The role of the $\Delta(1232)$ on the longitudinal response in the inclusive electron scattering reaction

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Abstract

We have performed a many-body calculation of the longitudinal nuclear $(e,e')$ reaction employing a Second RPA (SRPA) formalism which contains the $\Delta(1232)$. More explicitly, our scheme contains RPA correlations as well as Hartree-Fock and second order self-energies, where an accurate evaluation of exchange terms is achieved. Using this formalism we have evaluated the longitudinal response function for $^{40}\text{Ca}$. We give final results at momentum transfers ranging from 300 up to 500 MeV/c, obtaining a good agreement with data.

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1 INTRODUCTION

The quasi-elastic peak in inclusive electron scattering for medium and heavy nuclei has been paid much attention in recent years. The good description of the cross section in the former work of Monitz et al. [1] was suddenly broken when the longitudinal and transverse responses were experimentally separated by Meziani et al. [2]. Even though the total cross section is well reproduced by a Fermi gas, this simple model overestimate data in the longitudinal channel, while it does the opposite in the transverse one. More recently Williamson et al. [3] have presented new measurements with an increase (decrease) of the longitudinal (transverse) nuclear response. Still, the discrepancy remains.

Many theoretical efforts have been developed to overcome this problem. Let us resume the main ones. One group ascribes the problem to modifications of the nucleon properties within the nuclear medium by the employment of small effective masses, swelling of nucleons, abnormal nucleon form factors, etc. (see Ref. [4]). Even though this approach is successful for the longitudinal channel, it fails in the transverse one. The other group, to which belongs the present work, is based on the many-body theory.

Many-body calculations were performed in finite nucleus [5]-[14] or in nuclear matter [15]-[30]. Fortunately, surface effects for heavy and medium nuclei in the energy-momentum region of interest are not very important and the nuclear matter formalism is a good approximation for the nuclear system, once a variable Fermi momentum or the local density approximation is used. On the other side, the many-body problem is very complex: initial and final state interactions, meson exchange currents, etc., should be considered, which implies a huge numerical effort, even in nuclear matter.

Perhaps the simplest way of introducing nuclear correlations is the RPA (see Refs. [6], [11], [20] and [21]). In the same line, a step further in difficulty is the SRPA [7], [24], which opens decay channels beyond one particle-one hole (1p1h) states and the Extended RPA (ERPA) ([8], [19] and [22]), adding more complex ground state correlations than in the RPA. Others approaches are the Correlated Basis Function (CBF) [9], the Green function method [10] and the one Boson Loop Expansion (BLE) [29], [30]. Looking at these formalism in terms of Feynman diagrams, all these approaches should converge to the same result, once the proper number of diagrams are considered. In spite of the sophistication of all these models, a clear understanding of the nuclear response at any momentum transfer is still elusive.
In some recent works the role of the $\Delta(1232)$ on the longitudinal channel was explored in some recent works. Gil et al. [28] study both channels within an elaborate formalism which includes two-body corrections to the excitation operator, initial and final state interaction and the $\Delta$. The $\Delta$ was considered by modifications in the external operator, self-energy insertions and in the nuclear interaction by means of the ring series with the $\Delta$ (polarization effect). The $\Delta$ is also incorporated in the longitudinal response in the works of Cenni et al. [29] and Amore et al. [30] where the Boson Loop Expansion is employed. In Amaro et al. [18] (and also in Ref. [30]), it is addressed that a $\Delta$ – hole pair can be directly excited in the longitudinal channel. In Ref. [21] we have also examined both longitudinal and transverse responses with the explicit inclusion of the $\Delta$. In that work, however, we concentrate on the transverse channel, with special attention on the RPA-exchange terms, concluding that an accurate evaluation of them is important. In that early work, the $\Delta$ has entered into the longitudinal response only through the second order self-energy, as there were no RPA terms in the longitudinal channel due to the particular election of the residual interaction.

In the present work we have developed a SRPA formalism which employs a more elaborate residual interaction than in our previous work [21]. The SRPA contains the RPA correlations together with the Hartree-Fock and second order self-energies. Exchange terms were carefully taken into account and the $\Delta$ was incorporated both in the many-body calculation and by its direct excitation.

The work is organized as follows. In Section II, we present the formalism. In Section III we analyze the effect of each ingredient in our model and analyze results for $^{40}Ca$. Finally, in Section IV some conclusions are given.
2 FORMALISM

Even though the main ingredients of our formalism were already presented in Refs. [21] and [24], here we have preferred to show a complete version of it. This is because there are some elements which are particular to the nuclear longitudinal response with the $\Delta$ which should be discussed.

The nuclear response function to an electromagnetic probe in the longitudinal channel is defined as,

$$ R_L(q, \omega) = -\frac{1}{\pi} \text{Im} < |O_L^\dagger G(\omega)O_L| >, \tag{1} $$

where $q$ represents the magnitude of the three momentum transfer by the electromagnetic probe $O_L$, $\omega$ is the excitation energy and $| >$ is the uncorrelated nuclear ground state. Ground state correlations beyond RPA are not analyzed in this work. The polarization propagator is given by,

$$ G(\omega) = \frac{1}{\omega - H + i\eta} - \frac{1}{\omega + H - i\eta}, \tag{2} $$

where $H$ is the nuclear Hamiltonian. As usual, $H$ is separated into a one-body part, $H_0$, and a residual interaction $V$.

We present now two projection operators $P$ and $Q$. The action of $P$ is to project into the ground state, the one particle-one hole ($ph$) and one delta-one hole ($\Delta h$) configurations. While $Q$ projects into the residual $n_p$ particle-$n_h$ hole-$n_\Delta$ delta configurations. More explicitly,

$$ P = | >>< | + P_N + P_\Delta, \tag{3} $$

with

$$ P_N = \sum_{1p1h} |1p1h><1p1h|, \tag{4} $$

$$ P_\Delta = \sum_{1\Delta1h} |1\Delta1h><1\Delta1h| \tag{5} $$

and

$$ Q = \sum_{n_h \geq 2} \sum_{0 \leq n_p \leq n_h} |n_p p (n_h - n_p)\Delta n_h h><n_p p (n_h - n_p)\Delta n_h h|, \tag{6} $$

where we have introduced $P_N$ and $P_\Delta$ for convenience. It is easy to verify that $P + Q = 1$, $P^2 = P$, $Q^2 = Q$, and $PQ =QP = 0$ and also, $P_i P_j = \delta_{ij} P_i$ and $P_i Q =QP_i = 0$ ($i = N, \Delta$).
We insert now the identity into eq. (1). Note that the external operator can connect the uncorrelated ground state only to the $P$ space if we use a one body operator for $O_L$, we have,
\[ R_L(q, \omega) = -\frac{1}{\pi} \text{Im} \langle O_L^\dagger P G_{PP}(\omega) P O_L \rangle, \]
where $G_{PP} \equiv PGP$. It is easy to see that,
\[ G_{PP}(\omega) = \frac{1}{\omega - H_{PP} - \Sigma_{PQP} + i\eta} - \frac{1}{\omega + H_{PP} + \Sigma_{PQP} - i\eta}, \]
with obvious definitions for $H_{PP}$, etc.

