Power Coverage Analysis of Cellular Networks With Energy Harvesting and Microwave Power Transfer-Based Power Sharing

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Abstract: This paper studies the performance of hyperdense large-scale cellular networks with energy harvesting. To combat the randomness of renewables, the base stations (BSs) with the harvested power larger than the predefined power target are assumed to cooperatively share their surplus power to the BSs with insufficient energy via directed microwave power transfer (MPT). To evaluate the performance of the proposed cooperative power sharing scheme, a new performance metric, namely the power coverage probability, which is defined as the probability that a BS satisfies its respective power target, is introduced and then characterized by applying tools from stochastic geometry. It is shown that in the hyperdense regime, the power coverage probability of cellular networks with energy harvesting and MPT based power sharing depends on the statistical mean of the harvested renewables, the power target, the power sharing distance, and the MPT path-loss exponent. Simulations are provided to validate our analysis.

Index Terms: Energy harvesting, multi-cell cooperation, microwave power transfer, stochastic geometry, power coverage probability.

I. INTRODUCTION

Hyper densification of cellular network deployment is considered to be the key enabling technology for 5G system to meet the 1000x data rate challenge [1]–[3]. With densely deployed base stations (BSs), the reduction of cell size considerably increases the spatial reuse of the cellular network while decreases the transmission distance of each data link. Though deploying more cells significantly improves the spectrum efficiency, the corresponding energy consumption grows dramatically with the increased BSs. Renewable energy, envisioned as a promising alternative to the traditional fossil fuel based power generation, provides an effective way of reducing energy costs in dense cellular networks [4]. One of the most challenging issues of cellular networks powered by renewable energy lies in the stochastic and intermittent nature of the renewable sources [5]. In this paper, motivated by the law of large numbers, we smooth out such fluctuations by enabling cooperative power sharing among the BSs.

Point process theory [6], [7] has been widely applied in the study of large-scale wireless networks with energy harvesting. In [8], Huang characterized the tradeoff between the energy arrival rate and the spatial throughput of mobile ad hoc networks with energy harvesting. In [9], Che et al. investigated the optimization problem of bidirectional energy harvesting and information transmission in large-scale communication networks with wireless charging. In [10], Lee et al. studied the performance of opportunistic energy harvesting in large-scale cognitive radio networks and maximized the throughput of secondary network subject to outage constraints. In [11], Dhillon et al. developed a tractable model for K-tier downlink heterogeneous networks with energy harvesting and characterized the availability region for a set of general uncoordinated BS operational strategies. In [12], Huang considered deploying power beacons in cellular networks to enable microwave power transfer (MPT) for mobile recharging and derived the tradeoffs between the network parameters. In [13], Sakr et al. investigated the performance of device to device (D2D) communications with energy harvesting and characterized the outage probabilities for both D2D and cellular users. It is worth noting that in the above prior works [8]–[13], with energy storage units, the randomness of the harvested energy was smoothed out by exploiting the time diversity of the
generated renewables. In [14], Huang et al. proposed a novel energy field model and characterized the performance of downlink cellular networks with on-site/distributed energy harvesters. In [15], Khan et al. evaluated the transmission success probability of a large-scale cluster based wireless networks with energy harvesting, and captured the trade-off between link-level performance and density of receivers served. In [16], Wu et al. investigated a novel cooperative transmission strategy for energy harvesting based small cell basestations, and derived the cell load distributions, the average user capacity, and the coverage probability via Gamma approximation. In [17], Guo et al. investigated a multi-cell network with cooperative NOMA and energy harvesting, and derived the closed-form expressions of the coverage probability, ergodic rate, and energy efficiency. It was pointed out in [14]–[17] that the fluctuating effect of the harvested energy can also be counteracted by exploiting the spatial diversity of the generated renewables.

In this paper, a novel framework is provided to analyze the performance of large-scale cellular networks with energy harvesting. The locations of BSs are modeled as a homogeneous Poisson point process (HPPP). With energy harvesting module and energy storage module, each BS is able to collect and store ambient renewables for data transmissions in the subsequent time slot. The power\(^1\) harvested by the BSs are expected to exceed their respective power targets such that the outage constraints at the intended mobile users can be satisfied. It is worth noting that due to the fluctuating effect of the renewables, the harvested power is unstable and may falls below the predefined power target. To combat such randomness, it is assumed that the BSs with the harvested power larger than the predefined power target cooperatively share their surplus power to the BSs with the harvested power lower than the predefined power target via directed MPT [12]. Thus, different from that in [8]–[13], we stabilize the harvested power by exploiting the spatial diversity, instead of time diversity, of the generated renewables. Further, different from [14]–[17], we investigate the MPT based power sharing for power transfer. In this paper, we mainly focus on the hyperdense regime of the cellular networks with energy harvesting, which is a typical scenario of 5G cellular systems. To evaluate the performance of the proposed cooperative power sharing scheme, a new performance metric, namely the power coverage probability, which is defined as the probability that a BS satisfies its respective power target, is introduced and then characterized by applying tools from stochastic geometry. It is shown that in the hyperdense regime, the power coverage probability of cellular networks with energy harvesting and MPT based power sharing depends on the mean value of the harvested renewables, the power target, the power sharing distance, and the MPT path-loss exponent. Simulations are provided to validate our analysis. The main contributions of this paper are summarized as follows:

- We consider a hyperdense large-scale cellular network with BSs powered by energy harvesting, where the locations of BSs are modeled as a Poisson point process (PPP) on \(\mathbb{R}^2\). To cope with the randomness of the harvested renewables, a MPT based power sharing scheme is proposed. Particularly, it is designed that the BSs with the harvested power larger than the predefined power target cooperatively share their surplus power to the BSs with the harvested power lower than the predefined power target via directed MPT.
- To analyze the performance of the proposed MPT based power sharing scheme, a novel framework is developed with stochastic geometry. We first derive the mean and variance of the received shared power via MPT at a BS with the respective harvested power lower than the predefined power target. Then, based on the results, we further derived the upper and lower bounds on the power coverage probability by applying the Cantelli’s inequality.
- Extensive simulations are provided to verify our analysis. An implication of our analytical and simulated results is that: for cellular networks with energy harvesting and MPT based power sharing, it is more beneficial to increase the density of BSs than expanding the power sharing distance to counteract the randomness of the renewables.

The remainder of this paper is organized as follows. The system model and performance metrics are described in Section II. The power coverage probability of the cellular network with energy harvesting and MPT based power sharing is characterized in Sections III. Simulation results are presented in Section IV. Finally, we conclude our paper in Section V.

II. MODEL AND METRIC

A. SYSTEM MODEL

We consider a hyperdense large-scale cellular network powered by renewable energies on \(\mathbb{R}^2\), where the locations of BSs are modeled as a HPPP with density \(\lambda\). Time is assumed to be slotted. In each time slot, the BSs are designed to collect and store ambient renewable energies for data transmissions in the subsequent time slot. It is worth mentioning that compared with the traditional cellular network with stable power supply, the cellular network with energy harvesting suffers considerable performance loss due to unreliable and spatially fluctuating renewable sources. To combat such randomness of renewables, in this paper, motivated by the law of large numbers, we enable the BSs to cooperatively share their surplus power via directed MPT.

Let \(P_k\) be the renewable power harvested by the \(k\)-th BS in time slot \(t\). To capture the fluctuating effect of renewables, it is assumed that \(P_k\) follows an exponential distribution with mean given by \(P\). Further, to simplify the analysis, the renewable power harvested by BSs at different locations

\(^1\)In the following, due to the fact that power is the amount of energy consumed per unit time, we simply denote energy as power by abuse of notation.
are assumed to be independent. The power targets, i.e., the transmission powers, for BSs in the cellular network with energy harvesting are assumed to be the same and denoted by $\eta_b$. Let $\Pi_b$ be the set of BSs with the harvested power larger than the predefined power target $\eta_b$ and let $\Psi_b$ be the corresponding locations. In addition, let $\Pi_b^c$ be the set of BSs with the harvested power less than the predefined power target $\eta_b$ and let $\Psi_b^c$ be the corresponding locations. Then, with coloring theorem [7], it can be easily verified that $\Psi_b$ and $\Psi_b^c$ follow two independent HPPPS with densities $\lambda_b = \lambda \cdot \beta_b$ and $\lambda_b^c = \lambda \cdot (1 - \beta_b)$, respectively, where $\beta_b = \Pr\{P_k \geq \eta_b\}$.

The power $P_k$ harvested by the $k$-th BS is expected to exceed its respective power target $\eta_b$ such that the transmission outage constraints at the intended mobile users are satisfied. It is worth noting that due to the randomness of the renewables, $P_k$ is unstable and thus may falls below the predefined power target $\eta_b$. To make all the BSs reach their respective power target $\eta_b$, in this paper, it is assumed that the $i$-th BS in $\Pi_b$ located at $X_i$ equally share its surplus power $P_i - \eta_b$ to the BSs in $\Pi_b^c(X_i, d)$ via directed MPT as illustrated in Fig.1, where $\Pi_b^c(X_i, d)$ denotes the set of BSs in $\Pi_b^c$ within a distance of $d$ from $X_i$ and $d$ denotes the power sharing distance of cellular networks with energy harvesting. Particularly, denoting $M_l$ as the counting measure of $\Pi_b^c(X_i, d)$, the power shared by the $i$-th BS in $\Pi_b$ located at $X_i$ for each BS in $\Pi_b^c(X_i, d)$ is given by

