Abstract—From the perspective of physical layer security enhancement, energy-efficient intelligent reflecting surface (IRS) is introduced to assist the investigated secure wireless system, where a base station (BS) equipped with an uniform linear antenna (ULA) array sends confidential signals to a single-antenna legitimate user in the presence of a single-antenna eavesdropper. We aim to maximize the secrecy rate of the multi-antenna communication systems subject to the maximum transmit power constraint and the unit modulus constraints. To achieve this goal, the beamforming vector at the BS and the passive phase shifts at the IRS are jointly optimized. In this letter, to tackle the resulting non-convex optimization problem, we proposed a low-complexity algorithm based on quadratic transform and manifold optimization techniques, the convergence of which is guaranteed theoretically. Simulation results demonstrate that the proposed algorithm can provide a high-quality solution and outperform the researched benchmark schemes.

Index Terms—Intelligent reflecting surface ; secrecy rate; quadratic transform; manifold optimization; passive phase shifts

I. INTRODUCTION

With the dramatic evolution of radio frequency (RF) micro electromechanical systems (MEMS) and composite material, it becomes possible to adjust the phase shifts of reflected signals cost-effectively, even in real time [1]. This development has promoted a mass of applications of an emerging wireless communication device named intelligent reflecting surface (IRS) both in civil fields [2]. Specifically, an IRS is composed of a large number of reprogrammable, passive, reflecting units capable of interacting with the incident signal without a dedicated RF processing. By appropriately tuning phase shifts of the reflectors, the reflected signals can be deliberately strengthened or impaired at designated users. Additionally, it is worth noting that IRS consumes much less energy compared to the well known massive multiple-input-multiple-output (MIMO) technology due to the operating mechanism that it merely reflects signals without injecting excess power [3]. Furthermore, unlike the conventional relays or transceivers, IRSs possess the property of functioning under full-duplex mode without worrying self-interference and only transmit original information sent from the BS without generating new one. These advantages make IRS a promising paradigm to enhance physical layer security for 5G and beyond wireless systems efficiently.

Previous efforts have been devoted to IRS-aided systems in terms of physical layer security enhancement. In [4], the authors proposed a block coordinate descent (BCD) based algorithm to maximize the secrecy rate of an IRS-assisted multi-input-single-output (MISO) system. Each passive unit is regarded as one block and optimized in sequence during each iteration, thus it is less propitious for large-scale IRSs. The same goal is achieved in [5], where the IRS phase shifts are optimized in parallel utilizing semidefinite relaxation (SDR) and Gaussian randomization techniques. However, the SDR based algorithm is practically infeasible due to its high computational complexity.

In this letter, we proposed a computationally-efficient algorithm exploiting fractional programming and manifold optimization techniques to maximize the secrecy rate of an IRS-aided secure wireless system. To cope with the non-convex optimization problem, we first use quadratic transform to decouple the numerator and denominator of the objective function, then adopting manifold optimization method to resolve the unit modulus constraints in parallel with low-complexity. Simulation results show that significant gains can be acquired with respect to system secrecy rate through the proposed algorithm, as compared to the scenario without deploying the IRS.

