Recent advances in the theory of nuclear forces

R Machleidt¹, D R Entem²

¹ Department of Physics, University of Idaho, Moscow, Idaho 83844, U. S. A.
² Nuclear Physics Group, University of Salamanca, E-37008 Salamanca, Spain

E-mail: machleid@uidaho.edu

Abstract. After a brief historical review, we present recent progress in our understanding of nuclear forces in terms of chiral effective field theory.

1. Historical perspective

The theory of nuclear forces has a long history (cf. table 1). Based upon the seminal idea by Hideki Yukawa [1], first field-theoretic attempts to derive the nucleon-nucleon (NN) interaction focused on pion-exchange. While the one-pion exchange turned out to be very useful in explaining NN scattering data and the properties of the deuteron [2], multi-pion exchange was beset with serious ambiguities [3, 4]. Thus, the “pion theories” of the 1950s are generally judged as failures—for reasons we understand today: pion dynamics is constrained by chiral symmetry, a crucial point that was unknown in the 1950s.

Historically, the experimental discovery of heavy mesons [5] in the early 1960s saved the situation. The one-boson-exchange (OBE) model [6, 7] emerged which is still the most economical and quantitative phenomenology for describing the nuclear force [8, 9]. The weak point of this model, however, is the scalar-isoscalar “sigma” or “epsilon” boson, for which the empirical evidence remains controversial. Since this boson is associated with the correlated (or resonant) exchange of two pions, a vast theoretical effort that occupied more than a decade was launched to derive the $2\pi$-exchange contribution of the nuclear force, which creates the intermediate range attraction. For this, dispersion theory as well as field theory were invoked producing the Paris [10, 11] and the Bonn [7, 12] potentials.

The nuclear force problem appeared to be solved; however, with the discovery of quantum chromo-dynamics (QCD), all “meson theories” were relegated to models and the attempts to derive the nuclear force started all over again.

The problem with a derivation from QCD is that this theory is non-perturbative in the low-energy regime characteristic of nuclear physics, which makes direct solutions impossible. Therefore, during the first round of new attempts, QCD-inspired quark models [13] became popular. These models were able to reproduce qualitatively some of the gross features of the nuclear force. However, on a critical note, it has been pointed out that these quark-based approaches were nothing but another set of models and, thus, did not represent any fundamental progress. Equally well, one may then stay with the simpler and much more quantitative meson models.

A major breakthrough occurred when the concept of an effective field theory (EFT) was introduced and applied to low-energy QCD. As outlined by Weinberg in a seminal paper [14],...
Table 1. Seven Decades of Struggle: The Theory of Nuclear Forces

| Year   | Theory and Milestones |
|--------|------------------------|
| 1935   | Yukawa: Meson Theory   |
| 1950’s | One-Pion Exchange: o.k.|
|        | Multi-Pion Exchange: disaster |
| 1960’s | Many pions ≡ multi-pion resonances: \( \sigma, \rho, \omega, \ldots \) |
| 1970’s | The One-Boson-Exchange Model |
|        | Refine meson theory:    |
| 1980’s | \( QCD \)               |
|        | Quark Cluster Models    |
| 1990’s | Nuclear physicists discover \( EFT \) |
|        | Weinberg, van Kolck     |
| and beyond | Back to Meson Theory! |
|        | \( But, with Chiral Symmetry \) |

One has to write down the most general Lagrangian consistent with the assumed symmetry principles, particularly the (broken) chiral symmetry of \( QCD \). At low energy, the effective degrees of freedom are pions and nucleons rather than quarks and gluons; heavy mesons and nucleon resonances are “integrated out”. So, the circle of history is closing and we are back to Yukawa’s meson theory, except that we have learned to add one important refinement to the theory: broken chiral symmetry is a crucial constraint that generates and controls the dynamics and establishes a clear connection with the underlying theory, \( QCD \).

It is the purpose of the remainder of this paper to describe the EFT approach to nuclear forces in more detail.

