Extra Dimensions and Strong Neutrino-Nucleon Interactions Above $10^{19}$ eV: Breaking the GZK Barrier

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Abstract: Cosmic ray events above $10^{20}$ eV are on the verge of confronting fundamental particle physics. The neutrino is the only candidate primary among established particles capable of crossing 100 Mpc intergalactic distances unimpeded. The magnitude of $\nu N$ cross sections indicated by events, plus consistency with the Standard Model at low-energy, point to new physics of massive spin-2 exchange. In models based on extra dimensions, we find that the $\nu N$ cross section rises to typical hadronic values of between 1 and 100 mb at energies above $10^{20}$ eV. Our calculations take into account constraints of unitarity. We conclude that air-showers observed with energies above $10^{19}$ eV are consistent with neutrino primaries and extra-dimension models. An upper bound of 1-10 TeV on the mass scale at which graviton exchange becomes strong in current Kaluza-Klein models follows.

1 Introduction

The energy of extra-galactic proton cosmic rays should not exceed the GZK bound [1]. The bound, about $10^{19}$ eV, is based on the known interactions of nucleon primaries with the photon background of intergalactic space. The GZK bound is tantamount to an upper limit on cosmic ray energies, inasmuch as nuclei and photons have lower energy cutoffs [2]. Yet an experimental puzzle exists, as evidence for air shower events with energies above the GZK bound has steadily accumulated over the last 35 years [3]. There seem to be inadequate sources nearby to account for such events, and the sources are almost certainly extragalactic [4]. A number of showers with energies
reliably determined to be above $10^{20}$ eV have been observed in recent years [5], deepening the puzzle.

A completely satisfactory explanation of the so-called GZK-violating events is still lacking. Models have been constructed to explain the puzzle by invoking “conventional” extragalactic sources of proton UHE acceleration, as reviewed recently in Ref.[6]. Other models introduce exotic primaries such as magnetic monopoles [7] or exotic sources such as unstable superheavy relic particles [8], or appeal to topological defects [9]. Except for monopoles all of the proposed sources must be within 50-100 Mpc to evade the GKZ bound, a requirement which is difficult to satisfy.

The $10^{20}$ eV events potentially pose a confrontation between observation and fundamental particle physics. Except for the neutrino, there are no candidates among established elementary particles that could cross the requisite intergalactic distances of about 100 Mpc or more [10, 11]. The neutrino would be a candidate for the events, if its interaction cross section were large enough, but this requires physics beyond the Standard Model. The neutrino-nucleon total cross section $\sigma_{\text{tot}}$ is the crucial issue: Flux estimates of UHE neutrinos produced by extra-galactic sources and GZK-attenuated nucleons and nuclei vary widely, but suffice to account for the shower rates observed. Very significantly, there is a hint of correlations in direction of the events, both with one another and candidate sources [12]. Event-pointing toward distant sources, if confirmed, would require a neutral, long-lived primary, reducing the possibilities for practical purposes to the neutrino plus new physics to explain the cross section.

Current understanding of the UHE Standard Model $\sigma_{\text{tot}}$ is based on small-x QCD evolution and $W^\pm, Z$ exchange physics [13]. This physics is extremely well understood and has been directly tested up to $s = 10^5$ GeV$^2$ with recent HERA-based updates [14]. These calculations are then extrapolated to the region of $10^{20}$ eV primary energy, leading to cross sections in the range $10^{-4} - 10^{-5}$ mb, far too small to explain the GZK-violating air shower events. The observationally indicated cross section is completely out of reach of the extrapolations of $W$- and $Z$- exchange mechanisms.

Since the neutrino-nucleon cross section at $10^{20}$ eV has never been directly measured, it is quite reasonable to surmise that new physical processes may be at work. Total cross sections at high energies are dominated by characteristics of the t-channel exchanges. The growth of $\sigma_{\text{tot}}$ with energy, in turn, is directly correlated with the spin of exchanged particles. Exchange of new
(W- or Z-like) massive vector bosons would produce $\sigma_{tot}$ growing at the same rate as the standard one, failing to explain the puzzle. If the data indicates a more rapid growth with energy, one is forced to consider higher spin, with the next logical possibility being massive spin-2 exchange. We reiterate that this deduction is data-driven; if data indicates (a) correlations with source directions, and (b) $\sigma_{tot}$ in the mb and above range, there are few options other than neutrinos interacting by massive spin-2 exchanges.

