Energetic optimization considering a generalization of the ecological criterion in traditional simple-cycle and combined cycle power plants

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Abstract

The fundamental issue in the energetic performance of power plants, working both as traditional fuel engines and as combined cycle turbine (gas-steam), lies in quantifying the internal irreversibilities which are associated with the working substance operating in cycles. Several irreversible models of energy converters have been studied under different contexts of non-equilibrium thermodynamics, their purpose is to find objective thermodynamic functions that determine more realistic bounds for real energy converters and at the same time reduce energy losses per cycle. As these objective functions characterize specific operation regimes, we focus our attention in one generalization of the so-called ecological function in terms of a generalizing parameter (ε) which can be related not only to a large number of accessible operation modes, but also to parameters representing both internal and external irreversibilities in models of power plants. In this work, we conclude the characteristic loops in the power output-efficiency space can be sketched, for either of the two irreversible heat engine models considered with heat leak. We found out that from a set of loops related to the physical constraints, the above systems work as heat engines and their operating conditions lie in 3 particular zones. The optimum zone featured by high power output and high efficiency is characterized through a relation between the irreversibility degrees and the constraints-parameters.

Keywords: Performance characteristics of energy conversion systems, Irreversible heat engine models, Characterization of configuration spaces, Fossil fuels and nuclear power

1 Introduction

In recent decades, various areas of the knowledge related to production of non fossil fuels and sustainable generation of energy, have invested efforts in combining innovative operation cycles with recovery waste heat systems [1, 2, 3]. As a result, numerous studies have been developed to determine energy conversion processes that reflect the best commitment between the maximum useful power generated and the maximum achievable efficiency [4, 5, 6, 7]. In general, the study of thermal engines has allowed not only to design more sophisticated engines but also to focus the attention on a more flexible operation, since the emissions during the combustion have become increasingly low. The type of thermal engines (energy converters), known as power plants have diversified and evolved due to social needs, whether for ecological [4] or economic [8, 9] reasons. In this context, the combined cycle power plants in whose construction coexist two thermodynamic cycles from the same source of heat, have turned out to have a greater amount of available energy. Although in practice, the amount of non useful energy generated compared to the used one continues to be a big problem, it can be modulated (reduced) by paying attention to the operation regimes [10] [11] [12] with which this type of power plants can be operated. These regimes are normally associated with operation and design parameters that measure qualitatively the internal and external irreversibilities.

There are several branches of non equilibrium thermodynamics [13, 14] in which physical models have been established to understand the performance of energy converters. In particular, we have used the approximation of Finite Time Thermodynamics (FTT) [15, 16, 17], this one has emulated energy conversion processes by means
of objective thermodynamics functions in several irreversible power plant models [18, 20, 19, 21, 22]. Within this context, different parameters related to their construction and the operation of the energy converters play a fundamental role to characterize objective functions which achieve a good trade off between process variables (such as power output, efficiency and dissipation) [23, 24, 25]. One of the most used models within the FTT context is the endoreversible Curzon and Ahlborn model (CA model) [26]. This model allows us to establish upper limits for the operation modes that a thermal engine can undergoes, considering not only external irreversibilities (heat transfer laws) but also the ones inside the working substance [27].

In 1994, as an extension to the CA model hypothesis (endoreversibility) [28], S. zkaynak et al [19] and J. Chen et al [20], introduced a modification, the non-endoreversibility parameter $R$. It takes into account the irreversibility degree of internal processes in the working substance. Another way of introducing internal irreversibilities is through the so-called uncompensated Clausius heat $r$ [29], which qualitatively measures the non recovered amount of heat during the operation cycles. Later, Fischer and Hoffmann [21] considered an additional term in this model (the so-called Novikov engine with heat leak [30]), they showed that from a simple model, the complex behavior of real heat engines is characterized by means of a dynamical simulation of an Otto engine, including the energy losses due to heat conduction and frictional losses. This way of introducing dissipative effects was first proposed by Bejan, Gordon and Huleihil [31, 32, 33], they verified that adding the information of a heat bypass, a more real behavior is reproduced in the operation of thermal machines (the well known loops in the Power output versus Efficiency space).

In a recent paper [7], the energetics of the CA endoreversible model was studied by means of a generalization of the so-called objective function Efficient power [25, 34]. Because of there are physically accessible operation points for lots of thermal engines, that can be reached without having to modify their interior design. With the help of the extremal properties which have the generalization of the known objective functions within the context of FTT, the best performance conditions can be found, in terms of the design and construction parameters of each machine. In such a way that the conditions are obtained for each energy converter (power plants) to operate in an energetic zone with high output power and high efficiency. In the case of Ecological function, it was shown that its optimization leads us to get a better economic performance than the Efficient Power one [34], since it has implicit idea of obtaining the highest possible power output and the highest efficiency at the lowest energetic costs (at lower entropy production).

In this work, we have considered two irreversible CA models (the non-endoreversible and irreversible one) and, by means of a generalization of the Ecological function [35] we identified three well-defined operation zones (ZI, ZII and ZIII) in the characteristic loops obtained for irreversible heat engine models. Those zones are completely characterized in the Power output-Efficiency configuration space by the generalization parameter ($\epsilon$), by highlighting their main features. The paper is organized as follows: in Section 2, we describe mathematically the irreversible CA models, as well as the conditions to reach the optimum points for both regimes of maximum power output and maximum efficiency. In Section 3, we study the constraints that must be satisfied the parameters associated with the irreversibilities in both irreversible models to establish the operation zones of the power plants. We also show that the $\epsilon$-parameter can modify its operation. Finally, in Section 4 we present our conclusions.

## 2 Energetic description of a CA–type heat engine

In this section, we present a thermodynamic analysis of a Curzon-Ahlborn-like irreversible engine extended as show in Fig. [1] which consists of two energy reservoirs: the first one at temperature $T_h$ and the other one at temperature $T_c$, where $T_h > T_c$. As is well known, the CA model includes two auxiliary energy reservoirs with working temperatures $T_{hw}$ and $T_{cw}$, with $T_h > T_{hw} > T_{cw} > T_c$ and they are in contact with a working substance operating in cycles. As in the typical CA model for a heat engine, this variant incorporates two thermal conductances $\alpha$ and $\beta$ that reflect the existence of natural heat flows through the materials that make up the heat exchangers. Besides, in this version of the CA–heat engine model, another thermal conductance ($\delta$) and internal irreversibilities in the working substance are taken into account, by means of an appropriate parameter, which makes this model emulates a behavior closer to what is happening in real heat engines.

As a first approximation to this CA model, we consider a linear heat transfer law (Newtonian). Therefore, the heat fluxes are given by:

$$ Q_h = \alpha T_h \left( 1 - a_h \right), $$

$$ Q_c = T_h \frac{\alpha \tau}{\gamma} \left( \frac{1}{a_c} - 1 \right) $$
Figure 1: Sketch of a modified CA heat engine

and

\[ Q_{hl} = T_h \delta (1 - \tau), \quad (3) \]

where \( \gamma = \alpha/\beta \) and \( \tau = T_c/T_h \) (0 < \( \tau < 1 \)); \( a_h = T_{hw}/T_h \) is defined as the high reduced temperature and \( a_c = T_c/T_{cw} \) as the low reduced temperature.

