Duality in Bipolar Fuzzy Number Linear Programming Problem

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ABSTRACT
We develop a linear ranking function for ordering bipolar fuzzy numbers and study its properties. Using this ranking function, we solve a bipolar fuzzy linear programming problem. Then, we present the dual of the problem and establish several duality results. Also, we presented an application of bipolar fuzzy number in real life problem.

ARTICLE HISTORY
Received 7 September 2017
Revised 16 August 2017
Accepted 22 August 2018

KEYWORDS
Fuzzy linear programming problem; bipolar fuzzy number; duality; ranking function

1. Introduction

In a human decision making, there is a bipolar judgmental thinking on a negative side and a positive side; for instance, see [1]. In bipolar information, two types of information (as positive and negative) must be distinguished [2,3]. Positive information is given by observation or experimentation. But, negative information represents impossibility. This domain has recently invoked several interesting research areas such as psychology [4], image processing [5], human reasoning [6] and graph theory [7]. Zhang [8] initiated the concept of bipolar fuzzy set as a generalization of fuzzy set. He defined bipolar fuzzy set as an extension of fuzzy set whose range of membership degree is $[-1, 1]$. Akram [7,9] used the concept of bipolar fuzzy set in graph theory. Broumand [10] introduced the concept of bipolar-valued fuzzy sub-algebras of BCK/BCI-algebras and investigated some of their useful properties. Zhou and Li [1] presented the concepts of bipolar fuzzy $h$-ideals and normal bipolar fuzzy $h$-ideals. Then, they investigated characterizations of bipolar fuzzy $h$-ideals by means of positive $t$-cut, negative $s$-cut, homomorphism and equivalence relation. Some other works on bipolar fuzzy sets can be found in [11–15].

There are several methods for comparison of unipolar fuzzy numbers based on ranking functions [16,17] and most convenient methods for solving linear programing problems are based on the concept of ranking functions [16,18,19,20,21,22]. Inspired by ranking functions of unipolar fuzzy numbers, we develop a ranking function for bipolar fuzzy numbers. Then, we solve a bipolar fuzzy linear programming problem by using a certain ranking function and we give duality results for bipolar fuzzy linear programing problems.
In Section 2, we review the fundamental notions of bipolar fuzzy sets. Then, we propose a linear ranking function to order bipolar fuzzy numbers. In Section 3, we investigate and characterize several properties for the bipolar fuzzy number linear programming problem by using a linear ranking function. In Section 4, we introduce the dual of a bipolar fuzzy number linear programming problem. In Section 5, we present an application of bipolar fuzzy number in a real life problem. We give our concluding remarks in Section 6.

2. Preliminaries
2.1. Definitions and Notations

Here, some necessary definitions and relevant results of bipolar fuzzy sets are given.

**Definition 2.1:** [8] Let $X$ be a nonempty set. A bipolar fuzzy set $\tilde{B}$ in $X$ is an object with the following form

$$\tilde{B} = \{(x, \mu^P_B(x), \mu^N_B(x))| x \in X\},$$

where, $\mu^P_B(x) : X \rightarrow [0, 1]$ and $\mu^N_B(x) : X \rightarrow [-1, 0]$.

**Definition 2.2:** [11] Given $\tilde{B} = ((\mu^P_B(x), \mu^N_B(x))$ a bipolar-valued fuzzy set and $(s, t) \in [-1, 0] \times [0, 1]$, the sets $B^P_t = \{x \in X| \mu^P_B(x) \geq t\}$ and $B^N_s = \{x \in X| \mu^N_B(x) \leq s\}$ are respectively called the positive $t$-cut of $\tilde{B}$ and the negative $s$-cut of $\tilde{B}$, and for every $k \in [0, 1]$, the set:

$$B_k = B^P_k \cap B^N_k$$

is called the $k$-cut of $\tilde{B}$.

We next define a bipolar triangular fuzzy number.

