Multiplicity dependence of strange and multi-strange hadrons in p–p, p–Pb and Pb–Pb collisions at LHC energies using Tsallis-Weibull Formalism

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The transverse momentum ($p_T$) distribution of strange hadrons ($K^0_S$ and Λ) and multi-strange hadrons (Ξ and Ω) measured in p–p, p–Pb, and Pb–Pb collisions at LHC energies have been studied for different multiplicity classes using Tsallis-Weibull (or q–Weibull) formalism. The distribution describes the measured $p_T$ spectra for all multiplicity (or centrality) classes. The multiplicity dependence of the extracted parameters are studied for the mentioned collisions systems. The $\lambda$ parameter was observed to increase systematically with the collision multiplicity and follows a mass hierarchy for all collision system. This characteristic feature indicates that $\lambda$ can be associated to the strength of collectivity for heavy ion collisions. It can also be related to strength of dynamic effects such as multi-partonic interactions and color reconnections which mimic collectivity in smaller systems. The non-extensive $q$ parameter is found to be greater than one for all the particles suggesting that the strange particles are emitted from a source which is not fully equilibrated.

I. INTRODUCTION

The study of collisions of relativistic heavy ions at RHIC and LHC energies allow us to understand the formation of the new phase of matter created at extreme conditions of high temperature and high density. The key observables studied in heavy ion collisions to characterize the produced system are often studied in smaller collision systems such as proton-nucleus ($p$–$A$) and elementary proton-proton ($p$–$p$) collisions to disentangle the effects of initial and final stages of the collision. The recent observations in high multiplicity $p$–$p$ and $p$–$Pb$ collisions at LHC energies such as the enhanced production of strange particles, the long-range correlations ("ridge" structure) in the near-side region of the two-particle correlations, hardening of spectra of identified particles which become more pronounced for massive particles and few more are reminiscent of creation of a deconfined medium as produced in $A$–$A$ collisions.

One of the proposed signatures of the formation of this novel state is the enhanced production of strange and multi-strange hadrons. The strange valence quarks are predominantly produced in the parton-parton scattering processes (like flavor creation and flavor excitation) as they are not present in the initial state of colliding nuclei. The abundance of strange partons in the deconfined medium results in the production of more strange particles compared to $p$–$p$ collisions. The yield of strange particles in central and mid-central collisions has been well described by statistical hadronization models assuming a grand canonical ensemble approach. However, in smaller systems like $p$–$p$ collisions, the relative abundance of strange hadrons is reduced compared to $A$–$A$ collisions. This observation was attributed to the effects of canonical suppression which imposes a local conservation of strangeness quantum number in a canonical ensemble formalism while producing strange hadrons. However, this mechanism could not explain the strangeness production in peripheral collisions at RHIC where a smaller system is expected to be produced. The system produced in $p$–$A$ collisions can be considered to be intermediate between the heavy-ion and the $p$–$p$ collisions in terms of particle multiplicity and volume of the system. Therefore, the study and comparison of particle production in these systems is crucial in order to interpret the effects of canonical formalism and hadronization in general. The recent observation of strangeness enhancement in high multiplicity $p$–$Pb$ and $p$–$p$ collisions indicated towards the progressive lifting of canonical suppression with increase in particle multiplicity. Therefore, it is worth investigating the transverse momentum ($p_T$) distribution of the produced strange hadrons created in such collisions within different statistical frameworks to understand the strange particle production as the integrated effects of dynamics of particle production from initial stages of collision leaves its footprints in the $p_T$ spectra. Generally, Tsallis statistics is used to describe the $p_T$ distribution in $p$–$p$ collisions and the flow parameter is introduced in

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Tsallis distribution to explain the hadronic spectra in heavy ion collisions to account effects of collectivity[10][13]. Recently it was shown that the Tsallis–Weibull (or q–Weibull) distribution described the $p_T$ spectra in $p\text{--}p$, $p\rightarrow A$ and $A\rightarrow A$ collisions for the measured $p_T$ range[14]. In this work, the Tsallis–Weibull ($q$–Weibull) distribution has been used to describe the multiplicity dependence of strange and multi-strange hadron production in $p\text{--}p$, $p\rightarrow Pb$ and $Pb\rightarrow Pb$ collisions at LHC energies. The parameters which characterize the distribution are studied as a function of charged particle density and centrality to understand their evolution with collisions dynamics. This study allows to extend the $q$–Weibull formalism to identified strange particles in order to gain a clear physical interpretation of the associated parameters.

