Magnetically-Induced Buckling of a Whirling Conducting Rod with Applications to Electrodynamic Space Tethers

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Abstract We study the effect of a magnetic field on the behaviour of a slender conducting elastic structure, motivated by stability problems of electrodynamic space tethers. Both static (buckling) and dynamic (whirling) instability are considered and we also compute post-buckling configurations. The equations used are the geometrically exact Kirchhoff equations. Magnetic buckling of a welded rod is found to be described by a surprisingly degenerate bifurcation, which is unfolded when both transverse anisotropy of the rod and angular velocity are considered. By solving the linearised equations about the (quasi-) stationary solutions, we find various secondary instabilities. Our results are relevant for current designs of electrodynamic space tethers and potentially for future applications in nano- and molecular wires.

Keywords Rod mechanics · Kirchhoff equations · Magnetic buckling · Degenerate pitchfork bifurcations · Hopf bifurcation · Spinning electrodynamic tether

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1 Introduction

A straight current-carrying wire held in tension between pole faces of a magnet is well known to buckle into a (roughly) helical configuration at a critical current (see Fig. 1). A photograph of this phenomenon is shown in Sect. 10.4.3 of Woodson and Melcher (1968), where a linear stability analysis is carried out for a simple string model. (A string here is meant to be a perfectly flexible elastic wire.) The problem was studied by Wolfe (1983) by means of a rigorous bifurcation analysis for a (non-linearly elastic) string suspended between fixed supports and placed in a uniform magnetic field directed parallel to the undeformed wire. It was found that an infinite number of solution branches bifurcate from the trivial straight solution, much like in the Euler elastica under compressive load. Minimisation of the potential energy indicated that the first branch of bifurcating solutions is stable while all other branches are unstable.

In a subsequent paper, Wolfe (1985) extended the analysis to a uniformly rotating (whirling) string and showed again the existence of bifurcating branches of whirling non-trivial solutions. This result was further extended by Healey (1990) using equivariant bifurcation theory in order to deal with the symmetries of the problem, which caused the bifurcations to be degenerate.

Wolfe also considered a conducting rod in a uniform magnetic field (Wolfe 1988). In addition to extension, a rod can undergo flexure, torsion, and shear, and for the case of welded boundary conditions, it was found that in certain cases bifurcation occurs with the usual infinity of non-trivial equilibrium states. All the works cited above were content with showing the existence of bifurcating solutions and did not study their post-buckling behaviour. The Hamiltonian structure of the equations for a rod in a magnetic field was investigated in Sinden and van der Heijden (2008) where it was shown that in the case of an isotropic, inextensible, and un-shearable rod the equations are completely integrable.

The study of strings and rods in a magnetic field is of great interest to space tethers. Although space tethers in the last 20 years or so have become a well-established concept in astrodynamics (Beletsky and Levin 1993), new designs continue to be proposed that hold great potential for future space applications. A space tether is

Fig. 1 Experimental setup for a conducting wire