Gravity Waves from Chain Inflation

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Chain inflation proceeds through a series of first order phase transitions, which can release considerable gravitational waves (GW). We demonstrate that bubble collisions can leave an observable signature for future high-frequency probes of GWs, such as advanced LIGO, LISA and BBO. A "smoking gun" for chain inflation would be wiggles in the spectrum (and consequently in the tensor spectral index) due to the multiple phase transitions. The spectrum could also be distinguished from a single first order phase transition by a small difference in the amplitude at low frequency.

A second origin of GWs in chain inflation are tensor modes from quantum fluctuations; these GW can dominate and be observed on large scales. The consistency relation between scalar and tensor modes is different for chain inflation than for standard rolling models and is testable by Cosmic Microwave Background experiments. If inflation happened through a series of rapid tunnelings in the string landscape, future high frequency probes of GW can shed light on the structure of the landscape.

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Several experiments are underway to detect the stochastic gravitational background from early universe cosmology. These gravity waves (GWs) propagate almost freely throughout the entire history of the universe and thus can be a direct source of information about the universe at very early times. GW experiments include the ground-based LIGO [1] and VIRGO [2], taking data now, and the upcoming space-based LISA [3], with possible launch within 20 or 30 years.

One source of GWs is inflation [5], a quasi-exponential growth phase of the universe which can explain its homogeneity and isotropy as well as the generation of density perturbations required for structure formation. There are two types of contributions to GW from inflation. In slowly rolling models, quantum fluctuations of spacetime lead to contributions proportional to the height of the inflaton potential. In tunneling models, there is an additional contribution due to bubble collisions of true vacuum bubbles at the end of the phase transition [14, 16, 19]. In this paper we consider the model of chain inflation, which gives rise to both types of contributions.

In chain inflation [8], the universe tunnels rapidly through a series of first order phase transitions. One can imagine a multidimensional potential containing many minima of varying energy. The universe starts out in a high-energy minimum, and then sequentially tunnels down to bowls of ever lower energy until it reaches the bottom. During the time spent in any one of these minima, the universe inflates by a fraction of an e-fold. After many hundreds of tunneling events, the universe has inflated by the 60 (or so) e-folds required to resolve the cosmological problems. In one variant of chain inflation, the tunneling corresponds to quantized changes in four form fluxes [9] such as may be found in the string landscape. Other variants have been discussed in [10, 11], but see [12]. At each stage, the phase transition is rapid enough that bubbles of true vacuum intersect one another and percolation is complete. Thus the failure of "old" inflation [8] to reheat is avoided, and "graceful exit" is achieved.

In the zero temperature limit, the nucleation rate \( \Gamma \) per unit spacetime volume at a phase transition for producing bubbles of true vacuum in the sea of false vacuum through quantum tunneling is \( \Gamma(t) = A e^{-S_E} \), where \( S_E \) is the Euclidean action for the bounce solution extrapolating between false and true vacua [13] and \( A \) is a determinantal factor. For a first order phase transition, with Einstein gravity, it has been shown that the probability of a point remaining in the false vacuum is given by

\[
p(t) \sim \exp(-\frac{4\pi}{3} \gamma H t),
\]

where \( \gamma \) is defined as

\[
\gamma \equiv \frac{\Gamma}{H^4}.
\]

Writing equation (11) as \( \exp(-t/\tau) \), the lifetime of the field in the false vacuum is \( \tau = \frac{3}{4\pi\gamma H} \). The number of e-foldings for the tunneling event is

\[
\chi = \int H dt \sim H \tau = \frac{3}{4\pi\gamma}.
\]

\( \gamma \) has to be greater than critical value, \( \gamma_c \), where

\[
\gamma_c = 9/4\pi,
\]

\( \tilde{\gamma} \).
to achieve percolation and thermalization. This corresponds to having an upper bound on the number of e-foldings that can be obtained in each stage of inflation:

$$\chi \leq \chi_c = \frac{1}{3}. \quad (5)$$

The number of e-foldings required for successful inflation depends on the energy scale of inflation and the reheating temperature. For a model with energy scale $\sim$ GUT scale, about 60 e-foldings are required. This corresponds to having at least $\sim 200$ tunneling events for GUT scale chain inflation.

