Compact Thermal Model of the Pulse Transformer Taking into Account Nonlinearity of Heat Transfer †

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Abstract: This paper presents a compact nonlinear thermal model of pulse transformers. The proposed model takes into account differentiation in values of the temperatures of a ferromagnetic core and each winding. The model is formulated in the form of an electric network realising electrothermal analogy. It consists of current sources representing power dissipated in the core and in each of the windings, capacitors representing thermal capacitances and controlled current sources modelling the influence of dissipated power on the thermal resistances in the proposed model. Both self-heating phenomena in each component of the transformer and mutual thermal couplings between each pair of these components are taken into account. A description of the elaborated model is presented, and the process to estimate the model parameters is proposed. The proposed model was verified experimentally for different transformers. Good agreement between the calculated and measured waveforms of each component temperature of the tested pulse transformers was obtained. Differences between the results of measurements and calculations did not exceed 9% for transformers with a toroidal core and 13% for planar transformers.

Keywords: nonlinear thermal model; SPICE; pulse transformer; thermal phenomena; self-heating; modelling; measurements

1. Introduction

Pulse transformers are an important component of switched-mode power converters [1–3]. These transformers have simple structure, and they consist of two kinds of components, i.e., a ferromagnetic core and at least two windings. During the operation of the considered device, an increase in temperature of each transformer component is observed [4–6]. This increase is a result of thermal phenomena occurring in the pulse transformer, such as self-heating in each component of the transformer and mutual thermal interaction between each pair of these components [7–9].

Knowing the core temperature and the windings temperatures is important from the point of view of electrical and magnetic properties of a pulse transformer. As is shown in [10,11], the temperature significantly changes the characteristics of ferromagnetic materials used to make a transformer core and causes a change in the resistances of the windings. In particular, an excessive increase in temperature can lead to damage in the insulation of the windings or can reduce the magnetic permeability of the core [8,11,12]. Additionally, an increase in the temperatures of electronic components causes a decrease in their lifetime [13,14].
In order to calculate the temperature waveforms of electronic components with thermal phenomena taken into account, a thermal model of these components is indispensable [15,16]. In many papers [10,11,17–25], thermal models of transformers are described, but they have disadvantages. One group of thermal models is microscopic models, which make it possible to calculate temperature distribution in the considered component. For example, in [11,18], the finite element method is used to determine the temperature distribution in a transformer, but at the same time the distribution of the wasted power per unit of volume in the transformer is assumed. Another group of thermal models is compact thermal models, which take into account only one temperature characterising the whole device [17].

Reference [26] presents three-dimensional (3-D) numerical compact thermal models of planar transformers. The DC thermal network of the cited model is inspired by the Delphi method, which allows obtaining shorter time of calculations than the finite volume method. The presented results do not illustrate the influence of dissipated power on the temperatures of the core and the windings. Additionally, this model is dedicated to the ANSYS software, and it is difficult to implement it in other software, e.g., in SPICE.

The similar approach to modelling thermal properties of electronic components is presented in [27,28]. The model presented in [27] uses homogenisation techniques to reduce calculation requirements. The advantage of this model is the reduction of the number of thermal factors to 6–8, and good agreement between the results of calculations and measurements can still be obtained. Unfortunately, the mentioned model could be used in the software dedicated only to a 3-D thermal analysis. In turn, the approach to the thermal modelling of components of electric machines presented in [28] requires time-consuming measurements of the tested prototype. In the paper [28], some temperature waveforms of the tested machine are presented.

Reference [29] describes a simplified form of an electrothermal model dedicated to planar magnetic components (inductors and transformers), which operate in the space industry. An important part of this model is the thermal model dedicated to the ANSYS program. Due to the fact that a lot of factors are taken into account, the network representation of the proposed model is complex and contains three subcircuits representing the thermal network of the transformer windings, the thermal network of the transformer core and the thermal network of the connection between the windings and the Printed Circuit Board (PCB).

References [30,31] are dedicated to parameters estimation of thermal models of electronics devices in the Delphi-inspired form. To this end, genetic algorithms are used. Unfortunately, for multiple heat sources in such models, calculations are time-consuming, and the total simulation time can reach 800 h.

Reference [32] presents a thermal model of a planar transformer. The form of this model is obtained on the basis of computation fluid dynamics. The structure of this model is adequate for planar transformers only. Unfortunately, in the paper [32], no results of experimental verification of this model are presented.

