)-smooth.
We next analyze the impact of rebates. If we assume no participant overbids, then payments (and hence rebates) are necessarily bounded by values, and we obtain a similar conclusion to Lemma~\ref{lem:linear}. Moreover, we show in the supplement that an overbidding assumption is necessary for second-price rosca, as it is often the case for auctions with second-price payments. The overbidding assumption we require is as follows:

**Definition 7.** Action profile $a$ satisfies no-overbidding if $B_t(a) \leq v_i \cdot x_i(a)$ for every participant $i$.

**Theorem 2.** Let $M$ be a strongly IR, single-item auction that is $(\lambda, \mu_1, \mu_2)$-smooth, with $\lambda \leq 1$. With quasilinear participants, every no-overbidding Nash equilibrium of the corresponding auction rosca with rebates has PoA at most $(2 + \mu_1 + \mu_2)/\lambda$.

**Proof.** Lemma~\ref{lem:linear} implies that the rosca is $(\lambda, 1 + \mu_1, \mu_2)$-smooth before rebates. We can therefore write:

$$\sum_i u_i^V(a) \geq \sum_i u_i^V(a^*_t, a_{-i}) \geq \sum_i (v_i \cdot x_i(a^*_t, a_{-i}) - \hat{p}_t(a^*_t, a_{-i})) \geq OPT(v) \cdot (1 + \mu_1) \sum_i \hat{p}_t(a)$$

$$\geq \lambda \sum_i B_t(a) \geq \lambda OPT(v) \cdot (1 + \mu_1 + \mu_2) \sum_i B_t(a) \geq \lambda OPT(v) \cdot (1 + \mu_1 + \mu_2) \sum_i v_i \cdot x_i(a)$$

Since both $\sum_i u_i^V(a)$ and $\sum_i v_i \cdot x_i(a)$ are equal to equilibrium welfare, the result follows.

**Corollary 1.** For quasilinear participants, any Nash equilibrium of the first-price rosca satisfying no-overbidding has PoA at most $3e/(e - 1)$.

**Corollary 2.** For quasilinear participants, any Nash equilibrium of the second-price rosca satisfying no-overbidding has PoA at most 3.

### 3.3 Relaxing No-Overbidding

The no-overbidding assumption in the previous section rules out behavior where participants overbid in early rounds to induce others to bid high in later rounds, thereby resulting in high rebates. When this behavior is extreme, participants’ payments could conceivably far exceed their values, which in turn complicates the smoothness-based approach. The following example gives a Nash equilibrium of a first-price rosca where overbidding leads to welfare loss.

**Example 1.** Consider three participants, with $v_1 = (1, 0, 0)$, $v_2 = (2, 2, 0)$, and $v_3 = (2, 2, 0)$. The following behavioral strategies form a Nash equilibrium. Participant 1 bids 2 in round 1. Participants 2 and 3 bid 1 in round 1. If participant 1 bids less than 2 in round 1, participants 2 and 3 bid 0 in round 2. Otherwise, they bid 2. The optimal welfare is then 4, but the equilibrium welfare is 3.\footnote{This example does not satisfy the refinement of subgame perfection, though our welfare guarantees do not need this restriction.}

Despite the loss exhibited in Example~\ref{ex:linear} we can obtain a constant price of anarchy for first-price rosca without an overbidding assumption. Lemma~\ref{lem:linear} below shows that overbidding cannot drive payments much higher than equilibrium welfare. The lemma extends the following logic: In equilibrium, the participant who wins in the final round has no competition, and is therefore making zero payments. Consequently, the participant who wins in the second-to-last round cannot expect any rebates from round $n$, and therefore has no incentive to overbid. This, in turn, limits the rebates due the participant who wins the round before that, and so on. These limits on rebates limit the extent of overbidding that might occur. Throughout this section, we index participants such that in round $t$, the winner is participant $t$.

**Lemma 4.** Fix a Nash equilibrium of a first-price rosca. Then:

$$\hat{p}^*_t \leq v_t' + \frac{1}{n-t} \sum_{t'=t+1}^n v_t' \left(\frac{n}{n-t}\right)^{t'-t-1}.$$ 

We provide the proof in the supplementary materials.

**Theorem 3.** In any Nash equilibrium of the first-price rosca, the PoA is at most $(2e + 1)/e - 1$.

The result follows from summing the bounds on $\hat{p}_t(a)$ from Lemma~\ref{lem:linear} which can be arranged to obtain an upper bound of $e \sum_i v_i \cdot x_i(a)$ of the total gross payments. The theorem then follows from applying smoothness as before.

### 3.4 Extension to Nonlinear Utilities

In Appendix~\ref{app:nonlinear} we extend the price of anarchy results above beyond quasilinear utilities. With arbitrary convex cost for payments $C$, the setting comes to resemble hard budgets, for which the price of anarchy is known to be poor. We parametrize our results by upper ($\beta$) and lower ($\alpha$) bounds on the slope $C'$. We give performance guarantees which scale linearly with the ratio $\beta/\alpha$. For up-front rosca, our bounds are unconditional, while for sequential rosca, we assume an analogous no-overbidding condition to the quasi-linear version.

### 4 Swap Roscas

Several common rosca formats eschew competition between participants in allocating pots. Examples include rosca based on random lottery allocations or those based on seniority or social status (Anderson, Baland, and Moenepel 2009; Kostved and Lyk-Jensen 1999). To improve total welfare, it is common practice for participants to engage in an aftermarket by buying or selling their assigned allocations when it is mutually beneficial, i.e., by swapping rounds in the rosca (Mequanent 1996).

In this section, we formally define these swap roscas and show that, for participants with quasilinear utilities ($C(p) = p$), this aftermarket is guaranteed to converge to an outcome that yields at least half of the optimal welfare. We then present experimental results showing that this guarantee is often better, even for strictly convex $C$.\footnote{This example does not satisfy the refinement of subgame perfection, though our welfare guarantees do not need this restriction.}
4.1 Theoretical Analysis

As is common in the literature and in practice, we assume that the after-market occurs via a series of two-agent swaps (Mequanent 1996; Bouman 1995b; Ardener 1964). We assume these swaps can occur at any round \( t \). We denote by \( p^t \) the vector of payments for round \( t \), which are initialized to 0 for each round and updated as swaps occur. A swap occurs if and only if it is utility-improving for two participants under some set of payments. Formally:

**Definition 8.** Given initial allocation \( x \) and payments \( p^t \) at round \( t \), a swap is given by a pair of participants \( i, i' \) assigned to rounds \( j, j' \), respectively, and a payment \( \hat{p} \). A swap is valid if \( v_{i}^t - C(p_i^t + \hat{p}) > v_{i'}^t - C(p_{i'}^t) \) and \( v_{i}^{t'} - C(p_{i}^{t'} - \hat{p}) > v_{i'}^{t'} - C(p_{i'}^{t'}) \).

Upon executing a swap, set \( x_i^t, x_{i'}^t \leftarrow 0 \), \( x_i^{t'}, x_{i'}^{t'} \leftarrow 1 \), \( p_i^t \leftarrow p_i^t + \hat{p} \), and \( p_{i'}^t \leftarrow p_{i'}^t - \hat{p} \).

Note that with quasilinear participants, all valid swaps must strictly improve allocative efficiency since \( C(p) = p \).