Notations: In this letter, matrices and vectors are denoted by bold-faced capital and bold-faced lower-case characters, respectively. $\mathbb{C}^{M \times N}$ represents the linear space comprised of all $M \times N$ complex-valued matrices. $\mathbf{0}_N$ is a $N$-dimensional vector whose entries are all zero while $\mathbf{1}_N$ is a $N$-dimensional all one vector. $\mathbf{I}_M$ indicates an $M$-dimensional identity matrix. $(\cdot)^*,(\cdot)^{-1}$ and $(\cdot)^H$ stand for the conjugate, the inversion and the conjugate-transpose operations, respectively. $\mathbf{A}_{i,j}$ is the $(i,j)$th element of matrix $\mathbf{A}$. $\text{diag}(a_1,\cdots,a_N)$ denotes a $N \times N$ diagonal matrix with $a_1,\cdots,a_N$ as its diagonal entries. unit(a) normalizes all components of vector a. $\| \cdot \|$ and $\| \cdot \|$ represent the absolute value of a complex scalar and the $\ell_2$ norm of a vector, respectively. $\mathbb{E}\{ \cdot \}$ returns the expectation of a random variable. $\Re\{ \cdot \}$ extracts the real part of a complex number, and $\odot$ represents the Hadamard product.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider an IRS-assisted multi-antenna system consisting of one BS, one legitimate user (Bob), one eavesdropper (Eve) and one IRS, as illustrated in Fig. 1. The BS is equipped with $M$ antennas arranged in uniform linear array (ULA), while Bob and Eve are assumed to be single-antenna receivers. An IRS composed of $N$ low-cost passive reflectors is deployed to provide a reliable link between the BS and Bob. IRS
adjusts the phase shift of each reflecting unit through smart controller dynamically depending on the wireless propagation conditions. Note that it is rational to assume that the power of signals reflected by the IRS two or more times is negligible due to severe transfer loss and thus neglected. The baseband equivalent channels between the BS and the IRS, between the BS and Bob, between the IRS and Bob, and between the IRS and Eve are represented by $H_{TI} \in \mathbb{C}^{N \times M}$, $h_{TB} \in \mathbb{C}^{M \times 1}$, $h_{IE} \in \mathbb{C}^{M \times 1}$, $h_{IB} \in \mathbb{C}^{N \times 1}$, $h_{IE} \in \mathbb{C}^{N \times 1}$, respectively. We further assume that all channels involved experience frequency-flat fading and are perfectly known at the BS and the IRS.

Based on the above definitions, the received baseband signals at Bob and Eve can be written respectively as:

$$
y_B = (h_{TB}^H \Phi H_{TI} + h_{TB}^H)w_s + n_B,$$

$$
y_E = (h_{IE}^H \Phi H_{TI} + h_{IE}^H)w_s + n_E,$$

where $\Phi = \text{diag}(e^{j\theta_1}, e^{j\theta_2}, \ldots, e^{j\theta_N})$ represents the phase shift matrix of IRS, $\theta_i \in [0, 2\pi]$ denotes the phase shift induced by the $i$-th element on the IRS. $w \in \mathbb{C}^{M \times 1}$ is the beamforming vector at the BS satisfying $\|w\|_2^2 \leq P_{\text{max}}$, where $P_{\text{max}}$ represents the total transmit power of the BS. Without loss of generality, the Bob-oriented signal is given by $s$, which is a complex variable with $E\{|s|^2\} = 1$. $n_B$ and $n_E$ denote the complex Additive White Gaussian Noises (AWGN) with zero mean and variances $\sigma_B^2$ and $\sigma_E^2$, respectively. Therefore, the achievable secrecy rate from the BS to Bob can be expressed as:

$$
R_S(w, \Phi) = [R_B - R_E]^+,
$$

where $[x]^+ = \max(0, x)$, and

$$
R_B = \log_2 \left(1 + \frac{|(h_{TB}^H \Phi H_{TI} + h_{TB}^H)w|^2}{\sigma_B^2}\right),
$$

$$
R_E = \log_2 \left(1 + \frac{|(h_{IE}^H \Phi H_{TI} + h_{IE}^H)w|^2}{\sigma_E^2}\right),
$$

represent the achievable rates of Bob and Eve, respectively. Since the optimal secrecy rate of the considered system must be non-negative, the $[\cdot]^+$ operation can be omitted without loss of optimality.

