Integral Inequalities for Mappings Whose Derivatives Are s-Convex in the Second Sense and Applications to Special Means for Positive Real Numbers

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Abstract In this paper, the authors establish a new type integral inequalities for differentiable s-convex functions in the second sense. By the well-known Hölder inequality and power mean inequality, they obtain some integral inequalities related to the s-convex functions and apply these inequalities to special means for positive real numbers.

Keywords: s-convexity, Hermite-Hadamard Inequality, Bullen’s inequality, Special Means.

Cite This Article: MEVLÜT TUNÇ, and SEVIL BALGEÇTİ, “Integral Inequalities for Mappings Whose Derivatives Are s-Convex in the Second Sense and Applications to Special Means for Positive Real Numbers.” Turkish Journal of Analysis and Number Theory, vol. 4, no. 2 (2016): 48-53. doi: 10.12691/tjant-4-2-5.

1. Introduction

1.1. Definitions

Definition 1. [10] A function \( \varphi : I \rightarrow \mathbb{R} \) is said to be convex on \( I \) if inequality

\[
\varphi(tx + (1-t)y) \leq t \varphi(x) + (1-t) \varphi(y)
\]  

holds for all \( x, y \in I \) and \( t \in (0,1] \). We say that \( \varphi \) is concave if \( -\varphi \) is convex.

Definition 2. [8] Let \( s \in (0,1] \). A function \( \varphi : (0,\infty) \rightarrow [0,\infty] \) is said to be s-convex in the second sense if

\[
\varphi(tx + (1-t)y) \leq t^s \varphi(x) + (1-t)^s \varphi(y),
\]

for all \( x, y \in (0,b] \) and \( t \in (0,1] \). This class of s-convex functions is usually denoted by \( K^2_s \).

Certainly, s-convexity means just ordinary convexity when \( s = 1 \).

1.2. Theorems

Theorem 1. The Hermite-Hadamard inequality: Let \( \varphi : I \subseteq \mathbb{R} \rightarrow \mathbb{R} \) be a convex function and \( u, v \in I \) with \( u < v \). The following double inequality:

\[
\varphi\left(\frac{u+v}{2}\right) \leq \frac{1}{v-u} \int_u^v \varphi(x) \, dx \leq \frac{\varphi(u) + \varphi(v)}{2}
\]  

is known in the literature as Hadamard’s inequality (or Hermite-Hadamard inequality) for convex functions. If \( \varphi \) is a positive concave function, then the inequality is reversed.

Theorem 2. [6] Suppose that \( \varphi : [0,\infty) \rightarrow [0,\infty) \) is an s-convex function in the second sense, where \( s \in (0,1] \), and let \( a,b \in [0,\infty) \), \( a < b \). If \( \varphi \in L^1([0,1]) \), then the following inequalities hold:

\[
2^{s-1} \frac{u+v}{2} \leq \frac{1}{v-u} \int_u^v \varphi(x) \, dx \leq \frac{\varphi(u) + \varphi(v)}{s+1}.
\]

The constant \( k = \frac{1}{s+1} \) is the best possible in the second inequality in (1.4). The above inequalities are sharp. If \( \varphi \) is an s-concave function in the second sense, then the inequality is reversed.

Theorem 3. Let \( \varphi : I \subseteq \mathbb{R} \rightarrow \mathbb{R} \) be a convex function on the interval \( I \) of real numbers and \( a,b \in I \) with \( a < b \). The inequality

\[
\frac{1}{v-u} \int_u^v \varphi(x) \, dx \leq \frac{1}{2} \left[ \frac{\varphi(u) + \varphi(v)}{2} \right].
\]

is known as Bullen’s inequality for convex functions [[5], p.39].