II. THE Q-WIEBULL DISTRIBUTION

The natural evolving processes where the dynamic evolution is governed by fragmentation and sequential branching are described by Weibull distribution[15][16]. The production of hadrons in non-perturbative domain has an inherent cascade branching fragmentation and therefore Weibull distribution successfully described the multiplicity distribution of charged particles in hadron–hadron and lepton collisions[17][18]. The Weibull-distribution is incorporated with Tsallis statistics to obtain q–Weibull distribution.

Mathematically, the q–Weibull distribution is obtained by replacing the exponential factor in Weibull distribution by its equivalent q-exponential[19].

$$P_q(x; q, \lambda, k) = \frac{k}{\lambda} \left( \frac{x}{\lambda} \right)^{k-1} e_q\left( \frac{x}{\lambda} \right)$$

where

$$e_q\left( \frac{x}{\lambda} \right) = (1 - (1 - q)\left( \frac{x}{\lambda} \right)^k)^{\frac{1}{1-q}}$$

As observed in previous analysis of $p_T$ spectra of charged particles in heavy ion collisions[14], the value of the non-extensive $q$ parameter is related to the deviation of the system from thermal equilibrium. In general, $q > 1$ value is attributed to the presence of intrinsic fluctuations in the system and depends upon the observable being measured. The $\lambda$ parameter can be associated with the mean $p_T$ in hadronic collisions or the collective expansion velocity of hadrons in heavy ion collisions. The $k$ parameter can be related to system dynamics associated with the type of collisions.

III. ANALYSIS AND DISCUSSION

In a previous study[14], the q-Weibull distribution was successful in explaining the $p_T$ distribution of charged particles in $p\text{--}p$, $p\rightarrow A$, $A\rightarrow A$ collisions for the measured $p_T$ range for most of the beam energies studied. In this work, the analysis has been extended to identified strange and multi-strange hadrons in these collision systems. The $p_T$ distribution of strange and multi-strange hadrons in different multiplicity classes were fitted with q-Weibull function in $p\rightarrow p$, $p\rightarrow Pb$[2, 21], and $Pb\rightarrow Pb$[22][23] collisions at beam energies of 7 TeV, 5.02 TeV, and 2.76 TeV, respectively as measured by ALICE experiment.

Figure 1 shows the invariant yield of $K^0_S$, $\Lambda^0$, $\Xi^\pm$ and $\Omega^\pm$ as a function of $p_T$ for $p\rightarrow p$ collisions at $\sqrt{s} = 7$ TeV for ten different multiplicity classes. The definition of multiplicity classes and the associated values of $(dN_{ch}/d\eta)_{\eta<0.5}$ based on measurement by ALICE experiment[10] is shown in Table I. The spectra is observed to be nicely described by q–Weibull function for all multiplicity classes as shown by solid lines in the figure. The evolution of values of the extracted fit parameters with respect to the mean charged particle density $(\langle dN_{ch}/d\eta]_{\eta<0.5})$ corresponding to different multiplicity classes is shown in Figure 2. The values of the extracted fit parameters together with the $\chi^2/ndf$ are shown in Table II. The $q$ parameter is observed to be greater than one for the measured strange and multi-strange hadrons for all multiplicity classes. These values are consistent with the non-equilibrium scenario present in $p\rightarrow p$ collisions. The $\lambda$ parameter which is related to the mean $p_T$ shows a gradual decrease from higher multiplicity classes to lower ones. It can also be noted that the values are higher for massive hadrons compared to the less massive ones. This observation seems interesting and is reminiscent of the onset of collective motion as observed in heavy ion collisions. However, other mechanisms like increase of multipartonic interactions and color reconnections in final stages as predicted by fragmentation models[24][26] can play an important role in this observation. The $k$ value which was predicted to be related to the dynamical processes contributing to nonequilibrium scenario (dominant in initial stages) is almost constant for the selected hadrons. However,
the values are systematically higher for events with higher multiplicity.

