I. INTRODUCTION

When mirrors are closely placed, the attractive or repulsive force between them is observed. This phenomenon, known as the Casimir effect \[1\], is explained by the fact that the vacuum state of the electromagnetic (EM) field in the presence of the mirrors is modified from that of the free space, and the vacuum fluctuation energy depends on the positions of the mirrors. On the other hand, when the mirrors move very rapidly, quantum state of the EM field cannot adiabatically follow the instantaneous vacuum state for each position of the mirrors, resulting in the creation of photons. Such excitation of the quantum field by non-adiabatic change of the vacuum state \[2, 3, 4\] is referred to as the dynamical Casimir effect (DCE), and there have been numerous investigations into this subject \[5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32\], e.g., spectral properties of created photons \[3\], radiation pressure on a moving mirror \[3, 4, 5\], squeezing in the radiation field \[6, 7\], effective Hamiltonian for polaritons \[8\], non-adiabatic movement of the mirrors excites cavity polaritons and this phenomenon may be called the DCE of polaritons. In this paper, we derive the effective Hamiltonian for polaritons, apply it to the moving-mirror problem, and compare the result with that based on the external boundary conditions \[12\].

The aim of the present paper is to formulate the DCE in a moving-matter system in terms of the quantized field-matter theory. The EM field attenuates by coupling with the matter field inside the mirrors, and therefore no external boundary conditions are required. In other words, the EM field confined in the resonator is dressed by the matter field inside the mirrors, forming cavity polaritons. Non-adiabatic movement of the mirrors excites the cavity polaritons and this phenomenon may be called the DCE of polaritons. This paper is organized as follows. In Sec. II, the moving-mirror problem is briefly reviewed. In Sec. III, we formulate quantum theory of field-matter interacting systems, in which matter is allowed to move. In Sec. IV, we derive the effective Hamiltonian for polaritons in the moving-mirror system, and apply it to the one-dimensional case. Final section presents the summary of this paper, and some complicated algebraic manipulations are relegated to appendices.

II. BRIEF REVIEW OF THE MOVING-MIRROR PROBLEM

We briefly review the moving-mirror problem \[3\] to make the present paper self-contained and to fix the notation. The simplest system consists of two perfectly reflecting mirror plates placed in parallel as illustrated in Fig. 1. The mirror at the origin \(z = 0\) is fixed and the other at the position \(z = L(t)\) is allowed to move. The system is assumed to be uniform in the \(x\) and \(y\) directions, and we consider only one component of the vector...
potential, say the $x$ component $A_x(z, t)$, without loss of generality. The vector potential in the Coulomb gauge obeys the wave equation as (we omit the subscript $x$ of $A_x$ from now on)

$$\frac{\partial^2 A_z(t, t)}{c^2 \partial t^2} = \frac{\partial^2 A_z(t, t)}{\partial z^2}, \quad (1)$$

and the boundary conditions are imposed as $A(0, t) = A(L(t), t) = 0$, which guarantee that the transverse components of the electric field vanish at the surfaces of the mirrors in their rest frames. The field operator of the vector potential $A(z, t)$ in the Heisenberg representation can be expanded as

$$\hat{A}(z, t) = \sum_n \sqrt{\frac{\hbar}{2\omega_n}} \left[ \hat{a}_n(t) f_n(z, t) + \hat{a}_n^\dagger(t) f_n^*(z, t) \right], \quad (2)$$

where $\hat{a}_n$ and $\hat{a}_n^\dagger$ are the annihilation and creation operators of photons of the $n$th mode.

One approach [3] to this problem is to fix $\hat{a}_n$ and $\hat{a}_n^\dagger$ to ones at $t = 0$, and evolve the function $f_n(z, t)$ as

$$\frac{\partial^2 f_n(z, t)}{c^2 \partial t^2} = \frac{\partial^2 f_n(z, t)}{\partial z^2}, \quad (3)$$

with the boundary conditions

$$f_n(0, t) = f_n(L(t), t) = 0, \quad (4)$$

which ensure that $\hat{A}(z, t)$ obeys the wave equation and the boundary conditions $\hat{A}(0, t) = \hat{A}(L(t), t) = 0$. When $L$ is constant, the function $f_n(z, t)$ is given by

$$f_n(z, t) = \sqrt{\frac{2}{\omega_n L}} e^{-i \omega_n t \sin k_n z}, \quad (5)$$

where $k_n = n \pi / L$ and $\omega_n = c k_n$. When the characteristic time of the mirror motion is much larger than $L(t)/c$, $f_n(z, t)$ adiabatically follows the mode function for each $L(t)$ as

$$f_n(z, t) \approx \sqrt{\frac{2}{\omega_n(t) L(t)}} e^{-i \int_0^t \omega_n(\tau) d\tau \sin k_n(t) z}, \quad (6)$$

where $k_n(t) = n \pi / L(t)$ and $\omega_n(t) = c k_n(t)$. When the mirror moves much faster, the adiabatic theorem breaks down, and $f_n(z, t)$ evolves in a more complicated manner. The time evolution of the system in this approach is thus not generated by a predetermined Hamiltonian but by the classical equation of motion and the boundary conditions, by which the time evolution of $\hat{A}(z, t)$ in the Heisenberg representation is obtained.

Another approach to the moving-mirror problem is the method of an effective Hamiltonian [11, 12, 14], where the function $f_n(z, t)$ is fixed to the mode function for each $L(t)$ as

$$f_n(z, t) = \sqrt{\frac{2}{\omega_n(t) L(t)}} \sin k_n(t) z, \quad (7)$$

and $\hat{a}_n(t)$ and $\hat{a}_n^\dagger(t)$ are time dependent. This time evolution is described by the Heisenberg equation as

$$i \hbar \frac{d\hat{a}(t)}{dt} = [\hat{a}(t), \hat{H}_a(t)], \quad (8)$$

where the effective Hamiltonian for an operator $\hat{O}$ is denoted by $\hat{H}_a(t)$. In the one-dimensional moving-mirror problem, the effective Hamiltonian for $\hat{a}_n$ and $\hat{a}_n^\dagger$ is obtained as [14] (see Appendix A for the derivation)

$$\hat{H}_a(t) = \sum_n \omega_n(t) \hat{a}_n^\dagger \hat{a}_n - \frac{i \dot{L}(t)}{4L(t)} \sum_n (\hat{a}_n^2 - \hat{a}_n^\dagger^2) + \frac{i \dot{L}(t)}{L(t)} \sum_{n \neq n'} (n + 1) n' \frac{nn'}{n^2 - n'^2} \sqrt{\frac{n'}{n}} (\hat{a}_n^\dagger + \hat{a}_n)(\hat{a}_{n'}^\dagger - \hat{a}_{n'}). \quad (9)$$

If the first term is dominant on the right-hand side of Eq. (8), i.e., $\dot{L}/L \ll \omega_n$ for any $n$, the time evolution operator approximately becomes

$$\exp \left[ -i \int_0^t d\tau \sum_n \omega_n(\tau) \hat{a}_n^\dagger \hat{a}_n \right],$$

showing the adiabatic theorem.