It is well known that the performance in several types of energy converters has become a topic of general interest, and its study has been carried out from different contexts [14, 36, 37]. FTT has proved to be a good tool for performing various energy analyzes, since it has allowed to establish mathematical relationships that contain information on the way in which energy exchanges take place between the system and its surroundings through phenomenological parameters. These relationships have derived in the main process variables such as power output, efficiency and dissipation. In this model, these process functions have the following form:

\[ P = Q_i - Q_O = (Q_h + Q_{hl}) - (Q_c + Q_{hl}), \quad (4) \]

\[ \eta = 1 - \frac{Q_c + Q_{hl}}{Q_h + Q_{hl}}, \quad (5) \]

and

\[ \Phi = T_c \sigma_T. \quad (6) \]

Where \( Q_i = Q_h + Q_{hl} \) and \( Q_o = Q_c + Q_{hl} \) correspond to the input and output heats respectively. While \( \sigma_T \) is the total entropy production of the heat engine and its surroundings; that is, \( \sigma_T = \sigma_e + \sigma_w \), where \( \sigma_e \) (the entropy produced by the surroundings) is given by:

\[ \sigma_e = \frac{Q_c + Q_{hl}}{T_c} - \frac{Q_h + Q_{hl}}{T_h}, \quad (7) \]

and \( \sigma_w \) (the entropy produced by the heat engine) is,

\[ \sigma_w = \frac{Q_h}{T_h} - \frac{Q_c}{T_c} + \sigma_s. \quad (8) \]

In the above equation, \( \sigma_s \) represents the entropy production of the working substance due to different dissipative processes such as; turbulence, friction, viscosity, etc. This term represents the irreversibilities of the working substance. To quantify this loss of energy, we used two schemes already mentioned above, the non-endoreversible and the irreversible models (uncompensated heat of Clausius) [38].
2.1 Non–endoreversible model with heat leak (NEHL)

In this model, the parameter of non-reversibility $R$ quantifies the irreversibilities of the working substance and is related through a heat flux \[19\] \[20\] as:

$$\sigma_s = (1 - R) \frac{Q_c}{T_{cw}}, \tag{9}$$

as every thermal engine operating in cycles satisfies that $\sigma_s = 0$, then by substituting Eq. \[9\] into Eq. \[8\] leads us for this model, the relationship between the reduced temperatures (high and low),

$$a_c(\gamma, R, a_h) = 1 - \frac{\gamma (1 - a_h)}{Ra_h}. \tag{10}$$

Thus, the heat flow $Q_c$ can be rewritten as:

$$Q_c(\alpha, \gamma, T_h, \tau, R, a_h) = \frac{Th\alpha\tau(1 - a_h)}{a_h(R + \gamma) - \gamma}. \tag{11}$$

On the other hand, the process functions obtained by substituting the heat fluxes given by Eqs. \[4\] \[5\] and \[11\] into Eqs. \[4\] \[5\] and \[6\] remain in function of $R$. In particular, for an energy converter to operate as a heat engine, it is required that all the process functions are defined positively. To guarantee the above, it is necessary that the parameters $R$, $\delta$ and $a_h$ fulfilled with certain restrictions. Firstly, $a_h$ must be bounded between,

$$\frac{\gamma + \tau}{\gamma + R} < a_h < 1. \tag{12}$$

Since in those points, the values of $P$ and $\eta$ are zero. In addition, heat fluxes and dissipation must be positive. For the inequality given by \[12\] to be kept, it is needed that $\tau < R \leq 1$.

In this model of energy converter, the parameters that allow to establish the configuration space \[7\] \[34\] are essentially the thermal conductance $\alpha$, the ratio between conductances $\gamma$, as well as the relationship between the temperatures of the reservoirs ($\tau$). These parameters assume any value within the intervals already defined, because of they control the energy flux that enters to the system. On the other hand, $\delta$ takes into accounts the amount of energy exchanged between external reservoirs restricting the performance of the converter. Finally, $a_h$ is associated with the operation modes to which the energy configuration of a converter can access. For this model, the process functions can be obtained by replacing Eqs. \[4\] \[5\] and \[11\] into Eqs. \[4\] \[5\] and \[6\] we get respectively,

$$P = Th\alpha(1 - a_h) \left(1 - \frac{\tau}{a_h(R + \gamma) - \gamma}\right), \tag{13}$$

$$\eta = \frac{\alpha(1 - a_h)[\gamma + \tau - a_h(R + \gamma)]}{[\gamma - a_h(R + \gamma)][\alpha(1 - a_h) + \delta(1 - \tau)]}, \tag{14}$$

and

$$\Phi = Th \left\{ \delta(1 - \tau)^2 - \frac{\alpha\tau(1 - a_h)[1 + \gamma - a_h(R + \gamma)]}{\gamma + a_h(R + \gamma)} \right\}. \tag{15}$$

Power output and efficiency have a value of high reduced temperature which maximizes them and therefore, this allows to characterize both the maximum power output and maximum efficiency regimes, respectively. These $a_h$ are given by:

$$a_{h^{MP}}(\gamma, \tau, R) = \frac{\gamma + \sqrt{R\tau}}{R + \gamma} \tag{16}$$

and

$$a_{h^{M\eta}}(\alpha, \delta, \gamma, \tau, R) = \frac{(R + \gamma)[\alpha\tau - \delta(1 - \tau)] - r_{\eta R}}{(R + \gamma)[\alpha\tau - \delta(R + \gamma)(1 - \tau)]}, \tag{17}$$

with $r_{\eta R}$ of the form:

$$r_{\eta R} = \sqrt{R\tau\delta(R + \gamma)(1 - \tau)[\gamma\delta(1 - \tau) - \alpha\tau + R\{\alpha + \delta(1 - \tau)\}]}. \tag{18}$$

The mathematical expressions \[16\] and \[17\] are the result of a derivative of power output and efficiency (obtained by substituting Eqs. \[4\] \[5\] and \[11\] into \[4\] and \[5\] with respect to $a_h$, in order to find the reduced temperatures that cancel the derivatives.
Although, the maximum power output regime exists within the non-endorreversible model \((\delta = 0)\), so that the maximum efficiency regime also appears in this model such that it is necessary to consider a heat leak term \([27, 33, 41]\).

### 2.2 Irreversible model with heat leak (IHL)

In this model, the parameter associated with internal irreversibilities is the Clausius uncompensated heat. So \(\sigma_s\) can be expressed in the form,

\[
\sigma_s = r \alpha, \tag{19}
\]

where \(r\) is defined as the quotient \(\sigma_s/\alpha\) which represents in some way, the irreversibility degree of the working substance.

For IHL model, the relationship between reduced temperatures is obtained by substituting Eq. \((19)\) into Eq. \((8)\), due to the working substance operates in cycles,

\[
a_c (a, \gamma, \tau, a_h) = 1 + \gamma (1 - r) - \frac{\gamma}{a_h}, \tag{20}
\]

in such a way, the heat flux \(Q_c\) is given by,

\[
Q_c (a, \delta, \gamma, \tau, T_h, r, a_h) = \frac{T_h \alpha \tau [a_h (1 - r) - 1]}{\gamma - a_h [\gamma (1 - r) + 1]}, \tag{21}
\]

Besides, the mathematical expressions for the process functions: power output \(P\), efficiency \(\eta\) and dissipation \(\Phi\), can be obtained replacing the Eqs. \((1)\) \((21)\) and \((8)\) into Eqs. \((4)\) \((5)\) and \((9)\) that is,

\[
P = T_h \alpha \left\{1 - a_h + \frac{\tau [1 - a_h (1 - r)]}{\gamma - a_h [1 + \gamma (1 - r)]}\right\}, \tag{22}
\]

\[
\eta = \frac{\alpha \tau [1 - a_h (1 - r)] + \alpha (1 - a_h) \{\gamma - a_h [1 + \gamma (1 - r)]}\}}{\delta (1 - \tau) + \alpha (1 - a_h) [\gamma - a_h (1 + \gamma (1 - r))]} \tag{23}
\]

and

\[
\Phi = T_h a_h^2 \alpha \tau [1 + \gamma (1 - r)] - a_h \left\{\alpha \tau (1 + \gamma) (2 - r) - \delta (1 - \tau)^2 [1 + \gamma (1 - r)]\right\} + \alpha \tau + \gamma \left[\alpha \tau - \delta (1 - \tau)^2\right] \tag{24}
\]

