Effect of the Wheel/Rail Contact Geometry on the Stability of Railway Vehicle

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Abstract: Regarding the hunting instability issue occurred on the high-speed vehicle, a CRH2C vehicle model is built for investigating the stability based on the wheel/rail interaction analysis. It is found that the Hopf bifurcation type is dependent on the equivalent conicity slope in the neighborhood of 3mm wheelset displacement. A decreasing equivalent conicity leads to a supercritical Hopf bifurcation, while an increasing one leads to a subcritical Hopf bifurcation. Furthermore, a wide wear band on the rail head can raise the equivalent conicity, which in turn decreases the nonlinear critical speed. It is concluded that the more concave of the equivalent conicity curve, the more nonlinear characteristics can be noticed on the wheel/rail interaction which leads to a severe performance on the nonlinear stability.

1. Introduction
During the high-speed train operation on track, some instability related issues on the bogie have been encountered, in which the lateral acceleration exceeds 8.0 m/s² in six continuous cycles. Figure 1 shows an example for lateral acceleration on the bogie of CRH2C high-speed train with a speed of 275 km/h in the high speed lines. This instability of the bogie decreases the maximum allowed speed of vehicle, and shows obvious effects on the operation order. To address this problem, many authors have done a lot of relevant research[1,2]. In this investigation, the bogie instability of the CRH2C is going to be investigated. The measured wheel and rail profiles are used to study the field wheel/rail contact geometry and the influence on vehicle stability.

Figure 1 Lateral Acceleration on the Bogie Frame of CRH2C
2. Railway Vehicle Model

The multi-body system model for the high-speed train CRH2C has been built in SIMPACK. The CRH2C experiences a low equivalent conicity, soft primary suspension and high yaw damping. The model consists of one car body and two bogies, in which the bogie has two wheelsets and four rotary arms. The car body, bogie frame and wheelset have six degrees of freedom (DOF), three translational and three rotational movements, and there is only one rotational degree of freedom relative to wheelset for one rotary arm. The vehicle model totally has 50 degrees of freedom, and the flexible deformation of all parts is neglected. The field measured wheel and rail profiles are used in the numerical simulation.

3. Effect of the Wheel/Rail Contact geometry on the Linear Stability

The linearized wheel/rail contact geometry, in general, can be expressed by three parameters, the equivalent conicity \( \lambda_e \), the contact angel \( \varepsilon \) and the roll angel \( \sigma \). In order to linearize the wheel/rail contact geometry, two general methods can be applied [3],

1. Linearize the non-linear wheel/rail contact geometry, which directly leads to the three parameters with respect to the wheelset lateral displacement.
2. Set a specific equivalent conicity, and the other two parameters are expressed as the functions of the equivalent conicity.

In general, the first method can give a more accurate result, while the value of the linearized parameters has a limited range because of the field profiles. This kind of limit is not sufficient to study the effect of the equivalent conicity on the vehicle dynamics behavior, and it is suggested that the second method should be used in this analysis.

In SIMPACK, the second linearized method is applied to represent the contact angel \( \varepsilon \),

\[
\varepsilon = 85\lambda_e \tag{1}
\]

Similarly, the roll angel \( \sigma \) is written as,

\[
\sigma = 0.2\lambda_e \tag{2}
\]

For a set of specific equivalent conicities, we can have the specific values of the contact angel and roll angel. The root-locus method is applied in the linear critical speed analysis for the high-speed train, in which the track gauge is 1435mm and the distance between backs of wheel flanges is 1353mm. Figure 2 shows the relation between the conicity and the linear critical speed of the CRH2C. It is shown that the linear critical speed is 870km/h, which decreases with the growth of equivalent conicity. While it is 478km/h if the non-linear method is applied, that is quite smaller than the linearized one.

![Figure 2 Relationships between the Equivalent Conicity and the Linear Critical Speed](image)

4. Effect of the Wheel/Rail Contact Geometry on the Nonlinear Stability

The nonlinear characteristics of the vehicle comes from the creep force, damping of dampers and wheel/rail contact geometry. In a general case, the hunting stability of the high-speed train is strongly dependent on the wheel/rail interaction.

