Theoretical Review of Rare B Decays

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We present an overview of the recent neutral current $B$ decays focusing on the deviations with respect to the Standard Model predictions that are observed in $b \to s \ell^+\ell^-$ transitions, and discuss their implications for new physics scenarios in a model-independent way by means of global statistical fits. The prospects for future discoveries in the individual channels, in particular using the theoretically clean observables are also addressed.

I. INTRODUCTION

Rare $B$ decays are important probes for physics beyond the Standard Model (SM) as they are loop-suppressed in the SM and are therefore sensitive to new physics (NP) parameters. As sensitive flavour observables, they are connected to fundamental questions in particle physics and in particular play a key role in understanding the underlying pattern of the SM. Furthermore, they can provide guidance for NP model-building.

In the recent years, there have been several experimental measurements showing deviations with respect to the SM predictions, that are commonly referred to as flavour anomalies. The angular observable $P_s$ of the $B \to K^+ \mu^+\mu^-$ decay [1] was the first one measured by LHCb collaboration in 2013 [2] presenting more than $3\sigma$ discrepancy with the SM. Updated measurements by LHCb have persistently confirmed this tension, which can be explained with short distance new physics contributions to this decay [3-4]. The overall $B \to K^+ \mu^+\mu^-$ angular observables are in agreement (see e.g. [5]) and the tension is also supported by the recent angular analysis of $B^+ \to K^+ \mu^+\mu^-$ [6]. Another decay indicating tension with the SM is $B_s \to \phi \mu^+\mu^-$ [7-9], in particular its branching ratio is measured to be below SM predictions. This trend is observed in several other $b \to s\ell^+\ell^-$ branching ratios such as $B \to K \mu^+\mu^-$ [10] and $B_s \to \Lambda \mu^+\mu^-$ [11]. However, the branching ratios are dependent on the local form factors and suffer from large theoretical uncertainties [12-24]. The angular observables on the other hand, while having a reduced sensitivity to the form factor uncertainties [25-26], receive still non-local contributions that are not fully under control in QCD factorisation [27]. Therefore, the significance of the deviations depend on the estimated size of the non-local effects. Recent theoretical progress has been achieved for a better control of these effects in Refs. [28-31].

Furthermore, a set of observables is defined in order to check lepton flavour universality violation (LFUV) in $b \to s\ell^+\ell^-$ decays as [32]

$$R_H = (B \to H \mu^+\mu^-)/(B \to H e^+e^-),$$

with $H = K^+, K^*, \phi$, etc. Such ratios, contrary to the observables mentioned above, are very clean and precisely known in the SM (see Ref. [33] for a recent study of the QED corrections for these observables). Deviations from the SM predictions are seen in the LFUV ratios for $R_K$ [34-36] and $R_{K^*}$ [37] that are measured to be below the SM predictions. Similarly, the recent measurements of $R_{K_S^0}$ and $R_{K^+}$ [38] although within $2\sigma$ of the SM predictions, show the same trend with the central values below the SM predictions.

Interestingly, modest signs of LFUV are also observed in flavour changing neutral current processes in the Kaon sector [39].

Another theoretically clean observable with an uncertainty of less than 5% is the branching ratio of $B_s \to \mu^+\mu^-$ which has been measured by several experiments. Here we consider the combination of ATLAS [40], CMS [41] and LHCb [42-43] as given in Ref. [44].

While the significance of the reported anomalies taken individually is around $2 - 3\sigma$, when considered collectively they can find a common NP explanation with a much larger significance in a global analysis [44-49].

II. THEORETICAL SETUP AND UNCERTAINTIES

The description of $b \to s\ell^+\ell^-$ transitions is based on the effective Hamiltonian

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ C_1 O_i + \sum_{i=7,9,10, Q_1, Q_2, T} (C_i O_i + C'_i O'_i) \right\},$$

with $G_F$, $V_{tb}$ and $V_{ts}$ are the Fermi constant and two CKM matrix elements, respectively, and the $O_i$ are local operators coming each with an associated Wilson coefficient $C_i$ which is calculable perturbatively for a particular high-energy physics model.