The objective in this letter is to maximize the secrecy rate $R_S$ by jointly optimizing the BS beamforming vector $w$ and the IRS phase shift matrix $\Phi$, which can be formulated as:

$$
\begin{align*}
\text{(P1):} & \quad \max_{w, \Phi} \quad 1 + \frac{1}{\sigma_B^2} \left| (h_{TB}^H \Phi H_{TI} + h_{TB}^H)w \right|^2 \\
& \text{s.t.} \quad \|w\|_2^2 \leq P_{\text{max}}, \quad \Phi_{i,i} = 1, \ i = 1, 2, \ldots, N,
\end{align*}
$$

where $\Phi_{i,i} = e^{j\theta_i}$, $\forall i$. Despite the conciseness of (P1), it is generally a NP-hard problem owing to the non-concave objective function where $w$ and $\Phi$ are coupled and the non-convex unit modulus constraints in (6). Thus there exists no standard method that is straightforwardly applicable in finding the tractable solution optimally. Thus, we proposed a low-complexity algorithm for dealing with the non-convex optimization problem (P1) in an alternative manner next.

### III. ALTERNATING OPTIMIZATION BASED ALGORITHM FOR SECURITY RATE MAXIMIZATION PROBLEM

In this section, we propose an effective algorithm resorting to alternative optimization (AO) as main methodology to design the active transmit beamformer at the BS and the passive phase shifts at the IRS alternatively. We first derive a closed-form $w$ with given $\Phi$, then handling unit modulus constraints based on fractional programming (FP) quadratic transform and manifold optimization when $w$ is fixed.

#### A. Optimal Transmit Beamforming Design With Given $\Phi$

By fixing $\Phi$, the optimization problem (P1) in terms of $w$ can be recast to (P1a)

$$
\begin{align*}
\text{(P1a):} & \quad \max_w \quad w^H X_B w + 1 \\
& \text{s.t.} \quad \|w\|_2^2 \leq P_{\text{max}},
\end{align*}
$$

where $X_B = \frac{1}{\sigma_B^2} (h_{TB}^H \Phi H_{TI} + h_{TB}^H)(h_{TB}^H \Phi H_{TI} + h_{TB}^H)$ and $X_E = \frac{1}{\sigma_E^2} (h_{IE}^H \Phi H_{TI} + h_{IE}^H)(h_{IE}^H \Phi H_{TI} + h_{IE}^H)$. The optimal solution of (P1a) is offered in Lemma 1.

**Lemma 1.** Given the fixed phase shift matrix $\Phi$ of the IRS, the optimal beamforming vector $w$ has the closed-form solution

$$
w^* = \sqrt{P_{\text{max}}} \lambda_{\text{max}}(\tilde{X}_E^{-1}\tilde{X}_B),
$$

where $\tilde{X}_i = X_i + \frac{1}{P_{\text{max}}} I_M$, $i \in \{B, E\}$, $\lambda_{\text{max}}(\tilde{X}_E^{-1}\tilde{X}_B)$ denotes the unit norm eigenvector corresponding to the largest eigenvalue of matrix $\tilde{X}_E^{-1}\tilde{X}_B$.

**Proof.** It has been proved in [7] that the optimal beamforming vector can be acquired if and only if the entire transfer power was allocated, i.e., $\|w^*\|_2^2 = P_{\text{max}}$. Hence, the optimization problem in (7) can be transformed into

$$
\begin{align*}
\text{(P1b):} & \quad \max_{\|w\|_2^2 = 1} \quad w^H \tilde{X}_B w \\
& \text{w.r.t.} \quad w^H \tilde{X}_E w^*.
\end{align*}
$$

where $\tilde{w} = w / \sqrt{P_{\text{max}}}$ is a normalized vector. Obviously, both $\tilde{X}_B$ and $\tilde{X}_E$ are positive definite matrices, thus invertible. Note that (P1b) is a generalized eigenvalue problem, whose optimal solution is given by (9).
B. Optimization of Phase Shift Matrix $\Phi$ With Given $w$

In this section, we optimize the phase shift matrix $\Phi$ by fixing $w$. Thus, the associated optimization problem can be expressed as

$$\text{(P1c): } \max_{\Phi} \left\{ \frac{1}{2} \left( h_{iE}^H \Phi H_{\Pi} + h_{T_0}^H \right) w^2 + \sigma_B^2 \right\} \text{ s.t. } |e^{i\theta_i}| = 1, \ i = 1, 2, \cdots, N, \tag{11}$$

