Neutrino Scattering Off Spin-Polarized Particles in Supernovae

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Abstract

We calculate neutrino and anti-neutrino scattering off electrons and nucleons in supernovae using a detailed Monte Carlo transport code incorporating realistic equations of state. The goal is to determine whether particles in a neutron star core, partially spin-aligned by the local magnetic field, can give rise to asymmetric neutrino emission via the standard parity breaking weak force. We conclude that electron scattering in a very high magnetic field does indeed give a net asymmetry, but that nucleon scattering, if present, removes the asymmetry.

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PSR 2224+65 has a measured transverse speed $\geq 800$ km/s, indicating that it has enough kinetic energy to leave the gravitational potential well of our galaxy, which requires a speed of 450 km/s [2]. It now appears that many pulsars have abnormally high proper speeds [3]. Since neutrino emission carries off 99% of the binding energy of the magnetized neutron star/pulsar formed in supernova explosions, it is particularly attractive to consider models of asymmetric neutrino core emission as the source of the pulsar kick, since then only a small fraction of $O(1\%)$ of that energy is required to be asymmetrically radiated. In this regard, several authors have proposed mechanisms to accomplish this. Bisnovatyi-Kogan [4] considers toroidal magnetic field configurations in a differentially rotating star. In the absence of dissipative processes, the neutron star returns to the state of rigid rotation by loosening the induced higher moment fields via magnetorotational explosion. However, it is not very clear how this gives rise to asymmetric neutrino luminosities. Kusenko and Segrè [5], using the approximation of the last scattering surface method, propose that neutrino oscillations induced by non-zero neutrino masses alter the shape of the neutrinosphere, and derive a necessary tau neutrino mass of $\sim 100$ eV. Grasso et al [6] propose neutrino asymmetries without neutrino mass requirements, based on weak violation of universality. We want to study the possibility of explaining the effect within the Standard Model, without relying on last scattering assumptions. Instead, we simulate the complete process of neutrino birth and transport within the core. We find that the last scattering surface assumption does not reproduce the neutrino transport properties. While this paper was being prepared, Lai and Qian [7] have considered the possibility that spin aligned nucleons could lead to asymmetric neutrino emission. We, in fact, find just the opposite: nucleons are parasitic and it is the electrons which have the potential to lead to asymmetric emission.

The spin-aligned fraction, $\eta \equiv [N(+) - N(-)]/[N(+)+N(-)]$, for non-relativistic non-degenerate particles (i.e. nucleons in proto-neutron star...
cores) with magnetic moment $\mu_M$ in a magnetic field $B$ at temperature $T$ is

$$\eta = \tanh \frac{\mu_M B}{kT},$$

(1)

while for relativistic ideal degenerate particles (i.e. electrons in neutron star cores) (with $T \ll E_F$, $E_F$ = Fermi energy),

$$\eta = \frac{(E_F + \mu_M B)^3 - (E_F - \mu_M B)^3}{(E_F + \mu_M B)^3 + (E_F - \mu_M B)^3}.$$  

(2)

A cooling proto-neutron star core at $T = 5 \text{ MeV}$, $B = 10^{14}$ Gauss, with baryon density of $5 \times 10^{38}$ baryons/cm$^3$, 3\% of it as protons, has $\eta_{\text{neutron}} = 1.2 \times 10^{-4}$ and $\eta_{\text{electron}} = .011$. Thus nucleons are hardly aligned at all, and electrons, due to the relativistic degeneracy, have also a rather small spin alignment. Only for gigantic fields $B \sim 2000$ GT will the ideal gas electrons be highly spin aligned in neutron stars. It is possible, however, that electrons do not form an ideal relativistic degenerate gas within neutron stars, but instead become nearly a perfect ferromagnetic spin aligned fluid, with a nearest neighbor Hamiltonian $\sum_{i,j} \vec{s}_i \cdot \vec{s}_j h_{i,j}$ with $h_{i,j} < 0$. This may happen since parallel spin alignments will tend to reduce the Coulomb repulsion between electrons, a major energy consideration at these high number densities. If such an electron ferromagnetic fluid were present, then electron spin alignment would be nearly 100\% even for $10^{14}$ Gauss. In our simulation, we just use an ideal electron gas, disregarding possible ferromagnetic effects.

