Symanzik Flow on HISQ Ensembles

Nathan Brown
brownnathan@wustl.edu

Advisor: Claude Bernard
Washington University in St. Louis

MILC Collaboration
Other Co-Authors: A. Bazavov, C. DeTar, J. Foley,
S. Gottlieb, U.M. Heller, J.E. Hetrick, J. Laiho, L. Levkova,
R.L. Sugar, D. Toussaint, R.S. Van de Water
Outline

1. Theoretical Overview
2. Results for HISQ Ensembles
3. Fitting and Extrapolation
4. Autocorrelation
Wilson/Symanzik Flow

- Wilson flow is a smoothing of the original gauge fields $U$ towards stationary points of the Wilson action $S$. [Lüscher, JHEP 1008 (2010) 071]
- Successive links $V(t)$ are updated in flowtime according to the diffusion equation,

$$
\frac{d}{dt} V(t)_{i,\mu} = - V_{i,\mu} \frac{\partial S(V)}{\partial V_{i,\mu}} , \quad V(t)_{i,\mu}(0) = U_{i,\mu} \quad \left[ \frac{dA_\mu}{dt} = D_\nu F_{\nu\mu} \right]
$$

- Cuts out high momenta noise, thereby suppressing statistical fluctuations and discretization effects at minimal computational cost
- Used the Symanzik improved action ($\approx 2x$ cost) to further reduce discretization errors.
Scale Setting

- The scale can be extracted through the flowtime $t[a^2]$.
- Define an improved, dimensionless quantity through the energy density $\langle E(t) \rangle$. [BMW (S. Borsanyi et al.), JHEP 1209 (2012) 010]

$$W(t) = t \frac{d}{dt} \left( t^2 \langle E(t) \rangle \right)$$

- In the continuum, the energy density $\langle E(t) \rangle$ is finite (at least to one loop order) when expressed in terms of renormalized quantities. [Lüscher, JHEP 1008 (2010) 071]
- Empirically, the combination $t^2 \langle E(t) \rangle$ varies linearly with $t$ for large flow times.
- The $w_0[a]$ scale is defined from the cutoff at 0.3.

$$w_0 = \sqrt{t_c}, \quad W(t_c) = 0.3$$

- The value of the cutoff is chosen to minimize discretization and finite volume effects.
Ensembles with $m_s \approx m_s^{\text{physical}}$

| $a$(fm) | $m_l/m_s$ | $n \times 3 \times nt$ | $N_{\text{run}}$ | $w_0/a$ (stat) [\%] |
|---------|-----------|-----------------------|-----------------|-----------------------|
| 0.15    | 1/5       | $16^3 48$             | 1021            | 1.1221 (06) [0.06\%] |
| 0.15    | 1/10      | $24^3 48$             | 1000            | 1.1381 (04) [0.04\%] |
| 0.15    | 1/27      | $32^3 48$             | 999             | 1.1468 (03) [0.03\%] |
| 0.12    | 1/5       | $24^3 64$             | 1040            | 1.3835 (07) [0.05\%] |
| 0.12    | 1/10      | $24^3 64$             | 1020            | 1.4020 (10) [0.07\%] |
| 0.12    | 1/10      | $32^3 64$             | 999             | 1.4047 (06) [0.05\%] |
| 0.12    | 1/10      | $40^3 64$             | 1001            | 1.4041 (04) [0.03\%] |
| 0.12    | 1/27      | $48^3 64$             | 34              | 1.4168 (10) [0.07\%] |
| 0.09    | 1/5       | $32^3 96$             | 102             | 1.8957 (16) [0.08\%] |
| 0.09    | 1/10      | $48^3 96$             | 151             | 1.9296 (09) [0.05\%] |
| 0.09    | 1/27      | $64^3 96$             | 53              | 1.9473 (11) [0.06\%] |
| 0.06    | 1/5       | $48^3 144$            | 127             | 2.8956 (26) [0.09\%] |
| 0.06    | 1/10      | $64^3 144$            | 46              | 2.9486 (31) [0.11\%] |
| 0.06    | 1/27      | $96^3 192$            | 49              | 3.0119 (18) [0.06\%] |
Non-Physical Strange Mass Ensembles

