A Hybrid Approach to Targeting Social Assistance∗

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Abstract
Proxy means testing (PMT) and community-based targeting (CBT) are two of the leading methods for targeting social assistance in developing countries. In this paper, we present a hybrid targeting method that incorporates CBT’s emphasis on local information and preferences with PMT’s reliance on verifiable indicators. Specifically, we outline a Bayesian framework for targeting that resembles PMT in that beneficiary selection is based on a weighted sum of sociodemographic characteristics. We nevertheless propose calibrating the weights to preference rankings from community targeting exercises, implying that the weights used by our method reflect how potential beneficiaries themselves substitute sociodemographic features when making targeting decisions. We discuss several practical extensions to the model, including a generalization to multiple rankings per community, an adjustment for elite capture, a method for incorporating auxiliary information on potential beneficiaries, and a dynamic updating procedure. We further provide an empirical illustration using data from Burkina Faso and Indonesia.

Key words: Bayesian inference; community-based targeting; proxy means testing; social assistance; targeting

JEL codes: C11; D04; I32; I38; O20

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1 Introduction

Effectively identifying the poor is critical to the success of many social assistance programs, particularly in developing countries. Proxy means testing (PMT) is a popular method for “targeting” social assistance and has been implemented in a wide variety of countries (Coady et al., 2004a; Fiszbein and Schady, 2009; Devereux et al., 2017). The standard form of PMT ranks potential beneficiaries on the basis of a weighted sum of sociodemographic characteristics, where the weights are coefficients from a regression model of income or expenditure estimated using household survey data (Kidd and Wylde, 2011; Kidd et al., 2017; Brown et al., 2018). PMT is easy to implement, relies only on verifiable indicators, and imposes limited costs on potential beneficiaries.

Though PMT has a number of strengths, it also has limitations. As a highly centralized form of targeting, PMT is inconsistent with commitments to participatory development. In particular, PMT conflicts with Target 16.7 of the Sustainable Development Goals, which aims to “[e]nsure responsive, inclusive, participatory and representative decision-making at all levels” (UNGA, 2015). PMT also neglects local information, as household surveys may have limited geographic coverage of program areas, disproportionate non-response from the poor, and may not capture all relevant individual or household features (Alderman, 2002; Carr-Hill, 2013; Bollinger et al., 2019). A further concern is that PMT may conflict with local preferences regarding beneficiary selection. Perhaps most importantly, the income-oriented definition of poverty used by PMT may be inconsistent with local definitions of poverty (Alatas et al., 2012; Han and Gao, 2019; Hillebrecht et al., 2020b).

Community-based targeting (CBT) is a leading alternative to PMT. CBT typically revolves around an exercise where community members or local leaders meet to rank potential beneficiaries in terms of need (Conning and Kevane, 2002). CBT is a highly participatory targeting method that emphasizes local information and preferences, but it is also subject to limitations. While CBT is often considered less costly than PMT, the cost savings are achieved by reducing reliance on remunerated labor (e.g., for data collection and processing) and imposing additional unremunerated costs on community members in the form of opportunity and psychological costs (Devereux et al., 2017)\(^1\). In addition to imposing costs on

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\(^1\) See Kidd et al. (2017) or Brown et al. (2018) for discussion of additional limitations of PMT.

\(^2\) The opportunity costs associated with CBT result from the fact that local officials and community members must engage in extended exercises for identifying needy households and thus forego devoting time to productive activities. In Burkina Faso, for example, Hillebrecht et al. (2020b) found that the CBT exercises took a half a day for each community on average. The psychological costs of CBT are less acknowledged, but no less important. In particular, community members may find it shameful or embarrassing to be publicly identified as poor during the community ranking exercises. They may also find it difficult to discuss private matters (e.g., their standard of living or the standard of living of others) in a public setting.
community members, it is well-known that CBT can suffer from elite capture, where local elites use their power to influence the beneficiary selection process (Bardhan and Mookherjee, 2006; Kilic et al., 2015; Han and Gao, 2019; Basurto et al., 2020).

In this paper, we develop a hybrid targeting method that incorporates many of the advantages of PMT and CBT while minimizing their main limitations. Specifically, we outline a Bayesian framework for targeting that resembles PMT in that beneficiary selection is based on a weighted sum of sociodemographic characteristics, but we instead propose calibrating the weights to community preference rankings. The ratio of any two weights then reflects the implied rate at which potential beneficiaries themselves substitute sociodemographic features when making targeting decisions. Our approach thus inherits the advantages of PMT (e.g., beneficiary selection is based on objective and verifiable information), but discards its less desirable aspects. That is, in contrast to PMT, our method broadly respects commitments to participatory development, privileges local information, and is consistent with preferences revealed through community ranking exercises.

Our hybrid approach also overcomes the main limitations of CBT. While CBT is costly to potential beneficiaries in the sense that many (if not all) community members must participate in community ranking exercises, our method can be calibrated with community ranking data from a small sample of communities. Program administrators can thus minimize the opportunity and psychological costs imposed on beneficiaries by conducting model-based prediction of community preference rankings in non-sampled locations. Regarding elite capture, an important feature of our approach is that it explicitly models the influence of elite connections on community rankings and provides a simple way to purge these influences from the final ranks. The procedure thus mitigates concerns about transmitting the influence of elite capture to non-sampled communities and can also be used to reconstruct unbiased rankings from sampled communities.

There are other features of our approach worth highlighting. First, our model provides a principled way to aggregate rankings when CBT exercises generate multiple rankings per community. In Niger, for example, each community constituted three committees to independently rank potential beneficiaries, but then the ranks were aggregated in an ad hoc manner to arrive at the final list of beneficiaries (Premand and Schnitzer, 2020). Second, as a fully Bayesian framework, we strategically use priors to regularize the model’s coefficients to improve out-of-sample predictions, to incorporate subjective beliefs on the quality of different rankers, and to provide an efficient way to update the model over time. Finally, our framework provides a way to introduce auxiliary data (e.g., information on incomes or

3Specifically, the final list of beneficiaries was based on each household’s average rank from the three committees. See Hillebrecht et al. (2020b) for another example of CBT with multiple rankings per community.
expenses) to improve performance when preference rankings are only generated for a small number of communities.

We illustrate our method using data from Burkina Faso and Indonesia. These datasets were previously used by Hillebrecht et al. (2020b) (Burkina Faso) and Alatas et al. (2012) (Indonesia) to compare the performance of alternative targeting methods, and therefore combine information from community ranking exercises with household survey data. We use the datasets to (1) examine how our method weights sociodemographic characteristics relative to PMT, (2) demonstrate the out-of-sample predictive performance of the method, and (3) illustrate the various model features mentioned above. We find that communities implicitly weight sociodemographic characteristics quite differently than PMT, often with practically-relevant differences in signs and magnitudes. We further find that our method performs well in terms of predicting community preference rankings, most notably achieving error rates that are lower than what PMT can achieve when predicting household expenditures.

This paper contributes to the methodological literature on targeting social assistance. A few recent studies have suggested using PMT with alternative estimands, including food expenditures (Basurto et al., 2020), dietary diversity (Premand and Schnitzer, 2020), and human capabilities (Henderson and Follett, 2022). Other recent work has proposed using alternative estimators to improve the predictive performance of PMT (McBride and Nichols, 2018; Brown et al., 2018). Finally, Alatas et al. (2012) and Stoeffler et al. (2016) examined the performance of a hybrid targeting approach where the selected beneficiaries were those with the lowest PMT scores among a larger group of households selected via CBT. For example, Alatas et al. (2012) had communities nominate 1.5 times the quota of beneficiaries and then narrowed the list using PMT.

Our hybrid approach is distinct from existing hybrid methods. Rather than using CBT rankings to narrow the pool of potential beneficiaries for which PMT is applied, we instead (1) estimate the model’s weights using community preference rankings and (2) use information on incomes or expenditures to improve the predictive accuracy of the model. That is, the distinguishing feature of our approach is that we use community revealed preferences as the estimand, whereas existing hybrid approaches focus on incomes or expenditures. While emphasizing community preferences aligns our method with commitments to participatory development, it is also better reflects local information and values (e.g., local definitions of poverty), which may improve community satisfaction with targeting outcomes and reduce the social unrest often associated with statistical targeting procedures.

4 Also see Barrett and Clay (2003), Elbers et al. (2007), and Coady and Parker (2009) for some discussion of the potential complementarities between different targeting approaches.

5 A number of papers find that local definitions of poverty are often distinct from standard income-oriented definitions of poverty. For examples of such findings, see Alatas et al. (2012) on earning capacity.
In what follows, Section 2 describes a basic version of our Bayesian framework for hybrid targeting and Section 3 presents various model extensions. Section 4 then discusses the data we use for our empirical illustration and Section 5 details the results of our analysis. Finally, Section 6 provides concluding remarks, including a discussion of the limitations of our framework.

2 Targeting Community Revealed Preferences

In this section, we first discuss the data requirements for our method and present a simple benchmark model in the probit regression. Next, we outline the most basic version of our Bayesian framework for targeting community revealed preferences, which assumes a singular ranking scheme for each community and no auxiliary information. Our focus in this section is on fixing intuition related to modeling ranked data and, in the next section, we will extend the model in a variety of ways.

2.1 Data requirements and benchmark model

Let $G = \{1, 2, 3, \ldots\}$ represent the set of all geographic units within a given country (e.g., neighborhoods, villages, districts, etc.). We will refer to these units as communities. As social assistance programs do not necessarily cover all communities within a country, we let $J \subseteq G$ denote the set of communities in program areas. We will index these communities by $j$ and index households within any community by $i$. We then assume that for all households in the program areas we observe a row vector of sociodemographic characteristics $x_{ij}$, which is obtained from a census of the program areas. The covariate vector $x_{ij}$ is common to all forms of PMT and is used for calculating the scores that determine program inclusion.

