Abstract

The $\rho\rho$ scattering has been studied by two groups which both claimed a dynamical generated scalar meson, most likely to be $f_0(1370)$. Here we investigate the influence of coupled-channels of pseudoscalar mesons, i.e., $\pi\pi$ and $\bar{K}K$, on this dynamical generated scalar state. With the coupled channel effect included, the pole and partial decay widths are found to be more close to PDG values for $f_0(1500)$.

1 Introduction

The chiral unitary approach, which has made much progress in the study of pseudoscalar meson-meson [1] and meson-baryon [2, 3] interactions, has been used to study the interaction of vector mesons among themselves. The first such study of the $S$-wave $\rho\rho$ interactions found that the $f_0(1370)$ and the $f_2(1270)$ could be dynamically generated [4]. The work found that the strength of the attractive interaction in the tensor channel is much stronger than that in the scalar channel, hence leads to a tighter bound tensor state than the corresponding scalar one.

The work [4] based on the assumption that the three momenta of the $\rho$ is negligibly small compared to its large mass. This assumption was questioned by a recent work [5] which pointed out the importance of relativistic effect for energies around $f_2(1270)$ well below $\rho\rho$ threshold. The $N/D$ method [6–10] was used to get the partial wave amplitudes which result a pole for the scalar state similar to Ref. [4] but no pole for any tensor state in contradiction with Ref. [4]. However, this conclusion was rebuked by Ref. [11] in which the non-relativistic assumption was dropped by evaluating exactly the loops with full relativistic $\rho$ propagators in solving the B-S equation for $\rho\rho$ scattering. Both scalar state and tensor state associated with $f_0(1370)$ and $f_2(1270)$, respectively, were found in consistence with the conclusion of Ref. [4].
From the studies of above two groups, obviously, for the energies around $f_2(1270)$ well below $\rho\rho$ threshold, there is strong model dependence for the interaction of two far off-mass-shell $\rho$ mesons. For the scalar state closer to the $\rho\rho$ threshold, the two groups got similar result rather model independently. In this paper we shall study the influence of coupled-channels of pseudoscalar mesons, i.e., $\pi\pi$ and $\bar{K}K$, on this dynamical generated scalar state. In the $\rho\rho - K\bar{K}$ coupling we consider the case of $K$ and $K^*$ exchange, while in the $\rho\rho - \pi\pi$ coupling we consider the case of $\pi$ and $\omega$ exchange.

2 Formalism

2.1 $\rho\rho \rightarrow \pi\pi$ with $\pi$-exchange

We investigate the coupled channel effect based on a chiral covariant framework [5]. We follow the formalism of the hidden gauge interaction which provides the $\rho\pi\pi$ coupling by means of the Lagrangian [12,13]

$$\mathcal{L}_{VP\pi} = -ig\langle V^\mu[P, \partial_\mu P] \rangle.$$

where the symbol $\langle \ldots \rangle$ stands for the trace in the $SU(3)$ space with the coupling constant $g = M_V/2f_\pi$ and $f_\pi = 93MeV$ the pion decay constant. The matrices $V_\mu$ and $P$ are given by

$$V_\mu = \left( \begin{array}{c}\frac{\rho_0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \\ \rho^- \quad \rho^+ \quad K^{*+} \\ -\frac{\rho_0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \quad K^{*0} \quad \phi \end{array} \right), P = \left( \begin{array}{c}\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} \\ \pi^- \quad \pi^+ \quad K^+ \\ -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} \quad K^0 \quad -\frac{2\eta}{\sqrt{6}} \end{array} \right).$$

To get the three different isospin amplitudes for $\rho\rho \rightarrow \pi\pi$ we need the knowledge of the transitions $\rho^+(p_1)^0(p_2)^+\pi^+(p_3)^0(p_4)^-$, $\rho^+(p_1)^+\rho^-(p_2)^-\pi^0(p_3)^0(p_4)^-$, etc.