The semileptonic part of the Hamiltonian (second term in the left hand side) accounts for the domi-
nant contribution and can be factorised into a leptonic and a hadronic piece. The latter can be described by seven independent form factors $S,V_{λ},T_{λ}$, with helicities $λ=±1,0$. The hadronic part of the Hamiltonian (first term in the left hand side) has a subleading contribution to $B→K^+μ^+μ^-$ from a virtual photon decaying into a lepton pair. This leads to non-factorisable contributions and appears in the vectorial helicity amplitudes

$$H_V(λ) = -i N^λ \{ C_9^{eff} V_λ - C_9' \bar{V}_λ \}
+ \frac{m_B^2}{q^2} \left[ \frac{2m_B}{m_B} \left( C_7^{eff} T_λ - C_7' \bar{T}_λ - 16π^2 N_λ \right) \right],$$

where the factorisable piece is described as the effective part of $C_9^{eff} (≡ C_9 + Y(q^2))$ and the non-factorisable piece is encoded in $N_λ(q^2)$. Leading order in QCD + $h_λ(q^2)$, with $h_λ$ denoting the unknown power corrections. The short-distance NP contributions due to $C_9$ (and/or $C_7$) can be mimicked by long-distance effects in $h_λ$. Hence, the estimation of the size of the hadronic contributions is crucial in determining whether the observed deviations are due to new physics.

It is possible to parameterise the power corrections by a polynomial with a number of free parameters to be fitted to the experimental data. Assuming a $q^2$-polynomial ansatz [51, 52] we can write

$$h_±(q^2) = h_±(0) + \frac{q^2}{1\, GeV^2} h_±^1 + \frac{q^4}{1\, GeV^4} h_±^2,$$

$$h_0(q^2) = \sqrt{q^2} \times \left( h_0^0 + \frac{q^2}{1\, GeV^2} h_0^1 + \frac{q^4}{1\, GeV^4} h_0^2 \right),$$

which is the most general ansatz for the unknown hadronic contributions (up to higher order powers in $q^2$) compatible with the analyticity structure assumed in Ref. [28].

By doing separate fits for NP and hadronic parameters, a statistical comparison between the two fits is possible using the Wilks’ theorem [53]. As was shown in Ref. [54], while NP explanation seems to be favoured, more data is needed to be able to make conclusive statements.

### III. GLOBAL FITS

Global statistical fits of the Wilson coefficients with the available data are a standard way to search for evidence for NP contributions, in a way that is agnostic to the specific NP model.

In order to check the coherence of the implication of the clean observables for new physics compared to the rest of the observables, we first present a fit to the former where we consider $\text{BR}(B_{s,d}→μ^+μ^-)$, $R_K, R_{K^*}$ as well as the recently measured $R_{K_S^0}$ and $R_{K^{*+}}$ [38].

| TABLE I: Fit to clean observables with the full data from 2021. |
|---------------------------------------------------------------|
| Only LFUV ratios and $B_{s,d}→\ell^+\ell^-$ |
| (χ^2_m = 34.25, nr. obs. = 12) |
| b.f. value | χ^2_m | Pull_{SM} |
|---------------------------------------------------------------|
| δC_9 | −2.00 ± 5.00 | 34.1 | 0.4σ |
| δC_9' | 0.83 ± 0.21 | 14.5 | 4.4σ |
| δC_9'' | −0.80 ± 0.21 | 15.4 | 4.3σ |
| δC_10 | 0.43 ± 0.24 | 30.6 | 1.9σ |
| δC_10' | −0.81 ± 0.19 | 12.3 | 4.7σ |
| δC_10'' | 0.66 ± 0.15 | 10.3 | 4.9σ |
| δC^{LL}_{LL} | 0.43 ± 0.11 | 13.3 | 4.6σ |
| δC^{LL}_{LL}'' | −0.39 ± 0.08 | 10.1 | 4.9σ |

The calculation of the observables and the $χ^2$ fitting is done with the SuperIso public program [55-59].

