Gedanken Experiments Involving Black Holes

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ABSTRACT

Analysis of several gedanken experiments indicates that black hole complementarity cannot be ruled out on the basis of known physical principles. Experiments designed by outside observers to disprove the existence of a quantum-mechanical stretched horizon require knowledge of Planck-scale effects for their analysis. Observers who fall through the event horizon after sampling the Hawking radiation cannot discover duplicate information inside the black hole before hitting the singularity. Experiments by outside observers to detect baryon number violation will yield significant effects well outside the stretched horizon.

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1. Introduction

One of the most interesting open questions in theoretical physics is whether the unitary evolution of states in quantum theory is violated by gravitational effects [1]. The debate has centered on the process of black hole formation and evaporation, where, at the semi-classical level, almost no information about the initial quantum state of the infalling matter appears to be carried by the final outgoing Hawking radiation. Our viewpoint in this paper is a conservative one as far as quantum theory is concerned. We shall assume that outside observers can apply standard quantum mechanical rules to the evolution of black holes. This requires us to adopt a more radical view of spacetime physics in the presence of a black hole, but, as we shall argue, one which does not conflict with any known laws of physics.

In reference [2] three basic postulates for the quantum evolution of black holes were proposed. The most important implication of these postulates is that there exists a unitary $S$-matrix for any such process. Furthermore, the information carried by the infalling matter re-emerges in the outgoing radiation and is not stored in a stable or long-lived remnant. A description by a distant observer, necessarily involves non-trivial physical processes taking place in the vicinity of the event horizon. The nature of these processes is such that organized energy and information is absorbed, thermalized and eventually radiated. Such behavior is commonly encountered in macroscopic systems. For example a cold piece of black coal which absorbs a coherent laser signal will burn and emit the signal encoded in thermal radiation. The implication is that the “stretched horizon” is described by an outside observer as a physical membrane with microphysical structure and that the usual thermodynamics of black holes follows from a coarse graining of the microscopic description.

This point of view is in apparent contradiction with the expectation that a freely falling observer encounters nothing unusual when crossing the event horizon of a large black hole. The duplication of information behind the horizon and in the Hawking radiation seems to violate the principles of quantum mechanics. However obvious logical contradictions only arise when one attempts to correlate the results of experiments performed on both sides of the event horizon. The principle of black hole complementarity [2] states that such contradictions never occur because the black hole interior is not in the causal past of any observer who can measure the information content of the Hawking radiation.

In this paper we shall illustrate the concept of black hole complementarity by considering
a number of gedanken experiments where one might expect contradictions to arise. The basic argument we apply was formulated most clearly by J. Preskill [3]. It says that apparent contradictions can always be traced to unsubstantiated assumptions about physics at or beyond the Planck scale. We do not offer a full “resolution” of the information paradox in this paper. Our aim is limited to challenging the commonly held view that, as there is no strong curvature or other coordinate invariant manifestation of the event horizon, an information paradox can be posed without detailed knowledge of the underlying short distance physics. We analyze gedanken experiments which test the hypothesis that the event horizon has no distinguishing feature to an observer crossing it in free fall, while all information about the quantum state of the infalling matter is returned to outside observers in the Hawking radiation. In each case we find that in order to expose a violation of this hypothesis energies of order the Planck scale or higher are required. We conclude that the information paradox can only be precisely formulated in the context of a complete theory of quantum gravity and that the issue of information loss cannot be definitively settled without such a theory. We can, however, look for clues to the nature of the final theory by making concrete assumptions about information loss and exploring their consequences in gedanken experiments.

The reference frame of an asymptotic observer and that of another observer in free fall approaching the event horizon of a black hole are very different. As time measured by the distant observer goes on, the boost relating the two frames becomes so enormous that an electron at rest in one frame would appear to have super-planckian energy in the other. As t’ Hooft has stressed we have no experimental experience of such energetic particles [4]. Their physical description may well be quite different from an ordinary Lorentz boost applied to a localized object [5]. We wish to emphasize that black hole complementarity does not mean a departure from the dictum that the laws of nature appear the same in different frames of reference. Rather, the assertion is that the description of the same physical reality may differ quite significantly between reference frames separated by a large boost parameter.

