Reanalysis of two eclipsing binaries: EE Aqr and Z Vul

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Abstract We study the radial-velocity and light curves of the two eclipsing binaries EE Aqr and Z Vul. Using the latest version of the Wilson & Van Hamme (2003) model, absolute parameters for the systems are determined. We find that EE Aqr and Z Vul are near-contact and semi-detached systems, respectively. The primary component of EE Aqr fills about 96% of its 'Roche lobe', while its secondary one appears close to completely filling this limiting volume. In a similar way, we find fill-out proportions of about 72 and 100% of these volumes for the primary and secondary components of Z Vul respectively. We compare our results with those of previous authors.

Keywords Variable stars- Binaries- Eclipsing binary

1 Introduction

Eclipsing binary stars studies often involve the combination of photometric (light curve) and spectroscopic (mainly, radial velocity curve) data. The analysis of the light-velocity curves enable astronomers to obtain absolute physical parameters describing the system and its components. The physical parameters derived from the photometric and spectroscopic data can help to improve our understanding of physical processes in stars. In view of the importance of EB-EA type binary stars in the problem of mass exchange in binaries and in the theory of their evolution, the study of these systems play an important role in finding of the complex initial phases of stellar evolution. In order to understand the physical nature of EB and EA type binaries, two candidates, EE Aqr and Z Vul, were selected based on the following criteria: a) the shape of the light curve, b) their photoelectric measurements are not enough and also somewhat the analysis results derived by the previous authors appear to differ from each other. The first, we state their history in literature and then describe our solution method.

1.1 EE Aqr

The binary system EE Aqr (HD213863, BD-20\textdegree6454, SAO 191236, \( \alpha_{2000} = 22^h34^m41^s.87, \delta_{2000} = 19\textdegree51'34''\)) was discovered to be variable by Strohmeier & Knigge (1960). Strohmeier et al. (1962) derived its eclipsing behavior with a orbital period of 0.51 d and classified it as an Algol (EA) type eclipsing binary. Later, the photoelectric observations were obtained by Williamson (1974), Padalia (1979) and Covino et al. (1988). Williamson (1974) who showed the light curves to be more like that of \( \beta \) Lyrae (EB) type with a period of 0.5089951 d and analyzed these using the method of Russell & Merrill (1952). Thus preliminary photometric orbital parameters of the system were derived. Slightly different orbital parameters were determined by Padalia (1979) for the system using the same method. Russo & Solazzo (1982) found many inconsistencies in applying the Russell and Merrill (1952) method of solution. Therefore they reanalyzed both \( V \) light curves of Williamson (1974) and Padalia (1979) using the Wilson & Devinney (1971) model. They obtained the same results for
both light curves while the Russell-Merrill model does not. They found a solution as a semi-detached system. The system has also been studied spectroscopically by Hilditch & King (1988), from which radial-velocities were measured for both components by means of the cross-correlation code VCross. The first simultaneous solution of photometric and radial-velocity curves was performed by Covino et al. (1990) based on the Wilson & Devinney (1971) model. Their solution indicated that EE Aqr might be a contact configuration not yet in thermal equilibrium. From 1990 to now, no analysis of light curves and new photometric observations has been reported.

The period variations of the system has studied by Srivastava (1987) who found irregular changes of the order of $10^{-6}$ $d$ and many observations of the times of minima (Mallama (1980), Covino et al. (1988), Deeg et al. (2003)) so far.

The spectral classes of the components of EE Aqr were given as F0 (Williamon (1974)), FV+A (Padalia (1979)), A8V+(K3-K4)/(Russo & Sollazzo (1982)).

