Primordial black holes from D-parity breaking in SO(10) grand unified theory

Sasmita Mishra$^a$,* and Urjit A. Yajnik$^b$

$^a$National Institute of Technology Rourkela, Sundargarh, Odisha, 769 008, India
$^b$Indian Institute of Technology Bombay, Powai, Mumbai, 400 076, India

E-mail: mishras@nitrkl.ac.in, yajnik@iitb.ac.in

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Abstract. The growing evidence of gravitational waves from binary black hole mergers has renewed the interest in study of primordial black holes (PBH). Here we study a mechanism for the formation of PBH from collapse of pseudo-topological domain walls which form out of equilibrium during inflation and then collapse post inflation. We apply the study to domain wall formation due to D-parity embedded in a supersymmetric grand unified theory (GUT) based on SO(10) and compare the abundance of resulting PBH with the existing constraints. Thus the macroscopic relics can then be used to constrain or rule out a GUT, or demand a refinement of the theory of PBH formation in this class of GUTs.

Keywords: primordial black holes, Cosmic strings, domain walls, monopoles, particle physics - cosmology connection, physics of the early universe

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*Corresponding author.
1 Introduction

The LIGO and Virgo collaborations have confirmed the direct detection of gravitational waves [1, 2] and binary black hole mergers. The detection has also confirmed the existence of a population of stellar-mass black holes with masses \( \sim (10 - 100)M_\odot \). In the catalogs the Advanced LIGO and Advanced Virgo gravitational wave detectors [3, 4] along with KAGRA detector [5] have reported candidates consistent with the coalescence of binary black holes. This has revived the interest in the origin of massive black holes [6], in particular in the study of primordial black hole (PBH) [7–10] while the possibility of any of them being a member of the observed collapsing black hole binaries is an open question [7]. The existence of PBH was anticipated by Zel’dovich, Novicov [11] and Hawking [12]. The PBHs are interesting as they could in principle be small enough for Hawking radiation to be important [13]. The black holes radiate thermally and evaporate with a time scale \( \tau = 10^{64}(M/M_\odot)^3 \text{yr} \). So the PBHs with mass \( < 10^{15} \text{g} \) would have completely evaporated by the present epoch. Whereas the PBHs with mass \( \sim 10^{15} \text{g} \) would be evaporating today contributing to the cosmological \( \gamma \)-ray background, Galactic \( \gamma \)-ray background, \( \gamma \)-ray bursts, radio bursts or some of them might have clustered within Galactic halo. The PBHs with mass \( > 10^{15} \text{g} \) would still survive today to be detectable by their gravitational effects. Such PBHs could also act as non-baryonic dark matter candidates owing to their origin in radiation dominated era. They could also act as seeds of supermassive black holes in the centres of galaxies. In GUT inspired theories, the PBH can provide restrictions on the parameters of GUT [18] and on the form of fluctuations present in the very early Universe [19]. Also, they can act as probes for supersymmetry [20] and cosmology and high energy physics [21]. They may play an important role in the generation of cosmological structures. The PBH formation scenarios rely on the collapse of inhomogeneities [14] which has been more recently treated in [15], while PBH formation from the collapse of topological defects has also been proposed (see for instance [16, 17]). In this paper we study the formation of PBHs from the collapse of topological pseudo-defect domain walls that form in the course of spontaneous gauge symmetry breaking phase transition.

Domain walls are topological defects that appear in theories with broken discrete symmetry. Early studies of SO(10) grand unified theory in the context of cosmology [22] identified the possibility of unstable domain wall formation in certain realisations of the model. This situation generalises to non-equilibrium conditions in the early Universe when some
scalar fields develop transitory condensates, but falling into one of several degenerate minima of the thermal effective potential. The vacua are related by some of the broken generators of the group, however the barrier separating them is finite and there need be no topological obstruction to transition from one vacuum to the other. Yet, the dynamical effects combined with causal horizons in the early Universe can give rise to domain walls. Such defects were studied and dubbed Topological Pseudo-Defects in [23]. As discussed in [22], the collapse of such pseudo-defects can result in PBH, and this would be an interesting signature of the GUT visible in the present Universe. For the case of SUSY SO(10) pseudo-defects, there is indeed a danger that some of them persist for energetic though not topological reasons. These can in principle degrade by the processes noted by Preskill and Vilenkin [24]. But long lived defects at the GUT scale can interfere with inflation and ruin the observed isotropy of Cosmic Microwave Background (CMB) by the Planck satellite collaboration [25]. Thus a mechanism involving additional ultraviolet operators for their destabilisation was proposed in [26]. We explore this setup here further for its potential to originate the PBH.

We consider the non-equilibrium scenario along the lines considered in [27–29]. Accordingly, Bunch-Davis fluctuations cause a scalar field to evolve, alternating between two or more of its degenerate minima, giving rise to pseudo-defect domain walls (DW). The pseudo-defects at this stage carry relatively small localised energy density due to the local minima being shallow. Over time the relevant wavelength of the scalar field, and the sizes of the domains grow large enough to exit the de Sitter horizon. The pseudo-defect structure remains imprinted on this fluctuation. Later, after reheating, the wavelengths re-enter the horizon and the DW re-enter, with large surface tension due to the remaining medium having cooled off substantially. They then start to shrink because of the surface tension. If the collapse continues till the entire energy of such closed wall gets concentrated within its gravitational radius then the PBH can form. No conflict arises with standard cosmology, as the pseudo-defects either become PBH, being topologically unstable, or degrade by processes mentioned previously.