![Figure 1: $\pi$-exchange diagram for $\rho^+\rho^- \rightarrow \pi^+\pi^-$](image)

Starting with the Lagrangian in Eq.(1) we can immediately obtain the amplitude $A_t(p_1,p_2,p_3,p_4)$ of $\rho^+(p_1)^0(p_2)^+\pi^+(p_3)^0(p_4)^-$ corresponding to Fig.1 as

$$A_t(p_1,p_2,p_3,p_4) = \frac{-8g^2}{(p_1-p_3)^2 - m_\pi^2} \epsilon_1 \cdot p_3 \epsilon_2 \cdot p_4.$$  

In this equation, the $\epsilon_i$ corresponds to the polarization vector of the $i$-th $\rho$. Each polarization vector is characterized by its three-momentum $p_i$ and third component of the
spin \( \sigma_i \). Explicit expressions of these polarization vectors are given by [5]

\[
\epsilon(p, 0) = \left( \begin{array}{c}
\gamma \beta \cos \theta \\
\frac{1}{2}(\gamma - 1) \sin 2 \theta \cos \phi \\
\frac{1}{2}(\gamma - 1) \sin 2 \theta \sin \phi \\
\frac{1}{2}(1 + \gamma + (\gamma - 1)) \cos 2 \theta
\end{array} \right),
\]

\[
\epsilon(p, \pm 1) = \mp \frac{1}{\sqrt{2}} \begin{pmatrix}
\gamma \beta e^{\pm i \phi} \sin \theta \\
1 + (\gamma - 1) e^{\pm i \phi} \sin^2 \theta \cos \phi \\
\pm i + (\gamma - 1) e^{\pm i \phi} \sin^2 \theta \sin \phi \\
\frac{1}{2}(\gamma - 1) e^{\pm i \phi} \sin 2 \theta
\end{pmatrix}.
\] (4)

where \( \beta = |p|/E_p \) and \( \gamma = 1/\sqrt{1 - \beta^2} \). The \( u \)-channel \( \pi \)-exchange amplitude \( A_u \) can be obtained from the expression of \( A_t \) by exchanging \( p_3 \leftrightarrow p_4 \). In this way,

\[
A_u(p_1, p_2, p_3, p_4) = A_t(p_1, p_2, p_4, p_3).
\] (5)

And now we write the tree-level amplitude for \( \rho \rho \rightarrow \pi \pi \) with \( \pi \)-exchange

\[
\begin{align*}
\rho^+(p_1) \rho^-(p_2) & \rightarrow \pi^+(p_3) \pi^-(p_4) \quad A_t, \\
\rho^+(p_1) \rho^-(p_2) & \rightarrow \pi^0(p_3) \pi^0(p_4) \quad A_t + A_u, \\
\rho^0(p_1) \rho^0(p_2) & \rightarrow \pi^+(p_3) \pi^-(p_4) \quad A_t + A_u, \\
\rho^0(p_1) \rho^0(p_2) & \rightarrow \pi^0(p_3) \pi^0(p_4) \quad 0
\end{align*}
\] (6)

In order to obtain the \( S \)-wave amplitude in isospin \( I = 0 \) channel we need the isospin eigenstates. We have

\[
|\rho \rho, I = 0\rangle = -\frac{1}{\sqrt{3}}|\rho^+(p_1) \rho^-(p_2) + \rho^-(p_1) \rho^+(p_2) + \rho^0(p_1) \rho^0(p_2)\rangle,
\]

\[
|\pi \pi, I = 0\rangle = -\frac{1}{\sqrt{3}}|\pi^+(p_1) \pi^-(p_2) + \pi^-(p_1) \pi^+(p_2) + \pi^0(p_1) \pi^0(p_2)\rangle.
\] (7)

where we have used the convention \( |\rho^+\rangle = -|1, 1\rangle \) and \( |\pi^+\rangle = -|1, 1\rangle \) of isospin. By taking into account Eq.(7) and the amplitudes in Eq.(6) we can now write the isospin \( I = 0 \) amplitude for \( \rho \rho \rightarrow \pi \pi \)

\[
T^{(0)}_\pi = 16g^2 \left( \frac{\epsilon_1 \cdot p_3 \epsilon_2 \cdot p_4}{m_\pi^2 - t} + \frac{\epsilon_1 \cdot p_4 \epsilon_2 \cdot p_3}{m_\pi^2 - u} \right).
\] (8)

where \( t = (p_1 - p_3)^2 \) and \( u = (p_1 - p_4)^2 \).