#### A. Clean observables

In Table I the one-dimensional NP fit to clean observables are provided. Compared to the NP fit to the clean observables in Ref. [44], the two recently measured LFUV ratios $R_{K_S^0}$ and $R_{K^{*+}}$ [38] as well as the $R_K$ measurement by Belle [60] in the [1,6] GeV^2 bin are included in the fit. While the hierarchy of the most favoured scenarios has remained the same, the addition of the two former measurements has resulted in an increase in the NP significance compared to [44] ($\sim 0.4 \sigma$ for NP in $C_9^{LL}$, $C_1^{LL}$ and 0.3σ for $C_9^{LL}$). The inclusion of $\text{BR}(B_s→μ^+μ^-)$ in this set of observables is crucial in breaking the degeneracy between NP in $C_9^{LL}$ and $C_9^{LL}$ for explaining the measured values of the ratios (see Fig. 1).

#### B. All observables except the clean ones

In Table II we give the one-dimensional NP fit to all observables except the LFUV ratios and $B_{s,d}→\ell^+\ell^-$. In this set, the significance of the NP fit is dependent on the assumption for the size of the non-factorisable power corrections. The results of Table II is given with the assumption of 10% power correction. Compared to Ref. [44], the previous LHCb results of the $B_s→ϕμ^+μ^-$ observables are replaced by the recent measurements with an integrated luminosity of 8.4 fb^{-1} [8, 9]. We also include the $F_H(B^+→K^+μ^+μ^-)$ measurement by CMS [61] as well as the $B→K^{*0}ℓ^+ℓ^-$ angular observable measurements by LHCb [62].

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shows no indication for NP in the electron sector, however, besides the agreement of the measurements in the electron sector with their SM predictions, it should be noted that in this set of observables the experimental data in the electron mode is much scarcer compared to the muon sector. Comparison of Tables III and IV shows that there is not a complete consistency between all favoured scenarios for each of the two datasets. However, there are scenarios such as NP in $\delta C^\mu_9$ for which not only there is a large significance for both datasets but also the best fit values agree within 1$\sigma$.

C. Global fit to all $b \to s\ell^+\ell^-$ observables

To get the global picture of the rare $B$-decays, we consider here all the relevant observables (consisting of the clean and the rest of the observables mentioned in the previous subsections). For the global fit, similar to the fits of subsection III B, we consider 10% uncertainty for the power corrections.

1. One-dimensional fit

In Table III the one-dimensional global fits of NP to all relevant rare $B$-decays are given. As expected, the scenario with highest significance corresponds to NP in $\delta C^\mu_9$, this is followed by NP in $\delta C^\mu_\text{LL}$ and the lepton flavour universal contribution $\delta C_9$. This pattern is similar to what was observed in Ref. [11], while now the significance has reduced very slightly ($\sim 0.1\sigma$) for $\delta C^\mu_9$ and $\delta C^\mu_\text{LL}$ and $\sim 0.4\sigma$ for $\delta C_9$. In Table III we have not considered scenarios with NP contribution to the electromagnetic dipole coefficient $C_7$ or $C_9$.

| All observables except LFUV ratios and $B_{s,d} \to \ell^+\ell^-$ ($\chi^2_\text{SM} = 253.3, \text{ nr. obs.} = 183$) | $\chi^2_{\text{min}}$ | $\text{Pull}_{\text{SM}}$ |
|---------------------------------|------------------|-----------------|
| $\delta C_9$ | $-0.93 \pm 0.13$ | 185.1 | 6.1$\sigma$ |
| $\delta C^\mu_9$ | $0.70 \pm 0.60$ | 220.5 | 1.1$\sigma$ |
| $\delta C^\mu_9$ | $-0.96 \pm 0.13$ | 182.8 | 6.2$\sigma$ |
| $\delta C_{10}$ | $0.29 \pm 0.21$ | 219.8 | 1.4$\sigma$ |
| $\delta C^\mu_{10}$ | $-0.60 \pm 0.50$ | 220.6 | 1.1$\sigma$ |
| $\delta C^\mu_{10}$ | $0.35 \pm 0.20$ | 218.7 | 1.8$\sigma$ |
| $\delta C^\mu_{\text{LL}}$ | $0.34 \pm 0.29$ | 220.6 | 1.1$\sigma$ |
| $\delta C^\mu_{\text{LL}}$ | $-0.64 \pm 0.13$ | 195.0 | 5.2$\sigma$ |

these updates, the hierarchy of the most favoured scenarios has remained the same, with the largest significance for NP in vector lepton current in the muon sector $\delta C^\mu_9$ or by lepton flavour universal NP in $\delta C_9$ followed by NP in the chiral basis $\delta C^\mu_{\text{LL}}$ in the muon sector. These three favoured scenarios are all now showing a reduction of $\sim 0.4\sigma$ compared to Ref. [11] which can be attributed mostly to the updated measurement of the $B_s \to \phi\mu^+\mu^-$ observables.

