Dissipation of non-linear circularly polarized Alfvén waves

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Abstract. We study propagating Alfvén waves by solving the time-dependent equations of magnetohydrodynamics (MHD) in one dimension numerically. In a homogeneous medium the circularly polarized Alfvén wave is an exact solution of the ideal MHD equations, and therefore it does not suffer from any dissipation. A high-amplitude linearly polarized Alfvén wave, on the other hand, steepens and forms current sheets, in which the Poynting flux is lost. In a stratified medium, however, a high-amplitude circularly polarized Alfvén wave can also lose a significant fraction of its Poynting flux.

1. Introduction

The acceleration of the fast solar wind is a long-standing problem in solar physics. It is likely that Alfvén waves play a significant role in this process (e.g. Leer et al. 1982). They can propagate over long distance, which allows them to reach the outer corona, because to lowest order they are incompressible and do not dissipate. However, dissipative damping is required at some point to avoid too high wind velocities (e.g. Holzer et al. 1983). A closer examination shows though that a linearly polarized Alfvén wave is compressible to second order, since the magnetic pressure, \( B^2_1/(2\mu_0) \), varies with half the wave length of the Alfvén wave itself (Alfvén & Fälthammar 1963). In a circularly polarized Alfvén wave, on the other hand, the magnetic pressure is constant along the wave, which is the physical reason why the circularly polarized Alfvén wave in a homogeneous medium is an exact solution to the nonlinear MHD equations.

Several groups have studied different mechanism through which Alfvén waves can dissipate. In two dimensions phase mixing due to a transverse gradient in the phase velocity (e.g. Heyvaerts & Prist 1983) can lead to a strong damping of the wave. This mechanism is not available in one dimension though, and one has then rather considered the nonlinear coupling of the Alfvén wave with other modes (e.g. Wentzel (1973); for circularly polarized Alfvén waves in particular see also Del Zanna et al. (2001)). Cohen & Kulsrud (1974) found in an approximate solution of the nonlinear MHD equations that the linearly polarized Alfvén wave steepens and can form current sheets. This result has
Figure 1. Magnetohydrodynamics waves propagating through a homogeneous medium and wave amplitude $\left(\frac{B_x}{B_0}\right)_0 = 0.01$. In a, b $\beta_{\text{mag}} = 17$ and in c, d $\beta_{\text{mag}} = 0.042$. We plot in a and c $\Delta \rho = \rho - \rho_0$ (solid line) and $v_z$ (dashed line) as a function of distance, $z$, at 2000 s. $\Delta \rho$ has been converted to velocity units by multiplying with $c_s/\rho_0$ in a and with $v_A/\rho_0$ in c, where $\rho_0$ is the background density. In b and d we plot $B_x$ (solid line) and $v_x$ (dashed line) as a function of distance $z$ at the same time. $B_x$ has been converted to velocity units by multiplying with $1/\sqrt{\mu_0 \rho_0}$.

later been confirmed by numerical simulations by Boynton & Torkelsson (1996) and Ofman & Davila (1997).

2. Results

In this paper we use the numerical code of Boynton & Torkelsson (1996) (see also Torkelsson & Boynton (1998)) to simulate propagating circularly polarized Alfvén waves. In the following we will consider both models with a constant background density, and stratified models. For all the simulations the temperature is $10^6$ K and the period of the waves is 300 s. The isothermal speed of sound is $1.29 \times 10^5$ m s$^{-1}$.

2.1. Unstratified models

For the unstratified models, we differentiate between two cases, when the magnetic field is weak, that is $\beta_{\text{mag}} = 17$ and $c_s > v_A$, and when it is strong that is $\beta_{\text{mag}} = 0.042$ and $v_A > c_s$. For both the runs $\rho = 1 \times 10^{-14}$ kg m$^{-3}$. We compare the runs in Fig. 1. If we compare our Fig 1a to Fig. 1 of Boynton & Torkelsson (1996) we notice that in both cases there is an acoustic precursor moving at a
speed close to $c_s$, but in their case the precursor is a sinusoidal oscillation in $\Delta \rho$ and $v_z$. Apparently the gradual increase of the magnetic pressure at the front of the linearly polarized Alfvén wave drives an oscillatory wave, while the sharp discontinuity of the magnetic pressure at the front of the circularly polarized wave works as a piston. The amplitude of the acoustic precursor in Fig. 1a scales with the square of the amplitude of the Alfvén wave.

