Charm Degrees of Freedom in the Quark Gluon Plasma

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Lattice QCD studies on fluctuations and correlations of charm quantum number have established that deconfinement of charm degrees of freedom sets in around the chiral crossover temperature, $T_c$, i.e. charm degrees of freedom carrying fractional baryonic charge start to appear. By reexamining those same lattice QCD data we show that, in addition to the contributions from quark-like excitations, the partial pressure of charm degrees of freedom may still contain significant contributions from open-charm meson and baryon-like excitations associated with integral baryonic charges for temperatures up to $1.2 T_c$. Charm quark-quasiparticles become the dominant degrees of freedom for temperatures $T > 1.2 T_c$.

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Nuclear modification factor and elliptic flow of open-charm hadrons in heavy-ion collision experiments are important observables that provide us with detailed knowledge of the strongly coupled quark gluon plasma (QGP) [1]. Most of the theoretical models that try to describe these quantities rely on the energy loss of heavy quarks via Landau dynamics [2][4]. However, the importance of possible heavy-light (strange) bound states inside QGP have been pointed out in Refs. [5][8]. In particular, presence of such heavy-light bound states above the QCD transition temperature seems to be necessary for the simultaneous description of elliptic flow and nuclear modification factor of $D_s$ mesons [5]. Presence of various hadronic bound states [9] as well as colored [10][11] ones in strongly coupled QGP created in heavy-ion collisions have also been speculated in other other contexts.

By utilizing various novel combinations of up to fourth order cumulants of fluctuations of charm quantum number ($C$) and its correlations with baryon number ($B$), electric charge and strangeness ($S$) lattice QCD studies [12] have established that charm degrees of freedom associated with fractional baryonic and electric charge start appearing at the chiral crossover temperature, $T_c = 154\pm9$ MeV [13][15]. Below $T_c$ the charm degrees of freedom are well described by an uncorrelated gas of charm hadrons having vacuum masses [12], i.e. by the hadron resonance gas (HRG) model. Similar conclusions were also obtained from lattice QCD studies involving the light up, down and strange quarks [16][17].

On the other hand, lattice QCD calculations have also shown that weakly interacting quasi-quarks are good descriptions for the light quark degrees only for temperatures $T \gtrsim 2 T_c$ [16][18][20]. The situation for the heavier charm quarks is also analogous. By re-expressing the lattice QCD results for charm fluctuations and correlations up to fourth order from Ref. [12] in the charm $(c)$ and up $(u)$ quark flavor basis, we show the $u-c$ flavor correlations, described as $\chi^{uc}_{mn} = (\partial^{m+n} p / \partial \hat{p}^m \partial \hat{p}^n)$ at $\mu_u = \mu_c = 0$ in Fig. [1]. Here, $p$ denotes the total pressure in QCD, $\mu_u$ and $\mu_c$ indicate the up and charm quark chemical potentials with $\hat{p}^{X} \equiv \mu_X / T$. In order to compare these lattice QCD data with resummed perturbation theory results, which are available only for zero quark masses, we normalize the off-diagonal flavor susceptibilities with the second order charm quark susceptibility $\chi^c_2 = (\partial^2 p / \partial \mu_c^2)$ calculated at $\mu_X = \mu_c = 0$. Such a normalization largely cancels the explicit charm quark mass dependence of the off-diagonal susceptibilities and enables us to probe whether the $u-c$ flavor correlations can be described by the weak coupling calculations. In the weak coupling limit $\chi^{u_{11}}_c , \chi^{c_{13}}_c$ and $\chi^{s_{13}}_c$ are expected to have leading order contributions at $O(\alpha_s^4)$ [21], where $\alpha_s$ is the QCD strong coupling constant. This contribution, strictly speaking, non-perturbative but can be calculated on the lattice using dimensionally reduced effective theory for high temperature QCD, the so-called electrostatic QCD (EQCD) [22]. Similarly, in the weak coupling picture, the leading contribution to $\chi^{u_{22}}_{uc}$ arises from the so-called plasmon term and starts at $O(\alpha_s^{3/2})$ [23]. Thus, it is generically expected $\chi^{u_{22}}_{uc} \gg \chi^{c_{13}}_c \sim 2 \chi^{s_{13}}_c$ in the weak coupling limit. As shown in Fig. [1] such an obvious hierarchy in magnitude of the off-diagonal susceptibilities is clearly absent in the lattice data for $T < 200$ MeV. However, for $T \gtrsim 200$ MeV these lattice results are largely consistent with the weak coupling calculations, indicating that the weakly coupled quasi-quarks can be considered as the dominant charm degrees freedom only above this temperature. The fact that for $T_c \lesssim T \lesssim 200$ MeV the charm degrees of freedom are far from weakly interacting quasi-quarks is also supported by lattice QCD studies of the screening properties of the open-charm mesons. In this temperature range the screening masses of open-charm mesons also turn out to be quite different from the expectation based on an uncorrelated charm and a light quark degrees of freedom [24].

