Calculation of the upper beam bending of a saw gin

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Abstract. The article presents the data of studies of the upper beam deflection of a saw gin. The displacement-based finite element method can be used to calculate the upper beam bending of a saw gin with variable thickness. It was determined that under the maximum load of 3004.43 N/m on the upper beam of the saw gin, the deflection reached 10.2 mm at L=1.327 m. Therefore, to reduce the deflection of the upper beam, it is necessary to increase the moment of inertia by changing its cross-section. To do this, it is necessary to create a new design of the upper beam of a saw gin, providing the greater value of the moment of inertia than $I=2.19\times10^{-5}$ m$^4$.

1. Introduction

In [1], the deflection and the angle of rotation of the saw shaft are determined using the method of equivalent diameters developed by B.N.Zhemochkin [2]. As a result, the shaft deflections for the existing design of the saw cylinder shaft were determined knowing the diameters of the saw shaft sections, their axial moments and reduction coefficients, the magnitudes of the transverse forces, bending moments and their differences at the transition points of the sections.

Various physical properties of the elements of building structures were calculated in [3]. In particular, the theory of the Fourier series was used when calculating the beam bending in building structures.

E.A.Kochetov and I.L.Sal [4] proposed to use the applied computer programs MathCAD, Solid Works, T-Flex, Microsoft Excel, etc. to solve the problem of the beam bending. From the listed Microsoft programs, Excel allows us to perform mathematical operations that can be used in the calculation of beam bending.

In [5], the problems of bending of a beam of variable thickness made of a nonlinear-deformable material under transverse load with various degrees of in homogeneity in its thickness are considered: inhomogeneous layers with degrading strength characteristics – to model the aggressive effect of media, inhomogeneous layers with improved strength characteristics – to model the technological hardening. The analysis of changes in stresses on the beam surface and at the boundary of the inhomogeneity front is performed for the supporting and central sections of the beam, and the corresponding graphical results are presented.

A.A.Limantsev and A.V.Denisenko [6] give the solution to the problem of determining the optimal design resistance for a steel I-beam as the least metal-intensive section. A definition and formulas for determining the specific characteristics of the section are given.
In [7] the problem of the pipeline bending when it is laid on a solid bottom is considered. The pipeline is modeled as a semi-infinite hinged elastic beam. Analytical expressions are obtained and the dependencies of internal forces and stresses arising in the pipeline on the distance to the bottom and the water pressure are built taking into account the pre-stressing.

In the abstract of A.A.Gavrilov's thesis [8], expressions were obtained for the main modes of bending and torsional vibrations for deflections, angles of rotation and twisting of the section, deplanation function, bimoment, bending moment, moments of pure and constrained torsion of a continuous beam, considering the secondary shear. The results obtained using specialized software systems for beam modeling based on volumetric finite elements gave 1-3% difference, which confirms the possibility of using the proposed method.

To determine the static characteristics of the saw gin cylinder, the displacements-based finite element method was used [9-12].

2. Materials and methods
The displacements-based finite element method can be used to reduce the amount of analytical computing when calculating the upper beam bending of a saw gin with variable thickness.

Figure 1 shows the design diagram of the upper beam of a saw gin and the loads acting on it. The calculated values of the moment of inertia and the parameters of the upper beam sections of the saw gin are shown in table 1. The external loads acting on the upper beam are as follows:

Uniformly distributed loads \( q_1, q_2, q_3 \) and \( q_4 \) are:

\[
q_1 = \frac{m_1}{L_1} = \frac{0.822}{0.0729} = 11.27 \frac{kg}{m} = 110.51 \frac{N}{m}
\]

\[
q_2 = \frac{m_3 + m_4}{L_2} = \frac{0.373 + 0.277}{0.06456} = 10.07 \frac{kg}{m} = 98.73 \frac{N}{m}
\]

\[
q_3 = \frac{m_4}{L_3} = \frac{0.668}{0.008} = 83.5 \frac{kg}{m} = 818.8 \frac{N}{m}
\]