A bipolar triangular fuzzy number is defined as a quadruple $\tilde{A} = (a^L, a^P, a^N, a^R)$ with positive and negative membership functions $\mu^P_A(x)$ and $\mu^N_A(x)$ as follows (see Figure 1):

$$\mu^P_A(x) = \begin{cases} \frac{x - a^L}{a^P - a^L}, & a^L \leq x < a^P \\ \frac{x - a^R}{a^P - a^R}, & a^P \leq x < a^R \\ 0, & \text{Otherwise} \end{cases}$$

$$\mu^N_A(x) = \begin{cases} \frac{-(x - a^L)}{a^N - a^L}, & a^L \leq x < a^N \\ \frac{-(x - a^R)}{a^N - a^R}, & a^N \leq x < a^R \\ 0, & \text{Otherwise}. \end{cases}$$

**Note 1:** In a bipolar triangular fuzzy number, $a^P$ and $a^N$ can be equal (see Figure 2).

**Note 2:** We denote the set of all bipolar triangular fuzzy numbers by $F(\mathbb{R})$. 
Using similar argument for fuzzy arithmetic, we now give the following proposition.

**Proposition 2.1:** Let \( \tilde{a} = (a^L, a^P, a^N, a^R) \) and \( \tilde{b} = (b^L, b^P, b^N, b^R) \) be two bipolar fuzzy numbers. Define:

\[
\begin{align*}
    x > 0, & \quad x \in \mathbb{R}, x\tilde{a} = (xa^L, xa^P, xa^N, xa^R), \\
    x < 0, & \quad x \in \mathbb{R}, x\tilde{a} = (xa^R, xa^N, xa^P, xa^L), \\
    \tilde{a} + \tilde{b} = (a^L + b^L, a^P + b^P, a^N + b^N, a^R + b^R).
\end{align*}
\]

**2.2. Bipolar Ranking Function**

There are several methods for solving unipolar fuzzy linear programing problems by using ranking function [16,18,19,20,21,22]. We can define ranking function on bipolar fuzzy numbers and use it for solving bipolar fuzzy number linear programing problems. A bipolar ranking function \( R^b : F(\mathbb{R}) \to \mathbb{R} \) maps bipolar fuzzy numbers into the real line, where a
natural order exists. Orders on \( F(\mathbb{R}) \) are defined as follows:

\[
\begin{align*}
\tilde{a} \geq_{R^b} \tilde{b}, & \quad \text{if and only if} \quad R^b(\tilde{a}) \geq R^b(\tilde{b}), \\
\tilde{a} \leq_{R^b} \tilde{b}, & \quad \text{if and only if} \quad R^b(\tilde{a}) \leq R^b(\tilde{b}), \\
\tilde{a} =_{R^b} \tilde{b}, & \quad \text{if and only if} \quad R^b(\tilde{a}) = R^b(\tilde{b}),
\end{align*}
\]

where \( \tilde{a}, \tilde{b} \in F(\mathbb{R}) \).

Inspired by a special version of the ranking function on unipolar fuzzy numbers proposed by Yager [17], we define ranking function on bipolar fuzzy numbers as follows:

\[
R^b(\tilde{a}) = \int_{-1}^{0} \frac{\left( \inf \tilde{a}_s + \sup \tilde{a}_s \right)}{2} ds + \int_{0}^{1} \frac{\left( \inf \tilde{a}_t + \sup \tilde{a}_t \right)}{2} dt.
\]

By using Definition 2.2 we have:

\[
\tilde{a}_t^p = [a^l + ta^p - ta^r, ta^p - ta^r + a^r]
\]

and

\[
\tilde{a}_s^n = [-sa^N + sa^L + a^r, -sa^N + sa^R + a^r].
\]

So,

\[
R^b(\tilde{a}) = \int_{-1}^{0} \frac{(-sa^N + sa^L + a^l - sa^N + sa^R + a^r)}{2} ds + \int_{0}^{1} \frac{(a^l + ta^p - ta^r + ta^p - ta^r + a^r)}{2} dt = \frac{(a^r + a^N + a^P + a^L)}{2}.
\]

Then, for bipolar triangular fuzzy numbers \( \tilde{a} = (a^l, a^p, a^N, a^r) \) and \( \tilde{b} = (b^l, b^p, b^N, b^r) \) we have:

\[
\tilde{a} \geq_{R^b} \tilde{b} \quad \text{if and only if} \quad \frac{(a^r + a^N + a^P + a^L)}{2} \geq \frac{(b^r + b^N + b^P + b^L)}{2}.
\]

