Buckling Problem Statement and Approaches to Buckling Problem Investigation of Structurally-Anisotropic Aircraft Panels Made from Composite Materials

L M Gavva¹, V V Firsanov¹ and A N Korochkov¹

¹ Moscow Aviation Institute, Moscow, Russia
E-mail: rva101@mail.ru

Received xxxxxx
Accepted for publication xxxxxx
Published xxxxxx

Abstract

Aircraft composite structure design in the field of production technology is the outlook research trend. New mathematical model relations for the buckling investigation of structurally-anisotropic panels comprising composite materials are presented. The primary scientific novelty is the further development of the theory of thin-walled elastic ribs related to the contact problem for the skin and the rib with an improved rib model. One considers the residual thermal stresses and the preliminary tension of the reinforcing fibers with respect to panel production technology. The buckling problem results in the boundary value problem when solving for the eighth order partial derivative equation in the rectangular field. The solution is designed by a double trigonometric series and by a unitary trigonometric series. The results of testing series are presented.

Keywords: panels made of composite materials, eccentric longitudinal and lateral set, force and technology temperature action, pre-critical stress state, buckling, bending mode, torsion mode

1. Introduction

The buckling problems of a flat rectangular multilayer panel made from polymer fiber composite materials with the eccentric longitudinal and lateral stiffening set are considered. The panels are subjected to the distributed constant compressive loading applied to the edges in the casing plane in the stationary temperature field. The boundary conditions at the two opposite sides of the contour are assumed to be the particular case with conformable boundary restrictions for the plane problem and problem of bending, the solution in closed form is designed by a trigonometric series.

One should take into consideration the technological factors occurring in the fabrication of composites, namely, residual thermal stresses arising during cooling after hardening and pre-stressed tension of reinforcing fibers that is performed in order to increase the bearing strength of the structure [1].

The refined statement of the buckling problems has been formulated subject to the pre-critical stress state in the compression of flat rectangular multiplied panels made of polymer fiber composite materials.

The papers [2–38] describe buckling problems of structurally-anisotropic composite panels: buckling problem statement, buckling problem statement with thermal and force loading, buckling problem statement subject to...
production technology, analytical methods to solve buckling problems, numerical methods to solve buckling problems, test investigations. The presented mathematical model is seemed to be new one.

The critical force calculations for the general bending mode of the thin-walled system buckling and the critical force calculations for the multi-wave torsion buckling are of the most actual interest in accordance with traditional design practices. In both cases, bending is integral with the plane stress state. The schematization of the panel as structurally-anisotropic has been proposed as a design model when the stress-strain state and critical forces of total bending mode of buckling are determined. For a multi-wave torsion buckling study, one should use the generalized function techniques.

The scientific novelty of this research reflects the further development of the theory of thin-walled elastic ribs related to the contact problem for the skin and rib with an improved rib torsion model. The aim of this study is the buckling problem statement and the approaches to solve this problem in view of the non-uniform pre-critical stress state and production technology. The buckling problem results in the boundary value problem when solving for the eighth order partial derivative equation in the rectangular field. The buckling problem statement and the proposed approaches to solve this problem are new and are of the interest from the design and manufacture of aircraft outlook specimen made from modern composite materials. The aim of this study is to confirm the authenticity of the presented mathematical model and to verify the research results.

2. Buckling of structurally-anisotropic composite panels. Problem Statement

Both cases, the buckling problem as the problem of the pre-critical stress state cannot be divided into a plane part and a plate bending.

The eighth order differential equation is resolved for the buckling problems (e.g. Firsanov and Gavva 2017). This equation is designed as an equilibrium differential equation with the effect of the reduced loading connected with the normal forces $N_x, N_y$ and tangential forces $N_{x\alpha}, N_{y\alpha}$.

