$|V_{ub}|$ determination by $B \to D_s \pi$

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Abstract

We investigate $\bar{B}^0 \to D_s^- \pi^+$ decay in perturbative QCD approach which has recently been applied to $B$ meson decays. $\bar{B}^0 \to D_s^- \pi^+$ decay (and its charge conjugated mode) can be one of the hopeful modes to determine $|V_{ub}|$ since it occurs through $b \to u$ transition only. We estimate both factorizable and non-factorizable contribution, and show that the non-factorizable contribution is much less than the factorizable one. Our calculation gives $\text{BR}(\bar{B}^0 \to D_s^- \pi^+) = (50 \sim 70) \times f_{D_s}^2 |V_{ub}V_{cs}|^2$.

Keywords: KM matrix; $V_{ub}$; non-leptonic $B$ decay; pQCD approach.

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The determination of all the elements of Kobayashi-Maskawa matrix $[1]$ is important for the consistency check of the standard model and a search for new physics. Much extensive experimental efforts have been done at $B$ meson dedicated facilities (B factories) to complete the determination of KM matrix elements to third generation. $|V_{ub}|$ has well been determined owing to heavy quark symmetry $[2]$ to the accuracy of less than 10% error. But $|V_{ub}|$ has not yet been determined so precisely $[3]$. The experimental error will be greatly reduced by the coming experiments in the very near future, while we need more effort to reduce the theoretical uncertainty. It is mainly due to hadronic effects which we do not yet have an precise method to calculate.

So far $b \to u$ semi-leptonic decays have been mainly used for the experimental determination of $|V_{ub}|$. It is interesting to investigate other modes involving non-leptonic decays to extract $|V_{ub}|$ for the consistency checks of the experimental value of $|V_{ub}|$ and the theoretical methods to estimate hadronic effects. More experimental information can be available and we can tune up the theoretical methods. We propose here $B^0 \to D_s^- \pi^+$ decay as one of good candidates to investigate. The final state $D_s$ meson is composed of $(s\bar{c})$ $(u\bar{d})$ quark state. The $b$ quark in $B^0$ meson cannot directly decay into $\bar{c}$ quark by $W^-$ emission as the color quantum number is different. Also no penguin, exchange nor annihilation contributions exist since no $q\bar{q}$ state presents in the final state. Therefore, the decay occurs through $b \to u$ transition only, which makes $B^0 \to D_s^- \pi^+$ a good mode to determine $|V_{ub}|$.

$B^0 \to D_s^- \pi^+$ and its charge conjugate mode are hopeful from experimental point of view also. Both $D_s$ and charged pion are relatively easy to be identified in the present experiments. Recently, BABAR and BELLE groups obtained the branching ratio, $\text{BR}(B^0 \to D_s^+ \pi^-) = 3.2 \pm 0.9 \pm 1.0 \times 10^{-5}$ (BABAR) $[4]$, 2.4 $^{+1.0}_{-1.0} \pm 0.7 \times 10^{-5}$ (BELLE) $[5]$. With increasing statistics at B factories we can expect that the branching ratio is fixed more precisely in the very near future.

The $B^0 \to D_s^- \pi^+$ decay occurs through the effective Hamiltonian,

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{ub} V_{cs}^* \left[ c_1(\bar{s}b)_{V-A}(\bar{u}c)_{V-A} + c_2(\bar{u}b)_{V-A}(\bar{s}c)_{V-A} \right] + \text{(h.c.)} \quad (1)$$

where $c_{1,2}$ are the Wilson coefficients obtained by solving renormalization group equations, and $O_{1,2}$ are 4-quark operators $[6]$. We need to estimate $\langle D_s^- \pi^+ | O_{1,2} | B^0 \rangle$ to obtain the branching ratio theoretically. So far, factorization ansatz $[7]$ has often been used to estimate this kind of 2-body decay hadron matrix elements;

$$A[B^0 \to D_s^- \pi^+] \simeq \frac{G_F}{\sqrt{2}} V_{ub} V_{cs}^* \left( c_2 + \frac{c_1}{N_{eff}} \right) \langle D_s^- (P_2) | (\bar{s}c)_{V-A} | 0 \rangle \langle \pi^+ (P_3) | (\bar{u}b)_{V-A} | B^0 (P_1) \rangle$$

