Phase diagram and Fermi-liquid properties of the extended Hubbard model on the honeycomb lattice

Wei Wu\textsuperscript{1} and A.-M.-S. Tremblay\textsuperscript{1,2}

\textsuperscript{1}Départment de Physique and RQMP, Université de Sherbrooke, Sherbrooke, Québec, Canada
\textsuperscript{2}Canadian Institute for Advanced Research, Toronto, Ontario, Canada

(Dated: December 20, 2013)

The Hubbard model and extended Hubbard model on the honeycomb lattice can be seen as prototype models of single layer graphene placed in a high dielectric constant environment that screens the Coulomb interaction. Taking advantage of the absence of a sign problem at half-filling, we study this problem with clusters up to 96 sites with the Determinant Quantum Monte Carlo Method as an impurity solver for the the Dynamical Cluster Approximation at finite temperatures. After determining the stability of the semi-metallic phase to interaction-induced spin-density wave (SDW), charge-density wave (CDW) and Mott insulating phases, we study the single particle dynamics of the Dirac fermions. We show that when spontaneous symmetry breaking is avoided, the semi-metallic phase is a stable Fermi liquid in the presence of repulsive interactions and that Kondo screening dominates the low temperature regime, even though there is a $\rho(\omega) = |\omega|$ type local density of states. We also investigate the impact of the correlation effects on the renormalization of the Fermi velocity $v_F$. We find that $v_F$ is not renormalized when only on-site repulsion $U$ is present, but that near-neighbor repulsion $V$ does renormalize $v_F$. This may explain the variations between different measurements of $v_F$ in graphene.

PACS numbers: 71.27.+a, 71.30.+h, 71.10.Fd

I. INTRODUCTION

Graphene, fabricated as a novel two dimensional material, is considered as a promising material for future electronics. In graphene physics, the Fermi velocity $v_F$ is an important quantity that determines various fundamental physical properties of this system.\textsuperscript{3} Generally speaking, the renormalization of $v_F$ from its bare value reflects the effective interaction strength in graphene.

In recent years, great progress has been achieved to experimentally measure $v_F$ in graphene. There are still puzzling questions however about the measured values of $v_F$ placed on different substrates. For graphene in vacuum, namely suspended graphene, D. C. Elias \textit{et al}\textsuperscript{2} found that $v_F$ increases logarithmically as one approaches the Fermi level, eventually reaching $v_F \approx 3.0 \times 10^6 m/s^{-1}$, more than triple the bare value near the charge neutrality point. This finding apparently confirms the occurrence of strong correlation effects in this system. This validates random phase approximation calculations\textsuperscript{3} based on the usual assumption that due to the poor dielectric screening, Coulomb interaction between electrons are long range and rather strong.\textsuperscript{4} Similar logarithmically renormalized $v_F$ is also observed in graphene on hexagonal boron nitride (hBN) surface\textsuperscript{5} though the largest $v_F$ detected there is only about $1.3 \times 10^6 m/s^{-1}$. Nevertheless, there are many other experiments that actually endorse weak or barely enhanced $v_F$ in graphene, regardless of the different substrate dielectric constants.\textsuperscript{6–8} In particular, by using angle-resolved photoemission spectroscopy (ARPES) method, a nearly ideal linear band structure was observed for isolated multilayers of graphene on SiC substrate\textsuperscript{5}, with a $v_F = 1.0 \pm 0.05 \times 10^6 m/s^{-1}$. This implies that correlation effects are irrelevant for the single particle dynamics in this system.

In this paper, we study this problem using the Hubbard and extended Hubbard model on the honeycomb lattice. One first needs to find the region of stability of the semi-metallic phase in parameter space, so we first determine the phase diagram. Most recently, both analytical and numerical studies of the phase diagram mainly focused on the Mott transition and possible emergent exotic quantum states.\textsuperscript{10–14} In particular, much attention has turned to the controversial topic of the existence of a $Z_2$ spin liquid phase laying between the semi-metal and the antiferromagnetic state.\textsuperscript{12–14} Here we focus on the different phase transitions and single particle dynamics of the interacting Dirac fermions that are relevant to the low-energy physics of single-layer graphene. We find in passing that the $Z_2$ spin-liquid is preempted by an antiferromagnetic quantum critical point.

Taking advantage of the absence of a sign problem at half-filling, we attack these problems by solving a cluster extension of dynamical mean field theory for clusters up to 96 sites using mostly a large scale determinant quantum Monte Carlo (DQMC) method as an impurity solver.