A study of density perturbations from chain inflation has been done by [10, 13]; the former set of authors found that the right perturbations to match data can be generated. In this letter we calculate the gravity spectrum from chain inflation. Since chain inflation proceeds through first order phase transitions, one expects a considerable amount of GWs from chain inflation.

The spectrum of gravity waves (GWs) from multibubble collisions at a single phase transition (PT) with energy difference $\delta$ between false and true vacua was worked out numerically in [16]. In this paragraph we review their results, and in the next section generalize to the multiple PT of chain inflation. We note that an energy difference $\delta$ can released into gravity waves, with this fraction

$$\epsilon = \left( \frac{2.8}{f_{\text{max}}} \right)^{-1.8} f \leq f_{\text{max}},$$

$$\epsilon = \left( \frac{\beta}{f_{\text{max}}} \right) f \geq f_{\text{max}}.$$

The peak of the spectrum is at

$$f_{\text{max}} \approx 0.2\beta, \quad (7)$$

where

$$I_{\text{max}} \equiv \frac{E_{\text{GW}}}{\epsilon} (f_{\text{max}}) \approx 6 \times 10^{-2} \left( \frac{H}{\beta} \right), \quad (8)$$

$$\beta \equiv \frac{d \ln \Gamma(t)}{dt} = \frac{4\pi\gamma H}{3} = \frac{H}{\chi}. \quad (9)$$

Redshifting the frequency and energy of gravitational radiation as $a^{-1}$ and $a^{-4}$ respectively, at the current epoch the peak frequency, $f_0$, and the corresponding logarithmic contribution of GWs to critical density at the peak are given by [16]:

$$f_0 \approx 3 \times 10^{-10} \chi \left( \frac{g_*}{100} \right)^{1/6} \left( \frac{100}{g_*} \right)^{1/3}, \quad (10)$$

$$\Omega_0(f_0) h^2 \equiv \frac{1}{2\pi \rho_c} dE_{\text{GW}}(f_0) \approx 10^{-6} \chi^2 \left( \frac{100}{g_*} \right)^{1/3} \left( \frac{T_*}{1 \text{ GeV}} \right), \quad (11)$$

Here $\Omega_0$ is the GW energy density in units of the critical density $\rho_c$ required to close the universe and $h$ is today’s Hubble constant in units of 100 km/s/Mpc. $T_*$ is the temperature increase right after a single phase transition,

$$T_* = \left( \frac{300}{g_* \pi^2} \right)^{1/4}, \quad (12)$$

and the total number of relativistic degrees of freedom at temperature $T_*$ is taken to be $g_* \approx 100$. We note that in the past few years, the GW resulting from bubble collisions have been reexamined by [17] and [18]. The former find that the fall-off at the low-frequency tail $f^{-1}$, rather than $f^{-1.8}$; this result leads to a smaller fall-off of GW amplitude and hence improves detectability. To be conservative, we will mainly stick to the result of [16] and compare with the consequences of [17] as needed.

**Spectrum of GWs from Bubble Collisions:** We now generalize these results to a series of PT to obtain GW from a series of bubble collisions in chain inflation. Let us assume that the total number of minima that the universe passes through during chain inflation is $N$. As we will see its precise value will not have any impact on our final result. Then we will have $N - 1$ PTs during the course of chain inflation. The energy difference between two consecutive minima is assumed to be the constant value of $\epsilon$. We also assume that $\gamma$ does not change and it satisfies the condition $\gamma < \chi_c$. Assuming instant reheating after each PT, the temperature of the universe rises after each stage by an amount $T\gamma$, where $T\gamma$ is given in equation (12). The universe’s temperature redshifts by a factor of $\exp(-\gamma)$ during each bout of chain inflation. Thus the temperature of the universe at the end of chain inflation is

$$T\gamma = T\gamma (1 + e^{-\chi} + \cdots + e^{-(N-1)\chi}) = T\gamma \left( \frac{1 - e^{-N\chi}}{1 - e^{-\chi}} \right). \quad (13)$$

