Superfluidity of “dirty” indirect excitons and magnetoexcitons in two-dimensional trap

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(Dated: February 8, 2020)

Abstract

The superfluid phase transition of bosons in a two-dimensional (2D) system with disorder and an external parabolic potential is studied. The theory is applied to experiments on indirect excitons in coupled quantum wells. The random field is allowed to be large compared to the dipole-dipole repulsion between excitons. The slope of the external parabolic trap is assumed to change slowly enough to apply the local density approximation (LDA) for the superfluid density, which allows us to calculate the Kosterlitz-Thouless temperature $T_c(n(r))$ at each local point $r$ of the trap. The superfluid phase occurs around the center of the trap ($r = 0$) with the normal phase outside this area. As temperature increases, the superfluid area shrinks and disappears at temperature $T_c(n(r = 0))$. Disorder acts to deplete the condensate; the minimal total number of excitons for which superfluidity exists increases with disorder at fixed temperature. If the disorder is large enough, it can destroy the superfluid entirely. The effect of magnetic field is also calculated for the case of indirect excitons. In a strong magnetic field $H$, the superfluid component decreases, primarily due to the change of the exciton effective mass.

PACS numbers: 71.35.Lk, 73.20.Mf, 73.21.Fg, 71.35.-y

Key words: coupled quantum wells, superfluidity, indirect excitons, Bose-Einstein condensation of excitons
I. INTRODUCTION

Superfluidity in a system of spatially indirect excitons (with spatially separated electrons and holes) in coupled quantum wells (CQW) has been predicted by Lozovik and Yudson, and several subsequent theoretical studies have suggested that this should be manifested as persistent electric currents, quasi-Josephson phenomena and unusual properties in strong magnetic fields. In the past ten years, a number of experimental studies have focused on observing these behaviors. One of the appeals of this system is that the electron and hole wavefunctions have very little overlap, so that the excitons can have very long lifetime (> 20 µs), and therefore they can be treated as metastable particles to which quasiequilibrium statistics apply. Also, when the spatial separation between the electrons and holes is large enough, the interactions between the excitons are entirely dipole-dipole repulsive.

In real experiments disorder plays a very important role. Although the inhomogeneous broadening linewidth of typical GaAs-based samples has been improved from around 20 meV to less than 1 meV, the disorder energy is still not always small compared to the exciton-exciton repulsion energy. At a typical exciton density of a few times 10^{10} cm^{-2}, the interaction energy of the excitons is several meV. On the other hand, the typical disorder energy of 1 meV is low compared to the typical exciton binding energy of 5 meV, so that the excitons can be viewed as stable particles. Typical thermal energies at liquid helium temperatures are \( k_B T = 0.2 - 2 \) meV. Previous work has shown that in the low-temperature limit, in the case of no trapping potential, the density \( n_s \) of the superfluid component in CQW systems and the temperature of the superfluid transition (the Kosterlitz-Thouless temperature \( T_{c,24} \)) decrease with increasing amplitude of the random field (the general description of “dirty” boson problem is presented in Ref. [26]).

In the present paper we consider the phase transition in the system of the excitons trapped in a confining potential, in which case true Bose condensation is possible. Experimentally, an in-plane harmonic potential is possible in GaAs quantum well structures by using inhomogeneous stress. The Bose condensation in this case is similar to that for Bose atoms in trap. The slope of the external parabolic trap is assumed to change slowly enough to apply the local density approximation (LDA) for the superfluid density and Kosterlitz-Thouless temperature at each local point of the trap. This assumption allows
us to substitute the exciton density profile in the Thomas-Fermi approximation\textsuperscript{34} into the expressions for the superfluid density and Kosterlitz-Thouless temperature obtained for the system without the confinement, found in Ref. \textsuperscript{25}. We also analyze the phase transitions in strong magnetic field, applying the effective Hamiltonian approach developed in Ref. \textsuperscript{35}. 

The paper is organized in the following way. In Sec. \textsuperscript{II} the local density profile, superfluid density and Kosterlitz-Thouless transition in the trapped indirect exciton system with disorder are obtained. The temperature dependence of the minimal total number of excitons for superfluidity at various levels of disorder is calculated. In Sec. \textsuperscript{III} the local density profile and the phase transitions of the trapped dirty system in magnetic field are obtained. In Sec. \textsuperscript{IV} we present our conclusions.

