Thermoelectric signature of individual skyrmions

Alexander Fernández Scarioni 1*, Craig Barton 2, Héctor Corte-León 2, Sibylle Sievers 1, Xiukun Hu 1, Fernando Ajejas 3, William Legrand 3, Nicolas Reyren 3, Vincent Cros 3, Olga Kazakova 2, and Hans W. Schumacher 1

1) Physikalisch-Technische Bundesanstalt, Braunschweig, Germany

2) National Physical Laboratory, Teddington, United Kingdom

3) Unité Mixte de Physique, CNRS, Thales, Université Paris-Saclay, 91767 Palaiseau, France
Abstract

We experimentally study the thermoelectrical properties of individual skyrmions in Pt/Co/Ru multilayers. Skyrmions are nucleated by current pulse injection in micropatterned Pt/Co/Ru stripes. In-situ magnetic force microscopy (MFM) is used to characterize the number, size, and spatial distribution of the skyrmions. A transverse thermal gradient from a microheater allows to generate an anomalous Nernst effect (ANE) signal proportional to the out-of-plane magnetization of the wire, which is detected electrically. To derive the thermoelectrical signature of individual skyrmions, the stray field of the MFM tip is used to delete individual skyrmions one by one while the respective thermoelectrical signals are compared. The observed signature is attributed to the ANE of the skyrmion’s spin structure. We demonstrate the thermoelectrical detection and counting of individual skyrmions and study the field dependence of their ANE signature. Possible topological contributions to the experimentally observed thermoelectrical signature are discussed. Such thermoelectrical characterization allows for non-invasive detection and counting of skyrmions and enables fundamental studies of topological thermoelectric effects.
Skyrmions are nano-scale topologically non-trivial spin structures which are inherently robust due to their particular topology and can be driven efficiently by electrical currents\textsuperscript{1,2}. Their electrical characterization and manipulation have been investigated intensely over recent years\textsuperscript{3-12}. However, only very few studies have addressed their thermoelectrical properties\textsuperscript{13-15}. In conventional spintronic materials thermoelectric studies have led to important discoveries such as the tunneling magneto-Seebeck effect or spin heat accumulation\textsuperscript{16}, and have enabled nano-scale detection of domain wall motion\textsuperscript{17}. For skyrmions such studies could shine light on topological contributions to the Nernst effect\textsuperscript{18} and might provide new tools for skyrmion detection and manipulation. Recently Iguchi et al.\textsuperscript{19} demonstrated the imaging of individual skyrmions with thermoelectric microscopy, which is an ultraviolet laser-assisted scanning technique. In this Letter, we experimentally study the thermoelectrical signature of individual skyrmions with an all-electrical single-shot technique and attribute it unambiguously to the anomalous Nernst effect (ANE) of the skyrmion’s spin structure. Beyond this fundamental insight into the thermoelectrical properties of topological spin structures our findings enable non-invasive characterization, detection and counting of skyrmions in magnetic microdevices.

**Results and Discussions.** In our experiments we use micron-wide stripes made of Pt/Co/Ru multilayers which exhibit perpendicular magnetic anisotropy (PMA) and interfacial Dzyaloshinskii-Moriya interaction (DMI) as shown in Figure 1a. A Pt microheater adjacent to the microstripe is used to induce a transverse in-plane thermal gradient $\nabla T_x$ in the microstripe. $\nabla T_x$ is perpendicular to the PMA magnetization $\mathbf{M}$ resulting in an ANE voltage $V_{\text{ANE}}$ in the longitudinal direction between the two contacts which can be expressed by:

$$V_{\text{ANE}} = -N_{\text{ANE}} \mu_0 \langle \nabla T_x \rangle M_z \quad (1).$$
where $N_{ANE}$ is the ANE coefficient per magnetic moment, $\mu_0$ is the vacuum permeability, $l$ is the wire length, $\overline{\nabla T_x}$ is the averaged $\nabla T_x$ and $M_z$ is the averaged $z$-component of the magnetization between the contacts. The condition for saturation is denoted by $M_z = M_s$, the saturation magnetization.

The ANE in the lithographically patterned devices was first characterized by ANE hysteresis measurements. In Figure 1b we show, a hysteresis loop of $V_{ANE}$ as function of the perpendicularly applied out-of-plane field $\mu_0 H_z$. The $z$ component of the averaged magnetization, $M_z$, shows the expected reversal behavior of PMA samples with maze domains. These maze domains are observed in the image of the magnetic configuration of the non-patterned thin films recorded by Magnetic Force Microscopy (MFM) at zero applied magnetic fields (inset of Figure 1h). With increasing applied field, regions of reversed magnetization expand in area until full saturation is reached. This behavior is confirmed by MFM micrographs of the microstripe in applied fields (Figure 1c, d). However, not only maze domains can be observed but also individuals skyrmions (as marked by an arrow in Figure 1e) can be stabilized during the reversal loops.

