On the Debye effect in a semiconductor and an electrolyte

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Abstract

It is shown that the standard expression for the diffusive flux in a semiconductor can be transformed to the flux from the Einstein-Smoluchowski equation after making use of the "Einstein identities". Later, it is possible to use a transformation introduced in: S. Piekarski, "On the modified Fick law and its potential applications" (J.Tech.Phys., 44, 2, 125-131, 2003). In this formalism, it is possible to formulate simple models of a semiconductor and an electrolyte and to obtain the alternative description of a Debye effect (by the Debye effect we mean here the screening of the point particle potential). Final results are compared with the standard analysis.

1 Introduction

According to Peierls ([1], p.204, Eq.(10.21)), the electric current $J$ in a semiconductor is given by

$$ J = -\sigma \frac{\partial \phi}{\partial x} - D \frac{\partial n}{\partial x}, $$

where $\sigma$ is the conductivity, $D$ is the diffusion coefficient, $\phi$ is the electric potential and $n$ is the concentration of the electrons. The conductivity $\sigma$ and the diffusion coefficient $D$ are related by the "Einstein identity"

$$ D = \frac{kT \sigma}{e n} = kT u, $$

(2)
where \( u = \sigma / ne \) is independent of the electron density, and is called the "mobility" (compare also [5], p.199).

From the "Einstein identity" (2) one can determine the conductivity \( \sigma \) as a function of the temperature and the diffusion coefficient

\[
\sigma = \frac{e D n}{kT}.
\]  

(3)

After inserting (3) into (1) one arrives at

\[
J = -\sigma \frac{\partial \phi}{\partial x} - D \frac{\partial n}{\partial x} = -\frac{e D n}{kT} \frac{\partial \phi}{\partial x} - \frac{D \partial n}{\partial x} = -D \left[ \frac{\partial n}{\partial x} + \frac{e n}{kT} \frac{\partial \phi}{\partial x} \right].
\]  

(4)

It can be seen that (4) is a flux from the Einstein - Smoluchowski equation (up to minor differences in the conventions for signs and notations) [2,3,8,9]. For the constant temperature field, this flux can be written in the following form [2,3]

\[
J = -D \exp \left[ -\frac{e \phi}{kT} \right] \frac{\partial}{\partial x} \left\{ n \exp \left[ \frac{e \phi}{kT} \right] \right\}.
\]  

(5)

Let us show this explicitly

\[
-D \exp \left[ -\frac{e \phi}{kT} \right] \frac{\partial}{\partial x} \left\{ n \exp \left[ \frac{e \phi}{kT} \right] \right\} =
\]

\[
-D \exp \left[ -\frac{e \phi}{kT} \right] \exp \left[ \frac{e \phi}{kT} \right] \frac{\partial}{\partial x} n - D \exp \left[ -\frac{e \phi}{kT} \right] n \frac{\partial}{\partial x} \exp \left[ \frac{e \phi}{kT} \right] =
\]

\[
-D \frac{\partial}{\partial x} n - D \exp \left[ -\frac{e \phi}{kT} \right] \exp \left[ \frac{e \phi}{kT} \right] n \frac{\partial}{\partial x} \left[ \frac{e \phi}{kT} \right] =
\]

\[
-D \frac{\partial}{\partial x} n - D n e \frac{\partial}{kT} \frac{\partial}{\partial x} \phi = -D \left[ \frac{\partial}{\partial x} n + \frac{n e}{kT} \frac{\partial}{\partial x} \phi \right] = J.
\]  

(6)

It should be stressed again that the temperature \( T \) in (5) and (6) is constant.

