THz wave emission from intrinsic Josephson junctions controlled by surface impedance and in-plane magnetic field: Numerical study

Yoshihiko Nonomura
Computational Materials Science Unit, National Institute for Materials Science, Tsukuba, Ibaraki 305-0047, Japan
E-mail: nonomura.yoshihiko@nims.go.jp

Abstract. Motivated by recent experiments on THz wave emission from intrinsic Josephson junctions without or in in-plane magnetic field, effect of surface impedance \( Z \) is investigated numerically. Without external field, dynamical phase transitions between the McCumber-like state and various \( \pi \)-phase-kink states occur as the bias current and \( Z \) are varied. Emission is optimized for \( Z \approx 50 \) in the \( n = 1 \) mode. In this mode dependence of emission intensity on in-plane field shows a crossover behavior around \( Z = 50 \): monotonic decrease for \( Z > 50 \) and two characteristic peaks for \( Z < 50 \), each of which is consistent with a recent experiment.

1. Introduction
Over a decade of struggles to observe THz wave emission from intrinsic Josephson junctions in in-plane magnetic field, [1] evident emission was observed without external magnetic field. [2] Then, the uniform McCumber-like state [3] and the symmetry-breaking \( \pi \)-phase-kink states [4] were proposed as possible emission states. Recently emission in in-plane magnetic field was investigated again and there appeared two experiments reporting monotonic decrease of emission intensity [5] or some characteristic peaks in the field profile. [6] In the present study we show a unified viewpoint based on numerical simulations of a single model characterized by the surface impedance \( Z \). That is, as \( Z \) is varied, there occurs a dynamical phase transition between the above two types of emission states without external magnetic field, [7] and a dynamical crossover between the above two types of field profile is reproduced in in-plane magnetic field.

2. Model and implementation
In the present study the external field \( B_y \) (including \( B_y = 0 \)) is applied along the \( y \) axis, and Josephson vortices are driven by the dc bias current \( J \) along the \( c \) axis. Electromagnetic waves are emitted from both edges into the dielectrics. Thermal fluctuations are neglected, and Josephson vortices are straight along the \( y \) axis. Then, the system can be reduced in two dimensions with \( x \) and \( c \) axes described by the following differential equations, [8]

\[
\frac{\partial^2 \psi_{l+1,l}}{\partial x^2} = (1 - \zeta \Delta^{(2)}) \left( \partial_x E'_{l+1,l} + \beta E'_{l+1,l} + \sin \psi_{l+1,l} - J' \right), \quad \frac{\partial \psi_{l+1,l}}{\partial c} = (1 - \alpha \Delta^{(2)}) E'_{l+1,l}, \quad (1)
\]

where the superconducting layers are labeled by \( l \), and the electric field in the insulating layer and the gauge invariant phase difference between the \( l \)-th and \( (l+1) \)-th superconducting layers are...
chosen as basic quantities. The magnetic field in the insulating layer is obtained from \( \partial_x \psi_{l+1,l} = (1 - \zeta \Delta^{(2)}) B'_{l+1,l} \), with the operator \( \Delta^{(2)} \) defined in \( \Delta^{(2)} X_{l+1,l} \equiv X_{l+2,l+1} - 2 X_{l+1,l} + X_{l,l-1} \). In the above formulas the following scaled quantities in the MKSA unit are used:

\[
\begin{align*}
    x' &= \frac{x}{\lambda_c}, & t' &= \omega_p t, & E'_{l+1,l} &= \frac{\sigma_c}{\beta J_c} E_{l+1,l}, & B' &= \frac{2\pi \lambda_c d}{\phi_0} B, & J' &= J/J_c, \\
    \zeta &= \frac{\lambda_{ab}^2}{s d}, & \alpha &= \frac{\epsilon_c \mu^2}{s d}, & \beta &= \frac{\sqrt{\epsilon_c} \sigma_c \lambda_c}{\epsilon_c c}, & \omega_p &= \frac{c}{\sqrt{\epsilon_c} \lambda_c}, & J_c &= \frac{\phi_0}{2\pi \mu_0 \lambda_c^2 d},
\end{align*}
\]

with the penetration depths \( \lambda_{ab} = 0.4 \mu \text{m} \) and \( \lambda_c = 200 \mu \text{m} \), thickness of superconducting layers \( s = 3 \AA \) and that of insulating layers \( d = 12 \AA \), Debye length \( \mu = 0.6 \AA \), dielectric constant of the junction \( \epsilon' = \epsilon_c/\epsilon_0 = 10 \) with permittivity of the junction \( \epsilon_c \) and that of vacuum \( \epsilon_0 \), plasma frequency \( \omega_p \), conductivity \( \sigma_c \), critical current \( J_c \) and vacuum permeability \( \mu_0 \), following the BSCCO parameters. [8] They give \( \alpha = 0.1 \) and \( \beta = 0.02 \) is chosen here.

In spite of treating several hundreds of junctions in experiments, the periodic boundary condition along the c axis is introduced, which corresponds to infinite junctions. Simulated number of junctions in the present study is \( N = 4 \). Instead of considering dielectrics on edges, the dynamical boundary condition [9] is introduced. For infinite number of junctions, this boundary condition is simplified [10] as the relation between dynamical part of boundary fields:

\[
\begin{align*}
    \partial_x \psi_{l+1,l} &= B'_{\text{ext}}, & \partial_x \psi_{l+1,l} &= \langle E'_{l+1,l} \rangle + \tilde{E}'_{l+1,l} ; & \tilde{E}'_{l+1,l} &= \mp Z \tilde{E}'_{l+1,l}, & Z &= z \sqrt{\epsilon'_d/\epsilon'_{cl}},
\end{align*}
\]

with the surface impedance \( Z \). Typical values of the surface factor is \( z \approx \lambda/L_z \) with the wavelength of emitted electromagnetic wave \( \lambda \) and thickness of the sample \( L_z \), [11] and the dielectric constant of dielectrics \( \epsilon'_{cl} = 1 \) (for vacuum). The static part of electric field \( \langle E'_{l+1,l} \rangle \) is determined self-consistently, starting from a value lower than the actual one. This condition corresponds to the upward current sweep which results in strong emission, as will be shown later. Width of the junction \( L_x = 86 \mu \text{m} \) is divided into 80 numerical grids, and numerical calculations are performed on the basis of the RADAU5 ODE solver. [12]

