Mathematical modelling to predict the roughness average in micro milling process

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Abstract. Surface roughness plays a very important role in micro milling process and in any machining process, because indicates the state of the machined surface. Many surface roughness parameters that can be used to analyse a surface, but the most common surface roughness parameter used is the average roughness ($R_a$). This paper presents the experimental results obtained at micro milling of the C45W steel and the ways to determine the $R_a$ parameter with respect to the working conditions. The chemical characteristics of the material were determined from a spectral analysis, chemical composition was measured at one point and two points, graphical and tabular. A profilometer Surtronic 3+ was used to examine the surface roughness profiles; the effect of independent parameters can be investigated and can get a proper relationship between the $R_a$ parameter and the process variables. The mathematical model were developed, using multiple regression method with four independent variables $D$, $v$, $a_p$, $f_z$; the analysis was done using statistical software SPSS. The ANOVA analysis of variance and the $F$-test was used to justify the accuracy of the mathematical model. The multiple regression method was used to determine the correlation between a criterion variable and the predictor variables. The prediction model can be used for micro milling process optimization.

1. Introduction

Micromilling process is one of the most modern methods of manufacture because it gathers all the claims of accuracy and ultra-accuracy, layout, shape, etc. The micro milling process is particularly attractive because of its relatively large material removal rates compared to e.g. EDM and LBM, and its flexibility in producing different component sizes, shapes, features, and the ability to machine a variety of materials, including most metals and plastics. It is suitable for machining complex 3D micro-structures with high aspect ratios, and is mentioned specifically for its applicability for the fabrication of moulds for micro-forming processes [1].

In manufacturing industries, manufacturers focused on the quality and productivity of the product. To increase the productivity of the product, computer numerically machine tools have been implemented during the past decades. Surface roughness is one of the most important parameters to determine the quality of product. The surface roughness has an important property in any machining process and in micro milling it is really critical as the product needs to be of a very high surface quality [2].

A large number of studies were reported concerning analytical and mechanical models for the prediction of the micro milling surface roughness. After assessing comprehensive studies previously made by fellow researchers, Jie Yi et al. [3] summarized their findings and conclusions. Lou et al. examined the multiple regression models for finished surface prediction [4]. Taguchi statistical...
method was used by Yang and Chen [5] and Ghani et al. [6] for optimum milling parameters. Recently, Ben Fredj et al. developed surface roughness prediction model using design of experiment method and the neural network [7]. Thepsonthi and Özel [8] studied the chip flow and tool wear of Ti-6Al-4V titanium alloy in micro-end milling. Kopač et al. used signal-to-noise response method to analyze the surface roughness with turning [9]; Benardos and Vosniakos introduced the prediction method to predict surface roughness in machining [10]. Kiswanto et al. [11] researched about the effect of spindle speed, feed rate, and machining time to the surface roughness and burr formation of aluminum alloy 1100 in micro milling operation. Kant and Sangwan [12] used artificial neural network coupled with Genetic Algorithm for predictive modelling and optimization of machining parameters to minimize surface roughness. Campatelli et al. [13] used the response surface method to optimize the process parameters for minimizing power consumption in the milling of carbon steel. The research work mentioned above by Jie Yi et al. [3] used the conventional machine tool.

This paper presents the authors’ contribution regarding a mathematical model has been developed to evaluate the surface quality due to predict the surface roughness of micro milling C45W steel. This study uses the roughness average $R_a$, the most common surface roughness parameter, to evaluate surface roughness. The mathematical model was developed using response surface with four independent variables: $D$, $v$, $a_p$, $f_z$; the analysis was done using statistical software SPSS and Data Fit. The analysis of variance (ANOVA) is used to check the adequacy of the regression model and $F$-test is used to find the significant parameters. The results can be used in further research in order to increase the surface quality and the micro milling process productivity.

2. Methodology and experimental conditions

In this study, the material used for the experiments is C45W steel, which has the following applications: holder blocks for injection molding, mould and toolmaking attachments, P-Plates, bolsters, tools for blow mouldings. The chemical composition is given in table 1, respectively figure 1. The chemical characteristics of the material were determined from a spectral analysis; chemical composition was measured at one point and two points, graphical and tabular [14].

| Element       | AN series | Net         | [wt.%]  | [norm. wt. %] | [norm. at.%] | Error in % |
|---------------|-----------|-------------|---------|---------------|--------------|------------|
| Iron          | 26        | K-series    | 139642  | 92.02499      | 94.36551     | 2.359766   |
| Carbon        | 6         | K-series    | 2302    | 3.880144      | 3.97883      | 0.736494   |
| Manganese     | 25        | K-series    | 2120    | 1.088467      | 1.116151     | 0.985998   |
| Silicon       | 14        | K-series    | 560     | 0.526129      | 0.539511     | 0.932275   |
|               | Sum:      |             | 97.51973| 100           | 100          |            |

