CHIRAL SYMMETRY
AND
MESON-EXCHANGE CURRENTS

Mannque Rho

Service de Physique Théorique, CEA Saclay,
F-91191 Gif-sur-Yvette, France

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ABSTRACT

The meson-exchange current in nuclei, a long-standing problem in nuclear physics, is described in modern theory of strong interactions, namely, QCD expressed in terms of effective chiral Lagrangian field theory. Some old results are given a modern interpretation and some new results are predicted. Among the topics treated are an accurate calculation of the radiative np capture process at thermal energy, the enhanced axial charge transition in heavy nuclei, Brown-Rho (BR) scaling in dense medium induced by vacuum changes, dropping meson masses and “mended symmetry” in relativistic heavy-ion processes and the link between the physics of dilute and dense hadronic systems through a mapping of effective chiral Lagrangians to Landau Fermi-liquid theory.

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In a recent *Nature* article, Weinberg, in summarizing the present understanding of what is meant by “elementary particles” and field theory, wrote [1]: *We have come to understand that particles may be described at sufficiently low energies by fields appearing in so-called effective quantum field theories, whether or not these particles are truly elementary. For instance, even though nucleon and pion fields do not appear in the Standard Model, we can calculate the rates for processes involving low-energy pions and nucleons by using an effective quantum field theory of pion and nucleon fields rather than of quark and gluon fields. .... When we use a field theory in this way, we are simply invoking the general principles of relativistic quantum theories, together with any relevant symmetries; we are not making any assumption about the fundamental structure of physics.*

In this article, I would like to illustrate how this statement can be applied to nuclear physics. I will start with a well-known case of the lightest nucleus, namely, two-nucleon system, go to heavy nuclei and then finally to dense matter with a density higher than nuclear matter density.

Consider a process involving two-nucleon systems. The classic case studied for almost half a century is the process

\[ n + p \rightarrow d + \gamma \]  

(1)

at thermal energy, with the relative momentum in the center of mass system \( p \simeq 3.4451 \times 10^{-5} \text{ MeV} \). This process was explained within 10% accuracy by Austern already in 1953 [2] and the remaining 10% discrepancy was explained in terms of meson-exchange currents by Riska and Brown in 1974 [3]. I will now describe how one can completely understand this process in an effective chiral Lagrangian formalism [4].

For this process, we can start with a theory defined in the vacuum, that is to say, in matter-free space since the two-body system is a dilute one. From the QCD point of view, the essential physics is dictated by the quark condensate in the vacuum \( \langle \bar{q}q \rangle \) since apart from the small up and down quark masses, the masses and couplings of the relevant degrees of freedom, i.e., light-quark hadrons, are dictated by the spontaneously broken chiral symmetry. This is because the length scale is set by the quark condensate in the vacuum. Thus the relevant theory is the one that one should be able to write down in matter-free space.

Now following the general strategy of effective field theories, we first have to identify the relevant degrees of freedom that we would like to treat explicitly and put all irrelevant degrees of freedom into the constants appearing in the Lagrangian. Since the process involved (1) is a very low-energy process, we take explicitly as relevant degrees of freedom the proton and neutron fields denoted as a doublet \( N \) together with the Goldstone excitations of spontaneously broken chiral symmetry, namely, the pions \( \pi^i \). For the moment, other heavy degrees of freedom such as the baryon resonance \( \Delta \), the vector mesons \( \rho \) and \( \omega \) and the scalar \( \sigma \) will be integrated out so that they will not figure explicitly in the theory. They will of course figure somewhere in the theory and I will show where later.
The next step is to write down the most general Lagrangian consisting of the $N$ and $\pi$ fields consistent with the general properties Weinberg is referring to above, notably, spontaneously broken chiral symmetry. The Lagrangian will contain, in addition to bilinears in the $N$ field, terms involving $4N$, $6N$ etc. to all orders in power of fermion fields suitably coupled to Goldstone pions and since light-quark masses are not zero, though small, there should be terms involving quark mass terms. In nuclear physics at low energies, the nucleon can be considered as heavy. When the nucleon is treated in that way, the leading Lagrangian, when expanded, can be written as

$$\mathcal{L} = N^\dagger \left(i\partial^0 + \frac{\nabla^2}{2M}\right)N - \frac{g}{f_\pi} \nabla_\pi \cdot \tau N + \frac{1}{2}(\partial^0 \pi)^2 - \frac{1}{2}(\nabla \pi)^2 - \frac{1}{2}m_\pi^2(\pi)^2 + \frac{C_1}{f_\pi^2}(N^\dagger N)^2 + \frac{C_2}{f_\pi^2}(N^\dagger \sigma N)^2 + \cdots$$

(2)

where $\pi$ is the triplet pion field. Counting rules can be devised in such a way that one can do a systematic expansion in some momentum scale $Q$ being probed that is in some sense small compared to the typical chiral scale $\Lambda$ which can be taken to be roughly of the mass of the heavy particles that have been integrated out. If this is effectuated to all orders, then according to the axiom given by Weinberg [1], we are in principle doing a full theory.

