A graphical solution in CATIA for profiling end mill tool which generates a helical surface

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Abstract. The generation of a helical flute, which belongs to a helical cylindrical surface with constant pitch, can be made using end mill tools. The tools on this type are easiest to make than the side mills and represent a less expensive solution. The end mill profiling may be done using the classical theorems of surfaces enveloping, analytical expressed, as Olivier theorem or Nikolaev method. In this paper is proposed an algorithm, developed in the CATIA design environment, for profiling such tool’s type. The proposed solution is intuitive, rigorous and fast due to the utilization of the graphical design environment capabilities. Numerical examples are considered in order to validate the quality of this method.

1. Introduction
The complex helical surface generation of a helical flute (the case of helical teeth, cutting tool’s flutes, compressor rotors, helical pumps rotors) may be realized with end mill tools.

The tool’s construction is simple and less expensive.

Usually, the end mill axis is perpendicular to the helical surface axis. The helical flute may be generated in a single position of the tool only that its normal section is symmetrical regarding the tool’s axis. This is a limitation of this generating process.

A specific problem of the generation by enwrapping with end mill is the determination of the tool’s profile. There a used the Olivier or Gohman theorems [1-3], in analytical form, if are known the parametrical equations which describe the helical surface.

More, the complementary theorems, as the “minimum distance” method, can be use [4]. Also, it is specific for this problem the Nikolaev theorem [6], as general solution for the issue of generation be enveloping.

Analytical solutions are applicable if the helical surface’s flank is known in analytical form.

The development of the graphical Auto-CAD or CATIA design environment allows approaching these problems using specifically capabilities [5], [8-10]. More, using CNC machines these problems can be addressed using this type of machine-tools [7], [11].

In this paper is proposed an algorithm, based on the specific theorem of the “minimum distance”, developed in CATIA graphical design environment. The algorithm leads to rigorous solutions and very intuitive. The specific issues of tool’s profile discontinuities may be approached using the proposed method.
There are presented numerical applications in order to validate the method.

2. The method of “minimum distance”

The method of “minimum distance” [2, 7] is a complementary method based on a theorem specific to this method. The contact between the cylindrical helical surface with constant pitch and the future revolution surface is examined in planes perpendicular to the axis of the revolution surface.

The specific theorem is: The contact line (characteristic curve) between a cylindrical helical surface with constant pitch and a revolution surface is the geometric locus of the points belong to the helical surface where, in plane perpendicular to the revolution surface’s axis, the distance to the curve which represents the intersection between the helical surface with these planes, is minimum, see figures 1.a and 1.b.

In figure 1.a and 1.b it is presented the case of generating of the helical surface with the end mill tool.

The reference systems are defined as in figure 1.

$XYZ$ is the reference system joined with the helical surface (the $Z_2$ axis overlapped to the $V$ axis of the cylindrical helical surface with constant pitch, $\Sigma$);

$X_2Y_2Z_2$ – reference system associated with the end mill tool (the $X_2$ axis overlapped with the $\bar{A}$ axis of the $S$ surface).

According to the theorem, in a plane perpendicular to the tool’s axis $\bar{A}$, is determined onto the $\Sigma$ surface a curve belong to this $- C_\Sigma$.

\[ P \quad X_2 = H, \quad (H - \text{arbitrary variable}). \quad (1) \]

![Figure 1](attachment:figure1.png)

**Figure 1.** End mill tool: a). position of end mill tool’s axis; characteristic curve; b). the form of peripheral surface of end mill tool.

The equations assembly of the helical surface, in principle in form:

\[
\begin{align*}
X_2 &= X(u)\cos \varphi - Y(u)\sin \varphi; \\
\Sigma Y_2 &= X(u)\sin \varphi + Y(u)\cos \varphi; \\
Z_2 &= p \cdot \varphi,
\end{align*}
\]

(2)

which, together with the plane $P$ (see (1)) determine the curve $C_\Sigma$, by eliminating the $\varphi$ parameter, with equations on form:
\[ X_2 = H; \]
\[ C_{\Sigma} \]
\[ Y_2 = Y_2(u_H); \]
\[ Z_2 = Z_2(u_H). \]

