LETTER TO THE EDITOR

C-deformation of supergravity

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Abstract

A four-dimensional supergravity toy model in an arbitrary self-dual gravi-photon background is constructed in Euclidean space, by freezing out the gravi-photon field strength in the standard $N = (1, 1)$ extended supergravity with two non-chiral gravitini. Our model has local $N = (1/2, 0)$ supersymmetry. Consistency of the model requires the background gravi-photon field strength to be equal to the self-dual (bilinear) anti-chiral gravitino condensate.

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1. Introduction

As was shown by Ooguri and Vafa in [1], the superworldvolume of a supersymmetric D-brane in a constant Ramond–Ramond type flux gives rise to the remarkable new structure in the corresponding superspace, which is now called non-anticommutativity (NAC). The non-anticommutativity means that the fermionic superspace coordinates are no longer Grassmann (i.e., they no longer anticommute), but satisfy a Clifford algebra. In other words, the impact of the RR flux on the D-brane dynamics can be simply described by the non-anticommutativity in the D-brane superworldvolume. In its turn, the non-anticommutativity in superspace can be easily described by the (Moyal–Weyl type) non-anticommutative star product among superfields, which gives rise to the NAC deformed supersymmetric field theories with partially broken supersymmetry [2–4]. When gluino background is added, one can deform the anticommutation relation of the spinors in the D-brane worldvolume in order to recover full supersymmetry [1].

As regards a D3-brane with its four-dimensional worldvolume, a ten-dimensional (self-dual) five-form flux upon compactification to four dimensions gives rise to the (self-dual) gravi-photon flux [1]. All recent studies of the NAC supersymmetric field theories after the pioneering papers [2–4] were limited to rigid supersymmetry, i.e. without gravity. In this letter we investigate the impact of a self-dual gravi-photon flux on supergravity.

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The simplest supergravity model with a gravi-photon is the pure $N = (1, 1)$ (or $N = 2$ in the Lorentz case) supergravity unifying gravity with electromagnetism. Therefore, the easiest thing to do is to ‘freeze out’ the gravi-photon field in that supergravity model to some self-dual value of its field strength. Of course, such a condition would break $N = (1, 1)$ supersymmetry, so we should like to investigate the residual supersymmetry, if any, and then cut the theory properly, in our search for a supergravity model with lower local supersymmetry but with a non-vanishing self-dual gravi-photon background. It is the task that we pursue in this paper. The Euclidean signature appears to be crucial here, similarly to the rigid NAC supersymmetric field theory [3, 4]. We use the component formulation of supergravity [5, 6], as regards rigid $N = 1/2$ supersymmetric field theories in superspace (see, e.g. [2–4, 7]).

Our paper is organized as follows. In section 2 we briefly introduce our notation. In section 3 we formulate the standard (Euclidean) four-dimensional $N = (1, 1)$ supergravity in our notation. Section 4 is devoted to our results. Our conclusion is section 5.

2. About our notation

Our notation is based on the standard review about supergravity [5] with the four-dimensional spacetime signature $(+, +, +, +)$. We use lower case Greek letters for (curved space) vector indices, $\mu, \nu, \ldots = 1, 2, 3, 4$, and early lower case Latin indices for (target space) vector indices, $a, b, \ldots = 1, 2, 3, 4$, early capital Latin letters for (anti)chiral spinor indices (dotted or undotted), $A, B, \ldots = 1, 2$, and middle lower case Latin letters for indices of the $O(2)$ internal symmetry rotating two gravitini, $i, j, \ldots = 1, 2$.

Raising and lowering of spinor indices is performed with the help of two-dimensional Levi–Civita symbols,

$$\psi_A = \psi^B \epsilon_{BA}, \quad \psi^A = \epsilon^{AB} \psi_B, \quad \tilde{\psi}_A = \tilde{\psi}_B \epsilon^{BA}, \quad \tilde{\psi}_A = \epsilon^{AB} \tilde{\psi}_B,$$

while we have

$$\epsilon_{AB} = -\epsilon_{BA}, \quad \epsilon^{AB} \epsilon^{BC} = -\delta^A_C, \quad \epsilon_{12} = \epsilon^{12} = -\epsilon_{i\bar{i}} = -\epsilon^{i\bar{i}} = 1.$$  

(1)

Chiral and anti-chiral (with bars) spinors are independent in Euclidean space with the signature $(4, 0)$, as well as in Atiyah–Ward space with the signature $(2, 2)$ [3, 9]. As a rule, we omit contracted spinor indices for simplicity in our equations, by using the notation

$$x \psi \equiv x^A \psi_A = -x^A \bar{\psi}_A = \bar{x} \bar{\psi}_A = x^A \bar{\psi}_A = \bar{x} \psi_A = x^A \bar{\psi}_A = \bar{x} \psi_A = x^A \bar{\psi}_A.$$

