Multinomial method for pricing Lookback Option

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Abstract. One way to avoid or minimize risk due to price fluctuation of asset price in the market is through option trading. Lookback option is a contract that gives a holder right but not the obligation to buy/sell a certain asset at minimum/maximum price during certain time. Lookback option has two types, namely fixed strike lookback and floating strike lookback. The method used was multinomial method. The parameters used are initial asset price, strike price, risk-free interest rate, time to maturity, and volatility. The method has been applied, and obtained that the increasing of the order of the multinomial method do not give significant changes to the price of the option. Another conclusion we get is for fixed strike price, the put is more expensive than the call, different with floating strike, the call is more expensive than the put.

1. Theory and Model Description

1.1 Option
Option has been mentioned in [1], it is the right, but not the obligation, between holder (buyer) and writer (seller), where writer gives holder the right to buy or to sell a certain asset for a certain price (strike price) at a certain date. The term asset means any financial object, [2] gives examples of assets, they are shares in a company, currencies, and commodities, such as gold, oil or electricity.

Based on the right, option can be divided into two types, namely call and put option. [2] explains that call option means a holder buys a right to writer to buy a certain asset, while put option gives a right to sell a certain asset during a certain period. To buy option, the holder must pay option price to the writer, and the payment is mandatory. Whereas the writer must sell or buy the asset to holder if holder decides to exercise their right. Moreover, there are two kind of option based on time to exercise, they are european style and american style. European style means option can be exercised at maturity time only, while american style can be exercise at any time, as long as option has not expired.

Another kind of option besides standard option, is exotic option. It involves more complex derivatives and terms. There are many kinds of exotic options, namely asian option, lookback option, exact option, barrier option, exchange option, and compound option.

1.2 Binomial method
Binomial method is the simplest method of lattice in predicting asset price movement by using a simple discrete model. [2] explains that by letting \( \delta = \frac{T}{N} \) denote the spacing between successive time points, where \( T \) is the expiry date. So asset price will be considered at times \( t_i = i\delta \), for \( 0 \leq i \leq M \).

[2] mentioned that there are three assumptions in the binomial method, which are
Since the initial asset price, $S_0$, is known, during the time period $\delta$, $S_t$ can change into two possible asset prices, which are $S_u d$ and $S_u u$, with $0 < d < 1 < u$. Where $u$ is the factor of asset price moves up and $d$ is the factor of asset moves down.

- The probability of an asset moves up is $p$ while the probability of a downward movement occurs is $1-p$.
- The expectation of continuous $S_t$ is $\mathbb{E}[S_t] = S_0 \exp(r\delta)$.

Based on the assumption (1) and (2), the expectation of discrete asset price $S_n$, is $\mathbb{E}[S_{n,\delta}] = pS_n u + (1-p)S_n d$. By equating the expection of continous and discrete $S_n$, obtained:

$$p = \frac{e^{r\delta} - d}{u - d}$$

Because $p$ is probability with $0 < p \leq 1$, then $d < e^{r\delta} < u$.

To get the values of parameter $u$ and $d$, it is necessary to combine between the variance of continous and discrete asset price, which are respectively stated as follows:

- Discrete Models: $\text{Var}[S_{n,\delta}] = S_n^2 \left[ pu^2 + (1-p)d^2 - \exp(2r\delta) \right]$  

- Continous Model: $\text{Var}[S_t] = S_t^2 e^{2\sigma^2 \delta} (e^{2\sigma^2 \delta} - 1)$

To find out particular solution, by combining equation (2) dan (3), we may set $ud = 1$ and solve to find that

$$u = \exp\left(\sigma\delta^{1/2}\right), \quad d = \frac{1}{u} = \exp\left(-\sigma\delta^{1/2}\right)$$

### 1.3 Multinomial method

Multinomial method is also one of the lattice method to predict asset price movement starting from today to maturity time. To understand further about multinomial model, here are the following of the differences in the binomial tree and multinomial tree from asset price $S_0$, introduced in [3].

Multinomial lattice is transformation from binomial lattice by removing some intraday periods. In this method, it is required some probability of asset price up and down for every node.