Let us see what this scheme means for nuclear interactions. In so doing I will uncover what is called “chiral filter phenomenon” in nuclear processes. A physical amplitude involving $E_N$ external nucleon lines can be written as

$$A \sim Q^\nu F(Q/\Lambda)$$

(3)

where $F$ is a slowly varying function of the dimensionless quantity $Q/\Lambda$. Given that $Q$ is small compared with the chiral scale, the idea is to calculate to the highest order possible in $\nu$ and sum the terms to the order calculated. If one can do this to all orders, as mentioned, then one is doing the full theory. With the chiral Lagrangian that we are concerned with, the counting rule can be readily deduced by looking at the Feynman diagrams. One finds

$$\nu = 4 - 2C + 2L - (E_N/2 + E_{ext}) + \sum_i V_i \bar{\nu}_i$$

(4)

where $C$ stands for the number of clusters, $E_N$ the number of external incoming and outgoing nucleon lines, $E_{ext}$ the number of external fields, $L$ the number of loops and $V_i$ the number of vertices of type $i$ and

$$\bar{\nu}_i = d_i + \frac{n_i}{2} + e_i - 2$$

(5)

with $d_i$ the number of derivatives, $n_i$ the number of nucleon lines and $e_i$ the number of external fields entering the $i$th vertex.

In this paper we will be concerned with at most one slowly varying external field, so we will have $E_{ext} = 1$. Obviously $e_i = 1$ will appear only once if at all. For two-body
exchange currents, we are concerned with an irreducible graph with $E_N = 4$ and $C = 1$. So the important quantities in (5) are the number of loops $L$ and the $\tilde{\nu}$. For a given $L$, therefore, only $\tilde{\nu}$ matters. The particular structure of chiral symmetry requires that

$$\tilde{\nu}_i \geq 0. \quad (6)$$

We can now state the “chiral filter phenomenon” which will figure prominently in what follows. In the form it was first stated [5], the general argument that follows from effective chiral Lagrangians was not used. What the statement says is that while nuclear forces involve long-range and short-range interactions on the same footing and hence have to be taken into account at the same time, the response to a slowly varying electroweak field screens short-distance physics, thereby causing the effect of soft-pion mediated process to show up prominently (unless accidentally suppressed by kinematics). One can see this to the lowest order in the chiral counting. Take the two-nucleon potential generated by one-pion exchange and the one generated by a contact four-Fermi interaction in Eq.(2). The one-pion exchange involves two vertices each of which has index $\tilde{\nu}_i = 0$ since there is one derivative in the pion-nucleon coupling and two nucleons attached to the vertex, $n_i = 2$. Higher-order terms in derivative will bring in higher power and be suppressed. So at the zero-loop level, it is this tree term that matters. But the same is true with the four-Fermi interaction with zero loops as there is no derivative at the vertex but four nucleons enter with $n_i = 4$, so again $\tilde{\nu} = 0$. The one-pion-exchange potential is the longest-range one in two-nucleon systems. Now four-Fermi interactions represent the short-range part of the potential in which heavy mesons representing the degrees of freedom lying above the chiral scale $\Lambda$ have been integrated out. Thus we conclude that as far as the chiral counting is concerned both the longest-range and shortest-range potentials contribute on an equal footing and cannot be separated.

Consider attaching an external field to the same two-body system. Attaching an external field to the one-pion-exchange term does not modify the index $\tilde{\nu}$ since $e_i + d_i = 1$, hence we still have $\tilde{\nu} = 0$ in the leading order but the four-Fermi interaction term requires one derivative in addition to $e_i = 1$ leading to an index $\tilde{\nu} \geq 2$. Therefore in contrast to the nucleon-nucleon potential, short-range interaction terms are naturally suppressed relative to the one-pion-exchange term when attached to an external field [6]. This is true for a slowly varying electroweak current in general.

This “chiral filtering” is both a good news and a bad news. It is a good news in that meson-exchange currents can be under control with the dominance of Goldstone pions without interference from poorly-understood short-range degrees of freedom. It is a bad news since the pion dominance means that unless accidentally suppressed, pions will not allow us to learn short-distance physics through exchange currents. To some who would like to see a “smoking gun” of quark-gluon degrees of freedom in nuclear processes, this is a pity.

