Abstract: Simultaneous wireless information and energy transfer based on radio frequency (RF) energy harvesting can transmit power and information at the same time and can use the same wireless signal to decode the information and to charge the battery. In order to improve spectrum efficiency of energy-constrained networks, we consider combining cooperative relay with simultaneous information and energy transfer in this paper. The optimal relay power allocation and relay selection algorithms based on statistical channel state information (CSI) and perfect CSI are proposed, respectively. Specifically, the system first selects the relay nodes that can correctly decode the source information and, then among them, selects the node with the best second hop channel condition. We derive the exact analytical outage probability of the proposed algorithms by using order statistical tool. The analytical results are compared and verified by Monte Carlo simulations. It is observed that the outage probabilities of our proposed algorithms based on statistical CSI and perfect CSI are all lower than that of the corresponding traditional max–min algorithms.

Keywords: SWIPT; cooperative relay; relay selection; order statistics; RF energy harvesting

1. Introduction

The next generation wireless network represented by 5G is expected to greatly increase wireless data rates, bandwidth, coverage, and connectivity, with a massive reduction in round trip latency and energy consumption. Compared to 4G networks, 5G will provide 110 Gbps of data rates, about 10 times higher, with almost 10 times less latency and almost 100% availability [1]. However, energy efficiency (EE) and spectrum efficiency (SE) are still the key problems to be solved by the next generation networks. In order to improve EE and SE, cooperative relay technology (CoR) is one of the necessary options [2]. Cooperative relay is a kind of technology that completes communication tasks through the mutual aid of nodes. The relay node is used to forward the information between the source node and the destination node to obtain the spatial diversity. As a result, the relay nodes can increase the coverage area of the node, can mitigate channel fading, can reduce the transmission power, and can increase the system bandwidth. However, as forwarding messages consumes additional energy, the battery in a relay node is depleted more quickly than in a non-relay node. Since the relay nodes are limited by the battery capacity, their service life inherently depends on the battery life. Fortunately, the power-constrained cooperative relay can be complemented by the radio frequency energy harvesting technology (RF-FH), which stimulates a new round of academic research and leads to a new revolution in wireless network technology [3].

RF-EH has made great progress in recent years [3] and enables relay nodes to convert radio signals received from hybrid access points (AP) or base stations (BS) into electrical energy to increase
the node’s power. Simultaneous wireless information and power transfer (SWIPT) based on RF-EH can transmit power and information at the same time, so that the same wireless signal can be used to decode information and to charge the battery [4–6]. In theory, the SWIPT receiver can extract energy and decode the information from the same waveform [7]. However, due to the constraints of the circuit, such as the different power sensitivity of the receiving antennas, its realization is practically impossible. It should be noted that energy harvesting requires $-10$ dBm power sensitivity, while information decoding requires as lower as $-60$ dBm [8]. For practical implementation, the structures where SWIPT can adopt includes [9] time switching, power splitting, antenna switching, and integrated information-decoding and energy-harvesting. Introducing SWIPT into CoR can solve the problem of limited energy of relay nodes, thereby providing higher energy efficiency (EE) and spectrum efficiency (SE) for next generation wireless networks.

1.1. Related Works

The relay selection in cooperative relay network is to select the best relay node from many relays in order to improve the overall Quality of Service (QoS) of the network and minimize energy consumption. Several relay selection mechanisms are proposed in References [6,10,11]. However, the relay selection problem in energy harvesting and relay networks is still in its infancy. In Reference [12], the relay selection in power splitting-based energy-harvesting full-duplex relay networks is investigated to minimize the outage probability and to maximize the sum capacity respectively. Starting with Reference [13], the researchers found a significant difference between a relay network with and without energy harvesting. According to their model, relays are chosen based on the average energy harvesting rate, transmission power, and number of relays in the system based on channel state information (CSI). The relay selection problem in SWIPT-based CoR networks was investigated by in Reference [14]. Based on their results, it was found that the availability of relay CSI significantly enhances the overall performance of the system and that there is a trade-off between the number of relay participants and the EH efficiency of the relay.