One of the advantages of the effective Hamiltonian ap-
The Lagrangian for this system is then given by

\[ i\hbar \frac{\partial}{\partial t} \psi(t) = \hat{H}_a(t) \psi(t). \]  

(10)

In this case, we note that the operators \( \hat{a}_n \) and \( \hat{a}^\dagger_n \) can be interpreted as the annihilation and creation operators of photons of the \( n \)th mode defined in the interval \( 0 \leq z \leq L(t) \). Another advantage of the effective Hamiltonian approach is that we can study the state evolution in the approach is that we can study the state evolution in the

\[ \frac{\partial}{\partial t} \psi(t) = \hat{H}_a(t) \psi(t). \]

(10)

and the Hilbert space on which it operates are defined of the

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(10)

The third term induces pair creation and annihilation of photons and energy transfer between different modes.

In the above approaches, the boundary conditions that the filed operator \( \hat{A}(z,t) \) vanishes at \( z = 0 \) and \( z = L(t) \) are imposed. Because of these boundary conditions, the canonical commutation relation \( [\hat{A}(r,t), \hat{E}(r',t)] = -i\hbar/\varepsilon_0 \delta_T(r-r') \) does not hold at the boundaries, where \( \delta_T \) is the transverse delta function. The field operator \( \hat{A} \) and the Hilbert space on which it operates are defined only within the interval \( 0 \leq z \leq L(t) \), and then the Hilbert space varies accordingly.

III. FORMULATION OF THE FIELD-MATTER INTERACTING SYSTEMS

A. Field representation of systems

We start from a classical microscopic model, in which polarizable atoms in the matter interact with the EM field. We suppose that the \( n \)th atom consists of an electron with charge \(-e\) and mass \( m_e \) at the position \( r_i \) and an ion with charge \(+e\) and mass \( M \) at the position \( R_i \). The relative vector between the electron and the ion is denoted by \( \mathbf{x}_i = r_i - R_i \), and the center-of-mass vector is \( \mathbf{X}_i = (M R_i + m_e r_i)/(M + m_e) \approx R_i \). We assume that the center-of-mass vectors \( \mathbf{X}_i(t) \) are given functions of time when the matter is moved. The kinetic energy of the \( n \)th atom is expressed as

\[ m_e \xi_n^2/2 + \frac{M \xi_n^2}{2} = (M + m_e) \xi_n^2/2 + m \mathbf{x}_n^2/2, \]

where \( m = M m_e/(M + m_e) \approx m_e \) is the reduced mass. Furthermore the electrons and ions are assumed to be bounded by the effective potential \( m \Omega^2 \mathbf{x}_n^2/2 \).

The Lagrangian for this system is then given by

\[ L = \int dx \left[ \frac{\varepsilon_0}{2} E^2(r,t) - \frac{1}{2 \mu_0} B^2(r,t) \right] + \sum_i \left[ \frac{m}{2} \mathbf{x}_n^2 - \frac{\Omega^2}{2} \mathbf{x}_n^2 \right] - e \sum_i [\phi(r_i,t) - \phi(r_i,t)] + e \sum_i [\mathbf{R}_i \cdot \mathbf{A}(r_i,t) - \mathbf{r}_i \cdot \mathbf{A}(r_i,t)], \]  

(11)

where \( \mathbf{E} = -\nabla \phi - \partial \mathbf{A}/\partial t \) and \( \mathbf{B} = \nabla \times \mathbf{A} \). The Euler-Lagrange equations are obtained as

\[ \varepsilon_0 \nabla \cdot \mathbf{E}(r,t) - e \sum_i [\delta(|r - R_i|) - \delta(|r - r_i|)] = 0, \]  

(12a)

\[ \varepsilon_0 \frac{\partial \mathbf{E}(r,t)}{\partial t} = \frac{1}{\mu_0} \nabla \times \mathbf{B}(r,t) + e \sum_i \left[ \mathbf{R}_i \delta(|r - R_i|) - \mathbf{r}_i \delta(|r - r_i|) \right] = 0, \]  

(12b)

\[ \mathbf{x}_i = -\Omega^2 \mathbf{x}_i - \frac{e}{m} \mathbf{E}(r_i,t) + \mathbf{r}_i \times \mathbf{B}(r_i,t) \]  

(12c)

\[ -\frac{e}{M} \left[ \mathbf{E}(R_i,t) + \mathbf{R}_i \times \mathbf{B}(R_i,t) \right]. \]

The first and second equations are the Maxwell equations, and the third one describes the motion of charged particles in the EM field.

We rewrite the above particle picture of the polarizable atoms in terms of the field picture. When the difference between adjacent polarizations \( |\mathbf{x}_{i+1} - \mathbf{x}_i| \) is much smaller than the characteristic amplitudes of \( |\mathbf{x}_i| \) and \( |\mathbf{x}_{i+1}| \), namely the polarizations change smoothly in the lattice scale, we can replace \( \mathbf{x}_i \) with the polarization field \( \mathbf{X} \) as

\[ \mathbf{x}_i(t) \Rightarrow \mathbf{X}(r,t). \]  

(13)

The density of the polarizable atoms is replaced as

\[ \sum_i \delta(|r - \mathbf{X}_i(t)|) \Rightarrow \rho(r,t). \]  

(14)

The polarization field \( \mathbf{X} \) and the density \( \rho \) vanish outside the matter. The velocity of the matter \( \mathbf{X}_i(t) \) is denoted by \( \mathbf{v}(r,t) \), which is defined only inside the matter. The time derivative \( \mathbf{x}_i(t) \) should be replaced by \( d\mathbf{X}(r,t)/dt \) with

\[ \frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v}(r,t) \cdot \nabla. \]  

(15)

In the present paper, for simplicity, we consider the case in which the matter is allowed to undergo only translational motion. Rotations and deformations of the matter complicate extremely the formulation and are not considered. In this case, the velocity of the matter is uniform in each object, i.e., \( \nabla \mathbf{v}(r,t) = 0 \). For example, in the case of Fig. 1, \( v(z,t) = 0 \) for \( z \leq 0 \) and \( v(z,t) = L(t) \) for \( z \geq L(t) \). The density \( \rho(r,t) \) becomes a function of \( r - vt \) with \( v \) the velocity of each object, giving

\[ \frac{d \rho(r,t)}{dt} = \left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \rho(r,t) = 0. \]  

(16)

Using Eqs. (13) and (14), the first summation in the

\[ \frac{m}{2} \int dr \rho(r,t) \left[ \left( \frac{d\mathbf{X}(r,t)}{dt} \right)^2 - \Omega^2 \mathbf{X}^2(r,t) \right]. \]  