The distance from the \( C_{\Sigma} \) current point to the \( X_2 \) axis (\( A \)) is:

\[ d = \sqrt{Y_2^2(u) + Z_2^2(u)}. \]  (4)

The condition that this distance be minimal is given by the equation:

\[ X_2(u) \cdot X_{2u} + Y_2(u) \cdot Y_{2u} = 0. \]  (5)

The assembly of equations (1) and (5) determines, for each value of the \( H \) parameter a value of the \( d \) distance, (4), representing the minimum distance to the \( \overline{A} \) axis (\( X \)) – the axis of the end mill. The axial section of the end mill is:

\[ H = X_2(u_H); R = (d_{\min})_H. \]  (6)

where \( u_H \) is the value of the \( u \) parameter from the \( C_{\Sigma} \) curve depending to the \( H \) arbitrary variable.

3. The end mill tool for generating the rotor with three lobs in the construction of the Roots compressors

3.1. The frontal profiles of the three-lobs rotor

It is presented, in figure 2, the crossing section of the three-lobs rotor from the construction of the Roots compressor [16].

The roots compressor is composed by two three-lobs rotors. The geometry of these rotors is presented in figure 2 and includes: circle’s arcs, with radius \( r \), on the zones \( AC \) and \( BD \) and an epicycloids’ arc on the \( CB \) portion.

The reference systems are defined as follows:

- \( x_0/y_0/z_0 \) is the reference system joined with the \( C_1 \) centrode;
- \( x_1/y_1/z_1 \) — reference system joined with the \( C_2 \) centrode;
- \( XYZ \) — reference system joined with the helical flank (the \( X \) axis is the symmetry axis of the gap);
- \( X_2/Y_2/Z_2 \) — reference system joined with the rotor.

- The circle arc \( AC \):

\[ x_1 = R_r + r \cos \theta_1; \quad y_1 = r \sin \theta_1. \]  (7)

They are defined \( \theta_{\min} = 0 \) and \( \theta_{\max} \) from intersecting condition between the circle (7) and the circle with radius \( R_r \),

\[ (x_1 - A_{12})^2 + y_1^2 = R_r^2; \quad A_{12} = 2R_r; \]  (8)

\[ \theta_{\max} = \arccos \frac{r}{2R_r}. \]  (9)
- The epicycloids are $CB$ is described in the relative motion of the two centrodes, $C_1$ and $C_2$, with radius $R_r$.

The movement of the $x_2y_2$ reference system regarding the $x_1y_1$ reference system is described by the coordinates transformation

$$x_1 = \omega_3 (\phi_1) \left[ \omega_3^T (-\phi_2) \right] \cdot x_2 - \left( A_{12} \right),$$

(10)

where: $\omega_3(\phi_1)$ and $\omega_3(\phi_2)$ are the rotation matrix around the $O_1$ and $O_2$ origins; $A_{12}$ – is the distance between the rotation axes.

$$A_{12} = 2R_r.$$

(11)

The $x_2$ matrix is given by

$$x_2 = \begin{bmatrix} x_{2c_2} \\ y_{2c_2} \end{bmatrix},$$

(12)

where $x_{2c_2}$, $y_{2c_2}$, are the coordinates of the $c_2$ point (in figure initially overlapped with $C$) onto the $C_2$ centrod.

The coordinates of the $c_2$ point is determined from the equations assembly:

$$c_2 \begin{cases} x_2^2 + y_2^2 = R_r^2, \quad C_2 \text{ centrod}; \\ (x_2 - R_r)^2 + y_2^2 = r^2, \quad \text{circle with radius } r. \end{cases}$$

(13)

From (13) result:

$$x_{2c_2} = \frac{2R_r^2 - r^2}{2R_r}; \quad y_{2c_2} = \sqrt{R_r^2 - \left[ \frac{2R_r^2 - r^2}{2R_r} \right]^2}.$$  

(14)

The relative motion (10) and the rolling condition of the $C_1$ and $C_2$ centrodes,

$$R_r \phi_1 = R_r \phi_2,$$

(15)

determine the epicycloids $CB$ in the $x_1y_1$ reference system:
\[ CB \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \cos \Phi & - \sin \Phi & 0 \\ \sin \Phi & \cos \Phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{1_{AC}} \\ y_{1_{AC}} \\ 0 \end{bmatrix} + p \cdot \Phi \cdot \vec{k} \]  