Some studies have shown that multiple relays perform better than a single relay in terms of energy and bandwidth [6,15,16]. Based on EH cooperative relay, a relay selection strategy is proposed to minimize the transmission power for a conventional wireless network [10]. In Reference [17], the relay selection based on the battery capacity level is proposed in wireless powered cooperative networks with spatially random relays. In Reference [18], a strategy of partial relay selection and global optimal selection is proposed, which corresponds to source-relay channel selection and end-to-end channel quality optimization (max–min algorithm), respectively.

1.2. Our Contribution

Max–min relay selection algorithm proposed in Reference [18] has been widely used in traditional cooperative relay systems. However, for energy harvesting relays, there exist the following problems: (1) A fixed power splitting ratio may cause some nodes to have too low energy to decode information owing to deep channel fading, which may cause Source-Relay (S-R) link interruption, and (2) the relay node with the best S-R link quality does not necessarily have the best Relay-Destination (R-D) link. Even the relay selection method based on the poor link between the two hops is also too arbitrary. None of the above solutions have chosen the global optimal relay.

In this paper, we propose an optimal relay selection strategy based on dynamic power allocation as well as statistical and perfect CSI. The proposed protocol can further improve the communication performance and reduce the global outage probability. Our main contributions include the following:

- A dynamic power allocation optimization strategy based on channel quality and target rate is proposed to ensure that relay nodes provide relay services on the premise of successful decoding source information.
- The optimal relay selection based on statistical and perfect CSI are proposed. The exact analytical outage probability of the proposed algorithms are derived by using order statistic tool.
The performance of the proposed algorithm is verified by numerical simulation and Monte Carlo Simulation. The results show that the performance of the proposed scheme is better than that of traditional max–min algorithm.

This paper is organized as follows. We introduce the system model and the main assumptions in Section 2. In Section 3, we present the relay selection policies based on statistical CSI and analyze their outage performance. Section 4 presents the relay selection schemes based on perfect CSI and derives the corresponding outage probability. Numerical results and discussions are presented in Section 5, followed by our conclusions in Section 6.

2. System Model

We consider a decode-and-forward (DF) cooperative network with a source node $S$, a destination node $D$, and $N$ energy harvesting candidate relays, where the relays are labeled as $R_i$, $i = 1, 2, \ldots, N$, as shown in Figure 1. All nodes are equipped with a single antenna and operate in a half-duplex mode in the common frequency band. It is assumed that the source node has no energy constraint and transmits at a constant power. It is also assumed that there is no direct link between source and destination node due to obstacles or severe attenuation. We denote the channel fading coefficient between $S$ and $R_i$ and between $R_i$ and $D$ by $h_i$ and $g_i$, respectively. Without loss of generality, we assume that all channels experience independent flat quasi static Rayleigh fading, i.e., $h_i \sim \mathcal{CN}(0,1)$ and $g_i \sim \mathcal{CN}(0,1)$.

![Figure 1. System model](image)

We assume that DF relays between $S$ and $D$ are performed for each fixed length of time $T$ and are divided into two phases. Assuming the network is perfectly synchronized, the source node broadcasts the signal in the first phase. The relay node uses an energy harvesting receiver architecture to collect signals and energy. Using the power splitting technique, the relay divides the received signal into two streams according to power splitting ratio (PSR) $\rho_i$. Note that $\rho_i$ is used to decide how much observation flow will be directed to the energy harvesting circuit. For example, the choice of $\rho_i = 0$ means that all observations will flow to the detection circuit, and $\rho_i = 1$ means that the energy harvesting circuit receives all of the observation flow. Accordingly, the $\sqrt{\rho_i}$ part of the received signal in the relay is used for energy harvesting and the $\sqrt{1-\rho_i}$ part of the received signal is used for information decoding. In the second phase, the relay node with decoded information forwards the signal to the destination node and consumes all the harvested energy. The operation structure of the relays is shown as Figure 2.
According to the above model, the incident signal at the antenna of relay $R_i$ is as follows [2]:

