A Novel SVM-Based Edge Detection Method

WU Peng¹, CHEN Qichao²

¹College of Mechanical and Electronic Engineering, Northeast Forestry University, Harbin, China
²School of Electrical Engineering and Automation, Harbin Institute of Technology, Harbin, China

Abstract

This paper presents a new, simple and effective edge detection algorithm based on support vector machine (SVM), because of the disadvantages in the traditional image edge detection methods, such as rough edge, noise edge and inaccurate edge location. Based on least squares SVM with Gaussian radial basis function kernel, a set of the new gradient operators and the corresponding second derivative operators are obtained. The experimental results indicate that the proposed edge detector is greatly improved comparing with the traditional edge detection methods.

Keywords: edge detection; least squares support vector machine; Gaussian radial basis function kernel

1. Introduction

The process of image edge detection is based on the hypothesis that directs the edge is a point where the image intensity has sharp intensity transitions [1-4]. Important regions of interest are separated by different level of pixel intensity value. Upon this assumption, many edge detectors have been proposed. Most of them depend on the local pixel intensity gradient, done by differencing [3, 6] as a calculation of convolution of weighted matrix called local gradient mask. This group consists of well-known edge detector, such as Sobel, Roberts, Prewitt, Robinson, Kirsch, Frei-Chen [1, 5-7]. Another interesting principle of edge detection in [8] is done by approximation of circular masks and associating each image point with a local area of similar brightness. Their major drawbacks are high sensitivity to noise and disability to discriminate edges versus textures. Because of these limitations more advance edge detectors have been proposed which do not only detect edges but also try to connect neighboring edge points into a contour. In this way, many authors have developed different edge detectors based on the scale space
active contours [9], morphological operations [14] and also gradient values [11]. Among all, the fundamental one is Canny edge detector [8], which is fast, reliable, robust and generic, but the accuracy is not satisfactory, because of the parameters, which is the weakest point in the procedure. For this reason Canny was extended to the time-scale plane in [15].

The purpose of this paper is to present a new efficient edge detection algorithm based on the hybrid of gradients and zero crossings obtained by convolving the image with the corresponding operators. The approach is used the least squares SVM (LS-SVM) with a typical and most frequently studied kernel function, Gaussian radial basis function.

The organization of the work is as follows. In Section II the LS-SVM is introduced. The algorithm and theory for edge detections is presented in Section III. The application of the presented algorithm and the experimental result are given in Section IV. The conclusion is given in Section V.

2. The Theory of LS-SVM

In theory of SVM, the Vapnik’s standard SVM classifier is following

$$\min_{\omega,a,\xi} J(\omega,a,\xi) = \frac{1}{2} \|\omega\|^2 + C \sum_{i=1}^{N} \xi_i$$

s.t. 

$$g_i[\omega^T \phi(x_i) + a] \geq 1 - \xi_i, \quad \xi_i > 0, i = 1,2,\ldots,N, C > 0$$

(1)

Where $C$ is a positive constant parameter used to control the tradeoff between the training error and the margin. The dual of the system (1) via Karush-Kuhn-Tucker (KKT) conditions leads to a well-known convex quadratic programming (QP) problem.

In the LS-SVM theory, the Vapnik’s standard SVM classifier has been modified into the following LS-SVM formulation

$$\min_{\omega,a,e} J(\omega,a,e) = \frac{1}{2} \omega^T \omega + \gamma \frac{1}{2} \sum_{i=1}^{N} e_k^2$$

s.t. 

$$g_k[\omega^T \phi(x_k) + a] = 1 - \xi_k, k = 1,2,\ldots,N$$

(2)

We note that the passage from Eq.(1) to Eq.(2) involves replacing the inequality constraints by equality constraints and a squared error term similar to ridge regression. The corresponding Lagrangian for Eq.(2) is

$$L(\omega,a,e;\alpha) = J(\omega,a,e) - \sum_{k=1}^{N} \alpha_k \{g_k[\omega^T \phi(x_k) + a] - 1 + e_k\}$$

(3)

