Precise Determinations of the CKM Matrix
From CP-Asymmetries in B-Decays and $K_L \rightarrow \pi^0 \nu \bar{\nu}$

ANDRZEJ J. BURAS

Technische Universität München, Physik Department
D-85748 Garching, Germany

Max-Planck-Institut für Physik
– Werner-Heisenberg-Institut –
Föhringer Ring 6, D-80805 München, Germany

Abstract

We point out that the measurements of the CP asymmetries in neutral B-decays together with a measurement of $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and the known value of $|V_{us}|$ can determine all elements of the Cabibbo-Kobayashi-Maskawa matrix without essentially any hadronic uncertainties. In particular the strong dependence of $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ on $|V_{cb}|$ ($|V_{cb}|^4$) can be used to determine this element precisely. Consequently $|V_{ub}|$, $|V_{td}|$ and $|V_{ts}|$ can also be determined. We derive a set of formulae which facilitate this study and we present a numerical analysis of several future scenarios. Accurate determinations of all parameters are expected from the B-factories and the Main Injector era at Fermilab including KAMI. An impressive accuracy should be reached in the LHC era. An analysis using $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ instead of $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ shows that although the determination of $|V_{cb}|$ and $|V_{td}|$ this way is less accurate, still useful results for these elements can be obtained.

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The Cabibbo-Kobayashi-Maskawa matrix \([1, 2]\) which parametrizes the weak charged current interactions of quarks contains four parameters which must be determined by comparing the theoretical branching ratios with the experimental data. One of these parameters, the element \(V_{us}\), is known from the decays \(K^+ \rightarrow \pi^0 e^+ \nu\), \(K_L \rightarrow \pi^- e^+ \nu\) and semileptonic hyperon decays with a high precision: \(|V_{us}| \equiv \lambda = 0.2205 \pm 0.0018\) \([3, 4]\). Recent critical discussions of this determination and of the related element \(|V_{ud}|\) can be found in \([5]\). The present determinations of the remaining three parameters are subject to theoretical uncertainties resulting from our inability to perform precise non-perturbative calculations of various hadronic matrix elements of weak currents and local four-quark operators which enter the relevant formulae.

It has been stressed in numerous papers \([3, 6, 7]\) that CP asymmetries in \(B^0 \rightarrow f\), where \(f\) is a CP eigenstate can determine two parameters \(\rho\) and \(\eta\) in the Wolfenstein parametrization \([8]\) without essentially any hadronic uncertainties. A recent analysis of the related unitarity triangle can be found in \([9]\), where a high accuracy of this determination at future B-factories has been emphasized. Here we would like to point out that the measurement of the purely short distance CP violating decay \(K_L \rightarrow \pi^0 \nu \bar{\nu}\) \([10]\) together with the CP asymmetries in question, allows a clean and precise determination of the fourth parameter \(|V_{cb}|\) (or \(A\)). The argument is as follows.

The last year calculations \([11, 12]\) of next-to-leading QCD corrections to \(Br(K_L \rightarrow \pi^0 \nu \bar{\nu})\) reduced the theoretical uncertainty due to the choice of the renormalization scales present in the leading order expressions \([13]\) below \(\pm 1\%\). Because the relevant hadronic matrix element of the weak current \(\bar{s} \gamma_\mu (1-\gamma_5) d\) can be measured to better than \(1\%\) in the leading decay \(K^+ \rightarrow \pi^0 e^+ \nu\), the resulting theoretical expression for \(Br(K_L \rightarrow \pi^0 \nu \bar{\nu})\) is only a function of \(\eta, V_{cb}\) and \(m_t = \sqrt{m_t(m_t)}\), the running top quark mass at the \(m_t\) scale. The very weak dependence on the QCD scale \(\Lambda_{\overline{MS}}\) can be safely neglected. Now as shown below \(\eta\) will be known with high precision from future B-factories \([14]\), HERA-B \([15]\), Tevatron with the Main Injector \([16]\) and LHC B-physics experiments \([17, 18]\). Similarly Tevatron and LHC will offer precise determinations of \(m_t\). Consequently \(|V_{cb}|\) can be determined by measuring \(Br(K_L \rightarrow \pi^0 \nu \bar{\nu})\). Since in addition \(Br(K_L \rightarrow \pi^0 \nu \bar{\nu})\) is a very sensitive function of \(|V_{cb}| (|V_{cb}|)^4\), the latter element of the CKM matrix can be extracted from \(K_L \rightarrow \pi^0 \nu \bar{\nu}\) with a precision comparable or
even better than achieved directly in tree level semi-leptonic B-decays. To this
end $Br(K_L \to \pi^0 \nu \bar{\nu})$ must be known to within 5%. Since the present standard
model estimate of $Br(K_L \to \pi^0 \nu \bar{\nu})$ is in the ball park of $(20 - 40) \cdot 10^{-12}$, the
sensitivity of $O(10^{-12})$ aimed for at KAMI [13] and KEK [20] should allow to
achieve this goal.

