Research on single-epoch RTK positioning method based on Doppler relative velocimetry

Wei Li, Chenglin Cai*, Yuzhen Deng, Qian Wu, Zhiqiang Zhang
School of Xiangtan University, Xiangtan, China

*Corresponding author e-mail: caichenglin@xtu.edu.cn

Abstract. In the traditional single-epoch kinematic positioning, the positioning performance is relatively poor due to the less prior constraint information and lower success rate of ambiguity fixation. Accordingly, this paper proposes a single-epoch kinematic positioning method using Doppler relative velocity information constraint, that is, using Doppler observation information of rover station and base station for relative velocity measurement, and then using rover’s prior coordinates and velocity information to predict the coordinates of the current epoch, considering the advantages of higher accuracy of this method for coordinate update and more robustness of traditional single-point positioning, the corresponding strategy for single-epoch ambiguity resolution is further introduced to improve the positioning performance. The experimental results show that the proposed method can improve the accuracy of the float solution and ambiguity fixation rate, especially when the satellite geometry is poor and the observation conditions are obscured.

1. Introduction

GNSS signals are subject to frequent interference and occlusion when vehicles are driven in urban environments, and traditional multi-epoch RTK positioning is subject to problems such as cycle-slip and ambiguity mistransmission, which degrade the positioning performance, while RTK single-epoch positioning is not affected by these problems because each epoch’s element is independent of each other [1-5]. However, there will be a large number of parameters in the single-epoch solution process, with less a priori constraint information, resulting in a low ambiguity fixation rate, especially in urban high-rise environments with severe satellite signal occlusion. Currently, most GNSS receivers are capable of outputting Doppler observations, but their role in positioning is often neglected. Therefore, it is necessary to study how to effectively utilize Doppler observations and apply them to RTK single-epoch dynamic positioning.

Most of the current RTK positioning methods use the Kalman filter for parameter estimation to get the positioning float solution and then use integer search to get the integer ambiguity resolution to achieve high accuracy positioning. In the prediction stage of the Kalman filter, the most applied method is to use the pseudo-range single-point positioning result as the coordinate value of the state vector and then perform the coordinate update, but because the pseudo-range single-point positioning is susceptible to interference and low accuracy, which makes the accuracy of the float solution after filtering also relatively low and has a great impact on the success rate of ambiguity fixation [6]. For this reason, this paper uses Doppler observations for coordinate updating, and the main principle is to estimate the
coordinates of the current epoch by solving for the coordinate information of the previous epoch and the Doppler relative velocity information. This method has theoretically higher accuracy compared to single-point positioning coordinate updating [7], where the most critical aspect is how to effectively use the velocity information, i.e., to determine the stochastic model of the coordinate prediction values in the dynamic positioning model. The literature [8] uses raw Doppler observations for velocimetry with an accuracy of about 0.1 m/s and a slightly larger error. The literature [9] uses the average velocity of multiple epochs as the velocity of the receiver, and although this improves the accuracy, it does not reflect the instantaneous motion velocity of the carrier, and the measurement results have a delay and may differ significantly from the actual velocity in a highly dynamic environment.

Therefore, this paper uses the Doppler observations of GNSS to solve the receiver velocity parameters and combines the coordinate parameters of the previous epoch to achieve the coordinate update of the current epoch and introduces the corresponding ambiguity resolution strategy. The experimental results show that the Doppler relative velocimetry can also accurately measure the carrier's motion speed when the GNSS signal is blocked by trees and tall buildings, and the success rate of ambiguity fixation is improved after the coordinate update with the new method.

2. RTK single-epoch positioning algorithm based on Doppler relative velocimetry

When using the Kalman filter for parameter estimation, the accuracy of the initial state can directly affect the accuracy of the parameter update, and then also has a direct impact on the success rate of subsequent ambiguity fixation. In the process of RTK single-epoch dynamic positioning, compared with the traditional method of using pseudo-range single-point positioning for carrier coordinate parameter updating, this paper proposes a more accurate coordinate updating strategy, that is, using Doppler observation information of the rover station and the base station for relative velocimetry to obtain the accurate velocity information of the rover station, and then using this velocity information to constrain the coordinate updating and finally the ambiguity state parameter estimation. The corresponding data processing strategy is also given.