$$= if_{D_s} \frac{G_F}{\sqrt{2}} V_{ub} V_{cs}^* \left( c_2 + \frac{c_1}{N_{eff}} \right) M^2_B F_0(M_{D_s}^2), \quad (3)$$
where $N_{\text{eff}}$ is the effective color number and $f_{Ds}$ is the $D_s$ meson decay constant. Pion mass is neglected. The $B \rightarrow \pi$ transition form factors are defined as follows;

$$
\langle \pi^+(P_3)|\bar{u}\gamma_\mu b|B^0(P_1)\rangle \equiv \left[(P_1 + P_3)_\mu - \left(\frac{M_B^2}{q^2}\right)q_\mu\right] F_1(q^2) + \left(\frac{M_B^2}{q^2}\right)q_\mu F_0(q^2),
$$

where $q = P_1 - P_3$. Form factors are to be obtained from another theory or experimental data. The deviation of $N_{\text{eff}}$ from the number of color, $N_C = 3$, accounts for so-called non-factorizable contributions. With the amplitude given by eq.(3) the $B^0 \rightarrow D^- \pi^+$ branching ratio is calculated as

$$
\text{BR}(B^0 \rightarrow D^- \pi^+) = \tau(B^0) \left(\frac{1 - r^2}{32\pi}\right) G_F M_B^3 \left(c_2 + \frac{c_1}{N_{\text{eff}}}\right)^2 F_0(M_{Ds})^2 f_{Ds}^2 |V_{ub}V_{cs}|^2,
$$

where $r \equiv M_{Ds}/M_B$. Below we first give estimations of the branching ratio based on the factorization ansatz by using the $B \rightarrow \pi$ transition form factor from light-cone sum rules and lattice QCD. Then we calculate $B^0 \rightarrow D^- \pi^+$ amplitude by using perturbative QCD (pQCD) approach to estimate non-factorizable contribution. pQCD approach has been applied to estimate pion electro-magnetic form factor, $B \rightarrow D$, $B \rightarrow \pi$ transition form factors and several decay amplitudes of 2-body decays of B meson ($D\pi$, $\pi\pi$, $K\pi$ and $K^*\gamma$) [8]. The results of pQCD approach nicely agree with experimental data. The advantage of pQCD approach lies in the point that the non-factorizable contribution can be calculated based on well-established perturbative QCD technique for heavy meson decays. We show that the non-factorizable contribution is about 10% or less of the factorizable one in $B \rightarrow D_s\pi$, so that naive factorization estimation gives a reasonable prediction for this decay mode.

In the evaluation of eq.(5) we consider the Wilson coefficients at two scales, $\mu = M_B$ and $M_B/2$, to estimate the ambiguity coming from the choice of the scale;

| $c_1(\mu)$ | $M_B$ | $M_B/2$ |
|------------|-------|--------|
| -0.274     |       | -0.393 |
| 1.12       |       | 1.19   |

The $B \rightarrow \pi$ transition form factor based on light-cone sum rules calculation can be parametrized as

$$
F_0(q^2) = \frac{F_0(0)}{1 - a_F(q^2/M_B^2) + b_F(q^2/M_B^2)^2},
$$

with $F_0(0) = 0.305$, $a_F = 0.266$ and $b_F = -0.752$ [10]. With the calculation by lattice QCD [11] we make a fit for $B \rightarrow \pi$ transition form factor as

$$
F_0(q^2) = \frac{F_0(0)}{1 - c_F(q^2/M_B^2)},
$$

3
with $F_0(0) = 0.310$, $c_F = 0.760$. The estimated value of $\text{BR}(\bar{B}^0 \rightarrow D_s^- \pi^+)/|f_{D_s}^2|V_{ub}V_{cs}|^2|$ is given in Table 1. By adopting $f_{D_s} = 0.241 \pm 0.032$ GeV obtained by taking average of several experimental data [12], we can summarize the naive factorization estimation as

$$\text{BR}(\bar{B}^0 \rightarrow D_s^- \pi^+)/|V_{ub}V_{cs}|^2 = \begin{cases} 2.0 \sim 4.0 & \text{(lattice } F_0) \\ 1.8 \sim 3.5 & \text{(sum rules } F_0) \end{cases}.\quad (8)$$

The ambiguity from the choice of $N_{eff}$ and $\mu$ is about 10% and 5%, respectively. The major ambiguity lies in $f_{D_s}$ value.