The model and method are described in the following section. The main results are presented in three subsections of Sec. III: First we find phase diagrams as a function of temperature and interaction strength, second we show that in the semi-metallic regime, the low temperature behavior is that of a Fermi liquid, and finally, we study the effects of interactions on the Fermi velocity $v_F$. The results are summarized and briefly discussed in Sec. IV.
II. MODEL AND METHOD

We investigate the extended Hubbard model on the honeycomb lattice defined by the Hamiltonian,

\[ H = -t \sum_{<i,j>,\sigma} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} + \sum_{i,j,\sigma,\bar{\sigma}} V_{ij} n_{i\sigma} n_{j\bar{\sigma}} - \mu \sum_i c^\dagger_{i\sigma} c_{i\sigma}, \]

where \( c^\dagger_{i\sigma} \) and \( c_{i\sigma} \) are creation and annihilation operators for fermions with site index \( i \) and spin index \( \sigma \), \( n_{i\sigma} = c^\dagger_{i\sigma} c_{i\sigma} \) is the density operator, \( t \) is hopping amplitude between nearest-neighbor sites \( \langle i,j \rangle \), while \( U \) is the on-site repulsion and \( V_{ij} \) is the Coulomb repulsion between occupied sites. We assume \( i > j \) to eliminate double counting of non-local repulsion \( V \). The standard Hubbard model corresponds to \( V_{ij} = 0 \). In our study, due to the limitations of the determinant quantum Monte Carlo (DQMC) impurity solver, we only cope with nearest-neighbor (NN) and the next-nearest-neighbor (NNN) repulsions. The chemical potential \( \mu \) is always chosen as \( -U/2 - \sum_j V_{jj} \) to fulfill the half-filling condition. We work in units where hopping \( t \) is unity.

The tight-binding limit \( (U,V_{ij} = 0) \) of this model contains the famous massless Dirac fermions at low energies which are extensively employed as a first approximation for studies of single layer graphene.\(^{1,17}\)

Here, we mainly employ the dynamical cluster approximation (DCA) method\(^{18}\) combined with large-scale DQMC impurity solver\(^{19}\) throughout our study. The prominent merit of DQMC as an impurity solver is that the quasi-linear scaling of computing time with respect to the inverse temperature\(^{19}\) \( \beta \) allows one to reach large system sizes at low temperature.

III. RESULTS AND ANALYSIS

We first present our results for the phase diagram and then discuss in the remaining two subsections the Fermi liquid regime and the influence of interactions on Fermi velocity renormalization.
A. Phase transitions

The finite temperature phase diagram for the standard Hubbard model on the honeycomb lattice, i.e., when $V_{ij} = 0$ and $U \neq 0$, is shown in Fig. 2. It was obtained with a 96-site DCA calculation. The transition point to the paramagnetic Mott insulator at zero temperature is estimated to be $U_{\text{Mott}} \approx 4.2t$ when paramagnetic conditions are artificially imposed. If we lift this restriction and allow for a spin density wave (SDW) transition, we find that it appears at $U_c^{\text{SDW}} \lesssim 3.6t$. A summary of estimates for the zero-temperature phase transitions found with other methods can be found in Refs. 11 and 22.

For the SDW phase transition our value $U_c^{\text{SDW}} \lesssim 3.6t$ is a bit smaller than the large scale quantum Monte Carlo results $U_c^{\text{SDW}} \approx 3.8$ in Ref. 13 and $U_c^{\text{SDW}} \approx 3.87$ in Ref. 14. The differences in the $U_c^{\text{SDW}}$ is a direct consequence of the mean-field nature of dynamical mean-field theory methods. We expect that $U_c^{\text{SDW}}$ for the semi-metal to antiferromagnetic transition would systematically approach the large scale QMC value upon increasing the cluster size of DCA.

In Ref. 13 it was found with large scale quantum Monte Carlo calculations that $U_{\text{Mott}} \approx 3.4$. This suggested a possible regime between $U_{\text{Mott}} \approx 3.4$ and $U_c^{\text{SDW}}$ where a spin-liquid phase could exist. In our calculation, the spin-liquid phase is pre-empted by the appearance of antiferromagnetism since our estimate for the critical value $U_{\text{Mott}} \approx 4.2t$ is larger than that found for the antiferromagnetic phase $U_c^{\text{SDW}} \lesssim 3.6t$. The non-existence of a zero-temperature spin-liquid phase for this model is in agreement with the results of larger scale Quantum Monte Carlo calculations$^{14}$ and with other cluster-in-a-bath calculations.$^{23}$ Convergence with cluster size is, however, subtle for this problem.$^{11,24}$

Since screening in graphene is not expected to be very effective because of the low density of states at the Fermi level, it is useful to consider the phase diagram in the presence of both on-site interaction $U$ and nearest-neighbor inter-site interactions $V$. This reveals a competition between the antiferromagnetic SDW phase and the staggered charge density wave (CDW) phase that has non-equivalent electron density on the A/B sublattices and hence also breaks chiral symmetry, like the antiferromagnetic phase.

