COSMOLOGICAL ORIGIN OF THE STELLAR VELOCITY DISPERSIONS IN MASSIVE EARLY-TYPE GALAXIES

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1. INTRODUCTION

The line-of-sight velocity dispersion of the stars in an elliptical galaxy with luminosity \( L > L_a \) is typically \( \sigma_r \sim 200 \text{ km s}^{-1} \), while elliptical or cD galaxies with twice this velocity dispersion are exceedingly rare. This is equivalent to a fairly sharp bound on the mass that is gathered within the luminous parts of the largest galaxies. One might expect this striking effect to have a simple explanation. Our proposal follows a simple path through the LCDM model for cosmology and structure formation.

We begin in the next section by considering the simple case where dissipative processes in the baryons are ignored: all matter is treated as collisionless and initially cold. As discussed in §2.1, the standard picture for mass clustering in the LCDM cosmology predicts that in this case the extreme mass concentrations characteristic of the luminous parts of giant elliptical galaxies have comoving number density as a function of velocity dispersion that is strikingly similar to what is observed for these galaxies today.

Reality has to be more complicated than this, because baryons must dissipatively settle to form stars that make appreciable contributions to the mass within the effective radii of ellipticals, \( R_e \). As we will discuss, if stellar mass were simply added to the cold dark matter (CDM) present in these cores the velocity dispersion within the characteristic effective radius \( R_e \sim 10 \text{ kpc} \) would be unacceptably large. Our proposed remedy involves two postulates. The first is that the density profile in a dark matter halo acts as an attractor or fixed point in the sense of nonlinear dynamics (Syer & White 1998): the formation of a new halo tends to erase memory of the conditions in previous generations of halos, including the distortion caused by the addition of stellar “collisionless matter” to the regions. This requires our second postulate, that the bulk of the stars formed when the mass concentrations characteristic of the luminous parts of the giant elliptical galaxies were still being assembled.

We have a measure of when assembly on the scale of the optical parts of the largest galaxies was close to complete, from the number density of mass peaks with mass greater than \( M_e \) inside a centered sphere with physical radius \( R_e \sim 10 \text{ kpc} \). At fixed comoving number density \( n(M_e, t) \), the mass \( M_e \) increases with increasing time at a redshift \( z \sim 10 \), because the dense regions of the halos are still being assembled then, while near the present epoch \( M_e \) is close to constant, because the dense central regions of normal galaxies are not much affected by the ongoing growth of the halo through the addition of matter at much larger radii. We calculate that the transition is about at redshift \( z_f \sim 6 \). Thus, within this model we must postulate that the bulk of the stars in a giant elliptical formed and were assembled into a first approximation to the present-day galaxy at \( z_f \sim 6 \).

The situation is still more complicated by the evidence that the density profile within the effective radius of a present-day giant elliptical differs from the standard estimates of the inner density profiles of pure cold dark matter halos. This requires yet another hypothesis, that star formation at low redshift has rearranged the stellar mass distribution. As we discuss in §3 there is evidence for modest recent star formation in the central parts of giant ellipticals, perhaps in part due to the recycling of mass shed by evolving stars.

Three points may be of particular interest. First, stars that form at high redshift behave thereafter dynamically as dark matter particles. The numerical experiments reviewed in §2.3 suggest the density profiles in the subsequent generations of halos are not much affected by the special initial conditions of this new collisionless matter. The resulting displacement of dark matter by stars could help resolve observational challenges to the predicted central mass distributions in large galaxies. And an attractive byproduct is that the early formation of giant early-type galaxies fits a considerable variety of observations (as reviewed in Peebles 2002).

Second, the central parts of the most massive halos might be expected to stop evolving as they become very much denser than the mean density of newly collapsing halos. The point is
widely discussed, as by Navarro, Frenk & White (1997; hereafter NFW) and in more detail by Wechsler et al. (2002). Indeed, this stable core concept was the basis for the estimates of the redshift of galaxy formation in Partridge & Peebles (1967). Within our schematic model for halo formation this concept leads us to prefer a form for the characteristic inner halo mass density profile in the ΛCDM cosmology that is intermediate between

$$\rho(r) = \frac{\rho_0}{(r/r_0)^{3/2}[1+(r/r_0)^3/2]}.$$  \hspace{1cm} (1)