\section{II. THE DENSITY PROFILE AND THE LOCAL KOSTERLITZ-THOULESS PHASE TRANSITION IN LDA}

We consider a slowly changing parabolic potential with the characteristic length of inhomogeneity $l$ much greater than the excitonic de-Broglie wavelength $\lambda = \hbar / \sqrt{2M\mu}$:

$$l \gg \frac{\hbar}{\sqrt{2M\mu}}, \quad (1)$$

where $M = m_e + m_h$ is the mass of indirect exciton ($m_e$ and $m_h$ are the electron and hole masses, respectively), and $\mu$ is the chemical potential of the system with dipole-dipole repulsion, given in the ladder approximation by\textsuperscript{36} (here and below, $\hbar = 1$)

$$\mu = \frac{8\pi n}{2M \log \left( \frac{\epsilon^2}{8\pi s^2 n M^2 e^4 D^4} \right)}, \quad (2)$$

where $e$ is the charge of an electron, $D$ is the distance between quantum wells, $s$ is the spin degeneracy, $\epsilon$ is the dielectric constant, $n$ is the two-dimensional (2D) exciton density without confinement (we consider the dilute exciton gas $n\rho^2 \ll 1$, where $\rho$ is the exciton radius).

Following earlier work\textsuperscript{37} one can write the random field felt by an indirect exciton in coupled quantum wells, induced by fluctuations in the widths of electron and hole quantum wells, in the form $V(\mathbf{r}_e, \mathbf{r}_h) = \alpha_e [\xi_1(\mathbf{r}_e) - \xi_2(\mathbf{r}_e)] + \alpha_h [\xi_3(\mathbf{r}_h) - \xi_4(\mathbf{r}_h)]$, where separate parameters are introduced for the random fields felt by the electrons and holes which make up
the exciton, since in the standard coupled quantum well structure the electron and hole are spatially separated in two different wells. Here $\alpha_{e,h} = \partial E_{e,h}^{(0)} / \partial d_{e,h}$, where $d_{e,h}$ is the average width of the electron or hole quantum well, and $E_{e,h}^{(0)}$ is the lowest energy of the electron and hole confined state; $\xi_1$, $\xi_2$, $\xi_3$ and $\xi_4$ are the fluctuation amplitudes of the upper and lower interfaces of the two quantum wells. We assume that fluctuations on different interfaces are statistically independent, whereas fluctuations of a specific interface are characterized by Gaussian correlation function

$$\langle \xi_i(r_1) \xi_j(r_2) \rangle = g_i \delta_{ij} \delta(r_2 - r_1), \quad \langle \xi_i(r) \rangle = 0,$$

where $g_i$ is proportional to the squared amplitude of the $i$th interface fluctuation.

As shown in earlier work, in the low frequency and long wavelength limits, the effect of this type of disorder is to give a single parameter $Q$ which is an imaginary self-energy of the exciton due to the disorder. The Green’s function of the center of mass of the isolated exciton at $T = 0$ in the random field is given in the momentum-frequency domain within the second order Born approximation by

$$G^{(0)}(p, \omega) = \frac{1}{\omega - \varepsilon_0(p) + \mu + iQ(p, \omega)}, \quad (3)$$

where $\mu$ is the chemical potential of the system, $\varepsilon_0(p) = p^2/2M$ is the spectrum of the center mass of the exciton in the “clean” system, and the parameter $Q$ is determined by the disorder as

$$Q(p, \omega) = Q = \frac{\alpha_e^2(g_1 + g_2) - \alpha_h^2(g_3 + g_4)}{16\pi^4} M; \quad m_e = m_h; \quad \alpha_h \left( \frac{m_h}{m_e} \right)^2 \ll \alpha_e,$$

$$Q(p, \omega) = Q = \frac{\alpha_e^2(g_1 + g_2)}{64\pi^4} M, \quad m_h \gg m_e; \quad \alpha_h \left( \frac{m_h}{m_e} \right)^2 \ll \alpha_e. \quad (4)$$

