Testing the hadro-quarkonium model on the lattice

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Motivation

**LHCb pentaquark candidates**

\[ P_c^+(4380) \ (J^P = \frac{3}{2}^-) \] and \[ P_c^+(4450) \ (J^P = \frac{5}{2}^+) \] from \( \Lambda_b \to J/\psi \ p \ K \)  
[ LHCb: R. Aaij et al, 1507.03414, 1604.05708 ]

Conjecture of attractive forces between charmonium and \( pp \) systems  
[ Brodsky, Schmidt and de Teramond, PRL64 (90) 1011 ]

Many interpretations
Hadroquarkonia

5 \( (4 q, 1 \bar{q}) \) quark systems are very difficult to study directly on the lattice

20 MeV binding energy for charmonium-nucleon system for \( m_\pi \approx 800 \) MeV [NPLQCD Collaboration: S. R. Beane et al, 1410.7069]

Here we test a particular model instead. \textbf{Hadro-quarkonia: quarkonia bound “within” ordinary hadrons} [S. Dubynskiy and M. Voloshin, 0803.2224]

Examples of close-by charmonium-baryon systems:
\( J^P = \frac{3}{2}^- \) : \( m(\Delta) + m(J/\psi) \approx 4329 \) MeV vs. 4380 MeV (width 200 MeV)
\( J^P = \frac{5}{2}^+ \) : \( m(N) + m(\chi_{c2}) \approx 4496 \) MeV vs. 4450 MeV (width 40 MeV)
The hadro-quarkonium model can be tested in the static limit. To leading order in (p)NRQCD, quarkonia can be approximated by the non-relativistic Schrödinger equation with a static potential $V_0(r)$. Does this become more or less attractive, when light hadrons are “added”?

Create a zero-momentum projected hadronic state $|H\rangle$ at the time 0.
Let it propagate to $\delta t$, create a quark-antiquark “string”.
Destroy this at $t + \delta t$ and the light hadron at $t + 2\delta t$: 

![Diagram showing the process of creating and destroying a hadronic state](image-url)
Correlation function

\( \overline{Q}Q \) binding energy shift “within” a hadron \( H \)

We compute

\[
C_H(r, \delta t, t) = \frac{\langle W(r, t)C_{H,2pt}(t + 2\delta t) \rangle}{\langle W(r, t) \rangle \langle C_{H,2pt}(t + 2\delta t) \rangle}
\]

where we average over the spatial positions of the Wilson loop \( W(r, t) \) and over the hadronic sink positions.

The shift of the binding potential is obtained from

\[
\Delta V_H(r, \delta t) \equiv V_H(r, \delta t) - V_0(r) = - \lim_{t \to \infty} \frac{d}{dt} \ln[C_H(r, \delta t, t)]
\]

and extrapolating \( \delta t \to \infty \)
Lattice details

We analyse the $N_f = 2 + 1$ CLS ensemble C101 ($96 \times 48^3$ sites) [Bruno et al, 1411.3982]:

$m_\pi \approx 223$ MeV, $m_K \approx 476$ MeV, $Lm_\pi \approx 4.6$, $L \approx 4.1$ fm, $t_0/a^2 = 2.9085(51)$, $a = 0.0854(15)$ fm from $\sqrt{8t_0}$ extrapolated to physical point [G.S. Bali et al, 1606.09039] and $\sqrt{8t_0} = 0.4144(59)(37)$ fm [Borsanyi et al, 1203.4469]

High statistics: 1552 configs, separated by 4 MDUs, times 12 hadron sources (1 forward, 1 backward, 10 forward and backward propagating $\Rightarrow$ 22 2-point functions).

Wilson loops at all positions and in all directions.

Hadronic two-point functions have improved overlap with the ground state. We measure $\Delta V_H$ for $\pi$, $K$, $\rho$, $K^*$ and $\phi$ mesons; for $N$, $\Sigma$, $\Lambda$ and $\Xi$ octet baryons with $JP = \frac{1}{2}^\pm$; and for $\Delta$, $\Sigma^*$, $\Xi^*$ and $\Omega$ decuplet baryons with $JP = \frac{3}{2}^\pm$. 
Static potential $V_0(r)$

Using the methods of [Donnellan, FK, Leder and Sommer 1012.3037] variational basis with $0, 5, 7, 12$ spatial HYP levels; open boundary conditions: average between $t = 24$ and $t = 72$; Sommer scale $r_0/a = 5.890(41)$
QQ binding energy shift “within” a pion

[ M. Alberti et al, in preparation ]

We can resolve small energy differences
Details of the pion analysis

We do a linear fit in $t$ to $\ln[C]$, here for $\delta t = 5a$ and $r = 6a \approx r_0$
For a baryon of parity $P$, parity $-P$ is taken in the forward propagator and parity $P$ in the backward propagator.
Details of the nucleon analysis

Linear fit in $t$ to $\ln[C(r = 6a \approx r_0, \delta t = 5a, t)]$
The baryon decuplet

Candidates for pentaquark states

We measure decuplet baryons with helicity $\pm \frac{3}{2}$. Quarkonium in S-wave has $J^P = 0^-$ and $1^-$. Combining $0^-$ or $1^-$ quarkonium with a $\frac{3}{2}^+$ decuplet baryon gives a $J^P = \frac{3}{2}^-$ state. Combining $1^-$ quarkonium with a $\frac{3}{2}^-$ decuplet baryon gives a $J^P = \frac{5}{2}^+$ state.
Motivation

Hadroquarkonia in the static limit

Lattice results

Conclusions and outlook

$\bar{Q}Q$ binding energy shift “within” a $\Delta(\frac{3}{2}^+)$

Polarisation: correlation of baryon polarised in $z$ direction with Wilson loops in $z$ direction
$\bar{Q}Q$ binding energy shift “within” a $\Sigma^*(\frac{3}{2}^+)$
The signal for $\frac{3}{2}^-$ decuplet baryons is noisier. Also they do not match the mass of the $\frac{5}{2}^+$ pentaquark.
Summary

Conclusions

- Modifications of the static potential $V_0$ in the presence of light hadrons are found $\Delta V_H(r) < 0$ and are few MeV at $r = 0.5$ fm.

- Solving the Schrödinger equation with $V_0$ and $V_0 + \Delta_H V$ gives a stronger binding of charmonium $1S$ state by $-1$ MeV to $-2.5$ MeV, of $1P$ state by $-2.5$ MeV to $-4.5$ MeV and of $2S$ state by $-2.5$ MeV to $-5.5$ MeV. Somewhat inconsistent with the original hadro-charmonium.

- Interestingly, there is a similar attraction in all of the channels investigated so far.

Outlook

study of finite volume effects