Now let $K \subseteq G$ represent the set of communities conducting community ranking exercises and index these communities by $k$. Note that CBT requires $K = J$ or that all program areas perform the ranking exercises. As mentioned above, however, our model only requires that the community ranking exercises be conducted in a sample of communities, which will most plausibly be taken from program areas (i.e., $K \subseteq J$). In specifying $K \subseteq G$, we nevertheless do not rule out the possibility of sampling from non-program areas. We assume

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Gao (2019) on multidimensional poverty, and [Hillebrecht et al. (2020)] on assets (among other dimensions). Alatas et al. (2012) find that targeting procedures that are in line with community preferences (i.e., CBT) tend to improve community satisfaction, though Premand and Schnitzer (2020) find that satisfaction can be undermined if community members believe that CBT exercises are manipulated for personal gain. Finally, it is well-documented that PMT is often associated with social unrest due to lacking transparency, inaccuracy, or the fact that it conflicts with local definitions of poverty (Cameron and Shah, 2014; Kidd et al., 2017; Sumarto, 2021).
that each exercise ranks households in ascending order from most to least needy and denote each household’s rank by $z_{ik}^6$. Finally, we assume that we observe the (same) vector of sociodemographic characteristics for all households in the sampled areas, which we denote by $x_{ik}$.

At a minimum, we then observe $x_{ij}$, $x_{ik}$, and $z_{ik}$. Recall that our objective is to develop a PMT-like method where the estimand is community revealed preferences. PMT typically consists of two stages: (1) an estimation stage that calibrates a statistical model and (2) a prediction stage that calculates the scores that determine program inclusion based on the statistical model. For example, the first stage in the standard form of PMT uses household survey data to regress income or expenditure per capita on some observed sociodemographic traits. The estimated coefficients are then used in a second stage to predict income or expenditure per capita for potential beneficiaries, and it is these scores that determine program inclusion. While we retain this simple two-stage procedure, we deviate from the standard form of PMT by proposing a different first stage where the weights are calibrated to community preference rankings.

To this end, first consider a simple benchmark model in the form of a probit regression. Let $d_{ik} = I(z_{ik} \leq q_k)$ where $I(\cdot)$ is the indicator function and $q_k$ represents the beneficiary quota for community $k$. The variable $d_{ik}$ is thus a binary variable that captures program inclusion, which we can model in terms of a continuous latent variable $\tilde{d}_{ik}$ as follows:

$$
\tilde{d}_{ik} = x_{ik} \beta + \epsilon_{ik}
$$

where $d_{ik} = I(\tilde{d}_{ik} > 0)$, $\beta$ denotes a column vector of first-stage coefficients (including an intercept), and $\epsilon_{ik} \sim N(0,1)$. The above is often estimated via maximum likelihood using the following likelihood function:

$$
P(d \mid \beta) = \prod_k \prod_i (\pi_{ik})^{d_{ik}} (1 - \pi_{ik})^{1-d_{ik}}
$$

where $\pi_{ik} \equiv P(d_{ik} = 1 \mid x_{ik}) = P(\tilde{d}_{ik} > 0 \mid x_{ik}) = \Phi(x_{ik} \beta)$ and where $\Phi(\cdot)$ is the standard normal cumulative distribution function. The likelihood function can then be maximized with respect to $\beta$ and the estimated coefficients $\hat{\beta}$ can be used to calculate out-of-sample score estimates for all potential beneficiaries as $x_{ij} \hat{\beta}$.

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6 Note that here we assume that each community only generates a single list of ranks. Below we consider extensions that accommodate multiple rankings per community.

7 Note that prediction with the probit regression often aims to estimate the probability that a given observation takes on the value of one. In our case, this prediction would be calculated as $\Phi(x_{ij} \hat{\beta})$. Given that $\Phi(\cdot)$ is a monotonic function and that we are only interested in relative ranks, we can ignore this additional step.
While simple, the probit model has important limitations for estimating community revealed preferences. First, the probit model requires dichotomizing the community rankings and thus discards information: Beneficiaries and non-beneficiaries near the threshold are treated very differently, while all households within the two groups are treated identically. Second, the probit model is subject to the issue of complete or quasi-complete separation (i.e., when covariates provide perfect or near-perfect predictions), which typically requires ad hoc solutions to obtain finite maximum likelihood estimates. Finally, the maximum likelihood estimation procedure implicitly assumes a uniform prior on all parameters, thus neglecting potentially relevant information that may improve out-of-sample predictions and therefore targeting performance.

2.2 A Bayesian framework

Here we outline a basic framework for modeling community rankings that overcomes many of the limitations of the probit regression. The [Thurstone 1927] order statistics model is among the most popular for modeling ranked data. The Thurstone model relies on a latent variable formulation similar to the probit regression, though the latent variable in the Thurstone model determines the ranks rather than a dichotomized rank variable. That is, the ranks are modeled directly, meaning that the Thurstone model not only makes full use of the available information, but is also not subject to the problem of separation. We specifically consider a class of the Thurstone model called the Thurstone-Mosteller-Daniels (TMD) model, which also like the probit assumes that the noise associated with the latent variable follows a normal distribution (Thurstone, 1927; Mosteller, 1951; Daniels, 1950).

Let $\tilde{z}_{ik}$ denote a continuously-valued latent variable that determines the observed ranks $z_{ik}$. We will further use $\tilde{z}_k$ and $z_k$ to denote the vectors of latent variables and observed rankings for each community, respectively. We then model $\tilde{z}_{ik}$ as follows:

$$\tilde{z}_{ik} = x_{ik}\delta + \eta_{ik}$$

where $\text{rank}(\tilde{z}_k) = z_k$, $\delta$ is a column vector of coefficients, and $\eta_{ik} \sim \mathcal{N}(0, 1)$. Note that in the case of ranked data, the parameter vector $\delta$ does not include an intercept, as ranks are invariant to the location shifts generated by intercepts. Similar to the probit model, our objective is then to estimate $\delta$, which we can then use to calculate the scores for all potential beneficiaries as $x_{ij}\hat{\delta}$.

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8For example, one common solution is to simply drop the variable in question, which is often unsatisfactory given the variable’s predictive power. See Zorn (2005) for further discussion.
The likelihood is defined as the probability of observing the ranks generated by the communities. For community \( k \), this probability can be represented by \( P(z_k | \delta) \). To calculate the likelihood, we must take the conditional likelihood \( P(z_k | \tilde{z}_k) \) weighted by the density \( P(\tilde{z}_k | \delta) \) and integrate out the latent variables. Since the event \( z_k \) occurs if and only if \( \text{rank}(\tilde{z}_k) = z_k \), we can write \( P(z_k | \tilde{z}_k) = I[\text{rank}(\tilde{z}_k) = z_k] \). We then have the following expression for the likelihood function:

\[
P(z | \delta) = \prod_k P(z_k | \delta)
= \prod_k \int P(z_k | \tilde{z}_k) P(\tilde{z}_k | \delta) \, d\tilde{z}_k
= \prod_k \int \left\{ I[\text{rank}(\tilde{z}_k) = z_k] \prod_i \phi(\tilde{z}_{ik} - x_{ik} \delta) \right\} \, d\tilde{z}_k
\]

(4)

where \( \phi(\cdot) \) is the standard normal density function and each integral’s dimension is equal to the number of ranked individuals in each community.

Maximum likelihood estimation of the above model is non-trivial because of the form of the likelihood. In particular, evaluation of the likelihood requires numerical approximation of the integral, which can only be done accurately when the number of ranked entities is quite small (generally less than 15) \cite{Alvo2014}. To avoid these limitations, we require an alternative estimation procedure that circumvents the integration. Bayesian methods have become increasingly popular for estimating Thurstonian models because they provide a straightforward way to obtain simulation-based estimates of the unknown parameters \cite{Yao1999, Johnson2013}. Recently, \cite{Li2016} developed a unified Bayesian framework for estimating a wide array of Thurston-type models and we draw on this work extensively in what follows.

While maximum likelihood estimation seeks to find the parameter values that maximize the likelihood of the data, Bayesian inference instead examines the probability distribution of the parameters conditional on the data (known as the posterior). The posterior can be derived as follows:

\[
P(\delta | z) = \frac{P(z \cap \delta)}{P(z)} = \frac{P(z | \delta) P(\delta)}{P(z)} \propto P(z | \delta) P(\delta)
\]

(5)

where \( \delta \) represents the unknown parameters and \( z \) represents the data. The first equality above is an application of the conditional probability rule and the second equality is Bayes’ rule. Bayes’ rule states that the posterior is proportional to the likelihood multiplied by the prior distribution of \( \delta \), where the prior describes our beliefs about the location of \( \delta \).
before observing the data. We can thus think of the posterior as a compromise between the information contained in the likelihood and the prior (with more weight given to the likelihood as more data is obtained).

Bayes’ rule formalizes the process through which we update our beliefs about $\delta$ after observing the data. Priors are central to this process and when we decide on the form of $P(\delta)$ we are balancing the uncertainty of the parameters against the bias. One could choose “non-informative” priors that will provide (unbiased) results identical to frequentist estimation methods, such as maximum likelihood. Alternatively, one can place weakly informative priors on $\delta$ based on information available a priori (further discussed below). By the bias-variance trade-off, this will reduce the error variability on the parameter estimates at the expense of increasing bias. A critical feature of weakly informative priors in the context of targeting is that reducing parameter variability can improve out-of-sample prediction by mitigating overfitting (Gelman et al., 2013, 2017).

