BLACK HOLES IN YOUNG STELLAR CLUSTERS

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ABSTRACT

We present theoretical models for stellar black hole (BH) properties in young, massive star clusters. Using a Monte Carlo code for stellar dynamics, we model realistic star clusters with N ≃ 5 × 10^5 stars and significant binary fractions (up to 50%) with self-consistent treatments of stellar dynamics and stellar evolution. We compute the formation rates and characteristic properties of single and binary BHs for various representative ages, cluster parameters, and metallicities. Because of dynamical interactions and supernova (SN) kicks, more single BHs end up retained in clusters compared to BHs in binaries. We also find that the ejection of BHs from a cluster is a strong function of initial density. In low-density clusters (where dynamical effects are negligible), it is mainly SN kicks that eject BHs from the cluster, whereas in high-density clusters (initial central density ρ_c(0) ∼ 10^5 M⊙ pc^{-3} in our models) the BH ejection rate is enhanced significantly by dynamics. Dynamical interactions of binary systems in dense clusters also modify the orbital period and eccentricity distributions while increasing the probability of a BH having a more massive companion.

Key words: galaxies: starburst – galaxies: star clusters: general – methods: numerical – stars: kinematics and dynamics

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1. INTRODUCTION

Young massive clusters (YMCs) and super star clusters (SSCs) are dense systems of young stars, which have received considerable interest over the past decade. Typically, YMCs are younger than ~100 Myr, more massive than ~10^5 M⊙ and have a density higher than ~10^3 M⊙ pc^{-3} (Portegies Zwart et al. 2010). Observationally, YMCs are found in a variety of environments including the Milky Way and the Local Group, starburst galaxies, and interacting galaxies, SSCs (≳10^5 M⊙) populate the most massive and luminous end in the continuum of YMCs. Elmegreen & Efremov (1997) and others have argued that the upper end of the luminosity function of globular clusters (GCs) is very similar to that observed for YMCs, suggesting that a universal formation mechanism might be responsible for the formation of star clusters in all environments and at all epochs.

In addition to providing predictions for X-ray sources in YMCs and SSCs, the theoretical study of primordial black hole (BH) populations in clusters can provide realistic initial conditions for investigating the long-term dynamical evolution of GCs. Mass segregation in GCs has often been studied previously using simple two-component models, where the BHs are treated as a tracer population of heavy objects in a background of equal-mass, lighter stars (e.g., Fregeau et al. 2002; Breen & Heggie 2013). Mass segregation is a consequence of the tendency toward equipartition of energy among stars of different masses. As the cluster evolves, the more massive objects (here BH remnants) concentrate lower in the gravitational potential well, i.e., closer to the cluster center, whereas the lighter stars gain energy and are displaced outward. While previous studies have shed light on this dynamical process in the context of simplified models, a more realistic study of BHs in GCs clearly requires a treatment of the full stellar mass spectrum (Morscher et al. 2013).

Past theoretical work on BHs in clusters has also been done using direct N-body simulations; these are always limited to small-N systems, however, in a range more appropriate for modeling open clusters. Portegies Zwart & McMillan (2000) investigated the formation of BH–BH binaries in star clusters through dynamical encounters, with a focus on predicting the merger rate for these binaries, which are key sources of gravitational waves detectable by the current generation of laser interferometers such as LIGO (Aasi et al. 2013). They found that, in their models with N ∼ 10^3, BHs sink to the core within ~10 Myr. In the core these BHs acquire binary companions; the binaries harden quickly through superelastic encounters with other stars or BHs and are ultimately ejected from the cluster. Aasi et al. (2013) predicted that the ejected BH binaries have very short orbital periods and high eccentricities, and will coalesce within a few gigayears, contributing significantly to the potential LIGO detection rate.

Mackey et al. (2007) studied the effect of stellar mass BHs on the dynamical evolution and structural parameters of Magellanic Cloud clusters using larger N-body simulations with N ∼ 10^5. However, their simulations did not include primordial binaries and assumed simple limiting cases of natal kicks imparted to BHs. They argued that both mass loss from early stellar evolution and longer-term heating by BH ejections and interactions can produce a core expansion with age, in agreement with observations of Magellanic Cloud clusters. More recently, Mapelli et al. (2013) used direct N-body simulations with N ∼ 10^4 to study the dynamics of stellar BHs in young star clusters with different metallicities, focusing on the implications for X-ray binaries (XRBs). We will return to this study and compare it to our results for much larger clusters in Sections 2 and 4.

Chandra observations of starburst galaxies uncovered large numbers of bright point X-ray sources (Fabbiano et al. 2001; Kaaret et al. 2001). Optical and infrared observations of these sources often revealed massive young clusters or SSCs associated with them (Zezas et al. 2002). From previous theoretical studies and the spectral and temporal variability of these
sources, it is now believed that many of these sources are bright XRBs formed in star clusters (Kaaret et al. 2001). From observations of the Antennae it has been concluded that the luminosity \( L_X \) of these X-ray sources can be divided into three classes (Zezas & Fabbiano 2002). The lowest luminosity sources with \( L_X < 3 \times 10^{38} \text{ erg s}^{-1} \) are thought to be supernova (SN) remnants or neutron star (NS) binaries from their steep X-ray spectra. Power-law spectra of sources with luminosities exceeding the Eddington luminosity for NS binaries \( \left( 3 \times 10^{39} \text{ erg s}^{-1} < L_X < 10^{39} \text{ erg s}^{-1} \right) \) resemble those of Galactic BH XRBs. Out of the 18 sources in this luminosity range in the Antennae, 10 sources are thought to be associated with a young star cluster (age \( < 100 \text{ Myr} \)). If \( L_X > 10^{39} \text{ erg s}^{-1} \), the sources are classified as ultraluminous X-ray sources (ULXs), possibly associated with more massive stellar BHs (MSBHs; Belczynski et al. 2004) or even intermediate-mass BHs (IMBHs). Out of the 49 X-ray sources detected in the Antennae, 18 are ULXs.

Kaaret et al. (2004) found significant statistical association between X-ray point sources and young stellar populations in the three starburst galaxies (M82, NGC 1569, and NGC 5253) they studied: the X-ray point sources are at distances of 30–100 pc from the young star clusters with which they are associated. However, there is an apparent lack of X-ray sources coincident with the clusters. Sepinsky et al. (2005) tried to explain this observation by arguing that a significant number of X-ray sources can be ejected from the parent cluster due to SN kicks. However, they used a simple population synthesis approach to study the effects of SN kicks on XRB populations, neglecting any cluster dynamics.

*Chandra* X-ray observations of the Galactic young cluster Westerlund 1 (Wd 1) have revealed a large population of X-ray sources (Clark et al. 2008). Wd 1 has an age of 4–5 Myr and is estimated to have a high binary fraction (Skinner et al. 2006; Clark et al. 2011). Observations of Wolf–Rayet and OB stars in Wd 1 have suggested a binary fraction approaching unity (Crowther et al. 2006) for massive stars (with initial masses \( > 45 M_{\odot} \)). In spite of the rich X-ray point source population in Wd 1, there is a lack of bright X-ray sources (\( L_X > 10^{38} \text{ erg s}^{-1} \)) in this cluster. This is in spite of the young cluster age, which would seem to imply the existence of many high-mass XRBs accreting from stellar winds of massive main-sequence (MS) companions. Several possible reasons have been offered to explain this discrepancy (Clark et al. 2005; Negueruela & Clark 2005), all involving combinations of binary stellar evolution and dynamics.