$$\hat{P}_i = \frac{P_i - \eta_b}{M_l + 1},$$

where $M_l$ is a Poisson distributed random variable with density given by $\lambda_m = \lambda_b^c \pi d^2$. It is worth noting that in each time slot, for BSs in $\Pi_b$, all the remaining power $\eta_b$ will be utilized for data transmission in the next time slot.

The propagation loss of directed MPT is modeled as [12]

$$l(r) = r_+^{\alpha},$$

where $r$ denotes the power sharing distance, $2 < \alpha < 4$ denotes the directed MPT path-loss exponent, and

$$r_+ = \max(1, r).$$

Let $Y_j$ be the location of the $j$-th BS in $\Pi_b^c$. In addition, let $\Pi_b(Y_j, d)$ be the set of BSs in $\Pi_b$ within a distance of $d$ from $Y_j$ as illustrated in Fig.1(b). Then, based on (1) and (2), we obtain the total shared power received at the $j$-th BS in $\Pi_b^c$ as

$$S_j = \sum_{i \in \Pi_b(Y_j, d)} \frac{P_i - \eta_b}{M_i + 1} |X_i - Y_j|_+^{-\alpha},$$

where $X_i$ denotes the location of the $i$-th BS in $\Pi_b(Y_j, d)$. Let $N_j$ be the counting measure of $\Pi_b(Y_j, d)$. Then, it can be easily verified that $N_j$ follows a Poisson distribution with density given by $\lambda_n = \lambda_b \pi d^2$. It can be also verified that for $\forall p, q \in \Pi_b(Y_j, d), p \neq q$, $M_p$ and $M_q$ are dependent. With (4), it thus follows that the total power (the harvested power plus the shared power) received by the $j$-th BS in $\Pi_b^c$ at the tagged time slot is given by

$$T_j = P_j + S_j,$$

where $P_j$ denotes the renewable power harvested by $Y_j$. To simplify the analysis, it is worth noting that in each time slot, if $T_j \geq \eta_b$, the $j$-th BS in $\Pi_b^c$ will use $\eta_b$ for data transmission in the next time slot and discard the remaining power.

B. PERFORMANCE METRIC

To evaluate the performance of the proposed cooperative power sharing scheme, we introduce a new performance
metric, namely the power coverage probability, for cellular networks with energy harvesting and MPT based power sharing, which is specified as follows.

**Power Coverage Probability:** For cellular networks with energy harvesting and MPT based power sharing, the power coverage probability, denoted by \( \tau \), is defined as the probability that a BS satisfies its respective power target. Particularly, denoting \( P \) as the harvested power of the BS, \( \eta \) as the corresponding power target, and \( S \) as the total shared power received at the BS, the power coverage probability for cellular networks with energy harvesting and MPT based power sharing is given by

\[
\tau = \Pr \{ P + I (P < \eta) \cdot S \geq \eta \},
\]

where \( I(\cdot) \) denotes the indicator function.

**III. CHARACTERIZATION OF \( S_j \)**

In this section, we characterize the mean and variance of \( S_j \).

**Lemma 1:** For cellular networks with energy harvesting and MPT-based power sharing, the mean value of \( S_j \) received at the \( j \)-th BS in \( \Pi^b_j \) is given by

\[
\mathbb{E}[S_j] = \frac{\beta_b}{1 - \beta_b} \cdot P \cdot \frac{1 - e^{-\lambda(1-\beta_b)\pi d^2} \cdot \alpha - 2d-(\alpha-2)}{\alpha - 2},
\]

where \( \beta_b = \Pr \{ P_k \geq \eta_b \}. \)

**Proof:** See Appendix A. \( \Box \)

**Remark 1:** It can be easily verified that \( \mathbb{E}[S_j] \) is an increasing function of \( \beta_b \) when \( \beta_b \) is small, while a decreasing function of \( \beta_b \) when \( \beta_b \) is large.

**Remark 2:** It can be also verified that \( \mathbb{E}[S_j] \) is a decreasing function of \( d \), which is intuitively expected since a long range of power transfer via MPT may suffer a considerable loss of energy.

**Remark 3:** Further, it can be verified that \( \mathbb{E}[S_j] \) is an increasing function of \( \lambda \), which is due to the fact that the energy loss of MPT based power transfer decreases with the transmission \( d \) and thereby \( \lambda \).

