I investigate the difference between the quasiparticle properties in two dimensional (2D) and three dimensional (3D) s-wave superconductors. Using the original BCS model for the pairing interaction and direct Coulomb interaction I show that quasiparticle interactions lead to a stronger energy dependence in the single-particle self-energies in 2D than in 3D superconductors. This difference arises from the presence of the low lying collective mode of the order parameter in the 2D case which ensures that oscillator strength in the response function is at low frequencies, $\sim \Delta$. This strong quantitative difference between 2D and 3D superconductors points to the importance of treating quasiparticle interactions in low dimensional superconductors rather than assuming that renormalizations remain unchanged from the normal state.
Electronic properties in the superconducting state have generally been investigated using mean-field approximations. In these calculations direct interactions among quasiparticle in the ordered state determined in the mean field approximation are ignored. These interactions arise from the residual interactions neglected in the mean field approximation but have been ignored with the assumption that any renormalization of quasiparticle properties determined in the normal state remain unchanged in the superconducting state. However quasiparticle interactions in the ordered state have been shown to renormalize the superconducting gap, change the temperature dependence of the gap, and lead to finite quasiparticle lifetimes. The corresponding feature in the tunneling conductance across an superconductor-insulator-superconductor (SIS) junction at 3∆ in addition to the peak at 2∆ expected from the mean field approximation. The corresponding feature in the spectral density has also been identified in ARPES data on BSCCO.

The question which I address in this paper is whether the 2D nature of the cuprate superconductors is responsible for the comparatively large effects of quasiparticle interactions in the cuprates compared to conventional 3D superconductors. In one dimension quantum fluctuations of the order parameter are sufficient to destroy long-range order and in two-dimensions thermal fluctuations destroy the order. Given this the effects of quantum fluctuations on superconducting properties would be expected to be more important in 2D than in 3D. This is investigated using a model Hamiltonian for 2D and 3D s-wave superconductors at zero temperature which includes the Coulomb interaction between electrons and the pairing interaction originally introduced by BCS which is characterized by a magnitude $V_0$ for electrons within an energy $\omega_D$ of the Fermi surface.

The long-range nature of the Coulomb interaction leads to a qualitative difference in the quasiparticle properties of 2D and 3D s-wave superconductors. The long-range nature of the Coulomb interaction results in the renormalization of the Goldstone mode associated with the phase of the superconductor order parameter in three dimensions so that the collective mode is at the plasmon energy. As a result the collective mode has no influence on superconducting properties. In 2D the collective mode remains gapless just as the plasmon mode does for the 2D electron gas in the normal state. There is experimental evidence for low lying collective modes in the cuprate superconductors and calculations also support the existence of low energy collective modes in layered superconductors.

This makes it possible for the collective mode to have a strong influence on quasiparticle properties in the cuprates unlike the case in conventional 3D superconductors.

Most models of the electronic properties of the cuprate superconductors have emphasized their layered nature and have employed 2D or quasi-2D Hamiltonians. These models have stressed short-range correlations either through an on-site Hubbard repulsion, in the weak or strong-coupling limits, or phenomenologically through residual short-range antiferromagnetic order which survives in the doped materials. Except in references the effect of quasiparticle interactions in the superconducting state have been ignored and the importance of long-range correlations coming from the Coulomb interaction in quasi-2D systems, such as the layered cuprate superconductors, have not been considered.

The single-particle self-energies are calculated here for 2D and 3D s-wave superconductors and it is found that their magnitude in 2D is roughly ten times the value in 3D. The single-particle self-energies also have a stronger frequency dependence in 2D than in 3D which leads stronger signatures of quasiparticle interactions in the tunneling conductance of 2D superconductors compared to those in 3D superconductors. I briefly discussed these effects in 2D d-wave superconductors and in quasi-2D superconductors.