As in the previous model, both power output and efficiency are at the same time objective functions with a maximum and both are zero at the same high reduced temperature values. While, dissipation function is a decreasing function with respect to the \(a_h\) variable. Analogously, the zeros of \(P\) and \(\eta\) functions define the interval of values for \(a_h\) that allow the energy converter to work as a heat engine,

\[
\frac{1 + \gamma (2 - r) + \tau (1 - r) - n_i}{2[1 + \gamma (1 - r)]} \leq a_h \leq \frac{1 + \gamma (2 - r) + \tau (1 - r) + n_i}{2[1 + \gamma (1 - r)]}, \tag{25}
\]

where \(n_i\) is given by,

\[
n_i = \sqrt{(1 - \tau)^2 - 2r (1 + \tau) (\gamma + \tau) + r^2 (\gamma + \tau)^2}, \tag{26}
\]

because of \(\tau\) is related to the thermal gradient that promotes the heat flow inside the converter and therefore, this limits the available energy in the system to access certain operation modes, its effect is reflected in the values of \(a_h\). Likewise, for \(a_h\) to satisfy the inequality Eq. \((25)\), the following condition must be fulfilled simultaneously:

\[
0 < r < \frac{(1 - \sqrt{\tau})^2}{\tau + \gamma}, \tag{27}
\]

with \(\gamma > 0\). On the other hand, the value that \(\delta\) can takes is generally positive. Nevertheless, it is always desirable that this thermal conductance must be lower than the \(\alpha\) value, so that a greater entering heat flux to the system is guaranteed, i.e, the inequality must be kept \(0 \leq \delta < \alpha\). In the following section, we establish the necessary conditions for the parametric curves \((P \text{ vs } \eta\) and \(P \text{ vs } \Phi)\) in order to they are compatible with the characteristic operation mode.
In this scheme, as in the NEHL model, there are high reduced temperatures values that allow us to maximize the power output and efficiency, to attain both the maximum power output and maximum efficiency regimes. These reduced temperatures are given by,

$$ a^M_P (\gamma, \tau, r) = \frac{\gamma + \sqrt{\tau}}{1 + \gamma (1 - r)}, $$

(28)

and

$$ a^M_\eta (\alpha, \delta, \gamma, \tau, r) = \frac{\alpha \tau + \gamma \left\{ \alpha \tau (1 - r) - \delta (1 - \tau) [1 + \gamma (1 - r)] \right\} - r_{\eta r}}{\left[ 1 + \gamma (1 - r) \right] \left\{ \alpha \tau (1 - r) + \delta (1 - \tau) [1 + \gamma (1 - r)] \right\}} $$

(29)

with $r_{\eta r}$ given by:

$$ r_{\eta r} = \sqrt{\tau \left[ 1 + \gamma (1 - r) \right] \left\{ \delta^2 (1 - \tau)^2 [1 + \gamma (1 - r)] + r \alpha^2 \tau + \alpha \delta (1 - \tau) [1 - r (\gamma - \tau)] - \tau \right\}}. $$

(30)

Similarly, the $a_h$ given by Eqs. 28 and 29 that equalize to zero the mathematical expressions for the derivatives of power output and efficiency are obtained by replacing Eqs. 1, 3 and 21 into 4 and 5. Under this model, it is important to note that the maximum efficiency regime exists without the need to consider a heat leakage.

3 Configuration and energetic reconfiguration of some power plants

During the operation of several thermal engines a great number of parameters associated with the exchange of energy joined in, which together give rise to a configuration space. In this space there is an infinity of operation modes compatible with the performance of every energy converter [27, 34] which is formed by a unique combination of phenomenological parameters, and are related to each other through the model of each converter that is being analyzed. The irreversible models used in this work incorporate the most representative elements in the energy transfer such as the thermal conductances ($\alpha$, $\delta$ and $\beta$), as well as the $\gamma$ parameter [7, 34]. Another important parameter is the temperatures ratio between the external reservoirs ($\tau$) related to the capacity of the system to promote an effective heat flow and the parameters associated with the internal irreversible degree ($R_xr$).

When the behavior of the $P$ vs $\eta$ curves in the NEHL and IHL models is analyzed, a particular curve (loop) can be observed, in which three operating zones are well distinguished (see Figs. 2 a and b). In each zone there are specific operation modes that allow the energy converter to achieve a unique performance. The operation modes located in zone I (ZI) are usually characterized by an $a_h$ such that $a_h \in (a_{h0}, a^M_P)$, where $a_{h0}$ is the value of the high reduced temperature from which the converter starts to work as a heat engine and $a^M_P$ represents the high reduced temperature value of the maximum power output regime, these modes have high dissipation (HD) and low
Table 1: Operating reported data of some power plants: temperatures of energy reservoirs \((T_h, T_c)\), power output \((P)\) and efficiency \((\eta)\). Two nuclear (Almaraz II and Cofrentes plants), two mono-cycle (West Thurrock and Larderello plants) and two of combined cycle (Toshiba and Alstom plants).

| Plant          | PWR, Spain, 83 | West Thurrock (WT) | Toshiba (T) |
|----------------|----------------|--------------------|-------------|
| Almaraz II (A)|                 |                    |             |
| (BWR, Spain, 84) | 562 289 | 523 353 | 1398 0.48 |
| Cofrentes (C)  |                 |                    |             |
| (BWR, Spain, 84) | 600 290 | 838 298 | 1573 303 |
| Toshiba (T)    |                 |                    |             |
| (109FA, 04)    | 1.044 0.35      | 1.240 0.36         | 0.342 0.48 |
| Alstom (Al)    |                 |                    |             |
| (ka26-1)       | 0.150           | 0.16               | 0.410 0.57 |

efficiency \((LE)\). The operation modes in zone II \((ZII)\) are distinguished by \(a_h \in \left[ a_h^{MP}, a_h^{MQ} \right]\) where \(a_h^{MQ}\) is the high reduced temperature value of the maximum efficiency regime. In this zone, the performance of the converters reach high power output \((HP)\), good efficiency \((HE)\) and a moderate dissipation. On the other hand, when the operation mode of a converter is in zone III \((ZIII)\), it is usually associated with \(a_h \in \left( a_h^{MQ}, a_h^{K} \right]\), where \(a_h^{K}\) is the upper bound for the high reduced temperatures that restricts every energy converter to operate as a heat engine, the operation modes in \(ZIII\) have low dissipation and low power output \((LD\) and \(LP)\). It is understood by \(HE\) an efficiency greater than the efficiency of maximum power output regime. In \(HD\) stay the highest dissipation values bounded by the maximum power output regime, while \(LD\) refers to dissipation values less than the dissipation of the maximum efficiency regime. Every power output greater than the power output in the maximum efficiency regime can be considered as \(HP\), this behavior is shown in Fig. 2a and b. Another particularity is observed in Fig. 2c, an energy converter can have the same power output for two different efficiency values. Similarly, for a given efficiency, a converter can develop two completely different power output values.

In Table 1, some reported operating data are shown for some power plants (temperatures of the reservoirs, power output and efficiency values). With these data and using any of the CA–type irreversible models, it is possible to mark off with greater precision the values in the configuration space that assure an energy generating plant to operate as a heat engine, and simultaneously give the necessary conditions to study the quality in its operation.

### 3.1 Energetic configuration of NEHL and IHL models

When energetic performance of different power plants under NEHL and IHL models is analyzed, it is needed more information than the provided in Table 1 to establish the appropriate configuration space, in which can be found all of the particular operation modes of any power plants. While efficiency and power output depict a point in the configuration space, there is a great number of curves (which correspond to a particular combination of construction parameters) that allow the converter to reach specific values of \(P\) and \(\eta\). Due to the above, it is possible to give conditions on the parameters that are not reported during the operation of the power plants, using the operating bounds imposed by each model.

As the thermal conductances \((\alpha, \beta, \gamma, \delta)\) must be defined positive, then \(\gamma > 0\). Within the IHL model \(r \in [0, \infty)\), in the other one \((NEHL)\) \(R \in (0, 1]\). The value of \(\tau\) and \(a_h\) are in the range \((0, 1]\), no matter the irreversible model.

#### 3.1.1 NEHL model

In this model, we have determined the parameters \(\delta, R\) and \(a_h\) require extra–thermodynamic constraints to fully characterize a compatible point with a operation mode in the configuration space. As \(R\) quantifies to some degree the irreversibilities of the working substance during the operation of a heat engine, it can also be associated with the energy dissipated by operation cycle. \(\delta\) parameter modulates the amount of energy that leaks from the system and therefore plays no role in the operation of the converter. With the inclusion of these irreversibilities, it is guaranteed the parametric curves \(P\) vs \(\eta\) form the well–known loop. Figs. 3a and 3b reflect the use of \(a_h(\eta)\) as the parametric variable. Since it is always possible to isolate \(a_h\) from the expression for efficiency (Eq. 14). When
Figure 3: Curves of power output vs efficiency for a non-endoreversible model with heat leak and different configurations, in each of them some parameters have been varied, on the basis of (I) \( \alpha = 1, \gamma = 3, \tau = 0.5, \delta = 0.001, R = 0.9 \) y \( T_h = 500K \). In c) we show how Eqs. 31 (solid line) and 32 (dashed line) allow to describe the operation modes of the curve (I) in their respective zones.