The interesting possibility is that the same condensate which gives rise to pseudo-defects also acts as the inflaton. For Minimal Supersymmetric SO(10) GUT (MSGUT) a scenario with an embedded inflaton has been considered in a variety of contexts [30–36]. However, for the considerations in this paper we will leave this possibility open and consider the occurrence of the pseudo-defects in conjunction with a generic inflation, even if the inflaton can be identified with scalar or gauge fields [37] within the same GUT. Further, several of the above mentioned scenarios rely on gauge singlet superfields. Thus we leave embedding of the inflaton within the GUT to later, and focus on the wall collapse scenario of PBH formation.

The plan of the paper is as follows. In section 2 we discuss a generic model of formation of PBHs under non-equilibrium conditions. In section 3 and its subsections we discuss generic DW formation due to the structure of SO(10) followed specifically by the role of D-parity in MSGUT SO(10) model and apply the non-equilibrium formalism to obtain our main results. Section 4 contains the summary and conclusions. In appendix A we summarise the observational constraints on pre-galactic black hole abundances against which we have compared our results.

## 2 The non-equilibrium initial conditions

In this section we discuss a mechanism for appearance of DWs in a theory of a complex scalar field and subsequent PBH formation in an inflationary universe as considered in [27–29].
Although first discussed in the context of axions, the whole scenario is fortuitously applicable to SO(10) due to the discrete parity flipped local minima available in the latter. Consider the pseudo-Nambu-Goldstone bosons (PNG) arising from symmetry breaking of a complex scalar field. Its potential is chosen to be eq. (2.1)

\[ V(\varphi) = \lambda(|\varphi|^2 - f^2/2)^2, \tag{2.1} \]

signalling the breakdown of its U(1) symmetry at a scale \( f \). The minima of the potential in this case are degenerate, with possible vacuum expectation values \( \varphi = \frac{f}{\sqrt{2}} \exp(i\phi/f) \). Further, it is assumed that the residual axionic degree of freedom \( \phi \) develops a periodic potential, as for example due to instanton effects,

\[ \delta V = \Lambda^4(1 - \cos \theta), \tag{2.2} \]

where \( \theta = \phi/f \). In this dynamics of two successive second order phase transitions it is assumed that \( f \gg \Lambda \) and also, \( \Lambda \ll H \), where \( H \) is the Hubble parameter during inflation.

At the first stage the spontaneous breaking U(1) symmetry fills the Universe with global U(1) strings. In the second stage of symmetry breaking which happens at the post-inflation, the phase \( \theta \) of the complex scalar field acquires minima at the points \( \theta_{\text{min}} = 0, \pm 2\pi, \pm 4\pi \ldots \) and the field acquires mass \( m_0 = 2f/\Lambda^2 \) but also the extended configurations develop kinks as argued next. The field \( \theta \) obeys the standard equation of motion,

\[ \ddot{\theta} - \nabla^2 \theta + 3H\dot{\theta} + \frac{d\delta V}{d\theta} = 0. \tag{2.3} \]

This equation permits two interesting regimes. At the scale of the field theory, i.e., length scales \( \ll H^{-1} \), we can ignore the \( H\dot{\theta} \) term and we get kink-like or DW solutions where the field interpolates between two adjacent vacua. The domain wall solution perpendicular to x-axis between two vacua \( \phi_1 = 0 \) and \( \phi_2 = 2\pi f \) is given by,

\[ \phi_{\text{wall}} = 4f \arctan \left( \exp \left( \frac{x - x_0}{d} \right) \right), \tag{2.4} \]

where \( x_0 \) is the position of the centre of the wall and the width of the wall is set by \( d \approx f/2\Lambda^2 \). The surface energy density of the wall is then taken to be \( \sigma \approx 8f\Lambda^2 \). At early stages while \( H \) dominates we can ignore the axion potential of (2.2). During this time however, the fluctuations of \( \theta \) in regions separated by causal horizon can differ randomly to the extent of

\[ \delta \theta = \frac{H}{2\pi f}. \tag{2.5} \]

This gives rise to two crucial steps on which the present scenario relies:

- Such fluctuations have wavelengths ranging in size from the particle horizon during inflation \( H^{-1} \) to the inflation horizon \( e^{(N_{\text{max}} - N)}H^{-1} \), where \( N \) is the number of e-folds remaining before the end of inflation. The fluctuations continue to evolve during inflation, with the phase \( \theta \) making quantum jumps of \( \delta \theta \) during every e-fold and in every space volume with characteristic size of the order \( H^{-1} \). When the wavelength of the fluctuation becomes larger than the \( H^{-1} \) the average amplitude of this fluctuation freezes out, with the differences (2.5) imprinted on them.
• Subsequent to inflation and reheating, the Universe begins to cool and the local minima of the potential (2.2) begin to have significance. The fluctuations subside and the large scales begin to re-enter the Friedmann-Lemaître-Robertson-Walker (FLRW) horizon. The Hubble scale now drops rapidly as $1/2t$, and $\delta V$ signals the formation of kink-like or DW solutions where the field interpolates between two adjacent vacua. As mentioned earlier, the surface energy density of such walls is $\sigma \simeq 8f\Lambda^2$. If the energy stored $E$ in the closed DWs fall within its gravitational radius $r_g = 2E/M_{Pl}^2$, they could collapse to the black holes due to their surface tension.