Until now, the formulation is quite general. The next step is to adopt a model for the external operator, the nuclear Hamiltonian and the nuclear states. The nuclear interaction is discussed in the next section, while the nuclear structure problem is the main issue of this section. Before this, we show a model for $O_L$, which is divided into two contributions,
\[ O_L = O_{ph} + O_{\Delta h}, \]
where the action of $O_{ph}$ ($O_{\Delta h}$) is to create a $ph$ ($\Delta h$) pair. The matrix elements for $O_{ph}$ and $O_{\Delta h}$ were taken from Refs. [17] and [18], respectively,
\[ \langle 1p1h | O_{ph} | \rangle = G_{pE}(q, \omega) \frac{\kappa}{\sqrt{T}} \chi_s^{\dagger} \chi_t \frac{1 + \tau_3}{2} \chi_s \chi_h \]
and
\[ \langle 1\Delta1h | O_{\Delta h} | \rangle = G_{\Delta}(q, \omega) \frac{\sqrt{1 + \tau' \tau}}{m^2} \chi_s^{\dagger} \chi_t^{\dagger} \cdot (h \times q) \ T_3 \chi_s \chi_h \]
where the $s$’s and the $t$’s stand for the spin and isospin quantum numbers, $h$ is the momentum carried by the hole, $\kappa = q/(2m)$, $\tau = (q^2 - \omega^2)/(4m^2)$, $\tau' = \frac{m}{m_{\Delta}} (\tau + (\frac{m_{\Delta} - m}{2m_{\Delta}})^2)$, and $m$ ($m_{\Delta}$) is the nucleonic ($\Delta$) mass. In eq. (12) the Pauli matrices $\sigma$ and $\tau_3$ were replaced by the corresponding transitions matrices $S$ and $T_3$ [31]. The electromagnetic form factors are,
\[ G_{pE}^p(q, \omega) = \frac{1}{(1 + 4.97\tau)^2} \]
and
\[ G_{\Delta}(q, \omega) = 2.97 \ G_{pE}^p(q, \omega) (1 - \frac{\omega^2 - q^2}{3.5(GeV/c)^2})^{-1/2}. \]
In the longitudinal channel there is no interference term between $O_{ph}$ and $O_{\Delta h}$. This is at variance with the case of the transversal one and it is a consequence of the presence of the hole momentum in $O_{\Delta h}$. This property allows to rewrite Eq. (7) into two terms,

$$R_L = R_{ph} + R_{\Delta h},$$

where

$$R_{ph} = -\frac{1}{\pi} Im \langle |O_{ph}^\dagger P_N G_{P_NP_N}(\omega) P_N O_{ph}| >$$

and

$$R_{\Delta h} = -\frac{1}{\pi} Im \langle |O_{\Delta h}^\dagger P_\Delta G_{P_\Delta P_\Delta}(\omega) P_\Delta O_{\Delta h}| > .$$

In the next two subsections we analyze separately $R_{ph}$ and $R_{\Delta h}$.

### 2.1 $R_{ph}$ contribution

The starting point of this subsection is Eq. (16) where our concern is to understand this expression in terms of its physical ingredients. To start with, we replace $P_N$ by its explicit expression,

$$R_{ph} = -\frac{1}{\pi} Im \sum_{1p1h,1p'1h'} < |O_{ph}^\dagger 1p1h| < 1p1h | \frac{1}{\omega - H_{P_NP_N} - \Sigma_{P_NQP_N} + i\eta} |1p'1h' > \times < 1p'1h'|O_{ph}| >$$

where for simplicity we have shown only the forward going contribution. We consider now the energy denominator of Eq. (18). For $H_{P_NP_N}$ we have,

$$< 1p1h |H_{P_NP_N}| 1p'1h' > = \delta_{ph,p'h'} (E_{1p1h}^{(0)} + \Sigma_{1p1h}^{HF}) + V_{1p1h,1p'1h'},$$

where $E_{1p1h}^{(0)}$ is the kinetic energy of the particle-hole pair, $\Sigma_{1p1h}^{HF}$ is the Hartree-Fock self-energy contribution and $V_{1p1h,1p'1h'}$ is the residual interaction between two particle-holes pairs. More explicitly, we can define the energy of a $1p1h$ pair up to first order in the residual interaction as,

$$E_{1p1h}^{(1)} \equiv E_{1p1h}^{(0)} + \Sigma_{1p1h}^{HF} = \varepsilon^{(1)}(p) - \varepsilon^{(1)}(h),$$

where in addition, we have introduced the energy of a single nucleon with momentum $p$ as,

$$\varepsilon^{(1)}(p) \equiv \frac{p^2}{2m} + \Sigma^{HF}(p).$$
The analytical expression for $\Sigma^{HF}(p)$ depends on the interaction employed and it is discussed in the next section.

The action of the $\Sigma^{PNQP}$ self-energy is to connect the $PN$-space with the $Q$ one. This means that $\Sigma^{PNQP}$ opens the decay channel of a $1p1h$ pair into a $2p2h$ configuration, a $1\Delta1p2h$ one and also more complex configurations like $2\Delta2h$.

The matrix elements of $\Sigma^{P_NQP}$ are written as,

$$<1p1h|\Sigma^{P_NQP}|1p'1h'> \propto \delta_{ph,p'h'}\Sigma^{P_NQP}_{1p1h}(h,q,\omega)$$

where we have kept only the diagonal terms of the self energy. This approximation is based on numerical reasons already discussed in Ref. [21]. Also, in this work we have studied in detail $\Sigma^{P_NQP}_{1p1h}$ splitting it into four components, depending on the character of the intermediate configuration $Q$, as,

$$\Sigma^{P_NQP}_{1p1h}(h,q,\omega) = \sum_{i=1}^{4} \Sigma^{P_NQ_iP_N}_{1p1h}(h,q,\omega).$$

where $Q_1$ represents a $1p1h$ bubble attached to a $1p1h$, $Q_2$ is a $1\Delta1h$ bubble attached to a $1p1h$, $Q_3$ is a $1p1h$ bubble attached to a $1\Delta1h$ and finally, $Q_4$ is a $1\Delta1h$ bubble attached to a $1\Delta1h$. Diagrams of each of the $Q_i$ can be found in Fig. 6 of Ref. [21]. Their analytical expressions are also found in the same reference.

