Spin transfer and polarization of antihyperons in lepton induced reactions

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Abstract

We study the polarization of antihyperon in lepton induced reactions such as $e^+e^- \rightarrow \bar{H} + X$ and $l + p \rightarrow l' + \bar{H} + X$ with polarized beams using different models for spin transfer in high energy fragmentation processes. We compare the results with the available data and those for hyperons. We make predictions for future experiments.

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I. INTRODUCTION

The polarizations of hyperons have been widely used to study various aspects of spin effects in high energy reactions, in particular, the spin-dependent fragmentation functions for their self spin-analyzing parity violating decay \[1\]. One of the important aspects in this connection is the spin transfer in high energy fragmentation processes. Here, it is of particular interest to know whether SU(6) wave-function or the results drawn from polarized deeply inelastic lepton-nucleon scattering and related data should be used in connecting the polarization of the fragmenting quark and that of the produced hadrons. Clearly such study can provide us with useful information on hadronization mechanism and the spin structure of hadron.

Theoretical calculations of hyperon polarizations in different reactions have been carried out using different models \[2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\]. The results show, in particular, that it is possible to differentiate different pictures by measuring the polarizations of different hyperons in $e^+e^-$ annihilations, polarized deeply inelastic lepton-nucleon scattering at high energies, and in the high transverse momentum regions in polarized $pp$ collisions. They show also that the decay contributions to $\Lambda$ polarization are usually high and should be taken into account in the calculations, and that the contamination from the fragmentation of remnant of target in deeply inelastic scattering at relatively low energies such as the CERN NOMAD or DESY HERMES energies are very important. Both of them have to be taken into account in comparing theoretical predictions with the experimental results.

Presently, data in this connection are already available for $\Lambda$ polarization in $e^+e^-$ annihilation at the $Z^0$ pole \[16, 17\], in deeply inelastic scattering using neutrino beam \[18\] and that using electron or muon beam \[19, 20, 21\]. But the amount of data and the statistics of them are still not high enough to judge which picture is better. More complementary measurements are necessary and some of them are underway. In this connection, it is interesting to note that the NOMAD and COMPASS Collaboration at CERN has measured not only the polarization of $\Lambda$ but also that of $\bar{\Lambda}$ \[18, 21\]. Since the polarization of antihyperon in semi-inclusive deeply inelastic lepton-nucleon scattering is definitely more sensitive to the nucleon sea, it is thus instructive to make more detailed
study in this direction not only to get more information on spin transfer in fragmentation but also on nucleon spin structure.

In this paper, we make calculations of the polarizations of antihyperons in lepton induced reactions such as \( e^+e^- \rightarrow Z^0 \rightarrow \bar{H} + X \), \( e^- + p \rightarrow e^- + \bar{H} + X \) and \( \nu_\mu + p \rightarrow \mu^- + \bar{H} + X \). In Sec. II, we present the general formulas used in these calculations and summarize the key points of different models for spin transfer in fragmentation. We present the results for antihyperon polarizations in Sec. III and compare them with those for hyperons and available data. We give a short summary and an outlook in Sec. IV.

II. THE CALCULATION FormULAS

As have been shown in [6, 7, 8, 9], to calculate hyperon polarization in high energy reactions, it is necessary to take also the decay contribution into account. In this section, we present the general formulas for such calculations with the contribution from decay. We first present the formulas for a pure quark fragmentation in Sec. II A, then the formulas for hyperon and antihyperon polarization in Sec. II B. We summarize the key points of a few models for spin transfer in fragmentation processes in Sec. II C.

A. Calculation formulas for a pure quark fragmentation

We first present the formulas for a pure quark fragmentation which are the basis for different reactions. We present the formulas for \( q_f \rightarrow H + X \), \( q_f \rightarrow \bar{H} + X \), \( \bar{q}_f \rightarrow H + X \) and \( \bar{q}_f \rightarrow \bar{H} + X \), respectively, and the relations between them obtained from charge conjugation symmetry.

1. \( q_f \rightarrow H + X \)

Now we consider the fragmentation process \( q_f \rightarrow H + X \), where we use the subscript \( f \) to denote the flavor of the quark, \( H \) to denote hyperon. We use \( D^H_f(z) \) to denote the fragmentation function, which is defined as the number density of \( H \) produced in the fragmentation of \( q_f \), where \( z \) is the fraction of momentum of \( q_f \) carried away by \( H \). We do not study the transverse momentum dependence in this paper so an integration over
transverse momentum is understood. In general, if the contribution from decay is taken into account, $D_f^H(z)$ can be written as two parts, i.e.,

$$D_f^H(z) = D_f^H(z; \text{dir}) + D_f^H(z; \text{dec}), \quad (1)$$

where $D_f^H(z; \text{dir})$ and $D_f^H(z; \text{dec})$ denote the directly produced part and the decay contribution respectively.

It is clear that the decay contribution can be calculated from the following convolution,

$$D_f^H(z; \text{dec}) = \sum_j \int dz' K_{H,H_j}^H(z, z') D_{f}^{H_j}(z'), \quad (2)$$

where the kernel function $K_{H,H_j}^H(z, z')$ is the probability for $H_j$ with a fractional momentum $z'$ to decay into an $H$ with $z$ and anything. If we consider, as usual, only the $J^P = (1/2)^+$ octet and $J^P = (3/2)^+$ decuplet baryon production, most of the decay processes are two body decay. For an unpolarized two body decay $H_j \to H + M$, the kernel function $K_{H,H_j}^H(z, z')$ can be calculated easily. In this case, the magnitude of the momentum of the decay product in the rest frame of $H_j$ is fixed and it has to be isotropically distributed. By making a Lorentz transformation of this isotropic distribution to the moving frame of $H_j$, we obtain the result for $K_{H,H_j}^H(z, z')$ as given by

$$K_{H,H_j}^H(z, \vec{p}_\perp^i; z', \vec{p}'_\perp^i) = \frac{N}{E^*_j} Br(H_j \to H_i M) \delta(p_i \cdot p_j - m_j E^*_i), \quad (3)$$

where $Br(H_j \to HM)$ is the corresponding decay branching ratio, $N$ is a normalization constant, $E^*_i$ is the energy of $H_i$ in the rest frame of $H_j$ which is a function of the masses $m_j$, $m_i$ and $m_M$ of $H_j$, $H_i$ and $M$.

Similarly, in polarized case, we have

$$\Delta D_f^H(z) = \Delta D_f^H(z; \text{dir}) + \Delta D_f^H(z; \text{dec}), \quad (4)$$

where $\Delta D_f^H(z) = D_f^H(z, +) - D_f^H(z, -)$, and the + or − denotes that the produced $H$ is polarized in the same or opposite direction as the initial quark $q_f$. $\Delta D_f^H(z; \text{dir})$ and $\Delta D_f^H(z; \text{dec})$ are the corresponding quantities for directly produced $H$ and decay contribution. For decay contribution, we have

$$\Delta D_f^H(z; \text{dec}) = \sum_j \int dz' t_{H,H_j}^D K_{H,H_j}(z, z') \Delta D_f^{H_j}(z'), \quad (5)$$
where \( t_{H,H}^D \) is the spin transfer factor for the decay process \( H_j \to H + M \). \( t_{H,H}^D \) is a constant which is independent of the process where \( H_j \) is produced. It is completely determined by the decay process. For different decay processes, \( t_{H,H}^D \) can be found, e.g., in Table II of [6].

The unknowns left now are \( D_f^H(z; \text{dir}) \) and \( \Delta D_f^H(z; \text{dir}) \). They are determined by the hadronization mechanism and the structure of hadrons. Presently, we can calculate \( D_f^H(z; \text{dir}) \) using a hadronization model or using a parametrization of fragmentation functions. For \( \Delta D_f^H(z; \text{dir}) \), there are also different models in literature [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. We will briefly summarize a few of them in Sec. II C.

