Searching for the CP-Violation Associated with Majorana Neutrinos

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Abstract

The effective Majorana mass which determines the rate of the neutrinoless double beta ($\beta\beta^0$-decay, $|\langle m \rangle|$ , is considered in the case of three-neutrino mixing and massive Majorana neutrinos. Assuming a rather precise determination of the parameters characterizing the neutrino oscillation solutions of the solar and atmospheric neutrino problems has been made, we discuss the information a measurement of $|\langle m \rangle| \gtrsim (0.005-0.010)$ eV can provide on the value of the lightest neutrino mass and on the CP-violation in the lepton sector. The implications of combining a measurement of $|\langle m \rangle|$ with future measurement of the neutrino mass $m_{\nu_e}$ in $^3$H $\beta-$decay experiments for the possible determination of leptonic CP-violation are emphasized.
1 Introduction

Experiments on atmospheric and solar neutrinos have produced convincing evidence of neutrino oscillations [1, 2, 3, 4, 5, 6, 7]. Ongoing and planned experiments, including long baseline ones, aim to determine the parameters for these oscillations. Assuming mixing of only three neutrinos, these are the magnitudes of the elements of the $3 \times 3$ unitary lepton mixing matrix - the Maki-Nakagawa-Sakata-Pontecorvo (MNSP) mixing matrix - a CP-violating phase, and the mass-squared difference parameters, say, $\Delta m^2_{31}$ and $\Delta m^2_{21}$. In principle, long baseline experiments at neutrino factories can distinguish the alternatives of (i) hierarchical neutrino mass spectrum and of (ii) neutrino mass spectrum with inverted hierarchy [11]. If we number (without loss of generality) the neutrinos with definite mass in such a way that $m_1 < m_2 < m_3$, case (i) corresponds to $\Delta m^2_{31} \equiv \Delta m^2_{atm} \gg \Delta m^2_{21} \equiv \Delta m^2_{sol}$, while in case (ii) we have $\Delta m^2_{31} \equiv \Delta m^2_{atm} \gg \Delta m^2_{21} \equiv \Delta m^2_{sol}$, where $\Delta m^2_{atm}$ and $\Delta m^2_{sol}$ are the values of the neutrino mass-squared differences inferred from the atmospheric and solar neutrino data. These experiments cannot determine, however, the actual neutrino masses, that is, the value of the lightest neutrino mass $m_1$. Furthermore, assuming the massive neutrinos are Majorana particles, as we will in this paper, there are two more parameters, two Majorana CP-violating phases, associated with the MNSP mixing matrix [12] (see also [13]).

The neutrino oscillation experiments cannot provide information on the Majorana CP-violating phases [12, 14] as well. This paper is concerned with the prospects and problems in determining or constraining these three parameters, assuming the others have been well determined. The mass $m_1$ is of interest, e.g., in cosmology since massive neutrinos at present are the only non-baryonic dark matter constituents known. Knowing the neutrino mass spectrum is fundamental for understanding the origin of the neutrino masses and mixing. The Majorana CP-violating phases indicate the relation between CP violation and lepton number violation; a major goal is to identify any possibility of detecting this CP-violation.

2 Neutrinoless Double $\beta$-Decay and $^3$H $\beta$-Decay Experiments

The process most sensitive to the existence of massive Majorana neutrinos (coupled to the electron in the weak charged lepton current) is the neutrinoless double beta ($((\beta\beta)_{0\nu})$) decay (see, e.g., [15, 16]). If the $((\beta\beta)_{0\nu})$ decay is generated only by the left-handed (LH) charged current weak interaction through the exchange of virtual massive Majorana neutrinos, the probability amplitude of this process is proportional in the case of Majorana neutrinos having masses not exceeding a few MeV to the so-called “effective Majorana mass parameter”, $|<m>|$ (see, e.g., [17]). A large number of experiments are searching for $(\beta\beta)_{0\nu}$-decay of different nuclei at present (a rather complete list is given in [16]). No indications that $(\beta\beta)_{0\nu}$-decay takes place have been found so far. A stringent constraint on the value of the effective Majorana mass $|<m>|$ was obtained in the $^{76}$Ge Heidelberg-Moscow experiment [18]:

$$|<m>| < 0.35 \text{ eV}, \quad 90\% \text{ C.L.}$$

(1)

Taking into account a factor of 3 uncertainty associated with the calculation of the relevant nuclear matrix element (see, e.g., [17, 16]) we get

$$|<m>| < (0.35 \div 1.05) \text{ eV}, \quad 90\% \text{ C.L.}$$

(2)

The IGEX collaboration has obtained [19]:

$$|<m>| < (0.33 \div 1.35) \text{ eV}, \quad 90\% \text{ C.L.}$$

(3)

A sensitivity to $|<m>| \sim 0.10 \text{ eV}$ is foreseen to be reached in the currently operating NEMO3 experiment [20], while the next generation of $((\beta\beta)_{0\nu})$-decay experiments CUORE, EXO, GENIUS, MOON [21, 22, 23, 24], aim at reaching a sensitivity to values of $|<m>| \sim 0.01 \text{ eV}$, which are considerably smaller than the presently existing most stringent upper bounds (2) and (3).
The results of the $^3$H $\beta$-decay experiments studying the electron spectrum, which measure the electron (anti-)neutrino mass $m_{\nu_e}$, are of fundamental importance, in particular, for getting information about the neutrino mass spectrum. The Troitzk $^{25}$ and Mainz $^{26}$ experiments have provided stringent upper bounds on $m_{\nu_e}$:

$$m_{\nu_e} < 2.5 \text{ eV} \quad [25], \quad m_{\nu_e} < 2.9 \text{ eV} \quad [29] \quad (95\% \text{ C.L.}).$$

There are prospects to increase substantially the sensitivity of the $^3$H $\beta$-decay experiments and probe the region of values of $m_{\nu_e}$ down to $m_{\nu_e} \sim (0.3 - 0.4) \text{ eV} \quad [27] \quad \text{(the KATRIN project).}$

It is difficult to overestimate the importance of the indicated future $(\beta\beta)_{0\nu}$-decay and $^3$H $\beta$-decay experiments for the studies of the neutrino mixing: these are the only feasible experiments which can provide information on the neutrino mass spectrum and on the nature of massive neutrinos. Such information cannot be obtained $^{[2]} \quad [12, 14]$, as we have indicated, in the experiments studying neutrino oscillations. The measurement of $|<m>| \gtrsim 0.02 \text{ eV}$ and/or of $m_{\nu_e} \gtrsim 0.4 \text{ eV}$ can give information, in particular, on the type of neutrino mass spectrum $^{[30, 31, 32]}$. As we will discuss, it is only by combining a value of $|<m>|$ and a value of, or a sufficiently stringent upper limit on, $m_{\nu_e}$ one might hope to detect Majorana CP-violation.