At first sight it is probably surprising that we use a rare K-meson decay to
determine $|V_{cb}|$. The natural place to do this are of course tree level B-decays.
On the other hand using unitarity and the Wolfenstein parametrization with
$|V_{cb}| = A \lambda^2$ it is clear that $|V_{cb}|$ gives the overall scale $A$ of the top quark
couplings $V_{td}$ and $V_{ts}$ which are the only CKM couplings in $K_L \to \pi^0 \nu \bar{\nu}$. From
this point of view it is very natural to measure the parameter $A$ in a short distance
process involving the top quark and using unitarity of the CKM matrix to find
the value of $|V_{cb}|$. Moreover this strategy in contrast to tree-level B-decays is
free from hadronic uncertainties. On the other hand one should keep in mind
that this method contains the uncertainty from the physics beyond the standard
model which could contribute to short distance processes like $K_L \to \pi^0 \nu \bar{\nu}$. We
will return to this below.

The aim of this letter is to present this strategy for a clean determination of
$\varrho$, $\eta$ and $|V_{cb}|$ in explicit terms and to calculate the accuracy for these three
parameters which we think should be achieved in the first decade of the next
millennium. As byproducts we predict very accurate determinations of the top
quark couplings $|V_{td}|$ and $|V_{ts}|$ and of the ratio $|V_{ub}/V_{cb}|$ which in our
opinion are superior to any other determinations. We derive a set of formulae
which should be useful in future studies of CP asymmetries. We also present an
analysis in which $Br(K^+ \to \pi^+ \nu \bar{\nu})$ instead of $Br(K_L \to \pi^0 \nu \bar{\nu})$ is used. We find
that although the determination of $|V_{cb}|$ and $|V_{td}|$ this way is less accurate,
still useful results for these elements can be obtained.

Our discussion of the Cabibbo-Kobayashi-Maskawa matrix will be based on
the standard parametrization [21], which can equivalently be rewritten in terms
of the Wolfenstein parameters $\lambda$, $A$, $\varrho$, $\eta$ through the definitions [3]
\[
s_{12} \equiv \lambda \quad s_{23} \equiv A \lambda^2 \quad s_{13} e^{-i\delta} \equiv A \lambda^3 (\varrho - i\eta)
\] (1)
Due to the resulting simplifications, the Wolfenstein parametrization [8] is par-
ticularly useful when an expansion in $\lambda = |V_{us}| = 0.22$ is performed. Including
next-to-leading terms in $\lambda$ implies that the apex of the reduced unitarity triangle defined through

$$\bar{\vartheta} + i \bar{\eta} \equiv -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}$$  \hspace{1cm} (2)$$

is with an error of less than 0.1% given by

$$\bar{\vartheta} = \vartheta (1 - \frac{\lambda^2}{2}) \quad \bar{\eta} = \eta (1 - \frac{\lambda^2}{2})$$  \hspace{1cm} (3)$$

and not by $(\vartheta, \eta)$ as usually found in the literature. Working in the Wolfenstein parametrization such a treatment is required if we aim at a determination of the CKM parameters with a high precision.