**Definition 9.** A swap rosca starts from an initial allocation \( x \) and initial payments \( p = \{p^1\}_{i=1}^n \) for each participant round \( t \). At each round \( t = 1, \ldots, n \), participants execute valid swaps and we update the allocation and payment accordingly. We do so until there are no valid swaps.

Note that for non-linear \( C \), new swaps may become valid moving from round \( t \) to \( t+1 \), as each new round’s payments reset to 0. For quasilinear participants, however, Definition 9 executes all swaps in round 1. In this case, the resulting allocation is guaranteed to be stable to pairwise swaps.

**Definition 10.** An allocation \( x \) is swap-stable if for all participants \( i, i' \) assigned to \( j, j' \), we have that \( v_{i}^t + v_{i'}^{t'} \geq v_{i}^{t'} + v_{i'}^{t'} \).

For quasilinear participants, swap-stability is guaranteed regardless of the initial allocation. Convergence of the swap process follows from the fact that the total allocated value \( \sum_i v_i \cdot x_i \) strictly increases each swap and that the number of allocations is finite.

**Theorem 4.** For quasilinear participants, the welfare approximation for every swap rosca is at most 2.

**Proof.** Without loss of generality, assume that the welfare-optimal allocation assigns each participant \( i \) to be allocated the pot in round \( i \), so the optimal welfare is \( \sum_i v_i \). Now let \( \pi(i) \) denote the round when participant \( i \) is allocated the pot in the swap rosca’s final allocation, and \( \pi^{-1}(i) \) the participant allocated the pot in round \( i \). Note that \( \pi \) and \( \pi^{-1} \) are bijections. Furthermore, under quasilinear utilities, all payments between participants are welfare-neutral, and hence the rosca welfare is given by \( \sum_i v_i \).

For any participant \( i \), note that swap-stability implies

\[ v_i^{\pi(i)} + v_{\pi^{-1}(i)} \geq v_i^i + v_{\pi^{-1}(i)} \geq v_i^i. \]

Summing over all participants \( i \), we get

\[ \sum_i v_i^{\pi(i)} + \sum_i v_{\pi^{-1}(i)} \geq \sum_i v_i^i. \]

Since \( \pi \) and \( \pi^{-1} \) are bijections, both sums on the lefthand side are equal to the rosca welfare, and the righthand side is the optimal welfare, giving us a 2-approximation.

Example 1 in the appendix shows that this bound is tight.

4.2 Experimental Results

The results presented so far partially rationalize the prevalence of auction and swap rosicas. However, two limitations prevent a comprehensive view of rosicas’ allocative efficiency. First, the worst-case nature of our theoretical results give little detail about outcomes in typical instances. Second, our results hold only under quasilinear utilities, which may be less realistic for extremely vulnerable participants.

This section complements our theoretical results with computational experiments that shed light on these latter questions for swap rosicas. We simulate swap rosicas under natural instantiations of participants’ values, and with participants’ costs for payments taking a well-studied but non-linear form. We find that the approximation ratio of these rosicas in more typical scenarios is significantly better than the worst-case ratio, even after relaxing quasilinearity. Our experiments also allow us to study the way rosca performance changes as participants’ values for their payments become more convex. In particular, we use constant relative risk aversion (CRRA) utilities, given by

\[ C(p; W, a) = (1 - a)^{-1} (W^{1-a} - (W - p)^{1-a}), \]

where the parameter \( W \) represents the participants starting wealth, and \( a \) governs the convexity of the function, with \( a = 0 \) being quasilinear. For \( a > 0 \), CRRA utilities have a vertical asymptote at \( p = W \), as participants are unable to spend beyond their means. We choose \( W \) to be less than many of our participants’ maximum values for the rosca pot. This is intended to capture that most participants cannot afford the durable good without the rosca (Anderson and Baland 2002). Note that as \( a \to 1 \), \( C(p; W, a) \to \ln(W) - \ln(W - p) \). We choose CRRA utilities because they are standard for modeling preferences for wealth in economics (see, e.g., Romer 1996).

We give two sets of experimental results. In each, we run 9- and 30-person rosicas (typical sizes for small- and medium-sized rosicas), and compare three quantities: the optimal welfare under our selected value profile, the expected approximation ratio of a random allocation before any swaps, and the approximation ratio for a swap rosca run from a random allocation. Our swap rosicas are simulated according to the description in Section 4. For a pair of participants \( i \) and \( j \) for whom there exists a valid swap, there are generally many payments which will incentivize a swap and we choose the smallest such payment.

4.3 Experiment: CRRA Utilities

Our first experiment fixes a profile of participant values and studies the performance of swap rosicas as the convexity parameter \( a \) and starting wealth \( W \) vary. The value profile,
comprised of 9 participants, features 6 with cutoff values of the form $\psi_{i} = \pi$ for all $i \leq t$ for some $t$, and three participants with values which are roughly linearly decreasing in time. The average maximum value among cutoff participants is 5, which matched the average value for linearly decreasing values. We give all value profiles explicitly in the supplement. We consider values of $a$ ranging from 0 (quasi-linear) to 2 (very convex), focusing on smaller values, as larger values of $a$ tend to represent very similar, extreme functions. We take $W$ in the range $\{1, \ldots, 5\}$, as this puts participants’ wealth levels generally below their values for the rosca pot. Welfare values are averaged over 10,000 simulation runs, each starting with a random initial allocation that participants can pay to improve through swaps. Results for this simulation can be found in Table I.

Table 1: Swap Rosca Performance Under Different CRRA Parameters (OPT = 45, random baseline ratio = 1.601)

| $a$ | 1   | 2   | 3   | 4   | 5   |
|-----|-----|-----|-----|-----|-----|
| 0   | 1.035 | 1.035 | 1.035 | 1.035 | 1.035 |
| 0.1 | 1.121 | 1.119 | 1.070 | 1.067 | 1.063 |
| 0.2 | 1.122 | 1.121 | 1.080 | 1.074 | 1.074 |
| 0.3 | 1.122 | 1.119 | 1.118 | 1.086 | 1.081 |
| 0.5 | 1.122 | 1.124 | 1.127 | 1.119 | 1.120 |
| 0.75| 1.122 | 1.122 | 1.123 | 1.121 | 1.121 |
| 1   | 1.124 | 1.122 | 1.121 | 1.123 | 1.122 |
| 1.5 | 1.123 | 1.123 | 1.122 | 1.122 | 1.123 |
| 2   | 1.122 | 1.125 | 1.123 | 1.122 | 1.123 |

Across all values of $W$, the approximation ratio of swap rosca generally worsens (increases) as the level of convexity $a$ increases. Intuitively, this is likely due to the fact that since $C$ is convex, a participant receiving payments for a swap values them less than the participant offering the payments. Consequently, swaps are less likely to occur, even if they would lead to improved allocative efficiency. Meanwhile, the effect of $W$ depends on the level of convexity $a$. When $0 < a < 0.5$, participants with higher wealth $W$ have more money to spend on swaps, making swaps more likely to occur and hence improve allocative efficiency. Thus, approximation ratios improve (decrease) with higher $W$. However, as convexity increases, the disincentive to swap caused by convexity overcomes the benefit of having greater wealth with which to pay for swaps, and the approximation ratios no longer change with $W$. For all parameter values chosen, however, swap rosca led to a marked improvement over the approximation ratio from random allocation alone, suggesting that even under extreme convexity, participants are able to identify local improvements to social welfare. We also repeat this experiment with a 30-participant rosca using similar value profiles and observe the same trends. We present the results in the supplement.