I show in Figure 1 the results of the most recent calculation to the next-to-next
Figure 1: Total capture cross section $\sigma_{\text{cap}}$ (top) and $\delta$’s (bottom) vs. the hard-core radius $r_c$ that represents the uncertainty due to short-distance physics unaccounted for in low-order chiral perturbation theory. The solid line represents the total contributions and the experimental values are given by the shaded band indicating the error bar. The dotted line gives $\delta_{\text{tree}}$, the dashed line $\delta_{\text{tree}} + \delta_{1\pi}$, the dot-dashed line $\delta_{\text{tree}} + \delta_{1\pi} - \frac{\pi}{2}$, and the solid line the total ratio, $\delta_{2B}$. 

$\pi$ and the solid line the total ratio,
order in the chiral expansion $^{[4]}$. I should explain in some detail what went into this calculation. In calculating the cross section, imagine that we have the process occurring between a proton and a neutron interacting to all orders through the chiral Lagrangian of the form $(2)$. The initial state is the scattering state in $^1S_0$ and the final state is the bound (deuteron) $^3S_1$ state with a small D-state admixture. The electromagnetic current – which is predominantly magnetic dipole ($M1$) – connects the initial and final states. Both the bound state and the scattering state with a large scattering length (with $a_s = -23.75$ fm) are not amenable to a straightforward chiral perturbation expansion because of infrared divergences and nobody knows how to calculate this in a systematic chiral expansion. However this difficulty can be circumvented by realizing that what we are interested in is the meson-exchange current contribution relative to the single-particle matrix element for which we can take the most realistic wave functions available for the initial and final states. Such wave functions are indeed in the market, namely those computed from the Argonne $v_{18}$ potential $^{[7]}$. This procedure is not exactly a systematic chiral expansion since the infrared-divergent reducible diagrams are summed to all orders (in the form of solving the Schrödinger equation) while irreducible graphs are computed to the NNL order only. However the NNL order essentially saturates the irreducible graphs within the uncertainty associated with the short-distance part of the wave function, which is of the order of less than 1%, so the scheme is consistent as far as this calculation is concerned.

To the extent that the wave functions are very accurate, the single-particle matrix element will also be. One can gauge this by looking at the prediction for the $^1S_0$ scattering length and the static properties of the deuteron, all of which are remarkably accurately given by the Argonne $v_{18}$ potential. It is therefore convenient to look only at the exchange-current corrections relative to the single-particle matrix element. The dominant one-pion exchange matrix element is $\mathcal{O}(Q^2)$ relative to the single-particle matrix element. One-loop radiative corrections are further suppressed by the same order. The ratios of the matrix elements relative to the single-particle matrix elements are denoted $\delta$ in Figure $^{[1]}$. The four-Fermi interactions found to be suppressed by the “chiral filtering” enter at the range of uncertainty associated with the short-range correlation in the wave functions given by the hard-core radius $r_c$ in Figure $^{[0]}$. In fact, the suppression of the zero-range operators due to the correlation function represents in an indirect way this part of physics$^{[2]}$. The $1\pi(\omega)$ and $1\pi(\Delta)$ correspond to one-pion exchange contributions with vertex corrections due to the counter terms resonance-saturated with an $\omega$ and a $\Delta$, respectively. The $2\pi$ corresponds to the two-pion exchange one-loop correction. The total 2-body correction is denoted $\delta_{2B}$.

Now calculated with the single-particle matrix element alone, the cross section comes out to be

$$\sigma_{imp} = 305.6 \text{ mb} \quad (7)$$

$^2$Work is in progress$^{[8]}$ to eliminate the ad hoc use of the hard-core cutoff in the wave function by a systematic chiral expansion in the Hamiltonian and in the wave function following a method recently discussed in a lecture by Lepage$^{[9]}$. 

5
which differs by about 9\% from the experimental value

$$\sigma_{\text{exp}} = 334.2 \pm 0.5 \text{ mb.}$$  \hfill (8)

The chiral Lagrangian treatment, taking into account the short-range uncertainty given above, gives an accurate result, accounting fully for the missing 9\%,

$$\sigma_{\text{theory}} = 334 \pm 3 \text{ mb} \quad \hfill (9)$$

where the quoted error represents theoretical uncertainty associated with short-distance physics.

Perhaps much less solid theoretically but more spectacular is the electrodisintegration of the deuteron, a sort of inverse process to the np capture,

$$e + d \rightarrow e + n + p.$$  \hfill (10)

If one applies the same formalism as in the np capture, it is found that the meson-exchange current effect, while small in the np capture, becomes big because of a substantial cancellation at finite momentum transfer between the S-state and D-state components of the deuteron wave functions. The chiral expansion to the next-to-next order described above turns out to work well up to a large momentum transfer of order $\sim 1 \text{ GeV}$ \hfill [10]. While it has not been checked in detail that other corrections remain negligible here, the important presence of the mesonic current is clearly exhibited in this process. It remains to prove that higher order chiral corrections are indeed suppressed at large momentum transfers involved.