Where the $\alpha_k$ is the Lagrange multipliers, the optimality condition leads to the following $(N+1) \times (N+1)$ linear system

$$\begin{bmatrix} 0 & G^T \\ G & ZZ^T + \gamma^{-1} I \end{bmatrix} \begin{bmatrix} a \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(4)

where
3. Proposed Theory and Algorithm

3.1 Gray level intensity surface

In SVM, the underlying image intensity surface $f$ of a small neighborhood in image can be approximated by a combination of a set of support vectors. In LS-SVM form, $f$ can be represented as

$$
f(x) = \sum_{k=1}^{N} \alpha_k g_k K(x, x_k) + a
$$

where $\alpha_k$ and $a$ are based the solution to Eq.(4), $K(x, x_k)$ are the kernel functions. It is worth noting, the Eq.(4) can be rewritten as follows

$$
\begin{bmatrix}
0 & 1 \\
1 & \Omega
\end{bmatrix}
\begin{bmatrix}
a \\
\alpha
\end{bmatrix} =
\begin{bmatrix}
0 \\
G
\end{bmatrix}
$$

where $\Omega = K(x, x_j) + \gamma^{-1}I$, $G = [g_1, \ldots, g_N]^T$.

Indeed the second row of Eq.(6) gives $1a + \Omega \alpha = G$ and together with the first row gives the explicit solution

$$
a = 1^T \Omega^{-1} G, \quad \alpha = \Omega^{-1} (G - 1a)
$$

It is observed that the $\Omega$ in Eq.(7) is only related to the input vector and parameters $\gamma$ and kernel functions. For the two dimensional images, let $R$ and $C$ be the index sets of the neighborhood that satisfy the symmetric conditions, i.e., and $r \in R$ and $c \in C$ imply $-r \in R$ and $-c \in C$.

In this way, when the kernel function and its parameters are known, values $A$ and $B$ defined over $R \times C$ can be used as constants and be pre-calculated by

$$
A = \Omega^{-1}, \quad B = \frac{1^T \Omega^{-1}}{1^T \Omega^{-1}1}
$$

then the solution Eq.(7) can be given by

$$
a = BG, \quad \alpha = A(G - 1a)
$$

3.2 Derivatives of the intensity surface

Many characteristics of the gray level intensity function in Eq.(5) are determined by the type of kernel function $K(x, x_j)$. Every kernel has its advantages and disadvantages. Numerous possibilities of kernels satisfying Mercer’s theorem exist and the selection of kernel function is very important. Although, there are many kernel functions that can be used. There are two parameters $\gamma$ and $\sigma$ in RBF kernel.
based LS-SVM. The parameter $\sigma$ is very important and can be constant in the image processing within the limited error tolerance. Parameters $\gamma$ in LS-SVM controls the solution insensitivity to the error, when the $\gamma$ is infinite, a solution with least squared error is attained. In our study, the $\gamma$ should be infinite.

Let $S$ be a symmetric 2-D neighborhood defined on $\mathbb{R} \times \mathbb{C}$, and $f(r,c)$ be the observed intensity value at $(r,c) \in S$. The intensity estimation function Eq.(5) of the LS-SVM with RBF kernel over $S$ can be rewritten as follows

$$f(r,c) = \sum_{k=1}^{N} \alpha_k \exp\left\{-\frac{(|r-r_k|^2 + |c-c_k|^2)}{\sigma^2}\right\} + a$$ (10)

Where the $\alpha_k$ and $b$ are the solution in Eq.(9). Evaluating the first and second row and column partial derivatives at point $(r,c)$ yields the one-order and second-order directional derivatives

$$\frac{\partial f}{\partial r} = -\sum_{k=1}^{N} \frac{2\alpha_k}{\sigma^2} (r-r_k) \times \exp\left\{-\frac{(|r-r_k|^2 + |c-c_k|^2)}{\sigma^2}\right\}$$

$$\frac{\partial f}{\partial r} = -\sum_{k=1}^{N} \frac{2\alpha_k}{\sigma^2} (c-c_k) \times \exp\left\{-\frac{(|r-r_k|^2 + |c-c_k|^2)}{\sigma^2}\right\}$$