1.2 Z Vul

The variability of Z Vul (HD181987, $\alpha_{2000} = 19^h 21^m 39^\circ.11, \delta_{2000} = 25^\circ 34'29'' .45$) was discovered to be an eclipsing binary by Herschel (It has been reported by Astbury, 1909). Plaskett (1920), Petrie (1950), Roman (1956), Popper (1957b) and Cester et al. (1977) determined the spectral type of the components as B3, B4+B6, B5V+A, B3-4V+A2-3III and B2V+A1III respectively. The first researches on the system presented its eclipsing behavior with an orbital period slightly less than 2.5 $d$ and classified it as an Algol (EA) type eclipsing binary. Cester et al. (1977) analyzed Broglia’s photometric observations (1964) using the Wilson program (Wood, 1972) and determined slightly different masses, radii and luminosities for the system with those found in the literature. They found a solution as a semi-detached system.

Peters (1994) presented the first IUE observations to investigate the effect of the cross-section and temperature of the primary on the mass transfer and mass loss in the system. They found that the primary involves weak winds, originating from the gas stream will strike the primary instead of forming an accretion disk. The lack of H-alpha emission in system supports this proposition.

The first results from far-UV observations obtained by Peters & Polidan (1997) about the nature of the circum stellar material in the system. Their observations showed no infalling from a gas stream. So it is consistent with this fact that the system is sufficiently close and can not establish a disk.

It is recognized that depend on applying present models, we can obtained the same or different results of analyzing light-velocity curves and thus it follows an low or high understanding of stellar evolution. Therefor it is need to research a tool for a better comprehension of the binaries structure and evolutive stage.

In this study the authors adopted the more realistic close binary model based on the Roche model. In order to obtain accurate solutions in this situation, the light curves were analyzed by means of the latest the Wilson’s computer code. The computing code used was developed by the Wilson & Van Hamme code (2003)(here after WV) to determine the parameters of the system. We selected it as our analysis research tool both for its intrinsic virtues and because of some improvements in comparison with earlier code (Wilson & Devinney, 1971). The fourth revision (2003) is improved for example based on Kurucz’s new atmospheres, log g as a parameter (allowing for handling giants, sub-giants, etc., in addition to main sequence stars) so that temperature ranges vary according to log g together with 19 abundances (relative to the sun). Hence we felt it useful to reanalyze these systems and get the most reliable elements with the model of WV, which was developed especially to handle close systems and compare our finding with results of the previous authors.

All the data we analyze in the paper are taken from the literature so that these data appeared to be free from any peculiarities reported by the previous researches. We consider photometric observations of Williamon (1974) (here after WI) and spectroscopic observations of Hilditch & King (1988) (here after HK) for EE Aqr and the photometric observations of Broglia (1964) (here after BR) and spectroscopic observations of Popper (1957b) (here after PO) for Z Vul.

The present paper is organized as follows. The assumptions are described in section 2. Photometric solutions of light curves is given in section 3. Spectroscopic solutions of radial velocities curves in section 4. Absolute elements for the primary and secondary components in section 5 and in section 6, we give conclusion.

2 Assumptions

The latest 2003 version of the Wilson program was applied for photometric and spectroscopic solutions. The method assumes the star surfaces to be equipotential and uses a set of curve-dependent or curve-independent parameters that can be adjusted by $LC$ and $DC$ programs: the orbital inclination $i$, surface potentials $\Omega_1, \Omega_2$, the mean surface effective temperatures $T_{1,2}$, the mass ratio $q = m_2/m_1$, the bandpass luminosities $L_{1,2}$, the
Initially, the light curve program (LC) was implemented in mode 2 with no third light (based on the
spectroscopic observations, HK and PO) corresponding to detached configuration by choosing $i, T_2$ (while
the temperature of the primary component was assumed from spectroscopic or color data), $\Omega_{1,2}$ and $L_1$
as adjustable parameters in UBV filters. The symmetry between the maximums in the light curves indicate
the lack of star spots on the components. After a few runs of the LC program an initial set of values was
employed as input parameters for the DC program. After a few runs with the differential correction program,
we could not obtain the values of the physical parameters with less error, thus making change to mode 5 so that
expected from the earlier workers. By several iterations of the LC and DC programs and adjusting the parameters
$i, T_2, \Omega_1$ and $L_1$ for each filter, finally the solution evolved into a nearcontact configuration for EE Aqr
and semi-detached one for Z Vul. The results of light curve solutions with the final elements are given in Ta-
bles 3 and 4. The theoretical light curves computed with these results are shown in Figures 1 and 2. The
agreement between the observed (solid circles) and the theoretical light curves (continuous lines) is quite good.