4.4 Experiment: Distributional Diversity

Our second set of experiments, discussed in more detail in the supplement, varies the distribution of values across the population of participants, again for 9- and 30-person rosca.

This allows us to study the way the distribution of need across a population impacts rosca welfare. We find that performance is insensitive to wide inequality in values of participants in the population.

5 Discussion and Conclusion

Roscas are complex and varied social institutions, significant for their integral role in allocating financial resources worldwide. In this work, we focus specifically on the allocative efficacy of rosca as lending and saving mechanisms. We derive welfare guarantees for rosca under a variety of allocation protocols and show that many commonly-observed rosca provide a constant factor welfare approximation to the optimal allocation. This guarantee, we believe, gives partial explanation for the ubiquity of rosca. In addition to these specific results, our work also serves as proof of concept for the potential for techniques from algorithmic game theory to help us better understand rosca and, more generally, how communities self-organize to create opportunity. We highlight ideas for further exploration below.

First, our work modeled the savings aspect of rosca, though rosca are also used as insurance when participants experiencing unanticipated needs may bid to obtain the pot earlier than they may have otherwise planned (Calomiris and Rajaraman 1998; Klønner 2003b, 2001). There remain many gaps in our understanding of rosca when participants’ values and incomes evolve stochastically over time.

Another challenge is understanding the tension between allocative efficiency and wealth inequality. Participants with valuable investment opportunities might not bid as aggressively if their low wealth causes them to value cash highly. This is exacerbated when participants experience income shocks, which is often experienced by economically vulnerable individuals (Abebe, Kleinberg, and Weinberg 2020; Nokhiz et al. 2021). Ethnographic work shows that altruism plays a significant role in alleviating this tension (Klønner 2008, Sedai, Vasudevan, and Pera 2021). Roscas often serve a dual role of community-building institutions. Consequently, participants tend to observe signals about each others’ shocks, and act with mutual aid in mind (Klønner 2008; Mequenem 1996).

Though rosca often work outside formal institutions, studies show that “rosca enforcement” is not often an issue. For instance, (Smets 2003; Van den Brink and Chavas 1997) show that early recipients of the pot rarely default, in part due to strong community norms and standards. These considerations often go unaccounted for in theoretical studies of rosca. A deeper understanding of community norms and standards can shed more light on rosca enforcement mechanisms and robustness.

Finally, there are many questions on how aspects of the population and environment govern the performance of rosca: i.e., under what conditions would one prefer one type of rosca over another? Similarly, how do rosca perform when their members evolve over time, e.g., with some participants joining part way through the rosca and potentially holding more leverage? Likewise, rosca formation is
known to be crucial, with many rosacas preferring individuals with similar socio-economic backgrounds. Modeling and examining the rosca formation process can improve our understanding of the interaction between the rosca formation process and their functionality, efficacy, and robustness.

References
Abebe, R.; Kleinberg, J. M.; and Weinberg, S. M. 2020. Subsidy Allocations in the Presence of Income Shocks. In AAAI, 7032–7039.

Adams Dale, W.; and Canavesi, M. L. 1992. Rotating savings and credit associations in Bolivia. Adams and Fitchett, Informal Finance in Low-Income Countries.

Alabi, G.; Alabi, J.; and Akrobo, S. T. 2007. The Role Of Susu A Traditional Informal Banking System In The Development Of Micro And Small Scale Enterprises (MSEs) In Ghana. International Business & Economics Research Journal (IBER), 6(12).

Amankwah, E.; Gockel, F. A.; Eric, O.-A.; and Nubuoer, A. 2019. Pareto superior dimension of rotating savings and credit associations (ROSCAs) in Ghana: Evidence from Asunafo North Municipality of Ghana. Journal of Economic Library, 6(4): 287–309.

Anderson, S.; and Baland, J.-M. 2002. The economics of rosacas and intrahousehold resource allocation. The Quarterly Journal of Economics, 117(3): 963–995.

Anderson, S.; Baland, J.-M.; and Moene, K. O. 2009. Enforcement in informal saving groups. Journal of development Economics, 90(1): 14–23.

Ardener, S. 1964. The comparative study of rotating credit associations. The Journal of the Royal Anthropological Institute of Great Britain and Ireland, 94(2): 201–229.

Ardener, S.; and Burman, S. 1995. Money-go-rounds: The importance of rotating savings and credit associations for women.

Aredo, D. 2004. Rotating savings and credit associations: characterization with particular reference to the Ethiopian Iqquh. Savings and Development, 179–200.

Baland, J.-M.; Guirkinger, C.; and Hartwig, R. 2019. Now or later? The allocation of the pot and the insurance motive in fixed rosacas. Journal of Development Economics, 140: 1–11.

Besley, T.; Coate, S.; and Loury, G. 1993. The economics of rotating savings and credit associations. The American Economic Review, 792–810.

Besley, T.; Coate, S.; and Loury, G. 1994. Rotating savings and credit associations, credit markets and efficiency. The Review of Economic Studies, 61(4): 701–719.

Bouman, F. 1995a. Rosca: On the Origin of the Species/Rosca: sur L’Origine du Phenomène. Savings and Development, 117–148.

Bouman, F. J. 1995b. Rotating and accumulating savings and credit associations: A development perspective. World development, 23(3): 371–384.

Calomiris, C. W.; and Rajaraman, I. 1998. The role of ROSCAs: lumpy durables or event insurance? Journal of development economics, 56(1): 207–216.

Chevée, A. 2021. Mutual Aid in north London during the Covid-19 pandemic. Social Movement Studies, 1–7.

Dobzinski, S.; and Paes Leme, R. 2014. Efficiency guarantees in auctions with budgets. In International Colloquium on Automata, Languages, and Programming, 392–404. Springer.

Fang, H.; Ke, R.; and Zhou, L.-A. 2015. Rosca meets formal credit market. Technical report, National Bureau of Economic Research.

Feldman, M.; Immorlica, N.; Lucier, B.; Roughgarden, T.; and Syrgkanis, V. 2016. The price of anarchy in large games. In Proceedings of the forty-eighth annual ACM symposium on Theory of Computing, 963–976.

Hartline, J.; Hoy, D.; and Taggart, S. 2014. Price of anarchy for auction revenue. In Proceedings of the fifteenth ACM conference on Economics and computation, 693–710.

Kabuya, F. I. 2015. The rotating savings and credit associations (ROSCAs): Unregistered sources of credit in local communities. IOSR Journal of Humanities and Social Science, 20(8): 95–98.

Kesselheim, T.; Kleinberg, R.; and Tardos, E. 2015. Smooth online mechanisms: A game-theoretic problem in renewable energy markets. In Proceedings of the Sixteenth ACM Conference on Economics and Computation, 203–220.

Kesselheim, T.; and Kodric, B. 2018. Price of Anarchy for Mechanisms with Risk-Averse Agents. In 45th International Colloquium on Automata, Languages, and Programming (ICALP 2018). Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik.

