On the occurrence of Kosterlitz-Thouless behavior in cuprate superconductors

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The observation of the characteristic Kosterlitz-Thouless behavior requires the attainment of the two dimensional limit where the correlation-length anisotropy, $\gamma = \xi_{ab}/\xi_c$, diverges. Our findings strongly suggest that the failure of several experiments on films and single crystals to observe any trace of KT-behavior is attributable either to inhomogeneities or doping by means of chemical substitution.

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Since the pioneering work of Kosterlitz and Thouless (KT) on the KT transition in the two-dimensional (2D) XY model, much efforts have been devoted to observe the universal behavior characteristic of the KT transition, as the universal jump of the superfluid density, measured in $^4$He superfluid films, or the non-linear I-V characteristic, observed in thin films of conventional superconductors. Signatures of KT physics can be expected also in layered superconductors with weak inter-layer coupling. Potential candidates are underdoped cuprate superconductors where the anisotropy increases with reduced transition temperature $T_c$. Recent studies of the I-V characteristic, the frequency dependent conductivity, the Nernst signal, the nonlinear magnetization, and of the resistance have been interpreted as signatures of KT behavior. On the other hand, several experiments failed to observe any trace of the universal jump in the superfluid density at $T_c$.

In this context it is important to recognize that the existence of the KT-transition (vortex-antivortex dissociation instability) in $^4$He films is intimately connected with the fact that in such films the interaction energy between vortex pairs depends logarithmically on the separation between them. As shown by Pearl, vortex pairs in thin superconducting films (charged superfluid) have a logarithmic interaction energy out to the characteristic length $\lambda_{2D} = \lambda_{ab0}/d$, beyond which the interaction energy falls off as $1/r$. Here $\lambda_{ab}$ is the in-plane penetration depth of the bulk and $d$ is the film thickness. As $\lambda_{2D}$ increases the diamagnetism of the superconductor becomes less important. Consequently, as $\lambda_{2D}$ increases, the vortices in a thin superconducting film become progressively like those in $^4$He films and according to this $\lambda_{2D} > L_s = \min[W, L]$ is required, where $W$ and $L$ denote the width and the length of the perfect sample. Since real systems, and in particular the cuprate superconductors are inhomogeneous, the correlation length $\xi$ cannot grow beyond the lateral extent $L_{ab}$ of the homogeneous regions. This begs the question of why one should see critical behavior at all if there is no true phase transition in finite-size systems. The answer is that critical behavior can be seen in an intermediate temperature regime, provided that $L_{ab} < L_s$ is sufficiently large in order that the fluctuation dominated regime is attained.