Also through the numerical analysis it turns out that the dependence of the self-energy over the hole momentum is not very strong. This allows us to make an average over it,

$$\Sigma^{P_NQP}_{1p1h}(h,q,\omega) \approx \Sigma^{P_NQ_iP_N}_{1p1h}(q,\omega)$$

where

$$\Sigma^{P_NQ_iP_N}_{1p1h}(q,\omega) \equiv \frac{1}{3}\int d^3h \theta(k_F - |h|) \Sigma^{P_NQ_iP_N}_{1p1h}(h,q,\omega)$$

and $k_F$ is the Fermi momentum. Now Eq. (18) reads,

$$R_{ph} = -\frac{1}{\pi} Im \sum_{1p1h,1p'1h'} <|\mathcal{O}^{\dagger}_{ph}|1p1h><1p1h| \frac{1}{\omega - \delta_{ph,p'h'}E^{(2)}_{1p1h} - V_{1p1h,1p'1h'} + i\eta}$$

$$\times|1p'1h'?><1p'1h'|\mathcal{O}_{ph}>$$

where

$$E^{(2)}_{1p1h} = E^{(0)}_{1p1h} + \Sigma^{HF}_{1p1h} + \Sigma^{P_NQP}_{1p1h}(q,\omega).$$
The non-diagonal term of the interaction \( V_{1p1h,1'p'1'h'} \), is what makes Eq. (26) hard to evaluate. Due to this reason, it is convenient to show first the result without \( V_{1p1h,1'p'1'h'} \). We define,

\[
R^{(i)}_{ph} \equiv -\frac{1}{\pi} \text{Im} \sum_{1p1h} |<1p1h|O_{ph}|>^2 \frac{1}{\omega - E^{(i)}_{1p1h} + i\eta}
\]  

(28)

where the \( 1p1h \) energies \( E^{(i)}_{1p1h} \) with \( i = 0, 1 \) and 2, were already defined in Eqs. (19, 21) and (27), respectively. For \( i = 0 \) we obtain the free response,

\[
R^{(0)}_{ph}(q, \omega) = \sum_{1p1h} |<1p1h|O_{ph}|>|^2 \delta(\omega - (\frac{(h + q)^2}{2m_p} - \frac{h^2}{2m_h}))
\]  

(29)

where \( h + q \) is the particle momentum and we have allowed the particles and the holes to have different masses. Performing the summation over spin and isospin and making the conversion of sums over momenta into integrals we have,

\[
R^{(0)}_{ph}(q, \omega) = \frac{3\pi^2 A}{k_F^3} (G^p_{E}(q, \omega))^2 \frac{K^2}{\tau} \mathcal{L}(q, \omega, m_p, m_h)
\]  

(30)

where \( A \) is the mass number and \( \mathcal{L}(q, \omega, m_p, m_h) \) is a modified Lindhard function defined in Appendix A. The Goldstone diagram of \( R^{(0)}_{ph} \) is drawn as the first one in the r.h.s. of Fig. 1.

The response function for \( i = 1 \) is,

\[
R^{(1)}_{ph}(q, \omega) = \sum_{1p1h} |<1p1h|O_{ph}|>|^2 \delta(\omega - (\varepsilon^{(1)}(|h + q|) - \varepsilon^{(1)}(h)))
\]  

(31)

The expression within the \( \delta \)-function is a transcendental equation due to the presence of \( \Sigma^{HF} \) and is solved numerically. Alternatively, in Ref. [27] it was discussed that the \( R^{(1)}_{ph} \) response is well reproduced by \( R^{(0)}_{ph} \) but using two effective masses (one for particles and one for holes), and an energy shift. Adopting this approximation, we can write,

\[
R^{(1)}_{ph}(q, \omega) = \frac{3\pi^2 A}{k_F^3} (G^p_{E}(q, \omega))^2 \frac{K^2}{\tau} \mathcal{L}(q, \bar{\omega}, m_p, m_h)
\]  

(32)

where \( \bar{\omega} = \omega + \Delta \omega \) and the values for \( m_p, m_h \) and \( \Delta \omega \) depend on the interaction and on the momentum transfer. The first three diagrams contributing to \( R^{(1)}_{ph} \) are the first three ones of Fig. 1. The fact that the Hartree-Fock self-energy is in the energy denominator implies that this contribution is sum up to infinite order.

Finally we consider \( R^{(2)}_{ph} \). At variance with the case of \( R^{(1)}_{ph} \) where the the Hartree-Fock self-energy is a pure real function of the particle (or hole) momentum, the \( \Sigma_{ph}^{PNP} \) self-energy
has a real and an imaginary part. It is easy to see that $R^{(2)}_{ph}$ can be written as,

$$R^{(2)}_{ph}(q, \omega) = -\frac{1}{\pi} \int_0^\infty dE \frac{I_m \Sigma^{PN}_{QP}N(q, \omega)}{(E - \omega + R e \Sigma^{PN}_{QP}N(q, \omega))^2 + (I m \Sigma^{PN}_{QP}N(q, \omega))^2},$$

(33)

The lower order contributions which comes entirely from $\Sigma^{PN}_{QP}$, are shown in the second line of Fig. 1 (in upward order of $i$).

We turn now to the discussion of $R_{ph}$ with the inclusion of the $V_{1p1h, 1p'1h'}$ term. We write down from Eq. (26) the matrix element of the polarization propagator,

$$G(\omega)_{1p1h, 1p'1h'} = \frac{1}{\omega - \delta_{ph,p'h'} - E^{(2)}_{1p1h} - V_{1p1h, 1p'1h'} + i\eta},$$

(34)

where as mentioned above, we show for simplicity only the forward going contribution. To treat this, the standard Dyson equation is employed

$$G(\omega)_{1p1h, 1p'1h'} = G^{(2)}(\omega)_{1p1h} + G^{(2)}(\omega)_{1p1h} V_{1p1h, 1p'1h'} G(\omega)_{1p1h, 1p'1h'},$$

