An incipient consistency identification scheme of failure mechanism based on improved grey theory

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Abstract. Accelerated degradation test (ADT) is an effective technique to quantitatively evaluate the life characteristics of high-reliability and long-life products. And the consistency of failure mechanism under different accelerated stress levels is the premise of ADT. In order to identify the consistency of failure mechanism as early as possible and improve the identification accuracy, an incipient consistency identification method of failure mechanism based on improved grey theory is proposed in this paper. Firstly, we reconstruct the background value through the calculation of the integral in the interval. Then the new information priority principle is introduced to improve the construction of initial values in the conventional grey theory. Subsequently, combined with the equal dimension model, the integrated scheme is employed to analyze the enhancement testing data of a resistor in a switching power supply to identify the mutation point of failure mechanism. Compared with the conventional GM (1,1) model, the proposed method has smaller residual, and another identification based on degradation model is also constructed to verify the feasibility and creditability of the proposed method.

1. Introduction

With the improvement of design concept and manufacturing technology, more and more equipment and products show the properties of “high-reliability and long-life”. Hence, in order to quantitatively evaluate the reliability and lifetime, accelerated degradation test (ADT) has been paid more and more attention, because we can accelerate the degradation and failure process of products, and the performance degradation information obtained in this process is also very meaningful to estimate lifetime under the constant stress [1-5]. However, the consistency of failure mechanism under different accelerated stress levels is the premise of ADT, which greatly influence the selection of the ultimate stress and the final test results [6-7].

So far, consistency identification methods of failure mechanism can be summarized into four aspects. The first part is based on the theory that activation energy of failure is constant under different accelerated stress levels. Guo et al [8] has applied this theory to identify the failure mechanism consistency of a diode successfully. This kind of method also has some disadvantages: 1) Whether the activation energy changes with the failure mechanism is indefinite; 2) The method is only suitable for some certain classical acceleration models which contain the activation energy.

The second kind of identification method is mainly based on the failure analysis via some certain instruments like scanning electron microscope [9]. This direct technique is pretty effective, but the cost of the experiment is often very expensive. Meanwhile, the same failure mechanism corresponds
to multiple failure modes, so the consistency of failure mechanism can not only be determined by observation.

The third mean is based on some statistical methods. If the failure mechanism of the product is consistent, the acceleration coefficient is also a constant which is independent of the reliability. That is to say, the lifetime distribution parameters are always constant. The failure mechanism consistency conditions of 19 kinds of common lifetime distributions are summarized in [10]. However, this identification method also has some transparent shortcomings: 1) The physical meanings between failure mechanism and lifetime distribution parameters is indefinite; 2) This method needs a long-time test and a large number of test data, so it can't provide accurate identification in the early stage of engineering practice.

To address the above-mentioned problems, the fourth kind of method based on grey theory has been applied maturely [11-12]. GM (1,1) combining equal dimension model can be employed to predict the test data, and when the residual between the actual value and predicted value is abruptly changed, the failure mechanism is exactly changed. However, when the original feature sequence changes violently, the prediction error of this model will be greatly increased [13]. Based on the above discussion, an incipient consistency identification method of failure mechanism based on improved grey theory is proposed in this paper to identify the consistency of failure mechanism as early as possible and exactly improve the identification accuracy. We reconstruct the background value through the calculation of the integral in the interval and introduce the new information priority principle to improve the construction of initial values in the new proposed algorithm. Case validation combined with the comparative analysis highlight the superiority of the proposed method.

2. Methodology

2.1. The algorithm of improved grey theory

Grey theory is a kind of systematic scientific theory to study the uncertainty of small sample and poor information. And grey prediction is an important part of grey theory, which is mainly used for uncertainty prediction problems. GM (1,1) is the basic model of grey prediction technology, and its modeling principle is summarized as follows [11]:

The sequence of performance parameters obtained in reliability enhancement test can be described as:

\[ \gamma^{(i)}(0) = (\gamma^{(i)}(1), \gamma^{(i)}(2), ..., \gamma^{(i)}(i), ..., \gamma^{(i)}(n)) \]

\[ i = 1, 2, ..., n \]  

where \( \gamma^{(i)}(1) \) denotes the initial performance parameter, \( \gamma^{(i)}(n) \) means the parameter of the highest accelerated stress level.

