The resonance $Y(4660)$ as a vector tetraquark and its strong decay channels

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(Dated: May 15, 2018)

The spectroscopic parameters and partial widths of the strong decay channels of the vector meson $Y(4660)$ are calculated by treating it as a bound state of a diquark and antidiquark. The mass and coupling of the $J^{PC} = 1^{--}$ tetraquark $Y(4660)$ are evaluated in the context of the two-point sum rules method by taking into account the quark, gluon and mixed condensates up to dimension ten. The widths of the $Y(4660)$ resonance’s strong $S$-wave decays to $J/\psi f_0(980)$ and $\psi(2S)f_0(980)$, as well as to $J/\psi f_0(500)$ and $\psi(2S)f_0(500)$ final states are computed. To this end, strong couplings in the relevant vertices are extracted from QCD sum rules on the light-cone supplemented by the technical methods of the soft approximation. The obtained result for the mass of the resonance $m_Y = 4677^{+71}_{−63}$ MeV, and prediction for its total width $\Gamma_Y = (64.8^{+10.8}_{−10.4})$ MeV are in nice agreements with the experimental information.

I. INTRODUCTION

Last fifteen years were very fruitful for hadron physics due to valuable information on properties of the hadrons collected by numerous experimental collaborations, and owing to new theoretical ideas and predictions that extended boundaries of our knowledge about the quark-gluon structure of elementary particles. An observation of the resonances that may be interpreted as four and five quarks’ states is one of most interesting discoveries to be mentioned among these achievements. Strictly speaking, existence of the multiquark states does not contradict the fundamental principles of QCD and was foreseen in the first years of QCD [1], but only results of the Belle Collaboration about the narrow resonance $X(3872)$ placed the physics of multiquark hadrons on a firm basis of experimental data [2]. Now experimentally detected and theoretically investigated four-quark resonances form a family of the particles known as $XYZ$ states [3,4].

The resonance $Y(4660)$ which is the subject of the present study was observed for the first time by the Belle Collaboration in the process $e^+e^- \rightarrow \gamma\psi(2S)\pi^+\pi^−$ via initial state radiation (ISR) as one of two resonant structures in the $\psi(2S)\pi^+\pi^−$ invariant mass distribution [5, 6]. The second state discovered in this experiment received the label $Y(4360)$. The analysis carried out there showed that these structures cannot be interpreted as known charmonium states. The measured mass and total width of the resonance $Y(4660)$ are [6]

\[
\begin{align*}
  m_Y &= 4652 \pm 10 \pm 8 \text{ MeV}, \\
  \Gamma_Y &= 68 \pm 11 \pm 1 \text{ MeV}.
\end{align*}
\]

The state $Y(4630)$, which is usually identified with the $Y(4660)$, was detected in the process $e^+e^- \rightarrow \Lambda_c^+\Lambda_c^−$ as a peak in the $\Lambda_c^+\Lambda_c^−$ invariant mass distribution [7]. Making assumption on a resonance nature of this peak it mass and width were found equal to $m_Y = 4634^{+5}_{−3}(\text{stat.})^{+3}_{−8}(\text{sys.})$ MeV and $\Gamma_Y = 92^{+40}_{−23}(\text{stat.})^{+10}_{−21}(\text{sys.})$ MeV, respectively. Independent confirmation of the $Y(4630)$ state came from the BaBar Collaboration [8], where the same process $e^+e^- \rightarrow \gamma\psi(2S)\pi^+\pi^−$ was studied and two resonant structures were fixed in the $\pi^+\pi^−\psi(2S)$ invariant mass distribution. Their mass and width confirm that these structures can be identified with resonances $Y(4630)$ and $Y(4360)$. Besides two resonances under discussion there are also states $Y(4260)$ and $Y(4390)$ which together constitute the family of at least four $Y$ hidden-charmed particles with $J^{PC} = 1^{--}$.

The numerous theoretical articles claiming to interpret the $Y(4660)$ and $Y(4360)$ embrace variety of models and schemes available in high energy physics. Thus, attempts were made to consider the new resonance $Y(4660)$ as an excited state of conventional charmonium: in Refs. [9] and [10] it was interpreted as the excited $5^3S_1$ and $6^3S_1$ charmonia, respectively. In order to explain the experimental information on the resonance $Y(4660)$ it was examined as a compound of the scalar $f_0(980)$ and vector $\psi(2S)$ mesons [11, 12] or as a baryonium state [13, 14]. The hadro-charmonium model for these resonances was suggested in Ref. [15].

The most popular models for the states $Y(4360)$ and $Y(4660)$, however, are the diquark-antidiquark models which suggest that these resonances are tightly bound states of a diquark and a antidiquark with required quantum numbers. Within this picture the resonance $Y(4360)$ was analyzed in Ref. [16] as an excited $1P$ tetraquark built of an axial-vector diquark and antidiquark, whereas $Y(4660)$ (and also $Y(4630)$) was found to be the $2P$ state of scalar diquark-antidiquark. Calculations there were carried out in the context of the relativistic diquark picture. The resonance $Y(4360)$ was interpreted as a radial excitation of the tetraquark $Y(4008)$ in Ref. [18]. The similar idea but in the framework of QCD sum rules method was realized in Ref. [19]: the $Y(4660)$ was considered as the $P$-wave $[cs][\bar{c}\bar{s}]$ state and modeled by $C_{75} \otimes D_{a75} C$ type interpolating current, where $C$ is the
II. MASS AND COUPLING OF THE VECTOR TETRAQUARK $Y(4660)$

We calculate the mass and coupling of the resonance $Y(4660)$ within QCD two-point sum rules approach by introducing the correlation function

$$\Pi_{\mu\nu}(p) = i \int d^4 x e^{ipx} \langle 0 | T \{ J_\mu(x) J_\nu^\dagger(0) \} | 0 \rangle, \tag{2}$$

and using technical tools which are standard for such kind of problems. We interpolate the resonance $Y(4660)$ employing the $C_\upgamma \gamma_5 \gamma_\mu C$ type current that is given by the expression

$$J_\mu(x) = \epsilon \overline{c} [ s^5_T(x) C \gamma_5 c(x) \overline{s}_d(x) \gamma_\mu \gamma_5 C \overline{c}^T(x) + s^5_T(x) C \gamma_\mu \gamma_5 c(x) \overline{s}_d(x) \gamma_5 C \overline{c}^T(x) ], \tag{3}$$

where $\epsilon = \epsilon_{abc} \epsilon_{adc}$, and $a$, $b$, $c$, $d$ and $e$ are color indices.