There has been some recent discussion in the literature indicating that no asymmetry can arise from neutrino multiple scattering if the neutrinos are in thermal equilibrium $[8]$. We fully agree with this statement, although it is not the case we are considering here. The neutrinos that produce the asymmetry do not reach thermal equilibrium, due to a non-negligible neutrino mean free path comparable to the temperature gradient scale. In fact, too little or too many collisions, leads to zero neutrino asymmetry, but we find that a moderate number of collisions per neutrino of $\sim 4000$ leads to the effects reported here.
The probability $P(\theta_4, \phi_4)d\theta_4d\phi_4$ that a neutrino or anti-neutrino will scatter into the solid angle $d\Omega_4 = \sin \theta_4 d\theta_4 d\phi_4$ is

$$P(\theta_4, \phi_4)d\theta_4d\phi_4 = \frac{|M(p_1, p_2, p_3, p_4)|^2 \sin \theta_4}{\int |M(p_1, p_2, p_3, p_4)|^2 d\Omega_4} d\theta_4 d\phi_4,$$

where $M(p_1, p_2, p_3, p_4)$ is the scattering amplitude for a (anti) neutrino with incoming momentum $p_2$ and outgoing momentum $p_4$, off a target with initial and final momenta $p_1$ and $p_3$ respectively. For scattering of neutrinos off electrons, the spin-dependent matrix element squared is

$$|M(e_1\nu_2 \rightarrow e_3\nu_4)|^2 = 16G_F^2 \left[ L_e^2 [(p_4 \cdot p_3)(p_2 \cdot p_1) - m_e(p_4 \cdot p_3)(p_2 \cdot s_1)] ight]$$

$$+ R_e^2 [(p_4 \cdot p_1)(p_3 \cdot p_2) + m_e(p_3 \cdot p_2)(p_4 \cdot s_1)]$$

$$- L_e R_e [m_e^2(p_4 \cdot p_2) + m_e(p_2 \cdot p_1)(p_4 \cdot s_1) - m_e(p_4 \cdot p_1)(p_2 \cdot s_1)]$$ (4)

where the weak couplings $L_e$ and $R_e$ are shown in Table 1, and $s_1$ is the polarization four-vector of the initial electron, which satisfies $p_1 \cdot s_1 = 0$ and $s_1 = -1$. For antineutrino scattering, the respective expression is the same as the one above after exchanging $p_2 \leftrightarrow p_4$.

For nucleon scattering, Ref. [3] has given the scattering probability

$$P(\theta_4, \phi_4)d\theta_4d\phi_4 = \frac{d\sigma}{d\Omega_4} \sin \theta_4 d\theta_4 d\phi_4,$$ (5)

where the cross section for neutrino-nucleon scattering is:

$$\frac{d\sigma}{d\Omega_4} = \frac{G_F^2 E_\nu}{4\pi^2} \left\{ c_v^2 + 3c_a^2 + (c_v^2 - c_a^2) \cos \theta + 2\eta c_a [(c_v - c_a) \cos \theta_{out} + (c_v + c_a) \cos \theta_{in}] \right\}$$

(6)

Here $\eta$ is the polarization factor of Eq. (1), $c_v$ and $c_a$ are given in Table 1, $\theta$ is the scattering angle, and $\theta_{in}$ and $\theta_{out}$ are the angles between the nucleon
spin and the incoming or outgoing neutrino momentum, respectively. In Table 2 we give the star parameter ranges used in our simulations, including the assumed large magnetic fields associated with magnestars.

We now add the astrophysics. The density of matter in the interior of neutron stars increases from \(10^{25}\) baryon/cm\(^3\) in the surface layer to \(\sim 10^{39}\) baryon/cm\(^3\) near the center. However, more than 95% of the mass of a typical neutron star is compressed to baryon densities exceeding \(10^{38}\) cm\(^{-3}\) and, to an excellent approximation, the supernova core can be described as hadronic matter at constant density. The different equations of state (EOS) give rise to different fractions of electrons present, \(f_e = n_e/n_B\), where \(n_e\) and \(n_B\) are the electron and baryon densities respectively. Thus we describe the EOS landscape by two parameters: \(n_B\) and \(f_e\). We set the mass of the core to 1.8 \(M_\odot\). The thermodynamic parameters are the temperature \(T\) and the interior magnetic field. We use constant interior magnetic fields. With respect to neutrino production, we only consider neutrino-anti-neutrino thermal pairs, as 90% of the neutrino emission is of thermal origin.

