\textbf{PT Symmetry and PT-Enhanced Quantum Sensing in a Spin-Boson System}

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Open systems, governed by non-Hermitian Hamiltonians, evolve fundamentally differently from their Hermitian counterparts and facilitate many unusual applications. Although non-Hermitian but parity-time (PT) symmetric dynamics has been realized in a variety of classical or semiclassical systems, its fully quantum-mechanical demonstration is still lacking. Here we ingeniously engineer a highly controllable anti-Hermitian spin-boson model in a circuit quantum-electrodynamical structure composed of a decaying artificial atom (pseudospin) interacting with a bosonic mode stored in a microwave resonator. Besides observing abrupt changes in the spin-boson entanglement evolution and bifurcation transition in quantum Rabi splitting, we demonstrate super-sensitive quantum sensing by mapping the observable of interest to a hitherto unobserved PT-manifested entanglement evolution. These results pave the way for exploring non-Hermitian entanglement dynamics and PT-enhanced quantum sensing empowered by nonclassical correlations.

Hermiticity, as a fundamental postulate in the formalism of canonical quantum mechanics, ensures the realness of any observable so as to coincide with measurement outcomes. This is especially exemplified by the energy operator or Hamiltonian in the Schrödinger equation, which delineates the evolution of a closed quantum system, isolated from its surrounding environment, to be unitary and probability-preserving. To account for the inevitable interaction between the system and environment, the quantum master equation by tracing over reservoir modes describes the resultant dynamics of such an open system as a mixture of numerous trajectories, a stochastic process interrupted by random quantum jumps in a non-Hermitian Hamiltonian. However, for a controllable open system without quantum jumps, its dynamical evolution can be governed by a non-Hermitian but parity-time (PT) symmetric Hamiltonian $H$, which commutes with the joint parity ($P$) and time-reversal ($T$) operators, $[PT, H] = 0$. Counterintuitively, a Hamiltonian $H$ of such could also display entirely real eigenspectra above some phase-transition threshold or exceptional point (EP) \cite{1, 2}. This spectral bifurcation stems from the existence of a parameter in $H$ which governs characteristic features of eigenvalues and eigenfunctions. Altering this parameter prompts real eigenspectra to coalesce and become complex conjugate pairs, heralding the spontaneous breakdown of PT symmetry and the occurrence of a nontrivial phase transition \cite{2, 3}. These striking attributes have enabled the application of quantum critical phenomena to high-precision metrology \cite{4–8} and the emergence of exotic topological effects absent in Hermitian systems \cite{9}.

Over the past decade, PT symmetry has been realized in various classical platforms \cite{9, 10} by the interplay between gain and loss. Yet, its observation in a quantum setting is highly challenging, due to the lack of proper quantum techniques. Thanks to the recent advances in quantum control technologies, we have witnessed a growing effort on performing PT symmetry with single qubits in different schemes ranging from optics \cite{11–14} and ultracold atoms \cite{15}, to nitrogen-vacancy centers in diamond \cite{16–18}, trapped ions \cite{19, 20}, and superconducting circuits \cite{21–23}. Despite the impressive achievements behind these experiments, their underlying physics in fact can be well interpreted by a semiclassical model, in which either the degree of freedom of light or matter is treated classically in their system Hamiltonians. This leaves a full quantum-mechanical PT symmetry yet to be demonstrated. We also notice that

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there are experimental endeavors to observe EPs in composite structures involving two interacting bosonic modes [24–26], a photonic mode coupled to a molecular reservoir [27], and flying atoms across a cavity [28], but the associated non-Hermitian entanglement dynamics remains yet to be investigated.

Here, we report an observation of quantum optical $PT$ symmetry in a distinct spin-boson system, whose evolution trajectory is ruled by a non-Hermitian Hamiltonian. Specifically, in a superconducting circuit we utilize a microwave field mode, stored in a bus resonator ($R_b$), as a quantum beam splitter (QBS) to act onto a dissipative artificial atom or pseudospin (test qubit, $Q_t$). The anti-Hermiticity is introduced by coupling $Q_t$ to a readout cavity ($R_r$) with a fast photonic decay. Thanks to the tunable competition between $R_b$ and $R_r$, the scheme enables us to observe peculiar quantum effects inaccessible to a classical BS. One example of such is the spin-boson entanglement evolution (readout jointly by $Q_t$ and an ancilla qubit, $Q_a$, via joint quantum state tomography), with the concurrence oscillating above the EP but monotonically growing to a steady value below the EP. This bifurcation behavior persists in quantum Rabi splitting (QRS), whose spectra undergo a real-to-complex phase transition across the EP. The superior coherence controllability further makes the system a remarkable candidate for developing a $PT$-enhanced quantum sensor with eminent sensitivity leveraged by entanglement.

The evolution of our $Q_r$-QBS-coupled system obeys the effective $PT$-symmetric Hamiltonian,

$$H_S = \hbar \Omega \left( a^\dagger |g\rangle \langle e| + a |e\rangle \langle g| \right) - \frac{i}{2} \hbar \kappa |e\rangle \langle e|,$$

where $|e\rangle$ ($|g\rangle$) denotes the upper (lower) energy level of $Q_t$ with a decay rate $\kappa$, $a^\dagger$ ($a$) represents the creation (annihilation) operator of the bosonic mode or QBS, and $\Omega$ stands for the spin-boson coupling strength. Distinct from previous semiclassical models [11–23], the above Hamiltonian $H_S$ bears a $U(1)$ symmetry – invariant under the number-preserving transformation $e^{i\varphi (a^\dagger + a)|e\rangle \langle e|} (\varphi \in \mathbb{R})$. This property helps to restrict the system dynamics within the reduced Hilbert subspace $\{ |e, n - 1\rangle, |g, n\rangle \}$ for a definite initial excitation number $n$. From Eq. (1), one gets the following paired eigenstates and eigenspectra,

$$|\Phi_{n+}\rangle = N_n^+ \left( \cosh \frac{\theta}{2} |e, n - 1\rangle + i \sinh \frac{\theta}{2} |g, n\rangle \right),$$

$$|\Phi_{n-}\rangle = N_n^- \left( -i \sinh \frac{\theta}{2} |e, n - 1\rangle + \cosh \frac{\theta}{2} |g, n\rangle \right),$$

$$\omega_{n\pm} = \pm \omega_n = \pm \sqrt{n\Omega^2 - \kappa^2/16},$$

with the normalization coefficients $N_{n\pm}$ and $\tan \theta = 4\sqrt{n\Omega}/\kappa$. Starting with the initial state $|g, n\rangle$, the $Q_r$-QBS state evolves in time as

$$|\psi(t)\rangle = N_n \left\{ [\omega_n \cos (\omega_n t) + \kappa \sin (\omega_n t)/4] |g, n\rangle - i\Omega \sqrt{n} \sin (\omega_n t) |e, n - 1\rangle \right\}.$$

Again, $N_n$ is a normalization constant. From Eqs. (2)-(4), the physics now becomes apparent: when $4\sqrt{n}\Omega > \kappa$, the system is in the unbreaking $PT$-phase region, such that quantum Rabi oscillations (QROs) between $|g, n\rangle$ and $|e, n - 1\rangle$ occur with real eigenspectra, indicating coherent periodic excitation exchange in $Q_t$ and QBS; conversely, for $4\sqrt{n}\Omega < \kappa$, the spontaneous breakdown of $PT$ symmetry halts the population oscillations to a steady value, because of the imaginary eigenspectra. The abrupt phase transition happens at the EP ($4\sqrt{n}\Omega = \kappa$) with the coalesce of $|\Phi_{n\pm}\rangle$ and $\omega_{n\pm}$ for the QRS disappearance.

One striking feature of our system is that $Q_t$ and QBS are entangled with each other by jointly sharing $|\Phi_{n\pm}\rangle$ and $\omega_{n\pm}$, and neither of them has its own state and eigenenergy. This is in stark contrast to previous $PT$-studies, where associated energy gap is produced by a classical BS with use of classical drive or optics. Given the presence of $PT$ symmetry, the $Q_r$-QBS entanglement dynamics is fundamentally transformed, as evidenced by the concurrence [29],

$$C(t) = |\sin [2\phi(t)]|,$$

with $\phi(t) = \arctan [4\sqrt{n}\Omega \tan (\omega_n t)/\left( 4\omega_n + \kappa \tan (\omega_n t) \right)]$. A close examination on Eq. (5) reveals that, above the EP, the bipartite entanglement develops in a way significantly different from QROs. Other than possessing twice the oscillating frequency, $C(t)$ becomes more sensitive to experimental control parameters. For instance, $\sin (2\phi(t))$ varies as $0 \rightarrow \sqrt{2}\Omega$, but $\sin (2\phi(t))$ rapidly grows as $0 \rightarrow 0.6$, over an order of magnitude larger than the population variation in $|g, n\rangle$-state. This parameter-sensitive nature hence offers a new mechanism for developing novel quantum sensors.