(21)

From condition:

\[ x_1^2 + y_1^2 = R_r^2, \]  

(17)

results

\[ x_{1_{AC}}^2 + y_{1_{AC}}^2 + A_{12}^2 + 2x_{1_{AC}} \cdot y_{1_{AC}} \sin(2\phi_1 - 2\phi_2) - 2x_{1_{AC}} A_{12} \cos \phi_{1_{max}} + 2y_{1_{AC}} A_{12} \sin \phi_{1_{max}} = R_r^2. \]  

(18)

- The BD circle arc:

\[ x_1 = R_r \cos \left( \frac{\pi}{3} - r \cos \left[ \frac{2\pi}{3} - \theta_2 \right] \right); \quad y_1 = R_r \sin \left( \frac{\pi}{3} - r \sin \left[ \frac{2\pi}{3} - \theta_2 \right] \right); \]  

(19)

\[ \theta_{z_{max}} = 0; \quad \theta_{z_{max}} = \arccos \left[ \frac{r}{2R_r} \right]. \]  

(20)

3.2. The helical flank’s surfaces of the three-lobed rotor
The two rotors are cylindrical helical surfaces with constant pitch, described by transformations such as

\[ \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \cos \Psi & - \sin \Psi & 0 \\ \sin \Psi & \cos \Psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{1_{AC}} \\ y_{1_{AC}} \\ 0 \end{bmatrix} + p \cdot \Psi \cdot \vec{k} \]  

(22)

for clockwise worm with \( p \) helical parameter.

The flank with the generatrix AC, see (7):

\[ x_1 = \left[ R_r + r \cos \theta_1 \right] \cos \Psi - r \sin \theta_1 \sin \Psi; \quad y_1 = \left[ R_r + r \cos \theta_1 \right] \sin \Psi + r \sin \theta_1 \cos \Psi; \quad z_1 = p \Psi, \]  

(23)

with \( \Psi \) – angular parameter of rotation around the \( z_1 \) axis.

The helical flank with the generatrix an epicycloids arc CB, see (16):

\[ \begin{align*}
  x_1 &= x_{1_{AC}} \cos(2\phi_1) + y_{1_{AC}} \sin(2\phi_1) - A_{12} \cos \phi_1 \cos \Psi - \ldots \\
  y_1 &= x_{1_{AC}} \sin(2\phi_1) + y_{1_{AC}} \cos(2\phi_1) + A_{12} \sin \phi_1 \sin \Psi; \\
  z_1 &= -x_{1_{AC}} \sin(2\phi_1) + y_{1_{AC}} \cos(2\phi_1) + A_{12} \sin \phi_1 \cos \Psi \sin \Psi + \ldots \\
  z_1 &= p \Psi.
\end{align*} \]  

(24)
The helical flank with $BD$ generatrix:

\[
x_i = \left[ R, \cos\left(\frac{\pi}{3}\right) - r \cos\left(\frac{2\pi}{3} - \theta_2\right) \right] \cos \Psi' - \left[ R, \sin\left(\frac{\pi}{3}\right) - r \sin\left(\frac{2\pi}{3} - \theta_2\right) \right] \sin \Psi';
\]

\[
y_i = \left[ R, \cos\left(\frac{\pi}{3}\right) - r \cos\left(\frac{2\pi}{3} - \theta_2\right) \right] \sin \Psi' + \left[ R, \sin\left(\frac{\pi}{3}\right) - r \sin\left(\frac{2\pi}{3} - \theta_2\right) \right] \cos \Psi';
\]

\[
z_i = p \Psi'.
\]

### 3.3. The Nikolaev theorem

For a validation criterion of the graphical method, an analytical method is accepted as profiling method of the side mill or end mill, based on a fundamental method — the method of helical motion decomposition, known as the Nikolaev theorem [11], figure 3.a.

The reference systems are defined as follows:

- $XYZ$ is the reference system associated with the helical flanks of the three-lobed rotor; the $Z$ axis overlapped to the $\bar{V}$ axis of the helical surface;
- $X_2Y_2Z_2$ — associated with the side mill, the $\bar{A}$ axis, revolution axis perpendicular to the helix with the maximum diameter of the composed surface.

