Magnetic Properties of HTSC with Weak Interlayer Coupling.

Gregory M. Braverman
Max-Planck-Institut für Kernphysik Heidelberg, Germany.

Magnetic properties of layered high temperature superconductors with a weak interlayer coupling in the region of critical fluctuations are investigated in framework of the Ginzburg–Landau approach. The sample magnetization is calculated perturbatively to the second order in the interlayer coupling constant. The first order correction can be incorporated into the non-interacting expression for the magnetization with the corresponding shift of the critical temperature. Only the second order contribution modifies thermodynamics, leading to violation of the 2D scaling law and to disappearance of the magnetization crossing point. Magnetic field suppresses the interlayer interaction, and at sufficiently high values of the applied field 2D scaling is recovered.

I. INTRODUCTION.

Scaling properties of layered high temperature superconductors (HTSC) around the mean–field transition line \(H_{c2}(T)\) have been under intensive experimental and theoretical study during the last decade. The layered YBa\(_2\)Cu\(_3\)O\(_{7-\delta}\) compounds exhibit 3D properties. All thermodynamic and transport quantities obey the 3D scaling law as a function of the scaling variable \(B(T - H_{c2}(H))/\langle H \rangle^{2/3}\), when the applied magnetic field is perpendicular to the layers plane. In the case of Bi\(_2\)Sr\(_2\)Ca\(_3\)Cu\(_3\)O\(_{10}\) experimental results show 2D scaling behavior with respect to the 3D scaling variable \(A(T - H_{c2}(H))/\langle H \rangle^{1/2}\). The fact that the phenomenon is observed in the vicinity of the mean–field upper critical field indicates that the problem can be studied in the framework of the Ginzburg–Landau (GL) theory with the order parameter projected onto the space of the lowest Landau levels (LLL). First calculations of the free energy scaling function were done perturbatively. The non-perturbative approach for pure 2D systems was developed by Tesanović et al. It was shown that around the mean–field transition line only the fluctuations of the total amplitude of the order parameter play important role in the superconductor thermodynamics. The remaining part, fluctuations in the position of the vortices enters through the dimensionless Abrikosov geometric factor \(\beta_A\). In the most realistic cases this quantity weakly depends on the vortex configuration and can be taken as a constant from the very beginning. This approximation was generalized for the layered 2D and 3D systems by Tesanović and Andreev. They showed that the system dimensionality depends on the strength of the interlayer coupling constant. When coupling is absent, the system becomes pure two dimensional. In the limit of strong interlayer coupling, when the superconducting correlation length \(\xi_c\) along the \(c\)-axis direction becomes much larger then the effective interlayer separation \(s\) the 3D description becomes appropriate. In the intermediate case \(\xi_c < s\) the system becomes quasi two–dimensional for which scaling is impossible.

In this Paper we study quasi two dimensional layered superconductor in the region of the critical fluctuations.

In this region we use GL-LLL description, which allows us to build a solvable model. The interlayer coupling is assumed to be small and therefore can be taken into account perturbatively. Within this approach we calculated the sample magnetization to the second order in the interlayer coupling constant. If this constant is extremely small one can consider only the first order correction. We show that because of the nearest neighboring coupling this correction leads to shift of the critical temperature, preserving the form of usual 2D scaling function of the magnetization. Only the next order correction destroys scaling and leads to disappearance of the magnetization crossing point. The effective interlayer coupling constant turns out to be proportional to the square root of the inverse applied field, which results in effective suppression of the interlayer interaction in high field region and, therefore, leads to recovering of the 2D scaling law.

II. THE MODEL.

Consider a layered type II superconductor in the region of critical fluctuations around its mean-field transition line \(T_{c2}(H)\) (or \(H_{c2}(T)\)). The applied magnetic field \(H\) is assumed to be normal to the layers plane: \(H \parallel \hat{c}\). Then the superconductor thermodynamics at the temperature \(T\) can be described by the following partition function:

\[
Z \propto \int D[\Psi] \int D[A] \exp \left\{ -\frac{\mathcal{F}[\Psi,A]}{k_B T} \right\},
\]