A new data sequence is generated by accumulation:

\[ \gamma^{(i)}(k) = \sum_{i=1}^{k} \gamma^{(i)}(i) \quad k = 1, 2, ..., n \]  

After the performance degradation sequence is accumulated, its randomness is weakened. \( \eta^{(i)} \) is the background value of GM (1, 1) model, and the mean value calculation of adjacent data can be obtained:

\[ \eta^{(i)} = \{ \eta^{(i)}(2), \eta^{(i)}(3), ..., \eta^{(i)}(n) \} \]  

\[ \eta^{(i)}(k) = \frac{1}{2} (\gamma^{(i)}(k) + \gamma^{(i)}(k-1)), k = 2, 3, ..., n \]

The mean value model is expressed as:

\[ \gamma^{(i)}(k) + a \eta^{(i)}(k) = b \]  

When we use GM (1,1) model, it is considered that \( \gamma^{(i)} \) satisfies the first order linear differential equation:
where \( a, b \) represent the development coefficient and grey action, respectively. The development coefficient reflects the development trend of predicted value, and the grey action reveals the inherent changes in raw data. The above-mentioned parameters \( a \) and \( b \) can be calculated by the least square method. The prediction model of degradation series can be expressed as follows:

\[
\hat{\gamma}^{(1)}(k) = (\gamma^{(1)}(1) - \frac{b}{a})e^{-\alpha(k-1)} + \frac{b}{a}, k = 1, 2, ..., n
\]  

(8)

The prediction sequence is obtained:

\[
\hat{\gamma}^{(0)}(k) = (1 - e^a)(\gamma^{(1)}(1) - \frac{b}{a})e^{-\alpha(k-1)}
\]  

(9)

where \( \hat{\gamma}^{(0)}(k) \) is the restored value sequence of the original data.

However, the changing trend of feature sequence is usually unknown and random. As the construction of background value in conventional GM (1,1) model is via the mean value calculation of adjacent data, so it can’t fit the various trend conditions, which will significantly influence the identification accuracy [13]. Consequently, we reconstruct the background value through the calculation of the integral in the interval as:

\[
\eta^{(1)}(k) = \int_{k-1}^{k} \hat{\gamma}^{(1)}(t)dt
\]  

(10)

It is assumed that \( \gamma^{(1)}(t) = Ae^{Bt} \), \( A, B \) are the coefficients to be determined. The corresponding background value expression can be constructed as follows:

\[
\eta^{(1)}(k) = \frac{\hat{\gamma}^{(1)}(k) - \hat{\gamma}^{(1)}(k-1)}{\ln \hat{\gamma}^{(1)}(k) - \ln \hat{\gamma}^{(1)}(k-1)}, k = 2, 3, ..., n
\]  

(11)

Meanwhile, the conventional grey model is mainly dependent on the first component. If the new degradation feature data is not used effectively, the prediction error will increase [13]. Hence, the new information priority principle is introduced to improve the construction of initial values.

According to the new information priority principle, the \( n \)th component of the first accumulation generating sequence \( \hat{\gamma}^{(1)} \) is taken as the initial value of the grey model to improve the GM (1,1) model. The optimized data prediction model is expressed:

\[
\hat{\gamma}^{(1)}(k) = (\gamma^{(1)}(n) - \frac{b}{a})e^{-\alpha(n-k-1)} + \frac{b}{a}
\]  

(12)

In this paper, \( \gamma^{(1)}(n) \) is the initial value of the grey model, and \( \eta^{(1)}(k) = \frac{\hat{\gamma}^{(1)}(k) - \hat{\gamma}^{(1)}(k-1)}{\ln \hat{\gamma}^{(1)}(k) - \ln \hat{\gamma}^{(1)}(k-1)} \) is the background value of the grey model.

