The effects of coherent neutrino mixing modulation by dark matter.

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Abstract. If there was a coupling between neutrinos and Dark Matter, then there could be an induced modulation of the neutrino-mixing spectrum analogous to the one produced by the MSW effect. The possible results of such a modulation have previously been investigated for the case where coherent effects cannot be isolated and the resulting pattern is de-coherent. This presentation is on an investigation that included coherent effects, which are relevant for very high-energy neutrinos as the wave-packets remain coherent over cosmological distances. The investigation found that observable coherent effects are produced by potentials several orders of magnitude smaller than the minimum potential required to produce an observable de-coherent effect, with the amount being baseline dependent. This extends the range of observability down to levels where an interaction is likely to exist, if it exists at all. The significant challenges in making such an observation will be covered.

This work discussed here is presented in more detail in [1], which is still awaiting publication.

Neutrinos are electro-magnetically neutral leptons with a very small mass, which was proven by the discovery of neutrino oscillations [2, 3]. This is where a neutrino produced in one of the three flavour states enters into a quantum superposition of the three mass states which then propagate at different velocities due to having different masses. These velocity differences produce interference effects which cause the flavour content of the wavepacket to change in an oscillatory manner. If the neutrinos pass through a medium with which they have a coupling that distinguishes between species (meaning that different species have different couplings), a shift in the mixing pattern will occur [4].

It is known that the bulk of the matter in the universe and in our galaxy is comprised of a currently unknown form dubbed Dark Matter [5]. If there is a coupling between Dark Matter and neutrinos, then there could potentially be an observable shift if the interaction is strong enough [6]. Previous work on this subject focused exclusively on de-coherent astrophysical neutrinos where the wave-packets have separated and oscillations no longer occur (although there still is a shift from the original flavour composition). This is because neutrinos lose their coherence over time and resolving coherent effects across astronomical distances is extremely difficult [7].

The neutrino mixing equation is:

\[ f_\beta = f_\alpha \times ((W_{\beta,1} W_{1,\alpha}^\dagger)^2 + (W_{\beta,2} W_{2,\alpha}^\dagger)^2 + (W_{\beta,3} W_{3,\alpha}^\dagger)^2 + 2W_{\beta,1} W_{1,\alpha}^\dagger W_{\beta,2} W_{2,\alpha}^\dagger \cos(\frac{\Delta M_{12}^2}{2E} L) + \ldots \]
\[
+2W_{\beta,1}W_{1,\alpha}^\dagger W_{\beta,3}W_{3,\alpha}^\dagger \cos(\frac{\Delta M_{13}^2}{2E}L) + 2W_{\beta,2}W_{2,\alpha}^\dagger W_{\beta,3}W_{3,\alpha}^\dagger \cos(\frac{\Delta M_{23}^2}{2E}L),
\]

where \(f_\alpha\) is the emission flavour vector (i.e. a normalized vector containing the three flavour fractions), \(f_\beta\) is the flavour vector at the detector and \(W\) is the matrix translating between the flavour basis and the effective mass basis. (A note about the effective mass basis: The effective mass basis is simply the one in which the Hamiltonian in the presence of a medium is diagonal. It is called the effective mass basis because, if it is calculated, then the equations for oscillations in a vacuum can be used with simple substitution [4].) For de-coherent neutrinos the baseline dependent terms are removed.

The size of a potential effect for de-coherent neutrinos scales with energy, i.e. the minimum potential required to produce a noticeable effect decreases as energy increases [6]. The minimum observable potential for 1 PeV neutrinos just over \(10^{-20}\) eV, for any species. Given the mass density of Dark Matter in the near vicinity of the Solar System and presuming that it is within a few orders of magnitude the same throughout the galaxy, producing a potential this size would require either an (energy-dependent) interaction much stronger than the Weak Interaction or a Weak-scale coupling to sub-axion scale particles. While neither is strictly speaking outside the realm of possibility, these would both be quite irregular. Therefore, coherent effects were investigated despite of the great difficulties in making a coherent measurement to see what kind of improvement could be obtained.

