Abstract. In the course of a decade, Einstein singlehandedly overthrew the centuries-old
Newtonian framework and gave the world a radically new demonstrably deeper understand-
ing of gravity. It does not take much to get experts and nonexperts to gush over the sheer brillian-
tce and monumental originality of Einstein’s accomplishment in fashioning both the special and the general theories of relativity. There is no doubt that both theories capture the imagination. The anti-intuitive properties of the special theory of relativity and its deep philosophical implications, the bizarre and dazzling predictions of the general theory of relativity: the curvature of spacetime, the exotic characteristics of black holes, the bewildering prospects of gravitational waves, the discovery of astronomical objects as quasars and pulsers, the expansion and the (possible) recontraction of the universe..., are all breathtaking phenomena. In this paper, we give a philosophical non-technical treatment of both the special and the general theory of relativity together with an exposition of some of the latest physical theories. We then give an outline of an axiomat-
ic approach to relativity theories due to Andreka and Nemeti that throws light on the logical structure of both theories. This is followed by an exposition of some of the bewildering results established by Andreka and Nemeti concerning the foundations of mathematics using the notion of relativistic computers. We next give a survey on the meaning and philosophical implications of the the quantum theory and end the paper by an imaginary debate between Einstein and Neils Bohr reflecting both Einstein’s and Bohr’s philosophical views on the quantum world.

The paper is written in a somewhat untraditional manner; there are too many foot-
notes. The reason behind this is the following: ignoring the footnotes, the paper is intended to be complete in itself. The footnotes, on the other hand, deal mainly with the more technical issues of both the special and general theories of relativity together with the intricate concepts of the general theory of relativity and its connection to other modern physical theories. In order not to burden the reader with all the details, we have collected the more advanced material (mostly of philosophical nature) in the footnotes. We think that this makes the paper easier to read and simpler to follow. In fact, the paper without the footnotes should be understood by anyone having a good scientific or a philosophical background. The paper in full is addressed more to experts.
0. Einstein, the Man and the Philosopher

Most of the material presented here may be found in [9] and [14].

Still there are moments when one feels free from one's own identification with human limitations and inadequacies. At such moments, one imagines that one stands on some spot of a small planet, gazing in amazement at the cold yet profoundly moving beauty of the eternal, the unfathomable; life and death flows into one, and there is neither evolution nor destiny; only being. Einstein

Joy in looking and comprehending is nature's most beautiful gift. Einstein

Nature in its simple truth is greater and more natural than any creation of human hands, than all the illusions created by the spirit. Robert Mayer as quoted in [9].

For more than two centuries, Newton's system had been regarded as the ultimate solution to the fundamental problems of science; as the final and preordained picture of the world. This estimation is reflected in Alexander Pope's verse:

Nature and Nature's law lay hid in night;
God said, let Newton be and all was light.

Then came Einstein with his theory of relativity, and some wit added the lines:

But not for long. Let Einstein be! the devil said;
And lo, it was dark: the light has fled.

Albert Einstein, the most famous scientist of the twentieth century, perhaps the greatest genius of all time, whose title to fame is uncontested and whose impact on modern scientific thought is unchallenged, was a man of touching simplicity and modesty. He was always surprised by the masses' infatuation with his theories, because they could not possibly mean anything to them. Those who could understand his work did not crowd around him like primitives at the feet of their idol. They read, criticized and discussed him.

Einstein never sought fame, fortune or glory. He did nothing to outclass others and certainly nothing to please others. He strove to understand and resolve the laws of nature. This, he did, more like an amateur. He was an original and a deep thinker, who presented the world with a diamand mine. His ideas have led directly to the era of atomic power and the dawning of the space age. He can be described as the Copernicus of the twentieth century.

In his book, “Principia Mathematica” - the most influential scientific book of all times - Newton was able to lay down the fundamental laws of mechanics in the language of differential calculus using the notion of rate of change. He then succeeded to deduce Kepler's laws and generalize Galileo's ideas in his three famous laws of nature. Newton was thus the first to realize that the universe operated according to strict mathematical laws!
Einstein did not create his own legend as so many others believe they should. When asked to give a portrait of himself towards the end of his life he wrote: “Of what is significant in one’s own existence one is hardly aware, and it certainly should not bother the other fellow. What does a fish know about the water in which he swims all his life.”

Einstein was never a good pupil. His grades were mediocre and he never reached the top of his class. His family thought he was retarded as he did not talk until the age of three. Strangely enough, what attracted him, apart from mathematics and physics, was religious matters. His father, a far from orthodox Jew, sent him to a Catholic institution. There he acquired a great interest in “divinity”, the biblical legends and the epic of Christ.

At the age of ten, he left primary school for the Lutpoid Gymnasium in Munich. Again, he showed no more brilliance than at primary school. It was thanks to books of popular science, however, that the young Albert felt a passion for knowledge awake in him. His favorite classical writers were Hume, Schiller and Goethe.

At an earlier age, his mother had made him take violin lessons. His rather mild enthusiasm was transformed gradually into a passion for music. Music became his favourite past-time. From the age of fourteen, he took part in domestic concerts. Mozart’s music played the same role in Einstein’s life as Euclidean geometry did in his scientific development.

Einstein came up against a wall of refusals when he tried to obtain a post as an assistant on the completion of his studies. His views were too original with an independent character that terrified his professors. He also had the misfortune to be born a Jew. Although officially there was no anti-Semitism in Switzerland, racial prejudice, as elsewhere, was not unknown. To earn a living, Einstein was forced to accept a regular job as an engineer in the Patent Office in Bern. Einstein’s life in Bern can be compared with Newton’s Woolsthorpe period during the plague of 1665 – 1667, when he had to go away to Cambridge. It was in Woolsthorpe that Newton developed his ideas on differential calculus, universal gravitation and the breaking down of light into monochromatic rays. It was in Bern that Einstein developed the theory of Brownian motion, the photon theory (the photo-electric effect) and the special theory of relativity. Each one of these contributions could have earned him a Nobel price.

Also to be noted is Einstein’s attitude towards philosophical literature. He ascribed purely aesthetic value to many philosophical works. At the same time, he ascribed great philosophical and scientific value to works of fiction. His attitude was that of

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3 In the current literature, the special theory of relativity is attributed to Lorentz, Poincare and Einstein. We believe that this is not very fair. Though both Lorentz and Poincare contributed to the theory, it was Einstein who formulated the theory on solid physical grounds, revolutionizing Newton’s absolute space and time.

4 Einstein received his Nobel price as a reward on “his work on the photo-electric effect and his contributions to theoretical physics”. Both his special and general theories of relativity were not yet properly understood!
a sympathetic listener accepting a philosophical point of view with an indulgent or ironical - as the case might be - smile. His attitude towards the 18-th and 19-th century philosophy may be summed in the following way: Einstein’s problem was whether or not it was possible to deduce from observation of physical phenomena the causal relationship between them. Hume’s answer is negative: It is impossible to penetrate into the causality of observable phenomena. Human understanding should be restricted to the phenomena themselves. However, Einstein’s conception of the real world of matter as the cause of sense impressions and of the cognisability of the objective laws of motion were not in the least shaken by reading Hume. Einstein proceeds from the idea that a series of observable phenomena does not determine unequivocally the nature of the causal relationships between them. Hence the picture of causal relationship is to some degree deduced independently of direct observations. Einstein speaks of the free construction of concepts expressing causal relationships. Does this mean that such concepts are a priori or conceptual, or that causal concepts are arbitrary as a whole? The answer is no. The causal connections of processes may be expressed by means of different kinds of constructions. In this sense, their choice is arbitrary. But they must be in agreement with observation; and it is our duty to select the construction which agrees best.

When Einstein was trying to formulate both his special and general theories of relativity\(^5\), he did not study mathematics, nor did he supplement his scientific training by looking at the latest experiments on the speed of light. Rather, he read about Ernst Mach, Immanuel Kant and David Hume. His essential thinking was philosophical, thinking deeply about the meaning of science, the problem of knowledge and the philosophical meaning of spacetime. In this way, he was brought to ask some fundamental questions.\(^5\)

\(^5\)The theory of general relativity is based on a very simple idea, which can be explained to a teenager. One must only imagine the experience of falling and recall that those who fall have no sensation of weight. In the hands of Einstein, this everyday fact became the opening to a profound shift in our way of understanding the world: while you can abolish the effects of gravity locally, by freely falling, this can never be done over a large region of spacetime. Therefore, while curved space(time) can be approximated by a patchwork of small flat regions, these regions will always have discontinuities where we try to join them at their edges. This could be taken to mean that the overall space is curved. To put it in a more suggestive and provocative way: the very fact of this failure to join smoothly is the curvature of space.

\(^6\)At the center of the discussion about quantum theory was a great debate between Einstein and the great Danish physicist Neils Bohr\[^8\]. Nearly every time they met, from their first meeting until Einstein’s death, they argued about the meaning of quantum theory. Einstein believed that the goal of physics was to construct a description of the world as it would be in our absence. For Einstein, probabilistic theories were extremely interesting and important and beautiful. But they were neither fundamental physical theories, nor objective. They were subjective theories, theories which we have to introduce because of the fragmentary character of our knowledge. On the other hand, Bohr believed that this was impossible. For him, physics was an extension of the common language, which people used to apprise each other of the results of their observations of nature. Einstein’s view of nature was classical, believing that it is possible to construct an objective picture of the world, and thereby capture something of the eternal transcendent reality behind nature. Bohr, however, believed in the principle that properties are only defined relationally, and that physics is an aspect of our relation with the world. Einstein revealed his uneasiness and discomfort with the quantum theory in his two famous sayings: “God does not play dice with nature” and “God is subtle but not malicious”\[^8\],\[^9\]. According to Einstein, he had spent hundred times more effort in trying to understand the quantum theory as
Einstein always began with the simplest possible idea when confronting a problem, no matter how complicated. Then, by describing how he saw the problem, he put it into the appropriate context. This intuitive approach was like painting a picture. One could even speak of the music of his work; following Einstein’s train of thought in attacking any problem is like listening to a musical piece in which every note is uniquely determined by the dominant theme. In fact, one aspect of Einstein’s genius was his astounding ability to confront the deepest of problems in the most simple, direct and crystal-clear manner throwing a penetrating light on such intricate and difficult issues by the visionary lucidity of his ideas.

No one can deny that both the special and general theories of relativity shook the foundations of physics at their time, giving us a new understanding of the world around us. Both theories went far beyond anything that was well established in the world of physics. The special theory of relativity, developed by Einstein in 1905, gave rise to a truly major revolution in our notions of space and time. The effects of this revolution on the known laws of physics were soon felt, as theorists began to reformulate them within a framework compatible with special relativity. The attempt to reformulate the theory of gravitation produced a second major revolution in our notions of space and time; one that was even much more radical than that produced by the special theory of relativity: Einstein’s general theory of relativity, developed in 1915 [4]. Though other scientists took part in the discovery and development of the special theory of relativity, the theory of general relativity was unquestionably the work of one man; Albert Einstein. In fact, the special theory of relativity would have been discovered sooner or later even in the absence of Einstein. The ideas in the scientific community were ripe and mature enough to give birth to the special theory of relativity. However, the general theory of relativity was totally unaccepted. In fact, thanks to Einstein, it could have taken another hundred years or more for the general relativity to be discovered! Moreover, most probably more than one person would have contributed to the theory.

1. The Special Theory of Relativity

Henceforth space by itself and time by itself are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality. Hermann Minkowski [8]

opposed to his general theory of relativity. A debate between Einsein and Bohr will be given at the end of the paper.

It should be noted that the contribution of Einstein to SR is more significant and more important than any contribution due to other scientists (Lorentz, Poincare, ...!)

One of Einstein’s teachers at the Institute of Technology in Zurich, Switzerland, was the mathematician Hermann Minkowski. Two years after the revolutionary paper of Einstein on the nature of space and time, Minkowski made an extremely important contribution to the theory developed by his former student. He suggested a geometric representation for the theory of special relativity (SR) that explained all exotic predictions of SR in a natural and elegant way. His idea was simple. Since the Lorentz transformation upon which SR is based involves a transformation of both space and time, one may treat time just like another dimension of space; a forth dimension as it were. This very fruitful idea of a four dimensional space became known as Minkowski space $M^4$. Addition of a fourth dimension,
Space is different for different observers, time is different for different observers; spacetime is the same for all observers. John Wheeler

Special theory of relativity is based on the following two principles:

(a) The principle of the constancy of the speed of light: the speed of light is always the same independent of the motion of the source.

(b) The principle of relativity: All laws of physics are invariant relative to inertial observers.