(35)

where we have used $G^{(2)}(\omega)_{1p1h}$ defined as,

$$G^{(2)}(\omega)_{1p1h} = \frac{1}{\omega - E^{(2)}_{1p1h} + i\eta},$$

(36)

instead of the free polarization propagator $G^{(0)}(\omega)_{ph}$. By successive iterations Eq. (35) leads to a series which can be summed up only in some few particular cases: when exchange terms are neglected or when $V_{1p1h, 1p'1h'}$ is a contact or a separable interaction. In the case that exchange terms are neglected the series is known as ring series and the sum of it as ring function. The puzzle is then, how to deal with exchange terms. For a general force, exchange terms should be evaluated numerically order by order and in practice, this can be done up to second order in the residual interaction. Third and higher order contributions, imply a quite involve numerical task due to the number of diagrams and the dimensionality of the integrals required to evaluate each diagram. However, exchange terms of a contact interaction can be easily included by a re-definition of the constants entering into the interaction. Using these facts, in Ref. [20] we tackle this problem as follows: we re-write the residual interaction by summing and subtracting a contact force, $V^{C}_{1p1h, 1p'1h'}$,

$$V_{1p1h, 1p'1h'} = V^{C}_{1p1h, 1p'1h'} + V^{F}_{1p1h, 1p'1h'},$$

(37)
where $V_{ip1h,lp'1h'}^F \equiv V_{ip1h,lp'1h'} - V_{ip1h,lp'1h'}^C$ contains the finite range part of the interaction. Varying $V_{ip1h,lp'1h'}^C$, the interaction $V_{ip1h,lp'1h'}^F$ is adjusted in order to ensure a fast convergence of the exchange terms. More specifically, the different constant terms of $V_{ip1h,lp'1h'}^F$ (that is, the ones of $V_{ip1h,lp'1h'}^C$\footnote{Eventually, $V_{ip1h,lp'1h'}^C$ can contain constant terms, but these terms can not be changed. Only the ones of $V_{ip1h,lp'1h'}^C$ can be varied, as this interaction is an artificial device to guarantee the fast convergence of exchange terms evaluated with $V_{ip1h,lp'1h'}^F$ instead of $V_{ip1h,lp'1h'}$.}), are varied until the second order exchange contributions are almost negligible compared with the first order ones.

The polarization propagator of Eq. (35) can now be drawn as

$$G(\omega)_{ip1h,lp'1h'} = G(\omega)_{ip1h} + G(\omega)_{ip1h,lp'1h'} + G(\omega)_{ip1h,lp'1h'}^C, \quad (38)$$

where,

$$G_{ip1h}^C = G_{ip1h}^{(2)} + G_{ip1h,lp1h}^{(2)} V_{ip1h,lp1h}^C + G_{ip1h,lp1h}^{(2)} V_{ip1h,lp1h}^C G_{ip1h,lp1h}^{(2)} + ... \quad (39)$$

$$G_{ip1h,lp'1h'}^C = G_{ip1h,lp'1h'}^{(2)} V_{ip1h,lp'1h'}^C + G_{ip1h,lp'1h'}^{(2)} V_{ip1h,lp'1h'}^C V_{ip1h,lp'1h'}^C G_{ip1h,lp'1h'}^{(2)} + ... \quad (40)$$

$$G_{ip1h,lp'1h'}^{CF} = G_{ip1h,lp'1h'}^{(2)} V_{ip1h,lp'1h'}^C G_{ip1h,lp'1h'}^{(2)} + G_{ip1h,lp'1h'}^{(2)} V_{ip1h,lp'1h'}^C G_{ip1h,lp'1h'}^{(2)} + ... \quad (41)$$

Inserting Eq. (38) into Eq. (26) one can define three different contributions to the response function, $R_{ph}^C$, $R_{ph}^F$, and $R_{ph}^{CF}$, associated to $G_{ip1h}^C$, $G_{ip1h,lp'1h'}^F$, and $G_{ip1h,lp'1h'}^{CF}$, respectively. In the case of $R_{ph}^C$, since $V_{ip1h,lp'1h'}^C$ is a pure contact interaction, the direct and exchange terms are equivalent and they can both be summed up to infinite order by evaluating the ring series of Eq. (39). In the case of $R_{ph}^F$, Eq. (40) is only considered up to second order in $V_{ip1h,lp'1h'}^F$, while for $R_{ph}^{CF}$, Eq. (41) is also evaluated up to infinite order in the $V_{ip1h,lp'1h'}^C$ interaction keeping terms up to second order in $V_{ip1h,lp'1h'}^F$. Our final result is then,

$$R_{ph} = R_{ph}^C + R_{ph}^F + R_{ph}^{CF} \quad (42)$$
Explicit expressions for the different exchange terms needed to build up the $R_{ph}^F$ contribution are very similar to the corresponding ones in the transverse channels reported in Ref. [20]. There are minor differences in the multiplying constants, the spin and isospin sums and the energy $E_{1p1h}^{(0)}$ should be replaced by $E_{1p1h}^{(2)}$. For this reason, we do not reproduce them. We neither show the ring series nor the direct terms as they can be found in many references (see for example Ref. [32]). If in Eq. (42) we replace $G^{(2)}(\omega)_{1p1h}$ by the free propagator $G^{(0)}(\omega)_{1p1h}$, the RPA approximation is obtained. The first order direct contribution to the RPA is drawn as the fourth diagram of the first line in Fig. 1, while the next diagram is the corresponding exchange term. Our approximation, where self-energies are included in $G^{(2)}(\omega)_{ph}$, is usually known as Second RPA (SRPA).

2.2 $R_{\Delta h}$ contribution

At variance with $R_{ph}$, the particular character of $O_{\Delta h}$ does not allow a $1\Delta 1h$ bubble to propagate as a RPA-series. In this case, only the self-energy modifies the free response. The expression for $R_{\Delta h}$ is obtained following closely the steps done in the last sub-section to arrive to $R_{ph}^{(2)}$. Obviously, one should replace $P_N$ by $P_{\Delta}$ and do some others minor changes. Before presenting the expression for $R_{\Delta h}^{(2)}$, we would like to discuss briefly the lower order contributions to the $R_{\Delta h}$ response shown in Fig. 2: the first diagram in the $r.h.s.$ of the first line is the free response. The second and third diagrams represent the Hartree-Fock self-energy over a hole line. There is no Hartree term for the $\Delta$ due to the absence of a scalar-isoscalar term in the $V_{1\Delta 1h,1\Delta'1h'}$ interaction. The fourth diagram in the first line of this figure is the Fock contribution to the $\Delta$. Finally, in the second line, we show the diagrams with self-energy insertions $\Sigma_{P_{\Delta}QP_{\Delta}}$.

Now we briefly outline the main expressions required to obtain $R_{\Delta h}^{(2)}$. In Eq. (17) we substitute $P_{\Delta}$ by its explicit expression,

$$R_{\Delta h} = -\frac{1}{\pi} Im \sum_{1\Delta 1h,1\Delta'1h'} <|O_{\Delta h}^\dagger|1\Delta 1h><1\Delta 1h|\frac{1}{\omega - H_{P_{\Delta}P_{\Delta}} - \Sigma_{P_{\Delta}QP_{\Delta}} + \eta}|1\Delta'1h'|> <1\Delta'1h'|O_{\Delta h}|>$$

(43)

The energy denominator of Eq. (43) is approximated as,

$$<1\Delta 1h|H_{P_{\Delta}P_{\Delta}} + \Sigma_{P_{\Delta}QP_{\Delta}}|1\Delta'1h'> \approx \delta_{\Delta h,\Delta' h'}(E_{1\Delta 1h}^{(0)} + \Sigma_{1\Delta 1h}^{HF} + \Sigma_{1\Delta 1h}^{P_{\Delta}QP_{\Delta}}).$$

(44)
It is convenient to define the energies,

\[ E_{1\Delta h}^{(0)} = \frac{p^2}{2m_\Delta} - \frac{\hbar^2}{2m} + m_\Delta - m, \]

\[ E_{1\Delta h}^{(1)} = E_{1\Delta h}^{(0)} + \Sigma^{\Delta}(p) - \Sigma^{HF}(\hbar), \]

\[ E_{1\Delta h}^{(2)} = E_{1\Delta h}^{(1)} + \Sigma^{P_\Delta Q\Delta}_1(q, \omega) \] (45)

where expression for \( \Sigma^{\Delta}(p) \) is given in Appendix B. We have adopted the same approximations for \( \Sigma^{P_\Delta Q\Delta}_1 \) as for \( \Sigma^{P_N Q\Delta}_1 \). Explicit expressions for both self-energies can be found in Ref. [21].