Compact thermal models of transformers are described in [10,11,17,18,26]; however, differences between the core and windings temperatures are typically not taken into account in the mentioned models. In addition, in the models described in [24,25], the dependence of the dissipation efficiency of heat generated in the device on the power dissipated in the transformer is omitted [33].

It is widely known [7,15,16,34–36] that some factors, such as ambient temperature, cooling systems or power dissipated in electronic devices, influence the efficiency of heat removal from the devices. Reference [37] presents a nonlinear thermal model of a planar transformer. In this model, the influence of power dissipated in particular components of the transformer on the efficiency of heat removal is taken into account.

This paper, which is an extended version of Reference [38], proposes a compact nonlinear thermal model (CNTM) of a pulse transformer based on a thermal model of a planar transformer described in [19]. In comparison to [19], which presents a linear thermal model, the nonlinearity of the heat transfer process is taken into account. In comparison to [38], a detailed description of the nonlinear
thermal model of the transformer and the new results of measurements and calculations illustrating the effectiveness of this model for transformers including different ferromagnetic cores are presented in this paper. The model proposed by the authors’ takes into account self-heating phenomena in all components of the transformer and mutual thermal couplings between each pair of these components.

The selected thermal models of transformers given in the literature are discussed in Section 2. The form of a nonlinear thermal model is presented in Section 3. The obtained results of measurements and calculations proving the effectiveness of the elaborated model are shown in Section 4. The advantage of this nonlinear thermal model over a linear thermal model is experimentally confirmed for the selected transformers containing cores with different shapes and made of different ferromagnetic materials. For all models described in the following sections, thermal resistance and thermal capacitance are given in K/W and J/K, respectively, whereas all temperatures are expressed in Celsius degrees.

2. Selected Thermal Models of Transformers in the Literature

A network representation of classical linear thermal models of transformers [10,39] is shown in Figure 1. It can be seen that only one internal temperature of the whole transformer is used. In this model, differences in the temperatures of the core and the windings are not taken into account.

![Network representation of the thermal model described in [10,39](https:// Energies 2020, 13, 2766)](https:// Energies 2020, 13, 2766)

In the presented model, the controlled current source $G_1$ represents the sum of the power dissipated in the core and in the windings of the transformer. Thermal capacitance is represented by capacitor $C_{th}$, while $R_{th1}$ is the thermal resistance characterising heat convection and $R_{th2}$ is thermal resistance characterising the heat radiation from the surface of the examined device. Voltage source $T_a$ is the ambient temperature, and the voltage in node $T_j$ is the temperature of the transformer.

In [17,33], a thermal model of magnetic components (transformers and inductors) is proposed. This model enables calculating the temperature difference between the core and ambient temperature ($\Delta T_C$) and the temperature difference between the windings and ambient temperature ($\Delta T_W$) by taking into account self-heating and mutual thermal couplings between the core and the windings. The network representation of this model is shown in Figure 2.

![Network representation of the thermal model in [17,33](https:// Energies 2020, 13, 2766)](https:// Energies 2020, 13, 2766)

However, in this model, only single thermal time constants for the windings ($R_{thW}$ and $C_{thW}$) and for the core ($R_{thC}$ and $C_{thC}$) are taken into account. Current sources $G_{PC}$ and $G_{PW}$ describe power losses in the core and in the windings, respectively; whereas current sources $G_{PC1}$ and $G_{PW1}$ model the influence of mutual thermal couplings between the core and the winding on the temperature differences $\Delta T_W$ and $\Delta T_C$. 

The thermal model of a transformer proposed in [17], of which the diagram is presented in Figure 3, makes it possible to calculate the temperature of the core \( T_C \) and the temperatures of both windings \( T_W \) by taking into account self-heating and mutual thermal couplings between the core and the windings. This model has the form of RC Foster networks excited by current sources representing the powers dissipated in the core (\( P_{thC} \) and \( P_{thW1} \)) and in the windings (\( P_{thW} \) and \( P_{thWC1} \)).

In order to take into account thermal couplings between the core and the windings, controlled current sources \( P_{thWC1} \) and \( P_{thCW1} \) are applied. The voltage source \( T_a \) represents ambient temperature. In the described model, common RC networks are used to model self-heating and mutual thermal couplings between the components of the transformer.