The method used to evaluate the nonlinear stability is summarized as [4]: (1) nonlinear critical speed, which exists two approaches in the numerical simulation, one is no excitation and another is
applied with lateral sinewave excitation. In the case of no excitation, the vehicle runs on the ideal straight track and keeps increasing the speed until the limit cycle is reached. Then decrease the speed until the bogie become stable again, and the specific speed is defined as the nonlinear critical speed. In the case of only lateral sinewave excitation, the speed that the vehicle will not come back to stable after going over the excitation is defined as the nonlinear critical speed. (2) The irregularities method, in which the vehicle goes over the straight track with measured irregularities. In this method, the root mean square (R.M.S) of the lateral forces on the bogie frame, and the peak value (simplified method) or R.M.S of the lateral acceleration on the bogie frame are used to evaluate the nonlinear stability. In the simplified method of bogie acceleration, the definition of the stability limit is six continuous wave amplitude of 8.0 m/s².

In this paper, the nonlinear critical speed is evaluated by a method without excitation on ideal track.

4.1 Effect of the Wheel/Rail Contact Geometry on the Hopf Bifurcation Type

In the initial stage, the linear stability analysis is commonly applied in the vehicle system dynamics application[5]. With the development of the computer science, the nonlinear problems can be solved by numerical solution successfully. In the recent years, the vehicle system dynamics intends to be studied by the nonlinear Hopf bifurcation theory after considering the nonlinear characteristics.

According to the previous research[6,7], the common Hopf bifurcations can be classified as subcritical and Hopf bifurcation (figure 3(a)) and supercritical Hopf bifurcation (figure 3(b) and (c)). In the three figures, the dash curve is unstable solution, in which the limit cycle is dependent on the amplitude of the excitation. Whereas, the solid curve corresponds to the stable solution. The speed A is then defined as the linear critical speed in figure 3(a), and B is regarded as the nonlinear critical speed. For the solution on the curve AB, the limit cycle is stable if the excitation amplitude is above the curve AB and the stable limit cycle is the curve BC. For the excitation amplitude lower than the curve AB, then there will be no stable limit cycle. In figure 3(b), the curve AD and BC are stable limit cycle, in which the AD has small amplitude and the BC has larger amplitude, moreover, the curve BD is unstable. For the solutions in figure 3(c), the nonlinear and linear critical speed have the same value, A. In this case, the limit cycle increase with the rise in the vehicle speed. In the railway industry, the nonlinear critical speed \( V_{cr} \) is often evaluated.

The nominal equivalent conicity \( \lambda_e \), the equivalent conicity at a lateral displacement of 3mm, is generally used in the wheel/rail contact geometry analysis. In this investigation, the nonlinear conicity curve is obtained by using the quasi-linearized harmonic calculation method. Besides the \( \lambda_e \), O’Polach proposed a slope \( \lambda_N \) of the equivalent conicity in the neighborhood of 3mm lateral displacement to present the nonlinear characteristics of the wheel/rail contact geometry. The literature [8] shows the calculation procedure of \( \lambda_N \). In this numerical simulation, the worn wheel and rail profile are used to generate a set of wheel/rail contact geometry, and the Hopf bifurcation results are numerically summarized with various wheel/rail contact geometry, see Tab. 1.
Table 1 Hopf Bifurcation Type with Various Wheel/rail Contact Geometry

| Cases | Nominal Equivalent Conicity | Equivalent Conicity $10^{-3}$ | Hopf bifurcation           |
|-------|-----------------------------|-------------------------------|-----------------------------|
| 1     | 0.054                       | 0.867                         | subcritical Hopf bifurcation|
| 2     | 0.091                       | 1.998                         | subcritical Hopf bifurcation|
| 3     | 0.095                       | 2.376                         | subcritical Hopf bifurcation|
| 4     | 0.122                       | 4.525                         | subcritical Hopf bifurcation|
| 5     | 0.129                       | 9.588                         | subcritical Hopf bifurcation|
| 6     | 0.213                       | -18.120                       | supercritical Hopf bifurcation|
| 7     | 0.260                       | -23.717                       | supercritical Hopf bifurcation|
| 8     | 0.260                       | 119.791                       | subcritical Hopf bifurcation|

It can be seen from the Tab. 1 that the Hopf bifurcation type is dependent on the equivalent conicity slope. At the lateral displacement of 3 mm, the subcritical Hopf bifurcation is obtained for a positive slope, and it is the supercritical Hopf bifurcation for the negative slope of conicity. For a further investigation, the last two cases in Tab. 1 are used to evaluate the relation between the equivalent conicity slope and the Hopf bifurcation type. In this two cases, the conicities have a same value of 0.26, and they are noted as 26A and 26B respectively.