As schematic in Fig. 1, the experiment involves a 5.582-GHz bus resonator ($R_b$) with 13-µs photonic lifetime and two frequency-tunable superconducting qubits, one for $Q_t$ and the other for $Q_a$ (used to prepare/map out the $R_b$ initial/final state at each measurement). The $PT$ observation relies on manipulating the $\kappa/\Omega$ ratio, achieved by periodically modulating the QRS’ spectral gap [30, 31] via two parametric modulations $f_q = f_0 + \sum_{j=1, 2} \varepsilon_j \cos (2\pi n_j t)$.
with $f_0 = 5.920 \text{ GHz}$. Precisely speaking, Mod-1 (modulation frequency $\nu_1 = 368 \text{ MHz}$ and amplitude $\varepsilon_1 = 89 \text{ MHz}$) was adopted to shape the $Q_t$ decay rate by coupling to a $6.656-\text{GHz}$ readout bad resonator ($R_b$) at the second upper sideband. This coupling $\Omega$, in turn establishes an artificial dissipation channel to $Q_1$ and raises its decay rate from $0.08 \text{ MHz}$ to $1.54 \text{ MHz}$. Meantime, Mod-2 ($\nu_2 = 338 \text{ MHz}$) was chosen to couple $Q_t$ to $R_b$ at the first red sideband but with changeable $\varepsilon_2$ for adjusting $\Omega$.

Before measurement, both ($Q_t, Q_b$) and $R_b$ were initialized at their ground states. The QBS was sequentially prepared by applying a $\pi$-pulse ($R^+_b$) to excite $Q_a$ to its excited state, implementing an on-resonance swapping operation to transfer this excitation to $R_b$, and biasing $Q_a$ back to its idle frequency. Subsequently, Mod-1 and Mod-2 with the pulse sequence sketched in Fig. 1B were turned on to couple $Q_t$ to $R_b$ for splitting the $Q_t$ state. After a preset interaction time interval $t$, switching off both modulations effectively decoupled $Q_t$ from $R_b$ and $R_r$, due to the large detuning. Under these procedures, the $Q_t-Q_b$ trajectories defined by $H_S$ (1) fall well into the single-excitation Hilbert subspace $\{|e, 0\rangle, |g, 1\rangle\}$ and showcase their features via QROs. To implement $PT$-regulated QROs, one way is to first transfer the QBS state to $Q_a$ (with which it resonates) for a $\pi/2\Omega$-time window and then bias $Q_a$ back to its idle frequency ready for next state readout. To ensure, the targeted subspace mentioned above, the joint detection $|g,g\rangle$-events were discarded by post-selecting the population evolution of interest. As a representative, Fig. 2 presents the typical QRO data attained from the joint probability, $P_{|g,g\rangle} = N_n^2 |\omega_n \cos (\omega_n t) + \kappa \sin (\omega_n t)/4|^2$, of detecting $Q_t$ in $|g\rangle$ and $Q_a$ in $|e\rangle$ by altering $\Omega$ and $t$. Above the EP ($\Omega = 2\pi \times 0.058 \text{ MHz}$), obviously, both the QROs’ profiles and periods are vitally redefined by the anti-Hermiticity, as compared to their Hermitian counterparts. Nonetheless, these results agree well with the numerical simulations (see Supplementary Material). Besides, $Q_a$ here plays a radically differing role from the previous experiments [11, 16–18, 23], where it was introduced as an artificially engineered environment to $Q_t$ so that $PT$ arose only when projecting its state out of the $Q_t$’s subspace (ascribed by a semiclassical non-Hermitian Rabi Hamiltonian).

To reveal the $Q_t$-$Q_a$ entanglement transition, next we performed the joint state tomography on the dual-qubit evolution after state transfer. Experimentally, tomography measurements were done by correlating the Bloch vectors of the two qubits, individually measured along different axes. Among them, the $z$-component is directly available from the state readout, while the $x$- and $y$-components often require the $y$- and $x$-rotations, respectively, prior to the state readout. Unfortunately, such operations would violate the excitation-number conservation, rendering it impossible to distinguish jump events from no-jump ones. To overcome this obstacle, by post-selection we abandoned the elements pertaining to $|g,g\rangle$ in the two-qubit density matrix constructed from all measurement outcomes. In this way, we ended up with the desired reduced density matrix describing the two-qubit dynamics in the subspace $\{|e, g\rangle, |g, e\rangle\}$. Consequently, the concurrence $C$ computed from this matrix shall exhibit compelling characteristics beyond the Hermitian case. Indeed, as experimentally verified in Fig. 3A, $C$ derived from this procedure abides by a distinctive way of evolving over $\Omega$ and $t$. To visualize entanglement manifestations before and after the phase transition, we especially selected two such cases (dashed red lines) and replotted the corresponding $C(t)$ in Figs. 3B and C. Apparently, for $\Omega = \kappa$ the $PT$-phase is intact and $C(t)$ oscillates periodically. In contrast, for $\Omega = 0.3\kappa$, the system resides in the breaking-phase region and $C(t)$ ceases to oscillate. We are aware that the $C(t)$ evolution is more or less accompanied with unwanted fluctuations, which mainly stem from the off-resonant couplings that are neglected when deriving $H_S$ (1). This is confirmed by full numerical simulations (light blue lines) based on the original Hamiltonian.

With the measured $C$ above, we are now ready to extract the system’s eigenspectra so as to compare with the theoretically deduced $\omega_n \pm \Delta$ (3). In practice, we retrieve the real and imaginary parts of each eigenspectrum $\omega_n$ by numerically fitting every individually measured $C(t)$ with a sinusoidal form given in Eq. (5). By this way, the extracted real and imaginary parts of $\omega_n$ are accordingly illustrated in Figs. 4A and B. As a comparison, we present the theoretical eigenspectra computed by using $H_S$ and the original Hamiltonian. Note that above the EP, the paired eigenstates $|\Phi_{n \pm}\rangle$ possess different real eigenenergies generated by quantum Rabi splitting [32, 33], a pure quantum phenomenon that can lead to intriguing effects including photon blockade [34, 35] and quantum phase transition [36, 37]. By continuing to lower $\varepsilon_2$, the QRS-induced energy gap steadily narrows down and vanishes until reaching the EP. After crossing the EP, the two levels are virtually re-split with an imaginary spectral gap, which increases by decreasing $\varepsilon_2$ (or $\Omega$). This real-to-imaginary eigenenergy transition in the course of QRS has no classical analog and is a direct consequence of the spin-boson entanglement.

As aforementioned, a tiny change in $\Omega$ results in a dramatic variation in $C(t)$. This drastic change prompts us to search its potential in quantum sensing. Though $PT$-enhanced sensing has been demonstrated mostly with classical schemes [5, 6] and recently with single photons [11], these experiments focused on probing some geometric quantities. We here demonstrate a complementary $PT$-enhanced quantum sensor enabled by encoding the physical quantity into the yet-to-be-measured non-Hermitianally evolved entanglement. Working in the unbroken phase regime, we are interested in the scenario where for a preset coupling $\Omega_0$, the ideal system will return to its initial state $|g, 1\rangle$ without the entanglement formation, if assuming the interaction time $\tau$ to meet $\omega_1 \tau = \pi$. However, a slight perturbation $\Delta \Omega$ on $\Omega_0$ would shift $\omega_1$ by a marked amount, $\delta \omega_1 \approx \Omega_0 \delta \Omega/\omega_1$, and conversely brings the entanglement into the bipartite
evolution. Alternatively, the emergence of the two-qubit entangled state becomes a powerful probe for designing a \( P \bar{T} \)-type quantum sensor. To evaluate its performance, it is necessary to look at the susceptibility in terms of the response of \( C \) to a small variation \( \delta \Omega \) around \( \Omega_0 \). As such, we define the susceptibility as

\[
\chi_C = \frac{\partial C}{\partial \Omega} \approx \frac{2\pi \Omega_0^2}{\omega_1},
\]

for \( \delta \omega_1 \tau \ll 1 \) and \( |\delta \Omega/\Omega_0| \ll 1 \). \( \chi_C \) suggests that at the expense of a prolonged process (incurring more noise), the smaller \( \omega_1 \), the greater \( \chi_C \) and the sensitivity. In reality, a trade-off takes place between the maximally achievable \( \chi_C \) and experimental imperfections. To test the idea, in one series of measurements we implemented such a protocol by fixing \( \Omega_0 = 2\pi \times 0.217 \) MHz and \( \tau = 1.4 \) \( \mu \)s. The recorded \( C \) is shown in Fig. 5A, where the data were taken with \( \Delta \Omega = 2\pi \times 4 \) kHz. Due to the imperfection of the artificial dissipation channel originating from the limited ratio of the \( R_c \)'s photonic decay rate over the \( Q_1-R_c \) coupling, the experimental turning point \( \Omega' = 2\pi \times 0.200 \) MHz in gray line is drifted away from \( \Omega_0 \) in solid orange curve. After \( \Omega_0' \), \( C \) approximately follows a linear scaling within the domain \([\Omega_0', \Omega_N']\) where \( N = 6 \) and \( \Omega_N' = \Omega_0' + N \Delta \Omega = 2\pi \times 0.224 \) MHz. By operating in this region, one reads \( \chi_C = \left| \frac{C(\Omega'_N) - C(\Omega'_0)}{(\Omega'_N - \Omega'_0)} \right| / \chi_C \). The sensitivity at \( \Omega \) is then determined by the noise-to-signal ratio \( S_C(\Omega) = \delta C(\Omega)/\chi_C \), where \( \delta C(\Omega) \) is the standard deviation of \( C(\Omega) \). Therefore, the averaged sensitivity in \([\Omega_0', \Omega_N']\) is

\[
S_C = \frac{1}{N + 1} \sum_{j=1}^{N} S_C(\Omega'_j) = 2\pi \times (0.646 \pm 0.239) \text{ kHz}
\]