From the preceding discussion it is clear that the weakly interacting charm quasi-quarks cannot be the only carriers of charm quantum number for $T \lesssim 200$ MeV. Such an observation naturally raises the question whether charm excitations associated with baryon number zero and one, exist in QGP for $T_c \lesssim T \lesssim 200$, along with the charm quasi-quark excitations carrying
Also shown, as dashed lines, are the results of dimensionally generalized charm susceptibilities, the lattice QCD data of Ref. [12] on up to fourth order. To check our postulates against the temperature range of interest and we thus neglect their contributions to QCD thermodynamics in the temperature range of interest and we thus neglect their contributions to QCD thermodynamics. Furthermore, as discussed in Ref. [12], the doubly and triply charged baryons are too heavy to have any significant contributions to QCD thermodynamics in the temperature range of interest and we thus neglect their contributions. With these simplifications the partial pressure of the open-charm sector, $p^C$, can be written as

$$p^C(T, \mu_C, \mu_B) = p_{q}^C(T) \cosh(\mu_C + \mu_B/3) + p_{M}^C(T) \cosh(\mu_C + \mu_B) + p_{B}^C(T) \cosh(\mu_C),$$

where $p_{q}^C$, $p_{M}^C$ and $p_{B}^C$ denote the partial pressure of the quark-like, meson-like and baryon-like excitations, respectively, and $\mu_B$ and $\mu_C = \mu_S$ represents the baryon and charm chemical potentials.

Using combinations of up to fourth order baryon-charm susceptibilities it is easy to isolate the partial pressures of $p_{q}^C$, $p_{M}^C$ and $p_{B}^C$ appearing in Eq. (2). For example, $p_{q}^C = \chi^C_{22} - \chi^C_{13} + \chi^C_{22} + \chi^C_{22}$ and $p_{B}^C = \chi^C_{22} + \chi^C_{22}$ and $p_{M}^C = \chi^C_{22} + \chi^C_{22} + \chi^C_{22}$. The contributions of these partial pressures compared to total charm pressure $p^C(T, 0, 0) = \chi^C_{22}$ is shown in Fig. 2 (top). For $T < T_c$ the partial pressure of mesons, $p_{M}^C$ and the partial pressure of baryons, $p_{B}^C$ agree with the corresponding partial pressures from the HRG model including all the experimentally observed as well as additional quark model predicted but yet unobserved open-charm hadrons with vacuum masses [12]. The contributions of $p_{M}^C$ and $p_{B}^C$ remain significant till $T \lesssim 200$ MeV. In fact, for $T \lesssim 180$ MeV the combined contributions of $p_{M}^C$ and $p_{B}^C$ exceeds the contribution from $p_{q}^C$. With increasing temperatures $p_{M}^C$ and $p_{B}^C$ deviate from the HRG model predictions. This indicate that these charm meson and baryon-like excitations can no longer be considered as vacuum charm mesons and baryons. This is in line with the lattice QCD studies on spatial correlation functions of open-charm mesons [24], which show significant in-medium modifications of open-charm mesons already in the vicinity of $T_c$. The partial pressure of quark-like excitations is quite small for $T \sim T_c$ and becomes the dominant contribution to $p^C$ only for $T > 200$ MeV.