Recent theoretical progress has opened up the fascinating possibility of massive spin-2, t-channel exchange in the context of large “extra” dimensions \[15\], while the fundamental scale can be related to a string scale of order several TeV \[16\]. In this context the Kaluza-Klein (KK) excitations of the graviton act like a tower of massive spin-2 particle exchanges. We will show that large UHE neutrino cross sections, sufficient to generate the observed showers, are a generic feature of this developing framework \[17\]. At the same time the new contributions to $\sigma_{tot}$ at energies below center of mass energy $\sqrt{s}$ of 500 GeV is several orders of magnitude below the Standard Model component. In fact, the new physics we propose to explain the puzzle of the GZK-violating events is consistent with all known experimental limits.

2 The $\nu N$ Cross Section with Massive Spin-2 Exchange

The low-energy, 4-Fermi interaction total cross section, $\sigma_{tot}$, grows like $s^4$ over many decades of energy. Perturbative unitarity implies that at an invariant cm energy $\sqrt{s}$ large compared to the exchange mass $m_W$, the growth rate slows to at most a logarithmic energy dependence. The shift from power-law to logarithmic growth is seen to occur in the Standard Model. There is a second effect, that above 100 TeV the total number of targets (quark-antiquark pairs) grows roughly like $(E_\nu)^{0.4}$. This fractional power, in turn, leads to a formula for $\sigma_{tot} = 1.2 \times 10^{-5} mb(E_\nu/10^{18} eV)^{0.4}$ as a reasonable approximation to the Standard Model calculation \[14\].

Exchange of additional spin-1 bosons cannot produce faster growth with energy than just described. However, a massive spin-2 exchange grows quite quickly with energy on very general grounds. A dimensionless spin-2 field gets its couplings from derivatives, which translate to factors of energy. Thus
the naive cross section grows like $E^3_\nu$, in the “low-energy” regime.

These general features are exemplified in the Feynman rules for this regime developed by several groups [18]. To be consistent with the literature, we will describe the interaction as “graviton” exchange, implying the standard picture of a tower of spin-2 $KK$ modes. The parton level $\nu$ gluon differential cross section is given by,

$$\frac{d\hat{\sigma}^{Gg}}{d\hat{t}} = \frac{\pi \lambda^2 \hat{u}}{2M^8_S \hat{s}^2} \left[2\hat{u}^3 + 4\hat{u}^2\hat{t} + 3\hat{u}\hat{t}^2 + \hat{t}^3\right]$$

Here $M_S$ is the cutoff on the graviton mass, and $\lambda$ is the effective coupling at the scale $M_S$ that cuts off the graviton $KK$ mode summation. The magnitude of parameter $\lambda$ has been lumped into the scale parameter $M_S$, hence $\lambda = \pm 1$ for our purposes [19]. In Eqs. (1) and (2) we take $-\hat{t} \ll M^2_S$, which leads to the simple factor $1/M^8_S$. This suffices for our extrapolation, but the full $\hat{t}$ dependence is used in the partial wave projections to check the unitarity constraint (discussed momentarily). The corresponding parton level $\nu$ quark differential cross section is given by,

$$\frac{d\hat{\sigma}^{Gq}}{d\hat{t}} = \frac{\pi \lambda^2}{32M^8_S \hat{s}^2} \left[32\hat{u}^4 + 64\hat{u}^3\hat{t} + 42\hat{u}^2\hat{t}^2 + 10\hat{u}\hat{t}^3 + \hat{t}^4\right]$$

We include the contribution of the two valence quarks as well as the $\bar{u}, \bar{d}$ and $s$ sea quarks. The $Z$-graviton interference terms are included with negative $\lambda$, though their contribution is very small compared to other terms. The negative sign gives a slight enhancement for the final result of $\sigma_{tot}$. Collider physics and astrophysics constrain the effective scale $M_S$ to be above 1 TeV, with lower number of dimensions leading to stronger constraints.