(2)

As regards, the (anti)self-dual parts of an antisymmetric tensor $K_{\mu \nu}$, we define

$$K_{\mu \nu}^\pm \equiv \frac{1}{2} (K_{\mu \nu} \mp \frac{1}{2} \epsilon_{\mu \nu \rho \sigma} K^{\rho \sigma}) \quad \epsilon_{1234} = 1, \quad e = \det e_\mu^a.$$

(3)

The sigma matrices in our notation are given by

$$(\sigma)_{AB} = (\bar{\sigma}, iI)^{AB}, \quad (\sigma_a)_{AB} = (\bar{\sigma}, -iI)_{AB},$$

while their triple (totally antisymmetric) product is given by

$$\sigma_{abc} = \frac{1}{6} \sigma^{AB} e_{\alpha\beta} e_{\gamma}\sigma^C_{AB}.$$

(4)

(5)

(6)

where $\bar{\sigma}$ are Pauli matrices and $I$ is a unit matrix.

Flat and curved vector indices are related by a vierbein $e_\mu^a$ and its inverse $e_a^\mu$, as usual, e.g., $\sigma_\mu = e_\mu^a \sigma_a$ and $\sigma_a = e_a^\mu \sigma_\mu$, etc.

1 Chiral and anti-chiral spinors are related by complex conjugation in Minkowski spacetime.
3. \( N = (1, 1) \) supergravity

Our starting point is the pure \( N = (1, 1) \) extended supergravity in four Euclidean dimensions. The \( N = (1, 1) \) supergravity multiplet unifies a graviton field \( e_\mu \), two non-chiral gravitino fields \( \psi_\mu \), and a graviphoton gauge field \( A_\mu \). The \( N = (1, 1) \) supergravity action was first constructed in [6] by Noether procedure. When using our notation with chiral and antichiral gravitini, and making the \( O(2) \) internal symmetry manifest, the \( N = (1, 1) \) supergravity Lagrangian of [6] reads

\[
\mathcal{L} = -\frac{e}{2k} R(e, \omega) + ie_\mu \sigma^{\mu \nu} D_\nu(\omega) \bar{\psi}_\mu - \frac{e}{4} F_{\mu \nu} F^{\mu \nu} - \frac{\epsilon k}{2\sqrt{2}} \left( \psi_\mu \gamma^\nu \bar{\psi}_\mu \right) \epsilon^{\mu \nu} [F_{\mu \nu} + \tilde{F}_{\mu \nu}] - \frac{\epsilon k}{2\sqrt{2}} \left( \bar{\psi}_\mu \gamma^\nu \bar{\psi}_\mu \right) \epsilon^{\mu \nu} [F_{\mu \nu} + \tilde{F}_{\mu \nu}]^*,
\]

where we have used the standard definitions [5],

\[
F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad \tilde{F}_{\mu \nu} = F_{\mu \nu} + \frac{\kappa}{\sqrt{2}} \left( \psi_\mu \gamma^\nu \psi_\mu + \bar{\psi}_\mu \gamma^\nu \bar{\psi}_\mu \right) \epsilon^{\mu \nu},
\]

of the graviphoton field strength \( F_{\mu \nu} \) and its supercovariant extension \( \tilde{F}_{\mu \nu} \). The dimensional parameter \( \kappa \) is the gravitational coupling constant.

The spinor covariant derivative \( D_\mu(\omega) = D_\mu \) contains the independent spin connection \( \omega^a_{\mu b} \) that is supposed to be fixed as a function of the vierbein and gravitini by solving its algebraic equation of motion, \( \delta S/\delta \omega = 0 \) (it is known as the 1.5 order or Palatini formalism), as usual [5].