$$y_{SR_i} = \sqrt{\frac{P_S}{1 + d_{SR_i}^2}} h_i x,$$

where $P_S$ is the transmission power of the source node. The source signal, labeled by $x$, is supposed to have normalized power, i.e., $mE(|x|^2) = 1$. The $d_{SR_i}$ is the distance between $S$ and the $i$th relay $R_i$, and $\alpha$ is the path loss coefficient. In this paper, a bounded path loss model is used to ensure that the path loss is greater than 1 even if the transmission distance is extremely short. Otherwise, it will cause the paradox that the received power is greater than the transmitted power, thus violating the law of conservation of energy.

The harvested energy at the relay can be expressed as follows:

$$E_i = \frac{T \eta P_S |h_i|^2}{2(1 + d_{SR_i}^2)},$$

where $\eta$ is the RF energy harvesting efficiency. The remaining signal is converted into baseband signals for information decoding. Accordingly, the baseband signal observed by the relay is as follows:

$$y_{SR_i} = \sqrt{1 - \rho_i} \sqrt{\frac{P_S}{1 + d_{SR_i}^2}} h_i x + w_{R_i},$$

where $w_R \sim CN(0, \sigma^2_{R_i})$ is the noise introduced from the $i$th relay’s antenna. The instantaneous received signal-to-noise ratio (SNR) at $R_i$, following Equation (3), can be obtained as [2]

$$\gamma_{SR_i} = \frac{(1 - \rho_i)P_S |h_i|^2}{\sigma^2_{R_i}(1 + d_{SR_i}^2)}.$$  

Since the relays employ power splitting structure, it is necessary to ensure that the relay can decode the information correctly and, then, the remaining power is used for energy harvesting. We suppose the targeted rate for successfully decoding information at the relay is $R$. According to the rate of half-duplex cooperative communication, $R = \frac{1}{2} \log_2(1 + \gamma_{SR_i})$, we get the power splitting factor as

$$\rho_i = 1 - \frac{\tau(1 + d_{SR_i}^2)}{P_S |h_i|^2},$$

where $\tau = 2^{2R} - 1$. As $\rho_i$ should be between 0 and 1, we further derive that

$$\rho_i = \min \left( \max \left( 1 - \frac{\tau(1 + d_{SR_i}^2)}{P_S |h_i|^2}, 0 \right), 1 \right).$$
In the second phase of transmission, the relay transmits the received signal to \( D \) by using the energy harvested in the first phase. The transmitting power of \( R_i \) is

\[
P_{R_i} = \frac{E_i}{T/2} = \frac{\eta \bar{\rho}_i P_S |h_i|^2}{1 + d_{SR_i}^a}.
\]

(7)

Then, the signal observed by the destination node is

\[
y_{R_iD} = \sqrt{P_{R_i}} \frac{g_i \hat{x} + w_D}{1 + d_{SR_i}^a}.
\]

(8)

where \( w_D \sim \mathcal{C}\mathcal{N}(0, \sigma^2_D) \) represents the noise introduced by the antenna of \( D \) and \( \hat{x} \) is the re-encoded signal. Accordingly, the received SNR at the destination node is

\[
\gamma_{R_iD} = \frac{\eta \bar{\rho}_i P_S |h_i|^2 |g_i|^2}{\sigma^2_D (1 + d_{SR_i}^a)(1 + d_{R_iD}^a)}.
\]

(9)

3. Relay Selection Based on Statistical Channel State Information

For many practical scenarios, it is convenient to get the statistics of a wireless channel, which is determined by the distance between the transmitter and receiver [2]. Note that we assume here that the channel attenuation mainly comes from the dissipation of the signal over distance and the stochastic small-scale fading. Therefore, the greater the distance between the transceivers, the smaller the average gain of the channel. Conversely, the smaller the distance between the transceivers, the larger the average channel gain. In this section, we propose a relay selection strategy by using the statistical channel state information (S-CSI). Since we choose relays based on average channel gain rather than instantaneous channel gain, this will inevitably result in performance loss. Therefore, this is actually a suboptimal selection method. However, the advantage of using this method is that the instantaneous state information of the channel is not needed. The specific steps are as follows.

