The impact of meridional circulation on stellar butterfly diagrams and polar caps

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ABSTRACT

Observations of rapidly rotating solar-like stars show a significant mixture of opposite-polarity magnetic fields within their polar regions. To explain these observations, models describing the surface transport of magnetic flux demand the presence of fast meridional flows. Here, we link sub-surface and surface magnetic flux transport simulations to investigate (i) the impact of meridional circulations with peak velocities of \( \leq 125 \text{ m s}^{-1} \) on the latitudinal eruption pattern of magnetic flux tubes and (ii) the influence of the resulting butterfly diagrams on polar magnetic field properties. Prior to their eruption, magnetic flux tubes with low field strengths and initial cross sections below \( \sim 300 \text{ km} \) experience an enhanced poleward deflection through meridional flows (assumed to be poleward at the top of the convection zone and equatorward at the bottom). In particular flux tubes which originate between low and intermediate latitudes within the convective overshoot region are strongly affected. This latitude-dependent poleward deflection of erupting magnetic flux renders the wings of stellar butterfly diagrams distinctively convex. The subsequent evolution of the surface magnetic field shows that the increased number of newly emerging bipoles at higher latitudes promotes the intermingling of opposite polarities of polar magnetic fields. The associated magnetic flux densities are about \( 20\% \) higher than in the case disregarding the pre-eruptive deflection, which eases the necessity for fast meridional flows predicted by previous investigations. In order to reproduce the observed polar field properties, the rate of the meridional circulation has to be on the order of \( 100 \text{ m s}^{-1} \), and the latitudinal range from which magnetic flux tubes originate at the base of the convective zone (\( \lesssim 50^\circ \)) must be larger than in the solar case (\( \lesssim 35^\circ \)).

Key words: stars: magnetic fields – stars: activity – stars: interior – stars: rotation – stars: spots – stars: imaging

1 INTRODUCTION

On the Sun, dark spots are exclusively found within an equatorial belt between about \( \pm 40^\circ \) latitude. In contrast, stellar surface brightness maps, secured with the technique of Doppler imaging (Collier Cameron 2001, and references therein), show that other cool stars frequently have large high-latitude and polar spots, often in conjunction with low-latitude features as well (Strassmeier 2002, and references therein). Due to specific requirements of the observing technique the targets so far are stars rotating more rapidly than the Sun. Although the time base of surface maps acquired for individual stars is yet not sufficiently long to conclusively discern long-term activity properties, their behaviour seems to digress from the solar 11-year spot/22-year magnetic cycles and polar field properties. In fact, many of the younger and more active stars show no apparent cycle insofar as chromospheric indicators such as Ca II H & K can be used as a proxy for magnetic activity (Baliunas et al. 1995; Donahue 1996). There is, as yet, also no example of a star undergoing a ‘Maunder minimum’. Different activity and cycle signatures may however be present or more pronounced, such as apparent preferred longitudes in the spot distribution or the ‘flip-flop’ phenomenon (Jetsu et al. 1994; Berdyugina 2004; Moss 2004; Korhonen & Elstner 2005).

An important characteristic difference to the solar-like magnetic field distribution is the significant mixture of magnetic flux of opposite polarity within polar regions, as observed with the Zeeman-Doppler Imaging technique (Semel 1989; Donati & Brown 1997). In particular the strong flux intermingling in the polar regions is in contrast to the Sun (Donati et al. 2003), where the high-latitude magnetic field is essentially unipolar throughout the majority of the activity cycle. On rapidly rotating stars, the associated polar magnetic flux densities are sufficiently high and persistent to cause dark polar caps lasting over a large number of stellar rotation periods (e.g. Jeffers et al. 2005).

The magnetic activity signatures of cool stars are ascribed to the emergence of magnetic flux generated by sub-surface dy-
dynamo mechanisms. Scenarios for dynamo operation are based on the magneto-hydrodynamic interaction between convective motions and (differential) rotation. Yet there is no complete theory, which unifies the amplification, storage, transport, eruption, and (possibly cyclic) re-generation of magnetic flux consistently, for an extensive review on stellar dynamo theory see Ossendrijver (2003). Dynamo mechanisms inside the convective envelope leave characteristic imprints on the observable activity signatures in the stellar atmosphere, which provide constraints for the underlying processes.

In the following, we focus on the transport of magnetic flux both below and on the stellar surface. Magnetic flux tube models (e.g. Moreno-Insertis 1986; Choudhuri & Gilman 1987; D’Silva & Choudhuri 1992; Fan et al. 1994; Schüssler et al. 1994, 1996) describe the evolution of magnetic flux, concentrated in strands of magnetic field lines, inside the convection zone until their eruption on the stellar surface. Although they exclude aspects concerning the (cyclic) generation of magnetic fields, they successfully reproduce characteristic properties of emerging bi-polar spot groups like, in the case of the Sun, their latitude of emergence, their relative velocities and topologies, the asymmetries between the preceding- and following spot group, and Joy’s law (e.g. D’Silva & Choudhuri 1993; Caligari et al. 1993). Further applications of the flux eruption model comprise cool stars with different rotation rates, stellar masses, and evolutionary stages as well as components of close binary systems (Granzer et al. 2006; Holzwarth & Schüssler 2001, 2003; Holzwarth 2004). Frequently observed starspots at higher latitudes of rapid rotators, for example, can be explained by the poleward deflection of rising flux tubes prior to their eruption on the stellar surface (e.g. Schüssler & Solanki 1992). A persistent magnetic flux eruption at high latitudes implies however the availability (and possibly generation) of large amounts of magnetic flux over a similar latitudinal range inside the stellar convection zone, which would be in dissent with the solar case, where the production of magnetic flux is anticipated to be most efficient at low latitudes (e.g. Choudhuri & Dikpati 1999).

Surface flux transport models (e.g. DeVore et al. 1984; Wang et al. 1998; van Ballegooijen et al. 1998; Schröder & Title 2001; Mackay & Lockwood 2002; Mackay et al. 2004; Baumann et al. 2004) follow the evolution of the radial magnetic field component on the stellar surface under the combined effects of magnetic flux emergence, differential rotation, meridional flow, and supergranular diffusion. Using empirical properties of bi-polar regions (i.e. latitudinal migration of emergence rates and tilt angles) they successfully reproduce major features of the solar cycle like the reversal of the polar field. Considering moderately rotating stars, Schröder & Title (2001) showed that an enhancement of the rate of flux emergence produces dark polar caps of a single magnetic polarity, surrounded by a flux ring of opposite polarity. More recently, Mackay et al. (2004) showed that additional enhancements of both the latitudinal range of flux emergence and the meridional flow velocity are required to successfully reproduce a significant mixture of magnetic polarities at high latitudes on rapid rotators.

The surface flux transport models may accurately describe the surface evolution of the magnetic field, but they do not address the question of whether the assumed properties of erupting magnetic flux are consistent with the requirement of high meridional flow velocities. In particular, how the latitudinal distributions of flux eruption are affected by enhanced rates of meridional flows. We therefore link our studies on the pre-eruptive and the post-eruptive evolution of magnetic flux to consistently quantify the impact of meridional circulations on the butterfly diagram and the polar magnetic field properties of rapidly rotating stars. The main aim of this investigation is to validate the assumptions of Mackay et al. (2004) about extended latitudinal ranges of magnetic flux emergence. Yet the relation between observable activity properties and sub-surface transport mechanisms also makes it possible to infer empirical constraints for specific dynamo properties. In Sect. 2 we investigate the rise of magnetic flux tubes through the convection zone prior to their emergence on the stellar surface to determine how the latitudinal eruption pattern depends on the strength of the circulation. The results are used in Sect. 3 to determine the impact of meridional circulations on stellar butterfly diagrams. In Sect. 4 we use the specified flux emergence latitudes within surface flux transport simulations to determine the requirements for the reproduction of observed stellar magnetic field properties. In Sect. 5 we discuss our results and their implications for possible dynamo scenarios. Our conclusions are summarised in Sect. 6.