There are factorizable and non-factorizable contribution in $\bar{B}^0 \rightarrow D_s^- \pi^+$. When a gluon connects the spectator $\bar{d}$ quark and a quark in $D_s$ meson in Fig. 1, the contribution cannot be factorized as in eq. (3). The non-factorizable contribution is taken into account by changing the color number from $N_C = 3$ to $N_{eff}$ in the calculation based on the factorization ansatz. However, the number to be taken as $N_{eff}$ is theoretically unclear. We have to rely on fit of $N_{eff}$ by using experimental data other than $B \rightarrow D_s \pi$. But, we cannot simply adopt the $N_{eff}$ in other decays because the topology of diagrams contributing to other decays is not necessary same as in the case of $\bar{B}^0 \rightarrow D_s^- \pi^+$ decay. Here we calculate $\bar{B}^0 \rightarrow D_s^- \pi^+$ amplitude in pQCD approach [8] to estimate the non-factorizable contribution based on QCD.

Some of representative diagrams contributing to $\bar{B}^0 \rightarrow D_s^- \pi^+$ in pQCD approach is shown in Fig. 2. The point is that in 2 body decays of $B$ meson the spectator quark has

| $N_{eff}$ | $\mu = M_B$ | $\mu = M_B$ | $\mu = M_B/2$ | $\mu = M_B/2$ |
|---|---|---|---|---|
| lattice $F_0$ | 51.1 | 46.7 | 53.9 | 47.4 |
| sum rules $F_0$ | 44.0 | 40.1 | 46.4 | 40.8 |

Table 1: $\text{BR}(\bar{B}^0 \rightarrow D_s^- \pi^+)/|f_{D_s}^2|V_{ub}V_{cs}|^2$.
Figure 2: Some diagrams contributing to $\bar{B}^0 \rightarrow D_s^- \pi^+$ in pQCD. Gluon is shown in dashed line.

Figure 3: Momentum flow in the diagrams contributing to $\bar{B}^0 \rightarrow D_s^- \pi^+$ in pQCD.
to obtain high momentum to form a meson with one of the emitted rapid quarks from b quark decay. The high momentum is carried by a gluon, so the perturbative QCD treatment is possible. The non-perturbative nature is put into meson wave functions. For more details refer the papers in [8]. There is another approach of calculating non-leptonic 2-body decays of B meson by using perturbative QCD, the so-called QCD factorization method [9]. One of the major differences between pQCD approach and QCD factorization lies in the treatment of the transverse momentum. It is taken into account in the pQCD approach while neglected in the QCD factorization. Without the transverse momentum the diagrams given in fig.2 have infrared singularities which makes the results of the calculation implausible. The calculation based on pQCD approach can be done in a similar way to calculate $B \to D\pi$ amplitude given in Li and Melić [8]. The assignment of momenta is shown in Fig.3 where $P^\pm = \max (|0\sqrt{r_x}|, |0\sqrt{r_y}|)$ and $P^\pm = \max (|0\sqrt{r_x}|, |0\sqrt{r_y}|)$ and $k_1 = (0, x_1(M_B/\sqrt{2}), k_{1T})$, $k_2 = (0, x_2(M_B/\sqrt{2}), k_{2T})$, $k_3 = (x_3(1-r^2), (M_B/\sqrt{2}), 0, k_{3T})$ in light-cone coordinate. We obtain

$$\Gamma[B^0 \to D_s^- \pi^+] = \frac{G_F^2}{128\pi} |V_{ub}V_{cs}|^2 M_B^3 \frac{(1-r^2)^3}{r} |f_{Ds}\xi_{int} + M_{int}|^2. \quad (9)$$

The factorizable contribution, $f_{Ds}\xi_{int}$, is given as

$$\xi_{int} = 16\pi C_F \sqrt{r} M_B^2 \int_0^1 dx_1 dx_3 \int_0^{1/\Lambda} b_1 db_1 b_3 db_3 \phi_B(x_1, b_1) \times \alpha_s(t_{int}) \left( c_2(t_{int}) + \frac{c_1(t_{int})}{N_C} \right) \exp[-S_B(t_{int}) - S_\pi(t_{int})] \times \left[ (1 + x_3(1-r^2))\phi_A(x_3) + r_0 \left\{ \frac{1 + r^2}{1 - r^2} - 2x_3 \right\}\phi_P(x_3) 
- r_0(2x_3 - 1)\phi_T(x_3) h(x_1, x_3, b_1, b_3, m_{int}) 
+ [x_1 r^2 \phi_A(x_3) + 2r_0(1 - \frac{r^2}{1 - r^2}) x_1 \phi_P(x_3)] h(x_1, x_3, b_1, b_3, m_{int}) \right], \quad (10)$$