The Bayesian approach to Thurstonian models circumvents integration through a procedure called data augmentation, which facilitates computation by treating the latent variables as quantities to be estimated (Tanner and Wong, 1987; Gelfand and Smith, 1990). The augmented posterior can be written as follows:

$$
P(\delta, \tilde{z} | z) \propto P(z | \tilde{z}) P(\tilde{z} | \delta) P(\delta)
= \left\{ \prod_k \mathbb{I}\left[ \text{rank}(\tilde{z}_k) = z_k \right] \right\} \left\{ \prod_k \prod_i \phi(\tilde{z}_{ik} - x_{ik}\delta) \right\} N(\mu_\delta, \Sigma_\delta).
$$

where $P(z | \tilde{z})$ is the likelihood, the distribution $P(\tilde{z} | \delta)$ follows directly from our assumptions on $\eta$, and we use the multivariate normal distribution for the prior on $\delta$. In our application of this basic model, we set $\mu_\delta = 0$ and $\Sigma_\delta = 2.5^2 \times I$ where $I$ denotes an identity matrix. The prior on $\delta$ reflects the simple belief that most of the covariates are unlikely to be useful, but that a few may have a strong relationship with community revealed preferences. The prior thus regularizes the coefficients to avoid overfitting without resorting to complicated procedures like cross-validation.

The objective is then to draw samples from the posterior and use the samples to calculate quantities of interest. We use Markov Chain Monte Carlo (MCMC) methods, specifically the well-known Gibbs sampler, to draw samples from the posterior (Yao and Böckenholt, 1999; Johnson and Kuhn, 2013; Li et al., 2016). Details of the sampling procedure can be found in Appendix A. After obtaining samples, we use the posterior mean of $\delta$ as the weights assigned to the sociodemographic characteristics. That is, letting $b = 1, 2, \ldots, B$ index draws from the posterior, we calculate $\hat{\delta} = \frac{1}{B} \sum_{b=1}^{B} \delta^b$. As mentioned, the final step is to generate
the scores for each potential beneficiary as \( x_{ij} \hat{\delta} \), and these scores can be used to determine program inclusion through a rank-ordering of the potential beneficiaries.

It is useful at this point to note how our approach is distinct from PMT. Recall that the standard form of PMT calibrates the weights by using household survey data to regress income or expenditures on the observed sociodemographic characteristics (Brown et al., 2018). The ratio of any two weights in this case reflects the rate at which the corresponding sociodemographic features can be substituted while maintaining a given level of income or expenditures. In contrast, we use community preference rankings as the estimand, meaning that the weight ratios associated with our method capture the rate at which community members themselves substitute sociodemographic features while maintaining a given household rank. That is, rather than imposing an income-oriented definition of poverty on the targeting algorithm, our approach attempts to reflect community preference orderings regarding beneficiary selection and thus accommodates alternative definitions of poverty.

3 Extending the Bayesian Framework

In this section, we introduce a number of extensions to our framework for targeting community revealed preferences. We first extend the framework to accommodate multiple ranking schemes per community. We then discuss the issue of elite capture and present a simple way to remove the influence of elite capture from the final ranks. Next, we consider how auxiliary information, namely household survey data, can be used to improve the predictive performance of the model. Finally, we outline a procedure for dynamically updating the model’s parameters as new information becomes available.

3.1 Multiple rankings

Some recent implementations of CBT have generated multiple ranking schemes per community. As mentioned, Premand and Schnitzer (2020) document a CBT exercise conducted in Niger that had each community form three committees: one committee of local leaders, one committee of non-leader women, and one committee with a mixed group of non-leaders. Each committee conducted its ranking of potential beneficiaries independently and then the final ranks for each community were calculated by averaging each household’s rankings across the three lists. To cite another example, Hillebrecht et al. (2020) discuss a CBT exercise conducted in Burkina Faso where each community nominated three key informants to rank
potential beneficiaries. The actual beneficiaries were then selected via an algorithm that prioritized households based on agreements across the three lists.\(^9\)

To extend our model to accommodate multiple rankers, let \(z_{ik}^r\) denote the rank of household \(i\) in community \(k\) by ranking entity \(r\), and let \(\tilde{z}_{ik}^r\) denote the corresponding latent variable. Further, let \(\tilde{z}_k^r\) and \(z_k^r\) represent ranker-specific vectors of latent variables and observed rankings for each community, respectively. Similar to Li et al. (2016), we can then rewrite Eq. (3) to accommodate multiple rankers as follows:

\[
\tilde{z}_{ik}^r = \alpha_{ik} + x_{ik}\delta + \eta_{ik}^r
\]

where now \(\text{rank}(\tilde{z}_k^r) = z_k^r\), \(\alpha_{ik}\) is a household-specific intercept, and \(\eta_{ik}^r \sim N(0, \omega_r^{-1})\). The term \(\omega_r\) represents the precision of the noise in the model, which is allowed to vary across rankers. That is, the model implicitly aggregates alternative ranking schemes according to their relative precision or quality, as more reliable rankers disproportionately inform \(\delta\).\(^10\)

One can incorporate subjective beliefs about ranker qualities through the use of priors. Again following Li et al. (2016), we specify a simple prior where \(\omega_r\) can take on three values: 0.5, 1, and 2 corresponding to low-quality, mediocre, and reliable rankers, respectively. More specifically, we let

\[
\omega_r = \begin{cases} 
0.5 & \text{with probability } \lambda_1^r \\
1 & \text{with probability } \lambda_2^r \\
2 & \text{with probability } \lambda_3^r 
\end{cases}
\]

where prior beliefs are incorporated through the choice of \(\lambda_1^r\), \(\lambda_2^r\), and \(\lambda_3^r\) for each ranker. The full posterior for the multiple-ranker model is presented in Appendix A. Once again, we use the samples from the posterior to generate the scores for potential beneficiaries as \(x_{ij}\).\(^11\)

\(^9\)That is, any household that was ranked among the poorest by all three informants was selected as a beneficiary, then households ranked among the poorest by two informants, and so on. See Hillebrecht et al. (2020b) for detailed discussion.

\(^{10}\)This contrasts with the aggregation methods used by Hillebrecht et al. (2020b) and Premand and Schnitzer (2020), which assume that all rankers are equally reliable. It is nevertheless possible that different individuals or groups generate rankings of differing qualities. For example, in Premand and Schnitzer (2020), the ranking exercises were conducted by committees of leaders and non-leaders. Non-leaders may have better information about the living standards of community members than leaders, perhaps due to more frequent interactions or because community members conceal relevant information from leaders.

\(^{11}\)In generating out-of-sample predictions, we set the random effects equal to their expected value, which is zero in this case. As such, the random effects only influence the scores through their effect on \(\delta\).
3.2 Elite capture

As mentioned, CBT can be subject to elite capture whereby local elites influence the beneficiary selection process in order to privilege relatives or friends. Evidence of elite capture has been found in China (Han and Gao, 2019), Ethiopia (Caeyers and Dercon, 2012), India (Besley et al., 2012; Panda, 2015), Malawi (Kilic et al., 2015; Basurto et al., 2020), and Tanzania (Pan and Christiaensen, 2012), to name a few. While there are instances where elite capture has been found to be negligible or non-existent (Alatas et al., 2012; Bardhan et al., 2010; Schüring, 2014), where it is a potential threat to the targeting process, it may be useful to have a way to mitigate its influence. We thus outline a method for removing the influence of elite connections on the scores for potential beneficiaries.

Let \( x_{ik} = [x_{sik}, x_{eik}] \) where \( x_{sik} \) is a row vector of sociodemographic characteristics and \( x_{eik} \) is a row vector capturing elite connections for household \( i \) in community \( k \). While there could be a variety of ways to specify \( x_{eik} \), a simple specification could consist of a single indicator variable that captures whether or not any household member holds a leadership position or is related to anybody in a leadership position (Alatas et al., 2012). We can then define the corresponding vector of coefficients as \( \delta = [\delta^s, \delta^e] \) and apply the models outlined in Eqs. (3) or (7) to retrieve estimates of \( \delta \). For well-specified models, the inclusion of \( x_{eik} \) removes any confounding influence of elite connections on \( \delta^s \) and, as such, we can generate unbiased scores for potential beneficiaries as \( x_{sij} \hat{\delta}^s \) (i.e., we set \( \delta^e = 0 \) when computing the scores).

3.3 Auxiliary information

In the event that household survey data is available, one can use this auxiliary data to improve the predictive accuracy of the model. To this end, let \( M \subset G \) represent the set of communities sampled for some household survey and index these communities by \( m \). We assume that the household survey contains information on household incomes or expenditures and denote the associated variable by \( y_{im} \). We further assume that the household survey includes sociodemographic information for all households and denote the row vector of such characteristics as \( x_{im} \). We then specify the following model:

\[
y_{im} = \phi + x_{im} \gamma + \psi_{im}
\]

where \( \phi \) is the intercept, \( \gamma \) is a column vector of parameters, and \( \psi_{im} \sim \mathcal{N}(0, \sigma^2_{\psi}) \). Note that the above model is equivalent to the standard form of PMT.
We can then specify a new (joint) likelihood as follows:

\[
P(z, y \mid \tilde{z}, \theta) = P(z \mid \tilde{z})P(y \mid \phi, \gamma, \sigma^2_\psi)
= \left\{ \prod_k \prod_r I[\text{rank}(\tilde{z}_k^r) = z_k^r] \right\} \left\{ \prod_m \prod_i N(y_{im} \mid \phi + x_{im}\gamma, \sigma^2_\psi) \right\}
\]

where \(\theta = \{\alpha, \delta, \omega, \phi, \gamma, \sigma_\psi\}\) denotes a vector of all fixed parameters. The first component of the above is the likelihood of the ranked data and the second component is the likelihood associated with the auxiliary data. The full posterior, including a description of all priors, is presented in Appendix A. A critical feature of this model is the priors we place on \(\delta\) and \(\gamma\). In particular, we let \(\delta, \gamma \sim N(\mu, \Sigma)\) such that both parameter vectors are drawn from the same multivariate normal distribution with mean vector \(\mu\) and covariance matrix \(\Sigma\). The auxiliary data is thus permitted to inform \(\mu\) and \(\Sigma\), which in turn informs \(\delta\). Below we show that this procedure improves the predictive accuracy of the targeting scores \(x_{ij}\hat{\delta}\) (or \(x_{isj}\hat{\delta}_s\) when adjusting for elite capture).

