Performance enhancement of a low frequency vibration driven 2-DOF piezoelectric energy harvester by mechanical impact

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Abstract. We present a low frequency vibration driven 2-DOF piezoelectric energy harvester with increased performance, in terms of both bandwidth and output power, by mechanical impact. It consists of two series spring-mass systems (positioned in a parallel manner) one of which responds to low frequency vibration, engages with the harvester base stopper periodically by piecewise linear impact, and transfers a secondary shock to the second spring-mass system comprising of power generating element. It introduces a non-linear frequency up-conversion mechanism which, in turn, generates increased output power within a wide range of applied frequency. A 2-DOF prototype harvester without stopper shows two narrow resonant peaks and delivers maximum 2.11μW peak power to its matched load resistance at 17Hz frequency and 0.5g acceleration. On the other hand, it offers a -3dB bandwidth of 15Hz (9Hz-24Hz) and delivers maximum 202.4μW peak power to its matched load resistance at the same operating condition when a stopper is placed below the primary mass at 0.5mm distance. Generated power increases up to 449μW as the acceleration increases to 1g.

1. Introduction
During last few decades, A lots of efforts have been deployed to develop piezoelectric energy harvesters to scavenge power effectively from ambient vibration for powering low-power-consumption small wireless electronics [1,2] so that these harvesters can be an alternative to eliminate the need for recharging or replacing their external power sources such as batteries. Generally, a vibration energy harvester generates maximum power when its resonant frequency matches ambient vibration frequency and power flow decreases with the decrease in resonant frequency; the ambient vibrations, unfortunately, are of low frequencies with eccentric nature which may drift over time [3,4]. A conventional piezoelectric vibration energy harvester is designed as a single-degree-of-freedom (SDOF) model in the form of a single mass-loaded cantilever beam (made of either a piezoelectric bimorph or a piezoelectric layer attached to a non-piezoelectric layer) which is efficient at its 1st resonance mode; 2nd and higher resonance modes with comparatively low response levels occur in excessively high frequencies and are generally ignored. This instinctive behaviour results in a narrow operating bandwidth making itself inefficient for power harvesting from a wide range of low frequency vibrations. In order to overcome the narrow bandwidth problem, a number of researchers have demonstrated broadband harvester by using a multi-resonant piezoelectric beam, a second
piezoelectric beam as stopper, two-mass cantilever (piezoelectric) beam constituting a 2-degree-of-freedon (2-DOF) system, and many more [5-7].

A conventional 2-DOF piezoelectric vibration energy harvester comprising two spring-mass systems generates two close resonant peaks that increase its effective bandwidth than a 1-DOF energy harvester [8]. It also increases the output power by increasing the strain produced in the piezoelectric element because the spring-mass system with non-piezoelectric element works as a dynamic magnifier [9]. However, its power generation capability is still limited in low frequency vibration (below 30Hz). Also, such 2-DOF harvesters cannot be considered as true wideband because the 2nd resonance peak is still considerably smaller and quite far away from the 1st resonance peak, as well as contain a deep valley between two peaks. In order to overcome these constraints, we have proposed a mechanical impact based 2-DOF piezoelectric energy harvester that generates much higher output power from wideband low frequency vibration as compared to its without-stopper counterpart. It uses a flexible dynamic magnifier that not only decreases the resonant frequency but also increases the displacement limit in order to enhance dynamic magnification. The dynamic magnifier impacts on the harvester base stopper by piecewise linear motion during its vibration, even at a wide range of low frequencies; delivers secondary shock and introduces non-linear frequency up-conversion mechanism to the power generating element which, in turn, increases the output power significantly by increasing the strain and decreasing the source resistance within it.