Since a charm-quark-like excitation does not carry a strangeness quantum number, the excitations carrying both strangeness and charm quantum numbers are a much cleaner probe of the postulated existence of the charm hadron-like excitations. In this sub-sector, the pressure can be partitioned into partial pressures of $|C| = 1$ meson-like excitations carrying strangeness $|S| = 1$ and $C = 1$ baryon-like excitations with $|S| = 1, 2$, i.e.

$$p_{C,S=1}^C(T, \mu_B, \mu_S, \mu_C) = p_{C,S=1}^C(T) \cosh(\mu_S + \mu_C) + \sum_{j=1}^{2} p_{C,S=j}^C(T) \cosh(\mu_B - j \mu_S + \mu_C).$$

Thus, the partial pressures of the strange-charm hadron-like excitations can be obtained as: $p_{C,S=1}^C = \chi_{13}^C$, $p_{C,S=2}^C = \chi_{22}^C - 3 \chi_{112}^C$, and $p_{B}^C = (2 \chi_{22} + \chi_{22} - \chi_{112}^C)/2$. In Fig. 2 (bottom) we show the fractional contributions of these partial pressures towards the total charm partial pressure $p^C(T) = \chi^C_{22}$. Even in this sub-sector, contributions from the hadron-like excitations are significant for $T \lesssim 200$ MeV. However, partial pressure for the $S = 2$ charm baryon-like excitations is negligible.
that has to hold if the model is correct. If we consider contributions from charm meson and baryon-like excitations to the total charm partial pressure ($p^C$). (Bottom) Fractional contributions of partial pressures of charm-strange meson-like ($p^{C,S\neq1}$), charm-singly-strange baryon-like ($p^{C,S=1}$) and charm-doubly-strange baryon-like ($p^{C,S=2}$) excitations to the total charm partial pressure ($p^C$). The solid lines show the corresponding partial pressures obtained from HRG model including additional quark model predicted charm hadrons (see text).

Having shown that there can be significant contributions from charm meson and baryon-like excitations to the charm partial pressure in QGP, it is important to ask whether the addition of only these charm degrees of freedom besides the charm quark-like excitations is sufficient to describe all available lattice QCD results for up to fourth order charm susceptibilities. As discussed previously in Ref. [12], the constraints $\chi^C = \chi^C_2$, $\chi^{BC} = \chi^{BC}_3$, $\chi^{SC} = \chi^{SC}_3$ are due to negligible contributions from $|C| = 2,3$ hadron-like states and they do not provide any independent constraint specific to our proposed model. The remaining four independent fourth order generalized charm susceptibilities, $\chi^C_2$, $\chi^{BC}_3$, $\chi^{BC}_2$ and $\chi^{BC}_3$ allow us to define the three partial pressures, $p^C_q$, $p^C_M$ and $p^C_B$ and one constraint

$$c_1 \equiv \chi^{BC}_3 - 4\chi^{BC}_2 + 3\chi^{BC}_3 = 0,$$  

that has to hold if the model is correct. If we consider the strange-charm sub-sector, we have six generalized susceptibilities $\chi^{BC,S}_{13}$, $\chi^{BC,S}_{22}$, $\chi^{SC}_{13}$, $\chi^{SC}_{121}$, $\chi^{BC}_{121}$, and $\chi^{BC}_{211}$. We can use three of these to estimate the partial pressures $p^{C,S=1}_M$, $p^{C,S=1}_B$ and $p^{C,S=2}_B$ defined above, while the remaining ones will provide three additional constraints that can be used to validate our proposed model. These constraints can be written as:

$$c_2 \equiv 2\chi^{BC}_{121} + 4\chi^{BC}_{112} + \chi^{SC}_{122} - 2\chi^{SC}_{13} + \chi^{SC}_{31} = 0, (5a)$$

$$c_3 \equiv 3\chi^{BC}_{112} + 3\chi^{BC}_{121} - \chi^{SC}_{13} + \chi^{SC}_{31} = 0, (5b)$$

$$c_4 \equiv \chi^{BC}_{211} - \chi^{BC}_{112} = 0. (5c)$$

Note that the above constraints hold trivially for a free charm-quark gas. It is assuring that our proposed model also smoothly connects to the HRG at $T_c$. In Fig. 3 we show the lattice QCD data for $c_i$'s. Despite large errors on the presently available lattice data all the $c_i$'s are, in fact, consistent with zero. Note that, since a possible strange-charm di-quark-like excitation will carry $|C|=|S|=1$ but $|B|=2,3$, the lattice QCD data being consistent with the constraint $c_4 = 0$ actually tells us that the thermodynamic contributions of possible di-quark-like excitations are negligible in the deconfined phase of QCD.