Einstein chose between mechanics and electrodynamics in favour of electrodynamics. He demonstrated that Maxwell’s electrodynamics is entirely consistent with the

time, gave a geometrical meaning to that of an event, a localization of a material particle at a given point at a given instant. Four dimensional geometry and the concept of four dimensional spacetime were then used to develop the laws governing the motion of bodies. In Newtonian mechanics, a vector $a$, for example, the momentum of a particle, is represented by three components $a = (a_x, a_y, a_z)$. In Minkowski four dimensional space, a vector $A$ has three space components in addition to the time component $A = (A_t, A_x, A_y, A_z)$. The union of space and time represented by a Minkowski space gives us a new and deeper understanding of the laws of dynamics. For example, it turns out that the four velocity of any particle $v = (v_t, v_x, v_y, v_z)$ representing its motion in spacetime has norm (length) $||v|| = c$ (c is the speed of light). This means that all bodies move at the speed of light in spacetime. Accordingly, any particle at rest with respect to some observer moves only in time at the speed of light with respect to such an observer; a photon, on the other hand, moves only in space at the speed of light relative to all observers. The larger the motion in space, the slower the motion in time and vice-versa. In general, the four velocity of any body has different “projections” in space and time as revealed to different observers. The energy and the momentum of a particle also acquire a totally different and much clearer interpretation than that in the Newtonian framework. Again, they form a single four vector; the four momentum $P = (E, p_x, p_y, p_z)$ whose norm is constant $||P|| = c^2$. The energy represents the time component of $P$, whereas $(p_x, p_y, p_z)$ are the space components. Similar to the notion of the four velocity, the four momentum of a particle may be viewed as a four dimensional vector of constant length having different components with respect to different observers. Both the notions of four velocity and four momentum have the following simple and elegant geometric representation: each may be viewed as a “rotating” four vector in Minkowski space having constant magnitude. Indeed, the rotation causes the projections of the four vector on the time and space axis to vary; this is precisely the four vector viewed from the perspective of different observers. The constancy of the magnitude of the four vector means that the four vector transcends any particular observer.

Finally, to express the principle of relativity mathematically, we need some basic definitions. For $u \in M^4$, we have $u = (u_t, u_1, u_2, u_3)$, where $u_t$ is the time component and $u_i, \ i = 1, 2, 3$ are the space components. The Minkowski distance or Minkowski metric between two elements $u, v \in M^4$ is defined by $\mu(u, v) := u_t v_t - u_1 v_1 - u_2 v_2 - u_3 v_3$. A Lorentz transformation is a linear transformation $L : M^4 \rightarrow M^4$ that preserves the Minkowski metric: $\mu(u, v) = \mu(L(u), L(v))$. A Poincare transformation is a Lorentz transformation combined with a translation. Expressed in the geometric language of Minkowski space, the principle of relativity can be stated as follows: Laws of nature are invariant (retain their form) under a Poincare transformation.

If one admits that there is an ultimate velocity of signal propagation in nature, its absolute value must be the same in all inertial frames. In fact, all these frames are equivalent according to the principle of relativity. It is impossible to suggest a physical experiment to detect the difference between them. Had the velocity of interaction transmission been different in different inertial frames, it would have been possible to distinguish between them. This is impossible, however, provided the principle of relativity is assumed to be universal. It follows from this that the velocity of light in vacuo must be the same in all inertial frames of reference.

The actual equations that James Maxwell wrote for the electromagnetic field explain, among other
above two principles. Moreover, he constructed a new mechanics, necessarily different from Newton’s to conform with them. He thus showed that Maxwell’s theory is correct being compatible with his new theory, but Newton’s theory must be modified.

In his special theory of relativity, Einstein demonstrated a number of bizarre effects: masses increase, clocks run slower, moving pairs of clocks get out of synchronism (time is path dependent), rods shrink when physical systems move at speeds close to that of light and last, but not least, that simultaneity is frame dependent. An immediate consequence of this is that the same phenomenon will have different appearances relative to observers who move at different speeds. However, Einstein stressed the importance of an underlying unity to nature; no matter how varied these appearances may be, the same

things, how light and radio waves can travel through empty space. Suppose that oscillations are set up in an electrical field. The changing (oscillating) electrical field now acts to generate an oscillating magnetic field; located at right angles to the electrical field. In turn, the variations in the magnetic field are the source of oscillations in the electrical field. Accordingly, the whole thing works like a feedback loop. The entire disturbance moves through vacuum of space at the speed of light and is experienced as light or radio waves.

The special theory of relativity postulates than any physical object which has (non-zero rest) mass cannot move with a velocity greater or equal to the velocity of light. A direct, though dazzling, consequence of this postulate is that the mass of an object grows without bound as its speed approaches that of light (the mass here is understood to be a physical quantity that measures the resistance of the particle to change its state of motion when acted upon by a force). This could be seen from the following reasoning: When we apply a (constant) force to a particle, it gains acceleration and so its speed increases. According to the above mentioned principle, its velocity cannot exceed that of light. Therefore, the mass of the particle, increases in such a way, so that a further increase in its velocity (due to the application of the force) is counterbalanced by the increase in the mass of the particle. In other words, if the original force contributes to a positive increase in the velocity of the particle, then the increase in the mass of the particle may be viewed as a hindering force - opposing the original force - that contributes to a “negative increase” in the velocity of the particle. Both forces, the accelerating force and the de-accelerating force (resulting from the growth of the mass of the particle) combine, so that when the velocity of particle reaches a “critical stage” (acquiring a velocity comparable to that of light), the hindering force becomes dominant and the particle starts moving with a negative acceleration, that is, the rate of change of its velocity starts to decrease. As the particle accelerates, it picks up momentum (due to its increase of mass) and not speed!

One of the fundamental concepts that acquired a new meaning within the framework of special relativity is the notion of “simultaneity”. According to Newtonian mechanics, it was considered absolutely obvious that events which are simultaneous in one frame are simultaneous in all frames. It is easy to see, however, that this statement contradicts the principle of the constancy of the speed of light. Indeed, consider two bodies $K$ and $K'$ (which we take as our frames of reference) with their corresponding clocks. Assume that $K$ moves with velocity $v$ relative to $K'$ along the straight line joining their centers. We place two bodies $M$ and $N$ along this line (in the direction of motion) so that they are rigidly joint to $K'$ at equal and opposite distances from its center. Let us consider the same process in both frames, namely the emission of a light signal from the center of body $K'$ and its reaching bodies $M$ and $N$. In the frame $K'$, the light signals reach $M$ and $N$ at the same instant $t'$, since both are at equal and opposite distances from its center. In other words, the two events would appear to be simultaneous in the frame $K'$. In the frame $K$, however, $M$ moves towards the light signal, whereas $N$ moves in the same direction as the signal. Consequently, if $t$ and $t'$ are the time taken by the light signal to reach $M$ and $N$ respectively, then $t$ would be less than $t'$. In other words, the light signal reaches $M$ before it reaches $N$. The two events are not simultaneous in $K$. What appears to be simultaneous in $K'$ is no longer so in $K$. Simultaneity depends on the reference frame. Accordingly, the notion of “now” in the physical world is simply meaningless!
underlying law must hold for all observers. Einstein was saying that the mathematical structure of a physical law must not change as we go from one observer to the other. Laws of nature have the same form relative to all inertial observers. In fact, the essence of the special theory of relativity may be summed in the following statement, due to John Wheeler: space is different for different (inertial) observers, time is different for different (inertial) observers; spacetime is the same for all (inertial) observers.

The second postulate of relativity theory states that “Laws of nature remain invariant relative to all inertial reference frames”. Any physical law retains its structure or external form independently of the observer. Though the physical quantities involved in one and the same physical law may vary from one frame to another, they would always combine in such a way through the same mathematical pattern. The separate components, namely, the parameters that constitute a physical law may appear different relative to different observers, however, the overall pattern, that is, the general mathematical form of the law remains invariant with respect to all observers. For example, Newton’s law of motion remains valid as a law that describes the motion of a particle under the action of a force. However, each physical quantity appearing in Newton’s law acquires a drastically new meaning in the framework of special relativity theory. The formulation of such a law combines the following concepts: the force acting on the particle, the momentum of the particle (the product its mass and velocity) and its rate of change. Each of these quantities are not absolute, but change from one reference frame to the other (for example, the mass of a moving body is always greater than its rest mass). However, and this is the important point, all observers, no matter what position they stand relative to the particle, would describe the change in motion of the particle as the result of the action of a force acting on the particle. Though, the force differs relative to each observer, the mathematical formula that expresses such a law acquires an absolute status in so far as it transcends the relative position of the subject. Relativity theory, contrary to what is believed, is an objective theory per-exellence in the sense so far discussed. Such an objective character of the theory may be better apprehended if one borrows Einstein’s own words to describe it as the “the theory of invariants.”. Indeed, it is true that the special relativity theory does introduce the position of the observer, making it play a fundamental role in its very formulation. However, it also succeeds, inspite of emphasizing the role played by the subject, to give a clear, unambiguous meaning of an objective physical world that transcends the very existence of the subject. In fact, the second postulate of the theory can be regarded as a definition of the meaning of objectivity in the classical sense.

2. The General Theory of Relativity

Some of the material discussed here may be found in [3].

In 1919, Einstein’s nine-year old son Edward asked him: “Daddy, why are you so famous?” Einstein laughed and then explained quite seriously: “You see, son, when a blind bug crawls along the surface of a sphere it doesn’t notice that its path is curved. I was fortunate enough to notice this”.
I hold in fact (1) That small portions of space are in fact of a nature analogous to little hills on a surface which is on the average flat; namely, that the ordinary laws of geometry are not valid in them (2) That this property of being curved or distorted is continually being passed on from one portion of space to another in the manner of a wave (3) That this variation of the curvature of space is what really happens in that phenomenon which we call the motion of matter. William Clifford as quoted in [15].

It is a matter of fact that Leibniz applies his principle successfully to the problem of motion and that he arrived at relativity of motion on logical grounds..... The famous correspondence between Leibniz and Clarke... reads as though Leibniz had taken the arguments from expositions of Einstein’s theory. Hans Reichenbach, “The philosophical significance of relativity”.

There is no doubt that the intellectual leap accomplished by Einstein to move from the special to the general theory of relativity is one of the greatest in the history of human thought. There are five principles which, explicitly or implicitly, guided Einstein in his attempt to generalize his special theory of relativity to a more general theory that encompasses gravity.[13]

We first recall these five principles, then we elaborate as we go along.

(1) Mach’s principle
(2) Principle of equivalence
(3) Principle of covariance
(4) Principle of minimal coupling
(5) Correspondence principle

The status of these principles has been the source of much controversy. For example, the principle of covariance (laws of nature should be expressed in the language of tensors) is considered by some authors to be empty, whereas others claim that it is possible to derive general relativity more or less from this principle! The status of this principle, we believe, lies somewhere in between these two extreme views. One can say that tensors are one possible mathematical formalism (among other approaches)[14] formulating Einstein’s

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[13]The logical core of the special theory of relativity (the relativity of space and time) is simply in direct conflict with the logical core of Newton’s theory of gravitation (based on the assumption that space and time are absolute). It is strange that no one other than Einstein struggled to modify Newton’s theory to make it compatible with his special theory of relativity. This actually took Einstein eight years of hard work!

[14]One such approach is due to Istvan Nemeti and Hajnal Andreka, the Hungarian Professors at the mathematical institute of Hungarian Academy of Science, together with their student Judit Madarasz. They have succeeded to formulate the special theory of relativity in an axiomatic method based on a transparent relatively simple set of axioms. Towards general relativity, they played with models where observers are allowed to accelerate. At the limit, using ultra products or non-standered analysis, they
general theory of relativity. There is a fairly general agreement, however, that among the five principles, the equivalence principle is the key principle. It has a special status. Indeed, the mere fact that Einstein built the theory on this principle justifies its importance. One source of the different views concerning the other four principles arises from the fact that these principles are more of a philosophical nature. Accordingly, they are not on an equal footing with the equivalence principle in so far as they invoke controversy.

2.1 Mach’s Principle

The writings of Ernst Mach, the great nineteenth-century physicist and philosopher, had deeply influenced the young Albert. Mach not only made significant contributions to the foundations of physics, but his writings also deeply influenced the logical positivists. obtained two notions which are two sides of the same coin. One is gravity, or rather the logistic formulation of a four dimensional manifold (curved spacetime) which is basically a geometry in the very broad sense of the word. The other is acceleration, viewed as a delicate patching of instantaneous inertial frames. The duality between gravity and acceleration, in short, the equivalence principle, is thus formulated as a typical adjoint situation that pops up in different parts of mathematics [10].

This approach to general relativity uses the machinery of algebraic logic in formulating Einstein’s field equations avoiding the use of tensors! The goal of the project is to prove strong theorems of the special and the general theory of relativity from a small number of easily understandable and convincing axioms. In doing so, the authors try to eliminate all tacit assumptions from relativity and replace them with explicit and crystal clear axioms in the spirit initiated by Tarski in his first order axiomatization of geometry [17].

15The idea of abolishing gravitational effects by the process of free fall was considered by Einstein as the “happiest thought of my life” [4]. Einstein was aware that such a simple idea would be a breakthrough in generalizing his special theory of relativity to a more general theory of gravity: gravity cannot be abolished globally, but only locally!