Clearly, a more detailed theoretical study of BHs in young star clusters is necessary to explain the statistics, spatial distributions, and luminosities of these X-ray sources. Our goal in this paper is to model, in a self-consistent manner, the populations of BHs in YMCs and SSCs, taking into account both stellar dynamics and stellar evolution. This is only a first step: detailed modeling of X-ray sources is beyond the scope of this study, which will focus instead on understanding the effects of dynamics in large clusters and the overall properties of all single and binary BHs, independent of whether they might be active X-ray sources or not. The initial conditions and stellar evolution prescriptions in our work include a full global initial mass function (IMF), significant binary fractions, and realistic natal kick prescriptions for BHs based on our current understanding of core collapse SNe.

Our paper is organized as follows. In Section 2 we discuss in more detail and summarize some of the previous theoretical work addressing this problem. In Section 3 we provide a detailed description of our stellar dynamics code and our stellar evolution prescriptions. In Section 4 we present and discuss our results. Finally, a summary and conclusions are presented in Section 5.

## 2. PREVIOUS STUDIES

### 2.1. BHs in Young Starburst Environments

Belczynski et al. (2004, hereafter B04) studied young populations of BHs as observed in starburst galaxies using a population synthesis approach. Their simulations did not explicitly take into account that the starburst was happening within a clustered environment. They investigated, for many representative models, the numbers of BH systems produced as well as their physical properties (e.g., binary period and BH mass distributions). They also studied, in great detail, the evolutionary channels leading to these different properties. They found that, soon after (within \( \sim 5 \text{ Myr} \)) the initial starburst most BHs were single, with only \( \sim 10\% \) in binaries. Single BHs were also formed from binary progenitors, when, e.g., the binary disrupted following a SN explosion, or underwent a merger following Roche lobe overflow (RLOF) and dynamically unstable mass transfer. In the initial few megayears, the most common BH binary systems were BH–MS systems. After the cluster had evolved for a significant period of time (\( > 20 \text{ Myr} \)) the most common BH binary systems were BH–BH systems. In B04, the overall mass distribution of the BH population was not affected much by most initial parameters, with the exception of metallicity. As expected, the highest-mass BHs were found in models with the lowest metallicity. In general, BH masses were found to be within the range \( 7 M_{\odot} \) to \( 25 M_{\odot} \) for both single and binary BHs. Moreover, single BHs set the overall shape of the BH mass distribution.

B04 also found that the orbital periods for the binaries in their models cover a broad range, \( P_{\text{orb}} \sim 0.1–10^6 \text{ days} \). The distribution had two distinct peaks, one at small periods, centered around 10 days, and another around \( 10^6 \text{ days} \). Moreover the orbital period distribution, like the mass distribution, was dependent on metallicity. With increasing metallicity the shorter-period BH binaries were suppressed, while more of the longer-period binaries survived.

Belczynski et al. (2006, hereafter B06) extended the population synthesis approach of B04 by including the possibility that BHs could be ejected from their original star cluster, depending on the recoil speed following SN explosions. They varied the assumed cluster potential and escape speed in their models over the full relevant range, taking into account that smaller clusters have escape speeds as low as \( \sim 10 \text{ km s}^{-1} \), while the largest clusters have escape speeds as high as \( \sim 100 \text{ km s}^{-1} \). B06 found that a significant fraction of BHs could be ejected from their cluster. At early times the number of BHs ejected increased with time as progressively less massive stars formed BHs which received larger kicks, removing them from the cluster. At later times (\( > 15 \text{ Myr} \)), no more BHs were ejected. As previously seen in B04, the orbital period of the retained BH population was bimodal, with only short-period binaries being ejected from their cluster (long-period binaries are more prone to disruption).

### 2.2. Ultraluminous X-Ray Sources (ULXs)

Observations of extragalactic ULXs can be interpreted in a number of ways. In this section we focus on the possible implications of the various interpretations for BHs in clusters.

The extremely high luminosities of some sources exceed the Eddington luminosity of most stellar mass BHs and hence
are incompatible with simple models of BH XRBs assuming isotropic emission. Anisotropic or beamed emission has been discussed extensively in the literature as a possible explanation for this apparent super-Eddington luminosity. King et al. (2001) have argued that ULXs might correspond to a phase of rapid mass transfer during the lifetime of the XRB. They have also demonstrated that this assumption along with mild beaming can explain ULXs with luminosity \( L_X < 10^{40} \text{ erg s}^{-1} \). Still there are some difficulties in explaining ULXs with luminosity \( L_X > 10^{40} \text{ erg s}^{-1} \) as a BH of moderate mass requires extreme beaming fractions to explain this high luminosity. In addition, this accreting BH scenario requires a very massive companion \( (q = M_2/M_1 > 1 \text{, where } M_1 \text{ is the mass of the BH and } M_2 \text{ is the mass of the companion}). \) Finally, there have been observations which claimed that emission from the ULXs might be isotropic (an isotropic nebula was observed around the ULX in Holmberg II (Kaaret et al. 2004) and ULX M81 X-9 (Miller et al. 2004)). Thus, whether or not an anisotropic emission can explain all ULXs remains quite uncertain.

Another suggestion from the X-ray spectrum is that the brightest ULXs observed in young starbursts environments might harbor a more massive BH with \( M_{BH} \sim 10^2-10^3 M_\odot \) (Colbert & Mushotzky 1999; Mushotzky 2000; van der Marel 2003; Miller et al. 2004), a so-called IMBH. Theoretically, runaway collisions during an early episode of core collapse (Gürkan et al. 2004; Portegies Zwart et al. 2004; Freitag et al. 2006; Goswami et al. 2012) provide a way to form IMBHs in young star clusters. An IMBH formed in the dense cluster environment can dynamically acquire a binary companion, but subsequent mass transfer leading to ULX formation is expected to occur only rarely (Blecha et al. 2006). The steep hard X-ray emission of the ULXs observed, is found to be consistent with that of an accreting BH in its soft (high) state. Observations of the accretion disk spectra also provide evidence for the existence of IMBHs (Mitsuda et al. 1984; Miller et al. 2003, 2004).

However, high-mass \( (\sim 80 M_\odot) \) stellar BHs formed in low metallicity environments also explain observations of some ULXs (Gonçalves & Soria 2006; Stobbart et al. 2006; Copperwheat et al. 2007; Mapelli et al. 2009; Zampieri & Roberts 2009). Mapelli et al. (2011) investigated the formation of MSBHs in star clusters. According to stellar evolution calculations (Fryer 1999; Heger & Woosley 2002; Heger et al. 2003), a star with a final mass \( M_{\text{fin}} > 40 M_\odot \) immediately before collapse could avoid a SN explosion and directly collapse to a BH of almost the same mass. Mapelli et al. (2011) simulated star clusters with a Salpeter IMF and King density profiles. They introduced a MSBH with a binary companion in their simulations assuming that such a massive BH had formed in the cluster at sufficiently low metallicity. They assumed that the MSBH should be in a binary since both its progenitor and the MSBH itself were among the most massive stars in the cluster (Kulkarni et al. 1993). They chose the mass of the MSBH to be a constant \( 50 M_\odot \). Moreover, the mass of the binary companion was chosen to be in the range \( 10 M_\odot < M_{\text{companion}} < 50 M_\odot \) in order to explain ULX luminosities (Patruno et al. 2005) with stable mass transfer (Rappaport et al. 2005). The separation \( a \) (0.1 AU < \( a < 10 \text{ AU} \)) was chosen such that the binary was not easily ionized during the evolution of the cluster. Instead they found that a large number of MSBHs were ejected from the cluster in less than 10 Myr because of close interactions. Moreover, the MSBHs ejected from the cluster retained their binary companions. Previous population synthesis studies such as that of Linden et al. (2010) suggested that MSBHs could hardly become RLOF high-mass XRBs due to the absence of natal kicks in their formation pathway. However, dynamical studies in Mapelli et al. (2011) suggested that these MSBHs formed through direct collapse could pass through RLOF high-mass X-ray binary (HMXB) phase since dynamical interactions had similar effects as natal kicks. Mapelli et al. (2011) concluded that dynamical interactions changed the orbital parameters of these MSBHs, favoring the occurrence of mass transfer.