Another place where the chiral filtering is visibly operative is in axial charge transitions in nuclei, i.e,

$$J^+ \leftrightarrow J^- \quad \Delta T = 1.$$  \hfill (11)

If $J = 0$, this process is analogous to a pion decaying into the vacuum carrying an interesting information on the “vacuum property” of nuclear ground state. Warburton \hfill [11] studied extensively this class of transitions in light, medium and heavy nuclei, and obtained an important result that as one goes to heavier nuclei the effect of the pion exchange in the axial charge matrix elements becomes stronger. He defined a quantity called $\epsilon_{\text{MEC}}$

$$\epsilon_{\text{MEC}} = \frac{M_{\text{exp}}}{M_{\text{sp}}}$$  \hfill (12)

where $M_{\text{exp}}$ is the experimentally measured matrix element of the axial charge operator and $M_{\text{sp}}$ is the theoretical matrix element of the single-particle axial charge operator calculated with the best possible nuclear wave functions available in the literature. Since this quantity involves both experimental and theoretical quantities, it is not quite what one would call experimental value. There is an inherent uncertainty associated with the single-particle matrix element. What is however quite significant in the study of Warburton is that unlike
in the case of electromagnetic exchange currents the effect of the chiral-filtered pions can be enormous. Indeed in light nuclei, $\epsilon_{MEC}$ is around 1.5, that is, the exchange correction is 50% in the matrix element. This is a huge correction. What is more significant is that in heavy nuclei, the effect is even more dramatic. In lead region, Warburton found

$$\epsilon_{MEC} = 1.8 \sim 2.0.$$  \hfill (13) 

The range is the uncertainty involved in the theoretical single-particle matrix element alluded to above.

There is a simple way of getting the enhancement (13). This can be done by combining the chiral filter mechanism together with what is known as “BR scaling” in dense medium which I shall now explain. The idea involves once more the general philosophy of effective Lagrangians but extrapolated further into the regime where matter is dense and where direct measurements are not readily available.

If physics does not change drastically from light to heavy nuclei, one may start with a Lagrangian like (2) and then compute systematically the effect of the medium by suitably accounting for additional scales brought in by matter density. There are efforts to do this sort of calculations. Here I will consider approaching from the other extreme (say, a “top-down” approach) where possible nonperturbative effects associated with the medium are taken into account \textit{ab initio} in a manner consistent with the notion of chiral effective theories. This has an advantage in that physics under extreme conditions such as the state of dense matter encountered in compact neutron stars and relativistic heavy-ion collisions can be treated on the same footing. Viewed in this way, calculating the enhancement (13) will be a low-order calculation whereas starting from (2) would require “high-order” calculations.

In writing down Eq. (2), I emphasized that it has particles whose parameters are defined in the absence of background matter. Now consider a particle, a fermion or a boson, propagating in a medium consisting of matter in a bound state like in the interior of a very heavy nucleus. For this, I can start with a Skyrme type Lagrangian containing only meson fields. Imagine having a realistic Lagrangian of such type containing not only pions as in the original Skyrme Lagrangian but also vector mesons and other heavy mesons. A nucleon with this Lagrangian comes out as a soliton, “skyrmion,” with baryon number $B = 1$ which is just the winding number of the soliton. The same Lagrangian in principle can describe the deuteron, triton ... and $B = \infty$ nucleus, all arising from the same Lagrangian. At present we do not know how to write such a Lagrangian and hence we do not know how to compute, for instance, the binding energy, the equilibrium density of nuclear matter and nuclear matrix elements of currents. What is known is that the deuteron and nuclear force can be reasonably understood even from a drastically simplified skyrmion Lagrangian[12].

Given a realistic chiral Lagrangian of the skyrmion type, the question one can ask is: How does a hadron propagate in a medium defined by a density $\rho$, say ? The most obvious thing to do is then to write an effective Lagrangian that has all the right symmetries of
the original theory, QCD, but suitable in the background defined in the presence of a medium. In QCD, the quantity that reflects the background or the “vacuum” is the quark condensate and since the background is changed, we expect that the condensate would be suitably changed. Let me denote the modified condensate by putting an asterisk

\[ \langle \bar{q}q \rangle^\ast \neq \langle \bar{q}q \rangle_0, \quad \rho \neq 0. \]  

(14)

Since the condensate is modified, all the associated quantities such as light-quark hadron masses, the pion decay constant \( f_\pi \) etc. will be suitably modified. I will denote them with an asterisk on top. By following the strategy of preserving the same symmetry present in matter-free space except that asterisked parameters enter, it is possible to establish the scaling

\[ m_V^\ast/m_V \approx M_N^\ast/m_N \approx m_\sigma^\ast/m_\sigma \approx f_\pi^\ast/f_\pi \approx \left( \langle \bar{q}q \rangle^\ast/\langle \bar{q}q \rangle_0 \right)^n \]  

(15)

where the subscripts \( V, \sigma, \) and \( N \) stand, respectively, for (light-quark) vector meson, scalar meson and nucleon fields and the index \( n \) is some power that depends on specific models (for the simplest Skyrme model, \( n = 1 \), for the NJL model, \( n = 1/2 \) etc.). An effective Lagrangian of the type (2) with its parameters given by (15) can then describe, at tree order, fluctuations around the state defined by density \( \rho \).