On p.204 of [1] Peierls writes that the concentration of electrons is given by the "Boltzmann formula" if the system is in equilibrium in the spatially varying electric potential \( \phi = \phi(x) \). This property can be shown immediately after putting the flux (5) equal to zero:

\[
J = -D \exp \left[ -\frac{e \phi}{kT} \right] \frac{\partial}{\partial x} \left\{ n \exp \left[ \frac{e \phi}{kT} \right] \right\} = 0.
\]  

(7)
Since the diffusion coefficient is assumed to be different from zero, the above condition implies that
\[
\frac{\partial}{\partial x} \left\{ n(x) \exp \left[ \frac{e\phi(x)}{kT} \right] \right\} = 0 \tag{8}
\]
and therefore
\[
n(x) = C \exp \left[ -\frac{e\phi(x)}{kT} \right], \quad C > 0. \tag{9}
\]

It is worth to discuss also the explicit form of the Einstein-Smoluchowski equation for the electrically neutral matter; in [2, 3], the following convention has been used:
\[
\frac{\partial}{\partial t} p(x, t) = \frac{\partial}{\partial x} D'(T, x) \left\{ \frac{\partial}{\partial x} p(x, t) + \frac{p(x, t)}{kT} \frac{\partial}{\partial x} U(x) \right\}, \tag{10}
\]
where \( U(x) \) is a potential for the external force acting on the diffusing molecule, \( p \) is the concentration of the diffusing molecules, and \( D'(T, x) \) is the diffusion coefficient. The equation (10) can be generalized to the equation for diffusion of the electrically charged matter after the replacement
\[
U(x) \to U(x) - q\phi(x, t) \tag{11}
\]
where \( q \) is the electric charge of a diffusing molecule and \( \phi(x, t) \) denotes the electric potential [4]. The flux of diffusing molecules, corresponding to the equation
\[
\frac{\partial}{\partial t} p(x, t) = \frac{\partial}{\partial x} D'(T, x) \left\{ \frac{\partial}{\partial x} p(x, t) + \frac{p(x, t)}{kT} \frac{\partial}{\partial x} [U(x) - q\phi(x, t)] \right\}, \tag{12}
\]
is given by
\[
J' = D'(T, x) \left\{ \frac{\partial}{\partial x} p(x, t) + \frac{p(x, t)}{kT} \frac{\partial}{\partial x} [U(x) - q\phi(x, t)] \right\}. \tag{13}
\]
In turn, the electric flux, corresponding to the flux of molecules (13), is equal to the product of the flux of molecules (13) and the electric charge of a single molecule \( q \):
\[
J_e' = qD'(T, x) \left\{ \frac{\partial}{\partial x} p(x, t) + \frac{p(x, t)}{kT} \frac{\partial}{\partial x} [U(x) - q\phi(x, t)] \right\}. \tag{14}
\]
In particular, if the non-electric potential $U(x)$ is constant

$$U(x) = U_0 = \text{const.} \quad (15)$$

the electric flux (14) takes the form

$$J'_e = qD'(T, x) \left\{ \frac{\partial}{\partial x} p(x, t) + \frac{p(x, t)}{kT} \frac{\partial}{\partial x} [U_0 - q\phi(x, t)] \right\} =$$

$$qD'(T, x) \left\{ \frac{\partial}{\partial x} p(x, t) + \frac{p(x, t)}{kT} \frac{\partial}{\partial x} [-q\phi(x, t)] \right\} =$$

$$qD'(T, x) \frac{\partial}{\partial x} p(x, t) - q^2 D'(T, x) \frac{p(x, t)}{kT} \frac{\partial}{\partial x} \phi(x, t). \quad (16)$$

The electric flux (16) is identical to the electric flux from the Peierls formula under the conditions

$$-\frac{eDn}{kT} \frac{\partial}{\partial x} \phi(x, t) = -\sigma \frac{\partial}{\partial x} \phi(x, t) = -q^2 D'(T, x) \frac{p(x, t)}{kT} \frac{\partial}{\partial x} \phi(x, t) \quad (17)$$

and

$$-D \frac{\partial}{\partial x} n = qD'(T, x) \frac{\partial}{\partial x} p(x, t), \quad (18)$$