To extract the temperature distribution and $\overline{\nabla T_x}$ finite element modelling (detailed in Supporting Information) of the device and experimental conditions was implemented\textsuperscript{20-21}. The resulting temperature distribution for a microheater current $I_{heater}$ of 4.3 mA is shown in Figure 1f. The average thermal gradients along the x,y,z axes in the device are found to be $\overline{\nabla T_x} = 4.2$ K/µm, $\overline{\nabla T_y} = 2.9$ mK/µm and $\overline{\nabla T_z} = 0.2$ mK/µm, respectively. Given the order of magnitude difference between $\overline{\nabla T_x}$ on one hand, and $\overline{\nabla T_y}$ and $\overline{\nabla T_z}$ on the other hand, we neglect the latter two contributions to the ANE in further analysis. In Figure 1g, we show the simulated surface temperature profile across the microstripe along the dashed line in Figure 1f. Inside the microstripe (inset), the temperature $T_{stripe}$
increases by about 8.6 K. Using the microstripe as a resistive thermometer, the averaged $T_{\text{stripe}}$ can be experimentally determined as function of $I_{\text{heater}}$ (see the Supporting Information). As shown in Figure 1h (black circles), $T_{\text{stripe}}$ increases quadratically with $I_{\text{heater}}$ as expected for Joule heating and the experimentally determined temperature increase of 8.6 K for the maximum $I_{\text{heater}} = 4.3$ mA agrees well with the simulations.

Similar ANE hysteresis loops as in Figure 1b were measured for a range of input heat currents $I_{\text{heater}} = 0.2 - 4.3$ mA. In Figure 1h, we show the ANE amplitude $V_{\text{ANE}}^{\text{max}}$ (red squares) derived from subtracting the measured saturated $V_{\text{ANE}}^{\text{sat}}$ for positive and negative fields. Again, $V_{\text{ANE}}^{\text{max}}$ scales quadratically with $I_{\text{heater}}$ and thus proportionally with $\nabla T_x$ confirming the thermoelectrical origin. Using eq.(1), the measured $V_{\text{ANE}}^{\text{max}}$ with the average $\nabla T_x$ from the temperature calibration/simulations and $\mu_0M_s = 1.4$ T from magnetometry data, a Nernst coefficient of $N_{\text{ANE}} = 8.1$ nV/KT is derived.

To controllably study the thermoelectrical response of individual skyrmions we first generate skyrmions in the microstripe by current induced nucleation\textsuperscript{4,7,11,22-24}. MFM images of the microstripe before and after application of a single rectangular current pulse with nominal duration of 200 ns are displayed in Figure 2a-d. The pulse current densities $j$ ranged from $3.34 \times 10^{11}$ A/m$^2$ to $3.97 \times 10^{11}$ A/m$^2$. The magnetic fields were in the range $\mu_0H_z = 0.49 - 40.7$ mT. Before the pulse, an initial maze domain state was obtained by applying a magnetic field above the saturation field and then sweeping down the field to the desired value. It is found that pulse application results in various end states comprised of maze domains and skyrmions, depending on the relative intensities of $\mu_0H_z$ and $j$. The states can be classified by the following three categories: maze domains (Figure 2a), coexistence of skyrmions and maze domains (Figure 2b), or only insolated skyrmions (Figure 2c, d). The panel in Figure 2e summarizes the final
magnetization state as a function of $\mu_0 H_z$ and $j$ for the specific sample used in this experiment. The dotted line marks the region where only skyrmions are nucleated (blue triangles). The skyrmions density is controlled by the pulse amplitude and the external magnetic field, allowing the generation of single-digit numbers of skyrmions between the contacts as shown in Figure 2d. Repeated measurements did not reveal preferential nucleation sites but a rather stochastic skyrmion distributions in the microstripe.