A Monte Carlo neutrino transport code for neutrino scattering off polarized targets (electrons only or both electrons and nucleons) was written to determine if the parity violation in the weak force can lead to asymmetric neutrino emission from the surface, and thus possibly explain the large proper motion of pulsars. Because of Fermi degeneracy, essentially only those electrons near the Fermi level participate in scattering. Since the electrons do not interact via the hadronic force (and the star is electrically neutral) they might be described as an ideal relativistic degenerate Fermi gas with Fermi momentum \(p_F = 6.10464 \times 10^{-11}(f_e n_B)^{1/3}\) MeV/c, with \(n_B\) in 1/cm\(^3\).

| Star Parameters         |
|-------------------------|
| \(B\) | 0.1 \(\rightarrow\) 5000 [GTesla] |
| \(T\) | 1 \(\rightarrow\) 100 [MeV]    |
| \(n_B\) | \(5 \times 10^{37} \rightarrow 1.16 \times 10^{39}\) [baryon/cm\(^3\)] |
| \(f_e\) | 0.001 \(\rightarrow\) 0.5 |

Table 2: The physical parameters involved in the asymmetry calculation.
(but see above qualifications concerning possible electron ferromagnetism).

An experimental constraint on the EOS is that the radius of the neutron star must be smaller than 14 - 16 km \[[1]\].

The simulation goes as follows: a random flavor $\nu$-$\bar{\nu}$ pair is born at a random location within the core, with a spherically random direction and each particle with an energy equal to $3T$, where $T$ is the temperature at that position in the star. A discrete temperature gradient was imposed. For this purpose, four concentric shells of inner and outer radiae $n \frac{R_{\text{star}}}{4}$ and $n + 1 \frac{R_{\text{star}}}{4}$ respectively ($n = 0, 1, 2, 3$) were considered. The temperature in each shell was set to $\frac{6-n}{3}T_s$, where $T_s$ is a temperature scale parameter. Thus for birth location in the $n$th shell the neutrino energy was $E_{\nu,\bar{\nu}} = (6 - n)T_s$.

For a uniform temperature in a constant density core, the $\nu$-$\bar{\nu}$ pair emission probability per unit volume is also uniform; the production point thus has a radius $r \sim R_{\text{star}} \times \xi^{1/3}$, where $\xi \in [0,1]$ is a uniformly distributed random variable. Instead, with negative temperature gradients (hotter center region), the emission is also higher at the center, skewing the distribution towards $r \sim R_{\text{star}} \times \xi$. We used here $r = 0.4R_{\text{star}} \times \xi$, with the phenomenological factor 0.4 to take into account $\nu$-$\bar{\nu}$ production only within the interior core.

After birth, the location of the next scattering event is statistically generated from the distribution given by the mean free path $\ell = 1/(\sum_i n_i \sigma_i)$ ($n_i$ is the $i$-type target density –electrons or nucleons– and $\sigma_i$ the cross section) \[[2]\]. If this is outside the radius, the neutrino energy and momentum components are accrued as ejected flux. If the scattering position is still within the core, a randomly directed electron (in the case of electron collisions) from the Fermi surface is picked, with its spin direction parallel or anti-parallel to the magnetic field, determined from the spin probability distribution function. The probability of parallel/antiparallel spin is defined in terms of $\eta$ from Eq. (2) as $P_\pm = (1 \pm \eta)/2$. The direction of the outgoing neutrino is then generated according to the distribution given by the differential cross section (Eq. 3) and a Pauli check is made. Then the outgoing neutrino seeks the next scattering event. This is continued until the neutrino leaves the star.
Finally, the history of the neutrino’s birth partner is followed. This is done for $N$ neutrinos.

The asymmetry is defined as the accrued ejected momentum over the accrued ejected energy:

$$\text{Asymmetry} = \sqrt{\frac{\sum p_x^2 + \sum p_y^2 + \sum p_z^2}{\sum E_i}} \rightarrow \frac{|\sum p_z|}{\sum E_i},$$  \hspace{1cm} (7)

where $z$ is the direction of the magnetic field\[13\]. We verified that the other momentum components, $\sum p_x$ and $\sum p_y$, fall within the statistical fluctuations, while $\sum p_z$ stands out as the signal. The variance of the statistical fluctuations increases as $\sim \sqrt{N}$, so the asymmetrical statistical ‘background’ decreases $\sim 1/\sqrt{N}$. To obtain a $O(1\%)$ positive signal with a $\sim .1\%$ error thus requires $N \sim 10^6$. In the actual runs, one million pairs were followed, giving $N = 2 \times 10^6$. The ‘background’ statistical asymmetry was calculated by performing a run with random spin orientation at each scattering location, which, for the samples of a million pairs gave a typical magnitude of 0.05%.

For runs including both electron and nucleon scattering, the probability of electron scattering is $P_{\text{electron}} = \Sigma_{\text{electron}}/(\Sigma_{\text{electron}} + \Sigma_{\text{proton}} + \Sigma_{\text{neutron}})$, where $\Sigma_i = n_i$ (particle density) $\times \sigma_i$ (scattering cross section). Similar probabilities exist for scattering off neutrons and protons.