2. Chiral perturbation theory and the hierarchy of nuclear forces

The chiral effective Lagrangian is given by an infinite series of terms with increasing number of derivatives and/or nucleon fields, with the dependence of each term on the pion field prescribed by the rules of broken chiral symmetry. Applying this Lagrangian to NN scattering generates an unlimited number of Feynman diagrams. However, Weinberg showed [15] that a systematic expansion exists in terms of \( (Q/\Lambda_{\chi})^\nu \), where \( Q \) denotes a momentum or pion mass, \( \Lambda_{\chi} \approx 1 \text{ GeV} \) is the chiral symmetry breaking scale, and \( \nu \geq 0 \) (cf. figure 1). This has become known as chiral perturbation theory (\( \chi PT \)). For a given order \( \nu \), the number of terms is finite and calculable; these terms are uniquely defined and the prediction at each order is model-independent. By going to higher orders, the amplitude can be calculated to any desired accuracy.

Following the first initiative by Weinberg [15], pioneering work was performed by Ordóñez, Ray, and van Kolck [16, 17] who constructed a NN potential in coordinate space based upon \( \chi PT \) at next-to-next-to-leading order (NNLO; \( \nu = 3 \)). The results were encouraging and many researchers became attracted to the new field. Kaiser, Brockmann, and Weise [18] presented the first model-independent prediction for the NN amplitudes of peripheral partial waves at NNLO.
Epelbaum *et al.* [19] developed the first momentum-space NN potential at NNLO, and Entem and Machleidt [20] presented the first potential at N$^3$LO ($\nu = 4$).

In $\chi$PT, the NN amplitude is uniquely determined by two classes of contributions: contact terms and pion-exchange diagrams. There are two contacts of order $Q^0$ [$O(Q^0)$] represented by the four-nucleon graph with a small-dot vertex shown in the first row of figure 1. The corresponding graph in the second row, four nucleon legs and a solid square, represent the seven contact terms of $O(Q^2)$. Finally, at $O(Q^4)$, we have 15 contact contributions represented by a four-nucleon graph with a solid diamond.

Now, turning to the pion contributions: At leading order [LO, $O(Q^0)$, $\nu = 0$], there is only the well-known static one-pion exchange, second diagram in the first row of figure 1. Two-pion exchange (TPE) starts at next-to-leading order (NLO, $\nu = 2$) and all diagrams of this leading-order two-pion exchange are shown. Further TPE contributions occur in any higher order. Of this sub-leading TPE, we show only two representative diagrams at NNLO and three diagrams at N$^3$LO. While TPE at NNLO was known for a while [16, 18, 19], TPE at N$^3$LO has been calculated only recently by Kaiser [21]. All $2\pi$ exchange diagrams/contributions up to N$^3$LO are summarized in a pedagogical and systematic fashion in Ref. [22] where the model-independent results for NN scattering in peripheral partial waves are also shown.

Finally, there is also three-pion exchange, which shows up for the first time at N$^3$LO (two loops; one representative $3\pi$ diagram is included in figure 1). In Ref. [23], it was demonstrated that the $3\pi$ contribution at this order is negligible.

One important advantage of $\chi$PT is that it makes specific predictions also for many-body forces. For a given order of $\chi$PT, 2NF, 3NF, . . . are generated on the same footing (cf. figure 1).

### Figure 1. Hierarchy of nuclear forces in $\chi$PT. Solid lines represent nucleons and dashed lines pions. Further explanations are given in the text.
Table 2. $\chi^2$/datum for the reproduction of the 1999 $np$ database below 290 MeV by various $np$ potentials. ($\Lambda = 500$ MeV in all chiral potentials.)

| Bin (MeV) | # of data | $N^3$LO$^a$ | NNLO$^b$ | NLO$^b$ | AV18$^c$ |
|-----------|-----------|-------------|-----------|---------|---------|
| 0–100     | 1058      | 1.06        | 1.71      | 5.20    | 0.95    |
| 100–190   | 501       | 1.08        | 12.9      | 49.3    | 1.10    |
| 190–290   | 843       | 1.15        | 19.2      | 68.3    | 1.11    |
| 0–290     | 2402      | 1.10        | 10.1      | 36.2    | 1.04    |

$^a$Reference [20]. $^b$Reference [26]. $^c$Reference [27].

At LO, there are no 3NF, and at next-to-leading order (NLO), all 3NF terms cancel [15, 24]. However, at NNLO and higher orders, well-defined, nonvanishing 3NF occur [24, 25]. Since 3NF show up for the first time at NNLO, they are weak. Four-nucleon forces (4NF) occur first at $N^3$LO and, therefore, they are even weaker.