### 3 A Brief Summary of the Formalism

As it is well known, the explanation of the atmospheric and solar neutrino data in terms of neutrino oscillations requires the existence of 3-neutrino mixing in the weak charged lepton current:

$$\nu_{\ell L} = \sum_{j=1}^{3} U_{\ell j} \nu_{j L}, \quad \text{(5)}$$

where $\nu_{\ell L}$, $\ell = e, \mu, \tau$, are the three left-handed flavour neutrino fields, $\nu_{j L}$ is the left-handed field of the neutrino $\nu_j$ having a mass $m_j$ and $U$ is the MNSP neutrino mixing matrix $^{[8, 9]}$. If $\nu_j$ are Majorana neutrinos with masses not exceeding few MeV, as will be assumed in what follows, the effective Majorana mass $|<m>|$ of interest can be expressed in the form

$$|<m>| = |m_1| |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i\alpha_{31}} \quad \text{(6)}$$

where $\alpha_{21}$ and $\alpha_{31}$ are the two Majorana CP-violating phases $^{[9]} \quad [12]$ (see also $^{[13]}$). If CP-invariance holds, one has $^{[33, 34, 35]} \quad \alpha_{21} = k\pi, \quad \alpha_{31} = k'\pi, \quad k, k' = 0, 1, 2, ...$. In this case

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1, \quad \text{(7)}$$

represent the relative CP-parities of the neutrinos $\nu_1$ and $\nu_2$, and $\nu_1$ and $\nu_3$, respectively.

The quantities relevant for eq. (1) to be determined in neutrino oscillation experiments in the case of three-neutrino mixing are $\Delta m^2_{\text{atm}}, \Delta m^2_{\odot}$, the mixing angle, $\theta_\odot$, constrained by the solar neutrino data, and the mixing angle, $\theta$, determined from the probability that the atmospheric neutrino oscillations involve $\nu_e$. At present $\theta$ is limited by the data from the CHOOZ $^{[36]}$ and Palo Verde $^{[37]}$ experiments, but in the future it should be determined, e.g., in long baseline neutrino oscillation experiments $^{[38, 11, 33]}$.

We can number (without loss of generality) the neutrino masses in such a way that $m_1 < m_2 < m_3$. The neutrino masses $m_{2,3}$ can be expressed in terms of the lightest neutrino mass $m_1$ and, e.g., $\sqrt{\Delta m^2_{21}}$ and $\sqrt{\Delta m^2_{32}}$ (see, e.g., $^{[10, 11, 12]}$):

$$m_2 = \sqrt{m_1^2 + \Delta m^2_{21}}, \quad \text{(8)}$$

---

$^2$Cosmological and astrophysical data provide information on the sum of the neutrino masses. The current upper bound reads (see, e.g., $^{[28]}$ and the references quoted therein): $\sum_j m_j \lesssim 5.5 \text{ eV}$. The future experiments MAP and PLANCK may be sensitive to $^{[29]} \sum_j m_j \approx 0.4 \text{ eV}$.

$^3$We assume that the fields of the Majorana neutrinos $\nu_j$ satisfy the Majorana condition: $C(\nu_j)^T = \nu_j$, $j = 1, 2, 3$, where $C$ is the charge conjugation matrix.
\[ m_3 = \sqrt{m_1^2 + \Delta m_{21}^2 + \Delta m_{32}^2} \, . \] (9)

For \( \Delta m_{\text{atm}}^2 \) inferred from the neutrino oscillation interpretation of the atmospheric neutrino data we have:

\[ \Delta m_{\text{atm}}^2 = \Delta m_{31}^2 = \Delta m_{21}^2 + \Delta m_{32}^2 \, , \] (10)

In the case of normal neutrino mass hierarchy,

\[ \Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \, , \] (11)

and

\[ |U_{e1}| = \cos \theta_{\odot} \sqrt{1 - |U_{e3}|^2} \, , \quad |U_{e2}| = \sin \theta_{\odot} \sqrt{1 - |U_{e3}|^2} \, , \quad |U_{e3}|^2 = \sin^2 \theta. \] (12)

For the inverted neutrino mass hierarchy one has [43]:

\[ \Delta m_{\odot}^2 \equiv \Delta m_{32}^2 \, , \] (13)

and

\[ |U_{e2}| = \cos \theta_{\odot} \sqrt{1 - |U_{e1}|^2} \, , \quad |U_{e3}| = \sin \theta_{\odot} \sqrt{1 - |U_{e1}|^2} \, , \quad |U_{e1}|^2 = \sin^2 \theta. \] (14)

In our analysis we will consider values of \( m_1 \) varying from 0 to 2.9 eV - the upper limit from the \(^3\text{H} \beta\)-decay data, eq. (4). As \( m_1 \) increases from 0, the three neutrino masses get closer in magnitude. For \( m_1 > 0.2 \) eV, the neutrino masses are quasi-degenerate and the differences between the cases of hierarchical spectrum and the spectrum with inverted hierarchy essentially disappear.

Given the values of \( \Delta m_{\odot}^2 \), \( \theta_{\odot} \), \( \Delta m_{\text{atm}}^2 \) and of \( \theta \), the effective Majorana mass \( \langle m \rangle \) depends, in general, on three parameters: the lightest neutrino mass \( m_1 \) and on the two CP-violating phases \( \alpha_{21} \) and \( \alpha_{31} \). It depends also on the “discrete ambiguity” expressed in eqs. (11) - (14) and related to the two possible types of neutrino mass spectrum - the hierarchical and that with inverted hierarchy. As is obvious from eqs. (8) - (11) and (13), the knowledge of \( m_1 \) would allow to determine the neutrino mass spectrum.