2. \( \bar{q}_f \to H + X \)

For \( \bar{q}_f \to H + X \), we have

\[
D_f^H(z) = D_f^H(z; \text{dir}) + D_f^H(z; \text{dec}),
\]

\[ D_f^H(z; \text{dec}) = \sum_j \int dz' K_{H,H_j}(z, z') D_f^{H_j}(z'), \]

\[ \Delta D_f^H(z) = \Delta D_f^H(z; \text{dir}) + \Delta D_f^H(z; \text{dec}), \]

\[ \Delta D_f^H(z; \text{dec}) = \sum_j \int dz' t_{H,H_j}^D K_{H,H_j}(z, z') \Delta D_f^{H_j}(z'). \]

3. \( q_f \to \bar{H} + X \)

For \( q_f \to \bar{H} + X \), we have

\[
D_f^{\bar{H}}(z) = D_f^{\bar{H}}(z; \text{dir}) + D_f^{\bar{H}}(z; \text{dec}),
\]

\[ D_f^{\bar{H}}(z; \text{dec}) = \sum_j \int dz' K_{H,H_j}(z, z') D_f^{H_j}(z'), \]

\[ \Delta D_f^{\bar{H}}(z) = \Delta D_f^{\bar{H}}(z; \text{dir}) + \Delta D_f^{\bar{H}}(z; \text{dec}), \]

\[ \Delta D_f^{\bar{H}}(z; \text{dec}) = \sum_j \int dz' t_{H,H_j}^D K_{H,H_j}(z, z') \Delta D_f^{H_j}(z'). \]

Here we assume that the charge conjugation symmetry is applicable to the decay process so that the relations \( K_{\bar{H},H_j}(z, z') = K_{H,H_j}(z, z') \) and \( t_{\bar{H},H_j}^D = t_{H,H_j}^D \) are valid. We assume
also the validity of the charge conjugation symmetry for the fragmentation functions and obtain further the following relations:

\[ D_{\bar{f}}^H(z; i) = D_f^H(z; i), \]  
\[ \Delta D_{\bar{f}}^H(z; i) = \Delta D_f^H(z; i), \]  
for total, the direct \((i = \text{dir})\) and decay \((i = \text{dec})\) parts respectively.

4. \( \bar{q}_f \rightarrow \bar{H} + X \)

For \( \bar{q}_f \rightarrow \bar{H} + X \), we use charge conjugation symmetry and obtain that

\[ D_{\bar{f}}^H(z; i) = D_f^H(z; i), \]  
\[ \Delta D_{\bar{f}}^H(z; i) = \Delta D_f^H(z; i), \]  
for total, the direct \((i = \text{dir})\) and decay \((i = \text{dec})\) parts respectively. Hence, in the following, we need only to write out the formulas for \( q_f \rightarrow H + X \) and \( \bar{q}_f \rightarrow H + X \). Those for the other two cases are obtained from charge conjugation symmetry.

Independent of the models, we expect the following qualitative features for \( D_f^H(z; \text{dir}) \) and \( \Delta D_f^H(z; \text{dir}) \). Since hadrons containing the initial quark, i.e. the first rank hadrons in Feynman-Field type of cascade fragmentation models, usually carry a large fraction of momentum of the initial quark, we expect that, for large \( z \)

\[ D_{q_f}^H(z; \text{dir}) \gg D_{\bar{q}_f}^H(z; \text{dir}), \]  
\[ |\Delta D_{q_f}^H(z; \text{dir})| \gg |\Delta D_{\bar{q}_f}^H(z; \text{dir})|. \]  
But for small \( z \), they can be comparable. Since \( P_H \) due to spin transfer is expected to be significant for large \( z \), we should see that

\[ |P_{H}^{q_f \rightarrow HX}(z)| \gg |P_{H}^{\bar{q}_f \rightarrow HX}(z)|. \]  
This implies that the qualitative behavior of \( P_H \) in a given reaction is determined mainly by quark fragmentation and that of \( P_H \) is determined mainly by anti-quark fragmentation.
B. Polarization of hyperon or antihyperon in lepton induced reactions

To calculate the polarization of hyperon or antihyperon in a given reaction, we need to sum over the contributions from the fragmentation of quarks and anti-quarks of different flavors. For example, for \( A + B \to H + X \), we have

\[
\frac{d\sigma}{dx_F}(AB \to HX) = \sum_f [\frac{d\hat{\sigma}}{d\alpha}(AB \to q_fX) \otimes D_f^H(z) + \frac{d\hat{\sigma}}{d\alpha}(AB \to \bar{q}_fX) \otimes D_{\bar{f}}^H(z)],
\]

(21)

\[
\frac{d\Delta\sigma}{dx_F}(AB \to HX) = \sum_f [\frac{d\hat{\sigma}}{d\alpha}(AB \to q_fX)P_f(\alpha)\otimes\Delta D_f^H(z) + \frac{d\hat{\sigma}}{d\alpha}(AB \to \bar{q}_fX)P_f(\alpha)\otimes\Delta D_{\bar{f}}^H(z)],
\]

(22)

where \( x_F \) is the fractional momentum carried by the produced \( H \); \( \hat{\sigma} \) denotes the cross section for the production of \( q_f \) or \( \bar{q}_f \) in \( A + B \) collisions and \( \alpha \) denotes the kinematic variables describing cross section; \( P_f(\alpha) \equiv (\Delta d\hat{\sigma}/d\alpha)/(d\hat{\sigma}/d\alpha) \) is the polarization of \( q_f \), and similar for \( P_{\bar{f}} \). In general, they can be dependent on some kinematic variables hence we use \( \otimes \) to denote convolutions. Or equivalently, we have,

\[
N(x_F, H) = \sum_f [R_f(\alpha) \otimes D_f^H(z) + R_{\bar{f}}(\alpha) \otimes D_{\bar{f}}^H(z)],
\]

(23)

\[
\Delta N(x_F, H) = \sum_f [R_f(\alpha)P_f(\alpha) \otimes \Delta D_f^H(z) + R_{\bar{f}}(\alpha)P_{\bar{f}}(\alpha) \otimes \Delta D_{\bar{f}}^H(z)],
\]

(24)

\[
N(x_F, \bar{H}) = \sum_f [R_f(\alpha) \otimes D_f^H(z) + R_{\bar{f}}(\alpha) \otimes D_{\bar{f}}^H(z)],
\]

(25)

\[
\Delta N(x_F, \bar{H}) = \sum_f R_f(\alpha)P_f(\alpha) \otimes \Delta D_f^H(z) + R_{\bar{f}}(\alpha)P_{\bar{f}}(\alpha) \otimes \Delta D_{\bar{f}}^H(z),
\]

(26)

where we use \( N(x_F, H) \) and \( N(x_F, \bar{H}) \) to denote the number density of \( H \) and that of \( \bar{H} \) at a given momentum fraction \( x_F \) produced in the reaction respectively, \( \Delta N(x_F, H) \) and \( \Delta N(x_F, \bar{H}) \) to denote the corresponding differences in the polarized case. \( P_f \) and \( P_{\bar{f}} \) are respectively the polarization of \( q_f \) and that of \( \bar{q}_f \); \( R_f \) and \( R_{\bar{f}} \) are the fractional contributions of \( q_f \to h + X \) and \( \bar{q}_f \to h + X \) to the whole hadronic events. They are related to \( d\hat{\sigma} \) by

\[
R_f(\alpha) = \frac{1}{\sigma_{\text{inel}}} \frac{d\hat{\sigma}}{d\alpha}(AB \to q_fX),
\]

(27)

where \( \sigma_{\text{inel}} \) is the total inelastic cross section for hadron production in \( A + B \) collisions. The final result for the polarization is given by,

\[
P_H(x_F) = \frac{\Delta N(x_F, H)}{N(x_F, H)}.
\]

(28)
\[ P_H(x_F) = \frac{\Delta N(x_F, \bar{H})}{N(x_F, H)}. \]  

(29)

For different reactions, the fragmentation functions \( D \)'s and \( \Delta D \)'s are assumed to be universal, but the results of \( R_f, R_{\bar{f}}, P_f \) and \( P_{\bar{f}} \) are different. These differences lead to different results for the polarization of hyperons and/or antihyperons. In the following, we summarize the formulas in different lepton induced reactions, respectively, and discuss the qualitative features of these results. The numerical results are given in Sec. III.