At present there is no systematic derivation of such an effective Lagrangian from first-principle arguments. As such, the scaling (15) is not a relation that can be used in any Lagrangian field theory dealing with nuclear matter. It should be considered as a particular parameterization with a given Lagrangian of the type I have been considering. Thus the quantities with such scaling can have meaning only as parameters of a specific theory and it would be too hasty to identify them as “physical” masses and constants. The only quantity that is physically meaningful is the measurable one.

One way to “derive” the scaled Lagrangian is to look for a non-topological soliton of the effective action arising from a high order chiral perturbation theory. As suggested by Lynn [14], it could be a “chiral liquid” that defines the Fermi sea with a given Fermi momentum \( k_F \). One can identify this as a “chiral-scale” decimation in the renormalization group approach mentioned below, with the cutoff set at the chiral scale \( \Lambda \sim \Lambda_\chi \sim 1 \) GeV. Once such a “chiral liquid” state is obtained, then the scaling parameters will be defined in fluctuations around the chiral liquid state in what I would call “Fermi-liquid scale decimation.” I will return to this matter.

Let us now go back to Warburton’s \( \epsilon_{MEC} \) in heavy nuclei for which we will take \( \rho \approx \rho_0 \). Suitably coupling the axial current to a BR effective Lagrangian, one can calculate and find

\[ \epsilon_{MEC} = \Phi^{-1}(1 + R) \]  

(16)

where

\[ \Phi := f_\pi^\ast/f_\pi \approx m_V^\ast/m_V \cdots \]  

(17)
and $R$ is the ratio of the matrix elements of the meson-exchange axial charge operator over the single-particle axial charge operator. The meson-exchange operators are given in chiral perturbation theory to the next-to-next-to leading order in the chiral expansion as in the electromagnetic case, again dominated by the pions due to the chiral filter as explained in [5]. The ratio $R$ does not depend much on how one calculates the matrix elements, that is, nuclear model-independent, and depends only slightly on density. For heavy nuclei, it comes out to be $R \approx 0.5 \sim 0.6$. The quantity we need to compute $\epsilon_{MEC}$ is $\Phi$, the only quantity that knows that nuclear matter “vacuum” is different from the matter-free vacuum. There are two ways known to get this quantity – and this is not given by the strategy of effective chiral Lagrangian field theory. One is to use the Gell-Mann-Oakes-Renner formula for the pion embedded in nuclear medium, the other is to do a QCD sum-rule calculation for the vector-meson mass in medium. While both quantities in medium are not without ambiguity, they nonetheless give the same answer. The result by the latter method at $\rho = \rho_0$ is [17]

$$m_V^*/m_V = \Phi(\rho) = 0.78 \pm 0.08, \quad \rho = \rho_0.$$  \hspace{1cm} (18)

With this value, (16) gives

$$\epsilon_{MEC} = 1.9 \sim 2.0.$$  \hspace{1cm} (19)

This agrees with Warburton’s analyses. As emphasized before, one could of course calculate corrections to the chiral-filtered pionic contribution without invoking BR scaling but instead using a vacuum-defined chiral Lagrangian and explicitly incorporating other degrees of freedom (such as an effective scalar meson $\sigma$ and light-quark vector mesons) and get the required enhancement [18]. The two methods must be equivalent to leading order at nuclear matter density.

It is easy to generalize the formalism to $SU(3)$ flavors and study fluctuations in the strangeness directions. For instance, one could look at the production of kaons in dense medium in heavy-ion collisions. Once the ground state is defined in terms of BR scaling chiral Lagrangians, fluctuations are then automatic at tree level, combining both flavor $SU(3)$ symmetry and chiral symmetry. How this can be done is discussed in [15]. Some of the predictions made in this way have been tested by experiments recently performed at GSI (e.g., FOPI and KaoS) and are fairly well confirmed [19]. Extended smoothly beyond nuclear matter density, the theory can make predictions on possible phase transition with condensation of kaons at a density $\rho \sim 3\rho_0$ in compact-star matter like in nucleon stars with a fascinating consequence on the formation of small black holes and on the maximum mass of stable neutron stars etc. [20].