4 Spectroscopic Solutions

Using the radial-velocity data from HK and PO, we derived the orbital elements of the systems. Adjustable
parameters were the following: the orbital semi-major axis $a$, the radial velocity of the binary system center
of mass $V_z$. The results are given in table 5. Using the final elements of the objects, the theoretical radial-
velocity curves are shown in Figure 3. The agreement between the observed (solid circles) and the theoretical
radial-velocity curves (continuous lines) is quite good.

5 Absolute Elements

Using the obtained results of the light and radial-velocity curves, we calculate the absolute parameters

| Table 1 | The limb darkening coefficients for EE Aqr |
| Parameters | Filter B | Filter U | Filter V |
| $x_1(bol)$ | 0.640 | 0.640 | 0.640 |
| $y_1(bol)$ | 0.255 | 0.255 | 0.255 |
| $y_2(bol)$ | 0.153 | 0.153 | 0.153 |
| $x_1$ | 0.690 | 0.795 | 0.788 |
| $x_2$ | 0.800 | 0.802 | 0.834 |
| $y_1$ | 0.291 | 0.328 | 0.279 |
| $y_2$ | -0.014 | -0.411 | -0.189 |

| Table 2 | The limb darkening coefficients for Z Vul |
| Parameters | Filter B | Filter U | Filter V |
| $x_1(bol)$ | 0.762 | 0.762 | 0.762 |
| $x_2(bol)$ | 0.703 | 0.703 | 0.703 |
| $y_1(bol)$ | 0.090 | 0.090 | 0.090 |
| $y_2(bol)$ | 0.072 | 0.072 | 0.072 |
| $x_1$ | 0.434 | 0.482 | 0.509 |
| $x_2$ | 0.584 | 0.615 | 0.682 |
| $y_1$ | 0.233 | 0.228 | 0.275 |
| $y_2$ | 0.290 | 0.249 | 0.336 |
Table 3  The photometric parameters of *EE Aqr*

