Novel time-domain asymptotic-numerical solution for forward transient scattered magnetic field from a coated metal cylinder

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Abstract A novel time-domain (TD) asymptotic-numerical solution (novel TD-ANS), which provides basic knowledge of engineering applications such as a nondestructive testing of reinforced concrete structures, is proposed for the forward transient scattered magnetic field when an ultra-wideband (UWB) pulse wave is incident on a metal cylinder covered with a homogeneous medium layer (coated metal cylinder). The TD-ANS is represented by multiple reflected and surface diffracted ray (multiple RSD) series solution in the TD. The TD-ANS is useful in understanding forward transient scattering phenomena because it can reproduce any part of a response waveform. The accuracy and validity of the TD-ANS are confirmed by comparing with the reference solution.

Keywords: time-domain asymptotic-numerical solution (TD-ANS), forward transient scattering, coated metal cylinder, ultra-wideband (UWB) pulse wave

Classification: Electromagnetic theory

1. Introduction

Keller has presented the geometrical theory of diffraction (GTD) [1] to analyze with high accuracy the high-frequency (HF) electromagnetic wave scattered fields from a conducting cylinder. Thereafter, the GTD [1] was widely used in the application area such as the design of antennas, the electromagnetic wave propagation, and the radar cross section (RCS) [2, 3]. However, it is difficult for Keller’s GTD to calculate the scattered field in the transition region (TR) adjacent to the shadow boundary (SB). Pathak [4] and Pathak et al. [5] proposed the uniform GTD (UTD) applicable in the TR including the SB. It was shown that the result calculated from the UTD agrees excellently with the exact solution calculated from the eigenfunction expansion [4, 5], the numerical solution calculated from the method of moment (MoM) [4, 5, 6], and the experimental results [5].

The UTD solution in [4, 5] has been extended to the one for the HF scattered fields from an impedance cylinder [7, 8]. Kim et al. [7] and Syed et al. [8] derived the UTD solutions for the impedance cylinder by applying the impedance boundary conditions [9]. Sasamori et al. extended from the UTD solution in [4, 5] to the one for the scattering from a dielectric cylinder [10]. Hussar et al. [11] has indicated that the UTD shadow region solution in [4, 5] becomes increasingly inaccurate with the increasing frequency. To overcome the difficulty, Hussar et al. [11], Ishihara et al. [12], and Ida et al. [13] proposed the improved uniform representations for the HF scattered field by a conducting cylinder.

Ida et al. [14, 15] have derived the frequency-domain (FD) uniform asymptotic solutions for the HF scattered fields from an impedance cylinder and a dielectric cylinder by extending the asymptotic solutions in [13]. It was shown that the FD extended UTD and the FD modified UTD solution derived by retaining the high-order terms in the integrals for the scattered fields can be applied in the deep shadow region in which the UTD solutions proposed in [4, 5, 7, 8, 9, 10] produce the substantial errors. Also, the time-domain (TD) extended UTD and the TD modified UTD solution for the transient scattered field were derived by assuming the widely used Gaussian-type modulated pulse source [14, 16] and by applying the saddle point technique [17] in evaluating the inverse Fourier transform [16].

The authors have pointed out that the UTD shadow region solution proposed in [7] produces substantial errors in the deep shadow region when an observation point is located in the nearby region relatively from the surface of a circular cylinder with thin lossy coatings. To circumvent this difficulty, we proposed the FD uniform asymptotic solution (FD-UAS) [18, 19]. By extending the FD-UAS, we derived the TD asymptotic-numerical solution (TD-ANS) [20].

The authors have also derived the FD extended UTD series solution [21, 22] for the scattered field by a coated conducting cylinder including the scattering phenomena inside of a coating medium. The FD extended UTD series solution is represented by a combination of a direct geometric optical ray (DGO) and a multiple reflected surface diffracted ray (multiple RSD) series [21, 22]. The FD extended UTD series solution is applicable in the TRs near the geometrical boundaries (GBs) and the deep shadow regions far away from the GBs. We developed the TD-ANS [23, 24] for the transient scattered field by extending the FD extended UTD series solution in [21, 22].