(in 17), the “Einstein identity” (3) has been taken into account). It implies that

$$n = p$$

and

$$e = q.$$  

From the relation

$$-D = qD'(T, x) \quad (19)$$

one can see that there is a difference in notational conventions for the diffusion coefficients. In the Peierls’ version, the charge of the carrier is inserted into the definition of the diffusion coefficient while in (14) they are explicitly separated. It seems useful to compare both fluxes with the balance of electric charge, derived from Maxwell equations. In general, the electric flux is a part of Maxwell equations

$$\text{rot} H = j + \frac{\partial}{\partial t} D_e \quad (20)$$

$$\text{rot} E = -\frac{\partial}{\partial t} B \quad (21)$$
\[
div B = 0 \tag{22}
\]
\[
div D_e = \rho_s. \tag{23}
\]

After taking a divergence of (20) and the time derivative of (23) one arrives at the identities
\[
0 = \div j + \frac{\partial}{\partial t} \div D_e \tag{24}
\]
and
\[
\frac{\partial}{\partial t} \div D_e = \frac{\partial}{\partial t} \rho_s. \tag{25}
\]

Comparing of (24) and (25) gives
\[
\frac{\partial}{\partial t} \rho_s + \\div j = 0. \tag{26}
\]

From the form of (26) one can see that the sign convention applied in (14) is opposite to that used in (20) (compare [2, 3]). In the rest of this paper we shall use the notational convention used in (14), but without the prime in the notation for the diffusion constant.

In order to discuss the Debye effect in a semiconductor, let us assume that the electric charge \( \rho_s \) is a sum of the electric charge of the negative carriers \( en(x, t) \) and the electric charge density of the static "background" \( -e\rho_b \). The processes of ionization and recombination are neglected. Since we are interested in the Debye effect, we assume that, besides the constant background charge, there is a point charge \( \eta e \delta(x) \) at \( x = 0 \). Therefore, the final expression for \( \rho_s \) is
\[
\rho_s = en(x, t) - e\rho_b + \eta e \delta(x) \tag{27}
\]
while the explicit form of the diffusive flux in Maxwell equations is
\[
j = -eD(x, T) \exp \left[ -\frac{n\phi}{kT} \right] \frac{\partial}{\partial x} \left\{ n \exp \left[ \frac{n\phi}{kT} \right] \right\}. \tag{28}
\]

Under these assumptions, Maxwell equations become
\[
\rot H = -eD \exp \left[ -\frac{e\phi}{kT} \right] \frac{\partial}{\partial x} \left\{ n \exp \left[ \frac{e\phi}{kT} \right] \right\} + \frac{\partial}{\partial t} D_e, \tag{29}
\]
\[
\rot E = -\frac{\partial}{\partial t} B, \tag{30}
\]

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\[ \text{div} B = 0, \quad (31) \]

\[ \text{div} D_\varepsilon = e n (x, t) - e \rho_b + \eta e \delta (x), \quad (32) \]

and the balance of the electric charge becomes

\[
\frac{\partial}{\partial t} [e n (x, t) - e \rho_b + \eta e \delta (x)] + \text{div} \left\{-e D \exp \left[-\frac{e \phi}{kT}\right] \frac{\partial}{\partial x} \left\{n \exp \left[\frac{e \phi}{kT}\right]\right\}\right\} =
\]

\[
\frac{\partial}{\partial t} e n (x, t) + \text{div} \left\{-e D \exp \left[-\frac{e \phi}{kT}\right] \frac{\partial}{\partial x} \left\{n \exp \left[\frac{e \phi}{kT}\right]\right\}\right\} = 0 \quad (33)
\]

For simplicity, one can assume that

\[ D_\varepsilon = \varepsilon_0 \varepsilon_r E, \quad (34) \]

\[ B = \mu_0 \mu_r H, \quad (35) \]

and

\[ B = 0. \quad (36) \]

In turn, (30) and (36) imply that

\[ \text{rot} E = 0. \quad (37) \]