16The geometric language of the general theory of relativity is 4-dimensional Riemannian geometry. Though it is the best known theory for studying gravitational interaction, so far, it suffers from some problems. Examples of these problems are: the horizon problem, the initial singularity, the flatness of the rotation curve of spiral galaxies [15], the Pioneer 10, 11-anomaly [15] and the interpretation of supernovae type-Ia observation [5]. Some of these problems are old (for example, the initial singularity and the horizon problems), while others have been discovered in the last ten years or so. In formulating general relativity (and possible generalizations), some authors prefer to use more general geometric structures (cf. [1, 2, 21], [20], [19], [21], [25]). The author of this paper, together with Professor Nabil L. Youssef and Professor Mamdouh I. Wanas, constructed a unified field theory (UFT) unifying gravity and electromagnetism and possibly other interactions in a much richer context than Riemannian geometry ([22], [23]). Unlike the classical general theory of relativity, which is formulated on the base manifold \( M \) (spacetime manifold of dimension 4), the UFT is constructed in the tangent bundle \( TM \) of \( M \) (a manifold of dimension \( 2 \times 4 = 8 \)). These “extra degrees of freedom” make the UFT “wide enough” to describe electromagnetic interactions on a geometric basis giving matter a geometric origin. It is still an open question, though, whether the UFT is capable of achieving a geometric description of other micro-phenomena.

17A school of thought that played an important role in the development of science. They believed that the reason for the lack of progress in the domain of philosophy is that most philosophers use ambiguous, abstruse and vague concepts in tackling fundamental questions. As such, they regarded most such doctrines of thought as having a negative influence on scientific knowledge hindering its progress. Considering mathematics and physics as the only exact sciences, the logical positivists struggled to eliminate “metaphysical” ideas from the world of science and philosophy. They claimed that such ideas were not well defined sometimes even paradoxical and thus may lead to unresolved contradictions.
He was the kind of philosopher who was always asking troublesome questions. Mach had been particularly puzzled by the meaning of certain properties like linear and angular momentum in an otherwise empty space. In fact, Einstein specifically referred to this question of Mach in the first pages of the great paper “The Foundations of the General Theory of Relativity”. What, Mach had asked, would it mean to say that a planet is rotating in empty space; that is, if there was nothing else in the universe against which this rotation can be measured? For example, if the sky were totally empty, how would we ever know that the earth spins on its axis? It is possible to answer that the earth bulges at the equator because of the effect of centrifugal force. Because we can observe this bulge, it implies that the earth must indeed be spinning. But what exactly is this centrifugal force, and how and why does it arise? If we choose a set of axis that rotates at exactly the same speed as the planet, then everything would appear stationary in this otherwise empty space. Where does this force have its origin? What does it mean for there to be a direction of spin when nothing else is present?

In short, Mach’s principle means that there is no meaning to the concept of absolute motion, but only that of relative motion. In a populated universe, it is the interaction

*Their motto was: define your terms before using them!* In fact, Einstein in his early days was largely influenced by such a school. His paper on the special theory of relativity was written to a large extent in the spirit of the above motto. Years later, though, Einstein regarded logical positivism as somewhat rigid and unflexible. He realized that metaphysical ideas could be useful and effective in the development of scientific thought. In formulating new ideas, one needs a margin of ambiguity. Ideas should not be strictly defined before hand, they are not “born” in the best possible form, but acquire articulation and exactness as they evolve. They are dynamic with no well defined boundaries.

Sad to relate, Mach in his old age, after his insights had been incorporated by Einstein into a successful theory, refused to accept relativity.

Roger Penrose asked the same question while constructing his twister theory. Quantum theory gives a meaning to the spin of the electron. It claims that an electron can spin in one of two alternative directions: “up” or “down”. But what meaning would these alternatives have when the universe is totally empty? What would the difference be between a spin up and a spin down? We should expect such differences to manifest themselves only when a number of other reference points are present. If the distinction between a spin up and a spin down is to have meaning within a quantum theory set in empty space, this seems to imply that rather than living in some sort of a general background space, electrons actually create their own spacetime. Each electron would therefore have an associated “primitive space” - possibly at this stage nothing like our own spacetime at all. However, as large numbers of electrons combine, Penrose conjectured, it is possible that such individual “protospaces” to fuse together, giving rise to a collective space; a sort of shared spatial relationship which may begin to resemble our own space. Penrose was actually using Mach’s ideas now in the broader context of a quantum theory of gravity: the properties of space(time) are both a reflection and a result of the properties of the objects living in space(time).

Leibniz, among Newton’s critics, saw most deeply why Newton’s conception of absolute space and time could not ultimately succeed. The argument Leibniz makes for his relational point of view is one of the most important in the whole history of philosophizing about nature. We cannot do better than reproduce his words: “I am granted this important principle that nothing happens without a sufficient reason why it should be thus rather than otherwise...I say then that if space were an absolute being, there would happen something for which it would be impossible that there should be a sufficient reason, and this is contrary to our axiom. This is how I prove it. If we suppose that space is something in itself, other than the order of bodies among themselves, it is impossible that there should be a reason why God, preserving the same position for bodies among themselves, should have arranged bodies in space thus, and not otherwise, and why everything was not put the other way round (for instance) by
between the matter in the universe - other and above the gravitational interaction - which is the source of inertial effects. In our universe, the bulk of the matter resides in what is called the fixed stars. An inertial frame is a frame in some privileged state of motion relative to the average motion of the fixed stars. More specifically, each and every body is coupled to the whole universe through its interaction with all other bodies.

Leibniz taught us to reject any reference to a-priori and immutable structure, such as Newton’s absolute space and time. But he did not tell us what to replace them with. Mach did, for he showed us that every use of such an absolute entity hides an implicit reference to something real and tangible that has so far been left out of the picture. What we feel pushing against us when we accelerate cannot be absolute space, for there is no such thing. It must somehow be the whole of the matter of the universe. Einstein took a third step in the transformation from an absolute to a relational conception of space and time. In this step, the absolute elements, identified by Mach as the distance galaxies, are tied into an interwoven, dynamical cosmos. The final result is that the geometry of space and time - which was for Newton absolute and eternal - became dynamical, contingent and lawful.

Let us now return to the relational viewpoint of space and time. Space and time are merely bookkeeping devices for conveniently summarizing relationships between objects and events in the universe. The location of an object in space has meaning only in comparison with another. Space and time are the vocabulary of these relations and nothing more. The above ideas may be succinctly summarized as follows:

\[ M_1: \text{Matter distribution determines the geometry. By the geometry of the universe is meant the privileged paths which particles and light rays travel.} \]

\[ M_2: \text{If there is no matter there is no geometry.} \]

\[ M_3: \text{A body in an otherwise empty universe should possess no inertial properties.} \]

2.2 The Equivalence Principle

The equivalence principle may be summarized in the following four principles:

\[ P_1: \text{The motion of a gravitational test particle in a gravitational field is independent of its mass and composition.} \]

This is known as the strong form of the equivalence principle. In Newtonian theory, it is an observational result; a coincidence. It is possible and compatible with Newton’s theory of gravity that if we look closer, with an accuracy greater than 1 in \(10^{12}\), different bodies would undergo different accelerations when placed in a gravitational field. In general relativity, however, it forms the essential logical core of the theory; if it falls then so does the theory.

\[ \text{changing east and west.} \]

\[ ^{21}\text{A particle which experiences a gravitational field without altering the field.} \]
$P_2$: The gravitational field is coupled to everything.

This is known as the weak form of the equivalence principle. It makes explicit the assumption that matter both response to, and is a source of gravity. In other words, *no body is shielded from a gravitational field*. However, it is possible to remove gravitational effects (and hence regain special relativity) by considering a local inertial frame; a frame in a state of free-fall. This naturally leads to the following:

$P_3$: There is no local experiment that distinguishes a non-rotating free-fall in a gravitational field from uniform motion in space in the absence of a gravitational field.

Einstein noticed another *coincidence* in Newtonian theory. All *inertial forces* are proportional to the mass of the body experiencing them. The gravitational force has the same property. This led Einstein to conclude that gravitational (inertial) effects are inertial (gravitational) effects. Put differently, *gravitation is an effect that arises from not using an inertial frame*. Consequently, we have

$P_4$: A frame linearly accelerated relative to an inertial frame in special relativity is locally identical to a frame at rest in a gravitational field.

The last two versions of the equivalence principle can be **vividly** clarified by considering Einstein’s famous lift experiment.$^{24}$

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$^{22}$A reference frame is inertial in a certain region of space and time when throughout that region of spacetime and within some specific accuracy, every test particle originally at rest with respect to that frame remains at rest, and every free test particle initially in motion with respect to that frame continues its motion without change in speed and direction.

$^{23}$This means that gravity and acceleration are actually *indistinguishable*. The distinction between who is accelerating and who is not may be thought of as part of the intrinsic structure of space and time. For Newton, it was absolute. Mach and Einstein made it dynamical; the distinction can be made differently at different places and at different times.

$^{24}$Einstein always loved simple examples, pictures that one might consider with suspicion on account of their apparent naïveté; for example, of trains, boxes, rooms and lifts. One aspect of Einstein’s genius was his astounding ability to express deep and subtle ideas through the invention of “simple” and sometimes apparently naïve thought experiments. Einstein devised a thought experiment that illuminated one of the most important predictions of the special theory of relativity; namely, the **equivalence** of mass and energy. (Einstein demonstrated that the energy $E$ of a particle and its mass $m$ are not independent quantities but are related by the equation $E = mc^2$, where $c$ is the speed of light. This is probably the most famous equation in modern physics!). Consider two identical particles (i.e. having the same rest mass) moving relative to an observer in a straight line towards each other with equal opposite velocities, say $v$ and $-v$. As the mass depends on the (square of the) velocity, both particles will have the same mass $m$ relative to the observer, which is, according to the predictions of the special theory of relativity, greater than their rest mass (if $m_o$ is the rest mass of each particle, then $m = m_o/(1 - (v/c)^2)^{1/2}$, where $c$ is the speed of light). Since both particles move with the same velocity in two opposite directions, the total momentum of the system vanishes (the momentum is a vector quantity which depends on the direction of motion. Hence, if the momentum of the first particle is $mv$, then the momentum of the second particle, owing to the fact that it is moving in the opposite direction, is given by $-mv$. The total momentum measured by the observer is thus found to be $mv - mv = 0$). The law of **conservation** of momentum tells us that the total momentum of a closed system, i.e. any system of particles in which no external forces act, does not change with time. This implies that the total momentum of the system under consideration before, after and at the instant of collision remains identically zero. In particular, at
Case 1: The lift is placed in a rocket ship in a part of the universe far removed from gravitational influence. The rocket is accelerated forward with constant acceleration $g$ relative to an inertial frame. The observer in the lift releases a body from rest and (neglecting the influence of the lift) sees it fall to the floor with acceleration $g$.

Case 2: The rocket engine is switched off so that the lift undergoes uniform motion relative to the inertial observer. A released body is found to remain at rest relative to the observer.

Case 3: The lift is next placed on the surface of the earth, whose rotational and orbital motion are ignored. A released body is found to fall to the floor with acceleration the instant of collision, the two particles appear to the observer to be momentarily at rest, and so he must be observing their rest mass $m_0$. Accordingly, the total mass of the system before (and after) collision appears to be greater than its mass at the instant of collision. This, however, violates the principle of conservation of mass. In fact, the difference in mass, according to the principle of equivalence of mass and energy, is lost or transformed into the energy of deformation and the internal frictional forces resulting from the collision of the two particles. A glaring example of a (very simple!) physical situation in which mass is transformed into energy. It should be noted, though, that the previous arguments are not a proof of the equivalence of mass and energy, but merely a manifestation of such a principle.

Another brilliant thought experiment was introduced by Einstein in his passage from the special to the general theory of relativity. Einstein devised a thought experiment showing that the curving of space(time) is in fact demanded when (constant) accelerated motion is combined with (the predictions of) the special theory of relativity (SR + ACC $\rightarrow$ CUR) [16]. Take the case of a rotating disc such as the platform of a merry-go-round. Contemplation of the seemingly simple mechanics of such a disc actually provides deep conceptual insight. It is a stepping stone for the transition from the special to the general theory of relativity. In fact, this thought experiment was a turning point in Einstein’s success to generalize his special theory of relativity to a theory of gravitation. We are assured that the special theory of relativity is valid relative to an inertial frame such as the non-rotating frame outside the merry go round, which we denote by $R$. Relative to $R$, and using meter sticks at rest in $R$, the diameter $D$ and the circumference $C$ of the merry-go-round have the Euclidean ratio $C/D = \pi$. In the rotating frame $R'$, however, points on the rotating platform are accelerating. But if the platform is large enough, then a point on the rotating circumference $C'$ moves almost in a straight line for a short period. We can therefore have a meter stick that moves exactly in a straight line (with constant velocity) move along with this point in such a way that it has for that instant exactly the same velocity as the point on $C'$. In this way, we can calibrate meter sticks at rest on $C'$ with meter sticks moving uniformly in $R$. This permits us to transfer our standard of lengths from the inertial frame $R$ to the non-inertial frame $R'$. According to the predictions of the special theory of relativity, the uniformly moving meter stick will be contracted by a factor $1/\gamma$, where $\gamma = 1/(1-(v/c)^2)^{1/2}$. Therefore the standard at rest on $C'$, which is a copy of it, will be also contracted relative to the one at rest in $R$. This means that the observer in $R'$ will measure along his circumference $C'$ with a shorter meter stick than the observer in $R$. Consequently, the observer in $R'$ finds the circumference $C'$ to be larger than $C$ by a factor $\gamma$. On the other hand, the radial distance on the platform will not be contracted because it is perpendicular to the direction of motion. The diameter $D'$ as measured by the observer in $R'$ will thus be the same as the diameter $D$ measured in $R$. Conclusion: the ratio of circumference to diameter in the rotating frame $R'$ is $C'/D' = \gamma C/D = \gamma \pi$, which is larger than plane Euclidean geometry permits. This led Einstein to propose the following amazing idea: the curving of space is the (explanation for the) violation of ordinary Euclidean geometry. The above argument concerning length measurements can easily be repeated for time measurements. One replaces the meter stick at rest on $C'$ by a clock, and one compares that clock with one that is uniformly moving relative to $R$. One finds again, using the same reasoning as above, that the clock at rest in $R'$ goes slower than the clock at rest in $R$ (time dilation). Moreover, the larger the platform, the slower will be the clock on its circumference. Again time is wrapped: its rate of passage differs from one location to another.
Case 4: Finally, the lift is placed in an evacuated lift shaft and allowed to fall freely towards the center of the earth. A released body is found to remain at rest relative to the observer.