**Lemma 2.1:** Ranking function (3) is linear.

**Proof:** We need to show \( R^b(k\tilde{a} + \tilde{b}) = kR^b(\tilde{a}) + R^b(\tilde{b}) \), for any \( k \in \mathbb{R} \), and \( \tilde{a} = (a^l, a^p, a^N, a^r) \) and \( \tilde{b} = (b^l, b^p, b^N, b^r) \) any two bipolar triangular fuzzy numbers. Assume \( k > 0 \) (for \( k < 0 \) is similar). From Proposition 2.1, we have

\[
k\tilde{a} + \tilde{b} = (ka^l + b^l, ka^p + b^p, ka^N + b^N, ka^r + b^r),
\]

and from (3), we get,

\[
R^b(k\tilde{a} + \tilde{b}) = \frac{ka^l + b^l + ka^p + b^p + ka^N + b^N + ka^r + b^r}{2} = \frac{k(a^l + a^p + a^N + a^r)}{2} + \frac{(b^l + b^p + b^N + b^r)}{2} = kR^b(\tilde{a}) + R^b(\tilde{b}).
\]

\( \blacksquare \)
Lemma 2.2: Let \( R^b \) be defined as (3). Then,
1. \( \bar{a} \geq_{R^b} \bar{b} \) if and only if \( \bar{a} - \bar{b} \geq_{R^b} \bar{0} \) if and only if \( -\bar{b} \geq_{R^b} \bar{a} \).
2. If \( \bar{a} \geq_{R^b} \bar{b} \) and \( \bar{c} \geq_{R^b} \bar{d} \) then \( \bar{a} + \bar{c} \geq_{R^b} \bar{b} + \bar{d} \).

Proof: It is straightforward, by using (3).

3. Linear Programming Problem with Bipolar Triangular Fuzzy Numbers

A bipolar triangular fuzzy number linear programming problem (BTFNLPP) is defined to be

\[
\begin{align*}
\text{(BTFNLPP)}: \quad & \min \bar{z} = R^b \bar{c}^T x, \\
& \text{s.t.} \quad \bar{A} x = R^b \bar{b}, \\
& \quad x \geq 0,
\end{align*}
\]

where, \( \bar{b} \in F(\mathbb{R}^m) \), \( \bar{A} \in F(\mathbb{R}^{m \times n}) \), \( \bar{c} \in F(\mathbb{R}^n) \) are given, \( x \in \mathbb{R}^n \) is to determined, and \( R^b \) is a linear ranking function as defined by (3), where \( R^b(\bar{c}) \in \mathbb{R}^n \), \( R^b(\bar{b}) \in \mathbb{R}^m \) and \( R^b(\bar{A}) \in \mathbb{R}^{m \times n} \).

The following result gives an alternative formulation of (4).

Proposition 3.1: Problem (4) is equivalent to

\[
\begin{align*}
\min \bar{z} = \bar{c}^T x, \\
\text{s.t.} \quad R^b(\bar{A}) x = R^b(\bar{b}), \\
x \geq 0.
\end{align*}
\]

Proof: Since \( R^b \) is a linear ranking function, we have:

\[
R^b(\bar{c}^T x) = R^b \left( \sum_{j=1}^{n} \bar{c}_j x_j \right) = \sum_{j=1}^{n} R^b(\bar{c}_j) x_j = R^b(\bar{c})^T x,
\]

where \( R^b(\bar{c}) = (R^b(\bar{c}_1), \ldots, R^b(\bar{c}_n))^T \in \mathbb{R}^n \). On the other hand

\[
R^b \left( \sum_{j=1}^{n} \bar{a}_j x_j \right) = \sum_{j=1}^{n} R^b(\bar{a}_j) x_j = \sum_{j=1}^{n} R^b(\bar{a}_j) x_j.
\]

So, if we denote the \( i \)-th row of \( \bar{A} \) by \( \bar{a}_i \), we have

\[
R^b(\bar{A} x) = (R^b(\bar{a}_1 x), \ldots, R^b(\bar{a}_m x))^T = \left( R^b \left( \sum_{j=1}^{n} \bar{a}_1 x_j \right), \ldots, R^b \left( \sum_{j=1}^{n} \bar{a}_m x_j \right) \right)^T
\]

\[
= \left( \sum_{j=1}^{n} R^b(\bar{a}_1 x_j), \ldots, \sum_{j=1}^{n} R^b(\bar{a}_m x_j) \right)^T = (R^b(\bar{a}_1) x, \ldots, R^b(\bar{a}_m) x)^T = R^b(\bar{A}) x.
\]
4. Duality

Similar to the duality theory in linear optimization (see, for example, Luenberger and Ye [23]), for every \textit{BTFNLPP} there is an associated dual \textit{BTFNLPP} (\textit{DBTFNLPP}) which satisfies some important properties.