A disadvantage of thermal models of transformers described in this section is they do not take into account differentiations of windings temperatures, nonlinearities of thermal properties and thermal couplings occurring between the components of the transformer. Therefore, in the following Section, the authors’ nonlinear thermal model of the transformer is proposed. In this model, nonlinearities of thermal phenomena and thermal couplings between the components of the transformer are taken into account.

3. Proposed Nonlinear Thermal Model of Pulse Transformers

As is shown in [17,19], temperature distributions on each of the windings and on the core in pulse transformers are practically uniform. Therefore, their thermal properties can be described with the use of a compact thermal model [7,19]. On the other hand, the temperatures of the core and the windings can be significantly different from each other [7]. Therefore, in such models, the differences of the core and windings temperatures should be taken into account. In each transformer component, a self-healing
The nonlinear thermal model of the transformer worked out by the authors has the form of a subcircuit dedicated for SPICE. This form is inspired by the linear thermal model of a planar transformer proposed in [19] and by the nonlinear thermal model of semiconductor devices described in [37,40]. In the new model, the differences in the temperatures of components in the transformer (the core and all the windings) are taken into account. These temperatures result from self-heating phenomena in every component and mutual thermal couplings between each pair of the components of the transformer. The presented model makes it possible to calculate the temperature waveforms of the core and each winding by taking into account both the mentioned phenomena. When formulating the considered model, based on the results of measurements shown in [7], changes in the powers dissipated in each component influence only the values of transformer thermal resistances in the thermal model, whereas thermal capacitances do not depend on power. The network representation of the proposed model is presented in Figure 4.

In this network, nine circuits can be distinguished. Three of them located on the left-hand side of Figure 4 are used to calculate the temperature waveforms of particular components of the transformer—the primary winding $T_{W1}$, the secondary winding $T_{W2}$ and the core $T_C$. These temperatures (given in °C) correspond to the voltages (given in V) in the nodes and are denoted as $T_{W1}$, $T_{W2}$ and $T_C$, respectively. Current sources $I_{C1}$, $I_{W1}$ and $I_{W2}$ model power losses in the core and the two windings, respectively.

Circuits including capacitors and controlled current sources model self-transient thermal impedances of the primary winding ($C_{W11}$, …, $C_{W1n}$ and $G_{W11}$, …, $G_{W1n}$), of the secondary winding ($C_{W21}$, …, $C_{W2n}$ and $G_{W21}$, …, $G_{W2n}$) and of the core ($C_{C1}$, …, $C_{Cn}$ and $G_{C1}$, …, $G_{Cn}$), respectively. The voltages on these circuits (given in V) correspond to differences (given in °C) between the temperatures of the transformer components and ambient temperature, resulting from self-heating phenomena.

The controlled voltage sources $E_1$, $E_2$ and $E_3$ represent the influence of mutual thermal couplings between the components of the transformer on the temperatures of these components. Capacitors represent the thermal capacitances of all the elements in the heat flow path, whereas the controlled current sources represent nonlinear thermal resistances between the elements in the heat flow path.

The output voltage of the controlled voltage source $E_1$ is equal to the sum of voltages in nodes $T_{W11}$ and $T_{WC1}$, the output voltage of the controlled voltage source $E_2$ is equal to the sum of voltages in nodes $T_{W21}$ and $T_{WC2}$, and the output voltage of the controlled voltage source $E_3$ is equal to the sum of voltages in nodes $T_{CW1}$ and $T_{CW2}$. Voltage sources $V_1$, $V_2$ and $V_3$ represent ambient temperature.
Six other subcircuits are used to model mutual thermal couplings between each pair of the transformer components. Current sources in these subcircuits represent power (given in W) dissipated in particular components of the transformer ($I_{W12}$ and $I_{W1C}$ in the primary winding, $I_{W21}$ and $I_{W2C}$ in the secondary winding and $I_{C1}$ and $I_{C2}$ in the core). Networks containing capacitors and controlled current sources connected to the above-mentioned current sources model mutual transient thermal impedances between each pair of components of the transformer.

