Joint Resource Allocation and Antenna Selection In the Uplink of OFDMA Networks

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Abstract—The problem of joint sub-channel, power control and antenna selection in the uplink of OFDMA networks is investigated. The corresponding optimization problem is a non convex mixed integer non-linear problem (MINLP). To tackle the problem, an iterative two-step optimization problem is devised. First, Having fixed the selected antenna as well as the associated power of each user, a sub-channel assignment problem using the so-called Hungarian method is proposed. Second, the joint power and antenna selection problem is extended into a more elegant form through adding a penalty term to the objective function forcing the relaxed boolean variables associated with antenna selection problem to take binary values. To this end, the problem is restated as a difference of two concave function (DC programming) and is effectively solved in an iterative manner. The foregoing procedure is iteratively repeated until approaching a fixed network throughput. Simulation results demonstrate the superiority of the proposed method w.r.t. the exhaustive search with fixed power control mechanism.

Index Terms—MINLP, Sub-channel assignment, Hungarian algorithm, Boolean variable, DC programming.

I. INTRODUCTION

Spectral efficiency, flexibility in resource allocation, and low implementation cost have turned deploying OFDMA systems to the main trend in the broadband wireless industry. Power control and sub-channel assignment are among the most challenges for managing the transmission policy in downlink and uplink directions. Moreover, thanks to the new communication technologies, users are employing multiple antenna handsets, and controlling the use of antennas is another optimization tool in the operators hands. The concepts of sub-channel assignment and power control are the main contributions of many papers, dealing with the emerging wireless networks.

Many papers have considered sub-channel assignment, together with power control in wireless networks [11]–[13]. In [3], downlink beamforming and antenna selection, alongside with uplink power allocation with the goal of minimizing the total transmit power of the network are considered. The authors of [2] proposed a coordinated joint uplink user scheduling and power control algorithm across multiple cells in a wireless cellular network, where the problem is converted into a fractional form by using some approximations. Afterwards, it is shown that the problem can be cast as a sum-of-ratio problems, by decoupling the problem on a per-cell basis, and finally, a distributed iterative algorithm is proposed to tackle the problem. In [5], sub-channel assignment and power control are considered in terms of maximizing the minimum rate of the network’s users. The resulting optimization problem is solved iteratively, while its major shortcoming is the exponential complexity growth of the solution. The concept of joint sub-channel assignment and power control is considered in [8] in terms of maximizing the throughput of a heterogeneous networks. The problem is converted to a convex form by adding a penalty term to the objective function and relaxing the integer variable, and the solution is based on DC programming. The provided method is novel in terms of jointly optimizing the sub-channel assignment and power control, however, it suffers from poor performance in high SNR region compared with [13].

To the best of our knowledge, there has not been any efforts in the uplink in an OFDMA networks including antenna selection. It is worth mentioning that antenna selection is shown to achieve the benefits of using multi antennas in terms of achieving diversity gain, however, with much lower complexity. Therefore, this motivated us to focus on the joint antenna selection, sub-channel assignment and power control mechanism in the uplink of an OFDMA network. As will be discussed in the forthcoming sections, the underlying optimization problem is a non-convex mixed integer non linear problem (MINLP), where similar to what is done in [4]–[7], a two-step optimization procedure is being employed. First, having fixed the power of each user, the sub-channel assignment problem using the so-called Hungarian method is effectively solved. Second, for the chosen sub-channels, the joint antenna selection and power control mechanism are cast as an optimization problem where through relaxing the boolean variables corresponding to the antennas of each user, and adding a penalty term to the original problem to make sure that these variables take binary values, the extended problem is reformulated as a Difference of two Concave function (DC) problem with an iterative sub optimal solution.