**Definition 4.1:** Using the notation of (4) define
\[
\text{(DBTFNLPP)} \left\{ \begin{array}{l}
\max \bar{u} = \bar{y}^T \bar{b} \\
\text{s.t.} \ y^T \bar{A} \leq \bar{c}^T \varepsilon \\
\end{array} \right.
\]

(6)

Indeed, \text{(DBTFNLPP)} is the dual of \text{(BTFNLPP)} and appropriate duality results can be established. Next, we show the weak duality result.

**Lemma 4.1:** Dual of DBTFNLPP is BTFNLPP.

**Proof:** This can be shown by changing the inequalities in (6) into equalities and an appropriate change of the \(y\) variables followed by an application of Proposition 3.1. ■

**Theorem 4.1:** If \(x_0\) and \(y_0\) are respectively feasible solutions to BTFNLPP and DBTFNLPPs, then \(\bar{c}^T x_0 \geq \bar{R} \bar{b}^T y_0\).

**Proof:** Multiplying \(\bar{A} x_0 = \bar{b}\) on the left by \(y_0^T\), we have \(y_0^T \bar{A} x_0 = \bar{R} y_0^T \bar{b}\) and multiplying \(y_0^T \bar{A} \leq \bar{c}^T \varepsilon\) on the right by \(x_0 \geq 0\), we have \(y_0^T \bar{A} x_0 \leq \bar{c}^T x_0\). So, we get \(y_0^T \bar{b} \leq \bar{c}^T x_0\). ■

**Note 3:** Similar to some duality results for unipolar fuzzy number linear programming problem, the value of the ranking function for the bipolar fuzzy value of the objective function at any feasible solution to BTFNLPP is always bigger than or equal to the value of the ranking function for the bipolar fuzzy value of the objective function for any feasible solution to DBTFNLPP. Also, if \(x_0\) and \(y_0\) are feasible solutions to BTFNLP and DBTFNLP problems, respectively, and \(y_0^T \bar{b} = \bar{c}^T x_0\), then \(x_0\) and \(y_0\) are optimal solutions to their respective problems and if any one of the BTFNLPP or DBTFNLPP is unbounded, then the other problem has no feasible solution. Next, we define a basic solution and then establish the strong duality result.

**Definition 4.2:** Let \(A = [a_{ij}]_{m \times n} = R^b(\bar{A})\). Assume rank\((A)\) = \(m\), and partition \(A\) as \([B, N]\) where \(B, m \times m\), is nonsingular. It is obvious that rank\((B)\) = \(m\) and \(B = R^b(\bar{B})\), where \(\bar{B}\) is the fuzzy matrix in \(\bar{A}\) corresponding to \(B\). Let \(y_j\) be a solution of \(By = a_j\). A basic solution is
\[
x_B = (x_{B_1}, \ldots, x_{B_m})^T = B^{-1} b, x_N = 0,
\]
where \(B = (\bar{b}_1, \ldots, \bar{b}_m) = (a_{B_1}, \ldots, a_{B_m})\) with \(B_i\) being the index corresponding to the \(i\)-th column of \(B\), that is, \(\bar{b}_i = a_{B_i}\) and \(b = R^b(\bar{B})\). If \(x_B \geq 0\), then the basic solution is feasible and the corresponding fuzzy objective value is \(\bar{z} = (\bar{c}_B^T x_B)\), where
\[
\tilde{c}_B = (\tilde{c}_{B_1}, \ldots, \tilde{c}_{B_m})^T.
\]
Now, corresponding to every nonbasic variable \(x_j, 1 \leq j \leq n, j \neq B_i, i = 1, \ldots, m\) define
\[
\tilde{z}_j = R^b \tilde{c}_B^T y_j = R^b \tilde{c}_B^T B^{-1} a_j.
\]
Theorem 4.2: Assume the BTFNLPP is non-degenerate (see [23]). A basic feasible solution \( x_B = B^{-1}b, x_N = 0 \) is optimal to (4) if and only if \( \tilde{z}_j \leq R^b \tilde{c}_j \), for all \( j, 1 \leq j \leq n \).

