From horseshoe to quasi-satellite and back again: the curious dynamics of Earth co-orbital asteroid 2015 SO₂

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Abstract Earth co-orbitals of the horseshoe type are interesting objects to study for practical reasons. They are relatively easy to access from our planet and that makes them attractive targets for sample return missions. Here, we show that near-Earth asteroid (NEA) 2015 SO₂ is a transient co-orbital to the Earth that experiences a rather peculiar orbital evolution characterised by recurrent, alternating horseshoe and quasi-satellite episodes. It is currently following a horseshoe trajectory, the ninth asteroid known to do so. Besides moving inside the 1:1 mean motion resonance with the Earth, it is subjected to a Kozai resonance with the value of the argument of perihelion librating around 270°. Contrary to other NEAs, asteroid 2015 SO₂ may have remained in the vicinity of Earth’s co-orbital region for a few hundreds of thousands of years.

Keywords Celestial mechanics · Minor planets, asteroids: general · Minor planets, asteroids: individual: 2015 SO₂ · Planets and satellites: individual: Earth

1 Introduction

In the Solar System, co-orbital asteroids share the orbit of a planet, going around the Sun in almost exactly one orbital period of their host planet, i.e., move inside the 1:1 mean motion resonance with a planet. These minor bodies are not truly captured by the gravity of the host planet as natural satellites are, but from the planet’s point of view they loop around, sometimes in remarkably stable orbits.

Co-orbital motion is characterised by a suitable angular variable, the resonant or critical angle, that in this case is the difference between the mean longitudes of the asteroid and its host planet or relative mean longitude. The mean longitude of an object —planet or minor body— is given by \( \lambda = M + \Omega + \omega \), where \( M \) is the mean anomaly, \( \Omega \) is the longitude of the ascending node, and \( \omega \) is the argument of perihelion (see, e.g., Murray & Dermott 1999). For these objects, the value of the critical angle librates or oscillates; for a regular passing body, the relative mean longitude circulates or takes any value in the range \((0, 2\pi)\). Even if some of them can experience relatively close flybys, co-orbital asteroids tend to be small, transient visitors and they do not pose a particularly high hazard for their host planets.

Dynamically speaking, the co-orbital state is surprisingly rich and it includes three elementary configurations —quasi-satellite or retrograde satellite, Trojan or tadpole, and horseshoe— and a multiplicity of hybrid arrangements or compound orbits (for instance, combination of quasi-satellite and tadpole orbits, Morais & Morbidelli 2002). Recurrent transitions between the main resonant states are possible (Namouni et al. 1999; Namouni & Murray 2000). The quasi-satellite dynamical state is observed when the value of the relative mean longitude, \( \lambda_r \), oscillates around \( 0^\circ \) (see, e.g., Mikkola et al. 2006). If the libration is around \( \pm 60^\circ \) we call the object a Trojan and it follows a tadpole orbit (see, e.g., Murray & Dermott 1999); such an object is classified as an \( L_4 \) Trojan when the value of \( \lambda_r \) librates around \(+60^\circ\), or as an \( L_5 \) Trojan if the value oscillates around \(-60^\circ\) (or \(300^\circ\)). The Trojan configuration is the most prevalent co-orbital state among long-term, stable co-orbitals. However, the most usual co-orbital dynamical state among transient co-orbitals is the one characterised by a libration amplitude larger than \(180^\circ\) around a value of \( \lambda_r = 180^\circ \) and often enclosing \(\pm 60^\circ\); such objects follow horseshoe orbits (see, e.g., Murray & Dermott 1999). At regular intervals and before moving...
away, a horseshoe librator follows a corkscrew-like trajectory in the vicinity of its host planet for a certain period of time, a decade or more in the case of the Earth; these episodes are recurrent.