If

\[ E = -\nabla \phi (x, t) \quad (38) \]

then the remaining equations take the form

\[
0 = -e D \exp \left[-\frac{e \phi}{kT}\right] \frac{\partial}{\partial x} \left\{n \exp \left[\frac{e \phi}{kT}\right]\right\} + \frac{\partial}{\partial t} \varepsilon_0 \varepsilon_r [\nabla \phi], \quad (39)
\]

and

\[
-\varepsilon_0 \varepsilon_r \nabla^2 \phi = e n (x, t) - e \rho_b + \eta e \delta (x). \quad (40)
\]

In order to describe the process of dynamical screening one has to impose the corresponding initial and boundary conditions. Let us assume that the support of the diffusion coefficient (or, strictly speaking, the tensor of the diffusion coefficients) is restricted to the interior of the sphere with a radius
The important class of solutions of (39) and (40) consists of stationary solutions for which the time derivative is put equal to zero

\[
0 = -eD \exp \left( \frac{-e\phi}{kT} \right) \frac{\partial}{\partial x} \left\{ n \exp \left( \frac{e\phi}{kT} \right) \right\}, \quad \frac{\partial \phi}{\partial t} = 0, \quad (41)
\]

\[-\varepsilon_0 \varepsilon_r \nabla^2 \phi = e n (x) - e \rho_b + \eta e \delta (x). \quad (42)\]

The condition (41) can be solved to give

\[n (x) = C \exp \left( \frac{-e\phi (x)}{kT} \right), \quad C > 0 \quad (43)\]

(compare (7) – (9)). In principle, the value of the positive parameter \(C\) can be determined from the initial conditions.

Now, (43) can be inserted into (42):

\[-\varepsilon_0 \varepsilon_r \nabla^2 \phi (x) = eC \exp \left( \frac{-e\phi (x)}{kT} \right) - e \rho_b + \eta e \delta (x). \quad (44)\]

This equation depends explicitly on the three parameters \(C, \rho_b\) and \(\eta\). The fourth parameter is the radius \(R\) of the sphere in which the transport processes take place. In the more detailed analysis one should use the dimensionless variables, taking into account the spatial scales of the system (for example, the radius \(R\)), in the way similar to that used in the Chapman - Enskog expansion, but in our preliminary discussion we simply assume that

\[\exp \left( \frac{-e\phi (x)}{kT} \right) \approx 1 - \frac{e\phi (x)}{kT}. \quad (45)\]

After inserting (45) into (44) one arrives at

\[-\varepsilon_0 \varepsilon_r \nabla^2 \phi (x) = eC \left[ 1 - \frac{e\phi (x)}{kT} \right] - e \rho_b + \eta e \delta (x). \quad (46)\]

It is well-known that the solution of the modified Poisson equation

\[(\nabla^2 - \mu^2) \phi = -4\pi e \delta (r) \quad (47)\]

is of the form

\[\phi (r) = \frac{e}{r} \exp (-\mu r). \quad (48)\]
Some solutions of (46) can be determined from the solution (48) of the modified Poisson equation (47) after the corresponding change of variables. Let us describe it in detail.