We begin in Section 2 by comparing measurements made by different observers in Rindler space. In Section 3 we turn our attention to black holes and discuss gedanken experiments in which outside observers attempt to establish the non-existence of a physical membrane at the stretched horizon. We consider both static geometries and a simple model of black holes formed by gravitational collapse. In Section 4 we analyze a more interesting class of
experiments involving observers who, after sampling the Hawking radiation, cross the event horizon and attempt to observe duplicate information. These observers have to wait outside the black hole until some of the information is returned and in Section 5 we give an estimate of the time required. In Section 6 we discuss the issue of time-reversal in black hole evolution and Section 7 contains our concluding remarks.

2. Experiments in Rindler Space

Before considering finite mass black holes it is instructive to consider the simpler geometry of Rindler space which can be viewed as the exterior region close to the horizon in the limit of infinite black hole mass. It is also isomorphic to the region of flat four-dimensional Minkowski space defined by $|z| > |t|, z > 0$, shown in Figure 1.

We introduce Rindler coordinates $R, \omega$ as follows,

\[
t = R \sinh \omega,
\]
\[
z = R \cosh \omega.
\] (2.1)

Note that $R$ is a spacelike coordinate and $\omega$ is time-like. The Rindler line element is

\[
d s^2 = -R^2 d \omega^2 + dR^2 + dx^2 + dy^2.
\] (2.2)

The Rindler Hamiltonian generates translations of $\omega$, which are Lorentz boosts of Minkowski space. At time $t = 0$ it is given by

\[
H_R = \int_{-\infty}^{\infty} dx dy \int_0^\infty dz z T^{00}(x, y, z),
\] (2.3)

where $T^{00}$ is the Minkowski space energy density.

In classical physics the evolution of fields in Rindler space can be described in a self contained fashion without any reference being made to the regions of Minkowski space that have been excised in Figure 1. Events which take place in the region $z - t > 0, z + t < 0$ can be summarized by initial conditions at $\omega = -\infty$, and the other excised regions are not in the causal past of the Rindler space.
FIGURE 1.

Rindler space as a wedge of Minkowski space: surfaces of constant Rindler time are straight lines through the origin while surfaces of fixed Rindler position are hyperbolas. There is a future (past) event horizon at \( z-t=0 \) (\( z+t=0 \)).

When dealing with quantum fields, the space of states for a Rindler observer consists of the functionals of fields defined on the positive \( R \) axis. This restriction requires the usual Minkowski space vacuum to be described in Rindler space as a density matrix for a mixed state. The appropriate density matrix was obtained by Unruh [6] and is given by

\[
\rho = \exp \left( -2\pi H_R \right). \tag{2.4}
\]

The form of the Rindler density matrix corresponds to a thermal state with a temperature \( T_R = \frac{1}{2\pi} \). Although, (2.4) is merely a rewriting of the Minkowski vacuum, the thermal effects are nevertheless quite real to an accelerated observer. For example any standard thermometer at rest in Rindler coordinates at position \( R \) will record a temperature \( \frac{1}{2\pi R} \) *.

* The factor of \( \frac{1}{R} \) in the temperature comes from the conversion between Rindler coordinate time and the proper time registered by a clock attached to the thermometer.
The Minkowski observer attributes the same effect to the thermometer’s uniform acceleration of magnitude $\frac{1}{R}$. More generally, observers at rest in Rindler coordinates will describe the Minkowski vacuum as a state with position-dependent proper temperature. For large values of $R$ the acceleration of such an observer is small and the proper temperature goes to zero asymptotically, while Rindler observers near the edge of the Rindler wedge experience enormous acceleration and measure a temperature which diverges as $|z| \to |t|$.