The starting point of the present analysis is a Hamiltonian, Eq.(1), describing a system of fermions interacting via Coulomb and pairing potentials,

\begin{equation}
H = \sum_{\vec{k},\sigma} \xi_{\vec{k}\sigma} c_{\vec{k}\sigma}^\dagger c_{\vec{k}\sigma} + \sum_{\vec{k},\vec{q}} V(\vec{k},\vec{q}) c_{\vec{k}+\vec{q}\uparrow}^\dagger c_{-\vec{k}+\vec{q}\downarrow} c_{-\vec{k}\downarrow} c_{\vec{k}\uparrow} + \frac{1}{2} \sum_{\vec{k},\vec{q},\alpha,\beta} U_{\alpha\beta} c_{\vec{k}\sigma}^\dagger c_{-\vec{k}+\vec{q}\beta}^\dagger c_{-\vec{k}\beta} c_{\vec{k}\alpha} \tag{1}
\end{equation}

where $\xi_{\vec{k}} = \frac{\hbar^2}{2m} - e_F$, $e_F = \frac{p_F^2}{2m}$, $p_F$ is the Fermi momentum, $U_{\alpha\beta} = \frac{2\pi e^2}{\epsilon_{\beta}[\epsilon_{\alpha}]}$ for 2D and $\frac{4\pi e^2}{\epsilon_{\beta}[\epsilon_{\alpha}]}$ for 3D, where $\epsilon_\alpha$ is the dielectric constant of the material, and $V(\vec{k},\vec{q})$ is the same pairing interaction as introduced by BCS, $V(\vec{k},\vec{q}) = -V_0 \Theta(w_D - |\xi_{\vec{k}}|)\Theta(w_D - |\xi_{\vec{k}-\vec{q}}|)$. $w_D$ is a cutoff energy for the pairing interaction which is free parameter in the current calculation and was taken to be the Debye energy by BCS. The Hamiltonian is written in terms of quasiparticle operators, $\gamma_{\vec{k}\alpha}$, which create elementary excitations of a mean field s-wave superconducting ground state, by substituting $c_{\vec{k}\alpha}$ with $u_{\vec{k}} \gamma_{\vec{k}\alpha}^\dagger + v_{\vec{k}} \gamma_{\vec{k}\alpha}$ and $c_{\vec{k}+\vec{q}}$ with $u_{\vec{k}+\vec{q}} \gamma_{\vec{k}+\vec{q}\alpha}^\dagger - v_{\vec{k}+\vec{q}} \gamma_{\vec{k}+\vec{q}\alpha}$ where $u_{\vec{k}}^2 = 1 - v_{\vec{k}}^2 = \frac{1}{2}(1 + \frac{\xi_{\vec{k}}}{\epsilon_{\vec{k}}})$ are the usual coherence factors, $E_{\vec{k}} = \sqrt{\xi_{\vec{k}}^2 + \Delta^2}$ is the quasiparticle spectrum and $\Delta$ is determined by the weak coupling gap equation. The transformed Hamiltonian has three vertices $\gamma_{\vec{k}+\vec{q}\alpha}^\dagger \gamma_{\vec{k}\sigma}^\dagger \gamma_{\vec{k}+\vec{q}\sigma}^\dagger$, $\gamma_{\vec{k}+\vec{q}\alpha} \gamma_{\vec{k}\sigma} \gamma_{\vec{k}+\vec{q}\sigma}$, $\gamma_{\vec{k}\alpha} \gamma_{\vec{k}\sigma} \gamma_{\vec{k}+\vec{q}\sigma}$ and their hermitian conjugates. The matrix element associated with these overlaps combines with the coherence factors which are different for the Coulomb and pairing interactions because of their different dependence on spin. The single-particle self-energy for the $\gamma_{\vec{k}\alpha}^\dagger \gamma_{\vec{k}\sigma}$ and $\gamma_{\vec{k}\alpha} \gamma_{\vec{k}\sigma}^\dagger$ propagators for superconducting quasiparticles with the
mean field spectrum, \( E_0 \), are calculated at zero temperature in the generalized random phase approximation (GRPA). In the GRPA ring diagrams are summed in which the vertices are dressed by the pairing interaction. This leads to an effective interaction given by \[ V_e(\vec{q}, \omega) = \frac{U_q}{1 - U_q \Pi(\vec{q}, \omega)} \] (2)

where

\[
\Pi(\vec{q}, \omega) = A_{00}(\vec{q}, \omega) - \frac{V_0 C_{20}(\vec{q}, \omega)}{1 + V_0 B_{20}(\vec{q}, \omega)},
\]