To controllably define the number of skyrmions between the contacts for the ANE device, we used a single skyrmion annihilation procedure using an MFM tip. Here, the confined stray magnetic field gradient around the tip apex allows the precise local annihilation of single skyrmions. In Figure 3a, we show an MFM image after nucleation of five skyrmions in an applied magnetic field of +34.84 mT. For a given external applied magnetic field, i.e. +11.3 mT, the local total field (applied field + stray field of the tip) under the MFM tip is always below the skyrmion annihilation field (between 50 mT and 60 mT). As such this allows us to image the skyrmions without significant tip-sample interaction. In contrast, when the applied field is exceeds +24 mT, the total local field under the MFM tip is greater than the annihilation threshold. The assistance of such a bias field leads to a precision methodology to delete individual skyrmions at the nanoscale. Two examples of this selective annihilation process are shown in two sequences of MFM images in Figure 3a-c and Figure 3d-f. In Figure 3a-c, the number of skyrmions is first reduced from five (a) to three (b) and finally to one (c). In Figure 3d-f, the number of skyrmions is first reduced from four (d) to two (e) and finally to one (f).

Importantly, our experimental setup allows simultaneous in-situ MFM characterization and annihilation of the skyrmions along while the thermoelectrical characterization are
performed. This allows for the first time to measure the thermoelectrical signature of individual skyrmions, analogous to previous studies of the thermoelectrical signature of an individual magnetic domain wall in a nanowire\textsuperscript{21} or anomalous Hall effect detection of skyrmions\textsuperscript{9}. The measured ANE signal from the skyrmion configurations shown in Figure 3a-c and Figure 3d-f is plotted in in Figure 3g as red dots and blue triangles respectively. Here, denoted $\Delta V_{\text{ANE}}$ as the difference in the measured ANE signal away from the saturation condition ($\Delta V_{\text{ANE}} = V_{\text{ANE}} - V_{\text{ANE}}^{\text{sat}}$) as function of detected skyrmions between the two contacts. The data for zero skyrmions correspond to the saturated signal $V_{\text{ANE}}^{\text{sat}}$, where, by definition, $\Delta V_{\text{ANE}} = 0$. The error bars reflect the standard deviation of the experimental data. We demonstrated that $\Delta V_{\text{ANE}}$ shows a linear dependence with the number of skyrmions that are present in the sensing region. From fitting the data with a linear regression (supposing all skyrmions of the same shape) with zero interception an average ANE voltage signature per skyrmions of $V_{\text{ANE}}^{\text{sky}} = 4.6 \pm 0.2$ nV is derived for the given experimental conditions, i.e. $\mu_0 H_z = +11.3$ mT, $\overline{\nabla T_x} = 4.2$ K/µm.

Following eq. (1), the measured $\Delta V_{\text{ANE}}$ should be proportional to $M_z$ in the area between the two contacts. This area is proportional to the skyrmion area and as such we observed a reduction in, $\Delta V_{\text{ANE}}$. We estimate the effective skyrmion area $A_{\text{sky}}$ of each individual skyrmion from MFM measurements; by fitting a 2D Gaussian function with parameters $\sigma_1$ and $\sigma_2$ and an in-plane rotation angle $\theta$ to the measured MFM signal. The angle $\theta$ takes into account circularly asymmetric skyrmions (i.e. without rotational symmetry). $A_{\text{sky}}$ is calculated as an ellipse using the two sigma values of the Gaussian fit (see the Supporting Information). The lower left inset to Figure 4a shows a zoom-in view of an MFM image of a typical skyrmion with the derived skyrmion area $A_{\text{sky}}$ marked by the white line. The upper right inset shows a profile through one
skyrnion extracted from an experimental MFM phase signal (black) and from the Gaussian fit (red) demonstrating very good agreement. The main panel in Figure 4a displays \( \Delta V_{\text{ANE}} \) collected from three nucleation and erasure sequences as function of the total reversed area skyrmion \( A_{\text{tot}} = \sum_{i=0}^{n} A_{\text{sky},i} \) summed over all \( n \) skyrmions inside the microstripe. Again, a linear behavior is observed. Note that the orange line is not a fit to the data, but it is the calculated ANE signal considering the average thermal gradient \( \langle \nabla T \rangle_x \), the estimated \( N_{\text{ANE}} \) and the total reversed skyrmion area \( A_{\text{tot}} \). It demonstrates the very good description of the signal by the ANE response of the reversed skyrmion area without considering any topological contributions. Note that the thermal simulations of Fig. 1f suggest a spatial variation of \( \nabla T_x \) over the width of the wire which should result in dependence of the ANE response on the skyrmion’s \( x \)-position inside the wire (see the Supporting Information). From the good agreement of the experimental data with the with the above calculated ANE signal we however deduce that the spatial variation of \( \nabla T_x \) is overestimated by the simulations.