Two different potential configurations were tested: one where the potential is diagonal in the neutrino flavour basis (the flavour-state interaction) as would occur with Weak interactions and one that is diagonal in the mass basis (the mass-state interaction) which corresponds interactions predicted by certains models that give the neutrino its mass through couplings with a Dark Matter candidate (sometimes referred to as Scotogenic models [9]). The second case is of interest because de-coherent effects cannot exist with those types of interactions, regardless of the coupling strength. In both cases the coherent effects behaved in the same manner.

Included are plots of the deviations from the vacuum electron neutrino detection probability \((f_\beta - f_{\beta,0})\) induced by various potentials.

**Figure 1.** The de-coherent flavour state interaction at an energy of 1 PeV.

**Figure 2.** The coherent flavour state interaction at an energy of 1 PeV with a baseline of 1 kpc = \(3 \times 10^{19}\) m.

It was found that the minimum for coherent effects scales with baseline in much the same way as the de-coherent portion scales with energy. At a baseline of approximately 1 kpc, the minimum is just over \(10^{-26}\) eV; an improvement over the de-coherent effect of six orders of magnitude. Unfortunately, for the mass state interaction, this is above the limit set by cosmological observables, which is \(\approx 10^{-38}\) eV [8, 1]. This limit is for energy independent interactions (the type that scotogenic interactions would be), which is the form of interactions that cosmological observables are most sensitive to.
Figure 3. The coherent flavour state interaction with a different energy (290 TeV) and baseline ($4.384 \times 10^{25}$ m) for comparison.

Figure 4. The coherent mass state interaction at an energy of 1 PeV with a baseline of 1 kpc = $3 \times 10^{19}$ m.

Also plotted were the difference patterns for all three flavours in both interaction cases plotted against baseline. The small amplitude oscillations are for the mass case and the large amplitude oscillations are for the flavour case. The two potentials ($V_{11}$ and $V_{22}$) were both set to $10^{-26}$ eV.

Figure 5. This plot shows the long range behaviour of the potential differences.

Figure 6. This shows the potential differences at a displacement of 16.2 kpc from the source with the baseline in AU.

As stated before, the challenges in resolving coherent effects are highly nontrivial. The requirement on baseline, $L$, is that it be less than the coherence length:

$$L \ll L_{\text{coh}} = \sigma_X \frac{V_G}{\Delta V} \approx \sigma_X \frac{2E^2}{\Delta m^2}$$

where $\sigma_X$ is the wavepacket size, $V_G$ is the group velocity, $\Delta V$ is the difference in wavepacket velocities, and the ultra-relativistic approximations, $V_G \approx 1$ and $\Delta V \approx \frac{\Delta m^2}{2E}$, have been used. ($\Delta m^2$ is the larger of the two mass-squared differences, as that is the dominant factor. Also, the convention $\hbar = c = 1$ has been used.)

Using the equivalence between wavepacket size and energy uncertainty:

$$\sigma_X \approx \frac{V}{\sigma_E} \approx \frac{1}{\sigma_E},$$

a relationship between baseline and energy resolution can be established:

$$\frac{\Delta m^2}{4\pi E^2} \ll \sigma_E \ll \frac{2E}{L\Delta m^2}.$$  

At 1 PeV and with a baseline of 1 kpc, the requirement on energy resolution is $\sigma_E \ll \frac{1}{10^6}$, with a similar requirement on baseline resolution. (The derivation of the coherence conditions comes
from [7] and are covered in more detail in [1].) That puts the making of coherent measurements well out of reach for current technology.

In conclusion, attempting to observe an astro-physical neutrino-Dark Matter coupling by utilising a matter effect observation is, barring unforeseen developments of a spectacular nature, highly impractical.

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