Up to this point, I have not discussed how the ground state, namely nuclear matter, comes out in this description. I shall now “map” the chiral Lagrangian with BR scaling treated in mean field to Landau Fermi-liquid theory of nuclear matter developed by Migdal [21]. The idea is based on two observations. The first is that relativistic mean-field theory for nuclear matter is known to be interpretable as equivalent to Landau Fermi-liquid theory.
For instance, Walecka’s mean field theory has been shown to be one such theory\(^\text{[22]}\). The second is that Landau Fermi-liquid theory is a renormalization-group fixed point theory\(^\text{[23]}\). This observation allows one to formulate a many-body problem from the point of view of effective field theories which is clearly what is needed to go further into the unknown regime of high density. In a recent paper, Brown and I\(^\text{[24]}\) argued that the BR scaled chiral Lagrangian in a simplified form, when treated at the mean field level, is equivalent to a Walecka-type mean field theory. It is therefore quite logical that the BR scaled chiral Lagrangian mean-field theory is equivalent to Landau Fermi-liquid fixed point theory. The relevant arguments linking various elements of the theory are found in\(^\text{[15]}\).

The crucial link is found at the stage of the second – “Fermi-liquid” – decimation that integrates out excitations of the scale \(\bar{\lambda}\) around the Fermi surface defined by the Fermi momentum \(k_F\) and then does the rescaling. The main ingredient is the renormalization group-flow result that there are two fixed-point quantities in the theory\(^\text{[23]}\). One of them is the effective mass of the nucleon \(m_N^*\) and the other is the Landau interaction \(F\). Both are defined at the Fermi surface. By Galilean invariance, the effective mass is related to the Landau parameter \(F_1\) as

\[
m_N^*/m_N = 1 + \frac{1}{3} F_1 = \left(1 - \frac{1}{3} \bar{F}_1\right)^{-1} \tag{20}\]

where \(\bar{F}_1 := (m_N/m_N^*)F_1\). Now using that the Walecka model is equivalent to Landau Fermi liquid, we deduce that \(\bar{F}_1\) gets a contribution from the \(\omega\) channel, say, \(\bar{F}_1^\omega\). Due to chiral symmetry, there is also the Goldstone pion contribution through a Fock term to \(\bar{F}_1\) which can be explicitly calculated. Thus

\[
\bar{F}_1 = \bar{F}_1^\omega + \bar{F}_1^\pi \tag{21}\]

with

\[
\bar{F}_1^\omega = 3(1 - m_N/M_N^*) = 3(1 - \Phi^{-1}) \tag{22}\]

and

\[
\bar{F}_1^\pi = -\frac{9f_{\pi NN}^2 m_N}{8\pi^2 k_F} \left[ \frac{m_\pi^2 + 2k_F^2}{2k_F^2} \ln \frac{m_\pi^2 + 4k_F^2}{m_\pi^2} - 2 \right] \tag{23}\]

where \(f_{\pi NN} \approx 1\) is the nonrelativistic \(\pi N\) coupling constant. Note that \(^\text{[23]}\) is precisely determined once the Fermi momentum is given, say, \(\approx -0.153\) at normal matter density. The important point here is that the effective mass gets contributions from the (BR) scaling parameter \(^\text{[23]}\) and the pion. The pion comes in as a perturbative correction to the nonperturbative “vacuum” contribution given by \(\Phi\). The effective (Landau) mass \(^\text{[20]}\) is therefore

\[
m_N^*/m_N = \left(\Phi^{-1} - \frac{1}{3} \bar{F}_1^\pi\right)^{-1} \tag{24}\]
which at \( \rho = \rho_0 \) predicts

\[
\frac{m_N^*}{m_N(\rho_0)} \approx 0.69. \tag{25}
\]

This is a genuine prediction which is supported by the orbital gyromagnetic ratio in heavy nuclei, discussed below. It is also consistent with the QCD sum rule calculation of the nucleon mass in medium\(^{23}\),

\[
(m_N^*/m_N)_{\text{QCD}}(\rho_0) = 0.69^{+0.14}_{-0.07}. \tag{26}
\]

Now having the relation between the fixed point \( m_N^* \) and \( \Phi \) (plus the calculable pionic term), we can derive various interesting and highly nontrivial relations applicable to long-wavelength processes\(^{26}\). For instance, the EM convection current for a nucleon on the Fermi surface which can be written down on the basis of \( U(1) \) gauge invariance can be derived from our chiral Lagrangian:

\[
J = g_l \frac{\mathbf{P}}{m_N} \tag{27}
\]

where \( g_l \) is the orbital gyromagnetic ratio given by

\[
g_l = \frac{1 + \tau_3}{2} + \delta g_l \tag{28}
\]

with

\[
\delta g_l = \frac{4}{9} \left[ \Phi^{-1} - 1 - \frac{1}{2} \tilde{F}_1 \right] \tau_3. \tag{29}
\]

I should stress that this relation is highly non-trivial for several reasons. First of all, the isoscalar current is given by \( J^{(0)} = \mathbf{p}/2m_N \), so the scaling mass \( M_N^* \) does not figure in the current (this is an equivalent to “Kohn theorem” in condensed matter physics) and secondly the many-body nature of the system is manifested only in the isovector part through \( \delta g_l \).