We consider time evolution as a count down in terms of $N$ the number of e-folds remaining before the end of inflation, beginning with value $N_{\text{max}}$. For the generic initial value of the classical field $\theta$ whose initial value is $\theta_{N_{\text{max}}}$, one may assume $\theta_{\text{in}} < \pi$. The presence of the de Sitter-like phase gives rise to characteristic quantum fluctuations. The initial domain containing phase $\theta_{N_{\text{max}}}$ increases its volume in $e^3$ times after one e-fold and contains $e^3$ separate causally disconnected domains of size $H^{-1}$. For each domain, after e-folds $N_{\text{max}} - N$,

$$\theta_{N_{\text{max}} - N - 1} = \theta_{N_{\text{max}} - N} \pm \delta \theta.$$  \hspace{1cm} (2.6)

In half of the domains the phase evolve towards $\pi$ and in the other half it moves towards zero. The process repeats during each e-fold. The size distribution of the phase value $\theta$ can be given as a Gaussian,

$$P(\bar{\theta},l) = \frac{1}{\sqrt{2\pi}\sigma_l}\exp\left(-\frac{(\theta_{N_{\text{max}}} - \bar{\theta})^2}{2\sigma_l}\right),$$  \hspace{1cm} (2.7)

where

$$\sigma_l = \frac{H^2}{4\pi^2 f^2}(N_{\text{max}} - N_l).$$  \hspace{1cm} (2.8)

The final stage ends with a collective dynamics of bubble walls as a whole. The preferred situation is the fluctuations commence with $\theta_{N_{\text{max}}} < \pi$ so that the present particle horizon contains the phase $\theta_{\text{min}} = 0$. The islands with $\theta > \pi$ will move to the final state $\theta_{\text{min}} = 2\pi$. When the size of the domain wall becomes causally connected, it will start to self collapse acquiring spherical form due to self collapse. The typical size of a closed DW is the correlation length of the last phase transition $L_c = 0.3M_{Pl}/\sqrt{g^*}\Lambda^2$, where $g^*$ is the effective number of degrees of freedom in the plasma. This relates the characteristic epoch of the Universe with the mass scale of the black holes being formed.

To calculate the mass spectrum of PBHs, one has to calculate the size distribution of domains that contains phase at the range $\theta > \pi$. Suppose $V(\theta, N)$ is the volume that has been filled with average phase $\bar{\theta}$ at $N$ e-folds before the end of inflation, then at the $N - 1$ e-folds before the end of inflation,

$$V(\bar{\theta}, N - 1) = e^3V(\bar{\theta}, N) + (V_U(N) - e^3V(\bar{\theta}, N))P(\bar{\theta}, N - 1)\delta \theta,$$  \hspace{1cm} (2.9)

where $V_U \approx e^{3N}H^{-3}$ is the volume of the Universe at N e-fold. The physical size that leaves the horizon during e-fold number $N (N \leq N_{\text{max}})$ reads,

$$l = H^{-1}e^N.$$  \hspace{1cm} (2.10)

This scale becomes comparable to the FLRW particle horizon at the moment

$$t_h = H^{-1}e^{2N}.$$  \hspace{1cm} (2.11)
We begin with a brief recapitulation of [22] where the possibility of DW formation in a non-supersymmetric SO(10) model was considered in detail. Consider the symmetry breaking pattern of Spin(10),

\[
\text{Spin}(10) \xrightarrow{M_{10}} \text{Spin}(6) \times \text{Spin}(4) \xrightarrow{M_{126}} \text{SU}(3) \times \text{SU}(2) \times \text{U}(1) \xrightarrow{M_{15}} \text{SU}(3) \times \text{U}(1)
\] (3.1)

The scales of symmetry breaking obtained from one-loop renormalization-group analysis are

\[
M_z \simeq 1.9 \times 10^{15} \text{ GeV}, \quad M_R \simeq 4.4 \times 10^{13} \text{ GeV}.
\] (3.2)

The first stage of symmetry breaking is achieved by using the 54-dimensional scalar field acquiring vacuum expectation value (vev) and topologically stable \(Z_2\) strings arise,

\[
\langle \phi_{54} \rangle = \phi_0(2, 2, 2, 2, 2, 2, -3, -3, -3, -3).
\] (3.3)

However, the stability group of this vev contains more elements than that of \(H_0 = \text{SU}(6) \times \text{SU}(4)\) which can be understood in the following way. Consider the generators of Spin(10) as

\[
T_{ij} = \frac{\sigma_{ij}}{2} = \frac{[\Gamma_i, \Gamma_j]}{4i}, i, j = 1, 2 \ldots 10, \text{ where } \Gamma_i's \text{ are generalised Dirac matrices in 10 dimension.}
\]

The generators of the subgroup \(H_0\) of are then \(T_{ab}, 1 \leq a, b \leq 6, \text{ and } T_{a\beta}, 7 \leq a\beta \leq 10.\) Consider one element of Spin (10), \(e^{i\theta T_{07}}\) where \(\theta\) is the angle of rotation in the 6 – 7 plane. Under this transformation, the 6 – 7 submatrix of \(\langle \phi_{54} \rangle\) transform as,

\[
e^{i\theta T_{07}} \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} e^{-i\theta T_{07}} = \begin{pmatrix} \frac{1}{2} + \frac{5}{2} \cos 2\theta & -\frac{5}{2} \sin 2\theta \\ -\frac{5}{2} \sin 2\theta & \frac{1}{2} - \frac{5}{2} \cos 2\theta \end{pmatrix}.
\] (3.4)

The \(\langle \phi_{54} \rangle\) is left invariant for \(\theta = n\pi, n \in \mathbb{Z}\).