Before transmission, we first put the relays that can successfully decode the source information into a set \( \Omega \) according to the channel statistics. The set \( \Omega \) is defined as

\[
\Omega \triangleq \{ i \in \Omega : \gamma_{SR_i} |\rho_i=0 = \frac{P_S}{\sigma^2_{SR_i} (1 + d_{SR_i}^a)} > \tau \}.
\]

(10)

Next, we select the relay from set \( \Omega \) that has the highest average rate of \( R - D \) link. The optimal relay \( \tilde{R}_s \) then forwards the information to the destination node. The signal-to-noise ratio of the optimal relay can be expressed as

\[
\gamma_{\tilde{R}_sD} = \frac{\eta \bar{\rho}_i P_S}{\sigma^2_D (1 + d_{SR}^a)(1 + d_{R_iD}^a)}.
\]

(11)

and

\[
\bar{\rho}_i = 1 - \frac{\tau (1 + d_{SR_i}^a)}{P_S}.
\]

(12)

We now analyze the outage probability of relay selection strategy based on S-CSI.

\[
\overline{P}_{out} = \Pr(\gamma_{SR_i} |\rho_i=0 < \tau) + \Pr(\gamma_{R_iD} < \tau, \gamma_{SR_i} |\rho_i=0 > \tau).
\]

(13)

Note that we assume all channels experience Rayleigh fading; thus, the channel gains \( X = |h_i|^2 \) and \( Y = |g_i|^2 \) follow exponential distribution, i.e., \( X \sim \exp(1), Y \sim \exp(1) \) [19]. It is worth noting that the assumption of channel gain with unit mean is only for easy derivation. Channel gains with other
We observe from Equation (9) that the distribution of \( \gamma_{SR} \) where (15) comes from the assumption that \( \epsilon \) where

\[
\Omega = \{ i \in \Omega : \gamma_{SRi} \neq 0 > \tau \}.
\]

Then, from the set \( \Omega \), we select the relay \( R_s \), which maximizes the instantaneous rate of R-D link. The optimal relay’s signal-to-noise ratio can be expressed as \( \gamma_{R,D} = \max \{ \gamma_{R,D,i} : i \in \Omega \} \). To obtain the outage probability analytically, we need to calculate the \( n \)th order statistics of SNRs with independent nonidentical distributions, which is different from the works in References [16,21].

In order to derive the performance of this scheme, we need to sort the received SNR at the relay and at the destination. The \( \gamma_{SR_i} \), for power splitting factor \( \rho_i = 0 \) is denoted as

\[
\gamma'_{SR_i} = \frac{P_S |h_i|^2}{\sigma^2_{h_i} (1 + d_{SR_i}^\alpha)}. 
\]

For different relays, the \( \gamma'_{SR_i} \) follows exponential distribution with different expectation \( m_i \), where

\[
m_i = P_S / [\sigma^2_{h_i} (1 + d_{SR_i}^\alpha)].
\]

We label the cumulative distribution function (CDF) of the \( n \)th order statistics of \( N \) independent exponential random variables as \( F_{(n)}(x_i), x_i \sim \text{Exp}(1/m_i) \).