Eq.(46) is equivalent to
\[
\left( \nabla^2 - \frac{e^2 C}{kT \varepsilon_0 \varepsilon_r} \right) \phi(x) = -\frac{e}{\varepsilon_0 \varepsilon_r} [C - \rho_b] - \frac{\eta e}{\varepsilon_0 \varepsilon_r} \delta(x). \tag{49}
\]
Therefore, one can introduce the notation
\[
\mu^2 = \frac{e^2 C}{kT \varepsilon_0 \varepsilon_r}. \tag{50}
\]
Now, we would like to change the variables in
\[
(\nabla^2 - \mu^2) \phi(x) = -\frac{e}{\varepsilon_0 \varepsilon_r} [C - \rho_b] - \frac{\eta e}{\varepsilon_0 \varepsilon_r} \delta(x), \tag{51}
\]
transforming it into the modified Poisson equation. To this aim, we insert
\[
\phi(x) = \phi'(x) + \alpha \tag{52}
\]
into the l.h.s. of (51):
\[
(\nabla^2 - \mu^2) [\phi'(x) + \alpha] = (\nabla^2 - \mu^2) \phi'(x) - \mu^2 \alpha = -\frac{e}{\varepsilon_0 \varepsilon_r} [C - \rho_b] - \frac{\eta e}{\varepsilon_0 \varepsilon_r} \delta(x). \tag{53}
\]
Eq.(53) is equivalent to
\[
(\nabla^2 - \mu^2) \phi'(x) = \mu^2 \alpha - \frac{e}{\varepsilon_0 \varepsilon_r} [C - \rho_b] - \frac{\eta e}{\varepsilon_0 \varepsilon_r} \delta(x), \tag{54}
\]
and the value of \(\alpha\) can be determined from the condition that the sum of first two terms on the r.h.s. of (54) should vanish. It means that
\[
\mu^2 \alpha - \frac{e}{\varepsilon_0 \varepsilon_r} [C - \rho_b] = 0 \tag{55}
\]
what after insertion of (50) into (55) gives
\[
\alpha = \frac{kT}{eC} [C - \rho_b]. \tag{56}
\]
Therefore, our equation (51) takes the form

\[ \nabla^2 - \mu^2 \phi' (x) = -\frac{\eta e}{\varepsilon_0 \varepsilon_r} \delta (x). \]  

(57)

In order to transform (57) into the "modified Poisson equation" one can make the following transformation

\[ \nabla^2 - \mu^2 \phi' (x) = -\frac{\eta e}{\varepsilon_0 \varepsilon_r} \delta (x) = -4\pi \frac{\eta e}{4\pi \varepsilon_0 \varepsilon_r} \delta (x). \]  

(58)

After comparing (58) and (47) one sees that

\[ \phi' (x) = -\frac{\eta e}{4\pi \varepsilon_0 \varepsilon_r x} \exp \left[ -\left( \frac{e^2 C}{kT \varepsilon_0 \varepsilon_r} \right)^{\frac{1}{2}} x \right] \]  

(59)

and therefore

\[ \phi (x) = \phi' (x) + \alpha = -\frac{\eta e}{4\pi \varepsilon_0 \varepsilon_r x} \exp \left[ -\left( \frac{e^2 C}{kT \varepsilon_0 \varepsilon_r} \right)^{\frac{1}{2}} x \right] + \frac{kT}{eC} [C - \rho_b]. \]  

(60)

A similar discussion can be made in the case of an electrolyte.

Again we start with the Maxwell equations but in this case of an electrolyte we introduce two different kind of carriers, one charged with a negative charge and the second one charged with a positive charge. The density of the negative carriers is denoted \( n(x,t) \) while the density of the positive carriers is denoted \( p(x,t) \). We also introduce two different fluxes

\[ j_n = -eD_n \exp \left[ -\frac{e\phi}{kT} \right] \frac{\partial}{\partial x} \left\{ n \exp \left[ \frac{e\phi}{kT} \right] \right\} \]  

(61)

and

\[ j_p = eD_p \exp \left[ \frac{e\phi}{kT} \right] \frac{\partial}{\partial x} \left\{ p \exp \left[ -\frac{e\phi}{kT} \right] \right\}. \]  

(62)

Also in this case we introduce the point charge \( \eta e \delta (x) \) fixed at the point \( x = 0 \). We also assume that the static background charge is absent in this case and the processes of ionization and recombination are absent. Therefore, Maxwell equations are in this case

\[ \text{rot} H = -eD_n \exp \left[ -\frac{e\phi}{kT} \right] \frac{\partial}{\partial x} \left\{ n \exp \left[ \frac{e\phi}{kT} \right] \right\} + \]  

\[ \text{rot} H = -eD_p \exp \left[ \frac{e\phi}{kT} \right] \frac{\partial}{\partial x} \left\{ p \exp \left[ -\frac{e\phi}{kT} \right] \right\}. \]
\[ eD_p \exp \left[ \frac{e\phi}{kT} \right] \frac{\partial}{\partial x} \left\{ p \exp \left[ -\frac{e\phi}{kT} \right] \right\} + \frac{\partial}{\partial t} D_e \] (63)

\[ \text{rot} E = -\frac{\partial}{\partial t} B \] (64)

\[ \text{div} B = 0 \] (65)

\[ \text{div} D_e = en(x,t) - ep(x,t) + \eta e\delta(x). \] (66)