The CP asymmetries in $B_{d,s}^0$-decays to CP eigenstates determine $\sin(2\phi_i)$ where $\phi_i = \alpha, \beta, \gamma$ are the angles in the unitarity triangle defined by (2). Strategies involving several channels are sometimes necessary in order to remove hadronic uncertainties. $\sin(2\phi_i)$ can be expressed in terms of $(\bar{\vartheta}, \bar{\eta})$ as follows

$$\sin(2\alpha) = \frac{2\bar{\eta}(\bar{\eta}^2 + \bar{\vartheta}^2 - \bar{\vartheta})}{(\bar{\vartheta}^2 + \bar{\eta}^2)((1 - \bar{\vartheta})^2 + \bar{\eta}^2)}$$  \hspace{1cm} (4)$$

$$\sin(2\beta) = \frac{2\bar{\eta}(1 - \bar{\vartheta})}{(1 - \bar{\vartheta})^2 + \bar{\eta}^2}$$  \hspace{1cm} (5)$$

$$\sin(2\gamma) = \frac{2\bar{\vartheta}\bar{\eta}}{\bar{\vartheta}^2 + \bar{\eta}^2}$$  \hspace{1cm} (6)$$

Next

$$\text{Br}(K_L \to \pi^0 \nu \bar{\nu}) = \kappa_L \eta^2 |V_{cb}|^4 X^2(x_t)$$  \hspace{1cm} (7)$$

where $x_t = m_t^2/M_W^2$,

$$\kappa_L = \frac{3\alpha^2 \text{Br}(K^+ \to \pi^0 e^+ \nu) \tau(K_L)}{2\pi^2 \sin^4 \Theta_W \tau(K^+)} = 0.354 \cdot 10^{-4}$$  \hspace{1cm} (8)$$

and

$$X(x) = \eta_X \cdot \frac{x}{8} \left[ -\frac{2 + x}{1 - x} + \frac{3x - 6}{(1 - x)^2} \ln x \right] \quad \eta_X = 0.985$$  \hspace{1cm} (9)$$

Here $\eta_X$ is the NLO correction calculated in \[11, 12\]. With $m_t \equiv m_t(m_t)$ the QCD factor $\eta_X$ is practically independent of $m_t$.

Using (4-8) we can express $\bar{\vartheta}$, $\bar{\eta}$ and $|V_{cb}|$ in terms of

$$a \equiv \sin(2\alpha) \quad b \equiv \sin(2\beta) \quad c \equiv \sin(2\gamma)$$  \hspace{1cm} (10)$$
and \( Br(K_L \rightarrow \pi^0\nu\bar{\nu}) \). There are several solutions. We give first only the solution which is favoured on the basis of what we already know about the CKM matrix. Using (4-6) we express \( \bar{\rho} \) in terms of \( \bar{\eta} \) and \( a, b, \) and \( c \) respectively:

\[
\bar{\rho} = \frac{1}{2} - \sqrt{\frac{1}{4} - \bar{\eta}^2 + \bar{\eta}r_-(a)} \quad , \quad \bar{\rho} = 1 - \bar{\eta}r_+(b) \quad , \quad \bar{\rho} = \bar{\eta}r_-(c)
\]

where we have introduced

\[
r_{\pm}(z) = \frac{1}{z}(1 \pm \sqrt{1 - z^2}) \quad z = a, b, c
\]

For the pairs \((a, b)\), \((b, c)\) and \((a, c)\), assuming \( \bar{\eta} \neq 0 \), we can then determine \( \bar{\eta} \) with the result

\[
\bar{\eta} = \frac{r_-(-a) + r_+(b)}{1 + r^2_+(b)} = \frac{1}{r_+(b) + r_-(c)} = \frac{r_-(-a) + r_-(c)}{1 + r^2_-(c)}
\]

respectively. Using \((7)\) and \((8)\) we can next determine \( |V_{cb}| \) as follows:

\[
|V_{cb}| = \lambda^2 \left[ \frac{\sqrt{B_L}}{\eta X(x_t)} \right]^{1/2}
\]

\[
B_L = \frac{Br(K_L \rightarrow \pi^0\nu\bar{\nu})}{1.94 \cdot 10^{-10}}
\]

