Heavy hybrid stars from multi-quark interactions

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January 22, 2014

Abstract

We explore the possibility of obtaining heavy hybrid stars within the framework of the two flavor Nambu–Jona-Lasinio model with 8-quark interactions in the scalar and in the vector channel. The main impact of the 8-quark scalar channel is to reduce the onset of quark matter, while the 8-quark vector channel acts to stiff the equation of state at high densities. Within the parameter space where the 4-quark vector channel is small, and the 8-quark vector channel sizeable, stable stars with $2M_\odot$ and above are found to hold quark matter in their cores.

PACS: 26.60.Kp, 12.38.Lg, 97.60.Jd, 25.75.Ag, 12.39.Ki, 11.30.Rd
1 Introduction

One of the most severe constraints of the equation of state (EoS) of QCD at extreme densities comes from the recent $2M_\odot$ mass determinations of PSR J1614-2230 \cite{1} and PSR J0348-0432 \cite{2} having important consequences for the existence of quark matter in compact stars \cite{3, 4, 5, 6}.

In order to obtain heavy hybrids stars, quark matter should be stiff, with a low onset \cite{7}. This can be achieved in the MIT bag model by perturbative corrections to the EoS and small bag pressures \cite{8}, while within the Nambu–Jona-Lasinio (NJL) model large vector channel is used for stiffness, while shift in the vacuum energy \cite{9, 10} or introduction of superconductivity ensures low onset \cite{11}. A significant influence on the maximum mass is expected by the nature of the phase transition itself \cite{12, 13, 14}.

We propose an alternative scenario by taking into account the fact that NJL is a non-renormalizable effective model, valid up to some scale $\Lambda$. As we approach this scale, say by increasing the chemical potential, higher dimensional operators should become important. It has been pointed out \cite{15, 16} that including higher scalar interactions can reduce the critical temperature in the NJL model, bringing it in closer agreement with lattice results at finite temperature \cite{17}. For further work on higher dimensional operators in the context of the NJL model see \cite{18, 19, 20, 21, 22, 23, 24}.

Our aim is to make an initial study of the effect of multiquark interactions on occurrence of quark matter in $2M_\odot$ stars. In this work we will use the NJL model parametrization of Ref. \cite{17} where the critical temperature at zero chemical potential is fitted to lattice QCD. Additionally, we will also introduce the 8-quark vector channel, as a natural candidate for playing a relevant role at large densities reached in the cores of compact stars.

Our main results are that with scalar 8-quark interactions provided by Ref. \cite{17} we are able to obtain stable hybrid stars even with small vector coupling in the 4-quark and zero vector coupling in the 8-quark channel. Second, we demonstrate that the mass of the star can be increased up to and above $2M_\odot$ with the 8-quark vector interaction, while still keeping the 4-quark vector interaction low.

2 NJL model with 8-quark interactions

We work within the framework of a $N_f = 2$ NJL model defined as follows

$$\mathcal{L} = \bar{q}(i\partial - m)q + \mu_q\bar{q}\gamma^0q + \mathcal{L}_4 + \mathcal{L}_8,$$  \hspace{1cm} \hspace{1cm} (1)
where $\mu_q$ is the quark chemical potential and $m$ is the current mass. The interaction terms are

$$L_4 = \frac{g_{20}}{\Lambda^2}[(\bar{q}q)^2 + (i\gamma_5 \tau q)^2] - \frac{g_{02}}{\Lambda^2}(\bar{q}\gamma_\mu q)^2,$$

$$L_8 = \frac{g_{40}}{\Lambda^8}[(\bar{q}q)^2 + (i\gamma_5 \tau q)^2]^2 - \frac{g_{04}}{\Lambda^8}(\bar{q}\gamma_\mu q)^4 - \frac{g_{22}}{\Lambda^4}(\bar{q}\gamma_\mu q)^2[(\bar{q}q)^2 + (i\gamma_5 \tau q)^2],$$

where $\Lambda$ is the model cutoff. In the mean-field approximation the Lagrangian becomes

$$L_{MF} = \bar{q}(i\partial - M)q + \tilde{\mu}_q \bar{q}\gamma^0 q - U,$$

where

$$M = m + 2\frac{g_{20}}{\Lambda^2} \langle \bar{q}q \rangle + 4\frac{g_{40}}{\Lambda^8} \langle \bar{q}q \rangle^3 - 2\frac{g_{22}}{\Lambda^4} \langle \bar{q}q \rangle \langle q^\dagger q \rangle^2,$$