### 2.1 Unitarity

The complete theory of massive $KK$ modes is not yet developed, making it impossible to know the exact cross sections at asymptotic energies. The situation is analogous to the case of the 4-Fermi theory before the Standard Model. By observing the $s^4$ growth of $\sigma_{tot}$ it was possible to deduce a massive vector exchange long before a consistent theory existed. In much the same way, present data indicate a spin-2 exchange while the analogous complete “standard model” of gravitons does not yet exist. Unlike the electroweak
case, in either the data-driven or extra-dimensions scenario we must face a strongly interacting, non-perturbative problem in the high energy, \( s \gg M_S^2 \). Perturbative unitarity breaks down as a host of new channels opens up in that regime. The low-energy effective theory remains an accurate description within a particular domain of consistency. Extrapolation of the 4-Fermi predictions to higher energies is possible by matching the consistent, low-energy description with the asymptotic demands of unitarity. Similarly, we resolve the difficulties of massive graviton exchange in the high energy regime by matching the \( \sqrt{s} < M_S \) predictions, where the perturbative calculation is under control, to the \( \sqrt{s} \gg M_S \) non-perturbative regime.

We proceed by first evaluating the theory’s partial wave amplitudes to find the highest energy where the low energy effective theory is applicable. Taking the case \( \nu + q \rightarrow \nu + q \), and including the full \( Q^2 \) dependence of the propagator, we find that the unitarity bound on the \( J = 0 \) projection of the helicity amplitude \( T_{++} \) gives the strongest bound. For example, with the number of extra dimensions \( n = 2 \) we find \( \sqrt{s} \leq 1.7M_S \), while with \( n = 4 \) we find \( \sqrt{s} \leq 2.0M_S \). As mentioned earlier, the most attractive value of \( M_S \) is in the TeV range.

The invariant energies of the highest energy cosmic rays are approximately 1000 units of the scale \( M_S \sim 1 \text{ TeV} \), well beyond the low-energy regime. A phenomenological prescription consistent with unitarity is clearly necessary to extrapolate the low energy amplitudes. We now turn to describing and motivating three different asymptotic forms that span reasonable possibilities: \( \log(s) \), \( s^1 \) and \( s^2 \) growth of \( \sigma_{\text{tot}}(s) \). There is no guarantee a priori that any should extrapolate from low to high energy with \( M_S \sim 1 \text{ TeV} \) and produce hadronic-size cross sections at \( 10^{20} \text{ eV} \). As we shall see, surprisingly, they all do!

As a first version of an extrapolation model, we use a well known result from general features of local quantum field theory. The Froissart bound \cite{21}, reflecting the unitarity constraint on cross sections from exchange of massive particles, dictates that \( \sigma_{\text{tot}} \) grows no more rapidly than \( (\log(\hat{s}/M_S^2))^2 \). The bound is an asymptotic one, and strictly speaking incapable of limiting behavior at any finite energy; moreover, the bound is probably violated in the case of graviton exchange. It is quite conservative to use the Froissart bound, with its mild logarithmic growth in \( s \), as a first test case. In terms of
the differential cross section, we have at high energy
\[ \frac{d\hat{\sigma}}{dt} \rightarrow \frac{\text{const}}{t M_S^2} \log(\hat{s}/M_S^2) \] (3)

We then propose the following interpolating formula which reproduces Eq. (1) in the low energy limit and Eq. (3) in the high energy limit
\[ \frac{d\hat{\sigma}^{Gg}}{dt} = \frac{\pi \lambda^2}{2M_S^2(M_S^2 + \hat{s})^2(M_S^2 - \hat{t})} \frac{\hat{u}}{s^2} \left[ 2\hat{u}^3 + 4\hat{u}^2 \hat{t} + 3\hat{u} \hat{t}^2 + \hat{t}^3 \right] \times \left[ 1 + \xi \log(1 + \hat{s}/M_S^2) \right] \] (4)

The \( \nu \)-quark parton level cross section is similarly extrapolated to high energies. We have introduced the parameter \( \xi \), which we will allow to vary between 1 and 10. It cannot be much larger than 10 since then the low energy cross section gets modified, violating consistency. We use these parton-level expression to calculate \( \sigma_{\text{tot}} \). We convolute the parton-level cross section with CTEQ 4.6 parton distribution functions, which give a continued growth in the \( UHE \) regime from the small-x effect. As pointed out previously, the cross section can be expected to grow to “strong interaction” magnitudes, where parton coalescence and string effects ultimately come into play.