It is interesting to consider temperature dependent phenomena such as baryon number violation at different locations in Rindler space. At first sight one might expect significant B-violation due to the ‘t Hooft anomaly [7] when the local proper temperature reaches the weak scale. However, this is not so. The temperature only reaches or exceeds a given value, $T$, over a region of proper size $T^{-1}$ in the $R$ direction. The gauge field configurations which ordinarily lead to B-violation at high temperature have a much larger size of order $\frac{1}{g^2 T}$ and therefore cannot contribute. Configurations of scale $\frac{1}{T}$ contribute with the usual exponential suppression, $\exp\left(-\frac{4\pi}{g^2}\right)$. On the other hand, when the local temperature reaches the GUT scale, $T_{\text{GUT}}$, $X$ bosons of wavelength $T_{\text{GUT}}^{-1}$ will mediate B-violation, suppressed only by powers of the coupling $g$.

Suppose that a Rindler observer wishes to test the prediction that baryon number violation takes place within a distance of order $T_{\text{GUT}}^{-1}$ from the edge of Rindler space. The observer prepares a sealed “bucket” with walls that can transmit heat but not baryon number$. The baryon number in the bucket is measured and recorded at the beginning of the experiment. The bucket is then lowered toward $R = 0$ and retrieved after its bottom edge reaches $R = T_{\text{GUT}}^{-1}$, as indicated in Figure 2. The baryon number is remeasured and compared with its original value. According to the Rindler observer the thermal effects can induce a change in $B$. The same phenomenon is described by a Minkowski observer as due to the intense acceleration disturbing the virtual processes involving $X$-bosons inside the bucket.

By contrast, a Minkowski observer would see no baryon number violation for a freely falling bucket which falls through the Rindler horizon at $z - t = 0$. This does not contradict the result of the previous experiment for several reasons. First of all, the absence of $B$-

$\dagger$ The infinite extension in the x, y directions does not affect this.

$\ddag$ Although the manufacture of such buckets from conventional matter may be impossible, their existence does not violate the rules of relativistic quantum field theory.
violation cannot be reported back to the Rindler observer. One might try to communicate the absence of B-violation at the bottom of the freely falling bucket during the time that the bottom is between $R = T_{GUT}^{-1}$ and $R = 0$. To do so will require the message sent by the bucket to be composed of quanta of frequency at least $T_{GUT}$ as seen in the frame of the bucket. Therefore the processes, by which the message is sent, will themselves lead to baryon number violation. If lower frequencies are used the uncertainty principle will insure that the message will be lost behind the Rindler horizon.

![Figure 2](image-url)  

**FIGURE 2.**

*A gedanken experiment to test baryon nonconservation near the Rindler horizon.*

Furthermore there is a real sense in which baryon number violation does take place in the frame of the Rindler observer even for the freely falling bucket. B-violation is of course continuously taking place in short lived processes in the freely falling frame of the bucket. However everyday observations average over much longer time scales and do not resolve these virtual effects. Consider now averaging the baryon number over a finite resolution of the Rindler time $\omega$. As the event horizon is approached a finite time interval in the freely
falling frame becomes indefinitely stretched in Rindler time. A virtual fluctuation, where an X-boson is emitted causing a transition from a down quark to an electron on one side of the horizon and reabsorbed on the other side, appears to the Rindler observer as a violation of baryon number. The recombination is never observed in Rindler space.

Even closer to $R = 0$ we encounter temperatures approaching the Planck scale where the laws of nature are unknown. Therefore we can only deal with this region phenomenologically. From the point of view of a distant Rindler observer it can be replaced by a membrane or stretched horizon with certain properties such as electrical conductance and vanishing reflectivity [8]. Most importantly it has a proper temperature of planckian magnitude. It provides the hot boundary which the rest of the Rindler space is in equilibrium with.

Let us consider matter in a particular quantum state which falls through the Rindler horizon. Rindler observers can describe this process in the following way. The infalling matter approaches the stretched horizon and is absorbed. At this point the Rindler observer may conjecture the existence of Planck scale degrees of freedom on the stretched horizon which become correlated with the initial quantum state of the absorbed matter and store the information among a large number of thermalized degrees of freedom. The quantity of information that can be indefinitely stored is infinite because the area of the stretched Rindler horizon is infinite. To a distant observer this is indistinguishable from information loss.