For the received SNR \( \gamma_{R,D} \) at the destination, it is much more complicated to analyze. We observe from Equation (9) that the distribution of \( \gamma_{R,D} \) has no closed expression. Although the

\[
Pr(\gamma_{SR} \mid |\eta_i| = 0 < \tau) = Pr\left( \frac{P_S |h_i|^2}{\sigma^2_{h_i} (1 + d_{SR}^\alpha)} < \tau \right) = Pr\left( X < \tau / \epsilon_1 \right) = 1 - \exp(-\tau / \epsilon_1),
\]

where \( \epsilon_1 = \frac{P_S}{\sigma^2_{h_i} (1 + d_{SR}^\alpha)} \). The second item in right-hand side of Equation (13) is given by

\[
Pr(\gamma_{R,D} < \tau, \gamma_{SR} \mid |\eta_i| = 0 > \tau) = Pr(\gamma_{R,D} < \tau) Pr(\gamma_{SR} \mid |\eta_i| = 0 > \tau) = Pr(X < \tau / \epsilon_2) Pr(X < \tau / \epsilon_1)
\]

\[
= \exp(-\tau / \epsilon_1) \int_0^\infty \left( 1 - \exp\left(-\frac{\tau}{\epsilon_2 y}\right) \right) dy
\]

\[
= \exp(-\tau / \epsilon_1) \left( 1 - 2\sqrt{\tau / \epsilon_2 K_1(2\sqrt{\tau / \epsilon_2})} \right).
\]

By integrating Equations (14) and (16) into Equation (13), we obtain the outage probability for the S-CSI based relay selection method as

\[
\hat{P}_{out} = 1 - 2\exp(-\tau / \epsilon_1) \sqrt{\tau / \epsilon_2 K_1(2\sqrt{\tau / \epsilon_2})}.
\]

4. Relay Selection Based on Perfect Channel State Information

If the global CSI is available at the source and the relays, we propose an optimal relay selection method to further improve the outage performance. In this scheme, we first put the relays that successfully decode the source information into a candidate set \( \Omega \) in the first phase of transmission. The set \( \Omega \) is given by

\[
\Omega = \{ i \in \Omega : \gamma_{SRi} \neq 0 > \tau \}.
\]

means can be absorbed into the path loss model. The first item in right-hand side of Equation (13) can be calculated as follows:

\[
Pr(\gamma_{SR} \mid |\eta_i| = 0 < \tau) = Pr\left( \frac{P_S |h_i|^2}{\sigma^2_{h_i} (1 + d_{SR}^\alpha)} < \tau \right) = Pr(X < \tau / \epsilon_1) = 1 - \exp(-\tau / \epsilon_1),
\]

where \( \epsilon_1 = \frac{P_S}{\sigma^2_{h_i} (1 + d_{SR}^\alpha)} \). The second item in right-hand side of Equation (13) is given by

\[
Pr(\gamma_{R,D} < \tau, \gamma_{SR} \mid |\eta_i| = 0 > \tau) = Pr(\gamma_{R,D} < \tau) Pr(\gamma_{SR} \mid |\eta_i| = 0 > \tau) = Pr(X < \tau / \epsilon_2) Pr(X < \tau / \epsilon_1)
\]

\[
= \exp(-\tau / \epsilon_1) \int_0^\infty \left( 1 - \exp\left(-\frac{\tau}{\epsilon_2 y}\right) \right) dy
\]

\[
= \exp(-\tau / \epsilon_1) \left( 1 - 2\sqrt{\tau / \epsilon_2 K_1(2\sqrt{\tau / \epsilon_2})} \right).
\]

where (15) comes from the assumption that \( SR_i \) link and \( R_id \) link are independently realized and the symbol \( K_n(x) \) stands for the \( n \)th order modified Bessel function of the second kind [20]. The symbol \( \epsilon_2 \) stands for

\[
\epsilon_2 = \frac{\eta_\gamma P_S}{\sigma^2_{h_i} (1 + d_{SR}^\alpha)}.
\]