For the slowly changing confinement potential $U(r) = \gamma r^2/2$ ($\gamma$ is the curvature of the confinement potential applied to the center of mass of the exciton taken from Ref. [27], and $r$ is the distance between the center of mass of the exciton and the center of confinement), assuming the inequality Eq. (1) holds, the Thomas-Fermi approximation can be applied in the thermodynamic equilibrium. In the Thomas-Fermi approximation, the chemical potential of the system at each local point equals the sum of the coordinate-dependent local chemical potential and the external field. Since for the “dirty” system the chemical potential enters to the interacting Green’s function as $\sqrt{\mu^2 - Q^2}$ (which represents the chemical potential of the “dirty” infinite [unconfined] excitonic system), we obtain the Thomas-Fermi equation for the local density profile $n(r) = n_0 - \Delta n(r)$, (where $n_0 = n(r = 0)$ is the exciton density
at the point of the minimum of the parabolic potential)

\[ \sqrt{\mu^2[n_0] - Q^2} = \sqrt{\mu^2[(n_0 - \Delta n(r))] - Q^2 + U(r)}, \]  

(5)

where \( \mu[n(r)] \) represents the contribution to the chemical potential of the “clean” system due to dipole-dipole repulsion at each local point, as found by substituting the coordinate-dependent local density \( n(r) \) into Eq. (2). In the very dilute limit \( (n_0 \rho^2 \to 0, \ \rho \) being the exciton radius) for the weak confinement potential \( (\Delta n(r) \ll n_0 \text{ and } \mu^2[n_0] \Delta n(r)/[(\mu^2[n_0] - Q^2)n_0] \ll 1) \) we get the approximate exciton density profile

\[ \Delta n(r) = \frac{\gamma}{2} \sqrt{\frac{\mu^2[n_0] - Q^2}{\mu^2[n_0]} n_0 r^2}. \]  

(6)

The exciton density profiles at different \( n_0 \) and \( Q \) at \( T = 0 \) are represented in Figs. 1 and 2.

We apply the local density approximation (LDA) to obtain the local superfluid density \( n_s(r) \) and the Kosterlitz-Thouless phase transition temperature \( T_c(r) \) at each point \( r \) of the trap. The LDA can be used for a potential which is flat on the characteristic length scale \( l \) much greater than the average distance \( R \) between vortices of the normal phase, where \( R \) is estimated as

\[ \ln \frac{R}{\rho} \sim \exp \left\{ \left( \ln \frac{T_c}{T - T_c} \right)^{1/2} \right\}, \quad T \to T_c^+. \]  

(7)

This estimation works at temperatures above \( T_c \), and diverges at \( T = T_c \). So we can use the LDA at temperatures sufficiently close to \( T_c \), but it breaks down at \( T_c \). In the LDA framework, the trapped system has at each point in space exactly the same properties as an infinite system, using the local density \( n(r) \) instead of the average density \( n \) for the infinite system. Substituting the local density profile Eq. (6) into the expressions for \( n_s \) and \( T_c \) for the infinite system, we obtain for the local superfluid density

\[ n_s(r) = n(r) - s \frac{3 \zeta(3)}{2\pi} \frac{T^3}{c_s^2[n(r), Q] M} - s \frac{n(r)Q}{2Mc_s^2[n(r), Q]}, \]  

(8)

and for the local Kosterlitz-Thouless temperature

\[ T_c(r) = \left[ \left( 1 + \frac{32}{27} \left( \frac{M \theta_o(r)}{\pi n'(r)} \right)^3 + 1 \right)^{1/3} - \left( \frac{32}{27} \left( \frac{M \theta_o(r)}{\pi n'(r)} \right)^3 + 1 - 1 \right)^{1/3} \right] \frac{T_c^0(r)}{2^{1/3}}, \]  

(9)

where \( c_s^2(r) = \sqrt{\mu^2[n(r)] - Q^2}/M \) is the local velocity of sound and \( s \) is the spin degeneracy, equal to 4 for excitons in GaAs quantum wells. Here \( T_c^0 \) is an auxiliary quantity, equal to
the temperature at which the superfluid density vanishes in the mean-field approximation (i.e., \( n_s(T_c) = 0 \)),

\[
T_c^0(r) = \left( \frac{2\pi n'(r) e_4(r) M}{3\zeta(3)} \right)^{1/3}. \tag{10}
\]