### 2.2 \( \rho \rho \rightarrow \pi \pi \) with \( \omega \)-exchange

One needs the \( \rho \omega \pi \) coupling which is provided within the framework [14] of the hidden gauge formalism by means of the Lagrangian

\[
\mathcal{L}_{\nu \nu \pi} = \frac{G'}{\sqrt{2}} \epsilon_{\mu \nu \alpha \beta} \langle \partial_\mu V_\alpha \partial_\nu V_\beta P \rangle
\] (9)

with

\[
G' = \frac{3g^2}{4\pi^2 f} \quad g' = -\frac{G_V m_\rho}{\sqrt{2} f^2}.
\] (10)
where $G_V = 55\text{MeV}$ and $f = 93\text{MeV}$. At this point we can write down the amplitude of $\rho^+(p_1)\rho^-(p_2) \rightarrow \pi^+(p_3)\pi^-(p_4)$ with $\omega$-exchange corresponding to Fig.2 as in the $\pi$-exchange case

$$B_t = \frac{-G^2}{(p_1 - p_3)^2 - m^2_\omega} \left( p_3 \cdot p_4 p_1 \cdot \epsilon_2 p_2 \cdot \epsilon_1 + p_1 \cdot p_2 p_4 \cdot \epsilon_1 p_3 \cdot \epsilon_2 + p_1 \cdot p_4 p_2 \cdot p_3 \epsilon_1 \cdot \epsilon_2 
- p_2 \cdot p_3 p_1 \cdot \epsilon_2 p_4 \cdot \epsilon_1 - p_1 \cdot p_4 p_2 \cdot \epsilon_1 p_3 \cdot \epsilon_2 - p_1 \cdot p_2 p_3 \cdot p_4 \epsilon_1 \cdot \epsilon_2 \right). \tag{11}$$

$$\begin{array}{c}
\rho^+(p_1) \\
\omega \\
\rho^+(p_2)
\end{array} \quad \begin{array}{c}
\pi^+(p_3) \\
\pi^+(p_4)
\end{array}$$

Figure 2: $\omega$-exchange diagram for $\rho^+\rho^- \rightarrow \pi^+\pi^-$

And the $u$-channel $\omega$-exchange amplitude $B_u(p_1, p_2, p_3, p_4)$ can be obtained from the expression of $B_t$ as the case in $\pi$-exchange by exchanging $p_3 \leftrightarrow p_4$, thus

$$B_u(p_1, p_2, p_3, p_4) = B_t(p_1, p_2, p_4, p_3). \tag{12}$$

Next we write the tree-level amplitude for $\rho\rho \rightarrow \pi\pi$ with $\omega$-exchange

$$\begin{align*}
\rho^+(p_1)\rho^-(p_2) &\rightarrow \pi^+(p_3)\pi^-(p_4) \quad B_t, \\
\rho^+(p_1)\rho^-(p_2) &\rightarrow \pi^0(p_3)\pi^0(p_4) \quad 0, \\
\rho^0(p_1)\rho^0(p_2) &\rightarrow \pi^+(p_3)\pi^-(p_4) \quad 0, \\
\rho^0(p_1)\rho^0(p_2) &\rightarrow \pi^0(p_3)\pi^0(p_4) \quad B_t + B_u. \tag{13}
\end{align*}$$

Then using Eq.(7) we can get the $I = 0$ amplitude

$$T^{(0)}_\omega = \frac{-G^2}{(p_1 - p_3)^2 - m^2_\omega} \left( p_3 \cdot p_4 p_1 \cdot \epsilon_2 p_2 \cdot \epsilon_1 + p_1 \cdot p_2 p_4 \cdot \epsilon_1 p_3 \cdot \epsilon_2 + p_1 \cdot p_4 p_2 \cdot p_3 \epsilon_1 \cdot \epsilon_2 
- p_2 \cdot p_3 p_1 \cdot \epsilon_2 p_4 \cdot \epsilon_1 - p_1 \cdot p_4 p_2 \cdot \epsilon_1 p_3 \cdot \epsilon_2 - p_1 \cdot p_2 p_3 \cdot p_4 \epsilon_1 \cdot \epsilon_2 \right) + (p_3 \leftrightarrow p_4). \tag{14}$$