In the second stage of symmetry breaking the \(126\) develops vacuum expectation value and thereby breaking \(H\) down to \(\text{SU}(3) \times \text{SU}(2) \times \text{U}(1).\) A \(Z_2\) subgroup of \(H_0 = \text{Spin}(6) \otimes \text{Spin}(4)\) is also left unbroken by \(\langle \phi_{126} \rangle.\) This leads to the formation of \(Z_2\) strings [38]. This transition also breaks the discrete charge-conjugation symmetry. As a result domain walls are formed that separate the vacua related by charge conjugation. The domain walls are topological pseudo-defects [23], and can be bounded \(Z_2\) strings.

Assume the \(\langle \phi_{126} \rangle\) lies along the \((\mathbb{I}, 1, 3)\) direction on the semi-infinite plane \(y = 0^+, x > 0 (\varphi = 0).\) Then,

\[
\langle \phi_{126} \rangle (\varphi) = \exp \left[ \frac{i \varphi}{2} (T_{23} + T_{67}) \right] \langle \phi_{126} \rangle (\varphi = 0), 0 \leq \varphi \leq 2\pi.
\] (3.5)

The above equation means that \(\langle \phi_{126} \rangle (\varphi = 2\pi)\) lies along the \((10, 1, 3)\) direction which is the charge conjugate of the \((\mathbb{I}, 1, 3)\) direction. Thus the expectation values of the \(126\) does not return to its original value after a full rotation around the string. This leads to the existence of a physical domain wall along \(y = 0, x \geq 0\) semi-infinite plane. The wall is bounded by the string along z-axis.

These are important illustrative considerations. However the more realistic theory to consider is a supersymmetric one, due to the stability of the hierarchy between its high scale and the Standard Model scale. This is what we take up in the next subsection.
3.1 The supersymmetric SO(10) GUT

We now review the Minimal Supersymmetric Grand Unified Theory model within which the scenario of section 2 can be realised. One of the major problems of GUTs and specifically SUSY SO(10) is the compatibility of small neutrino masses and mixings with the high scale of unification [39–44]. However, it is shown in [45–48] that this is achievable within reasonably economical models, including accommodating inflation in [33, 36], still retaining proton decay suppressed.

An interesting issue not noted by the proposers of the model is that due to the presence of $D$-parity within the theory, it has degenerate vacua, one of them possessing the Standard Model (SM), SU(2)$_L$ as subgroup of its stability group, and the other having instead SU(2)$_R$ as a subgroup. This would lead to the formation of DW, analysed in [23] and shown to be topological pseudo-defects. They arise due to the topology of the vacuum manifold. However they are not stable due to the simply connected nature of the original gauge group Spin(10). Preskill and Vilenkin [24] have analysed metastable topological defects in generic and more general cases. These occur in theories with more than one stage of symmetry breaking. The pseudo-defects, we have proposed occur after a single stage of breaking, but due to the Kibble mechanism of finiteness of the causal horizon. The mechanism for their destabilisation could be one of those discussed by [24], specifically the formation of holes formed by thermal and quantum tunnelling. Another possibility for their destabilisation through soft breaking of Spin(10) symmetry was considered in [26].

For the duration that such DW exist, they give rise to an effective potential of the type eq. (2.2), though not a periodic one. As we shall see, the axion of the toy model of section 2 is now a collective degree of freedom connecting symmetry related distinct vacua related by the $D$-parity. While we study a specific model [39, 50], the $D$-parity considerations presented here easily generalise to the case of other supersymmetric SO(10) models with $D$-parity, [40–44], generically arising from the fermions symmetrically placed in the $16$.

The heavy Higgs part of the renormalisable superpotential of SO(10) MSGUT is as follows [49, 50]:

$$W = \frac{m}{4!} \Phi_{ijkl} \Phi_{ijkl} + \frac{\lambda}{4!} \Phi_{ijkl} \Phi_{klmn} \Phi_{mnij} + \frac{M}{5!} \Sigma_{ijklm} \Sigma_{ijklm} + \frac{\eta}{4!} \Phi_{ijkl} \Sigma_{ijmno} \Sigma_{ijmno}, \tag{3.6}$$

where $\Phi$ is the 4-index anti-symmetric representation $210$, $\Sigma$ is the 5-index self-dual anti-symmetric representation $126$, and $\bar{\Sigma}$ is the 5-index anti-self-dual anti-symmetric representation $1\bar{26}$ of SO(10). In the above expression, all indices range independently from 1 to 10. The $\Sigma$ is needed to give the large mass to the $\nu^c$ state, and $\bar{\Sigma}$ to preserve SUSY while breaking the gauge symmetry. An additional 10-plet is desirable for light fermion masses but its required vev is not large and can be ignored for the present purpose. Below, for brevity of notation, we shall use 0 to denote the index 10 in our expressions. Of these, the $\Phi$ multiplet is needed for desirable gauge symmetry breaking,

$$\text{Spin}(10) \xrightarrow{M_R} G, \tag{3.7}$$

where $G$ is generically MSSM for judicious choice of the 210 vacuum expectation value (vev), and $M_R$, the traditional way to denote the scale of right handed symmetry breaking will be identified with the scale $f$ here. However, special values of parameters exist for which $G$ can be one of the intermediate symmetry groups such as SU(5) or Pati-Salam, or the left-right
The symmetric group with $U(1)_{B-L}$. We summarise here the results of \cite{49, 50}, also recapitulated in \cite{23, 26}.