Hollabaugh and Everhart (1973) predicted the existence of asteroids moving in long-lasting horseshoe orbits associated with the Earth. None was found until 3753 Cruithne (1986 TO) was identified by Wiegert et al. (1997) as an asteroidal companion to our planet following a horseshoe orbit. Nearly two decades later, the number of known asteroids which are Earth co-orbitals remains relatively small. Our planet hosts one Trojan —2010 TK\textsubscript{7} (Connors et al. 2011)—, four quasi-satellites —164207 (2004 GU\textsubscript{9}) (Connors et al. 2004; Mikkola et al. 2006; Wajer 2010), 277810 (2006 FV\textsubscript{35}) (Wiegert et al. 2008; Wajer 2010), 2013 L\textsubscript{X28} (Connors 2014), and 2014 OL\textsubscript{339} (de la Fuente Marcos & de la Fuente Marcos 2014)—, and eight horseshoe librators —3753 (Wiegert et al. 1997, 1998), 85770 (1998 UP\textsubscript{1})\textsuperscript{1} \& 54509 YORP (2000 PH\textsubscript{5}) (Wiegert et al. 2002; Margot & Nicholson 2003), 2001 GO\textsubscript{2} (Wiegert et al. 2002; Margot & Nicholson 2003; Brasser et al. 2004), 2002 AA\textsubscript{29} (Connors et al. 2002; Brasser et al. 2004), 2003 YN\textsubscript{107} (Brasser et al. 2004; Connors et al. 2004), 2010 SO\textsubscript{16} (Christou & Asher 2011), and 2013 BS\textsubscript{45} (de la Fuente Marcos & de la Fuente Marcos 2013). Most of these objects are not large enough to cause widespread damage if they strike our planet, but there are some outliers. While objects such as 2003 YN\textsubscript{107}, 2002 AA\textsubscript{29} or 2001 GO\textsubscript{2} are probably not large enough to reach the ground —in the rare event of a collision—with a significant fraction of their original kinetic energy remaining, others, e.g., 3753 (\(H = 15.7\) mag) or 85770 (\(H = 20.5\) mag) are significantly larger than either the Tunguska (see, e.g., Yeomans 2006) or Chelyabinsk (Mikkola et al. 2015) impacts and may cause considerable destruction over populated and/or coastal areas. Based on their current orbital solutions, the 13 objects cited are present-day—but transient—Earth co-orbital asteroids; the values of their respective \(\lambda\)\textsubscript{\text{\textsubscript{L}}} librates or oscillate as described above although some of them are in the process of transitioning between co-orbital states or follow compound orbits. These co-orbitals are often considered rare curiosities, but their dynamical behaviour and affordable accessibility from the Earth (e.g. Stacey & Connors 2009) make them very good candidates for future interplanetary space activities such as in situ study, sample return missions, or even commercial mining (e.g. Lewis 1996; Elvis 2012, 2014; García Yáñez et al. 2013; Harris & Drube 2014).

Here, we show that the recently discovered asteroid \(2015\) SO\textsubscript{2} is performing the corkscrew motion characteristic of Earth co-orbitals of the horseshoe type. This object is the 14th known Earth co-orbital and the 9th horseshoe librator. This paper is organised as follows. In Sect. 2, we present the available data on \(2015\) SO\textsubscript{2} and the methodology followed in this study. The dynamical evolution of this minor body is studied in Sect. 3. The details of the mechanism triggering the transitions between the horseshoe and the quasi-satellite dynamical states are explored in Sect. 4. In Sect. 5, we consider the impact of errors on our results. A discussion is presented in Sect. 6 and in Sect. 7 we summarise our conclusions.

2 Asteroid \(2015\) SO\textsubscript{2}: data and methodology

Asteroid \(2015\) SO\textsubscript{2} was discovered on 2015 September 21 by B. Mikuž and S. Mатичич observing with the 0.6-m f/3.3 Cichocki telescope at the Crni Vrh Observatory in Slovenia (Mikuž et al. 2015). It had a \(R\) magnitude of 19.4 when first observed. It is a small object with \(H = 23.9\) mag, which translates into a diameter in the range 50–111 m for an assumed albedo of 0.20–0.04. Its orbit is moderately well determined with 84 observations acquired during 9 d (see Table \ref{tab:1} and it is typical of a minor body that moves co-orbitally with the Earth. The source of the Heliocentric Keplerian osculating orbital elements and uncertainties in Table \ref{tab:1} is the JPL Small-Body Database\footnote{http://ssd.jpl.nasa.gov/sbdb.cgi} and they are referred to the epoch JD2457200.5, i.e. a time prior to its discovery and subsequent close encounter with our planet. At that time, the object was an Apollo asteroid moving in an orbit with a value of the semi-major axis \(a = 1.00079\) AU, very close to that of our planet (0.99957 AU), relatively low eccentricity, \(e = 0.11\), and moderate inclination, \(i = 9.2\). Since the close encounter with the Earth, \(2015\) SO\textsubscript{2} has become an Aten asteroid.