(Ghigna et al. 2000; see also Moore et al. 1999; hereafter called the Moore form) and the NFW form

$$\rho(r) = \frac{\rho_0}{(r/r_0)(1+r/r_0)^2}.$$  \hspace{1cm} (2)

Third, the cutoff in $\sigma_e$ or $M_e$, respectively. The primeval mass density fluctuation spectrum is taken to be scale-invariant (that acts like $\delta_m(r)$). Klypin et al. (2001) find that this is the typical ratio of concentration parameters when the virial radius and the radius at maximum circular velocity are constrained (Bryan & Norman 1998; Barkana & Loeb 2001) to be of the same order of magnitude between the Moore form and the NFW form

$$\rho(r) = \frac{\rho_0}{(r/r_0)^{3/2}[1+(r/r_0)^3/2]}.$$  \hspace{1cm} (1)

We use the Moore and NFW density profiles in equations (1) and (2) to extrapolate from $V_c$ at the virial radius to the circular velocity at an effective radius $R_e$, for giant ellipticals, and we divide the circular velocity by the factor $2^{1/2}$ to estimate the line-of-sight velocity dispersion. This is a crude approximation: our halos are not isothermal and the stellar velocities need not be isotropic. However, since redistribution of the stellar mass is likely to occur (see §3), and some ellipticals are known to have anisotropic velocity distributions, the sake of simplicity we prefer not to refine the calculation. In our approximations the stellar velocity dispersion at radius $R_e$ is

$$\sigma_e = V_c \left\{ \frac{\ln [(1 + (c x)^2)]}{2 \ln [1 + (c^2)]} \right\}^{1/2}, \quad x = r/R_v,$$  \hspace{1cm} (5)

for the Moore profile, and

$$\sigma_e = V_c \left\{ \frac{\ln [(1 + (c x) - c (1 + c)]}{2 \ln [c (1 + c) - (1 + c)]} \right\}^{1/2},$$  \hspace{1cm} (6)

for the NFW profile. In both models we choose a single value for the concentration parameter: for NFW, $c = c_{\text{NFW}} \equiv (R_v/R_e)$ = 4, which is typical of the results from fits of the NFW profile to numerical simulations of the more massive newly collapsed ΛCDM halos (NFW; Wechsler et al. 2002), and for the corresponding Moore profile $c = c_{\text{NFW}}/1.72$, which is the adjustment recommended by Klypin et al. (2001). To avoid confusion we remind the reader that we are considering the rarest most massive halos that tend to have collapsed close to the redshift

$\rho_0$ Klypin et al. (2001) find that this is the typical ratio of concentration parameters when the virial radius and the radius at maximum circular velocity are constrained to be the same in the two functional forms.
at which they are identified. We suggest such halos tend to grow by the addition of matter to the outer envelope, causing the break radius $r_e$ to increase in rough proportion to the virial radius, $R_v \sim c r_e$ with $c$ constant. 

Figure 1 shows the evolution of the mass $M_e(t)$ within a central physical radius $R_e = 10$ kpc of halos defined by a fixed value of the comoving number density $n(>M)$. Here and throughout the paper, number densities are comoving and normalized to the present epoch. The number densities belonging to the curves in the figure are in the range of the Sloan Digital Sky Survey (SDSS) measurements for giant early-type galaxies.

Both models for the halo density profile, NFW in the upper panel and Moore in the lower panel, predict similar behavior at $z \gtrsim 6$ because the mass distributions at radii larger than the maximum circular velocity radius are quite similar. Both models indicate that the mass $M_e(< 10$ kpc) at fixed comoving number density does not evolve much at $z < 4$. This is in line with the idea noted above that the central core of a very massive halo stabilizes dynamically at late times, when the core is much denser than newly collapsing halos. New mergers tend to add mass to the outer halo envelope at impact parameters $\gg 10$ kpc (due to the expansion of the universe), mostly in much smaller halos. In the NFW form the mild decline of $M_e(< 10$ kpc) at low redshifts is a result of the smaller power law index at $x \ll 1$. This requires either that late mergers, at virial radii of a few hundred kpc, tend to lower the densities in the inner 10 kpc of the most massive galaxies, or that late assembly happens to produce new galaxies with $\sigma \gtrsim 200$ km s$^{-1}$ at $R_e \sim 10$ kpc. Since both options seem unlikely to us we conclude that a form closer to Moore is more useful within our approximations. We emphasize that we cannot judge which halo model would be more useful under a better approximation to how halos form.