The phase diagram is displayed in Fig. 3. When $U, V \gg t$, the physics is controlled mostly by the competition between $U$ and $V$. In that limit, as expected from simple classical arguments on potential energy, a phase transition between SDW and CDW occurs around $U \sim zV$, where $z$ is the coordination number of the lattice, namely $z = 3$ here. Consistent with this result, one has $U_c \approx 4V$ on the 2d square lattice$^{25}$ and $U_c \approx 2V$ on the 1d chain$^{26}$. However, for small $U$ and $V$, the hopping term $t$ takes part in the competition, resulting in a CDW to semi-metal (SM) transition at $U < zV$. The phase transition ends at $U = 0$, $V = 0.45$, where the charge density fluctuations solely drive a CDW transition.

It is notable that the phase transitions involving the CDW, namely the SDW/CDW and SM/CDW phase transitions, are all of first-order while the SM/SDW transition is continuous. This behavior is reminiscent of the analogous phase transitions on the 2d square lattice$^{27}$ but is different from the 1d chain case$^{26–28}$ where the transition eventually becomes continuous as $U, V$ decrease. Note that in mean-field theory$^{29}$ the SM/CDW transition was found to be continuous, in contrast with our more reliable calculation.

B. Fermi liquid

While the critical point for the Mott transition in Fig. 2 control a large area of the bad insulator and bad metal region at finite temperatures, it actually does not engulf the whole small $U$ regions of the phase diagram. In other words, a semi-metallic state persists at low temperatures against the formation of local moments when $U$ is sufficiently small that the Mott and the SDW transitions are avoided.

Is this semi-metallic state a Fermi liquid? It has been argued that owing to the linearly vanishing low energy density of states, the Hubbard model on the honeycomb lattice may possess an under-screened fixed point$^{30}$. This
FIG. 4. (Color online) $Z_T(k)$ as a function of temperature $T$ for different $U$ of the Hubbard model. Polynomial fitting of the $U_c = 4.2$ line suggests where the system becomes insulator. For $U < 4.2$, the quasi-particle residue is finite hence suggesting a Fermi liquid state. A 24-site DCA + CTQMC calculation was also used at $U = 3.5$ to reach the very low-temperature properties of $Z_T(k)$.

FIG. 5. (Color online) A sketch of the density of states of the conduction band of the effective Anderson impurity model obtained from DMFT on the honeycomb lattice. 300 bath sites are used to parameterize the effective Anderson impurity model. The spectrum of bath-level energies is displayed with a broadening factor $\delta = 0.08$.

argument seems to agree with numerical renormalization group analysis,\textsuperscript{31,32} which shows that when an Anderson impurity site couples to a pseudogap conduction band with density of states $\rho \sim |\omega|^r$, the local moments on the impurity sites cannot be fully screened out even at zero temperature when $r > \frac{1}{2}$, because the itinerant electrons are exhausted. Based on this argument, a non-Fermi liquid should be expected on the honeycomb lattice, since at low-energies it has $r$ equals 1.

However, our DCA simulations unambiguously suggest the existence of a Fermi liquid at low temperatures. In order to clarify this paradox for the single particle behavior of the Dirac fermions, we investigate the following physical quantity\textsuperscript{33},

$$Z_T(k) = \frac{1}{1 - \frac{\text{Im} \Sigma(k, \pi T)}{\pi T}},$$

which becomes the quasiparticle renormalization factor in the limit $T \to 0$. For a Fermi liquid phase, this quantity stays finite at zero temperature, in contrast with the marginal Fermi liquid or Mott insulator where $Z_T(k)$ goes to zero in the limit $T \to 0$. Fig. 4 shows $Z_T(k)$ as a function of $T$ in the semi-metallic phase (no SDW allowed) for various values of $U$ at the Dirac point $K$. From this, we can see that for $U < 4$ the quantity $Z_T(k)$ varies slowly as the temperature decreases reflecting the Fermi liquid characteristic. By further increasing $U$, $Z_T(k)$ becomes dramatically suppressed as temperature decreases and at $U_{\text{Mott}} = 4.2$ one observes a sharp drop to zero, suggesting a Mott transition for a value of $U$ which is in good agreement with the position of the transition point shown on the phase diagram Fig. 2, where the emerging finite single-particle gap was used to determine $U_{\text{Mott}}$.