There are 5 MSSM singlet vev components from the 3 fields of eq. (3.6). For our purpose the vev at the highest scale that emerges towards the end of inflation matters, and we shall study the case that results in the SM gauge group. To recognize the SM singlets, the decomposition of Higgs supermultiplets required for SO(10) symmetry breaking to MSSM in terms of Pati-Salam gauge group $(SU(4)_C \times SU(2)_L \times SU(2)_R)$ is given as \cite{50},

$$
210 = (15, 1, 1) + (1, 1, 1) + (15, 1, 3) + (15, 3, 1) + (6, 2, 2) + (10, 2, 2) + (10, 2, 2),
$$
$$
126 = (10, 1, 3) + (10, 3, 1) + (6, 1, 1) + (15, 2, 2),
$$
$$
\underline{126} = (10, 3, 1) + (10, 1, 3) + (6, 1, 1) + (15, 2, 2).
$$

(3.8)

There are three independent components from 210 which preserve the SM, and are designated as $p$ from $P(1, 1, 1)$, $a$ from $A$ (irreducible singlet of $(15, 1, 1)$) and $w$ from $\Omega_R^{0}$ ($(113^{-})$ of $(15, 1, 3)$). Similarly we designate $\sigma$ as vev of $\Sigma_R^{+}$ ($(\bar{1}13^{-})$ of $(10, 1, 3)$) from 126 and $\tilde{\sigma}$ as the vev of $\Sigma_R^{+}$ ($(\bar{1}13^{+})$ of $(10, 1, 3)$) from $\underline{126}$. The details of how these fields are defined in terms of components having SO(10) indices breaking them in SO(6) $\otimes$ SO(4) indices is elaborated in the appendix of \cite{23}.

The superpotential in terms of these vevs is given by,

$$
W = m(p^2 + a^2 + 6w^2) + 2\lambda(a^3 + 3pw^2 + 6aw^2) + M\sigma\tilde{\sigma} + \eta\sigma\tilde{\sigma}(p + 3a - 6w).
$$

(3.9)

The $F$ flatness and $D$ flatness conditions impose five conditions on these, which are at most quadratic in any one vev and trilinear in them at the highest order. The solution values can be parameterised in terms of one parameter $x$, satisfying a cubic equation, whose roots can be expressed in terms of the parameters $m$, $\lambda$, $M$ and $\eta$ of the superpotential. The parameters of the superpotential and the possible vev’s are then related as \cite{49, 50},

$$
a = \frac{m x^2 + 2x - 1}{\lambda (1 - x)}; \quad p = \frac{m x(5x^2 - 1)}{\lambda (1 - x)^2},
$$
$$
\sigma\tilde{\sigma} = \frac{2m^2 x(1 - 3x)(1 + x^2)}{\eta \lambda}; \quad w = -\frac{m}{\lambda} x,
$$

(3.10)

where $x$ is the solution of following cubic equation,

$$
8x^3 - 15x^2 + 14x - 3 = -\frac{\lambda M}{\eta m} (1 - x)^2.
$$

(3.11)

As $x$ is varied, it corresponds to different values of these basic parameters, and their choices signal different patterns of symmetry breaking. Thus in addition to the generic case of MSSM, one obtains a list of possible larger symmetry groups depending on the value of $x$ as enumerated below.

1. For $x = 1/2$ and if $\lambda M/\eta m = -5$, it gives SU(5) minimum.

2. For $x = 0$ and if $\lambda M/\eta m = 3$, this results in $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ minimum.

3. For $x = \pm i$ and if $\lambda M/\eta m = -3 (1 \pm 2i)$, it gives $SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ symmetry.
Figure 1. The barrier along the trajectory $\mathcal{P}(\alpha)$ in MSGUT, with parameters appearing in eq. (3.6) are $\eta = 0.1, \lambda = 0.01$ blue curve, left panel and $\eta = 0.1, \lambda = 0.001$, red curve, right panel. These provide an upper bound on actual barrier height, for a generic set of parameters when MSSM can result, with $x = 1/4$ in eq. (3.11).