Klonner, S. 2001. How Roscas perform as insurance.

Klonner, S. 2002. Understanding Chit Funds: Prize Determination and the Role of Auction Formats in Rotating Savings and Credit Associations. New Haven, USA.

Klonner, S. 2003a. Buying fields and marrying daughters: An empirical analysis of ROSCA auctions in a south Indian village. Available at SSRN 400220.

Klonner, S. 2003b. Rotating savings and credit associations when participants are risk averse. International Economic Review, 44(3): 979–1005.

Klonner, S. 2008. Private information and altruism in bidding ROSCAs. The Economic Journal, 118(528): 775–800.

Kovsted, J.; and Lyk-Jensen, P. 1999. Rotating savings and credit associations: the choice between random and bidding allocation of funds. Journal of Development Economics, 60(1): 143–172.

Kuo, P.-S. 1993. Loans, bidding strategies and equilibrium in the discount-bid rotating credit association. Institute of Economics, Academia Sinica.

La Ferrara, E. 2002. Inequality and group participation: theory and evidence from rural Tanzania. Journal of Public Economics, 85(2): 235–273.
Mequanent, G. 1996. The Role of Informal Organizations in Resettlement Adjustment Process: A Case Study of Iqubs, Idris, and Mahabers in the Ethiopian Community in Toronto. *Refuge: Canada’s Journal on Refugees*, 30–40.

Mesch, D.; Osili, U.; Skidmore, T.; Bergdoll, J.; Ackerman, J.; and Sager, J. 2020. COVID-19, Generosity, and Gender: How Giving Changed During the Early Months of a Global Pandemic.

Nokhiz, P.; Ruwanpathirana, A. K.; Patwari, N.; and Venkatasubramanian, S. 2021. Precarity: Modeling the Long Term Effects of Compounded Decisions on Individual Instability. In *Proceedings of the 2021 AAAI/ACM Conference on AI, Ethics, and Society*, 199–208.

Oguljuba, K.; Jumare, F.; and Stiegler, N. 2013. Challenges of microfinance access in Nigeria: Implications for entrepreneurship development.

Pasha, S. A. M.; and Dayrra, A. D. D. 2016. Role of ‘Iqqubs’ in Private Business Start Up and Development of Smes with Reference to Arba Minch, Ethiopia. *Journal of Social Welfare and Management*, 8(2).

Raccanello, K.; and Anand, J. 2009. Health expenditure financing as incentive for participation in ROSCAs. *Revista Desarrollo y Sociedad*, (64): 173–206.

Romer, D. 1996. *Advanced macroeconomics*. McGraw-Hill advanced series in economics. New York: McGraw-Hill Companies. ISBN 9780070536678.

Roughgarden, T. 2009. Intrinsic robustness of the price of anarchy. In *Proceedings of the forty-first annual ACM symposium on Theory of computing*, 513–522.

Roughgarden, T.; Syrgkanis, V.; and Tardos, E. 2017. The price of anarchy in auctions. *Journal of Artificial Intelligence Research*, 59: 59–101.

Sedai, A. K.; Vasudevan, R.; and Pena, A. A. 2021. Friends and benefits? Endogenous rotating savings and credit associations as alternative for women’s empowerment in India. *World Development*, 145: 105515.

Smets, P. 2000. ROSCAs as a source of housing finance for the urban poor: An analysis of self-help practices from Hyderabad, India. *Community Development Journal*, 35(1): 16–30.

Syrgkanis, V.; and Tardos, E. 2013. Composable and efficient mechanisms. In *Proceedings of the forty-fifth annual ACM symposium on Theory of computing*, 211–220.

Travlou, P. 2020. Kropotkin-19: A Mutual Aid Response to COVID-19 in Athens. *Design and Culture*, 1–10.

Van den Brink, R.; and Chavas, J.-P. 1997. The microeconomics of an indigenous African institution: the rotating savings and credit association. *Economic development and cultural change*, 45(4): 745–772.
Roscas are well-documented as both pervasive and effective in promoting positive economic, social, and even health outcomes. Beyond works already mentioned, Raccanello and Anand (2009) document the use of rosca to finance healthcare expenditures and build wealth in Mexico. Aredo (2004) demonstrates the flexible and varied nature of rosca in Ethiopia. Pasha and Davra (2016) show that ekub are an engine of small business finance in the city of Arba Minch, and that private businesses actually prefer raising money from rosca than from formal financial institutions. Amankwah et al. (2019) and Alabi, Alabi, and Akrobo (2007) study rosca in Ghana. Ogujiuba, Jumare, and Stiegler (2013) in Nigeria and Kabuya (2015) in Eswatini. Alabi, Alabi, and Akrobo (2007) show from evidence in Ghana that people joined rosca for their perceived efficiency, and that rosca facilitated small-scale business enterprises.

Many studies analyze composition and participation differences across age, ethnic, gender, and socioeconomic lines. Adams Dale and Canavesi (1992) and Ardener and Burman (1995) show that rosca participation is higher among women than men. Anderson and Baland (2003) shows how employed married women in particular in Kenya use rosca to save, protecting earnings from their husbands’ more immediate-minded spending. Roscas are known to often include members from similar socioeconomic backgrounds. Aredo (2004). Nonetheless, Klonner (2008) shows that intragroup diversity is associated with higher rates of bidder altruism and more efficient intra-rosca allocations.

Economists have also studied the interaction of rosca with formal credit markets. For instance, Besley, Coate, and Loury (1994) show that while credit markets are more efficient than rosca, there are situations in which one can expect a higher ex ante expected utility in rosca than formal credit markets. Relatedly, Fang, Ke, and Zhou (2015) show that in cases where formal credit markets are present but imperfect, rosca and credit markets can complement one another, thereby improving social welfare.

A different line of work studies rosca as a form of insurance. Klonner (2001) develops the first such model of rosca, comparing their performance to risk-sharing contracts. In this model, rosca can serve as a financial intermediary and benefit risk-averse participants. This work also analyzes the risk-sharing performance of several bidding rosca run simultaneously. The conclusion is that this set-up matches the performance of linear risk-sharing contracts while boasting greater enforceability. Other studies of rosca as insurance include Baland, Gurkinger, and Hartwig (2019), and Calomiris and Rajaraman (1998).

This work is most closely related existing analyses of rosca’s efficiency. Besley, Coate, and Loury (1993, 1994) introduce the first theoretical model of rosca, and a strong focus these and of subsequent studies is on providing comparative welfare guarantees, e.g. between different types of rosca, or between rosca and alternative financial institutions. These results typically require strong assumptions, e.g. on homogeneity values of either across participants or over time. For example, Besley, Coate, and Loury (1993, 1994) show that both the random and bidding rosca are inefficient, but do not give bounds on this inefficiency. Kovsted and Lyk-Jensen (1999) analyze differences between random and bidding rosca, again under the assumption that people are saving for a large purchase. They allow for some heterogeneity in people’s access to credit, and again provide a comparative welfare analysis between bidding and random rosca. Our work paints a more complete picture by giving quantified bounds on rosca’s inefficiency, even in the face of heterogeneity of participants’ values across agents and across time.