Before proceeding the discussions, it is instructive to check the sensing performance by exploiting the population variation in \( P_{ge} \). Under the same experimental conditions, Fig. 5B shows the measured \( P_{ge} \) to be most susceptible to a minute disturbance near \( \Omega = 2\pi \times 0.152 \) MHz. In the same way, we found a smaller susceptibility at this coupling strength, \( \chi_P = 9.37 \pm 0.24 \text{ MHz}^{-1} \), and a worse averaged sensitivity, \( \overline{S}_P = 2\pi \times (1.24 \pm 0.01) \text{ kHz} \). Nevertheless, integrating entanglement with \( P \bar{T} \) symmetry for quantum metrology remarkably differentiates our scheme from previous EP- [4–8] or entanglement-based [38, 39] proposals.

In conclusion, we successfully constructed a non-Hermitian spin-boson model with both \( P \bar{T} \) and \( U(1) \) symmetries in a superconducting circuit, featuring a decaying \( Q_1 \) swapping quanta with a bosonic QBS mode. We witnessed that the introduction of \( P \bar{T} \) symmetry radically reshapes quantum Rabi oscillations and entanglement evolution, compared to their Hermitian counterparts. We further utilized the observed sharp change to develop a quantum sensor empowered by \( P \bar{T} \) symmetry and entanglement. Other than the employed superconducting circuit, our model can also be realized in other platforms such as cavity QED and trapped ions [40]. Our work opens a door to uncover unorthodox physics inherent in quantum composite systems and innovate anti-Hermiticity-enabled quantum sensing technologies.

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FIG. 1: Quantum-optical $\mathcal{PT}$ symmetry in a spin-boson system. (A) Experimental schematic and parametric modulations. In a superconducting circuit, the system employs a decaying artificial atom or pseudospin (test qubit, $Q_t$) interacting with a bosonic mode stored in a microwave bus resonator ($R_b$). An ancilla qubit ($Q_a$) is applied to prepare and map out the initial and final state of $R_b$. Under two sinusoidal modulations, $Q_t$ is coupled to a readout resonator ($R_r$) at the second blue sideband by the first modulation (modulation frequency $\nu_1 = 368$ MHz and amplitude $\varepsilon_1 = 89$ MHz) with modulated coupling strength $\Omega_r$. The strong photonic decay of $R_r$ mediates $\Omega_r$ and effectively introduces $Q_t$ a dissipation rate $\kappa$. Meanwhile, $Q_t$ is also coupled to $R_b$ at the first red sideband by the second modulation ($\nu_2 = 338$ MHz and variable $\varepsilon_2$) with coupling strength $\Omega$ tuned by $\varepsilon_2$. (B) Pulse sequence. Initially, $Q_t$ and $Q_a$ are in their ground states at respective idle frequencies $5.992$ GHz and $5.900$ GHz. Then, $Q_a$ is driven to the excited state $|e\rangle$ by a $\pi$-pulse ($R_x^\pi$), followed by a swapping operation to map its state to $R_b$. Subsequently, two sinusoidal modulations are applied to modulate $Q_t$ around the central frequency $f_0 = 2\pi \times 5.920$ GHz. By altering $\varepsilon_2$, $\mathcal{PT}$ symmetry is revealed by performing quantum state tomography on the two-qubit entangled state.
FIG. 2: *PT*-symmetric quantum Rabi oscillations. By changing coupling strength $\Omega$ and interaction time interval $t$, $PT$ symmetry apparently transforms the evolution of the joint probability $P_{ge}$ of detecting $Q_t$ in $|g\rangle$-state and $Q_a$ in $|e\rangle$-state.

FIG. 3: *PT*-symmetric concurrence $C(t)$. (A) By varying $\Omega$ and $t$, the $PT$-symmetric concurrence $C$ is inferred from the renormalized post-projected joint density matrix of the $Q_t$-$Q_a$ entanglement evolution limited to one excitation in measurement. As a representative of the unbroken and broken $PT$ phase (dashed red lines), their respective $C(t)$-evolutions are correspondingly plotted in (B) and (C) with ($\Omega = \kappa, \Omega = 2\pi \times 0.24$ MHz) and ($4\Omega = 0.3\kappa, \Omega = 2\pi \times 0.02$ MHz). The orange solid (light blue) curves are the numerical simulations using the effective (original) Hamiltonian.

FIG. 4: Eigenspectra $2\omega_1$ of $PT$ symmetry obtained by measuring quantum Rabi splitting. The spectral gap created by quantum Rabi splitting originates from the eigenspectral difference of the two entangled eigenstates of the $Q_t$-$Q_a$ system by sharing a single microwave photon. (A) and (B) respectively present the real and imaginary part of $\omega_1$ extracted from measured concurrence $C$. Similarly, the orange (blue) lines are results simulated with the effective (original) Hamiltonian.
FIG. 5: (A) Quantum sensing enhanced jointly by $\mathcal{PT}$ symmetry and entanglement. As an exemplar, the measured concurrence $C(\Omega)$ with $\Omega_0 = 2\pi \times 0.217$ MHz and $t = 1.4$ $\mu$s is most susceptible to $\Omega$ after the turning point $\Omega_0' = 2\pi \times 0.200$ MHz. Within the pink shadow region ($\Omega \in 2\pi \times [0.200, 0.224]$ MHz), the approximately linear trend of $C$ leads to the averaged sensitivity $S_C = 2\pi \times (0.646 \pm 0.239)$ kHz. With the same parameters, in contrast, (B) shows quantum sensing enhanced only by $\mathcal{PT}$ symmetry via measuring the jointing probability $P_{g,e}(\Omega/\Omega_0)$ of detecting the $Q_1$ in $|g\rangle$-state and $Q_a$ in $|e\rangle$-state, giving rise to a worse averaged sensitivity $S_P = 2\pi \times (1.24 \pm 0.01)$ kHz in the narrower pink shady area ($\Omega \in 2\pi \times [0.152, 0.156]$ MHz).
Supplementary Materials for
PT Symmetry and PT-Enhanced Quantum Sensing in a Spin-Boson System

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1. ARTIFICIAL DISSIPATION CHANNEL ENGINEERING FOR TEST QUBIT ($Q_t$)

The effective non-Hermitian but $\mathcal{PT}$-symmetric Hamiltonian $H_{S}$, given by Eq. (1) in the main text, describes an interesting spin-boson system with a tunable coupling strength $\Omega$ realized by parametric modulations. A simple calculation shows that the exceptional point (EP) locates at $\Omega = \kappa/4$. If $\kappa$ is not large enough, the $\mathcal{PT}$-broken region will become relatively narrow, thus adversely making the experimental measurement highly challenging. To overcome this hurdle, it is instructive in experiment to employ the readout resonator ($R_{r}$) as a bifunctional unit, besides the fast non-demolition measurements, also used as a decay channel, on the test qubit ($Q_t$). Under this design, the difficulty of our experiment gets substantially reduced since the coupling ($\Omega_{r}$) to $R_{r}$ introduces $Q_i$ an effective decay rate and automatically expands the $\mathcal{PT}$-broken region. To this end, we first apply a single-tone parametric modulation (Mod-1) to enhance the $Q_t$ decay rate $\kappa$ by establishing an interaction between $Q_t$ and $R_{r}$ through the following Hamiltonian,

$$H_{qr} = \sum_{n=-\infty}^{\infty} J_{n}(\mu_1)\hbar g_{r}|e^{i 2\pi (f_{r} - f_{0} - n\nu_1)}t_{r}^{\dagger}|g\rangle\langle e| + H.c.$$.  \hspace{1cm} (S1)

