Observations of complex frequency comb structure in a harmonically-pumped femtosecond optical parametric oscillator

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Abstract. Various schemes allow femtosecond optical parametric oscillators to produce pulses at harmonics of their pump laser repetition frequency, each apparently offering the possibility of generating widely-spaced, tunable frequency combs. Using a 100-MHz Ti:sapphire pump laser, we have compared two alternative optical parametric oscillator architectures, both leading to 300-MHz pulses but one configured in a cavity three times shorter than the pump laser and the other in a cavity one-third longer. Heterodyne measurements between the pump and each of these two systems show that they possess different carrier-envelope offset characteristics, with implications on the coherence and stabilization of the resulting combs.

1. Introduction
Synchronously-pumped optical parametric oscillators (OPOs) are used to convert femtosecond or picosecond pump pulses to signal and idler pulses of longer wavelengths in a resonant cavity whose optical length matches that of the pump laser [1]. The broadband wavelength coverage offered by OPOs has made them attractive as a source of laser frequency combs, with applications in precision spectroscopy [2,3] and coherent pulse synthesis [4]. Extending these OPO frequency combs to high repetition rates would allow their use in applications where it is important to be able to resolve individual comb modes, such as in astronomical spectrograph calibration [5,6]. Strict synchronous pumping of an OPO limits its repetition frequency to exactly that of its pump laser, however operating the OPO at a harmonic of the pump repetition frequency is possible by employing a cavity whose length is an integer [7–9] or integer fraction [10–13] of the pump laser length. Here, we theoretically and experimentally compare two alternative routes to obtaining 300-MHz operation from an OPO pumped by a 100-MHz femtosecond pump laser, and show important practical differences in their frequency-comb characteristics, which have so far remained unexplored.

2. Harmonic operation of synchronously-pumped optical parametric oscillators
For a fundamentally modelocked pump laser of round-trip optical length $L_{PUMP}$, harmonic operation of a signal-resonant synchronously-pumped OPO is achieved when the OPO cavity length is

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\[ L_{\text{OPO}} = L_{\text{PUMP}} \times \frac{n}{Q} \]  

where \( n \) and \( Q \) are positive integers and \( n/Q \) is an irreducible fraction [12]. Configured in this way, the OPO produces signal pulses at a repetition frequency \( Q \) times higher than that of the pump laser (\( f_{\text{REP}} \)), and fundamental synchronous pumping is obtained when \( n = Q = 1 \). As illustrated in Figure 1, two categories of harmonic operation (\( Q > 1 \)) can be identified, depending on the choice of \( n \).

**Figure 1.** Schematic depiction of harmonic operation of a synchronously-pumped OPO, configured as a standing-wave cavity. (a) an OPO (blue) \( 1/Q \) times the pump cavity length (green); (b) an OPO (orange) \( n/Q \) times the pump cavity length (green). For clarity, the OPO gain medium is not shown.

2.1. Simple harmonic pumping

When \( n = 1 \), the OPO cavity is shorter than the pump cavity and a single pulse is resonant, experiencing gain when it coincides with a pump pulse every \( Q \) round-trips of the OPO cavity, as depicted in Figure 1a. Operation in this regime presents design challenges in obtaining good mode-matching at the small spot sizes needed to achieve the necessary intensities for parametric oscillation, in turn requiring the use of very small radius-of-curvature intracavity optics.

2.2. Vernier harmonic pumping

The case of \( n > 1 \) (Figure 1b) corresponds to \( n \) independent, equidistant pulses resonating in the OPO cavity, with each pulse interacting with a pump pulse every \( Q \) round trips, outputting an interleaved pulse train with a repetition frequency \( Qf_{\text{REP}} \). If \( Q > n \) the OPO is shorter than the pump cavity, but if \( Q < n \) the OPO is longer. As demonstrated by Kokabee et al [14], designs where both \( Q \) and \( n \) are large allow high harmonic repetition frequencies to be obtained from a cavity which is sufficiently long to permit optimum pump-signal mode-matching and the insertion of dispersion-compensating optics.

3. Frequency-comb structure of harmonically-pumped optical parametric oscillators

It is well established that the Fourier transform of a regularly spaced train of pulses produced by a modelocked laser corresponds to a comb of equally spaced frequencies, separated by their repetition frequency [15]. The same is true for a strictly synchronously pumped OPO, which produces distinct pump, signal and idler frequency combs which all share the spacing of the pump repetition frequency.
The frequency comb structure of both short and extended cavity OPOs producing pulse trains at harmonics of the pump repetition frequency $Q_{\text{REP}}$ will be examined theoretically.

In the case of simple harmonic pumping ($n = 1$ and $Q > 1$) there is a single circulating pulse in the OPO that experiences gain after every $Q$ round trips (Figure 2b), with ring-down losses writing sidebands at $\pm f_{\text{REP}}$ onto comb lines spaced at $Qf_{\text{REP}}$. When $n > 1$ and $Q > 1$ there are $n$ circulating pulses that each experience gain after every $Q$ round trips; such a configuration can be considered as $n$ OPOs operating in a single cavity (Figure 2c). Each distinct OPO comb has the same CEO frequency, which is determined by the OPO cavity dispersion. As with the simple harmonic case, sidebands are written onto the comb lines due to ring-down losses. Here, instability in the sideband structure is expected as the phases of the circulating pulses are unrelated; the initial phase of each pulse is random, determined as each signal pulse train builds up from quantum noise.