These findings are further confirmed by measurements of the ANE signal of a single skyrmion as function of the applied magnetic field, which modifies the skyrmion size. Five MFM images of a single skyrmion between the ANE measurement contacts taken at increasing applied magnetic fields ranging from \(+11.3\) to \(+23.0\) mT show the shrinking of the skyrmion size (see Figure 4b), before it is annihilated in a field above \(+24\) mT (not shown). Again, the reversed skyrmion area was derived from the MFM data and the ANE response was measured for all field configurations. The variation \( \Delta V_{\text{ANE}} \) as function of the skyrmion area of Figure 4b (blue triangles) is plotted in Figure 4c. The red dots and black squares correspond to data from subsequent single skyrmion magnetic field dependent measurements (see the Supporting Information). Again, the
orange line describes the calculated ANE signal and describes well the single skyrmion data within the experimental measurement uncertainty.

It has been observed that when spin polarized electrons transverse a topological non-trivial spin texture, such as a magnetic skyrmion they can accumulate a Berry phase\(^{26}\) which can be viewed analogous to a Aharonov-Bohm phase resulting from a fictitious magnetic field antiparallel to the uniform magnetization\(^{27}\). Experimentally this accumulated Berry phase can result in a topological Hall effect\(^{28}\) or a topological Nernst effect\(^{18}\) as observed in the skyrmion phase of bulk MnGe\(^{13}\) and MnSi\(^{15}\) as well as in Mn\(_{1.8}\)PtSn thin films\(^{14}\).

This topological contribution \(V_{\text{ANE}}^{\text{top}}\) to the measured Nernst signal \(\Delta V_{\text{ANE}}\) should scale with the total topological charge of the skyrmions inside the microwire. For the chiral system under investigation, in principle, each skyrmion carries the same topological charge independent of the exact surface area. Hence the topological contribution should scale with absolute number \(n\) of skyrmions between the contacts and not with their area \(A_{\text{sky}}\). In the data in Figure 4c, this should result in a discontinuity at zero size (no skyrmion vs. single skyrmion), corresponding to the topological contribution of a single skyrmion. However, such discontinuity is not be observed within the measurement noise level.

Finally, an attempt was made to extract a possible topological Nernst contribution in our data. To do this we first calculate \(\delta V_{\text{ANE},i} = \Delta V_{\text{ANE},i+1} - \Delta V_{\text{ANE},i}\) being the change of \(\Delta V_{\text{ANE},i}\) when changing the number of skyrmions (and thus the topological charge of the system) by one. \(\delta V_{\text{ANE},i}\) is derived for all data sets in the study and is plotted in Figure 4d as function of the relative area change for each change in the skyrmion number \(\Delta A_{\text{sky},i} = A_{\text{sky},i+1} - A_{\text{sky},i}\). We evidence that the predicted ANE signal, depicted
by the orange line, describes well the data within the experimental uncertainty. However, by considering a linear fit to the data, we observe a nonzero intercept at zero area (no skyrmions in the microstripe). The presence of a nonzero intercept indicates a potential topological contribution $V_{\text{ANE}}^{\text{top}}$ to the skyrmion Nernst signature independent of $A_{\text{sky}}$. The derived interception of $1.2 \pm 0.6$ nV is about 4 times smaller than the ANE contribution per skyrmion, but different from zero value within $2\sigma$ confidence. Based on this dataset, a topological contribution thus cannot be ruled out. However, considering the statistical uncertainty of our data, further studies are required to unambiguously conclude on the nature of the topological Nernst signature of individual skyrmions. For the metallic multilayers studied here, both the electron’s mean free path and the spin diffusion length are of the order of few nanometers and thus significantly smaller than the average skyrmion diameter. This provides one hypothesis for the absence of a strong topological Nernst contribution in our data.

In conclusion, we have measured and characterized the thermoelectrical signature of individual nanoscale skyrmions in Pt/Co/Ru multilayers allowing a non-invasive all-electrical detection and for individual counting of skyrmions in magnetic devices. The thermoelectric signature is well explained by ANE of the reversed magnetization area of the individual skyrmions. Only weak indications of an additional topological Nernst contribution are present in the data. Future experiments using material systems with smaller skyrmion diameters or longer mean free paths of the conduction electrons could allow to unequivocally identify the topological Nernst contribution of individual skyrmions.
Methods. The multilayer stacks of Ta 5/Pt 8/(Co 1.0/Ru 1.4/Pt 0.6)x10/Pt 2.4 (numbers are thickness in nm) were deposited at room temperature by direct current sputtering under an argon pressure of 2.5 mbar on a thermally oxidized Si substrate. The microstripes are patterned by electron beam lithography and argon etching. Sputter deposited contacts and microheater of 5 nm Ta and 95 nm Pt were patterned by lift-off. To ensure low contact resistances in-situ low energy argon ion etching was performed prior to deposition. The microheater has a width of 200 nm and the contacts a width of 500 nm.