We can gain insight into this apparent contradiction between the under-screened fixed point that exists in the Anderson impurity problem and our Fermi-liquid result by studying the basic difference between the effective cluster model of DCA and the pseudogap Anderson/Kondo model. Without loss of generality, we consider a single-site DMFT effective model\textsuperscript{34} which can be parameterized as an impurity site coupled homogeneously to a conduction band. The DMFT self-consistent equation requires the local density of states on the impurity site to be just the same as that of the lattice model, thus on Fermi level it must hybridized strongly to the conduction band to obtain a semi-metallic behavior in the local density of states. As a result, the effective conduction band of the DMFT impurity model must have a large density of states at the Fermi level (see Fig.5) which is in contrast to the pseudogap Anderson impurity model. Consequently, we conclude that the under-screened fixed point scenario of pseudogap Anderson impurity model does not apply to the Hubbard model on the honeycomb lattice.

In order to find out whether the Fermi liquid in this many-body system can be broken down by correlation effects, we plot in Fig. 6 the imaginary part of the self-energy $\Sigma(i\omega_n)$ at the Dirac point $K$ as a function of Matsubara frequency. Due to particle-hole symmetry, the real part of $\Sigma_K(i\omega_n)$ vanishes in our study while the imaginary part should scale linearly in the low-energy region if the system preserves the Fermi-liquid behavior. We investigate the effect of on-site $U$, nearest-neighbor $V$ and next-nearest-neighbor $V'$ interactions on the self-energy.
We begin with Fig. 6a, where only $U$ differs from zero. In this case, the system is governed by spin fluctuations and hence competition between Kondo screening and RKKY correlations dominates the physical properties. This is reflected by the evolution of the self-energy, as we now show. As the interaction increases, there is a gradual transition from the Fermi liquid ($U = 2.0, 3.5, 4.0$) state to a crossover region ($U = 4.2$), to a bad insulator ($U = 4.4$) state and finally to a Mott insulator at $U = 4.5$. The signature of the transition to the Mott insulating state is not as sharply defined as in our earlier approach.

To study the effects of the charge density fluctuation on the single-particle dynamics, we consider the effect of $V$ in Fig. 6b and Fig. 6c. No breakdown of Fermi-liquid behavior is observed for all of the parameters, as long as the SDW/CDW phase transitions are avoided. From Fig. 6c we also see clearly that even in the presence of both $U$ and $V$, we recover a Fermi liquid behavior. We also checked for on-site Hubbard interaction, $U = 2$ and $U = 3.5$, that stronger inter-site interaction $V$ decreases the magnitude of the self-energy. Note that when $U, V$ are weak in Figs. 6b,c one observes kinks on the self-energy caused by the charge-density fluctuations. These kinks eventually fade out when $U, V$ become strong. Fig. 6d shows that when $V'$, namely next-nearest-neighbor repulsion, is turned on to destroy the CDW order, the Fermi liquid immediately recovers.

Finally, note that although the RPA calculations with full Coulomb potential suggest a marginal Fermi liquid (MFL) in graphene, a more sophisticated RG analysis shows that the MFL is ultimately avoided when the running of the interaction parameters is taken into ac-

---

**FIG. 6.** (Color online) Imaginary part of the self-energies as a function of Matsubara frequency at the Dirac point $K$. (a) For the Hubbard model ($V = V' = 0$) at $T = 1/20$. (b) For various values of the nearest-neighbor repulsion $V$ at $T = 1/20$ (c) For both on-site $U$ and nearest-neighbor repulsion $V$ at $T = 1/20$ (d) For on-site $U$, nearest-neighbor repulsion $V$ and next-nearest-neighbor repulsion $V'$ at $T = 1/30$.

C. Fermi velocity and correlation effects

In graphene the Fermi velocity $v_F$ is an important physical quantity which defines the low-energy effective theory of the Dirac fermions. For the non-interacting case, the Fermi velocity $v_F$ is related to the local density of states (LDOS) via

$$\rho(E) = \frac{3\sqrt{3}a^2|E|}{\pi} \times \frac{1}{v_F^3}$$

(2)

where $a$ is the lattice constant. The Fermi velocity $v_F$ is usually estimated as $10^6 m/s$ for the tight-binding model. Recent studies suggest that the Fermi velocity is renormalized in suspended graphene. In this subsection, we investigate how interactions renormalize this velocity.