In the discussion which follows we use the best fit value for \( \Delta m_{\text{atm}}^2 \), obtained in the analysis of the atmospheric neutrino data in [43].

\[ (\Delta m_{\text{atm}}^2)_{BFV} = 2.5 \times 10^{-3} \, \text{eV}^2 \, . \] (15)

In what regards the parameters \( \Delta m_{\odot}^2 \) and \( \theta_{\odot} \), in most of the discussion we assume they lie in the region of the large mixing angle (LMA) MSW solution of the solar neutrino problem, although we comment briefly on how our conclusions would change in the cases of the LOW - quasi-vacuum oscillation (LOW-QVO) solution and of the small mixing angle (SMA) MSW solution. The most recent analyses [46, 47, 48, 49] show that the current solar neutrino data, including the SNO results, favor the LMA MSW and the LOW-QVO solutions. To illustrate our discussion and conclusions we use the best fit value of \( \Delta m_{\odot}^2 \) found in [47],

\[ (\Delta m_{\odot}^2)_{BFV} = 4.5 \times 10^{-5} \, \text{eV}^2 \, , \] (16)

three values of \( \cos 2\theta_{\odot} \) from the LMA solution region [49] and two values of the mixing angle \( \theta \), constrained by the CHOOZ and Palo Verde data.

\(^4\)For the values of \( \Delta m_{\text{atm}}^2 \) obtained in [55], one has neutrino mass spectrum with hierarchy (with partial hierarchy) or with inverted hierarchy (partial inverted hierarchy) for [4] \( m_1 \ll 0.02 \) eV (0.02 eV \( \leq m_1 \leq 0.2 \) eV).

\(^5\)In our further discussion we assume \( \cos 2\theta_{\odot} \geq 0 \), which is favored by the analyses of the solar neutrino data [41, 47, 48, 49]. The modification of the relevant formulae and of the results in the case \( \cos 2\theta_{\odot} < 0 \) is rather straightforward.
4 Constraining or Determining the Lightest Neutrino Mass $m_1$

and/or the Majorana CP-Violating Phases

If the $\beta\beta$-decay of a given nucleus will be observed, it would be possible to determine the value of $|<m>|$ from the measurement of the associated life-time of the decay. This would require the knowledge of the nuclear matrix element of the process. At present there exist large uncertainties in the calculation of the $\beta\beta$-decay nuclear matrix elements (see, e.g., [3], [4]). This is reflected, in particular, in the factor of $\sim (2 - 3)$ uncertainty in the upper limit on $|<m>|$, which is extracted from the experimental lower limits on the $\beta\beta$-decay half life-time of $^{76}\text{Ge}$. The observation of a $\beta\beta$-decay of one nucleus is likely to lead to the searches and eventually to observation of the decay of other nuclei. One can expect that such a progress, in particular, will help to solve completely the problem of the sufficiently precise calculation of the nuclear matrix elements for the $\beta\beta$-decay. Taking the optimistic point of view that the indicated problem will be resolved in one way or another, we will not discuss in what follows the possible effects of the currently existing uncertainties in the evaluation of the $\beta\beta$-decay nuclear matrix elements on the results of our analysis.

In this Section we consider the information that future $\beta\beta$-decay and/or $^3\text{H}$ $\beta$-decay experiments can provide on the lightest neutrino mass $m_1$ and on the CP-violation generated by the two Majorana CP-violating phases $\alpha_{21}$ and $\alpha_{31}$. The results are summarized in Fig. 1 (normal neutrino mass hierarchy) and in Fig. 2 (inverted hierarchy).

We shall discuss first the case of $\Delta m^2_{\text{atm}} \equiv \Delta m^2_{21}$ (eqs. (11) - (12)).

4.1 Normal Mass Hierarchy: $\Delta m^2_{\odot} \equiv \Delta m^2_{21}$

If $\Delta m^2_{\odot} = \Delta m^2_{21}$, for any given solution of the solar neutrino problem LMA MSW, LOW-QVO, SMA MSW, as can be shown, $|<m>|$ can lie anywhere between 0 and the present upper limits, given by eqs. (3) and (4). This conclusion does not change even under the most favorable conditions for the determination of $|<m>|$, namely, even when $\Delta m^2_{\text{atm}}$, $\Delta m^2_{\odot}$, $\theta_{\odot}$ and $\theta$ are known with negligible uncertainty, as Fig. 1 indicates. The further conclusions that are illustrated in Fig. 1 are now summarized. We consider the case of the LMA MSW solution of the solar neutrino problem.

Case A. An experimental upper limit on $|<m>|$, $|<m>| < |<m>|_{\exp}$, will determine a maximal value of $m_1$, $m_1 < (m_1)_{\max}$. The latter is fixed by the equality:

$$
(m_1)_{\max} : (m_1 \cos^2 \theta_{\odot} - \sqrt{m_1^2 + \Delta m^2_{\odot} \sin^2 \theta_{\odot}})(1 - |U_{e3}|^2) \pm \sqrt{m_1^2 + \Delta m^2_{\text{atm}} |U_{e3}|^2} = |<m>|_{\exp}.
$$

(17)

Given $m_1 \neq 0$ and $\Delta m^2_{\odot}$, the sign of the last term in the left-hand side of the inequality depends on the value of $\cos 2\theta_{\odot}$: the positive sign corresponds to $\cos 2\theta_{\odot} < \Delta m^2_{\odot} \sin^2 \theta_{\odot}/m_1^2$ (i.e., to $\cos 2\theta_{\odot} \geq 0$), while the negative sign is valid for $\cos 2\theta_{\odot} > \Delta m^2_{\odot} \sin^2 \theta_{\odot}/m_1^2$.

For the quasi-degenerate neutrino mass spectrum one has $m_1 \gg \Delta m^2_{\odot}, \Delta m^2_{\text{atm}}, m_1 \approx m_2 \approx m_3 \approx m_{\nu_e}$, and up to corrections $\sim \Delta m^2_{\odot} \sin^2 \theta_{\odot}/(2m_1^2)$ and $\sim \Delta m^2_{\text{atm}} |U_{e3}|^2/(2m_1^2)$ one finds:

$$
(m_1)_{\max} \approx \frac{|<m>|_{\exp}}{\cos 2\theta_{\odot}(1 - |U_{e3}|^2) - |U_{e3}|^2}.
$$

(18)

If $|\cos 2\theta_{\odot}(1 - |U_{e3}|^2) - |U_{e3}|^2|$ is sufficiently small, the upper limit on $m_{\nu_e}$ obtained in $^3\text{H}$ $\beta$-decay experiments could yield a more stringent upper bound on $m_1$ than the bound following from the limit on $|<m>|$.