We now turn to some simple gedanken experiments that the Rindler observer can do to try to discover whether the infalling information is actually stored at the stretched horizon. Our first example involves an observer who approaches and examines the stretched horizon and then returns to report the results (see Figure 3). Any observer who penetrates all the way to the stretched horizon will have to undergo Planck scale acceleration to return. As a result this experiment cannot be analyzed in terms of known physics and therefore it cannot at present be used to rule out the Rindler observer’s hypothesis that information is stored at the stretched horizon.

Next consider an experiment in which a freely falling observer, who passes through the event horizon, attempts to continuously send messages to the outside reporting the lack of substance of the membrane. First suppose his messages are carried by radiation of bounded frequency in the free-falling frame. Because the observer has only a finite proper time before crossing the Rindler event horizon only a finite number of bits of information can be sent.
The last few bits get enormously stretched by the red shift factor and are drowned by the thermal noise. Therefore there is in a sense a last useful bit. If the carrier frequency is less than the Planck frequency the last useful bit will be emitted before the stretched horizon is reached. In order to get a message from behind the stretched horizon the observer must use super-planckian frequencies. Again the experiment cannot be analyzed using conventional physics.

FIGURE 3.
To approach the stretched horizon and return, an observer must undergo Planck scale acceleration.

In both these experiments, efforts made to investigate the physical nature of the stretched horizon are frustrated by our lack of knowledge of Planck scale physics. J. Preskill has speculated that this is a general feature of all such experiments and as a result there may be no well posed information paradox in black hole physics [3].
3. Experiments Outside Black Holes

The event horizon of Rindler space can be viewed as a black hole event horizon in the limit of infinite mass. Any attempt by external observers to determine the physical presence of an information storing membrane outside a black hole event horizon will run into planckian obstacles just as in Rindler space. The discussion of the gedanken experiments of the previous section carries over to this case as the reader can easily check. In this case we define the stretched horizon as the time-like surface where the area of the transverse two-sphere is larger than at the null event horizon by order one in Planck units. With this definition the proper acceleration of a point on the stretched horizon at fixed angular position is approximately one Planck unit. For a static Schwarzhild geometry the proper spatial distance from the stretched horizon to the event horizon is also about one Planck unit.

Two important differences do occur when one considers black holes of finite mass. The first is that the red shift factor between the stretched horizon and infinity is finite and an asymptotic observer sees finite temperature radiation. This leads to the evaporation of the black hole. For large black holes the evaporation is very slow and one can approximate the evolving geometry by a static one for the purpose of discussing messages sent by observers, who come close to or enter the stretched horizon. The second difference, granting that the stretched horizon can store quantum information, is that the storage capacity is finite and given by the Bekenstein entropy [9]. This implies that the quantum correlations of the initial state are returned in the Hawking radiation [2]. Furthermore, a stretched horizon which carries finite entropy cannot hold onto the quantum information for an indefinite length of time. We shall discuss further the concept of a finite information retention time in the following sections.

Now consider a non-static geometry corresponding to the formation of a large black hole by infalling matter. The simplest case to study involves a thin spherical shell of massless matter [6,10]. The geometry is constructed by matching a flat spacetime and a massive Schwarzhild solution across a radial null surface as in Figure 4.

In order to discuss gedanken experiments performed by outside observers it is convenient to define coordinates which cover the exterior of the black hole. Near $I_\pm$ the geometry approaches flat spacetime. We introduce polar coordinates in this region denoted by $(t, r^*, \theta, \varphi)$. 
The combinations $u = t - r^*$, $v = t + r^*$ are null coordinates, and can be continued by extrapolating the light cones to finite locations. $r^*$ and $t$ can then be defined in the entire region outside the event horizon as

$$t = \frac{v + u}{2},$$

$$r^* = \frac{v - u}{2}.$$  \hspace{1cm} (3.1)

The global event horizon shown in Figure 5 is the null surface $u = \infty$ which determines the boundary of the black hole region.