The deflection function $w(x,y)$ is coupled with the potential function $\Phi(x,y)$. For the composite panel of orthotropic structure the left-hand part of the eighth order differential equation is limited by the even-numbered partial derivatives of $\Phi(x,y)$ but the odd-numbered partial derivatives in the right-hand part are connected with the shear

$$
K_{xx} \frac{\partial^4 \Phi}{\partial x^4} + K_{yy} \frac{\partial^4 \Phi}{\partial y^4} + K_{xy} \frac{\partial^4 \Phi}{\partial x \partial y^2} + K_{yx} \frac{\partial^4 \Phi}{\partial y \partial x^2} + K_{x\alpha} \frac{\partial^4 \Phi}{\partial x \partial x_{\alpha}} + K_{y\alpha} \frac{\partial^4 \Phi}{\partial y \partial x_{\alpha}} + K_{x\alpha y} \frac{\partial^4 \Phi}{\partial x \partial y \partial x_{\alpha}} + K_{y\alpha x} \frac{\partial^4 \Phi}{\partial y \partial x \partial x_{\alpha}} = \left[ \left( N_{x\alpha} + N_{y\alpha} \right) R_{x\alpha} \frac{\partial^2 \Phi}{\partial x \partial x_{\alpha}} + \left( N_{x\alpha} + N_{y\alpha} \right) R_{y\beta} \frac{\partial^2 \Phi}{\partial y \partial x_{\alpha}} + \left( N_{x\alpha} + N_{y\alpha} \right) R_{x\alpha y} \frac{\partial^2 \Phi}{\partial x \partial y \partial x_{\alpha}} + \left( N_{x\alpha} + N_{y\alpha} \right) R_{y\alpha x} \frac{\partial^2 \Phi}{\partial y \partial x \partial x_{\alpha}} \right]
$$

(1)

$x = x/a, \ y = y/b$ are the dimensionless coordinates related to the panel length $a$ and to its width $b$.

The coefficients $R_{ij}, i = 4,3, \ldots, 0; j = 0,1, \ldots, 4$ in the relation formulas and the coefficients $K_{ij}, i = 8,7, \ldots, 0; j = 0,1, \ldots, 8$ in the resolving equation (1) are the constant values, which depend on the elastic characteristics of the material and geometrical structure parameters.

All components of the stress-strain state including the inner force factors are related with the potential function $\Phi(x,y)$ as

$$
N_x = L_{x\alpha} \Phi - N_{x\alpha} - N_{x\alpha}^H, \quad N_y = L_{y\alpha} \Phi - N_{y\alpha} - N_{y\alpha}^H
$$

$$
N_{x\alpha} = L_{x\alpha\alpha} \Phi - N_{x\alpha\alpha} - N_{x\alpha}^H, \quad N_{y\alpha} = L_{y\alpha\alpha} \Phi - N_{y\alpha\alpha} - N_{y\alpha}^H
$$

(2)

The linear differential operator for the orthotropic structure, for example

$$
L_{x\alpha} = P_{60} \frac{\partial^6 \Phi}{\partial x^6} + P_{42} \frac{\partial^6 \Phi}{\partial x^4 \partial y^2} + P_{34} \frac{\partial^6 \Phi}{\partial x^3 \partial y^3} + P_{16} \frac{\partial^6 \Phi}{\partial x \partial y^5},
$$

The thermal forces and moments are $N_{y\alpha}^T, N_{x\alpha}^T, N_{x\alpha}^{T\alpha}, N_{y\alpha}^{T\alpha\alpha}$, the tension forces and moments of the composite fibers are $N_{x\alpha}^M, N_{y\alpha}^M, N_{x\alpha}^{M\alpha}, N_{y\alpha}^{M\alpha\alpha}$. The coefficients $P_{ij}, i = 6,4,2,0; j = 0,2,4,6$ as the coefficients in the relation formulas, depend on the elastic characteristics of the material and geometrical structure parameters.