By construction, the action \( S = \int d^4 x \mathcal{L} \) of equation (7) is invariant under the following transformation rules of local \( N = (1, 1) \) supersymmetry:

\[
\delta e_\mu = -\frac{i\kappa}{2} \left( \bar{\epsilon}^a \sigma^a \psi_\mu + \epsilon^a \sigma^a \bar{\psi}_\mu \right), \quad \delta A_\mu = -\frac{1}{\sqrt{2}} \epsilon^{\mu \nu} \left( \epsilon_\nu \psi_\mu + \bar{\epsilon}_\nu \bar{\psi}_\mu \right),
\]

\[
\delta \psi_\mu = -\frac{1}{\kappa} D_\mu \epsilon^i - \frac{i}{\sqrt{2}} \epsilon^i \tilde{F}_{\mu \nu} \sigma^\nu \bar{\epsilon}_i, \quad \delta \bar{\psi}_\mu = \frac{1}{\kappa} D_\mu \bar{\epsilon}_i + \frac{i}{\sqrt{2}} \bar{\epsilon}_i \tilde{F}_{\mu \nu} \sigma^\nu \epsilon_i,
\]

\[
\delta F_{\mu \nu} = -\frac{1}{\sqrt{2}} \epsilon^{\mu \nu} \left[ D_\mu \left( \epsilon \psi_\nu \right) + D_\nu \left( \epsilon \psi_\mu \right) \right],
\]

where \( \epsilon^i \) and \( \bar{\epsilon}_i \) stand for the infinitesimal chiral and anti-chiral anticommuting spinor parameters of local \( N = (1, 1) \) supersymmetry, respectively.

The supercovariant graviphoton field strength \( \tilde{F}_{\mu \nu} \) transforms covariantly under the transformations (9) by construction (i.e., without derivatives of the supersymmetry parameters),

\[
\delta \tilde{F}_{\mu \nu} = -\frac{1}{\sqrt{2}} \epsilon^{\mu \nu} \left[ \epsilon^i D_\mu \left( \psi_\nu \right) + \bar{\epsilon}_i D_\mu \left( \bar{\psi}_\nu \right) \right] + \frac{i\kappa}{2} \tilde{F}_{\lambda [\mu} \left( \bar{\psi}_\nu \psi_\lambda \right) + \frac{i\kappa}{2} \tilde{F}_{\lambda [\mu} \left( \epsilon \sigma^\lambda \psi_\nu \right). \tag{10}
\]

4. \( C \)-deformation

We now impose the condition

\[
\tilde{F}_{\mu \nu}^+ = C_{\mu \nu}(x), \quad \tilde{F}_{\mu \nu}^- = 0, \tag{11}
\]

where \( \tilde{F}_{\mu \nu} \) is the covariantized graviphoton field strength (8), and \( C_{\mu \nu}(x) = C_{\mu \nu}^+(x) \) is an arbitrary given function. We have chosen \( \tilde{F}_{\mu \nu} \) instead of \( F_{\mu \nu} \) in equation (11) because \( \tilde{F}_{\mu \nu} \) is a tensor under the local supersymmetry (section 3), though it turns out to be unimportant (see the end of this section). Also, we do not require the background \( C_{\mu \nu}(x) \) to be constant,
because it turns out to be unessential too (cf a coordinate-dependent deformation of rigid $N = 1/2$ supersymmetry in superspace [8]).

Of course, the condition (11) is not compatible with the full $N = (1,1)$ local supersymmetry (9). The consistency condition

$$\delta \hat{F}_{\mu \nu} = 0,$$

has, however, the residual $N = (1/2,0)$ local supersymmetry with the infinitesimal spinor parameter $\epsilon^1(x)$, when choosing

$$\epsilon^2 = \bar{\epsilon}^1 = \bar{\epsilon}^2 = 0 \quad \text{and} \quad \psi^2_\mu = 0.$$  

Note that then $\delta \psi^2_\mu = 0$ is automatically satisfied, while the Lagrangian (7) takes the form

$$L = -e^2 \kappa^2 R(e, \omega) - i e \psi_\mu \sigma^{\mu \rho \sigma} D_\rho(\omega) \bar{\psi}_\sigma - \frac{e}{4} C_{\mu \nu} C^{\mu \nu},$$

(18)

where

$$C_{\mu \nu} = \frac{\kappa}{\sqrt{2}} [\bar{\psi}_\mu \bar{\chi}_\nu]^\dagger.$$  

(19)

By our construction, the supergravity action $S = \int d^4 x \ L$ with the Lagrangian (18) is invariant under the $N = 1/2$ local supersymmetry with the transformation laws

$$\delta e^\mu_\mu = -\frac{i \kappa}{2} \epsilon^\alpha_\mu \bar{\psi}_\alpha, \quad \delta \psi_\mu = \frac{1}{\kappa} D_\mu \epsilon, \quad \delta \bar{\psi}_\mu = 0, \quad \delta \bar{\chi}_\mu = 0.$$  

(20)

Note that equations (8) and (16) also imply

$$F_{\mu \nu} = 0.$$  

(21)

