An Adaptive GNSS Attitude Determination Algorithm Based on Multiplicative Error Quaternion

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Abstract. To further improve the multi-antenna instantaneous attitude observation performance and optimize the ambiguity and cycle slip processing in attitude determination using GNSS carrier phase observations, an adaptive attitude estimation algorithm based on multiplicative error quaternion is proposed in this paper. The algorithm integrates the attitude constraint information by using state and observation models, and combines the adaptive factors and robust equivalent weights with cycle slips, thereby improving the ambiguity float solution and its covariance structure. The simulation results show that the proposed algorithm can increase the yaw estimation precision and the data processing flexibility while ensuring outstanding real-time performance.

1. Introduction

GNSS attitude observation technology has gradually become a research focus in the GNSS application field due to its advantages of high precision, strong real-timeliness, small size, low cost and short initialization time[1-2]. Since the GNSS attitude observation system contains priori constraint information, it is important for data processing researchers to resolve the instantaneous integer ambiguity and enhance the attitude observation precision on the basis of the constraint information [3]. Currently, attitude observation method is mainly multi-baseline observation way, take the MC-LAMBDA (Multivariate Constrained Least-square Ambiguity Decorrelation Adjustment) for example. The MC-LAMBDA method has low ambiguity search efficiency, which affects the instantaneous attitude observation performance. In terms of the processing of GNSS attitude observation data, the single-epoch least-square method is adopted to avoid cycle slips and ineffective utilization of observation information[4-5]. The filtering method strikes an effective balance between historical information and current observation information, thereby playing an important role in GNSS instantaneous attitude observation[6-8].

This paper presents a carrier phase observation model and state model based on multiplicative error quaternion. In addition, it introduces an adaptive filter to estimate ambiguity, thereby eliminating cycle slips. The algorithm effectively improves the ambiguity float solution by using the constraint information and historical information, and realizes the quick search of fixed solutions by using the LAMBDA method. Finally, simulation experiments are carried out to verify the effectiveness of the algorithm.

2. Multiplicative error quaternion filter model

Based on the attitudes of visible satellites observed by GNSS antennas $j_1$ and $i$, the observation
equation in matrix form can be summarized as

$$
L_j = A_{j1}\phi + M_{j1}\nabla \Delta N_{j1} + e_{j1, \nabla \Delta \phi} \quad (1)
$$

Where

$$
L_j = L_{q, j} - B_j - \left[ G_{q, j} C_n b_{ji} \right]; \quad A_{j1} = G_{q, j} C_n \left[ -\left( b_{ji} \times \right) \right]; \quad G_{q, j} \text{ is the observation design matrix constituted by the direction cosine}
$$

$$
LA M Ne
$$

$$
(1)
$$

$$
M_{j1} \text{ is the coefficient matrix of the ambiguity} \quad N_{j1}; \quad C_n \text{ is the attitude matrix. The setting details are shown in reference [9]. When a multiplicative error quaternion } \delta Q = [\delta q_0, \delta q]^T, \text{ carrier phase observation equation of the baseline multiplicative error quaternion can be obtained under the condition of small error angles}
$$

$$
L_j = A_{j1}\delta q + M_{j1}\nabla \Delta N_{j1} + e_{j1, \nabla \Delta \phi} \quad (2)
$$

It can be inferred from the relations among quaternion differential equation, attitude matrix and attitude quaternion that

$$
\delta \hat{Q} = \frac{1}{2} \omega_{nb} \otimes \delta Q - \frac{1}{2} \delta Q \otimes \hat{\omega}_{nb} = \frac{1}{2} \omega_{nb} \otimes \delta Q - \frac{1}{2} \delta Q \otimes \hat{\omega}_{nb} - \frac{1}{2} \delta \omega_{nb} \delta Q \quad (3)
$$

Where \( \omega_{nb} \) is the projection of the angular velocity of the carrier coordinate system relative to the local horizontal coordinate system in the carrier coordinate system; \( \otimes \) represents quaternion multiplication; \( \hat{\delta Q} = Q \otimes \hat{Q} \) (\( \hat{Q} \) is the conjugate quaternion of \( \hat{Q} \)). According to the quaternion multiplication principles, the state model in multiplicative error quaternion form can be expressed as

$$
\delta \hat{q} = (\hat{\omega}_{nb} \times) \delta q - \frac{1}{2} \delta \omega_{nb} \otimes \delta Q \quad (4)
$$

