Experimental studies of the car-trailer system when passing by a suddenly appearing obstacle in the aspect of active safety of autonomous vehicles

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Abstract. Experiment studies focus on increasing active safety (steerability and stability) of the combination of vehicles (car with a trailer). The aim of the conducted studies is increasing active safety in the motion of autonomous vehicles with trailer via creating algorithm of functioning of a system which facilitates forecasting of the possibility of dangerous behaviour occurrence for the vehicles combination during complex road manoeuvres at a high velocity. Within this scope the system will ensure collecting data from the environment, data selection (detecting the situation which requires reaction), as well as joining the information which require decision making about the further steps of the current manoeuver. As a result, corresponding control signals will be sent to the controllers.

The starting point in this work are synchronous measurements of physical quantities for the car and trailer. They enabled the analysis of trends in changes of relative position between the car and the centre-axle trailer during avoiding a suddenly appearing obstacle. The analysis results of these measurements enable pointing out the symptoms which will signal the possibility of danger in the movement of the combination (e.g. dropping a trailer). A special feature of the study is that the mass of a pulling car is comparable to the trailer’s mass (1800 kg and 1840 kg).

The values of indexes which characterize the changes of quantities describing the movement of the combination of vehicles during avoiding a suddenly appearing obstacle have been measured. The subject of analysis was a difference in time of occurrence of characteristic indexes’ values among the physical quantities describing the motion of the car and the vehicle. The results of the study confirmed that in the course of measured quantities such characteristic values can be detected, whose occurrence for the trailer is delayed by 0.2 to 0.5 s with respect to the car driving at a velocity of 60 km/h. This time span is sufficient for a corrective measure done by the controller of the autonomous car.
1. Introduction

Kinematics of the combination of vehicles is much more complex than one of a single vehicle. Ensuring a safe ride is based on the analysis of the movement track for both vehicles and their mutual interaction which results from the kinematic bond in the connection link joining the vehicles. In the classical process of controlling the system of vehicles the interactions between a driver and vehicles, as well as driver’s experience and ability to foresee the trajectory are of great importance. Ensuring the vehicle movement stability, for instance, in a situation, in which it is necessary to make a defence manoeuver as a response to a sudden occurrence of an obstacle onto the road at the distance lower than a stopping distance of the vehicles. A main problem is that in such a situation there are no catch-all solutions.

A controller used for this purpose will be the system of processing information defined by the measured physical quantities. It should ensure collecting data from the environment, data selection (detecting the situation which requires reaction), and then recognition of one reference model for the further ride during the current manoeuver and, consequently, sending corresponding signals to the vehicle controller. The threats in such a situation result from the complexity of mutual interaction of the features of at least few elements: car, trailer, controller, surrounding (for instance, condition of the road, meteorological conditions) as well as the existing trajectory course.

As a result of developing systems facilitating the driver, new systems appeared: Electronic Stability Control (ESC), and in some cars with a trailer - Trailer Stability Assist (TSA), which reacts upon the excessive deviation of the trailer. Another system - Electronic Saving Sway Control (ESSC) equipped with the gyroscope system detects when the trailer sways or throws. It reacts with the braking force which depends on the velocity. The expectations placed upon the systems of reacting to the suddenly occurring obstacle in front of a mobile platform or an autonomous vehicle with a trailer are much higher. Also here the results from the experimental, analytic and simulation studies obtained so far are used, the starting point being the studies on car behaviour in the rotary motion, [2, 3, 8, 9] among others. It is stated that at a high velocity and small distance from the obstacle, an avoidance is a more effective manoeuver than braking [9, 11]. In the works [2, 3] it has been pointed out that there is a time delay in the reaction of a car to the turning of a steering wheel and this delay is particularly visible in the movement of the trailer (semi-trailer).

The basics for the analysis of kinematics of the car-trailer combination have been taken from the works [1, 4, 5, 8, 10], and the possibility of generating the reference model for the trajectory of combination of vehicles was presented in the work [6]. Reacting upon disturbances in the movement of autonomous platform model via adaptive control of controller parameters was used in the work [7]. Such hints are important but we still lack the models which enable estimation of forecast of the trailer behaviour during continuing of the started manoeuver of passing by and on this basis they will signal in advance about the possible critical situation in the further movement of the vehicles’ combination.