### 3.4 Dynamic updating

Using data from Burkina Faso, Hillebrecht et al. (2020a) examined the dynamic targeting performance of PMT and CBT in terms of identifying the consumption poor. While the authors found that PMT initially outperformed CBT, the performance of PMT deteriorated faster than that of CBT, to the extent that CBT outperformed PMT after just one year. One reason for the dynamic success of CBT was that communities implicitly weighted sociodemographic characteristics in a way that better predicted future poverty. Though our method might be expected to inherit the forward-looking nature of community rankings, the fact that both PMT and CBT were subject to non-negligible deterioration suggests that targeting mechanisms need to be regularly updated.

While many poverty-alleviation programs using PMT conduct periodic updates (Kidd and Wylde, 2011; Kidd et al., 2017), the procedure for updating the PMT algorithm generally neglects important information. In particular, re-estimating PMT weights by applying frequentist estimation methods (e.g., OLS) to new data implicitly assumes uniform priors on all weights, when in fact relevant information is available from previous implementations. The implication of this “memoryless” updating is that it sacrifices predictive accuracy. In contrast, our Bayesian framework naturally accommodates updating, as the posterior from any given period can be used as the prior in some subsequent period. The Bayesian approach thus provides a coherent way to dynamically update the targeting algorithm as new information becomes available.
To illustrate, let \( P(\theta \mid z_{1:t+1}) \) denote the posterior distribution of the model’s parameters \( \theta \) conditional on all data available up to some time period \( t + 1 \). We can then show that the posterior \( P(\theta \mid z_{1:t+1}) \) is proportional to the likelihood of some new data \( P(z_{t+1} \mid \theta) \) multiplied by the posterior from the previous period \( P(\theta \mid z_{1:t}) \):

\[
P(\theta \mid z_{1:t+1}) \propto P(z_{1:t+1} \mid \theta) P(\theta) = P(z_{t+1} \mid \theta) P(z_{1:t} \mid \theta) P(\theta) \propto P(z_{t+1} \mid \theta) P(\theta \mid z_{1:t}).
\]

That is, using the posterior from a previous period as the prior in a subsequent period is mathematically equivalent to conducting the analysis using data from all periods. The above assumes that all data contribute equally relevant information, which may be questionable given the results of Hillebrecht et al. (2020a). To overweight more recent information, one can simply diffuse \( P(\theta \mid z_{1:t}) \) around the first moments, thus weakening the influence of previous data on the posterior. See Appendix A for additional discussion.

4 Data

We illustrate our hybrid targeting framework using data from Burkina Faso and Indonesia. Our primary goal is to understand the out-of-sample predictive performance of our model and, as such, we split each dataset into three mutually exclusive samples: (1) a test sample that we use to evaluate the performance of the model, (2) a training sample that we use to estimate the model, and (3) an auxiliary sample that we reserve for augmenting the training sample to estimate the model that incorporates auxiliary information (see Section 3.3). In what follows, we discuss the data from both countries and outline our approach for splitting each sample.

4.1 Burkina Faso

Hillebrecht et al. (2020b) examined the performance of alternative targeting methods using data from the department of Nouna in the northwest of Burkina Faso. With the objective of increasing enrollment rates in a community-based health insurance scheme, the Burkinabé Ministry of Health in partnership with a local NGO offered a 50 percent discount on the premium for the poorest 20 percent of households in a number of villages and urban neighborhoods in Nouna. To identify beneficiaries, each village or neighborhood conducted CBT

\[\text{[Footnote]}\]

The data is not publicly available and was provided to us by the authors.
exercises in the years 2007, 2009, and 2011. Hillebrecht et al. (2020b) focused on the 2009 campaign where CBT exercises took place in 58 different communities, including 36 villages and 22 neighborhoods in Nouna Town.

The CBT exercises began with a public meeting that included focus group discussions and an election of three local informants that were charged with ranking all community members in terms of the poverty and wealth criteria identified by the focus groups. The three local informants were then physically separated from the assembly to complete the ranking exercises. The final beneficiaries were then selected based on agreements across the three rank orderings. First, all households ranked among the poorest 20 percent by all informants were automatically eligible. Second, all households ranked among the poorest 20 percent by two informants were deemed eligible, unless the number of households exceeded the quota, in which case the informants were consulted to narrow the list. Finally, if necessary, the informants were consulted to select the remaining households from those that were ranked among the poorest 20 percent by one informant.

The Nouna Household Survey was also conducted in 2009 and it includes information on a sample of 655 households in the 58 communities where the CBT exercises were conducted. Due to some missing information on key variables, we only use 608 observations from this dataset. Merging the household survey with the rankings from the CBT exercises, we then have information on the rank, monthly per capita consumption, and sociodemographic characteristics (e.g., household demographics, dwelling characteristics, and assets) of each household. We split this sample equally into our testing and training samples. For our auxiliary sample, we use data from the 2007 round of the Nouna Household Survey (654 households), which is the same sample that Hillebrecht et al. (2020b) used to calibrate their PMT model. The reader is referred to their paper for more detailed discussion.

4.2 Indonesia

Alatas et al. (2012) conducted a targeting experiment in three provinces of Indonesia – North Sumatra, South Sulawesi, and Central Java – that were selected to be geographically and ethnically representative of the country\textsuperscript{13}. The researchers randomly selected 640 villages across the three provinces and then selected one subvillage in each to participate in the experiment. The subvillages contained 54 households on average. Subvillages were then randomly assigned to three “treatment” arms, each of which entailed a different procedure for selecting beneficiaries to receive a one-time cash payment of Rp. 30,000 (approximately

\textsuperscript{13}The data is publicly available on the Harvard Dataverse (Alatas et al. 2013).
$3 and roughly equal to the daily wage for the average laborer). While the beneficiary quota was different for each subvillage, on average about 30 percent of households were selected.

The treatments assigned to the subvillages included PMT, CBT, and a hybrid approach where CBT was used to select 1.5 times the quota of beneficiaries and then the list was narrowed using PMT. Both the CBT and hybrid subvillages thus conducted full community ranking exercises where residents met to rank households from “poorest” to “most well-off.” In these exercises, a facilitator presented community members with a randomly ordered set of index cards, each of which displayed the name of one household. Through public discussions, the community members then ordered the households by placing the index cards on a string hung from a wall. The lowest-ranking households were then selected to receive benefits according to the predetermined quota for each subvillage.

Prior to the targeting exercises in late 2008, an independent survey firm gathered data on a number of households from each subvillage, including the subvillage head and a random sample of eight households. The sample consists of a total of 5,756 households, of which we use 5,004 due to missing information for some variables. The targeting exercises were conducted shortly after the data were collected and the resulting household ranks were recorded for each household. In addition to the rank for each household, we observe monthly expenditure per capita (Rp. 1000s) and rich sociodemographic information, including household demographics, dwelling characteristics, and assets. For additional details on the dataset, the reader is referred to Alatas et al. (2012).

Regarding splitting the dataset, we take all households that conducted community ranking exercises – namely, the 3,375 households from the CBT and hybrid communities – and split them equally into training and testing samples. Note that our interest is in predicting community rankings, so for the hybrid communities we discard the rankings generated by the hybrid procedure and focus on the underlying rankings from the community targeting exercises. Finally, for the auxiliary sample, we use the households assigned to the PMT treatment (1,629 households). The subvillages assigned to the PMT treatment did not conduct community ranking exercises, so we do not have the necessary information on these households to include them into either the testing or training samples.

5 Results

In this section, we present the results from our analysis of the data from Burkina Faso and Indonesia. We first present some baseline results that focus on comparing our hybrid model

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14 All data and code used for our empirical illustration can be found at [https://github.com/LendieFollett/Hybrid-Targeting](https://github.com/LendieFollett/Hybrid-Targeting)
to competing statistical targeting methods, namely PMT and the benchmark probit model. We then consider in more detail our various model extensions, including our generalization to multiple rankers, our adjustments for elite capture, the inclusion of auxiliary information, and dynamic updating.

5.1 Baseline results

We have argued that PMT neglects local preferences by imposing an income- or expenditure-oriented definition of poverty on communities. To illustrate the implications of this issue, Figures 1 and 2 provide a representation of how our hybrid model weights sociodemographic characteristics relative to PMT. For the Burkinabé data (Figure 1), we apply the hybrid model outlined in Section 3.1 because this dataset includes information from multiple rankers. For the Indonesian data (Figure 2), we use our most basic hybrid model outlined in Section 2.2. We use all available observations to estimate each model and the included covariates are taken directly from the specifications used in Hillebrecht et al. (2020b) and Alatas et al. (2012) for the Burkinabé and Indonesian data, respectively.15

To compare coefficients across the hybrid and PMT models, we standardize the estimated coefficients by dividing each by the average of the absolute value of the coefficients from each model. With this standardization, the sign of each effect reflects the sign of the associated coefficient whereas the magnitude captures the average marginal rate of substitution (MRS). The average MRS tells us how many units, on average, each covariate must change to compensate for a one-unit increase in the covariate of interest while holding the outcome constant. We view the average MRS as a measure of variable importance, reflecting the relative sensitivity of the model to a given covariate. This interpretation requires that all covariates are placed on a similar scale and we accomplish this by dividing all continuous variables by two times their standard deviation (Gelman, 2008). See Appendix B for detailed discussion.