2 SUB-SURFACE EVOLUTION AND ERUPTION OF MAGNETIC FLUX TUBES

2.1 Basic scenario

Magnetic activity signatures in the atmosphere of cool stars are ascribed to the eruption of magnetic flux tubes, which are generated by sub-surface dynamo processes inside the convective envelope (e.g. Ossendrijver 2003; Schüssler 2005, and references therein). The amplification of the magnetic field is expected to take place at the tachocline, a region of strong shear flows at the interface to the radiative core of the star. Helioseismological observations indicate that the tachocline is slightly prolate (e.g. Charbonneau et al. 1993; Basu & Antia 2001). At the equator, the bulk of the tachocline is right beneath the convection zone, whereas at higher latitudes a substantial part of it is located inside the convection zone. In the dynamically unstable stratification of the convection zone, magnetic flux is subject to magnetic buoyancy, which leads to its rapid loss through eruption (Parkes 1975; Moreno-Insertis 1983). In the case of the Sun, for example, a magnetic flux tube can traverse the convection zone within several weeks. The generation of high field strengths requires magnetic fields to persist in the amplifying region over time scales comparable with those of supposed dynamo processes. This requirement leads to the conjecture of magnetic flux being stored inside the superadiabatic overshoot region beneath the convection zone (Spruit & van Ballegooijen 1982; van Ballegooijen 1982). Inside this stably stratified region, magnetic flux tubes are perturbed through overshooting gas plumes penetrating from the convective envelope above. If its magnetic field strength is sufficiently large, a displaced flux tube is liable to a buoyancy-driven instability (cf. Parkes 1966; D’Silva & Choudhuri 1993; Schüssler et al. 1994). Once an unstable, growing flux loop is properly located inside the superadiabatic convection zone, it rapidly rises to the stellar surface. Coriolis forces, induced by the internal plasma flow along the flux tube, cause an asymmetric evolution of the preceding- and following legs of the tube (relative to the direction of stellar rotation), which twists the upper part of the rising loop (e.g. Caligari et al. 1993). Upon eruption on the stellar photosphere, this twist entails a tilt between the polarity centres of the emerging bipole and the parallels on the stellar surface. Depending on their size and average field strength, the erupted flux tubes produce a spectrum of magnetic activity.
signatures, from small magnetic knots and pores to bi-polar spot
groups and active regions (e.g. Schrijver & Zwaan 2003). Newly
emerged surface magnetic flux dynamically disconnects from the
sub-surface parts of the parent flux tube (Schrijver & Title 1995;
Schüssler & Rempe 2005); in the case of the Sun this process is
expected to take place after a few days in a depth less than 10 Mm
below the surface. The associated upflow of entropy-rich plasma
inside the decapitated flux tube, from its anchor rooted in the over-
shoot region into the upper convection zone, weakens the magnetic
field of the tube stumps and promotes their disintegration through
turbulent diffusion driven by magneto-convective motions. The dis-
solving magnetic field may be transported downwards to the sta-
tile overshoot region through meridional circulation and convective
pumping (e.g. Tobias et al 2001), providing the seed field for fur-
ther amplification or a new activity cycle.

There is currently no detailed model for the removal of ‘old’
flux from the lower convection zone prior to the production of mag-
netic flux of the opposite polarity in the successive cycle. Neither
it is clear how, if at all, the transformation of toroidal to poloidal
magnetic fields through the twisting of erupting flux loops is rela-
ted to the re-generation of magnetic flux at the bottom of the convec-
tion zone. The lack of models prevents a prediction of the amount
and location of magnetic flux as a function of the meridional flow
speed. The value of the dimensionless drag coefficient, \( C_D \),
depends on the radius, \( a \), of the tube considered in relation
to the spectrum of length scales of the convective motions in its
vicinity. The value of the effective turbulent viscosity depends on
the specific model description of the convective energy transport.
Since it is typically much larger than the molecular viscosity, the
drag coefficient is anticipated to be of order one.

In addition to the interaction with its environment, realised
through the lateral pressure equilibrium (i.e. magnetic buoyancy)
and the hydrodynamic drag, the evolution of a magnetic flux tube
also depends on the magnetic curvature force and the Coriolis
force. In rapidly rotating stars, the interplay between the last two
forces typically causes a deflection of rising flux loops to higher lati-
itudes (e.g. Choudhuri & Gilman 1987; Schüssler & Solanki 1992;
Caligari et al 1993).

2.2 Model setup

2.2.1 Thin magnetic flux tubes

The first part of the investigation is carried out in the framework of
the thin flux tube approximation (Spruit 1981). For this approxima-
tion to be applicable, the radius of the flux tube must be smaller
than all other relevant length scales like, for example, the scale
height of the pressure or the superadiabaticity, the local radius of
curvature of the magnetic field, or the wavelength of perturbations
propagating along the tube. Assuming an ideal plasma with infinite
conductivity, the magnetic flux over a tube’s circular cross section
is conserved.

Due to the relatively short signal timescale across the tube’s
radius, the magnetic flux tube is in instantaneous pressure equi-
librium with its environment, with the sum of the gas and magnetic
pressure inside the tube balancing the gas pressure of the field-free
external plasma,

\[
p_e = p + \frac{B^2}{8\pi} \tag{1}
\]

The total pressure changes smoothly across the tube’s surface (e.g.
Stix 1989). For magnetic fields inside the convection zone the mag-
netic pressure is typically smaller than the gas pressure (i.e.
\( B^2/(8\pi) \ll p \)). Since the flux tube is impenetrable for the exter-
nal plasma, perpendicular motions relative to the tube’s axis cause
the external plasma to flow around the tube. The reaction of this
distortion on the flux tube dynamics is quantified in terms of a hy-
drodynamic drag,

\[
\vec{F}_D = \rho_e C_D \frac{\nabla p}{\pi a} \vec{v}_\perp \vec{v}_\perp, \tag{2}
\]

with \( \rho_e \) being the density of the external medium. The drag
force tries to reduce the perpendicular velocity difference, \( \vec{v}_\perp =
(\vec{v}_e - \vec{v})_\perp \), between the external and internal motions, \( \vec{v}_e \)
and \( \vec{v} \), respectively. The value of the dimensionless drag coefficient,
\( C_D \), depends on the radius, \( a \), of the tube considered in relation
to the spectrum of length scales of the convective motions in its
vicinity. The value of the effective turbulent viscosity depends on
the specific model description of the convective energy transport.
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forces typically causes a deflection of rising flux loops to higher lati-
itudes (e.g. Choudhuri & Gilman 1987; Schüssler & Solanki 1992;
Caligari et al 1993).

2.2.2 Stellar stratification and meridional circulation

We consider the evolution of magnetic flux tubes in a 1 \( M_\odot \)
star, described by a spherically symmetric model of the current
Sun (i.e. \( R_* = R_\odot = 6.96 \times 10^8 \) cm). The outer convec-
tion zone extends down to about 0.72 \( R_\odot \) and comprises at its
lower boundary a superadiabatically stratified overshoot region of
about 10 \( R_\odot \) depth. The stellar rotation period is taken to be 6 \( d \)
(\( \Omega = 1.212 \times 10^{-5} \) s\(^{-1}\)); for the sake of simplicity differential
rotation is neglected.