(1)

where \(\mathcal{F}[\Psi,A]\) is the GL free energy functional of the layered system with a nearest neighboring Josephson coupling between pancake vortices given by

\[
\mathcal{F}[\Psi,A] = s \sum_n \int d^2r \left\{ \alpha_0 |\Psi_n|^2 + \frac{\beta}{2} |\Psi_n|^4 + \gamma_{ab} |\theta_n \Psi_n|^2 + \gamma_c |\Psi_n - \Psi_{n+1}|^2 + \frac{(H - B_n)^2}{8\pi} \right\}
\]

(2)

and \(k_B\) is the Boltzmann constant. The quantity \(s\) is an effective interlayer spacing, \(\alpha_0 = a(T - T_0)\) and \(\beta = \text{const}\) are the first and the second GL coefficients correspondingly. The third GL coefficient \(\gamma\) is assumed
to be anisotropic, where the quantities $\gamma_{ab}$ and $\gamma_c$ define its value in the layer plane and $\hat{c}$ direction correspondingly. In what follows, we refer to the quantity $\gamma_c$ as an interlayer coupling constant. The quantity $\Psi_n(r)$ is the order parameter of the $n$th layer and $B_n(r) \parallel \hat{c}$ is the magnetic induction induced in the $n$th layer of the superconductor. Two-dimensional gauge invariant gradient $\partial_n$ is defined as:

$$\partial_n = -i\hbar \frac{\partial}{\partial r} - \frac{2e}{c} A_n(r),$$

where $A_n(r) = \nabla \times B_n(r)$. The sample magnetization is given by

$$M = \frac{1}{4\pi N_L} \sum_n \int d^2r \left\{ \frac{F}{k_B T} \right\},$$

where $N_L$ is the number of layers. In the weak coupling regime, it is convenient to introduce renormalized critical temperature:

$$T_c = T_{c0} - 2\frac{\gamma_c}{a} \equiv T_{c0} - \delta T_c.$$  

This modifies the expression (3) for the free energy, which now reads as

$$F[\Psi, A] = s \sum_n \int d^2r \left\{ |\Psi|^2 + \frac{\beta}{2} |\Psi|^4 + \gamma_{ab} |\nabla \Psi|^2 - \gamma_c \Psi^\ast_{n+1} (\Psi_{n-1} + \Psi_{n+1}) + \frac{(H - B_n)^2}{8\pi} \right\},$$

where $\alpha = a(T - T_c)$. Thus, in order to solve the problem one has to calculate extremely complicated integrals over the order parameter $\Psi$ and the vector potential $A$ appearing in the expressions (3) and (4) for the partition function and the magnetization. However, there are number of simplifications, which can be done.

In the limit of large values of the GL parameter $\kappa$ ($\kappa \sim 100$ for the most HTSC) one can neglect fluctuations of the magnetic induction $B_n$. Then we minimize the last expression (3) for the GL free energy with respect to the vector potential $A_n$, as it is done in the case of conventional superconductor. This leads to the set of decoupled (with respect to the layer index) GL equations for the vector potential $A_n$ and the order parameter $\Psi_n(r)$. In general, the order parameter can be expanded over the electron eigenfunctions of the Landau levels in the applied magnetic field $H$. However, close to the mean-field transition line $H_{c2}(T) = -\alpha(T)c/(2\hbar e \gamma_{ac})$ one can restrict oneself to the zeroth Landau level only. This approximation can be used, at least, if $H > 1/3H_{c2}(T)$, until the first Landau level becomes important. Recently, it was shown that the lowest Landau level (LLL) approximation works good even if $H \ll H_{c2}(T)$ for $\kappa \gg 1$. In the LLL approximation the equations, described above, can be solved analytically. After substituting these solutions into the expression (3) for the free energy, we finally obtain:

$$F = s S \sum_n \left\{ \alpha \left(1 - \frac{H}{H_{c2}}\right) |\Psi_n|^2 + \frac{\beta}{2} |\Psi_n|^4 - \gamma_c \Psi^\ast_n (\Psi_{n-1} + \Psi_{n+1}) \right\},$$

where the bar means averaging over the layer area $S$. In this case only the integrals over the order parameter $\Psi$ remain in the expression for the partition function (4):

$$Z \propto \int D[\Psi] \exp \left\{ -\frac{F[\Psi]}{k_B T} \right\},$$

where the quantity $F[\Psi]$ is given now by (3). In the LLL approximation the sample magnetization (3) can be calculated using the following formula:

$$M = \frac{\beta H_{c2}}{8\pi \alpha^2} \sum_n \int d^2r \left\{ \frac{|\Psi|^2}{|\Psi|^4} \right\},$$