Figures 1 and 2 show that communities implicitly weight sociodemographic characteristics quite differently from PMT. Regarding dwelling characteristics, we see some stark contrasts, particularly with respect to roof type. For both Burkina Faso (“roof is concrete, metal, or tile”) and Indonesia (“concrete or corrugated roof”), we find that our model positively weights roofing type whereas PMT counterintuitively estimates negative weights. As dwelling characteristics are easily observable indicators of living standards, it is natural that

15More specifically, we use all available observations to estimate our hybrid model and then estimate the PMT model using the same sample. The sociodemographic characteristics used by Hillebrecht et al. (2020b) are listed in the repository provided to us by the authors. We have added the variable household size to their specification. The characteristics used by Alatas et al. (2012) are presented in their Table 11. For ease of interpretation, we omit a small number of quadratic and interaction terms from their specification.
communities give higher ranks or scores to households with higher-quality dwellings. In the case of roof type, PMT thus appears to be at odds with such preferences by prioritizing households with high-quality roofing, all else equal. We view this results as a fairly clear illustration of how PMT can be inconsistent with local preferences.

In terms of demographic characteristics, we find some pronounced differences in how communities weight education. For Burkina Faso, the hybrid model indicates that the average MRS for “any member with tertiary education” is 5.14, meaning that a one-unit increase in this variable must be compensated for by a relatively large 5.14 unit change in the other covariates on average. In contrast, PMT is associated with a relatively small negative effect of -0.22. We similarly find some notable disagreements related to advanced education in the Indonesian data where the effects on “highest education in HH is senior high or higher” also differ in sign across the two models. Regarding the other demographic variables, it is worth

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16Recall that those households with the lowest ranks or scores are selected for program inclusion.
highlighting the differing weights placed on marital status, namely “head is widowed” in the Burkinabé results and “head is married” in the Indonesian results. The PMT results here once again appear inconsistent with local preferences.

Figures 1 and 2 also show some notable differences in how communities weight asset variables. Most interestingly, we find that communities implicitly disagree with the PMT weights on some livestock-related variables, namely “owns bullock” in the Burkinabé data and “owns caribou/cow” in the Indonesian data. For example, in the Burkinabé data, the effect on “owns bullock” from the hybrid model is 1.49 whereas that from the PMT model is -0.05. While the effect magnitudes are quite different, it is worth emphasizing that PMT places a negative weight on livestock ownership in this case, which is surprising given that livestock are important for income-generating activities. The Indonesian data show a similar result. Taken together, the results from Figures 1 and 2 thus show that PMT conflicts with community targeting preferences, particularly with respect to important traits like roofing type, education, and livestock ownership.
Figure 3 presents results related to the out-of-sample predictive performance of our model where the estimand of interest is community preference rankings. We measure performance through exclusion and inclusion error rates (Coady et al., 2004b; Stoeffler et al., 2016; Brown et al., 2018). The exclusion error rate is defined as the share of the truly poor (i.e., the households actually selected by the communities) not selected for program participation by any given statistical targeting method. The inclusion error rate is defined as the share of households selected by a given targeting method that are not truly poor (i.e., those households not selected by the communities). We set the number of households selected by any method to be equal to the number of truly poor and this implies that the exclusion and inclusion error rates will be identical. We will thus simply refer to the error rate in what follows.

Panel (a) in Figure 3 uses the data from Burkina Faso and displays average error rates across 30 replications for varying numbers of communities sampled from the training data (see Section 4 for discussion of our training-test splits). That is, for a given number of training communities, we repeatedly sample communities from the training sample and for each replication we use the estimated model to calculate error rates in the test sample. We conduct this procedure for three models: our hybrid model for multiple rankers, the benchmark probit, and PMT. Panel (b) conducts the same exercise, but uses the data from Indonesia. Note that the Indonesian data uses a single ranking scheme, so here we use the

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17 More specifically, we set the number of households selected from any community to be equal to the number of truly poor households in that community. Following from the discussion in Section 4, we then select approximately 20 percent of households when using the data from Burkina Faso and 30 percent of households when using the Indonesian data.
basic hybrid model outlined in Section 2.2. Further note that we use different community numbers across the two applications because of the differing sample sizes.\footnote{Recall from Section 4 that the Burkinabé data contains roughly 11 households per community and the Indonesian data contains about nine households per community. A simple way to translate communities sampled into households sampled is then to multiply the number of communities sampled by 10.}

The expected error rate under purely random targeting is 80 percent for Burkina Faso and 70 percent for Indonesia.\footnote{Once again, the Burkinabé community ranking exercises selected the poorest 20 percent of households and the Indonesian exercises selected the poorest 30 percent. A purely random targeting method that selects the same proportion of households will achieve an expected error rate of one minus that proportion.} All the methods presented in Figure 3 thus outperform purely random targeting in all cases, at least on average. Given our previous results showing that communities implicitly weight sociodemographic traits differently from PMT, it is unsurprising that PMT performs relatively poorly across all sample sizes. While the simple probit model outperforms PMT in the vast majority of cases, it never outperforms our hybrid model. Recalling our discussion in Section 2.1, this is also an expected result because the probit model is subject to a number of limitations (e.g., loss of information by dichotomizing the community rankings). One conclusion from Figure 3 is then that, on average, our hybrid model outperforms random targeting, PMT, and the probit model for all sample sizes.

Consider in more detail the error rates achieved by the hybrid model. For Burkina Faso (Indonesia), we find error rates ranging from 0.53 to 0.41 (0.36 to 0.26) for the smallest and largest sample sizes, respectively. While we have seen that the hybrid model outperforms PMT in terms of predicting community rankings, we can also consider how well the hybrid model performs relative to PMT’s ability to predict household expenditures. To this end, we have conducted similar experiments with both datasets to calculate the error rates achieved by PMT in terms of predicting expenditures out of sample.\footnote{That is, for varying training sample sizes, we fit an OLS model of (log) expenditure per capita on all sociodemographic characteristics and then use the estimates to calculate out-of-sample error rates. A household’s true poverty status in this experiment is determined by their expenditure per capita rather than their rank from the community exercises. Given that these experiments are less computationally demanding, we use 1,000 replications for each sample size.} For Burkina Faso (Indonesia), we find that PMT achieves error rates ranging from 0.65 to 0.57 (0.40 to 0.30) for the smallest and largest sample sizes, respectively. The hybrid model thus also outperforms PMT on its own terms, with error rate differences of 12-16 percentage points for Burkina Faso and about four percentage points for Indonesia.

### 5.2 Model extensions

As discussed in Section 3.1, our model not only accommodates multiple rankers, but also aggregates alternative ranking schemes according to their relative precision or quality. While the data from Burkina Faso includes multiple rankers, there is no fundamental heterogeneity
across the rankers given that each community simply selected three informants to complete the ranking exercises. Other implementations of community ranking exercises, however, have featured distinct ranker types, which we might expect to differ systematically in terms of quality. Recall the exercises conducted in Niger that consisted of three different ranker types: local leaders, non-leader women, and a mixed group of non-leaders (Premand and Schnitzer, 2020). To show how our multiple-ranker model adjusts for ranker quality, we conduct a simple experiment with the Burkinabé data where we artificially create a non-informative ranker by randomly shuffling one ranking scheme from each community.

The results from this exercise are presented in Figure 4 and correspond to one run of the model using all available data. Panel (a) plots the posterior means of each ranker’s precision $\omega$ where higher values indicate higher-quality ranking schemes. Recalling that $\omega$ can take on three values – 0.5, 1, and 2 for low-quality, mediocre, and reliable rankers, respectively – we see that the precision for the shuffled ranker ($\omega_3$) clearly indicates low quality. Panel (b) illustrates the implications of this adjustment: For each community we calculate the rank correlation between the aggregated ranking generated by our model and the rankings of the non-informative ranker and one informative ranker. Plotting the correlations associated with the non-informative ranker on that for the informative ranker for each community, we see that the vast majority of points fall below the diagonal. Higher-quality rankers are thus more heavily weighted by our model.

Turning to our adjustments for elite capture, the Indonesian data includes a variable for elite connections. This binary variable takes on the value of one for a household if (1) any member of the household held a formal leadership position in the village, (2) at least two other households in the village identified the household as having a member that held a formal
or an informal leadership position, or (3) the household was connected by blood or marriage to any household meeting the first two conditions (Alatas et al., 2012). In accordance with Section 3.2, we augment our basic hybrid model with this elite connections variable and then set the coefficient to zero when calculating the targeting scores. To reiterate, the purpose of this procedure is to remove any confounding influence of elite connections from the targeting scores.

Estimating the model with all observations gives a standardized coefficient estimate on the elite connections variable of -0.04 with a 95 percent credible interval of -0.46 to 0.34. While the effect size is relatively small and imprecisely estimated, the negative sign suggests some tendency for connected households to be privileged in the community ranking exercises, all else equal. To see how the addition of the elite connections variable affects the

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21 This result contrasts with Alatas et al. (2012), who find that connected households are less likely to be treated (see their Table 7). The primary reason for our differing results is that we use the rankings as our outcome variable whereas Alatas et al. use a dichotomized ranking variable. Specifically, we find
coefficients on the remaining covariates. Figure 5 plots the standardized coefficients from the hybrid model with ("Hybrid-EC") and without ("Hybrid") the elite capture correction. Note the subtle differences regarding the occupational variables, namely those related to whether the household head works in the industrial, service, or agricultural sector. We find that omitting the elite connections variable imparts a downward bias on these coefficients, presumably because connected households are more likely to be employed.

