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To cite this article: V I Petrenko et al 2018 IOP Conf. Ser.: Mater. Sci. Eng. 450 042016

View the article online for updates and enhancements.
The method of forming a geometric solution of the inverse kinematics problem for chains with kinematic pairs of rotational type only

V I Petrenko, F B Tebueva, M M Gyrchinsky, V O Antonov and J A Shutova

Institute of information technologies and telecommunications, North-Caucasus Federal University, 1 Pushkina street, Stavropol, 355009, Russia

Abstract. The goal of the article is to provide researchers with a method that allows to form a geometric solution to the inverse kinematics problem for any kinematic chain containing kinematic pairs of rotational type only by performing a specified sequence of simple steps. The developed method is based on the Denavit-Hartenberg representation and analytic geometry. The input data for the geometric solution of the inverse kinematics problem obtained with its help are the Cartesian coordinates of the chain nodes. The essence of the method is to identify typical modules in the configuration of the kinematic chain and to apply the derived formulas for calculating the generalized coordinates.

1. Introduction

Robotic systems are mainly designed to function in a volumetric space, so many tasks, such as planning a path and trajectories, as well as some types of optimization, are more convenient to solve in Cartesian coordinates [1-2]. However, the management of robotic systems is carried out in generalized coordinates, so the necessary aspect is the application of solutions of the inverse kinematics problem. There are various ways to solve the inverse problem of kinematics. Conventionally, they can be divided into numerical [3-7] and analytical [8-12]. Numerical methods for solving the inverse problem of kinematics make it possible to obtain solutions that are optimal with given criteria with an acceptable degree of accuracy, but their main drawback is the high computational complexity of the algorithms for their implementation. Analytical methods have low computational complexity, but they are applicable only to a small number of specific tasks, and they also require a considerable amount of time, due to the lack of universal methods for their formation.

The availability of the developed method would significantly reduce the time required to develop an analytical solution to the inverse kinematics problem. The purpose of this article is to develop a complete method for forming a solution of the inverse kinematics problem for kinematic chains, including joints of the rotational type only.

2. Problem statement

As an example, we will consider an anthropomorphic manipulator with seven degrees of mobility. The kinematic scheme of this manipulator is shown in figure 1. Figure 2 shows the coordinate systems associated with the links.
We will introduce the concept of a nodal point. Nodal points will be understood as the points lying at the beginning of one or several coordinate systems associated with links. Let us denote the nodes as $N_i$, where $i = 0, 1, 2, \ldots$. Numbering is performed from the base of the chain to its end. For the kinematic chain shown in figures 1 and 2, the nodal points are as follows: $N_0$ - coinciding with the origin $O_0 – O_2$ (link 1 has zero length); $N_1$ - coinciding with the origin of coordinates $O_3 – O_4$, etc.

We will also introduce the concept of a module. Module is an integral part of the considered kinematic chain. The modules are numbered from 1 to $n – 1$, where $n$ is the number of chain nodes. Modules are introduced sequentially, starting from the base, and include kinematic pairs and links that simultaneously satisfy the following conditions:

1. Kinematic pairs and links of the $i$-th module lie up to the $i$-th nodal point, counted from the base.
2. A change in the generalized coordinates of the kinematic pairs of the $i$-th module leads to a change in the Cartesian coordinates of the $i$-th nodal point.
3. Kinematic pairs and links of the $i$-th module do not belong to the previous modules.

Let’s call the node point $N_{i-1}$ as initial for the $i$-th module, and a $N_i$ will be final.

For the considered kinematic chain of an anthropomorphic manipulator, three modules can be distinguished:

1. the first module consisting of joints 1 and 2, as well as links 1, 2 and 3;
2. the second module consisting of joints 3 and 4, as well as links 4 and 5;
3. the third module, consisting of joints 5, 6 and 7, as well as links 6 and 7.

If the module has two adjacent links, and the joint connecting them does not belong (for example, links 2 and 3 belong to the first module, and joint 3 does not), when considering this module, the links are merged into one.