From Eq. (43) we define now,

\[ R_{\Delta h}^{(i)} = -\frac{1}{\pi} Im \sum_{1\Delta h} |<1\Delta h|\mathcal{O}_{\Delta h}|>^2 \frac{1}{\omega - E_{1\Delta h}^{(i)} + i\eta} \] (46)

After some algebra it is easy to find,

\[ R_{\Delta h}^{(0)}(q, \omega) = \frac{8\pi^2 A k_F}{m^4} (G_\Delta(q, \omega))^2 (1 + \tau') \mathcal{L}_\Delta(q, \omega, m_\Delta, m_h) \] (47)

\[ R_{\Delta h}^{(1)}(q, \omega) = \frac{8\pi^2 A k_F}{m^4} (G_\Delta(q, \omega))^2 (1 + \tau') \mathcal{L}_\Delta(q, \bar{\omega}, m_\Delta, m_h) \] (48)

\[ R_{\Delta h}^{(2)}(q, \omega) = -\frac{1}{\pi} \int_0^\infty dE \frac{Im\Sigma^{P_\Delta Q\Delta}_1(q, \omega)}{(E - \omega + Re\Sigma^{P_\Delta Q\Delta}_1(q, \omega))^2 + (Im\Sigma^{P_\Delta Q\Delta}_1(q, \omega))^2}, \] (49)

where in analogy with the \( ph \)-case, we have employed a modified function \( \mathcal{L}_\Delta(q, \omega, m_\Delta, m_h) \), which is defined in Appendix A.
3 RESULTS

In this section we show explicit results for the nuclear quasi-elastic longitudinal response. The properties of medium mass nuclei in the energy-momentum region of interest are reasonably described by non-relativistic nuclear matter once a proper Fermi momentum is used [16]. In particular, we consider the response of $^{40}Ca$ using a Fermi momentum $k_F = 235\ \text{MeV}/c$.

For the residual interaction $V_{1p1h,1p'1h'}$, we have made use of the parameterization of the Bonn potential [33], as presented in Ref. [27]: it is expressed as the exchange of $\pi, \rho, \sigma$ and $\omega$ mesons, while the $\eta$ and $\delta$-mesons are neglected. Also from this reference, we have taken the expressions for the Hartree-Fock self-energy $\Sigma_{1p1h}^{HF}$. When at least one $\Delta$ is involved, we have adopted the model interaction already used in Ref. [21], which for convenience is reproduced in Appendix B. Employing these interactions, the first step is to solve Eq. (26) (and the corresponding one in the $\Delta$ sector, $R^{(1)}_{\Delta h}$), in order to fix the effective masses and energy shifts for particles, holes and deltas. The results for several momentum transfers is shown in Table I.

Our main concern is to understand the longitudinal response and in particular the effect of the $\Delta$ over it. As already stated in the last section, the $\Delta$ affects the longitudinal response via two mechanisms: by the direct excitation of a $\Delta h$-pair and through the self-energies $\Sigma_{1p1h}^{PNQ_i}$ and $\Sigma_{1\Delta h}^{P_\Delta Q_i P_\Delta}$ with $i = 2, 3$ and 4. The first effect is considered in the $R_{\Delta h}$ response function. In Fig. 3 we present results for $R^{(0)}_{\Delta h}, R^{(1)}_{\Delta h}$ and $R^{(2)}_{\Delta h}$ at momentum transfer $q = 400\ \text{MeV}/c$. The comparison of $R^{(1)}_{\Delta h}$ with $R^{(0)}_{\Delta h}$ shows that the effect of the Hartree-Fock self-energies is not relevant. At variance, the influence of the $\Sigma_{1\Delta h}^{P_\Delta Q_i P_\Delta}$ is very significant: it produces an important redistribution of the intensity to lower energies. The $\Sigma_{1\Delta h}^{P_\Delta Q_i P_\Delta}$ self-energy has a real and an imaginary part and both terms are connected through a dispersion relation. The effect of the imaginary part is to spread the response, while the real part shifts the energy position of the peak to lower values. These effects are very strong because $\Sigma_{1\Delta h}^{P_\Delta Q_i P_\Delta}$ increases with energy and the $\Delta h$ peak is high in energy. It will be shown soon that when we compare the magnitude of $R_{\Delta h}$ with $R_{ph}$ and with data, the $R_{\Delta h}$ response is as a whole negligible. However, $R_{\Delta h}$ had been discussed for completeness.

In Fig. 4 we analyze the effect of the different contributions needed to built up the $R_{ph}$ response. In each panel, we have drawn the free response, $R^{(0)}_{ph}$, the total response $R_{ph}$, and the experimental points. This was done as a guidance to understand the behaviour of each term separately. Let us resume the meaning of dashed lines for each panel:
Panel (a): dashed lines represents the $R_{ph}^{(1)}$ response. Which means to study the effect of the Hartree-Fock self-energy.

Panel (b): we show the RPA response; that is, in Eq. (42) we have employed $G^{(0)}(\omega)_{ph}$ instead of $G^{(2)}(\omega)_{ph}$.

Panel (c): here we isolate the effect of the $\Sigma_{1p1h}^{PNQ1PN}$ self-energy. In this panel, we have evaluated Eq. (33) using a re-defined energy, $\tilde{E}_{1p1h}^{(2)} = E_{1p1h}^{(0)} + \Sigma_{1p1h}^{PNQ1PN}(q, \omega)$.

Panel (d): we evaluate Eq. (42) excluding the $\Delta$ terms of the self-energy $\Sigma_{1p1h}^{PNQ1PN}$ (which means to use $\Sigma_{1p1h}^{PNQ1PN}$ alone).