It is clear that $P_3$ implies that cases 1 and 3 are indistinguishable, whereas $P_4$ implies that cases 2 and 4 are indistinguishable.

To sum up, the equivalence principle asserts that it is impossible to tell whether one is in a room which is freely falling in a gravitational field or in a room moving uniformly in deep space. It also asserts that a room which is accelerating steadily in deep space, with the same acceleration as falling bodies at the surface of the earth have, is indistinguishable from a room sitting on the surface of the earth.

### 2.3 The Principle of Covariance

The principle of relativity, which lies at the heart of special relativity, tells us that *all physical laws must be the same regardless of the constant-velocity relative motion that individual observers might experience*. This is a symmetry principle, because it means that nature treats all such observers identically - symmetrically. Through the *equivalence principle* - the cornerstone of the general theory of relativity - Einstein significantly extended this symmetry principle by showing that *the laws of physics are actually identical for all observers regardless of their state of motion: an accelerated observer is perfectly justified in declaring himself to be at rest and claiming that the force he feels is due to a gravitational field*. Once gravity is included in the framework, *all possible observational vantage points are on a completely equal footing*. Einstein thus proposed the following as the logical completeness of the principle of special relativity:

Laws of nature should have the **same** form relative to **all** observers.

Observers are intimately tied up with their reference systems. If an observer can discover a physical law, than any other observer (no matter what kind of motion he is experiencing) will discover the same law. In other words, any coordinate system should

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25 Many beautiful effects follow from this simple principle, such as the bending of light and the slowing down of clocks in a gravitational field [12].

26 Just as the symmetry between all possible observational vantage points in general relativity requires the existence of the gravitational force, developments relying on work of Hermann Weyl in the 1920’s and Chen-Ning Yang and Robert Mills in the 1950’s showed that gauge symmetries require the existence of yet other force fields. Certain kinds of force fields, according to Yang and Mills, will provide perfect compensation for “shifts in force charges”, thereby keeping the physical interactions between the particles completely unchanged. For the case of the gauge symmetry associated with shifting quark-colour charges, the required force is none other than the strong force. A similar discussion applies to the weak and electromagnetic forces, showing that their existence, too, is bound up with yet other gauge symmetries - the so-called weak and electromagnetic gauge symmetries. And hence, all four forces are directly associated with principles of symmetry. [6]
do. The situation is different in special relativity. If spacetime is curved, then there are no frames that qualify as being inertial everywhere. Free falling frames of reference are coordinate systems whose axis are straight only in the vicinity of a point locally. If extended beyond this region, they have no properties that would distinguish them from other curvilinear coordinate systems. In a curved spacetime manifold any coordinate system is as suitable as any other. It is not so much that any coordinate system should work, but rather that the theory should be invariant under a coordinate transformation. This is expressed mathematically as follows:

The equations of physics should be expressed in the language of tensors.

2.4 Principle of Minimal Gravitational Coupling

The minimal gravitational coupling is a simplicity principle. We should not add unnecessary terms in the transition from the special to the general theory of relativity. For example, in the context of the special theory of relativity, the energy-momentum tensor satisfies the conservation law

$$ T^{ab}_{;b} = 0, $$

where “;” denotes partial differentiation. The simplest generalization of the above equation in curved spacetime is the following tensorial equation

$$ T^{ab}_{;b} = 0, $$

where “;” denotes covariant differentiation.

Adding new terms containing the curvature tensor to the last equation will not alter the form of the first equation. For example, we can write

$$ T^{ab}_{;b} + g^{be} F^a_{bcd} T^{cd}_{;e} = 0 $$

\[27\] Technically speaking, because the connection is integrable and the metric is flat, there exists a canonical coordinate system, namely Minkowski coordinates. In a curved spacetime, that is, a manifold with a non-flat metric, there is no canonical coordinate coordinate system.

\[28\] This statement should be treated with caution. In many applications there would be a preferred coordinate system. Many problems possess symmetries. In this case, it is advisable to adapt the coordinate system to the underlying symmetry.

\[29\] Any tensorial equation remains invariant (retain its form) under a change of coordinate.

\[30\] Application of equivalence principle gives same equation in a local Lorentz frame. Since the connection coefficients vanish at the origin of any local Lorentz frame, the above equation may be written in any reference frame in the form $T^{ab}_{;b} = 0$, where “;” denotes covariant differentiation (unlike partial differentiation, the covariant derivative of a tensor is a tensor). Thus we arrive at the following remarkable result: The laws of physics written in component form change in passage from flat spacetime to curved spacetime by a mere replacement of all commas by semi-colons. No change in physics; change due to a switch in reference frames from Lorentz to non-Lorentz!

\[31\] We face here a technical problem. In what order should the derivatives be written when applying the commas goes to semi-colon rule? Interchanging derivatives makes no difference in flat spacetime. In curved spacetime, however, it produces terms that couple to curvature! This is a non-trivial issue.
This is because the Riemann curvature tensor $R^a_{bcd}$ - the mathematical object describing gravity, vanishes in the context of special relativity. We can thus add terms explicitly containing the curvature tensor when passing from the special to the general without violating the form of the equations obtained in the special relativity. Accordingly, Einstein implicitly applied the following principle:

Principle of minimal gravitational coupling: No terms explicitly containing the curvature tensor should be added in making the transition from the special to the general theory of relativity.

### 2.5 The Correspondence Principle

Any new theory must be consistent with any established earlier theory. Thus general relativity must agree with both special relativity (in the absence of gravitation) and Newton’s theory of gravity (in the limit of weak gravitational field and low velocities). In fact, general relativity has as distinct approximate theories:

(a) Special relativity: General relativity has two distinct kinds of correspondence with special relativity. The first is the limit of vanishing curvature (flat spacetime). In this case, one can introduce a global inertial frame and recover completely the special theory of relativity. The second is local rather than global. In a local inertial frame, all the laws of physics take on their special relativistic form (the equivalence principle).

(b) Newtonian theory: In the limit of weak gravitational fields, small velocities (compared to the speed of light), and small pressures and density of matter, general relativity reduces to the Newtonian theory of gravity.

(c) Post Newtonian theory: When first order relativistic corrections are added to the Newtonian theory, that is when Newtonian theory is nearly valid, one deals with Post Newtonian theory of gravity.

(d) Linearized theory: In the limit of weak gravitational fields, but possibly large velocities and pressures, general relativity reduces to the linearized theory of gravity.

### 2.6 Black Holes

We now discuss one of the most important predictions of Einstein’s general theory of relativity. The most dramatic prediction of Einstein’s theory are black holes. A detailed and excellent treatment of the geometry of black holes may be found in [12]. See also [7].

The misleading name (invented by John Wheeler) does not refer to holes, but to regions in space where the gravitational pull is extremely strong, stronger than a certain critical value. The critical value can be characterized as follows: One of the important predictions of the general theory of relativity is that light rays are bent by
Actually, the bending depends not only on the mass $m$ but also on the distance $d$ from the center of that mass at which the light ray passes. The smaller the distance, the larger is the bending. In fact, the bending depends on the ratio $m/d$. As stars burn out, as they deplete energy so that they cannot radiate light any more, the gravitational pull of its matter collapses the star to a very compact “sphere” of extremely high density. As a result, the ratio $m/d$ increases tremendously: A collapsed star can bend light rays a great deal more strongly because the rays pass by it at a much closer range. If a “dead” star collapses to such a small size that the light rays reaching its surface are bent into the interior of the star so that they cannot escape again, it is called a black hole. If light rays get caught so does everything else (travelling at lower speeds). Now a black hole does not send out radiation of its own; therefore if it absorbs all radiation reaching it and reflects none, it is necessarily invisible. The region of space appears entirely black.

We now consider the collapse of a spherically symmetric non-rotating star until its surface reaches its Schwarzchild radius. As long as the star remains spherically symmetric, its external field is given by the Schwarzchild vacuum solution. If signals are sent out an observer on the surface of the star at regular intervals according to his clock, then as the surface of the star reaches the Schwarzchild radius, a distance observer will receive these signals with an ever-increasing time gap between them. In particular, the bending predicted by general relativity is greater than that predicted by Newton’s gravitation.

This is true only in the context of the classical general theory of relativity. Stephen Hawking, the brilliant Cambridge physicist, argued that close to a black hole, the extreme degree of curvature of spacetime actually creates elementary particles. In more detail, taking the quantum theory into consideration, Hawking showed that black holes actually radiate light, so that they are not too black after all! The idea is simple though the calculations are quite tedious. The uncertainty principle, the cornerstone of the quantum theory, tells us that space is a teeming rolling frenzy of virtual particles, momentary erupting into existence and subsequently annihilating one another. This jittery quantum behaviour also occurs on the event horizon of a black hole. Hawking realized, however, that the gravitational might of the black hole can inject energy into a pair of virtual photons. This process tears them just far enough so that one gets sucked into the hole. With its partner having disappeared into the black hole, the other photon of the pair no longer has a partner with which to annihilate. Instead, the remaining photon gets an energy boost from the gravitational force of the black hole and, as its partner falls inward, it gets shot outward, away from the black hole. The combined effect of such a process, happening over and over again around the horizon of the black hole, appears as a steady stream of outgoing radiation. Black holes have temperature; hence have entropy. A partial unification of relativity and quantum theories accomplished by Hawking gave rise to a radically new and totally unexpected description of black holes: “black” holes can actually “glow”! In fact, Hawking’s discovery says that the gravitational laws of black holes are nothing but a rewriting of the laws of thermodynamics in an extremely exotic gravitational context! This was Hawking’s bombshell in 1974. (Incidently, the above phenomenon is an indication that the uncertainty principle (UP) is, in a sense, not a “limitative” result after all. GR without the UP predicts that black holes are completely invisible. On the other hand, GR with UP (as studied by Hawking) predicts that black holes can glow: a combination of general relativity with (the statistical nature of) the quantum theory (applied to a “black” hole) gives the following amazing and astounding result: $GR + UP \implies$ Black holes have entropy. They can glow).

The Schwarzchild radius of the earth is 1 cm and that of the sun is 3 km.

Technically, this is known as Birkhoff’s theorem: Let the geometry of a given region of spacetime be spherically symmetric and be a solution to Einstein field equations in vacuum. Then that geometry is necessarily a piece of the Schwarzchild geometry.
the signal sent off at \( r = 2m \) will never escape from \( r = 2m \), and all successive signals will ultimately be dragged back to the singularity at the center. No matter how long the distant observer waits, it will be only possible to see the surface of the star as it was just before it plunged through the Schwarzschild radius. It follows that there is a great difference in the description of motion of both observers. From the point of view of the *falling* observer, it takes only a *finite* time to reach \( r = 2m \) or even \( r = 0 \). However, relative to the *far away* observer, safely outside the event horizon, it takes an *infinite* amount of time for the falling observer to reach \( r = 2m \). The whole range of \( r \) values, from \( r = 2m \) to \( r = 0 \) is perfectly good physics, and physics that the falling observer is going to see and explore, but physics that the outside observer never will see and never can see. This phenomena is a clear example of what may be called “infinite time dilation”. Note that this occurs in the context of stationary (static) black holes. It is an inevitable consequence of Einstein’s field equations.\(^{36}\)

Andreka and Nemeti showed that if one takes into account the laws of the general theory of relativity, more specifically, the infinite time dilation in strong gravitational fields of (rotating) black holes, then one can imagine “thought experiments” in which Church thesis is no longer valid! In these “thought experiments”, one can compute *non computable functions (in the old sense)* and one can prove that ZFC is consistent! So was David Hilbert, right after all?\(^{37}\) Ironically, it was Kurt Godel who started considering such thought experiments involving rotating black holes with strong gravitational fields.\(^{38}\) Obviously, Godel was not concerned with violating his own incompleteness results!\(^{39}\)

\(^{36}\)The phenomenon of infinite time dilation also shows that different observers in the context of GR may not see the same set of events. Indeed, the falling(in) observer passes through the horizon of the black hole reaching the singularity at the center of the black hole in a *finite* time according to his clock (realistically, of course, the falling observer will be torn to pieces (crushed!) as he gets closer and closer to the horizon). On the other hand, the far-away observer will never “see” *beyond* the horizon of the black hole. All events interior to the horizon simply do *not* belong to his world view. From his “perceptive”, the far-away observer will “think” that the falling observer is coming to a *halt* as he approaches the horizon of the black hole and will never see him crossing the horizon.