**Lemma 2:** For cellular networks with energy harvesting and MPT-based power sharing, the variance of \( S_j \) received at the \( j \)-th BS in \( \Pi^b_j \) can be approximated by

\[
\text{Var}[S_j] \approx \frac{\beta_b}{1 - \beta_b} \cdot \frac{2P^2 \cdot \left(1 - e^{-\lambda(1-\beta_b)\pi d^2}\right)}{\lambda \cdot (1 - \beta_b) \pi d^4} \cdot \frac{\alpha - 2d-(\alpha-2)}{\alpha - 1}.
\]

**Proof:** See Appendix B. \( \Box \)

**Remark 4:** It can be easily verified that \( \text{Var}[S_j] \) is a decreasing function of \( d \), which is due to the fact the fluctuation of \( \text{Var}[S_j] \) decreases with the energy loss of MPT.

**Remark 5:** It can be also verified that \( \text{Var}[S_j] \) is a decreasing function of \( \lambda \), which is due to the law of large number.

With Lemmas 1 and 2, in the following section, we characterize the power coverage probability of the cellular networks with MPT-based power sharing.

**IV. POWER COVERAGE PROBABILITY**

In this section, we analyze the power coverage probability of the cellular networks with MPT based power sharing. It is worth noting that thanks to the homogeneity of the studied cellular network with energy harvesting and MPT based power sharing, the average power coverage performance of the cellular network can be characterized by the coverage performance experienced at the \( j \)-th BS. With this fact and (6), we obtain the power coverage probability of cellular networks with energy harvesting and MPT-based power sharing as

\[
\tau = \Pr \{ P_j < \eta_b \} \cdot \Pr \{ P_j + S_j \geq \eta_b \} \cdot (1 - \beta_b) \cdot \Pr \{ P_j + S_j \geq \eta_b \} + \beta_b
\]

where

\[
\varphi_b = \Pr \{ P_j + S_j \geq \eta_b \} \cdot \Pr \{ P_j < \eta_b \}
\]

\[
= \Pr \{ P_j + S_j \geq \eta_b \} \cdot \Pr \{ P_j < \eta_b, S_j \geq \eta_b \} \cdot \Pr \{ S_j \geq \eta_b \}
\]

\[
= \Pr \{ P_j + S_j \geq \eta_b \} \cdot \Pr \{ P_j < \eta_b, S_j \geq \eta_b \} \cdot \Pr \{ S_j \geq \eta_b \}
\]

\[
= \Pr \{ P_j + S_j \geq \eta_b \} \cdot \Pr \{ P_j < \eta_b, S_j \geq \eta_b \} \cdot \Pr \{ S_j \geq \eta_b \}
\]

\[
= \Pr \{ P_j + S_j \geq \eta_b \} \cdot \Pr \{ P_j < \eta_b, S_j \geq \eta_b \} \cdot \Pr \{ S_j \geq \eta_b \}
\]

**Remark 6:** It can be easily verified that the power coverage probability \( \tau \) contains two parts. The first part calculates the power coverage probability of BSs with the harvested energy less than the predefined target \( \eta \). In this case, the respective BSs may satisfy the power target \( \eta_b \) if and only if \( P_j + S_j \geq \eta_b \). The second part calculates the power coverage probability of BSs with the harvested energy larger than the predefined target \( \eta \), which is straightforward from the proposed MPT based power sharing scheme.

In the following, we first derive \( \beta_b \) and \( \varphi_b \), and then based on which characterize \( \tau \).

**Lemma 3:** For cellular networks with energy harvesting and MPT-based power sharing, we obtain \( \beta_b \) as

\[
\beta_b = e^{-\varphi_b \cdot \eta_b}. \quad (11)
\]

**Remark 7:** It can be verified that \( \beta_b \) is a decreasing function of \( \eta_b \) while an increasing function of \( P \).

**Proof:** See Appendix C. \( \Box \)

In the following, we first characterize \( \Pr \{ S_j \geq \eta_b \} \), and then based on which characterize \( \varphi_b \).