2.1. Doppler relative velocimetry model

The Doppler relative velocimetry algorithm is a method of differencing the Doppler observation equations of the rover station and the base station, and then solving for the rover station velocity and the receiver clock difference as unknown parameters. The Doppler observation equations of the rover station and the base station can be expressed as follows:

\[ f_b^i = \frac{(v_b^i - v_r^i)I_b}{\lambda} + \alpha_b + \beta^i \]  \hspace{1cm} (1)

\[ f_r^i = \frac{(v_r^i - v_r^i)I_r}{\lambda} + \alpha_r + \beta^i \]  \hspace{1cm} (2)

Where \( f_b^i \) and \( f_r^i \) are the Doppler observations of the base station and rover station, respectively; \( v_b \) is the velocity of the base station, which is fixed at a place with a good observation environment and has a velocity of 0; \( v_r^i \) denotes the velocity of the \( i \)th satellite, which can be calculated from the navigation file; \( v_r^i \) denotes the user velocity to be estimated, with the three directional components of \( v_x^i, v_y^i, v_z^i \), \( I_b \) and \( I_r \), respectively denotes the directional cosine of the line between the rover and the base and the \( i \)th satellite; \( \alpha_b \) and \( \alpha_r \) denotes the error caused by the clock difference of the rover and the base, respectively; \( \beta^i \) denotes the error term caused by the change of the clock difference of the \( i \)th satellite, ionospheric error, tropospheric error. In the case of a short baseline, this error is equal for the receiver of the base station and the receiver of the rover station; \( \lambda \) is the wavelength of the GNSS signal, because it is a single-frequency receiver, there is only one wavelength.

Differentiating the two observation equations gives the Doppler relative velocimetry equation.

\[ I_r v_r + \lambda (\alpha_r - \alpha_b) = (I_r - I_b)v_r^i + \lambda (f_r^i - f_b^i) \]  \hspace{1cm} (3)
The parameters to be estimated include the user velocity $v$ and the receiver clock drift, a total of four parameters, which can be solved from Doppler observations of more than 4 satellites according to the least-squares estimation, and the observation equation is:

$$Ax = b$$  \hspace{2cm} (4)

$$x = \begin{bmatrix} v_x \\ v_y \\ v_z \\ \lambda(a - a_h) \end{bmatrix}$$  \hspace{2cm} (5)

$$A = \begin{bmatrix} e_{x,l} & e_{y,l} & e_{z,l} & 1 \\ M & M & M & M \end{bmatrix}$$  \hspace{2cm} (6)

where $b$ is the residual of the observation equation, and $v_x$, $v_y$, and $v_z$ denote the velocity of the carrier in the three directions, respectively.

The coordinate parameters of the current epoch are updated according to the state vector $X$ solved by the previous epoch and the velocity parameters, Eq. 1

$$\hat{X}_k = \hat{X}_{k-1} + v \cdot T$$  \hspace{2cm} (7)

$$X = [x, y, z, N_{b(n-1)}]^T$$  \hspace{2cm} (8)

$$v = [v_x, v_y, v_z, E_{b(n-1)}]^T$$  \hspace{2cm} (9)

The equation gives a functional model for coordinate updating using Doppler relative velocity information, where $T$ denotes the data sampling interval time of the GNSS receiver, $k$ is the number of ephemeris elements, $n$ is the number of observed satellites, $N$ is the ambiguity of each satellite, $x$, $y$, and $z$ denote the three-dimensional coordinates of the carrier, and $E$ denotes the unit matrix.

2.2. Velocity-constrained RTK single-epoch element positioning algorithm

Before the ambiguity is fixed, it is usually necessary to use the Kalman filter to get the float solution of ambiguity, and then LAMBDA to fix the ambiguity and get the integer solution of ambiguity. In this process, the accuracy and convergence speed of the float solution directly determine the speed and reliability of the ambiguity resolution. The process of solving the float solution by Kalman filter is as follows.

$$K_g = P_k^{-1}H^T(HP_k^{-1}H^T)^{-1}$$  \hspace{2cm} (10)

$$\hat{X}_k = \hat{X}_k + K_g V_k$$  \hspace{2cm} (11)

$$P_k = (I - K_g H)P_k^{-1}$$  \hspace{2cm} (12)

$$V_k = y_k - H \hat{X}_k$$  \hspace{2cm} (13)

Where $K_g$ is the Kalman gain, $P_k$ is the covariance matrix; $I$ is the unit matrix; $H$ is the observation matrix; $V_k$ is the innovation matrix, representing the measurement noise; $y_k$ is the vector consisting of the observations of the receiver.