Different from the optimization problem (P1a), it is quite hard to derive an optimal solution of (P1c) directly owing to the non-convex constraint (12). To make it more tractable, we define $\theta = [e^{i\theta_1}, \cdots, e^{i\theta_N}]^H$. Using the equality $h_{iE}^H \Phi H_{\Pi} = \theta^H \text{diag}(h_{iE}^H) H_{\Pi}$, $k \in \{B, E\}$, we reformulate (P1c) as

$$\text{(P1c'): } \max_{\theta} \left\{ \frac{1}{2} \left( \theta^H \alpha_B + \tilde{\alpha}_B \right)^2 + \sigma_B^2 \right\} = \varphi_1(\theta) + \varphi_2(\theta), \tag{13}$$

$$\text{s.t. } |e^{i\theta_i}| = 1, \ i = 1, 2, \cdots, N, \tag{14}$$

where $\alpha_B = \text{diag}(h_{iE}^H) H_{\Pi} w$, $\tilde{\alpha}_B = \text{diag}(h_{iE}^H) H_{\Pi} w$, $\alpha_E = h_{iE}^H w$, and

$$\varphi_1(\theta) = \frac{1}{2} \left( \theta^H \alpha_B + \tilde{\alpha}_B \right)^2 = \left| \theta^H \alpha_B + \tilde{\alpha}_B \right| + \sigma_E^2. \tag{15}$$

$$\varphi_2(\theta) = \frac{1}{2} \left( \theta^H \alpha_E + \tilde{\alpha}_E \right)^2 + \sigma_E^2. \tag{16}$$

Note that $\varphi_2(\theta)$ can be considered as a constant function of $\theta$. Hence, (P1c') belongs to multiple-ratio FP problem, whose objective function can be translated to (17) based on the quadratic transform technique [8]

$$f_1(\theta, y_1, y_2) = 2R \left| y_1^* (\theta^H \alpha_B + \tilde{\alpha}_B) - |y_1|^2 (\theta^H \alpha_E + \tilde{\alpha}_E) \right|^2 + \sigma_B^2 + (2R |y_2| - |y_2|) \left| \theta^H \alpha_E + \tilde{\alpha}_E \right|^2 + \sigma_E^2, \tag{17}$$

where $y_1, y_2 \in \mathbb{C}$, are the introduced auxiliary variables. Then, the equivalent optimization problem becomes

$$\text{(P1d): } \max_{\theta, y_1, y_2} f_1(\theta, y_1, y_2) \tag{18}$$

$$\text{s.t. } |e^{i\theta_i}| = 1, \ i = 1, 2, \cdots, N, \tag{19}$$

Since $f_1(\theta, y_1, y_2)$ is a quadratic concave function of $y_1$ and $y_2$ for a given $\theta$ [8], the optimal solutions of the auxiliary variables can be attained by setting $\partial f_1/\partial y_{1,2} = 0$, i.e.,

$$y_1^* = \frac{\theta^H \alpha_B + \tilde{\alpha}_B}{|\theta^H \alpha_E + \tilde{\alpha}_E|^2 + \sigma_E^2}, \tag{20}$$

$$y_2^* = \frac{\sigma_B}{|\theta^H \alpha_E + \tilde{\alpha}_E|^2 + \sigma_E^2}. \tag{21}$$

In this way, the remaining work is to optimizing $\theta$ with given $y_1^*, y_2^*$. Note that the term $|\theta^H \alpha_E + \tilde{\alpha}_E|^2$ can be further recast to

$$|\theta^H \alpha_E + \tilde{\alpha}_E|^2 = \theta^H \alpha_E \alpha_E^H \theta + 2R \left| \tilde{\alpha}_E^H \theta \right|^2 + |\tilde{\alpha}_E|^2. \tag{22}$$