Mapelli et al. (2013) investigated the evolution of young clusters (\( \sim 100 \text{ Myr} \)) using direct N-body simulations. They explored 3000–4000 \( M_\odot \) clusters with varying metallicities. Their models included metallicity-dependent stellar evolution recipes (Hurley et al. 2000), binary evolution using the Starlab code (Portegies Zwart & Verbunt 1996), and natal kicks imparted to the stellar remnants as implemented in Starlab (Portegies Zwart & Verbunt 1996). They found that three-body encounters, and especially exchange interactions, play an important role in the evolution of the massive (>25 \( M_\odot \)) BHs in all environments. Almost 75% of the massive (>25 \( M_\odot \)) BH population that were in a binary at some point, acquired a binary companion through dynamical exchanges. However, for lower mass BHs, this percentage was reduced to 20%. Moreover, they also found that all the BHs with a companion overflowing its Roche lobe, acquired their companions through dynamical exchange interactions. Clearly, these results suggest that a complex interplay between binary evolution and cluster dynamics is responsible for the formation of XRBs in these systems.

3. NUMERICAL SIMULATIONS

To study the dynamical evolution of BHs in young star clusters we have used our Cluster Monte Carlo (CMC) code, based on the classic work of Hénon (1971), as implemented and described in Fregéau & Rasio (2007, and references therein). To model stellar evolution CMC employs the single star evolution formulae of Hurley et al. (2000) and the binary star evolution (BSE) formulae of Hurley et al. (2002). We have made some modifications to the formulae used for determining the masses of compact remnants and to stellar wind prescriptions, which we outline in Sections 3.3.1 and 3.3.3. We also note that Hurley et al. (2002) derived their fitting formulae based on detailed models for stellar masses up to 100 \( M_\odot \). We have extended this to include stars up to 150 \( M_\odot \) in some of our simulations, covering the full range of theoretically predicted stellar masses from star formation (Weidner & Kroupa 2004).

3.1. Initial Conditions

Our basic cluster model starts with a King model with \( W_0 = 5 \) and initial \( N \) in the range \( 2 \times 10^3-5 \times 10^3 \) (typical to the extragalactic YMCs). In our simulations, we vary the initial binary fraction from \( f_b = 0\% \) to \( f_b = 50\% \). A binary system is described initially by four parameters: the mass of the primary \( (M_1) \) (the initially more massive component), the mass ratio \( q = M_2/M_1 \), where \( M_2 \) is the mass of the secondary (initially less massive component), the semi-major axis \( a \) of the orbit, and the orbital eccentricity \( e \). We assume that the initial distributions of these parameters are independent. For both single stars and binary system primaries we adopt the Kroupa IMF in the mass range \( M_1 = 0.08–100 M_\odot \). The masses of the secondary stars in the binary systems, are sampled from the mass ratio \( q \) assumed to be constant between 0 and 1. We have chosen two different initial binary separation distributions for this study, namely,
Table 1

Summary of all the Simulations with Metallicity $Z = 0.001$

| Name | $N_f$ | $f_b$ | $r_v$ | $\rho_c(0)$ | $t_{rh}(0)$ | Dynamics | $f_{\text{binj}}$ at 100 Myr |
|------|-------|-------|-------|-------------|-------------|----------|-----------------------------|
|      |       | (%)   | (pc)  | ($10^3$ $M_\odot$ pc$^{-3}$) | (10$^5$ yr) | (y/n)    | (~%)                        |
| A    | $5 \times 10^5$ | 50    | 1.25  | 1.3          | 2.0         | y        | 57                           |
| B    | $5 \times 10^5$ | 50    | n     | n            | n           | n        | n                            |
| A1   | $2 \times 10^5$ | 0     | 1.25  | 0.26         | 2.0         | y        | 0                            |
| B1   | $2 \times 10^5$ | 0     | 1.74  | 0.10         | 3.3         | y        | 0                            |
| C1   | $2 \times 10^5$ | 0     | 3     | 0.02         | 7.5         | y        | 0                            |
| D1   | $2 \times 10^5$ | 0     | n     | n            | n           | n        | n                            |
| A2   | $2 \times 10^5$ | 10    | 1.25  | 0.26         | 2.0         | y        | 0                            |
| B2   | $2 \times 10^5$ | 10    | 1.74  | 0.10         | 3.2         | y        | 20                           |
| C2   | $2 \times 10^5$ | 10    | 3     | 0.02         | 7.3         | y        | 15                           |
| D2   | $2 \times 10^5$ | 10    | n     | n            | n           | n        | n                            |
| A3   | $2 \times 10^5$ | 30    | 1.25  | 0.32         | 1.8         | y        | 33                           |
| B3   | $2 \times 10^5$ | 30    | 1.74  | 0.12         | 3.1         | y        | 25                           |
| C3   | $2 \times 10^5$ | 30    | 3     | 0.02         | 7.0         | y        | 15                           |
| D3   | $2 \times 10^5$ | 30    | n     | n            | n           | n        | n                            |
| A4   | $2 \times 10^5$ | 50    | 1.1   | 0.32         | 1.8         | y        | 32                           |
| B4   | $2 \times 10^5$ | 50    | 1.74  | 0.12         | 2.9         | y        | 20                           |
| C4   | $2 \times 10^5$ | 50    | 3     | 0.02         | 6.7         | y        | 15                           |
| D4   | $2 \times 10^5$ | 50    | n     | n            | n           | n        | n                            |
| I1   | $4 \times 10^5$ | 0     | 1.25  | 0.52         | 2.6         | y        | 0                            |
| J1   | $4 \times 10^5$ | 0     | 1.64  | 0.23         | 3.9         | y        | 0                            |
| K1   | $4 \times 10^5$ | 0     | 3     | 0.04         | 9.7         | y        | 0                            |
| L1   | $4 \times 10^5$ | 0     | 3     | n            | n           | n        | n                            |
| I2   | $4 \times 10^5$ | 10    | 1.25  | 0.52         | 2.6         | y        | 43                           |
| J2   | $4 \times 10^5$ | 10    | 1.64  | 0.24         | 3.8         | y        | 30                           |
| K2   | $4 \times 10^5$ | 10    | 3     | 0.04         | 9.4         | y        | 23                           |
| L2   | $4 \times 10^5$ | 10    | n     | n            | n           | n        | n                            |
| I3   | $4 \times 10^5$ | 30    | 1.25  | 0.61         | 2.4         | y        | 45                           |
| J3   | $4 \times 10^5$ | 30    | 1.64  | 0.26         | 3.7         | y        | 34                           |
| K3   | $4 \times 10^5$ | 30    | 3     | 0.04         | 9.0         | y        | 20                           |
| L3   | $4 \times 10^5$ | 30    | n     | n            | n           | n        | n                            |
| I4   | $4 \times 10^5$ | 50    | 1.25  | 0.66         | 2.3         | y        | 45                           |
| J4   | $4 \times 10^5$ | 50    | 1.40  | 0.30         | 3.4         | y        | 32                           |
| K4   | $4 \times 10^5$ | 50    | 1.64  | 0.49         | 8.5         | y        | 22                           |
| L4   | $4 \times 10^5$ | 50    | n     | n            | n           | n        | n                            |
| M1   | $5 \times 10^5$ | 0     | 1.25  | 0.98         | 2.4         | y        | 0                            |
| N1   | $5 \times 10^5$ | 0     | 1.40  | 0.38         | 3.7         | y        | 0                            |
| O1   | $5 \times 10^5$ | 0     | 1.64  | 0.24         | 4.7         | y        | 0                            |
| P1   | $5 \times 10^5$ | 0     | 3     | n            | n           | n        | n                            |
| M2   | $5 \times 10^5$ | 40    | 1.25  | 1.13         | 2.14        | y        | 56                           |
| N2   | $5 \times 10^5$ | 40    | 1.40  | 0.46         | 3.34        | y        | 40                           |
| O2   | $5 \times 10^5$ | 40    | 1.64  | 0.30         | 4.22        | y        | 13                           |
| P2   | $5 \times 10^5$ | 40    | n     | n            | n           | n        | n                            |
| N4   | $5 \times 10^5$ | 50    | 1.40  | 0.45         | 3.37        | y        | 36                           |
| O4   | $5 \times 10^5$ | 50    | 1.64  | 0.20         | 4.2         | y        | 22                           |