4. For $x = 1/3$ and if $\lambda M/\eta m = -2/3$, it results in the flipped SU(5) $\times$ U(1) minimum.

5. For $x = 1/4$ and if $\lambda M/\eta m = 5/9$, it results in MSSM minimum.

However the theory contains the well known operator, the $D$-parity, flipping these vev’s to other degenerate vev’s which flip SU(2)$_L \leftrightarrow$ SU(2)$_R$, a variant of eq. (3.4),
\[
D = \exp(-i\pi J_{23}) \exp(i\pi J_{67}).
\] (3.12)

Under the action of D-parity the components of the representations identified above transform as
\[
P \to -P; \quad A \to A; \quad \Omega^0_R \to \Omega^0_L, \quad \Sigma^-_R \to -\Sigma^+_L; \quad \Sigma^+_R \to -\Sigma^-_L,
\] (3.13)
in an intuitively obvious notation, whose details are given in the appendix of [23]. This model was studied in [23, 26] where it was shown that exactly the same values of $a$, $\omega$, $p$, $\sigma$ and $\tilde{\sigma}$ ensure F-flatness at the original vevs as well as at the $D$-flipped vevs. While topologically the same, the two possible subgroups get distinguished in Physics due to chirality of fermion content. We next need to identify the barrier that will mimic the potential of eq. (2.2) and determine the scale corresponding to $\Lambda$. Due to the large parameter space, and the large number of field components, it is difficult to visualise paths that lead from one such vacuum to another vacuum. However the simplest path is the one that joins the two vacua directly, passing through the symmetric point where all the vevs are zero. Thus the vector basis from which the tensor representations are constructed is multiplied by,
\[
\mathcal{P}(\alpha) = \cos\{\alpha(J_{23} - J_{67})\},
\] (3.14)
with $\alpha \in [0, \pi]$ and the corresponding $\alpha$ dependent vevs are calculated. We have checked that this choice gives a better upper bound on the height of the barrier than the circuitous path considered in [23]. In figure 1 we show two representative barriers recalculated along the lines of [23], separating an MSSM vacuum and its $D$-parity flipped analogue. The plots are for the case $x = 1/4$ of the MSGUT SO(10) model [49, 50] so that $m/M = 5/9$. Thus the scale $\Lambda$ which is fourth-root of the barrier height can easily lie in the range $\sim f/100$ for $\lambda < 10^{-3}$. 

Figure 2. The PBH mass spectrum for different values of the initial phase \( \theta_{\text{in}} \). Here we have used \( N_{\text{max}} = 60, H = 10^{13}\text{GeV}, f = 10 \times H \) and \( \Lambda = f/100 \).

This barrier is only an upper bound and the actual barrier can be smaller, justifying the assumption that the field remains free to be redistributed over the barrier due to fluctuations eq. (2.5).

3.2 PBH creation and comparison with constraints

We assume that inflation takes place due to presence of a flat direction in the scalar field potential of SO(10), or due to an independent inflaton field. The parameter \( \alpha \) of \( P(\alpha) \) of section 3.1 is now identified with the axionic degree of freedom \( \theta \) of section 2 and is assumed to follow the equation of motion given in eq. (2.3). During each e-folding the value of \( \theta \) changes as given in eqs. (2.5) and (2.6). Here the initial value of \( \theta, \theta_{N_{\text{max}}} \equiv \theta_{\text{in}} \) can commence with any value which can not be fixed by theory. We take three different values \( \theta_{\text{in}} = 0.5, 0.6 \) and 0.7 to illustrate our results and compare with observational constraints. Eventually as the length scale re-enter the horizon after inflation and the fluctuations encoding kink solutions freeze in and DW form. The surface energy density contributing to the PBH mass is calculated as (see for instance [51])

\[
\sigma = 8f\Lambda^2.
\]  

The one undetermined value \( \theta_{\text{in}} = \theta_{N_{\text{max}}} \) is a free parameter and the results are indeed sensitive to the choice of this value. We considered three representative values for it which give interesting results. The resulting spectrum of black holes is as shown in figure 2.

These results must then be transferred to the present epoch and compared with observational constraints. In appendix A we provide a summary of the constraints on PBH with which we compare our results. The constraints discussed there are taken from [52, 53] and more detailed references therein. In figure 3 we present the comparison of our results with the constraints. For \( \theta_{\text{in}} = 0.5 \), both evaporation and non-evaporation constraints are satisfied,
Figure 3. Evaporation and non-evaporation constraints: in the left panel the shaded area is a rough exclusion region sketched corresponding to the figures 4, 5 and 7 of reference [53] and corresponds to evaporation constraints. In the right panel the exclusion region corresponds to the figure 10 of reference [53] and corresponds to non-evaporation constraints. The different colours correspond to different initial values of the phase, $\theta_{\text{in}}$. In the vertical axes the quantities $\beta'$ and $f(M_{\text{PBH}})$ are calculated using equations (A.3) and (A.9) respectively.

while for $\theta_{\text{in}} = 0.6$ and 0.7, the evaporation constraint is satisfied but for the non-evaporation constraint, the larger mass end of the spectrum can be seen to be already in conflict.

Realistically, the parameter $\theta_{\text{in}}$ probably has a randomised distribution and all the values sufficiently close to the barrier should be implemented to obtain a comprehensive spectrum of resulting PBH. We expect that such a more detailed study could constrain the scales $f, H$ and $\Lambda$ as well as demand a better theory for $\theta_{\text{in}}$ to confirm avoidance of late Universe constraints.