Here, $H.c.$ means Hermitian conjugate, $J_{n}(x)$ is the $n$-th Bessel function of the first kind, $a_r$ ($a_r^{\dagger}$) denotes the annihilation (creation) operator for $R_{r}$, $g_{r}$ stands for the coupling strength between $Q_t$ and $R_{r}$, $f_{0}$ ascribes the central frequency of the $Q_t$ under Mod-1, $f_{r}$ represents the $R_{r}$-frequency, $\nu_1$ tells the modulation frequency of Mod-1, and $\mu_1 = \epsilon_{1}/\nu_1$ with $\epsilon_{1}$ being the modulation amplitude. From Eq. (S1), one can see that when $f_r - f_0 - n\nu_1 = 0$, an ideal Rabi oscillation occurs between the adjacent $Q_t$-$R_{r}$ states $|e,0_{r}\rangle$ and $|g,1_{r}\rangle$ in response to the $n$-th order sideband modulation. After eliminating the Stark shift terms, the effective Hamiltonian (S1) is reduced to

$$H_{qr} = \hbar \Omega_{r}(a_{r}^{\dagger}|g\rangle\langle e| + a_{r}|e\rangle\langle g|)$$, \hspace{1cm} (S2)

where $\Omega_{r} = J_{n}(\mu_1)g_{r}$ is the aforementioned coupling strength between $Q_t$ and $R_{r}$. By taking into account the dissipation, the overall $Q_t$-$R_{r}$ dynamics governed by the Lindblad master equation follows

$$\hbar \frac{\partial \rho}{\partial t} = -i[H_{qr},\rho] + \hbar \gamma_q \sigma_- \rho \sigma^+ + \hbar \gamma_r a_r \rho a_r^{\dagger}$$, \hspace{1cm} (S3)

where $\sigma^- = |g\rangle\langle e|$ and $\sigma^+ = |e\rangle\langle g|$ are the pseudospin transition operators, $\gamma_q$ and $\gamma_r$ being the intrinsic decay rates of $Q_t$ and $R_{r}$, respectively, and $H_{qr}$ accordingly takes the form

$$H_{qr}' = H_{qr} - \frac{i}{2} \hbar (\gamma_q |e\rangle\langle e| + \gamma_r a_r^{\dagger}a_r)$$. \hspace{1cm} (S4)

FIG. S1: Experimental results under single-tone parametric modulation (Mod-1). (A) Scanning map of the population $P_{e}$ after 1-$\mu$s parametric modulations. (B) Temporal evolution of the population $P_{e}$ under different single-tone modulations with parameters marked in (A). Here, “zpa” is an abbreviation for the z pulse amplitude.
Limited by the available power and the sampling rate of the Z control board, experimentally there exists an upper bound on the available modulation frequency $\nu_1$. In the current setup, the $Q_t$ frequency set at the sweet point of $f_q = 6.009$ GHz is largely detuned from the $R_r$ frequency of $f_r = 6.656$ GHz. Given these circumstances, we find that the second-order modulation is most preferable in our work. Figure S1 shows the experimental results under Mod-1. We start with preparing $Q_t$ in the $|e\rangle$ state, followed by Mod-1 for a 1-$\mu$s duration and terminated by measuring the $Q_t$ state population right after the modulation. By this way, the measured population $P_e$ of $Q_t$ in $|e\rangle$ is provided in Fig. S1A, where the bright areas labeled as 1, 2 and 3 correspond to the $Q_t$-$R_r$ interaction, the $Q_t$-$R_b$ interaction, and the $Q_t$-$Q_a$ interaction in turn. As an example, in Fig. S1B the red curve displays the retrieved $P_e$ evolving over time with use of the corresponding parameters marked by the red star in Fig. S1A. At this position, no photon bounces back from $R_r$ and the induced $Q_t$ overall decay rate is $\kappa = 1.54$ MHz. On the other hand, varying $\varepsilon_1$ or $\nu_1$ in Mod-1 will result in the shift of $\kappa$. This has been confirmed in Fig. S1B, where the remaining three curves (blue, gray and brown) present different $P_e$ trajectories at the other three locations highlighted in Fig. S1A. In addition, to illustrate how $\kappa$ would change along with the variation of $\nu_1$, we carry out a series of measurements on $P_e$ by altering the modulation time interval as well as the frequency offset $\delta f$ from $\nu'_1$ by fixing the Mod-1 amplitude at $zpa_1 = 1.3$. The recorded experimental data is given in Fig. S2, where the green hollow squares are the extracted decay rates $\kappa$ and the green solid line is the fitting curve (symmetric about $\delta f = 0$ MHz).

**FIG. S2:** Time-evolved population $P_e$ and its corresponding decay rate $\kappa$ by varying frequency offset $\delta f$ from $\nu'_1$.

2. CONTROLLABLE COUPLING BETWEEN THE TEST QUBIT ($Q_t$) AND BUS RESONATOR ($R_b$)

In the experiment, we have also applied a second-tone parametric modulation (Mod-2) to have a controllable coupling between $Q_t$ and $R_b$. Working in the rotating frame, one can perform a similar analysis on delineating this coupling by the following Hamiltonian,

$$H_{qb} = \sum_{n=-\infty}^{\infty} \hbar J_n(\mu_2) g_b [e^{i2\pi(f_b-f_0-n\nu_2)t}a|g\rangle \langle e| + H.c.],$$

(S5)

where $a$ ($a^\dagger$) is the annihilation (creation) operator of the bosonic mode in $R_b$, $g_b$ is the coupling strength between $Q_t$ and $R_b$, $f_b$ is the $R_b$ frequency, and $\mu_2 = \varepsilon_2/\nu_2$ with $\varepsilon_2$ and $\nu_2$ being the modulation amplitude and frequency of Mod-2. Since our system involves two-tone modulations (Mod-1 and Mod-2), it is preferable to set up the $Q_t$-$R_b$ coupling ($\Omega$) by utilizing the first-order sideband modulation (satisfying $f_b - f_0 + \nu_2 = 0$ and $n = -1$) rather than the high-order harmonics. In this way, the coupling is reduced to a compact form, $\Omega = J_1(\mu_2) g_b$. In Fig. S3, we showcase some representative $P_e$’s measured by altering the modulation amplitude $zpa_2$. 
FIG. S3: Time-evolved population $P_e$ by varying the $Q_t$-$R_b$ coupling strength $\Omega$ through tuning the modulation amplitude $zpa_2$.

3. QUANTUM RABI OSCILLATIONS AND ENTANGLEMENT EVOLUTION MODULATED BY $\mathcal{PT}$ SYMMETRY

Our spin-boson system employs two resonators $R_b$ and $R_r$, and each of them is connected to a single Xmon that only takes into account the lowest two energy levels. The application of the two-tone modulation scheme effectively couples $Q_t$ to these two resonators $R_r$ and $R_b$ with the corresponding coupling strength $\Omega_r$ and $\Omega_b$, as discussed in the preceding sessions. Summarizing these together leads us to the total Hamiltonian characterizing the interaction of the whole system as follows,

$$
H = 2\pi \hbar \left[ f_e(t)|e\rangle\langle e| + f_b a^\dagger a + f_r a_r^\dagger a_r \right] + \hbar \gamma_q (a^\dagger |g\rangle\langle e| + a|e\rangle\langle g|) + \hbar \gamma_r (a^\dagger_r |g\rangle\langle e| + a_r|e\rangle\langle g|).
$$  

(S6)

In Eq. (S6), $f_e(t)$ is the modulated $Q_t$ transition frequency, whose format is determined by the two-tone parametric modulations (see below for details). Similarly, one can evaluate the general dynamics of the system by the Lindblad master equation,

$$
\hbar \frac{\partial \rho}{\partial t} = -i[H,\rho] + \hbar \gamma_q \sigma^- \rho \sigma^+ + \hbar \gamma_r a_r^\dagger \rho a_r.
$$  

(S7)
A. The parametric modulation protocol

In a superconducting circuit, the parametric modulation can be done by modulating the flux. In our work, the two-tone modulation protocol is implemented by periodically tuning an external flux of the form

$$\Phi_{ext}(t) = \Phi_0 + \Phi_1 \cos(2\pi \nu_1 t) + \Phi_2 \cos(2\pi \nu_2 t),$$  \hspace{1cm} (S8)

to modulate the $Q_t$ transition frequency. Here, $\Phi_0$ is the parking flux, $\Phi_1$ and $\nu_1$ are the modulation amplitude and frequency of tone-$j$ ($j = 1, 2$). On the other hand, an Xmon qubit, enabled by a superconducting quantum interference device (SQUID), modifies the Josephson energy $E_J$ with the external flux $\Phi_{ext}$ as

$$E_J(t) = E_J \sum \cos \left[ \frac{\Phi_{ext}(t)}{\Phi_0} \right],$$  \hspace{1cm} (S9)

where $\Phi_0 = h/(2e)$ is the flux quantum. Such a modified Josephson energy $E_J(t)$ accordingly transforms the $Q_t$ transition frequency into the following form,

$$2\pi \hbar f_c(t) \approx \sqrt{8E_c E_J(t)} - E_c,$$  \hspace{1cm} (S10)

with $E_c$ representing the charge energy. In the light of the sinusoidal function in $E_J(t)$ (S9), Eq. (S10) can be simplified by Fourier series expansion, that is,

$$f_c(t) = f_0 + \sum_{k=1}^{\infty} \varepsilon_k \cos \left( f_k t \right).$$  \hspace{1cm} (S11)