$$\tilde{\mu}_q = \mu_q - 2\frac{g_{02}}{\Lambda^2} \langle q^\dagger q \rangle - 4\frac{g_{04}}{\Lambda^8} \langle q^\dagger q \rangle^3 - 2\frac{g_{22}}{\Lambda^4} \langle \bar{q}q \rangle^2 \langle q^\dagger q \rangle,$$

and the classical potential

$$U = \frac{g_{20}}{\Lambda^2} \langle \bar{q}q \rangle^2 + 3\frac{g_{40}}{\Lambda^8} \langle \bar{q}q \rangle^4 - 3\frac{g_{22}}{\Lambda^4} \langle \bar{q}q \rangle^2 \langle q^\dagger q \rangle^2 - \frac{g_{02}}{\Lambda^2} \langle q^\dagger q \rangle^2 - \frac{g_{04}}{\Lambda^8} \langle q^\dagger q \rangle^4.$$

Integrating out the quark degrees of freedom, the full thermodynamic potential takes the following form

$$\Omega = U - 2N_f N_c \int \frac{d^3p}{(2\pi)^3} \left\{ E - \log[1 + e^{-\beta(E - \tilde{\mu}_q)}] + \log[1 + e^{-\beta(E + \tilde{\mu}_q)}] \right\},$$

where $E = \sqrt{\mathbf{p}^2 + M^2}$ and $\beta = 1/T$.

The model is solved by minimizing the thermodynamic potential with respect to the mean-fields $X = \langle \bar{q}q \rangle, \langle q^\dagger q \rangle$, i.e.

$$\frac{\partial \Omega}{\partial X} = 0.$$
\( \mu_q > 0 \). If we for the moment fix \( \zeta = g_{40}/g_{20}^3 \) as a measure of the strength of 8-quark scalar channel then

\[
\bar{g}_{20}(\zeta) = \frac{g_{20}}{1 + \frac{1}{4\zeta}W(-4\zeta e^{2-4\zeta})},
\]

where \( W \) is the Lambert \( W \)-function. Note now that as \( \zeta \) is increased \( \bar{g}_{20} \) is decreasing, finally to reach \( g_{20}^c \) as \( \zeta \to \infty \).

The point of this analysis is that in this case already the critical chemical potential for 2nd order phase transition, given in the chiral limit as

\[
\mu_c^\varepsilon = \Lambda \sqrt{1 - \frac{g_{20}}{g_{20}^c}},
\]

drops to zero if we replace \( g_{20} \to \bar{g}_{20} \) and take \( \zeta \to \infty \). In a realistic scenario, i.e. with finite \( g_{40} \), the critical chemical potential will be reduced. This is the mechanism by which the 8-quark scalar interaction reduces the onset.

The vector channel strengths are treated as free parameters, quantified by

\[
\eta_2 = \frac{g_{02}}{g_{20}}, \quad \eta_4 = \frac{g_{04}}{g_{40}}.
\]

We will be interested in a particular region of the vector channel couplings where the \( g_{02} \) coupling is kept small, while \( g_{04} \) is increased. The reason behind our choice is as following. Since heavy hybrid stars require a stiff EoS, a repulsive vector coupling should be present. As the vector coupling renormalizes the chemical potential, it delays the onset of quark matter. This leads to a scenario where the hadronic mantle becomes too large for the pressure in the quark core to be able to hold the star against gravitational collapse. Therefore, the appearance of quark matter in such a scenario usually makes the star unstable. An attractive channel like e.g. superconductivity needs to be invoked in order to lower the onset [11]. We point out that an alternative microscopic picture is possible with multiquark interactions: whereas stiffening of the EoS is provided by the 8-quark vector channel interaction, lowering of the onset is accomplished by introducing a sizeable 8-quark scalar channel interaction and keeping the 4-quark vector channel interaction small.