In order to obtain the LDOS in real frequency space, we employ the maximum entropy method (MEM) to perform the numerical analytical continuation of the imaginary-frequency local Green’s function $G(i\omega_n)$. In Fig. 7a, we first show the LDOS for DCA with only the on-site interaction. We observe that while the van Hove singularity is rounded and the high-energy LDOS decreases in magnitude, spreading beyond the non-interacting bandwidth, the LDOS in the vicinity of Fermi level stays constant as the interaction strength increases. In other words, when only the Hubbard interaction is considered, large cluster DCA calculations demonstrate that $v_F$ is insensitive to the interaction strength, sticking to the tight-binding model value of...
$10^6 \text{m/s}$. This is in agreement with the previous cellular DMFT (CDMFT) result\textsuperscript{10} and RG analysis\textsuperscript{38}. We note that studies with DCA on smaller clusters\textsuperscript{10} suggest that $v_F$ decreases as the Hubbard interaction increases. In contrast, CDMFT is able to find the correct invariable $v_F$ even in the case of small clusters\textsuperscript{10}. This observation is in agreement with previous benchmark calculations that suggest that on small clusters, CDMFT converges faster than DCA for local quantities such as the LDOS\textsuperscript{39,40}. Limitations of DCA have been discussed in this context in Ref. \textsuperscript{11}. In Fig. 7b, we compare the LDOS when only nearest-neighbor repulsion $V = 0.3$ is taken into account with the LDOS for $U = 3.5, V = 0$. As we can see, for $U = 3.5$ the LDOS almost superposes with the non-interacting result at low-energies, whereas the $V = 0.3$ curve is significantly below the non-interacting result, suggesting an increased $v_F$. This dramatic difference between the correlation effects of local and non-local repulsions on the single-particle dynamics leads us to conclude that the observed increase of $v_F$ in suspended graphene\textsuperscript{2} should be attributed to the long range Coulomb potential rather than to the local Hubbard repulsion.

IV. SUMMARY AND DISCUSSION

Using clusters as large as 96 sites with the Dynamical Cluster Approximation, we have found the phase diagram of correlated electrons on the honeycomb lattice at half-filling. SDW, CDW and Fermi-liquid semi-metallic phases are present when on-site interaction $U$ and nearest-neighbor repulsion $V$ are taken into account.

As a function of temperature and on-site interaction $U$ (for $V = 0$) the semi-metal turns into a state where the A and B sub-lattices order antiferromagnetically with respect to each other at low temperature when $U$ increases. At finite $T$, this antiferromagnetic transition should be interpreted as a crossover to the renormalized classical regime since in two dimensions antiferromagnetic long-range order can occur only at $T = 0$, as required by the Mermin-Wagner theorem. The zero-temperature Mott transition is masked by the antiferromagnetic phase so that there is no spin-liquid regime.

At low temperature, the effect of a large enough near-neighbor repulsion $V$ is to induce a staggered CDW phase where A and B sublattices exhibit an excess of either electrons or holes, depending on the sublattice. The transition line between the SDW and staggered CDW phases is roughly determined by $U \approx 3V$ when $U$ is significantly larger than the hopping amplitude $t$. The transition to the CDW phase is always first order but the semi-metallic to SDW transition is continuous.

We have also investigated in detail the properties of the Fermi-liquid phase. As temperature decreases, the high-temperature bad metal phase screens off the local moments that eventually evolve into a Fermi-liquid phase at low temperature, despite the semi-metallic nature of the density of states. We have shown that this occurs because the self-consistent bath is in fact metallic. This Fermi-liquid state is stable even in the presence of non-local Coulomb interactions, at least up to next-nearest-neighbor repulsion $V'$, as long as the mutual competitions between repulsive potentials do not order the system.

Moreover, we demonstrated that the short-ranged interaction $U$ does not lead to a measurable renormalization of the Fermi velocity $v_F$ close to Fermi level. However, $v_F$ is clearly renormalized in the presence of $V$. This could explain the variations of $v_F$ observed in different experiments on graphene. Since $v_F$ is not sensitive to the short-ranged repulsion, graphene placed in high dielectric-constant environment should have a Fermi velocity less effected by correlation effects than graphene in vacuum or on a substrate with small dielectric constant $\epsilon$, which is expected to have a strongly renormalized $v_F$.