While there are evidently some coefficient changes after introducing the elite connections variable, there are few sign changes and the effect sizes remain similar for most covariates. This suggests that the out-of-sample predictive performance of the two models should be similar. Figure 6 displays average error rates across 30 replications, once again varying the number of communities sampled from the training data. In comparing the hybrid model with and without the elite capture correction, we find the expected result that the average error rates are similar across the two models. Note, however, that the corrected model performs slightly worse than the uncorrected model for some sample sizes. This is also an expected result given that the corrected model is constrained by setting to zero the coefficient on the elite connections variable.

Figure 7 presents results from the inclusion of auxiliary information into our hybrid model (see Section 3.3 for discussion). Panel (a) presents the results from the Burkinabé data where the auxiliary component of the model is estimated using data from the 2007 Nouna Household Survey (654 households). Panel (b) shows the results using the Indonesian data where the a qualitatively similar result to Alatas et al. when we regress a dichotomized ranking variable on elite connections (and all other covariates) using our sample. This suggests that the treatment of the outcome variable explains the differing results.

The relationship between elite connections and employment in any sector follows from the fact that we find a negative relationship between elite connections and a household’s ranking. That is, the relationship between elite connections and employment in any sector must be positive to impart a downward bias on the coefficients.
auxiliary component of the model is estimated using data from those households randomly assigned to the PMT treatment (1,629 households). For both countries, we then examine the out-of-sample predictive performance of this model (“Hybrid-AI”) relative to our basic hybrid model (“Hybrid”) for alternative training sample sizes. We find that including auxiliary information leads to non-negligible improvements in error rates for both countries, particularly for the smallest training sample sizes. For example, for Burkina Faso, with five communities in the training sample, we see that the error rate is reduced by roughly seven percentage points.

Finally, Figure 8 presents results associated with the dynamic updating procedure discussed in Section 3.4. We focus on the data from Burkina Faso because it is longitudinal. Similar to the previous exercises, we examine the predictive performance of the dynamically-updated model (“Hybrid-DU”) relative to the basic hybrid model (“Hybrid”) for alternative training sample sizes. In these simulation exercises, the basic hybrid model uses default priors whereas the dynamically-updated model uses priors derived from the previous period’s posterior distribution. Recalling that the Burkinabé training and testing samples correspond to the year 2009, the priors for the dynamically-updated model are thus derived from the posterior estimated using 2007 data. That is, we use the auxiliary sample (654 households) to inform the priors used in the updating procedure.

\[23\] Note that the sample size for the auxiliary model remains constant across all experiments. We once again use 30 replications for each experiment.

\[24\] Recall that the auxiliary model informs $\delta$ indirectly through informing $\mu$ and $\Sigma$. When the training sample size is small, the auxiliary model is relatively more informative because the auxiliary sample size is fixed. As the training sample size grows, the auxiliary model becomes relatively less informative and the error rates from the model with auxiliary information eventually converge to those of the basic hybrid model.

\[25\] Each run of the model uses the same priors estimated from the full auxiliary sample (i.e., the auxiliary sample size does not change as we vary the training sample size). Even though these experiments and the auxiliary information experiments both draw on the auxiliary sample, they use the auxiliary sample in
Figure 8: Out-of-sample predictive performance of hybrid model with dynamic updating.

Figure 8 shows that dynamic updating can lead to notable improvements in error rates, especially when the training sample size is small. For example, with five communities in the training sample, we find that dynamic updating reduces the average error rate by about 12 percentage points. As the training sample size increases, the two models converge in terms of targeting performance because the influence of the priors diminishes as the sample size grows. Note that these results suggest that dynamic updating can be helpful for economizing on data collection costs. Specifically, the dynamically-updated model applied to the smallest sample size performs comparably to the basic model applied to the largest sample size. Relative to “memoryless” updating procedures, dynamic updating thus allows social assistance programs to recertify targeting algorithms using a fraction of the sample size without sacrificing targeting performance.

6 Conclusions

PMT and CBT are two of the leading methods for targeting social assistance in developing countries. While PMT relies only on verifiable information and imposes limited costs on potential beneficiaries, it is a highly centralized form of targeting that neglects local information and preferences. In contrast, CBT emphasizes local information and preferences by decentralizing the targeting process, but as a consequence it can be costly to potential beneficiaries and subject to elite capture. We have thus proposed a hybrid targeting framework that incorporates many of the advantages of PMT and CBT while minimizing their main limitations. Our method resembles PMT in that beneficiary selection is based on a weighted sum of verifiable sociodemographic features. We nevertheless incorporate local information different ways. Most notably, the auxiliary information experiments focus on household expenditure data in the auxiliary sample whereas the dynamic updating experiments use the household ranking data.
and preferences via weights that reflect the implied rate at which potential beneficiaries themselves substitute sociodemographic features when making targeting decisions.

There are a few features of our method worth reiterating. First, the method only requires community ranking data from a sample of eligible communities, thereby reducing the costs imposed on potential beneficiaries relative to CBT. Second, we have extended our basic model to adjust for elite capture by explicitly modeling the influence of elite connections on community rankings and purging these influences from the beneficiary selection process. We have further extended the model to accommodate multiple rankers per community, auxiliary information, and dynamic updating. Finally, we have relied heavily on Bayesian methods, which facilitate the estimation of Thurstone-type models and enable many of our model extensions (e.g., dynamic updating). Our Bayesian framework additionally provides a simple way to regularize the model’s coefficients to improve out-of-sample predictions.

We have illustrated our method using data from Burkina Faso and Indonesia. The estimates from our hybrid model show that communities implicitly weight sociodemographic characteristics quite differently than PMT, with notable differences in sign and magnitude on features like roofing type, advanced education, and livestock ownership. We also found that our method performs well in terms of predicting community preference rankings out of sample. Specifically, we found that our most basic model outperforms all benchmark models and reaches error rates as low as 41 and 26 percent for the Burkinabé and Indonesian data, respectively. These error rates are lower than what the standard PMT can achieve when predicting household expenditures out of sample. Lastly, we demonstrated that further error rate reductions are possible when augmenting the basic model with auxiliary information or dynamically-updated priors, especially for smaller sample sizes.

Our results provide some guidance on the sample sizes necessary for applications of our method. The Indonesian data allowed us to consider a wider range of sample sizes and we found that error rates stabilize at around 100 communities in that dataset (see Figures 3 and 7). Given that the Indonesian data contains roughly nine households per community, this suggests that a sample size of about 900 households is sufficient for the initial implementation of the method. If samples of this size are not feasible for the initial implementation (e.g., due to budget limitations), we recommend using the model with auxiliary information to improve targeting performance at smaller sample sizes (see Figure 7). For repeated implementations, we recommend using our dynamic updating procedure, which can help economize on data collection costs in subsequent rounds. In particular, our results using the data from Burkina Faso suggest that 15 communities or about 165 households is sufficient to recertify the targeting procedure (see Figure 8).26

26Recall that the data from Burkina Faso contains roughly 11 households per community.
We conclude by discussing a few limitations of our method. First, as is often the case with Bayesian methods, we use MCMC techniques to sample from the posterior, which can be computationally demanding and may require fine-tuning to achieve convergence. Second, our method has some unique data needs, at a minimum requiring both (1) data on community preference rankings for a sample of communities and (2) a census capturing sociodemographic information for all eligible households. It is nevertheless unclear whether our method is more or less costly than PMT or CBT, as a detailed understanding of relative costs will depend on the context and specifics of the implementation. Finally, we have relied on linear functional forms in all the models presented in this paper, which likely restricts the predictive performance of our method. In future work, we hope to extend the model to accommodate more flexible functional forms (e.g., regression trees).

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27 We are specifically referring to total costs, including administrative costs, data collection costs, and the costs imposed on potential beneficiaries. Recall that our method has one particular cost advantage, which is that it will reduce the costs imposed on potential beneficiaries relative to CBT.
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Appendix A

This appendix presents the computational details needed to implement our hybrid model. We first outline an MCMC procedure for sampling from the joint posterior of our most general model specification (i.e., the model that accommodates multiple rankers and auxiliary information). This initial presentation uses default priors that can be used for any implementation of the method. We then detail our dynamic updating procedure and show how the posterior from any given implementation can be used to inform the priors for subsequent implementations.

MCMC specifics

The joint posterior distribution for the most general form of our hybrid model can be written as follows:

\[
P(\tilde{z}, \alpha, \delta, \omega, \gamma, \sigma^2, \psi, \mu, \Sigma | z, y) \\
\propto P(z | \tilde{z}) P(y | \gamma, \sigma^2) P(\tilde{z} | \alpha, \delta, \omega) P(\alpha) P(\gamma, \delta | \mu, \Sigma) P(\mu) P(\Sigma) P(\omega) P(\sigma^2) \\
\approx \left\{ \prod_{k,r} \mathbb{I}(\text{rank}(\tilde{z}_k^r) = z_k^r) \right\} \left\{ \prod_{i,m} N(y_{im} | x_{im} \gamma, \sigma^2) \right\} \\
\times \left\{ \prod_{r,i,k} N(\tilde{z}_ik^r | \alpha_{ik} + x_{ik} \delta, \omega_r^{-1}) \right\} \left\{ \prod_{i,k} N(\alpha_{ik} | 0, 1) \right\} \\
\times N(\delta, \gamma | \mu, \Sigma) N(\mu | 0, 1) \text{Scale-inv-}\chi^2(\Sigma | 1, 1) \\
\times \left\{ \prod_r \text{Multinomial}(\omega_r | \alpha_r) \right\} \text{Scale-inv-}\chi^2(\sigma^2 | 1, 1).
\]

We cannot sample from the above joint posterior directly. We thus use an MCMC Gibbs sampler where we sequentially sample from each conditional posterior. For our applications, we run the algorithm for 4,000 iterations, but keep only the last \(B = 2,000\) iterations, discarding the initial burn-in samples. After initializing the parameters at reasonable starting values, we use Algorithm 1.