Our results apply techniques from auction theory and the price of anarchy literature. In particular, we make extensive use of smoothness, formalized in Roughgarden (2009) and adapted to auctions in Syrgkanis and Tardos (2013). Smoothness is a sufficient condition for approximately-optimal equilibrium welfare, and is preserved by combination with other smooth mechanisms. In addition, smoothness-derived guarantees generalize beyond standard quasilinear, full-information settings to learning outcomes and Bayes-Nash equilibria, revenue Hartline, Hoy, and Taggart (2014), large games Feldman et al. (2016), risk-averse agents, Kesselheim and Kodric (2018), and more. Roughgarden, Syrgkanis, and Tardos (2017) give an excellent survey. Notably, because of the way payments are redistributed, smoothness is not sufficient for approximately optimal welfare in rosca. To derive our welfare results, we instead combine smoothness with extra analysis to control the impact of redistribution on welfare.

B Missing Proofs and Examples

B.1 Swap Rosca Example

The example below shows that the bound from Theorem 1 is tight.

Example 2. Suppose we have three participants with value vectors: \( v_1 = (0, 0, 0) \), \( v_2 = (1, 0, 0) \), and \( v_3 = (1, 1, 0) \). The optimal allocation allocates the pot to participant 2 in round 1, participant 3 in round 2, and participant 1 in round 3 for total welfare \( \text{OPT}(v) = 2 \). However, the initial allocation that allocates the pot to participant 1 in round 2, participant 2 in round 3, and participant 3 in round 1 is swap-stable and has welfare 1.

B.2 Proof of Lemma

Lemma 3. Let \( M \) be a strongly individually-rational single-item mechanism. If \( M \) is \((\lambda, \mu_1, \mu_2)\)-smooth for \( \lambda \leq 1 \) and \( \mu_1, \mu_2 \geq 0 \), then its round-robin composition is \((\lambda, \mu_1 + 1, \mu_2)\)-smooth as long as \( v_1^t \geq v_1^{t+1} \) for all \( t \) and \( i \).

Proof. Given value profile \( v \), an optimal allocation is an assignment from participants \( i \) to pots \( t^*_i \). For values \( v \) and action profile \( a \) in the sequential mechanism, we construct a deviation \( a_i \) for each participant \( i \) in the following way: participant \( i \) simulates their equilibrium strategy up until round
Then, in round \( t^*_i \), they play their smoothness deviation for \( M \) on value profile \( \hat{v} \) and action \( a^*_i \) for \( i \), where \( \hat{v}_i = v^*_i \) and 0 for all other participants. They play \( \perp \) in all subsequent rounds. Note that, in simulating \( \alpha_i \), participant \( i \) may win in some round \( t_i \leq t^*_i \), in which case the mechanism excludes them from future rounds.

Let \( S_{EQ} \) denote the set of participants whose deviations cause them to win before \( t^*_i \) and let \( S_{OPT} \) denote the set of participants who do not win before \( t^*_i \), hence are able to play their smoothness deviation in round \( t^*_i \). For participants in \( S_{OPT} \), the choice of smoothness deviation for round \( t^*_i \) implies:

\[
\vec{v}_i \cdot \hat{x}_i(a^*_i, a_{-i}) - \hat{p}_i(a^*_i, a_{-i}) \\
\geq \lambda \cdot v^*_i - \mu_1 \sum_k \hat{p}_k^t(a) - \mu_2 \sum_k B_k^t(a),
\]

where for any period \( t \), \( B_k^t(a) \) is participant \( k \)'s bid if they win in round \( t \) and 0 otherwise.

For participants in \( S_{EQ} \), if \( t_i \) denotes the round they win in equilibrium, then

\[
\vec{v}_i \cdot \hat{x}_i(a^*_i, a_{-i}) - \hat{p}_i(a^*_i, a_{-i}) = v^*_i - \hat{p}_i(a) \geq v^*_i - \hat{p}_i(a),
\]

where the inequality follows from the fact that values are nonincreasing over time.

Summing over participants, we obtain:

\[
\sum_i (\vec{v}_i \cdot \hat{x}_i(a^*_i, a_{-i}) - \hat{p}_i(a^*_i, a_{-i})) \\
\geq \lambda \cdot \text{OPT}(\vec{v}) - \mu_1 \sum_{i \in S_{OPT}} \sum_k \hat{p}_k^t(a) - \sum_{i \in S_{EQ}} \hat{p}_i(a) \\
- \mu_2 \sum_{i \in S_{OPT}} \sum_k B_k^t(a).
\]

The second and third terms on the righthand side of this inequality are each at least the total payments of the sequential mechanism. For the last term, we can write

\[
\sum_{i \in S_{OPT}} \sum_k B_k^t(a) \leq \sum_i \sum_k B_k^t(a) = \sum_i B_i(a),
\]

where the first line follows from the fact that participants' bids are nonnegative and the second line from the fact that participants win in at most one round. We therefore obtain

\[
\sum_i (\vec{v}_i \cdot \hat{x}_i(a^*_i, a_{-i}) - \hat{p}_i(a^*_i, a_{-i})) \\
\geq \lambda \cdot \text{OPT}(\vec{v}) - (\mu_1 + 1) \sum_i \hat{p}_i(a) - \mu_2 \sum_i B_i(a)
\]

giving us the desired inequality.

The following example demonstrates the necessity of our no-overbidding assumption.

**Example 3.** Suppose participant 1 has value \((10, 0)\) and participant 2 has value \((0, 0)\). If participant 2 bids 10 in round 1, then participant 1 cannot win without incurring negative utility. Participant 2 thus wins for free and hence no rebates are distributed. The equilibrium welfare here is 0, whereas the optimal welfare is 10.

\[\text{B.3 Proof of Lemma 4}\]

**Lemma 4.** Fix a Nash equilibrium of a first-price rosca. Then:

\[
\hat{p}^t_i \leq v^*_i + \frac{1}{n - 1} \sum_{t' = t + 1}^n v^*_{t'} (\frac{n - 1}{n - t})^{t' - t - 1}.
\]

**Proof.** In what follows, we index bidders so that in the optimal allocation, participant \( i \) wins in round \( t \).

We suppress dependence of payments and allocations on the profile of equilibrium bids when it is clear from context. We argue by strong induction on the number of rounds remaining.

As our base case, note that participant \( n \) could choose to bid 0 and incur nonnegative utility from round \( v^*_n \). Their continuation utility for playing according to their equilibrium strategy is \( v^*_n - \hat{p}^n_n \geq 0 \), which yields the desired inequality for the base case.

Now assume that for all rounds from \( t + 1 \) onward, the desired inequality holds, participant \( t \) could choose to bid 0 in all rounds starting with \( t \). The continuation utility from this deviation is at least 0. Meanwhile, in equilibrium, \( t \) is receiving \( v^*_t - \hat{p}^t_t + \sum_{t'' = t + 1}^n \hat{p}^t_{t''} (n - 1) \geq 0 \) from future rounds.