The rest of the paper is organized as follows. The system model of an OFDMA network in the uplink direction is introduced in section II. In section III the sub-channel assignment based on the Hungarian method is investigated. The power control and antenna selection procedures for the chosen sub-channels are determined in section IV. In section V the iterative algorithms which makes use of the results of two latter sections, in order to reach an optimal solution is provided. Section VI presents the simulation results, associated with the proposed method and section VII concludes the paper with findings.
II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider the uplink channel of an OFDMA network where each cell is influenced by at most one co-channel cell, thereby scheduling can be done on a two-cell basis. Each cell $B = \{1, 2\}$ is serving $I$ users with $M$ available sub-channels. $I_b$ denotes the set of users in the cell $b \in B$, where each user is equipped with $A = \{1, 2, \ldots, A\}$ antennas. Considering the set of available sub-channels as $\mathcal{M} = \{1, 2, \ldots, M\}$, $h_{ib,m}^{(m)}(a)$ and $h_{ib,m}^{(m)}(a)$ denote the direct and cross uplink channel coefficients from the $i^{th}$ user in the $b^{th}$ cell to the $m^{th}$ sub-channel from the $a^{th}$ antenna to the cells $b$ and $b'$, respectively. Let's $p_{ib}^{(m)}(a)$ denote the transmit power, $s_{ib}^{(m)}$ and $x_{ib}^{(m)}(a)$ binary variable indicators to, respectively, represent the $i^{th}$ user in the $b^{th}$ cell being served over the $m^{th}$ sub-channel, and the $a^{th}$ antenna of this user is being selected. Also, the vector of power, sub-channel and antenna variables are defined as, $p_{ib} = [p_{ib}^{(1)}(1), \ldots, p_{ib}^{(M)}(1), p_{ib}^{(1)}(2), \ldots, p_{ib}^{(M)}(2), \ldots, p_{ib}^{(1)}(A), \ldots, p_{ib}^{(M)}(A)]$, $s_{ib} = [s_{ib}^{(1)}, \ldots, s_{ib}^{(M)}(1), \ldots, s_{ib}^{(M)}(A)]$, and $x_{ib} = [x_{ib}^{(1)}(1), \ldots, x_{ib}^{(1)}(A), \ldots, x_{ib}^{(M)}(1), \ldots, x_{ib}^{(M)}(A)]$, respectively.

The main objective pursued in the current work is to assign sub-channel(s), set the transmit power(s), and select the best antenna of each user, such that the total network throughput is maximized. Noting that the antenna selection and sub-channel assignment variables are boolean, the data rate of the $i^{th}$ user in the $b^{th}$ cell over the $m^{th}$ sub-channel when the $a^{th}$ antenna is selected can be written as,

$$r_{ib}^{(m)}(a) = \log_2 \left( 1 + \frac{x_{ib}^{(m)}(a) p_{ib}^{(m)}(a) h_{ib,m}^{(m)}(a)^2}{\sigma^2 + \sum_{b' \neq b} \sum_{a' \in A} x_{ib}^{(m)}(a) p_{ib}^{(m)}(a) h_{ib,m}^{(m)}(a')^2} \right)$$

where $\sigma^2$ is the variance of additive white Gaussian noise. Furthermore, the equality (b) is due to the boolean nature of the variables $x_{ib}^{(m)}(a)$ and $s_{ib}^{(m)}$. $R_{ib}^{(m)}(a)$ is defined as follows,

$$R_{ib}^{(m)}(a) = \log_2 \left( 1 + \frac{p_{ib}^{(m)}(a) h_{ib,m}^{(m)}(a)^2}{\sigma^2 + \sum_{b' \neq b} \sum_{a' \in A} x_{ib}^{(m)}(a) p_{ib}^{(m)}(a) h_{ib,m}^{(m)}(a')^2} \right)$$

Thus the optimization problem can be cast as,

$$\max_{x \in \mathbb{X}, p \in \mathbb{P}} \sum_{b \in B} \sum_{i \in I_b} \sum_{m \in \mathcal{M}, a \in A} x_{ib}^{(m)}(a) s_{ib}^{(m)} R_{ib}^{(m)}(a)$$

s.t. $C_1 : \sum_{a \in A, m \in \mathcal{M}} x_{ib}^{(m)}(a) p_{ib}^{(m)}(a) \leq p_{max}, \forall b \in B, \forall i \in I_b$

$C_2 : p_{ib}^{(m)}(a) \geq 0, \forall b \in B, \forall i \in I_b, \forall m \in \mathcal{M} \forall a \in A$

$C_3 : \sum_{i \in I_b} s_{ib}^{(m)}(a) = 1, \forall b \in B, \forall m \in \mathcal{M}$