(17)
The interaction terms in the Lagrangian (11) can be rewritten as

\[
\sum_i \phi(R_i, t) - \phi(r, t) - \dot{R}_i \cdot A(R_i, t) + \dot{r}_i \cdot A(r, t)
\]

\[
= \sum_i \int dr \left[ \phi \left( r - \frac{m}{M} x_i, t \right) - \phi \left( r + \frac{m}{m_e} x_i, t \right) \right] 
- \dot{R}_i \cdot A \left( r - \frac{m}{M} x_i, t \right) + \dot{r}_i \cdot A \left( r + \frac{m}{m_e} x_i, t \right) \times \delta(r - \Xi_i) 
\approx \sum_i \int dr \left[ -x_i \cdot \nabla \phi(r, t) + \dot{x}_i \cdot A(r, t) \right] 
+ \Xi_i \cdot \left[ x_i \cdot \nabla A(r, t) \right] \delta(r - \Xi_i),
\]

where in the last line we assumed that \( \phi \) and \( A \) are slowly varying functions in the scale of \( |x_i| \), and ignored the second and higher order of \( |x_i| \). Applying the replacements (13) and (14) to Eq. (18) yields

\[
\int dr \rho(r, t) \left\{ -X(r, t) \cdot \nabla \phi(r, t) + \frac{dX(r, t)}{dt} \cdot A(r, t) \right\} 
+ v(r, t) \cdot [X(r, t) \cdot \nabla] A(r, t)
= \int dr \rho(r, t) X(r, t) \cdot [E(r, t) + v(r, t) \times B(r, t)] 
+ \frac{d}{dt} \int dr \rho(r, t) X(r, t) \cdot A(r, t),
\]

where the second integral in the second line can be ignored because the total derivative term in the Lagrangian is irrelevant in the dynamics. Thus, from Eqs. (17) and (14), the Lagrangian for the system is obtained as

\[
L = \int dr \left\{ \frac{\varepsilon_0}{2} E^2(r, t) - \frac{1}{2 \mu_0} B^2(r, t) 
+ \frac{m}{2} \rho(r, t) \left[ \left( \frac{dX(r, t)}{dt} \right)^2 - \Omega^2 X^2(r, t) \right] 
- e \rho(r, t) X(r, t) \cdot [E(r, t) + v(r, t) \times B(r, t)] \right\}
\]

We note that this Lagrangian reduces to the one used in the static and uniform dielectrics (30), when the matter is fixed and the density \( \rho \) is uniform.

The conjugate momenta of \( A \) and \( X \) are given by

\[
\Pi(r, t) \equiv \frac{\delta L}{\delta \dot{A}(r, t)} = -\varepsilon_0 E(r, t) + e \rho(r, t) X(r, t),
\]

\[
Y(r, t) \equiv \frac{\delta L}{\delta \dot{X}(r, t)} = \frac{m \rho(r, t) dX(r, t)}{dt}.
\]

The Euler-Lagrange equation for \( \phi \) reads

\[
\nabla \cdot D(r, t) = 0,
\]

where \( D = -\Pi \) can be regarded as the electric displacement with polarization \(-\varepsilon_0 X\). Adopting the Coulomb gauge \( (\nabla \cdot A = 0) \) in Eq. (22), we can write the electric potential \( \phi \) as

\[
\phi(r, t) = -\frac{e}{\varepsilon_0} \frac{1}{\sqrt{2}} \nabla \cdot [\rho(r, t) X(r, t)],
\]

where \( e \phi = \int f(r')/(4\pi |r - r'|) \). The Euler-Lagrange equations for \( A \) and \( X \) are obtained as

\[
\varepsilon_0 \frac{\partial E(r, t)}{\partial t} = \frac{1}{\mu_0} \nabla \times B(r, t) + e \rho(r, t) \frac{dX(r, t)}{dt} 
- e v(r, t) \nabla \cdot [\rho(r, t) X(r, t)],
\]

\[
\frac{m}{2} \frac{d^2 X(r, t)}{dt^2} = -m \Omega^2 X(r, t) 
- e [E(r, t) + v(r, t) \times B(r, t)].
\]

Equation (25) corresponds to the Maxwell equation \( \nabla \times B/\mu_0 = J + \partial D/\partial t \) with current \( J = e \nu \nabla \cdot (\rho X) - e (v \cdot \nabla) \rho X \), which satisfies \( \nabla \cdot J = 0 \). Equation (26) describes the dynamics of moving in the EM field. Using Eqs. (20) to (23), we obtain the Hamiltonian as

\[
H = \int dr \left[ \Pi(r, t) \cdot \frac{\partial A(r, t)}{\partial t} + Y(r, t) \cdot \frac{\partial X(r, t)}{\partial t} \right] - L
= \int dr \left\{ \frac{1}{2 \varepsilon_0} ||\Pi(r, t) - e \rho(r, t) X(r, t)||^2 + \frac{1}{2 \mu_0} B^2(r, t) 
+ \frac{1}{2m \rho(r, t)} Y^2(r, t) + \frac{m \Omega^2}{2} \rho(r, t) X^2(r, t) 
+ e \rho(r, t) X(r, t) \cdot [v(r, t) \times B(r, t)] 
- Y(r, t) \cdot [v(r, t) \cdot \nabla X(r, t)] \right\}.
\]

Equations (21), (22), (25), and (26) are derived as the canonical equations of this Hamiltonian.

We note that the Hamiltonian (27) reduces to the macroscopic model described in terms of the dielectric constant \( \varepsilon \) (19, 21, 22, 23, 24) when the dynamics of polarization can be eliminated. The Hamiltonian in this model is given by

\[
H = \int dr \left[ \frac{1}{2} (E \cdot D + B \cdot H) 
+ \frac{1}{2} \left[ E \cdot D + \frac{1}{\mu_0} B^2 \right] \right] 
+ O(v^3),
\]

where we used the relations for moving medium (23)

\[
D = \varepsilon E + (\varepsilon - \varepsilon_0) v \times B + O(v^2),
\]

\[
B = \mu_0 H + (\varepsilon - \varepsilon_0) E \times v + O(v^2).
\]

If we neglect the left-hand side of Eq. (27), we get

\[
X = -\frac{e}{m \Omega^2} (E + v \times B).
\]
This relation and \( D = \varepsilon_0 E - e\rho X \) yield Eq. (29a) with \( \varepsilon = 1 + e^2/\varepsilon_0 m \Omega^2 \). Substituting Eq. (29) into our Hamiltonian (27), and dropping the terms including \( Y \), we obtain Eq. (28). Our model, therefore, reduces to the above dielectric model, when the dynamics of polarization is neglected. This corresponds to the case in which the relevant frequencies of the EM field are much smaller than the characteristic frequencies of matter \( \Omega \) and \( \omega^\prime \).

**B. Quantization: polaritons**

We find out the normal mode of the field-matter coupled equations, in which the positions of the matter are fixed, i.e., \( \mathbf{v} = 0 \). We denote this fixed matter configuration as \( \mathcal{M} \), and the mode functions and frequencies depend on it: \( A_n (r, \mathcal{M}), \cdots, \omega_n (\mathcal{M}) \), where \( n \) is the index of the mode. For brevity, we omit the argument \( \mathcal{M} \) below. Substitution of \( (\hbar/2\omega_n)^{1/2} A_n (r) e^{-i\omega_n t}, \) \( i(\hbar\omega_n/2)^{1/2} \Pi_n (r) e^{-i\omega_n t}, \) and \( (\hbar/2\omega_n)^{1/2} Y_n (r) e^{-i\omega_n t} \) into \( \mathbf{A}, \mathbf{\Pi}, \mathbf{X}, \mathbf{Y} \) in Eqs. (21), (22), (23), and (26) for \( \mathbf{v} = 0 \) yields the normal-mode equations

\[
\Pi_n (r) = -\varepsilon_0 A_n (r) + e \left( 1 - \nabla \frac{1}{\varepsilon_0 \nabla^2} \right) [\rho(r) X_n (r)],
\]

\[
\omega_n^2 \Pi_n (r) = \frac{1}{\mu_0} \nabla^2 A_n (r),
\]

\[
\omega_n^2 \Pi_n (r) = m_\omega^2 \rho(r) X_n (r),
\]

\[
\omega_n^2 \Pi_n (r) = \left[ m\Omega^2 \rho(r) + \frac{e^2}{\varepsilon_0} \mu^2(r) \right] X_n (r)
\]

where we used Eq. (24). We take \( A_n, \Pi_n, X_n, \text{and} Y_n \) to be real without loss of generality. These mode functions can be shown to satisfy the orthonormal relation (see Appendix B)

\[
\int dr \left[ A_n (r) \cdot \Pi_n (r) - Y_n (r) \cdot X_n (r) \right] = -\delta_{nn'}. \tag{32}
\]