A different constraint for unitarizing would be \( s^2 \) growth. Regge theory would suggest \( s^2 \) growth for spin-2 exchange at small, fixed \( t \), which (in fact) occurs in this theory when the \( t \) values are restricted self-consistently. Thus the use of \( s^2 \) growth makes a comparatively mild alteration of the perturbative predictions and follows from eikonal unitarization of Reggeized graviton exchange [22]. Another unitarization procedure indicates a linear growth in \( s \) [23], which represents a case intermediate to the other two. These cases serve to establish a fair range of possibilities for models of unitarization.

For the \( s^1 \) and \( s^2 \) models, the extrapolation form of Eq. (1) we choose is
\[ \frac{d\hat{\sigma}^{Gg}}{dt} = \frac{\pi \lambda^2}{2M_S^2(M_S^2 + \hat{s})^p(M_S^2 - \beta \hat{t})} \frac{\hat{u}}{s^2} \left[ 2\hat{u}^3 + 4\hat{u}^2 \hat{t} + 3\hat{u} \hat{t}^2 + \hat{t}^3 \right] \], (5)

where \( p = 1,0 \) for \( s^1,s^2 \). A similar extrapolation is applied to the cross section formula for the \( \nu \) quark parton case, Eq. (2).

Note that we use the detailed perturbative low energy calculations that follow from [18] to anchor the low energy end. As we discuss below, we also
explore the allowed range of $M_S$ values, corresponding to a range of numbers of extra large dimensions. As far as we know, these consistency features in the present context have not previously appeared in the literature.

An essential feature of spin-2 exchange, complementary to the growth of $\sigma_{\text{tot}}$ with energy in the UHE region, is suppression of observable effects in the low energy regime of existing data. Turning the problem around to a “data driven” view, the energy dependence of the new physics must be so strong that the millibarn-scale total cross sections at $\sqrt{s} \geq 10^3$ TeV are suppressed well below the Standard Model values below 1 TeV. This also follows in our approach.

3 Results

Results of the calculations based on our models (Eqs. 4 and 5) are given in Fig. 1. For the $ln(s)$ model, shown by the dotted line, we find that for incident neutrino energy of order $10^{12}$ GeV, $\sigma_{\text{tot}}$ is roughly 0.5 mb, with the effective cutoff scale $M_S = 1$ TeV and the choice $\xi = 10$. This cross section is remarkably consistent with the low end of the range required to explain the GZK violating events. Looking at higher values of $M_S$, we find that the $\sigma_{\text{tot}}$ falls so steeply as a function of $M_S$ that $M_S \approx 1$ TeV is required for this model to be a viable explanation of cosmic ray events with $E \geq 10^{19}$ eV.

The results of the calculations with the $s^1$ model are shown in the dash-dotted line ($\beta = 0.1$) and short-dashed line ($\beta = 1.0$). The value of $M_S$ is fixed at 1 TeV; $\beta$ values of 0.1 and 1.0 are chosen to illustrate that this intermediate growth model easily produces hadronic size cross sections above the GZK cutoff. We find that raising the cutoff $M_S$ much above 2 TeV suppresses the cross section well below the range required for the GZK events, independently of the value of $\beta$.

The long-dashed curve (Fig. 1) shows the case with $s^2$ growth for values $M_S = 3$ TeV and $\beta = 1.0$. The growth of $\sigma_{\text{tot}}$ to tens of millibarns at the highest energies shows the viability of this model even at $M_S$ values above 1 TeV.

The key feature we emphasize about Fig. 1 is that $\sigma_{\text{tot}}$ somewhere in the range 1-100 mb is obtained at the highest energies for reasonable parameter choices for all three unitarization models. As a significant corollary, we find that if $M_S$ is larger than 1 - 10 TeV (depending on the model), then the
low scale gravity models fail to increase $\sigma_{tot}$ by enough to explain the GZK-violating events. Thus the analysis serves to put an upper bound on the cutoff mass $m_1$.