2. Harvester design

Figure 1 shows the schematic structure of the proposed harvester. It is consisted of two parts: a primary spring-mass system comprising a flexible primary beam (32×11×1 mm$^3$ polycarbonate) with a primary mass (Aluminium, 6.1 gram) attached to one end and, a relatively smaller and rigid secondary beam (28×6×0.6 mm$^3$ PZT bimorph) with a small secondary mass (NdFeB cylinder, 1.05 gram) attached to one end, other end being clamped on the primary mass in a parallel manner over the primary beam, constituting secondary spring-mass system. The primary spring-mass system acts as a dynamic magnifier. In order to enhance its performance, a base stopper is placed at the bottom and the overall system is clamped to the anchor (Aluminium) in such a position that a small gap (1 mm) exists between the fender (lower part) of the primary mass and the harvester base as shown in figure 1(b). The overall spring-mass system as in figure 1(a) constitutes a 2-DOF system. While low frequency periodic excitation with sufficiently large acceleration is applied, the overall system starts vibrating as a 2-DOF system. On the other hand, while the primary mass engages the stopper, it results in a retardation of vibration motion and frequency response diverges from normal behaviour into non-linearity, broadening operating frequency bandwidth.

![Figure 1. Schematics of the proposed 2-DOF energy harvester (a) without stopper and (b) with stopper.](image-url)
The dynamic equation of motion for the conventional 2-DOF system (figure 1(a)) can then, be expressed as [7]

\[
\begin{align*}
    m_2 \frac{d^2 x_2}{dt^2} + k_2(x_2 - x_1) + c_m \left( \frac{dx_2}{dt} - \frac{dx_1}{dt} \right) &= 0 \\
    m_1 \frac{d^2 x_1}{dt^2} + k_1(x_1 - x_0) &= k_2(x_2 - x_1) + c_m \left( \frac{dx_2}{dt} - \frac{dx_1}{dt} \right)
\end{align*}
\]

(1)

in which \(m_1\), \(k_1\) and \(x_1\) are the mass, stiffness and displacement of the primary spring-mass system, respectively; \(m_2\), \(k_2\) and \(x_2\) are the mass, stiffness and displacement of the secondary spring-mass system, respectively; \(x_0\) is the displacement of the vibrating base; and \(c_m\) is the mechanical damping.

On the other hand, in the stopper-engaged 2-DOF system (figure 1(b)), the impact mechanism induces a secondary shock to the secondary spring-mass system. As a result, the secondary mass vibrates at its own natural frequency (higher than that of the primary spring-mass system) with exponential decay. The average stress on the piezoelectric layer is increased due to larger displacement of the secondary mass resulting in increased output voltage or power as compared to the earlier one. In the time interval between consecutive impacts, the dynamic equations of motion can be given as

\[
\begin{align*}
    m_1 \frac{d^2 x_1}{dt^2} + k_1 x_1 + k_2 (x_1 - x_2) + c_m \left( \frac{dx_1}{dt} - \frac{dx_2}{dt} \right) &= m_1 a \sin \omega_d t \\
    m_2 \frac{d^2 x_2}{dt^2} + k_2 (x_2 - x_1) + c_m \left( \frac{dx_2}{dt} - \frac{dx_1}{dt} \right) &= \delta_2(t - \tau)
\end{align*}
\]

(2)

in which \(\omega_d\) and \(a\) are the applied angular frequency and acceleration, respectively. \(\delta_2(t - \tau)\) is the impulsive force generated by the secondary shock (while impact occurs) at time \(t \geq \tau\). Then the response relative displacement of the secondary mass can be expressed by Duhamel's integral as [8]

\[
x_2 = \frac{1}{m_2 \omega_d^2} \int_{t - \tau}^{t} p(\tau) e^{-\zeta_2 \omega_d (t-\tau)} \sin(\omega_d (t-\tau)) d\tau
\]

(3)

where \(\omega_d\) is the damped natural frequency, \(p(\tau)\) is the arbitrarily varying excitation force at time \(\tau\), and \(\zeta_2\) is the damping ratio of the secondary beam.