By integrating Equations (14) and (16) into Equation (13), we obtain the outage probability for the S-CSI based relay selection method as

\[
\hat{P}_{out} = 1 - 2\exp(-\tau / \epsilon_1) \sqrt{\tau / \epsilon_2 K_1(2\sqrt{\tau / \epsilon_2})}.
\]
distribution of $|h_i|^2$ and $|g_i|^2$ is known, distribution of $\rho_i$ has no closed form since it is a bounded piecewise function according to Equation (6). For simplicity, we here set $\rho_i$ as a static value $\rho$ and we will examine the effects of varying $\rho$ on the outage performance by simulations in Section 5. The expression of $\gamma_{R,D}$ can be simplified to

$$\gamma_{R,D} = n_i |h_i|^2 |g_i|^2, (i \in \Omega),$$  

(20)

where $n_i = \eta p D / |\rho^2 |D (1 + d_{SR}^2) (1 + d_{SR}^2)$. Since $|h_i|^2$ and $|g_i|^2$ both follow exponential distribution with unit expectation, it is easy to derive the CDF of their products as follows:

$$G(z_i) = 1 - 2 \sqrt{\frac{z_i}{n_i}} K_1(2 \sqrt{\frac{z_i}{n_i}}),$$  

(21)

where $K_n(x)$ is the modified Bessel function of the second kind. The proof is given in Appendix A. Having the above preliminary, we can obtain the outage probability of the optimal relay selection method based on perfect CSI as follows:

$$P_{out} = \Pr(|\Omega| = 0) + \sum_{n=1}^{N} \Pr(|\Omega| = n) \min_{\Omega[n]} \prod_{j=1}^{N} G_j(\tau),$$  

(22)

where $S[n]$ denotes the $n$-subset of set $\{1, 2, 3, \ldots, N\}$ and $\Pr(|\Omega| = n)$ denotes the probability that the candidate set $\Omega$ has $n$ elements. The probability can be obtained as

$$\Pr(|\Omega| = n) = \begin{cases} F_{(N)}(\tau), & n = 0 \\ F_{(N-n)}(\tau) (1 - F_{(N-n+1)}(\tau)), & 0 < n < N \\ F_{(1)}(\tau), & n = N \end{cases}$$  

(23)

for $n = 0, 1, 2, \ldots, N$. $F_{(n)}(x)$ denotes the CDF of $n$th order statistics of $N$ independent random variables $\gamma_{S,R}$. The proof is given in Appendix B. For $N$ random variables with nonidentical distributions, $F_{(n)}(x)$ does not have a closed expression [22]. However, we can compute the distribution of order statistics by using Lemma 1 [23].

Lemma 1. Suppose $X_1, \ldots, X_n$ are independently distributed, $X_k$ having distribution function $F_k(x)$ ($k = 1, \ldots, n$). The distribution function of the $r$th order statistic $X_{(r)}$ ($r = 1, \ldots, n$) is

$$F_{(r)}(x) = \sum_{k=r}^{n} \frac{1}{k!(n-k)!} \text{per}(B_k), t \in \mathbb{R},$$

where $B_k$ is defined as a square matrix constructed by repeated rows of $\{F_k(x)\}_{k=1,\ldots,n}$ and $\{1 - F_k(x)\}_{k=1,\ldots,n}$,

$$B_k = \begin{pmatrix} F_1(x) & \cdots & F_n(x) \\ 1 - F_1(x) & \cdots & 1 - F_n(x) \end{pmatrix}_{k \times (n-k)},$$

where the notation $k$ here means that the respective row has been repeated $k$ times. $\text{per}(A)$ stands for the permanent of matrix $A$.

5. Numerical Results and Discussions

In this section, we verify the analytical results by Monte Carlo simulations. Two relay selection algorithms including statistical channel state information-based methods and perfect channel state information-based methods are compared with the max–min algorithm with respect to outage probability performance. At the same time, we also compare the impact of the target information rate on the outage probability in the two relay selection algorithms. Finally, it is difficult to derive the
outage probability analytically due to the introduction of dynamic PSR; we study the impact of static PSR setting on the system performance.

Without loss of generality, we consider placing all nodes in a square area of unit area. The coordinates of the source \( S \) and the sink \( D \) are \((0, 0.5)\) and \((1, 0.5)\), respectively. All relay nodes are random distributed in this square area. In all simulations, the energy harvesting efficiency are assumed to be 1, the path loss factor is set 3, and power of the noise introduced in all antennas and circuits is 1, i.e., \( \sigma^2_R = \sigma^2_D = 1 \).