In the following, we study the short-term dynamical evolution of this recently discovered near-Earth asteroid (NEA). Our analysis is based on results of direct \(N\)-body calculations that use the most updated ephemerides and include perturbations from the eight major planets, the Moon, the barycentre of the Pluto-Charon system, and the three largest asteroids. The extensive numerical integrations presented here have been performed using the Hermite scheme described by Makino (1991) and implemented by Aarseth (2003). The standard version of this direct \(N\)-body calculations
code is publicly available from the IoA web site. Non-gravitational forces, relativistic and oblateness terms are not included in the calculations; for further details, see de la Fuente Marcos & de la Fuente Marcos (2012). Initial conditions (positions and velocities in the barycentre of the Solar System) have been obtained from the Jet Propulsion Laboratory (JPL) HORIZONS system (Giorgini et al. 1996; Standish 1998) and they are referred to as the JD 2457200.5 epoch (2015-June-27.0), which is the $t = 0$ instant in our figures.

Although the current orbit of 2015 SO$_2$ is relatively poor, if enough test orbits are studied, the best estimates of the past and future dynamical evolution of this object can be determined. This assumption is based on the widely accepted idea that statistical results of an ensemble of collisional $N$-body simulations are accurate, even though individual simulations are not (see, e.g., Boekholt & Portegies Zwart 2015). For values of the standard deviation of the orbital elements as moderate as those in Table 1 if consistent behaviour is systematically found within reasonable limits, the dynamical nature of the object can be firmly established. In addition to the calculations completed using the nominal orbital elements in Table 1, we have performed 50 control simulations with sets of orbital elements obtained from the nominal ones within the accepted error limits (up to 9σ) that reflect the observational uncertainty in astrometry (see Sect. 5 for details). In the figures when an orbit is labelled ‘±nσ’, where $n$ is an integer, it has been obtained by adding (+) or subtracting (−) $n$-times the uncertainty from the orbital parameters (the six elements) in Table 1. All the control orbits exhibit consistent behaviour within a few hundred years of $t = 0$, which is the Lyapunov time — time-scale for exponential divergence of initially close orbits — for this object.

### Table 1

| Parameter                  | Value                                      |
|----------------------------|--------------------------------------------|
| Semi-major axis, $a$ (AU)  | $1.00079\pm0.00004$                       |
| Eccentricity, $e$          | $0.10758\pm0.00009$                       |
| Inclination, $i$ (°)       | $9.198\pm0.010$                           |
| Longitude of the ascending node, $\Omega$ (°) | $183.029\pm0.002$                   |
| Argument of perihelion, $\omega$ (°) | $289.28\pm0.02$                  |
| Mean anomaly, $M$ (°)      | $174.47\pm0.03$                           |
| Perihelion, $q$ (AU)       | $0.89312\pm0.00005$                       |
| Aphelion, $Q$ (AU)         | $1.10845\pm0.00005$                       |
| Absolute magnitude, $H$ (mag) | $23.9$                               |

### 3 Dynamical evolution

In order to study the dynamics of 2015 SO$_2$ we have performed $N$-body calculations in both directions of time for 50 kyr using the physical model described above. Figure 1, top panel, shows the evolution of the relative mean longitude, $\lambda_r$, during one period of the horseshoe orbit. The value of $\lambda_r$ behaves as expected of a classical horseshoe librating (see, e.g., Murray & Dermott 1999). The motion of 2015 SO$_2$ over the time range (-170, 57) yr as seen in a coordinate system rotating with the Earth in space (middle panel) and projected onto the ecliptic plane (bottom panel) is plotted in Fig. 4 (nominal orbit in Table 1). Asteroid 2015 SO$_2$ is an Earth co-orbital; it is performing the corkscrew motion characteristic of Earth co-orbitals of the horseshoe type (see Fig. 1 middle panel). All the investigated control orbits (within ±9σ) exhibit the same behaviour within the timeframe mentioned above. Based solely on the number of simulations performed, we estimate the probability of 2015 SO$_2$ being a present-day horseshoe librating to the Earth at $>99.9\%$. The motion in Fig. 4 is the result of the superposition of a $\sim 1$ yr epicyclic motion describing a kidney-like path and a slow oscillation in mean longitude with a half period close to 113 yr (the time to reach the other end of the horseshoe). The shape of the epicycle-like path is affected by the value of the eccentricity of the orbit. Minor bodies subjected to this type of dynamics may have had their origin in the Earth-Moon system (see, e.g., Margot & Nicholson 2003) although other sources in the main asteroid belt are possible. As pointed out by Connors et al. (2004) the effects derived from orbital chaos severely limit our ability to determine the source of these NEAs.