Case B. A measurement of $|<m>| = (|<m>|)_{\exp} \geq 0.02$ eV would imply that $m_1 \geq 0.02$ eV and thus a neutrino mass spectrum with partial hierarchy or of quasi-degenerate type [31]. The
lightest neutrino mass will be constrained to lie in the interval, \((m_1)_{\text{min}} \leq m_1 \leq (m_1)_{\text{max}}\), where \((m_1)_{\text{max}}\) and \((m_1)_{\text{min}}\) are determined respectively by eq. (17) and by the equation:

\[
(m_1)_{\text{min}} : \quad (m_1 \cos^2 \theta_\odot + \sqrt{m_1^2 + \Delta m_\odot^2} \sin^2 \theta_\odot)(1 - |U_{e3}|^2) + \sqrt{m_1^2 + \Delta m_{\text{atm}}^2} |U_{e3}|^2 = (|<m>|)_{\text{exp}}.
\]  

(19)

The limiting values of \(m_1\) correspond to the case of CP-conservation. For \(\Delta m_\odot^2 \ll m_1^2\), (i.e., for \(\Delta m_\odot^2 \lesssim 10^{-4} \text{ eV}^2\), \((m_1)_{\text{min}}\) to a good approximation is independent of \(\theta_\odot\), and for \(\sqrt{\Delta m_{\text{atm}}^2 |U_{e3}|^2} \ll m_1\), which takes place in the case we consider as \(|U_{e3}|^2 \lesssim 0.05\), we have \((m_1)_{\text{min}} \equiv (|<m>|)_{\text{exp}}\). For \(|U_{e3}|^2 \ll \cos 2\theta_\odot\), which is realized in the illustrative cases in Fig. 1 for \(|U_{e3}|^2 \lesssim 0.01\), practically all of the region between \((m_1)_{\text{min}}\) and \((m_1)_{\text{max}}\), \((m_1)_{\text{min}} < m_1 < (m_1)_{\text{max}}\), corresponds to violation of the CP-symmetry. If \(|U_{e3}|^2\) is non-negligible with respect to \(\cos 2\theta_\odot\), e.g., if \(|U_{e3}|^2 \approx (0.02 - 0.05)\) for the values of \(\cos 2\theta_\odot\) used to derive the right panels in Fig. 1, one can have \((m_1)_{\text{min}} < m_1 < (m_1)_{\text{max}}\) if CP-symmetry is violated, as well as in two specific cases of CP-conservation. One of these two CP-conserving values of \(m_1\), corresponding to \(\eta_2 \equiv -\eta_3 \equiv 1\), can differ considerably from the two limiting values (see Fig. 1).

A measured value of \(|<m>\) satisfying \((|<m>|)_{\text{exp}} < (|<m>|)_{\text{max}}\), where \((|<m>|)_{\text{max}} \equiv m_1 \equiv m_\nu_{\text{e}}\), in the case of a quasi-degenerate neutrino mass spectrum, and \((|<m>|)_{\text{max}} \equiv (\sqrt{\Delta m_\odot^2 \sin^2 \theta_\odot})(1 - |U_{e3}|^2) + \sqrt{\Delta m_{\text{atm}}^2 |U_{e3}|^2}\) if the spectrum is hierarchical (i.e., if \(m_1 \ll m_2 \ll m_3\)), would imply that at least one of the two CP-violating phases is different from zero: \(\alpha_2 \neq 0\) or \(\alpha_3 \neq 0\); in the case of a hierarchical spectrum that would also imply \(\alpha_2 \neq \alpha_3\). In general, the knowledge of the value of \(|<m>|\) alone will not allow to distinguish the case of CP-conservation from that of CP-violation.

Case C. It might be possible to determine whether CP-violation due to the Majorana phases takes place in the lepton sector if both \(|<m>|\) and \(m_\nu_{\text{e}}\) are measured. Since prospective measurements are limited to \((m_\nu_{\text{e}})_{\text{exp}} \gtrsim 0.35 \text{ eV}\), the relevant neutrino mass spectrum is of quasi-degenerate type (see, e.g., [31]). In this case one has \(m_1 > 0.35 \text{ eV}, m_1 \equiv m_2 \equiv m_3 \equiv m_\nu_{\text{e}}\). And

\[
|<m>| \sim m_\nu_{\text{e}} \left| \cos^2 \theta_\odot (1 - |U_{e3}|^2) + \sin^2 \theta_\odot (1 - |U_{e3}|^2) \right| e^{i\alpha_2 - |U_{e3}|^2 e^{i\alpha_3}}.
\]  

(20)

If we can neglect \(|U_{e3}|^2\) in eq. (20) (i.e., if \(\cos 2\theta_\odot \gg |U_{e3}|^2\), a value of \(m_\nu_{\text{e}} \equiv m_1\), satisfying \((m_1)_{\text{min}} < m_\nu_{\text{e}} < (m_1)_{\text{max}}\), where \((m_1)_{\text{min}}\) and \((m_1)_{\text{max}}\) are determined by eqs. (19) and (17), would imply that the CP-symmetry does not hold in the lepton sector. In this case one would obtain correlated constraints on the CP-violating phases \(\alpha_2\) and \(\alpha_3\) [31, 33]. This appears to be the only possibility for demonstrating CP-violation due to Majorana CP-violating phases in the case of \(\Delta m_\odot^2 \equiv \Delta m_{\text{atm}}^2\) under discussion. In order to reach a definite conclusion concerning CP-violation due to the Majorana CP-violating phases, considerable accuracy in the measured values of \(|<m>|\) and \(m_\nu_{\text{e}}\) is required. For example, if the oscillation experiments give the result \(\cos 2\theta_\odot \leq 0.3\) and \(|<m>| = 0.3 \text{ eV}\), a value of \(m_\nu_{\text{e}}\) between 0.3 eV and 1.0 eV would demonstrate CP-violation. However, this requires better than 30\% accuracy on both measurements. The accuracy requirements become less stringent if the upper limit on \(\cos 2\theta_\odot\) is smaller.

If \(\cos 2\theta_\odot > |U_{e3}|^2\) but \(|U_{e3}|^2\) cannot be neglected in (20), there exist two CP-conserving values of \(m_\nu_{\text{e}}\) in the interval \((m_1)_{\text{min}} < m_\nu_{\text{e}} < (m_1)_{\text{max}}\). The one that can significantly differ from the extreme values of the interval corresponds to a specific case of CP-conservation - to \(\eta_2 = -\eta_3 = -1\) (Fig. 1).