In terms of the mode functions, we can expand the time evolution of the fields as

\[
\mathbf{A}(r, t) = \sum_n \sqrt{\frac{\hbar}{2\omega_n}} A_n (r) \left( b_n e^{-i\omega_n t} + \text{c.c.} \right), \tag{33a}
\]

\[
\mathbf{\Pi}(r, t) = \sum_n i \sqrt{\frac{\hbar\omega_n}{2}} \Pi_n (r) \left( b_n e^{-i\omega_n t} - \text{c.c.} \right), \tag{33b}
\]

\[
\mathbf{X}(r, t) = \sum_n i \sqrt{\frac{\hbar\omega_n}{2}} X_n (r) \left( b_n e^{-i\omega_n t} - \text{c.c.} \right), \tag{33c}
\]

\[
\mathbf{Y}(r, t) = \sum_n \sqrt{\frac{\hbar}{2\omega_n}} Y_n (r) \left( b_n e^{-i\omega_n t} + \text{c.c.} \right), \tag{33d}
\]

where \( b_n \) is the complex amplitude of each mode, and c.c. indicates the complex conjugate of the previous term. Substituting the expansions (33) into the Hamiltonian (27) for \( \mathbf{v} = 0 \), denoted by \( H_{v=0} \), and using Eqs. (31), we obtain

\[
H_{v=0} = \sum_n \hbar\omega_n b_n^\dagger b_n. \tag{34}
\]

Following the standard quantization procedure, we replace the c-numbers \( b_n \) and \( b_n^\dagger \) with the Bose operators \( \hat{b}_n \) and \( \hat{b}_n^\dagger \) satisfying the commutation relation \( \left[ \hat{b}_n, \hat{b}_n^\dagger \right] = \delta_{nn'} \). The elementary excitations created by \( \hat{b}_n^\dagger \) can be regarded as polaritons, since they are linear combinations of the photon and polarization fields. We should note that the operators \( \hat{b}_n \) also depend on the matter configuration \( \mathcal{M} \), and are to be expressed as \( \hat{b}_n (\mathcal{M}) \) in full detail. The field operators in the Schrödinger representation can be expanded as

\[
\hat{A}(r) = \sum_n \sqrt{\frac{\hbar}{2\omega_n}} \hat{A}_n (r) (\hat{b}_n + \hat{b}_n^\dagger), \tag{35a}
\]

\[
\hat{\Pi}(r) = \sum_n i \sqrt{\frac{\hbar\omega_n}{2}} \hat{\Pi}_n (r) (\hat{b}_n - \hat{b}_n^\dagger), \tag{35b}
\]

\[
\hat{X}(r) = \sum_n i \sqrt{\frac{\hbar\omega_n}{2}} \hat{X}_n (r) (\hat{b}_n - \hat{b}_n^\dagger), \tag{35c}
\]

\[
\hat{Y}(r) = \sum_n \sqrt{\frac{\hbar}{2\omega_n}} \hat{Y}_n (r) (\hat{b}_n + \hat{b}_n^\dagger). \tag{35d}
\]

In the above argument, we assumed the discrete spectrum of polaritons. When the spectrum is continuous, the continuous index of the mode, such as wave number \( k \), should be used instead of \( n \), and the summation \( \sum_n \) should be replaced by an appropriate integral.

**IV. FIELD-MATTER FORMALISM OF THE MOVING-MIRROR PROBLEM**

**A. The effective Hamiltonian**

In the previous section, the polaritons were derived for fixed matter configuration \( \mathcal{M} \). The number states of the polaritons are eigenstates of the Hamiltonian for the system with \( \mathcal{M} \), and thus suitable for orthogonal set of bases in the Fock space of polaritons. When the matter configuration transforms to \( \mathcal{M}' \), definition of polaritons alters accordingly, and the number states of the polaritons in \( \mathcal{M}' \) should be used as new bases. As a result of change of bases, the state vector undergoes unitary transformation. Thus, when the matter configuration continuously transforms as \( \mathcal{M}(t) \) and we insist on using the Fock state bases in instantaneous matter configuration, the state vector undergoes extra evolution in addition to the usual time evolution. This representation (the bases follow the
eigenstates of the time-dependent Hamiltonian instantaneously) is often employed in the adiabatic approximation \[38\]. The effective Hamiltonian describing such state evolution can be derived by a few ways \[11, 12, 14, 29\] that are equivalent each other, and here we follow the one in Ref. \[29\].

The Schrödinger equation is written by

\[
\hat{H} \langle \mathcal{M}(t) | \psi(t) \rangle = \hat{H}(\mathcal{M}(t)) | \psi(t) \rangle
\]

where \( \hat{H}_{v=0} \) is the part that does not include the velocity of the matter explicitly, and \( \hat{K} \equiv \hat{H} - \hat{H}_{v=0} \). Expanding the state vector as \( |\psi(t)\rangle = \sum_i c_i(t) |\psi_i(\mathcal{M}(t))\rangle \) with the eigenvectors satisfying

\[
[\hat{H}_{v=0}(\mathcal{M}(t)) - E_i(\mathcal{M}(t))] |\psi_i(\mathcal{M}(t))\rangle = 0,
\]

Eq. (36) becomes

\[
\frac{i\hbar}{\partial t} \psi_i(t) = E_i c_i(t) + \sum_j \langle \psi_i | \hat{K} | \psi_j \rangle c_j(t)
\]

\[
-\frac{i\hbar}{\partial \mathcal{M}}(\mathcal{M}(t)) \sum_j \langle \psi_i | \partial_{\mathcal{M}} | \psi_j \rangle c_j(t)
\]

\[
= \sum_j \langle \psi_i | \hat{H}^{\text{eff}} | \psi_j \rangle c_j(t),
\]

where we omit the argument \( \mathcal{M}(t) \) for brevity. The symbol \( \partial/\partial \mathcal{M} \) indicates differentiation with respect to the positions of the matter, e.g., \( \partial/\partial L \) in the situation of Fig. 1. The matrix element of the effective Hamiltonian is thus given by

\[
\langle \psi_i | \hat{H}^{\text{eff}} | \psi_j \rangle = E_i \delta_{ij} + \langle \psi_i | \hat{K} | \psi_j \rangle
\]

\[
-\frac{i\hbar}{\partial \mathcal{M}}(\mathcal{M}(t)) \langle \psi_i | \partial_{\mathcal{M}} | \psi_j \rangle.
\]