In Eqs. (9) and (10), \( n'(r) \) is

\[
n'(r) = n(r) - s \frac{n(r) Q}{2Me_s^2(r)}. \tag{11}
\]

For the trapped excitonic system we find the dependence of the critical minimal total number \( N_c \) of particles where the global superfluidity exists on the temperature \( T \) at different \( Q \). The total number of particles corresponding to fixed \( n_0 \) is given by

\[
N = 2 \int_0^{r_0} n(r) d^2r, \tag{12}
\]

where, again, the exciton density profile \( n(r) = n_0 - \Delta n(r) \) (\( \Delta n(r) \) is given by Eq. (11)); \( r_0 = \mu(2/(\gamma\sqrt{\mu^2 - Q^2}))^{1/2} \) is the root of the equation \( n(\pm r_0) = 0 \).

Expressing \( n(r) \) in terms of \( N \) using Eq. (12), and substituting \( n(r) \) in Eq. (9), we calculate the critical total number \( N_c(T) \) of excitons corresponding to Kosterlitz-Thouless phase transition (see Fig.3). Our calculations show that at fixed temperature \( T \), it takes a greater total number of excitons \( N_c \) to achieve superfluidity as disorder increases.

## III. TRAPPED INDIRECT MAGNETOEXCITONS IN COUPLED QUANTUM WELLS IN RANDOM FIELD

We now consider indirect excitons in a strong magnetic field perpendicular to the quantum wells in the presence of the disorder, extending previous work on a homogeneous gas of excitons in a magnetic field.\(^{35}\) In this case we neglect the transitions between different Landau levels of the magnetoexciton, including transitions caused by scattering from the slowly varying spatial confinement potential \( U(r_e, r_h) = \frac{1}{2}(r_e^2 + r_h^2) \) and the random field potential. We also neglect nondiagonal matrix elements of the Coulomb interaction between a paired electron and hole. The region of applicability of these two assumptions is defined by the inequalities\(^{37}\) \( \omega_c \gg E_b, \omega_c \gg \sqrt{\left\langle V_{e(h)}^2 \right\rangle_{av}} \), where \( \omega_c = eH/m_e-h \), and \( m_e-h = m_e m_h/(m_e + m_h) \) is the exciton reduced mass in the quantum well plane; \( E_b \) is the magnetoexciton binding energy in an ideal “pure” system as a function of magnetic field \( H \) and the distance between electron and hole quantum wells \( D \): \( E_b \sim e^2/\epsilon r_H \sqrt{\pi/2} \) at \( D \ll r_H \) and \( E_b \sim e^2/\epsilon D \) at
Here, $r_H = (eH)^{-1/2}$ is the magnetic length. $\langle \ldots \rangle_{av}$ denotes an average over the fluctuations of the random field. The characteristic length of inhomogeneity is assumed to be much greater than the magnetic length: $l \gg r_H$. We consider the characteristic length of the random field potential $L$ to be much shorter than the average distance between excitons $r_s \sim 1/\sqrt{n}$ ($L \ll 1/\sqrt{n}$, where $n$ is the total exciton density) similar to Ref. [25].

It can be shown [35] that the effective Hamiltonian $\hat{H}_{\text{eff}}$ of the system of indirect “dirty” magnetoexcitons at small momenta is identical to the Hamiltonian of indirect “dirty” excitons without magnetic field but with magnetic mass $m_H$ instead of $M = m_e + m_h$. In strong magnetic fields at $D \gg r_H$ the exciton magnetic mass is $m_H \approx D^3 \epsilon / (e^2 r_H^4)$ [39]. Therefore, for the trapped system in strong magnetic field we can apply the expressions for the exciton local density profile, local superfluid density and local temperature of the Kosterlitz-Thouless phase transition for the “dirty” trapped (Sec. II) system without magnetic field using random field $V_{\text{eff}}$, and an effective external field of confinement $U_{\text{eff}}$ instead of $U_e(r) + U_h(r)$, respectively. The effective parameter of disorder $Q$ for the system of magnetoexcitons is given by [35]

$$Q(p, \omega) = Q = \frac{\alpha_e^2 (g_1 + g_2) + \alpha_h^2 (g_3 + g_4)}{64\pi^4} m_H.$$  \hspace{1cm} (13)

and the effective field of confinement has the form

$$U_{\text{eff}}(R) = \frac{1}{\pi r_H^2} \int \exp \left( -\frac{(R - r)^2}{r_H^2} \right) \left[ U_e(r) + U_h(r) \right] dr.$$  \hspace{1cm} (14)