1. \( e^+e^- \rightarrow H(\text{or } \bar{H}) + X \)

At high energies, due to the contribution through weak-interaction at the \( e^+e^- \) annihilation vertex such as \( e^+e^- \rightarrow Z^0 \rightarrow q_f\bar{q}_f \), the initial \( q_f \) and \( \bar{q}_f \) are longitudinally polarized. The magnitude of the polarization of \( q_f \) and that of \( \bar{q}_f \) are the same but the sign are different. They are constants at a given center of mass (c.m.) energy of \( e^+e^- \) system. Also the relative weights for the contributions of different flavors are constants when averaging over the different jet (or initial quark) directions. They are the same for quarks and for anti-quarks. Namely, we have

\[ P_{e^+e^-}^f = -P_{\bar{e}^+\bar{e}^-}^f, \]  

(30)

\[ R_{e^+e^-}^f = R_{e^+e^-}^{\bar{f}}. \]  

(31)

and \( R_{e^+e^-}^f \) is given by

\[ R_{e^+e^-}^f = \frac{\sigma(e^+e^- \rightarrow q_f\bar{q}_f)}{\sum_{f} \sigma(e^+e^- \rightarrow q_f\bar{q}_f)}. \]  

(32)

For example, for reactions at the \( Z^0 \)-pole, we neglect the contribution from the annihilation via virtual photon. We have \( P_f = -0.67 \) for \( f = u \) or \( c \) and \( P_f = -0.94 \) for \( f = d, s, \) or \( b \). Hence, we have

\[ N_{e^+e^-}^f(z, H) = \sum_f R_{e^+e^-}^f [D_f^H(z) + D_{\bar{f}}^H(z)], \]  

(33)

\[ \Delta N_{e^+e^-}^f(z, H) = \sum_f R_{e^+e^-}^f P_{e^+e^-}^f [\Delta D_f^H(z) - \Delta D_{\bar{f}}^H(z)]. \]  

(34)

For antihyperons, we have

\[ N_{e^+e^-}^f(z, \bar{H}) = \sum_f R_{e^+e^-}^f [D_f^H(z) + D_{f}^H(z)], \]  

(35)
\[ \Delta N^{e^+e^-}(z, \bar{H}) = \sum_f R_{f}^{e^+e^-} P_{f}^{e^+e^-}[\Delta D_{f}^{H}(z) - \Delta D_{f}^{H}(z)]. \] (36)

We see that,
\[ N^{e^+e^-}(z, \bar{H}) = N^{e^+e^-}(z, H), \] (37)
\[ \Delta N^{e^+e^-}(z, \bar{H}) = -\Delta N^{e^+e^-}(z, H). \] (38)

This means that, under the charge conjugation symmetries for the fragmentation functions and decay processes, we have
\[ P_{\bar{H}}^{e^+e^-}(z) = -P_{H}^{e^+e^-}(z). \] (39)

This result is true independent of model for fragmentation functions. It is a direct consequence of the charge conjugation symmetries for fragmentation and decay processes. Since we do not have the contaminations from the initial state such as the structure of nucleon in this reaction, this is an ideal place to test such charge conjugation symmetries.

2. \( e^- + N \rightarrow e^- + H(\text{or } \bar{H}) + X \)

At sufficiently large energies and momentum transfer \( Q^2 \), hadron produced in the current fragmentation region of semi-inclusive deeply inelastic lepton-nucleon scattering such as \( e^- + N \rightarrow e^- + H(\text{or } \bar{H}) + X \) can be considered as a pure product of fragmentation of the struck quark. In this case, the cross sections are given by
\[ \frac{d^3\sigma}{dx dy dz}(eN \rightarrow eHX) = \frac{d\sigma_{\text{Mott}}}{dy} \sum_f e_f^2[q_f(x, Q^2)D_{f}^{H}(z) + \bar{q}_f(x, Q^2)D_{\bar{f}}^{H}(z)], \] (40)
\[ \frac{\Delta d^3\sigma}{dx dy dz}(eN \rightarrow eHX) = \frac{d\sigma_{\text{Mott}}}{dy} \sum_f e_f^2[P_{f}^{e^+e^-}(x, y)q_f(x, Q^2)\Delta D_{f}^{H}(z) + P_{f}^{e^+e^-}(x, y)\bar{q}_f(x, Q^2)\Delta D_{\bar{f}}^{H}(z)], \] (41)

where \( \sigma_{\text{Mott}} \) is the Mott cross section; \( x \) is the usual Bjorken \( x \); \( y \) is the fractional energy transfer in the rest frame of the nucleon; and \( z \) is the fraction of momentum of struck \( q \) carried by \( H \). The polarization of the struck quark \( q_f \) and that of anti-quark \( \bar{q}_f \) are given by
\[ P_{f}^{e^+e^-}(x, y) = \frac{P_{L}^{(e)}D_{L}(y)q_f(x) + P_{L}^{(N)}q_f(x)}{q_f(x) + P_{L}^{(e)}D_{L}(y)P_{L}^{(N)}\Delta q_f(x)}, \] (42)
\[ P_{f}^{e^+e^-}(x, y) = \frac{P_{L}^{(e)}D_{L}(y)\bar{q}_f(x) + P_{L}^{(N)}\bar{q}_f(x)}{\bar{q}_f(x) + P_{L}^{(e)}D_{L}(y)P_{L}^{(N)}\Delta \bar{q}_f(x)}, \] (43)
for longitudinally polarized case, where $P_L^{(e)}$ and $P_L^{(N)}$ are polarizations of the incident electron and nucleon, respectively; $D_L(y)$ is the longitudinal spin transfer factor in $eq \to eq$. It is given by

$$D_L(y) = \frac{1 - (1 - y)^2}{1 + (1 - y)^2}.$$  \hfill (44)

In the transversely polarized case, we have

$$P_{fT}^{eN}(x, y) = P_T^{(N)} \frac{\delta q_f(x)}{q_f(x)} D_T(y).$$ \hfill (45)

$$P_{fT}^{e\bar{N}}(x, y) = P_T^{(N)} \frac{\delta \bar{q}_f(x)}{\bar{q}_f(x)} D_T(y).$$ \hfill (46)

where the transversal spin transfer factor $D_T(y)$ in $eq \to eq$ is given by

$$D_T(y) = \frac{2(1 - y)}{1 + (1 - y)^2}.$$ \hfill (47)

We see that the relative contributions of different flavors are given by

$$R_f^{eN}(x, y) = \frac{1}{\sigma_{inel}} \frac{d\sigma_{\text{Mott}}}{dy} e_f^2 q_f(x, Q^2),$$ \hfill (48)

$$R_f^{e\bar{N}}(x, y) = \frac{1}{\sigma_{inel}} \frac{d\sigma_{\text{Mott}}}{dy} e_{\bar{f}}^2 \bar{q}_f(x, Q^2).$$ \hfill (49)

From Eqs.(42)–(49), we see that both $P_f$ and $R_f$ depend on $x$ and $y$. We see also that, in general, $P_f^{eN}(x, y) \neq P_f^{e\bar{N}}(x, y)$, $R_f^{eN}(x, y) \neq R_f^{e\bar{N}}(x, y)$, they are equal only if $q_f(x, Q^2) = \bar{q}_f(x, Q^2)$ and $\Delta q_f(x, Q^2) = \Delta \bar{q}_f(x, Q^2)$.

For the corresponding number densities of $H$ or $\bar{H}$ in $e + N \to e + H$ or $\bar{H} + X$, we have

$$N^{eN}(z, H) = \sum_f \int dx dy \left[ R_f^{eN}(x, y) D_f^H(z) + R_f^{eN}(x, y) D_f^\bar{H}(z) \right],$$ \hfill (50)

$$N^{eN}(z, \bar{H}) = \sum_f \int dx dy \left[ R_f^{eN}(x, y) D_f^H(z) + R_f^{e\bar{N}}(x, y) D_f^\bar{H}(z) \right],$$ \hfill (51)

$$\Delta N^{eN}(z, H) = \sum_f \int dx dy \left[ R_f^{eN}(x, y) P_f^{eN}(x, y) \Delta D_f^H(z) + R_f^{eN}(x, y) P_f^{e\bar{N}}(x, y) \Delta D_f^\bar{H}(z) \right],$$ \hfill (52)

$$\Delta N^{eN}(z, \bar{H}) = \sum_f \int dx dy \left[ R_f^{e\bar{N}}(x, y) P_f^{e\bar{N}}(x, y) \Delta D_f^H(z) + R_f^{e\bar{N}}(x, y) P_f^{eN}(x, y) \Delta D_f^\bar{H}(z) \right],$$ \hfill (53)

Now, we compare the results for $\bar{H}$ with those for $H$, and we see the following qualitative features in the two different cases.
In the first case, we consider reactions at very high energies so that small $x$-contribution dominates. In this case, we can neglect the valence-quark contributions with good accuracy, i.e., $q_f(x, Q^2) \approx q_{f,s}(x, Q^2)$ and $\Delta q_f(x, Q^2) \approx \Delta q_{f,s}(x, Q^2)$, where the subscript $s$ denote “sea.” If we assume the charge conjugation symmetry in nucleon sea, i.e., $q_{f,s}(x, Q^2) = \bar{q}_{f,s}(x, Q^2)$, and $\Delta q_{f,s}(x, Q^2) = \Delta \bar{q}_{f,s}(x, Q^2)$, we expect only a very small difference between quark and anti-quark distributions, i.e., $q_f(x, Q^2) \approx \bar{q}_f(x, Q^2)$, and $\Delta q_f(x, Q^2) \approx \Delta \bar{q}_f(x, Q^2)$. Hence, we expect that, $P_{en}(x, y) \approx P_{\bar{e}n}(x, y)$, $R_{en}(x, y) \approx R_{\bar{e}n}(x, y)$, and finally $P_H(z) \approx P_{\bar{H}}(z)$.