The buckling problem for a structurally-anisotropic composite panel is a nonlinear one according to equation (1) and connection formulas (2). The linearization method is used to determine the critical forces. First, one considers the stress-strain state of the structure at compression, namely, the pre-critical main stress state being complicated as it is not divided into the plane problem and bending problem according to the mathematical model proposed. It is necessary to determine the distribution law of the normal and shear inner forces caused by the external loading. Then one considers the buckling problem as the proper value problem to obtain the additional displacement of the basis surface.
3. Pre-critical stress-strain state of structurally-anisotropic composite panels in compression

We consider the pre-buckling stress state of a flat rectangular composite panel with the eccentric stiffening set being orthotropic one. The panel is subjected to the uniform distributed normal compressive loading with \( P \) intensity applied to the lateral opposite sides in the skin plane. Boundary restrictions satisfy the hinging condition in respect to bending and the sliding constraint condition in the tangential direction for the plane problem when the panel is loaded by the shear force flows along its longitudinal edges.

The normal forces \( N_x \), corresponding to the pre-critical stress state of the stiffened composite panel compressed along the \( x \) axis are distributed as

\[
N_x = P \sum_{i=1}^{m} \left[ \sum_{a=1}^{a_i} \left( N^x_a \right) \right] \left( \lambda_i, \lambda_x \right) \left( \frac{N^y_i}{P} \right) \sin(i\pi y) \quad (3)
\]

\( \left( N^x_i \right) \) are the coefficients of single trigonometric series for the normal forces \( N_x \), known after the determination of the constants of single trigonometric series, \( \left( N^y_i \right) \), \( \left( N^x_i \right) \), are the coefficients of single trigonometric series for the thermal and tension forces.

Here \( \lambda_x = \frac{\pi}{a} \lambda_y, \lambda_y = \frac{\pi}{b} \lambda_x \), \( \lambda_x \) are the roots of the corresponding characteristic polynomial and are calculated using the MATLAB operating environment.

The variations of the transverse forces \( N_y \) and shear forces \( N_{y_1}, N_{x_1} \) are obtained analog to (3).

One can estimate the influence of production technology factors on the bearing strength of structurally-anisotropic composite panels if the non-uniform pre-critical stressed state is considered, boundary conditions are non-conformable, and the solution is formed by a unitary trigonometric series.

4. Buckling of structurally-anisotropic composite panels subject to pre-critical stress state

The linear buckling problem is formulated as the proper value problem.

The general differential equation of the curved surface (1) determines the additional equilibrium state of the structure in view of the initial pre-buckling state.

The integral of this equation satisfying to the uniform boundary conditions may be approximated by a double trigonometric series. But it is not feasible to solve the problem in close form as the method is reduced with an infinite system of the linear algebraic equations. The one-member approximation is considered as the first solution.

The critical force formula is designed with the orthogonalization procedure of the general differential equation of the curved surface. The expression

\[
P = \frac{\sum_{i=1}^{m} \left( \frac{2}{b^2} \right) \left( \frac{1}{2 \pi} \right)^{2i} \left( \frac{1}{2 \pi} \right)^{2i} \left( \frac{1}{2 \pi} \right)^{2i} \left( \frac{1}{2 \pi} \right)^{2i} \left( \frac{1}{2 \pi} \right)^{2i}}{\sum_{i=1}^{m} \left( \frac{2}{b^2} \right) \left( \frac{1}{2 \pi} \right)^{2i} \left( \frac{1}{2 \pi} \right)^{2i} \left( \frac{1}{2 \pi} \right)^{2i} \left( \frac{1}{2 \pi} \right)^{2i} \left( \frac{1}{2 \pi} \right)^{2i}}
\]

provides the range of values \( P \) for additional possible deformation of the base surface at \( m = 1, 2, 3, \ldots, n = 1, 2, 3, \ldots \), \( m \) and \( n \) are the wave parameters.