In order to proceed further, we define the Abrikosov geometric factor for each layer separately: $\beta_A(n) = \frac{|\Psi|^2}{(\Psi^2)^2}$, and its average $\beta_A = N_L^{-1} \sum_n \beta_A(n)$. Following the refs we assume that the quantity $\beta_A(n)$ only slightly depends on the actual vortex configuration and therefore we put $\beta_A(1) = \beta_A(2) = \ldots = \beta_A$. Then we replace the quantities $|\Psi|^2$ by $\beta_A (|\Psi|^2)^2$ in the expression (3) for the free energy. With this replacement the model becomes exactly solvable:

$$Z \propto \int D[\Delta] \exp \left\{ -N_v \sum_n \left\{ \frac{1}{2} |\Delta_n|^2 + \frac{1}{4} (|\Delta_n|^2)^2 - \mu \Delta^\ast_n (\Delta_{n+1} + \Delta_{n-1}) \right\},$$

where $N_v = \Phi/\Phi_0$ is the number of vortices. The standard 2D scaling variable $x$ is given by

$$x = A \frac{T - T_{c2}(H)}{\sqrt{TH}},$$

where $A = \sqrt{s \Phi_0/(16\pi \kappa^2 \beta_A k_B)} H_{c2}'$ and $H_{c2}' = -dH_{c2}(T)/dT|_{T=T_c}$. The dimensionless coupling constant $\mu$ is

$$\mu = \frac{A \delta T_c}{2 \sqrt{TH}},$$

and the dimensionless order parameter $\Delta_n$ reads as

$$|\Delta_n|^2 = A \frac{2\beta \Delta^2}{a \sqrt{TH}} |\Psi_n|^2.$$
With these new variables the sample magnetization \( \hat{\mathbf{M}} \) can be expressed as
\[
\frac{M}{\sqrt{HT}} = \frac{k_B A}{s \Phi H_{c2}} N^{-1} \int d \ln Z.
\]

III. CALCULATION OF THE PARTITION FUNCTION.

In order to compute magnetization of the superconductor we, first, have to calculate the partition function \( \hat{\mathbf{M}} \). This involves evaluation of the integrals over the order parameter \( \Delta \). The main difficulty in such calculation is the quartic term appearing in the exponent of the right hand side of the formula (9). In order to decouple it, we introduce a set of additional integration variables \( \{ \lambda_n \} \):
\[
Z \propto \int \mathcal{D}[\Delta] \prod_n \int_{\Gamma_n} d \lambda_n \exp \left\{ -N_v \sum_n \left[ \lambda_n^2 + (x + i \lambda_n) | \Delta_n |^2 - \mu \Delta_n^* (\Delta_{n+1} + \Delta_{n-1}) \right] \right\},
\]
where
\[
\int_{\Gamma_n} \mathcal{D}[\lambda] = \prod_n \int_{\Gamma_n} d \lambda_n.
\]
The contours \( \Gamma_n \) are parallel to the real axis, standing on some distance from it, in order to insure convergence of the integrals over the order parameter \( \Delta_n \). The formula (11) is the result of use of the simplified version of the Hubbard-Stratonovich transformation, usually applied in the field theory. As it was explained above, the order parameter \( \Delta_n \) is the linear combination of the electron eigenfunctions of the lowest Landau level:
\[
\Delta_n (r) = \sum_{k=0}^{N_v} C_{nk} L_k (r),
\]
and
\[
L_k (r) = \frac{1}{\sqrt{k!}} \left( \frac{r}{l} \right)^k \exp \left\{ -ik \theta - \frac{r^2}{2l^2} \right\},
\]
where \( l \) is the magnetic length corresponding to the charge \( 2e \). Then the meaning of the integration over the order parameter becomes clear:
\[
\mathcal{D}[\Delta] \propto \prod_{n,k} \int d C_{nk}^* d C_{nk}.
\]
The integrals over these expansion coefficients in (11) are of the generalized gaussian type and can be evaluated analytically. As a result, we obtain:
\[
Z \propto \int \mathcal{D}[\lambda] \exp \left\{ -N_v \mathcal{L} \right\},
\]
The action \( \mathcal{L} \) is given by
\[
\mathcal{L} = \text{tr} \left[ \hat{\lambda}^2 + \ln \left( x \hat{I} + i \hat{\lambda} - \mu \hat{\gamma} \right) \right],
\]
where \( \hat{\lambda} \) is diagonal matrix, consisting from the elements \( \lambda_n \). The matrix \( \hat{\gamma} \) is the real symmetric matrix, arising as a result of the interlayer coupling:
\[
\gamma_{mn} = \delta_{m,n+1} + \delta_{m,n-1}.
\]
The integrals over the expansion coefficients \( C_{nk} \) converge, if along the integration contours \( \Gamma_n \) the following inequality is satisfied:
\[
\text{Re} (f_n) > 0,
\]
where \( f_n \) are eigenvalues of the complex symmetric matrix \( x \hat{I} + i \hat{\lambda} - \mu \hat{\gamma} \). In the thermodynamic limit \( N_v \to \infty \) the integrals over \( \lambda_n \) in the partition function (13) can be evaluated within saddle point approximation. It will be shown below that due to particular properties of the saddle-point manifold the condition (14) can be satisfied in the weak-coupling regime in rather large range of the scaling variable \( x \).