\( a_h(\eta) \) is substituted in the expression for power output (Eq. 13), we have two equations in terms of efficiency, given by

\[
P = T_h \eta \frac{R[\alpha(1 - \eta) + \delta(2 - \eta)(1 - \tau)] + \gamma \delta(2 - \eta)(1 - \tau) - \alpha \tau + r_{pR}}{2\alpha(R + \gamma)(1 - \eta)},
\]

and

\[
P = T_h \eta \frac{R[\alpha(1 - \eta) + \delta(2 - \eta)(1 - \tau)] + \gamma \delta(2 - \eta)(1 - \tau) - \alpha \tau - r_{pR}}{2\alpha(R + \gamma)(1 - \eta)}.
\]

With \( r_{pR} \),

\[
r_{pR} = \sqrt{[\gamma \delta \eta + R \eta(\alpha + \delta) - Ra]^2 - 2\tau \{ Ra^2 - \alpha R \eta(\alpha - \delta)(R - 1)(R + \gamma) + \delta \eta R(R + \gamma)[\gamma \delta + R(\alpha + \delta)] \}} + d_R.
\]

where,

\[
d_R = \tau^2[\alpha + \delta \tau(R + \gamma)]^2.
\]

Eq. 31 describes the points located in zone I and II, while Eq. 32 allows to plot the points situated in zone III (as shown in Fig. 3c).

As the reported data in Table I are not enough to generate the configuration space of each power plants, the model (NEHL) allows us to get some relationships between the parameters related to the main sources of irreversibilities (\( \delta, R \)) and the variables that modulate the energy input to the system (\( \alpha, \gamma \) and \( T_h \)). Because of the mathematical expressions for the process functions (\( P, \eta \) and \( \Phi \)), show a dependency on these parameters and their simultaneous variation, modifies the performance of a converter. In Figs. 3a and b is also observed how the relation \( P vs \eta \) is modified when one of the parameters changes. For instance, a change in \( \alpha \) or \( \gamma \) stands for a variation in power output, without reducing considerably the efficiency; if \( T_h \) is modified, the effect is only reflected in the power output. On the contrary, when the parameters \( \tau, \delta \) and \( R \) change, both \( P \) and \( \eta \) are significantly altered (see blue (III) and orange (VI) curves).

Therefore, it is convenient to establish a direct relationship between \( R \) and \( \delta \) with \( P \) and \( \eta \). With the Eqs. 31 and 32 it is guaranteed that a specific curve contains the corresponding operation mode with the power and efficiency reported by the plants that we are analyzing. Moreover, \( P \geq 0 \) for any operation zone then, from the Eqs. 31 and 32 it is concluded that the parameter \( \delta \) must be of the form:

\[
\delta_{Z1} = \frac{P(R + \gamma)(2 - \eta) - \eta \left[T_h \alpha(R - \tau) - \sqrt{P^2(R + \gamma)^2 + T_h^2\alpha^2(R - \tau)^2 - 2PT_h\alpha(R + \gamma)(R + \tau)}\right]}{2T_h \eta(R + \gamma)(1 - \tau)}
\]
or

\[
\delta_{Z2,3} = \frac{P (R + \gamma) (2 - \eta) - \eta \left[ T_h \alpha (R - \tau) + \sqrt{P^2 (R + \gamma)^2 + T_h^2 \alpha^2 (R - \tau)^2 - 2PT_h \alpha (R + \gamma) (R + \tau)} \right]}{2T_h \eta (R + \gamma) (1 - \tau)}.
\] (36)

The value of \(\delta_{Z1}\) given by Eq. 35 associates the mode of operation reported by any of the power plants with a particular energy configuration in \(ZI\), according to this model. Therefore, \(\alpha\) parameter must be bounded as follows:

\[
P (R + \gamma) \left( \sqrt{R} + \sqrt{\tau} \right)^2 \leq \alpha_{Z1} < \frac{P (R + \gamma) (1 - \eta)}{T_h \eta |R (1 - \eta) - \tau|}.
\] (37)

this condition is obtained by finding the points for which \(\delta\) is non-negative. Inequality 37 is positive when the parameter \(R\) is between,

\[
\frac{\tau}{(1 - \eta)^2} < R \leq 1.
\] (38)

Likewise, to be physically consistent this condition must be satisfied:

\[
0 < \tau < (1 - \eta)^2.
\] (39)

For the possible \(\delta_{Z2,3}\) values provided by eq. 36 which relate a characteristic operation regimen to a configuration in \(ZII\) or \(ZIII\) \((\delta_{Z2,3} > 0)\), then \(\alpha\) parameter must carry out,

\[
\alpha_{Z2,3} > \frac{P (R + \gamma) (1 - \eta)}{T_h \eta |R (1 - \eta) - \tau|},
\] (40)

since these \(\alpha\) values makes the parameter \(\delta_{Z2,3}\) equal to zero. In addition, to ensure that Eq. 40 is positive, the \(R\) parameter must be bounded by,

\[
\frac{\tau}{1 - \eta} < R \leq 1.
\] (41)

it should be noted that the lower bound of \(R\) is an asymptotic point, this makes tend \(\alpha\) to infinity. Therefore, \(R\) must satisfy the inequality 41 and so,

\[
0 < \tau \leq (1 - \eta).
\] (42)

Although Eq. 39 is contained in Eq. 42 when the relationship between \(\tau\) and \(\eta\) that satisfies exclusively the constraint 42 configurations appear for which the operation mode reported by the power plant is in \(ZI\).

### 3.2 IHL model

In this model, we define once again, under extra thermodynamic conditions, the constraints for the parameters \(a_h\), \(r\) and \(\delta\) in such a way they allow the construction of a curve in the configuration space compatible with a particular operation model. In this case, \(r\) reflects the internal irreversibilities of the system and therefore is associated with the dissipated energy during the operation of the heat engine. The conductance \(\delta\) can be interpreted as a control mechanism that regulates the incoming energy to the system. Although \(r\) is the only parameter that allows the model to characterize the loops in the \(P\) vs \(\eta\) plane, parameter \(\delta\) affects the maximum value that the efficiency can reach (Fig. 4 a). Therefore, some restrictions are imposed on the \(\delta\) parameter, which are directly related to the parametric equations:

\[
P = T_h \eta \left[ \alpha (1 - r\gamma) (1 - \eta) + \delta (1 - \tau) (2 - \eta) [1 + \gamma (1 - r)] - \alpha \tau (1 - r) - r_{1r} \right] \frac{1}{2 (1 - \eta) [1 + \gamma (1 - r)]}
\] (43)

and,

\[
P = T_h \eta \left[ \alpha (1 - r\gamma) (1 - \eta) + \delta (1 - \tau) (2 - \eta) [1 + \gamma (1 - r)] - \alpha \tau (1 - r) - r_{1r} \right] \frac{1}{2 (1 - \eta) [1 + \gamma (1 - r)]}
\] (44)

where Eq. 43 locate the operation modes found in I and II zones. While in Zone III the modes are described by Eq. 44 (see Fig. 4 b). With \(r_{1r}\),
Figure 4: Several configurations for a) power output vs efficiency plane of the IHL model. In each configuration the model’s parameters have been considered, taking as reference the loop I, built with the values of: $\alpha = 1$, $\delta = 0.001$, $\gamma = 3$, $\tau = 0.5$, $T_h = 500$ and $r = 0.001$. In b), loop I is shown with its characteristics zones described by Eqs. 43 (solid line) and 44 (dashed line).