$C_4 : \sum_{a \in A} x_{ib}^{(m)}(a) = 1, \forall b \in B, \forall i \in I_b$

$C_5 : x_{ib}^{(m)}(a) \in \{0, 1\}, C_6 : s_{ib}^{(m)} \in \{0, 1\}$

where the maximization is carried out over the vector of power $p \triangleq [p_{11}, \ldots, p_{1I}, p_{21}, \ldots, p_{2I}]^T$, and binary vectors $s \triangleq [s_{11}, \ldots, s_{1I}, s_{21}, \ldots, s_{2I}]^T$ and $x \triangleq [x_{11}, \ldots, x_{1I}, x_{21}, \ldots, x_{2I}]^T$. The constraint $C_1$ indicates that the total transmit power of each user is limited to the $p_{max}$. $C_2$ guarantees the positivity of the allocated power to each user. The constraints $C_3$ and $C_4$, respectively, ensure that each sub-channel is merely assigned to one user and each user makes use of one antenna. Finally, $C_5$ and $C_6$ indicate that the sub-channel indices and the antenna indicators are binary variables.

One can readily verify that the defined optimization problem in (3) involves continuous variables $p_{ib}^{(m)}(a)$ and boolean variables $s_{ib}^{(m)}$ and $x_{ib}^{(m)}$. Moreover, the objective function of (3) is not convex. In the following sections, we are going to address the solution of the aforementioned problem through the use of an iterative approach consisting of two approximations, which yields a close to optimal result. Sections III and IV describe the solution steps and Section V describes the iterative approach, incorporated to make use of the latter two sections to derive a close-to-optimal solution.

In this regard, for a given power $p_t^{i-1}$, $x_t^{i-1}$, section III attempts to solve a combinatorial sub-channel assignment to reach an optimal solution for high SNR regime and section IV tries to find the joint antenna selection variables $x^t$ and the allocated power $p^t$ through solving a non-convex mixed integer non linear problem for the chosen sub-channels.

III. SUB-CHANNEL ASSIGNMENT FOR A FIXED POWER AND ANTENNA ALLOCATION

This section tends to determine the sub-channel assignment for each user. For a given power and antenna selection $(p, x)$ the optimization problem is now simplified to,

$$\max_s \sum_{b \in B} \sum_{i \in I_b} \sum_{m \in \mathcal{M}, a \in A} x_{ib}^{(m)}(a) s_{ib}^{(m)} R_{ib}^{(m)}(a) (p, x)$$

s.t. $C_3 : \sum_{i \in I_b} s_{ib}^{(m)} = 1, \forall b \in B, \forall m \in \mathcal{M}$

$C_6 : s_{ib}^{(m)} \in \{0, 1\}$

Fig. 1. System model of the considered co-channel OFDMA network in the uplink direction, the solid lines denote the desired signals while the dashed lines represent the interfering signals.
It is assumed that all users use a fix power, for any active subchannel and operate at high SINR region. So, the achievable rate of $I$ can be approximated as,

$$\log_2(1+\text{SINR}) \approx \log_2(\text{SINR}) = \sum_{a \in A} \log_2(p_{ib}^{(m)(a)}|h_{ib,b}^{(m)(a)}|^2)$$

$$- \log_2(\sum_{a \in A} p_{ib}^{(m)(a)}|h_{ib,b}^{(m)(a)}|^2 + \sigma^2) \quad (5)$$

where the approximation comes from the fact that $\log_2(1 + x) \approx \log_2(x)$ for large values of $x$. As it is inferred from (5), the direct and cross channel gains incur positive and negative impacts on the network’s throughput. Attempting to determine the best sub-channels in each cell for the $i^{th}$ user, (5) is reformulated as follows,

$$\max_s \sum_{b \in B} \sum_{i \in I_s} \sum_{m \in M} \sum_{a \in A} s_{ib}^{(m)} (\sigma^2) \left( \log_2(p_{ib}^{(m)(a)}|h_{ib,b}^{(m)(a)}|^2) - \log_2(\sigma^2 + \sum_{a \in A} p_{ib}^{(m)(a)}|h_{ib,b}^{(m)(a)}|^2) \right)$$

s.t. $C_1 : \sum_{i \in I_s} s_{ib}^{(m)} = 1, \ m \in \{1, 2, ..., M\}, \ b \in \{1, 2\}$

$C_2 : s_{ib}^{m} \in \{0, 1\}$

(6)