For $N\chi \gg 1$, the final temperature of the universe is simplified to

$$T\gamma = \frac{T\gamma}{1 - e^{-\chi}}. \quad (14)$$

As stated above, every first order PT can release a substantial amount of GWs from bubble collisions whose profile, peak frequency and energy fraction at the peak frequency are given by eqs. (3), (7) and (8). After each PT energy density in the gravitational radiation decreases like $a^{-4}$, whereas the frequency redshifts like $a^{-1}$ [20]. The peak frequency and fraction of critical density from GWs at the peak frequency generated during the $m$-th PT redshifts as:

$$f_m = \left( \frac{a_m}{a_e} \right) \left( \frac{a_e}{a_0} \right) f_{\text{max}}, \quad (15)$$

$$\Omega h^2(f_m) = \left( \frac{a_m}{a_e} \right)^2 \left( \frac{a_e}{a_0} \right)^2 \frac{dI(f)}{d\ln f} \bigg|_{f=f_m}. \quad (16)$$
where $a_m$, $a_e$ and $a_0$ are respectively the scale factors at the end of $m$-th phase transition, at the end of chain inflation and today. The scale factor ratio $a_m/a_e$ is equal to $\exp[-(N-m-1)\chi]$. If we also assume that the evolution from the end of chain inflation up to today has been adiabatic [i.e. $S \propto a^3 g(T) T^3 = \text{const}$], we have

$$\frac{a_e}{a_0} \simeq 8 \times 10^{-14} \left( \frac{100}{g_*} \right)^{1/3} \left( \frac{1 \, \text{GeV}}{T_e} \right).$$

(17)

Thus

$$f_m = \frac{3 \times 10^{-8}}{\chi} e^{-(N-m-1)\chi} \left( \frac{g_*}{100} \right)^{1/6} \left( \frac{T_e}{1 \, \text{GeV}} \right)$$

(18)

$$\Omega h^2 (f_m) = 10^{-6} (1 - e^{-\chi})^4 \chi^2 e^{-4(N-m-1)\chi} \left( \frac{100}{g_*} \right)^{1/3}$$

(19)

The spectrum at other frequencies get redshifted such that the profile given in eq. (18) is preserved:

$$\Omega_m h^2 (f) = \begin{cases} 
\Omega h^2 (f_m) \left( \frac{f_m}{f} \right)^{2.8} & f \leq f_m \\
\Omega h^2 (f_m) \left( \frac{f_m}{f} \right)^{-1.8} & f \geq f_m,
\end{cases}$$

(20)

The total energy density in GW due to bubble collisions in chain inflation is obtained by summing all the contributions from the multiple tunneling events,

$$\Omega_{\text{bub}} h^2 (f) = \sum_{m=1}^{N-1} \Omega_m h^2 (f)$$

(21)

and is plotted in Fig. 1. The frequency range plotted is from $10^{-4}$ Hz (roughly the minimum frequency attainable by LISA and BBO) to $10^{2.5}$ Hz (roughly the maximum sensitivity frequency of Advanced LIGO [22]). In the figure, we take $N = 1000$ minima, $\epsilon^{1/4} = 10^8$ GeV, and $\chi = \chi_c = 1/3$ (the critical nucleation rate). For such parameters the tip of the spectrum is observable in Advanced LIGO which is sensitive within a frequency band of 1-1000 Hz to stochastic signals with $\Omega_{\text{GW}}h^2 \gtrsim 10^{-11}$. Part of the low-frequency tail of the spectrum could be seen at BBO too, which could probe GWs with much smaller amplitude, i.e. $\Omega_{\text{GW}}h^2 \gtrsim 10^{-17}$ for $f \sim (10^{-4} - 10^1)$ Hz.

One can see that the GW spectrum due to bubble collisions at frequencies smaller than the peak frequency is blue. This is in contrast with slow-roll inflation which can not produce a blue gravity spectrum on an extended range of scales, unless one violates the null energy condition $H > 0$. This is rooted in the sub-horizon mechanism of generating the GWs from bubble-collision in chain inflation. As we will see, chain inflation, like any other inflationary model, additionally produces a red tensor spectrum at large scales through super-horizon quantum fluctuations of the metric.