\[
\begin{align*}
\text{r}_{1r} &= \sqrt{\{\delta \eta (1 - \tau) [1 + \gamma (1 - r)] - \alpha (1 - \eta) [1 + \gamma (2 - r)] - \alpha \tau (1 - r)\}^2 + d_{1r}}, \\
\end{align*}
\]

and \(d_{1r}\) on the form of,

\[
\begin{align*}
d_{1r} &= 4 \alpha (1 - \eta) [1 + \gamma (1 - r)] \left[\gamma \delta \eta - \alpha \gamma (1 - \eta) - \tau (\alpha + \delta \tau \eta)\right].
\end{align*}
\]

Analogous to the NEHL model, to find a curve (loop) compatible with the efficiency and power output values reported in some power plants data within the configuration space, parameter $\delta$ has two cases according to the operation zone:

\[
\begin{align*}
\delta_{Z1} &= \frac{P (2 - \eta) [1 + \gamma (1 - r)] + \eta \{T_h \alpha [\tau + r (\gamma - \tau) - 1] - r_\delta\}}{2T_h (1 - \tau) [1 + \gamma (1 - r)]}, \\
\delta_{Z2,3} &= \frac{P (2 - \eta) [1 + \gamma (1 - r)] + \eta \{T_h \alpha [\tau + r (\gamma - \tau) - 1] + r_\delta\}}{2T_h (1 - \tau) [1 + \gamma (1 - r)]},
\end{align*}
\]

where,

\[
\begin{align*}
r_\delta &= \sqrt{\{P [1 + \gamma (1 - r)] + T_h \alpha (r \gamma - 1)\}^2 - 2T_h \alpha \tau \{T_h \alpha + [P (1 - r) + r T_h \alpha] [1 + \gamma (1 - r)]\} + T_h^2 \alpha^2 \tau^2 (1 - r)^2}.
\end{align*}
\]

To ensure that $\delta_{Z1}$ (in ZI) and $\delta_{Z2,3}$ (in ZII or ZIII) are positive, $\alpha$ and $r$ parameters must be constrained. For example, all of the operation modes in ZI must fulfill:

\[
\begin{align*}
P \left[\frac{1}{P - T_h \alpha \eta} + \frac{1 - \eta}{P \gamma (1 - \eta) + T_h \alpha \eta \tau}\right] < r < \frac{P (1 + \gamma) - T_h \alpha (1 - \sqrt{\tau})^2}{P \gamma - T_h \alpha (\gamma + \tau)},
\end{align*}
\]

this condition guarantees that $\delta_{Z1} > 0$. As long as,

\[
\begin{align*}
P \frac{(1 + \gamma)}{T_h (1 - \sqrt{\tau})^2} < \alpha_{Z1} < \frac{P \left[\gamma (1 - \eta) + \sqrt{\tau}\right]}{T_h \eta \sqrt{\tau} (1 - \sqrt{\tau})},
\end{align*}
\]

and at the same time $\delta_{Z1} > 0$.

In the same way, for every operation mode in ZII or ZIII, if $\delta_{Z2,3} > 0$ then by transitivity:
and
\[ 0 < r < P \left[ \frac{1}{P - T_h \alpha \eta} + \frac{1 - \eta}{P \gamma (1 - \eta) + T_h \alpha \eta \tau} \right], \]  
(52)
and
\[ 0 < \alpha_{Z2,3} < \frac{P (1 + \gamma) (1 - \eta)}{T_h \eta [1 - (\eta + \tau)]}. \]  
(53)

In the NEHL and IHL models, three particular operation zones are characterized. Although there are thermal engines that can operate in ZI or ZIII, it is desirable they operate within ZII. In the following, we will show that from a specific parametric variable, it is possible to find conditions to project operation modes outside of ZII to modes within it.

### 3.3 Energetic reconfiguration of NEHL and IHL models

Within the context of the FTT, it has been possible to define certain objective functions, this type of functions connected to the performance of an energy converter with a particular operation mode. Some examples are: the Power output function [20], the Ecological function \((E)\) [23], and in the case of NEHL and IHL models, the Efficiency function itself [39]. As the ecological function relates the power output and the dissipation of a system \((E = P - \Phi)\) in such a way that the compromise between these two process variables is maximum, then the well known ecological maximum regimen can be accessed. However, despite the fact that different objective functions have been characterized (Omega function \(\Omega\) [24], Efficient power \(P_\eta\) [25], etc.), when their characterized regimes are compared with the operation modes to which an energy converter can access, it is clear that there is a large number of them with neither associated objective function. For this reason, several generalization proposals have emerged for the aforementioned functions [35, 41], as an alternative to represent more physical attainable operation modes. One example is the generalization for the ecological function, in which the so-called generalization parameter \((\epsilon)\) is incorporated:

\[ E_G = P - \epsilon \Phi, \]  
(54)

this \(\epsilon\)-parameter generates a family of ecological functions. To pick and choose one of them different ways been used [10, 41]. Nevertheless, in [41] it has been shown through a Newtonian heat transfer law, the possibility of linking a generalization parameter with any operation mode.

Up to this point, we have given conditions so that different physically reachable operation modes are located on a loop described by a particular configuration space. However, it has not been specified under what conditions this operation mode could be in any of the operation zones. Nor have we mentioned that there is the possibility of finding conditions so that the operation modes that are not in ZII can be led to this zone; considering the following possibilities: maintaining the reported power output and improving the efficiency, retaining the reported efficiency and raising the power output or simply finding a new configuration where the reported power output and efficiency values are in ZII. To achieve these improvements, we have used the the ecological function’s generalization parameter [35], and thus characterize the operation mode in which the power plants of the Table 1 are working, we also explore other modes to which they could access. The above allows us to establish the so-called “improvement condition” [7, 41]. On the other hand, by means of the maximum efficiency regime, we find other conditions with which each power plant would be operating in ZII.

#### 3.3.1 NEHL model

The purpose of the energetic restructuring lies in finding conditions of certain elements (heat exchangers) for different power plants, such that they operate in ZII. In Fig. 5a, two different configurations are shown, for which West Thurrock plant (WT) can operate. In the first one, the operation mode with \(\alpha = 0.95GW/K, \gamma = 3, R = 0.75\) and \(\delta = 1.951MW/K\), is in ZIII. While the other one, whose configuration is \(\alpha = 0.078GW/K, \gamma = 3, R = 0.784\) and \(\delta = 910.525W/K\), it is shown that its associated operation mode lies in ZII. In Fig. 5b, for the reported data in Table 4 a configuration whose design parameters are: \(\alpha = 67.941MW/K, \delta = 1.75MW/K, \gamma = 3.5\) and \(R = 0.9\) (loop in solid line), the operation mode emulate the maximum efficiency regime. In Fig. 5b, the maximum power output regime is obtained when the construction parameters are: \(\alpha = 0.052GW/K, \delta = 196.02W/K, \gamma = 3.5\) and \(R = 0.9\), also in solid line. When these cases are compared, a fact is revealed: if some control parameters are varied, the energetic performance of this type of thermal engines can improve (better power output and efficiency). In addition, to know the necessary conditions which an operation mode can be in ZII or ZIII, it is necessary to have well defined the point that corresponds to the regime of maximum efficiency, because it separates both zones.
Figure 5: Characteristics loops for West Thurrock plant. In a) the loops exemplify two configurations, one of them lies in ZII (solid line). The other one (dotted line) represents a configuration whose operation mode is in ZIII. In b) it is shown a configuration (solid line) in which, the reported mode for WT plant represents the maximum efficiency. In dotted line, the same operation mode is located in ZIII with new features. In c) a specific configuration is sketched, in which the operation mode under new conditions for WT plant represents the maximum efficiency point (solid line). The other configuration contains the same operation mode is in ZIII (dotted line) with new features.