We therefore can upper bound \( \hat{p}^t_t \) by

\[
v^*_t + \frac{1}{n - 1} \sum_{t' = t + 1}^n v^*_{t'} (\frac{n - 1}{n - t})^{t' - t - 1},
\]

where \( \tau = t'' - t' - 1 \). Collecting coefficients on \( v^*_t \), gives us the following equivalent upper bound:

\[
\hat{p}^t_t \leq v^*_t + \frac{1}{n - 1} \sum_{t' = t + 1}^n v^*_{t'} \left( 1 + \frac{1}{n - 1} \sum_{j = 0}^{n - 1} (\frac{n - 1}{n - j})^{j - t - 1} \right)
\leq v^*_t + \frac{1}{n - 1} \sum_{t' = t + 1}^n v^*_{t'} \left( 1 + \frac{1}{n - 1} \sum_{j = 0}^{n - 1} \frac{(n - 1)^{j - t - 1}}{n - j} \right)
\leq v^*_t + \frac{1}{n - 1} \sum_{t' = t + 1}^n v^*_{t'} (\frac{n - 1}{n - t})^{t' - t - 1}.
\]

**Proof of Theorem 3** We first sum the bounds from Lemma [4][4]

\[
\sum_i \hat{p}_i(a) = \sum_{t = 1}^n \hat{p}^t_i(a)
\leq \sum_{t = 1}^n \left[ v^*_t + \frac{1}{n - 1} \sum_{t' = t + 1}^n v^*_{t'} (\frac{n - 1}{n - t})^{t' - t - 1} \right]
\leq \sum_{t = 1}^n \left( \frac{n}{n - 1} \right)^{t - 1} v^*_t
\leq e \sum_{t = 1}^n v^*_t = e \sum_i \vec{v}_i \cdot \hat{x}_i(a).
\]

\footnote{Note that this is different from their overall utility due to rebates in rounds 1 through \( n - 1 \). Also note that we can, in fact, say \( \hat{p}^n_n = 0 \), but the weaker argument above yields cleaner indexing without changing the end constant.}
By Lemma 3, the first-price rosca is $(1 - 1/e, 2, 0)$-smooth before rebates. Considering the same deviation strategy with rebates, we obtain,

$$
\sum_i u^v_i(a) \geq \sum_i u^v_i(a^*, a_{-i}) \\
\geq \sum_i (v_i \cdot x_i(a^*, a_{-i}) - \hat{p}_i(a^*, a_{-i})) \\
\geq (1 - 1/e)OPT(v) - 2\sum_i \hat{p}_i(a) \\
\geq (1 - 1/e)OPT(v) - 2e\sum_i v_i \cdot x_i(a)
$$

The result follows from noting that both $\sum_i u^v_i(a)$ and $\sum_i v_i \cdot x_i(a)$ represent the equilibrium welfare.

C. Extension to Nonlinear Utilities

In this appendix, we show how our auction results for up-front roscas and no-overbidding sequential rosca extend to the more general setting of nonlinear cost for money $C$. We consider the class of convex functions $C$ satisfying $C(0) = 0$ and $C'(x) \in [\alpha, \beta]$ on the range of relevant values for some $0 < \alpha \leq \beta$, and we prove bounds which degrade linearly in the ratio $\beta/\alpha$. If $C$ is allowed to be an arbitrary convex function, then it could serve as a hard budget; auctions with hard budgets are known to have unbounded PoA (Syrgkanis and Tardos 2013; Dobzinski and Paes Leme 2014). Under this regime we can take advantage of two facts which follow immediately from basic calculus:

**Lemma 5.** Let $C$ be convex and increasing and satisfy $C'(x) \in [\alpha, \beta]$ and $C(0) = 0$, with $0 < \alpha \leq \beta$.

Then:

a. For $x \geq 0$, $C(x) \in [\alpha x, \beta x]$.

b. For $x \leq 0$, $C(x) \in [\beta x, \alpha x]$.

As in the quasilinear case, we demonstrate our approach with up-front rosacas, then present the more involved analysis of sequential rosacas. Both the tradeoff step between utilities and payments and the upper bound on payments need to be adjusted to accommodate nonlinear utilities.

C.1. Up-Front Roscas

Under quasilinear utilities, it was unimportant to the analysis whether payments and rebates were made before round 1 or distributed across time. When utilities are nonlinear, recall that an participant $i$’s disutility for payments is broken into the sum $\sum_i C(p_i)$. Hence, the timing of payments and rebates could change payoffs dramatically. Our analysis will be agnostic to these timing considerations. Whatever the timing, we continue to denote by $p_i^t(b), \hat{p}_i(b)$, and $r_i^t(b)$ the net payments, gross payments, and rebates, respectively, of participant $i$ at time $t$ under bid profile $b$. Under any up-front rosca, participant $i$’s total payments are their bid. Hence, $\sum_i p_i^t(b) = b_i$ and $\sum_i r_i^t(b) = \sum_{j \neq i} b_j/(n-1)$.

Trading off payments and utilities becomes more complex. We extend the proof from Syrgkanis and Tardos (2013), parameterizing by a constant $\rho$:

**Lemma 6.** For any $\rho \in [0, 2/\beta]$ and any Nash equilibrium $b$ of an up-front rosca with values $v$:

$$
\sum_i u^v_i(b) \geq (1 - \frac{\rho^2}{2})OPT(v) - \frac{\rho^2}{\beta} \sum_i \hat{p}_i(b).
$$

**Proof.** Pick a player $i$, and consider the deviation bid $b_i^* \sim U[0, \rho v_i]$ where $t^*$ denotes round $i$ is allocated in the optimal assignment. Let $\hat{i}$ denote the participant who wins $t^*$ under $b$. Then we can lower bound the deviation utility $u^v_i(b_i^*, b_{-i})$ as:

$$
\int_0^{\rho v_i^t} (v_i x_i^t(b_i^*, b_{-i}) - \sum_i C(p_i^t(b_i^*, b_{-i}))) / \rho v_i^t db_i^t
\geq \int_0^{\rho v_i^t} (v_i x_i^t(b_i^*, b_{-i}) - C(b_i^*)) / \rho v_i^t db_i^t
\geq \int_0^{\rho v_i^t} (v_i x_i^t(b_i^*, b_{-i}) - \beta b_i^*) / \rho v_i^t db_i^t
= \int_{b_i^t}^{\rho v_i^t} \frac{1}{\rho} db_i^t - \frac{\beta v_i^t}{\rho}
= v_i^{t^*} - \frac{b_i^* - \beta v_i^{t^*}}{\rho}
$$

Summing the deviation utilities across participants yields the desired bound.

To upper bound the impact of rebates, we use:

**Lemma 7.** Let $b$ be a Nash equilibrium of an up-front rosca. Then for any participant $i$, $\hat{p}_i(b) \leq v_i \cdot x_i(b)/\alpha$.

**Proof.** Assume for some $i$ that $\alpha \hat{p}_i(b) > v_i \cdot x_i(b)$. Then $i$ could improve their utility by bidding $0$, which, in an up-front rosca, does not change their rebates: $\hat{r}_i(0, b_{-i}) = \hat{r}_i(b) > v_i \cdot x_i(b) - \alpha \hat{p}_i(b) + \hat{r}_i(b) \geq v_i \cdot x_i(b) - \sum_i C(p_i^t) + \hat{r}_i(b).