- $m'_s$ and $m_l$ are the sea quark masses
- $m_s$ is the physical strange quark mass

| $a$(fm) | $m_l/m_s$ | $m'_s/m_s$ | $n x^3 nt$ | $N_{run}$ | $w_0/a$ (stat) [%] |
|---------|-----------|-------------|-------------|-----------|-------------------|
| 0.12    | 0.10      | 0.10        | 32$^3$64    | 102       | 1.4833 (13) [0.09%] |
| 0.12    | 0.10      | 0.25        | 32$^3$64    | 204       | 1.4676 (11) [0.07%] |
| 0.12    | 0.10      | 0.45        | 32$^3$64    | 205       | 1.4470 (11) [0.08%] |
| 0.12    | 0.10      | 0.60        | 32$^3$64    | 107       | 1.4351 (20) [0.14%] |
| 0.12    | 0.175     | 0.45        | 32$^3$64    | 134       | 1.4349 (13) [0.09%] |
| 0.12    | 0.20      | 0.60        | 24$^3$64    | 255       | 1.4170 (10) [0.07%] |
| 0.12    | 0.25      | 0.25        | 24$^3$64    | 255       | 1.4336 (16) [0.11%] |

- Most ensembles have $\approx 1000$ configurations, so $N_{run}$ can still be increased considerably to improve statistics.
'est' stands for estimate for full ensemble run using conservative estimates of the autocorrelation length

Dashed vertical lines denote the lattice spacing for each ensemble; all scales are at these lattice spacings but data points are separated horizontally to make the comparison easier.
Naive Continuum Extrapolation
Including quark mass dependence allows us to include ensembles with $m_s \neq m_s^{\text{physical}}$ and correct for fine-tuning errors.

Using $M_{\pi}^2$ and $2M_{K}^2 - M_{\pi}^2$ as proxies for $m_l$ and $m_s$, included up to cubic powers in the quark mass.

To extrapolate to the continuum, included $k\alpha_s a^2$ and higher orders of $a^2$, up to $a^6$ ($k$ is a constant).

Due to the large range of $m_s$ covered by the full set of ensembles, some fits drop various ensembles with low values of $m_s$. 
Continuum, Physical Quark Mass Extrapolation

Only $m_s = m_s^{physical}$ ensembles are plotted, but fit includes all $m_s \leq m_s^{physical}$ ensembles.

Dotted lines are for actual masses run; solid lines are for re-tuned masses per legend.
Current Results for $w_0$

- Central fit has $\chi^2/dof = 7.5/10$, $p = 0.68$
- Found 78 different fits with $p > 0.01$; used the standard deviation of the fits’ extrapolated values to estimate the systematic uncertainty at $4e^{-4}$ fm
- There is also residual finite volume error in $f_\pi$ that cannot be corrected for, adding another systematic error of $2e^{-4}$ fm.
- **Preliminary Result:** $w_0 = 0.1712(3)(4)(2)(3)$ fm
  First is the statistical error, then systematic error from the continuum extrapolation, residual finite volume effects, and experimental value of $f_\pi$, respectively.
- As a sanity check, the naive fit through the four physical quark mass ensembles found $0.1711(3)(3)(2)$ fm. The naive fit is in good agreement with the improved fit.
Comparison (HPQCD, BMW)

BMW: [BMW (S. Borsanyi et al.), JHEP 1209 (2012) 010]
HPQCD: [HPQCD (R. J. Dowdall et al.), arXiv:1303.1670]
ETM and MILC values are preliminary
ETM did not provide an error estimate for $w_0$.
The BMW point is for their final, quoted value on the HEX smeared Wilson ensembles.
Integrated AutoCorrelation Length of $\langle E(t) \rangle$

Each solid line corresponds to the ensemble at the ratio $m_l/m_s = 1/10$ and specified $a$

Dashed lines correspond to the value of $w_0$ for each ensemble

The $a = 0.06\text{fm}$ ensemble ran with a larger separation between configurations; the low resolution yields noise at low autocorrelation lengths.
Our preliminary value of

\[ w_0 = 0.1712(3)(4)(2)(3) \text{ fm} \]

agrees with HPQCD within 1\(\sigma\), but deviates from BMW by 2.2\(\sigma\) compared to their final, HEX smeared Wilson result

\[ w_0 = 0.1755(18)(04) \text{ fm} \]

This deviation may be due to the difference in \(N_f\). However, ETM also used \(N_f = 2 + 1 + 1\) ensembles and found a central value even higher than that of BMW. But without an error estimate, the significance of this result is unclear.

Statistical errors are still being improved.

We found integrated autocorrelation lengths that are fairly large: up to 55 trajectories on the \(a = 0.06\) fm ensemble. These are comparable to but generally smaller than those found for twisted mass ensembles [ETM (A. Deuzeman, U. Wenger), PoS (Lattice 2012) 162].