Replacing Eq. 17 in the mathematical expression for the power output (Eq. 4) and by solving for \( \alpha \) parameter, this new solution helps us to find the first necessary transition condition between zones ZIII \( \rightarrow \) ZII.

\[
\alpha_{M\eta} = \frac{\delta\eta (R + \gamma) (1 - \tau) \left[ R (1 - \eta) + \tau + 2\sqrt{R\tau (1 - \eta)} \right]}{[\tau - R (1 - \eta)]^2}.
\]  

(55)

The other necessary condition arises from evaluating the efficiency function at the point of maximum efficiency (replacing eq. 17 in Eq. 5) and by solving for \( \delta \) we get,

\[
\delta_{M\eta} = \frac{P (R + \gamma) (R + \tau) + (R - \tau) \left[ \sqrt{P^2 (R + \gamma)^2 + T_h^2 \alpha^2 (R - \tau)^2 - 2PT_h\alpha (R + \gamma) (R + \tau) - T_h\alpha (R - \tau)} \right]}{2 (R + \gamma) (1 - \tau) \sqrt{P^2 (R + \gamma)^2 + T_h^2 \alpha^2 (R - \tau)^2 - 2PT_h\alpha (R + \gamma) (R + \tau)}}.
\]

(56)

By solving the equations system formed by eqs. 55 and 56, we get the values of \( \alpha \) and \( \delta \) that would represent the operation mode of WT plant under the maximum efficiency regime. Thus, there exists a particular \( \alpha^* = \alpha(R, P, \eta) \). So, for any \( \alpha \) bounded between \( \alpha_{ZII,ZIII} \) (see Eq. 40) and \( \alpha^* \), the operation mode reported in Almaraz II plant will always be stayed in ZII. On the other hand, if \( \alpha > \alpha_{\eta R} \) then it will be located in ZIII. Analogously, there are conditions for \( \alpha \) and \( \delta \) such that the operation mode reported during the running of the power plants, they represent the maximum power output regime. Thus, by replacing Eq. 16 in Eq. 4, we get a mathematical expression for \( \delta \); that is,

\[
\delta_{MP} = \frac{\alpha \left[ \sqrt{R + \gamma} (1 - \eta) + (\eta - 2) \sqrt{R\tau} \right]}{\eta (R + \gamma) (1 - \tau)}.
\]

(57)

in the same way, by replacing Eq. 16 in Eq. 5 we have,

\[
\alpha_{MP} = \frac{P (R + \gamma) \sqrt{R\tau}}{T_h \left( \sqrt{R\tau - R} \right) \left( \sqrt{R\tau - \tau} \right)}.
\]

(58)

The lack of information in the reported data, on the operation of the power plants, restricts knowing the number of energetic favourable configurations, i.e, there are plants that operate in unprofitable zones (ZI and ZIII) energetically. To identify in which operation zone each of the analyzed plants is located, we use the generalization
parameter of the Ecological Function (Eq. 54) as the parametric variable that allows to establish a relation between $a_h$, power output $y$, and efficiency. By using Eqs. 13 and 15 in the expression for ecological function (Eq. 54) and by taking the derivative with respect to the high reduced temperature, we get:

$$a_h^{MEG} = \frac{\gamma (1 + \epsilon \tau) + \sqrt{R\tau(1 + \epsilon)(1 + \epsilon \tau)}}{(R + \gamma)(1 + \epsilon \tau)}.$$  \hspace{1cm} (59)

This high reduced temperature that characterizes the maximum ecological generalization regime allows to establish a relationship between $\epsilon$-parameter and the processes variables $P$ and $\eta$. By replacing Eq. 59 in the expression for power output (see Eq. 13), we get:

$$\epsilon_{PR1} = \frac{(1 + \tau) n_{\epsilon p1} + (1 - \tau) [P (R + \gamma) - T_h \alpha (R + \tau)] \sqrt{n_{\epsilon p1}}}{2\tau T_h^2 \alpha^2 (R - \tau^2) (1 - R) + 2PT_h \alpha (R + \gamma) (R + \tau) - P^2 (R + \gamma)^2}.$$  \hspace{1cm} (60)

and

$$\epsilon_{PR2} = \frac{(1 + \tau) n_{\epsilon p1} - (1 - \tau) [P (R + \gamma) - T_h \alpha (R + \tau)] \sqrt{n_{\epsilon p1}}}{2\tau T_h^2 \alpha^2 (R - \tau^2) (1 - R) + 2PT_h \alpha (R + \gamma) (R + \tau) - P^2 (R + \gamma)^2},$$  \hspace{1cm} (61)

in analogous way, by replacing Eq. 59 into Eq. 14 for efficiency, we obtain

$$\epsilon_{\eta R1} = \frac{R^2 (1 + \tau) \left[ \alpha (1 - \eta) + \delta \eta (1 - \tau)^2 \right] + n_{\eta 1} - (1 - \tau) \left\{ R \left[ \alpha (1 - \eta) - \delta \eta (1 - \tau) \right] - \gamma \delta \eta (1 - \tau) + \alpha \tau \right\} r_{\eta 1}}{2\tau \left\{ \alpha^2 \tau \left( \tau + (1 - \eta)^2 \right) - \gamma \delta^2 \eta^2 (1 - \tau)^2 (1 + \tau) - \alpha \delta \eta \left[ \gamma (1 - \eta) - \tau \right] (1 - \tau)^2 \right\}},$$  \hspace{1cm} (62)

and

$$\epsilon_{\eta R2} = \frac{R^2 (1 + \tau) \left[ \alpha (1 - \eta) + \delta \eta (1 - \tau)^2 \right] + n_{\eta 1} - (1 - \tau) \left\{ R \left[ \alpha (1 - \eta) - \delta \eta (1 - \tau) \right] - \gamma \delta \eta (1 - \tau) + \alpha \tau \right\} r_{\eta 1}}{2\tau \left\{ \alpha^2 \tau \left( \tau + (1 - \eta)^2 \right) - \gamma \delta^2 \eta^2 (1 - \tau)^2 (1 + \tau) - \alpha \delta \eta \left[ \gamma (1 - \eta) - \tau \right] (1 - \tau)^2 \right\}}.$$  \hspace{1cm} (63)

Where $n_{\epsilon 1}$, $n_{\eta 1}$ and $r_{\eta 1}$ are given by:

$$n_{\epsilon p1} = P^2 (R + \gamma)^2 + T_h^2 \alpha^2 (R - \tau^2) - 2PT_h \alpha (R + \gamma) (R + \tau),$$  \hspace{1cm} (64)

$$n_{\eta 1} = 2R \left\{ \gamma \delta^2 \left( 1 - \tau \right)^2 (1 + \tau) - \alpha^2 \tau \left[ \tau + (1 - \eta)^2 \right] + \alpha \delta \eta \left[ \gamma (1 - \eta) - \tau \right] (1 - \tau)^2 \right\} + (1 + \tau) \left[ \alpha \tau - \gamma \delta \eta (1 - \tau) \right]^2$$  \hspace{1cm} (65)

and

$$r_{\eta 1} = \sqrt{[\gamma \delta \eta - R\alpha + R\eta (\alpha + \delta)]^2 - 2 \{ R\alpha^2 - \alpha \eta [R\alpha - \delta (1 - R) (R + \gamma)] + \delta \eta^2 (R + \gamma) \gamma \delta + R (\alpha + \delta) \}}.$$  \hspace{1cm} (66)

The above suggests the operating modes that lie in a configuration curve can be specified via the $\epsilon$-parameter (see Fig. 6). And according to each operation zone, they must be specified with some of the values: $\epsilon_{PR,\eta R1}$ or $\epsilon_{PR,\eta R2}$.

In Eq. 59 we have proved $a_h = a_h(\epsilon)$. That is, $\epsilon$ characterizes the operation zones analogously to the high reduced temperature, since in practice it is difficult to measure directly the working temperature ($T_{hw}$). Then, by replacing Eqs. 60 in Ineq. 12 as well as in Eqs. 16 and 17, can be obtained bounds for $\epsilon$, given by:

$$\frac{\tau - R}{R - \tau^2} < \epsilon < 0,$$  \hspace{1cm} (67)

in such a way that the operation mode is located in ZI. On the contrary, if it lies in:

$$0 \leq \epsilon \leq \frac{r_{\eta R}^2 - 2R\alpha \tau r_{\eta R} + R\tau \left[ R\alpha^2 + 2\alpha \kappa \delta + 2\kappa^2 \delta^2 \right] - \tau^2 \left( \alpha + \kappa \delta \right)^2 - \kappa^2 \delta^2}{\tau \left[ R\alpha^2 \delta^2 - R^2 r_{\eta R} + 2R\alpha \tau r_{\eta R} + R\tau \left[ \alpha^2 (1 - R) + 2\alpha \kappa \delta + \kappa^2 \delta^2 \right] - 2\kappa \delta \left[ \alpha + \delta \kappa \right] \right]}.$$  \hspace{1cm} (68)

the operation mode is in ZII. Finally, whether the value
loops are not equivalent to those of the NEHL model. However, the variation of the control parameters influence the heat engine performance. Under this model, the generated α is a system of equations that arises from replacing Eq. 28 into Eqs. 4 and 5 in the case of 52 and 53. Thus, we find conditions for α = 0 and γ = 0.