3. Chiral NN potentials

The two-nucleon system is non-perturbative as evidenced by the presence of shallow bound states and large scattering lengths. Weinberg [15] showed that the strong enhancement of the scattering amplitude arises from purely nucleonic intermediate states. He therefore suggested to use perturbation theory to calculate the NN potential and to apply this potential in a scattering equation (Lippmann-Schwinger or Schrödinger equation) to obtain the NN amplitude. We follow this philosophy.

Chiral perturbation theory is a low-momentum expansion. It is valid only for momenta $Q \ll \Lambda _\chi \approx 1$ GeV. Therefore, when a potential is constructed, all expressions (contacts and irreducible pion exchanges) are multiplied with a regulator function,

$$\exp \left[ - \left( \frac{p}{\Lambda} \right)^{2n} - \left( \frac{p'}{\Lambda} \right)^{2n} \right],$$

where $p$ and $p'$ denote, respectively, the magnitudes of the initial and final nucleon momenta in the center-of-mass frame; and $\Lambda \ll \Lambda _\chi$. The exponent $2n$ is to be chosen such that the regulator generates powers which are beyond the order at which the calculation is conducted.

The $\chi^2$/datum for the fit of the $np$ data below 290 MeV is shown in table 2, and the corresponding one for $pp$ is given in table 3. The $\chi^2$ tables show the quantitative improvement.

Table 3. $\chi^2$/datum for the reproduction of the 1999 $pp$ database below 290 MeV by various $pp$ potentials. ($\Lambda = 500$ MeV in all chiral potentials.)

| Bin (MeV) | # of data | $N^3$LO$^a$ | NNLO$^b$ | NLO$^b$ | AV18$^c$ |
|-----------|-----------|-------------|-----------|---------|---------|
| 0–100     | 795       | 1.05        | 6.66      | 57.8    | 0.96    |
| 100–190   | 411       | 1.50        | 28.3      | 62.0    | 1.31    |
| 190–290   | 851       | 1.93        | 66.8      | 111.6   | 1.82    |
| 0–290     | 2057      | 1.50        | 35.4      | 80.1    | 1.38    |

$^a$Reference [20]. $^b$See footnote [28]. $^c$Reference [27].
Figure 2. \( np \) phase parameters below 300 MeV lab. energy for partial waves with \( J \leq 2 \). The thick solid (dashed) line is the result by Entem and Machleidt [20] at \( N^3\text{LO} \) using \( \Lambda = 500 \) MeV (\( \Lambda = 600 \) MeV). The thin dotted and dash-dotted lines are the phase shifts at NLO and NNLO, respectively, as obtained by Epelbaum et al. [26] using \( \Lambda = 500 \) MeV. The solid dots show the Nijmegen multienergy \( np \) phase shift analysis [29], and the open circles are the GWU/VPI single-energy \( np \) analysis SM99 [30].

of the NN interaction order by order in a dramatic way. Even though there is considerable improvement when going from NLO to NNLO, it is clearly seen that \( N^3\text{LO} \) is needed to achieve an accuracy comparable to the phenomenological high-precision Argonne \( V_{18} \) potential [27]. Note that proton-proton data have, in general, smaller errors than \( np \) data which explains why the \( pp \) \( \chi^2 \) are always larger.

The phase shifts for \( np \) scattering below 300 MeV lab. energy are displayed in figure 2. What the \( \chi^2 \) tables revealed, can be seen graphically in this figure. The \( ^3P_2 \) phase shifts are a particularly good example: NLO (dotted line) is clearly poor. NNLO (dash-dotted line) brings improvement and describes the data up to about 100 MeV. The difference between the NLO and NNLO curves is representative for the uncertainty at NLO and, similarly, the difference between NNLO and \( N^3\text{LO} \) reflects the uncertainty at NNLO. Obviously, at \( N^3\text{LO} \) (\( \Lambda = 500 \) MeV, thick solid line) we have a good description up to 300 MeV. An idea of the uncertainty at \( N^3\text{LO} \) can be obtained by varying the cutoff parameter \( \Lambda \). The thick dashed line is \( N^3\text{LO} \) using \( \Lambda = 600 \) MeV. In most cases, the latter two curves are not distinguishable on the scale of the figures. Noticeable differences occur only in \( ^1D_2 \), \( ^3F_2 \), and \( \epsilon_2 \) above 200 MeV.

4. Conclusions
The EFT approach to nuclear forces is a modern refinement of Yukawa’s meson theory. It represents a scheme that has an intimate relationship with QCD and allows to calculate nuclear forces to any desired accuracy. Moreover, nuclear two- and many-body forces are generated on the same footing.