Figure 3 shows the description of $p_T$ spectra for strange and multi-strange hadrons in $p$–$Pb$ collisions at 5.02 TeV [2, 21] for different centrality classes. The variation of $q$-Weibull parameters as a function of $\langle dN_{ch}/d\eta \rangle$ is shown in Fig 4. The $p_T$-distribution of $K^0_S$, $\Lambda^0$, $\Xi^-$ and $\Omega^-$ in $Pb$–$Pb$ collisions at 2.76 TeV [22] in different centrality classes is shown in Fig 5. Figure 6 shows the centrality ($N_{part}$, number of participating nucleons in a collision) dependence of parameters for different strange hadrons. Table I and Table II shows the value of $\langle dN_{ch}/d\eta \rangle_{\eta<0.5}$ for different centrality classes. The $\langle dN_{ch}/d\eta \rangle$ values are also consistently higher for massive hadrons in a common radial flow velocity (as per transverse expansion scenario in hydrodynamic evolution) which increases with centrality and shows a mass hierarchy. This observation is important and thus $\lambda$ can be related to collective velocity of particles. The parameter $k$ shows a slight decrease as we move from central to peripheral collisions in both $Pb$–$Pb$ and $p$–$Pb$ collisions. This indicates that the value of $k$ parameter increases with onset of processes leading to non-equilibrium conditions. The deviation from local equilibrium can be quantified by the non-extensive parameter, $q$. The $q$ values show a deviation from one which decreases slightly from central to peripheral collisions. A detailed investigation of other identified particles in terms of system size and beam energies is required to obtain a clear and consistent interpretation of the parameters.

### IV. SUMMARY

The study of production of strange and multi-strange particles in different systems have become quite relevant with the recent observation of strangeness enhancement in high multiplicity $p$–$p$ collisions at LHC energies. The $q$–Weibull statistics has been applied for the first time to comprehend the $p_T$ distribution of strange and multi-strange hadrons in different colliding systems ($p$–$Pb$, $p$–$Pb$ and $Pb$–$Pb$) for a broad range of multiplicity classes. The $q$-Weibull function successfully describes the $p_T$ distribution for all ranges of $p_T$ measured. The evolution of the parameters of the distribution extracted from the fit were studied as a function of multiplicity for all the collision systems. The $\lambda$ parameter which was conjectured to be associated with the collective expansion velocity was observed to systematically increase with the collision centrality in $Pb$–$Pb$ and $p$–$Pb$ collisions. A mass hierarchy was also observed i.e. the particles with higher mass had larger values of $\lambda$. These features are consistent with the presence of collectivity (radial flow) in the medium formed and hence one can attribute $\lambda$ parameter to the collective velocity. A similar behavior for the variation of $\lambda$ with multiplicity was also observed for $p$–$p$ collisions. For $p$–$p$ collisions, this parameter can be related to strength of other mechanisms like multi-partonic interactions and color reconnections which mimic features of collectivity. The $q$ parameter which is related to the degree of deviation from conditions of thermal equilibrium was almost a constant with respect to centrality for all collision systems studied. The values consistently deviated from one for all the collision systems and were also higher for $Pb$–$Pb$ and $p$–$Pb$ collisions indicating that the system from which the strange hadrons are emitted is not fully equilibrated. The values of $k$ shows a slight variation with centrality where the values decrease while moving from high multiplicity to lower ones for certain particles. It can be related to certain dynamical features of collision which indicates the presence of processes which are dominant in initial stages of collision.