The above problem is solved through the assignment method. In a typical assignment problem [16], $I$ machines are opted to handle $M$ jobs, where the cost of doing the $i^{th}$ job on the $m^{th}$ machine is $c_{im}^{(m)}$. The goal is to effectively assign the jobs to the machines in a way that the total cost is minimized. Let $C_{ib}^{(m)}$ be the cost of assigning the $m^{th}$ sub-channel to the $i^{th}$ user in the $b^{th}$ cell. Defining the $M \times M$ cost matrix $C_{ib}^{(m)}$ associated with $[\sigma]$, its corresponding $m^{th}$ row and $n^{th}$ column element is set to,

$$C_{ib}^{(m)} = \sum_{a \in A} \log_2(p_{ib}^{(m)(a)}|h_{ib,b}^{(m)(a)}|^2) - \log_2(\sigma^2 + \sum_{a \in A} p_{ib}^{(m)(a)}|h_{ib,b}^{(m)(a)}|^2) \quad (7)$$

By using the Hungarian method\footnote{Which is known to be a classical method to solve these kind of assignment problems.}, the best sub-channels are assigned to the users through the use of a combinatorial algorithm in a polynomial time [16]. Now, taking (7), the problem (6) can be expressed as follows,

$$\min_s \sum_{b \in B} \sum_{i \in I_s} \sum_{m \in M} \sum_{a \in A} -x_{ib}^{(a)} s_{ib}^{(m)} C_{ib}^{(m)}$$

s.t. $C_1 : \sum_{i \in I_s} s_{ib}^{(m)} = 1, \ m \in \{1, 2, ..., M\}, \ b \in \{1, 2\}$

$C_2 : s_{ib}^{m} \in \{0, 1\}$

(8)

where $s_{ib}^{(m)}$ is boolean variable, which ensures that each sub-channel is assigned to only one user. It should be noted that the assignment problem can be extended to a more general case without imposing any constraint on the number of sub-channels and users. The main advantage of this method is having low complexity. For the case of $I < M$, one can add $M - I$ virtual users with the corresponding $-\infty$ (a relatively large negative value) values for the added users, where $I$ out of $M$ sub-channels would be assigned to current $I$ users. Then for the remaining unassigned $M - I$ sub-channels, one can run another optimization problem as follows. To set the number of unassigned sub-channels $(M - I)$ equal to that of current $M$ users, one can add $I$ virtual sub-channels with corresponding $-\infty$ cost values for current $M$ users to make a square cost matrix again. Then, solve the new assignment problem to choose the best users for the remaining sub-channels. For the case of $M < I$, one can add $M - I$ virtual sub-channels with the corresponding $-\infty$ cost values for the current users to make a square cost matrix and then solve the assignment problem. Again, the actual sub-channels are assigned to the best $M$ users.

IV. OPTIMAL JOINT ANTENNA SELECTION AND POWER ALLOCATION STRATEGY FOR A FIXED SUB-CHANNEL ALLOCATION

In this section, we attempt to jointly find the optimal antenna selection and power control, for the assigned sub-channels from the previous section, to maximize the uplink throughput of the network. To this end, (3) is reformulated to a more mathematically tractable form. Considering that $C_1$ is a non-convex constraint due to the multiplication of the boolean term of $x_{ib}^{(a)(m)}$ to the affine parameter $p_{ib}^{(m)(a)}$, the constraint is restated as follows,

$$C_{1a} : \sum_{a \in A} \sum_{m \in M} p_{ib}^{(m)(a)} \leq p_{max},$$

$$C_{1b} : 0 \leq \hat{p}_{ib}^{(m)(a)} \leq x_{ib}^{(a)(m)} p_{iba} \quad (9)$$

The inequalities of (9) are analyzed in two following cases,\

**Case1**, if $x_{ib}^{(a)(m)} = 0$, the $m^{th}$ sub-channel or $a^{th}$ antenna is not assigned to the $i^{th}$ user of the $b^{th}$ cell. Therefore, its associated transmission power is equal to zero.

**Case2**, if $x_{ib}^{(a)(m)} = 1$, the transmission power for the $m^{th}$ sub-channel and $a^{th}$ antenna is $p_{max}$.