The meridional flow pattern, \( \vec{v}_m \), is superposed on the stellar
structure, anticipating that its influence on the underlying hydro-
static stratification is negligibly small. We follow the approach of
van Ballegooijen & Choudhuri (1988) and describe the meridional
circulation analytically through the poloidal velocity components
\( v_{p,r} \) and \( v_{p,\theta} \)

\[
v_{p,r} = u_{0} (1 + \xi^2) F \sin^m \theta \left[(m+2) \cos^2 \theta - \sin^2 \theta \right] \tag{3}
\]

\[
v_{p,\theta} = -u_{0} \frac{G}{\sin \theta}(1 + \xi^2) \left(1 - c_1 \xi^n + c_2 \xi^{n+k} \right), \tag{4}
\]

where \( F(\xi) \) and \( G(\theta) \) are univariate functions of the radius, \( r \),
and the co-latitude, \( \theta \), respectively, with \( \xi = R_*/r - 1 \); a more
detailed description of this flow model is given in Appdx. A. The
circulation is parameterised through the location of its lower bound-
ary, \( r_b \), and the dimensionless quantities \( n \), \( m \), and \( k \), which also
define the coefficients \( c_1 \) and \( c_2 \) in Eqs. (3) and (4), respec-
tively. We locate the maximum of the latitudinal flow velocity at
co-latitude \( \theta_M = 53^\circ \) to obtain a solar-like surface flow. Since the
radial profiles of stellar meridional circulations are as yet virtually
unknown, we adopt for the radial parameters \( r_b \) and \( k \) the values
used by van Ballegooijen & Choudhuri (1988). Thus, the assumed
meridional flow pattern is quantified through the set of parameters
\( r_b = 0.7 R_*, n = 1.5, k = 0.5, m = 0.76 \). It is composed of
a single-cell circulation (per hemisphere) with a poleward flow at the
stellar surface and an equatorward flow at the bottom of the convec-
tion zone (Fig. 4). The amplitude of the circulation, \( u_{0} \approx -2.47 v_M \) [cf. Eq. (11)], depends on the choice of the peak
flow velocity, \( v_M \), on the stellar surface.

2.2.3 Flux tube equilibria

The magnetic flux tubes start their evolution from a consistently de-
ned initial configuration. For this we assume a mechanical equi-
librium, which implies that their orientation is parallel to the equa-
torial plane and that their radius of curvature, \( R_0 = r_0 \cos \lambda_0 \), is
constant, where \( r_0 \) and \( \lambda_0 \) are the equilibrium radius and latitude, respectively. In the absence of meridional flows, flux rings in mechanical equilibrium are non-buoyant and the magnetic curvature force (pointing toward the axis of stellar rotation) is balanced by inertia and Coriolis forces (pointing away from the rotation axis). The Coriolis force is caused by a plasma flow inside the flux tube, pregrade to the stellar rotation. The properties of the mechanical equilibrium are described by [Spruit & van Ballegooijen 1982; Ferriz-Mas & Schüssler 1993, 1995]; for different approaches see [Choudhuri 1989; Moreno-Insertis et al. 1992; Fan et al. 1994; Caligari et al. 1998; Rempel 2003]. In the presence of an equatorward meridional flow (with the relative direction \( \vec{e}_x \) perpendicular to the tube axis) the drag force, Eq. (B3), tries to push the flux ring to lower latitudes. For a stationary mechanical equilibrium to exist (Appdx. B), the component of the Eq. (2), tries to push the flux ring to lower latitudes. For a stationary inertia and Coriolis forces (pointing away from the rotation axis).

\[
\rho_0 = \frac{C_D v_x^2}{\pi a_0 (\vec{g}_{eff} \cdot \vec{e}_z)} < 1
\]

between the external and internal density, where \( \vec{g}_{eff} \) is the effective gravitational acceleration defined in Eq. (B2). The component of the drag force perpendicular to the rotation axis opposes the magnetic curvature force so that the internal flow velocity, \( v_0 \), required to balance the tension force, is

\[
v_0 = \Omega R_0 \left( \sqrt{1 + \frac{c_A^2 - c_D^2}{(\Omega R_0)^2}} - 1 \right),
\]

where \( c_A \) is the Alfvén velocity and \( c_D \) the modification caused by the drag force in the presence of meridional circulation, given in Eq. (B8). The Alfvén velocity introduces the magnetic field dependence in the equilibrium condition.

At the assumed equilibrium radius, \( r_0 = 5.07 \cdot 10^{10} \) cm, in the middle of the overshoot region, the stellar stratification is characterised through the pressure scale height \( H_p = 5.52 \cdot 10^{10} \) cm, the gravitational acceleration \( g = 5.06 \cdot 10^4 \) cm s\(^{-2}\), the density \( \rho_e = 0.154 \) g cm\(^{-3}\), and the superadiabaticity \( \delta = -9.77 \cdot 10^{-7} \). For the meridional circulation defined above, the density contrast and internal flow velocity required for a stationary mechanical equilibrium are shown in Figs. 1 and 2, respectively.

2.3 Flux tube simulations

The stability properties of magnetic flux tubes depend on the stellar stratification in its environment (including the meridional flow topology) and the magnetic field strength. The determination of instability criteria and characteristic growth times requires a specific linear stability analysis similar to [Spruit & van Ballegooijen 1982; Van Ballegooijen & Choudhuri 1988; Ferriz-Mas & Schüssler 1995], which is however beyond the scope of the present paper. Investigations in the case of the Sun and other cool stars consistently show that the evolution of flux tubes with initial radii larger than \( \sim 1000 \) km are only marginally affected by the drag force, and that magnetic field strengths \( \gtrsim 10^5 \) G are required to initiate rising flux loops whose properties upon eruption at the surface are in agreement with observations (e.g. [Choudhuri & Gilman 1983; Fan et al. 1994; Schüssler et al. 1994; Schrijver & Zwaan 2000]. We accomplish simulations for the meridional flow pattern defined above with peak flow velocities \( v_{\text{shear}} \leq 125 \) m s\(^{-1}\).

The simulations start with the perturbation of an equilibrium flux ring located in the middle of the overshoot region within the range of latitudes \( \lambda_0 = 5 \sim 75^\circ \). The initial perturbation consists of a superposition of harmonic displacements with low wave numbers and amplitudes of a few percent of the local pressure scale height. The adiabatic evolution of individual flux tubes is followed using a non-linear Lagrangian scheme described in more detail in [Moreno-Insertis 1986] and [Caligari et al. 1995]. Owing to the lateral pressure balance, Eq. (1), the summit of a rising flux loop expands as the external pressure decreases. The simulations stop just beneath the stellar surface \( r > 0.98 R_\star \), where the underlying thin flux tube approximation becomes inapplicable. In the uppermost part of the convection zone the rise of the tube summit is dominated by strong magnetic buoyancy so that the tube trajectory is almost radial. The final point of the simulations thus determines the location of the emergence of a bi-polar spot group.

**Figure 1.** Density contrast of a magnetic flux ring in mechanical equilibrium at latitude \( \lambda_0 \) and depth \( r_0 = 5.07 \cdot 10^{10} \) cm. The peak velocity of the meridional flow is \( v_M = 0, 25, 75, 100, 125 \) m s\(^{-1}\) (top to bottom) and the radius of the flux tube \( a_0 = 10^7 \) cm.

**Figure 2.** Flow velocity inside a magnetic flux ring in mechanical equilibrium at latitude \( \lambda_0 \) and depth \( r_0 = 5.07 \cdot 10^{10} \) cm. The meridional flow velocity is \( v_M = 0, 25, 75, 100, 125 \) m s\(^{-1}\) (top to bottom), the radius of the flux tube \( a_0 = 10^7 \) cm, and the field strength \( B_0 = 15 \cdot 10^4 \) G. Positive (negative) values correspond to flow velocities faster (slower) than the stellar rotation.
2.4 Latitudinal distributions of magnetic flux eruption

The interplay between Coriolis and magnetic tension force results in a deflection of rising flux loops to higher latitudes. Without a meridional circulation the deflection is less than \(\Delta \lambda = \lambda_e - \lambda_0\), with \(\lambda_e\) being the eruption latitude on the stellar surface. The deflection is greatest for flux tubes which originate from low initial latitudes and decreases for higher starting latitudes. In the presence of meridional flows the poleward deflection is considerably larger and shows a characteristic dependence on the initial latitude of the flux ring (Fig. 3). Flux tubes originating from low to intermediate latitudes (here, around \(\lambda_0 \sim 25^\circ\)) are most affected by poloidal flows, whereas those starting from equatorial or high latitudes are in contrast only slightly more deflected than in the absence of an external circulation (Fig. 4). This latitude-dependent deflection resembles closely the initial density contrast and internal velocity variation (Figs. 2 and 3, respectively), suggesting that the strong deflection is partly caused by the equilibrium conditions.