| Parameters | This work B Filter (Johnson) | This work U Filter (Johnson) | This work V Filter (Johnson) | Williamon (1974) | Padalia (1979) | Russo and Sollazzo (1982) | Hilditch and King (1988) | Covino et al. (1990) | Covino et al. (1990) |
|------------|-----------------------------|------------------------------|----------------------------|-------------------|---------------|----------------------|-----------------------|-------------------|---------------------|
| $i$        | 80.5074 ± 0.2975            | 81.3365 ± 0.4360             | 78.9509 ± 0.2379           | 73.71             | 74.00         | 76.0                 | 80.5                  | 80.0              | 80.2                |
| $q$        | 0.32                        | 0.32                         | 0.32                       | 0.503             | 0.500        | 0.4                  | 0.32                  | 0.332             | 0.327               |
| $T_1$      | 7060 ± 116                  | 7060 fixed                   | 7060 fixed                 | 8000              | 8000         | 8000                 | 7060                  | 7230              | 7227                |
| $T_2$      | 4200 ± 116                  | 3952 fixed                   | 4173 fixed                 | 3002              | 3003         | 2.78                 | 2.619                 | 2.568             |                    |
| $\Omega_1$| 2.6178 ± 0.0118             | 2.6141 ± 0.0106              | 2.6030 ± 0.0116            | 3.002             | 3.003        | 2.78                 | 2.619                 | 2.568             | 2.619               |
| $\Omega_2$| 2.5100 ± 0.0118             | 2.5100 fixed                 | 2.5100 fixed               | 2.882             | 2.875        | 2.68                 | 2.538                 | 2.561             | 2.568               |
| $L_1/(L_1 + L_2)$ | 0.9832             | 0.9950                      | 0.9716                     | 0.966             | 0.964        | 0.951                | 0.952                 | 0.959             | 0.959               |
| $L_2/(L_1 + L_2)$ | 0.0168             | 0.0050                      | 0.0284                     | 0.034             | 0.036        | 0.049                | 0.048                 | 0.041             | 0.041               |
| $\log g_1$(CGS) | 4.33                 | 4.33                       | 4.33                       | 4.33              | 4.33         | 4.33                 | 4.33                  | 4.33              | 4.33                |
| $\log g_2$(CGS) | 4.24                 | 4.24                       | 4.24                       | 4.24              | 4.24         | 4.24                 | 4.24                  | 4.24              | 4.24                |
| $r_1(pole)$ | 0.4303 ± 0.0021            | 0.4310 ± 0.0019              | 0.4330 ± 0.0021            | 0.394             | 0.394        | 0.434                | 0.43                  | 0.44              | 0.44                |
| $r_1(point)$ | 0.5121 ± 0.0056            | 0.5139 ± 0.0051              | 0.5194 ± 0.0059            | 0.469             | 0.467        | 0.52                 | 0.52                  | 0.54              | 0.54                |
| $r_1(side)$ | 0.4568 ± 0.0027            | 0.4577 ± 0.0025              | 0.4603 ± 0.0027            | 0.415             | 0.414        | 0.46                 | 0.46                  | 0.47              | 0.47                |
| $r_1(back)$ | 0.4765 ± 0.0033            | 0.4775 ± 0.0030              | 0.4806 ± 0.0033            | 0.437             | 0.435        | 0.48                 | 0.48                  | 0.49              | 0.49                |
| $r_2(pole)$ | 0.2659 ± 0.0025            | 0.2659 ± 0.0017              | 0.2659 ± 0.0032            | 0.300             | 0.299        | 0.262                | 0.27                  | 0.26              | 0.26                |
| $r_2(point)$ | 0.3853 ± 0.0121            | 0.3853 ± 0.0121              | 0.3853 ± 0.0121            | 0.406             | 0.405        | 0.36                 | 0.36                  | 0.33              | 0.33                |
| $r_2(side)$ | 0.2769 ± 0.00080           | 0.2769 ± 0.0031              | 0.2769 ± 0.0034            | 0.281             | 0.312        | 0.312                | 0.312                 | 0.30              | 0.30                |
| $r_2(back)$ | 0.3096 ± 0.0027            | 0.3096 ± 0.0027              | 0.3096 ± 0.0027            | 0.345             | 0.345        | 0.345                | 0.345                 | 0.30              | 0.30                |
| $\Omega_1$ | 2.5100 ± 0.0118            | 2.5100 fixed                 | 2.5100 fixed               | 2.882             | 2.875        | 2.68                 | 2.538                 | 2.561             | 2.568               |
| $\Omega_2$ | 2.3114 ± 0.0027            | 2.3114 ± 0.0018              | 2.3114 ± 0.0034            | 0.0172            | 0.0234       | 0.0156               |                      |                   |                    |
Table 4 The photometric parameters of \textit{Z Vul}.