Given the numerical value \((18)\) at nuclear matter density, we get

\[
\delta g_l = 0.23\tau_3. \tag{30}
\]

This should be compared with \( \delta g_l^{\text{proton}} = 0.23 \pm 0.03 \) obtained from a dipole sum rule in \(^{209}\)Bi\(^{27}\) and with \( \delta g_l^{\text{proton}} \approx 0.33, \delta g_l^{\text{neutron}} \approx -0.22 \) obtained from an analysis of the magnetic moments in the \(^{208}\)Pb region\(^{28}\).

The deviation of the nucleon effective mass from the “universal” scaling factor \( \Phi \), \((24)\), arises from the presence of the Goldstone pions. In the skyrmion description, the difference arises from the fact that aside from the known current algebra term, an additional term – Skyrme quartic term – is needed for stabilizing the soliton that metamorphoses into the physical nucleon. Expressed in terms of physical variables, the difference can be attributed to the fact that the axial coupling constant \( g_A \) can scale in nuclear medium\(^{13, 23}\). It turns out that

\[
\frac{m_N^*}{m_N} = \left( \frac{g_A^*}{g_A} \right)^{\frac{1}{2}} \Phi. \tag{31}
\]
Comparing with (24), we find that
\[ g_A^* A g_A = (1 + \frac{1}{3} F_1^2)^2 = (1 - \frac{1}{3} \Phi \bar{F_1}^2)^{-2}. \] (32)

For nuclear matter $\rho \approx \rho_0$, this predicts
\[ g_A^* (\rho_0) \approx 1. \] (33)

This is quite close to what is found in nature. The same result was obtained many years ago in terms of the Landau-Migdal parameter $g_N'$ in the $\Delta N$ channel, which has recently been interpreted as a counter term in higher order chiral expansion. The relationship between these different interpretations is not yet understood and remains to be clarified.

This close agreement of the chain of predictions with experiments can be taken to confirm the validity of the notion that the scaling (15) – initially introduced as a vacuum change – is associated with the Fermi liquid fixed point in many-body interactions. I will later use this observation to propose a dual description between QCD and hadronic variables. More immediately this result should allow us to write down an effective chiral Lagrangian with the scaling (15) that in the mean-field approximation reproduces exactly the above Fermi liquid structure. The corresponding Lagrangian is
\[ \mathcal{L}_{BR} = \bar{N} (i \gamma_\mu (\partial^\mu + ig_A^* \omega^\mu) - M_N^* + h \phi) N - \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (\partial_\mu \phi)^2 + \frac{m_\omega^2}{2} \omega^2 - \frac{m_s^2}{2} \phi^2 \] (34)

where we have retained only the $\omega$ field and the effective scalar field $\phi$ in the meson sector eliminating the pion field from the chiral Lagrangian since we are to interpret it as an effective one to be considered only in the mean field. I have written this Lagrangian in analogy to Walecka’s original linear $\sigma \omega$ model but it would be better to consider it as a Lagrangian that obtains in a chirally invariant way from one with 2-Fermi and 4-Fermi interactions using massive auxiliary fields $\omega$ and $\phi$. Treated in the standard manner as for the Walecka model, this effective Lagrangian describes nuclear matter fairly accurately. For instance with the physical masses, $m_N = 939$ MeV, $m_\omega = 783$ MeV and $m_s = 700$ MeV and the parameters, $h = 6.62$, $g_\omega = 15.8$ and assuming the scaling
\[ \Phi(\rho) = (1 + 0.28 \rho / \rho_0)^{-1} \] (35)

Clearly the formula (32) cannot be valid beyond a certain density $\rho_0$. It appears that $g_A^* \approx 1$ is a fixed point.

This should be understood in the sense of the effective action in the mean field sense, with $\delta S^{eff}/\omega_*, \phi_* = 0$ for $S^{eff} = \int d^4x \mathcal{L}_{BR}(x)$. Fields not figuring in the mean field such as pion field are not explicit in here. However once the mean field is defined, fluctuations into strange and non-strange directions can be described by restoring pion, kaon,... fields in a way consistent with chiral symmetry. Certain consistency conditions require that $g^*_\omega/g_\omega$ scale like $\Phi$ whereas no such conditions exist for the scalar constant $h$. 

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normalized so that the known value \( \rho = \rho_0 \) is reproduced at \( \rho = \rho_0 \), we get the binding energy \( B \), the equilibrium Fermi momentum \( k_F \) and the compression modulus \( K \):

\[
B = 16.0 \text{ MeV}, \quad k_F = 257 \text{ MeV}, \quad K = 296 \text{ MeV}.
\]

The corresponding effective mass of the nucleon at the minimum is

\[
m_N^* = M_N^* - \hbar \langle \phi \rangle^* = 0.62 m_N
\]

which should be compared with (25) and (26). The nuclear matter property so obtained is quite close to that obtained from an effective chiral Lagrangian constructed based on naturalness condition [32].