In general, $\Pi_{\mu\nu}(p)$ has the following Lorentz decomposition

$$\Pi_{\mu\nu}(p) = \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right) \Pi_V(p^2) - \frac{p_\mu p_\nu}{p^2} \Pi_S(p^2), \tag{4}$$

where the invariant functions $\Pi_V(p^2)$ and $\Pi_S(p^2)$ are contributions of the vector and scalar states, respectively. Because we are interested only in the analysis of $\Pi_V(p^2)$ it is convenient to choose such a structure in Eq. (4) which accumulate effects due to only the vector particles. It is seen, that such Lorentz structure is $g_{\mu\nu}$. In fact, the terms proportional to $p_\mu p_\nu$ are forming owing to both the vector and scalar particles.

Deriving of the sum rules for the mass $m_Y$ and coupling $f_Y$ proceeds through two main stages: at the first step we express the correlation function in terms of the physical parameters of the tetraquark $Y(4660)$ which give rise to the function $\Pi_{\mu\nu}^{\text{phys}}(p)$. At the next phase we calculate $\Pi_{\mu\nu}(p)$ employing the interpolating current $J_\mu(x)$, contract relevant quark fields and replace the obtained propagators with their explicit nonperturbative expressions. As a result of these manipulations we get $\Pi_{\mu\nu}^{\text{phys}}(p)$ that depends on the various quark, gluon and mixed vacuum condensates. By invoking assumptions about the quark-hadron duality we can equate the functions $\Pi_{\mu\nu}^{\text{phys}}(p)$ and $\Pi_{\mu\nu}^{\text{OPF}}(p)$ to each other, fix invariant amplitudes corresponding to the chosen Lorentz structure, and after well known operations extract required sum rules.

Let us begin from the phenomenological side of the sum rules, i. e. from function $\Pi_{\mu\nu}^{\text{phys}}(p)$. We assume that the tetraquark $Y(4660)$ with the chosen quark content and diquark-antidiquark structure is the ground-state particle in its class. Then by introducing into Eq. (2) the full set of corresponding states, performing the integration over $x$ and isolating contribution to $\Pi_{\mu\nu}^{\text{phys}}(p)$ of the ground-state we obtain (for brevity, in formulas we use $Y \equiv Y(4660)$)

$$\Pi_{\mu\nu}^{\text{phys}}(p) = \left(\frac{\langle 0 | J_\mu(Y(p)) Y(p) | J_\nu^\dagger(0) \rangle}{m_Y^2 - p^2} + \ldots, \tag{5}$$

$$\Pi_{\mu\nu}^{\text{phys}}(p) = \left(\frac{\langle 0 | J_\mu(Y(p)) Y(p) | J_\nu^\dagger(0) \rangle}{m_Y^2 - p^2} + \ldots, \tag{5}$$

charge conjugation matrix. The tetraquark $\left[c s \overline{c} \overline{s}\right]$ with interpolating current $C_\upgamma \gamma_5 \gamma_\mu C$ was used in Ref. [20] to treat $Y(4660)$, and the mass of this state was evaluated employing QCD sum rules approach in a nice agreement with experimental data. There are many other interesting models of the vector resonances details of which can be found in the reviews (see, Refs. [3, 4]).

In general, the vector tetraquarks with different $P$ and $C$ parities can be built of using the five independent diquark fields with spin 0 and 1 and different $P$-parities [21]. This implies an existence of numerous diquark-antidiquark structures, and as a result different interpolating currents with the same quantum numbers $J^{P C} = 1^{--}$. Apart from works mentioned above, the masses of the vector tetraquarks with $J^{P C} = 1^{--}$ and quark content $[c s] [\overline{c} \overline{s}]$ ( $[c q] [\overline{c} \overline{q}]$ ) were computed in Ref. [21] using the two-point sum rules method and different interpolating currents, excluding ones with derivatives. The resonance $Y(4660)$ was investigated also in Refs. [22–24] by considering it as a tetraquark with $[c q] [\overline{c} \overline{q}]$ or $[c s] [\overline{c} \overline{s}]$ quark content and using the interpolating currents of $C_\upgamma \gamma_\mu C$ and $C \gamma_5 \gamma_\mu C$ types.

In the present work we treat the $Y(4660)$ resonance as the vector tetraquark with $[c s] [\overline{c} \overline{s}]$ content and calculate its mass and total width. The mass and coupling of $Y(4660)$ are computed by utilizing the two-point sum rules approach. The QCD sum rules method allows one to evaluate parameters of the hadrons, and is one of the powerful nonperturbative methods for investigation of their features [21–23]. It is suitable for studying not only conventional hadrons, but also multiquark systems. In our computations we take into account vacuum condensates up to dimension ten, which lead to reliable predictions for quantities of interest.

Another problem addressed in the present article is investigation of the $Y(4660)$ state’s strong decays. Some of possible decay channels of the vector tetraquarks were written down in Ref. [21]. Our aim is to evaluate the width of the main $S$-wave decays $Y \rightarrow J/\psi f_0(980)$, $Y \rightarrow \psi(2S) f_0(980)$, $Y \rightarrow J/\psi f_0(500)$ and $Y \rightarrow \psi(2S) f_0(500)$ of the resonance $Y(4660)$ and estimate its full width that can be confronted with existing data. To this end, we employ QCD sum rules on the light-cone (LCSR) in conjunction with a technique of the soft approximation [27, 28]. This approach was elaborated in Ref. [29] and used successfully to investigate strong decays of numerous tetraquarks.