For the ANE voltage measurements, the thermal gradient was generated by ac currents with a frequency of 2024 Hz applied to the microheater by a commercial ac current source (Keithley K6221). $V_{\text{ANE}}$ was detected at the second harmonic using a lock-in amplifier (Stanford SR-830). Different frequencies were tested, the frequency of 2024 Hz yielding optimum results for in-situ MFM and ANE measurements.

MFM measurements were performed in a NT-MDT Aura SPM in ambient temperature. Magnetic fields were applied by a built-in electromagnet. Commercial low magnetic moment MFM tips (NT-MDT MFM_LM) were used. Before performing the measurements, the magnetic tip was magnetized in the positive direction out-of-plane with respect to the sample (see Fig. 1a), so that the dark contrast areas in our measurements correspond to the negative out-of-plane magnetization and the light parts the opposite direction. The MFM images were taken with an estimated oscillating amplitude of 170 nm of the cantilever and a lift height of 50 nm. In all the MFM measurements related to ANE data a current was applied to the microheater while the MFM images were obtained.
Figure 1. Device characterization and properties. (a) scanning electron micrograph of the device employed in this study. (b) ANE hysteresis measurement: showing the measured ANE voltage as a function of the applied out-of-plane field. The arrows mark the corresponding domain states imaged by MFM shown in c - e. (c-e) MFM measurements of the microstripe for increasingly negative values of the total out-of-
plane magnetic field. Here the total field is the sum of the applied field and the tip stray field at the sample surface. (c), (d) show maze domains with different densities, respectively. (e) shows a single skyrmion inside the microstripe (white arrow). (f) Simulated temperature distribution for $I_{heater} = 4.3$ mA. (g) Simulated temperature profile along the dashed line in (f). The temperature drops from 478K at the microheater to about 305 K at the microstripe center (inset picture). (h) measured temperature rise (black circles) as a function of applied current and ANE amplitude (red squares) in the microstripe vs. $I_{heater}$. The inset shows an MFM image of the film before patterning in zero field.

**Figure 2.** Skyrmion nucleation in the microstripe. (a-d) MFM measurements of the microstripe before and after application of a 200-ns-long current pulse through the microstripe. Under variation of the applied current density $j$ and the magnetic field $\mu_0H_z$, different final magnetization states can be generated. (a) $j = 3.55 \times 10^{11} \text{A/m}^2$, $\mu_0H_z = 11.28 \text{mT}$ no skyrmion is created and only perturbation of maze domains is observed. (b) $j = 3.76 \times 10^{11} \text{A/m}^2$, $\mu_0H_z = 11.28 \text{mT}$ observation of the coexistence skyrmions and maze domains. (c) $j = 3.86 \times 10^{11} \text{A/m}^2$, $\mu_0H_z = 17.16 \text{mT}$ only skyrmions are observed (high density). (d) $j = 3.97 \times 10^{11} \text{A/m}^2$, $\mu_0H_z = 23.05 \text{mT}$ only skyrmions (low density) are observed. (e) Phase diagram for skyrmion generation
determined from several nucleation experiments with varying pulse current density and applied fields for the same device. The dotted line represents a guide to the eye indicating the parameter range suitable for skyrmion nucleation.