The transient electromagnetic wave scattering problem for a metal cylinder covered with a dielectric medium has recently received a lot of attention in the area of nondestructive testing which investigates the degree of the corrosion of rebar in steel reinforced concrete structures [25, 26]. Roqueta et al. [25] presented a nondestructive corrosion damage detection method for reinforced concrete structures based on the analysis of the electromagnetic signature of the steel rebar corrosion. Nishimoto et al. [26] analyzed transient

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DOI: 10.1587/elex.17.20200246
Received July 15, 2020
Accepted August 17, 2020
Publicized September 4, 2020
Copyedited September 25, 2020

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scattering by a metal cylinder covered with inhomogeneous lossy material approximated by a cylindrical multilayered medium. However, only a limited number of papers have been reported on the analysis and interpretation methods for forward transient scattering problems.

In this study, by developing the conventional FD extended UTD series solution in [21, 22], we propose a novel TD-ANS for the forward transient scattered magnetic field [27] when an ultra-wideband (UWB) pulse wave [28] is incident on a two-dimensional (2-D) metal cylinder covered with a homogeneous medium layer (2-D coated metal cylinder). The TD-ANS is useful in understanding forward transient scattering phenomena because it can reproduce any part of a response waveform. The accuracy and validity of the TD-ANS are confirmed by comparing with the reference solution arriving at the observation point P from a counterclockwise direction (n = 0) (a clockwise direction (n = −1)).

2.2 FD extended UTD series solution

The FD extended UTD series solution for the z-component of a forward scattered magnetic field \( H_z^r(\rho_0, \phi_0, \rho, \phi = \phi_0 + 180^\circ; \omega) \) is given by [22]:

\[
H_z^r(\omega) = \sum_{n=1}^{N_n} \sum_{p=0}^{N_p} \text{RSD}_p^n(\omega) (1)
\]

where \( \text{RSD}_p^n(\omega) \) represents the \( p \) times RSD solution arriving at the observation point \( P \) from a counterclockwise direction (\( n = 0 \)) and RSD solution arriving at the observation point \( P \) from a clockwise direction (\( n = −1 \)).

In the case of the forward scattering, a pair of RSD solutions are constructed so as to be strengthened each other. Therefore, the sum of RSD solutions is simplified as follows [27]

\[
\sum_{n=1}^{N_n} \sum_{p=0}^{N_p} \text{RSD}_p^n(\omega) = 2\text{RSD}_{p=0}^n(\omega) = 2\text{RSD}_{p=−1}^n(\omega). (2)
\]

Substituting the relational expression in (2) into the FD solution in (1) yields a new FD extended UTD series solution in (11) for a forward transient scattered magnetic field [27] taking 13 when numerical parameters in the caption of Fig. 3 are applied to a schematic model shown in Fig. 2.

HF asymptotic analyses for a forward scattered magnetic field in the FD have been discussed in [22] in detail. In Section 2.2, only the final result needed in the TD asymptotic-numerical analysis method in Section 3.2 is summarized.

2. Formulation and forward scattering in the FD

2.1 Formulation

Fig. 1 shows a 2-D coated metal cylinder with radius \( \rho = a \) covered with a homogeneous medium \( 2(\varepsilon_2, \mu_2) \) of thickness \( t(= a − b) \), coordinate systems \((x, y, z)\) and \((\rho, \phi)\), and magnetic line source \( Q(\rho_0, \phi_0) \) and observation point \( P(\rho, \phi, \phi = \phi_0 + 180^\circ) \), located in a surrounding medium \( 1(\varepsilon_1, \mu_1) \). Notation \( \varepsilon_2 \) denotes a complex permittivity of the medium and is given by \( \varepsilon_2 = \varepsilon_2 = \varepsilon + \text{i}\sigma/\omega \) where \( \sigma \) is a conductivity and \( \mu_0 \) is a permeability of free space. We assume that the layer of the metal medium is thick (\( t > \lambda_0 \)) as compared with the wavelength \( \lambda_0 \) of a central angular frequency in a pulse source function (see (5)).

Fig. 2 shows GBs (\( G_{n,p} \), \( n = 0, −1; p = 0, 1, 2, \ldots, N_n (= 13) \)) and forward scattering phenomena when a UWB pulse wave radiated from the source is incident on a coated metal cylinder from the counterclockwise (\( n = 0 \)) and clockwise directions (\( n = −1 \)), respectively. Notation \( p \) denotes the number of times of reflection on a metal cylinder defined by radius \( \rho = b \). While, integers \( N_n, n = 0, −1 \), express a truncation number of a multiple RSD series solution in (1) for a forward scattered magnetic field [22]. Each of the truncation numbers \( N_n, n = 0, −1 \), of a multiple RSD series solution in (11) for a forward transient scattered magnetic field [27] takes 13 when numerical parameters in the caption of Fig. 3 are applied to a schematic model shown in Fig. 2.
for the forward scattered magnetic field:

\[ H^z_5(\omega) \sim H^z_c, \text{ extended UTD series} (\omega) = 2 \sum_{p=0}^{N_{RSD}=0} \text{RSD}_p^{\text{ref}(\omega)}. \]  

(3)

The readers can find the explicit representation for the \( \text{RSD}_p^{\text{ref}(\omega)} \) in [22].