| Parameters       | This work B Filter (Johnson) | This work U Filter (Johnson) | This work V Filter (Johnson) | Popper (1957b) B Filter (Johnson) | Cester et al. (1977) B Filter (Johnson) | Cester et al. (1977) U Filter (Johnson) | Cester et al. (1977) V Filter (Johnson) |
|------------------|-----------------------------|------------------------------|----------------------------|-----------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|
| $i$              | 88.6106 ± 0.0666            | 88.5185 ± 0.0453            | 88.6478 ± 0.0772            | 88                                | 88.5                                   | 88.9                                   |
| $q$              | 0.43 fixed                  | 0.43 fixed                  | 0.43 fixed                  | 0.43                              | 0.43                                   | 0.43                                   |
| $T_1$            | 19840 fixed                 | 19840 fixed                 | 19840 fixed                 | 19850                             | 19850                                  | 19840                                  |
| $T_2$            | 10909 ± 25 ± 32 ± 27        | 9882 ± 0.0106 ± 0.0083 ± 0.0119 | 10810 ± 25 ± 32 ± 27        | 9000                              | 10290                                  | 9410                                   |
| $\Omega_1$      | 3.7975 ± 0.0106 ± 0.0083 ± 0.0119 | 3.8512 ± 0.0106 ± 0.0083 ± 0.0119 | 3.7434 ± 0.0106 ± 0.0083 ± 0.0119 | 3.7975                             | 3.8512                                  | 3.7434                                  |
| $\Omega_2$      | 2.7387 fixed                | 2.7387 fixed                | 2.7387 fixed                | 2.7387                            | 2.7387                                  | 2.7387                                  |
| $L_1/(L_1 + L_2)$| 0.8151 0.8869 0.7889        | 0.962 0.930 0.954           |                             |                                   |                                        |                                        |
| $L_2/(L_1 + L_2)$| 0.1849 0.1131 0.2111        | 0.038 0.070 0.046           |                             |                                   |                                        |                                        |
| log $g_1$(CGS)   | 3.87 3.89 3.86              |                              |                             |                                   |                                        |                                        |
| log $g_2$(CGS)   | 3.49 3.49 3.49              |                              |                             |                                   |                                        |                                        |
| $r_1$(pole)      | 0.2954 ± 0.0009 ± 0.0007 ± 0.0011 | 0.2908 ± 0.0009 ± 0.0007 ± 0.0011 | 0.3002 ± 0.0009 ± 0.0007 ± 0.0011 | 0.2954                             | 0.2908                                  | 0.3002                                  |
| $r_1$(point)     | 0.3086 ± 0.0011 ± 0.0008 ± 0.0013 | 0.3031 ± 0.0011 ± 0.0008 ± 0.0013 | 0.3144 ± 0.0011 ± 0.0008 ± 0.0013 | 0.3086                             | 0.3031                                  | 0.3144                                  |
| $r_1$(side)      | 0.3011 ± 0.0010 ± 0.0007 ± 0.0012 | 0.2962 ± 0.0010 ± 0.0007 ± 0.0012 | 0.3063 ± 0.0010 ± 0.0007 ± 0.0012 | 0.3011                             | 0.2962                                  | 0.3063                                  |
| $r_1$(back)      | 0.3058 ± 0.0011 ± 0.0008 ± 0.0012 | 0.3006 ± 0.0011 ± 0.0008 ± 0.0012 | 0.3113 ± 0.0011 ± 0.0008 ± 0.0012 | 0.3058                             | 0.3006                                  | 0.3113                                  |
| $r_2$(pole)      | 0.2881 ± 0.0004 ± 0.0004 ± 0.0005 | 0.2881 ± 0.0004 ± 0.0004 ± 0.0005 | 0.2881 ± 0.0004 ± 0.0004 ± 0.0005 | 0.2881                             | 0.2881                                  | 0.2881                                  |
| $r_2$(point)     | 0.4142 ± 0.0018 ± 0.0016 ± 0.0021 | 0.4142 ± 0.0018 ± 0.0016 ± 0.0021 | 0.4142 ± 0.0018 ± 0.0016 ± 0.0021 | 0.4142                             | 0.4142                                  | 0.4142                                  |
| $r_2$(side)      | 0.3004 ± 0.0004 ± 0.0004 ± 0.0005 | 0.3004 ± 0.0004 ± 0.0004 ± 0.0005 | 0.3004 ± 0.0004 ± 0.0004 ± 0.0005 | 0.3004                             | 0.3004                                  | 0.3004                                  |
| $r_2$(back)      | 0.3330 ± 0.0004 ± 0.0004 ± 0.0005 | 0.3330 ± 0.0004 ± 0.0004 ± 0.0005 | 0.3330 ± 0.0004 ± 0.0004 ± 0.0005 | 0.3330                             | 0.3330                                  | 0.3330                                  |
| $\Omega_{in}$   | 2.7387 ± 0.0004 ± 0.0004 ± 0.0005 | 2.7387 ± 0.0004 ± 0.0004 ± 0.0005 | 2.7387 ± 0.0004 ± 0.0004 ± 0.0005 | 2.7387                             | 2.7387                                  | 2.7387                                  |
| $\Omega_{out}$  | 2.4781 ± 0.0004 ± 0.0004 ± 0.0005 | 2.4781 ± 0.0004 ± 0.0004 ± 0.0005 | 2.4781 ± 0.0004 ± 0.0004 ± 0.0005 | 2.4781                             | 2.4781                                  | 2.4781                                  |
| $\Sigma (o-c)^2$| 0.0039 0.0057 0.0031        |                              |                             |                                   |                                        |                                        |
of the systems. The results are listed in tables 6 and 7. We determine the absolute dimensions using the following formulae:

\[
\frac{M_{1}/M_\odot}{\sin^3 i} = f_1(M_1, M_2, i)(1 + q)^2, \quad (2)
\]

\[
f_1(M_1, M_2, i) = \frac{(M_2 \sin i)^3}{(1 + M_1 + M_2)^2}, \quad (3)
\]

\[
\frac{M_2/M_\odot}{q(M_1/M_\odot)} = \Omega_1, \quad (4)
\]

\[
\frac{R_{1,2}/R_\odot}{(1 + q)^{1/3}r(side)} = 4.207(1 + q)^{1/3}P^2, \quad (5)
\]

\[
\frac{L_{1,2}}{L_\odot} = \left(\frac{R_{1,2}/R_\odot}{T_{1,2}/T_\odot}\right)^{1/4}, \quad (6)
\]

\[
\frac{\rho_{1,2}r_\odot}{\rho_\odot} = \left(\frac{0.01344M_{1,2}}{[(1 + M_1 + M_2)P^2r_{1,2}^{3/2}]}, \quad (7)
\]

\[
(M_{bol})_{1,2} = 42.36 - 10\log T_{1,2} - 5\log(R_{1,2}/R_\odot). \quad (8)
\]

### 6 Conclusion

Comparing the new photometric-spectroscopic solutions of the systems with those of given in literature, the present authors can get following conclusions:

1-In our study, we may conclude that the systems are near-contact and semi-detached for EE Aqr and Z Vul respectively. The fillout for the components can be calculated from the following formula:

\[
\text{fillout}_{1,2} = \frac{\Omega_{1,2}}{\Omega_{1,2}} \times 100. \quad (9)
\]

Also from the following formulae we calculate \(K_{1,2}\) and \(a_{1,2}\sin i\) for each of the systems:

\[
K_{1,2} = \frac{2\pi a_{1,2}\sin i}{P}, \quad (10)
\]

\[
a_1 \sin i = \sqrt{\frac{G P^2 f_1(M_1, M_2, i)}{4\pi^2}}, \quad (11)
\]

\[
a_2 \sin i = \frac{a_1 \sin i}{q}. \quad (12)
\]

The derived values of the parameters are given in tables 8 and 9. According to our results, EE Aqr is a near-contact system which the primary and secondary components filling are almost 96 ± 1 and 100 percent of their respective critical Roche lobes. Also Z Vul is a semi-detached system that the primary and secondary components filling are almost 72 ± 1 and 100 percent of their respective critical Roche lobes. The fillout percentage was not accurately determined in previous workers (86 ± 3 and 94 ± 3 percent according to HK for the primary and secondary components of EE Aqr, respectively; no information for ones of Z Vul). It seems that fillout obtained from our solutions is appropriate for highly evolved system.

2- According to our synthetic light curves, EE Aqr is \(\beta\)-Lyrae type and Z Vul is an \(\text{Algol}\) type system. Also according to the obtained parameters of the systems, we have drawn the configuration of the components using the Binary Maker 2.0 (BradStreet, 1993) software, which are shown in figures 4 and 5.

3- The new values of the velocity amplitudes and the gama velocity differ only slightly from earlier researches.