where \( \eta \) is to be found from \((3)\) and \((13)\). Note that the factor in front of \( \lambda^2 \) gives the parameter A in the Wolfenstein parametrization. Finally using

\[
X(x_t) = 0.65 \cdot x_t^{0.575}
\]

which reproduces the function \( X(x_t) \) to an accuracy of better than 0.5% for \( 150 \text{ GeV} \leq m_t \leq 190 \text{ GeV} \) we find a useful formula

\[
|V_{cb}| = 39.1 \cdot 10^{-3} \sqrt{\frac{0.39}{\eta} \left[ \frac{170 \text{ GeV}}{m_t} \right]^{0.575} \left[ \frac{Br(K_L \rightarrow \pi^0\nu\bar{\nu})}{3 \cdot 10^{-11}} \right]}^{1/4}
\]

We note that the weak dependence of \( |V_{cb}| \) on \( Br(K_L \rightarrow \pi^0\nu\bar{\nu}) \) allows to achieve high accuracy for this CKM element even when \( Br(K_L \rightarrow \pi^0\nu\bar{\nu}) \) is known within \( 5 - 10\% \) accuracy.

Equations (11)-(14) together with \((3)\) are the main formulae which we will use to determine \( \rho, \eta \) and \( |V_{cb}| \). First however we would like to discuss the remaining solutions.

There are four solutions for \( \bar{\rho} \) coming from \((4)\) at fixed \( \sin(2\alpha) \) and \( \bar{\eta} \). They are given by the first formula in \((4)\) with \( r_{\pm}(a) \) and with \( \pm \) in front of the square root. The solutions with + in front of the square root can be excluded by
imposing $|V_{ub}/V_{cb}| \leq 0.10$ in accordance with the data on semi-leptonic B-decays [22]. Next there are two solutions for $\bar{\rho}$ coming from (5) given by the second formula in (11) with $r_\pm(b)$. The solution with $r_-(b)$ violates $|V_{ub}/V_{cb}| \leq 0.10$ and consequently there is only one acceptable solution coming from (5). Finally there are two solutions for $\bar{\rho}$ from (6) given by the last formula in (11) with $r_\pm(c)$.

Retaining the allowed solutions for $\bar{\rho}$ we find then the generalization of (13) with $(r_\pm(a), r_+(b))$, $(r_+(b), r_\pm(c))$ and $(r_\pm(a), r_\pm(c))$, in an obvious notation, respectively. One can then check numerically by varying $(a, b, c)$ in the full range that the unique solution with $(r_- (a), r_+(b))$, $(r_+(b), r_-(c))$ and $(r_- (a), r_-(c))$ as given in (11) and (13) is obtained if one requires

$$\eta > 0.20, \quad -0.20 \leq \varrho \leq 0.25, \quad 0.06 \leq \frac{V_{ub}}{V_{cb}} \leq 0.10$$

This range is favoured by a recent analysis of the unitarity triangle [4] and the data on B-decays [22]. We will discuss only this solution in what follows. In the future when $(a, b, c)$ will be measured one will have to use a similar strategy to select a unique solution by means of other measurements. Once this has been done, a precise determination of CKM parameters within this solution will be possible along the lines discussed here. We will illustrate this on examples below.

We now turn to a numerical investigation of the formulae above. In (8) and (14) we use [21]

$$\lambda = 0.22 \quad \alpha = 1/128 \quad \sin^2 \Theta_W = 0.23$$

$$\text{Br}(K^+ \to \pi^0 e^+ \nu) = 4.82 \cdot 10^{-2} \quad \tau(K_L)/\tau(K^+) = 4.18$$

and assume that the $O(1\%)$ uncertainties in these numerical values will be reduced in the coming years to the level that they can be neglected. We will also neglect the small residual scale ambiguity in $X(x_t)$ [12] which can effectively be taken into account by introducing an additional error $\Delta m_t \leq \pm 1 \text{ GeV}$.