Based on the above observation and state models, the filter algorithm based on carrier phase observation estimates at time \( k \) can be obtained

$$
X_k = \Phi_{k, k-1} X_{k-1} + W_k = \left[ I - (\phi \times) \right]_{k-1} 0 \right] \phi \right] + \left[ w_\phi \right] \quad (5)
$$

$$
L_k = A_k X_k + e_k \quad (6)
$$

Where \( X \) is the state vector; \( \Phi_{k, k-1} \) is the state transition matrix between time \( k \) and time \( k-1 \); \( W \) is the state noise; \( e \) is the observation noise.

3. Adaptive GNSS attitude determination algorithm

The state prediction information can be trusted when attitude estimation precision in the previous epoch is high. Otherwise, an adaptive factor should be adopted to reduce the contributions of state information. The three parameters attitude error, angular velocity and ambiguity belong to different categories, which indicate the necessity of adaptively robust filter with classified factors. The adaptive filter with classified factors can be expressed as

$$
\hat{X}_k = \left[ A_k \bar{P}_k A_k + \bar{P}_{s_1} \right]^{-1} \left[ A_k \bar{P}_k L_k + \bar{P}_{s_1} \hat{X}_k \right] \quad (7)
$$

$$
P_{\hat{X}_k}^{-1} = \left[ A_k \bar{P}_k A_k + \bar{P}_{s_1} \right]^{-1} \quad (8)
$$

Where \( \bar{P}_k \) is the robust equivalent weight matrix; \( \bar{P}_{s_1} \) is the adaptive weight matrix of state prediction (\( \bar{P}_{s_1} = a^2 \bar{P}_k \bar{P}_k \)); \( a_s \) is the adaptive factor matrix (\( a_s = \text{diag}[\alpha_\phi, \alpha_\omega, \alpha_N] \)). The setting details are shown in reference [10]. Since \( \omega \) is an indirect observation, there is no need for
adaptive processing. In addition, $\alpha_a = 1$. $\alpha_N$ can adjust the weight of priori information. When the maximum cycle slip estimate is $\hat{d}N$, the adaptive factor can be selected in consideration of the noise caused by cycle slip detection as

$$\alpha_N = \frac{\sigma_N^2}{dN^2 + \sigma_{DT}^2}$$  (9)

Where $\sigma_N$ is the standard deviation for ambiguity prediction and determined by the set state noise; $\sigma_{DT}$ is the quadratic mean deviation of cycle slip detection. The larger the cycle slip estimate is, the smaller the adaptive factor of ambiguity will be. Generally, $\alpha_N < 1$. (in this paper) should be extremely small, so as to avoid cycle slip detection and therefore facilitate instantaneous calculation. In this paper, the priori information is reduced to an extremely small value, thereby being free from the influence of cycle slips.

Commonly used error discriminate statistics in adaptive filter include state discrepancy and prediction residual. However, it is impossible to obtain sufficiently accurate values of the two parameters before the ambiguity is fixed. Considering that the attitude error prediction performance is essentially determined by the nominal solution (attitude filtering precision in the previous epoch), its reliability can be reflected by the corresponding ratio value to a certain extent. $\alpha_\phi$ can be represented by piecewise function as

$$\alpha_{\phi,k} = \begin{cases} 
0 & \text{Rat}_{k-1} \leq c_0 \\
\text{Rat}_{k-1} - c_0 & c_0 \leq \text{Rat}_{k-1} < c_1 \\
1 & \text{Rat}_{k-1} \geq c_1 
\end{cases}$$  (10)

Where $\text{Rat}$ is the ratio value; $c_0 = 2$; $c_1 = 3$.

4. Simulation experiment

The simulation experiments were carried out on a simulation platform established in MATLAB 2014a, producing a total of 1400 epochs of observation data. The GNSS data reception frequency was 1Hz. Since most of the ionospheric and tropospheric errors were eliminated by using the differences method, the residual errors were multipath errors and thermal noise errors. The two independent baselines, whose baseline carrier coordinates were $[0.0 \ 10.0 \ 0.0]^T$ and $[15.0 \ 30.0 \ 0.0]^T$, were composed of three GNSS antennas separately. The satellite cut-off elevation angle was 15°. Figure 1 displays the number of visible satellites and the value of PDOP (Position Dilution of Precision) during the observation period. Yaw changes of the simulated carrier are shown in Figure 2. It can be seen from Figure 2 that yaw changes take place in the range of 1700s ~ 2400s.