3. TD-ANS for forward transient scattering

3.1 Forward transient scattered magnetic field integral

The integral \( y(\rho_0, \phi_0, \rho, \phi = \phi_0 + 180^\circ; t) (\equiv y(t)) \) for a forward transient scattered magnetic field from a coated metal cylinder can be expressed by the inverse Fourier transform of the product of a FD forward scattered magnetic field \( H^z_5(\omega) \) and the frequency spectrum \( S(\omega) \) of a pulse source function \( s(t) \) [23, 24, 27]:

\[ y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H^z_5(\omega)S(\omega)\exp(-\text{i}\omega t) d\omega. \]  

(4)

We assume a truncated Gaussian-type modulated pulse source function \( s(t) \) [24, 27] defined by

\[ s(t) = \begin{cases} \exp\left[-\text{i}\omega_0(t-t_0) - \frac{(t-t_0)^2}{2(d^2)}\right] & \text{for } 0 \leq t \leq 2t_0 \\ 0 & \text{for } t < 0, t > 2t_0 \end{cases} \]  

(5)

where \( \omega_0 \) denotes a central angular frequency, and \( t_0 \) and \( d \) are constant parameters. The frequency spectrum \( S(\omega) \) of \( s(t) \) in (5) is given by

\[ S(\omega) = 2d\sqrt{\pi}\Re\left[\text{erf} \beta(\omega)\right] \exp[i\omega_0 t_0 - d^2(\omega - \omega_0)^2] \]  

(6)

\[ \beta(\omega) = \frac{t_0}{2d} - id(\omega - \omega_0) \]  

(7)

where the error function \( \text{erf} z \) [24, 27, 29] is defined as

\[ \text{erf} z = \frac{2}{\sqrt{\pi}} \int_{0}^{z} \exp(-t^2) dt. \]  

(8)

After substituting the FD exact solution \( H^z_5(\omega) \) obtained from the eigenfunction expansion [7] and the \( S(\omega) \) in (6) into the integral \( y(t) \) in (4), by applying the fast Fourier transform (FFT) numerical code [30] to the \( y(t) \), we can obtain a reference solution \( y_{\text{reference}}(t) \). The response waveform of \( y(t) \) is obtained from the real part of \( y(t) \), namely, \( \Re[y(t)] \).

In Section 4, the reference solution \( \Re[y_{\text{reference}}(t)] \) is used to confirm the accuracy, validity, and usefulness of the TD-ANS proposed in Section 3.2.

3.2 TD-ANS for forward transient scattered magnetic field

Replacing \( H^z_5(\omega) \) in (4) with \( H^z_c, \text{ extended UTD series} (\omega) \) in (3) gives the following concise expression [27]:

\[ y(t) \sim y_{\text{TD-ANS}}(t) = 2 \sum_{p=0}^{N_{RSD}=0} y_{\text{RSD}p}^{\text{ref}(t)} \]  

(9)

where \( y_{\text{RSD}p}^{\text{ref}(t)} \) denotes the \( p \) times RSD pulse element in the TD arriving from a counterclockwise direction \( n = 0 \) and is expressed by

\[ y_{\text{RSD}p}^{\text{ref}(t)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{RSD}_p^{\text{ref}(\omega)}S(\omega)\exp(-\text{i}\omega t) d\omega. \]  

(10)

The RSD pulse element \( y_{\text{RSD}p}^{\text{ref}(t)} \) represented by integral form in (10) is computable numerically by applying the FFT numerical code [30]. The TD-ANS for the forward transient scattered magnetic field can be derived by substituting the numerical result of \( y_{\text{RSD}p}^{\text{ref}(t)} \) into (9). Therefore, the interpretation method using the \( y_{\text{TD-ANS}}(t) \) can reproduce any part of the response waveform calculated from appropriate RSD pulse series elements.

4. Numerical results and discussions

In this section, we perform numerical calculations required to assess the accuracy, validity, and usefulness of the TD-ANS for the forward transient scattered magnetic field developed in Section 3.2 when a UWB pulse wave is incident on a coated metal cylinder.