Combining Lemmas 5 and 6 and choosing $\rho$ as follows:

$$
\rho = \frac{\beta + \sqrt{\beta(2\alpha + \beta)}}{\beta(\alpha + \beta + \sqrt{\beta(2\alpha + \beta)})}
$$

yields the following price of anarchy bound:

**Theorem 5.** With cost for money $C$ satisfying $C'(x) \in [\alpha, \beta]$ every Nash equilibrium of an up-front rosca has PoA at most:

$$
PoA \leq \frac{\alpha + \beta + \sqrt{\beta(2\alpha + \beta)}}{\alpha}
$$

Note that the bound degrades linearly with the ratio $\beta/\alpha$, as promised. Moreover, taking $\alpha = \beta = 1$ slightly improves on the bound from Theorem 1; several of the other constants in the paper can be improved through similar optimization of the bid deviations. We eschewed such optimization in favor of readability.
C.2 Sequential Roscas

We take a smoothness-based approach to sequential rosca. We will use the following generalization, adapted to non-linear utilities:

Definition 11. Let $M$ be a sequential single-bid auction. We say $M$ is $(\lambda, \mu_1, \mu_2)$-smooth if for every value profile $v$ and action profile $a$, there exists a randomized action $a^*_i(a_i, v)$ for each $i$ such that:

$$
\sum_i u^v_i(a^*_i(a_i, v), a_{\neq i}) \\
\geq \lambda \OPT(v) - \mu_1 \sum_i \hat{p}_i(a) - \mu_2 \sum_i C(B_i(a)),
$$

where $B_i(a)$ is $i$’s bid in the round where they win, or 0 if no such round exists.

First-and second-price auctions are smooth, via extensions of the arguments from Syrgkanis and Tardos (2013).

Lemma 8. The single-round second-price auction is $(1, 0, 1)$-smooth.

Proof. Let $h$ be the index of the participant with the highest value. Have this bidder bid $C^{-1}(v_h)$, while the remaining participants bid 0. Let $i^*$ denote the index of the highest bidder in equilibrium, and $i$ the index of the highest bidder other than $h$. (We may have $i^* = i$.) If $h$ wins bidding $C^{-1}(v_h)$, their utility is $v_h - C(a_{i^*}) = \OPT - C(a_{i^*}) \geq \OPT - C(a_h)$. Otherwise, they lose and earn utility 0, while $a_i \geq C^{-1}(v_h)$. It follows that $\OPT - C(a_{i^*}) = v_h - C(a_{i^*}) \leq v_h - C(C^{-1}(v_h)) = 0$. This implies the desired smoothness bound.

Lemma 9. The single-item first-price auction is $((1 - 1/e^3)\beta^{-1}, 1, 0)$-smooth.

Proof. The highest valued participant, say index $h$, can deviate to submitting a randomized bid $a^*_h$ drawn from the distribution with density function $f(x) = \frac{1}{v_h - \beta}$ and support $[0, (1 - 1/e^3)v_h/\beta]$. The utility of the highest bidder participant from this deviation is:

$$
\hat{u}^v_h(a^*_h, a_{\neq h}) = \int_{\max_i \neq h a_i} \frac{1}{v_h - \beta} (v_h - C(x)) f(x) dx \\
\geq \int_{\max_i \neq h a_i} \frac{1}{v_h - \beta} (v_h - \beta \cdot x) f(x) dx \\
= \left(1 - \frac{1}{e^3}\right) \frac{v_h}{\beta} - \max_i a_i.
$$

Since all other bidders can get utility at least 0 by e.g. bidding 0, the stated smoothness bound holds.

Under the generalized definition of smoothness, the round-robin composition result, Lemma 3, holds without modification. Hence, we obtain:

Corollary 3. First-price rosca are $((1 - 1/e^3)\beta^{-1}, 2, 0)$-smooth, and second-price rosca are $(1, 1, 1)$-smooth.

Under non-linear utilities, the appropriate notion of over-bidding states that no participant bids in a way that could yield negative utility. This upperbounds their bids by an amount that depends on $C$:

Definition 12. Action profile $a$ satisfies no-overbidding if $C(B_i(a)) \leq v_i \cdot x_i(a)$ for every participant $i$.

Under no-overbidding, values and payments cannot exceed utilities by too much, as the following two Lemmas state.

Lemma 10. In any no-overbidding action profile $a$ of sequential rosca based on a strongly IR single-item auction:

$$
\sum_i v_i \cdot x_i(a) \leq \frac{\beta}{\pi} \sum_i u^v_i(a).
$$

Proof. By definition, $\sum_i u^v_i(a) = \sum_i v_i \cdot x_i(a) - \sum_i \sum_t C(p_t^i(a))$. The last term can be further decomposed as $\sum_i \sum_t C(p_t^i(a)) = \sum_i \sum_t C(-\hat{p}_t(a)) + \sum_i \sum_t C(p_t^i(a))$. By the no-overbidding assumption, $\sum_i v_i \cdot x_i(a) \geq \sum_i \sum_t C(p_t^i(a))$. Hence, the ratio of values to utilities is upper bounded by the ratio of disutility from payments to utility from rebates. That is:

$$
\frac{\sum_i v_i \cdot x_i(a)}{\sum_i u^v_i(a)} \leq \frac{\sum_i \sum_t C(-\hat{p}_t(a))}{-\sum_i \sum_t \sum_i C(p_t^i(a))}.
$$

The lemma then follows from noting:

$$
\sum_i \sum_t C(-\hat{p}_t(a)) \geq \alpha \sum_i \sum_t \hat{p}_t(a)
$$

Lemma 11. In any no-overbidding action profile $a$ of sequential rosca based on a strongly IR single-item auction: $\sum_i \hat{p}_t(a) = \alpha \sum_i u^v_i(a)$.

Proof. By no-overbidding, $\sum_t C(p_t^i(a)) \leq v_i \cdot x_i(a)$. We may therefore write:

$$
\sum_i \hat{p}_t(a) = \sum_i \hat{r}_t(a) \\
\leq \alpha \sum_i \sum_t -C(-\hat{r}_t^i(a))
$$

Theorem 6. Let $M$ be a strongly IR, single-item auction that is $(\lambda, \mu_1, \mu_2)$-smooth, with $\lambda \leq 1$. With quasilinear participants, every no-overbidding Nash equilibrium of the corresponding auction rosca with rebates has PoA at most $(1 + \mu_1 \alpha^{-1} + \mu_2 \beta \alpha^{-1})/\lambda$. 

\end{document}
Proof. The theorem follows from the inequalities below, justified after their statement:

\[ \lambda \text{OPT} \leq \sum_i u_i^\gamma(a_i^\gamma(a_i, v), a_{-i}) + \mu_1 \sum_i \tilde{p}_i(a) + \mu_2 \sum_i \tilde{C}(B_i(a)) \]

\[ \leq \sum_i u_i^\gamma(a_i^\gamma(a_i, v), a_{-i}) + \frac{\mu_2}{\alpha} \sum_i u_i^\gamma(a) + \mu_2 \sum_i \tilde{C}(B_i(a)) \]

\[ \leq \sum_i u_i^\gamma(a_i^\gamma(a_i, v), a_{-i}) + \frac{\mu_2}{\alpha} \sum_i u_i^\gamma(a) + \frac{\mu_2 \beta}{\alpha} \sum_i u_i^\gamma(a) \]

The first inequality follows from smoothness. The second follows from Lemma [11] and the fourth from Lemma [10]. The third inequality follows from no-overbidding. Applying the promised linear dependencies:

\[ \text{Corollary 4. Any Nash equilibrium of the second-price rosca satisfying no-overbidding has PoA at most } 1 + 2\beta/\alpha. \]

\[ \text{Corollary 5. Any Nash equilibrium of the first-price rosca satisfying no-overbidding has PoA at most } (1 + 2\beta/\alpha)(1 - 1/e). \]

Note that our first-price result requires the no-overbidding assumption. We leave open whether the less restrictive analyses of Section 3.3 can be extended in some way to the non-linear setting as well.