Figure 3 compares the performance of the relay selection algorithm based on statistical CSI and max–min algorithm [18]. The simulation results show that the obtained analytical results are in good agreement with the simulation results. It is also observed that the performance of the proposed algorithm is significantly better than that based on the max–min algorithm, especially in a region with high transmitting SNR.

![Figure 3](image)

**Figure 3.** Outage probability for statistical channel state information (CSI)-based relay selection and max–min relay selection with relay number \( N = 5 \), \( R = 2 \text{ bit/s/Hz} \), and the transmitting signal-to-noise ratio (SNR) varying from 0 to 50 dB.

In Figure 4, we compare the impact of the system target information rate \( R \) on the outage probability in the relay selection algorithm based on statistical CSI, taking into account the impact of the transmitting SNR. The simulation and numerical results show that the system performance is very sensitive to the variation of \( R \), which indicates that the performance gain brought by the relay is limited for systems with high quality of service requirements. At the same time, it can be found from the figure that improving the transmitting SNR can significantly improve the performance of the outage probability, while this will reduce the energy efficiency of the system.

Similar to Figure 3, Figure 5 compares the outage probability of relay selections based on perfect CSI and max–min algorithm. The results show that the analytical results are basically consistent with Monte Carlo simulations. At the same time, the performance of the proposed algorithm is significantly better than the max–min algorithm, especially in the region of higher transmitting SNR. It is worth noting that the max–min algorithm here is also based on perfect CSI, while the max–min algorithm considered in Figure 3 is based on statistical CSI.
In Figure 4, we attempt to analyze the effect of varying target rate on the outage probability of the relay selection algorithm based on perfect CSI. Both simulation and numerical results show that the system performance decreases sharply when the target rate increases. Especially for the case of lower transmit SNR, such as SNR = 20 dB with $R = 3$ bit/s/Hz, the system becomes almost unavailable.

Finally, in order to observe the impact of static PSR design on system performance, we depict the case where the analytical outage probability varies with PSR in Figure 7. Here, we consider the selection algorithm based on perfect CSI because it is more difficult to analyze the exact results in this case. At the same time, as a comparison, we also describe the approximate analytical results and simulation results based on dynamic PSR. The figure shows that the static PSR design has a large impact on the outage probability performance, which is significantly inferior to the dynamic PSR design. The approximate analytical result performance is slightly better than the simulation result because the approximate results are analyzed based on the average performance. As a result, the simulation results obtained with fewer relay nodes are inferior to the expectation from a probabilistic perspective.
Figure 6. Outage probability for perfect CSI-based relay selection with the target rate varying from 1 to 3 bit/s/Hz for $N = 5$ and where the transmitting SNRs are 20 and 30 dB, respectively.

Figure 7. Outage probability for perfect CSI-based relay selection with the static power splitting ratio (PSR) varying from 0 to 1, with the target rate $R = 2$ bit/s/Hz, and with $N = 5$: For comparison, the analytical and simulation results with dynamic PSR are also plotted.

By comparing Figure 3 and Figure 5, we find that the algorithm based on perfect CSI is obviously superior to the algorithm based on statistical CSI. While obtaining channel state information requires a large communication overhead, for most cases, algorithms based on statistical CSI are still more practical. However, relay selection based on perfect CSI can be used as the upper limit of other methods.

6. Conclusions

Aiming at the problems of low energy efficiency and spectrum efficiency of traditional cooperative networks, this paper proposes a routing algorithm based on statistical and perfect CSI for SWIPT enabled relays. The algorithm is divided into two steps. The first step is to select the relay that can successfully decode the source information to form a candidate set. The second step is to select the node with the highest statistical or instantaneous SNR from the candidate set as the optimal relay. We obtained the exact outage probability expressions of the two algorithms and verified them by
Monte Carlo simulations. The results show that the performances of the two algorithms proposed in this paper are better than that based on traditional max–min algorithm.