Notes. Here $N$, $r_v$, $\rho_c(0)$, and $t_{rh}(0)$ are the initial number of stars, the initial virial radius, the initial core density, and the initial half mass relaxation time, respectively. $f_b$ denotes the primordial binary fraction in the cluster while $f_{\text{binj}}$ denotes the fraction of all BHs in binaries that were ejected from the cluster. All initial models start with a Kroupa IMF (mass range $0.08$–$150 M_\odot$). All the simulations have an Arzoumanian et al. (2002) kick distribution and Hurley et al. (2002) wind prescription. The maximum neutron star mass in these simulations is, $M_{\text{max,NS}} = 3 M_\odot$. The prescription for the masses of the compact remnants are as implemented in B06. The percentage values ($f_{\text{binj}}$) are calculated after averaging the binary BH ejection fraction over five simulation results.

(1) Ivanova et al. (2005; similar to that assumed in the studies of B04 and B06), which includes many soft binaries, and (2) the Hurley et al. (2005) prescription. For eccentricities we initially assume a thermal distribution.

In our models we have also considered a range of metallicities: $Z = 0.0002$, $Z = 0.001$, and $Z = 0.02$ (solar). For our first set of simulations (Table 1) we have used a metallicity $Z = 0.001$, typical of old GCs. We consider two “standard” models in this set: model A and model B, both initiated with $N = 5 \times 10^5$ and binary fraction $f_b = 50\%$. The number of BHs formed in a cluster is a function of the initial number of stars in the cluster ($N$) and the IMF, and in our cluster simulations (with no primordial binaries and typical Kroupa IMF) we expect $N_{\text{BH}} = 5 \times 10^{-4}N$ BHs to form after 10 Myr. For these set of
simulations we have used the orbital period distribution (Ivanova et al. 2005), wind mass loss prescription and the prescription for the calculation of masses of remnant objects (Belczynski et al. 2005; discussed in more detail in Section 3.3.3), similar to B06 for easier comparison. Model A represents a dense cluster ($\rho_c(0) = 3 \times 10^5 M_\odot pc^{-3}$) with a virial radius ($r_v$) of 1.25 pc. Model B on the other hand represents a zero-density cluster, and we have studied model B with a pure population synthesis approach, i.e., explicitly turning off dynamics in our simulations. In Tables 4 and 5 we have listed the subpopulations of all BHs (both single BHs and BHs in binaries) for models B and A, respectively. For rest of our model simulations in Table 1, we vary the initial central density from as high as $\rho_c(0) = 1.3 \times 10^5 M_\odot pc^{-3}$ to $\rho_c(0) = 2 \times 10^2 M_\odot pc^{-3}$ for different initial $N$.

A strong correlation between the metallicity and the number density low-mass X-ray binaries (LMXBs; $L_X > 10^{36}$ erg s$^{-1}$), in GCs, has been discussed for the Milky Way, M31 (Bellazzini et al. 1995), and NGC 4472 (Kundu et al. 2002). It has also been observed that the metal poor Large Magellanic Cloud has a lower ratio of LMXBs to HMXBs than the Milky Way, which is metal rich (Cowley 1994; Iben et al. 1997). Furthermore, ULXs have been observed in environments with a wide range of metallicities. Winter et al. (2007) found solar abundances for the ULXs (in 11 nearby galaxies, spiral galaxy NGC 4559, and in M33) they investigated with XMM-Newton spectra. However, a fraction of ULXs are often associated with extremely low metallicity environments, especially in galaxies with high specific star formation rates (Pakull & Mirioni 2002; Zampieri et al. 2004; Soria et al. 2005; Swartz et al. 2008; Mapelli et al. 2009, 2010; Prestwich et al. 2013). Hence, in our second set of simulations (Table 2), we compare the primordial BH populations for solar metallicity to those in extremely low metallicity clusters (with $Z = 0.0002$). For the simulations in Table 2 with low metallicity, as well as with solar metallicity, we have adopted the Belczynski et al. (2010) treatment of compact object formation. All the models have been initiated with $N = 5 \times 10^5$ stars varying the initial core density ($\rho_c(0) \sim 1.3 \times 10^5-5 \times 10^3 M_\odot pc^{-3}$) and the primordial binary fraction ($f_b \sim 0\%–50\%$).

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**Table 2**

| Name | $N$ | $f_b$ (%) | $r_v$ (pc) | $\rho_c(0)$ ($10^3 M_\odot pc^{-3}$) | $t_{fbf}$ (10$^4$ yr) | Dynamics ($y/n$) | $f_{kepler}$ at 100 Myr (%$\pm$) |
|------|-----|-----------|------------|-----------------|---------------------|----------------|-------------------------------|

**Orbital period distribution:** Hurley et al. (2005)

**Metallicity: $Z = 0.0002$**

| Name | $N$ | $f_b$ (%) | $r_v$ (pc) | $\rho_c(0)$ ($10^3 M_\odot pc^{-3}$) | $t_{fbf}$ (10$^4$ yr) | Dynamics ($y/n$) | $f_{kepler}$ at 100 Myr (%$\pm$) |
|------|-----|-----------|------------|-----------------|---------------------|----------------|-------------------------------|

**Metallicity: $Z = 0.02$**

**Notes.** Same as Table 1 except that all the simulations have a Maxwellian kick distribution with $\sigma = 265$ Km s$^{-1}$ and wind prescription as well as remnant masses of NSs and BHs as implemented in Belczynski et al. (2010). The maximum neutron star mass in these simulations is $M_{max,NS} = 2.5 M_\odot$. 