In the second case, we consider reactions where large $x$ contributions dominate. In this case, valence-quark contribution plays an important role and there should be significant differences between quark and anti-quark distributions — thus significant differences between $P_H$ and $P_{\bar{H}}$. To get some feeling of these differences, we make the following rough estimations.

We denote $R_{en}(x, y) = R_{en,v}(x, y) + R_{en,s}(x, y)$, (where the $v$ and $s$ in the subscripts denote valence or sea contribution), and expect the following approximate relations, if we neglect the flavor-dependence in the quark distribution functions:

$$R_{en,v}(x, y) \approx 8R_{en,v}(x, y),$$  \hspace{1cm} (54)

$$R_{en,s}(x, y) \approx 4R_{en,s}(x, y) \approx 4R_{en,s}(x, y),$$  \hspace{1cm} (55)

$$R_{en,s}(x, y) \approx 4R_{en,s}(x, y) \approx 4R_{en,s}(x, y).$$  \hspace{1cm} (56)

These approximate relations can be used to give us some guidances of the qualitative features of hyperon and antihyperon polarizations in the reaction. For example, for $e^- + p \rightarrow e^- + \Lambda + X$, we note further that, $D^\Lambda_u(z) = D^\Lambda_d(z)$ and for large $z$ both of them have the strangeness suppressions factor $\lambda \approx 0.3$ relative to $D^\Lambda_s(z)$ and that both $u$ and $d$ have, if any, small and negative contributions to the spin of $\Lambda$ but $s$ has large and positive contribution. Hence, for reactions where the valence-quark contributions dominate for hyperon production, we expect $u \rightarrow \Lambda + X$ dominates in large $z$ region, and $P_\Lambda$ should be small. But for $\bar{\Lambda}$, there is no contribution from valence-quark to the first rank particles; we expect that, for large $z$, $\bar{u} \rightarrow \bar{\Lambda} + X$ and $\bar{s} \rightarrow \bar{\Lambda} + X$ give comparable contributions to $\bar{\Lambda}$ production and $P_{\bar{\Lambda}}$ should be mainly determined by $\bar{s} \rightarrow \bar{\Lambda} + X$. This implies that,
in this case, $P_\lambda$ should be positive and the magnitude is much larger than $P_\lambda$ at large $z$ values.

We can see that the qualitative features are independent of the models for spin transfer but depend strongly on the kinematic region.

3. $\nu_\mu + N \rightarrow \mu^- + H$ (or $\bar{H}$) + $X$

For charged current neutrino reaction, we have

$$\frac{d^3\sigma}{dxdydz}(\nu_\mu N \rightarrow \mu^- HX) = \sum_{f,f'} \frac{d\hat{\sigma}_0}{dy}(\nu_\mu q_f \rightarrow \mu^- \bar{q}_{f'}) q_f(x, Q^2) D^H_{f'}(z)$$

$$+ \sum_{f,f'} \frac{d\hat{\sigma}_0}{dy}(\nu_\mu \bar{q}_f \rightarrow \mu^- \bar{q}_{f'}) \bar{q}_f(x, Q^2) D^H_{f'}(z).$$

We do not consider top production. In this case we have two Cabbibo favored elementary processes for quarks. The differential cross sections are given by

$$\frac{d\hat{\sigma}_0}{dy}(\nu_\mu d \rightarrow \mu^- u) = \frac{d\hat{\sigma}_0}{dy}(\nu_\mu s \rightarrow \mu^- c) = \frac{G^2 x_s}{\pi} \cos^2 \theta_c,$$  \hspace{1cm} (57)$$

and two Cabbibo suppressed elementary processes with cross sections,

$$\frac{d\hat{\sigma}_0}{dy}(\nu_\mu d \rightarrow \mu^- c) = \frac{d\hat{\sigma}_0}{dy}(\nu_\mu s \rightarrow \mu^- u) = \frac{G^2 x_s}{\pi} \sin^2 \theta_c,$$  \hspace{1cm} (58)$$

where $\theta_c$ is the Cabbibo angle. Similarly for anti-quarks, we have two Cabbibo favored elementary processes,

$$\frac{d\hat{\sigma}_0}{dy}(\nu_\mu \bar{u} \rightarrow \mu^- \bar{d}) = \frac{d\hat{\sigma}_0}{dy}(\nu_\mu \bar{c} \rightarrow \mu^- \bar{s}) = \frac{G^2 x_s}{\pi} (1 - y)^2 \cos^2 \theta_c,$$  \hspace{1cm} (59)$$

and two Cabbibo suppressed processes,

$$\frac{d\hat{\sigma}_0}{dy}(\nu_\mu \bar{u} \rightarrow \mu^- \bar{s}) = \frac{d\hat{\sigma}_0}{dy}(\nu_\mu \bar{c} \rightarrow \mu^- \bar{d}) = \frac{G^2 x_s}{\pi} (1 - y)^2 \sin^2 \theta_c,$$  \hspace{1cm} (60)$$

This means that

$$\frac{d^3\sigma}{dxdydz}(\nu_\mu N \rightarrow \mu^- HX) = \frac{G^2 x_s}{\pi} \left\{ [d(x, Q^2) \cos^2 \theta_c + s(x, Q^2) \sin^2 \theta_c] D^H_u(z) \right.$$  

$$+ [d(x, Q^2) \sin^2 \theta_c + s(x, Q^2) \cos^2 \theta_c] D^H_{c}(z) \right.$$  

$$+ (1 - y)^2 [\bar{u}(x, Q^2) \cos^2 \theta_c + \bar{c}(x, Q^2) \sin^2 \theta_c] D^H_{d}(z) \right.$$  

$$+ (1 - y)^2 [\bar{u}(x, Q^2) \sin^2 \theta_c + \bar{c}(x, Q^2) \cos^2 \theta_c] D^H_{s}(z) \right\},$$  \hspace{1cm} (61)$$
We note further that the quarks in the final state of $\nu_\mu q \rightarrow \mu^- q'$ are longitudinally polarized with polarization $P_q = -1$, while the anti-quarks in the final state of $\nu_\mu \bar{q} \rightarrow \mu^- \bar{q}'$ are longitudinally polarized with $P_{\bar{q}} = 1$. We obtain that

$$
\frac{d^3\Delta\sigma}{dxdydz}(\nu_\mu N \rightarrow \mu^- H X) = \frac{G^2xs}{\pi} \left\{ -[d(x, Q^2) \cos^2 \theta_c + s(x, Q^2) \sin^2 \theta_c] \Delta D^{H}_{u}(z) \\
\quad - [d(x, Q^2) \sin^2 \theta_c + s(x, Q^2) \cos^2 \theta_c] \Delta D^{H}_{c}(z) \\
\quad + (1 - y)^2[\bar{u}(x, Q^2) \cos^2 \theta_c + \bar{c}(x, Q^2) \sin^2 \theta_c] \Delta D^{H}_{d}(z) \\
\quad + (1 - y)^2[\bar{u}(x, Q^2) \sin^2 \theta_c + \bar{c}(x, Q^2) \cos^2 \theta_c] \Delta D^{H}_{s}(z) \right\}, (62)
$$