The \( P \) with \( a^* \) is the critical force calculated with the main uniform pre-buckling stress state (e.g. Firsanov and Gavva 2017)

\[
P = \frac{\pi}{b^2} \left( \frac{m^2}{c^2} \right) + K_{ao} \left( \frac{m^2}{c^2} \right) + K_{bo} \left( \frac{m^2}{c^2} \right) + K_{co} \left( \frac{m^2}{c^2} \right) + K_{ao} \left( \frac{m^2}{c^2} \right) + K_{bo} \left( \frac{m^2}{c^2} \right) + K_{co} \left( \frac{m^2}{c^2} \right)
\]

for the general bending mode of buckling, the panel side ratio \( c = 2a/b \), \( a, b \) are the panel half-length and width, correspondingly.

The formula for the critical loading \( P \) of the multi-wave torsion buckling problem coincides to this formula within the coefficients \( K_{*o} \) and \( K_{*p} \). The coefficients \( K_{*o}, i, j = 0, 1, 2, 4, 6, 8 \) and \( K_{*p}, i, j = 0, 2, 4 \), are determined by generalized stiffness characteristics while stiffness averaging for the elements of longitudinal set up to the skin is replaced with the discrete characteristics

\[
\frac{1}{\epsilon_1} \rightarrow 2 \sum_{i=1}^{m} \sin^2 (n\pi y_i)
\]

\[
\frac{2}{\epsilon_1} \rightarrow 2 \sum_{i=1}^{m} \cos^2 (n\pi y_i)
\]

\( \epsilon_1 \) is the stringer distance; \( y_i \) is the coordinate of the discrete stringer.

It is allowable to estimate the influence of production technology factors on the bearing strength of the structurally-anisotropic composite panels if the complicated pre-critical stress state is considered and the solution is derived in a single approximation of the trigonometric series. One considers the residual thermal stresses arising during cooling after hardening and the pre-stressed tension of the reinforcing fibers with respect to panel production technology.

The step by step method is used to determine the critical forces. The critical force \( P \) with \( a^* \) is calculated with the uniform pre-buckling stress state is proposed as an initial first approach.
5. Buckling of structurally-anisotropic composite panels subject to uniform pre-critical stress state. General boundary problem

One considered the general boundary problem at two opposite sides for buckling of structurally-anisotropic composite panels.

The integral of the equation (1) satisfying to the boundary conditions of general type along lateral edges may be approximated by a single trigonometric series.

This case the step by step method is used to determine the critical forces according to Figure 1.

6. Buckling of structurally-anisotropic composite panels subject to pre-critical stress state. Results and Discussion

A computer program package is developed using the MATLAB operating environment. The computer program package has been utilized for the determination of the critical forces subject to the uniform and non-uniform pre-buckling stress state and for multi-criteria optimization of the design of structurally-anisotropic aircraft composite panels.

The results of determining the critical parameters for rectangular carbon plastic panels eccentrically stiffened and compressed in the longitudinal direction are presented as an example in view of the uniform pre-buckling stress state.

The testing series of uniform compressed stiffened composite panels for carrying the objects to the moment of stability loss (Figure 2) have been made using the special fixture for the proposed mathematical model verification.

The refined theoretical results and the experimental data are in agreement qualitatively with respect to buckling modes and quantitatively with respect to critical forces within 19 - 20% if pre-buckling stress state is considered uniform. The stringer cross section of the panel specimen is non-symmetric, but the numerical results were obtained for the panels with the symmetric cross section of the stringer. Thus, it also confirms the authenticity of the presented mathematical model.

7. Conclusion

Since the solution is obtained by analytical methods, the calculation time is minimal. This is of interest from the perspective of practical design using parametric analysis. The results of the stress analysis calculations, as well as the results of the buckling analysis calculations, offer...
opportunities for reducing and optimizing the weight characteristics of aircraft elements.

**Acknowledgements**

The study was performed in the framework of the RFFI (the project № 17-08-00849/17).