(3)

\[ A_{00}(\vec{q}, \omega) = \sum_k \frac{2 m^2 (\vec{q} - \vec{k})^2 |E_{\vec{p}, \vec{q}} + E_{\vec{q}, \vec{k}}|^2}{\omega^2 - (E_{\vec{p}, \vec{q}} + E_{\vec{q}, \vec{k}})^2}, \]

\[ B_{20}(\vec{q}, \omega) = \sum_k \frac{\omega^2 - (E_{\vec{p}, \vec{q}} + E_{\vec{q}, \vec{k}})^2}{\omega^2 - (E_{\vec{p}, \vec{q}} + E_{\vec{q}, \vec{k}})^2} \] and \[ C_{20}(\vec{q}, \omega) = \sum_k \frac{(E_{\vec{p}, \vec{q}} + E_{\vec{q}, \vec{k}})^2}{E_{\vec{p}, \vec{q}} E_{\vec{q}, \vec{k}} \omega^2 - (E_{\vec{p}, \vec{q}} + E_{\vec{q}, \vec{k}})^2}. \]

The second term gives the collective mode contribution. Collective modes have been investigated extensively for 3D superconductors \[ 15 \] and more recently for 2D and quasi-2D superconductors \[ 17, 19–22 \]. As is well-known the energy of the collective mode, \( \omega_{\vec{q}} \), in 2D in contrast to 3D. \( \Sigma_{\gamma, \gamma}(\vec{q}, E) = \sum_{\vec{q}} \Theta(x) n^2(\vec{p}, \vec{q} - \vec{p}) V_e^2(\vec{q}, x) \), \( \Sigma_{\gamma, \gamma}''(\vec{q}, E) = \sum_{\vec{q}} \Theta(x) m(\vec{p}, \vec{p} - \vec{q}) n(\vec{p}, \vec{p} - \vec{q}) V_e''(\vec{q}, x) \)

(4)

where \( x = E - E_{\vec{p} - \vec{q}} \), \( \Theta(x) \) is zero for \( x < 0 \) and is equal to one otherwise. The imaginary part of the effective interaction is

\[
V_e''(\vec{q}, \omega) = \frac{U_q^2 \Pi''(\vec{q}, \omega)}{|1 - U_q \Pi(\vec{q}, \omega)|^2} + U_q \delta[1 - U_q \Pi(\vec{q}, \omega)]
\]

(5)

The calculated imaginary part of \( \Sigma_{\gamma, \gamma}(\vec{p}, E) \) is shown for a 2D s-wave superconductor in Figure 1 for the parameter values \( N(0)V_0 = -0.5 \), \( \omega_{\vec{q}} = c_F/10 \), giving \( \Delta = 0.0275 c_F \), and \( c_F p_F = 25.12 A^{-1} \). In Figure 1 the dashed curve is the contribution due to scattering off the collective mode and the solid curve is the sum of this contribution and the contribution due to scattering from the continuum. The collective mode contribution in 2D has a threshold at \( E \approx E_F \) since the collective mode is confined to \( |\vec{q}|^2 \sim p_F (\frac{\Delta}{E_F})^2 \) and leads to quasiparticle decay at energies below the \( 3\Delta \) threshold for the continuum contribution. Using the same parameters for the 3D case as in the 2D case, one finds that the energy dependence of the continuum contribution to \( \Sigma_{\gamma, \gamma}'(\vec{p}, E) \) for \( E > 3\Delta \) is determined by the increase in phase space for decay. The magnitudes of this contribution to the self-energies in 2D are roughly an order magnitude larger than in 3D pointing to the greater importance of quantum fluctuations in 2D compared to 3D. There is no contribution from the collective mode in 3D so that the strong energy dependence at \( E \approx 3\Delta \) is absent in 3D. \( \Sigma_{\gamma, \gamma}'(\vec{p}, E) \) is very similar in magnitude and frequency dependence for \( E_p \) and \( E \) less than \( 3\Delta \) but is reduced in magnitude compared to \( \Sigma_{\gamma, \gamma}'(\vec{p}, E) \) at higher values of \( E_p \) and \( E \). The real part of \( \Sigma_{\gamma, \gamma}'(\vec{p}, E) \) is