The short-term dynamical evolution of the nominal orbit in Table 1 is presented in Fig. 2. Figure 2 panel D, shows that 2015 SO$_2$, like other horseshoe librators, alternates between the Aten and Apollo dynamical classes. Prior to its close encounter with the Earth on 2015 September 30, Earth’s gravity was decreasing the asteroid’s orbital energy in such a way that its semi-major axis was changing from $\sim 1.006$ AU (Apollo-type orbit) to a value of $\sim 0.994$ AU (Aten-type orbit). As an Aten, it will reach the other end of the horseshoe path in about 113 years from now, going from the leading side of the Earth to the trailing side. At that time, Earth’s gravity will increase the asteroid’s orbital energy so its current value of the semi-major axis of $\sim 0.994$ AU will become $\sim 1.006$ AU. Figure 2 panel A, shows that 2015 SO$_2$ has only experienced relatively distant close encounters with our planet during the time interval displayed (4000 yr), well beyond the value of the radius of the Hill sphere of the Earth, 0.0098 AU.
The figure also shows that although 2015 SO$_2$ currently follows a horseshoe orbit with respect to the Earth, it will become a quasi-satellite of our planet (the value of $\lambda_r$ librating around $0^\circ$) in about 350 yr (see panel C). This dynamical state will persist for about 150 yr.

Figure 2 panel G, shows that the argument of perihelion of 2015 SO$_2$ oscillates around $-90^\circ$ or $270^\circ$. Therefore, this object reaches perihelion when it is farthest from the ecliptic plane, i.e., close encounters with inner planets are not possible at perihelion or aphelion. When an object exhibits this behaviour, it is said to be affected by the Kozai mechanism or trapped in a Kozai resonance (Kozai 1962). Because of the Kozai resonance, both eccentricity and inclination oscillate with the same frequency but out of phase (see Fig. 2 panels E and F); when the value of the eccentricity reaches its maximum the value of the inclination is the lowest and vice versa ($\sqrt{1-e^2}\cos i \sim$ constant, see Fig. 2 panel B). Although certainly present as verified by the behaviour of the invariant, the Kozai resonance is not very strong.

Figure 3 central panels, shows the results of the entire integration. We confirm that 2015 SO$_2$ does not experience particularly close encounters with the Earth; a large number of transitions from horseshoe to quasi-satellite and back again are observed. The object exhibits Kozai-like behaviour throughout the entire time interval with its nodes confined between Earth’s aphelion and perihelion most of the time. The distance between the Sun and the nodes for a prograde orbit is given by

$$r = a(1 - e^2)/(1 \pm e \cos \omega) ,$$

where the "+" sign is for the ascending node (where the orbit crosses the Ecliptic from South to North) and the "−" sign is for the descending node. Longer integrations not shown here indicate that it may have reached Earth’s co-orbital region 500–200 kyr ago and it could leave in another 100–200 kyr to follow orbits interior to that of the Earth. Based on the integrations performed, the Earth’s co-orbital region extends from $\sim 0.994$ AU to $\sim 1.006$ AU.