Our choice of the phenomenologically interesting parameter space will have an effect of stiffening the quark matter at higher densities, while at the same time keeping the transition density low. Due to this restriction we will also put \( g_{22} = 0 \) by hand. Namely, the operator controlled by the size of \( g_{22} \) can be important at moderate \( \mu_q \) only if the vector mean-field is sizeable, which will not be the case for the parameter region of small \( g_{02} \) in which we are interested in. In addition, it will not be important at high \( \mu_q \) since there the scalar mean-field is zero.
500 1000 1500

Figure 1: (Color online) On the left panel we show the EoS of hybrid matter. Thick black curve is the hadronic contribution. Flat region corresponds to Maxwell construction. We fix $\eta_2 = 0.05$. Quark matter curves are denoted as following: the orange, dashed line accounts for $\eta_4 = 0.0$, dash-dotted, green line for $\eta_4 = 1.0$, dash-double-dotted, indigo line for $\eta_4 = 2.0$ and double-dash-dotted, magenta line for $\eta_4 = 5.0$. Right panel shows the speed of sound of quark matter with the same labelling.

3 Equation of state

By evaluating the full thermodynamical potential at the minimum, we obtain the quark matter equation of state as

$$p_q = -\Omega + \Omega_0,$$  \hspace{1cm} (14)

where the constant $\Omega_0$ is introduced to yield zero pressure in the vacuum. The quark number density $n_q$ and energy density $\epsilon_q$ are defined as follows

$$n_q = -\frac{\partial p_q}{\partial \mu_q}, \quad \epsilon_q = -p_q + n_q \mu_q.$$  \hspace{1cm} (15)

Beta equilibrium is taken into account by the weak processes $d \to u + e^- + \bar{\nu}_e$, $\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$, implying $\mu_u = \mu_d + \mu_e$, $\mu_\mu = \mu_e$, where the neutrino chemical potential is set to zero. Finally, the baryon chemical potential, and the baryon density are $\mu_B = 3\mu_q = 2\mu_d + \mu_u$ and $n_B = n_q/3$, respectively.

For the nuclear matter we choose the DD2 EoS\[25\,26\]. The transition from nuclear to quark matter is provided by the traditional Maxwell construction. Therefore, the EoS is obtained by requiring local charge neutrality.
The full pressure in the quark phase takes into account the contribution of electrons and muons
\[ p(\mu_q) = p_u(\mu_u) + p_d(\mu_d) + p_e(\mu_e) + p_\mu(\mu_\mu), \] (16)
where \( p_{e,\mu} \) is the pressure of a free electron (muon) gas. We emphasize that the Maxwell construction is merely the simplest possible choice of constructing the phase transition - the actual nature of the QCD phase transition at small temperature and finite densities is unknown.

The hybrid EoS are given on Fig. 1 for \( \eta_{02} = 0.05 \) and on Fig. 2 for \( \eta_{02} = 0.1 \) and a range of \( \eta_4 \). Owing to 8-quark scalar interactions, and small 4-quark vector interactions, onset of quark matter is rather low: for \( \eta_{02} = 0.05 \) and \( \eta_4 = 0.0 \) it is under \( p \approx 100 \text{ MeV/fm}^3 \). Even a drastic increase of \( \eta_4 \) has barely any influence on the onset: this is only natural, since there is extra suppression due to high dimensionality of the corresponding operator.

The influence of \( \eta_4 \) is best seen by inspecting the speed of sound: while small or almost vanishing vector interactions yield the relativistic value \( c_s^2 \approx 1/3 \), the speed of sound significantly increases with \( \epsilon \) already for \( \eta_4 = 1.0 \), see right panels of Figs. 1 and 2. This gradual stiffening of the EoS can also be seen as a microscopic mechanism of a postulated scenario of a medium-dependent parameter \( \eta_{2} \) [27]. Assuming that the vector mean-field is given by the density of massless fermions, an approximative formula for the speed of sound \( c_s^2 = \partial p/\partial \epsilon \) as a function of the quark chemical potential can be

Figure 2: (Color online) Same as the previous Fig. but for \( \eta_{02} = 0.1 \).
shown to hold

\[
\begin{align*}
\frac{c_s^2}{3} & \simeq \frac{1}{3} + \frac{32 g_{02} \mu_4^2}{6 \pi^2 \Lambda^2} + \frac{512 g_{02}^2 \mu_4^4}{4 \pi^4 \Lambda^4} + \frac{75776 g_{02}^3 \mu_4^6}{24 \pi^6 \Lambda^6} \\
& + \frac{3768320 g_{02}^4 \mu_4^8}{48 \pi^8 \Lambda^8} + \frac{8192 g_{04} \mu_4^8}{48 \pi^8 \Lambda^8}.
\end{align*}
\] (17)

illustrating that the \( g_{04} \) term starts to be important only at higher chemical potentials. From (17) we also see that, in principle, when vector couplings are introduced, the speed of sound is not limited from above. This is the reason why the EoS with \( \eta_4 = 5.0 \) turns acausal already at \( \epsilon \sim 2000 \text{ MeV/fm}^3 \). Therefore, results obtained for this extreme scenario are given for illustrative purposes.