We will also introduce the following notations. $N_i^j$ is point radius vector of $N_i$ in $j$-th coordinate system. $N_i^{jx}, N_i^{jy}, N_i^{jz}$ are radius vector projections of $N_i^j$ on axis $x, y, z$ of $j$-th coordinate system. $\text{atan2}(x, y)$ is arctangent function of two arguments, taking into account the quadrant of the angular argument:

$$\text{atan2}(x, y) = \begin{cases} \arctg(y/x), & \text{если } x \geq 0, y \geq 0; \\ \arctg(y/x), & \text{если } x < 0, y < 0; \\ \arctg(y/x) - \pi, & \text{если } x < 0, y > 0; \\ \arctg(y/x) + \pi, & \text{если } x > 0, y > 0. \end{cases} \quad (1)$$
The input data for the method being developed are the Denavit-Hartenberg parameters of the circuit under consideration. The input data for the solution of the inverse kinematics problem obtained with its help are the Cartesian coordinates of the nodes and the rotation matrix of the coordinate system associated with the final node point relative to the zero coordinate system in the case of redundant modules.

Depending on the task, the solution of the problem in Cartesian coordinates can be carried out in different coordinate systems. Therefore, for consistency, we will assume that by the time the developed method is used, all the necessary Cartesian coordinates are transformed into a zero coordinate system.

3. Method development

Since the i-th module uniquely determines the mutual position of the nodes $N_{i-1}$ and $N_i$, the complex task of converting Cartesian coordinates into generalized for the entire kinematic chain can be reduced to a number of simpler problems of this transformation for each module. Let us consider solutions to the inverse kinematics problem for various modules.

3.1. Single joint module

Let us consider an arbitrary module with one joint and having number $i$. The kinematic diagram of this module is shown in figure 3.

![Figure 3. Geometric ratio of a single articulation module.](image)

The module consists of a series-connected rotary joint and a link. Let the node $N_{i-1}$ lie at the origin of the attached coordinate system $O_b$, and the node $N_i$ lie at the origin of the coordinate system $O_c$, $b < c$. The only generalized coordinate for this module is the angle $\theta_c$. Since the joint axis does not coincide with the segment $N_{i-1}N_i$ (one of the conditions for the inclusion of a joint in the module), the Denavit-Hartenberg parameter $a_c$ is not equal to 0.

If $a_c > 0$, the direction of the $x_c$ axis coincides with the direction of the projection of the radius vector $O_c^b$ on $x_bO_b y_b$ plane. If $a_c < 0$, the direction of the $x_c$ axis is opposite to the direction of the projection of the radius vector $O_c^b$ on $x_bO_b y_b$ plane. Thus, the formula for calculating the angle $\theta_c$ takes the form:

$$\theta_c = \text{atan2}(\text{sign}(a_c) \cdot O_c^b y_b, \text{sign}(a_c) \cdot O_c^b x_b).$$

(2)

3.2. Two joint module

Let us consider the module that includes two joints. Let the module realize the movement $N_i$ relative to $N_{i-1}$ and include kinematic pairs, with numbers $b \neq c$, $b < c$, $N_{i-1}$ will coincide with $O_b$, and $N_i$ with $O_d$. 
We will prove that the parameters $d_c$ and $a_c$ have zero value. Let us suppose that this is not the case, and $d_c \neq 0$ or $a_c \neq 0$. In this case, $O_c$ does not coincide with $O_b$, and another node point must be entered. In this case, the considered module will include not two, but three nodal points, which contradicts the rules of its formation. Thus, $O_b$ and $O_c$ coincide, i.e. $d_c = 0$ and $a_c = 0$. Also, the parameter $a_d \neq 0$ (otherwise the joint axis $c$ would coincide with the segment $N_{i-1}N_i$ and the joint would not be included in the module under consideration).

Based on the proven assertions, the module consists of two joints with coinciding origins of coordinates, and a link connecting the nodes $N_{i-1}$ and $N_i$, as shown in figure 4.