**References**

[1] Gavva L M and Endogur A I 2018 Statics and buckling problems of aircraft structurally-anisotropic composite panels with the influence of production technology IOP Conference Series: Materials Science and Engineering 312 012009

[2] Setoodedeh A R and Karami G 2003 Compos. Struct. 60 245–53

[3] Gangadhara P B 2008 Int. J. Mech. Sci. 13 1326–33

[4] Mittelstedt C and Schroder K U 2010 Compos. Struct. 92 2830–44

[5] Yshii L N, Lucena Neto E, Monteiro F A C and Santana R C 2018 Journal of Engineering Mechanics 144 04018061

[6] Ragb O and Mathuly M S 2017 International Journal of Computational Methods in Engineering Science and Mechanics 6 292–301

[7] Castro S G P and Donadon M V 2017 Composite Structures 160 232–47

[8] Shukla K K and Nath Y 2002 Trans. ASME. J. Appl. Mech. 5 684–92

[9] Chen C S, Lin C Y and Chen R D 2011 Int. J. Mech. Sci. 53 51–8

[10] Matsunaga H 2005 Compos. Struct. 68 439–54

[11] Cetkovic M 2016 Composite Structures 142 238–53

[12] Cetkovic M and Gyorgy L 2016 Structural Integrity and Life 1 43–8

[13] Kettaf F Z, Benguediab M and Tounsi A 2015 Periodica Polytechnica Mechanical Engineering 4 164–8

[14] Naik N S and Sayyad A S 2019 Journal of Thermal Stresses 55 559–79

[15] Chen X, Dai S and Xu K 2001 Ziran keexue ban 41 77–9 83

[16] Pandey R, Shukla K K and Jain A 2009 Commun. Nonlinear Sci. and Numer. Simul. 4 1679–90

[17] Vescovini R and Dozio L 2015 Compos. Struct. 127 35668

[18] Kazemi M 2015 Arch. Appl. Mech. 85 1667–77

[19] Yeter E, Erklig A and Bulut M 2014 Compos. Struct 118 19–27

[20] Abramovich H, Weller T and Bisagni C 2008 J. of Aircraft 45 (2) 402–13

[21] Huang L, Sheikh A H, Ng C T and Griffith M C 2015 Compos. Struct. 122 41–50

[22] Guo M W, Harik I E and Ren W X 2002 Int. J. Solid and Structure. 11 3039–55

[23] Thankam V S, Singh G, Rao G V and Rath A K 2003 Compos. Struct. 59 351–9

[24] Tenek L T 2001 AIAA Journal 39 (3) 546–8

[25] Tran L V, Wahab M A and Kim S E 2017 Composite Structures 179 35–9

[26] Kumar S, Kumar R, Mandal S and Ranjan A 2018 International Journal of Civil Engineering and Technology 6 586–94

[27] Kumar S, Kumar R, Mandal S and Rahul A K 2018 Open Civil Engineering Journal 1 468–80

[28] Zarei A and Khosravifard A 2019 Composite Structures 209 206–18

[29] Castro S G P, Donadon M V and Guimaraes T A M 2019 Composite Structures 209 67–78

[30] Falzon B G, Stevens K and Davies G O 2000 Compos. A 5 459–68

[31] Park O, Hafika R T, Sakar B V, Starnes J H and Nagendra S 2001 J. Aircraft 2 379–87

[32] Rouse M and Assadi M 2001 J. Aircraft 5 950–5

[33] Ungbhakorn V and Singhatanadgul P 2003 Compos. Struct. 59 455–65

[34] Baker D J 2000 J. Aircraft 1 37 138–43

[35] Zhao W, Xie Z, Wang X, Li X and Hao J 2019 Mechanics of Advanced Materials and Structures 3 215–23

[36] Bai R, Bao S, Lei Z, Liu D and Yan C 2018 Ocean Engineering 160 382–8

[37] Kumar S, Kumar R and Mandal S 2018 International Journal of Civil Engineering and Technology 6 1324–32

[38] Sanchez M L, De Almeida S F M and Carrillo J 2017 Revista Latinoamericana de Metalurgia y Materiales 1 45–9

[39] Firsanov V V and Gavva L M 2017 Structural Mechanics of Engineering Constructions and Buildings 4 66–76