ACKNOWLEDGMENTS

W. W. would like to thank Ansgar Liebsch for useful discussions about the parameterization algorithms of DCA. This work was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC), and by the Tier I Canada Research Chair Program (A.-M.S.T.). Simulations were performed on computers provided by CFI, MELS, Calcul Québec and Compute Canada.

1. A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, Rev. Mod. Phys. 81, 109 (2009).
2. D. Elias, R. Gorbachev, A. Mayorov, S. Morozov, A. Zhukov, P. Blake, L. P. I. Grigorieva, K. Novoselov, F. Guinea, and A. Geim, Nature Physics 7, 701 (2011).
3. S. Das Sarma, E. H. Hwang, and W.-K. Tse, Phys. Rev. B 75, 121406 (2007).
4. T. O. Wehling, E. Šašošču, C. Friedrich, A. I. Lichtenstein, M. I. Katsnelson, and S. Blügel, Phys. Rev. Lett. 106, 236805 (2011).
5. J. Chae, S. Jung, A. F. Young, C. R. Dean, L. Wang, Y. Gao, K. Watanabe, T. Taniguchi, J. Hone, K. L. Shepard, P. Kim, N. B. Zhitenev, and J. A. Strocio, Phys. Rev. Lett. 109, 116802 (2012).
6. J. Martin, N. Akerman, G. Ulbricht, T. Lohmann, J. Stret, K. Von Klitzing, and A. Yacoby, Nature Physics 4, 144 (2007).
7. M. Yankowitz, J. Xue, D. Cormode, J. D. Sanchez-Yamagishi, K. Watanabe, T. Taniguchi, P. Jarillo-Herrero, P. Jacoby, and B. J. LeRoy, Nature Physics 8, 382 (2012).
J. P. Reed, B. Uchoa, Y. I. Joe, Y. Gan, D. Casa, E. Fradkin, and P. Abbamonte, *Science* **330**, 805 (2010).

M. Sprinkle, D. Siegel, Y. Hu, J. Hicks, A. Tejeda, A. Taleb-Ibrahimi, P. Le Fèvre, F. Bertran, S. Vizzini, H. Enriquez, S. Chiang, P. Soukiassian, C. Berger, W. A. de Heer, A. Lanzara, and E. H. Conrad, *Phys. Rev. Lett.* **103**, 226803 (2009).

W. Wu, Y.-H. Chen, H.-S. Tao, N.-H. Tong, and W.-M. Liu, *Phys. Rev. B* **82**, 245102 (2010).

A. Liebsch and W. Wu, *Phys. Rev. B* **87**, 205127 (2013).

S. Sorella, Y. Otsuka, and S. Yunoki, *Scientific reports* **2** (2012).

R. Blankenbecler, D. J. Scalapino, and R. L. Sugar, *Phys. Rev. D* **24**, 2278 (1981).

E. Loh Jr and J. Gubernatis, Modern Problems of Condensed Matter Physics **32**, 177 (1992).

E. Khatami, C. R. Lee, Z. J. Bai, R. T. Scalettar, and M. Jarrell, *Phys. Rev. E* **81**, 056703 (2010).

A. N. Rubtsov, V. V. Savkin, and A. I. Lichtenstein, *Phys. Rev. B* **72**, 035122 (2005).

M. Hettler, A. N. Tahvildar-Zadeh, M. Jarrell, T. Pruschke, and H. R. Krishnamurthy, *Phys. Rev. B* **58**, R7475 (1998).

M. Aichhorn, H. G. Evertz, W. von der Linden, and M. Potthoff, *Phys. Rev. B* **70**, 235107 (2004).

B. Fourcade and G. Sponken, *Phys. Rev. B* **29**, 5089 (1984).

S. Raghu, X.-L. Qi, C. Honerkamp, and S.-C. Zhang, *Physical Review Letters* **100**, 156401 (2008).

R. Bulla, T. Pruschke, and A. Hewson, *Journal of Physics: Condensed Matter* **9**, 10463 (1997).

K. Chen and C. Jayaprakash, *Journal of Physics: Condensed Matter* **7**, L491 (1995).

C. Gonzalez-Buxton and K. Ingersent, *Phys. Rev. B* **57**, 14254 (1998).

S. Arya, P. Sriluckshmy, S. Hassan, and A.-M. S. Tremblay, Unpublished (2014).