In [4], Dragomir and Agarwal obtained inequalities for differentiable convex mappings which are connected to Hadamard’s inequality, as follow:

Theorem 4. Let \( f : I \subseteq \mathbb{R} \rightarrow \mathbb{R} \) be a differentiable mapping on \( I \), where \( a,b \in I \), with \( a < b \). If \( f' : I \rightarrow \mathbb{R} \) is convex on \( [a; b] \), then the following inequality holds:

\[
\left| \frac{f(a) + f(b)}{2} - \frac{1}{v-u} \int_a^b f(x) \, dx \right| \leq \frac{b-a}{8} \left[ \left| f'(a) \right| + \left| f'(b) \right| \right].
\]
In [11], Pearce and Pečarić obtained inequalities for differentiable convex mappings which are connected with Hadamard’s inequality, as follow:

**Theorem 5.** Let \( f : I \subseteq \mathbb{R} \to \mathbb{R} \) be differentiable mapping on \( I^* \), where \( a,b \in I \) with \( a < b \). If \( |f|^q \) is convex on \( [a,b] \) for some \( q \geq 1 \), then the following inequality holds:

\[
\frac{f(a)+f(b)}{2} - \frac{1}{b-a} \int_a^b f(x)dx \leq b-a \left[ \left( \frac{a+b}{2} \right)^q + \left( \frac{a+3b}{4} \right)^q \right].
\]

If \( |f|^q \) is concave on \( [a,b] \) for some \( q \geq 1 \), then

\[
\frac{f(a)+f(b)}{2} - \frac{1}{b-a} \int_a^b f(x)dx \leq b-a \left[ \left( \frac{a+b}{2} \right)^q + \left( \frac{a+3b}{4} \right)^q \right].
\]

In [1], Alomari, Darus and Kirmacı obtained inequalities for differentiable \( s \)-convex and concave mappings which are connected with Hadamard’s inequality, as follow:

**Theorem 6.** Let \( f : I \subseteq [0,\infty) \to \mathbb{R} \) be differentiable mapping on \( I^* \) such that \( f' \in [a,b] \), where \( a,b \in I \) with \( a < b \). If \( |f|^q \), \( q \geq 1 \) is concave on \( [a,b] \) for some fixed \( s \in (0,1] \), then the following inequality holds:

\[
\frac{f(a)+f(b)}{2} - \frac{1}{b-a} \int_a^b f(x)dx \leq b-a \left[ \left( \frac{a+b}{2} \right)^q + \left( \frac{a+3b}{4} \right)^q \right].
\]

In [12], Tunç and Balğcı obtained inequalities for differentiable convex functions which are connected with a new type integral inequality, as follow:

**Lemma 1.** Let \( f : J \to \mathbb{R} \) be a differentiable function on \( J^* \). If \( f' \in L[a,b] \), then

\[
\frac{1}{b-a} \int_a^b f(x)dx - \frac{1}{2} \left( \frac{bf(a) - af(b)}{b-a} + f(\frac{a+b}{2}) \right)
\]

\[
= \frac{1}{4} \left[ (b+1-t)a f' \left( \frac{1-t}{2}b + \frac{1+t}{2}a \right) dt + (a+1-t)b f' \left( \frac{1-t}{2}a + \frac{1+t}{2}b \right) dt \right]
\]

for each \( t \in [0,1] \) and \( a,b \in J \).

**Theorem 7.** [12] Let \( f : J \to \mathbb{R} \) be a differentiable function on \( J^* \). If \( |f'| \) is convex on \( J \), then

\[
\frac{1}{b-a} \int_a^b f(x)dx - \frac{1}{2} \left( \frac{bf(a) - af(b)}{b-a} + f(\frac{a+b}{2}) \right)
\]

\[
\leq \frac{5}{48} a + \frac{7}{48} b \left| f'(a) \right| + \frac{7}{48} a + \frac{5}{48} b \left| f'(b) \right|
\]

for each \( a,b \in J \).

**Theorem 8.** [12] Let \( f : J \to \mathbb{R} \) be a differentiable function on \( J^* \). If \( |f|^q \) is convex on \( [a,b] \) and \( q > 1 \) with \( \frac{1}{p} + \frac{1}{q} = 1 \), then

\[
\frac{1}{b-a} \int_a^b f(x)dx - \frac{1}{2} \left( \frac{bf(a) - af(b)}{b-a} + f(\frac{a+b}{2}) \right)
\]

\[
\leq \frac{1}{4^{1+\frac{1}{q}}} L_\rho (a,b)
\]

\[
\left[ \left| f'(b)^q \right| + \left| f'(a)^q \right| \right] \frac{1}{q} + \left[ \left| f''(a)^q \right| + \left| f''(b)^q \right| \right] \frac{1}{q} \cdot
\]