The most important contributions to the GW arise towards the very end of inflation, during the last few PTs. The peak frequency of the total spectrum is at $f_{N-1}$ with amplitude $\Omega_{\text{bub}} h^2 (f_{N-1})$. The contribution from much earlier PTs—which have a smaller peak frequency— are (nearly) invisible, because they are screened by the low-frequency tail of the spectrum of subsequent PTs due to the redshift factor $e^{-4(N-m-1)\chi}$. That is why the number of minima, $N$, does not change the final results as long...
as \( N \chi \gg 1 \). However closer to the last bouts of chain inflation the effect of redshift factor diminishes; thus the effect of PTs that happened toward the end of chain inflation are in fact observable in the spectrum as wiggles at \( f \ll f_{\text{peak}} \).

Focusing on the frequencies around the tip of the spectrum for chain inflation reveals a richer structure: wiggles in the GW signal due to different PT giving contributions at different frequencies. Since the resolution of Fig. 1 is inadequate to show these wiggles, Fig. 2 illustrates the tensor spectral index \( n_T \equiv d \ln \Omega_{\text{bub}} h^2 / 2 \pi d \ln f \) close to the peak frequency \( f_{N-1} \). This signature of wiggles near the peak from chain inflation can be used to distinguish it from an arbitrary single first order phase transition that happens to have the same peak frequency and amplitude at the peak frequency. Such wiggles can be a "smoking gun" for chain inflation. However, we caution that the details of the wiggles have not yet been computed precisely, e.g. we have not considered the effects of having the \( m+1 \)th transition start before the \( m \)th is complete. Further work will be required to nail down the details.

There is a second way to differentiate multiple vs. single tunneling events; i.e. to differentiate chain inflation from a single first order PT with the same peak frequency and amplitude. For both models to produce the same GW, the potential height \( V_0 \) of a single PT would have to be related to the energy difference between vacua \( \epsilon \) of chain inflation in the following way: \( V_0 \sim \epsilon / (1 - \exp(\chi))^4 \) (where \( \chi \) is the number of \( \epsilon \)-folds per PT in chain inflation). The difference in the overall amplitude of the spectrum can be seen (with poor resolution) in Fig.1: matching both spectra at \( f_{\text{max}} \), one finds that the single PT has an overall amplitude that is lower than that of chain inflation by up to 3% at frequencies below the max \( f < f_{N-1} \). The reason for this difference is the gradual decrease of the tensor spectral index at frequencies close to but smaller than \( f_{N-1} \). This small difference in the amplitude at the low frequency end can in principle be detected.

Three parameters in chain inflation determine the GW. The peak frequency in Eq. 15 depends on \( \chi \) (the number of \( \epsilon \)-folds per stage), \( \epsilon \) (energy difference between any two vacua), and (very weakly) \( N \) (the total number of minima); while the GW energy density in Eq. 15 depends on \( \chi \) and (again very weakly) \( N \). Since the peak frequency and energy density scale exponentially with \( -\chi N \) where \( \chi N \gg 1 \), the \( N \)-dependence can be ignored. The largest GW amplitude is found for the largest allowed value of \( \chi \), i.e., \( \chi = \chi_e = 1/3 \). For the remainder of this paragraph we investigate this case and vary \( \epsilon \). The GW amplitude remains the same for all values of \( \epsilon \), and the spectrum just shifts to a different frequency. For \( 10^{7.5} \text{ GeV} \lesssim \epsilon^{1/4} < 10^{9.5} \text{ GeV} \), at least a part of the spectrum from bubble collisions falls into the advanced LIGO sensitivity region. For values of \( 10^{9.2} \text{ GeV} \lesssim \epsilon \lesssim 10^{7.5} \text{ GeV} \), the peak frequency shifts to the BBO region. Finally, for \( 10^{1.5} \text{ GeV} \lesssim \epsilon^{1/4} \lesssim 10^{5.2} \text{ GeV} \), the peak frequency shifts to LISA region. The gravitational wave signature of chain inflation from bubble collision cannot be detected in these experiments for \( \epsilon^{1/4} \lesssim 10 \text{ GeV} \) and \( \chi = 1/3 \). Alternatively, if we follow the results of [17], scales as low as \( \epsilon^{1/4} \sim \text{few } 0.01 \text{ GeV} \) for \( \chi = 1/3 \) should be detectable. For \( \chi = 1/3 \) and \( \epsilon^{1/4} \ll \text{few eV} \), the amplitude of the spectrum from bubble collisions on large \( \sim \text{horizon} \) scales becomes bigger than the observational bound from WMAP, \( \Omega_{\text{GW}} h^2 < 10^{-15} \). This puts a lower bound on the value of \( \epsilon^{1/4} \gtrsim 10 \text{ eV} \), assuming that the nucleation rate takes the critical value, \( \gamma = 0.017 \).