So that the reported operation mode of WT (Tab. 1) can represent α = 0 and γ = 0. Which also show the possibility of performing transitions between zones ZII and ZIII. In a) it is shown that Almaraz II plant can go from operating in ZIII (ε_α2 > ε_MPO) to a point ε_α1 that represents its performance in ZII maintaining the same efficiency and improving its power output. In b), Larderello plant works in ZI (ε_p1 < ε_MPO) and the point ε_p2 symbolizes its operation in ZII, which has the same power output and improve its efficiency. In c) Toshiba’s plant operation is schematized in ZIII (ε_α2 < ε_MPO) and the point ε_p2 shows its performance in ZII improving both the power as efficiency.

\[
\frac{r^2_{\eta R} - 2 R \alpha \tau r_{\eta R} + R \tau \left[\tau \left( R \alpha^2 + 2 \alpha \kappa \delta + 2 \kappa^2 \delta^2 \right) - \tau^2 (\alpha + \kappa \delta)^2 - \kappa^2 \delta^2 \right]}{\tau \left( R \kappa^2 \delta^2 - R^2 r_{\eta R} + 2 R \alpha \tau r_{\eta R} + R \tau \left[\tau \left( \alpha^2 (1 - R) + 2 \alpha \kappa \delta + \kappa^2 \delta^2 \right) - 2 \delta \kappa (\alpha + \delta \kappa) \right) \right]} < \epsilon < \frac{R - \tau}{\tau (1 - R)}, \tag{69}
\]

the operation mode will be located inside of ZIII. These dimensions for ε are obtained after replacing the boundary conditions: eqs. 12 and 17 in eq. 59.

### 3.3.2 IHL model

Another way to find the restructuring conditions for the "optimal" operation of power plants is through the IHL model. Similarly as in the previous section and, considering the new set of variables α, γ, r and δ, there are for example, two different configurations for West Thurrock plant (Fig. 7). In the first one (dashed line, Fig. 7), \(\alpha = 0.82 GW/K\), \(\gamma = 3\), \(r = 0.0005\) and \(\delta = 2.43 MW/K\), therefore, the mode of operation is in ZIII. While in the second configuration: \(\alpha = 0.0012 GW/K\), \(\gamma = 3\), \(r = 0.0046\) and \(\delta = 1.13 MW/K\), the operation mode is located in ZII. Which also show the possibility of performing transitions between zones ZII and ZIII. Anew, the variation of the control parameters influence the heat engine performance.

Under this model, the generated loops are not equivalent to those of the NEHL model. However, \(MP\) and \(M\eta\) points have the same problem to be characterized. So that the reported operation mode of WT (Tab. 1) can represent \(MP\) regime, it is necessary that \(\alpha = 0.045 GW/K\), \(\gamma = 3\) and \(\delta = 2.726 KWM/W\) (Fig 7, solid line), while the \(M\eta\) regime needs \(\alpha = 0.045 GW/K\), \(\gamma = 3\) and \(\delta = 1.96 KMW/K\) (Fig. 7; solid line). In addition, these values belong to the intervals given by Ineqs. 52 and 53 Thus, we find conditions for \(\alpha\) and \(\delta\) that allow them to reach \(MP\) and \(M\eta\) operation modes. The result is a system of equations that arises from replacing Eq. 28 into Eqs. 4 and 5 in the case of \(MP\) regime. Thereby, the \(\alpha\) value must be:

\[
\alpha^{MP} = \frac{P [1 + \gamma (1 - r)]}{T_h [1 - r \gamma - 2 \sqrt{\tau + \tau (1 - r)}]} \tag{70}
\]

besides, it must be replaced in,

\[
\delta^{MP} = \frac{(1 - r \gamma) (1 - \eta) - \sqrt{\tau (2 - \eta) + \tau (1 - r)}}{\eta (1 - \eta) [1 + \gamma (1 - r)]} \tag{71}
\]

For \(M\eta\) regime, the system of equations to solve is:

\[
\alpha^{M\eta} = -\frac{\delta \eta [1 + (1 - r) (1 - \tau)] [1 - r \gamma (1 - \eta) - \eta + \tau (1 - r) + 2 \sqrt{\tau (1 - \eta)}]}{r^2 [\gamma (1 - \eta) + \tau^2] + (1 - \eta - \tau)^2 - 2 \tau [\gamma (1 - \eta) + \tau] (1 - \eta + \tau)} \tag{72}
\]
and,
\[
\delta \eta_{a} = \alpha \frac{P^2 n_{a1}^2 \left[ r (\gamma - \tau) + \tau - 1 \right] + T_h \alpha n_{a2} (r_{a1} + T_h \alpha n_{a3}) - P n_{a1} \left[ 1 + \tau + r (\gamma + \tau) \right] (r_{a1} + 2 T_h \alpha n_{a3})}{2 (1 - \tau) n_{a1} \left\{ P^2 n_{a1}^2 - 2 P T_h \alpha n_{a1} [1 + r (\gamma + \tau)] + T_h^2 \alpha^2 n_{a2} \right \}}
\]

where,
\[
n_{a1} = 1 + \gamma (1 - r),
\]
\[
n_{a2} = (1 - \tau) (r \gamma + \tau - r \tau - 1)^2 - 2 r (1 + \tau) (\gamma + \tau) + r^2 (\gamma + \tau)^2,
\]
\[
n_{a3} = r \gamma + \tau - r \tau - 1
\]

and
\[
r_{a1} = \sqrt{\left\{ P \left[ 1 + \gamma (1 - r) \right] - T_h \alpha \left[ 1 - r \gamma \right] \right \}^2 - 2 T_h \alpha \left\{ T_h \alpha + P \left[ 1 - r \right] + r \alpha T_h \left[ 1 + \gamma (1 - r) \right] \right \} - T_h^2 \alpha^2 \tau^2 (1 - r)^2}.
\]

It is a fact not all of the power plants have the same conditions to work in the "optimal" operation zone (ZII). Those conditions can be found through the Ecological function’s generalization parameter (Eq. 54), which establishes two functional relations ($\epsilon_{Pr,\eta r 1}$ and $\epsilon_{Pr,\eta r 2}$) between $a_{h}^{MEG}$ value and the control parameters ($\tau$, $\gamma$ and $r$), given by:
\[
\alpha_{h}^{MEG} = \frac{\gamma (1 + \epsilon r) + \sqrt{(1 + \epsilon r)(1 + \gamma (1 - r))}}{(1 + \epsilon r)(1 + \gamma (1 - r))}
\]

which can be obtained analogously to the NEHL model, by replacing the expressions for power output (eq. 22) and dissipation (eq. 24) in eq. 54. Likewise, to get the parametric relations $\epsilon = \epsilon (P, r)$, we replaced Eq. 28 in Eqs. 4 and 5 and we solved them and get,
\[
\epsilon_{r 1} = -(1 + \tau) r_{a1} \frac{r_{a1} - (1 - \tau) \left\{ P \left[ 1 + \gamma (1 - r) \right] + T_h \alpha \left[ r \gamma + \tau - (1 + \tau) \right] \right \}}{2 \tau \left\{ P \left[ 1 + \gamma (1 - r) \right] + T_h \alpha \left[ r \gamma + \tau \right] \right \} \left\{ P \left[ 1 + \gamma (1 - r) \right] + T_h \alpha \left[ r \gamma + \tau - 2 (1 + \tau) \right] \right \}}
\]
\[
\epsilon_{r 2} = -(1 + \tau) r_{a1} \frac{r_{a1} + (1 - \tau) \left\{ P \left[ 1 + \gamma (1 - r) \right] + T_h \alpha \left[ r \gamma + \tau - (1 + \tau) \right] \right \}}{2 \tau \left\{ P \left[ 1 + \gamma (1 - r) \right] + T_h \alpha \left[ r \gamma + \tau \right] \right \} \left\{ P \left[ 1 + \gamma (1 - r) \right] + T_h \alpha \left[ r \gamma + \tau - 2 (1 + \tau) \right] \right \}},
\]