\[
\Sigma_{\gamma, \gamma}'(\vec{p}, E) = -\frac{1}{2} \sum_{\vec{q}} \left[ n^2(\vec{p} - \vec{q}, \vec{p}) \left( V_e'(\vec{q}, E - E_{\vec{p} - \vec{q}}) + \int_0^\infty \frac{d\omega}{\pi} \frac{2(E - E_{\vec{p} - \vec{q}}) V_e'(\vec{q}, \omega)}{(E - E_{\vec{p} - \vec{q}})^2 - \omega^2} \right) \right] - m^2(\vec{p} - \vec{q}, \vec{p}) \left( V_e'(\vec{q}, E + E_{\vec{p} - \vec{q}}) + \int_0^\infty \frac{d\omega}{\pi} \frac{2(E + E_{\vec{p} - \vec{q}}) V_e'(\vec{q}, \omega)}{(E + E_{\vec{p} - \vec{q}})^2 - \omega^2} \right)
\]

(6)

There is an analogous expression for \( \Sigma_{\gamma, \gamma}'(\vec{p}, E) \). \( \Sigma_{\gamma, \gamma}'(\vec{p}, E) \) is plotted in Figure 2 for a 2D superconductor. The \( E \) dependence of \( \Sigma_{\gamma, \gamma}'(\vec{p}, E) \) for \( E \leq 4\Delta \) is responsible for the "strong-coupling" features in the calculated \( g_{SIS}(eV) \) which will be discussed below. The frequency dependence of \( \Sigma_{\gamma, \gamma}'(\vec{p}, E) \) is almost exactly the same as in \( \Sigma_{\gamma, \gamma}'(\vec{p}, E) \).
except that the variation in magnitude with frequency is about a factor of 8 smaller. As in the case of the imaginary part of these self-energies the magnitudes of the real parts are roughly an order of magnitude greater in 2D compared to 3D and in contrast to the 2D case there is only a very weak frequency dependence.

The same calculation can be carried out for a 2D d-wave superconductor which arises in many models of superconductivity of the cuprates. The long range Coulomb interaction leads to quantitatively the same collective mode behavior [19] and so of weight in the effective quasiparticle interactions at low frequencies is also present for 2D d-wave superconductors.

Experimental quantities which measure the convolution of two superconducting densities of states such as the tunneling current across an SIS junction or optical conductivity of a superconductor can be sensitive probes of the effects of quasiparticle interactions. The tunneling current is calculated using the Hamiltonian introduced by Cohen et al. [20], \( H_T = \sum_{R,L} T_{RL} \psi_R^\dagger \psi_L + h.c. \), which describes the destruction of a normal state electron on the left side of the junction and the creation of a normal state electron on the right side, where \( \sigma \) is the spin of the electron. The tunneling matrix element, \( T_{RL} \), is between the normal electron states on the two sides of the junction. In the absence of spin-flip scattering in the barrier the matrix element is spin independent and for tunneling between s-wave conductors, discussed here, the matrix can be taken to be a constant. In anisotropic cases in which there are strong bandstructure effects and also depending on the nature of the junction the tunneling matrix element can be strongly momentum dependent. This is clearly seen in the break junction experiments of Hartge et al. [21]. For a discussion of the modeling of the tunneling matrix in the context of the cuprates see reference [22] and references therein.