Now we express the effective Hamiltonian by using the instantaneous creation and annihilation operators \( \hat{b}_n(\mathcal{M}(t)) \) and \( \hat{b}_n(\mathcal{M}(t)) \) (we omit the argument \( \mathcal{M}(t) \) below). The first term on the right-hand side of Eq. (38) corresponds to \( \sum_n \hbar \omega_n \hat{b}_n^\dagger \hat{b}_n \) in the effective Hamiltonian. Substituting the field expansions (35) into the velocity-dependent part

\[
\hat{K} = \int dr \left\{ e \rho(r, t) \mathbf{X}(r, t) \cdot [\mathbf{v}(r, t) \times \mathbf{B}(r, t)] - \mathbf{Y}(r, t) \cdot [\mathbf{v}(r, t) \cdot \nabla] \mathbf{X}(r, t) \right\},
\]

we obtain

\[
\dot{\mathcal{K}} = i\hbar \sum_{n,n'} F_{nn'}^{(1)} (\hat{b}_n^\dagger + \hat{b}_n) (\hat{b}^\dagger_{n'} - \hat{b}_{n'}),
\]

where we defined

\[
F_{nn'}^{(1)} = \frac{1}{2} \sqrt{\frac{\omega_n}{\omega_{n'}}} \int dr \left\{ e \rho(r, t) \left[ \mathbf{v}(r, t) \cdot \left\{ \mathbf{X}(r, t) \cdot \nabla \right\} \mathbf{A}(r) \right] - \mathbf{Y}(r) \cdot \nabla \mathbf{X}(r, t) \right\}.
\]

The last term on the right-hand side of Eq. (39) is treated as follows. Differentiating the eigenvalue equation (37) with respect to \( \mathcal{M} \), we find

\[
\langle \psi_i | \frac{\partial}{\partial \mathcal{M}} | \psi_j \rangle = -\frac{\langle \psi_i | \frac{\partial \mathcal{H}^{\text{eff}}}{\partial \mathcal{M}} | \psi_j \rangle}{E_i - E_j}
\]

for \( i \neq j \), and we take the eigenstates as \( \langle \psi_i | \frac{\partial}{\partial \mathcal{M}} | \psi_j \rangle = 0 \).

From Eqs. (39) and (43), a term \( \hat{b}_n \hat{b}_n^\dagger \) in \( \frac{\partial \mathcal{H}^{\text{eff}}}{\partial \mathcal{M}} \), for example, corresponds to \( i \hbar \omega_n \hat{b}_n^\dagger \hat{b}_n / (\omega_n - \omega_{n'}) \) in the effective Hamiltonian, since \( E_i - E_j = \hbar (\omega_n - \omega_{n'}) \) for non-vanishing matrix element. The correspondences of terms in \( \frac{\partial \mathcal{H}^{\text{eff}}}{\partial \mathcal{M}} \) to those in the effective Hamiltonian are therefore obtained as

\[
(\hat{b}_n + \hat{b}_n^\dagger)^2 \rightarrow \frac{i}{2 \omega_n} (\hat{b}_n^\dagger)^2 - \hat{b}_n^2,
\]

\[
(\hat{b}_n \pm \hat{b}_n^\dagger)(\hat{b}_n \mp \hat{b}_n^\dagger) \rightarrow \frac{i}{\omega_n + \omega_{n'}} (\hat{b}_n \hat{b}_n^\dagger - \hat{b}_n^\dagger \hat{b}_n)
\]

\[
\pm \frac{i}{\omega_n - \omega_{n'}} (\hat{b}_n^\dagger \hat{b}_n - \hat{b}_n \hat{b}_n^\dagger).
\]

In the Hamiltonian \( 29 \), the density \( \rho \) depends on the matter configuration, and then

\[
\mathcal{K}(\mathcal{M}) \frac{\partial \mathcal{H}_{v=0}}{\partial \mathcal{M}} = \int dr \frac{\partial \rho(r, t)}{\partial t} \left\{ \frac{e}{\varepsilon_0} \left[ \Pi(r, t) - e \rho(r, t) \mathbf{X}(r, t) \right] \cdot \mathbf{X}(r, t) - \frac{1}{2 m \rho^2(r, t) \mathbf{Y}^2(r, t) + m \Omega^2/2} \mathbf{X}^2(r, t) \right\}.
\]
right-hand side of Eq. (39) becomes in the effective Hamiltonian as

$$i\hbar \sum_{n}(\frac{1}{2}F^{(2)}_{nn} - \frac{1}{4}F^{(2)}_{nn}) (\hat{b}^\dagger_n - \hat{b}_n^2)$$

$$+ i\hbar \sum_{n\neq n'} \sqrt{\omega_n \omega_{n'}} \left[ (F^{(2)}_{nn'} - \frac{1}{2}F^{(3)}_{nn'}) \frac{1}{\omega_n + \omega_{n'}} (\hat{b}^\dagger_n \hat{b}^\dagger_{n'} - \hat{b}_{n'} \hat{b}_n) - \left( \frac{F^{(2)}_{nn'} + \frac{1}{2}F^{(3)}_{nn'}}{\omega_n - \omega_{n'}} \right) (\hat{b}^\dagger_n \hat{b}_{n'} - \hat{b}_{n'} \hat{b}_n) \right], \tag{46}$$

where we defined

$$F^{(2)}_{nn'} = \int d\mathbf{r} \frac{\partial \rho(\mathbf{r}, t)}{\partial t} \left\{ \frac{e}{2\varepsilon_0} \left[ \Pi_n(\mathbf{r}) - e\rho(\mathbf{r}, t)X_n(\mathbf{r}) \right] \cdot X_n(\mathbf{r}) - \frac{m\Omega^2}{4} X_n(\mathbf{r}) \cdot X_n(\mathbf{r}) \right\}, \tag{47a}$$

$$F^{(3)}_{nn'} = \frac{1}{\omega_n \omega_{n'}} \int d\mathbf{r} \frac{\partial \rho(\mathbf{r}, t)}{\partial t} \frac{1}{2m\mu^2(\mathbf{r}, t)} Y_n(\mathbf{r}) \cdot Y_{n'}(\mathbf{r}). \tag{47b}$$

Rearranging the terms in Eqs. (41) and (46), we obtain the effective Hamiltonian for polaritons as

$$\frac{\hat{H}^{\text{eff}}(t)}{\hbar} = \sum_n \omega_n \hat{b}^\dagger_n \hat{b}_n + i \sum_n C_{nn'} \left( \hat{b}^\dagger_n - \hat{b}_n^2 \right) + i \sum_{n\neq n'} C_{nn'} (\hat{b}^\dagger_n + \hat{b}_n) (\hat{b}^\dagger_{n'} - \hat{b}_{n'}) \tag{48}$$

with

$$C_{nn'} = \begin{cases} F^{(1)}_{nn'} + \frac{1}{2}F^{(2)}_{nn'} - \frac{1}{4}F^{(3)}_{nn'} \frac{\sqrt{\omega_n \omega_{n'}}}{\omega_n - \omega_{n'}} \left[ \omega_n (F^{(2)}_{nn'} + F^{(3)}_{nn'}) + \omega_{n'} F^{(3)}_{nn'} \right] & (n = n') \tag{49} \\
\frac{F^{(1)}_{nn'} - \frac{1}{2}F^{(2)}_{nn'}}{\sqrt{\omega_n \omega_{n'}}} & (n \neq n'). \end{cases}$$