Our further discussion is analogous to the previously discussed case of a semiconductor. We still assume (34), (35), (36) and obtain (37) as a consequence. Then, we assume (38) and obtain, as the counterparts of (39) and (40), equations

\[ 0 = -eD_n \exp \left[ -\frac{e\phi}{kT} \right] \frac{\partial}{\partial x} \left\{ n \exp \left[ \frac{e\phi}{kT} \right] \right\} + eD_p \exp \left[ \frac{e\phi}{kT} \right] \frac{\partial}{\partial x} \left\{ p \exp \left[ -\frac{e\phi}{kT} \right] \right\} + \frac{\partial}{\partial t} [-\varepsilon_0 \varepsilon_r \nabla \phi] \] (67)

and

\[ -\varepsilon_0 \varepsilon_r \nabla^2 \phi(x,t) = en(x,t) - ep(x,t) + \eta e\delta(x). \] (68)

We discuss the particular solutions of (67) and (68), with the time derivative equal to zero and both fluxes equal to zero

\[ -eD_n \exp \left[ -\frac{e\phi}{kT} \right] \frac{\partial}{\partial x} \left\{ n \exp \left[ \frac{e\phi}{kT} \right] \right\} = 0 \] (69)

\[ eD_p \exp \left[ \frac{e\phi}{kT} \right] \frac{\partial}{\partial x} \left\{ p \exp \left[ -\frac{e\phi}{kT} \right] \right\} = 0. \] (70)

Similarly as before, we solve (69) and (70) and insert the results into (68), arriving at

\[ -\varepsilon_0 \varepsilon_r \nabla^2 \phi(x) = eC_n \exp \left[ -\frac{e\phi}{kT} \right] - eC_p \exp \left[ \frac{e\phi}{kT} \right] + \eta e\delta(x) \] (71)

this time. Both exponents from (71) are linearized according to the rules

\[ \exp \left[ -\frac{e\phi}{kT} \right] \approx 1 - \frac{e\phi}{kT} \] (72)

\[ \exp \left[ \frac{e\phi}{kT} \right] \approx 1 + \frac{e\phi}{kT} \] (73)
and therefore one arrives at
\[-\varepsilon_0 \varepsilon_r \nabla^2 \phi (x) = eC_n \left[ 1 - \frac{e\phi}{kT} \right] - eC_p \left[ 1 + \frac{e\phi}{kT} \right] + \eta \varepsilon \delta (x). \quad (74)\]

This equation is equivalent to
\[\left[ \nabla^2 - \frac{e^2}{\varepsilon_0 \varepsilon_r kT} [C_n + C_p] \right] \phi (x) = - \frac{e}{\varepsilon_0 \varepsilon_r} [C_n - C_p] - \frac{\eta e}{\varepsilon_0 \varepsilon_r} \delta (x). \quad (75)\]

Similarly as in the case of a semiconductor \((50)\), one defines the parameter \(\mu^2\) which is now
\[\mu^2 = \frac{e^2}{\varepsilon_0 \varepsilon_r kT} [C_n + C_p] \quad (76)\]
and after inserting \((76)\) into \((75)\) one arrives at
\[\left[ \nabla^2 - \mu^2 \right] \phi (x) = - \frac{e}{\varepsilon_0 \varepsilon_r} [C_n - C_p] - \frac{\eta e}{\varepsilon_0 \varepsilon_r} \delta (x). \quad (77)\]