![Figure 3.](image)

**Figure 3.** a). The position of the side mill relative to the reference systems associated with the helical surface of the rotor’s lobe for the compressor. 

b). The unfold of the helix with the radius $R_e$ and $P_E$ helical pitch.

The unfold line of the helix from the cylinder with the radius $R_e$ is presented in figure 3.b.

\[
tg \alpha = \frac{2\pi R_e}{P_E} ; \quad R_e = R_e + r ,
\]

or, with notation

\[
P_E = 2\pi \cdot p ;
\]

with $p$ — helical parameter,
\[
\alpha = \arctg \left( \frac{R}{p} \right).
\]  

(28)

The Nikolaev theorem assumes that the vectors \( \overrightarrow{N}_\Sigma \), \( \overrightarrow{A} \) and \( \overrightarrow{r}_1 \) are on the same plane,

\[
\left( \overrightarrow{N}_\Sigma, \overrightarrow{A}, \overrightarrow{r}_1 \right) = 0,
\]  

(29)

where: \( \overrightarrow{N}_\Sigma \) is the normal to the \( \Sigma \) composed surface of the flute of three-lobed rotor;
\( \overrightarrow{A} \) - axis versor of the future side mill tool;
\( \overrightarrow{r}_1 = -\overrightarrow{d} + \overrightarrow{r} \) - position vector of the side mill, regarding the \( O_2 \) origin of the \( X_2Y_2Z_2 \) reference system joined with the tool’s axis.

In order to simplify the further expression of the side mill’s profile, it is proposed that the \( X \) axis be the symmetry axis for the gap of the helical flute, figure 4, see conditions (23), (24), (25); see also figure 2 – the crossing section of the Roots compressor.

**Figure 4.** The position of frontal profile’s arcs of rotor regarding the reference system \( XYZ \).

The reporting of the rotor’s frontal profile, and also of the rotor’s helical surfaces, at the \( XYZ \) reference system (with \( X \) axis as symmetry axis of the helical flute) assume the coordinates transformation, see figure 4:

\[
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix} = \begin{pmatrix}
\cos \left( \frac{\pi}{3} \right) & -\sin \left( \frac{\pi}{3} \right) & 0 \\
\sin \left( \frac{\pi}{3} \right) & \cos \left( \frac{\pi}{3} \right) & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
x_1 \\
y_1 \\
z_1
\end{pmatrix}.
\]  

(30)

In this way, the helical flank’s equations, in the new reference system, see (23)-(25), become:
- for the helical flank with generatrix \( AC \),
\[
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix} = \begin{pmatrix}
\cos\left(\frac{\pi}{3}\right) & -\sin\left(\frac{\pi}{3}\right) & 0 \\
\sin\left(\frac{\pi}{3}\right) & \cos\left(\frac{\pi}{3}\right) & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
\left(R_{r} + r \cos \theta_{1}\right) \cos \Psi - r \sin \theta_{1} \sin \Psi \\
\left(R_{r} + r \cos \theta_{1}\right) \sin \Psi + r \sin \theta_{1} \cos \Psi \\
p^{\Psi}
\end{pmatrix},
\]

which, after developments, leads to the following forms:

\[
\begin{align*}
X &= \left[\left(R_{r} + r \cos \theta_{1}\right) \cos \Psi - r \sin \theta_{1} \sin \Psi\right] \cos\left(\frac{\pi}{3}\right) - ... \\
Y &= \left[\left(R_{r} + r \cos \theta_{1}\right) \sin \Psi + r \sin \theta_{1} \cos \Psi\right] \sin\left(\frac{\pi}{3}\right); \\
\Sigma_{AC} &= \left[\left(R_{r} + r \cos \theta_{1}\right) \sin \Psi + r \sin \theta_{1} \cos \Psi\right] \cos\left(\frac{\pi}{3}\right); \\
Z &= p^{\Psi};
\end{align*}
\]