\(^{37}\)Five days before Einstein presented his field equations in its final form, Hilbert animated by Einstein’s earlier work, independently discovered how to obtain them from an action principle\(^{12}\).

\(^{38}\)In the 1949 volume celebrating Einstein’s seventieth birthday, Godel presented work that sparked research aimed at finding exact solutions to Einstein’s field equations that were more complex than any previously known. In particular, Godel’s solution allowed *closed time like world lines*, that is, it allowed time to be *cyclic*! The solution was genuinely puzzling and raised very deep questions concerning the nature of time. In fact, Einstein regarded it as too absurd and thought that it should be excluded on physical grounds.

\(^{39}\)Church thesis is the assumption that the class of recursive functions coincide with the class of computable functions. Godel proved that any recursive axiomatization (via Godel numbering) of arithmetic is not complete. This is known as his first incompleteness theorem. His second incompleteness theorem is even more profound, with devastating philosophical implications. It states that any strong enough system (like Peano arithmetic ZFC) cannot prove its own consistency, unless it is inconsistent in which case it can prove anything. So there is always the possibility that ZFC turns out to be inconsistent, though highly unlikely. Returning to recursive and computable functions, Turing gave an equivalent definition of computable functions as those functions that a Turing machine can compute. Other rigorously defined classes of functions, that turn out equivalent to recursive function are given by the Russian mathematician Markov, and Church via his invention of the so called lambda calculus. In fact the Lambda calculus was an outcome of Church’s attempts to violate Godel’s first incompleteness theorem,
Andreka and Nemeti tell us that the above mathematical results (which were thought to be well established) are context dependent; they depend on the physical nature of our world. They are an outcome of a Newtonian world, a world in which time is absolute. By changing the underlying laws of physics, it is consistent with general relativity that one can compute non-computable functions in the older “Newtonian” framework, and one can prove ZFC consistent.

2.7 Gravitational Radiation

In electromagnetic theory, the acceleration of a charged particle produces electromagnetic radiation (that is, light, radio waves, X-rays, and so on). One can think of electromagnetic radiation as “ripples” in the electric and magnetic fields which propagate through spacetime at the speed of light.

In general relativity in a nearly flat region of spacetime one finds closely analogous behaviour. Acceleration of a mass produces gravitational radiation which can be thought of as ripples in the spacetime curvature, again propagating at the speed of light. In a strongly curved region of spacetime the distinction between these ripples and the background curvature of spacetime is unclear and one cannot give a precise definition of gravitational radiation \[12\] However, outside a strongly curved region (for example, far away from a black hole) the notion of gravitational radiation is unambiguous. Astrophysical events which are likely to produce large amounts of gravitational radiation are

(a) Supernovas or other gravitational collapse phenomena\[41\]
(b) Accretion of a star into a black hole at the center of a galaxy or star cluster.
(c) Coalescence of neutron stars and/or black holes in a close binary orbit. Hence, the detection of a burst of gravitational radiation may give us information about phenomena probably involving a black hole.

2.8 The Twin Paradox

According to the first postulate of the special theory of relativity, the velocity of light is the same for all observers. From the constancy of the speed of light follows the relativity of time. It is not true that there is an “objective time” spanning the whole universe.

via a system that Kleene proved inconsistent. Markov’s procedure is now termed Markov’s algorithms highlighting the algorithmic character of computing recursive functions. Today a recursive function is one that a computer can determine. A historical comment: When Godel proved his first incompleteness result he dealt only with the now called primitive recursive functions, which he attributed the name recursive to them. This is a class that is strictly smaller than that of the now called recursive functions, which Church introduced as an attempt to violate Godels first incompleteness theorem thinking that it does not apply to the wider class of the now called general recursive functions. Godel however showed that his reasoning applies to this wider class.

\[40\]This is one of the difficult technical problem in general relativity \[12\].
\[41\]A supernova should produce a large burst of neutrinos.
“Time is not an absolute physical quantity flowing by itself with no relation to anything external, only due to its internal structure,” as Newton believed, but varies from one inertial frame to the other. One and the same event, when observed from different frames, may appear to occur during different time intervals. This is exactly what is meant by relativity of time. In particular, the time of an event measured by an observer $S$ relative to which the event is stationary, called proper time, would be less than that measured by another observer $S'$ moving relative to the same event. Every particle has its own proper time as measured by an observer moving with it. Moreover, the time measured by the observer moving with the particle (relative to which the particle is at rest) would be less than that measured by other observers (relative to which the particle is moving). In brief, proper time is the time registered by a particle using its own clock. All observers agree on the proper time. Proper time, unlike “coordinate time”, is an absolute physical quantity; it measures the rate of aging of the particle in motion: the faster the particle moves, the slower it ages! Accordingly, clocks associated with moving particles are slowed down and so a photon of light does not age.

All kinds of objections were raised against the special theory of relativity due to its

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42 More specifically, let $k$ and $m$ be two inertial observers with relative constant velocity $v$. Suppose that $e_1$ and $e_2$ are two events occurring at the same location with respect to $k$. Then, owing to the motion of $m$ relative to $k$, the two events necessarily occur at different locations with respect to $m$. Let $t_k = time_k(e_1, e_2)$ and $t_m = time_m(e_1, e_2)$ be the time measured between the two events by both $k$ and $m$, respectively. Then $t_k$ is the proper time (the two events occurred at the same location relative to $k$), $t_m$ is the coordinate time (the two events occurred at distinct locations relative to $m$) and $t_k$ is strictly less than $t_m$: such a difference increasing with the increase of the magnitude of the relative velocity $v$. In fact, using the constancy of the speed of light, Einstein presented the following ingenious thought experiment to show that time is not absolute but varies from one frame of reference to the other. He argued as follows: We take $k$’s frame of reference to be a train moving with uniform velocity $v$ relative to $m$ who is standing on the platform. Observer $k$ holds a torch in his hand. The ceiling of his carriage has a mirror attached to it. He turns on his torch sending out a light signal towards the mirror. Let $e_1, e_2$ be the sending and the reception of the light signal, respectively. Relative to $k$, the light ray is reflected back from the mirror reaching him at the same location from where he has sent it. Assuming that the mirror is at a perpendicular distance $d$ from where $k$ is sitting, the light ray, according to $k$, has travelled the distance $d_k = 2d$ going vertically up and down. The two events occurred at the same location with respect to $k$. On the other hand, because the train is moving relative to $m$, the sending and the reception of the light ray take place at distinct locations according to him. This implies that the path traversed by the light ray is larger than $2d$. In fact $d_m = 2(3d^2 + \frac{1}{2}(vt_m)^2)^{1/2}$. Indeed, the light ray traverses a “triangular” path with base $z = vt_m$ and height $d$. Since the speed of light is the same for both observers, it follows that $t_k = d_k/c < t_m = d_m/c$. In fact, a very elementary calculation based on Pythagoras theorem shows that $t_k = (1-(v/c)^2)^{1/2}t_m$. One of the most important and revolutionary ideas in the history of science, namely, the relativity of time, is deduced by Einstein from a very simple thought experiment that can be easily understood by a teenager!

43 The length of an object also loses its absolute nature. Moving objects tend to contract in their direction of their motion. Any object, for example, a rod of proper length $l_o$ (its length in its rest state.) would appear to have length $l$ less than $l_o$ when viewed by another observer relative to which it is moving; such a contraction depending on the (square of the) velocity. In fact, $l = (1-(v/c)^2)^{1/2}l_o$. Consequently, the distance between points in space and the interval between points in time are not, as Newton claimed, well defined unambiguous concepts, but depend on the position of the observer. As Einstein says “It is neither the point in space nor the instant in time, at which something happens, that has physical reality, but only the event itself. There is no absolute relation in space, no absolute relation in time, but only absolute relation in space and time.”
anti-intuitive character and revolutionary ideas. One of the earliest and most persistent objections centered around what was referred to as the twin paradox. This paradox was in fact introduced by Einstein himself in 1905, in his paper on special relativity. The twin paradox has caused the most controversy - a controversy which raged on and off for over 50 years. The paradox is usually described as a thought experiment involving twins. Both twins synchronize their watches. One of them then gets into a spaceship and makes a long trip through space. Assuming that he has travelled with a speed comparable to the speed of light time dilation would be large. Accordingly, he finds himself much younger than his twin brother who has stayed behind on earth as he has aged much less than his twin brother. In fact, if the spaceship travels just under the limiting speed of light, time within the spaceship will proceed at a much lower rate; judged by the earth time (the stay-at-home twin’s clock), the trip may take more than one thousand years, whereas judged by the travelling twin’s clock, the trip may take only a few decades! (the faster the spaceship, the less the travelling twin ages!) This involves no paradox; being actually a direct consequence of the predictions of the special theory of relativity. The paradox, however, becomes apparent when we describe this thought experiment from the “perspective” of the general theory of relativity. Indeed, according to the general relativity, there is no absolute motion of any sort; no preferred frame of reference. It is always possible to take any moving object as a “fixed” frame of reference. This seems to suggest that the situation is totally symmetric. By taking the travelling twin as the fixed frame (now the earth makes a long journey away from the ship and back again), we are led to conclude that the stay-at-home twin is necessarily younger than the travelling twin! However, there is an important difference between the status of the two twin brothers; the stay-at-home twin moves on a geodesic of spacetime while the travelling twin does not. Thus the situation is not symmetric after all. One concludes from the above arguments that the twin paradox, though anti-intuitive, is only an apparent and not a genuine paradox. Moreover, it is perfectly compatible with the predictions of both the special and the general theories of relativity.

2.9 A Glimpse of Einstein’s Field Equations

In a nutshell, Einstein’s field equations say that matter curves spacetime and curved spacetime tells matter how to move. In more detail, Einstein’s equation can be read as follows: matter - represented by the energy momentum tensor - curves spacetime (such curvature given by the Einstein tensor) and curved spacetime tells matter how to move, namely, on timelike geodesics of the resulting geometric structure of spacetime.

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44 Accordingly, Einstein has specified the mechanism by which gravity is transmitted: the wrapping of spacetime. Einstein tells us that the gravitational “pull” holding the earth in orbit is not, as Newton claimed, a mysterious instantaneous action of the sun; rather, it is the wrapping of the spatial fabric induced by the sun.

45 In curved spacetime, bodies having non-zero rest mass move on paths that maximize proper time choosing the “straightest” or “longest” path compared to all nearby paths. Such a property was described by the English philosopher and mathematician Bertrand Russel as cosmic “laziness”. In fact, geodesics in curved four dimensional spacetime are the analogue of straight lines in flat spacetime. Geodesics appear to us as curved paths (for example, the trajectories of planets around the sun) because we tend to separate between space and time. Had we been four dimensional creatures, we would
Einstein’s equation representing such an interaction is prominently non-linear: matter distorts spacetime geometry, the resulting (distorted) geometry constrains the motion of matter, which, in turn, affects the geometry and so on and so forth. The left hand-side of Einstein’s equation is a purely geometric entity, whereas the right hand-side is more of an empirical or physical nature. The equation is not to be regarded as an identity; the Einstein’s tensor does not define the energy momentum tensor. Rather, the equation represents a dynamic feedback loop between two distinct yet compatible entities. This dynamic relation between matter and geometry; this feedback loop, is coded in the following very elegant (and simple!) equation

\[ G_{ab} = 8\pi T_{ab}. \]

On the right, stands the source of curvature, namely, the energy-momentum tensor. On the left, stands the receptacle of curvature in the form of what one wants to know, the metric coefficients twice differentiated (the Einstein tensor). The equation is in line with Mach’s principle as expressed in \( M_1 \); the matter distribution \( T_{ab} \) determines the geometry \( G_{ab} \), and hence is a source of inertial effects. Here we are reading the field equations from right to left. We want to determine the metric coefficients from a given energy momentum tensor, that is, the spacetime geometry corresponding to a given distribution of matter. Conversely, we may regard the field equations as defining an energy momentum tensor corresponding to a given spacetime geometry. In this case, we are reading the field equations from left to right. It was originally thought that this is a productive way of determining energy-momentum tensors. However, this rarely turned out to be very effective as the resulting energy-momentum tensors violated some essential physical constraints.