One can investigate the dependence of the GW on the value of \( \chi \). We assumed so far that the nucleation rate is...
at the critical value so that \( \chi = 1/3 \). For smaller values of \( \chi \), the peak frequency shifts to higher values whereas the amplitude of gravitational waves decreases. Since the peak frequency of the spectrum occurs at \( f_{N-1} \), for a fixed value of \( \epsilon \), the peak frequency varies like \( \frac{1}{\chi(1-e^{-\chi})} \).

The dependence of the amplitude of the spectrum on \( \gamma \) is more involved, but as the amplitude at the peak frequency is \( \propto \Omega_b h^2 (f_{N-1}) \), one would expect that the amplitude would depend on \( \chi \) as \( \chi^2 (1-e^{-\chi})^4 \). One should note that for values of \( \chi \) that do not satisfy the condition \( N_\chi > 1 \), the dependence of the peak frequency and its amplitude on \( \chi \) are respectively given as \( \frac{1-e^{-N\chi}}{\chi(1-e^{-\chi})} \) and \( \frac{\chi^2 (1-e^{-\chi})^4}{(1-e^{-\chi})} \).

**Spectrum of GWs from Quantum Fluctuations:**

In addition to GW from bubble collisions, chain inflation, like any other inflationary model, can produce a tensor spectrum from quantum fluctuations of the space-time. The amplitude of such quantum fluctuations is proportional to the overall amplitude of the potential, \( V \):

\[
P_T(k) = \frac{2}{3\pi^2 M_P^2} \frac{V}{M_P^2} \left( \frac{H}{2\pi} \right)^2,
\]

where the expression should be calculated at the moment of horizon-crossing. The relevant scale for comparison with CMB experiments corresponds to \( \sim N_\epsilon \) e-foldings before the end of inflation, where \( N_\epsilon \) is the minimum number of e-foldings required to solve the horizon problem (\( N_\epsilon \sim 60 \) for GUT scale inflation); we take the corresponding potential height to be \( V_\star \). \( \Omega_Q h^2 \) can be related to the amplitude of primordial tensor perturbations through [10]:

\[
\Omega_Q h^2 = A_{GW} P_T = A_{GW} r P_S,
\]

where \( A_{GW} = 2.74 \times 10^{-6} \left( \frac{100}{\Omega} \right)^{1/3} \) and \( P_S = 2.45 \times 10^{-9} \), at \( k_* = 0.002 \text{Mpc}^{-1} \) and WMAP+BAO+SN constrain \( r < 0.22 \) (95\% C.L.) [28]. In chain inflation, the slope of the tensor power spectrum, \( n_T \) relates to \( r \) through the relation

\[
r = -\frac{8n_T}{3}.
\]

which is different from the well-known consistency relation \( r = -8n_T \) for standard single-field slow-roll inflation (n.b., a different setup that violates the consistency relation is described in [24]). The extra factor of 3 in the denominator of the right hand side (RHS) is due to the fact that in chain inflation, the scalar spectrum is [10]

\[
P_S = \frac{H^2}{8\pi^2 M_P^2 \epsilon'}
\]

where \( \epsilon' \) is the equivalent of the slow-roll parameter as defined in [10]. Thus for a given \( r \), the tensor spectrum of chain inflation which arises from quantum fluctuations is redder with respect to its slow-roll counterpart. For example, for \( r = 0.01 \), one finds \( \Omega_Q h^2 = 6.58 \times 10^{-17} \) at \( k_* = 0.002 \text{Mpc}^{-1} \) and \( n_T = -3.75 \times 10^{-3} \) for chain inflation. The total gravity spectrum from chain inflation will be

\[
\Omega_{tot} h^2 = \Omega_{hub} h^2 + \Omega_Q h^2.
\]