The result (21) may prompt us to choose another constraint,

$$F^+_{\mu \nu} = C_{\mu \nu}(x), \quad F^-_{\mu \nu} = 0,$$

(22)

from the very beginning of this section, instead of equation (11). Then its consistency with local supersymmetry,

$$\delta F_{\mu \nu} = 0,$$

(23)
under the transformations (9) again has a solution (13), with an arbitrary infinitesimal parameter \( \epsilon^1(x) \). The Lagrangian (7) then takes the form

\[
L = -\frac{e}{2\kappa^2}R(e, \omega) - ie\psi_1^\dagger \sigma^{\mu\rho\sigma} D_\rho(\omega) \bar{\psi}_1^\dagger - \frac{e}{4} C_{\mu\nu} C^{\mu\nu}
\]

\[
- \frac{e}{2} \left[ 2C_{\mu\nu} + \left( \frac{\kappa}{\sqrt{2}} (\bar{\psi}_1^\dagger \psi_1^\dagger) \epsilon^{ij} \right) \right] \frac{\kappa}{\sqrt{2}} (\bar{\psi}_k^\dagger \psi_l^\dagger) \epsilon^{kl}.
\]

(24)

The algebraic equation of motion of the field \( \bar{\psi}_2^\dagger \gamma^\beta \),

\[
\left[ C_{\mu\nu} + \left( \frac{\kappa}{\sqrt{2}} (\bar{\psi}_2^\dagger \psi_1^\dagger) \epsilon^{ij} \right) \right] g^{\gamma\nu} \bar{\psi}_1^{\dagger} \theta^\beta = 0,
\]

(25)

now has a solution

\[
C_{\mu\nu} = -\frac{\kappa}{\sqrt{2}} (\bar{\psi}_2^\dagger \psi_1^\dagger) \epsilon^{ij} = e \epsilon_{\mu\nu\rho\sigma} \frac{\kappa}{\sqrt{2}} (\bar{\psi}_2^\dagger \psi_1^\dagger) \epsilon^{ij}.
\]

(26)

When using the notation (17), we arrive at the \( N = 1/2 \) supergravity Lagrangian

\[
L = -\frac{e}{2\kappa^2}R(e, \omega) - ie\psi_2^\dagger \sigma^{\mu\rho\sigma} D_\rho(\omega) \bar{\psi}_2^\dagger + \frac{e}{4} C_{\mu\nu} C^{\mu\nu}
\]

(27)

subject to the constraint

\[
C_{\mu\nu} = -\frac{\kappa}{\sqrt{2}} [\bar{\psi}_2^\dagger \theta(\epsilon)]^\dagger,
\]

(28)

which are both invariant under the same \( N = 1/2 \) local supersymmetry transformations (20).

In this case we have

\[
\tilde{F}_{\mu\nu} = 0
\]

(29)

instead of equation (21).

5. Conclusion

The new model we constructed is, of course, a toy model with local \( N = 1/2 \) supersymmetry. Nevertheless, we found that an \( N = 1/2 \) supergravity is possible, while it can be very simple, like the C-deformed \( N = 1/2 \) supersymmetric gauge theory constructed in the NAC-deformed superspace [4]. Perhaps, the most remarkable feature of our construction is the very simple relation it implies between the expectation value of \( C_{\mu\nu}^+ \) and that of the self-dual product of two anti-chiral gravitini—see equations (19) and (28). Another approach to a construction of a C-deformed \( N = 1/2 \) supergravity in four Euclidean dimensions can be based on the NAC deformed \( N = (1/2, 1/2) \) superspace, by imposing the NAC relation on the chiral superspace coordinates, \( [\theta^A, \theta^B] = C^{AB} \), with \( C^{\mu\nu} = C^{AB} \epsilon_{BC} (\sigma^{\mu\nu})_A \). When using the Moyal–Weyl-type star product (\( \star \)) for the supergravity superfields [4] and applying the explicit superfield star-product summation formulae of [7], it may be possible to get another C-deformed \( N = 1/2 \) supergravity action in components (in a Wess–Zumino gauge). However, the last procedure seems to be much more complicated [10], and we do not claim that the result is going to be equivalent to our model constructed above.

A non-vanishing gravitino condensate is the well-known tool for spontaneous breaking of local supersymmetry, which may lead to a natural solution to the hierarchy problem in elementary particles physics (see e.g., the pioneering paper [11] for the dynamical mechanism due to gravitational instantons in quantum gravity, and [12] for its possible realization in superstring theory). Because of the relations (19) and (28), the C-deformation may be the viable alternative mechanism of spontaneous local supersymmetry breaking. To explore its physical consequences, one has to go beyond our toy model of C-deformed pure supergravity by adding some matter content [10].
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