4- The absolute elements of both components determine the evolutionary state of the systems. Entering the results in the mass-luminosity (M-L), \(M_{bol}\)-mass (M-M), mass-radius (M-R) and H-R diagrams (the diagrams are displayed in figures 6 and 7, it appears that EE Aqr contains two main-sequence stars with B2IV+K4III spectral type while the primary of Z Vul is still a main-sequence object, and the secondary component is on its way to the giant stage with B2V+B9V spectral type for the primary and secondary respectively (according to tables of Straizys & Kurilienė, 1981). The results are tabulated in table 10 and compared with the results of the pervious authors.

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### Table 5 The spectroscopic parameters

| Parameters | EE Aqr  | Z Vul  |
|------------|---------|--------|
| \(V_7 (Km/s)\) | -1.6786 | -22.7690 |
| \(a(R_\odot)\) | 4.1240 | 15.8411 |

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### Table 6  The absolute elements of the binary system EE Aqr

| Parameters | This Work | This Work | This Work | Russo and Sollazzo | Hilditch and King (1988) | Covino et al. (1990) |
|------------|-----------|-----------|-----------|---------------------|-------------------------|----------------------|
|            | Filter B  | Filter U  | Filter V  | (Johnson)           | (Johnson)               | (Johnson)            |
| $M_1/M_⊙$  | 2.761     | 2.761     | 2.761     | 1.9                 | 2.2                     | 2.14                 |
| $M_2/M_⊙$  | 0.883     | 0.883     | 0.883     | 0.95                | 0.71                    | 0.70                 |
| $L_1/L_⊙$  | 6.55      | 6.55      | 6.55      | 9.16                |                         | 7.8                  |
| $L_2/L_⊙$  | 0.32      | 0.25      | 0.32      | 0.47                | 0.33                    |                      |
| $R_1/R_⊙$  | 1.88      | 1.88      | 1.88      | 1.58                | 1.75                    | 1.79                 |
| $R_2/R_⊙$  | 1.18      | 1.18      | 1.18      | 1.21                | 1.07                    | 1.06                 |
| $ρ_1/ρ_⊙$  | 0.4123    | 0.4099    | 0.4030    |                     |                         |                      |
| $ρ_2/ρ_⊙$  | 0.5922    | 0.5922    | 0.5922    |                     |                         |                      |
| $M_1(\text{bol})$ | 2.55      | 2.55      | 2.53      |                     |                         |                      |
| $M_2(\text{bol})$ | 5.82      | 6.08      | 5.85      |                     |                         |                      |
| $f_1(M_1, M_2, i)$ | 0.0498    | 0.0501    | 0.0490    |                     |                         |                      |

### Table 7  The absolute elements of the binary system Z Vul

| Parameters | This Work | This Work | This Work | Popper (1957b) | Cester et al. (1977) |
|------------|-----------|-----------|-----------|----------------|---------------------|
|            | Filter B  | Filter U  | Filter V  | (Johnson)     | (Johnson)           |
| $M_1/M_⊙$  | 6.209     | 6.209     | 6.209     | 5.4           | 5.4                 |
| $M_2/M_⊙$  | 2.670     | 2.670     | 2.670     | 2.3           | 2.3                 |
| $L_1/L_⊙$  | 2631.37   | 2543.85   | 2720.37   | 1850          | 2818                |
| $L_2/L_⊙$  | 252.75    | 170.21    | 243.75    | 175           | 162                 |
| $R_1/R_⊙$  | 4.77      | 4.69      | 4.85      | 4.7           | 4.5                 |
| $R_2/R_⊙$  | 4.89      | 4.89      | 4.89      | 4.7           | 4.6                 |
| $ρ_1/ρ_⊙$  | 0.0571    | 0.0600    | 0.0543    |               |                      |
| $ρ_2/ρ_⊙$  | 0.0247    | 0.0247    | 0.0247    |               |                      |
| $M_1(\text{bol})$ | -3.96     | -3.93     | -4.00     | -3.5          |                     |
| $M_2(\text{bol})$ | -1.42     | -0.99     | -1.38     | -1.0          |                     |
| $f_1(M_1, M_2, i)$ | 0.2412    | 0.2412    | 0.2412    |               |                     |