Using the modified version of the constraint $C_1$ in (2), one can easily verify that the non-convex term of $x_{ib}^{(a)(m)}$ is erased from (3), and the problem is cast as,

$$\max_{x, p} R(x, p) = \sum_{b \in B} \sum_{i \in I_s} \sum_{m \in M} \sum_{a \in A} \hat{R}_{ib}^{(m)(a)}$$

subject to $C_1 : \sum_{a \in A} x_{ib}^{(a)} = 1$

$$C_2 : \sum_{a \in A} \sum_{m \in M} p_{ib}^{(m)(a)} \leq p_{max}$$

$$C_3 : 0 \leq \hat{p}_{ib}^{(m)(a)} \leq x_{ib}^{(a)(m)} p_{iba}$$

$$C_4 : x_{ib}^{(a)} \in \{0, 1\}$$

(10)
where \( \tilde{R}_{ib}^{(m)(a)} \) is stated below,
\[
\tilde{R}_{ib}^{(m)(a)} = \log_2 \left( 1 + \sum_{b' \neq b} \sum_{l \in I_b, a' \in A} \left( \frac{1}{P_{ib}^{(m)(a)}} \left| h_{ib,b'}^{(m)(a')} \right|^2 \right)^2 \right).
\]

The optimization problem of (10) is non-convex MINLP, which is very difficult to solve. Concerning to provide a tractable solution to this problem, the constraints \( R_1 \) and \( R_2 \) are added to the optimization problem (10).
\[
R_1 : \quad 0 \leq x_{ib}^{(a)} \leq 1
\]
\[
R_2 : \quad \sum_{b} \sum_{i} \sum_{a} (x_{ib}^{(a)} - (x_{ib}^{(a)})^2) \leq 0.
\]
The constraints \( R_1 \) and \( R_2 \) do not change the result of (10).

Considering \( Q \) as the feasible set, spanned by the constraints \( C_1 - C_4 \), the optimization problem (10) is written as follows,
\[
\max_{x, p} \quad R(x, p) \quad \text{s.t.} \quad x, p \in Q, R_1, R_2 \quad (13)
\]

Where, \( R(x, p) \) represents the objective part of (10). So, the problem of (13) is a continuous optimization problem with respect to \( x \) and \( p \). However, we tend to obtain the boolean solution for \( x_{ib}^{(a)} \). To this end, a penalty term is added to the objective function and the boolean variable is relaxed, and the problem is modified to form,
\[
\max_{x, p} \quad L(x, p, \mu) \quad \text{s.t.} \quad x, p \in Q, R_1 \quad (14)
\]

where \( L(x, p, \mu) \) is the lagrangian associated to (13), and is obtained as follows,
\[
L(x, p, \mu) \triangleq R(x, p) - \mu \left( \sum_{b} \sum_{i} \sum_{a} (x_{ib}^{(a)} - (x_{ib}^{(a)})^2) \right)
\]
The coefficient \( \mu \) is a constant value which acts as a penalty component. For very greater than unity values of \( \mu \), the optimization (14) is equivalent to (13), and solving the dual form of \( \min_{\mu} \max_{x, p} L(x, p, \mu) \) equal to solve the primal form of \( \max_{x, p} \min_{\mu} L(x, p, \mu) \).

Now, the optimization problem (14) can be expressed as follows,
\[
\max_{x, p} \quad \sum_{b \in B} \sum_{i \in I_b} \sum_{m \in M} \sum_{a \in A} F(p, x) - G(p, x) \quad (16)
\]
subject to \( C_1 - C_4 \)

Bearing the log(.) nature of the objective in mind, the network’s throughput can be expressed as the difference of two logarithms, which the first term is carrying signal, noise and interference components and the second term is carrying the latter two. Concurrently, we define,
\[
F(x, p) \triangleq \sum_{b \in B} \sum_{i \in I_b} \sum_{m \in M} \sum_{a \in A} \left( \log_2 \left( \frac{1}{P_{ib}^{(m)(a)}} \left| h_{ib,b'}^{(m)(a')} \right|^2 \right)^2 \right)^2 + \sigma^2
\]
\[
G(x, p) \triangleq \sum_{b \in B} \sum_{i \in I_b} \sum_{m \in M} \sum_{a \in A} \left( \log_2 \left( \frac{1}{P_{ib}^{(m)(a)}} \left| h_{ib,b'}^{(m)(a')} \right|^2 \right)^2 \right)^2 - \mu(x_{ib}^{(a)}) \quad (17)
\]

\( G(x, p) \) is a reverse convex function \([3], [9], [10]\), which is known as a reverse convex function \([3], [9], [10]\).