4 From horseshoe to quasi-satellite and back again: the mechanism

Figure 2 panel C, shows that in general 2015 SO$_2$ follows a horseshoe orbit with respect to our planet but has periods of quasi-satellite behaviour; in 4000 years of simulated time we observe three instances of the quasi-satellite phase with an average duration of 150
Fig. 3  Same as Fig. 2 but for the nominal orbit (same data as in Fig. 2 but over a longer time span) and two representative examples of orbits that are the most different from the nominal one among those integrated up to ±3σ deviations (see text for details). Venus’ and Mars’ aphelion and perihelion distances are also shown in panel H.
**Fig. 2** Time evolution of various parameters for the nominal orbit during the time interval (-2000, 2000) yr. The distance from the Earth (panel A) with the value of the radius of the Hill sphere of the Earth, 0.0098 AU (dashed line). The parameter $\sqrt{1-e^2 \cos i}$ (panel B). The resonant angle, $\lambda_r$ (panel C). The orbital elements $a$ (panel D), $e$ (panel E), $i$ (panel F), and $\omega$ (panel G). The distances to the descending (thick line) and ascending nodes (dotted line) are plotted in panel H; Earth’s aphelion and perihelion distances are also shown.

Asteroid 2015 SO$_2$ is not the first transient Earth co-orbital of the horseshoe type that suffers repeated transitions to the quasi-satellite dynamical state, Brasser et al. (2004) have documented this behaviour for 2001 GO$_2$, 2002 AA$_{29}$ and 2003 YN$_{107}$. These transitions had been predicted and explained by Namouni (1999) and Christou (2000), and depend on the influence of other planets, in particular the giant planets and Venus, owing to the overlapping of multiple secular resonances. This is not surprising as Ito & Tanikawa (1999) pointed out that the terrestrial planets share the effect of the secular perturbation from Jupiter. In a follow-up work, Tanikawa & Ito (2007) further extended this analysis concluding that, regarding the secular perturbation from Jupiter, the inner planets are a planetary group or collection of loosely connected mutually dynamically dependent planets. The effects of the overlapping secular resonances persist even outside the region where $\lambda_r$ librates and the dynamics of some passing bodies —which are Kozai librators because their argument of perihelion oscillates— is still controlled by them (de la Fuente Marcos & de la Fuente Marcos 2015a). Brasser et al. (2004) noticed that, in the cases studied by them, the transition to the quasi-satellite phase was possible both at the leading and at the trailing sides of the Earth, although most transitions took place at the trailing side of our planet.

Figure 2, panel C, shows that quasi-satellite episodes start when the nodes are farthest from each other (see panel H) and also farthest from the path of the Earth. In the Solar System and for a minor body moving in an inclined orbit, like 2015 SO$_2$, close encounters with major planets are only possible in the vicinity of the nodes. Therefore, the start of the quasi-satellite phase of the co-orbital motion coincides with the time when the average gravitational effects of the Earth on the object are the weakest possible. The transition back to the horseshoe phase is triggered when both nodes are close to Earth’s perihelion; then, encounters with the Earth-Moon system are possible at both nodes and the sustained action of these encounters increases the asteroid’s orbital energy eventually triggering the transition and returning the minor body to a horseshoe trajectory. In our simulations, the transition from horseshoe librator to quasi-satellite always takes place when the orbital energy of the asteroid increases after having followed a trajectory of the Aten type for nearly 100 yr. In other words, all the observed transitions from horseshoe to quasi-satellite took place at the trailing side of the Earth, never at the leading side. During the quasi-satellite phase, the orbital eccentricity is the highest and the inclination, the lowest; the value of the argument of perihelion decreases as predicted by Namouni.
5 Impact of errors on the short-term dynamical evolution

It may be argued that the orbit of 2015 SO₂ currently available is based on a short arc and, therefore, this makes our conclusions rather weak or even entirely questionable. However, not all orbital solutions with short data-arc spans are equally poor. The object discussed here follows a relatively stable orbit that is only directly perturbed by the Earth–Moon system. In addition to the integrations performed making use of the nominal orbital parameters in Table 1, we have computed 50 control simulations with sets of orbital elements obtained from the nominal ones within the quoted uncertainties and assuming Gaussian distributions for them up to ±9σ. Representative results of these calculations are displayed in Figs. 3, 4 and 5. The orbital evolution of 2015 SO₂ within a few thousand years of t = 0 is not too different from that in Fig. 2 even if deviations in the values of the orbital elements as large as ±3σ or even ±6σ are considered. For the extreme case of ±9σ, the overall dynamical behaviour is still consistent: the object remains within Earth’s co-orbital zone for the entire integration. These results clearly indicate that our conclusions are robust and reliable.