The latitudinal distribution patterns obtained for different meridional flow velocities are qualitatively similar, with the amplitude of the pattern increasing with the peak flow velocity (Figs. 4 and 5). Whereas flux tubes with smaller radii show a characteristic deflection pattern with a maximum at intermediate latitudes, flux tubes with radii larger than 200 – 300 km yield an eruption pattern in which the poleward deflection continuously decreases from low to high initial latitudes (Fig. 4b). Flux tubes with larger initial radii show deflections only slightly larger than in the case without a meridional circulation. For magnetic flux tubes with high field strengths the latitudinal deflection is weaker (Fig. 4b), since the relative impact of the magnetic buoyancy on the dynamical evolution is larger. Their trajectory through the convection zone is more radial, with minor latitudinal deflections only.

Figure 3. Latitudinal trajectories of the summit of rising flux loops with initial field strength \(B_0 = 15 \cdot 10^4\) G and tube radius \(a_0 = 10^7\) cm for maximal flow velocities \(v_M = 10, 75,\) and 100 m \(\cdot\) s\(^{-1}\) (grey shaded lines). The flow of the meridional circulation (solid lines, \(n = 1.5, m = 0.76, k = 0.5, r_b = 0.7 R_\ast\)) is poleward at the surface and equatorward at the bottom of the convection zone. The shaded region at the bottom of the convection zone marks the overshoot region.

Figure 4. Latitudinal deflection, \(\Delta \lambda = \lambda_e - \lambda_0\), of erupting flux tubes in the presence of meridional flows with different peak flow velocities (Panel a, for \(B_0 = 15 \cdot 10^4\) G and \(a_0 = 10^7\) cm); initial tube radii (Panel b, for \(v_M = 100\) m \(\cdot\) s\(^{-1}\) and \(B_0 = 15 \cdot 10^4\) G); initial magnetic field strengths (Panel b, for \(v_M = 100\) m \(\cdot\) s\(^{-1}\) and \(a_0 = 10^7\) cm). Gaps in the curves for high flow velocities and low magnetic field strengths are due to stable flux ring equilibria for the respective set of parameters.

Figure 5. Dependence of the poleward deflection, \(\Delta \lambda = \lambda_e - \lambda_0\), on the meridional flow velocity, \(v_M\).

2.5 Comments

Thin magnetic flux tubes with weak average field strengths are found to be susceptible to meridional circulations. In particular flux tubes originating from low to intermediate latitudes are subject to significant poleward deflections during their rise to the stellar surface. We conjecture that this latitude-, size-, and field strength-dependent deflection causes the bulk of low-flux elements to erupt at higher latitudes than more substantial flux tubes. Compared to stars with weak or no meridional flows, this separation effect causes
a bias in the overall surface distribution of erupting magnetic flux toward higher latitudes.

The enhanced susceptibility of smaller magnetic flux elements inside the convection zone to meridional flows is in basic agreement with the properties of magnetic flux features observed on the solar surface. There, smaller flux elements are readily transported to higher latitudes, whereas new active regions and sunspots (in their entity) hardly participate in the poleward motion ($\lambda \approx 10 \, \text{m} \cdot \text{s}^{-1}$). However, after their fragmentation through supergranular motions, the dissolving remnants of initially large flux concentrations are swept polewards as well.

3 STELLAR BUTTERFLY DIAGRAMS

The latitudinal distribution of erupting flux tubes depends in a non-linear way on the magnitude of the meridional circulation as well as on the magnetic flux and original latitude of the tube inside the overshoot region. In the following, we consider the emergence pattern of flux tubes with an initial magnetic field strength $B_0 = 15 \cdot 10^4 \, \text{G}$ and an initial radius $a_0 = 100 \, \text{km}$ as representative for the full range of parameters considered in Sect. 2.

Surface flux transport models require a description of the statistical properties of newly emerging bipoles during an activity cycle, including their emergence rates, latitudes, sizes, fluxes, and tilt angles. Since the observational data base on rapidly rotating solar-like stars is insufficient to create empirical butterfly diagrams, we adopt the solar butterfly diagram as a qualitative template, which we extrapolate in latitude and scale according to the results in Sect. 2. The solar butterfly diagram is characterised by emergence latitudes between about $40^\circ$ latitude at the start of a cycle and $5^\circ$ at its end. The width of the wing of each cycle is about $10^\circ$ at the beginning of the cycle and decreases to $5^\circ$ at the end. The cycle period is $11 \, \text{yr}$, excluding one-year overlaps of successive cycles (van Ballegooijen et al. 1998; Mackay et al. 2004).

3.1 Latitudes of emergence vs. latitudes of origin

Based on the simulation results in Sect. 2, we associate latitudinal ranges of magnetic flux emergence, $\lambda_e$, with latitudinal ranges of origin, $\lambda_0$, of the parent flux tubes in the overshoot region. For the reference (i.e. solar) butterfly diagram, we refer to the case $v_M = 0 \, \text{m} \cdot \text{s}^{-1}$, since the weak solar meridional circulation with a peak velocity of $\sim 10 \, \text{m} \cdot \text{s}^{-1}$ has little effect on the sub-surface evolution of rising flux tubes. In this case, the relationship between $\lambda_e$ and $\lambda_0$ is virtually linear (Fig. 6, solid line/asterisks), and we ascribe the emergence of bipolar regions on the Sun to magnetic flux tubes which originate from within the range of latitudes $\lambda_0 \approx 35^\circ - 5^\circ$. Further assignments between latitudes of emergence and latitudes of origin are summarised in Table 1. We shall refer to the small ($\lambda_0 = 35^\circ - 5^\circ$), intermediate ($47^\circ - 3^\circ$), and large ($58^\circ - 3^\circ$) ranges of originating latitudes as case S, I, and L, respectively.

Table 1. Correspondence between the latitudes of origin, $\lambda_0$, and latitudes of emergence, $\lambda_e$, of flux tubes with an initial field strength of $B_0 = 15 \cdot 10^4 \, \text{G}$ and radius $a_0 = 100 \, \text{km}$ (see Fig. 6).

| Case | $\lambda_0[^{\circ}]$ | $v_M = 0 \, \text{m} / \text{s}$ | 50 m/s | 75 m/s | 100 m/s |
|------|----------------------|-----------------|--------|--------|--------|
| S    | 35 – 5               | 40 – 10         | 44 – 10 | 47 – 13 | 54 – 15 |
| I    | 47 – 3               | 50 – 10         | 54 – 8  | 47 – 9  | 60 – 9  |
| L    | 58 – 3               | 60 – 10         | 61 – 9  | 63 – 9  | 65 – 9  |

3.2 The wings of the butterfly

In contrast to the solar reference case, strong meridional circulations imply non-linear relationships between the eruption latitudes of magnetic flux tubes and the latitudes of their origin (Fig. 6) and, consequently, distortions of the wing shapes of stellar butterfly diagrams. To transform the solar template into a stellar butterfly diagram, we determine the ratio between the latitudes of emergence, $\lambda_e$($v_M$), subject to meridional flows and the latitudes of emergence, $\lambda_e$($0$), of the reference case with vanishing meridional flow. The deflection ratios $\lambda_e(v_M)/\lambda_e(0)$ in Fig. 6 represent mappings, which describe the non-linear stretching of the solar template. In addition to the poleward displacement of flux emergence at intermediate latitudes also a slight equatorward displacement occurs at low latitudes.