Eq. (14) is valid if the characteristic length $l$ of inhomogeneity of the trapping potential $U(r_e, r_h)$ is much greater than the magnetoexciton mean size $r_{\text{exc}} \approx r_H$. In a strong magnetic field we obtain $U_{\text{eff}}(R) = U(R) = \gamma R^2 / 2$. Therefore, magnetic field does not change the effective trapping potential.

Since magnetic field affects the effective Hamiltonian only by replacing the excitonic mass $M$ by effective the magnetic mass $m_H$ [35], the magnetoexciton local density profile can be obtained by substitution the effective disorder parameter $Q$ (Eq. (13)) into Eq. (6) and effective magnetic mass $m_H$ instead of $M$ and $s = 1$ into the expression for $\mu$ (Eq. (2)) and Eq. (6). As we can see from this substitution, the magnetoexciton local density at a fixed local point $r$ decreases as magnetic field increases proportional to the magnetic mass $m_H \sim H^2$ when $D \gg r_H$. The negative correction to the local exciton density is also an increasing function of the interwell separation $D$ because $m_H \sim D^3$. The local superfluid
density and the local Kosterlitz-Thouless phase transition temperature can be obtained by replacing $M$ by $m_H$ and substitution of the magnetoexciton local density profile in Eqs. (8), (9), (10) and (11). As we can see from Eq. (13), increasing magnetic field $H$, corresponding to higher $Q$, results in higher minimal total number $N_c$ required to observe superfluidity at fixed $T$.

IV. DISCUSSION

As we can see from Eq. (9), the critical temperature $T_c$ for superfluidity at $r = 0$ is the same as that of an infinite system with $n = n_0$. Since at $r > 0$ the local exciton density decreases, as we go farther from the minimum, the Kosterlitz-Thouless critical temperature and superfluid density will decrease. In general, there will be a superfluid centered at $r = 0$ and a normal phase outside this area; the size of the superfluid region will shrink as $T$ increases.

The disorder is found to deplete the superfluid, making the superfluid region smaller than it would be in the absence of disorder, according to Eq. (11) (see Figs. 1 and 2). A “clean” system is always superfluid at $T = 0$, but a “dirty” system will have a quantum phase transition to the superfluid state at some critical density (see Fig. 3) even at $T = 0$.

In a strong magnetic field $H$, the correction of the local density is found to increase with magnetic field as $H^2$ and with interwell separation $D$ as $D^3$. As magnetic field $H$ increases, the minimum total number $N_c$ required to observe superfluidity increases at fixed $T$. The critical effective disorder parameter increases with magnetic field as $H^2$ at $T = 0$.

Acknowledgements

O. L. B. wishes to thank the participants of the First International Conference on Spontaneous Coherence in Excitonic Systems (ICSCE) in Seven Springs PA for many useful and stimulating discussions. Yu. E. L. was supported by the INTAS grant. D. W. S. and R. D. C. have been supported by the National Science Foundation.

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FIG. 1: Exciton density profile at $T = 0$ for different $n_0$ at the disorder parameter $Q = 0.58\, \text{meV}$; interwell distance $D = 14\, \text{nm}$; $\gamma = 40\, \text{eV/cm}^2$; dielectric constant $\epsilon = 13$. The parameters of the system are taken from Ref. [27].
FIG. 2: Exciton density profile at $T = 0$ for different disorder parameters $Q$ (in units of $meV$) at $n_0 = 3 \times 10^{11} cm^{-2}$; interwell distance $D = 14 nm$; $\gamma = 40 eV/cm^2$; dielectric constant $\epsilon = 13$. The parameters of the system are taken from Ref. [27].
FIG. 3: Critical total number of particles $N_c(T)$ corresponding to the Kosterlitz-Thouless phase transition as a function of temperature $T[K]$ for different disorder parameters $Q[meV]$ at interwell distance $D = 14nm$; $\gamma = 40eV/cm^2$; dielectric constant $\epsilon = 13$. The parameters of the system are taken from Ref. [27].