4 Conclusion

As of now we have broad proposals of how early universe fluctuations could have given rise to pre-galactic black holes as discussed for instance in [9]. A suggestive model has been detailed in [27–29]. In this version, black holes are created from the collapse of closed domain walls formed during a second order phase transition which creates topologically non-trivial profiles for the fluctuations due to strongly non-equilibrium initial conditions. The non-equilibrium initial conditions are generated and also preserved till late epoch by the inflationary dynamics. The spontaneous symmetry breaking freezes in during late stages of inflation. Thus the walls which emerge as definite semi-classical objects only at a late epoch are preserved from destruction by wall collisions and formation of holes bounded by strings in them. Such walls eventually become causally connected and begin to contract due to their surface tension. This ends with the formation of black holes with a mass set by the mass scale of the wall when it enters the horizon. Thus the size distribution of closed walls gets converted into the mass spectrum of resulting primordial black holes.

Among GUTs that can provide a detailed model for such a process, a supersymmetric GUT satisfying proton decay constraints and stable against its hierarchy with the electroweak scale is an appealing one to consider. In particular, the SO(10) model accommodates right chiral neutrino states making neutrino masses natural, while also encompassing an elegant accidental symmetry $D$-parity, that can exchange left and right chiralities. This discrete
symmetry when embedded in a continuous one-parameter group \( U(1)_D \) provides the conditions of the scenario envisaged in [27, 28]. Here we have worked out the implications to PBH abundance from the mass scales typical of this MSGUT \( SO(10) \) based on an earlier study of the topological pseudo-defects in this theory [23]. A minimal non-renormalisable extension of the model also ensures eventual removal of the DW as shown in [26]. Interestingly, the DW which are usually considered a nuisance to cosmology are now seen to leave behind relics with macroscopic, stellar scale masses, as a signature of the GUT era. Thus the study suggests a variety of conclusions, viz., for the PBH forming scenario, for the GUT and for the statistical distribution of initial fluctuations. As the knowledge of black hole spectrum improves, one can either constrain the parameters of such GUTs or even rule them out, or seek refinement of the mechanisms for such PBH creation.

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A Summary of constraints on PBH

The PBHs smaller than about \( 10^{15} g \) would have evaporated by now and could have many cosmological consequences. PBHs larger than \( 10^{15} g \) are unaffected by Hawking radiation and are also attractive because of the possibility that they provide the dark matter content of the Universe. The critical mass for which \( \tau \) equals the age of the Universe,

\[
M_* \approx 5 \times 10^{14} \text{ g.} \tag{A.1}
\]

For evaporating PBHs the value of the fraction of the Universe in PBHs at the formation time, \( t_i \) is related to their number density at \( t_i \) and \( t_0 \) is given by,

\[
\beta(M) = \frac{M n_{\text{PBH}}}{\rho(t_i)} \approx 7.98 \times 10^{-29} \gamma^{-1/2} \left( \frac{g_{*i}}{106.5} \right)^{1/4} \left( \frac{M}{M_\odot} \right)^{3/2} \left( \frac{n_{\text{PBH}}(t_0)}{1 \text{ Gpc}^{-3}} \right). \tag{A.2}
\]

Since \( \beta \) always appears in combination with \( \gamma \) \( g_{*i}^{-1/4} h^{-2} \) a new parameter can be defined as

\[
\beta'(M) \equiv \gamma^{1/2} \left( \frac{g_{*i}}{106.75} \right)^{-1/4} \left( \frac{h}{0.67} \right)^{-2} \beta(M) \tag{A.3}
\]

where \( \gamma \sim 0.2 \) depends on the details of gravitational collapse. The strongest limit comes from \( \gamma \)-ray, \( \beta(10^{15} g) \leq 10^{-28} \) over all mass ranges. Some of the constraints coming from possible evaporation of lighter black holes (in the mass range \( 10^9 \text{ g} - 10^{14} \text{ g} \)) are listed below.

- The effects of PBH evaporation in CMB for masses smaller than \( 10^9 \text{ g} \)

\[
\beta' \approx 10^{-5} \left( \frac{M}{10^9 \text{ g}} \right)^{-1}, \quad (M < 10^9 \text{ g}). \tag{A.4}
\]
• The PBHs in the mass range $10^{11} g < M < 10^{13} g$ produce distortion in the CMB spectrum,
\[ \beta' \approx 10^{-16} \left( \frac{M}{10^{11} g} \right)^{-1} (10^{11} g < M < 10^{13} g). \]  
(A.5)

• The PBHs evaporating after the time of recombination results in small scale anisotropies,
\[ \beta'(M) \approx 3 \times 10^{-30} \left( \frac{f H_0}{0.1} \right)^{-1} \left( \frac{M}{10^{13} g} \right)^{3.1} (2.5 \times 10^{13} g \leq M \leq 2.4 \times 10^{14} g). \]  
(A.6)

• From reionisation of 21 cm signature
\[ \beta'(M) \leq 3 \times 10^{-29} \left( \frac{M}{10^{14} g} \right)^{7/2} M > 10^{14} g. \]  
(A.7)

Next we list the constraints associated with PBHs which are too large to have evaporated by the present epoch. The current density parameter $\Omega_{\text{PBH}}$ associated with unevaporated PBHs that form at a redshift $z$ is roughly related to $\beta$ by,
\[ \Omega_{\text{PBH}} \simeq \beta \Omega_r (1 + z) \beta \left( \frac{t}{1 \text{ s}} \right)^{-1/2} \sim 10^{18} \beta \left( \frac{M}{10^{5} g} \right)^{-1/2}, \]  
(A.8)

where $\Omega_r \sim 10^{-4}$ is the density parameter of the cosmic microwave background. The factor $(1 + z)$ arises because the radiation density scales as $(1 + z)^4$ and the PBH density scales as $(1 + z)^3$. For non-evaporating PBHs the constraints are expressed as constraints on the fraction of dark matter in the PBHs denoted as $f(M)$,
\[ f(M) = \frac{\Omega_{\text{PBH}(M)}}{\Omega_{\text{CDM}}} \approx 3.79 \Omega_{\text{PBH}} = 3.81 \times 10^8 \beta' \left( \frac{M}{M_\odot} \right)^{-1/2}. \]  
(A.9)

The constraints are summarised here.