When Einstein had created his general theory of relativity, he is supposed to have said that while the left hand side had been curved in marble, the right hand side was built out of straw\(^1\). The left hand side of Einstein’s equations referring to the actual geometry of spacetime is surely one of the great insights of science. The right hand side describing how the mass and energy produces this curvature did not follow with such elegance as the geometric part of the field equations\(^1\). Most physical theories nowadays,\(^4\)

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\(^{(1)}\) Einstein always hoped to extract the matter content of spacetime from its geometrical properties. He always regarded the fact that the right-hand side of his famous field equations containing the phenomenological tensor \( T_{ab} \), an essentially non-geometric entity, a blemish on his theory. In fact, Einstein was rather sceptical about the full field equations and regarded the vacuum field equations, namely \( G_{ab} = 0 \), as more fundamental.
namely, string theory, twister theory... are attempts to fully understand the right hand side of Einstein’s field equations trying to establish a discrete description of (the geometry of) space and time that might lead to a smooth merging of both quantum and relativity theories.

We note that the field equations show how the stress energy of matter generates an average curvature in its neighbourhood. It governs the external spacetime curvature of both a static and dynamic source, the generation of gravitational waves (ripples in the curvature of spacetime) by stress energy in motion, the external (and internal) spacetime geometry of a (static and rotating) black hole and, last but not least, the expansion and the contraction of the universe.

In conclusion, the field equations not only described the dynamics of the universe...
on large scale, but also predicted mind-boggling physical phenomena (black holes, event horizon, time wraps, gravitational waves, ...etc) that opened up new horizons for research in the world of physics; phenomena that are till this very moment (almost a century after the discovery of general relativity) the subject of intensive investigation and thorough scrutiny!

3. An Axiomatic Approach to Relativity Theories

Logical axiomatization of physical theories is far from being a new idea. It goes back to such leading mathematicians and philosophers as Hilbert, Godel, Tarski, Reichenbach, Carnap, Suppes, ... and many other eminent scientists. There are many examples showing the benefits of such an axiomatic approach when applied to the foundation of mathematics. Accordingly, it is natural and useful to apply such a method to physical theories; in particular to spacetime theories like both the special (SR) and the general (GR) theories of relativity.

According to the English philosopher and mathematician Bertrand Russell, mathematics does not tell you what is but what will be if, a physical theory is supposed to tell you what is. Accordingly, the process of axiomatization in the realm of mathematics and physics serves different purposes. In the world of mathematics, we are free to choose any set of axioms; statements that we assume without proof, as long as they are non-contradictory. We don’t care whether such axioms or their logical consequences describe any properties of the external world. What really concerns us is the compatibility of the chosen axioms and their consistency (their independance is also important for aesthetic considerations). On the other hand, the status of axioms in the world of physics is different. We are not totally free in our choice of the axioms since our aim is to describe certain regularities of nature. As such we are constrained by the laws imposed on us by the outside world. The domain of application of our chosen axioms is somewhat limited and in some sense fixed.

50 This is only true in the realm of classical physics. In the micro world, matters are more complicated; what we observe is not nature itself, but rather nature as revealed to us through our methods of questioning. The last part of this paper discussing Einstein’s view on the quantum theory will tackle such issues.

51 In mathematics, any theorem that we prove is correct in an absolute sense. According to the Austrian philosopher Karl Popper (regarded as one of the most important philosophers in the twentieth century), the nature of physical knowledge is different. We can’t prove the “correctness” of a physical theory. In fact, any physical theory is necessarily incomplete. It is valid only in a certain restricted domain. The strength of a physical theory is measured by its informative content. The more it tells us about the external world, the more it is powerful and the more it can be potentially “proved wrong”. Accordingly, physical theories can only be falsified and not verified. A single phenomenon that escapes the explanatory power of some physical theory indicates that the theory is necessarily a special case of a more general one that captures such a phenomenon and enlarges the domain of applicability of the original theory. Progress in the physical world is thus accomplished by the replacement of “weaker” theories by “stronger” ones, that is, theories that have more “informative” content. To sum up, the notion of falsification states that our physical knowledge develops by the overthrow of hitherto well-established theories to be replaced by “better”ones, that is theories that have a larger and wider scope thus have more explanatory power. In the language of Darwin, physical theories compete and the ones surviving are the fittest!
The role played by the axioms in the formulation of a physical theory is thus more intricate than their role in the formulation of a mathematical theory. The axioms chosen for a physical theory should be simple, logically transparent, illuminating, intuitively clear and easy-to-believe. All surprising, bizarre, exotic, unexpected or unusual predictions of the theory are to be described, not by the axioms themselves, but rather by theorems derived from the axioms (which we may refer to as “fancy” theorems).

The process of axiomatizing a physical theory provides both precision and rigour for the theory. In fact, the utility of the axiomative system chosen to describe a physical theory could be measured by its ability to describe exotic and strange phenomena from clear and self-evident axioms. Another important advantage that could be gained from the axiomatization of a physical theory is to be able to derive most, if not all, the interesting and unpredictable phenomena of the theory from the smallest number of independent and simple axioms.

Both theories of relativity have consequences that are anti-intuitive and defy common sense. Most of their predictions are bizarre and far from being evident. For these reasons, it is interesting and important to try to construct an axiomatic approach to these spacetime theories. This might make us better comprehend the exotic phenomena predicted by both theories and throw light on the meaning of the apparently paradoxical predictions occurring in them. In fact, by playing with the axioms, such an approach may actually lead to the prediction of physical phenomena that transcend the domain of applicability of these spacetime theories. Indeed, one of the aims of axiomatizing a physical theory is not only to reveal its physical content, but also, if possible, to enlarge its scope and its power of predictability. For example, by weakening or adjusting the axioms describing GR, we might be able to throw light on the logical structure of the more general five-dimensional Kaluza-Klien theory unifying gravity and electromagnetism on a geometric basis. This may even open the door for dealing with other physical theories having more informative content and stronger explanatory power. One possible way to accomplish this is that the questions addressed when investigating the logical structure of relativity theories should not be only the “how” questions but, more importantly, the

52For example, in the context of SR, the property that no inertial observer moves faster or equal to the speed of light should be derived as a theorem and not be put as an axiom.

53Postulating the existence of an extra curled up space dimension, the Polish mathematician Theodor Kaluza argued that gravity is carried by ripples in the familiar three space dimensions, while electromagnetism is carried by ripples involving the new, curled up dimension. The reason for the unobservability of the fifth dimension (its compactness) was suggested by the Swedish physicist Oskar Klien. Today the theory is called the 5D Kaluza-Klien theory. (Kaluza sent his paper to Einstein in 1919. Einstein regarded Kaluza’s assumption of the existence of “an extra curled up space dimension” not very convincing. Kaluza did not publish his paper until two years later when Einstein became more at ease with Kaluza’s assumption giving him the green light to publish it. In fact, Kaluza’s “wilde” idea was way far beyond his time; namely, “unifying” apparently distinct physical phenomena by “jumping into a new dimension”. It seemed “crazy” even for Einstein himself. Though it failed experimental tests, this idea of postulating the existence of extra curled up space dimension was to be revived again in the context of (super)string theory. Modern versions of the Kaluza-Klien theory go up to ten space dimension (six of which are curled) and one time dimension. These “extra degrees of freedom” make such theories have the potential flexibility to merge all known forces of nature (including gravity!) into one harmonious framework.
"why" questions: What is believed and why? Which axioms are responsible for what predictions? What happens if we weaken some of the axioms? Can we change (some of) the axioms and at what price?

In their attempt to axiomatize GR, Andreka and Nemeti first present streamlined axiomatic system for SR. From this simple and naturel set of axioms all paradigamic effects of SR are derived: moving clocks slow down, moving rods shrink and moving pairs of clocks get out of synchronism... etc, thereby capturing all exotic predictions of SR. The transition from SR, which is based on the notion of inertial observers, to GR is partially accomplished by introducing the more general notion of accelerated observers.

In the context of accelerated observers, the twin paradox is derived as a theorem being a logical consequence of the more flexible set of axioms describing accelerated observers. The elimination of the difference between inertial and accelerated observers on the level of axioms leads to an axiomatization of the spacetime of GR. In fact, using the equivalence principle as a guide, the axioms chosen for GR are in a (well defined) sense a localization of the axioms describing SR. The models of GR are locally Lorentzian smooth manifolds in which every local chart is equipped with a locally Minkowskian metric. 54 Roughly, these locally Lorentzian manifolds $M_L \text{ are constructed as follows: The points of } M_L$ stand for the events; an event being nothing but a set of bodies, $I$ is an indexing set numbering the observers, $U_i$ stands for the frame of reference of the observer $i$ and, finally, the transition functions $\phi_{ij}: U_j \rightarrow U_i$ are the world view transformation between the $i$-th and $j$-th observers. Each local chart $U_i$ is equipped with a (local) Minkowski metric $g_i$. These local metrics $g_i’s$ are then “lifted” to the whole manifold by constructing the tangent space $T_e(M)$ at each event $e$. The process of “lifting” these local metrics produces a global metric defined on the entire Lorentzian manifold.

The three theories - special relativity, accelerated observers and general relativity - are formulated in first order logic (FOL). There are at least two reasons for this. First, to avoid any tacit assumptions so that all concepts dealt with are clear and well defined.

54 Technically, a topological manifold $M$ of dimension $n$ is a topological space that is Hausdorff (distinct points can be seperated by disjoint open sets) and locally Euclidean: every point $p \in M$ has a neighbourhood homeomorphic to some open subset of $R^n$; i.e., for each $p \in M$, there exists a pair $(U, \phi)$ where $U$ is open in $M$ and $\phi: U \rightarrow \phi(U)$ is a homeomorphism (both $\phi$ and its inverse $\phi^{-1}$ are continuous). The pair $(U, \phi)$ is called a local chart. We say that $(U, \phi)$ and $(V, \psi)$ are $C^\infty$ compatible if whenever $U \cap V \neq \phi$, then the transition function $\psi \circ \phi^{-1}: \phi(U \cap V) \rightarrow \psi(U \cap V)$ and its inverse are $C^\infty$ as mappings of open subsets of $R^n$. A $C^\infty$ differentiable (smooth) structure on $M$ is a family $\mathcal{U} = (U_i, \phi_i)_{i \in I}$, with $U_i$ open, such that (a) the $U_i$’s cover $M$. (b) the $(U_i, \phi_i)_{i \in I}$ are $C^\infty$ compatible. (c) $\mathcal{U}$ satisfies the following maximality property: every $(U, \phi)$ which is $C^\infty$ compatible with each and every element of $\mathcal{U}$ belongs to $\mathcal{U}$. A smooth manifold is a topological manifold equiped with a smooth structure.

55 For physical considerations, $M_L$ is not necessarily Hausdorff.

56 Having the notion of a tangent space, the concept of a tensor can be introduced in the axiomatic language of GR; a multilinear map of the cross product of the tangent space with itself and its dual to the field $Q$. The field $Q$ plays the role of the real numbers $R$, namely representing the "quantities". The axioms satisfied by $Q$ are weaker than those satisfied by the real field $R$. In fact, the "quantity" part $(Q, +, \cdot, <)$ is a Euclidean ordered field; an ordered field in the sense of abstract algebra in which every positive number has a root.
Secondly, by Godel’s incompleteness, FOL has a complete inference system which higher-order logics do not have.

4.1 Breaking the Turing Barrier

We here give a very concise summary of the construction of relativistic computers breaking the Turing barrier. Using Malament-Hogarth spacetimes and other general relativistic phenomena, Andreka and Nemeti have succeeded to construct (what they refer to as) relativistic computers. Relativistic computers are based on the property of time dilation. As previously mentioned, among the interesting and bizarre predictions of the general theory of relativity is that time slows down in strong gravitational fields.

A “stationary” (Schwarzchild) black hole has “one event horizon”. The event horizon acts as a one-way membrane surface; one can pass through the horizon, but once inside one can never leave. The event horizon represents the boundary of all events which can be observed by an external observer; if one crosses this boundary, one becomes trapped behind it. The Schwarzchild event horizon is absolute because it seals off all internal events from every external observer. A “rotating” black hole, unlike a stationary one, has “two event horizons”. The gravitational pull of stationary (rotating) black holes grows without limit as one approaches the (outside) event horizon.

Assume now that two observers, which we denote by $H$ and $L$ respectively, are hovering near the outside event horizon, with $H$ being higher up. Then $K$’s clock runs slower than $H$’s clock since he is experiencing a stronger gravitational field. Moreover, as $L$ moves towards the horizon, this discrepancy between the ticking of both clocks gets larger and larger. In fact, by lowering $L$ appropriately, we can actually control this “time lag”. Now, if a programmer $P$ gets very close to the outside event horizon while leaving his computer $C$ “higher up”, then in a few days time relative to the programmer, the computer does a few million’s year’s job! Accordingly, one can reach an “infinite speed up” by lowering $P$ to the right position; hence breaking the “Turing barrier”. The above mentioned thought experiment, however, cannot be carried out in a Schwarzchild, that is, a non-rotating black hole. This is because either $L$ will be destroyed from the gravitational might or some photon sent by $H$ would not reach him. A possible way out is to choose a slowly rotating black hole. The rotation of the black hole induces a repelling effect - a centrifugal force in the language of Newtonian mechanics, that counter-balances the strong gravitational pull of the black hole. In this way, $L$ can slow down as desired without being crushed.