Case D. A measured value of \(m_\nu_{\text{e}}, (m_\nu_{\text{e}})_{\text{exp}} \gtrsim 0.35 \text{ eV}, (m_\nu_{\text{e}})_{\text{exp}} > (m_1)_{\text{max}}\), where \((m_1)_{\text{max}}\) is determined from the upper limit on \(|<m>|\), eq. (17), in the case the \((\beta\beta)_{0\nu}\)-decay is not observed, might imply that the massive neutrinos are Dirac particles. If \((\beta\beta)_{0\nu}\)-decay has been observed and \(|<m>|\) measured, the inequality \((m_\nu_{\text{e}})_{\text{exp}} > (m_1)_{\text{max}}\), with \((m_1)_{\text{max}}\) determined from the upper limit or the value of \(|<m>|\), eq. (17), would lead to the conclusion that there exist
contribution(s) to the \((\beta\beta)_{0v}\)-decay rate other than due to the light Majorana neutrino exchange (see, e.g., [2] and the references quoted therein) that partially cancels the contribution from the Majorana neutrino exchange.

A measured value of \(|<m>|\), \(|<m>|_{\text{exp}} \gtrsim 0.01 \text{ eV}\), and a measured value of \(m_{\nu e}\) or an upper bound on \(m_{\nu e}\) such that \(m_{\nu e} < (m_1)_{\text{min}}\), where \((m_1)_{\text{min}}\) is determined by eq. (19), would imply that there are contributions to the \((\beta\beta)_{0v}\)-decay rate in addition to the ones due to the light Majorana neutrino exchange (see, e.g., [2]), which enhance the \((\beta\beta)_{0v}\)-decay rate and signal the existence of new \(\Delta L = 2\) processes beyond those induced by the light Majorana neutrino exchange in the case of left-handed charged current weak interaction.

**Case E.** An actual measurement of \(|<m>|\) \(\lesssim 10^{-2} \text{ eV}\) is unlikely, but it is illustrated in Fig. 1 to show the interpretation of such a result. There always remains an upper limit on \(m_1\). As \(|<m>|\) decreases, there appears a finite lower limit on \(m_1\) as well. Both the upper and the lower limits on \(m_1\) approach asymptotic values which depend on the values of \(\Delta m^2_{\odot}\), \(\Delta m^2_{\odot}\), \(\cos 2\theta_{\odot}\), and \(|U_{e3}|^2\), but are independent of \(|<m>|\) (Fig. 1). For \(\cos 2\theta_{\odot} > 2|U_{e3}|^2\), the maximum and minimum asymptotic values of \(m_1\) are determined by the expressions:

\[
m_1^{(+)} = \left( -\eta_{31} \sqrt{\Delta m^2_{\odot} |U_{e3}|^2 \cos^2 \theta_{\odot}} \right. \\
\left. \pm \sqrt{\Delta m^2_{\odot} |U_{e3}|^4 \cos^4 \theta_{\odot} - (\Delta m^2_{\odot} |U_{e3}|^4 - \Delta m^2_{\odot} \sin^4 \theta_{\odot} \cos 2\theta_{\odot}) \cos^{-1} 2\theta_{\odot} \right).
\]  \hspace{1cm} (21)

For the maximum asymptotic value we have \((m_1)_{\text{max}} = m_1^{(+)}\) with \(\eta_{31} = -1\). If further \(\Delta m^2_{\odot} |U_{e3}|^4 \times \cos^4 \theta_{\odot} \gg |(\Delta m^2_{\odot} |U_{e3}|^4 - \Delta m^2_{\odot} \sin^4 \theta_{\odot} \cos 2\theta_{\odot})|\) which requires \(|U_{e3}|^2 \approx (0.02 - 0.05)\), the expression for the asymptotic value of interest is given approximately by \((m_1)_{\text{max}} \approx 2 \sqrt{\Delta m^2_{\odot} |U_{e3}|^2} \times \cos^2 \theta_{\odot}/\cos 2\theta_{\odot}\) and is typically in the range \((m_1)_{\text{max}} \approx (0.7 - 3.0) \times 10^{-2} \text{ eV}\) (Fig. 1, right panels). If, however, \(\Delta m^2_{\odot} \sin^4 \theta_{\odot} \gg \max(\Delta m^2_{\odot} |U_{e3}|^4, \Delta m^2_{\odot} |U_{e3}|^4 \cos^4 \theta_{\odot}/\cos 2\theta_{\odot})\), one typically finds: \((m_1)_{\text{max}} \approx (0.3 - 1.0) \times 10^{-2} \text{ eV}\) (Fig. 1, left panels).

For the minimum asymptotic value of \(m_1\) we have \((m_1)_{\text{min}} = m_1^{(-)}\) with \(\eta_{31} = 1\) if \(\Delta m^2_{\odot} \sin^4 \theta_{\odot} > \Delta m^2_{\odot} |U_{e3}|^4\), and \((m_1)_{\text{min}} = m_1^{(-)}\) with \(\eta_{31} = -1\) if \(\Delta m^2_{\odot} \sin^4 \theta_{\odot} < \Delta m^2_{\odot} |U_{e3}|^4\).

Over certain interval of values of \(|<m>|\), which depends on \(|U_{e3}|^2\), on the values of the difference of the Majorana CP-violating phases, \((\alpha_{31} - \alpha_{21})\), and on \(\cos 2\theta_{\odot}\), the lower limit on \(m_1\) goes to zero, as is shown in Fig. 1. This interval, \(|<m>|_{-} \leq |<m>| \leq |<m>|_{+}\), is given by \(|<m>|_{-} = \left| \sqrt{\Delta m^2_{\odot} \sin^2 \theta_{\odot} (1 - |U_{e3}|^2)} \right| \pm \left| \sqrt{\Delta m^2_{\odot} |U_{e3}|^2} \right|\), and has a width of \(2 \sqrt{\Delta m^2_{\odot} |U_{e3}|^2}\). For a given \(|<m>|\) from the indicated interval we have \(0 \leq m_1 \leq (m_1)_{\text{max}}\), with \((m_1)_{\text{max}}\) determined by eq. (17). Further, the limiting value of \(m_1 = (m_1)_{\text{max}}\), as well as at least one and up to three internal values of \(m_1\) from the interval \(0 < m_1 < (m_1)_{\text{max}}\) in the simplified case we are analyzing are CP-conserving (Fig. 1). The remaining values of \(m_1\) from the interval \(0 < m_1 < (m_1)_{\text{max}}\) are CP-violating.

It should be noted also that one can have \(|<m>| = 0\) for \(m_1 = 0\) in the case of CP-invariance if \(\eta_{21} = -\eta_{31}\) and the relation \(\sqrt{\Delta m^2_{\odot} \sin^2 \theta_{\odot} (1 - |U_{e3}|^2)} = \sqrt{\Delta m^2_{\odot} |U_{e3}|^2}\) holds. Finally, there would seem to be no practical possibility to determine the Majorana CP-violating phases.