### 2.3 $\rho\rho \rightarrow K\bar{K}$ with $K$-exchange

The $\rhoKK$ coupling is provided in the same Lagrangian in Eq.(1), so we can immediately write down the amplitude of $\rho^+(p_1)\rho^-(p_2) \rightarrow K^+(p_3)K^-(p_4)$ with $K$-exchange corresponding to Fig.3

$$C_t(p_1, p_2, p_3, p_4) = -4g^2 \frac{\epsilon_1 \cdot p_3 \epsilon_2 \cdot p_4}{(p_1 - p_3)^2 - m^2_K} \tag{15}$$

and the $u$-channel

$$C_u(p_1, p_2, p_3, p_4) = C_t(p_1, p_2, p_4, p_3). \tag{16}$$

Then we can obtain the tree-level amplitudes for $\rho\rho \rightarrow K\bar{K}$ with $K$-exchange as the
following
\[\begin{align*}
\rho^+(p_1)\rho^-(p_2) &\rightarrow K^+(p_3)K^-(p_4) \quad C_t, \\
\rho^+(p_1)\rho^-(p_2) &\rightarrow K^0(p_3)\bar{K}^0(p_4) \quad C_u,
\end{align*}\]
\[\rho^0(p_1)\rho^0(p_2) \rightarrow K^+(p_3)K^-(p_4) \quad \frac{C_t + C_u}{2}, \quad \rho^0(p_1)\rho^0(p_2) \rightarrow K^0(p_3)\bar{K}^0(p_4) \quad \frac{C_t + C_u}{2}. \quad (17)\]

Similar to Eq.(7) we need the isospin \(I = 0\) eigenstate for \(|K\bar{K}\rangle\). We have
\[|K\bar{K}\rangle = -\frac{1}{\sqrt{2}}\left(|K^+(p_1)K^-(p_2) + K^0(p_1)\bar{K}^0(p_2)\right). \quad (18)\]
where we use the convention \(|K^\pm\rangle = -|\pm_{\frac{1}{2}, \frac{1}{2}}\rangle\) of isospin. By using the isospin wave functions we can obtain for \(I = 0\)
\[T_K^0 = 2\sqrt{6}g^2\left(\frac{\epsilon_1 \cdot p_3 \epsilon_2 \cdot p_4}{m_K^2 - t} + \frac{\epsilon_1 \cdot p_3 \epsilon_2 \cdot p_3}{m_K^2 - u}\right). \quad (19)\]
with \(t\) and \(u\) the usual Mandelstam variable. We can see that the Eq.(19) is similar to the Eq.(8). The former can be obtained from the latter just by substituting \(16 \rightarrow 2\sqrt{6}\) and \(m_\pi \rightarrow m_K\).

2.4  \(\rho\rho \rightarrow K\bar{K}\) with \(K^*\)-exchange

As for the \(\rho KK^*\) coupling, we use the Lagrangian in Eq.(9). Then we get the amplitude for \(\rho^+(p_1)\rho^-(p_2) \rightarrow K^+(p_3)K^-(p_4)\) with \(K^*\)-exchange corresponding to the Fig.4 as
\[D_t = \frac{-G^2/2}{(p_1 - p_3)^2 - m_{K^*}^2} \left(p_3 \cdot p_4 \epsilon_1 \cdot p_2 \cdot \epsilon_2 + p_1 \cdot p_2 p_4 \cdot \epsilon_1 p_3 \cdot \epsilon_2 + p_1 \cdot p_4 p_2 \cdot p_3 \epsilon_1 \cdot \epsilon_2ight)
- p_2 \cdot p_3 p_1 \cdot \epsilon_2 p_4 \cdot \epsilon_1 - p_1 \cdot p_4 p_2 \cdot \epsilon_1 p_3 \cdot \epsilon_2 - p_1 \cdot p_2 p_3 \cdot p_4 \epsilon_1 \cdot \epsilon_2. \quad (20)\]
and the \(u\)-channel
\[D_u(p_1, p_2, p_3, p_4) = D_t(p_1, p_2, p_4, p_3). \quad (21)\]