**Proof:** Let \( x = (x_B^T, x_N^T)^T \) be an optimal solution to (4), where \( x_B = B^{-1}b, x_N = 0 \), corresponding to an optimal basis \( B \). Then, the corresponding optimal objective value is

\[
\tilde{z} = R^b \tilde{c}^T x = R^b \tilde{c}_B^T x_B + \tilde{c}_N^T x_N
\]

Now, since \( x \) is feasible, we have \( x \geq 0 \), and based on Definition 4.2,

\[
b = Ax = Bx_B + Nx_N.
\]

Hence, we can rewrite (9) as follows:

\[
x_B = B^{-1}b - B^{-1}Nx_N.
\]

Substituting (10) in (8), we obtain

\[
\tilde{z} = R^b \tilde{c}^T x = R^b \tilde{c}_B^T x_B + \tilde{c}_N^T x_N
\]

Thus,

\[
\tilde{z} = R^b \tilde{z}^* - \sum_{j=1, j \neq B_i}^{n} (\tilde{z}_j - \tilde{c}_j) x_j.
\]

Now, from (11) it is obvious that if there is a nonbasic variable \( x_j \) with \( \tilde{z}_j > R^b \tilde{c}_j \), then we can enter \( x_j \) into the basis and obtain \( \tilde{z}^* > R^b \tilde{z} \) (since the problem is non-degenerate and \( x_j > 0 \) in the new basis). This is contrary to \( \tilde{z}^* \) being optimal and hence we must have \( \tilde{z}_j \leq R^b \tilde{c}_j \) for all \( j, 1 \leq j \leq n \). The converse of the theorem is similar to Theorem 3.1 in [19].

Theorem 4.3: If any one of the BTFNLPP or DBTFNLPP has an optimal solution, then both problems have optimal solutions and the two optimal value of ranking functions for the fuzzy objective values are equal.

**Proof:** Assume that BTFNLPP has an optimal solution, \( \text{rank}(R^b(\tilde{A})) = m \) and \( B = R^b(\bar{B}) \) is the basic matrix and \( (x_B^*, x_N^*)^T = \begin{bmatrix} B^{-1}b \\ 0 \end{bmatrix} \) forms the basic optimal solution corresponding to BTFNLPP. Since \( x^* \) is an optimal solution, according to Theorem 4.1, we have

\[
\tilde{c}_B^T B^{-1} a_j - \tilde{c}_j \leq R^b \tilde{0}, j = 1, \ldots, n,
\]

and thus,

\[
R^b(\tilde{c}_B^T B^{-1} A) - R^b(\tilde{c})^T \leq R^b(\tilde{0})^T,
\]
where by letting \( y^* = R_b(c^T B^{-1}) = c^T B^{-1} \), we can write \( y^*^T \tilde{A} \leq R_b \tilde{c}^T \). Thus, \( y^* \) is a feasible solution to DBTFNLPP. Based on Definition 4.2, we have

\[
y^* R_b(b) = y^*^T b = c^T B^{-1} b = c^T B x^* = c^T B x^* + c^T x^*_N = R_b(\tilde{c}) x^*.
\]

Hence,

\[
y^*^T \tilde{b} = R_b \tilde{c}^T x^*.
\]

Due to Lemma 3.1, the converse of the theorem follows similarly. ■

**Theorem 4.4:** For any BTFNLPP and its corresponding DBTFNLPP, exactly one of the following statements is true.

1. Both have optimal solutions \( x^* \) and \( y^* \) with \( \tilde{c}^T x^* = R_b y^*^T \tilde{b} \).
2. One problem is unbounded and the other is infeasible.
3. Both problems are infeasible.

**Proof:** 1 and 2 are proven in Theorem 4.2 and 3.1 respectively and we give an example for 3. ■

**Example 4.1:** Consider BTFNLPP and its corresponding DBTFNLPP as follows:

\[
(\text{BTFNLPP}) \begin{cases} 
\min \tilde{z} = R_b(-6, -4, 4, 10)x_1 + (-14, -4, 3, 5)x_2 \\
\text{s.t.} \\
(-8, 0, 2, 4)x_1 + (-3, 0, 1, 4)x_2 = R_b(1, 2, 3, 4), \\
(-5, 0, 1, 6)x_1 + (-5, 0, 1, 2)x_2 = R_b(2, 4, 6, 8), \\
x_1, x_2 \geq 0,
\end{cases}
\]

\[
(\text{DBTFNLPP}) \begin{cases} 
\max \tilde{u} = R_b(1, 2, 3, 4)w_1 + (2, 4, 6, 8)w_2 \\
\text{s.t.} \\
(-8, 0, 2, 4)w_1 + (-5, 0, 1, 6)w_2 \leq R_b(-6, -4, 4, 10), \\
(-3, 0, 1, 4)w_1 + (-5, 0, 1, 2)w_2 \leq R_b(-14, -4, 3, 5), \\
x_1, x_2 \geq 0,
\end{cases}
\]

We clearly see that both problems are infeasible.