We conclude from Figure 1 that the approximate redshift at which the assembly of the mass concentrations characteristic of giant elliptical galaxies nears completion is

$$z_f \sim 6.$$ (7)

This is close to the half-mass point in the Moore case, and just before the peak in the NFW case.

Figure 2 shows the comoving number density of dark matter halos, $n(>\sigma_e)$, at redshift $z = 4$, as a function of the one-dimensional velocity dispersion $\sigma_e$ computed between two bracketing radii, $R_e = 6$ kpc and $R_e = 16$ kpc. These are one standard deviation above and below the mean effective (half-light) radius $R_e \sim 10$ kpc of giant ellipticals at $\sigma > 300$ km s$^{-1}$ in the SDSS sample (Bernardi et al. 2001). There is a smaller spread of values of $n(>\sigma_e)$ between the two radii in the Moore form, because the density profile is closer to the limiting isothermal case. A more complete computation would convolve the probability distribution $dn/\sigma_e$ at fixed $R_e$ with a model for the distribution of $R_e$, and would take account of the distribution in values of the concentration index $c$, but we leave that for future work.

The choice of redshift in Figure 2, $z = 4$, is slightly past the characteristic epoch $z_f$ at which structure formation nears completion in the rare massive objects we are considering, and it is close to the redshifts reached in deep rest-frame optical galaxy surveys (Rudnick, Rix, & Franx 2001; Cimatti et al. 2002). In the Moore model the distribution $n(>\sigma_e)$ is not very sensitive to time at $z < 4$. The NFW model predicts a slight decrease with increasing time, which we are suggesting is an artifact of the slightly too shallow inner power-law slope (within our approximations).

The solid line in Figure 2 shows the fitting formula for the measured abundance of early-type galaxies as a function of velocity dispersion in the SDSS sample (Sheth et al. 2002). The standard deviation of the measurement error, $\delta n_e \sim 25$ km s$^{-1}$, is small enough not to appreciably broaden the steep observed drop of the distribution function at $n(>\sigma_e) \sim 10^{-7}$ Mpc$^{-3}$.

The comparison of the SDSS data to our model depends on the inner power-law index $\alpha \equiv -(d\log n/d\log \sigma)$ (Syer & White 1998) present an elegant argument for the value of $\alpha$: if the primeval power spectrum varies as $P(k) \propto a^2k^\alpha$, where $a(t)$ is the cosmological expansion factor and $k$ is the comoving wavenumber, then stable clustering indicates $\alpha = (9 + 3n)/(5 + n)$. Subramanian, Cen, & Ostriker (2000) and Ricotti (2002) have checked this relation against numerical simulations. In the $\Lambda$CDM model, the value of the effective index $n \equiv d\log P(k)/d\log k$ increases with mass scale. At the wavenumbers characteristic of the halo masses of interest, from $10^{15} M_\odot$ to $10^{13} M_\odot$, the primeval power spectrum yields values of this index between $n \sim -2.1$ and $-1.4$, implying $\alpha$ between 0.93 and 1.33. Because this range is intermediate between the NFW and Moore inner slopes, our model should best be considered as intermediate between these two cases in Figures 1 and 2. It is encouraging that the data support this intermediate regime across several orders of magnitude in galaxy number density. Before considering the possible significance of this result we must deal with the loading of the dark halos by the settling of baryons.

2.2. Halo Loading by Star Formation

In the SDSS sample, the ellipticals with $\sigma_e \gtrsim 300$ km s$^{-1}$ have mass-to-light ratio $M/L_\star \sim 6$ solar units within $R_e$ (Bernardi et
and scaled radii and velocities. The velocity dispersion \( \sigma_e \) before compression is larger than in Figure 2 because it is computed at a larger radius, \( R_e = (1 + \eta) R_e \). And the observed velocity dispersion is larger than \( \sigma_e \) by another factor \( 1 + \eta \). This results in quite unacceptable velocity dispersions unless \( \eta \) is much less than unity, which does not seem likely.

We are not able to judge whether a more violent addition of stellar mass could have a less severe effect on \( \sigma_e \), but the indication from equations (8) and (9) is that the loading of the dark halos by baryon settling could be a serious problem for the \( \Lambda \)CDM model. We turn now to a possible remedy.