Examples of stellar butterfly diagrams subject to meridional circulations with different peak velocities are shown in Fig. 7 (for case I). Figure 7a depicts a stellar butterfly diagram subject to a weak meridional circulation (i.e. the solar template linearly stretched to the latitudinal range $50^\circ - 10^\circ$), whereas the Figs. 7b & c show the result of the non-linear transformation of the latitudinal pattern of emerging bipoles caused by higher flow velocities. The diagrams show that for strong meridional circulations the number of bipoles emerging at higher latitudes is increased: the higher the flow velocity, the more convex is the temporal evolution of the stellar butterfly diagram. Qualitatively similar results are obtained for the cases S and L.
4 SURFACE EVOLUTION AND INTERMINGLING OF MAGNETIC BIPOLES

4.1 Basic scenario and empirical constraints

After the dynamical disconnection of newly emerged bipoles from their parent flux tubes, the small-scale evolution of magnetic surface features is governed by local magneto-convective motions and interactions with ambient magnetic flux concentrations. Advection of flux with the same (opposite) polarity through the convective turnover of supergranular cells leads to an enhancement (annihilation) of magnetic flux. Averaged over length scales larger than supergranular cells (in the case of the Sun about 30 Mm) the evolution of the magnetic field resembles a dispersion process [Leighton 1964]. The lifetime of individual magnetic surface features depends on the diffusion time scale. The evolution of the global magnetic field topology, in turn, depends on the relation between the lifetimes of merging flux features and the characteristic transport timescales of the differential rotation and the meridional circulation. Differential rotation shears and diverges bipolar spot groups spread out over a range of latitudes. The increasing distance between the centres of opposite flux polarities reduces the rate of mutual flux annihilation, whereas the associated decrease of the tilt angle reduces the bias in the latitudinal distribution of magnetic polarities (see also Appdx. C). Merging magnetic surface features are transported to higher latitudes by the poleward directed meridional circulation and pile-up in polar regions. Owing to the biased latitudinal polarity distribution, the polar flux is dominated by one polarity during half of the magnetic cycle. With the reversal of the polarity of newly emerging bipolar groups after half a magnetic cycle (Hale’s law), magnetic flux of the opposite polarity is predominantly transported toward the pole, which annihilates and substitutes the previous flux conglomeration. This alternating process implies a stable oscillation of the net surface flux over successive activity cycles, without a persistent accumulation of magnetic flux. For a review about the development and limitations of surface flux transport model see [Sheeley 2005].

In contrast to the case of the Sun, where the polar magnetic field is virtually unipolar, with magnetic flux densities of about 10 Mx cm\(^{-2}\), the surface magnetic fields observed on young active stars exhibit a high degree of intermingling of magnetic flux polarities [Donati & Collier Cameron 1993; Donati et al. 1999, 2003] and dark polar caps [Strassmeier 2002], indicative for strong magnetic field strengths [Schrijver & Title 2001] found that flux emergence rates thirty times larger than on the Sun entail polar flux densities of 300–500 Mx cm\(^{-2}\), which are deemed to be sufficiently large to suppress convective upflows of energy, entailing the formation of dark spots (e.g. Priest 1982). In their simulations, the polar magnetic flux is unipolar and surrounded at lower latitudes by a ring of magnetic flux of the opposite polarity. Focusing on the intermingling of polar magnetic flux, Mackay et al. (2004) identified two key parameters for the generation of high degrees of polarity mixture: magnetic bipoles have to emerge at latitudes up to 50°–70°, and the peak value of the meridional flow has to be over 100 m s\(^{-1}\).

The simulations of Mackay et al. (2004) disregard the feedback of strong meridional circulations on the latitudinal pattern of flux eruption. This feedback is now provided through the consistently determined stellar butterfly diagrams in Sect. 3. In the following, we use the Mackay et al. model to investigate the impact of the convex wing structures on the polar magnetic field properties.

4.2 Surface flux transport model

The surface flux transport simulations of Mackay et al. (2004) follow the radial magnetic field component on the stellar surface regarding the combined influence of flux emergence, differential rotation, meridional flow, and supergranular diffusion. Using spherical harmonics up to degree 63, the spatial resolution is about 30 Mm. The simulations are based on the
extrapolation of solar transport and flux emerging properties (Gaizauskas et al. 1983; Wang & Sheeley 1983; Harvey & Zwaan 1993; Schrijver & Harvey 1994; Tian et al. 1999) to more rapidly rotating stars (see also Schrijver & Title 2001).

The strength and profile of the differential rotation of rapidly rotating stars is similar to the solar case (e.g. Collier Cameron et al. 2002). We therefore adopt the solar differential rotation profile given by Snodgrass (1983), which implies the characteristic shear timescale $\tau_\Omega = 0.25 \, \text{yr}$. The meridional flow profile is given by $v_p(\lambda) = -v_M \sin (\pi \lambda / \lambda_p)$, whereby the flow velocity vanishes near the pole above $\lambda_p = 75^\circ$ (Hathaway 1996). The characteristic timescale for the latitudinal flux transport is $\tau_{\text{lat}} = R_\odot / v_M$. Lacking appropriate empirical constraints, it is assumed that the supergranular convective profile and turn-over timescales of rapidly rotating stars are solar-like, implying a diffusion coefficient of $D = 450 \, \text{km}^2 \cdot \text{s}^{-1}$ (van Ballegooijen et al. 1998) and a diffusion timescale for magnetic features with length scale $l$ of $\tau_D = l^2 / D$ ($= 34 \, \text{yr}$ for $l = R_\odot$).

Following Schrijver & Title (2001), we take the net amount of magnetic flux emerging during each cycle to be thirty times the solar value, that is $3.15 \times 10^{26} \, \text{Mx}$. New flux is injected into the evolving surface magnetic field distribution at random longitudes in the form of $13,200$ bipolar magnetic regions, whose average tilt angles vary as $\lambda_p / 2$ (Wang & Sheeley 1983). In each successive cycle the polarity of the preceding and following flux of the bipolar alternates according to Hale’s law. The simulations cover a time span of four activity cycles to verify that stable oscillations of the polar field from one cycle to the next are obtained. For further details about model assumptions, input parameters, and initial conditions see Mackay et al. (2004). Sect. 2-4.

In the following, we focus on the total (unsigned) magnetic flux,

$$\Phi_{\text{tot}}(t) = \int |B_r(R, \theta, \phi, t)| \, dS ,$$

the total (unsigned) polar magnetic flux in each hemisphere,

$$\Phi_{\text{polar}}(t) = \int |B_r(R, \theta, \phi, t)| \, dS ,$$

the imbalance of positive and negative magnetic flux in the polar regions,

$$\delta_{\text{polar}}(t) = \int B_r(R, \theta, \phi, t) \, dS ,$$

and the polar magnetic flux in the northern hemisphere,

$$\Phi_{\text{n, polar}}(t) = \int B_r(R, \theta, \phi, t) \, dS .$$

The principal quantities of the analysis will be mean magnetic flux densities, that is the respective magnetic flux divided by the surface over which it is calculated (see Mackay et al. 2004). The last quantity, Eq. (10) is a measure for the degree of intermingling of opposite polarity elements at high latitudes.

![Figure 10](image)

**Figure 10.** Panel a: Cycle-averaged ratio between the positive and negative magnetic flux within the northern polar cap as a function of the meridional flow velocity, $v_M$, for originating latitudes $\lambda_0 = 35^\circ$ (triangles); $47^\circ$ (diamonds); and $57^\circ$ (asterisks). Panel b: Cycle-averaged polar flux densities.