Substituting (20)-(22) into (17), after some basic algebraic manipulations, the objective function associated with $\theta$ can be represented as

$$f_2(\theta) = f_1(\theta, y_1^*, y_2^*) = 2R \left| (\theta^H \alpha_B + \tilde{\alpha}_B + (y_2^*)^2 \sigma_B \right|^2 + \left| y_1^* \right|^2 \left| (\theta^H \alpha_E + \tilde{\alpha}_E + \sigma_E^2) \right|^2 - \left| y_1^* \right|^2 \left| (\theta^H \alpha_E + \tilde{\alpha}_E + \sigma_E^2) \right|^2 + \left| y_2^* \right|^2 \left| (\tilde{\alpha}_E^H \theta \right|^2 + |\tilde{\alpha}_E|^2 \right)$$

$$= -\theta^H \Omega \theta + 2R \theta^H \gamma + C, \tag{23}$$

where

$$\Omega = ((y_1^*)^2 + |y_2^*|^2) \alpha_B \alpha_E^H, \tag{24}$$

$$\gamma = (y_1^*) \alpha_B - (|y_1|^2 + |y_2|^2) \tilde{\alpha}_B \alpha_E, \tag{25}$$

$$C = 2R \left| (\tilde{\alpha}_B^H \tilde{\alpha} + (y_2^*)^2 \sigma_B \right| - \left| y_1^* \right|^2 + |y_2^*|^2 \left| (\tilde{\alpha}_E^H \theta \right|^2 + |\tilde{\alpha}_E|^2 \right). \tag{26}$$

The phase shift optimization problem can be reformulated as

$$\text{(P1e): } \max_{\theta} f_2(\theta) \tag{27}$$

$$\text{s.t. } |e^{i\theta_i}| = 1, \ i = 1, 2, \cdots, N, \tag{28}$$

Visibly, $\Omega$ is a positive semidefinite matrix, and $f_2(\theta)$ is a quadratic concave function of $\theta$. Therefore, (P1e) is a quadratically constrained quadratic program (QCQP) which is similar as that in [1] and [3]. Unlike [1] and [3], in this letter, we handle the non-convex unit modulus constraints via manifold optimization tools [10] effectively.

We note that the unit modulus constraints in (P1) defines a complex circle manifold [10] which can be characterized as

$$\mathbb{CCM} = \{ \theta \in \mathbb{C}^{N \times 1} : |\phi_1| = |\phi_2| = \cdots = |\phi_N| = 1 \}, \tag{29}$$

where $\phi_i = e^{i\theta_i}, \ i = 1, \cdots, N$. Thus, the search space of $\theta$ is scaled down to a Riemannian sub-manifold of $\mathbb{C}^N$ containing the product of $N$ complex circles in the complex plane. We note that the unit modulus constraints are automatically satisfied when deciding the optimal $\theta$ over $\mathbb{CCM}$ in each iteration. Therefore, (P1e) can be ultimately characterized as an unconstrained optimization problem shown below

$$\text{(P1f): } \min_{\theta \in \mathbb{CCM}} f_1(\theta) = \theta^H \Omega \theta - \theta^H \gamma - \gamma^H \theta \tag{30}$$

where the irrelevant constant term $C$ in (23) is ignored.

Appreciably, the minimization optimization process over a manifold can be carried out based on gradient descent methods, and it has been certified that optimization techniques suitable for Euclidean space have counterparts on Riemannian manifolds [11]. Now, the main ideas of manifold optimization are interpreted sequentially.

Firstly, we need to find the line-search direction for a given point $\theta_k$. Hereby, we introduce a concept widely used in manifold optimization named tangent space. For each point on $\mathbb{CCM}$, the tangent space is the set including all tangent vectors passing through it tangentially. The tangent space for $\mathbb{CCM}$ at point $\theta_k$ is denoted by

$$T_{\theta_k} \mathbb{C} = \{ x \in \mathbb{C}^N : R \{ x \} = 0 \}, \tag{31}$$

Among all vectors in $T_{\theta_k} \mathbb{C}$, the one yielding the steepest increment of $f_2$ is called the Riemannian gradient, i.e., $\text{R-grad}_{\theta_k} f$. Specifically, the Riemannian gradient at the current point $\theta_k$ can be calculated by orthogonally projecting the Euclidean
gradient $\text{E-grad}_{\theta_{k}}f$ onto $T_{\theta_{k}}\mathcal{C}$ [12], which is represented as

$$\text{R-grad}_{\theta_{k}}f = \text{E-grad}_{\theta_{k}}f - \Re [\text{E-grad}_{\theta_{k}}f \circ \theta_{k}^*] \circ \theta_{k},$$