**Theorem 9.** [12] Let \( f : J \to \mathbb{R} \) be a differentiable function on \( J^* \). If \( |f|^q \) is convex on \( [a,b] \) and \( q \geq 1 \), then

\[
\frac{1}{b-a} \int_a^b f(x)dx - \frac{1}{2} \left( \frac{bf(a) - af(b)}{b-a} + f(\frac{a+b}{2}) \right)
\]

\[
\leq \frac{1}{4^{1+\frac{1}{q}}} \frac{1}{A} \rho (a,b)
\]

\[
\left[ \left| f'(b)^q \right| + \left| f'(a)^q \right| \right] \frac{1}{q} \cdot
\]

For recent results and generalizations concerning Hadamard's inequality and concepts of convexity and \( s \)-convexity see [1-12] and the references therein.

Throughout this paper we will use the following notations and conventions. Let \( J = [0,\infty) \subseteq \mathbb{R} = (-\infty, \infty) \), and \( u,v \in J \) with \( 0 < u < v \) and \( f' \in L[u,v] \) and

\[
A(u,v) = \frac{u+v}{2},
\]

\[
G(u,v) = \sqrt{uv},
\]

\[
L_p(u,v) = \left( \frac{v^{p+1} - u^{p+1}}{(p+1)(v-u)} \right)^{\frac{1}{p}}
\]

be the arithmetic mean, geometric mean, generalized logarithmic mean for \( u,v > 0 \) respectively.

## 2. Inequalities for \( s \)-convex Functions and Applications

**Theorem 10.** Let \( f : J \to \mathbb{R} \) be a differentiable function on \( J^* \). If \( |f'| \) is \( s \)-convex on \( [a,b] \) for some fixed \( s \in (0,1] \), then
If
\[ \int_{b-a}^{b} f(x) dx - \frac{1}{2} \left( b f(a) - a f(b) + f\left(\frac{a+b}{2}\right) \right) \]
\[ \leq \frac{b (s^{2s+1} + s + 2) + a (2^{2s+2} - s - 2)}{2^{s+2} (s+1)(s+2)} \int' (a) \]
\[ + \frac{a (s^{2s+1} + s + 2) + b (2^{2s+2} - s - 2)}{2^{s+2} (s+1)(s+2)} \int' (ab) \]
for each \( x \in [a,b] \).

Proof. Using Lemma 1 and from properties of modulus, and since \( \int' \) is \( s \)-convex on 1, then we obtain
\[ \int_{b-a}^{b} f(x) dx - \frac{1}{2} \left( b f(a) - a f(b) + f\left(\frac{a+b}{2}\right) \right) \]
\[ \leq \frac{1}{4} \int_{0}^{1} (tb + (1-t)a) \int' \left( \left(\frac{1-t}{2}\right) b + \left(\frac{1+t}{2}\right) a \right) dt \]
\[ + \frac{1}{4} \int_{0}^{1} (ta + (1-t)b) \int' \left( \left(\frac{1-t}{2}\right) a + \left(\frac{1+t}{2}\right) b \right) dt \]
\[ \leq \frac{1}{4} \int_{0}^{1} (tb + (1-t)a) \int' (b) \left(\frac{1-t}{2}\right) \int' (a) \left(\frac{1+t}{2}\right) dt \]
\[ + \frac{1}{4} \int_{0}^{1} (ta + (1-t)b) \int' (a) \left(\frac{1-t}{2}\right) \int' (b) \left(\frac{1+t}{2}\right) dt \]
\[ \leq \frac{1}{4} \int_{0}^{1} (tb + (1-t)a) \int' (b) \left(\frac{1-t}{2}\right) \int' (a) \left(\frac{1+t}{2}\right) dt \]
\[ + \frac{1}{4} \int_{0}^{1} (ta + (1-t)b) \left(\frac{1-t}{2}\right) \int' (a) \left(\frac{1+t}{2}\right) \int' (b) dt \]
\[ + \frac{1}{4} \int_{0}^{1} (ta + (1-t)b) \left(\frac{1-t}{2}\right) \int' (a) \left(\frac{1+t}{2}\right) \int' (b) dt \]
\[ = \frac{1}{2^{s+2} (s+1)(s+2)} \int' (b) \]
\[ + \frac{1}{2^{s+2}} \left( \frac{bs + b + a}{(s+1)(s+2)} \right) \int' (a) \]
\[ + \frac{1}{2^{s+2}} \left( \frac{a s + a + b}{(s+1)(s+2)} \right) \int' (b) \]
\[ = \frac{1}{2^{s+2}} \left( \frac{a s + a + b}{(s+1)(s+2)} \right) \int' (b) \]