This effective Hamiltonian for the polaritons with moving matter is the main result of the present paper. It is interesting to note that the effective Hamiltonian for polaritons [43] and that for photons based on the external boundary conditions [3] have a similar form with respect to the creation and annihilation operators, suggesting that Eq. (48) reduces to Eq. (4) in some limiting case. This will be explicitly shown in Sec. VII B for one-dimensional case. It is also suggested that the squeezed state of polaritons will be generated by oscillation of the matter at an appropriate frequency by analogy with the case of photons [43]. The important difference between Eqs. (48) and (4) is that the time evolution is fully described in a common Hilbert space in Eq. (48) in contrast to Eq. (4) in which the Hilbert space changes by the mirror motion.

### B. One-dimensional case

In order to compare our effective Hamiltonian (48) with Eq. (4), we consider the one-dimensional moving-mirror problem as illustrated in Fig. 1. We assume that the system is uniform in the $x$ and $y$ directions, and consider only the $x$ components of the vector fields $\hat{A}_x(\hat{z})$, $\hat{\Pi}_x(\hat{z})$, $\hat{X}_x(\hat{z})$, and $\hat{Y}_x(\hat{z})$ without loss of generality (we omit the subscript $x$ below). The normal-mode equations [31] reduce to

$$\Pi_n(z,t) = -\varepsilon_0 A_n(z,t) + e\rho(z,t) X_n(z,t), \tag{50a}$$

$$\Pi_n(z,t) = \frac{1}{\mu_0 \omega_n^2(t)} A''_n(z,t), \tag{50b}$$

$$Y_n(z,t) = m\omega_n^2(t)\rho(z,t)X_n(z,t), \tag{50c}$$

$$Y_n(z,t) = \rho(z,t) [m\Omega^2 X_n(z,t) + eA_n(z,t)]. \tag{50d}$$

From these equations, we obtain

$$A''_n(z,t) + \frac{\omega_n^2(t)}{e^2} \varepsilon_n(z,t) A_n(z,t) = 0, \tag{51}$$

where

$$\varepsilon_n(z,t) \equiv 1 - \frac{e^2 \rho(z,t)}{\varepsilon_0 m} \frac{1}{\omega_n^2(t) - \Omega^2} \tag{52}$$

can be regarded as the dielectric constant. Thus, in our effective Hamiltonian, the dispersion relation is included. If the reservoir is taken into account, the Kramers-Kronig relations will be satisfied as shown in Ref. [36] for static dielectrics.

The properties of polaritons in the matter significantly depend on the sign of the dielectric constant $\varepsilon_n$. When $\varepsilon_n$ is negative, the wave function of the polariton decays in the matter, and then polaritons localize between mirrors and the energy spectrum is discrete. This condition is given by $\Omega^2 < \omega_p^2 < \Omega^2 + \omega_p^2$, where $\omega_p \equiv (e^2 \rho/\varepsilon_0 m)^{1/2}$ is the plasma frequency. When $\varepsilon_n$ is positive, the wave function extends indefinitely inside the matter and the energy spectrum is continuous.
Let us consider the case in which the matter is uniform, i.e., \( \rho(z, t) = \rho \theta(-z) + \rho \theta(z - L(t)) \), where \( \rho \) is the polarization density in the matter and \( \theta(z) \) is the Heaviside function. In this case, Eq. (51) can be solved, and when \( \varepsilon_n \) is negative in the matter, the solutions are given by

\[
A_n(z, t) = \begin{cases} 
\alpha_n(t)e^{\alpha_n(t)z} \cos \left( \frac{k_n(t)L(t)}{2} \right) & (z \leq 0) \\
\alpha_n(t) \cos k_n(t) \left[ z - \frac{L(t)}{2} \right] & (0 < z < L(t)) \\
\alpha_n(t)e^{-\alpha_n(t)[z-L(t)]} \sin \left( \frac{k_n(t)L(t)}{2} \right) & (z \geq L(t)) 
\end{cases}
\]

for the even number of nodes, and

\[
A_n(z, t) = \begin{cases} 
\alpha_n(t)e^{\alpha_n(t)z} \sin \left( \frac{k_n(t)L(t)}{2} \right) & (z \leq 0) \\
\alpha_n(t) \sin k_n(t) \left[ z - \frac{L(t)}{2} \right] & (0 < z < L(t)) \\
\alpha_n(t)e^{-\alpha_n(t)[z-L(t)]} \sin \left( \frac{k_n(t)L(t)}{2} \right) & (z \geq L(t)) 
\end{cases}
\]

for the odd number of nodes, where \( k_n \equiv \omega_n/c, \kappa_n \equiv \frac{\varepsilon_n^{1/2}}{k_n} \), and we take the label \( n \) to be the number of nodes. (The solutions for the continuous spectrum are given in Appendix C.) From the orthonormal relation \( \int dz \left[ A_n(z)\Pi_{n'}(z) - Y_n(z)X_{n'}(z) \right] = -\delta_{nn'} \), the normalization constant \( \alpha_n \) is obtained by

\[
\alpha_n^2(t) = \left[ \varepsilon_0 \left( \frac{L(t)}{2} + \frac{1}{\kappa_n(t)} \frac{\omega_n^2(t)}{\omega_n^2(t) - \Omega^2} \right) \right]^{-1},
\]

where the sign of \( \alpha_n \) is taken to be \( A_n(0, t) > 0 \). The eigenvalues \( \omega_n(t) \) are determined so that the solutions are smoothly connected at \( z = 0 \) and \( z = L(t) \), giving the eigenvalue equation

\[
\tan \left( \frac{\omega_n(t)L(t)}{2c} \right) = \pm |\varepsilon_n(t)|^{1/2} = \pm \left( 1 - \frac{\omega_n^2(t)}{\omega_n^2(t) - \Omega^2} \right)^{1/2},
\]

where the signs + and − correspond to the solutions (53a) and (53b), respectively. Using Eqs. (52), (57), (53), (53), and \( \partial \rho(z, t)/\partial t = -\rho \dot{L}(t)\delta(z - L(t)) \), we obtain the coefficient (53) as

\[
C_{nn}(t) = \frac{1}{8}\varepsilon_0 \dot{L}(t) \alpha_n^2(t),
\]

and

\[
C_{nn'} = \frac{1}{2}\varepsilon_0 \dot{L}(t) \left[ \frac{(-1)^{n+n'}\alpha_n\alpha_{n'}}{\omega_n^2 - \omega_{n'}^2} - \frac{1}{\kappa_n + \kappa_{n'}} \right] \sqrt{\frac{\omega_n}{\kappa_n + \kappa_{n'}}}
\]

\[
\times \left( \kappa_n\omega_n^2 \sqrt{\frac{1 + |\varepsilon_n|}{1 + |\varepsilon_{n'}|}} + \kappa_{n'}\omega_{n'}^2 \sqrt{\frac{1 + |\varepsilon_{n'}|}{1 + |\varepsilon_n|}} \right)
\]

for \( n \neq n' \), where we omit the argument \( t \) for brevity.