The idea developed here allows one to explore what happens when matter is compressed to a density greater than normal. This is a relevant issue for on-going experiments in relativistic heavy-ion collisions and for understanding such compact stars as neutron stars. Suppose one would like to probe the regime where \( \rho > \rho_0 \). Instead of approaching this regime with a Lagrangian defined at \( \rho = 0 \) as is done conventionally, I would like to consider fluctuations around \( \rho \approx \rho_0 \) with the effective Lagrangian defined at that point.

The advantage in doing this is that even if fluctuating around the \( \rho = 0 \) vacuum were a strong-coupling process and hence required a high-order calculation, fluctuations around the ground state at \( \rho = \rho_0 \) could be weak-coupling allowing for a tree-order or at most a next-to-leading calculation. Indeed the recent elegantly simple explanation of the CERES dilepton data [33] by Li, Ko and Brown (LKB) [34] is a nice example of such an application. Here one is probing hadronic matter at a density \( \sim 3 \rho_0 \) at some high temperature. In the LKB approach, the dileptons measured in the experiments are interpreted as arising from mesons in a heat bath with their masses scaled as (15). The result is consistent with a quasiparticle picture for both nucleons and mesons in a heat bath.

The argument developed to link effective chiral Lagrangians and Fermi-liquid theory is manifestly tailored for very low-energy excitations for which Landau quasiparticle picture is valid. For instance, in describing nuclear matter ground state, the heavy meson fields whose parameters scale as (15) are way off-shell. In matter-free space their masses are comparable to the chiral scale \( \Lambda_{\chi} \) and hence one might naively think that processes involving excitations of such particles on-shell could not be handled reliably by the argument based on chiral symmetry used here. Now what is observed in the CERES experiment is highly excited modes, involving hundreds of MeV. In particular the “\( \rho \) meson” which plays an important role in the description of [34] is near its mass shell \textit{albeit at a scaled mass} and moving in the medium with certain momentum. Thus it may be puzzling that the quasiparticle picture for the mesons works so well. One would have expected that even within the given scheme, higher loop graphs (e.g., widths) and explicit momentum dependences should enter importantly. This puzzle is further highlighted by the equally successful explanation of the same process by a description that is based on standard many-body approach starting from
a theory defined at zero density \[35\] which relies on the mechanism that in medium, the width of the \( \rho \) meson increases\[36\] or the \( \rho \) “melts.”

One possible solution to this puzzle is that as alluded already, the description based on BR-scaling chiral Lagrangians and the one based on many-body hadronic interactions are “dual” in the sense that they represent the same physics\[37\]. What the CERES data are telling us is that this duality may be holding in the heat bath and that the two descriptions may be mapped to each other\[37\]. This may be understood in terms of a “mended symmetry”\[38\]. As interpreted in \[39\], the mended symmetry argument goes as follows. While in matter-free space, chiral symmetry is non-linearly realized with the massive scalar degree of freedom purged from the low-energy sector, as density increases, a scalar, say, the \( \sigma \), comes down to join the triplet of the pion to “mend” the \( O(4) \) symmetry of the chiral \( SU(2) \times SU(2) \) and to become the fourth component of the four vector. That is, in dense medium, the non-linear \( \sigma \) model is “mended” to the linear \( \sigma \) model with the masses BR-scaling\[39\]. How this can happen in nuclear dynamics with the broad scalar in matter-free space becoming a local field in dense medium is described in \[40\]. Now the vector mesons, as long as they are still heavy, can be introduced much like the nucleon as matter fields with their masses scaling as \( (m) \). As density increases further and approaches the critical density for the chiral transition, then the vector mesons become light and the Georgi vector symmetry\[41, 40\] would be “mended.” This interpretation clearly puts more significance on the symmetry consideration than on the complex dynamics (e.g., used in \[42\]), in conformity with what has been established in QCD at long wavelength, namely, that in low-energy regime, it is chiral symmetry of QCD that governs the physics of hadrons. I suspect that it is this aspect that is at the root of the duality we see in the CERES data. It would be extremely interesting to see whether this dual description continues to hold true when heavy mesons are probed in cold nuclear matter as in Jefferson Laboratory or denser (somewhat warm) matter as in HADES.

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\[5\] The duality I am referring to here resembles in some sense the “quark-hadron” duality much discussed in the literature. For example, in the (1+1)-dimensional 't Hooft model, one can describe the non-leptonic decay width of a heavy-light meson (e.g., \( B \) meson) in terms of either the sum of exclusive partial widths or the tree-level partonic width. In the infinite mass limit of the meson, the two descriptions are equivalent to each other, the difference arising at the level of 1 over the mass\[42\].
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