This article is structured in the following manner: In the section [11] we calculate the mass $m_Y$ and coupling $f_Y$ of the vector $Y(4660)$ resonance using the two-point sum rule method and include into analysis the quark, gluon and mixed condensates up to dimension ten. The obtained results for these parameters are applied in Sec. [13] to evaluate strong couplings and widths of the $Y(4660)$ state’s partial $S$-wave decays. In Section [14] we present our conclusions. The Appendix contains technical details of calculations.
where $m_Y$ is the mass of $Y(4660)$, and dots show contribution of higher resonances and continuum. We simplify this formula by introducing the matrix element

$$\langle 0| J_\mu | Y(p) \rangle = m_Y f_Y \varepsilon_\mu$$

with $f_Y$ and $\varepsilon_\mu$ being the coupling and polarization vector of the resonance $Y(4660)$, respectively. After some simple calculations we get

$$\Pi_{\mu\nu}^{\text{Phys}}(p) = \frac{m_Y^2 f_Y^2}{m_Y^2 - p^2} \left( -g_{\mu\nu} + \frac{p_{\mu}p_{\nu}}{p^2} \right) + \ldots$$

It is evident that $\Pi_{\mu\nu}^{\text{Phys}}(p^2) = m_Y^2 f_Y^2 / (m_Y^2 - p^2)$ is the invariant amplitude that can be used later to derive sum rules.

In order to find $\Pi_{\mu\nu}^{\text{OPE}}(p)$ we follow the recipes that have just been outlined above and express it in terms of the quark propagators

$$\Pi_{\mu\nu}^{\text{OPE}}(p) = i \int d^4x e^{ipx} \varepsilon_\mu \varepsilon_\nu \varepsilon^{*} \varepsilon'^{*} \left\{ \text{Tr} \left[ \gamma_5 \bar{S} \gamma^\mu S_c'(x) \gamma_5 S_c(0) \right] \right\}$$

where

$$\bar{S}_c(x) = CS_T^c(x)C,$$

and $S_{c(s)}(x)$ is the heavy $c$-quark (the light $s$-quark) propagator.

The expressions of the quark propagators are well known and therefore we do not provide them here explicitly (see, for example Appendix in Ref. [31]). We calculate $\Pi_{\mu\nu}^{\text{OPE}}(p)$ by taking into account various vacuum condensates up to dimension ten and write the QCD counterpart of the phenomenological function $\Pi_{\mu\nu}^{\text{OPE}}(p^2)$ in terms of the corresponding spectral density $\rho(s)$

$$\Pi_{\mu\nu}^{\text{OPE}}(p^2) = \int_{M^2}^{\infty} \frac{\rho(s)}{s - p^2} ds,$$

where $M^2 = (m_c + m_s)^2$. Now to extract the required sum rules we equate these invariant amplitudes to each other, the Borel transformation to both sides of obtained expression to suppress contributions arising from higher resonances and continuum, and perform the continuum subtraction by utilizing the assumption about the quark-hadron quality. The second equality can be derived from the first one by acting on it by the operator $d/d(-1/M^2)$: these two equalities can be used to extract the sum rules for $m_Y$ and $f_Y$

$$m_Y^2 = \frac{\int_{M^2}^{s_0} ds d\rho(s) s e^{-s/M^2}}{\int_{M^2}^{s_0} d\rho(s) s e^{-s/M^2}},$$

$$f_Y^2 = \frac{1}{m_Y^2} \int_{M^2}^{s_0} d\rho(s) e^{(m_Y^2 - s)/M^2}.$$
Numerical computations confirm that at $M^2_{\text{min}}$ the convergence of the operator product expansion is fulfilled with high accuracy, and $R(4.8 \text{ GeV}^2) = 0.017$, which is estimated employing the sum of last three terms, i.e. DimN = Dim8 + Dim9 + Dim10. Moreover, at $M^2_{\text{min}}$ the perturbative contribution amounts to more than 74% of the full result considerably overshooting the nonperturbative effects. The pole contribution is $\text{PC} = 0.16$, which is typical for sum rules involving multiquark aggregations. It is worth noting that PC at $M^2_{\text{min}}$ reaches its maximal value and becomes equal to 0.78.

In Figs. 1 and 2 we plot the sum rules’ predictions for $m_Y$ and $f_Y$, which visually demonstrate their dependence on the used values of $M^2$ and $s_0$. It is seen, that the dependence of the mass and coupling on the Borel parameter is very weak: the predictions for $m_Y$ and $f_Y$ demonstrate a high stability against changes of $M^2$ inside of the working interval. But $m_Y$ and $f_Y$ are sensitive to the choice of the continuum threshold parameter $s_0$. Namely this dependence generates a main part of uncertainties in the present sum rules, which, nevertheless remain within standard limits accepted for such kind of computations. From these studies we extract the mass and coupling of the resonance $Y(4660)$ as

$$m_Y = 4677^{+71}_{-63} \text{ MeV},$$
$$f_Y = (0.99 \pm 0.16) \cdot 10^{-2} \text{ GeV}^4. \quad (15)$$

Our result for $m_Y$ is in a reasonable agreement with experimental data [6]. The predictions for $m_Y$ and $f_Y$ will be used as the input parameters in the next section when investigating strong decays of the $Y(4660)$ resonance.
III. STRONG DECAYS OF THE RESONANCE \(Y(4660)\)

The strong decays of the tetraquark \(Y(4660)\) can be fixed using the kinematical restriction which is evident from Eq. (15). Because we are interested in \(S\)-wave decays of \(Y(4660)\) the spin in these processes should be conserved. Another constraint on possible partial decay modes of the \(Y(4660)\) tetraquark is imposed by \(P\)-parities of the final particles. Performed analysis allows us to see that partial decays to \(J/\psi f_0(980)\), \(\psi(2S)f_0(980)\) and \(J/\psi f_0(500)\), \(\psi(2S)f_0(500)\) are among important decay modes of \(Y(4660)\).

The \(Y(4660)\) resonance’s decays contain in the final state the scalar mesons \(f_0(980)\) and \(f_0(500)\), which we are going to treat as diquark-antidiquark states. The interpretation of the mesons belonging to the light scalar nonet as four-quark systems is not new and starts from analysis of Refs. \[3, 32\]. In the model suggested recently in Ref. \[33\] the isoscalar mesons \(f_0(980)\) and \(f_0(500)\) are considered as mixtures of the basic tetraquark states \(L = [ud] [ud]\) and \(H = ([su][su] + [ds][ds]) / \sqrt{2}\).