In addition, the occurrence of KT-behavior in single crystals and thick films of cuprate superconductors requires that the anisotropy $\gamma = \xi_{ab}/\xi_c$ tends to diverge, whereupon the 2D limit is approached. $\xi_{ab,c}$ denote the correlation length in the $ab$-plane and along the $c$-axis. In a variety of cuprate superconductors this behavior is well described by $\gamma(T_c) = \gamma(T_{cm})/[1 - (1 - T_c/T_{cm})^{1/2}]$, where $\gamma(T_{cm})$ denotes the anisotropy at optimum doping where $T_c = T_{cm}$. In Fig.1 we depicted the resulting $T_c$ dependence of the anisotropy for YBa$_2$Cu$_3$O$_{7-\delta}$. The dashed line is $\gamma(T_c) = 2\gamma(T_{cm})/T_c$, the limiting behavior in the 2D-limit. When this limit is attained, a 2D quantum superconductor to insulator (QSI) transition with dynamic critical exponent $z = 1$ is expected to occur. Here $T_c$, the zero temperature in-plane ($\xi_{ab}(0)$) and $c$-axis ($\xi_c(0)$) correlation length, the in-plane magnetic penetration depth ($\lambda_{ab}(0)$) and the anisotropy scale as $T_c \propto 1/\lambda_{ab0}(0) \propto 1/\xi_{ab}(0) \propto 1/\gamma$, and $T_c\lambda_{ab0}(0) \propto \xi_c(0) \propto d_c$. $d_c$ denotes the thickness of the superconducting slabs, becoming independent in the 2D-limit. In a variety of cuprate superconductors traces of this behavior, in particular of $T_c \propto 1/\lambda_{ab0}(0)$, have been observed below $T_{cm}$ in the regime where the anisotropy scales roughly as $\gamma(0) \propto 1/T_c$. However, recent measurements of $\lambda_{ab}$ and $\lambda_{ab0}$ on YBa$_2$Cu$_3$O$_{6+x}$ single crystals, extending to much lower $T_c$'s reveal that the anisotropy does not attain the 2D-limit when $T_c$ is reduced further by chemical substitution. Their data yield $\gamma(T = 0) = \lambda_{c0}/\lambda_{ab0}(0) \simeq 70$ for $T_c$ from 5 to 15 K, instead of the characteristic 2D behavior $\gamma(T = 0) \propto 1/T_c$ shown in Fig.1. Indeed, their data is consistent with 3D-QSI critical behavior, namely $T_c \propto \lambda_{ab,c}(0)^{-2z/(z+1)} \propto \lambda_{a,b,c}(0)^{-1}$ with $z \gtrsim 1$. $z$ denotes the dynamic critical exponent of the quantum transition. Consistency with $T_c \propto \lambda_{ab0}(0)^{-1}$ was also observed by Zuev et al. in thick YBa$_2$Cu$_3$O$_{6+2x}$ films.

In contrast, considerable evidence for KT-behavior stems from resistance and mobile areal carrier density measurements in very thin (3-4 unit cells thick) underdoped NdBa$_2$Cu$_3$O$_{7-\delta}$ films near the 2D-QSI tran-
situation using the electric field effect technique\cite{10}. To estimate the $T_c$’s, ranging from 0.5 to 10K, consistency with the characteristic KT-behavior of the resistance, $\rho = \rho_0 \exp(-bt^{-1/2})$\cite{[1]}, was established, where $t = T/T_c - 1$ and $\rho_0$ and $b$ are material dependent parameters. Furthermore the measurements of the electric field induced changes of $T_c$ and the areal carrier density $n_{2D}$ uncovered the relationship, $T_c \propto n_{2D}^{\nu}$ with $\nu = 1$, the signature of a 2D-QSI-transition\cite{24,25}. $\nu$ is the critical exponent of the zero temperature in-plane penetration depth, $\xi_{ab}(0) \propto n_{2D}^{\nu}$. Together with the quantum counterpart of the Nelson-Kosterlitz (NK)-relation\cite{2} it implies $T_c \propto n_{2D} \propto 1/\lambda_{ab}^2(0)$, characteristic for a 2D-QSI transition with $z\nu = 1$. In this context it is interesting to note that a magnetic field tuned 2D-QSI transition with $\nu = 1.37 \pm 0.1$ was also observed in YBa$_2$Cu$_3$O$_{7-\delta}$ single crystals with $T_c \approx 2K$\cite{24}. Apparently, the reduction of $T_c$ by means of chemical substitution does not fully decouple the superconducting sheets. This provides a key anchor point for the understanding of the phase diagram of underdoped cuprate superconductors - distinct quantum critical points in the chemical substitution and electric field effect or magnetic field tuned case.

Finally we turn to the measurements of Rüfenacht et al.\cite{15}. They reported the temperature and electric-field dependence of $d/\lambda_{ab}^2$, obtained by capacitively charging an epitaxially grown ultrathin (two-unit-cell-thick) LSCO film in the underdoped regime ($x \approx 0.1$) with an electrostatic field applied across a gate insulator with a high dielectric constant. In Fig.\ref{fig:a} we show $\lambda_{2D} = \lambda_{ab}^2/d$ vs. $T$ derived from their data. The solid line is the Nelson-Kosterlitz line,