- for the helical flank generated by the epicycloids arc \(CB\) (see equations (24)) from movement

\[
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix} = \begin{pmatrix}
\cos\left(\frac{\pi}{3}\right) & -\sin\left(\frac{\pi}{3}\right) & 0 \\
\sin\left(\frac{\pi}{3}\right) & \cos\left(\frac{\pi}{3}\right) & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
x_{i}\left(\phi_{1}\right) \\
y_{i}\left(\phi_{1}\right) \\
p^{\Psi}
\end{pmatrix},
\]

with \(x_{i}(\phi_{1}), y_{i}(\phi_{1})\) given by (24), results:

\[
\begin{align*}
X &= x_{i}\left(\phi_{1}\right) \cos\left(\frac{\pi}{3}\right) - y_{i}\left(\phi_{1}\right) \sin\left(\frac{\pi}{3}\right); \\
\Sigma_{CB} &= \left[x_{i}\left(\phi_{1}\right) \sin\left(\frac{\pi}{3}\right) + y_{i}\left(\phi_{1}\right) \cos\left(\frac{\pi}{3}\right); \\
Z &= p^{\Psi};
\end{align*}
\]

- for the helical flank with generatrix \(BD\) :
\[
X = x_1(\theta_2) \cos \left( \frac{\pi}{3} \right) - y_1(\theta_2) \sin \left( \frac{\pi}{3} \right);
\]
\[
Y = x_1(\theta_2) \sin \left( \frac{\pi}{3} \right) + y_1(\theta_2) \cos \left( \frac{\pi}{3} \right);
\]
\[
Z = p \Psi,
\]
with \(x_1(\theta_2), y_1(\theta_2)\) given by (25).

The Nikolaev theorem (29) assumes to know the directrix parameters of normals to the helical surfaces (32)-(34). In this way, from (32) results:

\[
\overrightarrow{N}_{x'_{\varphi}} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \hat{X}_{\varphi} & \hat{Y}_{\varphi} & 0 \\ \hat{X}_{\varphi} & \hat{Y}_{\varphi} & p \end{bmatrix},
\]
where:

\[
\hat{X}_{\varphi} = \left( R_r + r \cos \theta_1 \right) \left( \cos \Psi - \sin \theta_1 \cos \Psi \right) \cos \left( \frac{\pi}{3} \right) - \ldots
\]
\[
\left( R_r + r \cos \theta_1 \right) \cos \Psi + r \sin \theta_1 \left( \sin \Psi \right) \sin \left( \frac{\pi}{3} \right);
\]
\[
\hat{Y}_{\varphi} = \left( R_r + r \cos \theta_1 \right) \left( -\sin \Psi \right) - \sin \theta_1 \cos \Psi \sin \left( \frac{\pi}{3} \right) + \ldots
\]
\[
\left( R_r + r \cos \theta_1 \right) \cos \Psi + r \sin \theta_1 \left( \sin \Psi \right) \cos \left( \frac{\pi}{3} \right);
\]
\[
\hat{Z}_{\varphi} = 0;
\]

\[
\dot{X}_{\varphi} = \left( R_r + r \cos \theta_1 \right) \left( \cos \Psi - \sin \theta_1 \cos \Psi \right) \cos \left( \frac{\pi}{3} \right) - \ldots
\]
\[
\left( R_r + r \cos \theta_1 \right) \cos \Psi + r \sin \theta_1 \left( \sin \Psi \right) \sin \left( \frac{\pi}{3} \right);
\]
\[
\dot{Y}_{\varphi} = \left( R_r + r \cos \theta_1 \right) \left( -\sin \Psi \right) - \sin \theta_1 \cos \Psi \sin \left( \frac{\pi}{3} \right) + \ldots
\]
\[
\left( R_r + r \cos \theta_1 \right) \cos \Psi + r \sin \theta_1 \left( \sin \Psi \right) \cos \left( \frac{\pi}{3} \right);
\]
\[
\dot{Z}_{\varphi} = p.
\]

Similarly, for the helical surface (24) and (34), the flank with the generatrix epicycloids arc \(CB\):

\[
\overrightarrow{N}_{x'_{\varphi}} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \hat{X}_{\varphi} & \hat{Y}_{\varphi} & \hat{Z}_{\varphi} \end{bmatrix};
\]

\[
\hat{X}_{\varphi} = \hat{x}_{i_{\varphi}} \cos \left( \frac{\pi}{3} \right) - \hat{y}_{i_{\varphi}} \sin \left( \frac{\pi}{3} \right); \quad \hat{Y}_{\varphi} = \hat{x}_{i_{\varphi}} \sin \left( \frac{\pi}{3} \right) + \hat{y}_{i_{\varphi}} \cos \left( \frac{\pi}{3} \right); \quad \hat{Z}_{\varphi} = 0.
\]