At \( N^3\text{LO} \) [20], the accuracy is achieved that is necessary and sufficient for reliable nuclear
structure calculations. First calculations applying the N$^3$LO potential have produced promising results [31, 32, 33, 34, 35, 36].

The 3NF at NNLO is known [25] and has had first successful applications [31]. The 3NF and 4NF at N$^3$LO is presently under construction.

In summary, the stage is set for many years of exciting nuclear structure research that is more consistent than anything we had before.

Acknowledgments
This work was supported by the U.S. National Science Foundation under Grant No. PHY-0099444, by the Spanish Ministerio de Ciencia y Tecnología under Contract No. BFM2001-3563, and the Junta de Castilla y León under Contract No. SA-104/04.

References
[1] Yukawa H 1935 Proc. Phys. Math. Soc. Japan 17 48
[2] 1956 Prog. Theor. Phys. (Kyoto) Supplement 3
[3] Taketani M et al. 1952 Prog. Theor. Phys. (Kyoto) 7 45
[4] Brueckner K A et al. 1953 Phys. Rev. 90 699
Brueckner K A et al. 1953 Phys. Rev. 92, 1023
[5] Erwin A R et al. 1961 Phys. Rev. Lett. 6 628
Maglić B C et al. 1961 Phys. Rev. Lett. 7, 178
[6] 1967 Prog. Theor. Phys. (Kyoto) Supplement 39
Bryan R A and Scott B L 1969 Phys. Rev. 177 1435
[7] Machleidt R 1989 Adv. Nucl. Phys. 19 189
[8] Stoks V G J et al. 1994 Phys. Rev. C 49 2950
Machleidt R 2001 Phys. Rev. C 63 024001
[9] R. Vinh Mau R 1979 Mesons in Nuclei, Vol. I, ed M Rho and D H Wilkinson (North-Holland, Amsterdam) p 151
[10] Lacombe M et al. 1980 Phys. Rev. C 21 861
[11] Machleidt R et al. 1987 Phys. Rep. 149 1
[12] Myhrer F et al. 1988 Rev. Mod. Phys. 60 629
[13] Weinberg S 1979 Physica 96A 327
[14] Weinberg S 1991 Nucl. Phys. B363 3
[15] Ordóñez C, Ray L and van Kolck U 1996 Phys. Rev. C 53 2086
[16] van Kolck U 1999 Prog. Part. Nucl. Phys. 43 337
[17] Kaiser N et al. 1997 Nucl. Phys. A625 758
[18] Epelbaum E et al. 2000 Nucl. Phys. A671 295
[19] Entem D R and Machleidt R 2003 Phys. Rev. C 68 041001
[20] Kaiser N 2001 Phys. Rev. C 64 057001
Kaiser N 2002 Phys. Rev. C 65 017001
[21] Entem D R and Machleidt R 2002 Phys. Rev. C 66 014002
[22] Kaiser N 1999 Phys. Rev. C 61 014003
Kaiser N 2000 Phys. Rev. C 62 024001
[23] van Kolck U 1994 Phys. Rev. C 49 2932
[24] Epelbaum E et al. 2002 Phys. Rev. C 66 064001
[25] Epelbaum E et al. 2002 Eur. Phys. J. A15 543
[26] Wiringa R B, Stoks V G J and Schiavilla R 1995 Phys. Rev. C 51 38
[27] Since Ref. [26] provides only the np versions of the NLO and NNLO potentials, we have constructed the pp versions by incorporating charge-dependence and minimizing the pp $\chi^2$.
[28] Stoks V G J, Klomp R A M, Rentmeester M C M and de Swart J J 1993 Phys. Rev. C 48 792
[29] SAID, solution SM99 (Summer 1999).
[30] Nogga A et al. 2004 Nucl. Phys. A737 236
[31] Kowalski K, Dean D J, Hjorth-Jensen M and Papenbrock T 2004 Phys. Rev. Lett. 92 132501
[32] Dean D J and Hjorth-Jensen M 2004 Phys. Rev. C 69 054320
[33] Navrátil P and Caurier E 2003 arXiv:nucl-th/0311036
[34] Forssen C, Navrátil P, Ormand W E and Caurier E 2004 arXiv:nucl-th/0412049
[35] Fujii S, Okamoto R and Suzuki K 2004 Phys. Rev. C 69 034328