All self-transient and mutual transient thermal impedances can be described by Equation (1) [7,15]:

$$Z_{th}(t) = R_{th} \cdot \left[ 1 - \sum_{i=1}^{N} a_i \cdot \exp \left( -\frac{t}{\tau_{thi}} \right) \right],$$

where $R_{th}$ is the thermal resistance, $a_i$ is the coefficient (without unit) corresponding to thermal time constant $\tau_{thi}$ (given in s), and $N$ is the number of thermal time constants.

The dependence of $R_{th}$ on the dissipated power is described as:

$$R_{th} = R_{th0} \cdot \left[ 1 + \alpha \cdot \exp \left( -\frac{p - p_0}{b} \right) \right],$$

where $R_{th0}$ denotes the minimum value of the thermal resistance, $p$ denotes the power dissipated in a heating component of the transformer, $\alpha$ is the parameter without unit, and $p_0$ and $b$ are model parameters given in W.

Changes in values of thermal resistance are modelled by the controlled current source $G_i$. The output current of the controlled source is described as:

$$G_i = \frac{V_{Gi}}{(a_i \cdot R_{th})},$$

where $V_{Gi}$ denotes the voltage on the current source $G_i$.

Thermal capacitances are given as:

$$C_i = \frac{\tau_{thi}}{(a_i \cdot R_{th})}.$$  

The values of the model parameters are estimated using the results of measurements of self- and mutual transient thermal impedances existing in the transformer thermal model. Measurements of such parameters at different powers dissipated in the core and in the windings of the tested transformer are realised by the method described in [7]. The values of parameters in Equation (1) are estimated for each transient thermal impedance using the method described in [15,41]. Next, parameters $\alpha$, $p_0$ and $b$ in Equation (2) are estimated for self-thermal and mutual thermal resistance in the transformer model by using local estimation [15].

4. Results

In order to verify the presented model and its practical use, measurements and calculations of the temperature waveform of each component of the tested transformers, which contained ferromagnetic cores with different shapes and sizes and were made of different materials, were performed. A planar transformer with a ferrite core and transformers with ring cores made of different materials were measured and modelled as examples.

The planar transformer, of which the cuboidal core dimension was $22 \text{ mm} \times 16 \text{ mm} \times 9 \text{ mm}$, was made of ferrite material 3F3 and contained windings in the form of printed paths on laminate FR-4, which was $1 \text{ mm}$ thick. The primary winding contained three turns with a width of $2.5 \text{ mm}$, and the secondary winding contained four turns with a width of $1 \text{ mm}$. Transformers with a toroidal core contained identical primary and secondary windings. On each of them, 20 turns of copper wire in the enamel with a diameter of $0.8 \text{ mm}$ were wound. The toroidal core had an external diameter equal
to 26 mm, the internal diameter equal to 16 mm and a width equal to 11 mm. Cores made of toroidal powdered iron (RTP), ferrites (RTF) and nanocrystals (RTN) were used for toroidal transformers.

The values of the parameters in the thermal model were estimated for transformers containing ferromagnetic cores with different shapes and dimensions and made of different ferromagnetic materials.

The values of the parameters in Equation (2), which represents the proposed compact nonlinear thermal model of the transformer (CNMT), are summarized in Table 1 (for a toroidal transformer) and Table 2 (for a planar transformer). The values of the parameters presented in Tables 1 and 2 describe the model of the network representation shown in Figure 4.

Table 1. Values of the parameters in Equation (2) for a toroidal transformer with a toroidal powdered iron core (RTP).

| Thermal Resistance | $R_{th0}$ [K/W] | $\alpha$ | $p_0$ [W] | $b$ [W] |
|--------------------|-----------------|----------|-----------|----------|
| $R_{thW1}$         | 12.75           | 10       | 0         | 1.13     |
| $R_{thW1W2}$       | 8.6             | 10       | 0         | 1.3      |
| $R_{thW1C}$        | 9.9             | 10.5     | 0         | 1.2      |

Table 2. Values of the parameters in Equation (2) for a planar transformer.

| Thermal Resistance | $R_{th0}$ [K/W] | $\alpha$ | $p_0$ [W] | $b$ [W] |
|--------------------|-----------------|----------|-----------|----------|
| $R_{thW1}$         | 26              | 0.27     | 1         | 2        |
| $R_{thW1W2}$       | 15              | 0.733    | 1         | 5.5      |
| $R_{thW1C}$        | 11              | 0.636    | 0.5       | 3        |
| $R_{thC}$          | 11.5            | 0.435    | 10        | 10       |
| $R_{thCW1}$        | 5.1             | 0.96     | 10        | 9.9      |
| $R_{thCW2}$        | 2.4             | 2.125    | 10        | 10       |