### 2.2. Equal dimension model

In the development process of grey system, there will be some random disturbances or driving factors entering the system, which will affect the development of the system successively. Although GM (1,1) model can be used for a long-term prediction, only one or two data points after \( \gamma^{(1)}(n) \) has practical significance and high accuracy for prediction. With the development of the system, the information of the old data will be gradually reduced. The old information will be removed in time while the new information is being supplemented. The modeling sequence can better reflect the current characteristics of the system. When using equal dimension and GM(1,1) model, we first employ GM (1,1) model to predict an initial value, and add it to the known data, remove the oldest data, maintain the equal dimension of the sequence, then establish GM (1,1) model to predict the next value [12]. Before prediction, the dimension of the model must be determined. In order to achieve the highest accuracy of the model, the optional dimension is generally 4 to 11 [12]. The selection of the best dimension can be determined through numerical experiments.
2.3. An incipient consistency identification scheme of failure mechanism based on improved grey theory and equal dimension model

In order to identify the mutation point of failure mechanism as early as possible in engineering practice, and determine the ultimate stress level in ADT, we need to make full use of the performance data obtained from the reliability enhancement test. And the specific steps of this proposed scheme can be summarized as follows:

1. According to the test requirements, a reasonable reliability enhancement test scheme is designed, which includes the initial value of stress, holding time under each stress, temperature step and the final stress level;
2. Select the appropriate dimension, and then establish the equal dimension model;
3. The proposed model is used to predict the enhanced test data, and the predicted value is compared with the actual observed value and then determined whether the residual changes significantly;
4. If the residual changes significantly, it means that the development trend of the data has changed, and the failure mechanism of the product has also changed exactly.

The flowchart of this proposed failure mechanism consistency identification scheme can also be displayed in Fig. 1.

![Figure 1. The flowchart of this proposed failure mechanism consistency identification scheme](image)

3. Case validation

In this section, a reliability enhancement test of a resistor in switch power supply is conducted to verify the proposed method. And the comparison with conventional GM (1,1) model highlights the superiority of the improved method. Furthermore, a long-term accelerated degradation experiment under several certain stress levels are also presented, and then a consistency identification method of failure mechanism based on degradation model is also established to illustrate the feasibility and creditability of the proposed method.

3.1. Reliability enhancement test design

In the reliability enhancement test, the resistance value of the resistor is recorded as the performance degradation parameter. And in this test, we adopt the off-line measurement function, i.e. restore the resistor to ordinary temperature. As the failure of electronic products occurring at high temperature
often returns to normal state when the temperature drops to the ordinary value, so we set the holding
time at each temperature level as 24 hours. By referring to the device manual of this resistor, the initial
temperature of the enhancement test is set at 90 ℃, the temperature step is set to 10℃, and the
maximum temperature stress is 190℃. Temperature changing rate is not more than 5℃/min. The test
device diagram and stress profile of this enhancement test are displayed in Fig. 2 and Fig. 3,
respectively.

![Monitoring and testing system]

**Figure 2.** The test device diagram

![Stress profile of enhancement test]

**Figure 3.** The stress profile of enhancement test

### 3.2. Consistency identification of failure mechanism via improved grey theory and equal dimension model

The degradation data of the resistor obtained from the enhancement test are shown in Table 1.

| Order | Temperature (℃) | Resistance value (kΩ) |
|-------|-----------------|------------------------|
| 1     | 90              | 5.0903                 |
| 2     | 100             | 5.0908                 |
| 3     | 110             | 5.0916                 |
| 4     | 120             | 5.0919                 |
| 5     | 130             | 5.0926                 |
| 6     | 140             | 5.0934                 |
| 7     | 150             | 5.0950                 |
| 8     | 160             | 5.0974                 |
| 9     | 170             | 5.1094                 |
| 10    | 180             | 5.1077                 |
| 11    | 190             | 5.1099                 |
11 groups of enhancement test data were obtained. Select the dimension of the equal dimension model as 4, calculate the development coefficient a and grey action b, respectively, which is shown in Table 2.

**Table 2.** The calculation of development coefficient and grey action

| Order | Development coefficient | Grey action |
|-------|-------------------------|-------------|
| 1-4   | 0.00010802              | 5.0901      |
| 2-5   | 9.8194e-05              | 5.0908      |
| 3-6   | 0.00014727              | 5.0908      |
| 4-7   | 0.00023559              | 5.0907      |
| 5-8   | 0.00043178              | 5.0899      |
| 6-9   | 0.00043178              | 5.0847      |
| 7-10  | 0.00096949              | 5.0919      |

We use the data 1-4 to establish the prediction model, and predict the value of the fifth degradation parameter, then compare with the actual observed value, calculate the residual, and so on. The mutation point of residual implies the failure mechanism has changed here. The predicted values, observed values and residuals of the resistor are displayed in Table 3.