where $\exp[-S_B(t_{int}) - S_\pi(t_{int})]$ is Sudakov factor [8], $m_{int} = (1 - r^2)M_B^2$, $\Lambda = \Lambda_{QCD}$ and $t_{int} = \max(\sqrt{x_1 m_{int}}, \sqrt{x_3 m_{int}}, 1/b_1, 1/b_3)$. The parameter $b_i (i = 1 \sim 3)$ is the conjugate variable of $k_{iT}$. The function $h(x_1, x_3, b_1, b_3, m)$ is defined as

$$h(x_1, x_3, b_1, b_3, m) = S_t(x_3) K_0(\sqrt{x_1 x_3 m b_1}) \times \left[ \theta(b_1 - b_3) K_0(\sqrt{x_3 m b_1}) I_0(\sqrt{x_3 m b_3}) + \theta(b_3 - b_1) K_0(\sqrt{x_3 m b_3}) I_0(\sqrt{x_3 m b_1}) \right], \quad (11)$$

where

$$S_t(x) = \frac{2^{1+c} \Gamma(c+3/2)}{\sqrt{x} \Gamma(c+1)} [x(1-x)]^c \quad (c = 0.3 \sim 0.4), \quad (12)$$
which comes from threshold resummation\[13\]. The non-factorizable contribution is given as

\[
\mathcal{M}_{\text{int}} = -32\pi \sqrt{2N_C}C_F \sqrt{r} M_B^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_{Ds}(x_2, b_2) \\
\times \alpha_s(t_d) \frac{c_1(t_d)}{N_C} \exp[-S(t_d)|_{b_3=b_1}] \\
\times \left[ [(x_1 - x_2)(1 + r^2)\phi_A(x_3) + r_0 \{ x_3 - \frac{(x_1 - x_2)r^2}{(1 - r^2)} \} \phi_P(x_3) \\
+ r_0 \{ x_3 + \frac{(x_1 - x_2)r^2}{(1 - r^2)} \} \phi_P(x_3) \right] S_{\text{int}}^{(1)} h_d^{(1)}(x_i, b_i) \\
+ \left[ \{ 1 - x_1 - x_2 + (1 - r^2)x_3 \} \phi_A(x_3) - r_0 \{ x_3 - \frac{(2 + x_1 + x_2)r^2}{(1 - r^2)} \} \phi_P(x_3) \right] S_{\text{int}}^{(2)} h_d^{(2)}(x_i, b_i), \tag{13}
\]

where \(N_C = 3, t_d = \max(DM_B, \sqrt{|D_1^2|}M_B, \sqrt{|D_2^2|}M_B, 1/b_1, 1/b_2),\)

\[
D^2 = x_1 x_3(1 - r^2), \\
D_1^2 = (x_1 - x_2)x_3(1 - r^2), \\
D_2^2 = (x_1 + x_2)r^2 - (1 - x_1 - x_2)x_3(1 - r^2). \tag{14}
\]

The functions \(h_d^{(1)}\) and \(h_d^{(2)}\) are defined as

\[
h_d^{(j)} = \left[ \frac{\theta(b_1 - b_2)}{2} K_0(DM_B b_1) I_0(DM_B b_2) \\
+ \frac{\theta(b_2 - b_1)}{2} K_0(DM_B b_2) I_0(DM_B b_1) \right] \\
\times \left( \frac{K_0(D_j M_B b_2)}{\sqrt{|D_j^2|} M_B b_2} \right) \text{ for } D_j^2 \geq 0 \\
\times \left( \frac{i\pi H_0^{(1)}(\sqrt{|D_j^2|} M_B b_2)}{\sqrt{|D_j^2|} M_B b_2} \right) \text{ for } D_j^2 < 0. \tag{15}
\]

The factor \(S_{\text{int}}\) accounts for threshold resummation in non-factorizable contribution investigated in the work by Li and Ukai\[14\]. The effects of this factor shall be discussed later.