![Figure 1. Visible satellite and PDOP](image1)

![Figure 2. Simulation yaw](image2)
The experiments involved three different solution schemes: Scheme 1 was the single-epoch least-square method; Scheme 2 was the filter method in Literature [6]; Scheme 3 was the adaptive attitude estimation method proposed in this paper. During attitude calculation, the LAMBDA method was adopted for ambiguity fixed solution search. The successfully fixed ratio threshold was set to 3. The ambiguity fixing results are shown in Table 1. Figures 3 and 4 demonstrate the yaw estimates and ADOPs (Ambiguity Dilution of Precision) respectively.

| Scheme | Number | Fix rate | Time-consuming(s) |
|--------|--------|----------|-------------------|
| 1      | 1148   | 82%      | 18.6              |
| 2      | 1165   | 83.2%    | 15.3              |
| 3      | 1287   | 91.9%    | 12.6              |

The results show

(1) All the three schemes can achieve GNSS attitude determination and complete ambiguity processing. In addition, the success rates of ambiguity fixing are all greater than 80% and the yaw calculated are roughly the same. But ADOP has obvious advantages in Scheme 3.

(2) Scheme 2 enjoys a higher success rate of ambiguity fixing than Scheme 1, since the filter method can effectively utilize the historical and constraint information to improve the ambiguity fixing efficiency. The adaptive method in Scheme 3 is based on adaptive factors to utilize historical and constraint information. The method improves the overall structure, greatly reduces the ADOP value and effectively resists the impacts of gross errors, thereby significantly increasing the success rate of ambiguity fixing.

(3) The fixed solution curve is smoother than the float solution curve in Scheme 3. In the meantime, the results are more consistent with the simulation results, indicating that the proposed algorithm is significantly superior to traditional algorithms in filtering precision.

(4) In comparison with the least-square method, the adaptive method is more time-consuming in calculating float solutions. However, it is superior in solution precision. As a result, the adaptive method in scheme 3 enjoys quicker ambiguity search and higher overall efficiency.

![Figure 3. Yaw of different schemes](image)
Figure 4. ADOP of different schemes

Table 2. Analysis of RMSE and calculation time

| Scheme | 1400 epochs | | | 3600 epochs | | |
|---|---|---|---|---|---|---|
| | Yaw (°) | Position(m) | Time (s) | Yaw (°) | Position(m) | Time (s) |
| 1 | 3.323 | 5.026 | 19.1 | 4.291 | 7.031 | 40.2 |
| 2 | 3.181 | 4.940 | 15.8 | 3.994 | 6.823 | 34.1 |
| 3 | 2.975 | 4.122 | 12.9 | 3.698 | 6.013 | 27.3 |

To further verify the performance of the algorithm proposed, data of different epoch lengths was selected for testing. It can be seen from Table 2 that the yaw estimation precision gradually decreases with the increase in data amount. Scheme 3 was always superior to the other two schemes. Specifically, the attitude estimation precision of Scheme 3 was 11% higher than that of Scheme 1 when there were 1400 sets of data. The difference was even more pronounced when the number of datasets reached 3000 (about 14%). Due to the correlation between position solution and attitude precision, attitude angle precision has a significant impact on positioning, which indicates that Scheme 3 has better stability in long-term navigation calculation. In addition, it consumes less time and enjoys higher instantaneous attitude observation precision.

5. Conclusion

This paper proposes an adaptive attitude observation algorithm based on multiplicative error quaternion, so as to further study the instantaneous high-precision attitude observation technology based on GNSS carrier phase observation. The method can directly calculate error quaternion. The state and observation models are established by using the filtering method, which makes full and effective use of the constraint and historical information. The filtering method also reduces the weight of ambiguity, thereby improving the ambiguity float solution and its covariance structure. The LAMBDA method is adopted for quick fixed solution search. It increases the flexibility, reliability and efficiency of GNSS attitude observation data processing. In summary, this paper provides engineering reference for future researches on the application of the multi-antenna GNSS attitude observation system.

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