For each of the steps in Algorithm 1, the specified conditional posteriors are available in closed form. For several of the steps, it will be useful to represent the model specification in matrix form. Let \(X^K\) represent the matrix of sociodemographic characteristics that corresponds to the set of communities \(K\) conducting community ranking exercises. Further, let \(X^M\) represent the matrix of sociodemographic features that corresponds to the set of
Algorithm 1: Gibbs algorithm to sample approximately from \( P(\tilde{z}, \alpha, \delta, \omega, \gamma, \sigma^2_\psi, \mu, \Sigma \mid z, y) \)

For \( b \) in 1 to \( B \) do:

1. Draw \( \tilde{z}^b \sim P(\tilde{z} \mid z, \alpha^{b-1}, \delta^{b-1}, \omega^{b-1}) \)
2. Draw \( \alpha^b \sim P(\alpha \mid \tilde{z}^b, \delta^{b-1}, \omega^{b-1}) \)
3. Draw \( \delta^b \sim P(\delta \mid \tilde{z}^b, \alpha^b, \omega^{b-1}, \mu^{b-1}, \Sigma^{b-1}) \)
4. Draw \( \gamma^b \sim P(\gamma \mid y, \sigma^2_\psi, \mu^{b-1}, \Sigma^{b-1}) \)
5. Draw \( \omega^b \sim P(\omega \mid \tilde{z}^b, \alpha^b, \delta^b) \)
6. Draw \( \sigma^2_\psi^b \sim P(\sigma^2_\psi \mid y, \gamma^b) \)
7. Draw \( \mu^b \sim P(\mu \mid \delta^b, \gamma^b, \Sigma^{b-1}) \)
8. Draw \( \Sigma^b \sim P(\Sigma \mid \gamma^b, \delta^b, \mu^b) \)

end do.

Communities \( M \) in the auxiliary component of the model. We can then write:

\[
\tilde{z} = X^R \alpha + X^K \delta + \eta \\
y = \phi + X^M \gamma + \psi
\]

where \( z^r_k = \text{rank}(\tilde{z}^r_k) \), \( X^R \) is a random-effect design matrix, \( \eta \sim \text{MVN}(0, \Omega) \), and \( \psi \sim \text{N}(0, \sigma^2_\psi I) \). Below we detail each of the conditional posteriors.

**Sampling \( \tilde{z} \):** The conditional posterior distribution of \( \tilde{z} \) can be written as:

\[
P(\tilde{z} \mid \cdot) \propto P(z \mid \tilde{z}) P(\tilde{z} \mid \alpha, \delta, \omega) \\
\propto \left\{ \prod_{k,r} I(\text{rank}(\tilde{z}^r_k) = z^r_k) \right\} \left\{ \prod_{r,i,k} N(\tilde{z}^r_{ik} \mid \alpha_{ik} + x_{ik} \delta, \omega_{\cdot k}^{-1}) \right\}.
\]

The above implies a truncated normal conditional posterior for each element of \( \tilde{z} \), where the bounds of the truncated normal are such that the original rank dictated by \( z \) is preserved. Elements of \( \tilde{z}^r_{ik} \) are drawn from their conditional posteriors sequentially, starting with the lowest ranked household within a community by ranker \( r \) and ending with the highest ranked household.

We represent the vector of ranks from ranker \( r \) in community \( k \) as \( z^r_k = [z^r_{i_1k} \leq z^r_{i_2k} \leq \ldots] \), where \( i_1 \) represents the index of the lowest-ranked household within community \( k \), \( i_2 \) is the second lowest-ranked household, and so on. We then draw for \( h = 1, 2, 3, \ldots \) the following:

\[
\tilde{z}^r_{ih,k} \mid z, \tilde{z}^r_{ih-1,k}, \alpha, \delta \sim N(\alpha_{ik} + x_{ik} \delta, \omega_{k}^{-1}, \tilde{z}^r_{ih-1,k}, \tilde{z}^r_{ih+1,k})
\]

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where here \( N(\cdot, \cdot, \cdot) \) denotes a truncated normal distribution where the first two parameters represent the mean and variance, respectively, and the last two parameters represent the lower and upper bound, respectively.

**Sampling** \( \alpha \): The conditional posterior distribution of \( \alpha \) can be written as:

\[
P(\alpha | \tilde{z}, \delta, \omega) \propto P(\tilde{z} | \alpha, \delta, \omega) P(\alpha) = \left\{ \prod_{r,i,k} N(\tilde{z}_{ik}^r | \alpha_{ik} + x_{ik}\delta, \omega^{-1}_r) \right\} \left\{ \prod_{i,k} N(\alpha_{ik} | 0, 1) \right\}.
\]

We can then draw from the conditional posterior as follows:

\[\alpha | \tilde{z}, \delta, \omega \sim N(\overline{m}_\alpha, \overline{V}_\alpha)\]

where

\[\overline{V}_\alpha = [(X^R)^T \Omega^{-1} X^R + I]^{-1}\]

and

\[\overline{m}_\alpha = (\tilde{z} - X^K \delta)^T \Omega^{-1} X^K \overline{V}_\alpha\]

represent the covariance matrix and mean of the multivariate normal distribution, respectively.

**Sampling** \( \delta \): The conditional posterior distribution of \( \delta \) can be written as:

\[
P(\delta | \cdot) \propto P(\tilde{z} | \alpha, \delta, \omega) P(\delta | \mu, \Sigma) = \left\{ \prod_{r,i,k} N(\tilde{z}_{ik}^r | \alpha_{ik} + x_{ik}\delta, \omega^{-1}_r) \right\} N(\delta | \mu, \Sigma).
\]

We can then draw from the conditional posterior as follows:

\[\delta | \mu, \Sigma, \tilde{z}, \omega \sim N(\overline{m}_\delta, \overline{V}_\delta)\]

where

\[\overline{V}_\delta = [(X^K)^T \Omega^{-1} X^K + \Sigma^{-1}]^{-1}\]
and

\[ \mathbf{m}_\delta = [(\tilde{z} - X^R \alpha)^T \Omega^{-1} X^K + \mu^T \Sigma^{-1}] V_\delta \]

represent the covariance matrix and mean of the multivariate normal distribution, respectively.

**Sampling \( \gamma \):** The conditional posterior distribution of \( \gamma \) can be written as:

\[
P(\gamma \mid \cdot) \propto P(y \mid \gamma, \sigma^2_\psi) P(\gamma \mid \mu, \Sigma) = \left\{ \prod_{i,m} N(y_{im} \mid x_{im} \gamma, \sigma^2_\psi) \right\} N(\gamma \mid \mu, \Sigma).
\]

We draw from the conditional posterior as follows:

\[
\gamma \mid \mu, \Sigma, \tilde{z}, \omega \sim N(\mathbf{m}_\gamma, \mathbf{V}_\gamma)
\]

where

\[
\mathbf{V}_\gamma = \left[ (X^M)^T \left( \frac{1}{\sigma^2_\psi} I \right) X^M + \Sigma^{-1} \right]^{-1}
\]

and

\[
\mathbf{m}_\gamma = \left[ y^T \left( \frac{1}{\sigma^2_\psi} I \right) X^M + \mu^T \Sigma^{-1} \right] V_\gamma
\]

represent the covariance matrix and mean of the normal distribution, respectively.

**Sampling \( \omega \):** In the case of heterogeneous rankers, where we model a distinct \( \omega_r \) for each ranker \( r \), the conditional posterior probability that \( \omega_r \) assumes the fixed value \( w_l \) can be written as:

\[
P(\omega_r = w_l \mid \cdot) \propto P(\omega_r) P(\tilde{z} \mid \alpha, \delta, \omega_r = w_l)
\]

\[
= a_{rl} \left\{ \prod_{i,k} N(\tilde{z}_{ik}^r \mid \alpha_{ik} + x_{ik} \delta, w_l^{-1}) \right\}
\]

where \( a_{rl} \) is the multinomial prior probability that ranker type \( r \) has a quality weight equal to \( w_l \) such that \( l = 1, 2, 3 \). Thus, the conditional posterior distribution of \( \omega_r \) is

\[
\omega_r \sim \text{Multinomial}(\bar{a}_{r1}, \bar{a}_{r2}, \bar{a}_{r3})
\]
where

\[ \bar{a}_{rl} = \frac{a_{rl} \prod_{i,k} N(\tilde{z}_{ik}^r | \alpha_{ik} + x_{ik}\delta, w_l^{-1})}{a_{r1} \prod_{i,k} N(\tilde{z}_{ik}^r | \alpha_{ik} + x_{ik}\delta, w_1^{-1}) + a_{r2} \prod_{i,k} N(\tilde{z}_{ik}^r | \alpha_{ik} + x_{ik}\delta, w_2^{-1}) + a_{r3} \prod_{i,k} N(\tilde{z}_{ik}^r | \alpha_{ik} + x_{ik}\delta, w_3^{-1})}. } \]

In our applications, we set \((w_1, w_2, w_3) = (0.5, 1, 2)\) and \((a_{r1}, a_{r2}, a_{r3}) = (1/3, 1/3, 1/3)\) for each ranker \(r\). In the next section, we provide an alternative specification based on the model with dynamic updating.