The Hartree-Fock self-energy $\Sigma_{1p1h}^{HF}$ spread the free response and moves the quasi-elastic peak to higher energies, as shown in Panel (a). This result was already reported in others works (see for example Ref. [27]). The effect of the RPA (see Panel (b)), depends on the character of the interaction. In the longitudinal channel, the direct contribution to the RPA (ring series), selects the isospin scalar and isospin vector central terms of the interaction (usually named as $F$ and $F'$). For our election of the interaction, these terms are repulsive, which is reflected in the response. Turning now to Panel (c), we see that the action of the $\Sigma_{1p1h}^{PNQ1PN}$ self-energy is similar to the one in the $\Delta$-sector. However, the magnitude of the spread and energy shift are smaller. Perhaps, the most interesting result of this figure is the one in Panel (d). The comparison of our final result with the dashed line, indicates that the $\Delta$ is very important in the longitudinal channel, not because of it direct excitation, but due to its indirect effect through the self-energy.

In Fig. 5 we compare our final result with data for several momentum transfer $q$, ranging from 300 up to 500 MeV/c. We have obtained a good agreement at low values of $q$. Although the agreement for $q = 450$ and 500 MeV/c is not so good, the accordance with the total intensity is satisfactory. At this point it is important to mention that the values of the parameters entering into the residual interaction when the $\Delta$ is involved are quite uncertain. This problem was already discussed in Ref. [21] where we showed that self-energy contributions which contains the $\Delta$, change considerably due to this ambiguity. Eventually, and in view of the result of Panel (d) in Fig. 4, a more accurate interaction could improve the agreement for high values of $q$. In addition, the $ph$-interaction itself is not unambiguous. Let us quote Ref. [27] where it has been pointed out that the bare nucleon-nucleon interaction employed is too repulsive to reproduce the correct binding energy and this shortcoming might also affect the RPA spectrum (overestimating the hardening of the response). In summary,
although our results are encouraging and in fact we have obtained perhaps one of the best agreements for the longitudinal response in a wide region of $q$; these results are conditioned by the election of the residual interaction which is yet unknown.

Concerning a comparison with other approaches, it is interesting to comment on two works already mentioned in the Introduction, where the longitudinal response with the inclusion of the $\Delta$ is considered. The first one is due to Gil et al. (Ref. [28]), where both the longitudinal and transverse responses are evaluated. This work is very complex in the sense that most of the many-body mechanisms where incorporated: mesons exchange currents, polarization, initial and final state interactions and the $\Delta$. The comparison with data is done only at two momentum transfers: $q = 300$ MeV/c for $^{12}\text{C}$ and $q = 410$ MeV/c for $^{40}\text{Ca}$. The last results are shown for energies ranging between 40 and 220 MeV. Even if there is a small tendency to overestimate data, these results certainly represents an improvement over previous ones. Unfortunately, final values for $^{40}\text{Ca}$ are shown for only one momentum transfer and in a narrow energy region. In addition, the role of the $\Delta$ in the longitudinal channel is not discussed in particular. The second work was done by Amore et al. (Ref. [30]), where they have studied the effect of the $\Delta$ over the longitudinal response using the BLE. As in our case, they have considered both the direct excitation of a $\Delta h$-pair and the effect of the $\Delta$ over the response through many-body contributions. They have also incorporated relativistic effects and done the comparison with $^{12}\text{C}$ data for $q = 400, 500$ and $600$ MeV/c. Their final results underestimate the experimental points. Using a different formalism and interaction, this work agrees with our results in two main issues: the direct excitation of a $\Delta h$-pair is not significant and the many-body effect of the $\Delta$ is important.

The main difference between our scheme and those of Refs. [28] and [30] is the way in which we deal with the Pauli exchange terms. In several works (see for instance Refs. [21], [23]), we have determined that exchange terms are important and need to be accurately evaluated. If we now incorporate the transverse channel into the discussion, there are many contributions like meson exchange currents and ground state correlations beyond the RPA, which also need to be considered. However, in the longitudinal channel these terms are less relevant than in the transverse one. Moreover, the interaction utilized in Refs. [28] and [30] is different than ours (in fact, each work employs a different one). This point is particularly important not only because of the sensibility of the results over the interaction, but also because antisymmetrization establishes constrains over the interaction (see Ref. [34]).
us be more specific about these constrains. When an antisymmetric matrix element is evaluated, the antisymmetrization operator can act either over the initial and final state or over the interaction itself. It is possible to build up an 'antisymmetric' interaction where the prescription is to evaluate direct matrix elements with this force. This is equivalent to employ the non-antisymetrized interaction with direct and exchange matrix elements. When the antisymmetric interaction is employed, the different channels of the interaction (for instance, $F$, $F'$, $G$ and $G'$ in a Landau-Migdal type interaction), are not independent: a change in one affects all the others. As the RPA longitudinal and transverse responses are dominated by different channels of the interaction, the just mentioned constrains should be carefully considered when dealing with both channels. Alternatively, the employment of any non-antisymetrized interaction within a scheme which accurately evaluates exchange terms is equivalent, as just mentioned. In a forthcoming work we will discuss this issue in detail [26].
4 CONCLUSIONS

In the present work we have analyzed the longitudinal response of the inclusive electron scattering process. We have presented a formalism developed for non-relativistic nuclear matter, which includes correlations of the SRPA type. The $\Delta$-degree of freedom was incorporated via two mechanisms: the direct excitation of a $\Delta h$-pair and through self-energy contributions which contains the $\Delta$. Our scheme puts special emphasis on the evaluation of Pauli exchange terms.

By the employment of an effective Fermi momentum we have evaluated the response for $^{40}$Ca at several momentum transfers, obtaining a good agreement with data. These results were obtained as a consequence of the addition of several ingredients, which basically can be grouped into two sets: the opening of new decay channels (as $2p2h$, $1p1\Delta 2h$, etc.; represented by the second order self-energies) which reduce the height of the quasi-elastic peak and spread the response, but moves the peak towards lower energies. And in the second group we put the Hartre-Fock self-energy and the RPA, which moves the peak to higher energies, but do not provide the required reduction of the intensity. In both cases the action of the exchange terms and the election of the interaction is very important.

In many works, it is mentioned that the longitudinal response seems more elusive than the transverse one, even though the underlying physics is more complex in the last case. In view of our results, we think that the just mentioned assertion should be revised. Referring this affirmation we recall that, $i)$ the first experimental results which separates the longitudinal and transverse channels [2], underestimate the longitudinal response. $ii)$ the free Fermi response strongly differs with data only in the longitudinal case. $iii)$ A very simple model, which uses the ring approximation with a $\pi + \rho$ plus the Landau-Migdal $g'$-interaction gives a good adjustment for the position and height of the transverse peak. The conjunction of these elements certainly shows the transverse channel as more understandable. However, a word of caution should be given: this simple model provides no intensity in the dip region and when all possible mechanisms (mesons exchange currents, initial and final state interactions, etc.) are incorporated, the resulting response shows a tendency to overestimate data. Our model is more complex than the ring series with the $g'+\pi + \rho$-interaction. But it is simpler than the more elaborate ones for the transverse channel already mentioned. These points suggest that it is precisely the transverse channel the most difficult to be understood.