Figure 4 plots the equivalent conicity for the specific cases, in which the harmonic method is used to calculate the equivalent conicity. It shows that the equivalent conicity goes down (negative slope) for 26A and goes up (positive slope) for 26B around the lateral displacement at 3 mm. Figure 5 illustrates the nonlinear critical speed based on no excitation method. The case 26A experiences a small amplitude and it decreases to zero when the vehicle speed continuously decreasing, and the nonlinear critical speed is 352km/h. While the case 26B has a large limit cycle amplitude, and it comes to stable state sharply at 378km/h.

![Figure 4 Equivalent Conicity Curve](image1.png)
For the wheel/rail contact geometry in the case of 26A, the limit cycle can be obtained under various running speed when the vehicle travelling on the straight track without irregularity. Same numerical procedures can be performed on the case of 26B, and the solid (stable) solution can be obtained. For the dashed (unstable) solution, the DOE module in SIMPACK is applied for calculating the unstable solution for a specific running speed. It can be seen that, the case of 26A belongs to the supercritical Hopf bifurcation, while the type is subcritical Hopf bifurcation for the case of 26B. Moreover, the nonlinear critical speed in the supercritical Hopf bifurcation is smaller than that in the subcritical Hopf bifurcation at the same equivalent conicity.

4.2 Effect of the Rail Wear on the Nonlinear Stability

After the wheel and rail wear, different wheel/rail contact geometry can be encountered compared to the initial state. The rail profile, also, is not exactly the same as the designed one during grinding. Various rail profiles can be noticed on track. In this case, four different worn wheel profiles and several worn rail profiles are matched to different wheel/rail contact situation, and the corresponding nonlinear critical speeds are obtained. Four worn wheel profiles are measured from the left wheel on leading wheelset of the high-speed train CRH2C-2103. These profiles are referred as 2103-0101-Z1~Z4, with an interval of running distance of 50,000km.

Firstly, the standard rail profile CHN60, referred as TB60_standard is matched with the four measured wheel profiles respectively. Figure 7 plots the equivalent conicities in this case. The nominal equivalent conicities locate in 0.07~0.18, and the critical speeds are 428km/h, 420km/h, 410km/h and 415km/h, respectively, for the case of 2103-0101-Z1~Z4.
The wear band on the rail top can be used to evaluate the wheel/rail contact condition effectively, and it can be used to judge the rail wear state. The gauge corner wear is a general form, which firstly found on curves. The abnormal grinding at rail gauge corner leads to gauge corner wear loss the same as the gauge corner wear. Figure 8 illustrates the rail wear on an actual railway line, in which the gauge corner is over grinded which resulting in a narrow contact band on the rail top. The measured rail profile in this case is referred as TB60_wear(abnormal grinding). Another case happened on some specific section of track, where the rail profile is referred as TB60_wear(gauge corner). Figure 9 plots the two worn rail profiles with respect to the standard one. It can be clearly seen the difference near the gauge corner among these profiles. Then the equivalent conicity for the combination of TB60_wear(abnormal grinding) and worn wheels is shown in figure10(a), and figure10(b) presents the contact spot for the match between rail profile TB60_wear(abnormal grinding) and wheel profile 2103-0101-Z4. It shows that the equivalent conicity is low and changes slowly with the lateral displacement of the wheelset in figure10(a). Moreover, a narrow contact band on the rail top can be obtained in figure10(b), which shows good agreements with the field measurement in figure8.
The numerical simulations imply that the nonlinear critical speeds are all over 500km/h for the cases of TB60_wear (gauge corner) matching with aforementioned four worn wheel profiles. Similarly, the nonlinear critical speeds are more than 800km/h for the cases of matching with the rail profile TB60_wear (abnormal grinding). In the case of the hunting stability, the gauge corner wear rises the nonlinear critical speed, but it leads to a narrow wear band on the rail top which can result in severe wear on the rail and wheel. This will lead to a concave wear both on the rail and wheel, which inevitably increase the maintenance cost.

In the field measurement, another type of wear band noticed on track is wide contact band on the rail top, as shown in figure11. The measured rail profile with wide contact band is defined as TB60_wear (wide contact band), and the resultant equivalent conicities are demonstrated in figure12. The nominal equivalent conicities locate in 0.06~0.27 which are greater than the cases of standard rail profile. Nonlinear characteristic in the equivalent conicity curve can be noticed obviously. Then the nonlinear critical speeds are 393km/h, 385km/h, 382km/h and 376 km/h, respectively. From the simulation results, it can be concluded that the equivalent conicities rise for the wide contact band, then the critical speeds and hunting stability go down.