3. Experimental results and discussion

Figure 2 illustrates the schematic diagram of the complete experimental setup along with the photograph of the harvester prototype without stopper and with stopper under test. The frequency response of the peak-to-peak open circuit voltage of the prototype with stopper at various gap
distances along with the prototype without stopper is shown in figure 3. The prototype without stopper shows two narrow resonant peaks at 17Hz and 23Hz, and that with stopper shows non-linear, wideband frequency response. It shows a maximum 15Hz -3dB bandwidth that reduces as the gap distance increases. When the primary mass impacts on the stopper, the effective stiffness of the primary beam changes, allowing the primary mass to deviate from its normal motion and enables the resonance to extend over a wider range. Besides, the peak-to-peak open circuit voltage from the prototype with stopper is found to be more than double than that of the prototype without stopper at the frequency of its 1st resonance. An impulsive force during impact increases the stress on the PZT beam that allows to increase the voltage.

![Figure 4](image1.png)  
**Figure 4.** Load voltage and power of the proposed 2-DOF energy harvester (a) without stopper and (b) stopper with 0.5mm gap at 17Hz and 0.5g.

As shown in figure 4, the load voltages across various load resistances have been measured to determine the maximum power delivering capability of the proposed device. It was done at the 1st resonant frequency (17Hz) of the prototype without stopper for convenient comparison. The generated power is experimentally equal to $V_{pp}^2/4R_s$; where $V_{pp}$ is the peak-peak load voltage across the load resistance $R_l$. Results show that the prototype with stopper delivers maximum 202.4μW peak power to a 30kΩ load resistance at 0.5g acceleration when the stopper was placed below at 0.5mm gap distance. On the other hand, the prototype without stopper delivers 2.51μW peak power to its optimal load resistance, 370kΩ at the same operating condition. Mathematically, the optimal $R_l$ can be determined by

$$R_l = R_s = \frac{1}{\omega_n C_p}$$  \hspace{1cm} (4)

where $R_s$, $\omega_n$, and $C_p$ are the source resistance, natural frequency, and capacitance of the piezoelectric beam, respectively. The frequency of the secondary (piezoelectric) beam is up-converted after the impact which, in turn, reduces the source resistance.

![Figure 5](image2.png)  
**Figure 5.** Instantaneous voltage waveforms of the proposed 2-DOF energy harvester (a) without stopper and (b) stopper with 0.5mm gap at 17Hz and 0.5g.

Figure 5 shows the instantaneously generated voltage waveform across the corresponding optimal load resistances of the harvester prototype without stopper and prototype with stopper at 17Hz frequency and 0.5g acceleration. The peak-to-peak voltage of the second one right after an impact is quite high, as compared to the first one, but decays exponentially with time due to damping and becomes almost zero before the next impact occurs. The up-converted frequency of the decaying waveform is 375Hz, obtained by Fast Fourier Transform (FFT) analysis. As seen, both the waveforms contain sub-
harmonics, resulting in waving of the corresponding outputs. It happens because the flexible primary spring-mass system vibrates at low frequency with large displacement along with the secondary spring-mass system (power generating element). However, generated voltage and power increases in both cases as the acceleration increases as shown in figure 6. A maximum power of 449 µW and 11.2µW was delivered to corresponding optimal load resistances of the prototype with and without stopper, respectively at an acceleration of 1g. All experimental results reveals that the proposed impact based 2-DOF piezoelectric vibration energy harvester performs better, in terms of both bandwidth and power, than the conventional one.

4. Conclusions
An impact based low frequency driven 2-DOF piezoelectric vibration energy harvester has been proposed and demonstrated that exhibits significantly improved performance in terms of both output power and wideband operation in low frequency vibration energy harvesting, compared to its conventional counterpart. It offers maximum 15Hz -3dB bandwidth and 202.4µW peak power at 0.5g which is increased up to 449µW at 1g acceleration. Piecewise linear impact of the primary mass on the mechanical stopper transforms an impulsive force to the secondary power generating element, introducing a frequency up-conversion mechanism and non-linear motion of the primary mass as well. Non-linearity results in wideband operation and frequency up-conversion allow the power to increase by increasing stress and decreasing source resistance in the generating element. Experimentally obtained results show that it can be used in automobile applications. Use of flexible beam material and mechanical stopper offers its reliable operation even when subjected to shock vibration. Optimization through further analysis and fabrication through MEMS process can further improve its performance.

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