#### 4.3 Surface magnetic field properties of rapid rotators

We consider weak and strong meridional flows of the case I to detail our results (Fig. 8). For both flow velocities the mean surface flux density, $\Phi_{\text{surf}}/(4\pi R^2)$, is in phase with the activity cycle emergence rate (Fig. 8a & i). If the meridional flow velocity is high, then more magnetic flux is pushed to higher latitudes, where the proximity of opposite bipole polarities increases the flux annihilation rate. In this case, the mean surface flux densities are consequently smaller than for slow meridional circulations. The resulting variation of the mean polar flux density (i.e. beyond $70^\circ$ latitude) is shown in Figs. 8 & g. In the case of weak meridional flows this quantity shows a noticeable phase shift relative to the mean surface flux density, with a cycle-averaged polar flux density of $\sim 192 \, \text{G}$. In the case of fast flows the polar field rises and falls in phase, with a significantly higher cycle-averaged polar flux density of $\sim 270 \, \text{G}$. Breaking up the polar flux density imbalance (Figs. 8 & h) according to the contributions of positive and negative flux (Fig. 8k & l) shows that for slow meridional flow velocities the polar field is unipolar, with a field reversal midway through the cycle. For fast flow velocities, in contrast, both magnetic polarities are present throughout the cycle and rise and fall in strength together. The ratio between the positive and negative flux densities in the polar caps is thus in both cases different (Fig. 8j & j). For weak flows there is hardly any intermingling of flux of opposite polarities at high latitudes, except for a limited time span during field reversal, when little magnetic flux is located within the polar cap. For strong meridional flows, in contrast, the intermingling of magnetic flux with opposite polarities is about $65\%$, averaged over an activity cycle.

Snapshots of maps showing the radial magnetic field component (Fig. 8) illustrate that in the case of weak flows the magnetic flux in polar regions is dominated by one polarity only, whereas in the case of fast flows the field is intermingled.

#### 4.3.1 Parameter study

For the latitudinal ranges I and L, the degree of intermingling of opposite polarities within the polar regions exceeds $20\%$ for meridional flow velocities beyond about $40 \, \text{m} \cdot \text{s}^{-1}$ (Fig. 10b). The fact that in case I the intermingling is higher than in case L indicates that merely increasing both the meridional flow velocity and the latitudinal range of flux origin does not produce per se higher degrees of intermingling. Instead, there exists an optimal combination of latitudes of origin and meridional flow velocities for which the intermingling within polar caps peaks.
Meridional flows, butterfly diagrams, and polar caps

Figure 8. Results of the surface flux transport simulations for meridional circulations with a weak flow velocity of 11 m·s\(^{-1}\) (Panels a–e), and a strong flow velocity of 100 m·s\(^{-1}\) (Panels f–j); the associated butterfly diagrams are shown in Figs. 7a and 7c, respectively, with generating flux tubes originating from within the range of latitudes \(\lambda_0 = 47 - 3^\circ\) (Case I). Panels a & e: mean surface flux density, \(\Phi_{\text{tot}}/(4\pi R^2)\); panels b & g: mean polar flux density of the northern (solid) and southern (dotted) hemispheres; panels c & h: signed polar flux densities in the northern (solid) and southern (dotted) hemisphere, with dashed lines representing the initial values; panels d & i: positive (solid) and negative (dotted) polar flux density of the northern hemisphere; panels e & j: ratio between the positive and negative polar flux density of the northern hemisphere. Each quantity is determined once every 27 days. Dashed-dotted lines mark the start of a new cycle. All flux densities are in Mx·cm\(^{-2}\).

The associated polar flux densities are on the order of 200 – 300 Mx·cm\(^{-2}\) (Fig. 10b), with values increasing (decreasing) with the meridional flow velocity above (below) \(v_M \sim 50\) m·s\(^{-1}\). This behaviour is due to the different transport- and annihilation-timescales (see Appdx. C). In the case of strong meridional flows, the advection of magnetic flux to higher latitudes is more efficient than the decrease caused by flux annihilation as the magnetic surface features are transported poleward. The same argument holds for the increase of the polar flux density with larger latitudinal ranges of flux origin, since for bipoles emerging at higher latitudes the shorter migration time towards the pole implies less flux annihilation. For meridional flow velocities around 100 m·s\(^{-1}\) the polar flux densities become sufficiently large (\(\sim 300\) G) to suppress convection.

In case S, the polar magnetic flux is unipolar throughout the activity cycle, without a significant intermingling of opposite polarities. The average polar flux densities of \(\approx 300\) G are almost independent of the meridional flow velocity.

In summary, we find that a combination of intermediate ranges of originating latitudes (here, \(\lambda_0 = 47 - 3^\circ\)) with fast meridional flow velocities (\(v_M \sim 100\) m·s\(^{-1}\)) is optimal for the production of both strong intermingling of polarities and high field strengths in polar regions. This suggests that in rapidly rotating solar-like stars magnetic flux originates from latitudinal ranges somewhat larger than in the case of the Sun.

4.3.2 Effect of the pre-eruptive poleward deflection

Mackay et al. (2004) disregard the influence of meridional circulations on the sub-surface evolution of magnetic flux. In their investigation the reference (i.e. solar) butterfly diagram is linearly stretched to different latitudinal ranges, leaving the structure of the wings unaltered. According to our results in Sect. 5.2, the eruption pattern within a given range of emergence latitudes depends on the meridional flow velocity, with more bipoles emerging at higher latitudes the faster the flow velocity. We compare the two approaches in the case of a meridional circulation with a flow velocity of 100 m·s\(^{-1}\). Keeping other model parameters unchanged, including the pre-eruptive poleward deflection entails about 20% higher
Figure 9. Maps of the radial magnetic field component in the case of weak (10 m·s$^{-1}$, left column) and strong (100 m·s$^{-1}$, right column) meridional flow velocities. The associated butterfly diagrams (cf. Fig. 7b & c) are generated by magnetic flux tubes which originate from within the latitudinal range $\lambda_0 = 47° - 3°$. Each set shows snapshots after year 1 (cycle minimum, Panels a & e), 5 (before maximum, Panels b & f), 8 (after maximum, Panels e & g), and 11 (declining phase, Panels d & h). White and black shadings represent positive and negative magnetic flux densities, respectively, saturating at $\pm300 \text{Mx} \cdot \text{cm}^{-2}$.

polar flux densities than in the original approach of Mackay et al. This result applies to all ranges of generating latitudes (Table 2). The reason for this increase lies in the more convex structure of the wings of the butterfly (cf. Fig. 7). The larger number of emerging bipoles at higher latitudes over an activity cycle implies shorter transport times (on average) toward the pole during which less flux annihilation occurs.
5 DISCUSSION

The present work relates observed properties of magnetic flux on the stellar surface to the sub-surface evolution of the originating magnetic fields and, therefore, to the underlying dynamo processes inside the convective envelope. Our results extend the findings of Mackay et al. (2004), who identified high latitudes of magnetic flux emergence and high meridional flow velocities as the key ingredients for a strong mixing of opposite polarities in polar regions of rapidly rotating stars. We find that the enhanced pre-eruptive poleward deflection of rising flux tubes inside the convection zone provides a consistent explanation for the required larger latitudinal range for newly emerging bipoles.

Within the framework of our model assumptions, a solar-like butterfly diagram with flux emergence up to 40° latitude is generated by erupting magnetic flux tubes, which originate at the bottom of the convection zone from latitudes ≤ 35°. Owing to the slight prolateness of the solar tachocline (e.g. Charbonneau et al. 1999), these low latitudes coincide in the case of the Sun well with the region of strong shear flows, efficient magnetic field amplification, and field storage. For more rapidly rotating stars, the butterfly structures producing the optimal levels of polar flux intermingling are generated by erupting flux tubes, which originate from higher latitudes, ≤ 50°, as well. This suggest the existence of a larger latitudinal range of efficient dynamo operation than in the case of the Sun, implying a possibly less prolate tachocline.