Next we list the tree-level amplitudes for \(\rho\rho \rightarrow K\bar{K}\) with \(K^*\)-exchange as the following:
\[\begin{align*}
\rho^+(p_1)\rho^-(p_2) &\rightarrow K^+(p_3)K^-(p_4) \quad D_t, \\
\rho^+(p_1)\rho^-(p_2) &\rightarrow K^0(p_3)\bar{K}^0(p_4) \quad D_u, \\
\rho^0(p_1)\rho^0(p_2) &\rightarrow K^+(p_3)K^-(p_4) \quad \frac{D_t + D_u}{2}, \\
\rho^0(p_1)\rho^0(p_2) &\rightarrow K^0(p_3)\bar{K}^0(p_4) \quad \frac{D_t + D_u}{2}. \quad (22)\]
Using Eqs.(7) and (18) we obtain the $I = 0$ amplitude

$$T_{K^*}^{(0)} = \frac{\sqrt{6}G^2/4}{(p_1 - p_3)^2 - m_{K^*}^2} (p_3 \cdot p_4 p_1 \cdot \epsilon_2 p_2 \cdot \epsilon_1 + p_1 \cdot p_2 p_4 \cdot \epsilon_1 p_3 \cdot \epsilon_2 + p_1 \cdot p_4 p_2 \cdot p_3 \epsilon_1 \cdot \epsilon_2)
- p_2 \cdot p_3 p_1 \cdot \epsilon_2 p_4 \cdot \epsilon_1 - p_1 \cdot p_4 p_2 \cdot \epsilon_1 p_3 \cdot \epsilon_2 - p_1 \cdot p_2 p_3 \cdot p_4 \epsilon_1 \cdot \epsilon_2) + (p_3 \leftrightarrow p_4).$$

which can be obtain from Eq.(14) by substituting $1 \rightarrow \sqrt{6}/4$ and $m_\omega \rightarrow m_{K^*}$.

2.5 Partial-wave decomposition

In term of these amplitudes with isospin $I = 0$, we can calculating the partial-wave amplitudes in the $\ell SJI$ basis [5], denoted as $T_{\ell S,h\bar{S}}^{(JI)}(s)$ for the transition $(\ell SJI) \rightarrow (\ell SJI)$

$$T_{\ell S,h\bar{S}}^{(JI)}(s) = \frac{Y_0^h(\hat{z})}{\sqrt{2}N(2J + 1)} \sum_{\sigma_1,\sigma_2,\sigma_m} \int dp'' Y_\ell^{m}(\hat{p}'')^*(\sigma_1 \sigma_2 M|s_1 s_2 S)
\times (M \bar{M}|\ell SJ)(\bar{\sigma}_1 \bar{\sigma}_2 \bar{M}|\bar{s}_1 \bar{s}_2 \bar{S})(0 \bar{M} \bar{M}|\bar{\ell} \bar{S} J)T^{(I)}(p_1, p_2, p_3, p_4)$$

with $M = \sigma_1 + \sigma_2$ and $\bar{M} = \bar{\sigma}_1 + \bar{\sigma}_2$. And $N$ accounts for identical particles, for example

$N = 2$ for $\rho \rho \rightarrow \pi \pi$,
$N = 1$ for $\rho \rho \rightarrow K\bar{K}$.  

By using Eq.(24) we can calculate the partial-wave projected tree-level amplitudes of Eqs.(8), (14), (19) and (23) with quantum number $I, \ell, S = 0, 0, 0$. We denote $T_{00,00}^{(00)}$ by $V$ for simplicity and we have

for $\rho \rho \rightarrow \pi \pi$ with $\pi$-exchange

$$V_\pi = \frac{2g^2}{\sqrt{3}} \left( \frac{m_\rho^2 - 4m_\pi^2}{s - 4m_\pi^2} \ln \frac{s - 2m_\rho^2 + \sqrt{s - 4m_\pi^2 \sqrt{s - 4m_\pi^2}}}{s - 2m_\rho^2 - \sqrt{s - 4m_\pi^2 \sqrt{s - 4m_\pi^2}}} + \frac{s}{m_\rho^2} \right).$$