Calculations performed using this new model led to reasonable predictions for the mass and full width of the \(Y(4660)\) resonance to the \(f_0(980)\) and \(f_0(500)\) mesons within the same framework, because both of them interact with \(Y(4660)\) through their \(H\) components.

We concentrate on the decays \(J/\psi f_0(980)\) and \(\psi(2S)f_0(980)\), and calculate the strong couplings \(g_{YJ/\psi f_0(980)}\) and \(g_{Y\psi f_0(980)}\) corresponding to the vertices \(YJ/\psi f_0(980)\) and \(Y\psi(2S)f_0(980)\), respectively. For these purposes we employ the LCSR method and consider the correlation function

\[
\Pi_{\mu \nu}(p, q) = i \int d^4x e^{ipx} \langle f_0(q) | T \{J^\psi_\mu(x), J^\nu_\nu(0)\} | 0 \rangle, \tag{16}
\]

where \(J^\psi_\nu(x)\) and \(J^\nu_\nu(x)\) are the interpolating currents to \(Y(4660)\) and \(J/\psi\), respectively. The current \(J_\nu(x)\) has been defined in Eq. (3), whereas \(J^\nu_\nu(x)\) is given by the expression

\[
J^\nu_\nu(x) = \bar{c}_i(x)i\gamma_\mu e_\mu(x). \tag{17}
\]

In the vertices \(p, q\) and \(p' = p + q\) are the momenta of \(J/\psi\) or \(\psi(2S)\), \(f_0(980)\) and \(Y(4660)\), respectively.

In order to derive the sum rules for \(g_{YJ/\psi f_0(980)}\) and \(g_{Y\psi f_0(980)}\) we first calculate \(\Pi_{\mu \nu}(p, q)\) in terms of the physical parameters of involved particles. It is not difficult to get

\[
\Pi_{\mu \nu}^{\text{Phys}}(p, q) = \frac{\langle 0 | J^\psi_\mu(p) | J/\psi(p) f_0(q) \rangle | Y(p') \rangle}{p^2 - m_J^2} \times \frac{\langle Y(p') | J^\nu_\nu(0) \rangle}{p^2 - m_Y^2} + \frac{\langle 0 | J^\psi_\mu(p) | (2S)(p) \rangle}{p^2 - m_{2S}^2} \times \frac{\langle (2S)(p) f_0(q) | Y(p') \rangle}{p^2 - m_Y^2} \times \langle Y(p') | J^\nu_\nu(0) \rangle \ldots, \tag{18}
\]

where \(m_J\) and \(m_\psi\) are the masses of the mesons \(J/\psi\) and \(\psi(2S)\), respectively. The dots in Eq. (18) denote a contribution of the higher resonances and continuum states. As is seen, \(\Pi_{\mu \nu}^{\text{Phys}}(p, q)\) contains two terms and corresponds to the "ground-state+first radially excited state + continuum" scheme.

Further simplification of \(\Pi_{\mu \nu}^{\text{Phys}}(p, q)\) can be achieved by employing the matrix element (3) and new ones from Eq. (19)

\[
\langle 0 | J^\psi_\mu(p) | J/\psi(p) f_0(q) \rangle = f_J m_J \varepsilon_\mu,
\]

\[
\langle 0 | J^\psi_\mu(p) | (2S)(p) \rangle = f_\psi m_\psi \varepsilon_\mu, \tag{19}
\]

as well as by introducing two elements that describe the vertices

\[
\langle J/\psi(p) f_0(q) | Y(p') \rangle = g_{YJ/\psi f_0(980)} [(p \cdot p') \times (\varepsilon^* \cdot \varepsilon') \times (p^* \cdot p') \times (\varepsilon^* \cdot \varepsilon') \times (p \cdot \varepsilon') (p' \cdot \varepsilon^*). \tag{20}
\]

In the expressions above \(f_J(f_\psi)\) is the \(J/\psi(\psi(2S))\) meson’s decay constant, and \(\varepsilon_\mu\) and \(\varepsilon^*_v\) are the polarization vectors of the \(J/\psi(\psi(2S))\) mesons and the resonance \(Y(4660)\), respectively.

Then the correlation function takes the following form

\[
\Pi_{\mu \nu}^{\text{Phys}}(p, q) = \frac{g_{YJ/\psi f_0(980)} f_J m_J f_Y m_Y}{(p^2 - m_Y^2)} \left( -p' \mu \nu \right) + \frac{m_Y^2 + m_2^2}{2 g_{\mu \nu}} \times \frac{g_{Y\psi f_0(980)} f_\psi m_\psi f_Y m_Y}{(p^2 - m_Y^2)} \left( -p' \mu \nu \right) \times \frac{m_\psi^2 + m_\psi^2}{2 g_{\mu \nu}} + \ldots. \tag{21}
\]

We extract the sum rules for the strong couplings using the invariant functions corresponding to the structure \(~g_{\mu \nu}\). The correlation function \(\Pi_{\mu \nu}(p, q)\) contains inside of the \(\mathcal{T}\)-operation a tetraquark and a conventional meson currents, therefore this situation does not differ considerably from analysis of the tetraquark-meson-meson vertices elaborated in Ref. [29]. These vertices can be investigated using the \(q \to 0\) limit of the full LCSR method, which is known as the "soft-meson approximation" [28, 30]. This approximation was applied numerously to study decays of the tetraquarks, for example, in Refs. [31, 32].
In soft approximation $p = p'$ and invariant function $\Pi_{\text{Phys}}(p^2, p^2)$ that in the general case depends on two variables reduces to $\Pi_{\text{Phys}}(p^2)$. In this approach we replace $1/[(p'^2 - m_1^2) (p^2 - m_2^2)]$ by the double pole factor $1/[(p^2 - m_1^2)^2]$, where $m_1^2 = (m_1^2 + m_2^2)/2$. The same is true also for the second term in Eq. (21) with clear replacement $m_1^2 \to m_2^2 = (m_1^2 + m_2^2)/2$. Then the Borel transformation of the $\Pi_{\text{Phys}}(p^2)$ reads

\[
\text{B} \Pi_{\text{Phys}}(p^2) = g_{Yf(980)} f_{IM} f_{M} f_{Y} m_{Y} m_{1} m_{2} e^{-m_{1}^2/M^2} M^2
\]