Based on the results of Schröder & Title (2001), we assumed a magnetic flux emergence rate thirty times larger than the solar value to produce polar flux densities sufficiently strong to entail dark caps. Dynamo operation is generally expected to be more efficient at higher rotation rates, mainly entailing an increase of the surface filling factor of magnetic features (e.g. Saar 1991). The larger latitudinal range of dynamo operation suggested by our results complies with this expectation of a larger amount of magnetic flux generated during an activity cycle. Considering a possible concomitant increase of the average magnetic field strength, we note that strong-field flux tubes are less susceptible to meridional circulations and therefore hamper the formation of high flux densities and high degrees of flux intermingling in polar regions. We also find that flux tubes with initial radius \( a_0 \geq 1000 \text{ km} \), which are expected to lead to the formation of average sunspots, are only marginally affected by meridional flows. Since the magnitude of the drag force scales \( \propto v_{M}^2 / a \), a meridional flow velocity about three times faster would correspond roughly to a flux tube radius about ten times smaller, entailing a noticeable change of the eruption latitude. In view of the uncertainties of the sub-surface flow profile, this estimate indicates that also larger magnetic flux elements could be affected by meridional circulations.

By using the solar butterfly diagram as an empirical template, we circumvent the treatment of the magnetic field amplification and dissolving mechanisms inside the stellar convection zone, which as yet lack a consistent theoretical description. This approach, however, implies that the activity signatures of rapidly rotating stars are due to a solar-like dynamo. The assumption of an \( \alpha \Omega \)-dynamo mechanism has important implications, for example, on the period, and even the mere existence, of the activity cycle. The 11 yr activity cycle assumed here sets the time scale for the pile-up of magnetic flux in polar regions. If we, for example, increase the cycle length, then we would obtain qualitatively similar results by increasing the total amount of magnetic flux emerging per cycle. A conclusive parameter study of different ratios between eruption, transport, and diffusion time scales is however beyond the scope of this work; a discussion of the impact of selected surface transport model parameters is given by Mackay et al. (2004).

Our work supplements earlier investigations, which considered the evolution of the weak solar magnetic field subject to a prescribed meridional flow pattern and a dynamo wave at the bottom of the convection zone (e.g. Dikpati & Choudhuri 1994, Choudhuri et al. 1995, Choudhuri & Dikpati 1999). All approaches, however, suffer from the same lack of knowledge about the sub-surface profiles of meridional circulation and their dependence on the rotation, mass, structure, and evolutionary stage of the star. Owing to the density difference between the top and the bottom of the convection zone, the equatorward flow is slower than the surface flow, and the larger the radial extent of the equatorward flow region, the lower the associated velocities will be. The latitudinal flow velocity at the bottom of the convection zone possibly has an influence on the propagation velocity of dynamo waves and, consequently, on the period of the activity cycle. But in absence of an appropriate theory, this impact cannot be quantified. The circulation model applied here assumes a ratio of 2:1 between the upper convection zone flowing poleward and the lower convection zone flowing toward the equator. A different radial profile would not only affect the equator- and poleward drift of rising flux tubes on their way to the surface, but also their equilibrium and stability properties inside the overshoot region. The former aspect has to be investigated in a parameter study considering both meridional flow and differential rotation profiles as well as different stellar rotation periods. Furthermore, specifically determined tilt angles and eruption time scales of newly emerging bipoles have to be consistently included in the surface flux transport model.

Compared with a mere linear stretching of the solar butterfly diagram to higher latitudes (e.g. Mackay et al. 2004), the additional convexity of the butterfly wings promotes the intermingling of polar flux even more, which eases the necessity for high poleward flow velocities. Although the meridional flow velocities of \( \sim 100 \text{ m} \cdot \text{s}^{-1} \) are lower than the values suggested by Mackay et al. (2004), they are yet still about ten times larger than in the case of the Sun. Whereas in the case of the Sun, with \( v_{M} \approx 11 \text{ m} \cdot \text{s}^{-1} \), the bias in the latitudinal size distribution of newly emerging bipoles is probably neither discernible nor relevant, convex butterfly structures and the size distributions of emerging bipoles may become observable on rapidly rotating stars once the temporal and spatial resolutions are sufficiently high. Strong meridional circulations on the order of \( 100 \text{ m} \cdot \text{s}^{-1} \) are also expected to cause a shift of the stellar eigenfrequencies, in addition to the splitting of eigenfrequencies caused by differential rotation (M Roth, priv. comm.) Depending on the sub-surface profile of the flow, a meridional circulation of \( 100 \text{ m} \cdot \text{s}^{-1} \) would cause in the case of the Sun a frequency shift of a few tenth of \( \mu \text{Hz} \). Albeit small, this effect would be observable on the Sun. Given sufficiently long and complete time sequences, astroseismological observations may provide con-
6 CONCLUSION

Strong meridional circulations enhance the poleward deflection of rising magnetic flux tubes inside stellar convection zones. Flux tubes which comprise small amounts of magnetic flux and which originate between low to intermediate latitudes within the overshoot region are particularly susceptible to meridional flows. The resulting latitudinal distribution of newly emerging bipoles renders the wing structure of stellar butterfly diagrams distinctively more convex than in the solar case. The larger amount of magnetic flux emerging at higher latitudes supports the formation of high magnetic flux densities and a significant intermingling of opposite-polarity magnetic flux in polar regions, which is in agreement with recent observational results of rapidly rotating solar-like stars.

The strong pre-eruptive deflection of magnetic flux tubes provides a consistent explanation for the required high latitudes of flux emergence identified by previous investigations [Mackay et al. 2004]. The synergetic combination of pre-eruptive flux tube deflection and post-eruptive bipole transport yields higher values for the average polar flux densities than in the previous approach, and thus eases the necessity for high meridional flow velocities. The functional dependence of the polar field properties on the extent and structure of the stellar butterfly diagram makes it possible to conjecture about potential regions of efficient dynamo operation at the bottom of the convection zone. In rapidly rotating cool stars, we suppose the latitudinal range of magnetic flux production to extend to higher latitudes (\( \lesssim 50^\circ \)) than in the case of the Sun (\( \lesssim 35^\circ \)).

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APPENDIX A: MERIDIONAL FLOW MODEL

We use the analytical model of van Ballegooijen & Choudhuri (1988) for a parametrized description of the meridional circulation inside the convection zone (see also Dikpati & Choudhuri 1994). The poloidal flow pattern, \( \vec{v}_p = (\nabla \times \Psi \vec{e}_\phi) / \rho_c \), is expressed in terms of the scalar function, \( \Psi \), to ensure that the stationary continuity equation is consistently fulfilled. Under the assumption of a separation of variables, the stream function \( \Psi(r, \theta) \) is

\[
\Psi(r, \theta) = u_0 R^2 \rho_c(r) \cdot F(r) \cdot G(\theta),
\]

(A1)

is expressed in terms of the univariate functions

\[
F(r) = \left( -\frac{1}{n+1} + \frac{c_1}{2n+1} \xi^n - \frac{c_2}{2n+k+1} \xi^{n+k} \right) \xi^n \quad \text{(A2)}
\]

\[
G(\theta) = \sin^{m+2} \theta \cos \theta, \quad \text{(A3)}
\]

with the rescaled radial coordinate

\[
\xi = \frac{R}{r} - 1. \quad \text{(A4)}
\]

The coefficients

\[
c_1 = \frac{(2n+1)(n+k)}{(n+1)k} \xi_{\theta}^{-n} \quad \text{and} \quad c_2 = \frac{(2n+k+1)n}{(n+1)k} \xi_{\phi}^{-(n+k)} \quad \text{(A5)}
\]

depend on the location of the lower boundary, \( \xi_b = R_b / r_b - 1 \), of the flow pattern, with \( r_b \) being a free model parameter. The poloidal components of the flow velocity are

\[
v_{p,r} = -u_0 \frac{R}{r} \cos \theta \cdot F \frac{\partial F}{\partial \theta} \quad \text{and} \quad v_{p,\theta} = -u_0 R^2 \sin \theta \cdot F_{\theta} \frac{\partial F}{\partial r} \frac{d \ln \rho_c}{dr} \quad \text{(A7)}
\]

of the circulation is determined through the maximal flow velocity, \( v_M \).