Due to the nonlinear flux dependence of $f_c(\Phi_{ext})$, the average value of the $Q_t$ transition frequency $f_0$ is shifted from $f_c(\Phi)$ by a certain amount, which can be measured by a Ramsey interferometer. In this experiment, the operation point during the modulation is chosen at the sweet point of $Q_t$, where $\Phi_{ext} = 0$ and $\Phi = 0$. For this special case, Eq. (S11) readily takes the following expression,

$$f_c(t) \approx f_0 + \varepsilon_1 \cos \left( 4\pi \nu_1 t \right) + \varepsilon_2 \cos \left[ 2\pi \left( \nu_1 + \nu_2 \right) t \right] + \varepsilon_3 \cos \left( 4\pi \nu_2 t \right) + \varepsilon_4 \cos \left[ 2\pi \left( \nu_1 - \nu_2 \right) t \right],$$  \hspace{1cm} (S12)

by keeping these four dominant Fourier components while neglecting all the rest higher-order harmonic terms. If $\Phi_2 \ll \Phi_1$, the Fourier coefficients will satisfy $\varepsilon_1, \varepsilon_2 \gg \varepsilon_3, \varepsilon_4$. By applying this inequality to Eq. (S12) and ignoring the last two smaller terms, we finally arrive at

$$f_c(t) \approx f_0 + \varepsilon_1 \cos \left( 4\pi \nu_1 t \right) + \varepsilon_2 \cos \left[ 2\pi \left( \nu_1 + \nu_2 \right) t \right].$$  \hspace{1cm} (S13)

By setting $\nu_1 = 2\nu_1'$ and $\nu_2 = \nu_1' + \nu_2'$, we can get the modulation amplitude $\varepsilon_j$ and frequency $\nu_j$ of Mod-$j$ ($j = 1, 2$) as discussed in the previous sessions. In the experiment, we manipulate $\varepsilon_k$ by adjusting the z-pulse amplitude ($zpa$). Now, substituting Eq. (S13) into Eq. (S6) and working in the interaction picture would transform the Hamiltonian $H$ (S6) into

$$H_1 = \hbar g_b \left[ e^{i2\pi(f_c-f_0)t} \left| \mu_1 \sin(2\pi \nu_1 t) + \mu_2 \sin(2\pi \nu_2 t) \right| a^\dagger |g\rangle \langle e| + H.c. \right] + \hbar g_r \left[ e^{i2\pi(f_c-f_0)t} \left| \mu_1 \sin(2\pi \nu_1 t) + \mu_2 \sin(2\pi \nu_2 t) \right| a^\dagger |g\rangle \langle e| + H.c. \right],$$

where $\mu_1 = \varepsilon_1/\nu_1$ and $\mu_2 = \varepsilon_2/\nu_2$. By assuming $\Delta_b(r) = f_b(r) - f_0$, the above interaction Hamiltonian can be recast by a compact form,

$$H_1 = \hbar g_b e^{i2\pi \Delta_b t} \left| \mu_1 \sin(2\pi \nu_1 t) + \mu_2 \sin(2\pi \nu_2 t) \right| a^\dagger |g\rangle \langle e| + \hbar g_r e^{i2\pi \Delta_r t} \left| \mu_1 \sin(2\pi \nu_1 t) + \mu_2 \sin(2\pi \nu_2 t) \right| a^\dagger |g\rangle \langle e| + H.c..$$  \hspace{1cm} (S14)

Using the Jacobi-Anger expansion

$$e^{i\mu \sin \theta} = \sum_{-\infty}^{\infty} J_n(\mu) e^{in\theta},$$  \hspace{1cm} (S15)

with $J_n(x)$ being the nth Bessel function of the first kind, the interaction Hamiltonian (S14) then becomes

$$H_1 = \hbar g_b \left[ \sum_{m,n=-\infty}^{\infty} J_n(\mu_1) J_m(\mu_2) e^{-i2\pi(n\nu_1 + m\nu_2 - \Delta) t} a^\dagger |g\rangle \langle e| + H.c. \right] + \hbar g_r \left[ \sum_{m,n=-\infty}^{\infty} J_n(\mu_1) J_m(\mu_2) e^{-i2\pi(n\nu_1 + m\nu_2 - \Delta) t} a^\dagger |g\rangle \langle e| + H.c. \right].$$  \hspace{1cm} (S16)
From Eq. (S16), we notice that when the modulation frequency satisfies $\nu_1 = \frac{1}{2} \Delta_r \ (\nu_2 = -\Delta_b)$ which corresponds to the second (first)-order sideband modulation for the coupling between $Q_t$ and $R_r$ ($R_b$), a swap $|e, 0_{\tau(b)}\rangle \leftrightarrow |g, 1_{\tau(b)}\rangle$ is available. In this way, we arrange our two-tone modulation protocol and confine the system’s energy-level structure to the configuration shown in Fig. S4 or Fig. 1 in the main text.

### B. $\mathcal{PT}$-symmetric entanglement dynamics

The entanglement associated with the non-Hermitian dynamics of a quantum system can be characterized by the Wootters’ concurrence [1]. In this work, we define a general pure state of our spin-boson system in the basis $\{|g, n-1\}, |g, n\}, |e, n-1\}, |e, n\}$ as

$$|\psi\rangle = A_0|g, n-1\rangle + A_1|g, n\rangle + A_2|e, n-1\rangle + A_3|e, n\rangle,$$

(S17)

where each joint state vector is defined as $|x, y\rangle \equiv |x\rangle \otimes |y\rangle$ with $|x\rangle \in \{|e\}, |g\}\}$ and $|y\rangle$ belonging to the photon-number-limited subspace $\{|n-1\}, |n\}\}$. $A_k$ ($k = 0, 1, 2, 3$) is the complex amplitude and satisfies the normalization condition $\sum_k |A_k|^2 = 1$. The density matrix of such a pure state (S17) can be computed by $\rho = |\psi\rangle \langle \psi|$. Assuming a spin-flipped operator $\sigma_y$ and quasi-spin-flipped operator $\bar{\sigma}_y$ acting respectively on the Hilbert subspaces $\{|e\}, |g\}\}$ and $\{|n-1\}, |n\}\}$, the defined spin-flipped state $\bar{\rho}$ [1] can be calculated as $\bar{\rho} = (\sigma_y \otimes \bar{\sigma}_y)\rho^* (\sigma_y \otimes \bar{\sigma}_y)$, where the asterisk denotes the operation of complex conjugation. With these preparations, the concurrence is readily identified from [1]

$$\mathcal{C}(\rho) = \max \left\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right\},$$

(S18)

where $\lambda_j$ ($j = 1, 2, 3, 4$) are the four eigenvalues, in decreasing order, of the non-Hermitian matrix $\rho \cdot \bar{\rho}$. For the given pure state (S17), after some algebra, we find $\lambda_1 = 4||A_0|^2|A_3|^2 - 2\text{Re}(A_0A_1^*A_3^*A_3) + |A_1|^2|A_2|^2|A_3|^2|$ while $\lambda_2 = \lambda_3 = \lambda_4 = 0$. Obviously, the corresponding concurrence (S18) will be determined by $\mathcal{C}(\rho) = 2||A_0|^2|A_3|^2 - 2\text{Re}(A_0A_1^*A_2^*A_3) + |A_1|^2|A_2|^2|A_3|^2|^{1/2}$. By comparing Eq. (S17) with Eq. (4) (whose normalization coefficient is $\mathcal{N}_n = 2e^{-nt/4} (i \sin \theta_n/\theta_n)$) in the main text, we immediately have

$$A_1(t) = \mathcal{N}_n \left[\omega_n \cos (\omega_n t) + \kappa \sin (\omega_n t)/4\right],$$

(S19)

$$A_2(t) = -i\mathcal{N}_n \Omega \sqrt{n} \sin (\omega_n t),$$

(S20)

and $A_0 = A_3 = 0$. This gives rise to the concurrence (which is used to characterize the temporal evolution of the spin-boson entanglement),

$$\mathcal{C}(t) = |\sin[2\phi(t)]|,$$

(S21)

where $\phi(t)$ can be attained from

$$\tan \phi(t) = \frac{4 \sqrt{n} \Omega \sin (\omega_n t)}{4\omega_n \cos(\omega_n t) + \kappa \sin (\omega_n t)}.$$  

(S22)

One can see that Eq. (S21) is the same Eq. (4) given in the main text.

