Predictions on the $\alpha$-cluster structure in $^{104}$Te

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Abstract. The present work analyzes the $\alpha$ + core structure in $^{104}$Te using the local potential model. The $\alpha$ + core interaction is described by a nuclear potential of $(1 + \text{Gaussian}) \times (\text{W.S.} + \text{W.S.}^3)$ shape. The energy levels, total $\alpha$ widths and rms intercluster separations are determined for the ground state band and compared with a previous calculation which uses a double-folding potential. The two potential forms produce similar spectra between the $0^+$ and $14^+$ states. The antistretching effect is predicted for the $^{104}$Te ground state band, as is observed in previous $\alpha$ + core calculations in intermediate mass nuclei. An $\alpha$-decay half-life $T_{1/2,\alpha} \approx 3$ ns is predicted for $^{104}$Te in the $\alpha$-decay energy $Q_\alpha \approx 5.36$ MeV using an $\alpha$ preformation factor $P = 1$. The calculated $T_{1/2,\alpha}$ value is compatible with the recently reported experimental result on $\alpha$-decay of $^{104}$Te.

1. Introduction

The $\alpha$-cluster model has been able to describe satisfactorily properties as energy levels, $\alpha$ emission widths, electromagnetic transition rates and $\alpha$ elastic scattering data for light, intermediate and heavy nuclei [1–4]. In this context, there is a great interest in the $^{104}$Te nucleus, since an $\alpha$-decay experimental observation of this nucleus could indicate the presence of the $\alpha$+$^{100}$Sn structure, as well as reinforce the influence of the double shell closure $N, Z = 50$ in the nuclear structure of this mass region. Recently, the observation of the $^{108}$Xe $\rightarrow$ $^{104}$Te $\rightarrow$ $^{100}$Sn $\alpha$-decay chain was reported [5], including the measurement of the $\alpha$-decay energy and half-life of $^{104}$Te. This experimental information motivates the comparison with nuclear models applied to $^{104}$Te.

The work of P. Mohr [6] discusses the $\alpha$-decay in Te isotopes and shows predictions on the $\alpha$ + core structure in $^{104}$Te, such as energy levels and the $\alpha$-decay half-life, using a double-folding potential for the nuclear $\alpha$ + core interaction. In the present work, the $\alpha$ + core structure in $^{104}$Te is analyzed with a nuclear potential of $(1 + \text{Gaussian}) \times (\text{W.S.} + \text{W.S.}^3)$ shape, which was successful in the study of the same structure in $^{46}$Cr and $^{54}$Cr [2]. It is intended to verify the
similarities or differences between the results obtained in this work and previous calculations as in Ref. [6].

2. The Model
The $\alpha$-cluster model regards the total nucleus as an $\alpha$-particle orbiting an inert core. Therefore, the $^{104}\text{Te}$ nucleus is assumed as an $\alpha+^{100}\text{Sn}$ system. The $\alpha+$ core interaction is described through a local potential

$$V(r) = V_N(r) + V_C(r)$$

(1)

containing the nuclear and Coulomb terms. The Coulomb potential $V_C(r)$ is that of an $\alpha$-particle interacting with an uniformly charged spherical core of radius $R$. The intercluster nuclear potential is proposed as

$$V_N(r) = -V_0 \left[1 + \lambda \exp \left(-\frac{r^2}{\sigma^2}\right)\right] \left\{\frac{b}{1 + \exp[(r - R)/a]} + \frac{1 - b}{[1 + \exp[(r - R)/3a]]^3}\right\},$$

(2)

where $V_0$, $\lambda$, $a$, and $b$ are fixed parameters, and $R$ and $\sigma$ are variable parameters. The description of the ground state band of $^{104}\text{Te}$ is obtained with the fixed values $V_0 = 220$ MeV, $a = 0.65$ fm, $b = 0.3$, and $\lambda = 0.14$, while $R$ and $\sigma$ are fitted specifically for $^{104}\text{Te}$. The $R$ and $\sigma$ values employed for the ground state band are: $R = 5.708$ fm and $\sigma = 0.44$ fm. The values of $V_0$, $\lambda$, $a$, and $b$ were adjusted previously to describe the ground state bands of the nuclei $^{20}\text{Ne}$, $^{44}\text{Ti}$, $^{94}\text{Mo}$, and $^{212}\text{Po}$, in which the $\alpha$-cluster structure is recognized in preceding studies [1, 3, 4]. The parameter $R$ was fitted with $10^{-3}$ fm precision to give the bandhead closest to the value $E(0^+) = 5.354$ MeV, which is predicted by Mohr [6] in a study of the $\alpha+$ core potentials in neighboring Te isotopes. The fit criterion resulted in the energy $E(0^+) = 5.364$ MeV for the present calculation.

According to the Pauli principle, the nucleons of the $\alpha$-cluster must lie in shell-model orbitals outside the core. This restriction is defined by the global quantum number $G = 2N + L$, where $N$ is the number of internal nodes in the radial wave function and $L$ is the orbital angular momentum. In the case of the $\alpha+^{100}\text{Sn}$ system we have $G \geq 16$, where $G = 16$ corresponds to the ground state band. This value is determined from the Wildermuth condition [7]. The $\alpha+$ core system is solved numerically to obtain the properties of the resonant states. The energy eigenvalues provide the levels of the spectrum and the associated wave functions are used to calculate other nuclear properties.