| Multiplicity class | $\langle dN_{ch}/d\eta \rangle$ |
|-------------------|---------------------------------|
| I                 | 21.5 ± 0.6                      |
| II                | 16.5 ± 0.5                      |
| III               | 13.5 ± 0.4                      |
| IV                | 11.5 ± 0.3                      |
| V                 | 10.1 ± 0.3                      |
| VI                | 8.45 ± 0.25                     |
| VII               | 6.72 ± 0.21                     |
| VIII              | 5.40 ± 0.17                     |
| IX                | 3.90 ± 0.14                     |
| X                 | 2.26 ± 0.12                     |

TABLE I. The values of $\langle dN_{ch}/d\eta \rangle_{\eta<0.5}$ for different centrality classes for $p$–$p$ collisions at $\sqrt{s} = 7$ TeV [1].
FIG. 1. (Color online) $p_T$ distribution of strange and multi-strange hadrons in $p$–$p$ collisions at $\sqrt{s} = 7$ TeV for $|\eta| < 0.8$ as measured by ALICE experiment at LHC [3]. The solid lines are the q-Weibull fits to the data points. The data points are properly scaled for visibility.

V. ACKNOWLEDGEMENT

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| $(dN_{ch}/d\eta)$ | $k$   | $\lambda$ | $q$  | $\chi^2/ndf$ |
|-----------------|-------|---------|------|-------------|
| $K_S^0$         | 21.3 ± 0.6 | 1.84 ± 0.02 | 0.68 ± 0.007 | 1.31 ± 0.005 | 0.8 |
|                 | 16.5 ± 0.5 | 1.80 ± 0.02 | 0.65 ± 0.007 | 1.31 ± 0.005 | 0.54 |
|                 | 13.5 ± 0.4 | 1.81 ± 0.02 | 0.64 ± 0.006 | 1.31 ± 0.004 | 1.3 |
|                 | 11.5 ± 0.3 | 1.79 ± 0.02 | 0.62 ± 0.006 | 1.30 ± 0.004 | 1.65 |
|                 | 10.1 ± 0.3 | 1.79 ± 0.02 | 0.61 ± 0.006 | 1.30 ± 0.004 | 2.03 |
|                 | 8.45 ± 0.25 | 1.80 ± 0.02 | 0.58 ± 0.005 | 1.30 ± 0.004 | 1.09 |
|                 | 6.72 ± 0.21 | 1.77 ± 0.02 | 0.55 ± 0.005 | 1.30 ± 0.004 | 0.69 |
|                 | 5.40 ± 0.17 | 1.78 ± 0.02 | 0.54 ± 0.001 | 1.32 ± 0.001 | 1.7 |
|                 | 3.90 ± 0.14 | 1.79 ± 0.02 | 0.51 ± 0.005 | 1.30 ± 0.005 | 0.9 |
|                 | 2.26 ± 0.12 | 1.73 ± 0.03 | 0.48 ± 0.006 | 1.27 ± 0.005 | 2.13 |
| $\Lambda + \bar{\Lambda}$ | 21.3 ± 0.6 | 1.99 ± 0.08 | 1.08 ± 0.02 | 1.26 ± 0.019 | 0.18 |