For anti-hyperon, we have

$$
\frac{d^3\sigma}{dxdydz}(\nu_\mu N \rightarrow \mu^- \bar{H} X) = \frac{G^2xs}{\pi} \left\{ [d(x, Q^2) \cos^2 \theta_c + s(x, Q^2) \sin^2 \theta_c] D^{H}_{u}(z) \\
\quad + [d(x, Q^2) \sin^2 \theta_c + s(x, Q^2) \cos^2 \theta_c] D^{H}_{c}(z) \\
\quad + (1 - y)^2[\bar{u}(x, Q^2) \cos^2 \theta_c + \bar{c}(x, Q^2) \sin^2 \theta_c] D^{H}_{d}(z) \\
\quad + (1 - y)^2[\bar{u}(x, Q^2) \sin^2 \theta_c + \bar{c}(x, Q^2) \cos^2 \theta_c] D^{H}_{s}(z) \right\}, (63)
$$

$$
\frac{d^3\Delta\sigma}{dxdydz}(\nu_\mu N \rightarrow \mu^- \bar{H} X) = \frac{G^2xs}{\pi} \left\{ -[d(x, Q^2) \cos^2 \theta_c + s(x, Q^2) \sin^2 \theta_c] \Delta D^{H}_{\bar{u}}(z) \\
\quad - [d(x, Q^2) \sin^2 \theta_c + s(x, Q^2) \cos^2 \theta_c] \Delta D^{H}_{\bar{c}}(z) \\
\quad + (1 - y)^2[\bar{u}(x, Q^2) \cos^2 \theta_c + \bar{c}(x, Q^2) \sin^2 \theta_c] \Delta D^{H}_{\bar{d}}(z) \\
\quad + (1 - y)^2[\bar{u}(x, Q^2) \sin^2 \theta_c + \bar{c}(x, Q^2) \cos^2 \theta_c] \Delta D^{H}_{\bar{s}}(z) \right\}, (64)
$$

From these equations, we expect that, unlike that in $e^- N$ scattering, there should be a significant difference for the polarization of hyperon and that for the corresponding antihyperon. This is because, in $\nu_\mu + N \rightarrow \mu^- + H(\text{or} \ \bar{H}) + X$, (i) hyperons in the current fragmentation region are mainly from the fragmentation of $u$ and $c$ quarks but the antihyperons are from $\bar{d}$ and $\bar{s}$; and (ii) the helicities of the struck quarks are opposite to those of the anti-quarks.

To see the qualitative features more explicitly, we now make a qualitative analysis by taking the following approximations. We keep only Cabbibo favored processes and neglect $\Delta D^{H}_{f}(z)$ compared to $\Delta D^{H}_{f}(z)$. Under these approximations, we have

$$
\frac{d^3\Delta\sigma}{dxdydz}(\nu_\mu N \rightarrow \mu^- H X) \approx -\frac{G^2xs}{\pi} \cos^2 \theta_c [d(x, Q^2) \Delta D^{H}_{u}(z) + s(x, Q^2) \Delta D^{H}_{c}(z)], (65)
$$
\[
\frac{d^3\Delta\sigma}{dx dy dz}(\nu_\mu N \to \mu^- \bar{H}X) \approx \frac{G^2_F}{\pi} \bar{s}(1 - y)^2 \cos^2 \theta_c \bar{u}(x, Q^2) \Delta D^H_d(z) + \bar{c}(x, Q^2) \Delta D^H_s(z). 
\] (66)

We see that the polarization of hyperons in this reaction is mainly determined by \(\Delta D^H_d(z)\) and \(\Delta D^H_s(z)\) while those for antihyperons are determined by \(\Delta D^H_d(z)\) and \(\Delta D^H_s(z)\). We thus expect the following qualitative features:

1. For \(\Lambda\), we note \(\Delta D^\Lambda_d(z) = \Delta D^\Lambda_u(z) < 0\) and the magnitude is very small, while \(\Delta D^\Lambda_s(z) > 0\) and the magnitude is large. There is a significant contribution from \(\Lambda_c\) decay to \(\Lambda\), but the decay spin transfer in \(\Lambda_c \to \Lambda X\) is unclear. Hence it is very difficult to make any estimate on \(P_\Lambda\). But for \(\bar{\Lambda}\), we expect that the qualitative feature of \(P_\bar{\Lambda}\) in \(\nu_\mu + N \to \mu^- + \bar{\Lambda} + X\) is mainly determined by \(\Delta D^\Lambda_s(z)\). The results should be positive and the magnitude is large for large \(z\).

2. For \(\Sigma\) production, we recall that, from isospin symmetry, \(\Delta D^{\Sigma^+}_u = \Delta D^{\Sigma^+}_d\) is positive and large, while \(\Delta D^{\Sigma^+}_d = \Delta D^{\Sigma^+}_u\) is very small. The spin transfer from charmed baryon decay to \(\Sigma\) is unknown but such decay contribution is relatively small compared to \(\Lambda\). We thus expect that \(P_{\Sigma^+}\) is negative and the magnitude is large for large \(z\). But the magnitude of \(P_{\Sigma^-}\) is very small.

We note further that \(\Delta D^{\Sigma^\pm}_s\) is negative and the magnitude is smaller than \(\Delta D^{\Sigma^\mp}_d\) [half of it in \(SU(6)\) and similar in DIS picture]. But there is a strange suppression for \(D^{\Sigma}_d\) compared to \(D^{\Sigma}_s\). We expected the contributions from the two terms in Eq.(66) to \(P_{\Sigma^+}\) are opposite in sign with similar magnitudes. These two contributions partly cancel each other and the final results for \(P_{\Sigma^+}\) should be small in magnitude and very sensitive to the quark distribution functions. For \(P_{\Sigma^-}\), the contribution from \(\Delta D^{\Sigma^-}_d = \Delta D^{\Sigma^-}_d\) is very small and the results are mainly determined by \(\Delta D^{\Sigma^+_s}\), which is negative and larger at large \(z\).

3. For \(\Xi\) production, charmed baryon decay contribution can be neglected. \(P_{\Xi}\) is mainly determined by \(-\Delta D^\Xi_u\). Since \(\Delta D^\Xi_u(z, \text{dir})\) is negative but the decay contribution from \(\Xi^0\), i.e. \(\Delta D^\Xi_u(z, \text{dec})\), is positive, and the magnitude of polarization of the latter is larger, the final results for \(P_{\Xi^0}\) can be quite sensitive to the hadronization model. But in general, we expect both the magnitude of \(P_{\Xi^0}\) and that of \(P_{\Xi^-}\) to be small.

For \(\bar{\Xi}^0\), the contribution from the first term of Eq.(66), i.e. that from \(\Delta D^\Xi_u\), is much
smaller than that from the second term since $\Xi^0$ does not contain $d$ as a valence-quark. For $\bar{\Xi}^+$, there is a contribution from the first term of Eq.(66), i.e. that from $\Delta D_{d}^{\Xi^-}$, but the magnitude is smaller than $\Delta D_{s}^{\Xi^-}$ [half of it in SU(6)]. Furthermore, because of the strangeness suppression, $D_{d}^{\Xi^-}$ also is suppressed compared to $D_{s}^{\Xi^-}$. We thus expect that $|\Delta D_{d}^{\Xi^-}| \ll \Delta D_{s}^{\Xi^-}$ too. Hence for both $\Xi^0$ and $\Xi^+$, we expect that $P_{\Xi}$ is mainly determined by $\Delta D_{s}^{\Xi}$ and results should be positive and large for large $z$.

These qualitative features are independent of models for spin transfer and can be checked by experiments.