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Finally, we considered cluster models with a higher binary fraction for massive stars, as in the Wd 1 cluster (Table 3). Wd 1 is the most massive young clusters in our Galaxy. The binary fraction in this cluster is thought to be 100% among massive stars, with masses above 45 $M_\odot$ (Clark et al. 2008). In our models we assume that all stars with mass $>45 M_\odot$ are initially in binaries. The binary fractions quoted in Table 3 for these simulations refer to the binary fraction of less massive stars ($<45 M_\odot$). Here our goal was to study the effects of the higher binary fraction for massive stars. We used a metallicity of $Z = 0.001$ and the Hurley et al. (2005) orbital period distribution. All other initial conditions are the same as in our first set of simulations. Here also we consider two representative models, W1 with high density ($\rho_c \sim 1.3 \times 10^5 M_\odot$ pc$^{-3}$) and W2, with no dynamics.

### 3.2. Comparison with Population Synthesis

In this paper we study the initial population of BHs in dense star clusters with full dynamics, along with a realistic stellar evolution model. Previous studies like those of B04 and B06 have been done with a population synthesis approach, without taking into account the important role played by stellar dynamics in determining the numbers and characteristics of BHs formed in dense clusters.

B04 and B06 used a Kroupa IMF with mass spectrum from 4 $M_\odot$ to 150 $M_\odot$. A higher minimum mass in the IMF simply saves computational time as the low-mass end of the IMF does not affect the population synthesis results. However, in dynamical simulations the cluster potential is computed by summing up the potential due to each star. Since it is the lower mass stars that dominate the total mass, these stars will also dominate the cluster potential and very much affect the dynamics. For that reason, in our simulations we take into account the low-mass end of the IMF and adopt a standard Kroupa IMF with $M_{\text{min}} = 0.08 M_\odot$ and $M_{\text{max}} = 100 M_\odot$, representative of real clusters.

B06 did not have to explicitly consider models with different binary fractions since their results could be easily generalized by simply weighing differently the numbers obtained for a population of single stars and for a population of binaries. In contrast, in our dynamical simulations for dense clusters, binary interactions play a major role and results cannot be rescaled: the primordial binary fraction in our simulations must be set as an initial condition for each model. We will show results for specific primordial binary fractions $f_b$ ranging from 0% to 50%.

### 3.3. Stellar Evolution Assumptions Affecting Compact Objects

Our Monte Carlo code includes all necessary physics such as two-body relaxation, an explicit treatment of all stellar collisions, and direct integration of close encounters using Fewbody (Fregeau et al. 2004). In this section we discuss the modifications that we have implemented in BSE.

#### 3.3.1. Masses and Radii of Remnants

An important feature of BH evolution in clusters that impacts heavily on their stellar dynamics is the remnant mass function. The remnant masses of NSs and BHs in this paper are calculated as in Belczynski et al. (2008), whose method we briefly summarize here.

Belczynski et al. (2008) determine the masses of NSs and BHs by using information on the final CO ($M_{\text{CO}}$) and FeNi ($M_{\text{FeNi}}$) core masses, from the results of detailed simulations by Hurley et al. (2000) and Timmes et al. (1996), combined with the knowledge of the pre-SN mass of the star. For a given initial zero-age main-sequence (ZAMS) mass ($M_{\text{ZAMS}}$), $M_{\text{FeNi}}$ is obtained as follows:

$$ M_{\text{FeNi}} = \begin{cases} 1.50 & M_{\text{CO}} < 4.82(M_{\text{ZAMS}} < 18 M_\odot), \\ 2.11 & \begin{array}{l} 4.82 \leq M_{\text{CO}} < 6.31(18 M_\odot), \\ \leq M_{\text{ZAMS}} < 25 M_\odot, \end{array} \\ 6.9 M_{\text{CO}} - 2.26 & \begin{array}{l} 6.31 \leq M_{\text{CO}} < 6.75(25 M_\odot), \\ \leq M_{\text{ZAMS}} < 30 M_\odot, \end{array} \\ 0.37 M_{\text{CO}} - 0.07 & M_{\text{CO}} \geq 6.75(M_{\text{ZAMS}} \geq 30 M_\odot). \end{cases} $$

For sufficiently large progenitor masses, some of the envelope material may not be ejected in the SN explosion and instead falls back onto the newly formed remnant. The mass fraction of the stellar envelope falling back is denoted by $\xi_{fb}$ in this paper. For stars with $M_{\text{ZAMS}} \leq 20 M_\odot$, $\xi_{fb} = 0$ and only the stellar core influences the resultant remnant mass. For stars with $M_{\text{ZAMS}} \geq 42 M_\odot$, the whole pre-SN star is assumed to collapse directly and form a BH, i.e., $\xi_{fb} = 1$. Stars in the intermediate mass range with $20 M_\odot < M_{\text{ZAMS}} < 42 M_\odot$ undergo core collapse with $0 < \xi_{fb} < 1$. The point to note here is that the outcome of the core collapse depends on the collapsing core and not on $M_{\text{ZAMS}}$.

For standard wind mass loss assumptions (Hurley et al. 2002) with solar metallicity, the masses of the compact objects are obtained as

$$ M_{\text{rem.bar}} = \begin{cases} M_{\text{FeNi}} & M_{\text{CO}} < 5 M_\odot, \\ M_{\text{FeNi}} + \xi_{fb}(M_{\text{fin}} - M_{\text{FeNi}}) & 5 M_\odot \leq M_{\text{CO}} < 7.6 M_\odot, \\ M_{\text{fin}} & M_{\text{CO}} \geq 7.6 M_\odot. \end{cases} $$

The mass range for “no fall back” ($\xi_{fb} = 0$), “partial fall back” ($0 < \xi_{fb} < 1$), and direct collapse ($\xi_{fb} = 1$) are estimated from core collapse SN simulations by Fryer (1999) and the subsequent analysis of Fryer & Kalogera (2001).
The gravitational mass of a NS is obtained from the baryonic mass using the expression derived by Lattimer & Yahil (1989),

\[ M_{\text{rem, bar}} - M_{\text{rem}} = 0.075 M_{\text{rem}}^2 \]  

(1)

and the radius of all NSs is simply set to 10 km. The gravitational mass of a BH is set to \( M_{\text{rem}} = 0.9 M_{\text{rem, bar}} \) and the BH “radius” is assumed to be the Schwarzschild radius, \( R_{\text{BH}} = 2GM_{\text{BH}}/c^2 \).

As implemented in Belczynski et al. (2008; see their Section 2.3.1) we also allow low-mass NSs to form via the capture of electrons onto \(^{24}\text{Mg}\), \(^{24}\text{Na}\), and \(^{20}\text{Ne}\) nuclei within “electron capture supernovae,” which affects the low-mass end of the remnant mass function. Moreover the masses of compact object can increase through accretion in binary systems as discussed in Belczynski et al. (2008, see their Section 5.7). In our standard model, to facilitate comparison with Belczynski et al. (2008), we define a BH to be any compact object with mass exceeding the maximum NS mass, \( M_{\text{max, NS}} = 3 M_\odot \). For completeness we also use \( M_{\text{max, NS}} = 2.5 M_\odot \) in some simulations.