**Lemma 4:** For cellular networks with energy harvesting and MPT-based power sharing, \( \Pr \{ S_j \geq \eta_b \} \) is upper and lower bounded by

\[
\epsilon \leq \Pr \{ S_j \geq \eta_b \} \leq 1, \quad \text{if } \mathbb{E}[S_j] \geq \eta_b,
\]

\[
0 \leq \Pr \{ S_j \geq \eta_b \} \leq \epsilon, \quad \text{if } \mathbb{E}[S_j] < \eta_b.
\]

\[\epsilon \leq \Pr \{ S_j \geq \eta_b \} \leq 1, \quad \text{if } \mathbb{E}[S_j] \geq \eta_b,
\]

\[0 \leq \Pr \{ S_j \geq \eta_b \} \leq \epsilon, \quad \text{if } \mathbb{E}[S_j] < \eta_b,
\]
where
\[
\epsilon = \frac{\text{Var}[S_j]}{\text{Var}[S_j] + (\eta_b - \mathbb{E}[S_j])^2},
\]
and
\[
\tilde{\epsilon} = 1 - \epsilon = 1 - \frac{\text{Var}[S_j]}{\text{Var}[S_j] + (\eta_b - \mathbb{E}[S_j])^2}.
\]

**Proof:** See Appendix D. \(\square\)

**Remark 8:** It is worth noting that only the mean and variance of \(S_j\) can be derived. As such, only upper and lower bounds on \(\mathbb{P}\{S_j \geq \eta_b\}\) can be obtained.

**Remark 9:** It is also worth noting that the derived upper and lower bounds on \(\mathbb{P}\{S_j \geq \eta_b\}\) are piecewise functions, where the segmentation point depends on the values of \(\mathbb{E}[S_j]\) and \(\eta_b\).

Based on Lemma 4, we characterize \(\phi_b\) in the following lemma.

**Lemma 5:** For cellular networks with energy harvesting and MPT-based power sharing, \(\phi_b\) is upper and lower bounded by
\[
\begin{align*}
\left\{ \begin{array}{ll}
\epsilon \leq \phi_b \leq 1, & \text{if } \mathbb{E}[S_j] \geq \eta_b, \\
0 \leq \phi_b \leq \frac{\mathbb{E}[S_j]}{\eta_b} + \left(1 - \frac{\mathbb{E}[S_j]}{\eta_b}\right) \cdot \epsilon, & \text{if } \mathbb{E}[S_j] < \eta_b.
\end{array} \right.
\end{align*}
\]

**Proof:** See Appendix E. \(\square\)

**Remark 10:** Similar as that of \(\mathbb{P}\{S_j \geq \eta_b\}\), only upper and lower bounds on \(\phi_b\) can be obtained. Also, the derived upper and lower bounds on \(\phi_b\) are piecewise functions with the segmentation point depends on the values of \(\mathbb{E}[S_j]\) and \(\eta_b\).

Based on Lemmas 3 and 5, we characterize \(\tau\) in the following theorem.

**Theorem 1:** For cellular networks with energy harvesting and MPT-based power sharing, \(\tau\) is upper and lower bounded by
\[
\begin{align*}
\left\{ \begin{array}{ll}
\gamma \leq \tau \leq 1, & \text{if } \mathbb{E}[S_j] \geq \eta_b, \\
e^{-\frac{\eta_b}{\lambda}} \leq \tau \leq \nu, & \text{if } \mathbb{E}[S_j] < \eta_b,
\end{array} \right.
\end{align*}
\]
where
\[
\gamma = \tilde{\epsilon} \cdot \left(1 - e^{-\frac{\eta_b}{\lambda}}\right) + e^{-\frac{\eta_b}{\lambda}},
\]
and
\[
\nu = \left(\frac{\mathbb{E}[S_j]}{\eta_b} + \left(1 - \frac{\mathbb{E}[S_j]}{\eta_b}\right) \cdot \epsilon\right) \cdot \left(1 - e^{-\frac{\eta_b}{\lambda}}\right) + e^{-\frac{\eta_b}{\lambda}}.
\]

**Proof:** Based on Lemmas 3 and 5, with (9), (16) can be immediately obtained. This thus completes the proof of Theorem 1. \(\square\)

Based on Theorem 1, we characterize \(\lim_{\lambda \to \infty} \tau\) in the following corollary.

**Corollary 1:** For cellular networks with energy harvesting and MPT-based power sharing, we have
\[
\lim_{\lambda \to \infty} \tau = \begin{cases} 
1 & \text{if } \mu_c \geq \eta_b, \\
\frac{1}{e^{\frac{\eta_b}{\lambda}} - 1} \cdot \frac{P}{\alpha - 2d^{-\alpha - 2}} & \text{if } \mu_c < \eta_b,
\end{cases}
\]
where
\[
\mu_c = \lim_{\lambda \to \infty} \mathbb{E}[S_j]
\]
\[
= \frac{1}{e^{\frac{\eta_b}{\lambda}} - 1} \cdot \frac{P}{\alpha - 2d^{-\alpha - 2}}.
\]

**Proof:** See Appendix F. \(\square\)

### V. NUMERICAL RESULTS

In this section, to validate our analytical results and demonstrate the benefit of MPT based multi-cell cooperation, extensive simulations are presented. Throughout this section, unless specified otherwise, we set \(P = 5, \eta_b = 0.1,\) and \(\alpha = 3.\) Further, the power sharing distance \(d\) is measured in meter.