D Other Extensions for Auction Results

The smoothness framework is known to be robust to variations in equilibrium assumptions, and we inherit this robustness to a significant degree. First, in rosacas and especially sequential ones, it is reasonable to assume that participants may not best respond perfectly. A more natural notion may be some form of ε-best response, where participants maximize their utility up to an additive ε error. All our results hold under this generalization, with the welfare guarantees similarly degrading by an additive, \( O(\epsilon n) \) factor. Second, we may also want to study rosacas under incomplete information, with each participant’s values being drawn according to a prior. Smoothness-based welfare results typically extend to such settings, and ours do as well in large part. In particular, Theorems [1] and [2] both hold under any Bayes-Nash equilibrium where participants’ value vectors are drawn independently of one another. For Theorem [2] this can be derived directly by mimicking the proof of Theorem 4.3 in Syrgkanis and Tardos (2013). The extension to Theorem [1] holds by additionally observing that in any Bayes-Nash equilibrium of up-front rosca, no participant has an incentive to overbid.

Unfortunately, our proof of Lemma [4] seems to rely on the full-information assumption. We leave it as an open question whether a Lemma [4] can be extended beyond full-information environments. Either way, among all auction settings, rosacas, where participants are typically members of tight-knit communities, are maybe the best candidate for assuming full information.

E Experiment: CRRA Utilities (Extended)

Table 2: Value Profiles for 9-participant Roscas Used in CRRA Utilities Experiment

| Profiles (Values in Rounds 1-9) |
|--------------------------------|
| 2 0 0 0 0 0 0 0 0 |
| 2 2 2 2 2 0 0 0 0 |
| 5 5 0 0 0 0 0 0 |
| 5 5 5 5 5 5 5 5 |
| 8 8 8 0 0 0 0 0 |
| 8 8 8 8 8 8 8 0 |
| 8 8 8 5 5 2 2 2 |
| 8 8 6 6 4 4 2 2 0 |
| 8 7 6 5 4 3 2 1 0 |

The value profiles for our CRRA Utilities experiment in Section 4.3 are provided in Table 2. Recall that these nine value profiles feature six with cutoff values of the form \( v_i^t = \pi \) for all \( i \leq t \) and three participants with values that are roughly linearly decreasing in time. The average maximum value among cutoff participants is 5, which matched the average value for linearly decreasing values.

We repeat this experiment using a 30-participant rosca with qualitatively similar value profiles as those used in the 9-participant setting: the 30 value profiles feature 20 with cutoff values of the form \( v_i^t = \pi \) for all \( t \leq i \) and 10 participants with values that are roughly linearly decreasing in time. The average maximum value among cutoff participants is 15.5, which is very close to the average value for linearly decreasing values (15). Table 3 presents the results of our CRRA Utilities experiment from Section 4.3 repeated in a 30-participant rosca. Once again, we observe the same trends as in the 9-participant rosca experiment, providing further evidence to strengthen the observations and claims in Section 4.3.

F Experiment: Distributional Diversity

Tables 4 and 5 summarize our second set of experiments, which vary the composition of the population of participants in both 9-participant and 30-participant rosacas. More specifically, we consider seven different configurations of value profiles, described briefly below and provided for the 9-participant rosca in Table 4 (qualitatively similar profiles were again used for the 30-participant rosca, as in our CRRA...
Utilities experiments. All participants have cutoff values, with participant $i$’s cutoff at $t = i$. We then vary (a) the distribution of magnitudes for participants’ values and (b) the correlation of an participant’s value with their cutoff round. Distributions labeled “-dec” have values which are negatively correlated with the cutoff round, and “-inc” have positively correlated values. The instance “pointmass” gives all participants constant value up to their cutoffs. The remaining instances can be described by the distribution of participants’ constant values before their cutoffs: “unif ” has a distribution uniform on $\{1, \ldots, 9\}$, “pareto” a pareto distribution, and “unim” a unimodal distribution with its mode at 4. We further consider both quasilinear and CRRA utilities.

We first note that as in the previous experiment, swap rosca provided across-the-board improvements over both the worst-case ratios and those of random allocation, and that under CRRA utilities, the social welfare degraded slightly compared to quasilinear. Beyond these observations, we can glean further insights from pairwise comparisons. Distributions labeled “-dec” represent settings where higher-valued participants have more urgent need for allocation. Both random allocation and swap rosca (under CRRA utilities) perform poorly on these instances compared to their “-inc” counterparts, whereas swap rosca (under quasilinear utilities) achieved better (lower) performance ratios. Swap rosca welfare were roughly double those of the random allocations (i.e., swap rosca achieved performance ratios that were half those of random allocations). A second informative set of comparisons is between “pointmass” and the “unim” distributions. Under both quasilinear and CRRA utilities, the approximation ratio seems to be driven by the correlation between urgency and value much more than the level of inequality in needs: if the latter was the main concern, both “unim” distributions would see worse performance. Swap rosca seem well-equipped to coordinate allocation among heterogeneous participants.

**G Simulation Code**

All code used for simulations can be found at: [github.com/cikeokwu/swap_rosca_sims](https://github.com/cikeokwu/swap_rosca_sims)
Table 5: Swap Rosca Performance Under Diverse Value Distributions (9 participants). CRRA parameter values $W = 4$, $\alpha = .5$

| Profile   | OPT | Random | Quasilinear | CRRA |
|-----------|-----|--------|-------------|------|
| pointmass | 32  | 2.002  | 1.220       | 1.220|
| unif-dec  | 45  | 2.452  | 1.000       | 1.327|
| unif-inc  | 36  | 1.351  | 1.030       | 1.031|
| pareto-dec| 35.994 | 2.828  | 1.000       | 1.451|
| pareto-inc| 34.580 | 1.485  | 1.096       | 1.106|
| unim-dec  | 36  | 2.337  | 1.116       | 1.310|
| unim-inc  | 35.2 | 1.708  | 1.138       | 1.141|

Table 6: Swap Rosca Performance Under Diverse Value Distributions (30 participants). CRRA parameter values $W = 4$, $\alpha = .5$

| Profile   | OPT | Random | Quasilinear | CRRA |
|-----------|-----|--------|-------------|------|
| pointmass | 116 | 2.002  | 1.218       | 1.218|
| unif-dec  | 465 | 2.819  | 1.000       | 1.385|
| unif-inc  | 435 | 1.452  | 1.043       | 1.058|
| pareto-dec| 399.499 | 4.000  | 1.000       | 1.757|
| pareto-inc| 396.165 | 1.323  | 1.066       | 1.061|
| unim-dec  | 400 | 2.553  | 1.098       | 1.347|
| unim-inc  | 398.667 | 1.636  | 1.114       | 1.114|