$$k_B T_c = \frac{\pi}{2} \frac{\Phi_0}{16\pi^2} \frac{d}{\lambda_{ab}^2(T_c)} ,$$

where $d/\lambda_{ab}^2(T)$ jumps to zero at $T_c^-$. In view of the lateral extent of the film, 0.25 cm, and the expected KT-transition around $T \approx 1.8K$, the condition, $\lambda_{2D} >> \min[W, L]$, for the occurrence of KT-behavior is fairly satisfied. In a thin-film superconductor vortices interact logarithmically out to a distance on the order of $\lambda_{2D}$, at which point the interaction approaches a constant. Thus, because $\lambda_{2D}(T \approx 1.8K)$ exceeds the lateral extent of the film, the system does not exhibit what amounts to an intrinsic finite-size effect\cite{27}. Nevertheless, there is no signature of the characteristic jump. Instead there is an extended tail which appears to preclude a sharp transi-

![Figure 1](image1.png)

**FIG. 1:** $\gamma(T_c) = \gamma(T_{cm})/(1 - (1 - T_c/T_{cm})^{1/2})$ for YBa$_2$Cu$_3$O$_{7-\delta}$ with $\gamma(T_{cm}) = 7$ and $T_c = 93K$. The dashed curve is $\gamma(T_c) = 2\gamma(T_{cm}) T_{cm}/T_c$, the leading behavior in the 2D-limit. The dotted line is $\gamma = 70$.

![Figure 2](image2.png)

**FIG. 2:** a) $\lambda_{2D} = \lambda_{ab}^2/d$ vs. $T$ derived from the data of Rüfenacht et al.\cite{15}. The solid line is the Nelson-Kosterlitz-line given by Eq.\ref{eq:1}. b) $\ln(d/\lambda_{ab}^2)$ vs. $T$ derived from the data of Rüfenacht et al.\cite{15}. The solid lines are Eq.\ref{eq:2} with $\ln(A) = 13.5, 21, 10.6$ and $B_{L/ab} = 1.68$ $(K^{-1})$ for the gate voltages $V = -3, 0, +3$ (V), respectively.

- $\gamma(T_{cm}) = 7, \quad T_c = 93K$
- $\gamma(T_c) = 2\gamma(T_{cm}) T_{cm}/T_c$, the leading behavior in the 2D-limit.
- $\gamma = 70$
tion at higher temperatures. From the studies of finite 2D-XY-models it is known that the superfluid density remains finite above the transition temperature of the infinite counterpart. More precisely, it decreases smoothly and the tail increases with reduced lateral system size $L_{ab}$. If the pronounced tails observed by Rüfenacht et al. are attributable to an inhomogeneity induced finite size effect due to the limited extent $L_{ab}$ of the homogeneous regions, their temperature dependence should be of the form

$$\frac{d}{\lambda_{ab}^2(T)} \simeq A \exp(-BL_{ab}).$$

A glance to Fig. 2, showing $\ln(d/\lambda_{ab}^2)$ vs. $T$, reveals that the observed tails are remarkably consistent with this finite size behavior and a unique value for $BL_{ab}$. Apparently, the field induced modulation of $d/\lambda_{ab}^2$ and $\lambda_{2D} = \lambda_{ab}^2/d$ does not affect $L_{ab}$. This confirms that the rounding stems from an inhomogeneity induced finite size effect.

To summarize, our findings strongly suggest that the failure of several experiments to observe any trace of the universal jump in the temperature dependence of the superfluid density is attributable to the failure to attain the 2D limit. While in YBa$_2$Cu$_3$O$_{6+x}$ single crystals and thick films chemical substitution makes it impossible to attain this limit, the observation of KT-behavior in ultrathin films requires rounding stems from an inhomogeneity induced finite size effect due to the limited extent of the films and unique homogeneity. Furthermore, the observation of distinct quantum superconductor to insulator transitions in the chemical substitution and electric field effect or magnetic field tuned case, provides a key anchor point for the understanding of the phase diagram of underdoped cuprate superconductors.

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