$$\frac{\partial^2 f}{\partial r^2} = -\sum_{k=1}^{N} \frac{4\alpha_k}{\sigma^2} (1 - \frac{2}{\sigma^2}(r-r_k)^2) \times \exp\left\{-\frac{(|r-r_k|^2 + |c-c_k|^2)}{\sigma^2}\right\}$$

$$\frac{\partial^2 f}{\partial c^2} = -\sum_{k=1}^{N} \frac{4\alpha_k}{\sigma^2} (1 - \frac{2}{\sigma^2}(c-c_k)^2) \times \exp\left\{-\frac{(|r-r_k|^2 + |c-c_k|^2)}{\sigma^2}\right\}$$ (11)

with the derivatives of the point $(r,c)$ on the intensity surface, the gradient vector at points $(r,c)$ can be defined as $\nabla f(r,c) = [Y_r, Y_c]^T = [\partial f / \partial r, \partial f / \partial c]$, the gradient and directional angle of the vector is $\text{mag}(\nabla f) = [Y_r^2 + Y_c^2]^{1/2}$, $\phi(r,c) = \arctan(Y_c / Y_r)$. The second order derivative value of the point $(r,c)$ can be defined as $\nabla^2 f = \partial^2 f / \partial r^2 + \partial^2 f / \partial c^2$.

### 3.3 Gradient and second order derivative operators

Although we can calculate the gradient or second order derivative values of the image with Eq.(11), the computational complexity is still large and it is difficult to apply a large size image. Let us continue to observe the solution Eq.(9) of the LS-SVM defined over $\mathbb{R} \times \mathbb{C}$ and the derivatives Eq.(11) of the pixels in the neighborhood. The Eq.(14) can be rewritten as follows

$$\alpha = A(I - B)G, \quad I^T = [1, \cdots, 1]$$ (12)

The directional derivatives Eq.(11) can be rewritten as follow
\[
\frac{\partial f}{\partial r} = F_1\alpha = F_1A(I-1)BG = F_r G \\
\frac{\partial f}{\partial c} = F_2\alpha = F_2A(I-1)BG = F_c G \\
\frac{\partial^2 f}{\partial r^2} = F_{11}\alpha = F_{11}A(I-1)BG = F_{rr} G \\
\frac{\partial^2 f}{\partial c^2} = F_{22}\alpha = F_{22}A(I-1)BG = F_{cc} G
\] (13)

where

\[
F_1 = [f_{11}, \ldots, f_{1N}], \quad f_{1k} = -\frac{2}{\sigma^2}(r-r_k) \times \exp \left\{-\left(\frac{|r-r_k|^2+|c-c_k|^2}{\sigma^2}\right)\right\}, \quad k = 1, \ldots, N
\]

\[
F_2 = [f_{21}, \ldots, f_{2N}], \quad f_{2k} = -\frac{2}{\sigma^2}(c-c_k) \times \exp \left\{-\left(\frac{|r-r_k|^2+|c-c_k|^2}{\sigma^2}\right)\right\}, \quad k = 1, \ldots, N
\]

\[
F_{11} = [f_{111}, \ldots, f_{11N}], \\
F_{22} = [f_{221}, \ldots, f_{22N}]
\]

\[
f_{11k} = -\frac{2}{\sigma^2}(1-\frac{2}{\sigma^2}(r-r_k)^2) \times \exp \left\{-\left(\frac{|r-r_k|^2+|c-c_k|^2}{\sigma^2}\right)\right\}, \quad k = 1, \ldots, N
\]

\[
f_{22k} = -\frac{2}{\sigma^2}(1-\frac{2}{\sigma^2}(c-c_k)^2) \times \exp \left\{-\left(\frac{|r-r_k|^2+|c-c_k|^2}{\sigma^2}\right)\right\}, \quad k = 1, \ldots, N
\]