\[
q_4 = \frac{(m_0 + m_5) g + \frac{M_x}{r}}{L_4} = \frac{(35.044 + 89.655) \cdot 9.806 + \frac{974.4}{0.16}}{2.434} = 3004.43 \frac{N}{m}
\]
where \( m_0 = m_1 + m_2 + m_3 + m_6 + m_7 + m_8 + m_{10} + m_{11} + m_{12} = 35,044 \) kg are the masses of the beam between the supports; \( m_1 = m_{12} = 0.822 \) kg; \( m_2 = m_{11} = 0.373 \) kg; \( m_3 = m_{10} = 0.277 \) kg; \( m_4 = m_7 = 0.668 \) kg; \( m_5 = m_8 = m_9 = 5.525 \) kg; \( m_6 = m_0 = 5.527 \) kg are the masses of the elements, respectively; \( m_r = 139-0.645 \) kg=89.655 kg are the grate weights; \( L_1 = 0.0729 \) m is the length of the first element; \( L_2 = 0.06456 \) m is the length of the second element; \( L_r = 0.008 \) m is the length of the third element; \( L_e = 2.434 \) m is the length of the upper beam between the supports; \( M_s = 974.4 \) N-m is the rated technological load on the grate from the saw cylinder; \( r = 0.16 \) m is the radius of the cylinder saw blade.

| Coordinates, m | Element mass (m.), kg | Axial moment of inertia, m^4 | EI | EF |
|-------------|----------------------|-----------------------------|-----|----|
| beginning | end | length | 0.822 | 0.000001728 | 138240 | 115200000 |
| 0.0729 | 0.1100 | 0.0371 | 0.373 | 0.000001728 | 138240 | 102884957 |
| 0.1100 | 0.1375 | 0.0275 | 0.277 | 0.000001728 | 138240 | 102884957 |
| 0.1375 | 0.1455 | 0.0080 | 0.668 | 0.0000022430 | 1794443 | 852890080 |
| 0.1455 | 0.5393 | 0.3938 | 5.525 | 0.00001686 | 134889 | 143357200 |
| 0.5393 | 0.9332 | 0.3939 | 5.527 | 0.00001686 | 134889 | 143357200 |
| 0.9332 | 1.3270 | 0.3938 | 5.525 | 0.00001686 | 134889 | 143357200 |
| 1.3270 | 1.7208 | 0.3938 | 5.525 | 0.00001686 | 134889 | 143357200 |
| 1.7208 | 2.1147 | 0.3939 | 5.527 | 0.00001686 | 134889 | 143357200 |
| 2.1147 | 2.5085 | 0.3938 | 5.525 | 0.00001686 | 134889 | 143357200 |
| 2.5085 | 2.5165 | 0.0080 | 0.668 | 0.000022430 | 1794443 | 852890080 |
| 2.5165 | 2.5440 | 0.0275 | 0.277 | 0.00001728 | 138240 | 102884957 |
| 2.5440 | 2.5811 | 0.0371 | 0.374 | 0.00001728 | 138240 | 102884957 |
| 2.5811 | 2.6540 | 0.0729 | 0.822 | 0.00001728 | 138240 | 115200000 |

Let us outline the methodology for calculating the upper beam of a saw gin. The calculation was conducted using a beam element with six degrees of freedom (figure 2, c). This element (figure 2, a) is obtained by the arrangement of a beam finite element (figure 2, c) and a finite element of a uniaxial stress state (figure 2, b).

Consider the construction of the stiffness matrix of a beam finite element. The relationship between the loads and displacements of the beam under bending can be obtained by considering the energy functional under bending [13 - 16]:

\[
I = \frac{1}{2} \int_{L} \left[ EI \left( \frac{d^2w}{dx^2} \right)^2 - 2f(x)w \right] dx \tag{5}
\]

Where \( E \) is the modulus of elasticity; \( I \) is the moment of inertia of the cross-section; \( f \) is the distributed load per unit length.