![Figure 4. Geometric relations of the module with two joints.](image)

Denote the intersection point of the axes $z_c$ and $x_d$ as $A$. Then, when rotating around the axis $z_b$, point $A$ will describe a circle, which can be defined by the following equations:

\[
\begin{align*}
(x^2 + y^2 &= [d_d \cdot \sin(\alpha_d)]^2, \\
z &= -d_d \cdot \cos(\alpha_d).
\end{align*}
\]  

(3)

On the other hand, point $A$ is removed from the node $N_i$ at a distance $a_d$, which can be written as the equation of a sphere:

\[(x - N_{i,x})^2 + (y - N_{i,y})^2 + (z - N_{i,z})^2 = a_d^2.\]  

(4)

Solving together equations (3) and (4), one can find (generally two) variants of the location of a given point as the intersection of a plane, a cylinder and a sphere. For the options found, the generalized coordinates can be calculated using the following formulas:

\[
\begin{align*}
\theta_d &= \text{atan2}(A_y, A_x); \\
N_f^c &= T_b N_i^b; \\
\theta_c &= \text{atan2}(N_{i,y}, N_{i,x}).
\end{align*}
\]  

(5)

3.3. Module with three or more joints

Let us consider a module with three joints, which includes joints with the numbers $b$, $c$ and $d$, $b < c < d$, $N_{i-1}$ coincides with $c O_b$, $N_i$ coincides with $c O_c$ (figure 5).

This module is redundant, so if coordinates are known only for $N_i$, the inverse kinematics problem has an infinite number of solutions and must be solved using numerical methods. This article considers the case when, in addition to the coordinates, the desired orientation of the coordinate system associated with the node point $N_i$ is also known:

\[
T_e = \begin{bmatrix} R_e & 0 \\ 0 & 1 \end{bmatrix}
\]  

(6)

where $R_e$ is the rotation matrix of the coordinate system associated with the node $N_i$ relative to the coordinate system connected with $N_{i-1}$. 
Let us denote the intersection point of $z_d$ and $x_e$ axes as $B$. This point can be found by the known orientation of $x_e$ axis and the manipulator parameter $a_e$:

$$B = N_1 - T_e \cdot (a_e \ 0 \ 0 \ 1)^T.$$  \hspace{1cm} (7)

The angle $\theta_e$ does not affect the position of point $B$, so it can be temporarily excluded from consideration and reduce the inverse kinematics problem for a module with three junctions to a simpler problem for a module with two joints, the end node of which is point $B$. For this we introduce a virtual link, connecting points $O_c$ and $B$, having the following parameters:

$$d_B = l \cdot \cos(\alpha_e),$$
$$a_B = l \cdot \sin(\alpha_e),$$  \hspace{1cm} (8)

where $l$ is link length in the considered module with three joints.

Thus, the task is reduced to an equivalent problem for a module with two joints. A similar technique can be applied to the module with four or more joints.

3.4. **Method algorithm**

Based on the above calculations, the formation of the solution of the kinematics inverse problem based on the geometric approach for a chain consisting of joints of only rotational type can be performed by the following algorithm:

1. Form a Denavit-Hartenberg representation.
2. Break the kinematic chain into modules.
3. Include in the general solution of the inverse problem of kinematics formulas for each module.

When performing the above algorithm, each kinematic pair will go into one of the modules. When examining the modules sequentially, a formula will be drawn up for calculating the angle of rotation of each joint. Thus, a complete geometric solution of the inverse problem of kinematics for the considered chain will be formed.

4. **Conclusion**

Analytical methods for solving the inverse kinematics problem have a relatively low computational complexity, but are poorly formalized, and therefore require considerable time for their development. Thus, the development of universal methods that allow to form an analytical solution of the inverse problem of kinematics is relevant.

This article has developed a method for forming a solution to the inverse kinematics problem based on a geometric approach for kinematic chains consisting of joints of the rotational type only. The input to the method is the Denavit-Hartenberg representation of the circuit. The input data for the formed solution of the inverse problem of kinematics are the Cartesian coordinates of the nodes of the kinematic chain. When applied to redundant kinematic chains, it is also necessary to specify the desired orientation at the nodes of the chain, otherwise there are an infinite number of solutions, and the numerical solution of the inverse kinematics problem becomes relevant.
The developed method allows to form a geometric solution of the inverse problem of kinematics for any kinematic chains, including joints of the rotational type only. Despite the high prevalence of such manipulators, the method does not consider many other possible configurations of manipulators. Therefore, it is necessary to expand the developed method for working with translational manipulators, and manipulators of mixed type.

Acknowledgements
The study was carried out as part of a research project on the topic “Development of a software and hardware complex of a control system based on solving an inverse problem of dynamics and kinematics” within the framework of the FCPIR 2014-2020 (unique identifier RFMEFI57517X0166) with financial support from the Ministry of Education and Science of the Russian Federation.

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