The aim of the studies is providing the data to model a temporary trajectory of the trailer’s motion. The global trajectory will join all temporary trajectories. They will be used in the concept of system forecasting the dangerous behaviour of the trailer in the combination of vehicles, especially during complex road manoeuvers at a high velocity. Within this scope the system should have an algorithm to analyse the information necessary to create the forecast of the further course of currently performed manoeuver. In a case when a moving obstacle suddenly occurs in a plane of movement (configuration area) of the autonomous vehicle, the basic problem is to find an unobstructed trajectory of movement of the combination of vehicles. So far experimental studies have been conducted. Their results are the basis for the analysis of the changes in the trajectory course for the pulling vehicle and configuration of vehicles in the combination during passing by the suddenly occurring obstacle. These results will be used to look for the symptoms which signal more and more changes in the location of vehicles which may pose a threat for the trailer movement (e.g. throwing, trailer’s turning over).
The subject of the studies was a passenger car with a truck-centre axle trailer. In this combination the mass of the pulling car is comparable to the mass of the trailer \(m_1=1800\) kg, \(m_2=1840\) kg. The trailer has been linked to the vehicle with the use of ball-and-socket joint, which is typical in the motor vehicles. The kinematic link used between the vehicles transfers longitudinal, transverse and vertical forces (Fig.1)

Figure 1. Combination of studied vehicles and principal dimensions in millimetres.

Sensors’ systems placed in the car and trailer enabled synchronous measurements of the following physical quantities:

- acceleration vector components: \(a_{x1}, a_{y1}, a_{z1}\);
- angular velocity of the bodywork’s turn around longitudinal axle \(P_i\), transverse \(Q_i\) and vertical \(R_i\) of the local coordinate system;
- angle of rotation around the axle: longitudinal \(\Phi_i\), transverse \(\Theta_i\) and vertical (angle of deviation) \(\Psi_i\);
- velocity \((v)\) and angular rotation of the steering wheel \(\delta_{h}\).

Index \(i\) means respectively: 1-car, 2-trailer, 3-additional point on the trailer.

2. Direct measurements’ results

Figure 3 shows a variant of the analysed road situation used during experimental studies.
To register the signals two synchronised measuring systems (MP - Motion Pack) have been used, in which the satellite signal receiver has been integrated with in-built inertial sensors. Measuring signals have been registered in the local coordinate systems connected to the centre of the car mass $O_1$ and in the local coordinate system connected with the point $O_3$ of the trailer (Fig. 2). Some measurements’ results have been presented in Fig. 4.

![Figure 3. Variant of the analysed road situation.](image)

Both figures present the course of the same physical quantities, and the lines in the pictures are marked with the symbols listed in the text above. Increasing the velocity from 37 to 60 km/h caused not only the increase in the extreme values of the observed quantities, but most of all, change in the relation between these, what will be used in the further analysis.

3. Kinematics of the combination car-trailer

During the analysis of the experimental studies results we used assembly kinematics model, in which we used right hand coordinate systems: global and local, presented in the Fig.2.

The coordinate systems used:
- local coordinate systems, fixed with the bodywork of the car and a trailer. The system $O_1x_1y_1z_1$ has a start point $O_1$ in the centre of the car 1 mass, axle $O_1x_1$ is parallel to the longitudinal axle of the vehicle. Coordinate systems $O_2x_2y_2z_2$ and $O_3x_3y_3z_3$ are connected to the bodywork of the trailer, and have a start point in $O_2$ (middle point of the trailer’s mass) and $O_3$, axles $O_2x_2O_3x_3$ are parallel to the longitudinal axle of the trailer. In the local coordinate systems $O_1x_1y_1z_1$ and $O_3x_3y_3z_3$ acceleration vector components $(a_{xi}, a_{yi}, a_{zi})$ and bodywork angular velocity vector components $(P_i, Q_i, R_i)$ of a car and a trailer have been registered;

![Figure 4. Measurement results of the physical quantities describing the motion of vehicles.](image)
differentiation of the (4), after transformations we calculated: will also use the accelerations for the point O