When solving the positioning, the observations of the base station and the rover station need to be double-differenced. For shorter baselines, errors such as ionospheric and tropospheric delays can be neglected, and the positioning accuracy in the dynamic case is generally of centimeter-level. Therefore, in single-epoch dynamic positioning, the parameters to be estimated include three coordinate parameters and $n-1$ double-difference ambiguity parameters, and $n$ is the number of satellites jointly observed by the base station and the rover station. Then, the updated coordinates and double-difference ambiguity
parameter estimates, and variance array of the current ephemeris can be obtained by using the Kalman filter method [10-12]. The double-difference ambiguity parameter after the current ephemeris filtering is a float solution, and the ambiguity can be fixed using the classical LAMBDA method [13-15], in which the ambiguity fixation rate mainly depends on the accuracy of the variance array corresponding to the float solution.

Since Doppler velocimetry is introduced in the coordinate update, it is equivalent to increasing the observation value. Theoretically, the coordinate accuracy obtained by the new method should be improved compared with the traditional single-point positioning coordinate updating method, but considering that there may be anomalies such as large fluctuations of Doppler velocimetry errors in realistic tests, while the updating accuracy of single-point positioning coordinates may be more stable at this time, this paper proposes an ambiguity resolution strategy that takes into account the advantages of both. Firstly, the coordinates are updated by using the Doppler velocity information, and the corresponding ambiguity resolution is subjected to LAMBDA search and Ratio test, if it passes the Ratio test, the fixed ambiguity parameters and coordinate parameters can be obtained, otherwise, the traditional single-point positioning coordinate update method is adopted; LAMBDA searches and Ratio test is repeated, if the 2nd time also passes the Ratio test, then the traditional single-point positioning coordinates update method is used to obtain the final ambiguity fixed solution, otherwise the Ratio values of the two calculations are compared to determine the final ambiguity resolution; if the Ratio value of the second calculation is relatively large, then the single-point positioning coordinates are used to update the corresponding float solution, otherwise the Doppler velocity coordinates are still used to update the corresponding float solution. Otherwise, the Doppler velocity coordinates are used to update the float solution. The procedure is shown in Fig.1.

![Diagram](image_url)

**Figure 1. Ambiguity solving strategy.**

The ambiguity solution method in this paper uses the Ratio quadratic test, and the theoretical success rate of ambiguity fixation will be improved and the positioning accuracy will rise compared with the traditional methods of using single point positioning coordinates or updating the solution ambiguity parameters using only Doppler velocimetry information from rover stations.
3. Experiment and Analysis

3.1. Static testing
In the static experiment, the true value of the velocity of the rover station is zero because it is stationary so that the accuracy of Doppler single station velocimetry and Doppler relative velocimetry in an ideal environment can be analyzed. The static test uses two u-blox M8T single-frequency receivers to perform static data measurements in an open environment. The test data were observed by both GPS and BDS systems, with a static observation duration of 10 min, a sampling frequency of 1 Hz, and an altitude cutoff angle set to 10°. In the static test, the receiver is at rest and the Doppler relative velocimetry is compared with the true value of velocity 0 to determine the observation noise level. The velocity error plots for the three velocimetric methods were obtained by comparing them with the true value of 0. The results are shown in Figures 1 to 3 (where the horizontal axis represents the number of epochs and the vertical axis represents the velocity in m/s).

Figure 2. Static speed measurement comparison.

(1) The two types of velocimetric errors shown in Fig. 2 are relatively small under static conditions with a good observation environment, and there is no roughness in either of the two velocimetric methods. The observation noise fluctuates randomly within a certain range, and there is no obvious systematic deviation from the 10 min observation data.