As illustrative examples, let us consider the following three scenarios:

\textbf{Scenario I}

$$\sin(2 \alpha) = 0.40 \pm 0.08 \quad \sin(2 \beta) = 0.70 \pm 0.06$$

$$\text{Br}(K_L \to \pi^0 \nu \bar{\nu}) = (3.0 \pm 0.3) \cdot 10^{-11} \quad m_t = (170 \pm 5) \text{ GeV}$$

\textbf{Scenario II}

$$\sin(2 \alpha) = 0.40 \pm 0.04 \quad \sin(2 \beta) = 0.70 \pm 0.02$$

$$\text{Br}(K_L \to \pi^0 \nu \bar{\nu}) = (3.00 \pm 0.15) \cdot 10^{-11} \quad m_t = (170 \pm 3) \text{ GeV}$$
Table 1: Determinations of various parameters in scenarios I-III with $F_B \sqrt{B_B}$ given in MeV. The errors have been symmetrized so that sometimes the central values do not exactly correspond to the central values of the input parameters.

**Scenario III**

$$\sin(2\alpha) = 0.40 \pm 0.02 \quad \sin(2\beta) = 0.70 \pm 0.01$$

$$Br(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (3.00 \pm 0.15) \cdot 10^{-11} \quad m_t = (170 \pm 3) \text{ GeV}$$

The accuracy in the scenario I should be achieved at B-factories [14], HERA-B [15], at KAMI [19] and at KEK [20]. Scenarios II and III correspond to B-physics at Fermilab during the Main Injector era [16] and at LHC [17, 18] respectively. At that time an improved measurement of $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ should be aimed for.

The values of $m_t$ assumed here are in the ball park of the most recent results of the CDF collaboration [23]. Since in accordance with the QCD corrections in [12] we use here the current top quark mass at the scale $m_t$, our values correspond to $m_t^{phys} = (177 \pm 3) \text{ GeV}$. This should be compared with $m_t^{phys} = (174 \pm 16) \text{ GeV}$ reported by CDF [23].

The results that would be obtained in these scenarios for $\rho$, $\eta$, $|V_{cb}|$, $|V_{ub}/V_{cb}|$, $|V_{td}|$, $|V_{ts}|$ and $\sin(2\gamma)$ are collected in table 1. To this end we have used

$$|V_{td}| = |V_{cb}| \lambda \sqrt{(1 - \bar{\vartheta})^2 + \bar{\eta}^2} \quad \frac{|V_{ub}|}{|V_{cb}|} = \lambda \sqrt{\vartheta^2 + \eta^2}$$

and the standard expression for $|V_{ts}|$ [21].
Table 2: Determinations of various parameters in scenarios I’-III’.

In table 2 we show the results for the corresponding scenarios I’-III’ in which $\sin(2\gamma)$ is used instead of $\sin(2\alpha)$. Here we set

$$\sin(2\beta) = \begin{cases} 
0.60 \pm 0.06 \quad \text{(I')} \\
0.60 \pm 0.02 \quad \text{(II')} \\
0.60 \pm 0.01 \quad \text{(III')} 
\end{cases}$$

$$\sin(2\gamma) = \begin{cases} 
0 \pm 0.08 \quad \text{(I')} \\
0 \pm 0.04 \quad \text{(II')} \\
0 \pm 0.02 \quad \text{(III')} 
\end{cases}$$

Furthermore we use $m_t = 180$ GeV and $Br(K_L \to \pi^0\nu\bar{\nu}) = 3.2 \cdot 10^{-11}$ with the errors as in scenarios I-III. We are aware of the fact that the accuracy for $\sin(2\gamma)$ assumed in scenarios II’ and III’ will be difficult to achieve [6, 24] but we show the results of this exercise to further motivate the efforts in this field. For $\rho$ and $\eta$ we give three decimal places in order to show better the differences between various cases. One can easily check that other possible solutions for $(\rho, \eta)$ can be excluded. Choosing for instance the solution $(r_+(a), r_+(b))$ in scenarios I-III one finds $(\rho, \eta) \approx (-1.6, 1.1)$ and $|V_{ub}/V_{cb}| \approx 0.43$ which is much too large. Similarly the solution $(r_+(b), r_+(c))$ in scenarios I’-III’ gives $(\rho, \eta) \approx (1.0, 0)$ and $|V_{ub}/V_{cb}| \approx 0.22$ which is also too large. In the same manner all solutions discussed above except for the one presented in the tables can be excluded.