Figure 3. ANE signature of individual skyrmions. (a-c) and (d-f), are two sequences of MFM images of the tip induced annihilation process of individual skyrmions. In both cases skyrmions were nucleated in a field of ~35 mT. The applied current density was of \( j = 3.97 \times 10^{11} \text{ A/m}^2 \) (a-c) and \( j = 3.86 \times 10^{11} \text{ A/m}^2 \) (d-f). Tip induced annihilation was carried out in an applied field of ~ 24 mT. (g) \( \Delta V_{\text{ANE}} \) as function of the number of skyrmions inside the wire are between the ANE contacts corresponding to the skyrmion configurations (a–f). The red dots correspond to (a-c); the blue triangles correspond to (d-f). The green line is a linear fit through the data, yielding an average Nernst signature of 4.6 ± 0.2 nV per skyrmion.
Figure 4. Dependence of the skyrmion ANE signal on the area of reversed magnetization. The effective magnetized area of the individual skyrmions $A_{\text{sky}}$ is calculated by fitting the measured stray field distribution to the stray field distribution of a model skyrmion (see Supplement Information). The total effective reversed area $A_{\text{tot}}$ off all skyrmions is calculated by summing up all individual skyrmion areas between the contacts. (a) ANE voltage as a function of $A_{\text{tot}}$. Different symbols correspond to three nucleation/annihilation sequences. The orange line represents the expected ANE voltage calculated for the given reversed magnetization area. Lower left inset: MFM data of a single skyrmion with the derived $A_{\text{sky}}$ marked by the white circle. Upper right
inset: section through the MFM phase signal (black) of the same skyrmion and Gaussian-fit data (red) showing good agreement. (b) MFM measurements of a single skyrmion between the two ANE measurement contacts as a function of the magnetic field. The upper skyrmion is outside the measurement area and, therefore, does not contribute to the ANE signal. (c) ANE voltage as a function of $A_{\text{sky}}$ calculated from measurements of a single skyrmion as function of magnetic field. Red dots, blue triangles and black squares represent data of three different skyrmions from three different nucleation sequences. The orange line is an expected ANE signal from the reversed magnetization area. (d) Change of the ANE voltage due to the change of the skyrmion number by one. $\delta V_{\text{ANE}}$ is plotted as function of the reversed area difference. Orange line is the predicted ANE signal considering the reversed area of $A_{\text{tot}}$. The red line is linear fit of the data allowing for a non-zero intersection.
Supplementary information of the Manuscript:
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1) Physikalisch-Technische Bundesanstalt, Braunschweig, Germany

2) National Physical Laboratory, Teddington, United Kingdom

3) Unité Mixte de Physique, CNRS, Thales, Université Paris-Saclay, 91767 Palaiseau, France
COMSOL simulation parameters

Thermal modeling of the microheater and surrounding device topology was performed using FEM modelling with COMSOL. The Joule heating module was implemented to estimate the temperature rise and thermal gradient within the ANE device. The geometry and dimensions of the modeled sample match that of the device. Temperature dependent bulk material parameters, e.g. thermal conductivity $\kappa$ and heat capacity $C_p$, were taken from references [1-5], as listed in the Table S1. The temperature coefficient of the resistance $\alpha = \frac{1}{R} \frac{dR}{dT}$ of the microstripe and the microheater were calibrated by temperature dependent resistance measurements (see. below).

| Material      | $C_p$ (J/kg K) | $\rho$ (kg/m$^3$) | $\kappa$ (W/m K) | $\alpha$ (1/K) |
|---------------|----------------|-------------------|------------------|---------------|
| Si            | $C_{p,\text{Si}}(T)$ [1] | 2329             | $\kappa_{\text{Si}}(T)$ [2] |
| SiO$_2$       | $C_{p,\text{SiO}_2}(T)$ [3] | 2203             | $\kappa_{\text{SiO}_2}(T)$ [4] |
| Pt            | $C_{p,\text{Pt}}(T)$ [5] | 21450            | 0.0015           |
| Microstripe   | 130            | 8000             | 8.7e-4           |

Table S1: Material parameters used in the COMSOL simulations.

Temperature change generated by the microheater

The microstripe and the microheater could be used as temperature sensors exploiting their resistance change, induced by the Joule heating of the microheater. To this end, their temperature coefficients of resistance are calibrated. The ANE device used for the actual ANE-measurements does not allow 4-wire measurement of the microheater. Therefore, in the same fabrication process as for the ANE device a separate device with the same geometry was fabricated but with additional contacts. During the resistance measurements, both devices, were put in a temperature-controlled air bath
unit, where temperatures could be stabilized in the range from 288 K to 314 K. The resistance for both the microstripe and the microheater shows a linear dependency in the temperature range calibrated (Fig. S1a), as expected for metals. The resulting temperature coefficients of resistance $\alpha$, were determined as $8.7 \times 10^{-4}$ for the microstripe and the microheater $1.5 \times 10^{-3}$, respectively.

The known temperature coefficient allows to backward calculate the temperature from a given resistance value.

By this means, the temperature increase in the microheater and microstripe were experimentally determined for different microheater currents, as shown in Fig. S1b for an ambient temperature of 295K. The temperature dependence shows the expected quadratic dependency on the microheater current expected for Joule heating. For the current of 4.3 mA used during the ANE measurements an average increase of the temperature of 8.06 K and 165.1 K are found for the microstripe and the microheater, respectively.