5.1 Impact of errors: classical treatment

A more detailed analysis of the effects of errors on our short-term results is performed here by studying how the changing initial parameters of the test orbits influence the variation of the osculating orbital elements over time. Two additional sets of 100 shorter control simulations are discussed here. As in the previous calculations, the initial orbital elements of each control orbit have been computed varying them randomly, within the ranges defined by their mean values and standard deviations. For example, a new value of the orbital eccentricity has been found using the expression \( e_t = \langle e \rangle + n \sigma_e r_i \), where \( e_t \) is the eccentricity of the test orbit, \( \langle e \rangle \) is the mean value of the eccentricity from the available orbit (Table 1), \( n \) is a suitable integer (for instance, 3, 6 or 9), \( \sigma_e \) is the standard deviation of \( e \) (Table 1), and \( r_i \) is a (pseudo) random number with normal distribution in the range −1 to 1.

Figure 6 shows the short-term evolution of the orbital elements \( a, e, i, \Omega \) and \( \omega \) of the object studied here. The thick black curves show the average results of the evolution of 100 control orbits computed as described above. The thin red curves show the ranges (minimum and maximum) in the values of the parameters at a given time. Figure 6 left-hand and central panels, shows the results for 1σ and 6σ spreads, respectively, in the initial values of the orbital elements and confirms once more that even if our study is based on a relatively short-arc orbit, its results are robust and reliable. Although our conclusions rest on a 9-day observational arc for 2015 SO₂, we can reliably state that this object follows a horseshoe trajectory with respect to the Earth. In general, Fig. 6 shows that the value of the Lyapunov time of 2015 SO₂ is very asymmetrical (about 200 yr versus several hundred years); the orbital evolution is significantly more chaotic into the future than it was in the past.

5.2 Impact of errors: MCCM treatment

Sitarski (1998, 1999, 2006) has pointed out that the procedure used above is equivalent to considering a number of different virtual minor planets moving in similar orbits, but not a sample of test orbits incarnated from a set of observations obtained for a single minor planet. The correct statistical alternative is to consider how the elements affect each other, applying the Monte Carlo using the Covariance Matrix (MCCM) approach (Borovitsyna et al. 2001; Avdyushev & Banschikova 2007), or to follow the procedure described in Sitarski (1998, 1999, 2006).

As a consistency test, we have used an implementation of the MCCM approach to recompute the orbital evolution of this object generating control orbits with initial parameters from the nominal orbit adding random noise on each initial orbital element making use of the covariance matrix (for details, see de la Fuente Marcos & de la Fuente Marcos 2015b). Figure 6 right-hand panels, shows the results of these simulations and, in general, the difference is not very significant. The Lyapunov time asymmetry is also confirmed.

6 Discussion

If we compare the orbital evolution of 2015 SO₂ as depicted in Figs. 3, 4 and 5 with that of other horseshoe librators of the Earth, for instance 2001 GO₂,
Fig. 4  Same as Fig. 3 but for the nominal orbit (same data as in Fig. 3) and two representative examples of orbits that are the most different from the nominal one among those integrated up to $\pm 6\sigma$ deviations (see text for details).
Fig. 5  Same as Fig. 3 but for the nominal orbit (same data as in Fig. 3) and two representative examples of orbits that are the most different from the nominal one among those integrated up to ±9σ deviations (see text for details).
Fig. 6  Time evolution of the orbital elements $a$, $e$, $i$, $\Omega$ and $\omega$ of 2015 SO$_2$. The black thick curve shows the average evolution of 100 control orbits, the red thin curves show the ranges in the values of the parameters at the given time. Results for a 1σ spread in the initial values of the orbital elements (left-hand panels), a 6σ spread (central panels), and using MCCM (see the text, right-hand panels).