+ $g_{Yf(980)} f_{\psi} m_{\psi} f_{Y} m_{Y} m_{2} e^{-m_{2}^2/M^2} M^2$ . . . . (22)

At the next step one has to find expression of the correlation function in terms of the quark propagators. After some calculations we get

\[
\Pi_{\mu\nu}^{OPE}(p, q) = \int d^4 x e^{ipx} e^{\gamma_{\mu}(\bar{\phi}S_{\gamma}(x)\gamma_{\nu}) \times \bar{\phi}S_{\gamma}(x)\gamma_{\rho} \sigma_{\lambda\rho}(\bar{\phi}S_{\lambda}(x)\gamma_{\alpha}))_{\alpha\beta} \times \langle f_{0}(0) | \bar{\phi}S_{\alpha}(0) s_{\beta}^{q}(0) | 0 \rangle, \tag{23}
\]

where $\alpha$ and $\beta$ are the spinor indices.

The matrix element $\langle f_{0}(0) | \bar{\phi}S_{\alpha}(0) s_{\beta}^{q}(0) | 0 \rangle$ has to be rewritten in a form suitable for further analysis. To this end, we apply the expansion

\[
\bar{\phi}S_{\alpha}(0) s_{\beta}^{q}(0) \rightarrow \frac{1}{12} \delta_{\beta d} \bar{\gamma}^{\beta} \delta_{\alpha} \left( \mathbf{1} \Gamma^{d} s \right), \tag{24}
\]

where $\Gamma^{d} = 1, \gamma_{5}, \gamma_{\lambda}, i \gamma_{5} \gamma_{\lambda}, \sigma_{\lambda\rho}/\sqrt{2}$ form the full set of Dirac matrices, and express $\Pi_{\mu\nu}^{OPE}(p, q)$ in terms of the local matrix elements of the scalar meson $f_{0}(980)$. Calculations prove that the matrix elements with $\Gamma^{d} = 1, \gamma_{5}$ and $i \gamma_{5} \gamma_{\lambda}$, i.e. ones that lead to traces with odd number of $\gamma_{5}$ matrices are identically equal to zero. The matrix elements in Eq. (21) with $\gamma_{5}$ and $\sigma_{\lambda\rho}/\sqrt{2}$ should be proportional to $q_{5}$ and $q_{5} \gamma_{\rho}$, because only the momentum of $f_{0}(980)$ has the required Lorentz index. But in the soft approximation $q = 0$, and therefore these elements do not contribute to $\Pi_{\mu\nu}^{OPE}(p, q)$. In the matrix element with $\sigma_{\lambda\rho}/\sqrt{2}$ components $\sim g_{5}$ may lead to some effects, but in the present work we neglect them. We also ignore matrix elements $\sim G$ with insertions of the gluon field strength tensor, contributions of which in the soft approximation, as a rule vanish. Hence, the only matrix element that we take into account is

\[
\langle f_{0}(980) | \bar{\phi}(0) s(0) | 0 \rangle = \lambda_{f}, \tag{25}
\]

which forms the correlation function $\Pi_{\mu\nu}^{OPE}(p, q = 0)$. The $\lambda_{f}$ and the similar matrix element $\langle f_{0}(980) | \bar{\phi}(0) s(0) | 0 \rangle = \lambda_{f}$ can be computed using the two-point sum rule method details of which are presented in the Appendix.

After standard calculations for the Borel transformed correlation function $\Pi_{\mu\nu}^{OPE}(M^2)$ we find

\[
\Pi_{\mu\nu}^{OPE}(M^2) = \frac{\lambda_{f}}{24 \pi^2} \int_{4m^2_{s}}^{\infty} \frac{ds}{s} \sqrt{s(s - 4m^2_{s})}
\]

\[
x(s + 8m^2_{s}) + \lambda_{f} \int_{0}^{1} \frac{dz e^{-4m^2_{s}/s^2} F(z, M^2)}{s^2} \tag{26}
\]

where the first term is the perturbative contribution, whereas the nonperturbative effects are encoded by the second term. The function $F(z, M^2)$ in Eq. (26) has the following form

\[
F(z, M^2) = - \frac{\langle \alpha_{s} G^2 / \pi \rangle m_{2}^4}{72 M^4} \frac{1}{Z} \left[ m_{c}^2 (1 - 2Z) - M^2 Z (3 - 7Z) \right]
\]

\[
- \frac{\langle g_{Y}^{3} \rangle}{45 \cdot 2^{9} \pi^{2} M s Z^{5}} \times \left\{ m_{c}^2 (1 - 2Z) (9 - 11Z) + 2m_{c}^4 M^{2} Z^{2} \times [-42 + Z (122 + 9Z)] - 2 M^{6} Z^{3} \times [6 - Z (22 - 9Z)] + m_{c}^4 M^{2} Z (-11 + 11Z - 190Z^{2}) \right\} + \frac{\langle \alpha_{s} G^2 / \pi \rangle}{648 M_{0}^{10} Z^{3}} \left[ m_{c}^4 - m_{c}^2 M^{2} \times (1 + 4Z) + 2 M^{2} Z (2 - Z) \right], \tag{27}
\]

where $Z = z(1 - z)$.