![Fig. S1: Temperature coefficient of resistance calibration. a. 4-wire resistance measurements from the microstripe (black circles) and microheater (red squares) as a function of temperature. b. Temperature of the microstripe (black circles) and microheater (red squares) as a function of the current applied to the heater.](image-url)
Skyrmion nucleation experiment and field dependency of single skyrmion

As described in the main text, three independent skyrmion nucleation and deletion sequences were performed using the same microstripe. In Fig. S2a-c we show, three complete sets of MFM data taken following skyrmion nucleation and subsequent deletion steps. The nucleation parameters are the following: $3.97 \times 10^{11} \text{ A/m}^2$ at $+34.84 \text{ mT}$, $3.97 \times 10^{11} \text{ A/m}^2$ at $+34.84 \text{ mT}$ and $3.86 \times 10^{11} \text{ A/m}^2$ at $+34.84 \text{ mT}$ for images a,b and c respectively. For each sequence the first image from the left shows the MFM image performed directly after the skyrmion nucleation process, and the images following to the right show subsequent MFM measurements of consecutive deletion steps.

With a single skyrmion in the microstripe, as shown by the final image in the sequences, Fig. S2a-c, the magnetic field was increased to investigate the single skyrmion field dependency. In Fig. S3a-c we show the corresponding MFM measurements in the different applied fields.
Fig. S2: **a-c**, MFM measurements of the three independent sequences of skyrmion nucleation (leftmost image) and deletion by the MFM tip (subsequent images). The applied field is +11.28 mT. All the measurements were performed in the same microstripe.
Fig S3.: a-c, MFM images of a single skyrmion with increasing the applied field for three independent nucleation sequences.
Fit for the calculated stray field of single skyrmions

The local magnetic configuration in the sample was probed using an MFM in 'lift mode' at a distance of \( z = 220 \text{ nm} \) (lift height + cantilever oscillation amplitude) to the sample surface, mapping the cantilever phase shift \( \Delta \phi \).

To determine the area of the skyrmions from the phase shift images, we fitted the measured \( \Delta \phi \) from the cross-section (along two axes, x and y) of every skyrmion data with a Gaussian function as:

\[
G(x, y) = B \exp \left\{ -\frac{[(x-x_c) \cos \theta -(y-y_c) \sin \theta]^2}{2\sigma_x^2} + \frac{[(x-x_c) \sin \theta + (y-y_c) \cos \theta]^2}{2\sigma_y^2}\right\} \tag{1}
\]

where \( x_c \) and \( y_c \) are the center position of the Gaussian, \( \sigma_x \) and \( \sigma_y \) control the width of the Gaussian bell, \( B \) is the amplitude of the Gaussian bell and \( \theta \) is an in-plane rotation angle. The fit model allows elliptical skyrmions with an in-plane rotated long axis.

From the Gaussian fit the correspondent sigma values for the x cross-section (\( \sigma_x \)) and y cross-section (\( \sigma_y \)) of the skyrmion are obtained.

This fitting model was verified by employing the same fitting procedure to a calculated MFM phase signal obtained from a simulated skyrmion. To obtain the simulated MFM image we followed the Tip Transfer Function (TTF) approach [6,7]. The stray field distribution of a typical skyrmion at the MFM measurement height was calculated from its magnetization distribution as resulting from micromagnetic simulations according to [8].

The radius of the simulated skyrmion was in the same range as the experimentally observed radius. In the micromagnetic model the following experimental parameters were included: exchange constant \( A = 10 \text{ pJ/m} \) obtained from domain wall profile as
imaged by Lorentz TEM, Dzyaloshinskii-Moriya interaction (DMI) constant \( D = 1.19 \text{ mJ/m}^2 \) determined with Brillouin light scattering spectroscopy, saturation magnetization \( M_s = 1100 \text{ kA/m} \) and perpendicular anisotropy constant \( K_u = 1109 \text{ kJ/m}^3 \) determined by SQUID measurements. The stray field of the skyrmion was obtained by calculating the cartesian components of the skyrmion magnetization, \( m_x(x,y) \), \( m_y(x,y) \), and \( m_z(x,y) \), and thereof their respective magnetic stray fields at a distance \( z \) using the transfer function approach proposed by Schendel et al. [9].

The TTF was obtained by measuring a well characterized Co/Pt reference sample.