As a final comment, we would like to stress that the many-body problem in inclusive
quasi-elastic electron scattering is quite involved. Any result is strongly dependent on the nuclear interaction, which is full of unknown coupling constants and parameters. Recent improvements (see for instance, Ref. [28]), show that we are closer to the solution of the puzzle, which necessarily implies a complex many-body calculation together with an adequate election of the nuclear interaction. In any case, the simultaneous study of several momentum transfers should be given. In this sense, we think that the present contribution is a step further in this direction. Clearly, our next objective is the inclusion of ground state correlations and meson exchange currents in order to analyze the transverse channel.
APPENDIX A

In this appendix we give the explicit expression for $\mathcal{L}(q, \omega, m_p, m_h)$ and $\mathcal{L}_\Delta(q, \omega, m_\Delta, m_h)$. It is convenient to change variables using the dimensionless quantities $Q = q/k_F$, $h = h/k_F$, $\nu = \omega/2\varepsilon_F$, $\lambda_p = m/m_p$, $\lambda_h = m/m_h$, $\lambda_\Delta = m/m_\Delta$, $c = \frac{m}{m_\Delta \lambda_h}$ and $\delta = \frac{m^2 \lambda_h - \lambda_\Delta}{k_F^2 \lambda_\Delta^2}$, $\varepsilon_F = k_F^2/(2m)$ is the Fermi energy and $m$ is the nucleon mass.

\[
\mathcal{L}(Q, \nu, \lambda_p, \lambda_h) = mk_F \int \frac{d^3h}{(2\pi)^3} \theta(|h + Q| - 1)\theta(1 - |h|)\delta(\nu - (\lambda_p(h + q)^2 - \lambda_h h^2)) \tag{50}
\]

If $\lambda_p = \lambda_h = 1$,

\[
\mathcal{L}(Q, \nu, 1, 1) = \frac{mk_F}{(2\pi)^2} \frac{1}{Q} \nu \tag{51}
\]

for $Q \leq 2$ and $0 \leq \nu \leq Q - \frac{1}{2}Q^2$,

\[
\mathcal{L}(Q, \nu, 1, 1) = \frac{mk_F}{(2\pi)^2} \frac{1}{2Q} (1 - \left(\frac{\nu}{Q} - \frac{Q}{2}\right)^2) \tag{52}
\]

for $Q \leq 2$ and $Q - \frac{1}{2}Q^2 \leq \nu \leq Q + \frac{1}{2}Q^2$ or $Q \geq 2$ and $-Q + \frac{1}{2}Q^2 \leq \nu \leq Q + \frac{1}{2}Q^2$ and $\mathcal{L}(Q, \nu, 1, 1) = 0$ otherwise.

If $\lambda_p \neq \lambda_h$,

\[
\mathcal{L}(Q, \nu, \lambda_p, \lambda_h) = 0 \tag{53}
\]

when $|\lambda_h - \lambda_p + 2\nu - \lambda_p Q^2|/2\lambda_p Q > 1$ or $\nu < (\lambda_p - \lambda_h)/2$,

\[
\mathcal{L}(Q, \nu, \lambda_p, \lambda_h) = \frac{mk_F}{(2\pi)^2} \frac{1}{2Q} (2\nu + (\lambda_h - \lambda_p))/(\lambda_h \lambda_p) \tag{54}
\]

for $Q \leq 2$ and $0 \leq \nu \leq (\lambda_p - \lambda_h(Q - 1)^2)/2$,

\[
\mathcal{L}(Q, \nu, \lambda_p, \lambda_h) = \frac{mk_F}{(2\pi)^2} \frac{1}{2Q} \left[ (\lambda_h - \lambda_p)^2 - Q^2 \lambda_p (\lambda_h + \lambda_p) + 2\nu (\lambda_h - \lambda_p) + 2Q \lambda_p \sqrt{\lambda_h \lambda_p Q^2 - 2\nu (\lambda_h - \lambda_p)} \right] \frac{1}{\lambda_p (\lambda_h \lambda_p)^2} \tag{55}
\]

for $Q \geq 2$ or $\nu \geq (\lambda_p - \lambda_h(Q - 1)^2)/2$.

Now, we give the expression for $\mathcal{L}_\Delta(Q, \nu, \lambda_\Delta, \lambda_h)$,

\[
\mathcal{L}_\Delta(Q, \nu, \lambda_\Delta, \lambda_h) = mk_F \int \frac{d^3h}{(2\pi)^3} \theta(1 - |h|) (h^2Q^2 - (h \cdot Q)^2) \delta(\nu - (c(h + q)^2/2 - h^2/2 + \delta)) \tag{56}
\]
It is convenient to re-defined variables as,

\[
\begin{align*}
    a_1 &= \frac{1 - c}{2} \\
    a_2 &= -c|Q| \\
    a_3 &= \nu - \frac{cQ^2}{2} - \delta
\end{align*}
\]

then, we get,

\[
L_\Delta(Q, \nu, \lambda_\Delta, \lambda_h) = 0
\]

if \(|(a_1 + a_3)/a_2| > 1\) and

\[
L_\Delta(Q, \nu, \lambda_\Delta, \lambda_h) = \frac{2\pi}{c^2Q} \left\{ -a_1^2/6 + (a_2^2 - 2a_1a_3)/4 - c^2/2 + \\
+ a_1^2 h_{\min}^6/6 - (a_2^2 - 2a_1a_3) h_{\min}^4/4 + c^2 h_{\min}^2/2 \right\}
\]

if \(|(a_1 + a_3)/a_2| \leq 1\), with \(h_{\min} = |(a_2 - \sqrt{a_2^2 - 4a_1a_3})/(2a_1)|\).
APPENDIX B

In this appendix we show the residual interaction $V$ when at least one $\Delta$ is involved and we present the explicit expression for the Fock self-energy for the $\Delta, \Sigma^F_{\Delta}(p)$.