Proposition 1. Let \( a, b \in J^s, \ 0 < a < b \) and \( s \in (0,1) \) then
\[ L_p^s (a,b) + \frac{(s-1)G^2 (a,b)}{2} \left( \frac{ab^{s-1} + a^{s-1}b}{(s+1)(s+2)} \right) \]
\[ \leq \frac{1}{2^{s+2} (s+1)(s+2)} \int' (a) \]
\[ + \frac{1}{2^{s+2}} \left( \frac{ab^{s+1} + b^{s+1}}{(s+1)(s+2)} \right) \int' (b) \]
\[ \leq \frac{1}{2^{s+2} (s+1)(s+2)} \int' (a) \]
\[ + \frac{1}{2^{s+2}} \left( \frac{ab^{s+1} + b^{s+1}}{(s+1)(s+2)} \right) \int' (b) \]

Proposition 2. Let \( a, b \in J^s, \ 0 < a < b, \ s \in (0,1) \) then
\[ L_p^s (a,b) - \frac{s G^2 (a,b)}{2} \left( \frac{ab^{s-1} + a^{s-1}b}{(s+1)(s+2)} \right) \]
\[ \leq \frac{1}{2^{s+2} b^s} \left( \frac{a (s^{2s+1} + s + 2) + b (2^{2s+2} - s - 2)}{(s+1)(s+2)} \right) \]
\[ + \frac{1}{2^{s+2} a^s} \left( \frac{a (s - 2^{s+2} + 2) - b (s^{2s+1} + s + 2)}{(s+1)(s+2)} \right) \]

Remark 1. In (2.1), (2.2), if we take \( s \to 1 \), then (2.1), (2.2) reduces to (1.10), [[12], Proposition 2], respectively.

Theorem 11. Let \( f : J \to \mathbb{R} \) be a differentiable function on \( J^s \). If \( \int' \) is \( s \)-convex on \( [a,b] \) for some fixed \( s \in (0,1) \) and \( q > 1 \) with \( \frac{1}{p} + \frac{1}{q} = 1 \), then
\[ \int_{b-a}^{b} f(x) dx - \frac{1}{2} \left( b f(a) - a f(b) + f\left(\frac{a+b}{2}\right) \right) \]
\[ \leq \frac{L_p (a,b)}{4^{(s+1)^q}} \left( \int' (a)^q + \left( \int' (b)^q \right)^{\frac{1}{q}} \right) \]
(2.4)
for each \( x \in [a, b] \).

**Proof.** Using Lemma 1 and from properties of modulus, and since \( |f'| \) is s-convex on \( J \), then we obtain

\[
\left| \frac{1}{b-a} \int_a^b f(x) dx - \frac{af(b) - bf(a)}{2(b-a)} \right| + \frac{1}{2} f\left( \frac{a+b}{2} \right) \leq \frac{1}{4} \int_0^1 \left( (b + (1-t)a) f'\left( \frac{1-t}{2} \right) b + \frac{1+t}{2} a \right) dt \tag{2.5}
\]

then (2.4), \( \int_0^1 \left( \frac{1}{b-a} \right) f(x) dx + \frac{af(b) - bf(a)}{2(b-a)} - \frac{1}{2} f\left( \frac{a+b}{2} \right) \]

\[
= \frac{1}{4} \int_0^1 \left( (b + (1-t)a) f'\left( \frac{1-t}{2} \right) b + \frac{1+t}{2} a \right) dt.
\]