![Mean Value of Sj versus power sharing distance d for lambda = 0.01, 0.1, 1 respectively.](image)

Fig. 2 plots the analytical and simulated results on the mean value of \(S_j\) for \(\lambda = 0.01, 0.1, 1\), respectively. It is observed that the simulation results match with analytical results. It is also observed that the mean value of \(S_j\) is a decreasing function of the power sharing distance \(d\), which is intuitively expected since the energy transformed via MPT may suffer a considerable loss due to the long range transmission. Further, it is observed that the mean value of \(S_j\) is an increasing function of \(\lambda\), which is due to the fact that the total energy loss of MPT based power transfer decreases with \(d\) and thereby \(\lambda\).

Fig. 3 plots the analytical and simulated results on the variance of \(S_j\) for \(\lambda = 0.01, 0.1, 1\), respectively. It is observed that the simulation results match with analytical results. It is also observed that the variance of \(S_j\) is a decreasing function

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3It worth noting that the simulation results are fluctuating around the analytical results, which is due to the the central limit theorem.
of the power sharing distance $d$, which is due to the fact that the fluctuation of $S_j$ vanishes as $d$. Further, it is observed that the variance of $S_j$ is a decreasing function of $\lambda$, which is due to the law of large number.

Figs. 4, 5, 6 shows the analytical and simulated results on power coverage probability of cellular networks with energy harvesting and MPT based power sharing versus the power sharing distance $d$, for $\lambda = 0.01, 0.1, 1$, respectively. It is observed that the derived upper and lower bounds on power coverage probability are valid. It is also observed that the power coverage probability is a decreasing function of $d$, which is due to the fact the energy loss via MPT increases dramatically with $d$.

Fig. 7 compares the simulated values of power coverage probability of cellular networks with energy harvesting and MPT based power sharing for $\lambda = 0.01, 0.1, 1, \infty$. It is observed that the power coverage probability of cellular networks with energy harvesting and MPT based power sharing is an increasing function of $\lambda$, which is mainly due to the fact that the distance of the power transfer decreases with $\lambda$. Particularly, the tight upper bound on the power coverage probability is a piecewise function of $d$, which can be achieved by increasing $\lambda$ to $\infty$. Further, the segmentation
point of the power coverage probability depends on the mean value of $S_j$ and $\eta_b$. An implication of the above observations is that: for cellular networks with energy harvesting and MPT based power sharing, it is more beneficial to increase the density of BSs than expanding the power sharing distance $d$ to counteract the randomness of the renewables.

VI. CONCLUSION

This paper has studied the performance of hyperdense large-scale cellular networks with energy harvesting. To combat the randomness of renewables, the BSs with the harvested power larger than the predefined power targets are assumed to cooperatively share their surplus power to the BSs with the harvested power lower than the predefined power targets via directed MPT. By applying tools from stochastic geometry, we first derived the mean and variance of $S_j$ to capture the statistics of the shared power. Then, based on the obtained results, we characterize the power coverage probability of the studied large-scale cellular network with energy harvesting and MPT based power sharing. Finally, extensive simulations are provided to verify our analysis. An implication of our analytical and simulated results is that: for cellular networks with energy harvesting and MPT based power sharing, it is more beneficial to increase the density of BSs than expanding the power sharing distance to counteract the randomness of the renewables. It is hoped that the results in this paper will provide new insights to the practical design of power sharing schemes via MPT in cellular networks with energy harvesting.

APPENDICES

APPENDIX A

PROOF OF LEMMA 3.1

Proof: For cellular networks with energy harvesting and MPT-based power sharing, it can be easily verified that

$$E[S_j] = E\left[ \sum_{i \in \Pi_d(y_j, d)} \frac{P_i - \eta_b}{M_i + 1} \left| |X_i - Y_j\right|^{\alpha} \right]$$

$\equiv (a) \pi \lambda \beta_b \cdot \int_0^1 P_i - \eta_b \cdot \frac{M_i + 1}{P_i - \eta_b} \cdot r - \alpha \cdot r \cdot dr$