The saddle point equation is found from the condition \( \delta \mathcal{L} = 0 \) and reads as
\[
\text{tr} \left\{ \left[ 2 \lambda + i \left( x \hat{I} + i \hat{\lambda} - \mu \hat{\gamma} \right)^{-1} \right] \delta \hat{\lambda} \right\} = 0. \]
The position of the saddle point depends now on the temperature \( T \) and the applied field \( H \) via two parameters \( x(H,T) \) and \( \mu (H,T) \). In this case, 2D scaling of the magnetization (18) becomes impossible, as it was predicted in the ref. If the coupling is weak \( 2 \mu < | x + i \lambda | \) (this inequality is similar to Tesanović-Andreev criterion (19) for quasi 2D systems), the second term in the left hand side of the equation (13) can be expanded in the powers of \( \mu \). Then the saddle point equation (16) can be rewritten as follows:
\[
2 \lambda_n + \frac{i}{x + i \lambda_n} + \frac{i \mu^2}{(x + i \lambda_n)^2} \frac{2x + i (\lambda_n-1 + \lambda_{n+1})}{(x + i \lambda_{n-1})(x + i \lambda_{n+1})} + o(\mu^4) = 0.
\]
The terms of the order of \( \mu \) and \( \mu^3 \) drop out in the right hand side of the last equation, since \( \text{tr} (\hat{\gamma}^{2n+1}) = 0 \). The structure of the saddle point equation is such that, at least, up to the second order in \( \mu \) the saddle point solution for \( \lambda \) is proportional to the unit matrix, namely
\[
\lambda_1 = \lambda_2 = \ldots = \lambda.
\]
Then the equation (16) can be rewritten in the following simple form:
\[
2 \lambda + \frac{i}{x + i \lambda} + \frac{2 \mu^2}{(x + i \lambda)^3} + o(\mu^4) = 0.
\]
As soon as the saddle point solution is found, the sample magnetization \( M \) can be calculated using the following simple relation:

\[
\frac{M}{\sqrt{HT}} = -\frac{2Ak_B}{s\Phi_0 H c_2} i\lambda.
\]  

(18)

From the last equation we conclude that the physically meaningful solution for \( \lambda \) lies on the imaginary axis. Further, using the Rayleigh-Ritz theorem, one can show that both matrices \( 2I - \hat{\gamma} \) and \( 2I + \hat{\gamma} \) are positive definite. Then the eigenvalues of the matrix \( \hat{\gamma} \) belong to the interval \([-2, 2]\). In this case, the conditions (14) for convergence of the integrals over expansion coefficients \( C_k \) in the partition function can be written as:

\[
2\mu < |x + i\lambda|,
\]  

(19)

which is automatically satisfied in the weak coupling regime.