The optimization problem of (16) is still non-convex. In order to handle with difficulty and non-convex optimization problem, we can apply successive convex approximation \([5]\) to obtain a locally optimal solution and we reformulate the problem in (16) as:
\[
\max_{x, p} \quad \sum_{b \in B} \sum_{i \in I_b} \sum_{m \in M} \sum_{a \in A} F(p, x) - G(p, x)
\]
\[
- \nabla_p G(p_{n-1}, x_{n-1}).(p - p_{n-1})
\]
\[
- \nabla_x G(p_{n-1}, x_{n-1}).(x - x_{n-1})
\]
subject to \( C_1 - C_4 \)

Where the Taylor approximation is being used to turn the problem into a convex form. \( n \) denotes the iteration number, \( \nabla_p \) and \( \nabla_x \) denote the gradient with respect to \( p \) and \( x \), respectively. An iterative approach to solve the above D.C programming is devised, based on the interior point method, which its convergence is proved in the proposition 1. The solution begins from an initial point that lies in the feasible region, and continues until a settling point is derived. The optimization problem of (19) is solved iteratively based on the method, presented in Algorithm 1. In order to prove the convergence behavior of the algorithm, one should note that \( G(p, x) \) is a concave function, and its corresponding gradient is also super gradient \([3]\). So, we have,
\[
G(p, x) \leq G(p_{n-1}, x_{n-1}) + \nabla_p G(p_{n-1}, x_{n-1}).(p - p_{n-1}) + \nabla_x G(p_{n-1}, x_{n-1}).(x - x_{n-1})
\]

Thus, one can conclude that,
\[
G(p_{n+1}, x_{n+1}) \leq G(p_{n}, x_{n}) + \nabla_p G(p_{n}, x_{n}).(p_{n+1} - p_{n}) + \nabla_x G(p_{n}, x_{n}).(x_{n+1} - x_{n})
\]

**Proposition 1** Incorporating the Algorithm 1 at the end of each iteration, the solution of (19) will be improved as follows.
\[
F(x_{n+1}, p_{n+1}) - G(x_{n+1}, p_{n+1}) \geq F(x_{n}, p_{n}) - G(x_{n}, p_{n})
\]
\[
- \nabla_p G(x_{n}, p_{n}).(p_{n+1} - p_{n}) - \nabla_x G(x_{n}, p_{n}).(x_{n+1} - x_{n})
\]
\[
= \max_{x, p} F(x, p) - G(x_{n}, p_{n}) - \nabla_p G(x_{n}, p_{n}).(p_{n} - p_{n}) - \nabla_x G(x_{n}, p_{n}).(x_{n} - x_{n})
\]
\[
= F(x_{n}, p_{n}) - G(x_{n}, p_{n})
\]

Based on the Proposition 1 we observed that objective function of (19) takes larger values as each iteration.
Algorithm 1 Proposed Method based on Hungarian method and Successive Convex Approximation (Hungarian-D.C Programming)

1- For a fixed power \( p^{t-1} \) and selected antenna \( x^{t-1} \), find optimal sub-channel assignment \( s^t \) based on the Hungarian method

2- For a fixed sub-channel \( s^t \), solve the optimization problem of (19) find optimal \( p^t, x^t \)

2-1 Initialize \( n = 0 \), penalty factor \( \mu \gg 1 \) and feasible set vector \( p^0 \), and \( x^0 \)

3- Repeat

3-1 Solve optimization problem of (19) and store the intermediate \( x^n \) and \( p^n \)

3-2 Set \( n = n + 1 \) and \( p^n = p^* \) and \( x^n = x^* \)