After inserting \((52)\) into the l.h.s. of \((77)\) one arrives at
\[\left[ \nabla^2 - \mu^2 \right] \phi' (x) - \mu^2 \alpha = - \frac{e}{\varepsilon_0 \varepsilon_r} [C_n - C_p] - \frac{\eta e}{\varepsilon_0 \varepsilon_r} \delta (x). \quad (78)\]

The above formula is an analogy to \((53)\). Now, \((78)\) is equivalent to
\[\left[ \nabla^2 - \mu^2 \right] \phi' (x) = \mu^2 \alpha - \frac{e}{\varepsilon_0 \varepsilon_r} [C_n - C_p] - \frac{\eta e}{\varepsilon_0 \varepsilon_r} \delta (x) \quad (79)\]
and the value of \(\alpha\) is determined here from the condition that the sum of first two terms on the r.h.s. of \((79)\) should vanish. Therefore
\[\mu^2 \alpha = - \frac{e}{\varepsilon_0 \varepsilon_r} [C_n - C_p] \quad (80)\]

what after insertion of \((76)\) into \((80)\) gives
\[\alpha = \frac{kT [C_n - C_p]}{e [C_n + C_p]} \quad (81)\]

Similarly as in the case of a semiconductor, one can write our equation in the form
\[\left[ \nabla^2 - \mu^2 \right] \phi' (x) = -4\pi \frac{\eta e}{4\pi \varepsilon_0 \varepsilon_r} \delta (x) \quad (82)\]
and arrive at the solution

$$
\phi(x) = \frac{\eta e}{4\pi \varepsilon_0 \varepsilon_r x} \exp \left[ - \frac{kT}{e} \left( \frac{C_n - C_p}{C_n + C_p} \right) x \right] + \frac{kT}{e} \frac{C_n - C_p}{C_n + C_p}.
$$

(83)

It is worth to compare our approach with that described in Jackson’s monograph [6] on p.342:

"A nonrigorous derivation of the screening effect described above was first given by Debye and Huckel in their theory of electrolytes. The basic argument is as follows:

..." A nonrigorous derivation of the screening effect described above was first given by Debye and Huckel in their theory of electrolytes. The basic argument is as follows. Suppose that we have a plasma with a distribution of electrons in thermal equilibrium in an electrostatic potential $\Phi$. Then they are distributed according to the Boltzmann factor $e^{-\frac{H}{kT}}$ where $H$ is the electric Hamiltonian. The spatial density of electrons is therefore

$$
n(x) = n_0 \exp \left[ -\frac{e\Phi}{kT} \right].
$$

(84)

Now we imagine a test charge $Ze$ placed at the origin in this distribution of electrons with its uniform background of positive ions (charge density $-en_0$). The resulting potential $\Phi$ will be determined by Poisson’s equation

$$
\nabla^2 \Phi = -4\pi Ze \delta(x) - 4\pi en_0 \left[ \exp \left( -\frac{e\Phi}{kT} \right) - 1 \right].
$$

(85)

If

$$
\frac{e\Phi}{kT}
$$

is assumed small, the equation can be linearized:

$$
\nabla^2 \Phi - k_D^2 \Phi \simeq -4\pi Ze \delta(x)
$$

(86)

where

$$
k_D^2 = \frac{4\pi n_0 e^2}{kT}
$$

(87)

...Equation (87) has the spherically symmetric solution:

$$
\Phi(r) = Z e^{-k_D r}
$$

(88)
showing that the electrons move in such a way to screen out the Coulomb field of a test particle in a distance of the order of $\frac{1}{k_D}$.

It can be seen that the approach described by Jackson contains less parameters. The other difference is that in our approach one can state the boundary conditions while Jackson considers an infinite system.

It is also worth to stress that in our approach it is possible to describe the relaxation of the system towards the stationary state.

2 Conclusions

It would be interesting to solve our equations without linearisations and to compare our results with the experimental results (for isothermal conditions).

In principle, our approach can be applied also to the systems with the temperature changing in space and time (compare [2, 3, 4]). We hope to discuss that in future.

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