3.3.2. Supernova Kicks

During SN explosions the remnant may receive a significant velocity kick due to asymmetries in the explosion (Lyne & Lorimer 1994). This can be of considerable importance to a star in a binary, as the kick imparted to the remnant might result in disrupting the binary. It has been argued that all NSs and possibly also BHs (see, e.g., Gualandris et al. 2005; Frigos et al. 2009; Janka 2013) receive such natal kicks. In our simulations we give natal kicks to both NSs and BHs during SNe, following the prescriptions in Hurley et al. (2002) and Kiel & Hurley (2009). For better comparison with B06 we have chosen to use the Arzoumanian et al. (2002) kick velocity distribution in our standard models A and B. This is an empirical distribution based on the observed proper motions of single radio pulsars in the field. It is a two-component velocity distribution with characteristic velocities of 90 and 500 km s\(^{-1}\). For rest of the simulations in this paper we have used the Hobbs et al. (2005) distribution, which is a Maxwellian with standard deviation \( \sigma = 265 \text{ km s}^{-1} \). Both NS and BH kicks are drawn from these distributions; however, only compact objects formed with “no fall back” receive full kicks, while for compact objects receiving “partial fall back” we limit any kick velocity \( (v_k) \) drawn from these distributions by the mass fraction of the stellar envelope that falls back \( (v_k = (1 - \xi_{\text{FB}}) \times v_{\text{SN}}) \).

3.3.3. Wind Mass Loss

For our standard models and for simulations with \( Z = 0.001 \) we have used the wind mass loss prescriptions from Ivanova et al. (2005) and Hurley et al. (2002), respectively. For our simulations with \( Z = 0.002 \) we have modified the wind prescription as done in Belczynski et al. (2010), based on the work of Vink et al. (2001). With this modified wind prescription in their simulated cluster models (using population synthesis), Belczynski et al. (2010) found that very massive BHs (with masses up to \( \sim 100 M_\odot \)) could be formed at these extremely low metallicities.

4. RESULTS

4.1. Properties of Single and Binary BHs

B06 investigated the evolution of single and binary BHs formed in a cluster with a population synthesis approach (but taking into account ejections of BHs from their parent cluster by SN kicks), using the StarTrack code. Here we first examine whether we can reproduce some general results of B06 using BSE with our updates to its stellar evolution prescriptions.

Figure 1 shows the total (both retained and ejected) number of single and binary BHs formed in a cluster with \( N = 5 \times 10^5 \) stars and a 50% primordial binary fraction, as the cluster evolves until the end of the simulation at 100 Myr. Model B represents a low-density cluster in which dynamics does not play a significant role. We have investigated model B with an approach similar to B06, for easier comparison.

Clear agreement between B06 and our model B is seen for the trend in the evolution of the total number of BHs over time. Figure 1 shows that the total number of single BHs increases over time during the initial 20 Myr, while the number of binary BHs decrease. The binary BHs decrease in number because of stellar evolution events such as SN explosions, which tend to disrupt binaries, and common envelope phases (CEs), which can make binaries merge. Figure 1 also shows that, as the cluster evolves beyond 20 Myr, the total number of single BHs and BH binaries becomes constant (as in B06) because the cluster is depleted of the massive progenitor stars which form BHs.

Next, we want to compare these population synthesis results with our dynamical simulations for a dense cluster (see Tables 4 and 5). Model A represents a dense star cluster in which stellar dynamics is important (in contrast to model B). One of the results of including stellar dynamics with regards to BH numbers, is that at late times (>20 Myr) the total (retained and ejected) number of single and binary BHs is no longer constant (see Figure 1, solid lines). Moreover, we also observe an increase in the population of single BHs (by \( \sim 10\% \) compared to model B) and a decrease in the number of BHs in binaries, since all wide binaries are disrupted to form single BHs in dense cluster environments.

Also from Figure 1, we see that the early phase (up to 20 Myr) of evolution of a dense cluster mimics a low-density cluster, with respect to the increasing trend in the number of BHs. This shows the dominant importance of stellar and binary evolution at early times. After 20 Myr, the number of single BHs and BHs in binaries in model A never becomes constant. Clearly dynamical
Table 4
Black Hole Populations Retained In/Ejected from Standard Model B

| Type     | 10 Myr (0.05 $t_{\text{rh}}(0)$) | 20 Myr (0.10 $t_{\text{rh}}(0)$) | 40 Myr (0.20 $t_{\text{rh}}(0)$) | 60 Myr (0.29 $t_{\text{rh}}(0)$) | 80 Myr (0.39 $t_{\text{rh}}(0)$) | 100 Myr (0.49 $t_{\text{rh}}(0)$) |
|----------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| BH–MS    | 34/7                             | 28/9                             | 19/9                             | 15/9                             | 14/9                             | 13/9                             |
| BH–HG    | 0/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              |
| BH–RG    | 0/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              |
| BH–CheB  | 4/0                              | 1/0                              | 0/0                              | 1/0                              | 1/0                              | 1/0                              |
| BH–AGB   | 0/0                              | 1/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              |
| BH–He    | 1/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              |
| BH–WD    | 0/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              |
| BH–NS    | 0/0                              | 0/0                              | 1/0                              | 1/0                              | 1/0                              | 1/0                              |
| BH–BH    | 34/2                             | 38/3                             | 38/3                             | 38/3                             | 38/3                             | 38/3                             |
| Total    | 171/0                            | 218/10                           | 230/10                           | 230/10                           | 230/10                           | 230/10                           |

Note. Here $t_{\text{rh}}(0)$ is the initial half-mass relaxation time.

Table 5
Black Hole Populations Retained In/Ejected from Standard Model A

| Type     | 10 Myr (0.05 $t_{\text{rh}}(0)$) | 20 Myr (0.10 $t_{\text{rh}}(0)$) | 40 Myr (0.20 $t_{\text{rh}}(0)$) | 60 Myr (0.29 $t_{\text{rh}}(0)$) | 80 Myr (0.39 $t_{\text{rh}}(0)$) | 100 Myr (0.49 $t_{\text{rh}}(0)$) |
|----------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| BH–MS    | 11/5                             | 10/8                             | 6/8                              | 5/8                              | 5/8                              | 9/8                              |
| BH–HG    | 0/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              |
| BH–RG    | 0/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              |
| BH–CheB  | 0/0                              | 1/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              |
| BH–AGB   | 0/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              |
| BH–He    | 0/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              |
| BH–WD    | 0/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              |
| BH–NS    | 0/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              | 0/0                              |
| BH–BH    | 5/3                              | 3/3                              | 3/5                              | 6/8                              | 3/11                             | 5/12                             |
| Total    | 16/8                             | 14/11                            | 9/13                             | 11/16                            | 8/19                             | 15/20                            |
| Total    | 226/2                            | 280/8                            | 270/22                           | 241/40                           | 233/49                           | 214/57                           |

Note. Here $t_{\text{rh}}(0)$ is the initial half-mass relaxation time.

Figure 2. Number of single BHs retained in the cluster (black symbols) and ejected from the cluster (red symbols) within different time intervals for our standard models. Model A (solid lines) represents a dense cluster and model B (dashed lines) represents a zero-density cluster (simulated with a population synthesis approach).

(A color version of this figure is available in the online journal.)