|                 | 16.5 ± 0.5 | 1.95 ± 0.08 | 1.00 ± 0.018 | 1.25 ± 0.02 | 0.15 |
|                 | 13.5 ± 0.4 | 1.91 ± 0.08 | 0.95 ± 0.017 | 1.25 ± 0.02 | 0.10 |
|                 | 11.5 ± 0.3 | 1.93 ± 0.08 | 0.91 ± 0.017 | 1.25 ± 0.02 | 0.14 |
|                 | 10.1 ± 0.3 | 1.89 ± 0.08 | 0.88 ± 0.017 | 1.25 ± 0.02 | 0.16 |
|                 | 8.45 ± 0.25 | 1.89 ± 0.09 | 0.85 ± 0.016 | 1.25 ± 0.02 | 0.24 |
|                 | 6.72 ± 0.21 | 1.91 ± 0.09 | 0.79 ± 0.016 | 1.26 ± 0.02 | 0.17 |
|                 | 5.40 ± 0.17 | 1.81 ± 0.09 | 0.75 ± 0.003 | 1.24 ± 0.02 | 1.56 |
|                 | 3.90 ± 0.14 | 1.92 ± 0.11 | 0.70 ± 0.02 | 1.26 ± 0.02 | 0.40 |
|                 | 2.26 ± 0.12 | 1.97 ± 0.16 | 0.60 ± 0.03 | 1.28 ± 0.02 | 0.43 |
| $\Xi^− + \Xi^+$ | 21.3 ± 0.6 | 1.96±0.12 | 1.29±0.03 | 1.23±0.03 | 0.74 |
|                 | 16.5 ± 0.5 | 2.05±0.13 | 1.19±0.02 | 1.26±0.03 | 0.38 |
|                 | 13.5 ± 0.4 | 1.83±0.13 | 1.11±0.02 | 1.21±0.03 | 0.53 |
|                 | 11.5 ± 0.3 | 1.90±0.13 | 1.08±0.025 | 1.23±0.03 | 0.17 |
|                 | 10.1 ± 0.3 | 1.97±0.14 | 1.04±0.026 | 1.25±0.03 | 0.16 |
|                 | 8.45 ± 0.25 | 1.89±0.14 | 1.0±0.02 | 1.23±0.03 | 0.35 |
|                 | 6.72 ± 0.21 | 1.91±0.14 | 0.96±0.03 | 1.23±0.03 | 0.39 |
|                 | 5.40 ± 0.17 | 1.85±0.11 | 0.91±0.02 | 1.23±0.027 | 1.10 |
|                 | 3.90 ± 0.14 | 1.97±0.19 | 0.83±0.04 | 1.26±0.034 | 0.68 |
|                 | 2.26 ± 0.12 | 1.83±0.27 | 0.70±0.07 | 1.24±0.04 | 0.34 |
| $\Omega^− + \Omega^+$ | 18.9 ± 0.6 | 1.98±0.8 | 1.54±0.1 | 1.196±0.03 | 0.74 |
|                 | 12.5 ± 0.5 | 1.90±0.7 | 1.38±0.09 | 1.2±0.03 | 0.38 |
|                 | 9.3 ± 0.4 | 1.86±0.8 | 1.37±0.09 | 1.15±0.03 | 0.53 |
|                 | 6.06 ± 0.3 | 1.87±0.13 | 1.03±0.01 | 1.24±0.03 | 0.17 |
|                 | 3.08 ± 0.3 | 1.87±0.14 | 0.84±0.026 | 1.27±0.03 | 0.16 |

TABLE II. The values of $k$, $\lambda$, $q$ and $\chi^2/ndf$ obtained from the fits of $p_T$ distributions using $q$-Weibull function in p–p collisions at $\sqrt{s} = 7$ TeV for different multiplicity classes as measured by ALICE experiment at LHC [12,13].