For a better understanding of the effect of wide contact band and standard band on the vehicle lateral stability, the lateral acceleration on the bogie frame is simulated in SIMPACK with track irregularities applied. In the numerical simulation, the field measured irregularities on Beijing-Tian high-speed railway line is used. Two numerical scenarios are considered, in the first case, the track segment in 0~1000m has a rail profile TB_standard, and TB60_wear (wide contact band) is applied for the another segment in 1000~2000m. For the other scenario, the rail profiles on the two segments are...
switched by each other. The running speed is 400 km/h, and the filtered accelerations (0.5–10Hz) on the bogie frame are plotted in figure 13.

![Figure 13 Lateral Acceleration on the Bogie Frame](image)

Figure 13 Lateral Acceleration on the Bogie Frame

It can be seen from figure 13 that the first scenario has a larger acceleration amplitude in 0–1000m, and there exists an unstable condition since the limit of six continuous cycles have a value exceeds the limit 8.0m/s². For the second scenario, the acceleration on the bogie frame has a larger value on the track segment of 1000–2000m. Then it can be concluded that the wide contact band on rail leads to a larger lateral vibration on the bogie which resulting in unstable of vehicle.

Conclusions can be obtained after summarizing the results obtained above, (1) a wide contact band leads to a lower critical speed. (2) a negative conicity slope results in supercritical Hopf bifurcation and lower nonlinear critical speed than subcritical Hopf bifurcation one’s even with the same equivalent conicity around 3 mm. (3) Above all, the case of a wide contact band and negative slope of equivalent conicity in the neighborhood of 3mm displacement may lead to a severe hunting instability issue at relatively low speed.

4.3 Effect of the Equivalent Conicity on the Nonlinear Stability

The equivalent conicity is the commonly used parameter to evaluate the wheel/rail contact geometry, and the equivalent conicity curve is classified two types to show the influence of the equivalent conicity on the hunting stability of vehicle. One type of equivalent conicity is close to a linear relation and shown in figure 14(a), in which a linear relation of the conicity is noticed for the wheelset lateral displacement within 6 mm. Another type of equivalent conicity curve is concave, as shown in figure 14(c), in which a nonlinear characteristic is quite significant. Figure 14(b) corresponds to the nonlinear critical speed in figure 14 (a). It shows that the critical speed decreases clearly with the conicity growth, and it is 349 km/h for the conicity 0.35. Similarly, figure 14(d) corresponds to the nonlinear critical speed in figure14 (c), and there is no clear relation between them. For all of the equivalent conicities, the critical speeds are less than 300km/h.

Then it can be concluded that, a strong nonlinear wheel/rail contact geometry leads to a severe nonlinear stability performance. It can be explained that a concave conicity curve results in a strong nonlinear wheel/rail interaction situation. When the model takes time integration, the system converges very slowly, which causes the lower nonlinear critical speed of the vehicle. Consequently, in the wheel reprofiling procedure, it is suggested to consider the conicity value and its nonlinear characteristics with respect to the wheelset lateral displacement.
5. Conclusion
A high-speed train model has been built for the high-speed train CRH2C and it was used to study the effect of the wheel/rail contact geometry on the nonlinear critical speed, which is used to evaluate hunting stability of vehicle. Following conclusions can be obtained,

1. For the positive conicity slope at the lateral displacement of 3 mm, a subcritical Hopf bifurcation is obtained. While a supercritical Hopf bifurcation can be obtained for the case of negative conicity slope.

2. The vehicle has a good nonlinear stability performance in the case of gauge corner wear on rail, but it causes concentrated wear on the rail top and rises the maintenance cost. A wide contact band leads to a higher equivalent conicity and lower nonlinear critical speed. That is the main reason for the lateral hunting instability of the bogie.

3. When a linear characteristic is assigned for the equivalent conicity with respect to the wheelset lateral displacement, then vehicle stability decreases with the increasing of the equivalent conicity. For the case of a concave conicity curve, the nonlinear wheel/rail contact geometry becomes highly discrete. The lateral stability shows no clear relations with the equivalent conicity, and a severe stability is obtained.

4. The nonlinear characteristics of equivalent conicity curve should be considered during the wheel reprofiling procedure.

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