4 Hybrid stars

![Figure 3](image.png)

Figure 3: (Color online) Left panel shows the \( M - R \) and the right panel the \( M - \epsilon_c \) diagram of compact star solutions within the model. We fix \( \eta_2 = 0.05 \). The detachment from the hadronic branch given by the thick, black curve and the maximum mass depends on \( \eta_4 \). Hybrid star branches appear in the same line styles as the corresponding EoS on Fig. 1. Hatched regions mark experimental constraints from two heaviest sources, lower, red is [1], while higher, blue is [2].
The static, spherically symmetric stars are obtained as solutions of the Tolman-Oppenheimer-Volkoff (TOV) equations for the EoS shown in the previous section. On Figs. 3 and 4 we display the resulting mass-radius and mass-central energy density diagrams for $\eta_2 = 0.05$ and $\eta_2 = 0.1$, respectively.

Due to very early onset of quark matter, on Fig. 3 we are able to obtain stable stars with pure quark matter in their cores even for very small vector coupling. Such a scenario is not easy to achieve in the NJL model with only fourth order scalar and vector operators [11, 28], see however [10]. The strong vector interaction increases the speed of sound and make quark matter stiff, but at the same time it appears at too high energy densities. Higher dimensional vector operator stiffens the EoS and at the same time does not influence the onset significantly, giving a mechanism for $2M_\odot$ hybrid stars, see Fig. 3. Alternatively, one can still increase $\eta_2$ a bit, keep $\eta_4$ zero, and still obtain stable configurations within the experimental window, as shown on Fig. 4.

In order to cover the present experimental window provided by PSR J1614-2230 and PSR J0348-0432 we have found that $\eta_4 \approx 1.5 - 2.0$ is sufficient, depending on the strength of $\eta_2$. In the case where $\eta_2 = 0.1$ and $\eta_4 = 2.0$ we reach beyond the current experimental region and predict masses of around $M \approx 2.1M_\odot$. The extreme case with $\eta_4 = 5.0$ leads to a significant increase in the mass, yielding $M \approx 2.35M_\odot$.

### 5 Conclusions

Observing heavy compact stars offers a promising perspective on constraining the cold, dense EoS beyond saturation density. A compelling discrimination of many possible scenarios of dense matter will be possible once precise measurements of also star radii become available [29]. The data of both masses and radii will enable Bayesian inversion of the TOV equation leading to the EoS [30].

We have studied one possible scenario where multi-quark interactions coming from higher dimensional operators in the NJL model might play an important role at large densities. A sizeable 8-quark scalar channel is introduced in order to achieve a low onset. With a small 4-quark vector coupling onset is still low, while the relatively large 8-quark coupling is used to gradually stiffen the EoS at high densities. Within this parameter space we were able to fulfill and go beyond the $2M_\odot$ constraint.

Given the unclear status of the strength of the vector channel coupling [31], and the basically free choice of the parameters $g_{04}$ and $g_{22}$ used in this work, lattice QCD measurements at finite $T$ [32, 33, 34] and at imag-
Figure 4: (Color online) Line styles are the same as in the previous Fig. except $\eta_2 = 0.1$.

Binary chemical potential \cite{35} can be used to better constrain the model. Such a setup may prove to be valuable for investigation of the existence of strangeness in the cores of compact stars, to which we will devote our forthcoming investigations.

Acknowledgments

We acknowledge many illuminating discussions with D. Blaschke and thank D. E. Alvarez-Castillo for help with the TOV code. The author thanks the Yukawa Institute for Theoretical Physics, Kyoto University, where part of this work were performed during the YITP-T-13-05 on “New Frontiers in QCD”. This work is supported by the University in Zagreb under Contract No. 202348 and by the COMPSTAR Network.

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