Let the beam under consideration (figure 1) be divided into \( S \) elements of the type shown in figure 2. a. Then the energy functional (5) can be written as the sum of the energy functionals of each element:

\[
I = \sum_{S} I^{(S)} \tag{6}
\]

At the end points of the element, it is necessary to determine the transverse \( (w_1 \) and \( w_2 \)) and angular \( (\theta_1 \) and \( \theta_2 \)) displacements. The latter equal to the tangent of the angle of inclination of the neutral axis, i.e.

\[
\theta_1 = \frac{dw}{dx} \bigg|_{x=0} , \quad \theta_2 = \frac{dw}{dx} \bigg|_{x=L} \tag{7}
\]

We write the vector of nodal displacements in the following form

\[
\{q\} = \begin{bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{bmatrix}^T \tag{8}
\]
Since there are four degrees of freedom to represent the displacement field inside a finite element, it is necessary to take a third-degree polynomial

\[ w(x) = a_1x^3 + a_2x^2 + a_3x + a_4 = \begin{bmatrix} x^3 & x^2 & x & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \]  

(9)

The polynomial coefficients are determined from the boundary conditions

\[ \begin{align*} 
\varepsilon(0) &= w_1; \quad \varepsilon'(0) = \theta_1; \\
\varepsilon(L) &= w_2; \quad \varepsilon'(L) = \theta_2; 
\end{align*} \]  

(10)

Calculating (9) for each of the four conditions (10) we obtain the system of algebraic equations

\[ \begin{bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & a_1 \\ 0 & 0 & 1 & 0 & a_2 \\ L' & L' & L & 1 & a_3 \\ 3L' & 2L & 1 & 0 & a_4 \end{bmatrix} \]  

(11)

Solving it for the vector \{a\} and substituting it into (9), we obtain

\[ w = [N][q] \]  

(12)

where

\[ [N] = \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix}; \]

\[ N_1 = (1 + 2\xi^3 - 3\xi^4); \quad N_2 = -x(\xi - 1)^2; \]

\[ N_3 = (3\xi^2 - 2\xi^3); \quad N_4 = -x(\xi - 2\xi^2). \]  

(13)

Here \( \xi = x/L \).

A form function has the feature of taking a unit value at the node for which it is built. Substituting (13) into (5), we obtain
\[ I^{(s)} = \sum_{j=1}^{4} \sum_{i=1}^{4} K_{ij}^{(s)} q_j q_i - 2 \sum_{i=1}^{4} P_i^{(s)} q_i. \]  

(14)

where

\[ K_{ij}^{(s)} = \frac{1}{E} \left[ E I N_i'' N_j'' \right] dx, \quad P_i^{(s)} = \frac{1}{E} f(x)^{(s)} N_i dx \]  

(15)

in these expressions \( N_i'' \), \( i = 1, 4 \) are

\[ N_i'' = \frac{6}{L^3} (2 \xi - 1), \quad N_i'' = -\frac{2}{L} (3 \xi - 2), \quad N_i'' = -\frac{2}{L} (3 \xi - 1) \]  

(16)

If function \( f(x) \) is a uniformly distributed load, then it can be taken out of the integral sign.

It is known that the beam construction can exist only when the energy in it is minimal. The necessary conditions for the minimum of the functional are

\[ \frac{\partial I^{(s)}}{\partial q_i} = 0, \quad i = 1, \ldots, 4 \]  

(17)

Hence we obtain \( \sum_{j=1}^{4} K_{ij}^{(s)} q_j^{(s)} = r_i \) or in matrix form

\[ [K]^{(s)} [q]^{(s)} = [r] \]  

(18)

where \([K]^{(s)}\) is the stiffness matrix of the finite element \( s \), \([r]\) - is the vector of equivalent nodal forces. The stiffness matrix elements have the following meanings:

\[ [K]^{(s)} = \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & 6L & 4L \end{bmatrix} \]  

(19)

The total energy (functional) of the system is obtained by summing the energies of all its elements, i.e. (6), then for the entire system we can write

\[ [K] [Q] = [R] \]  

(20)

where \([R]\) is the vector of reduced nodal loads.

As indicated, a finite element of a uniaxial stress state is used to account for axial deformations. Consider the construction of the stiffness matrix of this element.

The change in displacements in the element is expressed as a linear function along the beam axis.

\[ u = \alpha_1 + \alpha_2 x = \begin{bmatrix} 1 & x \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \]  

(21)

The linear function was chosen because the element has two nodes, and therefore, the linear function gives an unambiguous value since only one straight line can be drawn through two points.