For completeness let us note that the interaction cross section with CMB photons is completely negligible because the cm-energy is about $10^{-13}$ that of a proton target. On the other hand, the mean free path of UHE neutrinos with $E_\nu > 10^{19}$ eV and mb-scale cross sections is of the order of a few kilometers in the upper atmosphere. This is still much larger than the mean free path of other particles such as protons at the same energy. If the ultrahigh energy showers are indeed initiated by neutrinos, the $\log(s)$ and $s^1$ models predict considerable vertical spread in the point of origin of the shower. The $s^2$ model is capable of much larger $\sigma_{tot}$.

Experimental extraction of cross sections is complicated by shower development fluctuations, but it would be useful to make all possible efforts to bound the mean free path of the primaries responsible for GZK-violating events. There is a further, interesting variant of the “double-bang” $\nu_\tau$ signal [24]: secondary events may come from a neutrino dumping part of its energy in a first collision, undergoing a second collision a few kilometers later. This process may serve to separate primaries with mb-scale cross sections from protons or nuclei, which have much shorter mean free paths.

### 3.1 Signatures of Massive spin-2 Exchange at Km-Scale Neutrino Detectors

It is also interesting to consider alternate signatures of the $\nu - N$ neutral-current cross section exceeding the Standard Model. The cosmic neutrino detectors, AMANDA [26] and RICE [27], for example, are expected to explore the TeV-10 PeV energy regime. The cross sections we find exceeding those of the Standard Model might be tested in these experiments. Of particular interest is the angular distribution of events. The diffuse background cosmic ray neutrino flux is expected to be isotropic. We can, therefore, measure the deviations of $\sigma_{tot}$ from SM predictions by measuring the ratio of upward- to downward-moving events. This ratio is plotted in Fig. 2 as a function of the incident neutrino energy. The plots show a few representative choices

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1Our requirement is consistent with estimates of lower bounds as discussed in [24], for example.
of $M_S$ and extrapolation parameter $\beta$, using the $s^1$ model for illustration. The up/down ratio starts to deviate very strongly from the SM value for an incident neutrino energy greater than about 5 PeV. Beyond about 5 PeV the ratio falls very sharply to zero. This in principle can be measured at RICE [27], which is sensitive to precisely the energy regime of 100 TeV to 100 PeV. The event rate in this energy region is also expected to be significant. Note that only the neutral-current events are affected. A more detailed, but also attractive extension of this technique is UHE Earth tomography, recently shown practicable with existing flux estimates, and also capable of measuring $\sigma_{tot}$ indirectly [28]. The graviton-exchange predictions are not especially sensitive to the precise value of $\beta$ in this region, but do depend strongly on the cutoff scale $M_S$. Large scale detectors such as RICE will be able to explore the range of $M_S < 2$ TeV.

We conclude by reiterating that the highest energy cosmic rays are on the verge of confronting fundamental particle physics. Exciting projects under way, including AGASA, Hi-RES, and AUGER should be able to collect enough data to resolve the issues. The puzzle of GZK violating events can be experimentally resolved by establishing neutral primaries via angular correlations. That feature would imply new physics of neutrinos, with the cross section then indicating massive spin-2 exchange rather directly. Experimentally bounding the primary interaction cross section below that of protons is another viable strategy in the $s^1$ and log($s$) scenarios.

Current limits on $KK$ modes of extra dimension models predict sufficiently large $\nu - N$ cross sections to produce such interactions, and are consistent with observation of events in the upper atmosphere at primary energies greater than $10^{20} - 10^{21}$ eV. Depending on the observational outcome, then, the subject matter could become very important, and further development is well warranted.

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References
[1] K. Greisen, *Phys. Rev. Lett.* **16**, 748 (1966); G. T. Zatsepin and V. A. Kuzmin, Sov. Phys. JETP Lett. **4**, 78 (1966).

[2] J. Puget, F. Stecker and J. Bredekamp, *Astrophys. J.* **205**, 638 (1976); J. Wdowczyk, W. Tkaczyk and A. Wolfendale, *J. Phys. A* **5**, 1419 (1972).