Tables 1 and 2 show very clearly the potential of CP asymmetries in B-decays and of $K_L \to \pi^0\nu\bar{\nu}$ in the determination of CKM parameters. It should be stressed that this high accuracy is not only achieved because of our assumptions about future experimental errors in the scenarios considered but also because $\sin(2\alpha)$
and \( \sin(2\gamma) \) are very sensitive functions of \( \rho \) and \( \eta \) and \( Br(K_L \to \pi^0\nu\bar{\nu}) \) depends strongly on \( |V_{cb}| \). In particular as seen in table 2 the determination of \( \sin(2\gamma) \) may offer a very precise measurement of \( \rho \).

The accuracy in scenarios I and I’ corresponds roughly to the cases considered in [9], where however \( K_L \to \pi^0\nu\bar{\nu} \) has not been discussed. There also the prospects of the determination of the unitarity triangle using \( B_d^0 - \bar{B}_d^0 \) mixing and the parameter \( \varepsilon_K \) have been analyzed. The results of [9] show that in such an analysis it will be very difficult to determine \( \rho \) and \( \eta \) to better than \( \Delta \rho = \pm 0.10 \) and \( \Delta \eta = \pm 0.05 \) which should be contrasted with the accuracy expected here.

Similarly because of theoretical uncertainties it is at present difficult to imagine that in the tree level B-decays a better accuracy than \( \Delta |V_{cb}| = \pm 2 \cdot 10^{-3} \) and \( \Delta |V_{ub}/V_{cb}| = \pm 0.01 \) could be achieved.

In tables 1 and 2 we have also shown the values of the non-perturbative parameters \( B_K \) and \( F_B\sqrt{B_B} \) which can be extracted from the data on \( \varepsilon_K \) and \( B_d^0 - \bar{B}_d^0 \) mixing once the CKM parameters have been determined in the scenarios considered. To this end \( x_d = 0.72 \) and \( \tau(B) = 1.5 \) ps have been assumed. The errors on these two quantities should be negligible at the end of this millennium. Note that the resulting central values for \( B_K \) in tables 1 and 2 are close to the lattice [25] and \( 1/N \) [26] results respectively.

We have checked that similar patterns of uncertainties emerge for different central input parameters. Needless to say when the scenarios presented here will become a reality one will have to make sure that the uncertainties present in the input parameters in (18) and (19) have been reduced to the desired level.

It is instructive to investigate whether the use of another short distance decay \( K^+ \to \pi^+\nu\bar{\nu} \) instead of \( K_L \to \pi^0\nu\bar{\nu} \) would also give interesting results for \( V_{cb} \) and \( V_{td} \). \( K^+ \to \pi^+\nu\bar{\nu} \) is CP conserving and receives also contributions from internal charm exchanges. This introduces the dependence on \( \Lambda_{MS} \) and \( m_c \) but otherwise this decay is known to be theoretically very clean. In particular the long distance contributions to \( K^+ \to \pi^+\nu\bar{\nu} \) have been considered in [27, 28, 29] and found to be very small: two to three orders of magnitude smaller than the short distance contribution at the level of the branching ratio. Moreover in contrast to \( K_L \to \pi^0\nu\bar{\nu} \), the decay \( K^+ \to \pi^+\nu\bar{\nu} \) could be observed already in the coming years at AGS in Brookhaven. Using the expressions in [30] and [3] we find instead
\[ |V_{cb}| = \lambda^2 \left[ \frac{\sqrt{B_+ (\eta^2 + b_1^2) - b_0^2 \eta^2 - b_0 b_1}}{(\eta^2 + b_1^2) X(x_t)} \right]^{1/2} \]  

(25)

where

\[ B_+ = \frac{Br(K^+ \to \pi^+ \nu \bar{\nu})}{4.64 \cdot 10^{-11}} \]