The perturbative term in Eq. (26) is calculated as an imaginary part of the relevant term in $\Pi_{\mu\nu}^{OPE}(p, q = 0)$, and afterwards the Borel transformation are carried out. The Borel transformation of the nonperturbative contribution is computed directly from $\Pi_{\mu\nu}^{OPE}(p, q = 0)$ and contains vacuum condensates up to dimension eight. By equating $\text{B} \Pi_{\text{Phys}}(p^2)$ to $\Pi_{\mu\nu}^{OPE}(M^2)$ and performing the continuum subtraction we find an expression that depends on two unknown variables $g_{Y} f_{f(980)}$ and $g_{Y} \psi f_{f(980)}$. Let us note that continuum subtraction in the perturbative part is done by $\rightarrow s_{0}$ replacement. The nonperturbative contribution preserves its original version, because all terms in Eq. (27) are proportional to inverse powers of the Borel parameter $M^2$, and in accordance with accepted methodology (see, Ref. [28]) they should be left in an unsubtracted form. The second equation necessary for our purposes can be derived by applying the operator $d/d(1/M^2)$ to both sides of this expression. These two equalities allow us to find sum rules for both $g_{Y} f_{f(980)}$ and $g_{Y} \psi f_{f(980)}$, explicit formulas of which are cumbersome to be presented here.

The width of the decay process, for example $Y \rightarrow \psi(2S) f_{0}(980)$ can be found by means of the formula

\[
\Gamma(Y \rightarrow \psi(2S) f_{0}(980)) = \frac{g_{Y} \psi f_{f(980)} m_{\psi}^{2}}{24 \pi} \times \Lambda \left( 3 + \frac{2 \Lambda^{2}}{m_{\psi}^{2}} \right), \tag{28}
\]

where $\Lambda = \Lambda(m_{Y}, m_{\psi}, m_{f_{0}})$ and

\[
\Lambda(a, b, c) = \sqrt{a^{4} + b^{4} + c^{4} - 2 (2ab^{2} + a^{2}c^{2} + b^{2}c^{2})} / 2a.
\]
The numerical computations of the strong couplings are performed using the values of the different vacuum condensates (see, Sec. II), as well as spectroscopic parameters of the mesons $J/\psi$ and $\psi(2S)$ (in units of MeV): $m_J = 3096.900 \pm 0.006$ and $f_J = 411 \pm 7$, $m_\psi = 3868.097 \pm 0.005$ and $f_\psi = 279 \pm 8$. The parameters of the resonance $Y(4660)$ have been found in the present work, and for the mass of the $f_0(980)$ meson we use its experimentally measured value $m_{f_0} = 990 \pm 20$ MeV. The parameters $M^2$ and $s_0$ are varied inside of the regions: $M^2 = (4.9 - 6.8)$ GeV$^2$ and $s_0 = (23.2 - 25.2)$ GeV$^2$.

The obtained results for the strong couplings read

$$|g_{YJf_0(980)}| = (0.22 \pm 0.07) \text{ GeV}^{-1},$$
$$|g_{Y\psi f_0(980)}| = (1.22 \pm 0.33) \text{ GeV}^{-1}. \quad (29)$$

Then widths of the corresponding partial decay channels become equal to (in units of MeV):

$$\Gamma(Y \to J/\psi f_0(980)) = 18.8 \pm 5.4,$$
$$\Gamma(Y \to \psi(2S)f_0(980)) = 30.2 \pm 8.5. \quad (30)$$

Analysis of the remaining two decays does not differ from previous ones and leads to predictions

$$|g_{YJf_0(500)}| = (0.07 \pm 0.02) \text{ GeV}^{-1},$$
$$|g_{Y\psi f_0(500)}| = (0.18 \pm 0.05) \text{ GeV}^{-1}, \quad (31)$$

and (in MeV)

$$\Gamma(Y \to J/\psi f_0(500)) = 2.7 \pm 0.7,$$
$$\Gamma(Y \to \psi(2S)f_0(500)) = 13.1 \pm 3.7. \quad (32)$$

The total width of the $Y(4660)$ resonance estimated using these four strong decay channels

$$\Gamma_Y = (64.8 \pm 10.8) \text{ MeV} \quad (33)$$

is in a nice agreement with the experimental value $68 \pm 11 \pm 1$ MeV. For the total width of the $Y(4660)$ resonance the Particle Data Group provides the world average $\Gamma_Y = 72 \pm 11$ MeV [31]. This is higher than the result of Ref. [2], nevertheless within uncertainties of theoretical calculations and errors of experimental measurements the prediction obtained here is compatible with the world average, as well. One has also to take into account that the diquark-antidiquark model for the $Y(4660)$ implies existence of the $S$-wave decay channels $Y(4660) \to D^{\pm}_sD^{\mp}_s(2460)$ and $Y(4660) \to D^{*\pm}_sD^{\mp}_s(2317)$ that also contribute to $\Gamma_Y$, and may improve this agreement.

**IV. CONCLUSIONS**

In the present work we have calculated the mass and full width of the vector resonance $Y(4660)$ by interpreting it as the diquark-antidiquark state with quantum numbers $J^{PC} = 1^{-+}$. Our results for its mass and width are in a nice agreement with data. We have found the full width of this resonance by taking into account its $S$-wave strong decays $Y \to J/\psi f_0(500)$, $Y \to \psi(2S)f_0(500)$, $Y \to J/\psi f_0(980)$ and $Y \to \psi(2S)f_0(980)$. However, the process $Y(4660) \to \psi(2S)\pi^+\pi^-$ is the only decay mode of the state $Y(4660)$ observed experimentally. It is known that the dominant decay channels of the $f_0(500)$ and $f_0(980)$ mesons are processes $f_0 \to \pi^+\pi^-$ and $f_0 \to \pi^0\pi^0$. Therefore, the chains $Y(4660) \to \psi(2S)f_0(980) \to \psi(2S)\pi^+\pi^-$ and $Y(4660) \to \psi(2S)f_0(500) \to \psi(2S)\pi^+\pi^-$ explain a dominance of observed final state in the decay of the resonance $Y(4660)$. In the tetraquark model, as we have seen the width of the channel $Y(4660) \to J/\psi f_0(980)$ is sizable. Additionally, the final states $\psi(2S)\pi^0\pi^0$ and $J/\psi\pi^0\pi^0$ should also be detected. But neither $J/\psi\pi^+\pi^-$ nor $\pi^0\pi^0$ final states were observed in the $Y(4660)$ decays. It is worth noting that most of aforementioned final particles were discovered in decays of the vector resonance $Y(4260)$: its partial decays to $J/\psi\pi^+\pi^-$ and $J/\psi\pi^0\pi^0$, as well as to $J/\psi K^+K^-$ were seen experimentally. Therefore, more accurate measurements may reveal these modes in decays of the resonance $Y(4660)$, as well.