As in the static case a stretched horizon can be defined as a time like surface just outside the event horizon. This can be achieved in a number of ways. The method proposed in [2] is as follows. At a point on the global event horizon construct the radial null ray which does not lie in the horizon. That ray intersects the stretched horizon at a point where the area of the transverse two-sphere has increased by an amount of order a Planck unit relative to its value at the corresponding point on the event horizon. An important new feature of the
collapse geometry as compared to an eternal black hole is that both the event horizon and
the stretched horizon extend into the flat spacetime region inside the infalling matter shell.
The stretched horizon begins (see Figure 5) at the same finite value of $v$ at which the event
horizon initially forms. At this point the area of the stretched horizon is approximately one
Planck unit and it is not useful to extend its definition beyond that.

![Figure 5](image)

**FIGURE 5.**
Location of horizons in a collapse geometry. The dashed line represents the global
event horizon.

An important property of the stretched horizon is that the proper acceleration is about
unity in Planck units everywhere on it, independent of time. This means that the stretched
horizon can again be viewed as a hot boundary with planckian temperature. The red shift
factor between the portion of the stretched horizon below $v = 0$ and infinity can be computed.
The result is that the quanta reaching infinity have energy of order $\frac{1}{M}$ where $M$ is the total
infalling mass. This agrees with other calculations. One might object that an observer in the
flat spacetime region, who is unaware of the incoming shock wave, should not detect radiation
coming from the stretched horizon. This, however, depends on the state of motion of the
observing apparatus. In particular, if the apparatus is at rest with respect to the tortoise
coordinates (3.1) then it is subject to a proper acceleration and will register radiation.

We are interested in testing the hypothesis that, from the outside viewpoint, infalling quantum information is stored on the stretched horizon and subsequently emitted during the evaporation process. More precisely, we would like to show that any such test runs up against our ignorance of Planck scale physics. Gedanken experiments involving messages sent by probes which enter the black hole will be frustrated by the Planck scale red shift factor from the stretched horizon to infinity, in precisely the same way as in the case of a static geometry. If the signal frequency is below the Planck frequency the last useful bit of information will be emitted before the probe reaches the stretched horizon. This is true even for a probe which is sent into the black hole in the flat region of spacetime inside the imploding shell. Similarly, probes which approach the stretched horizon, and then escape to infinity, will necessarily experience planckian acceleration at some point with unforeseeable consequences.

4. Experiments Performed Inside Black Holes

In this section we shall consider a class of experiments which explore quantum correlations between events on both sides of the event horizon. In particular, we are interested in contradictions which arise from the apparent duplication of quantum information. If the Hawking radiation is to faithfully encode the infalling information, while the infalling matter crosses the event horizon without disruption, one might be led to infer such duplication [2]. This would violate the linearity of quantum mechanics. So long as any single observer can never learn the results of experiments performed on both copies of the quantum state we are not led to logical contradiction. It is clear that an observer, who never enters the black hole, can know nothing of experiments performed inside. We must, however, also consider observers who enter the black hole and insist that they do not experience any violation of quantum mechanics. Indeed, for all we know, we ourselves could be living inside the event horizon of a gigantic black hole at this very moment.

Let us first consider a simple experiment, indicated in Figure 6, involving measurements made on both sides of the event horizon of a large black hole of mass $M$. A pair of “spins” \textsuperscript{*} are prepared in a singlet state at point $A$ and then one of them is carried into the black hole.

\textsuperscript{*} By spin we mean an internal label not coupled to a long range gauge or gravitational field.
Two spin measurements are made at points $B$ inside the event horizon and $C$ outside. The results are then transmitted to an observer at $D$ who can establish the correlation between them.† No Planck scale physics enters into the analysis of this experiment but neither does it lead to any paradox since no observation was made on the Hawking radiation.

![Diagram of a gedanken experiment involving correlations on both sides of the event horizon.](image)

**FIGURE 6.**
A gedanken experiment involving correlations on both sides of the event horizon. A pair of correlated “spins” are created at $A$. The spins are measured at $B$ and $C$ and the results are communicated to an observer at $D$.

We turn now to a class of experiments distinguished by the existence of an observer who first performs measurements on outgoing Hawking radiation and then falls through the event horizon. Typically the observer then receives a signal from some system which previously fell through. In this way the observer may collect duplicate information which could potentially lead to contradictions.