We have included twist 3 component into wave functions;

\[
B(P) = [P + M_B] \gamma_5 \phi_B(x), \tag{16}
\]

\[
D_s(P) = \gamma_5 [P + M_{Ds}] \phi_{Ds}(x), \tag{17}
\]

\[
\pi^+(P) = \gamma_5 [P \phi_A(x) + m_0 \phi_P(x) - m_0(\not{P} \not{\mu} - v \cdot n) \phi_T(x)], \tag{18}
\]

where \(m_0 \equiv M_n^2/(m_u + m_d), r_0 \equiv m_0/M_B, v = (1, 0, 0_T)\) and \(n = (0, 1, 0_T)\) in light-cone coordinate. We have adopted the following functions for \(B\) meson and pion.
\begin{align}
\phi_B(x, b) &= N_B x^2 (1 - x)^2 \exp \left[ -\frac{1}{2} \left( \frac{x M_B}{\omega_B} \right)^2 - \frac{1}{2} (\omega'_B b)^2 \right], \\
\phi_A(x) &= \frac{3 f_\pi}{\sqrt{2N_C}} x (1 - x) [1 + a_2 C_2^{3/2} (1 - 2x) + a_4 C_4^{3/2} (1 - 2x)], \\
\phi_P(x) &= \frac{f_\pi}{2 \sqrt{2N_C}} [1 + a_{2p} C_2^{1/2} (1 - 2x) + a_{4p} C_4^{1/2} (1 - 2x)], \\
\phi_T(x) &= \frac{f_\pi}{2 \sqrt{2N_C}} (1 - 2x) [1 + 6a_{2t} (10x^2 - 10x + 1)].
\end{align}

where \(N_B\) is normalization constant to have \(\int_0^1 \phi_B(x, 0) dx = f_B/2\sqrt{6}\), and \(C_j^k\) is a Gegenbauer polynomial: \(C_2^{1/2}(x) = (1/2)(3x^2 - 1)\), \(C_4^{1/2}(x) = (1/8)(35x^4 - 30x^2 + 3)\), \(C_3^{3/2}(x) = 3x\), \(C_2^{3/2}(x) = (3/2)(5x^2 - 1)\), \(C_4^{3/2}(x) = (15/8)(21x^4 - 14x^2 + 1)\). The parameters in the pion wave functions are given in ref.\[13\]:

\[a_2 = 0.44, \quad a_4 = 0.25, \quad a_{2p} = 30\eta_3, \quad a_{4p} = -30\eta_3\omega_3, \quad a_{2t} = 5\eta_3 - \frac{1}{2}\eta_3\omega_3,\]

with \(\eta_3 = 0.015\) and \(\omega_3 = -3.0\). As for \(D_s\) meson wave function we take:

\[\phi_{D_s} = N_D x (1 - x) [1 + \frac{c_d}{3} C_4^{3/2}(1 - 2x)].\]

By analyzing \(B \to \pi\) and \(B \to D\) form factors we have obtained a set of parameters consistent with experimental data and other theoretical predictions\[13, 16\];

\[\omega_B = \omega'_B = 0.4\] for \(B\) meson and \(c_d = 0.7\) for \(D\) meson.

The \(D_s\) meson wave function is thought to be similar to \(D\) meson wave function by \(SU(3)\) flavor symmetry. The parameter \(c_d\) are varied in the numerical analysis to see their effects;

\[c_d = 0.5, \quad 0.7, \quad 0.9.\]

The numerical results on the \(B^{0} \to D_s^- \pi^+\) branching ratio is given in Table 2 by taking \(c = 0.35\) and \(S_{int}^{(1,2)} = 1\), i.e. without the threshold resummation effect in non-factorizable contribution. The results show that the branching ratio of this mode is almost insensitive to the \(D_s\) meson wave function for a reasonable range of parameter \(c_d\)\[16\].