2.3. The Attractor Hypothesis

Numerical simulations of the growth of halos out of pure dark matter suggest that the strongly nonlinear part of the density profile is not very sensitive to initial conditions. A dramatic example in Navarro, Frenk & White (1996) shows a numerical simulation that evolves through an expansion factor of just \( 1 + z_f = 5.5 \). Because the initial density fluctuations are not large this in effect significantly truncates the small-scale initial power spectrum, yet it produces close to standard halo density profiles. This is demonstrated in more detail, along with the effects of other modifications of the shape of the primeval power spectrum, by NFW and Eke et al. (2001).

We apply this indication of a dynamical attractor effect (Syer & White 1998) to the case where the small-scale mass clustering has been increased by dissipative settling of the baryons, rather than truncated. Our working assumption is that stars that form prior to the assembly of the core simply replace the dark matter that was supposed to be there at late times. This is a conjecture: we are not aware of any numerical checks of this case. We note that this conjecture could in principle explain the observed absence of a cusp in the central dark matter distribution of nearby galaxies and galaxy clusters (the so-called ‘central cusp problem’).

3. Discussion

Our analysis does not do justice to the precise SDSS measurements of the abundance of early-type galaxies as a function of the stellar velocity dispersion: within our approximations that would require consideration of the sensitivity of the computed \( n(> \sigma_e) \) to the slope and normalization (\( \sigma_8 \)) of the primeval power spectrum; the distribution of values of the concentration parameter \( c \); functional forms intermediate between Moore and NFW (e.g., Power et al. 2002); the distribution of values of galaxy effective radii \( R_e \); and the conversion from the circular velocity at \( R_e \) to the stellar velocity dispersion \( \sigma_e \), which depends on a model for how the stars populate the halo.

Within the spread of possibilities offered by all these parameters, we can only conclude from the exploratory analysis presented here that our model seems to be capable of accounting for the observed upper bound on the mass concentrated in the largest galaxies.

Our model depends on the hypothesis that halo formation can erase the effect of dissipative settling of the baryons. We are not aware of a direct test by numerical simulations; a check would be feasible and useful. Also open for discussion, and much more difficult to test, is our assumption that star formation in the neighborhood of a giant elliptical is concentrated in the dense regions that end up in or near \( R_e \sim 10 \) kpc. Even if star formation were confined to dense regions, mergers would cause diffusion of stars away from \( R_e \) (Johnston, Sackett, & Bullock 2001). Diffusion could account for the extended optical halos of large ellipticals (Arp & Bertola 1971); numerical simulations might show whether the amount of diffusion is acceptable at the high redshifts of formation in our model.

Our analysis assumes that star formation in giant ellipticals is close to complete before their assembly at \( z_f \sim 6 \). This certainly seems consistent with the short cooling times in the central regions of the most massive halos. And this early star formation seems to be required in the \( \Lambda \)CDM model, because late star formation would produce unacceptably large velocity dispersions in giant ellipticals, as discussed in §2.2. Peacock et al. (1998) present another consideration that leads to a similar value for \( z_f \). They start from the observed comoving number density of giant ellipticals and their stellar velocity dispersion which they set equal to the velocity dispersion at the virial radius, \( V_c / \sqrt{2} \). Based on the Press-Schechter mass function, they also infer a formation redshift \( z_f \sim 6 \). Although in general \( \sigma_e \) is not equal to \( V_c / \sqrt{2} \), the approximation is acceptable here because \( r_t \sim 10 \) kpc at \( z \sim 6 \) (see Fig. 1).

There has been considerable discussion of observational constraints on the formation redshift of giant ellipticals (Kauffmann, Charlot, & White 1996). In the recent deep K-band survey of Cimatti et al. (2002) the counts of galaxies at \( 2 \lesssim z \lesssim 3 \) are consistent with early formation of very luminous galaxies. This is in line with the persuasive case by Dunlop et al. (1996) and Waddington et al. (2002) that some giant ellipticals formed at \( z_f \gtrsim 4 \), and with the evidence that many giant galaxies formed not much later than that (Zirm, Dickinson, & Dey 2002; Saracco et al. 2002). Other arguments for early formation of late-type galaxies are reviewed in Peebles (2002).