While replacing Eq. 29 in Eq. 4 and Eq. 5. Now, we obtain $\epsilon = \epsilon (\eta, r)$,

Figure 7: Loops representing different configurations for the operation of West Thurrock (WT) plant. In a) the existence of two configurations is shown, one operation mode is located in ZII (solid line) and the other one belongs to ZIII (dashed line). In b) there is a configuration for which the operation mode reported by WT represents the maximum power output regime under the appropriate conditions (solid line). Finally, in c) the same operation mode is observed, representing the maximum efficiency regime under other conditions (solid line).
\[ \epsilon_{\eta 1} = -\frac{2 \delta^2 \eta^2 (1-\tau)^2 (1+\tau) + n_{e1} + n_{e2} - (1-\tau) \{ \delta \eta \delta_1 (1-\tau) - \alpha [1-\eta + \tau - r (\tau + \gamma [1-\eta])] \} r_{\eta 2}}{2 \tau [\alpha (2-r\gamma) (1-\eta) - \delta \eta \delta_1 (1-\tau) + \alpha \tau (2-r)] \{ r\gamma [\delta \eta - \alpha (1-\eta)] - \delta \eta (1+\gamma) (1-\tau) - r \tau (\alpha + \gamma \delta \eta) \}} \] (81)

\[ \epsilon_{\eta 2} = -\frac{2 \delta^2 \eta^2 (1-\tau)^2 (1+\tau) + n_{e1} + n_{e2} - (1-\tau) \{ \delta \eta \delta_1 (1-\tau) + \alpha [1-\eta + \tau - r (\tau + \gamma [1-\eta])] \} r_{\eta 2}}{2 \tau [\alpha (2-r\gamma) (1-\eta) - \delta \eta \delta_1 (1-\tau) + \alpha \tau (2-r)] \{ r\gamma [\delta \eta - \alpha (1-\eta)] - \delta \eta (1+\gamma) (1-\tau) - r \tau (\alpha + \gamma \delta \eta) \}} \] (82)

where,

\[ n_{e1} = -\alpha^2 \left\{2r [\gamma (1-\eta) + \tau] (1+\tau - \eta) + \gamma (1-\eta + \tau)] (1+\tau) + (1-\tau) \left[ \tau^2 - (1-\eta)^2 \right] \right\} \] (83)

\[ n_{e2} = -2\alpha \delta \eta [1 + \gamma (1-r)] \{ \tau (1-r) + (1-r\gamma) (1-\eta) \} (1-\tau^2) \] (84)

\[ r_{\eta 2} = \sqrt{\frac{2 \delta^2 \eta^2 (1-\tau)^2 + 2 \alpha n \delta_1 \delta \eta (1-\tau) \{ (1-\eta) (1-r\gamma) + \tau (1-r) \} + n_{e3}}{2 \tau [\alpha (2-r\gamma) (1-\eta) - \delta \eta \delta_1 (1-\tau) + \alpha \tau (2-r)] \{ r\gamma [\delta \eta - \alpha (1-\eta)] - \delta \eta (1+\gamma) (1-\tau) - r \tau (\alpha + \gamma \delta \eta) \}} \] (85)

and

\[ n_{e3} = \alpha^2 \left\{ (1-\eta - \tau)^2 - 2r [\gamma (1-\eta)] (\tau + 1-\eta) + r^2 [\gamma (1-\eta) + \tau]^2 \right\} \] (86)

The obtained expression for \( \epsilon \) contains involved elements in the construction of the energy converter (\( \alpha, \delta, \gamma, T_h, \) and \( \tau \)), as well as elements that describe a specific operation mode (\( P \) and \( \eta \)). Analogously to the NEHL model, the conditions to identify the operation zones ZI, ZII and ZIII are presented respectively as follows:

\[ 2 \left\{ 2 - n_i - \tau [2 + n_i - \tau] + r^2 [\gamma + \tau]^2 + r [\gamma + \tau] [n_i - 2 (1+\tau)] \right\} \rho \{ 3 + n_i - \tau + r (\gamma + \tau) \} \{ 1 + n_i - \tau + r (\gamma + \tau) \} \leq 0 < \epsilon < 0, \] (87)

\[ 0 \leq \epsilon \leq \epsilon_{\eta \eta} \] (88)

and

\[ \epsilon_{\eta \eta} < \epsilon < 2 \left\{ 1 + n_i + \tau [n_i + \tau - 2] + r^2 [\gamma + \tau]^2 - r [\gamma + \tau] [2 + n_i + 2r] \right\} \rho \{ 3 + n_i + \tau + r (\gamma + \tau) \} \{ 1 - n_i - \tau + r (\gamma + \tau) \} \] (89)

where \( \epsilon_{\eta \eta} \) is:

\[ \epsilon_{\eta \eta} = \frac{\alpha \{ 2r_{\eta \tau} - \delta [1-\tau] [1 + \gamma (1-r)] [1 + \tau - r (\gamma + \tau)] - \alpha \tau [1 - \tau + r^2 (\gamma + \tau) - r (1 + \gamma + 2r)] \} \} \{ r\eta + \delta [1-\tau] [1 + \gamma (1-r)] - \alpha \tau [2 - r] \} \{ r\eta + \delta [1-\tau] [1 + \gamma (1-r)] - \alpha \tau \} \] (90)

As in the previous subsection, these limits for the values of \( \epsilon \)-parameter were obtained after replacing \( a_h \) (Eq. 25) into Eqs. 16 and 29.

4 Conclusions

As is well known, the characteristic loops of performance of a real thermal engine are not produced in the analysis of the power output versus efficiency curves of an endoreversible engine model. To emulate one of these loops, it is necessary to incorporate a heat leak between the two temperature reservoirs. However, in the irreversible model these loops arise without the need to incorporate the short circuit due to the exchange of energy and its surroundings is always present. Therefore, a more complete model must incorporate such dissipative element. In this work, we characterize the restrictions of the different parameters involved in the irreversible models that permit the physical configuration achievable for the energy converter. Likewise, we have shown that a particular mode of operation corresponds to a set of configurations that can be found from different ways. The parameters \( \alpha \) and \( \delta \) generate a greater effect on the behavior of the converter, due to their relationship with heat exchangers that allow the heat flows with the surroundings. In addition, the constraints on \( R \) and \( r \) parameters were analyzed because of they estimate to a certain irreversibility degree of each energy converter here studied. Besides, the relationship between the parameters associated with design-construction and the internal irreversibility degrees was obtained.
Figure 8: Curves that exemplify a configuration (red) which contains the operation mode reported by each plant, every black loops give an account of a specific configuration obtained from the improvement conditions under IHL model, this condition allows the plants to operate at ZII. In a) it is shown that Almaraz II can go from operating in ZIII ($\epsilon_{\Pi 2} > \epsilon_{MPO}$) to a point $\epsilon_{\Pi 1}$ that represents its performance in ZII maintaining the same efficiency and improving its power output. In b), Larderello works in ZI ($\epsilon_{P 1} < \epsilon_{MPO}$) and the point $\epsilon_{P 2}$ symbolizes its operation in ZII, which has the same power output and improves its efficiency. In c) Toshiba’s operation is schematized in ZIII ($\epsilon_{\Pi 2} < \epsilon_{MPO}$) and the point $\epsilon_{P 2}$ shows its performance in ZII improving both the power as efficiency.

On the other hand, we show the importance of the $\epsilon$-parameter to improve conditions in the performance of some power plants by using both non-endoreversible ($R$-parameter) and irreversible ($r$-parameter) models. By using the $\epsilon$-parameter in both maximum power output and maximum efficiency regimes allow us to classify the operation mode along the characteristic loops. In addition, through $\epsilon$-parameter we can obtain the achievable conditions so that an energy converter always operates in the optimal performance region (ZII) (see Figs. 6 and 8).

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