The effect of the threshold resummation is also investigated. The threshold resummation parameter \(c\) in eq.(12) for the factorizable contributions is varied within the range consistent with the \(B \to \pi\) form factor analysis\[13\]. As for the non-factorizable contribution we take \(S_{int}^{(1)} = S_t(x_3)\) and \(S_{int}^{(2)} = 1\) following the arguments given in ref.\[13\]. The parameter \(c\) is taken to be the same with that for factorizable contributions for the
Table 2: BR($B^0 \rightarrow D_s^- \pi^+)$/|$V_{ub}V_{cs}$|^2 and the ratio of non-factorizable contribution to factorizable one for different $D_s$ meson wave function parameter $c_d$. $f_{D_s}$ should be given in GeV.

| $c_d$ | 0.5 | 0.7 | 0.9 |
|-------|-----|-----|-----|
| $|\mathcal{M}_{\text{int}}|/f_{D_s}\xi_{\text{int}}$ | 0.093 | 0.088 | 0.082 |
| BR/($f^2_{D_s}|V_{ub}V_{cs}|^2$) | 60 | 59 | 59 |

Table 3: (a) BR($B^0 \rightarrow D_s^- \pi^+)$/|$V_{ub}V_{cs}$|^2 and the ratio of non-factorizable contribution to factorizable one for different threshold resummation parameter $c$ without threshold resummation in non-factorizable contribution. (b) same as (a) with threshold resummation in non-factorizable contribution.

| $c$ | 0.30 | 0.35 | 0.40 |
|-----|-----|-----|-----|
| $\mathcal{M}_{\text{int}}/f_{D_s}\xi_{\text{int}}$ | 0.079 $- 0.022i$ | 0.085 $- 0.024i$ | 0.090 $- 0.026i$ |
| BR/($f^2_{D_s}|V_{ub}V_{cs}|^2$) | 68 | 59 | 52 |
| $\mathcal{M}_{\text{int}}/f_{D_s}\xi_{\text{int}}$ | 0.069 $+ 0.018i$ | 0.074 $+ 0.025i$ | 0.077 $+ 0.031i$ |
| BR/($f^2_{D_s}|V_{ub}V_{cs}|^2$) | 66 | 58 | 51 |

Simplicity of the calculation. Two kinds of calculations have been made with or without threshold resummation factor in the non-factorizable contribution. The results are shown in Table 3. It is found that the branching ratio varies about 15% depending on the choice of the parameter $c$. But this parameter is just a numerical convenience to fit the true form of the following threshold resummation factor by eq.(12);

$$S_t(x) = \int_{a-i\infty}^{a+i\infty} \frac{dN}{2\pi i} \frac{J(N)}{N} (1 - x)^{-N},$$

where $a$ is an arbitrary real constant larger than all the real parts of the poles in the integrand and

$$J(N) = \exp \left[ \frac{1}{2} \int_0^1 dz \frac{1}{1 - z} \int_{1-z}^{(1-z)^2} \frac{d\lambda}{\lambda} \gamma_K(\alpha_s) \right]$$

with $\gamma_K(\alpha_s)$ being the anomalous dimensions[14, 16]. Thus the parameter $c$ is absent in the more careful (but complicated) treatment of numerical calculation where the above equations are used. So this ambiguity does not lead to a true theoretical error. The effect of the existence of the threshold resummation in non-factorizable contribution is found to be small in this decay. This is because non-factorizable contribution is small, 10% or less in amplitude, in comparison with the factorizable one in this decay mode. But the effects of the threshold resummation in non-factorizable part can be significant in another modes such as color-suppressed decay like $B^0 \rightarrow D^0 \pi^0$[18].
The prediction by pQCD approach is given considering the ambiguity discussed before as

\[
\text{BR} \left( \overline{B^0} \to D_s^- \pi^+ \right) = (50 \sim 70) \times f_{Ds}^2 |V_{ub}V_{cs}|^2 \quad (27) \\
= (2.4 \sim 4.6) \times |V_{ub}V_{cs}|^2. \quad (28)
\]

If we take the central values of parameters and the experimental data of \( B \to D_s \pi \) branching ratio given by BABAR and BELLE, we obtain,

\[
|V_{ub}| = (3 \pm 1) \times 10^{-3}, \quad (29)
\]

which is in good agreement with the value of \(|V_{ub}|\) obtained in \( b \to u \) semi-leptonic decay\[13\]. This agreement gives a support to our treatment of \( B \) meson decays in pQCD approach. It also implies that the naive factorization ansatz works well also in this decay mode\[17\] since our calculation has shown the dominance of factorizable contribution. The ambiguity will be reduced when \( f_{Ds} \) value is fixed more precisely in the future experiments. In the near future high statistics of \( B \) meson decay data will be available, then we can make pQCD prediction more precise by fitting parameters of the wave functions with rich experimental data. Then \(|V_{ub}|\) can be determined as precise as those from semi-leptonic decay by using the branching ratio of \( \overline{B^0} \to D_s^- \pi^+ \).

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