Our postulate that stars replace dark matter in a near universal form for the net mass distribution in a giant elliptical may have some bearing on the observation that the varying mix of baryonic and cold dark matter as a function of radius in some ellipticals adds up to a simple form for the total mass density, \( \rho \sim r^{-2} \) (Romanowsky & Kochanek 2001; Koopmans & Treu 2002; but for an exception see Sand, Treu, & Ellis, 2002; and see van Albada & Sancisi 1986 for the analog of this curious “conspiracy” in spiral galaxies). The interpretation must be more complicated than a universal mass density run, however, because the power law is steeper than Moore. The complication may result from the rearrangement of the stellar mass distribution by recycling of mass shed by evolving stars and from the addition of mass by low levels of merging and accretion. The spectra of large early-type galaxies show evidence of ongoing star formation (Jørgensen 1999; Trager et al. 2000; Menanteau, Abraham, & Ellis 2001), amounting to a few tens of percent of the total at \( z < 1 \). If this reflects the recycling of baryons
in stars it might be expected to move baryonic mass to smaller radii, making the density run steeper, as needed.

Merging at low redshift, driven by dynamical friction, is an important element in semi-analytic models for galaxy formation (Cole et al. 2000), and it is observed, as in the recent capture of a spiral by the nearby giant elliptical Centaurus A (NGC 5128), which has increased the mass in stars and gas in this elliptical by about 10% (Israel 1998). The Centaurus elliptical has many more late-type satellites and group members (Côté et al. 1997). But arguing against substantial growth by accretion is the evidence that the abundance of iron group elements relative to α elements is higher in younger late-type galaxies than in ellipticals, in line with the idea that the stars in ellipticals formed too rapidly for appreciable enrichment of iron from type Ia supernovae (Thomas et al. 1999; Pagel 2001).

We will be following with interest constraints from the chemistry on the amount stellar mass that merging have added to the centers of giant ellipticals, and the effect on the central mass density run.

Our model predicts that the most massive ellipticals reside in very rich clusters of galaxies. It would be interesting to see the results of a simple test, a comparison of the spatial autocorrelation functions of massive ellipticals and of rich clusters with the same comoving number density.

The proposed early formation of the giant ellipticals may help account for the luminous quasars at z ∼ 6 found by SDSS (Fan et al. 2001). If these objects are radiating isotropically at the Eddington limit then in the standard quasar model they are powered by black holes with mass $M_{\text{BH}} \gtrsim 10^9 M_\odot$, which is close to the largest masses inferred for central compact objects in present-day galaxies. The ΛCDM model does have initial conditions for the formation of these massive black holes and their host galaxies at z ∼ 6 (Barkana & Loeb 2002; Wyithe & Loeb 2002). The issue for our purpose is whether this early assembly is the dominant mode of formation of the giant ellipticals, as we are proposing, or whether these galaxies and their central black holes grew by a hierarchy of mergers at redshifts well below 6 (Haehnelt & Kauffmann 2000). In the latter case one might expect that the SDSS black holes have since grown considerably more massive than $10^9 M_\odot$. That would not naturally fit the correlation of $\sigma_e$ with $M_{\text{BH}}$ in present-day galaxies (Merritt & Ferrarese 2001; Tremaine et al. 2002) together with the sharp cutoff in $\sigma_e$.

Our conclusion from these considerations is that there is no serious observational problem but instead some possible encouragement for the idea that the giant ellipticals formed at high redshift.

The reader may have noticed that we are arguing for early formation of the most massive galaxies in a cosmology, ΛCDM, that usually is associated with the formation of massive galaxies at low redshift (see, e.g. Figure 13 in Baugh et al. 1998). We offer two considerations. First, Figure 1 indicates that, within commonly accepted approximations to structure formation in this cosmology, there is little late time addition to the mass concentrated within 10 kpc of the centers of the most massive galaxies. Perhaps the accretion at low redshifts seen in numerical simulations of this cosmology requires some modification of this statement, or perhaps it requires some modification of the model that would also bring it into agreement with the void phenomenon (Peebles 2001). Second, our early formation scenario may apply to the giant early-type galaxies and the late scenario to massive late-type galaxies. But the circular velocities of spiral galaxies also show a strong upper cutoff (Giovanelli et al. 1986, and references therein), and one would surely hope to find a common explanation for this striking effect in both types of galaxies.

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