As an illustrative example, indicated in Figure 7, consider an experiment in which a pair of particles is prepared in a “spin” singlet. One member $a$ of the pair is sent into a black hole along with an apparatus $A$ which can measure the spin and send out signals. The other

† The single spin pair could be replaced by an ensemble of identically prepared pairs in order to measure statistical quantum correlations.
member \( b \) remains outside. For definiteness we assume that the energy associated with the apparatus is small compared to the black hole mass \( M \) and that it is initially at rest outside the black hole.

**FIGURE 7.**

A gedanken experiment in which an observer \( O \) measures information in Hawking radiation before falling into the black hole. A spin \( a \) has previously crossed the horizon and is measured by apparatus \( A \). A message is sent from \( A \) to \( O \) before \( O \) hits the singularity.

An observer \( O \) who has been hovering outside the hole makes measurements on the Hawking radiation. Assuming that all infalling information is eventually radiated, a measurement can be performed on the radiation which is equivalent to a determination of any component of the original spin. Meanwhile the infalling spin \( a \) has been measured by the apparatus \( A \) which accompanied it. From the point of view of an external observer the “spin in the Hawking radiation” \( h \) must be correlated with the member \( b \) of the original pair which remained outside the black hole. If the spin \( b \) is measured along any axis, then the Hawking spin \( h \) must be found anti-aligned if it too is measured along the same axis. On the other hand the original spin \( a \) which fell through the horizon was also correlated to the other member of the pair \( b \). It would seem that the two separate spins (\( a \) and \( h \)) are correlated with a third (\( b \)) so as to be anti-aligned with it. This violates the principles of
quantum mechanics.

In reference [2] it was argued that no logical contradiction could be derived from this circumstance because no observer could know the results of the measurements of both the spin behind the horizon and the Hawking spin. An observer who measures the Hawking spin can, however, subsequently fall through the horizon carrying a record of his measurements, as was pointed out to us by J. Preskill [3]. If a message can be sent to that observer by the apparatus which measures the infalling spin then he clearly discovers a violation of quantum mechanics. The resolution of this difficulty was suggested by Preskill himself. We shall see that if the observer $O$ falls through after enough time has elapsed for the relevant quantum information to be gathered and if he is to receive the message from $A$ before hitting the singularity, the message must be sent in quanta of energy far beyond the Planck scale.

In the following section we shall discuss the length of time a piece of information remains on the stretched horizon before being emitted. We shall give plausible arguments that this information retention time is typically of order $M^3$. For now, we assume that a distant observer will have to conduct experiments on the outgoing Hawking radiation for a period $\tau \sim M^3$ before information can be recovered. During this time the mass of an evaporating black hole significantly decreases (the total lifetime is also of order $M^3$) but we shall ignore this for the time being and make use of a static Schwarzschild geometry. It will become apparent that the effect of evaporation is to strengthen our conclusion.

Since we wish to discuss observations made inside a black hole it is convenient to use Kruskal coordinates which extend past the event horizon,

\begin{align}
U &= -\exp\left(\frac{r^* - t}{4M}\right), \\
V &= \exp\left(\frac{r^* + t}{4M}\right). 
\end{align}

The Schwarzschild line element becomes

\begin{equation}
\begin{split}
ds^2 &= -\frac{32M^3}{r}\exp\left(\frac{-r}{8M}\right) dUdV.
\end{split}
\end{equation}

The future event horizon is at $U = 0$ and the singularity is at $UV = 1$. The geometry is shown in Figure 7. It is evident that the value of $U$ where the observer $O$ runs into the singularity becomes very small if the observer delays for a long time entering the black hole. This in turn
constrains the time which the apparatus $A$ has available to emit its message. Let us choose the origin of our tortoise time coordinate so that the apparatus passes through the stretched horizon at $V = 1$. The observer $O$ will go through the stretched horizon after a period of order $M^3$ has passed in tortoise time, i.e. at $\log V \sim M^2$. Since the singularity is at $UV = 1$ the message from $A$ must be sent before the apparatus reaches $U \sim \exp (-M^2)$. Near $V = 1$ this corresponds to a very short proper time $\tau \sim M^2 \exp (-M^2)$. The uncertainty principle then dictates that the message must be encoded into radiation with super-planckian frequency $\omega \sim M^{-2} \exp (M^2)$. The back-reaction on the geometry due to such a high-energy pulse would be quite violent. It is apparent that the apparatus $A$ cannot physically communicate the result of its measurement to the observer $O$ in this experiment.