4. $\bar{\nu}_\mu + N \to \mu^+ + H(\text{or } \bar{H}) + X$

For charged current reactions with anti-neutrino beam such as $\bar{\nu}_\mu + N \to \mu^+ + H(\text{or } \bar{H}) + X$, the contributing elementary processes are the following. We have two Cabbibo favored elementary processes for quarks with the differential cross sections as given by

$$\frac{d\sigma_0}{dy}(\bar{\nu}_\mu u \to \mu^+ d) = \frac{d\sigma_0}{dy}(\bar{\nu}_\mu c \to \mu^+ s) = \frac{G^2xs}{\pi} (1 - y)^2 \cos^2 \theta_c, \quad (67)$$

and two Cabbibo suppressed elementary processes with cross sections,

$$\frac{d\sigma_0}{dy}(\bar{\nu}_\mu u \to \mu^+ s) = \frac{d\sigma_0}{dy}(\bar{\nu}_\mu c \to \mu^+ d) = \frac{G^2xs}{\pi} (1 - y)^2 \sin^2 \theta_c. \quad (68)$$

Similarly for anti-quarks, we have two Cabbibo favored elementary processes,

$$\frac{d\sigma_0}{dy}(\bar{\nu}_\mu \bar{d} \to \mu^+ \bar{u}) = \frac{d\sigma_0}{dy}(\bar{\nu}_\mu \bar{s} \to \mu^+ \bar{c}) = \frac{G^2xs}{\pi} \cos^2 \theta_c, \quad (69)$$

and two Cabbibo suppressed processes,

$$\frac{d\sigma_0}{dy}(\bar{\nu}_\mu \bar{d} \to \mu^+ \bar{c}) = \frac{d\sigma_0}{dy}(\bar{\nu}_\mu \bar{s} \to \mu^+ \bar{u}) = \frac{G^2xs}{\pi} \sin^2 \theta_c. \quad (70)$$

Hence, we obtain that

$$\frac{d^3\sigma}{dx dy dz}(\bar{\nu}_\mu N \to \mu^+ H X) = \frac{G^2xs}{\pi} \left\{ [\bar{d}(x, Q^2) \cos^2 \theta_c + \bar{s}(x, Q^2) \sin^2 \theta_c] D_{u}^H(z) + [\bar{d}(x, Q^2) \sin^2 \theta_c + \bar{s}(x, Q^2) \cos^2 \theta_c] D_{d}^H(z) + (1 - y)^2 [u(x, Q^2) \cos^2 \theta_c + c(x, Q^2) \sin^2 \theta_c] D_{s}^H(z) \right\}, \quad (71)$$
\[
\frac{d^3\sigma}{dx dy dz}(\bar{\nu}_\mu N \rightarrow \mu^+ H X) = \frac{G^2 x s}{\pi} \left\{ [\bar{d}(x, Q^2) \cos^2 \theta_c + \bar{s}(x, Q^2) \sin^2 \theta_c] D_u^H(z) + [\bar{d}(x, Q^2) \sin^2 \theta_c + \bar{s}(x, Q^2) \cos^2 \theta_c] D_e^H(z) + (1 - y)^2[u(x, Q^2) \cos^2 \theta_c + c(x, Q^2) \sin^2 \theta_c] D_d^H(z) + (1 - y)^2[u(x, Q^2) \sin^2 \theta_c + c(x, Q^2) \cos^2 \theta_c] D_s^H(z) \right\}, \quad (72)
\]

We note further that the quarks in the final state of $\bar{\nu}_\mu q' \rightarrow \mu^+ q'$ are longitudinally polarized with polarization $P_{q'} = -1$, while the anti-quarks in the final state of $\bar{\nu}_\mu \bar{q} \rightarrow \mu^+ q'$ are longitudinally polarized with $P_{q'} = 1$. We obtain that,

\[
\frac{d^3\Delta\sigma}{dx dy dz}(\bar{\nu}_\mu N \rightarrow \mu^+ H X) = \frac{G^2 x s}{\pi} \left\{ [\bar{d}(x, Q^2) \cos^2 \theta_c + \bar{s}(x, Q^2) \sin^2 \theta_c] \Delta D_u^H(z) + [\bar{d}(x, Q^2) \sin^2 \theta_c + \bar{s}(x, Q^2) \cos^2 \theta_c] \Delta D_e^H(z) - (1 - y)^2[u(x, Q^2) \cos^2 \theta_c + c(x, Q^2) \sin^2 \theta_c] \Delta D_d^H(z) - (1 - y)^2[u(x, Q^2) \sin^2 \theta_c + c(x, Q^2) \cos^2 \theta_c] \Delta D_s^H(z) \right\}, \quad (73)
\]

\[
\frac{d^3\Delta\sigma}{dx dy dz}(\bar{\nu}_\mu N \rightarrow \mu^+ \bar{H} X) = \frac{G^2 x s}{\pi} \left\{ [\bar{d}(x, Q^2) \cos^2 \theta_c + \bar{s}(x, Q^2) \sin^2 \theta_c] \Delta D_u^H(z) + [\bar{d}(x, Q^2) \sin^2 \theta_c + \bar{s}(x, Q^2) \cos^2 \theta_c] \Delta D_e^H(z) - (1 - y)^2[u(x, Q^2) \cos^2 \theta_c + c(x, Q^2) \sin^2 \theta_c] \Delta D_d^H(z) - (1 - y)^2[u(x, Q^2) \sin^2 \theta_c + c(x, Q^2) \cos^2 \theta_c] \Delta D_s^H(z) \right\}, \quad (74)
\]

We compare these results with those for $\nu_\mu + N \rightarrow \mu^- + H(\text{or } \bar{H}) + X$ presented in last subsection. We see that the results for $\bar{\nu}_\mu + N \rightarrow \mu^+ + H(\text{or } \bar{H}) + X$ are the same as the corresponding results $\nu_\mu + N \rightarrow \mu^- + H(\text{or } \bar{H}) + X$ under the exchange $q_f(x, Q^2) \leftrightarrow \bar{q}_f(x, Q^2)$. Hence, if we consider the reactions at very high energies where small $x$ contribution dominates so that $q_f(x, Q^2) \approx \bar{q}_f(x, Q^2)$, we have

\[
P_{\mu N}^H(z) \approx P_{\mu N}^\bar{H}(z), \quad (75)
\]

\[
P_{\mu N}^H(z) \approx P_{\mu N}^\bar{H}(z). \quad (76)
\]

As we have emphasized before, to test different models for spin transfer in fragmentation, it is important to have high energy so that the results in the current fragmentation region can be considered as purely from the struck quark (anti-quark) fragmentation.
In this case, there is no new result for anti-neutrino charged current interactions compared with those for the corresponding neutrino reactions. The differences come from the valence-quark contributions which are small at very high energies where small $x$ dominate. In view of this and the difficulties in performing such experiments in the near future, we will not discuss this reaction in the next section.

C. Models for $\Delta D^H_f(z)$

There exist many different approaches for $\Delta D^H_f(z)$ in literature. We summarize the key points of some of them in the following.

1. Calculation of $\Delta D^H_f(z)$ according to the origin of $H$

In $\Delta D^H_f(z)$ has been calculated according to the origins of $H$. The produced $H$’s are divided into the following four categories: (A) those are directly produced and contain $q_f$; (B) decay products of polarized heavy hyperons; (C) those are directly produced and do not contain $q_f$; (D) decay products of unpolarized heavy hyperons. This is to say that we divide further

$$D^H_f(z; \text{dir}) = D^{H(A)}_f(z) + D^{H(C)}_f(z); \quad (77)$$

$$D^H_f(z; \text{dec}) = D^{H(B)}_f(z) + D^{H(D)}_f(z). \quad (78)$$

For the polarized case,

$$\Delta D^H_f(z; \text{dir}) = \Delta D^{H(A)}_f(z) + \Delta D^{H(C)}_f(z); \quad (79)$$

$$\Delta D^H_f(z; \text{dec}) = \Delta D^{H(B)}_f(z) + \Delta D^{H(D)}_f(z). \quad (80)$$

It is assumed that,

$$\Delta D^{H(A)}_f(z) = t^{F}_{H,f} D^{H(A)}_f(z), \quad (81)$$

$$\Delta D^{H(C)}_f(z) = \Delta D^{H(D)}_f(z) = 0. \quad (82)$$

Here, $t^{F}_{H,f}$ is a constant and is taken as,

$$t^{F}_{H,f} = \Delta Q_f / n_f \quad (83)$$
where $\Delta Q_f$ and $n_f$ are the fractional contribution of spin of quark with flavor $f$ to the spin of $H$ and the number of valence-quarks of flavor $f$ in $H$. Clearly, $Q_f$ is different in the SU(6) picture from those drawn from polarized deeply inelastic scattering (DIS) data. They are given in e.g. Table I of [6].