3. Numerical results without external magnetic field

The dynamical phase diagram in the \( Z-J \) plane for \( 1 \leq Z \leq 10 \) without external magnetic field is drawn in Fig. 1. [7] Various phases observed in in-plane magnetic field for larger \( Z \)
Figure 2. Z dependence of maximum emission intensity in the $n = 1$ mode.

Figure 3. In-plane field dependence of the maximum value of emission intensity in the $n = 1$ mode for $Z = 10$. Field-induced dynamical phase transitions between the (a) $\pi$-phase-kink state, (b) incommensurate-phase-kink state and (c) in-phase state are displayed in typical snapshots of gauge-invariant phase difference in all junctions.

appear for lower $Z$. In the retrapping (R) state voltage does not appear for finite currents. In the McCumber-like (M) state the gauge-invariant phase difference shows completely in-phase motion as displayed in Fig. 1(a). In the $n \pi$-phase-kink ($K_n$) state the gauge-invariant phase difference in each junction has $n \pm \pi$ phase kinks as shown in Fig. 1(b) (for $n = 1$). In the $m$ incommensurate-phase-kink ($I_m$) state the gauge-invariant phase difference in each junction has $m$ fractional phase kinks as shown in Fig. 1(c) (for $m = 2$). Although a set of one $\pm 3\pi/2$ kink and three $\mp \pi/2$ kinks is the only possibility of the incommensurate-phase-kink state for $N = 4$, various sets of incommensurate phase kinks become possible for larger $N$. Without external magnetic field, such kinks appear only in plus-minus pair, and $m$ is always even. While only $K_n$ phases are stabilized for larger $Z$, emission at the cavity resonance point becomes stronger. The strongest emission is obtained for $Z \approx 50$ in the $n = 1$ mode as shown in Fig. 2.

4. Numerical results in in-plane magnetic field
In principle, purpose of simulations in in-plane magnetic field is to draw a three-dimensional version of the dynamical phase diagram given in Fig. 1 with an extra axis of the in-plane field. However, strong emission in the in-plane field takes place in a rather narrow range of current, and it is justified to display only the data for the largest current which gives the maximum emission. In Fig. 3, maximum value of emission intensity is plotted versus in-plane field in the $n = 1$ mode for $Z = 10$. Dips of intensity correspond to dynamical phase transitions between
Figure 4. In-plane field dependence of the maximum value of emission intensity in the $n = 1$ mode for $Z = 30, 50$ and 70. Vertical lines represent transition fields.

the (a) $\pi$-phase-kink, (b) incommensurate-phase-kink, and (c) in-phase states characterized by different configurations of the gauge-invariant phase difference as shown in Figs. 3(a)-(c).

Finally, $Z$ dependence of field profile of the maximum value of emission intensity is studied. In order to compare with controversial experiments explained in the introduction, parameter search around the optimal emission without external magnetic field is required as displayed in Fig. 4. For $Z = 30$ (circles), there exist a wide peak with sharp edges in lower field and a broad peak in higher field in addition to the zero-field peak. The latter broad peak is also observed for $Z = 70$ (triangles), the broad peak around $B_y = 0.015T$ does not exist anymore, and this field is still in the incommensurate-phase-kink region. The lower-field peak is broader and directly connected to the dip at the phase boundary between the $\pi$- and incommensurate-phase-kink states. Crossover between these two different field profiles occurs around $Z = 50$ (squares), where the higher-field peak just disappears and the shape of lower-field peak becomes complicated. When the number of junctions $N$ is increased, the onset field of the $\pi$-phase kink state is proportional to $N^{-1}$ and the emission dip is expected to cancel with the zero-field peak, and then experimental situations [5, 6] are reproduced. Details will be reported elsewhere.

Acknowledgement. This work was partially supported by Grant-in-Aids for Scientific Research (C) No. 20510121 from JSPS.

References
[1] For example, M.-H. Bae et al., Phys. Rev. Lett. 98, 027002 (2007).
[2] L. Ozyuzer et al., Science 318, 1291 (2007); K. Kadowaki et al., Physica C 468, 634 (2008).
[3] H. Matsumoto et al., Physica C 468, 654, 1899 (2008); T. Koyama et al., Phys. Rev. B 79, 104522 (2009).
[4] S. Lin and X. Hu, Phys. Rev. Lett. 100, 247006 (2008); A. E. Koshelev, Phys. Rev. B 78, 174509 (2008).
[5] U. Welp et al., on the APS March Meeting 2009, D34-1.
[6] K. Yamaki et al., Physica C 470, 5804 (2010).
[7] Y. Nonomura, Phys. Rev. B 80, 140506(R) (2009).
[8] M. Tachiki et al., Phys. Rev. B 71, 134515 (2005).
[9] L. N. Bulaevskii and A. E. Koshelev, Phys. Rev. Lett. 97, 267001 (2006).
[10] S.-Z. Lin et al., Phys. Rev. B 77, 014507 (2008).
[11] A. E. Koshelev and L. N. Bulaevskii, Phys. Rev. B 77, 014530 (2008).
[12] http://www.unige.ch/~hairer/software.html