[3] J. Linsley, *Phys. Rev. Lett.* **10**, 146 (1963); World Data Center for Cosmic Rays, Catalogue for Highest Energy Cosmic Rays, No. 2, Institute of Physical and Chemical Research, Itabashi, Tokyo (1986); Efimov, N.N. et al., *Astrophysical Aspects of the Most Energetic Cosmic Rays*, M. Nagano and F. Takahara, Eds., (World Scientific, Singapore, 1991) pg. 20;

[4] M. Takeda et al., astro-ph/9902239.

[5] D. Bird et al., *Phys. Rev. Lett.* **71**, 3401 (1993); Astrophys. J. **424**, 491 (1994); M. Takeda et al, *Phys. Rev. Lett.* **81**, 1163 (1998) and astro-ph/9902239

[6] Acceleration of Ultra High Energy Cosmic Rays, R. Blandford, astro-ph/9906026

[7] T. Kephart and T. Weiler, *Astropart. Phys.* **4**, 271 (1996).

[8] V. Berezinsky, *Phys. Rev. Lett.* **79**, 4302 (1997); V. Kuzmin and I. Tkachev, Phys. Rep. [hep-ph/9902239];

[9] S. Yushida and M. Teshima, *Prog. Theor. Phys.* **89**, 833 (1993); G. Sigl, D. Schramm and P. Bhattacharjee, *Astropart. Phys.* **2**, 401 (1994)

[10] T. Weiler, hep-ph/9910316; D. Fargion, B. Mele and A. Salis, *Astrophys. J.*, 517 (1999), astro-ph/9710029; G. Gelmini and A. Kusenko, hep-ph/9908277.

[11] G. Domokos and S. Nussinov, *Phys. Lett. B* **187**, 372 (1987); J. Bordes, H.-M. Chan, J. Faridani, J. Pfaunder and S.-T. Tsou, hep-ph/9711433 G. Domokos and S. Kovesi-Domokos, *Phys. Rev. Lett.* **82**, 1366 (1999); H. Goldberg and T. Weiler, *Phys. Rev. D* **59**, 113005 (1999); J. Elbert and P. Sommers, Astrophys. J. **441**, 151 (1995).

[12] J. Elbert and P. Sommers, Ref. [11]; G. R. Farrar and P. Biermann, *Phys. Rev. Lett.* **81**, 3579 (1998); Y. Uchihori, M. Nagano, M. Takeda, M. Teshima, J. Lloyd-Evans, and A.A. Watson, *Astropart. Phys.* **13**, 151 (2000).

[13] Yu. Andreev, V. Berezinsky and A. Smirnov, *Phys. Lett. B* **84**, 247 (1979); D. W. McKay and J. F. Ralston, *Phys. Lett. B* **167**, 103, 1986. M. H. Reno and C. Quigg, *Phys. Rev. D* **37**, 657 (1987).
[14] G. M. Frichter, D. W. McKay and J. P. Ralston, *Phys. Rev. Lett.* **74**, 1508-1511 (1995); J. P. Ralston, D. W. McKay, and G. M. Frichter, in *International Workshop on Neutrino Telescopes*, (Venice, Italy, 1996), edited by M. Baldo-Ceolin; astro-ph/9606007; R. Gandhi, C. Quigg, M. H. Reno, I. Sarcevic, *Astropart. Phys.* **5**, 81 (1996); R. Gandhi, C. Quigg, M. H. Reno, I. Sarcevic, *Phys. Rev. D* **58** (1998) 093009.

[15] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett. B* **429**, 263 (1998). Some precursors of the model are V. Rubakov and M. Shaposhnikov, *Phys. Lett. B* **125**, 136 (1983); B. Holdom, Nucl. Phys. B **233** 413 (1983); M. Visser, Phys. Lett. B **159**, 22 (1985); E. Squires, Phys. Lett. B **167**, 286 (1986); J. Hughes, J. Liu and J. Polchinski, Phys. Lett. B **180**, 370 (1986); G. W. Gibbons, D. L. Wiltshire, Nucl. Phys. B **287** 717 (1987); K. Akama, Prog. Theor. Phys. **78**, 184 (1987); A. Barnaveli and O. Kancheli, *Sov. J. Nucl. Phys.* **51**, 573 (1990); I. Antoniadis, Phys. Lett. B **246** (1990) 377; I. Antoniadis, C. Muñoz and M. Quiros, *Nucl. Phys. B* **397**, 515 (1993); I. Antoniadis, K. Benakli and M. Quiros, *Phys. Lett. B* **331**, 313 (1994).