• The constraints on PBHs with very low mass come from the femtolensing of $\gamma$-ray bursts. Assuming the bursts are at a redshift $z \sim 1$, the limit is given by,
\[ f(M) < 0.1 \left( 5 \times 10^{16} g < M < 10^{19} g \right). \]  
(A.10)

• The microlensing observations of stars in Magellanic clouds combining MACHO and EROS results can be given as,
\[ f(M) < \begin{cases} 
1 & 6 \times 10^{25} g < M_{\text{PBH}} < 3 \times 10^{34} g, \\
0.1 & 10^{27} g < M_{\text{PBH}} < 3 \times 10^{33} g, \\
0.04 & 10^{30} g < M_{\text{PBH}} < 3 \times 10^{32} g.
\end{cases} \]  
(A.11)

The OGLE-IV provided stronger limits in the high mass range
\[ f(M) < \begin{cases} 
0.2 & 4 \times 10^{32} g < M_{\text{PBH}} < 2 \times 10^{34} g, \\
0.09 & 4 \times 10^{32} g < M_{\text{PBH}} < 1 \times 10^{33} g, \\
0.06 & 10^{32} g < M_{\text{PBH}} < 4 \times 10^{32} g.
\end{cases} \]  
(A.12)
• The studies of the microlensing of quasars were found to exclude all the dark matter being in objects with limit $10^{30} g < M_{\text{PBH}} < 6 \times 10^{34} g$ and $f(M_{\text{PBH}}) < 1$.

• The vulnerability of disruption of binary star systems with wide separation with PBHs constrains the abundance of halo PBHs. The resulting constraints are,

$$f(M) < \begin{cases} \frac{M_{\text{PBH}}}{5 \times 10^{33} g} \left(\frac{1}{10^{35} g} \right) & 5 \times 10^{35} g < M_{\text{PBH}} < 1 \times 10^{36} g, \\ 0.4 & 1 \times 10^{36} g < M_{\text{PBH}} < 1 \times 10^{41} g. \end{cases}$$

(A.13)

• The survival of globular clusters against tidal disruption by passing PBHs gives the limit,

$$f(M) < \begin{cases} \frac{M_{\text{PBH}}}{3 \times 10^{37} g} \left(\frac{1}{10^{39} g} \right) & 3 \times 10^{37} g < M_{\text{PBH}} < 1 \times 10^{39} g, \\ 0.03 & 1 \times 10^{39} g < M_{\text{PBH}} < 1 \times 10^{44} g. \end{cases}$$

(A.14)

• Halo objects can overheat the stars in the Galactic disc and provides the limits,

$$f(M) < \begin{cases} \frac{M_{\text{PBH}}}{3 \times 10^{39} g} \left(\frac{1}{10^{42} g} \right) & M_{\text{PBH}} < 3 \times 10^{42} g, \\ 0.03 & M_{\text{PBH}} > 3 \times 10^{42} g. \end{cases}$$

(A.15)

• The survival of galaxies in clusters against tidal disruption by giant clusters PBHs gives the following limit,

$$f(M) < \begin{cases} \frac{M_{\text{PBH}}}{7 \times 10^{42} g} \left(\frac{1}{10^{44} g} \right) & 7 \times 10^{42} g < M_{\text{PBH}} < 1 \times 10^{44} g, \\ 0.4 & 1 \times 10^{44} g < M_{\text{PBH}} < 1 \times 10^{47} g. \end{cases}$$

(A.16)

• The PBHs can not accrete appreciably in the radiation dominated era. They might do so in the period after decoupling that can be analysed through Bondi-type analysis. The simultaneous accretion and emission of radiation can have significant effect on the thermal history of the Universe. The emission of X-rays from accreting PBHs would produce measurable anisotropies and spectral distortions on the CMB. The constraints that are be obtained from WMAP data are given by,

$$f(M) < \begin{cases} \frac{M_{\text{PBH}}}{30 \times 10^{43} g} \left(\frac{1}{10^{37} g} \right) & 30 \times 10^{43} g < M_{\text{PBH}} < 10^{37} g, \\ 10^{-5} & 10^{37} g \leq M_{\text{PBH}} < 10^{44} g, \\ M_{\text{PBH}}/M_{l=100} & M_{\text{PBH}} > 10^{44} g. \end{cases}$$

(A.17)

where the last expression corresponds to having one PBH on the scale associated with the CMB anisotropies for $l = 100$ modes; $M_{l=100} \approx 10^{49} g$. However the limits associated with accretion of PBHs depend on large number of astrophysical parameters and qualitative features such as disc or spherical accretion. So the limits may be less secure than the previous ones.

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