We have employed the following $V_{1\Delta h,1h'\Delta'}$ interaction (this interaction is required in the evaluation of the fourth diagram in the first line of Fig. 2 and in the second one of the second line, already called as $Q_2$),

$$V_{1\Delta h,1h'\Delta'}(p) = \frac{f_{\pi\Delta N}^2}{\mu_\pi^2} \Gamma_{\pi\Delta N}^2(p) (\tilde{g}'_{\Delta N}(p) \sigma \cdot S + \tilde{h}'_{\Delta N}(p) \sigma \cdot \hat{p} S \cdot \hat{p}) \tau \cdot T \tag{60}$$

where,

$$\tilde{g}'_{\Delta N}(p) = \frac{g'_{\Delta N}}{\Gamma_{\pi\Delta N}^2(p)} \frac{C_{\rho\Delta N} p^2}{p^2 + \mu_\pi^2},$$

$$\tilde{h}'_{\Delta N}(p) = -\frac{p^2}{p^2 + \mu_\pi^2} + \frac{\Gamma_{\rho\Delta N}^2(p)}{\Gamma_{\pi\Delta N}^2(p)} \frac{C_{\rho\Delta N} p^2}{p^2 + \mu_\rho^2}, \tag{61}$$

and

$$\Gamma_{\pi\Delta N,\rho\Delta N}(p) = \frac{\Lambda_{\pi\Delta N,\rho\Delta N}^2 - (\mu_{\pi, \rho})^2}{\Lambda_{\pi, \rho}^2 + p^2}, \tag{62}$$

Analogous expressions are used for $V_{1\Delta h,1h'\Delta'}$ (needed in the evaluation of the first diagram of the second line in Fig. 2, called $Q_1$), $V_{1\Delta h,1\Delta',1h'}$ ($Q_3$) and $V_{1\Delta h,2\Delta}$ ($Q_4$). For each of these interactions, the coupling constants and parameters should be replace by their corresponding $NN$, $N\Delta$ and $\Delta\Delta$ values. Also Pauli matrices must be replaced by the corresponding transitions matrices $S$ and $T$ or $S$ and $\tilde{T}$, the $3/2-3/2$ spin matrices (see ref. [29]); depending on the character of the mesonic vertex.

For the parameters entering into our theory we have set at 140 MeV (770 MeV) the mass of the pion (rho meson). The pion coupling constant $f_{\pi NN}^2/4\pi=0.081$, $f_{\pi\Delta N}=2 f_{\pi NN}$ and $f_{\pi\Delta}=\frac{4}{3} f_{\pi NN}$. For the rho meson we have used $C_{\rho NN} = C_{\rho\Delta N} = C_{\rho\Delta\Delta} = 2.18$. The different mesons cut-offs at the vertices are set to $\Lambda_{\pi NN} = 1300$ MeV/c, $\Lambda_{\rho NN} = 1750$ MeV/c while all the remaining cut-offs are set to 1000 MeV/c. For the Landau Migdal parameter $g'_{\Delta N}$ and $g'_{\Delta\Delta}$ we have taken the values 0.55 and 0.40, respectively.

We present now the expression for $\Sigma^F_{\Delta}(p)$,

$$\Sigma^F_{\Delta}(p) = -\frac{2\pi k_F f_{\pi\Delta N}^2}{3} \frac{\mu_\pi^2}{\mu_\pi^2} \left\{ (3g'_{\Delta N} - 1) v_0(\Lambda_{\pi\Delta N}, \mu_\pi, p) - 2 C_{\rho\Delta N} v_0(\Lambda_{\rho\Delta N}, \mu_\rho, p) \
+ v_c(\Lambda_{\pi\Delta N}, \mu_\pi, p) + 2 C_{\rho\Delta N} v_c(\Lambda_{\rho\Delta N}, \mu_\rho, p) \right\} \tag{63}$$
where we have introduced the functions \( v_0 \) and \( v_c \), defined as,

\[
\begin{align*}
    v_0(\Lambda, \mu, \p) &= \int \frac{d^3k}{(2\pi)^3} \theta(1 - |k|) \left( \frac{\Lambda^2 - \mu^2}{\Lambda^2 + (k - \p)^2} \right)^2 \\
    v_c(\Lambda, \mu, \p) &= \int \frac{d^3k}{(2\pi)^3} \theta(1 - |k|) \frac{\mu^2}{\mu^2 + (k - \p)^2} \left( \frac{\Lambda^2 - \mu^2}{\Lambda^2 + (k - \p)^2} \right)^2
\end{align*}
\]

which can be easily integrated.
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Table I: Effective masses and energies shifts employed in $R^{(1)}_{\text{ph}}(q, \omega)$ and $R^{(1)}_{\Delta h}(q, \omega)$. We have used dimensionless quantities $\lambda_{m_i} \equiv m_i/m_i$, where $i = p, h$ or $\Delta$.

| $q$ [MeV/c] | $R^{(1)}_{\text{ph}}$ | $R^{(1)}_{\Delta h}$ |
|-------------|------------------------|----------------------|
|             | $\lambda_p$ | $\lambda_h$ | $\Delta \omega$ [MeV] | $\lambda_\Delta$ | $\lambda_h$ | $\Delta \omega$ [MeV] |
| 300.        | 1.33        | 1.54        | 11.0                | 1.08        | 1.11        | 4.0                |
| 325.        | 1.32        | 1.54        | 11.0                | 1.06        | 1.10        | 3.5                |
| 350.        | 1.31        | 1.54        | 9.0                 | 1.05        | 1.09        | 3.0                |
| 400.        | 1.28        | 1.54        | 9.0                 | 1.05        | 1.08        | 3.0                |
| 450.        | 1.32        | 1.56        | 9.0                 | 1.04        | 1.07        | 2.5                |
| 500.        | 1.37        | 1.59        | 7.0                 | 1.04        | 1.07        | 2.0                |
Figure Captions

**Figure 1**: Goldstone diagrams stemming from Eq. (42). In every diagram two wavy lines represent the external probe with momentum and energy \( q \) and \( \omega \), respectively. The dashed line represents the residual interaction. Continuous lines represent either a particle or a hole, while a double-continuous line represents the \( \Delta \). Only forward-going contributions are shown.

**Figure 2**: The same as Fig. 1, but for Eq. (49).

**Figure 3**: The \( R_{\Delta h} \) response at momentum transfer \( q = 400 \) MeV/c. The short-dashed line represents the free response of Eq. (47), the long-dashed line the \( R_{\Delta h}^{(1)} \) response of Eq. (48), while the continuous line is the final result \( R_{\Delta h}^{(2)} \) of Eq. (49).

**Figure 4**: Different contributions to the \( R_{\text{ph}} \) response. In each panel the short-dashed line represents the free response, the continuous line is the final result of Eq. (42) and the meaning of the long-dashed line for each panel is explained in the text. The momentum transfer is 400 MeV/c for all panels. Data was taken from Ref. [3].

**Figure 5**: The \( R_{\text{ph}} \) response at different momentum transfer. Short-dashed line are the free responses and the continuous ones are the sum of \( R_{\text{ph}} \) plus \( R_{\Delta h} \). The momentum transfer in each panel is: (a) 300 MeV/c, (b) 325 MeV/c, (c) 350 MeV/c, (d) 400 MeV/c, (e) 450 MeV/c and (f) 500 MeV/c. Data was taken from Ref. [3].
\[ R^{ph} = \quad \]
$R^{\Delta h} = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} + \text{Diagram 5} + \text{Diagram 6} + \text{Diagram 7} + \text{Diagram 8} + \ldots$
Fig. 3
Fig. 5