The results we present in our tables and figures correspond to a neutron star with 1.8 solar masses, radius 8.3 km and electron/nucleon ratio $f_e = 0.12$. This size and structure is representative of most realistic equations of state which incorporate nuclear repulsion at short distances. Typically, the heaviest neutron star for such equations of state is over two solar masses (see \textit{e.g.} R. Bowers \textit{et al.} in Ref [10]).

Several critical details arose. Firstly, it is a highly non-trivial problem in computational physics to draw scattering samples from Eqs. (3–4). We use the ‘rejection method’\[14\], generalized to two random variables. Secondly, due to the billions of random numbers used, we discovered that a popular random number generator, Knuth’s algorithm \[15\], performed unsatisfacto-
| $T_S$ (MeV) | $B$ (GT) | target particles | Asymmetry     |
|-----------|--------|------------------|---------------|
| 10        | 200    | electrons        | $(0.18 \pm 0.05)\%$ |
| 10        | 4000   | electrons        | $(0.47 \pm 0.05)\%$ |
| 10        | 4000   | nucleons & electrons | $(0.04 \pm 0.05)\%$ |

Table 3: Asymmetry in neutrino flux for different magnetic field and scattering targets, for a neutron star of 1.8 $M_\odot$, radius 8.3 km, and electron/nucleon fraction 0.12

rily. We finally used the ‘gold-plated’ random number generator ran2, of Ref. [16], which performed without problems.

In Fig. 1 we display the results of a positive signal, corresponding to the case of extremely high magnetic field ($\sim 10^3$ GT) and scattering with the electron gas only. Subtle physics effects are at play here. First, looking at Eqs. (3–4), we see that the spin terms are multiplied by $m_e$, the electron mass; thus, it would seem (wrongly) that relativistic electrons have little parity violation. Secondly, from Fig. 1, the asymmetry increases with temperature, reaching O(1%) at $\sim 17$ MeV. These two observations are related: the magnitude of the polarization four-vector for electrons traveling parallel or anti-parallel to the magnetic field is proportional to the relativistic $\gamma$ factor and so for such electrons $m_e s \sim E$; as the temperature increases, the Pauli window increases allowing a greater energy range of allowed transitions. Thus the asymmetry is expected to be an increasing function of temperature.

Now, if we include neutrino scattering with nucleons as well as with electrons, the results change dramatically. Elastic scattering of neutrinos off hadrons occurs via the neutral current. The small magnetic moment of the nucleons gives very little polarization, so we would expect that allowance of nucleon scattering would tend to remove the neutrino emission asymmetry. Moreover, the nucleon density is much larger than the electron density, so that nucleon scattering and its effects should dominate. In Table 3, we present the results of the nucleon scattering inclusion, showing that this is indeed the case: even at extremely high magnetic fields, the inclusion of nucleon scattering washes out the asymmetry, as shown in the last row of
Table 3. Here we should recall that the statistical fluctuation of the simulations are about 0.05% in the asymmetry, which is already larger than the signal obtained.

Our numerical results support the hypothesis that neutrino scattering off spin-polarized electrons is a viable mechanism for achieving high pulsar proper velocities in the case where extremely high magnetic fields exist in the core, and whenever scattering with nucleons can be neglected, without resorting to any non-standard physics or neutrino oscillation mechanism, but if nucleon scattering is present, the asymmetry disappears. Now, since the nucleon density is necessarily larger than the electron density, to neglect nucleon scattering in favor of electron scattering does not seem plausible, unless there is an additional mechanism, like superfluid phases in the core, that could block nucleon scattering.

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[12] To incorporate the Pauli exclusion Principle, one should use the cross section which includes the Fermi factor in the final state. An analytical expression for this cross section requires a rather cumbersome calculation. Alternatively and to avoid this calculation, we simulate the exclusion within the code: (i) use the mean-free-path without Pauli blocking to obtain the next chord distance; (ii) after sampling the scattering probability distribution, if the simulated scattered electron falls outside the Fermi sphere, the event is accepted as such; otherwise, it is considered as forward scattering (i.e. the electron maintains its momentum untils the next scattering event). It is straightforward to prove the equivalence of these two treatments.
By symmetry, $z$ is the only preferential direction.

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Figure 1: Asymmetry vs. Temperature, for a star with baryon density $n_B = 8 \times 10^{38} \text{ 1/cm}^3$, radius $R_{\text{star}} = 8.3 \text{ km}$, electron fraction $f_e = 0.12$, and magnetic field $B = 4000 \text{ GTesla}$.