Figure 2. OPO frequency comb structure. (a) strictly synchronous pumping; (b) simple harmonic pumping; (c) Vernier harmonic pumping.
4. Experiment

We experimentally tested the description presented above by using the configuration illustrated in Figure 3. A 100-MHz Ti:sapphire laser (Griffin-I, KM Labs) was used to pump two PPKTP OPOs with both operated at 300 MHz, one in a cavity three times shorter than the pump laser ($n = 1$ and $Q = 3$) and the other in a cavity four-thirds longer ($n = 4$ and $Q = 3$). Practical measurement of the OPO comb CEO frequency normally proceeds by heterodyning a portion of the pump supercontinuum with common wavelengths in the OPO sum-frequency or second-harmonic outputs [16–18]. In our measurement, second-harmonic (SHG) light from the OPO was interfered with a pump supercontinuum generated in a 20-cm length of photonic crystal fiber (PCF; NKT Photonics, NL-2.0-750), producing a heterodyne beat observable on an avalanche photodiode (APD; Hamamatsu, C5331-11). Fast-photodiode measurements of the second-harmonic signal output were monitored to confirm the OPO repetition frequency was 300 MHz.

![Figure 3](image)

Figure 3 Experimental schematic. APD, avalanche photodiode; IF, interference filter; OC, output coupler; PBS, polarizing beam splitter; PCF, photonic crystal fiber; POL, polarizer.

Radio-frequency (RF) spectra detected by the APD were recorded from the two OPO cavities and are shown in Figure 4a,c. Pairs of sidebands encoding the difference between the pump and OPO CEO frequencies were observed centered around multiples of $f_{REP}/n$. For the short OPO cavity ($n = 1$ and $Q = 3$) sidebands were observed at multiples of 100 MHz ($f_{REP}/1$). In the extended OPO cavity ($n = 4$ and $Q = 3$) sidebands were observed at multiples of 25 MHz ($f_{REP}/4$). The dense RF spectrum produced when $n > 1$ presents greater practical challenges in isolating the beat frequency required for CEO locking than are encountered when $n = 1$. RF spectra of fast-photodiode measurements of the second-harmonic signal output appear in Figure 4b,d, showing that in the time domain the intensity of the fields produced by the two OPOs is equivalent.

The experimentally observed RF spectra can be explained as follows. The heterodyne signal between the pump super-continuum and the second-harmonic of the OPO comb contains frequencies described by the general equation

$$
(Nf_{REP} + f_{CEO}^p) \pm (Mf_{CEO}^{2s} + f_{CEO}^{2s})
$$

(2)
where $N$ and $M$ are integers; $f_{\text{REP}} = 1/T$ is the repetition frequency of the pump pulses and $f_{\text{CEO}}^p$ is their CEO frequency; and $f_{\text{REP}}^{2s} = Q/nT$ is the repetition frequency of the OPO signal second-harmonic pulses (equal to that of the fundamental signal pulses) and $f_{\text{CEO}}^{2s}$ is their CEO frequency. Equation (2) can therefore be rewritten and its terms separated into those associated with the CEO and repetition frequencies:

$$\left( \frac{nN \pm Q}{n} f_{\text{REP}} + (f_{\text{CEO}}^p + f_{\text{CEO}}^{2s}) \right). \quad (3)$$

For the experimental value of $f_{\text{REP}} = 100$ MHz, (3) predicts that the heterodyne signal when $n = 4$ and $Q = 3$ should contain pairs of frequencies $f_{\text{CEO}} \pm f_{\text{CEO}}^{2s}$ centered around integer multiples of 25 MHz, as observed experimentally. By comparison, when $n = 1$ and $Q = 3$, pairs of CEO frequencies are predicted at multiples of 100 MHz, again as observed experimentally.

![Figure 4](image_url) RF spectra from differently constructed 300-MHz harmonic OPOs. a, b, beat frequency and fast photodiode measurements for $n = 1$ and $Q = 3$. c, d, beat frequency and fast photodiode measurements for $n = 4$ and $Q = 3$. Dashed lines indicate multiples of 25 MHz.

5. Conclusions

We have investigated the frequency comb structure associated with two different approaches to obtaining harmonic OPO operation, presenting a general theoretical description of the comb structure produced under harmonic pumping and showing that it predicts the heterodyne frequencies produced experimentally when interfering the pump and OPO combs. The dense heterodyne beats produced when $n > 1$ can be expected to present practical problems when implementing CEO locking, therefore the method of choice for obtaining a harmonic OPO frequency comb should be simple harmonic pumping ($n = 1$) of an OPO cavity which is $Q$ times shorter than its pump laser.

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