With the support of (1) the velocities (4) have been transported to the global coordinate system,

where: $x$, $y$, $z$

After integration of expressions (3) we achieve progressive velocity components of the point O

Progressive and rotational movement. On this basis we have achieved:

Movement of the autonomous vehicle is identified, among others, with reference to the road infrastructure, whose location is described in the global coordinate system. So the transformation of the results of local acceleration vector components’ measurements ($a_{xi}$, $a_{yi}$, $a_{zi}$) to the global coordinate system ($a_{Xi}$, $a_{Yi}$, $a_{Zi}$) is necessary. For instance, acceleration of a car and a trailer for small angles $\Phi_i$, $\Theta_i$ expressed in the global coordinate system is described with the simplified relation.

\[
\begin{bmatrix}
a_{Xi} \\
a_{Yi} \\
a_{Zi}
\end{bmatrix} \approx \begin{bmatrix}
\cos \Psi_i & 0 & 0 \\
0 & \cos \Psi_i & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
a_{xi} \\
a_{yi} \\
a_{zi}
\end{bmatrix}
\]

(1)

In order to define a trajectory of movement of the centre of trailer’s mass ($O_3$) one needs to make the following calculations. In these calculations we used the following relation between angular velocities described in both global and local coordinate system:

\[
\begin{bmatrix}
\Phi_i \\
\Theta_i \\
\Psi_i
\end{bmatrix} = \begin{bmatrix}
1 & \sin \Phi_i \tan \Theta_i & \cos \Phi_i \tan \Theta_i \\
0 & \cos \Phi_i & -\sin \Phi_i \\
0 & \sin \Phi_i \cot \Theta_i & \cos \Phi_i \cot \Theta_i
\end{bmatrix} \begin{bmatrix}
P_i \\
Q_i \\
R_i
\end{bmatrix}, \text{ so for small angles } \Phi_i, \Theta_i, \Psi_i \approx \begin{bmatrix}
P_i \\
Q_i \\
R_i
\end{bmatrix}
\]

(2)

Trailer 2 makes progressive and rotational movement. Progressive velocity of $O_3$ is $V_{O3}$ (where $U_3$, $V_3$, $W_3$ – velocity vector components of the point $O_3$ in the local coordinate system). Angular velocity of the trailer $\Omega_{n}$ (where $P_3$, $Q_3$ and $R_3$ – angular velocity vector components with relation to the local coordinate system). The accelerations measured in point $O_3$ are a sum of accelerations resulting from the progressive and rotational movement. On this basis we have achieved:

\[
\begin{align*}
\dot{U}_3 &= a_{x3} + V_3 \cdot R_3 - W_3 \cdot Q_3 \\
\dot{V}_3 &= a_{y3} - U_3 \cdot R_3 + W_3 \cdot P_3 \\
\dot{W}_3 &= a_{z3} + U_3 \cdot Q_3 - V_3 \cdot P_3
\end{align*}
\]

(3)

After integration of expressions (3) we achieve progressive velocity components of the point $O_3$ ($U_3$, $V_3$, $W_3$) and then we treat them as the date for further calculations. Progressive velocities $V_{O2}$ of the point $O_2$ in the local coordinate system $O_3x_2y_2z_2$ we calculate from the formula:

\[
\begin{bmatrix}
U_2 \\
V_2 \\
W_2
\end{bmatrix} = \begin{bmatrix}
U_3 \\
V_3 \\
W_3
\end{bmatrix} + \begin{bmatrix}
i & j & k \\
0 & P_3 & Q_3 \\
x_2 & y_2 & z_2
\end{bmatrix} \begin{bmatrix}
P_3 + Q_3z_2 - R_3y_2 \\
V_3 - P_3z_2 + R_3x_2 \\
W_3 + P_3y_2 - Q_3x_2
\end{bmatrix}
\]

(4)

where: $x_2$, $y_2$, $z_2$ – coordinates of the point $O_2$ in the coordinate system $O_3x_3y_3z_3$