One may speculate on the nature of these charm hadron-like excitations and, in particular, why their partial pressures vanish gradually with increasing temperature. A likely explanation may be that with increasing temperature the the spectral functions of these excitations gradually broaden. A detailed treatment of thermodynamics of quasi-particles with finite width was developed in Refs. [21, 24]. It was shown that broad asymmetric spectral functions lead to partial pressures that are considerably smaller than those obtained with zero width quasi-particles of the same mass, and for sufficiently large width the partial pressures can be made arbitrarily small. Thus, the smallness of the partial pres-
ure of charm quark-like excitations for $T \sim T_c$ may imply that they have a large width for those temperatures, while the widths of the charm hadron-like excitations increase with the temperatures and these excitations become very broad for $T \gtrsim 200$ MeV. Such a gradual melting picture is also consistent with the gradual changes of the screening correlators of open-charm meson-like excitations with increasing temperature [24].

Finally, one may wonder whether the rich structure of the up to fourth order generalized charm susceptibilities can be described only in terms of the charm quasi-quarks without invoking presence of any other type of charm degrees of freedom. In terms of charm quasi-quarks alone, the charm partial pressure will be $p_c^2/T^4 = 6/\pi^2 \tilde{m}_c^2 K_2(\tilde{m}_c) \cosh(\mu_C + \mu_B/3)$, where $\tilde{m}_c = m_c/T$ with $m_c$ being the mass of the charm quasi-particle. The lattice QCD results for the charm susceptibilities, for example, the non-vanishing values of $\chi^{SC}_{mn}$, can only be described if the charm quasi-quark mass depends on the chemical potentials of all the quark flavors, i.e. $m_c \equiv m_c(T,\mu_\ell,\mu_S,\mu_C)$. For simplicity, one may imagine Taylor expanding $m_c$ in terms of the chemical potentials and treat these coefficients as parameters for fitting all the lattice QCD results on the generalized charm susceptibilities. Obviously, such a quasi-quark model will contain at least as many tunable parameters as the number of susceptibilities. Moreover, in order to satisfy various other constraints observed in the lattice QCD data, such as $\chi^C_4 = \chi^C_2$, $\chi^{BC}_{11} = \chi^{BC}_{13}$, these parameters must also be very finely tuned. For example, in order to satisfy the constraint $c_4 = 0$ the coefficients of the $O(\mu_B^2\mu_S^2\mu_C)$ term of $m_c$ must be equal to coefficient of the $O(\mu_B\mu_S\mu_C^2)$ term. Even if one chooses to use such finely tuned parameters for the chemical potential dependence of the quasi-quark mass, the charm partial pressure is not guaranteed to go smoothly over to the HRG values, as observed in the lattice data.

To conclude, using the lattice QCD results for up to fourth order generalized charm susceptibilities [12] we have shown that the weakly coupled charm quasi-quarks becomes the dominant charm degrees of freedom only above $T \gtrsim 200$ MeV. To investigate the nature of charm degrees of freedom in the intermediate temperature regime, $T_c \lesssim T \lesssim 200$ MeV, we postulated the presence of non-interacting charm meson and baryon-like excitations in QGP, along with the charm quark-like excitations. We have shown that such a picture is consistent with the presently available lattice QCD results. We have isolated the individual partial pressures of these excitations and found that just above $T_c$ open-charm meson and baryon-like excitations provide the dominant contribution to the thermodynamics of charm sector. We also do not observe presence of di-quark like excitations in the $s\rightarrow c$ sector at these temperatures. Our study hints at possible resonant scattering of the heavy quarks in the medium till around $1.2 T_c$ as first advocated in Ref. [28]. These findings may have important consequences for the heavy quark phenomenology of heavy-ion collision experiments, especially in understanding the experimentally observed ellitic flow and nuclear modification factor of heavy flavors at small and moderate values of transverse momenta [5–8, 23, 29].

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