(32)

where the Euclidean gradient of $f_{i}(\theta)$ can be obtained as

$$\text{E-grad}_{\theta_{k}}f = 2(U_{\theta_{k}} - \gamma),$$

(33)

In this letter, the conjugate gradient (CG) algorithm is transplanted from the Euclidean space to $CCM$ directly due to its fast convergence rate. In particular, the update principle of the CG algorithm in the Euclidean space is defined as [13]

$$\zeta_{k+1} = -\text{E-grad}_{\theta_{k+1}}f + \beta_{k}\zeta_{k},$$

(34)

where $\zeta_{k}$ is the search direction at point $\theta_{k}$ and $\beta_{k}$ denotes the Polak-Ribière parameter. Since $\zeta_{k}$ and $\zeta_{k+1}$ lie in two different tangent spaces $T_{\theta_{k}}C$ and $T_{\theta_{k+1}}C$, respectively. Integration in (34) needs to be accomplished with the aid of another significant operation in manifold optimization called transport, which is written as [11]

$$T_{\theta_{k}}c \rightarrow T_{\theta_{k+1}}c(\zeta_{k}) = \zeta_{k} - \Re [\zeta_{k} \circ \theta_{k+1}^*] \circ \theta_{k+1},$$

(35)

Analogous to (34), the update principle of the Riemannian gradient on $CCM$ can be acquired by the following equation

$$\zeta_{k+1} = -\text{R-grad}_{\theta_{k+1}}f + \beta_{k}T_{\theta_{k}}c \rightarrow T_{\theta_{k+1}}c(\zeta_{k}),$$

(36)

With the search direction $\zeta_{k}$ and step size $\beta_{k}$ at hand, the resulting point on the tangent space $T_{\theta_{k}}C$ can be decided by

$$\theta_{k+1}' = \theta_{k} + \mu_{k}\zeta_{k},$$

(37)

where $\mu_{k}$ is the line search step size. Note that, $\theta_{k+1}' \notin \mathcal{C}\mathcal{C}M$, therefore, the next point $\theta_{k+1}$ should be obtained through retraction, which is a mapping operation from the tangent space to the manifold. In this letter, the retraction operator of $\theta_{k+1}'$ is defined as

$$\theta_{k+1} = \mathcal{R}(\theta_{k+1}') = \text{unit}(\theta_{k+1}),$$

(38)

The concrete steps of CG based manifold optimization algorithm and the whole procedures of the AO based algorithm is summarized in Algorithm 1 and Algorithm 2, respectively.

**Proposition 1.** Algorithm 2 is ensured to converge, if the optimized phase shift vector $\theta$ in the $i$-th iteration satisfies

$$f_{2}(\theta^{(i)}) \geq f_{2}(\theta^{(i-1)}),$$

(39)

*Proof.* The value of $f_{2}$ is monotonically non-decreasing after each iteration if proposition 1 is satisfied. Since $f_{2}$ is a quadratic concave function, Algorithm 2 is ensured to converge. \hfill \Box

Note that the convergence of Algorithm 1 is guaranteed by quadratic termination [13]. Thus, $f_{3}$ is monotonically decreasing after each iteration, which means that $f_{2}$ is on the opposite situation. Hence, proposition 1 is automatically satisfied after each iteration and the proposed algorithm is guaranteed to converge.