With the support of (1) the velocities (4) have been transported to the global coordinate system, achieving velocity components of the point $O_2$ in the global coordinate system. After a singular integration we achieved the trajectory of the motion of the point $O_2$. As in the further calculations we will also use the accelerations for the point $O_3$, we formulated the equation (5). On the basis of the differentiation of the (4), after transformations we calculated:
Applying transformation from expression (1), we transposed described in (5) component acceleration of the centre of mass to the global coordinate system using relative acceleration components in point O₂ in the local coordinate system O₃x₃y₃z₃ obtained from differentiation of the expression (4). The example of calculation results is presented in Fig. 5a and 5b in a form of course of the movement trajectory for the centre of car and trailer mass.

\[
\begin{align*}
a_{x2} &= \dot{U}_2 - V_2 \cdot R_3 + W_2 \cdot Q_3 \\
a_{y2} &= \dot{V}_2 + U_2 \cdot R_3 - W_2 \cdot P_3 \\
a_{z2} &= \dot{W}_2 - U_2 \cdot Q_3 + V_2 \cdot P_3
\end{align*}
\] (5)

Figure 5. Movement trajectories of the centre of mass for the car (a) and trailer (b).

4. Analysis of calculation results of quantities describing changes in the movement of vehicles’ combination during passing by the obstacle

Applying the model of vehicles’ combination kinematics and the relations (1)-(5) we determined the courses of the quantities of our interest in the global coordinate system. Figure 6 presents:

- angular rotation of the steering wheel \(\delta_0\);
- angular rotation of the car around the vertical axle (angle of deviation) \(\Psi_1\);
- acceleration and transverse displacement \(a_{Y1}\) and \(Y_1\) of the centre of mass of the car;
- differences in acceleration vector components \(a_{Y1} - a_{Y2}\) and transverse offset \(Y_1 - Y_2\);
- angle of deviation of trailer’s towing bar \(\delta_\theta = \Psi_1 - \Psi_2\).

Figure 6. Course of measured and calculated quantities.
Angles $\delta_H$ and rotation (deviation) of the vehicle $\Psi_1$ in this analysis are treated as conditions which have a significant influence on the movement of the trailer. On the basis of the courses shown in Fig. 6, we have agreed that:

- time delay $\Delta T$ of the car seen in a form of angle $\Psi_1$ change as a reaction upon the rotation of the car’s steering wheel is significant; and equals 0.9 s at the velocity $v=37$ km/h and 0.7 s at the velocity $v=60$ km/h and it decreases with the increase of $v$;
- time delay for extreme values in the courses $\Psi_1$ and a difference of $Y_1-Y_2$ equals, on average, 0.20 s at the velocity of $v=37$ km/h and 0.18 at $v=60$ km/h and it characterizes a quick reaction seen in a transverse displacement of the trailer in relation to the car, as the response to the turnover of a bodywork of the car by the angle of $\Psi_1$;
- time delay for extreme values in the courses $\delta_H$ and a difference of $\Psi_1-\Psi_2$ equals, on average, 0.28 s at the velocity of $v=37$ km/h and 0.25 s at $v=60$ km/h and shows a short reaction time of the trailer to the increase of the angle $\delta_H$ in the car, seen in a course of difference $\Psi_1-\Psi_2$.

Fig. 7 presents the way the results from Fig. 6 are used to determine the values:

- $t_1$, $t_2$, $t_3$ – characteristic time values, determined on the basis of the analysed course;
- growth rate of the observed quantity, marked as $tg\beta$.

![Figure 7. Example of determining of the characteristic values.](image)

Example results of the quantities calculations, which on this stage referred to the initial phase of the process of passing by the obstacle, so the phase of steering wheel rotation angle growth are shown in Table 1. The analysis of the initial phase is justified as we look for the forecast of the further course of the already started manoeuvre.