Since \( |f'|^q \) is s-convex, by the Hölder inequality, we have

\[
\left| \frac{1}{b-a} \int_a^b f(x)^q dx - \frac{af(b)^q - bf(a)^q}{2(b-a)^q} \right| + \frac{1}{2} f\left( \frac{a+b}{2} \right)^q \leq \frac{1}{4} \int_0^1 \left( (b + (1-t)a)^q f'\left( \frac{1-t}{2} \right) b + \frac{1+t}{2} a \right) dt \tag{2.6}
\]

and

\[
\left\| \int_0^1 f(x)^q dx - \frac{af(b)^q - bf(a)^q}{2(b-a)^q} \right\|_q \leq \frac{1}{4} \left\| \int_0^1 \left( (b + (1-t)a)^q f'\left( \frac{1-t}{2} \right) b + \frac{1+t}{2} a \right) dt \right\|_q \tag{2.7}
\]

\[
\int_0^1 \left( (b + (1-t)a)^q f'\left( \frac{1-t}{2} \right) b + \frac{1+t}{2} a \right) dt \leq \frac{1}{4} \left\| \int_0^1 \left( (b + (1-t)a)^q f'\left( \frac{1-t}{2} \right) b + \frac{1+t}{2} a \right) dt \right\|_q \tag{2.8}
\]

If expressions (2.6)-(2.8), we obtain

\[
\int_0^1 \left( \frac{1}{b-a} \right) f(x)^q dx + \frac{af(b)^q - bf(a)^q}{2(b-a)^q} - \frac{1}{2} f\left( \frac{a+b}{2} \right)^q \]

\[
= \frac{1}{4} \int_0^1 \left( (b + (1-t)a)^q f'\left( \frac{1-t}{2} \right) b + \frac{1+t}{2} a \right) dt.
\]

The proof follows from (2.4) applied to the s-convex function \( f(x) = x^s \) and \( f'(|x|) = sx^{s-1} \).

**Proposition 3.** Let \( a, b \in J^* \), \( 0 < a < b \) and \( s \in (0,1] \), then

\[
\int_0^1 \left( \frac{1}{b-a} \right)G^2(a,b)L^2_{s-2}(a,b) - A^s(a,b) \frac{1}{2} \leq \frac{L_p (a,b)}{2^{2q+x}(s+1)^{q}} \tag{2.9}
\]

\[
\int_0^1 \left( \frac{1}{b-a} \right) G^2(a,b)L^2_{s-2}(a,b) - A^s(a,b) \frac{1}{2} \leq \frac{L_p (a,b)}{2^{2q+x}(s+1)^{q}} \tag{2.10}
\]

**Proposition 4.** Let \( a, b \in J^* \), \( 0 < a < b \) and \( s \in (0,1] \), then

\[
\int_0^1 \left( \frac{1}{b-a} \right)G^2(a,b)L^2_{s-2}(a,b) - A^s(a,b) \frac{1}{2} \leq \frac{L_p (a,b)}{2^{2q+x}(s+1)^{q}} \tag{2.10}
\]

**Remark 2.** In (2.4), (2.9), if we take \( s \rightarrow 1 \), then (2.4), (2.9) reduces to (1.11), [12], Proposition 5, respectively.

**Theorem 12.** Let \( f : J \rightarrow \mathbb{R} \) be a differentiable function on \( J^* \). If \( f' \) is s-convex on \( [a,b] \) for some fixed \( s \in (0,1] \) and \( q > 1 \), then
\[
\frac{1}{b-a} \int_a^b f(x) \, dx + \frac{1}{2} \left( \frac{af(b) - bf(a)}{b-a} - f\left(\frac{a+b}{2}\right) \right) \leq \frac{1}{(2^{2q+2} (s+1)(s+2))^{1-\frac{1}{q}}} \left[ \frac{f''(a)\|f\|_q^{\frac{1}{q}}}{(s+1)(s+2)} + \frac{f''(b)\|f\|_q^{\frac{1}{q}}}{(s+1)(s+2)} \right] \]

\[
(2.11)
\]

**Proof.** From Lemma 1 and using the well-known power mean inequality and since \(f''(x)\) is \(s\)-convex on \([a,b]\), we can write

\[
\frac{1}{b-a} \int_a^b f(x) \, dx + \frac{1}{2} \left( \frac{af(b) - bf(a)}{b-a} - f\left(\frac{a+b}{2}\right) \right) = \frac{1}{4} \left( \int_0^1 (tb+(1-t)a) \, dt \right)^{1-\frac{1}{q}}
\]

\[
\times \left[ \int_0^1 (tb+(1-t)a) \left( \frac{(1-t)b+1+t\frac{a+b}{2}}{2} \right)^q \, dt \right]^{\frac{1}{q}}
\]