Comparing the values of the parameters describing thermal resistances in the thermal model of the toroidal transformer, it is obvious that the values of parameters $\alpha$, $p_0$ and $b$ were nearly the same for all considered thermal resistances. Differences were observed in the values of the parameter $R_{th0}$. This meant that courses $R_{thW1}(p_{W1})$, $R_{thW1W2}(p_{W1})$ and $R_{thW1C}(p_{W1})$ were nearly parallel. In contrast, big differences were observed between the values of the parameters describing the dependences of thermal resistances in the thermal model of the planar transformer on powers $P_{W1}$ and $P_{C}$.

The values of thermal capacitances were estimated with the use of the method described in [15]. As an example, in Table 3, the values of these parameters obtained for a planar transformer are summarised.

Table 3. Values of the parameters $a_i$ and $\tau_{thi}$ of the selected self-transient and mutual transient thermal impedances in a thermal model of a planar transformer.

| Parameter | $Z_{thW1}(t)$ | $Z_{thW1C}(t)$ | $Z_{thW1W2}(t)$ | $Z_{thC}(t)$ | $Z_{thCW1}(t)$ | $Z_{thCW2}(t)$ |
|-----------|---------------|----------------|-----------------|--------------|----------------|----------------|
| $a_1$     | 0.274         | 0.271          | 0.626           | 0.297        | 0.224          | 0.18           |
| $a_2$     | 0.448         | 0.456          | 0.374           | 0.676        | 0.776          | 0.82           |
| $a_3$     | 0.225         | 0.273          | 0.027           |              |                |                |
| $a_4$     | 0.053         |                |                |              |                |                |
| $\tau_{th1}$ [s] | 350.33         | 498.84         | 350.72          | 432.04       | 678.02         | 1067.99        |
| $\tau_{th2}$ [s] | 60.22          | 103.66         | 192.79          | 139.67       | 194.39         | 221.5          |
| $\tau_{th3}$ [s] | 14.31          | 16.26          | 17.11           |              |                |                |
| $\tau_{th4}$ [µs] | 40             |                |                |              |                |                |

It can be clearly seen that, in the considered self-transient and mutual transient thermal impedances, different numbers of thermal time constants occurred. In self-transient thermal impedances, three or four thermal time constants were used, whereas mutual transient thermal impedances were described.
with two or three thermal time constants. The values of the considered thermal time constants were in a range from 40 \( \mu \)s to over 1000 s.

For example, by means of Equation (2), the measured (indicated by points) and modelled (indicated by lines) dependences of thermal resistances in the thermal model of the transformer on the power dissipated in one of the components of the transformer are shown in Figures 5 and 6.

Figure 5 illustrates the influence of power \( P_{W1} \) dissipated in the primary winding of the transformer containing a toroidal core made of powdered iron on the thermal resistance of the winding \( R_{thW1} \) (blue colour) and on mutual thermal resistances between this winding and the secondary winding \( R_{thW1W2} \) (black colour), as well as those between the core and the primary winding \( R_{thW1C} \) (red colour).

Figure 6a illustrates the influence of power dissipated in the primary winding of the planar transformer on the thermal resistances \( R_{thW1} \) (blue colour), \( R_{thW1W2} \) (black colour) and \( R_{thW1C} \) (red colour). Figure 6b illustrates the influence of power dissipated in the core of the transformer on the thermal resistance of the core \( R_{thC} \) (blue colour) and the mutual thermal resistances between the core and both windings \( R_{thCW1} \) (red colour) and \( R_{thCW2} \) (green colour).

As it is obvious in Figures 5 and 6, it is possible to accurately model the measured dependences of the considered thermal resistance on the dissipated power using Equation (2). Visible differences between self-thermal and mutual thermal resistances were observed for both considered transformers. These thermal resistances differed from one another. It is worth noticing that the considered differences were bigger for the planar transformer than for the toroidal transformer. Changes in dissipated power can cause changes in thermal resistance even by 25%.

**Figure 5.** Measured (indicated by points) and modelled (indicated by lines) dependences of the selected thermal resistances in the thermal model of the transformer with a toroidal core made of powdered iron on the power dissipated in the primary winding.