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| Centrality(%) | $\langle dN_{ch}/d\eta \rangle$ | $k$ | $\lambda$ | $q$ | $\chi^2/ndf$ |
|--------------|-------------------------------|-----|--------|-----|-------------|
| $K_S^0$      | 0-5% 45 ± 1                  | 1.32 ± 0.03 | 0.42 ± 0.007 | 1.17 ± 0.008 | 0.4 |
|              | 5-10% 36.2 ± 0.8              | 1.30 ± 0.03 | 0.41 ± 0.007 | 1.17 ± 0.008 | 0.5 |
|              | 10-20% 30.5 ± 0.7             | 1.32 ± 0.03 | 0.39 ± 0.007 | 1.18 ± 0.008 | 0.3 |
|              | 20-40% 23.2 ± 0.5             | 1.3 ± 0.03  | 0.37 ± 0.007 | 1.18 ± 0.009 | 0.2 |
|              | 40-60% 16.1 ± 0.4             | 1.3 ± 0.03  | 0.35 ± 0.007 | 1.18± ± 0.009 | 0.1 |
|              | 60-80% 9.8 ± 0.2              | 1.29 ± 0.03 | 0.32 ± 0.007 | 1.18 ± 0.009 | 0.1 |
|              | 80-100% 4.4 ± 0.1             | 1.34± ± 0.04 | 0.29 ± 0.007 | 1.20 ± 0.009 | 0.3 |
| $\Lambda + \bar{\Lambda}$ | 45 ± 1 0-5% | 1.79 ± 0.10 | 0.88 ± 0.03 | 1.19 ± 0.02 | 0.1 |
|              | 5-10% 36.2 ± 0.8              | 1.77 ± 0.11 | 0.86 ± 0.03 | 1.25 ± 0.02 | 0.1 |
|              | 10-20% 30.5 ± 0.7             | 1.74 ± 0.11 | 0.83 ± 0.03 | 1.25 ± 0.02 | 0.05 |
|              | 20-40% 23.2 ± 0.5             | 1.69 ± 0.12 | 0.77 ± 0.03 | 1.25 ± 0.023 | 0.05 |
|              | 40-60% 16.1 ± 0.4             | 1.68 ± 0.12 | 0.71 ± 0.03 | 1.25 ± 0.02 | 0.1 |
|              | 60-80% 9.8 ± 0.24             | 1.62 ± 0.14 | 0.62 ± 0.04 | 1.25 ± 0.03 | 0.1 |
|              | 80-100% 4.4 ± 0.1             | 1.60 ± 0.19 | 0.52 ± 0.03 | 1.26 ± 0.03 | 0.2 |
| $\Xi^- + \Xi^+$ | 0-5% 45 ± 1 | 1.65± ± 0.09 | 1.03± ± 0.03 | 1.13± ± 0.02 | 0.3 |
|              | 5-10% 36.2 ± 0.8              | 1.65± ± 0.09 | 0.98± ± 0.03 | 1.15± ± 0.02 | 0.5 |
|              | 10-20% 30.5 ± 0.7             | 1.68± ± 0.09 | 0.97± ± 0.02 | 1.15± ± 0.02 | 0.2 |
|              | 20-40% 23.2 ± 0.5             | 1.65± ± 0.09 | 0.91± ± 0.02 | 1.16± ± 0.02 | 0.1 |
|              | 40-60% 16.1 ± 0.4             | 1.60± ± 0.11 | 0.82± ± 0.03 | 1.16± ± 0.02 | 0.3 |
|              | 60-80% 9.8 ± 0.2              | 1.62± ± 0.12 | 0.75± ± 0.034 | 1.18± ± 0.02 | 0.1 |
|              | 80-100% 4.4 ± 0.1             | 1.60± ± 0.19 | 0.52± ± 0.03 | 1.26± ± 0.03 | 0.2 |
| $\Omega^- + \Omega^+$ | 0-5% 45 ± 1 | 1.68± ± 0.2 | 1.22± ± 0.08 | 1.13± ± 0.03 | 0.6 |
|              | 5-10% 36.2 ± 0.8              | 1.66± ± 0.3 | 1.25± ± 0.08 | 1.12± ± 0.03 | 0.4 |
|              | 10-20% 30.5 ± 0.7             | 1.64± ± 0.3 | 1.23± ± 0.08 | 1.09± ± 0.03 | 0.2 |
|              | 20-40% 23.2 ± 0.5             | 1.60± ± 0.1 | 1.1± ± 0.08 | 1.13± ± 0.03 | 0.1 |
|              | 40-60% 16.1 ± 0.4             | 1.60± ± 0.1 | 1.06± ± 0.03 | 1.12± ± 0.03 | 1.6 |
|              | 60-80% 9.8 ± 0.2              | 2.69± ± 0.7 | 0.94± ± 0.03 | 1.13± ± 0.03 | 0.5 |
|              | 80-100% 4.4 ± 0.1             | 2.25± ± 0.1 | 0.68± ± 0.03 | 1.37± ± 0.03 | 0.16 |