for $\rho \rho \rightarrow \pi \pi$ with $\omega$-exchange

$$V_\omega = \frac{G^2s}{2\sqrt{3}} \left( \frac{m_\omega^2}{s - 4m_\pi^2 \sqrt{s - 4m_\rho^2}} \ln \frac{s + 2m_\omega^2 - 2m_\pi^2 - 2m_\rho^2 + \sqrt{s - 4m_\pi^2 \sqrt{s - 4m_\rho^2}}}{s + 2m_\omega^2 - 2m_\pi^2 - 2m_\rho^2 - \sqrt{s - 4m_\pi^2 \sqrt{s - 4m_\rho^2}}} - 1 \right).$$
for $\rho \rho \to K \bar{K}$ with $K$-exchange

$$V_K = \frac{g^2}{2} \left( \frac{2(m^2_\rho - 4m^2_K)}{\sqrt{s - 4m^2_\rho} \sqrt{s - 4m^2_K}} \ln \frac{s - 2m^2_\rho + \sqrt{s - 4m^2_K} \sqrt{s - 4m^2_\rho}}{s - 2m^2_\rho - \sqrt{s - 4m^2_K} \sqrt{s - 4m^2_\rho}} + \frac{s}{m^2_\rho} + 2 \right).$$

(28)

for $\rho \rho \to K \bar{K}$ with $K^*$-exchange

$$V_{K^*} = \frac{G'^2 s}{4} \left( \frac{m^2_{K^*}}{\sqrt{s - 4m^2_{K^*} \sqrt{s - 4m^2_\rho}}} \ln \frac{s + 2m^2_{K^*} - 2m^2_K - 2m^2_\rho + \sqrt{s - 4m^2_K} \sqrt{s - 4m^2_\rho}}{s + 2m^2_{K^*} - 2m^2_K - 2m^2_\rho - \sqrt{s - 4m^2_K} \sqrt{s - 4m^2_\rho}} - 1 \right).$$

(29)

3 Results and discussion

We label the three channels, $\rho \rho$, $K \bar{K}$ and $\pi \pi$ as 1, 2 and 3, respectively. With the channel transition amplitudes $V_\pi$, $V_\omega$, $V_K$ and $V_{K^*}$ given in last section, we calculate the full amplitude and its pole positions by using the Bethe Salpeter equation in its on-shell factorized form [4, 5]

$$T = \frac{V}{1 - VG}. \quad (30)$$

$G$ is a diagonal matrix made up by the two-point loop function [4, 5]

$$G_{jj}(s) = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2 - m^2_j)((P - q)^2 - m^2_j)}$$

(31)

with $P$ the total four-momentum of the meson-meson systems and $q$ the four-momentum of one intermediate meson. The channel is labelled by the subindex $j$. By using dimensional regularization the integration can be recast as

$$G_{jj}(s) = \frac{1}{(4\pi)^2} \left( a(\mu) + \log \frac{m^2_j}{\mu^2} + \sigma \log \frac{\sigma + 1}{\sigma - 1} \right) \quad (32)$$

with

$$\sigma = \sqrt{1 - \frac{4m^2_j}{s}} \quad (33)$$

or using a momentum cutoff $q_{\text{max}}$ as

$$G_{jj}(s) = \frac{1}{2\pi^2} \int_0^{q_{\text{max}}} dq \frac{q^2}{w(s - 4w^2 + i\epsilon)} \quad (34)$$

where $w = \sqrt{q^2 + m^2_j}$. The integral can be done algebraically

$$G_{jj}(s) = \frac{1}{(4\pi)^2} \left\{ \sigma \log \frac{1 + \frac{m^2_j}{q_{\text{max}}^2}}{\sigma} - 2 \log \left[ \frac{q_{\text{max}}}{m_j} \left( 1 + \sqrt{1 + \frac{m^2_j}{q_{\text{max}}^2}} \right) \right] \right\} \quad \text{(35)}$$