(26)

\[ b_0 = (1 - \frac{\lambda^2}{2}) P_0(K^+) \]

and

\[ b_1 = 1 - \rho + \lambda^2 (2 \rho - \rho^2 - \eta^2 - \frac{1}{2}) + O(\lambda^4) \]

(27)

Here \( P_0(K^+) \) represents the charm contribution to \( K^+ \to \pi^+ \nu \bar{\nu} \) calculated including next-to-leading QCD corrections in [31]. For 200 \( MeV \leq \Lambda_{\overline{MS}} \leq 350 \ MeV \)

\[ 1.25 \ GeV \leq m_c \leq 1.35 \ GeV \]

the residual uncertainty due to the choice of the renormalization scale \( \mu \) has been taken into account. Here \( m_c \) stands for the running charm quark mass at the \( m_c \) scale.

We again consider scenarios I-III with \( Br(K^+ \to \pi^+ \nu \bar{\nu}) = (1.0 \pm 0.1) \cdot 10^{-10} \) for the scenario I and \( Br(K^+ \to \pi^+ \nu \bar{\nu}) = (1.0 \pm 0.05) \cdot 10^{-10} \) for scenarios II and III in place of \( Br(K_L \to \pi^0 \nu \bar{\nu}) \) with all other input parameters unchanged. The results for \( \rho, \eta, |V_{ub}/V_{cb}| \) and \( \sin(2\gamma) \) remain of course unchanged. In table 3 we show the results for \( V_{cb}, V_{td} \) and \( F_B \sqrt{B_B} \). We observe that due to the uncertainties present in the charm contribution to \( K^+ \to \pi^+ \nu \bar{\nu} \), which was absent in \( K_L \to \pi^0 \nu \bar{\nu} \), the determinations of \( |V_{cb}|, V_{td} \) and \( F_B \sqrt{B_B} \) are less accurate. On the other hand, we note in the case of \( V_{td} \) that due to an approximate cancellation of the square root in (23) by the denominator in (25), the uncertainties due to the errors in \( \eta \) and \( \rho \) are considerably reduced. Consequently when compared with \( K_L \to \pi^0 \nu \bar{\nu} \) the reduction of the accuracy in \( V_{td} \) is less pronounced than in the case of \( V_{cb} \). If the uncertainties due to the charm mass and \( \Lambda_{\overline{MS}} \) are removed one day, only the uncertainty related to \( \mu \) will remain in \( P_0(K^+) \) giving \( \Delta P_0(K^+) = \pm 0.03 \) [31]. In this case the results in parentheses in table 3 would be found.

Let us finally summarize the main aspects of this letter.

We have pointed out that the measurements of the CP asymmetries in neutral B-decays together with a measurement of \( Br(K_L \to \pi^0 \nu \bar{\nu}) \) and the known value of \( |V_{us}| \) offer a precise determination of all elements of the Cabibbo-Kobayashi-Maskawa matrix without essentially any hadronic uncertainties. \( K_L \to \pi^0 \nu \bar{\nu} \) proceeds almost entirely through direct CP violation and is known to be a very
useful decay for the determination of $\eta$. However due to the strong dependence on $V_{cb}$ this determination cannot fully compete with the one which can be achieved using CP asymmetries in B-decays. As a recent analysis \cite{30} shows it will be difficult to reach $\Delta \eta = \pm 0.03$ this way if $|V_{cb}|$ is determined in tree level B-decays. Our strategy then is to find $\eta$ from CP asymmetries in B decays and use $K_L \to \pi^0 \nu \bar{\nu}$ to determine $|V_{cb}|$. To our knowledge no other decay can determine $|V_{cb}|$ as cleanly as this one.