First we consider the case in which the time scale of mirror motion is much larger than the inverse of the plasma frequency \( \omega_p^{-1} \). In this case, the transition between the discrete and continuous spectrum can be ignored, and then the continuous spectrum is irrelevant. In order to see the relation between our result and Eq. (4), we consider the case of metal, which is obtained by setting \( \Omega = 0 \). The coefficients (56) reduce to

\[
C_{nn}(t) = \frac{1}{4} \dot{L}(t) + \frac{2}{\kappa_n(t)},
\]

and

\[
C_{nn'}(t) = \frac{(-1)^{n+n'}\alpha_n\alpha_{n'}}{\omega_n^2 - \omega_{n'}^2} \sqrt{\frac{\omega_n^2}{\omega_{n'}^2}} \times \frac{\dot{L}(t)}{\sqrt{L(t) + \frac{2}{\kappa_n(t)}}} + \frac{2}{\kappa_{n'}(t)}
\]

for \( n \neq n' \). When \( \omega_n \ll \omega_p \), which corresponds to the case in which the penetration depth of the EM field is much smaller than its wavelength, \( \omega_n \) and the coefficients (56) can be expanded with respect to \( \eta(t) \equiv c/L(t)\omega_p \ll 1 \), giving

\[
\omega_n(t) = \frac{c}{L(t)} n\pi \left[ 1 - 2\eta(t) + 4\eta^2(t) \right] - 8 \left( 1 + \frac{n^2\pi^2}{24} \right) \eta^3(t) + O(\eta^4),
\]

\[
C_{nn}(t) = -\frac{\dot{L}(t)}{4L(t)} \left[ 1 - 2\eta(t) + 4\eta^2(t) \right] - 8 \left( 1 + \frac{n^2\pi^2}{24} \right) \eta^3(t) + O(\eta^4),
\]

\[
C_{nn'}(t) = \frac{\dot{L}(t)}{n^2 - n'^2} \sqrt{\frac{n'}{n}} \left[ 1 - 2\eta(t) + 4\eta^2(t) \right] - \frac{1}{3} (24 + m^2\pi^2) \eta^3(t) + O(\eta^4).
\]

If we identify the photon operators \( \hat{a}_n \) in \( \hat{H}^\text{eff}(t) \) [Eq. (4)] as the polaron operators \( \hat{b}_n \), we find

\[
\hat{H}^\text{eff}_b(t) = [1 - 2\eta(t) + 4\eta^2(t)]\hat{H}^\text{eff}(t) + O(\eta^3).
\]

When we neglect the terms of order \( O(\eta) \), the effective Hamiltonian for polaritons \( \hat{H}^\text{eff}_b(t) \) reduces to that based on the external boundary condition \( \hat{H}^\text{eff}(t) \), and therefore, our method reproduces the existing results of the moving-mirror problem in the limit of \( \eta \to 0 \). It is interesting to note that \( \hat{H}^\text{eff}(t) \) is proportional to \( \hat{H}^\text{eff}(t) \) up to the second order of \( \eta \). This physically indicates that the time scale is delayed by the factor \( \simeq 1 - 2\eta \) due to the coupling of the photon field with the matter field, i.e., the EM field drags electrons in the mirrors when it is excited. In other words, photons in the cavity are
dressed by plasmons in the cavity mirrors, forming the cavity polaritons.

Figure 2 shows the lowest two eigenfrequencies determined from Eq. (55) with \( \Omega = 0 \) and the coefficients \( C_{0i}L/L \) (dotted line) and \( C_{0i}L/L \) (dot-dashed line) as functions of \( \omega_p/\omega_c \), where \( \omega_c \equiv 2\pi c/L \).

When the time scale of mirror motion is comparable to \( \omega_p^{-1} \), transition between discrete and continuous spectrum occurs. The transition is significant when the mirror moves as \( \omega_p \) becomes small due to, e.g., decrease of the carrier density.

When the mirror vibrates at the frequency \( \omega_M > \omega_p - \omega_n \), which results in decay of the polaritons in the \( n \)th mode into continuous spectrum, namely, photons leak out of the cavity. When the mirror moves as \( L(t) = L_0 + \ell \sin \omega_M t \) with \( \ell \ll L_0 \), the time-dependent part of the effective Hamiltonian can be written by \( \hat{V} e^{i\omega_M t} + \hat{V} e^{-i\omega_M t} \), where \( \hat{V} \) is obtained by replacing \( \hat{L}(t) \) with \( \ell \omega_M/2 \) in \( \hat{H}_{\text{eff}}^{\text{th}} \). Using Fermi’s golden rule, the decay rate of a photon in the \( n \)th mode is estimated to be

\[
R_n(\omega_M) = \frac{2}{\hbar c^2} \sum_{i=1,2} |V_{kn}^{(i)}|^2 ,
\]

where \( V_{kn}^{(i)} \) is the matrix element of \( \hat{V} \) with respect to the \( n \)th mode in the discrete spectrum and the mode labeled by \( k = (\omega_n + \omega_M)/c \) and \( i = 1,2 \) in the continuous spectrum. Here we took the wave number \( k \) between mirrors as the mode index in the continuous spectrum. The explicit form of \( V_{kn}^{(i)} \) is given in Appendix C. We can show that when \( \Omega = 0 \) and \( \omega_n \ll \omega_p \ll \omega_M \) the decay rate \( R_n(\omega_M) \) reduces to \( n\ell^2 \omega_p^2/(4L_0^2 \omega_M) \).

In the effective Hamiltonian \( \hat{H}_{\text{eff}}^{\text{th}}(t) \), all the energy levels are commensurate (\( \omega_n = n\pi/L_0 \)), and the created photons make transition to higher levels unlimitedly as \( \omega_n \to \omega_{2n} \to \cdots \) due to the resonance. In our one-dimensional effective Hamiltonian \( \hat{H}_{\text{eff}}^{\text{th}}(t) \) \( \text{[Eq. (3)]} \), on the other hand, the transition stops at \( \approx \omega_p \) due to the incommensurate energy levels, or the decay into the continuous spectrum occurs, and thus the resonant enhancement of the DCE is to be naturally suppressed at \( \approx \omega_p \).
The equations of motion are given by
\[ \begin{align*}
\frac{\partial}{\partial t} A(z,t) &= -\dot{E}(z,t), \quad (A1a) \\
\frac{\partial}{\partial t} \dot{E}(z,t) &= -c^2 \frac{\partial^2}{\partial z^2} \dot{A}(z,t), \quad (A1b)
\end{align*} \]
and the boundary conditions are \( \hat{A}(0, t) = \hat{A}(L(t), t) = 0 \). The field operators are expanded as
\[ \hat{A}(z,t) = \sum_n \sqrt{\frac{\hbar}{2\varepsilon_0 \omega_n(t)}} \phi_n(z,t) [\hat{a}_n(t) + \hat{a}_n^\dagger(t)], \quad (A2a) \]
\[ \dot{E}(z,t) = \sum_n i \sqrt{\frac{\hbar \omega_n(t)}{2\varepsilon_0}} \phi_n(z,t) [\hat{a}_n(t) - \hat{a}_n^\dagger(t)], \quad (A2b) \]
where \( \omega_n(t) = n\pi c/L(t) \) and
\[ \phi_n(z,t) = \sqrt{\frac{2}{L(t)}} \sin \left( \frac{n\pi z}{L(t)} \right). \quad (A3) \]