From (24) the partial derivatives result:
\[ \dot{x}_{i1} = \begin{bmatrix} -2x_{c2} \sin(2\phi_i) + 2y_{c2} \cos(2\phi_i) + A_{12} \sin \phi_i \end{bmatrix} \cos \Psi + ... \]
\[ \dot{y}_{i1} = \begin{bmatrix} -2x_{c2} \cos(2\phi_i) - 2y_{c2} \sin(2\phi_i) + A_{12} \cos \phi_i \end{bmatrix} \sin \Psi; \]
\[ \dot{z}_{i1} = 0; \]
\[ \ddot{x}_i = -\begin{bmatrix} x_{c2} \cos(2\phi_i) + y_{c2} \sin(2\phi_i) - A_{12} \cos \phi_i \end{bmatrix} \sin \Psi - ... \]
\[ \ddot{y}_i = \begin{bmatrix} x_{c2} \cos(2\phi_i) + y_{c2} \cos(2\phi_i) + A_{12} \sin \phi_i \end{bmatrix} \cos \Psi; \]
\[ \ddot{z}_i = p. \]

The normal to the flank with generatrix \( BD \) circle arc, from (25) and (35) is given by
\[ \ddot{N}_{z_{av}} = \begin{bmatrix} \ddot{i} & \ddot{j} & \ddot{k} \\ \ddot{X}_{\theta_i} & \ddot{Y}_{\theta_i} & \ddot{Z}_{\theta_i} \\ \ddot{X}_\psi & \ddot{Y}_\psi & \ddot{Z}_\psi \end{bmatrix}, \] where:
\[ \ddot{X}_{\theta_i} = \ddot{x}_1 (\theta_2) \cos \left( \frac{\pi}{3} \right) \ddot{y}_1 (\theta_2) \sin \left( \frac{\pi}{3} \right); \quad \ddot{Y}_{\theta_i} = \ddot{x}_1 (\theta_2) \sin \left( \frac{\pi}{3} \right) \ddot{y}_1 (\theta_2) \cos \left( \frac{\pi}{3} \right); \quad \ddot{Z}_{\theta_i} = 0. \]
\[ \ddot{x}_i = -r \sin \left( \frac{2\pi}{3} - \theta_2 \right) \cos \Psi + r \cos \left( \frac{2\pi}{3} - \theta_2 \right) \sin \Psi; \]
\[ \ddot{y}_i = -r \sin \left( \frac{2\pi}{3} - \theta_2 \right) \sin \Psi + r \cos \left( \frac{2\pi}{3} - \theta_2 \right) \cos \Psi; \]
\[ \ddot{z}_i = 0; \]
\[ X_\psi = -\begin{bmatrix} R_x \cos \left( \frac{\pi}{3} \right) - r \cos \left( \frac{2\pi}{3} - \theta_2 \right) \end{bmatrix} \sin \Psi - \begin{bmatrix} R_x \sin \left( \frac{\pi}{3} \right) - r \sin \left( \frac{2\pi}{3} - \theta_2 \right) \end{bmatrix} \cos \Psi; \]
\[ Y_\psi = -\begin{bmatrix} R_x \cos \left( \frac{\pi}{3} \right) - r \cos \left( \frac{2\pi}{3} - \theta_2 \right) \end{bmatrix} \cos \Psi - \begin{bmatrix} R_x \sin \left( \frac{\pi}{3} \right) - r \sin \left( \frac{2\pi}{3} - \theta_2 \right) \end{bmatrix} \sin \Psi; \]
\[ Z_\psi = p. \]

The directional cosines of \( \overrightarrow{A} \) axis – the axis of further side mill, for a helical surface — clockwise worm, are:
\[ \vec{A} = -\sin \alpha \vec{j} + \cos \alpha \vec{k}, \]  
(47)

see relations (28) and figure 3.