\[
+ \frac{1}{4} \left( \int_0^1 (ta+(1-t)b) \, dt \right)^{1-\frac{1}{q}}
\]

\[
\times \left[ \int_0^1 (ta+(1-t)b) \left( \frac{(1-t)a+1+t\frac{a+b}{2}}{2} \right)^q \, dt \right]^{\frac{1}{q}}
\]

\[
(\frac{1}{4}) \left( \int_0^1 (tb+(1-t)a) \, dt \right)^{1-\frac{1}{q}}
\]

\[
\times \left[ \int_0^1 (tb+(1-t)a) \left( \frac{(1-t)^s (1-t)b+1+t\frac{a+b}{2}}{2} \right)^q \, dt \right]^{\frac{1}{q}}
\]

\[
+ \frac{1}{4} \left( \int_0^1 (ta+(1-t)b) \, dt \right)^{1-\frac{1}{q}}
\]

\[
\times \left[ \int_0^1 (ta+(1-t)b) \left( \frac{(1-t)^s (1-t)a+1+t\frac{a+b}{2}}{2} \right)^q \, dt \right]^{\frac{1}{q}}
\]

\[
(\frac{1}{4}) \left( \int_0^1 (tb+(1-t)a) \, dt \right)^{1-\frac{1}{q}}
\]

\[
\times \left[ \int_0^1 (tb+(1-t)a) \left( \frac{(1-t)^s f''(b)^q}{2} + \frac{1+t}{2} f''(a)^q \right) \, dt \right]^{\frac{1}{q}}
\]

\[
\leq \frac{1}{4} \left( \frac{a+b}{2} \right)^{1-\frac{1}{q}} \left[ \frac{f''(a)\|f\|_q^{\frac{1}{q}}}{(s+1)(s+2)} + \frac{f''(b)\|f\|_q^{\frac{1}{q}}}{(s+1)(s+2)} \right]
\]

\[
+ \frac{1}{4} \left( \frac{a+b}{2} \right)^{1-\frac{1}{q}} \left[ \frac{f''(a)\|f\|_q^{\frac{1}{q}}}{(s+1)(s+2)} + \frac{f''(b)\|f\|_q^{\frac{1}{q}}}{(s+1)(s+2)} \right]
\]

\[
(2.11)
\]

**Proposition 5.** Let \(a, b \in J^+\), \(0 < a < b\) and \(s \in (0,1]\), then

\[
J_s^2 (a,b) + (s-1)G^2 (a,b) J_s^2 (a,b) - \mathcal{A}^2 (a,b)
\]

\[
\leq \frac{\frac{1}{2}}{(2^{2q+2} (s+1)(s+2))^{1-\frac{1}{q}}}
\]

\[
\left[ \left( \frac{b}{(s+1)(s+2)} \right) \left( \frac{(as+a+b)\|f\|_q^{\frac{1}{q}}}{2} + \frac{f''(a)\|f\|_q^{\frac{1}{q}}}{2} \right) \right]^{\frac{1}{q}}
\]

\[
+ \left( \frac{b}{(s+1)(s+2)} \right) \left( \frac{(bs+b+a)\|f\|_q^{\frac{1}{q}}}{2} + \frac{f''(b)\|f\|_q^{\frac{1}{q}}}{2} \right) \right]^{\frac{1}{q}}
\]

\[
(2.12)
\]

**Proof.** The proof follows from (2.11) applied to the \(s\)-convex function \(f(x) = x^s\) and \(f''(x) = sx^{s-1}\).

**Proposition 6.** Let \(a, b \in J^+\), \(0 < a < b\) and \(s \in (0,1]\), then
\[ L_s^2(a,b) + sG^2(a,b)\frac{L_{1-s}^{-s}(a,b) - A^{-s}(a,b)}{2(1-s)} \leq \frac{A^{-q}(a,b)}{(2^{2q+s}(s+1)(s+2))^{\frac{1}{q}}} \]

**Proof.** The proof follows from (2.11) applied to the \( s \)-convex function \( f(x) = x^{1-s} \) and \( |f'(x)| = 1/x^2 \).

**Remark 3.** In (2.11), (2.12), if we take \( s \to 1 \), then (2.11), (2.12) reduces to (1.12), ([12], Proposition 8) respectively.

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