Furthermore, the complexity of the proposed algorithm is primarily induced by step 4 and 6, whose complexities are $O(M^{3})$ and no more than $O(N^{1.5})$ [11], respectively. Thus, the resulting complexity of Algorithm 2 is lower than $O(N_{h}(M^{3} + N^{1.5}))$, where $N_{h}$ is the iteration number of Algorithm 2, which is usually less than 10 with an accuracy of $\epsilon = 10^{-3}$ based on our simulations. Thus, the proposed algorithm is more computationally efficient than the SDR based algorithm [5].

**Algorithm 1 CG Based Manifold Optimization Algorithm**

1. Set the iteration index $k = 0$, convergence threshold $\epsilon$, and an initial point $\theta^{(0)}$.
2. Calculate $\zeta_{k} = -\text{R-grad}_{\theta_{k}}f$ based on (32)
3. **while** True **do**
   4. Calculate the step size $\mu_{k}$ according to [10, p. 62];
   5. Determine the next point $\theta_{k+1}$ using (37) and (38);
   6. Compute the Riemannian gradient based on (32) and (33);
   7. Perform transport operation to $\zeta_{k}$ according to (35);
   8. Find the Polak-Ribière parameter $\mu_{k}$ according to [10, Eq. 8.24];
   9. Compute the conjugate search direction $\zeta_{k+1}$ with (36).
   10. **if** $\|\text{R-grad}_{\theta_{k+1}}\| \leq \epsilon$ **then**
       11. Set $\theta^{*} = \theta_{k+1}$;
       12. break;
       13. **else**
       14. Set $\zeta_{k} = \zeta_{k+1}$, $k = k + 1$.
       15. **end if**
   16. **end while**
   17. Set $\theta^{*}$ as the main diagonal entries of matrix $\Phi^{*}$.

**Algorithm 2 AO Based Algorithm for (P1)**

1. Set $i = 0$, $w^{(0)} = h_{TI}^{H}/||h_{TI}||$, $\Phi^{(0)} = \text{diag}(1_{N})$, and $R_{S}^{(0)}(w^{(0)}, \Phi^{(0)})$.
2. **repeat**
   3. Set $i = i + 1$;
   4. With given $\Phi^{(i-1)}$, update $w^{(i)}$ with Lemma 1;
   5. With given $\Phi^{(i-1)}$ and $w^{(i)}$, calculate $y_{1}^{(i)}$ and $y_{2}^{(i)}$ via (20) and (21);
   6. Compute $\Phi^{(i)}$ exploiting Algorithm 1;
   7. Set $R_{S}^{(i)}(w^{(i)}, \Phi^{(i)})$;
   8. until $|B_{S}^{(i)} - B_{S}^{(i-1)}| \leq \epsilon$.
9. **Return** $\Phi^{*} = \Phi^{(i)}$ and $w^{*} = w^{(i)}$.

$O\left(N_{h}(M^{3} + N^{1.5})\right)$, where $N_{h}$ is the iteration number of Algorithm 2, which is usually less than 10 with an accuracy of $\epsilon = 10^{-3}$ based on our simulations. Thus, the proposed algorithm is more computationally efficient than the SDR based algorithm [5].

**IV. SIMULATION RESULTS**

We evaluate the performance of the proposed algorithm via numerical results, as compared to the following schemes:

(1) Heuristic: it firstly uses the maximum ratio transmission (MRT) at the BS towards Bob, i.e., $w = \sqrt{P_{\text{max}}}h_{TB}/||h_{TB}||$, then optimize phase shifts via Algorithm 1.
(2) Without IRS: it directly optimizes $w$ through Lemma 1.
(3) The SDR based algorithm proposed in [5].