The analysis of the Cases A - E for the LOW-QVO solution of the solar neutrino problem leads to the same qualitative conclusions as those obtained above for the LMA MSW solution. The conclusions differ, however, in the case of the SMA MSW solution and we will discuss them next briefly. An experimental upper limit on \(|<m>|\) (Case A) in the range \(|<m>|_{\text{exp}} \gtrsim 10^{-2} \text{ eV}\), would imply in the case of the SMA MSW solution, \(m_1 < |<m>|_{\text{exp}} (1 - 2|U_{e3}|^2)^{-1}\). For values of \(|<m>|_{\text{min}} \gtrsim 10^{-2} \text{ eV}\), the maximum and minimum values of \(m_1\) are extremely close: \((m_1)_{\text{min}} \approx |<m>|_{\text{exp}}\). As a result, a measurement of \(|<m>|\) (Case B) practically determines
the values of the relevant input parameters $\Delta m^2$, $\theta$, and $\sin^2\theta = |U_{e1}|^2$ can be obtained by the measurement of $|\Delta m^2|$. If both $|\Delta m^2| > \cos 2\theta > 0.02$ eV and $m_{\nu_e} > 0.35$ eV would be measured (Case C), the relation $m_1 \equiv (|\Delta m^2|)_{\text{exp}} \approx (m_{\nu_e})_{\text{exp}}$ should hold. The conclusions in the Cases D and E are qualitatively the same as for the LMA MSW solution.

4.2 Inverted Mass Hierarchy: $\Delta m^2_{\odot} \equiv \Delta m^2_{32}$

Consider next the possibility of a neutrino mass spectrum with inverted hierarchy, which is illustrated in Fig. 2. A comparison of Fig. 1 and Fig. 2 reveals two major differences in the predictions for $|\Delta m^2|$: i) even in the case of $m_1 < m_2 \equiv m_3$ (i.e., even if $m_1 < 0.02$ eV), $|\Delta m^2|$ can exceed $\sim 10^{-2}$ eV and can reach the value of $\sim 0.08$ eV [31, 33], and ii) a more precise determination of $\Delta m^2_{\text{atm}}$, $\theta$, and $\sin^2\theta$ can lead to a lower limit on the possible values of $|<m>|$. For the LMA and the LOW-QVO solutions, min$(|<m>|)$ will depend, in particular, on whether CP-invariance holds or not in the lepton sector, and if it holds - on the relative CP-parities of the massive Majorana neutrinos. All these possibilities are parametrized by the values of the two CP-violating phases, $\alpha_21$ and $\alpha_{31}$, entering into the expression for $|<m>|$.

The existence of a significant lower limit on the possible values of $|<m>|$ depends crucially in the cases of the LMA and LOW-QVO solutions on the minimal value of $|\cos 2\theta|$, allowed by the data: up to corrections $\sim 5 \times 10^{-3}$ eV we have for these two solutions (see, e.g., [13, 20, 31]):

\[
\text{LMA, LOW-QVO: } \min(|<m>|)_{\text{LMA}} \approx \sqrt{\Delta m^2_{\text{atm}}} |\cos 2\theta| (1 - |U_{e1}|^2) \pm 0(\sim 5 \times 10^{-3} \text{ eV})
\]

(22)

The min$(|<m>|)$ in eq. (22) is reached in the case of CP-invariance and $\eta_{21} = -\eta_{31} = \pm 1$. If $\cos 2\theta = 0$ is allowed, values of $|<m>|$ smaller than $\sim 5 \times 10^{-3}$ eV and even $|<m>| = 0$ would be possible. If, however, it will be experimentally established that, e.g., $|\cos 2\theta| > 0.20$, we will have min$(|<m>|) \geq 0.01$ eV if $\Delta m^2_{\text{atm}}$ and $|U_{e1}|^2$ lie within their 90% C.L. allowed regions found in [31] (i.e., $|U_{e1}|^2 < 0.055$, $\Delta m^2_{\text{atm}} = (1.4 - 6.1) \times 10^{-3}$ eV$^2$). According to the latest analysis of the solar neutrino data (including the SNO results) performed in [16], for the LMA MSW solution one has $\cos 2\theta > 0.30 (0.50)$ at 99% (95%) C.L.

For the SMA MSW solution one has in the case of $\Delta m^2_{\odot} = \Delta m^2_{32}$ under discussion:

\[
\text{SMA MSW: } \min(|<m>|)_{\text{SMA}} \approx |<m>| \approx \sqrt{\Delta m^2_{\text{atm}}} (1 - |U_{e1}|^2) \pm 0(\sim 5 \times 10^{-3} \text{ eV})
\]

(23)

where $|U_{e1}|^2$ is limited by the CHOOZ data. Using the current 99% (95%) C.L. allowed values of $\Delta m^2_{\text{atm}}$ and $|U_{e1}|^2$, derived in [31], one finds min$(|<m>|) \approx 0.030 (0.050)$ eV.

We shall discuss next briefly the implications of the results of future $(\beta\beta)_{0
u}$-decay and $^3$H $\beta$-decay experiments. We follow the same line of analysis we have used for neutrino mass spectrum with normal hierarchy. Consider the case of the LMA MSW solution of the solar neutrino problem.

**Case A.** An experimental upper limit on $|<m>|$, $|<m>| < |<m>|_{\text{exp}}$, which is larger than the minimal value of $|<m>|$, $|<m>|_{\text{exp}}$, predicted by taking into account all uncertainties in the values of the relevant input parameters ($\Delta m^2_{\text{atm}}$, $\Delta m^2_{\odot}$, $\theta_{\odot}$, etc.), $|<m>|_{\text{exp}} \geq |<m>|_{\text{min}}$, will imply an upper limit on $m_1$, $m_1 < (m_1)_{\text{max}}$. The latter is determined by the equality:

\[
(m_1)_{\text{max}} : \sqrt{m_1^2 + \Delta m^2_{\text{atm}} - \Delta m^2_{\odot}} \cos^2 \theta_{\odot}
\]

If, for instance, $|\cos 2\theta| \geq 0.30; 0.50$, then under the same conditions one will have min$(|<m>|) \approx 0.015; 0.025$ eV.
The term \( m_1 |U_{e1}|^2 \) enters with a plus (minus) sign if the difference between the two terms in the big round brackets in the left-hand side of the equation is negative (positive). For the quasi-degenerate neutrino mass spectrum \( (m_1 \gg \Delta m^2_\odot, \Delta m^2_{\text{atm}}, m_1 \equiv m_2 \equiv m_3 \equiv m_{\nu_e}, (m_1)_{\text{max}} \) is given by eq. (13) in which \( |U_{e3}|^2 \) is replaced by \( |U_{e1}|^2 \). Correspondingly, the conclusion that if \( |\cos 2\theta_\odot (1 - |U_{e1}|^2) - |U_{e1}|^2| \) is sufficiently small, the upper limit on \( m_1 \equiv m_{\nu_e} \), obtained in \(^3\text{H}\) \( \beta \)-decay, can be more stringent than the upper bound on \( m_1 \), implied by the limit on \( |<m>| \), remains valid.