We believe that the strategy presented here is the theoretically cleanest way to establish the precise values of the CKM parameters. The ratio $x_d/x_s$ of $B^0_d - \bar{B}^0_d$ to $B^0_s - \bar{B}^0_s$ mixings, $Br(K^+ \to \pi^+ \nu \bar{\nu})$ and the parity violating asymmetry ($\Delta_{LR}$) in $K^+ \to \pi^+ \mu \bar{\mu}$ \cite{32-34} are also theoretically rather clean and are useful in this respect. In particular as demonstrated in \cite{30}, $K^+ \to \pi^+ \nu \bar{\nu}$ together with $K_L \to \pi^0 \nu \bar{\nu}$ offers a respectable determination of $\sin(2\beta)$. However from the theoretical point of view the determinations of this type cannot compete with the strategy considered here. $x_d/x_s$ is subject to uncertainties related to SU(3) flavour breaking effects which will probably be difficult to bring below 5%. Moreover the dependence on $A$ cancels in this ratio and consequently $V_{cb}$ cannot be determined. $Br(K^+ \to \pi^+ \nu \bar{\nu})$ is more sensitive to $\Lambda_{MS}$ than $Br(K_L \to \pi^0 \nu \bar{\nu})$ and in addition receives a few % uncertainty from the error in the charm quark mass \cite{31}. As we have seen above these uncertainties lower the precision of the determination of $V_{cb}$ and $V_{td}$ compared to $K_L \to \pi^0 \nu \bar{\nu}$, although as shown in table 3 interesting results for these elements can still be obtained. Similar comments can be made about $\Delta_{LR}$ for which in addition possible long distance contributions at a few % level cannot be excluded \cite{33}.

|            | Central | I       | II      | III     |
|------------|---------|---------|---------|---------|
| $|V_{cb}|$/10^{-3} | 41.2    | ±4.3 (3.2) | ±3.0 (1.9) | ±2.8 (1.8) |
| $|V_{td}|$/10^{-3} | 9.1     | ±0.9 (0.7) | ±0.6 (0.4) | ±0.6 (0.4) |
| $F_B\sqrt{B_B}$ | 190     | ±17 (12) | ±12 (7) | ±12 (7) |

Table 3: Determinations of various parameters in scenarios I-III using $K^+ \to \pi^+ \nu \bar{\nu}$ instead of $K_L \to \pi^0 \nu \bar{\nu}$. The values in parentheses show the situation when the uncertainties in $m_c$ and $\Lambda_{MS}$ are not included.
On the other hand once $\rho$, $\eta$ and $|V_{cb}|$ (or $A$) have been precisely determined as discussed here, it is clear that $x_d/x_s$, $Br(K^+ \rightarrow \pi^+\nu\bar{\nu})$ and $\Delta_{LR}$ can be rather accurately predicted and confronted with future experimental data. Such confrontations would offer excellent tests of the standard model and could possibly give signs of a new physics beyond it.

Of particular interest will also be the comparison of $|V_{cb}|$ determined as suggested here with the value of this CKM element extracted from tree level semi-leptonic B-decays. Since in contrast to $K_L \rightarrow \pi^0\nu\bar{\nu}$, the tree-level decays are to an excellent approximation insensitive to any new physics contributions from very high energy scales, the comparison of these two determinations of $|V_{cb}|$ would be an excellent test of the standard model and of a possible physics beyond it. Also the values of $|V_{ub}/V_{cb}|$ from tree-level B-decays, which are subject to hadronic uncertainties larger than in the case of $V_{cb}$, when compared with the clean determinations suggested here could teach us about the reliability of non-perturbative methods. The same applies to the quantities like $x_d$ and the CP violating parameter $\varepsilon_K$ which are subject to uncertainties present in the non-perturbative parameters $F_B\sqrt{B_B}$ and $B_K$ respectively. In tables 1-3 we have shown the values of these parameters which can be extracted from the data in the scenarios considered.

It is also clear that once the accuracy for CKM parameters presented here has been attained, also detailed tests of proposed schemes for quark matrices \[35, 36\] will be possible.

Precise determinations of all CKM parameters without hadronic uncertainties along the lines suggested here can only be realized if the measurements of CP asymmetries in B-decays and the measurements of $Br(K_L \rightarrow \pi^0\nu\bar{\nu})$ and $Br(K^+ \rightarrow \pi^+\nu\bar{\nu})$ can reach the desired accuracy. All efforts should be made to achieve this goal.

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