$\equiv (b) \pi \lambda \beta_b \cdot \int_0^d P_i - \eta_b \cdot \frac{1}{P_i + 1} \cdot r - \alpha \cdot r \cdot dr$

$$= \pi \lambda \beta_b \cdot P \cdot \left[ \frac{1}{M_i + 1} \right] + \frac{1}{M_i + 1} \cdot \frac{2d - (\alpha - 2)}{\alpha - 2}$$

where (a) follows from the Campbell’s Theorem, (b) follows from the fact that

$$E\left[ P_i - \eta_b \right] = E\left[ \int_{\eta_b}^{\infty} \frac{e^{-\eta_b}}{\pi \beta_b} \cdot \frac{1}{\eta_b} dP_i \right]$$

(22)

and (c) follows from the fact that

$$E\left[ \frac{1}{M_i + 1} \right] = \sum_{M_j=0}^{\infty} \frac{\lambda_m}{M_i + 1} \cdot \frac{1}{M_i + 1}$$

(23)

This thus completes the proof of Lemma 1.

APPENDIX B

PROOF OF LEMMA 3.2

Proof: For cellular networks with energy harvesting and MPT-based power sharing, it can be easily verified that

$$Var(S_j) = Var\left[ \sum_{i \in \Pi_d(y_j, d)} \frac{P_i - \eta_b}{M_i + 1} \cdot |X_i - Y_j|^{\alpha} \right]$$

$\equiv (a) 2 \pi \lambda \beta_b \cdot \int_0^1 E \left[ \left( \frac{P_i - \eta_b}{M_i + 1} \right)^2 \right] \cdot r^\alpha \cdot r \cdot dr$

$\equiv (b) 2 \pi \lambda \beta_b \cdot 2P^2 \cdot E \left[ \left( \frac{1}{M_i + 1} \right)^2 \right] \cdot \frac{1 - d - (2\alpha - 2)}{\alpha - 1}$

(24)

where (a) follows from [20], and (b) follows from the fact that

$$E\left[ \left( P_i - \eta_b \right)^2 \right] = E\left[ \int_{\eta_b}^{\infty} \left( P_i - \eta_b \right)^2 \cdot \frac{1}{\eta_b} e^{-\eta_b} \cdot dP_i \right]$$

$$= 2P^2.$$
It is worth noting that
\[
\mathbb{E}\left[\left(\frac{1}{M_i+1}\right)^2\right] = \sum_{M_i=0}^{\infty} \frac{\lambda_m^M e^{-\lambda_m}}{M_i!} \cdot \left(\frac{1}{M_i+1}\right)^2
\]
\[= \frac{1}{\lambda_m^M} \sum_{M_i=1}^{\infty} \frac{\lambda_{M+1} e^{-\lambda_m}}{M_i!} \cdot \frac{1}{M_i+1}
\]
\[= \frac{1}{\lambda_m^M} \sum_{M_i=1}^{\infty} \frac{\lambda_{M+1} e^{-\lambda_m}}{M_i!} \cdot \frac{1}{M_i}
\]
\[\geq \frac{1}{\lambda_m^M} \cdot (1 - e^{-\lambda_m}) \cdot \frac{s(1, 1)}{\lambda_m}
\]
\[= \left(1 - e^{-\lambda_m}\right) \cdot \frac{s(1, 1)}{\lambda_m}
\]
where \(\lambda_m = \lambda (1 - \beta_m) \pi d^2\), (a) follows from [21], \(s(n, k)\) denotes the Stirling numbers of the first kind as
\[s(n, k) = (-1)^{n-k} \binom{n}{k}.
\]
and (b) follows from the fact that \(s(1, 1) = 1\). Then, with (24) and (26), (8) is immediately obtained. This thus completes the proof of Lemma 2. \(\square\)

**APPENDIX C**

**PROOF OF LEMMA 4.1**

**Proof:** For cellular networks with energy harvesting and MPT-based power sharing, given that the power targets for different BSs are equal to the same constant \(\eta_b\), it can be easily verified that

\[\beta_b = \Pr\{P_j \geq \eta_b\}
\]
\[= \int_{\eta_b}^{\infty} \frac{1}{F} \cdot e^{-\beta} dP_j
\]
\[= e^{-\eta_b/\beta}.
\]
This thus completes the proof of Lemma 3. \(\square\)

**APPENDIX D**

**PROOF OF LEMMA 4.2**

**Proof:** For cellular networks with energy harvesting and MPT-based power sharing, given that the power targets for different BSs are equal to the same constant \(\eta_b\), it can be easily verified that for \(\mathbb{E}[S_j] \geq \eta_b\),

\[1 \geq \Pr\{S_j \geq \eta_b\} \geq \frac{\Var[S_j]}{1 - \frac{\Var[S_j]}{\eta_b - \mathbb{E}[S_j]^2}}
\]
where \(a\) and \(b\) follow from the Cantelli’s inequality [22] that
\[\Pr\{X - \mathbb{E}[X] \geq \mathcal{A}\} \leq \frac{1}{1 - \frac{\Var[X]}{\Var[X] + \mathcal{A}^2}}, \quad \text{if } \mathcal{A} > 0.
\]
This thus completes the proof of Lemma 4. \(\square\)