The position vector \( \vec{r}_1 \) has components

\[ \vec{r}_1 = X\vec{i} + Y\vec{j} + Z\vec{k} - a\vec{i}, \]  
(48)

with \( X, Y, Z \) from (32), (34), (35).

In this way, the enwrapping condition is defined in the \( XYZ \) reference system, joined with the axis of the helical rotor.

In principle, the condition (29) determined for each helical surface from the construction of the helical flute, see (32), (34), (35), is a function dependent on two variable parameters:

\[ Q_1 (\theta_1, \Psi) = 0, \]  
(49)

for the surface with generatrix \( AC \);

\[ Q_2 (\phi_1, \Psi) = 0, \]  
(50)

for the surface with the generatrix \( CB \);

\[ Q_3 (\theta_2, \Psi) = 0, \]  
(51)

for the surface with generatrix \( BD \).

3.4. Characteristic curves

The equations assemblies (32) and (49); (34) and (50); (35) and (51) determine specific segments of the characteristic curves, the contact curves between the helical surface and the composed primary peripheral surface of the further side mill, in principle, under the form:

\[
\begin{align*}
C_{\xi_{AC}}: & \quad X = X (\theta_1); \\
C_{\zeta_{AC}}: & \quad Y = Y (\theta_1); \\
C_{\Xi_{BD}}: & \quad Z = Z (\theta_1);
\end{align*}
\]

(52)

By rotating the segments of characteristic curve around the \( \vec{A} \) axis — the axis of side mill — it is generated the composed primary peripheral surface which constitutes the primary peripheral surface of end mill: \( S_{dB}, S_{CB} \) and \( S_{BD} \).

In order to simplify the drawing of the characteristic curve’s segments, it is useful to represent the characteristic curves in the reference system joined with \( \vec{A} \) axis — the axis of the side mill (see figure 3) by a coordinates’ transformation under the form:

\[
\begin{align*}
X_2 = X - a; & \quad Y_2 = Y \cos \alpha + Z \sin \alpha; \\
Z_2 = \cos \alpha; & \quad Z_2 = -Y \sin \alpha + Z \cos \alpha.
\end{align*}
\]

(53)

In this way, the equations (32), (34), (35), with the transformation (53), will be defined in the \( X_2Y_2Z_2 \) reference system, joined with the \( \vec{A} \) axis — the axis of the further side mill tool.

Similarly, the characteristic curves (52), by means of the transformation of (53), will be reported to the \( X_2Y_2Z_2 \) reference system of the further side mill, in principle, under the form
The axial section of the revolution surface \( S_{AC} \), is determined from the equations under the form:

\[
\begin{align*}
S_{AC} & \quad \begin{cases} 
  X_2 = X_2(\theta_i); \\
  Y_3 = Y_3(\theta_i); \\
  Z_2 = Z_2(\theta_i). 
\end{cases} 
\end{align*}
\]

Similarly, (see (52)), for \( S_{A_{BC}} \) and \( S_{A_{BD}} \)— see figure 5, where is highlighted the singular point \( B \).

\[
\begin{align*}
H &= Z_2(\theta_i); \\
R &= \sqrt{X_2^2(\theta_i) + Y_3^2(\theta_i)}. 
\end{align*}
\]

4. The graphical method in CATIA, for profiling the end mill tool for generating the compressor worm

In figure 6 the axis position for the end mill tool’s surfaces, \( \overline{A} \), is presented, regarding the composed helical surface axis, \( \overline{V} \).

The issue of tools profiling, for this current case — the flank of Roots compressor rotor, can be solved according to the previously presented methodology, see section 2.

In figure 6, there are presented the plane sections \( X_2 = H \) together with planes perpendicular on the \( \overline{A} \) axis (command “INTERSECTION”) which determine on the helical surface with generatrix \( CB \) and \( BD \) (see figure 5) the \( \Sigma_{H_{BD}} \) and \( \Sigma_{H_{KC}} \) curves. The normals drawn from the \( \overline{A} \) axis to the planes \( X_2 = H \), from these curves determine the \( d_{\text{min}} \) minimal distances. The foot point of these normals, with coordinates \( X_2Y_3 \) represents, in planes \( X_2 = H \) (\( H \) — arbitrary), points from the characteristic curve \( C_{Z_{BC}}, C_{Z_{KC}} \).