APPENDIX B: FLUX TUBE EQUILIBRIUM

The mechanical equilibrium in the presence of meridional flows is determined in the frame of reference co-rotating with the star. For a homogeneous flux tube (i.e. vanishing derivatives with respect to the arc length), the stationary equation of motion is

\[
\rho \left( v^2 - \frac{B^2}{4\pi \rho} \right) \frac{1}{R} \vec{n} = - \nabla \left( p + \frac{B^2}{8\pi} \right) + \rho g_{eff} + 2\rho (\vec{v} \times \vec{\Omega}) \quad \text{,(B1)}
\]

where \( \vec{v} \) is the internal flow velocity, \( \vec{B} \) the magnetic field strength, \( \rho \) the density, \( p \) the gas pressure, \( R \) the local radius of curvature, \( \vec{n} \) the local normal to the tube, and \( g_{eff} \) the effective gravity.
the normal vector of the tube, \( \vec{\Omega} = \Omega \vec{e}_z \), the stellar rotation vector, and
\[
\vec{g}_{eff} = \vec{g} - \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) = g(r) \left( \vec{e}_r + \vec{e}_\theta \frac{\Omega^2 r}{g} \sin \theta \right)
\]  \( \text{(B2)} \)
the effective gravitational acceleration comprising the centrifugal contribution, which (for the model assumptions specified in Sect. 2.2.3) inside the overshoot region is of the order \( O \left( \Omega^2 r / g \right) \sim 10^{-4} \).
Anticipating the meridional circulation to be of minor importance for the stellar structure, the external stratification is approximately hydrostatic,
\[
\nabla p_e = \rho_e \vec{g}_{eff} .
\]  \( \text{(B3)} \)
Using the lateral pressure balance, Eq. \( \Omega \), the difference between Eqs. \( B1 \) and \( B3 \) yields
\[
\left( v^2 - \frac{B^2}{4 \pi \rho} \right) \kappa \vec{n} = \left( 1 - \frac{\rho_e}{\rho} \right) \vec{g}_{eff} + 2 \left( \vec{\vec{v}} \times \vec{\Omega} \right) + \vec{f}_D ,
\]  \( \text{(B4)} \)
with the hydrodynamic drag force, \( \vec{f}_D \), accounting for the influence of the distorted external flow on the dynamics of the flux tube. For toroidal magnetic flux tubes with a constant radius of curvature, the component of Eq. \( B4 \) tangential to the tube axis (i.e. the azimuthal component) vanishes per se. The bi-normal component (i.e. parallel to axis of stellar rotation, \( \vec{e}_z \)) implies the condition
\[
\left( 1 - \frac{\rho_e}{\rho} \right) \left( \vec{g}_{eff} \cdot \vec{e}_z \right) = - \frac{1}{\rho} \left( \vec{f}_D \cdot \vec{e}_z \right).
\]  \( \text{(B5)} \)
With the definition for \( \vec{f}_D \) in Eq. \( B2 \), this yields the density contrast given in Eq. \( C \), which is required to balance the influence of the drag force along the z-axis through buoyancy. The component perpendicular to the stellar rotation axis \( (\vec{e}_r = -\vec{n}) \),
\[
\left( v^2 - \frac{B^2}{4 \pi \rho} \right) \frac{1}{R} + \left( 1 - \frac{\rho_e}{\rho} \right) \left( \vec{g}_{eff} \cdot \vec{e}_r \right) + 2 \nu \frac{\Omega^2}{R} = - \frac{1}{\rho} \left( \vec{f}_D \cdot \vec{e}_r \right)
\]  \( \text{(B6)} \)
shows that a meridional flow toward the equator in the overshoot region supports the outward directed inertia and Coriolis force in balancing the magnetic tension force of the curved flux ring, where \( R = r \cos \lambda \) is the constant radius of curvature. From the alternative form of Eq. \( B6 \),
\[
\frac{v^2}{R^2} + \kappa \frac{c_D}{R} = \frac{c_A^2}{R}
\]  \( \text{(B7)} \)
follows the internal flow velocity given in Eq. \( C \), which is required to keep the flux ring in mechanical equilibrium. The contribution by the drag force is formally expressed in terms of the velocity
\[
\frac{c_D}{R_0} = \left( 1 - \frac{\rho_e}{\rho_0} \right) \left( \vec{g}_{eff} \cdot \vec{e}_r \right) + \frac{\rho_e}{\rho_0} \frac{C_D}{\pi} \frac{v_1^2}{a_0} \left( \vec{e}_\perp \cdot \vec{e}_r \right)
\]  \( \text{(B8)} \)
whereas the Alfvén velocity \( c_A = B^2 / (4 \pi \rho_0) \) contains the dependence on the magnetic field strength.

The particular susceptibility of magnetic flux rings located at low to intermediate latitudes (cf. Figs. 1 & 2) is likely a characteristic feature of the equilibrium properties. Neglecting centrifugal forces, the latitudinal variation of the density contrast, Eq. \( D \), is proportional to \( v_1^2 \cot \lambda \). Albeit the actual meridional flow profile inside the stars is unknown, the increase and decrease of the latitudinal flow velocity from the pole down the equator is to yield a peak at intermediate latitudes. In combination with the factor \( \cot \lambda \), which continuously increases toward the equator, the peak of the density contrast will be located at low to mid latitudes.

\section*{Appendix C: Evolution of Single Bipoles}

The surface flux transport simulations in Sect. 4.3 show that the intermediate range of originating latitudes (Case I, cf. Table 1) causes a higher degree of intermingling than the two cases S and L. To investigate this aspect in more detail, we analyse the relevant mechanisms in the simplified case of a single bipole evolving across the surface under the effect of meridional flow, differential rotation and supergranular diffusion. For a meridional flow velocity of \( v_M = 100 \, m \cdot s^{-1} \), the butterfly diagrams in Case S, I, and L have the mean latitude of bipole emergence \( \lambda_e = 21^\circ, 39^\circ, \) and \( 45^\circ \), respectively. In each of the three cases, a single bipole is inserted onto a (magnetically empty) stellar surface at the mean latitude of emergence. The initial tilt angle is \( \lambda_e / 2 \), whereas the initial total magnetic flux is in all cases the same (1.5 · 10^{23} Mx). The temporal evolution of the polar flux density and ratio of opposite polarities is shown Fig. C1. The higher the (mean) latitude of emergence, the sooner the bipole reaches the polar region, and the stronger is the remaining polar flux, due to the shorter time span available for flux annihilation as it is transported to the poles. The trailing flux region (here, of positive polarity) enters the polar region before the leading region (of negative polarity); when the latitude of emergence is small the timing when both polarities enter the polar region becomes similar. In contrast, the higher the latitude of emergence the less intermingling between magnetic flux of opposite polarity is obtained. As the entire flux is pushed into the polar region, the flux ratio eventually approaches unity. Using the area under each curve...
as a measure for the degree of intermingling, the results for the two lower mean latitudes of emergence are similar, whereas the degree of intermingling for $\bar{\lambda}_e = 45^\circ$ is overall considerably lower.

Since each bipole obeys Joy’s law, the trailing flux region is located at somewhat higher latitudes than the preceding flux region of the opposite polarity. The meridional circulation then pushes both magnetic flux regions in this form toward the pole. However, during the poleward transport the differential rotation however rotates the bipole. This causes more flux of each polarity to be located at a common latitude, which is a requirement for intermingling to occur once the bipole is pushed into the polar region. If a bipole emerges at high latitudes, then the differential rotation has not enough time to act before its entry into the polar region. Hence a higher degree of intermingling is obtained if the initial latitude of emergence is low. This is in contrast to the polar flux density, which is found to decrease with the initial latitudes of emergence. Consequently, there is a particular range of latitudes of emergence and poleward meridional flows, which results in both high degrees of intermingling and strong polar flux densities. Optimal values occur when the timescales of both meridional flow and differential rotation are similar.