The following result gives the complementary slackness conditions for optimality.

**Theorem 4.5:** The vectors \( x^* \) and \( y^* \) respectively feasible solutions to BTFNLPP and DBTFNLPP are optimal solutions of the corresponding problems if and only if

\[
(c^T - y^*^T \tilde{A})x^* = R_b \tilde{0}.
\]

**Proof:** For a primal optimal solution \( x^* \), we have

\[
\tilde{A}x^* = R_b \tilde{b},
\]

or

\[
R_b(\tilde{A})x^* = R_b(\tilde{b}),
\]
also, for a dual optimal solution \( y^* \), we have
\[
y^* \bar{A} \leq R_b \bar{c}^T,
\]
or
\[
y^* R^b(\bar{A}) \leq R^b(\bar{c})^T,
\]
which can be put in equality form by adding nonnegative surplus variables as follows:
\[
y^* R^b(\bar{A}) + v^T = R^b(\bar{c})^T
\]  
(13)

Using (12), we get,
\[
y^* R^b(\bar{A}) x^* = y^* R^b(\bar{b}),
\]  
(14)
\[
y^* R^b(\bar{A}) x^* + v^T x^* = R^b(\bar{c})^T x^*.
\]  
(15)

Subtraction of (15) from (14) yields,
\[
v^T x^* = R^b(\bar{c})^T x^* - y^* R^b(\bar{b}) = 0,
\]  
(16)
where the latter equality follows from Theorem 3.1. Hence, using (14)-(16), we obtain
\[
y^* R^b(\bar{b}) = R^b(\bar{c})^T x^* \rightarrow y^* R^b(\bar{A}) x^* = R^b(\bar{c})^T x^* \rightarrow (\bar{c} - y^* \bar{A}) x^* = R^b 0,
\]
which completes the proof for the 'only if' part. Conversely, \((\bar{c} - y^* \bar{A}) x^* = R^b 0\), gives
\[
\bar{c}^T x^* = R^b y^* \bar{A} x^* \rightarrow \bar{c}^T x^* = R^b y^* \bar{b},
\]
to complete the proof.

\[\square\]

5. Application of Proposed Method in Real Life Problems

Akram [9] studied an application of bipolar fuzzy set in graph theory. He used bipolar fuzzy set in a social group. Here, we demonstrate an application of bipolar fuzzy number in maximum weighted matching problem; matching problem has some applications in different fields such as scheduling [24] and network [25] problems. We consider each vertex as a person and weight of each edge between two vertices shows the influence of each person (vertex) to another person. In general, influence can be positive or negative. Suppose \( G = (V, E) \) is an arbitrary weighted graph, where \( V = \{1, \ldots, n\} \) is the vertex set of \( G \) and \( E \subseteq V \times V \) is the edge set of \( G \). The maximum weighted matching problem is:

\[
\begin{align*}
\min \sum_{e \in E} \tilde{w}(e) x(e) \\
\text{s.t.} \\
\sum_{e = (u,v) \in E} x(e) \leq 1, \quad \forall u \in V, \\
x(e) \in \{0, 1\}, \quad \forall e \in E,
\end{align*}
\]

where \( x(e) = 1 \) if two persons \( u \) and \( v \) are matched to each other, and \( x(e) = 0 \), otherwise, and \( \tilde{w}(e) \) is the weight of edge \( e \) (giving the influence of one person to another person), considered as a bipolar fuzzy number, since influence of a person cannot always be positive. We want to match every person to another person so that they have a stable relation.
6. Conclusions and Future Work

We presented a linear ranking function for ordering bipolar fuzzy numbers and studied some of its properties. We defined bipolar triangular fuzzy number linear programming problems and established its dual. We then proved the common duality results for the primal and dual problems. Similar to fuzzy primal simplex algorithms (see [21,26]), simplex, dual simplex [21], VNS algorithm [27,28] and primal-dual algorithms can be developed for solving bipolar fuzzy linear programming problems.

Acknowledgements

The first and second authors thank the Research Council of Ferdowsi University of Mashhad and optimization laboratory of Ferdowsi University of Mashhad and the third author thanks the Research Council of Sharif University of Technology for supporting this work.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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