**APPENDIX E**

**PROOF OF LEMMA 4.3**

**Proof:** For cellular networks with energy harvesting and MPT-based power sharing, given that the power targets for different BSs are equal to the same constant \(\eta_b\), it can be easily verified from (10) and Lemma 4 that for \(\mathbb{E}[S_j] \geq \eta_b\),

\[1 \geq \varphi_b \geq \Pr\{S_j \geq \eta_b\} \geq 1 - \varepsilon.
\]
Further, based on the proof of Lemma 5, it can be easily verified that

\[
\Pr \{ S_j \geq \eta_b \} + \Pr \{ S_j < \eta_b \} = \frac{E [S_j]}{\eta_b} + \frac{E [S_j]}{\eta_b} \cdot (1 - \Pr \{ S_j \geq \eta_b \}) + \Pr \{ S_j \geq \eta_b \} = \frac{E [S_j]}{\eta_b} + \frac{1}{\eta_b} \cdot \Pr \{ S_j \geq \eta_b \} \leq \frac{E [S_j]}{\eta_b} + \frac{1}{\eta_b} \cdot \epsilon,
\]

where (a) follows from the fact that

\[
\int_0^{\eta_b} S_j \cdot f(S_j)dS_j = \mathbb{E} [S_j] - \int_{\eta_b}^{\infty} S_j \cdot f(S_j)dS_j \leq \mathbb{E} [S_j] - \mathbb{E} [S_j] \int_{\eta_b}^{\infty} f(S_j)dS_j = \mathbb{E} [S_j] \cdot \Pr \{ S_j < \eta_b \}.
\]

This thus completes the proof of Lemma 5. □

APPENDIX F

PROOF OF COROLLARY 4.1

Proof: Based on Lemmas 1 and 2, for \( \eta_b \leq \mathbb{E} [S_j] \), it can be easily verified that

\[
\lim_{\lambda \to \infty} \epsilon = \frac{\text{Var} [S_j]}{\mathbb{E} [S_j]} + \left( \eta_b - \mathbb{E} [S_j] \right)^2 = 0.
\]

Thus, with Theorem 1, for \( \eta_b \leq \mathbb{E} [S_j] \), we obtain that

\[
1 \geq \lim_{\lambda \to \infty} \tau \geq (1 - \epsilon) \cdot \left( 1 - e^{-\eta_b / \tau} \right) + e^{-\eta_b / \tau} = 1.
\]

On the other hand, for \( \eta_b > \mathbb{E} [S_j] \), based on Lemma 4, we have

\[
0 \leq \lim_{\lambda \to \infty} \Pr \{ S_j \geq \eta_b \} \leq \lim_{\lambda \to \infty} \epsilon = 0.
\]

As such, for \( \eta_b > \mathbb{E} [S_j] \), it follows that

\[
\lim_{\lambda \to \infty} \Pr \{ S_j < \eta_b \} = 1.
\]

Further, based on the proof of Lemma 5, it can be easily verified that

\[
\lim_{\lambda \to \infty} \Pr \{ P_j + S_j \geq \eta_b, P_j < \eta_b, S_j < \eta_b \} \leq \lim_{\lambda \to \infty} \mathbb{E} \left[ \frac{S_j}{\eta_b} - 1 \right| S_j < \eta_b] = \lim_{\lambda \to \infty} \mathbb{E} \left[ \frac{e^{\tau / \lambda} - 1}{e^{\tau / \lambda} - 1} \right| S_j < \eta_b] = \frac{e^{\mu / \tau} - 1}{e^{\mu / \tau} - 1}.
\]

where (a) follows from Lemma 2 that

\[
\lim_{\lambda \to \infty} \mathbb{E} [S_j] = \lim_{\lambda \to \infty} \mathbb{E} \left[ (S_j - \mathbb{E} [S_j])^2 \right] = \lim_{\lambda \to \infty} \mathbb{E} \left[ (S_j - \mu_j)^2 \right] = 0.
\]

As such, with (9), (10) and (37), we obtain that

\[
\lim_{\lambda \to \infty} \tau = \frac{\mu_j}{\tau} - 1 \left( 1 - e^{-\eta_b / \tau} \right) + e^{-\eta_b / \tau} = e^{-\eta_b / \tau} \cdot e^{\mu_j / \tau}.
\]

This thus completes the proof of Corollary 1. □

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