If this expression is written for \( x=0 \) and \( x=L \) then we obtain

\[ \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & L \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = [C] [q], \]  

(22)

hence

\[ \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ L & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = [C]^{-1} [q]. \]  

(23)

Substituting (9) into (5), we obtain
\[ u = \alpha_1 + \alpha_2 x = [1 - \xi] \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = [N_1 \\ N_2] \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = [N]q. \]  

(24)

The deformation in the element is determined from the following expression

\[ \varepsilon = \frac{du}{dx} = \frac{1}{L} \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = [B]q. \]  

(25)

The stresses are:

\[ \sigma = E\varepsilon = E \cdot \frac{1}{L} \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = E[B]q. \]  

(26)

It is known that the work of external and internal forces on virtual displacements is equal.

Let \( d\{q\} \) be the virtual displacements of the finite element nodes. Then the deformations are written as

\[ d\varepsilon = [B](d\{q\}), \]  

(27)

The work done by the nodal forces is equal to the sum of the products of each force by the corresponding displacements, i.e. in matrix form

\[ (d\{q\})^T \{F\}. \]  

(28)

Similarly, the internal work of stresses per unit volume is

\[ d\varepsilon \cdot \sigma = [B](d\{q\}) \cdot \sigma. \]  

(29)

Equating the work of external forces to the total internal work obtained by integrating over the volume of the element, we obtain

\[ (d\{q\})^T \{F\} = (d\{q\})^T [B]^T \sigma dV \]  

(30)

This ratio is valid for any virtual displacement, so the coefficients on the right and left sides must be equal

\[ \{F\} = [B]^T E[B]q dV. \]  

(31)

The expression under the integral does not depend on the volume, so we can write

\[ \{F\} = [B]^T E[B]dV\{q\} = [B]^T E[B] A \cdot L \{q\}. \]  

(32)

Let us expand this expression

\[ \{F\} = [B]^T E[B] A \cdot L \{q\} = \frac{EAL}{L} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \{q\} = [k]\{q\}. \]  

(33)

With a relation between external forces and displacements for one element of the beam, it is possible to solve problems for any composite beams. The matrix \( k \) is called the stiffness matrix of the element.

Now, combining the matrices of the beam and uniaxial elements, we obtain

\[ [r] = \begin{bmatrix} \frac{EF}{L} & 0 & 0 & -\frac{EF}{L} \\ 0 & \frac{12EI}{L^2} & 6EI & 0 \\ -\frac{12EI}{L^2} & \frac{6EI}{L^2} & \frac{EI}{L^2} \end{bmatrix} \]  

(34)
We used this matrix to calculate the upper beam bending of a saw gin.

When calculating the upper beam of the saw gin the finite element method was implemented using the initial data given in table 1. The finite element is a rod 2-flat nodal element each node of which has two degrees of freedom (figure 2, a).

3. Results and discussion

Using the initial data (table 1), the stiffness matrix (34) and the system of linear equation (21), diagrams of bending moments (figure 3), shear forces (figure 4) and the deflection of the upper beam of the saw gin (figure 5) were plotted.

Figure 3. Diagram of bending moments.

Figure 4. Diagram of transverse forces.

Figure 5. Diagram of the upper beam deflection.
4. Conclusions
As a result of the calculated studies at various distributed loads (343.64 N/m, 879.16 N/m, 1222.798 N/m and 3004.43 N/m), the deflection diagrams of the upper beam of the saw gin (1.16; 2.97; 4.14 and 10.2 mm, respectively, at L = 1.327 m), the bending moments (253.85; 650.09; 904.35 and 2222.60 N/m, respectively, at L=1.327 m), the shear forces (415.28; 1047.99; 1454.0 and 3559.0 N, respectively, at L=2.544 m) were determined using the displacement-based finite element method.

It was determined that under the maximum load of 3004.43 N/m acting on the upper beam of the saw gin, the deflection reached 10.2 mm at L=1.327 m. One of the ways to reduce the upper beam deflection is to increase its moment of inertia. As a result of studying the influence of the moment of inertia of the upper beam (figure 6) on its deflection, it was found that the deflection of the upper beam decreases to 0.78 mm at I=2.19×10⁻⁵ m⁴ and the maximum load is 3004.43 N/m. Therefore, to reduce the deflection of the upper beam, it is necessary to increase the moment of inertia by changing its cross section. To do this, it is necessary to create a new design of the upper beam of the saw gin, providing the greater value of the moment of inertia than I=2.19×10⁻⁵ m⁴.

![Figure 6. Change in the upper beam deflection depending on its moment of inertia.](image-url)

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