The distance between the BS and the IRS is $d_{BI} = 50$ m. We set $M = 4$, $N = 32$ (if not specified), $P_{\text{max}} = 15$ dBm, and $\sigma_{B}^{2} = \sigma_{N}^{2} = -75$ dBm [6]. All channels are assumed to be Rayleigh fading channels. Both Bob and Eve are located on
a line in parallel to that connects the BS and the IRS. The vertical distance between these two lines is $d_v = 2$ m. The path loss model is $PL = PL_0 - 10 \cdot \log_10\left(\frac{d}{d_0}\right)$ dB, where $PL_0 = -30$ dB is the path loss at the reference distance $d_0 = 1$ m, $\zeta$ is the path loss exponent, $d$ is the horizontal distance between the BS and Bob. The path loss exponents of the BS-IRS link, the BS-Eve link, the IRS-Bob link, the IRS-Eve link and the IRS-Bob link are set to $\zeta_{BI} = 2.2$, $\zeta_{TE} = 3.5$, $\zeta_{TB} = 3.5$, $\zeta_{IB} = 2.5$ and $\zeta_{EB} = 2.5$, respectively. The convergence threshold is set to $\epsilon = 10^{-3}$. Over 500 channel realizations are averaged to obtain simulation results.

![Fig. 2. Secrecy Rate vs. The horizontal distance between The BS and Bob](image)

Fig. 2 illustrates the achieved secrecy rate versus the BS-Bob horizontal distance $d$ by fixing the BS-Eve distance $d_{TE} = 44$ m. As we can observe, the proposed algorithm achieves the best secrecy rate performance among the benchmark schemes. Severe performance loss can be noticed for the scenario without IRS as Bob moves far from the BS. The performance of the heuristic scheme is the worst most of the time, which is expected since the MRT beamforming vector is not orthogonal enough to the composite BS-Eve channel, and most power is beamed towards Bob, thus the potential of IRS can not be fully uncovered. Note that the secrecy rate is increasing when $d \in [40$ m, $50$ m] via the proposed algorithm, which demonstrates that IRS renders significant power enhancement gains when Bob is close to it.

In Fig. 3 we investigate the achievable secrecy rate by varying the maximum transmit power. Visibly, the proposed algorithm outperforms the other schemes. The secrecy rate increases slowly without the aid of the IRS, which reveals that using the optimal beamforming scheme alone achieves limit performance gain. Note that the performance gain of the heuristic scheme decreases gradually when $P_{\text{max}} \geq 10$ dBm, which is expected because higher achievable rate is obtained by Eve, hence less secrecy rate is reaped. More importantly, a more pronounced performance gain can be obtained as the increase of the maximum transmit power by jointly optimizing the beamforming vector at the BS and the phase shifts at the IRS, which suggests that deploying IRS is a promising way to enhance physical layer security of wireless communication systems.

![Fig. 3. Secrecy Rate vs. The Maximum Transmit Power](image)

In Fig. 3 we investigate the achievable secrecy rate by varying the maximum transmit power. Visibly, the proposed algorithm outperforms the other schemes. The secrecy rate increases slowly without the aid of the IRS, which reveals that using the optimal beamforming scheme alone achieves limit performance gain. Note that the performance gain of the heuristic scheme decreases gradually when $P_{\text{max}} \geq 10$ dBm, which is expected because higher achievable rate is obtained by Eve, hence less secrecy rate is reaped. More importantly, a more pronounced performance gain can be obtained as the increase of the maximum transmit power by jointly optimizing the beamforming vector at the BS and the phase shifts at the IRS, which suggests that deploying IRS is a promising way to enhance physical layer security of wireless communication systems.

![Fig. 4. Secrecy Rate vs. The Number of Reflecting Units](image)

Fig. 4 shows the secrecy rate versus the number of IRS’s reflecting units when $P_{\text{max}} = 15$ dBm and $d = 49$ m. It is observed that the secrecy rate achieved by the proposed algorithm increases significantly as the increment of $N$ since the signal reflected by the IRS becomes dominant at Bob. It can be also noticed that the performance gain of the proposed algorithm is larger than that of the heuristic scheme since the joint optimization of $w$ and $\Phi$.

V. CONCLUSION

This letter investigate the physical layer security enhancement problem in an IRS-aided multi-antenna system. A low complexity algorithm using quadratic transform and manifold optimization techniques were developed to deal with the non-convex optimization problem effectively. Simulation results confirmed the superiority of the proposed algorithm.

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