Figure 1. The spectral analysis of C45W steel – graphical values. [14]
The workpiece dimensions have 104.88 x 37.41 x 44.05 [mm]. The experiment is performed by using a CNC milling machine Concept Mill 55, with number of tool stations 8, tool swivel arm traverse speed 10 m/min (0.39 ipm), three-phase asynchronous motor, power rating 0.75 kW, speed range (infinitely variable) 150 – 3500 min-1, working feed and rapid traverse in X/Y/Z axes 0 – 2 m/min (0-78.74 ipm). The cutting tools that have been used are cylinder-frontal mills made of CMS, with three different diameters (ϕ=4, ϕ=8, ϕ=12 mm), produced by Gühring.

A total of 12 experiments were designed for study the effects of the input parameters on the surface roughness. Range of input parameters (various values for the cutting speed, depth of cut, feed per tooth, and three values for the mill diameter) were selected on the basis of the preliminary investigations. The output variable, the roughness average \( R_a \), was measured by Surtronic 3+ instrument, produced by Rank Taylor Hobson. This a contact diamond stylus profiler witch is moved across the peaks and valleys of the surface to be measured.

Table 2 presents the working parameters variation and the results of the measurements for the processed surfaces roughness.

| Exp. run | D [mm] | \( v_c \) [m/min] | \( a_p \) [mm] | \( f_z \) [mm/tooth] | \( R_a \) [\( \mu \)m] |
|----------|--------|-----------------|----------------|------------------|------------------|
| 1        | 4      | 25              | 0.15           | 0.12             | 1.47             |
| 2        | 4      | 25              | 0.10           | 0.12             | 1.66             |
| 3        | 4      | 20              | 0.15           | 0.15             | 1.39             |
| 4        | 4      | 20              | 0.10           | 0.15             | 1.54             |
| 5        | 8      | 50              | 0.15           | 0.12             | 0.36             |
| 6        | 8      | 40              | 0.15           | 0.15             | 0.33             |
| 7        | 8      | 50              | 0.10           | 0.12             | 0.33             |
| 8        | 8      | 40              | 0.10           | 0.15             | 0.38             |
| 9        | 12     | 75              | 0.15           | 0.06             | 2.40             |
| 10       | 12     | 60              | 0.15           | 0.075            | 2.29             |
| 11       | 12     | 75              | 0.10           | 0.06             | 2.80             |
| 12       | 12     | 60              | 0.10           | 0.075            | 2.46             |

The collected data were analysed using the multiple regression method, a collection of mathematical and statistical techniques which explores the linear correlations between two or more inputs or causes (independent variables) and a single output (dependent variable). A process contains the predicted surface, in our case \( R_a \), which depends on the input factors-tool diameter, cutting speed, cutting depth, feed per tooth, will affect the value of the surface roughness.

The relationship between surface roughness and machining parameters is modelled as follows:

\[
R_a = a_0 \cdot D_f^{a_2} \cdot v^{a_3} \cdot a_p^{a_4} \cdot f_z^{a_5}
\]  

(1)

The equation (1) has been linearized by using the logarithm.

\[
\lg R_a = \lg a_0 + a_1 \cdot \lg D_f + a_2 \cdot \lg v + a_3 \cdot \lg a_p + a_4 \cdot \lg f_z
\]  

(2)
After logarithm, the equation can be represented as follows:

\[ Y = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 \]  

(3)

Where \( Y \) is the predicted of the dependent variable (\( R_a \)), \( x_1, \ldots, x_4 \) are the independent variables, \( b_0 \) is the \( Y \) value when all of the independent variables are equal to zero, \( b_1, \ldots, b_4 \) are the regression coefficients estimated.

The coefficients and the \( b_0 \) value can be determined by the multiple linear regression analysis (we choose the backward elimination method), in SPSS software, ver. 13.0.

3. Results and discussions

The results of the experiment are shown in table 2. Four parameters (tool diameter, cutting speed, cutting depth, feed per tooth) are considered in this experiment. The experimental data obtained were introduced into PC and processed with the help of the SPSS ver. 13.0 software.

The collected data were analysed using parametric analysis of variance (ANOVA) to test the significance of the regression model and test the significant of the individual model coefficients.

The statistical model is shown in table 3.

**Table 3.** The model statistics summary.

| Model | R    | R²   | Adjusted R² | Std. Error of Estimate |
|-------|------|------|-------------|------------------------|
| 1     | .998 | .997 | .995        | .06223867              |

a Predictors: (constant), \( f_z, a_p, D_f, v \).