As previously stated, a slowly rotating black hole has two event horizons. The outer one is similar to that of a Schwarzchild black hole. The inner event horizon is the one that overcomes the gravitational force. Accordingly, it is possible for an observer $L$ to stay at a fixed distance from the center of the rotating black hole. In this way, $L$ can slow down as desired without being crushed.

\[\text{This can be actually deduced from the postulates of the special theory of relativity together with the equivalence principle: The special theory of relativity tells us that a clock associated with an accelerated observer slows down. Since gravity and acceleration are indistinguishable (the equivalence principle), it follows that clocks tick slower in a gravitational field.}\]
an infinite time dilation (speed-up) of the computer $C$ which lies safely away from the (outer) event horizon with respect to the programmer $P$ is accomplished. The creation of a computer that can compute tasks beyond the Turing limit can be achieved as follows. The programmer $P$ leaves earth in a spaceship towards a huge slowly rotating black hole. As $P$ is heading towards his target, $C$ checks one by one the theorems of set theory. If $C$ finds a contradiction, he sends a signal to $P$. Otherwise, he does nothing. Now, what happens to the programmer $P$ from $C$’s point of view. As the programmer $P$ approaches the event horizon, his clock will be ticking slower and slower relative to $C$’s clock. At the limit, that is, when $P$ reaches the inner horizon, his clock freezes, coming to a halt, so to speak, relative to $C$. From the point of view of $P$, however, the $C$’s clock appears to be running faster and faster. Moreover, assuming that the black hole is huge so that the tidal forces on the event horizon of the black hole are negligible, $P$ will safely cross the inner event horizon. Two things can occur: either $P$ receives a light signal from $C$ or not. In the latter case, $P$ will know that $C$ has found an inconsistency in ZFC set theory. Otherwise, $P$ concludes that ZFC is consistent. Finally, why the choice of a huge rotating black hole? There are actually two reasons for this: first, because the black hole is huge, the center of the black hole is relatively far from the event horizon. Secondly, and more importantly, the matter content, that is, the singularity is not a point as in the case of a static black hole, but is actually a ring. This is one of the fascinating properties of rotating black holes. These two features make $P$ comfortably pass through the middle of the ring without being crushed or torn apart!

We end this part of the paper by posing the following question: Andreka’s and Nemeti’s exotic results (computing non-computable functions, proving that ZFC is consistent, ... etc) are actually obtained in the context of the classical general theory of relativity. These results are a consequence of the property of infinite time dilation which occurs in the strong gravitational fields in the vicinity of rotating black holes. A natural question arises: Will these results still hold if we take the quantum theory into consideration. In this case, we will be dealing with (what we may refer to as) quantum relativistic black holes (QRBH). These behave in a manner categorically different than classical (rotating) black holes. For example, they emit radiation so they can actually glow. Will infinite time dilation still hold in the framework of QRBH? And if not, will Godel be right after all? If this is so, then there might be a way to relate the uncertainty principle (the corner stone of quantum theory) with the Godel’s undecidability (the corner stone of mathematical logic) near a quantum black hole. Essentially, both principles are derived from the notion of “self reference”: Heisenberg’s uncertainty relation is a consequence of the merging of subject (observer) and object (observed); whereas Godel’s undecidability results from a merging of subject (mathematics) and object (meta-mathematics). In the classical context of black holes, in which infinite time dilation occurs, Andreka and Nemati have shown that ZFC is consistent. In the quantum realm, that is, taking the uncertainty principle into considerations, it is not quite clear that the property of time dilation still holds so that ZFC may again be undecidable. Accordingly, the two most important limitative results discovered in the 20-th century may be an outcome of a more fundamental law describing the true physical nature of quantum black holes!

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58 According to Hawking, this occurs even if the black hole is not rotating!
4. On the Meaning of the Quantum Theory

One does not get an answer to the question, What is the state after an atomic collision? but only to the question, How probable is a given effect of the collision? From the standpoint of our quantum mechanics, there is no quantity which causally fixes the effect of a collision in an individual attempt. Should we hope to discover such properties later . . . and determine them in individual events? . . . I myself am inclined to renounce determinism in the atomic world, but this is a philosophical question for which physical arguments alone do not set standards. Max Born as quoted in [12].

In a way, quantum mechanics reminds us of the old wisdom that when searching for harmony in life, one must never forget that in the drama of existence we are ourselves players and spectators. Neils Bohr.

The spacetime continuum may be considered as contrary to nature on view of the molecular structure of everything which happens on a small scale..... Perhaps the success of the Heisenberg method points to a purely algebraic method of description of nature, that is the elimination of continuous functions from physics...At the present time, however, such a program looks like an attempt to breath in empty space. Albert Einstein, Out of My later Years.

My own view is that ultimately physical laws should find their most natural expression in terms of essentially combinatorial principles, that is to say in terms of finite processes such as counting... Thus, in accordance with such a view, should emerge some form of discrete or combinatorial spacetime. Roger Penrose, Magic without Magic

There is nothing wrong with the uncertainty principle. Einstein was simply confused. Stephen Hawking

I believe that no one truly understands quantum theory. Richard Feynman.

Chance and necessity, cause and effect, chaos and randomness are words commonly used in today’s world of physics reflecting the abstruse nature of the micro-world. The seemingly paradoxical laws of quantum mechanics, strangely enough, have given us a more exact model of our universe, in the sense that they have not only overcome contradictions which remained unresolved in the realm of classical physics, but also predicted unknown and bizzare phenomena giving us a radically new and deeper cognition of the world around us.

The statistical nature of the new-born theory at the turn of the century made most physicists sceptical towards it, believing that the theory was incomplete, and hence part of a more general theory based on non-probabilistic laws. Einstein was never at ease with the uncertainty principle, the cornerstone of quantum theory, expressing his doubts in his famous saying “God does not play dice with nature.”

According to Stephen Hawking, God not only plays dice with nature, but throws them in places where you cannot find them!

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of the great critics of quantum mechanics, believing in a deterministic reality based on exact objective laws. Neils Bohr, the great Danish physicist, not only showed no critical attitude towards the new theory, but was also interested in its philosophical implications, as proved by his “principle of complementarity”. The younger generation, not yet prisoners of the well-established theories and concepts of classical physics, were in a somewhat uncomfortable position. Baffled by the new discoveries on the one hand, and on the other, being more flexible than Einstein, they were able to cope brilliantly. “Heisenberg’s matrix-mechanics”, “Schrodinger’s wave equation,” and, finally, Dirac’s “bra-ket formulation” of quantum mechanics, which generalized and proved the mathematical equivalence of both approaches, marked the end of the first period (old quantum theory, which started with the discovery of the discrete nature of radiation by Max Plank) and was the beginning of a new era in quantum physics based on solid mathematical foundations.

The essence of the problem is that quantum mechanics is essentially a more complete theory than classical physics and yet describes the universe in a statistical manner. The idea of a statistical approach in describing natural phenomena was, of course, already introduced in the realm of classical physics, namely in the theory of thermodynamics, dealing with concepts like entropy and free energy. However, the meaning attributed to such a statistical analysis was of a categorically different nature than that of the new-born quantum theory. It was always assumed that, whenever probabilistic laws emerged in relation to any physical phenomenon in the classical domain, the element of indeterminism was purely due to a lack of knowledge; an incompleteness of the data necessary to describe the phenomenon under study. Physicists believed, at least in

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60 Technically, the mathematical framework of the quantum theory is an infinite dimensional Hilbert space equipped with the delta-Dirac function.

61 In the realm of classical physics, that is, in Newton’s and Maxwell’s theories, the property of a “wave” and a “particle” are simply mutually exclusive, representing “incomensurable” notions. A wave (resp. particle) cannot exhibit corpuscular (resp. wave) properties. Particles are discrete, being localized in space, whereas a wave is continuous and extended in space. Light, according to Maxwell’s theory is unquestionably a (special type of an electromagnetic) wave. It is subject to both interference and diffraction phenomena, properties that are undeniably allian to the behaviour of particles. On the other hand, particles as described in the Newtonian framework do not reveal any wave-like nature. It was only at the turn of the 20-th century that Einstein assumed a strange duality in the behaviour of light in order to understand the phenomena of the photo-electric effect. To explain the emission of electrons from the surface of (some kinds of) metals when being bombarded by light, Einstein postulated that light has a “grainy nature” and is composed of discrete particles which he called “quanta”. In doing so, Einstein was actually generalizing Plancke’s ad-hoc assumption proposed five years earlier on the nature of (electromagnetic) radiation. Contrary to the classical Maxwell’s theory, Planck had to assume that electromagnetic radiation is composed of discrete units to explain the phenomenon of the black body radiation in an attempt to resolve (what was known as) the ultra-violet catastrophe. These ideas were further developed by the French physicist de-Broglie and set in a much wider framework. An intuitive belief in the “symmetry” of nature made de-Broglie propose the following daring assumption: not only light, under certain circumstances, may reveal corpuscular nature (as Einstein has rightly discovered), but all matter objects are associated by (what he called) matter waves. Thus de-Broglie was actually extending the dualistic nature of light to all forms of matter! It was a few years later, through the work of Bohr, Born, Schrodinger, Heisenberg, Dirac and others that such a revolutionary conception was further elaborated in the probabilistic language of the quantum theory resulting in a subtle form of duality between matter and radiation.
principle, that it was always possible to overcome or to eliminate the statistical aspect once one was able to gather all the necessary information about the phenomenon in question. The lack of information, that is, the ignorance in relation to the system studied, is therefore due to a deficiency in the observer (subject) and not in the observed (object). In quantum mechanics, matters are more subtle and intricate. Statistical laws emerge from within, being an innate fundamental aspect of the theory. Having a probabilistic nature, quantum mechanics seems to give an incomplete picture of the world; a fuzzy description of reality. By definition, probabilistic laws can only predict what might be and not what is. When we talk quantum, the focus is on what we do and what we observe, rather than what is.

In the micro world, “subject and object”, “observed and observer” are not totally separated. As a result of this interaction, a kind of fuzziness evolves, an indeterministic factor that seems to dominate all micro phenomena. As one moves from the small to the large, as one enters the world of the macro, where the subject no longer affects the object, this fuzziness or blurriness gradually disappears. The uncertainty relation, according to the above argument, could be realized as the law that measures the degree of such merging; equivalently as a kind of limit beyond which the line of demarcation between subject and object becomes unclear and not well defined. However, this is not the whole story. Many questions remain unanswered, the most important of which, we believe, is the following: Is nature itself indeterministic, in the sense that the “grey zone”, an essential feature of the quantum world, is intrinsic in the universe, or is it due merely to our inability to know nature without meddling with it? In other words, does nature have a probabilistic essence, or does it only reveal itself to us through non-exact laws? Though some may argue that the above two questions are categorically different, we claim that the difference is of no relevance. Talking about the nature of the universe as a thing in itself is simply meaningless. What counts is what we (the subject) can know about nature (the object). We discover nature through ourselves. The laws of physics are defined only in relation to us, acquiring their meaning due to our existence. They are not floating in the cosmos, so to speak, but are out there to be extracted by us. The process of this extraction makes the universe, dead as it may seem, hit back revealing itself in a somewhat blurred form. As Heisenberg says, in his book, Philosophy and Physics: “What we observe is not nature itself, but rather nature exposed to our methods of questioning.”

Quantum mechanics made a revolutionary impact on our views towards the world and even towards ourselves. Scientists were forced to alter their beliefs and dogmas that were inherited from past ages. Words like “knowledge,” “understanding” and other fundamental concepts acquired a completely new meaning. Theories that were well established proved inadequate and incomplete, being only an approximation of reality. Scientists learnt not to take anything for granted, becoming more skeptical in dealing with new concepts and theories. The fascinating, if not unbelievable, consequences of the new theory made physicists more daring in their imagination. Any assumption, no matter how absurd, was admitted as long as it was compatible and consistent with the axiomatic scheme of quantum laws. In less than five years, after the mathematical foundations of the new theory were laid down, the progress achieved in the newly born
quantum theory was greater than that achieved in classical physics throughout the whole of the nineteenth century. It was a decisive period in the development of science; a time in which second rate physicists could come up with first rate ideas. Wild and crazy interpretations accompanied the new science, like the “many world interpretation” of quantum mechanics, the “absence” of cause and effect, and hence the possibility of time flowing backwards. Words like “ghost particles” and “tunnelling effects” were introduced, reflecting the abstruse and the (seemingly) paradoxical nature of the quantum world. The concept of a “well-defined path” was simply meaningless within the framework of the quantum world. The “wave particle” duality that dominated all atomic phenomena was given a philosophical interpretation in Neils Bohr’s “principle of complementarity”: although the wave-particle properties of atomic particles are apparently “mutually exclusive”, they are, nevertheless, “complementary” as opposed to...
to “contradictory”. An understanding of the nature of the electron could be only achieved by combining both aspects, the two together constituting the essence of the electron. In one context (experimental framework), electrons act like particles, while in another they reveal wave-like properties. In general, the “wave-particle” duality was a reflection, in the language of old physics, of a deeper conflict between the goals of “description” and “causality.” One can describe the world at any instant to any desired accuracy and produce a “snapshot,” so to speak, showing where everything is. The principle of complementarity states that such a “snapshot” could be only taken at the expense of forewearing any connection between its future “snapshots”. The sharper the snapshot, the looser its causal ties with the future. We must choose some compromise between an orderly, causal world which we cannot even visualize and a sharp picture which reflects only the instant it was taken.