For values of \( r \) detectable in the coming decades, the first term on the RHS dominates on large scales while the second term dominates on small scales, as can be seen in fig. [1]. The QUIET experiment [23] or the proposed CMBPOL [7] have the capability of detecting tensor modes down to at least \( r = 0.01 \) on Hubble scales, \( k = 0.002 \text{Mpc}^{-1} \) (\( f \sim 3.09 \times 10^{-18} \text{Hz} \)).

The scale of the transition from dominance of quantum-induced GW to bubble-induced GW depends on the values of \( V_\star \) (or, equivalently, \( r \) and \( \chi \)). For \( (r = 0.01, \epsilon = 10^6 \text{GeV}, \text{and } \chi = \frac{1}{2}) \), the transition occurs at \( k_t \approx 2.5 \times 10^{12} \text{ Mpc}^{-1} \) (\( f \approx 10^{-2.4} \text{ Hz} \)). Alternatively, for \( (r = 0.01, \epsilon = 10^2 \text{GeV}, \text{and } \chi = \frac{1}{4}) \), the transition takes place at \( k_t \approx 4 \times 10^6 \text{ Mpc}^{-1} \); see fig. [1]. For \( \epsilon = 10^3 \text{ GeV}, \text{and } \chi = \frac{1}{2} \), quantum fluctuations will always dominate on large scales as long as \( r > 1.39 \times 10^{-30} \) (i.e. \( V_\star > 8.73 \times 10^{15} \text{GeV} \)) [27]. For a given \( \chi \), the peak frequency shifts to lower frequencies as \( \epsilon \) decreases, whereas the amplitude of the peak frequency remains almost constant. For \( \chi = \frac{1}{2} \) and \( \epsilon^{1/4} \leq \text{few eV} \), the amplitude of the spectrum from bubble collision becomes bigger than the observational bound, \( \Omega_{GW} h^2 < 10^{-15} \). This puts a lower bound on the value of \( \epsilon^{1/4} \gtrsim 10 \text{ eV} \), assuming that the nucleation rate takes the critical value, \( \gamma = \frac{2}{\Pi} \).

### Conclusion

In this letter, we calculated the profile of GWs generated during chain inflation. Two mechanisms are responsible for generating the signal; bubble collisions (which dominate on small scales) and quantum fluctuations (which can dominate on large scales). We found three observational signatures for chain inflation. First, the signal could become detectable in upcoming large frequency probes of gravity waves: Advanced LIGO, LISA, or BBO. In fact, wiggles in the GW spectrum due to multiple phase transitions may be a "smoking gun" at frequencies below the peak. Second, the amplitude below the peak would be different from that of a single tunneling model. Third, the consistency relation between scalar and tensor modes from quantum fluctuations is different from the standard one for slow-roll inflation and may be tested in CMB experiments.

We also wish to comment on the connection with the landscape in string theory. If inflation happened through a series of rapid tunnelings in the string landscape, this work shows that future high frequency probes of GW can shed light on the structure of the landscape. Another possibility is that the many vacua in string theory may be connected by a combination of tunneling and rolling, alternating with one another. This case may generate the same GW discussed in this paper due to bubble collisions, as long as several of the last e-folds of inflation are due to tunneling. If there is rolling near the beginning of
inflation but tunneling near the end, then the quantum fluctuations would produce the standard result while the bubble collisions would be as described in this paper. A full analysis of the possibilities of alternating rolling and tunneling warrants another paper.

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[26] Notation: henceforth we will suppress the subscript 0 referring to values at the present epoch; e.g., $f_m$ is the frequency today.
[27] In chain inflation $r$ relates to the scale of inflation through $V_r$ through $V_r^{1/4} = 1.06 \times 10^{16}$ GeV $(a_{00})^{1/4}$.