In the superconducting circuit system, the diagonal elements of the density matrix can be directly obtained by measuring the population. In order to calculate the concurrence $\mathcal{C}$, however, one needs to have the complete information of the density matrix. In this regard, we perform quantum state tomography (QST) in our experiment to reconstruct the whole density matrix. For a single qubit, the QST measurement requires three separate operations ($I$, $X/2$, and $Y/2$) to individually get the $Z$-, $X$-, and $Y$-coordinate information of its state. Note that the quantum state needs to be re-prepared after each experimental run. Therefore, the joint state tomography on two qubits requires nine operations, thence demanding more copies of the targeted quantum state than when simply measuring the population. These extra consumed copies of the quantum state in turn provide additional information on the system dynamics, which is one of the sources of the sensitivity enhancement when using the entanglement concurrence rather than the population as an indicator.
Parameters & $Q_t$ & $Q_a$
\hline
Qubit idle frequency, $f_{d,j}$ & 5.992 GHz & 5.900 GHz \\
Coupling strength to the bus resonator $R_b, g_{b,j}/2\pi$ & 20.9 MHz & 20.3 MHz \\
Coupling strength to the readout resonator $R_r, g_{r,j}/2\pi$ & 41 MHz & 40 MHz \\
Energy relaxation time, $T_{1,j}$ & 12.6 $\mu$s & 12.4 $\mu$s \\
Ramsey dephasing time, $T_{2,J}^R$ & 2.8 $\mu$s & 2.6 $\mu$s \\
Dephasing time with spin echo, $T_{2,J}^{SE}$ & 12.9 $\mu$s & 8.6 $\mu$s \\
Frequency of readout resonator, $f_{r,j}$ & 6.656 GHz & 6.764 GHz \\
Leakage rate of readout resonator, $\gamma_{r,j}$ & 1/200 ns$^{-1}$ & 1/226 ns$^{-1}$ \\
$|g\rangle$ state readout fidelity, $F_{g,j}$ & 0.985 & 0.991 \\
$|e\rangle$ state readout fidelity, $F_{e,j}$ & 0.919 & 0.887 \\
\hline

TABLE S1: **Qubits characteristics.** In our experiment, both test qubit ($Q_t$) and ancilla qubit ($Q_a$) are initialized to the ground state $|g\rangle$ at their respective idle frequencies $f_{d,j}$ (j = t, a), at which we measured listed qubits coherence parameters (including their lifetime $T_{1,j}$, Ramsey Gaussian dephasing time $T_{2,j}$, and spin echo Gaussian dephasing time $T_{2,J}^{SE}$). $f_{d,j}$ is also the point at which the single-qubit rotations and corresponding state tomographies are performed. $g_{b,t}$ and $g_{b,a}$ are the coupling strengths for $R_b$ to $Q_t$ and $Q_a$, achieved by measuring quantum Rabi oscillations due to the interaction of the $Q_t/a$ transition with a $R_b$ photon. The fidelity for correctly recording each qubit’s state is $F_{k,j}$, characterized by extracting the state information of each readout resonator (whose frequency and leakage rate are $f_{r,j}$ and $\gamma_{r,j}$, respectively.)

4. EXPERIMENTAL SETUP AND DEVICE PARAMETERS

Our device consists of five frequency-tunable superconducting Xmon qubits, whose anharmonicities are about $\alpha_j \simeq 2\pi \times 240$ MHz. Each qubit has a microwave line (XY line) for driving its state transition as well as an individual flux line (Z line) for dynamically tuning its frequency. These two constituents make it flexibly on-and-off couple (with the coupling strength $g_{b,j}$) to $R_b$ with the bare frequency $f_b \simeq 5.582$ GHz and the energy relaxation time $T_r \simeq 13$ $\mu$s. Besides, each qubit is also dispersively coupled to its own readout resonator, whose frequency and leakage rate are $f_{r,j}$ and $\gamma_{r,j}$, respectively. All the readout resonators are coupled to a common transmission line for multiplexed readout of all qubits’ states. The readout measurement features single-shot and quantum nondestructive, and is achieved with the assistance of an impedance-transformed Josephson parametric amplifier (JPA) with a bandwidth of about 150 MHz. In the experiment, we choose two of them, one for the test qubit ($Q_t$) and the other for the ancilla qubit ($Q_a$), whose properties are characterized and listed in TABLE S1. In TABLE S1, their energy relaxation time $T_{1,j}$ (j = t, a), Ramsey Gaussian dephasing time $T_{2,j}$, and spin echo Gaussian dephasing time $T_{2,J}^{SE}$ are measured at their idle frequency $f_{d,j}$. Additionally, the probability of correctly reading out $Q_j$ in state $|k\rangle$ yields the readout fidelity $F_{k,j}$ for j = t, a. The detailed experimental setup including the whole electronics and wiring for the device control is summarized in Fig. S5 [5, 6], where the readout resonator for $Q_t$ is painted blue as an emphasis.

5. NUMERICAL SIMULATIONS

A. The off-resonant couplings in the modulations

To compare with the system dynamics described by the effective Hamiltonian $H_e$ (1) in the main text, we have done similar numerical simulations using the full Hamiltonian $H_I$ given by Eq. (S14). Some of the representative results are outlined in Fig. S6A. In these calculations, we use the Lindblad master equation to compute the temporal evolution of our system,

$$\hbar \frac{\partial \rho}{\partial t} = -i[H, \rho] + \sum_{k=q,b,r} \left( L_k \rho L_k^\dagger - \frac{1}{2} \{ L_k^\dagger L_k, \rho \} \right),$$

(S23)
FIG. S5: Schematic layouts of our circuit QED system and experimental setup. The superconducting circuit has five frequency-tunable Xmon qubits, two of which are used for the test qubit ($Q_t$) and the ancilla qubit ($Q_a$). Each qubit can be individually frequency-biased and frequency-modulated (through $Z$ line) and flipped (through $XY$ line). Thanks to such a flexible adjustability, every qubit can be controlled to couple to the bus resonator ($R_b$). The $XY$ control of each qubit is implemented through the mixing of the low-frequency signals (yielded by 2 Digital-to-analog converter (DAC)'s I/Q channels) and a Microwave source (MS) with 5.5-GHz carrier frequency; while the $Z$ control is fulfilled by two signals: one is produced by the Direct-current (DC) biasing line from a low frequency DC source, and the other is directly from the $Z$ control of a DAC. Meantime, every qubit has its own readout resonator which helps to produce the state information. Experimentally, this is accomplished by the mixing of the signals of 2 Analog-to-digital converter (ADC)'s I/Q channels and a MS with about 6.6-GHz frequency to output a readout pulse with 2 tones targeting 2 qubits’ readout resonators. In this work, one readout resonator ($R_r$) is used to construct a fast decay channel for $Q_t$. Both the employed DAC and ADC are field-programmable-gate-array (FPGA)-controlled to respond at the nanosecond scale. The output from the circuit, before being captured and demodulated by the ADC, is sequentially amplified by an impedance-transformed Josephson parametric amplifier (JPA, which is pumped by a 13.5-GHz MS and modulated by a DC bias), a high electron mobility transistor (HEMT), and room temperature amplifiers. Furthermore, few custom-made circulators, attenuators and filters are utilized at some specific locations of the signal lines to reduce the noise that may affect the operations of the device.
with the Lindblad dissipators $L_q = \sqrt{\hbar \gamma_q} \sigma^-$, $L_b = \sqrt{\hbar \gamma_b} a$, and $L_r = \sqrt{\hbar \gamma_r} a_r$. In our experiment, the effective decay rate $\kappa$ varies with the coupling strength $\Omega$. From Eq. (S16), we notice that when $\mu_2 \to 0$, the complicated coupling strength can be greatly simplified to $\Omega = J_2(\mu_1) g_r$, as the additional zeroth-order Bessel coefficient $J_0(\mu_2)$ turns out to be negligible. On the other hand, the limited available power and the shape of the Xmon energy bands become the primary reason for the deviation of the effective decay rate, and also lead to the actual output waveform drifting away from the desired one. Taking these into consideration, in our numerical simulations we decide to change the modulation amplitude $\varepsilon_1$ for each selected $\varepsilon_2$ while meantime to maintain a constant $f_0$. The values of $\varepsilon_1$ are

calculated using the two methods for a fixed $\Omega = 2\pi \times 0.24$ MHz.

chosen to optimize the effective decay rates shown in Fig. S11A. Fig. S6 displays the simulation results. Because of infinite terms in the Jacob-Anger expansion (S15), if the detuning in the off-resonant term is not large enough, the system will undergo notable high-frequency oscillations as revealed in the numerically simulated concurrence $C$ and population $P_{g,1}$ using $H_I$ and the target Hamiltonian for $\Omega = 2\pi \times 0.24$ MHz.

To have a further understanding of these off-resonant terms, we took a step further to analyze their roles in the high-frequency oscillations. According to our numerical simulations, we conclude that among these terms, the high-frequency oscillations disappear after removing some off-resonant terms in Fig. S6B. To make the point clear, in Fig. S6C we particularly pick up an example by comparing the numerically simulated $C$ and population $P_{g,1}$ using $H_I$ and the target Hamiltonian for $\Omega = 2\pi \times 0.24$ MHz: the orange solid lines represent the simulations with only the targeted term retained, while the blue dashed curves represent the results obtained by the full Hamiltonian. We can see that, indeed, the high-frequency oscillations disappear after removing the off-resonant terms in the computing.