TABLE III. The values of $k$, $\lambda$, $q$ and $\chi^2/ndf$ obtained from the fits of $p_T$ distributions using q-Weibull function in p–Pb collisions for different centrality classes at $\sqrt{s_{NN}} = 5.02$ TeV as measured by ALICE experiment at LHC [221].

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FIG. 2. (Color online) Variation of $k$ (Upper panel), \(\lambda\) (middle panel) and \(q\) (Lower panel) as a function of \(\langle dN_{ch}/d\eta \rangle\) for p–p collisions at \(\sqrt{s} = 7\) TeV.
FIG. 3. (Color online) $p_T$ distribution of strange and multi-strange hadrons in different multiplicity classes for p–Pb collisions at 5.02 TeV for $|\eta| < 0.8$ as measured by ALICE experiment at LHC [2, 21]. The solid lines are the $q$-Weibull fits to the data points. The data points are properly scaled for visibility.
FIG. 4. (Color online) Variation of $k$ (Upper panel),
$\lambda$ (middle panel) and $q$ (Lower panel) as a function of
$\langle dN_{ch}/d\eta \rangle$ for p–Pb collisions at $\sqrt{s} = 5.02$ TeV
FIG. 5. (Color online) $p_T$ distribution of strange and multi-strange hadrons for various centrality classes in Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV for $|\eta| < 0.8$ as measured by ALICE experiment at LHC [22, 23]. The solid lines are the $q$-Weibull fits to the data points. The data points are properly scaled for visibility.
FIG. 6. (Color online) Variation of $k$ (Upper panel), $\lambda$ (middle panel) and $q$ (Lower panel) as a function of $\langle N_{\text{part}} \rangle$ for Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV.
| Centrality(%) | $\langle N_{\text{part}} \rangle$ | $k$       | $\lambda$ | $q$     | $\chi^2/\text{ndf}$ |
|--------------|----------------|-----------|-----------|--------|----------------------|
| $K_S^0$      |                |           |           |        |                      |
| 0-5%         | 382.7 $\pm$ 3.0 | 2.46 $\pm$ 0.05 | 0.76 $\pm$ 0.01 | 1.33 $\pm$ 0.01 | 1.8 |
| 5-10%        | 329.4 $\pm$ 4.3 | 2.43 $\pm$ 0.05 | 0.75 $\pm$ 0.01 | 1.32 $\pm$ 0.01 | 1.5 |
| 10-20%       | 260.1 $\pm$ 3.8 | 2.39 $\pm$ 0.05 | 0.74 $\pm$ 0.01 | 1.32 $\pm$ 0.01 | 2.1 |
| 20-40%       | 157.2 $\pm$ 3.1 | 2.27 $\pm$ 0.05 | 0.71 $\pm$ 0.01 | 1.32 $\pm$ 0.01 | 1.2 |
| 40-60%       | 68.6 $\pm$ 2.0  | 2.07 $\pm$ 0.05 | 0.66 $\pm$ 0.01 | 1.30 $\pm$ 0.01 | 1.1 |