IV. MAGNETIC PROPERTIES.

In the previous section we derived the saddle point equation (13) for the layered superconductor under assumption that the interlayer coupling constant \( \mu \) is small. The sample magnetization \( M(H, T) \) is proportional to the saddle point solution and is given by the formula (15):

\[
M = \frac{M_0}{\sqrt{HT}} = \frac{Ak_B}{s\Phi_0 H c_2}(x - \sqrt{x^2 + 2}).
\]  

(20)

Actually, this "zeroth" order result includes the first order correction in \( \mu \) by means of the critical temperature shift (10). The second order correction to the saddle point solution is calculated as a small perturbation:

\[
\lambda(x, \mu) = \lambda_0(x) \left( 1 + 4\mu^2 \frac{i\lambda_0(x)}{x^2 + 2} \right).
\]  

(21)

Then the sample magnetization is given by

\[
M = M_0 \left( 1 - 2\mu^2 \frac{x - \sqrt{x^2 + 2}}{x^2 + 2} \right),
\]  

(22)

where \( M_0(H, T) \) is the magnetization of the "decoupled" sample given by the equation (20). Using the last formula, we plotted the quantity \( M/\sqrt{HT} \) as a function of \( 2D \) scaling variable \( x \) (see the figure 1) for five different values of the applied field \( H \) between 10 and 50 kOe. The interlayer coupling constant is chosen to be small, such that \( \delta T_c = 1K \) (see the eq. (14) for definition of \( \delta T_c \)). We used \( T_c = 111K, \kappa = 100, H c_2 = 40kOeK^{-1} \) and \( s = 2nm \), which are of the order of the typical experimental parameters. The whole range of the scaling variable \( x \) in the fig. 1 satisfies the applicability condition of the theory (19). As it was expected, the 2D scaling is destroyed, due to the dimensionless coupling constant \( \mu(T, H) \) appearing in the right hand side of the expression (22) for the magnetization.

![FIG. 1. The quantity \( M/\sqrt{HT} \) as a function of the 2D scaling variable \( x \) is plotted for \( \delta T_c = 1K \).](image)

![FIG. 2. The sample magnetization \( M \) as a function of the temperature \( T \) is plotted for \( \delta T_c = 1K \).](image)
which is the property of the scaling form \(20\) (see ref. 8). Actually, the crossing point is noticeably destroyed only in the low-field region. The higher the applied field, the better the crossing point is pronounced. This is the consequence of the specific form of the effective coupling constant \(\mu\) (see eq. (3)). The interaction correction to the magnetization in the formula \(24\) is proportional to the square of this constant and therefore is proportional to \(H^{-1}\). Then, at the large values of the applied field interlayer interaction is effectively suppressed. Indeed, one can treat the magnetization data plotted in the figure 2 as if they were obtained from the experiment. Then, we try to fit them to 2D scaling form varying phenomenological parameters \(T_c\) and \(H'_{c2}\), as it is usually done in experiments. The fitting results are given in figure 3. The best fit is obtained for \(T_c \approx 111.5K\). Like in the experiment, the scaling results are insensitive to the value of the phenomenological parameter \(\mu_0H'_{c2}\) in relatively large range of its values around \(4TK^{-1}\). It can be observed from the figure 3 that the scaling is satisfactory good for fields larger than \(30kOe\) and is destroyed in the weak-field region. It must be stressed that the mentioned above fit is made essentially by hand and without any error estimation. However, it can serve as demonstration of effective suppression of interlayer interaction in the high field region.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3.png}
\caption{2D scaling of the magnetization data, taken from the fig. 2}
\end{figure}

V. SUMMARY.

In this paper we considered HTSC assuming weak interlayer nearest neighboring coupling. In the region of the critical fluctuations, close to the mean-field transition line \(H_{c2}(T)\) the order parameter can be taken as a linear combination of the electron eigenfunctions of the lowest Landau level. This approximation together with the assumption that the Abrikosov geometric factor only weakly depends on the actual vortex configuration allows to reduce the problem to the simpler one, namely to the saddle point equation \(17\). This equation can be solved perturbatively. We calculated the magnetization of the superconducting sample to the second order in the effective coupling constant \(\mu(T,H)\). If the coupling is sufficiently weak, one can consider the first order correction only. It turns out that this correction does not modify the scaling properties of the sample, leading to the trivial renormalization of the critical temperature (see eq. (9)). Account of the second order correction leads to violation of the 2D scaling and destroys the magnetization crossing point in the low field region. At sufficiently high values of the applied field the interlayer interaction is effectively suppressed. This leads to recovering of 2D scaling with a well pronounced crossing point.

ACKNOWLEDGMENTS

The author would like to thank Sergey A. Gredeskul for helpful discussions.

The author gratefully acknowledges the MINERVA foundation for the financial support.

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