To have a further understanding of these off-resonant terms, we took a step further to analyze their roles in the high-frequency oscillations. According to our numerical simulations, we conclude that among these terms, the two terms involving $\{n = 0, m = 0\}$ and $\{n = -1, m = 0\}$ are the main causes of the high-frequency oscillations. As representative examples, Fig. S7 illustrates the effects on $C$ and $P_{g,1}$ after removing some off-resonant terms in Eq. (S16) when $\Omega = 2\pi \times 0.24$ MHz. Specifically, Fig. S7A shows $C$ and $P_{g,1}$ after excluding all higher-order terms with $|n|, |m| > 2$, where the high-frequency oscillations are still visible. In contrast, Fig. S7B (or C) eliminates only the $\{n = 0, m = 0\}$ (or $\{n = -1, m = 0\}$) term. As one can see, the high-frequency oscillations are significantly suppressed compared to those in Fig. S7A. What happens if one removes both terms in the simulations? Figure S7D gives the answer, that is, the high-frequency oscillations become basically invisible. The comparison between these simulations leads us to deduce that the high-frequency oscillations mainly stem from the contributions of the two terms, $\{n = 0, m = 0\}$ and $\{n = -1, m = 0\}$. 

FIG. S6: Color map of the simulated concurrence $C$ and normalized population $P_{N,1}$ versus the evolution time $t$ by varying $\Omega$. (A) Master equation simulation for the full Hamiltonian $H_I$ (S16). (B) Master equation simulation for the target Hamiltonian which keeps only the targeted resonant terms in Eq. (S16). (C) As a comparison, the simulated concurrence $C$ and population $P_{N,1}$ using $H_I$ and the target Hamiltonian for $\Omega = 2\pi \times 0.24$ MHz.
FIG. S7: Numerical analysis on the origin of appearance of high-frequency oscillations. The time-evolved concurrence \( C \) and normalized population \( P_{N|g,1}^N \) after removing (A) the higher-order terms \(|n|, |m| > 2\), (B) \{\( n = 0, m = 0 \)\} term, (C) \{\( n = -1, m = 0 \)\} term, and (D) both \{\( n = 0, m = 0 \)\} and \{\( n = -1, m = 0 \)\} terms in Eq. (S16).

B. The artificial dissipation channel

As illustrated in Fig. S11B, due to the \( Q_t-R_r \) coupling, we are aware that the dissipation profile of the artificial dissipation channel deviates from the reference curve, signaling the occurrence of some inconsistency between the ideal and practical cases. This inconsistency originates from the fact that there is still a little photon population residing in \( R_r \), which makes the condition \( \gamma_r \gg \Omega_r \) not met for the adiabatic elimination of \( R_r \) from the \( Q_t-R_b \) dynamics. Figure 3B and C in the main text contrast the evolution of the concurrence \( C(t) \) for \( \Omega = \kappa \) and \( \Omega = 0.3\kappa \), respectively. The displayed ideal and real situations in these two figures clearly show a noticeable difference. To demonstrate that this difference comes from the inconsistent dissipation lineshapes in the two cases, we replace the imperfect dissipation channel with an ideal one in the numerical simulations by only keeping the two-frequency modulations but disregarding the \( Q_t-R_r \) coupling. The relevant numerical simulations are presented in Fig. S8B by starting with the simplified Hamiltonian (S5). By comparing Fig. S8A with Fig. S8B, it is not difficult to find that the corresponding \( C \) and \( P_{N|g,1}^N \) based upon \( H_{S} \) and \( H_{qb} \) agree well with each other, except the largely-suppressed high-frequency oscillations appearing in the latter case. As a comparison, in Fig. S8C we plot \( C \) and \( P_{N|g,1}^N \) computed in the two cases for \( \Omega = 2\pi \times 0.24 \text{ MHz} \).

C. The influences of the \( Q_t \)'s dephasing

For the test qubit \( Q_t \), its Ramsey dephasing time and spin echo dephasing time, measured at the idle frequency of 5.992 GHz, are, respectively, \( T^*_{z,t} = 2.8 \mu s \) and \( T^*_{SE,t} = 12.9 \mu s \), while its dynamic frequency range under the two-tone parametric modulations is confined to \([5.831 \text{ GHz, 6.009 GHz}]\). In this work, we didn’t measure the real dephasing time for \( Q_t \) working within such a frequency range. In the numerical calculations, we include a dephasing term in Eq. (S23) with 12.9-\( \mu s \) (equivalent to \( T^*_{SE,t} \)) or 2.8-\( \mu s \) (equivalent to \( T^*_{z,t} \)) dephasing time to characterize the dephasing errors. The results are shown in Fig. S9, which indicates that the influences of the \( Q_t \)'s dephasing are indeed negligible. Thanks to the fact that the test qubit \( Q_t \) is coupled with the bosonic mode for excitation number exchange, the prolonged dephasing time seems to be more appropriate for fitting the numerical simulations. This is understandable because the coupling between \( Q_t \) and the bosonic mode somehow protects the \( Q_t \)'s dephasing during its dynamical process, which has been verified in the previous experiments \([7, 9]\).

6. \( \mathcal{PT} \)-ENHANCED SENSING

In this section, we would like to make a further clarification on how \( \mathcal{PT} \) symmetry can help boost the sensitivity of an entanglement-based quantum sensor as we have reported, for the first time, in the main text, in contrast to an only entanglement-based quantum sensor in the literature. In order to do this, let us recall the previously
**FIG. S8:** Temporally evolved concurrence $C$ and normalized population $P_N^{\Omega}$ for variable $\Omega$ using (A) the effective Hamiltonian $H_S$ (1) given in the main text, or (B) the simplified Hamiltonian $H_{qb}$ (S5) with the ideal dissipation but without the $Q_t-R_e$ coupling. (C) The comparison between (A) and (B), as an exemplar, for $\Omega = 2\pi \times 0.24$ MHz.

**FIG. S9:** The real and imaginary part of the $\mathcal{PT}$ eigenspectra $2\omega_1$ obtained by measuring quantum Rabi splitting, where the blue dots along with the error bars are experimental data. The orange lines are numerical simulations with use of the effective Hamiltonian $H_S$; while the blue lines are results simulated with the original Hamiltonian Eq. (S14) (A) without considering dephasing, (B) using 12.9-$\mu$s dephasing time, and (C) using 2.8-$\mu$s dephasing time.

relevant schemes without involving $\mathcal{PT}$ symmetry (and EP) but rather encoding the signal of interest into the population. This alternatively suggests that to have a fair comparison between the two different sensing protocols, we can perform a similar population evolution experiment with the same initial state as well as the same effective spin-boson coupling strength $\lambda'_0$ at the operation point, but without using the readout resonator to induce the artificial dissipation to $Q_t$. The experimental results are not shown here and may be published elsewhere. Instead, here we make a theoretical comparison between the two cases. For the entanglement-based only quantum sensing, we utilize the master equation to perform the numerical analysis on the population evolution in the corresponding Jaynes-Cummings system. Following the similar procedure, we can find an optimal interaction time to yield the optimal susceptibility, $\partial P_{g,1}/\partial \lambda$ maximized at the operating point. In Fig. S10, we present these numerical simulations in blue, compared to the theoretical results (in orange) for the $\mathcal{PT}$-involved case. As one can clearly see, $\mathcal{PT}$ symmetry significantly enhances the sensitivity of quantum sensing based solely on the entanglement.
FIG. S10: Comparison of quantum sensing enhanced jointly by $\mathcal{PT}$ symmetry and entanglement (orange) versus entanglement alone (blue). Specifically, (A) and (C) compare the population evolution and the sensitivity induced by the population change (i.e., the susceptibility $\chi_P$); similarly, (B) and (D) compare the concurrence and the sensitivity enabled by the concurrence change (i.e., the susceptibility $\chi_C$). As one can see, the $\mathcal{PT}$-manifested concurrence $C(\Omega)$ and population $P_{|g,e\rangle}$ (represented by the orange curves) for the interaction time interval $t = 1.4 \mu s$ demonstrate the highest sensitivity to the coupling strength $\Omega$ after the turning point $\Omega = 2\pi \times 0.200$ MHz. In contrast, with the same initial state and the same effective spin-boson coupling strength, the conventional Jaynes-Cummings model (in blue) gives rise to a poor performance with a worse sensitivity.