The fit with all observables except the clean ones

TABLE II: Fit to all except the clean observables with the full data from 2021.

| All observables except LFUV ratios and $B_{s,d} \to \ell^+\ell^-$ ($\chi^2_\text{SM} = 221.8, \text{ nr. obs.} = 171$) | $\chi^2_{\text{min}}$ | $\text{Pull}_{\text{SM}}$ |
|---------------------------------|------------------|-----------------|
| $\delta C_9$ | $-0.95 \pm 0.13$ | 185.1 | 6.1$\sigma$ |
| $\delta C^\mu_9$ | $0.70 \pm 0.60$ | 220.5 | 1.1$\sigma$ |
| $\delta C^\mu_9$ | $-0.96 \pm 0.13$ | 182.8 | 6.2$\sigma$ |
| $\delta C_{10}$ | $0.29 \pm 0.21$ | 219.8 | 1.4$\sigma$ |
| $\delta C^\mu_{10}$ | $-0.60 \pm 0.50$ | 220.6 | 1.1$\sigma$ |
| $\delta C^\mu_{10}$ | $0.35 \pm 0.20$ | 218.7 | 1.8$\sigma$ |
| $\delta C^\mu_{\text{LL}}$ | $0.34 \pm 0.29$ | 220.6 | 1.1$\sigma$ |
| $\delta C^\mu_{\text{LL}}$ | $-0.64 \pm 0.13$ | 195.0 | 5.2$\sigma$ |
the (pseudo)scalar coefficients \((C_{Q_1,2})\) since in a one-dimensional fit these get highly constrained, the former by \(B \to X_s\gamma\) and the latter by \(B_s \to \mu^+\mu^-\). We have also not given NP fits to contributions to right-handed quark currents \((\delta C_{ij}^p)\) which are not favoured by the data. However in general in a multidimensional fit the situation can change as for example pointed out for the pseudo(scalar) contributions which when varied together with \(C_{10}\) can admit a large range of values \([54]\).

2. Multidimensional fit

In general, UV-complete new physics models can contribute to several Wilson coefficients in the Weak effective theory, it is thus reasonable to make a multidimensional fit where more than one Wilson coefficient can be varied. Table IV presents the result of a twenty-dimensional fit to the rare \(B\) decay observables when varying all Wilson coefficients. Within this approach we avoid any look elsewhere effect (LEE). In general, LEE gets introduced when concentrating on a subset of observables, it also takes place if one and/or two specific NP directions are assumed a posteriori. However, this is avoided when assuming all possible Wilson coefficients. In principle, there might be insensitive coefficients and flat directions leading to an underestimation of the significance of the multidimensional fit. However, by considering likelihood profiles and correlation matrices we eliminate these coefficients, finding an “effective” number of degrees of freedom (for the 20-dimensional fit we find 19 effective dof). The significance of the 20-dimensional fit is 5.5\(\sigma\), similar to what was obtained in Ref. \([44]\). Sev-
eral of the Wilson coefficients are still only loosely constrained as less data with electrons in the final state is available than with muons in the final state.

TABLE IV: 20-dimensional fit to all observables with the full data from 2021.

| \(\delta C_7\) | \(\delta C_8\) |
| \(\delta C_9^{\mu}\) | \(\delta C_9^{\nu}\) | \(\delta C_9^{\tau}\) |
| \(\delta C_9^{e}\) | \(\delta C_9^{\mu e}\) | \(\delta C_9^{\nu e}\) |
| \(C_{Q_1}\) | \(C_{Q_2}\) |
| \(C_{Q_2}\) |

FIG. 2: Comparison of one-dimensional fits for NP contributions to \(C_9^\mu\) (top) and to \(C_9^\nu = -C_9^{10}\) (bottom). The darker (lighter) coloured lines correspond to 68\% (95\%) CL intervals. See the main text for the definition and the relevant reference for each group.