**Sampling \(\sigma_{\psi}^2\):** The conditional posterior distribution of \(\sigma_{\psi}^2\) can be written as:

\[
P(\sigma_{\psi}^2 | \cdot) \propto P(y | \gamma, \sigma_{\psi}^2) P(\sigma_{\psi}^2) = \left( \prod_{i,m} N(y_{im} | x_{im}\gamma, \sigma_{\psi}^2) \right) \text{Scale-inv-}\chi^2(\sigma_{\psi}^2 | 1, 1).
\]

We draw from the conditional posterior as follows:

\[
\sigma_{\psi}^2 | y, \gamma \sim \text{ Scale-inv-}\chi^2(\pi_{\sigma_{\psi}^2}, T_{\sigma_{\psi}^2}^2)
\]

where

\[
\pi_{\sigma_{\psi}^2} = 1 + \text{length}(y)
\]

and

\[
T_{\sigma_{\psi}^2}^2 = \sum_{im} (y_{im} - x_{im}\gamma)^2 + 1
\]

are the degrees of freedom and scale parameters, respectively.

**Sampling \(\mu\):** The conditional posterior distribution of \(\mu\) can be written as:

\[
P(\mu | \cdot) \propto P(\delta, \gamma | \mu, \Sigma) P(\mu) = N(\delta | \mu, \Sigma) N(\gamma | \mu, \Sigma) N(\mu | 0, 1).
\]

We can then draw from the conditional posterior as follows:

\[
\mu | \cdot \sim N(\bar{\mu}, V_{\mu})
\]
where
\[ \nabla_{\mu} = (2\Sigma^{-1} + I)^{-1} \]
and
\[ m_{\mu} = [(\delta + \gamma)^T \Sigma^{-1}] \nabla_{\mu} \]
denote the covariance matrix’ and mean of the normal distribution, respectively.

**Sampling \( \Sigma \):** In our formulation, \( \Sigma \) is a diagonal covariance matrix where each diagonal element \( \Sigma_{[p,p]} \) is the same. For simplicity, we refer to these diagonal elements as \( \Sigma \). The conditional posterior can be written as:
\[
P(\Sigma \mid \cdot) \propto P(\delta, \gamma \mid \mu, \Sigma) P(\Sigma) = N(\delta \mid \mu, \Sigma) N(\gamma \mid \mu, \Sigma) \text{ Scale-inv-}\chi^2(\Sigma \mid 1, 1).
\]

We can then directly sample from the following:
\[ \Sigma \mid \gamma, \delta, \mu \sim \text{Scale-inv-}\chi^2(\bar{\pi}_\Sigma, T^2_\Sigma) \]
where
\[ \bar{\pi}_\Sigma = 1 + \text{length}(\gamma) + \text{length}(\delta) \]
and
\[ T^2_\Sigma = \sum_p (\gamma_p - \mu_p)^2 + \sum_p (\delta_p - \mu_p)^2 + 1 \]
represent the degrees of freedom and scale parameters, respectively.

**Dynamic Updating**

We apply our dynamic updating procedure to the basic model outlined in Section 2. Though the joint posterior \( P(\theta \mid z_{1:t}) \) of this model is unavailable in closed form, we have posterior samples \( \{\theta^b : b = 1, \ldots, B\} \) that can be used to calculate the estimated posterior means \( \hat{E}(\theta \mid z_{1:t}) \), posterior variances \( \hat{V}(\theta \mid z_{1:t}) \), and other summaries. In dynamic updating, our goal is to use these types of posterior summaries to approximate \( P(\theta \mid z_{1:t}) \), thus mathematically representing our beliefs after having seen some data. One simplification that will aid in the
task of approximating a multivariate posterior distribution will be to assume independence. That is, we assume

\[ P(\delta, \omega) = P(\delta)P(\omega). \]

The above implies independence between parameters with different roles. We further assume independence within elements of vectorized parameters (e.g., \( P(\delta) = P(\delta_1)P(\delta_2) \cdots \)). In what follows, we present an approximation scheme for these parameters.

**Approximating** \( P(\delta | z_{1:t}) \): Elements of \( \delta \) are continuous and can be both positive and negative. Gelman et al. (2013) notes that for parameters for which the posterior distribution is unimodal and symmetric, the normal distribution can be a convenient approximation. Thus, for the \( p^{th} \) element of \( \delta \), a normal distribution with mean \( \hat{E}(\delta_p | z_{1:t}) \) and variance \( \hat{V}(\delta_p | z_{1:t}) \) would provide a reasonable approximation to the true \( P(\delta_p | z_{1:t}) \). That is, we may set

\[ \hat{P}(\delta_p | z_{1:t}) = N(\hat{m}_p, \hat{v}_p) \]

where

\[ \hat{m}_p = \frac{1}{B} \sum_{b=1}^{B} \delta_p^b \]

and

\[ \hat{v}_p = \frac{1}{B} \sum_{b=1}^{B} (\delta_p^b - \hat{m}_p)^2. \]

A less informative prior would maintain the same central moment \( \hat{E}(\delta | z_{1:t}) \) as the mean of the normal prior, but inflate the variance to something larger than \( \hat{V}(\delta | z_{1:t}) \) by multiplying by some constant greater than one.

**Approximating** \( P(\omega_r | z_{1:t}) \): The parameter \( \omega_r \) is discrete with a multinomial distribution. The hyperparameters \( a_{r1}, a_{r2}, \) and \( a_{r3} \) of the multinomial distribution represent the relative probabilities with which \( \omega_r \) takes on the possible values 0.5, 1, and 2. A reasonable approximation of \( P(\omega_r | z_{1:t}) \), then, might be

\[ \hat{P}(\omega_r | z_{1:t}) = \text{Multinomial}(a_1, a_2, a_3), \]
where

\[ a_1 = \frac{1}{B} \sum_{b=1}^{B} I(\omega_r^b = 0.5), \]

\[ a_2 = \frac{1}{B} \sum_{b=1}^{B} I(\omega_r^b = 1), \]

\[ a_3 = 1 - a_1 - a_2. \]

That is, we use the posterior samples to approximate the probability \( \omega_r \) takes on various values. A less informative prior would pull each of the probabilities \( a_1, a_2, \) and \( a_3 \) towards 1/3. Note that this is an especially important adjustment when \textit{a posteriori} \( \omega_r \) takes on a particular value with probability zero. For discretely-valued parameters, a prior mass of zero forces a posterior mass of zero, which may be more restrictive than necessary.
Appendix B

In this appendix, we detail our approach for comparing coefficients across models by calculating the average marginal rate of substitution (MRS) for each sociodemographic variable. For a linear model, the MRS for variable \( x_v \) with respect to variable \( x_p \) can be written as

\[
\left| \frac{\Delta x_v}{\Delta x_p} \right| = \left| \frac{\delta_p}{\delta_v} \right|
\]

where \( \delta_p \) and \( \delta_v \) represent the coefficients that correspond to \( x_p \) and \( x_v \), respectively. Rather than calculating the MRS for each pairwise comparison of covariates, we summarize the MRS for \( x_p \) by calculating the harmonic mean across all pairwise comparisons:

\[
\frac{\dim(x)}{\sum_v \left| \frac{\Delta x_p}{\Delta x_v} \right|} = \frac{\dim(x)}{\sum_v \frac{\delta_v}{\delta_p}}
\]

where \( \dim(x) \) represents the number of sociodemographic variables in the model.

Simplifying the right-hand side of the above equation, we then have

\[
\frac{\dim(x)}{\sum_v \frac{\delta_v}{\delta_p}} = \frac{\dim(x)}{\frac{1}{\delta_p} \sum_v |\delta_v|} = \frac{\dim(x)}{\frac{1}{\dim(x)} \sum_v |\delta_v|} = \frac{|\delta_p|}{\bar{\delta}}
\]

where \( \bar{\delta} \) represents the arithmetic mean of the absolute value of all coefficients in the model. Combining the previous equations, we can then write

\[
\frac{\dim(x)}{\sum_v \left| \frac{\Delta x_p}{\Delta x_v} \right|} = \frac{|\delta_p|}{\bar{\delta}}
\]

meaning that the (harmonic) mean marginal rate of substitution for a given variable \( x_p \) can be calculated by dividing the absolute value of the associated coefficient by the arithmetic mean of all coefficients. In terms of interpretation, the above tells us how many units, on average, each variable must change to compensate for a one-unit increase in \( x_p \) while holding the outcome variable constant.

Note that we summarize the MRS using the harmonic mean because the resulting calculations only require that we divide each coefficient by the arithmetic mean of all coefficients, which is computationally stable. An alternative approach would be to summarize the MRS using the arithmetic mean, which would require that each coefficient be divided by the harmonic mean of all coefficients. This alternative approach is computationally unstable in that it would require summing over the reciprocal of all coefficients, which is problematic if any
coefficient is near zero. While we could avoid averaging altogether by choosing one coefficient as the numeraire, the choice of coefficient would be arbitrary and we would not be able to interpret the MRS of the associated variable.

It is important to highlight a few additional details related to our calculations. First, recall that Bayesian methods provide a posterior distribution for all coefficients. As mentioned in Section 2.2, we summarize this distribution by using the posterior mean as our estimated coefficients. Second, we would like an understanding of the sign on each coefficient, so we simply ignore the absolute value function in the numerator and calculate $\delta_p / \bar{\delta}$. Finally, our calculations require that all variables are placed on a similar scale. To accomplish this, we follow Gelman (2008) and divide all continuous variables by two times their standard deviation. This follows from the observation that any binary variable with equal probabilities has a standard deviation of 0.5, meaning that a one-unit increase in any binary variable roughly corresponds to an increase of two standard deviations. The standardization of the continuous variables thus ensures that all variables are on roughly the same scale.