The decay contribution part $\Delta D_f^{H(B)}(z)$ is calculated using Eq.(5), i.e.,

$$\Delta D_f^{H(B)}(z) = \sum_j \int dz' t_{H,H_j}^{D} K_{H,H_j}(z,z')[\Delta D_j^{H(A)}(z') + \Delta D_j^{H(B)}(z')],$$

(84)

We should note that, in the Feynman-Field type of recursive cascade fragmentation model, $D_f^{H(A)}(z)$ is nothing else but the probability to produce a first rank $H$ with $z$. It is usually denoted by $f_q^H(z)$ in such fragmentation models, i.e.,

$$D_f^{H(A)}(z) = f_q^H(z).$$

(85)

It follows that,

$$D_f^{H(C)}(z) = D_f^H(z; \text{dir}) - f_q^H(z).$$

(86)

In this case, we have,

$$\Delta D_f^{H(A)}(z) = t_{H,f}^F f_q^H(z),$$

(87)

$$\Delta D_f^{H(B)}(z) = \sum_j \int dz' t_{H,H_j}^{D} K_{H,H_j}(z,z')[t_{H,f}^F f_q^H(z) + \Delta D_j^{H(B)}(z')].$$

(88)

In this model, for $\bar{q}_f \rightarrow HX$, we should have,

$$D_f^{H(A)}(z) = 0,$$

(89)

$$\Delta D_f^{H}(z) = 0.$$  

(90)

We recall that $f_q^H(z)$ is a quite essential input in such recursive cascade models. In most cases, there exist explicit expression for it. Hence, the $z$ dependence of $\Delta D$ in this model is determined by the fragmentation model for unpolarized case, which are empirically known from the experimental facts for unpolarized reactions. The only unknown is the spin transfer constant $t_{H,f}^F$ for type (A) hyperon which influences mainly the magnitudes of the polarizations of the hyperons. So the ingredients of this model can be tested separately by testing the $z$ dependence and magnitudes of the polarizations.

In [2, 3, 6, 7, 8, 9], calculations have been carried out using a Monte Carlo event generator JETSET for $e^+e^-$ annihilation and PYTHIA for $lp$ or $pp$ collisions based on Lund fragmentation model [24]. A set of formulas are given there which are more convenient for Monte Carlo calculations.
2. Calculation of $\Delta D^H_f(z)$ using Gribov relation

In [10, 11, 12, 13, 14], $\Delta D^H_f(z)$ has been calculated using the following proportionality relation, i.e., $D^H_f(z) \propto q^H_f(z)$ and $\Delta D^H_f(z) \propto \Delta q^H_f(z)$, which they referred as “Gribov relation” since it was first shown in [25] by Gribov and collaborator in 1971. In terms of the language given above, this relation, if true, should be valid for the directly produced part, i.e.

$$D^H_f(z; \text{dir}) \propto q^H_f(z), \quad (91)$$

$$\Delta D^H_f(z; \text{dir}) \propto \Delta q^H_f(z). \quad (92)$$

The decay contribution parts can be calculated using Eq.(5).

For $\bar{q}_f \rightarrow HX$, we have similar results, i.e.

$$D^H_{\bar{f}}(z; \text{dir}) \propto \bar{q}^H_f(z), \quad (93)$$

$$\Delta D^H_{\bar{f}}(z; \text{dir}) \propto \Delta \bar{q}^H_f(z). \quad (94)$$

In [10, 11, 12, 13, 14], as an approximation, decay contributions are not considered.

3. Other models

Other models for $\Delta D^H_f(z)$ have also discussed in the literature [4, 5]. They are essentially different combinations of the two models discussed above. We will not go to the details of these models but refer the interested readers to the references.

III. RESULTS AND DISCUSSIONS

By applying the formulas presented in the last section to different reactions, we obtain the results for the polarization of hyperons and antihyperons in these reactions. We use the model for $\Delta D^H_f(z)$ as described in the first subsection of Sec. II C, i.e. to calculate $\Delta D^H_f(z)$ according to the origin of $H$. The different contributions to $D^H_f(z)$ are calculated using a Monte Carlo event generator PYTHIA [23] based on Lund string fragmentation model [24]. The results for hyperon polarization are given in [3, 6, 7, 8, 9]. We present the results for antihyperons and compare them with those for the corresponding hyperon
in the following. As we have shown in Section IIB, the results for antihyperons in $e^+e^-$ annihilations differ from those for the corresponding hyperons only by a minus sign. We will not repeat them here. We will present the results for $e^- + N \rightarrow e^- + \bar{H} + X$ and those for $\nu_\mu + N \rightarrow \mu^- + \bar{H} + X$, respectively, in the following.

A. $e^- + N \rightarrow e^- + \bar{H} + X$

In Fig. 1, we show the results for antihyperons in $e^- + N \rightarrow e^- + \bar{H} + X$ with polarized beam or target. Clearly the situation is completely the same for $e^+ + N \rightarrow e^+ + \bar{H} + X$ or $\mu^+ + N \rightarrow \mu^+ + \bar{H} + X$, where one virtual photon exchange plays the dominate role. The incident energy of electron is taken as $E = 500$ GeV off a fixed target. We take this electron energy to guarantee that the energy is already high enough that we need only take the struck quark or anti-quark fragmentation into account when we look at the current fragmentation region. The results are shown as functions of $z$, where $0.2 < y < 0.9$, $Q^2 > 1$ GeV$^2$ and $W^2 > 1$ GeV$^2$. The results depend not much on the energy so long as $E$ is considerably large (say, larger than 200GeV).

From Fig. 1 we see that there is indeed little difference between the results for hyperons and those for antihyperons at such energies for reactions with polarized beams. This confirms the qualitative expectations presented in Sec.IIB. We also see that the polarizations are much larger in reactions with polarized beams than those for reactions where only the nucleon target is polarized. The results in the latter case are very sensitive to the parametrization of polarized quark (anti-quark) distribution functions used. The relatively large difference between the results for hyperons and those for the corresponding antihyperons reflects the differences in the polarized quark (anti-quark) distributions.

To compare the results with the results from COMPASS at CERN, we lowered the energy to $E = 160$ GeV and obtained the results as shown in the left column of Fig. 2. Our results show that, at this energy, the difference between the results for $\Lambda$ and those for $\bar{\Lambda}$ polarization is also quite small, in particular, in the $x_F \sim 0$ region. The difference can be a little bit significant only for larger $x_F$. According to this result, we should not see a large difference between $P_\Lambda$ and $P_{\bar{\Lambda}}$ as indicated by the COMPASS data.

As we have discussed in last section, in the theoretical framework described in this
paper, the following symmetries are supposed: (i) Charge conjugation symmetry in the
fragmentation functions and decay processes [see Eqs.(14)–(17)]; and (ii) charge conju-
gation symmetry in nucleon sea in particular $s(x) = \bar{s}(x)$ and $\Delta s(x) = \Delta \bar{s}(x)$. In this case,
the origins of the difference between the polarizations of hyperons and those of antihy-
perons can be only the contributions from the valence-quarks of the initial nucleons. Our
results show that the valence-quark contributions are already quite small at the COM-
PASS energy. To see this explicitly, we have examined the $x$ values of the struck quark
or antiquark for events with the kinematic constraints as imposed by COMPASS. (Here,
$x$ is the Bjorken $x$ used in describing deeply inelastic lepton-nucleon scattering.) The
results are shown in Fig.3. Here, in Fig.3(a) and 3(b), we show the $x$-distribution of the
struck quark and anti-quark that lead to the production of the hyperon and antihyperon,
respectively; and in 3(c) and 3(d), we show the average values of $x$ of such struck quarks
and anti-quarks as functions of $x_F$ of the produced hyperons or antihyperons. We see
that the $x$ value of the struck quark or anti-quark can be as small as 0.004. We see also
that $\langle x \rangle$ in this case is in general of the order of 0.01 for most of the $x_F$ values. In this
$x$ region, $q(x, Q^2)$ is already dominated by the sea quark distribution $q_s(x, Q^2)$. There
exists a valence-quark contribution $q_v(x, Q^2)$, but it is much smaller than $q_s(x, Q^2)$. This
is why the obtained $P_{\bar{H}}$ differs little from $P_H$.