The probability can be counted by using the equation below:
\[ P_{nm} = \binom{n}{m} p^{n-m} (1-p)^m, m = 0,1,2,...,n \]  

with,

- \( P_{nm} \) = probability for every node
- \( n \) = number of nodes
- \( m \) = located of node

After obtaining probability for every node, then it is necessary to do asset price modelling for every node using multinomial model, as illustrated in Figure 2.

1.4 Lookback Option

[1] mentioned that lookback option is an agreement between holder and writer, that provides holder a right to sell/buy asset at maximum/minimum price during period of the option. Lookback option is an exotic option where it’s payoff values does not only depend on the asset price at the end of the period but also on the asset price movement from previous period. Therefore, lookback option is also in one of path dependent derivative.

[4] explained that lookback option are classified into two types:

1.4.1 Fixed strike lookback.
- Fixed strike lookback call. The payoff depends on the maximum asset during the life of the option.
  \[ \text{Payoff} = \max (S_{\text{max}} - K, 0) \]  
- Fixed strike lookback put. The payoff depends on the minimum asset during the life of the option.
  \[ \text{Payoff} = \max (K - S_{\text{min}}, 0) \]  

1.4.2 Floating Strike Lookback.
- Floating Strike Lookback Call. The minimum asset during the life of the option and also the stock price at maturity time are needed to calculate the payoff.
  \[ \text{Payoff} = \max \left[ S(T) - S_{\text{min}}, 0 \right] \]  
- Floating Strike Lookback Put. The payoff depends not only on asset price at maturity time but also the maximum value of asset price.
  \[ \text{Payoff} = \max \left[ S_{\text{max}} - S(T), 0 \right] \]  

2. Results

Before calculating lookback option. It is important to set the input and calculate the parameters needed.
- \( T = 1 \) .
- \( r = 5\% \)
- \( S_0 = 75 \)
- \( K = 70 \)
- Volatility = 0.213
- Parameters \( u, d \) and \( p \)

By inputting various value of \( M \), then the parameters value obtained as follows:
Table 1. Parameters Values of $u, d$, and $p$.

| $M$  | $u$    | $d$    | $p$    |
|------|--------|--------|--------|
| 500  | 1.0096 | 0.9905 | 0.5029 |
| 1000 | 1.0068 | 0.9933 | 0.5020 |
| 2000 | 1.0048 | 0.9952 | 0.5014 |

By inputting all the values above, then the comparison of the option price using multinomial method is:

Table 2. Comparison Value of Lookback Option Price.

| $M$  | Fixed Strike Lookback Call | Fixed Strike Lookback Put | Floating Strike Lookback Call | Floating Strike Lookback Put |
|------|-----------------------------|---------------------------|--------------------------------|------------------------------|
| 500  | 16.9928                     | 66.5307                   | 70.3021                        | 13.2214                      |
| 1000 | 16.2974                     | 66.5832                   | 70.3548                        | 12.5256                      |
| 2000 | 15.8086                     | 66.5860                   | 70.3577                        | 12.0369                      |

In Table 2, $n$-order multinomial method are used to price fixed strike and floating strike lookback option. From the results above, we can see that there is no significant changes in the price of the option by increasing the order of multinomial.

In addition, for fixed strike lookback, the put option price is more expensive than the call. This was because the payoff for put option gains more value than the put, and it leads to bigger value of option. While for floating strike lookback, the call option price is more expensive than the put, with same reason with fixed strike lookback.

4. Conclusions

In this paper, multinomial method has been applied to predict asset price movement so that the option price can be calculated. According to the result above, we can see that this method increases the number of $S_{max}$ and downcreases number of $S_{min}$ significantly. Because number of payoff depends on both $S_{max}$ and $S_{min}$, and option price depends on its payoff, then the option price also changes accordingly. In addition, the increasing of the order of the multinomial method do not give significant changes to the price of the option. Another conclusion we get is for fixed strike price, the put is more expensive then the call, different with floating strike, the call is more expensive than the put.

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