Figure 2 shows the number of single BHs retained in (black symbols) and ejected from (red symbols) the cluster for models A and B. Again we compare the results from model B with those in B06, and then contrast them with model A, where dynamics is important. In model B, a small fraction (∼4%) of single BHs are ejected from the cluster, and only within the first 20 Myr. The ejected single BH population results from the single BHs in the cluster receiving SN kicks, and from the disruption interactions are responsible for these differences in the later phases of evolution. To investigate this further we now examine separately the retained and ejected populations of single BHs (Figure 2) and binary BHs (Figure 3), in more detail, for our two standard models.
4.2. Binary Ejection Fraction

We have seen in Figures 2 and 3 that, as a cluster evolves beyond \(\sim 10\) Myr, BH ejections become dominated by dynamical interactions rather than SN kicks. The dynamical interaction rate in a cluster depends on the central density. In our models we vary the initial density of the cluster by setting the initial virial radius \(r_v\) for the King model. Here we examine the dependence of the number of BHs ejected from the cluster on the density. In Figure 4, we plot the ejected fraction of BHs that are in binaries for Model A with \(\rho_c(0) = 1.3 M_\odot \text{pc}^{-3}\) (\(r_v = 1.25\) pc) and a zero-density cluster. We have included in Table 1 the BH ejection fraction from the different models, varying the initial density (from \(\rho_c(0) = 1.3 \times 10^5 M_\odot \text{pc}^{-3}\) to \(\rho_c(0) = 2.0 \times 10^3 M_\odot \text{pc}^{-3}\) for different \(N\)) and initial binary fraction. We concentrate mainly on binary BHs (binaries with one or both the stars as BHs) since these BHs will be the most massive BHs in the cluster and will sink toward the center faster than the single BHs. We find that the binary BH ejection fraction in a high-density cluster is significantly higher (by a factor of \(\sim 3\)) than in a low-density cluster. We find that, although the BH binary ejection fraction increases with increasing density, the primordial binary fraction does not play a significant role (see also Section 4.6).

4.3. Mass Distributions for Single and Binary BHs

After successfully reproducing the trend in the population of BHs, as suggested in B06, with our model B, we discuss model A in more detail. Mass distributions of single and binary BHs retained in the cluster (model A) at 10 Myr are shown in Figure 5. We find that the retained single BH population has three peaks: a first peak at 6–8 \(M_\odot\) (from stars of mass 40 \(M_\odot < M < 50 M_\odot\)), a second peak at 10–16 \(M_\odot\) (from stars of mass \(M > 50 M_\odot\)), and a third peak at 22–26 \(M_\odot\) (from stars of mass 25 \(M_\odot < M < 35 M_\odot\)). The maximum BH mass is \(\sim 26 M_\odot\) (maximum mass is dependent upon initial metallicity; see Sections 4.7 and 4.8). In Figure 6, we plot the mass distribution of BHs at 100 Myr. Low-mass BHs formed beyond 10 Myr, as shown in Figure 6, populate the extreme low-mass end of the BH mass spectrum. We find that a fraction of BHs in the mass range 15–24 \(M_\odot\) are missing (when compared...
to Figure 5). These BHs, being more massive, concentrate in the dense cluster core and get ejected through dynamical interactions within 100 Myr. We also looked into the mass distribution of different binary systems (BH–BH and BH–MS) retained in the cluster at 10 Myr (Figure 7). From Figure 7, we find that at 10 Myr most of the binaries are in BH–MS or BH–BH systems. Moreover, BH–MS systems are in general more massive than BH–BH systems.

4.4. Orbital Period Distribution

Figures 8(a) and (b) show the orbital period distribution ($P_{\text{orb}}$) of BH–BH systems and BH–MS systems for model A at 10 Myr and 100 Myr, respectively. For BH–MS systems, the mass ratio shown is the mass of the MS companion divided by the BH mass. For BH–BH systems, the ratio is the mass of the less massive BH divided by the mass of the more massive BH.

From Figure 8(a), we find that the orbital period distribution covers the range $P_{\text{orb}} \sim 10^{-10^5}$ days, with a peak around $P_{\text{orb}} \sim 10^3$ days. Systems ejected from the cluster within 10 Myr are mainly short-period binaries. These binaries have higher orbital speeds and hence suffer larger recoils through SN kicks during the formation of the compact object. The systems retained in the cluster at 10 Myr are mostly longer-period systems ($P_{\text{orb}} > 10^3$ days) with a few ($\sim 30\%$) shorter-period ones (higher mass BHs receiving low SN kicks).

In Figure 8(b) we see that $\sim 80\%$ of the binary systems in the retained population have short periods ($P_{\text{orb}} < 10^3$ days). Moreover, all the binaries that are ejected from the cluster after 20 Myr through dynamical interactions are also short-period systems. Thus, in contrast to what we saw in Figure 8(a), there is almost no difference at 100 Myr in the orbital period distribution between the retained and ejected systems. The long-period ($P_{\text{orb}} > 10^3$ days) binaries that do exist are low-mass BH–MS binaries and reside outside the core of the cluster, while the short-period binaries are more massive and concentrate in the core. This demonstrates that dynamical interactions of hard binaries with single stars or other binaries tend to make them harder, a well-known result (Heggie 1975). Figure 8(c) shows the orbital period distribution for BH binary systems in model B at 100 Myr. We see that in a low-density cluster with no dynamics only $\sim 5\%$ of the BH binary systems have $P_{\text{orb}} < 10^3$ days at 100 Myr.

4.5. Eccentricity Distribution

The eccentricity distributions for the binary systems in model A at 10 Myr and 100 Myr are shown in Figures 9(a) and (b), respectively. At 10 Myr (Figure 9(a)), the eccentricity ($e$) of the retained binary systems is evenly distributed between 0 and 1, while the eccentricity of the ejected population is distributed within a narrow range, $e \simeq 0.4–0.6$. However, at
In dense star clusters binaries play a key dynamical role. A strong dynamical interaction of a binary can disrupt it, exchange one of its members for an incoming star, cause its orbit to expand or shrink, modify its eccentricity, or cause two or more stars to physically collide. The dynamical friction timescale ($t_{df}$) of a star of mass $m$ is given by

$$t_{df} \sim \frac{\langle m \rangle}{m} t_{\text{fr}(0)},$$

where $\langle m \rangle$ is the average particle mass of the cluster and $t_{\text{fr}(0)}$ is the half-mass relaxation time. A high primordial binary fraction (when initial $N$ and $r_v$ is kept constant) increases $\langle m \rangle$ of the cluster, while decreasing $t_{\text{fr}(0)}$ (compare simulations M2 and A when $N = 5 \times 10^5$ and $r_v = 1.25$ pc). However, the increase in $\langle m \rangle$ is more than the decrease in $t_{\text{fr}(0)}$, increasing $t_{df}$ and hence the timescale for stars to mass-segregate toward the center of the cluster.

In Figures 10(a) and (b), we focus on the effects of exchange interactions of binary systems in dense clusters. In Figure 10(a), we plot the mass ratio of BH–MS systems (at 5 Myr, 10 Myr, 20 Myr, and 100 Myr) for model W1, which is a dense cluster and in Figure 10(b), for a zero density cluster (model W2). Comparing Figures 10(a) and (b) we find that at 5 Myr, when cluster evolution is dominated by SN mass loss, there is no difference in the overall mass ratio of BH–MS systems in the two clusters. As the cluster becomes dominated by dynamical interactions (>10 Myr), the BH–MS mass ratio becomes higher in the dense cluster (Figure 10(b)). However, we find that beyond 20 Myr, when the cluster is only left with very low-mass MS stars, exchange interactions once again do not affect the ratio of BH–MS systems.