Typical values of the cutoff $q_{\text{max}}$ are around 1 GeV. $G_{jj}(s)$ has a right-hand cut above the threshold $2m_j$. In order to make an analytical extrapolation to second Riemann sheet we make use of the continuity property
where the index (2) indicates the second Riemann sheet of $G_{jj}$. Then
\[
G_{jj}^{(2)}(\sqrt{s} + i\epsilon) = G_{jj}(\sqrt{s} - i\epsilon)
\]
and
\[
G_{jj}^{(2)}(\sqrt{s} + i\epsilon) = G_{jj}(\sqrt{s} - i\epsilon) = G_{jj}(\sqrt{s} + i\epsilon) - 2i\text{Im}G_{jj}(\sqrt{s} + i\epsilon)
\]
(37)

\[
= G_{jj}(\sqrt{s} + i\epsilon) + \frac{i|p|}{4\pi\sqrt{s}}
\]

Other potential of coupled-channels like $\pi\pi - K\bar{K}$ can be found in [1]. Our results are shown in Table 1 for various $q_{\text{max}}$ values. For comparison, the results for the $\rho\rho$ single channel without considering the coupled channel effects as in Ref. [5] are show in the second row. The 3 ~ 6 rows show the results including one coupled channel with the exchanged meson listed in the first column. For example the $\pi$ denotes the $\rho\rho - \pi\pi$ channel with $\pi$ exchange and so on. The 7-th row gives the results including all three coupled channels of $\rho\rho$, $\pi\pi$ and $K\bar{K}$.

| $q_{\text{max}}$(GeV) | 0.875  | 1.0   | 1.2   | 1.4   |
|----------------------|-------|-------|-------|-------|
| $\rho\rho$ only      | 1494.8| 1467.2| 1427.3| 1395.0|
| $\pi$                | 1530.0| 1519.5| 1501.5| 1488.6|
| $\omega$             | 1492.2| 1466.5| 1428.1| 1400.0|
| $\bar{K}$            | 1497.8| 1473.9| 1437.2| 1410.0|
| $K^*$                | 1489.6| 1463.3| 1424.5| 1396.1|
| 3-channels           | 1529.8| 1519.0| 1500.9| 1488.4|

Table 1: Pole position for coupled-channels

The results show that the influence of vector meson $\omega$ and $K^*$ exchanges is very small; the largest influence comes from the $\rho\rho - \pi\pi$ channel coupling by the pion exchange, which shifts up the pole mass and results in a sizable $\pi\pi$ decay width, comparable with relevant PDG values for $f_0(1500)$ [15]. For the $\rho\rho - K\bar{K}$ coupled-channel case we can see that the width is consistent with $f_0(1500)$ decaying into $K\bar{K}$ in PDG, which is about 8.9MeV. When taking into account all the three channels, the pole position is close to the results by considering only the pion exchange contribution. With $q_{\text{max}} = 1.4GeV$, the pole mass and partial decay widths to $\pi\pi$ and $K\bar{K}$ are roughly consistence with PDG values for $f_0(1500)$. The largest decay channel should be $4\pi$ either through $\rho\rho$ directly or by its cross talk with $\sigma\sigma$. Note that due to the binding energy of the molecule as well as the kinetic energy of $\rho$ inside the molecule, the $4\pi$ decay width through the decay of $\rho$ inside the $\rho\rho$ molecule can be smaller than the decay width of a single free $\rho$ meson. Similar effect was pointed out by Refs. [16,17] in their studies of $d^*(2380)$ as a $\Delta\Delta$ molecule which gets a decay width smaller than the decay width of a single free $\Delta$ state. This kind of effect was also observed by the study of other hadronic molecules [18,19].

In summary, the $\rho\rho$ scattering is revisited by including its coupled-channels of pseudoscalar mesons, i.e., $\pi\pi$ and $K\bar{K}$. It is found that the coupled-channel effect is important and shifts up the pole mass of the dynamically generated scalar state significantly. It makes the state to be more consistent with $f_0(1500)$ rather than $f_0(1370)$ as favored by the previous studies [4,5] without including these coupled channels. The $\rho\rho$ scattering has been extended to the S-wave interactions for the whole vector-meson nonet by two groups [20,21]. We expect similar significant coupled channel effects there.
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References

[1] J. A. Oller and E. Oset, Nucl. Phys. A 620, 438 (1997) Erratum: [Nucl. Phys. A 652, 407 (1999)] doi:10.1016/S0375-9474(99)00427-3, 10.1016/S0375-9474(97)00160-7 [hep-ph/9702314].