Fig. 3 shows a typical response waveform (or wave packet) observed in the forward direction. Numerical parameters used in computations are given in the caption of Fig. 3 and the time \( t \) is set at \( t = 0[s] \) when the UWB pulse wave is radiated from the source \( Q \). In this case, the GB\(_0^{[20]} \), \( p = 0,1,2,\cdots,13 \), around a counterclockwise direction \( n = 0 \) are located at \( |\phi - \phi_0| = 109.8^\circ, 114.9^\circ, 120.1^\circ, \cdots, 176.7^\circ \), and the GB\(_{11}^{[20]} \), \( p = 0,1,2,\cdots,13 \), around a clockwise direction \( n = 1 \) are located at \( |\phi - \phi_0| = 250.2^\circ, 245.1^\circ, 239.9^\circ, \cdots, 183.3^\circ \), respectively.

As shown in Fig. 3, the wave packet \( \Re[y_{\text{TD-ANS}}(t)] \) (black solid curve) is calculated from

\[ \Re[y_{\text{TD-ANS}}(t)] = 2 \sum_{p=0}^{N_{RSD}=13} \Re[y_{\text{RSD}p}^{\text{ref}(t)}] \]  

(11)

agrees very well with the reference solution \( \Re[y_{\text{reference}}(t)] \) (red dashed curve) in the whole region. Hence the accuracy of the TD-ANS is confirmed.

Fig. 3 Comparison of the response waveform (or wave packet) calculated from the TD-ANS in (11) with the reference solution. Numerical parameters used in the calculation are \( a = 5.0m, t_0 = 1.5m \) \( (\omega_0 = 0.059a, \gamma = 2\pi/(\omega_0\sqrt{|\mu_0\sigma|})) \), \( e_1 = e_0, e_2^2 = e_2 + i\sigma/\omega_0, e_2 = 3e_0, \sigma = 8.50 \times 10^{-5} \text{S/m} \). Location of the source point \( Q(\rho_0, \phi_0) = (1.4a, 0.0^\circ) \) and that of the observation point \( P(\rho, \phi) = (2.4a, 180.0^\circ) \). Pulse source \( s(t) \) in (5) with \( \omega_0 = 9.6 \times 10^5 \text{rad/s}, \omega_0 = 4.0 \times 10^8, \) and \( \delta = 7.0 \times 10^{-30}s \) (fractional bandwidth of \( S(\omega) \) in (6) is 0.452).
when both numerical parameters \((p, N_{n=0})\) and \((p, N_{n=1})\) in (11) are set as \((0, 5)\), namely,

\[
\text{Re}[\text{YTD-ANS}(t)] = \sum_{n=1}^{\infty} \sum_{p=0}^{5} \text{Re}[\text{YRSDF}_p(t)].
\]  

(12)

The calculation in (12) reproduces the former part of the \(\text{Re}[\text{Reference}(t)]\) in a high precision. In Figs. 5(b) and 5(c), we also show the response waveforms in (12) (black solid curve) calculated from RSD pulse series elements when both numerical parameters \((p, N_{n=0})\) and \((p, N_{n=1})\) in (11) are set as \((5, 8)\) (Fig. 5(b)),

\[
\text{Re}[\text{YTD-ANS}(t)] = \sum_{n=1}^{\infty} \sum_{p=1}^{8} \text{Re}[\text{YRSDF}_p(t)].
\]  

(13)

and are set as \((8, 13)\) (Fig. 5(c)),

\[
\text{Re}[\text{YTD-ANS}(t)] = \sum_{n=1}^{\infty} \sum_{p=1}^{13} \text{Re}[\text{YRSDF}_p(t)].
\]  

(14)

respectively. The calculations in (13) and (14) accurately replicate the middle part of the \(\text{Re}[\text{Reference}(t)]\) (Fig. 5(b)) and the latter part of the \(\text{Re}[\text{Reference}(t)]\) (Fig. 5(c)), respectively.

The above-mentioned discussions provide the usefulness of the interpretation method using the TD-ANS because the TD-ANS can reproduce any part of the response waveform (or wave packet) calculated from appropriate RSD pulse series elements. The analysis and interpretation methods using the TD-ANS provide us with basic knowledge of engineering applications such as nondestructive testing of reinforced concrete structures.

5. Conclusion

We have proposed the novel TD-ANS for the forward transient scattered magnetic field when the UWB pulse wave is incident on a coated metal cylinder. We verified that the TD-ANS provides the analysis and interpretation methods for the forward transient scattering phenomena. By comparing with the reference solution, we confirmed the accuracy and validity of the TD-ANS.

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