A situation with decays to $D_s$ mesons is more difficult, because in the tetraquark model there are not evident reasons for these channels of the $Y(4660)$ state to be highly suppressed or even forbidden. Decays to a pair of $D$ mesons were not seen in the case of the resonance $Y(4260)$, as well. It is quite possible that partial widths of decays to $D_s$ mesons are numerically small. But this is only an assumption which must be confirmed by explicit calculations. Further experimental investigations of the $Y(4660)$ resonance, more precise measurements can enlighten problems with its decays channels and, as a result, with its nature.

**ACKNOWLEDGEMENTS**

H. S. and K. A. thank TUBITAK for the partial financial support provided under Grant No. 115F183.

**Appendix:** The local matrix elements

In this Appendix we calculate the couplings $\lambda_f$ and $\lambda_{f'}$ (hereafter $f = f_0(500)$ and $f' = f_0(980)$) defined as the matrix elements of the current $J_\mu(x) = \bar{s}(x)s(x)$ sandwiched between the exotic meson and vacuum states

$$\langle f(q)|\bar{s}s|0\rangle = \lambda_f, \quad \langle f'(q)|\bar{s}s|0\rangle = \lambda_{f'}. \quad (A.1)$$

To this end, we explore the two-point correlation function (see, for example, Ref. [40])

$$\Pi^{f(f')}(q) = i \int d^4xe^{iqx}[\mathcal{M}(J^{f(\bar{f})}(x)J^{f(\bar{f})}_\mu(0))]|0\rangle, \quad (A.2)$$

where $J^{f(\bar{f})}(x)$ is the interpolating current for the scalar tetraquark $f$ or $f'$. In the two-angles mixing scheme
these currents are given by the expression \[34\]
\[
\begin{pmatrix}
J^I(x) \\
J^{I'}(x)
\end{pmatrix} = U(\varphi_H, \varphi_L)
\begin{pmatrix}
J^H(x) \\
J^L(x)
\end{pmatrix},
\] (A.3)
where \(U(\varphi_H, \varphi_L)\) is the mixing matrix
\[
U(\varphi_H, \varphi_L) = \begin{pmatrix}
\cos \varphi_H & -\sin \varphi_L \\
\sin \varphi_H & \cos \varphi_L
\end{pmatrix},
\] (A.4)
which is responsible also for the couplings’ mixing.

The currents \(J^I(x)\) and \(J^H(x)\) correspond to the basic states \(\mathbf{L} = [ud][}\overline{ud}]\) and \(\mathbf{H} = ([su][}\overline{su}] + [ds][}\overline{ds}] / \sqrt{2}\) and have the following forms
\[
J^H(x) = \frac{\bar{e}e}{\sqrt{2}} \left\{ [u^T_a(x)C\gamma_5 s_b(x)] [\vec{\tau}_c(x)\gamma_5 C\sigma^c_T(x)] \\
+ [\bar{d}^T_a(x)C\gamma_5 s_b(x)] [\vec{\tau}_c(x)\gamma_5 C\sigma^c_T(x)] \right\},
\] (A.5)
and
\[
J^L(x) = \frac{\bar{e}e}{\sqrt{2}} \left\{ [u^T_a(x)C\gamma_5 s_b(x)] [\vec{\tau}_c(x)\gamma_5 C\sigma^c_T(x)] \right\},
\] (A.6)
where \(\bar{e}e = \epsilon^{abc}d_{ab}d_{c}\).

As an example, let us write down all expressions for the \(f\) meson. In order to find the phenomenological side of the sum rule we use the "ground-state+continuum" scheme and get
\[
\Pi^{\text{Phys}}(q) = \frac{\langle 0| J^I(x) | f(q) \rangle \langle f(q) | J^{I'}(0) | 0 \rangle}{m_f^2 - q^2} + \ldots,
\] (A.7)
where the dots traditionally stand for the higher resonances and continuum. We continue using explicit expressions of the matrix elements \(\langle 0| J^I(x) | f(q) \rangle\) and \(\langle f(q) | J^{I'}(0) | 0 \rangle\). The former element has just been introduced by Eq. (A.1), and after some manipulations can be recast to the final form
\[
\langle 0| J^I | f(q) \rangle = m_f(F_H \cos^2 \varphi_H + F_L \sin^2 \varphi_L).
\] (A.8)
During this process we have used the current \(J^I\) as it is given in Eq. (A.3), and also the matrix elements
\[
\langle 0| J^I | f(p) \rangle = F_i^0 m_f, \quad i = H, L.
\] (A.9)
We also benefited from the suggestion made in Ref. \[34\] that the couplings \(F_i^0\) follow a pattern of state mixing which in the two-angles mixing scheme implies
\[
\begin{pmatrix}
F^H_f \\
F^H_I \\
F^L_f \\
F^L_I
\end{pmatrix} = U(\varphi_H, \varphi_L)
\begin{pmatrix}
F^H_f \\
F^H_I \\
F^L_f \\
F^L_I
\end{pmatrix},
\] (A.10)
where \(F_H\) and \(F_L\) may be formally interpreted as couplings of the "particles" |\(H\) and |\(L\).

Then we get
\[
\Pi^{\text{Phys}}(q) = \frac{\lambda_f m_f(F_H \cos^2 \varphi_H + F_L \sin^2 \varphi_L)}{m_f^2 - q^2} + \ldots
\] (A.11)
The following task is computation of \(\Pi^{\text{OPE}}(q)\) which leads
\[
\Pi^{\text{OPE}}(q) = \cos \varphi_H \Pi^{\text{OPE}}_0(q),
\] (A.12)
where
\[
\Pi^{\text{OPE}}_0(q) = i^2 \int d^4xe^{ipx} \frac{\epsilon_{dac}\epsilon_{dec}}{6\sqrt{2}} \langle q\bar{q} \rangle \\
\times \text{Tr} \left[ \gamma_5 \bar{S}_q(-x) \bar{S}_b(x) \gamma_5 \right].
\] (A.13)
The matrix element \(\lambda_f\) can be evaluated from the sum rule
\[
\lambda_f = \frac{\Pi^{\text{OPE}}_0(M^2, s_0) \cos \varphi_H}{m_f(F_H \cos^2 \varphi_H + F_L \sin^2 \varphi_L)},
\] (A.14)
where \(\Pi^{\text{OPE}}_0(M^2, s_0)\) is the Borel transform of the correlation function \(\Pi^{\text{OPE}}_0(q)\). The matrix element of the \(f'\) meson can be computed by means of the same expression with trivial replacements \(m_f \rightarrow m_{f'}, \lambda_f \rightarrow \lambda_{f'}, \cos \varphi_H \rightarrow \sin \varphi_H\) and \(\varphi_L \rightarrow \cos \varphi_L\).