3. Results
Applying the $\alpha$-cluster model to $^{104}\text{Te}$, we obtain the ground state band ($G = 16$) shown in figure 1 in comparison with the theoretical band calculated by Mohr [6] for the same nucleus. A similarity between the theoretical spectra calculated with the $(1 + \text{Gaussian}) \times (\text{W.S.} + \text{W.S.}^3)$ and double-folding potentials is observed, except for the spacing between the $14^+$ and $16^+$ levels. This result is gratifying, since the two potential forms were obtained independently.

Table 1 shows the results for the rms intercluster separations ($\langle R^2 \rangle^{1/2}$) and total $\alpha$-widths ($\Gamma_\alpha$) referring to the ground state band of $^{104}\text{Te}$. The $\alpha$-widths were calculated using the semiclassical approximation proposed in Ref. [8]. The $\alpha$-decay width calculated for the $0^+$ state is $\Gamma_\alpha \approx 1.51 \times 10^{-13}$ MeV in the $\alpha$-decay energy $Q_\alpha = 5.364$ MeV, using an $\alpha$ preformation factor $P = 1$; this width implies an $\alpha$-decay half-life $T_{1/2,\alpha} \approx 3.02$ ns. However, by applying a factor $P = 10\%$, as suggested by Mohr [6], $T_{1/2,\alpha}$ is increased to $\approx 30.21$ ns. For comparison, the $\alpha$-decay half-life obtained by Mohr with the double-folding potential is $T_{1/2,\alpha} \approx 5$ ns in the
Figure 1. Calculated energy levels for the ground state band \((G = 16)\) of the \(\alpha+^{100}\text{Sn}\) system in comparison with the theoretical band calculated by Mohr [6] which uses a double-folding potential as the nuclear \(\alpha+\) core interaction. The energy scale is given with reference to the \(\alpha+^{100}\text{Sn}\) threshold.

\(\alpha\)-decay energy \(Q_\alpha = 5.42\) MeV. The \(T_{1/2,\alpha}\) values obtained in this work with \(P = 10\%\) and the Mohr’s work are close with respect to the order of magnitude.

The recently published \(^{104}\text{Te}\) \(\alpha\)-decay experiment [5] obtained the measure \(T_{1/2,\alpha}^{\text{exp}} < 18\) ns in a decay energy \(Q_\alpha^{\text{exp}} = 5.1(2)\) MeV. Although the present calculation was made at an energy slightly above the experimental range, it should be noted that the calculated half-life with \(P = 1\) is consistent with the experimental data. Therefore the present calculation suggests a high \(\alpha\) preformation factor for the \(^{104}\text{Te}\) decay.

The calculated intercluster rms radii for \(^{104}\text{Te}\) indicate that the \(\alpha\)-cluster character is stronger for the first members of the ground state band. Such behavior (antistretching effect) has already been observed in the \(^{46}\text{Cr}\) and \(^{54}\text{Cr}\) ground state bands with the same \(\alpha+\) core potential shape [2]. Also, the antistretching effect is seen in other nuclei of the intermediate mass region with different \(\alpha+\) core potentials [1, 3].

4. Conclusions

The present work shows that the \((1 + \text{Gaussian}) \times (\text{W.S.} + \text{W.S.}^3)\) and double-folding potentials produce similar spectra for the \(\alpha+^{100}\text{Sn}\) system between the 0\(^+\) and 14\(^+\) states. This is a gratifying result and shows that the two potential forms are compatible in this aspect. The results obtained for rms intercluster separations indicate that the \(\alpha\)-cluster character is stronger for the first members of the ground state band, as is predicted in other intermediate mass nuclei with the \(\alpha+\) core structure. The \(\alpha\)-decay half-life predicted for \(^{104}\text{Te}\) is \(T_{1/2,\alpha} \approx 3.02\) ns in the \(\alpha\)-decay energy \(Q_\alpha \approx 5.36\) MeV, using an \(\alpha\) preformation factor \(P = 1\). This result suggests a high preformation factor for the \(^{104}\text{Te}\) \(\alpha\)-decay, being able to reach \(P \sim 1\).

The present work was developed mainly for a comparative study of the
Table 1. Calculated rms intercluster separations ($\langle R^2 \rangle^{1/2}$) and total $\alpha$-widths ($\Gamma_\alpha$) for the ground state band of $^{104}$Te. An $\alpha$ preformation factor $P = 1$ is applied. Note: $uE\nu = u \times 10^v$

| $J^\pi$ | $\langle R^2 \rangle^{1/2}$ (fm) | $\Gamma_\alpha$ (MeV) |
|--------|----------------|----------------|
| 0$^+$  | 5.255          | 1.51E−13       |
| 2$^+$  | 5.268          | 1.62E−11       |
| 4$^+$  | 5.238          | 2.50E−10       |
| 6$^+$  | 5.161          | 8.03E−10       |
| 8$^+$  | 5.059          | 1.09E−09       |
| 10$^+$ | 4.946          | 5.23E−10       |
| 12$^+$ | 4.836          | 7.28E−11       |
| 14$^+$ | 4.745          | 2.24E−12       |
| 16$^+$ | 4.695          | 7.58E−15       |

$(1 + \text{Gaussian}) \times (\text{W.S. + W.S.}^3)$ and double-folding nuclear potentials applied to $^{104}$Te. The recent experimental results of Ref. [5] should be analyzed in more detail in our forthcoming publication.

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