Another effect that relates to the valence-quark contribution and may cause a differ-
ence between $P_H$ and $P_{\bar{H}}$ in $e^- + N \rightarrow e^- + H$(or $\bar{H}$) + $X$ is the contribution from the
hadronization of the remnant of target nucleon. It has been pointed out first in [9] that
contribution of the hadronization of target remnant is important to hyperon production
even for reasonably large $x_F$ at lower energies. The effect has been confirmed by the
calculations presented in [15]. It has been shown that at the CERN NOMAD ener-
gies, contribution from the hadronization of nucleon target remnant dominates hyperon
production at $z$ around zero. It is impossible to separate the contribution of the struck
quark fragmentation from those of the target remnant fragmentation. Clearly, this effect
can be different for hyperon and antihyperon production since the target remnant contri-
bution comes mainly from the fragmentation of the valence diquark. It contributes quite
differently for hyperons and antihyperons. To see whether this effect plays an important
role at COMPASS energy, we have also calculated the contributions of target remnant
fragmentation. We found out that this contribution is already very small for $z > 0$ at the COMPASS energy. To see the influence on hyperon polarizations, we have also included the contributions in the results shown in Fig. 2(a) by using a valence-quark model for the quark polarization in the remnant of the target as described in [9]. We see that there is indeed some influence at $x_F \sim 0$ for the polarization of the hyperons but the effect is already very small at the COMPASS energy.

After having made these checks, we are quite confident that the valence-quark contributions are quite small at the COMPASS energy. Hence, under the conjugation symmetries in fragmentation function, decay processes and in nucleon sea, there should not be a large difference between $P_\Lambda$ and $P_{\bar{\Lambda}}$ at COMPASS energy. A significant difference could be a signature for the violation of such conjugation symmetries. In view of the recent discussions on the asymmetric strange and antistrange sea [28], it would be very significant to check whether similar asymmetry is also possible for the polarized case.

We also made the calculations at HERMES energy, i.e. at $E = 27.6\text{GeV}$. The results are shown in the right column of Fig. 2. We found out that at this energy, the major contribution comes mainly from the $x$-region of $0.02 < x < 0.8$. In this $x$ region, valence-quark plays an important role. Hence, we obtained a significant difference between the polarizations of antihyperons and those of hyperons in the current fragmentation region. We see, in particular, that the difference for $\Lambda$ and $\bar{\Lambda}$ is indeed as expected in the qualitative analysis given in Sec. II B 2.

We also calculated the contribution from target remnant fragmentation and found that it is important at HERMES energy. This can be seen clearly in Fig. 4 where different contributions to $\Lambda$ at this energy are shown. We see clearly that the contribution from target remnant fragmentation is much higher than that from the struck quarks in the region near $x_F = 0$. To show the influence of this effect on the polarizations, we calculate these contributions from target remnant fragmentation using a valence-quark model for the polarizations of the quarks in the remnant of the nucleon as described in [9]. The results are added to the struck quark fragmentation contribution and are shown in Fig. 2(b). We see that the contribution is very important in the $x_F \sim 0$ region. It is therefore meaningless to factorize the $\Lambda$ production cross section as one usually does at very high energies. On the other hand, the influence from target remnant fragmentation
to antihyperon polarization is negligibly small. This is another reason for the differences between the polarizations of hyperons and those of antihyperons shown in Fig. 2(b) in particular at the $x_F \sim 0$ region.

**B. $\nu_\mu + N \to \mu^- + \bar{H} + X$**

In Fig. 5, we show the results obtained for $\nu_\mu + N \to \mu^- + \bar{H}(\text{or } H) + X$ at an incident muon-neutrino beam energy $E = 500$ GeV off a fixed target. We calculated for proton and neutron target and different hyperons and antihyperons. To obtain the results for $\Lambda$, we simply take $t_{D,\Lambda,c} = 1$ as we did in [7]. From these figures, we see that the results for hyperons and those for antihyperons are indeed significantly different from each other. These features are quite different from that in electron nucleon collision where hyperon and antihyperon polarizations are essentially the same at very high energies. The qualitative features are the same as we obtained in the qualitative analysis using Eqs.(65) and (66) in the last section.

We also calculated the polarizations for antihyperons at the CERN NOMAD energies. The results are shown in Fig. 6. As it has been pointed out in [9], at this energy, contribution from the fragmentation of the remnant of target nucleon to hyperon production is very important. We also included this contribution from a rough estimation by using a valence-quark model for the polarization of the quarks in the target remnant as in [9]. For this reason, the polarizations obtained for hyperons at this energy differ significantly from those obtained at very high energies as shown in Fig. 5. But for antihyperons, this contribution is small thus the difference between the results and those shown in Fig. 5 is small. This is also one of the reasons for the differences between the results for hyperons and those for antihyperons, in particular, in the $x_F \sim 0$ region. The qualitative features are consistent with the NOMAD data and it will be interesting to make high statistics check in the future.

**IV. SUMMARY AND OUTLOOK**

In summary, we have calculated the polarizations of antihyperons in lepton induced reactions, in particular $l + p \to l' + \bar{H} + X$ at different energies with polarized beams,
using different models for spin transfer for fragmentation. The results show little difference between the polarization of antihyperon and that of the corresponding hyperon in \( e^- + N \rightarrow e^- + \bar{H} (or H) + X \) at COMPASS or higher energies when the charge conjugation symmetries for fragmentation, decay and nucleon sea are assumed. But there are in general large differences between those for antihyperons and the corresponding hyperons in neutrino induced charged current reactions since the flavors of dominant fragmenting quarks and those of the anti-quarks and their polarizations are different. A detailed discussion is given and the results can be tested by future experiments.

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FIG. 1: Comparison of the polarizations of antihyperons with those of the corresponding hyperons as functions of $x_F$ in $e^- p \to e^- \bar{H}(or \ H)X$ at $E_{e^-} = 500$ GeV. The solid and dashed lines denote the results obtained by using the SU(6) picture, while the dotted and dash-dotted lines denote those by using the DIS picture. The results for hyperons are the same as those presented in [7]. The quark distribution functions are taken from [26] and [27].
FIG. 2: Comparison of the polarizations of antihyperons with those of the corresponding hyperons as functions of $x_F$ in $\mu^+ p \to \mu^+ \bar{H}(or \; H)X$ at $E_{\mu^+} = 160$ GeV (in the left column) and that in $e^+ p \to e^+ \bar{H}(or \; H)X$ at $E_{e^+} = 27.6$ GeV (in the right column) when both the contributions from the fragmentation of the struck quark and that of the nucleon remnant are taken into account. The solid and dashed lines denote the results obtained by using the SU(6) picture, while the dotted and dash-dotted lines denote those by using the DIS picture.
FIG. 3: The $x$-distribution obtained at the COMPASS energy for the struck quark or anti-quark that leads to the production of the hyperon [shown in (a)] and antihyperon [shown in (b)], and the average values of $x$ of such struck quarks or anti-quarks as functions of $x_F$ of the produced hyperons [in (c)] and antihyperons [in (d)] respectively. The solid, dashed, thick-dotted, dash-dotted and thin-dotted lines are for $\Lambda$, $\Sigma^+$, $\Sigma^-$, $\Xi^0$ and $\Xi^-$ or the corresponding antihyperons respectively (where the thin-dotted and dash-dotted lines are almost coincide with each other).
FIG. 4: Different contributions to $\Lambda$ in $e^+p \to e^+\Lambda X$ at $E_{e^+} = 27.6$ GeV. Here, the solid, dotted, dashed and dash-dotted lines denote respectively (1) directly produced and contain the struck quark; (2) directly produced and contain a $u$-or $d$-quark in $(uu)_1$($ud)_1$; (3) decay products of hyperons which contain the struck quark; (4) decay products of hyperons which contain the $(uu)_1$($ud)_1$ or a $u$-or $d$ in the $(uu)_1$($ud)_1$.
FIG. 5: Comparison of the polarization of antihyperons with those of the corresponding hyperons as functions of $x_F$ in $\nu_\mu N \rightarrow \mu^- \bar{H} (or \ H) X$ at $E_\nu = 500$ GeV. The solid and dashed lines denote the results obtained by using the SU(6) picture, while the dotted and dash-dotted lines denote those by using the DIS picture.
FIG. 6: Polarizations of antihyperons compared with those of the corresponding hyperons as functions of $x_F$ in $\nu_\mu N \rightarrow \mu^- \bar{H} (or \ H) X$ at $E_{\nu} = 44$ GeV when both the contributions from the fragmentation of the struck quark and that of the nucleon remnant are taken into account. The solid and dashed lines denote the results obtained by using the SU(6) picture, while the dotted and dash-dotted lines denote those by using the DIS picture.