In Fig. S4 the simulated MFM signal cross-section along the x direction of the simulated skyrmion is shown. Here, the difference is a consequence of the asymmetry of the TTF. The data points obtained were then fitted by the Gaussian from equation 1, showing good qualitative agreement experimentally obtained data. To estimate the relationship between the sigma values from the Gaussian fit and the principal axes of the elliptical skyrmion \( r_{(x,y);\text{sky}} \), we compared the fit with the spatial distribution of the \( m_z \) component of the skyrmion from the simulations as plotted in Fig. S4. The resulting relation \( r_{\text{sky}} \approx \sqrt{2}\sigma \), as illustrated by the dashed line in Fig. S4 was used to calculate the skyrmion area as \( A_{\text{sky}} = 2\pi\sigma_x\sigma_y \).
Fig. S4: x cross-section through the simulated MFM phase data (black line) from a micromagnetic simulated skyrmion, Gaussian-fit data (red line) and the $m_z$ component of the skyrmion used in the simulations (blue line). Dashed black line indicates the cut off criterium used to calculate the skyrmion area representing $\sqrt{2}\sigma$.

**Influence in the thermoelectric voltage due to the thermal gradient along the microstripe**

The temperature increase and the thermal gradient simulations estimated with COMSOL show that the thermal gradient along the x direction is higher in amplitude compared to the other directions. For the calculations of the Nernst coefficient in the manuscript an average of the thermal gradient along the x direction was calculated and for the ANE voltage measurements as a function of the magnetization area of the skyrmions it was assumed that every skyrmion senses the same amplitude of this thermal gradient. In contrast the COMSOL simulations suggest that the thermal gradient significantly changes its amplitude along the x-direction as plotted in Fig. S5a. According to the ANE equation, such spatially varying gradient should result in x-position dependence of the skyrmion’s ANE voltage. However, the simple model based on a constant gradient inside the wire (orange line in Fig. 4a) shows a very good agreement with the experimental data. Therefore, we conclude, that the simulations
strongly overestimate the spatial variations in thermal gradient. One possible reason for these deviations are edge effects resulting from the unrealistic idealized square cross section of the microstripe in the simulation.

Nevertheless, we used the COMSOL results of the thermal gradient to estimate the upper limits of the uncertainty of the simulated data, complemented with possible error contributions from the calculated skyrmion area: For each skyrmion an effective local thermal gradient was obtained by averaging over its area using the COMSOL simulations with an assumed error of 10% due to position and shape inaccuracies. Additionally, an error is allowed in the calculation of the skyrmion area. To this end, we assumed an error in the relationship between the $\sigma$ of the fitted Gaussian and the effective skyrmion radius ($r_{\text{sky}} \approx \sqrt{2}\sigma$) of more than 12% ($r_{\text{sky}} \approx \sqrt{2} \pm 0.25\sigma$). In Fig. S6b we show how the effective skyrmion radius is thereby affected. The ANE voltage for every single skyrmion in this specific approach can be calculated using the following equation:

$$V_{\text{ANE}} = -N_{\text{ANE}} \mu_0 M_z \overline{\nabla T_{x,\text{local}}} A_{\text{tot}} / w$$  \hspace{1cm} (2)

where $\overline{\nabla T_{x,\text{local}}}$ represents the local average thermal gradient and $w = 2\mu m$ the width of the microstripe. The other parameters were previously obtained and mentioned in the manuscript. In Fig. S5c we show the comparison between the measured ANE voltage from the sequence shown in Fig. S2a depicted as the red dots and the simulated ANE voltage. Taking into account the above discussed error margins leads to a range of possible simulated ANE values that is shown here as the shaded region. We attribute the fact that the shaded region it mostly found at higher negative ANE values than the experimental data to an overestimation of the thermal gradient in the COMSOL simulations.
Fig. S5: a, cross-section of the temperature gradient along the x direction. The colormap represents the temperature gradient distribution along the x axis of the microstripe. b, the $m_z$ component of the skyrmion used in the simulations (blue line), the different dotted lines represent the radius calculated by $r_{sky} \approx \sqrt{x} \sigma$ where x is the relation that is obtained by comparing the simulated MFM image with the $m_z$ component of the skyrmion. c, measured ANE voltage as a function of the total effective reversed area (red dots) obtained from the nucleation and annihilation sequence depicted in Fig. S2a. The gray area depicts the calculated ANE voltage taking in to account the different positions of the skyrmions and its correspondent temperature gradient according to the COMSOL simulations, also the influence of the calculated magnetization area and an error for the temperature gradient from the simulations.
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