2002 AA$_{29}$, and 2003 YN$_{107}$ (Brasser et al. 2004) or 2013 BS$_{45}$ (de la Fuente Marcos & de la Fuente Marcos 2013) one rapidly realises that 2015 SO$_2$ is far more stable. Given the fact that the values of the semi-major axis of these objects are very similar, the source of the enhanced stability must be in the values of other orbital elements. Asteroids 2001 GO$_2$, 2003 YN$_{107}$ and 2013 BS$_{45}$ all move in orbits with $i < 5^\circ$; 2002 AA$_{29}$ has $i = 107.5^\circ$, but its eccentricity is rather low, 0.013. In contrast, 2015 SO$_2$ has $e = 0.11$ and $i = 92.2^\circ$. This combination of values of $e$ and $i$ appears to be particularly favourable regarding orbital stability. However, it places the object far from the Earth even at close approaches; in nearly two centuries around the current epoch, the distance of closest approach is 0.037 AU. If objects like 2015 SO$_2$ are numerous, then they will be intrinsically more difficult to discover than those similar to 2013 BS$_{45}$ that reach perigee at 0.013 AU. Asteroid 2013 BS$_{45}$ is far less stable than 2015 SO$_2$, compare Fig. 6 with fig. 2 in de la Fuente Marcos & de la Fuente Marcos (2013).

Almost certainly, the most stable known Earth’s co-orbital is 2010 SO$_{16}$ that stays as horseshoe librator for at least 120 kyr and possibly up to 1 Myr (Christou & Asher 2011), remaining in the same co-orbital configuration during this time span. Asteroid 2015 SO$_2$ may be almost as stable as 2010 SO$_{16}$ but it often switches co-orbital configuration. In addition, the value of its distance from the Earth at perigee remains under 0.5 AU for about 22 yr when it reaches the ends of its horseshoe orbit, every 113 yr. During its current stint as neighbour of our planet, its final perigee at < 0.5 AU will be on 2026 September 16. In other words, objects like 2015 SO$_2$ are characterised by relatively long favourable accessibility windows making the implementation of a hypothetical space mission easier. On the other hand, our analysis leaves the question of the possible origin of 2015 SO$_2$ open. It is clearly a transient co-orbital to our planet, but its dynamical behaviour is quite unusual. An origin in the Earth-Moon system cannot be entirely ruled out based on the available information alone; its orbital evolution was significantly more stable in the past.

Finally, it can be argued that presenting this object here is rather premature because its orbit is still not well known (see Table 1). However, this object is only directly perturbed by the Earth–Moon system and the time interval between closest approaches to the Earth–Moon system is currently 113 yr. It means that the characteristic time-scale between favourable visibility windows is over a century and that its orbit (see above) is quite stable. If one misses the opportunity of studying the object during the next few years, waiting for over a hundred years will be required to recover it. In addition, the analysis of the influence of errors on our conclusions clearly indicates that our study is justified and its conclusions robust. One of the objectives of this research is to bring this interesting minor body to the attention of the astronomical community, encouraging follow-up observations. Spectroscopic studies during its
next perigee should be able to confirm if an origin in the Earth-Moon system is plausible.

7 Conclusions

In this research, we have used N-body simulations and statistical analyses to study the orbital evolution of 2015 SO2. The main conclusions of our study can be summarised as follows:

- Asteroid 2015 SO2 currently follows a horseshoe trajectory with respect to the Earth (probability > 99.9%), the ninth asteroid known to do so.
- Asteroid 2015 SO2 is a transient co-orbital that experiences a rather peculiar orbital evolution characterised by alternating horseshoe and quasi-satellite episodes.
- Asteroid 2015 SO2 is subjected to a Kozai resonance; its argument of perihelion oscillates around −90° or 270°.
- Asteroid 2015 SO2 may have had its origin in the Earth-Moon system and it is one of the most stable Earth horseshoe librators discovered to date.

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References

Aarseth, S.J.: Gravitational N-Body Simulations, p. 27. Cambridge University Press, Cambridge (2003)
Avdyushev, V.A., Bantschikova, M.A.: Sol. Syst. Res. 41, 413 (2007)
Boekholt, T., Portegies Zwart, S.: Comput. Astrophys. Cosmol. 2, 2 (2015)
Bordovitsyna, T., Avdyushev, V., Chernitsov, A.: Celest. Mech. Dyn. Astron. 80, 227 (2001)
Brasser, R., Inmanen, K.A., Connors, M., Veillet, C., Wiegert, P., Mikkola, S., Chodas, P.W.: Icarus 171, 102 (2004)
Brown, P.G., et al.: Nature 503, 238 (2013)
Christou, A.: Icarus 144, 1 (2000)
Christou, A.A., Asher, D.J.: Mon. Not. R. Astron. Soc. 414, 2965 (2011)
Connors, M.: Mon. Not. R. Astron. Soc. 437, L85 (2014)
Connors, M., Chodas, P., Mikkola, S., Wiegert, P., Veillet, C., Inmanen, K.: Meteoritics Planet. Sci. 37, 1435 (2002)
Connors, M., Veillet, C., Brasser, R., Wiegert, P., Chodas, P., Mikkola, S., Inmanen, K.: Meteoritics Planet. Sci. 39, 1251 (2004)