In figure 7, it is presented the 3D model of the end mill tool’s primary peripheral surface, \( S_{BC}, S_{BD} \), and the position regarding the composed helical surface \( \Sigma \); \( \Sigma_{BC} \) and \( \Sigma_{DB} \).
Figure 6. End mill tool; relative position of $\vec{A}$ and $\vec{V}$ axes; $\Sigma_H$ curves; minimum distances in planes $X_2=H$.

The composed characteristic curve is presented in table 1.

|                  | $X_2$ [mm] | $Y_2$ [mm] | $Z_2$ [mm] | $X_2$ [mm] | $Y_2$ [mm] | $Z_2$ [mm] |
|------------------|------------|------------|------------|------------|------------|------------|
| $DB''$           | 9.319      | 0.000      | 0.000      | 17.435     | 8.319      | -3.743     |
|                  | 9.442      | 1.366      | -0.515     | 17.950     | 8.384      | -4.329     |
|                  | 9.808      | 2.692      | -1.018     | 18.500     | 8.590      | -4.863     |
|                  | 10.403     | 3.942      | -1.496     | 19.022     | 8.828      | -5.412     |
|                  | 11.206     | 5.084      | -1.940     | 19.515     | 9.076      | -5.983     |
|                  | 12.191     | 6.091      | -2.340     | 19.983     | 9.322      | -6.576     |
|                  | 13.330     | 6.944      | -2.688     | 20.427     | 9.562      | -7.189     |
|                  | 14.593     | 7.628      | -2.979     | 20.852     | 9.792      | -7.819     |
|                  | 15.949     | 8.134      | -3.206     | 21.260     | 10.010     | -8.465     |
|                  | 17.369     | 8.456      | -3.367     | 21.653     | 10.215     | -9.124     |

The axial section can be determined now, see figure 8 and table 2.

Figure 7. Helical surfaces $\Sigma_{BD}$, $\Sigma_{BC}$ and primary peripheral surface of tool, $S_{BC}$ and $S_{BD}$.

Table 1. The coordinates of characteristic curve – end mill tool – profile zone $D$-$B$-$C$.

Figure 8. Axial section of $CBD$ zone.
Table 2. Axial section.

| X₂ = H [mm] | R [mm] | X₂ = H [mm] | R [mm] |
|-------------|--------|-------------|--------|
| 9.319       | 0.000  | 17.435      | 9.123  |
| 9.442       | 1.460  | 18.027      | 9.491  |
| 9.808       | 2.878  | 18.568      | 9.930  |
| 10.403      | 4.217  | 19.076      | 10.408 |
| 11.206      | 5.441  | 19.555      | 10.915 |
| 12.191      | 6.525  | 20.011      | 11.443 |
| 13.330      | 7.446  | 20.446      | 11.988 |
| 14.593      | 8.189  | 20.864      | 12.547 |
| 15.949      | 8.743  | 21.265      | 13.117 |
| 17.369      | 9.102  | 21.653      | 13.697 |

It is obvious the discontinuity onto the tool’s profile. This is due to the presence of singular point, B, onto the rotor’s crossing section.

Table 3. The coordinates of characteristic curve – end mill tool – profile’s zone AC.

| X [mm] | Y [mm] | Z [mm] |
|--------|--------|--------|
| 3.065  | 29.158 | 20.000 |
| 4.503  | 28.946 | 19.020 |
| 6.044  | 28.555 | 18.281 |
| 7.603  | 27.928 | 17.785 |
| 9.083  | 27.040 | 17.476 |
| 10.406 | 25.908 | 17.272 |
| 11.524 | 24.568 | 17.103 |
| 12.411 | 23.067 | 16.910 |
| 13.060 | 21.458 | 16.652 |
| 13.472 | 19.792 | 16.293 |

5. Conclusions
The presented algorithm is based on the complementary theorem of the “minimum distance” and uses the capabilities of graphical modelling provided by CATIA design environment.

The algorithm allows determining the contact points between the helical surface and the revolution surface. These points are points onto the characteristic curve.

The solid model of the end mill tool can be generated and, starting from this, the axial section of this tool.

The numerical example proves the method quality.

The algorithm is simple, intuitive and leads to very rigorous results.

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