We note that the performance of the algorithm based on statistical CSI is significantly inferior to the algorithm based on perfect CSI while the acquisition of perfect CSI consumes more resources. Therefore, the tradeoff between these two algorithms is the future research direction of this work. In addition, for a relay network with multiple pairs of source and destination nodes, optimal selection of relays will be an interesting and challenging problem.

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**Appendix A. Proof of Equation (21)**

In this section, we derive the CDF of random variable \( Z = nXY \), where \( X \sim \exp(1) \), \( Y \sim \exp(1) \), and \( n \) is a positive real number.

\[
F_Z(z) = \Pr(Z \leq z) \\
= \Pr(X \leq \frac{z}{nY}) \\
= \int_{0}^{\infty} \left( \int_{0}^{\frac{z}{ny}} e^{-x}dx \right) e^{-y}dy \\
= \int_{0}^{\infty} (1 - e^{- \frac{z}{ny}}) e^{-y}dy \\
= 1 - \int_{0}^{\infty} e^{- \left( \frac{z}{ny} + y \right)} dy \\
= 1 - 2 \sqrt{\frac{z}{n}} K_1 \left( 2 \sqrt{\frac{z}{n}} \right), (A1)
\]

\( K_n(x) \) is the modified Bessel function of the second kind.

**Appendix B. Proof of Equation (23)**

In case of relay selection based on perfect CSI, the outage probability can be expressed as

\[
\mathcal{P}_{out} = \Pr(|\Omega| = 0) + \sum_{n=1}^{N} \Pr(\gamma_{R,D}^n < \tau|\Omega| = n), (A2)
\]

where \( \Pr(|\Omega| = n) \) denotes the probability that \( n \) relays are selected. For \( n = 0 \), which means no relay is selected in the first hop, \( \Pr(|\Omega| = 0) \) equals to the probability that the maximum of \( \gamma_{SR} \)'s is less than the threshold \( \tau \), i.e., \( \Pr(|\Omega| = 0) = F_{(N)|x}^{(N)}(\tau) \), where \( F_{(N)|x}^{(N)} \) is the distribution of maximum order statistic of the original SNRs. For \( n = N \), all the relays can decode the source information correctly; thus, \( \Pr(|\Omega| = N) \) equals the probability that the minimum of \( \gamma_{SR} \)'s is larger than the threshold \( \tau \), i.e., \( \Pr(|\Omega| = N) = 1 - F_{(1)|x}(\tau) \). For other \( n \)'s, \( \Pr(|\Omega| = n) \) equals the product of the probability that the \( (N-n) \)th order statistics is less than \( \tau \) and the probability that the \( (N-n+1) \)th order statistics is larger than \( \tau \), i.e., \( F_{(N-n)}^{(N-n)}(\tau)(1 - F_{(N-n+1)}^{(N-n+1)}(\tau)) \).

Now, we turn to calculating the second item of Equation (A2). As the relay links and the forward links are assumed to be independent, we can separate the \( \Pr(|\Omega| = n) \) from the conditional probability expression. While for different \( n \) there exist \( \binom{N}{n} \) combinations, we need to calculate the outage
probability of each forward link as $G_j(\tau)$ for $j \in (N_n)$. The system outaged when the maximum SNR of all of forward links is less than the threshold $\tau$; therefore, the outage probability is $\prod_{j=1}^{N_n} G_j(\tau)$. Note that there are $s[n]$ subsets in each $n$; if the weakest group of these subsets is outaged, then the system outaged. In conclusion, the second item of Equation (A2) can be derived as

$$\sum_{n=1}^{N} \Pr(\gamma_{n, D}^* < \tau | |\Omega| = n) = \sum_{n=1}^{N} \Pr(|\Omega| = n) \min_{s[n]} [\prod_{j=1}^{N_n} G_j(\tau)]. \quad (A3)$$

The conclusion of Equation (22) is proved.

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