From Eqs. (A2) and (A3), the annihilation operator is written by
\[ \hat{a}_n(t) = \int_0^{L(t)} dz \frac{1}{2} \phi_n(z,t) \left[ \sqrt{\frac{2\varepsilon_0 \omega_n(t)}{\hbar}} \hat{A}(z,t) - i \sqrt{\frac{2\varepsilon_0}{\hbar \omega_n(t)}} \hat{E}(z,t) \right]. \quad (A4) \]
Using Eqs. (A1), the time derivative of Eq. (A3) reads
\[ \frac{d\hat{a}_n(t)}{dt} = \int_0^{L(t)} dz \frac{1}{2} \phi_n(z,t) \left[ -\sqrt{\frac{2\varepsilon_0 \omega_n(t)}{\hbar}} \dot{\hat{E}}(z,t) + i \sqrt{\frac{2\varepsilon_0}{\hbar \omega_n(t)}} c^2 \frac{\partial^2}{\partial z^2} \hat{A}(z,t) \right] \]
\[ - \frac{\dot{L}(t)n\pi}{L^2(t)} \int_0^{L(t)} dz \frac{1}{2} \sqrt{\frac{2}{L(t)}} \cos \left( \frac{n\pi z}{L(t)} \right) \left[ \sqrt{\frac{2\varepsilon_0 \omega_n(t)}{\hbar}} \hat{A}(z,t) - i \sqrt{\frac{2\varepsilon_0}{\hbar \omega_n(t)}} \hat{E}(z,t) \right] \]
\[ = -i\omega_n(t) \hat{a}_n(t) - \frac{\dot{L}(t)}{2L(t)} \hat{a}_n^\dagger(t) - \frac{\dot{L}(t)}{L(t)} \sum_{n \neq n'} (-1)^{n+n'} \frac{n}{n^2 - n'^2} \left[ \sqrt{\frac{n}{n}} - \sqrt{\frac{n'}{n'}} \right] \hat{a}_{n'}(t) + \left( \sqrt{\frac{n}{n}} - \sqrt{\frac{n'}{n'}} \right) \hat{a}_{n'}^\dagger(t). \quad (A5) \]

Thus we find that the effective Hamiltonian gives this time evolution by the Heisenberg equation.

**APPENDIX B: ORTHOGONALITY OF THE MODE FUNCTIONS**

We give a proof of the orthogonality of the mode functions in Eq. (32). We consider the integral
\[ I_{nn'} = \int dr [A_{n'}(r) \cdot \Pi_n(r) - Y_{n'}(r) \cdot X_n(r)]. \quad (B1) \]
Substituting Eqs. (B1) and (B1') into \( \Pi_n \) and \( X_n \) in \( \omega_n^2 I_{nn'} \), we find
\[ \omega_n^2 I_{nn'} = \omega_{n'}^2 I_{n'n}. \quad (B2) \]
On the other hand, substituting Eqs. (B1) and (B1') into \( A_{n'} \) and \( Y_{n'} \) in \( I_{nn'} \), and using \( \nabla \cdot \Pi_n = 0 \), we find
\[ I_{nn'} = I_{n'n}. \quad (B3) \]
From Eqs. (B2) and (B3), we obtain \( (\omega_n^2 - \omega_{n'}^2)I_{nn'} = 0 \), i.e., \( I_{nn'} = 0 \) for \( \omega_n^2 \neq \omega_{n'}^2 \).

**APPENDIX C: CALCULATIONS FOR CONTINUOUS SPECTRUM IN ONE-DIMENSION**

When \( \varepsilon_n \) is positive in the matter, the energy spectrum is continuous, and we use the wave number \( k \) between mirrors as the mode index. There are two independent
solutions of Eq. (51) for given $k$ as

\[
A_k^{(1)}(z) = \begin{cases} 
\alpha_k \left( \frac{k}{\pi} \cos \frac{kL}{2} \sin \kappa z - \sin \frac{kL}{2} \cos \kappa z \right) & (z \leq 0) \\
\alpha_k \sin k(z - L/2) & (0 < z < L) \\
\alpha_k \left( \frac{k}{\pi} \cos \frac{kL}{2} \sin \kappa(z - L) + \sin \frac{kL}{2} \cos \kappa(z - L) \right) & (z \geq L),
\end{cases}
\]

\[
A_k^{(2)}(z) = \begin{cases} 
\beta_k \left( \frac{k}{\pi} \sin \frac{kL}{2} \sin \kappa z + \cos \frac{kL}{2} \cos \kappa z \right) & (z \leq 0) \\
\beta_k \cos k(z - L/2) & (0 < z < L) \\
\beta_k \left( -\frac{k}{\pi} \sin \frac{kL}{2} \sin \kappa(z - L) + \cos \frac{kL}{2} \cos \kappa(z - L) \right) & (z \geq L),
\end{cases}
\]

where $\kappa = \sqrt{1/2}k$, and the normalization constants

\[
\alpha_k = \left[ \frac{\pi\varepsilon_0 \sqrt{k}}{\varepsilon_k} \left( \frac{1}{\varepsilon_k} \cos^2 \frac{kL}{2} + \sin^2 \frac{kL}{2} \right) \right]^{-1/2},
\]

\[
\beta_k = \left[ \frac{\pi\varepsilon_0 \sqrt{k}}{\varepsilon_k} \left( \frac{1}{\varepsilon_k} \sin^2 \frac{kL}{2} + \cos^2 \frac{kL}{2} \right) \right]^{-1/2},
\]

are determined from the orthonormal relation

\[
\int dz [A_k^{(i)}(z)\Pi_k^{(j)}(z) - Y_k^{(i)}(z)X_k^{(j)}(z)] = -\delta(k - k')\delta_{ij}.
\]

We can see that the functions (13) and (41) are orthogonal each other. The matrix element $V_{kn}^{(1)}$ in Eq. (30) is obtained by the straightforward calculation as

\[
V_{kn}^{(1)} = (-1)^n \frac{i\hbar\varepsilon_0\omega_M}{4} \sqrt{\frac{k}{k_n}} \times \frac{\alpha_k |\alpha_n| \kappa_p(k + k_n)}{(k^2 + \kappa_n^2)(k^2 - \kappa_n^2)(k_n^2 - k_0^2)^{1/2}} \\
\times \left( \frac{k^2 \kappa_n \cos \frac{kL}{2} - k\kappa_n^2 \sin \frac{kL}{2}}{kL} \right),
\]

where $k_0 \equiv \Omega/c$, $k_p \equiv \omega_p/c$, and $V_{kn}^{(2)}$ is obtained by the replacement $\alpha_k \rightarrow \beta_k$, $\cos kL/2 \rightarrow -\sin kL/2$, and $\sin kL/2 \rightarrow \cos kL/2$.

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