An experimental upper limit on \( |<m>| \), which is smaller than the minimal possible value of \( |<m>|, |<m>|_{\text{exp}} < |<m>|_{\text{min}} \), would imply that either i) the neutrino mass spectrum is not of the inverted hierarchy type, or ii) that there exist contributions to the \((\beta\beta)_{0v}\)-decay rate other than due to the light Majorana neutrino exchange (see, e.g., [52]) that partially cancel the contribution from the Majorana neutrino exchange. The indicated result might also suggest that the massive neutrinos are Dirac particles.

**Case B.** A measurement of \( |<m>| = (|<m>|)_{\text{exp}} \approx \sqrt{\Delta m^2_{\text{atm}}} (1 - |U_{e1}|^2) \approx (0.04 - 0.08) \) eV, where we have used the 90\% C.L. allowed regions of \( \Delta m^2_{\text{atm}} \) and \( |U_{e1}|^2 \) from [31], would imply the existence of a finite interval of possible values of \( m_1 \), \((m_1)_{\text{min}} \leq m_1 \leq (m_1)_{\text{max}} \), with \((m_1)_{\text{max}} \) and \((m_1)_{\text{min}} \) given respectively by eq. (24) and by

\[
(m_1)_{\text{min}} : \quad m_1 |U_{e1}|^2 + \left( \sqrt{m_1^2 + \Delta m^2_{\text{atm}} - \Delta m^2_\odot} \cos^2 \theta_\odot \right. \\
+ \left. \sqrt{m_1^2 + \Delta m^2_{\text{atm}} \sin^2 \theta_\odot} \right) (1 - |U_{e1}|^2) = |<m>|_{\text{exp}}.
\]  

(25)

In this case \( m_1 \approx 0.04 \) eV and the neutrino mass spectrum is with partial inverted hierarchy or of quasi-degenerate type [31]. The limiting values of \( m_1 \) correspond to CP-conservation. For \( \Delta m^2_\odot \ll m_1^2 \), i.e., for \( \Delta m^2_\odot \lesssim 10^{-4} \) eV\(^2 \), \((m_1)_{\text{min}} \) is to a good approximation independent of \( \theta_\odot \) and we have: \( \sqrt{((m_1)_{\text{min}})^2 + \Delta m^2_{\text{atm}}}(1 - |U_{e1}|^2) \approx (|<m>|)_{\text{exp}} \).

For negligible \( |U_{e1}|^2 \) (i.e., \( |U_{e1}|^2 \lesssim 0.01 \) for the values of \( \cos 2\theta_\odot \) in Fig. 2), essentially all of the interval between \((m_1)_{\text{min}} \) and \((m_1)_{\text{max}} \), \((m_1)_{\text{min}} < m_1 < (m_1)_{\text{max}} \), corresponds to violation of the CP-symmetry. If the terms \( \sim |U_{e1}|^2 \) cannot be neglected in eqs. (24) and (25) (i.e., \( |U_{e1}|^2 \approx (0.02 - 0.05) \) for the values of \( \cos 2\theta_\odot \) in Fig. 2), there exists for a fixed \( |<m>|_{\text{exp}} \) two CP-conserving values of \( m_1 \) in the indicated interval, one of which differs noticeably from the limiting values \((m_1)_{\text{min}} \) and \((m_1)_{\text{max}} \) and corresponds to \( \eta_{21} = -\eta_{31} = 1 \) (Fig. 2).

In general, measuring the value of \( |<m>| \) alone will not allow to distinguish the case of CP-conservation from that of CP-violation. In principle, a measurement of \( m_{\nu_e} \), or even an upper limit on \( m_{\nu_e} \), smaller than \((m_1)_{\text{max}} \), could be a signal of CP-violation. However, unless \( \cos 2\theta_\odot \) is very small, the required values of \( m_{\nu_e} \) are less than prospective measurements. For example, as seen in Fig. 2, upper left panel, for \( \cos 2\theta_\odot = 0.1 \) and \( |<m>| = 0.03 \) eV, one needs to find \( m_{\nu_e} < 0.35 \) eV to demonstrate CP-violation.

If the measured value of \( |<m>| \) lies in the interval \( |<m>| - |<m>| \leq |<m>| \leq |<m>| + |<m>| \), where

\[
|<m>| = \sqrt{\Delta m^2_{\text{atm}} - \Delta m^2_\odot} \cos^2 \theta_\odot \pm \sqrt{\Delta m^2_{\text{atm}} \sin^2 \theta_\odot} \sqrt{1 - |U_{e1}|^2},
\]  

(26)

we would have \((m_1)_{\text{min}} = 0 \). The values of \( m_1 \) satisfying \( 0 \leq m_1 < (m_1)_{\text{max}} \), where \((m_1)_{\text{max}} \) is determined by eq. (24), correspond to violation of the CP-symmetry (Fig. 2).
Cases C. As Fig. 2 indicates, the discussions and conclusions are identical to the discussions and conclusions in the same cases for the neutrino mass spectrum with normal hierarchy, except that instead of eq. (20) we have

$$|<m>| \simeq m_{\nu e} |U_{e1}|^2 + \cos^2 \theta_\odot (1 - |U_{e1}|^2) e^{i \alpha_{21}} + \sin^2 \theta_\odot (1 - |U_{e1}|^2) e^{i \alpha_{31}},$$

(27)

($m_{1\text{max}}$ and ($m_{1\text{min}}$) are determined by eqs. (24) and (25), and $|U_{e1}|^2$ must be substituted by $|U_{e1}|^2$ in the relevant parts of the analysis.

Case D. If $m_{\nu e}$ is measured and ($m_{\nu e})_{\exp} \geq 0.35$ eV but the ($\beta\beta_{0\nu}$-decay is not observed or is observed and ($m_{\nu e})_{\exp} > (m_{1\text{max}}$, where ($m_{1\text{max}}$) is determined by eq. (17), the same considerations and conclusions as in the Case D for the normal hierarchy mass spectrum apply.