One can imagine a number of gedanken experiments which are variations on this theme. They all lead to the conclusion that attempts to evade black hole complementarity involve unjustified extrapolation far beyond the Planck scale. The evaporation of the black hole modifies the geometry in a manner which only makes the time available for $O$ to receive the message shorter.

5. Information Retention Time

We wish to define a measure of the “lifetime” of information stored in a thermalized system, $K$, such as a black hole stretched horizon or a thermal cavity. Suppose at time $t = 0$, the system $K$ is in some pure state $\psi$ which is a typical member of a thermal ensemble with energy $E$ and temperature $T$. Furthermore, let $N(E)$ be the number of possible states at energy $E$. To begin with, the system has zero fine grained entropy and coarse grained entropy equal to $\log N$.

We assume that the system $K$ radiates thermal radiation for a time $t$. At the end of this time $K$ is no longer in a pure state since it is correlated to the outgoing radiation field. Both the radiation and the system are described by density matrices and from these an entropy of entanglement can be defined [2]. This entropy is the same for both subsystems and is given by

$$S_E = -\text{Tr} \rho_K \log \rho_K .$$  \hspace{1cm} (5.1)

Eventually this entropy will return to zero when all the heat has been radiated and the system $K$ has reached its ground state.
The information contained in the radiation at time $t$ is defined to be

$$i(t) = S_{\text{max}} - S_{E}(t),$$

(5.2)

where $S_{\text{max}}$ is the thermal entropy of the radiation calculated as if it were emitted as conventional black body radiation. It is roughly equal to the number of emitted photons. D. Page’s calculation of the entropy of a subsystem [11] implies that the information $i(t)$ remains extremely close to zero until the number of emitted photons is of order $\frac{1}{2}\log N$. In other words no information is radiated until the thermal entropy of $K$ has decreased to half its original value. After this time the information in the radiation increases linearly with the number of emitted photons. Thus there is a considerable length of time before any information is released. In the case of a black hole of initial mass $M$ it is equal to the time it takes the horizon area to decrease to half its initial value. A simple calculation shows this information retention time to be $\tau_i \sim M^3$.

One may also consider the information retention time for a black hole which is being prevented from evaporating by an incoming energy flux. It seems likely that this would increase the information retention time because it would increase the number of available degrees of freedom among which the information is shared.

An information time of order $M^3$ leaves a comfortable margin in the analysis of the gedanken experiment in the previous section. Turning the argument around, one can use black hole complementarity to obtain a lower bound on the information retention time. As long as $\tau_i \gtrsim M \log M$ the observing apparatus $A$ will have to use at least Planck scale frequency to get a message to the observer $O$, who enters the black hole after making measurements on the Hawking radiation. If $\tau_i$ were to be any shorter, then one could detect a violation of quantum mechanics in such an experiment for a sufficiently massive black hole.
6. Experiments Involving Time Reversal

The time reversal of the formation and evaporation of a black hole has been a subject of controversy. Confusion about white holes and CPT easily arises when one contemplates information loss. This confusion is straightforwardly resolved in a theory with a quantum mechanical stretched horizon. Let us consider a black hole formed from the collapse of a diffuse cloud of elephants. The evaporation products consist of an outgoing train of thermal radiation whose temperature increases with time from $\frac{1}{8\pi M}$ initially to temperatures approaching the Planck scale at the end. The postulate of the validity of quantum mechanics predicts that if this radiation is time-reversed it will evolve back into an expanding diffuse cloud of elephants. We can consider such an experiment to be a test of the postulates of reference [2].