The abstract formulation of the new theory made it difficult to comprehend, even among physicists themselves. Based completely on pure mathematics, physicists realized the impossibility of “visualizing” the world of the small. Resorting to pictorial images in trying to understand atomic phenomena proved impossible. The meaning of the word “understanding” had to be revised and defined in more general terms. If “pictorial images” were meaningless in dealing with the atom, could mathematical rigour with its inner consistency be an alternative? And if so, does this mean that a “physical theory” is nothing more than its “mathematical model”? Paul Dirac, wrote in this respect: “The main object of physical sciences is not the provision of pictures, but is the formulation of laws governing phenomena and the application of these laws in discovering new phenomena. In the case of atomic phenomena, no picture can be expected to exist in the usual sense of the word picture, by which is meant a model functioning essentially on classical lines. One may, however, extend the meaning of the word picture to include any way of looking at the fundamental laws which make their consistency obvious. With this extension, one may acquire a picture of atomic phenomena by becoming familiar with the laws of quantum theory.”

phenomena has been the mechanism by which the boundaries of scientific knowledge expand. To be sure, it is the “essence” of scientific development. For example, in the world of the small, using the language of metaphores, one can say that “a wave is a particle”. Another clear and illuminating example in favour of Khun’s analysis is revealed in our (classical) understanding of electromagnetism. Throughout the eighteenth century, physicists dealt with electric and magnetic effects in nature as two completely separate phenomena described by mutually irrelevent physical theories. A partial unification of these two theories was first accomplished by Maxwell by discovering that a varying electric (magnetic) field gives rise to a varying (magnetic) field; both together giving rise to the notion of an electro-magnetic field whose evolution in space and time is mathematically described by Maxwell’s equations. If Maxwell had succeeded to show that a varying electric and magnetic field co-exist together to form one single entity, then Einstein, through his theory of special relativity (SR), went a step further. He was able to show that one and the same field may be either magnetic, electric or a combination of both when viewed from different “angles”; the nature of the field revealed being dependent on the state of motion of the (inertial) observer studying it. Accordingly, SR, not only showed a complete symmetry between electric and magnetic fields - a fact already recognized within the context of Maxwell’s theory, but also established a unification of two fragments into one whole through the metaphor “Electricity is magnetism and magnetism is electricity.
One of the interesting philosophical implications of quantum physics is the concept of “free will”. In the seventeenth century, it was believed that the universe is totally deterministic, a kind of gigantic clock, subject to Newtonian mechanics. Pierre Laplace, a French mathematician and physicist, reflected this belief by stating that if it were possible to express the equation of motion of every particle in the universe at some time $T$, then the state of the universe at any other time $T'$, by virtue of Newton’s laws of motion, would be completely determined. The present is, therefore, a mirror reflecting both the past and the future. This bizarre assumption was certainly a blow to the concept of free will. We, as a part of this gigantic clock, operated according to mechanical deterministic laws, and are hence totally “unfree” in our actions. The future being a direct outcome of the past, our life is nothing but a series of events each completely determined by its predecessors. This was a paradox and irony that has ever since haunted the modern epoch. To overcome this dilemma, philosophers assumed a kind of “duality” in man; the existence of “consciousness” outside nature. In that sense, the mind was governed by some mystical laws transcending the mechanistic universe. A separation between mind and matter was thus introduced, which remained ever since, expressing itself in different philosophical doctrines. This line of demarcation was never crossed: the “materialistic” world on one side (mind is a by-product of matter), the “idealistic” world on the other (mind precedes matter). Quantum theory abolishes this completely. Not only that the future is not an outcome of the present, but that the present itself is not completely specified or determined. Assuming that the human mind operates in accordance to quantum, that is, probabilistic laws, there will always be an element of “unpredictability” and “novelty” in its outcome. In other words, *intelligence cannot be tamed*.

We end the paper with the following philosophical debate between Albert Einstein and Niels Bohr.

### 4.2 A debate between Einstein and Bohr

Bohr: Quantum mechanics seems to accord a special role to an observer who is outside the system under study. The information that we as observers have about the quantum system is coded into the construction of the quantum state of the system. This is necessarily an abstract concept, not for something in nature, but for a mathematical entity that is invented to keep track of the information that one part of the universe can have about another part. The quantum state is not a property of the system it describes. It is the property of the boundary or the interface that separates that system from the rest of the universe, including the observer who studies it. Since a quantum

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Gödel’s undecidability gives a somewhat similar conclusion. Godel’s result: the consistency of arithmetic cannot be established by any metamathematical reasoning which can be represented within the formalism of arithmetic. Consequently, the concept of mathematical truth cannot be encapsulated in any formalistic scheme. Mathematical truth is something that goes beyond mere formalism. Stated differently, one can say that the essence of Godel’s result is the following: Human intelligence *transcends* mere formalism and mechanicizability. Again there is always an element of “unexpectedness” in the outcome of the human mind!
state changes when we make a measurement, I think that it is nothing but an encoding of what we know.

Einstein: I agree with you that the abstract state space used to represent the information we have is not necessarily something that is in complete correspondence with the system. If we ask a new question and gain a new information, we will represent it by a new state. This abrupt change is a reflection of change in our knowledge of the system and not a change in the system itself. However, the fact that it seems intrinsically impossible for one observer to have all the information that would be necessary to give a complete description of the world seems to me unacceptable. Heisenberg uncertainty says that we are allowed to know at most half the information that would be necessary to fully describe any physical system. I think that there is some crucial element missing in the “Heisenberg, Shrodinger, Dirac” formulation of the quantum theory as we understand it today. Nowadays, every Tom, Dick and Harry think they understand quantum theory. I think they are mistaken. I believe that the goal of physics is to construct a description of the world as it would be in our absence.

Bohr: I agree with you that in quantum mechanics we deal not only with a description of the system itself, but what we can know about it. We envision a situation in which the world is divided into two parts. On one side is the particular system under study. On the other side, are ourselves, as observers, whatever tools and instruments we intend to use in the study. This is very different from the description of the world in the classical context. There we are invited to imagine that mathematics gives a picture of reality in which the observer need not be glimpsed. While it is true that quantum theory doesn’t provide an objective picture of the world, this may actually be the great virtue of the theory. It frees us from the fiction of absolute observer, looking at everything from outside the world. We may miss the picture of the world given to us by classical physics. However, this idea of representing the whole universe as a collection of classical trajectories reflects a fictitious ideal that corresponds only very approximately to what is real.

Einstein: I fail to understand your subtle reasoning. You seem to suggest that the electron is a ghost. It is not here, it is not there, it is in a “state that is some mixture of here and there”. According to your view, only states and not transition of states is what makes sense in the context of the micro. I don’t like this fuzziness. You are basically saying that there is no world out there. There is only the abstract quantum physical description. The task of physics is not to find out what nature is. According to your view, physics is concerned with what we can say about nature and not nature itself. I simply don’t agree. If probability is the language of our world, then the problem is in the quantum theory and not in the quantum world. The wavefunction does not provide a complete description of physical reality. However, I do believe that we should leave open the question of whether or not such a description exists.

Bohr: I don’t see why you reject quantum theory. I believe that the logic of the micro world (and not the quantum theory alone) is bizzare. What quantum theory seems to tell us is the following: There is no such thing as an objective reality. Reality (whatever this
means) reveals itself through our methods of questioning. The electron in one context (experimental framework) may act like a particle and in another context it may act like a wave. The essence of the electron is the sum total of all its manifestations (revealed in different contexts). It is not only a wave, it is not only a particle. It is in some sense both and neither. We actually create the required property by choosing the suitable experimental context.

Einstein: The EPR thought experiment\(^{67}\) seems to suggest that quantum theory is somewhat incompatible with the central idea in special theory of relativity, namely that of causality. This notion of “action at a distance” makes me feel uneasy about the quantum theory. A change here producing “instantaneously” an affect there is too much for me to absorb. This means, as far that I understand, that quantum theory is a non-local theory. The property of entanglement\(^{68}\) - every object of the universe being coupled to every other object, which is the key idea in my general theory of relativity, seems to be an essential feature of the quantum world as well. We thus need a mathematical formulation of such a global notion. I still believe, I might be wrong of course, that Heisenberg’s uncertainty principle may be an approximation to a more general deterministic law provided that the notion of entanglement finds a clear unambiguous mathematical expression. In other words, a clear mathematical description of the notion of entanglement may possibly lead to the elimination of the probabilistic and fuzzy laws upon which the quantum theory is based which I still believe is not a genuine or a true reflection of how the micro-world operates. Accordingly, we would obtain a new formulation of the quantum theory based on exact non-probabilistic laws. In this more general theory, I imagine that a new kind of physics may emerge in which the world is described as a single entity so that the world around us no longer consists of a large number of autonomous atoms the properties of which owing nothing to the others. Instead, the world would be described as a vast, interconnected system of relations in such a way so that the properties of a single elementary particle or the identity of a point in space requires and reflects the whole rest of the universe.

Bohr: I agree with you that we do need a mathematical formulation of the notion of entanglement or quantum correlation (as it is technically known). However, I don’t see that if we do succeed in achieving this goal, then this may imply that the “uncertainty principle is an approximation to a more general deterministic law”, to use you own words. In fact, it is possible (and most probable) that the notion of entanglement turns out to be compatible with (if not a consequence of) the uncertainty relation. The “new

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\(^{67}\)The Einstein - Podelski - Rosen experiment (EPR for short) seems to suggest that at the quantum level, processes are involved that are somehow controlled at a global rather than the local scale of things. The remarkable piece of work carried out by the Irish physicist John Bell, in the early 1960’s proved that quantum theory is essentially non-local. What Bell did was to find a way to test directly the principle of locality. Bell found that, in certain cases, the predictions of any local theory must satisfy certain constraints, namely, the Bell inequalities. Quantum theory, being non-local, violates these constraints!

\(^{68}\)Whenever two systems interacted, it is more common to find them sharing properties in this way, that to find them in states such that each have definite individual properties. Quantum theory says quite generally that whenever two systems have interacted, their description is tied together in this way; “entangled”, no matter how far apart they may be.
physics” emerging in the context of this new theory (still based on the uncertainty principle), I imagine, should be interpreted as follows: No single observer can have complete knowledge of the world. This is basically what the uncertainty principle tells us. Accordingly, a *complete* description of the universe is only possible from the point of view of *many* observers. Briefly, “I cannot know everything, but we, in principle, can know everything”. We thus should have a large set of “quantum states”, each one of which describes the partial knowledge that an observer has about the other things in the universe including his knowledge of the information the other observers hold about the universe, without actually knowing the content of this information. All this is compatible with the limitations imposed by the uncertainty relation. The new theory, taking into account all possible views, must rely on some general principle that constrains how the different views may “alter”, while still being partial views of the “same world”. The “compatibility” of the different views; that is, the necessity that the views of the different observers should be in “harmony” with each other; cohere, so to speak, would, I, conjecture, be a natural consequence of the sought for matematization of the notion of entaglment. We again arrive at a “wholistic quantized theory” of the universe. This wholistic theory would be a generalization of our established quantum theory, though still having the uncertainty principle as its logical core. This again reflects my belief that a physical theory does not describe reality independently of our existence, but is merely a description of our relation with the external world. The property of every object is both a result and a reflection of its interaction with the rest of the cosmos.

Einstein: I certainly don’t agree with David Bohm’s hidden variable theory. I think his idea of a “pilot wave” is even more absurd than the notion of action at a distance. However, I do see some relevance in his “implicate order interpretation” of the quantum theory. I think that your Copenhagen interpretation - this dicotomy between subject and object - is not applicable if our “laboratory” is the whole universe. There is simply no place for an external observer. It is in this sense that the uncertainty principle, the corner stone of the “conventional” quatum theory, may be an approximation to a yet undiscovered hidden exact law that would establish my conviction that laws of physics transcend our own existence. A cosmological quantum theory, that is, a quantum theory of the whole universe, will necessarily abolish the idea of an external observer upon which your Copenhagenhagen interpretation is based. Such a seperation is possible as long as the system under study is a proper part of the universe. Obviously, any theory whose domain is the entire universe cannot make such a distinction; the line of demarcation between the subject (observer) and object (observed) will inevitably disapper. Then there might evolve a new order of reality; a deterministic non-probabilistic theory that reflects this smooth merging of the observed and the observer. Our conventional quantum theory would then be an approximation of such a theory when restricted to any proper part of the universe.

69Implicit order is a term coined by the physicist David Bohm to describe the sort of unfolded order that is characteristic of the quantum theory. It is to be contrasted with the explicit orders of Newtonian mechanics. Bohm believed that this implicate order has a universal importance and might be useful in understanding the nature of conciousness.
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