A measured value of $<|m>|$, ($<|m>|_{\exp} \geq 0.1$ eV, in the case when the measured value of $m_{\nu e}$ or the upper bound on $m_{\nu e}$ are such that $m_{\nu e} < (m_{1\text{min}}$, where ($m_{1\text{min}}$) is determined by eq. (25), would lead to the same conclusions as in the Case D for the normal hierarchy mass spectrum.

Case E. It is possible to have a measured value of $|<m>| \leq 10^{-2}$ eV in the case of the LMA MSW solution and neutrino mass spectrum with inverted hierarchy under discussion only if $\cos 2\theta_\odot$ is rather small, $\cos 2\theta_\odot \leq 0.2$. A measured value of $|<m>| < |<m>|_{\exp}^{m_{1\text{min}}}$ would imply that either the neutrino mass spectrum is not of the inverted hierarchy type, or that there exist contributions to the ($\beta\beta_{0\nu}$-decay rate other than due to the light Majorana neutrino exchange that partially cancel the contribution from the Majorana neutrino exchange.

The above conclusions hold with minor modifications (essentially of the numerical values involved) for the LOW-QVO solution as well. In the case of the SMA MSW solution we have, as is well-known, $\sin^2 \theta_\odot \ll 1$ and $\Delta m^2_{\odot} \ll 10^{-5}$ eV$^2$ (see, e.g., [47]). Consequently, the analog of eq. (18) in Case A reads $(m_{1\text{max}}) \simeq |<m>|_{\exp} (1 - 2|U_{e1}|^2)^{-1}$. The conclusions in the Cases B - D are qualitatively the same as in the case of neutrino mass spectrum with normal hierarchy. In particular, a measured value of $|<m>| > |<m>|_{\exp} \simeq \sqrt{\Delta m^2_{\odot}} (1 - |U_{e1}|^2)$, would essentially determine $m_{1\text{min}}$, $m_{1\text{min}} \simeq (|<m>|_{\exp})$. No information about CP-violation generated by the Majorana phases can be obtained by the measurement of $|<m>|$, or of $|<m>|$ and $m_{\nu e}$. If both $|<m>|$ and $m_{\nu e} \geq 0.35$ eV are measured, the relation $m_1 \simeq (|<m>|_{\exp}) \simeq (m_{\nu e})_{\exp}$ should hold. If it is found that $|<m>| = \sqrt{\Delta m^2_{\odot}} (1 - |U_{e1}|^2)$, one would have $0 \leq m_1 \leq (m_{1\text{max}}$, where $(m_{1\text{max}}$ is determined by eq. (24) in which effectively $\sin^2 \theta_\odot = 0$, $\cos^2 \theta_\odot = 1$, and $\Delta m^2_{\odot} = 0$. Finally, a measured value of $|<m>| < |<m>|_{\min} \simeq |<m>|_{\max} \simeq \sqrt{\Delta m^2_{\odot}} (1 - |U_{e1}|^2)$ would either indicate that there exist new additional contributions to the ($\beta\beta_{0\nu}$-decay rate, or that the SMA MSW solution is not the correct solution of the solar neutrino problem.

5 Conclusions

Neutrino oscillation experiments can never tell the actual neutrino masses (that is, the lowest mass $m_1$), whether neutrinos are Majorana, and, if so, whether there are Majorana CP-violating phases associated with the $\Delta L = 2$ neutrino mass. Neutrinoless double-beta decay experiments can, in principle, answer the first two questions, but cannot by themselves provide information about CP-violation. Here we have analyzed how, given optimum information from neutrino oscillation and ($\beta\beta_{0\nu}$-decay experiments, a measurement of neutrino mass from $^3$H $\beta$-decay -decay could, in principle, give evidence for Majorana CP-violating phases, even though no CP-violation would be directly observed. The indicated possibility requires quite accurate measurements and holds only for a limited range of parameters.

Note Added. After the completion of the present paper we became aware of the very recent work [53], where some of the topics we discuss are also considered but within a somewhat different approach.
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Figure 1: The dependence of $|\langle m \rangle|$ on $m_1$ in the case of $\Delta m^2_{\odot} = \Delta m^2_{21}$ (normal hierarchy of neutrino masses) for the LMA MSW solution of the solar neutrino problem. The three vertical left (right) panels correspond to $|U_{e3}|^2 = 0.01$ (0.05), while the two upper, the two middle and the two lower panels are obtained respectively for $\cos 2\theta_\odot = 0.10$; 0.30; 0.54. The figures are obtained for the best fit values of $\Delta m^2_{\text{atm}}$ and $\Delta m^2_{\odot}$, given in eqs. (15) and (16). In the case of CP-conservation the allowed values of $|\langle m \rangle|$ are constrained to lie on i) the solid line if $\eta_{21} = \eta_{31} = 1$, ii) on the dashed line if $\eta_{21} = -\eta_{31} = 1$, iii) on the dotted lines if $\eta_{21} = -\eta_{31} = -1$, and iv) on the dash-dotted lines if $\eta_{21} = -\eta_{31} = -1$. The region colored in grey (not including these lines) requires CP-violation (“just CP-violation” region).
Figure 2: The same as Fig. 1 for the inverted hierarchy, $\Delta m^2_{\odot} = \Delta m^2_{32}$. The three vertical left (right) panels correspond to $|U_{e3}|^2 = 0.005$ (0.05), while the two upper, the two middle and the two lower panels are obtained respectively for $\cos 2\theta_\odot = 0.10$; 0.30; 0.54. The figures are obtained for the best fit values of $\Delta m^2_{\text{atm}}$ and $\Delta m^2_{32}$, given in eqs. (13) and (14). If CP-invariance holds, the allowed values of $|\langle m \rangle|$ are constrained to lie on i) the solid line if $\eta_{21} = \eta_{31} = 1$, ii) on the dashed line if $\eta_{21} = \eta_{31} = -1$, iii) on the dotted line if $\eta_{21} = -\eta_{31} = 1$, and iv) on the dash-dotted lines if $\eta_{21} = -\eta_{31} = 1$ for $|U_{e3}|^2 = 0.05$ and on v) the solid line if $\eta_{21} = \eta_{31} = \pm 1$, vi) on the dotted line if $\eta_{21} = -\eta_{31} = \pm 1$ for $|U_{e3}|^2 = 0.005$. The region colored in grey (not including the indicated lines) requires CP-violation.