The model shows the high value of Adjusted \( R^2 \), 0.995 which meant that the correlation coefficient between the observed value of the dependent variable and the predicted value was high. The statistical model had a small value of standard error of the estimate, which was 0.6223867.

Table 4 shows the ANOVA test summary; the value of F is 538.083 and the significance value of F is zero, which is less than the critical value 5% (0.05), that indicate the prediction model has significant state, fitting well with the actual situation.

**Table 4.** ANOVA table for surface roughness.

| Model      | Sum of Squares | DF | Mean Square | F-value | Sig. |
|------------|----------------|----|-------------|---------|------|
| Regression | 8.337          | 4  | 2.084       | 538.083 | .00* |
| 1          | .027           | 7  | .004        |         |      |
| Residual   |                | 7  |             |         |      |
| Total      | 8.367          | 11 |             |         |      |

a Predictors: (constant), \( f_z, a_p, D_f, v \).

Table 5 presents the independent variables coefficients, in the B column. The B column show of how strongly each variable influences the surface roughness. The higher the beta value, the impact of the predictor variable on surface roughness is the greater.

By using the coefficients from table 5, have been obtained the multiple regression equation (the theoretical-experimental model):

\[ R_a = 4.108 \cdot D_f^{1.761} \cdot v^{-3.875} \cdot a_p^{0.207} \cdot f_z^{4.064} \]  

(4)

4
The deviation between predicted surface roughness and measured surface roughness values was within an error of 12.5%. The maximum errors value was for the experiment run 7 as shown in figure 2.

![Figure 2. Relative error in experimental points.](image)

The parameters coefficients give us an image of which parameter influence the most on the \( R_a \) parameter value. Taking into consideration the model obtained for the \( R_a \) and studying the size of the exponents and the values of Pearson coefficients correlation (table 6), we observed that the feed per tooth \( f_z \), has the most influence on the \( R_a \), with Pearson correlation coefficient 0.664 and significance value 0.009, followed in descending order by the cutting speed \( v \), tool diameter \( D_f \) and depth of cut \( a_p \).

### Table 6. Partial correlation matrix for experimental results.

| Pearson coefficient | Ra | Df | v | a_p | f_z |
|---------------------|----|----|---|-----|-----|
| Ra                  | 1.000 | .095 | .098 | -0.050 | -0.664 |
| Df                  | .095 | 1.000 | .971 | .000 | -.740 |
| v                   | .098 | .971 | 1.000 | .000 | -.795 |
| a_p                 | -.050 | .000 | .000 | 1.000 | .000 |
| f_z                 | -.664 | -.740 | -.795 | .000 | 1.000 |
| Sig.                | .385 | .381 | .438 | .009 |
| Ra                  | .385 | .381 | .000 | .500 | .003 |
| Df                  | .000 | .500 | .500 |
| v                   | .500 | .500 |
| a_p                 | .500 | .500 |
| f_z                 | .500 | .500 |
| Exp. run            | 12  | 12  | 12  | 12  | 12  |
The figure 3 presents the response surface graphs in experimental points and the response surface, where surface roughness is plotted versus the levels of tool diameter and feed per tooth, with a constant value of cutting speed \( v = 50 \text{ m/min} \) and depth of cut \( a_p = 0.1 \text{ mm} \), is presents in figure 4.

![Figure 3. The response surface for the experimental points.](image)

![Figure 4. The response surface for the regression model.](image)

In this case, the regression equation of response surface is:

\[
R_u = 5.39 + 1.49 \cdot D_f + 0.099 \cdot D_f^2 + 4.267 \cdot f_z
\]  

(5)

The coefficient of determination is 0.998, very satisfactory which mean the model adequacy is performed. It can be seen that the feed per tooth is a dominant influential factor on surface roughness; for this model, the tool diameter has a slight influence.

4. Conclusions

In this paper have been examined the influence of cutting speed, feed per tooth, tool diameter and depth of cut on the micro milling surface roughness.

Experiments have been performed on C45W steel, by using a CNC milling machine Concept Mill 55.

The research proposed the multiple regression method for predicts surface roughness.

The experimental data were analysed using parametric analysis of variance (ANOVA) to test the significance of the regression model; the prediction model has significant state.

The deviation between predicted surface roughness and measured surface roughness values was within an error of 12.5%.

The order of influence of the working parameters for the surface roughness is: feed per tooth \( f_z \), with the most influence, followed in descending order by the cutting speed \( v \), tool diameter \( D_f \) and depth of cut \( a_p \).

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