IV. COMPARISON OF GLOBAL FIT RESULTS

Here we present a comparison between global fits performed by the different fitting groups, based on the common work presented at the Flavour Anomaly Workshops \([63, 64]\), where the results of the following groups have been confronted:

- ACDMN: M. Algueró, B. Capdevila, S. Descotes-Genon, J. Matias, M. Novoa-Brunet \([45]\).
- AS: W. Altmannshofer, P. Stangl \([46]\).
- CFFPSV: M. Ciuchini, M. Fedele, E. Franco, A. Paul, L. Silvestrini, M. Valli \([47]\).
- HMMN: T. Hurth, F. Mahmoudi, D. Martínez-Santos, S. Neshatpour \([44]\).

Similar fits have also been performed in \([48, 65, 68]\). Fig. 3 shows the one-dimensional global fit results for \(C_9^\mu\) and \(C_9^\nu = -C_9^{10}\), and Fig. 3 shows the two-

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FIG. 3: Comparison of two-dimensional fits for NP contributions to \{C^9_\mu, C^{10}_\mu\} using the clean observables (top) and using all available \(b \rightarrow s\ell\ell\) data (bottom). The darker (lighter) coloured regions correspond to 68\% (95\%) CL fit results. See the main text for the definition and relevant reference for each group.

While there are differences in experimental inputs, form factors, assumptions on non-local matrix elements and statistical frameworks considered by different groups (see \cite{63, 64} for more details), Figs. 2 and 3 show a remarkable global agreement between different results and the robustness of the conclusions.

V. PROJECTIONS FOR CLEAN OBSERVABLES

The global fit to rare \(B\)-decay observables suggests several NP scenarios with quite large significances. However, these significances are dependent on the assumed size of non-factorisable power corrections. It is thus useful to check when the clean observables (which are theoretically very precisely predicted) can individually reach a 5\(\sigma\) significance. In Fig. 4, assuming the current best fit value from the clean observables in the one-dimensional fit to \(\delta C^9_\mu\) (\(\delta C^{10}_\mu\)) remains, in the upper (lower) plot we have shown the significance of \(R_K\) and \(R_{K^*}\) as a function of the luminosity with the upper and lower bound for each observable corresponding to two different assumptions on the systematic uncertainties \cite{69} (for further details see Ref. \cite{44}). From Fig. 4 it is clear that if the current fit to \(\delta C^9_\mu\) or to \(\delta C^{10}_\mu\) is the correct description of new physics, \(R_K\) can reach 5\(\sigma\) already with less than 20 fb\(^{-1}\) of data.

We also make projections for the two-dimensional fits to all clean observables assuming the current best fit values for \(\{C^9_\mu, C^{10}_\mu\}\) or \(\{C^9_\mu, C^{10}_\mu\}\) remain. The results are given in Fig. 5 for the planned LHCb up-
grades, for three benchmark points with 18, 50 and 300 fb$^{-1}$ integrated luminosities. The best fit points for each scenario has a Pull$_{SM}$ larger than 5σ already with 18 fb$^{-1}$ data.

VI. CONCLUSIONS

The updated NP fits to rare $B$ decays including the updated measurement of $B_s \to \phi \mu^+ \mu^-$ observables as well as the recent measurement of lepton flavour violating ratios $R_{K^{*}}$ and $R_{K_S}$ by LHCb follow the same trend as with the previous set of results favouring in particular new physics contributions in the Wilson coefficient $C_9$, with an increased significance. Interestingly, while the updated measurements have slightly changed the NP significance, the hierarchy of the preferred scenarios has remained the same. The projections for clean observables, show that if the current tensions remain, $R_K$ can establish NP with $5\sigma$ significance already with less than 20 fb$^{-1}$ of data. The main source of theory uncertainty in global fits is due to non-local hadronic contributions. However, different fits with different setups, inputs and statistical frameworks show a remarkable agreement so that the experimental observation of the discrepancy in these observables would be a clear sign of physics beyond the SM.

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