In numerical computations we have utilized the parameters of the \(f - f'\) system from Ref. \[34\], i.e. for the mixing angles we have used \(\varphi_H = -28^\circ.85 \pm 0^\circ.42\) and \(\varphi_L = -27^\circ.66 \pm 0^\circ.31\), whereas for the couplings \(F_H = (1.35 \pm 0.34) \cdot 10^{-3}\) GeV\(^2\) and \(F_L = (0.68 \pm 0.17) \cdot 10^{-3}\) GeV\(^4\) have been employed. The masses of the scalar particles \(m_f = (518 \pm 74)\) MeV and \(m_{f'} = (996 \pm 130)\) MeV have been borrowed from Ref. \[34\], as well. In calculations of \(\lambda_f\) the Borel and continuum threshold parameters have been chosen as \(M^2 = (0.75 - 1.0)\) GeV\(^2\) and \(s_0 = (0.8 - 1.1)\) GeV\(^2\), whereas in the case of \(\lambda_{f'}\) the form \(M^2 = (1.1 - 1.3)\) GeV\(^2\) and \(s_0 = (1.4 - 1.6)\) GeV\(^2\).

As a result we have found
\[
\lambda_f = (0.015 \pm 0.004)\text{ GeV}^2,
\]
\[
|\lambda_{f'}| = (0.052 \pm 0.013)\text{ GeV}^2,
\] (A.15)
which have been used in the section \[III\]
668, 1 (2017).
[5] X. L. Wang et al. [Belle Collaboration], Phys. Rev. Lett. 99, 142002 (2007).
[6] X. L. Wang et al. [Belle Collaboration], Phys. Rev. D 91, 112007 (2015).
[7] G. Pakhlova et al. [Belle Collaboration], Phys. Rev. Lett. 101, 172001 (2008).
[8] J. P. Lees et al. [BaBar Collaboration], Phys. Rev. D 89, 111103 (2014).
[9] G. J. Ding, J. J. Zhu and M. L. Yan, Phys. Rev. D 77, 014033 (2008).
[10] B. Q. Li and K. T. Chao, Phys. Rev. D 79, 094004 (2009).
[11] F. K. Guo, C. Hanhart and U. G. Meissner, Phys. Lett. B 665, 26 (2008).
[12] Z. G. Wang and X. H. Zhang, Commun. Theor. Phys. 54, 323 (2010).
[13] R. M. Albuquerque, M. Nielsen and R. Rodrigues da Silva, Phys. Rev. D 84, 116004 (2011).
[14] C. F. Qiao, J. Phys. G 35, 075008 (2008).
[15] G. Cotugno, R. Faccini, A. D. Polosa and C. Sabelli, Phys. Rev. Lett. 104, 132005 (2010).
[16] S. Dubynskiy and M. B. Voloshin, Phys. Lett. B 666, 344 (2008).
[17] D. Ebert, R. N. Faustov and V. O. Galkin, Eur. Phys. J. C 58, 399 (2008).
[18] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D 89, 114010 (2014).
[19] J. R. Zhang and M. Q. Huang, Phys. Rev. D 83, 036005 (2011).
[20] R. M. Albuquerque and M. Nielsen, Nucl. Phys. A 815, 53 (2009) Erratum: [Nucl. Phys. A 857, 48 (2011)].
[21] W. Chen and S. L. Zhu, Phys. Rev. D 83, 034010 (2011).
[22] Z. G. Wang, Eur. Phys. J. C 74, 2874 (2014).
[23] Z. G. Wang, Eur. Phys. J. C 76, 387 (2016).
[24] Z. G. Wang, arXiv:1803.05749 [hep-ph].
[25] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 147, 385 (1979).
[26] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 147, 448 (1979).
[27] I. I. Balitsky, V. M. Braun and A. V. Kolesnichenko, Nucl. Phys. B 312, 509 (1989).
[28] V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Ruckl, Phys. Rev. D 51, 6177 (1995).
[29] S. S. Agaev, K. Azizi and H. Sundu, Phys. Rev. D 93, 074002 (2016).
[30] S. S. Agaev, K. Azizi and H. Sundu, Phys. Rev. D 96, 034026 (2017).
[31] C. Patrignani et al. [Particle Data Group], Chin. Phys. C 40, 100001 (2016) and (2017).
[32] J. D. Weinstein and N. Isgur, Phys. Rev. Lett. 48, 659 (1982).
[33] H. Kim, K. S. Kim, M. K. Cheoun and M. Oka, arXiv:1711.08213 [hep-ph].
[34] S. S. Agaev, K. Azizi and H. Sundu, Phys. Lett. B 781, 279 (2018).
[35] S. S. Agaev, K. Azizi and H. Sundu, arXiv:1804.01726 [hep-ph].
[36] B. L. Ioffe and A. V. Smilga, Nucl. Phys. B 232, 109 (1984).
[37] S. S. Agaev, K. Azizi and H. Sundu, Phys. Rev. D 93, 114007 (2016).
[38] S. S. Agaev, K. Azizi and H. Sundu, Phys. Rev. D 95, 034008 (2017).
[39] S. S. Agaev, K. Azizi and H. Sundu, Phys. Rev. D 95, 114003 (2017).
[40] C. M. Zanetti, M. Nielsen and R. D. Matheus, Phys. Lett. B 702, 359 (2011).