7. ANALYSIS ON THE DEVIATION OF THE EFFECTIVE DECAY RATE

Ideally, one would expect that when the two-tone parametric modulations are applied, adjusting the amplitude of either modulation should not interfere with the interaction brought about by the other modulation. However, this does not quite match the reality. In fact, limited by the available power of the equipment and the energy band structures of the Xmon qubit, we find that when we increase the amplitude of either modulation, the averaged $Q_t$ frequency shift and the other modulation amplitude will be also changed accordingly, thus differentiating the interaction and effective decay rate from the originally designed during the experiment. To resolve this issue and have a quantitative understanding of the relationship between these variables, we particularly implemented a series of measurements on the population probability $P_e$ of $Q_t$ in $|e\rangle$ after 1-$\mu s$ parametric modulations, and presented the recorded data in Fig. S11A. In the experiment, $Q_t$ was initially prepared in $|e\rangle$, the Mod-1 modulation amplitude was set at $zpa_1 = 1.3$, and the Mod-2 modulation frequency was fixed at $\nu_2' = 155$ MHz. These parameters were selected to resemble the pulse used in the experiment as closely as possible while avoid the $Q_t$-$R_b$ interaction caused by the first-sideband modulation. After suppressing the $Q_t$-$R_b$ interaction, the deviation of the effective decay rate under the two-tone parametric modulations can be then measured in a meaningful way. Figure S11B shows the deviation effect on $P_e$ for the red dots highlighted in Fig. S11A. As one can see, these temporally evolved $P_e$’s deviate from the exponential decay reference line (red dashed) with 1.54-MHz decay rate. Moreover, the effective decay rate gets reduced as $zpa_2$ increases, albeit at a much slower rate in the $\mathcal{PT}$-phase broken region. This also explains the small difference between the experiment and the theory in Fig. 4 of the main text when the effective coupling $\Omega$ is large.
FIG. S11: Deviation of the effective decay rate due to the indirect coupling between two-tone parametric modulations. (A) Scanning map of the population $P_e$ of $Q_t$ in the $|e\rangle$-state after 1-$\mu$s two-tone sideband modulations, where the red dot size is apparently correlated with the effective decay rate. (B) Time-evolved $P_e$ for different $zp\alpha_2$ marked by red dots in (A).

8. Z-PULSE AMPLITUDE VS. THE COUPLING STRENGTH $\Omega$

The relation between the modulation amplitudes ($\varepsilon_1$ and $\varepsilon_2$) and the z-pulse amplitude ($zp\alpha$) is not straightforward when both modulations are applied simultaneously. Therefore, experimentally we turn to the modulation-induced Rabi oscillations between $Q_t$ and $R_b$ to determine the relation between $zp\alpha_2$ and the coupling strength $\Omega$. It becomes clear now from the previous discussions that Mod-1 used to enhance $Q_t$ decay rate has a fixed modulation amplitude and frequency, i.e., $zp\alpha_1 = 1.3$ and $\nu'_1 = 184$ MHz. To determine the relation between $zp\alpha_2$ and $\Omega$, the Mod-2 modulation frequency at which certain $zp\alpha_2$ can induce $Q_t$-$R_b$ resonator sideband resonance is to be first found. For each $zp\alpha_1$-$zp\alpha_2$ pair ($zp\alpha_1$ is always set to 1.3), the scanning plot of $P_e$ is shown in Fig. S12A, where the horizontal and vertical axes correspond to the modulation frequencies $\nu'_1$ and $\nu'_2$, respectively. Here, the population $P_e$ (shown in color) of $Q_t$ in the $|e\rangle$ state after performing a frequency modulation with a duration of 1-$\mu$s when the initial state of $Q_t$ is the $|e\rangle$ state. In particular, the intersection in Fig. S12 is the operating point of the experiment, where $Q_t$ is coupled to both $R_r$ (2nd-order sideband resonance) and $R_b$ (1st-order sideband resonance) by sideband modulations. Figure S12B (C) shows the Rabi oscillations between $Q_t$ and $R_b$ at the blue (green) dot location marked in Fig. S12A. The experimental parameters at these two positions are close to those at the operating point without worrying about the region where the qubit dissipation gets enhanced (due to the dominant coupling between $Q_t$ and $R_b$). The coupling strength $\Omega$ induced by the sideband modulations at this region can be derived from the fitted Rabi oscillation curves. Such a fitting process was performed for each $zp\alpha_1$-$zp\alpha_2$ pair to obtain the blue data points in Fig. S13. In the range of the experimentally chosen parameters, we find that the relation between $zp\alpha_2$ and $\Omega$ is approximately linear. This is evidenced by the solid straight line in Fig. S13, which is obtained by fitting those blue data points.

9. QUBIT READOUT CORRECTIONS

The fidelity matrix for calibrating the measured probabilities is defined as

$$\hat{F} = \begin{pmatrix} F_g & e_{ge} \\ e_{eg} & F_e \end{pmatrix},$$

(S24)
FIG. S12: Two-tone sideband modulations. (A) Scanning plot of population $P_e$ after 1-μs modulations for different modulation frequencies, $\nu_1'$ and $\nu_2'$. (B) and (C) Rabi oscillations at the blue and green dot positions in (A). Here, different curves correspond to different state’s population.

FIG. S13: Z-pulse amplitude vs. coupling strength $\Omega$. Blue and green dots mark the fitted coupling strength at the blue and green dot positions in Fig. S12A for the corresponding $zpa_z$.

where $F_j$ ($j = g, e$) represents the probability of the population that correctly reads the qubit in the $|j\rangle$ state in measurement, and $e_{jk}$ ($j, k = g, e$) stands for the error that describes the leakage probability from the state $|k\rangle$ to state $|j\rangle$. Akin to Eq. (S24), if we now define $\hat{P}_M$ as the measured probabilities and $\hat{P}_N$ as the natural probabilities of the system associated with its genuine states. The relationship between $\hat{F}$, $\hat{P}_M$, and $\hat{P}_N$ is established by the following simple identity,

$$\hat{P}_M = \hat{F} \cdot \hat{P}_N.$$  \hspace{1cm} (S25)

The above relation (S25) implies that the genuine states of the system can be mathematically reconstructed by performing the matrix inversion of $\hat{F}$, i.e., $\hat{P}_N = \hat{F}^{-1} \cdot \hat{P}_M$. The data used in our calibrations is extracted from the measured I-Q (in phase and quadrature) values, as shown in Fig. S14. In this work, the fidelity matrices for $Q_t$ and $Q_a$ are

$$\hat{F}_{Q_t} = \begin{pmatrix} 0.985 & 0.081 \\ 0.015 & 0.919 \end{pmatrix}, \quad \hat{F}_{Q_a} = \begin{pmatrix} 0.991 & 0.113 \\ 0.009 & 0.887 \end{pmatrix}. \hspace{1cm} (S26)$$
A joint measurement on $Q_t$ and $Q_a$ in the computational basis is corrected as
\[ \hat{P}_N = (\hat{F}_{Q_t} \otimes \hat{F}_{Q_a})^{-1} \cdot \hat{P}_M. \] (S27)

We only post-select the results associated with $|e, g\rangle$ and $|g, e\rangle$ after readout corrections.

FIG. S14: Qubit readouts for (A) $Q_t$ and (B) $Q_a$ with 6000 repetitions and 1.1-µs readout duration.

Due to the long duration of the experiment, the fidelity matrices obtained from the measurements at different moments may differ, and if the same fidelity matrix is used for all data, there is still a certain readout error. To characterize this error, we use 89 different fidelity matrices randomly measured between individual experimental runs for calibration and obtain 89 sets of data separately. The error bars are the standard deviation of these 89 sets of data.

10. CHARACTERIZATION OF THE SUSCEPTIBILITY IN QUANTUM SENSING

We note that the concurrence $C$ is inferred from the density matrix, which is reconstructed in a statistical manner. Consequently, a single measurement on the system does not give any information about $C$, and the standard deviation for $C$ extracted from the density matrix with a set of experimental data cannot be well defined. To reduce the error for measuring $C$, we repeat the quantum state reconstruction $N = 89$ times. Each time requires $3M$ samples, $M$ for diagonal elements and $2M$ for non-diagonal elements. In total, $M = 6000$ in our experiment. Then we calculate the concurrence for the density matrix reconstructed each time, and average the results over all runs of measurements: $\overline{C} = (1/N) \Sigma_{j=1}^{N} C_j$, where $C_j$ is the concurrence inferred from the density matrix reconstructed the $j$th time. Based on the thus-obtained mean concurrence, the standard deviation is calculated as $\delta C = \sqrt{\Sigma_{j=1}^{N} [(C - C_j)^2]/N}$.

For the susceptibility given in the main text, the detailed calculation procedure is: we first calculate the susceptibilities for these 89 groups of data according to the formula $\chi_{C_j} = |\{C_j(\Omega' N) - C_j(\Omega' 0)\}/(\Omega' N - \Omega' 0)|$; then average these 89 susceptibilities through $\overline{\chi_C} = (1/N) \Sigma_{j=1}^{N} \chi_{C_j}$ and calculate their standard deviations $\delta\chi_C = \sqrt{\Sigma_{j=1}^{N} [(\overline{\chi_C} - \chi_{C_j})^2]/N}$; the final result is got simply by $\chi_C = \overline{\chi_C} \pm \delta\chi_C$. The same procedure is also applied to calculate the susceptibility of the population for quantum sensing.

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