(2) From Table 1, we can see that the Doppler relative velocimetry method works better in the static and open environment, and the original Doppler velocimetry method is slightly worse than the Doppler relative velocimetry method, and the accuracy of both methods in the E and N directions is significantly higher than that in the U direction. 2 methods have poorer accuracy in the U direction, which is caused by the large component error of GNSS observations in the vertical direction.

Table 1. Static accuracy of two methods of speed measurement.

| Static       | Raw Doppler velocimetry (m/s) | Doppler relative velocity measurement (m/s) |
|--------------|-------------------------------|---------------------------------------------|
|              | E    | N    | U    | E       | N    | U     |
| RMS          | 0.0057 | 0.0044 | 0.0142 | 0.0014 | 0.0011 | 0.0045 |
| Average value| 0.0007 | 0.0001 | 0.0005 | 0.0001 | 0.0001 | 0.0001 |

3.2. Dynamic testing
The observation quality of GNSS data received by common single-frequency receivers differs between static and dynamic environments. To compare the speed measurement accuracy of different algorithms in dynamic environments, the following dynamic experiments are conducted in this paper. The dynamic test uses two u-blox M8T single-frequency receivers, one on the rooftop of the Information Building of
Xiangtan University and the other on a moving vehicle, and the test route is shown in Fig. 4. The test route is shown in Fig. 4. The test route is covered by trees, tall buildings, hillsides, and open roads, and the total length is about 1.26 km. The test data were observed by both GPS and BDS systems with a dynamic observation duration of 5 minutes, a sampling frequency of 1 Hz, and a height cutoff angle of 10°.

**Figure 3.** Experimental data acquisition, the red line is the vehicle motion trajectory

**Figure 4.** Number of satellites vs. PDOP value

| Dynamic                | No speed information assistance | Aided by speed information |
|------------------------|---------------------------------|-----------------------------|
| Fixed Success Rate     | 69.5%                           | 85.5%                       |
| Average Ratio Value    | 21.29                           | 12.39                       |

Analysis: Figure 4 shows the number of observable GNSS satellites and the corresponding position dilution of precision (PDOP) values. In some ephemeris, the number of visible satellites is reduced due to the blockage of the signal by trees or buildings, and the corresponding PDOP value is increased, which is not favorable for positioning.

To verify the effectiveness of the algorithm in this paper, the dynamic test data were processed by the following 2 methods. Method 1: Traditional single-point positioning coordinate update with the standard variance set to the a priori value 10. Method 2: Doppler relative velocity information coordinate update and the ambiguity solution strategy is shown in Figure 1.

The results of the single-epoch element Ratio values calculated by the above two methods are shown in Fig. 5. It can be found that the Ratio value of method 2 is improved compared with that of method 1. Combining the number of satellites and PDOP values in Fig. 4, it can be seen that the increase of the Ratio value is very obvious in the epoch element with a relatively small number of satellites, while in the epoch element with a relatively large number of satellites, the Ratio is also significantly better than that of the traditional method.
The red part represents the sequence of Ratio values corresponding to method 1, and the blue part represents the sequence of Ratio values corresponding to method 2. It can be found that the Ratio values corresponding to the Ratio single test in the last section of data are instead lower than those corresponding to the traditional single-point positioning coordinate update, which is due to the large deviation of the float solution positioning results, coupled with a certain deviation of the Doppler relative velocity measurement, which directly affects the accuracy of the coordinate update in the later epochs. According to the ambiguity solution strategy in Fig. 1, i.e., the Ratio quadratic test was performed, and the float solution results of the single-point positioning coordinate update were finally adopted, whose corresponding Ratio value sequence is shown in the blue part in Fig. 5.

4. Conclusion
This paper investigates the application algorithm of Doppler velocimetry in GNSS single-epoch dynamic positioning, proposes a coordinate update method for RTK single-epoch dynamic positioning assisted by Doppler relative velocimetry information and gives a single-epoch ambiguity parameter solution strategy based on this. The experimental results show that the proposed algorithm improves the ambiguity resolution accuracy, ambiguity fixation rate, and average positioning accuracy compared with the traditional single epoch dynamic positioning algorithm without velocity information coordinate constraint, especially in the case of a low number of observation satellites and high PDOP value. Currently, the algorithm is mainly validated in low dynamic conditions, and the next research work will be conducted in high dynamic or more complex environmental situations.

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