**Figure 6.** Measured (indicated by points) and modelled (indicated by lines) dependences of the selected thermal resistances in the thermal model of the planar transformer on the power dissipated in the primary winding (a) and in the core (b).
Calculations and measurements were performed at power dissipation in only one of the components of the tested transformers and at simultaneous dissipation of power in different components of the tested devices. Power dissipated in each component of the transformer always had a shape of a single rectangular impulse with a long duration time. The results of calculations obtained by means of the nonlinear thermal model were compared to the results of calculations performed by means of the linear thermal model described in [19] and the results of measurements performed with the use of a pyrometer. The windings and the core were excited with different powers. The successive figures (Figures 7–12) present the calculated and measured waveforms of the temperatures of the primary winding $T_{W1}$, the secondary winding $T_{W2}$ and the core $T_C$ of the tested transformers. In these figures, the results of measurements is represented by points, the results of calculations performed using the CNTM is represented by solid lines, and the results of calculations using the compact linear thermal model (CLTM) is represented by dashed lines [19].

Calculations were performed for the values of parameters describing the nonlinear thermal model of transformers according to the principles shown in Section 3. On the other hand, for the model from [19], the values of parameters estimated at the lowest measured values of power dissipated in each component of the transformers were used.

Figure 7 presents the measured and calculated waveforms of temperatures $T_{W1}$ and $T_C$ in the toroidal transformer with the RTP core at a dissipated power $P_{W1}$ of 2.83 W in the primary winding.

It can be observed that the nonlinear thermal model makes it possible to obtain very good agreement between the results of calculations and measurements. In contrast, the values of the considered temperatures obtained with the use of calculations performed with the linear thermal model were higher than the results of measurements by even 40 °C.

![Figure 7](image)

**Figure 7.** Measured and calculated temperature waveforms of the primary winding $T_{W1}$ and the core $T_C$ of a toroidal transformer with an RTP at a dissipated power $P_{W1}$ of 2.83 W in the primary winding.

Figure 8 shows the measured and calculated temperature waveforms of the primary winding $T_{W1}$, the secondary winding $T_{W2}$ and the core $T_C$ for a transformer containing an RTN. These waveforms were obtained, while the primary winding of the tested transformer was excited by a single rectangular pulse with duration times equal to 4000 s (Figure 8a) and 5000 s (Figure 8b). $P_{W1}$ was 1.05 W in the case presented in Figure 8a, and $P_{W1}$ was equal to 2.82 W in the case presented in Figure 8b.

As it is seen, very good agreement between the results of calculations performed with the use of the CNTM and measurements was achieved for both the values of power dissipated in this transformer. In contrast, for the linear thermal model, an excess of temperature of each component of the transformer over ambient temperature was overestimated by 10% at lower values of the considered powers and even 50% overestimated for higher values of the considered dissipated power.
By analysing the temperature waveforms of the core T_C, the primary winding T_W1 and the secondary winding T_W2 presented in Figure 9, it can be observed that the results of calculations performed by means of the nonlinear thermal model assured better agreement with the results of measurements than the results of calculations obtained by means of the model described in [19]. The obtained difference between the results of calculations performed with the use of the linear thermal model and the measurements results was 10%.

Figure 10 presents the measured and calculated waveforms of the temperature of the primary winding T_W1, the temperature of the secondary winding T_W2 and the core temperature T_C of the planar transformer by stimulating the primary winding with a single rectangular impulse having a duration time equal to 5500 s and P_W1 of 1 W (Figure 10a) and with a single rectangular impulse having a duration time of 5500 s and P_C of 2 W (Figure 10b).

As one can notice, the results of calculations by means of the nonlinear thermal model assured better agreement with the results of measurements than the results of calculations performed by means of the linear thermal model. In Figure 10a, it is visible that the difference between the waveforms of temperatures T_W1 and T_C obtained with the use of the considered models was more than 2 °C. The biggest difference between the results of calculations was observed for temperature T_W2. In Figure 10b, the considered differences were bigger than in the case presented in Figure 10a. These differences reached even 20 °C. When power was dissipated in the core only, the temperatures of both windings were nearly the same.
Figure 9. Measured and calculated waveforms of the temperature of the primary winding $T_{W1}$, the temperature of the secondary winding $T_{W2}$ and the temperature of the core $T_C$ at a dissipated power $P_{W1}$ of 0.77 W in the primary winding.