**Table 1.** Characteristic values, determined in the initial phase of the passing by of the obstacle

| Observed quantity | Time of the initial phase growth of the observed quantity, $t_1$ [s] | Growth rate (tg$\beta$) in the time, $t_2$ | Time of obtaining local extreme point, $t_3$ [s] |
|-------------------|----------------------------------|---------------------------------|-----------------|
| $\delta_H$        | 37 km/h 0.90 60 km/h 0.41        | $t_2=1.17$ s 37 km/h 0.61 s    | $t_3=1.71$ s 37 km/h 0.95 s |
| $\Psi_1$          | 37 km/h 1.29 60 km/h 0.72        | $t_2=1.89$ s 37 km/h 1.63 s    | $t_3=2.96$ s 60 km/h 1.72 s |
| $\Psi_1-\Psi_2$   | 37 km/h 1.15 60 km/h 0.59        | $t_2=1.50$ s 37 km/h 0.84 s    | $t_3=2.18$ s 60 km/h 1.23 s |
| $Y_1-Y_2$         | 37 km/h 1.1 60 km/h 0.7          | $t_2=2.12$ s 37 km/h 0.95 m/s  | $t_3=2.76$ s 60 km/h 1.73 m/s |
Taking the purpose of the studies into account, particular attention has been paid to:
- quantities which exhibit high sensitivity to the increase of velocity and rotation angle of the steering wheel;
- conditions which may be treated as reaction of the trailer to the behaviour of pulling vehicle;
- quantities whose time delay $\Delta T$ is small, which means they quickly react upon the change of $\delta_H$ and $\Psi_1$.

The calculations have shown that at the higher velocity, from 37 to 60 km/h and steering wheel rotation angle growth from 1.21 to 1.63 rad/s we observe the increase in growth rate of:
- bodywork rotation velocity $\Psi_2$ by 32.5%;
- angle of deviation of trailer’s towing bar velocity $\Psi_1 - \Psi_2$ by 24%;
- growth rate of the acceleration transverse component for the trailer $a_{y2}$ by 57.9%.

The measurements, calculations and analyses done allow for choosing the following indexes:
- steering wheel rotation angle $\delta_H$, driving velocity $v$ and growth rate of $\Psi_2$ and $a_{y2}$;
- growth rate of the change in transverse acceleration $a_{y1} - a_{y2}$ and angle of deviation of trailer’s towing bar $\Psi_1 - \Psi_2$.

The analysed indexes will be treated as symptoms of risk growth in the trailer’s movement. Their values grow fast, along with velocity growth of the vehicles’ combination and increase of rotation angle of the pulling car’s steering wheel. Particularly high sensitivity is very beneficial, and the course of changes will be determined on the basis of simulation studies of the kinematics of movement of the vehicles’ combination in different road conditions. These studies’ results will be the basis for models of function describing the relation between the steering wheel rotation angle and driving velocity in different road conditions.

Proposed functioning of the controller will be based on so called tracking by prediction, and a result will be the forecast estimation of trajectory deviation of the trailer from the car’s track. The calculated forecast of the trajectory with the connection of the analysis of the coming results of measurements for the trailer may allow for such a control which will enable sustaining of the trajectory with some margin of safety left. Thanks to such a coupling with the controller of a prediction function, the time until the start of correcting manoeuvre and manoeuvre which minimises the occurrence of critical situation in the trailer’s movement.

5. Summary and conclusions
These experimental studies allowed for selecting the symptoms sensitive to forces acting by the car onto the trailer and collecting significant information which will be used in the controller of the autonomous car’s trailer in order to forecast a dangerous situation during passing by a suddenly appearing obstacle. The starting point in the studies was defining the time difference in occurring the characteristic index values among physical quantities describing the movement of the car and the trailer. This difference limits the time span for estimation of the corrective action model taking into account the course of defensive manoeuvre done so far. For instance, time delay of the trailer’s reaction to the rotation of the car’s steering wheel $\delta_H$ in a form of turnover of the trailer by the angle $\Psi_2$ is 0.7-0.9 s, where observing a growth rate of the difference $\Psi_1 - \Psi_2$ allows to determine the reaction already after 0.3 s.
The results of the studies have enabled to point out the symptoms which will signal the possibility of a potential danger during the movement, e.g. dropping the trailer. Particularly important information can be achieved on the basis of observation of the growth rate of the difference in acceleration vector components $a_{Y1} - a_{Y2}$ and angle of deviation of trailer’s towing bar $\Psi_1 - \Psi_2$ (Fig. 6 and Table 1). This will be used during creating models for forecasting the occurrence of critical situation in the movement of autonomous vehicles.

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