[16] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett. B* **463**, 257 (1999); G. Shiu and S.-H.H. Tye, *Phys. Rev. D* **58**, 106007 (1998).

[17] For an s-channel point of view, see G. Domokos and S. Kovesi-Domokos, Ref. [11].

[18] T. Han, J.D. Lykken and R.-J. Zhang, *Phys. Rev. D* **59**, 105006 (1999); G.F. Giudice, R. Rattazzi and J.D. Wells, *Nucl. Phys. B* **544**, 3 (1998).

[19] P. Mathews, S. Raychaudhuri and K. Sridhar, *Phys. Lett. B* **455**, 115 (1999).

[20] Conventions are from A. Martin and T. Spearman, “Elementary Particle Theory”, (North-Holland, Amsterdam,1970).

[21] M. Froissart, *Phys. Rev.* **123**, 1053 (1961). An early treatment of unitarity bounds of the Reggeized graviton exchange in D dimensions is given in [22].

[22] I. Muzinich and M. Soldate, *Phys. Rev. D* **37**, 359 (1988).

[23] S. Nussinov and R. Shrock, *Phys. Rev. D* **59**, 105002 (1999).

[24] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Rev. D* **59**, 086004,1999; S. Cullen and M. Perelstein, hep-ph/9903422.

[25] J. Learned and S. Pakvasa, *Astropart. Phys.* **3**, 267 (1995).

[26] F. Halzen, for the AMANDA Collaboration, to be published in *Proceedings of International Cosmic Ray Conference 99* (Salt Lake City, 1999); Also see the AMANDA website [http://amanda.berkeley.edu/].
[27] G. M. Frichter, J. P. Ralston, and D. W. McKay, *Phys. Rev. D* **53**, 1684 (1996); G. Frichter, for the AMANDA and RICE Collaborations, to be published in *Proceedings of International Cosmic Ray Conference 99* (Salt Lake City, 1999); Also see the RICE websites [http://rice.hep.fsu.edu/](http://rice.hep.fsu.edu/) and [http://kuhep4.phsx.ukans.edu/iceman/biblio.html](http://kuhep4.phsx.ukans.edu/iceman/biblio.html)

[28] P. Jain, J. P. Ralston, and G. M. Frichter, *Astropart. Phys.* **12**, 193 (1999), [hep-ph/9902206](http://arxiv.org/abs/hep-ph/9902206).
Figure 1: The $\nu N$ cross section in the Standard Model (SM) compared to a theory with large extra dimensions and three different models for the unitarity extrapolation between perturbative to non-perturbative regimes. The dotted line shows the $\log(s)$ growth case with $M_S = 1$ TeV and $\xi = 10$. The short dashed and dash-dotted lines show $s^1$ growth with $M_S = 1$ TeV and $\beta = 1$ and 0.1 respectively. The long dashed line shows $s^2$ growth with $M_S = 3$ TeV and $\beta = 1$. The contribution from massive graviton exchange is negligible at low energies but rises above the SM contribution when $\sqrt{s} > M_S$, reaching typical hadronic cross sections at incident neutrino energies in the range $5 \times 10^{19}$ to $5 \times 10^{20}$ GeV. The HERA data point is shown for comparison. The approximate minimum value required for $\nu N$ cross-section, $\sigma = 1$ mb, is indicated by the horizontal straight line.
Figure 2: The ratio of upward- to downward-moving events at UHE cosmic ray neutrino detectors as a function of the incident neutrino energy. The solid curve corresponds to the SM prediction. Other curves include the contribution due to low scale gravity models with $s^1$ cross section growth for several different representative choices of the cutoff parameter $M_S$ and the dimensionless extrapolation parameter $\beta = 1$. Since the extrapolation is small, the result is not especially sensitive to the precise value of $\beta$. 

$E_V$ (TeV)

$\text{Rate(Up)/Rate(Down)}$

$M_S = 1 \text{ TeV}$
$\beta = 1$

$M_S = 1 \text{ TeV}$
$\beta = 2$

$M_S = 2 \text{ TeV}$
$\beta = 1$