Figure 10. Measured and calculated waveforms of the temperature of the primary winding $T_{W1}$, the temperature of the secondary winding $T_{W2}$ and the temperature of the core $T_C$ of the planar transformer at $P_{W1}$ of 1 W in the primary winding (a) and $P_C$ of 2 W in the core (b).
The results of measurements and calculations presented above corresponded to untypical situations, when power was dissipated in one of the transformer components only. Figures 11 and 12 present the results of measurements and calculations of temperature waveforms of components of the selected transformers obtained in the case when power was dissipated in both the components of the selected transformers.

Figure 11 shows the measured and calculated temperature waveforms of the primary winding $T_{W1}$, the secondary winding $T_{W2}$ and the core $T_C$ for the transformer containing the toroidal ferrite core (RTF) when it was excited by power dissipated simultaneously in the primary and secondary windings with a single rectangular pulse having duration times equal to 4500 s (Figure 11a) and 5500 s (Figure 11b). $P_{W1}$ was equal to 0.88 W and $P_{W2}$ was equal to 1 W in the case presented in Figure 11a, whereas $P_{W1}$ was equal to 2.4 W and $P_{W2}$ was equal to 2.7 W in the case presented in Figure 11b.

By analysing temperature waveforms of the core and the windings of the transformer containing an RTF (Figure 11), it is easy to observe that the difference between the results of calculations performed using the linear thermal model and the results of measurements was up to 20 °C. In contrast, the results of calculations made using the nonlinear thermal model showed very good agreement with the results of measurements. Differences between the results of measurements and calculations performed with the linear thermal model were bigger at higher powers dissipated in the components of the transformer.

Figure 12 presents the calculated and measured waveforms of the temperatures of each component of the planar transformer by simultaneously stimulating the core and the primary winding with a single rectangular impulse having a duration time equal to 4500 s, $P_{W1}$ of 2.3 W and $P_C$ of 2 W.

**Figure 11.** Measured and calculated temperature waveforms of the primary winding $T_{W1}$, the secondary winding $T_{W2}$ and the core $T_C$ with simultaneous power dissipation in the windings: (a) $P_{W1} = 0.88$ W and $P_{W2} = 1.05$ W; (b) $P_{W1} = 2.4$ W and $P_{W2} = 2.7$ W.
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As one can notice, the results of calculations by means of the nonlinear thermal model assured better agreement with the results of measurements than the results of calculations performed by means of the linear thermal model. The observed differences between the results of measurements and the results of calculations performed by means of the linear thermal model exceeded even 40 °C, whereas the difference between the results of calculations performed by means of the nonlinear thermal model and the results of measurements was no more than about 5 °C. It is also worth noticing that in the considered operation conditions the temperatures of the transformer components were low and did not exceed 10 °C.

Table 4 contains the values of the maximum errors of the temperatures of transformers components $\delta_{TW1}$, $\delta_{TW2}$ and $\delta_{TC}$ obtained using the CLTM and the CNTM of the transformer analysed in this section.

**Table 4.** Values of the maximum relative error of the temperatures of transformers components calculated with the CNTM and the compact linear thermal model (CLTM).