### 4.7. Simulations with Metallicity $Z = 0.0002$

The overall numbers of single and binary BHs formed in the cluster at very low metallicity do not differ much from those for $Z = 0.001$. However, clear differences are seen in the mass distributions. Figure 11(a) plots the mass distribution of single and binary BHs at 10 Myr for a cluster with $N = 5 \times 10^5$ stars, $r_v = 1.25$ pc, and $Z = 0.0002$. We find that the maximum mass for BHs reaches $50 M_\odot$ (cf. $M_{\text{max,BH}} \simeq 25 M_\odot$ for $Z = 0.001$).

At $Z = 0.001$, the most massive BHs are formed by stars with $M_{ZAMS} \sim 25–35 M_\odot$, as these stars do not undergo the strong luminous blue variable winds and are only subjected to weaker metallicity dependent winds. However, at $Z = 0.0002$ the progenitors of the most massive BHs are stars more massive than $50 M_\odot$. In fact, the mass distribution of BHs with $Z = 0.0002$ has only two peaks. Moreover, the 6–8 $M_\odot$ peak in the mass distribution of BHs from the high-mass stars (>50 $M_\odot$) found in simulations with $Z = 0.001$ is absent.

We find a dearth in the number of BHs ejected from the cluster in our simulations with $Z = 0.0002$ during the initial 10 Myr. In our simulations SN kicks are scaled with the progenitor masses of the BHs such that the more massive stars receive low-magnitude SN kicks and are retained in the cluster. As the cluster evolves beyond 10 Myr, we find that ~30% of the BHs are ejected through dynamical interactions, among which ~50% are in binaries.
Figure 10. Mass ratio of BH–MS systems in a cluster with $N = 5 \times 10^5$ stars and $r_v = 1.25$ pc (see Table 3 for model W1 and model W2). The BH–MS systems retained in the cluster at 5 Myr are shown here by black circles, at 10 Myr by red triangles, at 20 Myr by blue crosses, and at 100 Myr by green plus signs.  

(A color version of this figure is available in the online journal.)

Figure 11. Mass distribution of BHs for $Z = 0.0002$ with $N = 5 \times 10^5$ stars and $r_v = 1.25$ pc. Green lines represent single BHs while blue lines represent BHs in binaries.

(A color version of this figure is available in the online journal.)

4.8. Simulations with Solar Metallicities

The mass distribution of BHs for our models with $Z = 0.02$ at 10 Myr is shown in Figure 12(a). In these simulations the masses of the BHs are systematically lower than in the population of BHs with $Z = 0.001$. The BH mass distribution has two peaks and the high-mass peak (around $25 M_\odot$ in Figure 5 with $Z = 0.001$) is absent. However, in one of our simulations with $Z = 0.02$, we find a single BH with a mass of $30 M_\odot$; the progenitor star of this BH was formed in a collisional merger at around 4 Myr) during the resonant interaction of two binary systems (Fregeau et al. 2004). The mass distribution of BHs at 100 Myr is shown in Figure 12(b). The population of ejected BHs consists mainly of low-mass single BHs (with masses $\sim 10 M_\odot$) and a few ($\sim 10\%$) binary BH systems (unlike what we saw in simulations for $Z = 0.001$ and $Z = 0.0002$). Hence, comparing Figures 12(a) and (b), we note that some BHs in the second peak (10–15 $M_\odot$ BHs) of the mass distribution are missing in Figure 12(b).

5. SUMMARY AND CONCLUSIONS

Using our CMC code with full stellar evolution, we have investigated the formation and evolution of BHs in young massive stellar clusters. Our study extends and improves older work by B04 and B06 by including a full treatment of stellar dynamics. We find that, although stellar dynamics remains unimportant for the initial populations of BHs in low-density clusters, it can play a key role in dense clusters. Dynamical interactions between massive BH binaries and single stars or other BHs not only change the properties of these systems, but also alter the relative numbers of single and binary BHs retained in the cluster.
During the evolution of a dense cluster (in our models, $\rho_c \sim 1.3 \times 10^5 M_\odot pc^{-3}$), increasing numbers of BH binaries tend to get ejected from the cluster through dynamical interactions. In low-metallicity environments, dynamical interactions along with SN kicks, eject $\sim$60% of the BH binary systems from a dense cluster within the first $\sim$100 Myr of dynamical evolution. If we assume that the observable XRB systems of a YMC are represented by BH–MS binary systems, we find that in these low-metallicity high-density environments most bright XRBs should be observed near but outside their parent cluster, most having been ejected.

Analyzing the orbital period and eccentricity distributions of BH binary systems we find that BH binaries (BH–BH systems and more massive BH–MS systems) surviving in a dense cluster after $\sim$100 Myr, have $P_{\text{orb}} < 10^7$ days and $e > 0.2$. Only low-mass ($M_{\text{BH}} < 10 M_\odot$) BH–MS systems residing away from the core can have longer orbital periods.

We also find at very early times ($\sim$5 Myr) some collisional mergers (unlike the CE mergers mentioned in B04 and B06) of massive stars. Primordial binaries with massive components have the largest collision cross section and will contribute the most to these collisions. These early collisional mergers either increase the mass of the primary in the binary system, or form a massive single star (when all the participating stars merge together), eventually forming a more massive BH.

Exchange interactions are rare during the initial phase of the cluster evolution ($\sim$10 Myr), and become noticeable only at later times. In our simulations as the cluster evolves beyond 10 Myr, when dynamics becomes important, the most massive stars have already undergone SNe. Hence exchange interactions mainly happen among massive BH binaries, massive single BHs, and intermediate-mass MS stars which still exist in the cluster at that time. In most of these interactions a low-mass MS companion is exchanged for another more massive BH. A high primordial binary fraction (with all other input parameters remaining constant) leads to a higher rate of exchange interactions when dynamics is important (dense clusters older than $\sim$10 Myr).

However, a large fraction of primordial binaries increases the average particle mass in the cluster and hence the dynamical friction timescale. Thus, even with a very high primordial binary fraction for massive stars, as in our model W1, and a correspondingly higher rate of exchange interactions, we do not find a significant increase in BH–MS pairs, when compared to our model W2.

As expected, more massive BHs are formed at lower metallicities in our simulations. These more massive BHs concentrate in the cluster core through mass segregation and, in most of our models, $\sim$60% acquire a binary companion. The timescale for BHs to mass segregate and acquire a binary companion is always more than $\sim$10 Myr and the binary companion is almost always another BH. Hence, it is not clear whether the formation rate of XRBs is enhanced by exchange interactions in YMCs.

Comparing our population synthesis results (for low-density cluster models) with those of B04 and B06, we find that our stellar evolution prescriptions (BSE) agree reasonably with those of the StarTrack code. Our study with full dynamics and stellar evolution shows that dynamics play a major role in determining the numbers and properties of BHs in young star clusters. Dynamical interactions play an important role in: (1) ejecting more and more BHs from the cluster, (2) increasing the eccentricity of retained binary population, (3) decreasing the orbital period of the retained binary population, (4) increasing the probability of having a comparatively more massive companion through exchange interactions, and (5) forming massive BHs through early collisional mergers. All these effects can significantly modify the properties of BHs in binaries, including X-ray properties. Hence, we conclude that population synthesis studies are not adequate for analyzing dense young stellar clusters.

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