[2] E. Oset and A. Ramos, Nucl. Phys. A 635, 99 (1998) doi:10.1016/S0375-9474(98)00170-5 [nucl-th/9711022].

[3] J. A. Oller and U. G. Meissner, Phys. Lett. B 500, 263 (2001) doi:10.1016/S0370-2693(01)00078-8 [hep-ph/0011146].

[4] R. Molina, D. Nicmorus and E. Oset, Phys. Rev. D 78, 114018 (2008) doi:10.1103/PhysRevD.78.114018 [arXiv:0809.2233 [hep-ph]].

[5] D. Gülmez, U.-G. Meißner and J. A. Oller, Eur. Phys. J. C 77, no. 7, 460 (2017) doi:10.1140/epjc/s10052-017-5018-z [arXiv:1611.00168 [hep-ph]].

[6] J. A. Oller and E. Oset, Phys. Rev. D 60, 074023 (1999) doi:10.1103/PhysRevD.60.074023 [hep-ph/9809337].

[7] M. Albaladejo and J. A. Oller, Phys. Rev. C 84, 054009 (2011) doi:10.1103/PhysRevC.84.054009 [arXiv:1107.3035 [nucl-th]].

[8] M. Albaladejo and J. A. Oller, Phys. Rev. C 86, 034005 (2012) doi:10.1103/PhysRevC.86.034005 [arXiv:1201.0443 [nucl-th]].

[9] Z. H. Guo, J. A. Oller and G. Ríos, Phys. Rev. C 89, no. 1, 014002 (2014) doi:10.1103/PhysRevC.89.014002 [arXiv:1305.5790 [nucl-th]].

[10] J. A. Oller, Phys. Rev. C 93, 024002 (2016) doi:10.1103/PhysRevC.93.024002 [arXiv:1402.2449 [nucl-th]].

[11] L. S. Geng, R. Molina and E. Oset, Chin. Phys. C 41, no. 12, 124101 (2017) doi:10.1088/1674-1137/41/12/124101 [arXiv:1612.07871 [nucl-th]].

[12] M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, Phys. Rev. Lett. 54, 1215 (1985). doi:10.1103/PhysRevLett.54.1215

[13] M. Bando, T. Kugo and K. Yamawaki, Phys. Rept. 164, 217 (1988). doi:10.1016/0370-1573(88)90019-1

[14] H. Nagahiro, L. Roca, A. Hosaka and E. Oset, Phys. Rev. D 79, 014015 (2009) doi:10.1103/PhysRevD.79.014015 [arXiv:0809.0943 [hep-ph]].

[15] M. Tanabashi et al. [Particle Data Group], Phys. Rev. D 98, no. 3, 030001 (2018). doi:10.1103/PhysRevD.98.030001
[16] J. A. Niskanen, Phys. Rev. C 95, no. 5, 054002 (2017) doi:10.1103/PhysRevC.95.054002 [arXiv:1610.06013 [nucl-th]].

[17] A. Gal, Phys. Lett. B 769, 436 (2017) doi:10.1016/j.physletb.2017.03.040 [arXiv:1612.05092 [nucl-th]].

[18] Y. H. Lin and B. S. Zou, Phys. Rev. D 98, no. 5, 056013 (2018) doi:10.1103/PhysRevD.98.056013 [arXiv:1807.00997 [hep-ph]].

[19] Y. H. Lin, C. W. Shen and B. S. Zou, Nucl. Phys. A 980, 21 (2018) doi:10.1016/j.nuclphysa.2018.10.001 [arXiv:1805.06843 [hep-ph]].

[20] L. S. Geng and E. Oset, Phys. Rev. D 79, 074009 (2009) doi:10.1103/PhysRevD.79.074009 [arXiv:0812.1199 [hep-ph]].

[21] M. L. Du, D. Gülmez, F. K. Guo, U. G. Meissner and Q. Wang, Eur. Phys. J. C 78, no. 12, 988 (2018) doi:10.1140/epjc/s10052-018-6475-8 [arXiv:1808.09664 [hep-ph]].