| Core Type            | Power Dissipated | Model Type | $\delta_{TW1}$ [%] | $\delta_{TW2}$ [%] | $\delta_{TC}$ [%] |
|----------------------|------------------|------------|-------------------|-------------------|-----------------|
| RTP                  | PW1 = 2.3 W      | CLTM       | 48.8%             | -                 | 40.83%          |
|                      |                  | CNTM       | 8.16%             | -                 | 3.22%           |
|                      | PW1 = 1.05 W     | CLTM       | 5.49%             | 13.37%            | 2.86%           |
|                      |                  | CNTM       | 1.33%             | 5.22%             | 2.86%           |
|                      | PW1 = 2.82 W     | CLTM       | 19.2%             | 31.34%            | 8.4%            |
|                      |                  | CNTM       | 2.78%             | 9.05%             | 5.9%            |
| RTN                  | PW1 = 0.88 W     | CLTM       | 15%               | 13%               | 4%              |
|                      | PW2 = 1.05 W     | CNTM       | 4%                | 3.5%              | 2%              |
|                      | PW1 = 2.4 W      | CLTM       | 33%               | 16.5%             | 16%             |
| Toroidal ferrite core (RTF) | PW2 = 2.7 W  | CLTM       | 2%                | 2%                | 1%              |
|                      | PW1 = 0.77 W     | CNTM       | 10.5%             | 13.02%            | 10.74%          |
|                      |                  | CLTM       | 1.53%             | 6.25%             | 7%              |
| 3F3                  | PW1 = 1 W        | CLTM       | 4.18%             | 10.78%            | 4.20%           |
|                      |                  | CNTM       | 3.18%             | 3.13%             | 4.16%           |
|                      | PC = 2 W         | CLTM       | 30.6%             | 25%               | 20.7%           |
|                      |                  | CNTM       | 7.4%              | 6%                | 17.5%           |
|                      | PW1 = 2.3 W      | CLTM       | 24%               | 29.09%            | 47.32%          |
|                      | PC = 2 W         | CNTM       | 13.14%            | 4.47%             | 35.67%          |
It is clearly seen in Table 4 that the considered relative error of calculations of temperatures obtained using the CLTM achieved 48.8% for the primary winding temperature of the transformer with an RTP core whereas such an error of calculations of the primary winding temperature obtained using the CNTM was equal to 8.16% for the same transformer. It is also worth noticing that the value of the relative error of the temperatures of transformers components obtained using the CNTM did not exceed 9% for all the considered transformers containing a toroidal core made of different materials. It was also observed that an increase in power dissipated in the transformer components caused an increase of the value of the relative error of calculations for both the considered models.

5. Conclusions

This paper describes a new CNTM of a pulse transformer elaborated by the authors. It allows determining the waveforms of the temperatures of the core and each winding. Calculations were performed by taking into account a self-heating phenomenon and mutual thermal interaction between the transformer components. In our model, the dependences of self-thermal and mutual thermal resistances depend on power dissipated in the transformer components. The proposed model was verified experimentally for transformers including ferromagnetic cores with different shapes and made of different materials. High accuracy of this model and its advantage over the CLTM were also demonstrated. The results of calculations and measurements presented in this paper also confirmed that power dissipated in the transformer components influenced mostly thermal resistance and its influence on thermal capacity was neglected.

It was also observed that an increase in power dissipated in the transformer components caused an increase of the value of the relative error of calculations for both the considered models.

The presented results of calculations and measurements proved that the nonlinear thermal model of a pulse transformer proposed in this paper is able to accurately describe dynamic thermal properties of the transformer including cores with different dimensions and shapes and made of different ferromagnetic materials. It was also shown that using this model one can obtain accurate results of calculations at different conditions of power dissipation in the tested transformers.

It is also worth noticing that the obtained differences between the values of the temperatures of the transformer components can be equal to 50 °C. Such big differences justify the use different temperatures of the windings and the core in the thermal model. The linear thermal model makes it possible to obtain good agreement between the results of calculations and measurements only for very low powers dissipated in each component of the examined transformers. The nonlinear thermal model makes it possible to obtain good agreement between the results of calculations and measurements over a wide range of power dissipated in the windings or in the core of the examined transformers by considering the dependence of $R_{th}$ on power.

The CNTM of the pulse transformer proposed in this paper can be useful in designing switch-mode power supplies, and it can be used as a component of an electrothermal model of transformers.

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**Abbreviations and Notations**

- $R_{\text{th}}$: thermal resistance
- $C_{\text{th}}$: thermal capacitance
- $T_{W1}$: the temperature of the transformer primary winding
- $T_{W2}$: the temperature of the transformer secondary winding
- $T_{C}$: the temperature of the transformer core
- $T_{a}$: ambient temperature
- $Z_{\text{th}}(t)$: transient thermal impedance
- $\tau_{\text{th}}$: thermal time constant
- $a_{i}$: the dimensionless coefficient corresponding to a thermal time constant
- $N$: the number of thermal time constants
- RTP: toroidal powdered iron core
- RTN: toroidal nanocrystalline core
- RTF: toroidal ferrite core
- SPICE: Simulation Program with Integrated Circuits Emphasis
- CNTM: compact nonlinear thermal model
- CLTM: compact linear thermal model
- ANSYS: engineering simulation and 3D design software

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