Assuming a constant tunneling matrix element, the tunneling conductance across an SIS junction, \( g_{SIS} = \frac{\partial I(V)}{\partial V} \), is

\[
\propto \frac{N_0(\epsilon)N_0(eV - \epsilon) + N_1(\epsilon)N_1(eV - \epsilon)}{\mu^2} \nonumber
\]

\[
N_0(\epsilon) = \sum_p n_p^2 A_{\gamma\gamma}(\vec{p}, \epsilon) + v_F^2 A_{\gamma\gamma}(-\vec{p}, -\epsilon), \quad N_1(\epsilon) = \sum_p \frac{\Delta^2}{2\mu^2} \left[ A_{\gamma\gamma}(\vec{p}, \epsilon) + A_{\gamma\gamma}(-\vec{p}, -\epsilon) \right], \quad A(p, \epsilon) \text{ are spectral densities, and } V \text{ is the bias across the junction. In the case of optical conductivity there is also a combination of coherence factors in the expression and the bias would be replaced by the photon energy. The density of states in a 2D superconductor, } N(\epsilon), \text{ and } g_{SIS} \text{ between two 2D superconductors is compared with the mean field approximation, } N^{MF}(\epsilon) \text{ and } g^{MF}_{SIS}, \text{ in Figure 3. The negative shift in } \Sigma_{\gamma\gamma}(\vec{p}, \epsilon) \text{ with increasing } E_p \text{ leads to a piling up of states at } \epsilon \approx \Delta \text{ in } N(\epsilon) \text{ compared to } N^{MF}(\epsilon). \text{ This leads to a peak in the SIS tunneling current which is responsible for the form of } g_{SIS} \text{ at } eV \approx 2\Delta. \text{ The step feature at } eV \approx 4\Delta \text{ comes mostly from the form of } \Sigma_{\gamma\gamma}(\vec{p}, \epsilon) \text{ at } \epsilon \approx 3\Delta. \text{ If } \Sigma_{\gamma\gamma}(\vec{p}, \epsilon) \text{ had a step like dependence on } \epsilon \text{ a dip feature would be seen at } eV = 4\Delta \text{ analogous to the dip at } eV = 3\Delta \text{ seen in the d-wave case [23]. This form for the self-energy relies on bandstructure and enhanced quasiparticle interactions at frequencies } \sim \Delta \text{ compared to higher frequencies. Antiferromagnetic spin fluctuations due to short-range Coulomb correlations lead to this form for the self-energy. [24].}

The results of this calculation show that interactions among quasiparticles in low dimensional superconductors are much stronger than in conventional 3-D superconductors due to long range correlations from the Coulomb interaction. Low lying collective modes are also present in layered materials in which the Coulomb interaction is present between layers. [25] This suggests that in quasi-2D layered superconductors such as the cuprates the same enhancement of interactions among superconducting quasiparticles should occur. This offers an explanation for the magnitude of the dip feature seen in \( g_{SIS} \) on some of the cuprates [26] and suggests that quasiparticle interactions may also have important consequences in other classes of low dimensional superconductors. In particular in quasi-one dimensional systems the collective mode is known to be linear in the wavenumber [27] so that the low energy effective interaction should be enhanced above that of the quasi-2D case discussed here. This suggests that the effects of quasiparticles should be stronger in the quasi-one dimensional organic superconductors than in the cuprates.

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FIG. 1. Imaginary part of $\Sigma_{\gamma\gamma^+}(\vec{p}, E)$ for a 2D s-wave superconductor for different values of $E_p$ with $\epsilon_p F = 25.13 \text{Å}^{-1}$, $\omega_D = 0.1 e_F$ and $\Delta = 0.0273 e_F$. The dashed line is the contribution from the collective mode for each $E_p$. This is very small for $E_p = \Delta$ (dot-dash curve).

FIG. 2. Real part of the self-energy, $\Sigma_{\gamma\gamma^+}(\vec{p}, E)$, for a 2D s-wave superconductor for different momenta with $\epsilon_p F = 25.13 \text{Å}^{-1}$, $\omega_D = 0.1 e_F$ and $\Delta = 0.0273 e_F$.

FIG. 3. Comparison between $N(\epsilon)$ and $g_{SIS}(\epsilon)$ with the mean field approximation, $N^{MF}(\epsilon)$ and $g_{SIS}^{MF}(\epsilon)$ (dashed curves), for 2D superconductors. $N^{MF}(\epsilon)$ was calculated using a lorentzian with $\Gamma = \Delta/20$. 