Notice that, at the defined neighborhood center \((0,0)\), where \(r=0\) and \(c=0\), the \(F_1, F_2, F_{11}, F_{22}, A, B\) are all determined by the input vectors defined over \(R \times C\) and the kernel function, which can be pre-calculated. Therefore, \(F_r, F_c, F_{rr}, F_{cc}\) can be constants. At the same time, the second order derivative value of the center point \((0,0)\) can be rewritten as \(\nabla^2 f = \frac{\partial^2 f}{\partial r^2} + \frac{\partial^2 f}{\partial c^2} = F_{rr} G + F_{cc} G = [F_r + F_c] G = F_L G\), where the vector \(G = [g_1, \ldots, g_N]^T\) is defined by the image intensity values over \(R \times C\) neighborhood. Eq.(13) shows that each gradient and second order derivative values can be computed individually as a linear combination of the intensity values \(I(r,c)\). The weight associated with each \(I(r,c)\) for gradient value is determined by \(F_r, F_c\), the weight associated with each \(I(r,c)\) for second order derivative value is determined by \(F_L\). For a rectangular neighborhood, reshape the \(F_r, F_c, F_L, G\), then the gradient value is determined by \(F_r, F_c\), the weight associated with each \(I(r,c)\) for second order derivative value is determined by \(F_L\). For a rectangular neighborhood, reshape the \(F_r, F_c, F_L, G\), then the gradient weight kernels become the new gradient operator \(Y_r, Y_c\), and second derivative weight kernel becomes the new second order derivative operator...
the gradient and second order derivative values can be computed independently by convolving the image with the corresponding operators.

3.4 Hybrid edge detector

Most edge detectors, be the gradient-based methods or zero-crossing approaches, require convolving an image with a kernel to compute gradients or zero-crossing. Based on the results of the convolution, a decision is then made to whether a pixel is an edgel or not. As both the gradient value and the zero crossings can be simultaneously estimated, the new edge extraction method, using both the gradients and the zero crossings to locate the edge position, is developed. The procedure of the new edge detection approach mainly consists of three steps: gradient magnitude calculation and thresholding; second order derivative value calculation and looking for the zero crossings; A decision is then made to whether a pixel is an edgel or not based on the combination result of the threshold gradient and zero crossings.

It is worth noting, the proposed edge detector is a combination of the gradient-based method and the zero-crossing approach. The gradient-based methods give very little control over image noise and edge location. The second-order derivative approach tends to exaggerate noise twice as much. Some sort of noise suppression is need.

4. Experimental Result

To explore the utility and demonstrate the efficiency of the proposed edge detection approach, computer experiments on gray-level images are carried out. A summary of the images used in the present study is plotted in Fig.1. Lena, the cameraman and Barbara images are figure images. All of the images are of size 256×256. For simplicity, a fixed threshold is used in the experiments, although there adaptive thresholding techniques that could be implemented. We set the percentage to 25% for gradient thresholding and 85% for second derivatives zero crossing. Other standard edge detectors use themselves post-processing techniques.

Fig.1 A collection of sample images, all of the images are of size 256×256.

The computer experiments are conducted to test the proposed approach. The experiments are designed to compare the ability of the proposed method on extracting edges from clean image with the standard Canny detector and Sobel detector.
Fig. 2 Edge images obtained by different detectors. Column 1 is obtained by the proposed approach. Column 2 and 3 are respectively obtained by the standard Canny and Sobel detectors.

All the experiments are performed with Matlab 6.5 on the Pentium III 1GHZ PC.

Fig. 2 presents a comparison of the performance of 3 edge detectors on the sample images. Here, the first column is obtained by the proposed hybrid edge detectors with parameters \( R = \{-3, -2, -1, 0, 1, 2, 3\} \), \( c = \{-3, -2, -1, 0, 1, 2, 3\} \), \( m = 2 \), \( \sigma^2 = 1 \). The performances of the standard Canny and Sobel detectors are given in the 2 and 3 columns. We can see the impact of the parameter on the performance of proposed hybrid edge detectors. The proposed algorithm effectively detects more fine and fewer spurious structures than the Canny and Sobel approach.

5. Conclusion

The least squares support vector machine (LS-SVM) for edge detection is proposed in this paper. A number of gradient operator are obtained from the LS-SVM with RBF kernel, and the new edge detector, based on the combination of gradient and zero crossing, is presented. The performance of the proposed algorithm is compared with Canny and Sobel detectors. Experiments on images have been carried out by using LS-SVM with RBF kernel detects more fine and fewer spurious structures than the Canny and Sobel approach.

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