| 60-80%       | 22.3 $\pm$ 0.8  | 1.80 $\pm$ 0.05 | 0.57 $\pm$ 0.01 | 1.28 $\pm$ 0.01 | 0.5 |
| 80-90%       | 6.4 $\pm$ 4.2   | 1.61 $\pm$ 0.06 | 0.51 $\pm$ 0.01 | 1.26 $\pm$ 0.01 | 1.3 |
| $\Lambda + \bar{\Lambda}$ |                |           |           |        |                      |
| 0-5%         | 382.7 $\pm$ 3.0 | 2.73 $\pm$ 0.07 | 1.39 $\pm$ 0.02 | 1.26 $\pm$ 0.01 | 1.2 |
| 5-10%        | 329.4 $\pm$ 4.3 | 2.75 $\pm$ 0.07 | 1.36 $\pm$ 0.02 | 1.27 $\pm$ 0.02 | 1.3 |
| 10-20%       | 260.1 $\pm$ 3.8 | 2.70 $\pm$ 0.08 | 1.33 $\pm$ 0.02 | 1.27 $\pm$ 0.02 | 1.1 |
| 20-40%       | 157.2 $\pm$ 3.1 | 2.63 $\pm$ 0.08 | 1.26 $\pm$ 0.02 | 1.28 $\pm$ 0.02 | 1.1 |
| 40-60%       | 68.6 $\pm$ 2.0  | 2.47 $\pm$ 0.08 | 1.14 $\pm$ 0.02 | 1.28 $\pm$ 0.02 | 0.5 |
| 60-80%       | 22.3 $\pm$ 0.8  | 2.39 $\pm$ 0.09 | 1.00 $\pm$ 0.02 | 1.30 $\pm$ 0.02 | 0.9 |
| 80-90%       | 6.4 $\pm$ 4.2   | 1.84 $\pm$ 0.13 | 0.85 $\pm$ 0.03 | 1.21 $\pm$ 0.02 | 0.1 |
| $\Xi^- + \bar{\Xi}^-$ |               |           |           |        |                      |
| 0-10%        | 356.1 $\pm$ 3.6 | 2.64 $\pm$ 0.05 | 1.50 $\pm$ 0.01 | 1.22 $\pm$ 0.006 | 1.4 |
| 10-20%       | 260.1 $\pm$ 3.9 | 2.28 $\pm$ 0.04 | 1.46 $\pm$ 0.01 | 1.17 $\pm$ 0.006 | 1.8 |
| 20-40%       | 157.2 $\pm$ 3.1 | 2.31 $\pm$ 0.05 | 1.37 $\pm$ 0.02 | 1.21 $\pm$ 0.007 | 2.1 |
| 40-60%       | 68.6 $\pm$ 2.0  | 2.16 $\pm$ 0.05 | 1.30 $\pm$ 0.01 | 1.19 $\pm$ 0.01 | 0.7 |
| 60-80%       | 22.3 $\pm$ 0.8  | 2.13 $\pm$ 0.07 | 1.17 $\pm$ 0.01 | 1.21 $\pm$ 0.02 | 1.6 |
| $\Omega^- + \Omega^+$ |             |           |           |        |                      |
| 0-10%        | 356.1 $\pm$ 3.6 | 2.71 $\pm$ 0.18 | 1.67 $\pm$ 0.03 | 1.25 $\pm$ 0.03 | 1.1 |
| 10-20%       | 260.2 $\pm$ 3.9 | 2.36 $\pm$ 0.16 | 1.74 $\pm$ 0.03 | 1.17 $\pm$ 0.03 | 1.2 |
| 20-40%       | 157.2 $\pm$ 3.1 | 2.04 $\pm$ 0.16 | 1.62 $\pm$ 0.041 | 1.12 $\pm$ 0.03 | 1.3 |
| 40-60%       | 68.9 $\pm$ 2.0  | 2.41 $\pm$ 0.28 | 1.43 $\pm$ 0.043 | 1.24 $\pm$ 0.04 | 2.1 |
| 60-80%       | 22.3 $\pm$ 0.8  | 1.95 $\pm$ 0.32 | 1.41 $\pm$ 0.056 | 1.13 $\pm$ 0.05 | 2.6 |

TABLE IV. The values of $k$, $\lambda$, $q$ and $\chi^2/\text{ndf}$ obtained from the fits of $p_T$ distributions using $q$-Weibull function in Pb–Pb collisions for different multiplicity classes at $\sqrt{s_{NN}} = 2.76$ TeV as measured by ALICE experiment at LHC\textsuperscript{22, 23}.