Compare this with the result that would be obtained in the semi-classical approximation in which the time reversed average energy flux is used as a source of gravitation. In this case any initially formed small black hole will itself Hawking radiate and prevent the buildup to a large black hole. Furthermore, even if a large black hole were to form, there would be severe Boltzman suppression of finding even a single elephant in the final state. Thus a semiclassical analysis inevitably leads to information loss.

According to the view expressed in reference [2] the experiment described above is not fundamentally different from the explosion of a bomb among a herd of elephants. Again there is no chance of reconstructing the initial state from any crude approximation to the time reversed final state or its evolution. The conventional understanding of this type of experiment is that only a very tiny subset of configurations will evolve back to the kind of organized state that we started with. Very minute disturbances will cause the time-reversed state to evolve instead into a typical disorganized thermal state.

As with the other gedanken experiments described here, detailed analysis of the above black hole experiment cannot be carried out without the knowledge of Planck scale physics. This is evident from the fact that the first thing to happen in the time reversed process is the collision of Planck energy particles to form a small black hole. Our inability to follow even this early evolution renders any discussion of the remainder futile. While the assumption of a quantum-mechanical stretched horizon does not in itself provide a detailed dynamical evolution of the time reversed process it is unambiguous in predicting that a sufficiently
accurate construction of the time reversed state will lead to elephants emerging from a white hole.

The following extreme example illustrates how misleading semiclassical considerations can be. Consider a very high energy particle in an incoming S-wave in otherwise empty space. The wave function has support on a thin infalling shell. According to the usual semiclassical rules we first compute the expectation value of the energy momentum tensor. In this case it would describe a thin incoming spherical shell. We would then use that as a source in Einstein’s equations and solve for the combined evolution of the shell and the geometry. If the initial energy is well above the Planck scale a black hole will form in this approximation. The next step in a semi-classical calculation is to account for quantum effects, such as Hawking radiation, in the background geometry. When back-reaction is included, the semiclassical method further predicts that the black hole evaporates into thermal radiation. This is almost certainly not what happens. A single particle does not gravitationally attract itself and cannot form a black hole. This means that further corrections to the semi-classical approximation must lead to an entirely different picture of the final state. Once again our inexperience with Planck scale physics precludes detailed systematic improvement of the semi-classical picture. However in this rather trivial example it seems almost certain that the incoming S-wave in fact evolves into an outgoing S-wave state of a single particle.

7. Summary

In this paper we have analysed a number of gedanken experiments to illustrate the consistency of the following two hypotheses:

1) To an outside observer all information falling onto a black hole is stored and thermalized on the stretched horizon until it is radiated during evaporation. The thermalization involves processes which wash out baryon number.

2) To an infalling observer no exceptional phenomenon such as information bleaching or significant baryon number violation will take place upon crossing the horizon.

An observer hovering near the horizon experiences enormous proper acceleration and sees intense radiation emanating from the black hole. This observer can for most purposes replace the black hole by a hot membrane located just outside the event horizon. Our examples suggest that external experiments designed to detect whether quantum information is
stored at the stretched horizon cannot be analyzed using presently known physics. Furthermore experiments performed behind the horizon but far from the singularity cannot detect information duplication. This leads us to question whether Hawking’s information paradox can be well posed at present.

In addition to addressing the information question the stretched horizon concept provides insight into other puzzles. It demands that, to outside observers, baryon number violation appears to take place at a distance of order $M_{\text{GUT}}^{-1}$ from the event horizon and is not hidden behind the horizon.

The issue of time reversability of black hole processes is confusing in the context of information loss. By contrast, if it is admitted that a quantum-mechanical stretched horizon exists, standard statistical reasoning applies to black holes. Time reversed black holes or white holes are possible but no more likely than the random motion in a swimming pool ejecting a diver up to a spring board.

It is our view that black hole complementarity is not derivable from a conventional local quantum field theory. It seems more likely that it requires a radically different kinematical description of physics at very high energy, such as string theory [5]. Our point in this paper is that the revolutionary new elements need only become manifest at extreme energies where our present knowledge is insufficient. We may, however, be able to use black hole complementarity to guess some features of the new kinematics.

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