3-4 Until convergence of \( p, x \) and obtain the optimal solution

V. ITERATIVE APPROACH JOINT SUB-CHANNEL ASSIGNMENT, ANTENNA SELECTION AND POWER CONTROL

Opting to find the optimal power control, sub-channel assignment and antenna selection, the following iterative procedure is employed [5], [6]. At the initial point, (iteration number 0), the optimal sub-channel assignment for the maximum power in each sub-channel and arbitrary selected antenna for each user is derived, using the results of section [III]. After that, knowing the best sub-channel assignment, the best transmission power in each sub-channel and the best antenna for each user are derived by incorporating the results of section [IV]. Then, in the iteration number \( t = 1 \), the power allocation and antenna selection results of \( \{ p(0), x(0) \} \) lead to the sub-channel assignment of \( s(1) \) and consequently, \( \{ p(1), x(1) \} \) are derived. Continuing the above procedure, \( \{ p(t-1), x(t-1) \} \) result in \( s(t) \), and \( s(t) \) leads to \( \{ p(t), x(t) \} \). This iterative approach continues until no further improvement is made. The following proposition shows the gradual improvement of the proposed iterative algorithm.

Proposition 2 Algorithm 1 converges to a local optimum for (3).

Proof: The objective function of (3) can be expressed as follows,

\[
R(x, p, s) = \sum_{b \in B} \sum_{t \in T_a} \sum_{m \in M} \sum_{a \in A} x^t_{ib} s^t_{ib} P_{ib}^{(m)(a)}
\]

The following relationship between sub-channel allocation, joint antenna selection and power control is considered,

\[
R(x, p, s) \geq R(x(t-1), p(t-1), s(t))
\]

\[
= \max_s R(x(t-1), p(t-1), s)
\]

\[
\geq R(x(t-1), p(t-1), s(t-1))
\]

Therefore, the throughput of the network is increasing as the iterations go on.

This algorithm reduces the number of variables by half in each optimization subproblem and changes the original problem (3) into a mathematically tractable form to be solved. For the optimization problem of (3) we have 4MIA decision variables and \( 2I + 2IMA + 2M + 2I + 2AI + 2MI \) linear constraints. Therefore, the computational complexity has order of \( O((4MIA)^3(4I + 2IMA + 2M + 2AI + 2MI)) \) which is polynomial [10].

Discussion In the case of having more than one co-channel cell, by considering the feasible set of \( \{ p^{t-1}, x^{t-1} \} \), the optimal sub-channel assignment \( s^t \) can be found from the following optimization problem,

\[
\max_s \sum_{b \in B} \sum_{t \in T_a} \sum_{m \in M} \sum_{a \in A} x^t_{ib} s^t_{ib} P_{ib}^{(m)(a)} (p^{t-1}, x^{t-1})
\]

s.t. \( \sum_{b \in B} s^t_{ib} = 1 \quad \forall b \in B, \forall m \in M \)

\( s^t_{ib} \in \{0, 1\} \)

In order to find the solution for (24), instead of incorporating the Hungarian method, the optimal sub-channel assignment for each user should satisfy the \( \arg \max_{s \in S} R_{ib}^{(m)(a)}(p^{t-1}, x^{t-1}) \). 

VI. SIMULATION RESULT

In this section, we investigate the performance of the proposed algorithm for scheduling and power control in the uplink direction of a co-channel network. It is assumed that \( N = 6 \) orthogonal sub-channels are shared between 3 users in each cell which are equipped with two antennas. The distance between each user and its corresponding base station is 100m and the distance between each user and its corresponding co-channel base station is 500m. The wireless channel model is Rayleigh flat fading which includes a distance dependent path-loss component of \( 128.1 + 37.6 \log(d) \) dB (where \( d \) is in km) and a log normal shadowing component with 8dB standard deviation and background noise PSD of the user is set to \(-140\text{dbm} \) through simulation.

Monte carlo simulation is repeated by generating random realizations of the channel gains and obtain the average data rate of the network. Fig.1 illustrates the average sum rate versus maximum SNR of each user for different methods, where the maximum SNR is calculated by subtracting the maximum power of each user from direct path-loss and noise power.

According to Fig. 1, the proposed method reaches to a higher level of average throughput, compared with exhaustive search over all possible choices of sub-channel and antenna selection with equal power. Moreover, it can be seen that incorporating the proposed method in the sub-channel and antenna selection part, while employing full power closely follows the exhaustive search with full power, while both have a large gap to random scheduling which is being served as a benchmark throughout the simulations.
the optimal solution taken from time consuming exhaustive search.

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