### Table 8  Other elements of the binary system EE Aqr

| Parameters | This Work | This Work | This Work | Hilditch and King (1988) |
|------------|-----------|-----------|-----------|-------------------------|
|            | Filter B  | Filter U  | Filter V  | (Johnson)               |
| $a_1 \sin i$ | 0.9876    | 0.9899    | 0.9827    | 0.92                    |
| $a_2 \sin i$ | 3.0880    | 3.0951    | 3.0728    | 2.85                    |
| $K_1(\text{Km/s})$ | 98.2403   | 98.4678   | 97.7579   | 91.3                    |
| $K_2(\text{Km/s})$ | 307.1819  | 307.8929  | 305.6733  | 284                     |
| fillout 1(%) | 95.882    | 96.020    | 96.428    | 86                      |
| fillout 2(%) | 100.000   | 100.000   | 100.000   | 94                      |
Table 9  Other elements of the binary system \textit{Z Vul}

| Parameters | This Work Filter B (Johnson) | This Work Filter U (Johnson) | This Work Filter V (Johnson) | Plaskett (1920) | Popper (1957b) |
|------------|--------------------------------|-------------------------------|-------------------------------|----------------|----------------|
| $a_1 \sin i$ | 4.7716                        | 4.7714                        | 4.7717                        | Plaskett       | Popper         |
| $a_2 \sin i$ | 11.0963                      | 11.0958                      | 11.0964                      | Johnson        | Johnson        |
| $K_1 (Km/s)$ | 98.4167                       | 98.4127                      | 98.4182                      | 96.4           | 89.8           |
| $K_2 (Km/s)$ | 228.8649                      | 228.8557                      | 228.8685                      | 213.7           | 219.7           |
| $fillout \ 1(\%)$ | 72.118                        | 71.112                        | 73.161                        | Plaskett       | Popper         |
| $fillout \ 2(\%)$ | 100.000                      | 100.000                      | 100.000                      | Johnson        | Johnson        |

Table 10  Spectral type of \textit{EE Aqr} and \textit{Z Vul}

| Star        | This work (1974) | Williamon (1979) | Padalia (1979) | Russo and Sollazzo (1982) | Plaskett (1920) | Petrie (1950) | Roman (1956) | Poper (1957b) | Cester et al. (1977) |
|-------------|------------------|------------------|----------------|---------------------------|----------------|---------------|---------------|----------------|------------------------|
| \textit{EE Aqr} (primary) | F2 IV           | F0               | F V            | A                         | A8 V           | K3-K4         | B3            | B4            | B5 V                  |
| \textit{EE Aqr} (secondary) | K4 III          |                  | A              | A                         |                |               | B3            | B4            | B3-4 V                |
| \textit{Z Vul} (primary)    | B2 V            |                  | A              | A                         | K3-K4         |               |                | B3            | B2 V                  |
| \textit{Z Vul} (secondary)  | B9 V            |                  |                | A                         | A2-3 III      | A1 III        |                | B3            | A1 III                |
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Fig. 1  The observed and theoretical light curves of EE Aqr. Solid circles show the observed data and the theoretical light curves are shown by continuous lines.

Fig. 2  The observed and theoretical light curves of Z Vul. Solid circles show the observed data and the theoretical light curves are shown by continuous lines.
Fig. 3  The observed and theoretical radial-velocity curves of EE Aqr and Z Vul. Solid circles show the observed data and the theoretical radial-velocity curves are shown by continuous lines.

Fig. 4  The observed and theoretical radial-velocity curves of AV Del. Solid circles show the observed data and the theoretical radial-velocity curve is shown by continuous lines.
Fig. 5  Configuration of binary system $EE\ Aqr$

Fig. 6  Configuration of binary system $Z\ Vul$
Fig. 7  M-L and H-R diagram. The location of components are shown with star sign.

Fig. 8  M-M and M-R diagram. The location of components are shown with star sign.