**Table 3.** The predicted value, observed value and residual

| Order | Temperature at the predicted point (℃) | Predicted value | Observed value | Residual |
|-------|----------------------------------------|----------------|----------------|---------|
| 1-4   | 130                                    | 5.0925         | 5.0926         | -0.0001 |
| 2-5   | 140                                    | 5.0930         | 5.0934         | -0.0004 |
| 3-6   | 150                                    | 5.0941         | 5.0950         | -0.0009 |
| 4-7   | 160                                    | 5.0961         | 5.0974         | -0.0013 |
| 5-8   | 170                                    | 5.0998         | 5.1094         | -0.0096 |
| 6-9   | 180                                    | 5.1109         | 5.1077         | 0.0032  |
| 7-10  | 190                                    | 5.1128         | 5.1099         | 0.0029  |

In order to find the mutation point of failure mechanism more directly, the change trend of residual is presented in Fig. 4. And the residual generated by conventional GM (1,1) model is also plotted in this figure.

From Fig. 4, the residual value at 170℃ changes suddenly, which indicates the failure mechanism has changed. Meanwhile we can conclude that the residual value of improved GM (1,1) model is smaller than the conventional one, which implies that the improved model has a higher prediction accuracy.

![Figure 4. The change trend of residual](image-url)
3.3. Validation by ADT and degradation model

In order to illustrate the feasibility and creditability of the proposed method, a long-term accelerated degradation test under several certain stress levels are also constructed [14]. We selected 20 resistors with higher consistency, and the ADT were carried out under 5 accelerated stress levels, $T_1 = 90^\circ C, T_2 = 120^\circ C, T_3 = 130^\circ C, T_4 = 175^\circ C, T_5 = 190^\circ C$, respectively, and we place 4 resistors under each accelerated stress. The resistance values are measured and collected every 24 hours. And the test cycle is 55 days in total. The degradation increment at different accelerated stress levels can be described as Fig. 5.

![Degradation Increments of Resistors](image)

**Figure 5.** The degradation increment at different accelerated stress levels

Then we establish the degradation models under different stress levels using Wiener process. And the estimated value of drift parameters can be obtained by MLE, as displayed in Table 4.

**Table 4.** The estimated value of drift parameters by MLE

| Order | T1    | T2    | T3    | T4    | T5    |
|-------|-------|-------|-------|-------|-------|
| 1     | 7.5782| 5.9314| 7.7453| 17.9405| 24.0748|
| 2     | 7.0321| 6.0681| 8.7419| 17.7588| 22.4900|
| 3     | 7.1494| 7.3442| 8.2911| 17.2913| 23.7123|
| 4     | 8.6505| 7.1710| 8.6483| 17.3902| 24.4706|

According to the theoretical value of the drift parameters, when the salience level is 0.05, check the boundary of rejection area of $t$ distribution $t_{0.05}(18) = 2.101$, as shown in Table 5.

**Table 5.** Test results of statistics

| Item | T3    | T4    | T5    |
|------|-------|-------|-------|
| t statistic | 1.4892| 19.9257| 42.4945|
| H0   | Accept| Reject| Reject|

It can be concluded that the failure mechanism under $T_3 = 175^\circ C, T_4 = 190^\circ C$ is different from $T_3 = 130^\circ C$, which fully illustrates the feasibility and creditability of the proposed method.

4. Conclusions

In this paper, an incipient consistency identification method of failure mechanism based on improved grey theory is proposed to identify the consistency of failure mechanism as early as possible and improve the identification accuracy. We reconstruct the background value through the calculation of the integral in the interval, and then the new information priority principle is also introduced to improve the construction of initial values in the conventional grey theory. Case validation combined with the comparative analysis fully verify the feasibility and creditability of the proposed method.
Hence, this method is expected to serve as a potential improvement for the consistency identification in the case of less test data and limited test time.

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