When $v_A > c_s$ (Fig. 1c and 1d) a density enhancement propagates with the wave. $v_z$ and $\Delta \rho$ are then related by $v_z = v_A \frac{\Delta \rho}{\rho}$. While the right edge of the enhancement coincides with the front of the Alfvén wave, the left edge propagates with the velocity $c_s$. In the leftmost part of Fig. 1c we see the effect of the non-linear interaction between the acoustic wave and the Alfvén wave.

Boynton & Torkelsson (1996) found that when $B_1$ approaches $B_z$ in strength, the Alfvén wave starts to steepen into current sheets (Fig. 2a) which enhance the dissipation of the wave. The circularly polarized Alfvén wave does not develop such current sheets (Fig. 2b).

2.2. Stratified models

We discuss in this paper two models: one with $(\frac{B_x}{B_z})_0 = 0.1$ and one with $(\frac{B_x}{B_z})_0 = 1.0$, both have a background magnetic field $B_z = 3 \times 10^{-5}$ T.

At the bottom of the grid $v_A$ is smaller than $c_s$ and equal to $3.8 \times 10^4$ m s$^{-1}$, but it increases with $z$ and eventually becomes higher than $c_s$. Density is taken to be equal to $5 \times 10^{-13}$ kg m$^{-3}$ at the bottom of the corona and varies with $z$ as

$$\rho_0(z) = \rho_0(0) \exp\left(-\frac{R}{H} \frac{z}{R+|z|}\right)$$  \hspace{1cm} (1)

where $H_0 = 6.1 \times 10^7$ m is the scale height at $z = 0$, and $R$ is the solar radius.

The wave front of the Alfvén wave pushes matter upwards forming a density perturbation which behaves as an acoustic precursor as long as $v_A$ is smaller

Figure 2. Evolution of $B_z$ for linearly (a) and circularly (b) polarized Alfvén waves propagating through a homogeneous medium.
Figure 3. Magnetohydrodynamic waves propagating through a stratified medium with \(\left(\frac{B_y}{B_T}\right)_0 = 1\) and \(B_z = 3 \times 10^{-5} \, \text{T}\). a \(\Delta \rho = \rho - \rho_0\) (solid line) and \(v_z\) (dashed line) as a function of distance, \(z\) at time 4375 s. \(\Delta \rho\) has been converted to velocity units by multiplying with \(c_s/\rho_0\). b \(B_x\) (solid line) as a function of distance \(z\) at the same time. c The \(z\)-component of the Poynting flux vector at the same time.

than \(c_s\). When \(v_A\) becomes larger than \(c_s\) the Alfvén wave begins to overtake the precursor, which however persists and propagates forward with a speed close to the speed of sound (Fig. 3a). For a low amplitude Alfvén wave the density enhancement almost keeps its shape as it propagates while for a high amplitude wave the density enhancement changes gradually.

In Fig. 3c we see a decrease in the Poynting flux in front the acoustic precursor. Fig. 4 shows the time average over one Alfvén wave period of the Poynting flux and the integrated work it does via the Lorentz force. In the linearly polarized case the wave loses 70% of its Poynting flux below \(z = 0.4 \, R_\odot\) and very little of it is dissipated after that (see Boynton & Torkelsson (1996) Fig. 9) while in the circularly polarized case the wave loses very little of its Poynting flux below \(z = 0.5\), but it has lost almost half of it at \(z = 0.8 \, R_\odot\) by doing mechanical work on the plasma.

3. Conclusions

In constructing models of Alfvén wave driven stellar winds it is important to understand the mechanism by which the Alfvén waves are damped. A linearly polarized Alfvén wave of high amplitude can steepen and form current sheets
Figure 4. Time average over one Alfvén wave period of the Poynting flux (dashed line) and the integrated work done via the Lorentz force (solid line) for a circularly polarized Alfvén wave propagating through a stratified medium with \((\frac{B_x}{B_z})_0 = 1\).

even in a homogeneous medium, which leads to a quick damping of the wave. While a circularly polarized Alfvén wave does not dissipate its energy in a homogeneous medium, we have found that it may do so in a stratified medium. The dissipation sets in later than for a linearly polarized Alfvén wave though, but a significant fraction of its Poynting flux may still be spent on doing work on the medium through which the Alfvén wave propagates.

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