Connors, M., Wiegert, P., Veillet, C.: Nature 475, 481 (2011)
de la Fuente Marcos, C., de la Fuente Marcos, R.: Mon. Not. R. Astron. Soc. 427, 728 (2012)
de la Fuente Marcos, C., de la Fuente Marcos, R.: Mon. Not. R. Astron. Soc. 434, L1 (2013)
de la Fuente Marcos, C., de la Fuente Marcos, R.: Mon. Not. R. Astron. Soc. 445, 2985 (2014)
de la Fuente Marcos, C., de la Fuente Marcos, R.: Astron. Astrophys. 580, A109 (2015a)
de la Fuente Marcos, C., de la Fuente Marcos, R.: Mon. Not. R. Astron. Soc. 453, 1288 (2015b)
Elvis, M.: Nature 485, 549 (2012)
Elvis, M.: Planet. Space Sci. 91, 20 (2014)
García Várnos, D., Sanchez, J.P., McInnes, C.R.: Celest. Mech. Dyn. Astron. 116, 367 (2013)
Giorgini, J.D., et al.: Bull. Am. Astron. Soc. 28, 1158 (1996)
Harris, A.W., Drude, L.: Astrophys. J. 758, L4 (2014)
Hollabaugh, M., Everhart, E.: Astrophys. Lett. 15, 1 (1973)
Ito, T., Tanikawa, K.: Icarus 139, 336 (1999)
Kozai, Y.: Astron. J. 67, 591 (1962)
Lewis, J.S.: Mining the Sky: Untold Riches from the Asteroids, Comets, and Planets, p. 82. Addison-Wesley, Reading (1996)
Makino, J.: Astrophys. J. 369, 200 (1991)
Margot, J.L., Nicholson, P.D.: Bull. Am. Astron. Soc. 35, 1039 (2003)
Mikkola, S., Inmanen, K., Wiegert, P., Connors, M., Brasser, R.: Mon. Not. R. Astron. Soc. 369, 15 (2006)
Mikuž, B., et al.: MPEC 2015-S69 (2015)
Morais, M.H.M., Morbidelli, A.: Icarus 160, 1 (2002)
Murray, C.D., Dermott, S.F.: Solar System Dynamics, p. 97. Cambridge University Press, Cambridge (1999)
Namouni, F.: Icarus 137, 293 (1999)
Namouni, F., Murray, C.D.: Celest. Mech. Dyn. Astron. 76, 131 (2000)
Namouni, F., Christou, A.A., Murray, C.D.: Phys. Rev. Lett. 83, 2506 (1999)
Sitarski, G.: Acta Astron. 48, 547 (1998)
Sitarski, G.: Acta Astron. 49, 421 (1999)
Sitarski, G.: Acta Astron. 56, 283 (2006)
Stacey, R.G., Connors, M.: Planet. Space Sci. 57, 822 (2009)
Standish, E.M.: JPL Planetary and Lunar Ephemerides, DE405/LE405, Interoffice Memo. 312.F-98-048, Jet Propulsion Laboratory, Pasadena, CA, USA (1998)
Tanikawa, K., Ito, T.: Publ. Astron. Soc. Jpn. 59, 989 (2007)
Wajer, P.: Icarus 209, 488 (2010)
Wiegert, P.A., Inmanen, K.A., Mikkola, S.: Nature 387, 685 (1997)
Wiegert, P.A., Inmanen, K.A., Mikkola, S.: Astron. J. 115, 2604 (1998)
Wiegert, P., Connors, M., Chodas, P., Veillet, C., Mikkola, S., Inmanen, K.: American Geophysical Union, Fall Meeting 2002, P11A-0352 (2002)
Wiegert, P.A., DeBoer, R., Brasser, R., Connors, M.: J. R. Astron. Soc. Can. 102, 52 (2008)
Yeomans, D.K.: Bull. Am. Astron. Soc. 38, 1057 (2006)